Evolution of a spinor condensate: coherent dynamics, dephasing and revivals

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We present measurements and a theoretical model for the interplay of spin dependent interactions and external magnetic fields in atomic spinor condensates. We highlight general features like quadratic Zeeman dephasing and its influence on coherent spin mixing processes by focusing on a specific coherent superposition state in a $F=1$ $^{87}$Rb Bose-Einstein condensate. In particular, we observe the transition from coherent spinor oscillations to thermal equilibration.

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Multi-component Bose-Einstein condensates with spin degree of freedom, the so called spinor condensates, are experiencing rapidly growing attention. These ultra-cold quantum gas systems are interesting in several respects, e.g. they show intriguing static and dynamic magnetic properties, they represent a well controlled thermodynamic model system, and they promise unique insights into mesoscopic multi-component entanglement and decoherence processes.

Pioneering experiments on $F=1$ $^{23}$Na spinor condensates, found to be anti-ferromagnetic, and quasi-spin-1/2 systems in $^{87}$Rb have shown fascinating demixing dynamics, metastability and domain formation processes. Recently the spinor systems under study have been extended to $^{87}$Rb, which behaves ferromagnetic in the $F=1$ state and anti-ferromagnetic in $F=2$. In particular, these experiments also showed spinor oscillations and shifted the research interest towards dynamic spin conversion effects. Recent studies have demonstrated a magnetic field dependent oscillation amplitude and frequency, which was also theoretically assigned to an interesting resonance phenomenon. The coherent evolution in these experiments at short timescales is opposed to the incoherent thermodynamic behavior at long timescales, e.g. decoherence driven cooling, constant temperature BEC or temperature driven magnetization.

The coherent and thermodynamic regime are conceptually very different, as the first relies on the assumption of each atom in the ensemble being in the same superposition state, while there are presumably different and independent ensembles for each spin state in the second regime, i.e. here an atom is either in states $|F=1, m_F = \pm 1\rangle$ or $|F=1, m_F = 0\rangle$. It is still under discussion, how and at which point the crossover from the "pure state" ensemble to a "mixed state" arises and how this is connected to decoherence and dephasing mechanisms in spinor systems. This question can be generalized to the understanding of decoherence in arbitrary multi-component systems and its dependence on the number of constituents, which might have important consequences for quantum information theory. So far studies on the relative phases of different components have only been performed in quasi-spin 1/2 systems based on the preparation of mixtures with well defined phase, while investigations on spinor systems were essentially restricted to population-based state preparation without phase control.

In this paper we discuss dephasing and decoherence effects in spinor quantum systems. In particular we present measurements on the influence of finite temperature and external magnetic fields on the evolution of a $F=1$ $^{87}$Rb spinor condensate prepared in the $\zeta_+ = (\zeta_+, \zeta_0, \zeta_-)^T = (-1, -i\sqrt{2}, 1)^T/2$ state. In addition we develop an analytic description for the mean-field evolution and emphasize the importance of the quadratic Zeeman effect, which leads to periodic dephasing and rephasing of the spin expectation value. We theoretically predict and experimentally find dephasing induced spin dynamics with magnetic field dependent period and amplitude, which clearly shows the coherence of spin mixing in spinor condensates. An analysis of the data again independently confirms the $F=1$ state of $^{87}$Rb to be ferromagnetic.

I. THEORY

In order to simplify the discussion and concentrate on the spinor physics, we will restrict ourselves to the single mode approximation (SMA), which effectively factorizes the $F=1$ spinor order parameter as

$$\psi(r, t) = \begin{pmatrix} \psi_+(r, t) \\ \psi_0(r, t) \\ \psi_-(r, t) \end{pmatrix} = \sqrt{n(r)} \begin{pmatrix} \zeta_+(t) \\ \zeta_0(t) \\ \zeta_-(t) \end{pmatrix},$$

where the volume density $n(r)$ and spinor $\zeta$ are individually normalized to $\int d^3r n(r) = N$ and $||\zeta||^2 = 1$, respectively. This assumption is justified, if the spin dynamics is slow compared to the timescale of motion in the trap and if the spin dependent interaction is much smaller than the spin independent one.
In SMA, the mean-field equations of motion for the spinor are \[ i \partial_t \zeta = (H_z + H_{mf}) \zeta, \] \[ H_z = H_{Lz} + H_{CQZ} = -p F_z - q (1 - F_z^2), \] \[ H_{mf} = g_2 \langle n \rangle \sum_{\alpha=x,y,z} F_\alpha \zeta^\dagger F_\alpha, \] where \( p \propto B \) and \( q \propto B^2 \) are the coefficients of the linear and quadratic Zeeman effect respectively. For \(^{87}\text{Rb}, \) \( F=1, \) the Breit-Rabi formula allows to calculate them as \( p \approx -\mu_B B/2\hbar \) and \( q \approx \mu_B^2/\hbar \omega_{\text{hfs}} \) with the \(^{87}\text{Rb} \) ground state hyperfine splitting \( \omega_{\text{hfs}} \approx 2\pi \cdot 6.835 \text{ GHz}. \) The spin-dependent mean-field interaction is proportional to \( g_2 = 4\pi \hbar / m \times (a_{F=2} - a_{F=0})/3 \) and \( \langle n \rangle = \int d^3 r n^2(r)/N. \) We will identify the 3 \( \times \) 3 matrices \( \{F_x, F_y, F_z\} \) as the standard representation of the \( F = 1 \) angular momentum algebra. In principle, the mean-field Eqs. \( \ref{eq:mean-field} \) could be derived also from a Hamilton-Jacobi theory. The conservation of spin and energy follow naturally from rotational and time-translation symmetries. Due to these constraints, it is indeed possible to find integrable solutions to the full nonlinear equations of motion. In case of the initial state \( \zeta_0 \), the time-dependent spinor amplitudes can be written in terms of Jacobi elliptic functions: \[ \zeta_0(t) = \frac{s}{\sqrt{2}} \left[ \frac{1 - k \sn_k(\frac{t}{2})}{1 - k \sn_k^2(\frac{t}{2})} - i \frac{\cn_k(\frac{t}{2}) \dn_k(\frac{t}{2})}{1 + k \sn_k^2(\frac{t}{2})} \right], \] \[ \zeta_{\pm}(t) = \pm \frac{e^{\pm i\pi/2}}{2} \left[ \frac{\cn_k(\frac{t}{2}) \dn_k(\frac{t}{2})}{1 - k \sn_k^2(\frac{t}{2})} - i \frac{(1 + k) \sn_k(\frac{t}{2})}{1 + k \sn_k^2(\frac{t}{2})} \right], \] where \( s = \exp(-i(q_2\langle n \rangle - q)t/2) \) is a dynamic global phase and \( k = \frac{g_2\langle n \rangle}{q} \) is the ratio of interaction energy to quadratic Zeeman effect. For the directly measurable populations, the solution simplifies to \[ |\zeta_0(t)|^2 = (1 - k \sn_k^2(qt))/2, \] \[ |\zeta_{\pm}(t)|^2 = (1 + k \sn_k^2(qt))/4. \] For small \( |k| \ll 1, \) the Jacobi elliptic functions can be approximated by ordinary trigonometric ones, i.e., \( \sn_k(x) \approx \sin(x), \) \( \cn_k(x) \approx \cos(x), \) \( \dn_k(x) \approx 1. \) Thus, the populations of Eqs. \( \ref{eq:mean-field} \) oscillate with an amplitude given by \( k, \) and a period which is \( \pi/q \) for small \( |k|. \) This solution offers a clear way to determine the magnetic properties of the system as \( k \) is negative/positive for ferromagnetic/anti-ferromagnetic systems, resulting in an increase/decrease of \( |\zeta_0|^2 \) at the beginning of the oscillation (if \( q > 0). \)

At this point, we would like to emphasize the intricate connection between quadratic Zeeman dephasing and spin mixing dynamics: for zero quadratic Zeeman shift \( q = 0, \) the state \( \zeta_0 \) does not evolve according to Eq. \( \ref{eq:mean-field} \), except for a trivial Larmor precession around the magnetic field vector, which can be compensated by choosing a rotating frame of reference. The intuitive reason for this lies in the rotational symmetry of the spin interactions, leading to a conservation of magnetization. In a frame rotated by \( 90^\circ, \) the above state is just the fully stretched state \( \zeta = (0, 0, 1) \), which is an eigenstate of \( H_{mf} \) and consequently does not evolve. In fact, any rotated stretched state belongs to this class of stationary states of \( H_{mf}. \)

The quadratic Zeeman effect however destroys the fully stretched state, removing it from the class of stationary states, and thus allowing interaction driven spin dynamics. Two regimes can be distinguished in the resulting population oscillation: the interaction dominated regime at low magnetic fields, where the oscillation period is essentially given by the intrinsic spin interactions, and the quadratic Zeeman regime, where it is given by the respective beat period \( \pi/q. \) In both limits \( k \to 0 \) and \( q \to 0, \) the amplitude of population oscillations goes to zero. Interestingly the crossover between the two regimes shows a resonance-like structure, as also recently predicted for a more general case \( \ref{eq:mean-field}. \)

### II. EXPERIMENT

In our experimental setup described in \( \ref{eq:mean-field} \) we initially prepare degenerate \(^{87}\text{Rb} \) ensembles containing up to \( 10^6 \) atoms in the \( |F = 1, m_F = -1 \rangle \) state in an optical potential with trapping frequencies \( \omega_x : \omega_y : \omega_z \approx 1 : 7 : 40 \) and \( \omega_z = 2\pi \cdot 16 \ldots 25 \text{ s}^{-1}. \) Using a radio frequency pulse of typically \( 40 \mu\text{s} \) duration, we evolve the spinor into the superposition state \( \zeta(0) = \zeta_0^\dagger F \zeta \) from \( \mathcal{F} = -\varepsilon_z \) to \( \mathcal{F} = \varepsilon_y. \)

#### A. Phase evolution

Using the classical picture it is evident, that another \( \pi/2 \) radio frequency pulse will rotate the spin vector upward to \( \mathcal{F} = +\varepsilon_z \) or equivalently the spinor to \( \zeta = (1, 0, 0)^\top, \) transferring all the population into the \( |F = 1, m_F = +1 \rangle \) state. The addition of a delay time in between the two pulses results in the spin-1 analogon of the famous Ramsey experiment for spin-1/2 particles \( \ref{eq:mean-field} \), which is sensitive to the quantum mechanical phase evolution of the system. The last \( \pi/2 \)-pulse creates superpositions of the spinor components and thus transforms the phase information into population information inaccessible to detection. Thus, the state of the Ramsey spinor is \( \zeta_R(t) = \exp(-i\pi/2 F_z) \exp(i\omega_{RF}t F_z) \zeta(t) \) with \( \zeta(t) \) from Eqs. \( \ref{eq:mean-field}. \)

In the following we will analyze the time-dependent population distribution in such a Ramsey experiment in order to gain insights into dephasing and decoherence mechanisms. The corresponding analytical results in Fig. \( \ref{eq:mean-field} \) clearly show a magnetization oscillation with the detuning between the Larmor precession frequency
Gerlach separation and imaging of the individual spin components. In order to emphasize the effect of quadratic Zeeman dephasing instead of the trivial Larmor precession, we plot the magnitude of the magnetization vector instead of the magnetization itself and concentrate on the envelope of the corresponding data points [24]. In order to achieve the parameters used for Fig. 1 an offset field of $\approx 1.1$ G was applied, corresponding to a Larmor precession frequency of $|p| = 2\pi \cdot 772$ kHz and a quadratic Zeeman beat frequency $|q| = 2\pi \cdot 87.2$ Hz corresponding to a beat period of $\approx 5.7$ ms, i.e. operating in the quadratic Zeeman regime. In this case already the single particle picture discussed above clearly reproduces our experimental data: The magnetization is modulated in amplitude by the quadratic Zeeman effect with collapses and revivals corresponding to a beat period of $\pi/q$.

The timescale of the graph was chosen to capture the evolution from the coherent to the thermodynamic regime, which is reflected by the decreasing envelopes of magnetization and $m_F = 0$ population. In order to identify further dephasing and decoherence timescales we will now concentrate on the evolution of the $m_F = 0$ population, which is not visibly affected by quadratic Zeeman dephasing. The oscillation of both normal and condensed $m_F = 0$ populations is damped and initially tends toward its mean value of 0.25. This damping could be explained by random or spatial dephasing e.g. due to magnetic field gradients [24], or by breakup into small dynamical spin domains due to a dynamical instability [21]. On this timescale, dephasing should at least in principle be reversible. In contrast, the drifting apart of...
normal and condensed $m_F = 0$ population at $\approx 15$ ms marks the crossover to the thermodynamic regime on longer timescales. In this regime, the system tends to restore thermal equilibrium meaning in particular equal population (i.e. 0.33) of all spin components in the thermal cloud \cite{1, 2}. The main process here is redistribution of atoms between condensed fraction and thermal cloud within each spin component, leading to the diverging trend in $m_F = 0$ populations observed.

### B. Population evolution

In order to observe the effect of spin interaction beyond the single particle level, we investigated the resulting population evolution of the intermediate Ramsey state $\zeta_{\pm}$ directly, i.e. in a sequence of a $\pi/2$-pulse and a variable delay before detection. In contrast to the above Ramsey experiment this sequence is not sensitive to the relative phases of the spinor components but gives complementary information on the interaction driven population evolution.

The quadratic Zeeman dominated regime at high magnetic fields is particularly interesting for the investigation of coherence in interaction driven spin dynamics. Our theoretical results Eq. (4) predict that the quadratic Zeeman phase determines the direction of spin mixing dynamics, demonstrating the phase sensitivity of the process.

The results shown in Fig. 3 clearly confirm the coherence of spin dynamics in spinor Bose-Einstein condensates \cite{2}. Furthermore the positive initial slope of the population evolution of the $m_F = 0$ state confirms the ferromagnetic behavior of $^{87}$Rb in the $F=1$ hyperfine manifold \cite{3, 4}.

Comparison with theory (Eq. 4) shows an excellent agreement during the first 10 ms of evolution, in which a coherent behavior is expected considering the results of the Ramsey experiment. The crossover to the thermodynamic regime at later times goes along with damping of the population oscillations and a transfer of particles between the condensate fraction and normal component. The thermodynamic nature of this process also shows up as a temperature dependence with longer decoherence times for the lower temperature data. It is important to note, that the damping of the oscillations observed in this case is fundamentally different from the Ramsey experiment. The Ramsey experiment is sensitive to both spatial dephasing and thermal decoherence, while the "single $\pi/2$-pulse experiment" is only affected by thermal decoherence. The relatively long lasting $m_F = 0$ oscillations in this experiment thus corroborate our interpretation that the faster decay in the Ramsey experiment is not of thermal origin, but probably due to magnetic field dephasing or more complex effects \cite{21}.

The amplitude of the population oscillations is proportional to the spin dependent interactions via the parameter $k$ in Eq. (4). In order to compare the experimen-

![FIG. 3: Evolution of the $m_F = 0$ population in the condensate fraction for ensembles containing $\approx 10^5$ atoms with $25\ldots 30\%$ condensate fraction (upper figure) and $\approx 4 \cdot 10^5$ atoms with $75\ldots 80\%$ condensate fraction (lower figure), prepared in the state $\zeta_{\pm}$. The data points correspond to several measurements binned in 1 ms intervals and error bars representing ± the standard deviation. The blue/red/green curves correspond to measurements at different offset fields with quadratic Zeeman periods of $\pi/q \approx 23/11.7/5.7$ ms respectively. The lines correspond to the analytic solution \cite{8}, with parameters $q$ calculated from the magnetic offset field and $k$ obtained from fitting to the first 10 ms of data. Additional fit parameters account for detection errors (y-offset) and evolution during time of flight (x-offset).](image1)

![FIG. 4: Fit values of the $|\zeta_0|^2$ oscillation amplitude parameter $k$ of Eq. (4) versus the theoretical expectation for measurements with $25\ldots 30\%$ condensate fraction (squares) and with $75\ldots 80\%$ condensate fraction (triangles). The lines correspond to linear fits excluding the highest $k$ data points.](image2)
value and surprisingly show a clear temperature dependence. Possible explanations include the breakdown of the Thomas-Fermi approximation in the tightly confined vertical direction, trap anharmonicities or small scale domain formation and require further investigations.

In conclusion we have analyzed the interplay of external magnetic fields and interatomic interactions on the system evolution, giving analytic expressions in comparison to experimental data. The experimental data at finite temperature shows various dephasing/decoherence mechanisms, which we were able to separate using the theoretical model and adapted experimental sequences. These investigations pave the way to use spinor condensates for general studies of dephasing and decoherence phenomena in multi-component systems.

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[23] Note that the class of rotated stretched states does not form an eigenspace of $H_{\text{rat}}$, since $H_{\text{rat}}$ is an effective Hamiltonian which in turn depends on the spin state.
[24] We are able to follow the Larmor precession for 5 ms before shot-to-shot phase fluctuations corresponding to a magnetic field noise of $\approx 200\mu G$ lead to a random phase of the magnetization oscillation at the time of detection. This does not effect the detection of quadratic Zeeman dephasing of the coherent evolution, which can clearly be identified by looking at the envelope of the data points.
[25] It is interesting to note that the initially prepared spinor corresponds to the ferromagnetic ground state (a stretched state with $|F|^2 = 1$) while the fully dephased spinor corresponds to an anti-ferromagnetic ground state ($|F|^2 = 0$) of the spin interaction Hamiltonian $H_{\text{rat}}$. In the quadratic Zeeman regime, the spinor is thus periodically transformed between these two extremal states, which both show no spin dynamics. A comparison with the Ramsey experiment shows that the extrema of the $m_F = 0$ population oscillations indeed coincide with the nodes and antinodes of the Ramsey amplitude envelope.