Emission of Fermions from BTZ Black Holes

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Abstract

The emission rate of fermions from 2+1 dimensional BTZ black holes is shown to have a form which can be reproduced from a conformal field theory at finite temperature. The rate obtained for fermions is identical to the rate of non-minimally coupled fermions emitted from a five dimensional black hole, whose near horizon geometry is $BTZ \times M$, where $M$ is a compact manifold.

04.60.-m, 04.62.+v, 04.70.-m, 04.70.dy, 11.25.-w
I. INTRODUCTION

Understanding Hawking radiation from black holes using a unitary microscopic theory has been an outstanding and difficult problem to solve. Nevertheless, the progress made in the last few years suggests that resolution of the problem may be near. In a series of papers, it has been shown that the Hawking emission rates of a number of four and five dimensional black holes is reproducible from a 1+1 conformal field theory at finite temperature [1–5]. In case of the five dimensional black hole which is a solution of Type II B supergravity obtained by wrapping D5-branes and D1-branes on $T^4 \times S^1$, it is known that the entire configuration can be replaced by a long effective string along $S^1$ which is described by a conformal field theory at finite temperature. However, for other black holes like the rotating ones, the origin of the effective string is not well understood. Recently, new light has been shed regarding the origin of the underlying conformal field theory. It has been shown that the above black holes can be mapped to the asymptotically anti-de Sitter 2+1 dimensional BTZ black hole [1] times a compact manifold [3–5]. In particular, [3–4] the near-horizon geometries of these black holes have the form $\text{BTZ} \times M$, (or $\text{AdS}_3 \times M$ with global identifications in case of rotating black holes [10]) where $M$ is a compact manifold. The underlying microscopic theory of the BTZ black hole is known to be a 1+1 dimensional conformal field theory [11–13]. Hence the conjecture that near-horizon geometry explains thermodynamics of black holes, and the conformal field theory used to describe the $\text{BTZ} \times M$ geometry gives the microscopic theories of higher dimensional black holes. The conjecture is supported by the evidences that the near horizon BTZ black hole obtained has the same entropy as the higher dimensional black hole, and possesses the same decay rate for scalars as higher dimensional ones [4].

The 2+1 dimensional BTZ black hole is an interesting system to study by itself. The black hole is asymptotically $\text{AdS}_3$, and has two horizons with an ergosphere. For non-extreme BTZ black hole, there is a non-zero Hawking temperature, and an observer outside the outer horizon will detect thermal radiation. However, since the metric is asymptotically anti-de Sitter, the local temperature measured by any time-like observer decreases with distance at spatial infinity. The situation is different from flat space-time where temperature is constant at infinity. Moreover in flat space-time, the asymptotic observer measures a decay rate which is modified by the absorption coefficient or the greybody factor of the black hole. The greybody factor is defined as the ratio of the total number of particles entering the horizon and the incoming flux at spatial infinity. For asymptotically anti-de Sitter black holes, this is a little difficult to study, as spatial infinity for these black holes constitute a time-like surface through which information can enter and leave. The usual way to deal with this is to impose boundary conditions on fields such that the surface acts like a reflecting wall. In that case, the black hole is in equilibrium with thermal radiation and there is no
net flow of flux across any time like surfaces and the concept of greybody factor remains obscure. Nevertheless, an attempt can be made to define the Hawking rate in the same way as asymptotically flat space-times. Using this in $[14,15]$, it was shown that the decay rate for scalars has the same form as the higher dimensional black holes, and can be reproduced from a conformal field theory at finite temperature.

In this paper we address the issue of fermion emission from BTZ black holes. This is interesting to investigate by itself, as it is yet to be checked that the above stated result for scalars applies universally to all particle emissions from the BTZ black hole. Not only that, the result constitutes important evidence for the conjecture that near horizon geometry of higher dimensional black holes encode information about it’s thermodynamics. To define the Hawking emission rate for the BTZ black hole, we do not take an asymptotic observer, but an observer stationed at a radial distance $\rho \sim l \gg \rho_+$, where $l$ is related to the cosmological constant. A motivation for this is that at this position, the observer in the BTZ geometry measures a local temperature equal to the Hawking temperature of the black hole. This issue is discussed in section III. Also, for purposes of the greybody calculation, we assume that there is a flow of flux into the black hole. This is perhaps a relevant physical situation to consider if we want to answer questions about higher dimensional black holes as flux can flow into “near horizon geometry” (BTZ). Using the above assumptions, we show that the fermions in the BTZ geometry have exactly the same form of the Hawking emission rate as the non-minimally coupled fermions in five-dimensional black hole $[5]$. Indeed this strengthens the conjecture. The form obtained can be reproduced from a conformal field theory at finite temperature. Since a satisfactory derivation of higher dimensional fermion decay rates does not exist from the effective string picture $[4,5]$, this result is very useful. A microscopic derivation of the rate found here by using the conformal field theory describing the BTZ black hole will finally clear the issue. Moreover, three dimensional fermions are much easier to handle than the higher dimensional ones, and this calculation can be used to predict decay rates for higher dimensional fermions. In particular, the 4 dimensional black hole of M-theory also has BTZ as it’s near horizon geometry and, it is interesting to predict the nature of the fermions of 11-D supergravity compactified to 4-dimensions, which have the same decay rate as the fermions considered in three dimensions.

In the next section, we solve the equation of motion of the fermion in the BTZ background. We find that the equation of motion is exactly solvable, and yields a hypergeometric equation. We can choose the ingoing solution at the horizon, and determine the flux which flows down the hole. It should be mentioned that taking minimally coupled fermions instead of those considered here, does not give a meaningful result. In the following section, we derive the greybody factor. We determine the solutions in the region $\rho \sim l$ and deter-
mine the flux which flows towards the black hole horizon. Using this, we can calculate the greybody factor for the black hole, and hence the decay rate. In the third section, we give a comparison of the rate obtained here, and that obtained for higher dimensional black holes. In the last section, we include a brief discussion.

II. EQUATION OF MOTION OF THE FERMION IN BTZ BACKGROUND

Einstein’s gravity in 2+1 dimensions is essentially topological, and does not admit black hole solutions. However, gravity with a negative cosmological constant, has non-trivial solutions. The action for this is:

\[ S = \frac{1}{2\pi} \int d^3x \left( R + 2l^{-2} \right) \]  

Where \( R \) is the scalar curvature and \( \Lambda = -(1/l^2) \) is the cosmological constant; \( G = 1/8 \) according to the conventions of [3]. The space with constant negative curvature, is called anti-de Sitter space. This space is invariant under SO(2,2) group, which is larger than the usual Poincare group of flat space time. The appropriate covariant derivative for spin- half fields is:

\[ D = \gamma^\nu (\partial_\nu + \omega_\nu + g e_{\nu a} \gamma^a) \]  

Where, \( \omega_\nu \) is the spin connection, \( e_{\nu a} \) is the triad and \( g = 1/2l \) is related to the cosmological constant. The BTZ metric is derived by appropriate identifications of anti-de Sitter space time, and it’s asymptotic properties are the same as anti-de Sitter space [3]. Hence the above covariant derivative is relevant for our purposes. To write down the fermion equation in the BTZ background, we study the metric first. The BTZ metric in coordinates \( \rho, t \) and \( \phi \) is \((0 < \rho < \infty, 0 < \phi < 2\pi)\):

\[ ds^2 = -\frac{\Delta^2}{l^2 \rho^2} dt^2 + \frac{l^2 \rho^2}{\Delta^2} dr^2 + \rho^2 \left( d\phi - \frac{\rho + \rho -}{l \rho^2} dt \right)^2, \]  

\[ \Delta^2 = (\rho^2 - \rho_+^2) (\rho^2 - \rho_-^2). \]  

Clearly, the metric represents a rotating black hole, with two horizons at \( \rho_+ \) and \( \rho_- \). The angular momentum of the black hole is \( J = 2\rho_+ \rho_- / l \), and it’s mass is \( M = (\rho_+^2 + \rho_-^2) / l^2 \). The metric can be written in more convenient coordinates, with the radial coordinate \( \rho \) defined in terms of hyperbolic coordinate \( \mu \). The redefinition is:

\[ \rho^2 = \rho_+^2 \cosh^2 \mu - \rho_-^2 \sinh^2 \mu. \]
The spatial infinity corresponds to \( \tanh \mu \to 1 \). In this coordinate, the metric takes a form:

\[
ds^2 = -\sinh^2 \mu \left( \rho_+ \frac{dt}{l} - \rho_- d\phi \right)^2 + l^2 d\mu^2 + \cosh^2 \mu \left( -\rho_+ \frac{dt}{l} + \rho_+ d\phi \right)^2
\]

A convenient set of linear combinations of \( t \) and \( \phi \) gives us \( x^+ = \rho_+ t/l - \rho_- \phi \) and \( x^- = -\rho_- t/l + \rho_+ \phi \), and the killing directions of the metric are \( \partial_{x^+} \) and \( \partial_{x^-} \). The triads are chosen in an appropriate local Lorentz frame in the tangent plane.

\[
e^0_{x^+} = \sinh \mu \quad e^2_{x^-} = \cosh \mu \quad e^1_\mu = l.
\]

The non zero spin connections for this are:

\[
\omega_{x^+} = -\frac{1}{2l} \cosh \mu \sigma^{01} \quad \omega_{x^-} = \frac{1}{2l} \sinh \mu \sigma^{21},
\]

here \( \sigma^{ab} = 1/2[\gamma^a, \gamma^b] \). Using the above, we substitute them in (2) and obtain the fermion equation on the BTZ space-time as

\[
\gamma \frac{1}{l} \left( \partial_\mu + \frac{\sinh \mu}{2 \cosh \mu} + \frac{\cosh \mu}{2 \sinh \mu} \right) \psi + \gamma^0 \frac{\partial_{x^+} \psi}{\sinh \mu} + \gamma^2 \frac{\partial_{x^-} \psi}{\cosh \mu} + \frac{1}{2l} \psi = 0
\]

We take the representation of gamma matrices to be: \( \gamma^0 = i \sigma^2, \gamma^1 = \sigma^1, \gamma^3 = \sigma^3 \). The killing isometry requires that \( \partial_{x^\pm} \psi = -ik^\pm \psi \), where \( k^\pm \) are constants depending on the energy and azimuthal eigenvalues \( \omega \) and \( m \) respectively. In fact they can be determined, and \( k^+ = (\omega - m\Omega)/(2\pi l T_H) \) and \( k^- = (\rho_+ - T_H)/(2\pi l \rho_+ T_H) \). The Hawking temperature is \( T_H = (\rho_+^2 - \rho_-^2)/(2\pi l^2 \rho_+) \) and \( \Omega = J/(2\rho_+^2) \). We take the following form for the wavefunction:

\[
\psi = \frac{e^{-i(k^+ x^+ + k^- x^-)}}{\sqrt{\sinh \mu \cosh \mu}} \left( \begin{array}{c} \psi_1 \\ \psi_2 \end{array} \right)
\]

The radial equations for the two components are determined as

\[
\left( d_\mu - \frac{ik^+}{\sinh \mu} \right) \psi_2 = - \left( \frac{1}{2} - \frac{ik^-}{\cosh \mu} \right) \psi_1
\]

\[
\left( d_\mu + \frac{ik^+}{\sinh \mu} \right) \psi_1 = - \left( \frac{1}{2} + \frac{ik^-}{\cosh \mu} \right) \psi_2
\]

Interestingly, to separate the wave functions, we have to go to a different basis of wave functions. Let us call them \( \psi_1' \) and \( \psi_2' \), defined as,

\[
\psi_1 + \psi_2 = \left( 1 - \tanh^2 \mu \right)^{-1/4} \sqrt{1 + \tanh \mu} \left( \psi_1' + \psi_2' \right)
\]

\[
\psi_1 - \psi_2 = \left( 1 - \tanh^2 \mu \right)^{-1/4} \sqrt{1 - \tanh \mu} \left( \psi_1' - \psi_2' \right)
\]
We obtain the equations in coordinates $y = \tanh \mu$ for the $\psi'$ below as:

\begin{align}
(1 - y^2) d_y \psi'_2 - ul \left( \frac{k^+}{y} + k^- y \right) \psi'_2 &= - \left\{ 1 - ul(k^+ + k^-) \right\} \psi'_1 \\
(1 - y^2) d_y \psi'_1 + ul \left( \frac{k^+}{y} + k^- y \right) \psi'_1 &= - \left\{ 1 + ul(k^+ + k^-) \right\} \psi'_2
\end{align}

The equations are now very easily separable. The second order differential equation obtained from the above two equations can be cast in a simple form in the variable $y^2$ which we denote as $z$ for convenience.

\begin{align}
z(1-z) d^2 \psi'_1 + \frac{1}{2} (1-3z) dz \psi'_1 + \frac{1}{4} \left( \frac{-ulk^+ + l^2k^+^2}{z} + ulk^- + l^2k^-^2 - \frac{1}{1-z} \right) \psi'_1 &= 0
\end{align}

The solution to this equation is determined to be $\psi'_1 = z^m (1-z)^n F(\alpha, \beta; \gamma; z)$, where $F$ is a hypergeometric function. For the ingoing function, the constants are as follows: $m = 1/2 + ulk^+/2, n = -1/2$, and the hypergeometric parameters are: $\alpha = ul(k^+ + k^-)/2 + 1/2, \beta = ul(k^+ - k^-)/2$ and $\gamma = ulk^+ + 3/2$. The ingoing function is so chosen that at the horizon, the wave function has the dependence $\psi \sim exp(\frac{\omega}{4\pi T_H} \log z)$. The solution for the other component of the wave function can be determined easily now. It is:

\begin{align}
\psi_2 = & z^{ulk^+/(1-z)} \left( (-\gamma - 1)/\alpha \right) F(\alpha - 1, \beta; \gamma - 1; z),
\end{align}

where $\alpha, \beta, \gamma$ are constants as defined above. The flux for this function as shown in the next section is negative, indicating a flow into the black hole.

Thus we see that the fermion equation of motion in the BTZ background is exactly solvable. It is interesting to note that $n = 0$ corresponds to a minimally coupled fermion, and in that case $\gamma = \alpha + \beta$. The hypergeometric solution is not well behaved, and does not converge as $z \to 1$. There can be other kinds of couplings to the BTZ metric, and they will be interesting to investigate. The BTZ black hole is locally anti-de Sitter space, with global identifications. It will be interesting to see whether the solutions obtained here can be related to those obtained for $AdS_3$ in [16], modulo the global identifications.

III. GREY BODY FACTOR

The black hole grey body factor is also the absorption coefficient of the black hole. The geometry of the black hole provides a kind of potential barrier for the fields propagating on it. Only a fraction of the incoming flux at infinity is absorbed by the body, and rest is reflected back. In order to determine the total Hawking radiation rate of the observer, sitting far away from the black hole, we need to calculate this absorption rate. Indeed, as in ordinary quantum mechanics, the black hole absorption rate, which we denote by $\sigma_{abs}$ is related to the ratio of the ingoing flux at horizon and incoming flux at infinity. The
fermion flux into the horizon will be determined by the current which enters the horizon. Usually, to probe the black hole geometry, an incoming plane wave is taken at infinity. However, here, for our purposes, we take the incoming flux in the region $\rho \sim l \gg \rho_+$ as the incident flux on the black hole. This would correspond to an BTZ observer, sitting at finite $\rho$, detecting radiation. Though, the physics of this picture is not very clear, there are a number of reasons for choosing this. In curved space-time, an observer measures a thermal spectrum depending upon his local temperature, which is $T_H/\sqrt{g_{00}}$. In asymptotically flat space-time, $\sqrt{g_{00}} \to 1$ as $\rho \to \infty$. However, this is not the case in asymptotically anti-de Sitter space-time where $\sqrt{g_{00}} \sim \rho$ at spatial infinity. For small mass BTZ black holes, i.e. $\rho_+ \ll l$, it is easily seen that, $\sqrt{g_{00}} \to 1$ when $\rho \sim l$. This motivates the choice of the observer. Moreover, to compare our final answer with higher dimensional black hole rates, going infinitely away from the horizon would imply a modification of the near horizon BTZ geometry, and we are not interested in probing that region. As $\rho \sim l$, the black hole metric is same as asymptotically anti-de Sitter space. Solutions determined in this metric is also the same as that obtained in the vacuum solution of the black hole [17]. The metric is ($\rho \gg \rho_+$):

$$ds^2 = -\frac{\rho^2}{l^2} dt^2 + \frac{l^2}{\rho^2} d\rho^2 + \rho^2 d\phi^2$$

(17)

To, determine the wave functions we then solve the radial equations:

$$\left(\rho \frac{\partial}{\partial \rho} \pm \frac{\omega l^2}{\rho}\right) \psi_f^{i(1)} = -\left(\frac{1}{2} \pm \frac{im}{\rho}\right) \psi_f^{i(2)}$$

(18)

To separate this set, we go to a frame in which, $\psi_f^{i1} = \psi_f^1 + \psi_f^2$ and $\psi_f^{i2} = \psi_f^1 - \psi_f^2$. The equation can be exactly solved in this frame. The solutions are determined, in terms of Bessel functions,

$$\psi_f^{i1} = \sqrt{x} (A_1 J_0(\Lambda x) + i A_2 N_0(\Lambda x))$$

(19)

$$\psi_f^{i2} = \frac{i \sqrt{x}}{E} (A_1 J_1(\Lambda x) + i A_2 N_1(\Lambda x))$$

(20)

Where $J_n$ and $N_n$ are bessel functions of the first and second kind. $A_1$ and $A_2$ are arbitrary constants of integration. Also $x = 1/\rho, \Lambda = l\sqrt{\omega^2 - m^2}, E = l(\omega l + m)/\Lambda$. Note that the same solutions will survive when $\rho \to \infty$ in anti-de Sitter space. The interesting aspect about anti-de Sitter space is that $\rho \to \infty$ is a time like surface. Hence, it is necessary to specify boundary conditions, which are either Dirichlet or Neuman on the surface. These boundary conditions, also called reflective boundary conditions [18] can be realised in the set of functions defined in (20). On choosing $A_2 = 0$, this condition can be ensured for the above wavefunctions. It is easy to check that in that case $\sqrt{\rho} \psi = 0$ for $\rho \to \infty$. However, here we are not interested in making the wall totally impervious. Instead, we are forced to take $A_2$
have non-zero values if we want the wavefunction to match the wavefunction determined in (16) continued to $z \rightarrow 1$. This shows that, our choice of a net inflow of flux into the black hole ensures that we donot have reflecting boundary conditions at infinity. For asymptotic anti- de Sitter space, this might be related to the transparent boundary conditions defined in [18]. Before we can determine the greybody factor using this far solution, we need to match this with the ingoing wavefunction at the horizon since we want the wavefunction to be continuous in space-time. To do that, we continue the solution of (16) to $z \rightarrow 1$ [19].

Now, with the scalings and redefinitions, the radial wavefunction is:

$$
\psi_1 \rightarrow \frac{E_1 \sqrt{N(\rho_+ + \rho_-)}}{(2\rho)^{3/2}} \left\{ - (1 - 2\Psi(\alpha)\beta - 2\Psi(\beta + 1)\beta) + 4\beta \left( \log \left( \frac{\sqrt{N}}{\rho} \right) + C \right) \right\} \quad (21)
$$

$$
\psi_2 \rightarrow \frac{\sqrt{\rho_+ - \rho_-}}{\sqrt{2\rho}} E_1 \quad (22)
$$

Where,

$$
E_1 = \left( \frac{\Gamma(\gamma)\Gamma(\gamma - \alpha - \beta)}{\Gamma(\gamma - \alpha)\Gamma(\gamma - \beta)} \right), \quad (23)
$$

$\Psi$ are the digamma function (subsequently, we write $2\beta(\Psi(\alpha) + \Psi(\beta + 1)) \equiv \Psi(\alpha, \beta)$). Also $N = \rho_2^2 - \rho_1^2$, and $\Psi(1) = -C$ (euler’s constant). The factors $\sqrt{(\rho_+ \pm \rho_-)/N}$ enter as this wavefunction given in terms of $t, \rho, \phi$ coordinates is lorentz rotated from the wavefunction obtained in $x^+, z, x^-$ coordinates. Note in the above, we have taken, $\sqrt{N}/\rho \ll 1$ or in other words, $\sqrt{N} \ll l$. The wavefunctions obtained in (20), can be cast in a similar form when $\Lambda/\rho \ll 1$. (For $m = 0$, this indicates that $\omega l \ll 1$)

$$
\psi_f^1 \approx \frac{1}{\rho^{3/2}} \left( A_1 + \frac{2\pi A_2}{\pi} \left( \log \left( \frac{\Lambda}{\rho} \right) + C \right) \right) \quad (24)
$$

$$
\psi_f^2 \approx \frac{2A_2}{\pi E\Lambda \rho^{1/2}}. \quad (25)
$$

The asymptotic constants are determined from the above equations,

$$
A_1 = -\frac{\sqrt{N(\rho_+ + \rho_-)}}{2\sqrt{2}} E_1 (1 - \Psi(\alpha, \beta)) \quad A_2 = \frac{\pi E\Lambda}{2\sqrt{2}} \sqrt{\rho_+ - \rho_-} E_1 \quad (26)
$$

The fermionic flux is given by:

$$
\mathcal{F} = \sqrt{-g} J^\rho = \rho \bar{\psi} e^\rho \gamma^1 \psi \quad (27)
$$

(28)

The incoming flux at $\rho = l$ is determined as (The flux obtained from the above constants is multiplied by $l^2/N^2$ for normalisations)
\[ F^I = -\frac{l}{8N} |E_1|^2 (2 - 2\text{Re}\Psi(\alpha, \beta)) \]  

(29)

The absorption coefficient is defined as the ratio of total number of particles entering the horizon with the incoming flux at infinity \[20\]. The total number of particles entering the horizon is:

\[ P = -\int \sqrt{-g} J^\rho \rho^\rho d\phi \]  

(30)

This is equal to: \( A_H l / 4N |(\gamma - 1)/\alpha|^2 \), where \( A_H \) is the area of the horizon. The absorption coefficient is then determined as (for \( m = 0 \)):

\[ \sigma_{\text{abs}} = \frac{A_H}{1 - \text{Re}\Psi(\alpha, \beta)} \left( \left| \frac{\gamma - 1}{\alpha} \right|^2 \right) \left| \frac{\Gamma(\gamma)\Gamma(\gamma - \alpha - \beta)}{\Gamma(\gamma - \alpha)\Gamma(\gamma - \beta)} \right|^{-2} \]  

(31)

Here, \( \text{Re}\Psi(\alpha, \beta) = - (\omega/4T_L)\tanh (\omega/4T_R) - O(\beta^2) \) for \( m = 0 \). Using the expressions for the hypergeometric parameters as given in the earlier section, the greybody factor can be written in an interesting form:

\[ \sigma_{\text{abs}} = \frac{\omega A_H}{4T_L(1 + \omega/4T_L \tanh(\omega/4\pi T_R))} \frac{\exp(\omega/T_H) + 1}{(\exp(\omega/2T_L) - 1)(\exp(\omega/2T_R) + 1)} \]  

(32)

Where, the quantities \( T_L \) and \( T_R \) are defined by:

\[ \frac{1}{T_L} = \frac{1}{T_H} \left( 1 - \frac{\rho}{\rho_+} \right), \quad \frac{1}{T_R} = \frac{1}{T_H} \left( 1 + \frac{\rho}{\rho_+} \right) \]  

(33)

The extremal limit, defined by taking \( \rho_+ \to \rho_- \), corresponds to \( T_L \gg T_R \). Clearly, taking the above limit in equation (33), the absorption coefficient reduces to \( A_H / 2 \). The Hawking radiation rate will be now a product of thermal distributions, instead of being a single fermionic distribution. In fact, it is:

\[ \Gamma_H = \frac{\omega A_H}{4T_L} \frac{d^2k}{\exp(\omega/2T_L) - 1}(\exp(\omega/2T_R) + 1) \]  

(34)

This is precisely the form expected for emission rates from an underlying conformal theory at finite temperature \[3\]. The fermion in the bulk couples to operators of the 1+1 dimensional conformal field theory. The system is at finite temperature \( T_H \), which can be split into left and right temperatures such that \( 1/T_L + 1/T_R = 2/T_H \). The decay rate at finite temperature due to the coupling stated above is calculated to have the form of a product of left and right distributions. The fermions are associated with rightmoving temperature, indicating that fermions considered here couple to chiral conformal operator. Using the results of \[3\] it can be predicted that the conformal fermion couples to operator in the conformal theory of the form \( O^+O^- \), where \( O^+ \) is rightmoving, and has conformal weight \( 1/2 \) and \( O^- \) is leftmoving with weight \( 1 \). It will be interesting to determine the nature of the coupling as that fixes the coefficients exactly.
IV. COMPARISON WITH HIGHER DIMENSIONAL BLACK HOLES

One of the reasons behind the renewed interest in three dimensional black holes, is the fact that near horizon geometries of certain higher dimensional stringy black holes are BTZ times a compact manifold. Here we briefly review this mapping \[9,21\] and discuss the implications. The solution due to RR charged one branes and five branes wrapped on \(T^4 \times S^1\), and Kaluza Klein momenta along \(S^1\) in 10 dimensions, has a near horizon geometry \(BTZ \times T^4 \times S^3\). The radius of the \(S^3\) direction is \(l = r_1 r_5\), where \(r_1\) and \(r_5\) are related to the one brane and five brane charges respectively. The time, transverse radial distance \(\rho\) and \(S^1\) direction (\(\phi\)) constitute the BTZ black hole coordinates. The ordinary kaluza-klein reduction of the 10-D solution on \(T^4 \times S^1\) yields a 5-D black hole, which preserves \(N = 8\) supergravity. The entropy of the 5-D black hole is equal to the entropy of the near horizon BTZ black hole and scalar decay rate equals the decay rate for scalar emission from BTZ black holes. Here we make a comparison for fermion decay rates. In \[3\], it has been shown that the SUGRA fermions of \(N = 8\) supergravity, have a Hawking decay rate for the five dimensional black hole as

\[\Gamma^5_H = A^5_H \frac{\omega}{4 T_L} \frac{d^4k}{\left(\exp(\omega/2 T_L) - 1\right)\left(\exp(\omega/2 T_R) + 1\right)} \]

Clearly, our decay rate is identical to this decay rate. The temperatures of the left and right distributions are exactly the same as given in \((33)\) and \(A^5_H\) is the area of the horizon of the five dimensional black hole. A interesting point to note is that the rates can be matched up to exact coefficients if we choose to factor out the phase space factors of \(S^3\) and \(\phi = x_5/l\) (\(x_5\) is the \(S^1\) direction, with radius \(R\)) from the decay rates, as \(A^5_H / A^3_H = \pi l^3 / R\). However, an observer in five dimensional space detecting particles at infinity sees all the three dimensions of \(S^3\) as uncompactified. So, it is not clear what the above result implies. However, it can be said that our result confirms the observation about scalar decay rates. The range of frequency for both the calculations, \(\omega r_1 \ll 1\), is also same.

The exact matching observed above provides a basis to predict rates for non-minimally coupled fermions which propagate on the background of the four dimensional \(N=4\) SUGRA black hole obtained by compactifying M-theory (11-D supergravity) on \(T^6 \times S^1\). To identify the required fermion one requires to take the equation of motion in the 11 D Supergravity solution, and take the near horizon geometry limit as described above. All fermions which will couple in the same way as in equation \((2)\) in the BTZ part, can then be predicted to have the rate obtained in this paper. The metric in 11-Dimension is due to 3 M 5-branes wrapped on \(T^4 \times S^1\), i.e. directions \(x_4...x_{11}\) and a boost in the \(x_{11}\) (\(S^1\)) direction. The near horizon limit results in the metric splitting up into a \(BTZ \times S^2 \times T^6\), where \(S^2\) is the two sphere of the noncompact t,r,\(\theta,\phi\) dimensions of the four dimensional black hole. The radius
of the two sphere is \( R = l/2 = (r_1 r_2 r_3)^{1/3} \), where \( r_i \) are related to the charges of the black hole. As in the five dimensional case here \( \phi = x_{11}/R_{11}, \rho^2 = 2R_{11}^2 (r_0 + r_0 \sinh^2 \sigma') \) (\( R_{11} \) is the radius of \( x_{11} \)), and time form the BTZ coordinates. To find the relevant fermions which will have the rate as found in this paper, we start from the 11-D gravitino \( \psi_M \). Clearly gravitino with vector polarisation along \( x_{11} \) or the other \( x_1, x_2, x_3 \) directions will not satisfy our requirements. We take a representative \( \psi_5 \), as in the near horizon limit, all the torus directions are similar, apart from constant scalings. In this limit, since \( g_{ii} = \text{const}, i = 4..9 \), we can split the 11-D equation of motion as:

\[
\left( \mathcal{D}_3 + \frac{2}{l} \mathcal{D}_{11} \right) \psi = 0 \tag{36}
\]

Where \( \mathcal{D}_3 \) and \( \mathcal{D}_{11} \) are the dirac operators in the BTZ and the two sphere metrics respectively. To get simultaneous eigenstates of both the operators, we multiply by the two dimensional chirality matrix \( \Gamma_2 = i \Gamma_a \Gamma_b \). Thus for \( \Gamma_2 \mathcal{D}_{11} \psi = \lambda \psi \), the equation of the fermion in the near horizon limit is:

\[
\left( \mathcal{D}_3 + \frac{2 \lambda}{l} \right) \psi = 0 \tag{37}
\]

For \( \lambda = 1/4 \), we have the required fermion \( (\mathcal{D} \equiv \Gamma_2 \mathcal{D}) \). It is not very difficult to solve the eigenvalue equation stated above. The argument given here is heuristic, and we have not been careful about the supersymmetry preserved by the background metric. It is to be checked whether the fermion taken above falls in the \( N = 4 \) multiplet, as the four dimensional black hole preserves \( N = 4 \) super symmetry. However, it is an interesting calculation, and is under further investigation at present.

V. DISCUSSIONS

In this paper we have calculated emission rate of fermions from BTZ geometry, using techniques of asymptotically flat space-time calculations, like the greybody factor. However, since the physical situation we are interested in is when BTZ occurs as the near horizon geometry of higher dimensional black hole, this is justified. We show that indeed the BTZ calculation reproduces the rate of the non-minimally coupled fermions in the background of a five dimensional black hole whose near horizon geometry is \( BTZ \times S^3 \). The fact that the rate observed by a BTZ observer at \( \rho \sim l \) looks identical to that of an asymptotic observer in a five dimensional black hole is interesting. The physical implications of this are still not clear, but the answer might lie in the location of the degrees of freedom of the underlying conformal field theory. There are several ways to approach the problem. It is known that 2+1 gravity can be cast in the form of Chern Simon theory, which induces a conformal
field theory on the boundary. However, on inclusion of matter fields the theory is no longer topological, and the same conclusions cannot be drawn about the entropy. Hence, it is not clear how to study Hawking emission in the above framework. Recently, matter fields have been treated as a classical perturbation in the Chern Simons action, and the decay rate obtained for scalars \cite{22}. The agreement with the black hole decay rate is remarkable, and calls for further investigation. Apart from this, the BTZ black hole is asymptotically anti de Sitter, and has a conformal field theory living on its boundary \cite{12,13}. With the AdS/CFT correspondence, it is known now, that string theory on orbifolds of $AdS_3$ times a compact manifold $M$ is dual to a super conformal field theory whose target space is symmetric product of $M$ \cite{21}. In this matter fields are automatically included, and it will be interesting to study the decay rates, using this approach.

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