Large deformed structures in \( Ne - S \) nuclei near neutron drip-line

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The structure of Ne, Na, Mg, Al, Si and S nuclei near the neutron drip-line region is investigated in the framework of relativistic meanfield (RMF) and non-relativistic Skyrme Hartree-Fock formalisms. The drip-line of these nuclei are pointed out. We analysed the large deformation structures and many of these neutron rich nuclei are quite deformed. New magic number are seen for these deformed nuclei.

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I. INTRODUCTION

The structure of light nuclei near the neutron drip-line is very interesting for a good number of exotic phenomena. Nuclei in this region are very different in collectivity and clustering features than the stable counterpart in the nuclear chart. For example, the neutron magicity is lost for the N=8 nucleus for \(^{12}\text{Be}\) \(^1\) and N=20 for \(^{32}\text{Mg}\) \(^2\). The discovery of large collectivity of \(^{34}\text{Mg}\) by Iwasaki et al. \(^3\) is another example of such properties. The deformed structures and core excitations of Mg and neighboring nuclei and location of drip-line in this mass region is an important matter \(^4\). On the other hand, the appearance of N=16 magic number for \(^{24}\text{O}\) is well established \(^5\). The discovery of the two isotopes \(^{46}\text{Mg}\) and \(^{42}\text{Al}\), once predicted to be drip-line nuclei \(^6\) \(^7\) gives indication that the neutron drip-line is located towards the heavier mass region. The existence of neutron halo in \(^{11}\text{Li}\) is well established and the possibility of proton halo in \(^{8}\text{B}\) and the neutron halo in \(^{14}\text{Be}\) and \(^{17}\text{B}\) are very interesting phenomena for the drip-line nuclei. In addition to these above exciting properties, the cluster structure of entire light mass nuclei and the skin formation in neutron-drip nuclei provide us features for the study of light mass drip-line nuclei. Also, the exotic neutron drip-line nuclei play a role in many astrophysical studies. In this paper, our aim is to study the neutron drip-line for Ne-S isotopic chain in the framework of a relativistic mean field (RMF) and non-relativistic Skyrme Hartree-Fock formalism and analyse the large deformation of these isotopes.

The paper is organised as follows: The relativistic and non-relativistic mean field formalisms are described very briefly in Section II. The results obtained from the relativistic mean field (RMF) and Skyrme-Hartree-Fock (SHF) formalisms, and a discussion of these results, are presented in Section III. Finally summary and concluding remarks are given in Section IV.

II. THEORETICAL FRAMEWORK

Mean field methods have been widely used in the study of binding energies and other properties of nuclei \(^8\) \(^9\). Although the older version of the SHF and RMF models have some limitation to reproduce some of the observables, the recent formalisms are quite efficient to predict the bulk properties of nuclei not only near the stability valley, but also for the nuclei near the proton and neutron drip-lines. We use here two of the successful mean field models \(^8\) \(^10\) \(^11\) \(^12\) \(^13\) \(^14\) \(^15\) \(^16\) (Skyrme Hartree-Fock and the Relativistic Mean Field) to learn about the properties of drip-line nuclei \( Ne - S \).

A. The Skyrme Hartree-Fock (SHF) Method

There are many known parametrizations of Skyrme interaction which reproduce the experimental data for ground-state properties of finite nuclei and for the observables of infinite nuclear matter at saturation densities, giving more or less comparable agreements with the experimental or expected empirical data. The general form of the Skyrme effective interaction, used in the mean-field models, can be expressed as a density functional \( \mathcal{H} \) \(^10\) \(^11\), given as a function of some empirical parameters, as

\[
\mathcal{H} = \mathcal{K} + \mathcal{H}_0 + \mathcal{H}_3 + \mathcal{H}_{eff} + \cdots
\]

where \( \mathcal{K} \) is the kinetic energy term, \( \mathcal{H}_0 \) the zero range, \( \mathcal{H}_3 \) the density dependent and \( \mathcal{H}_{eff} \) the effective-mass dependent terms, which are relevant for calculating the properties of nuclear matter. These are functions of 9 parameters \( t_i, x_i \) \((i = 0, 1, 2, 3) \) and \( \eta \), and are given as

\[
\mathcal{H}_0 = \frac{1}{4} t_0 \left[ (2 + x_0) \rho^2 - (2x_0 + 1)(\rho_p^2 + \rho_n^2) \right],
\]

\[
\mathcal{H}_3 = \frac{1}{24} t_3 \rho^3 \left[ (2 + x_3) \rho^2 - (2x_3 + 1)(\rho_p^2 + \rho_n^2) \right],
\]

\[
\mathcal{H}_{eff} = \frac{1}{8} \left[ t_1(2 + x_1) + t_2(2 + x_2) \right] \tau \rho \\
+ \frac{1}{8} \left[ t_2(2x_2 + 1) - t_1(2x_1 + 1) \right] (\tau_p \rho_p + \tau_n \rho_n).
\]

The kinetic energy \( \mathcal{K} = \frac{\hbar^2}{2m} \tau \), a form used in the Fermi gas model for non-interacting fermions. The other terms, representing the surface contributions of a finite nucleus with \( b_4 \),...
and $b'_i$ as additional parameters, are
\[
\mathcal{H}_{S\rho} = \frac{1}{16} \left[ 3t_1 \left( 1 + \frac{1}{2} x_1 \right) - t_2 \left( 1 + \frac{1}{2} x_2 \right) \right] (\nabla \rho)^2
\]
\[
- \frac{1}{16} \left[ 3t_1 (x_1 + \frac{1}{2}) + t_2 (x_2 + \frac{1}{2}) \right]
\times \left[ (\nabla \rho_n)^2 + (\nabla \rho_p)^2 \right], \quad \text{and}
\]
\[
\mathcal{H}_{S\tilde{J}} = \frac{1}{2} \left[ b_4 \rho \tilde{J} + b'_4 (\rho n \tilde{J}_n + \rho p \tilde{J}_p) \right].
\]
(5)

Here, the total nucleon number density $\rho = \rho_n + \rho_p$, the kinetic energy density $\tau = \tau_n + \tau_p$, and the spin-orbit density $\tilde{J} = \tilde{J}_n + \tilde{J}_p$. The subscripts $n$ and $p$ refer to neutron and proton, respectively, and $m$ is the nucleon mass. The $\tilde{J}_q = 0$, $q = n$ or $p$, for spin-saturated nuclei, i.e., for nuclei with major oscillator shells completely filled. The total binding energy (BE) of a nucleus is the integral of the density functional $H$.

At least eighty-seven parametrizations of the Skyrme interaction are published since 1972 [12] where $b_i = b'_i = W_0$, we have used here the Skyrme SkI4 set with $b_i \neq b'_i$ [13]. This parameter set is designed for considerations of proper spin-orbit interaction in finite nuclei, related to the isotope shifts in Pb region and is better suited for the study of exotic nuclei. Several more recent Skyrme parameters such as SLY1, 10, SKX, SkI5 and SkI6 are obtained by fitting the Hartree-Fock (HF) results with experimental data for nuclei starting from the valley of stability to neutron and proton drip-lines [10, 13, 14, 17].

B. The Relativistic Mean Field (RMF) Method

The relativistic mean field approach is well-known and the theory is well documented [13, 16]. Here we start with the relativistic Lagrangian density for a nucleon-meson many-body system, as
\[
\mathcal{L} = -\bar{\psi}_i (i\gamma^\mu \partial_\mu - M) \psi_i + \frac{1}{2} \partial_\mu \sigma \partial^\mu \sigma - \frac{1}{2} m_\sigma^2 \sigma^2
\]
\[
- \frac{1}{3} g_{2\sigma} \sigma^3 - \frac{1}{4} g_3 \sigma^4 - g_\psi \bar{\psi}_i \psi_i \sigma - \frac{1}{4} \Omega^{\mu \nu} \Omega_{\mu \nu}
\]
\[
+ \frac{1}{2} m_\nu V^\nu V_\mu + \frac{1}{4} \epsilon_{\lambda \rho \sigma} (V_\lambda V^\rho V^\sigma)^2 - g_w \bar{\psi}_i \gamma^\mu \psi_i V_\mu
\]
\[
- \frac{1}{4} \tilde{B}^{\mu \nu} \tilde{B}_{\mu \nu} + \frac{1}{2} m_\rho \tilde{R}^\mu \tilde{R}_\mu - g_\rho \bar{\psi}_i \gamma^\mu \tilde{R}_\mu \psi_i
\]
\[
- \frac{1}{4} F^{\mu \nu} F_{\mu \nu} - i \bar{\psi}_i \gamma^\mu (1 - \tau_3) \frac{1}{2} \psi_i A_\mu.
\]
(7)

All the quantities have their usual well-known meanings. From the relativistic Lagrangian we obtain the field equations for the nucleons and mesons. These equations are solved by expanding the upper and lower components of the Dirac spinors and the boson fields in an axially deformed harmonic oscillator basis with an initial deformation. The set of coupled equations is solved numerically by a self-consistent iteration method.

The centre-of-mass motion energy correction is estimated by the usual harmonic oscillator formula $E_{c.m.} = \frac{1}{2} (41A^{-1/3})$. The constant gap BCS pairing is used to add the pairing effects for the open shell nuclei. It is to be noted that in the present work only intrinsic state solutions are presented. Each of these deformed intrinsic states is a superposition of various angular momenta states. To obtain the good angular momentum states and spectroscopic predictions for these nuclei near neutron drip-line we need to project out states of good angular momenta. Such calculation will be considered as a future extension of this work. The quadrupole moment deformation parameter $\beta_2$ is evaluated from the resulting proton and neutron quadrupole moments, as $Q = Q_n + Q_p = \sqrt{\frac{16\pi}{5}} (A \beta_2 / 2)$. The root mean square (rms) matter radius is defined as $\langle r_m^2 \rangle = \frac{1}{A} \int \rho(r_\perp, z) r^2 d\tau$; here $A$ is the mass number, and $\rho(r_\perp, z)$ is the deformed density. The total binding energy and other observables are also obtained by using the standard relations, given in [16]. We use the well known NL3 parameter set [18]. This set not only reproduces the properties of stable nuclei but also well predicts for those far from the $\beta$-stability valley. Also, the isoscalar monopole energy agrees excellently with the experimental values for different regions of the Periodic Table. The measured superdeformed minimum in $^{194}$Hg is 6.02 MeV above the ground state, whereas in RMF calculation with NL3 set, this number is 5.99 MeV [18]. All these facts give us confidence to use this older, though very much still in use, NL3 set for the present investigation.

III. RESULTS AND DISCUSSION

A. Ground state properties from the SHF and RMF models

There exists a number of parameter sets for the standard SHF and RMF Hamiltonians and Lagrangians. In some of our previous papers and of other authors [16, 13, 21] the ground state properties, like the binding energies (BE), quadrupole moment deformation parameters $\beta_2$, charge radii ($r_c$) and other bulk properties are evaluated by using the various non-relativistic and relativistic parameter sets. It is found that, more or less, most of the recent parameters reproduce well the ground state properties, not only of stable normal nuclei but also of exotic nuclei which are away from the valley of beta-stability. So, if one uses a reasonably acceptable parameter set the prediction of the results will remain nearly force independent. This is valid both for SHF and RMF formalisms. However, with a careful inspection of these parametrizations, some of the SHF and RMF sets can not reproduce the empirical data. In this context we can cite the deviation of isotopic shifts than the experimental data [22] for Pb nuclei while using SHF forces like, SkM* values [23]. However, the RMF sets reproduce the kink quite nicely [24]. On the other hand, most of the RMF sets over estimate the nuclear matter incompressibility. In general, the predictive power of both the formalisms are reasonably well and can be comparable to each other, which can be seen in the subsequent subsections. In addition to this, the general results SHF (SKI4) and RMF (NL3) forces are similar for the considered region. Thus in the sub-
TABLE I: The calculated ground state binding energy obtained from SHF and RMF theory are compared with the experimentally known heaviest isotope for Ne, Na, Mg, Al, Si and S [25].

| nucleus | RMF | SHF | Expt. | RMF | SHF |
|---------|-----|-----|-------|-----|-----|
| $^{30}$Ne | 215.1 | 210.6 | 211.2 | $^{34}$Na | 237.9 | 234.5 | 232.8 |
| $^{34}$Mg | 257.7 | 255.1 | 256.2 | $^{39}$Al | 285.2 | 281.9 | 283.1 |
| $^{41}$Si | 310.1 | 305.1 | 306.6 | $^{42}$S | 353.5 | 350.2 | 354.2 |

TABLE II: The predicted neutron-drip-line for Ne, Na, Mg, Al, Si and S in RMF (NL3) and SHF (SKI4) parameter sets are compared with prediction of infinite nuclear matter (INM) mass model [25], finite range droplet model (FRDM) [27] and experimental data [25] along with the number shown in parenthesis are the experimentally extrapolated values.

| nucleus | RMF | SHF | INM | FRDM | Expt. |
|---------|-----|-----|-----|-------|-------|
| Ne      | 34  | 34  | 36  | 32    | 30 (34) |
| Na      | 40  | 40  | 46  | 40    | 36 (40) |
| Mg      | 40  | 48  | 48  | 45    | 39 (42) |
| Al      | 54  | 54  | 53  | 50    | 45 (49) |
| Si      | 55  | 55  | 53  | 50    | 45 (49) |
| S       | 55  | 55  | 53  | 50    | 45 (49) |

sequent results during our discussion we will refer the results of RMF (NL3) calculations, except some specific cases. Thus the result of SHF (SKI4) are not displayed in Tables.

B. Binding energy and neutron drip-line

The ground state binding energy (BE) are calculated for Ne, Na, Mg, Al, Si and S isotopes near the neutron-drip-line. This is done by comparing the prolate, oblate and spherical solution of binding energy for a particular nucleus. For a given nucleus, the maximum binding energy corresponds to the ground state and other solutions are obtained as various excited intrinsic states. In Table I, the ground state binding energy for the heaviest isotopes for the nuclei discussed are compared with the experimental data [25]. From the Table it is observed that the calculated binding energies are comparable with SHF and RMF results. We have listed the neutron-drip-lines in Table II, which are obtained from the ground state binding energy for neutron rich Ne, Na, Mg, Al, Si and S nuclei. The nuclei with the largest neutron numbers so far experimentally detected in an isotopic chain till date, known as experimental neutron drip-line are also displayed in this Table for comparison. The numbers given in the parenthesis are the experimentally extrapolated values [25]. To get a qualitative understanding of the prediction of neutron drip-line, we have compared our results with the infinite nuclear matter (INM) [26] and finite range droplet model (FRDM) [27] mass estimation. From the table, it is clear that all the predictions for neutron-drip-line are comparable to each other.

The drip-lines are very important after discovery of the two isotopes $^{40}$Mg and $^{42}$Al [6] that here once predicted to be beyond the drip-line [7, 28]. This suggests that the drip-line is somewhere in the heavier side of the mass prediction which are beyond the scope of the present mass models [7, 28]. In

TABLE III: The calculated value of charge radius ($r_{ch}$), quadrupole moment deformation parameter $\beta_2$ and binding energy (BE) for Ne, Na and Mg nuclei in RMF (NL3) formalism. The maximum binding energy is the ground state solution and all other values are the intrinsic excited state solution. The radius $r_{ch}$ is in fm and the binding energy is in MeV.

| nucleus | $r_{ch}$ | $\beta_2$ | BE(MeV) |
|---------|----------|------------|---------|
| $^{20}$Ne | 2.970  | 0.535 | 156.7 |
| $^{21}$Ne | 2.953  | 0.516 | 165.9 |
| $^{22}$Ne | 2.940  | 0.502 | 175.7 |
| $^{23}$Ne | 2.913  | 0.386 | 181.8 |
| $^{24}$Ne | 2.890  | 0.278 | 188.9 |
| $^{25}$Ne | 2.907  | 0.272 | 194.2 |
| $^{26}$Ne | 2.926  | 0.277 | 199.9 |
| $^{27}$Ne | 2.945  | 0.247 | 203.9 |
| $^{28}$Ne | 2.965  | 0.225 | 208.2 |
| $^{29}$Ne | 2.981  | 0.161 | 211.2 |
| $^{30}$Ne | 2.998  | 0.100 | 215.0 |
| $^{31}$Na | 3.031  | 0.244 | 216.0 |
| $^{32}$Na | 3.071  | 0.373 | 218.6 |
| $^{33}$Na | 3.095  | 0.424 | 219.5 |
| $^{34}$Na | 3.119  | 0.473 | 220.9 |
| $^{35}$Na | 3.132  | 0.505 | 220.4 |
| $^{36}$Na | 3.146  | 0.539 | 220.3 |
| $^{20}$Na | 2.933  | 0.250 | 189.4 |
| $^{21}$Na | 2.938  | 0.258 | 200.3 |
| $^{22}$Na | 2.940  | 0.202 | 206.3 |
| $^{23}$Na | 2.946  | 0.157 | 212.5 |
| $^{24}$Na | 2.980  | 0.184 | 217.7 |
| $^{25}$Na | 3.012  | 0.205 | 223.4 |
| $^{26}$Na | 3.025  | 0.131 | 227.5 |
| $^{27}$Na | 3.043  | 0.074 | 232.5 |
| $^{28}$Na | 3.061  | 0.129 | 233.0 |
| $^{29}$Na | 3.082  | 0.179 | 234.3 |
| $^{30}$Na | 3.095  | 0.226 | 234.8 |
| $^{31}$Na | 3.108  | 0.270 | 235.9 |
| $^{32}$Na | 3.121  | 0.308 | 236.9 |
| $^{33}$Na | 3.135  | 0.345 | 238.4 |
| $^{34}$Na | 3.156  | 0.359 | 240.0 |
| $^{35}$Na | 3.180  | 0.375 | 241.8 |
| $^{20}$Mg | 3.043  | 0.487 | 194.3 |
| $^{21}$Mg | 3.009  | 0.376 | 202.9 |
| $^{22}$Mg | 2.978  | 0.273 | 212.5 |
| $^{23}$Mg | 3.015  | 0.310 | 220.2 |
| $^{24}$Mg | 3.048  | 0.345 | 228.7 |
| $^{25}$Mg | 3.055  | 0.289 | 234.3 |
| $^{26}$Mg | 3.062  | 0.241 | 240.5 |
| $^{30}$Mg | 3.131  | 0.599 | 237.7 |
| $^{31}$Mg | 3.075  | 0.179 | 245.1 |
| $^{32}$Mg | 3.090  | 0.119 | 250.5 |
| $^{36}$Mg | 3.212  | 0.462 | 264.9 |
| $^{38}$Mg | 3.227  | 0.492 | 266.3 |
| $^{39}$Mg | 3.237  | 0.473 | 267.8 |
| $^{40}$Mg | 3.247  | 0.456 | 269.7 |
this calculations the newly discovered nuclei $^{40}\text{Mg}$ and $^{42}\text{Al}$ are well within the prediction both in the SHF and RMF formalisms. Again a further comparison of the drip-line with RMF and SHF prediction, we find the drip-line predictions in both calculations are well comparable, except for a few exceptions in Na and Si as shown in Table II.

C. Neutron configuration

Analysing the neutron configuration for these exotic nuclei, we notice that, for lighter isotopes of Ne, Na, Mg, Al, Si and S the oscillator shell $N_{\text{osc}} = 3$ is empty. However, the $N_{\text{osc}} = 3$ shell gets occupied gradually with increase of neutron number. In case of Na, $N_{\text{osc}} = 3$ starts filling up at $^{35}\text{Na}$ with quadrupole moment deformation parameter $\beta_2 = 0.356$ and $-0.179$ with occupied orbits $[330]1/2^-$ and $[303]7/2^-$, respectively. The filling of $N_{\text{osc}} = 3$ goes on increasing for Na with neutron number and it is $[330]1/2^-$, $[310]1/2^-$, $[321]3/2^-$ and $[312]5/2^-$ at $\beta_2 = 0.472$ for $^{35}\text{Na}$. Again for the oblate solution the occupation is $[301]1/2^-$, $[301]3/2^-$, $[303]5/2^-$ and $[303]7/2^-$ for $\beta_2 = -0.375$ for $^{35}\text{Na}$. In the case of Mg isotopes, even for $^{30,32}\text{Mg}$, the $N_{\text{osc}} = 3$ shell have some occupation for the low-lying excited states near the Fermi surface for $^{30}\text{Mg}$ (at $\beta = 0.599$ with Be = 237.721 MeV the $N_{\text{osc}}=3$ orbit is $[330]7/2^-$ and for $^{32}\text{Mg}: \beta_2 = [330]1/2^-$, $BE=248.804\text{MeV at } \beta_2 = 0.471$). With the increase of neutron number in Mg and Si isotopic chain, the oscillator shell with $N_{\text{osc}} = 3$ gets occupied more and more. For most of the Si isotopes, the oblate solutions are the dominating ones than the low-lying prolate superdeformed states,
TABLE VI: The calculated value of charge radii ($r_{ch}$), quadrupole deformation parameter $\beta_2$ and binding energy (BE) for Ne, Mg, Si and S even-even nuclei in SHF (SK4) formalism. The maximum binding energy is the ground state solution and all other values are the intrinsic excited state solution. The radius $r_{ch}$ is in fm and the binding energy is in MeV.

| nucleus | $r_{ch}$ (fm) | $\beta_2$ | BE(MeV) | $r_{ch}$ (fm) | $\beta_2$ | BE(MeV) |
|---------|---------------|-----------|---------|---------------|-----------|---------|
| $^{20}$Ne | 3.029 | 0.5481 | 156.817 | 2.950 | -0.1356 | 154.474 |
| $^{22}$Ne | 3.005 | 0.5223 | 175.758 | 2.943 | -0.1989 | 172.758 |
| $^{24}$Ne | 2.952 | 0.2546 | 188.354 | 2.951 | -0.2541 | 188.538 |
| $^{26}$Ne | 2.953 | 0.1233 | 199.380 | 2.944 | 0.0060 | 199.389 |
| $^{28}$Ne | 3.013 | 0.1623 | 206.524 | 3.010 | -0.1334 | 206.433 |
| $^{30}$Ne | 3.054 | 0.0030 | 213.721 | 3.018 | 0.0078 | 213.720 |
| $^{32}$Ne | 3.103 | 0.3808 | 213.118 | 3.118 | 0.3716 | 213.215 |
| $^{34}$Ne | 3.179 | 0.4880 | 213.483 | 3.108 | -0.1462 | 209.695 |
| $^{36}$Ne | 3.203 | 0.6015 | 212.230 | 3.147 | -0.2789 | 208.770 |
| $^{24}$Mg | 3.128 | 0.5248 | 195.174 | 3.077 | -0.252 | 189.946 |
| $^{26}$Mg | 3.090 | 0.3623 | 212.885 | 3.079 | -0.2988 | 213.153 |
| $^{28}$Mg | 3.111 | 0.3419 | 228.997 | 3.056 | -0.1076 | 227.899 |
| $^{30}$Mg | 3.119 | 0.2022 | 240.328 | 3.117 | -0.1835 | 240.514 |
| $^{32}$Mg | 3.145 | 0.0000 | 252.033 | 3.145 | 0.0000 | 252.033 |
| $^{34}$Mg | 3.209 | 0.3263 | 255.067 | 3.175 | -0.1196 | 253.455 |
| $^{36}$Mg | 3.295 | 0.4884 | 201.512 | 3.252 | -0.289 | 257.634 |
| $^{38}$Mg | 3.265 | 0.4413 | 259.899 | 3.213 | -0.2124 | 255.368 |
| $^{40}$Mg | 3.321 | 0.4741 | 262.796 | 3.299 | -0.3538 | 260.200 |
| $^{28}$Si | 3.117 | 0.009 | 231.037 | 3.194 | -0.3494 | 233.590 |
| $^{30}$Si | 3.145 | 0.1477 | 252.146 | 3.168 | -0.2102 | 252.625 |
| $^{32}$Si | 3.179 | 0.007 | 269.479 | 3.199 | -0.1990 | 270.483 |
| $^{34}$Si | 3.216 | 0.9999 | 286.332 | 3.216 | -0.1196 | 253.455 |
| $^{36}$Si | 3.146 | 0.1549 | 292.418 | 3.241 | -0.099 | 292.425 |
| $^{38}$Si | 3.291 | 0.3051 | 298.173 | 3.279 | -0.1978 | 298.173 |
| $^{40}$Si | 3.325 | 0.2990 | 230.450 | 3.309 | -0.2817 | 230.369 |
| $^{42}$Si | 3.349 | 0.3592 | 307.399 | 3.334 | -0.3508 | 301.023 |
| $^{44}$Si | 3.334 | 0.2119 | 309.712 | 3.377 | -0.3031 | 311.601 |
| $^{46}$Si | 3.337 | 0.009 | 312.451 | 3.372 | -0.2405 | 313.508 |
| $^{48}$Si | 3.348 | 0.002 | 315.425 | 3.438 | -0.2893 | 313.995 |
| $^{50}$Si | 3.262 | 0.1491 | 241.434 | 3.178 | -0.1856 | 241.385 |
| $^{52}$Si | 3.271 | 0.200 | 208.173 | 3.256 | -0.1700 | 267.975 |
| $^{54}$Si | 3.288 | 0.1212 | 288.804 | 3.295 | -0.1566 | 289.304 |
| $^{56}$Si | 3.314 | 0.0031 | 309.619 | 3.314 | -0.0031 | 309.619 |

The ground and low-lying excited state deformation systems for some of the representative nuclei for Ne, Na, Mg, Al, Si and S are analysed. In Fig. 1, the ground state quadrupole deformation parameter $\beta_2$ is shown as a function of mass number for Ne, Na, Mg, Al, Si and S. The $\beta_2$ value goes on increasing with mass number for Ne, Na and Mg isotopes near the drip-line. The calculated quadrupole deformation parameter $\beta_2$ for $^{34}$Mg is 0.59 which compares well with the recent experimental measurement of Iwasaki et al [3] ($\beta_2 = 0.58 \pm 0.6$). Note that this superdeformed states in 3.2 MeV above the ground band. Again, the magnitude of $\beta_2$ for the drip-line nuclei reduces with neutron number N and again increases. A region of maximum deformation is found for almost all the nuclei as shown in the figure. It so happens in cases like, Ne, Na, Mg and Al that the isotopes are maximum deformed which may be comparable to superdeformed near the drip-line. For Si isotopes, in general, we find oblate solution in the ground configurations. In many of the cases, the low-lying superdeformed configuration are clearly visible and some of them are available in the Tables.

D. Quadrupole deformation

The ground and low-lying excited state deformation systems for some of the representative nuclei for Ne, Na, Mg, Al, Si and S are analysed. In Fig. 1, the ground state quadrupole deformation parameter $\beta_2$ is shown as a function of mass number for Ne, Na, Mg, Al, Si and S. The $\beta_2$ value goes on increasing with mass number for Ne, Na and Mg isotopes near the drip-line. The calculated quadrupole deformation parameter $\beta_2$ for $^{34}$Mg is 0.59 which compares well with the recent experimental measurement of Iwasaki et al [3] ($\beta_2 = 0.58 \pm 0.6$). Note that this superdeformed states in 3.2 MeV above the ground band. Again, the magnitude of $\beta_2$ for the drip-line nuclei reduces with neutron number N and again increases. A region of maximum deformation is found for almost all the nuclei as shown in the figure. It so happens in cases like, Ne, Na, Mg and Al that the isotopes are maximum deformed which may be comparable to superdeformed near the drip-line. For Si isotopes, in general, we find oblate solution in the ground configurations. In many of the cases, the low-lying superdeformed configuration are clearly visible and some of them are available in the Tables.

E. Shape coexistence

One of the most interesting phenomena in nuclear structure physics is the shape coexistence [29-31]. In many of the cases for the nuclei considered here near the drip-line isotopes, the ground state configuration accompanies a low-lying excited state. In few cases, it so happens that these two solutions are almost degenerate. That means we predict almost similar binding energy for two different configurations. For example, in the RMF calculation, the ground state binding energy of $^{24}$Ne is 189.093 MeV with $\beta_2 = -0.259$ and the binding energy of the excited low-lying configuration at $\beta_2 = 0.278$ is 188.914 MeV. The difference in BE of these two solutions is only 0.179 MeV. Similarly the solution of prolate-oblate binding energy difference in SK4 is 0.186 MeV for $^{30}$Mg with...
$\beta_2 = -0.183$ and $0.202$. This phenomenon is clearly available in most of the isotopes near the drip-line. To show it in a more quantitative way, we have plotted the prolate-oblate binding energy difference in Figure 2. The left hand side of the figure is for relativistic and the right side is the nonrelativistic SkI4 results. From the figure, it is clear that an island of shape coexistence solutions are available for Mg and Si isotopes. These shape coexistence solutions are predicted taking into account the intrinsic binding energy. However the actual quantitative energy difference of ground and excited configuration can be given by performing the angular momentum projection, which is be an interesting problem for future.

### F. Two neutron separation energy and new magic number

The appearance of new and the disappearance of known magic number near the neutron drip-line is a well discussed topic currently in nuclear structure physics [3, 32]. Some of the calculations in recent past predicted the disappearance of the known magic number N=28 for the drip-line isotopes of Mg and S [33, 34]. However, magic number 20 retains its magic properties even for the drip-line region. In one of our earlier publications [35] we analysed the spherical shell gap at N=28 in $^{44}$S and its neighboring $^{40}$Mg and $^{32}$Si using NL-SH [22] and TM2 parameter sets [36]. The spherical shell gap at N=28 in $^{44}$S was found to be intact for the TM2 and is broken for NL-SH parametrization. Here, we plot the two-neutron separation energy $S_{2n}$, of Ne, Mg, Si and S for the even-even nuclei near the drip-line (fig 3). The known magic number N=28 is noticed to be absent in $^{44}$S. On the other hand the appearance of steep 2n-separation energy at N=34 both in RMF and SHF calculation looks quite prominent, and this is just two units ahead than the experimental shell closure N=32 [37].

### G. Superdeformation and Low $\Omega$ parity doublets

The deformation-driving $m = 1/2 -$ orbits come down in energy in superdeformed solutions from the shell above, in contrast to the normal deformed solutions. The occurrence of approximate $1/2^+ 1/2^-$ parity doublets (degeneracy of $|m|^{\text{rel}}= 1/2^+ 1/2^-$ states) for the superdeformed solutions are clearly seen in Figs. 4 and 5 where excited superdeformed configurations for $^{32}$Mg and $^{34}$Mg and for $^{46}$Al and $^{47}$Al are given. For each nucleus we have compared the normal deformed ($\beta_2 \sim 0.1 - 0.3$) and the superdeformed configurations and analysed the deformed orbits.

The $1/2^+$ and $1/2^-$ states for the single particle levels are shown in Fig. 4 (for $^{32}$Mg and $^{34}$Mg). From the analysis of the results of this calculation, we have found a systematic behaviour of the low $\Omega$ (particularly $1/2^+$ and $1/2^-$) prolate deformed orbits for the superdeformed solutions. As repre-
sentive cases, we present here results for \((^{34}\text{Mg} - ^{32}\text{Mg})\) and \((^{47}\text{Al} - ^{46}\text{Al})\) and plot the \(1/2^+\) and \(1/2^-\) orbits for the superdeformed and normal deformed shapes of these nuclei. We notice from the plot of the orbits that there is occurrence of \(1/2^+\) and \(1/2^-\) orbits very closeby in energy for the superdeformed (SD) shape. Two such \(1/2^+\) and \(1/2^-\) doublet structures, marked in asterisk are shown in Figs. 4 and 5 for the SD solutions. Such \(1/2^+\), \(1/2^-\) degenerate orbits occur not only for the well-bound orbits but also for the unbound continuum states. As example, the doublet neutrons \([220]1/2^+\) and \([101]1/2^-\) states is 4 MeV apart in energy in the normal deformed prolate solutions tend to become degenerate in the SD solution \([220]1/2^+\) and \([101]1/2^-\) states (prolate) belonging to two different major shells, so close to each other in the superdeformed solution (shown in Figs. 4 for \(^{34}\text{Mg}\)). More such doublets are easily identified (Figs. 4 and 5) for superdeformed solutions of \(^{32},^{34}\text{Mg}\) and \(^{46},^{47}\text{Al}\). In fact it is noticed that the \(\Omega = 1/2\) states of unique parity, seen clearly well separated in the normal deformed solutions, get quite close to each other for the SD states, suggesting degenerate parity doublet structure. This can lead to parity mixing and octupole deformed shapes for the SD structures.\(^{[38]}\) Parity doublets and octupole deformation for superdeformed solutions have been discussed for neutron-rich Ba and Zr nuclei.\(^{[39]}\). There is much interest for the experimental study of the spectra of neutron-rich nuclei in this mass region.\(^{[40]}\) The highly deformed structures for the neutron-rich \(Ne - Na - Mg - Al\) nuclei are interesting and signature of such superdeformed configurations should be looked for.

IV. SUMMARY AND CONCLUSION

In summary, we calculate the ground and low-lying excited state properties, like binding energy and quadrupole deformation \(\beta_2\) using NL3 parameter set for Ne, Na, Mg, Si and S isotopes, near the neutron drip-line region. In general, we find large deformed solutions for the neutron-drip nuclei which agree well with the experimental measurement. We have done the calculation using the nonrelativistic Hartree-Fock formalism with Skyrme interaction SkI4. Both the relativistic and non-relativistic results were found comparable to each other for the considered mass region. In the present calculations a large number of low-lying intrinsic superdeformed excited states are observed for many of the isotopes and some of them are reported. The breaking of \(N=28\) magic number and the appearance of a new magic number at \(N=34\) appears in our calculations. A proper angular momentum projection may tell us the exact lowering of binding energy and it may happen that the superdeformed would be the ground band of some of the neutron-rich nuclei. Work in this direction is worth doing because of the present interest in the topic of the drip-line nuclei. In this study we find that, for the SD shape, the low \(\Omega\) orbits (particularly \(\Omega = 1/2\)) become more bound and show a parity doublet structure. Closelying parity-doublet band structures and enhanced electromagnetic transition rates are a clear possibility for the superdeformed shapes.

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