Permutation relative entropy in quantifying time irreversibility: loss of temporal asymmetries in epileptic EEG

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Permutation relative entropy is proposed to quantify time irreversibility in our nonlinear dynamics analysis of electroencephalogram (EEG). Ordinal patterns in multi-dimension phase space of time series are symbolized, and the probabilistic divergences of all symmetric ordinal pairs are measured by relative entropy as time irreversibility. Analyzing multi-channel EEG from 18 healthy volunteers and 18 epileptic patients (all in their seize-free intervals), the derived relative entropy of symmetrical ordinal patterns shows advantages to other time irreversible parameters and significantly distinguish two kinds of brain electric signals (the epileptic have lower temporal asymmetries than the healthy). Test results prove that it is effective to quantify time irreversibility by measuring probabilistic divergence of symmetrical ordinal patterns and validate our hypothesis that epilepsy has lasting impacts on brain’ nonlinear dynamics, leading to a decline in brain signals directional asymmetry or time irreversibility, and the losing temporal asymmetries stemming from our findings may contribute to the preclinical diagnosis of epilepsy.

I. INTRODUCTION

Complex systems with nonlinearity and nonstationarity yield statistical asymmetry under time reversal, therefore, time irreversibility is a fundamental property of nonlinear systems [1], and measurements for time irreversibility are feasible alternatives to other assessments for nonlinear complex system. A stochastic process is said to be time reversible if its probabilistic properties are invariant with respect to time reversal [2 3]. Statistically speaking, if \( \{X(t_1), X(t_2), \cdots, X(t_n)\} \) have same joint probability distributions to \( \{X(-t_1), X(-t_2), \cdots, X(-t_n)\} \) for every \( t \) and \( n \) and to \( \{X(t_{-1+m}), X(t_{-2+m}), \cdots, X(t_{-n+m})\} \), for every \( n \) and \( m \), the process is time reversible [3 5].

Moreover, if \( X = \{x_1, x_2, \cdots, x_t\} \) is time reversible or directional symmetry, the probability distributions of the \( X'_m = \{x_t, x_{t+\tau}, \cdots, x_{t+(m-1)\tau}\} \) for all \( m \) and \( \tau \) are symmetrical [4], which allow us to measure time irreversibility or directionality from multi-dimensional and higher order differential time series.

It is complicate to calculate the joint probability distributions of a process and all its variants, therefore, alternative approaches are taken to measure time irreversibility. Some evaluations, for calculative convince, target on the asymmetry of different components’ distributions of time series [7 11] and some others measure the asymmetry between the forward and its backward distributions [12 14]. These two kinds of temporal asymmetric measurements are in fact equivalent that for distribution \( \{x_1, x_2, \cdots, x_t\} \), its symmetric form \( \{x_{t}, x_{t-1}, x_{t-2}, \cdots, x_{1}\} \) and corresponding version in the reversible time series \( \{x_{t(-1)}, x_{t(-2)}, \cdots, x_{t(-t)}\} \) should be the same.

Our highly complicated brain, a collection of huge number of neurons interacting, coupling and synchronizing with each others, has arguably been a typical complex network with nonstationarity, nonequilibrium and nonlinearity [15 20]. Brain disorders or diseases, like the all-age affected epilepsy, have impacts on brain functionality and bring intriguing changes to brain nonlinear dynamics [20]. Serious brain dynamical disorders could be observed during epileptic seizures [21], however, the epileptic in their seizure-free intervals mostly show no significant difference to the healthy in daily behaviors, consciousness or brain activities. With major hallmark of recurrent seizures [22], epilepsy may cause lasting damage to the brain nonlinear dynamics, and the loss of functionality may lead some absence of time-reversal asymmetry. Therefore, we make hypothesis that the epileptic patients, despite in their seizure-free intervals, have...
lower EEG time irreversibility than healthy people, and we verify our assumption by measuring temporal asymmetry, quantified by permutation relative entropy, of two groups of epileptic and healthy participants.

II. PERMUTATION RELATIVE ENTROPY

If a process is time reversible, for all embedding dimension $m$ and delay factor $\tau$, the probability distributions of $m$-dimensional vectors $X^m = \{x_t, x_{t+\tau}, \cdots, x_{t+(m-1)\tau}\}$ are symmetric. Therefore we employ multi-dimensional phase space and use probabilistic asymmetry of ordinal pattern to quantify time irreversibility.

The permutative process, a kind of local symbolic time series analysis [14], is proposed as a chaotic deterministic measurement [23, 24] which can be directly applied to arbitrary real-world data [19, 26, 27]. Phase space is firstly reconstructed for different embedding dimensions $x^m(i) = (x(i), x(i+\tau), \cdots, x(i+(m-1)\tau))$, elements in each vector is reorganized according to their values $x_{m\tau}(j_i) \leq x_{m\tau}(j_2) \leq \cdots \leq x_{m\tau}(j_n)$ and its ordinal pattern is $\pi_i = \{j_1, j_2, \cdots, j_n\}$ which will be mapped onto a given alphabet. In ascending orders of 3-dimensional vectors, 6 order patterns and their symbols are illustrated in Fig. 1.

![FIG. 1. 6 order patterns, symbolized on the basis of alphabet {0, 1, 2, 3, 4, 5} (in parentheses), in ascending order of m=3 and τ=1. Motifs symbolized '0' and '5' are symmetrical, and other two symmetrical pairs are '1' and '4', and '2' and '3'.](image)

Each ordinal pattern $\pi_i = \{j_1, j_2, \cdots, j_n\}$ has its symmetric version $\pi_s = \{j_n, \cdots, j_2, j_1\}$, and we quantify temporal asymmetry by measure the divergences between all the symmetric ordinal pairs in probabilistic distributions. Relative entropy [25, 26] is applied to measure distances or divergences among probability distributions of symbolic motifs as Eq. (1) where $p(\pi_i)$ and $q(\pi_s)$ are probability distributions of symmetrical ordinal patterns.

$$\text{ReIrD} = \sum_i p(\pi_i) \log \frac{p(\pi_i)}{q(\pi_s)} \quad (1)$$

In measuring time asymmetry, some parameters target on the asymmetric distributions of ups $\Delta x^+$ and downs $\Delta x^-$ in time series, such as Porta index [7, 8], calculated as Eq. (2) and Costa parameter [9, 10] as Eq. (3), where $H$ is the Heaviside function.

$$P\% = \frac{N(\Delta x^-)}{N(\Delta x \neq 0)} \cdot 100 \quad (2)$$

$$A = \frac{\sum H(-\Delta x^-) - \sum H(\Delta x^+)}{N(\Delta x \neq 0)} \quad (3)$$

Considering continuous distributions of real-world signals and equal states are very rare, permutation relative entropy of $m=2$ and $\tau=1$ is mathematically equivalent to Porta and Costa indexes in measuring divergence between ups and downs. Porta index is $p(\Delta x^-) \ast 100$, and Costa index can be rewritten as Eq. (4).

$$\text{A} = \frac{\sum H(-\Delta x^-) - \sum H(\Delta x^+)}{N(\Delta x \neq 0)} = p(\Delta x^-) - p(\Delta x^+) = 2 \cdot P - 1 \quad (4)$$

We set $D = p(\Delta x^-)/p(\Delta x^+) = P/(1 - P)$ , the permutation relative entropy in Eq. (1) will be modified as Eq. (5).

$$\text{ReIrD} = \frac{p(\Delta x^-) \log p(\Delta x^-)}{p(\Delta x^+)} = P \log D \quad (5)$$

III. EEG TIME IRREVERSIBILITY

Epilepsy is characterized by recurrent epileptic seizures [21, 22], while we focus on the differences in brain activities nonlinear dynamics between the healthy and the epileptic in seizure-free intervals.

Multichannel non-intrusive collections of EEGs from Nanjing General Hospital of Nanjing Military Command are applied in our works. Following the International 10-20 system, we apply 16 location of scalp electrodes, namely Fp1, Fp2, F3, F4, C3, C4, P3, P4, O1, O2, F7, F8, T3, T4, T5 and T6, to collect EEG, and there are two channels of Eog (electrooculography) to remove eye movements artifacts. Eighteen healthy volunteers aged 27.61±12.45 years, range 14-47 years old, and eighteen epileptic patients aged 25.89±8.77 years, range 15-49 years old, contribute EEG collection in duration of about 1 min and with sampling frequency of 512 Hz. Participants are in idle states, and epileptic patients are all in their non-seizure intervals during brain activities monitoring.

Fig. 2 depicts EEGs from epilepsy and healthy people and their permutative probability distributions.

A. Permutation relative entropy of $m=2$

For the equivalence of permutation relative entropy ($m=2$) to Porta and Costa indexes, we compare their performances in this subsection.
FIG. 2. Exemplary EEG time series (channel of O2) and their symbolic order patterns’ probability distributions (m is 3, τ is 1). a) epileptic patient. b) healthy participant.

In quantifying temporal asymmetry of the two kinds of brain electrical activities, Porta and Costa indexes share the consistency that time irreversibility of the healthy EEGs are higher than that of the epileptic (seen Tab [I]).

TABLE I. Porta and Costa indexes of the healthy and epileptic EEGs (mean±std). We choose $P_{50} = |P - 50|$ as an alternative to Eq. (2).

|        | $P_{50}$ (Porta) | A (Costa)     |
|--------|------------------|---------------|
| Epilepsy | 0.646±0.225      | 0.013±0.006   |
| Healthy | 1.168±0.569      | 0.023±0.012   |
| p values | $1.9 \times 10^{-3}$ | $2.1 \times 10^{-3}$ |
TABLE II. Statistical analysis for permutation relative entropy of the two groups of EEGs.

| p value | \( \tau = 1 \) | \( \tau = 2 \) | \( \tau = 3 \) | \( \tau = 4 \) | \( \tau = 5 \) |
|---------|----------------|----------------|----------------|----------------|----------------|
| m=2     | 1.9 \times 10^{-3} | 1.7 \times 10^{-3} | 1.9 \times 10^{-3} | 1.7 \times 10^{-3} | 1.8 \times 10^{-3} |
| m=3     | 1.6 \times 10^{-3} | 1.5 \times 10^{-3} | 1.6 \times 10^{-3} | 1.2 \times 10^{-3} | 1.4 \times 10^{-3} |
| m=4     | \times 5.3 \times 10^{-4} | 1.4 \times 10^{-3} | 9.8 \times 10^{-4} | 1.4 \times 10^{-3} |

Showing in tab [II], ReIrD of \( m=3 \) and 4 have better distinctions than those of \( m=2 \) and Porta and Costa indexes, indicating the improvements by adjusting parameters of phase space to modify ordinal patterns. Among these analysis, permutation relative entropy of \( m=4 \) and \( \tau = 2 \) has the most significantly distinction between the two kinds of EEGs.

Theoretically there are \( m! \) order patterns in all, while when \( m \) becomes 5 or bigger, some ordinal patterns do not have their symmetric ones (like the cases of \( m=4 \) and \( \tau = 1 \)) which is a disadvantage of relative entropy [12,13]. So we suppose that the preferred embedding dimension in our EEGs’ time irreversibility analysis should be 3 or 4.

Nonlinear dynamics of brain electrical activities are effectively extracted by appropriate \( m \) and \( \tau \), and the asymmetric probabilistic distributions of ordinal patterns serve as a satisfied measurement for temporal asymmetry or time irreversibility. Due to the different structural information and nonlinear dynamics of different kinds of signals, whether our settings of phase space work well in other physiological signals should be validated by more representative researches.

For seizure brain electrical recordings, there are other literatures suggesting higher nonlinearity or time asymmetry than the normal healthy [17,30]. An explanation given by Costa et al. is multiple scale theory that these measurements focus on single scale while fail to consider the inherent multiple time scales [10,26,31,33]. The paradox may also due to hidden structural information in \( m \)-dimensional phase spaces [34,36] according to our researches. Phase space of different \( m \) and \( \tau \) reveal different levels of hidden information about the complex system and may lead different outcomes. Which is the main factor that determines the uncertainties, or there are other undiscovered theories should be responsible for these contradictory situations, more related works are needed to reveal underlying drivings for the paradox in nonlinear analysis.

IV. CONCLUSIONS

Time irreversibility is a fundamental property of nonlinear complex systems. It is feasible to quantify time irreversibility by measuring the probabilistic divergence of symmetric ordinal patterns, and the adjustable phase space make the permutation relative entropy have more adaptability and flexibility.

Taking contribution of permutation relative entropy, we effectively prove that epilepsy causes lasting brain dysfunction that time irreversibility of epileptic EEGs is lower than the healthy people. And because the brain signals collections are conducted during seizure free intervals of epileptic participants, the reliable distinguishings between two groups of volunteers suggests that it could serve as a diagnostic tool to shed light on preclinical diagnosis of epilepsy.

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