$B_s - \overline{B_s}$ mixing, B decays and R-parity violating supersymmetry

Soumitra Nandi $^1$ and Jyoti Prasad Saha $^2$

$^1$ Department of Physics, University of Calcutta, 92 A.P.C. Road, Kolkata 700009, India
$^2$ Institute of Mathematical Sciences, Chennai 600113, India

Abstract

We discuss the implications of the recent measurement of the $B_s - \overline{B_s}$ oscillation frequency $\Delta M_s$ on the parameter space of R-parity violating supersymmetry. For completeness, we also discuss the bounds coming from leptonic, semileptonic, and nonleptonic $B$ decay modes, and point out some possibly interesting channels at LHC.

Keywords: Supersymmetry, R-parity violation, $B_s - \overline{B_s}$ mixing, CP violation

1 Introduction

The $B_s - \overline{B_s}$ mass difference, recently measured by the D0 [1] and the CDF [2] collaborations, is given by,

$$17 \text{ ps}^{-1} < \Delta M_s < 21 \text{ ps}^{-1} \quad (D0), \quad \Delta M_s = (17.31^{+0.33}_{-0.18} \pm 0.07) \text{ ps}^{-1} \quad (CDF).$$

This result is consistent with the Standard Model (SM) prediction, which is estimated as $21.3 \pm 2.6 \text{ ps}^{-1}$ by the UTfit group [3] and as $20.9^{+4.5}_{-4.2} \text{ ps}^{-1}$ by the CKMfitter group [4]. The implications of $\Delta M_s$ measurements on the parameter space of New Physics (NP) have already been considered [5, 6, 7, 8]. However, given the hadronic uncertainties in the SM prediction, along with additional uncertainties when NP is included, the present measurement of $\Delta M_s$ does not provide a really strong constraint on NP [5, 6].

There have been some attempts to put bounds on the parameter space of R-parity conserving supersymmetry (RPC SUSY) from the $\Delta M_s$ data [9]. In this paper, we would like to put bounds on the R-parity violating (RPV) SUSY couplings. We will use not only the $\Delta M_s$ data but also the data on the leptonic, semileptonic, and nonleptonic branching ratios (BR) and CP asymmetries of $B$ and $B_s$ mesons that are affected by the particular RPV couplings. Such a work on $B^0$ mesons may be found in [10] and this is an extension of that work to the $B_s$ sector. For the relevant formulae, we refer the reader to [10].

It has been shown in [11] that RPV couplings involving sleptons ($\lambda$ and $\lambda'$ type) generate nonzero neutrino mass and one can put stringent constraints on them from the WMAP data [12]. The exact bounds depend on the relation of the mass matrices with the CKM matrix. Anyway, such a study forces us to consider only those couplings which can still be relatively large, and in this paper we derive better bounds on some of these product couplings than those coming from [11].

A major motivation for this study is the $B_s$ physics that is going to be probed at LHC-b, and even at CMS or ATLAS during the low-luminosity run of the Large Hadron Collider (LHC). The leptonic and semileptonic decays are clean and any enhancement over the SM expectations will signify some NP. Also, the phase $\chi$ in $B_s - \overline{B_s}$ mixing comes in the subleading order of the CKM matrix and is expected to be very small ($\sim 0.03$), so any CP-violating effect in $B_s$ decays not involving an up quark will be interesting.

Effects of RPV SUSY on B physics have been discussed extensively in the literature [13, 14, 15, 16, 17]. Constraints coming from neutral meson mixing have been discussed in [10, 18, 19]. However, in these papers,
the $B_s$ sector could not be dealt with, since only the lower bound on $\Delta M_s$ existed then. This paper, in a sense, is the completion of the series. All the computational details that have been taken into account in [10, 19] (e.g., the NLO QCD corrections for short-distance effects, inclusion of both SM and RPV) are incorporated in this paper.

The paper is arranged as follows. In Section 2 we outline the relevant formulae necessary for the analysis, and give the numerical inputs in Section 3. The analysis on $B_s - \bar{B_s}$ box and the decay processes is in Section 4, and we conclude and summarize in Section 5.

## 2 Basic inputs

### 2.1 $B_s - \bar{B_s}$ mixing

The off-diagonal element $M_{12}$ in the $2 \times 2$ effective Hamiltonian causes the $B_s - \bar{B_s}$ mixing. The mass difference between the two mass eigenstates $\Delta M_s$ is given by (following the convention of [20])

$$\Delta M_s = 2|m_{12}|,$$

(2)

with the approximation $|M_{12}| \gg |\Gamma_{12}|$ (this seems a good approximation even for the $B_s$ system). If we have $n$ number of NP amplitudes with weak phases $\theta_n$, one can write the mass difference between mass eigenstates as

$$\Delta M_s = 2(|M^{SM}_{12}|^2 + \sum_i |M_{12i}|^2 + 2|M^{SM}_{12}||\sum_i |M_{12i}| \cos 2(\theta_{SM} - \theta_i) + 2 \sum_i \sum_{j>i} |M_{12i}||M_{12j}| \cos 2(\theta_j - \theta_i)|^{1/2}. \tag{3}$$

For $B_s - \bar{B_s}$ system, the short-distance SM amplitude is

$$M^{SM}_{12} = \frac{(\bar{B_s} H_{eff} B_s)}{2m_B} = \frac{G_F^2}{6\pi^2}(V_{ts} V_{tb}^*)^2 \eta_{B_s} m_{B_s} f_{B_s} B_{B_s} m_W^2 S_0(x). \tag{4}$$

where generically $x = m^2/m_W^2$, $f_{B_s}$ is the $B_s$ meson decay constant, and $\eta_{B_s}$ and $B_{B_s}$ parametrize the short- and the long-distance QCD corrections, respectively. The function $S_0$ is given by

$$S_0(x) = \frac{4x - 11x^2 + x^3}{4(1-x)^2} - \frac{3x^3 \ln x}{2(1-x)^3}. \tag{5}$$

If the NP amplitude has a nonzero phase, then there will be an effective phase in $B_s - \bar{B_s}$ mixing amplitude, whose presence may be tested in the hadronic B factories. In the presence of NP, the general $\Delta F = 2$ effective Hamiltonian can be written as

$$\mathcal{H}_{eff}^{\Delta F=2} = \sum_{i=1}^5 c_i(\mu) \mathcal{O}_i(\mu) + \sum_{i=1}^3 \tilde{c}_i(\mu) \tilde{\mathcal{O}}_i(\mu) + H.c. \tag{6}$$

where $\mu$ is the regularization scale. The effective operators $\mathcal{O}_i$ and $\tilde{\mathcal{O}}_i$ are given in [10]. The Wilson coefficients $c_i$ at $q^2 = m_W^2$ include NP effects, coming from couplings and internal propagators. However, for most of the NP models, and certainly for the case we are discussing here, all NP particles are heavier than $m_W$ and hence the running of the coefficients between $m_W$ and $\mu = \mathcal{O}(m_b)$ are controlled by the SM Hamiltonian alone. In other words, NP determines only the boundary conditions of the renormalization group (RG) equations. For the evolution of these coefficients down to the low-energy scale, we follow Ref. [21], which uses, for $B^0 - \bar{B^0}$ mixing, $\mu = m_b = 4.6$ GeV. The expectation values of these operators between $B_s$ and $\bar{B_s}$ at the scale $\mu$ are analogous to those as given in [10]. The $B_{B_s}(\mu)$ parameters have been taken from [22].
2.2 R-parity violation

It is well known that in order to avoid rapid proton decay one cannot have both lepton number and baryon number violating RPV couplings, and we shall work with a lepton number violating model. This leads to both slepton (charged and neutral) and squark mediated decays, and new amplitudes for $B_s - \overline{B_s}$ mixing, where particles flowing inside the box can be (i) charged sleptons and up-type quarks, (ii) sneutrino and down type quarks, (iii) squarks and leptons. One or both of the scalar particles inside the box can be replaced by $W$ bosons, charged Higgs bosons and Goldstone bosons (in a non-unitary gauge) (see Fig. 1). We follow the usual practice of avoiding the so-called “pure SUSY” contributions to the box amplitudes, i.e., those coming from charginos, neutralinos or gluinos inside the loop. Not only the strongly interacting superparticles are expected to be heavier than the electroweak ones (and hence the contribution being suppressed), but also one can choose SUSY models where these contributions become negligible (e.g., alignment in the squark sector, or Higgsino-dominated lighter chargino, to kill off the respective boxes.) Since the current lower bound on the slepton mass is generally weaker than that on squark mass by a factor 2-3, the slepton mediated boxes have greater chance to be numerically significant.

We start with the superpotential

$$W_{\lambda'} = \lambda'_{ijk} L_i Q_j D^c_k,$$

(7)

where $i, j, k = 1, 2, 3$ are quark and lepton generation indices; $L$ and $Q$ are the $SU(2)$-doublet lepton and quark superfields and $D^c$ is the $SU(2)$-singlet down-type quark superfield respectively. Written in terms of component fields, this superpotential generates six terms, plus their hermitian conjugates:

$$\mathcal{L}_{\text{LQD}} = \lambda'_{ijk} \left[ \bar{\nu}_L \bar{d}_R d^c_L + \bar{d}_L \bar{d}_R u^c_L + (\bar{d}_R)^c \bar{\nu}_L \nu^c_L - \bar{\nu}_L d^c_R \bar{\nu}_L - \bar{d}_L u^c_R \nu^c_R - (\bar{d}_R)^c \nu^c_L \bar{d}_L \right] + \text{H.c.}$$

(8)

With such a term, one can have two different kind of boxes, shown in Fig. 1, that contribute to $B_s - \overline{B_s}$ mixing: first, the one where one has two sfermions flowing inside the loop, alongwith two SM fermions [23], and secondly, the one where one slepton, one $W$ (or charged Higgs or Goldstone) and two up-type quarks complete the loop [18]. It is obvious that the first amplitude is proportional to the product of four $\lambda'$ type couplings, and the second to the product of two $\lambda'$ type couplings times $G_F$. We call them L4 and L2 boxes, respectively, for brevity, where $L$ is a shorthand for $\lambda'$.

We will constrain only products of two $\lambda'$-type couplings at a time, and assume a hierarchical structure, i.e., only one product is, for all practical purpose, simultaneously nonzero (but can have a nontrivial phase). This may not be physically the most appealing scenario but keeps the discussion free from unnecessary complications. This product can in general be complex. Any product is bounded by the product of the bounds on the individual terms, which we call the direct product bound (DPB). Interesting bounds are those which are numerically smaller, and hence stronger, than the corresponding DPBs. The DPBs are mostly taken from [11], and we highlight those products which are more tightly constrained than their respective DPBs. The detailed formulae of the box amplitudes may be found in [10].

2.3 Semileptonic and leptonic decay channels

The RPV couplings that may contribute to $B_s - \overline{B_s}$ mixing should also affect various $B$ decay modes. Let us first consider the leptonic and semileptonic modes.

The expected BRs of leptonic flavour conserving $\Delta B = 1$ processes within SM are much below the experimental numbers (except $B \rightarrow K^{(*)} \ell^+ \ell^-$), so one can safely ignore the SM effects as well as the R-conserving SUSY effects to put bounds on the RPV couplings. (The final state leptons must be the same if the product coupling contributes to $B_s - \overline{B_s}$ mixing.) The leptonic decay modes are theoretically clean and free from any hadronic uncertainties. The semileptonic modes have the usual form-factor uncertainties.

To construct four-fermion operators from $\lambda'$ type couplings that mediate such semileptonic and leptonic $B$ decays, one needs to integrate out the squark or slepton field. The product RPV coupling may in general
Figure 1: $R$-parity violating contributions to $B_s - \overline{B_s}$ mixing. Figure (a) corresponds to $L_4$, while figure (b) to $L_2$ amplitudes (see text for their meanings). For $L_4$, there are similar diagrams with squarks and leptons (both charged and neutral), as well as diagrams with left-chiral quarks as external legs and quarks and sneutrinos flowing in the box. For $L_2$, there are diagrams where the $W$ is replaced by the charged Higgs or the charged Goldstone. The internal slepton can be of any generation, and so can be the internal charge $+2/3$ quarks, generically depicted as $u$.

be complex. However, since all the leptonic decays are one-amplitude processes (only RPV, for all practical purposes) there is no scope for CP-violation; one can only look at nonzero BRs. By the same argument, we can take all couplings to be real without any loss of generality.

The effective Hamiltonian is of the form [24]

$$ H_{RPV} = \frac{1}{2} B_{ijklm} \left[ \overline{u} \gamma^\mu P_L u \right] \left[ \overline{d} \gamma^\mu P_L d \right] + \frac{1}{2} B_{ijklm} \left[ \overline{\nu} \gamma^\mu P_L \nu \right] \left[ \overline{d} \gamma^\mu P_L d \right] + H.c., $$

$$ = -\frac{1}{2} C_{ijklm} \left[ \overline{\nu} \gamma^\mu P_L \nu \right] \left[ \overline{d} \gamma^\mu P_L d \right] + H.c., $$

(9)

where

$$ B_{ijklm} = \sum_{i=1}^{3} \lambda'^*_{ij} \lambda'_{km}, C_{ijklm} = \sum_{i=1}^{3} \lambda'^*_{ik} \lambda'_{lm}. $$

(10)

We take any one to be nonzero at a time. $m_{\tilde{L}}$ is the left-chiral up/down squark mass, taken to be degenerate.

The entire leptonic $B_s$ decay amplitude is solely due to new physics, as far as detectability is concerned. In RPV models, squark-mediated $\lambda'\lambda'$ type interactions are responsible for such purely leptonic decays. As already pointed out, the bounds are robust in a sense that they are free from any theoretical uncertainties (except for the decay constants of $B_s$), and do not depend on the phase of the RPV couplings. For the $B$ mesons, no such leptonic mode has yet been observed. The corresponding upper limits on the BRs are of the order of $10^{-7}$ for $\ell = \mu$ and $10^{-5}$ for $\ell = e$ modes. With 300 GeV squarks, from the bounds that one obtains here, a BR at most of the order of $10^{-8}$ can be expected. Thus, we do not expect to see such leptonic channels before the next-generation hadronic machine or super $e^+e^- B$ factories. However, a number of semileptonic modes $b \rightarrow s \ell^+ \ell^-$ have been observed and the BRs are at the SM ballpark.

The decay width of $B_s \rightarrow \ell^- \ell^+$ is given in [24]. For the semileptonic decays, we use

$$ \langle K(p_2)|\overline{\nu}_\mu |B(p_1)\rangle = P^\mu F_1(q^2) + q^\mu \frac{m_B^2 - m_{\nu}^2}{q^2} (F_0(q^2) - F_1(q^2)), $$

$$ \langle \phi(p_2, e)| V_\mu \mp A_\mu |B_s(p_1)\rangle = \frac{1}{m_B + m_\phi} \left[ -i V(q^2) \varepsilon_{\mu \nu \alpha \beta} \varepsilon^{* \nu} P^\alpha q^\beta \right]. $$
\[\pm A_0(q^2)(P \cdot q)\epsilon^*_\mu \pm A_\mp(q^2)(\epsilon^* \cdot p_1)P_\mu \]
\[\pm A_+(q^2)(\epsilon^* \cdot p_1)q_\mu\]

(11)

where \(m_{B_\gamma}\) and \(m_\phi\) are the meson masses, \(p_1(p_2)\) is the momentum of the initial (final) meson, \(\epsilon\) is the polarization vector of the vector meson \(\phi\), \(P = p_1 + p_2\), \(q = p_1 - p_2\), \(V_\mu = \tau_2\gamma_\mu q_1\), \(A_\mu = \tau_2\gamma_\mu\gamma_5 q_1\), \(V\), \(A_{0,\pm}\) and \(F_{1/0}(q^2)\) are the form factors. The values of these form factor are taken from [25, 26].

The RPV matrix elements for the decay mode \(B_{(s/d)} \rightarrow (\phi/K)l^+l^-\) (where \(l = e, \mu\)):
\[\mathcal{M}(B_{(s/d)} \rightarrow (\phi/K)\ell^+\ell^-) = \frac{1}{2} B_{33/2} \left( \ell^+ \gamma^\mu P_L \ell^- \right) \left( (\langle \phi/K | (s/d) \gamma_5 P_R b | B_{(s/d)}) \right)\]

(12)

### 2.4 Nonleptonic decay channels:

There are four types of slepton-mediated nonleptonic decays that can proceed through the relevant RPV couplings. Among them, \(b \rightarrow c\bar{s}s\) and \(b \rightarrow s\bar{s}s\) rates are bound to be undetectably small if these couplings are to be compatible with the neutrino mass bounds [11]. We, therefore, focus on the \(b \rightarrow u\bar{s}s\) and \(b \rightarrow d\bar{d}s\) type transitions. They mediate the channels \(B \rightarrow \pi K\), \(B \rightarrow \rho K\). The corresponding \(B_s\) decay channels do not have data at comparable level. Note that both BRs and CP asymmetries for these channels have been measured [27, 28]. While the data does not uniquely point to NP, the trend is encouraging.

We will use the Conventional Factorization (CF) model [26] to analyze the effect of RPV SUSY on the \(\pi K\) and \(\rho K\) channels. While the validity of such a simple approach may be questioned, it is not wildly off the truth, at least for these channels. The effective Hamiltonian reads
\[H_{RPV} = d^R_{jkn} \langle \bar{d}_n \gamma^\mu P_L d_j \rangle \langle \bar{d}_k \gamma_\mu P_R b \rangle_8 + d^L_{jkn} \langle \bar{d}_n \gamma^\mu P_L b \rangle_8 \langle \bar{d}_k \gamma_\mu P_R d_j \rangle_8\]

(13)

where
\[d^R_{jkn} = \sum_i \lambda^{j*}_{ij} \lambda^{*}_{kn3} \frac{m_{p_{L_i}}^2}{2m_{p_{L_i}}^2}, \quad d^L_{jkn} = \sum_i \lambda^{j*}_{ij} \lambda^{*}_{kn3} \frac{m_{p_{L_i}}^2}{2m_{p_{L_i}}^2}\]

(14)

Following the standard practice we shall assume that the RPV couplings are hierarchical \(i.e.,\) only one combination of the couplings is numerically significant. Let us assume, to start with, only \(d^R_{112}\) and \(u^R_{112}\) to be nonzero. The QCD corrections are easy to implement: the short-distance QCD corrections enhance the \((S - P) \times (S + P)\) RPV operator by approximately a factor of 2 while running from the slepton mass scale (assumed to be at 100 GeV) to \(m_b\) [29].

The RPV amplitude for \(B \rightarrow \pi K\) is given by
\[M_{\pi^+K^-} = \frac{1}{\sqrt{2}} [u^R_{112}(-R_1(A^{(1)}_{\pi K}) + A^{(2)}_{\pi K} \frac{1}{N_c}) + d^L_{112} \frac{1}{N_c} (A^{(2)}_{\pi K})]\]

(15)

\[M_{\pi^0K^0} = \frac{1}{\sqrt{2}} [d^L_{112}(-R_1(A^{(1)}_{\pi K}) + A^{(2)}_{\pi K} \frac{1}{N_c}) + u^R_{112} \frac{1}{N_c} (A^{(2)}_{\pi K})]\]

(16)

\[M_{\pi^-K^+} = d^L_{112} (R_1 A^{(1)}_{\pi K})\]

(17)

\[M_{\pi^0K^0} = u^R_{112} (-R_1 A^{(1)}_{\pi K})\]

(18)

The expressions for the \(B \rightarrow \rho K\) amplitudes will be similar to those shown above, with \(R_1\) replaced by \(R_2\) and \(A^{(1)}_{\pi K}\) replaced by \(A^{(1)}_{\rho K}\). We use the shorthand
\[R_1 = 2 \frac{m_{\rho^0}^2}{(m_u + m_s)(m_b - m_u)}, \quad R_2 = 2 \frac{m_{\rho^0}^2}{(m_u + m_s)(m_b - m_u)}\]

(19)
and

\[ A^{(1)}_{\pi K} = f_K F_0^{B \to \pi}(m_K^2)(m_B^2 - m_\pi^2), \quad A^{(2)}_{\pi K} = f_\pi F_0^{B \to K}(m_K^2)(m_B^2 - m_\pi^2), \]
\[ A^{(1)}_{\rho K} = 2f_K m_\rho A_0^{B \to \rho}(m_K^2)(\epsilon \cdot p_K), \quad A^{(2)}_{\rho K} = 2f_\rho m_\rho F_1^{B \to K}(m_\rho^2)(\epsilon \cdot p_K). \]  

(20)

3 Numerical Inputs

| Quantity | Value |
|----------|-------|
| \(\Delta M_s\) | \((17.31^{+9.18}_{-0.18} \pm 0.07)\) ps\(^{-1}\) |
| \(\gamma\) | \(50^\circ - 72^\circ\) |
| \(\eta_{B_s}\) | \(0.55 \pm 0.01\) |
| \(m_{B_s}^{MS}(m_{B_s}^{\overline{MS}})\) | \(166\) GeV |
| \(m_{B}^{MS}(m_{B}^{\overline{MS}})\) | \(4.23\) GeV |
| \(m_{b}(m_{b})\) | \(4.6\) GeV |
| \(m_{c}(m_{c})\) | \(1.3\) GeV |
| \(m_{d}(m_{d})\) | \(5.4\) MeV |
| \(m_{s}(2\) GeV\) | \(125\) MeV |
| \(F_0^{\sqrt{B_B}}|_{JLQCD}\) | \((0.245 \pm 0.021^{+0.003}_{-0.002})\) GeV |
| \(|V_{us}| \times 10^3\) | \(2.272^{+0.01}_{-0.01}\) |
| \(|V_{cs}| \times 10^3\) | \(9.73\) |
| \(|V_{ts}| \times 10^3\) | \(41.61^{+0.12}_{-0.78}\) |
| \(|V_{ub}| \times 10^3\) | \(4.4 \pm 0.3\) |
| \(|V_{cb}| \times 10^3\) | \(42.0 \pm 0.7\) |
| \(|V_{tb}| \times 10^3\) | \(9.99\) |

Table 1: Input parameters used for the numerical analysis, from [2, 4, 21, 30].

The major sources of the numerical inputs are: (i) the Heavy Flavor Averaging Group (HFAG) website [31] for the latest (summer 2006) updates on \(B\) physics; (ii) Particle Data Group 2006 edition [32]; and (iii) the inputs used in the CKMfitter package [4]. The quark masses and Wilson coefficients have been taken from [21, 33]. We use the following numbers.

The masses for all the mesons \(B^0\), \(B^-\), \(\pi\), \(\rho\), and \(K\) are the corresponding central values as given in [32]. The meson decay constants (in GeV) are:

\[ f_\pi = 0.133, \quad f_K = 0.158, \quad f_\rho = 0.210. \]  

(21)

The transition formfactors [34] at \(q^2 = 0\) are given by

\[ F_0(B \to K) = 0.38; \quad F_0(B \to \pi) = 0.33; \quad A_0(B \to \rho) = 0.28, \]  

(22)

and \(F_0(0) = F_1(0)\).

The quark masses have been evaluated in the \(\overline{MS}\) scheme. The pole mass for the top quark is about 5 GeV higher and the mass for the bottom quark is 4.6 GeV. The CKM elements are shown in Table 1.

The leptonic and semileptonic BRs for the \(B\) meson, which are of interest to us, are as follows [1, 32, 31]:

\[ \text{Br}(B \to K\ell^+\ell^-) < (0.57 \pm 0.07) \times 10^{-6} \quad (\ell = e/\mu); \]
\[ \text{Br}(B \to K e^+e^-) = (0.55 \pm 0.09) \times 10^{-7}; \]
\[ \text{Br}(B \to K \mu^+\mu^-) = (0.61 \pm 0.08) \times 10^{-7}; \]
\[
\begin{align*}
\text{Br}(B_s \to \phi^+\mu^-) &< 4.1 \times 10^{-6}; \\
\text{Br}(B_s \to \mu^+\mu^-) &< 1 \times 10^{-7}; \\
\text{Br}(B_s \to e^+e^-) &< 5.4 \times 10^{-5};
\end{align*}
\]

For the $\pi K$ and $pK$ modes, the data reads [32, 31, 35, 36, 37]:

\[
\begin{align*}
\text{Br}(B^0 \to \pi^-K^+) &= (18.9 \pm 0.7) \times 10^{-6} \\
\text{Br}(B^0 \to \pi^0K^0) &= (11.5 \pm 1.0) \times 10^{-6} \\
\text{Br}(B^+ \to \pi^+K^0) &= (24.1 \pm 1.3) \times 10^{-6} \\
\text{Br}(B^+ \to \pi^0K^+) &= (12.1 \pm 0.8) \times 10^{-6} \\
\text{Br}(B^+ \to \rho^-K^+) &= (9.9^{+1.6}_{-1.5}) \times 10^{-6} \\
\text{Br}(B^0 \to \rho^0K^0) &= (5.1 \pm 1.6) \times 10^{-6} \\
\text{Br}(B^+ \to \rho^0K^+) &= (4.23^{+0.56}_{-0.57}) \times 10^{-6} \\
A^{\text{dir}}_{\text{CP}}(B^0 \to \pi^\pm K^\mp) &= 0.115 \pm 0.018. 
\end{align*}
\]

To evaluate the QCD corrections, we take $\alpha_s(m_Z^2) = 0.1187$ [32], and take the SUSY scale $M_S = 500$ GeV. The precise value of this scale is not important, however, and we can take it to be at the squark mass scale (300 GeV) without affecting the final results. The exact evolution matrix can be found in [21] and [33]; for our purpose, it is sufficient to note that for the $B_s$ system, the operator $O_1$ is multiplicatively renormalized by a factor 0.820 at the scale $\mu = 2$ GeV, and the operator $O_4$ at $m_W$ changes to $2.83O_4 + 0.08O_5$. We again stress that theoretically the procedure is questionable for boxes with light quarks flowing in the loop. However, the numbers that we obtain are fairly robust and one can very well drop the NLO corrections altogether, if necessary, without compromising the results. Since the $O_2$ admixture is small, one can take the central values for these parameters without introducing too much error. The relevant $B$-parameters for $B_s$ system are taken from [22]

We take all sleptons to be degenerate at 100 GeV, and all squarks at 300 GeV. We also take $\tan \beta(\equiv v_2/v_1) = 5$ (very low values are excluded by LEP, and the numbers are not sensitive to the precise choice of $\tan \beta$), and the charged Higgs boson mass as 200 GeV (lower values are disfavored from $b \to s\gamma$).

# 4 Analysis

## 4.1 $B_s - \bar{B}_s$ mixing

For the $B_s$ system, the bounds are summarized in Table 2. When the product coupling is complex, we show only the real part, since the bound on the imaginary part is almost equal to this. The reason is easy to understand: the bounds are obtained when the RPV coupling has a phase opposite to that of the SM coupling, so that the interference is destructive. At the limit where the RPV coupling determines the mixing amplitude, the phase is irrelevant. The effect of this destructive interference is clear in Fig. 2(a) and Fig. 2(b). For a more detailed explanation, we refer the reader to [10].

The relative magnitudes of the bounds are also easy to understand. For example let us consider the bounds on $\lambda_{32}^2\lambda_{12}^4$ vis-a-vis $\lambda_{32}^3\lambda_{33}^4$. The relevant box diagrams have the same particle content; but the first one is proportional to $V_{ts}V_{cb} (\sim O(\lambda^4))$, and the second one to $V_{tb}V_{cs} (\sim 1)$. The relative suppression in $\lambda$ enhances the limit on the RPV coupling.

Though most of the bounds are of the same order in magnitude, these are, theoretically, an improvement over those obtained earlier [18, 23, 19]. We have taken into account all possible amplitudes (and the interference patterns play a nontrivial role), including the SM one, but have systematically neglected the pure supersymmetric
Table 2: Bounds on \( \lambda' \lambda' \) combinations from \( B_s - \bar{B}_s \) mixing. The table displays the magnitudes only, and not the signs.

| \( \lambda' \lambda' \) combination | Only real | Complex, real part |
|-----------------------------------|----------|--------------------|
| \((i32)(i33)\)                   | 1.01 \times 10^{-2} | 1.0 \times 10^{-2} |
| \((i22)(i23)\)                   | 8.2 \times 10^{-3}   | 8.0 \times 10^{-3} |
| \((i12)(i13)\)                   | 1.2 \times 10^{-2}   | 3.7 \times 10^{-2} |
| \((i22)(i33)\)                   | 1.8 \times 10^{-1}   | 1.7 \times 10^{-1} |
| \((i32)(i23)\)                   | 3.2 \times 10^{-4}   | 3.0 \times 10^{-4} |
| \((i22)(i13)\)                   | 3.47 \times 10^{-1}  | 3.45 \times 10^{-2} |
| \((i12)(i23)\)                   | 5.16 \times 10^{-2}  | 7.56 \times 10^{-2} |
| \((i32)(i13)\)                   | 1.4 \times 10^{-3}   | 1.3 \times 10^{-3} |
| \((i12)(i33)\)                   | 9.0 \times 10^{-1}   | 9.0 \times 10^{-1} |
| \((i23)(i33)\)                   | 1.66 \times 10^{-2}  | 5.1 \times 10^{-2} |
| \((i22)(i32)\)                   | 1.66 \times 10^{-2}  | 5.1 \times 10^{-2} |
| \((i21)(i31)\)                   | 1.66 \times 10^{-2}  | 5.1 \times 10^{-2} |

Figure 2: (a) Allowed parameter space for \( \lambda'_{12} \lambda'_{13} \) (b) The allowed parameter space of RPV phase for \( |\lambda'_{12} \lambda'_{13}| \), which can gives CP assymetries in the \( B_s \to K\pi \) decay.
boxes coming from gaugino exchange. The reason is that those boxes decouple in the heavy squark limit, and one can always take an RPV model embedded in a minimal supersymmetric theory where such FCNC processes are somehow forbidden. It was shown in [19] that the bounds are fairly robust even if one takes into account such SUSY contributions. Furthermore, the QCD corrections are implemented up to NLO.

### 4.2 Nonleptonic decay channels

As we have mentioned, the $B \to \pi K$ and $B \to \rho K$ numbers are encouraging for NP enthusiasts. However, the hadronic uncertainties are significant. Also, one must have a nonzero strong phase between the SM tree and the SM penguin amplitudes to explain the direct CP asymmetry data on $B \to \pi^+ K^-$. Thus, it is of importance to explore the data in conjunction with $B_s - \bar{B_s}$ mixing. In our analysis, we use the CF model, as mentioned, and vary the strong phase difference between the SM tree and the RPV amplitudes from $0$ to $2\pi$. To take into account the hadronic uncertainties, we (i) vary the SM amplitude from its CF value by $20\%$, and (ii) vary the RPV weak phase in the range $[0:2\pi]$. The allowed parameter space of RPV couplings, in the magnitude-phase plane, comes out with separate island-like structures. For example, $\lambda_{i21}'\lambda_{i31}'$ lies between $2.45 \times 10^{-3}$ to $2.6 \times 10^{-3}$ and the corresponding phase lies between $104^\circ$ to $120^\circ$, whereas $\lambda_{i13}'\lambda_{i12}'$ lies between $2.33 \times 10^{-3}$ to $2.82 \times 10^{-3}$ and the corresponding phase lies between $85^\circ$ to $105^\circ$. We quoted the highest value in Table 3. These bounds are much stronger than that coming from $B_s - \bar{B_s}$ mixing.

We have not considered the $B \to (\eta, \eta')K$ modes. Though they are mediated by $b \to s q \bar{q}$ ($q = u, d$) transitions, the BRs for those modes cannot be explained simultaneously by such a simple new physics structure [38]. Similarly, the mode $B \to \phi K^*$ has not been considered, since the longitudinal polarization anomaly for this mode cannot be explained without a contribution from tensor current but RPV SUSY do not provide for such tensor current structures, at least at the tree-level [39]. However, we note that unless a product coupling is at least of the order of $10^{-3}$, the RPV contribution is unlikely to affect the SM amplitude.

### 4.3 Leptonic and semileptonic decay channels

The detail procedure of the leptonic and semileptonic decay channels are given in [24]. While their bounds were for 100 GeV squarks, we scale the numbers for 300 GeV squarks. The bounds are shown in Table 4. Note that the channel $B \to K \mu^+ \mu^-$ gives the best bound. Also, there is no bound involving $s$ in the final state, but we can estimate the number of $B_s \to \tau^+ \tau^-$ decays. The relevant RPV coupling is $\lambda_{i31}'\lambda_{i32}'$. It can easily be checked that unless $i = 1$, the product is so constrained from neutrino mass [11] that even at LHC-b, there is no hope to detect a RPV signal in this channel. For $i = 1$, the coupling $\lambda_{i12}'\lambda_{i13}'$ should be less than $2.8 \times 10^{-3}$, which in turn translates into a bound on the BR to be less than $2.7 \times 10^{-6}$. Note that the SM expectation is about $7 \times 10^{-7}$.

| $\lambda\lambda\prime$ combination | Quark level | Meson level | Bound |
|-----------------------------------|-------------|-------------|-------|
| (i21)(i31)                        | $b \to d s$ | $B_d \to \pi^0 K^0$ | $2.45 \times 10^{-3}$ |
|                                  | $B^- \to \pi^0 K^-$ | |
| (i12)(i13)                        | $b \to d s$ | $B^- \to \pi^0 K^-$ | $2.82 \times 10^{-3}$ |
|                                  | $b \to u s$ | $B_d \to \pi^+ K^-$ | |
| (i22)(i32)                        | $b \to s s$ | $B_s \to \phi K^*$ | $2.33 \times 10^{-3}$ |

Table 3: Some of the possible nonleptonic transitions mediated by the RPV couplings discussed in the paper.
Table 4: Some of the possible leptonic and semileptonic transitions mediated by the RPV coupling relevant with $B_s - \bar{B_s}$ mixing are given here. It has shown that in many cases the bounds coming from this decays are much better then coming from mixing.

| $\lambda'\lambda'$ combination | Quark level | Meson level | Bound  |
|-------------------------------|-------------|-------------|--------|
| $(22)(23)$                    | $b \rightarrow s\mu^+\mu^-$ | $B_s \rightarrow \mu^+\mu^-$ | $7.7 \times 10^{-3}$ |
| $(22)(23)$                    | $b \rightarrow s\mu^+$ | $B_s \rightarrow \phi\mu^+\mu^-$ | $4.9 \times 10^{-3}$ |
| $(22)(23)$                    | $b \rightarrow s\mu^+$ | $B_s \rightarrow \mu^+\mu^-$ | $6.6 \times 10^{-4}$ |
| $(12)(13)$                    | $b \rightarrow s e^+e^-$ | $B \rightarrow K e^+e^-$ | $7.7 \times 10^{-4}$ |

Table 5: Bounds on real $\lambda'\lambda'$ combinations from $B_s - \bar{B_s}$ mixing and correlated decay channels. The DPBs, displayed in the last column, occur from neutrino constraints with no mixing scenario [11]. The product marked with a dagger is bounded from tree-level $B_s - \bar{B_s}$ mixing($\sim \mathcal{O}(10^{-6})$).

| $\lambda'\lambda'$ combination | Related process | Current bound | Previous bound |
|-------------------------------|-----------------|---------------|---------------|
| $(112)(113)$                  | $B_s \rightarrow Ke^+$ | $7.74 \times 10^{-4}$ | $1.52 \times 10^{-1}_\nu$ |
| $(112)(123)$                  | $B_s \rightarrow K e^+e^-$ | $7.74 \times 10^{-4}$ | $9.7 \times 10^{-9}_\nu$ |
| $(132)(133)$                  | $B_s \rightarrow Ke^+$ | $7.74 \times 10^{-3}$ | $6.0 \times 10^{-3}_\nu$ |
| $(212)(213)$                  | $B_s \rightarrow K\mu^+\mu^-$ | $6.57 \times 10^{-4}$ | $1.52 \times 10^{-1}_\nu$ |
| $(222)(223)$                  | $B_s \rightarrow K\mu^+\mu^-$ | $7.74 \times 10^{-4}$ | $9.7 \times 10^{-9}_\nu$ |
| $(232)(233)$                  | $B_s \rightarrow K\mu^+\mu^-$ | $7.74 \times 10^{-4}$ | $6.0 \times 10^{-3}_\nu$ |
| $(312)(313)$                  | $B \rightarrow \pi^+K^-\nu$ | $2.8 \times 10^{-3}$ | $1.52 \times 10^{-1}_\nu$ |
| $(322)(323)$                  | $B_s - \bar{B_s}$ | $8.2 \times 10^{-3}$ | $9.7 \times 10^{-6}_\nu$ |
| $(323)(333)$                  | $B_s - \bar{B_s}$ | $1.01 \times 10^{-2}$ | $6.0 \times 10^{-5}_\nu$ |
| $(12)(13)$                    | $B_s - \bar{B_s}$ | $1.8 \times 10^{-1}$ | $3.75 \times 10^{-8}_\nu$ |
| $(132)(123)^\dagger$         | $B_s - \bar{B_s}$ | $3.2 \times 10^{-4}$ | $1.56 \times 10^{-1}_\nu$ |
| $(12)(113)$                   | $B_s - \bar{B_s}$ | $3.47 \times 10^{-2}$ | $9.75 \times 10^{-6}_\nu$ |
| $(112)(123)$                  | $B_s - \bar{B_s}$ | $5.16 \times 10^{-2}$ | $1.52 \times 10^{-1}_\nu$ |
| $(32)(113)$                   | $B_s - \bar{B_s}$ | $1.4 \times 10^{-5}$ | $1.56 \times 10^{-1}_\nu$ |
| $(112)(113)$                  | $B_s - \bar{B_s}$ | $9.0 \times 10^{-1}$ | $5.8 \times 10^{-5}_\nu$ |
| $(23)(133)$                   | $B_s - \bar{B_s}$ | $1.66 \times 10^{-2}$ | $5.8 \times 10^{-5}_\nu$ |
| $(122)(132)$                  | $B_s - \phi K_s$ | $2.3 \times 10^{-3}$ | $1.0 \times 10^{-3}_\nu$ |
| $(121)(131)$                  | $B_s - \pi^0 K_s$ | $2.45 \times 10^{-3}$ | $1.56 \times 10^{-1}_\nu$ |
4.4 Comparison between bounds coming from mixing and decay

In Table 5 we summarize our results, displaying the bounds on all $\lambda'\lambda'$ type products that may be responsible for $B_s - \bar{B}_s$ mixing. We find that a number of them may have better bounds from semileptonic or nonleptonic decay modes. While the neutrino constraints are indeed tight, we obtain a tighter constraint for most of the products.

5 Summary and Conclusions

In this paper we have computed the bounds on the product couplings of the type $\lambda'\lambda'$ coming from $B_s - \bar{B}_s$ mixing. Though such an analysis is not new, we have implemented several features in the analysis which have not been taken into account in earlier studies. Previously there was a lower limit on $\Delta M_s$, here we have used the current bound on it and considered the exact expression for the box amplitude taking all possible processes, including that from SM. The QCD corrections to the amplitudes have been taken upto the NLO level. We have considered the possibility that the RPV product couplings may be complex. The analysis is done in the benchmark point $m_{H^+} = 200$ GeV, $\tan \beta = 5$, all sleptons degenerate at 100 GeV and all squarks degenerate at 300 GeV, and neglecting the pure MSSM contribution to the box amplitudes (by possibly applying some underlying FCNC suppression principle, like alignment of the squark mass matrices).

It is to be observed that in some cases, our bounds are actually weaker than those obtained earlier by saturating the mass difference with RPV alone. The reason is that destructive interference with the SM amplitude plays a very crucial role in determining the bounds, particularly when the phase of the RPV coupling is arbitrary. There is an intricate interplay among different amplitudes as can be seen in Fig. 2.

In some cases the bounds obtained from semileptonic $B$ decays are better. Of course, one can enhance the squark mass to a limit where these bounds become weaker then those obtained from the box (the latter is not much affected by decoupling the squarks), but such extremely massive squarks are not interesting, even for the LHC.

However, some of these couplings may affect the nonleptonic decay modes (which, being slepton mediated, cannot be suppressed by decoupling the squarks). For some cases, the bounds coming from such decays are tighter than those coming from mixing.

6 Acknowledgements

We would like to thank Rahul Sinha and Anirban Kundu for their useful comments and suggestions.

References

[1] V. Abazov [DO Collaboration], arXiv:hep-ex/0603029;

[2] CDF Collaboration, Phys. Rev. Lett. 97, 062003 (2006);

[3] M. Bona et al. [UTfit Collaboration], JHEP 0507, 028 (2005); updated results available at http://utfit.roma1.infn.it/.

[4] J. Charles et al. [CKMfitter Group], Eur. Phys. J. C 41, 1 (2005); updated results available at http://ckmfitter.in2p3.fr/.

[5] P. Ball and R. Fleischer, arXiv:hep-ph/0604249; A. Datta, arXiv:hep-ph/0605039.
[6] Z. Ligeti, M. Papucci and G. Perez, arXiv:hep-ph/0604112; Y. Grossman, Y. Nir and G. Raz, arXiv:hep-ph/0605028.

[7] M. Blanke, A. J. Buras, D. Guadagnoli and C. Tarantino, arXiv:hep-ph/0604057.

[8] K. Cheung, C. W. Chiang, N. G. Deshpande and J. Jiang, arXiv:hep-ph/0604223. X.-G. He and G. Valencia, arXiv:hep-ph/0605202.

[9] M. Ciuchini and L. Silvestrini, arXiv:hep-ph/0603114; J. Foster, K. i. Okumura and L. Roszkowski, arXiv:hep-ph/0604121; G. Isidori and P. Paradisi, arXiv:hep-ph/0605012. S. Khalil, arXiv:hep-ph/0605201. M. Endo and S. Mishima, arXiv:hep-ph/0603251. S. Baek arXiv:hep-ph/0605182.

[10] J.P. Saha and A. Kundu, Phys. Rev. D 70, 096002 (2004).

[11] B.C. Allanach, A. Dedes and H.K. Dreiner, Phys. Rev. D 60, 075014 (1999);

[12] M. Tegmark et al. (SDSS Collaboration), Phys. Rev. D 69, 103501 (2004).

[13] R. Barbieri and A. Masiero, Nucl. Phys. B267, 679 (1986).

[14] R. Barbieri, A. Strumia and Z. Berezhiani, Phys. Lett. B407, 250 (1997).

[15] K. Agashe and M. Graesser, Phys. Rev. D 54, 4445 (1996).

[16] S.A. Abel, Phys. Lett. B410, 173 (1997).

[17] J.-H. Jang, Y.G. Kim and J.S. Lee, Phys. Lett. B408, 367 (1997); Phys. Rev. D 58, 035006 (1998); J.-H. Jang, J.K. Kim and J.S. Lee, Phys. Rev. D 55, 7296 (1997); D. Guetta, Phys. Rev. D 58, 116008 (1998); K. Huitu et al., Phys. Rev. Lett. 81, 4313 (1998); D. Choudhury, B. Dutta and A. Kundu, Phys. Lett. B456, 185 (1999); G. Bhattacharyya and A. Datta, Phys. Rev. Lett. 83, 2300 (1999); G. Bhattacharyya, A. Datta and A. Kundu, Phys. Lett. B514, 47 (2001); D. Chakraverty and D. Choudhury, Phys. Rev. D 63, 075009 (2001); D. Chakraverty and D. Choudhury, Phys. Rev. D 63, 112002 (2001); H. Dreiner, G. Polesello and M. Thormeier, Phys. Rev. D 65, 115006 (2002); J.P. Saha and A. Kundu, Phys. Rev. D 66, 054021 (2002); A. Datta, Phys. Rev. D 66, 071702 (2002); A. Akeroyd and S. Recksiegel, Phys. Lett. B541, 121 (2002); B. Dutta, C.S. Kim and S. Oh, Phys. Rev. Lett. 90, 011801 (2003); A. Kundu and T. Mitra, Phys. Rev. D 67, 116005 (2003); B. Dutta et al., hep-ph/0312388, hep-ph/0312389.

[18] G. Bhattacharyya and A. Raychaudhuri, Phys. Rev. D 57, 3837 (1998).

[19] J.P. Saha and A. Kundu, Phys. Rev. D 69, 016004 (2004).

[20] A.J. Buras and R. Fleischer, hep-ph/9704376, also in Heavy Flavours II, World Scientific, Singapore (1997), ed. A.J. Buras and M. Lindner.

[21] D. Bećirević et al., Nucl. Phys. B634, 105 (2002).

[22] D. Bećirević et al., J. High Energy Physics 0204, 025 (2002);

[23] B. de Carlos and P. White, Phys. Rev. D 55, 4222 (1997).

[24] J.P. Saha and A. Kundu, in [17].

[25] C.Q.Geng and C.C.Liu, J. Phys. G 29, 1103 (2003);

[26] A. Ali, G. Kramer and C.-D. Lü, Phys. Rev. D 58, 094009 (1998).

[27] B. Aubert et al. (BaBar Collaboration), Phys. Rev. Lett. 89, 281802 (2002).

[28] K. Abe et al. (Belle Collaboration), hep-ex/0301032.

[29] J.L. Bagger, K.T. Matchev and R.J. Zhang, Phys. Lett. B412, 77 (1997); M. Ciuchini et al., Nucl. Phys. B523, 501 (1998).
[30] S. Aoki et al. [JLQCD coll.], Phys. Rev. Lett. 91 (2003) 212001. [arXiv:hep-ph/0307039].

[31] See http://www.slac.stanford.edu/xorg/hfag/, the website of the Heavy Flavor Averaging Group, for the Particle Data Group 2003 update of the rare decay (hadronic, charmless) data, averaged over BaBar, Belle and CLEO collaborations.

[32] Yao et al., (Particle Data Group Collaboration), Journal of Physics G 33, 1 (2006);

[33] M. Ciuchini et al., J. High Energy Physics 9810, 008 (1998).

[34] M. Wirbel, B. Stech and M. Bauer, Zeit. Phys. C 29, 637 (1985);
     hep-ph/0507253.

[35] R. Aleksan et al., hep-ph/0301165.

[36] B. Aubert et al. (BaBar Collaboration), Phys. Rev. D 65, 051101 (2002).

[37] K.F. Chen et al. (Belle Collaboration), Phys. Lett. B546, 196 (2002).

[38] A. Kundu, S. Nandi, and J.P. Saha, Phys. Lett. B622, 102 (2005);

[39] P.K. Das and K.C. Yang, Phys. Rev. D 71, 094002 (2005)
     S. Nandi and A. Kundu, J. Phys. G 32, 835 (2006).