T-duality and the Worldvolume Solitons of Five-Branes and KK-Monopoles

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ABSTRACT

We show that the fluxes of the various six-dimensional “gauge” theories are associated to below threshold bound states of D-branes with the NS-5-branes and KK-monopoles which preserve half of bulk supersymmetry. We then present the supergravity solutions that correspond to these bound states. In addition using the worldvolume solitons of IIA and IIB NS-5-branes and KK-monopoles, we investigate the sectors of the “gauge” theories that preserve one quarter of bulk supersymmetry. This leads to a generalization a supergravity solution which has the interpretation of two intersecting NS-5-branes at a 3-brane and to the construction of some of the worldvolume solitons of IIA and M-theory KK-monopoles. Furthermore, using the IIA/IIB T-duality of the bulk theories, we give the T-duality transformations of the worldvolume solitons of NS-5-branes and KK-monopoles. We find that the worldvolume 0-brane, self-dual string and 2-brane solitons of NS-5-branes appear in the same T-duality chain.
1. Introduction

It has been known for sometime that the boundary of a brane ending on another brane [1, 2] and intersection region of two or more branes are described by worldvolume solitons [3]. These solitons usually preserve a subgroup of the Poincaré group of the branes involved and therefore they have themselves a brane interpretation. Many such worldvolume solitons have been found in [4, 5, 6] and their supersymmetry has been investigated in [7].

More recently there is some interest in six-dimensional “gauge” theories which are found as limits of the superstring theory in the presence of \(n\) IIB NS-5-branes as the string coupling constant goes to zero. In this limit the theory at the bulk becomes free IIB string theory but interactions persist on the NS-5-branes leading to an interacting six-dimensional “gauge” theory [8, 9, 10]. The effective theory is described by a \((1,1)\) supersymmetric Yang-Mills multiplet with gauge group \(SU(n)\) (factoring out the centre of mass) which is a reduction of the ten-dimensional super-Yang-Mills multiplet to six dimensions. Some of the excitations of the six-dimensional “gauge” theory are due to IIA D-branes that lie within the NS-5-branes. All these excitations but one of this six-dimensional “gauge” theory can be identified with fluxes of the effective theory\(^\star\); the remaining degree of freedom signals that the six-dimensional theory has another sector which is not captured by the effective theory.

There are two possible IIA (or M-theory) duals to the above six-dimensional “gauge” theory associated with the IIB strings.

(i) Using T-duality along one of worldvolume directions of the IIB NS-5-brane leads to a IIA six-dimensional theory associated with the IIA NS-5-brane. The effective theory is described by a \(SU(n)\) generalization of \((2,0)\)-supersymmetric\(^\dagger\) tensor multiplet in six dimensions. Some of degrees of freedom of this theory are again due to IIA D-branes that lie within the IIA NS-5-brane. All these

\(^\star\) In the Coulomb branch of this theory the gauge group is broken to a product of \(U(1)\)'s.
\(^\dagger\) An action for the non-abelian \((2,0)\)-supersymmetric multiplet is not known.
degrees of freedom can be identified with the fluxes of the 3-form self-dual field strength and the fluxes of a 5-form field strength; the latter is dual to the scalar along the compact eleventh direction.\(^\dagger\)

\((ii)\) Using T-duality along a transverse direction of the IIB NS-5-brane leads to another six-dimensional “gauge” theory associated with the IIA Kaluza-Klein (KK) monopole. In this case the effective theory is described by the six-dimensional \((1,1)\) supersymmetric Yang-Mills multiplet which is the reduction of the \(N=1\) ten-dimensional one with gauge group \(SU(n)\) on a four-torus.

A larger class of six-dimensional theories can be found by starting from the \((p,q)\)-5-branes of IIB and then by T-dualizing in a transverse direction followed by a lifting to M-theory. The effective theory is described by a \((1,1)\)-supersymmetric Yang-Mills multiplet with gauge group \(SU(r)\) where \(r\) is the largest common divisor of \(p\) and \(q\) \([11]\).

In this letter, we shall show that all the configurations that involve D-branes that lie within NS-5-branes are bound states below threshold which preserve 1/2 of bulk supersymmetry. Consequently, the excitations of the six-dimensional “gauge” theories due to these bound states also preserve 1/2 of supersymmetry. We shall verify that all the required bound states exist as solutions of IIA and IIB supergravity theories preserving 1/2 of supersymmetry. We shall also present the worldvolume analogue of these bound states. In particular, we shall find solutions of the field equations of Born-Infeld (BI) actions\(^\S\) \([12-17]\) of IIB and IIA NS-5-branes which preserve 1/2 of bulk supersymmetry and have the fluxes as described in \([8]\).

The BI actions, apart from the solutions that preserve 1/2 of bulk supersymmetry, admit solitons which preserve 1/4 or less of supersymmetry. In the context

\[^{\dagger}\] The IIA NS-5-brane is viewed as the reduction of the M-5-brane along a transverse direction.

\[^{\S}\] The dynamics of a IIB NS-5-brane is described by a BI action as it is related to D-5-brane by S-duality. The field equations of M-theory five-brane, instead of the usual two-form BI field strength, involve a 3-form self-dual field strength.
of six-dimensional “gauge” theories, these solitons describe sectors which preserve $1/2$, $1/4$ or less of supersymmetry. We shall refer to these sectors as $1/2$-sector, $1/4$-sector and so on. We shall argue using supersymmetry that some of the worldvolume solitons cannot propagate in the bulk. We shall then explore the relation between worldvolume solitons and intersecting branes and compare the two pictures. As a result we shall find a new supergravity solution which has the interpretation of two IIA or IIB NS-5-branes intersecting on a 3-brane and corresponds to the 3-brane soliton of [7].

We shall also investigate the six-dimensional “gauge” theories based on the IIA and IIB KK-monopoles. As in the case of NS-5-branes, the $1/2$-sector is associated bound states of D-branes with the IIA and IIB KK-monopoles which are below threshold and preserve $1/2$ of bulk supersymmetry. We shall also use T-duality to describe the associated supergravity solutions. Then using the worldvolume action of the M-theory KK-monopole [19], we shall find some of worldvolume brane solitons of the IIA and M-theory KK-monopoles which preserve $1/4$ of bulk supersymmetry. In addition, viewing the worldvolume solitons of NS-5-branes and KK-monopoles as brane boundaries and intersections, we shall use the type II T-duality rules of the bulk theories to find the T-duality transformations of the worldvolume solitons. It turns out that worldvolume solitons that appear in the same T-duality chain have similar solutions. An example of this is the 0-brane, the self-dual string and the 2-brane worldvolume solitons of the IIA and IIB NS-5-branes.

This letter is organized as follows. In sections two and three, we investigate the $1/2$-and $1/4$-sectors of the six-dimensional theories associated with the NS-5-branes, respectively. In sections four and five, we examine the $1/2$-and $1/4$-sectors of the six-dimensional theories associated with the KK-monopoles, respectively. In section six, we give the T-duality transformations of the worldvolume solitons and finally in section seven we present our conclusions.
2. D-branes within the IIA and IIB NS-5-branes

2.1. Supergravity

The bound states that involve D-branes and NS-5-branes in IIA theory are $(0_D|5_S)_A$, $(2_D|5_S)_A$ and $(4_D|5_S)_A$. Similarly, the IIB D-brane/NS-5-brane bound states are $(1_D|5_S)_B$, $(3_D|5_S)_B$ and $(5_D|5_S)_B$. All these bound states preserve 1/2 of bulk supersymmetry provided that the associated D-brane is within the NS-5-branes and are below threshold. This can be easily seen by, for example, examining the supersymmetry projectors associated with the branes involved in the bound states (see [20, 21]). The energy of these bound states [22] is consistent with the formula given in [8] for computing the energy of the excitations of the six-dimensional “gauge” theories from D-branes within NS-5-branes.

All the supergravity solutions that correspond to the above bound states can be found using the T-duality chain

\[(0_D|5_S)_A \leftrightarrow (1_D|5_S)_B \leftrightarrow (2_D|5_S)_A \leftrightarrow (3_D|5_S)_B \leftrightarrow (4_D|5_S)_A \leftrightarrow (5_D|5_S)_B , \tag{1}\]

where T-duality operations are performed along NS-5-brane worldvolume directions. These solutions are controlled by one harmonic function which indicates that the positions of the branes involved in the superposition coincide. Some of these solutions have already being found, for example the $(2_D|5_S)_A$ has been given in [22] and the $(5_D|5_S)_B$ has been given in [23]. The $(4_D|5_S)_A$ is most easily constructed by reducing the M-theory five-brane at an angle (see [24-26]). The remaining solutions can be constructed from the T-duality chain and they will not be presented here.

\* In this notation, $(0_D|5_S)_A$ denotes the bound state of a D-0-brane with a solitonic (NS) 5-brane in IIA theory and similarly for the rest of the bound states.

\† We remark that if the D-branes are placed parallel but not within the NS-5-brane, then all supersymmetry is broken.
2.2. Worldvolume Solutions

The effective theory of the IIB NS-5-brane is described by a BI action which in the static gauge is

\[ I = \int d^6 u \sqrt{\det (\eta_{\mu\nu} + \partial_{\mu} X^i \partial_{\nu} X^j \delta_{ij} + F_{\mu\nu})}, \]  

(2)

where \( \{X^i; i = 1, \ldots, 4\} \) are the transverse coordinates of the 5-brane, \( \{u^{\mu}; \mu = 0, \ldots, 5\} \) are the worldvolume coordinates and \( F_{\mu\nu} \) is the BI field. The worldvolume solutions that corresponds to the bulk bound states (1) is

\[ F_{\mu\nu} = a^I \alpha_I \]
\[ X^i = c^i, \]  

(3)

where \( \{\alpha_I; I = 1, \ldots, 15\} \) is a basis of two forms on \( E^{(1,5)} \), and \( \{a^I; I = 1, \ldots, 15\} \) and \( \{c^i; i = 1, \ldots, 4\} \) are real constants. It is a simple application of the results of [20] to show that such configurations preserve \( 1/2 \) of the supersymmetry of the bulk. The solution (3) gives the fifteen fluxes \( \dagger \) in the counting of [8]. The missing flux corresponds to the D-5-brane within the NS-5-brane bound state. From the supergravity point of view it is easy to see why such a flux cannot be found within the effective theory. The supergravity solution is invariant under the six-dimensional Poincaré group of the worldvolume of the five branes. So the corresponding solution of the effective theory should have the same degree of symmetry but there are not exist such solutions of the effective theory apart from the vacuum one.

The field equations of the effective theory of IIA NS-5-brane can be found by reducing along a transverse direction those of M-5-brane [15-17]. We shall be interested in two classes of solutions which preserve \( 1/2 \) of bulk supersymmetry. In

\[ \dagger \] Upon compactification on a five-torus the fluxes are quantized.
the first case, we take the BI field to vanish but allow the transverse scalar along the eleventh direction to depend linearly on the worldvolume coordinates, i.e.

\[ f_3 = 0 \]
\[ X^{11} = b_\mu u^\mu + a \]
\[ X^i = c^i. \]  

(4)

Such solution preserves 1/2 of bulk supersymmetry and gives six fluxes as in [8].

For the other solution, we take all the transverse scalars to be constant and the BI field to be constant and self-dual, i.e.

\[ f_3 = a^I \omega_I \]
\[ X^i = c^i, \]  

(5)

where \( \{\omega_I; I = 1, \ldots, 10\} \) is a basis of constant self-dual 3-forms on \( E^{(1,5)} \). Such solution also preserves 1/2 of bulk supersymmetry and gives the remaining ten fluxes of [8].

3. The 1/4-sector and Brane Solitons

3.1. Worldvolume Solutions

To investigate the 1/4-sector of “gauge” theories, we shall present the worldvolume solutions of NS-5-branes that preserve 1/4 of bulk supersymmetry. In the case of IIB NS-5-brane these worldvolume solutions are the 0-brane, the 2-brane and the 3-brane. Let \( F_2 \) and \( X^1 \) be the BI field and one of the transverse scalars of the IIB NS-5-brane, respectively. The 0-brane and 2-brane worldvolume solutions are

\[ F_2 = -\frac{1}{4} dt \wedge dH_{(5)} \]
\[ X^1 = H_{(5)}, \]  

(6)
and
\[ F_2 = -\frac{1}{4} \star dH_{(3)} \]
\[ X^1 = H_{(3)} , \]  

respectively, where \( H_{(n)} \) is a harmonic function on \( \mathbb{E}^n \) and the Poincaré star operation is with respect to the flat metric on the three-dimensional subspace of the NS-5-brane transverse to the 2-brane soliton. For the 3-brane worldvolume soliton, the BI field vanishes. If we set \( z = u^4 + iu^5 \) and \( s = X^1 + iX^2 \), the 3-brane solution is determined by any holomorphic function \( s = s(z) \) or in an implicit form
\[ Z(s, z) = 0 . \]

These solitons are the same as those found in the worldvolume theory of the D-5-brane [4,5,7]. In fact they are related to them by S-duality.

The above worldvolume brane solitons do not propagate in the bulk. This is straightforward for the case of 0-brane and 2-brane solitons since there are no 0-branes and 2-branes in IIB theory. To see this for the case of the 3-brane soliton, we shall first argue that the 3-brane soliton cannot be identified with the IIB D-3-brane. To show this, let us suppose that the 3-brane soliton of the NS-5-brane is identified with the IIB 3-brane. If this were the case, we consider the configuration of a D-3-brane parallel to the NS-5-brane separated with a distance \( r \). As we have seen in the previous section, for \( r \neq 0 \) this configuration breaks all supersymmetry. For \( r = 0 \), the D-3-brane is within the NS-5-brane and 1/2 of bulk supersymmetry is preserved. Therefore a 3-brane worldvolume soliton of NS-5-brane and IIB D-3-brane within the NS-5-brane bound state preserve different fractions of bulk supersymmetry leading to a contradiction of the original hypothesis. So the 3-brane worldvolume soliton of NS-5-brane and the D-3-brane should no be identified. Since it is unlikely to exist another supersymmetric D-3-brane in IIB theory, because there is a unique 5-form field strength, the 3-brane worldvolume soliton does not propagate in the bulk.
Let $f_3$ be the self-dual 3-form field strength and $X^1$ be one of the transverse scalars of worldvolume theory of IIA NS-5-brane. There are two NS-5-brane worldvolume solitons preserving 1/4 of bulk supersymmetry, the self-dual string [7]

$$f_3 = \frac{1}{4} [dt \wedge d\rho \wedge dH(4) + \star dH(4)]$$

$$X^1 = H(4)$$

and the 3-brane whose solution is given as in (8), where $\{t, \rho\}$ are the coordinates of the string and the Poincaré star operation is with respect to the flat metric on the four-dimensional subspace of the NS-5-brane transverse to the string. Since there is no 3-brane in IIA theory, the 3-brane worldvolume soliton cannot propagate in the bulk. We have not being able to find a similar argument for the self-dual string. It is worth mentioning that the expressions for the solutions of the 0-brane, 2-brane and self-dual string worldvolume solitons are remarkably similar. As we shall show in section six, they are related by T-duality.

3.2. Supergravity

As we have already mentioned in the introduction, from the bulk perspective, the various brane solitons of the effective theories can be realized as brane boundaries or brane intersections. In the IIB case, the 0-brane, 2-brane and the 3-brane worldvolume solitons on the NS-5-brane are realized as follows: The 0-brane soliton is the boundary of a D-string on a NS-5-brane, $((0|1_D, 5_S)_B$ in the notation of [21]), the 2-brane soliton is the boundary of a IIB 3-brane on a NS-5-brane $(2|3_D, 5_S)_B$, and the 3-brane soliton is the intersection of two NS-5-branes on a 3-brane $(3|5_S, 5_S)_B$. The associated supergravity configurations* are derived from

* There is no supergravity solution that describes a brane ending on another one. However there are some solutions that resemble such configurations.
M-theory by the following chains of reductions and T-dualities:

\[(1|2, 5)_M \rightarrow (1|2_D, 5_S)_A \leftrightarrow (0|1_D, 5_S)_B\]

\[(1|2, 5)_M \rightarrow (1|2_D, 5_S)_A \leftrightarrow (2|3_D, 5_S)_B\]

\[(3|5, 5)_M \rightarrow (3|5_S, 5_S)_A \leftrightarrow (3|5_S, 5_S)_B .\]

In the IIA case, the self-dual string is the boundary of a D-2-brane ending on a NS-5-brane and the 3-brane soliton is the intersection of two NS-5-branes. The associated supergravity configurations can be constructed using the second and third chains above.

To illustrate further the relation between brane solitons and intersecting branes, we shall present the (IIA and IIB) supergravity solution with the interpretation of two intersecting NS-5-branes at a 3-brane which corresponds to the soliton (8). For this we shall take the overall positions of the two NS-5-branes to coincide. Let $F_3$ and $\phi$ be the 3-form field strength of the NS-NS sector and the dilaton, respectively. This solution can be written as

\[ds^2 = ds^2(\mathbb{E}^{(1,3)}) + H_1 ds^2(\mathbb{E}^2) + H_2 ds^2(\mathbb{E}^2) + H_1 H_2 ds^2(\mathbb{E}^2)\]

\[F_3 = \omega_1(\mathbb{E}^2) \wedge * dH_1 + \omega_2(\mathbb{E}^2) \wedge * dH_2\]

\[e^{2\phi} = H_1 H_2 ,\]

where $\omega_1$ and $\omega_2$ are the volume forms of the relative transverse directions, the Poincaré star operation is with respect to the flat metric of the overall transverse directions, and $H_1$ and $H_2$ are the real and imaginary parts of a holomorphic function of the overall transverse coordinates $(z, \bar{z})$, i.e. $Z(s, z) = 0$ where $s = (H_1 + iH_2)(z)$. In particular, if we set $t = e^{-s}$. The simplest choice is $s = -\sum_i q_i \log(z - z_i)$ or equivalently the zero locus of the polynomial $F(t, z) = t^2 - \Pi_i (z - z_i)^{2q_i}$. A more general configuration is given by the zero locus of the polynomial

\[F(t, z) = t^2 - 2B(z)t + \Lambda^{2N}\]
with scale $\Lambda$ used in [18], where

$$B(z) = z^N + u_{N-1}z^{N-2} + u_{N-2}z^{N-3} + \ldots + u_1 , \quad (13)$$

and $\{u_i; i = 1, \ldots, N - 1\}$ are the moduli parameters of a Riemann surface.

4. Branes within the IIA and IIB KK monopoles

From the bulk perspective, the IIA D-branes within the IIA KK-monopole bound states which preserve $1/2$ of supersymmetry can be found by acting with T-duality on the D-brane/NS-5-brane bound states of IIB theory along a direction transverse to the NS-5-brane as

$$\begin{align*}
(1D|5S)_B & \xleftrightarrow{T} (2D|0M)_A \\
(3D|5S)_B & \xleftrightarrow{T} (4D|0M)_A \\
(5D|5S)_B & \xleftrightarrow{T} (6D|0M)_A ,
\end{align*} \quad (14)$$

where $0_M$ denotes the KK-monopole [27, 28]. In all the above cases one of the directions of the D-2-brane, D-4-brane and D-6-brane is wrapped on the orbits of the $S^1$ isometry of the KK-monopole. This direction is transverse to the object (see for example [19]). The six-dimensional “gauge” theory associated with the IIA KK-monopole has excitations with sixteen degrees of freedom, one from $(6D|0M)_A$, five from $(2D|0M)_A$ and ten from $(4D|0M)_A$.

A BI type of action has been proposed for the IIA KK-monopole in [19] (we shall give more details in the next section). It can be easily seen that the constant configuration (3) is also a solution of the IIA KK-monopole worldvolume theory. Therefore, this solution has fifteen fluxes. The missing degree of freedom is due to the KK-monopole/D-6-brane bound state which does not correspond to a solution of the effective worldvolume theory.
In the IIB case, the bound states of D-branes with the IIB KK-monopole are T-duals of the D-brane/NS-5-brane bound states of IIA theory along a direction transverse to the NS-5-brane as
\[
(0_D|5_S)_A T \leftrightarrow (1_D|0_M)_B \\
(2_D|5_S)_A T \leftrightarrow (3_D|0_M)_B \\
(4_D|5_S)_A T \leftrightarrow (5_D|0_M)_B.
\]

In all the above cases one of the directions of the D-1-brane, D-3-brane and D-5-brane is wrapped on the orbits of the $S^1$ isometry of the KK-monopole. The associated six-dimensional “gauge” theory has excitations with sixteen degrees of freedom, one is due to $(1_D|0_M)_B$, ten are due to $(3_D|0_M)_B$ and five are due to $(5_D|0_M)_B$. In this case, it is not straightforward to give the worldvolume description of the above bound states since no BI type of action is known for the IIB KK-monopole. The bulk counting suggests though that it should be similar to that of the IIA NS-5-brane.

5. KK-monopole Worldvolume Solitons

An effective worldvolume action for the M-theory KK-monopole has been proposed in [19]. The construction of this action involves the use of an isometry of the background spacetime in such a way that the worldvolume theory has the desirable number of degrees of freedom. To describe the bosonic part of the worldvolume action for flat background, we first write the eleven-dimensional metric as
\[
ds^2(E^{(1,10)}) = ds^2(E^{(1,9)}) + dy^2,
\]
where $y$ is the coordinated adapted to the isometry and we choose the radius to be one. Then the action in the static gauge is
\[
I = \int d^7v \sqrt{|\det(\eta_{\alpha\beta} + \partial_\alpha X^r \partial_\beta X^s \delta_{rs} + F_{\alpha\beta})|},
\]
where, $\{v^\alpha; \alpha = 0, \ldots, 6\}$ are the worldvolume coordinates, $\eta$ is the seven-dimensional Minkowski metric, $F$ is the Born-Infeld field and $\{X^r; r = 1, 2, 3\}$ are the three
transverse scalars which are orthogonal to $y$. The solitons of this action are a 0-brane, a 3-brane and a 4-brane. The solutions are similar to those of (6)-(8) and we shall not give the expressions here. To find the action of the IIA KK-monopole, we simply use a double reduction by identifying one of the worldvolume directions, say $v^6$, with one of the spacetime ones, say $X^9$. This leads to the action

$$I = \int d^6u \sqrt{\left| \det \left( \eta_{\mu\nu} + \partial_\mu X^i \partial_\nu X^j \delta_{ij} + F_{\mu\nu} \right) \right|},$$

(18)

where \{\{X^i; i=1,\ldots,4\} = \{S, X^r; r=1,2,3\}, u^\mu = v^\mu\} for $\mu = 0,\ldots,5$ and $S$ is the $v^6$ component of the BI potential. This action is the same as that of the IIB NS-5-brane in the static gauge. The brane solitons on the worldvolume of the IIA KK-monopole are those of the IIB NS-5-brane. In particular, there is a 0-brane soliton, a 2-brane soliton and a 3-brane soliton. The expressions of these solutions are the same as those given in (6), (7) and (8).

From the bulk perspective, these solitons can be viewed as follows: (i) The 0-brane soliton is associated with the intersection of a D-2-brane with KK-monopole. The supergravity solution can be found using the T-duality chain

$$(0|1_D, 5S)_B \xrightarrow{T^\tau}(0|2_D, 0M)_A,$$

(19)

where the T-duality direction $\tau$ is chosen transverse to both the D-string and NS-5-brane. The metric of the resulting configuration in the string frame is

$$ds^2 = -H^{-\frac{1}{2}}dt^2 + H^{\frac{1}{2}}ds^2(\mathbb{R}^5) + H^{-\frac{1}{2}}V^{-1}(d\tau + \omega)^2$$

$$+ VH^{-\frac{1}{2}}dx^2 + VH^{\frac{1}{2}}dzd\bar{z},$$

(20)

where $H, V$ are the real parts of holomorphic functions $h, f$ in the coordinate $z$, respectively, and $\omega = \text{Im} f dx$. This metric is the superposition of a D-2-brane and a Gibbons-Hawking metric [29, 30] with the D-2-brane wrapped along the $\tau$
direction. (ii) The 2-brane soliton is associated with the intetersection of a D-4-brane with a KK-monopole. The supergravity solution can be found using the T-duality chain
\[ (2|3_D, 5_S)_B \overset{T}{\leftrightarrow} (2|4_D, 0_M)_A , \] (21)
where the T-duality direction is chosen transverse to both the D-3-brane and NS-5-brane. As in the previous case, the solution is determined by two holomorphic functions. (iii) The 3-brane soliton is the intersection of a IIA KK-monopole with a IIA NS-5-brane. The associated supergravity solution can be found using the T-duality chain
\[ (3|5_S, 5_S)_B \overset{T}{\leftrightarrow} (3|5_S, 0_M)_A , \] (22)
where the T-duality direction is chosen along a worldvolume direction of one of the NS-5-branes transverse to the 3-brane intersection. The solution is again determined by two holomorphic functions. The 3-brane soliton cannot propagate in the bulk since there is no 3-brane in IIA theory. All the above solutions preserve 1/4 of bulk supersymmetry.

As we have already mentioned in the previous section, the field equations of the worldvolume theory of IIB KK-monopole are not known. However it is expected that the field equations will closely resemble those of the IIA NS-5-brane. Therefore there should be a 3-brane and a string worldvolume solitons. From the bulk perspective, the 3-brane soliton can be realized as the intersection of a NS-5-brane with a IIB KK-monopole. The supergravity solution can be constructed by performing T-duality on the configuration of two IIA NS-5-branes intersecting on a 3-brane along a worldvolume direction of one of the 5-branes transverse to the intersection, i.e.
\[ (3|5_S, 5_S)_A \overset{T}{\leftrightarrow} (3|5_S, 0_M)_B . \] (23)
The string soliton can be realized as the intersection of D-3-brane with the IIB
KK-monopole. The supergravity solution can be found using the T-duality chain

\[(1|2_D, 5_S)_A \leftrightarrow (1|3_D, 0_M)_B\, , \tag{24}\]

where the T-duality direction is chosen transverse to both the D-2-brane and IIA NS-5-brane. The above two solutions preserve 1/4 of bulk supersymmetry and they are determined by two holomorphic functions.

6. T-duality and brane solitons

The association of worldvolume solitons as brane boundaries or brane intersections allows the investigation of their T-duality properties using the T-duality rules of IIA and IIB bulk theories. Therefore the T-duality transformations of the worldvolume brane solitons are already contained in the bulk T-duality chains in sections three and five. However, as we shall see, it is instructive to focus on the T-duality properties of the worldvolume solitons. Let \( p_{SA} \) and \( p_{KA} \) denote a p-brane worldvolume soliton of IIA NS-5-brane and KK-monopole, respectively, and similarly for IIB. Using the bulk T-duality chain

\[(0|1_D, 5_S)_B \leftrightarrow (1|2_D, 5_S)_A \leftrightarrow (2|3_D, 5_S)_B\, , \tag{25}\]

we find that

\[0_{SB} \leftrightarrow 1_{SA} \leftrightarrow 2_{SB}\, , \tag{26}\]

where the directions of the T-duality are along the worldvolume of the NS-5-brane. It is clear from this that under T-duality the 0-brane, 1-brane and 2-brane worldvolume solitons transform like D-branes. The relation of these solitons under T-duality also explains the similarity of the associated worldvolume solutions (6), (9)
and (8). Similarly, using the bulk T-duality chain

\[ (3|5_S, 5_S)_B \leftrightarrow (3|5_S, 5_S)_A , \]  

(27)

we find that

\[ 3_{SB} \leftrightarrow 3_{SA} . \]  

(28)

As we have seen the worldvolume solutions of $3_{SA}$ and $3_{SB}$ are the same.

Next using the bulk T-duality chains,

\[
(0|1_D, 5_S)_B \leftrightarrow (0|0_D, 0_M)_A \\
(2|3_D, 5_S)_B \leftrightarrow (2|2_D, 0_M)_A \\
(3|5_S, 5_S)_B \leftrightarrow (3|5_S, 0_M)_A ,
\]

(29)

we find

\[ 0_{SB} \leftrightarrow 0_{KA} \]
\[ 2_{SB} \leftrightarrow 2_{KA} \]
\[ 3_{SB} \leftrightarrow 3_{KA} . \]  

(30)

As we have already mentioned in the previous section the worldvolume solutions of the relevant worldvolume soliton pairs are identical.

Finally, the T-duality properties of the worldvolume solitons of the IIB KK-monopole are

\[ 1_{SA} \leftrightarrow 1_{KB} \]
\[ 3_{SA} \leftrightarrow 3_{KB} . \]  

(31)

The above T-duality transformations of the worldvolume solitons can be combined by performing T-duality along different directions.
7. Concluding Remarks

We have shown that the fluxes of the various six-dimensional “gauge” theories are associated to bound states below threshold of D-branes with the NS-5-branes and KK-monopoles. We then have used T-duality to construct the associated supergravity solutions and we found that they preserve $1/2$ of bulk supersymmetry. In addition, we have investigated the sectors of these theories that preserve $1/4$ of supersymmetry using the worldvolume solitons of IIA and IIB NS-5-branes and KK-monopoles. We have compared the worldvolume solitons with bulk configurations and found a new solution in IIA and IIB supergravity with the interpretation of two NS-5-branes intersecting on a 3-brane. We have also presented some of the worldvolume soliton solutions of M-theory and IIA KK-monopoles. In addition using the T-duality rules of IIA and IIB theories, we have found some of the T-duality properties of the worldvolume solitons and explained the similarities that some of the associated solutions have.

Most of our investigation was focused on the solitons that preserve $1/4$ of bulk supersymmetry. However many aspects of it can be easily extended to solitons that preserve $1/8$ or less supersymmetry [31]. Our investigation can also be extended to the non-abelian phase of the effective theories of the six-dimensional “gauge” theories. In particular, it is expected that there are soliton solution in this phase as well. For example, the effective theory associated with the IIA KK-monopole and the effective theory associated with the IIB NS-5-brane are expected to have instanton solutions which can be interpreted as worldvolume string solitons.

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