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ON ISOTOPY OF SELF-HOMEOMORPHISMS OF QUADRATIC INVERSE LIMIT SPACES

H. BRUIN AND S. ŠTIMAC

Abstract. We prove that every self-homeomorphism on the inverse limit space of a quadratic map is isotopic to some power of the shift map.

1. Introduction

The two most prominent families of unimodal maps are the family of quadratic maps $Q_a$, $a \in [1, 4]$, and the family of tent maps $T_s$, $s \in [1, 2]$. The inverse limit spaces of quadratic and tent maps share a lot of common properties. For example, if $f$ is a map from one of these families, then $0$ is a fixed point of $f$; the point $\bar{0} := (\ldots, 0, 0, 0)$ is contained in $\lim ([0, 1], f)$ and is an end-point. The arc-component $C$ of $\lim ([0, 1], f)$ which contains $\bar{0}$ is a ray converging to, but (provided $a < 4$ and $s < 2$) disjoint from the inverse limit of the core $\lim (\lbrack c^2, c^1], f)$, and $\lim ([0, 1], f) = C \cup \lim (\lbrack c^2, c^1], f)$, where the critical or turning point is denoted as $c$ and $c_k := f^k(c)$. If $c$ is periodic with (prime) period $N$, then $\lim (\lbrack c^2, c^1], f)$ contains $N$ end-points.

The relationships between quadratic and tent maps and between their inverse limits are mostly well understood. Each quadratic map $Q_a$ with positive topological entropy is semi-conjugate to a tent map $T_s$ with $\log s = h_{top}(Q_a)$, and this semi-conjugacy collapses (pre)periodic intervals to points [6]. If a quadratic map is not renormalizable and does not

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