Mathematical modeling of wind power plant capacity

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Abstract. The use of wind power plants is closely related to the issue of improving their efficiency. Accordingly, there is a need to build a model of turbine operation that allows you to evaluate the output characteristics of the installation and the influence of various parameters on them. In this paper, the simulation was performed using computational fluid dynamics (CFD) methods using the ANSYS Fluent program. The behavior of aerodynamic flows near the blades is also analyzed. As a model for describing the air flow, the k-ε model was chosen, for which the corresponding boundary conditions were selected. Simulations were performed for different air flow rates. The power curve obtained from the results of the aerodynamic flow simulation is compared with the power curve obtained from experimental data. Based on the simulation results, it is possible to predict the turbine power, depending on its initial parameters.

1. Introduction

In the modern world, due to the growing power consumption, there is a demand for environmentally friendly sources of electricity and the use of renewable natural resources for its production. Wind power has become one of the rapidly developing industries for generating such electricity. Indeed, wind is one of the most successful sources of "clean" energy, since wind power plants do not consume fossil fuels, and this reduces the production of carbon dioxide by wind turbines compared to thermal power plants.

The widespread use of wind power plants entails a number of issues related to improving its efficiency. So at present, the task of modeling the operation of a wind generator is becoming urgent, in order to determine the impact on its output characteristics of various design parameters and the environment.

1.1. The statistics of the development of the global wind energy

Wind generators as sources of electricity are becoming more and more widely used in developed countries today. For example, in a number of European countries, electricity generated by wind is from 10 to 20%, a world record was reached in Denmark in 2017 – 43%, in the United States – 40%. According to the World Wind Energy Association (WWEA) data for 2017, there is a steady increase in the total power generated by electric generators [1].
By the end of 2017, the total capacity of wind turbines installed around the world thus amounted to 539.291 MW, which according to some forecasts is able to cover more than 5% of all global electricity demand.

1.2. The problem of improving efficiency
Wind can be considered as a low-density energy flow. At the same time, increasing the efficiency of converting wind energy into mechanical energy of rotation of the rotor determines the economic benefits obtained from a single wind generator. To increase the overall performance of the wind turbine, it is necessary to understand the aerodynamics of the rotor, blades, and dynamic features of the mechanism.

There are three approaches for analyzing the behavior of air flows near the blade: full-scale testing, building analytical (or semi-empirical) models, and computational fluid dynamics (CFD). The first approach is the most complex and costly because it requires the availability of a ready-made wind turbines, but gives the most accurate results. The construction of analytical/semi-empirical models requires the imposition of various restrictions on the model to simplify it. Because of this, this approach is not universal. Ultimately, CFD is a good alternative way to model the behavior of air flows [2].

1.3. The design of the wind turbine
The block diagram of a horizontal wind generator is shown in figure 2. The foundation of the installation 1 prevents the installation from falling in a strong wind. The brake system 9 also protects the mechanism from strong gusts of wind or a hurricane. Wind energy spins the rotor on which the blades 11 are fixed, after which the energy is transmitted through the transmission 10 to the electric generator 7 [3].

Figure 1. Wind power development statistics according to WWEA [1].

Figure 2. The scheme of the wind generator [3]. 1 – foundation, 2 – power cabinet and control circuits, 3 – tower, 4 – ladder, 5 – rotary mechanism, 6 – gondola, 7 – electric generator, 8 – direction and speed tracking system, 9 – brake system, 10 – transmission, 11 – blades, 12 – system for changing the angle of attack of the blade, 13 – rotor cap. The performance of a wind generator is significantly affected by the number of its blades, and the number of blades increases the aerodynamic efficiency of the turbine. Switching from two-bladed systems to three-bladed systems can increase efficiency by 3%, but further
The performance of a wind generator is significantly affected by the number of its blades, and the number of blades increases the aerodynamic efficiency of the turbine. Switching from two-bladed systems to three-bladed systems can increase efficiency by 3%, but further increasing the number of blades increases efficiency slightly. In addition, the performance of power generation is affected by the design of the wind turbine blades.

2. Model building

The performance of a wind turbine can be characterized by three main parameters: power, torque, and thrust. Usually, the turbine power is selected as the key parameter, since it determines the amount of energy released by the turbine. The power is compared in theoretical calculations and experimental results. The ANSYS Fluent 18.0 software package was used for numerical simulation of gas-dynamic flow characteristics based on the simulation, and we made a conclusion about the total power of the wind generator. The main aspects of the constructed models are discussed in more detail in [12-17].

2.1. The choice of turbulence model and basic relations

As a result of the interaction of the air flow with the rotating blades, the flow, which previously could be considered laminar, turns to turbulent. Since turbulent air flows lead to high density and momentum fluctuations, it is necessary to take into account their influence on the flow behavior. As a rule, these fluctuations are high-frequency and small-sized, which is why they are extremely resource-intensive in modeling. The solution to the problem of modeling a turbulent flow is to choose a compromise between accuracy and calculation speed. In this case, instead of exact control equations, they are used averaged over time or some statistical ensemble. This approach allows us to use a larger grid for numerical modeling and not describe in detail small-size and high-frequency fluctuations. As a result, this approach leads to a modified system of equations that are less demanding on computing power.

In this work, we used one of the turbulence models presented in ANSYS Fluent – the k-ε model. This model is one of the most common, and it is well suited for cases where it is necessary to describe the turbulent flows of the core – the area removed from the walls. The model includes two augmented transport equations to represent the turbulent properties of a gas and is semi-empirical [2].

The k-ε model is based on modified equations for the transfer of kinetic turbulence $k$ and the rate of its dissipation $\varepsilon$. Its output assumes that the flow is completely turbulent. The kinetic energy of turbulence and the rate of dissipation can be obtained from the following transport equations [4]:

$$
\frac{d}{dt} (\rho k) + \frac{d}{dx} (\rho ku_i) = \frac{d}{dx} \left[ \left( \mu + \frac{\mu_t}{\sigma_k} \right) \frac{dk}{dx} \right] + G_k + G_b - \rho \varepsilon - Y_M + S_k;
$$

(1)

$$
\frac{d}{dt} (\rho \varepsilon) + \frac{d}{dx} (\rho \varepsilon u_i) = \frac{d}{dx} \left[ \left( \mu + \frac{\mu_t}{\sigma_\varepsilon} \right) \frac{d\varepsilon}{dx} \right] + C_{1\varepsilon} \frac{\varepsilon}{k} (G_k + C_{3\varepsilon} G_b) - C_{2\varepsilon} \rho \frac{\varepsilon^2}{k} + S_{\varepsilon};
$$

(2)

where $G_k$ – turbulent kinetic energy generated from average velocity gradients, $G_b$ – kinetic energy of buoyancy, $Y_M$ – contribution of fluctuating dilatation in compressible turbulence to the total dissipation rate, $C_{1\varepsilon}$, $C_{2\varepsilon}$, $C_{3\varepsilon}$ – experimentally defined constants, $\sigma_\varepsilon$, $\sigma_k$ – experimentally determined turbulent numbers for the kinetic energy of turbulence and its dissipation rate, respectively, $S_k$ and $S_{\varepsilon}$ – entry conditions, $\mu_t$ – turbulent viscosity, which is calculated as (3):

$$
\mu_t = \frac{\rho C_{\mu} k^2}{\varepsilon},
$$

(3)
where $C_{\mu} = \text{const}$.

2.2. Formulation of boundary conditions

After creating a three-dimensional model for the wind turbine rotor and setting the calculation area, you must specify the flow boundary conditions. To obtain adequate results, the correct setting of boundary conditions plays a significant role. They can be divided into the following categories:

- Conditions at the boundary with a solid area that will allow you to connect the air flow and the surface of the blade;
- Conditions imposed on the input speed. This boundary condition should be imposed on the area located as far away from the solid structure as possible. We will perform simulations for several speed values to get the dependence of the output power on the wind speed;
- Conditions imposed on the output stream. This condition does not specify the flow velocity or pressure details, but assumes that the gradient of all variables other than pressure is zero in the output flow region;
- The symmetry condition. It is assumed that the physical geometry and flow structure of interest has axial symmetry. It is assumed that the flow of all quantities across the symmetry boundary is zero, and the normal component of the velocity on the plane of symmetry is zero.
- Periodic conditions. Periodic boundary conditions imply that the geometry and physics we are interested in can be divided into periodically repeating regions. So, considering a turbine, you can get 1/3 of its surface, as shown in figure 3. This approach allows you to significantly simplify the model, since the number of grids can be reduced in favor of the quality of these grids. Moreover, if we initially consider the boundary of a cylindrical modeling area, then this boundary condition allows us to move to its segment.

As a result, the boundary conditions were formulated as follows: the input air flow velocity was selected sequentially from the following series: 5, 7, 10, 12, 20, 25 to simulate the output power of a wind turbine for different wind speeds, the turbulent intensity was set to 5%, and the ratio of turbulent viscosity was set to 10. Output characteristics: pressure 1 ATM. Conditions at the border with the blade – no sliding. The side borders are periodic with a period of 120°.

2.3. Getting the power value of a wind generator

To evaluate the efficiency of a wind generator depending on its capacity, it is convenient to enter a dimensionless parameter – the power factor $C_p$:

$$C_p = \frac{P_{\text{nom}}}{P_{\text{wind}}},$$

where $P_{\text{nom}}$ – rated power of the turbine, $P_{\text{wind}}$ – wind power.

Note that the maximum power factor $C_{p_{\text{max}}}$ is achieved if the wind speed on the rotor is 2/3 of the wind speed of the free flow. In this case $C_{p_{\text{max}}} = 0.5926$. This number is known as the Betz limit.

The decrease in wind speed between the free flow and the rotor plane, which prevents the flow from spreading, can be expressed in terms of the braking coefficient:

$$a = \frac{V_0 - V}{V_0};$$

$$V = V_0(1 - a);$$

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where \( V_0 \) – speed at the beginning of the flow simulation zone, \( V \) – flow rate in the area of the rotor blades.

The distribution of the air flow velocity is shown in figure 4, due to the continuity of the flow, its diameter should increase as the speed decreases, which occurs on the rotor plane. The pressure in the rotor area decreases and this contributes to the rotation of the blades [6].

\[ V_0 = V_1 (1 - 2a), \]  
(7)

\( V_3 \) – speed at the end of the simulation area.

Knowing the speed distribution in the flow tunnel you can find the power factor of the turbine as follows [6]:

\[ C_p = 4a(1-a)^2. \]  
(8)

Wind power can be found as follows:

\[ P_{wind} = \frac{\rho AV_0^3}{2}, \]  
(9)

where \( A \) – surface area of the rotor.

There are two approaches that allow you to find the power of a wind generator based on the results of CFD modeling. The first method is based on determining the torque \( T \) from the calculation results:

\[ P = T \omega, \]  
(10)

where \( \omega \) – angular speed of rotation of the rotor.

As a result:

\[ P = \rho Aa(1-a)^3V_0^3, \]  
(11)

The second method is based on obtaining the power factor based on the speeds at the beginning of the modeling area and at the end using formulas (5) – (8). The second method is preferred. It allows you to ignore the poor grid resolution near the rotor blades, while in the first method you need to know the torque in this area.
3. Simulation result
To check the adequacy of the calculation, a calculation was performed for the AWP 90/18 turbine, for which the power curve is known. To obtain the CFD calculation results, ANSYS Fluent 18.0 was used. Measurements were made for each speed from the proposed range 5, 7, 10, 12, 20, 25 m/s [5]. Rotor surface area \( A = 270 \text{ m}^2 \), gas density \( \rho = 1.225 \text{ kg/m}^3 \). The Results of modeling and calculation using the proposed method are shown in table 1 [9].

| Wind speed, m/s | Wind power, kW | The power turbine (CFD), kW | Power factor |
|----------------|----------------|-----------------------------|-------------|
| 3.5            | 7.09           | 1.47                        | 0.207       |
| 5              | 20.67          | 8.24                        | 0.398       |
| 7              | 56.72          | 20.20                       | 0.356       |
| 10             | 165.38         | 45.30                       | 0.274       |
| 12             | 285.77         | 50.46                       | 0.177       |
| 20             | 1323.00        | 86.72                       | 0.066       |
| 25             | 2583.984       | 93.822                      | 0.036       |

A graphical comparison of the experimental power and the power obtained by simulation is shown in figure 5. For low speeds, the output power of the turbine model is in good agreement with the experimental power. At medium speeds, when the air flow begins to separate along the blades, the model gives insufficiently accurate results, but they correspond to reality qualitatively. In the area of high speeds, the power and its growth rates are in good agreement with experimental data.

The simulation results showed that the method used can be used to predict the characteristics of turbines and create the best models in terms of power output. Note also that the power factor obtained from the simulation corresponds to the Betz limit. At high wind speeds, the power factor decreases.

Figure 5. Comparison of graphs of output power versus wind speed.

4. Conclusion
The use of alternative environmentally friendly sources of electric energy is an urgent problem in the modern world. The widespread use of wind power plants will reduce the amount of carbon dioxide emitted into the atmosphere. However, the use of such power sources requires solving the problem of increasing the efficiency of these installations.

The use of numerical methods can significantly reduce the cost and speed up the process of developing wind turbine designs. To solve this class of problems, you can use computational fluid dynamics, which is integrated into such computing packages as ANSYS Fluent. By varying the values of the input parameters, you can evaluate the effectiveness of various design solutions for a specific task of designing a wind generator and get qualitatively correct results.
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