Pilot Aided Channel Estimation for AFDM in Doubly Dispersive Channels

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Abstract—Affine frequency division multiplexing (AFDM) is a multi-chirp waveform and a promising solution for ultra-reliable communication under doubly dispersive channels. In this paper, we propose two pilot aided channel estimation schemes for AFDM, named single pilot aided (SPA) and multiple pilots aided (MPA) respectively. Pilot, guard and data symbols in the discrete affine Fourier transform (DAFT) domain are arranged appropriately at the transmitter. While at the receiver, channel estimation is performed with the aid of an estimation threshold and a mapping table. The bit error performance of AFDM applying the proposed channel estimation schemes shows only marginal loss compared to AFDM with ideally known channel state information. Moreover, extensions of the SPA scheme to multiple-input multiple-output (MIMO) and multi-user uplink/downlink are presented.

Index Terms—AFDM, DAFT domain, doubly dispersive channels, channel estimation.

I. INTRODUCTION

The beyond 5G / 6G wireless networks are expected to meet the requirements of reliable, low latency and wide range communication under high-speed scenarios. It is a huge challenge since the channel is no longer linear time-invariant as 4G systems. Under this time dispersive channels, the orthogonality between the subcarriers in orthogonal frequency division multiplexing (OFDM) is damaged greatly by the doppler shift, resulting in inter-carrier interference and intolerable performance loss.

Many new waveforms are explored recently for high-mobility communications. Orthogonal chirp division multiplexing (OCDM) [1], [2], which is based on the discrete Fresnel transform, outperforms OFDM in doubly dispersive channels. However, OCDM cannot achieve full diversity due to its unchangeable parameters. Orthogonal time frequency space (OTFS) modulation [3], a recently proposed two-dimensional modulation scheme, has shown the potential of tackling the dynamics of doubly dispersive channels with symbols multiplexed in the delay-doppler (DD) domain via symplectic finite Fourier transform. Given the sparse and stable property of the representation of channel characteristics, OTFS is resilient to delay-doppler shifts and outperforms OFDM significantly.

Affine frequency division multiplexing (AFDM), a recently discovered waveform, always attains full diversity due to parameters adjustment according to the DD profile of the channel [4], [5]. Symbols in AFDM are multiplexed on a set of orthogonal chirps which are tuned to adapt to the doubly dispersive channel characteristics, enabling a full and sparse delay-doppler representation of the channel in discrete affine Fourier transform (DAFT) domain [6], [7]. Low complexity minimum mean square error (MMSE) and maximal ratio combining (MRC) detectors have been proposed in [8]. Results in [4] [5] show that AFDM has similar outstanding performance just as OTFS but with lower complexity and advantage on less channel estimation overhead.

A fundamental building block in wireless communication systems is channel estimation, which acquires the accurate channel state information (CSI) during the transmission. The performance of detection at the receiver is deeply determined by how accurate the CSI is estimated. Embedded pilot channel estimation is mentioned in [5] with less explanation and no numerical results. To the best of our knowledge, this is the first work introducing the pilot aided channel estimation with systematic analysis in AFDM systems, regardless of single antenna or multiple antennas.

In this paper, we propose two pilot aided channel estimation schemes for AFDM, named single pilot aided (SPA) and multiple pilots aided (MPA) respectively. Channel estimation is performed with the aid of an estimation threshold and a mapping table that can extract the CSI perfectly from the received pilot symbols. Extensions of the SPA channel estimation scheme to multiple-input multiple-output (MIMO) and multi-user uplink/downlink scenarios are discussed. The simulation results show that the bit error performances based on the proposed schemes of SPA and MPA achieve that of AFDM with ideal CSI. The rest of this paper is organized as follows. Section II reviews some basic concepts of AFDM, which lays the foundations for the development of channel estimation schemes in Section III. Numerical results are presented in Section IV, followed by the conclusions in Section V.

II. BASIC CONCEPTS OF AFDM

In this section, we review some basic concepts of AFDM from [4] [5]. Fig. 1 shows the AFDM modulation/demodulation block diagrams.

Let \( x \) denote a vector of \( N \) quadrature amplitude modulation (QAM) symbols that reside on the DAFT domain. The \( N \) points inverse DAFT (IDAFT) is performed to map \( x \) to the time domain as
with $n = 0, \cdots, N - 1$, $N$ denoting the number of subcarrier, and $\Lambda_c = \text{diag} \left( e^{-i2\pi cn^2}, n = 0, 1, \ldots, N - 1 \right)$. Before transmitting $s$ into the channel, a chirp-periodic prefix (CPP) should be added with a length which is any integer greater than or equal to the value in samples of the maximum delay spread of the wireless channel with the following impulse response at time $n$ and delay $l$

$$g_n(l) = \sum_{i=1}^{P} h_i e^{-j2\pi \frac{n}{N} \delta (l - l_i)}$$

(2)

where $P$ is the number of paths, $h_i$ is the complex gain, $l_i$ and $\alpha_i$ are the delay and doppler normalized with sample period and subcarrier spacing respectively. While in this paper, we assume $l_i$ is a non-negative integer and $\alpha_i$ is an integer.

After transmitting through the channel, serial to parallel, discarding CPP and $N$ points DAFT are performed. Thus, the received time domain samples $y$ are transformed to DAFT domain samples $\tilde{y}$ with

$$y_m = \frac{1}{\sqrt{N}} \sum_{n=0}^{N-1} r_n e^{-i2\pi \left( c_2 m^2 + \frac{1}{N} mn + c_1 n^2 \right)}$$

(3)

where $m = 0, \ldots, N - 1$. As proven in [4], the input-output relation between $y$ and $x$ satisfies

$$y_p = \sum_{i=1}^{P} h_i e^{i2\pi \left( Nc_1 l_i^2 - q_i + Nc_2 (q_i - p^2) \right)} x_q + \tilde{w}, 0 \leq p \leq N - 1$$

(4)

where the noise in DAFT domain $\tilde{w} \sim \mathcal{CN} \left( 0, N_0 \right)$, $q = (p + \text{loc}_l) N$, $\text{loc}_l \triangleq \alpha_i + 2 N c_1 l_i$, $\alpha_i \in [-\alpha_{\text{max}}, \alpha_{\text{max}}]$ and $\alpha_{\text{max}}$ is the maximum doppler, $c_1 = \frac{2\alpha_{\text{max}} + 1}{2N}$ and $c_2$ is either an arbitrary irrational number or a rational number sufficiently smaller than $\frac{1}{2N}$. Equation (4) can be denoted in matrix form as

$$\tilde{y} = H_{\text{eff}} x + \tilde{w}$$

(5)

where the effective channel matrix in DAFT domain $H_{\text{eff}} = \Lambda_{c_2} F \Lambda_{c_1} H A_{\theta_0} H^T A_H$, with $F$ denoting the discrete Fourier transform (DFT) matrix, $H$ being the delay-time representation matrix of the channel, noise vector $\tilde{w} \sim \mathcal{CN} \left( 0, N_0 I \right)$.

### III. Pilot Aided Channel Estimation

In this section, we propose SPA and MPA channel estimation schemes by exploring the characteristics of the effective channel matrix $H_{\text{eff}}$. Both of the schemes are performed in the DAFT domain. Based on the received pilots, an estimation threshold and a mapping table are used to extract channel state information. Finally, the SPA scheme is extended to MIMO and multi-user uplink/downlink scenarios.

#### A. SPA and MPA schemes

In AFDM, there are $Q + 1$ possible non-zero entries representing $Q + 1$ possible paths in each row and column in the effective channel matrix $H_{\text{eff}}$, as shown by Fig.2, where $Q \triangleq (l_{\text{max}} + 1)(2\alpha_{\text{max}} + 1) - 1$, $l_{\text{max}}$ is the maximum delay spread normalized with sample period. It is important to notice that the indexes of possible non-zero entries are circulant in both row and column directions, which indicates that each row and column contains all the CSI.

Following the above analysis, a single pilot combined with guard symbols can be applied to perform channel estimation as presented by Fig.2, with the assumption that the pilot is placed in the 0-th position. Here we elaborate on how to arrange the guard symbols to protect the pilot symbols at the receiver from the interference of the other symbols. Firstly, identify the indexes of received pilot symbols, which is equal to sorting out the indexes of possible non-zero entries in the 0-th column from $H_{\text{eff}}$. In this case, it is $[0, \alpha_{\text{max}}]$ and $[N - Q + \alpha_{\text{max}}, N - 1]$. Secondly, find out all the other symbols that will affect the received pilot symbols and set them to be zeros. After that, the DAFT domain representation of the channel can be obtained at the receiver. The left blank slots in $x$ can be further explored. If data is placed in the blank slots, we have the embedded pilot scheme mentioned in [5].

Although the guard symbols in SPA can protect the received pilot symbols from other symbols, its estimation accuracy degrades under high noise conditions. In order to cope with
the interference from noise, multiple pilots can be inserted in the blank slots since all the columns in $H_{\text{eff}}$ contain the same CSI. With guard symbols arrangement similar to SPA, different groups of received pilot symbols corresponding to different pilots will not interfere with each other in MPA. Fig.3 displays an example of two pilots aided channel estimation. Pilot diversity gain can be extracted to enhance the channel estimation accuracy with the fact that the noise contained in different received symbols is independent identically distributed [9]. In order to perform channel estimation successfully, $N$ in SPA and MPA should satisfy

$$N_p(l_{\text{max}}+1)(2\alpha_{\text{max}}+1) \leq N$$

where $N_p$ denotes the number of used pilots and it is worth noting that no data symbols can be transmitted in the channel estimation frame when $N < (N_p + 1)(l_{\text{max}} + 1)(2\alpha_{\text{max}} + 1)$.

| $m$ | $t_i$ | $\alpha_i$ | $h_i$ |
|-----|------|------|------|
| $[0, \alpha_{\text{max}}]$ | $0$ | $-m$ | $y[m]$ |
| $[N - \alpha_{\text{max}}, N - 1]$ | $0$ | $N - m$ | $\bar{x}_p$ |
| $[N - \alpha_{\text{max}} - 2Nc_1d, N - \alpha_{\text{max}} - 2Nc_1(d - 1) - 1]$ | $d$ | $N - m - 2Nc_1d$ |

In the MPA scheme, multiple groups of received pilot symbols can be acquired to enhance the path detection accuracy. To avoid false detection under the condition of high noise and low threshold, a specific path should be detected as positive only when all the energy of the associated received pilot symbols exceed the threshold. When the threshold is set too high, a positive path detection should be made as long as at least one of the associated received pilot symbols possesses energy exceed the threshold to alleviate miss detection. The complex gain should be updated as the average of the corresponding complex gains obtained from different received pilot symbols groups in both cases.

Although thorough awareness of the DD profile of the channel is not necessary for equalization and detection in AFDM system [8], it is extremely important and useful in the field of integrated sensing and communication (ISAC). In the ISAC systems, the delay and doppler reflect the distance and velocity information of objects respectively and AFDM is a promising waveform [10].

### C. MIMO scenario

In the sequel, we apply the SPA scheme in the scenario of MIMO. For simplicity, we discuss the case of single pilot embedded in the data with the assumption that the $l_{\text{max}}$ and $\alpha_{\text{max}}$ are identical between the antennas at the transmitter (Tx) and antennas at the receiver (Rx).

![Fig. 3. Two pilots aided channel estimation.](image)

![Fig. 4. 3 × 3 MIMO-AFDM system.](image)
Let \( N_t \) and \( N_r \) denote the number of antennas in the transmitter and receiver respectively. In the transmitter, each of the Tx antennas places its pilot in a specific location surrounded by guard symbols. While in the receiver, each of the Rx antennas explores the \( N_t \) groups of received pilot symbols for channel estimation. Inspired by the mentioned channel estimation schemes above, we propose the following symbol arrangement for arbitrary \( N_t \geq 1 \) and \( N_r \geq 1 \) MIMO-AFDM system:

\[
x^{nt}[m] = \begin{cases} 
x^{nt}_p & m = (Q+1)(n_t-1) 
x^{nt}_d & m = (Q+1)N_t, ... , N - Q - 1 
0 & \text{otherwise}
\end{cases}
\]

where \( x^{nt}_p \) and \( x^{nt}_d \) denote the pilot and data symbols of the \( n_t \)-th Tx antenna respectively. An example of a \( 3 \times 3 \) MIMO-AFDM system is shown in Fig.4. We can see that the pilot symbols of the Tx antennas are sufficiently separated so that they do not interfere with each other at the Rx antennas.

The pilot and guard overhead of each transmit antenna in an \( N_t \times N_r \) MIMO-AFDM for channel estimation can be expressed as

\[
O_{AFDM} = (N_t+1)Q + N_t
\]

\[
= 2N_t l_{max} \alpha_{max} + 2l_{max} \alpha_{max} + 2N_t \alpha_{max} + N_t
\]

\[
= N_t l_{max} + 2 \alpha_{max} + l_{max} + N_t
\]

while the overhead of \( N_t \times N_r \) MIMO-OTFS system is [11]

\[
O_{OTFS} = ((N_t+1)l_{max} + N_t)(4 \alpha_{max} + 1)
\]

\[
= 4N_t l_{max} \alpha_{max} + 4l_{max} \alpha_{max} + 4N_t \alpha_{max} + N_t
\]

By comparing the overheads of AFDM and OTFS systems, we can conclude that AFDM maintains its advantage over OTFS in terms of overhead for channel estimation in MIMO system, which will be illustrated in the simulation. The extension of the MPA scheme to MIMO-AFDM system would be similarly straightforward.

### D. Multiuser scenario

As mentioned above, it is essential that we adopt the pilot aided estimation scheme in the scenarios of multiuser uplink and downlink. Thus, we also consider the single pilot embedded in the data, and assume that the \( l_{max} \) and \( \alpha_{max} \) are identical between the base station (BS) and different users.

1) Uplink: The arrangement of pilots for different users is similar to the MIMO case. Each user occupies only a non-overlapping portion of the rest of the slots for data transmissions. Zero guard symbols are inserted between the data symbols of different users to avoid inter-user interference (IUI). An example of three users case is shown in Fig.5(a)(b)(c).

2) Downlink: During the downlink transmission, only one pilot is needed in the Tx antenna of the BS since it can be used by all the users to estimate the channel from itself to the BS. Guard symbols are also required to avoid IUI, as an example of single-antenna BS with three users shown in Fig.5(d).

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![Fig. 5. Single antenna multiuser AFDM system.](image-url)
for each case, and the optimal choice in SNRp = 20 dB is 7, while that in SNRp = 30 dB is 8. This means, with the energy of the received pilot symbols stronger, a larger estimation threshold should be applied to avoid false detection, whereas a smaller estimation threshold should be adopted to alleviate miss detection as the energy of the received pilot symbols become weaker.

Fig.8 shows the BER versus SNRd under different SNRp with optimal estimation threshold for each case. The SPA scheme is applied, and we can observe that the BER performance enhances as the SNRp increases, which is reasonable since a more accurate channel estimation can be obtained. We can also notice that when SNRp = 40 dB, the BER performance of AFDM with estimated CSI is very close to AFDM with ideal CSI, which reveals the effectiveness of SPA scheme.

Fig.9 compares the embedded pilot, SPA and MPA scheme with SNRp = 20 dB. The estimation threshold $\zeta$ is set to be 4 for each case and the number of pilots used in MPA is two. We can observe that the embedded pilot scheme has the same performance as SPA, proving the effectiveness of the guard symbol arrangement in Section III-A. Moreover, with only one more pilot applied, the accuracy of channel estimation enhances distinctly since pilot diversity gain is explored to enhance the path detection accuracy.

Fig.10 shows the BER performance versus signal-to-noise ratio (SNR) of $2 \times 2$ MIMO-AFDM system with ideal CSI and estimated CSI applying SPA scheme. The BER performance of SISO-AFDM with ideal CSI is given as a reference. The op-
AFDM system. CSI, which verifies the effectiveness of SPA scheme in MIMO-with estimated CSI is very close to MIMO-AFDM when $\text{SNR}_{\text{P}} = 30 \, \text{dB}$, showing a large spectral efficiency gap between MIMO-AFDM and MIMO-OTFS.

Finally, Fig. 11 compares the BER performance between $2 \times 2$ MIMO-AFDM and $2 \times 2$ MIMO-OTFS systems, $N_{\text{SISO}} = 1024$, $Q_{\text{OTFS}} = 16$, $M_{\text{OTFS}} = 64$, QPSK. AFDM has the BER performance similar to $2 \times 2$ MIMO-OTFS. However, under this simulation configuration ($l_{\text{max}} = 20$, $\alpha_{\text{max}} = 4$ and $P = 9$), $Q = 188$ and $Q_{\text{AFDM}} = 566$ slots are required for each transmit antenna in $2 \times 2$ MIMO-AFDM system to perform channel estimation, corresponding to $55.27\%$ of the AFDM frame. The $2 \times 2$ MIMO-OTFS counterpart is $O_{\text{OTFS}} = 992$ (since $4\alpha_{\text{max}} + 1 > N_{\text{OTFS}}$, the data and pilot symbols can only be separated on the delay axis), corresponding to $96.88\%$ of the OTFS frame, which results in a large spectral efficiency gap between MIMO-AFDM and MIMO-OTFS.

V. CONCLUSION

In this paper, we have proposed two pilot-aided channel estimation schemes for AFDM. In particular, we arrange pilot, guard, and data symbols in the DAFT domain to sufficiently avoid interference between pilot and data symbols. The influence of estimation threshold and pilot power on BER are discussed with simulations. The results show that, by applying the proposed channel estimation scheme, AFDM with estimated CSI can achieve similar performance compared to AFDM with ideal CSI. Extensions of the SPA scheme to MIMO-AFDM and multi-user uplink/downlink have been presented, showing that AFDM maintains its advantage over OTFS in terms of overhead for pilot aided channel estimation in MIMO system.

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