Numerical Simulation of Successive Collision of Two Liquid Droplets with a Solid Wall

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(Received on December 11, 2000; accepted in final form on February 19, 2001)

Spray jet impingement is widely used to cool hot solids in iron- and steel-making processes. Numerous droplets impinge onto a solid at random and interact with each other. The present study treats the deformation behavior of two liquid drops one by one impinging coaxially onto a solid. The system of Navier–Stokes equations for incompressible fluid flow in the axisymmetric coordinate system is solved by means of a finite difference method. The effect of surface tension, gravity, and wettability between the liquid and the solid is taken into account. First, the deformation behavior of a single drop onto a solid is examined and compared to the experimental data for model validation. Then, the collision of two drops in tandem with the solid is simulated. The effect of the distance between two drops on the deformation behavior is studied. The physics of interaction phenomena of droplets is investigated theoretically.

KEY WORDS: spray cooling; numerical analysis; collision dynamics of droplets onto a solid; interaction of droplets.

1. Introduction

In iron and steel-making industries, spray jet impingement is widely used to cool hot solids. For instance, in the continuous casting process, the temperature of hot metals is controlled by means of the impingement of water sprays in the secondary cooling zone. Up to today, collision of a liquid droplet with a solid has been studied as fundamental research on the spray cooling.1–7) Akao et al.,1) Hatta et al.,2–4) Fujimoto et al.5) observed the deformation behavior of a droplet impinging onto a hot solid for various experimental conditions. Hatta et al.,6) Fujimoto and Hatta7) studied the collision of a droplet with a solid theoretically. These studies provide useful information on the physics of drop impingement.

In the actual flow of the spray cooling, numerous droplets impact the hot solid surface at random. The flow field is very complex in the vicinity of the hot solid because the interaction between droplets occurs very frequently. Thus, understanding the physics of droplet–droplet interaction phenomena is indispensable to comprehend the nature of the flow appearing in the actual spray cooling. However, little attention to the droplet–droplet interaction phenomena has been paid. Further, there are many factors affecting the collision behavior of droplets such as the impact velocity, number density of droplet, droplet diameter, and so on.8) The physics of phenomena remains unclear.

This study treats the collision of two droplets one by one with a solid at room temperature. The two droplets coaxially impinge on the solid with a certain time interval (see Fig. 1). The main purpose is to numerically understand the interaction phenomena of the droplets. The heat transfer between the droplets and the solid is neglected. The Navier–Stokes equations for incompressible fluid in the axisymmetric coordinate system are solved using a finite difference method. The effect of impact velocity, an interval between two incoming droplets on the deformation behavior of droplets is investigated. The flow structure is discussed in detail from a theoretical point of view.

2. Conservation Equations and Numerical Procedure

Figure 1 shows a schematic of the flow and coordinate system. Two water droplets coaxially impinge on a smooth solid surface one by one at impact velocity, $v_0$. The droplets are spherical in shape and their diameters, $D_p$, are identical. The first droplet impinges onto the dry solid at time $t=0$. The next droplet impinges onto the top of the
first droplet after a time interval, \( \tilde{t}_p \). In the present study, the effect of the time interval, \( \tilde{t}_p \), on the merged liquid motion is investigated in detail theoretically.

The liquid motion is assumed to obey the Navier–Stokes equations for incompressible viscous fluid in the axis symmetric system. The effects of gravity, viscosity, surface tension, and wettability between the liquid and the solid are taken into account. The wettability is specified by means of the contact angle at the liquid/solid interface. The heat transfer between the droplets and the solid is neglected. In addition, the effect of airflow surrounding the droplets is not taken into account. Since the liquid density (\( \approx 1000 \text{ kg/m}^3 \)) is approximately 830 times larger than the gas density (\( \approx 1.2 \text{ kg/m}^3 \)) at atmospheric pressure and temperature, the momentum of the surrounding airflow is negligibly small compared to the liquid. Thus, it is reasonable to consider that the liquid motion is little affected by the momentum transfer between the airflow and the liquid.

Every variable is normalized using the initial droplet diameter, \( D_p \), and/or the impact velocity of the first droplet, \( \tilde{v}_0 \), as the following forms,

\[
t = \tilde{t}/(\bar{D}_p/\tilde{v}_0), \quad r = r/\bar{D}_p, \quad z = z/\bar{D}_p
\]

\[
 u = \tilde{u}/\tilde{v}_0, \quad \bar{v} = \bar{v}/\tilde{v}_0, \quad \bar{p} = \tilde{p}/\tilde{v}_0^2 \]

where \( t, (r,z), (\tilde{u}, \tilde{v}), \) and \( \bar{p} \) represent the dimensionless time, coordinates, velocity components, and pressure, respectively. \( \tilde{\rho} \) denotes the liquid density. Also, overbars mean the dimensional variables. The dimensionless conservation equations are given as the following forms:

\[
\frac{\partial \tilde{u}}{\partial r} + \frac{\partial \tilde{v}}{\partial z} + \tilde{u} = 0 \quad \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdOTS (2)
\]

\[
\frac{\partial \tilde{u}}{\partial t} + \frac{\partial \tilde{u}^2}{\partial r} + \frac{\partial \tilde{v} \tilde{u}}{\partial z} + \tilde{u}^2 = -\frac{\partial \tilde{p}}{\partial r} + \frac{1}{\text{Re}} \left( \frac{\partial^2 \tilde{u}}{\partial r^2} + \frac{\partial^2 \tilde{u}}{\partial z^2} + \frac{\partial \tilde{u}}{\partial r} \right) \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdOTS (3)
\]

\[
\frac{\partial \tilde{v}}{\partial t} + \frac{\partial \tilde{u} \tilde{v}}{\partial r} + \frac{\partial \tilde{v}^2}{\partial z} + \tilde{v} \tilde{w} = -\frac{\partial \tilde{p}}{\partial z} + \frac{1}{\text{Re}} \left( \frac{\partial^2 \tilde{v}}{\partial r^2} + \frac{\partial^2 \tilde{v}}{\partial z^2} + \frac{\partial \tilde{v}}{\partial r} + \frac{\partial \tilde{w}}{\partial r} \right) \cdots \cdots \cdots \cdots \cdots \cdots \cdOTS (4)
\]

in which \( g \) indicates the dimensionless gravitational acceleration. \( \text{Re} \) represents the Reynolds number based on the pre-impact velocity of the first droplet and the droplet diameter defined as:

\[
\text{Re} = \frac{\bar{\rho} \tilde{v}_0 \bar{D}_p}{\bar{\mu}} \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdOTS (5)
\]

in which \( \bar{\mu} \) is the liquid viscosity.

The boundary conditions are explained next. At the symmetric boundary (\( r = 0 \)), the symmetric condition is imposed. No-slip condition is used at the liquid/solid boundary. At the free liquid surface, the following Laplace equation is used to determine the surface pressure.

\[
p_s - p_{sa} = \frac{1}{\text{We}} \left( \frac{1}{R_1} + \frac{1}{R_2} \right) \quad \cdots \cdots \cdots \cdots \cdots \cdOTS (6)
\]

in which \( p_s \) and \( p_{sa} \) are the pressure at the free liquid surface and the atmospheric pressure, respectively. \( R_1, R_2 \), and We are the principal radii of curvature at a free liquid surface, and the Weber number, respectively. The absolute values of \( R_1 \) and \( R_2 \) in Eq. (6) are given by

\[
(R_1, R_2) = \left[ \left( r_s + R_1^2 \right)^{1/2} z_s, \left( r_s + R_2^2 \right)^{1/2} z_s \right] \quad \cdots \cdots \cdots \cdOTS (7)
\]

in which \( (r_s, z_s) \) represent the coordinates at a free surface. The signs of \( R_1 \) and \( R_2 \) are dependent upon the shape of the local free liquid surface. The Weber number based on the pre-impact velocity of the first droplet and the droplet diameter is defined as:

\[
\text{We} = \frac{\bar{\rho} \tilde{v}_0^2 \bar{D}_p}{\bar{\sigma}} \quad \cdots \cdots \cdots \cdots \cdots \cdOTS (8)
\]

where \( \bar{\sigma} \) is the surface tension. At the contact line, which is defined by the free liquid surface at the solid, the advancing contact angle is specified when the contact line advances. In the present study, the advancing contact angle is set to 110 degrees.

The conservation equations are solved by a finite difference method.\(^{(9)}\) The explicit Euler scheme is used for the time integration. The advection terms are approximated using an upwind scheme.\(^{(10)}\) A second-order central differencing scheme is applied for the diffusion terms. The momentum equations are solved by a two-step projection method.

The time increment \( \Delta t \) is determined by the CFL (Courant–Friedrich–Levy) condition as:

\[
\Delta t = \text{Min} \left( \frac{f \Delta r}{\bar{u} + 2/\text{Re} \Delta r}, \frac{f \Delta z}{\bar{v} + 2/\text{Re} \Delta z} \right) \quad \cdots \cdots \cdots \cdOTS (9)
\]

in which the CFL number \( f \) is set to 0.1 in this study.

The non-uniform grid system is employed to compute the flow in the liquid. The grid point in the axial direction is given by

\[
z_{i+1} = z_i + \Delta z_i, \quad \Delta z_i = \frac{1}{1024} (1.01)^i, \quad i = 1, 2, 3, \ldots, \quad z_0 = 0 \quad \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdOTS (10)
\]

Also, the radial mesh size \( \Delta r \) is fixed to be 1/256.

The time evolution of the free liquid surface is tracked by the Lagrangian movement of numerous consecutive segments.\(^{(5)}\) The free surface positions at two consecutive time steps are formulated by

\[
r_{s+1} = r_s + \Delta t u_s, \quad z_{s+1} = z_s + \Delta t v_s \quad \cdots \cdots \cdots \cdOTS (11)
\]

in which \( n \) indicates a time step. The principal radii of the free liquid surface in Eq. (7) are calculated using the coordinates \( (r_s, z_s) \) at the free surface. The first and second derivatives appearing in Eq. (7) are determined by a finite difference approximation using some adjacent free surface
3. Results and Discussion

In order to validate the present numerical model, the comparison of numerical and experimental results is carried out. The collision of a single droplet into a solid is treated for the model validation, because it is difficult to perform the experiments for two droplets collision in tandem with a solid. Experiments are carried out in the case where water droplets with about 2.4 mm diameter impinges vertically onto a very smooth glass surface at the impact velocity 0.8–2.1 m/s. The time evolution of the droplet diameter at the liquid/solid interface is measured using a flash-photographic method. Experimental procedure and measurement method are identical to those in our previous paper.\textsuperscript{11) Figure 2} shows the comparison of the time evolution of the predicted dimensionless droplet diameter, $d$, with the experimental data. The diameter of circular wet area on the solid is chosen as the droplet diameter, as shown in the figure. The droplet diameter increases with time, and then reaches a maximum value. The spreading liquid diameter increases with the impact velocity. The numerical results agree very well with the experimental data for all cases.

Figure 3 represents the time evolution of velocity profile and pressure contour of the single droplet onto the solid for the case of $\bar{v}_0=2.1$ m/s. After the collision, a circular liquid film is formed around the bottom of the droplet and spreads radially along the solid surface. Then, the droplet is deformed into a thin circular disk. The ring structure is formed in the peripheral region. The predicted droplet deformation process agrees qualitatively with the photographic observations by some researchers.\textsuperscript{4,11,12) Incidentally, it is experimentally known that the droplet splashes on the solid surface if the impact velocity is large.\textsuperscript{12,13) The liquid drop breaks up into several secondary droplets during the collision. In such a case, the liquid motion is three-dimensional in nature and the present axisymmetric model is not applicable. Therefore, the splashing boundary is examined experimentally. Some experimental formulæ have been proposed to predict the splash boundary as a function of the Reynolds and Weber numbers. Mundo et al.\textsuperscript{13) showed the correlation of the splashing boundary for relatively smooth surfaces as follows,

$$\text{We}^{0.5}\text{Re}^{0.25}=57.7 \quad \text{..................}(12)$$

In the case of $\bar{v}_0=2.1$ m/s in Fig. 2, the value of $\text{We}^{0.5}\text{Re}^{0.25}$ is 100. Although the value is much larger than the Mundo’s critical, no splashing occurs in our experiments. Stow and Hadfield\textsuperscript{12) reported that the splashing is promoted by the surface roughness and that the splashing boundary defined by $\text{We}^{0.5}\text{Re}^{0.25}$ is given by a function of the surface roughness. They showed that the critical value of $\text{We}^{0.5}\text{Re}^{0.25}$ becomes large for smoother surface. Therefore, the reason why no splashing occurs even at $\text{We}^{0.5}\text{Re}^{0.25}>57.7$, might be due to the surface roughness of the solid. Since the solid surface used in the present experiments is very smooth, Mundo’s expression is not appropriate for the present conditions. In addition, it is confirmed that no splashing occurs even in the case of $\text{We}^{0.5}\text{Re}^{0.25}=164$ ($D_p=2.4$ mm, $\bar{v}_0=3.1$ m/s) in our experiments.

Since the validation of the numerical model is confirmed, the collision behavior of two droplets with a solid is examined. The case where two water droplets with 2.4 mm diameter impinge vertically onto the solid at the impact velocity 2.1 m/s is treated. The time interval, $t_p$ is set to zero. Figure 4 shows the time evolution of velocity profile and pressure contour of merged liquid after the collision. Corresponding Reynolds and Weber numbers are 4600 and 145, respectively. A high-pressure region appears near the impact point after the collision. Since the pressure at the free liquid surface is approximate to the atmospheric one, a
Therefore, a circular thin liquid large radial pressure gradient occurs near the solid surface. The stagnation pressure decreases with time \( t \leq 0.45 \), while the circular thin film spreads along the solid surface. The periphery of the circular film becomes roundish due to the surface tension effect. The deformation behavior of the first droplet shows very similar trends as the case of single droplet in this time stage (see also Fig. 3).

The pressure inside the second drop is small and unchanged in the time range \( t = 0.45 \). The second drop maintains the spherical shape. Also, the velocity profile inside the second drop is not changed. Since the liquid velocity in the upper part of the first droplet is almost equal to the impact velocity, the first drop slightly affects the liquid motion inside the second drop.

In the later time stage \( t > 0.45 \), a characteristic phenomenon appears. Liquid around the bottom of the second drop swells up \( t = 0.72, 0.90 \). For convenience, the circular swelling liquid is called ‘liquid crown’. The formation mechanism of the liquid crown is discussed. At \( t = 0.72 \), the pressure in the bottom part of the second drop is higher than that in the upper part. A relatively large pressure gradient normal to the free surface appears around the bottom of the second droplet. Thus, the liquid is accelerated obliquely upward, and swells up. The pressure near the stagnation point at \( t = 0.9 \) is apparently higher than that at \( t = 0.72 \) because of the inertia of the second droplet. This leads to the development of the liquid crown. After that, the crown moves radially outward. It reaches the periphery of the liquid at \( t = 2.0 \) because the spreading rate of the liquid crown is faster than that of the contact line.

Next, the case of \( t_d = 0.35 \) is examined. Figure 5 shows the time evolution of velocity profile and pressure contour of merged liquid after the collision. The pressure at the impact point of the second drop onto the first one becomes high at \( t = 0.35 \) because of the slight velocity difference between the second drop and the top part of the first drop. Liquid swells around the bottom of the second drop and it spreads radially with time. The deformation behavior of merged liquid shows trends similar to the case of \( t_d = 0.0 \).

Figure 6 depicts the numerical results for \( t_d = 1.3 \). The liquid film thickness of the first drop on the solid is very thin at the moment of the impact of the second drop. The circular liquid crown is higher and thinner than that in the previous two cases. Also, it is formed in earlier time stages. The circular liquid swelling is clearly seen at \( t = t_d = 0.2 \) in Fig. 6, while it does not for \( t_d = 0.35 \). It is confirmed that \( t_d \) affects the merged liquid motion.

Figure 7(a) shows the time evolution of dimensionless droplet height, \( h \), for \( t_d = 0.0, 0.35 \) and 1.3, respectively. The dimensionless droplet height, \( \tilde{h} \), is defined by the distance between the highest point of the merged liquid and the solid surface as shown in Fig. 7(b). It is found that the time evolution of the droplet height for \( t_d = 0.0 \) and 0.35 is quite similar in the entire time stages examined in the present study. The droplet heights for \( t_d = 0.0 \) and 0.35 coincide with the linear function \( h = 2 - t \) in the time range \( t < 0.8 \). This means that the droplet height decreases in proportion to the impact velocity with time. After that, the decreasing rate of \( h \) becomes small, and reaches a minimum height at \( t = 2.1 \). Subsequently, the droplet height increases a little.

It is seen from Figs. 3 and 4 that the liquid thickness at \( r = 0 \) decreases with time. It becomes smaller than the thickness of the circular liquid crown after \( t = 2.1 \) or so. \( h \) is determined by the liquid thickness at \( r = 0 \) in \( t < 2.1 \), while \( h \) is given by the thickness of the crown in the later time stages.
This is the reason why $h$ reaches a minimum value at $t=2.1$.

The time evolution of $h$ for $t_d=1.3$ shows similar trends for $t_d=0.0$ and $0.35$. The droplet height decreases, reaches a minimum value, and then increases. The minimum height is larger than other two cases, because the circular liquid crown develops so high. Also, the time reaching the minimum height is a little later.

**Figure 8** represents the comparison of the time evolution of $d$ for the case of single droplet with the case of two droplets. The definition of $d$ for the two droplets case is given in Fig. 7(b). The dashed and solid lines show the result for the single droplet and for two droplets ($t_d=0.35$), respectively. It is apparent that the droplet diameters at the solid surface for two cases are almost identical in the time range $t\leq2.1$. After $t=2.1$, the diameter for the case of two droplets becomes larger than the case of single droplet. Since the circular swelling liquid reaches the periphery at $t=2.1$ as shown in Fig. 4, the spreading rate of the contact...
Incidentally, when liquid droplets impacts onto a hot surface, the liquid/solid direct contact occurs first. Then, the vapor bubbles or films are formed at the liquid/solid interface due to boiling. The vapor film prevents the liquid/solid direct contact. The heat transfer from the hot solid to the droplets is dependent strongly upon the liquid/solid direct contact area before the vapor films are formed. Because the heat transfer between the vapor and the hot solid is much smaller than that between the liquid and the solid. Since the present simulations are carried out assuming isothermal conditions, the results are not directly applicable to understand the physics of the impact of droplets onto hot solid appearing in the actual spray cooling. However, the results shown in Fig. 8 provide some interesting information concerning heat transfer from the hot solid to the droplets. It is seen that the second droplet does not contribute to increase the liquid/solid direct contact area in the early time stages. Thus, the two droplets collision in tandem is not expected to promote the heat transfer between the liquid and the solid compared to the case of single droplet. In fact, it has been reported that the heat transfer coefficient between spray jet and a hot solid is not proportional to the number density of droplets. This discussion is consistent with the experimental results.

4. Conclusions

The collision behavior of two droplets one by one impacting coaxially onto the solid is studied numerically. The model prediction is validated by comparing with the experimental data for the case of single droplet. It is found that the deformation behavior of the first droplet shows trends similar to that for the case of single droplet. The circular liquid crown is formed around the bottom of the second droplet. It spreads radially, and then reaches the periphery. The time interval between the first and second droplets affects the size of the circular liquid crown. If the time interval is large, the crown becomes high and large.

Acknowledgement

This study has been supported by Grant-in-Aid for Scientific Research (B-2 (11450274)), (C-2 (11650739)) which is provided by the Ministry of Education, Science, Sports and Culture in Japan.

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