Formation of a layer of magnetorheological fluid on the surface of the moving object in the gradient magnetic field

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Abstract. The main forces that form the free surface of a layer of magnetorheological fluid (MRF) in contact with the surface of a moving object in the gradient magnetic field are determined. These forces are the force associated with the pressure caused by an external magnetic field, the force, determined by the ability of MRF material to be magnetized and magnetostrictive force. It is found that MRF layer in an inhomogeneous magnetic field retains its shape when the above mentioned forces are balanced by the elastic forces and surface tension.

1. Introduction

Devices for mechanical polishing of surfaces from different materials in the magnetic field are based on the use of fluid abrasive materials or pastes, which are modified by introduction of magnetically sensitive particles. For a more rough treatment using the lapping technology, magnetic fluids (MF) are usually applied [1, 2], and for the finishing of precision optical and semiconductor components using fluidic technology – magnetorheological fluids (MRF) are used [3-6]. Magnetic fluids contain colloidal ferromagnetic particles with the size, \( d < 10^{-8} \) m characterized by their noticeable Brownian motion and in experimental studies and physical-mathematical description they are considered [7] as a continuum medium. It is assumed that the external forces are applied to the centers of mass of the representative volumes in which a short-range order exists. The momentum, obtained by the particles of this volume for the time greater than the relaxation time (the time to reach an equilibrium state) is transmitted to the carrier fluid.

Unlike magnetic fluids magnetorheological fluid suspensions include a large multi-domain structure of ferromagnetic particles (size range 0.1–10 \( \mu \)m), has a low coercivity and low residual magnetization that allow one to classify them as superparamagnetic. Brownian motion in them has a much weaker impact on the particles then interaction between each other and with the external magnetic field. Approaching rigorously, a continuum approximation is inapplicable to them. However, up to the present time, due to the lack of data for accurate mathematical description of the kinetics of particle-particle interactions and the absence of any other reasonable physical and mathematical tools for the correct description of such media at shear deformation in the magnetic field, they are, especially in the case of medium- and highly concentrated suspensions are considered quasi-homogeneous and are subjected to a continuum approximation [8].

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Thus, it is assumed that the center of mass of the particle or aggregate is moving at an average speed of the fluid layer. Structures formed in the flowing MRF by the impact of the field of the dispersed particles is continuously broken by shear flow and are carried away by it, and are also continuously recovered in a short time from other fragments downstream. Kinetic processes, depending on the energy ratio of electrical and shear forces determine the overall performance characteristics of MRF fluidity.

As for the considered objectives soft ferromagnetic hysteresis-free particles are mainly used as a dispersed phase, and as a dispersion medium – oil or water; the degree of MRF magnetization is uniquely determined by the intensity of the magnetic field. Assuming that MRF during the time of experiment that is longer than the relaxation time, the rate of which is determined by the Stokes damping, but shorter that the time to establish a full balance, has the properties of the magnetic fluid and to its movement a steady-state approximation can be applied [7].

In the design of polishing devices it is necessary to keep the MRF layer as a ribbon on a moving support with a known cross section of the layer. The solution to this problem is possible by considering the equilibrium conditions of the bulk forces and the support reaction and Maxwell forces and surface tension, as in [9].

2. Forces impacting the layer of magnetorheological suspension in the magnetic field

Let us consider the peculiarities of formation of magnetorheological fluid layer on the surface of the machining tool-support in the finishing devices for various objects. Let MRF flow in a stream at a constant rate from the nozzle and get into the hydraulic system of a moving object-ribbon support of finite size in a region of a high gradient magnetic field. Its further movement takes place at a constant speed of the support as the layer with a free outer surface. Geometric parameters (the thickness, the shape of the free surface), and the adhesion to the surface to the support and tension on the free surface of the layer are defined by the ratio of support movement conditions, leakage rate in the initial pattern, gradient and impact area of the magnetic field. In our case, the non-uniform magnetic field created by a horseshoe magnet, placed so that the magnetic field lines cover the support and the entire MRF layer within it in planes perpendicular to the movement, as shown in figure 1.

![Figure 1](image_url)
The shape of the free surface of MRF layer in cross section relative to the speed of movement in the gradient magnetic field and especially its interaction with the bearing support are determined by the bulk and surface Maxwell forces induced by the magnetic field.

The induction $\vec{B} = \mu_0 \left( \vec{H} + \vec{J} \right) = \mu_0 \mu \vec{H}$ appearing in MRF is equal to the sum of the induction of the external field $\vec{H} = \frac{\vec{B}_0}{\mu_0}$, and the induction of the magnetization field, that is proportional to the magnetization $\vec{J}$ of the magnetic (MRF), and $\mu_0$ – the coefficient of the magnetic permeability of vacuum, $\mu = 1 + \chi$ – magnetic permeability of the magnetic, $\chi$ – its magnetic susceptibility. It is known that the intensity of the magnetic field, expressed through the induction and magnetic permeability of vacuum $\mu_0$, does not depend on the properties of the magnetic, in particular on its density $\rho$ [10]. For practical applications as particle’s material a superparamagnetic material with a narrow hysteresis loop (or its absence) is used, having a very small coercitive force and residual magnetization.

Holding MRF layer on the support is provided by the bulk force, applied to it from the magnet. The induction creates in MRF a field of bulk forces [10], density of which is described by the function:

$$\overline{f}(\vec{r}) = -\nabla \left[ \int_{V} \frac{d\vec{B}}{d\nu} d\vec{H} + \int \mu_0 \vec{J} d\vec{H} \right] + \mu_0 \vec{J} \nabla \vec{H} \quad ,$$

where $\nu$ – the volume of MRF in the unit of mass, in which the particle material is superparamagnetic.

It can be expressed through the gradient of the magnetic pressure as:

$$\overline{f}(\vec{r}) = -\nabla (P_s) - \nabla (P_{ht}) + \mu_0 \vec{J} \nabla (H)$$

The first term in (2) – is the gradient of magnetic striction pressure, which can be represented considering the average value of the induction as:

$$P_s = \int_{V} \nu \frac{d\vec{B}}{d\nu} d\vec{H} = \nu \frac{d}{d\nu} \frac{B(H)}{2} H = \mu_0 \nu \frac{d}{d\nu} \frac{J(H)H}{2} = -\frac{1}{2} \mu_0 H \rho \frac{d \mu(H)}{d\rho} \quad ,$$

where according to [9] for a weakly magnetic material we have:

$$\rho \frac{d \mu(H)}{d\rho} = \chi$$

The second term in (2) – is the gradient of magnetic pressure:

$$P_{ht} = \int \mu_0 J dH = -\frac{1}{2} \mu_0 H J$$

Regarding (2) and (3) the impacting force of the external magnetic field on the element of MRF is determined as:

$$\overline{f}(\vec{r}) = -\frac{1}{2} \mu_0 H^2 \left( \rho \frac{\partial}{\partial \rho} \nabla \mu - \nabla \mu \rho \right) + \frac{1}{2} \rho \mu_0 \vec{J} \nabla H^2 \quad .$$

In quasi-linear representation this force expressed through magnetic permeability, depending on the intensity of the magnetic field is written as:
\[ f(\vec{r}) = -\frac{1}{2} \mu_0 \nabla \left( H^2 \rho \frac{\partial}{\partial \rho} \mu \right) - \mu_0 H \nabla(J). \]  

(6)

And like gravity force it presses MRF layer to the region with the bigger value of the intensity of the field.

According to [9] bulk forces could be brought to the surface Maxwell tensions \( T_n \) relative to the unit normal \( \vec{n} \) to the surface element of the layer \( dS = \vec{n} \cdot dS \):

\[ \vec{T}_n = (\vec{B} \cdot \vec{n}) \vec{H} + \vec{n} \left[ \int \frac{dB}{d\nu} dH + \int B dH \right] \]  

(7)

In MRF like in other superparamagnetics

\[ \vec{T}_n = (\vec{B} \cdot \vec{n}) \vec{H} + \frac{1}{2} \vec{n} \left( 1 + \hat{b} \right) \vec{B} \cdot \vec{H}, \quad \hat{b} = -\rho \frac{\partial}{\partial \rho}, \]  

(8)

where \( \hat{b} \) determines the magnetic striction stress in MRF.

From this relation according to [10] it follows that in equilibrium the vector of intensity \( \vec{H} \) bisects the angle between the normal \( \vec{n} \) to the surface and tension vector \( \vec{T}_n \).

The condition of equilibrium of forces on the surface of the layer together with the boundary conditions of electrodynamics assumes the equality of the forces acting on each element of the surface, it means the equality of the sum of the Maxwell tensions, or the difference between the external pressures from the air \( P_0 \) and from the magnet \( P_1 \) [10] to the surface tension force of \( P_c \).

Given that

\[ P_1 = P_0 + P_{H} + P_s + \frac{1}{2} \mu_0 J_n^2, \]

where \( P_{H} \) — magnetic pressure (4), \( P_s \) — striction pressure (3), surface tension force \( P_c = 2\sigma \kappa \), and \( \sigma \) — coefficient of surface tension, \( \kappa \) — surface curvature, we have

\[ P_1 - P_0 = 2\sigma \kappa. \]  

(9)

In MRF besides the above mentioned forces there is also an impact of magnetization pressure:

\[ P_j = -\frac{1}{2} \mu_0 J_n^2, \]  

(10)

where \( J_n \) — normal to the surface of the layer component of magnetization which in the presence of hysteresis in ferromagnetic particles does not disappear after switching off the magnetic field.

From condition (9) after substituting expressions for the pressure we have the expression for the curvature of the free surface of MRF layer:

\[ \kappa = -\frac{\mu_0}{4\sigma} \left( \mu \mathcal{G} - \rho \frac{\partial}{\partial \rho} \mu \right) H^2. \]  

(11)

From it follows that the curvature of the surface is maximum there, where the maximum value of the field intensity is.

Resultant force of Maxwell tensions on the surface of the layer is equal to the resultant bulk forces, i.e.
\[ F = \int \vec{F} \cdot d\vec{S} = \int \hat{r} \cdot \nabla \left( \frac{\mu_0 H^2 \rho}{2} \frac{\partial}{\partial \rho} \mu \right) dV + \int \mu \hat{H} \cdot \nabla (\mu). \] (12)

It is directed to the area of a strong magnetic field and presses the layer to the surface of the object-support.

Consideration of the expressions for bulk and surface forces of Maxwell tensions, as well as the pressures applied to MRF layer in the gradient magnetic field, can further define the cross-sectional shape of the layer.

3. **Shape of the cross section of MRF layer in the gradient magnetic field**

When designing devices for finishing of precision optical or semiconductor components there is a need not only to keep MRF layer as a ribbon on a moving support, but also to satisfy the desired shape of the cross section of the layer.

The shape of the cross section of MRF layer in the gradient magnetic field of a permanent magnet under the conditions of applicability of the quasi-static approximation of a continuous medium is determined by the terms of the balance of forces on the surface (9). From (6), (12) according to [10] it is concluded that in the equilibrium state the vector of Maxwell tension \( \vec{Tn} \) near the surface of the layer is directed at the angle equal to twice the angle between the normal \( \vec{n} \) to the surface and the direction of the magnetic field intensity vector \( \vec{H} \). It can be represented as a vector sum of the normal and tangential components:

\[ \vec{Tn} = \vec{Tnn} + \vec{Tnt}. \] (13)

Let us consider the features of formation of the surface of MRF layer under conditions corresponding to the system of polishing device. According to the equilibrium condition (9) and (13), if the direction normal to the surface of the layer is close to the direction of the field intensity vector and the angle between the normal vector and the Maxwell tension \( \vec{Tn} \) is less than the right angle, its tangential component \( \vec{Tnt} \) causes a shift of MRF in the direction of the slope of the field intensity vector \( \vec{H} \) to the surface layer. In this zone, the volume of fluid increases and the normal to the surface of the layer deviates more than that of the initial extent from the direction of the field.

If the direction of the normal vector \( \vec{n} \) to the surface layer is close to the intensity vector \( \vec{H} \) of the perpendicular direction, the angle between the normal vector and the tension vector is more than that of the right angle. Then the tangent vector component of Maxwell tension causes the movement of MRF volume in the direction opposite to the slope of the intensity vector to the surface of the layer. This leads to an increase in the volume of fluid in this area and the deviation of the normal vector \( \vec{n} \) to the surface of the layer in the direction perpendicular to the field intensity vector \( \vec{H} \). This situation is more stable and the free surface of MRF layer takes a position along the magnetic field intensity vector, as shown in figure 2.

If the layer has a large volume, in which the point of contact of the object’s surface by the layer is in an area where the intensity vector \( \vec{H} \) crosses the support at an angle greater than the right angle relative to the normal to the support, than near this point on the surface of contact the normal to the surface of MRF layer is directed to the nearest pole of the magnet. Tension vector \( \vec{Tn} \) forms an angle with it that is less than the right angle, leading to compression of the layer along the surface of the object.

At the closest point on the free surface of the layer the vector \( \vec{Tn} \) is directed into the layer and compresses it in a direction opposite to the pole of the magnet. As a result the free surface of the layer extends along the lines of magnetic field intensity.
The compression of the layer in the side direction prevents constriction in its middle, which could occur under the influence of the Maxwell tension, compressing the layer in the middle section in the direction to the poles of a magnet. When the tension is balanced by the surface tension, an equilibrium shape of the section layer is settled in the form of a truncated oval with tops extended to the poles of the magnet (in the regions of the most powerful magnetic field), like the one shown in figure 2.

Thus, the analysis of the ratio of orientation patterns of the magnetic field lines, the surface tension forces and magnetic forces in the polishing device allows us considering the equilibrium conditions to determine the main factors affecting the features of the formation of the settled cross-section of MRF layer of the certain thickness in a gradient magnetic field, moving together with object-ribbon support. Knowing the shape of the free surface of the layer, the value of the surface tension force and the degree of its pressing to the object-support, depending on the magnitude and direction of the gradient of the magnetic field will optimize the design of the processing unit and operating data for the implementation of high-precision machining of the surfaces of various products.

4. Conclusion
The considered model of formation of magnetorheological fluid layer allows us to determine the shape of its cross section, if the distribution of the magnetic field intensity is known. Received laws define methodology for further calculations of MRF layer characteristics in different shear conditions, as well as in contact of the free surface with the surface of the outer layer (machining) object.

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