Flexural testing on carbon fibre laminates taking into account their different behaviour under tension and compression

M C Serna Moreno¹, A Romero Gutiérrez and J L Martínez Vicente

Universidad de Castilla-La Mancha, Instituto Investigaciones Energéticas y Aplicaciones Industriales & ETS Ingenieros Industriales, Av. Camilo José Cela s/n, 13071 Ciudad Real, Spain

¹E-mail: mariacarmen.serna@uclm.es

Abstract. An analytical model has been derived for describing the results of three-point-bending tests in materials with different behaviour under tension and compression. The shift of the neutral plane and the damage initiation mode and its location have been defined. The validity of the equations has been reviewed by testing carbon fibre-reinforced polymers (CFRP), typically employed in different weight-critical applications. Both unidirectional and cross-ply laminates have been studied. The initial failure mode produced depends directly on the beam span-thickness relation. Therefore, specimens with different thicknesses have been analysed for examining the damage initiation due to either the bending moment or the out-of-plane shear load. The experimental description of the damage initiation and evolution has been shown by means of optical microscopy. The good agreement between the analytical estimations and the experimental results shows the validity of the analytical model exposed.

1. Introduction

Three-point-bending tests are used extensively in mechanical characterization due to the simplicity of the specimen preparation and testing. It is an interesting way to subject a specimen to tension, compression and out-of-plane shear simultaneously for checking its structural integrity. Well documented formulas are available for analysing materials with equal tension and compression properties submitted to flexural testing [1,2]. However several structural materials have different tensile and compressive response and, so that, an analytical model which takes into account this aspect would be desirable.

Composites constructed from carbon fibres pre-impregnated by an epoxy polymer (pre-preg) generally present unconventional structural behaviour which needs thorough examination. In particular, the elastic responses under tension and compression in the principal material directions are both different. This fact was utilized by Jones [3] and, more recently, by other authors [4,5] for obtaining the elastic moduli and the maximum normal stresses in the tensile and compressive regions generated on unidirectional specimens submitted to flexural testing in parallel to the fibres direction. But none of the various analytical works found in the literature regarding cross-ply laminates in flexure took into account the dissimilar material behaviour in tension and compression, since these were mainly focused on different failure criteria, the displacements field or the flexural strength and modulus [6-13]. Nevertheless, the effect of this characteristic on the results of three point bending tests is notable. The deviation of the neutral axis from the mid-height plane and the non-symmetric strain and stress distributions are some of the consequences.
The analytical model stated in [14,15], based on the technique of the homogenised section, describes the mentioned features related to the tensile and compressive material response and predicts the first failure mode produced depending on the beam span-thickness relation. In this work we unify the nomenclature of both studies in order to highlight the predictive capacity of the model in any composite with no force coupling. Two unidirectional and three symmetric cross-ply laminates comprised by continuous carbon fibre-reinforced epoxy laminae were analysed in order to assess experimentally the mentioned theory. \([0\|s], [0\|s], [90\|0\|s], [0\|2\|90\|s]\) and \([0\|90\|s]\) composites were fabricated and tested, being the experimental observables the applied load and the maximum tensile strain in the most loaded section. Microscopy analysis was developed for reviewing whether flexural or interlaminar shear failure modes were obtained in the tests. The analytical estimation of the neutral axis position as well as the mode and region of damage initiation were in good agreement with the experimental observations and measurements.

2. Material

Composite plates were fabricated stacking several plies of pre-preg by means of a hand lay-up process and applying the curing cycle with a hot platen press. Afterwards, these plates were machined with the form of parallelepiped specimens in order to perform the experiments using a three-point bending tool with a span \(L = 25\) mm installed in a testing machine. Each ply consisted of unidirectional carbon-fibres IMA-12K in a resin matrix M21E from Hexcel™. The thickness of each ply after the curing process was \(\Delta = 0.25\) mm and these presented an orthotropic linear elastic behaviour whose properties in the principal directions under traction (super-index \(t\)) and compression (super-index \(c\)) are listed in table 1. The elastic moduli of both loading cases in the fibres direction (sub-index \(f\)) and in perpendicular to the fibres direction (sub-index \(m\)) are respectively \(E_f^t\), \(E_{m}^t\), \(E_f^c\) and \(E_{m}^c\). In parallel, \(f_f^t\), \(f_f^c\), \(f_m^t\) and \(f_m^c\) are the normal strengths in the principal directions under tensile and compressive loading. Lastly, the interlaminar shear strength of the material is defined as \(f_s\). Except the values of \(E_{m}^c\) and \(f_m^c\), the material properties have been determined in previous research by means of uniaxial [16] and flexural testing [14]. Four significant digits have been taken into account due to the accuracy of the measurement equipment. The value of \(E_{m}^c\) was calculated in [15] from the experimental results by means of the orthotropic relations and \(f_m^c\) was obtained from references [11-13].

| Table 1. Properties in principal directions depending on the loading case. |
|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|
| \(E_f^t\)      | \(E_f^c\)      | \(E_{m}^t\)    | \(E_{m}^c\)    | \(f_f^t\)      | \(f_f^c\)      | \(f_m^t\)      | \(f_m^c\)      | \(f_s\)         |
| (GPa)          | (GPa)          | (GPa)          | (GPa)          | (MPa)          | (MPa)          | (MPa)          | (MPa)          | (MPa)          |
| 177.6 ± 5.7    | 97.03 ± 3.03   | 11.84 ± 1.05   | 7.46*          | 2601. ± 121.   | 1536. ± 80.    | 56.07 ± 2.12   | 200**          | 46.65 ± 1.12   |

*Calculated [15]. **Estimation from [11-13].

3. Analytical model

A three point bending test consist in a bar of rectangular cross section resting on two supports, which is loaded and deflected by means of a loading nose acting in the midway between the supports [1,2]. Therefore a pinned–pinned specimen submitted to a central load \(P\) is studied, considering a span \(L\) and a rectangular cross section of width \(b\) and total thickness \(h\). It is assumed that no extensional-shear-bending–twisting coupling exists and then only normal stress \(\sigma\) (given in this work in absolute value) and out-of-plane shear stress \(\tau\) are produced. The neutral axis of the laminate is presumed to be deviated a distance \(d\) from the mid-height of the section due to the different tensile and compressive material behaviour. By way of example, figure 1a shows the cross section of a \([0\|2\|90\|s]\) laminate with total thickness \(h = 8\Delta\).
To overcome the analytical difficulty generated by the different response to tensile and compressive loading, the cross section is homogenised for unifying its elastic response [14,15] and handling with the problem by means of the linear classical bending theory of slender beams [17]. The rectangular cross section has to be examined for distinguishing the different regions submitted to tension and compression, which have different material response. Let us consider that the total section is divided in an even number of regions $N$ with different answer under loading. The variable $i$ (defined from 1 to $N$) numbers the regions from bottom to top, being $y_{i-1}$ and $y_i$ the limiting coordinates of each $i$-region and $h_i$ the maximum $y$-coordinate of each region in absolute value. Then, the cross section is modified in order to analyse it under the assumption that its whole material response is unique and equal to a reference behaviour. The original width of each region has to be reduced so as to maintain the equilibrium of the resultant forces, while the thickness is kept for conserving the original strain distribution. The highest elastic modulus in the loading direction is considered the one of reference. This value of reference divided by the elastic modulus of the $i$-region is defined as the moduli ratio $n_i$, being the modified width of the $i$-region defined as $b_i = b/n_i$. Once the geometrical modification has been developed (figure 1b), the static moment of the whole homogenised section about the neutral axis has to be zero. This fact is used for deducing the distance $d$ between the mid-height plane and the neutral axis using the relation given by equation (1). As well, the static moment of “half” of the modified area, $S_z$, can be obtained by means of equation (2).

$$
\sum_{i=1}^{N} \frac{\left(y_{i-1}^2 - y_i^2\right)}{n_k} = 0 \tag{1}
$$

$$
S_z = \sum_{i=1}^{N/2} b_i \frac{\left(y_{i-1}^2 - y_i^2\right)}{2} \tag{2}
$$

In order to determine the first failure mode and its location, $f_i$ is assumed to be the normal strength of each region in the loading direction and $f_s$ the interlaminar shear strength of the material. The top and bottom furthest fibres to the neutral plane are the most deformed due to the bending moment and, in case of initial flexural failure, the most likely to fail due to the normal stress distribution generated. From the two furthest regions to the neutral axis the less resistant is the one with lower normal failure strength, which is called the flexural failure region “$F$” and it has a normal failure strength $f_F$, an elastic moduli ratio $n_F$ and a maximum coordinate $h_F$. Meanwhile, in case of inter-laminar opening, the failure mode is expected to be initialized in the position of the neutral axis where the maximum out-of-plane shear stress is produced. Equations (3) and (4) show the threshold between both failure modes, obtained

$$
\sigma_{max} = f_F \frac{h_F}{n_F} \tag{3}
$$

$$
\tau_{max} = f_s \frac{h_F}{n_F} \tag{4}
$$

Figure 1. Laminate [02 902]s. (a) Limiting $y$-coordinates of each region in the actual section. (b) Corresponding width modifications in the homogenised section. (c) Normal stress and (d) out-of-plane shear stress distributions. Not to scale.
taking into account the relation $\frac{\tau^{\text{max}}}{\sigma^\text{max}_F}$ (figure 1c and figure 1d) between the maximum shear stress produced in the neutral axis and the maximum normal stress developed in the flexural failure region.

First flexural failure:

$$\frac{2n_F S}{bL_h_f} < \frac{f_T}{f_F}$$  \hspace{1cm} (3)

First out-of-plane shear failure:

$$\frac{2n_F S}{bL_h_f} > \frac{f_T}{f_F}$$  \hspace{1cm} (4)

4. Experimental observations and discussion

Three point-bending experiments were conducted by means of a standard fixture [1,2] installed in an universal testing machine. These tests were performed on specimens with the shape of rectangular prisms with slightly different dimensions due to the cutting process. The main limitations were related to their length and width, which were chosen for fulfilling the space requirements imposed by the three-point bending tool and for achieving an homogenous shear opening all along the width of the cross section. Regarding the thickness, the principal restriction was to employ a sufficient number of 0°-layers in order to obtain specimens rigid enough to avoid big displacements that could lead to non-linear effects. Five lay-ups were chosen to reproduce experimentally the different failure modes. Two unidirectional composites, [0]_5 and [0]_8, and three cross-ply laminates, [90; 0]_2S, [0; 90]_2S and [0; 90; 90]_2S, were fabricated and tested. A strain gage was located on the lower surface of the mid-span section (the cross section of the beam where the central load is applied) for registering the maximum tensile strain $\varepsilon_t$. Meanwhile, the applied load $P$ was acquired by means of the loading cell of the testing machine. The location of the failure regions was analysed by optical microscopy using a LEICA DR IRM microscope. Images of the damage initiation, taken in perpendicular direction to the most loaded section, are shown in figure 2 and figure 3 at magnification of 50x.

Figure 2. Damage initiation [14]: (a) [0]_5, (b) [0]_8. Magnified images 50x.

Figure 3. Damage initiation [15]: (a) [90; 0]_2S, (b) [0; 90]_2S, (c) [0; 90; 90]_2S. Magnified images 50x.
Due to the repetitively of the results, the experimental information at the instant of first failure for five representative specimens are listed in table 2 together with the evaluation of the analytical estimations. The comparison between the analytical estimations and the experimental observations is addressed in the next sub-sections.

Table 2. Experimental and analytical definitions in the moment of first failure [14,15].

| Material | $h$ (mm) | $b$ (mm) | $P$ (kN) | $m_{eq}$ | $d$ (mm) | $f_r$ (MPa) | $n_f$ | $h_f$ (mm) | $S_z$ (mm$^3$) | First damage$^*$ |
|----------|----------|----------|----------|----------|----------|------------|------|----------|-------------|----------------|
| [0]$_5$  | 1.25     | 9.80     | 0.60     | 10.00    | 0.37$\Delta$ | 1356. 1.83 | 0.57h | $h/2 + d = 0.57h$ | 0.09bh$^2$ | C             |
| [0]$_8$  | 2.00     | 9.86     | 1.20     | 8.86     | 0.60$\Delta$ | 1356. 1.83 | 0.57h | $h/2 + d = 0.57h$ | 0.09bh$^2$ | S             |
| [90; 0]$_5$ | 2.00  | 11.25    | 0.13     | 4.28     | 0.33$\Delta$ | 56.07 15.00 | 0.46h | $h/2 - d = 0.46h$ | 0.03bh$^2$ | T             |
| [0; 90]$_5$ | 2.00  | 10.35    | 1.13     | 6.52     | 0.84$\Delta$ | 1356. 1.83 | 0.60h | $h/2 + d = 0.60h$ | 0.07bh$^2$ | C             |
| [0; 90]$_8$ | 2.50  | 10.80    | 1.72     | 7.71     | 0.99$\Delta$ | 1356. 1.83 | 0.60h | $h/2 + d = 0.60h$ | 0.08bh$^2$ | S             |

$^*$C: Flexural failure (compression), T: Flexural failure (tension), S: Out-of-plane shear failure.

4.1. Unidirectional laminates

The existing normative for developing three point-bending tests on unidirectional composites [1,2] considers that the response of the material is the same under tension and compression and, in opposition to our study, it analyses a problem with symmetric normal and shear stress distributions. Nevertheless, the non-symmetrical failure modes found experimentally suggest the importance of the different loading response (figure 2a and figure 2b). Tests on unidirectional laminates were developed to compare the results of the standards with the theory derived in [14,15] which takes into account the difference in behaviour under tensile and compressive loading. The proposed laminates consists of five and eight plies ($h = 1.25$ mm and $h = 2$ mm, respectively), whose section contains two regions with different behaviour under tensile and compressive loading ($N = 2$). The highest elastic modulus in the loading direction was present in the region submitted to tension, which was taken as the reference for defining the moduli ratio of each region and developing the geometrical modifications. The lower normal strength in the loading direction was present in the region submitted to compression, so it was considered as the region where the first flexural failure could be initiated. The geometrical threshold given by equations (3) and (4) predicts that the thickness of the specimen has to be lower than $1.5$ mm for obtaining flexural failure and higher than $1.5$ mm for developing shear opening-modes. Therefore, the two different laminates should show experimentally opposite failure modes. It is important to highlight that, in the studied case, the value of this threshold ($h = 1.5$ mm) is 13% smaller than the classical limit ($h = 2L_f/f_r = 1.7$ mm) [18] obtained considering that the material presents the same response under tension and compression.

4.1.1. Laminate $[0]_5$. The 5-plies specimen presented initial flexural failure in the region submitted to compression due to the lower compressive strength in the fibres direction. The values of the experimental observables listed in table 2 correspond to the measurements when the loss of load-carrying capacity was observed. Typical compressive failure modes could be observed using optical microscopy (figure 2a). The dissimilar lengths of the regions submitted to tension and compression could be appreciated. Then the simplification of the response developed by the normative, which predicts equal dimensions for the tensile and compressive regions, does not reproduce completely the experimental observations. The length of the damaged region estimated analytically was $h_F = 0.57h = 712.5 \mu$m (table 2), which differs a 14% from the position of the mid-height plane of the section. After the moment of the load-carrying capacity loss, during the last stages of the loading process, the maximum shear stress could be able to achieve the shear strength of the material and then marginal shear failure modes could also be observed.
4.1.2. Laminate \([0]_2S\). The 8-ply specimens exhibited shear failure modes at the position of the neutral axis, at an average depth equal to 0.57 times the total thickness \(h\). This measurement coincides with the analytical estimation given in table 2, what confirms that the model developed reproduced the experimental results. The not-centred position of the interlaminar opening-mode can be appreciated in figure 2b for a representative specimen. The shear damage was initiated when the maximum shear stress achieved the interlaminar shear strength. Meanwhile, the maximum tensile and compressive normal stresses were lower than their correspondent normal strengths, thus only shear failure was induced. Once the experiment went on, the initial crack was propagated in the longitudinal direction of the specimen. Nevertheless, after the onset of the shear opening-mode, the loading continued over the upper part of the specimen delimited by the shear failure plane. Then the resistant layer had a lower effective thickness, in average smaller than the established limit \((h_f = 0.57h = 1.14 < 1.5 \text{ mm})\). This condition could be enough to trigger unexpected flexural failure modes after the stage of shear damage initiation.

4.2. Cross-ply laminates
In order to examine the correctness of the analytical technique in other symmetric and balanced composites, three different cross-ply laminates, \([90\_2]_3S\), \([0\_2]_3S\), and \([0\_2]_3S\), were experimentally tested. Different linear force-strain response and first failure load were appreciated. In particular, the strong influence of the stacking sequence is observed in the loading progression of the \([90\_2]_3S\) and \([0\_2]_3S\) laminates, being the force-strain slope and the first failure load of the \([90\_2]_3S\) specimen 5.62 times and 8.69 times higher than the ones of the \([90\_2]_3S\) lay-up (figure 2a). According to the analytical estimation given in table 2, what confirms that the model developed reproduced the experimental results. The not-centred position of the interlaminar opening-mode can be appreciated in figure 2b for a representative specimen. The shear damage was initiated when the maximum shear stress achieved the interlaminar shear strength. Meanwhile, the maximum tensile and compressive normal stresses were lower than their correspondent normal strengths, thus only shear failure was induced. Once the experiment went on, the initial crack was propagated in the longitudinal direction of the specimen. Nevertheless, after the onset of the shear opening-mode, the loading continued over the upper part of the specimen delimited by the shear failure plane. Then the resistant layer had a lower effective thickness, in average smaller than the established limit \((h_f = 0.57h = 1.14 < 1.5 \text{ mm})\). This condition could be enough to trigger unexpected flexural failure modes after the stage of shear damage initiation.

4.2.1. Laminate \([90\_2]_3S\). The top and bottom \(90^\circ\)-plies were the most deformed due to the bending moment. Among them, the bottom \(90^\circ\)-region submitted to tension was the furthest to the neutral axis with lower normal strength and it was considered as the region where the first flexural failure could be initiated. Using the analytical limitations given by equations (3) and (4), in this laminate first bending failure would be achieved if \(h < 11.6 \text{ mm}\), while first interlaminar shear failure would be produced if \(h > 11.6 \text{ mm}\). As the total thickness was \(h = 2 \text{ mm}\), this composite should fail experimentally due to the bending moment in the \(90^\circ\)-lower layers submitted to tensile loading. This result was confirmed experimentally and figure 3a shows the damage initiation in the so-called flexural failure region under tension. Not only one main crack could appear in the lower \(90^\circ\)-region of the mid-span section, but also several micro-cracks could emerge in its close vicinity at the same time. Observing the load-strain evolution, it could be inferred that the damage initiation is governed by the response of the first failure region. After the damage initiation, the bottom region lost its load-carrying capacity and the remaining lay-up \([90\_2]_3S\) supported the increasing external loading. Then the top \(90^\circ\)-region turned to be the most deformed with lower value of the normal strength of the material, being the following to fail when the normal compressive strength of the matrix was achieved. Afterwards the lasting core of the specimen, formed by four \(0^\circ\)-layers, was submitted to flexural loading. Due to the fact that the compressive normal strength in the fibres direction is lower than their tensile normal strength, the damage evolution continued in the upper \(0^\circ\)-plies submitted to compression and this led to the total failure of the specimen at a final load close to 0.96 kN.

4.2.2. Laminate \([0\_2]_3S\). Similar procedure was followed for studying the symmetric cross-ply \([0\_2]_3S\) (figure 1), which had the same number of plies and orientations than the laminate of the previous subsection. However, the different stacking sequence provoked the first failure due to the bending moment in the top \(0^\circ\)-layers submitted to compressive loading. In this case, the two top and bottom \(0^\circ\)-regions were the most deformed during the loading process. The level of the maximum normal stresses in both regions is sketched in figure 1c and a clear initial damage was produced in the top \(0^\circ\)-region.
The reason is that the tensile strength in the fibres direction is two times higher than their compressive strength, being the top 0°-layers the so called flexural failure region. The experimental observation was in agreement with the limitations given by equations (3) and (4), which stated that the conditions for developing first bending or interlaminar shear failure modes were $h < 2.08$ mm and $h > 2.08$ mm respectively. The total thickness of the laminate was $h = 2$ mm, so the analytical prediction was that the composite should fail experimentally due to the bending moment under compression in the location of the flexural failure region. In the moment of first failure, the maximum shear stress $\tau_{\text{max}}$ was just 14% lower than the interlaminar shear strength of the material $f_s$. Then, after the loss of the load-carrying capacity of the top 0°-plies, the remaining [90 90 90 90 0 0] laminate developed a fast shear failure in the 90°-region due to the closeness to the shear strength.

4.2.3. Laminate $[0_390^2\text{S}]$. Including to the previous laminate an extra 0°-layer on its top and its bottom should cause interlaminar shear failure in the position of the neutral axis, because according to equation (4) the condition for obtaining first delamination $h > 1.9$ mm was satisfied. The addition of both 0°-plies provoked the reduction of the maximum normal stresses in all the regions, while the maximum shear stress was increased. This growth was enough for achieving the value of the interlaminar shear strength $f_s$ while $\sigma_f^{\text{max}} < f_s$. The first opening-crack was initiated in the neutral axis (figure 3c), whose location has been analytically estimated at a distance $d = 0.99\Delta$ from the mid-height plane. The analytical position fitted perfectly with the experimental evidence, proving that the neutral axis was not centred in the mid-height of the section due to the different tensile and compressive material response. After the damage initiation, the experimental observations confirmed that the initial damage quickly expanded between the two lower 90°-plies. In figure 3e the las top 0°-ply does not appear due to the magnification used (50x).

5. Conclusions
Composites generally exhibit different stress–strain behaviour under tension and compression. The effect of this characteristic was examined in terms of three-point bending tests on unidirectional and cross-ply CFRP laminates. Using the technique of the homogenised section, the problem could be studied considering a homogeneous response of the material. Taking into account the different tensile and compressive material response, the full analytical descriptions of the normal and shear stress distributions allowed to describe the thickness condition which determined if the specimen would present either flexural or shear failure modes. The observations by optical microscopy confirmed that the initial damage quickly expanded between the two lower 90°-plies. In figure 3e the las top 0°-ply does not appear due to the magnification used (50x).

6. Acknowledgements
This work was financially supported by the University of Castilla-La Mancha.

References
[1] UNE-EN 2562:1997: Aerospace series. Carbon fibre reinforced plastics. Unidirectional laminates. Flexural test parallel to the fibre direction; 1997.
[2] UNE-EN ISO 14130:1999: Fibre-reinforced plastic composites. Determination of apparent interlaminar shear strength by short beam method; 1999.
[3] Jones R M 1976 J. Compos. Mater. 10 342–54.
[4] Mujika F, Carbajal N, Arrese A and Mondragón I 2006 Polym. Test. 25 766–71.
[5] Carbajal N and Mujika F 2009 Polym. Test. 28 150–6.
[6] Pagano N J 1976 J. Compos. Mater. 1 336–342.
[7] Turvey G J 1980 Int. J. Solids Struct. 16(5) 451–63.
[8] Turvey G J 1982 Fibre Sci. Technol. 16(1) 1–10.
[9] Kam T Y and Sher H F 1995 J. Compos. Mater. 29(4) 463–82.
[10] Mujika F 2012 J. Compos. Mater. 46(3) 259–74.
[11] Mechanical Properties of Carbon Fiber Composite Materials 2014, Fiber/Epoxy resin (120°C Cure), 1-800-811-2009, ACP Composites.
[12] Marín L, Trias D, Badalló P, Rus G and Mayugo J A 2012 Compos. Struct. 94 3321–6.
[13] Bunsell A R and Renard J 2005 Fundamentals of fibre reinforced composite materials CRC Press.
[14] Serna Moreno M C, Romero Gutiérrez A and Martínez Vicente J L 2016 Compos. Struct. 136 706–11.
[15] Serna Moreno M C, Romero Gutiérrez A and Martínez Vicente J L 2016 Compos. Struct. 146 62-8.
[16] Martínez Vicente J L, Serna Moreno M C, Caminero Torija M A and López Cela J J 2014, ECCM16, Seville (Spain).
[17] Garrido J A and Foces A 1994 Resistencia de Materiales Secretariado de Publicaciones, Universidad de Valladolid.
[18] Sideridis E and Papadopoulos G A 2004 J. Appl. Polym. Sci. 93 63–74.