A new twist on $Z \to b\bar{b}$

Pham Quang Hung

Department of Physics, University of Virginia, Charlottesville, Virginia 22901, USA

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Abstract

A new mechanism is proposed to explain the "anomaly" in $Z \to b\bar{b}$ resulting in a prediction of a new, non-sequential fourth family whose masses could all be below $M_W$, thus opening up an exciting prospect for near-future discoveries at LEP2 and possibly at the Tevatron.

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Precision tests of the Standard Model (SM) have reached a level where it "might" now be possible to look for indirect evidence of new physics and/or new degrees of freedom. One example is the apparent discrepancy between theory and experiment in the value of the ratio \( R_b \equiv \Gamma(Z \rightarrow b\bar{b})/\Gamma(Z \rightarrow \text{had}) \). This discrepancy which increases with \( m_t \), reaches the 2 \( \sigma \) level when \( m_t \) reaches 175 GeV. If one also includes the apparent disagreement between the QCD coupling \( \alpha_S \) determined at "low" energy and evolved to \( M_Z \) with that determined by the Z-lineshape, one is tempted to think that one might be already seeing some new kind of physics. It is therefore very crucial to confirm or disprove these so-called discrepancies. Let us nevertheless assume that they are there to stay and examine what kind of new physics that can be possible and what predictions that can be tested in the near future. (Even if the discrepancy were to disappear, this would put a severe constraint on this type of new physics.)

In this letter, a mechanism is proposed to explain the apparent increase of \( R_b \) and to make further predictions on other branching ratios, and ultimately on the new physics concerning the mechanism itself. This mechanism is based on the assumption that there is a new, heavy, non-sequential down quark \( (Q = -1/3) \) (part of a new family) with mass greater than 46 GeV and whose \( q\bar{q} \) bound state(s) mixes with the Z boson. By non-sequential, we mean that the fermions of the new family does not have (or has little) mass mixing with fermions of the other three generations. Other than this being a working assumption, a realization of this scenario is given at the end of the paper.

For this paper, we shall quote a few relevant observables \cite{1,2}: \( \Gamma_Z(\text{GeV}) = 2.4974 \pm 0.0027 \pm 0.0027(2.496 \pm 0.001 \pm 0.003 \pm [0.003]), R_b \equiv \Gamma(b\bar{b})/\Gamma(had) = 0.2202 \pm 0.0020(0.2155 \pm 0 \pm 0.0004), R_c \equiv \Gamma(c\bar{c})/\Gamma(had) = 0.1583 \pm 0.0098(0.171 \pm 0 \pm 0), \) where the numbers in parentheses are the standard model expectations, and \( R_e \equiv \Gamma(had)/\Gamma(e\bar{e}) = 20.850 \pm 0.067, R_\mu \equiv \Gamma(had)/\Gamma(\mu\bar{\mu}) = 20.824 \pm 0.059. \)

Let us denote this non-sequential family by \((\mathcal{R}, \mathcal{P})\) for the quarks and by \((\mathcal{N}, \mathcal{E})\) for the leptons. For reasons to be given below let us assume that the \((Q = -1/3)\) quark has a mass \( m_\mathcal{P} < m_\mathcal{R} \). We also assume that the up-type quark \( \mathcal{R} \) is heavy enough so that \( \mathcal{R}\mathcal{\bar{R}} \) bound
states are well above the $\mathcal{P}\mathcal{P}$ open threshold. The $\mathcal{P}\mathcal{P}$ bound states can be described by Richardson’s potential. Such an analysis has been carried out long ago by [3] for the $^3S_1$ $t\bar{t}$ bound states, but unfortunately in the now-obsolete range of $m_t \sim 40 – 50$ GeV. This analysis can however be applied to any quark in a similar mass range or higher, especially for our case where $m_\mathcal{P} > 46$ GeV. (The mass shift of the Z boson due to this mixing is negligible [3].)

$\mathcal{P}\mathcal{P}$ bound states which can mix with $Z$ are either vector, axial vector, or both. In what follows we shall neglect the mixing of $Z$ with the axial vector states since it goes like $\beta^3$ [4,3]. Consequently we shall focus only on the vector meson ($^3S_1$) bound states. In particular, we shall first examine the mixing of the ground state $1S$ with $Z$. In the mass range considered here, the ground state $1S$ is sufficiently far from open-$\mathcal{P}$ threshold so that the mass-mixing formalism can be applied. Denoting the $1S$ ($J^{PC} = 1^{--}$) state by $V^0$, the result of $V^0$ and $Z^0$ mixing is given in terms of the mass eigenstates [3]

$$|V\rangle = \cos \frac{\theta}{2} |V_0\rangle - \sin \frac{\theta}{2} |Z_0\rangle,$$

(1a)

$$|Z\rangle = \sin \frac{\theta}{2} |V_0\rangle + \cos \frac{\theta}{2} |Z_0\rangle,$$

(1b)

for the mass eigenvectors and where

$$\theta = sin^{-1}(\delta m^2/\Delta^2),$$

(2)

with $\Delta^2 = \left[\frac{(M^2_{V_0} - i\Gamma_{V_0}M_{V_0} - M^2_{Z_0} + i\Gamma_{Z_0}M_{Z_0})^2}{4} + (\delta m^2)^2\right]^{1/2}$. $\delta m^2$ is the off-diagonal element of the mass mixing matrix and is given by [3]

$$\delta m^2 = F_V\left[\left(\frac{g}{\cos \theta_W}\right)^4 \frac{\sin^2 \theta_W}{4} - \frac{1}{4}\right],$$

(3)

where $F_V = 2\sqrt{3} |\Psi(0)| \sqrt{M_{V_0}}$. The term inside the square brackets represents the vector coupling of the $\mathcal{P}$ quark to the $Z$ boson.

Let us assume that $M_V > M_Z$ and since present experiments are carried out on the $Z$ resonance, we need only to look at Eq. (1b) to see how the presence of $V_0$ modifies the
coupling of Z to "light" quarks and leptons. This, as we claim in this manuscript, is a possible source for the discrepancy seen in $\Gamma(b\bar{b})$. From Eq. (1b), one finds the physical Z couplings to a given fermion $f$ to be

$$g_{Zff}^{V,A} = \sin \frac{\theta}{2} g_{V0ff}^{V,A} + \cos \frac{\theta}{2} g_{Z0ff}^{V,A},$$

(4)

where $V$ and $A$ stand for vector and axial-vector couplings respectively.

The most obvious source of the coupling of $V_0$ to $f\bar{f}$ is via $\gamma$ and Z and evaluated at $s = M_Z^2$. The electroweak source alone however gives only a small change to $R_b$ and in the wrong direction, and this worsens when $V_0$ is close in mass to Z. A new and unconventional coupling of $P$ to $b$ (and to other normal fermions as well) is needed, not only to compensate for this small electroweak change but also to bring $R_b$ closer to its experimental value. To this end, let us write

$$g_{V0ff}^{V,A} = F_V G_f^{V,A} (s = M_Z^2) + g_{new,f}^{V,A},$$

(5)

where

$$G_f^{V}(M_Z^2) = e^2 Q_f Q_P M_Z^2 + \frac{g_f^V g_P^V}{\cos^2 \theta_W M_Z^2 - M_{Z0}^2 + i M_{Z0} \Gamma_{Z0}},$$

(6a)

and

$$G_f^{A}(M_Z^2) = \frac{g_f^A g_P^A}{\cos^2 \theta_W M_Z^2 - M_{Z0}^2 + i M_{Z0} \Gamma_{Z0}},$$

(6b)

and where $g_f^{V,A}$ is the vector (axial-vector) coupling of the Z boson to the fermion $f$, and $g_P^V = -(1 - (4/3)\sin^2 \theta_W)/4$ and $g_P^A = 1/4$. $Q_f$ and $Q_P(= -1/3)$ are the electric charges. $g_{new,f}$ is the coupling of $V_0$ to a fermion $f$ coming from some new physics. Since $M_Z^2 \simeq M_{Z0}^2$ and $M_{Z0} \Gamma_{Z0} \simeq 3 \times 10^{-2} M_Z^2$, we can safely neglect the $\gamma$ contribution in Eq. (6a) (it contributes negligibly to the present discussion).

For the mass range considered below (shown in the Figure), namely $m_P \simeq 48 GeV - 53 GeV$, $|\Psi(0)|$ is such that $|\delta m^2| \ll |M_{V0}^2 - i \Gamma_{V0} M_{V0} - M_{Z0}^2 + i \Gamma_{Z0} M_{Z0}|^2/2$ and consequently

$$\sin \frac{\theta}{2} \simeq \frac{\delta m^2}{M_{V0}^2 - M_{Z0}^2 + i \Gamma_{Z0} M_{Z0} - \Gamma_{V0} M_{V0}},$$

(7)
with $\cos^2 \frac{\theta}{2} \approx 1$. Typically, $\frac{\theta}{2} \approx 2 - 3 \times 10^{-2}$ and the deviation of $\cos^2 \frac{\theta}{2}$ from unity will be of order $10^{-4}$ and can be neglected considering the present level of precision.

The modified couplings of $Z$ to a fermion $f$ are now

$$\tilde{g}_f^V = (1 + \eta_{f,W}^V + \eta_{f,new}^V)g_f^V,$$ \hspace{1cm} (8a)

$$\tilde{g}_f^A = (1 + \eta_{f,W}^A + \eta_{f,new}^A)g_f^A,$$ \hspace{1cm} (8b)

where $W$ stands for electroweak and the $\eta$'s are complex numbers and are defined by

$$\eta_{f,w}^{V,A} = \sin \frac{\theta}{2} F_{V,A}^f(s = M_Z^2)/g_{f,w}^{V,A},$$ \hspace{1cm} (9a)

$$\eta_{f,new}^{V,A} = \sin \frac{\theta}{2} g_{new}^{V,A}/g_{f}^{V,A},$$ \hspace{1cm} (9b)

where the explicit forms for $\eta_{f,W}^{V,A}$ and $\eta_{f,new}^{V,A}$ can be obtained by using Eqs. (5, 6a, 6b, 7). For simplicity, we shall now assume that $g_{new,f}^V = g_{new} \neq 0$ and $g_{new,f}^A = 0$ so as to reduce the number of parameters and to study the implication of such an assumption. The introduction of a single new coupling is what we meant by universality earlier. We shall argue below why we expect such a behavior. (The inclusion of $g_{new,f}^A$ is quite straightforward.)

In computing the $Z$ widths using Eqs. (8a, 8b) and the range of mass mentioned earlier, one can safely neglect terms proportional to $(\text{Re} \, \eta)^2$ and $(\text{Im} \, \eta)^2$ since they turn out to be at least two orders of magnitude smaller than terms proportional to $\text{Re} \, \eta$ (assuming $g_{new,f}^V < 1$). (Considering the present level of precision, their inclusion is irrelevant to the present discussion.) With this remark in mind, the decay width for $Z \rightarrow f \bar{f}$ is now given by

$$\Gamma(Z \rightarrow f \bar{f}) = \Gamma_{f}^{SM}(1 + \delta_{new}^f),$$ \hspace{1cm} (10)

where $f = q, l$ and where

$$\delta_{new}^f = \frac{2((g_f^V)^2(\text{Re} \, \eta_{f,W}^V + \text{Re} \, \eta_{f,new}^V) + (g_f^A)^2 \text{Re} \, \eta_{f,W}^A)}{(g_f^V)^2 + (g_f^A)^2}.$$ \hspace{1cm} (11)

In Eq. (10), $\Gamma_{f}^{SM}$ contains various radiative correction factors as well as mass factors such as defined in Ref. (3). We find
\[
\Gamma(had) = \Gamma^{SM}(had) + \delta^{u}_{\text{new}}(\Gamma^{SM}_{u} + \Gamma^{SM}_{c}) \\
+ \delta^{d}_{\text{new}}(\Gamma^{SM}_{d} + \Gamma^{SM}_{s} + \Gamma^{SM}_{b}),
\]

(12a)

\[
R_f = \frac{R_{f}^{SM}(1 + \delta_{f}^{\text{new}})}{1 + \delta^{u}_{\text{new}}(R^{SM}_{u} + R^{SM}_{c}) + \delta^{d}_{\text{new}}(R^{SM}_{d} + R^{SM}_{s} + R^{SM}_{b})},
\]

(12b)

where \( R_f \equiv \Gamma(Z \rightarrow q_f \bar{q}_f)/\Gamma(had) \). The central theme of this paper is the use of \( R_b \) to obtain information on the model proposed here. By using Eq. (12b) for \( R_b \), one can extract the parameter \( R_{\eta_{b,new}}^{V} \) and consequently the common parameter \( sin^2\theta_{g_{\text{new}}} \) as a function of \( M_{V_0} \). This will then be used to make predictions on various ratios mentioned above and also on the total Z width. In particular, \( R_{e,\mu} \) will be used to constrain the range of allowed \( M_{V_0} \).

We shall use the following hadron ratios: \( R_b = 0.2202 \pm 0.0020, R_b^{SM} = 0.2155, R_c^{SM} = 0.1721, R_s^{SM} = 0.22, R_u^{SM} = 0.1722 \). The Standard Model predictions \([3]\) given here are for \( m_t = 170 \text{ GeV} \). A more extensive analysis using the range of \( m_t \) given by CDF and D0 will be given elsewhere. Notice that the Standard Model values quoted here are rather insensitive to the Higgs boson mass.

We predict: \( R_c = 0.165 \pm 0.003, R_s = 0.225 \pm 0.002, R_u = 0.165 \pm 0.003 \) to be compared with \( R_{c,\exp} = 0.1583 \pm 0.0098 \) (more than 1\( \sigma \) lower than the standard model prediction).

Notice that an increase in the ratio for a down-type quark corresponds to a decrease in the ratio for an up-type quark and vice versa. This happens because \( R_{\eta_{f,new}}^{V} \) is positive for \( f = u, c \) and negative for \( f = d, s, b \). (\( V_0 \) is a \( P\bar{P} \) bound state.)

Let us turn to \( R_{e,\mu} \). The experimental values are \( R_e \equiv \Gamma(had)/\Gamma(e\bar{e}) = 20.850 \pm 0.067, R_\mu \equiv \Gamma(had)/\Gamma(\mu\bar{\mu}) = 20.824 \pm 0.059 \) to be compared with the Standard Model expectations \((m_t = 170 \text{ GeV})\): \( R_e^{SM} = R_\mu^{SM} = 20.774, \ldots, 20.754 \) for \( m_H = 100, \ldots, 1000 \text{ GeV} \). Although these numbers are consistent within errors (except for \( R_e \)), experimentally there seems to be a tendency for an increase in these ratios. (Even when we take into account the spread in \( m_t, R_{e,\mu}^{SM} < 20.78.\)) Our predictions for \( R_e = R_\mu \) are shown in the Figure (curves labeled by 300 and 700) as a function of \( M_{V_0} \) and for two different values of \( m_H \), namely \( m_H = 300, 700 \text{ GeV} \). They are obtained by using \( R_{b,\text{min}} \) (the theoretical curves obtained by using \( R_{b,\text{max}} \).
The predicted regions lie above these curves. The two vertical lines represent the lower bounds on \( M_V \), namely \( M_{V_0} = 95.6, 103.25 \) GeV for \( m_H = 300, 700 \) respectively, coming from the \( \Gamma_Z \) constraint. The arrows indicate that the regions allowed by \( \Gamma_Z \) are to the right. Finally the two horizontal lines delimit the experimentally allowed region which we take to be the overlap between \( R_e \) and \( R_\mu \). There we take 20.883 as the maximum (from \( R_\mu \)) and 20.783 as the minimum (from \( R_e \)). We only show \( m_H \) up to 700 GeV to be consistent with the global fit although larger values are entirely possible. (For \( m_H = 1000 \) GeV, the lower bound on \( M_V \) is 95.2 GeV.)

The allowed regions are the ones bounded by the theoretical curves, the vertical lines and \( R_e, \text{exp} \). From the Figure one can see that the allowed region for \( m_H = 700 \) GeV is significantly larger than that for \( m_H = 300 \) GeV. This implies that if the resonance were to be found say at 96 GeV one would infer \( m_H > 700 \) GeV, while if it were found at 103.6 GeV one would have a looser bound, namely \( m_H > 300 \) GeV. A lower \( M_{V_0} \) implies a higher lower bound on \( m_H \). We can also conclude that, for \( m_H < 1000 \) GeV, the resonance mass which is compatible with all available data is bounded from below by 95.2 GeV. This corresponds to \( m_P \geq 48.5 \) GeV.

From \( R_\theta \), we can extract \( g_{\text{new}} \) as a function of \( M_{V_0} \) and use these values to compute \( \Gamma_{V_0} \). The dominant contribution to \( \Gamma_{V_0} \) turns out to come mainly from this \( g_{\text{new}} \) with \( \gamma, Z \), and three gluon processes contributing a small amount to the total width. For \( M_{V_0} > 95.2 \) GeV, the lower bound on \( \Gamma_{V_0} \) is found to be 1 GeV and increasing to approximately 8 GeV for \( M_{V_0} \approx 104 \) GeV. The expectation is in general a few GeVs for the width in our scenario while a standard heavy onium will have a width of at most a few MeVs. Notice that the shift in width due to the mixing with Z is small with respect to the above intrinsic width. A further prediction is the fact that, in our scenario, the new coupling is universal so that \( V_0 \) couples equally to quarks and leptons of both up and down types. This implies that \( \Gamma(V_0 \to l\bar{l}) = \Gamma_0 \) and \( \Gamma(V_0 \to q\bar{q}) = 3\Gamma_0 \). The prediction for the branching ratios is \( B_l = 1/24 \) and \( B_q = 1/8 \) where \( l = e, \mu, \tau, \nu,_{e,\mu,\tau} \) and \( q = u, d, s, c, b \).

Let us now turn to the other members of this non-sequential family, the \( R \) quark and the
leptons $N$ and $E$. This is where the S and T parameters come in. Since this new family is non-sequential, there is no reason to expect the mass splitting between up and down members to be "similar" to the other three families. We use the results of (a seven parameter fit) which show the allowed regions in the $T'_{\text{new}} - S'_{\text{new}}$ plane where $T'_{\text{new}} = T_{\text{new}} + T_{M_H}$ and $S'_{\text{new}} = S_{\text{new}} + S_{M_H}$. In view of the discrepancy between the SLD and the LEP asymmetries, we shall use as the allowed region the overlap of all data except the SLD asymmetries. In particular we would like to ask whether or not all of these new particles can be lighter than 80 GeV, an exciting scenario since they can all be accessible to LEP2 in that case. Since the possibilities are many, a few examples will be illuminating.

Let us take $m_P = 50$ GeV and $m_H = 700$ GeV. The Higgs contribution to S and T is given by $S_{M_H} = 0.045$ and $T_{M_H} = -0.132$. For $m_R = 60, 80$ GeV, one has $S_{\text{new, quark}} = 0.138, 0.107$ and $T_{\text{new, quark}} = 0.497, 0.686$ respectively. For the leptons, we shall assume that $N$ is a Majorana particle and that its mass (as well as that of $E$) is greater than 46 GeV. This is the only direct constraint one has on the leptons. We shall use two representative sets of values. 1) For $m_N = 46$ GeV, $S_{\text{new, lepton}} = 0.055, 0.027$ and $T_{\text{new, lepton}} = -0.004, 0.011$ for $m_E = 60, 80$ GeV respectively. We then get the following results in terms of $(S'_{\text{new}}, T'_{\text{new}})$ for the pair $(m_R, m_E)$. We obtain: $(0.238, 0.361)$ for $(60, 60)$ GeV, $(0.207, 0.55)$ for $(80, 60)$ GeV, $(0.21, 0.376)$ for $(60, 80)$ GeV, and $(0.179, 0.565)$ for $(80, 80)$ GeV. 2) For $m_N = 60$ GeV, $S_{\text{new, lepton}} = 0.073, 0.054$, $T_{\text{new, lepton}} = -0.013, -0.007$ for $m_E = 65, 80$ respectively. We obtain: $(0.256, 0.352)$ for $(60, 65)$ GeV, $(0.225, 0.541)$ for $(80, 65)$ GeV, $(0.237, 0.358)$ for $(60, 80)$ GeV, and $(0.206, 0.547)$ for $(80, 80)$ GeV. All of those values are inside the allowed region shown in [1].

From the (not-exhaustive)examples given above, it is clear that the S and T parameters certainly allow for the existence of this new, non-sequential family and that the T parameter tends to favor a value of $R$ mass lower than 80 GeV (and in no way should it be more than 90 GeV). This opens up the possibility that the whole family can be found by LEP2. First the R ratio would be 16/3 or at least 14/3 (if the $R$ quark mass is above 80 GeV). Secondly, there would be two narrow resonances: the first one being the $P \bar{P}$ bound state and the
second one being the $\mathcal{R}\mathcal{R}$ bound state. Since these quarks do not mix with those of the other three generations, $\mathcal{P}$ will be relatively stable and the search for $\mathcal{R}$ will be an interesting problem in hadron colliders. Finally there will be an unmistakable signature for the charged heavy lepton in LEP2. These issues will be explored elsewhere.

Finally we would like to say a few words about a possible origin of the coupling $g_{\text{new}}$. First solving for $g_{\text{new}}$ using Eq. (12), one has for example the following range: $g_{\text{new, min}} = 0.135 - 0.377$ for $M_{V_0} = 95.5 - 104.5$ GeV where $R_{b, \text{min}}$ has been used. (It turns out that the constraint coming from $R_{e, \mu}$ gives an allowed range for $g_{\text{new}}$ ranging from the previous $g_{\text{new, min}}$ to a slightly higher value for each $M_{V_0}$.) It is clear that the larger $M_{V_0}$ is (and consequently less mixing with $Z$) the larger $g_{\text{new}}$ should be in order to preserve the “anomaly” in $R_b$. The following discussion will be very speculative but helpful to illustrate a few interesting scenarios.

Let us now imagine there is a four-fermi coupling of the form: $(g^2_s/\Lambda^2)\mathcal{P}\gamma_\mu\mathcal{P}\bar{f}\gamma^\mu f$, where $f$ denotes any fermion of the first three generations. The translation of this coupling into $g_{\text{new}}$ is of course highly model-dependent. The simplest (and most likely easiest to be ruled out) scenario is to assume that the above coupling comes from the exchange of some vector boson with point-like coupling to the fermions. We would then identify $g_{\text{new}} \equiv (g^2_s/\Lambda^2)F_V$, meaning that it can be described by the wave function at the origin. If $g^2_s/4\pi = 1$ (a strong coupling scenario), we get $\Lambda = 163 - 104$ GeV for $M_{V_0} = 95.5 - 104.5$ GeV, while if $g^2_s/4\pi = 2.5$, we would get $\Lambda = 259 - 166$ GeV for the same range. These scales are ”uncomfortably” low. Another scenario is to assume that all fermions are composite (for instance they could be bound states of a scalar field and a fermion field). For definiteness, let us assume that only the fermionic constituents carry color. A four-fermi coupling given above would be diagramatically similar to the quark diagram for meson-meson scattering except that here we would have a scalar line instead of one of the two quark lines. It follows that $g_{\text{new}}$ is not necessarily given by the wave function at the origin. We shall assume that we can write $g_{\text{new}} \equiv (g^2_s/\Lambda^2)g^2_HF_V$ where $g^2_H$ represents the rescattering of the scalar components. If $g^2_s/4\pi = g^2_H/4\pi = 1, 2.5$, $\Lambda$ can be found to be respectively 580-371 GeV and 1.45-1.04 TeV.
for $M_{V_0} = 95.5 - 104.5$ GeV. Here $\Lambda$ would represent the compositeness scale, which as we have seen could be in the (low)TeV range. A model which we are currently investigating is similar in spirit to the Abbott-Farhi model except that the confining gauge group is not the electroweak group but a horizontal gauge group which we take to be $SU(2)_L \otimes SU(2)_R$. In this model, there remains a residual global horizontal $SU(2)$ with composite fermions forming a triplet (the three standard families) and a singlet (the non-sequential fourth family) under that group. A full discussion of the model is beyond the scope of this paper.

We have presented a simple scenario to explain the "anomaly" in $R_b$ and, as a consequence, we have made a number of predictions including the presence of a new, non-sequential fourth family whose masses could be all below $M_W$, an exciting prospect for near-future discoveries.

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FIGURES

FIG. 1. The allowed regions in the $R_e(\equiv \Gamma(Z \to had)/\Gamma(Z \to e\bar{e} or \mu\bar{\mu})) - M_{V_0}$ plane. The theoretical predictions lie above the curves labeled by 300 and 700 for two different values of $m_H$. The vertical lines with similar labels represent the regions (to the right) allowed by $\Gamma_Z$. The experimentally allowed regions lie between the two horizontal lines labeled by $R_{e,exp}$. The intersections of these lines represent the final allowed regions.
