Testing soft electroweak SUSY breaking from neutralino, chargino, and charged Higgs boson pairs production at linear colliders

M. Beccaria\textsuperscript{a,b}, H. Eberl\textsuperscript{c}, F.M. Renard\textsuperscript{d} and C. Verzegnassi\textsuperscript{e,f}

\textsuperscript{a}Dipartimento di Fisica, Università di Lecce
Via Arnesano, 73100 Lecce, Italy.
\textsuperscript{b}INFN, Sezione di Lecce
\textsuperscript{c}Institut für Hochenergiephysik der Österreichischen Akademie der Wissenschaften, A-1050 Vienna, Austria
\textsuperscript{d}Physique Mathématique et Théorique, UMR 5825
Université Montpellier II, P-34095 Montpellier Cedex 5.
\textsuperscript{e}Dipartimento di Fisica Teorica, Università di Trieste,
Strada Costiera 14, Miramare (Trieste)
\textsuperscript{f}INFN, Sezione di Trieste

We consider the production of neutralino, chargino, and charged Higgs boson pairs in the MSSM framework at future $e^+e^-$ colliders. We show that, for c.m. energies in the one TeV range and in a “moderately” light SUSY scenario, a combined analysis of the slopes of these production cross sections could lead to a strong consistency test involving the soft supersymmetric breaking parameters $M_1$, $M_2$, the Higgsino parameter $\mu$, and $\tan\beta$.

PACS numbers: 12.15.-y, 12.15.Lk, 13.10.+q, 14.80.Ly

It is a widespread hope within the members of the high energy elementary particle physics community that the upcoming experiments at Tevatron \textsuperscript{[1]} and at LHC \textsuperscript{[2]} will finally reveal the existence of supersymmetric particles, via direct production of sparticle-antisparticle pairs. In such an exciting hypothesis, the simplest available theoretical supersymmetric model, the MSSM, will acquire the same role that belonged to the SM after the W, Z discoveries, and a long and crucial period of precision tests will start, aiming to confirm, or to disprove, the main theoretical assumptions that were used in the practical construction of the model.

For the MSSM case, this program requires the construction of a linear $e^+e^-$ collider \textsuperscript{[3]} that explores the c.m. TeV energy region. This machine should be sufficiently accurate to provide the same kind of consistency tests of the model that were achieved in the hundred GeV region at LEP for the Standard Model, from detailed analyses of several independent one-loop virtual effects.

If such an ambitious project is to be carried on, a clean investigation strategy will undoubtedly be welcome, given the fact that the involved theoretical model has a not simple structure in which several independent parameters are implied. As a consequence of this feature, it might be appreciable to identify clean experimental measurements that, possibly, isolate in a relatively simple way a reduced subset of parameters and of theoretical assumptions behind them.

The aim of this short paper is to show that a dedicated combined analysis of the process of production of neutralino, chargino and charged Higgs boson pairs might lead to a relevant precision test of the theoretical assumptions that were used to fix, not only the basic supersymmetric sector (i.e. the chiral superpotential and the gauge multiplet), but also the gaugino component of the soft supersymmetry breaking of the model. This will be done in the paper by assuming a scenario of “moderately” light SUSY particles (i.e. the various masses are all below, roughly, 350 GeV) and a c.m. energy $\sqrt{s}$ in the one TeV region. Under these two assumptions, a simple asymptotic expansion of so called “Sudakov type” is effective \textsuperscript{[4]}. But the approach would not change for larger SUSY masses $< 700$ GeV since a related c.m. energy rescaling would still be possible, from our previous experience, up to $\sqrt{s}$ values of about 2 TeV, where the resummation to higher orders is still not necessary \textsuperscript{[4]}.

After this brief description of the strategy of our approach we are now ready to illustrate the practical details of our work. With this aim, we must recall, for the reader’s convenience, that the relevant details of an analogous procedure, limited to the combination of chargino and charged Higgs pairs production, have already appeared in a previous paper\textsuperscript{[5]}. It was shown there that from the combined analyses of the slopes of the total cross sections in a c.m. energy range of about 1 TeV, assuming a “light” SUSY scenario where all the relevant sparticle masses lie below $\sim 350-400$ GeV, a strong constraint on $\tan\beta$ (mostly produced by the Higgs cross section) and a strip in the $(M_2, \mu)$ plane (only produced by the chargino cross sections) were derivable. Here $M_2$ is the soft SUSY breaking wino mass, $\mu$ is the Higgsino mass parameter, and $\tan\beta = v_2/v_1$ is the ratio of the two Higgs doublets.
vevs. The novel process that we consider in this paper is neutralino pair production, and we shall show that its addition to the other processes will lead to highly improved constraints on the previous parameters, with the additional presence of $M_1$, the soft SUSY breaking mass. With this aim, we now briefly list the relevant theoretical formulae that we need for the analysis.

At the Born level, the neutralino pair production process $e^+ e^- \rightarrow \chi_i^0 \chi_j^0$ is described by three $s$, $t$, $u$ channel components of the scattering amplitude, using the following notations (the indices $a$, $b$ label the electron and the $i$-th neutralino chiralities):

$$A^{ab}_{ij} \equiv \frac{e^2}{s} S^{ab}_{ij} + \frac{e^2}{u} U^{ab}_{ij} + \frac{e^2}{t} T^{ab}_{ij} \quad (1)$$

with the tree level values for the $s$ channel

$$S^{LL}_{ij} = \frac{2 s_w^2}{4 s_w^2 c_w^2} O_{ij} \quad S^{LR}_{ij} = - \frac{2 s_w^2}{4 s_w^2 c_w^2} O^{*}_{ij} \quad (2)$$

$$S^{RL}_{ij} = \frac{1}{2 c_w} O_{ij} \quad S^{RR}_{ij} = - \frac{1}{2 c_w} O^{*}_{ij} \quad (3)$$

$$O_{ij} = Z^{N*N}_{3i} Z^{N}_{3j} - Z^{N*N}_{4i} Z^{N}_{4j} \quad (4)$$

and for the $t, u$ channels

$$U^{LL}_{ij} = - \frac{1}{4 s_w^2 c_w^2} (Z^{N*N}_{1i} s_w + Z^{N*N}_{2i} c_w)(Z^{N*N}_{1j} s_w + Z^{N*N}_{2j} c_w) \quad (5)$$

$$U^{RR}_{ij} = - \frac{1}{c_w} Z^{N*N}_{ij} Z^{N*N}_{1j} \quad (6)$$

$$T^{LR}_{ij} = \frac{1}{4 s_w c_w^2} (Z^{N*N}_{1i} s_w + Z^{N*N}_{2i} c_w)(Z^{N*N}_{1j} s_w + Z^{N*N}_{2j} c_w) \quad (7)$$

$$T^{RL}_{ij} = \frac{1}{c_w} Z^{N*N}_{ij} Z^{N*N}_{1i} \quad (8)$$

The quantities $Z^N_{ij}$ are the elements of the 4x4 mixing matrix, defined in a conventional way. For simplicity, we have neglected in the considered asymptotic region the electron mass in Eq.(1).

For the purposes of a precision test, it becomes mandatory to compute the following perturbative one-loop expansion of the scattering amplitude. This requires the calculation of a large number of Feynman diagrams. The resulting expressions are valid for any value of the c.m. energy $\sqrt{s} = (p_+ - p_-)^2$. The full one-loop result has been calculated recently. In this work we propose an alternative approach, valid for values of $\sqrt{s}$ “much” larger than all the SUSY sparticle masses involved in the process. In fact, our strategy should be now made very clear.

We assume a previous production of the charginos, of the charged Higgs bosons and of at least two neutralinos with mass $M_{\tilde{e}}^0, M_{\tilde{\chi}_2}^0$. Calling $M_{\text{SUSY}}$ the heaviest of the real and virtual SUSY particles that appear in the processes, and assuming a “reasonable” limit $M_{\text{SUSY}} \simeq 350 - 400$ GeV, we shall choose the value $\sqrt{s} \simeq 1$ TeV value to proceed with our approach. This is due to the fact that, in a previous paper only concerned with the charged Higgs pair production, we proved that in such a configuration an asymptotic energy logarithmic expansion of so called “Sudakov type” was reliable, with the only addition of a constant term to the leading quadratic and next-to-leading linear logarithmic terms. This conclusion allows to propose a determination of the SUSY parameters entering the Sudakov logarithms, based on measurements of the slope of the total cross section, in which the (complicated) constant terms cancel.

The approach that we shall follow in this paper, that was already used in the combined chargino-charged Higgs analysis, will assume that a similar expansion is valid, with only logarithmic and constant terms. To prove this statement would require a detailed comparison of the complete existing calculation with the assumed asymptotic expansion. This will be the goal of a forthcoming rigorous analysis. For the moment, we shall assume its validity as a working Ansatz, and we shall show the main relevant consequences that it will be able to produce.

After this preliminary discussion, we are now ready to write the relevant asymptotic expansion of the scattering amplitude at one loop, in which we shall only retain the leading quadratic and next-to-leading linear logarithmic terms. For this purpose, we shall use the following formal decomposition the separate $F = S, T$ or $U$ subamplitudes (the Born values are those listed in Eqs. (2-8)):

$$F^{ab}_{ij} = F^{ab}_{ij, \text{Born}} + F^{ab}_{ij, \text{Born}} c_{in} + F^{ab}_{ij, \text{fin}} + F^{ab}_{ij, \text{ang}} + F^{ab}_{ij, \text{RG}} \quad (9)$$

In the asymptotic expansion, two different kinds of logarithmic terms appear. The first ones are the standard linear ones of RG origin. They are well known and can be derived in a straightforward way replacing in the Born quantities the various bare couplings with running ones. For sake of completeness, we write the related formulae, noticing that they are only requested for the $s$-channel.
(purely Higgsino) component, as:

$$S_{ij}^{LL,RG} = -\frac{\alpha}{4\pi s_W^4} \log \frac{s}{M_W^2} [O_{ij}(\tilde{\beta}_0^0, \tilde{\beta}_0)]$$  \hspace{1cm} (10)$$

$$S_{ij}^{LR,RG} = \frac{\alpha}{4\pi} \log \frac{s}{M_W^2} [O_{ij}(\tilde{\beta}_0^0, \tilde{\beta}_0)]$$  \hspace{1cm} (11)$$

$$S_{ij}^{RR,RG} = -\frac{\alpha}{\pi} \log \frac{s}{M_W^2} [O_{ij}(\tilde{\beta}_0^0)]$$  \hspace{1cm} (12)$$

$$S_{ij}^{RL,RG} = \frac{\alpha}{4\pi s_W^4} \log \frac{s}{M_W^2} [O_{ij}(\tilde{\beta}_0^0)]$$  \hspace{1cm} (13)$$

where $\tilde{\beta}_0 = -1/4$ and $\tilde{\beta}_0^0 = -11/4$. The second type of logarithms which arise asymptotically is that of the “genuine weak” Sudakov terms. These are usually classified as logarithms of gauge non universal, gauge universal, and Yukawa origin. The gauge non universal, scattering angle dependent ones, stem from box diagrams and $t,u$ channel vertices and have the following expressions:

$$S_{ij}^{LL,ang} = -\frac{\alpha}{4\pi s_W^4} \log \frac{s}{M_W^2} [\log \frac{t}{s} (2Z_{2i} Z_{2j} + Z_{4i} Z_{4j}) - \log \frac{u}{s} (2Z_{3j} Z_{2i} + Z_{3i} Z_{2j})]$$  \hspace{1cm} (14)$$

$$S_{ij}^{LR,ang} = -\frac{\alpha}{4\pi s_W^4} \log \frac{s}{M_W^2} [\log \frac{t}{s} (2Z_{2i} Z_{2j} + Z_{3i} Z_{3j}) - \log \frac{u}{s} (2Z_{3j} Z_{2i} + Z_{3i} Z_{2j})]$$  \hspace{1cm} (15)$$

$$U_{ij}^{LL,ang} = \frac{\alpha}{4\pi s_W^4} \log \frac{s}{M_W^2} [\log \frac{s}{M_W^2} (2Z_{2i}(Z_{1i} s_W - Z_{2i} c_W) + Z_{2j}(Z_{1j} s_W + Z_{2j} c_W)]$$  \hspace{1cm} (16)$$

$$T_{ij}^{LR,ang} = -\frac{\alpha}{4\pi s_W^4} \log \frac{s}{M_W^2} [\log \frac{s}{M_W^2} (2Z_{2i}(Z_{1i} s_W - Z_{2i} c_W) + Z_{2j}(Z_{1j} s_W + Z_{2j} c_W)]$$  \hspace{1cm} (17)$$

The gauge universal ones are due both to the initial and to the final vertices. The initial contribution is fixed by the coefficients

$$c_L^{i,w} = \frac{\alpha (1 + 2c_w^2)}{16\pi s_W^4 c_W} \left[ \log \frac{s}{M_W^2} - \log^2 \frac{s}{M_W^2} \right]$$  \hspace{1cm} (18)$$

$$c_R^{i,w} = \frac{\alpha}{4\pi c_W} \left[ \log \frac{s}{M_W^2} - \log^2 \frac{s}{M_W^2} \right]$$  \hspace{1cm} (19)$$

The contribution from final neutralino legs is different for the s or u, t channels. For the s-channel we have higgsino components

$$S_{ij}^{ab,fin} = \sum_k \{ S_{ik}^{ab,Born} \cdot [e_{kj}^{fin gauge} + c_{kj}^{fin Yukawa}] + S_{kj}^{ab,Born} \cdot [e_{ki}^{fin gauge} + c_{ki}^{fin Yukawa}] \}$$  \hspace{1cm} (20)$$

where

$$c_{kj}^{fin gauge} = \left( \frac{\alpha}{\pi} \right) \left( \frac{1 + 2c_w^2}{32 s_W^4 c_W} \right) \left[ \log \frac{s}{M_W^2} - \log^2 \frac{s}{M_W^2} \right]$$  \hspace{1cm} (21)$$

$$\left( Z_{4i}^N Z_{4j}^N + Z_{3k}^N Z_{3j}^N \right) P_L + \left( Z_{4i}^N Z_{4j}^N + Z_{3k}^N Z_{3j}^N \right) P_R \right]$$

The logarithms of Yukawa origin are only due to final s-channel vertices with virtual heavy (b,t) quark-squark pairs. These are the only logarithmic terms that contain (also) the vevs ratio $\tan \beta$ and their expression is

$$c_{kj}^{fin gauge} = \left( \frac{\alpha}{\pi} \right) \left[ -\log \frac{s}{M_W^2} \left( \frac{3}{16 s_W^4 c_W} \right) \right]$$  \hspace{1cm} (22)$$

$$\left( Z_{4i}^N Z_{4j}^N + Z_{3k}^N Z_{3j}^N \right) P_L + \left( Z_{4i}^N Z_{4j}^N + Z_{3k}^N Z_{3j}^N \right) P_R \right]$$

To conclude, the final contribution to the u and t channel amplitudes is due to gaugino components for which there is no Yukawa contribution:

$$U_{ij}^{ab,fin} = \sum_k \{ U_{ik}^{ab,Born} \cdot [e_{kj}^{fin gauge}] + U_{kj}^{ab,Born} \cdot [e_{ki}^{fin gauge}] \}$$  \hspace{1cm} (23)$$

$$T_{ij}^{ab,fin} = \sum_k \{ T_{ik}^{ab,Born} \cdot [e_{kj}^{fin gauge}] + T_{kj}^{ab,Born} \cdot [e_{ki}^{fin gauge}] \}$$  \hspace{1cm} (24)$$

with

$$e_{kj}^{fin gauge} = -\frac{\alpha}{4\pi s_W^4 c_W} \left( Z_{3k}^N Z_{3j}^N P_L + Z_{3k}^N Z_{3j}^N P_R \right)$$  \hspace{1cm} (25)$$

Eqs. (10-25) represent the complete logarithmic contributions at one loop to the considered process, and are the main original result of this paper. We expect from our previous discussion their reliability for what concerns the calculation of the slope of the total cross section in the 1 TeV range. To perform the latter, one must first compute the expression of the differential cross section that is readily obtained from the above amplitude. After angular integration, one obtains the approximate (to next-to-leading logarithmic order) asymptotic expression of the total cross section. This will be sufficient to determine the effective asymptotic expression of the slope of the cross section, where by definition possible extra “next-to-next-to-leading” (i.e. constant) terms disappear. This
TABLE I: Input parameters and masses of charginos, and lightest neutralinos for the three input sets $S_1$, $S_2$, and $S_3$. The masses are expressed in GeV.

|      | $S_1$ | $S_2$ | $S_3$ |
|------|-------|-------|-------|
| $M_1$ | 78    | 100   | 200   |
| $M_2$ | 150   | 200   | 400   |
| $\mu$ | 263   | 200   | 100   |
| $\tan \beta$ | 30 | 30 | 30 |
| $\chi^\pm_1$ | 132 | 149 | 95 |
| $\chi^Z_1$ | 295 | 266 | 417 |
| $\chi^0_1$ | 75 | 92 | 82 |
| $\chi^0_2$ | 133 | 153 | 109 |

The mass expression will only depend on $\tan \beta$ and on the mixing parameters $Z^{SM}_{ij}$. The latter ones, in turn, can be expressed as functions of the supersymmetric parameters $M_1$, $M_2$, $\mu$ and $\tan \beta$. Thus, the final form of the slope will depend on these four parameters.

In order to achieve the maximal theoretical information using our approach, we have combined the theoretical expressions of the slope of the neutralino pair total cross section, given in this paper, with those of the chargino and charged Higgs pairs, given in Ref. 1. In practice, we have limited our analysis to the production of the two light observable final states ($\chi^0_1 \chi^0_2$, $\chi^0_1 \chi^0_2$) for neutralinos, combined with the production of all the three chargino pairs and of the charged Higgs pair. To work in a consistent way, we have chosen as relevant examples for the masses of the produced pairs three sets of values denoted by $S_1$, $S_2$, and $S_3$. The first, $S_1$, is the Tesla benchmark point RR2, with the two lightest neutralinos being respectively 95 % bino and 82 % wino. The set $S_2$ is a mixed scenario with neutralinos having non negligible gaugino and Higgsino components; $\chi^0_1$ is 86 % bino and 13 % higgsino, $\chi^0_2$ is 11 % bino, 48 % wino and 41 % higgsino. Finally, $S_3$ is a purely Higgsino one with the two lightest neutralinos being 92 % and 98 % higgsino like. The values of the input parameters as well as the masses of the two charginos and of the two lightest neutralinos are summarized in Tab. (1). We have performed a standard $\chi^2$ analysis assuming in the various scenarios 10-12 experimental points at energies ranging from 700-850 GeV (depending on the scenario) up to 1200 GeV and assuming an experimental accuracy of 1% for the cross sections of gauginos and 2% for that of Higgs bosons. For each scenario we have checked the condition $\sqrt{s} \gg M_{\text{SUSY}}$. The results of our analysis are shown in Figs.(1-3). From inspection of the figures one can draw the following conclusions. In Fig.(1) we consider the $S_1$ benchmark point. One notices that a combined analysis is able to generate closed contours in the planes of all possible six couples of parameters. This possibility is obviously existing for the first 3 cases i.e. $(M_1, M_2)$, $(M_1, \mu)$, $(M_1, \tan \beta)$ under the condition that the neutralino information is added to the remaining ones, that do not depend on $M_1$. Less obvious and more illustrative is the closure of the undetermined strip in the $(M_2, \mu)$ plane (drawn in the absence of neutralino data) when neutralino data are added to the remaining ones. This addition does not practically improve, on the contrary, the bounds on $(M_2, \tan \beta)$ and $(\mu, \tan \beta)$ obtained from chargino-Higgs data. One sees typical errors, under the assumed experimental conditions, of about 10 GeV for $M_1$, $M_2$, $\mu$ and of a relative $\sim 30 - 40$ % for $\tan \beta$. In

![FIG. 1: $S_1$ benchmark point. $1\sigma$ error bounds on the MSSM parameters $M_2$, $\mu$, $\tan \beta$. In this and in the following figures the crosses denote the values of the parameters in the specific benchmark.](image1)

![FIG. 2: $S_2$ benchmark point. $1\sigma$ error bounds on the MSSM parameters $M_2$, $\mu$, $\tan \beta$.](image2)
The size of errors is now of approximately $\sim 10$ GeV for $M_1$, $M_2$, and $\mu$, a relative 40-50% for $\tan\beta$.

Fig. (3) describes $S_3$, a typical Higgsino scenario. Here the new (negative) feature concerns the lack of boundary for $M_1$, as seen from the first three plots. This can be understood since, at the considered $M_1$, $M_2$, and $\mu$ values, the mixing parameters $Z_{ij}^N$ do not vary appreciably when $M_1$ increases. On the contrary, the three plots in the bottom line retain the typical features of the previous cases, with particular relevance of the neutralino role for the determination of lower limits for $\mu$ and $M_2$. The errors are now, typically, of about 20 GeV for $M_2$, $\mu$ and of a relative $\sim 10\%$ for $\tan\beta$.

The previous results should illustrate the aimed possible outcomes of the testing strategy that we are proposing in this preliminary qualitative paper. Our assumed physical inputs will necessarily be the masses of the (supposedly produced) (light) neutralinos, charginos and charged Higgs. Their values will depend on the four parameters $M_1$, $M_2$, $\mu$, $\tan\beta$ but, in general there will not be a 1-1 correspondence, and different sets of parameters might reproduce "essentially" the same masses. Within the errors that we have illustrated in our examples, these different sets can be discriminated via the $\chi^2$ analysis that we have proposed, and the “correct” set is selected by the true experimental data, if the latter are actually described by the model. If none of the candidate sets were compatible with the analysis, an indication would arise that some of the details of the model might need a suitable modification.

[1] see e.g. R. Ströhmer, Acta Physica Polonica B33 (2002) 3887; T. Kamon, hep-ex/0301019, published in SUSY02, "Hamburg 2002. Supersymmetry and Unification of fundamental Interactions, vol.1, p.138.
[2] see e.g. D.P. Roy, hep-ph/0303106 and F.E. Paige, hep-ph/0307342.
[3] F. Accomando et al., Phys. Rep. C299, 299 (1998).
[4] M. Beccaria, M. Melles, F. M. Renard, S. Trimarchi and C. Verzegnassi, Int. J. Mod. Phys. A18, 5069 (2003).
[5] M. Beccaria, H. Eberl, F. M. Renard and C. Verzegnassi, Phys. Rev. D69, 091301 (2004).
[6] W. Oeller, H. Eberl, W. Majerotto, Phys.Lett.B590, 273 (2004).
[7] M. Beccaria, F.M. Renard, S. Trimarchi, C. Verzegnassi, Phys. Rev. D 68, 035014 (2003).
[8] M. Beccaria, F.M. Renard, C. Verzegnassi, Phys. Rev. D64, 073008 (2001).
[9] S. Ambrosanio, G. Blair, P.M. Zerwas, in DESY ECFA 1998-99 LC Workshop, http://www.desy.de/conferences/ecfa-desy-lc98.html.
[10] N. Ghodbane, H-U. Martyn, APS/DPF/DPB Summer Study on the Future of Particle Physics, hep-ph/0201233 Proceedings of the Snowmass 2001 Summer Study, Snowmass CO (USA), June-July 2001;
[11] W. Oller, H. Eberl, W. Majerotto and C. Weber, Eur. Phys. J. C29, 563 (2003).