Robert B. Mann · Eduardo Martín-Martínez

Quantum Thermometry

Abstract In this review article we revisit and spell out the details of previous work on how Berry phase can be used to construct a precision quantum thermometer. An important advantage of such a scheme is that there is no need for the thermometer to acquire thermal equilibrium with the sample. This reduces measurement times and avoids precision limitations. We also review how such methods can be used to detect the Unruh effect.

1 Introduction

Introduction.– The science of thermometry is nearly 400 years old, dating back to the work of Galileo, Biancani, Sagredo, and Fludd. It was Ferdinando II de Medici, who constructed the first genuine thermometer, which consisted of sealed tubes with a bulb and stem that were partially filled with alcohol. This device was independent of air pressure, and so the expansion of the liquid within depended only on the temperature of its surrounding environment.

Standard commercially available thermometers are precise to approximately 0.01 Celsius degrees. Precision can be considerably improved by using resistance temperature detectors (RTDs). These devices exploit the fact that electrical resistance of platinum responds to temperature in a precisely known way: by 0.00385 ohm per ohm of resistance for each degree C of temperature change. The best such instruments available can resolve temperature differences as small as $10^{-7}$ degrees C. However all such devices require a given substance (alcohol, mercury, platinum) to come into thermal equilibrium with its environment.

Here we report on a class of proposed thermometers that make use of quantum effects to determine temperature. These devices make use of temperature-sensitive quantum effects to yield information about temperature. They do not need to come into thermal equilibrium with an environment. Furthermore, they are capable of considerable precision, of order $10^{-6}$ degrees C.

The primary quantum-mechanical effect we exploit is the geometric phase. Upon interacting with a quantum field, the state of a point-like quantum system with discrete energy levels (e.g. an atom) acquires a geometric phase [1] that is dependent on the state of the field. If the field is in a thermal state, this geometric phase encodes information about its temperature [2].
Our work was motivated by a recent proposal for measuring the Unruh effect at low accelerations that also exploits the geometric phase [3]. For several decades now it has been known that a (not quite) straightforward application of quantum field theory in relativistic settings implies that the vacuum state of a quantum field corresponds to a thermal state when described by uniformly accelerated observers, a phenomenon known as the Unruh effect [4,5]. Direct detection has continued to elude empirical scrutiny since the associated temperature of the thermal bath (the Unruh temperature) is smaller than a 1 Kelvin, even for accelerations as high as $10^{21}$ m/s$^2$. With current technology, sustained accelerations higher than $10^{26}$ m/s$^2$ are required to detect the effect [6], one of the main experimental goals of our time [7]. Observation of this phenomenon would settle long standing controversies concerning the very existence of the effect, and would also provide strong indirect empirical support for black hole evaporation and radiation [8]. Detection of the Unruh effect would have an immediate impact in many fields such as astrophysics [9], cosmology [10], black hole physics [11], particle physics [12], quantum gravity [13] and relativistic quantum information [14]. A number of proposals have been put forward to this end. These include analog systems such as fluids [15,16], Bose-Einstein condensates [17], optical fibers [18], slow light [19], superconducting circuits [20] and trapped ions [21]. The best case scenarios yield Unruh temperatures of the order of nanokelvin that remain very difficult to detect.

The key feature we exploit here is the conjunction of the geometric phase with methods from (relativistic) quantum information. The former effect, first noticed by Berry, is that when the parameters of the Hamiltonian of a quantum system are varied in a cyclic and adiabatic fashion, its eigenstates acquire a phase (in addition to the usual dynamical phase) [1]. The latter effect, noted more recently [14], is that acceleration degrades quantum entanglement. The conjunction of these two notions suggests that a point-like detector interacting with a quantum field can acquire a geometric phase due to its movement in space-time under certain conditions, inertial detectors acquire a phase different from that of accelerated ones (Fig. 1). Hence the phase encodes information about the Unruh temperature; as we shall see accelerations as low as $10^{17}$ m/s$^2$ can be detected in this manner. More generally, a detector moving through a thermal bath at one temperature will acquire a phase different from movement through a bath at another temperature, yielding a form of thermometry dependent only on quantum effects.

![Fig. 1 Trajectories for an inertial and accelerated detector.](image)

At this point our methods are best suited for measuring the temperature of radiation confined to a cavity. The thermometer is a 2-level atomic system that interacts with the radiation via a Hamiltonian whose properties we consider in the next section. We shall model the radiation as a massless scalar field and the 2-level system as an Unruh-de Witt detector. While this approach is restrictive, its applications
are conceivably quite broad. For example the temperature of a gas could be measured by putting it in thermal equilibrium with radiation, whose temperature could then be detected using our approach.

2 Hamiltonian Diagonalization

To illustrate how such quantum thermometry works, consider a massless scalar field in a cavity from the perspective of inertial observers moving in a flat (1 + 1)-dimensional spacetime. The field in the cavity is taken to be in a thermal state of a given temperature, either because it is in thermal equilibrium with an environmental reservoir, or due to the Unruh effect, in which uniformly accelerated observers is taken to be in a thermal state of a given temperature, either because it is in thermal equilibrium with radiation, whose temperature could then be detected using our approach.

To illustrate this, assume that the fundamental mode of length \( L \) cavity is in resonance with the energy gap of the atom. In order to neglect the contribution to the time evolution given by the interaction of the atom with the second, third and subsequent harmonics, the energy gap between the fundamental mode of the cavity and the second harmonic must be much larger than the gap between the ground and excited states of the atom. The gap between the different harmonics in the cavity is proportional to \( 1/L \), so for this single mode model to work we require that \( L \) is small enough to ensure there is a sufficiently large gap between the modes. To be sure that any of the effects described by this simplified model do not come from any spurious non-causal behaviour, we would have to make sure that the relevant times of evolution are much larger than the light-crossing time of the cavity. As we will see below, in our setting such characteristic times are of order \( \Omega^{-1} \) where \( \Omega \) is the atom gap. In our scenario, for the experimentally feasible values considered below, an atom gap of 1 Ghz yields an evolution time of 1 ns in the least favourable scenario we consider. The length of the cavity such that the light crossing time is precisely \( t_c = 1 \text{ ns} \) is \( L = 0.3 \text{ m} \). For time evolution scales to be much larger than the light crossing time of the cavity we need to consider a cavity of centimetres or millimetres, something very feasible from the experimental viewpoint. For the other cases proposed here the gaps as small as 1 Mhz; ensuring non-signaling implies that the cavity should only be smaller than hundreds of meters. This requirement is obviously easy to fulfill experimentally.

The procedure for computing the geometric phase is to first diagonalize the Hamiltonian (1). This can be done analytically and details are given in the appendix. The unitary operator that accomplishes this depends on the parameters \((u, v, s, p, \omega_a, \omega_b)\), each of which are functions of \( \lambda \) and the detector frequencies \( \Omega_a, \Omega_b \) — only three of these turn out to be independent, and we take them to be \((v, \omega_a, \omega_b)\).
The associated eigenstates of (1) are $U^\dagger |n_f n_d\rangle$, where $|n_f n_d\rangle$ are the eigenstates of $H_0(\omega_a, \omega_b) = \omega_a a^\dagger a + \omega_b b^\dagger b$ and $U = S_a S_b \hat{S}_b \hat{R}_a$. The subscripts $f$ and $d$ respectively refer to field modes (on which the $(a, a^\dagger)$ act) and detector modes (on which the $(b, b^\dagger)$ act). The operators

$$D_{ab} = \exp \left[ s(a^\dagger b - ab^\dagger) \right], \quad S_a = \exp \left[ \frac{1}{2} u (a^\dagger^2 - a^2) \right],$$

$$S_b = \exp \left[ \frac{1}{2} v (b^2 - b^\dagger^2) \right], \quad \hat{S}_b = \exp \left[ p (b^\dagger^2 - b^2) \right]$$

(2)

and $R_a = \exp \left( -i \varphi a^\dagger a \right)$ are the two-mode displacement, single-mode squeezing and phase rotation operators [23], respectively. Their action on the various creation and annihilation operators is given in the appendix.

The next step is to compute the geometric phase under cyclic evolution of the parameters $(v, \omega_a, \omega_b)$ for a detector (an inertial atom) interacting with an eigenstate of the Hamiltonian. The third step is to repeat this for an atom interacting with a field mode in a thermal state. Finally the (temperature-dependent) net geometric phase can be computed.

A schematic diagram is given in figure 2. Note that it is the displacement of the detector in space-time that generates a cyclic change in the Hamiltonian, with the phase $\varphi = k x - \Omega_a t$, of the field operators completing a $2\pi$ cycle in time $\Delta t \sim \Omega_a^{-1}$, where $(t, x)$ are Minkowski coordinates (a convenient choice for inertial observers).

Before the interaction between the field and the detector is switched on, the field is in the vacuum state and the detector in the ground state and so the system is in the state $|0_f 0_d\rangle$. We find that after the coupling is switched on the state of the system is

$$|0_f 0_d\rangle = \sum_{n_f, n_d} (n_f n_d |U| 00) U^\dagger |n_f m_d\rangle.$$

(3)
in the sudden switching approximation\footnote{ Suddenly switching on the coupling is known to be problematic since it can give rise to divergent results. However, in this case such problems are avoided because we are considering an effective (1 + 1) dimensional setting. In (3 + 1) dimensions these divergences can be treated by introducing a continuous switching function\cite{29}; the results are qualitatively the same.} In the coupling regimes we consider

\[
\langle n f m d | U | 00 \rangle = \langle n f m d | S_0 S_b D_{ab} S_b R_a | 00 \rangle \approx \delta_{n,0} \delta_{m,0}
\]

which can be demonstrated numerically. Hence for either cavity we have

\[
|\psi_{00}\rangle = \sum_{n,m} \langle n f m d | U | 00 \rangle U^\dagger |n f m d\rangle = U^\dagger |0 f 0_d\rangle + \mathcal{O}(\lambda^2)
\]

and so for small \( \lambda \) all changes are adiabatic. After the coupling is suddenly switched on and the state of the system is \( U^\dagger |0 f 0_d\rangle \), the movement of the detector in spacetime, which can be considered cyclic and adiabatic, generates a Berry phase.

### 3 Berry Phase Computation

The Berry phase \( \gamma \) acquired by the eigenstate \(|\psi(t)\rangle\) of a system whose Hamiltonian depends on \( k \) parameters \( R_1(t), \ldots, R_k(t) \) that vary cyclically and adiabatically is given by

\[
i \gamma = \oint \mathbf{A} \cdot d\mathbf{R}
\]

where

\[
\mathbf{A} = \begin{pmatrix}
\langle \psi(t) | \partial_{R_1} | \psi(t) \rangle \\
\langle \psi(t) | \partial_{R_2} | \psi(t) \rangle \\
\vdots \\
\langle \psi(t) | \partial_{R_k} | \psi(t) \rangle
\end{pmatrix}
\]

and \( \mathbf{R} \) is a closed trajectory in the parameter space \([1,30]\). We calculate the Berry phase acquired by an eigenstate of the Hamiltonian under cyclic and adiabatic evolution of parameters \((v, \varphi, \omega_a, \omega_b)\).

The only relevant parameter that will vary under time evolution is \( \varphi \). It is straightforward to see that the variation of the parameter \( \nu \) will not generate a Berry phase since

\[
A_\nu \propto \langle n f m d | S_a D_{ab} R_a \partial_\nu (R_a^\dagger D_{ab}^\dagger S_a^\dagger) | n f m d \rangle = \langle n f m d | S_a D_{ab} \partial_\nu (D_{ab}^\dagger S_a^\dagger) | n f m d \rangle = 0.
\]

Because there are no number operators inside the bra and the ket after derivation and action with all the operators, the only contribution to the Berry phase

\[
i \gamma_1 = \oint \mathbf{A} \cdot d\mathbf{R} = \int_0^{2\pi} d\varphi \ A_\varphi
\]

comes from the variation of the parameter \( \varphi \).

Now, since \( \partial_\varphi R_a^\dagger = iR_a^\dagger a^\dagger a \) and the other operators do not depend on \( \varphi \) we can readily compute

\[
A_\varphi = \langle n f m d | S_a S_b D_{ab} R_a \partial_\varphi (R_a^\dagger D_{ab}^\dagger S_a^\dagger) | n f m d \rangle = i \langle n f m d | S_a S_b D_{ab} R_a a^\dagger a R_a^\dagger D_{ab}^\dagger S_a^\dagger | n f m d \rangle = i \langle n f m d | S_b S_a D_{ab} a^\dagger a D_{ab}^\dagger S_a^\dagger | n f m d \rangle
\]

and making use of the relation \((7)\) in the appendix, we know that

\[
D(s, \phi) a^\dagger a D^\dagger(s, \phi) = a^\dagger a \cos^2 s + b^\dagger b \sin^2 s - \frac{1}{2} \sin 2s \left(a^\dagger b e^{i\phi} + b^\dagger a e^{-i\phi}\right)
\]

\[
\text{D}(s, \phi) a^\dagger a \text{D}^\dagger(s, \phi) = a^\dagger a \cos^2 s + b^\dagger b \sin^2 s - \frac{1}{2} \sin 2s \left(a^\dagger b e^{i\phi} + b^\dagger a e^{-i\phi}\right)
\]
\[ S_a(t, \theta) a^\dagger a S_a^\dagger(t, \theta) = a^\dagger a \cosh 2t - \frac{1}{2} (a^\dagger a^\dagger e^{-i\theta} + aa e^{i\theta}) \sinh 2t + \sinh^2 t \] (10)

and so we can successively commute the number operators and compute its integral over the parameter space. This yields the Berry phase

\[ \gamma_{I_{nf}} = 2\pi \left[ \frac{\omega_a n_d \cosh(2v) \sinh[2(C-v)]}{\omega_a \sinh[2(C-v)] + \omega_b \sinh(2v)} \right. \]

\[ + \frac{\omega_b n_f \sinh(2v) \cosh[2(C-v)]}{\omega_a \sinh[2(C-v)] + \omega_b \sinh(2v)} + T_{00} \] (11)

acquired by an eigenstate \( U |n_f n_d\rangle \), where

\[ T_{00} = \frac{\omega_a \sin^2 v \sinh[2(C-v)] + \omega_b \sinh(2v) \sinh^2(C-v)}{\omega_a \sinh[2(C-v)] + \omega_b \sinh(2v)} \] (12)

with \( C = \frac{1}{2} \ln(\omega_a/\omega_b) \) and \( \omega_a/\omega_b > e^{2v} \).

In the special case of the ground state \( (n_f = n_d = 0) \) we obtain

\[ \gamma_{I_{00}} = 2\pi T_{00}. \] (13)

Note that the ground state is non-degenerate and the gaps between energy levels are independent of time.

### 4 Thermometry from Geometric Phase

We here discuss how to utilize the geometric phase as a probe of thermal systems [2]. The idea is to use an atomic interferometer as a thermometer, measuring the temperature of a cold medium by comparison with a hotter thermal source of approximately known temperature. The geometric phase acquired by an atom interacting with a thermal state instead of an eigenstate of the Hamiltonian (see [31]) provides a measure of the temperature of the thermal state without any requirement that the atom comes into thermal equilibrium with the colder source.

Consider a bosonic medium at temperature \( T \) contained within a cavity. The density matrix is

\[ \rho_T = \bigotimes_{\omega} \frac{1}{\cosh^2 r_{\omega}} \sum n \tanh^{2n} r_{\omega} |n_{\omega}\rangle \langle n_{\omega}| \]

where

\[ \tanh r_{\omega} = \exp \left( -\frac{\hbar \omega}{2k_B T} \right). \]

The initial state of the field and atom the system is the mixed state \( |0\rangle \langle 0| \otimes \rho_T \). Upon adiabatically turning on the interaction, the system evolves into the state \( \rho = U^\dagger (|0\rangle \langle 0| \otimes \rho_f) U \).

The mixed state \( \rho \) acquires a geometric phase \( \gamma = \text{Re}(\eta) \), with

\[ e^{i\eta} = \sum_i \omega_i e^{i\gamma_i}, \]

(14)

after a a cycle of adiabatic evolution [31], where \( \gamma_i \) is the geometric phase acquired by the eigenstate \( |i\rangle \). Under one cycle of evolution for the state \( \rho_T \) we obtain

\[ e^{i\eta} = \frac{1}{\cosh^2 r_{\omega}} \sum n \tanh^{2n} r_{\omega} e^{i\gamma_{\omega n}} = \frac{e^{i\gamma_{I_{00}}}}{\cosh^2 r_{\omega} - e^{2\pi G} \sinh^2 r_{\omega}}, \] (15)

where

\[ G = \frac{\nu_f \sinh(2v) \cosh[2(C-v)]}{\nu_f \sinh[2(C-v)] + \nu_f \sinh(2v)}. \] (16)
and the parameters \((C, v, \nu_d, \nu_f)\) are as before, yielding
\[
\gamma_T = \text{Re} \eta = \gamma_{I_0} - \text{Arg} \left( \cosh^2 r_T - e^{2\pi i G} \sinh^2 r_T \right)
\]
for the acquired geometric phase. For two thermal environments at different temperatures the phase difference is
\[
\delta = \gamma_{T_1} - \gamma_{T_2} = \text{Arg} \left( 1 - e^{-\frac{\hbar \omega}{k_B T_1}} e^{-2\pi i G} \right) - \text{Arg} \left( 1 - e^{-\frac{\hbar \omega}{k_B T_2}} e^{-2\pi i G} \right)
\]
(18)

\[\begin{array}{cccc}
\hline
\text{Temperature (K)} & \text{Sensitivity} \frac{\Delta \delta}{\Delta T} \text{(a.u.)} \\
10^{-2} & 10^{-3} & 10^{-4} & 10^{-5} & 10^{-6} \\
10^0 & 10^1 & 10^2 & 10^3 & 10^4 \\
0.02 & 0.04 & 0.06 & 0.08 & 0.1 \\
\hline
\end{array}\]

Fig. 3 The geometric phase difference \(\delta\) between 2 detectors interacting with a cold and hot source of temperatures \(T_C\) and \(T_H\) respectively, as a function of the cold source temperature for different values of the atom gap and hot source temperature. From left to right: \(\Omega = 10^6\) hz, \(T_H = 1\) mK; \(\Omega = 10^7\) hz, \(T_H = 10\) mK; \(\Omega = 10^8\) hz, \(T_H = 0.1\) K; \(\Omega = 10^9\) hz, \(T_H = 1\) K. Coupling frequency: 1.2 KHz for all the cases. Red dashed lines are sensitivity curves for all the cases previously considered.

This phase difference can be quite large for realistic coupling values for atoms in cavities, as illustrated in fig. 3. Depending on the atomic gap, it is also very sensitive to a particular range of temperatures. We can thus tune the phase \(\delta\) to a particular temperature range; we find that \(\delta\) is quite sensitive to variations of the cold source but rather insensitive to changes in the hot source. This is shown in fig. 3 large variations in the hot source temperature translate into very small variations of the measured phase, providing us with a high precision thermometer. Furthermore, there is no need for the atomic (or multi-level system) probe to come into equilibrium with its thermal environment(s).

5 Detection of the Unruh Temperature

Our approach for constructing a large, high-precision thermometer using atomic interferometry techniques can be exploited to provide a new test of the Unruh effect. In this section we outline how this can be carried out.

A convenient choice of reference frame for computing the Berry phase in the case of an accelerating atom is to use Rindler coordinates \((\tau, \xi)\), for which \(\varphi = |\Omega_a| \xi - \Omega_a \tau\). The evolution is cyclic after a time \(\Delta \tau = \Omega_a^{-1}\). Adiabaticity can also be ensured in this case since the probability of excitation is negligible for the accelerations we consider \[32,4\]. Although \(H_T\) in \[1\] has the same form for both
Fig. 4 Relative error in the Berry phase (and therefore, the determination of the temperature for the cold source) as a function of the relative error in determining the temperature of the hot source. As we see, the setting is very robust: huge changes of temperature of the hot source translate into small changes in the phase δ.

inertial and accelerated detectors, in the inertial case $a, a^\dagger$ are Minkowski operators, whereas for the accelerated detector they correspond to Rindler operators.

To make this distinction clear, denote the respective Minkowski and Rindler operators by $U^\dagger_M$ and $U^\dagger_R$. The state of the field is not pure for accelerating observers but rather is mixed, a key distinction from the inertial case. In the basis of an accelerated observer, the state $|0_f\rangle\langle 0_f|$ transforms to the thermal Unruh state $\rho_f$ [41][43], and so before the field-detector interaction is turned on, the system is in the mixed state $\rho_f \otimes |0_d\rangle\langle 0_d|$. Upon suddenly switching on the interaction, a general state $|n_f0_d\rangle$ evolves, very close to a superposition of eigenstates $U^\dagger_R |i_fj_d\rangle$ where $N_f = i_f + j_d$ in the small λ coupling regime. We can ensure that the state of the joint system is $\rho_T = U^\dagger_R (\rho_f \otimes |0_d\rangle\langle 0_d|) U_R$ if we verify that the detector is still in its ground state (by making a projective measurement) immediately after switching on the interaction.

Calculating the mixed state Berry phase [31] we find

$$\gamma_a = \gamma_I - \text{Arg} \left( \cosh^2 q - e^{2\pi i G} \sinh^2 q \right)$$

where $\gamma_I$ is the inertial Berry phase, $q = \text{arctan} \left( e^{-\pi \omega_a c/a} \right)$ and

$$G = \frac{\omega_b \sinh(2v) \cosh[2(C - v)]}{\omega_a \sinh[2(C - v)] + \omega_b \sinh(2v)}$$

depends on the detector parameters.

We now compare the Berry phase acquired by the detector in the inertial and accelerated cases. After a complete cycle in the parameter space (with a proper time $\Omega^{-1}_a$) the phase difference between an inertial and an accelerated detector is $\delta = \gamma_I - \gamma_a$. The results are illustrated in figure 5 which plots the phase difference $\delta$ as a function of the acceleration for physically relevant atomic transition frequencies [33][32] coupled to the electromagnetic field (in resonance with the field mode they are coupled to) in the microwave regime (2.0 GHz). We consider three different coupling strengths: 1) $\lambda \simeq 34$ Hz, 2) $\lambda \simeq 0.10$ KHz, 3) $\lambda \simeq 0.25$ KHz.
The phase difference is large enough to be detected after a single cycle (about 3.1 ns). Evolving the system through more cycles will enhance the phase, since the effect is cumulative. The maximal phase difference achievable is $\delta = \pi$, corresponding to destructive interference. This can occur after 30000 cycles (95 µs) for an acceleration of $a \approx 4.5 \cdot 10^{17} \text{ m/s}^2$. For this magnitude of acceleration the atom will acquire a speed of $\approx 0.15c$ after a time $t \approx \Omega^{-1}$. Consequently the geometric phase acquired by the joint field/atom (more generally field/detector) state can be used as a tool to probe the Unruh effect for accelerations as small as $10^{17} \text{m/s}^2$.

6 Closing Remarks

There are several experimental challenges to be overcome in implementing quantum thermometry. The basic setup would be that of an interferometric experiment, as illustrated in figure [2].

For (inertial) quantum thermometry in general [2], it is necessary for weak adiabaticity to hold: there must be a near negligible probability of finding the atom in an excited state after one cycle of
evolution. Furthermore, this means that the atom does not have time to thermalize, and the hypotheses necessary to apply Berry’s formalism hold [3]. This requirement could fail for small atomic gaps, strong couplings, or high temperatures. In the latter case coherence loss will occur only for thermal sources at temperatures several orders of magnitude above the ones we are considering. When weak adiabaticity holds, the interaction time of the multi-level atom with the thermal state must be short enough so that the only change that atomic state acquires only a global phase (dynamical + geometrical). By solving the Schwinger equation (in the interaction picture)

\[
\frac{d}{dt} \rho = -i[H_I, \rho]
\]

numerically we find that the probability of finding the atom in the excited state cannot be distinguished from thermal noise for realistic values of the coupling \(\lambda\) after a short time \(\sim 10^4 \cdot 2\pi \Omega^{-1}\). Since in our scenario the atoms interact with the thermal bath only for very short times (1 cycle of evolution \(t \approx 2\pi \Omega^{-1}\)), weak adiabaticity holds, as illustrated in fig. [5]. Even in the worst case scenario (1 Mhz gap and 1 mK temperature) the probability of excitation is \(P \approx 10^{-3} \ll 1\), and values of \(P \approx 10^{-9} \ll 1\) are conceivable.

![Fig. 6](http://www.gebotech.de/pdf/LaserscaleGeneralCatalog_en_2010_04.pdf)

The quantum thermometer we propose has a rather sensitive target temperature range, typically about 3 orders of magnitude as shown in fig. [3]. The reference source temperature needs to be about 3 orders of magnitude larger (or smaller) than the target temperature, though hotter reference sources are preferred since they are easier to control.

To observe the Unruh effect, even though accelerations of \(10^{17}\)m/s are nine orders of magnitude smaller than other proposals [6], they are still formidable large, necessitating a compromise between the desired phase difference and feasibility of handling relativistic atoms. Since the phase accumulates independently of the sign of the acceleration, alternating periods of positive and negative acceleration could perhaps be exploited to reduce the atom’s final speed, and cancelling to some extent the dynamical phase difference between the paths in certain settings. For example, with current length metrology technology [2] the relative dynamical phase could be controlled with a precision \(\Delta \phi \approx 10^{-8}\), several

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\[2\] Laserscale, [http://www.gebotech.de/pdf/LaserscaleGeneralCatalog_en_2010_04.pdf](http://www.gebotech.de/pdf/LaserscaleGeneralCatalog_en_2010_04.pdf)
orders of magnitude smaller than the Berry phase acquired in one cycle. Recent work inspired by our quantum therometry approach has shown that it is possible to take both geometric and dynamical phases into account to build interferometric settings that are as precise as those we consider here [20]; no single-mode approximation is required.

Our approach can also be applied to Quantum Non-Demolition (QND) measurements. It can be shown that an atomic probe, on resonance with the target field mode we want to measure, can be sent through a cavity in a manner that does not alter the state of light in cavity whilst acquiring a non-negligible (and measurable) phase [20]. Known as ‘mode invisibility’, this technique allows for the effective distinction of Fock states containing very few photons via an interferometry setup similar to figure 2, in which one cavity contains a known state of light and the other one contains the unknown state of light that we want to probe. This method can be extended to coherent states of light that are experimentally more controllable and easier to prepare than Fock states, yielding information about some features of the Wigner function (such as the relative difference in the phase of a squeezing and a phase space displacement) [34].

Quantum therometry, while challenging, is at the edge of experimental feasibility. It opens up new ways to detect the Unruh effect and perhaps to probe other phenomena (for example Bose-Einstein condensates [35]) associated with relativistic quantum information. More generally, it can perhaps be used to probe a variety of field/atom systems that sensitively depend on one (or more) parameters. Work on these issues is in progress.

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### A Diagonalization of the Hamiltonian

Consider a point-like detector, endowed with an internal structure, which couples linearly to a scalar field \( \phi(x(t)) \) at a point \( x(t) \) corresponding to the world-line of the detector. The interaction Hamiltonian is of the form \( H_I \propto \hat{X}\hat{\phi}(x(t)) \) where we have chosen the detector to be modeled by a harmonic oscillator with frequency \( \Omega_d \). In this case the operator \( \hat{X} \propto (\hat{b}^\dagger + \hat{b}) \) corresponds to the detector’s position where \( \hat{b}^\dagger \) and \( \hat{b} \) are creation and annihilation operators.

Suppose that the detector couples only to a single mode of the field with frequency \( |k| = \Omega_a \). The field operator takes the form

\[
\hat{\phi}(x(t)) \approx \hat{\phi}_k(x(t)) \propto \left[ a e^{i(kx - \Omega_a t)} + a^\dagger e^{-i(kx - \Omega_a t)} \right]
\]

where \( a \) and \( a^\dagger \) are creation and annihilation operators associated with the field mode \( k \). The Hamiltonian is therefore given by eq. (1), which is

\[
H_T = \Omega_a a^\dagger a + \Omega_b \hat{b}^\dagger \hat{b} + \lambda(b + b^\dagger)[a^\dagger e^{i(kx - \Omega_a t)} + a e^{-i(kx - \Omega_a t)}]. \tag{20}
\]

where \( \lambda \) is the coupling frequency, and resembles an Unruh-DeWitt detector in the case where the atom interacts with a single mode of the field. In what follows we employ a mixed picture, in which the detector’s operators are time independent, in contrast to standard approaches that employ the interaction picture. The latter is the most convenient picture for computing transition probabilities, whereas we find the former mathematically more convenient for Berry phase calculations.

To diagonalize the Hamiltonian (20) we begin with a diagonal Hamiltonian of the form

\[
H_0 = \omega_a a^\dagger a + \omega_b \hat{b}^\dagger \hat{b}
\]

(21)

Our objective is to obtain the unitary transformation that diagonalises (20). We shall do this by finding the unitary transformation that transforms the Hamiltonian (22) into (20). The inverse operator is then the operator that diagonalizes (20). Once we obtain its eigenstates and eigenvalues we will be able to compute the geometrical phase acquired after cyclic evolution. Throughout we shall make use of the relation

\[
e^{B}A e^{-B} = \exp(a d_B) A = A + [B, A] + \frac{1}{2} [B, [B, A]] + \cdots
\]

(22)

where \( a d_B(A) \equiv [B, A] \).
Let us introduce the single mode squeeze operator

\[ S_a = \exp \left( a^* a - a^2 \right) \]

whose action on the creation/annihilation operators

\[
\begin{align*}
S_a^+ a S_a &= \cosh t + a^* e^{-i\phi} \\
S_a^+ a^* S_a &= a^* e^{-i\phi} + \cosh t
\end{align*}
\]

is straightforward to show upon setting \( \alpha = \frac{t}{2} e^{i\phi} \).

We first apply a 2 single mode squeeze to the Hamiltonian \( H_0 \) via

\[ H_{1s} = S_u(u, \theta_a) S_v(v, \theta_b) H_0 S^*_v(v, \theta_b) S^*_u(u, \theta_a) \]

obtaining

\[
H_{1s} = \omega_u \left[ a^* a \cosh 2u \sinh 2u \left( a^* a e^{-i\phi} + \cosh 2 \right) \right] + \omega_b \left[ b^* b \cosh 2v \sinh 2v \left( b^* b e^{-i\phi} + \cosh 2 \right) \right]
\]

where we have removed the constant term \( \sinh^2 u + \sinh^2 v \).

The 2-mode displacement operator is

\[ D(\chi) = \exp \left[ \chi a^* b - \chi^* a b \right] \]

and its action on the creation/annihilation operators is

\[
\begin{align*}
D^\dagger(s, \phi) a D(s, \phi) &= a^* \cos s + b^* e^{i\phi} \sin s \\
D^\dagger(s, \phi) a^* D(s, \phi) &= a \cos s + b^* e^{-i\phi} \sin s \\
D^\dagger(s, \phi) b D(s, \phi) &= b \cos s - a^* e^{-i\phi} \sin s \\
D^\dagger(s, \phi) b^* D(s, \phi) &= b^* \cos s - a^* e^{i\phi} \sin s
\end{align*}
\]

where we have defined \( \chi \equiv s e^{i\phi} \).

Computing the effect of the displacement on each of the 6 different operators in \( [24] \) we obtain

\[
\begin{align*}
D^\dagger(s, \phi) a^* a D(s, \phi) &= a^* \cos s + b^* b \sin^2 s + (1/2) \sin 2s \left( a^* b e^{i\phi} + b^* a e^{-i\phi} \right) \\
D^\dagger(s, \phi) b^* b D(s, \phi) &= a^* \sin^2 s + b^* b \cos^2 s - (1/2) \sin 2s \left( a^* b e^{i\phi} + b^* a e^{-i\phi} \right) \\
D^\dagger(s, \phi) a^* a^* D(s, \phi) &= a^* \cos s + b^* b e^{-2i\phi} \sin^2 s + a^* b^* e^{-i\phi} \sin 2s \\
D^\dagger(s, \phi) a a D(s, \phi) &= a^* e^{2i\phi} \cos s + b^* b \sin^2 s - a^* b^* e^{i\phi} \sin 2s \\
D^\dagger(s, \phi) b^* a D(s, \phi) &= b^* b \cos s + a^* a^* e^{2i\phi} \sin^2 s - a^* b^* e^{-i\phi} \sin 2s \\
D^\dagger(s, \phi) b a D(s, \phi) &= b^* b e^{2i\phi} \cos s + a^* a^* e^{-2i\phi} \sin^2 s - a^* b^* e^{i\phi} \sin 2s
\end{align*}
\]

Next we compute \( H_{1s,2d} = D^\dagger(s, \phi) H_{1s} D(s, \phi) \). Using \( [27] \) we find

\[
\begin{align*}
H_{1s,2d} &= \omega_u \left[ a^* a \cosh 2u \sinh 2u \left( a^* a e^{-i\phi} + \cosh 2 \right) \right] + \omega_b \left[ b^* b \cosh 2v \sinh 2v \left( b^* b e^{-i\phi} + \cosh 2 \right) \right] \\
&\quad + \frac{1}{2} \sinh 2u \left[ \left( a^* a \cos s + b^* b e^{-2i\phi} \sin^2 s + a^* b^* e^{-i\phi} \sin 2s \right) e^{-i\phi} \right] \\
&\quad + \left( a a \cos s + b b e^{2i\phi} \sin^2 s + b a e^{i\phi} \sin 2s \right) e^{i\phi} \\
&\quad + \frac{1}{2} \sinh 2v \left[ \left( b^* b \cos s + a^* a^* e^{-2i\phi} \sin^2 s - a^* b^* e^{i\phi} \sin 2s \right) e^{-i\phi} \right] \\
&\quad + \left( b b \cos s + a a e^{-2i\phi} \sin^2 s - a b e^{-i\phi} \sin 2s \right) e^{i\phi}
\end{align*}
\]
Regrouping terms we get

\[ H_{1s,2d} = g_1 a^\dagger a + g_2 b^\dagger b + g_3 a^\dagger b + g_4 b^\dagger a + g_5 a^\dagger a^\dagger + g_6 a b \]

+ \(g_7 b^\dagger b^\dagger + g_8 b b + g_9 a b^\dagger + g_{10} a b\) \hspace{1cm} (28)

where

\[
\begin{align*}
  g_1 &= \omega_a \cos^2 s \cosh 2u + \omega_b \sin^2 s \cosh 2v, \\
  g_2 &= \omega_a \sin^2 s \cosh 2u + \omega_b \cos^2 s \cosh 2v, \\
  g_3 &= \frac{1}{2} \sin 2s e^{i\phi} (\omega_a \cosh 2u - \omega_b \cosh 2v) \\
  g_4 &= \frac{1}{2} \left( \omega_a e^{-i\theta_a} \sinh 2u \cos^2 s + \omega_b e^{-i\theta_b} e^{2i\phi} \sinh 2v \sin^2 s \right) \\
  g_5 &= \frac{1}{2} \left( \omega_a e^{-i\theta_a} e^{-2i\phi} \sinh 2u \sin^2 s + \omega_b e^{-i\theta_b} \sinh 2v \cos^2 s \right) \\
  g_6 &= \frac{1}{2} \sin 2s \left( \omega_a e^{-i\theta_a} e^{-i\phi} \sinh 2u - \omega_b e^{-i\theta_b} e^{i\phi} \sinh 2v \right)
\end{align*}
\]

Applying a one mode rotation of the \(a\) operators

\[ R_a = \exp \left( - i \varphi a^\dagger a \right) \]

we find

\[
\begin{align*}
  R_a a R_a^\dagger &= e^{i\varphi} a \\
  R_a^\dagger R_a &= e^{-i\varphi} a^\dagger
\end{align*}
\] \hspace{1cm} (29)

yielding

\[
H_T = g_1 a^\dagger a + g_2 b^\dagger b + e^{i\varphi} g_3 a^\dagger b + e^{-i\varphi} g_4 b^\dagger a + e^{2i\varphi} g_5 a^\dagger a^\dagger + e^{-2i\varphi} g_6 a a^\dagger + e^{i\varphi} g_7 b^\dagger b^\dagger + e^{-i\varphi} g_8 b b + e^{i\varphi} g_9 a b^\dagger + e^{-i\varphi} g_{10} a b
\] \hspace{1cm} (31)

for the resultant Hamiltonian \(H_T = R_a^\dagger H_{1s,2d} R_a\).

Next we demand two conditions in order to reproduce the interaction Hamiltonian \([20]\). First we remove the squeezing terms \(a^\dagger a^\dagger\) of the field Hamiltonian. To do so, we fix

\[
\tan^2 s = \frac{\omega_a \sinh 2u}{\omega_b \sinh 2v}
\]

implying

\[
\begin{align*}
  g_1 &= \omega_a \cos^2 s \left[ \cosh 2u + \frac{\sinh 2u}{\tanh 2v} \right] \\
  g_2 &= \cos^2 s \left[ \frac{\omega_a^2 \sinh 4u}{2\omega_b \sinh 2v} + \omega_b \cos 2v \right] \\
  g_3 &= \frac{1}{2} \sin 2s e^{i\phi} (\omega_a \cosh 2u - \omega_b \cosh 2v) \\
  g_4 &= \frac{1}{2} \omega_a \cos^2 s \sinh 2u \left( e^{-i\theta_a} + e^{-i\theta_b} e^{2i\phi} \right) \\
  g_5 &= \frac{1}{2} \cos^2 s \left( \omega_a e^{i\theta_a} e^{-i\phi} e^{-2i\phi} + \omega_b \sinh 2v e^{-i\theta_b} \right) \\
  g_6 &= \frac{1}{2} \sin 2s \left( \omega_a e^{-i\theta_a} e^{-i\phi} \sinh 2u - \omega_b e^{-i\theta_b} e^{i\phi} \sinh 2v \right)
\end{align*}
\]
Setting $\theta_b = 2\phi + \theta_a - \pi$ yields

$$g_1 = \omega_a \cos^2 s \left[ \cosh 2u + \frac{\sinh 2u}{\tanh 2v} \right]$$
$$g_2 = \cos^2 s \left[ \frac{\omega_a^2 \sinh 4u}{2\omega_b \sinh 2v} + \omega_b \cos 2v \right]$$
$$g_3 = \frac{1}{2} \sin 2s e^{i\theta_b} (\omega_a \cosh 2u - \omega_b \cosh 2v)$$
$$g_4 = 0$$
$$g_5 = \frac{1}{2} e^{-i(2\phi + \theta_a)} \cos^2 s \left( \frac{\omega_b^2 \sinh 2u}{\omega_b \sinh 2v} - \omega_b \sinh 2v \right)$$
$$g_6 = \frac{1}{2} e^{-i(\theta_a + \phi)} \sin 2s (\omega_a \sinh 2u + \omega_b \sinh 2v)$$

and so the term corresponding to a squeezing of the field has been eliminated.

To reproduce the interaction part we require $g_3 = g_6$, implying

$$e^{i(2\phi + \theta_a)} (\omega_a \cosh 2u - \omega_b \cosh 2v) = (\omega_a \sinh 2u + \omega_b \sinh 2v)$$

Setting $\theta_a = 2n\pi - 2\phi$ gives

$$\omega_a \omega_b = \frac{\cosh 2v + \sinh 2v}{\cosh 2u - \sinh 2u} = \frac{e^{2v}}{e^{-2v}}$$

and as a consequence

$$u = \frac{1}{2} \ln \left( \frac{\omega_a}{\omega_b} \right) - v$$

Finally we need to demand that

$$\frac{\omega_a}{\omega_b} > e^{2v}$$

(32)

to ensure that $u > 0$.

Recapitulating, we started from the Hamiltonian $H_0$ and applied two 1-mode squeezing operators, a 1-mode displacement operator and a 1-mode rotation on the field operators

$$H_T = R_1^\dagger(\phi) D^\dagger(s, \phi) S_1^\dagger(u, \theta_a) S_1^\dagger(v, \theta_b) H_0 S_a(u, \theta_a) S_b(v, \theta_b) D(s, \phi) R_a(\phi)$$

(33)
yielding a Hamiltonian depending on 6 parameters. By fixing 4 of them

$$s = \arctan \left( \frac{\omega_a \sinh 2u}{\omega_b \sinh 2v} \right), \quad \theta_a = 2n\pi - 2\phi$$

(34)
$$\theta_b = 2\phi + \theta_a - \pi, \quad \omega_a \omega_b = \frac{\cosh 2v + \sinh 2v}{\cosh 2u - \sinh 2u} = \frac{e^{2v}}{e^{-2v}}$$

(35)

with the extra requirement for $v$ given by (32), we obtain the hamiltonian $H_T$

$$H_T = \Omega_a a^\dagger a + \Omega_b b^\dagger b + \lambda (b + b^\dagger)(a^\dagger e^{i(\phi + \varphi)} + a e^{-i(\phi + \varphi)}) + Z (b^\dagger b^\dagger + b b)$$

(36)

where

$$\Omega_a = \frac{\sinh 2v [\cosh [2(C - v)] + \frac{\sinh[2(C - v)]}{\tanh 2v}]}{\omega_a \sinh 2v + \omega_b \sinh [2(C - v)]}$$
$$\Omega_b = \sqrt{\frac{\omega_a \omega_b \sinh[2(C - v)] \sinh 2v}{\omega_b \sinh 2v + \omega_a \sinh [2(C - v)]}} [\omega_a \cosh [2(C - v)] - \omega_b \cosh 2v]$$

$$\lambda = \frac{\lambda}{\omega_a \omega_b \sinh[2(C - v)] \sinh 2v}{\omega_b \sinh 2v + \omega_a \sinh [2(C - v)]} [\omega_a \cosh [2(C - v)] - \omega_b \cosh 2v]$$
$$Z = \frac{1}{2} \sinh 2v \left( \frac{\omega_a^2 \sinh^2 [2(C - v)]}{\omega_b \sinh [2(C - v)]} - \omega_b \sinh [2(C - v)] \right)$$

(37)

$$\varphi = kx - \Omega_b t$$
with $C = \frac{1}{2} \ln \left( \frac{\omega}{\omega_0} \right)$ and where $2p = \tanh^{-1} \left[ -2Z/\Omega_b \right]$. The rotation is necessary to account for the time evolution on a given trajectory as it is completely decoupled from the rest of parameters. Actually for a particular choice of the displacement parameter phase $\phi$ (for example $\phi = 0$) we trivially get

$$H_T = \Omega_a a^\dagger a + \Omega_b b^\dagger b + \hat{\lambda}(b + b^\dagger)(a^\dagger e^{i\varphi} + a e^{-i\varphi}) + Z \left( b^\dagger b^\dagger + bb \right)$$

(38)

Applying another squeezing operator $S_b(p)$ (where $p$ is real) yields

$$S_b^\dagger S_b^\dagger b^\dagger b S_b S_a = b^\dagger b \cosh 2p + \frac{1}{2} \sinh 2p \left( b^\dagger b^\dagger + bb \right) + \sinh^2 p$$

(39)

and so the interaction Hamiltonian $H_T = S_b^\dagger(p)H_T S_b(p)$, after eliminating constant terms, is

$$H_T = \Omega_a a^\dagger a + \hat{\Omega}_b \left[ b^\dagger b \cosh 2p + \frac{1}{2} \sinh 2p \left( b^\dagger b^\dagger + bb \right) + \sinh^2 p \right]$$

$$+ \hat{\lambda}(\sinh q + \cosh q)(b + b^\dagger)(a^\dagger e^{i\varphi} + a e^{-i\varphi})$$

$$+ Z \left( fb \cosh^2 p + b^\dagger b^\dagger \sinh^2 p + b^\dagger b \sinh 2p \right)$$

$$+ b^\dagger b^\dagger \cosh^2 p + bb \sinh^2 p + b^\dagger b \sinh 2p$$

(40)

which can be rewritten as

$$H_T = \Omega_a a^\dagger a + \left( \hat{\Omega}_b \cosh 2p + 2Z \sinh 2p \right) b^\dagger b$$

$$+ e^\varphi \hat{\lambda}(b + b^\dagger)(a^\dagger e^{i\varphi} + a e^{-i\varphi})$$

$$+ (b^\dagger b^\dagger + bb) \left( Z \cosh 2p + \frac{\omega}{2} \sinh 2p \right)$$

(41)

(42)

Fixing a value of $p$ such that

$$2p = \tanh^{-1} \left( \frac{-2Z}{\omega} \right)$$

yields the Hamiltonian

$$H_T = \Omega_a a^\dagger a + \sqrt{\hat{\Omega}_b^2 - 4Z^2} b^\dagger b + e^\varphi \hat{\lambda}(b + b^\dagger)(a^\dagger e^{i\varphi} + a e^{-i\varphi})$$

We can rewrite this as an Unruh DeWitt hamiltonian

$$H_T = \Omega_a a^\dagger a + \Omega_b b^\dagger b + \lambda(b + b^\dagger)(a^\dagger e^{i\varphi} + a e^{-i\varphi})$$

(43)

where

$$\lambda = e^\varphi \hat{\lambda} \quad \Omega_b = \sqrt{\hat{\Omega}_b^2 - 4Z^2}$$

(44)

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