Chapter 5
Structural Design of HEPA Filter

With a certain amount of filtration efficiency with rated flow, the structural design of HEPA filter at present is how to obtain the minimum pressure drop with the corrugation angle (for the type with isolator), the corrugation height (for the type with isolator) or the line height (for the type without isolator), and the passage depth (for both types).

5.1 Flow State in the Passage of HEPA Filter

For the flow state in the passage (inlet and outlet) of ideal HEPA filter with isolator as shown in Fig. 5.1, Cheng Daiyun obtained the calculation result about the flow distribution in the passage on the basis of Bernoulli equation and modified momentum conservation equation, which is shown in Fig. 5.2 and Table 5.1 [1].

The conclusion obtained is that under the condition of low initial velocity, high corrugation, and high pressure drop, the flow in the passage is close to laminar flow with linear distribution of velocity (\(Re < 2,000\)), i.e.,

\[
\begin{align*}
v_1 & \approx v_0 \left(1 - \frac{x}{L}\right) \\
v_2 & \approx v_0 \frac{x}{L}
\end{align*}
\]

(5.1)

(5.2)

Otherwise, the nonlinear distribution will be obvious.

Experiment with Laser Doppler velocimeter has proved the above conclusion, which is shown in Fig. 5.3 [1]. In terms of outlet passage, the linear correlation coefficient of velocity distribution is more than 0.99.
Fig. 5.1  Model for passage of air filter

Fig. 5.2  Relationship between the velocity and distance in the passage

Table 5.1  Calculation result about the velocity distribution in passage of air filter with low initial velocity (m/s) and high corrugation (several millimeters)

| x/L | v_1/v_0 |
|-----|---------|
| 0.1  | 0.899   |
| 0.2  | 0.800   |
| 0.3  | 0.701   |
| 0.4  | 0.603   |
| 0.5  | 0.505   |
| 0.6  | 0.407   |
| 0.7  | 0.308   |
| 0.8  | 0.207   |
| 0.9  | 0.105   |
According to Chap. 4, the total pressure drop of air filter can be expressed as:

\[ \Delta P = \Delta P_1 + \Delta P_{2(1)} + \Delta P_{2(2)} + C \]

\[ C = \Delta P_3 + \Delta P_4 \] (5.3)

where

\( \Delta P_1 \) is the pressure drop of filter media;
\( \Delta P_{2(1)} \) is the frictional resistance of inlet passage;
\( \Delta P_{2(2)} \) is the frictional resistance of outlet passage;
\( C \) is the local resistance of both inlet and outlet passages;
\( \Delta P_3 \) is the local resistance of inlet passage;
\( \Delta P_4 \) is the local resistance of outlet passage.

The core purpose of structural design is to calculate the pressure drop. Foreign scholar has proposed other kind of the expression for the total pressure drop [2]. It did not reflect the influence of the flow rate, so the meaning is not clear. But in Chap. 4 we know that \( \Delta P_2 \) is related to the \( n \) power of the velocity, where the exponent \( n \) should be determined by experiment. It reflects the property, but it is not convenient for calculation. From the above theoretical and experimental analysis for the flow state in the air passage, the flow velocity is approximately linear. This finding provides the condition for obtaining the more accurate expression of the pressure drop in the passage, i.e., the frictional resistance in the structural resistance, with various structural parameters.

Cheng Daiyun investigated the flow state in the air passage, and obtained the pressure drop of filter material in filters with separator, as well as the frictional resistance. Then the derivation with some parameters was performed. On this basis, various kinds of pressure drop expressions with the actual process conditions of HEAP filters are derived.
5.2.1 Pressure Drop of Filter Media $\Delta P_1$

From Chap. 4, we know

$$\Delta P_1 = Av$$

where $A$ is the structural coefficient or the drag coefficient. It is equivalent with the pressure drop of filter media under the filtration velocity $v = 1$ m/s, (Pa·s/m).

The number of corrugation is $(\frac{a}{h+T} + 1)$ for filter media. According to the manufacturing process of air filter, the two corrugations near the frame are pasted to it. So the total area of filter media is $\Sigma f - 2f$.

From Fig. 5.4, we know:

$$\Delta P_1 = A \frac{Q}{\Sigma f} = A \frac{Q}{\left(\frac{a}{h+T} + 1 - 2\right)(b-2\delta)L} = A \frac{Q}{\left[\frac{a - (h + T)}{h + T}\right](b-2\delta)L} \quad (5.4)$$

where

$Q$ is the flow rate per second through air filter;
$a$ and $b$ are the side length of air filter and the net length after the frame thickness is subtracted;
$L$ is the depth of air passage, i.e., the length of corrugation line on the separator;
$f$ is the area of filter media on each corrugation;
$T$ is the thickness of filter media;
$\delta$ is the depth of adhesive at two ends of filters (including the altitude of glue on filter media) or the height of head glue.

Fig. 5.4 Calculation dimension for HEPA filter
For wooden frame, the depth of adhesive with new process can be as less as 5 mm. For iron frame, the surface of adhesive is within the thickness of frame. For both cases, there is no head glue. The height of glue can be thought to be 5 mm (it can be larger than this value for poor process). For the old process of gluing, the thickness of the head glue can be 15 mm or even more.

When labels $a'$ and $b'$ are introduced with the following expressions:

\[
\begin{align*}
    a' &= a - (h + T) \\
    b' &= b - 2\delta
\end{align*}
\]

then the above expression can be written as:

\[
\Delta P_1 = A \frac{Q(h + T)}{a' b' L}
\]

(5.5)

According to the detail investigation on air filter [3], the part of filter media near the top of the corrugation does not play a role in filtration. The effective filtration area reduces to 92%. So $\Delta P_1$ should be enlarged by $(1/0.92) = 1.087$ times. Then the above expression is rewritten as:

\[
\Delta P_1 = j A Q(h + T) \frac{1}{a' b' L} (\text{Pa})
\]

(5.6)

where $j = 1.087$.

### 5.2.2 Frictional Resistance of Air Passage $\Delta P_2$

From the Ref. [1], the flow inside the air passage is laminar. According to the fluid dynamics, the frictional resistance can be expressed as:

\[
\Delta P_{2(1)} = \frac{64}{\text{Re}} \frac{L}{D_p} \frac{\rho v^2}{2} \quad (\text{Pa})
\]

(5.7)

where

- $\Delta P_{2(1)}$ is the frictional resistance of inlet passage;
- $L$ is the depth of air filter (m);
- $D_p$ is the equivalent diameter of air passage (refer to the next section about corrugation angle) (m);
- $v$ is the velocity in the air passage (m/s);
- $\rho$ is the air density, 1.2 kg/m$^3$. 
Suppose the corrugation height is $h$ (m), and the corrugation angle is $90^\circ$, we can obtain the following expression:

$$D_p = \frac{4 \times (2h^2)}{2\sqrt{h^2 + h^2 + 2h}} = 0.83h \quad (5.8)$$

Suppose the nominal effective size of the cross section for air filter is $a \times b$ (m$^2$), where in practice $b$ is replaced by $b'$. The total number of air passage is $a/(T + h)$. The net height of passage is $h$. The total net height of all air passages is $(a/(T + h))h$. In all the air passage, one half of the area is inlet, and the other half is outlet. The passage area $F'$ for real inlet is:

$$F' = \frac{b' \Sigma h}{2} = \frac{b' a\ T + h h}{2} \quad (5.9)$$

On the other hand, there are hundreds of separators in a filter. The space they occupied in air passage cannot be neglected. So the actual value of $a$ is that when the total thickness of all separators are subtracted. Since the separator is tortuous and its length is $\sqrt{2}$ times more than the side length of air filter, the total thickness is $1.414e \frac{a}{T+h}$. So the value of $a$ is $a'' = a - 1.414e \frac{a}{T+h}$. The inlet velocity becomes:

$$v_0 = \frac{Q}{F'} = \frac{Q}{b'(a''/(T + h))h} = 2 \times \frac{Q}{a''b'} \times \frac{T + h}{h}$$

$$= 2 \times \frac{Q}{\left(a - 1.414e \frac{a}{T+h}\right)b'} \times \frac{T + h}{h} \quad (5.10)$$

When we set $(a'/a'') = i$, we know:

$$v_0 = 2 \times \frac{Q}{a' a'' b'} \times \frac{T + h}{h} = 2 \times \frac{Q i}{a' b'} \times \frac{T + h}{h} \quad (5.11)$$

Differential is performed on Eq. (5.7). When $Re = \frac{D_0 v}{\mu \rho}$ and $v = v_0 \frac{L-x}{L}$ are inserted, we obtain:

$$d(\Delta P_{2(1)}) = \frac{32\mu}{D_p^2} v_0 \left(\frac{L-x}{L}\right) dx$$

and

$$\Delta P_{2(1)} = \int_0^L d(\Delta P_{2(1)}) = \frac{16\mu v_0}{D_p^2} L \quad (5.12)$$

It is obvious that frictional resistance of inlet passage is equal with that of outlet passage, i.e.,
\[ \Delta P_{2(1)} = \Delta P_{2(2)} \]

So

\[ \Delta P_2 = 2\Delta P_{2(1)} = \frac{32\mu v_0 L}{D_p^2} \quad (5.13) \]

When the expressions of \( v_0 \) and \( D_p \) are inserted, we can get the following expression:

\[ \Delta P_2 = \frac{32\mu \frac{iQ}{a'b'} \frac{(T + h)}{h} L}{(0.83h)^2} = 93\mu \frac{iQ}{a'b'} \frac{(T + h)}{h^3} L \quad (5.14) \]

### 5.2.3 Local Resistance of Both Inlet and Outlet

The local resistance of inlet passage \( \Delta P_3 \) can be thought as the abrupt contraction resistance.

\[ \Delta P_3 = \xi_1 \frac{v_0^2 \rho}{2} \quad (Pa) \quad (5.15) \]

Where \( \xi_1 \) is the inlet drag coefficient, which can be found with the following value:

\[
\text{Net cross sectional area of the passage} = \frac{a''b'/2}{ab} = \frac{a''b'}{2ab}
\]

the inlet drag coefficient \( \xi_1 \) can be found with the above value.

The local resistance of outlet passage \( \Delta P_4 \) can be thought as the abrupt expansion resistance.

\[ \Delta P_4 = \xi_2 \frac{v_0^2 \rho}{2} \quad (Pa) \quad (5.16) \]

The outlet drag coefficient \( \xi_2 \) can be found with the value of \( a''b'/2ab \).

\[ C = \Delta P_3 + \Delta P_4 \quad (5.17) \]

### 5.2.4 Total Pressure Drop \( \Delta P \)

\[ \Delta P = \Delta P_1 + \Delta P_2 + \Delta P_3 + \Delta P_4 \]

\[ = jA \frac{Q}{a'b'} \frac{(T + h)}{L} + 93i\mu \frac{Q}{a'b'} \frac{(T + h)}{h^3} L + C \quad (5.18) \]
5.3 Optimal Height of Corrugation

When the crest is lower, the number of corrugation becomes large, which will cause the pressure drop of filter media lower, but the pressure drop of structure will increase. The conclusion of the case for higher crest is the opposite. Therefore, there is an optimal height of corrugation crest, which corresponds to the minimum pressure drop.

According to the common mathematic method, the first order and second order of the partial derivative of Eq. (5.18) can be performed with certain values of $Q$ and cross-sectional area, i.e.,

$$\frac{\partial (\Delta P)}{\partial h} = \frac{Q}{a'b'} \left[ \frac{jA}{L} + 93i\mu L \frac{h^3 - 3(T + h)h^2}{h^6} \right]$$  \hspace{1cm} (5.19)

$$\frac{\partial^2 (\Delta P)}{\partial h^2} = \frac{93i\mu LQ}{a'b'} \left( \frac{12T + 6h}{h^5} \right)$$  \hspace{1cm} (5.20)

Since $h > 0$, we know $\frac{\partial^2 (\Delta P)}{\partial h^2} > 0$. So when $\frac{\partial (\Delta P)}{\partial h} = 0$, there is a limit value for $\Delta P$, which is the minimum value. At the same time, the optimal height of $h$ is $h_0$, i.e.,

$$\frac{Q}{a'b'} \left[ \frac{jA}{L} + 93i\mu L \frac{h_0^3 - 3(T + h_0)h_0^2}{h_0^6} \right] = 0$$  \hspace{1cm} (5.21)

When both sides of the equation are multiplied with $\frac{a'b'}{Q}$, it can be simplified to:

$$\frac{jA}{L} + 93i\mu L \frac{h_0^3 - 3(T + h_0)}{h_0^6} = 0$$

$$\frac{h_0^4jA + 93i\mu L^2(h_0 - 3(T + h_0))}{h_0^4L} = 0$$

$$h_0^4jA + 93i\mu L^2h_0 - 279i\mu TL^2 - 279i\mu L^2h_0 = 0$$

$$h_0^4 - 186i\frac{\mu L^2}{jA}h_0 - 279i\frac{\mu L^2}{jA}T = 0$$  \hspace{1cm} (5.22)

Then the value of $h_0$ can be calculated with the trial method.

Now with the structural sizes and related parameters of A, C, and K type filters (the thickness of filter media is 0.25 mm), the parameters needed in the equation can be calculated. Then various pressure drops can be obtained, which is shown in Table 5.2. The optimal heights $h_0$ of corrugation crest can be derived, which are 3.3, 3.3, and 4.6 mm, respectively. The total pressure drops calculated with these data are indeed smaller than that with the original corrugation crest height.
| Type            | Known parameters | Calculation result | Measured value |
|-----------------|------------------|--------------------|----------------|
|                 | \( Q \) (m³/s)  | \( a \) (m)       | \( b \) (m)   | \( L \) (m) | \( h \) (m) | \( T \) (m) | \( e \) (m) | \( \mu \) (Pa·s) | \( A \) (Pa·s/m) | \( a' \) (m) | \( b' \) (m) | \( i \) | \( j \) | \( v_{01} \) (m/s) | \( \xi_1 \) | \( \xi_2 \) | \( \Delta P_{1} \) (Pa) | \( \Delta P_{2} \) (Pa) | \( C \) (Pa) | \( \Delta P \) (Pa) |
| To obtain the optimal corrugation crest height | A 0.2778        | 0.55              | 0.52           | 0.18        | 0.003       | 0.00025     | 0.0002    | 1.83 \( \times 10^{-5} \) | 3.3 \( \times 10^{3} \) | 0.5468      | 0.502       | 0.49     | 1.082 | 1.087 | 2.40 | 0.28 | 0.31 | 68 | 41  | 2 | 111 | 76 | 54 | 130 |
|                 | C 0.2778        | 0.55              | 0.52           | 0.18        | 0.005       | 0.00025     | 0.0002    | 1.83 \( \times 10^{-5} \) | 3.3 \( \times 10^{3} \) | 0.5468      | 0.52        | 0.49     | 1.038 | 1.087 | 2.30 | 0.28 | 0.31 | 110 | 14 | 2 | 126 | 105 | 53 | 158 |
|                 | K 0.2778        | 0.37              | 0.52           | 0.30        | 0.005       | 0.00025     | 0.0002    | 1.83 \( \times 10^{-5} \) | 3.3 \( \times 10^{3} \) | 0.3648      | 0.35        | 0.49     | 1.027 | 1.087 | 3.43 | 0.28 | 0.31 | 96 | 35 | 4 | 135 | 93 | 59 | 152 |
| \( h_0 \)       | A 0.0033        |                   |                |             |             |             |           |             |             |             | 0.543       | 0.508       | 0.49     |             |             |             |             | 74 | 34 | 2 | 110 |
|                 | C 0.0033        |                   |                |             |             |             |           |           |             |             | 0.543       | 0.506       | 0.49     |             |             |             |             | 74 | 34 | 2 | 110 |
|                 | K 0.00455       |                   |                |             |             |             |           |           |             |             | 0.36        | 0.348       | 0.49     |             |             |             |             | 88 | 42 | 4 | 134 |
Now taking the GB-01 HEPA filter as an example, the optimal corrugation crest heights with different drag coefficients of filter media can be calculated, which are shown in Table 5.3.

From the above calculation results, it is found that they are very close to the measured values. The key is that if some parameters in these examples are accurate. Taking the drag coefficient as an example, two different values can be calculated with the data shown in literatures. This is the important reason for the difference above.

Another reason is the error of measurement. For example, except that the corrugation crest height of C type filter is much larger than that of A type filter, other parameters are all the same; the structural pressure drop of C type should be much higher than that of A type, but in measurement the difference between them is only 1 Pa, which is not correct obviously. At present, the value of A is about $4.7 \times 10^3$ for filter media with efficiency “four 9,” and the total pressure drop of filter can reach 190 Pa (including that of inlet and outlet), which is very close to the actual situation.

Secondly, it can be seen that the crest height varies in a small range. For example, for conventional filters, this value is only between 3 and 4 mm.

Thirdly, the following derivation can be performed. Suppose $T = xh_0$, both sides of Eq. (5.21) can be multiplied with $(T + h_0)$, so that it can be simplified as:

$$Q \frac{a'b'}{L} \left[ \frac{jA}{L} + 93i\mu L \frac{-(3x+2)}{h_0^3} \right] = 0 \quad (5.23)$$

$$(T + h_0) \frac{Q}{a'b'} \frac{jA}{L} = (3x + 2)93iQ \mu \frac{T + h_0}{a'b'h_0^3} L \quad (5.24)$$

The left is the pressure drop of filter media $\Delta P_1$, and the right side means $(3x + 2)$ times of frictional resistance $\Delta P_2$. So it can be rewritten as:

$$\Delta P_1 = (3x + 2)\Delta P_2 \quad (5.25)$$

The above calculation shows that if $T = 0.1 \ h_0$, then $\Delta P_1 = 2.3\Delta P_2$. Therefore, for the conventional HEPA filter, when the structural resistance is smaller than half of the filter media resistance, the total resistance reaches the minimum. The crest height of this type is the optimal height with the condition of $L$. From Table 5.3, it can be seen that the conclusion is correct. So the opinion that the total pressure drop reaches the minimum when the resistance of filter media equals to that of structural is wrong.

| $A$ (Pa·s/m) | $T$ (m) | $e$ (m) | $h_0$ (m) | $\Delta P_1 + \Delta P_2$ (Pa) |
|-------------|--------|--------|-----------|-------------------------------|
| $3.3 \times 10^3$ | 0.00028 | 0.00017 | 0.0037 | 101.9 + 45.4 = 147.3 |
| $4.0 \times 10^3$ | 0.00028 | 0.00017 | 0.0036 | 111.2 + 50.3 = 161.5 |
| $4.5 \times 10^3$ | 0.00028 | 0.00017 | 0.0034 | 121.2 + 53.2 = 174.4 |
| $5.0 \times 10^3$ | 0.00028 | 0.00017 | 0.0033 | 130.7 + 56.9 = 187.6 |

Table 5.3 Optimal corrugation crest height $h_0$ of GB-01 HEPA filter ($484 \times 484 \times 220$, 1,000 m$^3$/h, with the head glue)
5.4 Optimal Depth

With the fixed value of crest height, the increase of passage depth of filter can increase the filtration area, which reduces the resistance of filter media. But at the same time the structural resistance along the air passage increases, so there is a smallest depth $L_0$ corresponding to the total resistance.

The first-order and second-order derivative of $L$ are performed for Eq. (5.18), i.e.,

$$\frac{\partial (\Delta P)}{\partial L} = \frac{Q}{a'b'} \left[ 93i\mu \frac{(T+h)}{h^3} - jA \frac{(T+h)}{L^2} \right]$$  (5.26)

$$\frac{\partial^2 (\Delta P)}{\partial L^2} = \frac{2jAQ(T+h)}{a'b'} \frac{1}{L^3}$$  (5.27)

Since $\frac{\partial^2 (\Delta P)}{\partial L^2} > 0$, $\Delta P$ is the minimum for the condition of $\frac{\partial (\Delta P)}{\partial L} = 0$, i.e.,

$$\frac{Q}{a'b'} \left[ 93i\mu \frac{(T+h)}{h^3} - jA \frac{(T+h)}{L^2} \right] = 0$$  (5.28)

$L = L_0$.

$$jA \frac{(T+h)}{L_0^2} = 93i\mu \frac{(T+h)}{h^3}$$  (5.29)

So we know

$$L_0 = \sqrt{\frac{jA h^3}{93i\mu}}$$  (5.30)

Values of $L_0$ for three kinds of filters in Table 5.2 can be obtained, which is illustrated in Table 5.4. It is shown that the structural resistance with $L_0$ is smaller than that with $h_0$. This also applies for the total resistance.

It is found that compared with Eqs. (5.6) and (5.14), both sides of Eq. (5.29) do not have the item $\frac{Q}{a'b'} L_0$. When both sides of Eq. (5.29) are multiplied this item, the following expression can be obtained:

$$\frac{Q}{a'b'} jA \frac{(T+h)}{L_0} = \frac{Q}{a'b'} 93i\mu \frac{(T+h)}{h^3} L_0$$  (5.31)

It means that left and right sides represent the filter media resistance and structural resistance with the optimal depth.
Conclusions can be obtained by the above calculation.
First of all, for the general HEPA filter its depth may increase.
Secondly, due to the extreme small resistance at inlet and outlet, as long as the filter media resistance equals to the structural resistance, the depth of this filter under the condition of \( h \) is optimal. It should be noted, the optimal values of \( L_0 \) and \( h_0 \) are calculated when \( h \) or \( L \) is fixed.

5.5 Corrugation Crest Angle

From the expression of the frictional resistance, with the fixed velocity and length, the influence of the equivalent diameter is the largest among all the influencing factors. When the corrugation crest height is required to be fixed, the corrugation angle will influence the equivalent diameter.

The traditional corrugation angle is \( 90^\circ \). The influence of angle can be expressed as (refer to Fig. 5.5) [4]:

With the corrugation angle \( 90^\circ \), the equivalent diameter
\[
    d = \frac{4F}{S} = \frac{4 \times h/2 \times 2h}{2\sqrt{2h + 2h}} = 0.83h.
\]

With the corrugation angle \( 60^\circ \), the equivalent diameter
\[
    d = \frac{4 \times h/2 \times 1.155h}{3 \times 1.155h} = 0.67h
\]

where
- \( F \) is the cross-sectional area of air passage;
- \( S \) is the perimeter of air passage.

Table 5.4 Optimal calculated depth \( L_0 \) of air passage

| Filter type | \( h \) (m) | Original \( L \) (m) | \( \Delta P \) with \( L \) (Pa) | \( L_0 \) (m) | \( \Delta P = \Delta P_1 + \Delta P_2 \) with \( L_0 \) (Pa) |
|-------------|-------------|---------------------|-----------------------------|-------------|----------------------------------|
| A           | 0.003       | 0.18                | 109                         | 0.23        | 53 + 63 = 106                    |
| C           | 0.005       | 0.18                | 124                         | 0.51        | 38.8 + 39.7 = 78.5               |
| K           | 0.005       | 0.3                 | 131                         | 0.51        | 56.5 + 59.5 = 116                |

Fig. 5.5 Schematic for calculation of corrugation angle
For the separator plate with poor quality, during the folding process the corrugation is lowered by the exerted force (this happens frequently) and the angle becomes \(120^\circ\), the new corrugation height becomes \(h' = 0.75\ h\), which is less than the original height by \(h' = 0.25\ h\). At this time, the equivalent diameter is:

\[
d = \frac{4 \times 0.75h \times 1.732 \times 0.75h}{2 \times 2 \times 0.75 + 2 \times 1.732 \times 0.75h} = 0.7h
\]

The equivalent diameter for the corrugation crest angle \(120^\circ\) and the same corrugation height \(h\) is:

\[
d = \frac{4 \times h/2 \times 2 \sqrt{3}h}{2 \times 2h \times 2 \sqrt{3}h} = \frac{6.928h}{7.454} = 0.928h
\]

The equivalent diameter for the corrugation crest angle \(150^\circ\) and the same corrugation height \(h\) is:

\[
d = \frac{4 \times h/2 \times 2 \times 3.732h}{2 \times 3.864h + 2 \times 3.732h} = \frac{14.928h}{15.192} = 0.983h
\]

The structural resistance is inversely proportional to \(d^2\) according to Eq. (5.13). When the corrugation angle is \(60^\circ\) and \(90^\circ\), the structural resistance is mainly dependent on the frictional resistance, i.e.,

\[
\frac{\Delta P_{2,60^\circ}}{\Delta P_{2,90^\circ}} = \frac{0.83^2}{0.67^2} = 1.53
\]

Similarly, the ratio of structural resistance between the case of corrugation crest angle \(120^\circ\) with less corrugation height \(h'\) to the above case with corrugation crest angle is \(90^\circ\) is:

\[
\frac{\Delta P_{2,120^\circ}}{\Delta P_{2,90^\circ}} = \frac{0.83^2}{0.7^2} = 1.41
\]

The ratio of structural resistance between the case of corrugation crest angle \(120^\circ\) with the same corrugation height \(h\) and the above case with corrugation crest angle is \(90^\circ\) is:

\[
\frac{\Delta P_{2,120^\circ}(h)}{\Delta P_{2,90^\circ}} = \frac{0.83^2}{0.928^2} = 0.8
\]

The ratio of structural resistance between the case of corrugation crest angle \(150^\circ\) with the same corrugation height \(h\) and the above case with corrugation crest angle is \(90^\circ\) is:
The following conclusions can be reached with the above analysis:

1. With the constant corrugation height, the larger the corrugation crest angle is, the smaller the structural resistance is.

2. When the corrugation crest angle increases to 150°, the equivalent diameter of the air passage is near the crest height. So it has little effect on increase of crest angle.

3. For the widely used partition board at present, which is not dipped with glue (gluing or adhesive spraying only happens on the surface or even one side of the surface), the texture is very soft and is not rigid. With very little pressure, the crest angle becomes larger and crest height decreases, which increases the structural resistance.

4. If the production process of separator improves, which means neither the past dipping method with serious pollution nor the chrome paper coated with glue will be used, instead one fixed process of crest height is adopted, the crest angle can increase from the traditional 90° to about 150°, which can reduce the structural resistance by about 30%.

5.6 Structural Parameters for Filters Without Separator

For the filter without separator, suppose the partition (line) height is $h$ and partition interval is $B$, the equivalent diameter of air passage is:

$$d = \frac{4hB}{2(h + B)}$$

Because the magnitude of $B$ is centimeter and $B \gg h$, so:

$$d \approx \frac{4Bh}{2B} = 2h \quad (5.32)$$

Since the linewidth occupies a very small proportion of the effective area, $j$ can be neglected in the above process of derivation. Since the corrugation number for filters without separators is more than that with separator, the filter media pasted onto the two ends can be neglected. So the value of $i$ can also be neglected. The total resistance can be obtained with Eqs. (5.14), (5.18), and (5.32) as follows:

$$\Delta P = \frac{AQ}{ab} \frac{(T + h)}{L} + 16\mu \frac{Q}{ab} \frac{(T + h)}{h^3}L + C \quad (5.33)$$
The expression to calculate the optimal crest height is:

\[ h_0^4 - 32 \frac{\mu L^2}{A} h_0 - 48 \frac{\mu L^2}{A} L = 0 \]  (5.34)

The expression of optimal depth is:

\[ L_0 = \sqrt[3]{\frac{Ah^3}{16\mu}} \]  (5.35)

The conclusion about the optimal match between the filter media resistance and the structural resistance is also valid for the filter without separator.

Let \( A = 3.5 \times 10^3 \) Pa·s/m, the depth of common filter without separator \( L = 0.08 \) m, and the thickness of filter material \( 0.003 \) m, we can obtain the following expression:

\[
h_0^4 = \frac{32 \times 1.83 \times 10^{-5} \times 0.08^2}{3.5 \times 10^3} h_0 - \frac{48 \times 1.83 \times 10^{-5} \times 0.08^2}{3.5 \times 10^3} \times 0.0003 = 0
\]

\[
h_0^4 - 1.07 \times 10^{-9} h_0 = 0.48 \times 10^{-12}
\]

With the trial method, we get \( h_0 = 1.14 \) mm. Insert it into the above equation, we obtain:

\[ 1.69 \times 10^{-12} - 1.22 \times 10^{-12} = 0.47 \times 10^{-12} \approx 0.48 \times 10^{-12} \]

Suppose the dimension of filter is 484 mm × 484 mm × 80 mm and the flow rate is 1,000 m³/h (the side width is 15 mm), the filter material resistance and structural resistance with \( h_0 = 1.14 \) mm can be calculated with Eq. (5.33) (the third item about inlet and outlet resistances is neglected):

\[
\Delta P_1 = \frac{3.5 \times 10^3 \times 0.2778}{0.454^2} \times \frac{(0.00114 + 0.0003)}{0.08} = 84.9 \text{ Pa}
\]

\[
\Delta P_2 = 16 \times 1.83 \times 10^{-5} \times \frac{0.2778}{0.454^2} \times \frac{(0.00114 + 0.0003)}{(1.14 \times 10^{-3})^3} \times 0.08 = 30.7 \text{ Pa}
\]

\[ \Delta P_1 + \Delta P_2 = 115.6 \text{ Pa} \]

According to Sect. 5.3, we know:

\[ x = \frac{T}{h} = \frac{0.0003}{0.00114} = 0.263 \]
So with Eq. (5.25), we know:

\[ \Delta P_1 = (3 \times 0.263 + 2) \Delta P_2 = 2.79 \Delta P_2 \]

and

\[ 2.79 \times 30.7 = 85.6 \text{ Pa} \]

It is reasonable to consider that there is no difference between this value and 84.9 Pa obtained beforehand. It proves that the resistance of filter media should match with the frictional resistance.

Because of the process that aims to reduce the installation height of filter, the thickness of filter without separator at present is usually less than 80 mm. The relationship between \( \Delta P \) and \( h \) can be calculated, which is shown in Fig. 5.6 \((A = 3.5 \times 10^3)\).

But now the height of partition line (the height for the adhesion of two lines) is 1.5 mm, instead of 1.14 mm. Other parameters are almost the same as before mentioned values. We can get:

\[ \Delta P_1 = 106 \text{ Pa} \]
\[ \Delta P_2 = 17 \text{ Pa} \]
\[ \Delta P_1 + \Delta P_2 = 123 \text{ Pa} \]

For the common products, \( A \) is about \( 5 \times 10^3 \). The theoretical total resistance is about 180 Pa, while the actual resistance of the product is higher than this value by 20%. Except for the uncertainty of the calculated parameters, there is also space for the discussion in theory.

But 80 mm is not the optimal depth for the case \( h = 1.5 \text{ mm} \). According to Eq. (5.35), the optimal depth for the case \( h = 0.0015 \text{ mm} \) should be:

\[ L_0 = \sqrt{\frac{3.5 \times 10^3 \times (1.5 \times 10^{-3})^3}{16 \times 1.83 \times 10^{-5}}} = 0.2 \text{ m} \]
After calculation, $\Delta P = 84.9$ Pa, which is smaller than the case $L = 0.08$ m by 38 Pa. Figure 5.7 shows the relationship between $\Delta P$ and $L$ for this kind of filter ($A = 3.5 \times 10^3$).

From the technical and economic point of view, it is still appropriate to choose the depth 0.08 m.

For the corrugated air filter without separator which is made of polypropylene fiber, $A = 0.3 \times 10^3$ while other parameters remain the same, we know:

| $h$ (m) | $L_0$ (m) | $\Delta P$ (Pa) |
|--------|-----------|-----------------|
| 0.0015 | 0.06      | 24.6            |
| 0.001  | 0.032     | 32.6            |

Figure 5.8 illustrates the relationship between $\Delta P$ and $L$ ($A = 3.5 \times 10^3$). It shows that $L = 0.06$ m is indeed the optimal value for $h = 0.0015$ mm.

For the sub-HEPA filter without separator when $L = 0.08$ m, the optimal crest height can be thought as $h_0 = 2.2$ m from Fig. 5.9 ($A = 3.5 \times 10^3$). When in practice $h = 1.5$ mm, the resistance is higher than that with optimal crest height by 4 Pa, where the difference is very small.

### 5.7 Calculation of Tubular Filter

If the resistance of filter is required to be lower, one method is to choose filter material with less resistance and the other one is to reduce the structural resistance. The tubular structure may meet the requirement of the second method. Now analysis for the structure of tubular sub-HEPA filter is performed as Chap. 4.

The frictional resistance of inlet passage is supposed to be equal to that of outlet passage, i.e.,

$$\Delta P_{2(1)} = \Delta P_{2(2)}$$
From the process point of view, the tubular diameter which is easy for process is determined at first. The appropriate number of tubes within the area $a \times b$ is chosen. Then the total resistance can be calculated with the following expression (refer to Fig. 5.10):

**Fig. 5.8** Relationship between $\Delta P$ and $L$ for sub-HEPA filter without separator ($h = 0.0015$ mm, $A = 3.5 \times 10^3$)

**Fig. 5.9** Relationship between $\Delta P$ and $h$ for sub-HEPA filter without separator ($h = 0.08$ mm)
\[ \Delta P = \Delta P_1 + 2\Delta P_{2(1)} + \Delta P_{2(3)} + \Delta P_{2(4)} \]  
(5.36)

where \( \Delta P_1 \) is the filter media resistance (Pa)

\[ \Delta P_1 = A \frac{Q}{n\pi d} = Av \]  
(5.37)

\( \Delta P_{2(1)} \) is the frictional resistance of inlet passage (Pa), which can be obtained with the method for corrugated filter

\[ \Delta P_{2(1)} + \Delta P_{2(2)} = 2\Delta P_{2(1)} = \frac{32\mu v_0}{d^2}L = \frac{86\mu v_0}{d^2}L \]

\( \Delta P_{2(3)} \) is the resistance of inlet perforated plate (Pa)

\[ \Delta P_{2(3)} = \zeta_1 \frac{u^2 \rho}{2} \]  
(5.38)

\( \Delta P_{2(4)} \) is the resistance of outlet (Pa).

In the above items, \( n \) is the number of tubes;

\( d \) is the diameter of tube, m;

\( v \) is the filtration velocity, m/s;

Fig. 5.10 Calculation parameters of tubular filter. 
1 filtration tube, 2 plug head, 3 vane, 4 frame
\( v_0 \) is the inlet velocity of tube, m/s;
\( d' \) is the equivalent diameter of air passage. For the case in Fig. 5.11, the vane is placed in the middle of the tube, where the tubular space is divided into two parts. For half circle, \( d' = 0.61d \);
\( u \) is the velocity at the weather side, m/s;
\( \zeta_1 \) is the inlet drag coefficient

\[
\zeta_1 = \frac{1 - c^2S^2}{c^2S^2}
\]  

(5.39)

where \( c \) is the contraction coefficient of the pore where air passes through. Usually it is 0.9. When the tubular diameter is as small as 15 mm, filter paper will become corrugated at the throat where the plug is placed. The air passage becomes narrower or even half when \( c \) is 0.5.

\( S \) is the porosity

\[
S = \frac{\text{Net total area of the pores}}{\text{Area of windward surface}} = \frac{F_1}{F_2}
\]  

(5.40)

where

\( F_1 \) is the net total area of pores, which is equivalent to the subtraction of the cross-sectional area of vane 0.001\( d_1 \) from the total area (which is dependent on the plug diameter \( d_1 \) (m), \( d_1 = d - 0.002 \) m).

\( F_2 \) is the area at the weather side, which is assumed the same as that of corrugated HEPA filter, i.e., 0.484 \( \times \) 0.484 m².

Since the outlet of filtration tube is flat, the actual outlet cross-sectional area is very close to that of air filter and it is gradually changed. \( \Delta P_{2(4)} \) can be neglected.

Taking the parameters of YGG low-resistance sub-HEPA filter introduced in Chap. 4 as an example, various resistances are calculated with the above method, which is shown in Table 5.5. From this table, the calculated result is very close to the experimental value, which means the calculation method is feasible. It is also shown that with the same dimension and flow rate, the structural resistance of tubular filter is much smaller than that of corrugated filter. At present, due to other reasons, the tubular HEPA filter made of polypropylene fiber has not appeared in the market.
Table 5.5 Calculation example of tubular filter

| Filtration tube (m) | Number of filtration tubes (#) | Tube length (l/h/m) | Opening area (m²) | Filtration velocity v (m/s) | Face velocity u (m/s) | Velocity v₀ in tube (m/s) | A (Pa·s/m) | C | ζ₁ | S | Calculation result (Pa) | Measured result (Pa) |
|-------------------|-------------------------------|---------------------|-------------------|--------------------------|---------------------|--------------------------|------------|---|-----|---|---------------------|---------------------|
| 0.019             | 330                           | 0.215               | 0.0692            | 0.066                    | 1.35                | 4.01                     | 5.5 × 10²  | 0.9| 10  | 0.336 | 36.3 3.7             | 10.9 50.9           |
| 0.019             | 330                           | 0.215               | 0.0692            | 0.066                    | 1.35                | 4.01                     | 7 × 10²    | 0.9| 10  | 0.336 | 46.2 3.7             | 10.9 60.8           |
| 0.019             | 347                           | 0.215               | 0.0728            | 0.0624                   | 1.35                | 3.81                     | 7.5 × 10²  | 0.9| 9   | 0.353 | 46.8 3.5             | 9.8 60.1            |
| 0.019             | 347                           | 0.215               | 0.0728            | 0.0624                   | 1.35                | 3.81                     | 5.6 × 10²  | 0.9| 9   | 0.353 | 34.9 3.5             | 9.8 48.2            |
| 0.019             | 347                           | 0.215               | 0.0728            | 0.0624                   | 1.35                | 3.81                     | 7.8 × 10²  | 0.9| 9   | 0.353 | 48.6 3.5             | 9.8 61.9            |
| 0.015             | 630                           | 0.215               | 0.0753            | 0.044                    | 1.35                | 3.69                     | 7 × 10²    | 0.9| 29.3| 0.336 | 31 5.5              | 31.9 68.4           |

Note: ΔP₁, ΔP₂, ΔP₃, ΔP represent pressure drop calculations and measured pressure drops respectively.
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