We investigate the $Q^2$ evolution of parton distributions at small $x$ values, obtained in the case of soft initial conditions. The contributions of twist-two and (renormalon-type) higher-twist operators of the Wilson operator product expansion are taken into account. The results are in very good agreement with deep inelastic scattering experimental data from HERA.

The measurements of the deep-inelastic scattering structure function (SF) $F_2$ in HERA have permitted the access to a very interesting kinematical range for testing the theoretical ideas on the behavior of quarks and gluons carrying a very low fraction of momentum of the proton, the so-called small $x$ region. The reasonable agreement between HERA data and the next-to-leading order (NLO) approximation of perturbative QCD that has been observed for $Q^2 \geq 2\text{GeV}^2$ (see and references therein) indicates that perturbative QCD could describe the SF evolution up to very low $Q^2$ values.

The standard program to study the behavior of quarks and gluons is carried out by comparison of data with the numerical solution of the DGLAP equations by fitting the parameters of the $x$ profile of partons at some initial $Q^2_0$. However, if one is interested in analyzing exclusively the small $x$ region ($x \leq 0.01$), there is the alternative of doing a simpler analysis by using some of the existing analytical solutions of DGLAP in the small $x$ limit (see, for example, ☢). The main ingredients of the study are:

- Both, the gluon and quark singlet densities are presented in terms of two components ($'+'$ and '$-'$) which are obtained from the analytical $Q^2$ dependent expressions of the corresponding ($'+'$ and '$-'$) parton distributions moments.
- The '$-'$ component is constant at small $x$, whereas the '$+'$ component grows at $Q^2 \geq Q^2_0$ as

$$\sim \exp \left( 2\sqrt{a_+ \ln \left( \frac{a_s(Q^2_0)}{a_s(Q^2)} \right) - \left( b_+ + a_+ \frac{\beta_1}{\beta_0} \right) \left( a_s(Q^2_0) - a_s(Q^2) \right) \ln \left( \frac{1}{x} \right) } \right),$$
where the LO term \( a_+ = 12/\beta_0 \) and the NLO one \( b_+ = 412 f/(27 \beta_0) \).
Here the coupling constant \( a_s = \alpha_s/(4\pi) \), \( \beta_0 \) and \( \beta_1 \) are the first two coefficients of QCD \( \beta \)-function and \( f \) is the number of active flavors.

We shortly compile below the main results of\(^\text{\textsuperscript{3}}\) at the leading order (LO) approximation and demonstrate some new (preliminary) results, where the contributions of higher-twist (HT) operators (i.e. twist-four ones and twist-six ones) of the Wilson operator product expansion are taken into account in the framework of renormalon model (see\(^\text{\textsuperscript{3}}\)). The importance of the HT contributions at small-\( x \) has been demonstrated in\(^\text{\textsuperscript{3}}\).

1. Our purpose is to show the small \( x \) asymptotic form of parton distributions in the framework of the DGLAP equation starting at some \( Q_0^2 \) with the flat function:

\[
f_a^{\tau,\pm}(Q_0^2) = A_a \quad (\text{hereafter } a = q, g),
\]

where \( f_a^{\tau,\pm} \) are the leading-twist (LT) parts of parton (quark and gluon) distributions (PD) multiplied by \( x \) and \( A_a \) are unknown parameters that have to be determined from data. Through this work at small \( x \) we neglect the non-singlet quark component\(^\text{\textsuperscript{3}}\).

The full small \( x \) asymptotic results for PD and SF \( F_2 \) at LO is:

\[
F_2(x, Q^2) = e \cdot f_q(x, Q^2), \quad f_a(x, Q^2) = f_a^+(x, Q^2) + f_a^-(x, Q^2),
\]

where the ‘+’ and ‘−’ components \( f_a^{\pm}(x, Q^2) \) are given by the sum

\[
f_a^{\pm}(x, Q^2) = f_a^{\tau,\pm}(x, Q^2) + f_a^{h\tau,\pm}(x, Q^2)
\]

of the LT parts \( f_a^{\tau,\pm}(x, Q^2) \) and the HT parts \( f_a^{h\tau,\pm}(x, Q^2) \), respectively.

The small \( x \) asymptotic results for PD, \( f_a^{\tau,\pm} \)

\[
f_g^{\tau,\pm}(x, Q^2) = \left( A_g + \frac{4}{9} A_q \right) \hat{I}_0(\sigma) e^{-h(1)s} + O(\rho),
\]

\[
f_q^{\tau,\pm}(x, Q^2) = \frac{f}{9} \left( A_g + \frac{4}{9} A_q \right) \rho \hat{I}_1(\sigma) e^{-\tau(1)s} + O(\rho),
\]

\[
f_q^{\tau,\pm}(x, Q^2) = A_q e^{-a(1)s} + O(x), \quad f_g^{\tau,\pm}(x, Q^2) = \frac{4}{9} f_q^{\tau,\pm}(x, Q^2),
\]

where \( \tau(1) = 1 + 20 f/(27 \beta_0) \) and \( a(1) = 16 f/(27 \beta_0) \) are the regular parts of \( d_+ \) and \( d_− \) anomalous dimensions, respectively, in the limit \( n \rightarrow 1 \). The

\( ^a \) We would like to note that new HERA data\(^\text{\textsuperscript{4}}\) show a rise of \( F_2 \) structure function at low \( Q^2 \) values \( (Q^2 \sim \text{1GeV}^2) \) when \( x \rightarrow 0 \) (see Fig.1, for example). The rise can be explained in a natural way by incorporation of HT terms in our analysis (see Eqs.\(^\text{\textsuperscript{3}}\))

\( ^b \) From now on, for a quantity \( k(n) \) we use the notation \( k(n) \) for the singular part when \( n \rightarrow 1 \) and \( \overline{k}(n) \) for the corresponding regular part.
Figure 1: The structure function $F_2$ as a function of $x$ for different $Q^2$ bins. The experimental points are from H1. The inner error bars are statistic while the outer bars represent statistic and systematic errors added in quadrature. The curves are obtained from fits at LO when the HT contributions have been incorporated.

The function $\tilde{I}_\nu (\nu = 0, 1)$ coincides with the modified Bessel function $I_\nu$ at $s \geq 0$ and the Bessel function $J_\nu$ at $s < 0$. Using the calculations $\ref{8}, \ref{9}$, we show the HT effect in the renormalon case. We present the results only for the terms proportional of some power of $\ln (1/x)$ (full expressions can be found in the last paper of $\ref{2}$), making the following substitutions in the corresponding LT results presented in Eqs.(4)-(6):

$$F_{\tau^2, (+)}(x, Q^2) \rightarrow F_{h\tau^2, (+)}(x, Q^2)$$

where $\Lambda_{2, a}$ and $\Lambda_{4, a}$ are magnitudes of twist-four and twist-six corrections.

$$f_{q^2, -}(x, Q^2) \rightarrow f_{h\tau^2, -}(x, Q^2)$$

2. With the help of the above equations we have analyzed $F_2$ HERA data at small $x$ from the H1 collaboration. We have fixed the number of active flavors $f=4$ and $\Lambda_{\overline{MS}}(n_f = 4) = 250$ MeV, which is a reasonable value extracted...
from the traditional (higher $x$) experiments. Moreover, we put $\Lambda_{1,a} = \Lambda_{2,a}$ in agreement with [10].

The results are shown on Fig. 1. We found very good agreement between our approach based on QCD and HERA data. The (renormalon-type) HT terms lead to the natural explanation of the rise of $F_2$ structure function at low values of $Q^2$ and $x$.

As a next step of our investigations, we plan to finish this study and to investigate HT contributions to PD and SF relations, observed in [11, 12].

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