Understanding the like-sign dimuon charge asymmetry in $p\bar{p}$ collisions

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The DØ collaboration has measured the like-sign dimuon charge asymmetry in $p\bar{p}$ collisions at the Fermilab Tevatron collider. The result is significantly different from the standard model expectation of CP violation in mixing. In this paper we consider the possible causes of this asymmetry and identify one standard model source not considered before. It decreases the discrepancy of the like-sign dimuon charge asymmetry with the standard model prediction, although it does not eliminate it completely.

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I. INTRODUCTION

The DØ Collaboration has measured $^{[3]}$ the like-sign dimuon charge asymmetry and the inclusive muon charge asymmetry in $p\bar{p}$ collisions at a center-of-mass energy $\sqrt{s} = 1.96$ TeV at the Fermilab Tevatron collider. After subtracting the known background sources, the inclusive muon charge asymmetry is found to be compatible with zero while the like-sign dimuon charge asymmetry significantly deviates from the standard model (SM) prediction. This deviation is usually interpreted as the anomalous charge asymmetry of $B^0$ and $\bar{B}_s^0$ semileptonic decays $^{[4]}$.

In this paper we consider other possible sources of the like-sign dimuon charge asymmetry, taking into account the constraint that the inclusive muon asymmetry is consistent with zero. We identify one significant contribution which was not accounted for previously. In addition, we discuss all other sources of the dimuon charge asymmetry and show that the measurements of the DØ collaboration put strong constraints on them. After presenting available experimental results related to the dimuon charge asymmetry in Section II, we consider in Section III one by one the contribution of different processes into this asymmetry. Our results are summarised in Section IV.

II. EXPERIMENTAL RESULTS

We use the results of Ref. $^{[4]}$ and express them in a model independent way as the charge asymmetry of the inclusive muon sample $a_S$, and the charge asymmetry of the like-sign dimuon sample $A_S$. We follow the notations and definitions of Ref. $^{[4]}$, where the asymmetries $a_S$ and $A_S$ are computed using the muons coming from the decays of $b$ and $c$ quarks and $\tau$ leptons, and from decays of short-living mesons such as $\phi$, $\omega$, $\eta$, $\rho^0$, $J/\psi$. The origin of the asymmetries $a_S$ and $A_S$ may be the semileptonic charge asymmetry of $B^0$ and $\bar{B}_s^0$ decays, as well as other $CP$-violating processes.

From the information presented in $^{[4]}$ we obtain

$$a_S = (-0.063 \pm 0.079(stat) \pm 0.141(syst))\%, \quad (1)$$

$$A_S = (-0.383 \pm 0.092(stat) \pm 0.102(syst))\%. \quad (2)$$

These values are obtained, respectively, by multiplying the results given in Eqs. (34) and (35) of Ref. $^{[4]}$. An important observation which can be derived from these results is that $a_S$ is compatible with zero, while $A_S$ is significantly different from zero.

The composition of the inclusive muon sample is presented, for convenience, in Table $^{[4]}$ taken from Ref. $^{[4]}$. The composition of the like-sign dimuon sample can be derived from the information given in Table $^{[4]}$ assuming that the two muons come from independent processes. This assumption can be applied because the requirement that the invariant mass of the two like-sign muons is greater than 2.8 GeV, which is used for selecting the dimuon pairs $^{[4]}$, suppresses decays with the two muons originating from the same $b$ quark. Since the oscillation of the $D^0$ meson is found to be small $^{[5]}$, the contribution of process $T_6$ in Table $^{[4]}$ to the like-sign dimuon sample is suppressed and, therefore, is neglected in the following discussion.

For our purposes it is convenient to classify the processes $T_3$ into three different categories denoted $T_{3d}(f_i)$, $T_{3s}(f_j)$ and $T_{36}$s. Processes $T_{3d}(f_i)$ correspond to the decays of $B^0$ or $\bar{B}^0$ to the final state $f_i$ containing two $c$ quarks and accessible to both $B^0$ and $\bar{B}^0$. At the quark level these decays correspond to the process $(bd) \rightarrow c\bar{c}d\bar{d}$ or $(bd) \rightarrow c\bar{c}d\bar{d}$. Similarly, processes $T_{3s}(f_j)$ correspond to the decays of $B^0_s$ or $\bar{B}^0_s$ to the final state $f_j$ containing two $c$ quarks and accessible to both $B^0$ and $\bar{B}^0$. These decays at the quark level are $(bs) \rightarrow c\bar{c} \bar{s}s$. All other decays of $B$ hadrons producing two charm quarks are flavour specific and included in the group denoted $T_{36}$. The weights of these processes are respectively $w_{3d}(f_i)$, $w_{3s}(f_j)$ and $w_{36}$. By definition,

$$w_3 = \sum_i w_{3d}(f_i) + \sum_j w_{3s}(f_j) + w_{36}. \quad (3)$$
TABLE I: Heavy-quark decays contributing to the inclusive muon and like-sign dimuon samples taken from Ref. [4]. The abbreviation “non-osc” stands for “non-oscillating,” and “osc” for “oscillating.” All weights are computed using MC simulation. $\chi_0 = 0.1260 \pm 0.0037$ is the time-integrated mixing probability [8].

| Process | Weight  |
|---------|---------|
| $T_1 \ b \rightarrow \mu^- X$ | $w_1 = 1$ |
| $T_{1a} \ b \rightarrow \mu^- X$ (non-osc) | $w_{1a} = (1 - \chi_0)w_1$ |
| $T_{1b} \ b \rightarrow \mu^- X$ (osc) | $w_{1b} = \chi_0 w_1$ |
| $T_2 \ b \rightarrow c \rightarrow \mu^+ X$ | $w_2 = 0.096 \pm 0.012$ |
| $T_{2a} \ b \rightarrow c \rightarrow \mu^+ X$ (non-osc) | $w_{2a} = (1 - \chi_0)w_2$ |
| $T_{2b} \ b \rightarrow c \rightarrow \mu^+ X$ (osc) | $w_{2b} = \chi_0 w_2$ |
| $T_3 \ b \rightarrow c \bar{c}q$ with $c \rightarrow \mu^+ X$ or $\bar{c} \rightarrow \mu^- X$ | $w_3 = 0.064 \pm 0.006$ |
| $T_4 \ \eta, \omega, \rho^0, \phi(1020), J/\psi, \psi' \rightarrow \mu^+ \mu^-$ | $w_4 = 0.021 \pm 0.002$ |
| $T_5 \ bb\bar{c}c$ with $c \rightarrow \mu^+ X$ or $\bar{c} \rightarrow \mu^- X$ | $w_5 = 0.013 \pm 0.002$ |
| $T_b \ \bar{c} \bar{c}$ with $c \rightarrow \mu^+ X$ or $\bar{c} \rightarrow \mu^- X$ | $w_6 = 0.675 \pm 0.101$ |

III. CONTRIBUTIONS TO THE LIKE-SIGN DIMUON CHARGE ASYMMETRY

We consider in this section some SM processes producing the like-sign dimuon charge asymmetry $A_S$, and the current experimental constraints of their contribution taken from [8]. Particles of physics beyond the SM may add new Feynman diagrams with loops in $B^0 \leftrightarrow B^0$ mixing, $B_s^0 \leftrightarrow B_s^0$ mixing, and in penguin decays. They are not discussed here.

The main source of the like-sign dimuon pairs produced in $p\bar{p}$ collisions is $b\bar{b}$ events. The background muons from $K \rightarrow \mu\nu$ and $\pi \rightarrow \mu\nu$ decays are excluded by definition from the asymmetries of $A_S$ and $A_g$, and are not discussed here. One of the $B$ hadrons from the $b\bar{b}$ pair decays to a “right sign” muon, i.e. to a muon with charge of the same sign as the charge of the initial $b$ or $\bar{b}$ quark at the time of the $p\bar{p}$ collision. To obtain a like-sign dimuon event, the other $b$-hadron must decay to a “wrong sign” muon.

A. CP violation in mixing of $B^0$ and $B_s^0$ mesons

The complex phase $\phi_{12}$ of the mass matrix of the $B_q^0$ ($q = d, s$) system produces the charge asymmetry $A_{q}^S$ of the “wrong sign” semileptonic $B_q^0$ decays defined as

$$a_{q}^S = \frac{\Gamma(B_q^0(t) \rightarrow l^+ X) - \Gamma(B_q^0(t) \rightarrow l^- X)}{\Gamma(B_q^0(t) \rightarrow l^+ X) + \Gamma(B_q^0(t) \rightarrow l^- X)}$$

(4)

The asymmetry $a_{q}^S$ is related to the phase $\phi_{12}$ as [8]

$$a_{q}^S = \frac{\Delta \Gamma_q}{\Delta m_q} \tan \phi_{12}.$$

(5)

The contributions of this asymmetry to the inclusive muon charge asymmetry and the like-sign dimuon charge asymmetry are

$$a_S^q(a_q^S) = c_b C_q a_{q}^S,$$

(6)

$$A_S^q(a_q^S) = C_b C_q a_{q}^S,$$

(7)

where the coefficients $c_b$, $C_b$, $C_d$ and $C_s$ are defined in Ref. [8]:

$$c_b = 0.061 \pm 0.007,$$

$$C_b = 0.474 \pm 0.032,$$

(8)

$$C_d = 0.594 \pm 0.022,$$

$$C_s = 0.406 \pm 0.022.$$  

(9)

The SM predictions of the phases $\phi_{12}^q$ and asymmetries $a_{q}^S$ are [8]:

$$\phi_{12}^q(\text{SM}) = -0.075 \pm 0.024,$$

(10)

$$\phi_{12}^d(\text{SM}) = +0.0038 \pm 0.0010,$$

(11)

$$a_{d}^S(\text{SM}) = -(4.1 \pm 0.6) \times 10^{-4},$$

(12)

$$a_{s}^S(\text{SM}) = +(1.9 \pm 0.3) \times 10^{-5}.$$  

(13)

Recently, the experimental measurements of both $a_{q}^d$ and $a_{q}^s$ became available. The measurements of $a_{q}^d$ are performed at $\Upsilon(4S)$ [2] and by the DO experiment [8]:

$$a_{q}^d = +0.0002 \pm 0.0031 \ (\Upsilon(4S)),  $$

(14)

$$a_{q}^d = +0.0068 \pm 0.0047 \ (DO).$$

(15)

Our combination of these values gives

$$a_{q}^d = +0.0022 \pm 0.0026.$$  

(16)

The measurements of $a_{q}^s$ are performed by DO [10] and LHCb [11] collaborations:

$$a_{q}^s = -0.0104 \pm 0.0074 \ (DO),$$

(17)

$$a_{q}^s = -0.0024 \pm 0.0063 \ (LHCb).$$

(18)

Our combination of these values gives

$$a_{q}^s = -0.0058 \pm 0.0048.$$  

(19)

Using the values [10] and [19] we obtain the allowed contributions to the inclusive muon and like-sign dimuon charge asymmetries from CPV in mixing of $B^0$ and $B_s^0$ mesons:

$$a_S(a_{q}^S) = (+0.008 \pm 0.009)%,$$

(20)

$$A_S(a_{q}^S) = (+0.062 \pm 0.073)%,$$

(21)

$$a_S(a_{q}^S) = (-0.014 \pm 0.012)%,$$

(22)

$$A_S(a_{q}^S) = (-0.111 \pm 0.093)%,$$

(23)

and their sum

$$a_S(a_{q}^S) = (-0.006 \pm 0.015)%,$$

(24)

$$A_S(a_{q}^S) = (-0.049 \pm 0.118)%.$$  

(25)

For comparison, the SM prediction is

$$A_S(a_{q}^S \text{ in SM}) = (-0.013 \pm 0.002)%.$$  

(26)
In addition, the estimate of $a^s_{\ell}$ can be extracted from the measurement of CP violation in the $B^0 \to J/\psi \phi$ decay. The corresponding phase $\phi_{s}^{\ell} e^{i s}$ is expected to change in the same way as the phase $\phi_{12}^{s}$ due to a new physics contribution $\phi_{s}^{\ell} [8,12]$

$$\phi_{s}^{\ell} = \phi_{s}^{\ell}(\text{SM}) + \phi_{s}^{\Delta}, \quad \phi_{s}^{\ell}(\text{SM}) = -\sin(2\beta) = -0.036 \pm 0.002. \quad (27)$$

The current world average value of $\phi_{s}^{\ell}$ is [2]:

$$\phi_{s}^{\ell} = -0.013^{+0.803}_{-0.090}. \quad (28)$$

From these expressions we get the following estimate of $a^s_{\ell}$ from $B_s^0 \to J/\psi \phi$ decay:

$$a^s_{\ell} \text{(from } B_s^0 \to J/\psi \phi) \approx -0.0001 \pm 0.0005. \quad (29)$$

However, we do not use the result (30) because Eq. (27) may be subject to large penguin corrections from new physics [13].

Thus, the current experimental results constrain the contribution of CPV in mixing of $B^0$ and $\bar{B}^0$ mesons to the measured like-sign dimuon charge asymmetry.

**B. CP violation in interference of $B^0$ decay with and without mixing**

The final states of the decay $B^0(\bar{B}^0) \to c\bar{c}d\bar{d}$ are accessible from both $B^0$ and $\bar{B}^0$. Therefore, the interference of decays to these final states with and without $B^0$ mixing results in CPV [8]. It turns out that this CPV produces a like-sign dimuon charge asymmetry. At the same time, its contribution is negligible in the inclusive muon charge asymmetry.

To demonstrate this, let us consider an example of the process producing a positive dimuon pair

$$\bar{p}p \to B^+ B^0 X, \quad B^+ \to \mu^+ X, \quad \bar{B}^0 \to D^+ D^-, D^+ \to \mu^+ X. \quad (31)$$

The state $f_i = D^+ D^-$ is a CP-even eigenstate accessible from both $B^0$ and $\bar{B}^0$ mesons. The $D^+ \to \mu^+ X$ decay produces a “wrong sign” muon, and contributes to the like-sign dimuon sample. The $D^- \to \mu^- X$ decay produces a “right sign” muon, and therefore does not contribute to the like-sign dimuon sample. The number of positive and negative muons from the decay $B^0 \to D^+ D^-$ is the same, therefore there is no contribution to the inclusive muon charge asymmetry from this decay.

The decay rate of the meson that is initially produced as a $B^0$ is [8]

$$\frac{d\Gamma(B^0 \to f_i)}{dt} \propto \exp(-\Gamma_d t) \times [1 + S_i \sin(\Delta m_d t) - C_i \cos(\Delta m_d t)]. \quad (32)$$

The term proportional to $S_i$ is due to CPV in the interference of decays with and without mixing to $D^+ D^-$, of the meson that is initially produced as $\bar{B}^0$. The term proportional to $C_i$ is due to the direct CPV in $B^0$ decay. Neglecting the loop contributions to the decay amplitude, the coefficients $S_i$ and $C_i$ in the SM are expressed as [13]

$$S_i = -\eta_i \sin(2\beta), \quad (33)$$
$$C_i = 0,$$

where $\eta_i$ is the CP eigenvalue of the final state $f_i$, and $\beta$ is the angle of the Unitarity Triangle [8]. The loop diagrams can change this estimate by a few percent [15,16]. For the $B^0 \to D^+ D^-$ final state $\eta_i = +1$. If $f_i$ is not a CP eigenstate, the coefficient $C_i$ may be non-zero, but experimentally for the decays considered here $C_i$ is negligible [8]. Therefore, we omit it in the following discussion. Integrating (32) we obtain the width of the decay to this final state

$$\Gamma(B^0 \to f_i) \propto 1 + S_i \frac{x_d}{1 + x_d^2}, \quad (34)$$

where $x_d = \frac{\Delta m_d}{\Gamma_d}$.

Now consider the CP-conjugate process producing a negative dimuon pair

$$\bar{p}p \to B^- B^0 X, \quad B^- \to \mu^- X, \quad B^0 \to D^+ D^-, D^- \to \mu^- X. \quad (35)$$

The decay rate of the meson that is initially produced as a $B^0$ is

$$\frac{d\Gamma(B^0 \to f_i)}{dt} \propto \exp(-\Gamma_d t) \times [1 - S_i \sin(\Delta m_d t)], \quad (36)$$

and the partial width is

$$\Gamma(B^0 \to f_i) \propto 1 - S_i \frac{x_d}{1 + x_d^2}. \quad (37)$$

The like-sign dimuon charge asymmetry from this process is

$$A_i = S_i \frac{x_d}{1 + x_d^2}. \quad (38)$$

Numerically the absolute value of this asymmetry is large, because $\sin(2\beta) = 0.679 \pm 0.020$ and $x_d = 0.770 \pm 0.008$ [8].

Let us now obtain the contribution of the decay channel $f_i$ with weight $w_{3d}(f_i)$ to the like-sign dimuon charge asymmetry $A_S$. This weight takes into account both the branching fraction of $B^0 \to f_i \to \mu X$ decay and detector-related efficiency of muon reconstruction. The weights for the various processes have been defined in Section [11]. The probability that an initial $b$ quark produces a “right sign” muon $\mu^-$ is [11]

$$P_b \propto 0.5 w_{3d}(f_i) \left[1 + S_i \frac{x_d}{1 + x_d^2}\right] + w_{4a} + w_{2b} + 0.5 (w_{3s} + w_{3Is} + w_4 + w_5). \quad (39)$$
The factor 0.5 in this expression corresponds to the statement that the number of positive and negative muons produced in the processes $T_3$, $T_4$ and $T_5$ is the same. The probability that an initial $\bar{b}$ quark produces a “wrong sign” muon $\mu^-$ is

$$P_b = 0.5w_{3d}(f_i) \left[ 1 - S_i \frac{x_d}{1 + x_d} \right] + w_{1b} + 0.5(w_{3s} + w_{3fs} + w_4 + w_5).$$

The number of $\mu^-\mu^-$ events is proportional to $P_b^2$. The number of $\mu^+\mu^+$ events is obtained by replacing $S_i$ by $-S_i$. The charge asymmetry from channels $f_i$ is then

$$A_S(f_i) = 0.5w_{3d}(f_i)S_i \frac{x_d}{1 + x_d} \frac{P_b - P_{\bar{b}}}{P_b P_{\bar{b}}}.$$  \hspace{1cm} (41)

Thus, CPV in interference of $B^0$ decay with and without mixing produces a like-sign dimuon charge asymmetry, while it does not contribute to the inclusive muon charge asymmetry. The possible final states produced in the $B^0\bar{B}^0 \to c\bar{c}d\bar{d}$ decay include $D^+D^-$, $D^{*+}D^-$, $D^+D^{*-}$, $D^{*+}D^{*-}$, $J/\psi\pi^0$, $J/\psi\eta$, $J/\psi\eta'_1$, etc. For many of these final states the value $S_i$ is measured experimentally \cite{8} and in all cases it is consistent with the SM value $S_i = -\sin(2\beta)$, which corresponds to the expectation of the dominance of the CP-even final states in the $B^0(\bar{B}^0) \to c\bar{c}d\bar{d}$ decay.

On the contrary, the contribution of CPV in the $b \to c\bar{c}s$ transition producing the CP eigenstates, like the $B^0 \to J/\psi K_S$ decay, should not contribute to the like-sign dimuon charge asymmetry, because for each CP-even final state there should be the corresponding CP-odd final state. For example, the contribution from the decay $B^0 \to J/\psi K_S$ is canceled by the decay $\bar{B}^0 \to J/\psi K_L$.

The weight $w_{3d}(f_i)$ can be obtained by using the measured branching fraction of $B^0 \to f_i$ decay. For example, the weight $w_{3d}(D^+D^-)$ can be computed using the following expression

$$w_{3d}(f_i) = f_d \alpha \frac{\text{Br}(B^0 \to D^+D^-)2\text{Br}(D^+ \to \mu X)}{\text{Br}(b \to \mu X)}.$$  \hspace{1cm} (42)

The coefficient $w_{3d}(f_i)$ by definition is normalised to the weight $w_1$, which is proportional to $\text{Br}(b \to \mu X)$. The value $\text{Br}(b \to \mu X)$ is the average branching fraction of the direct decay of all $B$ hadrons to muon weighted with the relative production rate of different $B$ hadrons at the hadron collider. Also by definition, the weight $w_{3d}(f_i)$ includes both decays $D^+ \to \mu^+X$ and $D^- \to \mu^-X$, hence the factors 0.5 in Eqs. (42) to (44) and the factor 2 in Eq. (42). The factor $f_d = 0.401 \pm 0.008$ \cite{8} is the fraction of $B^0$ plus $\bar{B}^0$ in the admixture of $b$-hadrons. The factor $\alpha$ is the ratio of detector acceptances of muons from $D^+$ and $B^0$ decay. Muons from $D^+$ and $B^0$ mesons have different detector acceptances because they have different kinematic distributions.

Using the results of Ref. \cite{4} and other experimental values from \cite{8} we estimate the coefficient $\alpha$ from the following expressions:

$$w_1 \propto \text{Br}(b \to \mu X),$$

$$w_3 \propto \text{Br}(b \to c\bar{c}X)\text{Br}(c\bar{c}q\bar{q}' \to \mu X)\alpha.$$  \hspace{1cm} (43)

Here $\text{Br}(b \to c\bar{c}X)$ is the branching fraction of $B$ hadron decays producing $c\bar{c}$ pair. We use the experimental value $\text{Br}(b \to c\bar{c}X) = 0.162 \pm 0.032$ which is obtained from $\text{Br}(B^+/B^0/B_s^+/b$-baryon mixture $\to c\bar{c}X) = 1.162 \pm 0.032$ \cite{8}. The quantity $\text{Br}(c\bar{c}q\bar{q}' \to \mu X)$ is the average branching fraction of the direct decay of all charmed hadrons to muon weighted with the relative production rate of different pairs of $c$ hadrons in $B$ hadron decay. Using the values of corresponding branching fractions for the $\bar{B}^0/B^0/B_s^+/b$-baryon mixture from Ref. \cite{8} we obtain

$$\text{Br}(b \to \mu X) = 0.107 \pm 0.003,$$

$$\text{Br}(c\bar{c}q\bar{q}' \to \mu X) = 0.164 \pm 0.032.$$ \hspace{1cm} (44)

From these expressions we obtain

$$\alpha = \frac{w_3}{\text{Br}(b \to c\bar{c}X)\text{Br}(c\bar{c}q\bar{q}' \to \mu X)} = 0.258 \pm 0.073.$$ \hspace{1cm} (45)

This estimate of $\alpha$ does not take into account different kinematic distributions for the various decays $c \to \mu X$. Therefore, a simulation of the DØ detector is required to obtain a more accurate value for $\alpha$.

Using Eqs. (42) and (46) we obtain

$$w_{3d}(f_i) = f_d w_3 \frac{\text{Br}(B^0 \to f_i)}{\text{Br}(b \to c\bar{c}X)\text{Br}(c\bar{c}q\bar{q}' \to \mu X)}.$$  \hspace{1cm} (47)

Using this observation, we estimate $A_S$ by several methods.

**Estimate 1.** Let us consider four measured decay channels \cite{8}: $D^+D^-$ with $S = -0.87 \pm 0.26$, $\Gamma_i/\Gamma = (2.11 \pm 0.31) \times 10^{-4}$, $D^*(2010)^+D^-$ with $S = -0.61 \pm 0.19$, $\Gamma_i/\Gamma = (6.1 \pm 1.5) \times 10^{-4}$; $D^*(2010)^-D^+$ with $S = -0.78 \pm 0.21$, $\Gamma_i/\Gamma \approx 6.1 \times 10^{-4}$ (our guess); and $D^{*+}D^{*-}$ with $S = -0.76 \pm 0.14$, $\Gamma_i/\Gamma \approx (8.2 \pm 0.9) \times 10^{-4}$. Using these numbers we obtain for the sum of these channels

$$\sum_i (S_i \text{Br}(B^0 \to f_i)\text{Br}(f_i \to \mu X)) = -0.00044 \pm 0.00011.$$  \hspace{1cm} (48)

Using this value and Eqs. (41) and (47), the contribution to $A_S$ from these 4 channels is

$$A_S(4 \text{ channels}) = (-0.028 \pm 0.011)\%.$$ \hspace{1cm} (49)

**Estimate 2.** For this estimate we assume that the final states $\bar{c}cd\bar{d}$ are mostly CP-even ($\eta_i = +1$), which...
is appropriate for $D^{(*)+}D^{(*)-}$ final states and confirmed by the experimental measurements. Then

$$
\frac{\text{Br}(B^0 \rightarrow c\bar{c}d\bar{d})}{(\text{Br}(b \rightarrow c\bar{c}s) + \text{Br}(b \rightarrow c\bar{c}d))} \approx V_{cd}^2.
$$

Eq. (41) becomes approximately

$$
A_S \approx -0.5w_3f_sV^2_{cd}\sin(2\beta)\frac{x_d}{1 + x_d^2}\frac{P_b - P_{\bar{b}}}{P_bP_{\bar{b}}},
$$

$$
\delta = \frac{\text{Br}(c\bar{c}d\bar{d} \rightarrow \mu X)}{\text{Br}(c\bar{c}q'q' \rightarrow \mu X)}.
$$

The factor $\delta$ takes into account the fact that the final state $c\bar{c}d\bar{d}$ contains more $D^\pm$ mesons than the generic $c\bar{c}q'q'$ state, and that the semileptonic branching fraction of $D^\pm$ meson is about 2.7 times larger than that of all other charm hadrons. Using the known branching fractions of $B$-meson decays, we estimate $\delta = 1.5\pm0.2$. Using the values from Table II we obtain the following estimate of $A_S$ from CPV in interference:

$$
A_S = (-0.089 \pm 0.015)\%.
$$

This value gives an upper bound of the $-A_S$ estimate, since in deriving it we assume that all $c\bar{c}d\bar{d}$ states are CP-even, which is clearly not the case.

**Estimate 3.** In the SM the CP violation in mixing of neutral $B$ mesons is small and the mass eigenstates of the $B^0$ system coincide with $CP$ eigenstates. In that case

$$
\Delta \Gamma = \Delta \Gamma_{CP} = \Gamma(B_{d_even}^0) - \Gamma(B_{d_odd}^0),
$$

where $\Gamma(B_{d_even}^0)$ ($\Gamma(B_{d_odd}^0)$) is the width of $B^0$ decay to the $CP$-even ($CP$-odd) final states, respectively. Assuming that this difference is saturated by the $B^0(B^0) \rightarrow c\bar{c}d\bar{d}$ transition, we obtain the following estimate:

$$
\sum_i (\text{Br}(B^0 \rightarrow f_i)S_i) = -\sin(2\beta)[\Gamma(B_{d_even}^0) - \Gamma(B_{d_odd}^0)] = -\sin(2\beta)\Delta \Gamma_{d}/\Gamma_d.
$$

In the SM framework $\Delta \Gamma_{d}/\Gamma_d = (42 \pm 8) \times 10^{-4}$. An alternative estimate of $\Delta \Gamma_{d}$ uses the SM relation

$$
\Delta \Gamma_{d}/\Delta m_d = \Delta \Gamma_s/\Delta m_s
$$

given in Ref. 8, page 1067. It results in a similar value $\Delta \Gamma_{d}/\Gamma_d = (44 \pm 6) \times 10^{-4}$. Substituting the expression (54) in Eqs. (11) and (41) we obtain

$$
A_S = (-0.045 \pm 0.016)\%.
$$

**Estimate 4.** In this estimate we use the measured value $|\Delta \Gamma_{d}/\Gamma_d| = 0.015 \pm 0.018$. We replace $\sum_i (\text{Br}(B^0 \rightarrow f_i)S_i)$ by $-\sin(2\beta)\Delta \Gamma_{d}/\Gamma_d$ and obtain

$$
A_S = (-0.16 \pm 0.20)\%.
$$

All these estimates of $A_S$ are consistent. In the following, we use the estimate (55). Comparing it with the experimental result (2), we conclude that CPV in interference of $B^0$ decay with and without mixing accounts for a part of the observed like-sign dimuon charge asymmetry. The experimental uncertainty on $\Delta \Gamma_d$ keeps open the possibility of a substantially larger contribution from this source. We also note that CPV in interference of $B^0$ is much larger than the SM prediction for CPV in mixing of $B^0$ and $B^0$ given in Eq. (41).

In Ref. 3 the like-sign dimuon charge asymmetry is measured in several sub-samples of events with an additional selection according to the muon impact parameter. This selection effectively changes the contribution of muons coming from the oscillated $B^0$ decays. The estimate (55) after applying this selection is also modified and the contribution from CPV in interference can be enhanced by selecting the dimuon events with large muon impact parameter. Therefore, the study of the dependence of the asymmetry $A_S$ on the muon impact parameter can provide an additional insight on this source of CPV.

**C. CPV in interference of $B^0$ decay with and without mixing**

Again we present several estimates.

**Estimate 1.** Four channels of interest are $D^{(*)+}_sD^{(*)-}_s$ with $\Gamma_s/\Gamma = (4.5 \pm 1.4)\%$. The CPV parameters $S_i$ have not been measured, but in the SM should be approximately

$$
S_i = -\sin(2\beta_s) \approx -0.036.
$$

Here, similarly to $B^0 \rightarrow D^{(*)+}_sD^{(*)-}_s$ decays, we assume that these final states are mainly CP-even. For these 4 decay channels we obtain

$$
A_S(D^{(*)+}_sD^{(*)-}_s) = (-0.0003 \pm 0.0001(stat))\%.
$$

**Estimate 2.** We assume that the final states $c\bar{s}c\bar{s}$ are mostly CP-even ($\eta_1 = +1$), which is appropriate for $D^{(*)+}_sD^{(*)-}_s$ final states. We take

$$
\frac{\text{Br}(B^0 \rightarrow c\bar{s}c\bar{s})}{(\text{Br}(b \rightarrow c\bar{c}s) + \text{Br}(b \rightarrow c\bar{c}d))} \approx V_{cs}^2.
$$

Then Eq. (41) becomes approximately

$$
A_S \approx -0.5w_3f_sV^2_{cs}\sin(2\beta_s)\frac{x_s}{1 + x_s^2}\frac{P_b - P_{\bar{b}}}{P_bP_{\bar{b}}} = (-0.0013 \pm 0.0002(stat))\%.
$$

The absolute value of estimate (60) can be considered as an upper bound on the contribution to $A_S$ from this source, because some of the $B^0 \rightarrow c\bar{s}c\bar{s}$ final states are not CP-even.
Estimate 3. It is known that the four decay channels $B_s \to D^{(*)+}D^{(*)-}$ do not exhaust the contributions to $\Delta \Gamma_s$ \cite{18}. To obtain an estimate of the like-sign dimuon charge asymmetry from CPV in interference of $B_s$ we replace $\sum_j \text{Br}(F_j \to f_j) S_j$ by $-\sin (2\beta_s) \Delta \Gamma_s/\Gamma_s$. We use the experimental value $\Delta \Gamma_s = 0.100 \pm 0.013 \text{ ps}^{-1}$ and obtain

$$A_s(B_s^0) = (-0.0009 \pm 0.0003 \text{ (stat)}) \%,$$

which can be compared with \cite{2}.

In conclusion, CPV in interference of $B_s^0$ decay with and without mixing is suppressed by the small values of $\sin (2\beta_s)$ and $x_s/(1+x_s^2)$.

D. Direct CPV in decay $b \to c \bar{c}q$ $(q = d$ or $s)$, followed by $\bar{c} \to \mu^-X$.

This type of CPV occurs due to the interference of the tree level and penguin diagrams with different strong phases and different weak phases. Let us consider, as an example, the decay $B^+ \to D^0D^+$, followed by $D^0 \to \mu^-X$. Its branching fraction is $(3.8 \pm 0.4) \times 10^{-4}$. The CP-violating asymmetry in this decay has not been measured, but should be less than $a = 0.1$ because the penguin diagram is suppressed by one loop. The like-sign dimuon charge asymmetry from this channel,

$$A_s = \frac{w_{a\bar{a}} \text{Br}(B^+ \to D^0D^+) \text{Br}(D^0 \to \mu^-X) P_b \text{Br}(b \to \mu X)}{P_b P_b \text{Br}(b \to \mu X)},$$

is less than $0.0002\%$. Considering also decays with $q = s$, which have a CP-violating asymmetry suppressed by one loop $\times x^2$, we conclude that the direct CP violation in the decays measured so far have a negligible contribution to the like-sign dimuon charge asymmetry.

E. Direct CPV in semileptonic decays of $b$ and $c$ quarks.

In this subsection we assume that the like-sign dimuon charge asymmetry is due solely to CPV in direct semileptonic decays of charged and neutral hadrons containing $b$ or $c$ quarks. This type of CP violation vanishes in the lowest order due to the CPT symmetry \cite{16}. The second order calculations give extremely small value for the asymmetry $A_s$ of the order of $10^{-9}$ \cite{18}. Despite such a strong theoretical constraint, the possibility of large contribution from this source is discussed in Ref. \cite{19}. There is no direct experimental measurements of this CPV in semileptonic $B$ decays, and the only experimental limitation can be derived from Eqs. \cite{11} and \cite{2}. Let $a_{(b)}$ ($a_{(c)}$) be the flavor averaged CP violating charge asymmetry of direct semileptonic decays of $b$ ($c$) quarks. The contributions of $a_{(b)}$ and $a_{(c)}$ to $A_s$ and $A_S$ are

$$a_s = \frac{w_1 a_{(b)} + (w_2 + w_3 + w_5 + w_6) a_{(c)}}{w_1 + w_2 + w_3 + w_4 + w_5 + w_6},$$

$$= 0.535 a_{(b)} + 0.454 a_{(c)},$$

and

$$A_s = \frac{w_{1a} P_b + w_{1b} P_b a_{(b)} + w_{2a} P_b + w_{2b} P_b + 0.5 (w_3 + w_5) (P_b + P_c) a_{(c)}}{P_b P_b} = 1.424 a_{(b)} + 0.525 a_{(c)}.$$

If the only asymmetry is $a_{(b)}$, then $A_s/A_S = 2.66$. If the only asymmetry is $a_{(c)}$, then $A_s/A_S = 1.16$. Taking $A_s$ from \cite{11}, we obtain the following estimates for the contribution of direct CPV to $A_S$:

$$A_S(\text{from } a_{(b)}) = (-0.17 \pm 0.43)\%,$$

$$A_S(\text{from } a_{(c)}) = (-0.07 \pm 0.19)\%.$$

Thus, the “closure test” \cite{11} begins to constrain the contributions of $a_{(c)}$ to the like-sign dimuon charge asymmetry $A_S$.

| TABLE II: Contributions to $A_S$ allowed by experiments. |
|----------------------------------------------------------|
| Process | allowed $A_S$ |
| A | Mixing of $B^0$ | $(-0.062 \pm 0.073)\%$ |
| A | Mixing of $B_s^0$ | $(-0.111 \pm 0.093)\%$ |
| B | Interference of $B^0$ | $(-0.045 \pm 0.016)\%$ |
| C | Interference of $B_s^0$ | $(-0.009 \pm 0.0003)\%$ |
| D | CPV in $b \to c\bar{c}q$ decays | $(-0.000 \pm 0.001)\%$ |
| E | $a_{(b)}$ in $b \to \mu X$ decays | $(-0.17 \pm 0.43)\%$ |
| E | $a_{(c)}$ in $c \to \mu X$ decays | $(-0.07 \pm 0.19)\%$ |

IV. CONCLUSIONS

We have considered several possible causes of the measured like-sign dimuon charge asymmetry $A_S$, and obtained their experimental constraints. A summary is presented in Table II. We find that standard model CP violation in interference of decays with and without mixing of $B^0$ to flavor non-specific states $f_i$, followed by the decay $f_i \to \mu X$, contributes

$$A_S(B^0) = (-0.045 \pm 0.016 \text{ (stat)}) \%$$

to the like-sign dimuon charge asymmetry $A_S$. CP violation in interference does not contribute to the inclusive muon charge asymmetry and therefore is compatible with the observation that $a_S$ is consistent with zero.

Among all other possible sources of the dimuon charge asymmetry, only the direct CP violation in semileptonic $b$- and $c$-hadron decays is not yet limited experimentally.
It is very small in the SM, but, until experimentally measured, this source of dimuon charge asymmetry cannot be excluded. Our estimate of this source is derived from the DØ measurements \(^\text{[1]}\) and \(^\text{[2]}\). The exclusive measurement of this type of \(CP\) violation is required to improve this constraint.

Taking into account the additional SM source of dimuon charge asymmetry \(^\text{[66]}\) identified in this paper, the combination of DØ measurements \(^\text{[1]}\) and \(^\text{[2]}\) becomes consistent with the SM expectation within 3 standard deviations. Still the difference between \(^\text{[2]}\), and \(^\text{[26]}\) and \(^\text{[66]}\), \((-0.32 \pm 0.14 \text{ (tot)})\%\), leaves some room for new physics \(CP\) violation in \(B^0\) and \(B^0_s\) mixing, in the interference of \(B^0\) and \(B^0_s\) decays with and without mixing, or in semileptonic decays of \(b\) and \(c\) hadrons. A deviation in the value of \(\Delta\Gamma_d\) from the SM prediction could also contribute to the difference between the observed and expected like-sign dimuon charge asymmetry.

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