Comparative Review of Structural Optimization Approaches on Rectangular Concrete Beams

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Abstract. Increasing population demands considerable infrastructure, given that embodied energy accounts for most of the carbon emissions of concrete structures, efficient use of materials is of vital importance. This can be achieved through Topology Optimization. In this investigation, a brief review of cable and arch analogy in topology optimization of prismatic concrete design space is presented. Topology Optimization was conducted for a design space with two boundary condition configurations that correspond to the two analogies, Cable and Arch. Concrete damage properties were introduced in the model to find the optimum topology for the corresponding design space. Various topological configuration were obtained for both approaches by varying optimizing parameters. Considering the manufacturing practicality of the form, an optimum topology was chosen from both analogy approaches. A comparative study conducted between the two approaches exhibited the efficacy of cable analogy over arch analogy approach. Comparatively, significant volume reduction was observed in case of arch analogy approach, however manufacturing feasibility, comparatively superior structural response and partially eliminating load-specific behavior, testifies the overall efficacy of cable analogy approach.

1. Introduction
Robert Hook ascertained that a hanging cable, if inverted and stiffened results in an equivalent arch. Naturally hanging cable is a pure tensile structure and the equivalent arch is a pure compression structure. These two structures are in equilibrium and are mirror equivalents of each other. These two mechanisms are often used in the design of bridges. Arch mechanism is used in Arch bridges and cable mechanism is used in suspension bridges. Hence its very essential to investigate these two approaches. These two mechanisms used to structurally optimize beam structures. Concrete being a quasi-brittle material is prone to abrupt failures and local defects due to its non homogeneous nature. This gives rise to the need of assessment of concrete structures using damage models [1][2][3][4][5]. The main objective of this research is review the structural response and feasibility of these two approaches at various configuration. In this study concrete damage plasticity was introduced in the design space (regular prismatic beam) before the optimization process. Moreover a comparative analysis of the two approaches was conducted and recommendations after various considerations were presented.

2. Structural Optimization
A brief introduction on the research and development of structural optimization is presented in this section. Structural optimization is classified into three types namely size, shape and topology optimization [6]. In context with the theme of this investigation topology optimization [7][8][9] will be
discussed further in this section. A Canonical Topology optimization minimum compliance problem can be framed in form of equation (1) and (2)

\[
\text{Maximize } C = \frac{1}{2} f^T u \\
V^* = \sum_{i=1}^{n} V_i X_i = 0 x_i
\]

Holes are introduced in the design space with the randomized locations or guided locations [10][11]. Adaptation of the shape of the holes depends upon the material properties as well as the boundary conditions. Location of the holes is determined efficiently by Topological derivatives as well as strain energy density approach, which also optimizes the shape of the holes introduced. Random locations of holes can be chosen in case of topological level set approach [12][13][14]. In order to find the location of these holes a level set function \( p(x) \) is used, which is defined as follows:

\[
p = \begin{cases} 
0 & \forall x \in \Omega, \phi < 0 \\
1 & \forall x \in \Omega, \phi \geq 0
\end{cases}
\]

Many approaches can be followed to re-arrange the shape of these holes. For this investigation Simple Isotropic Material Penalization (SIMP) approach was followed. This was first introduced by Bendsoe [15] and later extensively studied by Zho and Rozvany [16]. Material Penalization also imparts intermediate densities to the materials and hence leads to errors. To deal with intermediate densities, explicit penalization approaches are used [17][18]. Material penalization equation for SIMP approach is as follows.

\[
S = Q^p
\]

Application of topology optimization in concrete structures [19][20] is mostly avoided mostly because of the quasi brittle nature of concrete which make it very difficult to predict the structural response and mainly due to the manufacturability constraints given that, concrete structures are constructed on a large scale. Though researchers have conducted topology optimization of a planer structure and extruded the same for 3D loading [21]. However conducting topology optimization of the 3D design space results in much complex topologies. Also inclusion of reinforcement further complicates the optimization process, however researchers have attempted to tackle this problem with various approaches [22][23][24][25]. In this investigation topology optimization is conducted in a 3D design space and further manual refinements are done as per the manufacturability. Boundary conditions are maneuvered to obtain the cable and arch analogy arrangement

2.1. Design Space

| Type   | Length | Breadth | Depth |
|--------|--------|---------|-------|
| Arch   | 500    | 100     | 50    |
| Cable 1| 1000   | 100     | 50    |
| Cable 2| 1000   | 100     | 50    |
| Cable 3| 1000   | 100     | 50    |
| Cable 4| 1000   | 100     | 50    |

In this investigation two different approaches were followed in optimization of design space, arch and cable analogy. In case of arch analogy the design space was kept constant with dimensional
parameters as shown in Table 1. However, optimizing parameters were varied to obtain various topologies as discussed in [26]. Whereas in case of cable analogy variation in depth of the design space was used as a governing factor for optimization process as presented in section 2.6. The classification of the cable analogy specimens are tabulated in Table 1 and shown in Figure 1 (a).

![Cable Analogy Configurations](image)

**Figure 1.** (a) Cable analogy Configurations, (b) Boundary conditions for two analogies.

2.2. **Boundary Conditions**

Boundary Condition positioning determines the type of analogy followed by the optimization process as shown in Figure 1 (b). As evident, both types have hinged supports with arbitrary UDL Loading. Load intensity has no effect on the optimization process. However material properties play a crucial role in the SIMP method.

2.3. **Material Properties**

For both, arch and cable approach, concrete was assigned as design space material. concrete is mesh was established using 8 noded brick elements. The behavior of the concrete was modeled using Concrete damaged plasticity for 30 MPa concrete. Various parameters for the damage are tabulated in Table 2.

| $\psi^c$ | $\epsilon$ | $K_c$ | $f_{bd}/f_{ct}$ | Viscosity Parameter |
|---|---|---|---|---|
| 13 | 0.1 | 0.7 | 1.16 | 0.001 |

2.4. **Methodology**

Tosca Structures software package was used for optimization of the two approaches. First step involved freezing of the load regions which avoids optimization near the boundary conditions. SIMP optimization algorithm was used with a penalty factor of 3. Two other parameters that were used for classification of the optimized topologies were reduction rate (R%) and Number of smoothening cycles (NSC). Sensitivity options were used to obtain much more refined shape. Optimization criteria were defined as shown in Table 3 for convergence. A typical Convergence plot is shown in Figure 5.
### Table 3. Optimization Criteria

| Criterion                          | Value |
|-----------------------------------|-------|
| Objective Function delta criterion | 0.1   |
| Element Density delta criterion   | 0.005 |
| First design cycle evaluation criterion | 4     |

![Figure 2](image1.png)

**Figure 2.** (a) Convergence Plot for Optimization, (b) Stress Mises contours for arch analogy.

2.5. **Arch Analogy**

![Figure 3](image2.png)

**Figure 3.** Various Topologies at different configurations.
Arch analogy optimization is achieved by following boundary conditions as shown in Figure 1 (a). Von Mises stress contours for the given boundary conditions for arch analogy is shown in Figure 2 (b). The topology of the beam is optimized in three stages by varying ISO (Density value), R% and NSC as introduced in [26]. Most optimized topology is selected for analysis after various consideration such as volume reduction, structural performance and manufacturing constraints. The range of the topologies achieved under various configurations is exhibited in Figure 3.

2.6. Cable Analogy
Cable analogy optimization is achieved through boundary condition exhibited in Figure 1 (b). Topologies procured through SIMP algorithm for cable analogy provided non-practical results as shown Figure 4 (a). Hence a different approach was followed for the obtaining various configurations. The depth of the design space was varied and the cavities present inside were muted for manufacturing constraints. Optimization was run for the optimum depth. Configurations are shown in Figure 1 (a).

3. Results and Discussion
The most optimum topology for arch analogy was found to be S33 as shown in Figure 3, but considering the manufacturing constraints and the material behavior S11 was chosen as the most feasible. Finite element analysis was conducted on the re-engineering model of S11 and the results revealed a local failure at midspan as exhibited by Figure 5(a). Volume reduction of S11 was evaluated to be about 50 %. An experimental demonstration was also conducted which showed similar results in Figure 5(b). Thus this approach was deemed inefficient as the response of the optimized topology was load specific and prone to premature failures.

![Figure 4. (a) Cable analogy Optimized form, (b) Failure Pattern for Cable analogy specimen.](image)

On the other hand, cable analogy approach with muted cavities presented comparatively better results. Considering the volumetric and structural evaluation, specimen 1:2 proved to be the most optimum and finite element analysis was also conducted. Concrete damage parameters were defined and XFEM package present in Abaqus CAE, was used to determine the failure pattern. As evident from Figure 4 (b). Beam behaves flexurally and the predictable structural response was observed. Moreover volume reduction was assessed to be about 20-30%.

3.1 Discussion
From the above results it is evident that cable analogy approach with muted cavities proves to be more efficient figure 5. Though volume reduction is comparatively less than that of arch analogy approach, the flexural response and ability to adapt to wide variety of loading deems it practical. Manufacturing cable analogy beams with muted cavities is also feasible as shown by mark west [27] using fabric form work [28]. The topology obtained through cable analogy also corresponds to the bending moment shape of the given loading condition [29]. An extensive research has been conducted by John Orr [30] on these type of beams. Another advantage of cable analogy approach is the loading surface remains flat which in case of arch analogy approach has to be defined by providing the thickness of the freeze optimization near loading.
Figure 5. (a) Numerical failure Mode of arch specimen, (b) Experimental failure of Arch analogy Specimen.

4. Conclusions
In this investigation a comparative study was conducted between the arch analogy and cable analogy approach in concrete beams. Efficient of the two was decided based on various parameters considered in this study. Conclusion of this investigation are bulleted as follows.

- Penalty factor of 3 in SIMP algorithm resulted in fairly speedy convergence in both arch and cable boundary condition arrangement.
- After various consideration 2:1 specimen without cavities, from cable analogy and S11 from arch analogy approach were determined as the most optimum.
- Cable analogy with muted cavities exhibits a better performing optimized form as it responds fairly well for a range of load types. Arch analogy approach on the other hand produces a load specific behavior.
- Volume reduction in most optimum arch analogy specimen (S11) was found to be up to 50% which is greater than 20-30% of 2:1 cable analogy specimen. However, cable analogy was proffered over arch analogy approach due to the impractical manufacturing form work where formwork has to be rebuild completely for different configuration.
- Arch analogy beam specimen exhibited a local failure and the response was unpredictable while as cable analogy beams reveals a more predictable flexural failure.

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