Dark energy rest frame and the CMB dipole

Antonio L. Maroto

Dept. Física Teórica, Universidad Complutense de Madrid, 28040 Madrid, Spain

Abstract. If dark energy can be described as a perfect fluid, then, apart from its equation of state relating energy density and pressure, we should also specify the corresponding rest frame. Since dark energy is typically decoupled from the rest of components of the universe, in principle such a frame could be different from that of matter and radiation. In this work we consider the potential observable effects of the motion of dark energy and the possibility to measure the dark energy velocity relative to matter. In particular we consider the modification of the usual interpretation of the CMB dipole and its implications for the determination of matter bulk flows on very large scales. We also comment on the possible origin of a dark energy flow and its evolution in different models.

INTRODUCTION

The content of the universe can be appropriately described as a four-fluid system: baryons, radiation, dark matter and dark energy. Such fluids can be considered in good approximation to be decoupled from each other after recombination time. Each fluid carries its own rest frame. Thus for instance, the CMB radiation, being highly homogeneous allows us to define its rest frame by means of the CMB dipole anisotropy. Indeed, in the usual interpretation [1], the CMB dipole is due to the Doppler effect caused by the motion of the observer with respect to the last scattering surface. Therefore the radiation rest frame would be that in which an observer measures a vanishing dipole. This fact in turn has been used to calculate the velocity of our Local Group with respect to the CMB radiation, just by subtracting the velocity of the Sun with respect to the Milky Way and the Milky Way velocity with respect to the Local Group. The result is \( v_{LG-CMB} = 627 \pm 22 \text{ km s}^{-1} \) in the direction \((l, b) = (276 \pm 3^\circ, 30 \pm 3^\circ)\) in galactic coordinates.

The rest frame of (dark) matter is not so easily defined since we know that matter distribution is only homogeneous on very large scales. The presence of density inhomogeneities makes the velocity of a given matter volume to deviate from the pure Hubble flow. Such deviations \( v \) are known as peculiar velocities and they are related to the density perturbations \( \delta(x) \) by this simple expression to first order in perturbation theory:

\[
\nabla \cdot v = -H_0 \Omega_M^{0.6} \delta(x) \tag{1}
\]

According to the Cosmological Principle, the universe is homogeneous on very large scales and the amplitude of density perturbations on scales of size \( R \) will decline as we take larger and larger values of \( R \). This in turn means that the peculiar velocity of a matter volume of radius \( R \) will also decline for larger \( R \) (see Fig. 1), the matter rest frame eventually converging to the radiation frame on very large scales. In other words,
FIGURE 1. Bulk velocities vs. size $R$ for different peculiar velocity surveys centered around the Local Group in the CMB frame. The dot labelled by COBE denotes the velocity of the Local Group measured by COBE. The solid line corresponds to the expected $rms$ velocity in the standard $\Lambda$CDM cosmology, together with the 90% deviation in dashed lines. Figure from S. Zaroubi.

matter and radiation would share a common rest frame according to the Cosmological Principle.

However as shown in Fig. 1, this theoretical framework is not conclusively confirmed by observations. Indeed, in recent years several peculiar velocity surveys have tried to determine the volume size at which the streaming motion of matter with respect to the CMB vanishes. In the figure the results of different observations are compared with the $rms$ expected bulk velocity $V_b$ for standard $\Lambda$CDM model in a sphere of radius $R$. The results seem to agree with the theoretical expectations only at scales $R < 60h^{-1}$ Mpc. At larger scales, $R > 100h^{-1}$ Mpc, different data sets lead to different bulk velocities both in amplitude and direction. Moreover, there are indeed measurements in which large matter volumes are moving at speeds $> 600$ km s$^{-1}$ with respect to the CMB frame, several standard deviations away from the theoretical predictions. These results have been argued to be affected by systematic errors in distance indicators, but if confirmed by future surveys, a revision of some of the underlying ideas in Standard Cosmology would be required in order to understand the origin of such large flows.

Concerning the dark energy rest frame, it is usually assumed that dark energy does not cluster on small scales and therefore can be considered as a highly homogeneous fluid which is almost decoupled from the rest of components, its only interaction being of gravitational nature. In such a case there is no reason to expect that its rest frame should necessarily agree with the matter/radiation frame and therefore an observational determination of the dark energy bulk velocity would be needed. In the following
COSMOLOGY WITH MOVING DARK ENERGY

Let us therefore consider a cosmological scenario with four perfect fluids: baryons, radiation, dark matter and dark energy, whose equations of state read \( p_\alpha = w_\alpha \rho_\alpha \) with \( \alpha = B, R, DM, DE \). For the sake of generality, we will allow the dark energy equation of state to have a smooth dependence on redshift \( w_{DE}(z) \). The energy-momentum tensor of each fluid will take the form:

\[
(T^\mu_\nu)_\alpha = (\rho_\alpha + p_\alpha) u^\mu_\alpha u_\nu - p_\alpha \delta^\mu_\nu
\]  

Since we are only interested in the effects of fluids motion on the CMB dipole, it is sufficient to take into account the evolution of the largest-scale velocity perturbations, i.e. we will just consider the zero-mode equations. The presence of inhomogeneities will contribute to higher multipoles. Therefore, for this particular problem we can write:

\[
\rho_\alpha = \rho_\alpha(\eta),
\]
\[
p_\alpha = p_\alpha(\eta),
\]
\[
u^\mu_\alpha = \frac{1}{a}(1, v^i_\alpha(\eta))
\]

We will assume that \( \vec{v}^2_\alpha \ll 1 \) and we will work at first order in perturbation theory. In the particular case we are considering, the form of the space-time metric will be given by the following vector-perturbed spatially-flat Friedmann-Robertson-Walker metric:

\[
ds^2 = a^2(\eta) \left( d\eta^2 + 2S_i(\eta) d\eta dx^i - \delta_{ij} dx^i dx^j \right)
\]

Accordingly, the total energy-momentum tensor reads:

\[
T^0_0 = \sum_\alpha \rho_\alpha
\]
\[
T^0_i = \sum_\alpha (\rho_\alpha + p_\alpha)(S_i - v_{i\alpha})
\]
\[
T^i_0 = \sum_\alpha (\rho_\alpha + p_\alpha)v^i_\alpha
\]
\[
T^i_j = -\sum_\alpha p_\alpha \delta^i_j
\]

Notice that we are considering only the epoch after matter-radiation decoupling, assuming that dark energy is also decoupled and for that reason we will ignore possible energy and momentum transfer effects.

We now calculate the linearized Einstein equations using (4) and (5). They yield just the condition:

\[
S^i = \frac{\sum_\alpha (\rho_\alpha + p_\alpha)v^i_\alpha}{\sum_\alpha (\rho_\alpha + p_\alpha)}
\]
In General Relativity the combination \((\rho + p)\) appearing in (6) plays the role of inertial mass density of the corresponding fluid, and accordingly \(\vec{S}\) can be understood as the cosmic center of mass velocity. Notice that a pure cosmological constant has no inertial mass density.

On the other hand, the energy conservation equations are trivially satisfied, whereas from momentum conservation we see that the velocity of each fluid relative to the center of mass frame scales as:

\[
|\vec{S} - \vec{v}_\alpha| \propto a^{3w_a - 1}
\]

(7)

Notice that for dark energy the scaling properties will depend on the particular model under consideration [4].

Once we know the form of the perturbed metric, we can calculate the effect of fluids motion on photons propagating from the last scattering surface using standard tools [5]. The energy of a photon coming from direction \(n^\mu = (1, n^i)\) with \(\vec{n} = 1\) as seen by an observer moving with velocity \(u^\mu = a^{-1}(1, v^i)\) is given by \(E = g_{\mu\nu}u^\mu P^\nu\), i.e. to first order in the perturbation:

\[
E \simeq \frac{\varepsilon}{a} \left( 1 + \frac{d\delta x^0}{d\eta} + \vec{n} \cdot (\vec{S} - \vec{v}) \right)
\]

(8)

where \(\varepsilon\) parametrizes the photon energy and the perturbed trajectory of the photon reads \(x^\mu(\eta) = x^\mu_0(\eta) + \delta x^\mu\), with \(x^\mu_0 = n^\mu \eta\).

In order to obtain \(d\delta x^0/d\eta\), we solve the geodesics equations to first order in the perturbations, and for the 0-component we get \(d^2 \delta x^0 / d\eta^2 = 0\). By defining \(\hat{E} = aE\), the temperature fluctuation reads:

\[
\left. \frac{\delta T}{T} \right|_{\text{dipole}} = \frac{\hat{E}_0 - \hat{E}_{\text{dec}}}{\hat{E}_{\text{dec}}} \simeq \left. \frac{d\delta x^0}{d\eta} \right|_{\text{dec}} + \vec{n} \cdot (\vec{S} - \vec{v}) \left|_{\text{dec}}^0 \right.
\]

(9)

where the indices 0, \(\text{dec}\) denote the present and decoupling times respectively.

Today the only relevant contributions to the center of mass motion are those of matter and dark energy, radiation being negligible, so that:

\[
\vec{S}_{0} - \vec{v}_0 \simeq \frac{\Omega_M(\vec{v}_M - \vec{v}_0) + (1 + w^0_{\text{DE}})\Omega_{\text{DE}}(\vec{v}^0_{\text{DE}} - \vec{v}_0)}{1 + w^0_{\text{DE}}\Omega_{\text{DE}}}
\]

(10)

where \(w^0_{\text{DE}}\) is the present equation of state of dark energy and we have assumed that today the relative velocity of baryons and dark matter is negligible. For that reason we have used \(M\) to denote them both simultaneously. Notice that for a pure cosmological constant \(w^0_{\text{DE}} = -1\) and there would be no contribution from dark energy in such a case.

At decoupling, the universe is matter dominated and we can neglect the contribution to \(\vec{S}\) from dark energy. Therefore:

\[
\vec{S}_{\text{dec}} - \vec{v}_{\text{dec}} \simeq \frac{\Omega_{DM}(\vec{v}_{\text{dec}} - \vec{v}_B)}{\Omega_M(\vec{v}_{\text{DM}} - \vec{v}_B)}
\]
where the emitter velocity is nothing but the baryonic velocity \( \vec{v}_{\text{dec}} = \vec{v}^0_B \). As we will see below, when dark energy is absent, it is in the form of a pure cosmological constant or it is at rest with respect to radiation, we will have \( \vec{v}^0_{\text{DM}} = \vec{v}^0_B \) and this term vanishes.

According to this result, the CMB dipole has two different types of contributions: one is the usual Doppler effect due to the change of velocity between emitter and observer; the second contribution comes from the fact that the photon is propagating in an anisotropic medium which is changing in time. This second contribution is precisely given by the change in the velocity of the cosmic center of mass between emission and reception. The possibility that the dipole is not entirely due to a Doppler effect has been considered previously in the literature in different contexts (see [6]).

When all the components share a common rest frame then the previous result reduces to the usual expression for the dipole: \( \delta T / T \big|_{\text{dipole}} \simeq \vec{n} \cdot (\vec{v}^0_R - \vec{v}_0) \). However in general it is possible that an observer at rest with radiation \( \vec{v}_0 = \vec{v}^0_R \neq \vec{v}^0_M \neq \vec{v}^0_{\text{DE}} \) can measure a nonvanishing dipole according to (9).

In the absence of dark energy or in the case in which it is in the form of a pure cosmological constant (\( w_{\text{DE}} = -1 \)), dark energy would not contribute to the center of mass motion. Moreover, today the radiation contribution is negligible and accordingly the center of mass rest frame would coincide with the matter rest frame. This implies that the relative motion of matter and radiation today could not explain the existence of bulk flows on the largest scales, since the frame in which the dipole vanishes would coincide with the matter rest frame. Conversely, the existence of non-vanishing bulk flows would require the presence of moving dark energy with \( w_{\text{DE}} \neq -1 \).

Indeed, if moving dark energy is responsible for the existence of cosmic bulk flows on very large scales, then the amplitude and direction of such flows would provide a way to measure the relative velocity of matter and dark energy. As commented above, the bulk flow \( \vec{V}_b \) can be understood as the average velocity of a given matter volume with respect to an observer who measures a vanishing CMB dipole, i.e. \( \vec{V}_b = \vec{v}^0_M - \vec{v}_0 \). Such an observer has a velocity which can be obtained from (9), and accordingly:

\[
\vec{V}_b \simeq \frac{(1 + w^0_{\text{DE}}) \Omega_{\text{DE}}}{1 + w^0_{\text{DE}} \Omega_{\text{DE}}}(\vec{v}^0_M - \vec{v}^0_{\text{DE}}) + \frac{\Omega_{\text{DM}}}{\Omega_M}(\vec{v}^0_{\text{DM}} - \vec{v}^0_B)
\]

Notice that, according to these results, even if matter is at rest with respect to the CMB radiation, \( \vec{v}^0_M = \vec{v}^0_R \), it would be possible to have a non-vanishing flow \( \vec{V}_b \neq 0 \), provided dark energy is moving with respect to matter.

**A PRIMORDIAL DARK ENERGY FLOW?**

So far we have considered only the effects of the different components of the universe having different rest frames. However we have not discussed what is the origin of such a velocity offset. In standard cosmology, baryonic matter and radiation were coupled before recombination, and this was also so for dark matter, in the case in which it is in the form of weakly interacting massive particles. However the nature of dark energy is still a mystery and we ignore what is the type of interaction, if any, between dark energy and radiation/matter. For that reason, the primordial value of the dark energy rest
FIGURE 2. Qualitative evolution of fluids velocities in the cosmic center of mass frame for different dark energy models. The black full line corresponds to scaling dark energy. The various jumps arise because of momentum conservation in the corresponding transitions between different eras. The dotted black line is the $w_{DE} \approx -1$ model.

frame velocity with respect to radiation/matter should be considered as a completely free cosmological parameter, which could be determined only by observations.

Although the primordial value is unknown, its subsequent evolution is given by (7). In Fig. 2 we show the evolution of the velocities of the different fluids with respect to the center of mass frame. We compare two models for dark energy, one in which the equation of state is a constant close to $-1$ and a scaling [7] dark energy model in which the equation of state mimics that of the dominant component. We see that the velocity of radiation is almost constant throughout the cosmological evolution, whereas although matter is initially dragged by radiation, its velocity starts decaying as $a^{-1}$ after decoupling. Dark energy velocity is damped very fast for constant equation of state, whereas it could have appreciable amplitude today in scaling models.

Apart from the effects on the dipole, the metric anisotropies generated by the motion of dark energy can give rise in some cases to non-negligible quadrupole contributions and generate a net polarization of the CMB radiation. Such effects offer additional possibilities for the determination of the dark energy rest frame. These results will be presented elsewhere [8].

Acknowledgments: This work has been partially supported by DGICYT (Spain) under project numbers FPA 2004-02602 and FPA 2005-02327.
REFERENCES

1. P.J.E. Peebles and D.T. Wilkinson, *Phys. Rev.* **174**, 2168 (1968); P.J.E. Peebles, *Principles of Physical Cosmology*, Princeton University Press (1993); A. Kogut et al., *Astrophys. J.* **419**, 1 (1993)
2. A.G. Riess et al., *Astrophys. J.* **488**, L1 (1997); D.A. Dale et al., *Astrophys. J.* **510**, L11 (1999); M. Colles et al., *MNRAS*, **321**, 277 (2001); T.R. Lauer and M. Postman, *Astrophys. J.* **425**, 418 (1994); M.J. Hudson et al., *Astrophys. J.* **512**, L79 (1999); J.A. Willick, *Astrophys. J.* **522**, 647 (1999); M.J. Hudson et al., *Mon. Not. Roy. Astron. Soc.* **352**, 61 (2004); M.J. Hudson, *astro-ph/0311072*
3. S. Zaroubi, *astro-ph/0206052* Contribution to XIII Rencontres de Blois "Frontiers of the Universe", 17-23 June 2001
4. A.L. Maroto, *JCAP* **0605**:015, (2006); A.L. Maroto, *astro-ph/0605381*
5. M. Giovannini, *Int. J. Mod. Phys.* **D14**, 363, (2005)
6. R.A. Matzner, *Astrophys. J.* **241**, 851 (1980); B. Paczynski and T. Piran, *Astrophys. J.* **364**, 341 (1990); M.S. Turner, *Phys. Rev.* **D44**, 3737 (1991); J.F. Pascual-Sanchez, *Class. Quant. Grav.* **17**:4913-4918, (2000)
7. S. Capozziello, A. Melchiorri and A. Schirone, *Phys. Rev.* **D70**, 101301 (2004)
8. J. Beltrán and A.L. Maroto, in preparation.