The $\rho(2S)$, $\psi(2S)$ and $\Upsilon(2S)$ mesons in a double pole QCD Sum Rules

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We introduce the method of the double pole QCD sum rules and we study the $\rho(2S)$, $\psi(2S)$ and $\Upsilon(2S)$ mesons. We get the masses and decay constants of the mesons and show a prediction for the decay constant of the $\rho(2S)$ meson given by $(177 \pm 13)$ MeV.

PACS numbers: 14.40.Pq, 13.20.Gd

I. INTRODUCTION

In 1979, Shifman, Vainshtein and Zakharov (SVZ) created the successful method of QCD sum rules (QCDSR), which is widely used nowadays and they won the Sakurai prize in 1999 for this discovery. With this method, we can calculate many hadron parameters like: hadron’s mass, decay constant, coupling constant and form factors in terms of the QCD parameters like: quark masses, the strong coupling and non-perturbative parameters like quark condensate and gluon condensate. The main point of this method is that the quantum numbers and quarks hadron’s content are represented by an interpolating current, where we get a correlation function as a time order operation of this current. The determination of the ground state of hadron’s mass, we use the two point correlation function, where this correlation function has two point of view, the QCD side and the phenomenological side. On the QCD side, the correlation function can be written in terms of a dispersion relation, where the spectral density doesn’t have poles, on the other hand, the phenomenological side has a sum of the all states that coupling with the current. To represent the phenomenological side as a form of a dispersion relation the SVZ method uses an Ansatz that the spectral density can be represented by the form “pole + continuum”.

For the $\rho$ meson’s spectrum the proposal of SVZ is a good approach, due the large decay width of the $\rho(2S)$ or $\rho(1450)$ and the possible existence of a light $\rho(3S)$ or $\rho(1570)$ with decay width about 144 MeV, Fig. 1, that allow to approximate the excited states of this theory as a continuum, on the other hand, the $J/\psi$ meson’s spectrum is very different and a proposed type “pole+ pole+ continuum” seems more realistic, Fig. 1.

Alternative proposals for the phenomenological spectral density have been used in the QCDSR to study the case where the hadron ground state has a large decay width, where the pole is replaced by Breit-Wigner function. The case where the excited states are considered in QCDSR there are two methods: the Maximum Entropy Method and Gaussian Sum Rule with “pole+ pole+ continuum” Ansatz. In Maximum Entropy Method there are studies for the $\rho$ meson, nucleon, $J/\psi$ meson and $\Upsilon$ meson. In Gaussian Sum Rule there is a study for mixed states of the glueballs and scalar mesons.

In lattice QCD, the study of excited states is a recent area, where there are a lot of studies devoted to $\pi(2S)$ meson, $\rho$ meson excited states, charmonium, and exotic charmonium spectrum. In addition, the excited states have been studied recently by several approaches like: QCD’s Bethe-Salpeter Equation for $\pi(2S)$ and $\rho(2S)$, light-front quark model for $\rho(2S)$, $\eta_c(2S)$, $\psi(2S)$ and the bottomonium analogous. The $\psi(2S)$ has been studied in QCDSR as a hybrid meson using the “pole + continuum” Ansatz.

There are a lot of motivation to study the excited states that belong the charmonium spectrum. States recently discovered like $Y(4260)$ and $Y(4660)$ are a great example of this lack, considering theories like: $Y(4260)$ has been proposed as a bound state of $J/\psi + f_0$ and $Y(4660)$ has been interpreted as a bound state of $J/\psi(2S) + f_0$. show that $Y(4660)$ is an excited state of $Y(4260)$. Another point is that $Z^+(4430)$ would be an excited state of $X^+(3872)$ and $Z_{b0}^+(10610)$ would be an excited state of $X_{b0}^+(10100)$.

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The simple method “pole+ pole+ continuum” Ansatz was used in lattice QCD [15] and the authors have shown a problem in which the nucleon current coupling more with excited state than its ground state. In Gaussian Sum Rule the “pole+ pole+ continuum” Ansatz [4] was applied and they get the masses of the mixed states of the glueballs and scalar mesons.

In this paper, we study the excited state using the “pole+ pole+ continuum” Ansatz in QCD sum rules and we apply in three cases: the ρ(2S), ψ(2S) and Υ(2S) mesons and we calculate their masses and decay constants.

II. THE METHOD

To implement our method, we first use the invariant part of the generic correlation function and we apply the Borel transformation:

\[ B[\Pi^{phen}(q^2)] = \int_{0}^{\infty} ds \rho^{phen}(s) e^{-st}, \]  

where \( t = 1/M^2 \). We consider the following spectral density to the phenomenological side:

\[ \rho^{Phen}(s) = \lambda_1^2 \delta(s - m_1^2) + \lambda_2^2 \delta(s - m_2^2) + \rho^{Cont}(s) \theta(s - s'_0), \]

where \( m_1 \) is the mass for the ground state and \( m_2 \) is the mass for the first excited state and \( s'_0 \) mark the begin of the continuum states.

Inserting Eq. (2) in Eq. (1), we get the expression:

\[ \Pi^{phen}(\tau) = \lambda_1^2 e^{-m_1^2 \tau} + \lambda_2^2 e^{-m_2^2 \tau} + \int_{s'_0}^{\infty} ds \rho^{phen}(s) e^{-st}, \]

where \( \Pi^{phen}(\tau) = B[\Pi^{phen}(q^2)] \).

Using the quark hadron duality, we get the double pole QCD sum Rule,

\[ \lambda_1^2 e^{-m_1^2 \tau} + \lambda_2^2 e^{-m_2^2 \tau} = \Pi^{QCD}(\tau), \]

where,

\[ \Pi^{QCD}(\tau) = \int_{s'_0}^{s'_1} ds \ e^{-st} \rho^{OPE}(s) + \text{higher order condensates}, \]
where $s_0^{\text{min}}$ is a QCD parameter.

As usually done in QCDSR to obtain the mass of the hadron, we take the derivative of Eq. (4) with respect to $\tau$ and we get the new equation:

$$-m_1^2\lambda_1 e^{-m_1^2\tau} - m_2^2\lambda_2 e^{-m_2^2\tau} = \frac{d}{d\tau} \Pi^{QCD}(\tau).$$

We can observe that the equations Eq. (4) and Eq. (6) they form a equation’s system in the variables,

$$A(\tau) = \lambda_1^2 e^{-m_1^2\tau},$$

$$B(\tau) = \lambda_2^2 e^{-m_2^2\tau}.$$  

Solving the equation’s system Eq. (4) and Eq. (6) writings in terms of the functions $A(\tau)$ and $B(\tau),$ we easily get:

$$A(\tau) = \frac{D\Pi^{QCD}(\tau) + \Pi^{QCD}(\tau) m_2^2}{m_2^2 - m_1^2},$$

$$B(\tau) = \frac{D\Pi^{QCD}(\tau) + \Pi^{QCD}(\tau) m_1^2}{m_1^2 - m_2^2},$$

where: $DF(\tau) = \frac{d}{d\tau} F(\tau).$

To eliminate the dependence of the $\lambda_1$ coupling that appears in Eq. (9), we take a derivative of this equation with respect of $\tau$ and divide the result by Eq. (9). The result of this procedure is given by the Eq.(11). To eliminate $\lambda_2$ coupling the procedure is analogous and the result is given by the Eq.(12).

$$m_1 = \sqrt{-\frac{D\Pi^{QCD}(\tau) m_2^2 + D\Pi^{QCD}(\tau) m_1^2}{D\Pi^{QCD}(\tau) + \Pi^{QCD}(\tau) m_2^2}},$$

$$m_2 = \sqrt{-\frac{D\Pi^{QCD}(\tau) m_2^2 + D\Pi^{QCD}(\tau) m_1^2}{\Pi^{QCD}(\tau) m_1^2 + D\Pi^{QCD}(\tau)}.}$$

In the first view the Eq.(11) and Eq.(12) suggest a system of non-linear equations for the masses $m_1$ and $m_2,$ that could be extracted in independent way. On the other hand, using Eq. (11) to obtain an $m_2$ expression, it reproduces the same result given in Eq. (12). It is a very interesting result, that is impossible to isolate the masses $m_1$ and $m_2.$ In that way, our sum rule the mass of the ground state is a function of the mass of the excited state, where we use in our analysis the equation Eq.(11).

### III. SUBTRACTION OF THE CORRELATION FUNCTION

For the $q\bar{q}$ vector mesons, on the QCD side the correlation function has a form:

$$\Pi_{\mu\nu}(q) = i \int d^4x \ e^{iq\cdot x}\langle 0 \vert T\{j_\mu(x)j^\dagger_\nu(0)\} \vert 0 \rangle = (q_\mu q_\nu - q^2 g_{\mu\nu})\Pi_1(q^2),$$

where $j_\mu(x) = \bar{q}(x) \gamma^\mu q(x).$

On the phenomenological side we use:

$$\langle 0 \vert j_\mu(0) \vert V(q) \rangle = f_V m_V \epsilon_{\mu}^{(V)}(q),$$

where $f_V$ is the vector meson decay constant.
where \( f_V \) is the decay constant and \( m_V \) is the hadron’s mass. Inserting Eq. (14) in Eq. (13), we get:

\[
\Pi_{\mu\nu}(q) = (g_{\mu\nu} - m_V^2 g_{\mu\nu}) \frac{f_V^2}{m_V^2 - q^2} + \text{excited states contribution.} \tag{15}
\]

Comparing the Eq. (13) with Eq. (15) we notice that they have different structures. We choose the \( g_{\mu\nu} \) structure that it doesn’t depend on the choice of a gauge on the phenomenological side. We can write the invariant part of the correlators of the Eqs. (13) and (15), in a form:

\[
\Pi(q^2) = \frac{1}{0} ds \frac{\bar{\rho}(s)}{s - q^2}, \tag{16}
\]

Considering the subtracted correlator [29]:

\[
\bar{\Pi}(q^2) = \Pi(q^2) - \Pi(0), \tag{17}
\]

we get:

\[
\bar{\Pi}(q^2) = q^2 \int_0^{\infty} ds \frac{\bar{\rho}(s)}{s(s - q^2)}, \tag{18}
\]

where we use for the sum rule the function, \( \frac{\bar{\rho}(s^2)}{s^2} \). Comparing the \( \bar{\rho}(s) \) with spectral density given by Eqs. (1) and (5), we get:

\[
\rho_{\text{OPE}}(s) = \frac{\bar{\rho}(s)}{s} \quad \text{and} \quad \rho_{\text{Phen}}(s) = \frac{\bar{\rho}(s)}{s}.
\]

For the sum rules of \( \rho \), \( J/\psi \) and \( \Upsilon \) mesons we use the Ref. [29].

IV. RESULTS

In this work we use the following parameters: \( \alpha_s = 0.3 \), \( m_q = 4 \text{MeV} \), \( m_c = 1.3 \text{GeV} \), \( m_b = 4.5 \text{GeV} \), \( \langle \bar{q}q \rangle = -(0.23)^3 \text{GeV}^3 \) and \( \langle g_s^2 G^2 \rangle = 0.88 \text{ GeV}^4 \). For the study of the meson’s mass, is used a curve in two dimensions where the stability as a Borel mass is studied for three level curves: \( M_1 \), solid line, that it supplies a double pole that contributes with 90\% of Eq.(3), \( M_2 \), dash line, that contributes with 60\% and \( M_3 \), dash-point line, that contributes with 40\%.

This imposition that the pole contribution defined the Borel window is a recent progress of the QCDSR. The use of this criterion for the pentaquarks \( \Theta^+ (1540) \) and \( \Xi^{--} (1836) \), has been shown that the QCDSR not support a pentaquark state [30]. Recently the same criterion was used in Ref. [22] and they have shown that is impossible to interpret \( Y(4260) \) as a hadron molecule \( J/\psi - f_0 \).

A. \( \rho(2S) \) and \( \rho \) Sum Rule

Using the data of \( \rho \) meson’s spectrum Fig. (1), the \( s_0' = (m_2 + \Delta')^2 \) parameter has \( \Delta' = 0.4 \text{GeV} \). But this value supply unstable values for the decay constant. The best value for \( \Delta' \) is 0.1 GeV. In Fig. (2) we show in the level curves of the mass Eq. (11), where \( \rho \) meson’s mass, \( m_1 \) as a function of the \( \rho(2S) \) mass, \( m_2 \). We notice that with the increase of the Borel mass the curve of the mass converge to the curve with Borel mass \( M_3 \). Seemingly, none point seems privileged, but the mass difference, \( m_2 - m_1 \), it is only possible experimental value \( \Delta = 0.69 \text{GeV} \) for \( m_2 = 1.46 \text{GeV} \). In this case, we have \( m_1(m_2) = 0.77 \text{GeV} \). So for this sum rule it is necessary to do some estimate for the separation of the states 2S and 1S, to do the prediction of the mass.

For the calculation of the decay constant, we use the experimental values \( m_1 = 0.77 \text{GeV} \), \( m_2 = 1.46 \text{GeV} \) and \( \Delta' = 0.1 \text{GeV} \). In Fig. (3), we show the decay constant of the \( \rho \) meson, solid line and the decay constant of the \( \rho(2S) \) meson, dash line. We see the value for the \( \rho \) meson decay constant has a plateau on value 200 MeV. This value is in agreement with the experimental value, given by 216 MeV [22]. For
FIG. 2: The \( \rho \) mass as a function of the \( \rho(2S) \) mass for \( \Delta' = 0.1 \) GeV.

The \( \rho(2S) \) meson decay constant has a plateau with a value of 165 MeV. Considering uncertainty with respect to \( \Delta' \) parameter due large width of the \( \rho(2S) \) and uncertainty of the \( \rho(3S) \) mass, we varying this parameter in the range: \( \Delta' = (0.15 \pm 0.05) \) GeV, we have a prediction for its decay constant given by,

\[
f_{\rho(2S)} = (177 \pm 13) \text{ MeV.}
\]  

(19)

For the \( \rho \) meson decay constant, we get: \( f_\rho = (200 \pm 2) \) MeV.

FIG. 3: The decay constant of the \( \rho \) meson, solid line, and \( \rho(2S) \), dash line, as a function of the Borel mass with \( \Delta' = 0.1 \) GeV.

In Fig. 4 we show the pole contribution as a function of the Borel mass, comparing the sum rule with two poles for \( s_0' = (1.46 + 0.1)^2 \text{GeV}^2 \), dash line, with the sum rule of usual one pole sum rule for the \( s_0 = (0.77 + 0.7)^2 \text{GeV}^2 \), solid line. We see that the sum rule with two poles has a Borel window larger than the usual case.

In Fig. 5 we show the relative contribution of the \( \rho \) meson, dash line, and \( \rho(2S) \), solid line, that compose the double pole. We see that for low values for the Borel mass the contribution of the meson \( \rho \) it reaches 90\%, but for Borel mass \( M_3 \) it reaches a lowest contribution of the 67\%.

B. \( \psi(2S) \) and \( \psi \) Sum Rule

In the study of the \( J/\psi \), we use the data of \( J/\psi \) meson’s spectrum Fig. 1, where \( \Delta' = 0.3 \) GeV. In this case, we get the mass of the \( J/\psi \) for \( M_3 \) Borel mass, given by \( m_1(3.7) = 2.9 \) GeV, Fig. 2.

On the other hand, the calculation of the decay constant, we use the experimental values \( m_1 = 3.1 \) GeV, \( m_2 = 3.7 \) GeV and \( \Delta' = 0.3 \) GeV in Eqs. 9 and 10. In Fig. 6 we see the values for the decay constant.
FIG. 4: The pole contribution as a function of the Borel mass, comparing the two poles, dash line, and one pole sum rule, solid line.

FIG. 5: The relative contribution of the $\rho$ meson, dash line, and $\rho(2S)$, solid line.

FIG. 6: The $J/\psi$ mass as a function of the $\psi(2S)$ mass.

of the $J/\psi$ meson, solid line, has a good stability for the Borel mass above 3 GeV and its value is close the its experimental value, that is about 416 MeV. For the decay constant of the $\Psi(2S)$ meson, dash line, has the same stability window and its value is $(320 \pm 10)$ MeV. Considering $\Delta'$ parameter in the range: $\Delta' = (0.3 \pm 0.05)$ GeV, we get:

$$f_{\psi(2S)} = (327 \pm 37) \text{ MeV}. \quad (20)$$

This result is in agreement with the experimental value, given by $(297 \pm 3)$ MeV.

In Fig. 8 we show the contribution of the pole as a function of the Borel mass, comparing the sum rule with two poles for $s_0 = (3.7 + 0.3)^2 \text{ GeV}^2$, dash line with the usual sum rule for $s_0 = (3.1 + 0.56)^2 \text{ GeV}^2$, solid line. We see that the sum rule with two poles has a Borel window more larger than the usual case. In Fig. 9 we show the relative contribution of the $J/\psi$ meson, trace line, and $\psi(2S)$, solid line, that compose the double pole. We see that for low values for the mass of Borel the contribution of the meson $J/\psi$ it reaches 95% for the Borel mass $M_1$.

C. $\Upsilon(2S)$ and $\Upsilon$ Sum Rule

In the study of the $\Upsilon$, we use $\Delta' = 0.33 \text{ GeV}$. In this case, we get the mass of the $\Upsilon(1S)$ for $M_3$ Borel mass, given by $m_1(10.02) = 9.24 \text{ GeV}$, Fig. 10.

On the other hand, the calculation of the decay constant, we use the experimental values $m_1 = 9.46 \text{ GeV}$, $m_2 = 10.02 \text{ GeV}$ and $\Delta' = 0.33 \text{ GeV}$ in Eqs. (9) and (10). In Fig. 11 we see the values for the
FIG. 7: The decay constant of the $J/\psi$ meson, solid line, and $\psi(2S)$, dash line, as a function of the Borel mass with $\Delta' = 0.3$ GeV.

FIG. 8: The pole contribution as a function of the Borel mass, comparing the two poles, dash line, and one pole sum rule, $J/\psi$ meson, dash line, and $\psi(2S)$, solid line.

FIG. 9: The relative contribution of the $J/\psi$ meson, dash line, and $\psi(2S)$, solid line.

FIG. 10: The $\Upsilon$ mass as a function of the $\Upsilon(2S)$ mass.

decay constant of the $\Upsilon$ meson, solid line, has a good stability for the Borel mass above 5 GeV and its value is close to its experimental value, that is 714 MeV. For the decay constant of the $\Upsilon(2S)$ meson, dash line, has the same stability window and its value is,

$$f_{\Upsilon(2S)} = (530 \pm 30)\text{ MeV}. \quad (21)$$
This result is in agreement with the experimental value, given by $(497 \pm 3) \text{ MeV}$. In this case the uncertainty with respect to $\Delta'$ parameter does not exist, because the $\Upsilon(3S)$ is a narrow resonance, with full width of 20 KeV.

![Graph](image1)

FIG. 11: The decay constant of the $\Upsilon$ meson, solid line, and $\Upsilon(2S)$, dash line, as a function of the Borel mass with $\Delta' = 0.33 \text{ GeV}$.

In Fig. 12 we show the contribution of the pole as a function of the Borel mass, comparing the sum rule with two poles for $s'_0 = (10.02 + 0.33)^2 \text{ GeV}^2$, dash line with the usual sum rule for $s_0 = (9.46 + 0.56)^2 \text{ GeV}^2$, solid line. We see that the sum rule with two poles has a Borel window more larger than the usual case. In Fig. 13 we show the relative contribution of the $\Upsilon$ meson, dash line, and $\Upsilon(2S)$, solid line, that compose the double pole. We see that for low values for the mass of Borel the contribution of the meson $\Upsilon(2S)$ it reaches 92% for the Borel mass $M_1$.

![Graph](image2)

FIG. 12: The pole contribution as a function of the Borel mass, comparing the two poles, dash line, and one pole sum rule, solid line.

![Graph](image3)

FIG. 13: The relative contribution of the $\Upsilon$ meson, dash line, and $\Upsilon(2S)$, solid line.
V. CONCLUSIONS

In this work we have presented a new method to QCD sum rule with double pole and we have applied this method for three problems well established in QCD sum rules, the study of the $\rho$, $J/\psi$ and $\Upsilon$ mesons. We have shown that the obtaining of the hadron’s masses supply close values to the experimental masses and we have shown a prediction for the decay constant of the $\rho(2S)$ and we have obtained the decay constants of the $\psi(2S)$ and $\Upsilon(2S)$ and their ground states that are shown in Table I.

| TABLE I: Decay Constants of the 2S states and 1S states in MeV. |
|---------------------------------------------------------------|
| hadron | this work | Ref. [19] | Ref. [17] | Ref. [18] | lattice | experiment |
|--------|-----------|-----------|-----------|-----------|---------|------------|
| $\rho$ | 200 $\pm$ 2 | 216.37 | 268 | - | - | 216 $\pm$ 5 |
| $\rho(2S)$ | 177 $\pm$ 13 | 128 | 155 | - | - | - |
| $J/\psi$ | 416 $\pm$ 11 | - | - | 399 $\pm$ 4 | 416 $\pm$ 5 |
| $\psi(2S)$ | 327 $\pm$ 37 | - | - | 371 | 143 $\pm$ 8 | 295 $\pm$ 3 |
| $\Upsilon$ | 695 $\pm$ 11 | - | - | 546.6 | - | 714 $\pm$ 5 |
| $\Upsilon(2S)$ | 530 $\pm$ 30 | - | - | 583.2 | - | 497 $\pm$ 3 |

VI. ACKNOWLEDGEMENTS

We are indebted to Marina Nielsen, Fernando Navarra, Ricardo Matheus and Raphael Albuquerque for fruitful discussions, and also thanks the Professors Craig Roberts and Tom Steele for the contributions to this work.

This work has been partially supported by CAPES.

[1] M. A. Shifman, A. I. Vainshtein and V. I. Zakharov, Nucl. Phys. B 147, 385 (1979).
[2] S. H. Lee, K. Morita and M. Nielsen, Phys. Rev. D 78, 076001 (2008) [arXiv:0808.3168 [hep-ph]].
[3] P. Gubler and M. Oka, Prog. Theor. Phys. 124, 995 (2010) [arXiv:1005.2459 [hep-ph]].
[4] D. Harnett, R. T. Kleiv, K. Moats and T. G. Steele, Nucl. Phys. A 850, 110 (2011) [arXiv:0804.2195 [hep-ph]].
[5] K. Ohtani, P. Gubler and M. Oka, AIP Conf. Proc. 1343, 343 (2011) [arXiv:1104.5377 [hep-ph]].
[6] P. Gubler, K. Morita and M. Oka, Phys. Rev. Lett. 107, 092003 (2011) [arXiv:1104.4336 [hep-ph]].
[7] K. Suzuki, P. Gubler, K. Morita and M. Oka, [arXiv:1204.1173 [hep-ph]].
[8] T. Burch et al. [Bern-Graz-Regensburg Collaboration], Phys. Rev. D 70, 054502 (2004) [hep-lat/0405006].
[9] C. McNeile et al. [UKQCD Collaboration], Phys. Lett. B 642, 244 (2006) [hep-lat/0607032].
[10] J. J. Dudek, R. G. Edwards and D. G. Richards, Phys. Rev. D 73, 074507 (2006) [hep-ph/0601137].
[11] J. J. Dudek, R. G. Edwards, N. Mathur and D. G. Richards, Phys. Rev. D 77, 034501 (2008) [arXiv:0707.4162 [hep-lat]].
[12] L. Liu, S. M. Ryan, M. Peardon, G. Moir and P. Vilaseca, [arXiv:1112.1368 [hep-lat]].
[13] N. Mathur, Y. Chen, S. J. Chen, T. Draper, I. Horvath, F. X. Lee, K. F. Liu and J. B. Zhang, Phys. Lett. B 605, 137 (2005) [hep-ph/0306199].
[14] R. G. Edwards, J. J. Dudek, D. G. Richards and J. S. Wallace, Phys. Rev. D 84, 074508 (2011) [arXiv:1104.5152 [hep-ph]].
[15] D. B. Leinweber, Phys. Rev. D 51, 6369 (1995) [nucl-th/9405002].
[16] L. Liu, G. Moir, M. Peardon, S. M. Ryan, C. E. Thomas, P. Vilaseca, J. J. Dudek and R. G. Edwards et al., [arXiv:1204.5425 [hep-ph]].
[17] S. -x. Qin, L. Chang, Y. -x. Liu, C. D. Roberts and D. J. Wilson, Phys. Rev. C 85, 035202 (2012) [arXiv:1109.3459 [nucl-th]].
[18] T. Peng and B.-Q. Ma, [arXiv:1204.0863 [hep-ph]].
[19] D. Arndt and C.-R. Ji, Phys. Rev. D 60, 094020 (1999) [hep-ph/9905360].
[20] L. S. Kisslinger, Phys. Rev. D 79 (2009) 114026 [arXiv:0903.1120 [hep-ph]].
[21] A. Martinez Torres, K. P. Khemchandani, D. Gamermann and E. Oset, Phys. Rev. D 80, 094012 (2009) [arXiv:0906.5333 [nucl-th]].
[22] R. M. Albuquerque, M. Nielsen and R. R. da Silva, Phys. Rev. D 84 (2011) 116004 [arXiv:1110.2113 [hep-ph]].
[23] F. K. Guo, C. Hanhart and U. G. Meissner, Phys. Lett. B 665, 26 (2008) [arXiv:0803.1392 [hep-ph]].
[24] F. S. Navarra, M. Nielsen and J. M. Richard, J. Phys. Conf. Ser. 348, 012007 (2012) [arXiv:1108.1230 [hep-ph]].
[25] J. Beringer et al. (Particle Data Group), Phys. Rev. D 86, 010001 (2012).
[26] S. Godfrey and N. Isgur, Phys. Rev. D 32, 189 (1985).
[27] D. Ebert, R. N. Faustov and V. O. Galkin, Phys. Rev. D 79, 114029 (2009) [arXiv:0903.5183 [hep-ph]].
[28] O. Lakhina and E. S. Swanson, Phys. Rev. D 74 (2006) 014012 [hep-ph/0603164].
[29] P. Colangelo and A. Khodjamirian, In *Shifman, M. (ed.) : At the frontier of particle physics, vol. 3* 1495-1576 [hep-ph/0010175].
[30] R. D'E. Matheus, F. S. Navarra and M. Nielsen, Braz. J. Phys. 36, 1397 (2006).