Addressing the circularity problem in the $E_p - E_{iso}$ correlation of Gamma-Ray Bursts

Lorenzo Amati$^1$, Rocco D’Agostino$^{2,3}$, Orlando Luongo$^{4,5}$, Marco Muccino$^4$, Maria Tantalo$^4$

1 INAF, Istituto di Astrofisica Spaziale e Fisica Cosmica, Bologna, Via Gobetti 101, I-40129 Bologna, Italy
2 Dipartimento di Fisica, Università di Roma “Tor Vergata”, Via della Ricerca Scientifica 1, I-00133, Roma, Italy
3 Sezione INFN, Università di Roma “Tor Vergata”, Via della Ricerca Scientifica 1, I-00133, Roma, Italy
4 Istituto Nazionale di Fisica Nucleare, Laboratori Nazionali di Frascati, 00044 Frascati, Italy
5 Instituto de Ciencias Nucleares, Universidad Nacional Autónoma de México, AP 70543, México DF 04510, Mexico

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ABSTRACT

Context. Given their huge luminosity and redshift distribution extending up to $z \approx 9$, Gamma-Ray Bursts (GRBs) are potentially a very powerful tool for studying the geometry and the accelerated expansion of the universe. We here propose a new model-independent technique to overcome the circularity problem affecting the use of GRBs as distance indicators through the use of $E_p - E_{iso}$ correlation.

Aims. We calibrate the $E_p - E_{iso}$ correlation and find the GRB distance moduli that can be used to constrain dark energy models.

Methods. We use observational Hubble data to approximate the cosmic evolution through Bézier parametric curve obtained through the linear combination of Bernstein basis polynomials. In so doing, we build up a new data set consisting of 193 GRB distance moduli. We combine this sample with the supernova JLA data set to test the standard $\Lambda$CDM model and its $w$CDM extension. We place observational constraints on the cosmological parameters through Markov Chain Monte Carlo numerical technique. Moreover, we compare the theoretical scenarios by performing the AIC and BIC statistics.

Results. For the $\Lambda$CDM model we find $\Omega_m = 0.397^{+0.046}_{-0.048}$ at the 2$\sigma$ level, while for the $w$CDM model we obtain $\Omega_m = 0.34^{+0.13}_{-0.15}$ and $w = -0.86^{+0.36}_{-0.34}$ at the 2$\sigma$ level. Our analysis suggests that $\Lambda$CDM model is statistically favoured over the $w$CDM scenario.

Conclusions. The results of our numerical analysis are consistent with previous findings involving GRB data. Also, the values of $\Omega_m$ and $w$ for the $w$CDM model are in remarkable agreement with those obtained by the Dark Energy Survey collaboration. No evidence for extension of the $\Lambda$CDM model is found.

1. Introduction

The cosmic speed up is today a consolidate experimental evidence confirmed by several probes (Haridasu et al. 2017). Particularly, type Ia Supernovae (SNe Ia) have been employed as standard candles (Phillips 1993) to check the onset of cosmic acceleration (Perlmutter et al. 1998, 1999; Riess et al. 1998; Schmidt et al. 1998). Their importance lies in the fact that they may open a window into the nature of the constituents pushing up the universe to accelerate. Even though SNe Ia are considered among the most reliable standard candles, they are detectable up to $z \approx 2$ (Rønney et al. 2015). Thus, at intermediate redshifts the standard cosmological model, dubbed the $\Lambda$CDM paradigm, cannot be tested with SNe Ia alone. Consequently, higher redshift distance indicators, such as Baryon Acoustic Oscillations (BAO) (Percival et al. 2010; Aubourg et al. 2015; Luković et al. 2016), have been used to alleviate degeneracy among the $\Lambda$CDM paradigm and dark energy scenarios. In these respects, a relevant example if offered by Gamma-Ray Bursts (GRBs), which represent the most powerful cosmic explosions detectable up to $z = 9.4$ (Salvaterra et al. 2009; Tanvir et al. 2009; Cucchiara et al. 2011). Attempts to use GRBs as cosmic rulers led cosmologists to get several correlations between GRB photometric and spectroscopic properties (Amati et al. 2002; Ghirlanda et al. 2004; Amati et al. 2008; Schaefer 2007; Capozziello & Izzo 2008; Dainotti et al. 2008; Bernardini et al. 2012; Amati & Della Valle 2013; Wei et al. 2014; Izzo et al. 2015; Demianski et al. 2017a,b). The most investigated correlations involve the rest-frame spectral peak energy $E_{p}$, i.e. the rest-frame photon energy at which the $vF_v$ spectrum of the GRB peaks, and the bolometric isotropic-equivalent radiated energy $E_{iso}$, or peak luminosity $L_p$ (Amati et al. 2002; Yonetoku et al. 2004; Amati et al. 2008; Amati & Della Valle 2013; Demianski et al. 2017a,b). The use of GRBs in cosmology is unfortunately jeopardised by the circularity problem (see, e.g., Kodama et al. 2008), i.e. they cannot be seen as standard candles since their spectroscopic properties are established assuming a background cosmology. For example, calibrating GRBs through the standard $\Lambda$CDM model, the estimate of cosmological parameters of any dark energy framework inevitably returns an overall agreement with the concordance model.

In this paper, we propose a new model-independent calibration of the $E_p - E_{iso}$ correlation (the Amati relation see e.g., Amati et al. 2008; Amati & Della Valle 2013). We take the most recent values of observational Hubble Data (OHD), consisting of 31 points of Hubble rates got at different redshifts (see Capozziello et al. 2018, and references therein). These data have been obtained through the differential age method applied to pairs of nearby galaxies, providing model-independent measurements (Jimenez & Loeb 2002). We follow the strategy to fit OHD data using a Bézier parametric curve obtained through the linear combination of Bernstein basis polynomials. This treatment is a refined approximated method and reproduces Hubble’s rate at arbitrary redshifts without assuming an a priori cosmological model. We thus use it to calibrate the $E_{iso}$ values by means of a data set made of 193 GRBs with firmly measured redshift and spectral parameters taken from Demianski et al. (2017a) and
references therein. Detailed discussions of possible biases and selection effects can be found, e.g. in Amati & Della Valle (2013), Demianski et al. (2017a) and Dainotti & Amati (2018).

We find the best-fitting cosmological parameters by means of a Markov Chain Monte Carlo (MCMC) technique, combining the aforementioned 193 GRBs with the JLA SNe Ia compilation. In the numerical procedure, we assume uniform priors on the fitting parameters, taking $H_0$ to the best-fit value obtained from the model-independent analysis of the OHD data: $H_0 = (67.76 \pm 3.68) \, \text{km s}^{-1} \, \text{Mpc}^{-1}$, compatible with the current estimates by the Planck Collaboration (Aghanim et al. 2018) and Riess et al. (2018). We therefore calibrate the Amati relation, through a new data set in which we compute the corresponding GRB distance moduli $\mu_{\text{GRB}}$ and the corresponding 1$\sigma$ error bars, depending upon the uncertainties on GRB observables. Thus, using the above value of $H_0$ got from our analysis, we show that our results are in tension with the concordance paradigm (Aghanim et al. 2018) at $\geq 3\sigma$. As possible dark energy model, we test the $w$CDM model finding out that this scenario is disfavored with respect to the $w$CDM paradigm. We get evidence of such a conclusion through the use of the Akaike’s information criterion (AIC) and Bayesian information criterion (BIC).

The paper is divided into four sections. After this Introduction, in Sec. 2 we describe the main features of our treatment, using OHD data surveys over the Amati relation. In Sec. 3, we discuss our numerical outcomes concerning the use of our new data set. We thus get constraints over the free parameters of the $\Lambda$CDM and $w$CDM models. In Sec. 4, we draw conclusions and identify the perspectives of our work.

2. Model-independent calibration of the Amati relation

Calibrating the Amati relation represents a challenge due to the problem of circularity (see, e.g., Ghirlanda et al. 2004; Ghirlanda et al. 2006; Kodama et al. 2008; Amati & Della Valle 2013). In fact, in the $E_{\text{iso}}-E_{\text{bolo}}$ correlation, the cosmological parameters $\Omega_k$ and the Hubble constant $H_0$ enter in the $E_{\text{iso}}$ definition through the luminosity distance $d_L$, i.e., $E_{\text{iso}}(z, H_0, \Omega_k) \equiv 4\pi d_L^2(z, H_0, \Omega_k)S_{\text{bolo}}/(1 + z)$, where $S_{\text{bolo}}$ is the observed bolometric GRB fluence and the factor $(1 + z)^{-1}$ transforms the observed GRB duration into the source cosmological rest-frame one. The most quoted approach to the calibration of the Amati relation makes use of the SN Ia Hubble diagram, directly inferred from the observations, and interpolate it to higher redshift by using GRBs (see, e.g., Kodama et al. 2008; Liang et al. 2008; Demianski et al. 2017a,b). However, this method biases the GRB Hubble diagram by introducing the systematics of the SNe Ia.

Here, we propose an alternative calibration which makes use of the differential age method based on spectroscopic measurements of the age difference $\Delta t$ and redshift difference $\Delta z$ of couples of passively evolving galaxies that formed at the same time (Jimenez & Loeb 2002). This method implies that $\Delta z/\Delta t \equiv dz/dt$ and hence the Hubble function can be computed in a cosmology-independent way as $H(z) = -(1 + z)^{-1}\Delta z/\Delta t$. The updated sample of 31 OHD (see Capozziello et al. 2018) is shown in Fig. 1. To avoid the circularity problem, we approximate the OHD data by employing a Bézier parametric curve\textsuperscript{1} of degree $n$

$$ H_n(z) = \sum_{d=0}^{n} \beta_d H_0^d(z), \quad H_0^d(z) \equiv \frac{n!^z}{n!(n-d)!} \left(1 - \frac{z}{z_{\text{max}}}\right)^{n-d}, \quad (1) $$

where $\beta_d$ are coefficients of the linear combination of Bernstein basis polynomials $H_0^d(z)$, positive in the range $0 \leq z/z_{\text{max}} \leq 1$, where $z_{\text{max}}$ is the maximum $z$ of the OHD dataset. For $d = 0$ and $z = 0$, we easily identify $\beta_0 \equiv H_0$. To obtain a monotonic growing function, we use $n = 2$. The best fit with its $1\sigma$ and $3\sigma$ confidence regions are shown in Fig. 1. The best-fit parameters are $H_0 = 67.76 \pm 3.68, \beta_1 = 103.34 \pm 11.14$, and $\beta_2 = 208.45 \pm 14.29$ (all in units of km s$^{-1}$ Mpc$^{-1}$). The value of $H_0$ so obtained is compatible with the current estimate of the Planck Collaboration (Aghanim et al. 2018) and in agreement at the 1.49$\sigma$ level with the value measured by Riess et al. (2018).

Once the function $H_2(z)$ is extrapolated to redshift $z > z_{\text{max}}$, the luminosity distance is (see, e.g., Goobar & Perlmutter 1995)

$$ d_L(\Omega_k, z) = \frac{c}{H_0} \left(1 + \frac{z}{\Omega_k} \right) \left[ \sqrt{\Omega_k} \int_0^z \frac{d\Omega_k}{H_2(z')} \right], \quad (2) $$

where $\Omega_k$ is the curvature parameter, and $S_{\text{bolo}}(x) = \sinh(x)$ for $\Omega_k > 0$, $S_{\text{bolo}}(x) = x$ for $\Omega_k = 0$, and $S_{\text{bolo}}(x) = \sin(x)$ for $\Omega_k < 0$. We note that $d_L$ in Eq. (2) is not completely independent from cosmological scenarios, since it depends upon $\Omega_k$. However, supported by the most recent Planck results (Aghanim et al. 2018), which find $\Omega_k = 0.001 \pm 0.002$, we can safely assume $\Omega_k = 0$. In so doing, the dependency upon $\Omega_k$ identically vanishes and Eq. (2) becomes cosmology-independent:

$$ d_{\text{cal}}(z) = c(1 + z) \int_0^z \frac{dz'}{H_2(z')} \int_0^z \frac{d\Omega_k}{H_2(z')} \quad (3) $$

We are now in the position to use $d_{\text{cal}}(z)$ to calibrate the isotropic energy $E_{\text{iso}}^{\text{cal}}$ for each GRB fulfilling the Amati relation

$$ E_{\text{iso}}^{\text{cal}}(z) \equiv 4\pi d_{\text{cal}}^2(z)S_{\text{bolo}}(1 + z)^{-1}, \quad (4) $$

where the respective errors $\sigma E_{\text{iso}}^{\text{cal}}$ depend upon the GRB systematics on the observables and the fitting procedure (see confidence regions in Fig. 1). The corresponding $E_{\text{iso}}-E_{\text{iso}}^{\text{cal}}$ distribution is displayed in Fig. 2. Following the method by D’Agostini (2005), we fit the calibrated Amati relation by using a linear fit log($E_{\text{iso}}^{\text{cal}}/1\text{keV}$) = $q + m \log[E_{\text{iso}}/(\text{erg})] + 52$. We find the best-fit parameters $q = 2.06 \pm 0.03$, $m = 0.50 \pm 0.02$, and the extra-scatter

\textsuperscript{1} Bézier curves are easy to use in computation, are stable at the lower degrees of control points and can be rotated and translated by performing the operations on the points.
\[ \sigma_{ex} = 0.20 \pm 0.01 \text{ dex (see Fig. 2). The corresponding Spearman’s } \]

\[ \text{rank correlation coefficient is } \rho_s = 0.84 \text{ and the p-value from the two-sided Student’s } \]

\[ t-\text{distribution is } p = 2.42 \times 10^{-36}. \]

We can then compute the GRB distance moduli from the standard definition \( \mu_{\text{GRB}} = 25 + 5 \log(d_{\text{cal}}/\text{Mpc}). \] Using the fit of the calibrated Amati relation, we obtain

\[ \mu_{\text{GRB}} = 25 + \frac{5}{2} \left[ \frac{\log E_p - q}{m} - \log \left( \frac{4\pi S_{\text{iso}}}{1 + z} \right) + 52 \right], \] \[ (5) \]

where now \( S_{\text{iso}} \) has been normalized to erg Mpc\(^{-2}\) to obtain \( d_{\text{cal}} \)

\[ \text{in the desired units of Mpc. The attached errors on } \mu_{\text{GRB}} \text{ take into account the GRB systematics and the statistical errors on } q, \]

\[ m \text{ and } \sigma_{ex}. \] The distribution of \( \mu_{\text{GRB}} \) with \( z \) is shown in Fig. 3.

**Table 1.** Priors used for parameters estimate in the MCMC analysis.

| \( w \) | \( \Omega_m \) | \( M \) | \( \Delta M \) | \( \alpha \) | \( \beta \) |
|---|---|---|---|---|---|
| \(-0.5, -1.5\) | \(0.1\) | \(-20, -18\) | \(-1, 1\) | \(0.1\) | \(0.5\) |

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3. **Numerical results**

We here use our sample of GRBs to test cosmological models. In particular, we assume standard barotropic equation of state (EoS). Thus, for each fluid the pressure \( P_i \) is a one-to-one function of the density \( \rho_i \): \( P_i = w_i \rho_i \). As a consequence of Bianchi’s identity, one gets \( \rho_i + 3H \rho_i (1 + w_i) = 0 \) for each species entering the Einstein equations. Following the standard recipe, we here consider pressureless matter with negligible radiation and define current total density as \( \Omega_i = \rho_i / \rho_c \), with \( \rho_c \equiv 8\pi G / (3H_0^2) \) is the critical density, one can reformulate the Hubble evolution as:

\[ H(z) = H_0 \sqrt{\Omega_m (1 + z)^3 + \Omega_{DE} (1 + z)^{3(1+w)}}. \] \[ (6) \]
In the above relation, dark energy takes a net density given by \(\Omega_{DE} = 1 - \Omega_m\) to guarantee that \(H(z = 0) = H_0\), and \(\omega\) is the dark energy EoS parameter. In particular, Eq. (6) reduces to the \(\Lambda\)CDM model as \(\omega = -1\), whereas to the \(\omega\)CDM model when \(\omega\) is free to vary. The distance modulus is given by \(\mu_{GRB}(z) = 5 \log \left[\frac{d_L(z)}{M_{\odot}}\right] + 5\log(d_L(z)/\text{Mpc})\), where \(d_L(z)\) is given by Eq. (2) with \(\Omega_k = 0\). Thus, the likelihood function of the GRB data can be written as

\[
\mathcal{L}_{\text{GRB}} = \prod_{i=1}^{N_{\text{GRB}}} \frac{1}{\sqrt{2\pi \sigma_{\mu_{\text{GRB}}}}} \exp \left[ -\frac{1}{2} \left( \frac{\mu_{\text{GRB}}(z_i) - \mu_{\text{GRB}}(\omega)}{\sigma_{\mu_{\text{GRB}}}} \right)^2 \right],
\]

where \(N_{\text{GRB}} = 193\) is the number of GRB data points. To obtain more robust observational bounds on cosmological parameters, we consider a complete Hubble diagram by complementing the GRB measurements with the SN JLA sample (Betoule et al. 2014). The latter consists of 740 SN Ia in the redshift range 0.01 < \(z\) < 1.3. The distance modulus of each SN is parameterized as

\[
\mu_{SN} = m_B - M_B + \alpha X_1 - \beta C,
\]

where \(m_B\) is the \(B\)-band apparent magnitude, while \(C\) and \(X_1\) are the colour and the stretch factor of the light curve, respectively; \(M_B\) is the absolute magnitude defined as

\[
M_B = \begin{cases} 
M & \text{if } M_{\text{host}} < 10^{10} M_{\odot}, \\
M + A_M & \text{otherwise,}
\end{cases}
\]

where \(M_{\text{host}}\) is the host stellar mass, and \(M\), \(\alpha\) and \(\beta\) are nuisance parameters which enter along with cosmological parameters. The likelihood function of the SN data can be written as

\[
\mathcal{L}_{SN} = \frac{1}{(2\pi\sigma_{\mu_{SN}})^{1/2}} \exp \left[ -\frac{1}{2} \left( \frac{\mu_{\text{SN}} - \mu_{\text{SN}}(\omega)}{\sigma_{\mu_{SN}}} \right)^2 \right],
\]

where \(M\) is the 3SN \(\times N_{\text{SN}} = 2220 \times 2200\) covariance matrix with the statistical and systematic uncertainties on the light-curve parameters given in Betoule et al. (2014).

We thus perform a MCMC integration on the combined likelihood function \(\mathcal{L} = \mathcal{L}_{SN}\mathcal{L}_{\text{GRB}}\) by means of the Metropolis-Hastings algorithm implemented through the Monte Python code (Audren et al. 2013). In the numerical procedure, we assume uniform priors on the fitting parameters (see Table 1) and we take \(H_0\) as the best-fit value obtained from the model-independent analysis of the OHD data: \(H_0 = 67.74\) km s\(^{-1}\) Mpc\(^{-1}\). We summarize the results for the \(\Lambda\)CDM and \(\omega\)CDM models in Table 2. We show the marginalized 1\(\sigma\) and 2\(\sigma\) confidence contours in Fig. 4. One immediately sees that \(\Omega_m\) in the \(\Lambda\)CDM model is unusually high compared to previous findings which use SNe Ia and other surveys different from GRBs. In fact, our result is in tension with Planck’s predictions (Aghanim et al. 2018) at \(\geq 3\sigma\). However, our result is well consistent within 1\(\sigma\) with previous analyses which made use of GRBs (see, e.g. Amati & Della Valle 2013 for a review, and Izzo et al. 2015, Haridasu et al. 2017 and Demianski et al. 2017a,b for recent results). In addition, the tension is reduced as one considers the \(\omega\)CDM model, enabling \(w\) to vary. This does not indicate that \(\omega\)CDM is favoured with respect to the standard cosmological model. In fact, we immediately notice that \(w\) is consistent within 1\(\sigma\) with the \(\Lambda\)CDM case, i.e. \(w = -1\).

3.1. Statistical performances with GRBs

To test the statistical performance of the models under study, we apply AIC (Akaike 1974) and BIC (Schwarz 1978) criteria:

\[
\text{AIC} = 2p - 2\ln L_{\text{max}}, \quad \text{BIC} = p \ln N - 2\ln L_{\text{max}},
\]

where \(p\) and \(N\) are the number of free parameters in the model and the number of data points, respectively. Here, \(L_{\text{max}}\) is the maximum probability function calculated at the best-fit point. The best model is the one that minimizes both AIC and BIC values. We thus computed the AIC and BIC differences with respect to the reference \(\Lambda\)CDM flat scenario. While the AIC result indicates that the \(\Lambda\)CDM model is slightly favoured with respect to the \(\omega\)CDM model, the BIC value shows a more decisive evidence against the \(\omega\)CDM model (see Table 2). This is due to the more severe penalization of the BIC criterion that corrects for the larger number of free parameters with the logarithm of number of data.

4. Final outlooks and perspectives

In this work, we faced out the circularity problem in using GRBs as distance indicators. To do so, we employed the \(E_p - E_{iso}\) (“Amati”) correlation and we proposed a new technique to fix \(d_p\) in a model-independent way, using the OHD measurements. In particular, we considered the OHD data points and we approximated the Hubble function by means of a Bézier parametric curve obtained from the linear combinations of Bernstein’s polynomials. Assuming vanishing spatial curvature as suggested by Planck’s results, we were able to calibrate the Amati relation in a model-independent way. We thus obtained a new sample of distance moduli for 193 different GRBs (see Table 3).

We then used the new data sample to constrain two different cosmological scenarios: the concordance \(\Lambda\)CDM model, and the \(\omega\)CDM model, with the dark energy EoS parameter is free to vary. Hence, we performed a Monte Carlo integration through the Metropolis-Hastings algorithm on the joint likelihood function obtained by combining the GRB measurements with the SNe JLA data set. In our numerical analysis, we fixed \(H_0\) to the best-fit value obtained from the model-independent analysis over OHD data; i.e. \(H_0 = 67.74\) km s\(^{-1}\) Mpc\(^{-1}\). Our results for \(\Omega_m\) and \(w\) agree with previous findings making use of GRBs and our treatment candidates as a severe alternative to calibrate the Amati relation in a model-independent form. Finally, we employed the AIC and BIC selection criteria to compare the statistical performance of the investigated models. We found that the \(\Lambda\)CDM model is preferred with respect to the minimal \(\omega\)CDM extension. Although a pure \(\Lambda\)CDM model is statistically favoured, we note that the values of \(\Omega_m\) and \(w\) for the \(\omega\)CDM model are remarkably in agreement with those obtained by the Dark Energy Survey (DES) (Abbott et al. 2018). We can then conclude that no modifications of the standard paradigm are expected as intermediate redshifts are involved.

Future efforts will be dedicated to the use of our new technique to fix refined constraints over dynamical dark energy models. Also, we will compare our outcomes with respect to previous model-independent calibrations.

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Table 2. 95% confidence level results of the MCMC analysis for the SN+GRB data. The AIC and BIC differences are indicated with respect to the ACDM model.

| Model | $w$ | $\Omega_m$ | $M$ | $\Delta_{	ext{AIC}}$ | $\Delta_{	ext{BIC}}$ | $\alpha$ | $\beta$ |
|-------|-----|-------------|-----|---------------------|---------------------|---------|--------|
| ACDM  | -1  | 0.397±0.040 | -19.099±0.077 | -0.055±0.043 | 0.126±0.011 | -0.126±0.011 | 2.61±0.13 |
| wCDM  | -0.86±0.38 | 0.34±0.13 | -19.079±0.046 | -0.055±0.042 | 0.126±0.011 | 2.61±0.13  | 1.69 ± 1.28 |

Table 3. List of the full sample of GRBs used in this work and their redshift $z$ and measured $\mu_{GRB}$.

| GRB | $\mu_{GRB}$ | $z$ |
|-----|-------------|-----|
| 97028 | 0.695 | 43.76±0.77 |
| 970508 | 0.835 | 44.64±0.73 |
| 970218 | 0.958 | 43.21±0.71 |
| 971124 | 1.012 | 43.16±0.52 |
| 980613 | 1.096 | 46.06±1.11 |
| 980703 | 0.966 | 45.09±0.37 |
| 981226 | 1.11 | 44.03±1.13 |
| 990123 | 1.6 | 45.37±0.70 |
| 990506 | 1.3 | 47.34±0.57 |
| 990705 | 0.842 | 45.11±0.73 |
| 990712 | 0.434 | 48.15±0.44 |
| 990918 | 0.706 | 48.19±0.33 |
| 991216 | 1.02 | 43.36±0.52 |
| 991301 | 4.5 | 47.18±1.04 |
| 000210 | 0.846 | 44.82±0.34 |
| 000448 | 1.12 | 43.79±0.35 |
| 000911 | 1.06 | 47.56±0.78 |
| 001207 | 2.07 | 46.62±0.17 |
| 001516 | 2.03 | 48.52±0.54 |
| 002105 | 5.4 | 42.29±0.49 |
| 011112 | 0.36 | 44.03±0.70 |
| 011211 | 2.14 | 45.34±0.35 |
| 020104 | 0.706 | 45.13±0.52 |
| 020405 | 0.69 | 43.02±0.31 |
| 020813 | 2.15 | 43.73±0.67 |
| 030431 | 1.12 | 40.71±0.75 |
| 030200 | 0.25 | 39.31±0.18 |
| 030204 | 2.3 | 46.66±1.06 |
| 030326 | 0.45 | 45.23±0.55 |
| 030327 | 3.53 | 48.08±1.06 |
| 030352 | 1.52 | 43.38±0.43 |
| 030409 | 8.19 | 38.28±1.30 |
| 040204 | 2.65 | 46.09±0.50 |
| 040328 | 0.78 | 41.06±0.41 |
| 040318 | 1.51 | 42.91±2.16 |
| 040206 | 0.859 | 43.48±0.81 |
| 040106 | 0.716 | 44.19±0.57 |
| 040505 | 1.31 | 50.49±0.24 |
| 050318 | 1.443 | 44.42±0.52 |
| 050401 | 2.989 | 46.12±0.61 |
| 050416A | 0.653 | 41.8±0.54 |
| 050416B | 0.669 | 42.13±0.36 |
| 050503 | 2.821 | 47.76±0.37 |
| 050126 | 2.615 | 47.03±0.55 |
| 050129 | 0.994 | 44.09±0.84 |
| 050502C | 2.199 | 47.20±0.73 |
| 050102 | 0.809 | 43.30±0.82 |

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