Scalar mesons above and below 1 GeV

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Abstract

We show that two nonets and a glueball provide a consistent description of data on scalar mesons below 1.7 GeV. Above 1 GeV the states form a conventional $q\bar{q}$ nonet mixed with the glueball of lattice QCD. Below 1 GeV the states also form a nonet, as implied by the attractive forces of QCD, but of more complicated nature. Near the center they are $(q\bar{q})_3(q\bar{q})_3$ in S-wave, with some $q\bar{q}$ in P-wave, but further out they rearrange as $(q\bar{q})_1(q\bar{q})_1$ and finally as meson-meson states. A simple effective chiral model for such a system with two scalar nonets can be made involving two coupled linear sigma models. One of these could be looked upon as the Higgs sector of nonperturbative QCD.

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1 The Enigmatic Scalar Mesons

In the heavy flavour sector there are clearly established scalar mesons $c\bar{c}$ and $b\bar{b}$. They behave as canonical $^3P_0$ states which partner $^3P_{1,2}$ siblings. Their production (e.g. in radiative transitions from $^2S_1$ states) and decays (into $^1S_1$ or light hadrons) are all in accord with this. There is nothing to suggest that there is anything “exotic” about such scalar mesons.

For light flavours too there are clearly identified $^3P_{1,2}$ nonets which call for analogous $^3P_0$ siblings. However, while all other $J^{PC}$ combinations appear to be realised as expected, (apart from well known and understood anomalies in the $0^{-+}$ pseudoscalars), the light scalars empirically stand out as singular.

The interpretation of the nature of the lightest scalar mesons has been controversial for over thirty years. There is still no general agreement on where are the $q\bar{q}$ states, whether there is necessarily a glueball among the light scalars, and whether some of the too numerous scalars are multiquark, $K\bar{K}$ or other meson-meson bound states. These are fundamental questions of great importance in particle physics. The mesons with vacuum quantum numbers are known to be crucial for a full understanding of the symmetry breaking mechanisms in QCD, and presumably also for confinement.

In this paper we propose a resolution of the enigma of the scalar mesons.

Theory and data are now converging that QCD forces are at work but with different dynamics dominating below and above 1 GeV mass. The experimental proliferation of light scalar mesons is consistent with two nonets, one in the 1 GeV region (a meson-meson nonet) and another one near 1.5 GeV (a $q\bar{q}$ nonet), with evidence for glueball degrees of freedom. At the constituent level these arise naturally from the attractive interquark forces of QCD. Below 1 GeV these give a strong attraction between pairs of quarks and antiquarks in S-wave leading to a nonet which is “inverted” relative to the ideal nonets of the simple $q\bar{q}$ model. Conversely, above 1 GeV the states seeded by $^3P_0$ $q\bar{q}$ are present. The scalar glueball, predicted by lattice QCD in the quenched approximation, causes mixing among these states.

The phenomenon of multiplet doubling now requires a new effective model
for the light scalar spectrum. One possibility [1] is that two coupled linear sigma models may provide a first step for understanding such a proliferation of light scalar states. After gauging the overall symmetry one could then look at the lightest scalars as Higgs-like bosons for the nonperturbative low energy strong interactions.

Our main intuition on the strong interaction limit of QCD derives from Lattice QCD. This impacts on scalars in three ways:

(i) it gives a linear potential for $q\bar{q}$ systems [2] which implies a nonet of scalars in the 1.2-1.6 GeV region;

(ii) the lightest glueball, in the quenched approximation, is a scalar with mass $\sim 1.6$ GeV [3, 4, 5, 6, 7];

(iii) a strong attraction between $(qq)_{3}$ and $(\bar{q}\bar{q})_{3}$ in S-wave flavour nonet manifested below 1 GeV [8, 9, 10].

We shall argue that this guide enables us to decipher the scalar data. We shall summarise the phenomenology that appears to be consistent with this scenario and propose experimental tests.

We now expand on the three points above.

(i) The predicted linear potential is well established for heavy flavours, where data confirm it, and where “canonical” scalar $Q\bar{Q}$ are seen, as already mentioned. We are given a gift of Nature in that as one comes from heavy to light flavours, the linear potential continues phenomenologically to underpin the data: the S-P-D gaps are similar for $b\bar{b}$, $c\bar{c}$, and even $u\bar{d}$ as can be verified by looking in the PDG tables [11, 12]. Even though we have no fundamental understanding of why this is, we can nonetheless accept the gift and be confident that we can assign light flavoured mesons of given $J^{PC}$ to the required “slot” in the spectrum [13]. The resulting pattern leads one to expect that the lightest $J^{PC} 3P_{0}$ $q\bar{q}$ nonet should occur in the region above 1 GeV.

There is empirically a nonet (at least) in this region above 1 GeV and the identification of the $3P_{0}$ $q\bar{q}$ nonet should be apparent: there are candidates
in \( a_0(\sim 1450); f_0(1370); K(1430); f_0(1500) \) and \( f_0(1710) \). One immediately notes that if all these states are real there is an excess, precisely as would be expected if the glueball predicted by the lattice is mixing in this region.

(ii) Lattice QCD predictions for the mass of the lightest (scalar) glue ball are now mature. In the quenched approximation the mass is \( \sim 1.6 \text{ GeV} \). Flux tube models imply that if there is a \( q\bar{q} \) nonet nearby, with the same \( J^{PC} \) as the glueball, then \( G - q\bar{q} \) mixing will dominate the decay. This is found more generally and recent studies on coarse-grained lattices appear to confirm that there is indeed significant mixing between \( G \) and \( q\bar{q} \) together with associated mass shifts, at least for the scalar sector.

Furthermore the maturity of the \( q\bar{q} \) spectrum tells us that we anticipate the \( 0^{++} q\bar{q} \) nonet to occur in the 1.2 to 1.6 GeV region. Any such states will have widths and so will mix with a scalar glueball in the same mass range. It turns out that such mixing will lead to three physical isoscalar states with rather characteristic flavour content. Specifically: two will have the \( n\bar{n} \) and \( s\bar{s} \) in phase ("singlet tendency"), their mixings with the glueball having opposite relative phases; the third state will have the \( n\bar{n} \) and \( s\bar{s} \) out of phase ("octet tendency") with the glueball tending to decouple in the limit of infinite mixing. There are now clear sightings of prominent scalar resonances \( f_0(1500) \) and \( f_0(1710) \) and, probably also, \( f_0(1370) \). (Confirming the resonant status of the latter is one of the critical pieces needed to clinch the proof - see ref. and later). The production and decays of these states are in remarkable agreement with this flavour scenario.

A major question is whether the effects of the glueball are localised in this region above 1 GeV, as discussed by ref. or spread over a wide range, perhaps down to the \( \pi\pi \) threshold. This is the phenomenology frontier. There are also two particular experimental issues that need to be settled: (i) confirm the existence of \( a_0(1450) \) and determine its mass; (ii) is the \( f_0(1370) \) truly resonant or is it a \( t - \)channel exchange phenomenon associated with \( \rho \bar{\rho} \). We return to these resonances above 1 GeV later in the text.

Precision data on scalar meson production and decay are consistent with this and the challenge now centres on clarifying the details and extent of such mixing.
Were this the whole story on the scalar sector there would be no doubt that the glueball has revealed itself. However, there are features of the scalars in the region from two pion threshold up to \( O(1) \) GeV that have clouded the issue, in particular the existence and nature of the \( f_0(980) \) and \( a_0(980) \) mesons, and possibly a \( \sigma \) and a \( \kappa \) below 1 GeV. (While the \( \sigma \) is claimed in recent data, there are conflicting conclusions about the existence of the \( \kappa \) both in experiments\(^\text{[21]}\); and in phenomenology\(^\text{[22, 23]}\).)

There has been considerable recent progress here that enable a consistent picture to be proposed. We set the scene for this and return later to the experimental challenges.

(iii) QCD predicts that there is a strong attraction in S-wave between \((qq)^3\) and \((q\bar{q})^3\) (where superscripts denote their colour states) when in a flavour nonet\(^\text{[9, 10]}\). In addition long experience from meson-meson scattering has shown that at low energies there is attraction only in channels with nonexotic flavour quantum numbers, i.e. for flavour octets and singlets. This empirical attraction between such mesons matches that of the strong attraction between their (colour-rearranged) \((qq)^3(q\bar{q})^3\)

Thus as far as the quantum numbers are concerned these \((qq)^3(q\bar{q})^3\) states will be like two \( 0^{-+} \) \((q\bar{q})^1(q\bar{q})^1\) mesons in S-wave. In the latter spirit, Isgur and Weinstein\(^\text{[24]}\) had noticed that they could motivate an attraction among such mesons, to the extent that the \( f_0(980) \) and \( a_0(980) \) could even be interpreted as \( K\bar{K} \) molecules.

Thus the general conclusion from theory is that if there are resonances in addition to \( q\bar{q} \), one should expect them to form a nonet also, and that a meson-meson component can be substantial in their wave function. Thus we anticipate that below 1 GeV a nonet can occur with a compact \((qq)^3(q\bar{q})^3\) and a long range meson-meson \(((qq)^1(q\bar{q})^1)\) tail.

The relationship between these is being debated \(^\text{[25, 26, 27, 28, 29]}\), but while the details remain to be settled, there is a rather compelling message of the data as follows. Below 1 GeV the phenomena point clearly towards an S-wave attraction among two quarks and two antiquarks (either as \((qq)^3(q\bar{q})^3\), or \((q\bar{q})^1(q\bar{q})^1\)), while above 1 GeV it is the P-wave \( q\bar{q} \) that is manifested. There is a critical distinction between them: the “ideal” flavour pattern of
a $q\bar{q}$ nonet on the one hand, and of a $qq\bar{q}\bar{q}$ or meson-meson nonet on the other, are radically different; in effect they are flavoured inversions of one another. Thus whereas the former has a single $s\bar{s}$ heaviest, with strange in the middle and and $I=0$; $I=1$ set lightest (”$\phi; K; \omega, \rho$-like”), the latter has the $I=0$; $I=1$ set heaviest ($KK; \pi\eta$ or $s\bar{s}(u\bar{u} \pm d\bar{d})$) with strange in the middle and an isolated $I=0$ lightest ($\pi\pi$ or $u\bar{u}d\bar{d}$)[8, 9, 24].

The phenomenology of the $0^{++}$ sector appears to exhibit both of these patterns with $\sim 1$GeV being the critical threshold. Below 1 GeV the inverted structure of the four quark dynamics in S-wave is revealed with $f_0(980); a_0(980); \kappa$ and $\sigma$ as the labels. One can debate whether these are truly resonant or instead are the effects of attractive long-range $t$–channel dynamics between the colour singlet $0^{-+} K\bar{K}; K\pi; \pi\pi$, i.e., whether they are meson-meson molecules or $qq\bar{q}\bar{q}$. But the systematics of the underlying dynamics seems clear.

The phenomena are consistent with a strong attraction of QCD in the scalar S-wave nonet channels. The difference between molecules and compact $qq\bar{q}\bar{q}$ will be revealed (provided phase space effects are removed) in the tendency for the former to decay into a single dominant channel - the molecular constituents - while the latter will feed a range of channels driven by the flavour spin clebsch gordans. For the light scalars it has its analogue in the production characteristics.

The picture that is now emerging from both phenomenology[30, 31, 32] and theory[33] is that both components are present. As concerns the theory[33], think for example of the two component picture as two channels. One, the quarkish channel ($QQ$) is somehow associated with the ($qq)^3(q\bar{q})^3$ coupling of a two quark-two antiquark system, and is where the attraction comes from. The other, the meson-meson channel ($MM$) could even be completely passive (eg, no potential at all). There is some off diagonal potential which flips that system from the $QQ$ channel to $MM$. The way the object appears to experiment depends on the strength of the attraction in the $QQ$ channel and the strength of the off-diagonal potential. The nearness of the $f_0$ and $a_0$ to $KK$ threshold suggests that the $QQ$ component cannot be too dominant, but the fact that there is an attraction at all means that the $QQ$ component cannot be negligible. So in this line of argument, $a_0$ and $f_0$ must be
superpositions of four-quark states and $K\bar{K}$ molecules.

Continuing this argument to include a coupling to $q\bar{q}$ they can be superpositions of all 3 configurations ($q\bar{q}$, 4-quark, and meson-meson). However, for a scalar the $q\bar{q}$ system is in P-wave and naturally higher in energy. The $qq\bar{q}\bar{q}$ is strongly attracted in S-wave with no angular momentum barrier. Thus in a spatial picture one expects the four-quark S-wave state to form the core, while the outer regime (which extends to distances inversely proportional to $2m_K - m_{res}$) is composed of $K\bar{K}$, with any residual $q\bar{q}$ P-wave at intermediate range.

2 Heavy Flavours and Light Scalars

The working hypothesis is that the $q\bar{q}$ nonet, mixed with a glueball, is realised above 1 GeV and we now need to determine the flavour content of these states. In addition we need to confirm the picture of the $f_0(980)$ and light scalars below 1 GeV. New opportunities for improving data are coming from heavy flavour decays, in particular $D_s$ decays.

2.1 Scalars $\geq 1$ GeV

The decay $\psi \rightarrow \gamma\pi\pi$ compared with $D_s \rightarrow \pi\pi\pi$, and $\psi \rightarrow \gamma K\bar{K}$ compared with $D_s \rightarrow \pi K\bar{K}$ provide complementary entrees into the light flavoured $0^{++}$ mesons. Comparison with $D_s \rightarrow \pi K^*_0(1430)$ then enables us to "weigh" the flavour content of the nonets. In $\psi \rightarrow \gamma K\bar{K}$ Dunwoodie\(^\text{[35]}\) finds the $f_0(1710)$ as clear scalar, and this state could be sought in $D_s \rightarrow \pi K\bar{K}$ with enough statistics (in E687\(^\text{[36]}\) the $K^*\bar{K}$ band contaminates the 1710 region of the Dalitz plot). This could be a challenge for high statistics data e.g. with FOCUS. The major signal in the E687 data is the $\phi$; the $f_0(980)$ is just below threshold and it is not discussed whether any of the signal at threshold is due to this state. However, in the E791 data\(^\text{[37]}\) on $D_s \rightarrow \pi\pi\pi$ the $f_0(980)$ is very prominent (see next subsection), together with the $f_0(1370)$ and a possible (though unclaimed) hint of a shoulder that could
signal the \( f_0(1500) \). Dunwoodie’s analysis of \( \psi \to \gamma \pi\pi \) shows structure around 1400 MeV and with better statistics from BES and Cornell this should be verified and attempts made to resolve it into \( f_0(1370) \) and \( f_0(1500) \). The strength of \( f_0(1710) \) in these data should also be determined.

### 2.2 Scalars below 1 GeV

In the 22 different analyses on the \( \sigma \) pole position, which are included in the 2000 edition of the Review of Particle Physics \(^{38}\) under the entry \( f_0(400 – 1200) \) or \( \sigma \), most find a \( \sigma \) pole position near 500-i250 MeV. Also, at a recent meeting in Kyoto \(^{39}\) devoted to the \( \sigma \), many groups reported preliminary analyses, which find \( \sigma \) resonance parameters in the same region. Furthermore as we discuss in more detail below the \( \sigma \) has been claimed in \( D \to 3\pi \) and in \( \tau \) decay. There is also a very clear, although still preliminary, signal for a light \( \sigma \) (Breit-Wigner mass=390\(^{+60}_{-36}\) and width= 282\(^{+77}_{-50}\)) in a BES experiment\(^{40}\) on \( J/\psi \to \sigma \omega \to \pi \pi \omega \).

Barnes\(^{41}\) has warned that it may be premature to infer resonance parameters from such data. Only the low energy tails of the purported resonance phase shifts are actually in evidence in the charm data and crucial observation of a complete Breit-Wigner phase motion through 180\(^\circ\) has not been made. Especially the elastic \( K\pi \) phase shift as measured by LASS\(^{12}\) (but also the \( \pi\pi \) phase shift in many models) does not show evidence of a “complete” low mass scalar resonance. Therefore concluding that these resonances exist based on the charm data in isolation, which only covers part of the range of invariant mass that has already been studied in light hadronic processes, is unjustified. Barnes recommends that the charm decay analyses should include what is already known about phase shift analyses over the full mass range, e.g. from ref\(^{43}\).

In this connection it should be remembered that Breit-Wigner parameters and pole positions, can differ by several 100 MeV for the same data. (Also mass parameters which appear in phenomenological Lagrangian similarly differ considerably from these masses.) Therefore the uncertainty in mass determinations of a broad resonance like the \( \sigma \), or the \( \kappa \) is not only due to experimental uncertainties, but also depend on the definitions of mass used.
An interesting piece of data also comes from the CLEO analysis of $\tau \rightarrow a_1 \nu \rightarrow \sigma \pi \nu \rightarrow 3\pi \nu$ [14], which finds a $\sigma$ BW mass of approximately 555 MeV and a width of 540 MeV. Perhaps more importantly, their branching ratio for $a_1$ of $\Gamma_{\sigma \pi}/\Gamma_{\text{tot}} = .16$ should make S. Weinberg happy, since he urged people to look for this decay mode, and predicted a 50 MeV partial decay mode for $a_1 \rightarrow \sigma \pi$ in his “mended symmetry” paper of 1990 [15], at a time when the $a_1 \rightarrow \pi(\pi \pi)_{s\text{-wave}}$ was quoted to be essentially absent (0.7%). Although $\Gamma_{\text{tot}}(a_1)$ is very uncertain, 250-600 MeV, this agrees well with Weinberg’s estimate.

The recent experiments studying charm decay to light hadrons are opening up a new experimental window for understanding light meson spectroscopy and especially the controversial scalar mesons, which are copiously produced in these decays. We therefore discuss in more detail the recently measured $D \rightarrow \sigma \pi \rightarrow 3\pi$ and $D_s \rightarrow f_0(980)\pi \rightarrow 3\pi$ decays, where the $\sigma$, respectively the $f_0(980)$, is clearly seen as the dominant peak, and point out that these decays rates can be understood in a rather general model for the weak matrix elements. This indicates that the broad $\sigma(600)$ and the $f_0(980)$ belong to the same multiplet.

In particular we refer to the E791 study of the $D \rightarrow 3\pi$ decay [37] where it is shown how adding an intermediate S-wave structure with floating mass and width in the Monte Carlo program simulating the Dalitz plot densities, allows for an excellent fit to data provided the mass and the width of this scalar are $m_\sigma \simeq 478$ MeV and $\Gamma_\sigma \simeq 324$ MeV. In fact 46% of the $D^+ \rightarrow 3\pi$ Dalitz plot was explained by their $\sigma\pi$.

In a calculation by Gatto et al. [46] this hypothesis was checked adopting the E791 experimental values for its mass and width and using a Constituent Quark Meson Model (CQM) for heavy-light meson decays [17]. The $D \rightarrow \sigma \pi$ non-leptonic process was computed, assuming factorization [18], and taking the coupling of the $\sigma$ to the light quarks from the linear sigma model [49]. In such a way one is directly assuming that the scalar state needed in the E791 analysis could be the quantum of the $\sigma$ field of the linear sigma model. According to the CQM model and to factorization, the amplitude describing the $D \rightarrow \sigma \pi$ decay can be written as a product of the semileptonic amplitude $\langle \sigma | A^{\mu}_{(\bar{d}c)}(q) | D^+ \rangle$, where $A^{\mu}$ is the axial quark current, and $\langle \pi | A^{\mu}_{(\bar{u}d)}(q) | \text{VAC} \rangle$. 

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This computation indicates that the low mass S-wave enhancement described in the E791 paper can be consistently understood as the $\sigma$ of the linear sigma model. In a rather similar approach Paver and Riazuddin[50] also reach the same conclusion.

These models[51, 52] were also used to predict the related process $D_s \rightarrow f_0(980)\pi \rightarrow 3\pi$. The agreement with the data, where the $f_0(980)$ is dominant in the Dalitz plot, indicates that the $\sigma$ and the $f_0(980)$ belong to a similar flavour multiplet. Also when comparing the predicted rates of $\sigma$ and $f_0(980)$ in these charm decays to those predicted for $\rho$ and $\phi(1020)$ using the same models one finds a good agreement with the data.

Note however as we discussed above[41] that it may be premature to infer broad $\sigma$ and $\kappa$ resonance parameters from such data. Only the low energy tails of the purported resonance phase shifts are actually in evidence in the charm data and crucial observation of a complete Breit-Wigner phase motion through 180° has not been made.

But, whatever the true nature of this structure, there is clear evidence for a strong attractive enhancement at low $m(\pi\pi)$ in S-wave which we can denote by $\sigma$. Its dynamical origin and parameters remain to be settled. Further theoretical studies are now needed to propose alternative explanations of the E791 data. These are required to enable useful comparison of points of view on the nature of the $\sigma$.

One clear message from the E791 data is that $f_0(980)$ has strong affinity for $s\bar{s}$ in its production at short distances. Then it evolves building up a substantial virtual $K\bar{K}$ component at larger distances, and decays violating the OZI rule in a two step process, into $\pi\pi$ which is the only open channel.

There is a large amount of data on the production of the $f_0(980)$ which require in some cases a strong affinity for $s\bar{s}$ (e.g. the $D_s$ decays mentioned above), or for $n\bar{n}$ (the production in hadronic $Z$ decays has all the characteristics associated with well established $n\bar{n}$ states[52, 53]) and also data that require both components to be present ($\psi \rightarrow \omega f_0$ versus $\psi \rightarrow \phi f_0$). There are also new data from central production and on $\phi \rightarrow \gamma f_0(a_0)$ that touch on the relationship between $f_0 - a_0$. We shall now discuss these.
3 The light Scalars $f_0/a_0(980)$: Phenomena and Theory

3.1 $f_0/a_0$ mixing in central production

Further evidence that the dynamics of $f_0 - a_0(980)$ are strongly influenced by the $K\bar{K}$ threshold (see [54] for an early theoretical discussion and estimate of this effect) is the presence of strong mixing and violation of isospin or G-parity for these states. A more detailed review of ideas on $\phi \to \gamma f_0/\gamma a_0$ and the implications of the mixing hypothesis on these data is in ref.[55]. Emerging data from DAΦNE are in remarkable agreement with these predictions and add weight to the idea that these scalar mesons are compact $qq\bar{q}\bar{q}$ states with an extended meson-meson cloud “molecular” tail.

The $x_F$ distribution for the $a_0^0(980)$ in $pp \to ppa_{0,2}$ is the only state with $I = 1$ that is observed to have a $x_F$ distribution peaked at zero [50], and moreover the distribution for the $a_0^0(980)$ looks similar to the central production of states that are accessible to $P\bar{P}$ fusion, in particular $P\bar{P} \to f_0(980)$

The $\phi$ distribution for the $a_0(980)$ also looks very similar to that observed for the $f_0(980)$. Qualitatively this is what would be expected if part of the centrally produced $a_0^0(980)$ is due to $P\bar{P} \to f_0(980)$ followed by mixing between the $f_0(980)$ and the $a_0(980)$.

Ref [30] found that $80 \pm 25\%$ of the $a_0^0(980)$ comes from the $f_0(980)$ and upon combining this result with the relative total cross sections for the production of the $f_0(980)$ and $a_0^0(980)$ [30] they found the $f_0(980) - a_0(980)$ mixing intensity to be $8 \pm 3\%$. Achasov et al [54] predicted an order of magnitude $0.5 - 2\%$ for this mixing based upon the affinity of the scalars for the nearby $K\bar{K}$ threshold. The data appear larger even than this and add weight to the hypothesis that the $f_0(980)$ and $a_0(980)$ are siblings that strongly mix, and that the $a_0(980)$ is not simply a $3P_0 q\bar{q}$ partner of the $a_2(1320)$. This is consistent with the $0^{++}(QQ/\bar{MM})$ picture of these states and a natural explanation of these results is that $K\bar{K}$ threshold plays an essential role in the existence and properties.
Other lines of study are now warranted. Experimentally to confirm these ideas requires measuring the production of the $\eta\pi$ channel at a much higher energy, for example, at LHC, Fermilab or RHIC where any residual Reggeon exchanges such as $\rho\omega$ would be effectively zero and hence any $a_0(980)$ production must come from isospin breaking effects.

Other “pure” flavour channels should now be explored. Examples are $D_s$ decays where the weak decay leads to a pure I=1 light hadron final state. Thus $\pi f_0(980)$ will be (and is [37]) prominent, while the mixing results suggest that $\pi a_0$ should also be present at $8 \pm 3\%$ intensity. Studies with high statistics data sets now emerging from E791, Focus and BaBar are called for, and also studies of $J/\psi$ decays at Beijing, in particular to the “forbidden” final states $\omega a_0$ and $\phi a_0$ where ref[30] predicts branching ratios of $O(10^{-5})$. Decays of $f_1(1285) \rightarrow \pi\pi\eta$ should also be accompanied by $f_1(1285) \rightarrow \pi\pi\pi$ [54].

### 3.2 $\phi \rightarrow \gamma f_0/\gamma a_0$

The radiative decays of the $\phi \rightarrow \gamma f_0(980)$ and $\gamma a_0(980)$ have long been recognised as a potential route towards disentangling their nature. Isospin mixing effects could considerably alter some predictions in the literature for $\Gamma(\phi \rightarrow \gamma f_0(980))$ and $\Gamma(\phi \rightarrow \gamma a_0(980))$, and new data from DAΦNE promise to reveal their nature.

The magnitudes of these widths are predicted to be rather sensitive to the fundamental structures of the $f_0$ and $a_0$, and as such potentially discriminate amongst them. For example, if $f_0(980) \equiv s\bar{s}$ and the dominant dynamics is the “direct” quark transition $\phi(s\bar{s}) \rightarrow \gamma 0^+(s\bar{s})$, then the predicted b.r.($\phi \rightarrow \gamma f_0) \sim 10^{-5}$, the rate to $\phi \rightarrow \gamma a_0(q\bar{q})$ being even smaller due to OZI supression [57]. For $K\bar{K}$ molecules the rate was predicted to be higher, $\sim (0.4 - 1) \times 10^{-5}$ [57], while for tightly compact $qq\bar{q}\bar{q}$ states the rate is yet higher, $\sim 2 \times 10^{-4}$ [57, 58].

In the $K\bar{K}$ molecule and $qq\bar{q}\bar{q}$ scenarios it has uniformly been assumed that the radiative transition will be driven by an intermediate $K^+K^-$ loop ($\phi \rightarrow K^+K^- \rightarrow \gamma K^+K^- \rightarrow \gamma 0^{++}$). Explicit calculations in the literature
agree that this implies \([57, 58, 59, 60]\)

\[
b.r. (\phi \to f_0(980)\gamma) \sim (2 \pm 0.5)(10^{-4}) \times F^2(R) \quad (1)
\]

where \(F^2(R) = 1\) in point-like effective field theory computations, such as refs.\([58, 60]\). By contrast, if the \(f_0(980)\) and \(a_0(980)\) are spatially extended \(K\bar{K}\) molecules, (with r.m.s. radius \(R > O(\Lambda_{QCD}^{-1})\)), then the high momentum region of the integration in refs.\([57, 59]\) is cut off, leading in effect to a form factor suppression, \(F^2(R) < 1\).\([57, 61, 63]\). The differences in absolute rates are thus intimately linked to the model dependent magnitude of \(F^2(R)\).

If \(f_0\) and \(a_0\) have common constituents (and hence are “siblings”) and are eigenstates of isospin, then their affinity for \(K^+K^-\) should be the same and so \([57, 58, 60]\)

\[
\frac{\Gamma(\phi \to f_0\gamma)}{\Gamma(\phi \to a_0\gamma)} \sim 1 \quad (2)
\]

whereas the preliminary data find \([32]\)

\[
\frac{\Gamma(\phi \to \gamma f_0)}{\Gamma(\phi \to \gamma a_0)} = 4.1 \pm 0.4 \quad (3)
\]

or even, in their more recent report, find \([33]\)

\[
\frac{\Gamma(\phi \to \gamma f_0)}{\Gamma(\phi \to \gamma a_0)} = 6.1 \pm 0.6 \quad (4)
\]

On this ratio alone one might conclude evidence that the states are \(q\bar{q}\) and that the \(a_0\) is relatively suppressed due to its \(u\bar{u} - d\bar{d}\) content. However, this is not compatible with the intrinsically “large” branching ratios of \(\geq O(10^{-4})\).

Ref.\([31]\) noted that the \(\eta\pi\) signal in central production above, if described by an isospin mixing angle \(\theta\), would lead to the relative rates for \(\phi\) radiative

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to be
\[ \frac{\Gamma(\phi \to \gamma f_0)}{\Gamma(\phi \to \gamma a_0)} \sim \frac{g_{f_0K^+K^-}^2}{g_{a_0K^+K^-}^2} \equiv \cot^2 \theta = 3.2 \pm 0.8 \] (5)

It is intriguing that such a correlation appears to be realised. A problem though, as emphasised by ref. [62] is that the significant overlap of \( f_0 \) and \( a_0 \) states severely reduces such effects, such that one would expect the radiative ratio of unity to survive.

In order to use the individual rates to abstract magnitudes of \( F^2(R) \), and hence assess how compact the four-quark state is, a definitive accurate value for \( g_{fKK}^2/4\pi \) will be required. If for orientation we adhere to the value used elsewhere, \( g_{fKK}^2/4\pi \approx 0.6 \text{ GeV}^2 \), and impose the measured ratio above as an effective measure of the relative couplings to the \( K^+K^- \) intermediate state, then the results of ref. [57] are revised to

\[ \text{b.r.}(\phi \to \gamma f_0) + \text{b.r.}(\phi \to \gamma a_0) \leq (4 \pm 1)(10^{-4}) \] (6)

and

\[ \begin{align*}
\text{b.r.}(\phi \to \gamma f_0) &= (3.0 \pm 0.6)10^{-4} F^2(R) \\
\text{b.r.}(\phi \to \gamma a_0) &= (1.0 \pm 0.25)10^{-4} F^2(R)
\end{align*} \] (7)
(8)

Branching ratios for which \( F^2(R) \ll 1 \) would imply that the \( K^+K^-0^{++} \) interaction is spatially extended, \( R > O(\Lambda_{QCD}^{-1}) \). Conversely, for \( F^2(R) \to 1 \), the system would be spatially compact, as in \( q\bar{q}q\bar{q} \).

The preliminary data from KLOE are [32]

\[ \begin{align*}
\text{b.r.}(\phi \to \gamma f_0) &= (2.4 \pm 0.1)10^{-4} \\
\text{b.r.}(\phi \to \gamma a_0) &= (0.6 \pm 0.05)10^{-4}
\end{align*} \] (9)
(10)

which imply \( F^2(R) \sim 0.7 \pm 0.2 \), supporting the qualitative picture of a compact \( q\bar{q}q\bar{q} \) structure that spends a sizeable part of its lifetime at longer range in a two meson state, such as \( K\bar{K} \). Subsequently they have reported [33]
\[ b.r. (\phi \to \gamma f_0) = (4.5 \pm 0.2) \times 10^{-4} \]  
(11)

\[ b.r. (\phi \to \gamma a_0) = (0.75 \pm 0.07) \times 10^{-4} \]  
(12)

Caution is needed before over-interpreting these numbers. There are discrepancies between the magnitude of $S - K \bar{K}$ couplings in this experiment and some others. This may be connected to the fact that no $\pi\pi - KK$ coupled-channel analysis incorporating unitarity has yet been performed on the data. Also, the results depend upon the assumption that there is a large destructive interference in the data between the $f_0\gamma$ and $\sigma\gamma$, where the $\sigma$ is assumed to be described by a simple Breit-Wigner shape. The sensitivity to this assumption has not been discussed and needs to be assessed before strong conclusions about the $f_0\gamma$ branching ratio are made. Hence we delay detailed interpretation pending a more detailed analysis of these data. Certainly this process offers potentially significant insights into the nature of the scalar enhancements below 1GeV.

\section{Understanding the S-waves within a unitarized quark model (UQM)}

\subsection{The Nambu-Jona-Lasinio (NJL) model and the linear sigma model}

A light scalar-isoscalar meson (the $\sigma$), with a mass of twice the constituent $u,d$ quark mass, or $\approx 600$ MeV, coupling strongly to $\pi\pi$ is of importance in all Nambu–Jona-Lasinio-like (NJL-like) models for dynamical breaking of chiral symmetry. In these models the $\sigma$ field obtains a vacuum expectation value, i.e., one has a $\sigma$-like condensate in the vacuum, which is crucial for the understanding of all hadron masses, as it explains in a simple way the difference between the light constituent and chiral quark mass. Then most of the nucleon mass is generated by its coupling to the $\sigma$, which acts like an effective Higgs-like boson for the hadron spectrum.
The NJL model is an effective theory which is believed to be related to QCD at low energies, when one has integrated out the gluon fields. It involves a linear realization of chiral symmetry. After bosonization of the NJL model one finds essentially the linear sigma model as an approximate effective theory for the scalar and pseudoscalar meson sector.

About 30 years ago Schechter and Ueda\cite{64} wrote down the $U3 \times U3$ linear sigma model (broken by 2 quark mass terms and a $U1_A$ term) for the meson sector involving a scalar and a pseudoscalar nonet. This (renormalizable) theory has only 6 parameters, out of which 5 can be fixed by the pseudoscalar masses and decay constants; $m_\pi$, $m_K$, $f_\pi$, $f_K$, and a combination of $\eta$ and $\eta'$ masses (like $m_\eta^2 + m_{\eta'}^2$), which fixes the strength of the $U1_A$ breaking term.

The sixth parameter for the OZI rule violating 4-point coupling is small, and fixes the $\sigma - a_0$ splitting. One can then predict, with no free parameters, the other tree level scalar masses \cite{65}, which turn out to be not far from the lightest experimental masses, although the two quantities (say Lagrangian mass vs. second sheet pole mass) are not the same thing, but can differ for the same model and data by well over 100 MeV.

The important thing is that the scalar masses are predicted to be near the lightest experimentally seen scalar masses, and not in the $\sim 1.2$-1.6 GeV region where we expect the lightest $q\bar{q}$ scalars. The $\sigma$ is predicted \cite{63} at 620 MeV with a very large width ($\approx 600$ MeV) which agrees well with most data. The $a_0(980)$ is predicted at 1128 MeV, the $f_0(980)$ at 1190 MeV, and the $\kappa$ or $K_0^*(1430)$ at 1120 MeV. This is still surprisingly good considering that loops or unitarity effects must be large as we discuss next.

4.2 Unitarisation and S-waves in quark models

A few years ago one of us presented fits to the $K\pi$, $\pi\pi$ S-waves and to the $a_0(980)$ resonance peak in $\pi\eta$\cite{66}. A similar model was also presented by Van Beveren et al.\cite{70}. This involved, like the linear sigma model, the pseudoscalar nonet (taken from data) and a scalar nonet. It may be looked upon as a way of unitarising a chiral quark model for the scalars, and in a coupled channel framework it takes account of all the flavour related s-channel
two pseudoscalar thresholds. These are shown to distort any simple input bare scalar spectrum, through the mixing with the meson-meson continuum.

Also in this approach one must make simplifications; e.g. one neglects more distant singularities and assumes that crossed channel singularities can be represented by a simple form factor. But, a nice feature of such a model is that it simultaneously describes a whole scalar nonet and gives a good representation of a large set of relevant data, with a few physically well defined parameters.

Consistency with unitarity implies that when the effective coupling becomes large enough, twice as many poles can appear in the output spectrum as were put in as bare nonet masses. The new poles can then be interpreted as being mainly meson-meson bound states, but mixed with the states, which are put in.

At first\cite{25} the $\sigma$ was missed because only poles nearest to the physical region were looked for, and the possibility of the resonance doubling phenomenon, discussed below, was overlooked. Only later was it realised \cite{66} that two resonances can emerge although only one bare state is put in, i.e., one bare nonet can give rise to two nonets in the output, if the overall coupling is large enough. Then one had to look deeper into the second sheet and the broad $\sigma$ as the dominant singularity at low mass was found in the model. An advantage with this model was that in order to explore whether this pole was the relevant one, one could, within the model, decouple the effect of the $K\bar{K}$ threshold and find that the relevant singularity in the $n\bar{n}$ channel is indeed the broad $\sigma$.

In fact, it had been pointed out by Morgan and Pennington \cite{67} that for each $q\bar{q}$ state there are, in general, apart from the nearest pole, also image poles, usually located far from the physical region. As explained in more detail in Ref. \cite{66, 68}, some of these can (for a large enough coupling and sufficiently heavy threshold) come so close to the physical region that they make new resonances. And, in fact, there were more than four physical poles with different isospin, in the output spectrum of the UQM model, although only four bare states, of the same nonet, were put in!

In the I=1 channel two manifestations of the bare state were found, the
$a_0(980)$ and the $a_0(1450)$. Similarly for one input bare $s\bar{s}$ state, two poles the $f_0(980)$ and a heavier one, which at the time was assumed to be the $f_0(1370)$ were found, but the uncertainties of the model could in reality push the heavier state up in mass, e.g. to emerge within the $f_0(1500/1710)$ states. In the $K\pi$ channel a stronger overall coupling could within the model produce a virtual bound state $K\pi$ state near the threshold\cite{66}.

Cherry and Pennington \cite{69} have strongly argued against the existence of a light $\kappa$. With the presently known experimental $K\pi$ phase shifts\cite{42}, and the fact that it is essentially a single channel problem up to the $K_0^*(1430)$, this conclusion seems hard to avoid. But it should however, be remembered that the experimental phase shifts start at only at about $\sqrt{s} \approx 850$ MeV. New data on these phase shifts would be very welcome. As we have mentioned the E791 experiment see some evidence for a light $\kappa$ in $D^+ \rightarrow K^-\pi^+\pi^+$. The signal is much less evident than the $\sigma$ in $D \rightarrow 3\pi$, but the $\kappa$ improves their $\chi^2$ in the region dominated by the $K^*(890)$.

There are several authors\cite{70, 71, 72}, who within models support the existence of both the $\sigma$ and the $\kappa$, but the question of whether the $\kappa$ is truly resonant or whether it could be something like a virtual bound $K\pi$ state remains open, and the fact that there is activity with these quantum numbers appears to be established.

Although the details of any modelling can be criticized, the conclusion remains: after unitarisation, strong enough couplings can generate new bound states or resonances that were not present in the input or in the Born terms represented by an effective Lagrangian. In short, in addition to a conventional scalar nonet the unitarisation generates for large effective coupling another scalar nonet, which has a substantial component of meson-meson in its wave function. A more detailed QCD inspired UQM-like model, with better description of crossed channels, and more thresholds would be very welcome.

There are other effects that a model like the UQM explains, which are due to the fact that the inverse meson propagator is not just BW-like $(m_0^2 - s + ig^2\text{Im}\Pi(s))$, with constant $m_0$, but has an important cusp-like contribution $m_0^2 \rightarrow m_0^2 + g^2\text{Re}\Pi(s)$ to the mass term. For resonances decaying in an
S-wave near a threshold and with a large $g^2$ this makes a big difference in the mass, width and shape of the resonance. In many unitarisation schemes (e.g. often in K-matrix unitarisation) this requirement from analyticity is forgotten. We list here the most important effects.

(i) The large mass difference between the $K_0^*(1430)$ and the $a_0(980)$.

This arises as a secondary effect due to the large pseudoscalar mass splittings, and because of the large mass shifts coming from the loop diagrams involving the PP thresholds. The three thresholds $\pi\eta$, $K\bar{K}$, $\pi\eta'$ all lie relatively close to the $a_0(980)$. All three of them contribute to a large negative shift in mass, and to a large meson-meson component in $a_0(980)$, mainly $K\bar{K}$. On the other hand, for the $K_0^*(1430)$, the $SU3_f$ related thresholds ($K\pi$, $K\eta'$) lie far apart from the $K_0^*$, while the $K\eta$ nearly decouples because of the physical value of the pseudoscalar mixing angle. Therefore the $K_0^*(1430)$ is shifted down only slightly and furthermore remains essentially as $q\bar{q}$. Conversely, the $\kappa$ would be dominantly $K\pi$ (provided it forms a bound state near $K\pi$) and the $a_0(1430)$ is dominantly $u\bar{d}$.

(ii) The nearness of the $a_0(980)$ and the $f_0(980)$ to the $K\bar{K}$ threshold.

Because of the cusp in $m_0^2 + g^2 \text{Re}\Pi(s)$ at a threshold, a physical mass emerges just below the threshold for a wide range of $m_0^2$ values.

(iii) The narrowness of the $a_0(980)/f_0(980)$ peak width.

The cusp also changes the shape of the resonance peak to be considerably narrower than what a BW parameterization gives. This is the Flatté effect. In a space-like picture it is related to the fact that the large $K\bar{K}$ component in the wave function must convert near the origin to $\pi\eta$ or $\pi\pi$, which is the only open channel.

We now turn to the question of the scalar glueball. This will mix with the other scalars, whether below or above 1 GeV mass. The first issue therefore is how one might isolate such a state from data.
The folklore has been that to enhance glueball signals one should concentrate on production mechanisms where quarks are disfavoured: thus $\psi \rightarrow \gamma G[74]$, $p\bar{p} \rightarrow \pi + G$ in annihilation at rest [74, 75], and central production in diffractive (gluonic pomeron) processes, $pp \rightarrow pGP[75, 76]$. Contrasting this, $\gamma\gamma$ production should favour flavoured states such as $q\bar{q}$. Thus observing a state in the first three, which is absent in the latter, would be prima facie evidence.

Such ideas are simplistic. There has been progress in quantifying them and in the associated phenomenology. The central production has matured significantly in the last three years and inspires new experiments at RHIC, Fermilab and possibly even the LHC. These complementary processes collectively are now painting a clearer picture.

First on the theoretical front, each of these has threats and opportunities. (i) In $\psi \rightarrow \gamma G$ the gluons are timelike and so it is reasonable to suppose that glueball will be favoured over $q\bar{q}$ production. Quantification of this has been discussed in ref. [77] with some tantalising implications: (a) the $f_0(1500)$ and $f_0(1710)$ are produced with strengths consistent with them being $G - q\bar{q}$ mixtures, though there are some inconsistencies between data sets that need to be settled experimentally.

(ii) In $p\bar{p}$ the $q$ and $\bar{q}$ can rearrange themselves to produce mesons without need for annihilation. So although a light glueball may be produced, it will be in competition with conventional mesons and any mixed state will be produced significantly by its $q\bar{q}$ components.

(iii) In central production the gluons are spacelike and so must rescatter in order to produce either a glueball or $q\bar{q}$. Thus here again one expects competition. However, a kinematic filter has been discovered [78], which appears able to suppress established $q\bar{q}$ states, when the $q\bar{q}$ are in P and higher waves.

Its essence was that the pattern of resonances produced in the central region of double tagged $pp \rightarrow pMp$ depends on the vector difference of the transverse momentum recoil of the final state protons (even at fixed four momentum transfers). When this quantity ($dP_T \equiv |\vec{k}_{T1} - \vec{k}_{T2}|$) is
“large”, \((\geq O(\Lambda_{QCD}))\), \(q\bar{q}\) states are prominent whereas at “small” \(dP_T\) \((\leq O(\Lambda_{QCD}))\) all well established \(q\bar{q}\) are observed to be suppressed while the surviving resonances include the enigmatic \(f_0(1500)\), \(f_0(1710)\) and \(f_0(980)\).

The data are consistent with the hypothesis that as \(dP_T \rightarrow 0\) all bound states with internal \(L > 0\) (e.g. \(3P_{0,2}\) \(q\bar{q}\)) are suppressed while S-waves survive (e.g. \(0^{++}\) or \(2^{++}\) glueball made of vector gluons and the \(f_0(980)\) as any of glueball, or S-wave \(qq\bar{q}\bar{q}\) or \(KK\) state). Models are needed to see if such a pattern is natural. As the states that survive this cut appear to have an affinity for S-wave, this may be evidence for \(qq\bar{q}\bar{q}\) or \(qq\bar{q}\bar{q}\) (as for example the \(f_0(980)\)) or for \(gg\) content (as perhaps in the case of \(f_0(1500; 1710)\) and \(f_2(1930)\)). It would be interesting to study the production of known \(q\bar{q}\) states in \(e^+e^- \rightarrow e^+Me^-\) to see how they respond to this kinematic filter, and gain possible insights into its dynamics.

Following this discovery there has been an intensive experimental programme by the WA102 collaboration at CERN, which has produced a large and detailed set of data on both the \(dP_T\) and the azimuthal angle, \(\phi\), dependence of meson production (where \(\phi\) is the angle between the transverse momentum vectors, \(p_T\), of the two outgoing protons).

The azimuthal dependences as a function of \(J^{PC}\) and the momentum transferred at the proton vertices, \(t\), are also very striking. As described in ref.\(^{[55]}\) here again the scalar mesons appear to divide into two classes: \(f_0(980); f_0(1500); f_0(1710)\) which are all strongly peaked at small \(\phi\) and the \(f_0(1370)\) at large \(\phi\). Exactly what this phenomenon implies for the dynamics and structure of these scalar mesons remains to be solved.

One expects that there will be considerable mixing between the quenched glueball and the scalar mesons that were seeded by quarks. From a study of the \(0^-0^-\) decays of the \(f_0(1370; 1500; 1710)\), ref.\(^{[80]}\) conclude that

\[
\begin{array}{cccc}
\text{meson} & \text{meson} & \text{meson} & \text{meson} \\
& f_G & f_s & f_R \\
f_0(1710) & 0.39(0.03) & 0.91(0.02) & 0.15(0.02) \\
f_0(1500) & -0.65(0.04) & 0.33(0.04) & -0.70(0.07) \\
f_0(1370) & -0.69(0.07) & 0.15(0.01) & 0.70(0.07) \\
\end{array}
\]

This also intuitively is in line with the idea that \(G\) and \(s\bar{s}\) mix to give the
that $G$ and $n\bar{n}$ mix to give the $f_0(1370)$ and that that $G$, $s\bar{s}$ and $n\bar{n}$ mix to give the $f_0(1500)$ with a negative phase between the $n\bar{n}$-$s\bar{s}$ (“flavour octet tendency”). A new generation of experiments may enable flavour filtering in this extended nonet.

## 5.1 Scalar mesons and glueballs

Ref.[81] has shown that radiative transitions from excited vector mesons to the scalar sector may be experimentally accessible. In the simplest approach it assumed that there is no mixing among the scalars, so that the $f_0(1370)$ is pure $n\bar{n}$ and the $f_0(1710)$ is pure $s\bar{s}$.

The $\rho_D(1700) \rightarrow \gamma f_1(1285) \sim 1MeV$ is a benchmark for experiments to find (the $f_1$ is rather narrow which enables it to be seen in $4\pi$ final states). If this can be verified then they should look for the $\gamma f_0(n\bar{n}) \sim 0.9MeV$. If the $f_0(1370)$ is entirely $n\bar{n}$ then the $0.9MeV$ will go entirely into it. However, we anticipate glueball mixing into the 1370 (and other scalar mesons). If the glueball is light ($\sim 1300MeV$) it will mix strongly into the 1370 and dilute this $0.9MeV$ (it will be pushed into the other scalars, e.g. the 1500). Conversely, if the glueball is massive (up at 1700 MeV like Weingarten has argued) then the 1370 will remain rather pure $n\bar{n}$ and have a healthy radiative strength.

Ref.[81] studied the effect of glueball mixing into the scalars and found that the radiative transitions are potentially sensitive measures of this. The result of the mixing is that the bare $n\bar{n}$ and $s\bar{s}$ states contribute in varying degrees to each of the $f_0(1370)$, $f_0(1500)$, and $f_0(1710)$.

Three different mixing scenarios have been proposed: the bare glueball is lighter than the bare $n\bar{n}$ state (the light glueball solution); the mass of the bare glueball is between the bare $n\bar{n}$ state and the bare $s\bar{s}$ state (the middleweight glueball solution); and the mass of the bare glueball is greater than the mass of the bare $s\bar{s}$ state. The first two solutions have been obtained in [3, 5, 80] and the third has been suggested in [4, 7]. The effects of the mixing on the radiative decay widths of the $\rho(1700)$ and the $\phi(1900)$ to the three $f_0$ states are given in ref.[81] for each of these three cases. The relative
rates of the radiative decays of the $\rho(1700)$ to $f_0(1370)$ and $f_0(1500)$ change radically according to the presence of the glueball admixture. So for a light glueball the decay to $f_0(1370)$ is relatively suppressed whereas for a heavy glueball it is substantial. By contrast the effect on the decay to $f_0(1500)$ goes the other way. Further, the $\phi(1900)$ would give a large width for the decay to $f_0(1500)$ for a heavy glueball, but essentially zero for a light one. The $f_0(1710)$ will be prominent in the decays of the $\phi(1900)$ for all but the heaviest glueball. It is clear that these decays do provide an effective flavour-filtering mechanism.

Further, identifying the appropriate mixing scheme gives insight into the underlying physics of glueballs. The existing phenomenology from hadronic decays seems to favour a light glueball. Essentially, if the decays of the “bare” glueball are flavour-independent, then the observed flavour dependencies for the hadronic decays of the physical mesons require \cite{80} the glueball mass to be at the low end of the range preferred by quenched-lattice studies.

The resolution of the isoscalar-scalar problem is intimately connected with the isovector-scalar problem. The existence of any $a_0$ other than the $a_0(980)$ remains controversial. The different mixing schemes for the isoscalar-scalar mesons give rather different values for the mass of the bare $n\bar{n}$ state. This mass will be reflected in the mass of its isovector partner, the $a_0$. The width for the decay $\omega(1650) \rightarrow a_0\gamma$ is predicted to be large for a $q\bar{q}$ $a_0$, and is anticipated to be a well-defined decay. So this decay can provide independent information on the existence and properties of the $a_0(1450)$. Predicting the width for $\omega \rightarrow \gamma a_0(980)$ is now also a challenge.

6 Two coupled linear sigma models for two scalar nonets

As we have seen in the previous discussion there seems to be a proliferation of light scalar mesons. And as we have shown there is good evidence that the spectrum below 1.7 GeV includes two light nonets of scalar mesons, the heavier of which is the $q\bar{q}$ nonet expected from QCD or the quark model,
while the lighter is of more complicated structure, but also in a nonet. The glueball of lattice gauge theory is mixed into (at least some of) these states.

In order to have a realistic effective model at low energies for the scalars we need an effective chiral quark model, which includes all scalars and pseudoscalars, and where the chiral symmetry is broken by the vacuum expectation values of the scalar fields.

The simplest such chiral quark model is the $U(N_f)_L \times U(N_f)_R$ linear sigma model. Put as usual a scalar nonet into the hermitian part of a $3 \times 3$ matrix $\Phi$ and the associated pseudoscalar nonet into the antihermitian part. For two scalar nonets we need another such $3 \times 3$ matrix $\hat{\Phi}$. Let the scalar $q\bar{q}$ states above 1 GeV be in $\Phi$, while those below 1 GeV are in $\hat{\Phi}$. (In the approach of the unitarized quark models discussed previously the states $\hat{\Phi}$ can be generated by the unitarization.)

Then model both $\Phi$ and $\hat{\Phi}$ by a gauged linear sigma model, but with different sets of parameters $(\mu^2, \lambda \ldots)$ and $(\hat{\mu}^2, \hat{\lambda} \ldots)$.

\[
\mathcal{L}(\Phi) = \frac{1}{2} \text{Tr}[D_\mu \Phi D_\mu \Phi^\dagger] + \frac{1}{2} \mu^2 \text{Tr}[\Phi \Phi^\dagger] - \lambda \text{Tr}[\Phi \Phi^\dagger \Phi \Phi^\dagger] + \ldots
\]

\[
\hat{\mathcal{L}}(\hat{\Phi}) = \frac{1}{2} \text{Tr}[D_\mu \hat{\Phi} D_\mu \hat{\Phi}^\dagger] + \frac{1}{2} \hat{\mu}^2 \text{Tr}[\hat{\Phi} \hat{\Phi}^\dagger] - \hat{\lambda} \text{Tr}[\hat{\Phi} \hat{\Phi}^\dagger \hat{\Phi} \hat{\Phi}^\dagger] + \ldots
\]

One can add further terms, but these are not important for the present qualitative discussion. We have thus doubled the spectrum and initially we have two scalar, and two pseudoscalar multiplets, altogether 36 states for three flavours. We have not above included any flavour symmetry breaking nor the glueball $G$ or anomaly terms for simplicity. The glueball would mix with the singlet scalars through terms like $G \text{Tr}(\Phi + \Phi^\dagger)$.

Then it is natural to introduce a coupling between the two sets of multiplets, which can break the relative symmetry [1, 83]. The full effective Lagrangian for both $\Phi$ and $\hat{\Phi}$ thus becomes,

\[
\mathcal{L}_{\text{tot}}(\Phi, \hat{\Phi}) = \mathcal{L}(\Phi) + \hat{\mathcal{L}}(\hat{\Phi}) + \frac{\epsilon^2}{4} \text{Tr}[\Phi \hat{\Phi}^\dagger + h.c.]
\]

A similar scheme was discussed recently by Black et al. [71], who also emphasized, with explicit examples, that four-quark states of both the meson-meson
type and of the Jaffe type, \((\bar{q}q)_3(qq)_3\), can be constructed, which transforms in the same way under \(SU(3)_L \times SU(3)_R\) as is usually assumed for \(q\bar{q}\) within chiral models. E.g., for a lefthanded and righthanded diquark in the antisymmetric 3 representations of both colour and flavour one has

\[
L^{cC} = \epsilon^{cab} \epsilon^{CAB} q^T_{aA} C^{-1} \left( 1 + \frac{\gamma_5}{2} \right) q_{bB},
\]

\[
R^{cC} = \epsilon^{cab} \epsilon^{CAB} q^T_{aA} C^{-1} \left( 1 - \frac{\gamma_5}{2} \right) q_{bB}.
\]

Then the matrix \(\hat{\Phi}^d_c = (L^{cA})^d R^{dA}\) transforms under \(SU(3)_L \times SU(3)_R\) the same way as a conventional chiral \(q\bar{q}\) state (\(\Phi\)). Thus one may assume that \(\Phi\) includes the \(q\bar{q}\) nonets of scalars and pseudoscalars, while \(\hat{\Phi}\) stands for two extra chiral nonets of 4-quark states.

Now as a crucial assumption, let both \(\Phi\) and \(\hat{\Phi}\) have vacuum expectation values (VEV) \(v\) and \(\hat{v}\) even if \(\epsilon = 0\) \((v = \mu^2/(4\lambda) + \mathcal{O}(\epsilon^2), \hat{v} = \hat{\mu}^2/(4\hat{\lambda}) + \mathcal{O}(\epsilon^2))\). The simplest physical interpretation of these VEV’s is that \(v \propto < q\bar{q} >\) and \(\hat{v} \propto < qq\bar{q}\bar{q} >\). With a glueball one would expect that these would also have a gluonium component.

The two originally massless pseudoscalar nonets then mix through the \(\epsilon^2\) term, with a mixing angle \(\tan \theta = v/\hat{v}\), such that one nonet remains massless while the other nonet obtains a mass \(m^2_{\pi} = \epsilon^2(v^2 + \hat{v}^2)/v\hat{v}\). The mixing angle is determined entirely by the two vacuum expectation values, and is large if \(v\) and \(\hat{v}\) are of similar magnitudes, independently of how small \(\epsilon^2\) is as long as it stays finite.

On the other hand the scalar masses and mixings are only slightly affected if \(\epsilon^2/(\mu^2 - \hat{\mu}^2)\) is small. They are still close in mass to \(\sqrt{2}\mu\) and \(\sqrt{2}\hat{\mu}\) as in the uncoupled case. These would be the two scalar nonets we have discussed in the previous chapters.

In order that this should have anything to do with reality, one must of course get rid of the massless Goldstones. By gauging the overall axial symmetry \((D_{\mu} \Phi = \partial_{\mu} - ig[\lambda_i A_i \Phi + \Phi \lambda_i A_i])\) the Higgs mechanism absorbs the massless modes from the model, but these degrees of freedom enter instead
as longitudinal axial vector mesons and give these mesons (an extra) mass
\[ m_A^2 = 2g^2(v^2 + \hat{\nu}^2). \]
This is similar to the original Yang-Mills theory and the work of Bando et al.\cite{84} on hidden local symmetries. Then with \( \epsilon^2 \) proportional to the average chiral quark mass one can interpret the massive pseudoscalar nonet \( \pi_a \) as the physical light pseudoscalars. They would be mixtures of the two original pseudoscalar multiplets with a mixing angle \( \theta \).

The main prediction of this scheme is that one must have doubled the light scalar meson spectrum, as seems to be experimentally the case. Some more details are given in\cite{83}). Of course in order to make any detailed comparison with experiment one must also break the flavour symmetry, and unitarize the model. Especially the latter is not a simple matter since it is a strong coupling model, although in principle renormalizable.

The schizophrenic role of the pions in conventional models, as being at the same time both Goldstone bosons and \( q\bar{q} \) pseudoscalars, is here resolved in a particularly simple way: One has originally two Goldstone-like pions, out of which only one remains in the spectrum, and which is a particular linear combination of the two original pseudoscalar fields.

Both of the two scalar multiplets remain as physical states and one of these (formed by the \( \sigma(600) \) and the \( a_0(980) \) in the case of two flavours), or the \( \sigma, a_0(980), f_0(980) \) and the \( \kappa \) in the case of three flavours can then be looked upon as effectively a Higgs multiplet of strong nonperturbative interactions when a hidden local symmetry is spontaneously broken. The heavier scalar multiplet then being the \( q\bar{q} \) scalars augmented by the glueball.

7 Experimental Prospects

Establishing that gluonic degrees of freedom are being excited is now a real possibility.

The scalars below 1 GeV are too light to enable a simple distinction between loose molecules and compact four-quarks states to be felt in decays (except perhaps for the \( a_0(980) \) where the \( K\bar{K} \) and \( \eta\pi \) both couple strongly
and point to a significant compact four-quark feature). However, the production dynamics and systematics of these states is interesting and full of enigmas, which may be soluble if one adopts the four-quark/meson-meson bound state picture.

$D_s$ decays into $\pi f_0(980)$ clearly point to an $s\bar{s}$ presence in the $f_0(980)$. However, the production in $Z$ decays is rather non-strange-like$^5$. $\psi$ to $\omega f_0$ and $\phi f_0$ also points towards $n\bar{n}$ and $s\bar{s}$ structure in $f_0(980)$ and $a_0(980)$. The central production in pp shows that $f_0$ is strongly produced, akin to other $n\bar{n}$ states and much stronger than $s\bar{s}$ which appear to be suppressed in this mechanism. Furthermore, $f_0$ survives the $dk_T \to 0$ filter of ref.$^{[78]}$. The systematics of this appear to be driven by S-wave production: this would be fine for either a compact four-quark or molecule. We noted that there is also evidence for strong mixing between $f_0 - a_0$ associated with the nearby $K\bar{K}$ threshold. These phenomena fit more naturally with a $qq\bar{q}\bar{q}$/meson-meson attraction as the controlling dynamics.

Whenever S-wave dynamics can play a role it will override P-waves; so one expects $K\bar{K}$ S-wave production to drive the $f_0/a_0$ whenever allowed. This is indeed what happens in the $\phi \to \gamma f_0/\gamma a_0$; the “large” rate cries out for the $K^+K^-$ loop to drive it. A question is whether the $s\bar{s}n\bar{n}$ constituents of the intermediate state “between” the initial $\phi$ and final $f_0$ are able to fluctuate spatially enough to be identified as two colour singlet K’s, which then couple to the $f_0$, or whether they are a compact system in the sense of being confined within $\sim 1$fm. The former would have some form factor suppression of the rate; the latter would be more pointlike and larger rate. The emerging data are between these extremes, but nearer to the expectations for a compact $qq\bar{q}\bar{q}$ configuration. The $K\bar{K}$ would then be the long-range ($\geq O(\Lambda_{QCD})$) tail.

Knowledge on the $\gamma\gamma$ couplings is lacking and better data would be useful; however, it is not immediately clear how this probes the deep structure of the scalar mesons. We know that for the $2^{++}$ $\gamma\gamma$ reads the compact $q\bar{q}$ flavours; there is no 2-body S-wave competition in the imaginary part as $\rho\rho$ etc are too heavy. One would expect that for the $0^{++}$ the $K\bar{K}$ will dominate the $\gamma\gamma$ if there is a long-range $K\bar{K}$ component in the wavefunction. At the other extreme; were the state a pure compact four quark, then higher intermediate
states - KK, KK*,KK** etc - would all be present. Achasov has discussed these and a precise calculation has many problems, but the ratios of $\gamma\gamma$ to $f_0/a_0$ would probably be sensitive and more reliable.

The production by highly virtual $\gamma^*\gamma^*$ in $e^+e^- \rightarrow e^+e^- f_0/a_0$ could probe the spatial dependence of their wavefunctions. It would be especially instructive were the ratio to be strongly $Q^2$ dependent.

In summary, the theoretical frontier suggests that one can divide the phenomenology of scalars into those above and those below 1 GeV. We suspect that much of the confusion begins to evaporate if one adopts such a starting point. Empirically, signs of gluonic excitation are appearing: (i) in the form of hybrids with the exotic $J^{PC} = 1^{-+}$ now seen in various channels and more than one experiment; (ii) with $0^{-+}$ and $1^{-}$ signals in the 1.4 - 1.9 GeV region that do not fit well with conventional quarkonia and show features predicted for hybrids; (iii) in the form of the scalar glueball mixed in with quarkonia in the 1.3 - 1.7 GeV mass range. Theoretical questions about the latter are concerned with whether the effects of the glueball are localised above 1 GeV, or whether they are spread across a wider mass range, even down to threshold. Experimental questions that need to be resolved concern the existence and properties of the $f_0(1370)$ and $a_0(1450)$.

These questions in turn provoke a list of challenges for experiment.

(i) In $e^+e^-$, or vector meson photoproduction at Fermilab and Jefferson Laboratory high statistics studies of radiative decays of such states into the $f_0(980); f_1(1285); f_2(1270)$ could teach us much.

(ii) $\gamma\gamma$ couplings give rather direct information on the flavour content of $C=\pm$ states. Such information on the scalar mesons will be an essential part of interpreting these states. The $Q^2$ dependence of $\gamma^*\gamma^*$ in $e^+e^- \rightarrow e^+e^- f_0/a_0$ could probe the spatial dependence of these states. Complementing this $ep \rightarrow ep\pi\pi^0$ could probe their $\gamma^*\omega$ couplings.

(iii) Heavy flavour decays, in particular $D_s$ and $D$ into $\pi$ and associated hadrons can access the scalar states. Precision data are needed to disentangle the contributions of the various diagrams, whereby the flavour content of the scalars can be inferred. There is also a tantalising degeneracy between the
π_d(1.8) and the D, which may radically affect the Cabibbo suppressed decays of the latter. Hence precision data on such charm decays is warranted for reasons that go far beyond simply issues about scalar mesons.

(iv) Tau Charm Factories may at last appear. χ decays offer an entree into light flavoured states; the excitement about the scalar glueball mixing with the quarkonia nonet began when the precision data from p¯p annihilation at LEAR first emerged. Data at rest were beautiful and well analysed. Data in flight however tend to be more problematic, not least as one cannot so easily control knowledge of the incident partial wave. χ decays can access these phenomena, at c.m. energies up to 3.5 GeV, and from well defined initial J^{PC} states. χ_2 → f_2 + (ππ; K K; ηη) will favour the 0^{++} channel and flavour select n¯n and s¯s components if the f_2(1270) or f_2(1525) is selected respectively. The 1^{+} nonet is not flavour ideal[79] but f_1(1285) is narrow and potentially a clear signal against which the scalar hadrons can also be formed in S-wave. (The χ_1 → π + 1^{−+} also provides an entree into the exotic hybrid channel)

8 Conclusions

There seems to be growing experimental evidence for two light nonets of scalars, one in the 500-1000 MeV region (σ(600), a_0(980), f_0(980), κ(?)), and another one near 1.3-1.7 GeV (f_0(1370), f_0(1500)/f_0(1710), a_0(1450), K^0_0(1430)) where the “overpopulation” and systematics of the latter in particular fit with the notion that the scalar glueball of lattice QCD is mixing with these states. A linear sigma model has been proposed as a “toy model” for each multiplet, each one with its separate vacuum expectation value. Then after coupling these two models through a mixing term and gauging the overall symmetry, one can argue that one of the nonets (the lighter one) is a true Higgs nonet for strong interactions and the heavier one is strongly influenced by mixing with the scalar glueball.

A more detailed understanding requires a unitarized model whereby one can understand how the masses are shifted, the widths and mixings are distorted, and even why new scalar meson-meson resonances can be created in
nonexotic channels. These effects are important because of the large effective coupling, and because of the nonlinearities due to the S-wave cusps.

Finally, we note that no anomalies are anticipated, nor seen, for scalar mesons made from heavy flavours. While this remark may appear to be trivial, it reinforces the singular properties being manifested for the light scalars. Given that QCD effects, such as a glueball and strong attraction in S-waves are naturally expected in the light mass region, one may qualitatively conclude that these singular properties are in accord with theoretical prejudices. Isolating their dynamics in detail may therefore shed new light on the nature of confinement in QCD and of effective theories incorporating Higgs’ ideas.

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