Direction dependency of extraordinary refraction index

Mojca Čepič

Faculty of Education, University of Ljubljana, Kardeljeva pl.16, 1000 Ljubljana, Slovenia
Jožef Stefan Institute, Jamova 39, 1000 Ljubljana, Slovenia

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Abstract

A simple experiment is presented that enables qualitative and quantitative measurement of the extraordinary refractive index direction dependency in an uniaxial nematic liquid crystal. Three liquid crystalline cells were designed in which elongated molecules of nematic liquid crystal align in directions which enable to observe the variation of extraordinary refractive index as a function of the direction of light.
The most common experiment to demonstrate birefringence is the observation of doubled objects through the calcite crystal\(^1\). When a polarizer is placed behind the crystal or in front of it, one of the pictures dissapears if the polarizer direction coincides with one of the polarizability tensor eigenvectors. Although the doubling of pictures is persuasive for an educated physicist, an extensive explanation is needed for students to whom the phenomenon is presented for the first time.

Another experiment which demonstrates the splitting of the light beam into two perpendicularly polarized beams with different phase velocities uses a large birefringent crystal, thick enough that after the transmition of the incident nonpolarized beam, two separated beams can be observed as two light spots on a distant screen. Changing the direction of the incident light by rotating the sample, the direction dependency of the extraordinary refraction index in uniaxial crystals as well as the direction dependency of both indeces in biaxial crystals can be observed. Unfortunately, the accuracy of the quantitative measurements of both indeces is poor, since only a slight nonparallelism of the sample surfaces can results in huge changes of the transmitted light direction\(^2\). The easiest accesible crystals are biaxial (e.g. quuarz), which additionally complicates the comparison of the results with theoretical predictions.

In this paper a simple experiment is presented in which liquid crystal in the optically uniaxial nematic phase is used as a birefringent material. The experiment enables demonstration of the extraordinary refraction index direction dependency. In the undergraduate lab the experiment can also be used for accurate measurements of the extraordinary index in different directions. The characteristic experimental data is presented in the present paper.

Nematic liquid crystals are composed of elongated molecules with orientationally ordered long molecular axes. Due to the rapid molecular rotations around long molecular axis, the system is optically uniaxial. The orientational correlation length perpendicular to the molecular long axes extends to 500 \(\mu\text{m}\) in nematic liquid crystals\(^3\); therefore the well ordered samples have to be thinner than that. Since elongated molecules have a larger polarizability along the long molecular axis than perpendicular to it, the direction of the optical axis for a liquid crystal in a cell is known from the preparation of the glass coating. In thin samples, spatial separation of the ordinary and extraordinary beam can be obtained by the prismatic effect\(^4\).

In order to study the direction dependency of the extraordinary refraction index, three different wedge cells shown in Fig.1 were designed. The cells were approximately 1 cm long and half of the cm wide. The 200 \(\mu\text{m}\) thick foil was inserted and glued between the glasses in one of the narrower sides, while the other narrow side of the cell was glued directly together forming the wedge. Two of the cells had polymide coating rubbed along the wedge of the cell or perpendicular to it (Fig. 1 a,b). The third cell had a polymer coating which aligned long molecular axes perpendicular to the glass (Fig. 1c). To avoid a long description of molecular orientations in different cells, let us call the cell with molecular axes oriented along the wedge (Fig. 1a) the transverse cell, the cell with long axes oriented along the longer side of the cell (Fig. 1b) the longitudinal cell and the cell with molecular long axes perpendicular to the glass (Fig. 1c) the perpendicular cell. The names should remind the reader of the orientations of the long molecular axes and consequently of the direction of the extraordinary polarization and the optical axes. The cells were filled with the liquid crystal\(^5\) heated above the transition temperature from the nematic to the isotropic phase.
The capillary effect was used to fill the cell.

The phase velocity of light in the direction in an angle $\theta$ with the optical axis of an uniaxial crystal like a nematic liquid crystal is given by

$$v_e(\theta)^2 = v_o^2 \cos^2 \theta + v_{e,0}^2 \sin^2 \theta.$$  

Here $v_e$ is the phase velocity of light with the extraordinary polarization i.e. with the electric field oscillating in the incident plane. The phase velocity $v_{e,0}$ is the smallest phase velocity of the extraordinary polarized light in a negative birefringent uniaxial liquid crystal, when the light direction is perpendicular to the optical axis i.e. $\theta = 90^\circ$. The ordinary phase velocity $v_o$ is direction independent. Both refractive indeces can be obtained from the general definition of a refractive index as a ratio between the phase velocity of light in vacuum and the phase velocity in a transparent material:

$$n_e(\theta)^2 = \frac{n_{e,0}^2}{n_{e,0}^2 \cos^2 \theta + n_o^2 \sin^2 \theta}.$$  

where refraction indeces are marked with the same subscripts as the corresponding phase velocities. The light polarized in the incident plane therefore refracts differently that the light polarized perpendicular to the incident plane. The direction of the extraordinary light in the birefringent material therefore depends on the orientation of the optical axis as well as the incident angle.

The experimental setup is shown in Fig. 2. The wedge cell is fixed into the holder and it is put on the rotatable table with longer side parallel to the table surface. A laser pointer is used as a source of the nonpolarized light beam. In addition to the low price, laser pointer has another advantage over the He-Ne lasers, i.e. it is a nonpolarized quasi monochromatic light source. The dispersion of both refractive indeces is quite common in liquid crystals and the experimental study with a monochromatic light avoids this problem. The direction of the incident light is always in the incident plane perpendicular to the wedge. When passing through the wedge cell the nonpolarized incident beam splits into two perpendicularly polarized beams. After the transmission both beams have different directions due to the prismatic effect of the wedge and after a few tenths of a meter they become separated in space. On a distant screen two light dots appear (Fig. 2). We measure the position of the dots relative to the position of the direct beam dot, when light does not pass through the cell. Changing the incident direction of light by rotating the table, the position of dots changes and enable the calculation of both indeces.

In the transverse cell none of the indeces changes with the direction of light as long as the incident plane is parallel to the long side of the cell (Fig. 1a - below). We can calculate both of them from the Snell’s law.

$$\frac{\sin \alpha}{\sin \beta_o} = n_{e,0} \quad \text{and} \quad \frac{\sin \alpha}{\sin \beta_e} = n_o.$$  

In the expression (3) the angle $\alpha$ is the controlled incident angle and $\beta_o, \beta_e$ are the refraction angles of the ordinary and the extraordinary polarized light. Both refraction angles are obtained from the direction of refracted light $\gamma_o$ and $\gamma_e$ (see Fig. 3), which can be calculated from the positions of the light dots on the screen ($x_o$ and $x_e$), the distance $l$ between the cell
and the screen and the wedge angle \( \delta \) (Fig. 2) expressed in radians which is approximately \( d/h \) (Fig. 1 a). Using the identity

\[
\frac{\sin \alpha}{\sin \beta} = \frac{\sin(\gamma \pm \delta)}{\beta \mp \delta} = n \tag{4}
\]

the refracted beam direction \( \beta \) is found from

\[
\tan \beta = \frac{\delta \sin \alpha}{\sin \gamma \pm \sin \alpha}. \tag{5}
\]

In the expressions (4,5) the upper sign in \( \mp \), \( \pm \) stands for the beam direction given in Fig. 3 by solid line and the opposite sign stands for angles in the opposite direction (dashed line - Fig. 3) . The angle \( \delta \) of the wedge cells is small (\( \approx 1^\circ \)) therefore \( \sin \delta \approx \delta \), expressed in radians, and \( \cos \delta \approx 1 \). With the known direction \( \beta \), the value of the refraction index is given by the first part of Eq. (4). Expressions (4) and (5) are general expressions which can be used to obtain the refraction index when light passes through the thin wedge sample of any, not necessary birefringent, material and allows for evaluation of both, ordinary and extraordinary refraction indeces.

When light passes the transverse cell the oscillating electric field has always one component parallel to the long molecular axes and the other perpendicular to it. Both indeces are independent of the incident direction as seen in Fig 1a below. The cell can be used to demonstrate the splitting of the beam, to show the polarization of the ordinary and extraordinary beam and to make a quantitative measurement of the maximum value of the extraordinary refraction index and the value of the ordinary refraction index. In adition, measurements of the dependence of both refractive indeces on the incident angle, although they are constant, provide an estimation of the experimental accuracy.

In the perpendicular cell, the optical axes is perpendicular to the glass plate and the refraction angle \( \beta_e \) is equal to \( \theta \) (Fig. 1 b - below):

\[
\frac{\sin \alpha}{\sin \beta_e} = n_e(\beta_e) = n_e(\theta) \tag{6}
\]

where \( n_e \) is calculated from Eq. (4) and (5). When the incident light is perpendicular to the cell, the electric field is perpendicular to the long molecular axis so the light direction coincides with optical axis and the beam splitting does not occur. But as soon as the incident light is not perpendicular to the cell, there is a component of the electric field along the long molecular axis and consequently the value of the extraordinary index changes from the value of the ordinary index (Fig 1b below). When we rotate the cell, the beam splits and the single spots splits into two, demonstrating that the indeces are not equal anymore. From the measurements of the dot positions the behaviour of the extraordinary refractive index close to the optical axis can be calculated.

In the parallel cell, the optical axes is parallel to the glass and the refraction angle \( \beta_e = \frac{\pi}{2} - \theta \). The extraordinary refractive index is therefore given by:

\[
\frac{\sin \alpha}{\sin \beta_e} = \frac{\sin \alpha}{\sin(\frac{\pi}{2} - \theta)} = n_e(\beta_e) = n_e(\theta). \tag{7}
\]
The last of the three cells can be used to study the direction dependency of the extraordinary index when the difference of both indeces is close to the largest value. When incident direction of light is varied, the extraordinary index decreases, since the component of the oscillating electric field along the long molecular axis decreases (Fig. 1c below). Unfortunately, to show the decrease in the index, the evaluation of both indeces from the measurement of the dot positions is necessary.

There are few limitations in these three experiments which have to be considered. Although light with ordinary and extraordinary polarization can propagate in any direction, experimentally we are limited with the refraction of the incident light, since the light source is outside of the birefringent material. In the presented situation the ordinary beam direction was theoretically limited (for the light parallel to the cell surface) to 26° for $n_o = 1.5$ and to 22° for the extreme value of $n_{e,0} = 1.76$ in the geometry of the parallel cell. The cell size and the cell holder additionally limit the incident angle to the values of less than 50°. In Fig. 4 we show the combined results of the extraordinary index measurements in the paralell and the perpendicular cell and compare them to the theoretical expression (1).

To conclude, the experimental setup which enables a detailed study of the light behavior in an uniaxial birefringent material, is presented. In order to study the direction dependency of the extraordinary refraction index, three different liquid crystalline cells were designed. The transverse cell enables a demonstration of the splitting of a nonpolarized beam into the ordinary and the extraordinary beam. Their polarization can be shown of the extreme value for the extraordinary refraction index as well as the value of the ordinary refractions index can be measured. The transverse cell can also be used to estimate the measurement accuracy. The perpendicular cell enables the demonstration of the direction dependency of the extraordinary index, as well as its measurement, when the value is close to the value of ordinary refraction index. The parallel cell can be used for the quantitave measurement of the extraordinary index direction dependency close to its largest value.

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1E. Hecht, *Optics*, 3rd ed. (Addison Wesley Longman, 1998), p. 333.
2L. Kowalsky, private communications.
3P. G. de Gennes and J. Prost, *The Physics of Liquid Crystals*, 2nd ed., (Oxford Science Publications, 1995), p.98.
4D. K. Shenoy, ”Measurements of Liquid Crystal Refractive Indices”, Am. J. Phys. 62, 858-859 (1994).
5Nematic liquid crystals with appropriate properties are commercially available by Merck.
6M. Born and E. Wolf, *Principles of Optics*, 6th ed., (Pergamon Press, 1993), p.680.
FIGURES

FIG. 1. a) In the parallel cell molecular long axes are oriented along the wedge. If the incident plane is parallel to the longer side of the cell, the two indices do not change with the light direction (below). b) In the perpendicular cell molecular long axes are oriented perpendicularly to the cell glass. The extraordinary index increases with the increasing incident angle (below). c) In the longitudinal cell molecular long axes are oriented parallel to the longer side of the cell. The extraordinary index decreases from its maximum value with growing incident angle.

FIG. 2. Schematic presentation of the experimental setup.

FIG. 3. Geometry of the experiment. The light incident on the cell with the incident angle $\alpha$, refracts in the material by an angle $\beta$. The second incident angle is $\beta - \delta$ for the positive $\alpha$ (solid line) and $\beta + \delta$ for the negative $\alpha$ (dashed line). The light refracts again by an angle $\gamma$.

FIG. 4. The extraordinary refraction index at the angle $\theta$ close to 0 measured in the perpendicular cell and at $\theta$ close to $\pi/2$ measured in the longitudinal cell. The theoretical direction dependence for $n_{e,0} = 1.76$ and $n_o = 1.5$ is given for a comparison. For experiments the nematic liquid crystal E8 was used. Both indices $n_{e,0}$ and $n_o$ were measured in the parallel cell.