Non-Equilibrium Phenomena in Quantum Systems, Criticality and Metastability †

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Abstract: We summarise here some relevant results related to non-equilibrium quantum systems. We characterise quantum phase transitions (QPT) in out-of-equilibrium quantum systems through a novel approach based on geometrical and topological properties of mixed quantum systems. We briefly describe results related to non-perturbative studies of the bistable dynamics of a quantum particle coupled to an environment. Finally, we shortly summarise recent studies on the generation of solitons in current-biased long Josephson junctions.

Keywords: quantum open systems; non-equilibrium quantum phase transition; quantum metrology; quantum geometric information

1. Phase Transition in Dissipative Quantum Many Body Systems

Here we briefly summarise a method to characterise quantum phase transitions (QPT) in out-of-equilibrium quantum systems, which employs topological and geometric features of quantum systems [1]. The use of geometric and topological features for the study of QPT has been quite a successful approach, applied so far only in the context of equilibrium QPT [2–4].

A challenging new paradigm has recently been put forward by the discovery of novel types of QPT in reservoir-driven condensed matter systems [5–7]. A complete picture about the critical features of such systems, such as long-range order, existence order parameters, topological order, character of the fluctuation, is still lacking. Geometric concepts, such as geometric phase and geometric information may provide interesting new avenues to explore the latter.

In the context of non-equilibrium steady state (NESS) QPT, the evolution is driven by a Liouvillian superoperator $L(\lambda)$, which is parameterized by a set of (control) variables $\lambda \in \mathcal{M}$. These parameters determine, independently of the chosen initial state, the corresponding (unique) NESS $\rho(\lambda)$. Depending on $\lambda$, the NESS can display a variety of different properties potentially exhibiting NESS-QPTs. In analogy to equilibrium QPTs, also in NESS-QPTs, dramatic structural changes occurring in $\rho(\lambda)$ due to a criticality may be reflected in the statistical properties of the state and hence in the geometry of its manifold. A cyclic evolution of the NESS along the manifold can witness a singularity in the curvature and experience a geometric phase shift.
In the context of mixed quantum states, it is necessary to exploit unorthodox concepts of geometric phases (GP). Among the many possible definitions of the mixed state GP, the Uhlmann GP [8] stands out for its deep-rooted relation to information geometry and metrology [9], whose tools have been profitably employed in the investigation of QPT and NESS-QPT [10,11]. Motivated by this, we have introduced the mean Uhlmann curvature (MUC) and we have investigated its role in the characterisation of dissipative NESS-QPT. The MUC, defined as the Uhlmann GP per unit area of a density matrix evolving along an infinitesimal loop, has also a fundamental interpretation in multi-parameter quantum metrology: it marks the incompatibility between independent parameters arising from the quantum nature of the underlying physical system [12]. In this sense, the MUC is a measure of “quantumness” in the multi-parameter estimation problem, and its singular behaviour responds only to quantum fluctuations occurring across a phase transitions. Indeed, equilibrium phase transitions fall invariably into two markedly non-overlapping categories: classical phase transitions and quantum phase transitions. NESS-QPTs offer a unique arena where such a distinction fades off. The mean Uhlmann phase provides a method to reveal and quantitatively assess the quantum character of such critical phenomena.

Explicit examples [13] in the physically relevant class of quadratic Fermion models, coupled linearly to Markovian baths have been analysed [14]. Criticality, in the sense of diverging correlation length, has been found to be related to a non-analytic behaviour of the Uhlmann curvature. The relations between the behaviour of the Uhlmann curvature, the divergence of the correlation length, the character of the criticality and the dissipative gap are demonstrated. We have argued that this tool can shed light upon the nature of non-equilibrium steady state criticality in particular with regard to the role played by quantum vs classical fluctuations. [13]

Apart from the academic interest, this procedure have experimental relevance. Owing to its non-local character, the GP provides information on the criticality without the need to drive the system across the critical region. The latter is hard to physically implement as it is accompanied by multiple degeneracies that can take the system away from its steady state. Hence, it provides a way to probe criticalities in a physically appealing way. Moreover, such a non-local character can also be useful to probe critical phenomena such as topological phase transitions [15], which are undetectable by local order parameters [16,17].

2. Decoherence and Quantum Bistable Systems

A second line of research focuses on the study of the bistable dynamics of a quantum particle coupled to an environment. The main objective is the investigation of the quantum dynamics of a multilevel bistable system coupled to a bosonic heat bath beyond the perturbative regime. In this regime the effect of different spectral densities of the bath plays a considerable role. It is particularly relevant to consider the transition from sub-Ohmic to super-Ohmic dissipation. The approach of this type of investigations is based mainly on the real-time path integral technique of the Feynman-Vernon influence functional. Using this approach one finds that, in the crossover dynamical regime characterised by damped intrawell oscillations and incoherent tunnelling, the short time behaviour and the time scales of the relaxation starting from a non-equilibrium initial condition depend nontrivially on the spectral properties of the heat bath.

Real quantum systems are always in contact with noisy environments causing dissipation and decoherence. An ubiquitous model that describes dissipation by phononic or photonic environments is the celebrated Caldeira-Leggett model [18] in which the open system is linearly coupled to a reservoir of quantum harmonic oscillators. In the thermodynamical limit, the reservoir is a bosonic heat bath and is described by the spectral density function $J(\omega)$, generally assumed in the form $\omega^n$ with a high-frequency cut-off. The special case $s = 1$ gives the so-called Ohmic dissipation. In this case the quantum Langevin equation for the position operator of the particle has a memoryless damping kernel (frequency independent friction) and, in the classical limit $\hbar \to 0$, the heat bath reduces to a classical white noise source [19].
The usual approach to the tunneling dynamics in the presence of dissipation entails the use of a two-level system (TLS) approximation, which greatly simplifies its description. Within this approximation the Hilbert space of the particle is truncated to that spanned by the first two energy eigenstates, resulting in the so called spin-boson model.

The scope of this research is to consider the dynamics of a dissipative bistable system, beyond the TLS approximation, in a temperature regime in which the presence of the second energy doublet cannot be neglected [20–24]. To this end, a nonperturbative generalized master equation with approximated kernels can be derived within the Feynman-Vernon influence functional approach [25,26]. By means of the latter, it is possible to investigate a regime of dissipation which is beyond the perturbative Born-Markov approximation. A detailed study of the reduced dynamics has been performed in domains where the exponent \( s \) is in the crossover from the sub-Ohmic \((s < 1)\) to the super-Ohmic \((s > 1)\) regimes. It is also possible to explore the effects of varying cutoff frequency \( \omega_c \), which have also been subject of investigations. These investigations provide insight into the comparative role of the high-frequency and low-frequency modes on the open dynamics.

3. Dynamics of Solitons in Long Josephson Junctions under Noisy Environment

Another line of research concerns the studies of the generation of solitons in current-biased long Josephson junctions. In particular, the focus of investigation is on the properties of the solitons in relation to the superconducting lifetime and the voltage drop across the device.

In these investigations, the dynamics of the junction is modelled with a sine-Gordon equation driven by oscillating fields and subject to an external non-Gaussian noise. A wide range of \( \alpha \)-stable Lévy distributions can be considered as a noise source, with varying stability index \( \alpha \) and asymmetry parameter \( \beta \). In junctions longer than a critical length, the mean switching time (MST) from the superconductive to the resistive state assumes a value independent of the device length. This value is directly related to the mean density of solitons which move into or from the washboard potential minimum corresponding to the initial superconductive state. In particular one is able to observe: (i) a connection between the total mean soliton density and the mean potential difference across the junction; (ii) an inverse behaviour of the mean voltage in comparison with the MST, with varying the junction length; (iii) evidence of non-monotonic behaviours, such as stochastic resonant activation and noise-enhanced stability, of the MST versus the driving frequency and noise intensity for different values of \( \alpha \) and \( \beta \); (iv) finally, these non-monotonic behaviours are found to be related to the mean density of the solitons formed along the junction [27–30].

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