Analysis on high-altitude earth Orbit Satellite Determination

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Abstract. The difference is introduced between approx circular apogee orbit and approx circular perigee one by error transmitting at first. Then the characteristic of secant compensation is analysed when radar tracking object with high elevation. And two kinds of orbit force be pressed to, their perturbation influence and their earth-core angles are explained. And then the series of emulation results are shown including error data emulated with Monte Carlo method, the influence of the velocity increment from the ejecting force of spring while satellite-rocket separating and their perturbation influence and the length of influence of the data arc. Then decision analysis of Wald method and Bayesian statistics rule and the results from the two rule are introduced . So the suitable orbit determination decision is put forward from the decision method. Finally the result is tested reasonable and feasible via the real data. In the end it is useful to reference to make orbit decision in short injection of circular orbit far from the earth for calculating concurrently precise and timely.

1. Introduction

There are different results in same arc length of different kinds of orbit. Even to the same orbit there are different results in same arc length at different space position. So it is important to orbit that how long or how many points should be use. Also it is urgent to know that what precision should be. And a set of analysis method are explained in this paper.

2. Analysis of tracking characteristic

2.1. Analysis of error transmitting

It is obvious that different tracking equipments have different characteristic when tracking aircraft of space at different point. For example, when satellites are launched into schedule orbit at altitude about from 200 ~ 300 kilometers to 36000 kilometers the tracking distance changes from 300 ~ 3000 kilometers to 37000 ~ 40000 kilometers. And the location error gets obviously bigger, especially in REA single measuring frame the angle error alter great with larger rang.

\[
\begin{array}{c}
\sigma^x_z = \cos \theta \cos \phi \sigma^x + \sin \theta \sin \phi \sigma^y + \cos \phi \sigma^z
\\
\sigma^y_z = \sin \theta \sigma^x + \cos \theta \sigma^y
\\
\end{array}
\]

\(\sigma_x, \sigma_y, \sigma_z\) means error of measuring element,

\(\sigma_x, \sigma_y, \sigma_z\) means location error of aircraft.

2.2. Analysis of tracking with high elevation
When Radar tracking objective secant compensation[1] are added in azimuth spur track. When elevation is up to 70 degree, the Radar’s capability comes down in a sort of way, such as the largening dynamic lag error, the stronger influence of ship sway, the bigger random error with blowing up yawp and much more area of no sensitivity in electrocircuit exist. When radar running in azimuth direction its balance adjustment becomes worse. The influence of secant compensation not only aggravate the tracking balance but also make the remainder of error exaggerated when the radar is with high elevation. Even when elevation is up to 70 degree, the remainder of error exaggerated to 2.9 times refer to Table 1.

| E   | 45° | 60° | 70° | 75° |
|-----|-----|-----|-----|-----|
| SecE| 1.414 | 2.00 | 2.924 | 5.759 |

Table 1. Value of SecE

3. Analysis of orbital characteristic

3.1. Analysis of error transmitting
As a rule, the force element of the space craft is analysed firstly. Commonly, gravitation is the main force to all earth satellites. And other orbit perturbation force are used to get more accurate results of satellite of orbit determination, orbit prediction and orbit maneuver for researchers. For example, the atmospheric drag is the most important force of orbit perturbation for low earth orbit (LEO) comparing to the force of the solar light pressure which is almost ignored. But for geosynchronous-orbit satellite (GEO) the influence of the force of the solar light pressure is so obviously mighty that it is never to be dispensable. At the same time the force of the atmospheric drag is infirm enough to be neglectable.

3.2. Analysis of the perturbation influence of the force
Secondly, figures are listed for explaining the difference of the orbital variety between approx circular apogee orbit and approx circular perigee one. The semi-major axis of orbit of approx circular apogee orbit varies less than that of approx circular perigee orbit. View fig.1, the change of semi-major axis of orbit of approx circular apogee orbit is about 4 kilometres while that of approx circular perigee orbit is up to 10 kilometres within 24 hours. It is shown in fig.2 that the alteration of orbit elements is very small within 80 seconds.

![Figure 1. Variation of orbit determination within 24 hours](image-url)
Figure 2. Variation of orbit determination within 80 seconds

3.3. Analysis of measuring data arc

The data with different earth-core angle is with difference influence on the orbit determination. Figure 3 is shown that there is big difference between approx circular apogee orbit and approx circular perigee one with data arc from 60 seconds to 600 seconds. And the earth-core angle of approx circular perigee orbit is far away langer than that of approx circular apogee one in same hours.

Figure 3. Data arc and its earth-core angle

4. Analysis of orbit determination with emulational tracking data

4.1. Analysis of velocity increment

If the satellite gets 0.5 m/s velocity increment from the ejecting force of spring while satellite-rocket separating. It can change the orbit of the satellite. So it must be modified that when the results of orbit are calculated. At the same time we can infer that the attitude data influences the value of orbit determination refers to the table 2 and the table 3.

| order | Elevation (°) | Yaw (°) | Scroll (°) |
|-------|---------------|---------|------------|
| 1     | 151.6         | 1.8     | 91.1       |
| 2     | 150.5         | 14.9    | 83.8       |
| 3     | 149.3         | 20.8    | 80.3       |
| 4     | 148.4         | 24.3    | 78.0       |
Table 3. Orbit determination value from velocity increment

| order | Δa (km) | Δe (e-4) | Δi (º) | ΔΩ (º) | Δλ (º) |
|-------|---------|----------|--------|--------|--------|
| 1     | 11.3619 | -2.7     | -0.00527 | 0.000967 | 0.000852 |
| 2     | 12.8367 | -3.0     | -0.00342 | 0.000628 | 0.000108 |
| 3     | 13.2882 | -3.2     | -0.00251 | 0.00046 | -5.1E-05 |
| 4     | 13.4954 | -3.2     | -0.00193 | 0.000355 | -9.3E-05 |

4.2. Analysis of measuring data added error

Fifty groups of 80s observation data added errors are emulated from the standard orbit. Errors are created with the method of Monte Carlo. Then data are used to determine the results of the orbit. Finally the difference between the standard orbit and the results from the emulational data are listed in the table 4.

Table 4. Deviation of different emulational data

|       | Δa (km) | Δe (10-04) | Δi (º) | ΔΩ (º) | Δλ (º) |
|-------|---------|------------|--------|--------|--------|
| Mean  | 0.6     | -0.180     | 0.015722 | -0.006259 | -0.03034 |
| variance | 28.6   | 6.789     | 0.030131 | 0.005635 | 0.006559 |

4.3. Analysis of measuring data arc

Data arc of observation from 120s to 900s with errors are emulated. And every data arc includes fifty groups refer to 4.2. Then the trend of data is shown in figure 4 and figure 5.

Figure 4. Difference of orbit determination with different data arc

Figure 5. Difference of orbit determination with different data arc
5. Analysis of decision

5.1. Decision
Decision means the passing of judgment on an issue under consideration before it occurs. Decision can be classified into definite model and probability model.

5.2. Wald method
Wald method is a rule of selecting the good from the bad. It is also called Max-min rule. Under the rule the decision-maker will pick out the proceeding with the maximal income from the least. Premising that there are m kinds of spare plans and n sorts of nature the value of profit and loss is $F_{ij}$. And $F_{\text{min}}(a_i)$ stands for the least profit with the plan of i and the nature of $a_i$.

$$F_{\text{min}}(a_i) = \min_{\theta_j \in \Theta} F_{ij} = \min\{F_{i1}, F_{i2}, \ldots, F_{im}\}.$$  

If $F^*$ is the profit of optimization plan it can be

$$F^* = \max_{a_i \in d} F_{\text{max}}(a_i) = \max_{a_i \in d} \{F_{\text{max}}(a_1), F_{\text{max}}(a_2), \ldots, F_{\text{max}}(a_m)\}.$$  

The plan corresponding $F^*$ is the best choice according as the Wald rule.

If the function of profit and loss means the loss, the model is

$$F_{\text{max}}(a_i) = \max_{\theta_j \in \Theta} F_{ij} = \max\{F_{i1}, F_{i2}, \ldots, F_{im}\}, \quad F^* = \min_{a_i \in d} F_{\text{max}}(a_i) = \min\{F_{\text{max}}(a_1), F_{\text{max}}(a_2), \ldots, F_{\text{max}}(a_m)\}.$$

5.3. Bayes method
Bayes rules mean every nature will be assign certain probability to show its generant possibility. The Value of expectation is standard to all the possible results. And its weight is evaluated according to its probability.

First, the expected profit and loss is computed under every action, called $E(a_i)(i = 1, 2, \ldots, m)$, so

$$E(a_i) = \sum_{j=1}^{n} F_{ij} P(\theta_j)(j = 1, 2, \ldots, n).$$

As to the Wald method $F_{ij}$ is the value of profit and loss. And $P(\theta_j)$ is the probability of the nature of $\theta_j$. So there are m kinds of profit and loss if there are m kinds of preparing plan for selecting. So the plan with the max or min profit and loss is the recommended one based on the Bayes rule.

$E^* = \max_{a_i \in d} E(a_i), \quad E^* = \min_{a_i \in d} E(a_i).$

5.4. Decision examples
There are eight kinds of spare plan for selecting with nine orbit error zone in one plan. ‘a1’ is the result of orbit with 120 points, ‘a2’ is the result of orbit with 150 points, ‘a3’ is the result of orbit with 200 points, ‘a4’ is the result of orbit with 300 points, ‘a4’ is the result of orbit with 350 points, ‘a6’ is the result of orbit with 400 points, ‘a7’ is the result of orbit with 500 points, ‘a8’ is the result of orbit with 600 points. The zone from zero to 45 degree is divided into nine, shown in table 5.

| $a_i$ | 5   | 10  | 15  | 20  | 25  | 30  | 35  | 40  | 45  |
|------|-----|-----|-----|-----|-----|-----|-----|-----|-----|
| 1    | 0.25| 0.1 | 0.05| 0.25| 0.15| 0.1 | 0.1 | 0.1 | 0.1 |
| 2    | 0.2 | 0.05| 0.15| 0.3 | 0.15| 0.1 | 0.1 | 0.05| 0.05|
| 3    | 0.5 | 0.3 | 0.1 | 0.1 | 0   | 0   | 0   | 0   | 0   |
| 4    | 0.65| 0.3 | 0.05| 0   | 0   | 0   | 0   | 0   | 0   |
| 5    | 0.45| 0.55| 0   | 0   | 0   | 0   | 0   | 0   | 0   |
| 6    | 0.65| 0.35| 0   | 0   | 0   | 0   | 0   | 0   | 0   |
| 7    | 0.85| 0.15| 0   | 0   | 0   | 0   | 0   | 0   | 0   |
| 8    | 1.00| 0   | 0   | 0   | 0   | 0   | 0   | 0   | 0   |

### Table 5. Orbit determination arc decision
Based on the rule of Wald or Bayes, the plan ‘a8’ is the recommended one. But if we want to get the result with both suitable precision and the shortest data arc another plan will be the best. So we suppose the warp of 10 kilometers is the suitable precision, the effect is 0.8 when the time longer than 300 seconds because that will effect at a discount and the time less than 300 seconds is perfectible 1. Results of profit and loss is shown in table 6. From the table 6 the plan ‘a5’ is the most suitable one.

| Plan | at 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|------|-----|---|---|---|---|---|---|---|
| $F_{\infty}(a_8)$ | 45  | 45 | 20 | 15 | 10 | 10 | 10 | 5 |
| $E(a_8)$         | 17.5| 19.3| 9  | 7  | 7.8| 6.8| 5.8| 5 |
| assignment       | 0   | 0  | 0  | 1  | 1  | 1  | 1  | 1 |
| effect           | 1   | 1  | 1  | 0.8| 0.8| 0.8| 0.7| 0.7|

It is tested that the plan ‘a5’ is the most suitable one with real data. The error of orbit determination is 5 kilometers or so.

**Reference**

[1] En-dian H, Pei L and Rui C 2011 The Indetermination Factor Analysis of High Elevation Arc Segment Radar Traching J. Science Technology and Engineering. (Bei jing) 11 1671

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[3] Ying L 2005 Statistical Analysis of Decision (Tian Jin:tianjin university) chapter 4 pp 88–95