Models with phase slip in the hydrodynamics of granular media

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Abstract. Two-phase flows in granular media are widely used in the chemical and energy industries as they provide a high intensity of transfer processes. One of the key characteristics of such flows is the relationship between the pressure drop across the granular layer and the mass velocity of the two-phase mixture. To date, no single approach has been developed to calculate the characteristics of such flows. In this paper, we compare three models constructed given the difference in the velocities of the liquid and vapor phases. All models are used to solve both the direct problem of calculating the pressure drop for a given mass velocity and the inverse problem of calculating the mass velocity for a known pressure drop. The accuracy of the models was compared using two sets of experimental data with a combined pressure variation range of 0.6–15 MPa, a flow quality of 0.002–0.3, a mass velocity of 120–1100 kg/m²s, ball particle diameters of 2–4 mm, and a packed bed height of 50–355 mm. The calculation with the new gas-dynamic model demonstrated the best result in the entire range of experimental data.

1. Introduction
Among the multiphase flows through porous media, we can distinguish the flow of an evaporating steam-water mixture through a granular medium. Such processes occur during the operation of catalytic chemical reactors, nuclear reactors with particle fuel elements [1], in some highly efficient heat exchangers, as well as during water cooling of fragments resulting from an accident at a nuclear reactor. One of the important objectives of the study on such flows is to establish a relationship between the mass velocity of the mixture and the pressure drop in the granular layer.

An experimental study of the pressure drop in the steam-water mixture flowing through granular layers was carried out by Avdeev et al. [2,3]. The authors used a packed bed of spherical particles with a diameter of 2 mm, a height of 200 mm, and a mass rate of mixture filtration varied from 100 to 770 kg/m²s. The pressure and flow quality varied little along the packed bed height, however, from experiment to experiment they varied from 1.6 to 15 MPa and from 0.002 to 0.3, respectively. The studies on the flow of the steam-water mixture through the packed bed were also conducted at the High-temperature circuit facility. The experimental data are series obtained under constant inlet properties of the mixture with an incremental decrease in the outlet pressure until the critical flow conditions are reached when a further decrease in the outlet pressure does not increase the mass velocity [4], as well as the critical values of the pressure drop and mass velocity [5]. In this case, we varied both the inlet parameters of the mixture (pressure from 0.6 to 1.2 MPa, void fraction from 0.016 to 0.178, mixture filtration from 80 to 480 kg/m²s), and the packed bed parameters (particle diameter

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from 2 to 4 mm, granular bed height from 50 to 355 mm). Li et al [6], and Park et al [7] presented the results of experiments on the flow of a mixture of water and nitrogen (air) through the packed bed with low velocities of the liquid phase.

There are a number of steam-water mixture flow models based on single-phase flow models. The Lockhart-Martinelli and Darcy-Forchheimer models describe each phase by a nonlinear filtration law. In the Lockhart-Martinelli model, the pressure drop of the two-phase flow is associated with the pressure drop when only the vapor and liquid phases flow through the considered cross-section. Zeigarnik et al. [8, 9] were the first to apply the Lockhart-Martinelli method previously developed for two-phase flows in pipes to two-phase flow in porous media. Sorokin [10] specified the coefficients of this method for the two-phase flows through the packed bed. He noted that the resulting model does not take into account the compressibility of the mixture and can only be used under small pressure gradients, i.e. at mass velocities significantly lower than critical ones. In the generalized Darcy-Forchheimer model, the pressure drops for both phases in the mixture are considered to be the same. Equality of gradients is achieved by introducing the coefficients of relative permeability and passability that allow for the decrease in the fraction of the cross-section per each phase, as well as by taking account of the forces of interphase interaction in some implementations of the model. The generalized Darcy-Forchheimer model with various values of the coefficients and forces of interphase interaction has been used by many researchers, mainly as applied to the flow of an air-water mixture at low velocities [6, 7]. Tairov and Khan [11] built a model of the steam-water mixture flow through a packed bed based on the gas-dynamic model presented by Goldshlik [12]. The density and polytropic exponent of the mixture were obtained by generalizing the experimental data [3, 4] using equations that consider phase slip.

The paper presents the quality assessment of the above models involving the results of the experiments conducted at the High-temperature circuit facility [4] and the data obtained by Avdeev et al. [3]. The ranges of the considered pressures and mass velocities are 0.6 – 15.3 MPa and 150 – 1200 kg/m²s, respectively.

2. Models and methods

2.1. Model based on the Lockhart-Martinelli method (model 5)

According to the Lockhart-Martinelli technique [13, 14], originally proposed for the adiabatic flow of a two-phase mixture in pipes, the specific pressure drop is calculated using the formulas:

\[
\left(\frac{dP}{dy}\right)_{l} = \left(\frac{dP}{dy}\right)_{v} \Phi_{l}^{\mu}, \quad \left(\frac{dP}{dy}\right)_{v} = \left(\frac{dP}{dy}\right)_{l} \Phi_{v}^{\mu},
\]

where \((dP/dy)_{l}\), \((dP/dy)_{v}\) are pressure gradients when either only the liquid component or only the vapor component of the mixture flows through a complete pipe section. They were calculated using the modified Darcy equation [15]:

\[
\begin{align*}
\left(\frac{dP}{dy}\right)_{l} &= \alpha \left(\frac{\rho_{w}}{\rho}\right) (1-x) \mu' + \beta \left(\frac{\rho_{w}}{\rho}\right)^2 (1-x)^2, \\
\left(\frac{dP}{dy}\right)_{v} &= \alpha \left(\frac{\rho_{w}}{\rho}\right) x \mu'' + \beta \left(\frac{\rho_{w}}{\rho}\right)^2 x^2,
\end{align*}
\]

where \((\rho w)\) is the mixture filtration \(\alpha\) and \(\beta\) are viscous and inertial resistance coefficients, which are characteristics of the porous structure; \(\mu'\) and \(\mu''\) are the dynamic viscosities of liquid and vapor.
\[ \alpha = \frac{180(1-m)^2}{m^3d^2}, \quad \beta = \frac{1.5(1-m)}{m^3d}. \]  

The factors \( \Phi_j \) and \( \Phi_x \) are functions of the Martinelli parameter \( X \):

\[ \Phi_j = 1 + \frac{C}{X} + \frac{X}{1}, \quad \Phi_x = 1 + \frac{C X}{X + X^2}, \]  

\[ X = \left[ \left( \frac{dP}{dy} \right)_l \right] \left[ \left( \frac{dP}{dy} \right)_s \right]^{0.5}. \]  

Here, coefficient \( C \) depends on the flow conditions and fluid properties. Sorokin [10] obtained the following value of coefficient \( C \) for the steam-water mixture flow through the packed bed:

\[ C = \left( \frac{\beta/\alpha \rho^* \sigma}{(\rho w) \mu^*} \right)^{0.3}, \]  

where \( \sigma \) is the surface tension coefficient.

2.2. Model based on the Darcy-Forchheimer equations (model F)

Equations representing a generalization of the Darcy – Forchheimer equation for two-phase flows are usually written as follows [6]:

\[ -\left( \frac{dP}{dy} \right)_l - \rho \frac{g}{K \eta \rho} = \frac{\mu}{K \eta \rho} w_{s0} + \rho \frac{g}{\eta \rho} w_{s0} - \frac{F}{1 - \phi}; \]  

\[ -\left( \frac{dP}{dy} \right)_s - \rho \frac{g}{K \eta \rho} = \frac{\mu}{K \eta \rho} w_{s0} + \rho \frac{g}{\eta \rho} w_{s0} + \frac{F}{1 - \phi}. \]  

Here the velocities \( w_{s0} = (\rho w) x / \rho \) and \( w_{s0} = (\rho w) (1-x) / \rho \) represent the superficial velocities of the liquid and vapor phases through the packed bed cross-section. \( K_l \) and \( K_s \) are relative permeability for liquid and vapor phases; \( \eta_l \) and \( \eta_s \) are relative passability values of the porous medium for liquid and vapor phases; \( g \) is the acceleration of gravity. \( K \) and \( \eta \) were determined using the numerical Ergun coefficients:

\[ \frac{1}{K} = \frac{150(1-m)^2}{m^3 d^2}, \quad \frac{1}{\eta} = \frac{1.75(1-m)}{m^3 d}. \]  

In the system of differential equations, \( (dP/dy)_l, \) \( (dP/dy)_s \) represent the pressure gradients of the liquid and vapor phases in the mixture and should be equal to each other, which is ensured by the choice of coefficients \( K_l, K_s, \eta_l \) and \( \eta_s \). As shown in the review part of [6], there is a class of models based on equations (7) with zero interphase interaction force \( F \) and coefficients of the form

\[ K_l = (\phi^*)^2, \quad K_s = (1-\phi^*)^2, \quad \eta_l = (\phi^*)^2, \quad \eta_s = (1-\phi^*)^2. \]  

In this research, the best approximation of the experimental data [3] and [4] is obtained for \( a = 1 \) and \( b = 2 \). Thus, the pressure gradients satisfy the relationships
It is worth noting that \( \frac{w_a}{(1-\varphi)} \), \( \frac{w_a}{/\varphi} \) at \( \varphi^* \), equivalent to the void fraction, express the velocities of each phase, taking into account the fact that liquid occupies a fraction of the cross-section equal to \( \frac{(1-\varphi)}{\varphi^*} \), and steam occupies a fraction of the cross-section equal to \( \frac{\varphi}{\varphi^*} \). The magnitude \( \varphi^* \) is calculated taking into account the requirement that the pressure gradients in both phases should be equal. In the considered range of mass velocities \( (\rho w_a > 50 \text{ kg/m}^3\text{s}) \), the viscous and gravitational components are negligibly small compared to the inertial one, consequently equating the right-hand sides of equations (10) gives the following formula for \( \varphi^* \):

\[
\varphi^* = \left[ 1 + \left( \frac{\varphi^{**}}{\rho^*} \right)^{1/2} \left( \frac{1-x}{x} \right) \right]^{-1}.
\]

Relying on the definition of a slip ratio as a ratio of phase velocities, \( s = \frac{w_a}{w_{a0}} \), void fraction \( \varphi \), and flow quality \( x \) are related as follows:

\[
\varphi = \left[ 1 + s \left( \frac{\rho^{**}}{\rho^*} \right) \frac{1-x}{x} \right]^{-1}.
\]

Bearing in mind the universal expression (12), obtained for the model based on Darcy-Forchheimer equations, expression (11) corresponds to a slip ratio equal to

\[
s = \left( \frac{\rho^*}{\rho^{**}} \right)^{1/2}.
\]

### 2.3. Gas-dynamic model (model T)

To describe the critical and subcritical flow of the water-vapor mixture through the packed bed of fixed spherical particles, we developed a modification of the equations of gas dynamics of the granular layer. The basic equations [12] include:

- **equation of motion:**

\[
\frac{dP}{dy} = -\frac{3}{2} \frac{m(1-m)}{\psi d} \rho w_{a0}^2,
\]

where \( m \) is average porosity; \( w_a = w_{a0}/m \) is the volume-averaged velocity of the water-vapor mixture in the void space; \( d \) is spherical particle diameter; \( \psi = 0.508+0.56(1-m) \) is relative minimum flow section; \( y \) is a space variable:

- **continuity equation:**

\[
\rho w_m = \rho_1 w_{m1}.
\]

- **polytropic equation:**

\[
\frac{P}{P_1} = \left( \frac{\rho}{\rho_1} \right)^n.
\]
Integration of these equations along the packed bed height from 0 to $H$ gives the following expression

$$\frac{3}{2} m (1-m) \frac{H}{\psi} \left( \rho \nu_n \right)^2 \frac{d}{n+1} \frac{P}{\rho \nu_n} = 1 - \left( \frac{P}{P_i} \right) \frac{11}{1.}
$$

which has analytical solutions as to both outlet pressure $P_2$, and relative mass velocity $(\rho \nu_n)$. In the equations for the mixture density at the packed bed inlet

$$\rho_i = \rho_i^* \left( 1 - \varphi_i \right) + \rho_i^* \varphi_i,$$

the void fraction $\varphi_i$ is expressed by formula (12).

We determined polytropic index $n$ and slip ratio $s$, in equations (12, 16, 17), based on the approximation of experimental data of Avdeev et al. [3], and Tairov and Bykova [4], obtained in a range of inlet pressures – from 0.6 to 15.3 MPa, inlet flow quality $x_i$ – from 0.002 to 0.300, and mass velocity $\rho w_n$ – from 150 to 1200 kg / (m$^3$ s). We obtain the following relations for them:

$$n = 0.55 + 0.45 (1 - \exp (- x_i/0.237)),
$$

$$s = 1 + 11.4 \left( 1 - \Omega / P^{0.408} \right),
$$

$$\Omega = \left[ 1 + 0.1 (1 - x) / x \right]^{-1}.
$$

As follows from equation (19), the polytropic index does not depend on the packed bed parameters and inlet pressure but it is a function of flow quality only.

### 2.4. Computation procedure

In the original publications [13, 14, 10], equations (1, 2, 5) do not describe the pressure gradient, $(dP/dy)$, but rather a pressure drop along the entire packed bed height $H$, ($\Delta P/H$). Such an algebraic expression for the pressure drop is correct if the density and viscosity of the phases, as well as flow quality, vary little along the packed bed and the pressure profile is linear. In this study, all the models were applied to the data obtained in the experiments on reaching the critical flow [4] characterized by high mass velocities, outlet pressure close to atmospheric and a significant change in all properties of the steam-water mixture along the packed bed, therefore, equations (1, 2, 5) were used in a differential form.

We solved the differential equation of models S and F for changing the pressure along the flow numerically using the fourth-order Runge-Kutta method. The numerical scheme step size was changed automatically so that the total relative error of pressure drop calculation, which was evaluated using the third-order control term, did not exceed 0.001.

In the study of two-phase flows through packed beds, of practical importance are both the calculation of the pressure drop at a known mass velocity and inlet pressure, and the inverse problem of calculating the mass velocity at a known inlet and outlet pressures. In models S and F, where the solution to the direct problem is obtained by numerical integration, the inverse problem is also solved numerically by the secant method. Iterations continue until the relative change in the unknown variable becomes less than 0.001.

Owing to the use of the polytropic law of change in the density of the mixture, the gas-dynamic model gives analytical expressions for the direct and inverse problems equivalent to equation (17):

$$P_i = P_i \left[ 1 - \frac{3}{2} \left( \frac{1-m) \frac{H}{\psi} \left( \rho \nu_n \right)^2 \frac{d}{n+1} \frac{P}{\rho \nu_n} \right) \frac{n+1}{n} \right].
$$
Equation (22) is presented as follows:

$$\rho w_w = \left[ \frac{2 n}{3(n+1)} \frac{d}{H m (1-m)} P \rho \left( \frac{P_1}{P} \right)^{\frac{n+1}{n}} \right]^{\frac{1}{n+1}}.$$ 

3. Results and discussion

Each theoretical model presented in the paper for determining pressure loss in a two-phase flow contains individual values of empirical parameters, which can provide a satisfactory agreement with the experiment on a limited array of experimental data used in each case. One of the objectives of the study was to identify and compare the predictive capabilities of the models presented in section 2 using an extended amount of experimental data. The study employed the results of Tairov and Bykova [4] and the experimental data of Avdeev et al. [3]. The used experimental data cover the forced flow of the water-vapor mixture through fixed layers 50, 100, 200, 250, 355 mm high of spherical particles 2 mm, 2.123 mm and 4 mm in diameter, with the following inlet flow parameters: pressure $P_1 = 0.6 - 15.3$ MPa; flow quality $x_1 = 0.002 - 0.300$.

Figure 1 demonstrates the relative deviations of the pressure drop and mass velocity values calculated using each of the considered models from the experimental ones, depending on the relative mass velocity $\rho w_{m,ex} / \rho w_{m,cr,calc}$. The figure shows the proximity of the experimental conditions to the critical flow. It can be seen that, in the calculation of the pressure drop, model S gives underestimated values under the conditions close to critical, whereas model F overestimates the
pressure drop at low relative mass velocities. The relative deviation of the calculated values from the experimental values in models S and F reaches 60%, while model T gives the calculated values within 30% of the experimental values over the entire input data range. The relative deviations of the mass velocity for all models are 2–3 times lower than the relative deviations of the pressure drop. This is especially pronounced in the conditions close to critical. Moreover, the average relative deviation of the calculated values from the experimental ones in both the direct problem and the inverse problem for model T is approximately 1.5 times less than for models S and F.

Figure 2 shows a comparison of the slip ratios calculated by formulas (13) and (20) in models F and T for two pressure values, and figure 3 shows the corresponding relationships between the void fraction and flow quality calculated by formulas (11) and (12). As seen from the figures 2 and 3, the values of the slip ratio and the void fraction in the two models differ significantly. Empirically constructed formula (20) for the slip ratio takes into account the relationship between the slip ratio and void fraction and implicitly allows for the presence of interphase interaction forces. Therefore, in model T, the vapor velocity is lower while the void fraction is higher than in model F.

4. Conclusion
The focus of the paper is on the comparison of three models of the steam-water mixture flow through the close-packed layers of spherical particles, which assume that two phases have separate motion. The quality assessment of the two existing models and a new gas-dynamic model involved the results of the experiments conducted at the High-temperature circuit facility [4] and the data obtained by Avdeev et al. [3]. The ranges of the considered pressures and mass velocities are $0.6 \sim 15.3 \text{ MPa}$ and $150 \sim 1200 \text{ kg/m}^2\text{s}$. The findings demonstrate that the new gas-dynamic model presents more accurately the experimental data in a wide range of parameters. Another advantage of this model is the analytical form of expressions for pressure drop and mass velocity.

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