Effect Analysis of Polarization Aberration in the Non-line-of-sight Azimuth Transmission

Zhiyong Yang¹, Junchen Song¹*, Wei Cai¹, Danqiu Qiao¹, Gaoxiang Lu¹

¹Xi'an Research Institute of High-tech, Xi'an, Shanxi 710025, China
*corresponding: juicec_s@163.com

ABSTRACT

Focusing on the problem that the polarization aberration caused by the non-normal incidence of the polarized beam affects the accuracy of the azimuth transmission during the fiber coupling process of the non-line-of-sight azimuth transmission system, this paper starts from the principle of non-line-of-sight azimuth transmission. The polarization aberration relation of the lens-fiber combined interface is established based on the Fresnel formula for the attenuation difference between the horizontal and vertical electric vectors. Further, the azimuth solution model affected by polarization aberration is established. Numerical simulation results show that in non-normal incidence, no polarization aberration will occur when the polarization angle between the incident ray and incident surface is 0° or 90°. Otherwise, the polarization aberration changes toward the incident surface, and the azimuth transmission error will increase with the increase of the polarization aberration. Last, the optimization measures are proposed. This is of great significance for further improvement of the azimuth transmission system based on polarization-maintaining fibers, the selection of the instrument, and the improvement of the system measurement accuracy.

Keywords: Azimuth transmission, Lens, Polarization maintaining fiber, Polarization aberration, Fresnel formula
1. Introduction

The non-line-of-sight azimuth transmission technology based on polarization-maintaining fiber refers to realize the azimuth transmission between non mechanical connecting devices through using the polarization characteristics of light as the carrier of azimuth information according to Faraday’s magneto optical rotation effect\(^1\), polarization transmission performance of polarization maintaining fiber and Marius law. In the transmission process of polarized light, the polarization-maintaining fiber is coupled by lens. The increased optical interface will change the polarization of non-normal incident light wave\(^2\)\(^-\)\(^4\), resulting in polarization aberration.

Polarization aberration is the change of the polarization state of light wave when it passes through the optical system. It is formed at the optical interface and the propagation of light in the medium, and exists in all optical systems\(^5\)\(^-\)\(^8\). When linearly polarized light passes through Smith Prism, There will be different amplitude and phase in different vibration direction, and we will obtain the different polarization states\(^9\); For common optical lens, when the angle between the polarization direction of incident light and the incident surface is about \(\pm 45^\circ\), the value of the polarization aberration is the largest and positively correlated with the incident angle, and it will accumulate when passing through multiple lenses\(^10,11\); In addition, the optical coating and thin film will affect the amplitude and polarization of the light wave while improving the performance of the optical system, so polarization aberrations can be reduced by optimizing the interface transmittance or reflectivity coating\(^12\). In engineering field, the study of polarization aberration focuses on the optical system design, optical imaging, communications, and etc. Particularly, it has been applied in in the lithography technology. For example, with the increasing requirement for critical size (CD) and coverage control in high resolution lithography imaging, polarization aberration will affect the image position, the best focal plane displacement and deviation of CD\(^13\). The
influence of the polarization aberration on the quality of lithography imaging can be evaluated through analyzing relationships between the polarization aberration and the image position, the Optimal focal shift, and etc\textsuperscript{14}; In the imaging of high numerical aperture optical system, the polarization aberration is described by Zernike polynomials, and the distribution of polarization aberration in pupil is revealed\textsuperscript{15}; In coherent laser communication detection platform, the communication system and the transmitting system will change the polarization state of the signal light and reduce the mixing effect of the signal light and the local oscillator light due to the two-direction attenuation and phase delay aberration\textsuperscript{16}; Studies have shown that the non-collimating light in a fast spatial angular measurement system will generate polarization aberration when passing through a polarization prism. The azimuth measurement error will be the minimum when the azimuth angle is $0^\circ$, and the maximum when $90^\circ$. But when the azimuth angle is $90^\circ$, and it increases with the incidence angle increasing\textsuperscript{17}. The influence of the non-collimated incident light on the azimuth transmission accuracy of the non-line-of-sight azimuth transmission system needs to be studied because of the limitation of the performance index and installation error of the instrument.

In this paper, in order to solve the problem of polarization aberrations in non-line-of-sight azimuth transmission system, the relation of polarization aberrations of lens-fiber interface is derived by using the method of polarization ray tracing and Fresnel formula. Furthermore, based on the principle of non-line-of-sight azimuth transmission system, the azimuth calculation model affected by polarization aberrations is derived, whereby the variation law of polarization aberrations and the influence of polarization aberrations on azimuth transfer accuracy are obtained. The research results are of certain guiding significance for the improvement of non-line-of-sight azimuth transmission system, instrument selection and measurement accuracy.

2. System Principle
Figure 1 is a schematic diagram of the non-line-of-sight azimuth transmission system based on polarization-maintaining fiber. When the linearly polarized light passes through the magneto-optical material in the magneto-optical modulator, the Faraday magneto rotation effect will be produced due to the action of the same frequency alternating magnetic field generated by the sinusoidal excitation signal, so as to realize the modulation of the linearly polarized light. The modulated linearly polarized light is coupled into the polarization-maintaining fiber through a lens, and the polarization-maintaining property of the polarization-maintaining fiber is used to transmit the linearly polarized light carrying the azimuth information of the instrument. After the modulated linearly polarized light is output from polarization-maintaining fiber, the azimuth angle is calculated after signal detection and photoelectric conversion, that is, the azimuth information between Upper and lower instruments is obtained.

$$u_a = -U \cos \delta \sin \omega t - V \cos (2\omega t)$$

Where, $U = 2 \cdot k u_0 \cdot J_1 (m_J) \cdot \sin (2\alpha)$, $V = 2 \cdot k u_0 \cdot J_2 (m_J) \cdot \cos (2\alpha)$, $u_0 = \eta I_0 / 4$, $\eta$ is the photoelectric conversion coefficient, $I_0$ is the light intensity after passing through the polarizer, $k$ is the amplification factor, $J$ is the Bessel function of the first kind, $m_J$ is the
degree of magneto-optical modulation, \( \alpha \) is the azimuth angle, and \( \omega \) is the frequency of the sinusoidal excitation signal on the modulation coil, and \( \delta \) is the phase difference caused by the fiber polarization mode birefringence.

It is found that there are always two extreme points in the AC signal \( u_a \), whose abscissa do not change with the change of the azimuth angle, nor do them change with the change of the phase difference. The sampling integration circuit is used to collect the extreme points \( u_{A1} \) and \( u_{A2} \):

\[
\begin{align*}
    u_{A1} &= 2ku_0 \left[ J_2(m_f) \cos 2\alpha - J_1(m_f) \cos \delta \sin 2\alpha \right] \\
    u_{A2} &= 2ku_0 \left[ J_2(m_f) \cos 2\alpha + J_1(m_f) \cos \delta \sin 2\alpha \right]
\end{align*}
\]  

(2)

The extreme points \( u_{A1} \) and \( u_{A2} \) are processed by "subtraction division and sum" to obtain the azimuth angle solution model:

\[
\alpha = \frac{1}{2} \arctan \left[ \sec \delta \frac{J_2(m_f)}{J_1(m_f)} \frac{u_{A2} - u_{A1}}{u_{A2} + u_{A1}} \right]
\]  

(3)

In the non-line-of-sight azimuth transmission system, because of the increased optical interface brought by the coupling system and the polarization-maintaining fiber, the resulting polarization aberration will change the polarization state of the polarized light carrying the azimuth information. The following is to analyze the polarization aberration and the influence of the azimuth transmission accuracy of the lens-fiber combined interface during the coupling process.

3. Polarization Aberration

Take no account of other forms of energy loss, such as absorption and scattering, are not considered, the energy of incident light intensity can only be redistributed in reflected light and refrained light, while the total energy remains unchanged. Take the non-normal incident polarized light shown in Fig. 2 as an example. According to Fresnel formula, there are
differences in reflectivity and refractive index between the eigenstate $E_s$ electric field vector ($s$ component is perpendicular to the incident plane) and $E_p$ electric field vector ($p$ component is parallel to the incident plane) decomposed by the incident wave, so the optical interface is a weak attenuator when the incident light is non normal incidence, and both the reflected wave and the refracted wave will produce polarization images of poor quality; At the same time, the transparent medium has no effect on the phase of the refracted light, but it causes different phase jumps for the reflected light under the influence of different incident angle and refractive index. Therefore, optical interface also acts as a weak retarder.

For the metal surface, the refracted light will be absorbed by the free electrons, and the reflected light will force the electrons into the interface and pull them out because of the $E_{p\perp}$ component. The $p$ component will generate additional heat due to its effect on the free electrons, so there will be a lower reflectivity, so the reflected light will also produce polarization aberration.

![Figure 2. Light at non-normal incidence is polarized by all interfaces.](image)

### 3.1. Interface Polarization Aberration of Lens Fiber Combination

Take the thin lens coupled fiber as an example to trace the light beam emitted by the light source on the optical axis, as shown in Fig. 3(a) As shown in the figure, the section in the figure is the section of the light incident plane; $\nu$ is the angle between the light and the
optical axis (incident angle); $r_1$ and $r_2$ are the distances between the incident plane and the exit plane of the lens from the axis; $R$ is the spherical radius; $d$ is the thickness of the lens center; $n$ is the refractive index of the lens; $n_1$ and $n_2$ are the refractive indexes of the fiber core and cladding respectively. The relationship between the incident coordinate system and the $xy$ coordinate system is shown in Fig. 3(b), and $\phi$ is the angle between the incident plane and the horizontal plane.

According to the geometric relation and Snell’s law:

$$\begin{align*}
  r_i &\approx l \tan \nu \\
  \phi_1 &= \arcsin \left( \frac{r_i}{R} \right) \\
  i_1 &= \nu + \phi_1 \\
  i_2 &= \arcsin \left( \frac{\sin i_1}{n} \right) \\
  l_1 &= R - \sqrt{R^2 - r_1^2} \\
  l_2 &= R - \sqrt{R^2 - r_2^2} \\
  r_1 - r_2 &= (d - l_1 - l_2) \tan (\phi_1 - i_2) \\
  \phi_2 &= \arcsin \left( \frac{r_2}{R} \right) \\
  i_3 &= \phi_1 + \phi_2 - i_2 \\
  i_4 &= \arcsin (n \sin i_3) \\
  i_5 &= i_4 - \phi_2 \\
  i_6 &= \arcsin \left( \frac{\sin i_5}{n_1} \right)
\end{align*}$$

(4)
Figure 3. Polarized ray trace of thin lens. (a) Trace diagram; (b) Coordinate relationship.

The numerical aperture (NA) of the fiber determines whether the light can be received by the fiber and transmitted by total reflection, so \( \theta_s \) needs to meet the following conditions when coupling:

\[
\theta_s < 2 \arcsin (NA) 
\] (5)

Set \( a = \tan (\phi_i - i_2), b = r_i + (R + l_i - d) \) from Eq. (4), \( r_i - r_2 = (d - l_i - l_2) \tan (\phi_i - i_2) \), we obtain the real number solution \( r_2 = \frac{b + \sqrt{b^2 - (a^2 + 1)(b^2 - R^2 a^2)}}{a^2 + 1} \). The corresponding \( i_1, \ i_2, \ i_3, \ i_4 \) can be uniquely determined. According to Fresnel formula, the transmission coefficients of \( s \) and \( p \) components of incident light passing through the optical interface of lens and fiber are as follows:

\[
\begin{align*}
  t_{p1} &= \frac{2 \cos i_1}{n \cos i_1 + \cos i_2} \\
  t_{s1} &= \frac{2 \cos i_1}{\cos i_1 + n \cos i_2} \\
  t_{p2} &= \frac{2 n \cos i_3}{\cos i_3 + n \cos i_4} \\
  t_{s2} &= \frac{2 n \cos i_3}{n \cos i_3 + \cos i_4} \\
  t_{p3} &= \frac{2 \cos i_5}{n_1 \cos i_5 + \cos i_6} \\
  t_{s3} &= \frac{2 \cos i_5}{\cos i_5 + n_1 \cos i_6} \\
  t_{p4} &= \frac{2 n_1 \cos i_6}{\cos i_6 + n_1 \cos i_5} \\
  t_{s4} &= \frac{2 n_1 \cos i_6}{n_1 \cos i_6 + \cos i_5}
\end{align*}
\] (6)
Where, $t_{p1}$, $t_{s1}$ and $t_{p2}$, $t_{s2}$ are the transmission coefficients of the incident and outgoing surfaces of the lens respectively, $t_{p3}$, $t_{s3}$ and $t_{p4}$, $t_{s4}$ are the transmission coefficients of the incident and outgoing surfaces of the optical fiber respectively.

Take the linearly polarized light of $\begin{bmatrix} E_{ix1} \\ E_{iy1} \end{bmatrix} = \begin{bmatrix} \cos \alpha \\ \sin \alpha \end{bmatrix}$ as an example. The lens is made of isotropic material, whose propagation distance is short. Take no account of the transmission effect in the lens and set the components of the polarization field vector in the eigen coordinate system as $E_{ix1}$, $E_{iy1}$, $E_{on1}$, $E_{op1}$, and those in the $xy$ coordinate system as $E_{ox1}$, $E_{oy1}$, $E_{ot1}$, $E_{ot2}$ respectively, the following relationships will be obtained:

$$\begin{bmatrix} E_{op1} \\ E_{on1} \end{bmatrix} = \begin{bmatrix} \cos \phi & \sin \phi \\ -\sin \phi & \cos \phi \end{bmatrix} \begin{bmatrix} E_{ix1} \\ E_{iy1} \end{bmatrix}$$

(7)

So, the Jones matrix of the polarization aberration of the lens interface is as follows:

$$J_1 = \begin{bmatrix} t_{p2}t_{p1} \cos^2 \phi + t_{s2}t_{s1} \sin^2 \phi & (t_{p2}t_{p1} - t_{s2}t_{s1}) \sin \phi \cos \phi \\ (t_{p2}t_{p1} - t_{s2}t_{s1}) \sin \phi \cos \phi & t_{p2}t_{p1} \sin^2 \phi + t_{s2}t_{s1} \cos^2 \phi \end{bmatrix}$$

(8)

The Jones vector of the light emitted from the lens is:

$$\begin{bmatrix} E_{on1} \\ E_{ot1} \end{bmatrix} = \begin{bmatrix} (t_{p2}t_{p1} \cos^2 \phi + t_{s2}t_{s1} \sin^2 \phi) \cos \alpha + (t_{p2}t_{p1} - t_{s2}t_{s1}) \sin \alpha \sin \phi \cos \phi \\ (t_{p2}t_{p1} \sin^2 \phi + t_{s2}t_{s1} \cos^2 \phi) \sin \alpha + (t_{p2}t_{p1} - t_{s2}t_{s1}) \cos \alpha \sin \phi \cos \phi \end{bmatrix}$$

(9)

Then the polarization aberration of the output light of the lens is:

$$\Delta \alpha = \arctan \left( \frac{(t_{p2}t_{p1} \sin^2 \phi + t_{s2}t_{s1} \cos^2 \phi) \sin \alpha + (t_{p2}t_{p1} - t_{s2}t_{s1}) \cos \alpha \sin \phi \cos \phi}{(t_{p2}t_{p1} \cos^2 \phi + t_{s2}t_{s1} \sin^2 \phi) \cos \alpha + (t_{p2}t_{p1} - t_{s2}t_{s1}) \sin \alpha \sin \phi \cos \phi} \right) - \alpha$$

(10)
Because of the birefringence effect of the polarization-maintaining fiber, the transmission polarization in the fiber is unstable when the fiber is not incident to the axis. Consequently, when the fast and slow axes of the fiber coincide with the \( xy \) axis \((\gamma = 0)\), the Jones matrix of polarization aberration (interface term) and birefringence (propagation term) is:

\[
J_2 = \begin{bmatrix}
\cos \phi & -\sin \phi \\
\sin \phi & \cos \phi \\
0 & t_{s4}
\end{bmatrix}
\begin{bmatrix}
\cos \gamma & -\sin \gamma \\
\sin \gamma & \cos \gamma \\
0 & e^{-j\delta_{12}}
\end{bmatrix}
\begin{bmatrix}
\cos \gamma & \sin \gamma \\
-\sin \gamma & \cos \gamma \\
0 & t_{s3}
\end{bmatrix}
\begin{bmatrix}
\cos \phi & \sin \phi \\
-\sin \phi & \cos \phi \\
0 & 0
\end{bmatrix}
\]

\[
= \begin{bmatrix}
\cos \phi & -\sin \phi \\
\sin \phi & \cos \phi \\
0 & t_{s4}t_{s3} \\
\end{bmatrix}
\begin{bmatrix}
\cos \gamma & -\sin \gamma \\
\sin \gamma & \cos \gamma \\
0 & e^{-j\delta_{12}}
\end{bmatrix}
\begin{bmatrix}
\cos \phi & \sin \phi \\
-\sin \phi & \cos \phi \\
0 & t_{s3}
\end{bmatrix}
\begin{bmatrix}
\cos \gamma & \sin \gamma \\
-\sin \gamma & \cos \gamma \\
0 & 0
\end{bmatrix}
\]

(11)

It can be seen from Equation (11) that due to the phase change caused by birefringence and the amplitude change caused by the polarization aberration at the fiber interface, both the degree of polarization and the state of polarization will change during the transmission of polarized light by the polarization maintaining fiber. In this case, the polarization aberration at the lens fiber combination interface is analyzed without regard to the influence of birefringence \((\delta = 0)\). Equation (9) of the polarized light output by the lens is taken as the incident condition of the optical fiber, and the optical fiber exit polarized light vector is:

\[
\begin{bmatrix}
E_{ox2} \\
E_{oy2}
\end{bmatrix} = J_2 \begin{bmatrix}
E_{ax1} \\
E_{ay1}
\end{bmatrix}
\]

(12)

Then, the polarization aberration of the exiting light at the lens-fiber combination interface is:

\[
\Delta \alpha_2 = \arctan \left( \frac{E_{oy2}}{E_{ox2}} \right) - \alpha
\]

(13)

3.2. Simulation Analysis

Conditions are as follows. Choose the 532nm green semiconductor laser of a distance of 150mm (polarizer and magneto-optical modulator are installed between them); set the
modulation factor $m_f = 0.0087\text{rad}$; select the calcium fluoride biconvex lens (uncoated) of Thorlabs company; set the refractive index at 532nm as $n = 1.4354$; select the panda polarization maintaining fiber with working wavelength of 400-680nm; set the core refractive index $n_i = 1.4457$, and the numerical aperture is $NA = 0.12$.

![Polarization aberration is affected by incident angle and linear polarization angle](image)

**Figure 4.** Polarization aberration is affected by incident angle and linear polarization angle

According to Equation (13), when the incident plane is along the horizontal direction, the simulation of polarization aberration affected by the incident angle and linear polarization angle is shown in Fig. 4. When the incident angle is $0^\circ$, that is, the light is normal to the center of the lens, polarization aberration does not occur, and when the polarization direction of linear polarized light is $0^\circ$ or $90^\circ$, polarization aberration does not occur, either.

It can be seen from Figure 5(a) that the relationship between the polarization aberration and the polarization angle of linearly polarized light is approximately sinusoidal under non-normal incidence. Compared with the case of single lens polarization aberration in reference [10], the variation law of polarization aberration is the same after adding the fiber interface, but the variation degree is cumulative.

When the incident plane and the polarization vector of the incident light are close to $0^\circ$ or $90^\circ$ respectively, the polarization aberration of the incident plane and the polarization
vector of the incident light do not exist. This is because the eigenstates $E_S$ and $E_p$ are equal when the incident photoelectric vector $E$ is close to $45^\circ$ and the coupling between $E_S$ and $E_p$ is the most obvious after the influence of attenuation and delay, so the more the polarization aberration is affected.

![Graphs](image)

**Figure 5.** Analysis of polarization aberration in the interface.

(a) Influence of linear refractive index angle at different incident angles;
(b) Influence of incident angle at different linear polarization angles;
(c) Influence of incident distance at different linear polarization angles;
(d) Influence of linear polarization angle at a different coordinate relation.

Comparing Figure 5(a) with Figure 5(b), it can be seen that when the angle of the linear polarization is the same as the angle of the incident plane ($0^\circ$ or $90^\circ$), there is no polarization
aberration. In addition, the polarization aberration increases with the increase of the incident angle, and the polarization aberration always changes towards the direction of the incident plane (along the light propagation direction, the counterclockwise direction is positive), which is in line with the polarization characteristics of the refracted light in reference [18]. When the incident angle is constant and the polarization angle of the incident ray is 0°, 30°, 45° and 60° respectively, the variation of the polarization aberration affected by the incident distance is shown in Fig. 5(c), which is similar to Fig. 5(b). When the incident angle is constant, the farther the incident distance is, the larger the refraction angle is, the greater the difference of the transmission coefficients of s and p components is, and the greater the variation of the polarization aberration is due to the curvature of the convex lens.

Further, when the incident angle is 3mrad and the angle of the incident plane is 0°, 30°, 45° and 60° respectively, the variation of the polarization aberration affected by the polarization angle of the incident ray is shown in Fig. 5(d). The variation law of polarization aberrations is basically the same as that in Fig. 5(a). The curve moves the same distance as the angle of the incident plane (or the polarization angle of the incident ray) along the x-axis in a positive direction. (if the angle is negative, it will follow the negative direction). This is because when the incident coordinate system (sp system) does not coincide with the coordinate system, it is necessary to analyze the projection of the electric vector on the xy coordinate system, which results in the curve translation.

4. Azimuth Transmission Error

In section 2, taking the influence of birefringence of optical fiber into consideration, theoretically, the azimuth can be calculated. Based on the analysis in the section 3 and the Fresnel formula, the transmission coefficient \( t_p \neq t_s \), if the incident light is linearly polarized light, the output light will still be linearly polarized light, but its amplitude attenuation causes
the polarization angle to change, resulting in polarization aberration. The influence of azimuth transmission accuracy is analyzed.

4.1. Formula Derivation

Based on optical fiber polarization aberration and birefringence of Jones matrix Equation (11), set the light whose incident surface is along the x-axis direction \( \phi = 0 \), and incident Angle is \( \nu \) as a reference, and then rederive the other side of the solution model. The AC electrical signal obtained from the output light intensity after photoelectric conversion, straight filtering and Bessel expansion approximation processing is:

\[
u'_A = -U'\cos\delta\sin\omega t - V'\cos(2\omega t) + X'\cos(2\omega t) \tag{14}\]

Where, \( U' = k\eta I_0 \cdot pmqn \cdot J_1(m_f) \cdot \sin(2\alpha) \), 
\[
V' = \frac{1}{2} k\eta I_0 \cdot \left[ (mp)^2 + (nq)^2 \right] \cdot J_1(m_f) \cdot \cos(2\alpha),
\]
\[
X' = \frac{1}{2} k\eta I_0 \cdot \left[ (mp)^2 - (nq)^2 \right] J_5(m_f),
\]
\[
m = t_{p2}l_{p1}, n = t_{s2}l_{s1}, p = t_{p4}l_{p3}, q = t_{s4}l_{s3}.
\]

Derivation analysis of Equation (14) shows that there is still an extreme point \( u_A' \mid_{\omega t=\pi/2}, u_A' \mid_{\omega t=3\pi/2} \). With unchanged horizontal coordinate, the azimuth angle model is further deduced as follows:

\[
\alpha = \frac{1}{2} \arctan \left\{ \frac{\left[ (mp)^2 + (nq)^2 \right] J_2(m_f)}{\cos \delta \cdot (2mnq) J_1(m_f) \cdot \frac{u_{A2} - u_{A1}}{u_{A2} + u_{A1} + k\eta I_0 \left[ (mp)^2 - (nq)^2 \right] J_2(m_f)}} \right\} \tag{15}
\]

4.2. Simulation Analysis

According to Equation (15), the azimuth transmission error is affected by the incidence angle \( \nu \) and the azimuth angle \( \alpha \) when the phase difference is \( \delta \neq \pi/2 + k\pi \), as shown in Fig. 6. Compared with reference [1], taking the polarization aberration into consideration, the
azimuth angle calculated by the azimuth model is consistent with the original model. The accuracy of azimuth calculation is affected by the azimuth angle rather than the incidence angle, and the relative that error is still caused by omitting the higher order term of the Bessel function.

![Figure 6. Azimuth transmission error.](image)

Because of the divergence of the incident Angle of polarized beam and the unknown transmission coefficient of each optical interface in the actual azimuth measurement, Eq. (15) cannot be applied in the actual measurement. The original model Eq. (3) is selected to calculate the azimuth angle. At this point, when sampling integral is carried out for the extremum point of the output AC signal, polarization aberration term is included in the AC signal. The extremum point \( u_{\alpha' |\omega_{\alpha/2}} \) of Eq. (14) of AC signal is substituted into Eq. (3), which is simplified as follows:

\[
\alpha' = \frac{1}{2} \arctan\left( \frac{(2mnpq)\sin(2\alpha)}{(mp)^2 + (nq)^2} \right)
\]

(16)

Obviously, the azimuth Angle of the solution has changed, and the polarization aberration will affect the accuracy of the other direction transfer.
According to Equation (16), the simulation of azimuth transfer error affected by incident angle $\nu$ and azimuth angle $\alpha$ is shown in Fig.7(a), and the variation law is similar to Fig.4. When the azimuth angle is $0^\circ$ , the angle of the linear polarization is the same as that of the incident plane, and there is no polarization aberration, so the azimuth transfer error is $0^\circ$; when the azimuth angle is not $0^\circ$ , the polarization aberration increases with the increase of the incident angle, and the azimuth calculation error also increases. The simulation of the influence of the incident angle under different azimuth conditions is shown in Fig.7(b). Combined with Fig.5(b), it can be seen that when the incident plane is along the $x$–axis direction, the larger the azimuth, the more significant the influence of the
polarization aberration on the azimuth calculation error. In Section 3, the maximum azimuth transfer error is about \(118^\circ - 45^\circ\) azimuth angle of \(0.006\text{mrad}\) incidence angle.

Azimuth detection influenced by the polarization aberration is shown in Fig.8. After the optical fiber is output, based on the polarizer polarization angle, the linearly polarized light carrying the azimuth information produces polarization aberration change Angle \(\Delta \alpha\) (counterclockwise). Validate those polarization aberration and orientation calculating error under different azimuths, finding that all the calculated azimuths meet the equation \(\alpha' \approx \alpha + \Delta \alpha\) (omit Bessel function at the same time also lost part of the higher order term polarization aberration information). Lens - fiber optical interface polarization aberration is bearing the calculating error.

![Figure 8. Principle of azimuth detection affected by polarization aberration.](image)

As to the theoretical simulation of polarization aberration and azimuth transfer error, the following measures can be taken to optimize and improve the system: 1) When there’s enough installation space for each instrument, the distance between the laser and the coupling system can be reduced as far as possible; 2) Choose a laser with a better divergence Angle and a lens with a larger surface radius (a larger focal length); 3) Try to ensure that all instruments are on the same optical axis.
5. Conclusion

In this paper, the influence of polarization aberration on the incident light wave in the optical interface and the coupling process in the non-line-of-sight azimuth transmission system are discussed. Based on the tracing of polarized light and Fresnel formula, the relation of polarization aberration of lens - fiber composite interface is established. The influence of incidence angle, incidence distance, incidence polarization angle and incidence plane on polarization aberration is analyzed by numerical simulation. Furthermore, the azimuth calculation model under the influence of polarization aberration is derived, and the influence of polarization aberration on azimuth transmission accuracy is simulated and analyzed according to the actual situation. The simulation results show that: when the angle between the polarization angle and the incident plane is 0° or 90°, there is no polarization aberration, otherwise the polarization aberration is affected by the incident angle, the distance between the incident plane and other factors, and always changes towards the direction of the incident plane; the azimuth transfer error, which is also called the polarization aberration, increases with the increase of the incident angle, and the improvement measures are proposed according to the influencing factors. The research results have certain guiding significance for the improvement of non-line-of-sight azimuth transmission system based on polarization maintaining fiber, the selection of instrument and the improvement of measurement accuracy.

References

[1] Cai W, Zhao Z H & Yang Z Y.et al. A non-line-of-sight azimuth transmission technology based on polarization-maintaining fiber[J]. Acta Optica Sinica, 2020, 40(15):1512001.

[2] Sasián J. Polarization fields and wavefronts of two sheets for understanding polarization aberrations in optical imaging systems. Optical Engineering53(3): 035102. 
https://doi.org/10.1117/1.OE.53.3.035102(2014).

[3] Tu Y, Wang X & Li S.et al. Analytical approach to the impact of polarization aberration on
lithographic imaging. *Optics letters* **37**(11): 2061-2063.
https://doi.org/10.1364/OL.37.002061(2012).

[4] Huang W, Xu X & Xu M. *et al.* Polarization aberration function for perturbed lithographic lens. *Optical Design and Testing VI. International Society for Optics and Photonics* **9272**: 92720G.
https://doi.org/10.1117/12.2073629(2014).

[5] Sasián J. Polarization fields for understanding polarization aberrations in optical imaging systems. *Current Developments in Lens Design and Optical Engineering XIV. International Society for Optics and Photonics* **8841**: 88410C.
https://doi.org/10.1117/12.2026788(2013).

[6] Tu Y, Wang X & Li S. *et al.* Symmetric polarization aberration compensation method based on scalar aberration control for lithographic projection lens. *Optical Microlithography XXV. International Society for Optics and Photonics* **8326**: 83262T.
https://doi.org/10.1117/12.917909(2012).

[7] He W, Fu Y & Liu Z. *et al.* Three-dimensional polarization aberration functions in optical system based on three-dimensional polarization ray-tracing calculus. *Optics Communications* **387**: 128-134.
https://doi.org/10.1016/j.optcom.2016.11.046(2017).

[8] Wang J, Li Y. Three-dimensional polarization aberration in hyper-numerical aperture lithography optics. *Optical Microlithography XXV. International Society for Optics and Photonics* **8326**: 832624.
https://doi.org/10.1117/12.916299(2012).

[9] Lu J J, Yuan Q & Sun X P. *et al.* Research of the polarization aberration on Smith prism. *Physics Procedia* **19**: 447-455.
https://doi.org/10.1016/j.phpro.2011.06.191(2011).
[10] Shen L, Li S & Wang X. et al. Analytical analysis of the impact of polarization aberration of projection lens on lithographic imaging. *Journal of Micro-Nanolithography MEMS AND MOEMS* **14**(4): 043504. https://doi.org/10.1117/1.JMM.14.4.043504(2015).

[11] Yuan J, Li Z C & Yang H. The Polarization Aberration of Single Lens System. *Laser Journal* **36**(10): 21-23 https://doi.org/10.14016/j.cnki.jgzz.2015.10.021(2015).

[12] Chipman R A. Aberration Functions Due To Coatings. *Optical Interference Coatings* TC.9. https://doi.org/10.1364/oic.2016.tc.9(2016).

[13] Tu Y Y, Wang X Z & Li S K. et al. Analytical approach to the impact of polarization aberration on lithographic imaging. *Optics Letters* **37**(11): 2061-2063. https://doi.org/10.1364/OL.37.002061(2012).

[14] Shen L N, Wang X Z & Li S K. et al. General analytical expressions for the impact of polarization aberration on lithographic imaging under linearly polarized illumination. *Journal of the Optical Society of America* **33**(6): 1112-1119. https://doi.org/10.1364/JOSAA.33.001112(2016).

[15] Xu X R, Huang W & Xu M F. Orthonormal polynomials describing polarization aberration for M-fold optical systems. *Optics Express* **24**(5): 4906-4912. https://doi.org/10.1364/JOSAA.33.001112(2016).

[16] Yang Y F, Yan C X & Hu C H. et al. Polarization Aberration Analysis of Coherent Laser Communication System. *Acta Optica Sinica* **36**(11): 47-54. https://doi.org/10.3788/AOS201636.1106003(2016).

[17] Li C Y, Lu W G & Qiao L. Analysis and research of polarization aberration in rapid space angle measuring system. *Acta Physica Sinica* **67**(3): 75-82. https://doi.org/10.7498/aps.67.20171702(2018).

[18] Liao Y B. Polarized Optics[M]. Beijing: *Science Press*, 2003: 28-32.
**Acknowledgement**

This study was supported by the National Natural Sciences Foundation of China (grant no.61505254)

**Author contributions**

Zhiyong Yang and Junchen Song analyzed the equations and simulationed. Wei Cai and Gaoxiang Lu verificationed the results. Danqiu Qiao arranged the manuscript. All authors reviewed the manuscript.

**Competing interests**

The authors declare no competing interests.