Pseudogap phase in the U(1) gauge theory with incoherent spinon pairs

Xi Dai, Yue Yu, Tao Xiang and Zhao-bin Su

Institute of Theoretical Physics, Chinese Academy of Sciences, Beijing 100080, P.R.China

(March 24, 2022)

The pseudogap effect of underdoped high-$T_c$ superconductors is studied in the U(1) gauge theory of the t-J model including the spinon pairing fluctuation. The gauge fluctuation breaks the long range correlation between the spinons. The pairing fluctuation, however, suppresses significantly the low-lying gauge fluctuations and leads to a stable but phase incoherent spin gap phase which is responsible for the pseudogap effects. This quantum disordered spin gap phase emerges below a characteristic temperature $T^*$ which is determined by the effective potential for the spinon pairing gap amplitude. The resistivity is suppressed by the phase fluctuation below $T^*$, consistent with experiments.

The normal state behavior of high-$T_c$ cuprates has cast doubt on the conjecture that correlated electron metals are Fermi liquids in the absence of symmetry breaking. One anomalous feature which is difficult to understand is the spin gap or pseudogap behavior observed in underdoped materials. This phenomenon was first observed in the NMR measurement of deoxygenated YBCO, which showed that both the spin susceptibility and the resistivity [6], Hall coefficient [7], specific heat [8,9], optical properties [10], inelastic neutron scattering [17], and other thermodynamic or transport quantities [11].

At present there is no consensus as to the correct theory of the pseudogap effect. One interpretation is that the pseudogap is the energy gap of pre-formed Cooper pairs [11,12], and phase fluctuations in the underdoped regime prevent these pairs condensing until it has reached some lower temperature. This type of theory gives a qualitative account for the similarity between the pseudogap and the superconducting gap. But it is not clear if the phase fluctuation is really strong enough to suppress $T_c$ by one order of magnitude in the extremely underdoped limit.

A different interpretation is given by the resonant valence bond mean field theory based on the notion of charge-spin separation: electrons separate into spinons and holons, spinons have spin but no charge, holons have charge but no spin. In this theory, the spinons are predicted to form singlet pairs well above the superconducting $T_c$, with superconductivity setting in only when the holons become phase coherent at $T_c$. This theory provides a qualitative description of the high-$T_c$ phase diagram [21,22]. However, the spin gap phase is unstable against the fluctuation of the U(1) gauge field [23], which is introduced to enhance the constraints of no double occupancy.

In this paper, we study the effect of the spinon pairing fluctuation on the physical properties of the spin gap phase in the U(1) gauge theory of the t-J model. We propose that instead of the mean field spinon pairing phase with long range phase coherence, a local pairing phase of spinons which has no long range phase coherence will survive under the fluctuation of U(1) gauge field because it keeps the local U(1) symmetry. Then such a description can be viewed as the strong coupling version or the microscopic origin of the nodal liquid proposed by Balents,Fisher and Nayak(BFN) [25].

After integrating out the spinon and holon operators, we obtain an effective action for the gauge and pairing fields $S = S_\Delta + S_A$. Then the total action is $S = S_\phi + S_A$.

The effective action for the gauge field [20,23] is given by

$$S_A = \frac{T}{2} \sum_{i\nu_n} \int d^2q \Pi_{\mu\nu}(q, i\nu_n) A^\mu_q A^\nu_{-q},$$

where $A^\mu_q$ is the U(1) gauge field. The inverse of the gauge field propagator is

$$\Pi_{\mu\nu}(q, \nu) = \chi dq^2 - i\gamma_F \nu / q,$$

and

$$\frac{\Delta}{2} \sum_{\nu_n} \frac{d^2q}{(2\pi)^2} \Pi_{\mu\nu}(q, i\nu_n) A^\mu_q A^\nu_{-q},$$

where $A^\mu_q$ is the U(1) gauge field. The inverse of the gauge field propagator is

$$\Pi_{\mu\nu}(q, \nu) = \chi dq^2 - i\gamma_F \nu / q,$$
where $\gamma_F = 2n_e/k_F$ and $\chi_d = \chi_F + \chi_B$. $\chi_F$ and $\chi_B$ are the Landau diamagnetic susceptibilities of spinons and holons, respectively.

In order to treat the fluctuation of pairing order parameter, we assume that the dynamics of the spinon pairing order parameter is described by an effective Ginsburg-Landau theory with a minimal coupling with the gauge field, namely

$$S_\Delta = \int dx \left[ \kappa_\mu \Delta^*(x) (\partial_\mu - A_\mu(x))^2 \Delta(x) + \alpha|\Delta|^2 + \beta|\Delta|^4 \right]$$

(3)

where $\kappa_0 = \frac{1}{2m^*}, \kappa_{1.2} = \frac{1}{2m^*}$, $m^*$ and $c$ are the effective mass and velocity of spinon pairs, respectively. We further assume that $\alpha = a(T - T_0^*)$ and $\beta$ is independent of temperature. In $\alpha$, a is a constant and $T_0^*$ is the critical temperature of the spin gap phase without gauge fluctuation.

In the work of Ubbens and Lee, the phase fluctuation is ignored. The energy loss due to the gauge fluctuation is proportional to $\Delta^2$, while the energy gain from spinon pairing is proportional to $\Delta^2$. Thus for small $\Delta$ the first term always dominates and the phase coherent spin gap phase is unstable. This means that to study the phase fluctuation properly, we introduce the duality transformation, following reference. The phase variable $\phi$ and its derivative $\partial_\mu \phi$ are not curl-free. Thus there are vortices in the $\phi$ field and the cure of $\partial_\mu \phi$ introduces a vortex current operator $j^\nu_\mu = \epsilon_{\mu\nu\lambda} \partial_\lambda \phi$. To treat these vortices, a commonly used approach is to introduce a fictitious gauge field $a_\mu$, which is dual to the phase variable $\phi$, via the equation

$$\kappa_\mu (\partial_\mu \phi - A_\mu(x)) = \epsilon_{\mu\nu\lambda} \partial_\nu a_\lambda.$$  

(5)

Substituting the solution of $\phi$ from the above equation to the definition of the vortex current, one can relate $a_\mu$ to the vortex current operator

$$j^\nu_\mu = \epsilon_{\mu\nu\lambda} \partial_\nu \left[ \kappa_\lambda^{-1} \epsilon_{\alpha\mu\beta} \partial_\alpha a_\beta + A_\lambda \right].$$  

(6)

In the dual representation of $\phi$, vortices are treated as quantized particles and represented by a complex field $\Phi$. A dual Lagrangian of $\phi$ with minimal coupling between $a_\mu$ and $\Phi$ can be constructed as

$$L_\phi = \epsilon_{\mu\nu\lambda} \frac{1}{2\kappa_\mu(x)} (\partial_\nu a_\lambda - \partial_\lambda a_\mu)^2 + \epsilon_{\mu\nu\lambda} a_\mu \partial_\nu A_\lambda$$

$$+ \frac{\kappa_\mu(x)}{2} |(\partial_\mu - ia_\mu)\Phi|^2 - V_2\Phi^2,$$  

(7)

where the higher order terms of $\Phi$ are ignored. From the equation of motion of $L_\phi$, it can be shown that this Lagrangian has the desired property of the vortex current defined above. The bare mass of the vortex field $V_2$ depends on the superfluid density of spinon pairing $\rho(x)$ and can be estimated as follows. Close to the transition temperature $T_0^*$, the excitation energy of a single static vortex is approximately equal to $E_V \approx \sqrt{2V_2/\kappa_0}$. From the GL theory, we know that $E_V$ is also proportional to the spinon superfluid density $\rho(x)$. If we assume the spinon superconductivity is also type II, $E_V = \rho(x)$ and $g$ is a constant. Thus $V_2$ is approximately equal to

$$V_2 \approx \frac{1}{2} \rho^2 \kappa_0 = \frac{g^2}{4m^*c^2} \rho(x)^3,$$  

(8)

The highest power of $a_\mu$ in $L_\phi$ is 2. $a_\mu$ can therefore be rigorously integrated out from the Lagrangian. This leads to a self energy correction to the propagator of the gauge field $A_\mu$:

$$\Pi_{\mu\nu}(q, q') = \Pi_{\mu\nu}(q, \nu) \delta_{q, q'} + \frac{q_0 q'_0}{2} \sum_k \kappa_0(k) \Phi^2(-k + q' - q).$$  

(9)

By further integrating out the gauge field $A_\mu$, an effective action for the $\Phi(x)$ and $\rho(x)$ fields is obtained as

$$S[\Phi(x), \rho(x)] \approx -f_0 \left[ 1 + \int d^3x \frac{1}{2} \chi_0 \kappa_0(x) \Phi^2(x) \right]^{-\frac{1}{2}}$$

$$+ \int d^3x \left[ V_2 \Phi^2(x) + \alpha \rho(x) + \beta \rho(x)^2 \right],$$  

(10)

where $-f_0$ is the free energy of the system without spinon pairing.

In the dual representation of $\phi$, a phase incoherent state corresponds to a superfluid state of vortices with $<\Phi> = 0$, while a long range phase correlated state corresponds to a normal state of vortices with $<\Phi> = 0$. Thus to investigate the property of a quantum disordered spin gap phase, only the superfluid phase of vortices needs be considered. In this case the vortex field has a finite energy in its low-lying excitations, we can therefore take a saddle point approximation for the $\Phi$ field. The saddle point $\Phi_0$ is determined by the equation $\delta S[\Phi(x), \rho(x)]/\delta \Phi(x) = 0$. In the small $\Phi(x)$ limit and
using the slow varying condition of both $\kappa_0(x)$ and $\Phi(x)$, the equation becomes
\[ -\frac{4}{3} f_0 \left[ \frac{1}{2} \chi_d \kappa_0(x) \right]^{1/2} \Phi_0^2(x) + 2V_2(x) \Phi_0(x) = 0, \quad (11) \]

Due to the presence of the first term, $\Phi(x) = 0$ can not be the stable solution, which indicate that for any given configuration of $\rho(x)$ the effect of U(1) gauge field fluctuation will cause the condensation of vortices. In the description before duality, this means the quantum disorder phase is always more stable than the ordered phase. The spinons become ”superconducting” when $\chi'_d \to \infty$, then in our approach the pseudo gap phase is the middle state between the strange metal phase in which $\chi'_d$ keeps constant with the decrement of temperature and the mean field pairing phase with infinite $\chi'_d$. At the same time, the phase transition at $T^*_0$ predicted by the mean field theory becomes a crossover temperature $T^*$ below which the local minimum in the effective potential of $\rho$ moves away from zero.

Since the saddle point of $\Phi(x)$ is actually very large in small $\rho$ case, we must consider the saddle point equation in the large $\Phi(x)$ limit which leads to
\[ \Phi_0^2(x) = \frac{4 f_0}{3 \kappa_0(x) \chi_d V_2(x)} = 4 m^2 c^2 \sqrt{\frac{f_0}{3 g^2 \chi_d}} \rho^{-2}(x). \quad (12) \]

It can be shown that this quantum disorder phase is more stable than the ordered phase. In the BFN’s theory, the condensation of vortices is obtained by assuming $V_2 < 0$. In our case, however, $V_2$ is always positive and the condensation of vortices is caused by the U(1) gauge field fluctuation. In the superconducting phase, the U(1) gauge field is screened by the superfluid of holons, we find that $\Phi_0(x) = 0$.

Taking the saddle point approximation for the $\Phi$ field in $S[\Phi(x), \rho(x)]$, we then find the effective potential for the pairing gap amplitude
\[ S[\rho(x)] \approx -f_0 + \int dx \left[ (\alpha + \alpha_0) \rho + \beta \rho^2 \right]. \quad (13) \]

where $\alpha_0 = 2 \sqrt{g^2 f_0 / 3 \chi_d}$ is assumed to be weakly temperature dependent. The $\alpha_0$ term is the contribution of the gauge fluctuation. An important property revealed by this equation is that the contribution from the gauge fluctuation to the free energy is proportional to $\rho$, rather than $\rho^{5/6}$ as for the case in which the phase fluctuation vanishes. This means that the gauge fluctuation is greatly suppressed by the phase fluctuation. Since the contributions from both the gauge fluctuation and the pairing condensation are now proportional to $\rho$, the spin gap phase is therefore stable below a characterizing temperature $T^* = T_0^* - \frac{\alpha_0}{a}$, (14)

which is the solution of the equation $\alpha + \alpha_0 |T=T^* = 0$.

Well below $T^*$, $\rho$ becomes finite and its fluctuation can be omitted. In this case, the system we study is similar to the “”Nodal Liquid Phase” of BFN [23]. The only difference is that in our model the local pairs are not formed by electrons but by spinons.

The phase fluctuation modifies the Landau diamagnetic susceptibility. Under the saddle point approximation, the renormalized Landau diamagnetic susceptibility is approximately given by
\[ \chi^*_d \approx \chi_d + \frac{1}{2} \kappa_0 \Phi_0^2 \approx \chi_d (1 + u < \rho >), \quad (15) \]

where $u = \sqrt{3 g^2 / \chi_d f_0}$ and $< \rho > = \int_0^\infty dp e^{-\alpha (\alpha + \alpha_0) \rho - \alpha^2} m B \kappa_0$.

In the gauge theory, the electronic resistivity $R(T)$ is determined by the the transport scattering rate of holons $\tau_B^{-1}$. In the strange metal phase above $T^*$, $\tau_B^{-1}$ is determined by the Landau diamagnetic susceptibility $\chi^*_d$ [23]
\[ \tau_B^{-1} \approx \frac{k_B T}{m B \chi_d}. \quad (16) \]

Since $\chi^*_d = \chi^*_F + \chi^*_b$ is mainly determined by $\chi^*_F$ which is nearly temperature independent, $R(T)$ thus depends linearly on $T$ in this phase. In the spin gap phase, $\chi^*_d$ in $\tau_B^{-1}$ should be replaced by $\chi'_d$. Since $< \rho >$, and subsequently $\chi'_d$, increases rapidly with decreasing temperature in the spin gap phase, the temperature dependence of $\tau_B^{-1}$ is therefore changed. Near $T^*$, the deviation of $R(T)$ from its high temperature linear dependence is approximately given by
\[ \frac{R(T)}{C T^*} = \frac{\chi^*_d}{\chi_d} \approx 1 - u < \rho >, \quad (17) \]

where $C$ is the slope of $R(T)$ at high temperatures. From the previous result of $< \rho >$, we find that the leading temperature dependence of $R(T)$ calculated by us fits well with the experimental data of $YBa_2Cu_3O_{7-x}$ [3] as shown in Figure 1.

In this paper, we proposed that due to the strong fluctuation of the U(1) gauge field, the proper description of the pseudo gap phase is the quantum disordered spinon pairing state without the Bose-condensation of holons. The mean field spinon pairing state is proved to be unstable when the U(1) gauge field fluctuation is included [24]. While in our new description beyond mean field, the local U(1) symmetry which is broken in the mean field pairing state restores due to the phase disorder in the spinon pairing order parameter. Then the phase transition in the mean field description become a cross over from weak pairing fluctuation regime ($T > T^*$) to the strong fluctuation regime ($T < T^*$) in the present paper.
We further divide the pairing fluctuation to amplitude and phase part and treat them seperately. By integrating out the phase fluctuation by duality transformation, we obtain the effective potential for the amplitude. The crossover temperature can be determined by the minimum of the effective potential moving away from zero. For the temperature much lower than $T^*$, the minimum in the effective potential is far away from zero and the fluctuation of the amplitude is no longer be important. Then our result is quite similar with BFN except that in our case the local pairs are formed by spinons and the U(1) gauge field fluctuation plays a very important role to obtain the quantum disordered phase. For temperature near $T^*$, we can calculate the slope of the resistivity by considering the effect of amplitude fluctuation and our result fits very well with the experimental data.

Very recently Y. B. Kim and Z. Q. Wang proposed that the mean field spin gap phase can be stabilized by strong critical fluctuation of holons. Compared with their approach, ours works better in quite high temperature near $T^*$, where the diamagnetic susceptibility of holons $\chi_b$ can be viewed as constant.

[1] B. Batlogg, H. Y. Hwang, H. Takagi, R. J. Cava, H. L. Kao and J. Kwo, Physica C 235-240, 130 (1994).
[2] M. Takigawa et al, Phys. Rev. B43, 247 (1991).
[3] S. E. Barrett, et al Phys. Rev. B41, 6283 (1990).
[4] Takagi, et al, Phys. Rev. Lett. 69, 2975 (1992).
[5] T. Ito, K. Takenaka, S. Uchida, Phys. Rev. Lett. 70, 3995 (1993).
[6] D. N. Basov, et al, Phys. Rev. Lett. 77 4090 (1996).
[7] H. Y. Hwang, et al Phys. Rev. Lett. 72, 2636 (1994).
[8] J. Loram, et al, Phys. Rev. Lett. 71, 1740 (1993).
[9] J. Loram, et al, Physica C 235-240, 134 (1994).
[10] Ch. Renner, B. Revaz, J. Y. Genond, K. Kadowak and Fischer, Phsy. Rev. Lett. 80, 149 (1998).
[11] N. Miyakawa, et al, Phys. Rev. Lett. 80, 157 (1998).
[12] A. G. Loeser et al Science 273,325 (1996).
[13] D. S. Marshall et al Phys. Rev. Lett. 76, 4841 1996.
[14] J. M. Harris et al Phys. Rev. Lett. 79, 143 (1997).
[15] H. Ding et al Nature 382, 51 (1996).
[16] H. Ding et al Phys. Rev. Lett. 78, 2628 (1997).
[17] J.Rossat-Mignod, et al, Physica C 185-189, 86 (1991).
[18] V. Emery and S. Kivelson, Nature 374, 434 (1995).
[19] Mohit Randeria, preprint, cond-mat/9710224.
[20] M. U. Ubbens and P. A. Lee, Phys. Rev. B49, 6853 (1994).
[21] M. U. Ubbens and P. A. Lee, Phys. Rev. B46, 8434 (1992).
[22] G. Kotliar, Phys. Rev. B37, 3664 (1988); I. Affleck and J. B. Marston, Phys. Rev. B37,3774 (1988).
[23] P. A. Lee and N. Nagaosa, Phys.Rev.B46 5621 (1992).
[24] N. Nagaosa and P. A. Lee, Phys. Rev. B45, 966 (1992).
[25] L. Balents, M. P. A. Fisher and Nayak, cond-mat/9811236.
[26] P. G. de Gennes, Chapter 6, Superconductivity of Metals and Alloys, 1966.
[27] Y. B. Kim and Z. Q. Wang, cond-mat/9901003.
FIG. 1. Comparison between our theoretical results of $\rho(T)$ and the experimental data (square). The parameters used in fitting the experimental data are $T^* = 168 K$, $a = 19.1$, $\beta = 975 K$, $u = 0.67$. 