FULLY GENERAL RELATIVISTIC SIMULATIONS OF CORE-COLLAPSE SUPERNOVAE WITH AN APPROXIMATE NEUTRINO TRANSPORT

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ABSTRACT

We present results from the first generation of multi-dimensional hydrodynamic core-collapse simulations in full general relativity (GR) that include an approximate treatment of neutrino transport. Using an M1 closure scheme with an analytic variable Eddington factor, we solve the energy-independent set of radiation energy and momentum based on the Thorne’s momentum formalism. Our newly developed code is designed to evolve the Einstein field equation together with the GR radiation hydrodynamic equations. We follow the dynamics starting from the onset of gravitational core collapse of a 15 $M_\odot$ star, through bounce, up to about 100 ms postbounce in this study. By computing four models that differ according to 1D to 3D and by switching from special relativistic (SR) to GR hydrodynamics, we study how the spacial multi-dimensionality and GR would affect the dynamics in the early postbounce phase. Our 3D results support the anticipation in previous 1D results that the neutrino luminosity and average neutrino energy of any neutrino flavor in the postbounce phase increase when switching from SR to GR hydrodynamics. This is because the deeper gravitational well of GR produces more compact core structures, and thus hotter neutrino spheres at smaller radii. By analyzing the residency timescale to the neutrino-heating timescale in the gain region, we show that the criterion to initiate neutrino-driven explosions can be most easily satisfied in 3D models, irrespective of SR or GR hydrodynamics. Our results suggest that the combination of GR and 3D hydrodynamics provides the most favorable condition to drive a robust neutrino-driven explosion.

Key words: hydrodynamics – neutrinos – supernovae: general

Online-only material: color figures

1. INTRODUCTION

Core-collapse supernova simulations have been counted as one of the most challenging subjects in computational astrophysics. The four fundamental forces of nature are all at play; the collapsing iron core bounces due to strong interactions; weak interactions determine the properties of the stellar gas; electromagnetic interactions determine the energy and lepton number loss in the core via the transport of neutrinos; general relativity (GR) plays an important role due to the compactness of the proto–neutron star and also due to high velocities of the collapsing material outside. Naturally, such physical richness ranging from a microphysical scale (i.e., femtometer scale) to a macrophysical scale (i.e., meter scale) of strong/weak interactions to a macrophysical scale of stellar explosions has long attracted the interest of researchers, situating a world wide, multi-disciplinary collaboration to clarify the theory of massive stellar core collapse and the formation mechanisms of compact objects.

Ever since the first numerical simulation of such events (Colgate & White 1966), the neutrino-heating mechanism (Wilson 1985; Bethe & Wilson 1985), in which a stalled bounce shock could be revived via neutrino absorption on a timescale of several hundred milliseconds after bounce, has been the working hypothesis of supernova theorists for these ~45 years. However, the simplest, spherically symmetric (1D) form of this mechanism fails to blow up canonical massive stars (Rampp & Janka 2000; Liebendörfer et al. 2000; Thompson et al. 2003; Sumiyoshi et al. 2005). Pushed by mounting supernova observations of the blast morphology (e.g., Wang et al. 2001; Maeda et al. 2008; Tanaka et al. 2009 and references therein), it is now almost certain that the breaking of the spherical symmetry holds the key to solving the supernova problem. So far a number of multi-dimensional (multi-D) hydrodynamic simulations have shown that hydrodynamic motions associated with convective overturn (e.g., Herant et al. 1992; Burrows et al. 1995; Janka & Müller 1996; Fryer et al. 2002; Fryer 2004) and the Standing Accretion Shock Instability (SASI; e.g., Blondin et al. 2003; Scheck et al. 2004, 2006; Ohnishi et al. 2006, 2007; Foglizzo et al. 2006; Iwakami et al. 2008, 2009; Murphy & Burrows 2008; Fernández & Thompson 2009a, 2009b and references therein) can help the onset of the neutrino-driven explosion.

In fact, the neutrino-driven explosions have been obtained in the following first-principle two- (2D) and three-dimensional (3D) simulations in which the spectral neutrino transport is solved at various levels of approximations (e.g., Müller et al. 2011; Ott et al. 2011a; Kotake 2011 for recent status reports). Among them are the 2D neutrino-radiation-hydrodynamic simulations by Buras et al. (2006b, 2006a), and Marek & Janka (2009) who included one of the best available neutrino transfer approximations by the ray-by-ray variable Eddington factor method, by Bruenn et al. (2010) who included a ray-by-ray multi-group flux-limited diffusion (MGFLD) transport with the best available weak interactions, and by Suwa et al. (2010, 2011) who employed a ray-by-ray isotropic diffusion source approximation (IDSA; Liebendörfer et al. 2009) with a reduced set of weak interactions. By extending the 2D modules in Suwa et al. (2010) and Takiwaki et al. (2012) recently reported neutrino-driven explosion models in 3D for an 11.2 $M_\odot$ star. They pointed out whether 3D effects would help explosions or not is sensitive to the employed numerical resolutions (see also Hanke et al. 2011; Nordhaus et al. 2010). They argued that future peta- and exa-scale resources are at least needed to draw a robust conclusion of the 3D effects.

In addition to the 3D effects, impacts of GR on the neutrino-driven mechanism stand out among the biggest open questions in the supernova theory. It should be remembered that using...
newly derived Einstein equations (Misner & Sharp 1964), the consideration of GR was standard in the pioneering era of supernova simulations (e.g., May & White 1966). One year after Colgate & White (1966), Schwartz (1967) reported the first fully GR simulation of stellar collapse to study the supernova mechanism, and implemented a gray transport of neutrino diffusion in the 1D GR hydrodynamics. Using GR Boltzmann equations derived by Lindquist (1966), Wilson (1971) developed a 1D GR-radiation-hydrodynamic code including a more realistic (at the time) description of the collisional term than the one in Schwartz (1967). By performing 1D GR hydrodynamic simulations that included a leakage scheme for neutrino cooling, hydrodynamical properties up to the prompt shock stagnation were studied in detail (van Riper 1979, 1982; van Riper & Lattimer 1981). These pioneering studies, albeit using a much simplified neutrino physics than today, did provide a bottom line of our current understanding of the supernova mechanism (see Bruenn et al. 2001 for a complete list of references for the early GR studies). In the middle of the 1980s, Bruenn (1985) developed a code that coupled 1D GR hydrodynamics to the MGFLD transport of order (ν/c) including the so-called standard set of neutrino interactions. Since the late 1990s, the ultimate 1D simulations, in which the GR Boltzmann transport is coupled to 1D GR hydrodynamics, have been made feasible by Sumiyoshi-Yamada et al. (Yamada 1997; Yamada et al. 1999; Sumiyoshi et al. 2005, 2007) and by Liebendörfer-Mezzacappa-Bruenn et al. (Mezzacappa & Matzner 1989; Bruenn et al. 2001; Liebendörfer et al. 2000, 2001, 2004).

Among them, Bruenn et al. (2001) presented evidence that average neutrino energy of any neutrino flavor during the shock reheating phase increase when switching from Newtonian to GR hydrodynamics. They also pointed out that the increase is larger in magnitude compared to the decrease due to redshift effects and gravitational time dilation. By employing the currently best available weak interactions, Lentz et al. (2012) very recently reported the update of Bruenn et al. (2001). They showed that the omission of observer corrections in the transport equation particularly does harm to drive the neutrino-driven explosions. In these full-fledged 1D simulations, a commonly observed disadvantageous aspect of GR to drive neutrino-driven explosions is that the residency time of material in the gain region becomes shorter due to the stronger gravitational pull. As a result of these competing ingredients, in the end GR works disadvantageously to facilitate the neutrino-driven explosions in 1D. In fact, the maximum shock extent in the postbounce phase is shown to be 20% smaller when switching from Newtonian to GR hydrodynamics (e.g., Figure 2 in Lentz et al. 2012).

Among the most up-to-date multi-D models with spectral neutrino transport mentioned earlier, GR effects are at best attempted to be modeled by replacing the monopole term in the Newtonian potential with an effective Tolman–Oppenheimer–Volkov potential (Buras et al. 2006a, 2006b; Marek & Janka 2009; Bruenn et al. 2010). A possible drawback of this prescription is that a conservation law for the total energy cannot be guaranteed by adding an artificial term to the Poisson equation of self-gravity. Since the energy reservoir of the supernova engines is the gravitational binding energy, any potential inaccuracies in the argument of gravity would be better eliminated. There are a number of relativistic simulations of massive stellar collapse in full GR (e.g., 2D; Shibata & Sekiguchi 2005a or 3D; Shibata & Sekiguchi 2005b; Ott et al. 2007 and references therein) or using the conformally flatness approximation (CFC; e.g., Dimmelmeier et al. 2002; Cordero-Carrón et al. 2009). Although extensive attempts have been made to include microphysics such as by the Y_e formula (Liebendörfer 2005) or by a neutrino leakage scheme (Sekiguchi 2010), the effects of neutrino heating have yet to be included in them, which is a main hindrance to study the GR effects on the multi-D neutrino-driven mechanism.

In this paper, we present a new fully GR code for multi-D hydrodynamic supernova simulations in which an approximate neutrino transport is implemented. The code is a marriage of an adaptive-mesh refinement (AMR), conservative 3D GR magnetohydrodynamic (MHD) code developed by Kuroda & Umeda (2010), and the approximate neutrino transport code that we newly develop in this work. The spacetime treatment in our full GR code is based on the Baumgarte–Shapiro–Shibata–Nakamura (BSSN) formalism (see, e.g., Shibata & Nakamura 1995; Baumgarte & Shapiro 1999). The hydrodynamics can be solved either in full GR or in special relativity (SR), which allows us to investigate the GR effects on the supernova dynamics. We solve the energy-independent set of radiation moments up to the first order and evaluate the second-order momentum with an analytic variable Eddington factor (the so-called M1 closure scheme; Levermore 1984). This part is based on the partial implementation of the Thorne’s momentum formalism, which is recently extended by Shibata et al. (2011) in a more suitable manner applicable to the neutrino transport problem. Similar to the IDSA scheme (Liebendörfer et al. 2009), we conceptually divide the neutrinos into two parts, which are “trapped” and “free-streaming” neutrinos. By doing so, we model the source terms of the transport equations to be expressed in a simplified manner with the use of a multi-flavor neutrino leakage scheme (e.g., Rosswog & Liebendörfer 2003). Our newly developed code is designed to evolve the Einstein field equation together with the GR radiation hydrodynamic equations in a self-consistent manner while satisfying the Hamiltonian and momentum constraints. An AMR technique implemented in the 3D code enables us to follow the dynamics starting from the onset of gravitational core collapse of a 15 M⊙ star, through bounce, up to about 100 ms postbounce in this study. For the 15 M⊙ star, the neutrino-driven explosions are expected to take place later than ~200 ms postbounce at the earliest (e.g., Bruenn et al. 2010; Marek & Janka 2009). However, it is computationally too expensive to follow such a long-term evolution in our full 3D GR simulations. Albeit limited to the rather early postbounce phase, we would exploratory study possible GR effects in the multi-D neutrino-driven mechanism by comparing 1D to 3D results and by switching from SR to GR hydrodynamics.

This paper is organized as follows. In Section 2, after we introduce the model concept of the approximate GR transport scheme we summarize the governing equations of hydrodynamics and neutrino transport in detail. The main results are presented in Section 3. We summarize our results and discuss

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3 Schwartz (1967; p. 55) wrote, “In this calculation, the neutrino luminosity of the core is found to be 10^{54} erg s^{-1}, or 1/2 a solar rest mass per second!! . . . This is the mechanism which the supernova explodes.” The neutrino luminosity rarely becomes so high in the modern simulations, but it is surprising that the potential impact of GR on the neutrino-heating mechanism was already indicated in the very first GR simulation.

4 Very recently, Sumiyoshi & Yamada (2012) reported their success to develop the first multi-angle, multi-energy neutrino transport code in 3D.

5 Very recently, Müller et al. (2012b) reported explosions for 11.2 and 15 M⊙ stars based on their 2D GR simulations in CFC with detailed neutrino transport (Müller et al. 2010) similar to Buras et al. (2006b).
their implications in Section 4. Note that geometrized units are used throughout Sections 2 and 3, i.e., both the speed of light and the gravitational constant are set to unity: $G = c = 1$. Greek indices run from 0 to 3, Latin indices from 1 to 3.

2. BASIC EQUATIONS FOR GENERAL RELATIVISTIC NEUTRINO-RADIATION HYDRODYNAMICS

Our newly developed code consists of the three parts, in which the evolution equations of metric, hydrodynamics, and neutrino radiation are solved, respectively. As will be mentioned, each is solved in an operator-splitting manner, but the system evolves self-consistently as a whole satisfying the Hamiltonian and momentum constraints. Before going into details, we shortly describe the bottom line how to add radiation to GR hydrodynamics.

The starting point is the conservation of the total energy (fluid + radiation),

$$\nabla_a T^{\alpha\beta}_{(\text{total})} = \nabla_a T^{\alpha\beta}_{(\text{fluid})} + \nabla_a T^{\alpha\beta}_{(\nu\nu)} = 0,$$

where $T^{\alpha\beta}_{(\text{total})}$, $T^{\alpha\beta}_{(\text{fluid})}$, and $T^{\alpha\beta}_{(\nu\nu)}$ is the stress-energy tensor of the total energy, fluid, and neutrino radiation, respectively. Then Equation (1) can be decomposed as,

$$\nabla_a T^{\alpha\beta}_{(\text{fluid})} = -Q^{\beta},$$

and

$$\nabla_a T^{\alpha\beta}_{(\nu\nu)} = Q^{\beta},$$

where $Q^{\beta}$ represents the source terms that describe the exchange of energy and momentum between fluid and radiation. If $Q^{\beta}$ would be given, it is rather straightforward to evolve Equation (2) following standard procedures in numerical relativity. Accordingly, in this section we focus on how to determine the source terms $Q^{\beta}$ and the evolution equations of neutrinos (i.e., the concrete form of the left-hand side of Equation (3)).

In doing so, many terms related to GR will appear, such as $e^{\phi}$, $\beta^i$, etc. After we briefly summarize the BSSN formalism in the next section, we first present the transport equations in Section 2.2 and then the source terms in Section 2.3.

2.1. Metric Equations

We write the spacetime metric in the standard (3+1) form:

$$ds^2 = -\alpha^2 dt^2 + \gamma_{ij}(dx^i + \beta^i dt)(dx^j + \beta^j dt),$$

where $\alpha$, $\beta^i$, and $\gamma_{ij}$ are the lapse, shift, and spatial metric, respectively. The extrinsic curvature $K_{ij}$ is defined by

$$(\partial_t - L_\beta)\gamma_{ij} = -2\alpha K_{ij},$$

where $L_\beta$ is the Lie derivative with respect to $\beta^i$. The evolution of $\gamma_{ij}$ and $K_{ij}$ is governed by the Einstein equation $G_{\mu\nu} = 8\pi T_{\mu\nu}^{(\text{total})}$, where $G_{\mu\nu}$ is the Einstein tensor and $T_{\mu\nu}^{(\text{total})}$ is the total stress-energy tensor (e.g., Equation (1)).

We evolve $\gamma_{ij}$ and $K_{ij}$ using the BSSN formulation (Baumgarte & Shapiro 1999; Shibata & Sekiguchi 2005; Duez et al. 2006), in which the fundamental variables are

$$\phi \equiv \frac{1}{12} \ln[\det(\gamma_{ij})],$$

$$\tilde{\gamma}_{ij} \equiv e^{-\phi}\gamma_{ij},$$

and by the Gamma-driver shift,

$$\partial_t \beta^i = k \delta_t \tilde{\gamma}^i,$$

here we chose $k = 1$. For further information with code verification of the metric solver, see Kuroda & Umeda (2010). In addition, during calculations, we enforce following algebraic constraints every after the time updating to satisfy $\tilde{\gamma} = 1$ and $\Delta_{ij} = 0$ (Zlochower et al. 2005; Etienne et al. 2008)

$$\tilde{\gamma}_{ij} \rightarrow \tilde{\gamma}_{ij}\tilde{\gamma}^{-1/3},$$

$$\tilde{A}_{ij} \rightarrow \tilde{A}_{ij} - \frac{\tilde{\gamma}_{ij}}{3} \tilde{A}^k_k.$$

Having summarized the bottom line of the BSSN formalism, we are now ready to discuss the transport equations.

$\delta_t \phi$ is the Lie derivative with respect to $\beta^i$. The evolution of $\gamma_{ij}$ and $K_{ij}$ is governed by the Einstein equation $G_{\mu\nu} = 8\pi T_{\mu\nu}^{(\text{total})}$, where $G_{\mu\nu}$ is the Einstein tensor and $T_{\mu\nu}^{(\text{total})}$ is the total stress-energy tensor (e.g., Equation (1)).

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$$\tilde{A}_{ij} \rightarrow \tilde{A}_{ij} - \frac{\tilde{\gamma}_{ij}}{3} \tilde{A}^k_k.$$
2.2. Neutrino Transport Equations

To determine the transport equations in GR, we follow the truncated moment formalism (Thorne 1981), which was recently extended by Shibata et al. (2011), in a suitable form for the neutrino transport problem. The starting point is to define the radiation stress-energy tensor as

$$ T_{(v)}^{\alpha\beta} \equiv E_{(v)} n^\alpha n^\beta + F_{(v)}^{\alpha} n^\beta + F_{(v)}^{\beta} n^\alpha + P_{(v)}^{\alpha\beta}, \quad (21) $$

where $E_{(v)}$, $F_{(v)}$, and $P_{(v)}$ are the radiation energy, flux, pressure measured in the laboratory frame, respectively. Conversely, $E_{(v)}$, $F_{(v)}$, and $P_{(v)}^{\alpha\beta}$ are given by the stress-energy tensor as

$$ E_{(v)} = T_{(v)}^{\alpha\beta} n_\alpha n_\beta, \quad F_{(v)}^i = -T_{(v)}^{\alpha\beta} n_\alpha \gamma_i^\beta, \quad P_{(v)}^{ij} = T_{(v)}^{\alpha\beta} \gamma_i^\alpha \gamma_j^\beta. \quad (22) $$

In the following, radiation variables are all defined in the laboratory frame unless otherwise stated.

According to Shibata et al. (2011), the evolution equations of radiation energy and radiation flux in Equation (21) can be written as

$$ \partial_t (e^{\phi} E_{(v)}) + \partial_i [e^{\phi} (\alpha F_{(v)}^i - \beta^i E_{(v)})] = e^{\phi} (\alpha P_{(v)}^{ij} K_{ij} - F_{(v)}^i \partial_i \alpha - \alpha Q^i n_\mu), \quad (23) $$

and

$$ \partial_t (e^{\phi} F_{(v)}^i) + \partial_j [e^{\phi} (\alpha P_{(v)}^{ij} - \beta^j F_{(v)}^i)] = e^{\phi} [- E_{(v)} \partial_i \alpha + F_{(v)}^i \partial_i \beta^j + (\alpha/2) P^{ij}_1 \partial_j \gamma_k^i + \alpha Q^i \gamma_j^\mu], \quad (24) $$

where $Q^\mu$ denotes the source terms. For the three radiation variables ($E_{(v)}$, $F_{(v)}^i$, $P_{(v)}^{ij}$) in Equations (23) and (24), we have only two sets of the equation. Here we employ the so-called M1 closure (Levermore 1984), in which the radiation pressure is related to the radiation energy and flux by an analytical closure relation (i.e., $P_{(v)}(E_{(v)}, F_{(v)})$) as

$$ P_{(v)}^{ij} = \frac{3 \chi}{2} P_{\text{thin}}^{ij} + \frac{3(1 - \chi)}{2} P_{\text{thick}}^{ij}, \quad (25) $$

where $\chi$ represents the variable Eddington factor, $P_{\text{thin}}^{ij}$ and $P_{\text{thick}}^{ij}$ correspond to the radiation pressure in the optically thin and thick limit, respectively.

For the variable Eddington factor $\chi$, we employ the one proposed by Levermore (1984),

$$ \chi = \frac{3 + 4 \bar{F}^2}{2 + \sqrt{4 - 3 \bar{F}^2}}, \quad (26) $$

$$ \bar{F}^2 \equiv \frac{F^i F_i}{E^2}. \quad (27) $$

It can be readily checked that in the optically thick limit, $\chi \rightarrow 1/3$ because $F^i \rightarrow 0$, then $P_{\text{thick}}^{ij} \rightarrow P_{\text{thick}}^{ij}$, while in the optically thin limit, $\chi \rightarrow 1$ because $\bar{F}^2 \rightarrow 1$, then $P_{\text{thin}}^{ij} \rightarrow P_{\text{thin}}^{ij}$.

Following Audit et al. (2002) and Shibata et al. (2011), the following forms of $P_{\text{thin}}^{ij}$ and $P_{\text{thick}}^{ij}$ are adopted,

$$ P_{\text{thick}}^{ij} = \mathcal{J} \frac{\gamma_{ij} + 4 \nu_{ij} \gamma_{ij} u_k u_i}{3} + \nu_{ij} H^i u_k + \nu_{ik} H^k u_i, \quad (29) $$

and

$$ \mathcal{J} = E_{(v)} W^2 - 2 W F_{(v)}^i u_i + P_{(v)}^{ij} u_i u_j, \quad (30) $$

and

$$ H^\alpha = (E_{(v)} W - F_{(v)}^i u_i)(n^\alpha - W u^\alpha) + W h^\alpha_{\beta} F_{(v)}^i \partial_i u_j P_{(v)}^{ij}, \quad (31) $$

where $W = \alpha u^\mu$ is the Lorentz factor and $h_{\alpha\beta}$ is the projection operator defined by

$$ h_{\alpha\beta} \equiv g_{\alpha\beta} + u_\alpha u_\beta. \quad (32) $$

Having summarized the closed set of the two-moment transport equations, we are now going to discuss the source terms ($Q^\mu$) in the next section.

2.3. Source Terms

To model the source terms, we follow the idea of the IDSA scheme (Liebendörfer et al. 2009) in which neutrinos are conceptually divided into two parts, “trapped” and “free-streaming” neutrinos, respectively. We also utilize a methodology of multistatus neutrino leakage scheme (e.g., Rosswog & Liebendörfer 2003) to simplify the description of the source terms.

Figure 1 illustrates the concept of how to estimate the source terms. To describe the neutrino-matter coupling, we need to ask at least three actors, namely, “matter,” “trapped neutrino,” and “streaming neutrino,” to appear in the playground of the supernova core. The neutrino sphere is an important quantity to describe the relationship between them. The position of the
neutrino sphere\(^3\) at which the neutrino optical depth comes close to unity, is indicated by \(\tau_\nu = 2/3\) in Figure 1. The trapped neutrinos are always coupled with matter (through \(\beta\)-equilibrium) and their temperature is the same as that of matter. On the other hand, temperature of the free-streaming neutrinos cannot be determined locally owing to its decoupling nature from matter, which was the reason that we have to solve the evolution equations. Thus, these two represent a two extreme limit.

In Figure 1, trapped neutrinos are denoted by “\(v_{\text{trap}}\)” (in a diamond shape colored by gray), which is illustrated to shake hands with matter inside the neutrino sphere (inside the region enclosed by \(\tau_\nu = 2/3\)). There the trapped neutrinos dominate over the streaming neutrinos (denoted by “\(v_{\text{stream}}\)” in orange in Figure 1), which is opposite outside the neutrino sphere. This is illustrated in such a way that “\(v_{\text{trap}}\)” is bigger than “\(v_{\text{stream}}\)” inside the neutrino sphere, which is opposite outside the neutrino sphere. For the \(v_{\text{stream}}\) outside the neutrino sphere, the \(v_{\text{stream}}\) is drawn to have several tails, by which we intend to express that it can travel much more freely in the free-streaming regime.

In Figure 1, the three actors are connected by thick arrows (in blue or red), each of them is labeled by \(Q_{\nu,\text{diff}}\) (in blue), \(Q_{\nu,\text{intr}}\) (in blue), or \(Q_{\nu,H}\) (in red), representing the couplings in-between. The blue arrows (\(Q_{\nu,\text{diff}}^{\mu,C}\) and \(Q_{\nu,\text{intr}}^{\mu,C}\)) represent neutrino cooling, while the red arrow (\(Q_{\nu,H}^{\mu,C}\)) performs neutrino heating. The neutrino cooling means that energy is transferred from streaming neutrinos (or from trapped neutrinos) to streaming neutrinos that carry the imparted energy away from the system. On the other hand, the neutrino heating proceeds by energy transfer from streaming neutrinos to matter (see \(Q_{\nu,H}^{\mu,C}\) in Figure 1). Finally \(Q_{\nu,\text{diff}}^{\mu,C}\) in Figure 1 represents the cooling by neutrinos leaking out from opaque regions inside the neutrino sphere by diffusion.\(^8\)

Looking at Figure 1 again, the source term of Equations (23) and (24) can be readily defined as

\[
Q^\mu \equiv Q_{\nu,H}^{\mu,C} - Q_{\nu,\text{intr}}^{\mu,C},
\]

where each of the cooling (\(Q_{\nu,H}^{\mu,C}\)) and heating (\(Q_{\nu,\text{intr}}^{\mu,C}\)) term is calculated in the present scheme as

\[
Q_{\nu,H}^{\mu,C} = \sum_{v \in \{\nu_e, \nu_x\}} (1 - e^{-\beta_\nu \tau_\nu}) Q_{\nu,\text{diff}}^{\nu, \rightarrow v_e} \epsilon_\nu n_\nu d\epsilon_\nu dV(e_\nu),
\]

\[
Q_{\nu,\text{intr}}^{\mu,C} = \sum_{v \in \{\nu_e, \nu_x\}} e^{-\beta_\nu \tau_\nu} \epsilon_\nu \kappa_v (\mathcal{S}_\nu u^\mu - \mathcal{H}_\nu^\mu).
\]

Before we go into detail, we need to draw a caution that we introduced the concept of the streaming and trapped neutrinos only for the sake of (better) explanation of our approximate treatment. Actually, the sum of the trapped and streaming is transported by Equations (23) and (24) with the source terms described above. We design the source terms in such a way to connect the heating/cooling terms smoothly between the diffusion and free-streaming limit, which is basically similar to the concept of the M1 closure relation.

The cooling term (\(Q_{\nu,H}^{\mu,C}\)) consists of \(Q_{\nu,\text{diff}}^{\nu, \rightarrow v_e}\) and \(Q_{\nu,\text{intr}}^{\nu, \rightarrow v_e}\), which accounts for neutrino cooling by diffusion out of the neutrino sphere and the one determined locally outside the neutrino sphere, respectively (e.g., Figure 1). Following van Riper & Lattimer (1981), the terms of \(1 - e^{-\beta_\nu \tau_\nu}\) and \(e^{-\beta_\nu \tau_\nu}\) appearing in Equation (34) are introduced to smoothly connect the two quantities (\(Q_{\nu,\text{diff}}^{\nu, \rightarrow v_e}\) and \(Q_{\nu,\text{intr}}^{\nu, \rightarrow v_e}\)) for the semitransparent regime. Here, \(\tau_\nu\) represents the optical depth of neutrinos and \(\beta_\nu\) is a model parameter that we determine by the comparison with a spectral neutrino transport calculation (see Appendix A). With these terms bridging the two limits, it is easy to see that \(Q_{\nu,H}^{\mu,C}\) approaches to \(Q_{\nu,\text{diff}}^{\nu, \rightarrow v_e}\) for the diffusion limit (\(\tau_\nu \to \infty\)), and it does to \(Q_{\nu,\text{intr}}^{\nu, \rightarrow v_e}\) for the free-streaming limit (\(\tau_\nu \to 0\)).

Following the neutrino leakage scheme (e.g., Epstein & Pethick 1981; van Riper & Lattimer 1981; Kotake et al. 2003), the diffusion cooling rate (\(Q_{\nu,\text{diff}}\)) can be given as

\[
Q_{\nu,\text{diff}}[\text{erg cm}^{-3} \text{s}^{-1}] = \int \frac{e_\nu n_\nu}{T_{\nu,\text{diff}}(e_\nu)} d\epsilon_\nu,
\]

where the right-hand side of Equation (36) is simply proportional to the leakage of the neutrino energy density; \(\epsilon_\nu n_\nu\) (erg cm\(^{-3}\)) divided by the diffusion timescale \(T_{\nu,\text{diff}}\) [s]. More details to estimate these quantities as well as how to determine the neutrino sphere are summarized in Appendix A.

Striving for simplification of our modeling, we include a reduced set of neutrino-matter interactions (e.g., Table 1). Regarding the charged-current interactions, emission, and absorption of electron neutrinos by electrons (the first column in Table 1), emission and absorption of electron anti-neutrinos by proton (the second column), and emission and absorption of electron neutrinos by heavy nuclei (the third column), are included. For the neutral-current interactions, elastic scattering of all neutrino types of nucleons (the fourth and fifth columns in Table 1), and the coherent elastic scattering (the sixth column) are included. For the cross sections of these reactions, we employ the oness summarized in Burrows et al. (2006b) while omitting higher-order terms such as ion–ion correlations and weak magnetism for simplicity.

In computing the neutrino cooling rate (\(Q_{\nu,\text{intr}}\)), we furthermore consider the contribution from pair neutrino annihilation \(Q_{\nu, e^+\nu^-} \rightarrow v_e\) (Cooperstein et al. 1986), plasmon decay \(Q_{\gamma \nu_e \nu_x} \rightarrow v_e\) (Ruffert et al. 1996), and nucleon–nucleon bremsstrahlung \(Q_{NN,NN} \rightarrow NN\nu\) (Burrows et al. 2006b), which are also summarized in Itoh et al. (1996) and Sekiguchi (2010). Hence \(Q_{\nu,\text{intr}}\) can be expressed as

\[
Q_{\nu,\text{intr}} = Q_{\nu,\text{diff}} + Q_{\nu,\text{intr}}^h + Q_{\nu,\text{intr}}^e + Q_{\nu,\text{intr}}^\nu + \sum_{v \in \{\nu_e, \nu_x\}} 2(Q_{\nu, e^\pm v_e} + Q_{\nu, e^\nu v_x} + Q_{NN,NN, v_e}).
\]

Here defined for the average neutrino energy for simplicity.

Note that inside the neutrino sphere, neutrino heating locally balances with neutrino cooling by weak interactions due to \(\beta\)-equilibrium. So as a net, the diffusion-mediated cooling becomes dominant there.
where $Q_{\nu_i}$ and $Q_{e_i}$ represents the cooling rate by electron/positron capture on free nucleons and on heavy nuclei, respectively.

Concerning the neutrino heating ($Q^{\nu,H}$), we only include the dominant heating reactions in the gain region, which is absorption of electron/anti-electron neutrinos by free nucleons. Then $Q^{\nu,H}$ reads

$$Q^{\nu,H} = e^{-\beta \tau} \sum_{i=0}^{n} \int d\omega \omega_i (\mathcal{J}^{\nu,i} u^\mu - \mathcal{H}^{\nu,i} \mathcal{H}_{\nu,i}),$$

where $\omega$ denotes neutrino energy, $\kappa_{\nu,i}$ is the energy-dependent opacity for electron or anti-electron neutrinos (i.e., $i = e$ or $\bar{e}$; see Appendix A), and $\mathcal{J}_{\nu,i}$, $\mathcal{H}_{\nu,i}$ is the energy-dependent Eddington moments, respectively. Yielding to the prescription of the so-called light-bulb scheme (e.g., Nordhaus et al. 2010), a term of $e^{-\beta \tau}$ is introduced to vanish the neutrino heating smoothly as the opacity becomes higher inward down to the neutrino sphere. To take a gray approximation, we replace the energy integration in Equation (38) with the root-mean-squared (rms) energy of the streaming neutrinos ($\epsilon_{\nu,i}$; see Appendix A for the definition) as

$$\int d\omega \omega_i (\mathcal{J}^{\nu,i} u^\mu - \mathcal{H}^{\nu,i} \mathcal{H}_{\nu,i}) \approx (\epsilon_{\nu,i})^2 \kappa (\mathcal{J} u^\mu - \mathcal{H} \mathcal{H})^{\nu,i},$$

where $\kappa$ denotes the monochromatic opacity, in which the energy-dependence is replaced with the one of the rms energy (namely, $\kappa_i = \kappa \epsilon_{\nu,i}^2$). Since $\mathcal{J}$, $\mathcal{H}$ in Equation (39) is related to the variables in the laboratory frame by Equations (30) and (31), the two-moment equations of $E_\nu$ and $F_\nu$, with the source terms are finally closed. Having given explicit forms of $Q^{\nu}$, we are now moving on to summarize the GR hydrodynamic equations including the source terms in the next section.

2.4. Hydrodynamic Equations

From Equation (2), the hydrodynamic equations are written in a conservative form as

$$\partial_t \rho_\alpha + \partial_i (\rho_\alpha v^i) = 0,$$

$$\partial_t \tilde{S}_i + \partial_j (\tilde{S}_j v^i + \alpha \epsilon^{\nu,b} P_\nu P_j) = -\tilde{S}_0 \partial_i \alpha + \tilde{S}_0 \partial_i \beta^k + 2\alpha \epsilon^{\nu,b} S_k \partial_i \epsilon + \alpha \epsilon^{\nu,b} \rho_\alpha \epsilon^{\nu,b} \rho_\alpha,$$

$$\partial_t \tilde{\tau} + \partial_i (\tilde{S}_i v^i + \alpha \epsilon^{\nu,b} P_\nu P) = \alpha \epsilon^{\nu,b} K \tilde{S}^k / 3 + \alpha \epsilon^{\nu,b} \rho_\alpha \epsilon^{\nu,b},$$

where $\tilde{X} \equiv \epsilon^{\nu,b} X$, $\rho_\alpha \equiv \rho W \epsilon^{\nu,b}$, $S_i \equiv \rho h W u_i$, and $S_0 \equiv \rho h W^2 - p$, $\rho$ is the rest-mass density, $u_\mu$ is the four velocity of fluid, $h \equiv 1 + \epsilon + p / \rho$ is the specific enthalpy, $v^i \equiv u^i / u^0$, $\tilde{\tau} \equiv S_0 - \rho_\alpha$, $Y_e$ is the electron fraction, $\epsilon$ and $p$ is the internal energy and pressure, respectively (see Appendix B).

The delta function in the left-hand side of Equation (39) may be replaced by the Fermi-Dirac function. In the case, an additional factor of $F_3(\alpha,0) = 1 / 2$ can be multiplied, which could potentially enhance the impacts of neutrino heating.

From Equations (34) and (35), the source terms appearing in the right-hand side of Equations (41) and (42) can be explicitly written as

$$-Q^\mu \gamma_{\mu i} = -(Q^{n,C} - Q^{\nu,H}) \gamma_{\mu i}$$

$$= - \sum_{J \in \{e, \bar{e}, \gamma\}} \left[ (1 - e^{-\beta \tau}) Q_{\nu, \text{diff}} + e^{-\beta \tau} Q_{\nu, \text{int}} \right] H_i + \sum_{J \in \{e, \bar{e}, \gamma\}} e^{-\beta \tau} (\epsilon_{\nu,i})^2 \kappa_i (-WF_{\nu,i} + P_{\nu,i} u_k)$$

$$Q^{\mu} \rho_{\mu} = (Q^{n,C} - Q^{\nu,H}) \rho_\mu$$

$$= - \sum_{J \in \{e, \bar{e}, \gamma\}} \left[ (1 - e^{-\beta \tau}) Q_{\nu, \text{diff}} + e^{-\beta \tau} Q_{\nu, \text{int}} \right] W$$

$$+ \sum_{J \in \{e, \bar{e}, \gamma\}} e^{-\beta \tau} (\epsilon_{\nu,i})^2 \kappa_i (W E_\nu - F^3_{\nu,i} u_k).$$

From Equations (40)–(42) are closed. We employ the latest version of tabulated EOS by Shen et al. (1998) for heavy nuclei and uniform nuclear matter. Since Shen EOS contains contributions only from baryons, we need to add contributions from electron/positron, and photon (see Appendix B for more details).

3. INITIAL MODELS AND NUMERICAL METHODS

3.1. Initial Models

To assess GR and 3D effects on the neutrino-heating mechanism, we compute four models with a combination of SR or GR hydrodynamics in 1D or 3D, which we label as 1D-SR, 1D-GR, 3D-SR, and 3D-GR, respectively. We employ a widely used progenitor of a 15M☉ star (Woosley & Weaver 1995), model “s1557b2”). In our SR models, the spacetime metric is assumed to be flat (i.e., $\alpha = 1$, $\beta = 0$, $\gamma_{ij} = \delta_{ij}$, $\phi = 0$, $K_{ij} = 0$) and we also assume that the self-gravity acts instantaneously in a Newtonian way which is evaluated by solving the following Poisson equation:

$$\nabla^2 \Phi_N = 4\pi S_0.$$
source terms defined at the cell center (see Kuroda & Umeda 2010 for further details such as how to treat the self-gravity in the AMR structure and how to evolve the spacetime metric with GR hydrodynamics).

To construct 1D models in our Cartesian code, the following condition for the spacial velocity $u_i$ (and also for $S_i$) is enforced at every (hydro-) timestep,

$$ u_i = \frac{x^i u_j}{|x|^2} x^j. \quad (48) $$

As can be seen, this operation eliminates the non-radial components of the flow velocity and momentum. Although the artificial elimination could potentially lead to the shift of the kinetic energy into the thermal one, our 1D results (with and without neutrino heating/cooling) are in good agreement with the previous 1D results, as will be mentioned in Appendix C and Section 4. This suggests that the manipulation is not severely bad for the sake of this study. However, in our 1D-GR model, this artificial procedure violates the momentum constraint more or less, to which we will come back in Appendix C.

The 3D computational domain consists of a cube of $10,000^3$ km$^3$ volume in the Cartesian coordinates. In our 3D models, we set the maximum refinement AMR level ($L_{\mathrm{AMR}}$, i.e., refine the AMR boxes in the vicinity of the center) at 5 in the beginning and then increment it as the collapse proceeds. We define the criterion to increment $L_{\mathrm{AMR}}$ every time the central density exceeds $10^{12,13,13.5}$ g cm$^{-3}$ during the infall phase (see Kuroda & Umeda 2010 for more details). Each AMR level consists of $8^3$ AMR blocks with a nested structure and each AMR block has $8^3$ cubic cells. Roughly speaking, such structure corresponds to an angular resolution of $\sim$2 degree through the entire computational volume. Near the core bounce, an effective resolution becomes $\Delta x \sim 600$ m in the center of our 3D models. The numerical resolutions are summarized in Table 2. We note that the employed resolutions in the central region are almost similar to those in Ott et al. (2012) who very recently reported 3D GR results using AMR technique. However, our resolutions near the accretion shock surface ($r \sim 100$–150 km) are $\gtrsim$2 times coarser than their value $\Delta x \sim 900$ m.

### 3.2. Numerical Methods

Since the hydrodynamic and transport equations (Equations (40)–(42), and (23) and (24)) are all expressed in a hyperbolic form, they can be evolved by a standard high-resolution-shock-capturing scheme. We utilize the HLL (Harten–Lax–van Leer) scheme (Harten et al. 1983) to evaluate the numerical fluxes. A reconstruction of the primitive variables defined at the immediate left/right of the cell surface is performed by a monotonized central method (Van Leer 1977). The fastest (or right-going) and slowest (or left-going) characteristic wave speeds of fluid system, $\lambda_{\text{flu}}$, for $i (\in x, y, z)$ direction are obtained by solving the following second-order equation,

$$ A_{\text{flu}}^2 + 2B_{\text{flu}} + C = 0 \quad (49) $$

where

$$ A = \left( \frac{1}{c_s^2} - 1 \right) W^2 + 1, $$

$$ B = \beta^i - \left( \frac{1}{c_s^2} - 1 \right) (\alpha u^i - \beta^i) W^2, $$

$$ C = \left( \frac{1}{c_s^2} - 1 \right) W^2 (\alpha u^i - \beta^i)^2 + \beta^{i2} - \alpha^2 \gamma^i, $$

and $c_s$ is the sound velocity (see Appendix B.3).

Meanwhile, the fastest and slowest characteristic wave speeds of the radiation system, $\lambda_{\text{rad}}$, are assumed to have the same expression of the radiation pressure (Equation (25)) as

$$ \lambda_{\text{rad}} = \frac{3 \chi - 1}{2} \lambda_{\text{rad,thin}} + \frac{3(1 - \chi)}{2} \lambda_{\text{rad,thick}}, \quad (51) $$

where $\lambda_{\text{rad,thin}}$ and $\lambda_{\text{rad,thick}}$ are determined by $P_{\text{rad,thin}}^{ij}$ and $P_{\text{rad,thick}}^{ij}$ respectively. According to Shibata et al. (2011), the fastest (slowest) wave speed in the optically thick or thin limit is evaluated by taking maximum (minimum) values, that is,

$$ \left( -\beta^i + \frac{2 W^2 p^i \pm \sqrt{2 W^2 (2 W^2 + 1) - 2 W^2 p^2}}{2 W^2 + 1}, -\beta^i + p^i \right), $$

for the optically thick limit (where $p^i = \gamma^i u_j / u^j$) and

$$ \left( -\beta^i \pm \alpha \frac{F^i}{\sqrt{F_j F^j}}, -\beta^i + \alpha E \frac{F^i}{F_j F^j} \right), $$

for the optically thin limit, respectively. With these wave velocities regarding the fluid and radiation component ($\lambda_{\text{flu/rad}}$), we define the HLL flux (Anton et al. 2006) as

$$ F_{\text{HLL}} = \frac{\tilde{\lambda}_i F_L - \lambda_\rho F_R + \tilde{\lambda}_\rho (Q_R - Q_L)}{\tilde{\lambda}_\rho - \lambda_\rho}, \quad (54) $$

where $\tilde{\lambda} = \lambda / \alpha$, $L/R$ denotes the left/right states for the Riemann problem with $F_{L/R}$ and $Q_{L/R}$ representing the advection and conservative terms, respectively.

To ensure conservative laws at the interface of different AMR levels, we furthermore need to perform a “refluxing” procedure in estimating the numerical flux (see Kuroda & Umeda 2010). To evolve the BSSN terms, we adopt the fourth-order finite differences for the spatial derivatives and the fourth-order upwind differencing for the advection terms (Zlochower et al. 2005; Etienne et al. 2008) except at the AMR boundary. At the AMR boundary, we employed third-order upwind scheme for the advection terms. Numerical tests are presented in Appendix C, in which we first show a 1D adiabatic core-collapse test to validate the implementation of Shen EOS in the code, followed by the corresponding 1D tests including neutrinos.

### 4. RESULTS

First let us compare prebounce features among the four models (1D-SR, 1D-GR, 3D-SR, and 3D-GR) in Section 4.1, and move on to the postbounce phase in Section 4.2. In Section 4.3, we will discuss the 3D/GR effects on the neutrino-heating mechanism.
4.1. Infall and Bounce

We begin our comparisons with the infall, bounce, and immediate postbounce phase. As can be seen from Figure 2, collapse to bounce takes slightly less time in our GR models (137 ms) compared to the SR models (141 ms), and the central density $\rho_c$ at the bounce is approximately two times larger in the GR simulations than in the corresponding SR simulations (see the inset in Figure 2). For our non-rotating progenitor, the dynamics of collapsing iron core proceeds totally spherically until the stall of the bounce shock. This is the reason that the multi-D effects are invisible in the immediate postbounce phase. Hence, we focus on the comparisons between the 1D-SR and 1D-GR model in the rest of this subsection.

Figure 3 shows several snapshots of the lepton fraction ($Y_{\text{total}}$), electron fraction ($Y_e$), and electron-type neutrino fraction ($Y_{\nu_e}$) for the 1D-SR (left panel) and 1D-GR model (right panel), respectively. After neutrino trapping (i.e., at a central density of a few $10^{12}$ g cm$^{-3}$), the central lepton fraction (black lines) is shown to be conserved later on. In the trapped regions, the radial profile of the neutrino fraction (blue lines) is almost flat, while $Y_e$ shows a gradual increase to satisfy the $\beta$-equilibrium.

From Figure 4, it can be seen that the lepton fraction at bounce (right endpoint in density) is slightly larger for the 1D-GR model ($\sim 0.364$, solid line) compared to the 1D-SR model ($\sim 0.359$, dotted line). The slight suppression of deptonization is possibly because the neutrino opacity is effectively enhanced because of the more compact core in the GR model. Note that this trend is qualitatively in accord with the previous 1D results in which a spectral neutrino transport was solved (e.g., Lentz et al. 2012; Müller et al. 2010). Quantitatively, the central density ($\rho_c = 5.5 \times 10^{14}$ g cm$^{-3}$) at the bounce in our 1D-GR model is higher than the one ($\rho_c = 3.4 \times 10^{14}$ g cm$^{-3}$) obtained in a 1D full-fledged GR simulation by Sumiyoshi et al. (2005) who solved the 1D Boltzmann transport using the same progenitor and the same EOS. Regardless of big differences in the transport scheme as well as employed microphysics (i.e., very detailed versus very approximate), the central $Y_e \approx 0.3$ at the bounce happens to be very close with each other. Our results on the 1D-GR model are rather similar to Sekiguchi (2010). This is reasonable in the sense that the our neutrino transport relies partly on a multi-species leakage scheme using similar microphysical detail.

Figure 5 compares the mean energy of trapped neutrinos $\langle \varepsilon_{\nu} \rangle$ between the 1D-SR (left) and 1D-GR model (right) (see Appendix A for definition). The mean energy is shown to be
20% larger (maximally near the bounce) for the GR model compared to the SR counterpart. This is also because of the more compact core due to the GR hydrodynamics (Figure 6), leading to a more hotter neutrino sphere at smaller radii.

4.2. 3D Hydrodynamics in the Postbounce Phase

In the first 10 ms after bounce, the bounce shock turns into the so-called passive shock, which expands gradually outward with no positive radial velocities (Buras et al. 2006b). As shown in Figure 7, the average shock radii, until the passive expansion starts \((t_{pb} \lesssim 10 \text{ ms})\), remain almost close in all the models. It then diverges, which is more remarkable between the 3D models (solid lines). As will be discussed in the following, this is because multi-D effects (convection and SASI) sensitively affect the postshock hydrodynamic behaviors, also under the influence of the different treatment in gravity (SR versus GR).

First of all, let us compare the shock evolution among the 1D models in SR versus GR (1D-SR, thin dotted line or 1D-GR, thick dotted line). As expected, the shock radius is generally more compact for the 1D-GR model (thick dotted line). On the other hand, the maximum shock extent is observed to be almost the same with each other \((\langle R_{\text{shock}} \rangle \sim 130 \text{ km})\). Though we cannot unambiguously specify the reason, this trend was also seen in Müller et al. (2010), who compared the shock radii in 1D simulations with detailed neutrino transport in CFC versus the corresponding Newtonian model with the effective potential approach (e.g., right panel of their Figure 5). The maximum of \((\langle R_{\text{shock}} \rangle)\) in Figure 7 for our 1D models indicates the epoch when the passive expansion stops. Afterward \((t_{pb} \gtrsim 70 \text{ ms})\), the shock begins to shrink and a much more rapid recession is visible for the GR model (thick dotted line). The maximum shock radii and the shock recession timescale obtained here are again similar to those obtained in the previous 1D results for the same progenitor model with employing the Shen EOS (Sumiyoshi et al. 2005). The maximum shock extent was shown in Sumiyoshi et al. (2005) at a radius of \(\sim 150 \text{ km} \) at around 80–90 ms postbounce, which is at \(\sim 140 \text{ km} \) at \(\sim 70 \text{ ms} \) postbounce in our 1D-GR model. Considering the mentioned difference of neutrino transport as well as different hydrodynamics solvers, it may not be so unreasonable to say that our results show a relatively good agreement with Sumiyoshi et al. (2005). In addition, Sumiyoshi et al. (2005) showed that the shock propagation during the first \(\sim 200 \text{ ms} \) after bounce does not depend so much on the EOS. This should be the reason that our 1D-GR results, at least for the evolution of the shock radii, are quite similar to Lentz et al. (2012) and Müller et al. (2010).

Four snapshots in Figure 8 are helpful to characterize the postbounce features in our 3D-GR model. The top left panel is for \(t_{pb} \approx 10 \text{ ms} \), when the bounce shock stalls at a radius of \(\sim 90 \text{ km} \) (seen as a central blueish sphere). From the sidewall panels, the dominance of the \(\ell = 4 \) and \(m = 4 \) mode can be seen in the postshock region, which is a numerical artifact inherent to the use of Cartesian coordinates. Comparing the top left to top right panel in Figure 8, the size of the outer
sphere that marks the position of the shock (seen as greenish in the top right panel) becomes bigger, which is due to the passive expansion. At this stage, the gain region forms in which neutrino heating dominates over neutrino cooling (e.g., Janka 2001). The neutrino-driven convection gradually develops later on. The sidewall panels of the top right panel also indicate the growth of the postshock convection triggered by Rayleigh–Taylor instabilities. The entropy behind the standing shock becomes higher with time due to neutrino heating, which can be inferred from a yellowish bubble in the bottom left panel. The high entropy bubbles ($s[k_B/\text{baryon}] \gtrsim 10$) rise and sink behind the standing shock. The shock deformation is dominated by unipolar and bipolar modes, which may be interpreted as an onset of the SASI. The size of the neutrino-heated regions grows bigger with time in a non-axisymmetric way, which is indicated by bubbly structures with increasing entropy (indicated by reddish regions in the bottom right panel).

During our simulation time (100 ms after bounce), the shock radii can reach most farther out for our 3D-GR model (red line in Figure 7). In contrast, the shock has already shown a trend of recession in other models. Before we focus on the reason of it in the final section, let us next compare the activities of convective overturns as well as possible onset of the SASI that we only touched on above.

Figure 9 displays space–time diagrams of laterally averaged: (1) Brunt–Väisälä (BV) frequency $\omega_{\text{BV}}$ (top left panel), (2) the anisotropic velocity $V_{\text{aniso}}$ (top right), (3) the pressure perturbation $\Delta p$ in a logarithmic scale (bottom left), and (4) the net heating rate per baryon $Q_{\text{net}}$ (bottom right) for our 3D-SR (top four panels) and 3D-GR models (the other four panels), respectively. Each of the quantities is defined as

$$\omega_{\text{BV}} \equiv \text{sign}(C_L) \sqrt{|g_{\text{eff}} C_L|},$$  

(55)

where $g_{\text{eff}}$ represents effective gravitational acceleration that is estimated by taking a radial gradient of the potential, i.e., $g_{\text{eff}} = d\phi_{\text{NT}}/dr$ for SR models and $g_{\text{eff}} = d\alpha/dr$ for GR.
Figure 9. Panels (a)–(d) display angle averaged: (a) Brunt–Väisälä frequency ($\omega_{BV} \text{ ms}^{-1}$); (b) anisotropic velocity $V_{\text{aniso}}$ normalized by $10^9 \text{ cm s}^{-1}$; (c) normalized pressure perturbation $\Delta p$ (in a logarithmic scale); (d) net energy deposition rate per baryon $Q_{\text{net}}$ (GeV nucleon$^{-1}$ s$^{-1}$) for our 3D-SR (top four panels) and 3D-CR model (the rest four), respectively. Note that convectively unstable regions (i.e., $\omega_{BV} > 0$) are only shown in panel (a) and the white line represents the contour of $\omega_{BV} = 0$. In panel (d), color contour of negative value of $Q_{\text{net}}$ is saturated at $-0.3$. To guide the eye, a white arrow is inserted in panel (c) which points to a up-going pressure perturbation to the shock.

(A color version of this figure is available in the online journal.)
Comparing the black dotted line in Figure 10 (for 1D-GR model at $t_{pb} = 60$ ms) note that the shock is indicated by a white thin line quickly rising after models, respectively. $C_L$ is the Ledoux criterion

$$C_L \equiv -\frac{\partial P}{\partial Y_{\mu}} \left( \frac{\partial P}{\partial Y_{\mu}} \frac{ds}{dr} + \frac{\partial P}{\partial Y_{\mu}} \frac{dY_{\mu}}{dr} \right),$$

in which the neutrino contribution to entropy is taken into account where the $\beta$-equilibrium is satisfied (Buras et al. 2006b). Following Takiwaki et al. (2012), $V_{\text{aniso}}$ is estimated as

$$V_{\text{aniso}} = \sqrt{\langle \rho \left[(v_x - \langle v_x \rangle)^2 + v_y^2 + v_z^2 \right] \rangle / \langle \rho \rangle},$$

where $\langle A \rangle$ represents the angle average of quantity $A$. We define the normalized pressure perturbation $\Delta p$ and the net heating rate per nucleon $Q_{\text{net}}$ as

$$\Delta p \equiv \sqrt{\langle p^2 \rangle - \langle p \rangle^2} / \langle p \rangle,$$

and

$$Q_{\text{net}} \equiv \frac{Q_{\text{net}}}{\rho},$$

respectively. At first glance of Figure 9, one may not see any big differences between the 3D-SR (top four panels) and 3D GR models (the other), but indeed there are. Let us first discuss the properties of the four panels (a)–(d) taking the 3D-SR model as a reference and then proceed to focus on the differences between SR and GR.

From panel (a) (top left) showing the BV frequency for the 3D-SR model, one can depict three typical convectively unstable regions in the postbounce phase; prompt convection (greenish region at $t_{pb} \lesssim 20$ ms behind the shock), postshock convection (seen as a narrow horizontal stripe behind the shrinking shock, just behind the outer most boundary labeled by white line), and PNS convection (clearly seen as a thick horizontal stripe above the PNS at a radius of $\sim 10-20$ km after $t_{pb} \gtrsim 60$ ms).

In our 3D results, the PNS convection develops only very weakly before $\sim 60$ ms postbounce. This is due to the stabilizing effect by a positive entropy gradient (see the positive gradient persisting outside the PNS surface ($R \sim 10$ km) in the right panel of Figure 10). Afterward, the PNS convection gradually becomes vigorous with time as the negative lepton gradient nascent the PNS becomes remarkable (see the steepening slope of $Y_l$ near $R \sim 10-20$ km in the left panel of Figure 10).

Figure 10. Profiles of angle-averaged total lepton fraction (left) and entropy (right) at times for the 3D-GR model, when the postbounce time is as indicated in the plots. Note that the result of 1D-GR model at $t_{pb} = 60$ ms (black dotted curve) is shown for comparison.

(A color version of this figure is available in the online journal.)

In our 3D results, the PNS convection develops only very weakly before $\sim 60$ ms postbounce. This is due to the stabilizing effect by a positive entropy gradient (see the positive gradient persisting outside the PNS surface ($R \sim 10$ km) in the right panel of Figure 10). Afterward, the PNS convection gradually becomes vigorous with time as the negative lepton gradient nascent the PNS becomes remarkable (see the steepening slope of $Y_l$ near $R \sim 10-20$ km in the left panel of Figure 10). Comparing the black dotted line in Figure 10 (for 1D-GR model at $t_{pb} = 60$ ms) with the corresponding one (green line) for the 3D counterpart, the slope of the negative gradient is shown to become much smaller for the 3D model both in the profiles of lepton fraction (left panel) and entropy (right panel). This is a natural outcome of the convective overturns acting to wash out the local gradients.

From panel (b) in Figure 10 showing the anisotropic velocity, the postshock convection ($R \gtrsim 100$ km) is clearly seen as a reddish stripe running from top left to bottom right. Note that in the panel that the prompt convection can be also seen like a narrow prolate spheroid colored by red at $t_{pb} \lesssim 20$ ms with $10 \lesssim R \lesssim 60$ km. As can be seen, convective overturns operate above the PNS ($\sim 10-20$ km in radius) and below the shock $\sim 100$ km in radius. In-between, the region with smaller anisotropic velocity is formed (seen as a horizontal stripe colored by deep blue at a radius of 30–50 km after $t_{pb} \sim 50$ ms). By comparing to panel (d) (the net heating rate) to panel (c), the region is overlapped with the cooling region ($Q_{\text{net}} < 0$). The smaller anisotropic velocity there is because the infalling velocities in the cooling layer are so high that the convectively unstable material cannot stay there for long. Such a configuration has been already presented in 2D (Buras et al. 2006b) and 3D results (Takiwaki et al. 2012).

Here let us see panel (c) not for the 3D-SR model but for the 3D-GR model for convenience. The accreting flows should receive an abrupt deceleration near at the bottom of the cooling layer (the dark colored region in panel (d) (bottom right)), below which the regions are convectively stable (panel (b)). A strong pressure perturbation forms there (seen as a greenish horizontal stripe near $R \sim 30$ km in panel (c)). Subsequently the pressure perturbations propagate outward before they hit the shock (panel (c)), possibly leading to the formation of the next vortices. These features seem at least not to be inconsistent with the so-called advective-acoustic cycle (e.g., Foglizzo & Tagger 2000; Foglizzo 2002; Scheck et al. 2008, and references therein), which is also observed in our 3D-SR model (top four panels).

We now focus on the differences between the 3D-SR and 3D-GR models. Comparing the panel (a) of SR and GR, the unshocked core (regions below the PNS convection at $t_{pb} \gtrsim 50$ ms) is shown to be more compact for the GR model. Between the pair models, Figure 11 compares the maximum of the pressure perturbation that the advecting vortices form near in the vicinity of the deceleration regions. As can be seen, the pressure perturbation after the bounce is generally larger for the GR model (solid line) compared to the SR model (dotted line) in our simulation time. This is presumably because stronger gravitation pull in GR makes the position of the coupling

12 Note that the shock is indicated by a white thin line quickly rising after bounce and the passive shock stalls at a radius of $R \sim 150$ km.

13 Scheck et al. (2008) termed it as the “coupling radius” in which the coupling of vortices and acoustic waves occur.
radius deeper, leading to more energetic acoustic waves. It is not straightforward to say something very definitive only from the figure, but what we observed in our 3D-GR model (i.e., generation of stronger acoustic waves and the largest shock extent compared to the SR counterpart) does not seem, at least, unfavorable to drive neutrino-driven explosions. In the next section, we discuss in more detail how 3D and GR would potentially impact on the neutrino-heating mechanism.

4.3. 3D versus GR Effects on the Neutrino-heating Mechanism

Recalling that the neutrino-heating rate can be symbolically expressed as $Q^\nu \propto L_\nu \langle \epsilon^\nu \rangle$ (e.g., Janka 2001), we first analyze the neutrino luminosities ($L_\nu$) and the mean energies ($\langle \epsilon^\nu \rangle$) in the following. After that, we compare the dwell time to the neutrino-heating time in the gain region and discuss which one (3D-SR versus 3D-GR) is most likely to satisfy the criterion to initial the neutrino-driven explosions.

Figure 11 shows the evolution of the maximum of the pressure perturbation $\Delta p_{\text{max}}$ in our 3D-SR and 3D-GR models. In taking the maximum, we set the radial range as $20 \leq R \leq 50$ km to cover the coupling radius (see panel (c) in Figure 9).

After the neutronization burst ($t_{pb} \sim 10$ ms), the $\nu_e$ luminosity for the GR models slightly increases later on, while it stays almost constant for the SR models during the simulation time (green and blue lines). The $\bar{\nu}_e$ luminosity after 50 ms postbounce (right panel in Figure 12) is highest for the 3D-GR model (red line), which is also the case for the $\nu_x$ luminosity (left panel). Although the luminosities change with time, the luminosities generally yield to the following order:

1. for $\nu_e$, 3D-GR $> 1$DGR, 3D-SR $\sim 1$D-SR,
2. for $\bar{\nu}_e$, 3D-GR $> 1$DGR, 3D-SR $> 1$D-SR,
3. for $\nu_x$, 3D-GR $> 1$DGR, 3D-SR $> 1$D-SR.

To summarize, both 3D and GR work to raise the neutrino luminosities in the early postbounce phase. As can be seen from the left panel in Figure 12, GR maximally increases the $\nu_e$ luminosity up to $\sim 50\%$ (in 3D), while the maximum increase by 3D is less than $\sim 20\%$ (compare the $\bar{\nu}_e$ luminosity between the 3D-GR and 1D-GR model). These results indicate that compared to the spacial dimensionality, GR holds the key importance to enhance the neutrino luminosities.

By comparing our 1D-GR results with those in Sumiyoshi et al. (2005) again, the peak $\nu_e$ luminosity obtained here ($\sim 3.0 \times 10^{53}$ erg s$^{-1}$) is higher than their Boltzmann results ($\sim 1.8 \times 10^{53}$ erg s$^{-1}$), followed by a factor of two larger luminosities in all species of neutrinos at $t_{pb} \gtrsim 10$ ms for our model. This reflects a very approximate nature of our neutrino transport scheme. For example, $\alpha_\nu$ in Equation (A10), which regulates the neutrino diffusion timescale in our approximate scheme, should change with time in reality and can be determined only by solving a self-consistent neutrino transport. Concerning the rms energy, our 1D-GR models also show significantly

The spike in the $\nu_e$ luminosity corresponds to the so-called neutronization, when the shock propagates out through the $\nu_e$ sphere. The peak $\nu_e$ luminosity for the GR models is $L_{\nu_e} \sim 3 \times 10^{53}$ erg s$^{-1}$ (insensitive to 1D or 3D), which is slightly luminous compared to those in the SR models ($L_{\nu_e} \sim 2.9 \times 10^{53}$ erg s$^{-1}$). Using the same progenitor (Woosley & Weaver 1995), this trend is qualitatively similar to Bruenn et al. (2001). On the other hand, recent studies in which more detailed weak interactions are included in the Boltzmann transport have shown that the peak $\nu_e$ luminosity becomes $\sim 10\%$ smaller for the GR models (e.g., Lentz et al. 2012; Müller et al. 2010). This may carry an important message that the Boltzmann transport should be implemented in the full GR simulations to obtain a $\sim 10\%$ order accuracy, which is not insignificant when speaking about the neutrino-driven mechanism.

Figure 12. Neutrino luminosities of all neutrino types as a function of postbounce time (for $\nu_e$, $\nu_x$ (left panel), and for $\bar{\nu}_e$ (right panel), respectively.

(A color version of this figure is available in the online journal.)
higher energies (up to $\sim 30\%$ enhancement) compared to the Boltzmann results (e.g., Sumiyoshi et al. 2005; Müller et al. 2010; Lentz et al. 2012). Admitting that there is no doubt about the importance of implementing a more detailed transport scheme in our GR simulations, we think that our approximate neutrino transport is still useful for the sake of this study, in which we explore to discuss possible impacts of GR by comparing to the corresponding SR counterparts.

Top two panels in Figure 13 compare the angle average of the rms neutrino energy for $\nu_e$ (left panel) and $\bar{\nu}_e$ (right panel) after the breakout burst ($t_{pb} \gtrsim 10$ ms). As can be seen, the rms energies are highest for the 1D-GR model (black line), followed in order by 1D-SR, 3D-GR, and 3D-SR. In accord with the previous 1D results (Lentz et al. 2012; Müller et al. 2010; Liebendörfer et al. 2004; Bruenn et al. 2001), our 3D results (albeit limited to the early postbounce phase) support the expectation that the neutrino rms energies increase when switching from SR to GR hydrodynamics.

The reason for the higher neutrino energy in GR models is that the deeper gravitational well of GR produces more compact core structures, and thus hotter neutrino spheres at smaller radii. This is shown in the bottom panels in Figure 13 (compare the radii of the neutrino sphere between GR and SR models). The smaller neutrino energies for our 3D models compared to the corresponding 1D models (top panels) is due to their larger neutrino spheres (bottom panels). In our 3D models, the shock expands much further out assisted by convective overturns. (e.g., Figure 7), which also extends the positions of the neutrino spheres. The enlargement of the neutrino sphere in multi-D models is qualitatively consistent with the 2D post-Newtonian results by Buras et al. (2006b) including detailed neutrino transport.

As mentioned above, GR increases the neutrino luminosities and energies, while the 3D hydrodynamics works to make the neutrino energy smaller. What we discuss finally is whether the gain effects of GR could or could not overcome the possible loss effects of GR that should shorten the residency time of material in the gain region. Multi-D effects join in the game because they could potentially work against it to make the dwell time longer.

A widely prevailing indicator to diagnose the onset of the neutrino-driven explosions is the ratio of the residency timescale ($\langle t_{\text{res}} \rangle$) to the neutrino-heating timescale ($\langle t_{\text{heat}} \rangle$) in the gain region (e.g., Janka 2001; Thompson et al. 2005; Murphy & Burrows 2008). To estimate $\langle t_{\text{res}} \rangle$, we employ the effective advection timescale (Equation (8) in Buras et al. 2006b), in which $\langle t_{\text{res}} \rangle$ is determined by the crossing time of mass shell between shock and gain radii. The local heating timescale is estimated by the mass-weighted average of the local heating timescale,

$$\tau_{\text{heat}} = \frac{-\epsilon_{\text{bind}}}{\dot{Q}_{\nu,\text{total}}},$$

14 If this indicator is greater than unity, i.e., $\langle t_{\text{res}} \rangle/\langle t_{\text{heat}} \rangle > 1$, the neutrino heating proceeds fast enough to gravitationally unbind the fluid element, otherwise the matter is swallowed by the neutrino-cooling layer.
where we obey the Newtonian expression to estimate the local binding energy as
\[
\varepsilon_{\text{bind}} = \frac{1}{2} \rho u_i u_i + \phi_{\text{NT}}
\]
and the net neutrino-heating rate is calculated by
\[
\dot{Q}_{\nu,\text{total}} = \varepsilon_{\nu} + 1 \varepsilon_{\nu} - \varepsilon_{\text{bind}} > 0
\]
(see Equation (42)). In estimating the heating timescale, the numerical cells that satisfy both \( \varepsilon_{\text{bind}} < 0 \) and \( \dot{Q} > 0 \) are only taken into account.

As can be seen from Figure 14, the shock revival seems most likely to occur for the 3D-GR model (red line) in our simulation time, which is followed in order by 3D-SR, 1D-SR, and 1D-GR models. Thanks to a higher degree of freedom, the residency timescale becomes much longer for the 3D models than for the 1D models. In addition, the increase of the neutrino luminosity and rms energies due to GR (Figure 13) enhances the timescale ratio up to the factor of \( ~2 \) for the 3D-GR model (red line) compared to the SR counterpart (blue line). Therefore, our results suggest that the combination of 3D and GR hydrodynamics could provide the most favorable condition to trigger the neutrino-driven explosions.

As expected from Figure 14, the shock revival will never occur afterward for the 1D models that have already shown the sign of a rapid shock recession. On the other hand, the curves for the 3D models stay constant for the last 30 ms before our simulation terminates. For the 15 \( M_\odot \) progenitor employed in this paper, the neutrino-driven explosions are expected to take place later than \( ~200 \) ms postbounce at the earliest (Bruenn et al. 2010) and it could be delayed after \( ~600 \) ms postbounce (Marek & Janka 2009) as already mentioned. The parametric explosion models have shown that the earlier shock revival is good for making the explosion energy larger (e.g., Nordhaus et al. 2010). The onset timescale of the neutrino-driven explosions predicted in 2D models (Marek & Janka 2009; Bruenn et al. 2010; Suwa et al. 2010, 2011) could be shorter if the combination effects of GR and 3D would have been included. We anticipate that this can be a possible remedy to turn the relatively underpowered 2D explosions into the powerful ones. To draw a robust conclusion, the energy and angle dependence of the neutrino transport should be accurately incorporated in our full GR simulations with the use of more detailed set of weak interactions. This work is only the very first step toward the climax to investigate these fascinating issues.

5. SUMMARY

We presented the results from the first-generation multi-D core-collapse simulations in full GR that include an approximate treatment of neutrino transport. Using an M1 closure scheme with an analytic variable Eddington factor, we solved the energy-independent set of radiation energy and momentum based on the Thorne’s momentum formalism. To simplify the source terms of the transport equations, a methodology of multi-part neutrino leakage scheme was partly employed. Our newly developed code was designed to evolve the Einstein field equation together with the GR radiation hydrodynamic equations in a self-consistent manner while satisfying the Hamiltonian and momentum constraints. An AMR technique implemented in the 3D code enabled us to follow the dynamics starting from the onset of gravitational core collapse of a 15 \( M_\odot \) star, through the bounce, up to about 100 ms postbounce in this study. By computing four models that differ by 1D or 3D and by switching from SR to GR hydrodynamics, we studied how the spacial multi-dimensionality and GR would affect the dynamics in the early postbounce phase. Our 3D results support the anticipation in the previous 1D results that the neutrino luminosity and the average neutrino energy of any neutrino flavor in the postbounce phase generally increase when switching from SR to GR hydrodynamics. This is because the deeper gravitational well of GR produces more compact core structures, and thus hotter neutrino spheres at smaller radii. By analyzing the residency to the neutrino-heating timescale in the gain regions, we pointed out that the criteria to initiate neutrino-driven explosions could be most easily satisfied in the 3D models, irrespective of the SR or GR hydrodynamics. Keeping caveats in mind the omission of energy and angle dependence of the radiation fields and the use of reduced set of weak interactions in the present algorithm, our results indicated that the combination of 3D hydrodynamics and GR should provide the most favorable condition to drive a robust neutrino-driven explosion. On top of the omission of the spectral and angle dependence of the neutrino transport, we believe that the most urgent task is to replace the leakage scheme with more realistic modeling of the source terms.

In our 3D simulation, the numerical resolution behind the standing accretion shock is a few kilometers, which is not enough to capture the growth of SASI accurately (Sato et al. 2009). The numerical viscosity is expected to be large, especially in the vicinity of the shock, which may affect the growth of the SASI. It could also affect the growth of the turbulence in the postshock convectively active regions, which is very important to determine the success or failure of the neutrino-driven mechanism. To clearly see these effects of numerical viscosity, we need to conduct a convergence test in which a numerical gridding is changed in a parametric way (e.g., Hanke et al. 2011), although it is too computationally expensive to do so for our 3D-GR models at present. Some encouraging news is that we have access to a “K-computer,” which is the fastest in the world as of 2011 November. In the near future, we hope to report our 3D-GR models with much higher resolutions to check the convergence of the present results.

The most up-to-date neutrino transport code in core-collapse supernova simulations can treat the multi-energy and multi-angle transport in 2D (Ott et al. 2008) and even in 3D simulations (Sumiyoshi & Yamada 2012) but mostly in the Newtonian hydrodynamics (see, however, Müller et al. 2011, 2012b). As Schwartz (1967) originally pointed out by our exploratory results also support the importance of GR to draw a robust
conclusion to the supernova mechanism. The combined effects of GR and 3D\textsuperscript{15} should affect not only the supernova dynamics, but also the observational multi-messenger signatures (e.g., Kotake et al. 2012 for a recent review), such as gravitational waves (e.g., Müller et al. 2012a; Kotake et al. 2009a, 2009b, 2011; Ott et al. 2011b), neutrino emission (e.g., Abbasi et al. 2011; Marek et al. 2009; Lund et al. 2010), and explosive nucleosynthesis (e.g., Fujimoto et al. 2011; Thielemann et al. 2011). Continuing our efforts to improve the caveats mentioned above, we plan to study these fascinating subjects one-by-one in the near future.

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APPENDIX A

DETERMINATION OF NEUTRINO SPHERES

As illustrated in Figure 1, we must calculate the neutrino optical depth (τν) to determine the position of the neutrino spheres. It can be done by solving the following differential equation (actually, by a matrix inversion):

\[ x^i D_i \tau_\nu \bigg| r = -\kappa_\nu, \]  
(A1)

with an appropriate boundary condition (τν|_{r→∞} = 0). Here, \( \kappa_\nu \) represents the neutrino opacity of each species. Then, the neutrino sphere is determined where the optical depth exceeds 2/3 (τν = 2/3). For the matrix solver, we take the same scheme (see Appendix C.2).

\[ \kappa_{\nu_e} = \kappa_d(v_\nu e n) + \kappa_s(v_\nu A) + \kappa_s(v_\nu p) + \kappa_s(v_\nu A), \]

A2

and

\[ \kappa_{\bar{\nu}_e} = \kappa_s(\bar{\nu}_\nu e p) + \kappa_s(\bar{\nu}_\nu n) + \kappa_s(\bar{\nu}_\nu p) + \kappa_s(\bar{\nu}_\nu A), \]

A3

\[ \kappa_{\nu_x} = \kappa_s(v_\nu n) + \kappa_s(v_\nu p) + \kappa_s(v_\nu A), \]

A4

respectively (e.g., Ruffert et al. 1996). Here, the subindexes \( a \) and \( s \) denote absorption and scattering processes, respectively.

Detailed descriptions for the expressions of each opacity can be found in Bruenn (1985) and Burrows et al. (2006a).

Since we do not transfer the number density of neutrinos in the present scheme, we evaluate the emergent rms energy of neutrinos \( \varepsilon_{\nu,i} \) in the following way. Note here that the subscript \( i = \nu_e, \bar{\nu}_e \) denotes the neutrino species. We first project the neutrino sphere defined in the Cartesian grids to the spherical polar grids, which gives us the position of the neutrino sphere expressed in the polar grids as \( R_{\nu,i}(\theta, \phi) \). Then, we identify \( \varepsilon_{\nu,i} \) with the energy of neutrinos at the neutrino sphere assuming that they stream freely outward with possessing the information of the last scattering surface. Then, \( \varepsilon_{\nu,i} \) at arbitrary point (\( R, \theta, \phi \)) may be expressed as

\[ \varepsilon_{\nu,i}(R, \theta, \phi) \equiv \varepsilon_{\nu,i}(R_{\nu,i}(\theta, \phi), \theta, \phi). \]  
(A5)

Here \( \varepsilon_{\nu,i} \) in the right-hand side denotes the neutrino energy at \( R_{\nu,i}(\theta, \phi) \), which is estimated by

\[ \varepsilon_{\nu,i} \equiv k_B T \frac{F_3(\eta_{\nu_i}, 0)}{F_2(\eta_{\nu_i}, 0)}, \]  
(A6)

where \( F_k \) is the Fermi-Dirac integral and \( \eta_{\nu_i} \equiv \mu_{\nu_i}/k_B T \) is the degeneracy parameter with \( \mu_{\nu_i} \), \( T \), \( k_B \) representing the neutrino chemical potential, matter temperature, and Boltzmann constant, respectively.

A.1. Neutrino Diffusion Terms

Here we briefly summarize how to determine the diffusion term:

\[ Q_{\nu,\text{diff}} \equiv 4\pi \int \frac{v^2 n_{\nu,i}}{\alpha_{\nu,i} \tau_{\nu,i}^\text{diff}} d\epsilon_{\nu,i} \]  
(A7)

according to Ruffert et al. (1996), Rosswog & Liebendörf er (2003), and Sekiguchi (2010). In the above equation, depending on \( k = 1 \) or 0, one can obtain the energy (or number) diffusion rate. \( \alpha_{\nu,i} \) in Equation (A7) and \( \beta_{\nu,i} \) in Equations (34)–(35) are model parameters that affect the neutrino diffusion timescale and the position of the neutrino spheres, respectively. We adjust these parameters (\( \alpha_{\nu_e} = 2 \) and \( \beta_{\nu_e} = 3/2 \)) in such a way to fit the neutrino luminosity obtained in the 1D results using the IDSA scheme (see Appendix C.2). \( g_{\nu_e} = (g_{\nu_e} = 1) \) and \( g_{\nu_p} = 4 \) in Equation (A7) simply represents a multiplicity of neutrino species. \( n_{\nu} \) is the number density of neutrinos per each energy bin \( \varepsilon_{\nu,i} \) in thermal equilibrium with matter and expressed by the Fermi-Dirac distribution function as

\[ n_{\nu} = \frac{4\pi}{(hc)^3} \frac{\varepsilon_{\nu,i}^2}{1 + \exp\left(\frac{\varepsilon_{\nu,i} - \mu_{\nu,i}}{k_B T}\right)}. \]  
(A8)

where \( \mu_{\nu,i} \) is the chemical potential of neutrinos. We define the diffusion timescale of neutrinos as

\[ \tau_{\nu,i}^\text{diff} \equiv 3 \frac{\Delta x(\varepsilon_{\nu,i})}{c} \tau(\varepsilon_{\nu,i)}, \]  
(A9)

where \( \Delta x(\varepsilon_{\nu,i}) \) is assumed to be \( \Delta x(\varepsilon_{\nu,i}) \sim \tau(\varepsilon_{\nu,i})/\kappa(\varepsilon_{\nu,i}) \). We assume that the optical depth and opacity can be expressed as

\[ \tau_{\nu,i}(\varepsilon_{\nu,i}) \sim \varepsilon_{\nu,i}^2 \tau_{\nu,i}, \]

\[ \kappa_{\nu,i}(\varepsilon_{\nu,i}) \sim \varepsilon_{\nu,i}^2 \kappa_{\nu,i}, \]

\textsuperscript{15} It is worth mentioning that the MHD effects also have yet to be studied (Kotake et al. 2004; Takizaki et al. 2004, 2009; Burrows et al. 2007; Guilet et al. 2011; Kuroda & Umeda 2010; Obergaulinger & Janka 2011; Takizaki & Kotake 2011; see also Kotake et al. 2006 for collective references therein).
by neglecting the higher order correction terms of $\varepsilon_\nu$, $\tilde{\kappa}_\nu$, and $\tilde{\tau}_\nu$. They are energy-independent optical depth and opacity, respectively. Finally, the energy integration in Equation (A7) is rewritten as

$$Q_{v,\text{diff}} = g_\nu \frac{1}{\varepsilon_\nu} \frac{4\pi c}{3(\hbar c)^3} \frac{\varepsilon_\nu^k}{\varepsilon_\nu} \left( 1 + \exp \left( \frac{\varepsilon_\nu - E_r}{k_B T} \right) \right) d\varepsilon_\nu$$

$$= g_\nu \frac{1}{\varepsilon_\nu} \frac{4\pi c}{3(\hbar c)^3} \frac{\varepsilon_\nu^k}{\varepsilon_\nu} (k_B T)^k F_k(\eta_\nu, 0), \quad (A10)$$

where $\eta_\nu = \mu_\nu/k_B T$ is the degeneracy parameter of neutrino.

**APPENDIX B**

**IMPLEMENTATION OF EOS**

We employ the Shen EOS (Shen et al. 1998) based on the Thomas–Fermi approximation and a minimization of the free energy within a relativistic mean field theory. The available data is tabulated as a function of the three thermodynamic variables of density, temperature, and electron fraction as $(\rho, T, Y_e)$. We smoothly interpolate/extrapolate the original data as; $10^{-3} \leq \rho \leq 10^{10} \text{g cm}^{-3}$ with 200 equidistant intervals in a logarithmic scale, $10^6 \leq T \leq 10^{12} \text{K}$ with 200 equidistant intervals in a logarithmic scale, and $0.01 \leq Y_e \leq 0.55$ with 50 equidistant intervals in a linear scale. The interpolation is performed first by a bicubic interpolation in the $\rho$–$T$ plane and then by a cubic interpolation for $Y_e$ direction.

Thermodynamic variables such as the total pressure, internal energy, and entropy reads,

$$P(\rho, T, Y_e) = P_b + P_e + P_{\nu} + P_{\gamma}, \quad (B1)$$

$$\varepsilon(\rho, T, Y_e) = \varepsilon_b + \varepsilon_e + \varepsilon_{\nu} + \varepsilon_{\gamma}, \quad (B2)$$

$$s(\rho, T, Y_e) = s_b + s_e + s_{\nu} + s_{\gamma}, \quad (B3)$$

where subscripts $b$, $e$, $\nu$, and $\gamma$ denote the contributions from baryon, electron, positron, and photon, respectively.\(^{17}\)

**B.1. Supplement to the Original Shen EOS**

Since the original Shen EOS contains contributions only from baryons, we need to add the remaining contributions from leptons and photons. Although it can be done straightforwardly by using formulae given in, e.g., Blinnikov et al. 1996, we summarize them, for convenience, shortly in the following.

To construct the leptonic EOS, we have only to determine the electron chemical potential $\mu_e$ from a given data set of proton fraction, density, and temperature $(Y_p, \rho, T)$. This can be done by the charge neutrality condition $Y_p = Y_e$, where $Y_e$ is defined by

$$Y_e = \frac{n_{e^+} - n_{e^-}}{n_b}, \quad (B4)$$

where $n_{e^+}$ and $n_{e^-}$ are the number density of electrons/positrons and baryons with $n_b$ being the atomic mass unit. $n_{e^+}$ can be expressed by

$$n_{e^+} = \sqrt{\frac{m_e}{\hbar^2}} \beta^{3/2} \left[ F_{1/2}(n_{e^+}), \beta \right] + \beta F_{3/2}(n_{e^+}), \beta \right], \quad (B5)$$

where $\beta \equiv k_B T/m_e c^2$ and $\eta^{\pm} \equiv \mu_e/\beta k_B T$ with $k_B$, $m_e$, $T$, $\mu_e$ representing the Boltzmann constant, electron rest mass, temperature, and chemical potential of electron/positron, respectively (Blinnikov et al. 1996). $F_k(\eta, \beta)$ is the Fermi-Dirac integral,

$$F_k(\eta, \beta) = \int_0^{\infty} x^k(1 + \beta x/2)^{1/2} e^{-x - \eta} + 1 dx, \quad (B6)$$

where the useful analytical formula of the integral (with their derivatives) are given in Tooper (1969) and Miralles & van Riper (1996). Remembering that $\eta^- = -\eta^+$ is satisfied in the supernova cores due to high temperature ($\gtrsim 10^9$ K), one can find the solution of $\mu_e$ by Equations (B4) and (B5). Thus, the total pressure, specific internal energy, and entropy (per nucleon) can be readily calculated as

$$P_e = \frac{2\sqrt{2}}{3} \frac{m_e c^5}{\hbar^2 \pi^2} \beta^{5/2} \left[ F_{3/2}(\eta_e, \beta) + \frac{\beta}{2} F_{5/2}(\eta_e, \beta) \right], \quad (B7)$$

$$\varepsilon_e = \frac{2\sqrt{2}}{3} \frac{m_e c^3}{\hbar^2 \pi^2} \beta^{5/2} \left[ F_{3/2}(\eta_e, \beta) + \beta F_{5/2}(\eta_e, \beta) \right] \rho^{-1}, \quad (B8)$$

$$s_e = \frac{\rho \varepsilon_e + P_e - n_b \mu_e}{\rho T N_A k_B}, \quad (B9)$$

where $N_A$ is the Avogadro constant.

The contribution from photons is expressed as

$$P_\gamma = \frac{1}{3} a_r T^4, \quad (B10)$$

$$\varepsilon_\gamma = \frac{1}{\rho} a_r T^4, \quad (B11)$$

$$s_\gamma = \frac{4a_r T^4}{3\rho T N_A k_B}, \quad (B12)$$

where $a_r = 8\pi^5 k^4/(15c^3 h^2)$ denotes the radiation constant.

**B.2. Primitive Recovery**

Since we evolve hydrodynamic equations in a conservative form, we need to obtain primitive variables from the conservative ones. For the primitive recovery, we first solve the following simultaneous equations to obtain $Z = \rho h W^2$ and the Lorentz factor $W$ (Cerdà-Durán et al. 2008; Kuroda & Umeda 2010)

$$(Z^2 - S^2) W^2 - Z^2 = 0 \quad (B13)$$

$$\tau + D - Z + P(Z, W, Y_e/Y^*_e) = 0 \quad (B14)$$

for a given conservative set of variables $(\rho, S, \tau)$ and the electron/total lepton fraction $Y_e/Y^*_e$. In the above equations, $S^2 \equiv \gamma^4 S^2 S_i$ and $D \equiv \rho e/\epsilon^{\text{iso}}$. $P$ is the pressure and can be determined once the enthalpy $h = Z/DW$ and the rest-mass density $\rho = D/W$ are given. We iteratively solve these equations by the Newton–Raphson method until the sufficient convergence is achieved.
B.3. The Sound Velocity

As given in Shibata & Sekiguchi (2005b), the sound velocity is expressed as

\[
    c_s = \sqrt{\frac{1}{\hbar} \left( \frac{\partial P}{\partial \rho} + \frac{P}{\rho} \frac{\partial P}{\partial \varepsilon} \right)} , \quad (B15)
\]

where \( P \) and \( \varepsilon \) include the sum of contributions from baryon, electron, and photon. Regarding the partial derivatives of the thermodynamical variables, we take a finite differencing of Shen’s EOS table for the baryonic part, meanwhile we use analytical formulae of the Fermi-Dirac integrals given in Miralles & van Riper (1996) for the leptonic sector.

APPENDIX C
NUMERICAL TESTS

C.1. Core-collapse Tests with Shen EOS

We first present the 1D (in the same manner as 1D-SR/GR models by neglecting the non-radial matter velocity and momentum) core-collapse run without neutrinos to validate the implementation of Shen EOS instead of the phenomenological one taken in the original code (Kuroda & Umeda 2010). In the case of the adiabatic collapse, the so-called prompt explosion is expected to occur for the 15 \( M_\odot \) star (Woosley & Weaver 1995) as reported by Sumiyoshi et al. (2004) in their 1D-GR Lagrangian simulations using the same EOS.

Figure 15 shows the profiles of density (left panel) and radial velocity (right panel) between the GR (solid line) and SR (dashed line) model, respectively. As can be seen, the central density \( \rho_c \) (left panel) and the infall velocity (right panel) becomes higher for the GR model, which bounces at \( \rho_c = 4.5 \times 10^{14} \) g cm\(^{-3} \) with its inner-core baryon mass \( (M_{IC} = 0.91 \, M_\odot) \) being \( \sim 0.1 \, M_\odot \) smaller compared to the SR counterpart. After the bounce, the prompt shock propagates through the entire iron core (left panel in Figure 16) for both of the models. For the GR model, the shock reaches at a radius of 1000 km at \( \sim 20 \) ms after the bounce, with its explosion energy in the range of 1–1.5 \( \times 10^{51} \) erg (right panel in Figure 16), which is consistent with those obtained in Sumiyoshi et al. (2004).

As for the numerical accuracy, we monitor the violation of the average Hamiltonian constraint \( C_{hm} \) and the
Arnowitt–Deser–Misner mass (ADM mass) $M_{\text{ADM}}$. We adopt the following form for $C_{\text{hm}}$ (Shibata 2003)
\[
C_{\text{hm}} = \frac{1}{M_{\text{bar}}} \times \int \left[ \left( k_i^2 \bar{\Gamma}_i^j + e^2 \bar{\Gamma}_i^j + 5 \pi (S_0 + E) e^{-\phi} \right) \delta_{ij} - k_i \bar{\Gamma}_i \right] \, dx^3.
\tag{C1}
\]
where $M_{\text{bar}} \equiv \int \rho_0 \, d\chi^3$ is the proper rest mass and $\mathcal{H}$ is the Hamiltonian constraint,
\[
\mathcal{H} = \bar{D}_i \bar{D}_j e^{\phi} - \frac{e^2 \bar{R}}{8} + 2 \pi (S_0 + E) e^{-\phi}
+ \frac{e^5 \phi}{16 \pi} \left( \bar{A}_{ij} \bar{A}^{ij} - \frac{2}{3} K^2 \right) = 0.
\tag{C2}
\]
$M_{\text{ADM}}$ can be written as
\[
M_{\text{ADM}} = \int \left[ (S_0 + E) e^{-\phi} + \frac{e^2 \phi}{16 \pi} \times \left( \bar{A}_{ij} \bar{A}^{ij} - \frac{2}{3} K^2 - \tilde{\gamma}^{ij} \bar{R}_{ij} e^{-4\phi} \right) \right] \, dx^3.
\tag{C3}
\]
Every time after the number of the AMR blocks is increased by the AMR procedure and also after every restart of simulation, we enforce the Hamiltonian constraint by re-solving the following Poisson equation:
\[
\nabla_{\text{flat}}^2 \psi = \frac{\psi \bar{R}}{8} - 2 \pi (S_0 + E) \psi^{-1} - \frac{\psi^5}{8} \left( \bar{A}_{ij} \bar{A}^{ij} - \frac{2}{3} K^2 \right)
- f^{ij} \bar{D}_i \bar{D}_j \psi + \tilde{\gamma}^{ij} \bar{R}_{ij} \psi,
\tag{C4}
\]
until sufficient convergence is achieved. Here $\nabla_{\text{flat}}^2$ is the Laplacian in flat space, $\psi \equiv e^{\phi}$ and $f^{ij} \equiv \tilde{\gamma}^{ij} - \delta^{ij}$.

Here, we shortly comment on the side effect of this re-enforcement on the gravitational-wave content of the spacetime. The resulting change in $\psi$ is at most $\sim 0.01\%$ near the central region. This means that the third metric $\gamma_{ij}(\chi = 1 + \psi^2)$ is also altered at the level of $\sim 0.01\%$ after the re-setting of the Hamiltonian constraint. For the sake of this study, this is negligibly small. On the other hand, when we would deal with a much more massive progenitor, in which the spacetime could be strongly curved especially in the case of black hole formation, we may have to be much more careful about the re-enforcement procedure.

In Figure 17, we plot violation of the Hamiltonian constraint $C_{\text{hm}}$ (dash-dotted), baryon mass $M_{\text{bar}}$ (dashed), and the ADM mass (solid) for our 3D-GR model. Since our hydrodynamic equations are in conservative forms, the baryon mass is well conserved other than the inflowing materials through the outer computational boundary. In regard to the ADM mass, even though it shows fluctuations especially after the core bounce, the global trend shows similar behavior to $M_{\text{bar}}$ and the fluctuations are well below 1% and we thus consider our numerical scheme preserves conservative variables with sufficient accuracy. Next, in regard to the constraint conditions, $C_{\text{hm}}$ becomes larger in the postbounce phase, but this is not surprising considering the complicated nonlinear nature of the field equation and also the presence of the shock that makes the accuracy of the high-order shock-capturing scheme down to the first-order scheme inevitably. $C_{\text{hm}}$ is generally kept less than $10^{-3}$ in the postbounce phase.

In Figure 17, violation of the momentum constraint $C_{\text{mom}}$,
\[
c_{\text{mom}} = \frac{1}{M_{\text{bar}}} \int d\chi^3 \rho_0 \times \left| \frac{\partial_0 \bar{A}^{ij} + \bar{F}_{ij} \bar{A}_{i0} + 6 \bar{A}^{ij} \partial_0 \phi - \frac{2}{3} \bar{\gamma}^{ij} \partial_0 K - 8 \pi \bar{\gamma}^{ij} (S_i + F_i)}{\partial_0 \bar{A}^{ij} + \bar{F}_{ij} \bar{A}_{i0} + 6 \bar{A}^{ij} \partial_0 \phi + \frac{2}{3} \bar{\gamma}^{ij} \partial_0 K + 8 \pi \bar{\gamma}^{ij} (S_i + F_i)} \right|,
\tag{C5}
\]
is also plotted for our 1D-GR model. Note that only the $x$ component, $C_{\text{mom}}^{x}$, is shown since all other components of $C_{\text{mom}}$ show almost the same profiles. As denoted in the previous section, our manipulation of eliminating the non-radial components of fluid velocity could potentially violate the momentum constraint to a serious extent. On the other hand, the violation is shown to stay almost constant with time in the postbounce phase (see, $C_{\text{mom}}^{x}$ in Figure 17). Therefore, we think that the very simple way to construct 1D models in 3D simulations that we propose in this work would be quite useful.

\section*{C.2. Tests for Transport Scheme}

For numerical tests of our transport algorithm, we present the check for the trapped and streaming neutrinos, respectively. Note again that the sum of the streaming and trapped neutrinos is transported by the evolution equations (Equations (23) and (24)). The streaming part can be estimated by subtracting the trapped contribution from the sum, because the trapped part can be simply determined by local hydrodynamic quantities (i.e., density, temperature, and $Y_e$).

Figure 18 shows a comparison of the rms neutrino energy of the trapped neutrino and the one obtained by the IDSA scheme below the neutrino sphere. Since the IDSA can reproduce fundamental properties obtained in 1D full Boltzmann results (Liebendorfer et al. 2009), we think that the comparison with our approximate scheme with the IDSA is important. This test is done for a given background of density, temperature, and electron fraction profiles for several prebounce snapshots.

As can be seen, the assumption of $\beta$-equilibrium condition works well in the high density region and both of them show a quite similar profile there, in which neutrinos are essentially
trapped by matter. The agreement regarding the position of the neutrino sphere also certifies our estimate of the rms neutrino energy (Equation (A5)).

To check the properties of the streaming neutrinos, which is very relevant in the postbounce dynamics, we check the following two points: (1) whether the radiation energy flux falls proportional to $r^{-2}$ above the neutrino sphere and (2) whether the gain region, in which the net heating rate becomes positive, can be formed similar to previous studies. Figure 19 shows radial profiles of the radiation (energy) flux that is obtained by solving the closed set of the two-moments equations (Equations (23) and (24)).

As can be seen, the energy fluxes change with $r^{-2}$ (compare with the black line) irrespective of neutrino species outside the neutrino spheres. Note that the position of the neutrino spheres can be seen as the intersection point between the horizontal line ($\tau = 2/3$) and the solid lines. The radii of the neutrino spheres obey a canonical order $R_{\nu_e} < R_{\bar{\nu}_e} < R_{\nu_x}$. The emergent neutrino energy flux can be estimated by $4\pi r^2 \times (F_{r,\nu_e}, F_{r,\bar{\nu}_e}, F_{r,\nu_x}) \sim (9 \times 10^{52}, 8 \times 10^{52}, 4 \times 10^{52}) \text{ erg s}^{-1}$, which are all in good agreement with the luminosity defined by Equation (60) as plotted in Figure 12.

Figure 20 shows evolution of the net heating rate and radial velocity along the $x$-axis for our 3D-GR model at selected postbounce epochs. As the passive shock propagates (from top left to bottom right panels), the gain region also becomes larger. This reflects that the neutrino absorption on free nucleons predominantly takes place in the (enlarging) postshock region. The positive peak in the net heating rate is shown to be around 0.2 GeV nucleon$^{-1}$ s$^{-1}$ in the first 100 ms postbounce, which is in agreement with those in previous studies (e.g., Liebendörfer et al. 2000; Sumiyoshi et al. 2005; Ott et al. 2008).

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