Method Article

Modeling the impact of social media on the adoption of a new product by customers

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A B S T R A C T

Marketers and entrepreneurs need to keep up with the fast-paced changes that are happening in the business environment, or they might face the risk of becoming obsolete in the rapidly changing business environment. It is long gone the days when a conventional business model used to help grow fast and get success. With the emergence of social media, the role of consumer-to-consumer communication about the new products and the firms that produce them has been highly magnified in the marketplace. Social media advertisements are promising tools that affect the adoption of a new product. In this paper, a non-linear mathematical model is introduced for this study. To perceive the impact of social media advertisements on the adoption of a new product, we have considered three dynamic variables; namely, (i) non-adopting population, (ii) adopting population, and (iii) social media advertisements. The stability theory of differential equations has been used to study the model analytically. The computer generated figures are drawn in support of derived analytical results for a particular set of parameter values.

- We have proposed and analyzed an nonlinear mathematical model to study the impact of social media advertisements on adoption of new product.
- We have considered three dynamical variables; namely, non-adopting population, adopting population, and social media advertisements.
- The stability theory of differential equations has been used to study the model analytically.

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**Specifications Table**

| Description                        | Value                                                                 |
|------------------------------------|----------------------------------------------------------------------|
| Subject:                           | Mathematics and Statistics                                           |
| More specific subject area:        | Mathematical Modeling                                                 |
| Name of your method:               | Mathematical Modeling for adoption of new product                    |
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| Resource availability:             | The data that support the findings of this study are available within the article. |

**Method details**

Adoption of a new product is one of the most important business goals for any firm/company. Several research studies in the field of marketing prove that a user-friendly, well-structured and implemented product launch strategy enhances the success of a product [10]. The New Product Adoption (NPA) by any consumer is, in turn, affected by his/her perception, which depends on many factors, like the uniqueness of product, marketing strategies, reviews on social media, etc. According to a study in 2021 nearly 9 out of 10 (approximately 89 %) consumers worldwide make the efforts to read the review before buying product. Also, the adoption of a product by customers depends on the extent of individuals, earliest decision of adopting the product in his/her social group [21,24]. Generally, positive comments tend to prompt consumers to generate emotional trust, increase their confidence and trust in the product, and have an extremely persuasive effect on their decision to purchase. Almost every country is facing the problem of selling its own products within its borders. Social media advertisements can play a vital role in this direction by promoting newly launched products.

Notably, Generation Y (Boomerang Kids) increasingly embracing the use of social media [1,15]. The world’s most popular social media networks Facebook and Youtube had 2.910 and 2.562 billion active users [25] in January 2022. Some other social networks with monthly active users are shown in Fig. 1.

![Fig. 1. Some of the popular social networking sites around the world, with their monthly active users (January 2022)](image-url)
Online social networks have emerged to be the most propitious driving factor in the world of digital advertising [16], and have gained popularity as a predominant factor in marketing with the increase in success of a brand, product of service [7,11]. Social media offers a variety of benefits to firms, including enhancing their brand popularity, developing word-of-mouth marketing, growing sales, sharing information, and strengthening social support for their customers. By leveraging social media platforms, marketers can connect and engage customers wherever they are, whether it is LinkedIn, Twitter, Youtube, Facebook, Instagram, or some of the more recent platforms. Small business leaders use social media as a marketing tool to make their business more visible, viable, and sustainable, which is essential for surviving in the modern competitive landscape. In a survey, it is observed that 89% of the buyers are the people influenced by family and friends on social networking sites [3].

Since last century, the role of social media platforms in the field of marketing has become a subject of extensive research [2,5,18,20,22,23]. Herzenstein et al. [12] have mentioned that marketers spend quite good amount of time and efforts on understanding the adoption of a new product. Misra et al. [18] have formulated a mathematical model to investigate the role of social media in the promotion of online shopping. From the analytical finding of the proposed model, they conferred that the reduction in number of promotions on social media platforms negatively impact number of online shoppers. Also, Online reviewing plays a significant role in the purchase intentions of customers. customers after purchasing a product voluntarily post their reviews to help other potential customers [17]. Erkan and Evans [8] in their study proposed a information acceptance model to examine the impact of eWOM (electronic word of mouth) promotion on social media on customers’ purchasing behavior and claim that eWOM can influence the customers’ purchase intention. They validated their study through a survey of 384 university students. Moreover, with the new era of social networks, most customers rely on social networking sites, customers review sights, online discussion forums and blogs to exchange the information on products which help one customer to know about other customer’s experience of the adopted product as well as share their own experiences [1,6,14]. Rodrigues and Fonseca [20] in their study perceived viral marketing as viral epidemic which is directly related to the dissemination of promotional content. They used the SIR model to analyze the strategies of viral marketing. Farooq and Jan [9] outlined in their detailed research that companies need to utilize tag-based marketing on Facebook, and the feedback of social media users (customers) is crucial.

From the above studies, we can say that entrepreneurs can increase their revenue substantially if they incorporate social media into their business functions, especially in the 21st century, where technology is prevalent. Through social media sites, firms can influence the customer’s choice, and customers can influence other customers. Therefore, the following question must be answered in order to make an informed decision: How do word-of-mouth publicity and promotions on social media affect customers' perception towards a newly launched product? To answer this question, we formulate a three-dimensional model that incorporates both word-of-mouth promotions and companies’ advertisements through social media for the adoption of a new product by customers.

Mathematical model

In this section, to see the impact of promotions through social media on the adoption of a new product, we formulate a three dimensional mathematical model. This model is a kind of generalization of the Bass diffusion model, where we have incorporated social media advertisements in a constant size of population [2]. Here, we assume that the total population is constant, i.e., \(N\) and splits into two different classes: non-adopting population \(N_0(t)\) and adopting population \(N_a(t)\). Also, at time \(t\), let \(M(t)\) denotes the number of advertisements through social media.

In the modeling process, since constant population size is assumed in the shopping system, we assume that the per capita exit rate is balanced by the per capita entering rate into the shopping system. Thus, \(1/\mu\) represents the time period during which population is active in purchasing a product. At some point of time due to dissatisfaction or death of individuals, they leave the non-adopting class at a rate \(\mu N_a\) and adopting population class at a rate \(\mu N_0\). Due to communication between non-adopting population and adopting population, non-adopting population get influenced and move to the adopting population class at a rate \(\beta N_a N_0/N\). The online posts, social media advertisements, reviews from the individuals of adopting class, etc., are some of the factors that
motivate the non-adopting individuals to move in adopting class at a rate \( \lambda \frac{M}{P+M} N_0 \). Furthermore, the impact of promotions through social media on non-adopting population is represented by the a saturated type function \( M/(P + M) \), as social media has limited impact on the population. There is also a possibility that some individuals feel uncomfortable due to cost, features or any other factors, with the adoption of new product and switch from adopting population class to non-adopting population class, which is incorporated by the term \( \lambda_0 N_0 \). We also consider that some individuals from the non-adopting class are self-motivated to use the launched product, therefore join the adopting class at a rate \( \sigma N_0 \).

The reviews and posts including pictures of the adopted product from adopting population play a crucial role which impacts the purchase and adoption intent of the non-adopting population. Thus, we have considered that advertisements through social media proportionally increase with the adopting population, i.e. \( r N_0 \). But it is important to note that if a good number of reviews and posts are already present on the social sites, then the adopting population may feel reluctant to add their reviews on the site. In order to incorporate this aspect in the modeling of social media advertisements, we have considered the net growth in the promotions through social media because of individuals of adopting class is \( r \left( 1 - \theta \frac{N_0}{W+N_0} \right) N_0 \), where \( \theta \in (0,1) \). Here, we consider that social media advertisement increases for small adopting population at a rate \( r N_0 \) and as the adopting population increases, this growth rate decreases by the factor \( f(N_0) = r \theta \frac{N_0}{W+N_0} \). With due course of time, it may be possible that the some social media advertisements start disappearing from the social sites because of their incapability to woo the population for the adoption of the new product [13].

Thus, the promotions through social media reduce at a rate \( r_0 M \). Also, \( \eta \) is the rate of promotions on social media platforms broadcast by companies to influence the individuals of non-adopting class.

Keeping in mind the above assumptions, following system depicts the model dynamics:

\[
\begin{align*}
\frac{dN_a}{dt} &= \mu N - \lambda_0 \frac{M}{P+M} N_a - \beta \frac{N_0 N_a}{N} - \sigma N_a + \lambda_0 N_a - \mu N_a, \\
\frac{dN_a}{dt} &= \sigma N_a + \lambda_0 \frac{M}{P+M} N_a + \beta \frac{N_0 N_a}{N} - \lambda_0 N_a - \mu N_a, \\
\frac{dM}{dt} &= \eta N_0 + r \left( 1 - \theta \frac{N_0}{W+N_0} \right) N_0 - r_0 M, \\
\end{align*}
\]

with \( N_0(0) > 0, N_a(0) \geq 0 \) and \( M(0) \geq 0 \). Throughout this study, we assume that all the considered parameters of model system (1) are always positive and delineated in Table 1.

Now, we define new variables; namely \( n_a, N_a, m, p \) and \( \omega \) as fractions of the considered total population \( (N) \), which is active in purchasing of a product to proportionalize our system. Let \( n_a = \frac{N_a}{N}, N_a = \frac{N_a}{N}, m = \frac{M}{N}, p = \frac{p}{N} \) and \( \omega = \frac{W}{N} \) represent the proportionate variables of the proposed system.

| Parameter | Description |
|-----------|-------------|
| \( \beta \) | Rate of transmission from non-adopting to adopting class |
| \( \lambda \) | Promulgation rate of advertisements through social media among individuals in non-adopting class |
| \( \lambda_0 \) | Transfer rate from adopting to non-adopting class |
| \( \frac{1}{\pi} \) | Time period during which population is active in purchasing a product. |
| \( P \) | Half-saturation point for the impact of advertisements through social media on non-adopting class as it attains half of its maximum possible value \( \lambda N_0 \), when cumulative number of advertisements through social media to influence the population arrives at \( P \). |
| \( \sigma \) | Transfer rate of self-motivated individuals from non-adoptive class to adoptive class |
| \( \eta \) | Rate at which companies advertise their product |
| \( \theta \) | Decay rate of advertisements on social media due to increase in population of adoption class |
| \( W \) | Half saturation point for \( f(N_0) \) as it attains half of its maximum possible value \( r \theta \), when due to social media advertisements, individuals from adopting class arrive at \( W \). |
| \( r_0 \) | The diminution rate of advertisements broadcasting from social media platforms due to inefficiency to motivate the population |
| \( r \) | Growth rate of advertisements through social media |

Table 1: Parameter description for model system (1).
Thus, system (1) reduces to the following system:

\[
\begin{cases}
    \frac{dn_a}{dt} = \mu - \lambda \frac{m}{p+m} n_a - \beta n_a n_a - \sigma n_a + \lambda_0 n_a - \mu n_a, \\
    \frac{dn_a}{dt} = \sigma n_a + \lambda \frac{m}{p+m} n_a + \beta n_a n_a - \lambda_0 n_a - \mu n_a, \\
    \frac{dm}{dt} = \eta n_a + r \left(1 - \frac{n_a}{\omega + n_a}\right) n_a - r_0 m. 
\end{cases}
\]  

(2)

Let \((N_n^*, N_n^*, M^*)\) be the interior equilibrium of the non-reduced system (1) and \((n_a^*, n_a^*, m^*)\) be the interior equilibrium of the above model, then \(n_a^* = \frac{N_n^*}{n_a}, n_a^* = \frac{N_n^*}{N_n^*}\) and \(m^* = \frac{M^*}{N_n^*}\). As the total population in the system is constant over time (i.e., \(N_n + N_a = N\)), thus by incorporating the above proportions gives, \(n_a + n_a = 1\) for the reduced system (2), therefore the system (2) further reduces to the following two dimensional system:

\[
\begin{cases}
    \frac{dn_a}{dt} = \sigma (1 - n_a) + \lambda \frac{m}{p+m} (1 - n_a) + \beta (1 - n_a) n_a - \lambda_0 n_a - \mu n_a, \\
    \frac{dm}{dt} = \eta (1 - n_a) + r \left(1 - \frac{n_a}{\omega + n_a}\right) n_a - r_0 m. 
\end{cases}
\]  

(3)

Thus, studying the model system (3) doesn’t make any difference to the results and is similar to studying the proposed model system (1).

**Equilibrium analysis**

There is only one possible equilibrium \(E^*(n_a^*, m^*)\) for the model system (3), that can be established by examining the equations given below:

\[
\sigma (1 - n_a) + \lambda \frac{m}{p+m} (1 - n_a) + \beta (1 - n_a) n_a - \lambda_0 n_a - \mu n_a = 0. 
\]

(4)

\[
\eta (1 - n_a) + r \left(1 - \frac{n_a}{\omega + n_a}\right) n_a - r_0 m = 0. 
\]

(5)

From Eq. (5), we have

\[ m = f(n_a). \]

where \(f(n_a) = \frac{1}{\eta} \left[ \eta (1 - n_a) + r \left(1 - \frac{n_a}{\omega + n_a}\right) n_a \right] > 0. \]

Then, we have \(f(0) = \frac{\eta}{\eta_0} > 0\), and \(\frac{df(n_a)}{dn_a} = -\frac{1}{\eta_0} \left[ (\eta - r) + r\theta \left(1 + \frac{\omega}{\omega+n_a}\right) \left( \frac{n_a}{\omega+n_a} \right) \right] < 0\), if \(\eta > r\).

Now, from Eq. (4), we have

\[ G(n_a) = \sigma (1 - n_a) + \lambda \frac{f(n_a)}{p + f(n_a)} (1 - n_a) + \beta (1 - n_a) n_a - \lambda_0 n_a - \mu n_a. \]

(6)

One can easily note that

(i) \(G(0) = \sigma + \frac{f(n_a)}{p + f(n_a)} > 0\), (ii) \(G(1) = -(\lambda_0 + \mu) < 0\), and

(iii) \(G'(n_a) = -\sigma + \frac{p f(n_a) (1 - n_a) - \lambda f(n_a)}{(p + f(n_a))^2} \sigma (1 - n_a) - \frac{\lambda f(n_a)}{(p + f(n_a))^2} (1 - n_a) - \beta n_a < 0\). All these facts together imply that the equation \(G(n_a) = 0\) has a unique positive solution, if \(\eta > r\). Thus, model system (3) has unique equilibrium \(E^*(n_a^*, m^*)\).

**Stability analysis**

**Theorem 1.** The interior equilibrium \(E^*\) is locally asymptotically stable, whenever exists.
Proof. To determine the local stability behavior of the obtained interior equilibrium $E^*(n_a^*, m^*)$, the variational matrix for the model system (3) is given below:

$$V = \left( \begin{array}{cc} -(\sigma + \frac{\lambda m^*}{p + m^*}) & \frac{1}{n_a^*} - \beta n_a^* - \lambda p (1 - n_a^*)/(p + m^*)^2 \\ - (\eta - r) & - \frac{r\theta n_a^*}{(\omega + n_a^*)^2} - r_0 \end{array} \right).$$

Here,

$$\text{Trace}(V) = -(\sigma + \frac{\lambda m^*}{p + m^*}) \frac{1}{n_a^*} - \beta n_a^* - r_0 < 0, \quad \text{and}$$

$$\text{Det}(V) = \frac{r_0}{n_a^*} \left( \sigma + \frac{\lambda m^*}{p + m^*} \right) + r_0 \beta n_a^* + \frac{\lambda p (1 - n_a^*)}{(p + m^*)^2} \left( \eta - r \right) + \frac{r\theta n_a^*}{(\omega + n_a^*)^2} + \frac{r_0 n_a^*}{(\omega + n_a^*)} > 0, \quad \text{if} \ \eta > r.$$

The negative trace and positive determinant of matrix $V$, assures that matrix $V$ always has eigenvalues, which are negative or contains the negative real parts. Therefore, the equilibrium $E^*(n_a^*, m^*)$ is locally asymptotically stable. This completes the proof. □

Theorem 2. Model system (3) does not exhibit any limit cycle in the interior of the positive quadrant of the $n_a - m$ plane.

Proof. Considering the function

$$F(n_a, m) = \frac{1}{n_a m}.$$

In the positive quadrant of $n_a - m$ plane, this function is positive.

Now define

$$f_1(n_a, m) = \sigma (1 - n_a) + \lambda \frac{m}{p + m} (1 - n_a) + \beta (1 - n_a) n_a - \lambda_0 n_a - \mu n_a,$$

$$f_2(n_a, m) = \eta (1 - n_a) + r (1 - \frac{n_a}{\omega + n_a}) n_a - r_0 m.$$

Thus, we have

$$\Delta(n_a, m) = \frac{\partial}{\partial n_a}(f_1 F) + \frac{\partial}{\partial m}(f_2 F)$$

$$= - \left[ \left( \frac{\eta}{n_a} (1 - n_a) + r \left( 1 - \frac{n_a}{\omega + n_a} \right) \right) \frac{1}{m^2} \right]$$

$$< 0.$$

Thus, $\Delta(n_a, m)$ is not identically zero in the interior of the positive quadrant of the $n_a - m$ plane. Accordingly, we conclude that there is no closed trajectory and, therefore, interior of positive quadrant of the $n_a - m$ plane does not contain any limit cycle. □

Numerical simulations

To examine the feasibility of our analytical findings regarding the existence and stability of obtained equilibrium, we have conducted some numerical calculations using the parameter values given in Eq. (7). Since the realistic data is not available, we have inquisitively selected the hypothetical data, so that the existence condition is satisfied at least for a set of parameter values.

$$\lambda = 0.02, \ \beta = 0.003, \ \lambda_0 = 0.01, \ \mu = 0.006, \ r = 0.001, \ \theta = 0.5, \ \omega = 0.5, \ r_0 = 0.2, \ \eta = 0.06, \ \sigma = 0.03, \ p = 0.3.$$ (7)
For these parameter values, the components of equilibrium are obtained as:

\[ n^*_a \approx 0.7449, \quad m^* \approx 0.7912, \]

and the corresponding eigenvalues are \(-0.02191\) and \(-0.60065\). These negative eigenvalues conclude that the equilibrium is locally asymptotically stable. To show the global stability behavior of obtained equilibrium \(E^*\), we have plotted the solution trajectories in \(n_a - m\) plane taking different initial starts as shown in Fig. 2. This figure shows that the solution trajectories approach to the equilibrium \(E^*\) and validates its global stability behavior.

To see the impact of parameters \(\beta\), \(\eta\) and \(\lambda\) on the fraction of people adopting a new product and social media advertisements, we generate bar plots for their different values (Figs. 3, 4 and 5). From Fig. 3, we observe that with the increase in value of \(\beta\) from 0.01 to 0.05, the fraction of individuals in
Fig. 4. Bar plot of equilibrium values of $n_a$ and $m$ for different values of $\eta$, with the set $\eta = \{0.01, 0.019, 0.028, 0.037, 0.049\}$.

Fig. 5. Bar plot of equilibrium values of $n_a$ and $m$ for different values of $\lambda$, with the set $\lambda = \{0.01, 0.07, 0.13, 0.19, 0.25\}$.

Adopting class increases from 0.7642 to 0.841 as well as the fraction of social media advertisements decreases. Furthermore, with increasing rate of social media promotion by companies, i.e., $\eta$ and dissemination rate of social media advertisement $\lambda$, positively impact the fraction of individuals in adopting class, Figure 4 and 5, respectively. Additionally, we have varied the value of $\lambda$, $\eta$ and $\beta$, $r$ and generated the surface plots for fraction of people adopting new products and social media advertisements (Figs. 6 and 7, respectively). From Fig. 6, we can see that the fraction of adopting class increases with an increase in $\lambda$ and $\eta$. In contrast, the fraction of social media advertisements decreases with the increasing value of $\lambda$ of social media advertisements. Also, $\beta$ and $r$ increase social media advertisements as well as the fraction of individuals in adopting class, Fig. 7.
Fig. 6. Surface plot of (a) $n_x$ and (b) $m$ for different values of $\eta$ and $\lambda$. 
Fig. 7. Surface plot of (a) $n_a$ and (b) $m$ for different values of $\beta$ and $r$. 
Fig. 8. Plot showing the sensitivity of parameters $\lambda$, $\beta$, and $\eta$. 

(a) 

(b)
Sensitivity analysis

In this section, the aim is to vary the model parameters like \( \lambda \), \( \beta \) and \( \eta \) and assess the associated changes in model outcomes \([19,22]\). Therefore, sensitivity analysis is being conducted for the model system (3). For this, suppose \( V_B \left( \frac{\partial y(t,B)}{\partial B} \right) \) is the sensitivity function of dynamical variable \( V \) with respect to parameter \( B \). The change in the value of the variable \( V \) resulted from doubling a parameter is determined by the semi-relative sensitivity solution (i.e., \( BV_B(t,B) \)). Hence, the following equations are derived for sensitivity analysis:

\[
\begin{align*}
\dot{(n_a)_\lambda} &= -\sigma(n_a)\lambda + \frac{m}{p} (1 - n_a) - \frac{\lambda m}{p + m} (n_a)_\lambda + \lambda (1 - n_a) - \frac{pm}{a} (n_a)_\lambda + \beta (1 - n_a)(n_a)\beta - \beta n_a(n_a)\beta - (\lambda + \mu)(n_a)\beta, \\
\dot{m}_\lambda &= -\eta(n_a)\lambda + \frac{r(1 - \frac{\theta n_a}{\omega + n_a}) (n_a)_\lambda - r n_a \omega (n_a)_\lambda}{(\omega + n_a)^2} - r_0 m\lambda, \\
\dot{(n_a)_\beta} &= -\sigma(n_a)\beta - \frac{\lambda m}{p + m} (n_a)_\beta + \lambda (1 - n_a) - \frac{pm}{a} (n_a)_\beta + \beta (1 - n_a)(n_a)\beta - \beta n_a(n_a)\beta + (1 - n_a)n_a - (\lambda + \mu)(n_a)\beta, \\
\dot{m}_\beta &= -\eta(n_a)\beta + r(1 - \frac{\theta n_a}{\omega + n_a}) (n_a)_\beta - r n_a \omega (n_a)_\beta - r_0 m\beta, \\
\dot{(n_a)_\eta} &= -\sigma(n_a)\eta - \frac{\lambda m}{p + m} (n_a)_\eta + \lambda (1 - n_a) - \frac{pm}{a} (n_a)_\eta + \beta (1 - n_a)(n_a)\eta - \beta n_a(n_a)\eta - (\lambda + \mu)(n_a)\eta, \\
\dot{m}_\eta &= (1 - n_a) - \eta(n_a)\eta + r(1 - \frac{\theta n_a}{\omega + n_a}) (n_a)_\eta - r n_a \omega (n_a)_\eta - r_0 m\eta.
\end{align*}
\]

In order to conduct the semi-relative sensitivity analysis, we have chosen \( \lambda = 0.006 \), \( \beta = 0.03 \) and \( \eta = 0.08 \). The semi-relative sensitivity is depicted in Fig. 8 for both the dynamical variables \( n_a \) and \( m \) of the model system (3) with respect to parameters \( \lambda \), \( \beta \) and \( \eta \), and the change in dynamical variables when the value of a particular parameter gets doubled is shown in the Fig. 8. From Fig. 8, it can be observed that as the value of \( \lambda \) gets double, the fraction of people adopting a new product decreased by 0.75995. Further, the effect of doubling the value of the parameter \( \beta \) increases the fraction of people adopting a new product by 0.0946, whereas the fraction of social media advertisements increases by 1.387. Lastly, the impact of doubling the parameter \( \eta \) can also be seen as the fraction of people adopting a new product increases by 0.004223.

Conclusion

With the advancement of technology and internet, social media has been found to play a vital role in influencing customers’ behavior in adopting a new product. The boom in social media has drastically changed the traditional business and marketing strategies. It is ascertained that the impact of promotions on social media platforms is most significant on the awareness of the new product. It is a good source of information to make customers aware of any new product. Also, social media provides a platform to interconnect with other customers of the same product and develop reviews and recommendations. Social media has become a source of trust in adoption of different products. Based on these aspects, we have modeled the situation in order to investigate how social media promotions influence the adoption of a new product by customers. In the considered system, we have speculated that the peoples enter to non-adopting population automatically and then shift themselves from non-adopting class to adopting class due to word of mouth communication or getting influenced from social media advertisements or reviews from adopting class. The equilibrium analysis of the model tells that the model exhibits only one non-negative equilibrium that is stable in nature. The divergence criterion is used to prove the global stability behavior of this unique equilibrium. Further, numerical simulation has been also carried out to confirm the analytically obtained results and the simulation shows that the fraction of people adopting a new product increases with the increase in the promotion of that product on social media.

Social media marketing is important in establishing trust, attachment and priority for new products. It is evident from the model analysis that promotion through media have a substantial role in increasing the product’s sales. Also, it is noticed that the reduction in the advertisements of any
new product on social media platforms leads to the reduction in the population of adopting class. It is important for any organization that they keep in mind the requirements and expectations of customers before launching a new product. They need to be consistent in making efforts to woo the audience towards their products through social media advertisements. All in all, it is important to have an effective marketing strategy in order to have a successful adoption of any new product.

As we know that various factors, like per capita incom, memory of peoples, etc. affect the purchasing of any product and thus this model can also be studied using other approaches, like stochastic differential equations [26], fractional order differential equations [4], etc.

**Ethics statements**

Not applicable

**CRediT author statement**

All the authors of this article contributed equally in all sections.

**Declaration of Competing Interest**

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

**Data availability**

No data was used for the research described in the article.

**CRediT authorship contribution statement**

Jyoti Maurya: Formal analysis, Methodology, Software, Writing – original draft, Writing – review & editing. Kanishka Goyal: Writing – review & editing. A.K. Misra: Formal analysis, Methodology, Software, Writing – original draft, Writing – review & editing, Supervision, Validation.

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