Numerical simulation of turbulent flow and heat transfer in tube under injection of gas through permeable walls

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Abstract. Using a differential turbulence model, a numerical simulation of turbulent flow in a tube is carried out and the dependences of the flow and heat transfer characteristics on the intensity of the coolant injection through permeable walls are obtained. It is shown that the coefficients of friction and heat transfer decrease with increasing injection rate. The comparison of calculation results reflecting the main features of rather complicated processes of flow in tube under gas injection with well-known experimental data over a wide range of the Reynolds number and the intensity of injection show their satisfactory agreement.

1. Introduction
Using of permeable surfaces in various devices is due to necessity for thermal protection of the walls by injection. The relevance of the problem of thermal protection of walls can be judged from a large number of publications on the study of the boundary layer with injection. A fairly complete bibliography of the studies is contained in [1,2]. Amount of experimental data of the flow in tubes under injection is significantly less (see, for example, in [2]). As the most representative, it should be noted experimental studies [3, 4], where the results of investigation of turbulent flow and heat transfer in a permeable round tube with a uniform injection of gas through the wall were obtained. Among the calculation studies devoted to the investigation of turbulent flow in a round tube with uniform injection through permeable walls, a review of which is presented in [2], we should mention the paper [5] where a three-parameter model of turbulence as one of the variants of the model [6] was used.

The purpose of this paper is to perform a numerical study of flow and heat transfer in a porous tube with a gas injection through the wall using the differential turbulence model [6]. To solve the problem, the equations of continuity, motion, and energy were used in the narrow-channel approximation [7].

2. Formulation of the task
In this paper, the development of flow along the length of the tube is investigated for conditions corresponding to experiments [3, 4]. The entrance to a porous tube of diameter \( D = 35 \) mm and length \( L = 857 \) mm \((L/D \cong 24)\) is preceded by a section of stabilization where a developed turbulent flow is established at a given Reynolds number. Throughout the entire length of the porous tube, a uniform air injection was carried out with an intensity \( j_w^0 = (\rho u)_w / (\rho u)_0 \) determined by the conditions at the inlet.
To solve the problem, as in [7], the equations of continuity and motion in the narrow-channel approximation were used. The flow parameters at Reynolds numbers at the Re₀ = 2800-80000 input and the injection rate j₀ₘ = 0-0.0012 were calculated.

To calculate the turbulent friction in the equation of motion \( \rho \tau = -\rho <u'v'> \), a three-parameter differential turbulence model was used [6], in which the transfer equations are written for the turbulence energy \( E = 0.5 \sum <u_i'^2> \), the shear stress \( \tau = -<u'v'> \) and the parameter \( \omega = E/l^2 \) proposed by Kolmogorov (l is a transverse integral scale of turbulence).

The parameters of the task are Reynolds number Re₀ and the injection rate \( j₀ₘ \).

3. Calculation results

Figure 1 shows the velocity profiles in a number of sections along the length (x/D, where x is measured from the entrance to the porous tube) at Re = 80000, \( j₀ₘ = 0.0012 \). As one can see, the velocity profiles due to injection become more elongated along the length of the tube, which agrees with experiment [3].

![Figure 1](image-url)

Figure 1. Speed profiles \( u \) for a number of sections along the length of the tube Reₓ. Comparison between calculation (lines) and experimental [3] (dots) results.

The change in the coefficient of momentum flux \( \beta = \frac{1}{\rho_0} \left( \frac{u}{\bar{u}} \right)^2 \left( \frac{r}{R} \right)^2 \) along the tube length (Re = 80000), characterizing the deformation of the velocity profile, is shown in figure 2. As one can see, the character of the change in the \( \beta \) value along the length \( x/D \) agrees qualitatively with the experimental data [3]. The growth in the initial section is due to a decrease in the occupancy of the axial velocity profile (see figure 1) when it is rebuilt under the action of injection. The decrease of \( \beta \) value after the passage of the maximum is explained by the fact that the flow becomes quasi-stabilized and the velocity distribution in each section corresponds to the local intensity of the injection \( j₀ₘ = (\rho v)_w/\rho u \) (\( \bar{u} \) is the average velocity in this section of the tube), whose value falls along the length of the tube.
Figure 2. Variation of the coefficient of momentum flux $\beta$ along the length of the tube $x/D$ for a number of values of the injection rate (lines are calculations, dots are experimental data [3]).

Figure 3. Variation of the pressure gradient $dP/d\bar{x}$ along the length of the tube $x/D$ for a number of values of the injection rate (lines are calculations, dots are experimental data [3]).

The variation of the dimensionless longitudinal pressure gradient $d\overline{P}/d\bar{x}$ obtained in the calculations for $Re = 80000$ ($\overline{P} = P/(\rho u^2/2)$, $\bar{x} = x/D$) along the tube length essentially depends on the injection rate $j_w^0$ and is in good agreement with the experimental data [3] (see figure 3).

Figure 4. Curves of the relative value of the friction coefficient $C_f/C_{f0}$ on the intensity of injection $j_w^0$ in the cross-section of the tube $x/D = 6$ for two Reynolds numbers (lines are calculations, dots are experimental data [3]).

The local friction coefficient $C_f = 8\tau_w/\rho u^2$, determined from the local shear stress at the wall $\tau_w$ and the velocity head $\rho u^2$ in the porous tube, does not depend monotonically on the intensity of the injection $j_w^0$ with a minimum in the region $x/D = 4.8$. In figure 4 lines shows the calculated dependences of the relative friction coefficient $C_f/C_{f0}$ ($C_{f0}$ is the value of the friction coefficient at $j_w^0 = 0$) on the injection rate for $Re_0 = 28000$ and 80000. The results of the calculation are in good agreement with the experimental data [3] and indicate a significant decrease of the friction coefficient at injection.
Numerical study of heat transfer in a porous tube with injection was carried out for conditions corresponding to the experiment [4]. In [4] a hydrodynamically developed air flow at temperature $T = 300$ K entered the inlet section of the tube, and air heated to temperature $T = 330$ K was blown through the tube wall along a section of the relative length $L/D \equiv 24$.

The experiments were carried out for Reynolds numbers of the main flow at the input $Re_0 = (4-67) \cdot 10^3$ and for Reynolds numbers of injection $Re_w = j^0_w \cdot Re_0 = 25, 51, 101, 197$.

To determine the turbulent heat flux in the energy equation, the following hypothesis was used:

$$\rho q_T = -\rho c_p <v' T'> = \rho c_p \frac{\varepsilon_T}{Pr_T} \frac{\partial T}{\partial y}.$$  

Here $\varepsilon_T = \tau/\partial u/\partial y$ is the turbulent viscosity, $Pr_T = 0.9$ is the turbulent Prandtl number.

The results of calculations of the dimensionless heat transfer coefficient (Nusselt number) $Nu$ and comparison with the experimental data [4] are shown in figures 5, 6.

![Figure 5](image-url)

**Figure 5.** The variation of the Nusselt number $Nu$ along the length of injection section $x/D$ in the tube for two Reynolds numbers (lines are calculations, dots are experimental data [4]).

As one can see from figure 5, the agreement of the calculation results with the experiment [4] for Nusselt number at $x/D > 10$ can be considered satisfactory, which is also confirmed by the comparison between calculated and experimental data for Nusselt number dependences on the local Reynolds number $Nu(Re_x)$ for $x/D > 5-10$ (see in figure 6). The number of $Nu$ (figure 5) varies nonmonotonically along the length of the tube with a minimum in the region $x/D = 5-10$.

In figure 7 lines present the calculated dependences of the relative value of the Nusselt number $Nu/Nu_0$ ($Nu_0$ is the $Nu$ value at $j^0_w = 0$) on the Reynolds number $Re_w$ determined from the injection intensity at $x/D = 10$ for $Re_0 = 14000$ and 36000. The results of the calculation are in good agreement with the experimental data [4] and indicate a significant decrease in the value of the heat transfer coefficient at injection.

Thus, the performed calculations which showed satisfactory agreement with experiment in a wide range of Reynolds numbers and the injection rate provide a basis for using this calculation technique with a three-parameter turbulence model for further numerical studies of the effect of the injection intensity on the parameters characterizing the efficiency of heat exchangers with permeable cooling.
Figure 6. A generalization of the calculation and experimental results by the dependence \( \text{Nu} (\text{Re}_x) \). Lines are results of experimental data approximation [4], points are calculation for \( \text{Re}_0 = (14, 21, 36, 52, 67) \cdot 10^3 \).

Figure 7. Curves of the relative value of the Nusselt number \( \text{Nu}/\text{Nu}_0 \) due to the Reynolds number \( \text{Re}_w \) determined from the injection intensity at \( x/D = 10 \) for two Reynolds number \( \text{Re}_0 \) (lines are calculations, dots are experimental data [4]).

4. Conclusion

Using a differential turbulence model a numerical simulation of the turbulent flow in the tube was carried out and the dependences of the flow and heat transfer characteristics on the coolant injection intensity through the permeable walls along the tube was obtained. It was shown that the coefficients of friction and heat transfer decreased with increasing of injection intensity.

The comparison of calculation results reflecting the main features of quite complex processes of flow in the tube under injection with known experimental data over a wide range of Reynolds number and the intensity of injection, showed their satisfactory agreement. This provides a basis for using the developed calculation technique for performing a numerical study of flow and heat transfer in order to determine the main parameters characterizing the efficiency of heat exchangers with permeable cooling.

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