Paired and Stripe States in the Quantum Hall System

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We study a paired state at the half-filled Landau level using a mean field theory on the von Neumann lattice. We obtain a microscopic model which shows a continuous transition from the compressible stripe state to the paired state. The energy gap in the paired state is calculated numerically at the half-filled second Landau level.

Keywords: Quantum Hall system, BCS theory, Charge density wave, von Neumann lattice

In recent experiments, highly anisotropic states are observed in the half-filled third and higher Landau levels[1]. In the half-filled second Landau level the fractional quantum Hall effect (FQHE)[2] and transition to the highly anisotropic state are observed[3]. Theoretically the anisotropic states are regarded as stripe states[4] or unidirectional charge density wave states which are plausible in the Hartree-Fock approximation and numerical calculation of small systems[5, 6]. On the other hand, the FQHE state at the half-filled second Landau level is regarded as a paired state like a superconducting state[7, 8, 9, 10]. However the microscopic mechanism of the pairing is not understood yet. How is the paired state formed by the Coulomb interaction? Naively it seems that there is no possibility of formation of the paired state by the repulsive force in the mean field theory.

In the present work, we analyze the gap equation for the Coulomb interaction which is screened in the Landau level space and find a possibility of transition from the stripe state to a paired state by varying the screening length. We use a basis on the von Neumann lattice which is useful for the field theoretical study of the quantum Hall system[11, 12]. It was shown that the von Neumann lattice formalism is very useful tool for the anisotropic compressible state at the higher Landau levels[1, 1]. The effective potential on the von Neumann lattice becomes attractive for a small screening length which is on the order of the magnetic length. We present a microscopic model which shows a continuous transition from the compressible stripe state to the paired state.

Let us consider two-dimensional electron systems in the presence of a perpendicular uniform magnetic field $B$. The free particle energy is quenched to the Landau level energy $E_l = \hbar \omega_c (l + 1/2)$, $l = 0, 1, 2 \ldots$, where $\omega_c = eB/m$. We suppose that the electrons are spin-polarized and ignore the spin degree of freedom. For simplicity, we set $\hbar = c = 1$. The total Hamiltonian of the present system is

$$H = \int d^2r \psi^\dagger (r) \left( -i \nabla + eA \right)^2 \frac{\psi (r)}{2m} + \frac{1}{2} \int d^2r d^2r' : \delta \rho (r)V (r-r') \delta \rho (r') :,$$

where colons mean the normal ordering, $\nabla \times A = B$, $\delta \rho (r) = \psi^\dagger (r) \psi (r) - \rho_0$, $V (r) = q^2/r$, and $\rho_0$ is the uniform background density.

We project the system to the $l$th Landau level space. The electronic field $\psi$ is expanded by the Wannier basis $w_{l,X}(r)$ on the von Neumann lattice as

$$\psi (r) = \sum_{l,X} b_l (X) w_{l,X} (r).$$

The lattice spacing is given by $a = \sqrt{2\pi \hbar/eB}$. We set $a = 1$ for simplicity. $X$ is defined on the lattice sites. $b_l$ is the anti-commuting annihilation operator. The system is translationally invariant in the lattice space and in the momentum space[11]. The translational symmetry in the momentum space is called the K-invariance.

We apply the mean field approximation to the projected Hamiltonian. Let us consider the following mean fields, which are translationally invariant on the von Neumann lattice,

$$U_l (X - X') = \langle b_l^\dagger (X') b_l (X) \rangle,$$
$$U_l^{(+)} (X - X') = \langle b_l^\dagger (X') b_l^\dagger (X) \rangle,$$
$$U_l^{(-)} (X - X') = \langle b_l (X') b_l (X) \rangle.$$

These mean fields break the K-invariance and U(1) symmetry. Using these mean fields, we obtain a mean field Hamiltonian

$$H_{\text{mean}}^{(l)} = \sum_{XX'} \varepsilon_{l,X-X'} b_l^\dagger (X) b_l (X').$$
\[
\Delta_l (p) = - \int \frac{d^2 p_1}{(2\pi)^2} \frac{\Delta_l (p_1)}{2 E(p_1)} \tilde{c}_l (p_1 - p) e^{i \int_F^{p_1} 2 \alpha(k) dk},
\]
where \( \mu \) is the chemical potential and \( E_l (p) \) is the spectrum of the quasiparticle which is defined by \( E_l (p) = \sqrt{\xi_l (p)^2 + |\Delta_l (p)|^2} \), \( \xi_l (p) = \varepsilon_l (p) - \mu \). Note that the gauge field appears in the gap equation (5). The gauge field represents two unit flux on the Brillouin zone. The filling factor of the \( l \) th Landau level is denoted by \( \nu_l \) and total filling factor is given by \( \nu = l + m_\nu \).

Next, we analyze gap equation (5). It is shown that the screening effect plays important roles for the pairing mechanism. For simplicity, we omit the Landau level index \( l \) and use \( q^2/a \) as the unit of energy in the following.

Eq. (5) is rewritten as
\[
\Delta(p) = - \int \frac{d^2 k}{(2\pi)^2} \tilde{v}(k) e^{i k \cdot D} \frac{\Delta(p)}{2 E(p)},
\]
where \( D = (-i \frac{\partial}{\partial p_x} + 2 \alpha_x, -i \frac{\partial}{\partial p_y} + 2 \alpha_y) \). It is convenient to introduce eigenfunctions of the operator \( D^2 \), that is, \( D^2 \psi_n (p) = e_n \psi_n (p) \) with \( e_n = (2n + 1)/\pi \), \( n = 0, 1, 2, \ldots \). The index \( n \) labels the \( n \) th Landau level in the momentum space. The eigenfunctions are doubly degenerate and are given by
\[
\begin{align*}
\psi_n^{(2)} (p) &= \frac{2 \pi}{\sqrt{n!}} \frac{\pi}{2} \frac{1}{n^{n/2}} (D_x - i D_y)^n \psi_0^{(2)} (p), \\
\psi_n^{(3)} (p) &= \frac{2 \pi}{\sqrt{n!}} \frac{\pi}{2} \frac{1}{n^{n/2}} (D_x - i D_y)^n \psi_0^{(3)} (p),
\end{align*}
\]
where \( N_n = 1/\sqrt{2n-1} n! \), and \( \psi_0^{(2)}, \psi_0^{(3)} \) are written by theta functions as
\[
\begin{align*}
\psi_0^{(2)} (p) &= N_0 e^{-\frac{\pi^2}{4} \frac{1}{x^2}} \theta_2 \left( \frac{p_x + i p_y}{\pi} | 2 \right), \\
\psi_0^{(3)} (p) &= N_0 e^{-\frac{\pi^2}{4} \frac{1}{x^2}} \theta_3 \left( \frac{p_x + i p_y}{\pi} | 2 \right).
\end{align*}
\]
These eigenfunctions have the parity symmetry, \( \psi_n (-p) = (-)^n \psi_n (p) \). Therefore the gap potential can be expanded by \( \psi_{2n+1} (p) \). We can expand the gap potential as
\[
\Delta(p) = \sum_{n \geq 1, i = 1, 2, 3} c_n^{(i)} \psi_n^{(i)} (p), \quad (11)
\]
and use the screened potential to calculate \( \nu_l \) in the Hartree-Fock approximation.

The Coulomb potential is screened by the polarization \( \Pi(p) \) due to the Fermion loop diagrams. We approximate the screened potential as
\[
\hat{\nu}(p, m_{TF}) = 1/\hat{\nu}(p)^{-1} + m_{TF}, \quad (13)
\]
where \( m_{TF} = -\Pi(0) \) is the Thomas-Fermi mass. Calculating the one-loop diagram, \( m_{TF} \) is given by
\[
m_{TF} = \int_{\text{BZ}} \frac{d^2 p}{(2\pi)^2} \frac{|\Delta(p)|^2}{2 E(p)^3}. \quad (14)
\]
We use the screened potential \( \hat{\nu}(p, m_{TF}) \) in the gap equation. We also use the screened potential to calculate \( \varepsilon_l \) in the Hartree-Fock approximation.

The \( m_{TF} \) dependences of pseudopotentials \( F_n \) are plotted in Fig. (1) for \( l = 1 \). As seen in the figure, potentials change their signs at \( m_{TF} \approx 1.0 \). Thus, in the Hartree-Fock-Bogoliubov approximation, the stripe phase makes transition to the pairing phase for \( m_{TF} > 1.0 \).

We present an effective Hamiltonian which has a striped state as the normal state and paired state as the U(1) symmetry breaking state. We focus our argument on the half-filled second Landau level space, that is \( l = 1, \nu_1 = 1/2 \).

We truncate the expansions of Eq. (11) and calculate the self-consistent solution by iteration of numerical calculations until we obtain convergence. Considering only the lowest and next relevant terms,
the effective Hamiltonian for the quasiparticle in the striped and paired state is given by

\[ H_{\text{eff}} = \int_{BZ} \frac{d^2p}{(2\pi)^2} [\varepsilon_{\text{eff}}(p)a^\dagger(p)a(p) + \frac{1}{2}\Delta_{\text{eff}}(p)a^\dagger(-p)a^\dagger(p) + \frac{1}{2}\Delta_{\text{eff}}^*(p)a(p)a(-p)], \]  

(15)

where the effective hopping potential and effective gap potential are given by

\[ \varepsilon_{\text{eff}}(p) = -t_{\text{eff}} \cos p_y - t_{(0,3)} \cos 3p_y, \]  

(16)

\[ \Delta_{\text{eff}}(p) = c_{(3)}(3)\psi^\dagger \psi(p) + c_{(3)}\psi^\dagger \psi(p). \]  

(17)

\(a(p)\) is the anti-commuting annihilation operator in the momentum space. The hopping parameters \(t_{(0,3)}\) and gap potential \(\Delta_{\text{eff}}(p)\) depend on \(t_{\text{eff}}\). The magnitude of \(t_{\text{eff}}\) corresponds to the strength of the stripe order.

Chemical potential \(\mu\) is determined so that the filling factor \(\nu\) is equal to 1/2. Note that \(\mu\) includes the on-site term in \(H_{\text{mean}}\). We find that \(\mu\) is negative small number on the order of \(10^{-4}\) at most. The maximum value of the energy gap is \(0.027q^2/a = 0.01q^2/l_B(l_B = \sqrt{\hbar/eB})\) at \(t_{\text{eff}} = 0.03\). This value is the same order as Morf’s on-site. The excitation energy \(E(p)\) becomes small around \(p_y = \pm \pi/2\), which is the Fermi surface of the striped state. The transition to the stripe phase is continuous and very smooth. Near the transition point, the behavior of energy gap is approximated by \(t_{\text{eff}}e^{-2\pi t_{\text{eff}}/|F_1|}\), whose non-perturbative dependence on the coupling is well-known in the BCS theory. At \(m_{TF} = m_c \approx 1.4\), \(F_1\) behaves as \(\alpha(m_c - m_{TF})\) and the energy gap approaches to zero at \(t_{\text{eff}} = t_c \approx 0.2\). The energy gap is extremely small at \(0.1 < t_{\text{eff}} < t_c\). At \(t_{\text{eff}} > t_c\), the gap potential vanishes and the compressible striped state is realized. This state leads to the anisotropy of the magnetoresistance \(\hat{\alpha}\). Inspecting the energy spectrum of quasiparticle in the pairing phase, we find a crossover phenomenon at \(t_{\text{eff}} \approx 0.01\), that is, the minimum excitation energy \(\min(E(p))\) is placed around \(p_y = \pm \pi/2\) at \(0.01 < t_{\text{eff}} < t_c\), whereas at \(0 < t_{\text{eff}} < 0.01\), placed around \(p_y = 0\) and \(\pi\). We call the latter case the gap-dominant pairing phase. In this phase, the low-energy excitation occurs around zeros of the gap potential and the the energy gap is given by \(2\xi(0)\). The spectrum is close to the flatband and the stripe order is weakened. The \(t_{\text{eff}}\) dependence of the energy gap is summarized as

\[ \Delta E \propto \begin{cases} t_{\text{eff}}e^{-2\pi t_{\text{eff}}/|F_1|} & \text{for } 0 < t_{\text{eff}} < 0.01, \\ 0 & \text{for } 0.1 < t_{\text{eff}} < t_c, \\ t_{\text{eff}} & \text{for } t_c < t_{\text{eff}}. \end{cases} \]  

(18)

In conclusion, we obtain a microscopic model which shows a continuous transition from stripe to paired state. The gap potential has the p-wave-like pairing as seen in Eq. (11). Details of the calculation and numerical results are given in Ref. [13]. More experimental and theoretical studies are necessary for deciding the symmetry of the gap potential and understanding the underlying physics at the half-filled second Landau level.

This work was partially supported by the special Grant-in-Aid for Promotion of Education and Science in Hokkaido University provided by the Ministry of Education, Science, Sport, and Culture, the Grant-in-Aid for Scientific Research on Priority area (Physics of CP violation) (Grant No. 12014201), and the Grant-in aid for International Science Research (Joint Research 10044043) from the Ministry of Education, Science, Sports, and Culture, Japan.

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