Mathematical modelling of the spatial efficiency of car parks

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Abstract. One of the major urban concerns is the lack of parking places. The most appropriate indicator for assessing the spatial efficiency of the car parks is the unit surface. The largest space-saving resources to improve spatial efficiency belong to the large car parks in the distinct situations: for employees of a large company or for hypermarket customers. As a result, a mathematical model for access to the parking place by means of a single steering manoeuvre has been constructed, having the angle of inclination of the parking spaces as variable and the 95% percentiles corresponding to the overall current dimensions of the cars as parameters. In the case of customer parking, it was considered that it is necessary for the customers to have access to the trunk to pick up the purchased items from the shopping cart, so a specific arrangement of the vehicles on the parking places is required. The results confirmed the superiority, from the point of view of spatial efficiency, of the angled car parks as compared to the rectangular car parks. For the car parks angled at 45° - 60° it has been demonstrated that increases of more than 10% over rectangular parking spaces are achieved.

1. Introduction
It is generally acknowledged that one of the major urban concerns is the lack of parking places.

For any α angular layout of the parking places, the width l and the length L of the parking place must be ensured (Figure 1), as well as the width of the access lane, which can be organized as a two-way or as a one-way lane [7].

Figure 1. Parking options and parking facilities.

The dimensions of the parking place may be standardized or can be determined based on the statistical analysis of the vehicle dimensions: normally, for the length L and the width l of the parking place, the values corresponding to the 95% percentiles of these vehicle dimensions are obtained by statistical calculations (the 95% percentile is the value below 95% of the observed values).
To the left, the parking places are set to an angle of 90° (rectangular parking) and the access lane runs in both directions.

In the middle and to the right, the access is made on one way, parking can be performed on both sides and the parking places are designed angled. The difference lies only in the fact that in the middle figure the angled arrangement alternates from one access lane to the other, as well as the traffic directions, while in the figure on the right the parking places are arranged angled and interlaced (or in a “herringbone” pattern), but in both variants there is no unused space between the two adjacent motorcades, so they are equally effective in terms of space use.

Obviously, the width of the access lane (for face entry by performing a single manoeuvre, that of steering to the left or to the right) differs function of the traffic being two-way or one-way and depends on the angle under which the parking places are arranged, as can be seen from the figure below.

2. Problem Statement
In the case of very large parking places, such as those for supermarket customers or those for the employees of large corporations, the main criterion to be taken into account is to provide as many parking places as possible [6, 8], so the problem is to find the solution that offers maximum spatial efficiency (the area to be calculated for a parking place should be as small as possible, taking into account the areas of access to the parking place).

The most appropriate indicator for assessing the spatial efficiency of parking facilities is the unit surface – the surface of a parking place and the surface required for the vehicle’s access and manoeuvres. The largest resources for improving spatial efficiency belong to the large car parks [1, 4] met with in the following two distinct situations: for the employees of a large company or for the customers of a hypermarket.

It is obvious that, in order to find the best solution, one should analyse the car park with the parking places angled, at a specific angle and with one-way access, as is the case of the previously described “herringbone” parking.

Inevitably, the question arises: which angle gives maximum spatial efficiency, i.e. the most parking places for the same surface or which require the smallest area for a parking place?

3. Research conducted
The answer requires finding a mathematical model for the unit surface (for a parking unit). Since the length of the motorcades with parking places (or access lanes) is very large, the mathematical model can ignore the unused spaces next to the first and last parking places or can take them into account (these being two halves of two parking places, therefore equivalent to one parking place).

Literature provides a mathematical model for this calculation, widely publicized in the English media [7], which however takes into account the fact that cars are parked only to the right from the access lane.

As a matter of fact, organizing the car park with the places angled and with one-way access lanes allows parking on both left and right, which will certainly result in much higher values for spatial efficiency and, most likely, in another value for the optimal angle of inclination from the point of view of spatial efficiency).

As a result, a mathematical model for the surface area of a parking place (unit surface) will be developed according to the angle of inclination, having as parameters the length L and the width l of the parking place.

The vehicle is considered to be performing a single manoeuvre [2, 3, 5]: a circular turning (the radius being equal to half the turning circle between the walls, this being a geometric feature of the vehicle) through which it will go from the access lane to the parking place (Figure 2).

The unit surface (corresponding to a single parked vehicle) can be calculated, it has two components:

1 – surface of the access lane, with a width e \( L_\alpha \) and length \( l/sin\alpha \), divided by the two parking places (to the left and to the right) served:
Figure 2. Occupying the parking place through the steering manoeuvre.

In order to park under the angle $\alpha$, the width of the one-way lane will have to be:

$$L_\alpha = R_e - R_e \cdot \cos \alpha = R_e (1 - \cos \alpha)$$  \hspace{1cm} (2)

provided that its width is sufficient to allow movement, that is, at least equal to the width of the parking place:

$$L_\alpha = R_e (1 - \cos \alpha) \geq l$$  \hspace{1cm} (3)

or

$$1 - \cos \alpha \geq \frac{l}{R_e}$$  \hspace{1cm} (4)

i.e.

$$\cos \alpha \leq 1 - \frac{l}{R_e}$$  \hspace{1cm} (5)

And, as the cosine function is a strictly downward function on the $[0, 90^\circ]$ range, the following results:

$$\alpha \geq \arccos(1 - \frac{l}{R_e})$$  \hspace{1cm} (6)

2 – the surface occupied by a parked vehicle (located on either side of the access lane), consisting of a rectangle of dimensions $L$ and $l$ plus a rectangular triangle with the sides $l$ and $l/\tan \alpha$ (this is very useful for the car parks built near supermarkets because the customer can put the shopping cart there so as to be able to load the products in the car trunk) – Figure 3:

$$S_{parking.\ real} = L \cdot l + \frac{l}{\tan \alpha} \cdot \frac{l}{2} = L \cdot l + \frac{l^2}{2 \tan \alpha}$$  \hspace{1cm} (7)

As a result, the unit parking surface will be:

$$S_{unit} = \frac{R_e (1 - \cos \alpha)}{2 \sin \alpha} + L \cdot l + \frac{l^2}{2 \tan \alpha} = \frac{R_e l}{2 \tan \alpha} \left(\sqrt{1 + (\tan \alpha)^2} - 1\right) + L \cdot l + \frac{l^2}{2 \tan \alpha}$$  \hspace{1cm} (8)

where the known relation between $\sin \alpha$ and $\tan \alpha$ was used:
The variable is a trigonometric function, \( \tan \alpha \), and the other dimensions \((R_e, L, l)\) are data that can be adopted from the standards or can be set (usually the 95% percentile is set for each, i.e. the width, the length and the outer turning radius between the walls covering 95% of the vehicles of this type - cars).

The graph of the function can be drawn according to the angle \(\alpha\) at a convenient pace, but introducing numerical values for the three parameters (the values corresponding to the 95% percentiles for cars are adopted from the specialised literature: \(R_e = 8\) m, \(L = 4.9\) m; \(l = 2.6\) m).

The study is intended to be restricted only to the values of \(\alpha\) that satisfy the condition outlined above:

\[
\alpha \geq \arccos \left(1 - \frac{1}{R_e}\right) = \arccos \left(1 - \frac{2.6}{8}\right) = \arccos(0.675) = 47.5^\circ
\]  

For these values of the \(R_e, L\) and \(l\) parameters, the unit surface will be:

\[
S_{\text{unit}} = \frac{R_e l}{2 \tan \alpha} \left(\sqrt{1 + (\tan \alpha)^2} - 1\right) + L \cdot l + l^2 \frac{1}{2 \tan \alpha} \left(\sqrt{1 + (\tan \alpha)^2} - 1\right) + \frac{3.8}{\tan \alpha} + 12.74
\]  

For values of the \(\alpha\) angle between 30° and 90° (but also for angles of less than 48°, to better observe the traffic trend in the boundary area), Microsoft Excel calculates the \(S_{\text{unit}}\) unit surface values, at a pace of 5°.

It is found that the lowest value of the unit surface, which corresponds to the optimum value of the angle of inclination (for these values of the parameter \(L, l\) and \(R_e\)) is \(\alpha = 50^\circ\) (if a pace of 1° is used, a more precise result will be obtained: the minimum corresponds to the angle \(\alpha = 48^\circ\)). By representing the situation obtained graphically (Figure 4), the function is found to really present a minimum for \(\alpha = 50^\circ\), which also fulfils the aforementioned condition: \(\alpha \geq 47.5^\circ\).

Indeed, at the limit (with respect to the width of the access lane), the angle of inclination of 45° can be recommended. However, a solution quite close from the viewpoint of efficiency is the one with an angle of inclination of 60° (as can be seen on the graph), which additionally has the important advantage that the width of the access lane will have a much higher value than the limit, eliminating the feeling of unsafe traffic on the access lane.

\[
L_{\text{ax}} = R_e \left(1 - \cos \alpha\right) = 8 \left(1 - \cos 60^\circ\right) = 8 \left(1 - 0.5\right) = 8 \cdot 0.5 = 4\ m \gg l = 2.6\ m
\]  

It remains to be assessed how much is gained in point of number of parking places compared to the solution that is still used, with \(\alpha = 90^\circ\).

In the case of parking at 45°, it is found that the space required for parking 100 vehicles at 90° allows the parking of \(100 \times (23.14/20.42) = 113.28 = \text{about 113 vehicles at 45°, i.e. a 13% increase.}\)

Parking at 60° also ensures an increase almost as high: \(100 \times (23.14/20.69) = 111.81 = \text{about 112 vehicles, i.e. a 12% increase.}\)
Figure 4. Unit surface graph according to the angle of inclination $\alpha$.

Similarly, the angles of 50°, 55° provide a 12-13% increase in the number of parking places, so it can be said that the angles of inclination of 45°...60° produce increases in the parking capacity of 12...13%. However, the usual angles are those of 45° and 60°, which allow for an easier marking of the parking.

However, taking into account that, in the case of angled parking, for each motorcade of parked vehicles $\frac{1}{2} + \frac{1}{2} = 1$ parking place is lost, it results that 100 vehicles parked at 90° will correspond to 111-112 vehicles parked at 45° or 60°, which, nevertheless, means a sensible increase in the spatial efficiency of parking by 11-12% or with 55-60 parking places in a 500-lot rectangular car park.

It is to be noted that there are significant value differences with respect to the reference work [7], where the calculation algorithm followed only one motorcade of vehicles on the right side: the unit surface values are much lower (20.5 m² vs. 32 m² for the optimum resulting angle of 45°), and the spatial efficiency gain is about half (for 500 vehicles, there is space available for other 60 vehicles compared to 113 vehicles mentioned in the cited work).

As a result, for very large car parks, an important increase in the parking capacity is obtained by organizing them with the parking places angled, and the most suitable is the angle of 60°, which also provides a sufficiently wide access lane (4.0 m) and a capacity increase very close to the maximum possible.

Very large car parks are to be met with in two cases:
1. for customers at a supermarket, where each parking place is occupied and vacated several times a day, so that the distribution of the lots occupied throughout the car park is constantly changing;
2. for employees of a large company, when each parking place is occupied at the beginning of the work program and is vacated only at the end of the work schedule, so the distribution of the occupied places remains the same throughout the working hours.

As a result, there are two proposals for the organization of the car park with the lots angled at 60° (Figure 5):
• for the company, access will be made in the same direction, towards the entry, as the parking places will be occupied for the entire period of the work schedule, starting from the closest to the entrance to the company, as the employees arrive at the workplace (figure on the left);
• for the supermarket, access will be made in alternative ways, as the parking places will be occupied for a limited period of time and then vacated, so a customer will have to “look for” a free space and occupy it, being able to drive on consecutive access lanes (figure on the right).

Regarding the entry and exit to the car park, solutions differ here as well:
- for the company, cars will enter into the car park from the far side (because the employees will advance on the access lanes to the area with the parking places occupied), and the exit will be through the artery close to the company;
- for the supermarket, cars will enter into the car park from the near side (as customers will look for free parking places as close as possible to the supermarket), and the exit will be through the far artery.
4. Conclusions
Through the algorithm presented, the myth of the much higher efficiency of car parks angled to 45°, as compared to the rectangular ones, is confirmed, but to a lesser extent - by half, i.e. an increase of about 10%. This increase in parking capacity will only be achieved in the case of very large car parks, where the length of the parking motorcades is large enough.

Otherwise, when parked vehicle motorcades are not large, the gain is not that big, because the untapped triangular surfaces at the end of the motorcades are more numerous and affect the efficiency of angled parking.

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