What graph theory actually tells us about resting state interictal MEG epileptic activity

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1 EPILEPTIC PATIENTS

As presented in the paper the study was performed on 45 subjects: 15 patients suffering from frontal focal epilepsy (FE), 15 patients suffering from idiopathic generalized epilepsy (or presumed genetic, as the new terminology stands) (GE) and 15 healthy subjects (HS). A more detailed description of the patients could be seen in Table S1. Abbreviations in this table are as follows:

- **AGE**: age of the subject
- **ED**: Education (1: primary, 2: secondary, 3: university)
- **GD**: Gender (M: male, F: female)
- **LAT**: Hand laterality (L: left, R: right)
- **STA**: Age when epilepsy started
- **FAM**: Family history with epilepsy
- **FET**: Fetal distress
- **PRE**: Prematurity
- **LESION (RMI)**: (LF: left frontal, RF: right frontal, DNT: Dysembryoplastic neuroepithelial tumor)
- **DIAG**: Syndromic diagnosis (FLES: frontal lesional, FNOLES: frontal no lesional)
- **STATUS**: (C: convulsive, NC: no convulsive)
- **CRISIS**: (FOC: frontal focal, GEN: generalized)
- **FOCAL**: (PC: complex partial seizures: “frontal lobe absences”, PM: motor partial seizures (hemicorporal clonias), SL: detention of language, OC: oculocephalic version, CA: lift opposite arm (fencing position), AU: somatosensory aura)
- **TYPE of GE**: (IGE: Idiopathic Generalized Epilepsy, JME: Juvenile Mioclonic Epilepsy, JAE: Juvenile Absence Epilepsy)
- **GENERALIZED**: (TC: tonic-clonic, MIO: mioclonia, ABS: absence, GM: complex motor generalized during night)
- **M**: Number of antiepileptic drugs
- **AED1**: Name of AED1
- **AED2**: Name of AED2
- **Patients marked with * are the ones included in the study**
### Table S1: Frontal focal epilepsy (FE) and Generalized epilepsy (GE) subjects

| FE   | AGE | ED | GD | LAT | STA | FAM | FET | PBE | LESION (MRI)          | DIAG | STAT | CRISIS | FOCAL | GENERAL | M | AED1 | AED2      |
|------|-----|----|----|-----|-----|-----|-----|-----|-----------------------|------|------|--------|--------|---------|---|------|-----------|
| EPL 03* | 27  | 3  | F  | R   | 12  | No  | No  | Ne  | Parasagittal gliosis LF | FLES | C    | PC     | TC     | 2       |   |      | lamotrigin | fenobarbital |
| EPL 17* | 24  | 3  | M  | R   | 16  | No  | No  | Ne  | Venous angiomata LF + perifocal gliosis | FLES | No   | FOC | GEN | PC     | TM | TC   | 1     | levetiracetam | X  |
| EPL 18* | 37  | 3  | M  | R   | 18  | No  | No  | Ne  | Normal | FNOLES | NC | FOC | GEN | PC | TC | 2   | valproico | lamotrigin |
| EPL 19* | 46  | 3  | M  | R   | 43  | No  | No  | Ne  | Normal | FNOLES | NC | FOC | GEN | PC | TC | 1   | valproico | X  |
| EPL 20* | 59  | 2  | M  | R   | 47  | No  | No  | Ne  | Polymicrogyria LF | FLES | C    | FOC | GEN | SL, OC, PC | TC | 1   | lamotrigin | X  |
| EPL 21* | 52  | 2  | F  | R   | 34  | No  | No  | Ne  | Heterotopia RF | FLES | No   | FOC | GEN | PC | TC | 1   | carbamazepin | X  |
| EPL 22* | 29  | 2  | M  | R   | 14  | No  | No  | Ne  | Atrophy LF | FLES | No   | FOC | GEN | PC | GM | 2   | valproico | lacosamida |
| EPL 23* | 27  | 2  | M  | L   | 15  | No  | No  | Ne  | Diploid cortical RF | FLES | No   | FOC | GEN | PM | TC | 2   | valproico | lamotrigin |
| EPL 25* | 38  | 3  | M  | R   | 33  | Yes | No  | Ne  | Quistica subependiularis RF | FLES | No   | FOC | GEN | PM | GM | 1   | levetiracetam | valproico |
| EPL 26* | 23  | 2  | F  | L   | 17  | No  | No  | Ne  | Angiomata RF | FLES | No   | FOC | SL, PM | No | 2   | valproico | lamotrigin |
| EPL 27* | 46  | 2  | F  | R   | 43  | No  | No  | Ne  | Normal | FNOLES | No | FOC | GEN | OC | TC | 1   | levetiracetam | X  |
| EPL 28* | 21  | 3  | M  | R   | 20  | No  | No  | Ne  | Heterotopia LF | FLES | No   | FOC | GEN | CA, OC | TC | 1   | levetiracetam | X  |
| EPL 29* | 22  | 3  | M  | R   | 18  | No  | No  | Ne  | Venous angiomata LF | FLES | No   | GEN | SL | TC | 1   | valproico | X  |
| EPL 30* | 25  | 3  | M  | R   | 14  | No  | No  | Ne  | Normal | FNOLES | No | FOC | GEN | CA, OC | TC | 2   | valproico | lacosamida |
| EPL 31* | 22  | 2  | M  | R   | 19  | Yes | Yes | Yes | Angiomata RF | FLES | No   | GEN | No | TC | 2   | valproico | levetiracetam |
| EPL 32* | 69  | 3  | F  | R   | 50  | No  | No  | Ne  | Cavernous LF | FLES | C    | GEN | SL | TC | 1   | oxcarbazepin | X  |
| EPL 33* | 27  | 2  | F  | R   | 12  | No  | No  | Ne  | Normal | FNOLES | No | GEN |     | TC | 2   | lamotrigin | valproico |
| EPL 34* | 33  | 2  | F  | R   | 15  | No  | No  | Ne  | DNM in BF to cortical diplasia | FLES | No   | FOC | AU | No | 2   | propionalis | topiramato |
| EPL 35* | 58  | 2  | M  | R   | 19  | No  | No  | Ne  | Vascular malformations (hemangioma) | FLES | No   | FOC | SL | No | 2   | valproico | fenobarbital |

| GE  | AGE | ED | GD | LAT | STA | FAM | FET | PBE | LESION (MRI) | DIAG | MIO | CRISIS | TYPE | GENERAL | M | AED1 | AED2      |
|-----|-----|----|----|-----|-----|-----|-----|-----|--------------|------|------|--------|-------|---------|---|------|-----------|
| EPL 01* | 29  | 3  | M  | R   | 13  | No  | No  | Ne  | Normal | IGE | Yes | GEN | MIO | JME | TC, MIO | 1 | levetiracetam | X  |
| EPL 02* | 20  | 2  | M  | R   | 14  | No  | No  | Ne  | Normal | IGE | Yes | GEN | MIO | JME | TC, MIO | 1 | valproico | X  |
| EPL 04* | 27  | 2  | F  | R   | 17  | No  | No  | Ne  | Normal | IGE | Yes | GEN | MIO | JME | TC, MIO | 2 | valproico | lamotrigin |
| EPL 05* | 20  | 2  | M  | R   | 12  | Yes | No  | Ne  | Normal | IGE | Yes | GEN | MIO | JME | TC, MIO | 1 | valproico | X  |
| EPL 06* | 24  | 2  | F  | R   | 16  | No  | No  | Ne  | Normal | IGE | No  | GEN | MIO | IGE | TC | 1   | valproico | X  |
| EPL 07* | 31  | 2  | F  | R   | 12  | No  | No  | Ne  | Normal | IGE | Yes | GEN | MIO | IGE | TC, MIO | 2 | valproico | levetiracetam |
| EPL 08* | 23  | 2  | M  | R   | 12  | Yes | No  | Ne  | Normal | IGE | Yes | GEN | MIO | IGE | TC, MIO | 2 | valproico | lamotrigin |
| EPL 09* | 33  | 3  | F  | R   | 11  | Yes | No  | Ne  | Normal | IGE | Yes | GEN | MIO | JAE | ABS, TC | 1 | lamotrigin | X  |
| EPL 10* | 24  | 2  | F  | R   | 14  | Yes | No  | Ne  | Normal | IGE | No  | ABS | JAE | ABS, TC | 1 | etosuximida | X  |
| EPL 11* | 22  | 2  | F  | R   | 14  | Yes | No  | Ne  | Normal | IGE | No  | GEN | IGE | TC | 2   | lamotrigin | levetiracetam |
| EPL 12* | 25  | 2  | F  | R   | 13  | No  | No  | Ne  | Normal | IGE | Yes | GEN | MIO | JME | TC, MIO | 2 | levetiracetam | topiramato |
| EPL 13* | 31  | 3  | M  | R   | 18  | No  | Yes | Ne  | Normal | IGE | No  | GEN | IGE | TC | 1   | levetiracetam | X  |
| EPL 14* | 23  | 3  | F  | R   | 18  | Yes | No  | Ne  | Normal | IGE | Yes | GEN | MIO | IGE | TC, MIO | 1 | levetiracetam | X  |
| EPL 15* | 20  | 2  | F  | R   | 18  | No  | No  | Ne  | Normal | IGE | No  | GEN | JME | TC | 1   | levetiracetam | X  |
| EPL 16* | 29  | 3  | F  | R   | 14  | No  | No  | Ne  | Normal | IGE | Yes | GEN | MIO | JME | TC, MIO | 2 | levetiracetam | lamotrigin |
| EPL 17* | 40  | 3  | F  | R   | 15  | Yes | No  | Ne  | Normal | IGE | Yes | GEN | MIO | IGE | TC, MIO | 1 | valproico | X  |
| EPL 18* | 29  | 2  | M  | R   | 11  | Yes | No  | Ne  | Normal | IGE | No  | GEN | MIO | JME | TC, MIO | 2 | valproico | lacosamida |
| EPL 19* | 22  | 2  | F  | R   | 10  | Yes | No  | Ne  | Normal | IGE | Yes | GEN | MIO | JME | TC, MIO | 1 | levetiracetam | X  |
| EPL 20* | 25  | 2  | M  | R   | 14  | No  | No  | Ne  | Normal | IGE | Yes | GEN | MIO | IGE | TC, MIO | 2 | valproico | levetiracetam |
| EPL 21* | 40  | 2  | M  | R   | 15  | No  | No  | Ne  | Normal | IGE | Yes | GEN | MIO | IGE | TC, MIO | 1 | levetiracetam | X  |
2 DEALING WITH TWO ORTHOGONAL GRADIOMETERS

In this study, we used planar gradiometers instead of magnetometers, due to the reduced sensitivity to deep sources. Yet, as commented in the Method section of the paper, there is no consensus in the literature as to what is the best strategy to combine the information of the two orthogonal gradiometers in connectivity analyses. To compare the effect of different common strategies to deal with this problem on the temporal and spectral profile of the data, Figure S1 shows the time series and the power spectrum of each individual planar gradiometer \( g_x \) and \( g_y \), their first principal component \( g_{PC} \), and its absolute, and their amplitude (or RMS), defined as \( \text{RMS} = \sqrt{g_x^2 + g_y^2} \).

As expected, the temporal and spectral profile of \( g_{PC} \) clearly mirror those of both gradiometers, while at the same time explaining an important percentage of the total variance of both signals (panel D), and reducing the information to a single time series per sensor site. Instead, both its absolute value and the RMS, which are almost identical, modify the temporal course and most importantly the spectrum (panel C), which may be problematic to draw conclusions in the PS analysis. Therefore, we decided to use PCA as an objective method to combine the information from both planar gradiometers with minimum signal distortion in time and frequency domain.

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**Fig. S1: Dealing with planar gradiometers**

A) Signals obtained from two planar gradiometers (Grad1 and Grad2), and their principal component (Gpc), for 1000ms of recording in one channel. B) The same period of time for Gpc, its absolute value (Gpc abs) and the RMS. C) Total PSD for the average of the 102 sensors, 40 trials of 5s each for one subject. D) Histogram for the explained percentage of the total variance by the first principal component of the two gradiometers (Gpc) for the same dataset as in C.
3 STATIONARITY

One important (but often neglected) question, which must be tackled during the preprocessing of the data, is its stationarity. The human brain is a non-stationary system, but for most indices of brain activity from M/EEG at least approximate stationarity is required. In the case of connectivity indexes, stationarity is not always a pre-requisite for their calculation (e.g., those PS indexes where the phase obtained by means of the Hilbert Transform). But certainly, if a given time series is not stationary, the possibility of different brain states being mixed up in it cannot be ruled out. A recent result on spontaneous and evoked MEG activity (Kipiński et al., 2011) has shown that, whereas these data tend to be mean stationary, they are also mostly variance-nonstationary especially for long segments (i.e., longer than 1s). Here, we dealt with this issue by means of the KPSS test (Kwiatkowski et al., 1992), which has been successfully applied to multivariate M/EGG data in recent studies (González et al., 2013; Kipiński et al., 2011). This test calculates a statistic (ks), which is later used to decide whether it is possible to reject the null hypothesis of the stationarity of the data at the desired level of statistical significance (see (Kipiński et al., 2011; Kwiatkowski et al., 1992) for technical details). Since the statistics is calculated in univariate time series, we define:

$$ k_s = \frac{1}{N_s} \sum_{i=0}^{N_s} k_{s,i} $$

as the statistic for each multivariate ($N_s=102$ sensors) segment. Then, all the segments were sorted from the lowest to the highest values of the statistic (i.e., from the most to the least stationary segment), and select for further analysis the 40 most stationary segments out of the 100 a priori selected segments. We performed the KPSS analysis with the GCCA toolbox (Seth, 2010).

4 PHASE LOCKING VALUE

In this work we use the Phase Locking Value (PLV) as a measure of phase synchronization (PS) between signals (see, e.g., (Niso et al., 2013) for details). PS refers to a situation when the phases of two coupled oscillators synchronizes, even though their amplitudes may remain uncorrelated (Rosenblum et al., 1996). The PLV makes use only of the relative phase difference between two narrow-band signals (Lachaux, Rodriguez, Martinerie, & Varela, 1999) and it is defined as:

$$ PLV = \left\{ e^{i \phi_{rel}(t)} \right\} = \frac{1}{N} \sum_{n=1}^{N} e^{i \phi_{rel}(n,t)} = \sqrt{\cos \Delta \phi_{rel}(t)}^2 + \sin \Delta \phi_{rel}(t) $$

where $\langle . \rangle$ indicates time average. The PLV estimates how the relative phase is distributed over the unit circle and it has been computed using HERMES toolbox (Niso et al., 2013), available at http://hermes.ctb.upm.es/.
5 THE CHOICE OF AN OPTIMAL DENSITY OF LINKS

As commented in the manuscript (cfr. Section 2.8, Functional brain networks), the problem of finding an appropriate significance threshold for the links of a reconstructed functional brain network remains open. Most studies in the literature, however, take into account the results presented in (van Wijk et al., 2010), and use a fixed density approach, whereby the same number of links are used across groups and conditions. Recently, (Kim et al., 2013), analyzing epileptic brain networks from EEG data, have demonstrated that many brain network measures reach a maximum for a density slightly above the minimum value $k_{ER} = 2\ln N/N$ defined in the Erdős-Rényi model (Erdős and Rényi, 1961) for a random graph of $N$ nodes to become fully connected (in our case, $k_{ER} \approx 0.1$). Besides, (Zhang et al., 2011) have also used a relatively low density (i.e., less than 50% of the links) to investigate GE networks using fMRI and DTI, arguing that for such values “all functional (...) connectivity networks were fully connected and with a minimum number of spurious edges”. Thus, in this work we selected $k=0.35$, following these references. As shown in Figure S2, the results of the network measures singled out by the affinity propagation method (AP, see below) are robust for values of $k$ around the selected one.

Fig. S2: Results for different density of links for the FC matrix
Results for the three representatives of the complex networks groups (Global Efficiency, Eccentricity and Synchronizability), for density values $k=0.25$, $k=0.35$ (value actually used) and $k=0.45$. 
6 COMPLEX NETWORK INDICES

To characterize neural networks in this work we calculated many of the indices most commonly used for this purpose: the average degree \( \langle D \rangle \), the strength \( \langle S \rangle \), the clustering coefficient \( \langle C \rangle \), the transitivity \( \langle T \rangle \), the local efficiency \( \langle E_l \rangle \), the modularity \( \langle Q \rangle \), the characteristic path length \( \langle L \rangle \), the global efficiency \( \langle E_g \rangle \), the eccentricity \( \text{ecc} \), the network radius \( \text{rad} \) and diameter \( \text{diam} \), the betweenness \( \langle B \rangle \) and the eigenvalue spectral density \( \langle v \rangle \). Here we describe the details for each of them.

Before starting, some notation is explained for a given graph \( G \) with the adjacency matrix \( A \) (see (Rubinov and Sporns, 2010) for details):

- \( N \) is the set of all nodes in the network, and \( n \) is the number of nodes.
- \( L \) is the set of all links in the network, and \( l \) is number of links.
- \( (i, j) \) is a link between nodes \( i \) and \( j \), \( (i, j) \in N \).
- Links \( (i, j) \) are associated with connection weights \( w_{ij} \).
- \( a_{ij} \) is the connection status between \( i \) and \( j \): \( a_{ij} = 1 \) when link \( (i, j) \) exists (when \( i \) and \( j \) are neighbors); \( a_{ij} = 0 \) otherwise (\( a_{ii} = 0 \) for all \( i \)).

We compute the number of links as \( l = \sum_{i,j \in N} a_{ij} \) (to avoid ambiguity with directed links we count each undirected link twice, as \( a_{ij} \) and as \( a_{ji} \)). The sum of all weights in the network is \( l w \), and it is computed as \( l w = \sum_{i,j \in N} w_{ij} \). Henceforth, I assume that weights are normalized, such that \( 0 \leq w_{ij} \leq 1 \) for all \( i \) and \( j \).

All the following definitions are specifically for weighted undirected graphs.

6.1 Degree \( (D) \) and Strength \( (S) \)

The degree \( (D) \) of a node is the number of links connected to it.

\[
D = \frac{1}{N} \sum_{i=1}^{N} d_i
\]

(3)

The global degree of a network is the average of all its nodes’ degree. Note that connection weights are ignored in calculations. The mean network degree is most commonly used as a measure of density (see 6.2), or the total “wiring cost” of the network. The weighted variant of the degree, sometimes termed the strength \( (S) \), is defined as the sum of weights of links connected to the node.

\[
S_i = d_i^w = \sum_{j \in N} w_{ij}
\]

(4)

6.2 Density \( (K) \)

Density \( (K) \) is the fraction of present connections to possible connections. It ranges between 0 and 1, being 0 a totally disconnected network and 1 a totally regular network in which every node is connected to all the others.

\[
k_i = \frac{d_i}{N - 1} \quad K = \frac{1}{N} \sum_{i=1}^{N} k_i = \frac{D}{N(N-1)}
\]

(5)
6.3 Clustering coefficient (C)

The clustering coefficient (C) describes the likelihood that neighbours of a vertex are also connected. It is the fraction of triangles around a node, and is equivalent to the fraction of node’s neighbours that are neighbours of each other. It quantifies the tendency of network elements to form local clusters.

We used here the weighted version of this measure (Onnela et al., 2005), which characterizes local clustering as:

$$C_{i}^{w} = \frac{1}{n} \sum_{i \in N} C_{i}^{w} = \frac{1}{n} \sum_{i \in N} \frac{2 t_{i}^{w}}{k_{i} (k_{i} - 1)}$$

(6)

where $C_{i}$ is the clustering coefficient of node $i$ ($C_{i} = 0$ for $k_{i} < 2$), and the number of triangles $t_{i}^{w}$ around a node $i$, is defined by the geometric mean of triangles around it:

$$t_{i}^{w} = \frac{1}{2} \sum_{j, k \in N} \left( w_{ij} w_{ik} w_{jk} \right)^{1/3}$$

(7)

Triangles are important since they are directly related to the robustness and error tolerance of the network (Boccaletti et al., 2006).

The superscript ‘$w$’ denotes that the indexes are calculated for weighted networks.

6.4 Transitivity (T)

The transitivity (T) is the ratio of triangles to triplets in the network and it is an alternative to the clustering coefficient (e.g., Newman, 2003). Note that transitivity is not defined for individual nodes, it is only a global measure.

$$T_{w} = \frac{\sum_{i \in N} 2 t_{i}^{w}}{\sum_{i \in N} k_{i} (k_{i} - 1)}$$

(8)

6.5 Modularity (Q)

The optimal community structure is a subdivision of the network into non-overlapping groups of nodes in a way that maximizes the number of within-group edges, and minimizes the number of between-group edges. The modularity (Q) is a statistic that quantifies the degree to which the network may be subdivided into such clearly delineated subgroups or modules. We used a modification for weighted networks by (Newman, 2004):

$$Q_{w} = \frac{1}{l^{w}} \sum_{i, j \in N} \left[ w_{ij} - \frac{k_{i}^{w} k_{j}^{w}}{l^{w}} \right] \delta_{wj}$$

(9)

Roughly speaking, the greater the value of Q, the more modular a network is, i.e., the greater the density of within-group connections as compared to the between-group ones.
6.6 Characteristic path length (L)

To define this index, it is necessary to first introduce the notion of ‘path’. A path consists of a sequence of linked nodes that never visit a single node more than once. It is important, to differentiate it from ‘walks’, which are sequences of linked nodes that may visit a single node more than once. The characteristic path length, then, is defined as the average shortest path length in the network (Watts and Strogatz, 1998).

\[ L^* = \frac{1}{N} \sum_{i \in N} L_i^* = \frac{1}{n} \sum_{i \in N} \frac{\sum_{j \in N} d_{ij}}{n-1} \]  \hspace{1cm} (10)

where \( L_i \) is the average distance between node \( i \) and all other nodes. The shortest weighted path length between \( i \) and \( j \), is defined as:

\[ d_{ij} = \sum_{s_i \in S_{ij}} f (w_{si}) \]  \hspace{1cm} (11)

where \( f \) is a map (e.g., an inverse) from weight to length and \( g_{ij} \) is the shortest weighted path between \( i \) and \( j \). Here we followed the suggestion of (Boccaletti et al., 2006), and set the length of the edge connecting nodes \( i \) and \( j \) inversely proportional to the weight: \( d_{ij} = 1/w_{ij} \). The shortest path length quantifies the extent of average connectivity or the overall routing efficiency of the network (Achard and Bullmore, 2007).

6.7 Global Efficiency (Eg)

The global efficiency \( (E_g) \) is related to the previous index, as it is defined as the average inverse shortest path length in the network, hence they will be inversely correlated (Latora and Marchiori, 2001).

\[ E_g^* = \frac{1}{N} \sum_{i \in N} E_i^* = \frac{1}{N} \sum_{i \in N} \frac{\sum_{j \in N} \left( d_{ij} \right)^{-1}}{N-1} \]  \hspace{1cm} (12)

where \( E_i \) is the efficiency of node \( i \). It reflects the global efficiency of parallel information transfer in the network.

6.8 Local Efficiency (El)

The local efficiency \( (E_l) \) is the global efficiency (6.7) computed on node neighbourhoods, so it will be related to the clustering coefficient (6.3) (Latora and Marchiori, 2001).

\[ E_l^* = \frac{1}{N} \sum_{i \in N} E_{i_{vis}}^* = \frac{1}{N} \sum_{i \in N} \frac{\sum_{j,k \in N_i} \left( w_{ij} w_{ik} - \left[ d_{ik}^* (N_i) \right]^{-1} \right)^{1/3}}{k_i^* (k_i^* - 1)} \]  \hspace{1cm} (13)
where $E_{loc,i}$ is the local efficiency of node $i$, and $d_{jh}(N)$ is the length of the shortest path between $j$ and $h$, that contains only neighbours. The local efficiency can be understood as a measure of the fault tolerance of the network, indicating how well each subgraph exchanges information when the index node is eliminated (Achard and Bullmore, 2007).

### 6.9 Eccentricity (ecc), radius and diameter

The node eccentricity is the maximal shortest path length (see 6.6) between a node and any other node. The lower the eccentricity, the more central a vertex is in a network. The radius is the minimum eccentricity and the diameter is the maximum eccentricity (see above).

### 6.10 Betweenness (B)

Node betweenness centrality ($B$) is the number of all shortest paths in the network that contain a given node. Nodes with high values of betweenness centrality participate in a large number of shortest paths. It can be defined as in (e.g., (Freeman, 1979))

$$b_i = \frac{1}{(N-1)(N-2)} \sum_{h,j \in V, h \neq j} \frac{\rho_{hj}(i)}{\rho_{hj}}$$

where $\rho_{hj}$ is the number of shortest paths between nodes $h$ and $j$, and $\rho_{hj}(i)$ is the number of shortest paths between these two nodes that pass through node $i$. As an example of application to MEG data in epilepsy, this index has been recently used, along with the minimum spanning tree, to assess the outcome of surgery in a follow up study of lesional epilepsy patients (van Dellen et al., 2014).

### 6.11 Eigenvector Spectral centrality (v)

Eigenvector centrality ($v$) is a self-referential measure of centrality. It determines the importance of a node on the basis of its connections to other nodes, but also with respect to how those other nodes are connected (and so on). It weighs the connections of a node (Bonacich, 2007), so that when connected to a highly connected “hub” makes it more influential than being connected to many poorly connected peripheral nodes. In this way, eigenvector centrality takes the relation within the whole network into account and allows for the identification of hubs (Hardmeier et al., 2012).

### 6.12 Small worldness (SW)

Network small-worldness (Humphries and Gurney, 2008; Watts and Strogatz, 1998)

$$SW = \frac{C / C_{rand}}{L / L_{rand}}$$

(15)
where $C$ and $C_{\text{rand}}$ are the clustering coefficients, and $L$ and $L_{\text{rand}}$ are the characteristic path lengths of the respective tested network and a random network. Small-world networks often have $S \gg 1$.

Small-world property is characterized by a high local clustering of connections between neighboring nodes and short path lengths between any pair of nodes. It was after (Watts and Strogatz, 1998) introduced this concept, that it has been observed in many complex networks of nature, including social, economic and biological networks. The cortical networks of other mammalian brains as well as functional and structural human brain networks have been found to exhibit small-world properties (Li et al., 2009).

A real network would be considered as small-world if it meets the $C_r = \frac{C}{C_{\text{rand}}} \gg 1$ and $L_r = \frac{L}{L_{\text{rand}}} = 1$, in which $C_{\text{rand}}$ and $L_{\text{rand}}$ are the mean clustering coefficient and mean shortest path length of the random network (Watts and Strogatz, 1998). For the calculation of $C_{\text{rand}}$ and $L_{\text{rand}}$, we generated 50 random networks for each subject’s functional connectivity matrix by a randomization (using BCT toolbox (Rubinov and Sporns, 2010)), in which the original connectivity matrix was randomly permuted (randomly reshuffling links), while keeping the degree distribution of each node constant (Fornito et al., 2013; Maslov and Sneppen, 2002; Rubinov and Sporns, 2010). Then, the average across all 50 generated random networks was performed to obtain the mean $C_{\text{rand}}$ and $L_{\text{rand}}$ indexes, and thus the small-world indices $C_r$ and $L_r$ were calculated for the weighted FC networks.

### 6.13 Algebraic connectivity (ac)

Now we will introduce two parameters (algebraic connectivity, this section, and synchronizability, next section) based on the spectral graph theory, which studies the properties of graphs via the eigenvalues and eigenvectors of their associated graph matrices: the adjacency matrix ($A$) and the combinatorial Laplacian matrix ($L$), also known as Kirchhoff matrix (since it was originally proposed by Kirchhoff) and its variants. The connection between the Laplacian and the adjacency matrices is $L = D - A$, where $D$ is the diagonal degree matrix $D := D_i$, being $D_i$ the strength of node $i$.

We denote the set of eigenvalues of the adjacency matrix $A$ as $\lambda_1 \leq \lambda_2 \leq \cdots \leq \lambda_n$, where the largest eigenvalue $\lambda_n$ is called the spectral radius; and the eigenvalues of the Laplacian matrix $L$ as $0 = \mu_1 \leq \mu_2 \leq \cdots \leq \mu_n$. The eigenvalues of the adjacency matrix are real, while the eigenvalues of the Laplacian matrix are real and non-negative. (see (Boccaletti et al., 2006) for details).

The largest eigenvalue $\mu_n$ characterizes dynamic processes on networks such as virus spreading and synchronization processes. Its inverse, $1/\mu_n$, describes the threshold of the phase transition,
which specifies the onset of a remaining fraction of infected nodes and of locked oscillators, respectively, of both virus spread and synchronization of coupled oscillators in networks (Li et al., 2011).

The algebraic connectivity (ac), measures how difficult it is to tear a network apart (Fiedler, 1973). The algebraic connectivity is defined as the second smallest eigenvalue of the Laplacian matrix, $\mu_2$. It is greater than 0 if and only if the network is fully connected.

\[ ac = \mu_2 \]  \hspace{1cm} (16)

The magnitude of the algebraic connectivity is important for synchronization, and reflects also network robustness (resilience against damage).

6.14 Synchronizability (sync)

The term ‘synchronizability’ is often used for the ratio of Laplacian’s largest and second smallest eigenvalues $\mu_N$ and $\mu_2$, respectively. It gives the idea of the stability of a graph. The larger the ratio, the more difficult to synchronize the oscillators; the smaller it is, the more stable the network synchronization (Arenas et al., 2008).

\[ R = \frac{\mu_N}{\mu_2} \]  \hspace{1cm} (17)

This ratio is also referred to as the ‘paradox of heterogeneity’. It shows that (unweighted, undirected) networks with a more homogeneous degree distribution synchronize more easily than networks with a more heterogeneous degree distribution (Li et al., 2011; Mieghem, 2012). This index has been recently used in neuroscience (de Haan et al., 2012; Lehnertz et al., 2014; Li et al., 2011). Here, however, we will used the inverse (17) following (De Haan, 2012), to obtain an index ranging between 0 and 1, which is closer to 1 when the network is easier to synchronize.

\[ sync = \frac{1}{R} \]  \hspace{1cm} (18)

7 AFFINITY PROPAGATION CLUSTERING

The Affinity propagation (AP) is a recently described clustering data method that takes as input measures the similarity between pairs of data points and simultaneously considers all data points as potential exemplars, in contrast to traditional data clustering algorithms. Real-valued messages are exchanged between data points until a high-quality set of exemplars and corresponding clusters gradually emerges. It has been used to solve a variety of clustering problems and it was been shown to uniformly find clusters with much lower error than other methods, in less than one-hundredth the

\[ \text{http://www.psi.toronto.edu/index.php?q=affinity\%20propagation} \]
amount of time (Frey and Dueck, 2007; Givoni and Frey, 2009). In our particular application, AP presents three additional advantages: first, it works with similarities (i.e., correlations between the data, instead of distances); second, it is adaptive to the data; and third, it is not necessary to fix a priori the number of clusters, but the algorithm determines, in a purely data-driven way, the optimal number of clusters, thereby providing a direct answer to our question of how many different networks measures are enough to fully describe the data. This allows us to reduce the number of graph theoretical network measures to be further analysed in the epileptic study, by selecting only one representative from each cluster.

8 FALSE DISCOVERY RATE (FDR)

To correct for multiple comparisons, as a consequence of multiple hypothesis testing, we used the false discovery rate (FDR) method (Benjamini and Yekutieli, 2001; Genovese et al., 2002). FDR controls the expected proportion of incorrectly rejected null hypotheses (type I errors) among all rejected hypotheses. In this case we have used the type I FDR implementation (Genovese et al., 2002), since the results of the p values are very likely to be positively correlated, as the noise in the data is Gaussian with non-negative correlation across sensors (Benjamini and Yekutieli, 2001), and two close sensors (and the PLV between close sensors) are likely to reflect partially overlapping information. Table S2 summarizes the information regarding the statistics applied to the data in the study.

| GLOBAL | Power spectrum | Wilcoxon | FDR type I (q<0.05) |
|--------|----------------|----------|---------------------|
|        | Networks indices | Wilcoxon | FDR type I (q<0.1) |
| LOCAL | Power spectrum | Wilcoxon | FDR type I (q<0.05) |
|        | Networks indices | Wilcoxon | FDR type I (q<0.1) |
|        | NODES | Wilcoxon | FDR type I (q<0.1) |
|        | EDGES | Non parametric permutation test |

Table S2: Summary of the statistical tests applied to the data
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