Feature-Model-based Hybrid Adaptive Predictive Functional Control Algorithm

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Abstract. A new control algorithm was contributed to improving such bandwidth, robustness and real-time of servo tracking control system. The algorithm design is based on feature model-based of on-air plants by uniting with predictive functional control which was free of parametric varying and the parametric identification was updating by hybrid adaptive estimation which can reduce the overshoot and improve the response time.

1. Introduction
Industrial process and aerospace control require fast response, robustness, and on-line self-adjustment. To solve such problems, these control schemes have proposed, such as adaptive control [1, 2], adaptive robust control [3, 4], and predictive control [5]. However, the algorithms demand exact dynamics model, a quantity of computation effort, or complex hardware requirements. Thus, Richalet et al. [6] brought up predictive functional control with low-cost and better tracking accuracy performance, and has applied on industrial application [7, 8]. And Wu [9] presented characteristic model-based control scheme to prevent from varying-parameter and applied it on manned spacecrafts successfully. The authors are enlightened and introduce one new algorithm which constructs the plant’s model by its features, optimizes its performance with predictive functions and adopts hybrid adaptive law to estimate the model’s parameters according to [10, 11, and 12]. The objective of the new scheme is to eliminate the effect from parametric varying of the model and to improve the response-time and robustness of the system.

2 Algorithm of FHAPFC
In this section, we give one characteristic predictive mode, and then narrate the hybrid adaptive optimization law and error correction, accuracy analysis in detail. Figure 1 shows one control scheme of our proposed Feature-model-based Hybrid Adaptive Predictive Function Controller(that is called as FHAPFC).
2.1. Feature-model-based Predictive Model

One linear constant-coefficient system with high order is mini-phase system and shown as

\[ G(s) = \frac{b_n s^n + b_{n-1} s^{n-1} + \cdots + b_s + b_0}{s^n + a_{n-1} s^{n-1} + \cdots + a_s + a_0} \]  

which can be decomposed into

\[ G(s) = \frac{k}{s^2} + \sum_{i=1}^{\infty} \frac{k_i}{s + \lambda_i} + \sum_{i=1}^{\infty} \left( \frac{k_{i,p}}{s + \lambda_{i,p}} + \frac{k_{i,n}}{s + \lambda_{i,n}} \right) + \sum_{i=1}^{\infty} \frac{k_{o,i}}{(s + \omega_i)^2} \]  

Under the condition of certain sampling period \( T_s \), when the plant (2) holds or tracks the position and speed, its feature predictive model is made up of two parts which one is free response of the model with second-order time invariant difference equation:

\[ y_p(k+1) = \theta_1(k) y_p(k) + \theta_2(k) y_p(k-1) + \rho_o(k) u(k) \]  

and the other is its forced response which is the output under one new control input over a receding horizon \([0, H]\) where \( H \geq N_o \) (herein \( H = N_o \)),

\[ u(k+i) = \sum_{i=1}^{N_o} \alpha_s(k) u_{n,i}(i), i = 0, 1, \cdots, H - 1 \]  

where \( u_{n,i}(i) \) is the value of base function at \( t = iT \) and selected as step or ramp functions generally, thus

\[ u(k+i) = \alpha_i(k) + \alpha_{i+1}(k)i, i = 0, 1, 2, \cdots, H - 1 \]  

\[ \Delta u(k+H-1) = \Delta u(k+H-2) = \cdots = \Delta u(k+1) = \alpha_s(k) \]  

and the forced output is expressed as
\[ y_j(k + i) = \sum_{n=1}^{N_k} \alpha_n y_{in}(i) \]  

where \( T \) is the sampling period, \( \alpha_n \) is the unknown weighted coefficient, and \( y_{in} \) is the response under input \( u_{in} \) which can be off-line computed. Thus the output of the featured predictive model is expressed as

\[ y_{in}(k+i) = y_i(k+1) + y_j(k+i), i = 0, 1, \cdots, H - 1 \]  

If the plant (2) is stable, then

- Parameters \( \theta_i(k) \) and \( \rho_i(k) \) are slow time varying.
- The ranges of these parameters can be preset.
- The output of feature model is equivalent to one of the actual plant with same control input \( u \) during dynamic process (the appropriate sampling internal may ensure that the output error is within permitted accuracy), but the output of feature model is equal to the one of actual model during steady state.
- If static gain \( D = 1 \),

\[ \theta_1(\infty) + \theta_2(\infty) + \rho_0(\infty) = 1 \]  

The proof about above properties is referred to [14].

Let define two new variable vectors

\[ \Phi(k) = [y_i(k), y_i(k-1), u(k)]^T \]  
\[ \Gamma(k) = [\theta_i(k), \theta_2(k), \rho_0(k)]^T \]  

Therefore, with (9) and (10), (3) is rewritten as

\[ y_i(k+1) = \Phi(k)^T \Gamma(k) \]  

The parameter vector \( \Gamma(k) \) is required to estimate according to the following adaptive law.

2.2. Optimization and Control Law Equation

2.2.1. Parametric Estimation

Suppose \( \hat{\Gamma}(k) \) be the estimated value of \( \Gamma(k) \), the updating equation of \( \Gamma(k) \) is the form of Equ.(12)

\[ \hat{\Gamma}(k) = (1 - \delta) \hat{\Gamma}(k - 1) + \frac{\sigma \Phi(k) y(k)}{1 + \Phi(k)^T \Phi(k)} \]  

\[ \delta = \begin{cases} 
0, & \| \Gamma \| < M_0 \\
\delta_0 \frac{\| \Gamma \| - 1}{M_0}, & M_0 \leq \| \Gamma \| \leq 2M_0 \\
\delta_0, & \| \Gamma \| > 2M_0 
\end{cases} \]
where $\delta$, $a$ and $M_alpha$ are positive constants and to design. Herein, parameter $\delta$ introduced is to speed up self-tuning of parameters.

### 2.2.2. Reference Trajectory

Avoiding of dramatic change and overshoot of control input at the start, a reference trajectory is introduced with the form of

$$y_r(k+i) = y_d(k+i) - \beta^i (y_d(k) - y_r(k))$$

(14)

where $y_d$ is the desired point and set trajectory: $y_d(k+i) = y_d(k) + \sum_{j=1}^{N_d} y_d(k) i^j$ where $N_d$ is the order of polynomial and generally $N_d = 2$ , is set position, set speed, and set acceleration at $k$ , $\beta = \exp \left(-\frac{T_c}{T_r/3}\right) 0 < \beta < 1$ is the damped exponential, $y_p$ is the output of the model, and $T_r$ is the desired closed-loop response time which is more small and the performance quality is very good. The objective of the FHAPFC is to track the set point trajectory following the reference trajectory. The trajectory may be considered as the desired closed-loop behavior.

### 2.2.3. Optimization Index

The plant prediction output of the model is shown as

$$y_p(k+i) = y_m(k+i) + e_p(k+i)$$

(15)

where $e_p(k+i)$ is the predicted error between the plant and feature predictive model outputs. It is obtained through observations of the difference on a past horizon, by polynomial expression

$$e_p(k+i) = e_p(k) + \sum_{j=1}^{N_p} \mu_j(k) i^j$$

(16)

where $N_p$ is the order of error extrapolator, $\mu_j$ is the predictive error coefficient. If that is one first-order low-pass filter: $e(k+i) = e(k) + \mu_i$ and the predictive step is $N_p$, then

$$\min J_p = \min \left\{ \sum_{n=1}^{N_p} (e(k-m) + \mu_m e(k))^2 \right\}$$

(17)

Solving $\frac{\partial J_p}{\partial \mu_i} = 0$, we can obtain error weight coefficient,

$$\mu_i = \frac{\sum_{n=1}^{N_p} [m - (e(k) - e(k-m))]}{\sum_{n=1}^{N_p} m^2}$$

(18)

The objective of the proposed scheme is to minimize

$$J = \sum_{j=1}^{N_p} \left\{ y_p(k+j) - y_r(k+j) \right\}^2 + \sum_{j=1}^{N_d} \lambda \left\{ \Delta u(k+l-1) \right\}^2$$

(19)

in order to find out the optimized weighted values $\{\alpha_i\}$, the second part of the right of (19) is added to
smooth the start process before FHAPFC takes effect, and $N_s$, $N_e$ are optimized start instant and end instant respectively and to design in term of actual projects. Herein $N_e = N_s$. Over two coincidence points $N_s T_s$ and $N_e T_s$, $\alpha(k), \beta(k)$ are obtained by

$$\min J = \frac{1}{2} \left[ (y_r(k + N_s) - y_u(k + N_s) - e(k + N_s))^2 + (y_r(k + N_e) - y_u(k + N_e) - e(k + N_e))^2 \right] + \lambda \left( \alpha(k) - u(k) \right)^2 + \lambda \sum_{\alpha(k)}^N (\alpha(k))^2$$

(20)

The control law is expressed as

$$u(k) = \frac{C(k)D(k) - E(k)A(k)}{C(k)F(k) - E(k)B(k)}$$

(21)

where $A(k) = h_2(N_s)(G_1(N_s) - G_2(N_s)) + h_2(N_e)(G_1(N_e) - G_2(N_e))$, $B(k) = 2\rho_0 + h_1(N_s) + h_1(N_e)$, $C(k) = 2\lambda(N_s - 1) - (h_1(N_s) + h_1(N_e))$, $D(k) = [\rho_0 + h_1(N_s)](G_1(N_s) - G_2(N_s))$ and $E(k) = [\rho_0 + h_1(N_s)]h_2(N_s) + [\rho_0 + h_1(N_e)]h_2(N_e)$.

$$F(k) = \left[ \rho_0 + h_1(N_s) \right] + \left[ \rho_0 + h_1(N_e) \right]$$

$h_1(N_s)(j = 1, 2; q = S, E)$ is the response of base function control input. $y_r(k) = y_x(k) + \sum_{i=1}^{N_q} y_{q} y_{\alpha}(k) - \beta^{S}(y_{q}(k) - y_{\beta})(q = S, E) G_1(N_q) = \theta_1(y_{\alpha}(k) + \theta_2(k))y_u(k - 1)$

$$y_r(k + N_s) = G_1(N_s) + \rho_0 \alpha(k) + \alpha(k) h_1(N_s) + \alpha(k) h_2(N_s)$$

The desired trajectory and forced responses can be off-line computed which reduce complexity of computation. And the parametric identification can improve the accuracy being free of the parametric-varying of plant on work.

3. Simulation and Analysis

To verify the new algorithm, consider a plant described by $W(s) = \frac{1.5}{7s + 1}$. And give the responses under PID controller and FHAPFC controller by simulations. As shown as Figure 2 and 3, comparing with these outputs under two control schemes, we can obtain that the overshoot at the start of controller taking effect is reduced and the response time is shortened by FHAPF scheme. The advantages of the proposed method is which the main characters of the plant is still kept, base functions added into predictive control input can keep off unexpected disturbances, and parametric self-adjustment can speed up the convergence of those coefficients of the new predictive model.
4. Conclusion

It is clear that the proposed feature based model hybrid adaptive predictive functional control scheme can reduce the overshoot at the beginning of the controller taking effects compared with PID and the proposed controller, on the other side, the setup time is shortened. And the same time, the modified algorithm can self-adjust the model’s time-varying parameters from motion or working.

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