WHOLESALE PRICES AND COURNOT-BERTRAND COMPETITION

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ABSTRACT

This note considers the competing vertical structures framework with Cournot-Bertrand competition downstream. It shows that the equilibrium wholesale price paid by a Cournot (Bertrand)-type retailer is above (below) marginal costs of a corresponding manufacturer. This result contrasts with the one under pure competition downstream (i.e., Cournot or Bertrand), where the wholesale price is set below (above) marginal costs in case of a Cournot (Bertrand) game at the retail level.

Keywords: wholesale prices, vertical relations, Cournot-Bertrand model, two-part tariffs

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I. INTRODUCTION

The papers Alipranti et al. (2014) and Rozanova (2015) demonstrate that the equilibrium wholesale prices are set above (below) marginal costs of the manufacturers in the case of Bertrand (Cournot) competition downstream.1

Besides competition in quantities and in prices the literature considers the so called Cournot-Bertrand model (where some firms set quantities and others choose prices). The examples of these papers include Singh and Vives (1984), Tremblay and Tremblay (2011), Tremblay et al. (2011), and Tremblay et al. (2013).

The aim of the current note is to study the equilibrium wholesale prices in the situation of Cournot-Bertrand competition downstream. I show that in the case of two competing vertical structures the equilibrium wholesale price paid by the retailer choosing quantity (price) is set above (below) marginal costs of the manufacturer selling the good to the retailer under consideration. In other words, at the downstream level the Bertrand-type competitor has a

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1 Bonnano & Vickers (1988) proves the result in the setup with Bertrand competition downstream, two competing vertical structures and take-it-or-leave-it contract offers from the manufacturers to the retailers.
cost advantage over the Cournot-type firm. This outcome differs from the one in case of pure (i.e., Cournot or Bertrand) competition downstream, where equilibrium wholesale price is below (above) the manufacturer’s marginal costs when the retailers compete in quantities (prices).

II. MODEL

There are two manufacturers (1 and 2). Each of them sells a substitute product to its own retailer. At the downstream level retailers differ in their choice variables. Retailer 1 is a Cournot-type firm (i.e., its strategic variable is output, \( q_1 \)). Retailer 2 is a Bertrand-type firm (i.e., it sets price, \( p_2 \)).

The upstream firms 1 and 2 produce at constant marginal costs \( c_1 \) and \( c_2 \), respectively. The downstream firms do not have other costs than the spending on the input from the upstream firms.

The timing is as follows: at the first stage of the game, each upstream producer bargains with its retailer over the components of the two-part tariff contract, that is over a wholesale price, \( w_i \), and a fixed fee, \( F_i \), \( i = 1, 2 \). At the second stage, the downstream firms simultaneously set their strategic variables after observing each other’s contract terms.

The bargaining is modelled in the following way: it is assumed that each vertical structure (i.e., upstream firm and its downstream firm) solves the Nash bargaining problem. In vertical structure’s \( i = 1, 2 \) Nash bargaining problem, upstream firm \( i \) has bargaining power \( \beta_i \), while downstream producer’s \( i \) bargaining power is \( 1 - \beta_i \), where \( \beta_i \in (0, 1] \).

II.1 Consumer side

Utility of the consumer depends on the goods sold by the retailers and the expenditures on the composite commodity (i.e., \( T \)):

\[
U = U(q_1, q_2) + T,
\]

where \( q_i, i = 1, 2 \) is the quantity of commodity \( i \) consumed.

**Assumption 1.** \( \frac{\partial U}{\partial q_i} > 0; \frac{\partial^2 U}{\partial q_i \partial q_i} < 0, i = 1, 2. \)

**Assumption 2.** \( \frac{\partial^2 U}{\partial q_i \partial q_j} < 0, i, j = 1, 2; i \neq j. \)

From the utility maximization problem \( \max_{q_1, q_2} U(q_1, q_2) + T \) s.t. \( p_1 q_1 + p_2 q_2 + T \leq I \) (where \( I \) is the income of the consumer and \( p_i \) is the unit price of good \( i \)) we get the inverse demand functions for the goods 1 and 2:

\[
\begin{align*}
  p_1 &= \frac{\partial U}{\partial q_1} = D_1(q_1, q_2) \\
  p_2 &= \frac{\partial U}{\partial q_2} = D_2(q_1, q_2)
\end{align*}
\]

**Lemma 1.** \( \frac{\partial p_1}{\partial q_1} < 0; \frac{\partial p_2}{\partial q_2} < 0. \)

**Proof.** Differentiation of (2) with respect to \( q_1 \) and \( q_2 \) and application of Assumptions 1 and 2 give the result of Lemma 1. Q. E. D.

**Lemma 2.** \( \frac{\partial p_1}{\partial q_2} > 0, \frac{\partial p_2}{\partial q_1} < 0 \) and \( \frac{\partial p_2}{\partial q_1} < 0. \)
Proof. Differentiating (2) with respect to $p_2$ (taking into account the fact that $p_1$ and $q_2$ are the functions of $p_2$) we get:

$$\begin{align*}
\frac{\partial p_1}{\partial p_2} &= \frac{\partial D_1}{\partial q_2} \cdot \frac{\partial q_2}{\partial p_2} \\
1 &= \frac{\partial D_2}{\partial q_2} \cdot \frac{\partial q_2}{\partial p_2}
\end{align*}$$

(3)

From (3):

$$\begin{align*}
\frac{\partial p_1}{\partial p_2} &= \frac{\partial D_1}{\partial q_2} \\
\frac{\partial q_2}{\partial p_2} &= \frac{1}{\frac{\partial D_2}{\partial q_2}}.
\end{align*}$$

(4)

Due to Lemma 1 $\frac{\partial p_1}{\partial p_2} > 0$; $\frac{\partial q_2}{\partial p_2} < 0$.

Differentiating (2) with respect to $q_1$ we get:

$$\begin{align*}
\frac{\partial p_1}{\partial q_1} &= \frac{\partial D_1}{\partial q_1} + \frac{\partial D_1}{\partial q_2} \cdot \frac{\partial q_2}{\partial q_1} \\
0 &= \frac{\partial D_2}{\partial q_1} + \frac{\partial D_2}{\partial q_2} \cdot \frac{\partial q_2}{\partial q_1}.
\end{align*}$$

(5)

From (5)

$$\frac{\partial q_2}{\partial q_1} = -\frac{\frac{\partial D_2}{\partial q_1}}{\frac{\partial D_2}{\partial q_2}},$$

and it is negative due to Lemma 1.

Q. E. D.

II.2 Cournot-Bertrand competition

The payoff function of retailer 1 is $R_1 = (p_1(q_1, p_2) - w_1) \cdot q_1 - F_1$, where $(w_1, F_1)$ are the terms of the contract between retailer 1 and manufacturer 1.

Similarly the payoff of retailer 2 is $R_2 = (p_2 - w_2) \cdot q_2(q_1, p_2) - F_2$.

Assumption 3. ($q_1$ is a strategic complement to $p_2$)

$$\frac{\partial^2 R_1}{\partial q_1 \partial p_2} = \frac{\partial p_1}{\partial p_2} + q_1 \cdot \frac{\partial^2 p_1}{\partial q_1 \partial p_2} > 0.$$

Assumption 4. ($p_2$ is a strategic substitute to $q_1$)

$$\frac{\partial^2 R_2}{\partial p_2 \partial q_1} = \frac{\partial q_2}{\partial q_1} + (p_2 - w_2) \cdot \frac{\partial^2 q_2}{\partial p_2 \partial q_1} < 0.$$
II.3 Effect of wholesale prices on equilibrium

Lemma 3. Equilibrium output of the Cournot-type firm (i.e., \( q_1^* \)) is decreasing in \( w_1 \). Equilibrium price of the Bertrand-type firm (i.e., \( p_2^* \)) is increasing in \( w_1 \).

Formally: \( \frac{\partial q_1^*}{\partial w_1} < 0; \frac{\partial p_2^*}{\partial w_1} > 0. \)

Proof. Assuming internal solution, the second stage equilibrium is found from the system of the first-order conditions to the retailers’ maximization problems, that is:

\[
\begin{align*}
\frac{\partial R_1}{\partial q_1} &= 0 \\
\frac{\partial R_2}{\partial p_2} &= 0.
\end{align*}
\]

(6)

Differentiating (6) with respect to \( w_1 \) and rearranging the terms we get:

\[
\begin{align*}
\frac{\partial q_1^*}{\partial w_1} &= \frac{\partial^2 R_2}{\partial p_2^2} \cdot \frac{1}{\Omega} \\
\frac{\partial p_2^*}{\partial w_1} &= -\frac{\partial^2 R_2}{\partial q_1^2} \cdot \frac{1}{\Omega},
\end{align*}
\]

(7) (8)

where \( \Omega = \frac{\partial^2 R_1}{\partial q_1^2} \cdot \frac{\partial^2 R_2}{\partial q_2^2} - \frac{\partial^2 R_1}{\partial q_1 \partial q_2} \cdot \frac{\partial^2 R_2}{\partial q_1 \partial q_2}. \) Due to the fact that the second-order conditions to the retailers’ maximization problems must hold (i.e., \( \frac{\partial^2 R_1}{\partial q_1^2} < 0, \frac{\partial^2 R_2}{\partial q_2^2} < 0 \)) and due to Assumptions 3 and 4, \( \Omega > 0 \), while \( \frac{\partial^2 R_1}{\partial q_1 \partial q_2} < 0. \) Thus, \( \frac{\partial q_1^*}{\partial w_1} < 0 \) and \( \frac{\partial p_2^*}{\partial w_1} > 0. \) Q. E. D.

Lemma 4. Equilibrium output of the Cournot-type firm (i.e., \( q_1^* \)) and equilibrium price of the Bertrand-type firm (i.e., \( p_2^* \)) are increasing in \( w_2 \).

Formally: \( \frac{\partial q_1^*}{\partial w_2} > 0; \frac{\partial p_2^*}{\partial w_2} > 0. \)

Proof. Differentiating (6) with respect to \( w_2 \) and solving the resulting system for \( \frac{\partial q_1^*}{\partial w_2} \) and \( \frac{\partial p_2^*}{\partial w_2} \) we get:

\[
\begin{align*}
\frac{\partial q_1^*}{\partial w_2} &= \frac{\partial^2 R_1}{\partial q_1 \partial p_2} \cdot \left( \frac{\partial q_2}{\partial p_2} \right) \cdot \frac{1}{\Omega} \\
\frac{\partial p_2^*}{\partial w_2} &= \frac{\partial^2 R_1}{\partial q_1^2} \cdot \frac{\partial q_2}{\partial q_2^2} \cdot \frac{1}{\Omega}.
\end{align*}
\]

(9) (10)

As noted above \( \Omega > 0, \frac{\partial^2 R_1}{\partial q_1^2} < 0, \frac{\partial^2 R_1}{\partial q_1 \partial q_2} > 0 \) (due to Assumption 3). The term \( \frac{\partial q_2}{\partial p_2} < 0 \) due to Lemma 2. Thus, \( \frac{\partial q_1^*}{\partial w_2} > 0 \) and \( \frac{\partial p_2^*}{\partial w_2} > 0. \) Q. E. D.

III. EQUILIBRIUM WHOLESALE PRICES

Let \( (w_1^*, w_2^*) \) be the Subgame-Perfect Nash equilibrium wholesale prices that the retailers 1 and 2 pay to the manufacturers 1 and 2, respectively.
**Proposition 1.** \( w_1^* > c_1, \ w_2^* < c_2. \)

**Proof.** At the first stage of the game each vertical structure \( i = 1, 2 \) maximizes its Nash product with respect to \( w_i \) and \( F_i \). That is each vertical structure \( i \) solves the following optimization problem:

\[
\max_{w_i, F_i} \left[ \pi_i^U(w) + F_i - P_i^U \right]^{\beta_i} \left[ \pi_i^D(w) - F_i - P_i^D \right]^{1-\beta_i},
\]

(11)

where \( w = (w_1, w_2); \pi_i^U(w) \) is the profit of the upstream manufacturer \( i \) without taking into account the fixed fee, while \( \pi_i^D(w) \) is the downstream firm’s \( i \) payoff again without considering the fixed fee. \( P_i^U (P_i^D) \) is a disagreement payoff of an upstream (downstream) producer \( i \).

Maximizing (11) with respect to \( F_i \) we get:

\[
F_i = \beta_i \left( \pi_i^D(w) - P_i^D \right) - (1 - \beta_i) \left( \pi_i^U(w) - P_i^U \right).
\]

(12)

Plugging (12) into (11) we conclude that the equilibrium \( w_i \) is found from:

\[
\max_{w_i} \pi_i = \pi_i^U(w) + \pi_i^D(w).
\]

(13)

**Vertical structure 1:**

For the first vertical structure \( \pi_i^U(w) \) and \( \pi_i^D(w) \) are:

\[
\pi_i^U(w) = (w_1 - c_1) \cdot q_1(w)
\]

(14)

\[
\pi_i^D(w) = (p_1(q_1(w), p_2(w)) - w_1) \cdot q_1(w).
\]

(15)

Therefore (13) is:

\[
\max_{w_1} \Pi_1 = (p_1(q_1(w), p_2(w)) - c_1) \cdot q_1(w).
\]

(16)

Assuming internal solution the first-order condition to (16) is:

\[
\frac{\partial \Pi_1}{\partial w_1} = (w_1 - c_1) \cdot \frac{\partial q_1^*}{\partial w_1} + q_1 \cdot \frac{\partial p_1}{\partial p_2} \cdot \frac{\partial p_2^*}{\partial w_1} = 0.
\]

(17)

Assume that \( w_1^* = c_1 \). Then (17) becomes:

\[
\frac{\partial \Pi_1}{\partial w_1} \big|_{w_1=c_1} = q_1 \cdot \frac{\partial p_1}{\partial p_2} \cdot \frac{\partial p_2^*}{\partial w_1}.
\]

(18)

Due to Lemmas 2 and 3 \( \frac{\partial p_1}{\partial p_2} > 0, \frac{\partial p_2^*}{\partial w_1} > 0. \) Therefore \( \frac{\partial \Pi_1}{\partial w_1} \big|_{w_1=c_1} > 0 \) and \( w_1^* \neq c_1. \)

Assume \( w_1^* < c_1. \) Then due to Lemmas 2 and 3 \( \frac{\partial \Pi_1}{\partial w_1} \big|_{w_1<c_1} > 0. \) Therefore \( w_1^* \) cannot be below \( c_1. \) We conclude that \( w_1^* > c_1. \)

**Vertical structure 2:**

For the second vertical structure \( \pi_i^U(w) \) and \( \pi_i^D(w) \) are:

\[
\pi_2^U(w) = (w_2 - c_2) \cdot q_2(q_1(w), p_2(w))
\]

(19)

\[
\pi_2^D(w) = (p_2 - w_2) \cdot q_2(q_1(w), p_2(w))
\]

(20)

Therefore (13) takes the form:

\[
\max_{w_2} \Pi_2 = (p_2(w) - c_2) \cdot q_2(q_1(w), p_2(w))
\]

(21)

\(^3\)In this vertical structure the retailer sets output.

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Assuming internal solution the first-order condition to (21) is:

\[
\frac{\partial \Pi_2}{\partial w_2} = (p_2 - c_2) \cdot \frac{\partial q_2}{\partial q_1} + \left( w_2 - c_2 \right) \cdot \frac{\partial q_2}{\partial p_2} \cdot \frac{\partial p_2^*}{\partial w_2} = 0.
\] (22)

Assume that \( w^*_2 = c_2 \). Then (22) becomes:

\[
\frac{\partial \Pi_2}{\partial w_2} \bigg|_{w_2 = c_2} = (p_2 - c_2) \cdot \frac{\partial q_2}{\partial q_1} \cdot \frac{\partial q_1^*}{\partial w_2}.
\] (23)

Due to Lemmas 2 and 4 \( \frac{\partial \Pi_2}{\partial w_2} \bigg|_{w_2 = c_2} < 0 \). Therefore \( w^*_2 \neq c_2 \).

Assume that the equilibrium level of \( w_2 \) is above \( c_2 \). Then \( \frac{\partial \Pi_2}{\partial w_2} \bigg|_{w_2 > c_2} < 0 \) due to Lemmas 2 and 4. Therefore what we assumed is not true and \( w^*_2 < c_2 \). Q. E. D.

The intuition for the result formulated in Proposition 1 is straightforward: When vertical structure with a Cournot(Bertrand)-type firm downstream sets the wholesale price above(below) its upstream firm’s marginal costs it leads to the increase in price (reduction in quantity) of the competing downstream firm (compared to the situation where the wholesale price is equal to the marginal costs of the upstream firm). In other words, setting the equilibrium wholesale prices in the way described in Proposition 1 allows relaxing competition downstream.

IV. CONCLUDING REMARKS

The current note proves that in case of two competing vertical structures and Cournot-Bertrand competition downstream the equilibrium wholesale price paid by the Cournot(Bertrand)-type retailer is above(below) marginal costs of the corresponding manufacturer. This result suggests that if Cournot-Bertrand competition is considered in the context of vertically related markets, then the advantage that the Cournot-type firm has over the Bertrand-type producer in a one-tier framework\(^4\) may be reduced (or even eliminated).

It is important to underline that the observability of the contracts is crucial for obtaining the result presented in the current note. One may easily verify that in case of unobservable contracts (i.e., if the downstream firm \( j \) does not observe the contract terms between the upstream firm \( i \) and the downstream firm \( i \), where \( i, j = 1, 2, i \neq j \) before the second stage of the game) the equilibrium wholesale prices are equal to the marginal costs of the corresponding upstream firms (i.e., \( w^*_1 = c_1 \), \( w^*_2 = c_2 \)). The intuition for this result is the following: if the downstream firm \( j \) does not observe \( w_i \), then \( w_i \) cannot affect the behavior of the downstream firm \( j \).

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\(^4\)For the details of the advantage of the Cournot-type firm over the Bertrand-type competitor see Singh and Vives (1984), Tremblay and Tremblay (2011).
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