Numerically Robust Methodology for Fitting Current-Voltage Characteristics of Solar Devices with the Single-Diode Equivalent-Circuit Model

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Abstract

For experimental and simulated solar cells and modules discrete current-voltage data sets are measured. To evaluate the quality of the device, this data needs to be fitted, which is often achieved within the single-diode equivalent-circuit model. This work offers a numerically robust methodology, which also works for noisy data sets due to its generally formulated initial guess. A Levenberg–Marquardt algorithm is used afterwards to finalize the fitting parameters. The source code and an executable version of the methodology are published under https://github.com/Pixel-95/SolarCell_DiodeModel_Fitting on GitHub. This work explains the underlying methodology and basic functionality of the program.

1 Introduction

Experimental and simulated solar devices yield a certain current under a given applied voltage and illumination. Due to the underlying semiconductor physics of p-n junctions, this relation is highly non-linear [1]. Most of these current-voltage (IV) curves can be described by a simple electronic model with a lumped series and shunt resistance, a diode, and an ideal current source. The numerical challenge is to find the five fitting parameters for all electric components within the single-diode equivalent-circuit model from a set of experimental data given at \( n \) discrete voltages \( V_1, \ldots, V_n \) and their corresponding current values \( I(V_1), \ldots, I(V_n) \).

There exist many algorithms in literature [2, 3, 4, 5] and even their errors are analysed [6]. However, the goal of this work is to find a numerically robust method that works also for IV data perturbed by noise effects, such as data outliers, sparse data sets, or higher voltages in modules instead of cells and hence a significantly higher diode ideality factor. The following section will present the methodology of fitting, while the one after introduces an executable program capable of fitting measured data sets.

2 Methods

In general, the current-voltage relation[1] of regular solar devices at the temperature \( T \) as seen in Figure 1 can be described by the implicit relation [2]

\[
I(V) = -I_{ph} + I_{sh}(V) + I_d(V)
\]

\[
= -I_{ph} + \frac{V - I(V) \cdot R_s}{R_{sh}} + I_0 \cdot \left( \exp \left( \frac{q_e \cdot (V - I(V) \cdot R_s)}{n_d k_B T} \right) - 1 \right).
\]

Here, \( I_{ph} \) is the generated photo current, \( R_s \) and \( R_{sh} \) the lumped resistances in series and shunt, \( I_0 \) the reverse saturation current, \( n_d \) the diode ideality factor, \( k_B \) the Boltzmann constant, and \( q_e \) the elementary charge.

By using the Lambert W function [7] it can be converted into an explicit equation. [8, 9, 10]

\[
I(V) = \frac{n_d k_B T}{q_e R_s} \cdot W(f_{Lam}) + \frac{V - R_{sh} \cdot (I_{ph} + I_0)}{R_s + R_{sh}},
\]

Fig. 1: Single-diode equivalent-circuit model. While the photo current represents the generated current, the voltage-dependent shunt and diode currents flow in the opposing direction reducing the produced power.

1In this work, all currents are treated as absolute currents \( I \) with the SI unit Amperes (A). However, the same methodology also works for current densities \( j \) with the SI unit Ampere per square meter \( \left( \frac{A}{m^2} \right) \).
In Table 1, all variables used in this work are explained. The fitting process is divided into two sections. The first one being the initial guess for the start values of the fitting parameters and the second one being the actual fitting algorithm itself. Since IV characteristics are highly non-linear, getting a sophisticated initial guess is crucial for the subsequent fitting algorithm and therefore is the most important task. As already described in the introduction, the goal of the outlined initial guess is not to be as precise as possible, but to be as robust and universal as possible.

### 2.1.1 Initial Guess

The initial guess for the fitting algorithm is obtained by the following procedure.

1. As a first step, a cubic Savitsky-Golay filter with a window size of 9 is applied to the experimental data in order to smooth data and not be sensitive to outlier data and noise. Moreover, the current values are eventually multiplied by \(-1\) in order to put the data in the appropriate quadrant.

2. The maximum power point (MPP) is roughly estimated as the discrete data point with the maximum power calculated via \(P_i = V_i \cdot I_i\).

3. The open circuit voltage \(V_{oc}\) is estimated as the linear interpolation of last data point with a negative current and the first data point with a positive current. If there are only data points with negative currents it is calculated via \(V_{oc} = 1.2 \cdot V_{MPP}\).

4. The diode ideality factor is estimated to be \(n_d = 2 \cdot \frac{V_{oc}}{V_T}\).

5. All data points with a voltage below 20% of \(V_{oc}\) are fitted with a linear regression. The inverse slope is considered as the shunting resistance \(R_{sh}\) and the y-intercept as the photo current \(I_{ph}\).

6. The 5 data points with the largest voltage are fitted linearly. The inverse slope of the regression is taken as the initial guess for the series resistance \(R_s\).

7. Finally, the reverse saturation current is calculated by the diode equation \(I_0 = \frac{I_{sh}}{\exp(q_e V_{oc}/(n_d k_B T)) - 1}\) at \(V = V_{oc}\) and hence \(I(V_{oc}) = 0\) via \(I_0 = \frac{I_{sh} - V_{oc}/R_{sh}}{\exp(q_e V_{oc}/(n_d k_B T)) - 1}\).

### 2.1.2 Convergent Fitting Method

Starting from the initial guess described in the past section, the partial derivation with respect to all 5 fitting parameters are derived analytically via the Lambert W function. Using them, a Levenberg–Marquardt algorithm is used in order to perform a regression to all data points. Since the initial guesses for the photo current and the shunt resistance are typically accurate within a few percent, they are not fitted with the other parameters. Hence, only \(I_0\), \(n_d\), and \(R_s\) are fitted in this subprocedure. At this point, the regression curve usually fits the data points very well. However, in a second procedure, all 5 parameters are fitted with the Levenberg–Marquardt algorithm to give the program the chance to adapt every parameter at the same time.

### Table 1: Table of all variables and symbols in this work

| variable meaning | variable meaning |
|------------------|------------------|
| $V$ voltage      | $I_i$ $i$-th current of the experimental data set to be fitted |
| $V_{oc}$ open circuit voltage | $I_{ph}$ generated photo current |
| $V_{MPP}$ voltage at the maximum power point | $I_d$ current flowing across the diode |
| $I$ current      | $I_{sh}$ shunt current |
| $I_i$ $i$-th current of the experimental data set to be fitted | $I_{ph}$ generated photo current |
| $I_0$ reverse saturation current | $I_{sh}$ shunt current |
| $I_{MPP}$ current at the maximum power point | $P$ power |
| $I_i$ $i$-th current of the experimental data set to be fitted | $P_i$ $i$-th power of the experimental data set to be fitted (calculated from $V_i$ and $I_i$) |
| $P_{MPP}$ power at the maximum power point | $P_s$ series resistance |
| $R_s$ series resistance | $R_{sh}$ shunt resistance |
| $n_d$ diode ideality factor | $q_e$ elementary charge |
| FF fill factor    | $k_B$ Boltzmann constant |
| $W(x)$ Lambert W function | $\mathcal{L}_i$ function value of $W(x)$ in the $i$-th iteration of approximation |
| $R^2$ coefficient of determination | $n$ amount of experimental data points |

where $W(x)$ is the Lambert W function and

\[
f_{\text{Lam}} = \frac{q_e I_0 R_{sh} R_s}{n_d k_B T (R_s + R_{sh})} \cdot \exp \left( \frac{q_e R_{sh} \left( R_s \left( I_{ph} + I_0 \right) + V \right)}{n_d k_B T (R_s + R_{sh})} \right). \tag{3}
\]
2.2 Calculate Characteristic Data

Once all fitted diode parameters $I_{ph}$, $I_0$, $n_d$, $R_s$, and $R_{sh}$ are determined, the solar parameters open circuit voltage $V_{oc}$, short circuit current $I_{sc}$, and fill factor $FF$ need to be calculated. This section describes their direct determination from the fitting parameters.

2.2.1 Open Circuit Voltage $V_{oc}$

At the open circuit point, the current $I(V)$ vanishes. Therefore, the open circuit voltage $V_{oc}$ can be determined by Equation (1) via

$$V_{oc} = R_{sh}(I_{ph} + I_0) - \frac{n_d k T}{q_e} \cdot W(f_{Lam}^{oc})$$

with

$$f_{Lam}^{oc} = \frac{q_e I_0 R_{sh}}{n_d k T} \cdot \exp \left( \frac{q_e R_{sh}(I_{ph} + I_0)}{n_d k T} \right).$$

If the exponent in Equation (4) is larger than the maximum exponent of double value ($\sim 308$ for IEEE double precision) the Lambert W function $W(f_{Lam})$ is calculated via the approximation for large numbers given in Appendix A.

2.2.2 Short Circuit Current $I_{sc}$

The short circuit point is defined as the state without an externally applied voltage and therefore shorted contacts. Hence, $V = 0$ is be plugged into Equation (2) and can be solved via the Lambert W function yielding

$$I_{sc} = \frac{n_d k T}{q_e R_s} \cdot W(f_{Lam}^{sc}) - \frac{R_{sh}(I_{ph} + I_0)}{R_{sh} + R_s}$$

with

$$f_{Lam}^{sc} = \frac{q_e I_0 R_{sh} R_s}{n_d k T (R_{sh} + R_s)} \cdot \exp \left( \frac{q_e R_{sh} R_s (I_{ph} + I_0)}{n_d k T (R_{sh} + R_s)} \right).$$

2.2.3 Maximum Power Point MPP

In order to obtain the power $P$ from a current-voltage characteristic, the current $I$ is multiplied by the voltage $V$. The maximum power point (MPP) is defined as the point in the IV curve with the highest produced power. To calculate the MPP, Equation (2) is multiplied by $V$ and afterwards the maximum is determined via

$$\frac{dP(V)}{dV} = \frac{d(I(V) \cdot V)}{dV} = 0$$

by using the Newton-Raphson method. This yields the MPP voltage $V_{MPP}$, resulting in the MPP current $I_{MPP} = I(V_{MPP})$ via Equation (2) and MPP power $P_{MPP} = P(V_{MPP})$.

$$P_{MPP} = \frac{V_{MPP} \cdot I_{MPP}}{V_{oc} I_{sc}}.$$
Figure 3: Screenshot of the main menu of the program. On the left side, experimental data is inserted. The middle section contains the fitted diode parameters, the resulting solar cell parameters, all buttons and the logging output box. On the right side, a plot with the experimental data (black points), the fitted curve (dark red) and the area covering the MPP power (light red) are shown.

The source code and executable file can be found under [https://github.com/Pixel-95/SolarCell_DiodeModel_Fitting](https://github.com/Pixel-95/SolarCell_DiodeModel_Fitting) within a GitHub repository.

4 Conclusion

This work introduces a methodology of fitting current-voltage characteristics of solar devices with the single-diode equivalent-circuit model. Due to the general calculation of the initial guess it is applicable even to perturbed data sets of cells and modules. Afterwards a Levenberg–Marquardt algorithm is applied in order to convergently determine the required fitting parameters.

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A Calculating the Lambert W function

As the Lambert W function $W(x)$ [7] is defined via the inverse function of the transcendental equation

$$W(x) \cdot e^{W(x)} = x,$$

(12)

it is necessary to calculate it numerically. An efficient method to do so is Halley’s method [15] in order to iteratively approximate its value. The rough approximation

$$L_0 = \frac{3}{4} \log(x + 1)$$

(13)

is used as an initial guess for the function value $L$ and by using the first and second derivatives, the iteration procedure

$$L_{i+1} = L_i - \frac{L_i \cdot e^{L_i} - L_i}{e^{L_i} \cdot (L_i + 1) - (L_i + 2) \cdot \frac{L_i e^{L_i} - L_i}{2L_i + 2}}$$

(14)

can be derived. Equation [14] is iteratively executed $\left\lceil \frac{1}{4} \log_{10}(x) \right\rceil$ times with a minimum of four iterations. This ensures a sufficiently precise accuracy for double precision with a 52 bit long mantissa. [14]

For too large values of $x$, the numbers in equation [14] become too large to be handled by a regular double precision. Therefore, a simple approximation for large input values is used [16]. By using the auxiliary variable

$$w = \log(x)$$

(15)
without any physical meaning, the function value can be approximated via
\[ W(x) = w - \log(w) + \frac{\log(w)}{w} + \mathcal{O}\left(\left(\frac{\log(w)}{w}\right)^2\right). \] (16)

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