Determination of stresses in thin-walled steel fiber reinforced concrete roofs in the form of hyperbolic paraboloid

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Abstract. The authors have determined the absolute values of deformations of steel fiber reinforced concrete (SFRC) in a thin-walled shell in the form of a hyperbolic paraboloid that on the basis of which the experimental stresses arising in the shell are determined. The article presents the results of theoretical studies of stresses in thin-walled shells of steel fiber reinforced concrete in the form of hyperbolic paraboloid (hypar), which were determined on the basis of the developed calculation method based on the equations of the moment theory of shallow shells. To solve the basic equations, the authors generalized the method of double series of Fourier, which takes into account the curvature of torsion of the hypars. The solution for the non-fixed hypars with rectangular shape was found through the double series of Fourier with additional application of the iteration method. The coefficients in the series are determined analytically using recurrence formulas. An approach to the study of the stress-strain state of rectangular hypars with a fixed contour is proposed.

1. Introduction
The development of construction science is inextricably linked to the study of modern building materials [1-2] and constructions made of them [3-6]. Application of innovative materials and energy-efficient structures increases the basic strength characteristics of buildings and structures, terms of their maintenance-free operation, reduces the economic cost of their maintenance.

2. Analysis of publications
It is worth to note that in recent years the construction of industrial and public buildings and special structures has tended to use new design solutions for covering elements - thin-walled shells. A special place among the coating shells is occupied by the shells of negative Gauss curvature in the form of hyperbolic paraboloid. Due to architectural expressiveness and possibility of erection of various combinations of structural systems from them, hypars are most often found in the form of hangar coverings, gyms, exhibition halls and other large-span structures (Figure 1).

3. The purpose of research
Shells in the form of hyperbolic paraboloids have found practical use in construction later than shells of other types, so their static and rigidity characteristics were insufficiently studied by scientists and researchers.

Modern tendencies of buildings and constructions improvements are directed on replacement of designs from traditional materials by composite materials for which a number of specific features are
characteristic - among which, first of all, the raised crack resistance and bearing ability. One of the solutions in this direction is the use of concrete in thin-walled structures with the addition of reinforcing elements in the form of short steel pieces. Combination of rigid steel fibers with matrix (concrete) allows to localize the danger connected with brittle destruction of a matrix and thus to realize the basic properties of fibers: the big potential durability on a stretching and the raised modulus of elasticity.

4. Research results
During the experimental studies of structures made of steel fiber reinforced concrete in the construction laboratory of Lutsk NTU the authors obtained positive results of its properties [7-9]. It is worth to note that many scientists are currently engaged in the study of mechanical characteristics of steel fiber reinforced concrete and structures based on it and got the desired results [10-14].

Figure 1. General view of the thin-walled shell in the form of a hyperbolic paraboloid (a) and its practical application in the design of the covering: b – car showroom in Vestal, USA, c – railway station in Ochota, Poland; d – railway station in Predeal, Romania; e – library of medicine in USA

The paper describes the study of thin-walled shells of the negative Gaussian curvature covering in the form of a hyperbolic paraboloid from steel fiber reinforced concrete with next dimensions: \(a \times b = 2250\text{mm} \times 3500\text{mm}\), thickness: \(t = 30\text{mm}\) and angle lift \(f_1 = f_2 = 500\text{mm}\) (Figure 2). Experimental thin-walled hypar shells were made of C16/20 concrete. Percentage of fiber content in the SFRC shell was \(\mu\).
= 1.5% (it was used ∅1mm steel wavy fiber with 50mm long).

**Figure 2.** The scheme of the SFRC thin-walled shell

The method of calculation without torsional curvature is the most studied and developed for shallow shells. The studies carried out for the shells (according to Figure 2) have shown that the torsional curvature significantly affects the stress in the shell. In particular, it was found that at a fixed contour the stress in the hyperbolic paraboloid was more than 2 times lower than in the plate (in which the torsional curvature is zero).

For theoretical determination of deflections and stresses in hypar-shaped coverings, the equation of shallow shells with non-zero torsional curvature was used, for the solution of which the Fourier double row method was improved. Taking into account that the shell is rectangular in plan and its median surface occupies the region of 0 < x < a, 0 < y < b, the equation of shallow shells with non-zero torsional curvature is written in complex view:

$$
\left( \Delta^2 + i\frac{\mu}{h} \partial_\mu \right) F - \varepsilon \frac{\partial^2}{\partial x \partial y} F = q,
$$

$$
\mu = \sqrt{12(1-\nu^2)} , \ h - \text{ thickness, } q - \text{ shear load, } \Delta - \text{ Laplace operator, } \partial_\mu = k_2 \frac{\partial^2}{\partial x^2} + k_1 \frac{\partial^2}{\partial y^2},
$$

$$
\varepsilon = i2k_{12} \frac{\mu}{h} , k_1, k_2 - \text{ curvature in the direction of axes } O_x, O_y, k_{12} - \text{ torsional curvature},
$$

$$
V - \text{ Poisson's ratio, } i = \sqrt{-1} - \text{ imaginary unit.}
$$

Normal deflections W and the voltage function \( \varphi \) are determined by the entered complex function \( F \) by formulas: \( w = \frac{1}{D} \text{Re}(F) \), \( \varphi = \frac{\mu}{h} \text{Im}(F) \), \( D = \frac{Eh^2}{12(1-\nu^2)} \).

Two types of shell contour anchoring that are most often found in practice - hinged and fixed anchoring are considered in this article.

a) hinged anchoring (Figure 3, a). With this anchoring, it is assumed that the shell contour is additionally strengthened by rigid thin-walled elements.

b) fixed anchoring of the shell at rigid contour (Figure 3, b). With this anchoring, the shell contour does not shift in normal directions to the edges of the shell.

The cases, important for practice, when the shell contours are plane, are considered. The equations of the median surface of such hypar are written in the form:

$$
z = \frac{z_0}{ab} \sqrt{x}
$$
Figure 3. Shell anchoring scheme:

a – free anchoring, b – fixed anchoring

The three peaks of the hypar are in the plane \( z = 0 \), and the fourth one has a coordinate \((a, b, z_0)\). The curvatures in this shell will be: \( k_1 = 0, \ k_2 = 0, \ k_{12} = \frac{z_0}{ab} \).

To the equation (1) for a free anchored shell with a nonzero curvature of torsion \( k_{12} \) cannot be applied the method of Fourier rows that is known in the literature. Therefore, the solution of this equation is preliminarily depicted as a decomposition of the Taylor series by parameter \( \varepsilon \):

\[
F = F^{(0)} + \varepsilon F^{(1)} + \varepsilon^2 F^{(2)} + \ldots + \varepsilon^k F^{(k)}
\]

By substituting it into equation (1), we obtain the recurrence equations:

\[
\left( \Delta^2 + \frac{k_{12}}{h} \frac{\partial^2}{\partial k^2} \right) F^{(0)} = q, \quad \left( \Delta^2 + \frac{k_{12}}{h} \frac{\partial^2}{\partial k^2} \right) F^{(j)} = q^{(j)}, \quad q^{(j)} = \frac{\partial^2}{\partial x \partial y} F^{(j-1)}, \quad j = 1, 2, \ldots, k + 1, \quad (3)
\]

The functions entered are shown as double rows:

\[
F^{(k)} = \sum_{n,m=0}^\infty F_{nm}^{(k)} \sin(\alpha_n x) \sin(\beta_n y), \quad k = 0, 1, \ldots, \infty, \quad \alpha_n = \frac{\pi n}{a}, \quad \beta_n = \frac{\pi m}{b}.
\]

In this case, the boundary conditions of free anchoring are automatically met. After the transformations, we get recurrent formulas that allow us to calculate coefficients sequentially at \( k = 1, 2, \ldots, \infty \):

\[
F_{ij}^{(k)} = \frac{16}{ab} \sum_{n,m} \frac{nm}{(i^2 - n^2)(j^2 - m^2)} \varepsilon_{i+n} \varepsilon_{j+m} F_{nm}^{(k+1)}, \quad i, j = 1, 2, \ldots, \infty; \quad (4)
\]

\( \varepsilon_n = 1 \) at \( n \) – unpaired and \( \varepsilon_n = 0 \) at \( n \) – paired.

Coefficients \( F_{ij}^{(k)} \) in the formula (4) at \( k = 0 \), at which the calculations begin, when the load is evenly distributed, when \( q = Q = \text{const} \), are determined by the formula (5):
\[ F_{nm}^{(0)} = \frac{16Qe_n e_m}{\pi^2 n m A_{nm}}, \quad A_{nm} = \left( \alpha_n^2 + \beta_m^2 \right)^2 - i \frac{\mu}{h} \left( k_n \alpha_n^2 + k_m \beta_m^2 \right) \] (5)

After finding the torque and force coefficients, the formulas are displayed:

\[ M_x = \text{Re} \left( \sum_{n,m} \left( \alpha_n^2 + \nu \beta_m^2 \right) F_{nm} \sin \alpha_n x \sin \beta_m y \right), \quad M_y = \text{Re} \left( \sum_{n,m} \left( \nu \alpha_n^2 + \beta_m^2 \right) F_{nm} \sin \alpha_n x \sin \beta_m y \right) \]
\[ N_x = -\frac{\mu}{h} \text{Im} \sum_{n,m} \beta_m^2 F_{nm} \sin \alpha_n x \sin \beta_m y, \quad N_y = -\frac{\mu}{h} \text{Im} \sum_{n,m} \alpha_n^2 F_{nm} \sin \alpha_n x \sin \beta_m y, \] (6)
\[ F_{nm} = F_{nm}^{(0)} + \varepsilon F_{nm}^{(1)} + \varepsilon^2 F_{nm}^{(2)} + \ldots + \varepsilon^k F_{nm}^{(k)}. \] (7)

On the basis of the obtained formulas (4) - (7) the computer programs "Calculation of double curvature shells" (on the basis of the software complex "Matlab") [15-16] have been developed, which allow to determine the stresses and deflections in the shells made of steel fiber reinforced concrete in the form of a hyperbolic paraboloid at their free and fixed anchoring.

Calculations of deflections and stresses were conducted for SFRC, for which it was used \( E = 2 \times 10^4 \) MPa, \( \nu = 0.17, a = 2.25 \) m, \( b = 3.5 \) m. Taking into account its own weight and snow load, it is obtained \( Q = (1250 + 22000h) \).

Calculated with free anchoring of the shell contour (Figure 3, a) tensions \( \sigma_x, \sigma_y \) (values (+) relate to the upper surface of the shell, and values (−) – relate to the bottom one) in the shell cross section \( x = a / 2, x = 0.7a \) depends on \( y \) coordinate at \( h = 0.03m \) are shown in Figures 4, a and 4, b.

The stresses shown in these figures were quite large than allowable (experiments have shown that cracks do not occur in SFRC if the stresses do not exceed 2 MPa). Therefore, in order to reduce stress it is advisable to increase the thickness of the shell. The stresses at \( a = 2.25 \) m, \( b = 3.5 \) m and increased thickness \( h = 0.05 \) m are calculated in Figure 4, c and 4, d. It shows that the tension has decreased by about half and does not exceed the allowable tension.
Figure 4. Tensions in the shell at \( h = 0.03 \) m (a, b) and \( h = 0.05 \) m (c, d) at free anchoring.

At fixed anchoring of the hypar to a rigid contour (according to Figure 3, b) key functions after transformations are written in the form:

\[
\begin{align*}
    w &= \frac{4}{ab} \sum_{n,m=1}^{\infty} w_{nm}^x \sin(\alpha_n x) \sin(\beta_m y), \\
    \varphi &= \frac{4}{ab} \sum_{n,m=1}^{\infty} \varphi_{nm}^x \cos(\alpha_n x) \cos(\beta_m y), \\
    w_{nm}^\prime &= \frac{q_{nm}^x}{Dc_{nm}}, \\
    \varphi_{nm}^\prime &= 2k_{12} \frac{Eh}{D} \alpha_n \beta_n w_{nm}^\prime, \\
    q_{nm}^\prime &= \frac{16}{\pi^2 nm} Q, \text{ at } n, m - \text{unpaired}, \\
    \gamma_{nm} &= \alpha_n^2 + \beta_n^2, c_{nm} = \gamma_{nm}^2 + 4k_{12} \frac{Eh \alpha_n^2 \beta_n^2}{D} \\
    x &= \frac{a}{2}, x = 0.7a, x = 0.5a, x = 0.6a, x = 1.8a.
\end{align*}
\]
Figure 5. Tensions in the shell in cross sections $x=0.5a$ (a); $x=0.6a$ (b); $x=0.7a$ (c); $x=0.8a$ (d) at fixed anchoring

The calculated stresses at fixed anchoring in different cross sections along the shell $x = 0.5a; 0.6a; 0.7a; 0.8a$ depending on the coordinate are shown in Figure 5 (a, b, c, d).

According to the shell scheme shown in Figure 2, SFRC hypars were investigated (Fig. 6).

Figure 6. General view of the testing of SFRC shell in the shape of hypar at laboratory condition

5. Conclusions

For experimental determination of stresses in a SFRC hypars, it was determined deformations in a shell by using of strain gauge sensors (base 50 mm, resistance $R = 406.3$ Ohm) and measuring complex VNP-8. For this purpose, 50 sensors were fixed along the perimeter and along the diagonals of the shell (Fig. 6). Deformation values were detected by the fixed sensors and transformed into stress by using an experimentally determined stress-strain diagram ($\sigma$-$\varepsilon$) for SFRC.

The similarity of the experimentally obtained results with the data obtained using the developed computer programs that take into account the curvature of the torsion of the hypar under the action of operational loads is 92 - 94%.

On the basis of the conducted calculations it is possible to draw conclusions that at fixed anchoring of a shell contour in the form of a hyperbolic paraboloid there is a decrease in value of arising pressure more than twice in comparison with free anchoring. This can be explained by the fact that a significant part of
the load is absorbed (transferred to) the anchoring. At free anchoring the maximum stresses arise in the central part of the SFRC shell, whereas at fixed anchoring of the same shell - near the contour.

For this shell, the maximum stresses are considerably smaller than the allowable stresses for SFRC concrete. That is, using a rigid fixed anchoring at which ensures absence of displacement in a direction normal to a shell contour, allows to raise reliability of hypar roofs.

References
[1] Dvorkin L, Dvorkin O, Zhitkovsky V, Ribakov Y 2011 A method for optimal design of steel fiber reinforced concrete composition Materials & Design, 32 3254-3262
[2] Dvorkin L, Zhitkovsky V, Stepasyuk Y, Ribakov Y 2018 A method for design of high strength concrete composition considering curing temperature and duration Construction and Building Materials 186 731-739
[3] Krantovska O, Petrov M, Ksonshkevych L, Synii S, Sunak P 2018 Improved engineering method for calculating the strength of the supporting areas of reinforced concrete elements MATEC Web of Conferences, 230, 02014 1-9
[4] Ksonshkevych L, Krantovska O, Petrov M, Synii S, Uhl A 2018 Investigation of the structure of cement stone, obtaining and optimization of high-strength concrete on mechanically activated binder MATEC Web of Conferences, 230, 03010 1-8
[5] Kochkarev D, Azizov T, Galinska T 2018 Bending deflection reinforced concrete elements determination MATEC Web of Conferences, 230, 02012.
[6] Pavlikov A, Kochkarev D, Harkava O 2019 Calculation of reinforced concrete members strength by new concept. (Proceedings of the fib Symposium 2019: Concrete - Innovations in Materials, Design and Structures. Krakow, Poland)
[7] Andriichuk O, Babich Y 2017 Strength of elements with annular cross sections made of steel-fiber-reinforced concrete under one-time loads Materials Science, 52 (4), New York 509-513
[8] Andriichuk O, Babich V, Yasyuk I, Uzhehov S 2017 The influence of repeated loading on work of the steel fiber concrete drainage trays and pipes on the roads MATEC Web of Conferences, 116, 02001 1-9
[9] Andriichuk O, Babich V, Yasyuk I, Uzhehov S 2018 The impact of the reinforcement percentage on the stress-strain state of the bending steel fiber reinforced concrete elements MATEC Web of Conferences, 230, 02001 1-5
[10] Kinash R, Bilozir V 2015 Deformational calculation method of bearing capability of fiber-concrete steel bending elements (Czasopismo Techniczne)
[11] Shmyh R, Bilozir V, Vysochenko A, Bilozir V 2018 Carrying capacity of bending concrete elements reinforced by fibro and stripes taken from used PET bottles (International Scientific and Practical Conference World science)
[12] Oliveira M A B, Ramos E M L S, Oliveira D R C, Neto B B P 2018 Analysis of influence of concrete element format and properties steel fibers on flexural toughness (Matéria (Rio J.) 23 no.3 Rio de Janeiro, SciELO Brasil)
[13] Sameh Y 2018 Effect of steel fibers and gfrp sheet on the behavior of lightweight concrete specimens using waste lightweight sand bricks (International Journal of Engineering Research & Technology, IJERT (ISSN: 2278-0181), 7 03 69-75
[14] Islam M M, Dhar A, Patowary F, Asif J H, Rahman S, Das S S, Das S, Chowdhury M A, Siddique A 2015 Experimental investigation and finite element analysis on P-M interaction diagram of RC square columns made of steel fiber reinforced concrete (SFRC) (IABSE-JSCE Joint Conference on Advances in Bridge Engineering-III, Dhaka, Bangladesh) 192-200
[15] Pasichnyk R, Uzhehov S, Andriichuk O, Uzhehova O, Bozhynianik V 2015 Certificate of copyright registration № 59810 "Kompiuterna prohrama "Rozrakhunok obolonok"
[16] Babich Y, Andriichuk O, Uzhehov S 2018 Certificate of copyright registration № 59810 "Kompiuterna prohrama "Rozrakhunok obolonok dvoiakoi kryvyyny"