LETTERS AND COMMENTS

Reply to ‘Comment on “Why are very short times so long and very long times so short in elastic waves?”’

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Abstract

A reply to the comment of R Kwang-Hua Chu (2011 Eur. J. Phys. 32 L13) is given.

The idea that the collision time between particles is involved with the choice between isothermal and isentropic wave propagation is quite attractive and easy to apply at first glance. Unfortunately things are not so simple as other quantities play a role, so we believe that the collision time alone is not very helpful.

Consider the thermalization time $\tau(\lambda)$ of a wavelength as given in equation (8) of our paper [1]:

$$\tau(\lambda) \sim \frac{\lambda^2}{\chi},$$

where $\chi$ is the thermal diffusivity coefficient:

$$\chi = \frac{\kappa}{\rho c_v}.$$  \hfill (2)

We consider a solid, as treated in [2]. Express the heat conduction coefficient $\kappa$ in terms of the microscopic mean square velocity $v$, the mean free path $l$, density $\rho$ and specific heat at a constant volume $c_v$, as expressed in [2], equations (1.51) for a metal and (25.31) for a dielectric:

$$\kappa = \frac{1}{3}vl\rho c_v.$$  \hfill (3)

Substitution of this value for $\kappa$ into (2) yields the relation

$$\chi = \frac{1}{3}vl,$$  \hfill (4)

which, when substituted into (1), yields

$$\tau(\lambda) = \frac{3\lambda^2}{vl}.\hfill (5)$$

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Now we can eliminate the mean square velocity $v$ in favour of the mean collision time $\tau_{\text{coll}}$, the mean time elapsing between two successive collisions of the particle, see e. g. [2] (1.51),

$$v\tau_{\text{coll}} = l$$

(6)
to obtain

$$\tau(\lambda) = \frac{3\lambda^2}{l^2} \tau_{\text{coll}}.$$  

(7)

We now divide both members of the last equation into $T$, the period of the wave:

$$\frac{\tau(\lambda)}{T} = \frac{3\lambda^2}{l^2} \frac{\tau_{\text{coll}}}{T}.$$  

(8)

We know that the wave is isothermal when the ratio of the thermalization time to the wave period is very small and adiabatic when the same ratio is very large. In (8) we clearly see that even if the ratio of the collision time to the wave period is very small, so that one should expect an isothermal wave according to the comment, the ratio $\frac{\tau(\lambda)}{T}$ might be large and the wave might not be isothermal, depending on the ratio $\frac{3\lambda^2}{l^2}$. Equation (8) shows that we cannot easily read the isothermal or isentropic behaviour of a wave from the ratio of the collision time to the wave period only. The point is that it is not enough that the number of collisions during a period be large in order for a wavelength to be thermalized, but also that the collisions propagate heat from one end to the other of the wavelength. One has to take the ratio of the wavelength $\lambda$ to the mean free path $l$ (inverse of the Knudsen number $l/\lambda$) into account, which gives an indication of the numbers of collisions needed to propagate heat through the wavelength. When this ratio is very large the thermalization time is long, even if the collision time is small.

The statement ‘... Once any thermal (excess) excitations in the matter (composed of many particles) have a lifetime $\tau$ (cf the characteristic time [our italics] in [1]) which is very long such that $\omega \tau \gg 1$ (...) leaves us puzzled, as we state the opposite; see (11) of our paper and the following text. We, and the rest of the authors, state that when the ratio of the characteristic time $\tau_c$ (which is not the collision time) to the period $T$ is such that $\tau_c / T \gg 1$ holds, the wave is isothermal (many collisions, in Chu’s comment), not adiabatic (no or few collisions).

On temperature dependence, all the quantities we introduce are temperature dependent. As long as waves are isentropic (almost) under ordinary conditions, the temperature dependence is not truly important for our paper; it is very important though when one tries to implement the transition to isothermal waves, because this is fairly easily achieved around the critical temperature [3] as we clearly state in section 2.1.

References

[1] Parravicini G and Rigamonti S 2011 *Eur. J. Phys.* 32 169–74
[2] Ashcroft N W and Mermin N D 1976 *Solid State Physics* (New York: Holt, Rinehart and Winston)
[3] Bencivenga F, Cusolo A, Krisch M, Monaco G, Ruocco G and Sette F 2006 *Europhys. Lett.* 75 70–6