Probabilistic Binary Neural Networks

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Abstract

Low bit-width weights and activations are an effective way of combating the increasing need for both memory and compute power of Deep Neural Networks. In this work, we present a probabilistic training method for Neural Network with both binary weights and activations, called BLRNet. By embracing stochasticity during training, we circumvent the need to approximate the gradient of non-differentiable functions such as \( \text{sign}(\cdot) \), while still obtaining a fully Binary Neural Network at test time. Moreover, it allows for anytime ensemble predictions for improved performance and uncertainty estimates by sampling from the weight distribution. Since all operations in a layer of the BLRNet operate on random variables, we introduce stochastic versions of Batch Normalization and max pooling, which transfer well to a deterministic network at test time. We evaluate the BLRNet on multiple standardized benchmarks.

1 Introduction

Deep Neural Networks are notorious for having vast memory and computation requirements, both during training and test/prediction time. As such, Deep Neural Networks may be unfeasible in various environments such as on-body devices (such as hearing aids) due to heat dissipation, battery driven devices due to power requirements, embedded systems because of memory requirements, or real-time system in which constraints are imposed by a limited economical budget. Hence, there is a clear need for Neural Networks that can operate in resource limited environments.

One method for reducing the memory and computational requirements for Neural Networks is to reduce the bit-width of the parameters and activations of the Neural Network. This can be achieved either during training (e.g., Ullrich et al. (2017); Achterhold et al. (2018)) or using post-training mechanisms (e.g., Louizos et al. (2017), Han et al. (2015)). By taking the reduction of the bit-width for weights and activations to the extreme, i.e., a single bit, one obtains a Binary Neural Network. Binary Neural Networks have several advantageous properties, i.e., a 32\( \times \) reduction in memory requirements and the forward pass can be implemented using XNOR operations and bit-counting, which results in a 58\( \times \) speedup (Rastegari et al., 2016). Moreover, Binary Neural Networks are more robust to adversarial examples (Galloway et al., 2018).

Shayer et al. (2018) introduced a probabilistic training method for Neural Networks with binary weights, but allow for full precision activations. In this paper, we propose a probabilistic training method for Neural Networks with both binary weights and binary activations, which are even more memory and computation efficient. In short, we train a stochastic Binary Neural Network by leveraging both the local reparametrization trick (Kingma et al., 2015) and the Concrete distribution (Maddison et al., 2016; Jang et al., 2016). At test time, we obtain a single deterministic Binary Neural Network or an ensemble of Binary Neural Networks by sampling from the parameter distribution. An advantage of our method is that we can take samples from the parameter distribution indefinitely—without retraining. Hence, this method allows for anytime ensemble predictions and uncertainty estimates. The stochastic network has a clear Bayesian interpretation: the parameter
distribution $p(W)$ of the stochastic network is a variational approximation to the true posterior $p(W|X,Y)$, where $(X,Y)$ denote the data, assuming a uniform prior on the weights. This interpretation may be used for further pruning of the network or allows for the introduction of more sophisticated priors. Note that while in this work we only consider the binary case, our method supports any discrete distribution over weights and activations.

In the proposed method, binary activations are sampled as the very last operation in each layer. As such, any other operation that is normally applied to the pre-activation must be applied to random variables. One of the contributions of this paper is the definition of batch-normalization and max-pooling for random variables. Our experiments show that these operations transfer well to a non-stochastic operation in a deterministic network – after re-estimation of the batch norm statistics.

2 Binary Neural Networks

Binary and low precision neural networks have received significant interest in recent years. Most similar to our work, in terms of the final neural network, is the work on Binarized Neural Networks by [Hubara et al., 2016]. In this work a real-valued shadow weight is used and binary weights are obtained by binarizing the shadow weights. Similarly the pre-activations are binarized using the same binarization function. In order to back-propagate through the binarization operation the straight-through estimator [Hinton, 2012] is used. Several extensions to Binarized Neural Networks have been proposed which — more or less — qualify as binary neural networks: XNOR-net [Rastegari et al., 2016] in which the real-valued parameter tensor and activation tensor is approximated by a binary tensor and a scaling factor per channel. ABC-nets [Lin et al., 2017] take this approach one step further and approximate the weight tensor by a linear combination of binary tensors. Both of these approaches perform the linear operations in the forward pass using binary weights and/or binary activations, followed by a scaling or linear combination of the pre-activations. In [McDonnell, 2018], similar methods to [Hubara et al., 2016] are used to binarize a wide resnet [Zagoruyko and Komodakis, 2016] to obtain results on ImageNet very close to the full precision performance. Another method for training binary neural networks is Expectation Backpropagation [Soudry et al., 2014] in which the central limit theorem and online expectation propagation is used to find an approximate posterior. This method is similar in spirit to ours, but the training method is completely different.

2.1 Binary Weight Network using Local Reparameterization

In this section we describe the binary local reparameterization method by [Shayer et al., 2018] for a single layer in a neural network. Assume a layer with $K \times K$ dimensional stochastic binary weights $[B_{ij}] \sim p(B)$, such that

$$p(B_{ij} = -1) = \sigma(W_{ij}), \quad \text{and} \quad p(B_{ij} = +1) = 1 - p(B_{ij} = -1).$$

(1)

Since $B$ is a random variable, $z = Bh$ is also a random variable, where $h$ is the activation of the previous layer. From the (Lyapunov) Central Limit Theorem (CLT), it follows that $z$ is normally distributed, specifically:

$$z_i \sim \mathcal{N}(\mu_i, \sigma_i^2) = \mathcal{N}\left(\sum_{j=1}^{K} h_j \mathbb{E}[B_{ij}], \sum_{j=1}^{K} h_j^2 \mathbb{V}[B_{ij}]\right).$$

(2)

Hence, we obtain a distribution over pre-activations. From this distribution we can easily sample using the reparameterization trick [Kingma and Welling, 2014] to obtain a real-valued pre-activation, i.e.,

$$a_i = \mu_i + \sigma_i \odot \epsilon, \quad \text{where} \ \epsilon \sim \mathcal{N}(0,1).$$

(3)

The combination of the CLT approximation and sampling using the reparameterization trick is also known as the local reparameterization trick [Kingma et al., 2015]. Given the sample $a_i$ we can proceed as usual and apply batch normalization, max-pooling and non-linearities. At test time, instead of using the local reparameterization trick, a binary weight matrix $\hat{B} \sim p(B)$ is sampled and used for all test data, i.e., $a = \hat{B}h$. 

2
3 Binary Local Reparameterization Network

We introduce a binary local reparameterization network using both binary weights and binary activations. Even when using binary weights and binary inputs to a layer, the pre-activations can take on other values. Often, an activation function with a limited discrete co-domain – such as sign(·) – is applied to the pre-activations to constrain the activations of the network to some set of discrete values. Unfortunately, when using this, one must deal with a non-differentiable non-linear activation function. Our method is based on the observation that when these activation functions are applied to a normally distributed random pre-activation, the computation involves one or more evaluations of the cumulative density function (cdf) of the normal distribution, which is differentiable. A binary (or discrete) sample can then be obtained using the Concrete continuous relaxation of a discrete distribution. Although, this leads to biased (but low variance) gradients, it can be used to effectively optimize a network with discrete nodes (Maddison et al., 2016).

We extend the stochastic method for training binary weight networks of Shayer et al. (2018) to allow for binary activations. We assume a Bernoulli distribution over \{−1, +1\} for each parameter in the network and leverage the (Lyapunov) central limit theorem to obtain a normal distribution over the pre-activations in each layer. Subsequently, a binarization activation function is applied to these distributions in order to obtain a binary distribution over activations. We call a network using these methods a Binary Local Reparameterization Network, or BLRNet. A graphical overview of a BLRNet layer is given in Figure 1.

A consequence of applying the activation function to a random variable is that any operation normally applied between the linear operation and the activation function must also be applied to a random variable. For this reason, we introduce an interpretation of Batch Normalization (Ioffe and Szegedy, 2015) and max pooling that can be trained in a stochastic setting and applied in a deterministic setting. Pseudo code for the full forward pass of a single layer, including batch normalization and max pooling, is given in Algorithm 1.

3.1 Stochastic Binary Activation

Since the output of a linear operation using binary inputs is not restricted to be binary, it is required to apply a binarization (non-linear) operation to the pre-activation in order to obtain binary activations. Various works – e.g., Hubara et al. (2016) and Rastegari et al. (2016) – use either deterministic or stochastic binarization functions, i.e.,

\[
\begin{align*}
    b_{\text{det}}(a) &= \begin{cases} 
    +1 & \text{if } a \geq 0 \\
    -1 & \text{otherwise}
    \end{cases} \\
    b_{\text{stoch}}(a) &= \begin{cases} 
    +1 & \text{with probability } p = \text{sigmoid}(a) \\
    -1 & \text{with probability } 1 - p
    \end{cases}
\end{align*}
\]

However, in the present case there is no such distinction since the pre-activations are random variables: applying a deterministic binarization function to a random pre-activation results in a stochastic binary activation. Specifically, let \(a_i \sim \mathcal{N}(\mu_i, \sigma_i^2)\) be a random activation obtained using the CLT, then

\[
a_i^* = b_{\text{det}}(a_i) \sim \text{Bern}_\pm(q), \quad q = \Phi(0|\mu_i, \sigma_i^2),
\]

Figure 1: Graphical overview of a BLRNet layer. Given an input vector \(x_i\) and discrete distributions of the weights, a distribution over pre-activations \(a_i\) is obtained using the central limit theorem. This distribution is subsequently transformed using a discretization (or binarization) function, after which a discrete distribution over activations \(h_i\) is obtained. Samples from this distribution are the final result from a single layer in a BLRNet.
where $\Phi(\cdot|\mu, \sigma^2)$ denotes the cdf of $\mathcal{N}(\mu, \sigma^2)$, and $\text{Bern}_\pm(q)$ denotes a Bernoulli distribution on $\{-1, 1\}$, such that

$$
P(a_i^s = -1) = q, \quad \text{and} \quad P(a_i^s = 1) = 1 - q.
$$

During training, samples are drawn using the Concrete relaxation method [Maddison et al., 2016]. By following these steps, both the variance and the magnitude of the pre-activation are taken into account when constructing the binary activation distribution, whereas the stochastic activation function $b_{\text{stoch}}(a)$ only takes the magnitude into account. See Figure 2 for a graphical depiction of the stochastic binary activation.

![Figure 2](image.png)

Figure 2: Given a random variable and a deterministic binarization function, the probability associated with each bin of the discrete output distribution is computed using the cdf of the distribution of the input variable. Although a deterministic binarization function is used, a stochastic activation is obtained.

At test time, a single binary weight instantiation $B \sim p(B)$ is obtained from the weight distribution and used to compute the linear operation in a BLRNet layer. Subsequently, $b_{\text{det}}(\cdot)$ is applied as non-linear activation. Hence, at test time, a fully deterministic Binary Neural Network is obtained.

### 3.2 Batch Normalization and Pooling

Other than a linear operation and an (non-linear) activation function, Batch Normalization [Ioffe and Szegedy, 2015] and pooling are two popular building blocks for Convolutional Neural Networks. For Binary Neural Networks, applying Batch Normalization to a binarized activation will result in a non-binary result. Moreover, the application of max pooling on a binary activation will result in a feature map containing mostly +1s. Hence, both operations must be applied before binarization. However, in the BLRNet, the binarization operation is applied before sampling. As a consequence, the Batch Normalization and pooling operations can only be applied on random pre-activations. For this reason, we define these methods for random variables. Although there are various ways to define these operation in a stochastic fashion, our guiding principle is to only leverage stochasticity during training, i.e., at test time, the stochastic operations are replaced by their conventional implementations and learned parameters learned in the stochastic setting must be transferred to their deterministic counterparts.

#### 3.2.1 Stochastic Batch Normalization

Batch Normalization (BN) [Ioffe and Szegedy, 2015] — including an affine

**Algorithm 1:** Pseudo code for forward pass of single layer in BLRNet. $a_{l-1}$ denotes the activation of the previous layer, $B$ the random binary weight matrix, $\tau$ is the temperature used for the concrete distribution, $f(\cdot, \cdot)$ the linear transformation used in the layer, $\epsilon > 0$ a small constant for numerical stability, and $\gamma$ & $\beta$ are the parameters for batch normalization.

**Input:** $a_{l-1}, B \sim p(B)$, $\tau$, $f(\cdot, \cdot)$, $\epsilon$, $\gamma$, $\beta$

**Result:** Binary activation $a_l$

```
// CLT approximation
\mu = f(\mathbb{E}[B], a_{l-1});
\sigma^2 = f(\text{Var}[B], a_{l-1});

// Batch normalization
m = \text{channel-wise-mean}(\mu);
v = \text{channel-wise-variance}(\mu, \sigma^2, m);
\mu = (\mu - m) / \sqrt{v + \epsilon + \beta};
\sigma^2 = \gamma^2 \sigma^2 / (v + \epsilon);

// Max pooling
if max pooling required then
    n \sim \mathcal{N}(0, I);
    s = \mu + \sigma \odot n;
    \iota = \text{max-pooling-indices}(s);
    \mu, \sigma^2 = \text{select-variable-at-indices}(\mu, \sigma^2, \iota);
end

// Binarization and sampling
p \leftarrow \Phi(0|\mu, \sigma^2); \quad a_l \sim \text{BinaryConcrete}(1 - p, \tau);
return a_l
```
where \( \mathbf{a}_i \) denotes the pre-activation before BN, \( \hat{\mathbf{a}} \) the pre-activation after BN, and \( \mathbf{m} \) & \( \mathbf{v} \) denote the sample mean and variance of \( \{ \mathbf{a}_i \}^M_{i=1} \), for an \( M \)-dimensional pre-activation, respectively. In essence, BN translates and scales the pre-activations such that they are approximately zero mean and have unit variance, followed by an affine transformation. Hence, in the stochastic case, our aim is that samples from the pre-activation distribution after BN also have approximately zero mean and unit variance—to ensure that the stochastic batch normalization can be transferred to a deterministic binary neural network. This is achieved by subtracting the population mean from each pre-activation random variable and by dividing by the population variance. However, since \( \mathbf{a}_i \) is a random variable in the BLRNet, simply using the population mean and variance equations will result in a non-standard output. Instead, to ensure a standard distribution over activations, we compute the expected population mean and variance under the pre-activation distribution:

\[
\mathbb{E}_{p(\mathbf{a} | \mathbf{h}, \mathbf{b})}[\mathbf{m}] = \mathbb{E}
\left[
\frac{1}{M} \sum_{i=1}^{M} \mathbf{a}_i
\right] = \frac{1}{M} \sum_{i=1}^{M} \mathbb{E}[\mathbf{a}_i] = \frac{1}{M} \sum_{i=1}^{M} \mu_i
\]

\[
\mathbb{E}_{p(\mathbf{a} | \mathbf{h}, \mathbf{b})}[\mathbf{v}] = \mathbb{E}
\left[
\frac{1}{M-1} \sum_{i=1}^{M} (\mathbf{a}_i - \mathbb{E}[\mathbf{a}_i])^2
\right] = \frac{1}{M-1} \left\{ \sum_{i=1}^{K} \sigma_i^2 + \sum_{i=1}^{M} (\mu_i - \mathbb{E}[\mathbf{m}])^2 \right\},
\]

where \( M \) is the total number of activations and \( \mathbf{a}_i \sim \mathcal{N}(\mu_i, \sigma_i) \) are the random pre-activations. By substituting \( \mathbf{m} \) and \( \mathbf{v} \) in Equation 7 by Equation 8 and 9 we obtain the following batch normalized Gaussian distributions for the pre-activations:

\[
\hat{\mathbf{a}}_i = \frac{\mathbf{a}_i - \mathbb{E}[\mathbf{m}]}{\sqrt{\mathbb{E}[\mathbf{v}]} + \epsilon} \gamma + \beta \Rightarrow \hat{\mathbf{a}}_i \sim \mathcal{N}\left(\frac{\mu_i - \mathbb{E}[\mathbf{m}]}{\sqrt{\mathbb{E}[\mathbf{v}]} + \epsilon} \gamma + \beta, \frac{\gamma^2}{\mathbb{E}[\mathbf{v}]} \sigma_i^2\right).
\]

Note that this assumes a single channel, but is easily extended to 2d batch norm in a similar fashion as conventional Batch Normalization.

### 3.2.2 Stochastic Max Pooling

In the deterministic case, pooling applies an aggregation operation to a set of (spatially oriented) pre-activations. Here we discuss max pooling for stochastic pre-activations, however, similar considerations apply for other types of aggregation functions.

In the case of max-pooling, given a spatial region containing stochastic pre-activations \( \mathbf{a}_1, \ldots, \mathbf{a}_K \), we aim to stochastically select one of the \( \mathbf{a}_i \). Note that, although the distribution of \( \max(\mathbf{a}_1, \ldots, \mathbf{a}_K) \) is well-defined (Nadarajah and Kotz 2008), its distribution is not Gaussian and thus does not match one of the input distributions. Instead, we sample one of the input random variables in every spatial region according to the probability of that variable being greater than all other variables, i.e., \( \rho_i = p(\mathbf{a}_i > \mathbf{z}_{\{i\}}) \), where \( \mathbf{z}_{\{i\}} = \max(\{\mathbf{a}_j\}_{j \neq i}) \). \( \rho_i \) could be obtained by evaluating the CDF of \( \mathbf{z}_{\{i\}} - \mathbf{a}_i \) at 0, but to our knowledge this has no analytical form. Alternatively, we can use monte-carlo integration to obtain \( \rho \):

\[
\rho \approx \frac{1}{L} \sum_{l=1}^{L} \text{one-hot}(\arg\max s^{(l)}) , \quad s^{(l)} \sim p(\mathbf{a}_1, \mathbf{a}_2, \ldots, \mathbf{a}_K) = \prod_{i=1}^{K} \mathcal{N}(\mu_i, \sigma_i^2)
\]

where one-hot(\( i \)) returns a \( K \)-dimensional one-hot vector with the \( i \)th elements set to one. The pooling index \( i \) is then sampled from \( \text{Cat}(\rho) \). However, more efficiently, we can sample \( s \sim p(\mathbf{a}_1, \ldots, \mathbf{a}_K) \) and select the index of the maximum in \( s \), which is equivalent sampling from \( \text{Cat}(\rho) \). Hence, for a given max pooling region, it is sufficient to obtain a single sample from each normal distribution associated with each pre-activation and keep the random variable for which this sample is maximum. A graphical overview of this is given in Figure 3.

Other forms of stochastic or probabilistic max pooling were introduced by Lee et al. (2009) and Zeiler and Fergus (2013), however, in both cases a single activation is sampled based on the magnitude of the activations. In contrast, in our procedure we stochastically propagate one of the input distributions over activations.
3.3 Weight Initialization

The weights for a BLRNet are initialized using a pre-trained full precision network with the same architecture. This initializes the convolutional filters with more structure than a random initialization. This is desirable as in order to flip the value of a weight, the parameter governing the weight has to pass through a high variance regime, which can slow down convergence considerably.

We use the weight transfer method introduced by Shayer et al. (2018) in which the parameters of the weight distribution for each layer are initialized such that the expected value of the random weights equals the full precision weight divided by the standard deviation of the weights in the given layer. Since not all rescaled weights lay in the \([-1,1]\) range, all weight probabilities are clipped between \([0.05,0.95]\). This transfer method transfers the structure present in the filters of the full precision network and ensures that a significant part of the parameter distributions is initialized with low variance.

3.4 Deterministic Binary Neural Network

During training, a stochastic network is optimized. However, on hardware one wants to leverage all the advantages of a fully binary neural network. Therefore, we obtain one or multiple binary neural networks from the parameter distribution \(p(\mathbf{B})\) at test time. We consider two options: the MAP, or most likely, estimate denoted BLRNET-MAP, and an ensemble consisting of 2, 5, or 16 samples from the parameter distribution denoted BLRNET-\(x\). Note that, even when using multiple binary neural networks in an ensemble, the ensemble is still more efficient in terms of computation and memory when compared to a full precision alternative. The ensemble predictions are obtained by summing the log softmax probability for each member of the ensemble and selecting the class with the maximum resulting value.

Since the trained weight distribution is not fully deterministic, the sampling of individual weight instantiations will result in a shift of the batch statistics. As a consequence, the learned batch norm statistics no longer closely match the true statistics. This is alleviated by re-estimating the batch norm statistics based on (a subset of) the training set after weight sampling using a moving mean and variance estimator. We observed competitive results using as little as 5 batches from the training set. However, given the iid nature of the datasets considered in this work, these could be estimated more efficiently by directly computing the batch statistics using a smaller sample.

3.5 Bayesian Interpretation and Considerations

The BLRNet can be interpreted as a Bayesian neural network, i.e., \(p(\mathbf{B})\) is a variational approximation to the true posterior \(p(\mathbf{B}|\mathbf{Y}, \mathbf{X})\), where \(\mathbf{X}\) denotes the training inputs and \(\mathbf{Y}\) the training targets. In that case, assuming a uniform prior distribution on the binary weights, it can be optimized by maximizing the following variational lower bound:

\[
\mathcal{L}(\theta) = \mathbb{E}_{p(\mathbf{B})}[\log p(\mathbf{Y}|\mathbf{X}, \mathbf{B})] + \mathbb{H}[p(\mathbf{B})].
\]

(12)
This objective favors models with both high accuracy and uncertainty on the weights. This allows one to estimate prediction uncertainty originating from the model uncertainty. However, in the present case, we aim to obtain a single best predictive model. Therefore we deviate from the strict approximate Bayesian training and use the following objective:

\[
\hat{L}(\theta) = E_{p(B)}[\log p(Y|X, B)] - \beta||\sigma(W) \odot (1 - \sigma(W))||_1, \tag{13}
\]

where \(\sigma(W)\) contains the probabilities for the binary weight distributions. In contrast to the variational objective, this objective favors model with high accuracy and low uncertainty on the weight distributions. This regularizer is proportional to the variance of the weight distribution and therefore we refer to it as the variance regularizer. In Shayer et al. (2018) it is also used for the binary weight network and is called the beta parameter.

4 Experiments

We evaluated the BLRNet on the MNIST and CIFAR-10 benchmarks and compare the results to Binarized Neural Networks (Hubara et al., 2016), since the architecture of the deterministic networks obtained by training the BLRNet are equivalent.

4.1 Experimental Details

All BLRNetworks are trained using cross-entropy loss plus a weight decay term scaled by \(10^{-4}\) on the parameters of the final softmax layer and a variance regularizer on the parameters of the weight distributions rescaled by \(\beta = 10^{-6}\). Note that this training objective deviates from the variational lower bound as we aim to optimize the BLRNetwork to obtain a single best deterministic network, instead of obtaining a posterior that captures model uncertainty. The weights for all networks are initialized using the transfer method described in Section 3.3. All models are optimized using Adam (Kingma and Ba, 2014) with an initial learning rate of \(10^{-2}\), a batch size of 128, and a validation loss plateau learning rate decay scheme. We keep the temperature for the binary concrete distribution static at \(1.0\) during training. All models are implemented using PyTorch (Paszke et al., 2017). All models are optimized until convergence, after which the best model is selected based on a validation set.

For Binarized Neural Networks we use the training procedure described by Hubara et al. (2016), i.e., a squared hinge loss and layer specific learning rates that are determined based on the Glorot initialization method (Glorot and Bengio, 2010).

4.2 MNIST

The MNIST dataset consists of of 60K training and 10K test \(28 \times 28\) grayscale handwritten digit images, divided over 10 classes. The images are pre-processed by subtracting the global pixel mean and dividing by the global pixel standard deviation. No other form of pre-processing or data augmentation is used. For MNIST, we use the following architecture:

\[
\text{32C3} - \text{MP2} - \text{64C3} - \text{MP2} - \text{512FC} - \text{SM10}
\]

where \(X\) denotes a binary convolutional layer using \(3 \times 3\) filters and \(X\) output channels, followed by (stochastic) batch normalization and binarization of the activations, \(Y\) denotes a fully connected layer with \(Y\) output neurons, \(SM10\) denotes a softmax layer with 10 outputs, and \(MP2\) denotes \(2 \times 2\) (stochastic) max pooling with stride 2. Note that if a convolutional layer is followed by a max pooling layer, the binarization is only performed after max pooling. Results are reported in Table 1.

4.3 CIFAR-10

The CIFAR-10 (Krizhevsky and Hinton, 2009) dataset consists of 50K training and 10K test \(32 \times 32\) RGB images divided over 10 classes. The last 5,000 images from the training set are used as validation set. We perform two different experiments using CIFAR-10. In the first the images are only pre-processed by subtracting the channel-wise mean and dividing by the standard deviation.
Table 1: Test accuracy on MNIST and CIFAR-10 for Binarized NN ([Hubara et al., 2016]), BLRNet, and a full precision network (FPNet). BLRNet-map refers to a deterministic BLRNet using the map estimate, and BLRNet-X refers to an ensemble of X networks, each sampled from the same weight distribution. For the ensemble results both mean and standard deviation are presented obtained from sampling multiple ensembles from the weight distribution.

|                  | MNIST  | CIFAR-10 | CIFAR-10 (WHITE) |
|------------------|--------|----------|------------------|
| Binarized NN     | 99.17% | 88.17%   | 88.56%           |
| BLRNet-MAP       | 99.00% | 88.61%   | 88.96%           |
| BLRNet-2         | 99.09 ± 0.05% | 89.51 ± 0.25% | 89.78 ± 0.16%   |
| BLRNet-5         | 99.13 ± 0.03% | 90.66 ± 0.12% | 90.48 ± 0.13%   |
| BLRNet-16        | 99.15 ± 0.03% | 91.22 ± 0.08% | 90.82 ± 0.08%   |
| FPNET            | 99.48% | 92.36%   | 92.45%           |

(a) Error coverage curve for CIFAR-10.  
(b) Test set performance with and without BN re-estimation on CIFAR-10.

Figure 4: Error coverage curve and batch statistic re-estimation results for CIFAR-10.

The second experiment we perform ZCA-whitening on the images. For both, the same architecture as [Shayer et al., 2018] is used, i.e.,

\[
2 \times 128C3 - MP2 - 2 \times 256C3 - MP2 - 2 \times 512C3 - MP2 - 1024FC - SM10
\]

where we use the same notation as in the previous section. The Binarized Neural Network baseline uses the same architecture, except for one extra 1024 neuron fully connected layer. During training, the training set is augmented using random 0px to 4px translations and random horizontal flips. Results for both experiments – and ensembles – are reported in Table 1.

4.4 Effect of Batch Statistics Re-estimation

As discussed in Section 3.4, after sampling the parameters of a deterministic network the batch statistics used by Batch Normalization must be re-estimated. Figure 4b shows the results obtained using a various number of batches from the training set to re-estimate the statistics. This shows that even a small number of samples is sufficient to estimate the statistics.

4.5 Ensemble Based Uncertainty Estimation

As presented in Table 1 the accuracy improves when using an ensemble. Moreover, the predictions of the ensemble members can be used to obtain an estimate of the certainty of the ensemble as a whole. To evaluate this, we plot an error-coverage curve ([Geifman and El-Yaniv, 2017]) in Figure 4a. This curve is obtained by sorting the samples according to a statistic and computing the error percentage in the top x% of the samples – according to the statistic. For the Binarized Neural Network and BLRNet-MAP the highest softmax score is used, whereas for the ensembles the variance in the prediction of the top class. The figure suggests that the ensemble variance is a better estimator of network certainty, and moreover, the estimation improves as the ensemble sizes increases.
4.6 Abblation studies

We perform an abblation study on both the use of (stochastic) batch normalization and the use of weight transfer for the BLRNet on CIFAR-10. For batch normalization, we removed all batch normalization layers from the BLRNet and retrained the BLRNet on CIFAR-10. This resulted in a test set accuracy of 79.24%. For the weight initialization experiment, the BLRNet weights are initialized using the Xavier initialization scheme [Glorot and Bengio (2010)] and was trained on CIFAR-10. When using Xavier initialization, a test set accuracy of 75.07% was obtained. These results are also presented in Table 2. Moreover, the accuracy on the validation set during training is presented in Figure 5. Note that these numbers are obtained without sampling a binarized network from the weight distribution, i.e., local reparametrization and binary activation samples are used. The BLRNet that used both weight transfer and stochastic batch normalization results in a significant performance improvement, indicating that both stochastic batchnorm and weight transfer are necessary components for the BLRNet.

5 Conclusion

We have presented a stochastic method for training Binary Neural Networks. The method is evaluated on multiple standardized benchmarks and reached competitive results. The BLRNetwork has various advantageous properties as a result of the training method. The weight distribution allows one to generate ensembles online which results in improved accuracy and better uncertainty estimations. Moreover, the Bayesian formulation of the BLRNetwork allows for further pruning of the network, which we leave as future work.

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