Analysis of negative magnetoresistance. Statistics of closed paths. II. Experiment

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It is shown that a new kind of information can be extracted from the Fourier transform of negative magnetoresistance in 2D semiconductor structures. The procedure proposed provides the information on the area distribution function of closed paths and on the area dependence of the average length of closed paths. Based on this line of attack the method of analysis of the negative magnetoresistance is suggested. The method has been used to process the experimental data on negative magnetoresistance in 2D structures with different relations between the momentum and phase relaxation times. It is demonstrated this fact leads to distinction in the area dependence of the average length of closed paths.

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I. INTRODUCTION

The phenomenon of anomalous magnetoresistance at low temperature in “dirty” metals and doped semiconductors was explained by the theory of quantum corrections to the conductivity. The interference correction to the conductivity gives the main contribution to the negative magnetoresistance in 2D structures at low temperature and low magnetic field. A unique analytical expression for the magnetic field dependence of negative magnetoresistance has been found in Ref. [1]

\[ \Delta \sigma(B) = \sigma(B) - \sigma(0) = a G_0 \left( \Psi \left( 0.5 + \frac{B \tau_\varphi}{B \tau_\varphi} \right) - \ln \left( \frac{B \tau_\varphi}{B \tau_\varphi} \right) \right), \]  

(1)

where \( G_0 = e^2/(2\pi^2\hbar), \) \( B \tau_\varphi = \hbar c/(2eI^2), \) \( I \) is the mean free path, \( \Psi(x) \) is a digamma function, \( \tau \) and \( \tau_\varphi \) stand for the elastic scattering and phase breaking time, respectively. The parameter \( a \) is equal to unity for the non-interacting case. This expression was obtained in the diffusion approximation for the isotropic scattering by randomly distributed scatterers with a short-range potential. Nevertheless it is universally used in the analysis of experimental data to extract the phase breaking time and its temperature dependence through the fitting of experimental curves. It should be pointed out that \( \tau_\varphi \) determined in this way is a fitting parameter rather than the phase breaking time because some deviation of experimental curves from Eq. (1) takes place in almost without exception. This deviation may result from some correlations in distribution of scatterers, scattering anisotropy, or long-range potential fluctuations in real 2D systems. This may be a possible reason of the saturation of \( \tau_\varphi \) with decreasing temperature as it is seen in some experiments.

In the previous paper we have developed a new approach to the analysis of anomalous magnetoresistance which makes it possible to obtain the information about the statistics of closed paths from the magnetic field dependence of magnetoresistance. This approach provides the basis for a new method of analysis of negative magnetoresistance due to weak localization suppression. In the present paper we demonstrate the potentials of this method as it is applied for interpretation of concrete experimental results obtained for GaAs/InGaAs/GaAs quantum wells.

II. BASIS OF METHOD

The essence of the method is clear from Eq. (8) of the previous paper. One can see that the Fourier transform of negative magnetoresistance is given by

\[ \Phi(S) = \frac{1}{\Phi_0} \int_{-\infty}^{\infty} dB \, \delta \sigma(B) \cos \left( \frac{2\pi BS}{\Phi_0} \right) = 2\pi I^2 G_0 W(S) \exp \left( -\frac{L(S,l_\varphi)}{l_\varphi} \right), \]  

(2)

\( \Phi_0 = 2\pi c h/\epsilon \) is the elementary flux quantum, \( l_\varphi = v_F \tau_\varphi, \) \( W(S) \) and \( L(S,l_\varphi) \) are the area distribution function of closed paths and the area dependence of the average length of closed paths respectively, introduced in Section II of Ref. [1].

Thus, it is clearly seen from Eq. (2) that \( \Phi(S) \) contains the information on the area distribution function of closed paths \( W(S) \), and on the function \( L(S,l_\varphi) \). If \( l_\varphi \) tends to infinity when \( T \to 0 \), the extrapolation of \( \Phi(S,T) \) to \( T = 0 \) gives the value of \( 2\pi I^2 G_0 W(S) \).

To determine the area dependence of \( L \) we assume that for actual areas \( S \) is a power function of area, \( L(S,l_\varphi) = S^\beta f(l_\varphi) \). The numerical calculations of the
function $T(S, l_\varphi)$ (see Fig. 4 in Ref. 5) have shown that this assumption is valid in a wide range of $S$, $l_\varphi$. In the diffusion approximation (i.e. for $\tau/\tau_\varphi \ll 1$) the value of $\beta$ is about 0.67. It is clear that in this approximation the value of $\beta$ is independent of scattering anisotropy. Beyond the diffusion approximation the value of $\beta$ is lower and depends on $\tau/\tau_\varphi$ ratio.

To extract the value of $\beta$ from experimental data one can measure $\Delta\sigma(B)$ at two temperatures, i.e. at different $l_\varphi$, then find the function

$$ A(S) \equiv \ln \left[ \Phi(S, T_1) / \Phi(S, T_2) \right] = S^3 \left( f(l_{T1}) - f(l_{T2}) \right) $$

and finally determine $\beta$ from $A(S)$ curve.

![Diagram](image-url)

**FIG. 1.** (a) Magnetic field dependencies of $\Delta\sigma(B)/G_0$ for different temperatures for structures I (a) and II (b). Solid lines are the results of fitting by expression (3). Dashed lines are the differences between experimental and theoretical curves multiplied by 10 (a) and 20 (b). Insert in (b) shows the low field magnetoresistance at $T = 1.6$ K for structure II.

### III. EXPERIMENT

We have measured the conductivity in heterostructures n-GaAs/In$_{0.07}$Ga$_{0.93}$As/n-GaAs of two types. The heterostructures with 200 Å In$_{0.07}$Ga$_{0.93}$As quantum well, $\delta$-doped by Si in the centre, relate to the first type. The heterostructures with 50 Å In$_{0.07}$Ga$_{0.93}$As well and doped barriers relate to the second type. The $\delta$-doped by Si layers are arranged in them on both sides of the well at the distance 100 Å. In this paper we present the experimental results for two structures of different types which are refereed as structure I and structure II, respectively. The measurements carried out in wide ranges of magnetic fields (up to 6 T) and temperatures (0.4-40 K) show that in structures investigated only one size-quantized sub-band is occupied. The main contribution to the conductivity comes from the electrons in the quantum well of In$_{0.07}$Ga$_{0.93}$As. The electron density and mobility for structure I are $n = 1.2 \times 10^{12}$ cm$^{-2}$ and $\mu = 1.4 \times 10^{3}$ cm$^2$/V sec, respectively. For structure II they are the following $n = 2.5 \times 10^{11}$ cm$^{-2}$ and $\mu = 1.1 \times 10^{4}$ cm$^2$/V sec.

The magnetic field dependencies of the conductivity for both structures for low magnetic fields and different temperatures are shown in Fig. 4. The negative magnetoresistance is observed in the whole range of magnetic fields (up to 6 T). The main contribution to the negative magnetoresistance in the range $B < 0.5$ T for structure I and $B < 0.2$ T for structure II comes from the weak localization effect, while in the range $B > 1$ T for structure I and $B > 0.4$ T for structure II it results from the correction to the conductivity due to electron-electron interaction. Notice that the positive magnetoresistance due to weak antilocalization effect was observed in analogous structures for low magnetic fields in Refs. 6,7. This effect is observed when the spin relaxation time $\tau_s$ is less than the phase relaxation time. The positive magnetoresistance is absent in both our structures (for example, see the inset in Fig. 1b, where magnetoresistance of structure II for very low magnetic fields is shown). It should be mentioned that the conductivity of the structures studied in Ref. 6 was order of magnitude larger than that in our case therefore the phase relaxation time was longer, too. The phase relaxation time in our case lies in the range $(0.2 - 1.5) \times 10^{-11}$ sec (see below). Comparing this value with $\tau_s = (3 - 4) \times 10^{-11}$ sec determined in Ref. 6 we have $\tau_s > \tau_\varphi$ for our structures. Another reason for the absence of positive magnetoresistance in our case is the fact that, in contrast to structures investigated in Ref. 6, our structures are symmetric in the growth direction.

Usually the expression (4) is used to analyze the negative magnetoresistance, taking $a$ and $\tau_\varphi$ as fitting parameters. The solid curves in Fig. 4 have been obtained in this way and, at first glance, they are in good agreement with the experimental data in the range of $B < B_{yr}$. For structure I this procedure gives $a = 1$, $\tau_\varphi = 1.25 \times 10^{-11}$ sec for $T = 1.5$ K. However, the more detailed analysis reveals the difference between the theory and the experimental data (dashed curve in Fig. 4). As a consequence, the parameters $a$ and $\tau_\varphi$ vary in the range of $0.81 - 1.15$ and $(1.05 - 1.6) \times 10^{-11}$ sec, respectively, when the fitting procedure is undertaken in different intervals of $B$ within the range $0 < B < 0.5B_{yr}$. Thus, the accuracy of determination of $a$ and $\tau_\varphi$ values is $20 - 25\%$. The ratio $\tau/\tau_\varphi$ for this structure is 0.004–0.009 for the temperature range 1.5 – 4.2 K.

Analogous data treatment for structure II gives $a = 0.6 - 0.7$, $\tau_\varphi = 0.47 \times 10^{-11}$ sec for $T = 1.5$ K and the ratio $\tau/\tau_\varphi = 0.03 - 0.2$ for the temperature range 0.43 – 4.2 K. In the strict sense the expression (4) is not valid for this structure because the scattering potential is smooth and the scattering is anisotropic. Nevertheless it provides a good agreement with the experimental data (Fig. 1b). It is commonly believed that lower than unity value of $a$ results from the electron-electron interaction (Maki-Tompson term). However, below it is shown that
such value of $a$ in structure II is the result of failure of the diffusion approximation due to poor $\tau/\tau_\varphi$ ratio.

The temperature dependencies of $\tau_\varphi$ are plotted in Fig. 2 for both structures, and as is seen $\tau_\varphi \propto T^p$ with $p \simeq -1$. This means that the inelasticity of electron-electron interaction is the main mechanism of the phase relaxation.

Now we demonstrate new possibilities of analysis of experimental data provided by the method described above. It is obvious that the method is applicable for low magnetic fields, $B < B_c$, where the interference contribution to the negative magnetoresistance is dominant. As is seen from Eq. (2) the information about the statistics of closed paths can be obtained from the Fourier transform of $\delta \sigma(B) = \sigma(B) - \sigma(\infty)$. But the value $\Delta \sigma(B) = \sigma(B) - \sigma(0)$, not $\delta \sigma(B)$, is experimentally measured. It is easily shown that the Fourier transform of the experimental curve $\delta \sigma'(B) = \sigma(0) - \sigma(B_c) + \Delta \sigma(B)$ padded with zeros at $B > B_c$ is close to that of $\delta \sigma(B)$ at $S > \Phi_0/B_c$.

In Fig. 3 the Fourier transforms of $\delta \sigma'(B)$ for different temperatures are presented. The area range where the Fourier transform $\Phi(S)$ is shown is bounded. The maximum value of $S$ is determined by signal-to-noise ratio in the experimental curves $\Delta \sigma(B)$. The minimum value is determined by the range of magnetic field, $B < B_c$, where the weak localization effect is dominant. We assume that $B_c$ is about 0.5 T for structure I and 0.2 T for structures II.

As is seen from Eq. (3), the function $\log(A(S))$ must be linear with respect to $\log(S)$ with the slope $\beta$. It is seen from Fig. 4 that the corresponding curves are really close to straight lines for both structures, but the slopes are different: $\beta = 0.70 \pm 0.05$ for structure I and $\beta = 0.52 \pm 0.05$ for structure II. Thus, for structure I with $\tau/\tau_\varphi < 0.01$ the value of $\beta$ is close to that obtained in the improved diffusion approximation $\beta = 0.67$, but somewhat larger than the value $\beta = 0.62$ obtained in the numerical simulation.

In structure II the ratio $\tau/\tau_\varphi$ is significantly larger and the diffusion approximation fails. Besides, the fact that the impurities are arranged in the barriers in this struc-
ture leads to smooth scattering potential and anisotropic scattering. To our knowledge, there are no theoretical results for this case. However, it is valid to say that the anisotropy of scattering and smooth scattering potential do not change the statistics of closed trajectories with the lengths significantly larger than the mean free length. The numerical calculations beyond the diffusion approximation for isotropic scattering show that \( \beta = 0.55 \) at \( \gamma = 0.1 \). It is close to \( \beta \) value for structure I with \( \gamma = 0.2 - 0.03 \). Thus we believe that the main reason for small value of \( \beta \) in this structure is failure of diffusion approximation.

![Figure 5](image)

**FIG. 5.** (a) The temperature dependencies of \( \Phi \) at different \( S \) for structure II. The solid curves show the extrapolation of these dependencies to \( T = 0 \). (b) The area distribution functions for structure I (solid circles) and for structure II (open circles). The solid and dotted curves are the area dependencies of \( 2\pi l^2 W(S) \) obtained in the improved diffusion approximation for structure I and II, respectively.

The method put forward in this paper gives a possibility to determine the area distribution function \( W(S) \). The temperature dependencies of \( \Phi(S,T) \) for several \( S \) are plotted in Fig. 5a. The value of \( \Phi \) for a given \( S \) increases when \( T \to 0 \) due to increase of \( l_\phi \). Thus the extrapolation of curves \( \Phi(S,T)/G_0 \) to \( T = 0 \) (see Eq. (2)) gives the value of \( 2\pi l^2 W \) for the corresponding value of \( S \). The results of such a data treatment for both structures are shown in Fig. 5b.

In Fig. 5b the area distribution functions obtained within the improved diffusion approximation with parameters corresponding to the structures investigated are presented too. One can see that at low \( S \) the experimental area dependencies of \( 2\pi l^2 W \) are close to the theoretical ones for both structures. For \( S > (4 - 5) \times 10^{-10} \text{ cm}^2 \) the more rapid decreasing of the experimental curves is observed and for \( S \simeq 10^{-9} \text{ cm}^2 \) the experimental values of \( 2\pi l^2 W(S) \) is \( 3 - 5 \) times lower than the theoretical values.

There are two reasons for such a discordance: (i) the number of closed trajectories with large areas in real samples is smaller than the theoretical one due to, for instance, the long-range potential fluctuations; (ii) the saturation of the phase breaking length with decreasing temperature from \( T \simeq 1 \text{ K} \) has to lead to underestimation of the value of \( 2\pi l^2 W(S) \) for large \( S \) in the data processing described above. It should be noted that some evidence for \( l_\phi \) saturation was obtained only for temperatures \( T < 0.15 \text{ K} \). Our measurements were carried out at significantly higher temperature, \( T > 0.4 \text{ K} \). So, we believe that the rapid decreasing of \( 2\pi l^2 W(S) \) for \( S > (4 - 5) \times 10^{-10} \text{ cm}^2 \) (Fig. 5b) results from the shortage of large trajectories, rather than from the saturation of \( l_\phi \).

Thus, one can see that the area distribution functions of closed paths coincide practically for both structures, but the area dependencies of the average length of closed trajectories are distinguished. This distinction results from different \( \tau/\tau_\phi \) ratio. Just this fact leads to the lower than unity value of prefactor \( a \) in structure II rather than the electron-electron interaction.

**IV. CONCLUSION**

The new method of the analysis of negative magnetoresistance is used. This method provides a possibility to obtain an information on the statistics of closed paths. The experimental studies of negative magnetoresistance show that the area dependence of average length of closed paths depends on \( \tau/\tau_\phi \) ratio: \( \bar{L}(S) \propto S^{0.7} \) at \( \tau/\tau_\phi < 10^{-2} \); \( \bar{L}(S) \propto S^{0.5} \) at \( \tau/\tau_\phi \simeq 10^{-1} \). This fact leads to the lower than unity value of prefactor when one fits the experimental results to Hikami expression rather than contribution of electron-electron interaction (Maki-Tompson term). The experimental area distribution functions of closed paths are close to those obtained in improved diffusion approximation at low area, but distinct at large one. The shortage of large trajectories by long range potential fluctuation is a possible reason for such distinction.

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