Compressive Coded Random Access for Massive MTC Traffic in 5G Systems

Gerhard Wunder¹, Čedomir Stefanović², Petar Popovski², Lars Thiele³
¹Heisenberg Communications and Information Theory Group, Freie Universität Berlin
²Department of Electronic Systems, Aalborg University, Denmark
³Fraunhofer Heinrich Hertz Institute

Abstract—Massive MTC support is an important future market segment, but not yet efficiently supported in cellular systems. In this paper we follow-up on recent concepts combining advanced MAC protocols with Compressed Sensing (CS) based multiuser detection. Specifically, we introduce a concept for sparse joint activity, channel and data detection in the context of the Coded ALOHA (FDMA) protocol. We will argue that a simple sparse activity and data detection is not sufficient (as many papers do) because control resources are in the order of the data. In addition, we will improve on the performance of such protocols in terms of the reduction of resources required for the user activity, channel estimation and data detection. We will mathematically analyze the system accordingly and provide expressions for the capture probabilities of the underlying sparse multiuser detector. Finally, we will provide ‘structured’ CS algorithms for the joint estimation scheme and evaluate its performance.

I. INTRODUCTION

The Internet of Things (IoT) is a most promising 5G market segment and in the focus of all key players in the ICT domain. Even pessimistic forecasts predict several billions of connected devices. Major proliferation of the IoT will be naturally in the 5G wireless domain. Currently, IoT market is mainly served by short range capillary wireless technologies such as Bluetooth LE, ZigBee, and WiFi and proprietary (clean slate) low power wide area technologies such as SIGFOX, LoRA etc. There is only small share for cellular and there is clearly a need to act fast in this direction.

IoT requires support of scalable massive machine-type communication (MTC), which is essentially a sporadic traffic pattern generated by devices operating under tight (resource) constraints such as low cost, battery lifetime, computation capability etc. Such messages have typically very unfavorable control/data signaling ratio; recent proposals suggest 5G “one-shot” random access concepts where devices wake up and send data right away with no coordination whatsoever [1], [2]. The concept is depicted in Fig. 1. While this concept is quite appealing it comes with significant challenges:

(i) Temporal asynchronous access among different resources; spectral asynchronous access due to low-cost terminals; definition of shorter TTI’s and more granularity in allocating the physical resource blocks. This is the waveform challenge [1].

(ii) Relationship between the data and the control data (metadata); control signaling possibly in the order of data; per user resource control signaling becomes inefficient. This is the metadata challenge [1].

(iii) Throughput severely degraded due to collisions in random access unless successive cancellation is applied. This is the throughput challenge [3].

The challenges are depicted in Fig. 2. In this paper, we address the throughput challenge and follow-up on recent concepts combining advanced MAC protocols with Compressed Sensing (CS) based multiuser detection. Specifically, we introduce a concept for sparse joint activity, channel and data detection in the context of the Coded ALOHA (FDMA) protocol which we call Compressive Coded Random Access (CCRA) extending the work in [4], [5], [6]. We will argue that a simple sparse activity and data detection is not sufficient (as many papers do) because control resources are in the order of the data. In addition, we will improve on the performance of such protocols in terms of the reduction of resources required for the user activity, channel estimation and data detection. We will mathematically analyze the system accordingly and provide expressions for the capture probabilities of the underlying...
sparse multiuser detector. Finally, we will provide ‘structured’ CS algorithms for the joint estimation scheme and evaluate its performance.

**Notations.** $\|x\|_q = (\sum_i |x_i|^q)^{1/q}$ is the usual notion of $\ell_q$-norms and $|x| := \|x\|_2$. We denote with $\text{supp}(x) := \{i : x_i := \langle e_i, x \rangle \neq 0\}$ the support of $x$ in a given fixed (here canonical) basis $\{e_i\}_{i=1}^n$. The size of its support is denoted as $\|x\|_{\ell_0} := |\text{supp}(x)|$. $W$ is the (unitary) Fourier matrix with elements $(W)_{ij} = n^{-1/2}e^{-\sqrt{2\pi}ij/n}$ for $k, l = 0 \ldots n - 1$, hence, $W^{-1} = W^*$ where $W^*$ is the adjoint of $W$. We use $\hat{x} = Wx$ to denote Fourier transforms and $\odot$ means point-wise product. $I_n$ is the identity matrix in $\mathbb{C}^n$, $\text{diag}(x)$ is some arbitrary diagonal matrix with $x \in \mathbb{C}^n$ on its diagonal.

**II. CCRA Model**

For simplicity assume one time slot only and $n$ OFDM subcarriers. This is easily generalized to the case where there are multiple time slots, notably, within the coherence time so that channels are constant over these slots. Let $p_u \in \mathbb{C}^n$ be some signature from a given set $\mathcal{P} \subset \mathbb{C}^n$ and $x_u \in \mathcal{X}_u$ be an unknown (uncoded) data sequence from the modulation alphabet $\mathcal{X}_u^m$ both for the $u$-th user with $u \in \{1, \ldots, U\}$ and $U$ is the (fixed) maximum set of users in the systems. Note that in our system $n$ is a very large number, e.g. 24k. Due to the random zero-mean nature of $x_u$ we have $\frac{1}{n}E|p_u + x_u|^2 = 1$, i.e. the total (normalized) transmit power is unity. Provided user $u$ is active, we set:

$$\alpha := \frac{1}{n}\|p_u\|^2 \text{ and } \alpha' := 1 - \alpha = \frac{1}{n}E|\|x_u\|^2$$

Hence, the control signalling fraction of the power is $\alpha$. If a user is not active then we set both $p_u = x_u = 0$, i.e. either a user is active and seeks to transmit data or it is inactive. Let $h_u \in \mathbb{C}^s$ denotes the sampled channel impulse response (CIR) where $s \ll n$ is the length of the cyclic prefix. The most important assumptions in this paper are:

(i) Bounded support of $h_u$, i.e. $\text{supp}(h_u) \subseteq [0, \ldots, s - 1]$ due to the cyclic prefix

(ii) Sparsity of $h_u$ within $\text{supp}(h_u)$, i.e. $\|h_u\|_0 \leq k_1$

(iii) Sparse user activity, i.e. $k_2$ users out of $U$ in total are actually active.

Define $k := k_1k_2$.

Let $[h, 0] \in \mathbb{C}^n$ denote the zero-padded CIR. The received signal is then:

$$y = \sum_{u=0}^{U-1} \text{circ}([h_u, 0])(p_u + x_u) + e$$

$$y_B = \Phi_By$$

Here, $\text{circ}([h_u, 0]) \in \mathbb{C}^n$ is the circulant matrix with $[h_u, 0]$ in its first column. $\Phi_B$ denotes some measurement matrix (to be specified later on) typically referring to a frequency window $B$ of size $m := |B|$. All performance indicators depend strongly on the number of subcarriers in $B$ (control) and $B^C$ (data). The goal is clearly a small observation window $B$.

The AWGN is denoted as $e \in \mathbb{C}^n$ with $E(\|e\|^2) = \sigma^2 I_n$. For circular convolutions we have $\text{circ}([h, 0])p_B = \sqrt{n}W^*(h \odot \hat{p})$ so that:

$$y = \sum_{u=1}^{U} W^*(\{\sqrt{n}h_u \odot (\hat{p}_u + \hat{x}_u)\}) + \hat{e}$$

$$y_B = \Phi_By$$

where $e$ and $\hat{e}$ are statistically equivalent.

**A. Control signaling model**

For the CCRA scheme let us assume that users’ preambles ‘live’ entirely in $B$ while all data resides in $B^C$, so that $\text{supp}(p_u) \subseteq B \forall u$. We call this a common overloaded control channel [6]. Let $P_B : \mathbb{C}^n \rightarrow \mathbb{C}^m$ be the corresponding projection matrix, i.e. the submatrix of $I_n$ with rows in $B$. For identifying which preamble is in the system we can consider $\hat{y}$ and use the frequencies in $B$, i.e. $\Phi_B = P_BW$, so that:

$$y_B := P_B \sum_{u=1}^{U} \sqrt{n}h_u \odot (\hat{p}_u + \hat{x}_u) + P_B\hat{e}$$

For algorithmic solution, we can stack the users as:

$$y = \sum_{u=1}^{U} \text{circ}(h_u)(p_u + x_u) + e$$

$$y = D(p)h + C(h)x + e$$

where $D(p) := [\text{circ}(p_1), \ldots, \text{circ}(p_U)] \in \mathbb{C}^{n \times U \times n}$ and $C(h) := [\text{circ}([h_1, 0]), \ldots, \text{circ}([h_U, 0])] \in \mathbb{C}^{n \times U \times n}$ are the corresponding compound matrices, respectively $p = [p_1, p_2, \ldots, p_U]^T$ and $h = [h_1, h_2, \ldots, h_U]^T$ are the corresponding compound vectors. If we assume each user-channel vector $h_u$ to be $k_1$-sparse and $k_2$ active then $h$ is $k$-sparse.

For joint user activity detection and channel estimation exploiting the sparsity we can use the standard basis pursuit denoising (BPDN) approach:

$$h = \arg \min_h \|h\|_{\ell_1} \text{ s.t. } \|\Phi_B D(p)h - y\|_{\ell_2} \leq \epsilon$$

(1)

Moreover, several greedy methods such as CoSAMP exists for sparse reconstruction. After running the algorithm in eqn. (1) the decision variables $\|h_u\|_{\ell_2}^2, \forall u$, are formed, indicating that if $\|h_u\|_{\ell_2}^2 > \xi$ where $\xi > 0$ is some predefined threshold the user is considered active and its corresponding data is detected.
The iterative interference cancellation (IC) resembles iterative belief-propagation erasure decoding, allowing the use of the related theoretical concepts to analyze and design random access algorithms. However, the important differences have to be taken account, stemming from the nature of the physical layer operation:

(i) The received singleton slots are not always decodable, i.e. they are decodable with a certain probability, which depends on the received SNR, channel estimation etc.

(ii) The received collision slots may be decodable, depending on the multi-user detection capabilities. Further, the unbalance of received signal powers due to varying channels that users experience, may cause capture effect, where a subset of the collided users may be decoded as a result of a favorable SINR.

(iii) Cancellation of replicas, in general, is not ideal, due to imperfect channel estimation and/or channel variations among the slots where replicas occurred, operation of the physical layer etc., and leaves a residual interference power. This implies that, as the IC progresses, the residual interference accumulates in the affected slots, which may prevent further decoding of the remaining user packets.

Analytical modeling of the above is the main prerequisite to assess the performance of the random access algorithm, which in turn, allows for the design of the probability distribution that governs the choice slots, and which is typically optimized to maximize the throughput, i.e., the number of resolved packets per slot. In the context of application of coded slotted ALOHA to compressive-sensing based physical layer, some preliminary work can be found in [4]. Here we extend the approach, by taking into account a more detailed operation of the physical layer, which incorporates channel estimation and imperfect interference cancellation, as detailed in Section III-B.

### III. PERFORMANCE ANALYSIS

The performance analysis is split into activity detection/channel estimation and the data part, where coded random access is included.

#### A. User detection/channel estimation

In the data model we assume that fast fading effects are averaged out due to coding over subcarriers. Hence, user rates are ergodic and are calculated as expectations over the fading distributions. Achievable rates crucially depend on the receive powers (user position, slow fading effects), channel estimation errors and corresponding interference from colliding users then in [6]. The relevant expressions under erroneous channel estimation will be provided below.

Suppose user $u$ as well as colliding users $u(j) \in C_u, j = 1, \ldots, |C|$, which are detected before in some singleton slot have been assigned subcarriers $i \in B_u$. Due to the circular model each subcarrier has powers $E(\tilde{r}_{u,i}^2) = 1 - \alpha$, $|\hat{p}_{u,i}|^2 = \alpha$ and $E(\tilde{r}_{u,i}^2) = \sigma^2$. Denote the channel estimation error as $\hat{d}_{u,i} := \tilde{h}_{u,i} - \hat{h}_{u,i}$. Hence, the received signal is given by:

$$\hat{y}_{u,i} = (\sqrt{\tilde{n}}\tilde{h}_{u,i} + \hat{d}_{u,i})\tilde{x}_{u,i} + \tilde{e}_{u,i}$$
for singleton slots and
\[
\hat{y}_{u,i} = (\sqrt{n}h_{u,i} + \hat{d}_{u,i})\hat{x}_{u,i} + \sum_{j \in C} d_{u(j),i}\hat{x}_{u(j),i} + \hat{e}_{u,i}
\]
for collision slots. Suppose further we have calculated the probability of not detecting an active user \(P_{md}(\xi)\) ("missed detection"), and falsely detecting an inactive user \(P_{fa}(\xi)\) ("false alarm"). Define \(P_{md}(\xi) := 1 - P_{md}(\xi)\) \[9\]. Let the channel impulse response be \(k\)-sparse and use BPDN as the channel estimate. Further, let \(\Phi_B, m = |B|\) be a fixed measurement matrix with RIP constant \(\delta_{2k} < \sqrt{2} - 1\) and corresponding \(c_1(\delta_{2k})\). The achievable rate \(R(\alpha)\) per subcarrier for a particular user is lower bounded

- for singleton slots by:

\[
R(\alpha) \geq \mathbb{E}_{h \in \{||h|| > \xi\}} \left[ \log \left( 1 + \frac{(1 - \alpha)|h|^2}{\sigma^2} \right) \right] P_{md}(\xi)
\]

\[
- \log \left( 1 + \frac{(1 - \alpha)c_1(\delta_{2k})^2m}{\sigma^2\alpha n k_2} \right)
\]

- and for collisions slots by:

\[
R(\alpha) \geq \mathbb{E}_{h \in \{||h|| > \xi\}} \left[ \log \left( 1 + \frac{(1 - \alpha)|h|^2}{\sigma^2} \right) \right] P_{md}(\xi)
\]

\[
- \log \left( 1 + \frac{(|C| + 1)(1 - \alpha)c_1(\delta_{2k})^2m}{\sigma^2\alpha n k_2} \right)
\]

To prove we can extend the analysis in \[6\] in a straightforward manner. Note that the performance strongly depends on the scaling of \(c_1(\delta_{2k})^2\). From the CS literature upper and lower bounds are available (e.g. for CoSAMP see \[7\]), i.e. \(c_1(\delta_{2k}) = 4\sqrt{1 + \delta_{2k}/1 - (1 + \sqrt{2})\delta_{2k}}\) as well as bounds on the RIP constants \(\delta_{2k}\) \[8\], but these bounds are rather loose so numerical simulations are still necessary.

**B. Coded Slotted Aloha**

The analysis of coded slotted ALOHA is typically based on the and-or tree evaluation \[9\]. It is assumed that the graph representation can be unfolded in a tree, see Fig. 5 on which two operations are performed in succession:

(i) decoding of user packets in slots, corresponding to (a generalized) “and” operation, cf. \[10\], \[4\],

(ii) removal of replicas, corresponding to “or” operation.

Both operations are probabilistically characterized, in terms of probability of not decoding a user in a slot, denoted as \(p_i\), and not removing a replica \(q_i\). The tree structure allows for their successive updates, as reflected in the subscripts of \(p_i\) and \(q_i\). We also note that the analysis is asymptotic in nature, as in the non-asymptotic case, the graph representation contains loops, and the corresponding tree representation is only an approximation.

Before giving providing the expressions for \(p_i\) and \(q_i\), we introduce the following terminology. Denote the number of edges incident to slot/user node as slot/user node degree. Further, by edge-oriented slot degree distribution \(\omega_j, j \geq 1\) and \(\sum_j \omega_j = 1\), denote the probability distribution that a randomly chosen edge in the graph is connected to a slot node of degree \(j\) \[9\]. Similarly, by edge-oriented user degree distribution \(\lambda_k, k \geq 1\) and \(\sum_k \lambda_k = 1\), denote the probability distribution that a randomly chosen edge in the graph is connected to a user node of degree \(j\) \[9\]. Note that \(\lambda_k\) are subject to design of the random access algorithm, and that they implicitly determine \(\omega_j\). It could be shown that the probability update in slot node is:

\[
p_i = \sum_j \omega_j \sum_{t=0}^{j-1} \pi_{t,j} \left( j - 1 \right) q_{i-1}(1 - q_{i-1})^{j-t-1}, i \geq 1,
\]

where \(j\) is the slot degree, \(t\) is the number of interfering users that decreases through iterations via use of IC, \(\pi_{t,j}\) is the probability of decoding a user packet in the slot of degree \(j\) when \(t\) interfering packets have been cancelled, and where the combinatorial term \(\binom{j-1}{t-1}\) stems from the assumption that all colliding user packets in the slot are statistically a-priori
the same, in terms of probability of being decoded. Here it is important to note that the direct influence of the physical layer, i.e., receiver operation, as described in [II] is embedded in $\pi_{t,j}$. The probability update in user node is:

$$q_i = \sum_k \lambda_k \beta_i^{k-1}, \quad i \geq 1,$$

with the initial value $q_0 = 1$. Finally, the output of the evaluation is the probability that a user packet is decoded:

$$P_D = 1 - \lim_{i \to \infty} q_i.$$

**IV. SIMULATIONS**

An LTE-A 4G frame consists of a number of subframes with 20MHz bandwidth; the first subframe contains the RACH with one "big" OFDM symbol of 839 dimensions located around the frequency center of the subframe. The FFT size is 24578=24k corresponding to the 20MHz bandwidth whereby the remainder bandwidth outside PRACH is used for scheduled transmission in LTE-A, so-called PUSCH. The prefix of the OFDM symbol accommodates delays up to 100µs (or 30km cell radius) which equals 3000 dimensions. In the standard the RACH is responsible for user aquisition by correlating the received signal with preambles from a given set. Here, to mimic a 5G situation, we equip the transmitter with the capability of sending information in "one shot", i.e., in addition to user aquisition, channel estimation is performed and the data is detected. For this a fraction of the PUSCH is reserved for data packets of users which are detected in the PRACH. Please note the rather challenging scenario of only 839 subcarrier in the measurement window versus almost 24k data payload subcarriers.

In our setting, a limited number of users is detected out of a maximum set (here 10 out of 100). We assume that the delay spread is below 300 dimensions of which only a set of 6 paths are actually relevant. The pilot signalling is similar to [5] but modified to fit the data/pilot separation.

**V. CONCLUSIONS**

In this paper, we provided ideas how to enable random access for many devices in a massive machine-type scenario. In the conceptual approach as well as the actual algorithms sparsity of user activity and channel impulse responses plays an a pivotal role. We showed that using such framework efficient "one shot" random access is possible where users can send a message without a priori synchronizing with the network. Key is a common overloaded control channel which is used to jointly detect sparse user activity and sparse channel profiles. Such common control channel stands in clear contrast to dedicated control signalling per resource block, and is thus more efficient particularly for small resource blocks. Since each user also has channel state information for all subcarriers, there are additional degrees of freedom to place the resource blocks. We analyzed the system theoretically and provided a link between achievable rates and standard compressing sensing estimates in terms of explicit expressions and scaling laws. Finally, we supported our findings with simulations in an LTE-A-like setting allowing "one shot" sparse random access of 100 users in 1ms with good performance.

**REFERENCES**

[1] G. Wunder et al., “5GNOW: Non-Orthogonal, Asynchronous Waveforms for Future Mobile Applications,” IEEE Communications Magazine, vol. 52, no. 2, pp. 97–105, 2014.

[2] G. Wunder, H. Boche, T. Strohmer, and P. Jung, “Sparse Signal Processing Concepts for Efficient 5G System Design,” IEEE ACCESS, December 2015, to appear. [Online]. Available: http://arxiv.org/abs/1411.0435

[3] E. Paolini, C. Stefanovic, G. Liwa, and P. Popovski, “Coded Random Access: How Coding Theory Helps to Build Random Access Protocols,” IEEE Commun. Mag., vol. 53, no. 6, pp. 144–150, Jun. 2015.

[4] Y. Ji, C. Stefanovic, C. Bockelmann, A. Dekorsy, and P. Popovski, “Characterization of Coded Random Access with Compressive Sensing based Multi-User Detection,” in Proc. of IEEE Globecom 2014, Austin, TX, USA, Dec. 2014. [Online]. Available: www.arxiv.com/1404.2119

[5] G. Wunder, P. Jung, and C. Wang, “Compressive Random Access for Post-LTE Systems,” in IEEE International Conference on Communications (ICC’14) – Workshop on Massive Uncoordinated Access Protocols, Sydney, Australia, May 2014.

[6] G. Wunder, P. Jung, and M. Ramadan, “Coded Random Access Using A Common Overloaded Control Channel,” in IEEE Global Communications Conference (Globecom’14) – Workshop on 5G & Beyond - Enabling Technologies and Application, San Diego, USA, December 2015.

[7] S. Foucart, “Sparse recovery algorithms: sufficient conditions in terms of restricted isometry constants,” Approximation Theory XIII: San Antonio 2010, pp. 1–14, 2012. [Online]. Available: http://link.springer.com/chapter/10.1007/978-1-4614-0772-0_1

[8] M. Rudelson, R. Vershynin, and R. V. On, “On sparse reconstruction from Fourier and Gaussian measurements,” Communications on Pure and Applied Mathematics, vol. 61, no. 8, pp. 1025–1045, Nov. 2007.

[9] M. G. Luby, M. Mitzenmacher, and A. Shokrollahi, “Analysis of Random Processes via And-Or Tree Evaluation,” in Proc. of 9th ACM-SIAM SODA, San Francisco, CA, USA, Jan. 1998.

[10] C. Stefanovic, M. Momola, and P. Popovski, “Exploiting Capture Effect in Frameless ALOHA for Massive Wireless Random Access,” in Proc. of IEEE WCNC 2014, Istanbul, Turkey, May 2014.