Elastic modulus reduction method for stable bearing capacity analysis of dumbbell-shaped CFST arches

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Abstract: The elastic modulus adjustment procedures (EMAPs) can achieve much higher efficiency than incremental nonlinear finite element method (INFEM), however it could not take into account the stability effect on structures. In this paper, the elastic modulus reduction method (EMRM) was presented to estimate the stable bearing capacity of dumbbell-shaped concrete filled steel tube (CFST) arch. Firstly, the homogeneous generalized yield function (HGYF) was developed for dumbbell-shaped CFST members under combined action of axial force and bending moment by means of the fiber model technique, which was employed to determine the stable bearing capacity of these members. Secondly, the HGYF was adopted to define the element bearing ratio so that the EMRM was proposed for evaluating the stable bearing capacity of the dumbbell-shaped CFST arch by strategically reducing the elastic modulus of the highly stressed elements. Finally, results from the INFEM and test data of 3 dumbbell-shaped CFST arches and 30 dumbbell-shaped CFST members were adopted as examples to validate the proposed EMRM and the HGYF, which shows that the stability has a significant influence on the CFST arches and the proposed EMRM achieves higher accuracy and efficiency than the INFEM.

1. Introduction

Dumbbell-shaped concrete-filled steel tubes (CFST) are being increasingly used in the construction of bridges [1]. For long span CFST arches, stability is a key factor affecting the bearing capacity of arch.

Numerical method is a rather powerful and widely adopted tool for ultimate bearing capacity analysis of CFST arches, which include two main types of methods namely the elasto-plastic analysis method and the plastic limit analysis method. The former usually adopts incremental nonlinear finite element method (INFEM) [2-4], and estimates the ultimate bearing capacity of structures by increasing the external loads incrementally and nonlinear iterative analysis. The latter usually adopts the elastic modulus adjustment procedures (EMAPs), including the elastic compensation method [5], the m-Tangent method [6], the modified elastic compensation method [7] and the elastic modulus reduction method (EMRM) [8], which provide an alternative route for ultimate bearing capacity analysis of engineering structures. The EMAPs achieve much higher accuracy and efficiency through linear elastic iterative analysis than the INFEM, and were introduced into international pressure vessel design code [9].

In order to further improve the computing efficiency for EMAPs, Shi et al. [10], Yang et al [11], Martin and Dargush [12] introduced generalized yield function (GYF) into the EMAPs so as to reduce the discretized degrees of freedom in the finite element models. Though the EMAPs are usually convenient...
and simple to use with higher efficiency, they did not consider the influence of stability on the structures, which will overestimate the ultimate bearing capacity of structures.

In this paper, the homogeneous GYF was proposed for dumbbell-shaped CFST members under in-plane and out-of-plane loads by means of the fiber model technique, taking into consideration the stability effect. Then the EMRM with stability effect was developed for stable bearing capacity analysis of dumbbell-shaped CFST arch. Finally, the test data were adopted to demonstrate the efficiency and accuracy of the proposed method.

2. Homogeneous GYF for dumbbell-shaped CFST

A cross section of a dumbbell-shaped CFST member under axial load and bending moments is illustrated in Fig. 1 with outer diameter $D$ and thickness $t$ of the steel tube, and the height $H$ of the dumbbell-shaped section, where $h$ and $b$ are the height and width of the web of the cross section. An eccentric compressive load $P$ is applied to the section with orientation of $\theta$ and eccentricity of $e$. If $\theta = 0^\circ$ or $90^\circ$, the dumbbell-shaped CFST members are under unidirectional compression and bending moment. Otherwise, if $0 < \theta < 90^\circ$, the dumbbell-shaped CFST members are under axial compression and bi-axial bending moment.

![Fig. 1. Strain distribution in dumbbell-shaped CFST column section](image)

The generalized yield criterion is usually adopted to distinguish a fully plastic section of the dumbbell-shaped CFST member under combined actions of compression and bending, and gives:

$$f(n,m) = 1$$  \hspace{1cm} (1)

where $f(n,m)$ is the generalized yield function (GYF); $n$ and $m$ are dimensionless internal forces, and gives:

$$n = \frac{N}{N_u}, \quad m = \frac{M}{M_{uw}}$$  \hspace{1cm} (2)

where $N$ and $M$ are the axial force and bending moment on the section of dumbbell-shaped CFST member, respectively; $N_u$ and $M_{uw}$ are the fully plastic axial force and fully plastic bending moments of the section, respectively, which are found in literature $^{[3]}$ as:

$$N_u = 2N_c + N_w, \quad M_{uw} = 0.911\gamma W_{uc}f_{cy}$$  \hspace{1cm} (3)

$$N_c = A_c f_{cy}, \quad N_w = A_w f_y, \quad W_{uc} = \frac{3\pi D^4}{48h} + 12\pi D^2 (D + h)^2 + 16th^3$$  \hspace{1cm} (4)

in which, $N_c$ and $N_w$ are the fully plastic compressive strength of the circular CFST and that of the web, respectively; $\gamma$ is a sectional plastic development coefficient; $A_c$, $A_w$, and $W_{uc}$ denote the area of the circular CFST and that of the web, and the sectional modulus of the dumbbell-shaped CFST; $f_{cy}$ is the yield strength of the CFST; $\xi$ is the hoop ing coefficient; $f_y$ and $f_{ck}$ are the yield strength of circular steel and the compressive strength of core concrete, respectively.

Usually, the original GYF does not satisfy the requirement of proportional condition in limit analysis...
of structures, leading to unstable and inaccurate iteration when applied to the EMAPs [14]. Thus, it is essential to develop a homogeneous GYF (HGyF) to satisfy the proportional condition [11]. For this purpose, the fiber model technique [15] is adopted in this section to determine the plastic limit of P applied at different location (e, θ) when a cross section of the dumbbell-shaped CFST member reaches fully plasticity under combined action of compressive force and bending moments.

2.1. Collocation points determined by the fiber model technique

The behavior of the dumbbell-shaped CFST column is modeled by the fiber model technique. The cross-section is divided into fine fiber elements as shown in Fig. 1. It is assumed that there is no slippage between the steel tube and the concrete and the deflected shape of CFST columns is a part-sine, so that the cross section remains plane after deformation, resulting in a linear strain distribution. Fiber stresses are calculated from fiber strains using material relationships presented by Han [16]:

\[ y = \begin{cases} 
2x - x^2 & x \leq 1 \\
x & x > 1 
\end{cases} \]

(5)

where \( e_0 = \left(1300 + 12.5f_y + 800\xi^{0.2}\right) \times 10^{-6} \), \( \beta_i = 0.5 \left(2.36 \times 10^{-5}\right)^{0.25(1.00.55)} \left(f_y\right)^{0.5} \geq 0.12 \)

\[ y = \sigma / f_y \quad x = e / e_0 \]

Fig. 1 depicts the fiber strain distribution over the dumbbell-shaped section, where \( y_i \) is the coordinate of the fiber \( i \), \( \Phi \) is the curvature of the section, \( \varepsilon_i \) and \( \varepsilon_c \) are the strain at the \( i^{th} \) fiber element and the center of the section. In the nonlinear analysis of CFST column, the mid-height deflection of the CFST column under unidirectional compression and bending moment is gradually increased until the ultimate axial strength is obtained, then a fitting point can be obtained by means of Eq.(2). A total of 6000 numerical specimens are used to establish the HGyF for dumbbell-shaped CFST under eccentric compression on the basis of the full-factorial experimental design method. The outer diameter of the steel tube \( D \) is 108mm, and the thickness of the steel tube \( t \) and the web thickness keep the same, and range from 2mm to 6mm with an increment of 1mm. The width of web \( b \) takes 27mm, 54mm, 67.5mm and 81mm while the height of web \( h \) ranges from 16.2 to 70.2mm with an increment of 13.5mm. Meanwhile, the length of specimens are 400mm, 800mm, 1600mm, 2400mm and 3200mm, respectively. The characteristic value of yield strength \( f_y \) of the steel tube is taken as 235MPa, 345MPa, 390MPa and 420MPa, respectively. The cubic compression strength \( f_{cu} \) of the core concrete is taken as 40MPa, 50MPa and 60MPa, respectively. The parameters are listed in Table 1.

| \( L/mm \) | \( t/mm \) | \( b/mm \) | \( h/mm \) | \( f_{cu}/MPa \) | \( f_y/MPa \) | Number of the specimens |
|------|------|------|------|------|------|-----------------|
| 400  | 2~6  | 27~81| 16.2~70.2| 40~60 | 235~420 | 1200 |
| 800  | 2~6  | 27~81| 16.2~70.2| 40~60 | 235~420 | 1200 |
| 1600 | 2~6  | 27~81| 16.2~70.2| 40~60 | 235~420 | 1200 |
| 2400 | 2~6  | 27~81| 16.2~70.2| 40~60 | 235~420 | 1200 |
| 3200 | 2~6  | 27~81| 16.2~70.2| 40~60 | 235~420 | 1200 |

2.2. HGyF for dumbbell-shaped CFST members under unidirectional compression and bending

Based on the axial forces and bending moments of 1200 short columns with length of 400mm in Table 1 by means of the fiber model technique, the HGyF is developed for dumbbell-shaped CFST under unidirectional compression and bending without stability:

\[ f_M(n,m) = \sum_{i=0}^{M_1} a_i n^{M_1} \xi^i \]

(6)

where \( f_M(n,m) \) is the HGyF; \( M_1 \) is the order of power for the HGyF; \( a_i \) is the function of the hooping
coefficient which is fitted by polynomial; \( M_2 \) is the number of terms for \( a_i; b_j \) are undetermined coefficients.

When \( M_1=1 \), then the HGYF is defined as:

\[
\bar{f}(n,m) = n + m + \left( \sum_{j=0}^{M_2} b_j \xi^j \right) n^c\]

where \( c \) is the undetermined coefficient.

The axial forces and bending moments of the members can be obtained by means of the fiber model technique, then the fitting points on the HGYF can be got through Eq.(2) to Eq.(4). The HGYF is obtained by means of regression technique based on the fitting points \((n_k, m_k)\), \( k=1,2,3,\ldots,N_R \), and the undetermined coefficients in Eq.(6) and Eq.(7) can be obtained by the least squares method:

\[
\begin{align*}
\Pi &= \sum_{k=1}^{N_R} \left[ f(n_k, m_k) - \bar{f}(n_k, m_k) \right]^2, \\
\|\delta\| &= \sqrt{\frac{\Pi}{N_R - s_1}},
\end{align*}
\]

where \( \Pi \) and \( \|\delta\| \) are the residual sum of squares and the root mean square error, respectively, which are usually employed to measure the fitting accuracy; \( N_R \) is the number of fitting points; \( s_1 \) is the number of undetermined coefficients.

The root mean square errors of the HGYF under different orders are listed in Table 2, which shows that the fitting precision comes to the best when \( M_1=1 \). The fitting precision increases with \( M_2 \), but ranges little when \( M_2 \) is greater than 2. Therefore, \( M_1=1 \) and \( M_2=2 \) are chosen to develop the HGYF, and reads:

\[
\bar{f}(n,m) = n + m + \left( -0.604 + 0.098 \xi - 0.022 \xi^2 \right) n^{0.212} m^{0.788}
\]

### Table 2. Root mean square error

| \( M_2 \) | 1   | 2   | 3   | 4   |
|-----------|-----|-----|-----|-----|
| \( M_1 \) |     |     |     |     |
| 1         | 0.0318 | 0.1622 | 0.0900 | 0.0973 |
| 2         | 0.0289 | 0.1576 | 0.0740 | 0.0726 |
| 3         | 0.0282 | 0.1562 | 0.0701 | 0.0667 |
| 4         | 0.0279 | 0.1555 | 0.0685 | 0.0643 |

The yield surfaces of the HGYF for different hooping coefficients are illustrated in Fig. 2 versus the fiber model method results. It appears that the HGYF has good approximation to the fiber model method.

Based on the axial forces and bending moments of columns with length from 800mm to 3200mm, the HGYF was established taking into consideration the stability effect, and reads:

\[
\bar{f}(n,m) = n + m + \left( k_1 + k_2 \varphi + k_3 \varphi^2 \right) \left( -0.604 + 0.098 \xi - 0.022 \xi^2 \right) n^{0.212} m^{0.788}
\]

where \( k_1 \sim k_3 \) are undetermined coefficients; \( \varphi \) is the stability coefficient.

The fiber model is adopted to established the HGYF, then \( k_1, k_2 \) and \( k_3 \) are obtained by means of regression analysis, and reads:

\[
\bar{f}(n,m) = n + m + \left( -2.236 + 5.978 \varphi - 2.976 \varphi^2 \right) \left( -0.604 + 0.098 \xi - 0.022 \xi^2 \right) n^{0.212} m^{0.788}
\]
2.3. HGYF for dumbbell-shaped CFST under axial compression and bi-axial bending moment

When $0 < \theta < 90^\circ$, the dumbbell-shaped CFST members are under axial compression and bi-axial bending moment. The correlation curves of $N/N_u$-$M_y/M_{uy}$-$M_z/M_{uz}$ can be obtained by means of fiber model technique by changing the calculation parameters. The correlation curve of $M_y/M_{uy}$-$M_z/M_{uz}$ under different axial compression ratio is shown in Fig. 3.

![Fig. 3. The correlation curve of $M_y/M_{uy}$-$M_z/M_{uz}$ under different axial compression ratio](image)

It is suggested that the correlation curve is approximately expressed as follows by means of regression analysis:

$$\left( \frac{M_z}{M_{pz}} \right)^{1.5} + \left( \frac{M_y}{M_{py}} \right)^{1.5} = 1$$

(12)

where $M_z$ and $M_y$ are the bending moment in z and y direction, respectively; $M_{pz}$ and $M_{py}$ are the bending capacity under different axial forces.

The $M_{pz}$ and $M_{py}$ should be replaced by $M_{uz}$ and $M_{uy}$ for the members under pure bi-axial bending moment with $N=0$, and the interaction equation reads:

$$\left( \frac{M_z}{M_{uz}} \right)^{1.5} + \left( \frac{M_y}{M_{uy}} \right)^{1.5} = 1$$

(13)

where $M_{uz}$ and $M_{uy}$ are the full plastic bending moments in z and y direction, respectively. In which [17],

$$M_{wy} = 2M_{wc} + M_{uw}, \quad M_{uw} = \gamma W_{sc,c} f_{cy}, \quad M_{wc} = \gamma' W_{wc,w} f_{cy}', \quad W_{sc,c} = \pi D^3 / 32$$

(14)

where $M_{wc}$ and $M_{uw}$ are the full plastic bending moments of the circular CFST and the web CFST; $W_{sc,c}$ and $W_{sc,w}$ are the section modulus in bending of the circular CFST and the web CFST; $\gamma'$ is plastic coefficients under bending moments of the web CFST; $f_{cy}'$ is the compression strength of the web CFST.

When the dumbbell-shaped CFST members are under axial compression and bi-axial bending moment, the HGYF can be obtained by replacing the item $m$ in Eq.(11) by Eq.(13), and reads:

$$\overline{T}(n,m) = (-2.236 + 5.978\rho - 2.976\rho^2)(-0.604 + 0.098\xi - 0.022\xi^2)n^{0.212}\left(1.5 \sqrt{m_z^{1.5} + m_y^{1.5}}\right)^{0.788}$$

(15)

$$+n + \sqrt{m_z^{1.5} + m_y^{1.5}}$$

The results of the HGYF of Eq.(15) and the fiber model are shown in Fig. 4. It can be seen that the results of the HGYF are inclined to be safe in most cases.
3. Elastic modulus reduction method

The element bearing ratio (EBR) is adopted to represent the bearing state of elements and can be determined by the HGYF:

$$r_e^k = f(n,m)$$  (16)

where the superscript $e$ represents the element numbering and the subscript $k$ denotes the iteration step.

The reference EBR reads:

$$r_e^0 = r_{k}^{\text{max}} - d_k \times (r_{k}^{\text{max}} - r_{k}^{\text{min}})$$  (17)

where $r_{k}^{\text{max}}$ and $r_{k}^{\text{min}}$ are the maximum and minimum EBRs in the $k^{\text{th}}$ iteration, respectively. $d_k$ denotes the uniformity of the EBR:

$$d_k = \frac{r_{k}^{\text{max}} + r_{k}^{\text{min}}}{2} = \frac{1}{N_e} \sum_{i=1}^{N_e} r_{e_i}^k$$  (18)

where $N_e$ is the number of elements and $\overline{r_k}$ is the mean value of the EBRs.

$r_{k}^{\text{max}}$, $r_{k}^{\text{min}}$, $\overline{r_k}$, $d_k$, $r_e^0$ are the characteristic parameters for the distribution state of the EBR. The reference EBR, $r_e^0$, provides a criterion for distinguishing highly stressed elements. If $r_e^k > r_e^0$, the element $e$ is considered as highly stressed one in $k^{\text{th}}$ iteration, otherwise it is regarded as lowly stressed. Plastic damage will occur in the highly stressed elements and part of the internal force will transfer to lowly stressed elements, so as to improve the uniformity of the EBR in the structure.

The elastic modulus reduction strategy is established according to the strain energy conservation principle:

$$E_{e,k+1}^e = \begin{cases} E_k^e & r_e^k > r_e^0 \\ \frac{2(r_e^0)^2}{(r_e^0)^2 + (r_e^k)^2} E_k^e & r_e^k \leq r_e^0 \end{cases}$$  (19)

where $E_k^e$ and $E_{e,k+1}^e$ are the elastic moduli of the element $e$ in the $k^{\text{th}}$ and $(k+1)^{\text{th}}$ iterations, respectively. When $k=1$, $E_k^e$ takes its initial value $E^e$ as:

$$E^e = \frac{2E_w A_w^e + E_w A_w}{A_w}$$  (20)

where $A_w$, $A_w'$, $A_w''$ are the area of composite section, circular CFST and web CFST, respectively. $E_w$, $E_w'$ are the elastic moduli of the circular CFST and web CFST, respectively.

The limit load $P_k^e$ for $k^{\text{th}}$ iterative step is determined according to the maximum EBR, and reads:

$$P_k^e = P_0 / r_{k}^{\text{max}}$$  (21)

where $P_0$ denotes the initial load multiplier.

The above iteration process is repeated until the limit loads between two adjacent iterative steps meet the following convergence criterion:
where $\varepsilon$ is the allowable error.

If the criterion of convergence is satisfied at the $m^{th}$ iterative step, the ultimate bearing capacity of the CFST arch writes:

$$P_L = P_w^L$$  \hspace{1cm} (23)

The iteration process is illustrated by the flow chart in Fig. 5.

Fig. 5. Flow chart of EMRM

4. Analysis and validation

A number of CFST members and arches with test data were adopted as examples in this section to demonstrate the validity of the proposed HGYFs and the EMRM.

4.1. Concrete-filled steel tube members

A total of 30 dumbbell-shaped CFST specimens [18-20] under eccentric compression were collected and listed in Table 3.

| Source          | $L$/mm | $D$/mm | $t$/mm | $e$/mm | $f_y$/MPa | $f_{ck}$/MPa | Amount |
|-----------------|--------|--------|--------|--------|-----------|-------------|--------|
| Chen et al [21] | 800~3200 | 108    | 4      | 10~40  | 304.8     | 33.4        | 16     |
| Chen et al [22] | 800    | 108    | 4      | 10~40  | 429       | 29.8        | 8      |
| Sheng et al [23] | 800    | 108    | 4      | 10~40  | 429       | 29.8        | 6      |

The ultimate bearing capacity of each specimen was determined on the basis of the HGYF with the stability effect, whose result was denoted by $N_{uc}$, while the test data denoted by $N_{ut}$. The ratio of $N_{uc}$ and $N_{ut}$ reads:

$$R = \frac{N_{uc}}{N_{ut}}$$  \hspace{1cm} (24)

The results were illustrated in Fig. 6, and the calculated mean value $\mu$ and coefficient of variation $\delta$ of $R$ are listed in Table 4. From Fig. 6 and Table 4, it can be observed that the results of the HGYF agree well with the test data which demonstrates that the HGYF developed in this paper can achieve high accuracy.

In order to analyze the influence of stability coefficient on the ultimate bearing capacity of the dumbbell-shaped CFST members, results of the HGYF denoted by Eq.(9) are also listed in Table 4. It appears
that the results of the HGYF without stability coefficient are much larger than test data, the average error comes to 27%, which shows that the stability effect should be taken into consideration.

![Figure 6. Compression of HGYF and test results](image)

| Table 4. Mean value and coefficient of variation of R |
|---------------------------------------------|
| Results HGYF with stability | HGYF without stability |
| μ | 0.980 | 1.272 |
| δ | 0.105 | 0.116 |

4.2. Concrete-filled steel tube arches under in-plane load

Sheng [21] conducted two experiments on dumbbell-shaped CFST arches. The geometric configuration is shown in Fig. 7. The arch axis equation is of catenary curves with span length of $L=8.5m$. The outer diameter of the steel tube is 108mm, while the height and width of the web are 40mm and 10mm, respectively. The thickness of the tube and the web are both 4mm. The characteristic value of yield strength $f_y$ and elastic modulus $E_s$ of the steel are 378MPa and $2.1 \times 10^5$MPa, respectively. The characteristic value of the cubic compression strength $f_{cu}$ and elastic modulus $E_c$ of the core concrete are 39.38MPa and $27.3 \times 10^3$MPa, respectively.

![Figure 7. Geometric configuration of arches under plane loading](image)

The finite element models of the dumbbell-shaped CFST arches were developed on the basis of the beam189 in ANSYS package. Then the EMRM was employed to evaluate the ultimate bearing capacity of the CFST arches on the basis of the proposed HGYF. Results corresponding to different numbers of discretized elements are illustrated in Fig. 8, which tends to converge when the arches are discretized into only 8 beam elements.
The iteration processes of the EMRM are illustrated in Fig. 9. It is obvious that the EMRM results based on the HGYF are independent of the initial load, keeping constant under different initial loads and agreeing well with the test result, which demonstrate the accuracy of the proposed HGYF and the EMRM.

Furthermore, the results of the proposed EMRM, the INFEM as well as the equivalent beam-column method are compared in Table 5. It shows that the proposed EMRM agrees better with the test result under two cases with an error less than 2% compared with other methods. Meanwhile, it appears that although the INFEM and the equivalent beam-column method can achieve good accuracy in some cases, the results are not stable and have large errors sometimes. What is more, only 6 iterations and 5 seconds are needed for the proposed EMRM to obtain the ultimate bearing capacity of the arch on the basis of a computer with 3.6GHz CPU and 16G RAM, which validates the high accuracy and efficiency of the proposed EMRM.

In order to investigate the influence of stability on the ultimate bearing capacity of CFST arch, the results of the EMRM without stability effect were listed in Table 6 when the EBR was determined by the Eq.(9). It can be seen from Table 6 that the error comes to 31% when the stability coefficient is not taken into consideration, which validates the necessity of stability effect when estimating the ultimate bearing capacity of structures.

### Table 5. Ultimate bearing capacity of the arches of different methods/kN

| Loading position | Test | INFEM | Equivalent beam column method | EMRM |
|------------------|------|-------|-------------------------------|------|
|                  |      | Chen [22] | Li [23] | Chen [22] | Li [23] |                  |
| Crown            | 320  | 318.9  | 361.0  | 226.9  | 249.0  | 316.3  |
|                  |      | Error/% | 0.3    | 12.8   | 29.1   | 22.2   | 1.2    |
|                  | 1/4 span | 248.7  | 285.0  | 178.8  | 254.0  | 253.3  |
|                  |      | Error/% | 0.5    | 14.0   | 28.5   | 1.6%   | 1.3    |

| Loadposition | Test | With stability | Without stability |
|--------------|------|----------------|-------------------|
| Crown        | 320  | 316.3          | 31.0              |
| 1/4 Span     | 250  | 253.3          | 33.1              |

Fig. 8. Results under different finite element meshes

Fig. 9. Iteration processes of the EMRM based on the HGYF
4.3. Concrete-filled steel tube arch under spatial load

Li et al. [24] carried out an experimental study on a dumbbell-shaped CFST arch under spatial loadings. The length of the span is 8.5m, the height of the arch is 1.88m. The ultimate bearing capacity of the arch was measured through the test by applying vertically the in-plane concentrated loads at five points with equal distance along the span, meanwhile applying horizontally an out-of-plane concentrated load with 0.1 times of the in-plane load at the crown, as shown in Fig. 10. The outer diameter and wall thickness of the steel tube are 108mm and 4mm, respectively, with the height and width of the web are 40mm and 50mm, respectively. The characteristic value of the yield strength of steel \( f_y = 239.5 \text{MPa} \) and the elastic modulus \( E_s = 2.01 \times 10^5 \text{MPa} \). The characteristic value of the cubic compression strength \( f_{cu} \) and the elastic modulus \( E_c \) of the core concrete are 49.8MPa and 34.5\( \times 10^3 \text{MPa} \), respectively.

The finite element model is developed for the EMRM to evaluate the ultimate bearing capacity of the dumbbell-shaped CFST arch by means of the software package ANSYS. Results corresponding to different numbers of discretized elements are illustrated in Fig. 11, which tends to converge when the arches are discretized into 18 beam elements.

The iteration processes of the EMRM are illustrated in Fig. 12. It is obvious that the results based on the HGYF are independent of the initial load, keeping constant under different initial loads and agreeing well with the test result, which demonstrates the accuracy of the proposed HGYF.

Results of the INFEM, the EMRM and the test data are compared in Table 7. It can be observed that the results obtained by the INFEM and the EMRM are approximately equal to each other and agree well with the test data, validating the accuracy of the proposed method. However, the arch needs to be divided into 28037 elements when the INFEM [4] was adopted, which require a large amount of calculation time. In contrast, only 5 iterations and 3 seconds are needed for the proposed EMRM using a computer with 3.6GHz CPU and 16G RAM, which shows that the proposed EMRM has higher efficiency. When the stability effect is not taken into consideration, the results are listed in Table 7. It can be seen that the
calculation error is huge without taking the stability effect into account.

Table 7. Ultimate bearing capacity of the arch under spatial loading/kN

| Methods | Test | INFEM [4] | EMRM With stability | EMRM Without stability |
|---------|------|-----------|---------------------|-----------------------|
| Result  | 130  | 125.3     | 128.9               | 172.1                 |
| Error/% | —    | 3.62      | 0.85                | 32.4                  |

5. Conclusions
The HGYF and the EMRM were presented for estimating the ultimate bearing capacity of the dumbbell-shaped CFST arches. The following conclusions can be drawn:

1) The proposed HGYF satisfies the proportional condition for limit analysis, which achieves high accuracy. It is independent of the initial load when adopted for the EMRM.

2) The proposed EMRM achieves higher accuracy and efficiency than the INFEM and the equivalent beam-column method for ultimate bearing capacity analysis of dumbbell-shaped CFST arches under in-plane and out-of-plane loading conditions.

3) The proposed EMRM extends the EMAPs usually adopted for strength analysis to stable ultimate bearing capacity analysis.

Acknowledgements.
This work was supported by the National Natural Science Foundation of China (51738004), the Post-doctoral Science Foundation of China (2020M673095)

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