Investigations of Skin Nuclei in a Density Functional Approach

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Abstract.
A theoretical method incorporating the density functional theory and quasiparticle-phonon model is applied for systematic study of low-energy excitations of different multipolarities related to neutron or proton skins in stable and exotic nuclei. Furthermore, the unique character of the observed excitations identified with pygmy dipole or pygmy quadrupole resonances is confirmed in quasiparticle-random-phase-approximation plus multiphonon calculations. The achievements of the multiphonon approach for description of pygmy and other strengths are discussed.

1. Introduction

The observation of skin related dipole excitations in stable and unstable nuclei with neutron excess \cite{1, 2, 3, 4} reveals new insight into the dynamical properties of nuclear matter at low densities. An enhanced dipole strength, located close to the particle emission threshold was related to a new dipole mode named pygmy dipole resonance (PDR). Presently, the nature of the PDR is a subject of outstanding discussions. Nevertheless the most common theoretical interpretation associated it with oscillations of a weakly bound neutrons (for N/Z > 1) or protons (for N/Z ≈ 1) with respect to an isospin symmetric core \cite{3, 4, 5, 6, 7, 8, 9}.

A question, coming up immediately in this connection, is to what extent the presence of a neutron or proton skin will affect excitations of other multipolarities and parities and vice versa. Promising candidates are low-energy 1\textsuperscript{+}, 2\textsuperscript{+}, 3\textsuperscript{−} states etc., especially those in excess of the spectral distributions known from stable nuclei. Recently, quadrupole response functions were investigated, theoretically in neutron-rich Sn nuclei. A close connection of low-energy 2\textsuperscript{+} excitations and nuclear skins was found. These quadrupole states were related to pygmy quadrupole resonance (PQR) \cite{10}.

In recent experimental studies of (α, α'γ) reactions the isospin character of the PDR in \textsuperscript{124}Sn was investigated \cite{6}.

2. Theoretical Model

The model Hamiltonian resembles in structure the standard QPM model \cite{11} but in detail differs in the physical content in important aspects as discussed in Ref. \cite{8, 9}. Consequently the nuclear ground state is obtained in a fully microscopic Hartree-Fock-Bogoliubov (HFB) approach\cite{12} where dynamical effects beyond mean-field are taken into account. That goal is achieved by using the energy density functional theory (EDFT)\cite{9}.

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The nuclear excitations are described in terms of quasiparticle-random-phase-approximation (QRPA) phonons [11]:

\[ Q_{\lambda \mu}^+ = \frac{1}{2} \sum_{jj'} \left( \psi_{jj'}^{\lambda \mu} A_{\lambda \mu}^{+} (jj') - \varphi_{jj'}^{\lambda \mu} A_{\lambda \mu} (jj') \right), \]

where \( j = (nljm\tau) \) is a single-particle proton or neutron state; \( A_{\lambda \mu}^{+} \) and \( A_{\lambda \mu} \) are time-forward and time-backward operators, coupling two-quasiparticle creation or annihilation operators to a total angular momentum \( \lambda \) with projection \( \mu \) by means of the Clebsch-Gordan coefficients \( C_{j_{m}j_{m}'}^{\lambda \mu} = \langle j_{m}j_{m}'|\lambda \mu \rangle \). The excitation energies of the phonons and the time-forward and time-backward amplitudes \( \psi_{j_{1}j_{2}}^{\lambda \mu} \) and \( \varphi_{j_{1}j_{2}}^{\lambda \mu} \) in Eq. (1) are determined by solving QRPA equations [11].

Furthermore, the QPM provides a microscopic approach to multiconfiguration mixing [11]. For spherical even-even nuclei the model Hamiltonian is diagonalized by using of an orthonormal set of wave functions constructed from one-, two- and three-phonon configurations [13]:

\[ \Psi_{\nu}(JM) = \left\{ \sum_{i} R_{i}(J\nu) Q_{JM_{i}}^{+} + \sum_{\lambda_{1} \mu_{1} \lambda_{2} \mu_{2}} P_{\lambda_{1} \mu_{1} \lambda_{2} \mu_{2}}^{\lambda_{1} \mu_{1}} (J\nu) \left[ Q_{\lambda_{1} \mu_{1} \lambda_{1} \mu_{1}}^{+} \times Q_{\lambda_{2} \mu_{2} \lambda_{2} \mu_{2}}^{+} \right]_{JM} \right. \]

\[ + \sum_{\lambda_{3} \mu_{3} \lambda_{4} \mu_{4}} T_{\lambda_{3} \mu_{3} \lambda_{4} \mu_{4}}^{\lambda_{3} \mu_{3}} (J\nu) \left[ Q_{\lambda_{3} \mu_{3} \lambda_{3} \mu_{3}}^{+} \otimes Q_{\lambda_{4} \mu_{4} \lambda_{4} \mu_{4}}^{+} \right]_{IK} \otimes Q_{\lambda_{4} \mu_{4} \lambda_{4} \mu_{4}}^{+} \left. \right]_{JM} \left\} \Psi_{0}, \]

where \( R, P \) and \( T \) are unknown amplitudes, and \( \nu \) labels the number of the excited states.

In the procedure the exact commutation relations including the internal fermionic structure of the phonon operators are implemented. It is applied also for transition operators and transition matrix elements of multiphonon states [14]. This is a necessary condition in order to account for the Pauli principle.

3. Discussion

From QRPA calculations in Sn isotopes with \( N/Z = 1.2 \div 1.68 \), we observe a sequence of low-lying \( 1^- \) excitations with energies \( E^* = 5.5 \div 8 \) MeV. These states are related to oscillations of the least bound neutrons (the proton contribution is less than 1%) [8, 9]. We find similar results also in N=50, 82 isotones [3, 4] and in the newly calculated \( ^{86}\text{Kr} \) nucleus from N=50 chain. In the lightest \( ^{100\text{–}104}\text{Sn} \) isotopes, the lowest dipole QRPA states, located at \( E^* = 8.1 \div 8.3 \) MeV, are dominated by proton excitations involving quasibound proton states [9]. Electromagnetic breaking of isospin symmetry is the main reason for the persisting of low-energy dipole strength close to \( ^{100}\text{Sn} \). The observed sequences of low-energy neutron or proton dipole states are related to neutron or proton PDR, respectively.

In this connection, more detailed analysis of the dipole excitations is achieved by investigating proton and neutron transition densities in various energy and mass regions [4, 9]. The unique behavior of the dipole transition densities in the PDR region is found as a common feature of the lowest-lying dipole states in Z=50, N=50, 82 nuclei [3, 4, 9] making it meaningful to identify the PDR as a distinct and genuine excitation different from the giant dipole resonance (GDR). Furthermore, the change of the character of the transition densities from neutron to proton one is observed for lowest-mass tin isotopes thus confirming the transition of a neutron PDR to a proton PDR [9]. From QRPA calculations of total PDR strengths it is found to be closely related to the neutron skin thickness [3, 4, 5, 8, 9]. Namely, the PDR strength increases with the increase of N/Z ratio for both isotopic and isotonic chains. This result is further justified.
in our multiphonon QPM calculations presented in this paper. As an example case we consider the comparison between QRPA and QPM calculations of summed B(E1) strengths at different energies in $^{112,120}$Sn which is presented in Fig.1(right). In addition, calculations of the excited QRPA $1^-$ states and their fragmentation pattern obtained within two- and three-phonon QPM are shown in Fig.1(left)a,b,c, respectively.

According to analysis of QRPA transition densities and state vectors structure (see for details in Ref. [9]), the summed B(E1) strengths in the energy region $E^* \leq 8$ MeV refers to almost pure PDR strengths while for the regions $E^* \leq 8.5$ MeV and $E^* \leq 9$ MeV the GDR contribution is dominant. The comparison of QRPA and two- and three-phonon QPM excited $1^-$ states indicates that for the PDR region the coupling of QRPA PDR and GDR phonons to multiphonon states is very important. The result is a shift of E1 strength towards lower energies which is well visible in Fig.1(left)a,b,c and from Fig.1(right). Consequently in the energy interval $0 < E < 8$ MeV the QPM calculations give about twice as much total summed E1-strength than the QRPA calculations, as shown in Fig.1 (right). Differently, with the increase of the excitation energy toward the GDR the QRPA and QPM summed B(E1) transition strengths are of comparable size which suggests the prevalence of $p - h$ excitations of GDR type. Thus, our QPM calculations of $^{112}$Sn (N/Z=1.24) and $^{120}$Sn (N/Z=1.4) confirm experimental and QRPA observations of increasing PDR strength with the increase of N/Z ratios [3]. The comparison of $^{112}$Sn and $^{120}$Sn nuclei (see in Fig.1(right)) shows that the relative difference between the E1 strengths calculated for certain energy regions, both for the QRPA and QPM cases, reduces with the increase of the excitation energies toward GDR. This behavior is expected because of the weak dependence of GDR on N/Z ratios in tin isotopes.

From theoretical investigations of $2^+$ excitations in Sn nuclei we find a clustering of quadrupole states, at low-excitation energies $E^* = 2 - 4$ MeV which structure is dominated by neutron two-quasiparticle (2QP) excitations from the valence shells. The most important proton contribution in all isotopes is due to the $[1g_{9/2}2d_{5/2}]_\pi$ 2QP component, which, however, never exceeds 5% [10]. This dependence is illustrated for the $[2^+_1]_{QRPA}$ states in Fig.2(left). The figure reflects the change of the proton binding energy $\epsilon_b$ of the $g_{9/2}$ level when approaching the $N = Z$ limit [10].

Figure 1. (left) Calculations of electric dipole strength up to $E^* = 9$ MeV in $^{120}$Sn: a/QRPA, b/ two-phonon QPM and c/ three-phonon QPM; (right) Comparison of QRPA and three-phonon QPM B(E1) strengths summed up to $E^* = 8$ MeV (PDR region), 8.5 MeV (GDR region) and 9 MeV (GDR region), respectively.
In the neutron sector, the contributions follow closely the evolution of the shell structure as seen from Fig.2(left). In most cases the \([2^+_2]_{QRPA}\) state vectors are dominated by re-scattering contributions related to re-orientation of the s.p. angular momenta [10]. Theoretical results of summed B(E2) strength distributions in \(^{104÷134}\text{Sn}\) isotopes are presented in Fig.2(right). A sizable increase of B(E2) strength at \(E_x \approx 2-4\text{ MeV}\) is observed with increasing of \(N/Z\) ratio [10] associated with the mass dependence of the neutron 2QP transition matrix elements and decreasing neutron binding energy \(\epsilon_{b}\). For \(N/Z < 1.4\) the B(E2) transitions follow the increasing contribution of the protons, which are coupled directly by their physical charge to the electromagnetic field. Correspondingly, the connection of the low-energy \(2^+\) states with a PQR is clearly seen in the analysis of transition densities [10]. Results from QRPA proton and neutron quadrupole transition densities of the PQR mode in \(^{124}\text{Sn}\) are presented in Fig.3(left). Strong neutron oscillations at the nuclear surface play a dominant role in the PQR energy range which is in agreement with the observations from [10]. Similarly to the PDR, a transition from a neutron PQR to a proton PQR in \(^{104}\text{Sn}\) is observed for the mass region where the neutron skin reverses into a proton skin [10].

Investigations of B(E2) and B(M1) transition rates of low-lying \(2^+\) states in \(^{124}\text{Sn}\) in QPM presentation indicate differences from known collective states and scissors modes. The results on B(E2) transition rates are compared to experimental data in \(^{124}\text{Sn}\) in Fig. 3(right). A good agreement of the data with the calculated total B(E2) value related to PQR is obtained.

QPM calculations of low-energy E1 and M1 excitations in \(^{138}\text{Ba}\) [5] and \(^{90}\text{Zr}\) nucleus confirm unambiguously the predominantly electric character of the observed low-energy dipole strength in these nuclei. Furthermore, our recent QPM studies of the fine structure of M1 strength in \(^{90}\text{Zr}\) lead to interesting and new observations of the origin of the magnetic excitations as a composition of spin-flip and orbital modes of sizable sizes where the multiphonon counterparts play an important role. The results are of significant importance for the analysis of strength functions used in the \((n, \gamma)\) reactions data.
Figure 3. (left) QRPA calculations of quadrupole transition densities related to the main proton and neutron components in the state vectors of \([2^+_i]_{QRPA}\) states with excitation energies \(E^* = 2 - 5\,\text{MeV}\); (right) QPM calculations of \(B(E2)\) transitions of low-energy \(2^+\) excited states in \(^{124}\text{Sn}\) in comparison with experimental data [15, 16, 17].

4. Conclusions

In conclusion, systematic investigations of low-energy dipole and other multipole excitations in \(Z=50\) and \(N=50, 82\) nuclei reveal new modes of excitations related to PDR and PQR. A close correlation of PDR \(B(E1)\) and PQR \(B(E2)\) transition strengths related to \(N/Z\) ratios and thicknesses of a neutron or proton skin is found. The unique character of both modes is further confirmed by analysis of related dipole and quadrupole transition densities. A sizable increase of low-energy \(E1\) strength with respect to the pure QRPA PDR strength is obtained from multiphonon QPM calculations in \(^{112}\text{Sn}\) and \(^{120}\text{Sn}\). The result is related to contribution of multiphonon states and of the coupling to the GDR. As a general behavior we find an increase of the PDR with the neutron number.

The knowledge of the fine structure of the low-energy dipole strength by means of its multiphonon presentation is found essential for description of M1 resonances, dipole response functions and related corresponding neutron-capture cross sections relevant for nucleosynthesis.

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