ELASTIC PROTON-PROTON AND PROTON-ANTIPROTON SCATTERING: ANALYSIS OF COMPLETE SET OF HELICITY AMPLITUDES

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Abstract

The differential cross-sections are calculated for proton-proton and proton-antiproton elastic scattering using the phenomenological model based on the analytic parameterizations for global scattering parameters (total cross-section and $\rho$ - parameter), crossing symmetry and derivative relations. We confront our model predictions with experimental data in wide range of energy and momentum transfer. The suggested method may be useful for PAX Program (GSI) as well as for high-energy experiments at RHIC and LHC.

The elastic proton-proton and proton-antiproton interactions allow a unique access to a number of fundamental physics observables. Some important experimental $pp$ and $p\bar{p}$ data are drastically different in the energy region of $\sqrt{s} < 50$ GeV and become close each to other at higher energies approaching the asymptotical expectation. We have proposed earlier two analytical presentations for full set of helicity amplitudes for $p\bar{p}$ elastic scattering and have made predictions for $t$-dependences of some spin observables in first presentation [1]. In present paper we focus our attention on predictions for $pp$, $p\bar{p}$ elastic reactions in second approach.

We use the following analytic parameterization of averaged spin non-flip amplitude for elastic proton-proton scattering:

$$\Phi_+(s, t) = \sum_{i=1}^{3} A_i \delta_i \exp(-B_i(s, t) t/2),$$

where $A_i$ are free complex constant parameters, the slope parameters $B_i(s, t)$ are functions of $s$ and $t$, $\delta_i = 1$, $i = 1, 3$, and $\delta_2 = \exp(-i\beta\pi t/2)$ describes experimental data in the region of diffraction deep, $\beta$ - free parameter. We have approximated the experimental data for slope parameter in order to derive the analytic energy dependence for $B_i(s, t), i = 1, 2$ and the $B_3(s, t)$ was remained as a free parameter. The results for different approximations are shown on Fig. 1a, 1b for energy dependence of slope parameter at low and intermediate $t$ values correspondingly. We choose the following approximation for the slope parameter:

$$B_i(s, t) = B_0^i + a_1^i (s/s_1)^{a_2^i} + 2\alpha_i \ln(s/s_1)^{a_3^i}, \quad i = 1, 2,$$

where $s_1 = 1$ GeV$^2$, $a_1^2 = 1$ - fixed and $a_3^2 = 1.500 \pm 0.005$. One can see the parameterization [2] describes all experimental data quite reasonably for low $t$ domain ($\chi^2/ndf=2.54$).
The new experimental data are necessary at high (RHIC, LHC) energies for intermediate \( t \) values in order to derive more unambiguous energy dependence of slope parameter. But now the function (2) approximates this dependence reasonably \( (\chi^2/\text{ndf}=4.66) \) and predicts the values of \( B_2(s,t) \) in high energy domain which agree qualitatively with theoretical expectation \( B^{pp}(s,t) \approx B^{\bar{p}p}(s,t) \) at asymptotic energies. We have followed the standard way [3] and have assumed that the approximation (1) describes the spin non-flip helicity amplitude at \(|t| \geq 0 \) (GeV/c)^2. There are significant data set for total cross-section \( (\sigma^{pp}_\text{tot}) \) and \( \rho^{pp} = [\Re \Phi^+(s,t=0)] / [\Im \Phi^+(s,t=0)] \). We choose these two characteristics for present analysis in order to decrease the amount of free parameters in (1). The PAX project (GSI), in particular, plans to study the \( pp \) collisions at energies \( \sqrt{s} > 3 \) GeV. Therefore we have to investigate this energy domain at least in order to obtain the reasonable energy dependences of free parameters in spin non-flip amplitude.

We choose the following parameterization for proton-proton total cross-section:

\[
\sigma^{pp}_\text{tot}(s) = \sum_{j=1}^{3} \sigma^{pp}_\text{tot}(j),
\]

\[
(\sigma^{pp}_\text{tot}(j) = a_1 \left( \frac{s}{s - 4m_p^2} \right)^{a_2}; \quad (\sigma^{pp}_\text{tot}(j) = a_3 \frac{J_1(\xi)}{\xi}; \quad \xi = a_4(s/s_1 - a_5); \quad (\sigma^{pp}_\text{tot}(j) = Z^{pp} + B \ln^2(s/s_0) + Y_1^{pp}(s_1/s)^{y_1} - Y_2^{pp}(s_1/s)^{y_2}.
\]

The sum of first two terms is the modification of standard total cross section parameterization from [4] for \( \sqrt{s} \geq 5 \) GeV.

The different approximations are shown at Fig.2, the fit quality for [3] is \( \chi^2/\text{ndf}=6.95 \) when using all available experimental data. As seen the Donnachie - Landshoff (DL), Kang - Nikolecu (KN) and standard Particle Data Group (PDG) parameterizations do not describe the proton-proton total cross section at low energies. On the other side the suggested approach describes the \( \sigma^{pp}_\text{tot} \) at qualitative level reasonably but this fit is still statistically unacceptable. Therefore the problem of description the low energy data remains open.

**Figure 1.** Energy dependence of slope parameter for low (a) and medium (b) \(|t| \) values. The data are drawn from the Durham Database Group (UK). Thin solid line in (b) is a Regge model prediction from [2].
Based on the defined analytical parameterization for total cross-section (3) one can try to obtain the corresponding parameterization for \( \rho^{pp} \)-parameter from analyticity and the dispersion relations written in the derivative form. We use the following analytic parameterization for \( \rho^{pp} \)-parameter:

\[
\rho^{pp} = \frac{1}{2\sigma^{pp}_{tot}} \left[ 2\sigma^{pp}_{tot}A + \sum_{i=1}^{3} \left( \frac{K_i}{s} + \pi\delta_i \frac{d (\sigma^{pp}_{tot})}{d \ln (s/s_1)} \right) \right], \quad A = \lambda_1 \frac{J_1(\lambda_2(s/s_1 - \lambda_3))}{(\lambda_2(s/s_1 - \lambda_3))^{\lambda_4}},
\]

where the additional term \( \Lambda \) describes the low energy data, the \( \sigma^{pp}_{tot} \) are defined above. The first term and \( K_i, \delta_i, i = 1 - 3 \) can be derived from fit of experimental data. The fit quality for (4) is \( \chi^2/ndf = 7.8 \) for all experimental data. For comparison the fit quality is equal 54.4 for PDG parameterization, for example. There are a phase shift analysis results at energy lower than 5 GeV and we plan to look at these techniques and improve our description of the experimental data for low energies.

As seen from Fig. 2 the different models predict quite similar results for \( \sigma^{pp}_{tot} \) (Fig. 2b) and for \( \rho \) (Fig. 2d) at high energies, but they valid only above 10 GeV or so. These models differ at low energies \( \sqrt{s} < 5 \) GeV dramatically (Fig. 2a, 2c). Thus we approximated the global scattering parameters at qualitative level for all available energy domain and defined \( A_1 \).

The remaining parameters in (1) are defined by fit of experimental proton-proton data for differential cross-section \( d\sigma/dt \), in particular. We have used the method from [1] in order to obtain the full set of helicity amplitudes for proton-proton elastic scattering.

We have considered the data for \( pp \) differential cross-section in wide energy domain \( (\sqrt{s} \approx 2 - 62 \) GeV\) and for range of square of transfer momentum \( t \approx 10^{-2} - 10 \) (GeV/c)\(^2\). Experimental data and corresponding fits are shown on Fig. 3a for some initial energies. One can see that our parameterization describes experimental points well at any energies understudy and up to \( t \approx 9 \) (GeV/c)\(^2\) at quantitative level. Disagreement between the experimental data and approximation curves at high \( t \) is expected: the high \( t \) domain is described by power dependence inspired pQCD.

We have considered the large set of available experimental data for \( p\bar{p} \) differential cross-section. Analytic curves contradict with experimental data and some other models
Our approach describes experimental data fairly well at energies $\sqrt{s} \geq 19$ GeV at all $t$ values and it’s close to the modified additive quark (mAQ) model. But our approach contradicts to experimental data and Regge model predictions at low energies.

In summary, the new analytic approach for full set of helicity amplitudes for elastic $pp$ collisions allows to describe well proton-proton experimental differential cross section at $\sqrt{s} \simeq 2 - 62$ GeV and up to $t \sim 9$ (GeV/c)$^2$. Full set of helicity amplitudes for $p\bar{p}$ elastic scattering is derived based on the known helicity amplitude parameterization for $pp$ and crossing-symmetry. Analytic approach describes experimental $p\bar{p}$ data well at $\sqrt{s} \geq 19$ GeV and for low and intermediate $t$ value, $t < 1.5$ (GeV/c)$^2$.

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Figure 3. Differential cross sections for elastic $pp$ (a) and $p\bar{p}$ (b) scattering. A factor $10^{-2}$ between each successive energy is omitted. Experimental data are from the Durham Database Group (UK) for $pp$ and from [6, 7] for $p\bar{p}$. Solid lines are predictions of present work, other curves at (b): dashed - Regge-pole [6], dotted - mAQ [7] model prediction.