Abstract: Although the Higgs boson has been discovered, its self couplings are poorly constrained. It leaves the nature of the Higgs boson undetermined. Motivated by different Higgs potential scenarios other than the Landau-Ginzburg type in the standard model (SM), we systematically organize various new physics scenarios – elementary Higgs, Nambu-Goldstone Higgs, Coleman-Weinberg Higgs, and Tadpole-induced Higgs, etc. We find that double-Higgs production at the 27 TeV high energy LHC (HE-LHC) can be used to discriminate different Higgs potential scenarios, while it is necessary to use triple-Higgs production at the 100 TeV collider to fully determine the shape of the Higgs potential.
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1 Introduction

After a long wait of about half a century, in 2012, the Higgs boson was discovered at the Large Hadron Collider (LHC) by CMS and ATLAS Collaborations. With the discovery of this missing piece, all the particles of the standard model (SM) have now been discovered. Given the measured value of the Higgs mass, all the parameters in the SM are predicted. The main goal of the LHC machine now is to measure properties and interactions of the Higgs boson, as well as look for signatures of possible new physics beyond the SM. As of now, direct search for evidence of new physics (NP) has not yielded any thing of significance, new physics has been pushed to higher scales. On the other hand, precision measurements on various SM processes provide us an indirect way to probe new physics. The Higgs boson couplings to the gauge bosons and the SM fermions have been measured at the LHC through various production processes and decay modes, while the Higgs self couplings are not yet determined at the end of Run-2 LHC [1–8].

The self couplings of the Higgs boson, including the trilinear and quartic Higgs couplings, are nevertheless still mysteries. Experimentally, the trilinear and quartic Higgs couplings can be directly measured using double and triple-Higgs productions $pp \rightarrow hh$ and $pp \rightarrow hhh$ at the hadron colliders, respectively. The ATLAS and CMS Collaborations have been looking for the $hh$ signal in the data collected so far at the LHC, which only put very loose bound on the Higgs trilinear coupling. The $hhh$ signal has not yet been investigated with the Run-2 data. Evidently, it is quite challenging to measure the Higgs self couplings at the LHC, and this provides a strong motivation for building future high energy colliders.

Theoretically, there are still many unknowns about the Higgs boson, such as nature of the Higgs boson, origin of electroweak symmetry breaking (EWSB), shape of the Higgs potential, and strength of the electroweak phase transition, etc. All these questions can only be addressed after the Higgs self couplings are determined. So far the Higgs self couplings are not tightly constrained yet, thus the Higgs potential can be very different from the Landau-Ginzburg type in the SM. In this work, we systematically investigate various classes of new physics scenarios based on different types of Higgs potential. To be specific, we consider the following Higgs scenarios:

- **Elementary Higgs boson**, in which the Higgs boson is taken as an elementary scalar with rescaled self couplings (see, e.g. [9, 34]), the Higgs mass parameter is negative and thus triggers EWSB;

- **Nambu-Goldstone Higgs**, in which the Higgs boson is taken as a pseudo Nambu-Goldstone (PNG) boson [10, 11] emerging from strong dynamics at high scales (see Refs. [12–14] for comprehensive reviews);

- **Coleman-Weinberg (CW) Higgs**, in which EWSB is triggered by renormalization group (RG) running effects [15–17] with classical scale invariance;

- **Tadpole-induced Higgs**, in which EWSB is triggered by the Higgs tadpole [18, 19], and the Higgs mass parameter is taken to be positive.
In general, the Higgs potential could be organized according to their analytic structure. The key structure of the Higgs potential in each scenario is as follows

\[
V(H) \simeq \begin{cases} 
-m^2 H^\dagger H + \lambda (H^\dagger H)^2 + c_6 \lambda (H^\dagger H)^3, & \text{Elementary Higgs} \\
-a \sin^2(\sqrt{H^\dagger H}/f) + b \sin^4(\sqrt{H^\dagger H}/f), & \text{Nambu-Goldstone Higgs} \\
\lambda (H^\dagger H)^2 + \epsilon (H^\dagger H)^2 \log \frac{H^\dagger H}{\mu^2}, & \text{Coleman-Weinberg Higgs} \\
-\kappa^2 \sqrt{H^\dagger H} + m^2 H^\dagger H, & \text{Tadpole-induced Higgs}
\end{cases}
\]

where \( f \) denotes the decay constant of the NG Higgs boson, and \( \mu \) denotes the renormalization scale in case EWSB is triggered by radiative corrections, \( m^2, \lambda, c_6, \Lambda, a, b, \epsilon, \kappa \) are dimensionful or dimensionless parameters in each new physics scenario. The shapes of Higgs potential are schematically illustrated in Fig. 1, respectively. In both the elementary and Nambu-Goldstone Higgs cases, the Higgs potential could be expanded in power of \( H^\dagger H \), which could recover the Landau-Ginzburg effective theory description if a truncation on the series is a good approximation. In these two scenarios, the decoupling limit corresponds to the case when the new physics is much higher than the EW scale. However, there is no such simple power expansion in the scenarios of Coleman-Weinberg Higgs and Tadpole-induced Higgs. In all the above cases, the Higgs trilinear and quartic couplings could be quite different from the SM values.

![Figure 1](image)

**Figure 1**: Summary of the shapes of Higgs potential for different scenarios studied in this work.

All the scenarios above can be described in the effective field theory (EFT) framework. One of the most popular EFT frameworks is the SMEFT \cite{20–22}, which assumes new physics decouple at high scale, and EW symmetry is in the unbroken phase. The SMEFT up to dimensional-six operators is only suitable to describe the elementary Higgs scenarios, and the Nambu-Goldstone Higgs if the Higgs non-linearity effect is negligible \cite{23}. On the other hand, Coleman-Weinberg Higgs and Tadpole-induced Higgs cannot be described within the SMEFT. To unify all four scenarios in one framework, we utilize the EFT framework in the broken phase of EW symmetry, the Higgs EFT \cite{24–30}. Framed in the Higgs EFT, we summarize the general Higgs effective couplings in various scenarios, and parametrize the scaling behavior for the multi-Higgs production cross sections at various high energy hadron colliders.

We investigate different scenarios using the double-Higgs and triple-Higgs production at the high luminosity LHC (HL-LHC), high energy LHC (HE-LHC), and 100 TeV hadron
collider (FCC-hh). We compute cross sections, distributions, and discuss interesting interference effects. Some of these scenarios, the SM-Higgs, CW-Higgs, and Tadpole-induced Higgs scenarios have very different trilinear Higgs coupling. In these scenarios total cross sections are quite different. We find that different scenarios of Higgs potential can be distinguished via measuring double-Higgs production at HE-LHC and FCC-hh. We also consider the possibility of measuring the trilinear Higgs coupling, assuming certain accuracy for the measured cross section. In some of the scenarios, it is possible to measure the trilinear Higgs coupling quite precisely. The role of triple-Higgs production is to determine the shape of the Higgs potential by measuring quartic Higgs coupling. It would not be easy to observe this process even at FCC-hh. We determine the required luminosity, assuming the triple-Higgs production can be measured to certain accuracy at the FCC-hh, in order to measure the strength of the quartic Higgs coupling and the shape of Higgs potential.

The paper is organized as follows. In sections 2 and 3, we lay out the general framework of Higgs effective couplings and discuss various NP scenarios that could yield a different Higgs potential from the SM. In section 4, we consider theoretical constraints on the strength of Higgs boson self couplings, including tree-level partial wave unitarity and vacuum stability. In section 5, we consider the process $pp \rightarrow hh$ for its potential to discriminate various Higgs potential scenarios. In section 6, we examine the usefulness of the process $pp \rightarrow hhh$ to fully pin down the quartic Higgs coupling. In the last section, we conclude.

2 Higgs EFT Framework

In the EFT framework, new physics effect in the Higgs sector could be described using Higgs EFT and SMEFT in the broken and unbroken phase of electroweak symmetry, respectively. Higgs EFT could describe all the Higgs scenarios considered, while SMEFT is only suitable to describe NP models with decoupling behavior, such as the elementary Higgs scenario, and the Nambu-Goldstone Higgs scenario with negligible Higgs non-linearity.

2.1 Higgs EFT: Higgs in the Broken Phase

In the broken phase of electroweak symmetry, it is convenient to use the following Higgs EFT Lagrangian \cite{24–30} to describe the interactions of the top quark, the Higgs boson, and the Goldstone bosons eaten by the massive gauge bosons $W^\pm$ and $Z$. Only the $U(1)_{EM}$ symmetry is manifested (or equivalently, the SM gauge symmetry $SU(2)_L \times U(1)_Y$ is non-linearly realized) in the broken phase. Furthermore, the custodial symmetry $SU(2)_V$ should be respected when constructing the effective Lagrangian and the Higgs boson $h$ is taken as a custodial singlet in this framework. With the nonlinearly-realized symmetry $SU(2)_L \times SU(2)_R/SU(2)_V$, the leading Higgs EFT Lagrangian, in the limit of turning off

\footnote{In this work, we only care about Higgs couplings in double and triple-Higgs productions, the gauge-less limit, $g, g' \rightarrow 0$, could be taken.}
gauge couplings, is [24–30]

\[
\mathcal{L} = \frac{1}{2} (\partial^\mu h)^2 - V(h) + \frac{v^2}{4} \text{Tr}[(\partial_\mu U)^\dagger \partial^\mu U] \left( 1 + 2a \frac{h}{v} + b \frac{h^2}{v^2} + \cdots \right) \\
- \frac{v}{\sqrt{2}} (\bar{t}_L, \bar{b}_L) U \left( 1 + c_1 \frac{h}{v} + c_2 \frac{h^2}{v^2} + c_3 \frac{h^3}{v^3} + \cdots \right) \left( \begin{array}{c} y_t \nu_R \\ y_b \nu_R \end{array} \right) \text{h.c.} ,
\]
where \( V(h) \) is the Higgs potential, \( U \) is the Goldstone matrix of \( SU(2)_L \times SU(2)_R / SU(2)_V \)

\[
U = e^{i w^a \tau^a}. 
\]

Here \( v = 246 \text{ GeV} \) denotes the electroweak scale, \( \tau^a \) are the Pauli matrices (\( a = 1, 2, 3 \)), and \( w^a \) are the Goldstone bosons eaten by \( W^\pm, Z \). In general, the coefficients \( a, b, c_1, c_2 \) are independent unknown coefficients. The SM corresponds to \( a = b = c_1 = 1 \) and other couplings equal to zero. Note that the standard model gauge symmetry \( SU(2)_L \times U(1)_Y \) is the subgroup of \( SU(2)_L \times SU(2)_R \). After turning on the gauge coupling, one needs to replace the usual derivative with the gauge covariant derivative in the above equation as \( \partial_\mu \rightarrow D_\mu \), and the result in the unitary gauge can be easily obtained by setting \( U \rightarrow 1 \).

For convenience, we will work with the above effective Lagrangian with the gauge coupling being turned off.

To be specific, the general Higgs potential is denoted as

\[
V(h) = \frac{1}{2} m_h^2 h^2 + d_3 \left( \frac{m_h^2}{2v} \right) h^3 + d_4 \left( \frac{m_h^2}{8v^2} \right) h^4 + \cdots \\
≡ \frac{1}{2} m_h^2 h^2 + \frac{\lambda_3}{3!} h^3 + \frac{\lambda_4}{4!} h^4 + \cdots ,
\]
and accordingly the Goldstone matrix \( U \) can be parametrized as

\[
U = \sqrt{1 - \left( \frac{w_a \tau^a}{v} \right)^2 + i \frac{w_a \tau^a}{v}} ,
\]
where \( d_{3,4} \) are independent Higgs self-couplings with the SM limit \( d_3 = d_4 = 1 \). It is easy to see that the normalization of the Goldstone matrix satisfies the condition \( U^\dagger U = 1 \), thus the above parametrization for the Goldstone bosons is equivalent to the exponential one.

With this parameterization, the derivative-coupled interactions of the Goldstone bosons take a relative simple form as

\[
\text{Tr}[(\partial_\mu U)^\dagger \partial^\mu U] = \frac{2}{v^2} \partial_\mu w^a \partial^\mu w^a + \frac{2}{v^2} \left( \frac{w^a \partial_\mu w^a}{v^2 - \frac{w^2}{2}} \right)^2 ,
\]
which would give rise to the usual kinetic terms for \( w^a \) and their derivative-coupled interactions with the Higgs boson \( h \). In this work, we neglect the effects of heavy particles contributing to the contact interactions between gluons and the Higgs boson, \( h^a G^a_{\mu\nu} G^{a\mu\nu} \), as these effective couplings vanish when the heavy particles decouple. For simplicity, we assume these particles are heavy enough and thus the \( h^a G_{\mu\nu} G^{a\mu\nu} \) interactions can be safely negligible.
2.2 SM EFT: Higgs in the Unbroken Phase

Depending on nature of the Higgs boson, SMEFT could be a good general framework to parametrize the Higgs couplings. In the scenarios of Coleman-Weinberg Higgs and Tadpole-induced Higgs, the SMEFT cannot be utilized because of the non-decoupling behavior of new particles. On the other hand, the elementary Higgs and Nambu-Goldstone Higgs scenarios could be well described in the SMEFT framework, because of the decoupling feature of these new physics models. In the following, we present the SMEFT framework and provide the correspondence between the SMEFT and the Higgs EFT defined above.

From a bottom-up perspective, one can alternatively use higher dimensional operators of SMEFT to described new physics, if the new physics scale $\Lambda$ is much higher than the electroweak scale. The SM gauge symmetry $SU(2)_L \times U(1)_Y$ is manifested (or linearly-realized) in this case. Neglecting lepton-number violating operator at the dimension $D = 5$ (irrelevant to our study), the leading effective operators arise from dimension $D = 6$. The non-redundant set of $D = 6$ operators was laid out in Ref. [21], i.e. the Warsaw basis. There are totally 53 CP-even and 6 CP-odd effective operators at the $D = 6$ level. In this paper, we will focus on the CP-conserving case. By employing equations of motion, we can translate the $D = 6$ operators in Warsaw basis to the ones in the so-called strongly-interacting light Higgs (SILH) basis [22]; see the Rosetta package [31] for translating between different bases. The main difference between these two bases resides in the operators involving fermionic currents (in Warsaw basis) and the ones involving pure bosonic fields (in SILH basis); as $\sum_\psi Y_\psi O_{H\psi} \sim O_T, O_B$ and $O'_{Hq} + O'_{HL} \sim O_W$ where the sum is to sum over all fermions with $Y_\psi$ denoting the corresponding hypercharge of $\psi$ [32]. When considering $S$ parameter constraints, it is more convenient to use $O_B$ and $O_W$ instead of $O_{H\psi}$ or $O'_{Hq}, O'_{HL}$ as the latter operators can induce vertex corrections and modify the Fermi constant. Furthermore, the operators such as $O_{WW}$ and $O_{BB}$ (in the Warsaw basis) can be reparameterized by the linear combinations of the operators $O_{W,B,H,W,H,B,\gamma}$ (in the SILH basis) [32].

Regarding the processes of multi-Higgs production via gluon fusion, we list the following relevant $D = 6$ operators as

$$\mathcal{L}_{D=6} = \frac{c_H}{2\Lambda^2} \partial_\mu (H^\dagger H) \partial^\mu (H^\dagger H) - \frac{c_6}{\Lambda^2} \lambda (H^\dagger H)^3 - \left( \frac{c_t}{\Lambda^2} y_t H^\dagger H \bar{Q} L H c^t + \text{h.c.} \right) + \frac{\alpha_s}{4\pi} \frac{c_g}{\Lambda^2} H G_{\mu\nu}^a G^{a\mu\nu} + \frac{\alpha'}{4\pi} \frac{c_4}{\Lambda^2} H^\dagger H B_{\mu\nu} B^{\mu\nu}. \tag{2.6}$$

where $\lambda$ and $y_t$ are, respectively, the SM quartic Higgs coupling and the top Yukawa, $\alpha_s = g_s^2/4\pi$ and $\alpha' = e^2/4\pi$, $c_i (i = H, 6, t, g, \gamma)$ are unknown Wilson coefficients. It is worth pointing out there is another operator $O_T = \frac{1}{2} \left( H^\dagger \mathcal{D}_\mu H \right)^2$ that violates custodial symmetry at tree level, thus we neglect it in the following discussion. Further complication introduced by the flavor structure of the $D = 6$ Yukawa term will not be explored in this paper.

2.3 Relating SM EFT to Higgs EFT

Since the Higgs EFT does not care about the EWSB mechanism, it is more general description than the SM EFT. So we could identify the SM EFT Wilson coefficients with the Higgs EFT coefficients.
With appropriate field redefinition taken into account [33], we can match the Higgs-Goldstone couplings, Higgs-top couplings, and Higgs self couplings defined in Eqs. (2.1) and (2.3) to the Wilson coefficients in Eq. (2.6), as

\[ a = 1 - c_H \frac{v^2}{2\Lambda^2} + \mathcal{O}(\frac{1}{\Lambda^4}) , \]  
\[ b = 1 - c_H \frac{2v^2}{\Lambda^2} + \mathcal{O}(\frac{1}{\Lambda^4}) , \]  
\[ c_1 = 1 - c_H \frac{v^2}{2\Lambda^2} + c_t \frac{v^2}{\Lambda^2} + \mathcal{O}(\frac{1}{\Lambda^4}) , \]  
\[ c_2 = c_t \frac{3v^2}{2\Lambda^2} - c_H \frac{v^2}{2\Lambda^2} + \mathcal{O}(\frac{1}{\Lambda^4}) , \]  
\[ c_3 = c_t \frac{v^2}{2\Lambda^2} - c_H \frac{v^2}{6\Lambda^2} + \mathcal{O}(\frac{1}{\Lambda^4}) , \]  
\[ d_3 = 1 + c_6 \frac{v^2}{\Lambda^2} - c_H \frac{3v^2}{2\Lambda^2} + \mathcal{O}(\frac{1}{\Lambda^4}) , \]  
\[ d_4 = 1 + c_6 \frac{6v^2}{\Lambda^2} - c_H \frac{25v^2}{3\Lambda^2} + \mathcal{O}(\frac{1}{\Lambda^4}) . \]  

As we will see later, different Higgs couplings are usually correlated in the framework of specific models, and Higgs-Goldstone couplings are relevant to partial wave unitarity, while Higgs-top couplings and Higgs self couplings can be learnt from multi-Higgs production. With the purpose of probing the Higgs nature, we assume \( c_g \) and \( c_\gamma \) vanish in this paper for simplicity, although in general these two effective operators can be induced by heavy particles with nontrivial color or electric charges circulating in loops.

3 Various Higgs Scenarios

Contrary to the model independent discussions in the last section, we explicitly derive the Higgs effective couplings in some specific NP scenarios, i.e., the elementary Higgs, Nambu-Goldstone Higgs, Coleman-Weinberg Higgs, and Tadpole-induced Higgs. In order to identify the Higgs boson nature through Higgs self interactions, we will derive the cubic and quartic Higgs couplings for each scenario. Since different Higgs couplings are usually correlated for specific models in which the Higgs boson can have different nature, we will also present the relevant \( hVV(V = W^\pm, Z) \) and the \( ht\bar{t} \) and \( hht\bar{t} \) couplings if it is necessary.

3.1 Elementary Higgs Boson

For the case that the Higgs boson is an elementary scalar, we take the Ginzburg-Landau potential as the benchmark for the SM, and we implement the dimension-six operator \((H^\dagger H)^3\) in the potential to effectively describe the new physics contributions as shown in SMEFT. In scalar extensions, the singlet extension, the two Higgs doublet model, the real and complex triplets and quadruplet models [9, 34, 35] can all induce the \((H^\dagger H)^3\)
operator, which has been classified in Ref. [9] based on group theory construction. Similarly, integrating out new heavy fermions and gauge bosons at one-loop level could also induce the \((H^1 H)^3\) operator.

To be specific, the Ginzburg-Landau potential supplied by the contribution from the dimension-six operator \((H^1 H)^3\) is

\[
V = -\mu^2 H^1 H + \lambda(H^1 H)^2 + \frac{c_6}{\Lambda^2} \lambda(H^1 H)^3 \, ,
\]

(3.1)

where the Higgs doublet is \(H = 1/\sqrt{2} (0, v + h)^T\) in the unitary gauge, the Higgs boson mass term is \(m_h^2 = 2\lambda v^2 (1 + \frac{3c_6v^2}{2\Lambda^2})\), and the electroweak scale \(v\) is obtained by solving

\[
\mu^2 = \lambda v^2 (1 + \frac{3c_6v^2}{4\Lambda^2}) .
\]

(3.2)

In the SMEFT description, the Higgs trilinear and quartic couplings are

\[
d_3 = 1 + \frac{v^2}{\Lambda^2} - \frac{6v^2}{\Lambda^2} - c_H\frac{3v^2}{2\Lambda^2} + \mathcal{O}\left(\frac{1}{\Lambda^4}\right) ,
\]

(3.3)

\[
d_4 = 1 + \frac{6v^2}{\Lambda^2} - c_H\frac{25v^2}{3\Lambda^2} + \mathcal{O}\left(\frac{1}{\Lambda^4}\right) .
\]

(3.4)

Here \(c_H\), cf. Eq. (2.6), modifies the kinetic term of the Higgs field, which universally shift the Higgs couplings to electroweak gauge bosons. Thus the \(O_H\) operator is highly constrained. In some cases, the \(O_t\) operator is also generated from UV model, such as in the two Higgs doublet model case. It is constrained by the Higgs coupling measurements and the \(tth\) measurements. For the purpose of probing the Higgs-self couplings, we assume that the operator \((H^1 H)^3\) makes the most significant NP contribution and the other operators can be safely neglected.

### 3.2 Nambu-Goldstone Higgs Boson

The Higgs boson can be a pseudo Nambu-Goldstone boson [10, 11] arising from strong dynamics at the TeV scale. The pseudo Nambu-Goldstone Higgs corresponds to one of the broken generators for some spontaneously broken global symmetry \(G/H\), based on which all the operators, consistent with Higgs nonlinearity, can be systematically constructed [36, 37].

With its PNG nature, the general Higgs potential is approximately

\[
V(h) = -Af^4 \sin^2\left(\frac{h}{f}\right) + Bf^4 \sin^4\left(\frac{h}{f}\right) + \cdots .
\]

(3.5)

with higher terms being neglected, where \(A\) and \(B\) are the two coefficients whose values are determined by the specific dynamics responsible for generating the Higgs potential, and \(f\) denotes the decay constant. By naive dimensional analysis, the NP scale is expected to be at around \(4\pi f\). With the above notation, the coefficients \(A\) and \(B\) are positive. One can furthermore define a ratio between the electroweak scale and the scale \(f\) to denote the Higgs nonlinearity in this case. To be specific, the minimization condition of the Higgs potential
By expanding the Higgs potential in powers of $h$ after EWSB, we have

\[ V(h) = B f^2 \sin^2 \left( \frac{2\langle h \rangle}{f} \right) h^2 + B f \sin \left( \frac{4\langle h \rangle}{f} \right) h^3 + B \left( -\frac{1}{6} + \frac{7}{6} \cos \left( \frac{4\langle h \rangle}{f} \right) \right) h^4 + \cdots , \]  

Thus the Higgs mass is given by

\[ m_h^2 = 2 B f^2 \sin^2 \left( \frac{2\langle h \rangle}{f} \right) , \]  

and the trilinear and quartic Higgs couplings are respectively

\[ d_3 = \frac{B f \sin \left( \frac{4\langle h \rangle}{f} \right)}{\left( \frac{m_h^2}{2v} \right)} = \frac{1 - 2\xi}{\sqrt{1 - \xi}} , \]  

\[ d_4 = \frac{1}{6} B \left( -1 + 7 \cos \left( \frac{4\langle h \rangle}{f} \right) \right) \left( \frac{m_h^2}{8v^2} \right) = \frac{28\xi^2 - 28\xi + 3}{3 - 3\xi} , \]

where the ratio of $d_3$ and $d_4$ is obviously not one and depends on the parameter $\xi$.

Due to the Higgs nonlinear effects associated with its nature as a PNG, the Higgs couplings in the top sector, the $h\bar{t}t$, $hh\bar{t}$, $hhh\bar{t}$ couplings, and the Higgs couplings with electroweak gauge bosons can deviate from the SM values. Regarding the Higgs couplings in the top sector, the $h\bar{t}t$ and $hh\bar{t}$, $hhh\bar{t}$ couplings depend on the representation in which the top quark is embedded. As two benchmarks, we consider the minimal composite Higgs model (MCH or MCHM) [44, 45] where both the left-handed $t_L$ and the right-handed $t_R$ are embedded in the fundamental representation 5 of the global $SO(5)$ symmetry, and the composite twin Higgs model (CTH or CTHM) [46–48] where the left-handed $t_L$ is embedded in the fundamental representation 8 while the right-handed $t_R$ is a singlet of the global $SO(8)$ symmetry. The concrete results for the Higgs couplings are systematically derived in Ref. [23] and collected in Table 1.

### 3.3 Coleman-Weinberg Higgs Boson

Another theoretical attractive scenario is the Coleman-Weinberg Higgs, where the Higgs potential at the classical level is assumed to be scale invariant, i.e. only the quartic Higgs term

\[ \xi = \frac{v^2}{f^2} = \sin^2 \left( \frac{\langle h \rangle}{f} \right) = \frac{A}{2B} , \]  

(3.6)

It is nontrivial to realize a small $\xi$ (less than about 0.1) required by precision measurements of Higgs couplings and electroweak precision data. See, e.g., Ref. [38–41] for recent attempts to achieve this goal. It is also found experimentally challenging to extract out small $\xi$ values from observing Higgs coupling deviations at the LHC [42]. Note that the parameter $\xi$ is positive for compact cosets, while it is negative for non-compact cosets [43]. In this work, we only focus on compact cosets, as EWSB is hard to be triggered in models based on non-compact cosets.
is present at tree level [15–17]. However, with quantum corrections, the Higgs mass term is usually generated at the one loop level through the Coleman-Weinberg mechanism [15]. To be specific, the Higgs self couplings are essentially determined by the $\beta$-function of the quartic Higgs coupling $\lambda$, and the electroweak scale $v = 246$ GeV is generated at quantum level [17]. The $\beta$-function of the quartic Higgs coupling $\beta_\lambda$ is positive-definite, and accordingly the running quartic at the EW scale $\lambda(v)$ is negative [17], which corresponds to the minimum of the Higgs potential.

The general Coleman-Weinberg Higgs $h$ has the following potential

$$V(h) = Ah^4 + Bh^4 \log \frac{h^2}{\Lambda^2_{\text{GW}}},$$

(3.11)

where the coefficients are

$$A = \sum_i n_i \frac{m_i^4}{64\pi^2 v^4} \left(\log \frac{m_i^2}{v^2} - c_i\right), \quad B = \sum_i n_i \frac{m_i^4}{64\pi^2 v^4}.$$  

(3.12)

Here, the masses $m_i$ denote the masses of the particles circulating in the loop, which are defined in the vacuum background, $n_i$ denotes internal degrees of freedom, and $c_i$ is the renormalization-scheme dependent constant. Here the parameter $B$ is directly related to the $\beta$-function of the quartic Higgs coupling $\beta_\lambda$. The minimization condition $\frac{dV(h)}{dh} = 0$ leads to [16]

$$v = \langle h \rangle = \Lambda_{\text{GW}} \exp\left[-\frac{1}{4} - \frac{A}{2B}\right],$$

(3.13)

which causes relation between $A$ and $B$. At this minimum, the running quartic at the EW scale $\lambda(v)$ is negative. Since the VEV is determined from dimensionless parameters, this is one specific realization of the dimensional transmutation mechanism.

After expanding the above Higgs potential in powers of $h$ after EWSB, we have

$$V(h) \simeq 4B\langle h \rangle^2 h^2 + \frac{20}{3} B\langle h \rangle h^3 + \frac{11}{3} Bh^4 + \cdots.$$  

(3.14)

Here, all the Higgs self couplings are related to the parameter $B$ (or equivalently $\beta_\lambda$). Note that higher order terms, such as $h^5$, are neglected here. Therefore, the Higgs mass is

$$m_h^2 = 8B\langle h \rangle^2$$

(3.15)

and the trilinear and quartic Higgs couplings are, respectively,

$$d_3 = \frac{20}{3} B\langle h \rangle \left(\frac{m_h^2}{\sqrt{\mathcal{N}}^4}\right) = \frac{5}{3},$$

(3.16)

$$d_4 = \frac{11}{3} B \left(\frac{m_h^2}{8\langle h \rangle^2}\right) = \frac{11}{3}.$$  

(3.17)

---

3The specific form of the particles running in the loop is irrelevant at the one-loop order in the Higgs potential.

4For example, in the $\overline{\text{MS}}$ scheme, $c_i = \frac{5}{2}$ for gauge bosons while $c_i = \frac{3}{2}$ for scalars and fermions.
We note that the trilinear and quartic Higgs couplings are fixed at the one-loop order, small corrections to the above relations of $d_3$ and $d_4$ would appear only at the two-loop or higher orders \[17, 49, 50\].

### 3.4 Tadpole-Induced Higgs Boson

Another interesting scenario is the Tadpole-induced Higgs, namely the electroweak symmetry is spontaneously broken because of the existence of Higgs tadpole term. As a result, the Higgs self-couplings, both the Higgs trilinear and quartic couplings, can be largely suppressed with respect to the SM prediction. In such models, additional source of electroweak symmetry breaking other than the SM Higgs mechanism is needed. Specific realization of this class of model includes, e.g., bosonic technicolor model \[18, 51\]. In the typical technicolor models \[52, 53\], only the condensate of technifermions $\langle \bar{Q}_iQ_j \rangle \sim \Lambda_{\text{tech}}^4$ triggers EWSB, and thus it predicts no Higgs boson. However, this has been ruled out due to the discovery of the Higgs boson at the LHC. On the other hand, in the bosonic technicolor model, an elementary Higgs boson is also there to trigger EWSB with VEV $v_H$:

$$v_{EW}^2 \equiv v_H^2 + f^2,$$

(3.18)

where $f \equiv \Lambda_{\text{tech}}$. As both scalars can contribute the $W^\pm$ and $Z$ boson masses, the scale $f$ should be suppressed with respect to the electroweak scale $v_{EW}$, such that the $hVV (V = W^\pm, Z)$ couplings can be close to the SM predictions. This renders the scales $v_{EW} \simeq v_H \gg f$.

In low energy, the bosonic technicolor condensate could be parametrized as another effective scalar doublet field with the same quantum numbers of the Higgs doublet. For convenience, let us name this another doublet $\Sigma$ as the auxiliary doublet, and the $\Sigma$ field is interpreted as the condensate of technifermions $\Sigma \sim \langle \bar{Q}_iQ_j \rangle / \Lambda_{\text{tech}}^2$. The simplified Lagrangian \[19, 54\] for Tadpole-induced Higgs scenario is

$$\mathcal{L} = (D^\mu H)'(D_\mu H) + (D^\mu \Sigma)'(D_\mu \Sigma) - V(H, \Sigma)$$

(3.19)

where

$$V(H, \Sigma) = m_H^2 H^\dagger H - \left( \epsilon \Sigma^\dagger H + \text{h.c.} \right) - m_\Sigma^2 \Sigma^\dagger \Sigma + \lambda_S \left( \Sigma^\dagger \Sigma \right)^2.$$  

(3.20)

Note that the mass term of the Higgs doublet $H$ is positive, such that EWSB is not triggered by the $m_H^2 H^\dagger H$ term as the SM. To have Tadpole-induced mechanism dominant, the quartic $\lambda_H (H^\dagger H)^2$ should be sub-dominant (thus negligible) in the above Higgs potential. The vacuum structure is then parametrized as

$$\Sigma = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ f \end{pmatrix}, \quad H = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v_H \end{pmatrix}$$

(3.21)

where the VEV $f$ is obtained by the sector with auxiliary doublet alone, and $v_H$ is obtained by the competition of the $m_H^2 H^\dagger H$ term and the mixing term between the two scalar sectors. Such that

$$v_H = \frac{\epsilon f}{m_H^2}.$$  

(3.22)
More interestingly, the self couplings of the Higgs boson are highly suppressed in this class of model. Let us assume the Higgs state in the auxiliary scalar sector is heavy enough ($v_{EW} \ll m_{\Sigma}$) such that one can integrate out the auxiliary scalar and derive the tree-level effective potential for the Higgs boson. Because of the self interactions of the auxiliary scalar and the mixing between the auxiliary field and the Higgs boson, Higgs trilinear and quartic couplings are induced, as shown in Fig. 2. To be specific, we have the tree-level effective Higgs potential as

$$V(h) = \frac{1}{2} m_h^2 H^\dagger H - \epsilon f \sqrt{H^\dagger H} + \left(\frac{\epsilon^2}{m_{\Sigma}^2}\right)^2 \left(\sqrt{H^\dagger H}\right)^2 + \left(\frac{\epsilon}{m_{\Sigma}}\right)^3 \frac{m_{\Sigma}^2}{f} \left(\sqrt{H^\dagger H}\right)^3 + \left(\frac{\epsilon}{m_{\Sigma}^2}\right)^4 \frac{m_{\Sigma}^2}{4f^2} \left(\sqrt{H^\dagger H}\right)^4 + \cdots.$$  

(3.23)

In the above equation, all the terms of the self couplings of $h$ are suppressed if $m_{\Sigma}$ is sufficiently heavy, which in turn requires the self couplings of the auxiliary scalar field to be strong enough. For the physical Higgs field, one can perform a shift $h \rightarrow h + v_H$ after EWSB to remove the tadpole. In case when the term $\lambda_H (H^\dagger H)^2$ is present in the Higgs potential $V(H, \Sigma)$, Higgs self couplings can in general deviate from the prediction $d_3 \approx d_4 \approx 0$. In this work, we simply assume that the quartic $\lambda_H (H^\dagger H)^2$ vanishes, cf. Eq. (3.20).

3.5 Summary on Higgs Couplings

We collect all the relevant Higgs couplings in Table 1 for different NP scenarios, including the elementary Higgs (both the SM and SMEFT with the operator $O_6$), Nambu-Goldstone Higgs (MCH and CTH models), Coleman-Weinberg Higgs and Tadpole-induced Higgs. As we will see, these couplings are important for deriving theoretical constraints, including the partial wave unitarity and tree-level vacuum stability, and phenomenology study of the double-Higgs production $gg \rightarrow hh$ and triple-Higgs production $gg \rightarrow hhh$ at the LHC and future hadron colliders.

Below, we detail the specific assumptions made in each class of NP models for deriving the Higgs couplings listed in table 1.
relevant couplings | $a$ | $b$ | $c_1$ | $c_2$ | $c_3$ | $d_3$ | $d_4$
---|---|---|---|---|---|---|---
SM | $hVV$ | $hhVV$ | $h\bar{t}t$ | $hh\bar{t}t$ | $hhh$ | $hhhh$
SMEFT (with $O_6$) | 1 | 1 | 1 | 0 | 0 | 1 | 1
MCH$_{5+5}$ | $1 - \frac{\xi}{2}$ | $1 - 2\xi$ | $1 - \frac{3}{2}\xi$ | $-2\xi$ | $-\frac{7}{2}\xi$ | $1 - \frac{1}{2}\xi$ | $1 - \frac{3}{2}\xi$
CTH$_{8+1}$ | $1 - \frac{\xi}{2}$ | $1 - 2\xi$ | $1 - \frac{1}{2}\xi$ | $-\frac{1}{\xi}\xi$ | $-\frac{1}{6}\xi$ | $1 - \frac{3}{2}\xi$ | $1 - \frac{3}{2}\xi$
CW Higgs (doublet) | 1 | 1 | 1 | 0 | 0 | $\frac{5}{3}(1.75)$ | $\frac{11}{3}(4.43)$
CW Higgs (singlets) | 1 | 1 | 1 | 0 | 0 | $\frac{5}{3}(1.91)$ | $\frac{11}{3}(4.10)$
Tadpole-induced Higgs | $\simeq 1$ | $\simeq 1$ | $\simeq 1$ | 0 | 0 | $\simeq 0$ | $\simeq 0$

**Table 1:** Higgs couplings, defined in Eqs. (2.1) and (2.3), for the SM and different NP scenarios. For the Coleman-Weinberg Higgs scenario, we also present the Higgs self couplings in the parenthesis including the correction at the two-loop order, regarding two of the simplest conformal extensions for the scalar sector: SM Higgs doublet with another doublet [17], and SM Higgs doublet with two additional singlets [49].

- For the SMEFT scenario, we only include the $O_6$ operator, for simplicity, since almost all the other operators are (and will be further) constrained by precision Higgs coupling measurements.

- For the Nambu-Goldstone Higgs scenario, the deviation of Higgs self couplings only depends on the Higgs nonlinearity parameter $\xi$. We note that the value of $\xi$ has been constrained by the precision $hVV$ couplings to be around $\xi < 0.1$. To be concrete, we restrict ourselves in two specific benchmark models, MCH$_{5+5}$ and CTH$_{8+1}$. For consistency, we consider deviations for other Higgs couplings caused by Higgs nonlinear effects, but we neglect the contribution of composite states to Higgs couplings, by assuming that all the composite particles are heavy enough. The effects of composite particles in Higgs couplings have been systematically discussed in Ref. [23].

- For the Coleman-Weinberg Higgs scenario, we also simply assume all the other Higgs couplings, except Higgs self couplings $d_3$ and $d_4$, to be identical to the SM values. This can be achieved if extra scalar particles do not mix with the Higgs boson after EWSB. The Higgs self couplings are found to be universally $d_3 = \frac{5}{3}$ and $d_4 = \frac{11}{3}$ at the one-loop order, and their values at the two-loop order [17, 49] are also reported as in table 1.

- For the Tadpole-induced Higgs scenario, we approximate $d_3 = d_4 \simeq 0$, as they can be highly suppressed, though their exact values would depend on the self couplings of the auxiliary scalar field. We also simply neglect the mixing between the auxiliary doublet and the Higgs doublet, as it is required by the result of the precision $hVV$ coupling measurement.
4 Theoretical Constraints on Self-couplings

4.1 Tree-level Perturbative Unitarity

In this section, we aim to obtain the unitarity constraints on Higgs couplings defined after EWSB, especially the Higgs trilinear and quartic couplings. We adopt the method of coupled-channel analysis to obtain the optimal bound \([55, 56]\), and the most restrictive limit would come from the largest eigenvalue of the matrix for all the coupled scattering processes. For constraining the Higgs trilinear and quartic couplings, we therefore consider the electric-neutral channels for the scatterings between the top quark \((t)\), longitudinal \(W^\pm\) and \(Z\), and the Higgs boson at the energy \(\sqrt{s} \gg m_t, m_W, m_Z, m_h\). According to the Goldstone equivalence theorem, the longitudinal \(W^\pm\) and \(Z\) are equivalent to the Goldstone bosons \((w^a)\) when \(\sqrt{s} \to \infty\).

To be specific, the following coupled \(2 \to 2\) scattering processes at the tree level are considered

\[
\begin{align*}
t^{\lambda_1 \bar{P} \lambda_2} &\to w^{b \bar{P} \lambda_1} , \quad t^{\lambda_1 \bar{P} \lambda_2} \to w^{b \bar{P} \lambda_1} , \quad t^{\lambda_1 \bar{P} \lambda_2} \to h h , \\
w^{a \bar{P} \lambda_1} &\to w^{a \bar{P} \lambda_1} , \quad w^{a \bar{P} \lambda_1} \to w^{b \bar{P} \lambda_1} , \quad w^{a \bar{P} \lambda_1} \to h h , \\
h h &\to t^{\lambda_3 \bar{P} \lambda_4} , \quad h h \to w^{b \bar{P} \lambda_1} , \quad h h \to h h , \quad (4.1)
\end{align*}
\]

where \(\lambda_{1,2,3,4} = \pm\) denote the helicity of the initial-state and final-state top and anti-top quark, while \(a = 1, 2, 3\) and \(b = 1, 2, 3\) are the flavor indices for the initial and final state Goldstone bosons, respectively. It is worth noticing that the scattering process \(w^a w^a \to w^b w^b\) does not vanish only when \(a \neq b\).

In the isospin basis, the \(2 \to 2\) matrix element \(\mathcal{M}_{if}(\sqrt{s}, \cos \theta)\) can be decomposed into partial waves \((a_j)\) as

\[
\mathcal{M}_{if}(\sqrt{s}, \cos \theta) = 32\pi \sum_{j=0}^{\infty} \frac{2j + 1}{2} a_j(\sqrt{s}) P_j(\cos \theta) \quad (4.2)
\]

where \(P_j(\cos \theta)\) are the orthogonal Legendre polynomials. Therefore, partial waves are obtained as

\[
a_j(\sqrt{s}) = \frac{1}{32\pi} \int_{0}^{\pi} d\vartheta \sin \vartheta \ P_j(\cos \vartheta) \mathcal{M}_{if}(\sqrt{s}, \cos \vartheta) , \quad (4.3)
\]

which is required to be bounded at tree level as

\[
|\text{Re}(a_j)| < \frac{1}{2} , \quad (4.4)
\]

for satisfying partial wave unitarity. For the coupled channels listed above, the \(s\)-wave \((j = 0)\) scattering matrix at high energies, \(\sqrt{s} \gg m_t, m_W, m_Z, m_h\), is explicitly

\[
a_0(\sqrt{s}) = \frac{3}{16\pi} \frac{m_t}{v^2} \begin{pmatrix}
-(c_1^2 + 1)m_t & 0 & (1 - ac_1)\sqrt{3} & -2c_1\sqrt{3} \\
0 & -(c_1^2 + 1)m_t & (-1 + ac_1)\sqrt{3} & 2c_1\sqrt{3} \\
(1 - ac_1)\sqrt{3} & (-1 + ac_1)\sqrt{3} & \frac{s}{3m_t}(1 - a^2) & -\frac{s}{3m_t}(b - a^2) \\
-2c_2\sqrt{3} & 2c_2\sqrt{3} & -\frac{s}{3m_t}(b - a^2) & -d_4 m_h^2/m_t
\end{pmatrix} \quad (4.5)
\]
under the basis of

$$\left\{ t^+\bar{t}^+, t^-\bar{t}^-, \frac{1}{\sqrt{2}} w^aw^a, \frac{1}{\sqrt{2}} hh \right\}$$  \hspace{1cm} (4.6)$$

with the factors $\frac{1}{\sqrt{2}}$ accounting for identical particles in the initial and final states. Note that the states $t^+\bar{t}^-$ and $t^-\bar{t}^+$ do not contribute to the $s$-wave scatterings. Within specific models, we can diagonalize the scattering matrix in Eq. (4.5) numerically.

Elementary Higgs, CW Higgs, Tadpole-induced Higgs in $2 \rightarrow 2$ scatterings: The $s$-wave unitarity bounds on $d_3$ and $d_4$, obtained from the above $2 \rightarrow 2$ processes, are quite loose if the $hV_LV_L$ couplings ($V = W^\pm, Z$) equal to the SM predictions. This corresponds to the elementary Higgs, Coleman-Weinberg Higgs and Tadpole-induced Higgs. Moreover, many channels would further decouple when the $t\bar{t}hh$ contact interaction vanishes, and in this case we can solve the $s$-wave unitarity constraints on $d_4$ analytically. This leads to the result

$$\lim_{\sqrt{s} \to \infty} |a_0(\sqrt{s})| = \frac{|d_4|}{32\pi} \frac{3m_h^2}{v^2} < \frac{1}{2},$$  \hspace{1cm} (4.7)$$

roughly $|d_4| < 16\pi$. The constraint on $d_3$ could only be set at finite energy and it is only moderately bounded as $|d_3 - 1| < 5$ \cite{57}. Alternatively, this bound on $d_4$ can be translated into the bound on the Wilson coefficient $c_6/\Lambda^2$ for the case of SMEFT, which yields $|c_6| < \left( \frac{16\pi v^2}{3m_h^2} - 1 \right) \frac{\Lambda^2}{6\pi^2}.$

Figure 3: Unitarity constraints on the trilinear (left) and quartic (right) Higgs couplings for the Nambu-Goldstone Higgs scenario. The vertical axis denotes the unitarity-violating scale while the horizontal axis denotes the Higgs self interactions. The red line denotes the MCH, while the blue line denotes the CTH. The shaded region is excluded by unitarity.

Nambu-Goldstone Higgs in $2 \rightarrow 2$ scatterings: When $hV_LV_L$ couplings ($V = W^\pm, Z$) deviate from the SM values, the $s$-wave unitarity bound from $2 \rightarrow 2$ scatterings could be
quite stringent \(^5\). This applies to the case of the Nambu-Goldstone Higgs scenario due to Higgs nonlinearity. The unitarity violating scale is found to be \(\sqrt{s} \simeq 3\) TeV if the nonlinearity parameter is \(\xi \simeq 0.1\), which yields \(d_3 \simeq 1 - \frac{3}{2}\xi \simeq 0.85\). However, this unitarity bound could possibly be alleviated with appropriate composite resonances in the bosonic sector \(^5\). In this work, we do not consider them. In Fig. 3, we recast the unitarity constraints on Higgs self interactions \(d_3\) and \(d_4\), with \(\xi\) varying in the range \(0.01 < \xi < 0.15\) for the NG Higgs scenario.

CW Higgs and Tadpole-induced Higgs beyond \(2 \to 2\) scatterings: It is interesting to notice that a relatively stronger unitarity bound on the Higgs self-couplings can be obtained from \(2 \to n\) \((n > 2)\) processes if the Higgs potential is non-analytical \(^5, 6\), i.e. it corresponds to a pole in the Higgs potential when \(H^\dagger H \to 0\). This applies to the scenarios of Coleman-Weinberg Higgs and Tadpole-induced Higgs. The non-analytical Higgs potential would correspond to the non-decoupling behavior, such that the universal unitarity violating scale \(4\pi v \sim 3\) TeV is obtained, regardless of how much \(d_3\) and \(d_4\) deviate from the SM values \(^5, 6\). Schematically, for the high dimensional operator \(^6\)

\[
\mathcal{L}_{\text{int}} = \frac{\lambda_n}{n_1! \cdots n_r!} \phi_1^{n_1} \phi_2^{n_2} \cdots \phi_r^{n_r},
\]

the \(2 \to n\) \((n > 2)\) scattering process only matters when \(\lambda_n\) is an order-one coefficient \((\lambda_n \sim \mathcal{O}(1))\), i.e. unitarity requires the energy is bounded roughly as \(E < (1/\lambda_n)^{1/n}\) \(^6\). The stringent unitarity bound would come in the large \(n\) limit. Physically, \(\lambda_n \sim \mathcal{O}(1)\) is only possible in non-decoupling theories, because there is no large scale that is responsible for suppressing this coefficient \(\lambda_n\). On the other hand, one could expect the coefficient \(\lambda_n\) is highly suppressed by the cutoff scale in decoupling theories, then the unitarity bound from the \(2 \to n\) \((n > 2)\) process is very loose. Thus it is already enough to consider the conventional \(2 \to 2\) scatterings for decoupling theories.

Based on the above discussion, we summarize the tree-level partial wave unitarity bound in table 2 for each new physics scenario.

### 4.2 Tree-level Vacuum Stability

Even though the unitarity bound is not very tight for Higgs self couplings, the trilinear Higgs coupling \(d_3\) cannot be arbitrary large if the EW vacuum is required to be the global minimum. Based on the Higgs potential in Eq. (2.3), this requirement is formulated as

\[
\frac{dV(h)}{dh} = m_h^2 h + d_3 \frac{3m_h^2}{2v} h^2 + d_4 \frac{m_h^2}{2v^2} h^3 = 0.
\]

When \(9(d_3)^2 - 8d_4\) is positive or zero, the roots of the above equation are explicitly

\[
h_1 = 0; \quad h_2 = \frac{v \sqrt{9(d_3)^2 - 8d_4} - 3d_3}{2d_4}; \quad h_3 = \frac{v - \sqrt{9(d_3)^2 - 8d_4} - 3d_3}{2d_4}.
\]

\(^5\)The unitarity bound mainly results from the deviation of Higgs-Goldstone (eaten by EW gauge bosons in the unitary gauge) couplings. One can explicitly check the eigenvectors after diagonalizing the scattering matrix and find that the \(w^a w^a \to w^b w^b\) \((a \neq b)\) channel contributes the most to the eigenstate that violates the s-wave unitarity.
scenarios & unitarity constraints \\
SMEFT & $0 < c_6 < 1584$ for $\Lambda = 3$ TeV \\
NG Higgs & $\sqrt{s} < 4$ TeV for $\xi = 0.05$ \\
CW Higgs & $\sqrt{s} < 4\pi v \sim 3$ TeV \\
Tadpole Higgs & $\sqrt{s} < 4\pi v \sim 3$ TeV \\

Table 2. Tree-level unitarity constraints from the scatterings of the Higgs boson, the top quark, and longitudinal electroweak gauge bosons. For SMEFT and NG Higgs scenario, the bound is obtained from $2 \rightarrow 2$ scatterings. For CW Higgs and Tadpole-induced Higgs scenarios, the unitarity violating scale is roughly $4\pi v \sim 3$ TeV due to their non-decoupling nature of the theories [59]. Regardless of the deviation of Higgs self couplings, this value could be estimated from $2 \rightarrow n$ ($n > 2$) scatterings [60]. Note that we require $c_6$ to be positive, since the Higgs potential should be bounded from below.

In this case, $h_1 = 0$ corresponds to the EW vacuum, $h_3$ corresponds to another local minimum of the Higgs potential, while $h_2$ corresponds to the local maximum. Tree-level vacuum stability requires the EW local minimum to be the global minimum, i.e. $V(h_1) < V(h_3)$. When $9(d_3)^2 - 8d_4$ is negative, only one solution $h = 0$ exists for $dV(h)/dh = 0$, which corresponds to the only minimum of the Higgs potential. As a result, we obtain the tree-level vacuum stability bound on $d_3$ and $d_4$, as shown in Fig. 4 [6]. Consistent with Ref. [59], the conservative bound of Higgs trilinear coupling is obtained as $0 < \Delta_3 \equiv d_3 - 1 < 2$. Certainly this bound on $d_3$ can be slightly relaxed in case when $d_4$ is much larger than the SM values. As we see in Fig. 4, when $|\Delta_3| > 2$, $d_4$ is required be more than 10 times of the SM value. For illustration, we also mark several benchmark points for different Higgs scenarios.

5 Double-Higgs Production: Model Discrimination

In this section, we utilize the double-Higgs production cross section measurements to discriminate different Higgs scenarios, since these scenarios predict quite different trilinear Higgs couplings. At a high energy hadron collider, the gluon-gluon fusion channel is the dominant production mechanism for the double-Higgs boson production. This process has been widely considered in the literature for validating the SM cross section, measuring the Higgs trilinear coupling [61–78] and the $t\bar{t}hh$ coupling [79], to discriminate various NP scenarios [80–89]. It remains to be established that this processes can be seen at $5\sigma$ level at the LHC.

The ATLAS and CMS Collaborations have been looking for the $hh$ signal in the data collected so far at the LHC and have accordingly set upper limits on its production cross section [1–6]. Both collaborations have also examined the prospectes of detecting $hh$ signal at the high-luminosity LHC (HL-LHC) and the high-energy LHC (HE-LHC), respectively [7, 8].

\footnote{Given a consistent theory, $d_3$ and $d_4$ are usually correlated. Thus, we only focus on the region where both $d_3$ and $d_4$ are positive, rather than treating them as independent parameters.}
Certainly we notice higher powers of Higgs self couplings are relevant for stabilizing the EW vacuum for the NG Higgs scenario. Thus it is only an artifact that the NG Higgs scenario is in the shaded region, because of truncation of the full Higgs potential up to the order of $d_4$.

At the HL-LHC, without (with) systemic uncertainty, the signal can be measured at 31% (40%) accuracy relative to the standard model prediction with the significance $3.5\,\sigma$ ($3\,\sigma$), and the trilinear Higgs coupling can be constrained in the range $-0.1 < \frac{\lambda}{\lambda_{SM}} < 2.7$ and $5.5 < \frac{\lambda}{\lambda_{SM}} < 6.9$ ($-0.4 < \frac{\lambda}{\lambda_{SM}} < 7.3$). At the HE-LHC (27 TeV with 15 ab$^{-1}$ data), the signal can be measured at significance of $7.1\,\sigma$ and $11\,\sigma$, without systematic uncertainty, in the $b\bar{b}\gamma\gamma$ and $b\bar{b}\tau\tau$ channels, respectively [8]. A number of the above studies have performed detailed background analysis with optimized cut-based efficiency or with multivariate techniques. In this paper, we do not intent to perform detailed signal-to-background analysis. Instead, we utilize new-physics cross sections after primary cuts and obtain the double-Higgs production significance in new-physics scenarios, via recasting the SM cross section and backgrounds in the literature. As stated above, we mostly focus on double-Higgs production at the HE-LHC (27 TeV) and the FCC-hh (100 TeV) collider and explore possibility of distinguishing various scenarios and extracting the unknown Higgs couplings, especially trilinear Higgs coupling.

5.1 Cross Section and Distributions

With the effective Higgs couplings listed in Table 1, the total cross section for the double-Higgs production at hadron colliders can be written as

$$\sigma = c_1^4 \sigma_{b}^{SM} + c_1^2 d_3^2 \sigma_t^{SM} + c_1^3 d_3^2 \sigma_{bt}^{SM} + c_2^2 \sigma_{ttth} + c_1^2 c_2 \sigma_{tth} + c_1 d_3 c_2 \sigma_{t,ttth},$$

(5.1)
where we have six pieces for the form factors: three pieces are from the box contribution ($\sigma^\text{SM}_b$), the triangle contribution ($\sigma^\text{SM}_t$), and the interference of them ($\sigma^\text{SM}_{bt}$) for the SM-like diagrams, and the rest come from the new triangle contribution ($\sigma_{t\bar{t}hh}$), the interference of new triangle with the SM-like box ($\sigma_{b,t\bar{t}hh}$), the interference of new triangle with the SM-like triangle ($\sigma_{t,t\bar{t}hh}$). A representative set of diagrams, including triangle and box diagrams, are given in the Fig. 5 for illustration.

**Figure 5:** Different classes of diagrams for the $hh$ production. The third diagram occurs in models with $t\bar{t}hh$ coupling.

Methodology of the computation is discussed in Refs. [90, 91]. We use leading order CTEQ parton distribution functions, CT14lllo [92], and renormalization/factorization scale as $\sqrt{s}$. Numerical value for each form factor is listed in Table 3 at the 14, 27, and 100 TeV Proton-Proton colliders, respectively. To suppress the large QCD background, one needs to apply a large cut on the transverse momentum ($p_T$) of the Higgs boson, as discussed in the next paragraph. Therefore, the table also includes the cross sections with cut $p_T^h > 70$ GeV. No further detailed kinematic cuts are considered here, as we are not doing detailed signal-to-background study. These results are from leading order diagrams. The next-to-leading order (NLO) QCD corrections are large and can increase the cross section by about a factor of 1.7 [93]. Our numerical results in this section do not include this factor. For the 14 TeV collider, the SM cross section for no cut and $p_T^h > 70$ GeV are 17.2 fb and 15.4 fb, respectively. The corresponding values for the 27 TeV collider are 73.6 fb and 66.2 fb, respectively, which are about 4-5 times larger than the 14 TeV case. The cross section at 100 TeV collider are 830.1 fb and 756.8 fb, respectively, which are about 50 times large in comparison to the 14 TeV values.

As we see from Table 3, there is some interesting interference pattern between different classes of diagrams. This pattern can help us to understand the dependence of cross sections and distributions on various couplings. This will be discussed in more detail in the next section.

At the 27 TeV collider, for no cut the cross sections for Tadpole-induced Higgs model and Coleman-Weinberg model are 149.2 fb and 124.1 fb respectively, while with $p_T^h > 70$ GeV they are 44.2 fb and 40.3 fb respectively. For the $\xi = 0.05$ benchmark value, for no cut the cross section for the MCH and CTH models are 97.7 fb and 79.9 fb, while with $p_T^h > 70$ GeV they are 87.2 fb and 71.5 fb, respectively. The cross sections for 14 TeV and 100 TeV collider can be easily obtained using Eq. (5.1) and Table 3.
Table 3: Form factors as defined in Eq. (5.1) at the 14 TeV, 27 TeV, and 100 TeV Proton-proton colliders.

| Collider | $p_T^h$ | $\sigma_b^{SM}$ | $\sigma_t^{SM}$ | $\sigma_{b,t}^{SM}$ | $\sigma_{t,t}^{SM}$ | $\sigma_{tt}^{SM}$ |
|----------|---------|-----------------|-----------------|------------------|------------------|------------------|
| 14 TeV   | no cut  | 36.1            | 4.9             | -23.8            | -147.0           | 48.9             |
|          | $p_T^h > 70$ GeV | 29.6            | 2.9             | -17.1            | -122.4           | 36.3             |
| 27 TeV   | no cut  | 149.2           | 18.9            | -94.5            | -618.9           | 197.92           |
|          | $p_T^h > 70$ GeV | 124.1           | 11.6            | -69.6            | -524.5           | 151.1            |
| 100 TeV  | no cut  | 1607.6          | 184.3           | -961.8           | -6872            | 2077.3           |
|          | $p_T^h > 70$ GeV | 1370            | 118.8           | -732             | -5970            | 1645             |

Figure 6: Variation of the ratio of the new-physics cross section to that of the SM for $HH$ production with respect to the trilinear Higgs coupling $d_3$ as in the fundamental Higgs, Coleman-Weinberg Higgs and Tadpole-induced Higgs scenarios (upper row) and the parameter $\xi$ in Nambu-Goldstone Higgs scenario (lower row).

In Fig. 6, for illustration, we display the ratio of the new-physics to the SM cross sections in various Higgs potential scenarios at the 14 TeV LHC and the 27 TeV HE-LHC, respectively. In Fig. 7, this ratio is plotted in a 2-d plot as a function of $c_2$ and $d_3$ couplings. In this figure, the ratios for the SM Higgs, Coleman-Weinberg Higgs, and Tadpole-induced Higgs scenarios are marked. In the top row of Fig. 6, we see that the ratio of the cross sections increases for negative values and larger positive values of $d_3$. This behavior will be
explained below. The bottom row of the figure has the ratio as a function of the parameter \( \xi \) in the case of Nambu-Goldstone Higgs scenario. In the case of the CTH model, the cross section ratio slowly increases as \( \xi \) increases, and does not change much when the \( p_T^h > 70 \) GeV cut is imposed. The behavior of the cross section ratio in these models can be understood on the basis of interference pattern, as to be explained in the next section.

**Figure 7:** Cross section ratio \( \sigma/\sigma_{SM} \) as a function of \( c_2 \) and \( d_3 \); (a) without any cut, and (b) with the only kinematic cut \( p_T^h > 70 \) GeV. The standard model cross-sections, at the 27 TeV HE-LHC collider, for the above mentioned two cuts are 73.6 fb and 66.2 fb, respectively. For no cut the cross-sections for Tadpole-induced Higgs model and Coleman-Weinberg model are 149.2 fb and 124.1 fb, respectively, while with \( p_T^h > 70 \) GeV they are 44.2 fb and 40.3 fb, respectively. The magenta, blue, and cyan dots denote the ratios for Tadpole-induced Higgs model, the SM, and Coleman-Weinberg model, respectively.

In Fig. 8, we display the normalized hh invariant mass \( M(hh) \) and \( p_T^h \) distributions for the 14 TeV LHC and the 27 TeV HE-LHC. These distributions play role in determining suitable kinematic cuts to reduce the backgrounds. The upper row of Fig. 8 shows the normalized \( M(hh) \) distribution with a range of values of \( d_3 \). The case of \( d_3 \) shows an interesting two-peak structure in the normalized \( M(hh) \) distribution, arising from the competition between the triangle and box diagram contributions. We will come back to this discussion in the next section, around Fig. 9.

### 5.2 Interference Effects

As shown in Fig. 5, the trilinear Higgs coupling is present only in triangle diagrams. But as the box and triangle diagrams interfere, the trilinear Higgs coupling’s contribution to the cross section depends also on the box amplitude, and the interference is destructive. Some of the new Higgs potential scenarios would allow large deviations of the Higgs couplings,
and the cross section and distributions will change significantly. Moreover, the Nambu-Goldstone Higgs scenario also predicts non-zero $t\bar{t}hh$ coupling due to Higgs non-linearity, and there is correlation between the $t\bar{t}hh$ and $t\bar{t}h$ couplings. Because of this new $t\bar{t}hh$ interaction, two new triangle diagrams appear. These diagrams interfere with the triangle diagram containing trilinear Higgs coupling destructively, and with box diagram constructively. This happens as the $t\bar{t}hh$ coupling has a negative sign relative to $t\bar{t}h$ coupling in this scenario, as shown in Table 1. In the Table 3, we observe that for the SM couplings,

\footnote{In fundamental Higgs scenario, $t\bar{t}hh$ can also be induced via integrating out heavy particles, as listed in Eq. 2.13. Here for simplicity, we take the $t\bar{t}h$ coupling and the $hVV$ to be the SM ones, which eliminates the $t\bar{t}hh$ coupling.}

Figure 8: Normalized distributions for $hh$ production via gg fusion against partonic center of mass energy and $p_T$ of either Higgs. The case of $d_3 = 3$ shows an interesting feature, caused by the competition between the triangle and box diagram contributions, as explained in the text, around Fig. 9.
$d_3 = 1$, the pure box contribution is large, the pure triangle contribution is small, and the interference contribution is large and negative, i.e. destructive. This leads to the small total cross section of $pp \rightarrow hh$.

5.2.1 Interference effects without $\bar{t}thh$

Let us first consider scenarios without the $\bar{t}thh$ vertex. As one can see from Eq. (5.1), the pure triangle contribution depends quadratically on $d_3$, while the interference term depends linearly on it. However, the pure box contribution does not depend on $d_3$. For the negative values of $d_3$, the cross section keeps on increasing with more negative values of $d_3$, cf. Fig. 6, as both $\sigma^{SM}_t$ and $\sigma^{SM}_{bt}$ contributions increase. For the positive values of $d_3$, the cross sections first decreases, then increases, with larger values of $d_3$, as first $\sigma^{SM}_{bt}$ dominates and decreases the cross sections, then $\sigma^{SM}_t$ dominates which increases the cross sections. This explains the feature found in the upper row of Fig. 6.

![Figure 9](image.png)

**Figure 9**: Contribution of various classes of diagrams and their interference to the $M(hh)$ distribution of $hh$ production for $d_3 = 1$ and $d_3 = 3$. The triangle diagrams contribution and interference (negative) term get scaled by 9 and 3, respectively, when we go to $d_3 = 3$ from $d_3 = 1$. However, as “bx” does not depend on $d_3$, it remains the same. The peak of the total distribution gets shifted to left with increase in $d_3$ as the triangle diagram, being a s-channel one, contributes significantly near the threshold of $hh$ production.

To understand the feature found in Fig. 8, let us examine the contribution from each class of Feynman diagrams and their interference effect to the $M(hh)$ distribution. As shown in Fig. 9, the triangle diagram mostly contributes near the Higgs pair threshold, while the box diagram mainly contributes to the threshold of the top pair system. As $d_3$ increases, the contribution on the $M(hh)$ and $P^h_T$ distributions from the triangle diagram increases and eventually exceeds the box diagram when $d_3$ becomes very large. As $d_3 = 3$, both the triangle and box diagrams are sizeable, which, together with their interference effect, result in the two peaks in the $M(hh)$ and $P^h_T$ distributions, as shown in Fig. 8. Moreover, as we increase the minimum cut of the $p_T$ variable of the Higgs boson, which is
to suppress large QCD background further, the relative contribution of the pure triangle diagrams decreases more than the interference and the pure box term, as shown in Table 3. For the SM, under the $p_T^h > 70$ GeV cut on the final state Higgs bosons, the pure triangle contribution decreases by a factor of around 1.7, the magnitude of interference term by 1.4, and the pure box term by 1.2. This explains why, in Fig. 6, the minimum of the curves, where the pure triangle contribution starts to dominate over interference term, shifts to the right with the increase in $p_T^h$ cut. For the SM, since the triangle contribution is small, the reduction in the total cross section is not that steep with the increase in the minimum $p_T^h$, i.e., the total contribution decreases by a factor of 1.1 only. However, for larger positive $d_3$ values, the pure triangle contribution cannot be dominated by the negative interference as much as it used to do before applying any cuts. Thus, even though the cross section is large without any cut, imposition of some minimum $p_T^h$ cut would lead to a larger reduction in the cross section. For instance, for $d_3 = 10$, the total cross section is 288.9 fb with no $p_T$ cut; it reduces to 150.8 fb when a $p_T > 70$ GeV is applied, i.e. a reduction by a factor of 1.9. The cross section for any $d_3$, before and after cuts, can easily be obtained from Table 3.

At the 14 TeV HL-LHC, the double-Higgs production cross section is not large. Thus, in the case of the most promising final state signature ‘$bb\gamma\gamma$’, one can only have few tens of events, which only put very loose constraints on $d_3$. Nevertheless, the cross sections at the 27 TeV HE-LHC are about 5 – 6 times larger than that at the HL-LHC. Therefore, even for the rare decay signature of ‘$bb\gamma\gamma$’, one could have significant bound, and $d_3$ value can be determined within around 20% [8]. At this 27 TeV machine, there is a distinct possibility of distinguishing different Higgs potential models.

5.2.2 Interference effects with $t\bar{t}hh$

In the MCH and CTH models of Nambu-Goldstone Higgs scenario, in addition to the appearance of new $t\bar{t}hh$ vertex, the existing relevant vertices, such as $t\bar{t}h$ and $hhh$, also get modified from the SM ones, as shown in Table 1. In Ref. [23], a global fit on the MCH and CTH parameters was performed by using the available data from the LHC Run-2 data. The 95% CL limit on $\xi$ is obtained to be $\xi < 0.1$ for the MCH5 model. In our study, we will vary $\xi$ up to 0.1.

Fig. 6 shows the variation of the cross section with the parameter $\xi$. The rate of increase of the cross section in the MCH model is significantly larger than the CTH model. In both models, trilinear Higgs coupling is the same because of the universal form of the Higgs potential, but the $t\bar{t}h$ and $t\bar{t}hh$ couplings are different due to different fermion embeddings. From Table 3, we see that in the MCH model, the scaling of $(\sigma_{h}\,t\bar{t}hh - \sigma_{t\,t\bar{t}hh})$ is larger by a factor of four than that in the CTH model. The scaling of $\sigma_{t\bar{t}hh}$ is larger by a factor of 16 than in the MCH model. This term does not contribute much when $\xi$ is as small as 0.01 because of the $\xi^2$ scaling. It contributes significantly for $\xi = 0.1$ in the MCH model, while its contribution is still small in the CTH model. This explains the difference in the rate of increase of the cross section in the MCH model and the CTH model. Another feature found in Fig. 6 is that the rate of increase of the cross sections of the MCH and CTH models does not change noticeably with the $p_T^h$ cut. Finally, in Fig. 10, we show the importance of the $t\bar{t}hh$ coupling for increasing the cross section as a function of $\xi$. As the $\xi$ increases, even
Figure 10: Variation of different pieces of Eq. 5.1 with $\xi$ in MCH and CTH models at 14 TeV collider. The Magenta line (which shows the effect of $t\bar{t}hh$) crosses the blue line (which shows the effect of $t\bar{t}h$ and $hhh$ coupling) around $\xi = 0.06$.

though the contribution of the SM-like diagrams decreases, the total cross section increases due to the dominance of the $t\bar{t}hh$ contribution, most noticeably in the MCH model.

5.3 Model Discrimination and $\lambda_3$ Extraction

In this section, we investigate the ability of the 27 TeV HE-LHC and the 100 TeV FCC-hh collider to distinguish various new physics scenarios of Higgs potentials. At the HL-LHC, due to the limited cross section, it is difficult to constrain the trilinear Higgs coupling $d_3$. As the cross section of signal increases significantly at higher energy hadron colliders, the accuracy of measuring the total cross section, and thus the constraint on $d_3$, improves significantly.

It has been shown in the literature that the double Higgs boson production cross section of the SM, at the 27 TeV HE-LHC with integrated luminosity $15 \, \text{ab}^{-1}$, can be measured with the accuracy of 13.8% at the 1 $\sigma$ confidence level [74]. This accuracy would be further improved at the 100 TeV hadron collider with integrated luminosity $30 \, \text{ab}^{-1}$. Accordingly, the SM signal for double-Higgs production can be measured with the accuracy of 5% at the 1 $\sigma$ confidence level [74]. We use this information as the benchmark point and perform scaling to obtain the signal significance in various NP scenarios of Higgs potential. Using the fixed luminosity and recasting the backgrounds from Ref. [74], the significance is obtained via $Z = \Phi^{-1}(1 - 1/2p) = \sqrt{2}\text{Erf}^{-1}(1 - p)$ [94, 95], where $\Phi$ is the cumulative distribution of the standard Gaussian and Erf is the error function. In this case, the $Z$ value is

$$Z = \sqrt{2 \left[ n_0 \ln \frac{n_0}{n_1} + (n_1 - n_0) \right]}.$$  \tag{5.2}
Figure 11: Double-Higgs production at the 27 TeV hadron collider with integrated luminosity $15 \text{ ab}^{-1}$ (upper), and the 100 TeV hadron collider with integrated luminosity $30 \text{ ab}^{-1}$ (lower). The SM can be measured with the accuracy of 13.8% (at the $1\sigma$ confidence level) and 5% (at the $1\sigma$ confidence level), respectively, at the 27 TeV hadron collider and the 100 TeV hadron collider. The accuracy for other models are rescaled accordingly. The blue bars denote the expected accuracy for all the models.

Here, $n_0$ is the event number $n_0 = n_b + n_s$, where $n_b$ denotes the background event number and $n_s$ denotes the signal event number rescaled in each NP scenarios as

$$n_s \sim \frac{\sigma_{\text{SM after all cuts}}}{\sigma_{\text{NP after PT cuts}}} \sigma_{\text{after PT cuts}}.$$  \hspace{1cm} (5.3)

And $n_1 = n_b + n'_s$, with $n'_s$ being the signal event number that can be constrained at the $1\sigma$ confidence level, which can be obtained by solving Eq. (5.2) with $Z=1$ for a given $n_0$. With $n_s$ and $n'_s$, the relative accuracy for each NP scenario is obtained as $|n_s - n'_s|/n_s$. As expected, larger cross sections lead to smaller relative errors for different new physics models.
The results are shown in Fig. 11. At the 27 and 100 TeV colliders, the information on the total cross sections of double-Higgs production is already sufficient to distinguish new physics scenarios with different Higgs potentials. The following conclusions can be drawn:

- For SMEFT with non-vanishing $O_6 \sim (H^1 H)^3$ operator, the total cross section tends to be smaller than that of the SM. Because of the tree-level vacuum stability constraints discussed in Sec. 4, the Wilson coefficients of the $O_6$ operator is preferred to be positive, which leads to $d_3$ to be larger than one and in turn to the smaller cross section as shown in Fig. 6. It leads to the $1\sigma$ relative accuracy being $29.4\%$ at the 27 TeV HE-LHC, and $10.9\%$ at the 100 TeV hadron collider, respectively, for $d_3 = 2$.

- For Nambu-Goldstone Higgs, the total cross section tends to be larger than the SM prediction because of the presence of the contact $t\bar{t}hh$ coupling. Since the top quark can be embedded in different representations, we show different Nambu-Goldstone Higgs models can also be distinguished. The $1\sigma$ relative accuracy at $1\sigma$ confidence level is about $10\%$ at the 27 TeV HE-LHC, and about $5\%$ at the 100 TeV hadron collider, respectively, for $\xi \simeq 0.1$.

- The trilinear Higgs coupling in the Coleman-Weinberg Higgs scenario is universally predicted to be $d_3 = 5/3$. So, similar to SMEFT, models of Coleman-Weinberg Higgs also have smaller cross section with respect to the SM. The $1\sigma$ relative accuracy is
about 23% at the 27 TeV HE-LHC, and about 4.7% at the 100 TeV hadron collider, respectively.

- The trilinear Higgs coupling in the Tadpole-induced Higgs scenario is highly suppressed. Therefore Tadpole-induced Higgs models can have much larger cross section with respect to the SM, due to very small $d_3$. It turns out this scenario could be measured very well at both the 27 TeV HE-LHC (relative accuracy of 7.4% at the 1σ confidence level) and the 100 TeV hadron collider (relative accuracy of 2.7% at the 1σ confidence level), and it can be well discriminated from the SM.

After measuring the total cross section of the double-Higgs production up to certain precision, we would like to extract the information on $d_3$ for the given experimental precision. In Fig. 12, assuming the measured accuracy of the double-Higgs production cross section is 10% and 20% respectively, we extract the parameter range for the trilinear Higgs coupling $d_3$. We use $\tilde{d}_3$ to denote the scaled $d_3$. As shown in Fig. 12, we find the range are $0.86 < \tilde{d}_3/d_3 < 1.15 \cup 4.83 < \tilde{d}_3/d_3 < 5.12$ (0.73 < $\tilde{d}_3/d_3 < 1.31 \cup 4.67 < \tilde{d}_3/d_3 < 5.25$) if the accuracy is 10% (20%) for $d_3 = 1$, and $0.94 < \tilde{d}_3/d_3 < 1.07 \cup 1.92 < \tilde{d}_3/d_3 < 2.06$ (0.88 < $\tilde{d}_3/d_3 < 1.16 \cup 1.83 < \tilde{d}_3/d_3 < 2.11$) if the accuracy is 10% (20%) for $d_3 = 2$, respectively.

In Fig. 13, we show the parameter space of general effective couplings $c_2$ and $d_3$ (with fixed $c_1$) that can be constrained by the double-Higgs production at the 27 TeV HE-LHC, assuming the 1σ accuracy is 10% and 20%, respectively. The scaling factors of the trilinear Higgs coupling and the contact $t\bar{t}hh$ coupling are denoted as the ratio $\tilde{d}_3/d_3$ and $\tilde{c}_2/c_2$, respectively. Compared to the MCH model, the constrained regions in the CTH model are more steep as the absolute value of $c_2$ in the assumed CTH model is smaller than that in the assumed MCH model, cf. Table. 1. Hence cross section does not change much with the scaling of $c_2$. Overall, we see that the 27 TeV HE-LHC can already set strict bounds on these Higgs couplings.

6 Triple-Higgs Production: Shape Determination

In this section, we investigate the possibility and sensitivity to measure the quartic Higgs coupling, $d_4$, by using the $hhh$ production via gluon fusion, $gg \rightarrow hhh$. This process can help in a better understanding of the shape of the Higgs potential.

As discussed in the literature [96–103], measuring the quartic Higgs coupling in the three Higgs production channel is not easy even at the 100 TeV hadron collider. This is because the signal of triple-Higgs production $pp \rightarrow hhh$ is too small as compared to its backgrounds. Even worse, the contribution of the quartic coupling is over-shadowed by other couplings. This is because, the quartic coupling appears in a very diagrams which make very small contribution to the total cross section. The quartic Higgs coupling is only constrained in the ranges of $[-20, 30]$ (at the 2σ CL) by three Higgs production at the 100 TeV hadron collider with 30 fb$^{-1}$ data [99]. In another approach, there have been attempts to measure trilinear and quartic Higgs couplings indirectly using higher order loop corrections [75, 104, 105]. These indirect searches put quite loose bound on the quartic Higgs
coupling at future colliders, such as the double Higgs production at the future linear collider (ILC). A partial list of other related studies is included as Refs. [106–108].

To further pin down the quartic Higgs coupling, it is straightforward to utilize the triple-Higgs production channel at the 100 TeV hadron collider with high luminosity run. We calculate the triple-Higgs production cross sections with general parametrization of new physics effects in different scenarios. We consider five scenarios: Independent scaling of SM trilinear and quartic Higgs couplings, the SMEFT models with correlated trilinear and quartic Higgs coupling, the Nambu-Goldstone Higgs, Coleman-Weinberg Higgs and Tadpole-induced Higgs models. We shall first compute and discuss cross sections and distributions in these models, then we estimate how well the quartic Higgs coupling can be measured, assuming other couplings are already determined by other experiments. It is expected that one could determine the $t\bar{t}h$ coupling, trilinear Higgs coupling, and $t\bar{t}hh$ coupling more precisely before measuring the quartic Higgs coupling.

6.1 Cross Section and Distributions

As shown in Fig. 14, there are several basic classes of Feynman diagrams contributing to the process $gg \to hhh$, i.e. the pentagon-class diagrams, box-class diagrams, and triangle class diagrams. In the pentagon-class diagrams, there is no Higgs self-coupling; the main coupling is $t\bar{t}h$ coupling. In the box-class diagrams, trilinear Higgs coupling plays the major role. Only the triangle-class diagrams have dependence on both the trilinear and quartic Higgs couplings. However, only few diagrams depend on the quartic Higgs coupling 8. Besides, the relative contribution of the triangle-class diagrams is comparatively small. Because of this, the process $gg \to hhh$ is only moderately sensitive to quartic Higgs coupling. The cross section could change significantly only with large modification in the quartic Higgs coupling, and in this case, the trilinear Higgs coupling would also deviate from the SM accordingly.

![Figure 14: Different classes of diagrams for $hhh$ production in SM.](image)

Furthermore, as shown in Fig. 15, several new diagrams would appear if additional $t\bar{t}hh$ and $t\bar{t}hhh$ couplings are non-zero. This scenario is realized explicitly, e.g. in the Nambu-Goldstone Higgs case, because of the Higgs non-linearity. In these scenarios, there is strong connection between the $t\bar{t}h$ coupling with $t\bar{t}hh$ and $t\bar{t}hhh$ couplings. As we will

8To be specific, for each quark flavor in the loop, there are 24 pentagon-class diagrams, 18 box-class diagrams, and 8 triangle-class diagrams. Out of these 50 diagrams, only two triangle diagrams have dependence on quartic Higgs coupling.
σ = c₁^6 σ_{p,SM} + c₁^4 d₃^2 σ_{p,b}^SM + c₁^3 d₃^1 σ_{p,3t}^SM + c₁^2 d₃^1 σ_{p,4t}^SM + c₁^5 d₄^3 σ_{p,b}^SM + c₁^2 d₃^2 σ_{p,3t}^SM + c₁^4 d₄^3 σ_{p,4t}^SM +
+ (c₁^4 c₂ σ_{p,b−2t2h} + c₁^3 c₃ c₂ σ_{b−2t2h} + c₁^2 d₃^2 c₂ σ_{3t,b−2t2h} + c₁^2 d₄ c₂ σ_{4t,b−2t2h} + c₁^2 d₃^2 c₃ σ_{b−2t2h} +
+ c₁^2 c₃ c₂ σ_{p,t−2t2h} + c₁^2 c₃^2 c₂ σ_{b,t−2t2h} + c₁ c₂ c₃^3 σ_{3t,t−2t2h} +
+ c₁ c₂ c₃^2 d₃ σ_{b−2t2h,t−2t2h} + c₁ c₂^2 d₃^2 c₁ σ_{b,t−2t2h}) +
+ (c₁^3 c₃ σ_{p,t−2t3h} + c₁^2 c₃^2 c₃ σ_{b,t−2t3h} + c₁ d₃^2 c₃ σ_{3t,t−2t3h} +
+ c₁ d₄ c₃ σ_{4t,t−2t3h} + c₁ c₂ c₃ σ_{b−2t2h,t−2t3h} + c₂ d₃^2 c₃ σ_{b−2t2h,t−2t3h} + c₃^3 σ_{t−2t3h} ) ,

(6.1)

where individual contributions of the diagrams are separated, and one can explicitly read off their dependence on Higgs couplings.

We carry out the calculation in the way discussed in Refs. [90, 91]. We use FORM [109] to compute the trace of gamma matrices in the amplitude and to write the amplitude in terms of tensor integrals. These tensor integrals are computed using an in-house package, OVReduce [90], which implements the Oldenborgh-Vermaseren [110] technique of tensor integral reduction. Scalar integrals are computed using the package OneLOop [111]. We use leading order CTEQ parton distribution functions, CT14llo [92], and the renormalization (and factorization) scale as \( \sqrt{s} \). The numerical value of each individual term of the cross section is calculated and summarized in Table 4. Here we do not include the higher order QCD correction, which may lead to a K-factor (the ratio of next-to-leading to leading order cross section) of about 2 [112]. Due to the extremely small cross section of this process, the diagrams with \( t\bar{t}hh \) and \( t\bar{t}hhh \) couplings make very large contribution, which renders it more complicated to extract out the quartic Higgs coupling.

In the process of \( pp \rightarrow hhh \), there is strong destructive interference between different classes of diagrams. Interference between pentagon, box, and triangle diagrams plays a crucial role in dictating the cross section and distributions. Before we discuss the interference pattern and the extraction of quartic Higgs coupling, we first obtain the contribution of each class of diagrams to the total cross section. To be specific, the total cross section is

\[
\sigma = \sigma_p + \sigma_b + \sigma_{3t} + \sigma_{4t} + \sigma_{p,b} + \sigma_{b,3t} + \sigma_{b,4t} + \\
\sigma_{p,3t} + \sigma_{p,4t} + \sigma_{b,p,3t} + \sigma_{b,p,4t} + \\
\sigma_{b,3t,4t} + \sigma_{p,b,3t,4t} + \\
\sigma_{b,3t,4t} + \sigma_{p,b,3t,4t},
\]

Figure 15: New diagrams for \( hhh \) production in the presence of \( t\bar{t}hh \) and \( t\bar{t}hhh \) vertices.
and the large QCD backgrounds, we only present results at the 100 TeV hadron collider. Basic $p_T$ cuts are also implemented for each Higgs boson in the final state. At the 100 TeV collider, the SM cross-sections for no cut and $p_T > 70$ GeV cut are 2987 ab and 1710 ab, respectively. For clearness, we summarize the total cross sections of double and triple Higgs productions for the SM in Fig. 16 at the 14 TeV LHC, the 27 TeV HE-LHC and the 100 TeV hadron collider.

Table 4: Numerical values of various terms of Eq. 6.1 at the 100 TeV hadron collider.

| Parts | $p_T^h > 70$ GeV | $p_T^t > 70$ GeV | $p_T^t > 70$ GeV |
|-------|-----------------|-----------------|-----------------|
| $\sigma^{SM}_{p,\text{no cut}}$ | 7777 | 3526 | -41310 |
| $\sigma^{SM}_{b,\text{no cut}}$ | 4113 | 1542 | 39685 |
| $\sigma^{SM}_{t,\text{no cut}}$ | 92.2 | 26.0 | -3960 |
| $\sigma^{SM}_{b,\text{no cut}}$ | 46.57 | 22.52 | -3164 |
| $\sigma^{SM}_{p,\text{no cut}}$ | -8026 | -2873 | 130729 |
| $\sigma^{SM}_{b,\text{no cut}}$ | 381.5 | 7.5 | 1363 |
| $\sigma^{SM}_{p,\text{no cut}}$ | 133.5 | -49.5 | -13626 |
| $\sigma^{SM}_{b,\text{no cut}}$ | -985 | -298 | 2412 |
| $\sigma^{SM}_{b,\text{no cut}}$ | -673.3 | -266 | 1943 |
| $\sigma^{SM}_{t,\text{no cut}}$ | 121.5 | 45.0 | -66447 |
| $\sigma^{SM}_{t,\text{no cut}}$ | 21774 | 12329 | 21774 |

Figure 16: We summarize the total cross sections of the $pp \to hh$ and $pp \to hhh$ for the SM at the 14 TeV LHC, the 27 TeV HE-LHC and the 100 TeV hadron collider, respectively. The blue lines denote the cross sections without cut, and the red lines denote the ones with rudimentary cuts. Here we do not include the QCD K factors, which are known as around 1.7 [93] for $pp \to hh$ and around 2 [112] for $pp \to hhh$, respectively.

At the 100 TeV collider, for no cut the cross-sections for Tadpole-induced Higgs model and Coleman-Weinberg model are 7796 ab and 1272 ab, while with $p_T^h > 70$ GeV they are 3579 ab and 836 ab, respectively. For the $\xi = 0.05$ benchmark value, for no cut the cross-
section for the MCH and CTH models are 5033 ab and 3479 ab, while with $p_T^h > 70$ GeV they are 3302 ab and 2057 ab, respectively.

Based on these numerical values, we display the cross sections in the $(d_3, d_4)$ parameter plane in Fig. 17 and the $\xi$ dependence in Fig. 18, for different new physics scenarios, with or without including the contact $t\bar{t}hh$ and $t\bar{t}hh$ couplings. Fig. 17 shows the total cross section $\sigma$ as a function of the trilinear and quartic Higgs couplings, i.e. $d_3$ and $d_4$. We see there is significant increase in the cross section for zero or negative $d_3$. This is because, then the largest negative interference term between box and pentagon diagrams $\sigma_{SM}^{p,b}$ either vanishes or becomes positive. There is only marginal increase in the cross section for zero or negative value of $d_4$. In this figure, we also mark the SM, the Coleman-Weinberg Higgs scenario, the Tadpole-induced Higgs scenario by blue, cyan, and magenta dots, respectively. The orange line denotes the SMEFT with nonzero $O_6 \sim (H^\dagger H)^3$ operator under the linear expansion as in Eq. (2.13). The Nambu-Goldstone Higgs scenario is presented in Fig. 18, where all the Higgs couplings, and so the cross section, depend on the nonlinear parameter $\xi$. To be concrete, we consider two specific models, i.e. MCH and CTH models, and results are shown in Fig. 18. Compared to MCH, the cross section of the CTH remains close to the SM prediction (for the case of $\xi = 0$).

![Figure 17](image-url)

**Figure 17**: Cross section ratio $\sigma/\sigma_{SM}$ for the scaling of trilinear and quartic Higgs couplings for various cuts. At the 100 TeV collider, the standard model cross section for no-cut and $p_T > 70$ GeV cut are 2987 ab and 1710 ab, respectively. The blue, cyan, and magenta dots denote the SM, CW Higgs and Tadpole-induced Higgs model, respectively. The orange dashed line denotes the SMEFT (with non-vanishing $O_6$) for $d_3$ in the range of $[5/6,2.5]$.

To complete the discussion in this section, we present several basic distributions. In Fig. 19, we show the invariant mass, $M(hhh)$, distribution for various $d_3$ and $d_4$ values, and the normalized plots to examine the modification of the shape of the distributions. We observe contrasting behavior near the threshold of triple-Higgs boson production. In the case of $d_3$, there is larger increase in the cross section near threshold for its negative and
zero value, while decrease for positive values of $d_3$. The behavior is opposite in the case of $d_4$. Most of the increase is for smaller values of the invariant mass of triple-Higgs system, up to about 700 GeV, and it is near the threshold where the triangle diagram with quartic Higgs coupling is important.

Figure 18: Cross section ratio with parameter $\xi$ in the Minimal Composite Higgs (MCH) and Composite Twin Higgs (CTH) Models at the 100 TeV collider (FCC-hh).

Figure 19: Distributions with partonic center-of-mass energy $M(\text{hhh})$ for $hhh$ production via gluon-gluon fusion with different benchmark values of $d_3$ and $d_4$ at the 100 TeV collider. No cut on $p_T$ of Higgs bosons has been imposed.

6.2 Interference Effects

In this section, we investigate the interference patterns for the process of triple-Higgs production $pp \rightarrow hhh$, for better understanding variation of total cross section in different Higgs scenarios.

6.2.1 Interference without $t\bar{t}hh$ or $t\bar{t}hhh$

Let us first consider the cases without the $t\bar{t}hh$ and $t\bar{t}hhh$ couplings. There are 10 relevant terms, as shown in Eq. (6.1). The first four terms are always positive, and the rest of the six terms are interference terms and can be either positive or negative. It is worth reiterating the fact that the $d_4$ dependence of the cross section also depends on trilinear Higgs coupling.
$d_3$. As shown in the left panel of Fig. 20, the cross section first decreases and then increases within the range $-1 < d_3 < 6$. In addition, we show the variation of cross section, as the green band, with the quartic Higgs coupling $d_4$ varying within $0 < d_4 < 10$. In the right panel of Fig. 20, we explicitly see the variation of $\sigma/\sigma_{SM}$ with $d_4$, while $d_3$ is fixed. Although it is theoretically less plausible for a large $d_3$, e.g. $d_3 = 6$, hinted by vacuum stability, we still include this possibility here. In that case, the cross section only moderately varies with $d_4$ values. Hence, there will be degeneracy in $d_4$ determination if $d_3$ is around 5 to 6.

![Figure 20: Variation of Cross section ratio $\sigma/\sigma_{SM}$ with $d_3$ and $d_4$ at a 100 TeV collider.](image)

In the left figure, we see a band for $d_4$ in the range $[0,10]$. In the right figure, variation with $d_4$ for fixed $d_3$ is shown. The standard model cross section for no-cut and $p_T^h > 70$ GeV cut are 2987 ab and 1710 ab, respectively.

### 6.2.2 Interference with $t\bar{t}hh$ and $t\bar{t}hhh$

In this subsection, we discuss new physics scenarios in which the $t\bar{t}hh$ and $t\bar{t}hhh$ are non-vanishing, e.g. the Nambu-Goldstone Higgs scenario, and investigate the interference terms involving these couplings in details. In this scenario, all the Higgs couplings are correlated to the parameter $\xi$ due to Higgs non-linearity.

In the Fig. 21, we show the interference effect of the $t\bar{t}hh$ and $t\bar{t}hhh$ couplings in two specific NG Higgs models, i.e. the MCH and CTH models. As expected, in the case of CTH model, the contribution of these coupling remains to be very small, except at large value of $\xi$, where it is also not that significant. However, in the case of MCH model, both the $t\bar{t}hh$ and $t\bar{t}hhh$ couplings play important role. At larger value of $\xi$, the significant increase in the cross section is induced by these couplings. As $\xi$ increases, the contribution ($\sigma_{SM}$) of SM-like diagrams decreases due to smaller $t\bar{t}h$, $d_3$, and $d_4$ couplings, but the contribution of diagrams with $t\bar{t}hh$ and $t\bar{t}hhh$ couplings increase.

In Fig. 22, the ratios of the cross sections of MCH and CTH models with respect to the SM value is shown, and the ratios are varying with the parameter $\xi$. The green band shows variation of the ratios due to scaling of the quartic Higgs coupling, denoted by $\tilde{d}_4/d_4$. We see the variation due to quartic Higgs coupling scaling decreases with larger values of the parameter $\xi$, and the dashed line is for $d_4 = 1$. 

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**Figure 20**: Variation of Cross section ratio $\sigma/\sigma_{SM}$ with $d_3$ and $d_4$ at a 100 TeV collider. In the left figure, we see a band for $d_4$ in the range $[0,10]$. In the right figure, variation with $d_4$ for fixed $d_3$ is shown. The standard model cross section for no-cut and $p_T^h > 70$ GeV cut are 2987 ab and 1710 ab, respectively.


Figure 21: Cross section [in ab] with parameter $\xi$ in the Minimum Composite Higgs Model (MCH) and the Composite Twin Higgs Model (CTH). The magenta line shows the effect of $t\bar{t}hh$ coupling. In MCH model, it exceeds the "SM-like" effect ($\sigma_{\text{SM}}^{\text{Mod}}$) around $\xi = 0.05$. The blue line shows the effect of $t\bar{t}hhh$ coupling, which includes interference (which is negative for the shown range of $\xi$) of $t\bar{t}hh$ with $t\bar{t}hh$ as well.

Figure 22: Variation of the ratio of cross section to the SM value with $\xi$ and $d_4/d_4$ at the 100 TeV Proton-Proton collider. The band is obtained by varying $d_4/d_4$ in the range of [0,10] for the MCH and the CTH model, respectively. In the 2nd row and bottom row, how the cross section changes with $d_4/d_4$ for fixed $\xi$ have been shown for the two models. The standard model cross section for no-cut and $p_T^h > 70$ GeV cut are 2987 ab and 1710 ab, respectively.

6.3 Shape Determination and $\lambda_4$ Extraction

Here we investigate how to measure the quartic Higgs coupling at the 100 TeV hadron collider to validate various new physics scenarios. Similar to studying the double-Higgs production process, we do not perform any detailed collider analysis, but to utilize the
Table 5: The integrated luminosity required for the 5σ discovery of the SM and other new physics scenarios. Here, we take $d_3 = 2$ and $d_4 = 7$ for the SMEFT, and $\xi = 0.1$ for the MCH of NG Higgs scenario. These numbers for required luminosity are obtained without including the NLO QCD K factor which is about a factor of 2 [112]. By including this K factor, we expect the luminosity would be slightly reduced for discovering these NP scenarios.
Figure 23. Triple-Higgs production at the 100 TeV hadron collider with integrated luminosity 45000 ab$^{-1}$, such that the SM can be measured with the accuracy of 20% (at the 1σ confidence level). The accuracy for other models are rescaled accordingly. The blue bar denotes the prospected accuracy for testing the model.

and $\tilde{d}_4/d_4$ are respectively the scaling factors for the $t\bar{t}hh$ coupling and the quartic Higgs coupling, with other couplings are predicted and fixed in the given models. We do not include the accuracy plots for the Tadpole-induced Higgs scenario ($d_3 \simeq 0, d_4 \simeq 0$) because it is hard to pin down the quartic Higgs coupling in this scenario because of its tiny value.

Here, some conclusions are in order. For the SM, the scaling factor is constrained to be within the range of $0.3 < \tilde{d}_4/d_4 < 1.82 \cup 10.13 < \tilde{d}_4/d_4 < 11.66 \cup 0 < \tilde{d}_4/d_4 < 2.85 \cup 9.10 < \tilde{d}_4/d_4 < 12.28$, if the accuracy is 10% (20%). For the NP scenarios, we give a brief summary as follows.

- For the SMEFT, we note that the bound on the quartic Higgs coupling will be generally quite loose, unless cross sections can be measured with better than 10% accuracy. However, as shown in Fig. 20, the bounds on the quartic coupling $d_4$ could be tight when the trilinear coupling $d_3 \simeq 2 - 3$, in which case the $hh$ production cross section shows sizeable variation as $d_4$ value changes.

- For the Coleman-Weinberg Higgs, the bound on the quartic Higgs coupling $d_4$ is relatively tight, as the Higgs trilinear coupling is $5/3$.

- For the Nambu-Goldstone Higgs, we see the scaling factor $\tilde{c}_3/c_3$ could be constrained to be within the order of 10, but $\tilde{d}_4/d_4$ could only be constrained to the order of much larger than 10.

- For the Tadpole-induced Higgs, because the Higgs trilinear coupling $d_3$ could be highly suppressed, the dependence on the quartic Higgs coupling $d_4$ is very weak. On top of this, as $d_4$ is also suppressed in this scenario, only very large scaling factor $\tilde{d}_4/d_4$ is effective for varying the total cross section. This renders the precision determination of $d_4$ very difficult in this scenario.
Figure 24: Constraints on $\tilde{d}_4/d_4$ in various new physics models, when the cross section can be measured up to 10% and 20% accuracy, respectively. The parameter $d_4/d_4$ scales the quartic Higgs coupling in a given model.

7 Conclusion

The nature of the Higgs boson is still mysterious, for its potential is not well understood yet. In this paper, we consider several theoretically compelling new physics scenarios, in which the Higgs self couplings can be quite different from the SM prediction. To be specific, we have considered the elementary Higgs, Nambu-Goldstone Higgs, Coleman-Weinberg Higgs, and Tadpole-induced Higgs scenarios, with the trilinear and quartic Higgs couplings being either smaller or larger than the SM ones. Trilinear Higgs coupling is enhanced for the elementary Higgs scenario (with the preferred positive coefficient $c_6$ for the effective operator $(H^\dagger H)^3$) and Coleman-Weinberg Higgs scenario, while it is reduced for the Nambu-Goldstone Higgs scenario and Tadpole-induced Higgs scenario. Accordingly, the same pattern holds for the quartic Higgs coupling. We have also considered the Higgs non-linear effect in the Nambu-Goldstone Higgs scenario, and explored the relations among the $\bar{t}h$, $\bar{t}hh$, and $\bar{t}hhh$ couplings. Then, we investigate theoretical constraints on the Higgs self couplings via the partial wave unitarity and tree-level vacuum stability analyses. It turns out that the partial wave unitarity bound is not very tight for the $2 \rightarrow 2$ scatterings, if the Higgs couplings to the longitudinal electroweak gauge bosons $hW_L^\dagger W_L$, $hhW_L^\dagger W_L$ are not modified. The tree-level vacuum stability prefers the trilinear Higgs couplings to be within $0 < d_3 < 3$, even the quartic Higgs coupling can be 10 times larger than the SM value.

In general, the SMEFT and the Higgs EFT can be used to describe the Higgs boson
nature and parameterize Higgs interactions, depending on whether the SM gauge symmetry is linearly or nonlinearly realized. Thus the SMEFT is defined in the unbroken phase of the electroweak symmetry, while the Higgs EFT is defined in the broken phase. Comparing these two EFT frameworks, only the Higgs EFT can exhibit non-decoupling feature of new physics, this renders the Higgs EFT more general than the SMEFT. Among the new physics scenarios of different Higgs potential, the SMEFT can only describe the elementary Higgs and the Nambu-Goldstone Higgs, but the Higgs EFT can describe all the scenarios, including the Coleman-Weinberg Higgs and the Tadpole-induced Higgs scenarios.

Given the unique patterns of the Higgs self couplings predicted by various new physics scenarios, we investigate the possibility to distinguish different scenarios through the process of double-Higgs production $pp \rightarrow hh$ at the 27 TeV HE-LHC and the 100 TeV pp collider. We have studied in detail the total cross sections and various differential distributions, including the effect from distinct interference patterns, in each NP scenario. As a result, the cross section is reduced with respect to the SM one for the elementary Higgs, and the Coleman-Weinberg Higgs cases, while it is enhanced for the Nambu-Goldstone Higgs and Tadpole-induced Higgs cases. With larger cross sections, the corresponding uncertainties are reduced. Thus, one can distinguish different new physics scenarios at the 27 TeV HE-LHC, given the SM is expected to be measured with the accuracy of 14% at the 1\(\sigma\) confidence level. And the discrimination power is further improved at the 100 TeV hadron

Figure 25: Constraints on $\tilde{c}_3/c_3$ and $\tilde{d}_4/d_4$ if the cross section can be measured up to 10% and 20% accuracy, respectively, in the MCH and CTH models.
collider. Besides, we extract out the possible range of the trilinear Higgs couplings for several new physics scenarios, assuming the cross section is measured with 10% and 20% accuracy, respectively. They are shown in Figs. 11, 12, and 13.

To fully pin down the quartic Higgs coupling, we also need to investigate the triple-Higgs production \( pp \rightarrow hhh \) at future colliders. However, due to extremely small rate of the signal with respect to the backgrounds, one need very large luminosity, with the order of \( 10^4 \text{ ab}^{-1} \), cf. Table. 5, even at the 100 TeV hadron collider, to discover this process and precisely measure the quartic coupling. After investigating the interference patterns of the process \( pp \rightarrow hhh \), we find the dependence of the cross section on the quartic Higgs coupling is moderate because other couplings obscure the extraction of the quartic coupling. Thus, even when the total cross section can be relatively well measured with 10% and 20% accuracy, it is still not easy to measure the quartic Higgs coupling, cf. Figs. 23, 24, and 25. Therefore, we expect novel method on suppressing backgrounds, such as machine-learning technique, could help reducing the luminosity needed to measure the total cross section. Furthermore, it is well motivated to think how to improve the efficiency of extracting out the quartic coupling in the triple-Higgs production process. All these efforts are very crucial for a better understanding on the Higgs potential. They are however beyond the scope of this work.

Acknowledgments

J.-H.Yu is supported by the National Science Foundation of China under Grants No. 11875003 and the Chinese Academy of Sciences (CAS) Hundred-Talent Program. C.-P. Yuan is supported by the U.S. National Science Foundation under Grant No. PHY-1719914. C.-P. Yuan is also grateful for the support from the Wu-Ki Tung endowed chair in particle physics. L.-X. Xu is supported in part by the National Science Foundation of China under Grants No. 11635001, 11875072.

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