Internal stress behavior of the short ceramic fiber reinforced aluminum alloy under tensile deformation

Katsuyuki Funaki a,*, Shigeki Takago a, Kaname Fujii a, Toshihiko Sasaki b, Kazuo Kitagawa b, Yukio Hirose b, William A. Ellingson c

a Department of Machinery and Metal, Industrial Research Institute of Ishikawa, 2-1 Kuratsuki, Kanazawa 920-8203, Japan
b Graduate School of Natural Science & Technology, Kanazawa University, Kakuma-machi, Kanazawa 920-1192, Japan
c Energy Technology Division, Argonne National Laboratory, 9700 South Cass Avenue Argonne, IL 60439-4838, USA

Received 4 March 2005; received in revised form 1 August 2005; accepted 2 August 2005
Available online 26 October 2005

Abstract

The in situ measurement of phase stress under tensile deformation on an A6061 alloy reinforced with SiC whiskers (Al/SiCMMC: Metal Matrix Composite) was performed using the X-ray diffraction technique. In order to raise a preciseness of measurements, we applied a profile fitting technique to separate the nearby located diffraction peak. Tensile deformation on elastic to plastic range was applied by four points bending device and discussed internal stress behavior in the short ceramic fiber reinforced MMC. Phase stress in Al matrix was increased linearly up to 2800×10^6 in strain and then saturated immediately. On the other hand phase stress in SiC whiskers shows an unstable stress behavior. It was decreased at first because of the Poisson’s effect from Al matrix but reversed over 500×10^6 applied strain. The measured phase stress behavior in elastic region agreed with the calculations using micromechanics based on Eshelby/Mori–Tanaka model except for this unstable internal stress region. The macro stress behavior in plastic region was extremely small than that of the tensile test results. It supposed that the mechanism of strength is not so much the fiber reinforcing as the dispersion strengthening like the Orowan mechanism. Regarding the fatigue property, the Al/SiC MMC, this was lower than that of the A6061 alloy. On the Al/SiC MMC specimen, many micro void formations were observed around the fatigue crack tip even under the ΔKth of A6061. It was considered that these were caused by the high gradient of residual stress on composite process and the unstable stress behavior in low ΔK region.

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Keywords: Metal matrix composite; SiC whisker; X-ray diffraction; Peak separation; Profile fitting; Tensile deformation; Internal stress behavior; Phase stress; Micro mechanics; Fatigue crack propagation

1. Introduction

The aluminum alloy reinforced with ceramic short fibers (SF-MMC) has light weight and high wear-resistance properties. Since, it is able to plastically-work easily such as during extrusion, forging or rolling, it is expected there would be significant application for automobile and aero-space fields. However, this material consists of ductile matrix and brittle fibers. The interaction between these components will cause a complex internal stress under tensile state. On the other hand the SF-MMC shows the elastic–plastic behavior during tensile deformation in spite of including brittle fibers. This means strains in each phase in matrix and fibers are different from the macro strain of the SF-MMC. Therefore, it is interesting to measure the stress of each phase (called phase stress) in tensile state to analyze reinforcement and the fracture mechanism of the SF-MMC. In this study, we tried in situ measuring of phase stress in Al matrix and SiC whiskers using by X-ray diffraction technique [1–4] on elastic to plastic deformation range and discussed internal stress behavior of the SF-MMC by comparing experimental data with the calculation results by the micromechanics on Eshelby/Mori–Tanaka model [1,5]. In the X-ray stress analysis, high intensity diffraction and single peak are the necessary condition for precision 2θ measurement. It is well known that Fe Kα X-ray is suitable for the measurement of SiC ceramic. However, the diffraction of SiC phase is close to that of Al phase in the higher 2θ angle range. Therefore, the foot of two peaks is often overlapped and this makes difficulty to determine the base line of back ground noise. In order to raise a preciseness of measurements, profile fitting technique [6] was applied to the diffraction profile for
the purpose to separate the nearby located peak. In addition, peak separation of K\textsubscript{2} double line was performed to the SiC whiskers diffraction for avoiding K\textsubscript{2} and K\textsubscript{2} duplication owing to its narrow peak width. It is considered the internal stress effect caused by the interaction between whiskers and matrix more influences fatigue properties than the tensile strength. Therefore, we discussed from the viewpoint of fatigue crack propagation too.

2. Theory

2.1. X-ray stress measurement on dual phase material

Bragg angle, 2\theta_\text{\psi}, is obtained by diffraction experiment with characteristic X-ray. Subscripts \phi and \psi are angles which express the orientation of the strain, \epsilon_\text{\phi\psi}, as shown in Fig. 1.

According to Bragg’s law, we have

\[ \epsilon_\text{\phi\psi} = \frac{1}{2} (2\theta_0 - 2\theta_\text{\phi\psi}) \cot \theta_0 \] (1)

where 2\theta_0 indicates the diffraction angle at stress free. \epsilon_\text{\phi\psi} is a normal strain determined by X-ray method. The fundamental equation for the stress measurement is expressed as following equation [7], when both of matrix and inclusion phases have the isotropic elastic body.

\[ \epsilon_i^\text{\phi\psi} = \left( \frac{1 + \nu}{E} \right)^\text{ph} \left( \sigma_{11}^i \cos^2 \phi + \sigma_{12}^i \sin 2\phi + \sigma_{22}^i \sin^2 \phi - \sigma_{33}^i \right) \times \sin^2 \psi + \left( \frac{1 + \nu}{E} \right)^\text{ph} \sigma_{33}^i - \left( \frac{\nu}{E} \right)^\text{ph} \left( \sigma_{11}^i + \sigma_{22}^i + \sigma_{33}^i \right) + \left( \frac{1 + \nu}{E} \right)^\text{ph} \left( \sigma_{11}^i \cos \phi + \sigma_{22}^i \sin \phi \right) \sin 2\psi \] (2)

The symbol ‘i’ indicates a phase, for example i = M means matrix and i = f the second phase, E and \nu are Young’s modulus and Poisson’s ratio. The subscript ‘ph’ means the phase X-ray elastic constant as distinguished from the mechanical elastic constant.

When \phi = 0 Eq. (2) leads to

\[ \sigma_{11}^f - \sigma_{33}^f = \left( \frac{E}{1 + \nu} \right)^f \left( \frac{\partial \epsilon_\text{\phi\psi}(\phi = 0^\circ)}{\partial \sin^2 \psi} \right)^f \] (3)

Eq. (3) can be used to obtain primary stresses from X-ray diffraction data (2\theta_\text{\phi\psi} and sin\textsuperscript{2} \psi).

Stresses in individual phases, so-called phase stress, can be obtained directly by applying the X-ray stress measurement method [8]. Applying the above equations and measuring strain \epsilon_\text{\phi\psi} in \phi = 0, 45, 90, 180, 225 and 270°, we obtain all the tri-axial stress components \sigma_{ij}.

When residual stress generate in the dual phase composite, microscopic state of stress as shown in Fig. 2 is usually built up due to the misfit of physical and mechanical properties between the constituents in the material. The following equations can be deduced from the equilibrium conditions for micro stresses by defining \sigma_{ij}^M, \sigma_{ij}^f as micro stresses in matrix and in the second phase, \epsilon_{ij}^M, \epsilon_{ij}^f as phase stresses in matrix and in the second phase [9–11].

\[ \begin{align*}
\sigma_{ij}^M &= (1-f)\sigma_{ij}^M + f\sigma_{ij}^f \\
\sigma_{ij}^f &= \sigma_{ij}^M - \sigma_{ij}^0 \\
\sigma_{ij}^0 &= \sigma_{ij} - \sigma_{ij}^0
\end{align*} \] (4)

where f denotes the volume fraction of the second phase.

2.2. Theoretical solution of a phase stress

Using the Eshelby’s theory and the Mori–Tanaka method [12], Lin and Mura [13] arrived at the following equations when an ellipse inclusion with the volume fraction f is embedded in an isotropic matrix. (called Eshelby/Mori–Tanaka model)

\[ \sigma_i^M = \sigma_i^0 + \sigma_i^m = \sigma_i^0 - f C (S-I) [C - (C-C^*)] [S - f(S-I)]^{-1} \times [(C-C^*) C^{-1} \sigma_i^0 + C^* \Delta \sigma_i^0] \] (5)

\[ \sigma_i^f = \sigma_i^0 + \sigma_i^0 = \sigma_i^0 - (1-f) C (S-I) [C - (C-C^*)] [S - f(S-I)]^{-1} \times [(C-C^*) C^{-1} \sigma_i^0 + C^* \Delta \sigma_i^0] \] (6)
where $C$ and $C^*$ are mechanical elastic constants of the matrix and second phase. $S$ means the Eshelby’s tensor [14], $I$ is the unit tensor. $\Delta \epsilon_{ij}^p$ is a difference of the plastic strain between the matrix and the inclusions.

Next, mechanical elastic constants of the composite material are obtained by

\[
\begin{align*}
C^0 &= C + fD^{-1}(C - C^*)F^{-1} \\
D &= (C^* - C)S + C \\
F &= C + fCD^{-1}(S - I)(C - C^*)
\end{align*}
\]

(7)

For dual phase composites, which consist of both isotropic spherical inclusions and an isotropic matrix, $\sigma^M$ and $\sigma^I$ are expressed by

\[
\begin{align*}
\sigma^M_1 &= (A - B) \epsilon_1 - f \Delta \sigma^p + 3B \sigma_1^0 - 3B_f \Delta \epsilon_1^p \\
\sigma^M_2 &= (A - B) \epsilon_2 - f \Delta \sigma^p + 3B \sigma_2^0 - 3B_f \Delta \epsilon_2^p \\
\sigma^M_3 &= (A - B) \epsilon_3 - f \Delta \sigma^p + 3B \sigma_3^0 - 3B_f \Delta \epsilon_3^p
\end{align*}
\]

(8)

\[
\begin{align*}
\sigma^I_1 &= (A^* - B^*) \epsilon_1 - (1 - f) \Delta \sigma^p + 3B^* \sigma_1^0 - 3B_1(1 - f) \Delta \epsilon_1^p \\
\sigma^I_2 &= (A^* - B^*) \epsilon_2 - (1 - f) \Delta \sigma^p + 3B^* \sigma_2^0 - 3B_2(1 - f) \Delta \epsilon_2^p \\
\sigma^I_3 &= (A^* - B^*) \epsilon_3 - (1 - f) \Delta \sigma^p + 3B^* \sigma_3^0 - 3B_3(1 - f) \Delta \epsilon_3^p
\end{align*}
\]

(9)

where

\[
\begin{align*}
I_1 &= s_1^0 + s_2^0 + s_3^0, \\
\Delta \sigma^p &= (A_1 - B_1)(\Delta \epsilon_1^p + \Delta \epsilon_2^p + \Delta \epsilon_3^p), \\
A &= \frac{K - \alpha(K - K^*)}{3Q}, \\
B &= \frac{\mu - \beta(\mu - \mu^*)}{3R}, \\
A^* &= \frac{K}{3Q}, \\
B^* &= \frac{\mu^*}{3R}, \\
A_1 &= \frac{3(\alpha - 1)KK^*}{Q}, \\
B_1 &= \frac{2(1 - \mu)\mu^*}{R}, \\
\alpha &= \frac{1 + \nu}{3(1 - \nu)}, \\
\beta &= \frac{2(4 - 5\nu)}{15(1 - \nu)}, \\
K &= \frac{E}{3(1 - 2\nu)}, \\
K^* &= \frac{E^*}{3(1 - 2\nu^*)}, \\
\mu &= \frac{E}{2(1 + \nu)}, \\
\mu^* &= \frac{E^*}{2(1 + \nu^*)}, \\
Q &= K - [\alpha - f(\alpha - 1)](K - K^*), \\
R &= \mu - [\beta - f(\beta - 1)](\mu - \mu^*)
\end{align*}
\]

(10)

where $K$ and $\mu$ are bulk modulus and shear elastic stiffness of matrix. The symbol $*$ means second phase parameter as distinguished from that of matrix. Overall Young’s modulus and Poisson’s ratio of the composite material are as follows:

\[
\begin{align*}
\nu^0 &= \frac{-2[K - (1 - f)(K - K^*)\alpha]\mu R}{6[\mu - (1 - f)(\mu - \mu^*) \beta]K Q} + 2[K - (1 - f)(K - K^*)\alpha]\mu R
\end{align*}
\]

(11)

2.3. Profile fitting using by the model function

The parameters estimated by the profile separation are as follows: (1) number of peaks, (2) shape of peak profile (model function), (3) full width at half maximum intensity (FWHM), (4) peak position, (5) peak height, (6) back ground gradient. A known function was applied to the model function and the base line. Parameters of the full width at half maximum, the peak position and the peak height do not show linearity between the model function, so these parameters were optimized by using the non-linear least squares method. The modified Marquardt method [15] was used to obtain the minimum sum of residual squares quickly. X-ray diffraction profile is a function of Bragg angle for the diffraction intensity and can be express as Lorentz function $f_L(x)$ or Gaussian distribution $f_G(x)$ as follows [16]:

\[
f_L(x) = \frac{h}{\left[1 + \frac{(x - p)/w}{2}\right]^2}
\]

(12)

\[
f_G(x) = h \exp \left\{ \ln 2 \left[ \frac{(x - p)^2}{w} \right]^2 \right\}
\]

(13)

were $x$, $2\theta$; $p$, peak position; $h$, height of an X-ray diffraction peak; $w$, half width of FWHM.

Both functions were determined by three parameters, but these are not always fitting through the foot to the peak satisfactorily. Lorentz function is suited for the fitting all through profile approximately and Gaussian distribution appropriate for the peak top approximation [17]. In this experiment, in order to obtain accurate fitting, following pseudo-Voigt function $f_v(x)$ [18], synthesized function of $f_L(x)$ and $f_G(x)$, was used for the profile fitting.

\[
f_v(x) = \eta f_L(x) + (1 - \eta)f_G(x)
\]

\[
= \frac{\eta h}{1 + \left(\frac{x - p}{w}\right)^2} + (1 - \eta)h \exp \left[ -\ln 2 \left( \frac{(x - p)^2}{w} \right)^2 \right]
\]

(14)

where $\eta$ is a value varies 0 to 1 continuously with measurement condition. The characteristic X-ray Kz is double line which duplicates $\alpha 1$ and $\alpha 2$ radiations, therefore, these diffractions can be fit using pseudo-Voigt function as $f_1(x)$, $f_2(x)$ respectively. The following model function $f(x)$ for Kz double line is obtained.

\[
f(x) = f_1(x) + f_2(x)
\]

(15)

Supposing back ground level is primary function and using the model functions for the Al phase $g_{Al}(x)$ and SiC phase $g_{SiC}(x)$, the measured diffraction profile is approximated following model function $y(x)$ [19].

\[
y(x) = g_{Al}(x) + g_{SiC}(x) + a + bx
\]

(16)

where the back ground gradient $b$ is determined by linear least square method using ten data at both ends.
3. Experimental procedure

The Al/SiCw MMC specimens were prepared using a modified squeeze casting method [3] which permits three-dimensional infiltration of a molten A6061 alloy to the SiC whisker preform. Pressure in the mold reaches 100 MPa during solidification. After casting, the composites were heat-treated in T6 condition to make an improvement on their mechanical properties. The density of specimen and volume fraction (f) of SiC whiskers was measured by the Archimedes’ method using distilled water. The relative density of A6061 alloy to the theoretical value (ρ/ρ0) is more than 0.998, f was calculated to be approximately 0.15. The chemical composition of matrix alloy is shown in Table 1.

The tensile test was conducted by using rod type specimens with parallel area of 8 mm in diameter. The fracture toughness test was also carried out by using sub size compact tension (CT) specimens with 10 mm in thickness and a fatigue pre-crack was introduced in accordance with the JSME S-001 [20] by using servo-hydraulic closed loop testing machine. The result obtained at first fracture toughness test was not satisfied the condition of small scale yielding and consequently the starting point of instability crack (Ki) were monitored by the alternating current potential method.

The bar type specimens with 50×6×4 mm3 in dimension was used for the X-ray stress measurement. The surfaces of specimens were super finished by the electro-mechanical lapping technique. This finishing consists of the electro polishing and mechanical lapping using 1/4 μm diamond abrasive. The polishing and lapping were carried out by turn. There was no work stress after this finishing process.

X-ray diffractions obtained by Fe Kα radiation were recorded using an X-ray stress analyzer (Rigaku MSF-PSPC). The X-ray irradiated area was a circle of 4 mm in diameter at the center of the specimen. The average size of Al grain and SiC whiskers were 200 μm in diameter and 35 μm in length, respectively. Table 2 shows the characteristics of used SiC whiskers [21].

The conditions of X-ray measurement were given in Table 3. Measured profile was done a background canceling and then processed absorption and Lorentz correction. Profile fitting using the pseudo-Voigt function was conducted after that. The diffraction angle 2θ was determined by the peak top of fitting profile. These processes were performed consecutively by the original software. Phase stress was determined by the sin²ψ method using Eq. (3). On the X-ray elastic constant calculation, we applied Kroner model to lead from the elastic constant of monocrystal [22]. The obtained values were E=66.8 GPa, ν=0.358 for Al(400) and E=387 GPa, ν=0.194 for SiC(311), respectively.

Tensile deformation for the specimen was performed by using a modified four point bending device as shown in Fig. 3. Applied strain was read off by the stuck strain gauge on the backside and controlled by the screw installed at bottom.

4. Results

4.1. Characteristics of the specimen

Mechanical properties and fracture toughness of specimens are shown in Table 4. And Fig. 4 shows a metallurgical structure of the Al/SiCw MMC specimen. Bright part is whiskers and they are orientated at random to the loading axis. This composite shows a good wetting and observed no voids.

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### Table 1

| Chemical compositions of matrix alloy |
|-------------------------|---|
| Si | 0.51 |
| Fe | 0.30 |
| Cu | 0.31 |
| Mn | 0.02 |
| Mg | 0.88 |
| Cr | 0.14 |
| Zn | 0.02 |
| Ti | 0.15 |
| Al | Bal |

A6061

### Table 2

| Characteristics of used SiC whiskers |
|--------------------------|---|
| Average diameter (μm) | 0.5 |
| Average length (μm) | 35 |
| Average aspect ratio | 70 |
| Specific gravity (Mg/m³) | 3.2 |
| Elastic modulus (GPa) | 490 |
| Crystal structure | Cubic |
| Lattice parameter (nm) | a=b=c=0.43589 |

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4.2. Profile fitting of a diffraction using by non-linear least squares method

On the diffraction profile of Al/SiCw MMC measured by Fe Kα X-ray, the diffraction angles of Al(400) and SiC(311) locate nearby. Then foots of these peaks overlap each other. It causes error for determination of the peak angle because of lacking in the accuracy on the background canceling. In this experiment, the profile fitting technique by pseudo-Voigt function was used for the peak separation. In this process, the peak angle, intensity and FWHM were optimized by using the non-linear least squares method and the slope of base line for the back ground canceling was optimized by using the linear least squares method. Typical diffraction and its separation results are shown in Fig. 5. The profile fitting by pseudo-Voigt function shows good coincidence with the overall including foot of the peak and consequently this is applicable to the analysis of the material which has large back ground level such as a composite. On the other hand, whiskers in the composite are influenced by the magnitude of the tensile or compressive stress by the poisson effect during deformation on account of orienting at random to the loading axis. The ceramics whiskers have narrow FWHM because of the low disarray of lattice therefore, they tend to appear the duplicate profile arising from Kα double line when duty loading. Fig. 6 shows a typical diffraction from SiC(311) plane at 7000 ± 10 K6 strain applied (ε1′) and its separation results. The waves length used of Fe Kα are the double lines λ1 = 0.1936 nm, λ2 = 0.1940 nm and intensity ratio is 0.5. This profile is measured as a duplicating of Kα2 line. When applying the analysis using ordinary FWHM method, it includes error for determination of the peak angle. The peak separation technique of Kα double line can be obtained the peak angle with accuracy. As recognized from Fig. 5, SiC(311) diffraction is difficult to determine left side base on account of overlapping to the foot of Al(400) peak. The results comparing the data (circle) to the fitting profile (solid line) were revolved counterclockwise. This indicated that there was a separation of data noted by the base line horizontal correction. So it seems to cause a slight deviation between them. In reality, the fitting calculations are in agreement with measured raw data. You will recognize this to be true by the comparison of circle plots and solid line. Since, the peak separation of Kα double line is performed for all SiC diffractions to determine the peak angle.

4.3. θ vs. sin²ψ diagrams

The 2θ vs. sin²ψ diagrams of the MMC specimen under various strains are shown in Fig. 7. The diagram for Al(400) is Fig. 7(a) and SiC(311) diagram is Fig. 7(b). Lines between plots were obtained from the regression analysis by using a linear least squares method. In Fig. 7(a), three lines under ε1′ = 2400 × 10⁻⁶ have an intersection but the lines for ε1′ = 5000 × 10⁻⁶ and 7500 × 10⁻⁶ shift to right from the intersection owing to the plastic deformation. On the SiC diagram, all lines show good linearity and cross at one spot even at ε1′ = 7500 × 10⁻⁶. The line gradient M is decreased with increasing of applied strain and the intersection to the vertical axis 2θ = 0 is increased. Residual stress caused during a compounding process remains in the MMC since M is not zero but has a slight positive gradient even at a strain of ε1′ = 0.

### Table 4

| Material      | 0.2% proof stress σ0.2 (MPa) | Tensile strength σB (MPa) | Fracture elongation δ (%) | Young’s modulus E (GPa) | Fracture toughness K (Mpa/m) |
|---------------|-----------------------------|----------------------------|--------------------------|------------------------|-----------------------------|
| 6061 alloy    | 300                         | 320                        | 10.5                     | 68                     | 21.7                        |
| Composite     | 386                         | 481                        | 5.2                      | 94                     | 12.9                        |

f=0.15.

Fig. 4. Micro-photograph of Al/SiCw composite.

Fig. 5. Profile fitting and separation of diffraction.
5. Discussion

5.1. Phase stress in the short fiber reinforced MMC

Phase stresses $\sigma_{11} - \sigma_{33}$ for the Al matrix and SiC whiskers were determined from $M$ of $\sin^2 \psi$ diagrams and X-ray elastic constant calculated from monocrystal data using Eq. (3). Fig. 8 shows the relation between $\sigma_{11} - \sigma_{33}$ and $\epsilon_{11}$. For the purpose of comparison, stress–strain curve obtained by tensile test (SS curve) and macro stress $\sigma_{ij}^0$ calculated by Eq. (4) were also determined. In Al matrix, a tensile stress of 10 MPa remained and the phase stress increased linearly up to $\epsilon_{11} = 2800 \times 10^{-6}$ then saturated on account of the plastic deformation immediately. The gradient at the proportionate region is 67.5 GPa. This is very close to the Young’s modulus shown in Table 4. On the other hand a compressive stress of more than 100 MPa remained in SiC whiskers and phase stress showed an unstable stress behavior which decreased at first but reversed over $\epsilon_{11} = 500 \times 10^{-6}$. This unstable stress region, dispropor- tion between phase stress and applied strain, is caused by the large difference of Young’s modulus in Al and SiC. As concerns composite materials, it is considered that the deformation of small Young’s modulus phase takes precedence over the deformation of large Young’s modulus phase in low strain region. The instability of internal stress in SiC whiskers should be caused due to the reaction such as poisson effect from Al matrix. The gradient after the unstable stress region is 122 GPa. This is smaller than the Young’s modulus of SiC whiskers shown in Table 2. Although after the plastic deformation of Al matrix ($\epsilon_{11} = 2800 \times 10^{-6}$), while the gradient was decreased in half, phase stress in SiC whiskers still increased with $\epsilon_{11}$. Residual phase stresses in the MMC were considered to be that generated by the difference of thermal expansion coefficient during the compounding process or the heat treatment process. The intersection to the vertical line of macro stress $\sigma_{ij}^0$ was a slightly less to the zero point and $\sigma_{ij}^0$ was beneath the SS curve. The X-ray penetration depth in this experiment was 20 µm even using a perpendicular beam. The Al grain size is much larger than this and therefore, measured only a surface stress. Consequently, we must consider the stress relaxation near the free surface and this is a reason why $\sigma_{ij}^0$ was beneath the SS curve. And the gradient of $\sigma_{ij}^0$ was much smaller than the SS curve in the plastic deformation region of MMC. This means the mechanism of MMC strength is controlled by the dispersion hardening such as orowan mechanism rather than the fiber reinforcing.
Furthermore, although the SiC phase stress was still increasing in the plastic deformation region of MMC, the gradient of SiC whiskers was decreased in half. This means the binding force of composed whiskers was insufficient to restrict matrix deformation. According to this rough restriction, the stress burden of whiskers after plastic deformation will be reducing. This stress burden control system may make plastically-work possible.

5.2. Theoretical approach to the phase stress

Concerning the region which Al phase stress increased linearly \( (\varepsilon_{\text{Al}} = 2800 \times 10^{-6}) \), relationship between the theoretical phase stresses and applied stress \( \sigma_{11} \) are shown in Fig. 9. The theoretical solutions were obtained by the calculation using micromechanics on Eshelby/Mori–Tanaka model as Eqs. (8) and (9). Measurements in this experiment were also plotted in the figure. The values of \( \sigma_{11} = 0 \) on the calculation were estimated equal to residual stress. Both gradients of calculated Al matrix and macro stress are slightly larger than those of measured and extend their interval to measured with increasing of \( \sigma_{11} \) on account of the relaxation of \( \sigma_{33} \) near the surface. On the other hand calculated SiC phase stress disagreed with the measured accounts for the existence of unstable influence of the unstable stress region, the calculations were made at apposition using a parallel shift of 150 MPa downward. These calculations agreed with measured phase stress except for the unstable stress region. This means the measured phase stress is a reflection of internal stress increasing. In the case of inclusions, the relaxation of \( \sigma_{33} \) near the surface depends on the distance between them and its grain size. SiC whiskers which used in this experiment were 0.5 \( \mu \)m in diameter and dispersed in Al matrix fine and uniformly as shown in Fig. 4. X-ray penetration depth is deep enough to this size and consequently, \( \sigma_{33} \) relaxation influence may be small. In brief, X-ray stress measurement using Fe K\( \alpha \) radiation is useful for the evaluation of internal stress behavior on the secondary phase of MMC. However, it is interesting that whiskers which have high aspect ratio functioned to the same in the calculations as those using spherical inclusions in the matrix.

5.3. Disadvantage of internal stress against the fatigue crack propagation

From the Fig. 9, it is read out that the unstable stress behavior of SiC phase may be happened under 100 MPa of \( \sigma_{11} \). The large compressive residual stress caused by the mismatch of thermal expansion coefficient and the existence of the unstable stress region are considered that influenced something against the dynamic properties of the MMC. Fig. 10 shows the relation between the fatigue crack growth rate, \( da/dN \) and the range of stress intensity factor, \( \Delta K \), under the condition of stress ratio, \( R = 0.1 \), and repetition rate, \( f = 10 \) Hz. The sub size CT specimens were also used same as the fracture toughness test. \( \Delta K \) was decreasing gradually from the higher \( \Delta K \). The \( da/dN \) is uniquely related to \( \Delta K \) and relationship for the MMC has a larger gradient than that for A6061. And the lower range of stress intensity factor, \( \Delta K_{\text{th}} \) for the MMC is 6.8 MPa\( \sqrt{m} \). This value is smaller than 7.9 MPa\( \sqrt{m} \) of the Al alloy. The CT

![Fig. 9. Phase stress calculated by theoretical approach. Plots indicate experimental results.](image)

![Fig. 10. Fatigue crack propagation curve.](image)

![Fig. 11. Micro-voids around the crack tip.](image)
specimen surface observation around the crack tip of the MMC at \( \Delta K = 7.7 \text{ MPa}\sqrt{\text{m}} \) is shown in Fig. 11. Many micro voids were formed at the whisker–matrix interface in spite of under \( \Delta K_{\text{th}} \) for the Al alloy. It is considered that they were induced by the large gradient of residual stress at the whisker–matrix interface and the differences of the phase stress level in the instability stress region. Moreover, the coupling of micro voids makes it possible for the crack to propagate even under lower repeated stress. Therefore, the crack propagation resistance will be smaller than that of the Al alloy.

6. Conclusions

The main results obtained are summarized as follows:

(1) On the X-ray diffraction of Al/SiCw MMC, high intensity and isolated peak from Al(400) and SiC(311) planes were obtained using by Fe K\( \alpha \) radiation. On this occasion, although roots of the peaks overlapped each other, the profile fitting technique by pseudo-voigt function was useful to separate both peak and obtained each phase stress.

(2) The diffraction profile from SiC(311) tended to appear to duplicate the profile arising from K\( \alpha \) double line when duty loading. The K\( \alpha \) double line separation technique was also useful to determine the peak angle with accuracy.

(3) In the elastic deformation region, the phase stress in Al matrix increased linearly up to \( e_{11}^A = 2800 \times 10^{-6} \) and the gradient was very closely to the Young’s modulus. On the other hand phase stress in SiC whiskers showed an unstable stress behavior which decreased at first but reversed over \( e_{11}^A = 500 \times 10^{-6} \).

(4) The gradient of macro stress \( e_{11}^0 \) was much smaller than the stress – strain curve in the plastic deformation region. It was considered that the mechanism of the MMC strength was controlled by dispersion hardening such as orowan mechanism rather than fiber reinforcing.

(5) In the elastic deformation region, the phase stress in this MMC was equal to the calculated phase stress using spherical inclusions model.

(6) In the MMC specimen surface around the crack tip, many micro voids were formed at the whisker – matrix interface in spite of under \( \Delta K_{\text{th}} \) for the Al alloy. It is considered that they were induced by the large gradient of residual stress at the whisker – matrix interface and the differences of the phase stress level in the instability stress region.

Acknowledgements

The authors wish to acknowledge Dr H. Matsubara who was a previous researcher of AIST Cyubu for the cooperation in the preparation of the MMC and Dr T. Takahashi who was a previous researcher of AIST Touhoku for valuable comments and notes used in the profile fitting technique.

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