Adaptive inverse of piezoelectric bimorph actuator with Prandtl-Ishlinskii model for wide-band tracking control

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Abstract. A major defect of piezoelectric bimorph actuators is that the control accuracy is limited by the hysteresis nonlinearity, especially in the wide band. In this paper, an adaptive control method is proposed for hysteresis compensation. A comparison of PID control, inverse control with Prandtl-Ishlinskii (PI) model, adaptive inverse control with PI model, adaptive inverse control with correction network were performed in wide-band tracking application. The experimental results demonstrate that in low frequency conditions (0.1Hz~1Hz), the above methods have good control performances, and the tracking error is not much different, but under high frequency conditions (5Hz~20Hz), the tracking error of adaptive with PI model and adaptive inverse control with correction network are much better, the RMS tracking error of the piezoelectric bimorph actuators has been reduced from 0.3013 to 0.0489, the experimental results show it performs well in wide-band application.

1. Introduction
Piezoelectric bimorph actuators have a wide range of applications in modern society, such as microgripper, scanning electron microscope [1, 2]. However, hysteresis nonlinear is the inherent characteristic [3, 4]. This nonlinearity probably influences accuracy of positioning system, mainly in wide-band tracking applications [5, 6].

In order to eliminate these effects, establishing a hysteresis model is currently the most effective method to correct and compensate it. Xianfeng Song considered a novel modified Preisach model to identify and simulate the hysteresis phenomena observed in a piezoelectric stack actuator [7]. Wei Zhu considered an adaptive control which uses a piezo-actuated steering mirror to actively resist the jitter and achieve precise pointing [8]. In Reference [9], an inverse hysteresis compensation with generalized PI model was developed for one-dimensional piezoelectric scanner. A robust adaptive inverse control was proposed for nonlinear systems with PI hysteresis model [10]. In Reference [11], hysteresis operator is introduced and using Bayesian regularization training algorithm to train BP neural network to construct hysteresis model of piezoelectric ceramic actuator, the experimental results show that the hysteresis model of piezoelectric ceramic actuators constructed by BP neural network has more accurate identification capability.

In the existing literatures, piezoelectric stack actuators were usually applied in wide-band tracking control. Because its resonant frequency is very high, the frequency band for tracking control is usually far from the resonant frequency. Different from the stack actuators, the resonance frequency of piezoelectric bimorph actuator is much lower, some types are less than 100Hz. Due to the frequency band is close to the resonant frequency, the wide-band tracking control of piezoelectric bimorph
actuator is more complicated. In order to solve this problem, adaptive inverse control of piezoelectric bimorph actuator with Prandtl-Ishlinskii model is presented. A comparison of PID control, inverse control with Prandtl-Ishlinskii (PI) model, adaptive inverse control with PI model, adaptive inverse control with correction network were performed in wide-band tracking application. Different from the classic inverse control, the model parameters are updated with the environment changing. Compared to a single frequency, it is easy to develop an inverse control method and achieve better accuracy. In wide-band application, it is difficult to achieve better accuracy with classic control method. In this paper, an adaptive inverse control with correction network was proposed, and the experimental results show it performs well in wide-band application. The combination of adaptive inverse control and correction network can effectively improve accuracy in a wide-band tracking control.

2. Hysteresis model and control methods

2.1. Prandtl-Ishlinskii model

Prandtl-Ishlinskii model is a phenomenological model that used in modelling complex hysteretic nonlinear behaviour of piezoelectric actuators. This model is analytically invertible, and therefore can determine its accurate inverse model. Used as a feedforward controller in the control system, this model can identify system parameters online in real time.

Like Preisach model, the classical Prandtl-Ishlinskii model uses the classical play (or stop) operator with a density function to characterize the hysteretic behaviour of piezoelectric actuator. In this paper the Prandtl-Ishlinskii model with play operator was applied in adaptive control [12, 13]. It can be expressed as

\[ y(t) = qv(t) + \int_{0}^{R} p(r) F_r[v](t) \, dr \]  

(1)

where \( v(t) \) is the input, model \( q \) is a positive constant, \( R \) is the upper limit of threshold \( r \), \( p(r) \) is a density function, \( p(r) = 0 \) for \( r > R \), and is usually unknown and is expected to be identified with measured data. Let \( C[0, t_E] \) represent the space of piecewise monotone continuous functions. For any input \( v(t) \in C[0, t_E] \), the play operator is

\[ F_r[v](t) = f_r(v(t), F_r[v](t_i)), t_i < t < t_{i+1} \quad \text{and} \quad 0 \leq i \leq N - 1 \]  

(2)

and

\[ f_r(v, w) = \max(v - r, \min(v + r, w)) \]  

(3)

Its initial state is defined by

\[ F_r[v](0) = f_r(v(0), 0) \]  

(4)

where \( 0 = t_0 < t_1 < \cdots < t_N = t_E \) is partition of \([0, t_E]\) such that the function \( v \) is monotone on each sub-intervals \([t_i, t_{i+1}]\).

To manipulate Prandtl-Ishlinskii model in digital controller, Equation (1) is usually written as

\[ y(t) = \sum_{i=0}^{N} p_i F_r[v](t) \]  

(5)

where \( N \) is the number of the play operators in this summation. For the convenience of control programming, the minimum of the threshold values \( t_i, i = 0, 1, 2, 3 \cdots, \) is zero, and its maximum is set to be a half of the input range.

2.2. Adaptive control design

In the classic inverse control of hysteresis compensation, the density function \( p(r) \) is identified off-line. That is, the model parameters are obtained before the real-time control. However, the hysteresis will be changed with the environment or input frequency. In this case, the real-time control process has to be paused and identify the parameters again. In this paper, an adaptive inverse control is proposed for wide band application. The basic conception of inverse control is to establish a model which precisely
describe the inverse of hysteresis, then the inverse and hysteresis cancel out each other as shown in
Figure 1. Although the model is accuracy enough, the hysteresis changes with the environment. It will
degrade the performance of positioning accuracy, even cause the system instable. An adaptive control
method is proposed to update the model parameters in real-time control.

\[ v(t) \rightarrow \text{Inverse model} \rightarrow y(t) \rightarrow \text{Piezoelectric bimorph actuators} \rightarrow x(t) \]

**Figure 1.** Conception of inverse control.

With the inverse controller, the tracking error can be expressed as

\[ e(t) = v(t) - x(t) \]  \hspace{1cm} (6)

where \( v(t) \) is the desired displacement and the input of the inverse hysteresis model, \( x(t) \) is measured

displacement of piezoelectric bimorph actuator.

If model is precise enough, \( e(t) \) will equal to zero. Substitute Equation (1) into (6),

\[ e(t) = v(t) - H[y](t) \]
\[ = v(t) - H[\varphi(t) + \int_0^\Phi p(r) F_r[y](t)dr] \]  \hspace{1cm} (7)

where \( y(t) \) is the output of inverse controller and the input of piezoelectric bimorph actuator, \( H \)
represents the model of piezoelectric bimorph actuator, which can also be established with Prandtl-
Ishlinskii model,

\[ x(t) = cy(t) + \int_0^\Phi b(r) F_r[y](t)dr \]  \hspace{1cm} (8)

where \( c \) is a constant parameter, \( b(r) \) is the density function for direct model of piezoelectric bimorph
actuator.

For the control design, this Equation (8) should be written as

\[ x(t) = \sum_{i=0}^{N_2} b_i F_{r_i} [y](t) \]  \hspace{1cm} (9)

where \( N_2 \) is the number of play operators, \( b_i \) is density value corresponding to operator \( F_{r_i} \).

The derivative of model \( H \) is

\[ \frac{dx(t)}{dy(t)} = \sum_{i=1}^{N_2} \beta_i \]

(10)

and

\[ \beta_i = \begin{cases} b_i & \text{if } F_{r_i} [y](t) \neq F_{r_i} [y](t-\tau) \\ 0 & \text{if } F_{r_i} [y](t) = F_{r_i} [y](t-\tau) \end{cases} \]

(11)

where \( \tau \) is sampling period of the inverse controller.

In the classic inverse control, the density value \( p_i(t) \) is a constant, which is not changed in the real-
time control. In the adaptive control in density value will be updated by the adaptive control algorithm.
The derivative with respect to \( p_i(t) \) is

\[ \frac{dy(t)}{dp_i(t)} = \begin{cases} 1 & \text{if } F_{r_i} [y](t) \neq F_{r_i} [y](t-\tau) \\ 0 & \text{if } F_{r_i} [y](t) = F_{r_i} [y](t-\tau) \end{cases} \]

(12)

Then the derivative of error to density is

\[ \frac{de(t)}{dp_i(t)} = \frac{de(t)}{dy(t)} \frac{dy(t)}{dp_i(t)} \]

(13)
With Equations (10) and (12), Equation (13) can be calculated in real-time controller. The density value $p_i(t)$ is proposed to updated by the following

$$p_i(t) = p_i(t - \tau) + \alpha \frac{de(t)}{dp_i(t)}$$

(14)

where $\alpha$ is the adaptive rate.

The system structure of adaptive inverse control with correction network is shown in Figure 2. It describes the feedback process of the desired displacement error through the correction network and PI adaptive inverse. Where, the correction network transfer function describes the dynamic performance of the hysteresis model, which makes the model related to the input rate. By solving the amplitude ratio and phase difference of the input and output at different frequencies, frequency response data can be obtained to fit the correction network transfer function, it can be expressed as:

$$G(s) = \frac{1905s + 266700}{s + 8000}$$

(15)

![Figure 2. Structure of adaptive inverse control with correction network.](image)

2.3. Experiment setup

The experiment setup is shown in Figure 3. It consists of high-performance computer, power amplifier, displacement sensor, piezoelectric bimorph actuator and NI compactRIO-9030. The computer installed LabVIEW implements the adaptive control algorithm on cRIO-9030. The drive signal from embedded card on cRIO-9030 was amplified to drive the piezoelectric bimorph actuator. The motion of actuator is measured by displacement sensor (Type IL-s025). cRIO-9030 is a programmable real-time controller, which acquires the displacement signal, runs control algorithm and generates drive signal.

![Figure 3. Experiment setup.](image)
3. Experiment results

3.1. Without control
In this study, the hysteresis loops and tracking control errors of different methods are compared in a wide-band tracking control. The results of hysteresis in open-loop without control are shown in Figure 4 to Figure 8.

Figure 4. Hysteresis loop without control at 0.1Hz.

Figure 5. Hysteresis loop without control at 1Hz.

Figure 6. Hysteresis loop without control at 5Hz.

Figure 7. Hysteresis loop without control at 10Hz.

Figure 8. Hysteresis loop without control at 20Hz.
3.2. *Adaptive inverse control with correction network*
If the adaptive inverse control with correction network was applied, the tracking control error will be reduced. The results of hysteresis are shown in Figure 9 to Figure 13.

**Figure 9.** Hysteresis loop with adaptive inverse control and correction network at 0.1Hz.

**Figure 10.** Hysteresis loop with adaptive inverse control and correction network at 1Hz.

**Figure 11.** Hysteresis loop with adaptive inverse control and correction network at 5Hz.

**Figure 12.** Hysteresis loop with adaptive inverse control and correction network at 10Hz.

**Figure 13.** Hysteresis loop with adaptive inverse control and correction network at 20Hz.
3.3. **MAE tracking error and RMS tracking error**

Table 1 shows the MAE (Mean Absolute Error) of the different methods. It can be found that both ascending branches and descending branches of hysteresis loops have been taken into account. The results show that in low frequency (0.1Hz–1Hz), the tracking control error of the four methods are similar. With the frequency increases, the tracking average error of the inverse control and PID control increases rapidly. The proposed adaptive inverse control with correction network performs well in both low and high frequencies.

**Table 1. MAE of different methods.**

| Frequency (Hz) | Without control | Classic inverse control | PID control | Adaptive inverse control | Adaptive inverse control with correction network |
|---------------|-----------------|-------------------------|-------------|--------------------------|-----------------------------------------------|
|               | MAE results (mm)|                         |             |                          |                                               |
| 0.1Hz         | 0.0948          | 0.0458                  | 0.0283      | 0.0247                   | 0.0290                                        |
| 1Hz           | 0.1022          | 0.0462                  | 0.0328      | 0.0290                   | 0.0308                                        |
| 5Hz           | 0.1604          | 0.1137                  | 0.0812      | 0.0684                   | 0.0313                                        |
| 10Hz          | 0.2424          | 0.1850                  | 0.2232      | 0.0856                   | 0.0317                                        |
| 20Hz          | 0.4039          | 0.3512                  | 0.5591      | 0.1256                   | 0.0605                                        |

**Table 2. RMS tracking error of different models.**

| Frequency (Hz) | Without control | Classic inverse control | PID control | Adaptive inverse control | Adaptive inverse control with correction network |
|---------------|-----------------|-------------------------|-------------|--------------------------|-----------------------------------------------|
|               | RMS tracking error results(mm) |                         |             |                          |                                               |
| 0.1Hz         | 0.1184          | 0.0551                  | 0.0364      | 0.0310                   | 0.0364                                        |
| 1Hz           | 0.1196          | 0.0562                  | 0.0410      | 0.0366                   | 0.0380                                        |
| 5Hz           | 0.1850          | 0.1366                  | 0.1007      | 0.0836                   | 0.0384                                        |
| 10Hz          | 0.2730          | 0.2069                  | 0.2561      | 0.0860                   | 0.0382                                        |
| 20Hz          | 0.4459          | 0.3938                  | 0.6083      | 0.1433                   | 0.0702                                        |

In the calculation of RMS (Root Mean Square Error), both ascending branches and descending branches of hysteresis loops have been taken into account. Experimental results show that in low frequency (0.1Hz–1Hz), the tracking control error of the four methods are similar. As same as MAE, with the frequency increases, the tracking error of the inverse control and PID control increases rapidly. Adaptive inverse control and adaptive inverse control with correction network keep good performance. In high frequency conditions (5Hz–20Hz), the results of Adaptive inverse control with correction network are the best of these control methods.

The experimental results demonstrate that in low frequency band (0.1Hz–1Hz), the four methods have good control performances, and the tracking errors are close as shown in Table 2. In 1 Hz, the RMS tracking error of the PID control is 0.0410 mm and the error of the adaptive inverse control with correction network model is 0.0380 mm. But in high frequency band (5Hz–20Hz), the performance of adaptive with PI model and adaptive inverse control with correction network are much better, the RMS tracking error is only 0.0702 mm, which is much less than error of other three methods.

4. **Discussions and conclusions**

An adaptive inverse control was proposed PI model and with correction network. The model can describe the inverse of hysteresis, and its parameters can be updated with the environment changing. A comparison between classic inverse control, PID control, adaptive inverse control and adaptive inverse control with correction network were performed in wide-band tracking control. Experimental results show that classic inverse control and PID control is only suitable in low frequency or narrow band.
The proposed adaptive inverse control with correction network was performed well in wide-band tracking control. According to Reference [11], the relative error of the positive model is 0.0127 and the relative error of the inverse model is 0.014, only measured at 1 Hz. For a single frequency, it is easy to develop an inverse control method and achieve better accuracy. In wide-band application, it is difficult to achieve better accuracy with classic control method. In this paper, an adaptive inverse control with correction network was proposed, and the experimental results show it performs well in wide-band application. In this case, the proposed method maybe not very good at high frequency, but it works well in wide-band application.

The combination of adaptive inverse control and correction network can reduce the wide-band tracking error of piezoelectric bimorph actuators. The proposed method has the potential application in wide-band micro-positioning systems.

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