Superdeformation and alpha - cluster structure in $^{35}$Cl

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A superdeformed (SD) band has been identified in a non - alpha - conjugate nucleus $^{35}$Cl. It crosses the negative parity ground band above 11/2$^-$ and becomes the yrast at 15/2$^-$. Lifetimes of all relevant states have been measured to follow the evolution of collectivity. Enhanced B(E2), B(E1) values as well as energetics provide evidences for superdeformation and existence of parity doublet cluster structure in an odd-A nucleus for the first time in A $\simeq$ 40 region. Large scale shell model calculations assign (sd)$^{16}$(pf)$^3$ as the origin of these states. Calculated spectroscopic factors correlate the SD states in $^{35}$Cl to those in $^{36}$Ar.

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The superdeformed bands observed in the even-even nuclei in upper $sd$ shell have provided favourable condition to describe the collective rotation microscopically involving the cross-shell correlations [1,2]. Complementary descriptions in terms of particle-hole excitations in the shell model [2,3], and $\alpha$-clustering configurations within various cluster models [4,5] have been utilized to interpret the data. Till now no such band has been observed in non - alpha - conjugate odd-A, N$\neq$Z isotopes in this region [7]. If a nucleus clusterizes into fragments of different charge to mass ratios, the center of mass does not coincide with its center of charge. As a result a sizeable static E1 moment may arise in the intrinsic frame [8], resulting in several distinctive features in the spectra. Two adjacent opposite parity deformed $\Delta$I = 2 bands connected by strong E2 intra-band transitions in turn are connected by strong E1 inter-band transitions [8] forming an apparent $\Delta$I = 1 rotational band with alternating parity states. Since early seventies [9,10], a number of similar alpha-cluster bands have been studied extensively. In the spectrum of $^{19}F$, cluster-model calculations have shown that coupling of a proton hole in the $p$ shell coupled with four nucleons in the $sd$ shell (a proton hole coupled to $^{20}Ne$) gives rise to alpha-cluster bands. The lowest alpha + $^{15}N$ parity partner bands built on $K^\pi = 1/2^+$ ground-state band, lowest lying famous $K^\pi = 1/2^-$ at 110 keV and some other bands lying above 5 MeV have been observed. So far no similar clustering have been observed in odd A nuclei in the A $\simeq$ 40 region, where evidences of clustering have been manifested in even-even nuclei through superdeformation.

According to Ikeda [12], in the spectra of light nuclei, cluster like configurations would appear near the threshold energy needed for breakup into proper sub-nuclei. For the nucleus of our interest $^{35}$Cl, the threshold energy [13] to appear as a composite of $^{32}S$ and a triton (t) ($^{32}S + t$) is around 18 MeV. On the other hand, threshold for the decay of the composite system $^{35}$Cl into ($^{31}P + \alpha$) clusters is around 6.5 MeV. The SD rotational band observed in $^{36}$Ar has been shown to have cluster structure. So one may expect to find deformed cluster bands in the excitation spectra of $^{35}$Cl also generated by coupling a proton hole to SD states in $^{36}$Ar.

In this letter, we report the observation of a superdeformed band for the first time in the odd A $^{35}$Cl isotope. The reduced transition probabilities for all relevant transitions depopulating the states of the yrast negative parity band and related positive parity ones have been determined from lifetimes measured in the present experiment. Extracted B(E2)s and B(E1)s provide important information to probe the remnant of clustering in $^{35}$Cl. Large basis shell (LBSM) calculations have been done to understand the evolution of collectivity along this band. The predicted negative parity partner band of

![Partial level scheme of $^{35}$Cl](image-url)
the SD band in $^{36}\text{Ar}$ has also been reproduced within LBSM calculations for the first time. Spectroscopic factors have been used to correlate the SD states in $^{35}\text{Cl}$ to the cluster states in $^{36}\text{Ar}$ to establish the persistence of alpha-clustering features.

High spin states of $^{35}\text{Cl}$ have been populated through $^{12}\text{C(28Si,}\alpha p)^{35}\text{Cl}$ reaction in the inverse kinematics with an 110 MeV $^{28}\text{Si}$ beam at Inter University Accelerator Center (IUAC), New Delhi. The target was $^{12}\text{C}$ (50 $\mu$g/cm$^2$) evaporated on 18 mg/cm$^2$ Au backing. The $\gamma-\gamma$ coincidence measurement has been done using the multi detector array of thirteen Compton suppressed clover detectors (INGA setup). The relevant details of the experimental setup have been discussed in [14]. The detectors were placed at 148° (4), 123° (2), 90° (4), 57° (2) and 32° (1). The experimental data have been sorted into angle dependent symmetric (90° vs 90°) and asymmetric $\gamma-\gamma$ matrices to get the information about the gamma intensities, DCRO ratios and level lifetimes of this band.

In our earlier work [12], we have discussed about a few totally shifted gamma rays in the spectra emitted from states having lifetimes shorter than the characteristic stopping time ($\approx 10^{-13}$ s) of $^{35}\text{Cl}$ recoils in gold (Au) backing. Analysis of the present data confirmed [16] that four of them (2912 keV, 1862 keV, 2391 keV and 3734 keV) belong to the same sequence (marked as SD in Fig. 1), which is connected to the other states in $^{35}\text{Cl}$ through 1113, 1336, 2232 and 1693 keV transitions (Fig. 1). The lowest state of this sequence 5408 keV (11/2$^-$) is already known to be connected to the 3163 keV (7/2$^-$) through a strong 2244 keV transition [11]. This sequence from 3163 keV (7/2$^-$) state to 16306 keV (27/2$^-$) has been established as the yrast negative parity band. The three topmost transitions in the band show nearly perfect rotational behaviour by the almost linear increase in angular momentum with gamma - ray energy (rotational frequency) (Fig. 2). The plot shows a sharp break between 15/2$^-$ to 19/2$^-$ indicating a crossing between two weakly interacting bands of different configurations. The kinematic moment of inertia (KMOI) for the top three transitions in yrast band in $^{35}\text{Cl}$ scaled by the mass factor compares very well with those for SD bands in $^{36}\text{Ar}$ and $^{40}\text{Ca}$. The average KMOI, $\approx 8 \hbar^2$/MeV also compares well with the newly found candidate SD band in $^{28}\text{Si}$ ($\approx 6\hbar^2$/MeV) [6]. More dramatically, the inset of Fig. 2 shows that energies of alternating parity states, 15/2$^-$, 17/2$^+$, 19/2$^-$, 21/2$^+$, 23/2$^-$ and 27/2$^-$ follow a linear relation to the I(I+1) values exhibiting an apparent rotational band structure, which characterises nuclear molecular structure [3].

Lineshapes of all gamma rays in the yrast sequence (except the 2244 keV gamma), along with the 2614 keV gamma ray connecting the 17/2$^+$ state to 21/2$^+$, have been analysed to determine the lifetimes of the corresponding states. Due to the large recoil velocity in inverse kinematics and high energies of the relevant gammas, special care has been taken to choose the gating transitions such that the slow feeding components are eliminated. Gating from top could not be done due to poor statistics. However, while choosing a suitable gating transition from below, special care has been taken to eliminate the contribution of strong gamma peaks close to the peaks of interest. The spectra at mean angles of 148° and 90° relative to the beam axis were simultaneously fitted (Fig 3) using a modified version of the LINESHAPE code which included corrections for the broad initial recoil momentum distribution produced by the $\alpha$-particle evaporation. The initial momentum distribution of $^{35}\text{Cl}$ recoils has been obtained from statistical model code PACE4 [20]. Lineshapes of four gamma transitions (3734, 2391, 1862 and 2912 keV) were fitted simultaneously as members of a single band. The rotational cascade side-feeding has been considered, assuming 100% side feeding into the top of the band. During each line shape simulation, the background parameters, intensities

![FIG. 2: (a) Comparison of experimental and theoretical backbending plots for the yrast negative parity band in $^{35}\text{Cl}$. The figure shows the results of calculations with fixed $n\hbar\omega$ and mixed $n\hbar\omega$ truncations $\epsilon_{iz}$, Theo1: $[(1d_{5/2})^2(2s_{1/2}1d_{3/2})^6(pf)]^1$ $\oplus$ $(1d_{5/2})^2(2s_{1/2}1d_{3/2})^3(pf)^3]$; Theo2: $[(sd)^{18}(pf)^1$ $\oplus$ $(1d_{5/2})^2(2s_{1/2}1d_{3/2})^2(pf)^3]$. The amount of depression of the single particle energies (SPE) of $pf$ orbitals in each truncation scheme is also mentioned. (b) The energies (E(I)) of 15/2$^-$, 17/2$^+$, 19/2$^-$, 21/2$^+$, 23/2$^-$ and 27/2$^-$ states are plotted as function of I(I+1)
FIG. 4: $B(E2; I_i \rightarrow I_f)$ values for the yrst band in $^{35}$Cl compared with the results of shell model calculations.

of the contaminant peaks, and side-feeding quadrupole moments were allowed to vary, and the best fit set was obtained by using the $\chi^2$ minimization. The lineshape of the 1336 keV transition has also been fitted to get an independent estimate of the lifetime of the 10181 keV level. Using the relevant branching ratios [11], experimental $B(E2)$s were determined from these lifetimes (Fig. 4). Table I. The collectivity in this negative parity band evolves from single particle excitations ($B(E2) \approx 5$ W.u.) at lower spins, to a set of highly deformed SD states ($B(E2) \approx 20-33$ W.u.) in between, finally terminating at a state at $27/2^-$ with moderate deformation ($B(E2) \approx 13$ W.u.). The gamma transition (2614 keV) connecting the positive parity states also has comparable $B(E2)$ ($\approx 16$ W.u.) The decay-out transitions (1336 keV and 1113 keV) have relatively large B(E1) values, viz., 2.9 and 2.6 $\times 10^{-3}$ W.u., respectively. In $^{36}$Ar and $^{40}$Ca, with 4p-4h and 8p-8h excitations in the pf (N=3) shell, the deformation ($\beta_2$) in the SD bands were 0.45 [2] and $\approx 0.59$ [1], respectively. In $^{35}$Cl, 3p-3h excitations (discussed below) give rise to SD structure with comparable KMOI but relatively smaller deformation ($\approx 0.37$) (Table II) similar to the observation in heavier nuclei where the occupation numbers of high-N orbitals have been found to characterise SD bands.

Large basis shell model (LBSM) calculations have been done using the SDPFMW Hamiltonian [22] (as referred to within the OXBASH code package [23]). The SDPF.SM [1,2] Hamiltonian suitable for fixed $n\hbar\omega$ [3] excitation has been also used. The valence space consists of full $sd-pf$ orbitals for both protons and neutrons above the $^{16}$O inert core (see Ref. [12] for details). The negative (positive) parity spectra have been calculated with pure $n\hbar\omega = 1$ (0) and 3 (2) excitations (Fig. 2). The experimental gamma energies ($E_\gamma = E(I) - E(I-2)$) and corresponding $B(E2)$s for lowest state (I=11/2^-) and the upper three states (I=19/2^- to 27/2^-) agree reasonably well with the calculated values with 1p-1h and 3p-3h excitations, respectively. The results with SDPF.SM exhibit better agreement with the experimental data for spins $19/2^-$ to $27/2^-$ (Figs. 2, 3). However, for I=15/2^-, although the gamma energy agrees well with 1p-1h results, the experimental $B(E2)$ matches better to the theoretical $B(E2)$ values from 3p-3h calculations (Fig. 1). The reduction in experimental $B(E2)$ at $27/2^-$ is reproduced well by theory. This decrease indicates band termination at $27/2^-$, consistent with a proton hole coupled to the terminating spin (16^+) in the superdeformed band in $^{36}$Ar nucleus [2,3].

Configuration mixing between 1p-1h, 3p-3h configurations has been included for further improvement of the results. The set Theo1 has inert 1d$_{5/2}$ in 1p-1h excitation, whereas in Theo2, this orbital was active (Fig. 2). Inclusion of 5p-5h configurations has been found to be insignificant. It has been shown earlier that to reproduce the experimental data, the $sd-pf$ shell gap has to be decreased depending upon the particular truncation scheme involved [13,24]. In the experimental spectra (Fig. 1), two close lying 15/2^- states (energy difference is
\(\simeq 169\) keV are seen. The single particle energies (SPE) of the \(pf\) orbitals have been shifted downwards to reproduce the splitting between the two lowest \(15/2^-\) states. Results from mixed calculations show improvement in reproducing the gamma energy of \(15/2^-\) state (Fig. 2) deteriorating the agreement for \(B(E2;15/2^- \rightarrow 11/2^-)\) value. The reduced transition probabilities of decay-out E1 transitions (1336, 1113 keV) are reproduced well (Table I). Shell model calculations reproduce the transition energy and \(B(E2)\) of the 2614 keV transition reasonably well. The highest limit of experimental \(B(M1)\) for 2232 keV transition is larger than the predicted value. However, the calculated values of \(B(E2; 19/2_1^- \rightarrow 15/2_2^-)\) (1693 keV) and \(B(E2; 15/2_1^- \rightarrow 11/2_1^-)\) (2912 keV) both are severely underpredicted (Table I), indicating inadequacy of the configuration mixed calculations. In \(^{36}\text{Ar}\), \(^{30}\text{Ca}\) also \([2,3]\), the calculations failed to reproduce the transition probabilities for the states where different configurations interact to their maximum.

A simple phenomenological approach \([25]\) using two level mixing between pure 3p-3h and 1p-1h states have been used to determine the extent of configuration mixing existing in the states near the band crossing. In this calculation the 19/2^- state has been assumed to be a 3p-3h state (100%) with no mixing from 1p-1h (0%). Utilizing the transition matrix elements from pure 1p-1h and 3p-3h LBSM calculations, the experimentally observed \(B(E2; I \rightarrow I-2)\) values for the transitions with \(I < 19/2^-\) have been reproduced considering the the mixing coefficients of two component wavefunction for each state as variables (Table III). The shell model predictions deviate from the phenomenologically determined wavefunction structure only for the 11/2^- state. It is evident that this deviation leads to the large difference between the experimental and predicted (Theo1 and Theo2) \(B(E2)\) for the 15/2^- \(\rightarrow 11/2^-\) transition.

The SD band in \(^{36}\text{Ar}\) has been shown to originate from the band crossing of \((^{32}\text{S}(I^\pi = 0^+ - 8^+) + \alpha)\) cluster bands \([4]\). The existence of a negative-parity partner band of the SD band is also predicted \([4]\). Even in shell model picture, a negative parity partner of the SD band in \(^{36}\text{Ar}\) has been obtained in terms of 3p-3h excitation \(((sd)^{17}(pf)^3)\) in the \(pf\) shell. So far, it has not been verified experimentally. The parentage of the negative parity SD states in \(^{35}\text{Cl}\) in terms of a proton hole coupled to the alpha cluster SD states in \(^{36}\text{Ar}\) core, have been obtained from calculated spectroscopic factors (Theo1) (Fig. 3). The 15/2^- \(\rightarrow 27/2^-\) states in \(^{35}\text{Cl}\) are generated primarily from the cluster SD states, 10+ to 16+, whereas, the 17/2^- and 21/2^+ arise predominantly from the 11^- and 13^- states in the negative parity partner band in \(^{36}\text{Ar}\). The observed 17/2^+ and 21/2^+ states in \(^{35}\text{Cl}\) therefore provide indirect experimental evidence in favour of the existence of a negative-parity partner band of the SD band in \(^{36}\text{Ar}\) as predicted in Ref \([4]\) and present work.

Using the weak coupling model \([19]\), the interaction energy of (SD states in \(^{36}\text{Ar} + \pi 1f_{7/2}\) hole) which gives rise to the SD states in \(^{35}\text{Cl}\) has been calculated. The \(E(\pi 1f_{7/2})\) hole has been estimated from the excitation energy of the 7/2^- state in \(^{35}\text{Cl}\) at 3163 keV. For example, \(V_{int}(15/2^- \in ^{35}\text{Cl}) = [BE (10^+ \text{SD in } ^{36}\text{Ar}) - E(\text{hole in } 1f_{7/2})] - BE (15/2^- \text{SD in } ^{35}\text{Cl})\) These estimations show that to generate 15/2^- to 27/2^- SD states in \(^{35}\text{Cl}\), by coupling a proton hole with \(10^+\) to \(16^+\) SD states in \(^{36}\text{Ar}\), the interaction energies \([12]\) are 915 keV, 175 keV, -382 keV and -714 keV, respectively. These are small compared to a few MeV average particle-particle interaction, and in two of the cases are repulsive, which is favourable for cluster structure formation. The unequal masses of the underlying clusters give rise to dipole degree of freedom leading to the formation of an apparent rotational band containing alternating parity sequence (Fig 2) connected by strong E1 transitions. These E1 transitions are stronger than \(2 \times 10^{-3}\) W.u. indicating that the corresponding positive parity states are doublet partners \([5]\).

To summarize, a superdeformed rotational band with average transition quadrupole moment \(Q_t \simeq 0.75\) eb and kinematic moments of inertia \(\simeq 8h^2 / MeV\) has been identified for the first time in odd \(^{35}\text{Cl}\) which terminates at 27/2^- . The energies of alternating parity states, 15/2^-, 17/2^+, 19/2^-, 21/2^+, 23/2^- and 27/2^- connected by strong \(B(E1)\) values \( (> 2 \times 10^{-3}\) W.u.) and strong cross-over E2 transitions, follow a linear relation to the \(I(I+1)\) values providing a strong evidence in favour of cluster structure of these states. Results of LBSM calculations have shown that the SD band has been generated by 3p-3h excitations \(((sd)^{16}(pf)^3)\) configuration. A simple two level mixing calculation has been done to extract the wavefunction structure of these mixed states from the experimental \(B(E2)\) values. Weak coupling estimation in terms of \(^{36}\text{Ar}\) and a proton hole clearly identifies the origin of each SD state. The spectroscopic factors are calcu-
lated to estimate parentage of these SD states and their partners in terms of a proton hole coupled to the alpha cluster SD states in $^{36}\text{Ar}$ core. The observed $17/2^+$ and $21/2^+$ states in $^{35}\text{Cl}$ therefore provide indirect experimental evidence in favour of the existence of a negative-parity partner band of the SD band in $^{36}\text{Ar}$ as predicted in Ref [4] and present work.

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