Abstract—The sum-capacity for specific sub-classes of ergodic fading Gaussian two-user interference channels (IFCs) is developed under the assumption of perfect channel state information at all transmitters and receivers. For the sub-classes of uniformly strong (every fading state is strong) and ergodic very strong two-sided IFCs (a mix of strong and weak fading states satisfying specific fading averaged conditions) the optimality of completely decoding the interference, i.e., converting the IFC to a compound multiple access channel (C-MAC), is proved. It is also shown that this capacity-achieving scheme requires encoding and decoding jointly across all fading states. As an achievable scheme and also as a topic of independent interest, the capacity region and the corresponding optimal power policies for an ergodic fading C-MAC are developed. For the sub-class of uniformly weak IFCs (every fading state is weak), genie-aided outer bounds are developed. The bounds are shown to be achieved by treating interference as noise and by separable coding for one-sided fading IFCs. Finally, for the sub-class of one-sided hybrid IFCs (a mix of weak and strong states that do not satisfy ergodic very strong conditions), an achievable scheme involving rate splitting and joint coding across all fading states is developed and is shown to perform at least as well as a separable coding scheme.

Index Terms—Compound multiple access channel, ergodic capacity, ergodic fading, interference channel, polymatroids, separability, strong and weak interference.

I. INTRODUCTION

The interference channel (IFC) models a wireless network in which every transmitter (user) communicates with its unique intended receiver while causing interference to the remaining receivers. Gaussian interference channels model wireless networks consisting of two or more interfering transmit-receive pairs (links). The capacity region of Gaussian IFCs remains an open problem. In this paper, we focus on two-user fading Gaussian IFCs, and henceforth, use IFCs and Gaussian IFCs interchangeably. For two-user Gaussian nonfading IFCs, referred to in the literature as simply Gaussian IFCs, capacity results are known only for specific sub-classes identified uniquely by the relative strength of the cross-and direct links from each transmitter to the unintended and intended receiver, respectively, and/or the transmit powers. Specifically, the capacity region is known for strong Gaussian IFCs for which the strength of both cross-links are larger than that of the corresponding direct links and is achieved when both receivers decode both the intended and interfering messages [1]–[3]. A very strong IFC results when the sum-capacity of a strong Gaussian IFC is the sum of the interference-free capacities of the two links [2]. In contrast, weak IFCs are those for which the strengths of both cross-links are smaller than that of the corresponding direct links. The capacity region of weak IFCs remains open in general; however, for the class of one-sided weak IFCs in which the strength of one of the cross-links is zero, the weak sum-capacity is achieved by ignoring interference, i.e., by considering the interference as noise while decoding the desired signal at the interfered with receiver [4]. More recently, the sum-capacity of a class of noisy or very weak Gaussian IFCs has been determined independently in [5], [6], and [7] is shown to be achieved when both receivers ignore their interference. Outer bounds for IFCs are developed in [5]–[8] and [9] while several achievable rate regions for Gaussian IFCs are studied in [10].

The best known inner bound is due to Han and Kobayashi (HK) [3]. Recently, in [9], an HK-based scheme is shown to achieve every rate pair within 1 bit/s/Hz of the capacity region. In [11], the authors reformulate the HK region as a union of two sets to characterize the maximum sum-rate achieved by Gaussian inputs and without time-sharing. More recently, the approximate capacity of two-user Gaussian IFCs is characterized using a deterministic channel model in [12]. The sum-capacity of the class of nonfading multiple-input multiple-output (MIMO) IFCs is studied in [13].

Relatively fewer results are known for parallel or fading Gaussian IFCs. Parallel Gaussian IFCs (PGICs) where each parallel sub-channel is strong is considered in [14] for which the authors develop an achievable scheme using independent encoding and decoding in each parallel sub-channel. Sung et al. [15] present an achievable scheme for a class of one-sided PGICs that involves viewing each parallel sub-channel as

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an independent strong or weak one-sided IFC and coding appropriately for that sub-channel. Thus, for strong and weak parallel sub-channels, the interference is completely decoded and ignored, respectively. In this paper, we show that independent coding across fading states (viewed as sub-channels) is in general not sum-capacity optimal.

Recently, for PGICs, [16] determines the conditions on the channel coefficients and power constraints for which independent transmission across the parallel channels (often referred to as sub-channels) and treating interference as noise is optimal. In [17], techniques used for MIMO IFCs [13] are applied to study the optimality of coding independently in each sub-channel (often referred to as separability) of PGICs. It is worth noting that PGICs are a special case of ergodic fading IFCs in which each sub-channel is assigned the same weight, i.e., every sub-channel occurs with the same probability; furthermore, they can also be viewed as a special case of MIMO IFCs and thus results from MIMO IFCs can be directly applied to PGICs. For fading interference networks with three or more users, in [18], the authors develop an interference alignment coding scheme to show that the sum-capacity of a $K$-user IFC scales linearly with $K$ in the high signal-to-noise ratio (SNR) regime when all links in the network have similar channel statistics.

In this paper, we study ergodic fading two-user Gaussian IFCs and determine the sum-capacity and the corresponding optimal power policies for specific sub-classes which are defined in terms of the fading statistics. Noting that ergodic fading IFCs form a weighted collection of parallel IFCs (sub-channels), we identify four sub-classes that jointly contain the set of all ergodic fading IFCs. We develop the sum-capacity for two of them. For the third sub-class, we develop the sum-capacity when only one of the two receivers is affected by interference, i.e., for a one-sided ergodic fading IFC. For the fourth sub-class we present a general achievable scheme based on joint coding across fading states.

A natural question that arises in studying ergodic fading and parallel channels is the optimality of separable coding, i.e., whether encoding and decoding independently on each sub-channel (or fading state) is optimal in achieving one or more points on the boundary of the capacity region. For each sub-class of IFCs we consider, we address the optimality of separable coding, often referred to as separability, and demonstrate that in contrast to point-to-point [19], multiple-access [20], [21], and broadcast channels without common messages [22], separable coding is not necessarily sum-capacity optimal for ergodic fading IFCs.

The first of the four sub-classes is the set of ergodic very strong (EVS) IFCs in which each fading state (sub-channel) can be either weak or strong but averaged over all fading states the interference at each receiver is sufficiently strong that the two direct links from each transmitter to its intended receiver are the bottle-necks limiting the sum-rate. For this sub-class, we show that requiring both receivers to decode the signals from both transmitters is optimal, i.e., the ergodic very strong IFC modifies to a two-user ergodic fading compound multiple-access channel (C-MAC) in which the transmitted signal from each user is intended for both receivers [23]. To this end, as an achievable rate region for IFCs and as a problem of independent interest, we develop the capacity region and the optimal power policies that achieve them for ergodic fading C-MACs (see also [23]).

For EVS IFCs we also show that achieving the sum-capacity (and the capacity region) requires transmitting information (encoding and decoding) jointly across all fading states, i.e., separable coding in each fading state is strictly suboptimal. Intuitively, the reason for joint coding across fading states lies in the fact that, analogously to parallel broadcast channels with common messages [24], both transmitters in the EVS IFCs transmit only common messages intended for both receivers for which independent coding across the fading states or sub-channels becomes strictly sub-optimal. To the best of our knowledge this is the first capacity result for fading two-user IFCs with a mix of weak and strong states. For such mixed ergodic IFCs, recently, a strategy of ergodic interference alignment is proposed in [25], and is shown to achieve the sum-capacity in [26] for a class of $K$-user fading IFCs with uniformly distributed phase and at least $K/2$ disjoint equal strength interference links.

The second sub-class is the set of uniformly strong (US) IFCs in which in every fading state the resulting IFC is strong, i.e., the cross-links have larger fading gains than the direct links for each fading realization. For this sub-class, we show that the capacity region is the same as that of an ergodic fading C-MAC with the same fading statistics and that achieving this region requires joint coding across all fading states.

The third sub-class is the set of uniformly weak ( UW) IFCs for which in every fading state the resulting IFC is weak. As a first step, we study the one-sided uniformly weak IFC and develop genie-aided outer bounds. We show that the bounds are tight when the interfering receiver ignores the weak interference in every fading state. Furthermore, we show that separable coding is optimal for this sub-class. The sum-capacity results for the one-sided channel are used to develop outer bounds for the two-sided case; however, sum-capacity results for the two-sided case will require techniques such as those developed in [16] which determine the channel statistics and power policies for which ignoring interference and separable coding is optimal.

The final sub-class is the set of hybrid IFCs with at least one weak and one strong fading state but which do not satisfy the conditions for EVS IFCs (and by definition are also not US and UW IFCs). The capacity-achieving strategy for EVS and US IFCs suggest that a joint coding strategy across the fading states can potentially take advantage of the strong states to partially eliminate interference. To this end, for ergodic fading one-sided IFCs, we propose a HK-based general joint coding strategy that uses rate-splitting and Gaussian codebooks without time-sharing for all sub-classes of IFCs.

In comparison, a one-sided nonfading Gaussian IFC is either weak or strong and the sum-capacity is known in both cases. In fact, for the weak case the sum-capacity is achieved by ignoring the interference and for the strong case it is achieved by decoding the interference at the receiver subject to the interference. However, for ergodic fading one-sided IFCs, in addition to the UW and US sub-classes, we also have to contend with the hybrid and EVS sub-classes each of which has a unique mix of weak and strong fading states. The HK-based achievable strategy we propose applies to all sub-classes of one-sided IFCs and includes the capacity-achieving strategies for the EVS, US, and UW IFCs as special cases.
A sub-class of hybrid IFCs is the uniformly mixed (UM) two-sided IFCs in which a pair of fading states corresponding to the cross-link and direct link from one of the sources forms a strong one-sided IFC while the complementary pair from the other source is a weak one-sided IFC. For UM IFCs, we show that to achieve sum-capacity the transmitter that interferes strongly transmits a common message across all fading states and the transmitter interfering weakly transmits a private message across all fading states. The two different interfering links, however, require joint encoding and decoding across all fading states to ensure optimal coding at the receiver with strong interference.

Finally, a note on separability. In [27], Cadambe and Jafar demonstrate the inseparability of parallel interference channels using an example of a three-user frequency selective fading IFC. The authors use interference alignment schemes to show that separability is not optimal for fading IFCs with three or more users while leaving open the question for the two-user fading IFC. We addressed this question in [28] for the ergodic fading one-sided IFC and developed the conditions for the optimality of separability for EVS and US one-sided IFCs. In this paper, we readdress this question for all sub-classes of fading two-user IFCs. Our results suggest that in general both one-sided and two-sided IFCs benefit from transmitting the same information across all fading states (sub-channels), i.e., not independently encoding and decoding in each fading state, thereby exploiting the fading diversity to mitigate interference. While the definitions and proofs of the four sub-classes are formally developed in the sequel, we refer the reader to Fig. 1 for a pictorial summation of the different sub-classes, capacity results for specific sub-classes, and the optimality of joint versus separable coding in achieving these capacity results.

The paper is organized as follows. In Section II, we present the channel models studied. In Section III, we summarize our main results. The capacity region of an ergodic fading C-MAC is developed in Section IV. The proofs are collected in Section V. We discuss our results with numerical examples in Section VI and conclude in Section VII.

II. CHANNEL MODEL AND PRELIMINARIES

A. Channel Model

A two-sender two-receiver (also referred to as a two-user) ergodic fading Gaussian IFC consists of two source nodes $S_1$ and $S_2$, and two destination nodes $D_1$ and $D_2$ as shown in Fig. 2. Source $S_k$, $k = 1, 2$, uses the channel $n$ times to transmit its message $W_k$, which is distributed uniformly in the set $\{1, 2, \ldots, 2^{nR_k}\}$ and is independent of the message from the other source, to its intended receiver, $D_k$, at a rate $R_k$ bits per channel use. In each use of the channel, $S_k$ transmits the signal $X_k \in \mathbb{C}$ (is the complex domain) while the destination $D_k$ receives $Y_k \in \mathbb{C}$, $k = 1, 2$, such that for an input vector $X = [X_1 \ X_2]^T$, the channel output vector $Y = [Y_1 \ Y_2]^T$ is given by

$$Y = HX + Z$$

(1)

where $Z = [Z_1 \ Z_2]^T$ is a noise vector with entries that are zero-mean, unit variance, circularly symmetric complex Gaussian noise variables and $H$ is a random matrix of fading gains with entries $H_{m,k} \in \mathbb{C}$, for all $m, k = 1, 2$, such that $H_{m,k}$ denotes the fading gain between receiver $m$ and transmitter $k$. We use $h$ to denote a realization of $H$. We assume the fading process $\{H\}$ is stationary and ergodic but not necessarily Gaussian. Note that the channel gains $H_{m,k}$, for all $m$ and $k$, are not assumed to be independent; however, $H$ is known instantaneously at all the transmitters and receivers, i.e., just prior to the transmission in each use of the channel.

Over $n$ uses of the channel, the transmit sequences $\{X_{k,i}\}$ are constrained in power according to

$$\sum_{i=1}^{n} |X_{k,i}|^2 \leq nP_{k,\text{max}}, \text{ for } k = 1, 2.$$  

(2)

Since the transmitters know the fading states of the links on which they transmit, they can allocate their transmitted signal power according to the channel state information. The power policy $P_k(h)$ of user $k$ is a function of the fading gains $H(i) = h$ in channel use $i$ and is given by

$$E[|X_{k,i}|^2|H(i) = h] = P_k(h), \quad k = 1, 2.$$  

(3)

Denoting $P(h)$ as a length-two vector of policies for both users, (3) implies that $P(h)$ is a mapping from the fading state space consisting of the set of all fading states (instantiations) $h$ to the set of non-negative real values in $\mathbb{R}_{+}^2$. While $P(h)$ denotes the map for a particular fading state, we write $P(H)$ to explicitly describe the policy for the entire set of random fading states.

Fig. 1. Venn diagram representation of the four sub-classes of ergodic fading one-and two-sided IFCs.
Thus, we use the notation $P(H)$ when averaging over all fading states or describing a collection of policies, one for every $h$.

Remark 1: In general, the transmit sequences $\{X_{k,d}\}$ can be arbitrarily correlated subject to an average power constraint in (2). Thus, for any power policy $P_k(h)$, for all $k$, (3) defines the constraint on the diagonal elements of the covariance matrix of $\{X_{k,d}\}$.

For an ergodic fading channel, (2) then simplifies to

$$E[P_k(H)] \leq P_k^{\text{avg}}, \quad \text{for all } k = 1, 2,$$

(4)

where the expectation in (4) is taken over the distribution of $H$. We denote the set of all feasible policies $P(H)$, i.e., the power policies whose entries satisfy (4), by $P$. Finally, we write $P^{\text{avg}}_k$ to denote the vector of average power constraints with entries $P_k^{\text{avg}}, k = 1, 2$.

For the special case in which both receivers decode the messages from both transmitters, we obtain a C-MAC [see Fig. 2(a)]. A one-sided fading Gaussian IFC results when either $H_{1,2} = 0$ or $H_{2,1} = 0$ with probability 1 [see Fig. 2(b)]. Without loss of generality, we develop sum-capacity results for a one-sided IFC (Z-IFC) with $H_{2,1} = 0$. The results extend naturally to the complementary one-sided model with $H_{1,2} = 0$.

B. Notation

Before proceeding, we summarize the notation used in the sequel.

- Random variables (e.g., $H_{k,d}$) are denoted with uppercase letters and their realizations (e.g., $h_{k,d}$) with the corresponding lowercase letters.
- Bold font $\mathbf{X}$ denotes a random matrix while bold font $\mathbf{x}$ denotes a realization of $\mathbf{X}$.
- $I$ denotes the identity matrix.
- $|\mathbf{X}|$ and $\mathbf{X}^{-1}$ denote the determinant and inverse, respectively, of the matrix $\mathbf{X}$.
- $\mathbb{C}N(0, \Sigma)$ denotes a circularly symmetric complex Gaussian distribution with zero mean and covariance $\Sigma$.
- $\mathcal{K} = \{1, 2\}$ denotes the set of transmitters.
- $E(\cdot)$ denotes expectation; $C(x)$ denotes $\log(1 + x)$ where the logarithm is to the base 2; $(x)^+$ denotes $\max(x, 0)$; $I(\cdot; \cdot)$ denotes mutual information; $h(\cdot)$ denotes differential entropy; and $R_S$ denotes $\sum_{k \in S} R_k$ for any $S \subseteq \mathcal{K}$.

- Throughout the sequel, we will use the phrases fading state and sub-channel interchangeably.

We write $\mathcal{C}_\text{IFC}(P^{\text{avg}}_1, P^{\text{avg}}_2)$ and $\mathcal{C}_\text{C-MAC}(P^{\text{avg}}_1, P^{\text{avg}}_2)$ to denote the capacity regions of an ergodic fading IFC and C-MAC, respectively; for ease of notation, we simply write $\mathcal{C}_\text{IFC}$ and $\mathcal{C}_\text{C-MAC}$ in the sequel. Our definitions of average error probabilities, capacity regions, and achievable rate pairs $(R_1, R_2)$ for both the IFC and C-MAC mirror the standard information-theoretic definitions [29, Chap. 14].

Throughout the sequel we use the term waterfilling solution to denote the capacity achieving power policy for a fading point-to-point channel [19]. For multiple access channels, in addition to each user waterfilling over its fading link to the common destination, only the user with the best (largest) channel gain transmits in each channel use, i.e., the channel is used opportunistically to maximize the sum capacity, and the resulting power policies are called opportunistic waterfilling solutions [20].

In general, however, for multi-antenna (e.g., [30]) and multi-terminal (e.g., [31]) channels including the one studied here, the optimal power policy for each user is in general dependent on some or all of the fading links in the network and not just on a single link. The resulting policies have been referred to in the literature as generalized waterfilling (e.g., [30], [32], [33]); we adopt this terminology in the sequel for power policies that depend on more than one fading link.

C. Nonfading Gaussian IFCs: Preliminaries

Nonfading Gaussian IFCs (for which $H = h$ is not random) can be classified by the relative strengths of the interference to intended signals at each of the receivers. A (two-sided nonfading) strong Gaussian IFC is one in which the cross-link channel gains are larger than the direct link channel gains to the intended receivers [1], i.e.,

$$|h_{j,k}| \geq |h_{k,j}|, \quad \text{for all } j, k = 1, 2, j \neq k.$$  (5)

A strong Gaussian IFC, i.e., a Gaussian IFC for which (5) holds, is very strong if the cross-link channel gains dominate the transmit powers such that (see, for example, [1] and [2])

$$\sum_{k=1}^{2} C\left(|H_{j,k}|^2 P_k^{\text{avg}}(h)\right) < C\left(\sum_{k=1}^{2} |H_{j,k}|^2 P_j^{\text{avg}}(h)\right),$$

for all $j = 1, 2$.  (6)
One can verify in a straightforward manner that (6) reduces to the general conditions for a very strong IFC (see, for example, [1]) given by

\[
\begin{align*}
|h_{1,2}|^2 > |h_{2,2}|^2 (1 + |h_{1,1}|^2 P_{1k}^{avg}), & \quad (7a) \\
|h_{2,1}|^2 > |h_{1,1}|^2 (1 + |h_{2,2}|^2 P_{2k}^{avg}), & \quad (7b)
\end{align*}
\]

the very strong condition sets an upper bound on the average transmit power \( P_{k}^{avg} \) at user \( k \) as

\[
P_{k}^{avg} < \frac{1}{|h_{k,k}|^2} \left( \frac{|h_{k,j}|^2}{|h_{j,k}|^2} - 1 \right), \quad j \neq k, j, k \in \{1, 2\}. \quad (8)
\]

Note that this also requires \( |h_{k,j}|^2 > |h_{j,k}|^2 \) for all \( j, k \neq j, k \).

A Gaussian IFC is weak when

\[
|h_{k,k}| > |h_{j,k}|, \quad \text{for all } j, k = 1, 2, j \neq k. \quad (9)
\]

A Gaussian IFC is mixed when

\[
|h_{1,2}| \geq |h_{2,2}| \quad \text{and} \quad |h_{1,1}| > |h_{2,1}| \quad (10)
\]

or

\[
|h_{2,1}| \geq |h_{1,1}| \quad \text{and} \quad |h_{2,2}| > |h_{1,2}|. \quad (11)
\]

### D. Ergodic Fading Gaussian IFCs: Definitions

An ergodic fading Gaussian IFC is a set of fading states (parallel sub-channels), and thus, each fading state \( h \) can be either very strong, strong, or weak. Since a fading IFC can contain a mixture of different types of sub-channels, we introduce the following definitions to classify the set of all ergodic fading two-user Gaussian IFCs (see also Fig. 1). Unless otherwise stated, we henceforth simply write IFC to denote a two-user ergodic fading Gaussian IFC.

**Definition 1:** A uniformly strong IFC (US IFC) is one in which every fading state is strong, i.e., every realization \( h \) satisfies

\[
|h_{2,1}| \geq |h_{1,1}| \quad \text{and} \quad |h_{1,2}| \geq |h_{2,2}| \quad (12)
\]

**Definition 2:** An ergodic very strong IFC (EVS IFC) is one in which every fading state is either weak or strong such that

\[
\sum_{k=1}^{2} \mathbb{E} \left[ C \left( |H_{k,k}|^2 P_{k}^{avg}(H_{k,k}) \right) \right] < \mathbb{E} \left[ C \left( \sum_{k=1}^{2} |H_{j,k}|^2 P_{k}^{avg}(H_{k,k}) \right) \right],
\]

for all \( j = 1, 2 \),

\[
(13)
\]

where \( P_{k}^{avg}(H_{k,k}) \) is the optimal waterfilling policy that achieves the point-to-point ergodic fading capacity for user \( k \) in the absence of interference.

**Definition 3:** A uniformly weak IFC (UW IFC) is one in which every fading state is weak, such that the entries of every fading realization \( h \) satisfy

\[
|h_{1,1}| > |h_{2,1}| \quad \text{and} \quad |h_{2,2}| > |h_{1,2}| \quad (14)
\]

**Definition 4:** A uniformly weak one-sided IFC with \( H_{2,1} = 0 \) is one in which the entries of every fading realization \( h \) satisfy

\[
|h_{2,2}| > |h_{1,2}| \quad (15)
\]

**Definition 5:** A uniformly mixed IFC (UM IFC) is such that the entries of every fading realization \( h \) satisfy

\[
|h_{1,1}| > |h_{2,1}| \quad \text{and} \quad |h_{1,2}| \geq |h_{2,2}| \quad (16)
\]

i.e., receivers 1 and 2 see strong and weak interference in every fading instantiation, respectively. Alternately, a UM IFC can also be such that every fading realization \( h \) satisfies

\[
|h_{2,1}| \geq |h_{1,1}| \quad \text{and} \quad |h_{2,2}| > |h_{1,2}|. \quad (17)
\]

**Definition 6:** A hybrid IFC has at least one weak and one strong fading state such that the conditions in (6) are not satisfied when averaged over all fading states and for \( P_{k}(H) = P_{k}^{avg}(H_{k,k}) \).

**Definition 7:** A coding scheme for ergodic fading (or parallel) channels is separable if independent messages (data) are transmitted in every fading state.

**Definition 8:** A coding scheme for ergodic fading (or parallel) channels is inseparable if the same message (data) is transmitted (coded jointly) across all fading states.

### III. MAIN RESULTS

The following theorems summarize the main contributions of this paper. We collect the capacity results into two categories, namely, theorems for which nonseparable and separable coding schemes are optimal for IFCs. We also present an achievable scheme for a one-sided hybrid IFC which is the third sub-section of this section. The theorems for EVS and US IFCs depend on the capacity region of an ergodic fading Gaussian C-MAC; furthermore, since results for the C-MAC are also of independent interest, we begin with the C-MAC capacity theorem when presenting our results for the nonseparable case. The proof of the capacity region of the C-MAC and the details of determining the capacity achieving power policies are included in Section IV. The proofs for the remaining theorems, related to IFCs, are collected in Section V.

**A. Nonseparable Results**

1) Ergodic Fading C-MAC: An achievable rate region for ergodic fading IFCs results from requiring both receivers to decode the messages from both transmitters, i.e., by converting an IFC to a C-MAC. The following theorem summarizes the capacity region \( C_{\text{C-MAC}} \) of an ergodic fading C-MAC.

**Theorem 1:** The capacity region, \( C_{\text{C-MAC}}(P_{1}^{avg}, P_{2}^{avg}) \), of an ergodic fading two-user Gaussian C-MAC with average power constraints \( P_{k} \) at transmitter \( k, k = 1, 2 \), is

\[
C_{\text{C-MAC}}(P_{1}^{avg}, P_{2}^{avg}) = \bigcup_{P \in \mathcal{P}} \{ C_{1}(P(H)) \cap C_{2}(P(H)) \} \quad (18)
\]
where for \( j = 1, 2 \), we have

\[
C_j(P(H)) = \left\{ (R_1, R_2) : R_S 
\leq \mathbb{E} \left[ C \left( \sum_{k \in S} |H_{j,k}|^2 P_k(H) \right) \right], \text{ for all } S \subseteq K \right\}
\tag{19}
\]

The optimal coding scheme requires encoding and decoding jointly across all sub-channels.

**Remark 2:** The capacity region \( C_{\text{C-MAC}} \) is convex. This follows from the convexity of the set \( P \) and the concavity of the \( \log \) function.

**Remark 3:** \( C_{\text{C-MAC}} \) is a function of \( (P_{1}^{\text{avg}}, P_{2}^{\text{avg}}) \) due to the fact that the union in (18) is taken over all feasible power policies, i.e., over all \( P(H) \) whose entries satisfy (4).

**Remark 4:** In contrast to the ergodic fading point-to-point and multiple access channels, the ergodic fading C-MAC is not merely a collection of independent parallel channels; in fact encoding and decoding independently in each parallel channel is in general sub-optimal as demonstrated in Section IV.

**Corollary 1:** The capacity region \( C_{\text{US}} \) of an ergodic fading IFC is bounded as \( C_{\text{C-MAC}} \subseteq C_{\text{US}} \).

**2) Ergodic Very Strong IFCS:**

**Theorem 2:** The capacity region of an ergodic very strong IFC of Definition 2 is

\[
C_{\text{US}}(P_{1}^{\text{avg}}, P_{2}^{\text{avg}}) = \left\{ (R_1, R_2) : R_k \leq \mathbb{E} \left[ C \left( |H_{k}|^2 P_k(H) \right) \right] \right\}
\tag{20}
\]

The sum-capacity is

\[
\sum_{k=1}^{2} \mathbb{E} \left[ C \left( |H_{k,k}|^2 P_k(H) \right) \right]
\tag{21}
\]

where, for \( k = 1, 2 \), \( P_k(H) \) satisfies (13). The capacity achieving scheme requires encoding and decoding jointly across all sub-channels at the transmitters and receivers respectively. The optimal strategy also requires both receivers to decode messages from both transmitters.

**Remark 5:** In developing the proof in Section V we show that the condition in (13) is a result of the achievable strategy that simplifies the IFC to a C-MAC and, therefore, is a sufficient condition. For the special case of nonfading channel gains \( H = h \), and \( P_k(h) = P_{k}^{\text{avg}} \), (13) reduces to (7). In contrast, the fading averaged conditions in (13) imply that not every fading state needs to satisfy (7) and in fact, the ergodic very strong channel can be a mix of weak and strong fading states provided \( P_{k}^{(w,f)} \) satisfies (13).

**Remark 6:** The set of US IFCs for which the optimal water-filling policies for the two interference-free links satisfy (13) is strictly a subset of the set of EVS IFCs. Thus, in general, the sets of US and EVS IFCs are not disjoint.

**Remark 7:** As stated in Theorem 2, the capacity achieving scheme for EVS IFCs requires coding jointly across all fading states. Coding independent messages (separable coding) across the sub-channels is optimal only when every fading state is very strong, i.e., satisfies (7), at the optimal policy \( P_{k}^{(w,f)} \).

3) Uniformly Strong IFCS: In the following theorem, we present the capacity region and the sum-capacity of a uniformly strong IFC.

**Theorem 3:** The capacity region of a US IFC of Definition 1 is given by

\[
C_{\text{US}}(P_{1}^{\text{avg}}, P_{2}^{\text{avg}}) = C_{\text{C-MAC}}(P_{1}^{\text{avg}}, P_{2}^{\text{avg}})
\tag{22}
\]

where \( C_{\text{C-MAC}}(P_{1}^{\text{avg}}, P_{2}^{\text{avg}}) \) is the capacity of an ergodic fading C-MAC with the same channel statistics as the IFC. The sum-capacity is

\[
\max_{P(H) \in P} \min_{E[H(h)]} \left\{ \mathbb{E} \left[ C \left( \sum_{k=1}^{2} |H_{k,k}|^2 P_k(H) \right) \right] \right\},
\tag{23}
\]

The capacity achieving scheme requires encoding and decoding jointly across all sub-channels at the transmitters and receivers, respectively, and also requires both receivers to decode messages from both transmitters.

**Remark 8:** In contrast to the very strong case of Theorem 2, every sub-channel in a US IFC is strong.

**Remark 9:** The uniformly strong condition may suggest that separability is optimal. However, the capacity achieving scheme for the C-MAC requires joint encoding and decoding across all fading states. A strategy where each fading state (or parallel channel) is viewed as an independent IFC, as in [14], will in general be strictly sub-optimal. This is seen directly from comparing (23) with the sum-rate achieved by separable coding which is given by

\[
\max_{P(H) \in P} \min_{E[H(h)]} \left\{ \mathbb{E} \left[ C \left( \sum_{k=1}^{2} |H_{k,k}|^2 P_k(H) \right) \right] \right\},
\tag{24}
\]

The sub-optimality of separable coding follows directly from the fact that for two random variables \( A(H) \) and \( B(H) \), \( \mathbb{E}[\min(A(H), B(H))] \leq \min(\mathbb{E}[A(H)], \mathbb{E}[B(H)]) \) with equality if and only if for every fading instantiation \( h \), \( A(H) \) (resp. \( B(H) \)) dominates \( B(H) \) (resp. \( A(H) \)). Thus, independent (separable) encoding across the fading states is optimal only when, at the optimal power policy \( P_{k}^{(w,f)} \), the sum-rate in every sub-channel in (24) is maximized by the same sum-rate function.

4) Uniformly Mixed IFC: The following theorem summarizes the sum-capacity of a class of uniformly mixed two-sided IFC.
Theorem 4: For a class of uniformly mixed ergodic fading two-sided Gaussian IFCs of Definition 5 that satisfy (16) the sum-capacity is

$$\max_{P(H) \in \mathcal{P}} \left\{ \min \left\{ C \left( \sum_{k=1}^{2} \left| H_{1,k} \right|^{2} P_{k}(H) \right) \right\}, S^{(w,2)}(P(H)) \right\}$$

(25)

where

$$S^{(w,2)}(P(H)) = E \left[ C \left( \frac{\left| H_{2,1} \right|^{2} P_{2}(H)}{1 + \left| H_{2,1} \right|^{2} P_{1}(H)} \right) + C \left( \left| H_{1,1} \right|^{2} P_{1}(H) \right) \right].$$

(26)

The capacity achieving scheme requires encoding and decoding jointly across all sub-channels at the transmitters.

Remark 10: The sum-capacity for the uniformly mixed IFC for which (17) holds is given by (25) and (26) after interchanging the indices 1 and 2.

Remark 11: At the receiver with stronger interference in all fading states, the capacity achieving scheme requires joint coding at both transmitters just as for US IFCs. Thus, despite one of the receivers ignoring interference, separable encoding is not optimal here.

B. Separable Results

1) Uniformly Weak One-Sided IFC: The following theorem summarizes the sum-capacity of a one-sided uniformly weak IFC in which every fading state is weak.

Theorem 5: The sum-capacity of a uniformly weak ergodic fading Gaussian one-sided IFC of Definition 4 is given by

$$\max_{P(H) \in \mathcal{P}} \{ S^{(w,1)}(P(H)) \}$$

(27)

where $S^{(w,1)}(P(H))$ is obtained from (26) after interchanging the indices 1 and 2.

Remark 12: For the fading one-sided IFC in which $\left| h_{1,1} \right| > \left| h_{2,1} \right|$ and $h_{1,2} = 0$, the sum-capacity is given by (27) with the superscript 1 replaced by 2. The expression $S^{(w,2)}(P(H))$ is given by (26).

C. Achievable Schemes and Outer Bounds

1) Hybrid One-Sided IFC: Achievable Scheme Based on Joint Coding: For EVS and US IFCs, Theorems 2 and 3 suggest that joint coding across all fading states is optimal. Particularly for EVS IFCs, such joint coding allows one to exploit the strong states in decoding messages. Relying on this observation, we present an achievable strategy based on joint coding for all sub-classes of one-sided IFCs of Definition 4 ($H_{2,1} = 0$). The encoding scheme involves rate-splitting at user 2, i.e., user 2 transmits $u_{2} = (w_{2p}, w_{2c})$ where $w_{2p}$ and $w_{2c}$ are private and common messages, respectively and can be viewed as a Han-Kobayashi scheme with Gaussian codebooks and without time-sharing.

Theorem 6: The sum-capacity of a one-sided hybrid IFC (Definition 6 with $H_{2,1} = 0$) is lower bounded by

$$\max_{P(H) \in \mathcal{P}} \min_{\alpha H \in [0,1]} \{ S_{1}(\alpha H, P(H)), S_{2}(\alpha H, P(H)) \}$$

(28)

where

$$S_{1}(\alpha H, P(H)) = E \left[ C \left( \frac{\left| H_{1,1} \right|^{2} P_{1}(H)}{1 + \left| H_{1,1} \right|^{2} P_{1}(H)} \right) + E \left[ C \left( \left| H_{2,1} \right|^{2} P_{2}(H) \right) \right] \right]$$

(29)

and

$$S_{2}(\alpha H, P(H)) = E \left[ C \left( \left| H_{2,1} \right|^{2} \alpha H P_{2}(H) \right) \right] + E \left[ C \left( \left| H_{2,2} \right|^{2} \alpha H P_{2}(H) \right) \right]$$

(30)

such that $\alpha H$ is the power fraction allocated by user 2 in fading state $H$ and $\alpha H = 1 - \alpha H, \alpha H \in [0,1]$. For EVS one-sided IFCs, the sum-capacity is achieved by choosing $\alpha H = 0$ for all $H$ provided $S_{1}(0, P(u_{1}, P(H))) < S_{2}(0, P(u_{2}, P(H)))$. For US one-sided IFCs, the sum-capacity is given by (28) for $\alpha H = 0$ for all $H$. For UW one-sided IFCs, the sum-capacity is achieved by choosing $\alpha H = 1$ and maximizing $S_{2}(1, P(H)) = S_{1}(1, P(H))$ over all feasible $P(H)$. For a hybrid one-sided IFC, the achievable sum-rate is maximized by

$$\alpha H^{*} = \begin{cases} \alpha H \in (0,1), & \text{fading state } H \text{ is weak}, \\ 0, & \text{fading state } H \text{ is strong}. \end{cases}$$

(31)

and is given by (28) for this choice of $\alpha H^{*}$.

Remark 13: The optimal $\alpha H^{*}$ in (31) implies that in general for the hybrid one-sided IFCs joint coding the transmitted message across all sub-channels is optimal. Specifically, the common message is transmitted jointly in all sub-channels while the private message is transmitted only in the weak sub-channels.

Remark 14: The separation-based coding scheme of [34] is a special case of the above HK-based coding scheme and is obtained by choosing $\alpha H = 1$ and $\alpha H = 0$ for the weak and strong states, respectively. The resulting sum-rate is at most as large as the bound in (28) obtained for $\alpha H^{*} \in (0,1]$ and $\alpha H^{*} = 0$ for the weak and strong states, respectively.

Remark 15: In [35], a Han-Kobayashi based scheme using Gaussian codebooks and no time-sharing is used to develop an inner bound on the capacity region of a two-sided IFC.

2) Uniformly Weak IFC: Sum-Capacity Bounds: The sum-capacity of a one-sided uniformly weak IFC in Theorem 5 is an upper bound for that of a two-sided IFC for which at least one of two one-sided IFCs that result from eliminating a cross-link is uniformly weak. Similarly, a bound can be obtained from the sum-capacity of the complementary one-sided IFC. The following theorem summarizes this result.
Theorem 7: For a class of uniformly weak ergodic fading two-sided Gaussian IFCs for which the entries of every fading state \( \mathbf{h} \) satisfy

\[
|h_{1,1}| > |h_{2,1}| \quad \text{and} \quad |h_{2,2}| > |h_{1,2}|
\]

(32)

the sum-capacity is upper bounded as

\[
R_1 + R_2 \leq \max_{\mathbf{P}(\mathbf{H}) \in \mathcal{P}} \min \left( S^{(w,1)}(\mathbf{P}(\mathbf{H})), S^{(w,2)}(\mathbf{P}(\mathbf{H})) \right).
\]

(33)

Remark 16: For the nonfading case, the sum-rate bounds in (33) simplify to those obtained in [9, Theorem 3].

IV. COMPOUND MAC: CAPACITY REGION AND OPTIMAL POWER POLICIES

In this section, we prove Theorem 1 which establishes the capacity region of ergodic fading C-MACs and discuss the optimal power policies that achieve the points on the boundary of the capacity region. As stated in Corollary 1, an inner bound on the sum-capacity of an IFC can be obtained by allowing both receivers to decode both messages, i.e., by determining the sum-capacity of a C-MAC with the same internode links.

A. Capacity Region

The capacity region of a discrete memoryless compound MAC is developed in [36]. For each choice of input distribution at the two independent sources, this capacity region is an intersection of the MAC capacity regions achieved at the two receivers. The techniques in [36] can be easily extended to develop the capacity region for a Gaussian C-MAC with fixed channel gains. For the Gaussian C-MAC, one can show that Gaussian signaling achieves the capacity region using the fact that Gaussian signaling maximizes the MAC region at each receiver. Thus, the Gaussian C-MAC capacity region is an intersection of the Gaussian MAC capacity regions achieved at \( D_1 \) and \( D_2 \). For a stationary and ergodic process \( \{\mathbf{H}_t\} \), the channel in (1) can be modeled as a parallel Gaussian C-MAC consisting of a collection of independent Gaussian C-MACs, one for each fading state \( \mathbf{h} \), with an average transmit power constraint over all parallel channels.

We now prove Theorem 1 and Corollary 1 stated in Section III-A-1 which gives the capacity region of ergodic fading C-MACs.

Proof: We first present an achievable scheme. Consider a policy \( \mathbf{P}(\mathbf{H}) \in \mathcal{P} \). The achievable scheme involves requiring each transmitter to encode the same message across all sub-channels and each receiver to jointly decode over all sub-channels. Independent codebooks are used for every sub-channel. An error occurs at receiver \( j \) if one or both messages decoded jointly across all sub-channels is different from the transmitted message. Given this encoding and decoding, the analysis at each receiver mirrors that for a MAC receiver [29, 14, 3]. In particular, one can easily verify that for reliable reception of the transmitted message at receiver \( j \), the rate pair \((R_1, R_2)\) needs to satisfy the rate constraints in (19) where in decoding \( w_S = \{u_k : k \in S\} \) the mutual information collected in each sub-channel is given by \( C(\sum_{k \in S} |H_{j,k}|^2 P_k(\mathbf{H})) \), for all \( S \subseteq \mathcal{S} \). Thus, for any feasible \( \mathbf{P}(\mathbf{H}) \), the achievable rate region is given by \( C_1(\mathbf{P}(\mathbf{H})) \cap C_2(\mathbf{P}(\mathbf{H})) \). From the concavity of the logarithm function, the achievable region over all \( \mathbf{P}(\mathbf{H}) \) is given by (18).

For the converse, the proof technique mirrors the proof for the capacity of an ergodic fading MAC developed in [20, Appendix A]. For any \( \mathbf{P}(\mathbf{H}) \in \mathcal{P} \), one can use similar limiting arguments to show that for asymptotically error-free performance at receiver \( j \), for all \( j \), the achievable region has to be bounded as

\[
R_S \leq \mathbb{E} \left[ C \left( \sum_{k \in \mathcal{S}} |H_{j,k}|^2 P_k(\mathbf{H}) \right) \right], \quad j = 1, 2,
\]

(34)

The proof is completed by noting that, due to the concavity of the logarithm it suffices to take the union of the region over all \( \mathbf{P}(\mathbf{H}) \in \mathcal{P} \).

For continuously distributed fading channels, we begin our proof by quantizing the fading space to a countably finite number of fading states and using a single codebook comprised of codewords of length \( n \). Each entry of the length-\( n \) codewords is a vector whose entries, generated independently, are inputs to the sub-channels, one for each sub-channel. Allowing the quantization to become finer and finer and using limiting arguments as in [19] and [20] complete the proof.

Corollary 1 follows from the argument that a rate pair in \( \mathcal{C}_{\text{C-MAC}} \) is achievable for the IFC since \( \mathcal{C}_{\text{C-MAC}} \) is the capacity region when both messages are decoded at both receivers.

Remark 17: An achievable scheme in which independent messages are encoded in each sub-channel, i.e., separable coding, will in general not achieve the capacity region. This is due to the fact that for this separable coding scheme the achievable rate in each sub-channel is a minimum of the rates at each receiver. The average of such minima can at most be the minimum of the average rates at each receiver, where the latter is achieved by encoding the same message jointly across all sub-channels (see also Remark 9).

B. Sum-Capacity Optimal Power Policies

The capacity region \( \mathcal{C}_{\text{C-MAC}} \) is a union of the intersection of the pentagons \( C_1(\mathbf{P}(\mathbf{H})) \) and \( C_2(\mathbf{P}(\mathbf{H})) \) achieved at \( D_1 \) and \( D_2 \), respectively, where the union is over all \( \mathbf{P}(\mathbf{H}) \in \mathcal{P} \). The region \( \mathcal{C}_{\text{C-MAC}} \) is convex, and thus, each point on the boundary of \( \mathcal{C}_{\text{C-MAC}} \) is obtained by maximizing the weighted sum \( \mu_1 R_1 + \mu_2 R_2 \) over all \( \mathbf{P}(\mathbf{H}) \in \mathcal{P} \), and for all \( \mu_1 > 0 \) and \( \mu_2 > 0 \), subject to (34). In this section, we determine the optimal policy \( \mathbf{P}^*(\mathbf{H}) \) that maximizes the sum-rate \( R_1 + R_2 \) when \( \mu_1 = \mu_2 = 1 \). Using the fact that the rate regions \( C_1(\mathbf{P}(\mathbf{H})) \) and \( C_2(\mathbf{P}(\mathbf{H})) \), for any feasible \( \mathbf{P}(\mathbf{H}) \), are pentagons, in Figs. 3 and 4 we illustrate the five possible choices for the sum-rate resulting from an intersection of \( C_1(\mathbf{P}(\mathbf{H})) \) and \( C_2(\mathbf{P}(\mathbf{H})) \) (see also [31]).

Cases 1 and 2, as shown in Fig. 3 and henceforth referred to as inactive cases, are such that the constraints on the two sum-rates are not active in \( C_1(\mathbf{P}(\mathbf{H})) \cap C_2(\mathbf{P}(\mathbf{H})) \), i.e., no rate tuple on the sum-rate plane achieved at one of the receivers lies within or on the boundary of the rate region achieved at the other receiver. In contrast, when there exists at least one such rate tuple such that the two sum-rate constraints are active in \( C_1(\mathbf{P}(\mathbf{H})) \cap C_2(\mathbf{P}(\mathbf{H})) \).
to denote the optimal.

respectively. we also write simplify the optimization to with a larger sum-rate. Combining this with the to denote the sum-rate is first determined by maximizing the sum achieves its maximum outside , for every , .

there exists , and thus, are either open or half-open sets and we obtain an active case. This includes Cases 3a, 3b, and 3c shown in Fig. 4 where the sum-rate at is smaller, larger, or equal, respectively, to that achieved at . By definition, the active set also includes the boundary cases in which there is exactly one rate pair that lies within or on the boundary of the rate region achieved at the other receiver. There are six possible boundary cases ( , ) that lie at the intersection of an inactive case , 1, 2, and an active case , , respectively. Using these boundary cases from the active cases ensures that the sets do not share a boundary. This in turn implies that the power policies resulting in each case satisfy specific conditions that distinguish that case from all others. Using these disjoint cases and the fact that the rate expressions in (34) are concave (log) functions of simplifies the optimization to a convex optimization problem and allows us to develop closed form sum-capacity results and optimal policies for all cases as explained below.

We write , and , to denote the optimal policies for cases and ( , ) respectively. We also write , and to denote the sum-rate bound achieved for cases and ( , ) respectively, for some .

**Uniqueness of** **and** **:** Consider case . The optimal is first determined by maximizing the sum rate for this case over all . The resulting sum-rate optimal must satisfy the conditions for case , i.e., we require . If , the optimality of follows from the fact that the rate function for each case is strictly concave and that the sets and are disjoint for all and ( , ) as a result of which does not maximize the sum-rate for any other case. On the other hand, when , we now argue that achieves its maximum outside . The proof again follows from the fact that for all cases is a strictly concave function of for all . Thus, when , for every there exists a with a larger sum-rate. Combining this with the fact that the sum-rate expressions are continuous while transitioning from one case to another at the boundary of the open set , ensures that the maximal sum-rate is achieved by some . Similar arguments justify maximizing the optimal policy for each case over all .

**Fig. 3. Rate regions** and **and sum-rate for case 1 and case 2.**

**Fig. 4. Rate regions** and **and sum-rate for cases 3a, 3b, and 3c.**
the conditions for its case. The optimal $P^*(H)$ is given by this $P^{(i)}(H)$ or $P^{(lm)}(H)$.

The optimization problem for case $i$ or case $(l,m)$ is given by

$$
\max_{P(H) \in \mathcal{P}} S^{(i)}(P(H)) \quad \text{or} \quad \max_{P(H) \in \mathcal{P}} S^{(lm)}(P(H)) \\
\text{s.t.} \quad E[P_k(H)] \leq P_k^{\text{awg}}, \quad k = 1, 2 \\
P_k(H) \geq 0 \quad k = 1, 2, \text{ for all } H
$$

where

$$
S^{(1)}(P(H)) = \sum_{k=1}^{2} \mathbb{E}[|H_{k,k}|^2 P_k(H)] \\
S^{(2)}(P(H)) = \sum_{k=1}^{2} \mathbb{E}[|H_{k,k}|^2 P_k(H)] \\
S^{(3a)}(P(H)) = \mathbb{E}\left[C \left( \sum_{k=1}^{2} |H_{2,k}|^2 P_k(H) \right)^2 \right] \\
S^{(3b)}(P(H)) = \mathbb{E}\left[C \left( \sum_{k=1}^{2} |H_{1,k}|^2 P_k(H) \right)^2 \right] \\
S^{(3a)}(P(H)) = S^{(3a)}(P(H)) \\
S^{(3b)}(P(H)) = S^{(3b)}(P(H)) \\
S^{(lm)}(P(H)) = S^{(lm)}(P(H)), \text{ s.t.} \\
S^{(i)}(P(H)) = S^{(m)}(P(H)), \text{ for all } (l,m).
$$

Recall that case 3c results when the sum-rate bounds at both receivers are the same. We capture this constraint in (39) by setting $S^{(3a)}(\cdot)$ as $S^{(3a)}(\cdot)$ subject to the equality constraint on $S^{(3a)}(\cdot)$ and $S^{(3b)}(\cdot)$. Similarly, the condition for case $(l,m)$ that the sum-rates for cases $l$ and $m$ are equal is captured in (40).

The conditions for each case are (see Figs. 3–6) given below where for each case the condition holds true when evaluated at the optimal policies $P^{(i)}(H)$ and $P^{(lm)}(H)$ for cases $i$ and $(l,m)$, respectively. For ease of notation, we do not explicitly denote the dependence of $S^{(i)}$ and $S^{(lm)}$ on the appropriate $P^{(i)}(H)$ and $P^{(lm)}(H)$, respectively but use a subscript to indicate that the conditions are evaluated at the optimal policies for each case

$$
\text{Case 1: } S^{(1)}_{\max}(P^{(1)}(H)) = \min(S^{(3a)}, S^{(3b)}) \bigg|_{\mathcal{L}^{(1)}(H)} \\
\text{Case 2: } S^{(2)}_{\max}(P^{(2)}(H)) = \min(S^{(3a)}, S^{(3b)}) \bigg|_{\mathcal{L}^{(2)}(H)} \\
\text{Case 3a: } S^{(3a)}_{\max}(P^{(3a)}(H)) = \min(S^{(1)}, S^{(3b)}) \bigg|_{\mathcal{L}^{(3a)}(H)} \\
\text{Case 3b: } S^{(3b)}_{\max}(P^{(3b)}(H)) = \min(S^{(1)}, S^{(2)}) \bigg|_{\mathcal{L}^{(3b)}(H)} \\
\text{Case 3c: } S^{(3c)}_{\max}(P^{(3c)}(H)) = \min(S^{(3a)}, S^{(1)}, S^{(2)}) \bigg|_{\mathcal{L}^{(3c)}(H)} \\
\text{Case 1, 3a: } \min(S^{(1)}_{\max}(P^{(1)}(H)), S^{(3a)}_{\max}(P^{(3a)}(H))) \\
\text{Case 1, 3b: } \min(S^{(1)}_{\max}(P^{(1)}(H)), S^{(3b)}_{\max}(P^{(3b)}(H))) \\
\text{Case 1, 3c: } \min(S^{(1)}_{\max}(P^{(1)}(H)), S^{(3c)}_{\max}(P^{(3c)}(H))) \\
\text{Case 2, 3a: } \min(S^{(2)}_{\max}(P^{(2)}(H)), S^{(3a)}_{\max}(P^{(3a)}(H))) \\
\text{Case 2, 3b: } \min(S^{(2)}_{\max}(P^{(2)}(H)), S^{(3b)}_{\max}(P^{(3b)}(H))) \\
\text{Case 2, 3c: } \min(S^{(2)}_{\max}(P^{(2)}(H)), S^{(3c)}_{\max}(P^{(3c)}(H)))
$$

Recall that case 3c results when the sum-rate bounds at both receivers are the same. We capture this constraint in (39) by setting $S^{(3a)}(\cdot)$ as $S^{(3a)}(\cdot)$ subject to the equality constraint on $S^{(3a)}(\cdot)$ and $S^{(3b)}(\cdot)$. Similarly, the condition for case $(l,m)$ that the sum-rates for cases $l$ and $m$ are equal is captured in (40).

The conditions for each case are (see Figs. 3–6) given below where for each case the condition holds true when evaluated at the optimal policies $P^{(i)}(H)$ and $P^{(lm)}(H)$ for cases $i$ and $(l,m)$, respectively. For ease of notation, we do not explicitly denote the dependence of $S^{(i)}$ and $S^{(lm)}$ on the appropriate $P^{(i)}(H)$ and $P^{(lm)}(H)$, respectively but use a subscript to indicate that the conditions are evaluated at the optimal policies for each case

$$
\text{Case 1: } S^{(1)}_{\max}(P^{(1)}(H)) = \min(S^{(3a)}, S^{(3b)}) \bigg|_{\mathcal{L}^{(1)}(H)} \\
\text{Case 2: } S^{(2)}_{\max}(P^{(2)}(H)) = \min(S^{(3a)}, S^{(3b)}) \bigg|_{\mathcal{L}^{(2)}(H)} \\
\text{Case 3a: } S^{(3a)}_{\max}(P^{(3a)}(H)) = \min(S^{(3a)}, S^{(1)}, S^{(2)}) \bigg|_{\mathcal{L}^{(3a)}(H)} \\
\text{Case 3b: } S^{(3b)}_{\max}(P^{(3b)}(H)) = \min(S^{(3a)}, S^{(1)}, S^{(2)}) \bigg|_{\mathcal{L}^{(3b)}(H)} \\
\text{Case 3c: } S^{(3c)}_{\max}(P^{(3c)}(H)) = \min(S^{(3a)}, S^{(1)}, S^{(2)}) \bigg|_{\mathcal{L}^{(3c)}(H)} \\
\text{Case 1, 3a: } \min(S^{(1)}_{\max}(P^{(1)}(H)), S^{(3a)}_{\max}(P^{(3a)}(H))) \\
\text{Case 1, 3b: } \min(S^{(1)}_{\max}(P^{(1)}(H)), S^{(3b)}_{\max}(P^{(3b)}(H))) \\
\text{Case 1, 3c: } \min(S^{(1)}_{\max}(P^{(1)}(H)), S^{(3c)}_{\max}(P^{(3c)}(H))) \\
\text{Case 2, 3a: } \min(S^{(2)}_{\max}(P^{(2)}(H)), S^{(3a)}_{\max}(P^{(3a)}(H))) \\
\text{Case 2, 3b: } \min(S^{(2)}_{\max}(P^{(2)}(H)), S^{(3b)}_{\max}(P^{(3b)}(H))) \\
\text{Case 2, 3c: } \min(S^{(2)}_{\max}(P^{(2)}(H)), S^{(3c)}_{\max}(P^{(3c)}(H)))
$$

The optimal policy for each case is determined using Lagrange multipliers and the Karush-Kuhn-Tucker (KKT) conditions. The sum-capacity optimal $P^*(H)$ is given by that $P^{(i)}(H)$ or $P^{(lm)}(H)$ that satisfies the conditions of its case in (42)–(52).

Remark 18: For cases 1 and 2, one can expand the capacity expressions to verify that the conditions $S^{(i)} < \min(S^{(3a)}, S^{(3b)}), l = 1, 2,$ imply that $S^{(1)} < S^{(2)}$ and vice-versa. Therefore, if the optimal policy is determined in the order of the cases in (42)–(52), the conditions for cases
(1,3c) and (2,3c) are tested only after all other cases have been excluded. Furthermore, the two cases are mutually exclusive, and thus, (51) and (52) are simply redundant conditions written for completeness.

Remark 19: For the two-user case the conditions can be written directly from the geometry of intersecting rate regions for each case. However, for a more general $K$-user C-MAC, the conditions can be written using the fact that the rate regions for any $P(H)$ are polymatroids and that the sum-rate of two intersecting polymatroids is given by the polymatroid intersection lemma. A detailed analysis of the rate-region and the optimal policies using the polymatroid intersection lemma for a $K$-user two-receiver network is developed in [31].

C. Computing the Optimal Power Policies

The following theorem summarizes the form of $P^*(H)$ and presents an algorithm to compute it. The optimal policy maximizing each case can be obtained in a straightforward manner using standard constrained convex maximization techniques. The algorithm exploits the fact that each occurrence of one case excludes all other cases and the case that occurs is the one for which the optimal policy satisfies the case conditions. We refer the reader to [31, Appendix] for a detailed analysis.

Theorem 8: The optimal policy $P^*(H)$ achieving the sum-capacity of a two-user ergodic fading C-MAC is obtained by computing $L^{(i)}(H)$ and $L^{(lm)}(H)$ in order starting with cases 1 and 2, followed by cases 3a, 3b, and 3c, that is, $L^{(3a)} = L^{(3b)} = L^{(3c)} = \infty$, in that order, and finally the boundary cases $(l,m)$, in the order that cases $(l,3c)$ are the last to be optimized, with $P^*(H)$ being the optimal $P^*(H)$ for all cases. The case conditions and falls into one of the following three categories: Inactive Cases 1 and 2: The optimal policy for the two users is such that one user applies waterfilling over its interference-free link to one of the receivers while the other applies waterfilling over its link to the other receiver; Cases (3a,3b,3c): The optimal user policy for both users is opportunistic waterfilling over its link to destination 2 for case 3a and to destination 1 for case 3b. For case 3c, $P^*(H)$, for all $k = 1, 2$, takes an opportunistic generalized waterfilling form and depends on the channel gains of user $k$ at both receivers; Boundary Cases: The optimal user policies $P^*(H)$, for all $k = 1, 2$, takes an opportunistic generalized waterfilling form.

Remark 20: The sum-rate optimal policies for two-transmitter two-receiver ergodic fading channel where one of the receiver also acts as a relay is developed in [31]. The analysis here is very similar to that in [31], and thus, we briefly outline the proof of Theorem 8 below.

Proof: The optimal policy for each case can be determined using Lagrange multipliers and the Karush-Kuhn-Tucker (KKT) conditions. We consider the cases separately and explain the policies for each case.

Cases 1 and 2: From (36) and (37), since the sum-rates for cases 1 and 2 are a sum of the capacities of two point-to-point noninterfering links, the sum-rate optimization for these two cases simplifies to that for the classic ergodic fading channel. The optimal policies for each users is thus the classic point-to-point waterfilling solution [19] over its bottle-neck link, i.e., over the direct interference-free link to the receiver with the smaller (interference-free) ergodic fading capacity. Thus, for cases 1 and 2, each transmitter waterfills on the interference-free point-to-point links to its intended and unintended receivers, respectively. Thus, for case 1, $P_k^*(H) = P_k^{(1)}(H) = P_l^{in}(H_{ik})$, and for case 2, $P_k^*(H) = P_k^{(2)}(H) = P_l^{in}(H_{jk})$, $j, k = 1, 2, j \neq k$, where $P_k^{in}(H_{jk})$ is the waterfilling solution over an ergodic fading link whose link gain is the random variable $H_{jk}$.

Cases (3a,3b,3c): For cases 3a and 3b, from (38), $S_{(3a)}(\cdot)$ and $S_{(3b)}(\cdot)$ are the multiple access sum-capacities from both users to receivers 2 and 1, respectively. Thus, for these two cases, the optimal user policies $P_k^*(H)$, for all $k$, are the well-known opportunistic multiuser waterfilling solutions [20], [21] over the multiaccess links to receivers 1 and 2, respectively. We now briefly develop the optimization problem for case 3c and show how the solution has an opportunistic generalized waterfilling form. From (39) and (40), the KKT conditions for each case $x, x = i, (l,m)$, for all $i$ and $(l,m)$ are given as

$$f_k^x(P^*(h)) = \nu_k \ln 2 \leq 0, \text{ with equality for } P_k(h) > 0, \text{ for } k = 1, 2, \text{ for all } h (53)$$
where $\nu_k$, $k = 1, 2$, are dual variables (waterfilling levels) chosen to satisfy the power constraints in (35) and $f_k^{(2)}(\mathbf{h})$ is specific to each case. For case 3c, the functions $f_k^{(3c)}(\mathbf{h})$, $k = 1, 2$, satisfying the KKT conditions in (53) are given as

$$f_k^{(3c)}(\mathbf{h}) = (1 - \alpha)f_k^{(3a)}(\mathbf{h}) + \alpha f_k^{(3b)}(\mathbf{h})$$

where

$$f_k^{(3a)}(\mathbf{h}) = |h_{2,k}|^2 \left( \frac{1 + \sum_{k=1}^{2} |h_{2,k}|^2 P_k(\mathbf{h}) / \theta}{\sum_{k=1}^{2} |h_{1,k}|^2 P_k(\mathbf{h}) / \theta} \right),$$

$$f_k^{(3b)}(\mathbf{h}) = |h_{1,k}|^2 \left( \frac{1 + \sum_{k=1}^{2} |h_{1,k}|^2 P_k(\mathbf{h}) / \theta}{\sum_{k=1}^{2} |h_{2,k}|^2 P_k(\mathbf{h}) / \theta} \right),$$

and the Lagrange multiplier $\alpha$ accounts for the boundary condition

$$S^{(3a)}(\cdot) = S^{(3b)}(\cdot)$$

and the optimal policy $P^{(3c)}(\mathbf{H}) \in \mathcal{B}_{3c}$ satisfies this condition where $\mathcal{B}_{3c}$ is the set of $P(\mathbf{H})$ that satisfy (57). Using (53) it can be shown in a straightforward manner that the optimal user policies are opportunistic in form and are given by

$$f_1^{(3c)} / \nu_1 > f_2^{(3c)} / \nu_2 : \quad F_1^{(3c)}(\mathbf{h}) = \left( \text{root of } F_1^{(3c)} | P_2 = 0 \right)^+, \quad F_2^{(3c)}(\mathbf{h}) = 0$$

$$f_1^{(3c)} / \nu_1 < f_2^{(3c)} / \nu_2 : \quad F_1^{(3c)}(\mathbf{h}) = 0, \quad F_2^{(3c)}(\mathbf{h}) = \left( \text{root of } F_2^{(3c)} | P_1 = 0 \right)^+$$

$$f_1^{(3c)} / \nu_1 = f_2^{(3c)} / \nu_2 : \quad F_1^{(3c)}(\mathbf{h}) \text{ and } F_2^{(3c)}(\mathbf{h}) \text{ obtained using an iterative algorithm}$$

where we write

$$F_k^{(3c)} = f_k^{(3c)} - \nu_k \ln 2 \quad k = 1, 2.$$

Analogously to cases 3a and 3b, the scheduling conditions in (58) depend on both the channel states and the waterfilling levels $\nu_i$ at both users. The conditions in (58) also depend on the power policies, and thus, the optimal solutions are referred to as generalized waterfilling solutions. In [31] we show that the optimal user policies can be computed using an iterative algorithm which starts by fixing the power policy of one user, computing that of the other, and vice-versa until the policies converge to the optimal policy; the convergence proof hinges on the fact that the maximizing function $S^{(3c)}(P(\mathbf{H}))$ in (39) is a strictly concave function of $P_1(\mathbf{H})$ and $P_2(\mathbf{H})$ and is bounded from above because of the power constraints at the transmitters. The iterative algorithm is computed for increasing values of $\alpha \in (0, 1)$ until the optimal policy satisfies (57) at the optimal $\alpha^*$. Thus, for case 3c, $P_k^{(3c)}(\mathbf{H})$, for all $k$, takes an opportunistic generalized waterfilling form and depends on the channel gains for each user at both receivers.

Boundary Cases: A boundary case $(l, m)$ results when

$$S^{(l)}(\cdot) = S^{(m)}(\cdot) \quad l = 1, 2, \text{ and } m = 3a, 3b, 3c.$$  

(60)

Recall that $S^{(l)}(\cdot)$ and $S^{(m)}(\cdot)$ are sum-rates for an inactive case $l$, and an active case $m$, respectively. Thus, in addition to the constraints in (35), the maximization problem for these cases includes the additional constraint in (60). For all except the two cases where $m = 3c$, the equality condition in (60) is represented by a Lagrange multiplier $\alpha$. The two cases with $m = 3c$ have two Lagrange multipliers $\alpha_1$ and $\alpha_2$ to also account for both the equality condition in (60) and the condition $S^{(3a)} = S^{(3b)}$.

For the different boundary cases, the functions $f_k^{(3, l, m)}(\mathbf{h})$, $k = 1, 2$, satisfying the KKT conditions in (53) are given as

$$f_k^{(3, l, m)}(\mathbf{h}) = (1 - \alpha) f_k^{(l)}(\mathbf{h}) + \alpha f_k^{(m)}(\mathbf{h}),$$

$$k = 1, 2, m \neq 3c,$$

$$f_k^{(3, c)}(\mathbf{h}) = (1 - \alpha_1 - \alpha_2) f_k^{(l)}(\mathbf{h}),$$

$$\alpha_1 f_k^{(3a)}(\mathbf{h}) + \alpha_2 f_k^{(3b)}(\mathbf{h}),$$

(61)

(62)

For ease of exposition and brevity, we summarize the KKT conditions and the optimal policies for case $(1, 3a)$. It can be shown using (53) that the optimal user policies $P_k^{(1, 3a)}(\mathbf{h})$ are opportunistic in form and are given by

$$f_1^{(1, 3a)} / \nu_1 > f_2^{(1, 3a)} / \nu_2 : \quad P_1^{(1, 3a)}(\mathbf{h}) = \left( \text{root of } F_1^{(1, 3a)} | P_2 = 0 \right)^+, \quad P_2(\mathbf{h}) = 0$$

$$f_1^{(1, 3a)} / \nu_1 < f_2^{(1, 3a)} / \nu_2 : \quad P_1^{(1, 3a)}(\mathbf{h}) = 0, \quad P_2(\mathbf{h}) = \left( \text{root of } F_2^{(1, 3a)} | P_1 = 0 \right)^+$$

$$f_1^{(1, 3a)} / \nu_1 = f_2^{(1, 3a)} / \nu_2 : \quad P_1^{(1, 3a)}(\mathbf{h}) \text{ and } P_2(\mathbf{h}) \text{ solved jointly using an iterative algorithm}$$

(63)

where $F_k^{(1, 3a)} = f_k^{(1, 3a)} - \nu_k \ln 2$, for $k = 1, 2$. As in case 3c, the optimal policies take an opportunistic generalized waterfilling form and in fact can be obtained using an iterative algorithm as described for case 3c.

Remark 21: The iterative algorithm discussed as an approach to compute the generalized waterfilling solution can also be applied to determine the optimal policies for all cases (see, for example, [37] where an iterative waterfilling approach is applied for MIMO MACs).

D. Capacity Region: Optimal Policies

As mentioned earlier, each point on the boundary of $C_{\text{MAC}}(P_{1w}^{\text{avg}}, P_{2w}^{\text{avg}})$ is obtained by maximizing the weighted sum $\mu_1 R_1 + \mu_2 R_2$ over all $P(\mathbf{H}) \in \mathcal{P}$, and for all $\mu_1 > 0, \mu_2 > 0$, subject to (34). Without loss of generality, we assume
that \( \mu_1 < \mu_2 \). Let \( \mu \) denote the pair \((\mu_1, \mu_2)\). The optimal policy \( P^*(H, \mu) \) is given by

\[
P^*(H, \mu) = \arg \max_{P \in P} (\mu_1 R_1 + \mu_2 R_2) \text{ s.t. } (R_1, R_2) \in C_{C \rightarrow MAC}(P^1_{\text{avg}}, P^2_{\text{avg}})
\]

where \( \mu_1 R_1 + \mu_2 R_2 \), denoted by \( S^x(\mu, \mu)(H) \) for case \( x = \hat{i}, \hat{l}, m \), for all \( i \) and \( l, m \), for the different cases are given by

\[
S^{(1)}(\mu, \mu)(H) = \sum_{k=1}^{2} \mu_k E[C(H_{k,k})^2 P_k(H)]
\]

\[
S^{(2)}(\mu, \mu)(H) = \sum_{j=1}^{2} \sum_{k=1}^{2} \mu_k E[C(H_{j,k})^2 P_k(H)]
\]

\[
S^{(3)}(\mu, \mu)(H) = \mu_1 S^{(3)}(\mu, P(H)) = \mu_2 S^{(3)}(\mu, P(H))
\]

where \( \mu_1 R_1 + \mu_2 R_2 \), denoted by \( S^{(m)}(\mu, \mu)(H) \) for case \( m \).

The expressions for \( \mu_2 < \mu_1 \) can be obtained from (65) by interchanging the indexes 1 and 2 in the second term in the expression for \( S^{(3)}(\mu, P(H)), i = 3a, 3b \). From the convexity of \( C_{C \rightarrow MAC} \), every point on the boundary is obtained from the intersection of two MAC rate regions. From Figs. 3–6, we see that for cases 1, 2, and the boundary cases, the region of intersection has a unique vertex at which both rates are nonzero and thus, \( \mu_1 R_1 + \mu_2 R_2 \) will be tangent to that vertex. On the other hand, for cases 3a, 3b, and 3c, the intersecting region is also a pentagon and thus, \( \mu_1 R_1 + \mu_2 R_2 \), for \( \mu_1 < \mu_2 \), is maximized by that vertex at which user 2 is decoded after user 1. The conditions for the different cases are given by (42)–(52).

Note that for case 1, since the sum-capacity achieving policies also achieve the point-to-point capacity for each user to its intended destination, the capacity region is simply given by the single-user capacity bounds on \( R_1 \) and \( R_2 \).

The following theorem summarizes the capacity region of an ergodic fading C-MAC and the optimal policies that achieve it for \( \mu_1 < \mu_2 \). The policies for \( \mu_1 > \mu_2 \) can be obtained in a straightforward manner.

**Theorem 9:** The optimal policy \( P^*(H) \) achieving the sum-capacity of a two-user ergodic fading C-MAC is obtained by computing \( P^{(i)}(H) \) and \( P^{(l,m)}(H) \) starting with the inactive cases 1 and 2, followed by the active cases 3a, 3b, and 3c, in that order, and finally the boundary cases \( l, m \), in the order that cases \( l, 3c \) are the last to be optimized, until for some case the corresponding \( P^{(i)}(H) \) or \( P^{(l,m)}(H) \) satisfies the case conditions. The optimal \( P^*(H) \) is given by the optimal \( P^{(i)}(H) \) or \( P^{(l,m)}(H) \) that satisfies its case conditions and falls into one of the following three categories:

**Inactive Cases:** The optimal policies for the two users are such that each user waterfills over its bottleneck link. Thus, for cases 1 and 2, each user applies waterfilling on the (interference-free) point-to-point links to its intended and unintended receivers, respectively. Thus, for case 1, \( P_k^{(s)}(H) = P_k^{\text{uf}}(H_{k,k}) \), and for case 2, \( P_k^{(s)}(H) = P_k^{(i)}(H) = \mu_k P_k^{\text{uf}}(H_{j,k}), j = 1, 2, j \neq k \), where \( P_k^{\text{uf}}(H_{j,k}) \) for \( j, k = 1, 2 \), is defined in Theorem 2.

**Cases (3a, 3b, 3c):** In general, the optimal policies for all three cases are opportunistic generalized waterfilling solutions.

**Boundary Cases:** The optimal policies maximizing the constrained optimization of \( S^{(km)}(\mu_1, \mu_2)(H) \) are also opportunistic generalized waterfilling solutions.

**V. PROOFS**

**A. Proofs for Nonseparable IFCs**

1) Ergodic vs IFCs: Proof of Theorem 2: Converse: An outer bound on the sum-capacity of an interference channel is given by the sum-capacity of an IFC in which interference has been eliminated at one or both receivers. One can view it alternately as providing each receiver with the codeword of the interfering transmitter. Thus, from Fano’s and the data processing inequalities we have that the achievable rate must satisfy

\[
R_1 + R_2 - ne \leq I(X_1^m, Y_1^n(X_2^m, H^n)) + I(X_2^m, Y_2^n(X_1^n, H^n)) = I(X_1^m, Y_1^n[H^n]) + I(X_2^m, Y_2^n[H^n])
\]

where \( \epsilon \rightarrow 0 \) as \( n \rightarrow \infty \) and

\[
\hat{Y}_k = H_{k,k} X_k + Z_k, \quad k = 1, 2.
\]

The converse proof techniques developed in [19, Appendix] for a point-to-point ergodic fading link in which the transmit and received signals are related by (67) can be applied directly following (66b), and thus, we have that any achievable rate pair must satisfy

\[
R_1 + R_2 = 2 \sum_{k=1}^{2} E[C(H_{k,k}^2P_k^{\text{uf}}(H_{k,k}))].
\]

**Achievable Scheme:** Corollary 1 states that the capacity region of an equivalent C-MAC is an inner bound on the capacity region of an IFC. Thus, from Theorem 8 a sum-rate of

\[
\sum_{k=1}^{2} E[C(H_{k,k}^2P_k^{\text{uf}}(H_{k,k}))]
\]

is achievable when \( P^*(H) = P_k^{\text{uf}}(H_{k,k}) \) satisfies the condition for case 1 in (42), which is equivalent to the requirement that \( P_k^{\text{uf}}(H_{k,k}) \) satisfies (13).
Finally, since the achievable bound on the sum-rate in (69) also achieves the single-user capacities, the capacity region of an EVS IFC is given by (20).

Separability: Achieving the sum-capacity and the capacity region of the C-MAC requires joint encoding and decoding across all fading states. This observation also carries over to the sub-class of ergodic very strong IFCs. In fact, any strategy where each fading state is viewed as an independent IFC will be strictly sub-optimal except for those cases where every sub-channel is very strong at the optimal policy.

2) Uniformly Strong IFC: Proof of Theorem 3: Converse: In the proof of Theorem 2, we developed a genie-aided outer bound on the sum-capacity of ergodic fading IFCs. One can use similar arguments to write the bounds on the rates $R_1$ and $R_2$, for every choice of feasible power policy $P(\mathbf{H})$, as

\[
R_k \leq \mathbb{E} \left[ \log \left( 1 + \left| H_{k,k} \right|^2 P_k(\mathbf{H}) \right) \right], \quad k = 1, 2. \tag{70}
\]

\[
R_j \leq \mathbb{E} \left[ \log \left( 1 + \left| H_{j,k} \right|^2 P_k(\mathbf{H}) \right) \right], \quad j = 1, 2, j \neq k. \tag{71}
\]

where (71) follows from the uniformly strong condition in (12). We now present two additional bounds in which the genie reveals the interfering signal to only one of the receivers. Consider first the case in which the genie reveals the interfering signal at receiver 2. One can then reduce the two-sided IFC to a one-sided IFC, i.e., set $H_{21} = 0$.

For this genie-aided one-sided channel, from Fano’s inequality, we have that the achievable rate must satisfy

\[
n(R_1+R_2) - n \epsilon \leq I(X_1^n; Y_1^n|\mathbf{H}^n) + I(X_2^n; Y_2^n|\mathbf{H}^n), \tag{72a}
\]

We first consider the expression on the right-hand side of (72a) for some realization $\mathbf{h}^n$. We thus have

\[
I(X_1^n; Y_1^n|\mathbf{H}^n = \mathbf{h}^n) + I(X_2^n; Y_2^n|\mathbf{H}^n = \mathbf{h}^n)
\]

\[
= I(X_1^n; \mathbf{h}_{1,1}^{n}X_1^n + \mathbf{h}_{1,2}^{n}X_2^n + Z_1^n) + I(X_2^n; \mathbf{h}_{2,1}^{n}X_1^n + \mathbf{h}_{2,2}^{n}X_2^n + Z_2^n) \tag{73}
\]

where $\mathbf{h}_{j,k}^{n}$ is a diagonal matrix with diagonal entries denoted as $h_{j,k,i}$, for all $i = 1, 2, \ldots, n$, such that $h_{j,k,i}$ is the channel gain between transmitter $j$ and receiver $k$ in symbol time $i$. Consider the mutual information terms on the right-hand side of the inequality in (73). We can expand these terms as

\[
\begin{align*}
& h(\mathbf{h}_{1,1}^{n}X_1^n + \mathbf{h}_{1,2}^{n}X_2^n + Z_1^n) - h(\mathbf{h}_{1,2}^{n}X_2^n + Z_1^n) \\
& + h(\mathbf{h}_{2,1}^{n}X_1^n + \mathbf{h}_{2,2}^{n}X_2^n + Z_2^n) - h(Z_2^n) \tag{74a}
\end{align*}
\]

\[
\leq \sum_{i=1}^{n} \left( h(h_{1,1,i}X_{1,i} + h_{1,2,i}X_{2,i} + Z_{1,i}) - h(Z_{2,i}) \right) \tag{74b}
\]

where $(a)$ follows from the fact that conditioning does not increase entropy. For the uniformly strong ergodic IFC satisfying (12), i.e., $|h_{2,2,i}^{n}| \leq |h_{1,2,i}^{n}|$, for all $i = 1, 2, \ldots, n$, the third and fourth terms in (74b) can be simplified as

\[
\begin{align*}
- h \left( X_2^n + (\mathbf{h}_{1,2}^{n})^{-1} Z_1^n \right) \\
+ h \left( X_2^n + (\mathbf{h}_{2,2}^{n})^{-1} Z_2^n \right) \\
- \log \left( |h_{1,2}^{n}| \right) + \log \left( |h_{2,2}^{n}| \right) \\
= h \left( X_2^n + (\mathbf{h}_{1,2}^{n})^{-1} Z_1^n \right) \\
+ h \left( X_2^n + (\mathbf{h}_{2,2}^{n})^{-1} Z_2^n \right) \\
- \log \left( |h_{1,2}^{n}| \right) + \log \left( |h_{2,2}^{n}| \right) \\
= h \left( Z_1^n \right) + h \left( Z_2^n \right) \\
\end{align*}
\]

where $\hat{Z}_i \sim \mathcal{CN}(0, |\mathbf{h}_{1,2}^{n}|^2 - |\mathbf{h}_{2,2}^{n}|^2)$, for all $i$, and the inequality in (75c) can be obtained as

\[
I \left( \hat{Z}_i^n; X_2^n + (\mathbf{h}_{1,2}^{n})^{-1} Z_1^n + \hat{Z}_i^n \right) \tag{75a}
\]

\[
= H \left( \hat{Z}_i^n \right) - H \left( X_2^n + (\mathbf{h}_{1,2}^{n})^{-1} Z_1^n + \hat{Z}_i^n \right) \tag{75b}
\]

\[
\leq H(\hat{Z}_i^n) - H \left( (\mathbf{h}_{1,2}^{n})^{-1} Z_1^n + \hat{Z}_i^n \right) \tag{75c}
\]

\[
= I \left( \hat{Z}_i^n; (\mathbf{h}_{1,2}^{n})^{-1} Z_1^n + \hat{Z}_i^n \right) \tag{75d}
\]

where (76c) is due to the fact that mixing increases entropy, and (75e) results from combining the two channel gains entropy terms with the first mutual information term and is analogous to inverting the step in (75a).

Substituting (75e) in (74b), we thus have that for every instantiation, the $n$-letter expressions reduce to a sum of single-letter expressions. Over all fading realizations, one can thus write

\[
(R_1 + R_2) - n \epsilon \leq \frac{1}{n} \sum_{i=1}^{n} I(X_{1,i}X_{2,i}; Y_{1,i}|H = h_i) \tag{77}
\]

where $h_i$ is the fading realization in the $i$th use of the channel.

Our analysis from here on is similar to that for the fading MAC studied in [20, Appendix A], and thus, we omit it. Effectively, the analysis involves considering an increasing sequence of partitions (quantized ranges) $\mathcal{H}_k$, $k = 1, \ldots, T$, of the alphabet of $\mathbf{H}$, while ensuring that for each $k$, the transmitted signals are constrained in power. Taking limits appropriately over $n$ and $k$, as in [20, Appendix A], we obtain

\[
R_1 + R_2 - n \epsilon \leq \mathbb{E} \left[ C \left( \sum_{k=1}^{n} |H_{1,k}|^2 P_k(\mathbf{H}) \right) \right] \tag{78}
\]

where $P(\mathbf{H})$ satisfies (4).

One can similarly let $H_{1,2} = 0$ and show that

\[
R_1 + R_2 - n \epsilon \leq \mathbb{E} \left[ C \left( \sum_{k=1}^{n} |H_{2,k}|^2 P_k(\mathbf{H}) \right) \right]. \tag{79}
\]
Combining (70), (71), (78), and (79), we see that, for every choice of \( \mathcal{P}(\mathbf{H}) \), the capacity region of a uniformly strong ergodic fading IFC lies within the capacity region of a C-MAC for which the fading states satisfy (12). Thus, over all power policies, we have

\[
\mathcal{C}_{\text{C-MAC}}(P_1^{\text{avg}}, P_2^{\text{avg}}) \subseteq \mathcal{C}_{\text{C-IFC}}(P_1^{\text{avg}}, P_2^{\text{avg}}). \tag{80}
\]

**Achievable Strategy:** Allowing both receivers to decode both messages as stated in Corollary 1 achieves the outer bound. For the resulting C-MAC, the uniformly strong condition in (12) limits the intersection of the rate regions \( \mathcal{C}_{\text{C-MAC}}(\mathcal{P}(\mathbf{H})) \) and \( \mathcal{C}_2(\mathcal{P}(\mathbf{H})) \), for any choice of \( \mathcal{P}(\mathbf{H}) \), to one of cases 1, 3a, 3b, 3c, or the boundary cases \( (1, m) \) for \( m = 3a, 3b, 3c \), such that (70) defines the single-user rate bounds.

The sum-capacity optimal policy for each of the above cases is given by Theorem 8. Thus, the optimal user policies are single-user waterfilling solutions when the uniformly strong fading IFC also satisfies (13), i.e., the optimal policies satisfy the conditions for case 1. For all other cases, the optimal policies are opportunistic multiuser allocations. Specifically, cases 3a and 3b the solutions are the classical multiuser waterfilling solutions [20].

One can similarly develop the optimal policies that achieve the capacity region. Here too, for every point \( \mu_1 R_1 + \mu_2 R_2, \mu_1, \mu_2 \), on the boundary of the capacity region, the optimal policy \( P^*(\mathbf{H}) \) is either \( P^{(1)}(\mathbf{H}) \) or \( P^{(n)}(\mathbf{H}) \) or \( P^{(1,n)}(\mathbf{H}) \) for \( n = 3a, 3b, 3c \).

**Separability:** See Remark 9.

3) **Uniformly Mixed IFC:** Proof of Theorem 4: The proof of Theorem 4 follows directly from bounding the sum-capacity a UM IFC by the sum-capacities of a UW one-sided IFC and a US one-sided IFC that result from eliminating links one of the two interfering links. Achievability follows from using the US coding scheme for the strong user and the UW coding scheme for the weak user.

### B. Proofs for Separable IFCs

1) **Uniformly Weak One-Sided IFC:** Proof of Theorem 5: We now prove Theorem 5 on the sum-capacity of a sub-class of one-sided ergodic fading IFCs where every sub-channel is weak, i.e., the channel is uniformly weak. We show that it is optimal to ignore the interference at the unintended receiver.

**Converse:** From Fano’s inequality, any achievable rate pair \((R_1, R_2)\) must satisfy

\[
n(R_1 + R_2) - ne \leq I(X_1^n; Y_1^n|\mathbf{H}^n) + I(X_2^n; Y_2^n|\mathbf{H}^n). \tag{81a}
\]

We first consider the expression on the right-side of (81a) for some instantiation \( \mathbf{h}^n \), i.e., consider

\[
I(X_1^n; Y_1^n|\mathbf{h}^n = \mathbf{h}^n) + I(X_2^n; Y_2^n|\mathbf{h}^n = \mathbf{h}^n)
= I(X_1^n; h_{1,1} h_{1,1} X_1^n + h_{1,2} X_2^n + Z_1^n + h_{2,1} X_1^n + h_{2,2} X_2^n + Z_2^n)
= I(X_1^n; h_{1,1} h_{1,1} X_1^n + h_{1,2} X_2^n + Z_1^n)
+ I(X_2^n; h_{2,2} X_2^n + Z_2^n) \tag{82}
\]

where \( \mathbf{h}_{ij} \) is a diagonal matrix with diagonal entries \( h_{ij} \), for all \( i = 1, 2, \ldots, n \). Let \( N^n \) be a sequence of independent Gaussian random variables, such that

\[
\left[\begin{array}{c}
Z_{1,i} \\
N_i
\end{array}\right] \sim \mathcal{CN}\left(0, \begin{bmatrix} 1 & \rho_i \sigma_i \\ \rho_i \sigma_i & \sigma_i^2 \end{bmatrix}\right) \tag{83}
\]

and

\[
\rho_i^2 = 1 - \left(\frac{|h_{1,2,i}|^2}{|h_{2,2,i}|^2}\right) \tag{84}
\]

\[
\rho_i \sigma_i = 1 + \frac{|h_{2,2,i}|^2 P_{2,i}}{P_{2,i}}. \tag{85}
\]

Let \( X_{k,i}^n \sim \mathcal{CN}(0, P_{k,i}) \) for all \( i \). We bound (82) as follows:

\[
I(X_1^n; Y_1^n|\mathbf{h}^n) + I(X_2^n; Y_2^n|\mathbf{h}^n)
\leq I(X_1^n; Y_1^n, h_{1,1} X_1^n + h_{1,2} X_2^n + Z_1^n|\mathbf{h}^n) + I(X_2^n; Y_2^n|\mathbf{h}^n)
= h_{1,2} X_2^n + Z_2^n - h_{1,1} X_1^n + Z_1^n
= h_{1,2} X_2^n + Z_1^n
\]

\[
\sum_{i=1}^n h_{1,1,i} X_{1,i}^n + N_i - \sum_{i=1}^n h_{2,2,i} Z_{2,i}
\]

\[
= \sum_{i=1}^n \left\{ h_{1,1,i} X_{1,i}^n + N_i - h_{2,2,i} Z_{2,i} \right\}
= \sum_{i=1}^n \left\{ h_{1,1,i} X_{1,i}^n + N_i \right\}
\]

\[
\log \left( |h_{1,1,i}|^2 P_{1,i} + \sigma_i^2 \right) - h(\sigma_i)
\]

\[
+ \log \left( |h_{1,2,i}|^2 P_{2,i} + 1 \right)
\]

\[
- \log \left( |h_{1,2,i}|^2 P_{2,i} + (1 - \rho_i^2) \right)
\]

\[
+ \log \left( |h_{1,1,i}|^2 P_{1,i} + |h_{1,2,i}|^2 P_{2,i} + 1 \right)
\]

\[
- \left( |h_{1,1,i}|^2 P_{1,i} + \sigma_i \right) - 1 \left( |h_{1,1,i}|^2 P_{1,i} + \rho_i \sigma_i \right)^2
\]

\[
= \sum_{i=1}^n \log \left( |h_{2,2,i}|^2 P_{2,i} + 1 \right)
\]

\[
+ \sum_{i=1}^n \log \left( 1 + \frac{|h_{1,1,i}|^2 P_{1,i}}{|h_{1,2,i}|^2 P_{2,i}} \right)
\]

where (88) follows from the fact that conditioning does not increase entropy and that the conditional entropy is maximized by Gaussian signaling, which we denote for every channel use \( i \) by
a random variable \( X_{k,i}^* \sim N(0, P_{k,i}) \); (87) follows from applying chain rule for the resulting mutual information expressions in (86) and expanding the terms as difference of entropies; (89a) follows from (83) and (84) which imply that

\[
\text{var}\left(h_{1,i}^{-1} X_{1,i}^*|N_i\right) = 1 - \rho_{1,i}^2 = |h_{2,i}^2|^{-2}
\]

where \( \text{var} \) denotes the variance. Therefore, we have

\[
h\left(h_{2,i} X_{2,i}^* + Z_{2,i}^*\right) - h\left(h_{2,i} X_{2,i}^* + Z_{2,i}^*|N_i^n\right) \]
\[
= \log\left|h_{2,i}^2\right| - \log\left|h_{1,i}^2\right|
\]
\[
= \sum_{i=1}^n h\left(h_{2,i} X_{2,i}^* + Z_{2,i}ight) - h\left(h_{1,i} X_{2,i}^* + Z_{1,i}\right) N_i
\]

and (89c) follows from substituting (85) in (89b) and simplifying the resulting expressions.

Our analysis from here on is similar to that for the US IFC (see also [20, Appendix A]). Effectively, the analysis again involves considering an increasing sequence of partitions (quanti- zed ranges) \( I_k, k \in \mathbb{T}^+ \), of the alphabet of \( H \), while ensuring that for each \( k \), the transmitted signals are constrained in power. Taking limits appropriately over \( n \) and \( k \) and using the fact that the \( \log \) expressions in (89c) are concave functions of \( P_{k,i} \), for all \( k \), and that every feasible power policy satisfies (4), we obtain

\[
R_1 + R_2 - \epsilon \leq \mathbb{E}\left[C\left(\frac{|H_{2,i}|^2 P_1(H)}{1 + |H_{1,i}|^2 P_2(H)}\right)\right] + \mathbb{E}\left[C\left(\frac{|H_{1,i}|^2 P_2(H)}{1 + |H_{1,i}|^2 P_2(H)}\right)\right].
\]

An outer bound on the sum-rate is obtained by maximizing over all feasible policies and is given by (27) and (26).

**Achievable Strategy:** The outer bounds can be achieved by letting receiver 1 ignore (not decode) the interference it sees from transmitter 2. Averaged over all sub-channels, the sum of the rates achieved at the two receivers for every choice of \( P(H) \) is given by (92a). The sum-capacity in (27) is then obtained by maximizing (92a) over all feasible \( P(H) \).

**Separability:** The optimality of separate encoding and decoding across the sub-channels follows directly from the fact that the sub-channels are all of the same type, and thus, independent messages can be multiplexed across the sub-channels. This is in contrast to the uniformly strong and the ergodic very strong IFCs in which mixtures of different channel types in both cases is exploited to achieve the sum-capacity by encoding and decoding jointly across all sub-channels.

**Remark 22:** A natural question is whether one can extend the techniques developed here to the two-sided UW IFC. In this case, one would have four parameters per channel state, namely \( \rho_k(H) \) and \( \sigma^2_k(H), k = 1, 2 \). Thus, for example, one can generalize the techniques in [5, Proof of Th. 2] for a fading IFC with non-negative real \( H_{j,k} \) for all \( j, k \), such that \( H_{1,1} > H_{2,1} \) and \( H_{2,2} > H_{1,2} \), to outer bound the sum-rate by

\[
\mathbb{E}\left[C\left(\frac{|H_{1,1}|^2 P_1(H)}{1 + |H_{1,1}|^2 P_2(H)}\right)\right] + \mathbb{E}\left[C\left(\frac{|H_{2,2}|^2 P_1(H)}{1 + |H_{2,2}|^2 P_2(H)}\right)\right].
\]

This implies that for a given fading statistics, every choice of feasible power policies \( P(H) \) must satisfy the condition in (94). With the exception of a few trivial channel models, the condition in (94) cannot in general be satisfied by all power policies. One approach is to extend the results on sum-capacity and the related noisy interference condition for PGICs in [16, Proof of Th. 3] to ergodic fading IFCs. Despite the fact that ergodic fading channels are simply a weighted combination of parallel sub-channels, extending the results in [16, Proof of Th. 3] are not in general straightforward.

**C. Achievable Schemes and Outer Bounds**

1) **Hybrid One-Sided IFC: Proof of Theorem 6:** The bound in (28) can be obtained from the following code construction: user 1 encodes its message \( w_1 \) across all sub-channels by constructing independent Gaussian codebooks for each sub-channel to transmit the same message. On the other hand, user 2 transmits two messages \( (w_{2p}, w_{2c}) \) jointly across all sub-channels by constructing independent Gaussian codebooks for each sub-channel to transmit the same message pair. The messages \( w_{2p} \) and \( w_{2c} \) are transmitted at (fading averaged) rates \( R_{2p} \) and \( R_{2c} \), respectively, such that \( R_{2p} + R_{2c} = R_2 \). Thus, across all sub-channels, one may view the encoding as a Han Kobyashi coding scheme for a one-sided nonfading IFC in which the two transmitted signals in each use of sub-channel \( H \) are

\[
X_1(H) = \sqrt{P_1(H)} V_1(H), \quad \text{and} \quad X_2(H) = \sqrt{\alpha_H P_2(H)} V_2(H) + \sqrt{\alpha_H P_2(H)} U_2(H)
\]

where \( V_1(H) \), \( V_2(H) \), and \( U_2(H) \) are independent zero-mean unit variance Gaussian random variables, and for all \( H, \alpha_H \in [0, 1] \), and \( \alpha_H \) are the power fractions allocated for \( w_{2p} \) and \( w_{2c} \), respectively. Thus, over \( n \) uses of the channel, \( w_{2p} \) and \( w_{2c} \) are encoded via \( V_{2p} \) and \( U_{2c} \), respectively.
Receiver 1 decodes $w_1$ and $w_{2p}$ jointly and receiver 2 decodes $w_{2p}$ and $w_{2s}$ jointly across all channel states provided

$$R_{2p} \leq E \left[ C \left( H_{2s}^2 \alpha_{H} P(\mathbf{H}) \right) \right]$$  \hspace{1cm} (97a)

$$R_{2p} + R_{2s} \leq E \left[ C \left( H_{2s}^2 P(\mathbf{H}) \right) \right]$$  \hspace{1cm} (97b)

$$R_{1} \leq E \left[ C \left( \frac{H_{1s}^2 P(\mathbf{H})}{1 + H_{1s}^2 \alpha_{H} P(\mathbf{H})} \right) \right]$$  \hspace{1cm} (98a)

$$R_{2s} \leq E \left[ C \left( \frac{H_{1s}^2 \alpha_{H} P(\mathbf{H})}{1 + H_{1s}^2 \alpha_{H} P(\mathbf{H})} \right) \right], \text{ and}$$  \hspace{1cm} (98b)

$$R_{1} + R_{2s} \leq E \left[ C \left( \frac{H_{1s}^2 P(\mathbf{H})}{1 + H_{1s}^2 \alpha_{H} P(\mathbf{H})} \right) \right].$$  \hspace{1cm} (98c)

Using Fourier-Motzkin elimination, we can simplify the bounds in (97) and (98) to obtain

$$R_{1} \leq E \left[ C \left( \frac{H_{1s}^2 P(\mathbf{H})}{1 + H_{1s}^2 \alpha_{H} P(\mathbf{H})} \right) \right]$$  \hspace{1cm} (99a)

$$R_{2} \leq E \left[ C \left( \frac{H_{1s}^2 P(\mathbf{H})}{1 + H_{1s}^2 \alpha_{H} P(\mathbf{H})} \right) \right]$$  \hspace{1cm} (99b)

$$R_{2} \leq E \left[ C \left( \frac{H_{1s}^2 \alpha_{H} P(\mathbf{H})}{1 + H_{1s}^2 \alpha_{H} P(\mathbf{H})} \right) \right]$$  \hspace{1cm} (99c)

$$R_{1} + R_{2} \leq E \left[ C \left( \frac{H_{1s}^2 P(\mathbf{H})}{1 + H_{1s}^2 \alpha_{H} P(\mathbf{H})} \right) \right]$$  \hspace{1cm} (99d)

Combining the bounds in (99), for every choice of $(\alpha_{H}, P(\mathbf{H}))$, the sum-rate is maximized by the minimum of two functions $S_{1}(\alpha_{H}, P(\mathbf{H}))$ and $S_{2}(\alpha_{H}, P(\mathbf{H}))$, where $S_{1}(P(\mathbf{H}))$ is the sum of the bounds on $R_1$ and $R_2$ in (99a) and (99b), respectively, and $S_{2}(\alpha_{H}, P(\mathbf{H}))$ is the bound on $R_1 + R_2$ in (99d). The bound on $R_1 + R_2$ from combining (99a) and (99c) is at least as much as (99d), and, hence, is inferior.

The maximization of the minimum of $S_{1}(P(\mathbf{H}))$ and $S_{2}(\alpha_{H}, P(\mathbf{H}))$ can be shown to be equivalent to a minimax optimization problem (see, for example, [38, II.C]) for which the maximum sum-rate $S^*$ is given by three cases. The three cases are defined below. Note that in each case, the optimal $P^*(\mathbf{H})$ and $\alpha^*_{H}$ maximize the smaller of the two functions and, therefore, maximize both in case when the two functions are equal. The three cases are

$$S^* = S_{1}(\alpha^*_{H}, P^*(\mathbf{H})) < S_{2}(\alpha^*_{H}, P^*(\mathbf{H}))$$ \hspace{1cm} (100a)

$$S^* = S_{2}(\alpha^*_{H}, P^*(\mathbf{H})) < S_{1}(\alpha^*_{H}, P^*(\mathbf{H}))$$ \hspace{1cm} (100b)

$$S^* = S_{1}(\alpha^*_{H}, P^*(\mathbf{H})) = S_{2}(\alpha^*_{H}, P^*(\mathbf{H})).$$ \hspace{1cm} (100c)

Thus, for Cases 1 and 2, the minimax policy is the policy maximizing $S_{1}(P(\mathbf{H}))$ and $S_{2}(\alpha_{H}, P(\mathbf{H}))$ subject to the conditions in (100a) and (100b), respectively, while for Case 3, it is the policy maximizing $S_{1}(P(\mathbf{H}))$ subject to the equality constraint in (100c). We now consider this maximization problem for each sub-class. Before proceeding, we observe that, $S_{1}(\cdot)$ is maximized for $\alpha^*_{H} = 0$ and $P_{k}(\mathbf{H}) = P_{k}(\mathbf{H})$ for $k = 1, 2$. On the other hand, the $\alpha^*_{H}$ maximizing $S_{2}(\alpha_{H}, \cdot)$ depends on the sub-class.

Uniformly Strong: The bound $S_{2}(\alpha_{H}, P(\mathbf{H}))$ in (99d) can be rewritten as

$$E \left[ C \left( H_{1s}^2 \alpha_{H} P(\mathbf{H}) \right) \right] - E \left[ C \left( H_{1s}^2 \alpha_{H} P(\mathbf{H}) \right) \right]$$

$$+ E \left[ C \left( H_{1s}^2 P(\mathbf{H}) + H_{1s}^2 \alpha_{H} P(\mathbf{H}) \right) \right] + \left(101\right)$$

and thus, when $|h_{1,2}| > |h_{2,2}|$ for every fading instantiation, for every choice of $P(\mathbf{H})$, $S_{2}(\alpha_{H}, P(\mathbf{H}))$ is maximized by $\alpha^*_{H} = 0$, i.e., $w_{2} = w_{2s}$. The sum-capacity is given by (23) with $H_{2,1} = \infty$ (this is equivalent to a genie aiding one of the receivers thereby simplifying the sum-capacity expression in (23) for a two-sided IFC to that for a one-sided IFC). Furthermore, $\alpha^*_{H} = 0$ also maximizes $S_{1}(\alpha_{H}, P(\mathbf{H}))$. In conjunction with the outer bounds for US IFCs developed earlier, the US sum-capacity and the optimal policy achieving it are obtained via the minimax optimization problem with $\alpha^*_{H} = 0$ such that every sub-channel carries the same common information.

Uniformly Weak: For this sub-class of channels, it is straightforward to verify that for $\alpha^*_{H} = 0$ (100a) will not be satisfied. Thus, one is left with Cases 2 and 3. From Theorem 5, we have that $\alpha^*_{H} = 1$ achieves the sum-capacity of one-sided UW IFCs, i.e., $w_{2} = w_{2s}$. Furthermore, $S_{2}(1, P(\mathbf{H})) = S_{1}(1, P(\mathbf{H}))$, and thus, the condition for Case 2 is not satisfied, i.e., this sub-class corresponds to Case 3 in the minimax optimization. The constrained optimization in (100c) for Case 3 can be solved using Lagrange multipliers though the solution is relatively easier to develop using techniques in Theorem 5.

Ergodic Very Strong: As mentioned before, $S_{1}(\cdot)$ is maximized for $\alpha^*_{H} = 0$ and $P_{k}(\mathbf{H}) = P_{k}(\mathbf{H})$, $k = 1, 2$, i.e., when $w_{2} = w_{2s}$; and each user applies waterfilling on its intended link. From (100), we see that the sum-capacity of EVS IFCs is achieved provided the condition for Case 1 in (100) is satisfied. Note that this maximization does not require the sub-channels to be UW or US.

Hybrid: When the condition for Case 1 in (100) with $\alpha^*_{H} = 0$ is satisfied, we obtain an EVS IFC. On the other hand, when this condition is not satisfied, the optimization simplifies to considering Cases 2 and 3, i.e., $\alpha^*_{H} \neq 0$ for all $\mathbf{H}$. Using the linearity of expectation, we can write the expressions for $S_{1}(\cdot)$ and $S_{2}(\cdot)$ as sums of expectations of the appropriate bounds over the collection of weak and strong sub-channels. Let $S_{1}(w)(\cdot)$ and $S_{2}(s)(\cdot)$ denote the expectation over the weak and strong sub-channels, respectively, for $k = 1, 2$, such that $S_{k}(\cdot) = S_{k}(w)(\cdot) + S_{k}(s)(\cdot)\), $k = 1, 2$.

Consider Case 2 first. For those sub-channels which are strong, one can use (101) to show that $\alpha^*_{H} = 0$ maximizes $S_{2}(s)(\cdot)$. Suppose we choose $\alpha^*_{H} = 1$ to maximize $S_{2}(s)(\cdot)$. From the UW analysis earlier, $S_{2}(s)(1, P(\mathbf{H})) = S_{1}(1, P(\mathbf{H}))$ and, therefore, (100b) is satisfied only when $S_{2}(s)(0, P(\mathbf{H})) < S_{1}(0, P(\mathbf{H}))$. This requirement may not hold in general, and thus, to satisfy (100b), we require that $\alpha^*_{H} \in [0, 1]$ for those $\mathbf{H}$ that represent weak sub-channels. Similar arguments hold for Case 3 too thereby justifying (31) in Theorem 6.
Remark 23: The bounds in (97) are written assuming superposition coding of the common and private messages at transmitter 2. The resulting bounds following Fourier-Motzkin elimination remain unchanged even if we included an additional bound on $R_2$, at receiver 2 in (97).

2) Uniformly Weak IFC: Proof of Theorem 7: The proof of Theorem 7 follows directly from bounding the sum-capacity a UW IFC by that of a UW one-sided IFC that results from eliminating one of the interfering links (eliminating an interfering link can only improve the capacity of the network). Since two complementary one-sided IFCs can be obtained thusly, we have two outer bounds on the sum-capacity of a UW IFC denoted by $S^{(u1)}(\mathbf{P}(\mathbf{H}))$ and $S^{(u2)}(\mathbf{P}(\mathbf{H}))$ in (33), where $S^{(u1)}(\mathbf{P}(\mathbf{H}))$ and $S^{(u2)}(\mathbf{P}(\mathbf{H}))$ are the bounds for one-sided UW IFCs with $H_{2,1} = 0$ and $H_{1,2} = 0$, respectively.

VI. DISCUSSION

A. Computing the Optimal Policies

When the channel statistics are assumed to be known a priori, the optimal policies for those sub-classes for which the sum-capacity is known can be computed beforehand. Furthermore, since the transmitters are also assumed to know the instantaneous channel state information, allocation from the optimal policies which are a function of the fading states can be done in each time symbol. Computing the optimal policies for discrete channels is relatively straightforward using standard optimization techniques (as summarized briefly for the C-MAC model). For continuous fading models, a closed form expression may not always be easy to derive and numerical approaches may be needed. In the examples we present shortly, we quantize a Rayleigh fading channel into a large number (≈10,000) of discrete states and determine the optimal policies for this model.

B. Illustration of Results

We now present an example for which the channel states satisfy the EVS condition. Without loss of generality we assume that the direct links are nonfading. We assume that the cross-links are independent and identically distributed Rayleigh faded links, i.e., $H_{jk} \sim CN(0,\sigma^2/2)$ for all $j \neq k, j, k = 1, 2$. Thus, for the case in which the fading statistics and average power constraints $P^{avg}_k$ satisfy the EVS conditions in (13), it is optimal for transmitter $k$ to transmit at $P^{k}_{avg}$. Finally, we set $P^{avg}_1 = P^{avg}_2 = P^{avg}$. For computational ease, for the plots below we first quantize the Rayleigh fading channel to a large number of states (10,000) and evaluate the maximum average power $P^{avg}$ for which the EVS condition holds for a chosen $\sigma^2$.

From (8), we see that for a nonfading very strong IFC with a given channel gain, the very strong condition sets an upper bound on the average transmit power $P^{avg}_k$. In the ergodic case, not every fading state is required to be strong or very strong for the EVS conditions to be satisfied. However, one can expect that for the EVS condition to be satisfied (on average), the channel statistics must bound the average power analogously to the nonfading bound in (8). Our first plot seeks to understand the relationship between the fading variance $\sigma^2$ and the average transmit power $P^{avg}$. For every choice of the Rayleigh fading variance $\sigma^2$, we determine $P^{avg}_{max}$, the maximum $P^{avg}$, for which the EVS conditions in (13) hold. The resulting feasible $P^{avg}$ versus $\sigma^2$ region is plotted in Fig. 7(a). Our numerical results indicate that for very small values of $\sigma^2$, i.e., $\sigma^2 < 1.5$, where the cumulative distribution of fading states with $|H_{jk}| < 1$ is close to 1, the EVS condition cannot be satisfied by any finite value of $P^{avg}$, however small. As $\sigma^2$ increases thereby increasing the likelihood of $|H_{jk}| > 1$, $P^{avg}$ increases too. One can thus view $P^{avg}$ for the EVS IFCs in Fig. 7 as an equivalent fading-averaged bound. Also plotted in Fig. 7(b) is the EVS sum-capacity achieved at $P^{avg}_{max}$, the maximum $P^{avg}$ for every choice of $\sigma^2$.

A natural issue that arises is the rate loss resulting if sub-optimal schemes such as time-sharing and interference as noise were used. However, for interference as noise scheme, no effective algorithm exists to compute the sum-rate maximizing power policies and the problem is particularly intractable for a large number of fading states as approximated for the Rayleigh fading channel here. To this end, we compare the EVS sum-capacity with a relatively simpler achievable scheme of time-sharing. For the specific symmetric setup we consider, sharing the bandwidth equally between the two users maximizes the sum-rate. This is shown as the dashed curve in Fig. 7(b). As expected, the rate loss of time-sharing relative to the EVS sum-capacity is approximately half.

We next compare the effect of joint and separate coding for one-sided EVS and US IFCs. For computational simplicity, we consider a discrete fading model where the nonzero cross-link fading state takes values in a binary set $\{h_1, h_2\}$ while the direct links are nonfading unit gains. For a one-sided EVS IFC, we choose $(h_1, h_2) = (0.5, 3.5)$ and $P^{avg}_{1} = P^{avg}_{2} = P^{avg}_{max}$ where $P^{avg}_{max}$ is the maximum power for which the EVS conditions in (13) are satisfied. Note that only one of the conditions are relevant since it is a one-sided IFC. In Fig. 8, the EVS sum-capacity is plotted along with the sum-rate achieved by independent coding in each sub-channel as a function of the probability $p_1$ of the fading state $h_1$. Here independent coding means that each sub-channel is viewed as a nonfading one-sided IFC and the sum-capacity achieving strategy for each sub-channel is applied.

As expected, as $p_1 \to 0$ or $p_1 \to 1$, the sum-rate achieved by separable coding approaches the joint coding scheme. Thus, the difference between the optimal joint coding and the sub-optimal independent coding schemes is the largest when both fading states are equally likely. In contrast to this example where the gains from joint coding are not negligible, we also plot in Fig. 8 the sum-capacity and sum-rate achieved by independent coding for an EVS IFC with $(h_1, h_2) = (0.5, 2.0)$ for which the rate difference is very small. Thus, as expected, joint coding is advantageous when the variance of the cross-link fading is large and the transmit powers are small enough to result in an EVS IFC. In the same plot, we also compare the sum-capacity with the sum-rate achieved by a separable scheme for two US IFCs, one given by $(h_1, h_2) = (1.25, 1.75)$ and the other by $(h_1, h_2) = (1.25, 3.75)$. As with the EVS examples, here too, the rate difference between the optimal joint strategy and the, in general, sub-optimal independent strategy increases with increasing variance of the fading distribution.

One can similarly compare the performance of independent and joint coding for two-sided EVS and US IFCs. In this case,
the more general HK scheme needs to be considered in each fading state for the independent coding case. In general, the observations on separability for the one-sided IFC also extend to the two-sided IFC.

Finally, we demonstrate sum-rates achievable by Theorem 6 for a hybrid one-sided IFC. As before, for computational simplicity, we consider a discrete fading model where the cross-link fading states take values in a binary set \{h_1, h_2\} while the direct links are nonfading unit gains. Without loss of generality, we choose \( (h_1, h_2) = (0.5, 2.0) \) and assume \( P_1^{\text{avg}} = P_2^{\text{avg}} = P^{\text{avg}} \). The sum-rate achieved by the proposed HK-like scheme, denoted \( R^{\text{HK}}_{\text{sum}} \), is determined as a function of the probability \( p_1 \) of the weak state \( h_1 \). For each \( p_1 \), using the fact that a hybrid IFC is by definition one for which the EVS condition is not satisfied, we choose \( P^{\text{avg}}(p_1) = P^{\text{avgEV}}_{\text{HS}}(p_1) + 1.5 \) where \( P^{\text{avgEV}}_{\text{HS}}(p_1) \) is the maximum \( P^{\text{avgEV}} \) for which the EVS conditions hold for the chosen \( p_1 \) and \( (h_1, h_2) \).

In Fig. 9(a), we plot \( R^{\text{HK}}_{\text{sum}} \) as a function of \( p_1 \). We also plot the largest sum-rate outer bounds \( R^{\text{OB}}_{\text{sum}} \) obtained by assuming interference-free links from the users to the receivers. Finally,
for comparison, we plot the sum-rate $R_{\text{sum}}^{(\text{Ind})}$ achieved by a separable coding scheme in each sub-channel. This separable coding scheme is simply a special case of the HK-based joint coding scheme presented for hybrid one-sided IFCs in Theorem 6 obtained by choosing $\alpha_{H}^* = 0$ and $\alpha_{h}^* = 1$ in the strong and weak sub-channels, respectively. Thus, $R_{\text{sum}}^{(\text{Ind})} \leq R_{\text{sum}}^{(\text{HK})}$ as demonstrated in the plot. In Fig. 9(b), the fractions $\alpha_{h_1}^*$ and $\alpha_{h_2}^*$ in the $h_1$ (weak) and the $h_2$ (strong) states, respectively, are plotted. As expected, $\alpha_{h_2}^* = 0$; on the other hand, $\alpha_{h_1}^*$ varies between 0 and 1 such that for $p_1 \to 1$, $\alpha_{h_1}^* \to 1$ and for $p_1 \to 1$, $\alpha_{h_1}^* \to 1$. Thus, when either the weak or the strong state is dominant, the performance of the HK-based coding scheme approaches that of the separable scheme in [34].

VII. CONCLUSION

We have developed the sum-capacity of specific sub-classes of ergodic fading IFCs. These sub-classes include the ergodic very strong (mixture of weak and strong sub-channels satisfying the EVS condition), the uniformly strong (collection of strong sub-channels), the uniformly weak one-sided (collection of weak one-sided sub-channels) IFCs, and the uniformly mixed (mix of UW and US one-sided IFCs) two-sided IFCs. Specifically, we have shown that requiring both receivers to decode both messages, i.e., simplifying the IFC to a compound MAC, achieves the sum-capacity and the capacity region of the EVS and US (one-and two-sided) IFCs. For both sub-classes, achieving the sum-capacity requires encoding and decoding jointly across all sub-channels.

In contrast, for the UW one-sided IFCs, we have used genie-aided methods to show that the sum-capacity is achieved by ignoring interference at the interfered receiver and with independent coding across fading states. This approach also allowed us to develop outer bounds on the two-sided UW IFCs. We have combined the UW and US one-sided IFCs results to develop the sum-capacity for the uniformly mixed two-sided IFCs and have shown that joint coding is optimal.

For the final sub-class of hybrid one-sided IFCs with a mix of weak and strong sub-channels that do not satisfy the EVS conditions, using the fact that the strong sub-channels can be exploited, we have proposed a Han-Kobayashi based achievable scheme that allows partial interference cancellation using a joint coding scheme. Under the assumption of no time-sharing, we have shown that the sum-rate is maximized by transmitting only a common message on the strong sub-channels and transmitting a private message in addition to this common message in the weak sub-channels. Proving the optimality of this scheme for the hybrid sub-class remains open. However, we have also shown that the proposed joint coding scheme applies to all sub-classes of one-sided IFCs and, therefore, encompasses the sum-capacity achieving schemes for the EVS, US, and UW sub-classes.

Analogously with the nonfading IFCs, the ergodic capacity of a two-sided IFC continues to remain unknown in general. However, additional complexity arises from the fact that the fading states can in general be a mix of weak and strong IFCs. A direct result of this complexity is that, in contrast to the nonfading case, the sum-capacity of a one-sided fading IFC remains open for the hybrid sub-class. The problem similarly remains open for the two-sided fading IFC. An additional challenge for the two-sided IFC is that of developing tighter bounds for the uniformly weak channel. A related question that arises is whether unlike the sub-classes for which we have developed capacity results here, the channel phase information will be pertinent to the remaining sub-classes.

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