Off-diagonal helicity density matrix elements for heavy vector mesons inclusively produced in $NN$, $\gamma N$ and $\ell N$ interactions.

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Abstract:

Final state interactions in quark fragmentation may give origin to non zero values of the off-diagonal element $\rho_{1,-1}$ of the helicity density matrix of vector mesons $V$ produced in current jets, with a large energy fraction $x_\ell$; the value of $\rho_{1,-1}(V)$ is related to the hard constituent dynamics and tests unusual properties of it. Some recent data on $\phi$, $K^*$ and $D^*$ produced in $e^+e^-$ annihilations at LEP show such effects. Predictions are given here for $\rho_{1,-1}$ of heavy mesons produced in nucleon-nucleon, $\gamma$-nucleon and $\ell$-nucleon interactions.

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1 - Introduction

The elucidation of strong interactions requires a detailed understanding of long distance properties of QCD. The difficulty of this physics hinges on the non-perturbative nature of confinement phenomena. For instance, whereas jet production is quite well under theoretical control, the hadronization of quarks and gluons is still badly understood.

In Ref. [1] it was suggested how the coherent fragmentation of $q\bar{q}$ pairs created in $e^+e^- \rightarrow q\bar{q} \rightarrow V + X$ processes might lead to non-zero values of the off-diagonal element $\rho_{1,-1}(V)$ of the helicity density matrix of the vector mesons $V$; in Ref. [2] actual predictions were given for several spin 1 particles produced at LEP energies in two jet events, provided they carry a large fraction $x_E$ of the parent quark energy and have a small intrinsic $k_\perp$, i.e. they are collinear with the parent jet.

The values of $\rho_{1,-1}(V)$ are related to the values of the off-diagonal helicity density matrix element $\rho_{+--++}(q\bar{q})$ of the $q\bar{q}$ pair, generated in the $e^-e^+ \rightarrow q\bar{q}$ process [2]:

$$\rho_{1,-1}(V) \simeq [1 - \rho_{0,0}(V)] \rho_{+--++}(q\bar{q})$$  \hspace{1cm} (1)

where the value of the diagonal element $\rho_{0,0}(V)$ can be taken from data. The values of $\rho_{+--++}(q\bar{q})$ depend on the elementary short distance dynamics and can be computed in the Standard Model. Thus, a measurement of $\rho_{1,-1}(V)$, is a further test of the constituent dynamics, more significant than the usual measurement of cross-sections in that it depends on the product of different elementary amplitudes, rather than on squared moduli:

$$\rho_{+--++}(q\bar{q}) = \frac{1}{4N_{q\bar{q}}} \sum_{\lambda_-,\lambda_+} M_{+-\lambda_+\lambda_-}^* M_{+-\lambda_-\lambda_+},$$  \hspace{1cm} (2)

where the $M$’s are the helicity amplitudes for the $e^-e^+ \rightarrow q\bar{q}$ process. With unpolarized $e^+$ and $e^-$:

$$4N_{q\bar{q}} = \sum_{\lambda_-,\lambda_+} |M_{\lambda_+\lambda_-\lambda_+}|^2.$$  \hspace{1cm} (3)

At LEP energy, $\sqrt{s} = M_Z$, one has [3]

$$\rho_{+--++}(q\bar{q}) \simeq \rho_{+--++}^Z(q\bar{q}) \simeq \frac{1}{2} \frac{(g_V^2 - g_A^2)q}{(g_V^2 + g_A^2)q} \frac{\sin^2 \theta}{1 + \cos^2 \theta}. $$  \hspace{1cm} (4)

Eq. (1) is in good agreement with OPAL Collaboration data on $\phi$, $D^*$ and $K^*$, including the $\theta$ dependence induced by Eq. (4); however, no sizeable value of $\rho_{1,-1}(V)$ for $V = \rho, \phi$ and $K^*$ was observed by DELPHI Collaboration [3]. Further tests are then necessary.

We consider here other interactions in which the value of $\rho_{1,-1}(V)$ could be measured, namely $NN \rightarrow VX, \gamma N \rightarrow VX$ and $\ell N \rightarrow \ell VX$, with $V = \phi, D^*$ or $B^*$. The choice of a heavy vector meson implies the dominance in each of these cases...
of a particular elementary hard contribution, respectively $gg \to q\bar{q}$, $\gamma g \to q\bar{q}$ and $\gamma^* g \to q\bar{q}$, with $q = s, c$ or $b$. The hadronization process – the fragmentation of a $q\bar{q}$ pair – is then similar to the one occurring in $e^+e^-$ annihilations; however, the value of $\rho_{1,-1}(V)$ in these cases should be different from that observed in $e^-e^+ \to VX$ at LEP, due to a different underlying elementary dynamics, i.e. a different value of $\rho_{+,--}(q\bar{q})$. A measurement of $\rho_{1,-1}(V)$ in agreement with our predictions in these other processes would be an unambiguous test of both the quark hadronization mechanism and the real nature of the constituent interactions.

2 - $\rho_{1,-1}(V)$ in the process $NN \to V + X$

According to the QCD hard scattering scheme and the factorization theorem \[6, 7\] the helicity density matrix of a heavy hadron $V$ inclusively produced at large $p_T$ in the $AB \to VX$ process, with unpolarized initial particles, at leading twist and lowest order in the coupling constant, can be written as

$$\rho_{\lambda_V,\lambda'_V}(V) \ d\sigma^{AB\to VX} = \sum_{a,b,c,d,X} \sum_{\lambda_c,\lambda_d,\lambda'_{c'},\lambda'_{d'}} \ f_{a/A}(x_1) \ f_{b/B}(x_2) \ d\sigma^{ab\to cd} \ \rho_{\lambda_c,\lambda_d;\lambda'_{c'},\lambda'_{d'}}(cd) \ \ D_{\lambda_V,\lambda_X;\lambda_c,\lambda_d} \ \ D^*_{\lambda'_{V'},\lambda'_{X'};\lambda'_{c'},\lambda'_{d'}}$$

(5)

where the $f_{i/I}$ are the unpolarized quark or gluon number densities, $d\sigma$ is the unpolarized cross-section for the elementary interaction $ab \to cd$, and $\rho(cd)$ is the helicity density matrix of the $cd$ final partons. Finally, $D_{\lambda_V,\lambda_X;\lambda_c,\lambda_d}$ is the helicity amplitude for the fragmentation process $cd \to VX$;

$$D_{cd}^{\lambda_V,\lambda_X} = \sum_{\lambda_c,\lambda_d,\lambda'_{c'},\lambda'_{d'}} \ \rho_{\lambda_c,\lambda_d;\lambda'_{c'},\lambda'_{d'}} \ \ D_{\lambda_V,\lambda_X;\lambda_c,\lambda_d} \ \ D^*_{\lambda'_{V'},\lambda'_{X'};\lambda'_{c'},\lambda'_{d'}}$$

(6)

is the fragmentation function of the $cd$ state, with polarization described by $\rho(cd)$, into the hadron $V$ with helicity $\lambda_V$. If one chooses hadron $V$ collinear with, say, $c$, one has the usual fragmentation function

$$D_{c}^{V}(x_E) = \sum_{\lambda_V} D_{c,d}^{V,\lambda_V} ,$$

(7)

where $x_E$ is the energy or momentum fraction of parton $c$ carried by the observed hadron $V$.

Notice that we have considered the fragmentation process as $cd \to VX$, neglecting all possible interactions of the fragmenting quarks or gluons with the remnants of hadrons $A$ and $B$; we trust this to be a good approximation for all events in which two well defined large $p_T$ jets, originated by $c$ and $d$, are observed. Our considerations will be limited to these cases.

Eq. (6) can be evaluated in any frame; we consider it in the c.m. frame of interacting partons $a$ and $b$, which can be determined provided one measures the
two current jets total energy $\sqrt{s}$ and longitudinal momentum $p_L$

$$x_1 x_2 = \frac{\hat{s}}{s} \quad x_1 - x_2 = \frac{2p_L}{\sqrt{s}}$$

Our estimate for the helicity density matrix of $V$ will hold in such a frame; the two body decays of $V$, which allow to measure $\rho_{0,0}(V)$ and $\rho_{1,-1}(V)$, should be observed in the helicity rest frame [8] of $V$, as reached from the c.m. frame of $a$ and $b$.

In this frame the fragmentation process $cd \rightarrow VX$ is a c.m. forward process, similarly to what happens in $e^-e^+ \rightarrow q\bar{q} \rightarrow VX$ [2]. We can then use the same considerations introduced in Ref. [2] about the otherwise unknown hadronization mechanism: by selecting $V$ collinear to the parent jet, angular momentum conservation requires for the forward fragmentation amplitudes appearing in Eq. (5):

$$\lambda_X = \lambda_V - (\lambda_c - \lambda_d) = \lambda'_V - (\lambda'_c - \lambda'_d).$$

All other helicity configurations are suppressed by powers of $k_\perp/(x_E\sqrt{s})$, where $k_\perp$ is the transverse momentum of the observed hadron with respect to the parent jet. By choosing vector mesons with a sizeable fraction of the quark energy, say $x_E \gtrsim 0.5$ and a $k_\perp$ such that $k_\perp/(x_E\sqrt{s}) \ll 1$ we can safely neglect contributions from helicity fragmentation amplitudes which do not satisfy Eq. (9).

Let us now consider in particular the production of a vector meson, with a large $x_E$ value and a small $k_\perp$, for which the corresponding dominant hard process is $gg \rightarrow q\bar{q}$; this should apply to the production of $\phi (q = s, \bar{s})$, $D^* (q = c, \bar{c})$ and $B^*(q = b, \bar{b})$.

For the fragmentation process of a $q\bar{q}$ state Eq. (5) implies

$$\sum_{\lambda_q,\lambda_{\bar{q}},\lambda'_q,\lambda'_{\bar{q}},\lambda_X} \rho_{\lambda_q,\lambda_{\bar{q}};\lambda'_q,\lambda'_{\bar{q}}}(q\bar{q}) D_{1\lambda_X;\lambda_q} D^*_{-1\lambda_X;\lambda'_q} \approx \rho_{+--;+}(q\bar{q}) D_{10;+} \ D^*_{-10;+}$$

and [see Eqs. (6) and (7)]

$$D_q^V \approx \sum_X (1 + \alpha_q^V) |D_{10;+}|^2.$$  

In obtaining Eq. (11), following Ref. [2], we have neglected quark masses, we have assumed that $\pm$ helicity quarks do not fragment into $\mp 1$ helicity vector mesons and we have defined, for each quark flavour:

$$|D_{0-1;+}|^2 = \alpha^V |D_{10;+}|^2,$$

with $\alpha^V$ measured by

$$\alpha^V = \frac{\rho_{0,0}(V)}{1 - \rho_{0,0}(V)}.$$
We then obtain, exploiting parity invariance in quark fragmentation and with the same assumptions as in Ref. [2]:

\[
\rho_{1,-1}(V) \frac{d^4\sigma_{NN\to VX}}{dx_1 dx_2 d\cos\theta^* dx_E} \simeq \sum_X g(x_1) g(x_2) \frac{d\hat{\sigma}^{gg\to q\bar{q}}}{d\cos\theta^*} \rho^{gg\to q\bar{q}}_{1,-1-+}(q\bar{q};\theta^*) |D_{10,-+}(x_E)|^2 ,
\]

with

\[
\frac{d^4\sigma_{NN\to VX}}{dx_1 dx_2 d\cos\theta^* dx_E} \simeq \sum_X g(x_1) g(x_2) \frac{d\hat{\sigma}^{gg\to q\bar{q}}}{d\cos\theta^*} (1 + \alpha_q^V) |D_{10,-+}(x_E)|^2 .
\]

\(\theta^*\) is the production angle of \(q\) and \(V\) in the \(gg\) c.m. frame.

We then end up with the simple prediction, analogous to (1):

\[
\rho_{1,-1}(V) \simeq [1 - \rho_{0,0}(V)] \rho^{gg\to q\bar{q}}_{1,-1-+}(q\bar{q};\theta^*) .
\]

\(\rho^{gg\to q\bar{q}}_{1,-1-+}(q\bar{q};\theta^*)\) can be easily computed and is given, neglecting quark masses, by:

\[
\rho^{gg\to q\bar{q}}_{1,-1-+}(q\bar{q};\theta^*) = \frac{1}{2} \frac{\sin^2 \theta^*}{1 + \cos^2 \theta^*} .
\]

A similar result holds when taking quark masses into account and it would not change our following conclusions.

Eqs. (10) and (17) hold for vector mesons inclusively produced in \(NN\) interactions, selecting events in which: two large energy and large \(p_T\) current jets are observed, the vector meson carries a large fraction of the parent jet momentum with a small transverse component \(k_{\perp}\), and the dominant hard interaction is \(gg \to q\bar{q}\).

The main point to be observed is that both Eq. (1) and (16) have the same structure, but they differ in the dynamics of the elementary processes, given respectively by \(\rho^{Z\to q\bar{q}}_{1,-1-+}(q\bar{q})\), Eq. (4), and \(\rho^{gg\to q\bar{q}}_{1,-1-+}(q\bar{q})\), Eq. (17). Whereas the angular dependence is the same in both cases, the coefficients in front are negative for \(\rho^{Z\to q\bar{q}}_{1,-1-+}\) and positive for \(\rho^{gg\to q\bar{q}}_{1,-1-+}\), making a clear testable difference; while the values of \(\rho_{1,-1}(V)\) are negative at LEP, they should be positive in \(NN\) interactions. Notice that the value of \(\rho^{gg\to q\bar{q}}_{1,-1-+}(q\bar{q})\) is the same that one obtains for \(e^-e^+ \to q\bar{q}\) processes at low energies, where the weak effects are negligible and one only takes into account the electromagnetic contribution [4].

3 - \(\rho_{1,-1}(V)\) in the process \(\gamma N \to V + X\)

There are two classes of contributions for this process. In the first class, usually called the resolved photon case, the photon interacts through its partonic components; the reaction may thus be considered as a particular case of the previous one, \(AB \to VX\), the gluon content of the photon replacing the gluon content of the
The second class is defined as a direct interaction of the photon with the partons from the nucleon and the subprocess leading to a heavy meson production is then $\gamma g \rightarrow q\bar{q}$; under the same assumptions and kinematical conditions we have, analogously to Eqs. (14) and (15):

$$\rho_{1,-1}(V) \frac{d^3\sigma^{\gamma N \rightarrow V X}}{dx \, d\cos \theta^* \, dx} \simeq \sum_X g(x) \frac{d\tilde{\sigma}^{\gamma g \rightarrow q\bar{q}}}{d\cos \theta^*} (q\bar{q}; \theta^*) |D_{10;+}(x_E)|^2,$$

with

$$\frac{d^3\sigma^{\gamma N \rightarrow V X}}{dx \, d\cos \theta^* \, dx} \simeq \sum_X g(x) \frac{d\tilde{\sigma}^{\gamma g \rightarrow q\bar{q}}}{d\cos \theta^*} (1 + \alpha_q^V) |D_{10;+}(x_E)|^2.$$

where $\theta^*$ is the production angle of $q$ and $V$ in the $\gamma g$ c.m. frame.

Again, we end up with:

$$\rho_{1,-1}(V) \simeq [1 - \rho_{0,0}(V)] \rho_{+,-:--}(q\bar{q}; \theta^*),$$

where, neglecting quark masses:

$$\rho_{\gamma}^{g \rightarrow q\bar{q}}(q\bar{q}; \theta^*) = \frac{1}{2} \frac{\sin^2 \theta^*}{1 + \cos^2 \theta^*}.$$

Notice that the value of $\rho_{\gamma}^{g \rightarrow q\bar{q}}(q\bar{q})$ is the same as $\rho_{\gamma}^{g \rightarrow q\bar{q}}(q\bar{q})$: no trace of colour dynamics remains in the latter. This enables to add the two classes of contributions for $\gamma N$ interactions and get a simple clear conclusion: Eqs. (20) and (21) being the same as (16) and (17), the value of $\rho_{1,-1}(V)$ for $\phi$, $D^*$ or $B^*$ mesons with large $x_E$ and small $k_\perp$ inside a current jet, should be the same in $NN$ and $\gamma N$ interactions.

4 - $\rho_{1,-1}(V)$ in the process $\ell N \rightarrow \ell + V + X$

Such a process can be seen as a $\gamma^* N \rightarrow VX$ process; then one obtains, similarly to previous cases:

$$\rho_{1,-1}(V) \simeq [1 - \rho_{0,0}(V)] \rho_{\gamma^*}^{g \rightarrow q\bar{q}}(q\bar{q}) \cdot (q\bar{q}).$$

The difference with the previous two cases is in the value of $\rho_{\gamma^*}^{g \rightarrow q\bar{q}}(q\bar{q})$. Whereas in $NN$ and $\gamma N$ the initial $\gamma$ and $g$ were unpolarized, now the off-shell photons emitted by the lepton are polarized and one has:

$$\rho_{\lambda\lambda_p\lambda'_q\lambda'_q}(q\bar{q}) = \frac{1}{N_q} \sum_{\lambda_g,\lambda'_{\ell_g},\lambda_g} M_{\lambda\lambda_p\lambda_g}^{\lambda'_{\ell},\lambda_g} M_{\lambda'_{\ell},\lambda_g}^{\lambda_p,\lambda_g} \rho_{\lambda\lambda'}^{\gamma^*} \gamma_{DIS}(q\bar{q}),$$

where $\rho(\gamma^*)$ is the helicity density matrix of $\gamma^*$, which depends on the dynamics of its emission and the DIS kinematical variables:

$$\rho_{\lambda\lambda'}^{\gamma^*}(\gamma_{DIS}) = \frac{1}{N_{\gamma}} (-1)^{\lambda_{\gamma} + \lambda'_{\gamma}} \left[ 4l \cdot e^*(\lambda_{\gamma}) l \cdot e(\lambda'_{\gamma}) - Q^2 e^*(\lambda_{\gamma}) \cdot e(\lambda'_{\gamma}) \right]$$

(23)
where \( l \) is the initial lepton four-momentum and \( \epsilon(\lambda_{\gamma}) \) the photon polarization vector.

By computing Eq. (24) in the \( \gamma^* g \) c.m. frame one has:

\[
\rho(\gamma_{DIS}) = \frac{1}{2(2 - y)^2} \begin{pmatrix}
1 + (1 - y)^2 & iC(y)e^{-i\beta} & -2(1 - y)e^{-2i\beta} \\
-iC(y)e^{i\beta} & 4(1 - y) & iC(y)e^{-i\beta} \\
-2(1 - y)e^{2i\beta} & -iC(y)e^{i\beta} & 1 + (1 - y)^2
\end{pmatrix} \tag{25}
\]

where \( y = Q^2/(sx) \), \( x \) is the Bjorken variable and \( C(y) = \sqrt{2(1 - y)(2 - y)} \). \( \beta \) is the azimuthal angle of \( l \), having chosen the direction of \( \gamma^* \) as the positive \( z \)-axis. We choose the \( \gamma^* g \to q\bar{q} \) scattering plane as the \( xz \) plane.

By inserting the above expression of \( \rho(\gamma^*) \) into Eq. (22) gives our prediction for \( f_{1\perp 1}(V) \) in DIS, and its dependence on the variables \( y = Q^2/(sx) \), \( z = x/x_g \), the production angle \( \theta^* \) of \( V \) in the \( \gamma^* g \) c.m. frame and the measured value of \( \rho_{0,0}(V) \).
Such dependence can easily be tested experimentally. In Fig. 1 we plot the value of $\rho_{\gamma g \rightarrow q\bar{q}}$ as given by Eqs. (28) and (29), as function of $\theta^*$. We have fixed typical HERA values for the DIS variables, $Q^2 = 100$ (GeV/c)$^2$ and $x = 0.01$, which imply $y = 0.11$; the solid line corresponds to $z = 0.5$, the dashed line to $z = 0.02$ and the dotted line shows for comparison $\rho_{\gamma g \rightarrow q\bar{q}}$, Eq. (21), which corresponds to $z = 0$. In the first case – which means $x_g = 0.02$ – the value of $\rho_{\gamma g \rightarrow q\bar{q}}$ is negative, whereas in second case – $x_g = 0.5$ – is positive and close to the real $\gamma$ value. Thus, there is a huge variation, both in size and sign, of the value of $\rho_{1,1}$, which should be easily observable.

The same results hold for different values of $x$ and $Q^2$, provided the values of $y$ and $z$ do not change: different ranges of $y$ and $z$ are reachable at different accelerators. In general, at fixed values of $y$, the value of $\rho_{\gamma g \rightarrow q\bar{q}}$ differs mostly from the real $\gamma$ case when $z = 0.5$ and one recovers Eq. (21) when $z \rightarrow 1$ or $z \rightarrow 0$; at fixed values of $z$, instead, one reaches the real $\gamma$ limit when $y \rightarrow 1$.

### 5 - Conclusions

In this paper, we have derived the values of the off-diagonal element $\rho_{1,1}$ of the helicity density matrix of heavy vector mesons, with well defined properties, in jets produced through a selected hard interaction. Such values are crucially related to the nature of the partonic hard interactions and, as all quantities which depend on the product of different scattering amplitudes, they constitute a most sensitive test of the underlying dynamics. Our numerical results show clear and unmistakable differences among the values of $\rho_{1,1}$ of the same vector mesons obtained in different processes, like $e^+e^-$ annihilations, nucleon-nucleon and $\gamma$-nucleon processes and DIS.

Apart from the necessary approximations (like the dominance of definite subprocesses and the absence of any significant higher twist contributions) which may be controlled in the production cross section, the basic hypothesis underlying the whole approach is the factorizability of the hadronization process – even in its detailed off-diagonal helicity structure – from the hard subprocess amplitudes. In particular, the confirmation of our predictions, as given in Eqs. (16), (20) and (22), tests the absence of significant final state interactions between the jet containing the heavy vector meson and other jets such as for instance the backward jet containing the remnants of the target proton. The very fact of finding a non zero value of $\rho_{1,1}$, independently of its actual value which depends on the underlying dynamics, shows in a clear way what the hadronization mechanism should be, namely – in our cases – a $q\bar{q}$ interaction. This should be the case at least in $NN$ interactions, as the $q\bar{q}$ produced in the $gg$ fusion could be in a colour singlet state. Any experimental departure from the results obtained here – no significant deviation from zero of $\rho_{1,1}(V)$, even for events strictly selected in order to obey our validity conditions – would thus constitute a minor but real crisis towards a simple understanding of hadronization.

The measurements we suggest, and for which we give predictions, are simple and
clear; they should help in understanding some non perturbative aspects of strong interactions and their relation with perturbative ones. Such an issue is of great importance and we look forward to an experimental study of this question, which might well be pursued with existing data.

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Figure Caption

Fig. 1 : $\rho_{\gamma^* g \rightarrow q\bar{q}}(q\bar{q})$ as given by Eqs. (28) and (29), as function of $\theta^*$ for $Q^2 = 100$ (GeV/c)$^2$ and $x = 0.01$; the solid line corresponds to $z = 0.5$, the dashed line to $z = 0.02$ and the dotted line shows for comparison $\rho_{\gamma^* g \rightarrow q\bar{q}}(q\bar{q})$, Eq. (21), which corresponds to $z = 0$. 

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Fig. 1