On physical foundations and observational effects of cosmic rotation

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Abstract

An overview of the cosmological models with expansion, shear and rotation is presented. Problems of the rotating models are discussed, their general kinematic properties and dynamical realizations are described. A particular attention is paid to the possible observational effects which could give a definite answer to the question: is there evidence for the rotation of the universe?

1. INTRODUCTION

In his lecture “Spin in the Universe” [432], E.T. Whittaker has drawn attention to the following questions: “Rotation is a universal phenomenon; the earth and all the other members of the solar system rotate on their axes, the satellites revolve round the planets, the planets revolve round the Sun, and the Sun himself is a member of the galaxy or Milky Way system which revolves in a very remarkable way. How did all these rotary motions come into being? What secures their permanence or brings about their modifications? And what part do they play in the system of the world?”

The standard Friedman-Lemaître cosmological models are based on very idealized assumptions about the spacetime structure and matter source. The Friedman-Robertson-Walker metrics describe an exactly isotropic and homogeneous world filled with an ideal fluid (radiation and dust are the well known particular cases). Although these assumptions
are qualitatively supported by observations, it seems doubtful that the geometry of the universe and its physical contents are so finely tuned starting with the very Big Bang till the modern stage of the evolution. A reasonable step is to look for more general and realistic models with the assumptions of isotropy, homogeneity, and ideal fluid matter contents, relaxed. Here we will not discuss inhomogeneous cosmologies which are reviewed, e.g., in [238,239].

Already in the early years of the general relativity theory, attempts were made to construct (exact or approximate) solutions of the gravitational field equations with rotating matter sources (see, e.g. [225,11,231,407]). Lanczos [225] appears to be the first who considered the universe as the largest possible rotating physical system in a model of a rigidly rotating dust cylinder of an infinite radius. Dust density in this solution (which was later rederived by van Stockum [407]) diverges at radial infinity, which was a serious problem.

In 1946, Gamow [112] stressed a possible significance of the cosmic rotation for explaining the rotation of galaxies in the theories of galaxy formation, see also his discussion in [113]. Independently, Weizsäcker [130] initiated the study of the primordial turbulence and of its role in the formation of the universe’s structure. Soon after this, K. Gödel [116] had proposed a stationary cosmological model with rotation in the form (coordinates \{t, x, y, z\}, \(a = \text{const}\)):

\[
\begin{align*}
    ds^2 &= a^2 \left( dt^2 - 2e^x dt dy + \frac{1}{2} e^{2x} dy^2 - dx^2 - dz^2 \right).
\end{align*}
\]

In the Gödel’s model, the matter is described as dust with the energy density \(\varepsilon\), and the cosmological constant \(\Lambda\) is nontrivial and negative (i.e. its sign is opposite to that introduced by Einstein). The angular velocity \(\omega\) of the cosmic rotation in (1.1) is given by \(\omega^2 = \frac{1}{2a^2} = 4\pi G \varepsilon = -\Lambda\), with \(G\) as Newton’s gravitational constant. For many years this model became a theoretical “laboratory” for the study of rotating cosmologies, see e.g. [6,429,27,79,152,153,217,336–338,406,72]. The stationary solution (1.1) is distinguished among other spatially homogeneous models: the so-called Gödel theorem (complete proof is given in [300]) states that the only spatially homogeneous models with zero expansion and
shear are Einstein’s static universe and Gödel model (1.1).

Subsequently, Gödel [117] himself outlined a more physical expanding generalizations of (1.1), although without giving explicit solutions. Later a considerable number of exact and approximate models with rotation and with or without expansion were developed. At the end of this paper one can find a list of references which may be considered as a replacement of a detailed review of the previous work on rotating cosmology.

Here we would like to mention only briefly some earlier papers. Maitra [242] gave a generalization of van Stockum’s solution which is cylindrically symmetric inhomogeneous stationary dust-filled world with rotation and shear. Maitra formulated a simple criterion of absence of the closed timelike curves. Wright [437] presented another inhomogeneous cylindrically symmetric solution for dust plus cosmological term.

Dust and cosmological constant represented the matter source in the series of papers by Schücking and Ozsváth [295, 304], where the spatially homogeneous models with expansion, rotation and shear were considered. The closed Bianchi type IX solution (299, with stationary limit 296) has drawn a special attention. Most general anisotropic Bianchi type IX model was studied by Matzner [261, 262] who took ideal fluid plus collisionless massless particles (photons and neutrinos) as matter source. The approximate solution was given. Later, Rebouças and de Lima [359] described the exact analytic Bianchi type IX solution with expansion, shear and rotation for a source represented by dust with heat flow and cosmological constant. Analogously, Sviestins [412] presented an exact Bianchi type IX solution for ideal fluid plus heat flow. In an interesting recent paper, Grøn [127] gave a shear-free Bianchi type IX solution in absence of matter (but with cosmological constant) which can be considered as a rotating generalization of de Sitter spacetime. In fact, the latter is recovered asymptotically after the quick decay of the vorticity.

The wide spectrum of spatially homogeneous models includes approximate and exact solutions for all Bianchi types and different matter contents (from simple dust to sometimes quite exotic energy-momentum currents). To mention but a few, Ruzmaikina-Ruzmaikin [373], Demiański-Grishchuk [38], and Batakis-Cohen [36] presented approximate solutions
of Bianchi type V and VII with expansion, shear and vorticity for the ideal fluid in non-comoving coordinates. Bradley and Sviestins \[51\] added heat flow to the ideal fluid source in their analysis of a Bianchi type VIII model. In a series of papers, Fennelly \[104–108\] has studied Bianchi models of different types for homogeneous universes filled with perfect charged (conducting) fluid and magnetic field. A neutral ideal fluid plus electromagnetic field and null radiation represented the matter source in a shear-free expanding and rotating solution of Patel-Vaidya \[122\] which can be interpreted as a standard Friedman-Robertson-Walker cosmology with rotating matter \[130\]. More sophisticated matter source may include viscous fluid and scalar fields, see, e.g., recent papers \[246–256\]. More exact solutions, which we will not discuss in detail, \[7–9,36,51,283,354,356,357,370,412,85\] were reported in the literature. Most of these models were overviewed by Krasiński \[198–202\], more recent review of generalized cosmologies with shear and rotation see in \[206–208\].

It is worthwhile to notice that along with the constant and deep interest in the rotating cosmologies, historical development has revealed several difficulties which were considered by the majority of relativists as the arguments against the models with nontrivial cosmic rotation. One of the problems was the the stationarity of Gödel’s model \[1.1\]. Apparent expansion of the universe is usually related to the fact of the red shift in the spectra of distant galaxies, and thus all the standard cosmological models are necessarily non-stationary. It was discovered, however, that it is impossible to combine pure rotation and expansion in a solution of the general relativity field equations for a simple physical matter source, such as a perfect fluid \[93,97\]. There are two ways to overcome that difficulty: one should either take a more general energy-momentum or to add cosmic shear, the explicit examples were already mentioned above. The possibility of combining cosmic rotation with expansion was the first successful step towards a realistic cosmology.

Another problem which was discovered already by Gödel himself, was related to causality: the spacetime \[1.1\] admits closed timelike curves. This was immediately recognized as an unphysical property because the causality may be violated in such a spacetime, see relevant discussion in \[118,143,134,135,227,336,406\]. Considerable efforts were thus focused
on deriving completely causal rotating cosmologies. In his last work devoted to rotation, Gödel without proof mentions the possibility of positive solution of the causality problem \[117\]. First explicit solutions were reported later \[242,297\]. Maitra \[242\] formulated a simple criterion for the (non)existence of closed timelike curves in rotating metrics which we will discuss below. Eventually, it became clear that the breakdown of causality is not inevitable consequence of the cosmic vorticity, and many of the physically interesting rotating models were demonstrated to be completely causal. That was the second important step in the development of the subject.

The discovery of the microwave background radiation (MBR) has revealed the remarkable fact that its temperature distribution is isotropic to a very high degree. This fact was for a long time considered as a serious argument for isotropic cosmological models and was used for obtaining estimates on the possible anisotropies which could take place on the early stages of the universe’s evolution. In particular, homogeneous anisotropic rotating cosmologies were analyzed in \[142,81,33,32,180,435\], and very strong upper limits on the value of the cosmic rotation were reported. Unfortunately, usually in these studies the effects of vorticity were not separated properly form the cosmic shear.\(^1\) For the first time, in \[287,288\], the effects of pure cosmic rotation were carefully separated from shear effects. The most significant result is the demonstration that in a large class of spatially homogeneous rotating models the pure rotation leads neither to an anisotropy of the MBR temperature nor to a parallax effects, although the geometry remains completely causal. A typical representative of that class is the so called Gödel type metric with rotation and expansion (4.1) which is considered below. Most recent analyzes, based also on the COBE data on the anisotropy of MBR, see \[61,237,180\], again estimated not a pure rotation but rather a shear.

The last but not least problem is the lack of direct observational evidence for the cosmic

\(^1\)Even now in the literature one can often encounter statements like: “If the universe had vorticity, the microwave background would be anisotropic...” \[403\].
rotation. Attempting at its experimental discovery one should study possible systematic irregularities in angular (ideally, over the whole celestial sphere) distributions of visible physical properties of sources located at cosmological distances. Unfortunately, although a lot of data is already potentially accumulated in various astrophysical catalogues, no convincing analysis was made in a search of the global cosmic rotation. Partially this can be explained by the insufficient theoretical study of observational cosmology with rotation. To our knowledge, till the recent time there were only few theoretical predictions concerning the possible manifestations of the cosmic rotation. Mainly these were the above mentioned estimates of MBR anisotropies [142, 81, 32, 180] (also of the X-ray background anisotropies [435]) and the number counts analyses [225, 17, 31, 265, 103]. Besides that, it was pointed out in [11, 373] that, in general, parallax effects can serve as a critical observational test for the cosmic rotation.

Very few purely empirical analyses (without constructing general relativistic models) of the angular distributions of astrophysical data are available which interpreted the observed systematic irregularities as the possible effects of rotation. Of the early studies, let us mention the reports of Mandzhos and Tel’nyuk-Adamchuk [243, 245] on the preferential orientation of clusters of galaxies, Valdes et al [424] on the search of coherent ellipticity of galaxy images, and Andreasyan [16, 17] on flattening of distant galaxies.

Birch [46, 47, 82] (Jodrell Bank Observatory) reported on the apparent anisotropy of the distribution of the observed angle between polarization vector and position of the major axis of radio sources, and related this global effect (using heuristic arguments) to the nontrivial cosmic rotation. Subsequently, these observations were disputed from a statistical theory viewpoint [333, 13, 44], but the analysis using indirectional statistics [173] confirmed Birch’s results. It seems worthwhile to mention that the independently data from the Byurakan radio observatory [13, 14] indicated the same anisotropy along, approximately, the same direction. These observations have stimulated the interest in rotating models; nevertheless, no further observations were reported until the most recently, when the new data appeared [279, 279, 347, 348] claiming the same effect. Note however, that again the statistical signifi-
The theoretical analysis of the observational effects in the class of viable rotating cosmologies has shown that the cosmic rotation produces a typical dipole anisotropy effect on the polarization of electromagnetic waves. Considering the case of the distant radio sources, one finds the dipole distribution \( \eta = \omega_0 r \cos \theta \), see (7.32) below, for the angle \( \eta \) between the vector of polarization and an observed direction of the major axis of the image of a radio source. Here \( \omega_0 \) is the present value of the cosmic rotation, \( r \) is the (apparent area) distance to a source, and \( \theta \) is the spherical angle between the rotation axis and the direction to the source, \( Z \) is the red shift. The data of Birch and the new data of Nodland and Ralston may be extremely important for improving the direct estimates of vorticity.

The study of observational effects in rotating and expanding cosmological models is essentially based on the kinematical properties of spacetime. The next step is to investigate the dynamical aspects of cosmological models with rotation, in other words, to construct viable models as solutions of the gravitational field equations. The ultimate goal is to achieve a self-consistent description of the physical matter sources and to establish a complete scenario of a realistic evolution of the universe, including the most early stages of the big bang. Already Lemaître and Whittaker raised questions about the nature of a rotating “primeval atom” from which the universe is supposed to emerge. A related problem is the developments of matter irregularities which is relevant to the galaxy and large-scale structure formation. Gamov and Weizsäcker stressed the importance of the cosmic vorticity in the early universe. Silk was the first to demonstrate instability of rotating models to axial density perturbations and stability to perturbations in the plane of rotation. The idea that primordial vortical motions of cosmological matter gave rise to the formation of large-scale structure underlied the related study of the so-called whirl theory (cosmological turbulence) developed some time ago by Ozernoy and Chernin, see also. In the recent paper Li has demonstrated that the global cosmic rotation can provide a natural origin of the
rotation of galaxies, and the empirical relation \( J \sim M^{5/3} \) between the angular momentum \( J \) and the mass \( M \) of galaxies can be derived from the cosmological vorticity. [See Brosche 57–50 for empirical data, and Tassie 113 for the string-theoretical discussion].

Of the recent theoretical developments, it is worthwhile to mention the work 123 in which Grishchuk has demonstrated that the rotational cosmological perturbation can be naturally generated in the early universe through the quantum-mechanical “pumping” mechanism similar to the primordial gravitational waves. These perturbations were considered as the source of the large-angular-scale anisotropy of MBR, and comparison with observations was made. However, as was recognized by Grishchuk, the pumping mechanism only works when the primeval cosmological medium can sustain torque oscillations. No attempt was made to study the possible physical nature of the relevant cosmological matter. Exploiting the close relation between rotation and spin, one can try to attack this problem using the models of continua with microstructure. Self-consistent variational general relativistic theory of spin fluids was constructed in 289–291. The investigation of the development of small perturbations for the models with spinning cosmological matter 291 yielded a generalization and refinement of the result by Silk 392 about instability to perturbations along the spin.

Most of the dynamical realizations of rotating cosmological models were obtained in the framework of general relativity theory as the exact or approximate solutions of Einstein’s gravitational field equations in four dimensions. However, we can mention also the study of higher-derivative extensions of general relativity 1–2,57, solutions in 5 dimensions 218,365, in 3 dimensions 133,133,134, and in string-motivated effective cosmology 30,31. On the other hand, rotation, spin and torsion are closely interrelated in the Poincaré gauge theory of gravity 148,149,341,168,266, and hence it is quite natural to study cosmologies with rotation within the gauge gravity framework. Exact rotating solutions in Einstein-Cartan theory of gravity with spin and torsion were described, e.g., in 267,40,90,110,15,189,289,308,309,329,400,401. General preliminary analysis of the separate
stages of the universe’s evolution was made in our works \cite{164,166}, while in \cite{189,167} the complete cosmological scenarios are considered.

Summarizing the present state of the problem, it is noteworthy once again to quote Whittaker \cite{432}: “It cannot be said, however, that any of the mathematical-physical theories that have been put forward to explain spin (rotation) in the universe has yet won complete and universal acceptance; but progress has been so rapid in recent years that it is reasonable to hope for a not long-delayed solution of this fundamental problem of cosmology”.

2. STATIONARY COSMOLOGICAL MODEL OF GÖDEL TYPE

The Gödel metric \cite{111} represents a particular case of a wider family of stationary cosmological models described by the interval

\[ ds^2 = dt^2 - 2\sqrt{\sigma}e^{mx}dtdy - (dx^2 + ke^{2mx}dy^2 + dz^2), \]  

(2.1)

where \( m, \sigma, k \) are constant parameters. Clearly, \( \sigma > 0 \), and for definiteness, we choose \( m > 0 \).

The metric (2.1) is usually called the model with rotation of the Gödel type. Coordinate \( z \) gives the direction of the global rotation, the magnitude of which is constant:

\[ \omega = \sqrt{\frac{1}{2} \omega_{\mu\nu} \omega^{\mu\nu}} = \frac{m}{2} \sqrt{\frac{\sigma}{k + \sigma}}. \]  

(2.2)

As we see, vanishing of \( m \) or/and \( \sigma \) yields zero vorticity.

Let us describe the geometry of a Riemannian spacetime (2.1) in detail. The determinant of the metric is equal \((\text{det } g) = -(k + \sigma)e^{2mx}\), and hence we assume that \((k + \sigma) > 0\) in order to have the Lorentzian signature.

Technically, all calculations are simplified greatly when exterior calculus is used. It is convenient to choose at any point of the spacetime (2.1) a local orthonormal (Lorentz) tetrad \( h^a_\mu \) so that, as usual, \( g_{\mu\nu} = h^a_\mu h^b_\nu \eta_{ab} \) with \( \eta_{ab} = \text{diag}(+1, -1, -1, -1) \) the standard Minkowski metric. This choice is not unique, and we will use the gauge in which

\[ h^0_0 = 1, \quad h^0_2 = -\sqrt{\sigma}e^{mx}, \quad h^1_1 = h^3_3 = 1, \quad h^2_2 = e^{mx} \sqrt{k + \sigma}. \]  

(2.3)
Hereafter a caret denotes tetrad indices; Latin alphabet is used for the local Lorentz frames, $a, b, \ldots = 0, 1, 2, 3$. The local Lorentz coframe one-form is defined, correspondingly, by 

$$\vartheta^a := h^a_{\mu} dx^\mu. \quad (2.4)$$

The curvature two-form for (2.1) is computed straightforwardly:

$$R^{ab} = \omega^2 \vartheta^a \wedge \vartheta^b + \frac{km^2}{k + \sigma} \left( \delta^a_1 \delta^b_2 - \delta^a_2 \delta^b_1 \right) \vartheta^1 \wedge \vartheta^2 \quad \text{for} \quad a, b = 0, 1, 2. \quad (2.5)$$

It is worthwhile to note that the spacetime (2.1) has a formal structure of a product of the curved three-dimensional manifold with the curvature (2.5) times flat one-dimensional space ($z$-axis), which is reflected in the range of indices in (2.5).

The only nontrivial components (with respect to the local Lorentz frame (2.3)) of the Weyl tensor are as follows:

$$C_{0101} = C_{0202} = -C_{3232} = -C_{3131} = \frac{1}{2} C_{1212} = -\frac{1}{2} C_{0303} = \frac{m^2}{6} \left( \frac{k}{k + \sigma} \right). \quad (2.6)$$

One thus verifies that the model (2.1) belongs to type $D$ according to Petrov’s classification.

The first dynamical realization of the model (2.1) was obtained by Gödel [116]. In general relativity theory, metric satisfies Einstein’s field equations:

$$G_{ab} := R_{ab} - \frac{1}{2} g_{ab} R = \Lambda g_{ab} + \kappa T_{ab}. \quad (2.7)$$

Gödel [116] considered the simplest matter source: ideal dust with the energy-momentum tensor $T_{ab} = \rho u_a u_b$. Substituting the four-velocity of the co-moving matter $u^a = \delta^a_0$, and using the components of the Einstein tensor,

$$G_{00} = -\omega^2 \left( 1 + 4 \frac{k}{\sigma} \right), \quad G_{11} = G_{22} = \omega^2, \quad G_{33} = \omega^2 \left( 3 + 4 \frac{k}{\sigma} \right), \quad (2.8)$$

one finds the parameters of the famous Gödel solution [compare with (1.1)]:

$$\omega^2 = -\Lambda, \quad \kappa \rho = 2 \omega^2, \quad \frac{k}{\sigma} = -\frac{1}{2}. \quad (2.9)$$

As we see, both the cosmological constant $\Lambda$ and the parameter $k$ must be negative.
Without confining ourselves to a particular dynamical realization, we should study the whole class of stationary models (2.1). There is a wide symmetry group for that metric. Three evident isometries are generated by the Killing vector fields

\[ \xi(0) = \frac{\partial}{\partial t}, \quad \xi(1) = \frac{\partial}{\partial y}, \quad \xi(2) = \frac{\partial}{\partial z}. \]  

(2.10)

These are all mutually commuting. However, one can find two more Killing vectors. Most easily this can be done with the help of the suitable coordinate transformations.

Without touching \( z \) coordinate, let us transform the rest three \((t, x, y) \rightarrow (\tau, r, \varphi)\) as follows:

\[ e^{mx} = e^{\tau} \cosh(m\tau), \]  

(2.11)

\[ ye^{mx} = \frac{\sinh(m\tau)}{m\sqrt{k + \sigma}}, \]  

(2.12)

\[ \tan \left[ m\sqrt{\frac{k + \sigma}{\sigma}} (t - \tau) \right] = \sinh(m\tau). \]  

(2.13)

Straightforward calculation gives a new form of the metric (2.1):

\[ ds^2 = d\tau^2 - d\tau^2 - \frac{k \sinh(m\tau) + \sigma}{m^2(k + \sigma)} d\varphi^2 - dz^2 + \frac{2}{m} \sqrt{\frac{k + \sigma}{k + \sigma}} \sinh(m\tau) d\tau d\varphi \]  

(2.14)

\[ = \left( d\tau + \frac{1}{m \sqrt{k + \sigma}} \sinh(m\tau) d\varphi \right)^2 - \left( d\tau^2 + \frac{\cosh^2(m\tau)}{m^2} d\varphi^2 + dz^2 \right). \]  

(2.15)

We thus immediately find the fourth Killing vector, \( \partial_{\varphi} \). In the original coordinates it reads:

\[ \xi(3) = \frac{\partial}{\partial \varphi} = \frac{1}{m} \frac{\partial}{\partial x} - y \frac{\partial}{\partial y}. \]  

(2.16)

Alternatively, let us consider another transformation \((t, x, y) \rightarrow (\tau, r, \varphi)\) given by the formulae:
\[ e^{mx} = \cosh(mr) + \cos \varphi \sinh(mr), \quad (2.17) \]
\[ ye^{mx} = \frac{\sin \varphi \sinh(mr)}{m\sqrt{k + \sigma}}, \quad (2.18) \]
\[ \tan \left[ \frac{m}{2} \sqrt{\frac{k + \sigma}{\sigma}} (t - \tau) \right] = \frac{\sin \varphi}{\cos \varphi + \coth \left( \frac{mr}{2} \right)}. \quad (2.19) \]

The metric (2.1) in the new coordinates reads:
\[ ds^2 = d\tau^2 - dr^2 - \frac{4 \sinh^2 \left( \frac{mr}{2} \right) [k \cosh^2 \left( \frac{mr}{2} \right) + \sigma]}{m^2(k + \sigma)} d\varphi^2 - dz^2 \]
\[ + \frac{4}{m \sqrt{k + \sigma}} \sinh \left( \frac{mr}{2} \right) d\tau d\varphi \]
\[ = \left( d\tau - \frac{2}{m} \sqrt{\frac{\sigma}{k + \sigma}} \sinh \left( \frac{mr}{2} \right) d\varphi \right)^2 - \left( dr^2 + \frac{\sinh^2 (mr)}{m^2} d\varphi^2 + dz^2 \right). \quad (2.20) \]

Evidently \( \partial_\varphi \) is also the Killing vector field. In the original coordinates it looks somewhat complicate and can be represented as a linear combination of three vectors:
\[ \frac{\partial}{\partial \varphi} = \frac{1}{m} \sqrt{\frac{\sigma}{k + \sigma}} \xi(0) + \frac{1}{2m\sqrt{k + \sigma}} \xi(1) - \sqrt{k + \sigma} \xi(4), \quad (2.22) \]

where we denoted the *fifth* Killing vector:
\[ \xi(4) = \frac{\sqrt{\sigma} e^{-mx}}{m(k + \sigma)} \frac{\partial}{\partial t} + y \frac{\partial}{\partial x} + \frac{1}{2} \left[ \frac{e^{-2mx}}{m(k + \sigma)} - my^2 \right] \frac{\partial}{\partial y}. \quad (2.23) \]

Three of the five Killing vectors (2.10), (2.16), (2.23) satisfy commutation relations
\[ [\xi(3), \xi(1)] = \xi(1), \quad [\xi(3), \xi(4)] = - \xi(4), \quad [\xi(1), \xi(4)] = m\xi(3). \quad (2.24) \]

Thus \( \xi(1), \xi(3), \xi(4) \) form the closed subalgebra isomorphic to \( so(2,1) \). Two Killing vectors \( \xi(0), \xi(2) \) span the center of the isometry algebra, commuting with all its elements.

In order to get some insight into possible causal problems, it is helpful to compute squares of the Killing vectors. Straightforwardly we obtain:
\[ (\xi(0) \cdot \xi(0)) = 1, \quad (2.25) \]
\[ (\xi(1) \cdot \xi(1)) = -k e^{2mx}, \quad (2.26) \]
\[ (\xi(2) \cdot \xi(2)) = -1, \quad (2.27) \]
\[ (\xi(3) \cdot \xi(3)) = - \left( ky^2 e^{2mx} + \frac{1}{m^2} \right), \quad (2.28) \]
\[ (\xi(4) \cdot \xi(4)) = - \frac{k}{4} \left( e^{mx} my^2 + \frac{e^{-mx}}{m(k + \sigma)} \right)^2. \quad (2.29) \]
The vector field $\xi_{(0)}$ is always timelike, its integral curves coincide with the lines of coordinate time. The cases $k > 0$, $k < 0$ and $k = 0$ are essentially different, and it is reasonable to consider them separately.

For positive values of $k$ the four Killing vector fields $\xi_{(1)}, \ldots, \xi_{(4)}$ are strictly spacelike; the first three of them provide spatial homogeneity of the $t = \text{const}$ hypersurfaces. We will demonstrate the complete causality of the models with $k > 0$ later. In particular, the closed timelike curves are absent in these geometries.

For negative $k$, the vectors $\xi_{(1)}$ and $\xi_{(4)}$ become timelike, whereas $\xi_{(3)}$ can be both spacelike and timelike, depending on the values of spatial coordinates $x, y$. One can immediately verify the existence of timelike closed curves, which was first noticed already by Gödel \cite{116}. Indeed, consider the closed curve $\{t(\varphi), x(\varphi), y(\varphi)\}$, with $0 \leq \varphi \leq 2\pi$, given by the coordinate transformation (2.17)-(2.19) for $\tau = \text{const}, r = \text{const}, z = \text{const}$. From (2.20) we find that for large enough $r$, when $\cosh^2(\frac{mr}{2}) > |\frac{\sigma}{r}|$, such a curve is timelike. By construction, it is closed, and this may lead to the violation of causality. Although in \cite{72} it was shown that such curves are not geodesics, one can in principle imagine a motion of a physical particle along these curves under the action of certain forces. It is worthwhile to note that the form of the metric is similar in two different coordinate systems, compare (2.15) and (2.21). However, for $k < 0$ the coordinate $\varphi$ curve (with $\tau = \text{const}, r = \text{const}, z = \text{const}$) is also timelike for $\cosh^2(\frac{mr}{2}) > |\frac{\sigma}{r}|$, but it is not closed.

The special case $k = 0$ was studied in detail by Rebouças and Tiomno \cite{360}, see also \cite{364} and more recently \cite{368}. The Riemannian curvature is simplified greatly for $k = 0$. In particular, the Weyl tensor (2.6) vanishes, and thus four-dimensional metric is conformally flat. At the same time, (2.5) reduces to $R^{ab} = \omega^2 \vartheta^a \wedge \vartheta^b$, with $a, b = 0, 1, 2$. The last relation describes a three-dimensional manifold of constant curvature, i.e. (anti) de Sitter spacetime. This space has much wider isometry group than $k \neq 0$ case. In addition to the 5 Killing vectors (2.11), (2.16), (2.23), we easily find 2 more:

$$\xi_{(5)} = \sin(mt) \frac{\partial}{\partial t} - \cos(mt) \frac{\partial}{\partial x} + \frac{e^{-mx}}{\sqrt{\sigma}} \sin(mt) \frac{\partial}{\partial y}, \quad (2.30)$$
\[
\xi_{(6)} = \cos(mt) \frac{\partial}{\partial t} + \sin(mt) \frac{\partial}{\partial x} + \frac{e^{-mx}}{\sqrt{\sigma}} \cos(mt) \frac{\partial}{\partial y}.
\] (2.31)

Together with \(\xi_0\) these vector fields form the second closed subalgebra isomorphic to \(so(2, 1)\):

\[
[\xi_{(0)}, \xi_{(5)}] = m\xi_{(6)}, \quad [\xi_{(0)}, \xi_{(6)}] = -m\xi_{(5)}, \quad [\xi_{(5)}, \xi_{(6)}] = -m\xi_{(0)}.
\] (2.32)

The six Killing vectors \(\xi_{(A)}, A = 1, 3, 4, 0, 5, 6\) with the commutation relations (2.24), (2.32) describe the complete \(so(2, 2)\) algebra of isometries (algebra of the conformal group of a three-dimensional (anti) de Sitter spacetime) [368]. It is worthwhile to note that both new Killing vectors are spacelike:

\[
(\xi_{(5)} \cdot \xi_{(5)}) = (\xi_{(6)} \cdot \xi_{(6)}) = -1,
\] (2.33)

Starting from the metric in the form (2.21) with \(k = 0\) inserted, we can make a simple coordinate transformation

\[
(\tau, r, \varphi) \longrightarrow (\tau, r, \theta),
\]

redefining the angular coordinate,

\[
\varphi = \frac{m}{2} (\theta - \tau),
\] (2.34)

which brings the metric to an explicitly (anti) de Sitter form:

\[
ds^2 = \cosh^2 \left(\frac{mr}{\omega} \right) d\tau^2 - dr^2 - \sinh^2 \left(\frac{mr}{\omega} \right) d\theta^2 - dz^2.
\] (2.35)

Note that \(\omega = \frac{m}{2}\) for \(k = 0\).

### 3. CLASS OF SHEAR-FREE COSMOLOGICAL MODELS WITH ROTATION AND EXPANSION

In [287,187] we considered a wide class of viable cosmological models with expansion and rotation. Let us describe it here briefly. Denoting \(x^0 = t\) as the cosmological time and \(x^i, i = 1, 2, 3\) as three spatial coordinates, we write the space-time interval in the form
\[ ds^2 = dt^2 - 2a n_i dx^i dt - a^2 \gamma_{ij} dx^i dx^j, \] (3.1)

where \( a = a(t) \) is the scale factor, and

\[ n_i = \nu_a e_i^{(a)}, \quad \gamma_{ij} = \beta_{ab} e_i^{(a)} e_j^{(b)}. \] (3.2)

Here \((a, b = 1, 2, 3) \nu_a, \beta_{ab}\) are constant coefficients, while

\[ e^{(a)} = e_i^{(a)}(x) dx^i \] (3.3)

are the invariant 1–forms with respect to the action of a three-parameter group of motion which is admitted by the space-time (3.1). We assume that this group acts simply-transitively on the spatial \((t = \text{const})\) hypersurfaces. It is well known that there exist 9 types of such manifolds, classified according to the Killing vectors \(\xi^{(a)}\) and their commutators \([\xi^{(a)}, \xi^{(b)}] = C^{c}_{ab} \xi^{(c)}\). Invariant forms (3.3) solve the Lie equations \(\mathcal{L}_{\xi^{(b)}} e^{(a)} = 0\) for each Bianchi type, so that models (3.1) are spatially homogeneous.

We consider \((t, x^i)\) as comoving coordinates for the cosmological matter, that is the four-vector of average velocity of material fluid is equal \(u^\mu = \delta_0^\mu\). It is worthwhile to note that \(u^\mu\) is not orthogonal to hypersurfaces of homogeneity. Such a models are usually called “tilted”.

The kinematical characteristics of (3.1) are as follows: volume expansion is

\[ \vartheta = 3 \dot{a}/a, \] (3.4)

nontrivial components of vorticity tensor are

\[ \omega_{ij} = -\frac{a}{2} \hat{C}^{k}_{ij} n_k, \quad i, j = 1, 2, 3, \] (3.5)

and shear tensor is trivial,

\[ \sigma_{\mu\nu} = 0. \] (3.6)

Hereafter the dot (\(\dot{}\)) denotes derivative with respect to the cosmological time coordinate \(t\). Tensor \(\hat{C}^{k}_{ij} = e^k_{(a)}(\partial_i e^{(a)}_j - \partial_j e^{(a)}_i)\) is the anholonomity object for the triad (3.3); for I-VII Bianchi types values of its components numerically coincide with the corresponding structure.
constants $C^a_{bc}$. The list of explicit expressions for $\xi^{(a)}$, $\epsilon^{(a)}$, $C^a_{bc}$, $\hat{C}^k_{ij}$ for any Bianchi type is given in [287].

For completeness, let us mention that cosmological models (3.1) have nontrivial acceleration

$$u^\nu \nabla_\nu u_\mu = (0, \dot{a}, n_i).$$

(3.7)

We choose the constant matrix $\beta_{ab}$ in (3.2) to be positive definite. This important condition generalizes results of Maitra [242], and ensures the absence of closed time-like curves.

One can immediately see that space-times (3.1) admit, besides tree Killing vector fields $\xi^{(a)}$, a nontrivial conformal Killing vector

$$\xi_{\text{conf}} = a \partial_t.$$  

(3.8)

All models in the class (3.1) have a number of common remarkable properties [287,187]:

**Causality:** Space-time manifolds (3.1) are completely causal.

Indeed, following [242], let us suppose the spacetime (3.1) contains a closed curve $x^\mu(s)$ where $s$ is some parameter, $0 \leq s \leq 1$, which is timelike. The latter means that at any point on the curve the length of the velocity vector is strictly positive,

$$g_{\mu\nu} \frac{dx^\mu}{ds} \frac{dx^\nu}{ds} > 0.$$  

(3.9)

Since the curve is closed, $x^\mu(0) = x^\mu(1)$, there exists a value $s_0$ at which $\frac{dt}{ds} = 0$. At this point one has

$$g_{\mu\nu} \frac{dx^\mu}{ds} \frac{dx^\nu}{ds} \bigg|_{s=s_0} = -a^2 \beta_{ab} \epsilon_i^{(a)} \epsilon_j^{(b)} \frac{dx^i}{ds} \frac{dx^j}{ds},$$

(3.10)

which is negative when $\beta_{ab}$ is positive definite. This contradicts (3.9) and hence we conclude that our assumption was wrong and there are no closed timelike curves.

For example, the stationary Gödel type metric (2.1) represents a particular case of (3.1) with $a = 1$ and
\[ \nu_a = (0, \sqrt{\sigma}, 0), \quad \beta_{ab} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & k & 0 \\ 0 & 0 & 1 \end{pmatrix}, \quad e^{(a)}_i = \begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{mx} & 0 \\ 0 & 0 & 1 \end{pmatrix}. \] 

(3.11)

It is clear that \( \beta_{ab} \) in (3.11) is positive definite for \( k > 0 \). Negative values of \( k \), as we saw explicitly in Sec. 2, allow for the closed timelike curves. In particular, this is true for the original Gödel solution (2.9).

**Isotropy of MBR:** The Microwave Background Radiation (MBR) is totally isotropic in (3.1) for any moment of the cosmological time \( t \).

In the geometric optics approximation, light propagates along null geodesics, i.e. curves \( x^\mu(\lambda) \) with certain affine parameter \( \lambda \) and tangent vector \( k^\mu = dx^\mu/d\lambda \) which satisfy

\[ k^\nu \nabla_\nu k^\mu = 0, \quad k^\mu k_\mu = 0. \] 

(3.12)

As it is well known, see e.g. [95, 96], the red shift \( Z \), which reflects the dependence of frequency on the relative motion of source and observer, is given by the simple formula

\[ 1 + Z = \frac{(k^\mu u_\mu)_S}{(k^\mu u_\mu)_P}, \] 

(3.13)

where the subscripts “S” and “P” refer, respectively, to the spacetime points where a source emitted a ray and an observer detected it.

MBR is characterized by the black-body spectrum. One can show [95] that the temperature of the black-body radiation depends on the red shift,

\[ T_P = \frac{T_S}{1 + Z} = T_S \frac{(k^\mu u_\mu)_P}{(k^\mu u_\mu)_S}. \] 

(3.14)

In general, as was noticed, e.g., in [12, 81, 52, 33, 25], the observed temperature depends on the direction of observation through the ratio of \( (k^\mu u_\mu) \)'s. However, it is not the case for the class of shear-free models (3.1). Indeed, from (3.12) we easily prove that the scalar product of the wave vector with any Killing vector or conformal Killing vector
is constant along the ray. In particular, for the conformal Killing (3.8) we immediately obtain the first integral of the null-geodesics equations (3.12):

\[ k_\mu \xi^\mu_{\text{conf}} = \text{const} \implies a(t_P)(u^\mu k_\mu)_P = a(t_S)(u^\mu k_\mu)_S, \quad (3.15) \]

where we used the evident proportionality of the Killing vector to the average four-velocity of matter,

\[ \xi^\mu_{\text{conf}} = a(t) u^\mu. \quad (3.16) \]

Substituting (3.15) into (3.14), one finds

\[ T_P = \frac{T_S}{1 + Z} = T_S \frac{a(t_S)}{a(t_P)}. \quad (3.17) \]

Thus, in the class of cosmological models (3.1), MBR is completely isotropic, exactly like in the standard cosmology. As a consequence, the earlier strong restrictions of the global rotation derived from the isotropy of MBR [142, 81, 32, 33, 25], are not applicable to the class of models under consideration. At the same time, the result obtained by no means contradicts the earlier observations, all of which were made for cosmologies with shear. Here we have trivial shear (3.6), and thus one clearly concludes that it is shear, not vorticity, which is the true source of anisotropy of MBR.

**No parallaxes:** Rotation (3.5) does not produce parallax effects.

In a very interesting paper [141], Hasse and Perlick introduced the generalized notion of parallax in cosmology. Let \( S_1 \) and \( S_2 \) be any two sources, seen by an observer \( P \). Provided all the three objects do not have peculiar motion with respect to the matter with the average four-velocity \( u^\mu \), one says of a parallax effect when the angle between the rays coming from \( S_1 \) and \( S_2 \) to \( P \), changes in time. Six equivalent conditions were formulated in [141] which are sufficient for the absence of parallax. One can easily check that all of them are satisfied for the class of models (3.1). For example, according to the P(3) condition (in notation of [141]), the velocity \( u^\mu \) should be proportional to some
conformal Killing vector. Indeed, this is the case, (3.10). Alternatively, according to P(4) there should exist a function $f$ such that

$$\frac{1}{2} \mathcal{L}_u g_{\mu\nu} = (u^\lambda \partial_\lambda f) g_{\mu\nu} - u_{(\mu} \partial_{\nu)} f.$$ 

One verifies straightforwardly that $f = \ln a$. According to P(5), $u^\mu$ is shear-free and the one-form

$$\rho := \left( u^\nu \nabla_\nu u^\mu - \frac{\vartheta}{3} u^\mu \right) dx^\mu$$

is closed, $d\rho = 0$. From (3.7) and (3.4), we find $\rho = 0$, hence P(5) is trivially fulfilled. Finally, according to P(6), there should exist a “red shift potential”. One can prove that again the latter if given by the function $f = \ln a$.

We thus conclude, that the models (3.1) are parallax-free, and consequently, the value of vorticity cannot be determined from parallax effects, contrary to the claims of [373,417,418].

Summarizing, cosmological models with rotation and expansion (3.1) solve the first three problems of cosmic rotation, and the most strong limits on the cosmic rotation, obtained earlier from the study of MBR [142,81,32] and of the parallaxes in rotating world [417,373], are not true for this class of cosmologies. This class of metrics is rich enough, as it contains all kinds of worlds: open and closed ones with different topologies.

## 4. GÖDEL TYPE EXPANDING COSMOLOGICAL MODEL

To the end of this paper we will consider now the natural non-stationary generalization of the original Gödel metric (1.1) which has drawn considerable attention in the literature. This generalized model is obtained from (2.1) by introducing the time-dependent scale factor $a(t)$,

$$ds^2 = dt^2 - 2\sqrt{\sigma} a(t)e^{mx} dt dy - a^2(t)(dx^2 + ke^{2mx} dy^2 + dz^2).$$

(4.1)
Here, \( x^1 = x, x^2 = y, x^3 = z \), and the metric (4.1) evidently belongs to the family (3.1) with (3.11). As we saw already, the condition \( k > 0 \) guarantees the absence of closed time-like curves. The metric (4.1) is usually called the Gödel type model with rotation and expansion. Coordinate \( z \) gives the direction of the global rotation, the magnitude of which

\[
\omega = \sqrt{\frac{1}{2} \omega_{\mu \nu} \omega^{\mu \nu}} = \frac{m}{2a} \sqrt{\frac{\sigma}{k + \sigma}} \quad (4.2)
\]
decreases in expanding world [compare with (2.2)].

The three Killing vector fields are, recall (2.10) and (2.16),

\[
\xi_1 = \partial_y, \quad \xi_2 = \partial_z, \quad \xi_3 = \frac{1}{m} \partial_x - y \partial_y, \quad (4.3)
\]

These satisfy commutation relations

\[
[\xi_1, \xi_2] = \xi_2, \quad [\xi_1, \xi_3] = [\xi_2, \xi_3] = 0, \quad (4.4)
\]
showing that the model (4.1) belongs to the Bianchi type III.

As before, it is convenient to choose at any point of the spacetime (4.1) a local orthonormal (Lorentz) tetrad \( h^a_\mu \). For simplicity, we will work with the immediate replacement of (2.3):

\[
h^0_0 = 1, \quad h^0_2 = -a \sqrt{\sigma e^{mx}}, \quad h^1_1 = h^2_2 = a, \quad h^3_3 = a e^{mx} \sqrt{k + \sigma}. \quad (4.5)
\]

From the point of view of the Petrov classification, one can verify that the Gödel type model is of the type \( D \). Indeed, a straightforward calculation yields an obvious generalization of (2.6):

\[
C^0_0^0_0 = C^0_0^2_2 = - C^2_3^3_2 = - C^3_3^3_2 = \frac{1}{2} \quad C^1_1^2_2 = - \frac{1}{2} \quad C^0_0^3_3 = \frac{m^2}{6a^2(t)} \left( \frac{k}{k + \sigma} \right). \quad (4.6)
\]

5. DYNAMICAL REALIZATIONS

Several dynamical realizations (i.e. construction of exact cosmological solutions for the gravitational field equations) of the shear-free models (3.1), and in particular of the Gödel
type metric (4.1), are known. In the Einstein’s general relativity theory the Gödel type models were described in \[212,190,311–318,320\] with different matter sources, whereas the Bianchi type II and III solutions (3.1) were obtained in \[357,283\] for the imperfect fluid with heat flow and cosmological constant.

In this section, we will describe a model \[167\] in which the gravitational field dynamics is determined by the minimal quadratic Poincaré gauge model, which is the closest extension of the Einstein’s general relativity theory.

### A. Field equations

In the framework of Poincaré gauge theory, gravitational field is described by the tetrad \(h^a_\mu\) and the local Lorentz connection \(\Gamma^a_{b\mu}\). In general case, gravitational Lagrangian is constructed as an invariant contraction from the curvature tensor

\[
R^a_{b\mu\nu} = \partial_\mu \Gamma^a_{b\nu} - \partial_\nu \Gamma^a_{b\mu} + \Gamma^a_{c\mu} \Gamma^c_{b\nu} - \Gamma^a_{c\nu} \Gamma^c_{b\mu},
\]

(5.1)

and the torsion tensor

\[
T^a_{\mu\nu} = \partial_\mu h^a_\nu - \partial_\nu h^a_\mu + \tilde{\Gamma}^a_{b\mu} h^b_\nu - \tilde{\Gamma}^a_{b\nu} h^b_\mu.
\]

(5.2)

Note that here our notations for the connection, curvature and torsion are slightly different from \[149\]. In order to illustrate some common properties of cosmological models with rotation and expansion, here we consider the so called minimal quadratic Poincaré gauge gravity \[167\] which is the closest extension of Einstein’s general relativity (we may also call such a model the generalized Einstein-Cartan theory). The corresponding Lagrangian reads:

\[
L_g = -\frac{1}{16\pi G} \left[ R + b R^2 + a_1 T^a_{\mu\nu} T^\mu\nu + a_2 T^a_{\alpha\mu\nu} T_{\mu\alpha\nu} + a_3 T^a_{\mu} T^\mu \right].
\]

(5.3)

Here \(R = h_\mu^a h^{\nu b} R^a_{b\mu\nu}\) – the Riemann-Cartan curvature scalar, \(T^a_{\mu\nu} = h_\alpha^a T^a_{\mu\nu}\), and \(T^a_{\mu} = T^\lambda_{\mu\lambda}\) – is the torsion trace, \(b, a_1, a_2, a_3\) are the coupling constants.

Independent variation of the action (5.3) with respect to \(h^a_\mu\) and \(\tilde{\Gamma}^a_{b\mu}\) yields the field equations:
\[ R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R + 2bR \left( R_{\mu\nu} - \frac{1}{4} g_{\mu\nu} R \right) \]
\[ + a_1 \left( -\frac{1}{2} g_{\mu\nu} T_{\alpha\beta\gamma} T^{\alpha\beta\gamma} + 2T^\alpha_{\mu\beta} T^{\mu\beta} - 2T^\alpha_{\alpha\nu} T^{\mu\nu} - 2\nabla_\alpha T^{\mu\nu} - T^{\mu\nu}_{\alpha\beta} T^{\alpha\beta} \right) \]
\[ + a_2 \left( -\frac{1}{2} g_{\mu\nu} T_{\alpha\beta\gamma} T^{\beta\alpha\gamma} + T^\beta_{\mu\beta} T^{\mu\alpha} + T^\alpha_{\alpha\nu} T^{\mu\nu} - \nabla_\alpha T^{\mu\nu} + \nabla_\alpha T^{\alpha}_{\nu\mu} \right) \]
\[ + a_3 \left( \frac{1}{2} g_{\mu\nu} T^{\alpha}_{\alpha\nu} - \nabla_\nu T^{\alpha}_{\mu} + g_{\mu\nu} \nabla_\alpha T^{\alpha} \right) = \kappa \Sigma_{\mu\nu}, \quad (5.4) \]

\[- \frac{1 + 2bR}{2} \left( T^\alpha_{\mu\nu} + 2\delta^\alpha_{[\mu} T^{\nu]} \right) - 2b\delta^\alpha_{[\mu} \partial_{\nu]} R \]
\[ + (2a_1 - a_2) T^{\alpha}_{[\mu\nu]} + a_2 T^{\alpha}_{\mu\nu} + a_3 \delta^\alpha_{[\mu} T^{\nu]} = \kappa \tau^\alpha_{\mu\nu}, \quad (5.5) \]

where \( \kappa = 8\pi G \), with \( \tau^\alpha_{\mu\nu} \) and \( \Sigma_{\mu\nu} \) as the canonical tensors of spin and energy-momentum, respectively.

Equation (5.3) generalizes the well known algebraic relation between torsion and spin in the Einstein-Cartan theory. The tensors of torsion and spin may be decomposed into irreducible parts: trace \( T^\mu_{\mu} \) and \( \tau^\mu_{\mu} = \tau^\lambda_{\mu\lambda} \), pseudotrace \( \nabla^\alpha T^\mu_{\mu} = \frac{1}{2} \epsilon_{\mu\nu\alpha\beta} T^{\nu\alpha\beta} \) and \( \nabla^\alpha \tau^\mu_{\mu} = \frac{1}{2} \epsilon_{\mu\nu\alpha\beta} \tau^{\nu\alpha\beta} \), and trace-free parts \( T^\alpha_{\mu\nu} \) and \( \tau^\alpha_{\mu\nu} \):

\[ T^\alpha_{\mu\nu} = T^\alpha_{\mu\nu} + \frac{2}{3} T_{[\mu} \delta^\alpha_{\nu]} + \frac{1}{3} \epsilon^\alpha_{\mu\nu\lambda} \nabla^\lambda, \quad (5.6) \]
\[ \tau^\alpha_{\mu\nu} = \tau^\alpha_{\mu\nu} + \frac{2}{3} \tau_{[\mu} \delta^\alpha_{\nu]} + \frac{1}{3} \epsilon^\alpha_{\mu\nu\lambda} \tau^\lambda. \quad (5.7) \]

Easy to see that eq. (5.3) decomposes into three equations for irreducible components:

\[ (\mu_2 + 2bR) T^\mu_{\mu} + 3b \partial_\mu R = \kappa \tau^\mu_{\mu}, \quad (5.8) \]
\[ (\mu_1 - bR) \nabla^\mu T^\nu_{\mu} = \kappa \nabla^\nu T^\mu_{\mu}, \quad (5.9) \]
\[ (\mu_3 - bR) T^\alpha_{\mu\nu} = \kappa \tau^\alpha_{\mu\nu}, \quad (5.10) \]

where we denoted the following combinations of the coupling constants

\[ \mu_1 = 2a_1 - 2a_2 - \frac{1}{2}, \quad (5.11) \]
\[ \mu_2 = 1 - \frac{1}{2} \left( 2a_1 + a_2 + 3a_3 \right), \quad (5.12) \]
\[ \mu_3 = -\left( \frac{1}{2} + a_1 + \frac{a_2}{2} \right), \quad (5.13) \]
which play important role in the determination of mass for quanta of torsion and couplings in all spin sectors of quadratic Poincaré gauge gravity model.

The eq. (5.8) shows that in contrast to the other irreducible parts, the torsion trace is characterized by the differential and not merely algebraic coupling. The Riemann-Cartan curvature scalar plays a role of its “potential”. Contraction of (5.4) yields an equation for $R$,

$$- R + \frac{1}{2} (1 + 2 \mu_3) T_{\alpha\mu\nu} T^{\alpha\mu\nu} + \frac{1}{6} (1 + 2 \mu_1) \dot{\gamma} \mu T^\nu + 2 (1 - \mu_2) \left( \dot{\nabla}_\mu T^\nu - \frac{1}{3} T^\mu T^\nu \right) = \kappa \Sigma,$$

(5.14)

where $\Sigma = g_{\mu\nu} \Sigma^{\mu\nu}$ is the trace of the energy-momentum tensor. Inserting (5.8)-(5.10) into (5.14), we find a non-linear differential equation for the curvature scalar. As usual, hereafter we denote by tilde the purely Riemannian (torsion-free) geometrical objects and operators.

In view of specific nature of the torsion trace, it is convenient to separate its contribution to the eq. (5.4). Let us introduce

$$\hat{\hat{T}}_{\alpha\mu\nu} = \hat{T}_{\alpha\mu\nu} + \frac{1}{3} \epsilon_{\alpha\mu\nu\lambda} \dot{\gamma} \lambda,$$

(5.15)

and hereafter hat “” will denote the corresponding geometrical quantities constructed with the help of the Riemann-Cartan connection with the trace-free torsion (5.15). The symmetric part of (5.4) then is rewritten as

$$(1 + 2 b R) \left( \hat{R}_{(\mu\nu)} - \frac{1}{2} g_{\mu\nu} \hat{R} \right) + \frac{1}{2} b R^2 g_{\mu\nu} + \left( \frac{1 + 2 \mu_3}{12} \right) T^\lambda \hat{T}_{(\mu\nu)\lambda}

+ a_1 \left( - \frac{1}{2} g_{\mu\nu} \hat{T}_{\alpha\beta\gamma} \hat{T}^{\alpha\beta\gamma} + 2 \hat{T}^{\alpha\mu\beta} \hat{T}_{\alpha\nu} - \hat{T}_{\mu\alpha\beta} \hat{T}_{\nu}^{\alpha\beta} - 2 \dot{\nabla}_\alpha \hat{T}_{(\mu\nu)}^{\alpha} \right)

+ a_2 \left( - \frac{1}{2} g_{\mu\nu} \hat{T}_{\alpha\beta\gamma} \hat{T}^{\beta\alpha\gamma} + \hat{T}_{\alpha\beta\mu} \hat{T}^{\beta\alpha\nu} - \dot{\nabla}_\alpha \hat{T}_{(\mu\nu)}^{\alpha} \right)

+ \left( \frac{8 b R + 4 \mu_2}{3} \right) \left( \dot{\nabla}_{(\mu} T_{\nu)} - g_{\mu\nu} \nabla_\alpha T^\alpha - \frac{1}{6} g_{\mu\nu} T_\alpha T^\alpha \right) = \kappa \Sigma_{(\mu\nu)},$$

(5.16)

whereas the antisymmetric part of (5.4) is an identity in view of the geometric relation

$$2 R_{[\mu\nu]} + (\nabla_\alpha + T_\alpha) \left( T^{\alpha}_{\mu\nu} + 2 \delta^{\alpha}_{[\mu} T_{\nu]} \right) = 0,$$

(5.17)

and the angular momentum conservation law
\[(\nabla_{\tau} + T_{\alpha})_{\mu \nu} = \Sigma_{\mu \nu}, \] (5.18)

Inserting the torsion \( T_{\mu} \) and \( \nabla_{\tau} \) from (5.9) and (5.10) into (5.13), and further into (5.16), one can write, similarly to the Einstein-Cartan theory, the effective Einstein equations for the metric. Spin of matter will contribute to the so called modified energy-momentum tensor.

### B. Matter sources

We will describe cosmological matter by means of phenomenological model of a spinning fluid of Weyssenhoff and Raabe. This is a classical model of a continuous medium the elements of which are characterized, along with energy and momentum, by an intrinsic angular momentum (spin). Usually, hydrodynamical approach is considered as a good approximation to the description of realistic cosmological matter on the early, as well as on the later stages of universe’s evolution. It seems natural to take into account spin (proper angular momentum) of elements of cosmological fluid, i.e., particles on early stage and galaxies and clusters of galaxies on later stages. In general case, an element of the Weyssenhoff fluid is characterized by the tensor of spin density \( \tau_{\mu \nu} \), charge \( e \) and (proportional to spin) magnetic moment \( \chi \tau_{\mu \nu} \) (\( \chi \) is a constant). The consistent variational theory of spin fluid in the Riemann-Cartan spacetime, developed in [289], yields the following currents in the right-hand sides of the gravitational field equations (5.4) and (5.5):

\[
\Sigma_{\mu \nu} = -pg_{\mu \nu} + u_{\mu} \left[ u_{\nu} \left( \epsilon + p + \chi \tau_{\alpha \beta} F^{\alpha \beta} \right) + 2u^{\alpha} \nabla_{\beta} \tau_{\alpha \nu} \right] \\
- F_{\mu \alpha} F_{\nu}^{\alpha} + \frac{1}{4} g_{\mu \nu} F_{\alpha \beta} F^{\alpha \beta} - 2\chi \left( \tau_{\mu \alpha} \tau_{\nu}^{\alpha} + \tau_{\nu \beta} \tau^{\alpha \beta} u_{\alpha} u_{\mu} \right),
\] (5.19)

\[
\tau^{\alpha}_{\mu \nu} = u^{\alpha} \tau_{\mu \nu}, \] (5.20)

where \( u^{\mu} \) is fluid’s four-velocity, \( p \) is pressure, \( \epsilon \) is the internal energy density, \( F_{\mu \nu} = \partial_{\mu} A_{\nu} - \partial_{\nu} A_{\mu} \) is the electromagnetic field created by and interacting with the electrodynamical characteristics of the medium.

Dynamics of the fluid is given by the rotational and translational equations of motion:
\[(\nabla_\alpha + T_\alpha) \tau^\alpha_{\mu \nu} = u_\mu u^\lambda \nabla_\alpha \tau^\alpha_{\lambda \nu} - u_\nu u^\lambda \nabla_\alpha \tau^\alpha_{\lambda \mu} + \chi F^{\alpha \beta} \left[ (\delta^\beta_\mu - u^\beta u_\mu) \tau_{\alpha \nu} - (\delta^\beta_\nu - u^\beta u_\nu) \tau_{\alpha \mu} \right], \tag{5.21} \]
\[(\nabla_\nu + T_\nu) \Sigma^\nu_{\mu} - T^\alpha_{\mu \beta} \Sigma^\beta_{\alpha} - \tau^\nu_{\alpha \beta} R^{\alpha \beta}_{\mu \nu} = 0. \tag{5.22} \]

The self-consistent variational principle [289] yields also the Maxwell’s equations for the electromagnetic field and the conservation law of number of particles:

\[\tilde{\nabla}_\mu F^{\mu \nu} = e \rho u^\nu + 2 \chi \tilde{\nabla}_\mu \tau^{\mu \nu}, \tag{5.23} \]
\[\tilde{\nabla}_\mu (\rho u^\mu) = 0, \tag{5.24} \]

where \(\rho\) is the particle density.

The system of differential and algebraic equations (5.4), (5.5), (5.19)-(5.24) is complete, and one can determine all the gravitational and material physical variables from this system. It is worthwhile to remind that spin satisfies the Frenkel condition,

\[u^\mu \tau_{\mu \nu} = 0. \tag{5.25} \]

The specific features of dynamics of the Weyssenhoff spin fluid in the Riemann-Cartan spacetime were studied in detail in [289].

**C. Exact solution for the Gödel type universe**

Let us describe cosmological model with rotation and expansion which provides an exact solution of the gravitational field equations. We will restrict ourselves to the shear-free class of spatially homogeneous metrics (3.1), and for definiteness, a particular case – the Gödel type model (4.1) – will be considered in detail.

The field equations (5.4) and (5.5) with the sources (5.19), (5.20) are most conveniently formulated with respect to (nonholonomic) local Lorentz frame. Let us choose the coframe (4.5). Its inverse reads:

\[h_0^0 = 1, \quad h_2^0 = \sqrt{\frac{\sigma}{k + \sigma}}, \quad h_3^3 = h_1^1 = \frac{1}{a}, \quad h_2^3 = \frac{1}{ae^{mx} \sqrt{k + \sigma}}, \tag{5.26} \]
Direct calculation of the local Lorentz connection \( \tilde{\Gamma}_{b\mu}^a = h_a^\alpha h_b^\beta \tilde{\Gamma}_\beta^\alpha_{\mu} + h_a^\alpha \partial_\mu h_b^\alpha \) (with \( \tilde{\Gamma}_\beta^\alpha_{\mu} \) as the Christoffel symbols) yields for the metric \( 4.1\):

\[
\tilde{\Gamma}^0_{20} = \tilde{\Gamma}^1_{21} = \tilde{\Gamma}^3_{23} = \frac{a}{\sqrt{k + \sigma}}, \quad \tilde{\Gamma}^1_{22} = -\frac{m}{a};
\]

\[
\tilde{\Gamma}^2_{01} = \tilde{\Gamma}^2_{10} = -\tilde{\Gamma}^0_{12} = \frac{m}{2a} \sqrt{\frac{\sigma}{k + \sigma}},
\]

where \( \tilde{\Gamma}_{bc} = \tilde{\Gamma}_{b\mu} h_c^\mu \).

Since the direction of rotation (along the \( z \) axis) is apparently a distinguished one, it is natural to assume that the spin of the fluid and electromagnetic field are also oriented correspondingly along \( z \). For the medium with vanishing magnetic moment of its elements \( (\chi = 0) \) one finds from \( 5.21\)\)-(\( 5.25\)):

\[
\tau^i_{12} = \tau_0 a^{-3};
\]

\[
F^i_{12} = B_0 a^{-2},
\]

\[
\rho = \rho_0 a^{-3},
\]

where the integration constants are related by the consistency condition

\[
mB_0 \sqrt{\frac{\sigma}{k + \sigma}} = e \rho_0.
\]

We will now describe a solution of the gravitational field equations for which the Riemann-Cartan curvature scalar is assumed to be constant:

\[
R = \text{const}.
\]

Then in view of \( 5.25\), eqs. \( 5.8\)\)-(\( 5.10\)) yield the irreducible parts of torsion:

\[
T_\mu = 0, \quad \tilde{T}_\mu = \frac{\lambda_1}{2} \epsilon_{\mu\nu\alpha\beta} u^\nu \tau^{\alpha\beta}, \quad \mathbf{T}^\alpha_{\mu\nu} = \frac{\lambda_3}{3} \left( 2u^\alpha \tau_{\mu\nu} + u_\mu \tau^\alpha_{\nu} - u_\nu \tau^\alpha_{\mu} \right),
\]

where we denoted the constants

\[
\lambda_1 = \frac{\kappa}{\mu_1 - bR}, \quad \lambda_3 = \frac{\kappa}{\mu_3 - bR}.
\]
Nontrivial solution exists only when \( \mu_1 \neq bR \neq \mu_3 \), otherwise (5.9) and (5.10) lead to the vanishing spin. As for the eq. (5.8), one has two alternatives: if \( \mu_2 + 2bR \neq 0 \), then the torsion trace is zero, whereas if \( \mu_2 + 2bR = 0 \), then \( T_\mu \) is arbitrary. However in both cases the torsion trace completely decouples from the eq. (5.16), hence without restricting generality we can put \( T_\mu = 0 \).

Using (5.34), we can rewrite (5.15) as

\[
\hat{T}_\alpha^{\mu\nu} = \frac{1}{3} [(2\lambda_3 + \lambda_1) u^\alpha \tau_{\mu\nu} + (\lambda_3 - \lambda_1) u_\mu \tau^\alpha_\nu + (\lambda_1 - \lambda_3) u_\nu \tau^\alpha_\mu].
\] (5.36)

Substituting this expression into (5.16), one finds

\[
a_1 \left( -\frac{1}{2} g_{\alpha\beta} \hat{T}_{\alpha\beta\gamma} \hat{T}^{\alpha\beta\gamma} + 2 \hat{T}_\alpha^{\mu\beta} \hat{T}_\mu^{\alpha\beta} - \hat{T}_{\mu\alpha\beta} \hat{T}_\nu^{\alpha\beta} - 2 \nabla_\alpha \hat{T}(\mu\nu)^\alpha \right)
+a_2 \left( -\frac{1}{2} g_{\alpha\beta} \hat{T}_{\alpha\beta\gamma} \hat{T}^{\beta\alpha\gamma} + \hat{T}_{\alpha\beta\mu} \hat{T}_\nu^{\beta\alpha} - \nabla_\alpha \hat{T}(\mu\nu)^\alpha \right) = \frac{1}{18} (2\lambda_3 + \lambda_1) \left[ \lambda_1 (1 + 2\mu_1) - \lambda_3 (1 + 2\mu_3) \right] \tau_{\mu\alpha} \tau_\nu^\alpha
+ \left( \frac{\lambda_1 - 4\lambda_3}{36} \right) [\lambda_1 (1 + 2\mu_1) + 2\lambda_3 (1 + 2\mu_3)] u_\mu u_\nu \tau_{\alpha\beta} - \tau_{\alpha\beta}
+ \frac{1}{24} \left[ 4\lambda_3^2 (1 + 2\mu_3) - \lambda_1^2 (1 + 2\mu_1) \right] g_{\mu\nu} \tau_{\alpha\beta} \tau^{\alpha\beta} + \lambda_3 (1 + 2\mu_3) \nabla_\alpha \left[ u(\mu_\tau)^\alpha \right].
\] (5.37)

After these computations, one finally finds that when the constant in (5.33) is equal \( R = -1/2b \), the equations (5.16) are reduced to the algebraic relations between the pressure \( p \), energy density \( \epsilon \), spin \( \tau_{\alpha\beta} \) and Maxwell tensor \( F_{\alpha\beta} \). From these equations, in addition to (5.32) we find a new relation between the integration constants,

\[
B_0^2 = -m \tau_0 \sqrt{\frac{\sigma}{k + \sigma}}.
\] (5.38)

Without restricting generality, one can choose the parameter \( m \) to be positive, then the minus sign in (5.38) means that the spin and vorticity have the same direction. Thus \( \tau_0 < 0 \) [from (3.3), (4.5) and (5.26) we have \( \omega_{12} = -\frac{m}{2\alpha} \sqrt{\frac{\sigma}{k + \sigma}} \)]. In its turn, the direction of the magnetic field (sign of \( B_0 \)) depends on the sign of the charge \( e \).

With the help of (5.38) the equations (5.14) are reduced to the following two equations which describe the state of matter:
\[
p(t) = -\frac{1}{8b\kappa} + \frac{B_0^2}{2a^4} + \left(\frac{\lambda_1 - 4\lambda_3}{6}\right) \frac{\tau_0^2}{a^6}, \quad (5.39)
\]
\[
\epsilon(t) = \frac{1}{8b\kappa} + \frac{3B_0^2}{2a^4} + \left(\frac{\lambda_1 - 4\lambda_3}{6}\right) \frac{\tau_0^2}{a^6}, \quad (5.40)
\]

Finally, the evolution of the scale factor \(a(t)\) is given by the equation (5.33). Substituting (4.5), (5.26), and (5.28) in it, one finds
\[
\ddot{a} + \frac{\dot{a}^2}{a} - \frac{m^2(3\sigma + 4k)}{12ka^2} - \left(\frac{k + \sigma}{36k}\right) \left(4\lambda_3^2 - \lambda_1^2\right) \frac{\tau_0^2}{a^6} - \frac{k + \sigma}{12kb} = 0. \quad (5.41)
\]
As one can see, depending on the values of the coupling constants \(\lambda_3\) and \(\lambda_1\), torsion can either accelerate or prevent the cosmological collapse. We will consider the latter possibility, assuming, for concreteness, that \(4\lambda_3^2 \gg \lambda_1^2\).

Integrating (5.41), one gets
\[
\dot{a}^2 = \frac{m^2(3\sigma + 4k)}{12k} + \frac{a^2(k + \sigma)}{24kb} - \left(\frac{k + \sigma}{36k}\right) \left(4\lambda_3^2 - \lambda_1^2\right) \frac{\tau_0^2}{a^6} + \frac{\alpha}{a^2}. \quad (5.42)
\]

The integration constant \(\alpha\) is determined by the normalization of the scale factor \(a = 1\) at the moment of a bounce \((t = 0)\), when \(\dot{a} = 0\),
\[
\alpha = \left(\frac{k + \sigma}{36k}\right) \left(4\lambda_3^2 - \lambda_1^2\right) \tau_0^2 - \frac{m^2(3\sigma + 4k)}{12k} - \frac{k + \sigma}{24kb}. \quad (5.43)
\]

**D. Estimates of parameters of the cosmological model**

In order to integrate the equation (5.42), one needs to know the constant parameters which enter that equation, because their numeric essentially affect the form of solution. All constants are uniquely determined by the data which describe modern state of the universe: the equation of state \(p_t \approx 0\) (dust), energy density \(\epsilon_t\), Hubble constant \(H_t(= \dot{a}/a)\) and the deceleration parameter \(q_t(= -\ddot{a}/a\dot{a}^2)\) are estimated by
\[
0.15 < \frac{\kappa}{H_t} < 12,
\]
\[
4 \cdot 10^{-11} \text{ yr}^{-1} < H_t < 10^{-10} \text{ yr}^{-1},
\]
\[
0.01 < q_t < 1.
\]
(Hereafter the index \( t \) denotes modern values of the physical quantities).

Previously, \[46\], the angular velocity was estimated as relatively small \( \omega_t/H_t \approx 10^{-3} \).

New analyses of observational data suggests higher estimates for the global vorticity, more close to \( H_t \). For the qualitative analysis of the model let us choose

\[
q_t = 0.01, \quad \frac{k\epsilon_t}{H_t^2} = 0.18, \quad \frac{\omega_t}{H_t} = 0.1. \tag{5.44}
\]

Besides the estimates (5.44), one needs also the values of the spin-torsion coupling constants. There is a large ambiguity which affects the scale of the space at the moment of a bounce. If one assumes, cf. Trautman \[416\], that the cosmological collapse stops on the scale of order of 1 cm, then we get from (5.44) an estimate

\[
\frac{\kappa}{\lambda_1 - 4\lambda_3} \approx 2.3 \cdot 10^{27}. \tag{5.45}
\]

Comparison of the equations (5.41)-(5.43) with (5.44)-(5.45) yields the modern value of the spin density

\[
3 \cdot 10^8 \text{ g cm}^{-1} \text{ s}^{-1} < \tau_t < 7.6 \cdot 10^8 \text{ g cm}^{-1} \text{ s}^{-1}, \tag{5.46}
\]

the geometric parameters of the model

\[
\frac{k}{\sigma} \approx 71, \tag{5.47}
\]

the coupling constant

\[
 b^{-1} = 0.36 H_t^2, \tag{5.48}
\]

and the magnetic field

\[
10^{-6} \text{ G} < B_t < 2.6 \cdot 10^{-6} \text{ G}. \tag{5.49}
\]

The final integration of the equation (5.42) determines the law of evolution of the scale factor:

\[
4\sqrt{\alpha_3} H_t t = (\text{const}) + \ln Z(a^2) + \frac{2\mu}{k\sqrt{-B_1B_2}} \left[ \frac{\gamma_1}{\gamma_1 + \gamma_2} F(\varphi; \tilde{m}) - C_1 \Pi(\varphi; n, \tilde{m}) \right], \tag{5.50}
\]
where the integration constant (const) is such that \( a = 1 \) for \( t = 0 \), and we denote the function of the scale factor

\[
Z(a^2) = \frac{(a^2 + \gamma_1)^2 + \sqrt{a^2(a^2 - 1)(a^2 - u_1)(a^2 - u_2)}}{\gamma_1 + \gamma_2(\gamma_1 - \gamma_2 + 2a^2)} - \frac{C_1 + C_2}{2}.
\]

In eq. (5.50), the constant parameters are determined by:

\[
\alpha_1 = \frac{m^2(3\sigma + 4k)}{12kH_t^2}, \quad \alpha_2 = \left(\frac{k + \sigma}{k}\right)\left(\frac{4\lambda_3^2 - \lambda_1^2}{36}\right)\frac{\gamma_0^2}{H_t^2}, \quad \alpha_3 = \frac{k + \sigma}{12bkH_t^2};
\]

\( u_1 > u_2 \) are the roots of the quadratic equation

\[
u^2 + \left(1 + \frac{\alpha_1}{\alpha_3}\right)\nu + \frac{\alpha_2}{\alpha_3} = 0,
\]

from which one constructs

\[
\lambda_\pm = -u_1u_2 \left(\sqrt{1 - 1/u_1} \pm \sqrt{1 - 1/u_2}\right)^2,
\]

\[
B_1 = \frac{\lambda_+ (1 - \lambda_-)}{\lambda_+ - \lambda_-}, \quad C_1 = \frac{\lambda_- (\lambda_+ - 1)}{\lambda_+ - \lambda_-}, \quad B_2 = \frac{1 - \lambda_-}{\lambda_+ - \lambda_-}, \quad C_2 = \frac{\lambda_+ - 1}{\lambda_+ - \lambda_-}, \quad \gamma_1 = \sqrt{\frac{u_1u_2}{1 - \lambda_-}}, \quad \gamma_2 = \sqrt{\frac{u_1u_2}{1 - \lambda_+}}, \quad \mu^2 = -\frac{B_1}{C_1}, \quad \hat{k}^2 = \frac{\lambda_+}{\lambda_-}, \quad \hat{m}^2 = (\lambda_+ - \lambda_-) / \lambda_+, \quad n = B_1\hat{m}^2 = 1 - \lambda_-.
\]

Finally, \( F(\varphi; \hat{m}) \) and \( \Pi(\varphi; n, \hat{m}) \) are elliptic integrals of the first and the third kinds, respectively, with their argument given by

\[
\left(1 - \hat{k}^{-2}\right)\sin^2 \varphi = 1 - \mu^{-2} \left(\frac{a^2 - \gamma_2}{a^2 - \gamma_1}\right)^2.
\]

The complete picture of the cosmological evolution is obtained when one substitutes (5.44)-(5.49) into the above equations. In particular, one can verify that the age of the universe [estimated as the time since \( t = 0 \) till the modern stage characterized by (5.44)] is equal \( T \approx H_t^{-1} \), in a good agreement with observational data.

The analysis of the evolution of the scale factor \( a(t) \), see (5.50) and (5.42), shows that one can naturally distinguish several qualitatively different stages in the history of the universe. First stage is the shortest one and it describes a bounce in the vicinity of \( t = 0 \). There is no
initial singularity due to the dominating spin contribution in \((5.42)\). At this stage, the fluid source is characterized by the approximate equation of state of stiff matter

\[ p = \epsilon \approx \left( \frac{\lambda_1 - 4\lambda_3}{6} \right) \frac{\tau_t^2}{a^6} \]

The applicability of the classical (non-quantum) gravitational theory is guaranteed by the condition on the curvature invariant

\[ \bar{R}_{\alpha\beta\mu
\nu} \bar{R}_{\alpha\beta\mu
\nu} \ll l_{pl}^{-4}, \]

which is satisfied at the moment of the bounce. The duration of that stage is quite small \((\ll 1 \text{ s})\), since the spin term quickly decreases in \((5.42)\) with the growth of the scale factor.

Next comes the second stage when the scale factor increases like \(\sqrt{t}\), while the equation of state is of the radiation type, \(p \approx \epsilon/3\). This “hot universe” expansion lasts until the size of the metagalaxy approaches \(\approx 10^{27} \text{ cm}\). After this the “modern” stage starts with the effectively dust equation of state \(p_t \approx 0, \epsilon_t \approx (2b\kappa)^{-1}\). Scale factor still increases, but the deceleration of expansion takes place. The final stage depends on the value of the cosmological term, and either the future evolution enters the eternal de Sitter type expansion, or expansion ends and a contraction phase starts.

Our purpose in this section was to demonstrate the exact solution which avoids the principal difficulties of old cosmological models with rotation and expansion. Certainly, one cannot claim that the results obtained describe the real universe, and consequently we have limited ourselves to a test type computations for the physical and geometrical parameters, without trying to find the best estimates. The main difficulty of this particular dynamical realization, in our opinion, is presented by the magnitude of magnetic field \((5.49)\) which at the “modern” stage should be close to the upper limits established for the global magnetic field from astrophysical observations, see e.g. [24].

It seems worthwhile to note that \((5.46)\) gives a rather big value for the spin density \((\approx 10^{35}\hbar/\text{cm}^3)\). This is in a good agreement with the macroscopic interpretation of the cosmological spin fluid, the elements of which are spinning galaxies. Indeed, \(\tau_t/\rho_t \approx 10^{107}\hbar\) is close to the observed magnitude of the angular momentum of a typical galaxy. Moreover, for the total spin of the metagalaxy occupying the volume \(V \approx 10^{84}\text{ cm}^3\), one finds \(\tau_{tot} = \)
\[ \tau V \approx 10^{120} \hbar. \] This once again draws attention to the Large Number hypothesis of Dirac-Eddington and related questions, see [57–60, 63, 140] and [271–274].

6. NULL GEODESICS IN THE GÖDEL TYPE MODEL

Practically all the information about the structure of the universe and about the properties of astrophysical objects is obtained by an observer in the form of different kinds of electromagnetic radiation. Thus, in order to be able to make theoretical predictions and compare them with observations, it is necessary to know the structure of null geodesics in the cosmological model with rotation. All the models from the class \((3.1)\) have three Killing vectors and one conformal Killing vector field. Hence the null geodesics equations \((3.12)\) have four first integrals,

\[ q_0 = \xi^\mu_{\text{conf}} k_\mu, \quad q_a = -\xi^\mu_{(a)} k_\mu, \quad a = 1, 2, 3. \] \hspace{1cm} (6.1)

Solving \((6.1)\) with respect to \( k^\mu \), one obtains a system of ordinary first order nonlinear equations which can be straightforwardly integrated. Complete solution of the null geodesics equations in the Gödel type model is given in [186, 287], and here we present only short description of null geodesics in \((4.1)\).

To begin with, let us define convenient parametrization of null geodesics. Without losing generality (using the spatial homogeneity) we assume that an observer is located at the space-time point \( P = (t = t_0, x = 0, y = 0, z = 0) \). Now, arbitrary geodesics which passes through \( P \) is naturally determined by its initial direction in the local Lorentz frame of observer at this point. In the tetrad \((4.5)\) we may put

\[ k^a_P = (h^a_\mu k^\mu)_P = (1, \sin \theta \cos \phi, \sin \theta \sin \phi, \cos \theta), \] \hspace{1cm} (6.2)

where \( \theta, \phi \) are standard spherical angles parametrizing the celestial sphere of an observer. Then from \((3.8), (4.3), (6.2)\) and \((6.1)\) one finds the values of the integration constants

\[ q_0 = a_0, \]
\[ q_1 = a_0(\sqrt{\sigma} + \sqrt{k + \sigma \sin \theta \sin \phi}), \quad (6.3) \]
\[ q_2 = a_0 \cos \theta, \]
\[ q_3 = \frac{a_0}{m} \sin \theta \cos \phi. \quad (6.4) \]

Generic null geodesics, initial directions of which satisfy \( \frac{q_1}{a_0} = (\sin \theta \sin \phi + \sqrt{\frac{\sigma}{k+\sigma}}) \neq 0, \) are then described by

\[ e^{-mx} = \frac{\sqrt{\sigma} + \sqrt{k + \sigma \sin \theta \sin \Phi}}{\sqrt{\sigma} + \sqrt{k + \sigma \sin \theta \sin \phi}}, \quad (6.5) \]
\[ y = \frac{\sin \theta (\cos \Phi - \cos \phi)}{m(\sqrt{\sigma} + \sqrt{k + \sigma \sin \theta \sin \phi})}, \quad (6.6) \]
\[ z = \left( \frac{k + \sigma}{k} \right) \cos \theta \left[ t \int_{t_0}^{t} \frac{dt'}{a(t')} + \sqrt{\frac{\sigma}{k + \sigma}} \left( \frac{\Phi - \phi}{m} \right) \right], \quad (6.7) \]

where the function \( \Phi(t) \) satisfies the differential equation

\[ \frac{d\Phi}{dt} = -\frac{m}{a} \left( \frac{\sqrt{\frac{\sigma}{k+\sigma}} + \sin \theta \sin \Phi}{1 + \sqrt{\frac{\sigma}{k+\sigma}} \sin \theta \sin \Phi} \right), \quad (6.8) \]

with initial condition \( \Phi(t_0) = \phi. \)

For a detailed discussion of rays which lie on the initial cone \( (\sin \theta \sin \phi + \sqrt{\frac{\sigma}{k+\sigma}}) = 0 \) and different subcases of (6.5)-(6.8) see [287].

7. OBSERVATIONS IN ROTATING COSMOLOGIES

The qualitative picture of specific rotational effects which could be observed in the Gödel type model (4.1) is in fact independent of the dynamical behaviour of the scale factor \( a(t). \) To some extent the same is true also for the quantitative estimates, especially if one uses the Kristian-Sachs formalism [214,240] in which all the physical and geometrical observable quantities are expressed in terms of power series in the affine parameter \( s \) or the red shift \( Z. \) In this case the description of observable effects on not too large (although cosmological) scales involves only the modern values (i.e. calculated at the moment of observation \( t = t_0) \) of the scale factor \( a_0 = a(t_0), \) Hubble parameter \( H_0 = (\dot{a}/a)_P, \) rotation value \( \omega_0 = \omega(t_0), \) deceleration parameter \( q_0 = -(\ddot{a}/a^2)_P, \) etc.
In this section we will outline possible observational manifestations of the cosmic rotation in the Gödel type universe. Estimates for the value of vorticity and for the direction of rotation axis can be found from the recent astrophysical data, see Sects. 7C and 7D.

A. Kristian-Sachs expansions in observational cosmology

Kristian and Sachs in the fundamental paper [214] have developed universal technique for the analysis of the observational effects in an arbitrary cosmological model. Essentially, their method is based on the theory of null geodesic (ray) bundles in Riemannian spacetime. The main advantage of the Kristian-Sachs approach is in its generality which makes it possible to consider cosmological tests even without the knowledge of explicit solution of the gravitational field equations. This is particularly convenient when one is interested in revealing additional (as compared to the standard Hubble expansion) geometrical and physical properties of the universe. In this section we will apply the method of Kristian and Sachs to the rotating cosmological models.

Let \( k^\mu \) be a wave vector field which satisfies the null geodesics equation (3.12) in a Riemannian manifold. (The connection is torsion-free, and we will omit the tilde to simplify the notation). The starting point is the equation

\[
\ddot{\Delta}^\mu = - R^\mu_{\alpha \nu \beta} \Delta^\nu k^\alpha k^\beta,
\]

which determines the deviation vector \( \Delta^\mu := \delta x^\mu \) of the rays in a bundle. Hereafter dots denote covariant derivatives along a ray: \( (\dot{\cdot}) = k^\nu \nabla_\nu, \) e.g., \( \dot{\Delta}^\mu = D\Delta^\mu / ds = k^\nu \nabla_\nu \Delta^\mu, \) etc.

The crucial idea of Kristian and Sachs is to find a solution of (7.1) in the form of a series in the affine parameter \( s. \) Correspondingly, all the physical and geometrical quantities are ultimately represented by series in \( s. \) Let us give some basic results of that approach, without going into calculational details.

We will denote the spacetime points of an observer and a source by \( P \) and \( S, \) respectively. Let us start with the red shift defined by (3.13). A simple Taylor expansion of the right-hand side yields:
\[ Z = -s^{(1)} u + \frac{s^2}{2} u^{(2)} - \frac{s^3}{6} u^{(3)} + \ldots \]  

Here we denoted:

\[ s := s(k^\mu u_\mu)_P, \quad K^\mu = \left( \frac{k^\mu}{k^\nu u_\nu} \right), \quad (n)^{(u)} := \left( k^{\mu_1} \ldots k^{\mu_{n-1}} K^{\mu_n} \nabla_{\mu_1} \ldots \nabla_{\mu_{n-1}} u_{\mu_n} \right)_P. \]

Note that the value \( s \) describes a position of the observer on a null geodesic connecting \( S \) and \( P \), so that \( s = 0 \) corresponds to \( S \). The minus signs in (7.2) arise when we expand \((u^\mu k_\mu)_S\) at \( P \).

Let us consider a bundle of rays emitted from a point source \( S \). We are interested in \( \Delta^\mu(s) \) along a geodesic from \( S \) to \( P \). One has \( \Delta^\mu(0) \), and hence from (7.1), \( \ddot{\Delta}^\mu(0) = 0 \).

The vector \( \dot{\Delta}^\mu(0) \) is chosen to be orthogonal to \( k^\mu \) and \( u^\mu \) at \( S \), i.e. we are studying the cross-section of the bundle which is orthogonal to the central ray in the rest frame of the source. Note that such an initial condition provides orthogonality \( k^\mu \Delta_\mu = 0 \) at any value of \( s \), in view of (7.1).

The area distance is one of the central notions in the observational cosmology. If an observer \( P \) is viewing the distant source \( S \) with an intrinsic perpendicular area \( dA_S \) which subtends the solid angle \( d\Omega_P \) at \( P \), the area distance \( r \) between \( P \) and \( S \) is defined by

\[ dA_S = r^2 d\Omega_P, \]  

Another (reciprocal) area distance \( r_S \) between \( S \) and \( P \) is defined by \( dA_P = r_S^2 d\Omega_S \), where the solid angle \( d\Omega_S \) subtends the area \( dA_P \) at \( P \). The reciprocity theorem \([93,96,240]\) tells that these distances are related by

\[ r_S^2 = r^2 (1 + Z)^2. \]  

One can obtain \( r_S \) with the help of \( \Delta^\mu(s) \) which describes the behaviour of the bundle of rays.

The deviation equation (7.1) determines iteratively the third and higher covariant derivatives of \( \Delta^\mu \) at \( S \), and thus one construct the solution in the form of the Taylor series

\[ \Delta^\mu(s) = s \dot{\Delta}^\mu(0) + \frac{s^3}{6} \dddot{\Delta}^\mu(0) + \ldots. \]
In order to find \( r_S \), one needs actually only the length of \( \Delta^\mu \). Straightforward calculation yields:

\[
\Delta^\mu(s)\Delta_\mu(s) = s^2 \hat{\Delta}^\mu(0)\hat{\Delta}_\mu(0) \left[ 1 - \frac{s^2}{6} b - \frac{s^3}{12} \dot{b} + O(s^4) \right]_s + s^2 \hat{\Delta}^\mu(0)\hat{\Delta}^\nu(0) \left[ -\frac{s^2}{3} c_{\mu\nu} - \frac{s^3}{6} \dot{c}_{\mu\nu} + O(s^4) \right]_s.
\]

(7.5)

The spacetime curvature contributes via the expressions

\[
 b := R_{\mu\nu} k^\mu k^\nu, \quad c_{\alpha\beta} := C_{\alpha\mu\beta\nu} k^\mu k^\nu,
\]

(7.6)

higher order corrections are contained in the terms \( O(s^4) \).

After some algebra, one finds from (7.3):

\[
r^2_s = s^2 (k_\mu a_\mu)_s \left[ 1 - \frac{s^2}{6} b - \frac{s^3}{12} \dot{b} + O(s^4) \right]_s.
\]

(7.7)

Combining (3.13), (7.4), and (7.7), one can easily derive the series which represents the area distance \( r \) in terms of \( s \).

However, since the value of the affine parameter \( s \) for the source \( S \) and the observer \( P \) cannot be measured directly, the series in powers of \( s \) are not particularly useful. Instead, Kristian and Sachs proposed to invert the relation (7.2), and then substitute \( s = s(Z) \) in all other quantities. Direct calculation yields the following final series representation:

\[
1 + Z = 1 - r^{(1)} u + \frac{r^2}{2} (2) u^2 - \frac{r^3}{6} (3) u^3 + \frac{1}{2} B + \ldots,
\]

(7.8)

where, replacing (7.6), \( B := (R_{\mu\nu} K^\mu K^\nu)_P \). Inverting (7.8), one finds

\[
r = \frac{Z}{-u^{(1)}} \left[ 1 - \frac{Z (2) u}{2 (1) u^2} + \frac{Z^2}{2} \left( \frac{(2) u^2}{(1) u^4} - \frac{1}{6} \frac{(3) u}{(1) u^3} \frac{1}{12} \frac{B}{(1) u^2} \right) + O(Z^3) \right].
\]

(7.9)

**B. Classical cosmological tests**

Classical cosmological tests, such as apparent magnitude – red shift \((m - Z)\), number counts – red shift \((N - Z)\), angular size – red shift relations, and some other, reveal specific
dependence of astrophysical observables on the angular coordinates \((\theta, \phi)\) in a rotating world. Thus a careful analysis of the angular variations of empirical data over the whole celestial sphere is necessary.

The knowledge of null geodesics enables one to obtain the explicit form of the area distance \(r\) between an observer at a point \(P\) and any source \(S\). Each ray (a null geodesic) \(x^\mu(s; p^A)\) passing through \(P\) is labelled by the two parameters \(p^A := (\theta, \phi), \ A = 1, 2\) which have a simple meaning of the spherical angles fixing the direction of ray at \(P\). Thus in general, \(r\) is a function of the direction of observation, that is \(r = r(\theta, \phi)\). Besides this, \(r\) of course depends on the value of the affine parameter \(s\), or equivalently on the moment of \(t\) at which source radiates a ray detected by an observer at \(t_0\). The area of the cross-section of the bundle of rays is

\[
dA_S = \sqrt{\det G_{AB}} \, d\theta \, d\phi, \quad G_{AB} = g_{\mu\nu} \frac{\partial x^\mu}{\partial p^A} \frac{\partial x^\nu}{\partial p^B}.
\]

(7.10)

Since the solid angle in (7.3) is defined, as usually, by \(d\Omega_P = \sin \theta \, d\theta \, d\phi\), one easily obtains the general expression for the area distance between an observer \(P\) and a source \(S\) which lies on the geodesic labeled by the observational angles \((\theta, \phi)\) at the value \(s\) of the geodetic parameter,

\[
r(s; \theta, \phi) = \frac{\sqrt{\det G_{AB}(s; \theta, \phi)}}{\sin \theta}.
\]

(7.11)

The knowledge of the null geodesics (6.5)-(6.8) makes it possible to calculate area distances in the Gödel type universe explicitly. For example, using (6.5)-(6.8), one can find for the area distance along the axis of rotation the following exact result

\[
r^2(t; \theta = 0) = \frac{\sin^2 \left( \int_{t_0}^{t} dt' \omega(t') \right)}{\omega^2(t)}.
\]

(7.12)

The rotational effects are maximal in the directions close to the \(z\) axis, and quite remarkably the observations in the direction of rotation can be described by simple and clear formulas.

As for an arbitrary direction, exact formulas become very complicated and it is much more convenient to replace them by the Kristian-Sachs expansions. One can use the expansions (7.8) and (7.9) in the calculations of observable effects in rotating cosmologies.
All the angular dependent rotational contributions are contained in the coefficients of these expansions.

For the Gödel type cosmology (4.1) the classical \((m - Z)\) and \((N - Z)\) relations read as follows \([287,288,187]\).

**Apparent magnitude \(m\) versus red shift \(Z\):** The apparent magnitude of a source \(S\) with the energy flux \(L_P\), measured at \(P\), is defined by \(m = -\frac{5}{2} \log_{10} L_P\). Given the total output of the source \(L, L_S = L/4\pi\), the luminosity distance is \(D^2 := L_S/L_P = r^2(1 + Z)^4\), see \([95,96]\). Using (7.9), one finds for the Gödel type universe:

\[
m = M - 5 \log_{10} H_0 + 5 \log_{10} Z + \frac{5}{2} (\log_{10} e)(1 - q_0)Z
- 5 \log_{10} \left(1 + \sqrt{\frac{\sigma}{k+\sigma}} \sin \theta \sin \phi\right)
- \frac{5}{2} (\log_{10} e) \frac{\omega_0}{H_0} Z \sin \theta \cos \phi \left[\frac{\sqrt{\frac{\sigma}{k+\sigma}} + \sin \theta \sin \phi}{\left(1 + \sqrt{\frac{\sigma}{k+\sigma}} \sin \theta \sin \phi\right)^2}\right] + O(Z^2). \tag{7.13}
\]

Here \(M = -\frac{5}{2} \log_{10} L_S\) is the absolute magnitude of a source with an intrinsic luminosity \(L_S\).

**Number of sources \(N\) versus red shift \(Z\):** Let us assume that the sources of electromagnetic radiation are distributed homogeneously with the average density \(n(t)\). Neglecting the evolution of sources, one can count the number of images seen by an observer as follows. Consider the bundle with a solid angle \(d\Omega\) detected by an observer at \(P\). When moving along the central ray, the distance between the two infinitesimally close cross-sections is equal \(d\ell = (k^\mu u_\mu) ds\), and the volume between these sections is \(dA d\ell\), with \(dA = r^2 d\Omega\). Hence, the number of sources contained between these two sections is given by \(dN = -n_S(k^\mu u_\mu) r^2 d\Omega ds\). Integrating, and using (7.8)-(7.9):

\[
\frac{dN}{d\Omega} = \frac{n_0 Z^3}{3H_0^3 \left(1 + \sqrt{\frac{\sigma}{k+\sigma}} \sin \theta \sin \phi\right)^3} \left[1 - \frac{3}{2}(1 + q_0)Z
- 3 \frac{\omega_0}{H_0} \sin \theta \cos \phi \left(\frac{\sqrt{\frac{\sigma}{k+\sigma}} + \sin \theta \sin \phi}{\left(1 + \sqrt{\frac{\sigma}{k+\sigma}} \sin \theta \sin \phi\right)^2}\right) Z + O(Z^2)\right]. \tag{7.14}
\]
The \((N - Z)\) relation (7.14) describes the number of sources per solid angle \(d\Omega\) observed up to the value \(Z\) of red shift. One can estimate the global difference of the number of sources visible in two hemispheres of the sky, \(N^+, N^-\), by integrating (7.14). The result is

\[
\frac{N^+ - N^-}{N^+ + N^-} = \frac{1}{2} \sqrt{\frac{\sigma}{k + \sigma}} \left(3 - \frac{\sigma}{k + \sigma}\right) + O(Z^2).
\]  

(7.15)

It seems worthwhile to draw attention to the absence of a correction proportional to \(Z\) in (7.13). It is difficult to make a comparison of our results with [131, 263, 103] as they study stationary rotating models in which there is no red shift.

For some time classical cosmological tests were carefully carried out for standard models, but later it was recognized that evolution of physical properties of sources often dominates over geometrical effects. However, specific angular irregularities predicted in rotating cosmologies, (7.13), (7.14)-(7.15), may revive the importance of the classical tests.

C. Periodic structure of the universe

Recent analysis of the large–scale distribution of galaxies [54] has revealed an apparently periodic structure of the number of sources as a function of the red shift. Cosmic rotation may give a natural explanation of this fact [193]. The crucial point is the helicoidal behaviour of the null geodesics (6.5)-(6.7) in the Gödel type model in the directions close to the rotation axis. This yields a periodicity of the area distance as a function of red shift, and hence the visible distribution of sources turns out to be also approximately periodical in \(Z\).

This effect is most transparent for the direction of rays straight along the axis of rotation. The area distance is then given by (7.12). In order to be able to make some quantitative estimates, let us assume the polynomial law for the scale factor,

\[
a(t) = a_0 \left(\frac{t - t_\infty}{t_0 - t_\infty}\right)^\gamma,
\]  

(7.16)

which is naturally arising in a number of cosmological scenarios \((0 < \gamma < 1)\). Then one can derive (analogously to (7.14)) the distribution of number of sources per red shift per solid angle:
\[
\frac{dN}{d\Omega dZ} = \frac{n_0}{\omega_0^2 H_0 (1 + Z)^{1/\gamma}} \sin^2 \left( \frac{\gamma \omega_0}{(1 - \gamma) H_0} \left[ (1 + Z)^{(\gamma - 1)/\gamma} - 1 \right] \right).
\] (7.17)

Thus, the apparent distribution of visible sources is an oscillating function of red shift, with slowly decreasing amplitude. A similar generalized formula can be obtained for arbitrary directions, so that (7.17) is modified by additional angular dependence of the magnitude of successive extrema of distribution function.

The observational data [54] give for the distance between maxima the value $128h^{-1}$ Mps (where $H_0 = 100h$ km sec$^{-1}$Mps$^{-1}$). From this one can estimate the rotation velocity which is necessary to produce such a periodicity effect,

\[
\omega_0 \approx 74H_0.
\] (7.18)

This result does not depend on $\gamma$.

D. Polarization rotation effect

The cosmic rotation affects a polarization of radiation which propagates in the curved spacetime (4.1), and this produces a new observable effect which has been reported in the literature by Birch [46,47] and more recently by Nodland and Ralston [279,280]. In the geometrical optics approximation, polarization is described by a space-like vector $f^\mu$ which is orthogonal to the wave vector, $f^\mu k_\mu = 0$, and is parallelly transported along the light ray,

\[
k^\mu \nabla_\mu f^\nu = 0.
\] (7.19)

The influence of the gravitational field of a compact rotating massive body on the polarization of light was investigated by many authors, see e.g., [399,426,26,340,18,20,53,101,115,179,258]. The analysis of (7.19) for the rotating cosmologies shows that the global cosmic rotation also forces a polarization vector to change its orientation during the propagation along a null geodesics [279,257,210]. It is clear that this conclusion has physical meaning only when one can define a frame at any point of the ray with respect to which polarization rotates. Let us describe how this can be achieved.
As it is well known, gravitational field affects all the properties of an image of a source, such as shape, size and orientation \cite{37,286,95,96,389}. Like the rotation of the polarization vector, the deformation and rotation of an image depend both on the local coordinates and on the choice of an observer’s frame of reference. However, if one considers a combination of the two problems, then a truly observable effect arises which is coordinate and frame independent. Putting it in another way, one should calculate the influence of the cosmic rotation on the relative angle $\eta$ between the polarization vector and the direction of a major axis of an image \cite{194}. This problem was discussed in the papers \cite{314,141}, but in our opinion, their results are incomplete, in the sense that only null congruences converging at the point of observation were considered.

The relative angle $\eta$ can be most conveniently defined within the framework of the Newman-Penrose spin coefficient formalism. Namely, it is sufficient to construct a null frame $\{l, n, m, \overline{m}\}$ is such a way that $l$ coincides with the wave vector $k$, and the rest of the vectors are covariantly constant along $l$. Then we can consider $m$ as a polarization vector, and thus a deformation of an image of a source, calculated with respect to the frame $\{l, n, m, \overline{m}\}$, gives the observable relative angle $\eta$. Let us describe explicitly the null frame:

\begin{align*}
l &= \frac{a_0}{a} \left[ 1 + \sqrt{\sigma k + \sigma} \sin \theta \sin \Phi \right] \partial_t \\
&\quad + \frac{1}{a} \sin \theta \cos \Phi \partial_x + \frac{e^{-mx}}{a \sqrt{k + \sigma}} \sin \theta \sin \Phi \partial_y + \frac{1}{a} \cos \theta \partial_z \right], \\
\mathbf{n} &= \frac{a}{a_0} \left( \frac{1}{1 + \cos \theta} \right) \left[ \partial_t - \frac{1}{a} \partial_z \right], \\
\mathbf{m} &= e^{i\Phi} \sqrt{2} \left[ \sqrt{\sigma k + \sigma} + i \left( \frac{\sin \theta}{1 + \cos \theta} \right) e^{-i\Phi} \right] \partial_t \\
&\quad + \frac{i}{a} \partial_x + \frac{e^{-mx}}{a \sqrt{k + \sigma}} \partial_y - \frac{i}{a} \left( \frac{\sin \theta}{1 + \cos \theta} \right) e^{-i\Phi} \partial_z \right]; \\
\overline{\mathbf{m}} &= e^{-i\Phi} \sqrt{2} \left[ \sqrt{\sigma k + \sigma} - i \left( \frac{\sin \theta}{1 + \cos \theta} \right) e^{i\Phi} \right] \partial_t \\
&\quad - \frac{i}{a} \partial_x + \frac{e^{-mx}}{a \sqrt{k + \sigma}} \partial_y + \frac{i}{a} \left( \frac{\sin \theta}{1 + \cos \theta} \right) e^{i\Phi} \partial_z \right].
\end{align*}

Here
\[ \Psi(t, z) = z \frac{m}{2} \sqrt{\frac{\sigma}{k+\sigma}} + \Phi(t). \quad (7.24) \]

One should note that \((7.20)-(7.23)\) is a smooth field of frames which covers the whole space-time manifold. Direct computation proves that \((7.20)\) is the null geodesic congruence with an affine parametrization. We say that this congruence is oriented along a direction given by the spherical angles \((\theta, \phi)\) in the local Lorentz frame of an observer at \(P\), because a null geodesics \((6.5)-(6.8)\) belongs to this congruence.

It is straightforward to find the spin coefficients (we are using the definitions of \([71]\), and mark the spin coefficients with tildes in order to distinguish them from the other quantities in our paper):

\[
\tilde{\varepsilon} = 0, \quad \tilde{\kappa} = 0, \quad \tilde{\lambda} = 0, \quad \tilde{\nu} = 0, \quad (7.25)
\]

\[
\tilde{\rho} = -\frac{a_0}{a^2} \left[ \dot{a} \left( 1 + \sqrt{\frac{\sigma}{k+\sigma}} \sin \theta \sin \Phi \right) + \frac{m}{2} \left( \frac{k}{k+\sigma} \right) \frac{\sin \theta \cos \Phi}{\left( 1 + \sqrt{\frac{\sigma}{k+\sigma}} \sin \theta \sin \Phi \right)} \right]
+ i \frac{a_0 m}{a^2} \frac{\cos \theta}{2} \left( \frac{\sqrt{\frac{\sigma}{k+\sigma}} + \sin \theta \sin \Phi}{1 + \sqrt{\frac{\sigma}{k+\sigma}} \sin \theta \sin \Phi} \right), \quad (7.26)
\]

\[
\tilde{\sigma} = \frac{a_0 m}{a^2} \frac{1}{2} \left( \frac{k}{k+\sigma} \right) \frac{e^{2i\Psi} \sin \theta}{(1 + \sqrt{\frac{\sigma}{k+\sigma}} \sin \theta \sin \Phi)} \left( -\cos \Phi \left[ 2 \cos \theta - 1 \right] - 2 \cos^2 \Phi (\cos \theta - 1) \right) + i \sin \Phi \left[ \cos \theta - 2 \cos^2 \Phi (\cos \theta - 1) \right]. \quad (7.27)
\]

We do not write the other spin coefficients, because their values are irrelevant. Only one important step remains to be done: the spin coefficient \(\tilde{\pi} \neq 0\) in the frame \((7.20)-(7.23)\), and we need to make an additional Lorentz transformation

\[
l \rightarrow l, \quad n \rightarrow n + A^* m + A \overline{m} + A^* A l, \quad m \rightarrow m + A l, \quad \overline{m} \rightarrow \overline{m} + A^* l, \quad (7.28)
\]

where the function \(A\) satisfies equation \(l^\mu \partial_\mu A + \tilde{\pi}^* = 0\). This ensures that \(\tilde{\pi} = 0\) in a new frame, while remarkably the transformation \((7.28)\) does not change any of the spin coefficients \((7.23)-(7.27)\). Thus one obtains finally the field of null frames \(\{l, n, m, \overline{m}\}\) with the required properties: \(l\) is the null geodesic congruence with affine parametrization, while \(n, m, \overline{m}\) are covariantly constant along \(l\). The latter is equivalent to \(\tilde{\kappa} = \tilde{\varepsilon} = \tilde{\pi} = 0\).
As it is well known, deformation and rotation of an image along a null geodesics are described by the optical scalars [377]:

\[ \tilde{\vartheta} = -\text{Re}\tilde{\rho}, \quad \tilde{\omega} = \text{Im}\tilde{\rho}, \quad \tilde{\sigma}. \] (7.29)

We now assume, for definiteness, that the polarization vector \( f^\mu \) coincides with the vector \( m^\mu \) of the above null frame. Let an image of a source, as seen at the point corresponding to a value \( s = s_1 \) of the affine parameter, be an ellipse with the major axis \( p \) and the minor axis \( q \). Then one can straightforwardly obtain [194] for the angle of rotation of the major axis of the image at \( s_2 = s_1 + \delta s \),

\[ \delta \eta = -\tilde{\omega}\delta s - \frac{p^2 + q^2}{p^2 - q^2} \text{Im}\tilde{\sigma} \delta s. \] (7.30)

Integration along a ray gives the finite angle of rotation.

It is worthwhile to notice that exactly along the cosmic rotation axis the observer at \( P \) finds for the optical scalars

\[ \tilde{\vartheta}_P = H_0, \quad \tilde{\omega}_P = \omega_0, \quad \tilde{\sigma} = 0. \] (7.31)

Thus the effect of rotation of the polarization vector in this direction is the most explicit.

As for an arbitrary direction of observation, we finally find from (7.30) for \( k/\sigma \ll 1 \) with the help of the Kristian–Sachs expansions (7.7)-(7.9):

\[ \eta = \omega_0 r \cos \theta + O(Z^2). \] (7.32)

This result is in a good agreement with the observational data reported [16] on the dipole anisotropy of distribution of the difference between the position angles of elongation (the major axis) and polarization in a sample of 3CR radio sources. The estimate for the direction and the magnitude of the cosmic rotation, obtained in [288,187,194] from Birch’s data, reads

\[ \ell^\circ = 295^\circ \pm 25^\circ, \quad b^\circ = 24^\circ \pm 20^\circ, \] (7.33)

\[ \omega_0 = (1.8 \pm 0.8)H_0. \] (7.34)
Not entering into the discussion of the statistical significance of the results of Nodland and Ralston, we can make a comparison with the new data. In [272], the dipole effect of the rotation of the plane of polarization was reported in the form
\[ \beta = \frac{1}{2} \Lambda_s^{-1} r \cos \gamma, \]
where \( \beta \) is the residual rotation angle between the polarization vector and the direction of the major axis of a source (remaining after the Faraday rotation is extracted), \( r \) is the distance to the source, and \( \gamma \) is the angle between the direction of the wave propagation and the constant vector \( \vec{s} \). The analysis of the data for 160 radio sources yielded [272] the best fit for the constant \( \Lambda_s = (1.1 \pm 0.08) \frac{2}{3} \times 10^{15} \text{ m yr}^{-1} \) (with the Hubble constant \( H \)), and the \( \vec{s} \)-direction RA (21h±2h), dec (0° ± 20°). In an attempt to explain their observations, Nodland and Ralston concluded that it is impossible to understand such an effect within the conventional physics. Instead, they considered a modified electrodynamics with the Chern-Simons type term violating the Lorentz invariance [57], and related \( \Lambda_s \) to the coupling constant of that term. However, interpreting the effect as arising from the cosmic rotation, we immediately find

\[
\begin{align*}
I^\circ &= 50^\circ \pm 20^\circ, \\
\delta^\circ &= -30^\circ \pm 25^\circ, \\
\omega_0 &= (6.5 \pm 0.5) H_0.
\end{align*}
\]

This is larger than the estimate (7.34) obtained from Birch’s data [16]. Also the direction of the cosmic vorticity is different from (7.33). However, it is interesting to note that within the error limits, the two directions are orthogonal to each other.

It is clear that further careful observations and statistical analyses will be extremely important in establishing the true value of the cosmic rotation. In particular, it has been recently claimed in [68], that the magnitude of the polarization rotation effect should be one or two orders lower than reported in [272]. The value of the cosmic vorticity then should be reduced correspondingly.
8. CONCLUSIONS

In our discussion of the properties of rotating cosmologies we have paid special attention to the Gödel type model (4.1). However the main conclusions are true also for the whole class of metrics (3.1). One can notice, that in some cases the numerical estimates obtained for the value of the vorticity do not agree with each other, e.g., see (7.18), (7.34), and (7.36). Such inconsistencies can reflect the fact that too few empirical data were analyzed until now, so that a further detailed discussion is required in order to obtain the final convincing estimates. Already now certain modifications of the above simple models are necessary: It is clear, in the light of the modern COBE results [402,41,436], that the purely rotating models (3.1) should be replaced by the cosmologies with a nontrivial shear. A preliminary analysis of such generalizations shows that rotating models can be made compatible with the COBE data without destroying the rest of the rotational effects (in particular, without essential modification of the polarization rotation formulas). It may turn out, though, that some (or all) of the above mentioned effects are explained after all by some different physical (and geometrical) reasons, not related to the cosmic rotation. However, “whether our universe is rotating or not, it is of fundamental interest to understand the interrelation between rotation and other aspects of cosmological models as well as to understand the observational significance of an overall rotation” [369].

We believe that the cosmic rotation is an important physical effect which should find its final place in cosmology. In this paper we outlined one of the possible theoretical frameworks which can underlie our understanding of this phenomenon.

Acknowledgments. This work was partly supported by the Deutsche Forschungsgemeinschaft (Bonn) grant He 528/17-2. I thank Prof. K.-E. Hellwig for the kind invitation to the Colloqium on Cosmic Rotation at the Technische Universität Berlin. The warm hospitality of the organizers and highly interesting discussions with the participants are gratefully acknowledged.
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