Role of Chiral Symmetry in the Nuclear Many-Body Problem

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Abstract

The role of chiral (pion) dynamics in nuclear matter is reviewed. Contributions to the energy per particle from one- and two-pion exchange are calculated systematically, and it is demonstrated that already at order $k_F^4$ in the Fermi momentum, two-pion exchange produces realistic nuclear binding together with very reasonable values for the compressibility and the asymmetry energy. Further implications of these results are discussed.

1 Introduction

The present status of the nuclear matter problem is that a quantitatively successful description can be achieved, using advanced many-body techniques\textsuperscript{[1]}, in a non-relativistic framework when invoking an adjustable three-body force. Alternative relativistic mean field approaches, including non-linear terms with adjustable parameters, are also widely used for the calculation of nuclear matter properties and finite nuclei\textsuperscript{[2]}. At a more basic level, the Dirac-Brueckner method\textsuperscript{[3]} solves a relativistically improved Bethe-Goldstone equation with one-boson exchange $NN$-interactions.

In recent years a novel approach to the $NN$-interaction based in effective field theory (in particular, chiral perturbation theory) has emerged\textsuperscript{[4, 5]}. The key element is a power counting scheme which separates long- and short-distance dynamics. Methods of effective field theory have also been applied to systems of finite density\textsuperscript{[6]}.

The purpose of this presentation is to point out the importance of explicit pion dynamics in the nuclear many-body problem. While pion exchange processes are well established as generators of the long and intermediate range $NN$-interaction, their role in nuclear matter is less evident. The one-pion exchange Hartree term vanishes identically, and the leading Fock exchange term is small. Two-pion exchange mechanisms are commonly hidden behind a purely phenomenological scalar ("sigma"-) mean field which is fitted to empirical data but has no basic justification. This is an unsatisfactory situation which calls for a deeper understanding. We report on steps and thoughts in this direction, following ref.\textsuperscript{[7]}. Our approach is closely related to the work of Lutz et al. in ref.\textsuperscript{[8]}.\textsuperscript{1}

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Before passing on to our calculation it is useful to draw attention to the following fact. A simple but realistic parametrization of the energy per particle, \( \bar{E}(k_f) = E/A \), of isospin symmetric nuclear matter is given in powers of the Fermi momentum \( k_f \) as

\[
\bar{E}(k_f) = \frac{3k_f^2}{10M} - \alpha \frac{k_f^3}{M^2} + \beta \frac{k_f^4}{M^3},
\]

where the nucleon density is \( \rho = 2k_f^3/3\pi^2 \) as usual, and \( M = 0.939 \text{ GeV} \) is the free nucleon mass. The first term is the kinetic energy of a Fermi gas. Adjusting the (dimensionless) parameters \( \alpha \) and \( \beta \) to the equilibrium density, \( \rho_0 = 0.16 \text{ fm}^{-3} (k_{f0} = 1.33 \text{ fm}^{-1}) \) and \( \bar{E}_0 = \bar{E}(k_{f0}) = -16 \text{ MeV} \), gives \( \alpha = 5.27\) and \( \beta = 12.22\). The compression modulus \( K = k_{f0}^2(\partial^2 \bar{E}(k_f)/\partial k_f^2)_{k_{f0}} \) is then predicted at \( K = 236 \text{ MeV} \), well in line with empirically deduced values, and the density dependence of \( \bar{E}(k_f) \) using eq. (1) is remarkably close to the one resulting from the realistic many-body calculations of the Urbana group [8].

\section{Chiral in-medium perturbation theory}

The tool to investigate the implications of spontaneous and explicit chiral symmetry breaking in QCD is chiral perturbation theory. Observables are calculated within the framework of an effective field theory of Goldstone bosons (pions) interacting with the lowest-mass baryons (nucleons). The diagrammatic expansion of this low-energy theory in the number of loops has a one-to-one correspondence to a systematic expansion of observables in small external momenta and the pion (or quark) mass.

In nuclear matter, the relevant momentum scale is the Fermi momentum \( k_f \). At the empirical saturation point, \( k_{f0} \approx 2m_\pi \), so the Fermi momentum and the pion mass are of comparable magnitude at the densities of interest. This immediately implies that pions must be included as explicit degrees of freedom: their propagation in matter is relevant. Pionic effects cannot be accounted for simply by adjusting coefficients of local \( NN \) contact interactions.

Both \( k_f \) and \( m_\pi \) are small compared to the characteristic chiral scale, \( 4\pi f_\pi \approx 1.2 \text{ GeV} \), which involves the pion decay constant \( f_\pi = 0.092 \text{ GeV} \). Consequently, the equation of state of nuclear matter as given by chiral perturbation theory will be represented as an expansion in powers of the Fermi momentum. The expansion coefficients are non-trivial functions of \( k_f/m_\pi \), the dimensionless ratio of the two relevant scales inherent to the problem.

The chiral effective Lagrangian generates the basic pion-nucleon coupling terms: the Tomozawa-Weinberg \( \pi\pi NN \) contact vertex, \( (1/4f_\pi^2)(q_0^a - q_0^b)\gamma_\mu\epsilon_{abc}\tau_c \), and the pseudovector \( \pi NN \) vertex, \( (g_A/2f_\pi)q_a^\mu\gamma_\mu\gamma_5\tau_a \), where \( q_{a,b} \) denotes (outgoing) pion four-momenta and \( g_A \) is the axial vector coupling constant (we choose \( g_A = 1.3 \) so that the Goldberger-Treiman relation \( g_{\pi N} = g_AM/f_\pi \) gives the empirical \( \pi N \) coupling constant, \( g_{\pi N} = 13.2 \)).

The only new ingredient in performing calculations at finite density (as compared to evaluations of scattering processes in vacuum) is the in-medium nucleon propagator. For a relativistic nucleon with four-momentum \( p^\mu = (p_0, \vec{p}) \) it reads

\[
(p + M) \left\{ \frac{i}{p^2 - M^2 + i\varepsilon} - 2\pi\delta(p^2 - M^2)\theta(p_0)\theta(k_f - |\vec{p}|) \right\}.
\]

The second term is the medium insertion which accounts for the fact that the ground state of the system has changed from an "empty" vacuum to a filled Fermi sea of nucleons. Diagrams
can then be organized systematically in the number of medium insertions, and an expansion is performed in leading inverse powers of the nucleon mass, consistently with the $k_f$-expansion.

Our ”inward-bound” strategy [7] is now as follows. One starts at large distances (small $k_f$) and systematically generates the pion-induced correlations between nucleons as they develop with decreasing distance (increasing $k_f$). The present calculations are performed to 3-loop order (including terms up to order $k_f^5$) and incorporate one- and two-pion exchange processes. The procedure involves one single momentum space cutoff $\Lambda$ which encodes dynamics at short distances not resolved explicitly in the effective low-energy theory. This cutoff scale $\Lambda$ is the only free parameter which has to be fine-tuned. (Alternatively, and equivalently, one could use dimensional regularization and introduce short-distance physics through adjustable $NN$ contact terms).

We now outline the leading contributions to the energy per particle $\bar{E}(k_f)$. The kinetic energy including first order relativistic corrections is

$$\bar{E}_{\text{kin}}(k_f) = \frac{3k_f^2}{10M} \left(1 - \frac{5k_f^2}{28M^2}\right).$$

Terms of order $k_f^6$ are already negligibly small. At least from this perspective, nuclear matter is a non-relativistic system.

Nuclear chiral dynamics up to three-loop order introduces the diagrams, Fig. 1. They include the one-pion exchange (OPE) Fock term, iterated OPE and irreducible two-pion exchange.

Fig. 1: In-medium chiral perturbation theory: One-pion exchange Fock term (upper left), iterated one-pion exchange (upper middle and right) and examples of irreducible two-pion exchange terms. See ref. [7] for details.

Medium insertions are systematically applied on all nucleon propagators, and the relevant loop integrations yield results which can be written in analytic form for all pieces.

The OPE Fock term becomes

$$\bar{E}_{1\pi}(k_f) = \frac{g_A^2 m_\pi^3}{(4\pi f_\pi)^2} \left[ F \left( \frac{k_f}{m_\pi} \right) + \frac{m_\pi^2}{M^2} G \left( \frac{k_f}{m_\pi} \right) \right],$$

(4)
where $F$ and $G$ are functions of the dimensionless variable $k_f/m_\pi$. They are given explicitly in ref. [7]. All finite parts of iterated OPE and irreducible two-pion exchange are of the generic form

$$E_{2\pi}(k_f) = \frac{m_\pi^4}{(4\pi f_\pi)^4} \left[ g_A^4 M H_4 \left( \frac{k_f}{m_\pi} \right) + m_\pi H_5 \left( \frac{k_f}{m_\pi} \right) \right],$$

with the functions $H_{4,5}$ again given explicitly in ref. [7]. All power divergences specific to cutoff regularization are summarized in the expression

$$E_{\Lambda}(k_f) = \frac{\Lambda k_f^3}{(4\pi f_\pi)^4} \left[ -10 g_A^4 M + (3g_A^2 + 1)(g_A^2 - 1)\Lambda \right],$$

where the attractive and dominant first term in the brackets arises from iterated OPE. Note that this term could have been generated, equivalently, by a $NN$ contact interaction with appropriate coupling strength.

### 3 Results

#### 3.1 Nuclear matter equation of state

A striking feature of the chiral dynamics approach is the simplicity of the saturation mechanism for isospin-symmetric nuclear matter. Before turning to the presentation of detailed results, it is instructive first to discuss the situation in the exact chiral limit, $m_\pi = 0$. The basic saturation mechanism can already be demonstrated by truncating the one- and two-pion exchange diagrams at order $k_f^4$. We can make straightforward contact with the parametrization (1) of the energy per particle and identify the coefficients $\alpha$ and $\beta$ of the $k_f^3$ and $k_f^4$ terms, respectively. The result for $\alpha$ in the chiral limit is:

$$\alpha = \frac{10\Lambda}{M} \left( \frac{g_{\pi NN}}{4\pi} \right)^4 - \left( \frac{g_{\pi NN}}{4\pi} \right)^2,$$

where we have neglected the small correction proportional to $\Lambda^2$ in eq. (6). The strongly attractive leading term in eq. (7) is accompanied by the (weakly repulsive) one-pion exchange Fock term.

The $k_f^3$-contribution to $E(k_f)$ would lead to collapse of the many-body system. The stabilizing $k_f^4$-term is controlled by the coefficient (calculated again in the chiral limit)

$$\beta = \frac{3}{70} \left( \frac{g_{\pi NN}}{4\pi} \right)^4 (4\pi^2 + 237 - 24 \ln 2) - \frac{3}{56} = 13.55,$$

a unique and parameter-free result to this order. Here the two-pion exchange dynamics produces repulsion of just the right magnitude to achieve saturation: the result, eq. (8), is within 10% of the empirical $\beta = 12.2$. Adjustment of the short-distance scale $\Lambda$ between 0.5 and 0.6 $GeV$ easily leads to a stable minimum of $E(k_f)$ in the proper range of density and binding energy.

The full 3-loop chiral dynamics result for $E(k_f)$ in symmetric nuclear matter, using $m_\pi = 135 MeV$ (the neutral pion mass), is shown in Fig. 2 together with a realistic many-body calculation. The outcome is remarkable: with one single parameter $\Lambda = 0.65 GeV$ fixed to the value $E_0 = -15.3 MeV$ at equilibrium, perturbative pion dynamics alone produces an equation of state which follows that of much more sophisticated calculations up to about three times the density of nuclear matter. The predicted compression modulus is $K = 255 MeV$, well in line with the "empirical" $K = (250 \pm 25) MeV$ deduced in refs. [3, 10].
Fig. 2: Energy per particle, $\bar{E}(k_f)$, of symmetric nuclear matter derived from chiral one- and two-pion exchange (solid line) [7]. The cutoff scale is $\Lambda = 646 \text{ MeV}$. The dashed line is the result of ref. [8].

Fig. 3: Nuclear matter saturation point $(\bar{E}_0, \rho_0)$ at finite pion mass $m_\pi = 135 \text{ MeV}$ (solid line) and in the chiral limit $m_\pi = 0$ (dashed line) as function of the cutoff scale $\Lambda$ (given in MeV). The inserted rectangle corresponds to the empirical saturation point including its uncertainties.
It is interesting to examine the variation with the pion mass in this context. Fig. 2 displays the position of the nuclear matter saturation point \((\bar{E}_0, \rho_0)\), first in the chiral limit and then using the physical \(m_\pi\), along lines with varying short-distance scale parameter \(\Lambda\). Evidently, explicit chiral symmetry breaking by \(m_\pi\) is not a qualitatively decisive feature for saturation; it influences, however, the quantitative fine-tuning of \(\Lambda\).

### 3.2 Asymmetry energy

The specific isospin dependence of two-pion exchange should have its distinct influence on the behavior of asymmetric nuclear matter, with increasing excess of neutrons over protons. We introduce as usual the asymmetry parameter \(\delta = (\rho_n - \rho_p)/\rho = (N - Z)/(N + Z)\), keeping the total density \(\rho = \rho_n + \rho_p = 2k_f^3/3\pi^2\) constant. The proton and neutron densities are \(\rho_{p,n} = k_{p,n}^3/3\pi^2\) in terms of the corresponding Fermi momenta. Without change of any input, we have calculated \([7]\) the asymmetry energy \(A(k_f)\) defined by

\[
\bar{E}_{as}(k_p, k_n) = \bar{E}(k_f) + \delta^2 A(k_f) + \ldots
\]  

The result at nuclear matter density is \(A_0 = A(k_{f0}) = 33.8\,\text{MeV}\). This is in very good agreement with the empirical value \(A_0 = 33.2\,\text{MeV}\) derived from extensive fits to nuclide masses \([11]\).

Extrapolations to higher density work roughly up to \(\rho \simeq 1.5\,\rho_0\). At still higher densities, there are indications that non-trivial isospin dependence beyond one- and two-pion exchange starts to play a role. A similar statement holds for pure neutron matter which is properly unbound, but its predicted equation of state starts to deviate from that of realistic many-body calculations at neutron densities larger than 0.2\,fm\(^{-3}\).

### 3.3 Nuclear mean field from chiral dynamics

The in-medium three-loop calculation of the energy per particle defines the (momentum dependent) self-energy of a single nucleon in nuclear matter up to two-loop order. The real part of the resulting single particle potential in isospin-symmetric matter at the saturation point, for a nucleon with zero momentum, comes out as \([12]\)

\[
U(p = 0, k_{f0}) = -53.2\,\text{MeV},
\]  

using exactly the same one- and two-pion exchange input that has led to the solid curve in Fig. 2. The momentum dependence of \(U(p, k_{f0})\) can be rephrased in terms of an average effective nucleon mass \(M^* \simeq 0.8\,M\) at nuclear matter density, and the imaginary part of the potential for a nucleon-hole at the bottom of the Fermi sea is predicted to be about 30\,MeV. All these numbers are remarkably close to the empirically deduced ones.

### 4 Summary and outlook

Explicit pion dynamics originating from the spontaneously broken chiral symmetry of QCD is an important aspect of the nuclear many-body problem. In-medium chiral perturbation theory, with one single cutoff scale \(\Lambda \simeq 0.65\,\text{GeV}\) introduced to regularize the few divergent parts associated with two-pion exchange, gives realistic binding and saturation of nuclear matter already at three-loop order. At the same time it gives very good values for the compression
modulus and the asymmetry energy. These are non-trivial observations, considering that it all works with only one adjustable parameter which encodes unresolved short-distance dynamics. Of course, questions about systematic convergence of the in-medium chiral loop expansion still remain and need to be explored.

In view of the relevant scales in nuclear matter, the importance of explicit pion degrees of freedom does not at all come unexpected. Many of the existing models ignore pions, however. They must introduce purely phenomenological scalar fields with non-linear couplings and freely adjustable parameters in order to simulate two-pion exchange effects.

Finally, in order to discuss possible contacts with relativistic nuclear mean field phenomenology, the following working hypothesis suggests itself as a guide for further steps. Assume that the nuclear matter ground state represents a "shifted" QCD vacuum characterized by strong vector ($V$) and scalar ($S$) condensate fields acting on the nucleons, with $V \simeq -S \simeq 0.3\, GeV$ as suggested e. g. by in-medium QCD sum rules [13]. Such a scenario would not produce binding all by itself, but establish strong spin-orbit splitting and approximate pseudo-spin symmetry [14]. Binding and saturation would then result from the pionic (chiral) fluctuations around this new vacuum.

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