Deterministic Rendezvous at a Node of Agents with Arbitrary Velocities

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Abstract

We consider the task of rendezvous in networks modeled as undirected graphs. Two mobile agents with different labels, starting at different nodes of an anonymous graph, have to meet. This task has been considered in the literature under two alternative scenarios: weak and strong. Under the weak scenario, agents may meet either at a node or inside an edge. Under the strong scenario, they have to meet at a node, and they do not even notice meetings inside an edge. Rendezvous algorithms under the strong scenario are known for synchronous agents. For asynchronous agents, rendezvous under the strong scenario is impossible even in the two-node graph, and hence only algorithms under the weak scenario were constructed. In this paper we show that rendezvous under the strong scenario is possible for agents with restricted asynchrony: agents have the same measure of time but the adversary can arbitrarily impose the speed of traversing each edge by each of the agents. We construct a deterministic rendezvous algorithm for such agents, working in time polynomial in the size of the graph, in the length of the smaller label, and in the largest edge traversal time.

Keywords: rendezvous, deterministic algorithm, mobile agent, velocity.

1 Introduction

The background. We consider the task of rendezvous in networks modeled as undirected graphs. Two mobile entities, called agents, have different labels, start from different nodes of the network, and have to meet. Mobile entities may represent software agents in a communication network, or physical mobile robots, if the network is a labyrinth or a cave, or if it consists of corridors of a building. The reason to meet may be to exchange previously collected data or ground or air samples, or to split work in a future task of network exploration or maintenance.

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The task of rendezvous in networks has been considered in the literature under two alternative scenarios: *weak* and *strong*. Under the weak scenario [6, 7, 10], agents may meet either at a node or inside an edge. Under the strong scenario [8, 13, 17], they have to meet at a node, and they do not even notice meetings inside an edge. Each of these scenarios is appropriate in different applications. The weak scenario is suitable for physical robots in a network of corridors, while the strong scenario is needed for software agents in computer networks.

Rendezvous algorithms under the strong scenario are known for synchronous agents, where time is slotted in rounds, and in each round each agent can either wait at a node or move to an adjacent node. For asynchronous agents, where an agent decides to which neighbor it wants to move but the adversary totally controls the walk of the agent and can arbitrarily vary its speed, rendezvous under the strong scenario is impossible even in the two-node graph, and hence only algorithms under the weak scenario were constructed.

However, due to the fact that the strong scenario is appropriate for software agents in computer networks, and that such agents are rarely synchronous, it is important to design rendezvous algorithms under the strong scenario, restricting the asynchrony of the agents as little as possible. This is the aim of this paper. We consider mobile agents with asynchrony restricted as follows: agents have the same measure of time but the adversary can arbitrarily impose the speed of traversing each edge by each of the agents. We are interested in deterministic rendezvous algorithms for such agents.

**The model.** The network is modeled as a simple undirected connected graph. As in the majority of papers on rendezvous, we seek algorithms that do not rely on the knowledge of node labels, and we assume that the underlying graph is anonymous. Designing such algorithms is important because even when node labels exist, nodes may refuse to reveal them, *e.g.*, due to security or privacy reasons. It should be also noted that, if nodes had distinct labels, agents might explore the graph and meet in the smallest node, hence gathering would reduce to graph exploration. On the other hand, we make the assumption, again standard in the literature of the domain, that edges incident to a node $v$ have distinct labels in $\{0, \ldots, d - 1\}$, where $d$ is the degree of $v$. Thus every undirected edge $\{u, v\}$ has two labels, which are called its *port numbers* at $u$ and at $v$. Port numbers are visible to the agents. Port numbering is *local*, *i.e.*, there is no relation between port numbers at $u$ and at $v$. Note that in the absence of port numbers, edges incident to a node could not be distinguished by agents and thus rendezvous would be often impossible, as the adversary could prevent an agent from traversing some edge incident to the current node. Also, the above mentioned concerns of security and privacy that may prevent nodes from revealing their labels, do not apply to port numbers.

Agents $A_1$ and $A_2$ start at arbitrary different nodes of the graph. They cannot mark visited nodes or traversed edges in any way. The adversary wakes up the agents at possibly different times. Agents do not know the topology of the graph nor any bound on its size. They have clocks ticking
at the same rate. The clock of each agent starts at its wakeup, and at this time the agent starts executing the algorithm.

The adversary assigns different positive integer labels to both agents. Each agent knows a priori only its own label. Both agents execute the same deterministic algorithm whose parameter is the label of the agent. Moreover, for \( i = 1, 2 \), and for each edge \( e \) of the graph, the adversary assigns a positive real \( t(i, e) \). During the execution of an algorithm, an agent can wait at the currently visited node for a time of its choice, or it may choose a port to traverse the corresponding edge \( e \). In the latter case, agent \( A_i \) traverses this edge in time \( t(i, e) \), getting to the other end of the edge after this time. This very general modelling permits a lot of asynchrony: agents can have different velocities when traversing different edges, and an agent slower in one edge can be faster in another edge. This is motivated by the fact that congestion and bandwidth of different edges may be different, and that each of the agents can have a different traversing priority level on different edges. In particular, this general scenario includes the model of agents walking at possibly different constant velocities, that was used in \[9\] for the task of approach in the plane.

When an agent enters a node, it learns its degree and the port of entry. We assume that the memory of the agents is unlimited: from the computational point of view they are modeled as Turing machines.

The time of a rendezvous algorithm is the worst-case time between the wakeup of the earlier agent and the meeting at a node.

Our results. We construct a deterministic rendezvous algorithm working for arbitrary graphs under the strong scenario. Our algorithm works in time polynomial in \( n, \ell \) and \( \tau \), where \( n \) is the number of nodes of the graph, \( \ell \) is the logarithm (i.e., the length) of the smaller label, and \( \tau \) is the maximum of all values \( t(i, e) \) assigned by the adversary, over all edges \( e \) of the graph, and over \( i = 1, 2 \).

Related work. A survey of randomized rendezvous in various scenarios can be found in \[1\]. Deterministic rendezvous in networks was surveyed in \[15\]. In many papers rendezvous was considered in a geometric setting: an interval of the real line, see, e.g., \[4, 12\], or the plane, see, e.g., \[2, 3\]).

For deterministic rendezvous in networks, attention concentrated on the study of the feasibility of rendezvous, and on the time required to achieve this task, when feasible. For example, deterministic rendezvous with agents equipped with tokens used to mark nodes was considered, e.g., in \[14\]. Deterministic rendezvous of two agents that cannot mark nodes but have unique labels was discussed in \[8, 13, 17\]. All these papers were concerned with the time of rendezvous in arbitrary graphs. In \[8\] the authors showed a rendezvous algorithm polynomial in the size of the graph, in the length of the shorter label and in the delay between the starting times of the agents. In \[13, 17\] rendezvous time was polynomial in the first two of these parameters and independent of the delay. All the above papers assumed that agents are synchronous, and used the scenario called strong in the
Several authors investigated asynchronous rendezvous in the plane \cite{5, 11} and in network environments \cite{6, 7, 10}. In the latter scenario, the agent chooses the edge which it decides to traverse but the adversary totally controls the walk of the agent inside the edge and can arbitrarily vary its speed. Under this assumption, rendezvous under the strong scenario cannot be guaranteed even in very simple graphs, and hence the rendezvous requirement was weakened by considering the scenario called \textit{weak} in the present paper. In particular, the main result of \cite{10} is an asynchronous rendezvous algorithm working in an arbitrary graph at cost (measured by the number of edge traversals) polynomial in the size of the graph and in the logarithm of the smaller label. The scenario of possibly different fixed speeds of the agents was introduced in \cite{9}.

\section{Preliminaries}

Throughout the paper, the number of nodes of a graph is called its size. The following procedure, based on universal exploration sequences (UXS), is a corollary of the result of Reingold \cite{16}. Given any positive integer $n$, it allows the agent to visit all nodes of any graph of size at most $n$, starting from any node of this graph and coming back to it, using $P(n)$ edge traversals, where $P$ is some increasing polynomial. In the first half of the procedure, after entering a node of degree $d$ by some port $p$, the agent can compute the port $q$ by which it has to exit; more precisely $q = (p + x_i) \mod d$, where $x_i$ is the corresponding term of the UXS. In the second half of the procedure, the agents backtracks to its starting node.

A \textit{trajectory} is a sequence of nodes of a graph, in which each node is adjacent to the preceding one. Given any starting node $v$, we denote by $R(n, v)$ the trajectory obtained by Reingold’s procedure. The procedure can be applied in any graph starting at any node, giving some trajectory. We say that the agent \textit{follows} a trajectory if it executes the above procedure used to construct it. This trajectory will be called \textit{integral}, if the corresponding route covers all nodes of the graph. By definition, the trajectory $R(n, v)$ is integral, if it is obtained by Reingold’s procedure applied in any graph of size at most $n$, starting at any node $v$. Since in our algorithm an agent starts and ends each trajectory $R(n, v)$ at its node of origin $v$, we will write $R(n)$ instead of $R(n, v)$.

We will use the following terminology. The agent woken up earlier by the adversary is called the \textit{earlier} agent and the other agent is called the \textit{later} agent. If agents are woken up simultaneously, these appellations are given arbitrarily. Consider executions $E_1$ and $E_2$, respectively of procedures $\mathcal{P}_1$ and $\mathcal{P}_2$ by agents $A_1$ and $A_2$. Executions $E_1$ and $E_2$ are called \textit{overlapping}, if the time segments that they occupy are not disjoint.

We define the following transformation of labels. Consider a label $x$ of an agent, with binary representation $(c_1 \ldots c_r)$. Define the \textit{modified label} of the agent to be the sequence $M(x) =$
(c_{1c_1}c_2c_2\ldots c_tc_t01). Note that, for any x and y, the sequence M(x) is never a prefix of M(y). Also, M(x) \neq M(y) for x \neq y.

3 The algorithm and its analysis

Our solution is provided by Algorithm RV (shown in Algorithm 1 and its execution requires to call Procedure Phase(i) that is described in Algorithm 2. At a high level, Phase(i) consists of executions of R(2^i) and carefully scheduled waiting periods of various lengths, designed according to the bits of the modified label of the agent. The aim is to guarantee a period in which one agent stays still at a node and the other visits all nodes of the graph.

| Algorithm 1 RV |
|----------------|
| 1: i ← 1 |
| 2: while agents have not met do |
| 3: Execute Phase(i) |
| 4: i ← i + 1 |

| Algorithm 2 Phase(i) |
|----------------------|
| 1: Let x be the label of the agent and let M(x) = (b_1b_2\ldots b_s), where s is the length of M(x). |
| 2: /* Initialization */ |
| 3: Execute R(2^i) |
| 4: Wait for time 2^{i+3}(\sum_{k=1}^{i}(P(2^k))) |
| 5: /* Core */ |
| 6: j ← 1 |
| 7: while j \leq 2^i do |
| 8: if j > s or b_j = 0 then |
| 9: \hspace{1cm} Wait for time 2^{i+1}P(2^i) |
| 10: \hspace{1cm} Execute twice R(2^i) |
| 11: else |
| 12: \hspace{1cm} Execute twice R(2^i) |
| 13: \hspace{1cm} Wait for time 2^{i+1}P(2^i) |
| 14: j ← j + 1 |
| 15: /* End */ |
| 16: Wait for time 2^iP(2^{i+1}) |
| 17: Execute R(2^i) |

Now we analyse the correctness and the time complexity of Algorithm RV. We denote by G the graph in which the two agents evolve and by \alpha the smallest integer such that 2^\alpha upper bounds the following three numbers: the size n ≥ 2 of G, the length of the smaller modified label, and the parameter \tau.

By fixing an arbitrary integer i ≥ \alpha, the following proposition follows by induction on j, using the definitions of \alpha and of the polynomial P.

**Proposition 3.1.** For any positive integers i and j, such that i ≥ \alpha and i ≥ j, the integer T_{i,j} = 2^{i+j+3}(\sum_{k=1}^{i}(P(2^k))) upper bounds the time required by any agent to execute the sequence S_j = Phase(1), Phase(2), \ldots, Phase(j-2), Phase(j-1), R(2^j) in the graph G.

The following theorem proves the correctness of Algorithm RV.
Theorem 3.1. Algorithm RV guarantees rendezvous in G by the time the first of the agents completes the execution of Phase(\(\alpha + 1\)).

Proof. Assume by contradiction that the statement of the theorem is false. Note that when an agent finishes the first execution of \(R(2^\alpha)\) of Phase(\(\alpha\)) (line 3 of Algorithm 2) it has visited every node of \(G\) (because \(R(2^\alpha)\) is integral in \(G\), and thus the other agent has been woken up before the end of this execution, or else the agents would meet. The core of Phase(\(\alpha\)) (lines 6-14 of Algorithm 2) can be viewed as a sequence of \(2^\alpha\) blocks, where the \(j\)th block (for \(1 \leq j \leq 2^\alpha\)) corresponds to processing bit \(b_j\) of the modified label (or 0 if the length of the modified label is smaller than \(j\)). Each of these blocks in turn can be viewed as a sequence of 4 subblocks, each of which corresponds either to a waiting period of length \(2^\alpha P(2^\alpha)\), or to a single execution of \(R(2^\alpha)\). Let \(I_1, I_2, \ldots, I_{2^\alpha+2}\) (resp. \(J_1, J_2, \ldots, J_{2^\alpha+2}\)) be the sequence of the \(2^\alpha+2\) subblocks executed by agent \(A_1\) (resp. agent \(A_2\)) in the core of Phase(\(\alpha\)). We have the following claim.

Claim 3.1. For every \(1 \leq k \leq 2^\alpha+2\), \(I_k\) and \(J_k\) are overlapping.

To prove the claim, assume by contradiction that \(k = s\) is the smallest integer for which it does not hold. Without loss of generality, suppose that the first agent to complete its \(s\)th subblock is \(A_1\). If \(s = 1\), then in view of Proposition 3.1 when \(A_1\) starts and finishes \(I_1, A_2\) is executing the first waiting period of Phase(\(\alpha\)). Since \(I_1\) corresponds to \(R(2^\alpha)\), as the first bit of a modified label is always 1, we get a meeting by the end of the execution of \(I_1\) because \(2^\alpha \geq n\), which is a contradiction. So, \(s > 1\) and by the minimality of \(s\), \(I_{s-1}\) and \(J_{s-1}\) are overlapping. So, when \(A_1\) starts \(I_s\), \(A_2\) is executing \(J_{s-1}\) and when \(A_1\) completes \(I_s\), the execution of \(J_{s-1}\) has not yet been completed. This implies that the time required by \(A_1\) to execute \(I_s\) is shorter than the time required by \(A_2\) to execute \(J_{s-1}\). Hence, \(I_s\) cannot be a subblock corresponding to a waiting period, as each of these periods has length \(2^\alpha P(2^\alpha)\), which upper bounds the duration of every subblock corresponding to \(R(2^\alpha)\) (because \(\tau \leq 2^\alpha\)). Thus \(I_s\) corresponds to an execution of \(R(2^\alpha)\), and so does \(J_{s-1}\), as otherwise rendezvous would occur by the time \(I_s\) is completed, which would be a contradiction.

Consider the time lag between executions of \(I_s\) and \(J_s\). Let \(\theta_1 = t_2 - t_1\), where \(t_1\) (resp. \(t_2\)) is the time when \(A_1\) (resp. \(A_2\)) starts \(I_s\) (resp. \(J_s\)). The time required by \(A_1\) to execute \(R(2^\alpha)\) is at most \(\theta_1\), and \(A_1\) always executes \(R(2^\alpha)\) faster than \(A_2\) (note that the time required by \(A_1\) (resp. \(A_2\)) to execute \(R(2^\alpha)\) never changes because it is always executed from the initial node of \(A_1\) (resp. \(A_2\))). Moreover, in each block there are always four subblocks: either two waiting periods of \(2^\alpha P(2^\alpha)\) followed by two \(R(2^\alpha)\), or vice versa. Hence, in view of the fact \(I_s\) and \(J_{s-1}\) correspond to \(R(2^\alpha)\) we get the following feature. When considering each of the four positions that can be occupied by \(I_s\) in its corresponding block, the number of whole subblocks corresponding to \(R(2^\alpha)\) (resp. the waiting period of \(2^\alpha P(2^\alpha)\)) that remain to be executed by \(A_1\) from \(t_1\) is always at most the number of those that remain to be executed by \(A_2\) from \(t_1\). As a result, \(A_1\) is the first agent to finish
the core of \( \text{Phase}(\alpha) \) and there exists a difference of \( \theta_2 \geq \theta_1 \) between the times when \( A_1 \) and \( A_2 \) complete this core. To conclude the proof of the claim, we consider two cases: either \( \theta_2 \) is longer than the time \( A_1 \) needs to execute \( R(2^{\alpha+1}) \), or not.

In the first case, since \( A_1 \) executes \( R(2^\alpha) \) faster than \( A_2 \), it necessarily completes the end of \( \text{Phase}(\alpha) \) at least time \( \theta_2 \) ahead of \( A_2 \). As a consequence, it starts executing \( \text{Phase}(\alpha + 1) \), and in particular its first instruction \( R(2^{\alpha+1}) \), at least time \( \theta_2 \) ahead of \( A_2 \). This implies that \( A_1 \) completes the execution of the first instruction \( R(2^{\alpha+1}) \) of \( \text{Phase}(\alpha + 1) \) before \( A_2 \) starts it. Hence, \( A_1 \) starts the execution of the first waiting period of \( \text{Phase}(\alpha + 1) \) by the time \( A_2 \) starts the first execution of \( R(2^{\alpha+1}) \) in \( \text{Phase}(\alpha + 1) \). In view of Proposition 3.1, this leads to a meeting before any agent starts the core of \( \text{Phase}(\alpha + 1) \), which is a contradiction. In the second case, \( A_1 \) completes the last waiting period of \( \text{Phase}(\alpha) \) at a time \( \theta_2 \) ahead of \( A_2 \). Moreover, for any agent, the duration of the execution of this waiting period is at least \( \theta_2 \). Hence, while \( A_1 \) executes entirely the last instruction \( R(2^\alpha) \) of \( \text{Phase}(\alpha) \), \( A_2 \) is waiting (it executes the last waiting period of \( \text{Phase}(\alpha) \)). This leads to a meeting before any agent starts \( \text{Phase}(\alpha + 1) \), and hence we also get a contradiction in the second case, which proves the claim.

Note that the length of the smaller modified label is at most \( 2^\alpha \). Moreover, the modified labels are not prefixes of each other, hence they must differ at some bit. Thus, it follows from the claim that there exists a period during which an agent is waiting in \( \text{Phase}(\alpha) \) while the other entirely executes \( R(2^\alpha) \). Hence a meeting occurs before any agent starts \( \text{Phase}(\alpha + 1) \), which is a contradiction and proves the theorem.

According to Theorem 3.1, rendezvous occurs by the time the first of the two agents completes \( \text{Phase}(\alpha + 1) \), which occurs, by Proposition 3.1, before this agent has spent at most a time \( O(2^{\kappa \alpha}) \) since its wake up, for some constant \( k \). However, in view of the definition of \( \alpha \) and of the modified labels, we have \( 2^{\alpha-1} < \max(\tau, n, 2l + 2) \leq 2^\alpha \). Hence, we get the following theorem.

**Theorem 3.2.** The execution time of Algorithm \( RV \) is polynomial in \( n, \ell \) and \( \tau \).

### 4 Discussion of alternative scenarios

Our result shows that the time of rendezvous can be polynomial in \( n, \ell \) and \( \tau \), where \( n \) is the number of nodes of the graph, \( \ell \) is the logarithm of the smaller label, and \( \tau \) is the maximum of all values \( t(i, e) \) assigned by the adversary, over all edges \( e \) of the graph, and over \( i = 1, 2 \), and when the time is counted since the wakeup of the earlier agent. It is natural to ask, if it is possible to construct a rendezvous algorithm whose time depends on \( n, \ell \) and \( \tau' \), where \( \tau' \) is the minimum of all values \( t(i, e) \) assigned by the adversary, over all edges \( e \) of the graph, and over \( i = 1, 2 \). The answer is trivially negative, if time is counted, as we do, since the wakeup of the earlier agent. Indeed, suppose that there exists such an algorithm working in some time \( F(n, \ell, \tau') \). The adversary assigns
t(1, e) > F(n, ℓ, τ'), for the first edge e taken by the first agent, starts the first agent at some time \( t_0 \) and delays the wakeup of the second agent until time \( t_0 + t(1, e) \). Rendezvous cannot happen before time \( t_0 + t(1, e) \), which is a contradiction.

It turns out that the answer is also negative in the easier scenario, when time is counted since the wakeup of the agent that is woken up later. Consider even a simplified situation, where \( t_1 = t(1, e) \) is the same for all edges e, and \( t_2 = t(2, e) \) is the same for all edges e. In other words, each of the agents has a constant speed. Thus \( \tau = \max(t_1, t_2) \) and \( \tau' = \min(t_1, t_2) \). Call the agent for which \( t_i \) is larger the slower agent, and the other – the faster agent.

First notice that, if we show that any rendezvous algorithm must take time at least \( \tau \) since the wakeup of the later agent, the negative result follows, as the adversary can assign \( \tau > F(n, ℓ, \tau') \).

We now show that, indeed, for any rendezvous algorithm, there exists a behavior of the adversary for which this algorithm takes time at least \( \tau \) since the wakeup of the later agent.

Denote by \( \beta \) (resp. by \( \gamma \)) the waiting time between the wakeup of the faster (resp. slower) agent and the time when it starts the first edge traversal. Let \( d = |\beta - \gamma| \).

If \( \beta \geq \gamma \), the adversary wakes up the faster agent at some time \( t_0 \) and wakes up the slower agent at time \( t_0 + d \). Both agents start traversing their first edge at the same time \( t_0 + \beta \) and cannot meet before time \( t_0 + \beta + \tau \). Since both agents were awake at time \( t_0 + \beta \), our claim follows.

If \( \beta < \gamma \), the adversary wakes up the slower agent at some time \( t_0 \) and wakes up the faster agent at time \( t_0 + d \). Both agents start traversing their first edge at the same time \( t_0 + \gamma \) and cannot meet before time \( t_0 + \gamma + \tau \). Since both agents were awake at time \( t_0 + \gamma \), our claim follows. This concludes the justification that it is impossible to guarantee rendezvous in time depending on \( n, ℓ \) and \( \tau' \), even when time is counted since the wakeup of the later agent.

The above remark holds under the strong scenario considered in this paper. By contrast, the answer to the same question turns out to be positive in the weak scenario. This can be justified as follows. In [10] the authors showed a rendezvous algorithm with cost (measured by the total number of edge traversals) polynomial in \( n \) and \( ℓ \) under the weak scenario, assuming that agents are totally asynchronous. Let \( \mathcal{A} \) be this algorithm and let its cost be \( K(n, ℓ) \), where \( K \) is some polynomial. Consider algorithm \( \mathcal{A} \) in the simplified model mentioned above, where each of the agents has a constant speed, the speeds being possibly different, still under the weak scenario. Consider the part of the cost of the algorithm counted since the wakeup of the later agent. This cost is \( K'(n, ℓ) \), where \( K' \) is some polynomial. Of course, the behavior of the agents in which an agent never waits at a node and crosses each edge at its constant speed is a possible behavior imposed by a totally asynchronous adversary, and hence the result of [10] still holds (under the weak scenario). Hence, if time is counted from the wakeup of the later agent, the time of the algorithm from [10] is at most \( \tau' \cdot K'(n, ℓ) \), and therefore it is polynomial in \( n, ℓ \) and \( \tau' \).

To conclude, we get the following observation that shows a provable difference between the time
of rendezvous under the strong and the weak scenarios, even in the situation when rendezvous is possible under both of these scenarios. If time is counted from the wakeup of the later agent, and agents have constant, possibly different velocities, then rendezvous in time depending on $n$, $\ell$ and $\tau'$ cannot be guaranteed under the strong scenario, but there is a rendezvous algorithm working in time polynomial in $n$, $\ell$ and $\tau'$ under the weak scenario.

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