Mathematical Model of the Water Quality in Tlogo Ampel Watershed

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Abstract. The observation station of the study is placed at one point of Tlogo Ampel River in Wonokoyo Village, Kapongan Subdistrict, Situbondo Regency with location of 7°70’44” South Latitude and 114°08’99’ East Longitude. Tlogo Ampel River is one of the rivers that through the settlement of society and industry so that naturally experience of degradation of water quality. Sampling is done every month by the related institution that is Sampean River Basin Management Institute. To predict the value of water quality is needed mathematical model change the water quality of Tlogo Ampel watershed. The research parameters included physical properties (Temperature and TSS), organic chemical properties (pH, BOD, COD, DO, PO4P, NO3N) as well as microbiological properties (total coli and fecal coli). The results show the Polynomial equation of order 6 for parameters: temperature, TSS, pH, BOD, COD, DO, PO4P, NO3N, total coli and fecal coli respectively are: 

\[ y = 0.0002x^6 - 0.0078x^5 + 0.1203x^4 - 0.9031x^3 + 3.4485x^2 - 6.2299x + 27.595; \]
\[ y = -0.0027x^6 + 0.1391x^5 - 3.1236x^4 + 27.249x^3 - 108.22x^2 + 157.53x + 46.997; \]
\[ y = 0.0002x^6 - 0.0085x^5 + 0.1342x^4 - 1.0398x^3 + 4.0441x^2 - 7.4089x + 12.505; \]
\[ y = -0.0002x^6 + 0.0032x^5 + 0.0302x^4 - 0.9626x^3 + 6.8118x^2 - 16.907x + 17.482; \]
\[ y = 0.0005x^6 - 0.0328x^5 + 0.7417x^4 - 7.2595x^3 + 31.838x^2 - 55.407x + 47.365; \]
\[ y = -0.001x^6 + 0.0406x^5 - 0.6408x^4 + 4.9244x^3 - 18.785x^2 + 31.901x - 11.853; \]
\[ y = -1E-05x^6 + 0.0005x^5 - 0.0988x^4 + 0.0897x^3 - 0.3839x^2 + 0.6547x - 0.0902; \]
\[ y = 0.0004x^6 - 0.0143x^5 + 0.1694x^4 - 0.9131x^3 + 2.3917x^2 - 3.2352x + 4.1306; \]
\[ y = 0.0035x^6 - 0.269x^5 + 6.3886x^4 - 67.262x^3 + 337.4x^2 - 741.18x + 612.7; \]
\[ y = 0.0011x^6 - 0.0963x^5 + 2.2449x^4 - 22.035x^3 + 100.4x^2 - 193.42x + 140.76; \]

1. Introduction

The Tlogo Ampel Watershed is one of the watersheds which needs to be concerned because it is suspected to have decreased water quality, due to the high activity of the households, industry, livestock, and agriculture. This condition implies that the water quality as the embodiment of the environmental capacity of Tlogo Ampel Watershed needs to be studied, so that it can be known how far it benefits the lives of the society.

Hatala Nurleyla had conducted a research entitled Mathematical Model of Changes in the River Water Quality in the Cisadane Watershed by Using BOD, COD, and Fecal coli Parameters. This present study attempted to get the changes of the water quality of Tlogo Ampel Watershed taken at one observed station located at Wonokoyo Village, Kapongan District, Situbondo Regency by using regression analysis with the temperature, TSS, pH, BOD, DO, PO4P, NO3N, total coli and fecal coli parameters, so that it could provide the more comprehensive description in order to get the more...
accurate mathematical model to predict the decrease of the water quality due to the changes of physical, chemical, and biological properties [1].

The mathematical model tested is the form of extraction of the secondary data from the water quality data. Mathematical model as test of the data efficiency with the model produced. Based on the comparison, model can be accepted to describe the real condition, without any significant deviation between the model and the secondary data. If the model is rejected, there must be another model which is more suitable to describe the real data. By knowing the mathematical model of water quality declination, it is possible to predict the decline of the water quality at any time so that it can be a recommendation for the government and related stakeholders [2].

2. Literature Review
Literature review that will be used for this research are
2.1 Regression Analysis
2.1.1. The Definition of Regression
The regression equation is the correlation between two or more variables or observation values. According to the Center for Data and Statistics Processing, Agricultural Research and Development, states that regression can be defined in two forms that are: (1) the place of the average population value of a variable, (2) the function adjustment or curve to the data, if there is not enough available data so that there is only one Y value for each X value [3].

2.1.2 The Function of Regression
The function of regression equation are:

a. Data description, if the regression equation is in the data search stage and comparison.
b. Getting a causal relationship, can change the X level then observe the Y variable, then the regression equation Y and X explains the causal relationship between Y and X.
c. In the controlled trials, if there is a difficult factor to be controlled but is expected to affect the Y factor, the regression analysis is used as a comparative investigator.
d. The model formulation and pattern relationship of the variables \(X_1, X_2, X_3, \ldots, X_k\) with the Y variable, the regression to find the most appropriate relationship or model, only involves some of the variables \(X_1, X_2, X_3, \ldots, X_k\).

2.1.3 Regression Analysis Model
The regression analysis model of the water quality degradation model making of Tlogo Ampel Watershed, as follows:

1. Linear
Is a straight line equation written in the form of:

\[
y = ax + bx
\]

2. Polynomial
The general form of the polynomial equation:

\[
y = a_n x^n + a_{n-1} x^{n-1} + \ldots + a_1 x + a_0
\]

\(a_0, a_{n-1}, a_1, a_0\) are polynomial constants/coefficients; \(n\) is a non-negative integer; \(x\) is an independent variable with the value that can be used to predict \(y\) is a dependent variable

3. Exponential
The form of the exponential equation:

\[
f(x) = a^x.
\]

\(x\) is an independent variable with the value that can be used to predict; \(a\) is a positive constant; \(y\) is a dependent variable

There are three types of exponential functions, \(y = a^x\). If \(0 < a < 1\), the exponential function goes down; if \(a = 1\), the function is constant; and if \(a > 1\), the function goes up. These three
cases are shown in the figure below. Because \((1/a)^x = a^{-x}\), the graph \(y = (1/a)^x\) is the reflection of the graph \(y = a^x\) against the y axis.

4. Logarithm

If \(a > 0\) and \(a \neq 1\), the exponential function \(f(x) = ax\) is derivative and increase functions and therefore one-to-one. So, it has an inverse function \(f^{-1}\), which is called as logarithm function with the principal number \(a\), and denoted by \(\log\), if we use the inverse function formulation:

\[
\log_a x = y \iff a^y = x
\]

x is an independent variable with the value that can be used to predict; a is a positive constant; y is a dependent variable

2.2 Correlation, \(R^2\)

The regression equation obtained is used to estimate the equation produced by the data, then followed by assessing the good and badness of the model with the data. The assessment uses the Biggest Coefficient Relation Method, \(R^2\). \(R^2\) as the relation coefficient or determinant coefficient (determination). The closer \(R^2\) to 1, the better the data match with the model [4].

3. Working Methodology

3.1 Research Location

This research was conducted at Tlogo Ampel Watershed with the observed station located at Wonokoyo Village, Kapongan District, Situbondo Regency.

3.2 Tool and Material

The research tool used was Excel 2010 software. The materials used were: (1) the physical properties data (Temperature and TSS), (2) the organic chemical properties data (pH, BOD, COD, DO, PO4P and NO3N) and (3) the microbiological properties (total coli and fecal coli).

3.3 Research Methodology

3.3.1 The Framework of the Research Approach

The first framework of the research approach was the data collection to describe the water quality problem in Tlogo Ampel Watershed from the related agency. The data were in the form of the laboratory result analysis of the physical, chemical, and microbiological properties in the observed station of Tlogo Ampel Watershed. The result analysis was processed by using regression method and became the water quality degradation model of Tlogo Ampel Watershed according to the 3 parameters above and compared with the Indonesia Government Regulation Number 82 of 2001. The data of this research were from the secondary data of water quality in Tlogo Ampel Watershed from the Water Resources Management Unit of Bondowoso which had measured the water quality [5].

3.3.2 Data Analysis Method

1. Water Quality Analysis

The water quality data of Tlogo Ampel Watershed covered the analysis of physical, chemical, and microbiological properties according to the Indonesia Government Regulation Number 82 of 2001.

2. Regression Analysis

The analysis data method of the water quality in Tlogo Ampel Watershed used the regression analysis method of the 6th Polynomial Model, based on the MAXR concept, the 6th polynomial model was the most appropriate. The regression analysis method estimated the amount of water quality in Tlogo Ampel Watershed at the observed station, even though the observation was not administered in the station. The regression analysis used Microsoft Excel 2010 program and from the regression analysis model produced could be analyzed in more detail [6].

4. Experiment and Result

4.1 Physical Characteristic

4.1.1 Temperature (°C)

The temperature in the watershed changed due to the seasonal changes, daily changes and heat waste from the surrounding industries. Based on the measurement result in Tlogo Ampel Watershed obtained that the temperature in the period of January up to December 2017 ranged between 23.40-24.70 °C. The highest temperature reached in October and the lowest was in March and June. If based
on the Government Regulation Number 82 of 2001 concerning Management and Control of Water Pollution, this temperature was still in the standard limit of water quality. The water quality standard in the group I, II, and III indicated that the normal water temperature had deviation 3 of the surrounding temperature.

The temperature pattern distribution in the period of January up to December 2017 in Tlogo Ampel Watershed could be seen in the Figure 1. From the graph, regression was done to get the mathematical model by following the 6th polynomial equation. The mathematical model obtained was utilized to find the temperature value predicted as shown in the Figure 2.

\[ y = 0.0002x^6 - 0.0078x^5 + 0.1203x^4 - 0.9031x^3 + 3.4485x^2 - 6.2299x + 27.595 \]

The change of temperature value based on the MAXR method was 0.8296. The prediction result obtained by using mathematical model was not so much different from the observation temperature. The average of the prediction temperature was 23.94°C. The difference between the prediction temperature and the observation was only 4.2% so that this mathematical model could be used to find the prediction value of the temperature in Tlogo Ampel Watershed.

4.1.2 TSS (mg/L)

TSS was suspended solid that caused water turbidity because it could be directly dissolved and sediment. From the measurement result of TSS at Tlogo Ampel watershed in January-December 2017 period, it was about 20.80-255.40 mg/L. The highest TSS value was gotten in December and the lowest one in July.

The distribution pattern of TSS at Tlogo Ampel watershed in January-December 2017 period could be seen in Figure 3. From the graph, regression was done to get the mathematical model by using 6th polynomial equation. The mathematical model could be used to find the TSS value prediction like in Figure 4.

\[ y = -0.0027x^6 + 0.1591x^5 - 3.1236x^4 + 27.249x^3 + 108.22x^2 + 157.53x + 46.997 \]

The change of TSS value based on the MAXR method was 0.9052 and the mathematical model gotten was \[ y = -0.0027x^6 + 0.1591x^5 - 3.1236x^4 + 27.249x^3 + 108.22x^2 + 157.53x + 46.997 \]. The average value of TSS prediction was 55.91 mg/L and the TSS observation value was 59.77 mg/L. The deviation between the prediction and observation was not really different, it was 6.91%. It meant that the mathematical model was very good to find the value of TSS prediction in Tlogo Ampel watershed.
4.2 Chemical Characteristics

The chemical characteristics of Tlogo Ampel watershed that was used as the parameter consisted of PH, BOD, COD, DO, PO₄, P dan NO₃, N.

4.2.1 pH

From the measurement result of pH in the research location from January-December 2017, it was about 6.30-8.20. The fluctuation of pH value was caused by the organic and inorganic waste disposal of each period. The highest pH measurement was gotten in January and the lowest one in July. Based on the Government regulation (PP) No.82 Year 2001, the Tlogo Ampel watershed water pH was in the standard threshold of all water quality groups.

The result of pH measurement in Tlogo Ampel watershed was categorized as normal and it indicated that the aquatic biota was in good condition. The aquatic biota was very sensitive to the pH change and the favored pH was about 7-8.5. The pH distribution pattern in Tlogo Ampel watershed could be seen in Figure 5. The gotten mathematical model could be used to find the pH value prediction like in Figure 6.

![Fig 5. pH distribution of Tlogo Ampel watershed in January – December 2017 Period](image)

![Fig 6. The pH Observation and Prediction in January - December 2017 Period](image)

The pH value change based on MAXR method was 0.7461 and the mathematical model was $y = 0.0002x^6 - 0.0085x^5 + 0.1342x^4 - 1.0398x^3 + 4.0441x^2 - 7.4089x + 12.505$. The average pH prediction value was 6.53 and the TSS observation value was 6.91%. It meant that the gotten mathematical model was very good to find the pH value prediction in Tlogo Ampel watershed.

4.2.2 BOD (mg/L)

The result of BOD measurement in Tlogo Ampel watershed in January-December 2017 period was about 4.60-14.45 mg/L. The lowest BOD value was gotten in February and October while the highest was on in May. The high BOD value was caused by the activity of the society around the river who thrown their waste into the river so that it went spoiled and degraded by the microorganism. Based on the government regulation (PP) No.82 Year 2001, it could be known that the BOD content of Tlogo Ampel watershed was classified in I, II, III, and IV class, yet in July the content BOD surpassed the threshold so it was polluted.

The distribution pattern of BOD in Tlogo Ampel watershed could be seen in Figure 7, which showed it in January-December 2017 period. From the graph, regression was done to get the mathematical model by following 6th polynomial equation. The gotten mathematical model was used to find the BOD value prediction as shown in Figure 8.

The change of BOD value based on MAXR method was 0.3431 and the mathematical model was $y = -0.0002x^6 + 0.0032x^5 + 0.0302x^4 - 0.9626x^3 + 6.8118x^2 - 16.907x + 17.48$. The average of BOD value prediction was 6.13 mg/l and the BOD observation value was 6.69 mg/L. The deviation of the prediction and observation values was not really different, it was 8.97%. It meant that the gotten mathematical model was really good to find the BOD prediction value in Tlogo Ampel watershed.
4.2.3 COD (mg/L)

This parameter was used, generally, to measure the strength of water pollution in both the domestic and industrial waste. The chemical element measured was the total oxygen needed to oxidize the organic substances within the sample [7]. The measurement result of COD in Tlogo Ampel watershed in January-December 2017 period was about 11.76-34.21 mg/L. The lowest COD value was in October and the highest one in May. Generally, the COD value was increased because there was a human activity around the Tlogo Ampel watershed in which caused the increase of the oxidized material source of the waste.

The distribution pattern of COD in Tlogo Ampel watershed could be seen in Figure 9, it was in January-December 2017 period. From the graph, regression was done to get the mathematical model by following the 6th polynomial equation. The gotten mathematical model was used to find the COD prediction value as seen in Figure 10.

4.2.4 DO (mg/L)

This parameter indicated the aerobic condition of watershed area which contained pollution substances from wastewater, industrial waste as well as domestic waste. Based on the measurement result of dissolved oxygen in this parameter, it was about 2.61 - 7 mg/L which indicated that it still in the water quality standard 0-6 mg/L. Whereas, in June, it showed high DO value of > 6 mg/L which classified into class I in which the minimum value was 6 mg/L. The dissolved oxygen condition in Tlogo Ampel watershed was shown in Figure 11. The gotten mathematical model was used to find the DO prediction value as shown in Figure 12.

The change of DO value based on MAXR method was 0.6109 and the gotten mathematical model was $y = -0.001x^6 + 0.0406x^5 - 0.6408x^4 + 4.9244x^3 - 18.785x^2 + 31.901x - 11.853$. The average DO Prediction value was 5.38 mg/L and the DO observation value was 5.66 mg/L. The deviation of the
prediction and observation values was not really different, it was 5.19%. It meant that the gotten mathematical model was very good to find the DO prediction value in Tlogo Ampel watershed.

Fig 11. DO Distribution Pattern of Tlogo Ampel watershed in January – December 2017 Period

4.2.5 PO₄⁢P (mg/L)

The content of PO₄⁢P was influenced by the human activity especially on farming field. The effect of crops fertilization and others increased the PO₄⁢P content in Tlogo Ampel watershed. From the observation of PO₄⁢P content in January-December 2017, it was about 0.07-0.26 mg/L. Based on the data, it could be said that the PO₄⁢P content in Tlogo Ampel watershed was still in safe limit. The lowest PO₄⁢P value was in June because of the less farming process, while the highest one in February. The distribution pattern of PO₄⁢P could be seen in Figure 13 and the value between PO₄⁢P prediction and observation value could be seen in Figure 14.

Fig 13. PO₄⁢P Distribution Pattern of Tlogo Ampel watershed in January – December 2017 Period

The change of P₄O⁢P value based on the MAXR method was 0.7318 and the gotten mathematical model was $y = \frac{-0.0014x^4 + 0.0018x^3 - 0.0430x^2 + 0.5924x - 18.795}{31.00x^4 + 34.85x}$ $R^2 = 0.6409$

Fig 14. The PO₄⁢P Observation and Prediction in January December 2017 Period

4.2.6 NO₃⁢N (mg/L)

NO₃⁢N represented the final product of nitrogenous content oxidation. Therefore, the total of NO₃⁢N showed the rate of the transformation process to complete oxidation as well as the constancy. The analysis result of NO₃⁢N content in Tlogo Ampel watershed was about 0.14-7.93 mg/L. The lowest NO₃⁢N content was in May and the highest one in August. The values were categorized under the threshold of 10-20 mg/L. The distribution pattern of NO₃⁢N value could be seen in Figure 15 and the NO₃⁢N prediction and observation values could be seen in Figure 16.

The change of NO₃⁢N value based on the MAXR method was 0.3932 and the gotten mathematical model was $y = 0.0004x^6 - 0.0143x^5 + 0.1694x^4 - 0.9131x^3 + 2.3917x^2 - 3.2352x + 4.1306$. The average NO₃⁢N prediction value was 1.78 mg/L and the NO₃⁢N observation value was 2.38 mg/L. The deviation of the prediction and observation values was not really different so the gotten mathematical model was good to find the NO₃⁢N prediction value in Tlogo Ampel watershed.

Fig 15. NO₃⁢N Distribution Pattern of Tlogo Ampel watershed in January – December 2017 Period

Fig 16. The NO₃⁢N Observation and Prediction in January December 2017 Period
4.3 Microbiology Characteristics

The microbiology characteristics in Tlogo Ampel watershed that was used as the parameter consisted of total Coliform and Fecal Coliform.

4.3.1 Total coliform (population/100ml)

The Total coliform, commonly, was used as the bacteria indicator for food and water qualities. Coliform was founded a lot in warm-blooded animal’s feces but it could also find in the water, soil, and vegetation environments. The measurement result showed that the coliform concentration in Tlogo Ampel watershed was about 25-240 population/100ml. The lowest coliform content was in February and the highest one in June. The high of coliform value was caused by the high number of faeces domestic waste entranced the river. The distribution pattern of coliform value could be seen in Figure 17 and the coliform prediction and observation values could be seen in Figure 18. The value of total coliform was in water quality threshold.

4.3.2. Fecal Coli (population/100ml)

Escherichia coli bacterium is Fecal coli Bacteria which is commonly used as the sanitation parameter because it is the indicator of fecal pollution and pathogenic bacteria in water and food. The measurement result showed that the fecal coli concentration in Tlogo Ampel watershed was about 14-120 population/100ml. The lowest coliform content was in February and the highest one in October-November. The distribution pattern of fecal coli could be seen in Figure 19 and the fecal coli prediction and observation values could be seen in Figure 20. This fecal coli value was in water quality threshold, therefore, it could be said that Fecal coli did not contribute the pollution in Tlogo Ampel watershed.

The change of fecal coli value based on the MAXR method was 0.8125 and the gotten mathematical model was $y = 0.0011x^6 - 0.249x^5 + 6.388x^4 - 67.26x^3 + 337.4x^2 - 741.18x + 612.7$. The average total coliform prediction value was 116.3 population/100ml and the total coliform observation value was 113.17 population/100ml. The deviation of the prediction and observation values was not really different so the gotten mathematical model was good to find the NO3N prediction value in Tlogo Ampel watershed.
different so the gotten mathematical model was good to find the fecal coli prediction value in Tlogo Ampel watershed.

5. Conclusion
From the research results on mathematical model to obtain the prediction value of the water quality of the Kalibaru watershed, it was concluded:

1. The mathematical model of temperature and TSS parameter values were $y = 0.0002x^6 - 0.0078x^5 + 0.1203x^4 + 3.4485x^2 - 6.2299x + 27.595; y = -0.0027x^6 + 0.1591x^5 - 3.1236x^4 + 27.249x^3 - 108.22x^2 + 157.53x + 46.997.$

2. The mathematical model of pH, BOD, COD, DO, PO4P, NO3N parameter values were:
   - $y = 0.0002x^6 - 0.0085x^5 + 0.1342x^4 - 1.0398x^3 + 4.0441x^2 - 7.4089x + 12.505; y = -0.0025x^6 + 0.0002x^5 - 0.9626x^4 + 6.8118x^3 - 16.907x^2 + 17.482; y = 0.0005x^6 - 0.0328x^5 + 0.7417x^4 - 7.2595x^3 + 31.838x^2 - 55.407x + 47.365; y = -0.0016x^6 + 0.0406x^5 - 0.6408x^4 + 4.9244x^3 - 18.785x^2 + 31.901x - 11.853; y = -1E-05x^6 + 0.0005x^5 - 0.0098x^4 + 0.0897x^3 - 0.3839x^2 + 0.6547x - 0.0902; y = 0.0004x^6 - 0.0143x^5 + 0.1694x^4 - 0.9131x^3 + 2.3917x^2 - 3.2352x + 4.1306.$

3. The mathematical model of Total Coliform and Fecal Coli parameter values were:
   - $y = 0.0035x^6 - 0.2695x^5 + 6.3886x^4 - 67.262x^3 + 337.4x^2 - 741.18x + 612.7; y = 0.0011x^6 - 0.0963x^5 + 2.2449x^4 - 22.035x^3 + 100.4x^2 - 193.42x + 140.76.$

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