NEUTRINOS AND BBN (AND THE CMB)

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ABSTRACT

During Big Bang Nucleosynthesis (BBN), in the first $\sim 20$ minutes of the evolution of the Universe, the light nuclides, D, $^3$He, $^4$He, and $^7$Li were synthesized in astrophysically interesting abundances. The Cosmic Microwave Background Radiation (CMB) observed at present was last scattered some $\sim 400$ thousand years later. BBN and the CMB (supplemented by more recent, $\sim 10$ Gyr, Large Scale Structure data), provide complementary probes of the early evolution of the Universe and enable constraints on the high temperature/energy physical processes in it. In this overview the predictions and observations of two physical quantities, the baryon density parameter and the expansion rate parameter, are compared to see if there is agreement between theory and observation at these two widely separated epochs. After answering this question in the affirmative, the consequences of this concordance for physics beyond the standard models of particle physics and cosmology is discussed.

1. Introduction

Big Bang Nucleosynthesis (BBN) probes the evolution of the Universe during its first few minutes, providing a glimpse into its earliest epochs. The photons in the presently observed Cosmic Microwave Background Radiation (CMB) were last scattered at “recombination”, when the plasma of ions and electrons became predominantly neutral, some 400 thousand years later. Comparison of observations relating to BBN (the primordial abundances of the light elements) and the CMB (the temperature anisotropy spectrum) often provide unique tests of the consistency of the standard models of particle physics and cosmology, as well as enable the possibility of constraining alternate models of cosmology and of physics beyond the standard model of particle physics. In this article, based on my invited talk at the NO-VE IV International Workshop on: “Neutrino Oscillations in Venice”, the complementarity between BBN and the CMB (supplemented by the Large Scale Structure (LSS) data needed to break some degeneracies) is explored, concentrating on the universal baryon asymmetry and the expansion rate of the early, radiation-dominated Universe. In particular, do observations of the baryon density parameter and of the expansion rate parameter (both to be defined in more detail below) agree at $\sim 20$ minutes and $\sim 400$ thousand years and, if so, what does this tell us about non-standard physics and/or cosmology in the time interval between these two, so widely separated, epochs?

After introducing and defining the cosmological parameters in §1.1 and §1.2, their influence on the predictions of BBN (§2) and the CMB observations (§3) is discussed.
and compared to the standard model predictions and, with the observational data (§4). After establishing the concordance of the standard models of particle physics and cosmology, some general constraints on possible new physics in the interval between BBN and recombination are presented in §5. The results are summarized in §6. In my recent review of BBN\(^1\) these issues are discussed in more detail (including a more extensive list of references). My talk at NO-VE IV and this article are drawn from the recently published paper with V. Simha\(^2\).

1.1. The Baryon Density Parameter: \(\eta_B\)

During the much earlier evolution of the Universe than is considered here, a universal matter-antimatter (baryon-antibaryon) asymmetry was established by particle physics processes yet to be uniquely determined. Thereafter, in the evolution of the Universe up to the present, the number of baryons in a comoving volume is preserved, although the number density of baryons decreases as the Universe expands. During the same early evolution of the Universe, very rapid electromagnetic processes establish and maintain a Bose-Einstein (black body) distribution for the cosmic background photons. This guarantees that when the photons are in equilibrium or, when they are entirely decoupled, the number of CMB photons in a comoving volume is preserved (except for the new photons added when various particle-antiparticle pairs annihilate or, when unstable particles decay). In the standard models of cosmology and particle physics the numbers of baryons and of CMB photons in a comoving volume have been unchanged since \(e^\pm\) annihilation. The ratio of the numbers of baryons and CMB photons in a comoving volume in the post-\(e^\pm\) annihilation Universe provides us with a dimensionless, time-invariant parameter – in the context of the standard models of particle physics (baryon conservation) and cosmology (entropy conservation).

\[
\eta_B = \frac{n_B}{n_\gamma} \equiv 10^{-10} \eta_{10}.
\]  

\(\eta_{10}\) is related to the baryon mass density parameter \(\Omega_B\), the ratio of the baryon mass density to the critical mass density, and the reduced, present value of the Hubble parameter \(h\) (the Hubble “constant”: \(H_0 \equiv 100h \text{ km s}^{-1} \text{ Mpc}^{-1}\)) by\(^3\),

\[
\eta_{10} = 273.9 \Omega_B h^2.
\]  

\(\eta_{10}\) “measured” at BBN (~20 minutes) and inferred from the CMB (~400 thousand years later) should agree, enabling constraints to be placed on non-standard entropy production and/or baryon number violation.

1.2. The Universal Expansion Rate Parameter: \(S\)

In the standard model of cosmology, the expansion rate is described by the Hubble parameter \((H)\) which, in the early Universe, is related to the total energy density
through the Friedman equation by,

$$H^2 = \frac{8\pi G \rho_{\text{TOT}}}{3}.$$  \hfill (3)

During its early evolution, the Universe is “radiation dominated” ($\rho_{\text{TOT}} \to \rho_R$); that is, the total energy density is dominated by massless particles (e.g., photons) or relativistic, massive particles ($m \lesssim T$). Prior to $e^\pm$ annihilation, for temperatures $T \lesssim$ few MeV, the standard-model relativistic particles present are photons, $e^\pm$ pairs, and three flavors of left-handed neutrinos along with their right-handed antineutrinos. Counting degrees of freedom and accounting for the relativistic bosons and fermions,

$$\rho_R = \rho_\gamma + \rho_e + 3\rho_\nu = (1 + 7/8(2 + 3))\rho_\gamma = 43\rho_\gamma/8. \hfill (4)$$

As may be seen from eq. 3, any modification to the Friedman equation will change the expansion rate ($H \to H' \equiv SH$). For example, a non-standard, early-Universe expansion rate may be due to the presence of a non-standard energy density of relativistic particles ($\rho'_R \neq \rho_R$), or by a non-standard value of the gravitational constant $G' \neq G$), or by some form of new physics beyond the standard model which modifies the Friedman equation. A non-standard particle content may be conveniently parameterized by the equivalent number of “additional” neutrinos defined, prior to $e^\pm$ annihilation, by $\Delta N_\nu \equiv N_\nu - 3$, so that $\rho'_R \equiv \rho_R + \Delta N_\nu \rho_\nu$. $\Delta N_\nu$ and the expansion rate parameter $S$ are related by \[\]

$$S^2 \equiv \left(\frac{H'}{H}\right)^2 = \frac{\rho'_R}{\rho_R} = \frac{G'}{G} \equiv 1 + \frac{7\Delta N_\nu}{43} = 1 + 0.163\Delta N_\nu. \hfill (5)$$

However, any change in $H$ ($S \neq 1$) which is traceable to a term evolving like the radiation density (as the inverse fourth power of the scale factor) can equally well be parameterized by $\Delta N_\nu$. In this sense, $\Delta N_\nu \neq 0$ should be thought of as a proxy for any non-standard, early-Universe expansion rate ($S \neq 1$) and need have nothing at all to do with “extra” (or fewer!) neutrinos.

After $e^\pm$ annihilation the only relativistic particles present during the radiation dominated epoch are the photons (which redshift to become the presently observed CMB) and the neutrinos, which decoupled prior to $e^\pm$ annihilation. In the approximation that the neutrinos were fully decoupled at $e^\pm$ annihilation, the post-annihilation photons are hotter than the neutrinos by a factor of $T_\gamma/T_\nu = (11/4)^{1/3}$ and \[\]

$$S^2 \equiv \frac{\rho'_R}{\rho_R} \to 1 + 0.135\Delta N_\nu, \hfill (6)$$

where, in the post-BBN, pre-recombination Universe, $\Delta N_\nu \equiv N_\nu - 3$.

Since the standard-model neutrinos were not fully decoupled at $e^\pm$ annihilation, they share some of the energy/entropy released during $e^\pm$ annihilation. As a result, they are warmer than is predicted by the fully decoupled approximation, increasing
the ratio of the post-\(e^\pm\) annihilation radiation density to the photon energy density. In the standard model, this additional contribution to the total energy density can be accounted for by replacing \(N_\nu = 3\) with \(N_\nu = 3.046\). Any post-BBN deviations from the standard model that can be treated as equivalent to contributions from fully decoupled neutrinos can thus be accounted for by

\[
S^2 = \rho'_R / \rho_R \rightarrow 1 + 0.134\Delta N_\nu, \tag{7}
\]

where in the post-\(e^\pm\) annihilation Universe relevant for comparison with the CMB and LSS, \(\Delta N_\nu \equiv N'_\nu - 3.046\). Note that with these definitions, the standard model corresponds to \(\Delta N_\nu = 0\) in both the pre- and post-\(e^\pm\) annihilation Universe.

2. Overview Of Primordial Nucleosynthesis

The early, hot, dense Universe evolves through a brief epoch when it is a cosmic nuclear reactor. Since for “our” Universe, the entropy per baryon (nucleon) is very large, \(\sim 10^9\), the synthesis of the elements is delayed well beyond the time when the average thermal energy (\(\propto T\), the temperature of the radiation and of those particles in equilibrium with it) drops below the binding energy of the lightest nuclei, in particular, deuterium. As a result, even though nuclear reactions such as \(n + p \rightarrow D + \gamma\) begin very early, due to the large \(\gamma\)-ray background provided by the “blue-shifted CMB”, the “back reaction” \(D + \gamma \rightarrow n + p\), keeps the deuterium abundance very small, inhibiting the formation of any of the more complex nuclei until later, at \(t \sim 3\) minutes, when \(T \approx 0.08\) MeV. At this time the number density of those photons with sufficient energy to photodissociate deuterium is comparable to the number density of baryons and only now can the various two-body nuclear reactions begin to build more complex nuclei such as \(^3\)He (\(^3\)H), \(^4\)He, and \(^7\)Li (\(^7\)Be). The absence of a stable nuclide at mass-5 ensures that the primordial abundance of \(^7\)Li is much smaller than that of the other light nuclides and the similar gap at mass-8 guarantees negligible primordial abundances for any heavier nuclei. Less than \(\sim 20\) minutes later, when the temperature in the expanding, cooling Universe has dropped to \(T \approx 0.03\) MeV, the coulomb barriers in collisions between charged nuclei, combined with the absence of free neutrons (most of which have, by now, been incorporated into \(^4\)He, the most tightly bound of the light nuclei), nuclear reactions end and the primordial abundances of the light nuclei are frozen out.

For nucleosynthesis in the standard cosmology (SBBN), there is only one adjustable parameter, the baryon density parameter \(\eta_B\) (or, \(\eta_{10}\)). Observations which lead to a determination of the primordial abundance of any one of the light nuclides can determine \(\eta_{10}\), which then may be compared with its value inferred from the CMB. The internal consistency of SBBN can be checked by comparing the abundances of the other nuclides, predicted using this value of \(\eta_{10}\), with the observationally-inferred abundances. However, in contrast to the other light nuclides, the BBN-predicted
primordial abundance of $^4\text{He}$ is very insensitive to the baryon density parameter. Rather, the $^4\text{He}$ mass fraction, $Y_p$, depends on the neutron-to-proton ratio at BBN since virtually all neutrons available at that time are incorporated into $^4\text{He}$. In turn, the $n/p$ ratio depends on the competition between the charged-current weak interaction rate (normalized, for example, by the accurately known neutron lifetime) and the universal expansion rate, parameterized by $S$ (or $N_\nu$). Therefore, while D, $^3\text{He}$, and $^7\text{Li}$ are potential baryometers, $^4\text{He}$ provides a potential chronometer.

2.1. Deuterium

Of the light nuclides produced in astrophysically interesting abundances, deuterium is the baryometer of choice. As the Universe evolves and gas is cycled through succeeding generations of stars, deuterium is only destroyed, so that its post-BBN evolution is simple and monotonic. The abundance of deuterium observed anywhere, at any time in the evolution of the Universe, is never greater than its BBN abundance. If D is observed in systems of low metallicity and/or or at high redshift, where very little gas has been cycled through stars, its observationally inferred abundance should approach its primordial value. Furthermore, the BBN-predicted D abundance is sensitive to the baryon density parameter ($y_{\text{DP}} \equiv 10^5(D/H)_P \propto \eta_{10}^{-1.6}$), so that a $\sim10\%$ determination of $y_{\text{DP}}$, would lead to a $\sim6\%$ estimate of $\eta_{10}$.

Finding and observing suitable targets to determine $y_{\text{DP}}$ is telescope-intensive and subject to systematic errors. As a result, at present, observations and reliable determinations of D/H are available for only 7, high-redshift, low-metallicity, QSO Absorption Line Systems, in which neutral D and H are observed in absorption against background light sources (QSOs). See Pettini et al. 2008 for the most recent results and references to earlier work. Unfortunately, given the errors quoted for the individual D/H determinations, the dispersion among their central values is excessive (e.g., the reduced $\chi^2$ is $\gtrsim 3$), suggesting that the errors may have been underestimated and/or systematic effects contaminate one or more of the determinations. Until this is resolved with more and/or better data, I adopt for $y_{\text{DP}}$ the weighted mean of the individual D/H determinations, but inflate the errors to reflect the large dispersion (by multiplying them by the square root of the reduced $\chi^2$). The result is that $y_{\text{DP}} = 2.70^{+0.22}_{-0.20}$. For SBBN ($N_\nu = 3$), this implies that $\eta_{10} = 5.96^{+0.30}_{-0.33}$, a $\sim5\%$ determination of $\eta_{10}(\text{BBN})$. The SBBN D-inferred likelihood distribution of $\eta_{10}$ is shown as the dashed curve in Figure 1. Before comparing this SBBN-inferred value of the baryon density parameter, measured at $\sim20$ minutes, with the value inferred from the CMB at $\sim400$ thousand years, the self-consistency of SBBN is investigated.
The likelihood distributions of the baryon density parameter, $\eta_{10}$. The dashed (blue) curve shows the distribution inferred from SBBN ($N_\nu = 3$) and the adopted primordial abundance of deuterium. The solid (black) curve is the distribution of $\eta_{10}$ for $N_\nu = 3$ inferred from the CMB using the WMAP 5-year and small scale CMB data, the LSS matter power spectrum and SNIa data, and the HST Key Project value of $H_0$.

considering the relic abundances of $^3$He, $^4$He and $^7$Li.

2.2. Helium-3

For the D-inferred value of of $\eta_{10}$, the SBBN-predicted relic abundance of $^3$He is $y_{3P} \equiv 10^5 (^3\text{He}/\text{H})_P = 1.1 \pm 0.2$. The post-BBN evolution of $^3$He is much more complicated – and model dependent – than that of deuterium (or, of $^4$He; see below). When gas is cycled through stars D is first burned to $^3$He. In the cooler, outer layers of stars the prestellar D plus $^3$He is preserved. However, in the hotter stellar interiors $^3$He is burned away. In general, the more abundant, cooler, lower mass stars are net producers of $^3$He, but it is unclear how much of this stellar-synthesized $^3$He is actually returned to the interstellar medium (ISM). For further details and references see my review article\(^[1]\). The bottom line is that it is very difficult to use observations of $^3$He in chemically-evolved regions, such as the ISM of our Galaxy, to infer the relic abundance of $^3$He. Nonetheless, Bania, Rood & Balser\(^[7]\) adopt for an upper limit to the $^3$He primordial abundance, the abundance inferred from observations of the most distant (from the Galactic center), most metal poor (least chemically evolved?), Galactic H\(\,\uparrow\) region, $y_{3P} \lesssim 1.1 \pm 0.2$. This is in excellent agreement (within
the model-dependent uncertainties) with the SBBN prediction.

2.3. Helium-4

The SBBN-predicted $^4$He primordial mass fraction for the D-inferred value of $\eta_{10}$ is $Y_P = 0.2484 \pm 0.0007$. As gas is cycled through stars the post-BBN evolution of $^4$He is monotonic – hydrogen is burned to helium, increasing $Y$ (along with the abundances of the heavier elements such as, e.g., oxygen). For observations of gas anywhere, at any time in the post-BBN evolution of the Universe, $Y \gtrsim Y_P$. Since the BBN-predicted $^4$He relic mass fraction is very insensitive to the baryon density parameter, $^4$He is a poor baryometer.

In my opinion the observationally-determined value of $Y_P$ (and its error) is currently unresolved. The most recent analyses\textsuperscript{8,9,10}, using selected subsets of the available data, fail to find evidence for the expected correlation between the helium and oxygen abundances, calling into question the model-dependent extrapolations to zero metallicity they employ to infer the primordial abundance of helium. Here, the value suggested by me in my review\textsuperscript{1}\textsuperscript{1} is chosen for the subsequent discussion,

$$Y_P = 0.240 \pm 0.006. \quad (8)$$

The adopted error is an attempt to account for the systematic, as well as the statistical, uncertainties. While the central value of $Y_P$ adopted here is low, it is only slightly more than 1$\sigma$ below the SBBN-predicted central value; within $\sim 1.4\sigma$, the observations and predictions agree.

The recent data and analyses\textsuperscript{8,9,10} are, however, in agreement on a weighted mean of the post-BBN abundance, which can be used to provide an upper bound to $Y_P$. To this end an alternate constraint on (upper bound to) $Y_P < 0.251 \pm 0.002$ will also be used. At $\sim 2\sigma$, this suggests that $Y_P < 0.255$. This upper bound to $Y_P$ is entirely consistent with the SBBN-predicted abundance.

2.4. Lithium-7

For the D-inferred value of the baryon density parameter, the SBBN-predicted primordial abundance of lithium is $[\text{Li}]_P \equiv 12 + \log(Li/H)_P = 2.63^{+0.06}_{-0.07}$. Although the weakly-bound lithium nucleus is easily destroyed in the hot interiors of stars, theoretical expectations, supported by observational data, suggest that the overall post-BBN trend has been for lithium to increase its abundance in the Galaxy with time, and with increasing metallicity. The key data for probing the BBN $^7$Li yield are from observations of the surface abundances of the oldest, most metal-poor stars in the Galaxy. These lithium abundances are expected to form a plateau, the so-called “Spite plateau”, in a plot of $\text{Li}/H$ (or, of $[\text{Li}]$) versus metallicity. The surprise from recent observational data\textsuperscript{11,12} is the apparent absence of evidence for a lithium
plateau; the lithium abundance continues to decrease with decreasing metallicity. It is difficult to know how to use these data to infer the primordial abundance of $^7\text{Li}$. The lowest metallicity data suggest that $[\text{Li}]_p \lesssim 2.1 \pm 0.1$. There is clearly tension between this estimate (upper bound?) and the SBBN-predicted relic abundance which differ by a factor of three or more. The resolution of this conflict may be found in the stellar astrophysics: in the $\sim 13$ Gyr lifetimes of the oldest stars in the Galaxy where lithium is observed, the surface abundances of lithium (and other elements) may have been transformed by mixing surface material with that from the hotter interior where lithium has been destroyed. In support of this possibility, recent observations and stellar modeling by Korn et al.\textsuperscript{13} are of interest. They observe stars in a Globular Cluster which are of the same age and were born with the same metallicity. Comparing their observations of lithium and of heavier elements to models of stellar diffusion, they find evidence that both lithium and the heavier elements may have settled out of the atmospheres of these stars. Applying their stellar models to the data for this set of stars, they infer for the unevolved lithium abundance, $[\text{Li}] = 2.54 \pm 0.10$, in excellent agreement with the SBBN prediction. Before this can be claimed as the resolution of the conflict, more data on stars in other Globular Clusters is required. For a discussion of the proposed astrophysical solutions to this problem and further references, see\textsuperscript{11}.

2.5. BBN Summary

For SBBN, the only adjustable parameter is the baryon density parameter. Using deuterium, the baryometer of choice, to fix this parameter it is found (§2.1) that

$$10^{10} \eta_B(\text{SBBN}) = \eta_{10}(\text{SBBN}) = 5.96^{+0.30}_{-0.33}; \quad \Omega_B h^2(\text{SBBN}) = 0.0218^{+0.0011}_{-0.0012}. \quad (9)$$

This value of $\eta_{10}$ is then used to find the BBN-predicted abundances of the other light nuclides, $^3\text{He}$, $^4\text{He}$, and $^7\text{Li}$. When the predictions are compared to the observations, there is excellent agreement for $^3\text{He}$ (within very large uncertainties). The predicted and observationally-inferred primordial abundances of $^4\text{He}$ agree at a level of $\sim 1.4\sigma$. Lithium, however, poses a challenge; the predicted abundance exceeds the observationally suggested value by a factor of three or more. We’ll return to this problem below, after discussing the effect of a non-standard expansion rate on BBN and the CMB.

3. CMB Constraint On the Baryon Density Parameter

The relative heights of the odd and even “acoustic” peaks in the CMB temperature anisotropy spectrum are sensitive to the baryon density parameter. The CMB is a baryometer for the $\sim 400$ thousand year old Universe. The CMB determination of $\eta_B$ is largely uncorrelated with $S$ (or, $N_\nu$), so that the value determined for $N_\nu = 3$ is
consistent with that for $N_\nu \neq 3$. In our recent analysis, V. Simha and I, combining the WMAP 5-year data\(^{14}\), with other CMB experiment\(^{15}\), supplemented by LSS\(^{16}\) and SNIa\(^{17}\) data, and the HST determination of the Hubble parameter\(^{18}\), needed to break cosmological degeneracies, found\(^{2}\),

$$10^{10} \eta_B(\text{CMB}) = \eta_{10}(\text{CMB}) = 6.14^{+0.16}_{-0.11} ; \quad \Omega_B h^2(\text{CMB}) = 0.0224^{+0.0006}_{-0.0004}. \quad (10)$$

The CMB/LSS data determine the baryon density parameter (at $\sim 400$ thousand years) to an accuracy of $\lesssim 3\%$. The excellent agreement between the BBN- and CMB-inferred values of the baryon density parameter is displayed in Figure\(^{11}\) where the solid curve shows the CMB-determined likelihood distribution (for further details and references, see\(^{2}\)). Below, the implications of this excellent agreement for some classes of models of non-standard physics is discussed. First, though, let’s explore the effect on BBN and the CMB of a non-standard, early-Universe expansion rate.

4. Non-Standard Particle Content ($N_\nu \neq 3$) Or Expansion Rate ($S \neq 1$)?

If the assumption of the standard model expansion rate is relaxed, both BBN and the CMB are affected. For BBN, the largest effect is on the relic abundance of $^4\text{He}$, due to its sensitivity to the ratio of neutrons to protons at BBN, since virtually all neutrons available are incorporated into $^4\text{He}$, so that $Y_P \approx [2n/(n+p)]_{\text{BBN}}$. In the early Universe, prior to BBN, when the temperature drops below $\sim 0.8$ MeV, the rate of the charged-current weak interactions regulating the n/p ratio becomes smaller than the universal expansion rate, the Hubble parameter $H$. Thereafter, the n/p ratio deviates from (exceeds) its equilibrium value of $n/p = \exp(-\Delta m/kT)$, where $\Delta m$ is the neutron-proton mass difference. The value of n/p when BBN begins is therefore determined by the competition between the weak interaction rate and $H$. An increase in the expansion rate ($S \geq 1$; $\Delta N_\nu \geq 0$) leaves less time for neutrons to convert to protons, increasing the predicted $^4\text{He}$ primordial abundance. According to Kneller & Steigman\(^{19}\), a very good approximation to $Y_P$ in this case, updated to account for incomplete neutrino decoupling\(^{5}\), is\(^{12}\)

$$Y_P = 0.2485 \pm 0.0006 + 0.0016[(\eta_{10} - 6) + 100(S - 1)]. \quad (11)$$

In contrast, the BBN-predicted abundances of the other light elements are less sensitive to $S$. For example, for deuterium\(^{19}\),

$$y_D = 2.64(1 \pm 0.03) \left( \frac{6}{\eta_{10} - 6(S - 1)} \right)^{1.6}. \quad (12)$$

For the ranges in $\eta_{10}$ and $S$ where these fits are accurate (within the quoted errors), $4 \lesssim \eta_{10} \lesssim 8$ and $0.8 \lesssim S \lesssim 1.1$ ($0.8 \lesssim N_\nu \lesssim 4.3$), the $y_D$ and $Y_P$ isobundance contours in the $S - \eta_{10}$ plane are nearly orthogonal, so that the observationally-inferred primordial D and $^4\text{He}$ abundances serve to determine the $\{S, \eta_{10}\}$ pair. In
The 68% and 95% contours in the $N_\nu - \eta_{10}$ plane inferred from BBN using the D and $^4$He abundances adopted in §2.1 and §2.3 respectively. The “x” marks the “best fit” point at $\eta_{10} = 5.7$ and $N_\nu = 2.4$. The shaded area corresponds to the combination of $N_\nu$ and $\eta_{10}$ leading to a BBN-predicted $^4$He mass fraction in excess of the observationally-inferred upper bound of 0.255; see §2.3.

Figure 2 are shown the 68% and 95% contours in the $N_\nu - \eta_{10}$ plane inferred from BBN and the D and $^4$He abundances adopted above.

For BBN constrained by D and $^4$He, Simha and Steigman\cite{Simha99} find,

$$\eta_{10}(\text{BBN}) = 5.7 \pm 0.4, \quad N_\nu(\text{BBN}) = 2.4 \pm 0.4.$$  \hspace{1cm} (13)

Although the “best fit” value for $N_\nu$ is less than the standard model value of $N_\nu = 3$, BBN and the D and $^4$He abundances are in agreement with the standard model at the $\sim$68% confidence level. While this leaves the excellent agreement with $^3$He unchanged, it fails to resolve the “lithium problem”\cite{Simha99}.

The CMB temperature anisotropy power spectrum is affected by the radiation density primarily through the effect of $\rho_R$ in determining the epoch (redshift) of matter-radiation equality. The amplitudes of fluctuations on scales which enter the horizon when the Universe is still radiation dominated differ from the amplitudes on
those scales which enter the horizon later, when the Universe is matter dominated. Increasing the radiation content \((N_{\nu} \geq 3)\) delays matter-radiation equality, leaving less time before recombination, suppressing the growth of perturbations. The redshift of the epoch of matter-radiation equality, \(z_{\text{eq}}\), is related to the matter (baryonic plus non-baryonic) and radiation densities by,

\[
1 + z_{\text{eq}} = \frac{\rho_{\text{M}}}{\rho_{\text{R}}}.
\]  

(14)

Since \(\rho_{\text{R}}\) depends on \(N_{\nu}\), \(z_{\text{eq}}\) is a function of both \(N_{\nu}\) and \(\Omega_{\text{M}}h^{2}\), leading to a degeneracy between these two parameters. The CMB constrains \(z_{\text{eq}}\), so that any increase in \(N_{\nu}\) needs to be compensated by a corresponding increase in \(\Omega_{\text{M}}h^{2}\). As a result of this degeneracy, the CMB power spectrum alone can only lead to a very weak constraint on \(N_{\nu}\). Constraints on these parameters independent of the CMB are needed to break the degeneracy between them.

Since the luminosity distances of type Ia supernovae (SNIa)\(^{17}\) provide a constraint on a combination of \(\Omega_{\text{M}}\) and \(\Omega_{\Lambda}\) complementary to that from the assumption of flatness, they are of value in restricting the allowed values of \(\Omega_{\text{M}}\). In concert with a bound on \(H_{0}\)\(^{18}\), this helps to break the degeneracy between \(N_{\nu}\) and \(\Omega_{\text{M}}h^{2}\).

Another way to break the degeneracy between \(N_{\nu}\) and \(\Omega_{\text{M}}h^{2}\) is to use LSS measurements of the matter power spectrum in combination with the CMB power spectrum. The turnover scale in the matter power spectrum is set by the requirement that \(z_{\text{eq}}\) remain unchanged as \(N_{\nu}\) deviates from its standard-model value of 3. Since the baryon density is constrained by the CMB power spectrum, independently of \(N_{\nu}\), increasing the radiation density \((N_{\nu} > 3)\) requires a higher dark matter density in order to preserve \(z_{\text{eq}}\) (in a flat universe, \(\Omega_{\text{M}} + \Omega_{\Lambda} = 1\)). Between the epoch of matter-radiation equality and recombination the density contrast in the cold dark matter grows unimpeded, while the baryon density contrast cannot grow. Consequently, increasing \(N_{\nu}\) and \(\Omega_{\text{M}}h^{2}\) increases the amplitude of the matter power spectrum on scales smaller than the turnover scale corresponding to the size of the horizon at \(z_{\text{eq}}\). In this way, data from galaxy redshift surveys\(^{16}\) can be used to infer the matter power spectrum, thereby constraining \(\Omega_{\text{M}}h^{2}\) and \(N_{\nu}\). For these complementary datasets, Simha & Steigman\(^2\) obtain,

\[
\eta_{10}(\text{CMB}) = 6.1^{+0.2+0.3}_{-0.1-0.2},
\]  

(15)

and

\[
N_{\nu}(\text{CMB}) = 2.9^{+1.0+2.0}_{-0.8-1.4}.
\]  

(16)

The 68% and 95% contours in the \(N_{\nu} - \eta_{10}\) plane are shown in Figure\(^3\) where it is clear that, so far, the CMB is a better baryometer than a chronometer.

5. Comparing BBN With The CMB

At present the combined CMB and LSS data provide the best baryometer, determining the baryon density to better than 3%, but only a relatively weak chronome-
The 68% and 95% contours in the $N_\nu - \eta_{10}$ plane derived using the WMAP 5-year data, small scale CMB and SNIa data, and the HST Key Project prior on $H_0$ along with data from the LSS matter power spectrum (see the text).

After, still allowing a large range in $S$ ($0.87 \leq S \leq 1.14$ or, $1.5 \leq N_\nu \leq 4.9$ at 95% confidence). BBN ($D$ & $^4He$) provides a consistent, but tighter constraint on $S$ ($0.88 \leq S \leq 1.02$ or, $1.6 \leq N_\nu \leq 3.2$ at 95% confidence). As may be seen from the left hand panel of Figure 4, within the uncertainties, the CMB/LSS, which probes the Universe at $\gtrsim 400$ thousand years, is consistent with BBN, which provides a glimpse of the Universe at $\lesssim 20$ minutes. For example, the BBN abundances of $D$, $^3He$, and $^4He$, inferred using the CMB/LSS values of $\eta_{10}$ and $N_\nu$, are in excellent agreement, within the errors, with the observationally-inferred relic abundances. Lithium, however, remains a problem. The excellent agreement between BBN and the CMB permits constraints on any deviations from standard-model physics between BBN and recombination and/or the present epoch. For example, since baryons are conserved, $\eta_B$ relates the number of thermalized black body photons in a comoving volume at different epochs, constraining any post-BBN entropy production. The consistency of $\eta_B$ inferred from the CMB/LSS and from BBN implies\(^2\),

$$\frac{N_{\gamma}^{\text{CMB}}}{N_{\gamma}^{\text{BBN}}} = 0.92 \pm 0.07.$$  \hfill (17)

This ratio is consistent with no change in entropy between BBN and recombination at $\sim 1\sigma$, placing an interesting upper bound on any post-BBN entropy production.

For another example, late decaying particles might produce additional relativistic
Figure 4: (Left) In blue (solid), the 68% and 95% contours in the $N_\nu - \eta_{10}$ plane derived from a comparison of the observationally-inferred and BBN-predicted primordial abundances of D and $^4$He. In red (dashed), the 68% and 95% contours derived from the combined WMAP 5-year data, small scale CMB data, SNIa, and the HST Key Project prior on $H_0$ along with the LSS matter power spectrum data. (Right) The 68% and 95% joint BBN-CMB-LSS contours in the $N_\nu - \eta_{10}$ plane.

particles (radiation), but not thermalized, black body photons. Such deviations from the standard model radiation density can be parameterized by the ratio of the radiation density, $\rho'_R$, to the standard model radiation density, $\rho_R$ (or, to the photon energy density $\rho_\gamma$). In the post-$e^\pm$ annihilation universe relevant for this comparison of BBN with the CMB,

$$R = S^2 = \frac{\rho'_R}{\rho_R} = 1 + 0.134\Delta N_\nu.$$  \hspace{1cm} (18)

Comparing this ratio ($R$) at BBN and at recombination constrains any post-BBN production of relativistic particles.

$$\frac{R_{\text{CMB}}}{R_{\text{BBN}}} = 1.07^{+0.16}_{-0.13} \hspace{1cm} (19)$$

This ratio, too, is consistent with unity within 1\sigma, constraining any post-BBN relativistic particle production.

While a non-standard expansion rate ($S \neq 1$) has been parameterized in terms of an equivalent number of neutrinos ($N_\nu \neq 3$), we reiterate that a non-standard expansion rate need not be due to the presence of extra (or fewer) neutrinos. For example, deviations from the standard expansion rate could occur if the value of the early-Universe gravitational constant, $G$, were different in the from its present,
locally-measured value $G_0$. For the standard radiation density with three species of light, active neutrinos, the constraint on the expansion rate can be used to constrain variations in the gravitational constant, $G$. From BBN,

$$G_{\text{BBN}}/G_0 = S_{\text{BBN}}^2 = 0.91 \pm 0.07$$

and at the epoch of the recombination,

$$G_{\text{CMB}}/G_0 = S_{\text{CMB}}^2 = 0.99^{+0.13}_{-0.11}.$$  

Both are consistent with no variation in $G$ at the $\sim 1\sigma$ level.

Since the independent constraints on $\eta_B$ and $N_\nu$ from BBN and the CMB/LSS are in very good agreement, they may be combined to obtain the joint fit shown in the right hand panel of Figure 1. By comparing the two panels of Figure 4, it may be seen that the CMB/LSS drives the constraint on the baryon density parameter, while BBN is largely responsible for the bounds on $N_\nu$. At 68% and 95% confidence, Simha and Steigman find

$$\eta_{10} = 6.11^{+0.12+0.26}_{-0.13-0.27}; \quad \Omega_B h^2 = 0.0223^{+0.0004+0.0009}_{-0.0005-0.0010},$$

and

$$N_\nu = 2.5 \pm 0.4 \pm 0.7.$$  

While the best-fit value of $N_\nu$ is less than 3, this difference is not statistically significant.

6. Conclusions

In the standard models of cosmology and of particle physics the particle content is fixed (e.g., $N_\nu = 3$) and, baryon number has been conserved since the very earliest epochs. By comparing BBN with the CMB and LSS, baryon conservation can be tested between $\sim 20$ minutes (BBN) and $\sim 400$ thousand years. So, too, can deviations from the standard-model particle content ($N_\nu \neq 3$?), as well as deviations from the standard-model predicted early Universe expansion rate. In this talk and review, $\eta_B$ evaluated from BBN and from the CMB have been compared and shown to be in excellent agreement, consistent with the standard model expectation. BBN and the CMB have also been employed to compare $N_\nu$ (or, $S$) at these two, widely separated epochs and, to compare them to the standard-model expectation that $N_\nu = 3$ ($S = 1$). While the central values of $N_\nu$ evaluated at BBN and from the CMB differ from each other and from the standard-model value, they are, in fact, in agreement with each other, and with the standard-model value within the uncertainties. This concordance of the standard models of particle physics and cosmology permit constraints on “new” physics at the times (and energies) between BBN and the CMB (and, at present).
Prior to WMAP and the other ground- and balloon-based CMB experiments, BBN and deuterium provided the best cosmological baryometer, while $^4\text{He}$ provided the only early-Universe chronometer. The WMAP 5-year data, combined with other CMB and LSS data, now lead to a determination of the baryon density at the $\sim 2$-3% level, a factor of $\sim 2$ better than that from BBN. However, although the CMB/LSS constraint on $N_\nu$ has improved significantly and is consistent with that from BBN, it still remains weaker than the corresponding BBN constraint. For some time now BBN has established at high confidence that $N_\nu > 1$ at BBN, when the Universe was $\sim 20$ minutes old. The more recent CMB data, combined with other SNIa and LSS datasets, now confirm that $N_\nu > 1$ (or, $N_\nu > 2^{[21]}$) when the Universe was $\gtrsim 400$ kyr old.

It may be of interest to some readers that, since the NO-VE IV Workshop, V. Simha and I have presented the results of a similar analysis, updating the constraints from BBN and the CMB on the lepton asymmetry parameter.$^{[22]}$

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