**Microgravity/microscale double-helical fluid containment**

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Double-helical containment is a novel approach to open containment in microgravity (or at microscale). In contrast to axisymmetric containers, there is no length restriction on properly designed double-helical containers. Use of a double helix permits drainage to zero volume, an uncommon feature in microgravity; near-complete drainage is a key feature for any practically useful container. The fact that double-helical containers are open and tubular makes possible a broad range of applications that rely on accessible fluids.

The double-helical fluid containment and the stability of such volumes are examined in detail. Helical supports permit filamentary containers of infinite extent, but only double-helical containers are stable down to zero volume. The base case of symmetric supports (separation angle 180°) is considered in terms of symmetric volumes as well as asymmetric helicatenoid volumes. The distinct symmetric and asymmetric cases are then related by perturbing the angle of separation between the supports. The helicatenoid volumes may combine to form a dual helicatenoid volume. The dual helicatenoid is interesting in that it permits multiphase tube-like geometries.

Double-helical behaviours such as drainability are found to vanish at a critical separation angle 246.48°. With larger separation angles, double-helical containers behave like single-helical containers (360°), due to the dominance of the longer-span interface. A second shift in behaviour occurs at the limit angle 209.12°, where the regions of stability including the cylinder and the helicatenoids become more connected. The common structures of DNA appear to coincide with the limiting geometry at 209.12°. The most robust double-helical containers are therefore those with separation angles between approximately 210° and 240°. Experimental results roughly verify the volume maximum for near-symmetry, and more importantly verify stability to zero volume.

**Keywords:** microgravity; double-helical; containment; capillary; stability; DNA

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1. **Introduction**

The liquid containment in microgravity presents a special problem: ordinary open containers such as cups simply do not work, as without gravity they are difficult to drain and tend to leave stray fluid volumes (Thulasiram *et al.* 1992; Mittelmann & Zhu 1996; Concus & Finn 1998; Slobzhannin *et al.* 1999). Contact processes involving droplets are even more problematic in microgravity, as
gravity is routinely assumed for phase separations (cf. Cussler 1997). Liquid bridges may be used for open containment, but they become unstable prior to draining completely. Also, cylindrical liquid bridges become unstable when their length exceeds their circumference (Plateau 1863, 1864, 1865; Rayleigh 1879). Despite these limitations, axisymmetric liquid bridges in microgravity have been the subject of a broad range of study, from practical applications such as melt zone refining (Da-Riva & Meseguer 1978) to stability analysis (Myshkis et al. 1987; Lowry & Steen 1995; Lowry 2000a; Slobozhaini et al. 2002; May et al. submitted). Recent attempts to extend the maximum stable length of cylindrical liquid bridges have included stabilization by electric fields (Marr-Lyon et al. 2000) and acoustics (Marr-Lyon et al. 2001) as well as use of multiple liquid bridges to achieve greater compound length (Lowry 2000b; Patel et al. 2002; Terrier et al. submitted).

It has recently become apparent that helical supports are potentially versatile containers and tubes in microgravity (Lowry & Thiessen 2007). Helical containers can be infinite in extent, unlike liquid bridges, and they can therefore serve as tubes as well as open containers (Lowry & Thiessen 2007). It has been noted that the double helix \((N=2)\) alone has the property of continuous stability from the cylinder down to zero volume (Lowry & Thiessen 2007). Neither single-helical supports \((N=1)\) nor three or more \((N \geq 3)\) evenly spaced supports have this property (Lowry & Thiessen 2007; and there is no reason to expect unevenly spaced, drainable \(N \geq 3\) supports). The double-helical support therefore has the unique property of being an open, fully drainable microgravity container and conduit, and it is reasonable to expect that this property extends to microscale. However, the scope of work to date on helical interface stability does not extend to nanoscale, where surface tension is known to become curvature dependent (Kashchiev 2003; Wang & Yang 2005).

Helical interfaces are fluid surfaces formed when a helical boundary supports an interface, which may either surround a volume of fluid or exist as a solitary thin film. The helical support can take a variety of forms, but can be idealized as one or more coaxial helical thin wire springs. In the case of a double helix, there are two supporting springs, rotationally offset by some angle. The support, and thus the interface, is ideally of infinite extent. The helical interface adopts the helical symmetry of the support, as it stretches across the support. In axial profile, helical interfaces most resemble an infinite staggered series of liquid bridges (figures 1 and 2a). In azimuthal profile, the cross-section is less complete, as the pitch of the helix is not explicit (figure 2b). A helical volume of fluid supported by thin wires appears as a long slender filament of fluid, resembling a jet of fluid (figure 3).

There appears to be only one previous investigation of double-helical interface stability (Boudaoud et al. 1999). Boudaoud et al. investigated the stability of double-helical soap films both numerically and experimentally, finding excellent qualitative agreement. The work of Boudaoud et al. was motivated by qualitative similarities between the structure of DNA and that of helical surfaces of zero curvature. Owing to the nature of soap films, their work is limited to the interfaces of zero curvature, and they further limited their geometry to symmetric double helices (separation angle 180°). While the helicoid is a well-known minimal surface (zero curvature) in mathematics, often associated with
DNA, the helical structure of DNA is not particularly close to that of the helicoid. However, the structure of DNA is close to a family of zero-curvature ribbon surfaces that resemble the axisymmetric catenoid in profile. In this work, volumes related to such interfaces will be referred to as helicatenoids. Boudaoud et al. (1999) found a stability boundary that originates at a bifurcation between helicoids and helicatenoids, and which they concluded correlates loosely with the geometry of β-DNA.

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The bifurcation between helicoids and helicatenoids has been described in terms more closely related to the current work, by considering the simple intermediates in the mathematical transformation between unbounded catenoids and helicoids (May & Lowry submitted). The introduction of double-helical boundaries defines the intermediates as helicatenoids of zero curvature.

The stability of double-helical interfaces is determined using the method of Maddocks as applied to fluid interfaces by Lowry & Steen (1995). It has previously been determined that only the fixed pressure stability limit is relevant for helical interfaces of infinite extent (Lowry & Thiessen 2007). This greatly simplifies the stability analysis, as all stability limits then occur at turning points in pressure jump, which is proportional to the sum of the principal curvatures of the interface. Branches of equilibria and associated turning points have been determined numerically using a simple fourth-order Runge–Kutta shooting method with overall tolerances of the order of $10^{-10}$.

2. Helical parameters

(a) Helical coordinates

An ideal fixed contact line double-helical interface binds to two infinitely thin helical supports. Coordinates and metrics for an idealized double helix are shown in figure 4. The radius of the helix is normalized to 1 and the contact line is a uniform helix at $r=1$. A fixed contact line boundary condition is assumed. The formulation is limited to perfectly uniform boundaries of infinite extent. Helicosymmetry is assumed such that the helical interface exhibits helical symmetry about the axis of the helix. Non-helicosymmetry has previously been shown to be irrelevant to the stability of helical interfaces of infinite extent (Lowry & Thiessen 2007).

In this work, helicosymmetric interfaces are computed as cross-sectional shapes, based on Cartesian parametric coordinates (figure 2b)

$$\dot{x} = x(s)\cos(\alpha z) - y(s)\sin(\alpha z), \quad \dot{y} = y(s)\cos(\alpha z) + x(s)\sin(\alpha z), \quad \dot{z} = z,$$

where the arclength $s$ is defined such that $\dot{x}^2 + \dot{y}^2 = 1$, where dot superscripts denote derivatives by $s$ and the slope, $\alpha$, defines the tightness of the helical winding: $\alpha \to 0$ is an extremely loose winding, while $\alpha \to \infty$ is an extremely tight winding. The helicoid–helicatenoid bifurcation occurs at $\alpha = \lambda^{-1}$, where $\lambda = 0.66274342\ldots$ is the Laplace limit (May & Lowry submitted).
The pressure jump is proportional to the sum of principal curvatures, $\bar{\kappa}_0$, which is (cf. Lowry & Thiessen 2007 for derivation)

$$\Delta p = \bar{\kappa}_0 = \frac{\kappa(1 + \alpha^2 \tau^2) + \alpha^2(y\dot{x} - x\dot{y})}{g^{3/2}},$$

(2.1)

where the metric of the surface, $g$, is $g = (1 + \alpha^2 r^2 \tau^2)$.

The volume contained by a helical interface is expressed as the ratio of the actual volume (over the entire length of the helix) to that of a unit cylinder of identical length. In the case of double-helical interfaces, the volume is the sum of both volumes (note that in certain cases there are four interfaces and thus four volumes to sum) divided by that of the cylinder

$$V_1 = \int_0^{S_1} (x\dot{y} - \dot{x}y)ds, \quad V_2 = \int_0^{S_1} (x\dot{y} - \dot{x}y)ds \Rightarrow V = \frac{V_1 + V_2}{2\pi},$$

where $S_1$ and $S_2$ are the total arclengths of two interfaces. Cylindrical interfaces therefore contain a volume $V=1$. Thin film interfaces, which contain no volume, have $V=0$. The formulation permits either $V_1$ or $V_2$ to be negative, but it is physically impossible to have $V<0$. Results are presented only in terms of overall volume, $V$.

For a double-helical support, the two wires are separated in cross-section by some angle, $\theta$ (figure 5). The symmetric double helix has $\theta=180^\circ$, and asymmetric double helices are taken to have $180^\circ<\theta<360^\circ$, as the stability of

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Figure 4. Helical parameters defined via the example of a single interface on a double-helical support: $\alpha$, $L$ and $\hat{L}$.

Figure 5. Definition of $\theta$ on an $xy$ cross-section.
the double-helical interface generally depends on the larger span. The single helix therefore corresponds to the limiting case $\theta=360^\circ$.

In normalized helical coordinates, the axial distance between successive instances of the same wire, *pitch*, is $L = 2\pi/\alpha$. However, in helical coordinates, the length in the $z$-coordinate is less representative of an interface than the *span* across a cylindrical surface of unit radius perpendicular to the wires, $L = 2\pi/\sqrt{1+\alpha^2}$. As the span is orthogonal to the helix, it is a better measure of the actual separation of wires than the pitch. For this reason, in this work the slope is scaled in terms of normalized span, $1/\sqrt{1+\alpha^2}$.

### 3. Symmetric ($\theta=180^\circ$) double-helical volumes

As reported by Lowry & Thiessen (2007), the stability envelope for a single-helical volume is very similar to that for a fixed pressure, fixed contact line liquid bridge. In both the cases, there is a maximum stable volume and a minimum stable volume. However, for the single-helical case, there is also a previously unreported region of helicatenoid states, which extends to zero volume (figure 6e). The single-helical helicatenoids are filled shell shapes with small volumes of fluid trapped between two similar interfaces (figure 8d). The single-helical helicatenoids are loosely analogous to shell-form liquid bridges (Tsamopoulos et al. 1988), and the states are similarly disconnected—it is not possible to go from the cylindrical single-helical volume to a single-helical helicatenoid via any combination of changes in volume and/or pitch.

By contrast, the main stability envelope for a symmetric ($\theta=180^\circ$) double-helical volume extends to zero volume (figure 6a). The envelope actually extends past zero volume, symmetric across the $V=0$ axis. However, as negative volume states are non-physical, they are not considered here (see §5 for experimental verification of the stability of pseudo-negative volumes). The main portion of the envelope includes the $V=1$ cylindrical state for all pitches. Cylindrical interfaces of infinite axial extent are always stable with symmetric double-helical supports. In terms of overall stability, the region of stability for double-helical supports is extremely broad, including the entire range $V=0$–1 for looser pitches ($0<\alpha<\lambda^{-1}$) and most of this range for tighter pitches ($\alpha>\lambda^{-1}$). Figure 7 shows cross-sections for assorted $\theta=180^\circ$ volumes over a range of volume, illustrating the considerable stable range in volume.

For $\alpha<\lambda^{-1}$, the stable zero volume state is a helicoid (figure 6a). However, for $\alpha>\lambda^{-1}$, the stable zero volume states are helicatenoids rather than helicoids (May & Lowry submitted). Figures 8a(i) and 9b show the cross-section of a helicatenoid of non-zero volume, and figure 9a. The double-helical helicatenoid is more complex than the single-helical helicatenoid in that there are two supports. The cylindrical and helicoid interfaces, along with every interface in the main stability envelope, consist of two distinct interfaces bridging the two supports. In the main stability envelope, all such states are symmetrical (cf. figure 7). The single helicatenoid is asymmetric, comprising two distinct interfaces bridging the two supports (cf. figures 8a(i) and 9b). However, the asymmetry of the single helicatenoid permits a double case, which is symmetrical but which comprises four distinct interfaces bridging the two supports (cf. figure 8a(ii)). The helicoid and
Figure 6. Stability envelopes for (a) $\theta=180^\circ$, (b) $\theta=209^\circ$ (cylinder-helicatenoid(II) region), (c) $\theta=209^\circ$ (helicatenoid(I) and dual helicatenoid regions), (d) $\theta=246^\circ$ (solid lines) and $\theta=247^\circ$ (dashed lines), and (e) $\theta=360^\circ$.
helicatenoid regions are separated by the helicoid–helicatenoid bifurcation at $\alpha = \lambda^{-1}$. The dual helicatenoid is separated by the physically distinct presence of two extra interfaces, perfectly analogous to the distinction between single-helical helicatenoids and other single-helical interfaces (or between liquid bridges and shell-form liquid bridges). The overall effect is that there are three distinct regions of stability for double-helical volumes.

Near the helicoid–helicatenoid bifurcation, the dual helicatenoid stability region fits perfectly with the perimeter of the main cylindrical-helicoid stability region (figure 6a). Indeed, there is a region of overlap extending from the bifurcation at $\alpha = \lambda^{-1}$ to 4.25, where there are multiple equilibria having the same volume and slope. The overall effect is nearly continuous stability for $\lambda^{-1} < \alpha < 1.25$. However, in practical terms, the dual helicatenoid is not easily attainable relative to the simpler forms, as the dual helicatenoid has double the number of interfaces. Dual helicatenoids are considered here for completeness, but it will be shown below that the single helicatenoid is much more closely related to the cylindrical-helicoid states.

4. Asymmetric ($\theta > 180^\circ$) double-helical volumes

Even small deviations from $\theta = 180^\circ$ lead to significant shifts in the stability at singular points, such as the helicoid–helicatenoid bifurcation point (figure 10). Overall, the stability of double-helical volumes falls into three distinct regions away from $\theta = 180^\circ$. Figure 11 shows loci of extrema in $V$ (volume) and $\alpha$. Two volume extrema merge at $\theta = 209.12^\circ$ (B in figure 11), and the main cylindrical-helicoid stability region ruptures at $\theta = 246.48^\circ$ (A in figure 11). Thus there are
four regions of stability to be considered: \( \theta = 180^\circ \), \( 180^\circ < \theta < 209.12^\circ \), \( 209.12^\circ < \theta < 246.48^\circ \), and \( 246.48^\circ < \theta \leq 360^\circ \). The last case brings the transition fully from double-helical volume to single-helical limit, at \( 360^\circ \).

(a) Perturbations from symmetry: \( \theta = 180^\circ \)

The breaking of symmetry to \( \theta = 180^\circ + \epsilon \) results in two complementary angular spans of \( 180^\circ + \epsilon \) and \( 180^\circ - \epsilon \). In the main cylindrical-helicoid stability region, two interfaces each span one of the angles, and the overall effect tends to \( O(\epsilon) \). In the dual helicatenoid stability region, four interfaces span each angle as antisymmetric pairs, and again the overall effect tends to be \( O(\epsilon) \). However, in the asymmetric single-helicatenoid regions of stability, a pair of interfaces both
span the same angle. Two distinct overlapping stability regions appear from the single-symmetric stability region. The helicatenoids(I) that span the larger angle are stable over a larger range of volume, but exist in a distinct region of stability (figure 6c). The helicatenoids(II) that span the smaller angle are stable over a

Figure 9. Rendered images of (a) $V$ maximum of helicatenoid with $\theta=180^\circ$ and $\alpha=5$ and (b) $V$ maximum of dual helicatenoid with $\theta=180^\circ$ and $\alpha=5$.

Figure 10. Stability envelopes for $\theta=180^\circ$ (bold lines), $181^\circ$, $185^\circ$, $190^\circ$, $195^\circ$, $200^\circ$, $205^\circ$, and $209^\circ$ (dashed lines): cylinder-helicatenoid(II) boundaries in solid black lines, helicatenoid(I) in solid grey lines and dual helicatenoid in dotted grey lines.
smaller range of volume, but that region of stability merges with the main cylindrical-helicoid stability region as soon as the symmetry is broken (figures 10 and 6b). This isolation/merging occurs when the helicoid–helicatenoid bifurcation breaks into two turning points (May & Lowry submitted). The alteration of the stability region for $\theta \approx 180^\circ$ is not a linear perturbation; local to the helicoid–helicatenoid bifurcation there is a significant gain in stability even with a small degree of asymmetry.

There are two other significant effects of the breaking of symmetry: the cylinder becomes unstable for sufficiently small slope (large pitch) and the volume maximum decreases sharply as the symmetry is broken (figure 10). The destabilization of the cylinder is practically unimportant, as it affects only nearly parallel contact lines. The decrease in the absolute volume maximum is an interesting effect: when $\theta$ is varied from $180^\circ$, the maximum volume decreases rapidly. This singularity occurs well away from any special characteristic of the ideal double-helical case ($\theta = 180^\circ$), and thus was unexpected. It happens that the rapid decrease in volume is due to a rapid decrease in the volume contained by the short-span interface, well out of proportion to the change in angle. By considering the volumes spanned by each angle, in isolation, the cause of the singularity becomes apparent (figure 12). The stability limit at the volume maximum coincides with the pressure maximum.

Figure 11. Loci of extrema of $V$ and $\alpha$ in terms of $\theta$ and $\alpha$ with labelled point A convergence of local maximum and minimum $V$ for helicatenoid with $\theta = 209.12^\circ$, point B stability envelope rupture point with $\theta = 246.48^\circ$, point C1 $V$ maximum for helicatenoid with $\theta = 180^\circ$, point C2 $\alpha$ minimum for helicatenoid with $\theta = 180^\circ$, point C3 $V$ maximum from cylinder with $\theta = 180^\circ$, point C4 $V$ minimum from cylinder with $\theta = 180^\circ$, point D1 $V$ maximum for helicatenoid with $\theta = 360^\circ$, point D2 $\alpha$ minimum for helicatenoid with $\theta = 360^\circ$, point D3 $V$ maximum from cylinder with $\theta = 360^\circ$ and point D4 $V$ minimum from cylinder with $\theta = 360^\circ$. 

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which appears as a turning point on each curve of equilibria in figure 12. The absolute volume maximum occurs for $\theta=180^\circ$ at $\alpha=1.271$, but for increasing $\alpha$, it occurs at increasing $\alpha$. Furthermore, for $\theta=180^\circ$, the maximum double-helical volume is twice the volume spanned by a single interface, but for $\theta>180^\circ$, the volume is the sum of that spanned by $\theta$ and $360^\circ-\theta$. Figure 12 illustrates the overall effect of increasing $\theta$ on the absolute volume maximum: the slope increases slightly, increasing the maximum pressure at that point; but the increase in $\theta$ splits the equilibrium curve into two curves, for the long and short spans. Finally, the stability limit is governed by the longer span as the maximum pressure is smaller (and hence is reached first for that span). The effect on the shorter span is dramatic—so near to the pressure maximum, a small decrease in pressure changes the volume by a large amount. This volume shift is entirely dominant, and is proportional to $\sqrt{\theta-180^\circ}$. The strong volume dependence of the shorter span significantly restricts the maximum volume attainable for slight imperfections in angle (away from the ideal double-helical contact lines with $\theta=180^\circ$). In practical terms, the ideal absolute volume maximum is not attainable, so that a realistic absolute volume maximum for double-helical containment is probably closer to $V=2V_{\text{cyl}}$ than the greater volume suggested by theory.

A similar effect occurs for the dual helicatenoid at its volume maximum. The dual helicatenoid consists of helicatenoid(I) and helicatenoid(II) volumes, but its stability is governed by the long-span helicatenoid(I). As symmetry is broken, the

\[ \alpha = 1.271 \quad \theta = 180^\circ \]

\[ \alpha = 1.332 \quad 182^\circ \quad 180^\circ \quad 178^\circ \]

\[ V \quad 2.50 \quad 2.25 \quad 2.00 \quad 1.75 \quad 1.50 \]

\[ \tilde{\kappa}_0 \quad 1.38 \quad 1.40 \quad 1.42 \quad 1.44 \quad 1.46 \]

Figure 12. Equilibria in preferred coordinates of volume versus pressure (as sum of curvatures). Leftmost curve represents absolute volume maximum for $\theta=180^\circ$. Three curves on the right represent absolute volume maximum for pairing of $\theta=182^\circ$ and $178^\circ$ (and $\theta=180^\circ$ shown only to illustrate perturbation). Pressure maximum for $\theta=178^\circ$ represented by an open circle, as not actually attainable when matched with $\theta=182^\circ$.

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helicatenoid(II) volume rapidly decreases. The net effect is that as $\theta$ increases from $180^\circ$, the range of dual helicatenoid stability rapidly decreases, towards the helicatenoid(I) stability envelope.

(b) Assimilation of the helicatenoids: $180^\circ < \theta < 209.12^\circ$

For $180^\circ < \theta < 209.12^\circ$, there are three volume stability envelopes: the cylindrical-helicoid–helicatenoid(II); the helicatenoid(I); and the dual helicatenoid (figure 6b,c). The main cylindrical-helicoid–helicatenoid(I) stability region ranges over all $\alpha$ and up to roughly double the cylindrical volume. In contrast, the helicatenoid(I) and dual helicatenoid stability envelopes are restricted in volume, and share a minimum value of $\alpha$ that increases as $\theta$ increases.

For $\theta \approx 180^\circ$, the main stability region comprises the cylindrical-helicoid and helicatenoid(I) regions, which are joined by a narrow region of stability near the position of the critical helicoid ($V=0$ and $\alpha=\lambda^{-1}$). As $\theta$ increases, the bridge between the two regions becomes more substantial. Therefore, while the helicatenoid(I) volumes remain a discrete class of volume, they are accessible from the main cylindrical-helicoid volumes via careful manipulation of pitch and volume.

At $\theta = 209.12^\circ$, the maxima and minima in volume along the helicatenoid(II) stability envelope combine and disappear. Therefore, at this special angle, there is a stability limit that is independent of slope over some small range. Cross-sections of helicatenoids near this limit are shown in figure 13, and a simulated image of this limiting helicatenoid is shown in figure 16a. This angle might be of particular importance in applications where constant slope could not be assured at all points along the boundary, as otherwise regions of the volume where slope was slightly
higher or lower would always destabilize first. While this work cannot be applied directly to nanoscale, the existence of this angle strongly suggests that there is an ideal offset for nanoscale double-helical supports and molecules, which are likely to have fluctuating geometry due to thermodynamic effects.

\[(c)\] Robust double-helical stability: $209.12^\circ < \theta < 246.48^\circ$

When $\theta = 209.12^\circ$, the smallest local volume maximum and minimum on the helicoids stability envelope disappear (A on curve $C_1B_2C_3$ in figure 11). Beyond this point, an undivided cylindrical-helicatenoid(II) region of stability exists over a broad range of slope and volume. The broad stability is defined only by a volume maximum for smaller slopes, and by two discrete volume ranges for higher slopes (figure 16b,c illustrate the volume maximum and minimum from the cylinder). Stability over this range of $\theta$ is more constant and robust than that near $\theta = 180^\circ$ (cf. figures 10 and 14). Figure 15 illustrates how the short-span interface tends towards the cylinder as $\theta$ increases—stability in this range is entirely dominated by the large-span interface, as are shape and volume.

The minimum value of $\alpha$ for helicatenoids(I) and dual helicatenoids continues to increase with $\theta$ and the helicatenoid(I) and dual helicatenoid envelopes approach one another.

\[(d)\] Pseudo-single-helical volumes: $246.48^\circ < \theta \leq 360^\circ$

At $\theta = 246.48^\circ$, the stability envelope for the helicoids ruptures into two separate near-cylindrical and helicatenoid(II) envelopes (figure 6d). In this range of $\theta$, the stability is dominated by the near-cylindrical stability envelope, which
persists, relatively unchanged, over the entire range (figure 17). There is a slight increase in absolute maximum volume as the single-helical case ($\theta = 360^\circ$) is approached. Over this range of $\theta$, the stability, shape and volume are dominated by the long-span interface, as the short-span interface is practically cylindrical over the entire range of stability (except for helicatenoid(II) volumes, which are increasingly irrelevant).

The helicatenoid(II) envelope decreases in volume as $\theta$ increases, until it essentially disappears into $V=0$. However, the helicatenoid(II) volumes do remain viable over a very broad range of $\alpha$. The envelope is misleading, as the distance between the short-span contact lines decreases with increasing $\theta$, and

*Figure 15. Axial and azimuthal cross-sections with $V=0.5$, $\alpha=1$ and (a) $\theta=180^\circ$, (b) $\theta=210^\circ$, and (c) $\theta=240^\circ$.*

*Figure 16. Rendered images for $\theta=240^\circ$ and $\alpha=5$: (a) $V$ minimum from cylinder and (b) $V$ maximum from cylinder.*
the volume is roughly proportional to the distance between those contact lines. The discrepancy is due to the scaling by cylindrical volume, which is well removed from the helicatenoid(II) shapes. In the limit of \( q/360 \approx 8 \), the contact lines approach one another and the helicatenoid(II) volumes approach the case of a volume held between two parallel wires, as the helical twist is insignificant at sufficiently small scales.

In this range, the helicatenoid(II) portion of a dual helicatenoid has negligible volume, so that the dual helicatenoids are essentially helicatenoid(I) volumes plus tenuous films (cf. figure 8b(iii), c(iii)). In practical terms, the dual helicatenoid does not exist for larger \( q \).

The dual helicatenoid stability envelope approaches the near-cylindrical envelope as \( q \) increases and the two actually cross very near to \( q = 360 \) (the apparent cusp in figure 6e is actually a very slight overlap). A very narrow sliver of states exist where both dual helicatenoids and near-cylindrical volumes are possible for the same slope and volume. It is surprising that the coincidence is very close without being singular (such as at a single-cusp point), but since the envelopes cross there are actually two identical states on each envelope (figure 18 shows cross-sections for two corresponding states).

(e) Shape correspondence at stability limits

When \( 180^\circ < \theta < 360^\circ \), there are two dual helicatenoids. In the case of \( \theta = 180^\circ \) there is only one as a result of symmetry. In the limit of \( \theta \to 360^\circ \), the interface degenerates to being single (figure 8).

The longer-span, smaller radius interface of the dual helicatenoid at the volume maximum corresponds to longer-span, smaller radius interface at the near-cylindrical volume minimum, for the same value of \( \alpha \) and \( \theta \) (figure 19).
This correspondence of interfaces illustrates how the shape (and thus curvature/pressure) of the long-span interface governs the stability of the fluid interfaces rather than the overall fluid volume.

5. Experimental verification

Owing to the challenge of constructing double-helical systems with uniform contact lines, our initial experiments have been with single-helical contact lines spanning angles in the range of $\theta = 180^\circ$ (figure 20). The experiments have verified the general quality of the stability (figure 21), as well as verifying the stability of the fluid interfaces.

Figure 18. Axial and azimuthal cross-sections for $\alpha = 6.95$ and $\theta = 360^\circ$: (a) single-interface volume and (b) helicatenoid.

Figure 19. Compound cross-sections illustrating coincidence of interface at stability limit, with $\alpha = 5$ and (a) $\theta = 180^\circ$, (b) $\theta = 240^\circ$, and (c) $\theta = 360^\circ$. White and light-grey regions indicate dual helicatenoid volumes, while darker-grey region indicates filled volume at volume minimum from cylinder.

This correspondence of interfaces illustrates how the shape (and thus curvature/pressure) of the long-span interface governs the stability of the fluid interfaces rather than the overall fluid volume.
Figure 20. Single-interface support used in experiments, shown with ideal angle $\theta = 180^\circ$ over open portion (a) side view and (b) end view (cross-section).

Figure 21. Experimental photographs (N.B. actual apparatus is vertical, not horizontal): (a) $V=0.4$ (corresponds to $V=-0.2$ for double-helical volume), $\alpha_{\text{inner}}=0.96$, $\alpha_{\text{outer}}=1.20$, $\theta=184^\circ$ ($D_{\text{inner}}=7.56$ and $D_{\text{outer}}=9.53$ mm), (b–e) $\alpha_{\text{inner}}=0.75$, $\alpha_{\text{outer}}=0.94$, $\theta=198^\circ$ ($D_{\text{inner}}=7.59$ and $D_{\text{outer}}=9.54$ mm); (b) $V=0.9$, (c) $V=1.0$, (d) $V=1.1$, and (e) $V=1.5$. 

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stability to zero volume in a manner in which a true double-helical bound could not. In the single-helical apparatus as constructed, each interface exists in isolation, rather than paired as for double-helical volumes. Thus, the interface is free to move past the helicoid state \((V=0)\) and, as predicted, the interface remains stable past what would be zero volume in the double-helical case (cf. figure 21a).

Experiments were conducted with several titanium helical supports with selected \(\alpha\) and \(\theta\). The supports were closed at the ends by flat cylindrical end pieces, so that the contact line was necessarily non-ideal at the ends; however, the supports are all sufficiently long that there are several turns well removed from the end pieces (cf. figure 21). To achieve microgravity conditions, two fluids were used in a Plateau tank: water as the bath fluid and 2-fluorotoluene as the fluid contained by the helical support. The experiment tank was kept at approximately 27°C so that the fluids were neutrally buoyant. The resulting fluid interfaces were backlit and photographed using a digital camera.

6. DNA

The similarity between stability characteristics of double-helical films and the geometry of DNA has been raised by Boudouad et al. (1999). Although there is no reason to believe the coincidence is of any particular significance, it is presented here for completeness. Figure 22 illustrates how four of the five

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Figure 22. Geometries of A-, B-, C-, D- and Z-DNA versus the loci of double-helical extrema, in terms of \(\theta\) and \(\alpha\), showing rough cluster near limit point at \(\theta=209.12°\). Ranges of \(\alpha\) for DNA geometries are due to ambiguous contact radius at backbone molecules: largest slope based on largest reasonable outer radius and smallest slope on smallest reasonable inner radius.
configurations of DNA (geometries derived from data provided by Saenger (1984)) appear to cluster near the limit point at \( \theta = 209.12^\circ \). This may be suggestive of some abiotic molecular process where the backbone of DNA was sheathed in an oil or surfactant layer, or it may be purely coincidental—the molecular evolution of DNA lies well beyond the expertise of the authors.

7. Conclusions

Double-helical capillary volumes are extraordinarily robust microscale and microgravity containers, stable to infinite length and drainable to zero volume. The broadest range of robust double-helical stability exists for a rough range of \( \theta = 210^\circ - 240^\circ \). In the robust double-helical range, a single main stability envelope extends over the entire range of \( \alpha \), with stable volumes ranging from zero to about double that of a cylinder.

The stability of double-helical interfaces has been characterized by four ranges: the ideal case of \( \theta = 180^\circ \), and perturbations from that case; the near-ideal range of \( 180^\circ < \theta < 209.12^\circ \); the robust range of \( 209.12^\circ < \theta < 246.48^\circ \); and finally the pseudo-single-helical range of \( 246.48^\circ < \theta \leq 360^\circ \). Four regions of stability exist over the entire range of \( \theta \): near-cylindrical (also termed cylindrical-helicoid for smaller angles); helicatenoid(I); helicatenoid(II); and dual helicatenoid.

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