Heavy Quark Fragmentation

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Abstract

I present the main aspects of open charm production in $\pi^- p$ collisions in the context of the Lund String Fragmentation Model as implemented in the Monte Carlo program PYTHIA. The emphasis is on the transition from large to small strings and the dependence on model parameters. A modified version is presented and compared with experimental results both on asymmetries, single-charm spectra and correlations.
Introduction

Since the discovery of charm in 1974 numerous experiments on charm production at fixed target have been performed \[1\]. In recent years the precision has increased significantly \[2, 3, 4\], which enables a detailed comparison with theory. Perturbative QCD can describe some of the phenomenology of charm production, but not all \[3, \(x_F\)\] \[2, 3, 4\]. Most notably the asymmetry between leading and non-leading particles, which is negligible in NLO QCD, has been shown to increase with \(x_F\) \[2, 3, 4\]. Also the momentum spectra of produced D mesons are harder than the NLO QCD c quark predictions, especially for leading particles, see e.g. \[2, 6\]. These facts imply that nonperturbative effects are important in the production of charmed hadrons. In the string fragmentation approach both these aspects are included in the 'drag' effect, whereby a charm quark can gain momentum when it is connected to a beam remnant. The extreme case of this effect is the collapse of a small string into a single hadron, which gives rise to a dependence on the flavour contents of the beam.

Basics of string fragmentation

The Lund String Fragmentation Model \[7\] is best explored in e\(^+\)e\(^-\) annihilation, where the produced q and \(\bar{q}\) are connected by a linear force field with a string-like topology. The q\(\bar{q}\) production process is described by perturbative QCD, with a parton-shower approximation to higher orders. Radiated gluons are interpreted as 'kinks' on the string. The nonperturbative hadronization of a string proceeds via the production of q\(\bar{q}\) pairs in the colour force field, which arrange themselves to produce the observed hadrons. A strongly constrained fragmentation function can be derived from very general and physically intuitive assumptions about the fragmentation process \[8\]. Because of the non-negligible mass of the charm quark the fragmentation function has to be modified for heavy-flavour production \[9\]. This model has been implemented in the Monte Carlo program PYTHIA\[10\], which has been tuned to e\(^+\)e\(^-\) experiments to give a good description of available data.

Hadron-hadron collisions

When we carry this model over to hadron-hadron physics, we again divide the process into a perturbative and a nonperturbative part and assume factorization between the two. In addition we assume that the fragmentation process is universal, i.e. the hadronization of a colour singlet is independent of how it was produced. In a hadron-hadron collision, such as \(\pi^-p\), several ambiguities not present in e\(^+\)e\(^-\) annihilation are introduced. The main ones are presented in the following.

Structure of the incoming hadrons. The particles that participate in the collision are not point-like but have an internal structure. The longitudinal structure is parameterized by parton distribution functions (PDFs) which have been determined from other experiments such as deep inelastic scattering (DIS). These will not be discussed in the following, but in principle they give rise to some ambiguity, especially for small momentum fractions.

Structure of the beam remnant. When a parton has been picked out of a hadron, what is left continues in the direction of the beam and is called a beam remnant. If the beam remnant can be viewed as consisting of two or more objects its structure must be described. How this should be done is not known from first principles and has not
been studied much. This aspect is therefore parameterized in beam remnant distribution functions (BRDFs) and several variants are considered.

**Primordial transverse momentum.** The partons inside the hadron are confined to a transverse dimension less than 1 fm; therefore by the uncertainty principle the spread of the transverse momentum should be of the order $0.2$ GeV. This is modeled by adding a primordial $k_\perp$ to the partons going into the hard scattering process. In the default version of PYTHIA, $k_\perp$ is assumed distributed as a Gaussian with a width of 0.44 GeV. Several studies \cite{6, 11, 12} imply that this value is too small and a value of 1 GeV or more is needed to describe the data. This remains somewhat of a mystery and will not be resolved here.

**Small strings in hadronization. Quark masses.** When the colour topology of an event has been determined, every colour singlet is hadronized as a string would in $e^+e^-$. This works for strings with a mass of a few GeV or more. For strings with masses near the two-particle threshold, the standard string fragmentation approach cannot be used and some other scheme is needed. As we will see later this introduces a large dependence on the quark masses.

### String topologies and the ’beam drag’ effect

In a $\pi^-p$ collision we include charm production via the leading-order production mechanisms of quark and gluon fusion ($q\bar{q} \rightarrow c\bar{c}$ and $gg \rightarrow c\bar{c}$ respectively). The partons of the hard interaction and of the beam remnants are connected by strings, representing the confining colour field \cite{7}. Each string contains a colour triplet endpoint, a number (possibly zero) of intermediate gluons and a colour anti-triplet end. The string topology can usually be derived from the colour flow of the hard process. For instance, consider the process $u\bar{u} \rightarrow c\bar{c}$ in a $\pi^-p$ collision. The colour of the incoming $u$ is inherited by the outgoing $c$, so it will form a colour-singlet together with the proton remnant, here represented by a colour anti-triplet ud diquark. In total, the event will thus contain two strings, one $c$–ud and one $\bar{c}$–d (Fig. 1a). In $gg \rightarrow c\bar{c}$ a similar inspection shows that two distinct colour topologies are possible. Representing the proton remnant by a u quark and a ud diquark (alternatively d plus uu), one possibility is to have three strings $c$–\(\bar{\tau}\), $\bar{c}$–u and d–ud (Fig. 1b), and the other is the three strings c–ud, $\tau$–d and u–\(\bar{\tau}\) (Fig. 1c).

Other production mechanisms such as charm excitation and charm from parton showers (i.e. higher order effects) are not included in this study. Because of the relatively low virtuality of the hard process at fixed target energies the second mechanism is negligible, but it will become increasingly important at higher energies; at the LHC, e.g., this mechanism will dominate over the fusion mechanisms. Charm excited from the sea could give a non-negligible contribution, but we will not include it here.

Consider a colour singlet in Fig. 1 containing a charm endpoint. The hadronization of this string is performed in the CM system of the string. In that frame hadronization always results in a deceleration of the quark. After the rotation and boost to the hadron-hadron CM system, on the other hand, the net result of hadronization can be either an acceleration or a deceleration of the charm quark. This is interpreted as the beam remnant dragging the charm quark in the direction of the beam. This effect alone does not account for the asymmetry because of the cancellation between the diagrams in Fig 1b and c. We must therefore consider the flavour contents of the beam.
Cluster collapse of small strings

In $e^+e^-$ annihilation the string mass is fixed by the CM energy of the process. In a hadron-hadron collision, on the other hand, charmed strings can have any mass ranging from $m_q + m_c$ to $\sqrt{s}$. For string masses larger than some cut-off (here taken as $m_q + m_c + m_0$, with $m_0 \approx 1$ GeV and $q$ a light quark) the Lund string fragmentation approach can be used. The model assumes a continuous final-state phase space, and an iterative scheme is used which demands that at least two particles are produced from a string. A string with a mass smaller than the cut-off we call a cluster and it is hadronized in the following way:

**Cluster decay.** A $q\bar{q}$ pair is created, using standard flavour selection, from the force-field connecting the cluster endpoints, and two hadrons are produced. If kinematically possible the cluster will decay isotropically into these two hadrons.

**Cluster collapse.** If it is not kinematically possible for the cluster to decay into two hadrons, it will be forced to collapse into a single hadron under conservation of the flavour quantum numbers. Since the mass of the cluster most likely will not correspond to any physical state (e.g. D or $D^*$), the energy and momentum of the cluster must be slightly modified in order to put it on the hadronic mass shell.

There are two main ambiguities in this scheme. The first is that there is no clear separation between the two hadronization mechanisms, and the second is energy/momentum conservation in the cluster collapse.

To justify the cluster collapse approach we use an argument based on local duality, which has been shown to hold in $e^+e^-$ annihilation, DIS [13] and $\tau$ decay [14]. In $e^+e^-$ annihilation into hadrons around the $c\bar{c}$ threshold the observed cross section consists of peaks at $J/\psi$ and $\psi'$ and a continuum above the $DD^*$ threshold. The perturbatively calculated cross section, on the other hand, is continuous from $\sqrt{s} = 2m_c$ onwards. However, if the experimental cross section is suitably smeared, it approximately agrees with the perturbative one. Another way of stating this is that the integrated cross section (over $\sqrt{s}$) should be the same provided that the integration interval is suitably large. We use the same argument in the present case, by replacing $\sqrt{s}$ with $M_{\text{string}}$ and the $J/\psi$ and $\psi'$

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Figure 1: Examples of different string configurations in a $\pi^-p$ collision: (a) $u\pi \rightarrow c\bar{c}$ has a unique colour flow; (b,c) $gg \rightarrow c\bar{c}$ with the two possible colour flows. Dashed lines are strings.
peaks with $D$ and $D^*$. The duality argument could then be stated in the following way:

$$\int_{m_1}^{m_2} \frac{d\sigma_{\text{Partons}}}{dM_{\text{String}}} dM \approx \int_{m_1}^{m_2} \frac{d\sigma_{\text{Hadrons}}}{dM_{\text{String}}} dM$$

(1)

Fig. 2a shows how this looks using Pythia with standard parameters. The solid line is the mass distribution of produced clusters at the parton level and the dashed one is the produced hadrons. The clusters in the gray area have collapsed into single hadrons. The parton level distribution depends on many of the parameters such as the BRDF, primordial $k_\perp$, and quark masses. This is shown in Fig 2b. Consider e.g. an increase of the charm mass. The threshold will be shifted towards higher values and fewer clusters will be forced to collapse (the gray area is decreased). It is also possible to decrease the number of collapses from above by increasing the probability for a cluster above the $D\pi$ threshold to decay.

Fig. 3 shows the $x_F$ distributions for different production channels and different parameter sets. These parameters will be discussed in the following. The explanation of the leading particle asymmetry in this model is that $D^{+}$ cannot be produced from cluster collapse because it has no quark in common with the beam.

**Dependence on parameters**

In this section the different parameters that have already been introduced will be discussed in more detail. We start with the BRDFs. How the energy and momentum in the beam remnant should be split between the constituents is not known from first principles, so we consider two extreme cases. In the first (default) scenario, we use BRDFs similar to those in PDFs, where one object takes a small fraction of the available energy. In the other extreme, we use naive counting rules where the energy is shared democratically between the constituents. How this effects the distributions is shown in Fig 3. Most notably the dip around $x_F \sim 0.5$ in the cluster collapse distribution is smeared out when an even sharing is used. Also note that the asymmetry is much more sensitive to the BRDF in the proton region, where one of the constituents of the beam remnant is a diquark. In the tuned version we will use an intermediate energy sharing.
Figure 3: (a) $D^-$ and (b) $D^+$ meson production in a $\pi^- p$ collision at a $\pi^- \text{ beam momentum}$ of 500 GeV with different parameter sets. From top to bottom these are: default, using flat BRDFs and the new parameter set presented in this talk. The distributions are divided into different production channels: (i) Cluster collapse, light quark from p end, (ii) Cluster collapse, light quark from $\pi^-$ end, (iii) Cluster decay, light quark from p end, (iv) Cluster decay, light quark from $\pi^-$ end and (v) String fragmentation. (c) The resulting asymmetry.

We now come to the problem of energy/momentum conservation in the cluster collapse. To understand the dependence on this we consider two mechanisms that are of opposite nature:

**Old method:** 'far away'. In the first scheme, energy and momentum is shuffled to the parton in the event that is farthest from the cluster, in order to minimize the recoil. In this approach the D meson is often harder than the cluster.

**New method:** 'local'. In this new scheme we conserve energy locally by exchanging 'gluons' between the cluster and the string in the event that is closest to the cluster.

The details are presented in [11] and the conclusion is that the observables are not sensitive to the details of the energy/momentum conservation scheme, except for $x_F > 0.5$, where cluster collapse dominates but data is scarce.

Looking at Fig. 3 we see that the reason for the large asymmetry using the default parameters is that they allow many clusters to collapse for $x_F > 0$. The following set of parameters are inspired by the E791 collaboration and data from WA82 and they aim at decreasing the asymmetry by decreasing the number of clusters that collapse into one particle [13].
• Quark masses. The charm mass used in Pythia is by default set to the current algebra one (1.35 GeV). This is the value used in e.g. Higgs physics but it is far from obvious that this is the relevant mass in the present case. We therefore use constituent quark masses: $m_c = 1.5$ GeV; $m_d = m_u = 0.33$ GeV; $m_s = 0.5$ GeV. This will shift the threshold of the distribution of cluster masses and thus decrease the number of cluster collapses.
• Cluster decay. Increase the probability for a cluster to decay.
• Cluster collapse. Use new ‘local’ collapse mechanism.
• BRDF. We use an intermediate energy sharing that is more democratic, but not completely.
• Primordial $k_{\perp}$. The width of the Gaussian primordial $k_{\perp}$ is increased from 0.44 to 1.0 GeV. This allows cluster collapses between a charm quark and a beam remnant to occur also at fairly large values of $p_{\perp}$, thus leading to an essentially $p_{\perp}$-independent asymmetry. In addition, the $p_{\perp}$ kick added to charm quarks and beam remnants tends to increase the average invariant mass of the produced clusters, thereby reducing the number of cluster collapses.

Comparison with experiment

In this section we compare the model (with the new parameter set [15]) to some data. Fig. 4 shows the asymmetry as a function of $x_F$ and $p_{\perp}^2$ as well as single-charm spectra from WA82 for $D^+$ and $D^-$ individually. In this case the data is nicely described by the new parameter set.

Next we compare the model to the new correlation data from E791 [12]. Several correlations in events where a pair of $D\bar{D}$ mesons with rapidity in $-0.5 < y < 2.5$ is fully reconstructed are studied. The distributions in Fig. 5 show that the longitudinal correlations in the string model are generally different from the data and are better described by NLO QCD [12]. The transverse correlations on the other hand are better described by the model, mostly because of the increased primordial $k_{\perp}$.

It is clear that the correlation between charm quark pairs should be modified by hadronization, but the string model seems to produce D mesons that are too far from each other in momentum (rapidity). To attempt a description of the data we use the independent fragmentation approach where a charmed hadron is simply a scaled-down version of the charm quark. In this way the beam remnants do not affect the charm quarks at all. We use the fragmentation function of Peterson et. al. [16] with $\epsilon_c = 0.05$. Surprisingly the longitudinal correlations are quite nicely described by this scheme. Especially the rapidity distribution is interesting because NLO QCD also fails to describe the strong peaking at small rapidities seen in the E791 data, see Fig. 3. However, independent fragmentation of course fails to describe the single-charm spectra from WA82, falling way below the data and lacking any asymmetry at large $x_F$. Thus there seems to be a contradiction between the data on single-charm and correlations that we cannot explain. There are several possibilities that deserve to be investigated. There could be some cut in the data that we don’t understand, or problems with acceptance corrections at large $x_F$. Alternatively, some other mechanisms that we have not yet included (sea, intrinsic charm, parton showers) could give a sizable contribution.
Figure 4: Comparison between the new Pythia parameter set and data. Asymmetry as a function of (a) $x_F$ and (b) $p_T^2$. Single-charm spectra from the WA82 CERN experiment [2] for (c) D$^+$ and (d) D$^-$ (dashed line is standard parameters and the full line is the new).

Summary

To summarize we have described the string fragmentation approach to charm production in hadronic collisions. A number of uncertainties have been identified and studied in detail, in particular the transition from a continuous string-mass distribution to a discrete hadron-mass one. The conclusion is that the model can describe asymmetries, single-charm spectra and transverse correlations but not longitudinal correlations. Also, these data do not fully constrain the choice of model parameters. Further data on charm production in $\pi^-$p collisions may provide further information, as may charm production e.g. in ep collisions.

References

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Figure 5: $D\bar{D}$ correlations. (a) $x_F$; (b) $y$; (c) $p_{T}^2$; (d) $\Delta x_F = x_{F,D} - x_{F,\bar{D}}$; (e) $\Delta y = y_D - y_{\bar{D}}$; (f) $\Delta p_{T}^2 = |p_{T,1,D}^2 - p_{T,1,\bar{D}}^2|$. Experimental data are from the E791 experiment [12] compared to the default (dashed) and modified (full) PYTHIA predictions.

Figure 6: $D\bar{D}$ correlations compared to independent fragmentation. Correlations: (a) $x_F$; (b) $y$; (c) $\Delta x_F = x_{F,D} - x_{F,\bar{D}}$; (d) $\Delta y = y_D - y_{\bar{D}}$. Single-charm from WA82: (e) $D^+$ and (f) $D^-$. 
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PMAS(1,1)=PMAS(2,1)=0.33D0; PMAS(3,1)=0.5D0; PMAS(4,1)=1.5D0 (quark masses), MSTP(92)=3 (intermediate BRDFs), PARP(91)=1.D0 (width of Gaussian primordial $k_{\bot}$), PARP(93)=5.D0 (the maximum allowed $k_{\bot}$). The changes in cluster decay/collapse are not implemented in the standard distribution yet. Patches can be found on the PYTHIA homepage at [http://www.thep.lu.se/~torbjorn/Pythia.html](http://www.thep.lu.se/~torbjorn/Pythia.html)

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$A(x_F)$

- $X$ WA82@340 GeV
- $+$ E769@250 GeV
- $\circ$ E791@500 GeV