Triplicated Trinification

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Abstract

Gauge-coupling unification is just as successful in the standard model with six Higgs doublets as it is in the minimal supersymmetric standard model. However, the gauge couplings unify at $10^{14}$ GeV, which yields rapid proton decay in the $SU(5)$ model. I propose that the grand-unified gauge group is instead $SU(3)_c \times SU(3)_L \times SU(3)_R$, in which baryon number is conserved by the gauge interactions.
Nature appears to come in triplicate. The elementary fermions of nature come in three identical generations of quarks and leptons, distinguished only by their couplings to the Higgs field. We have no understanding of why nature chooses to triplicate itself.

In contrast, there is only a single Higgs field in the standard model, which is responsible for breaking the electroweak symmetry and generating the masses of all the particles. This is the simplest model of electroweak symmetry breaking, and it is consistent with all data. However, it seems odd that there should be only one Higgs field, when the fermion fields come in triplicate.

The gauge sector of the standard model, based on the symmetry $SU(3)_c \times SU(2)_L \times U(1)_Y$, does not come in triplicate. It was observed long ago that this gauge symmetry can be unified into an $SU(5)$ gauge group with a single gauge coupling [1]. When the $SU(5)$ gauge symmetry is broken at a high energy scale, the $SU(3)_c \times SU(2)_L \times U(1)_Y$ gauge couplings evolve to their low-energy values [2]. However, precision measurements reveal that the low-energy values of the gauge couplings are not consistent with $SU(5)$ grand unification.

As is well known, the minimal supersymmetric standard model nudges the relative evolution of the gauge couplings just enough to bring them into accord with $SU(5)$ grand unification [3, 4, 5]. The reason for this is three-fold. First, the relative evolution of the gauge couplings is unaffected when one adds a complete $SU(5)$ representation [2]. Since the fermions are in complete $SU(5)$ representations, the addition of their superpartners does not affect the relative evolution of the gauge couplings [6]. Second, the superpartners of the gauge bosons (which are not in complete $SU(5)$ representations) change only the unification scale, since they have the same gauge structure as the gauge bosons [6]. Third, the minimal supersymmetric standard model requires two Higgs doublets in order to generate masses for all the fermions. Since the Higgs field is not in a complete $SU(5)$ representation, the addition of a second Higgs doublet, as well as the superpartners of these two Higgs doublets, modifies the relative evolution of the gauge couplings. Thus it is the extension of the Higgs sector that is behind the successful $SU(5)$ unification of the gauge couplings in the minimal supersymmetric standard model [7, 8].

In the renormalization-group equations responsible for the evolution of the gauge couplings, a (chiral) fermion field counts twice as much as a (complex) scalar field with the same gauge quantum numbers, at least at leading order. Thus the successful $SU(5)$ unification of the gauge couplings in the minimal supersymmetric standard model, with its two Higgs doublets and their fermionic superpartners, can be mimicked by the standard model with six Higgs doublets. Since six is a multiple of three, this implies a triplication of the Higgs sector, in keeping with the triplication of the fermion fields.

Thus, with respect to the unification of gauge couplings, the six-Higgs-doublet standard model is on the same footing as the minimal supersymmetric standard model. However, the unification of the gauge couplings occurs at a lower scale in the six-Higgs-doublet model. The evolution of the gauge couplings is given at leading order by

\[ \frac{1}{\alpha_i(\mu)} - \frac{1}{\alpha_i(\mu')} = \frac{b_n}{2\pi} \ln \left( \frac{\mu'}{\mu} \right), \]  

where $b_n$ are the one-loop beta-function coefficients,

\[ b_n = -\frac{11}{3} C_2(G) + \frac{2}{3} \sum_{\text{fermions}} T(R) + \frac{1}{3} \sum_{\text{scalars}} T(R), \]  

\[ \text{Eq. } (1) \]  

\[ \text{Eq. } (2) \]
where \( C_2(SU(N)) = N \), \( T(R) = 1/2 \) for fermions and scalars in the fundamental or antifundamental representation of \( SU(N) \), and \( C_2(U(1)) = 0 \), \( T(R) = \frac{3}{5}Y^2 \) for fermions and scalars of hypercharge \( Y \) (in the convention \( Q = T_3L + Y \)). Equation (2) shows that (chiral) fermions count twice as much as (complex) scalars in the evolution of the couplings, as noted above. The unification scale, \( M_U \), may be obtained from the condition \( \alpha_3(M_U) = \alpha_2(M_U) \), which gives

\[
\frac{1}{\alpha_3(M_Z)} - \frac{1}{\alpha_2(M_Z)} = \frac{b_3 - b_2}{2\pi} \ln \left( \frac{M_U}{M_Z} \right).
\]

(3)

For \( N_H \) Higgs doublets, \( b_3 - b_2 = -11/3 - N_H/6 \). Using as inputs \( \alpha_3(M_Z) = 0.117 \) and \( \alpha_2(M_Z) = (\sqrt{7}/\pi)G_F M_W^2 = 0.034 \), Eq. (3) yields \( M_U \approx 10^{14} \) GeV for \( N_H = 6 \). This is much less than the unification scale of supersymmetric \( SU(5) \), \( M_U \approx 2 \times 10^{16} \) GeV [3, 4, 5]. The unified gauge coupling, obtained from Eq. (1), is \( \alpha_U(M_U) = \alpha_3(M_U) = 0.025 \), which is close to the value in the original non-supersymmetric \( SU(5) \) model [2].

I show in Fig. 1 the evolution of the gauge couplings from their low-energy values up to the unification scale, using Eq. (1). The input value of the hypercharge coupling is

\[
\alpha_1(M_Z) = (5/3)\alpha_2(M_Z) \tan^2\theta_W = 0.017,
\]

(4)

where the factor 5/3 is determined by the embedding of \( U(1)_Y \) in \( SU(5) \). The couplings meet around \( 10^{14} \) GeV, within the accuracy of a leading-order calculation.

A unification scale as low as \( 10^{14} \) GeV is disastrous — it yields rapid proton decay via the exchange of \( SU(5) \) gauge bosons. Therefore, the six-Higgs-doublet model cannot be embedded in a conventional \( SU(5) \) theory. One must modify the theory in some way to
adequately suppress proton decay.\footnote{The minimal supersymmetric SU(5) model is also tightly constrained by proton decay, due to one-loop processes involving superpartners \cite{10,11,12,13}.}

Rather than following this tack, I consider the possibility that the unified theory is something other than SU(5). In general, a unified theory based on a simple group leads to proton decay \cite{14}. However, a product group, supplemented with a discrete symmetry to enforce the equality of the gauge couplings, need not contain gauge bosons that mediate proton decay. The simplest theory of this type that has the same condition on the gauge couplings at the unification scale as SU(5) is SU(3)\(_c\) × SU(3)\(_L\) × SU(3)\(_R\), supplemented with a discrete cyclic symmetry Z\(_3\) that acts on the three groups. The condition on the gauge couplings at the unification scale is the same in these two theories because they are both subgroups of E\(_6\). The weak SU(2)\(_L\) group is a subgroup of SU(3)\(_L\), and U(1)\(_Y\) is a linear combination of U(1) subgroups contained in SU(3)\(_L\) and SU(3)\(_R\). This is the so-called “trinified” model \cite{15,16}. Remarkably, the triplication of the Higgs sector leads us to the triplication of the gauge sector.

The fermions of one generation are contained in the 27-dimensional representation of SU(3)\(_c\) × SU(3)\(_L\) × SU(3)\(_R\),

\[
27 = (3, \bar{3}, 1) + (\bar{3}, 1, 3) + (1, 3, \bar{3}).
\]

Quarks are contained in the (3, \bar{3}, 1) representation, antiquarks in the (\bar{3}, 1, 3) representation, and leptons and antileptons in the (1, 3, \bar{3}) representation. Thus baryon number is automatically conserved by the gauge interactions. The 27-dimensional representation decomposes under SU(3)\(_c\) × SU(2)\(_L\) × U(1)\(_Y\) as

\[
27 = 2(1, 1, 0) + (1, 2, \frac{1}{2}) + (3, 1, -\frac{2}{3}) + 2[(1, 2, -\frac{1}{2}) + (3, 1, \frac{1}{3})] + (1, 1, 1) + (\bar{3}, 1, -\frac{2}{3}) + (3, 2, \frac{1}{6}).
\]

This contains the familiar quarks and leptons, as well as additional fermions in the representations 2(1, 1, 0), (1, 2, \frac{1}{2}), (1, 2, -\frac{1}{2}), (3, 1, -\frac{1}{3}), (3, 1, \frac{1}{3}). These additional fermions are an unattractive feature, but since they are in a vectorlike representation of SU(3)\(_c\) × SU(2)\(_L\) × U(1)\(_Y\) they naturally acquire masses of order \(M_U\) when the unification-scale symmetry breaking occurs \cite{17}. Therefore they do not influence the evolution of the gauge couplings, unless their masses are less than \(M_U\). The two gauge-singlet fermions (dubbed “neutrettos” \cite{15}, although “sterile neutrinos” is also appropriate) do not affect the evolution of the gauge couplings regardless of their masses. Indeed, in the simplest models (discussed below), the two neutrettos acquire mass at one loop and are therefore lighter than \(M_U\) \cite{15,16}. The other additional fermions do not affect the relative evolution of the gauge couplings if their masses are less than \(M_U\) and nearly degenerate, because they fill out a complete 5 + \bar{5} representation of SU(5).

The details of the model depend upon on how the SU(3)\(_c\) × SU(3)\(_L\) × SU(3)\(_R\) symmetry is broken, first to SU(3)\(_c\) × SU(2)\(_L\) × U(1)\(_Y\) and then to SU(3)\(_c\) × U(1)\(_EM\), and on how this symmetry breaking is communicated to the fermions. Let us adopt the standard four-dimensional weakly-coupled field-theory framework in which the symmetry breaking is accomplished via the vacuum-expectation values of Higgs fields. In this framework, the symmetry breaking is communicated to the fermions via Yukawa couplings.
The simplest model that yields the desired pattern of symmetry breaking consists of two Higgs fields in the 27-dimensional representation \[|5, 16\]. However, in keeping with the triplication paradigm, it is natural to expect three or six such Higgs fields. Each 27-dimensional representation contains one \(SU(3)_c \times SU(2)_L \times U(1)_Y\) representation with the quantum numbers of the standard-model Higgs field and two with the quantum numbers of the conjugate Higgs field [see Eq. (6)]. As far as the evolution equations are concerned, these representations contribute equally. Whether there are two, three, or six 27-dimensional-representation Higgs fields, one must arrange for six of the Higgs doublets to have masses of order the weak scale and all the remaining \(SU(3)_c \times SU(2)_L \times U(1)_Y\) Higgs representations to have masses of order the unification scale. How this occurs depends upon the details of the symmetry-breaking sector. This evokes the usual gauge hierarchy problem: Why are some Higgs fields much lighter than others? Furthermore, since the model is non-supersymmetric, there is no mechanism to cancel quadratic divergences in the radiative corrections to the masses of the six weak-scale Higgs doublets.

Fermions acquire mass by coupling to these 27-dimensional Higgs fields. In general, these Yukawa couplings mediate proton decay via Higgs-boson exchange. However, it is possible to impose baryon number as a global symmetry. There are two types of Yukawa couplings allowed \[|5, 16\],

\[
\psi(3, \bar{3}, 1)\phi(1, 3, \bar{3}) + \psi(3, 1, 3)\phi(1, 3, \bar{3}) + \psi(1, 3, \bar{3})\phi(3, \bar{3}, 1) + \psi(3, \bar{3}, 1)\phi(3, 1, 3) \quad (7)
\]

and

\[
\psi(1, 3, \bar{3})\phi'(1, 3, 1) + \psi(3, \bar{3}, 1)\phi'(3, \bar{3}, 1) + \psi(3, 1, 3)\phi'(3, 1, 3). \quad (8)
\]

We assign to the fermion fields the canonical baryon numbers of 1/3 to \(\psi(3, \bar{3}, 1), \bar{1}/3\) to \(\psi(3, 1, 3), \) and zero to \(\psi(1, 3, \bar{3})\). In the first Yukawa coupling, one may assign baryon number zero to \(\phi(1, 3, \bar{3}), \) 1/3 to \(\phi(3, \bar{3}, 1), \) and \(-1/3\) to \(\phi(3, 1, 3)\). In the second Yukawa coupling, the assignments are zero to \(\phi'(1, 3, \bar{3}), \) \(-2/3\) to \(\phi'(3, \bar{3}, 1), \) and \(2/3\) to \(\phi'(3, 1, 3)\). If all 27-dimensional Higgs fields participate in only one Yukawa coupling or the other, and if the Higgs potential is constrained to conserve baryon number, then proton decay is absolutely forbidden \[|5, 16\].

In the minimal model, with just two 27-dimensional Higgs fields, it is not possible to impose baryon number and to generate a realistic fermion mass spectrum and Cabibbo-Kobayashi-Maskawa (CKM) mixing matrix. To conserve baryon number, one of these Higgs fields must participate in the first Yukawa coupling, the other in the second Yukawa coupling. With just one Higgs field participating in the first Yukawa coupling, the up and down quark mass matrices are proportional, yielding a unit CKM matrix and \(m_u/m_d = m_c/m_s = m_t/m_b \) \[|5, 16\]. Thus baryon number is necessarily violated in the minimal model. However, with three or more 27-dimensional Higgs representations, the imposition of baryon number symmetry is possible \[|5, 16\].

Although it is possible for the Yukawa couplings to conserve baryon number, this is a stronger requirement than is necessary. Since the Yukawa couplings of the first two generations (relevant for proton decay) are much smaller than gauge couplings, one can tolerate some Higgs-mediated baryon-number violation while respecting the lower bound on the proton lifetime.
The challenge is to find a model that fits nature, and to extract its predictions for physics beyond the standard model. In building such a model, there are pitfalls beyond proton decay to avoid. Models with multiple Higgs doublets generically have tree-level Higgs-mediated flavor-changing neutral currents [18], which are severely constrained experimentally. However, even if tree-level flavor-changing neutral currents are present, they may be sufficiently suppressed by small Yukawa couplings to avoid conflicting with experiment [19, 20], analogous to the suppression of Higgs-mediated proton decay mentioned above.

With the triplication of both the fermion and the Higgs sectors, it is tempting to look for family symmetries relating the fermion fields as well as the Higgs fields. A model with six Higgs doublets has recently been explored in Ref. [21, 22]. The approximate unification of gauge couplings was also noted in that work. The difficulties with Higgs-mediated flavor-changing neutral currents discussed above are exemplified in that study.

The unification scale of $10^{14}$ GeV is several orders of magnitude below the reduced Planck scale, $(8\pi G_N)^{-1/2} \approx 2 \times 10^{18}$ GeV. In weakly-coupled string theory, the string scale cannot be much less than the reduced Planck scale. Thus in this scenario, the model is an $SU(3)_c \times SU(3)_L \times SU(3)_R$ field theory between the grand-unified and string scales. This gauge group is a maximal subgroup of $E_6$, so it naturally arises in string theory [23, 24]. Furthermore, the fermion and Higgs representations arise at Kac-Moody level one, which corresponds to the simplest string models. The $SU(3)_c \times SU(3)_L \times SU(3)_R$ model may be embedded into a larger gauge group, such as $E_6$ [25], but this must be broken well above $10^{14}$ GeV in order to avoid rapid proton decay via gauge-boson exchange. A Higgs field in the 650-dimensional representation of $E_6$ could provide the desired breaking to $SU(3)_c \times SU(3)_L \times SU(3)_R$.

In strongly-coupled string theory, the string scale can be much less than the Planck scale [26]. Thus it is possible that the string scale and the unification scale are both $10^{14}$ GeV. This opens up additional possibilities for model building. If all three $SU(3)$ gauge symmetries are realized at Kac-Moody level one (or, more generally, at the same Kac-Moody level), then conformal symmetry equates their couplings [27], and obviates the discrete cyclic $Z_3$ symmetry. This is superior to string unification of the standard-model gauge group, where the normalization of the $U(1)_Y$ coupling may take any rational value, not necessarily the value $5/3$ needed for gauge-coupling unification [see Eq. (4)]. There are also additional possibilities for grand-unified symmetry breaking based on compactification of extra dimensions [28]. Similar mechanisms also operate in extra-dimensional field theories [29, 30].

A firm prediction of triplicated trinification, independent of the details of the model, is a weak-scale spectrum consisting of the standard model with six Higgs doublets. The model cannot be supersymmetric, because the fermionic superpartners of the six Higgs doublets would upset the unification of the couplings. Experiments at the Fermilab Tevatron and the CERN Large Hadron Collider should see some or all of the particles contained in these six Higgs doublets (eleven neutral scalars, five pairs of singly-charged scalars).

Should this model prove to be correct, it would mean that the supersymmetric SU(5) model is a red herring. Both the six-Higgs-doublet standard model and the supersymmetric standard model yield successful gauge-coupling unification due to the extension of the Higgs sector. The fermion content of the standard model fits perfectly into the $5+10$ of $SU(5)$, while the trinification model requires additional fermions to fill out the fermion representation. However, these additional fermions are in a vectorlike representation of the standard-model gauge group, so it is natural for them to have masses of order the grand-unified scale. Thus
the triplicated trinification model is a natural extension of the standard model.

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