The in-medium scale evolution in jet modification

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The in-medium modification of the scale dependence of the fragmentation function in dense matter, brought about by higher twist corrections to the Dokshitzer-Gribov-Lipatov-Altarelli-Parisi (DGLAP) evolution equations, is derived. A phenomenologically motivated resummation is outlined which incorporates the next-to-leading twist single gluon emission kernel along with the vacuum emission kernel and provides an in-medium virtuality evolution of the final fragmentation function of a hard jet propagating through dense matter. The concept of a fragmentation function is generalized to include a dependence on distance travelled in the medium. Following this, numerical implementations are carried out and compared to experimental results on the single inclusive suppression observed in Deep-Inelastic scattering (DIS) off a large nucleus.

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With the commissioning of the Large Hadron Collider (LHC) heavy-ion experiments, the study of hard probes in deconfined matter will have entered the deep perturbative domain: one expects jets with energies of several hundred GeV to be produced, plough through the dense deconfined matter and fragment far outside the medium [1]. With the start of the future Electron-Ion Collider (EIC), a similar step will be taken in the study of cold nuclear matter with hard jets. Current experiments at the Relativistic Heavy-Ion Collider (RHIC) [2, 3] and at DESY (HERMES) [4] have just begun to probe the perturbative regime. At RHIC this is believed to start above a rather high transverse momenta \( p_T \sim 6 - 7 \text{ GeV} \) (due to the expanding medium) [5]. In DIS, the extent of the perturbative region depends on the energy (\( \nu \)) lost by the electron, the fraction carried by the detected hadron and the size of the struck nucleus [6].

In the case of single-hadron-inclusive suppression in DIS on a large nucleus \( A \), the application of perturbative methods is based on the factorized cross section to produce a hadron with a fraction \( z \) of the photon light-cone momentum \( q^- \) [7]:

\[
\frac{d\sigma_i^h(Q^2)}{dz} = \sum_j F_i(x_i, Q^2)\sigma_{i\rightarrow j}(Q^2)\tilde{D}_j^h(z, Q^2). \tag{1}
\]

In this equation, \( F_i(x_i) \) is the parton distribution function for a parton \( i \) with momentum fraction \( x_i \), \( \sigma_{i\rightarrow j} \) is the hard cross section with the virtual photon to produce parton \( j \) and \( \tilde{D} \) is the medium modified fragmentation function to fragment into the hadron \( h \) after undergoing multiple scattering in the nuclear medium [8].

The multiple scattering of the hard parton in the medium generically has two parts, a perturbatively calculated contribution which represents the scattering and gluon radiation from the hard parton, and the non-perturbative distribution of the soft gluons off which the hard partons will scatter [9]. Both these parts in combination with the standard vacuum shower of gluons leading to the non-perturbative fragmentation into observable hadrons are included within the definition of the medium modified fragmentation function. The three different factorized parts in Eq. (1) will necessarily depend on the factorization scales (\( \mu_i \) for the initial state and \( \mu_f \) for the final) which in Eq. (1) have been chosen as the hard scale \( Q^2 \). So defined, the medium modified fragmentation functions will reduce to the standard vacuum fragmentation functions \( D(z, Q^2) \), as the extent of the medium in the final state is reduced.

As the energies of the process are increased, all three factorized functions in Eq. (1) change with \( Q^2 \). While volumes of work exist on the perturbatively calculable scale dependence of both the hard part and the nuclear structure functions (as well as vacuum structure and fragmentation functions) [10, 11, 12], little attention has been paid to the scale dependence of the medium modification of the fragmentation functions. In the case of DIS on a nucleon or in \( p-p \) collisions, i.e., in the absence of an extended medium, the vacuum fragmentation functions have a well known and perturbatively calculable dependence on the scale \( Q^2 \) of the process given by the DGLAP evolution equations [10]:

\[
\frac{\partial D_q^h(z, Q^2)}{\partial \log(Q^2)} = \frac{\alpha_s(Q^2)}{2\pi} \int_z^1 \frac{dy}{y} P_{q\rightarrow i}(y)D_q^h\left(\frac{z}{y}, Q^2\right). \tag{2}
\]

In the above equation, \( \alpha \) is the strong coupling constant and \( P_{q\rightarrow i} \) represents the probability for a parton \( q \) to split into an parton \( i \) which carries a fraction \( y \) of the original momentum and finally fragments into the hadron \( h \) (a sum over \( i \) is implied). To date, the analogue of such an evolution equation in a medium has not been given. It is the object of this Letter to carry out such an extension and to present the first numerical results of such an evolution in an extended dense medium. An alternative approach to this problem leading to a Monte-Carlo implementation has been proposed in Ref. [13].

It should be pointed out that the factorized form
Eq. (1) is an assumption that has not yet been proven to hold [14]. So far such a factorized form has been shown to hold (at leading twist) in the absence of a medium. Factorization of the hard part from the initial distribution has also been demonstrated to hold at all twist in the case of the totally inclusive cross section in DIS and up to next-to-leading twist for the single-hadron-inclusive cross section [14]. While it may reduce the rigor of the present approach, we persist with this form (in the absence of a more accurate formalism), seeking justification in phenomenological applications. While the LHC and EIC will engender energy scales that are higher by an order of magnitude, current experiments at RHIC and HERMES already sample a wide enough range of scales to provide a sufficient test for this formalism.

Imagine that a quark jet is produced in a hard collision with a large virtuality $M^2$. This virtuality is then reduced by a sequence of partonic emissions. The emitted partons will possess a lower virtuality than the parent. These will then decay further into lower virtuality branches and the process will continue. The subsequent branchings and the development of the partonic shower may be calculated using pQCD as long as the virtuality drops to some predetermined value $\mu^2$. Once the virtuality drops to some predetermined value $\mu^2$ (a few times $\Lambda_{QCD}^2$), a non-perturbative object such as a fragmentation function [14] needs to be introduced.

In order to incorporate the effect of a medium on this cascade process, the fragmentation function has to be generalized to include position dependence, i.e.,

$$D(z, M^2) \rightarrow D(z, M^2, q^-)^{\bar{\zeta}_f}_{\zeta_i}. \quad (3)$$

$\zeta_i$ denotes the location of the production of a hard quark with large negative light cone momentum $q^-$ and virtuality $M^2$ in an extended dense medium. The leading parton from the shower ensuing from such a quark is assumed to exit the medium at location $\zeta_f$. We will insist that the final exiting parton which fragments to produce the detected hadron possesses virtuality $m^2(\gg \Lambda_{QCD}^2)$ that is much larger than the transverse momentum gained by the jet in traversing the medium (we chose this to be $\gtrsim 1$ GeV for jets punching through large nuclei).

The virtuality drop from $M^2$ to $m^2$ may be achieved through any number of emissions. In the case of single emission without scattering, there is no dependence on location and the change in $D(z, M^2)$ is given as

$$\Delta D_q^h(z, M^2) = \frac{\alpha_s}{2\pi} \frac{m^2}{2\pi} \int \frac{d^2 l^2}{l^2} \int \frac{1}{z} dy P_{q\rightarrow i}(y) D^h_q \left(z, \frac{m^2}{y^2}\right). \quad (4)$$

The splitting function $P_{q\rightarrow i}(y)$ above contains the probability for a quark $q$ to radiate a gluon with momentum fraction $y$ and transverse momentum $l$. Virtual corrections which conserve unitarity are implicitly included.

The case of single gluon radiation accompanied by one scattering is given by the diagrams in Fig. 1. Only the relevant amplitudes are drawn. The gluons attached to the dark circles are meant to indicate gluons exchanged with the medium when the hard partons scatter off it. The light red (shaded) ellipses on the Feynman diagrams denote the propagators that are considerably more virtual than the remaining parton lines. To the five diagrams included, one must add contributions with no scattering as well as contributions denoting the interference of double scattering with no scattering diagrams and virtual diagrams (denoted as v.c.). The two diagrams in the top row of Fig. 1 where the initial parton with a large virtuality splits into two partons with lower virtuality, followed by a single soft scattering, is similar to the case in vacuum. The single soft scattering merely introduces a mild change in direction of propagation of the struck line. These diagrams are referred to as the vacuum-like contribution to in-medium induced radiation and their interference with the remaining diagrams as the vacuum-medium-induced interference contribution. The three diagrams in the bottom row represent the case where the scattering with a hard gluon in the medium leads to a rise in the virtuality, immediately lost by gluon emission [8].

The result of the interference of all such diagrams [which leads to the Landau-Pomeranchuck-Migdal (LPM) effect] yields the “medium dependent” correction to the in-medium modified fragmentation function. If the drop in virtuality from $M$ to $m$ were to occur solely through single scattering and single emission, then [3, 8],

$$\Delta D_q^h(z, M^2, q^-)^{\bar{\zeta}_f}_{\zeta_i} = \frac{\alpha_s}{2\pi} \frac{m^2}{M^2} \int \frac{dy}{y} \int \frac{1}{l^2} \frac{\bar{P}_{q\rightarrow i}(y)}{l^2} \int d\zeta \left(\frac{\alpha_s}{4\pi} \rho_g(\zeta, xT) D^h_q \left(z, \frac{m^2}{y}\right) \right. \times 2\pi \frac{\alpha_s}{N_c} \left. \left[2 - 2\cos \left(\frac{\zeta_i - \zeta_f}{2\pi}y(1 - y)\right)\right]\right) \int d\zeta. \quad (5)$$

In the equation above, $\rho_g(\zeta)$ is the gluon density at $\zeta$. The ($-$) superscript on the locations have been dropped for simplicity. The tilde on the splitting function indicates that the color factor and virtual correction are somewhat different from the case of the vacuum (requir-
ing the final outgoing line, after a scattering and an emission to be nearly on-shell restricts the range of \( y \), see Ref. [54] for details). In Eq. 6, \( \zeta \) represents the location of the scattering vertex of the hard parton off the medium in Fig. 1. Due to interference effects, the medium modified fragmentation function is now a function of the jet energy \( q^- \); it is no longer universal and strongly depends on the details of the medium as introduced by the density factor \( \rho_g(\zeta) \) and its space-time distribution. However, the functional dependence of the fragmentation function on the medium density is still universal and thus may be used to compute the medium modified fragmentation function in any medium where \( \rho_g(\zeta) \) is known.

Given that each radiation leads to a drop in the virtuality of the propagating parton, successive radiations are assumed to be strongly ordered in transverse momentum. While this is well known for the case of radiations in the vacuum, in a medium, the strong ordering of the virtualities will be broken if the parton encounters a hard scattering which will lead to a large transverse momentum radiation. At next-to-leading twist, as is the case of higher twist contributions as in Ref. [54], this hard scattering may occur at any later rescattering. In such an ordered scenario, we may write down the expression for the parton to lose virtuality from \( M^2 \) to \( m^2 \) in two ordered emissions as (we suppress \( q^- \) in the argument)

\[
\Delta D^b(g, M^2)\bigg|_{\zeta_1}^{\zeta_f} = \frac{\alpha_s}{2\pi} M^2 \int \frac{d^2l_1}{l_1^2} \int \frac{dy P_{q\to g}^z(y)}{y l_1^2} \int \frac{d\zeta}{\zeta} \times \rho_g(\zeta) \frac{2\pi\alpha_s}{N_c} \left[ 2 - 2\cos \frac{P^2_1(\zeta - \zeta_1)}{2q^-y(1-y)} \right] \frac{2\pi\alpha_s}{N_c} \left[ 2 - 2\cos \frac{P^2_1(\zeta - \zeta_1)}{2q^-y(1-y)} \right]
\]

Due to the drop in virtuality after the first emission, the location of the first scattering in the medium (whether soft or hard), is considered as the origin of the \( \zeta_1 \) integration and thus of the interference pattern connected with the second scattering. Alternatively stated, this means that the above expression focuses only on ladder diagrams both in the scattering and in the emitted gluon sector. The insistence on this is simply based on the dominance of terms which contain a strong ordering of virtualities in successive emissions.

In the strongly ordered \( l_{1\perp} \), or planar emission limit, the emission points for successive lower \( l_{1\perp} \) emissions are also strongly ordered. The extension to multiple emissions, may now be written down similar to Eq. (6). Differentiating the equation for \( D(z, M^2)\bigg|_{\zeta_1}^{\zeta_f} \) (containing multiple emissions) with respect to \( \log(M^2) \) leads to the “medium dependent part” of the evolution equations for the medium modified fragmentation function,

\[
\frac{\partial D^b_g(z, M^2 q^-)}{\partial \log(M^2)}\bigg|_{\zeta_1}^{\zeta_f} = \frac{\alpha_s}{2\pi} \int \frac{dy}{y} \int \frac{d\zeta}{\zeta} P(y)K_{q^-\to M^2}(y, \zeta) \times D^b_g\left(\frac{z}{y}, M^2 q^-\right)\bigg|_{\zeta_1}^{\zeta_f}.
\]

(7)

The single scattering kernel \( K \) is given as

\[
K = \frac{2\pi\alpha_s \rho(\zeta)}{N_c} \bigg[ 2 - 2\cos \left( \frac{M^2(\zeta - \zeta_1)}{2q^-y(1-y)} \right) \bigg].
\]

(8)

The full evolution equation for the medium modified fragmentation functions will include contributions from pure vacuum splitting functions as well as contributions from gluon fragmentation functions which have a similar in-medium evolution.

The simplest application of the formalism developed above is to compute the medium modified fragmentation function in the case of DIS on a large nucleus, where, at least, one hadron with a large forward momentum is detected in the final state. Experiments present the ratio of this with the vacuum fragmentation function at the same \( z \) and \( Q^2 \), called the nuclear attenuation factor [54]. Experimental results for three nuclei (\( N_e, K_f \) and \( X_e \)) are presented in Fig. 2. Assuming single scattering and single in-medium emission, the medium modified fragmentation function may be calculated using Eq. (6); the corresponding attenuation factor is the red dashed line in Fig. 2. In the case of multiple emissions, one will need to use the in-medium evolution equations of Eq. (7). In either case, the basic kernel \( K_{q^-\to M^2}(y, \zeta) \) is identical. To calculate this kernel, we introduce a nucleon density in the large nucleus as an input Ansatz. Due to the simplicity in analytic calculations, we use a hard sphere density distribution (with radius \( R_A \)); \( \rho(\zeta) = \rho_0 \theta(R_A - |\zeta|) \). While \( R_A \) depends on the nucleus, \( \rho_0 \) is a fit parameter dialed to obtain the best overall fit. We also approximate \( D(z)\bigg|_{\zeta_1}^{\zeta_f} \approx D(z)\bigg|_{\zeta_1}^{\zeta_f} \), which greatly speeds calculation.

As may be seen from the comparison with the data in Fig. 2, the fit, for the case of single scattering and single emission (red dashed lines), with smaller nuclei such as \( N_e \) is adequate. The comparison, progressively worsens as one proceeds to larger nuclei. This is clearly seen in the case for \( X_e \) where there seems to be almost a different slope with \( z \) between the calculations and the experimental results. In some ways, this is to be expected; as one proceeds to larger nuclei, the possibility of multiple scattering and multiple emission increases and the results of a formalism which only included single scattering and
Applications of the methods presented in this Letter to the case of high transverse momentum hadron production in heavy-ion collisions has recently been presented in Ref. [12], where Eq. (7) was evaluated in a three dimensional hydro-dynamically expanding medium. Due to the larger error bars in the data at RHIC, the improvement of the present formalism over the medium modified fragmentation functions calculated in the single emission limit (in an identical medium, using Eq. (5) [19]) are somewhat less evident.

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