Underground Cavity Detection Through Spectral Distortion of a GPR Signal

Caleb Leibowitz, Anthony J. Weiss, Fellow, IEEE
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Abstract—We present a novel method to detect underground cavities using cross-borehole ground-penetrating radar (GPR). We model the propagation of a GPR signal across a cavity and find that the spectrum of the signal will be distorted in a specific low-pass manner. Comparing the spectra of received signals with the predicted spectrum using a hypothesis-testing approach allows us to detect cavities with high probability. Our approach remains effective even when conventional methods fail; our method can also be combined with conventional methods in order to more successfully distinguish between cavities and geologic clutter.

I. INTRODUCTION AND RELATED WORK

We consider the problem of using cross-borehole ground-penetrating radar (GPR) to detect underground cavities. GPR has been applied to this problem since at least the 1980s [1], and methods exist to detect cavities and other anomalies using both zero-offset profiling (ZOP) and multiple-offset gathers (MOG). Most cavity-detection methods are based around travelt ime (e.g. [1]–[6]); since an electromagnetic (EM) wave will propagate more quickly in air than in soil, when a signal arrives at the receiver borehole earlier it may indicate that it propagated through a cavity. Imaging using full-waveform inversions (FWI) [7]–[11] is also increasingly used for cavity-detection applications. Other useful features include received energy [1], [12], [13] and time-frequency methods [14]–[16].

An interesting feature for cross-borehole GPR (not applied specifically to the cavity-detection problem) is offered by [17]. They would have liked to use the received energy as a feature for tomographic inversion. Unfortunately, they found that for a variety of reasons, direct measurements of the received energy are often unacceptably noisy (we have also found this to be true in our own measurements). To this end, we measure the centroid frequency of the received signal and use this quantity as a proxy for the attenuation of the signal.

We model the propagation of an EM wave through a cavity and show that this will tend to have a low-pass effect on the spectrum of the signal. This spectral distortion can be used as a feature for cavity detection. Indeed, we show that cavities can sometimes be detected using only the centroid frequency of [17]. However, having a model of what the spectrum of the received signal should look like both in the case in which the signal passed through a cavity and in the case in which it did not pass through a cavity, we can adopt a hypothesis-testing approach. As was found by [18], adopting a hypothesis-testing approach dramatically improves the efficacy of our method.

In the following section we model the propagation of an EM wave across a cavity. We subsequently present our method of cavity detection and validate it using both simulated and real GPR measurements. Finally, we conclude and offer directions for future research.

II. MODELING PROPAGATION ACROSS A CAVITY

Consider a plane wave propagating between two boreholes as shown in Figure 1. The wave propagates a distance $s_1$ through soil, across a cavity of width $a_1$ containing only free space, and a further distance $s_2$ through soil before arriving at the receiver borehole. At each interface a portion of the wave is transmitted and a portion is reflected. It is well known [19] that a plane wave with initial magnitude $E_0$, after propagating a distance $h$ through a medium, is given by

$$E = E_0 e^{jωt - γh} = E_0 e^{-αh} e^{j(ωt - βh)}$$

(1)

where $γ = α + jβ$ is the propagation constant, $α$ is the attenuation constant, and $β$ is the phase constant. Proceeding similarly to [20], we write equations for the waves shown in Figure 1, where the wave is measured at the tail of the arrow (the wave $w_0$ is ignored as it does not affect the field measured at the receiver borehole). Letting $E_{rc}$ denote the wave at the receiver borehole after propagation through a cavity, $k = ω/γ$ be the wavenumber of the wave in free space, $t_{sa}$ and $t_{as}$ be the coefficients of transmission from soil to air and from air to soil respectively, and $r_{as}$ be the coefficient of reflection.
from air to soil, and suppressing the $e^{j\omega t}$ time dependence as is customary, we write

$$E_{rc} = v_2 e^{-s_2 \gamma}$$  \hfill (2a)

$$v_2 = v_1 t_{as} e^{-jka_1}$$  \hfill (2b)

$$v_1 = v_0 t_{sa} e^{-\gamma s_1} + r_{as} w_1 e^{-jka_1}$$  \hfill (2c)

and

$$w_1 = r_{as} v_1 e^{-jka_1}.$$  \hfill (2d)

Solving and making explicit the $e^{j\omega t}$ time dependence, we find that if we let $v_0 = A_0 e^{j\omega t}$, and letting $d = s_1 + s_2 + a_1$ be the distance between the boreholes, then

$$E_{rc} = A_0 t_{sa} t_{as} e^{-jka_1} e^{-\gamma(d-a_1)} e^{j\omega t}$$  \hfill (3)

By linearity, it is clear that if our transmitted signal is a sum of various $A_n e^{j\omega t}$ (with $A_n$ possibly complex), then the wave at the Rx borehole will be given by a sum of solutions to (3).

It will be convenient to rewrite (3) in terms of the wave which would have been received in the absence of a cavity. Denoting this wave by $E_{ra} = A_0 e^{j\omega t} e^{-\gamma d}$, we can rewrite (3) as

$$E_{rc} = \frac{A_0 t_{sa} t_{as} e^{-jka_1} e^{-\gamma(d-a_1)} e^{j\omega t}}{1 - r_{as}^2 e^{-2jka_1}}.$$  \hfill (4)

Note that (4) is based on the following two assumptions:

- The wave can be treated as a plane wave (the far-field assumption).
- The effect of diffraction around the cavity can be neglected.

We attempted to validate (4) using GPRmax [21], [22] by simulating measurements in simple homogeneous soil. When we prevented diffraction around the cavity (by extending the cavity in the appropriate directions until it reached the perfectly-matched layers bounding the simulation), we found that (4) is indeed a reasonable model of the magnitude spectrum. However, (4) utterly fails to model both the phase spectrum and the received energy. We attribute this to the failure of the plane-wave assumption. Additionally, when we allow diffraction around the cavity, even small changes in depth (relative to the top or bottom of the cavity) can dramatically distort the received magnitude spectrum. While these distortions are difficult to model analytically, if we can sample multiple depths at which the signal passes through the cavity we can hope that these distortions will average out.

Having seen that we are unable to model the phase spectrum but must suffice with the magnitude spectrum, we can make two additional simplifications:

- While the Fresnel coefficients $t_{sa}$, $t_{as}$ and $r_{as}$ are complex functions of frequency, in practice we can approximate them as purely real constants over the relevant frequency range.
- We approximate $e^{-\gamma a} \approx e^{-\gamma a_{const}}$ - that is, we assume that the wave propagates through the distance $a$ of soil at the speed $v_{const}$ regardless of frequency and suffers only negligible attenuation while doing so.

### III. The Cavity-Detection Method

Let us assume that there exist either zero or one cavities and treat the problem as a hypothesis-testing problem. Let $Y_d(\omega)$ be the magnitude of the spectrum of the normalized (with respect to energy) signal received at depth $d$ and $G(\omega)$ be the magnitude of the spectrum received in the absence of a cavity. At each depth in a ZOP measurement there are two possibilities:

$$H_0 : Y_d(\omega) \propto G(\omega) + n(\omega)$$

$$H_1 : Y_d(\omega) \propto \frac{G(\omega) t_{sa} t_{as}}{1 - r_{as}^2 e^{-2jka_1}} + n(\omega)$$

The reason for using proportions rather than equalities is that, as mentioned above, our model is capable of predicting the received magnitude spectrum only up to a multiplicative constant. Additionally, we found, as did [17], that in practice our measurements of the received energy are unacceptably noisy. Having made the decision to work only with the proportions and having made the simplifying assumption that the Fresnel coefficients are independent of frequency, we can ignore the $t_{sa} t_{as}$ factor in $H_1$.

To use these two hypotheses for Neyman-Pearson-style hypothesis testing, it is necessary to first specify a distribution for the noise $n(\omega)$. As we work with magnitude spectra, the most natural choice for the distribution of $n(\omega)$ is probably the Rayleigh distribution. We have found, however, that that distribution will tend to lead to poor results. This is most probably because the noise is mostly *not* additive white Gaussian noise (AWGN) caused by e.g. thermal (Johnson-Nyquist) noise in the receiver and added to each measurement. Rather, the “noise” is mostly caused by the failure of the assumptions upon which we built our model and by the inhomogeneities of the soil. The Central Limit Theorem justifies the use of the normal distribution when the noise is due to the sum of these many different sources of error. Additionally, we note that the normal distribution is “well-behaved” in the sense that, no matter the value of the variance, the probability of an observation decreases monotonically with the distance of the observation from the mean. Use of the Rayleigh distribution, in contrast, might plausibly lead to the “failure mode” where the estimate of the scale parameter $\sigma$ is too far from the correct value; an observation might then be penalized for having too little noise. If the estimate of the scale parameter $\sigma$ is correct, then it is of course correct to penalize the observation for having too much noise. Still, if the performance of the algorithm is not robust with respect to penalize the observation for having too much noise. Still, if the performance of the algorithm is not robust with respect to penalize the observation for having too much noise, just as it is correct to penalize the observation for having too little noise. If the estimate of the scale parameter $\sigma$ is correct, then it is of course correct to penalize the observation for having too much noise. Still, if the performance of the algorithm is not robust with respect to penalize the observation for having too much noise, just as it is correct to penalize the observation for having too little noise.

This being the case, we make the following four additional assumptions:

- The noise $n(\omega)$ is normally distributed.
- The width of the cavity is at least approximately known.
- If a cavity is present, then the wave impinges upon the cavity at approximately normal incidence (or more generally, the angle of incidence is at least approximately known).

\footnote{Note that most formulations for dealing with diffraction, such as Fraunhofer or Fresnel diffraction, make use of the Kirchhoff approximation. This approximation, which states that the fields vanish at the obstructing screen, is not even approximately valid in this context.}
• The noise at each frequency is independent (the naïve assumption).

Note that the last assumption does not hold in practice; its purpose is to make the problem tractable.

We are almost ready to perform Neyman-Pearson hypothesis testing on the measurements received at each depth. All that remains is to estimate \( r_{as} \) and \( G(\omega) \). We find \( r_{as} \) by maximizing over all plausible values, and we find \( G(\omega) \) by a process inspired by cell-averaging Constant False Alarm Rate (CA-CFAR) radar. We slide a window over all depths where we expect a cavity might exist. At each step, we will assume that a cavity does not exist at all depths a sufficient distance from the window. From the measurements recorded at these depths we will estimate \( G(\omega) \). We will take the mean measurement of the depths in the window to be \( Y(\omega) \), and we can then find the likelihood for \( H_0 \) and the likelihood of \( H_1 \) for any \( r_{as} \).

The complete cavity-detection method is given as Algorithm 1.

Algorithm 1: CFAR-style algorithm for performing Neyman-Pearson hypothesis testing based on the shape of the magnitude spectra.

```plaintext
1 let \( V \) be the set of all depths from some minimum depth to some maximum depth;
2 let \( N \) be the set of all reasonable refractive indices for the soil;
3 slide a window over all depths in \( V \)
4 let \( W \) be the set of all depths in the window;
5 let \( T \) be the set of depths \( d \in V \) s.t.
\[
\min_{s \in W} |d - s| > \text{guardSize};
\]
6 \( \mu_{\text{train}}(\omega) := \frac{1}{|T|} \sum_{d \in T} Y_d(\omega); \)
7 \( \sigma_{\text{train}}(\omega) := \sqrt{\frac{1}{|T|} \sum_{d \in T} (Y_d(\omega) - \mu_{\text{train}}(\omega))^2}; \)
8 \( \mu_{\text{test}}(\omega) := \frac{1}{|W|} \sum_{d \in W} Y_d(\omega); \)
9 \( \log(L_0) = \sum_{\omega} \log(N(\mu_{\text{test}}(\omega), \mu_{\text{train}}(\omega), \sigma_{\text{train}}(\omega))); \)
10 \( \log(L_1) = \max_{n \in N} \sum_{\omega} \log(N(\mu_{\text{test}}(\omega), \|Z(\omega)\|, \sigma_{\text{train}}(\omega))); \)
where \( Z(\omega) = \frac{\mu_{\text{train}}e^{-jka}}{\mu_{\text{test}}e^{-jka}}; \)
11 score(center of window) = \( \log(L_1) - \log(L_0); \)
end
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Here, we take \( N(x, \mu, \sigma) \) to be the PDF of the Gaussian distribution with mean \( \mu \) and variance \( \sigma^2 \), evaluated at \( x \). Note that the maximum is taken over all \( n \in N \) in line 10 because the Fresnel coefficients are functions of the refractive index \( n \) of the soil.

We must consider only a finite number of frequencies \( \omega \). By default, we use all frequencies sampled by the FFT of the entire received signal. In certain circumstances, it might plausibly be advantageous to consider only a subset of the possible frequencies. In the limiting case of considering only a single frequency, the naïve assumption holds trivially; if only a small set of widely-spaced frequencies are considered it might hold approximately. Of course, in these cases we would be ignoring a large amount of potentially-useful information that resides in the portions of the spectrum not considered.

In this algorithm we attempt to determine whether a cavity exists at the depths in \( W \), while assuming that no cavity exists at any of the depths in \( T \). If we assume that each depth in \( T \) is an independent measurement of the true “background signal” contaminated by multivariate (not necessarily white) Gaussian noise, then the maximum likelihood estimator (MLE) for the background signal is \( \mu_{\text{train}} \) as calculated in line 6. This model is simplistic; in some cases where this model is less correct it might be possible for a more sophisticated method to more accurately estimate the background signal. In related research where we have used similar CFAR-style techniques to estimate the background signal we have sometimes found that we can improve results by taking the background signal to be the largest singular vector computed from the singular-value decomposition (SVD) of all background measurements. In this case the SVD method seems to interact poorly with the complexity of real soil.

IV. Evaluation and Results

We validated the performance of Algorithm 1 on several simulations, as well as on a series of real measurements. Using GPRmax, we first simulated cross-borehole GPR measurements across homogeneous soil with a relative permittivity of 10 and conductivity of 0.002 S/m, the parameters given by [23] for “Dry, sandy, coastal land.” A cavity (containing only free space) with a width of one meter, significantly extended in length (perpendicular to the line connecting the two boreholes), was placed between the depths of 3.85 and 5.5 meters below the surface. The transmitter and receiver boreholes were separated by six meters. ZOP measurements were simulated every 0.25 meters up to a depth of 10 meters. As seen in Figure 2, Algorithm 1 successfully detects the cavity. It is important to note, however, that perfectly homogeneous soil is unrealistically simple.

We further simulated a set of three ZOP profiles over soils characterized by strata. These simulations attempted to capture the complexity of real-world soils. Each stratum had dielectric properties that conformed to the fractal Peplinski model of [24]; the properties in each stratum were chosen randomly using a Brownian process based only on the properties of the previous stratum. The probability distributions for each property were normal and uniform distributions whose parameters were based upon the measurements in Table 1 in [25] and Table 4 in [26]. Each scenario was simulated both with and without a cavity. A complete description of these simulated stratified soils is available as “supplemental material” for this paper. The results of applying Algorithm 1 to these simulations are shown in Figure 3. While these realistic stratified soils are clearly more challenging, Algorithm 1 was mostly successful in detecting the cavities.

In each of the first two simulations, the cavity exists at the depth of the large peak in the likelihood ratio. On the third simulation, which is shown in Figure 4, our method...
Fig. 2. The results of applying Algorithm 1 to simulated homogeneous soil. The cavity exists between the depths of 3.85 and 5.3 meters below the surface.

Fig. 3. The results of applying Algorithm 1 to simulated stratified soil. The cavity exists at the depths of the major peaks of the likelihood ratios given in blue in the first two simulations. In the third (rightmost) simulation, the cavity exists at the depth of the smaller peak at around 4.3 meters below the surface.

Fig. 4. The received signals from the third pair of simulations in Figure 3. The simulations were identical other than the cavity which was simulated in the measurements shown on the right. The signals have been cropped to show the most relevant portions. The strange behavior of the signal at the shallowest and deepest depths is most likely a result of numerical artifacts and should have been ignored by Algorithm 1 due to its CFAR-style nature.

Fig. 5. The results of applying Algorithm 1 to real ZOP measurements. Blue measurements contained a cavity around 19 meters below the surface, while red measurements did not.

Finally, to show that our method can indeed detect real cavities, we carried out a series of real measurements at a test site characterized by sedimentary rock. A single cavity with a cross-section of around one square meter intersected two lines of boreholes. Eight ZOP profiles were taken between boreholes which did not bracket the cavity, while a further three profiles were taken between boreholes which did bracket the cavity. In each case, there were approximately seven meters between the boreholes and measurements were taken every 0.25 meters. The results of applying Algorithm 1 to these measurements are shown in Figure 5. Our method clearly succeeds in detecting the cavity. Note, however, that changing the parameters used in Algorithm 1 may result in a likelihood ratio at the anomaly at the deepest depths which can exceed the likelihood ratio at the true cavity. This anomaly is discussed further in the next section.

Our method compares favorably with standard cavity-detection methods based on traveltime. While the cavity from the homogeneous simulation in Figure 2 can be easily detected by the early arrival of the signal (not shown), the other examples are more complicated. As we’ve seen, the measurements in Figure 4 show a stratum in which the signal exhibits low traveltime. The measurements from Figure 5 exhibit similar
strata, as shown in Figure 6. Here there are two strata through which the signal exhibits low traveltime; one lies at around 13 meters below the surface, and the other lies at around 19 meters below the surface. Detection is complicated by the fact that the cavity lies in or adjacent to one of these strata. Only by comparison between adjacent profiles can the cavity be detected using traveltime alone, and this can only be done because there are only 7 meters between the two boreholes. As the distance between the boreholes grows, the difficulty of distinguishing the effect of the cavity from the effect of the stratum on traveltime increases; 7 meters is approximately the largest separation between the boreholes at which it is possible to do so.

V. DISCUSSION

It might appear that (4) predicts that propagation through a cavity will have an effect on the magnitude spectrum of the wave that is cyclical in frequency. However, the soil itself will only pass low frequencies. This property is reflected in the $e^{-\gamma d}$ factor in (3), where

$$\gamma = \alpha + j\beta = \omega \sqrt{\frac{\mu \epsilon}{2} \left( \frac{1 + \frac{\sigma^2}{\omega^2 c^2}}{1 + \frac{\sigma^2}{\omega^2 c^2}} \right)} + j\omega \sqrt{\frac{\mu \epsilon}{2} \left( \frac{1 + \frac{\sigma^2}{\omega^2 c^2}}{1 + \frac{\sigma^2}{\omega^2 c^2}} \right)}.$$

In practice, the only frequencies which are passed by the soil are those for which propagation through a cavity has a low-pass effect. It would then appear that rather than using Algorithm 1, cavities may be simply detected by searching for depths at which the received spectrum has an anomalously large amount of power in the lower frequencies relative to the higher frequencies.

Indeed, in some cases we can detect a cavity simply by computing the centroid frequency as defined by [17] at each depth in a gather of ZOP measurements. Note that [17] define the centroid frequency as $f_s = \int_{-\infty}^{\infty} f P(f) df$, where $P(f)$ is the magnitude of frequency $f$. The centroid frequencies at each depth of the profiles used in Figure 5 are shown in Figure 7. It can be seen that the profiles where the wave passed through the cavity do have anomalously low centroid frequencies at around 19 meters below the surface, which is the approximate depth of the cavity. However, as the centroid frequency is often lower when the signal did not pass through a cavity, it is clear that we cannot use the centroid frequency alone to detect cavities.

There is an interesting anomaly at the deepest depths of our real measurements. We see from Figure 6 that the signal exhibits a much greater traveltime at these depths, while we see from Figures 5 and 7 that the soil at these depths has some sort of low-pass effect on the spectra of the signals. Additionally, an anomalously low amount of energy is received at these depths (not shown), though as mentioned we have reason to suspect that the measurement of the signal’s energy is noisy. It is possible that these effects are simply the result of the properties of the soil at these depths. We conjecture, however, that these effects are the result of borehole drift. While [27] write that “It is well known that about 5 meters are deviated per every 100 meters in digging vertical boreholes,” we have seen that the actual deviation can differ from this figure by more than an order of magnitude in either direction. Furthermore, we have seen that the deviation is sometimes gradual and sometimes sudden (presumably caused by a refraction-like effect when drilling through different strata). It is therefore plausible that a sudden deviation in our boreholes occurs at these deepest depths; it is furthermore easy to see that random perturbations in the positions of our boreholes would tend to increase the distance between them.

If the distance between the boreholes were suddenly to increase, we would expect to see an increase in traveltime, a decrease in the amount of received energy, and a stronger low-pass effect on the spectrum of the signal; while the latter two effects might cause us to believe that a cavity lies at those depths, the first effect would disabuse us of that notion. Conversely, if the boreholes happened to draw closer together, then we would expect to see a smaller traveltime, which would cause us to suspect the existence of a cavity, along with a smaller low-pass effect (and greater received energy) which would allow us to conclude that a cavity does not in fact exist.

Recall that Algorithm 1 performs Neyman-Pearson-style hypothesis testing at each depth with the two hypotheses being either the existence of a cavity in soil otherwise identical to
the soil at the training depths, or alternatively that the soil is simply identical to that at the training depths. We could perhaps evaluate, as a third hypothesis, the possibility that the soil is identical to that at the training depths but that the distance between the boreholes has changed. However, this seems unlikely to be worth the additional complexity. We conclude that it is best to use both conventional traveltime-based methods and techniques such as Algorithm 1, as while both techniques may be susceptible to false positives, it is unlikely that the same phenomena would simultaneously cause false positives in both techniques.

Finally, we emphasize that while much of the power of our method comes from the fact that we compare the received signals with the signal predicted by our model, the likelihood ratio our method yields has little probabilistic significance. This is due to our use of the naïve assumption of the independence of the noise at each frequency. While this assumption may be appropriate for e.g. thermal noise, in practice the noise is probably mainly due to the simplifying assumptions used in our model; a failure of our model would likely have effects that are highly correlated between closely-spaced frequencies. This explains the unrealistically high likelihood ratios in the results we presented, and is similar to the overconfidence that are highly correlated between closely-spaced frequencies.

VI. CONCLUSION

We modeled the effect on the spectrum of a GPR signal of passing through a cavity and found that it will in general be a specific low-pass effect. Using this model, we developed a method to detect underground cavities using GPR ZOP measurements. Our method compares favourably to existing methods. Moreover, we conjecture that while both our method and existing methods may be susceptible to false positives, the phenomena which would tend to cause a false positive in one method would have the opposite effect on the other method. This implies that combining our method with traditional methods would lead to a more robust cavity-detection solution.

In this paper we used the spectral distortion implied by (4) as a feature to detect cavities. While it might seem more natural to have compared the entire signal predicted by (4) with the received signal, so far our attempts to do so have met with only limited success. This may be due to the inability of (4) to predict the phase of the received signal. We leave this as a topic for future research.

We’ve seen that our use of the naïve assumption destroys the probabilistic significance of the likelihood ratios our method returns. It would clearly be useful to have our method return accurate likelihoods for both hypotheses. We expect finding a more sophisticated replacement for the naïve assumption to be a fruitful topic for future research.

Using other features derived from (4) the way we have used the spectral distortion is an additional topic for future research. Preliminary research shows that the group dispersion implied by (4) is a strong candidate for such a feature. Significantly, the group dispersion experienced by a GPR signal appears to be relatively unaffected by small deviations in the distance between the boreholes.

Finally, in this paper we have considered only detection of the cavity using ZOP measurements. Using the feature considered here to localize the cavity horizontally or to perform a tomographic inversion remains an interesting problem. Most tomographic inversion algorithms are based around integrating some property of the medium, such as the slowness or opacity, over the path of the ray. The feature we have considered here, however, is influenced by the relative position of the two walls of the cavity; as it is in some sense a nonlocal property, using it as a feature for tomographic inversion is nontrivial. We expect this to be a fruitful direction for future research.

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Caleb Leibowitz received the B.Eng and M.Eng degrees from the Cooper Union for the Advancement of Science and Art in 2015, both in electrical engineering. He is currently pursuing a Ph.D. with the School of Electrical Engineering at Tel Aviv University.

Anthony J. Weiss (S’84–M’85–SM’86–F’97) received the B.Sc. degree from the Technion Israel Institute of Technology, Haifa, Israel, in 1973, and the M.Sc. and Ph.D. degrees from Tel Aviv University, Tel Aviv, Israel, in 1982 and 1985, all in electrical engineering. From 1973 to 1983, he was involved in research and development of numerous projects in the fields of communications, command and control, and emitter localization. In 1985, he joined the Department of Electrical Engineering-Systems, Tel Aviv University. From 1996 to 1999, he was the Department Chairman and the IEEE Israel Section Chairman. From 2006 to 2011, he was the Chairman of the School of Electrical Engineering, Tel Aviv University. He has authored or co-authored nearly 200 papers in professional magazines and conferences and holds 11 U.S. patents. His research interests include detection and estimation theory, signal processing, sensor array processing, and wireless networks. He was the Editor for the IEEE Transactions on Wireless Communications for several years. He was a recipient of the IEEE 1983 Acoustics, Speech, and Signal Processing Society’s Senior Award and the IEEE third millennium medal.