The traffic performance of the wheeled carrying liquid materials study

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Abstract. A feature of the wheeled vehicles capable of carrying liquid materials is the presence of the external disturbances in the form of interaction with the roadbed. This applies to peculiar transport vehicles, carrying liquid materials considered as non-Newtonian fluid. The internal disturbances in the form of the liquid material free surface interaction is the presence of the external disturbances in the form of interaction with the roadbed. Internal disturbances in the form of the wheeled concrete transport machine working vessel fluid free surface interaction should also be taken into account. The article describes the first part of the task - the interaction of the system with the roadbed. The simplest form of the system motion equations with rolling is in the accompanying coordinate system when written in the form of equations in quasi-coordinates. Based on the apparatus of nonholonomic mechanics, M. V. Keldysh built the theory of the rolling wheels with an elastic tire. Under certain assumptions, the above-mentioned theory in the article is extended to the case of curvilinear motion. These equations are represented as a system of first-order equations convenient for numerical solution.

1. Introduction
The wheeled concrete transport vehicles are specialized vehicles with variable mass geometry, among which the most widespread are the concrete mixers. Stability and controllability of the specified vehicles are a part of the main indicators of the vehicles’ safety at certification tests. The accumulated experience of CNIPstroydormash (Russia) testing of the mixers in terms of the test site in accordance with CD 37.001.005-86, GOST R 52302-2004 and the standard test procedure of the mixers shows their considerable complexity and insecurity [1, 2].
The calculation of the performance characteristics of the concrete mixers’ movement consists of two parts. In the first part, the forces of interaction of the mixer truck rolling wheel with the roadbed are considered. The second part deals with the characteristics of the roll, course and lateral stability of the mixer truck on a single or semi-trailer chassis. This article presents the first part of the study.
Let us consider the wheel mixer truck, rolling without slipping on a flat surface (Fig.1). The deformation of the pneumatic tire will be considered small and characterized by four parameters: the lateral displacement $Y_b$ of the contact area center relative to the track of the middle plane of the wheel on the road surface, the angle $\delta_b$ twisting pneumatics, the angle $\chi$ slope of the plane of the wheel to the road surface and the radial deformation $\eta$ pneumatics. Consideration of the parameters $Y_b$, $\delta_b$, $\chi$ is determined by the theory of the elastic pneumatics rolling developed by the academician M. V. Keldysh [3]. The need to take into account the radial deformation of the truck tire mixer is due to some significant changes in the load on the wheel due to the road irregularities and the mobility of the transported material.
2. Theoretical study

The position of the mixer truck wheel is determined by the values, \( X^*, Y^* \), \( \chi \), \( \alpha \) (Fig. 1). Here \( X^*, Y^* \) are the coordinates of the point \( K \) of the greatest slope line intersection drawn in the plane of the wheel through its center, with the plane of the road; \( \alpha \) – is the angle formed by the track plane of the wheel on the road and the axis of the fixed coordinate system. In accordance with the theory of academician M. V. Keldysh, the following conditions are imposed on the wheel rolling without slipping:

- tangent to the rolling line coincides with the axis of the contact area;
- the curvature of the rolling line is uniquely determined by the pneumatics deformation parameters \( Y^b, \delta^b, \chi \).

Rolling conditions without slippage can be written in the form of the following equations of nonholonomic bonds [3] superimposed on the \( i \)-th wheel of the mixer truck:

\[
\begin{align*}
\dot{X}^* \sin(\zeta + \delta^b) - \dot{Y}^* \cos(\zeta + \delta^b) - \dot{Y}_b &= 0; \\
\dot{\zeta}_i + \dot{\delta}_b + (\alpha_i Y_b + \beta_i \delta_b + \xi_i \chi_i) [X^* \cos(\zeta + \delta^b) + Y^* \sin(\zeta + \delta^b)] &= 0; \\
\end{align*}
\]

(1)

The reaction forces acting on the rolling wheel of the mixer truck are equivalent to the transverse force \( P_{bi} \), the vertical force \( P_{vi} \) applied to the point \( K_i \); the moment \( T_{bi} \) relative to the vertical axis and the moment \( T_{yi} \) relative to the horizontal axis. Expressions for \( P_{bi} \) and \( P_{vi} \) forces, \( T_{bi} \) and \( T_{yi} \) moments [3] have the form:

\[
P_{bi} = a_i \rho_{bi} - e_i \eta_i; \quad P_{vi} = -c_i \eta_i; \quad T_{bi} = b_i \delta^b; \quad T_{yi} = -d_i \zeta; \quad T_{yi} = +e_i Y^b.
\]

(2)

**Figure 1.** Interactions pneumatics with the roadbed: \( a \) – lateral movement; \( b \) – torsion; \( c \) – the inclination of the wheel plane.

In some cases, it is possible to simplify the equations of motion, in particular, the mixer truck, excluded from the ratio equations system (1). The order of the equations system describing the motion of the mixer truck on \( m \) wheels is reduced by \( 2m \) equations. Such a simplification is possible while neglecting transient processes in concrete mixer tires. Such neglect is acceptable if the time of the transition process in the tire \( \tau \) is short compared to the shortest time \( \tau \) of the transitional processes in the generalized coordinates describing the motion of the concrete mixer. In this case, it is possible to
use the assumption of movement with “high speed”, according to which $\dot{Y}_b$, $\delta_{bi}$ are negligible compared to the wheel speed $v_i$. The calculation carried out by the dependences [3] shows that for the speed of the concrete mixer truck more than 10 mps $\tau \leq 7 \cdot 10^{-3}$ s. The time $\tau$ of transients at the coordinates describing the movement of the mixer truck is not less than 0.1 s [4].

The condition of the removal hypothesis applicability is in the form: (in [5]),

$$\frac{mV^2}{2} > 2k_{\Delta},$$  \hspace{1cm} (3)

where $\Delta$ is the segment of the median line of the pneumatic tire involved in the deformation. For the concrete mixers, the limit value is $V \approx 10$ kmph. Thus, it is legitimate for the concrete mixers to use the “high speed” traffic assumption.

We write the equations of connections (1) in the coordinate system $0, X_i, Y_i, Z_i$ associated with the center of the $i$-th wheel mixer truck. The axis $0X_i$ is directed along the motion of the mixer parallel to its plane of symmetry, axis $0Y_i$ – to the left motion, $0Z_i$ axis – vertically upwards. The transformation is carried out in accordance with the dependencies:

$$\cos(\Omega - \gamma) \hat{X}_i = X_i, \quad \sin(\Omega - \gamma) \hat{Y}_i = Y_i, \quad \sin(\Omega - \gamma) \hat{Z}_i = Z_i,$$  \hspace{1cm} (4)

where $\Omega$ is the angle between the axes $0\hat{X}_i$ and $0X_i$.

Substituting the expression (4) in (1), taking into account that $\varsilon = \Omega + \nu$, (5)

$$\delta_{bi} + v_i \frac{\dot{Y}_i - \delta_{bi}}{v_i} = 0; \quad \dot{\delta}_{bi} + \alpha_i Y_{bi} + \beta \delta_{bi} + \xi \chi_i = 0.$$  \hspace{1cm} (6)

Note that $\dot{X}_i$ is equal to the speed of the $i$-th wheel $V_i$ and that the deformation $\delta_{bi}$ and the rotation angles $\nu_i$ controlled wheels are small. Neglecting the small second order, we obtain:

$$\delta_{bi} + v_i \frac{\dot{Y}_i - \delta_{bi}}{v_i} = 0; \quad \dot{\delta}_{bi} + \alpha_i Y_{bi} + \beta \delta_{bi} + \xi \chi_i = 0.$$  \hspace{1cm} (7)

In accordance with the theory, developed in [3], when driving at “high speed” we can neglect the speeds $\dot{Y}_b$, $\dot{\delta}_{bi}$ in comparison with $V_i$. So, we get the expression:

$$\delta_{bi} + v_i \frac{\dot{Y}_i - \delta_{bi}}{v_i} = 0; \quad \dot{\delta}_{bi} + \alpha_i Y_{bi} + \beta \delta_{bi} + \xi \chi_i = 0.$$  \hspace{1cm} (8)

It is possible to express the deformation characteristics $Y_b$, $\delta_{bi}$ from (8) in explicitly and substitute them in the expressions (2) for the forces acting on the mixer wheel. As a result, we obtain:

$$P_{bi} = -k_{1i}(\frac{\dot{Y}_i - v_i - \dot{\delta}_{bi}}{v_i}) - k_{2i}\chi_i = 0; \quad T_{bi} = k_{3i}(\frac{\dot{Y}_i - v_i}{v_i}), \quad T_{bi} = -k_{4i}\chi_i; \quad T_{bi} = -k_{5i}(\frac{\dot{Y}_i - v_i}{v_i});$$  \hspace{1cm} (9)

$$k_{1i} = \beta_i; \quad k_{2i} = e_i; \quad k_{3i} = a_i; \quad k_{4i} = d_i; \quad k_{5i} = e_i.$$  \hspace{1cm} (10)

The conditions (9) are obtained using the Keldysh theory of rolling elastic pneumatics under the assumption of “high speed” motion. We demonstrate the relationship between these equations and the equations for the forces acting on the rolling wheel of the mixer truck, adopted in the technical theory of trucks stability.
Deviations of the motion of an auto concrete mixer from a straight line will be considered small. In this case, the coordinates $X_M^*$, $Y_M^*$ of the center of the contact area of the point $M$ are expressed by the formulas:

$$X_M^* = X^* - Y_b \sin \varsigma \approx X^*; \quad Y_M^* = Y^* - Y_b \cos \varsigma \approx Y^* + Y_b,$$

where

$$\frac{dY_M'}{dS} = \frac{dY_M'}{V dt} = \frac{1}{V} (Y^* + Y_b).$$

The curvature of the rolling line:

$$R^{-1} = \frac{d^2 Y_M'}{dS^2} = \frac{d}{dS} (\varsigma + \delta_b) = \frac{1}{V} (\varsigma + \delta_b).$$

Under the assumption of the $\Omega$ small, the speed $\dot{Y}_b$ can be considered equal to $Y$. Speeds $\dot{Y}_b$ and $\dot{\delta}_b$ are neglected in comparison with $V_i$ in accordance with the assumption of movement with “high speed”. Then:

$$\frac{dY_M'}{dS} = \frac{\dot{Y}_b}{V}; \quad \frac{d^2 Y_M'}{dS^2} = \frac{\hat{\varsigma}}{V} \text{ and } \frac{\dot{\varsigma}}{V} + \frac{\dot{\varsigma}}{V} - \frac{dY_M'}{dS} = \frac{1}{dS} \left( 1 \frac{d^2 Y_M'}{dS^2} - \varsigma \right).$$

The coefficient $\beta_i$ in physical meaning has the dimension of length and is equal to [3]

$$\beta_i = \frac{r_j}{2}.$$

Taking into account (19), the expression (14) can be written as follows:

$$\frac{\dot{Y}}{V_i} + \frac{\dot{\varsigma}}{V_i} - \frac{\dot{v_i}}{V_i} = \frac{1}{r} \left( \frac{dY'_m}{dS} + \frac{1}{dS} \frac{d^2 Y'_m}{dS^2} - \varsigma \right).$$

where $r$ is the length of the rolling line section equal to the rolling radius of the wheel.

The expression in brackets can be considered as the first members of the power series of the function $Y_M'$. It should be noted that we can always put $Y_M' = 0$ at $S = 0$. It is also possible to ignore the second member of the series by limiting it to the linear member only (this can be done because $r < 1$). In this case, we obtain a classical definition of the car wheel withdrawal angle [6], adopted at small angles:

$$\delta_i = \frac{\dot{Y}}{V} - \dot{v_i}.$$

Substituting (21) in the expressions (9)-(11), we obtain the equations for the forces acting on the rolling wheel.

$$P_i = -k_1 \delta_i - k_2 \dot{\chi}_i; \quad T_i = k_3 \delta_i; \quad T_i = -k_4 \dot{\chi}_i - k_5 \delta_i.$$

The formulas (18) are widely used in the vehicles’ technical theory of stability and controllability [7]. The coefficients $k_1$, $k_2$, $k_3$, $k_4$, $k_5$ of the equations are generally nonlinear functions of the parameters $\chi$, $\delta$, and the wheel load $G_k$. They can be considered constant in the linear car model. The most significant effect on the motion mixer provides the lateral force resistance to the drift $-k_1 \delta_i$. In the concrete mixers, the normal load on the wheel during the movement varies in a fairly wide range [4]. The paper [7] proposes the following dependence of the drag coefficient $k_1$ on the normal load on the wheel $G_k$

$$k_1 = k_1 \left[ 2.4 \frac{G_k}{G_{opt}} - 1.8 \left( \frac{G_k}{G_{opt}} \right)^2 + 0.4 \left( \frac{G_k}{G_{opt}} \right)^3 \right].$$
where $k_y_{\text{max}}$ is the maximum value of the coefficient of resistance of the withdrawal as a function of the normal load; $G_{opt}$ is the value of the normal load at which $k_y_{\text{max}}$ is achieved. The expression (25) accurately approximates the dependence of the coefficient of resistance of the withdrawal of the normal load on the wheel in the interval $0 \leq G_{opt} / G_{opt} \leq 2$. When $G_{opt} / G_{opt} > 2$ physically unreasonable increase of the coefficient of resistance of withdrawal is observed. Therefore, the following dependence $k_1$ on $G_{ki}$, free from this drawback, is used later:

$$
k_1 = \begin{cases} 
  k_y_{\text{max}} \left[ 2.4 \frac{G_{ki}}{G_{opt}} - 1.8 \left( \frac{G_{ki}}{G_{opt}} \right)^2 + 0.4 \left( \frac{G_{ki}}{G_{opt}} \right)^{2.7} \right] & \text{for } npu \ 0 \leq \frac{G_{ki}}{G_{opt}} \leq 2 \\
  0.8 k_y_{\text{max}} \frac{G_{ki}}{G_{opt}} & \text{for } npu \frac{G_{ki}}{G_{opt}} > 2
\end{cases}
$$

(20)

The coefficients $k_2$, $k_3$, $k_4$, $k_5$ are further assumed to be constant. The normal load of $G_{ki}$ is calculated by the formula:

$$
G_{ki} = P_{vi} + G_{ki\text{ st}},
$$

(21)

where $G_{ki\text{ st}}$ is the gravity force on the $i$-th wheel of the mixer truck.

In the future, when determining the normal reaction of $P_{vi}$ acting on the wheel, the force of elastic hysteresis or viscous resistance is taken into account. The expression for the normal reaction takes the form:

$$
P_{vi} = -C_i(1 + j\varepsilon_i)\eta_i; \quad P_{vi} = -C_i \eta_i - k_i \dot{\eta}_i,
$$

(22)

where $\varepsilon_i$, $k_i$ are the coefficients of hysteresis losses and equivalent viscosity resistance, respectively. The expressions (18) and (22) define the forces acting on the rolling wheel of the concrete mixer truck.

The above-shown dependencies are considered from the standpoint of the established withdrawal, i.e. the curvilinear motion on a smooth surface. In real conditions of movement on the road irregularities, there are oscillations of the lateral force perceived by the wheel at a given angle of withdrawal. As a result, the lateral oscillations of the concrete mixer, caused by lateral compliance of the tires, can have a significant impact on the controlled movement of the concrete mixer. Therefore, in order to take into account, the lateral compliance of tires it is proposed [6] to introduce the concept of the contact area withdrawal angle $\delta_{ai}$, defining it as the angle between the plane of the wheel and the velocity vector of the contact area. Then:

$$
P_{bi} = -k_{i1} \delta_{ai} - k_{i2} \chi_i \dot{\chi}_i = C_i V_i (\delta_i - \delta_{ai}),
$$

(23)

where $C_i$ is the lateral stiffness of the pneumatics

Excluding $\delta_i$ from equations (29), we obtain:

$$
- \frac{k_u}{C_i V_i} \dot{\hat{\chi}}_i + P_{bi} = -k_{i2} \chi_i - k_{i2} \chi_i.
$$

(24)

Thus, the expressions (18), (22) and (24) determine the forces acting on the rolling wheel of the mixer truck.

In fig. 2, according to the obtained formulas, the dependences of the drag coefficients for the lateral withdrawal of k1 tires 11.00R20 and 9.00R20 on the normal load at tire pressures of 0.84 MPa and 0.65 MPa, respectively, are calculated, proving with the results of experimental studies of MADI, NAMI (Russia).
3. Summary
1. The wheeled concrete transport vehicles are the most vulnerable to interaction with the roadway due to additional disturbance from the side of the movable material (non-Newtonian fluid) during its transportation.
2. The use of the wheel rolling theory (elastic pneumatic), developed by the Academician Keldysh M.V., permits more precise consideration of the concrete-transporting vehicle wheels interaction with the roadbed.
3. The dependencies obtained from the developed formulas allow them to be used in the process of computer simulation of testing wheeled concrete transport machines, i.e. computational experiment, as close as possible to full-scale, which is not safe.

4. References
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