Determination of the Mandelstam – Brillouin Scatter Frequency Characteristic in Optical Fibers of Various Types

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Abstract. The results of researches of the Mandelstam – Brillouin scattering frequency characteristics in single-mode optical fibers of various types are presented in this paper. A technique for obtaining the spectral characteristics of Mandelstam – Brillouin scattering for predefined acoustic mode distributions in single-mode optical fibers is considered. The procedure for determining the frequency characteristics of the Mandelstam – Brillouin scattering for a given structure of the radial distribution of the optical fiber refractive index is discussed. The Mandelstam – Brillouin scattering frequency characteristics of single-mode optical fibers obtained by mathematical modeling and experimental studies are presented. A comparative analyze of obtained results is carried out.

Index Terms. Single-mode optic fiber, fiber core structure, Brillouin reflectometry, Mandelstam – Brillouin backscatter frequency characteristics, Brillouin frequency shift, classification of the optical fiber type.

1. Introduction
The timely detection of mechanically stressed sections in optical fibers (OFs) located in laid optical cables (OCs) is an important task of early diagnosis of the fiber physical state [1]. The Brillouin reflectometry method is applied for the timely detection of such fiber sections [1 – 5]. The Brillouin frequency shift (BSF – \( f_B \)) is determined by the main maximum of the Mandelstam – Brillouin backscatter spectrum (BSS), which requires obtaining and subsequent analysis of the frequency characteristics of the Mandelstam – Brillouin scattering (BS) in the OF. Currently, there are many fiber types, each of which is optimized for solving certain problems. Classification of the specific fiber type is an important task of early diagnosis of the physical state of the OC [6 – 8].

2. Theory. Statement of the problem
In order to obtain the frequency characteristics of BS (BSS graphs) in single-mode optical fibers with known distributions of acoustic modes for given fiber types, it is required to evaluate the parameters of
acousto-optical interaction in the fiber and determine the contribution of the frequency characteristics of the each lower-order acoustic modes to the formation of the final BSS.

The BS frequency characteristics (BSS graphs) is influenced by the dependence of the profile of acoustical modes and their directionality, which depend on the fiber layers structure and on the varieties and concentrations of various dopants in the OF core.

The properties of acoustic modes determine the nature of the hypersonic waves propagation in the OF and their interaction with the optic signal [1–8].

Wave processes of changes in the density of the propagation medium considering the existence of various acoustical modes in the OF, should be sought in the following form:

\[ \rho(z,t,r,\varphi) = \frac{1}{2} \sum_{m=1}^{M} \xi_m (r, \varphi) \rho_m (z,t) e^{i(\omega_m t - k_m z)} , \]

where \( m \) is an order of the acoustical mode, \( \rho_m (z,t) \) is the longitudinal change in the density of the medium along the propagation axis \( z \) over time \( t \) of the \( m^{th} \)-order acoustical mode, \( \xi_m (r, \varphi) \) is the distribution of the acoustical mode amplitude over the cross-section, \( \omega_m \) is a cyclic frequency, \( M \) is number of accounted acoustical modes, and \( k_m \) is a acoustic wave wavenumber [1, 5].

In single-mode OF, the main optical mode \( LP_0 \) (or \( HE_{11} \) in the different interpretation) is scattered in the back direction only by axially symmetrical acoustical modes [1–4, 9–15]. For describing the BS process in the single-mode fiber, it is required to consider the longitudinal and radial distribution, as well as the interaction of the main optical and lower order acoustical modes [4–8].

The axial symmetry of the fundamental optic mode distribution and its interaction with acoustical modes usually makes it possible to exclude a considerable number of different acoustical modes from the analysis \( (\xi_m (r, \varphi) \rightarrow \xi_{m_0} (r)) \).

To estimate the features of the acoustical mode for a specified refractive index function \( n(r) \), it is required to solve the system of equations for lower orders acoustical modes \( \xi_m (r) \) and the main optical mode \( F(r) \) [1, 5, 9, 12].

The scattering coefficient per unit length of the fiber (\( g_{am} \)) for the \( m^{th} \)-order acoustical mode is defined by the formula:

\[ g_{am} = p_{12} \omega L \beta_m b \int_{0}^{b} \rho_m (z,r) \xi_m (r) F^2 (r) rdr , \]

where \( p_{12} \) is the longitudinal acousto-optical factor \( (p_{12} \approx 0.27) \) [1, 5, 9], \( \omega_L = 2\pi f_L = 2\pi c / \lambda_L \) is the cyclic frequency of the laser, \( \beta_m \) is the phase coefficient for the \( m^{th} \)-order acoustic mode, \( b \) is the fiber cladding radius (acoustic modes penetrating from the fiber core into the cladding should also be considered), \( c \) is a light wave speed in the vacuum.

The contribution of each \( m^{th} \)-order acoustical mode is estimated by the formulas [1, 5, 9, 12]

\[ A_{Am} (f) = \frac{g_{an}^{2} \pi T_m}{(f - f_L + f_m)^2 + (\pi T_m)^2} = \frac{g_{bn}}{A_{Am} (f - f_L - f_m)^2 + (\Delta f_{Wbm} / 2)^2} = \frac{1}{A_{Am}} \frac{(\Gamma_m / 2)^2}{(f - f_L - f_m)^2 + (\Gamma_m / 2)^2} \] ,

where \( \Delta f_{Wbm} \) is the bandwidth of the BSS (Brillouin amplification) (at the half-power level – FWHM), \( \Gamma_m \) – the damping coefficient of each \( m^{th} \)-order acoustical mode [5, 9], \( g_{bm} = \frac{4\pi n_p^{2} p_{12}}{c\Sigma_{L(R)} B_{12} \Delta f_{Wbm}} \), \( A_{Am} \) is the factor of acousto-optic interaction \( (I_{Am} = A_{Af} / A_{Am}) \) or the overlap coefficient (the overlap integral in other interpretations [4, 12]), \( A_{Af} \) and \( A_{Am} \) corresponding effective acousto-optic interaction areas [1, 5, 9, 12], \( n_{ref} \) is an effective index of the medium refraction.

Different interpretations of \( I_{Am} \), as well as the variety of formulas (3), are associated with replacing the calculation of integral in works [1, 2, 9, 12] by averaging in work [5]. However, this does not affect the final frequency response of the BSS.

If the fiber structure (the number, thickness, physical characteristics and distribution \( n(r) \) of layers, the cross-section fiber profile kind, etc.) is known, then taking into account the formulas given in the
works [1 – 5, 10 – 12], it is possible to define the characteristics of acoustic modes, and then eventually evaluate the BS frequency graphs.

For example, as a result of modeling for G.652-fiber (Recommendation ITU–T G.652), the following values of the corresponding effective acousto-optical interaction areas and “peak” frequencies were obtained: \( A_{41} = 92 \, \mu m^2 \), \( A_{22} = 3918 \, \mu m^2 \), \( A_{43} = 4878 \, \mu m^2 \), \( A_{4p} = 84 \, \mu m^2 \), \( f_i = 10.86 \, GHz \), \( f_2 = 11.00 \, GHz \), \( f_3 = 11.14 \, GHz \).

For G.655-fiber (NZDSF) – \( A_{41} = 131 \, \mu m^2 \), \( A_{22} = 257 \, \mu m^2 \), \( A_{43} = 822 \, \mu m^2 \), \( A_{4p} = 73 \, \mu m^2 \), \( f_i = 10.66 \, GHz \), \( f_2 = 10.85 \, GHz \), \( f_3 = 11.04 \, GHz \).

For SMF-28 (“Corning” G.652) – \( A_{41} = 179 \, \mu m^2 \), \( A_{22} = 211 \, \mu m^2 \), \( A_{43} = 1542 \, \mu m^2 \), \( A_{4p} = 86 \, \mu m^2 \), \( f_i = 10.78 \, GHz \), \( f_2 = 10.87 \, GHz \), \( f_3 = 10.96 \, GHz \).

The research results have an agreement with the dependencies given in other works [2 – 5, 10, 13 – 15].

The BS frequency dependence (\( A(f) \)) in the OF (taking in consider the independence of the acoustic modes contribution [7 – 9, 11 – 13]) is determined by summing the spectral characteristics obtained for each accounted acoustical mode under analysis, using the following formula

\[
A(f) = A_{41}(f) + A_{22}(f) + A_{43}(f) + \ldots + A_{4p}(f),
\]

(4)

In further calculations of the BS frequency characteristics of the OF, the mode graphs which described above in formulas (3) are multiplied by the corresponding coefficient \( I_{\text{thm}} \) (or the like in formulas (3)) taking in consider the order and sign of the mode, and then they are summed over (4) to obtain the final BSS frequency profile.

Differences in the dependences of the lower-order acoustical modes in the interaction of optic and hyperacoustic waves have a significant impact on the BS characteristics (the value of the BFS, the number of “peaks” and their power level) [1 – 15].

3. A definition of the Mandelstam – Brillouin backscatter frequency characteristics in fibers of various types

Fig. 1 – Fig. 4 show the sequence of actions that allow you to obtain the frequency dependences of the BSS frequency profile of the next optic fiber types: G.652, G.653 (DSF), G.655 (NZDSF) and SMF-28 (G.652).

The values along the frequency axis (\( f \)) are indicated in GHz.

In each of these figures, the first graph (on the top) shows the frequency dependences of the velocities (\( v(f) \)) of the lowest-order acoustic modes \( (m = 1, 2 \text{ or } 3) \).

The straight line intersecting them \( (k(f) = (\lambda_f/n_{\text{ef}});f) \) is drawn from the form of the condition of “synchronism”, at which the wave front is back reversed [5, 10]:

\[
k_{\lambda} = 4\pi n_{\text{ef}} / \lambda_f,
\]

(5)

where \( n_{\text{ef}} \) is an effective refraction index of the medium.

The intersection points define the frequencies of the each mode “peaks” (\( f_i \)).

The second graph shows the frequency dependences of the losses (\( \Delta f(f) \)) of acoustical modes, which make it possible to determine their attenuation coefficients.

The third graph shows the frequency profile (\( s(f) \)) for each acoustical mode with damping and \( \Delta f_{\text{BSS}} \) (FWHM), taken in consider the formulas (3).

The fourth (at the bottom) graph shows the final BSS frequency profile.

The sequence of actions for obtaining the BS frequency characteristic of the G.652-OF is shown in Fig. 1. Influence of the frequency characteristics of the lower-order acoustical modes for the BSS formation in the DSF (G.653) is presented in Fig. 2.

Obtaining the BS frequency characteristic of the NZDSF (G.655) is given in Fig. 3.

The action sequence for obtaining the BS frequency characteristic of the SMF-28 is shown in Fig. 4.

Fig. 4 shows differences of the BFS and BSS parameters of the SMF-28 in comparison with the usual OF-G.652 (Fig. 1). The contribution of second- and third-order acoustic modes in these OFs forms the appearance of very small sideline “peaks” in the frequency range from 10.86 GHz to 11.06 GHz.
Figure 1. Influence of the frequency characteristics of the lowest acoustical modes for the formation of the BSS $s(f)$ in the conventional G.652 fiber.
Figure 2. Influence of the frequency characteristics of the lowest acoustical modes for the BSS formation in the DSF (G.653)
Figure 3. Obtaining the BS frequency characteristic of the NZDSF (G.655)
The introduction of dopants into the optical fiber makes it possible to change the mode distribution, which can be used to improve certain optical properties. For example, doping with GeO$_2$ (the concentration is 6%) of the core of an ordinary G.652-fiber allows increasing the power level introduced into the OF by increasing the threshold for the appearance of stimulated BS, which was implemented in the “Corning” SMF-28 [5, 12].

The modes of the second and third orders in DSF are comparable in level with the first (Fig. 2), therefore, in the BSS DSFs side maxima with a level close to the main peak are observed [6].

The levels of these modes in NZDSF are significantly reduced relative to the first (Fig. 3), while the spread of values in the highs and lows decreases. As a result, we observe relatively suppressed second and third peaks. When the profile $n(r)$ is formed with a shift of the maximum from the core axis towards the cladding (or the formation of the “falling section” $n(r)$ towards the OF axis [15]), the frequency of the main acoustic mode will decrease (and hence the BFS) [3, 4], and an increase in the influence of the next lowest acoustic modes will lead to an increase in the side “peaks” in the BSS.

Using fluorine for doping OF allows one to create complex profiles $n(r)$ and impart special properties to OF [12 – 14, 16]. For example, this makes it possible to obtain an OF with an “inverse” frequency.
dependence of dispersion (DCF), which is used to compensate for dispersion. A decrease in the radius of the main layer of the core followed by a strong dip $n(r)$ in front of the cladding leads to a significant decrease in the BFS (less than 10 GHz) [16], which is consistent with the above analysis of the characteristics of the BS.

There are also known examples demonstrating the “suppression” of the first acoustic mode in BSS relative to the second, which leads to a shift of the BFS to the high frequency region (over 11 GHz) [10–13].

After obtaining the distribution of BSS along the OF, the frequency of the “peak” (BFS – $f_B$) and its relative level ($I_B$) are determined. Revealing changes in the BFS along the fiber allows to determine the regions of the OF with increased longitudinal tension and changed temperature.

4. The experimental results

Fig. 5 – Fig. 8 show examples of the BS frequency characteristics, which were obtained using the Brillouin reflectometer (BOTDR) “Ando AQ 8603”.

Fig. 5 shows the BS frequency characteristic of the G.652-fiber.

![Figure 5](image)

**Figure 5.** A BS frequency characteristic of the G.652-fiber

A BS frequency characteristic of the DSF (G.653) is presented in Fig. 6.

![Figure 6](image)

**Figure 6.** A BS frequency characteristic of the DSF (G.653)

In each Fig. 5 – Fig. 8, the initial scanning frequency is shown in the lower left corner, in the lower right – the final one, in the upper right corner the BFS ($f_B$) and the level of the BSS “peaks” are presented, in the upper left – the width of the BSS (on 3 dB level) and a grid step along the power axis (5 dB between the “lines”) are given.

Fig. 7 presents the BS frequency characteristic of the NZDSF (G.655).

![Figure 7](image)

**Figure 7.** A BS characteristic of the NZDSF (G.655)

A BS frequency characteristic of the “Corning”-Ultra fiber (G.652) is presented in Fig. 8.

![Figure 8](image)

**Figure 8.** A BS characteristic of the “Corning”-Ultra fiber

There is a similarity of the BSS profiles (Fig. 5 – Fig. 8) with those obtained as a result of calculations using the above method Fig. 1 – Fig. 4.
5. Conclusion
Analysing of light-wave propagation in OFs, taking in consider the given method, allows defining the BS frequency characteristic of the various OF types.
Since the structure of various fiber types can have several layers with different optical and acoustical characteristics, the BFS should be determined for “effective” values using the following formula:

\[ f_b = \frac{2f_v \mu_{ad} n_d}{c} = \frac{2n_{ad} n_d}{\lambda}. \]  

(6)

With the known structure of the OF (the number, thickness, composition, physical parameters of layers, etc.), it is possible to estimate the acoustical mode distributions in the OF, which ultimately will make it possible to determine the frequency characteristics of the BS.
The obtained are useful for diagnosing the physical state of the fibers in the OC, which is necessary to ensure the effective functioning of optical telecommunication systems.

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