Polariton interactions in microcavities with atomically thin semiconductor layers

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We investigate the interactions between exciton-polaritons in \( N \) two-dimensional semiconductor layers embedded in a planar microcavity. In the limit of low-energy and low-momentum scattering, where we can ignore the composite nature of the excitons, we obtain exact analytical expressions for the spin-triplet and spin-singlet interaction strengths, which go beyond the Born approximation employed in previous calculations. Crucially, we find that the strong light-matter coupling enhances the strength of polariton-polariton interactions compared to that of the exciton-exciton interactions, with the latter vanishing in the zero-momentum limit. We furthermore show that the polariton interactions have a highly non-trivial dependence on the number of layers \( N \), and we highlight the important role played by the optically dark states that exist in multiple layers. In particular, we predict that the singlet interaction strength is stronger than the triplet one for a wide range of parameters in most of the currently used transition metal dichalcogenides. This has consequences for the pursuit of polariton condensation and other interaction-driven phenomena in these materials.

Microcavity exciton-polaritons (polaritons) are quasiparticles that arise from the strong coupling between semiconductor excitons (bound electron-hole pairs) and cavity photon resonances. Due to their excitonic component, polaritons interact with each other, in contrast to bare photons in vacuum. This interaction is the cornerstone of a variety of observed phenomena ranging from optical parametric scattering \([1]\) and bistability \([2]\), to Bose-Einstein condensation \([3, 4]\), superfluidity \([5]\) and the formation of quantized vortices \([6]\). Hence, semiconductor microcavities are fruitful platforms to investigate two-dimensional (2D) quantum fluids of light \([7–10]\).

Atomically thin semiconductors in the form of transition metal dichalcogenides (TMDs) have recently emerged as promising materials for realizing polaritonic phenomena at room temperature, due to the large exciton binding energies in TMD monolayers \([11–14]\). Furthermore, TMDs can be made nearly disorder free, unlike organic materials \([15]\), and they can be externally tuned using electrostatic gating \([14]\), which is an essential tool for any future optoelectronic devices \([16]\). Already, a strong exciton-photon (Rabi) coupling has been observed in both TMD monolayer \([17–20]\) and multilayer structures \([21, 22]\). In particular, the use of multilayer van der Waals heterostructures can generate large Rabi couplings \([23]\) as well as provide routes towards engineering other material properties \([24]\). However, it is an open and non-trivial question how the polariton-polariton interactions depend on experimental parameters such as the light polarization \([25]\) and the number of layers in these systems. The answer to this question impacts the highly active investigation of interaction-induced nonlinear optical properties \([26–30]\) and the ongoing quest \([31]\) to realize polariton condensation in pure TMD systems.

In this Letter, we address this question by studying the effective interactions between polaritons in a system of \( N \) identical 2D layers embedded in a planar microcavity. A key simplification of our work is to assume that the energy scale of polariton-polariton scattering is smaller than the exciton binding energy, thus allowing us to neglect the composite nature of the excitons and treat them as structureless bosons with contact interactions and mass \( m_X \). This is a reasonable assumption in the case of TMD layers, where the exciton binding energy is much larger than all other relevant energy scales \([14, 23]\).

Solving the scattering problem of two lower polaritons at zero momentum, we obtain the following exact expression for the polariton-polariton interaction strength:

\[
T_{\sigma\sigma'} = \frac{4\pi\hbar^2 X_0^4}{m_X N \ln \left( \frac{E_{\sigma\sigma'}^0}{2E_0^0} \right)} = \left\{ \begin{array}{ll}
\alpha_1, & \sigma = \sigma' \\
\alpha_2, & \sigma \neq \sigma'.
\end{array} \right.
\]

Here \( E_{\sigma\sigma'}^0 > 0 \) are the energies associated with the spin-triplet (\( \sigma = \sigma' \)) and spin-singlet (\( \sigma \neq \sigma' \)) exciton scattering lengths, where \( \sigma = \pm \) encodes the pseudo-spin (circular polarization) of the exciton (photon). \( X_0^2 \) and \( E_0^0 \) are, respectively, the exciton fraction and the energy (relative to the exciton energy) of the zero-momentum lower polariton, which depend on the number of layers \( N \) via the exciton-photon Rabi coupling. Crucially, Eq. (1) differs from the usual case of 2D quantum particles with short-range interactions, where the scattering vanishes in the zero-momentum limit \([32, 33]\). Hence the strong light-matter coupling enhances the polariton-polariton interaction strength with respect to the corresponding exciton-exciton interaction strength. Equation (1) is a key result of this work, which we derive in the following.

**Dark, bright and polariton states.**— We start with the single-polariton Hamiltonian that describes the coupling between the cavity photon and \( N \) monolayer excitonic modes:

\[
\hat{H}_0 = \sum_{\mathbf{k}, \sigma} E_k^C \hat{c}^\dagger_{\mathbf{k}\sigma} \hat{c}_{\mathbf{k}\sigma} + \sum_{\mathbf{k}, \sigma} \sum_{n=1}^N E_k^X \hat{x}^\dagger_{\mathbf{k}\sigma,n} \hat{x}_{\mathbf{k}\sigma,n} + \hbar g_R \sum_{\mathbf{k}, \sigma} \sum_{n=1}^N \left( \hat{x}^\dagger_{\mathbf{k}\sigma,n} \hat{c}_{\mathbf{k}\sigma} + \hat{c}^\dagger_{\mathbf{k}\sigma} \hat{x}_{\mathbf{k}\sigma,n} \right),
\]

\( \hbar g_R \) being the Rabi coupling.
where $\hat{c}_{k\sigma}$ ($\hat{c}_{k\sigma}^\dagger$) and $\hat{x}_{k\sigma,n}$ ($\hat{x}_{k\sigma,n}^\dagger$) are bosonic annihilation (creation) operators of cavity photons and monolayer excitons, respectively, with in-plane momentum $\hbar k$ and layer index $n$. The kinetic energies at low momenta are $E^C_k = \hbar^2 k^2/2mc + \delta$ and $E^X_k = \hbar^2 k^2/2m_X$, where $k \equiv |k|$ and we measure energies with respect to the exciton energy at zero momentum. Thus, $\delta$ is the photon-exciton detuning, while $m_C$ and $m_X$ are the photon and exciton masses, respectively. Here, for simplicity, we consider identical monolayers that are located at the maxima of the electric field within the cavity, so that both $E^X_k$ and the exciton-photon coupling $g_R$ are independent of $n$. However, it is straightforward to generalize our results to the case of a layer-dependent light-matter coupling.

The multilayer system is frequently described by a two-polariton modes (upper and lower branches) and $N-1$ dark states which are decoupled from light [42, 43]. The multilayer system is frequently described by a two-coupled-mode exciton-photon model with a renormalized Rabi coupling [9], but here, we keep track of the complete structure of the eigenstates. Since only the bright states [in-phase superpositions of all monolayer excitons, as depicted in Fig. 1(a)] couple to light, one can rewrite the Hamiltonian in the corresponding convenient basis:

$$\hat{H}_0 = \frac{\hbar}{2} \sum_{k,\sigma} \left[ E^C_k \hat{c}_{k\sigma}^\dagger \hat{c}_{k\sigma} + E^X_k \left( \hat{b}_{k\sigma}^\dagger \hat{b}_{k\sigma} + \sum_{l=1}^{N-1} \hat{d}_{k\sigma,l}^\dagger \hat{d}_{k\sigma,l} \right) \right]$$

$$+ \hbar g_R \sum_{k,\sigma} \left( \hat{b}_{k\sigma}^\dagger \hat{d}_{k\sigma} + \hat{c}_{k\sigma}^\dagger \hat{b}_{k\sigma} \right),$$

where $\hat{b}_{k\sigma}$ and $\hat{d}_{k\sigma,l}$ are the annihilation operators for bright and dark excitonic states, respectively, which are related to the bare monolayer exciton operators via the unitary transformation:

$$\hat{d}_{k\sigma,l} = \sum_{n=1}^{N} u_{ln} \hat{x}_{k\sigma,n}, \quad \hat{b}_{k\sigma} = \hat{d}_{k\sigma,N}$$

with $u_{ln} = \frac{1}{\sqrt{N}} e^{i2\pi nl/N}$ [44]. The multilayer nature of the bright states gives rise to an enhanced Rabi coupling $\hbar \Omega_R = \hbar g_R \sqrt{N}$, thus making it easier to access the strong-coupling regime in a multilayer structure. The diagonal form of the exciton-photon Hamiltonian is then:

$$\hat{H}_0 = \sum_{k,\sigma} \left[ E^C_k \hat{L}_{k\sigma}^\dagger \hat{L}_{k\sigma} + E^U_k \hat{U}_{k\sigma}^\dagger \hat{U}_{k\sigma} + \sum_{l=1}^{N-1} E^X_k \hat{d}_{k\sigma,l}^\dagger \hat{d}_{k\sigma,l} \right],$$

with $\hat{L}$ ($\hat{U}$) the lower (upper) polariton annihilation operators defined in the standard way:

$$\begin{pmatrix} \hat{L}_{k\sigma} \\ \hat{U}_{k\sigma} \end{pmatrix} = \begin{pmatrix} X_k & C_k \\ -C_k & X_k \end{pmatrix} \begin{pmatrix} \hat{b}_{k\sigma} \\ \hat{c}_{k\sigma} \end{pmatrix}.$$

Here $E^{U,L}_k$ are the polariton eigen-energies [see Fig. 1(c)],

$$E^{U,L}_k = \frac{1}{2} \left( E_k^X + E_k^C \pm \sqrt{(E_k^C - E_k^X)^2 + \hbar^2 \Omega_R^2} \right),$$

and $X_k, C_k$ are the Hopfield coefficients, corresponding to exciton and photon fractions:

$$X_k^2 = \frac{1}{2} \left( 1 + \frac{E_k^C - E_k^X}{E_k^U - E_k^L} \right), \quad C_k^2 = 1 - X_k^2.$$

Exciton-exciton interactions.— Since the layer spacing is typically larger than the exciton size, we may assume that the interactions between excitons only occur within the same layer. Furthermore, if the scattering energy is small compared to the exciton binding energy (as is the case in TMDs [14]), then we can describe the ex-
where the “bare” spin-dependent coupling strength $g_{\sigma\sigma'}$ is independent of layer index $n$ since we have assumed that the monolayers are identical. Also, we have set the monolayer area to 1. We emphasize that our approach is different from determining the exciton-exciton interaction strength within the Born approximation, as in previous works \[46–52\]. This approximation effectively estimates $g_{\sigma\sigma'}$ from the microscopic structure of the excitons, whereas here we solve the low-energy scattering problem exactly and treat $g_{\sigma\sigma'}$ as a bare parameter that must be related to experimental observations \[53\]. As such, we impose a cutoff $\Lambda$ on the relative scattering momentum, which we will send to infinity at the end of the calculation (for details, see the Supplemental Material \[54\]).

Transforming to the bright-dark-exciton basis using Eq. (4), the interaction term becomes:

$$
\hat{V} = \sum_{n=1}^{N} \sum_{k,k',q} \frac{g_{\sigma\sigma'}}{2} \hat{x}_{k+q+n, \sigma} \hat{x}_{k',-q+n, \sigma', n} \hat{x}_{k',n} \hat{\sigma}_{\sigma,n},
$$

where $\{l_j\} = \{l_1, l_2, l_3, l_4\}$. The kronecker delta ($\delta_{\mathcal{M}} = 1$ if $\mathcal{M} = 0$, $\delta_{\mathcal{M}} = 0$ otherwise) encodes the phase selection rule for binary scatterings illustrated in Fig. 1(b), where $\mathcal{M} = \text{Mod}[l_1 + l_2 - l_3 - l_4, N]$. It is worth noting that the interaction coupling constant is reduced by the factor of $1/N$ in the new bright-dark-states basis. Moreover, written in this form, Eq. (9) can involve a huge number of terms ($N^3$ for each spin channel). This highlights the complexity of the scattering processes which can occur in any multilayer structure in the strong-coupling regime.

**Two-polariton scattering.** To investigate polariton-polariton interactions, we consider the two-body scattering problem at zero-center-of-mass momentum. The two-particle states are $|A_{\sigma}, B_{\sigma'}, \mathbf{k}\rangle = \hat{A}_{\mathbf{k} \sigma}^\dagger \hat{B}_{\mathbf{k} \sigma'}^\dagger |0\rangle$, where the operators $\hat{A}$, $\hat{B}$ correspond to lower polaritons $\hat{L}$, upper polaritons $\hat{U}$ or dark-exciton operators $\hat{d}_l$, with $l = 1, 2, \ldots, N - 1$. To proceed, we employ the $T$-matrix operator, which is given by the Born series

$$
\hat{T}(E) = \hat{V} + \hat{V} \frac{1}{E - H_0 + i0} \hat{V} + \ldots
$$

where $E$ is the scattering energy and $+i0$ represents an infinitesimal positive imaginary part. The interaction strength for lower polaritons is then given by the matrix element

$$
T_{\sigma\sigma'}(k) = \frac{\langle L_{\sigma}, L_{\sigma'}, k' | \hat{T}(2E_{k}^L) | L_{\sigma}, L_{\sigma'}, k \rangle}{1 + \delta_{\sigma\sigma'}},
$$

with the on-shell condition $|k'| = |k| = k$. Here the normalization factor in the denominator accounts for scattering between identical particles. The Born approximation to the interaction strength corresponds to keeping only the first term in the series, which gives $g_{\sigma\sigma'} X_k^4/N$. However, higher order terms will significantly modify this result since they can involve scattering into dark intermediate states, as illustrated in Fig. 1(b).

Remarkably, we find that the low-energy polariton $T$ matrix takes the simple form \[54\]

$$
T_{\sigma\sigma'}(k) = \frac{X_k^4}{N \sigma\sigma' - \Pi(2E_{k}^{L})}.
$$

Here, the one-loop polarization bubble $\Pi(E)$ is extremely well approximated by that of $N$ exciton pairs:

$$
\Pi(E) \simeq N \sum_q \frac{1}{E - 2E_{q}^X + i0},
$$

since the exciton scattering is dominated by large momenta where the photon is far off resonant [see Fig. 1(c)]. This is a consequence of the photon mass being orders of magnitude smaller than the exciton mass, $m_c \ll m_X$. To obtain cutoff-independent results, we relate the bare couplings to physical observables as follows \[33\]

$$
\frac{1}{g_{\sigma\sigma'}} = - \sum_q \frac{1}{E_{\sigma\sigma'} + 2E_{q}^X},
$$

where we have introduced the physical energy scales $E_{\sigma\sigma'} = \frac{h^2}{2m_{\sigma\sigma'}}$ related to the 2D exciton $s$-wave scattering lengths $a_{\sigma\sigma'}$ and the two-exciton reduced mass $m_\sigma = m_X/2$. Note that the scattering parameters are intrinsic to the monolayer and are independent of $N$. In the singlet case, $E_{+} = E_{-} = E_B^{XX}$ corresponds to the binding energy of the biexciton (bound state of two excitons). Due to Pauli exclusion, there is no triplet biexciton state, but the triplet scattering length is well defined and is of the order of the 2D exciton Bohr radius $a_{\sigma\sigma} \sim a_B$ \[54\]; hence we have $E_{\sigma\sigma} \sim 2E_B^{X}$.

Inserting Eq. (14) into Eq. (12) and taking the limit $\Lambda \to \infty$, one obtains the cutoff-independent $T$ matrix

$$
T_{\sigma\sigma'}(k) = \frac{4\pi \hbar^2 C_{k}^4}{m_X N \text{ln} \left( \frac{E_{\sigma\sigma'}}{2E_{k}^{L}} \right)}.
$$

The limit $k \to 0$ finally yields $T_{\sigma\sigma'}$ in Eq. (1), which gives the lower polariton effective interaction “constants” for the triplet ($\alpha_1$) and singlet ($\alpha_2$) channels at zero momentum (a similar procedure can be used to examine the scattering between upper polaritons). In the absence of light-matter coupling, we recover the usual 2D two-body $T$ matrix for quantum particles in a monolayer \[32\],$4\pi \hbar^2 \left[ \text{ln} \left( \frac{E_{\sigma\sigma'}}{2E_{k}^{L}} \right) + i\pi \right]^{-1}$, which we see vanishes in the
Figure 2. Polariton interactions in several 2D materials. (a,e) Monolayer triplet interactions $\alpha_1$ and the corresponding singlet-triplet ratio $\alpha_2/\alpha_1$ as a function of the exciton-photon detuning $\delta$. (b-d) Multilayer triplet interactions as a function of $N$ for different $\delta$, and (f-g) the corresponding singlet-triplet ratio. Parameters are taken from Table I, with $m_0$ the free electron mass.

Implications for experiment.— Equations (1) and (15) show that the lower polariton interaction strength is enhanced compared to the monolayer exciton interaction strength because the strong light-matter coupling shifts the scattering energy in the low-momentum limit. In particular, Eq. (1) explains why the polariton blueshift in experiment scales linearly with condensate density [52], unlike for the case of standard 2D bosons [55]. Note that Eq. (1) is a low-energy expression that is valid in the regime $|E_L^0| \ll |E_X^0 + \delta|$, corresponding to the point where the lower polariton branch crosses the biexciton energy. Such resonances are present in Figs. 2(e-h), where we have plotted the singlet/triplet ratio ($\alpha_2/\alpha_1$) for each TMD system. Here we see that the sign and magnitude of $\alpha_2$ can be tuned by varying $\delta$ and/or $N$. Furthermore, these panels demonstrate that the singlet interaction is in general stronger than the triplet one for a wide range of experimental parameters. To our knowledge, this important feature has not been noticed previously. In particular, a large and negative $\alpha_2/\alpha_1$ can destabilize a polariton condensate, which possibly explains why condensation has been challenging to achieve thus far. Furthermore, the sizeable and tunable $\alpha_2$ in TMDs opens up the possibility of realizing strongly correlated phenomena such as polariton blockade [56], bipolariton superfluidity [57] and polaron physics [58].

The singlet resonance effectively corresponds to the polariton “Feshbach resonance” [63] experimentally investigated in single GaAs quantum wells [64, 65]. Note that GaAs-based microcavities are qualitatively different from the TMD case since they typically have $|\alpha_2/\alpha_1| < 1$ [66], except in a narrow parameter region close to the biexciton resonance [63], and moreover the exciton binding energy is not necessarily the largest energy scale — for example, for the case of 12 quantum wells, we have
On the other hand, neither TMDs nor GaAs quantum wells are expected to feature a triplet polariton Feshbach resonance. While Eq. (1) naively predicts a resonance at $|E_{01}^T| \approx E_{01}^X$, such a large energy scale goes beyond the validity of our model and requires a more precise description of the short-distance physics such as the composite nature of the excitons and the layer thickness. A full microscopic description of polariton interactions is beyond the scope of the present work.

Finally, we emphasize that most of the intermediate states appearing in the $T$ matrix are dark states [Fig. 1(b)] which lie at the exciton energy. In the present work, these are virtual states which only contribute to the final strength of polariton interactions. However, in principle they can be turned into real long-lived excitons via additional scattering processes, for example mediated by phonons [67, 68], and they can therefore populate an excitonic reservoir.

**Conclusions.**—We have derived exact analytical expressions for the polariton-polariton triplet and singlet interaction strengths in TMD layers embedded in a planar microcavity. Crucially, we have demonstrated that the strong exciton-photon coupling enhances the polariton interactions relative to those of bare excitons. Furthermore, we have analyzed the dependence on the number of layers and we have exposed the important role of optically dark states in multilayer polariton-polariton scattering. Our results suggest that the singlet interaction is stronger than the triplet one for a range of TMD heterostructures, which has important consequences for realizing polariton condensation and other interaction-driven phenomena in these systems. In particular, a large repulsive singlet interaction can lead to the formation of spin-polarized domains, and thus to spin-resolved ultra-low threshold lasing.

We are grateful to G. Li, F. M. Marchetti, E. Ostrovskaya, and M. Pieczarka for useful discussions. We acknowledge support from the Australian Research Council Centre of Excellence in Future Low-Energy Electronics Technologies (CE170100039). JL is also supported through the Australian Research Council Future Fellowship FT160100244.

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SUPPLEMENTAL MATERIAL: “POLARITON INTERACTIONS IN MICROCAVITIES WITH ATOMICALLY THIN SEMICONDUCTOR LAYERS”

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T-MATRIX

Here, we provide some details for the $T$-matrix calculation for the scattering between two lower polaritons. First, we recall the bosonic commutation rules:

$$\left[ \hat{A}_{k\sigma}, \hat{B}_{k'\sigma'} \right] = \delta_{AA'} \delta_{\sigma\sigma'} \delta_{kk'},$$

where $\hat{A}$, $\hat{B}$ can correspond to lower polaritons $\hat{L}$, upper polaritons $\hat{U}$ or dark-exciton operators $\hat{d}_l$, with $l = 1, 2, \ldots, N - 1$. The two-particle states with zero total momentum are defined as:

$$|A_{\sigma}, B_{\sigma'}, k\rangle = \hat{A}_{-k\sigma}^\dagger \hat{B}_{k\sigma'}^\dagger |0\rangle,$$

with the scalar product:

$$\langle A_{\sigma_1}, B_{\sigma_2', k_1} | C_{\sigma_2}, D_{\sigma_2', k_2} \rangle = \langle 0 | \hat{B}_{k_{1\sigma_1}} \hat{A}_{-k_{1\sigma_1}} \hat{C}_{k_{2\sigma_2}}^\dagger \hat{D}_{k_{2\sigma_2}}^\dagger |0\rangle = \delta_{BC} \delta_{AA'} \delta_{\sigma_1\sigma_2} \delta_{\sigma_1\sigma_2'} \delta_{k_1k_2}. $$

The non-interacting eigenvalues are given by

$$\hat{H}_0 |A_{\sigma}, B_{\sigma'}, k\rangle = \sum_j \sum_{q,s} E_j^{A}\hat{J}_{q,s}^A \hat{J}_{q,s}^{A*} |A_{\sigma}, B_{\sigma'}, k\rangle = \langle 0 | \hat{B}_{k_{1\sigma_1}} \hat{A}_{-k_{1\sigma_1}} \hat{B}_{k_{2\sigma_2}}^\dagger |0\rangle = (E_k^A + E_k^B) |A_{\sigma}, B_{\sigma'}, k\rangle,$$

where the sum on $J$ in the first line accounts for all the available single-particle energy states, and $E_j^A = E_j^X$ for all the dark states with $\hat{J} = \hat{d}_l$.

Now we consider the scattering between two lower polaritons, the first term of the Born series gives:

$$\langle L_{\sigma}, L_{\sigma'}, k' | \hat{V} | L_{\sigma}, L_{\sigma'}, k \rangle = X_{k'}^2 X_k^2 g_{\sigma\sigma'} N (1 + \delta_{\sigma\sigma'}).$$

(Notice that $g_{+-} = g_{-+}$, and $g_{++} = g_{--}$). Higher order terms in the $T$-matrix involve dark or upper polariton intermediate states. It is useful to introduce the matrix element between two lower polaritons and an arbitrary two-particle state

$$\langle A_{\sigma}, B_{\sigma'}, k' | \hat{V} | L_{\sigma}, L_{\sigma'}, k \rangle = \frac{g_{\sigma\sigma'} X_{k'}^2}{N} (1 + \delta_{\sigma\sigma'}).$$

and the completeness relation

$$1 = \frac{1}{2} \sum_q \sum_{A,B} \sum_{s,s'} |A_{s}, B_{s'}, q \rangle \langle A_{s}, B_{s'}, q|,$$

where $A = d_{l_1}, B = d_{l_2}, l_1, l_2 \neq N$.
which satisfies the usual property of the identity

\[ 1|C_\sigma, D_{\sigma'}, k) = \frac{1}{2} \sum_q A_k B_{k', q} |A_{\sigma, D_{\sigma'}, q} \langle A_\sigma, D_{\sigma'}, q|C_\sigma, D_{\sigma'}, k) \] (S11)

\[ = \frac{1}{2} \sum_q (|C_\sigma, D_{\sigma'}, q) \delta_{qk} + |D_{\sigma'}, C_\sigma, q) \delta_{q,-k}) \] (S12)

\[ = \frac{1}{2} (|C_\sigma, D_{\sigma'}, q) + |D_{\sigma'}, C_\sigma, -k)) \] (S13)

\[ = |C_\sigma, D_{\sigma'}, k) \] (S14)

Then the second-order term reads

\[ \langle L^\sigma, L^\sigma, k' | \tilde{V} - \frac{1}{E - H_0} \tilde{V} | L^\sigma, L^\sigma, k) = \langle L^\sigma, L^\sigma, k' | \tilde{V} \frac{1}{E - H_0} \tilde{V} | L^\sigma, L^\sigma, k) \] (S15)

\[ = (1 + \delta_{\sigma\sigma'}) \left( \frac{g_{\sigma\sigma'}}{N} \right)^2 X_k^2 X_{k'}^2 \] (S16)

\[ \times \sum_q \left[ \frac{X_q^4}{E - 2E_L^q} + \frac{C_q^4}{E - 2E_U^q} + \frac{2C_q^2}{E - 2E_L^q - E_U^q} + \sum_{l_1, l_2} \frac{N}{E - 2E_X^q} \right] \]

\[ = (1 + \delta_{\sigma\sigma'}) \frac{g_{\sigma\sigma'}X_k^4 g_{\sigma\sigma'}N}{N} \Pi(E), \] (S17)

where we have used \(|k'| = |k|\) in the last line. The one-loop polarization bubble, \(\Pi(E)\), is given by

\[ \Pi(E) = \sum_q \frac{X_q^4}{E - 2E_L^q} + \sum_q \frac{C_q^4}{E - 2E_U^q} + 2 \sum_q \frac{X_q^4C_q^2}{E - E_L^q - E_U^q} + (N - 1) \sum_q \frac{1}{E - 2E_X^q}, \] (S18)

where we have introduced the high-energy cutoff wavevector \(\Lambda\).

The generalisation to higher order terms leads to

\[ \langle L_\sigma, L_{\sigma'}, k' | \tilde{T}(E) | L_\sigma, L_{\sigma'}, k) = (1 + \delta_{\sigma\sigma'}) \frac{g_{\sigma\sigma'}X_k^4}{N} \left[ 1 + \frac{g_{\sigma\sigma'}}{N} \Pi(E) + \left( \frac{g_{\sigma\sigma'}}{N} \frac{\Pi(E)}{N} \right)^2 + \ldots \right] \] (S19)

\[ = (1 + \delta_{\sigma\sigma'}) \frac{N}{(g_{\sigma\sigma'}) - \Pi(E)}, \] (S20)

which after rearranging the prefactor, gives the formula (12) presented in the main text. Note that \(\Pi(E)\) also contains a factor \(N\). The integrals in \(\Pi(E)\) are dominated by the large wavevectors \(q\), where \(E_L^q \to E_X^q\), \(X_q^2 \to 1\), \(C_q^2 \to 0\) and one can neglect the second and third terms. Thus, \(\Pi(E)\) simply reduces to \(N \times \Pi_X(E)\). This approximation is valid because \(m_C \ll m_X\) as explained below.

\(m_C \ll m_X\) and \(\Pi(E)\) approximation

The approximation of the one-loop polarization bubble [Eq. (S18)] relies on the very large ratio between the cavity photon and the exciton masses (\(m_C/m_X \sim 10^{-4} - 10^{-5}\)). This approximation is equivalent to neglecting the low-q behavior of the integrand in:

\[ \Pi_{\rho}(E) = \frac{1}{2\pi} \int_0^\Lambda dq \left( \frac{X_q^4}{E - 2E_L^q} + \frac{C_q^4}{E - 2E_U^q} + 2 \frac{X_q^4C_q^2}{E - E_L^q - E_U^q} \right). \] (S21)

To analytically evaluate if this low-q behavior plays a role, one can approximate the integrand in two domains with the following replacements:

- \(q < q_0\): \(E_L^q \to \tilde{E}_L^q = \hbar^2 q^2/2m_L + E_0^L\), \(E_U^q \to \tilde{E}_U^q = \hbar^2 q^2/2m_U + E_0^U\), \(C_q \to C_0\), \(X_q \to X_0\)
- \(q > q_0\): \(E_L^q \to E_X^q\), \(C_q \to 0\), \(X_q \to 1\).
The continuity of the wavefunction at \( r = r_0 \) and the lower (upper) polariton effective masses \( m_L, (m_U) \) are defined as:

\[
q_0 = \frac{\sqrt{2m_\pi |E_0|}}{\hbar}, \quad m_L = \frac{m_C}{C_0^2}, \quad m_U = \frac{m_C}{X_0^2}.
\] (S22)

This gives

\[
\Pi_P(E) \simeq \frac{1}{2\pi} \int_0^\infty dq \left[ 2 \frac{X_0^4}{E - 2E_L} + \frac{C_0^4}{E - 2E_U} + 2 \frac{X_0^2C_0^2}{E - E_L - E_U} \right] + \frac{1}{2\pi} \int_{q_0}^\Lambda dq \left( \frac{1}{E - 2E_L} \right)
\]

\[
= \frac{m_C}{4\pi\hbar^2C_0^2X_0^2} \left[ X_0^4 \ln \left( \frac{2E_0^U - E}{E} \right) + C_0^4 \ln \left( \frac{2E_0^L - E}{E + 2E_0^L - 2|E_0|} \right) + 4X_0^4C_0^4 \ln \left( \frac{\delta - E)C_0^2X_0^2}{C_0^2(\delta - E) + |E_0|} \right) \right]
\]

For low-energy particles, the two-body scattering is dominated by the \( s \) wave contribution, and the center of mass wavefunction obeys the 2D radial Schrödinger equation [45]

\[
-\frac{\hbar^2}{2m_\pi} \frac{1}{r} \frac{\partial}{\partial r} \left( \frac{\partial \psi}{\partial r} \right) + U(r)\psi = E\psi,
\] (S26)

outside the potential, the general solution is a superposition of first and second kind Bessel functions

\[
\psi(r) = AJ_0(kr) + BY_0(kr),
\] (S28)

with \( k = \sqrt{2m_\pi E}/\hbar \). Then, using its asymptotic form, one introduces the scattering phase shift \( \delta_s \)

\[
\psi(r) \xrightarrow{r \to \infty} \sqrt{\frac{2}{\pi kr}} \left( A \cos(kr - \pi/4) + B \sin(kr - \pi/4) \right) = \sqrt{\frac{2}{\pi kr}} C \cos(kr - \pi/4 + \delta_s),
\] (S29)

with

\[
\tan(\delta_s) = -\frac{B}{A}.
\] (S30)

The continuity of the wavefunction at \( r = r_0 \) gives the relation:

\[
\cot(\delta_s) = -\frac{A}{B} = \frac{Y_0(kr_0)}{J_0(kr_0)}.
\] (S31)

Finally, taking the low-energy (low-\( k \)) limit one obtains

\[
\cot(\delta_s) = \frac{2}{\pi} \ln(ka_s),
\] (S32)
where we have introduced the 2D scattering length $a_s$

$$a_s = \frac{\gamma r_0}{2} \simeq 0.89 r_0,$$

(S33)

with $\gamma = 0.577...$ the Euler-Mascheroni constant.

The corresponding hard disc triplet exciton $T$-matrix reads:

$$T_{\sigma\sigma}(k) = \frac{2\pi \hbar^2}{m_r} \left[ \ln \left( \frac{E_{\sigma\sigma}}{E} \right) + i\pi \right]^{-1},$$

(S34)

with

$$E_{\sigma\sigma} = \frac{\hbar^2}{2m_r a_s^2}.$$  

(S35)

Note that a similar form for the low-energy $T$ matrix has been obtained in Ref. [70] which accounts for the composite (electron-hole) nature of the exciton in the limit of equal electron and hole masses.