Theoretical Aspects of HERA Physics

Stefano Forte$^\ast$ and Richard D. Ball$^\dagger$

$^\ast$INFN, Sezione di Roma III, via della Vasca Navale 84, I–00146 Roma, Italy
$^\dagger$Department of Physics and Astronomy, University of Edinburgh, Edinburgh EH9 3JZ, Scotland

Abstract. We discuss the theoretical underpinnings for the extraordinary success of perturbative QCD in the description of HERA data. In particular, we examine recent progress in the understanding of perturbative QCD at small $x$. We explain the relation between evolution equations in $Q^2$ and $x$, and how they can be used for simultaneous resummation of the relevant large logs at HERA. We show that while the HERA data can be understood within our current knowledge of the perturbative expansion of QCD, they pose stringent constraints on the perturbatively inaccessible behaviour of QCD in the Regge limit.

Presented at the workshop QCD@WORK
Martina Franca, Italy, June 2001

to be published in the proceedings

---

$^\ast$ On leave from INFN, Sezione di Torino, Italy
Theoretical Aspects of HERA Physics

Stefano Forte\textsuperscript{1} and Richard D. Ball\textsuperscript{1}\textsuperscript{†}

\textsuperscript{1}INFN, Sezione di Roma III, via della Vasca Navale 84, I–00146 Roma, Italy
\textsuperscript{†}Department of Physics and Astronomy, University of Edinburgh, Edinburgh EH9 3JZ, Scotland

PERTURBATIVE QCD AT HERA

QCD has been tested at HERA \cite{1, 2} over the last several years to an accuracy which is now comparable to that of tests of the electroweak sector at LEP: perturbative QCD turns out to provide an embarrassingly successful description of the HERA data, even in kinematic regions where simple fixed–order perturbative predictions should fail. This success is most strikingly demonstrated by the comparison with the data of the scaling violations of structure functions predicted by the QCD evolution equations \cite{3, 4}: the data agree with the theory over five orders of magnitude in both $x$ and $Q^2$.

The significance of this sort of result is somewhat obscured by the need to fit the shape of parton distributions at a reference scale, which might suggest that deviations from the predicted behaviour could be accommodated by changing the shape of the parton distribution. However, this is not true because of the predictive nature of the QCD result: given the shape of partons at one scale, there is no freedom left to fit the data at other scales. This predictivity is particularly transparent in the small $x$ region, where the fixed–order QCD result actually becomes asymptotically independent of the parton distribution, apart from an overall normalization. Indeed, the data for $\ln F_2$ plotted versus the variable $\sigma \equiv \ln \frac{x_0}{x} \ln \frac{\alpha_s(Q_0^2)}{\alpha_s(Q^2)}$ are predicted to lie on a straight line, with universal slope $2\gamma = 12/\sqrt{33} - 2n_f$ (double asymptotic scaling \cite{5, 6}). The predicted scaling is spectacularly borne out by the data, as shown in fig. 1: in fact, the data are now so accurate that one can see the change in slope when passing the $b$ threshold, and indeed double scaling is only manifest if one separates data in the regions where $\alpha_s$ runs with $N_f = 4$ from those with $N_f = 5$\textsuperscript{2}. Equally good agreement with fixed–order perturbation theory is seen when considering less inclusive observables.

This agreement of the data with fixed–order perturbative QCD computations is very surprising, in that the perturbative expansion receives contributions of order $\alpha_s \ln \frac{1}{x}$ so one would expect higher–order corrections to be non–negligible whenever $\alpha_s \ln \frac{1}{x} \gtrsim 1$, i.e. in most of the HERA region. As is well known, the resummation of leading $\ln \frac{1}{x}$ (LLx) contributions to gluon–gluon scattering, and thus to a wide class of hard pro-

\textsuperscript{1} On leave from INFN, Sezione di Torino, Italy
\textsuperscript{2} The fact that the observed slope is somewhat smaller than the predicted one, especially at low $Q^2$, is due to NLO corrections \cite{7} as well as corrections due to the “small” eigenvalue of perturbative evolution \cite{8}.
cesses, including small $x$ scaling violations of structure functions, is accomplished by means of the BFKL evolution equation [9, 10, 11]. Matching the BFKL approach to standard perturbative computation, however, is nontrivial [12, 13], while the BFKL equation itself seems to be unstable towards the inclusion of higher order corrections [14]. Hence, the main problem in understanding HERA physics, i.e. perturbative QCD at small $x$ is that of establishing “consistency of the BFKL approach with the more standard DGLAP [15, 16] evolution equations” [17], which embody the leading $\ln Q^2$ ($\mathrm{LLQ}^2$) resummation on which perturbative QCD is based. This problem is now solved [18, 19, 20], and on the basis of this solution it is possible to combine the available information on perturbation theory at small $x$, and use it to explain the unexpected success of fixed–order calculations.

**DUALITY**

Let us for definiteness consider the prototype problem of the description of small $x$ scaling violations of parton distributions. For simplicity, consider the case of a single parton distribution $G(x, Q^2)$, which can be thought of as the dominant eigenvector of perturbative evolution. Scaling violations are then described by the Altarelli-Parisi equation satisfied by $G(x, Q^2)$, and thus summarized by the Altarelli–Parisi splitting function $P(x, \alpha_s)$ [15].

The basic result which allows the study of scaling violations at small $x$ is duality of perturbative evolution [21, 19, 22], namely, the fact that, because the Altarelli-Parisi equation is an integro–differential equation in the two variables $t \equiv \ln Q^2 / \Lambda^2$ and $\xi \equiv 1/x$, it can be equivalently cast in the form of a differential equation in $t$ satisfied by the $x$–Mellin transform

$$G(N, t) = \int_0^\infty d\xi e^{-N\xi} G(\xi, t),$$

(1)
or a differential equation in $\xi$ satisfied by the $Q^2$–Mellin transform

$$G(\xi, M) = \int_{-\infty}^{\infty} dt \, e^{-Mt} \, G(\xi, t) \quad (2)$$

of the parton distribution. The pair of dual evolution equations are

$$\frac{d}{dt} G(N, t) = \gamma(N, \alpha_s) \, G(N, t) \quad (3)$$

$$\frac{d}{d\xi} G(\xi, M) = \chi(M, \alpha_s) \, G(\xi, M) \quad (4)$$

where eq. (3) is the standard renormalization–group equation, with anomalous dimension $\gamma(N, t)$, and eq. (4) is essentially the BFKL equation. Duality is the statement that the solutions of these two equations coincide to all perturbative orders, up to power suppressed corrections, provided their kernels are related by

$$\chi(\gamma(N, \alpha_s), \alpha_s) = N \quad (5)$$

This means that the BFKL and Altarelli-Parisi equations describe the same physics: it is the choice of the kernel to be used in the evolution equation which determines which is the large scale which is resummed. We can then discuss the construction and resummation of the kernel irrespective of the specific evolution equation where it is used, with the understanding that the kernel can be equivalently viewed as $\gamma(N, \alpha_s)$ or $\chi(M, \alpha_s)$, the two being related by eq. (5). Before doing this, we sketch how duality can be proven order by order in perturbation theory.

**Fixed coupling**

Perturbative duality is most easy to prove when the coupling does not run, since in this case the two scales $t$ and $\xi$ appear in the Altarelli–Parisi equation in a completely symmetric way. It is convenient to introduce the double–Mellin transform $G(N, M)$ of the parton distribution. The solution to the Altarelli–Parisi equation in $M, N$ space has the form (which can be e.g. obtained by performing an $M$–mellin transform eq. (2) of the solution to the renormalization–group eq. (3))

$$G(N, M) = \frac{G_0(N)}{M - \gamma(N, \alpha_s)} \quad (6)$$

where $G_0(N)$ is a boundary condition at a reference scale $\mu^2$.

The inverse Mellin transform of eq. (6) coincides with the residue of the simple pole in the $M$ plane of $e^M G(N, M)$, and thus its scale dependence is entirely determined by the location of the simple pole of $G(N, M)$ (6) , namely, the solution to the equation

$$M = \gamma(N, \alpha_s) \quad (7)$$
The pole condition Eq. (7) can be equivalently viewed as an implicit equation for $N$: $N = \chi(M, \alpha_s)$, where $\chi$ is related to $\gamma$ by eq. (5). Hence, the function

$$G(N, M) = \frac{F_0(N)}{N - \chi(M, \alpha_s)},$$

(8)

corresponds to the same $G(t, x)$ as eq.(6), because the location of the respective poles in the $M$ plane are the same, while the residues are also the same, provided the boundary conditions are matched by

$$G_0(N) = -\frac{F_0(\gamma(\alpha_s, N))}{\chi'(\gamma(\alpha_s, N))}.$$  

(9)

Eq. (8) is immediately recognized as the $N$-Mellin of the solution to the evolution equation (8) with boundary condition $F_0(M)$ (at some reference $x = x_0$), which is what we set out to prove. In general, the analytic continuation of the function $\chi$ defined by eq. (5) will be such that eq. (7) has more than one solution (i.e. $\gamma$ is multivalued). In this case, poles further to the left in the $M$ plane correspond to power–suppressed contributions, while poles to the right correspond to contributions beyond perturbation theory (they do not contribute when the inverse $M$–Mellin integral is computed along the integration path which corresponds to the perturbative region).

It is easy to see that upon duality the leading–order $\chi = \alpha_s \chi_0$ is mapped onto the leading singular $\gamma = \gamma_s(\alpha_s/N)$, and conversely the leading–order $\gamma = \alpha_s \gamma_0$ is mapped onto the leading singular $\chi = \chi_s(\alpha_s/M)$. In general, the expansion of $\chi$ in powers of $\alpha_s$ at fixed $M$ is mapped onto the expansion of $\gamma$ in powers of $\alpha_s$ at fixed $\alpha_s/N$, and conversely. So in particular at LLQ$^2$ it is enough to consider $\gamma_0$ or $\chi_0$, and at LLx it is enough to consider $\gamma_s$ or $\chi_s$. The running of the coupling is a LLQ$^2$ but NLLx effect, so beyond LLx the discussion given so far is insufficient.

**Running coupling**

The generalization of duality to the running coupling case is nontrivial because the running of the coupling breaks the symmetry of the two scales $\xi$ and $t$ in the Altarelli–Parisi equation. Indeed, upon $M$–Mellin transform (2) the usual one–loop running coupling becomes the differential operator

$$\hat{\alpha}_s = \frac{\alpha_s}{1 - \beta_0 \alpha_s \frac{d}{dM}} + \cdots,$$  

(10)

where $d\alpha_s/dt = -\beta_0 \alpha_s^2$.

Consider for simplicity the LLx $x$–evolution equation, i.e. eq. (4) with $\chi = \alpha_s \chi_0(M)$, and include running coupling effects by replacing $\alpha_s$ with the differential operator eq. (10). We can solve the equation perturbatively by expanding the solution in powers of $\alpha_s$ at fixed $\alpha_s/N$: the leading–order solution is given by eq. (8), the next–to–leading order is obtained by substituting this back into the equation and retaining terms up to
order $\beta_0 \alpha_s$, and so on [18]. We can then determine the associate $G(N,t)$ by inverting the $M$–Mellin, and try to see whether this $G(N,t)$ could be obtained as the solution of a renormalization group (RG) equation (3).

The inverse Mellin is again given by the residue of the pole of $e^{iM}G(N,M)$ in the $M$–plane, where $G(N,M)$ is now the perturbative solution. When trying to identify this with a solution to eq. (3) there are two potential sources of trouble. The first is that now the perturbative solution at order $(\alpha_s \beta_0)^n$ has a $(2n+1)$–st order pole. Therefore, the scale–dependence of the inverse Mellin is now a function of both $\alpha_s$ and $t$, whereas the solution of a RG equation depends on $t$ only through the running of $\alpha_s$. Hence it is not obvious that a dual anomalous dimension will exist at all. The second is that even if a dual $\gamma$ does exist, it is not obvious that it will depend only on $\chi$ and not also on the boundary condition $F_0(M)$ eq. (8); in such case, the running of the coupling in the $\xi$–evolution equation would entail a breaking of factorization.

However, explicit calculation shows that it is possible to match the anomalous dimension and the boundary condition order by order in perturbation theory in such a way that both duality and factorization are respected. Namely, the solution to the leading–twist running coupling $x$–evolution eq. (4) with kernel $\tilde{\alpha}_s \chi_0$ and boundary condition $G_0(M)$ is the same as that of the renormalization group eq. (3) with boundary conditions and anomalous dimension given by

$$\gamma(\alpha_s(t),\alpha_s(t)/N) = \gamma_s(\alpha_s(t)/N) + \alpha_s(t) \beta_0 \Delta \gamma_\alpha(\alpha_s(t)/N) + \lambda(\alpha_s(t) \beta_0)^2 \Delta \gamma_{\alpha\alpha}(\alpha_s(t)/N) + O(\alpha_s(t) \beta_0)^3$$

(11)

$$G_0(\alpha_s,N) = G_0(N) + \alpha_s \beta_0 \Delta^1 G_0(\alpha_s,N) + (\alpha_s \beta_0)^2 \Delta^2 G_0(N) + O(\alpha_s \beta_0)^3.$$  

(12)

where the leading terms $\gamma_s$ and $G_0(N)$ are given by eqs. (5) and (9) respectively. The subleading corrections are

$$\Delta \gamma_{\alpha\alpha} = -\frac{\chi_0''\chi_0}{2\chi_0^2}$$

(13)

$$\Delta^1 G_0(N) = \frac{2\chi_0^2 F_0 - \chi_0 F_0^0 (\chi_0'^3 + \chi_0'' F_0'')} {2\chi_0^3},$$

(14)

where all derivatives are with respect to the arguments of $\chi_0(M)$ and $F_0(M)$, which are then evaluated as functions of $\gamma_s(\alpha_s/N)$.

The sub–subleading correction to the anomalous dimension is

$$\Delta \gamma_{\alpha\alpha\alpha} = -\frac{\chi_0^3}{15\chi_0''} - \frac{16\chi_0'''}{24\chi_0^3},$$

(15)

and we omit the very lengthy expression for $\Delta^2 G_0(N)$. The fact that duality and factorization hold up to NNLLx is nontrivial, and suggests that they should hold to all orders. An all–order proof can be in fact constructed [23].

Once the corrections to duality eq. (12) are determined, they can be formally re-interpreted as additional contributions to $\chi$: namely, one can impose that the duality eq. (5) be respected, in which case the kernel to be used in it is an “effective” $\chi$, obtained
from the kernel of the $x$–evolution eq. (4) by adding to it running coupling corrections order by order in perturbation theory: $\chi_0$ will be free of such correction, $\chi_1$ will receive a correction

$$\Delta \chi_1 = \beta_0 \frac{1}{2} \frac{\chi_0(M)\chi_0'(M)}{\chi_0^2(M)}$$  \hspace{1cm} (16)$$

and so forth. Applying duality to the known one–loop anomalous dimensions $\gamma_0$ thus gives us the resummation of the all–order singular contributions $\chi(\alpha_s/M)$ to this effective $\chi$, which include the running coupling correction eq. (12) and its higher–order generalizations.

**RESUMMATION**

Because the first two orders of the expansion of $\chi$ in powers of $\alpha_s$ at fixed $M$ and of the expansion of $\gamma$ in powers of $\alpha_s$ at fixed $N$ are known, it is possible to exploit duality of perturbative evolution to combine this information into anomalous dimension which accomplish the simultaneous resummation of leading and next–to–leading logs of $x$ and $Q^2$. In fact, it turns out that both a small $M$ and a small $N$ resummation of anomalous dimensions are necessary in order to obtain a stable perturbative expansion, while unresummed anomalous dimensions leads to instabilities. Both sources of instability are generic consequences of the structure of the perturbative expansion, and could have been predicted before the actual explicit computation [14] of subleading small-$x$ corrections.

**Small M**

The perturbative expansion of $\chi$ at fixed $M$ is very badly behaved in the vicinity of $M \sim 0$: at $M = 0$, $\chi_0$ has a simple pole, $\chi_1$ has a double pole and so on. In practice, this spoils the behaviour of $\chi$ in most of the physical region $0 < M < 1$. Because $1/M^k$ is the Mellin transform of $\Lambda^2 Q^2 \ln^{k-1}(Q^2/\Lambda^2)$, these singularities correspond to logs of $Q^2$ which are left unresummed in a LLx or NLLx approach [24]. The resummation of these contributions may be understood in terms of momentum conservation, which implies that $\gamma(1, \alpha_s) = 0$ (note our definition of the $N$–Mellin transform (1), and also that $\gamma$ is to be identified with the large eigenvector of the anomalous dimension matrix). The duality eq. (5) then implies that a momentum–conserving $\chi$ must satisfy $\chi(0, \alpha_s) = 1$. This, together with the requirement that $\chi$ admits a perturbative expansion in powers of $\alpha_s$, implies that in the vicinity of $M = 0$, the generic behaviour of the kernel is

$$\chi_s \sim \frac{\alpha_s}{\alpha_s + \kappa M} = \frac{\alpha_s}{\kappa M} - \frac{\alpha_s^2}{(\kappa M)^2} + \frac{\alpha_s^3}{(\kappa M)^3} + \cdots$$  \hspace{1cm} (17)$$

where $\kappa$ is a numerical constant which turns out to be $\kappa = \pi/C_A$. Hence we understand that there must be an alternating–sign series of poles at $M = 0$, which sums up to a regular behaviour. In fact, we can systematically resum singular contributions to $\chi$ to
all orders in $\alpha_s$ by including in $\chi$ the terms $\chi_s(\alpha_s/M)$ derived from the leading order $\gamma_0(N)$, and similarly at next-to-leading order, and so on. Because the usual anomalous dimension automatically respects momentum conservation order by order in $\alpha_s$, in order to remove the small $M$ instability of the expansion of $\chi$ at fixed $M$, it is sufficient to improve the expansion by promoting it to a “double leading” expansion which combines the expansions in powers of $\alpha_s$ at fixed $M$ and at fixed $\alpha_s/M$ [19]. For example, at leading order $\chi = \alpha_s \chi_0(M) + \chi_s(\alpha_s/M) - \text{d.c.}$, where the subtraction refers to the double-counting of the $\alpha_s/M$ term which is present both in $\alpha_s \chi_0$ and in $\chi_s(\alpha_s/M)$. This expansion of $\chi$ is dual eq. (5) to an analogous expansion of $\gamma$, where at leading order $\gamma = \alpha_s \gamma_0(M) + \gamma_s(\alpha_s/M) - \text{d.c.}$, and so forth. Both expansions are well behaved at small $M$, i.e. large $N$. At this level, it is already clear that the impact of the inclusion of small-$x$ corrections is moderate: indeed, it turns out that the double-leading kernel is quite close to the usual two-loop kernel, except at the smallest values of $N$, i.e. in the neighbourhood of the minimum of $\chi(M)$ [19].

Small N

The improved double-leading expansion of the anomalous dimension still requires resummation at small $N$. This is because, even though the next-to-leading correction to the double-leading evolution kernel is small for all fixed $M$, it is actually large if $N$ is fixed and small. This in turn follows from the fact that the leading $\chi$ kernel has a minimum, so the small $N = \chi$ region corresponds by duality eq. (5) to the vicinity of the minimum where the kernel is almost parallel to the $\gamma = M$ axis.

At small $N$, unlike at small $M$, there is no principle like momentum conservation which may provide a fixed point of the expansion and thus fix the all-order behaviour. The only way out is thus to treat this all-order behaviour as a free parameter. Namely, we introduce a parameter $\lambda$ which is equal to the value of the all-order kernel $\chi$ at its minimum, and then we expand about this all-order minimum. In practice, this means that we reorganize the expansion of $\chi$ according to [18]
\[ \chi(M, \alpha_s) = \alpha_s \chi_0(M) + \alpha_s^2 \chi_1(M) + \ldots = \alpha_s \tilde{\chi}_0(M) + \alpha_s^2 \tilde{\chi}_1(M) + \ldots, \] (18)

where
\[ \alpha_s \tilde{\chi}_0(M, \alpha_s) \equiv \alpha_s \chi_0(M) + \sum_{n=1}^\infty \alpha_s^{n+1} c_n, \quad \tilde{\chi}_i(M) \equiv \chi_i(M) - c_i, \] (19)

and the constants $c_i$ are chosen in such a way that
\[ \lambda \equiv \alpha_s \tilde{\chi}_0(\frac{1}{2}) = \alpha_s \chi_0(\frac{1}{2}) + \Delta \lambda. \] (20)

is the all-order minimum of $\chi$. Of course, in practice phenomenological predictions will only be sensitive to the value of $\lambda$ in the region where very small values of $N$ are probed, i.e. at very small $x$. 
Using duality and the resummation discussed above, one can construct resummed expressions for anomalous dimensions and coefficient functions, and wind up with resummed expressions for physical observables which may be directly compared to the data. The need to resum the small $N$ behaviour entails that phenomenological predictions will necessarily depend on the parameter $\lambda$ eq. (20). When the resummed double–leading expansion is constructed, a further ambiguity arises in the treatment of double–counting terms. This ambiguity is related to the nature of the small $N$ singularities of the anomalous dimension, which control the asymptotic small $x$ behaviour. Specifically, according to the way the double–counting is treated, the $N = 0$ poles of the one– and two–loop result may survive in the resummed result (‘S–resummation’) or not (‘R–resummation’). Both alternatives are compatible with the known low–order information on the evolution kernel, and can be taken as two extreme resummation schemes which parametrize our ignorance of higher order perturbative terms. Since the resummed terms also have a cut starting at $N = \lambda$, whether or not these low–$N$ poles are present only makes a difference if $\lambda$ turns out to be small, $\lambda \lesssim 0.3$.

The $\chi^2$ and starting gluon slope for a fit [22] to the recent H1 data [4] for the deep–inelastic cross section are shown in figure 2, as a function of $\lambda$ and for the two different resummation prescriptions. It is clear that if the perturbative $N = 0$ poles do not survive the resummation (R resummation) then only a fine–tuned value of $\lambda \approx 0.2$ is acceptable, whereas if they do survive (S resummation) essentially any $\lambda \lesssim 0$ gives a good fit.

Figure 2 demonstrates that it is possible to accommodate the success of simple fixed–order approach within a fully resummed scheme, and in fact the resummed calculation is in somewhat better agreement with the data than the fixed order one. Even though the effects of the resummation are necessarily small (otherwise the success of the fixed order prediction could not be explained) they do have a significant impact in the extraction of
the parton distribution: the gluon comes out to be significantly more valence–like than in an unresummed fit. Hence, the use of resummed perturbation theory is crucial for the extraction of reliable parton distributions at small $x$.

From a theoretical point of view, we see that current data already pose very stringent constraints on the unknown high–orders of the perturbative expansion: only a rather soft high–energy behaviour of the deep-inelastic cross–section is compatible with the data. Further progress in the understanding of the Regge limit is likely to require either genuinely nonperturbative input, or an extension of the standard perturbative domain [18].

**ACKNOWLEDGMENTS**

A sizable part of this paper is based on work done in collaboration with G. Altarelli. S.F. thanks G. Nardulli for organizing a very stimulating workshop, and P. Nason for interesting discussions during the workshop. This work was supported in part by EU TMR contract FMRX-CT98-0194 (DG 12 - MIHT).

**REFERENCES**

1. Forte, S., hep-ph/9910397 (1999).
2. Chekelian, V., hep-ph/0107053 (2001).
3. Chekanov, S., et al., ZEUS Coll., hep-ex/0105090 (2001).
4. Adloff, C., et al., H1 Coll., hep-ex/0012053 (2000).
5. Ball, R. D., and Forte, S., Phys. Lett., B335, 77–86 (1994).
6. Rujula, A. D., et al., Phys. Rev., D10, 1649 (1974).
7. Forte, S., and Ball, R. D., Acta Phys. Polon., B26, 2097–2134 (1995).
8. Mankiewicz, L., Saalfeld, A., and Weigl, T., Phys. Lett., B393, 175–180 (1997).
9. Lipatov, L. N., Sov. J. Nucl. Phys., 23, 338–345 (1976).
10. Fadin, V. S., Kuraev, E. A., and Lipatov, L. N., Phys. Lett., B60, 50–52 (1975).
11. Kuraev, E. A., Lipatov, L. N., and Fadin, V. S., Sov. Phys. JETP, 44, 443–450 (1976).
12. Ball, R. D., and Forte, S., Phys. Lett., B351, 313–324 (1995).
13. Ellis, R., Hautmann, F., and Webber, B., Phys. Lett., B348, 582–588 (1995).
14. Fadin, V. S., and Lipatov, L. N., Phys. Lett., B429, 127–134 (1998).
15. Altarelli, G., and Parisi, G., Nucl. Phys., B126, 298 (1977).
16. Gribov, V. N., and Lipatov, L. N., Yad. Fiz., 15, 781–807 (1972).
17. McLerran, L., hep-ph/0104285 (2001).
18. Ball, R. D., and Forte, S., Phys. Lett., B465, 271–281 (1999).
19. Altarelli, G., Ball, R. D., and Forte, S., Nucl. Phys., B575, 313–329 (2000).
20. Ciafaloni, M., Colferai, D., and Salam, G. P., Phys. Rev., D60, 114036 (1999).
21. Ball, R. D., and Forte, S., Phys. Lett., B405, 317–326 (1997).
22. Altarelli, G., Ball, R. D., and Forte, S., Nucl. Phys., B599, 383–423 (2001).
23. Altarelli, G., Ball, R. D., and Forte, S., hep-ph/0109178 (2001).
24. Salam, G. P., JHEP, 07, 019 (1998).