The Effective $\eta' g^* g^*$ Vertex at Arbitrary Gluon Virtualities

A. Ali

Deutsches Elektronen-Synchrotron DESY, Hamburg
Notkestraße 85, D-22603 Hamburg, FRG

A.Ya. Parkhomenko

Department of Theoretical Physics, Yaroslavl State University, Sovietskaya 14,
150000 Yaroslavl, Russia

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The Effective $\eta' g^* g^*$ Vertex at Arbitrary Gluon Virtualities

A. Ali$^1$,† and A.Ya. Parkhomenko$^2$,‡

$^1$DESY, Hamburg, Germany
$^2$Yaroslavl State University, Yaroslavl, Russia

The effective $\eta' g^* g^*$ vertex is studied in the standard (Brodsky-Lepage) and modified hard scattering approaches for arbitrary gluon virtualities in the time-like and space-like regions. The contribution of the gluons in the $\eta'$-meson is taken into account, and the wave-function is constrained using data on the electromagnetic transition form factor of the $\eta'$ meson. Our results have implications for the inclusive decay $B \rightarrow \eta' X$ and exclusive decays, such as $B \rightarrow \eta'(K, K^*)$, and in hadronic production processes $N + N(\bar{N}) \rightarrow \eta' X$.

I. INTRODUCTION

The effective coupling involving two gluons and the $\eta'$-meson enters in a number of production and decay processes. For example, the inclusive decay $B \rightarrow \eta' X_s$ [1] and the exclusive decay $B \rightarrow \eta' K$ [2, 3], involve, apart from the matrix elements of the four-quark operators, the transitions $b \rightarrow s g^*$, followed by $g^* \rightarrow \eta' g$ [4, 5], $b \rightarrow s g g$ followed by $g g \rightarrow \eta'$ [6], as well as the transitions $g^* g^* \rightarrow \eta'$ and $g^* g \rightarrow \eta'$ [7]. Thus, a reliable determination of the vertex function, which is often called the transition form factor, $F(q_1^2, q_2^2, m^2_{\eta'})$ (here, $q_1^2$ and $q_2^2$ represent the virtualities of the two gluons) is an essential input in a quantitative understanding of these and related decays. Apart from the mentioned $B$-decays, the $\eta' g^* g^{(*)}$ vertex plays a role in a large number of processes, among them are the radiative decay $J/\psi \rightarrow \eta' \gamma$ and the hadronic production processes $N + N(\bar{N}) \rightarrow \eta' + X$, where $N$ is a nucleon. The QCD axial anomaly [8], responsible for the bulk of the $\eta'$ mass, normalizes the vertex function on the gluon mass-shell, yielding $F(0, 0, m^2_{\eta'})$. The question that still

*Electronic address: ali@x4u2.desy.de
†Electronic address: parkh@uniyar.ac.ru
remains concerns the determination of the vertex for arbitrary time-like and space-like gluon virtualities, $q_i^2$; $i = 1, 2$. A related aspect is to understand the relation between the $\eta' g^* g^*$ vertex and the wave-function of the $\eta'$-meson. Stated differently, issues such as the transverse momenta of the partons in the $\eta'$-meson and their impact on the $\eta' g^* g^*$ vertex have to be studied quantitatively.

While information on the $\eta' g^* g^*$ vertex is at present both indirect and scarce, its electromagnetic counterpart involving the coupling of two photons and the $\eta'$-meson, $F_{\eta'\gamma^*\gamma}$, more generally the meson-photon transition form factor, has been the subject of intense theoretical and experimental activity. In particular, the hard scattering approach to transition form factors, developed by Brodsky and Lepage [9], has been extensively used in studying perturbative QCD effects and in making detailed comparison with data [10]. A variation of the hard scattering approach, in which transverse degrees of freedom are included in the form of Sudakov effects in transition form factors [11, 12], has also been employed in data analyses. For a critical review and comparison of the standard (Brodsky-Lepage) and modified hard scattering (mHSA) approaches, see Refs. [13, 14]. We note that either of these approaches combined with data [15] constrains the input wave-function for the quark-antiquark part of the $\eta'$-meson. However, the gluonic part of the $\eta'$-meson wave-function is not directly measured in these experiments and will be better constrained in future experiments sensitive to the $\eta' g^* g^*$ vertex.

We have used the hard scattering approach to study the $\eta' g^* g^*$ vertex, incorporating the information on the wave-function and the mixing parameters entering in the $\eta - \eta'$ complex from existing data involving the electromagnetic transitions. The results and details of the calculation have been presented by us in Ref. [16]. Prior to our work, Muta and Yang [17] derived the $\eta' g^* g$ vertex, with one off-shell gluon in the time-like region, in terms of the quark-antiquark and gluonic parts of the $\eta'$ wave-function, taking into account the evolution equations obeyed by these partonic components. In Ref. [16], we also addressed the same issues along very similar lines. We first rederived the $\eta' g^* g$ vertex, pointing out the agreement and differences between our results and the ones in Ref. [17]. The differences have to do with the derivation of the leading order perturbative contribution to the gluonic part of the $\eta' g^* g$ vertex, and the use by Muta and Yang [17] of the anomalous dimensions derived in Ref. [18] in the evolution of the wave-functions. For the anomalous dimensions, we use instead the results derived in Refs. [19, 20], which are at variance with the ones given in
Ref. [18], but which have been recently confirmed by Belitsky and Müller [21]. Making use of the $\eta' \gamma^* \gamma$ data to constrain the $\eta'$-meson wave-function parameters, we find that the gluonic contribution in the $\eta'$-meson is very significant. We then extended our analysis to the case when both the gluons are virtual, having either the time-like or space-like virtualities. We studied the effects of the transverse momentum distribution involving the constituents of the $\eta'$-meson and took into account soft-gluon emission from the partons by including the QCD Sudakov factor, following techniques which were introduced in studies of the electromagnetic and transition form factors of the mesons [14, 22, 23]. We showed the improvements in the $\eta' g^* g^{(*)}$ vertex function due to the inclusion of the transverse-momentum and Sudakov effects, which is particularly marked in the space-like region, improving the applicability of the hard scattering approach to lower values of $Q^2$. These effects have a bearing on the hard scattering approach to exclusive non-leptonic decays [24]; the importance of the transverse-momentum and Sudakov effects in the decays $B \rightarrow \pi \pi$ and $B \rightarrow K \pi$ has also been recently emphasized in [25]. We also derived approximate formulae for the $\eta' g^* g^{(*)}$ vertex [16], which satisfy the axial-vector anomaly result for on-shell gluons and have the asymptotic behavior in the large-$Q^2$ domain, determined by perturbative QCD.

We summarize in this report the input, salient features of the derivation of the $\eta' g^* g^{(*)}$ vertex function, and some numerical results presented by us in Ref. [16], making comparison with the earlier work on this subject.

II. DEFINITION OF THE $\eta' g^* g^{(*)}$ VERTEX

The $\eta'$-meson, not being an SU(3)$_F$ flavor-octet meson state, has a gluonic admixture in its wave-function in addition to the usual quark-antiquark content. We take the parton Fock-state decomposition of the $\eta'$-meson wave-function as follows:

$$|\eta'\rangle = \sin \phi |\eta'_q\rangle + \cos \phi |\eta'_s\rangle + |\eta'_g\rangle,$$

where the SU(3)$_F$ symmetry among the light u, d and s quarks is assumed. Thus, $|\eta'_q\rangle \sim |\bar{u}u + \bar{d}d|/\sqrt{2}$ and $|\eta'_s\rangle \sim |\bar{s}s|$ are the quark-antiquark Fock states, where $\phi = 39.3^\circ \pm 1.0^\circ$ is the mixing angle [26], and $|\eta'_g\rangle \sim |gg|$ is the two-gluon Fock state. All other higher Fock states are ignored. Following Refs. [18, 19, 20], we express the wave function of the $\eta'$-meson
in terms of the eigenfunctions of the quark-antiquark $|q\bar{q}\rangle$ and gluonic $|g\bar{g}\rangle$ states:

$$\Psi = C \left[ \phi^{(q)}(x, Q) + \phi^{(g)}(x, Q) \right],$$  \hspace{1cm} (2)

$$C = \sqrt{2} f_q \sin \phi + f_s \cos \phi,$$  \hspace{1cm} (3)

where $f_q$ and $f_s$ are the decay constants of $|\eta'_q\rangle$ and $|\eta'_s\rangle$, respectively, and their present estimates are: $f_q = (1.07 \pm 0.02) f_\pi$, $f_s = (1.34 \pm 0.06) f_\pi$, where $f_\pi \simeq 131$ MeV is the pion decay constant \cite{26}. The eigenfunctions are then calculated for a given $Q^2$ by solving the evolution equations. As is well known, the results for both the quark-antiquark and gluonic components can be presented as infinite series involving Gegenbauer polynomials \cite{18, 19, 20}. Including the leading and next-to-leading terms in the expansion, the functions $\phi^{(q)}(x, Q)$ and $\phi^{(g)}(x, Q)$ are \cite{16}:

$$\phi^{(q)}(x, Q) = 6x\bar{x} \left\{ 1 + \left[ 6B_2^{(q)} \left( \frac{\alpha_s(Q^2)}{\alpha_s(\mu^2_0)} \right) \right]^{\frac{48}{\pi}} - \frac{B_2^{(g)}}{15} \left( \frac{\alpha_s(Q^2)}{\alpha_s(\mu^2_0)} \right) \right\} (1 - 5x\bar{x}) + \cdots, \hspace{1cm} (4)$$

$$\phi^{(g)}(x, Q) = x\bar{x}(x - \bar{x}) \left[ 16B_2^{(g)} \left( \frac{\alpha_s(Q^2)}{\alpha_s(\mu^2_0)} \right) + 5B_2^{(g)} \left( \frac{\alpha_s(Q^2)}{\alpha_s(\mu^2_0)} \right) \right] + \cdots, \hspace{1cm} (5)$$

where $x$ and $\bar{x} = 1 - x$ are the energy fractions of two partons in the $\eta'$-meson and $Q^2 > 0$ is the energy scale parameter. It is seen that in the limit $Q^2 \to \infty$ the quark wave-function (4) turns to its asymptotic form $\phi_{as}(x) = 6x\bar{x}$ (the same asymptotic behavior as the pion wave-function \cite{3} due to its quark-antiquark content), while the gluonic wave-function (5) vanishes in this limit, $\phi^{(g)} = 0$. The coefficients of the expansion of the wave-functions (4) and (5) are calculated by using perturbation theory and include the effective QCD coupling $\alpha_s(Q^2)$, which in the next-to-leading logarithmic approximation is given by \cite{27}

$$\alpha_s(Q^2) = \frac{4\pi}{\beta_0 \ln(Q^2/\Lambda^2)} \left[ 1 - \frac{2\beta_1}{\beta_0^2} \frac{\ln \ln Q^2/\Lambda^2}{\ln Q^2/\Lambda^2} \right],$$  \hspace{1cm} (6)

where $\beta_0 = 11 - 2n_f/3$, $\beta_1 = 51 - 19n_f/3$, $\Lambda = \Lambda_{\text{QCD}}$ is the QCD scale parameter, and $n_f$ is the number of quarks with masses less than the energy scale $Q$. In the energy region $m_c^2 < Q^2 < m_b^2$, where $m_c$ and $m_b$ are the charm and bottom quark masses, respectively, we have used the central value of the dimensional parameter corresponding to four active quark flavors, $\Lambda_{\text{MS}}^{(4)} = 280$ MeV \cite{27}.
III. THE $\eta' g^* g^*$ VERTEX FUNCTION IN THE BRODSKY-LEPAGE APPROACH

The invariant amplitude $\mathcal{M}$, corresponding to the effective $\eta' g^* g^*$ vertex, is the sum of the quark-antiquark $\mathcal{M}^{(q)}$ and gluonic $\mathcal{M}^{(g)}$ components:

$$\mathcal{M} = \mathcal{M}^{(q)} + \mathcal{M}^{(g)}.$$  \hspace{1cm} (7)

The quark-antiquark $F^{(q)}_{\eta' g^* g^*}$ and gluonic $F^{(g)}_{\eta' g^* g^*}$ components of the $\eta' g^* g^*$ vertex function $F_{\eta' g^* g^*}$ entering in the invariant amplitude are defined as follows [16]:

$$\mathcal{M}^{(q,g)} = -i F^{(q,g)}_{\eta' g^* g^*}(q_1^2, q_2^2, m_{\eta'}^2) \delta_{ab} \varepsilon^a_{\mu} \varepsilon^b_{\nu} q_{1\rho} q_{2\sigma},$$  \hspace{1cm} (8)

where $\varepsilon^a_{\mu}$ and $\varepsilon^b_{\nu}$ are the polarization vectors of the two gluons and $q_{1\rho}$, $q_{2\sigma}$ are their four-momenta.

The diagrams depicting the quark-antiquark and gluonic contents of the $\eta' g^* g^*$ vertex are shown in Fig. 1 and 2, respectively, and lead to the following vertex functions [16]:

$$F^{(q)}_{\eta' g^* g^*}(q_1^2, q_2^2, m_{\eta'}^2) = 4\pi \alpha_s(Q^2) \frac{C}{2N_c} \int_0^1 dx \phi^{(q)}(x, Q)$$

$$\times \left[ \frac{1}{xq_1^2 + \bar{x}q_2^2 - \bar{x}x m_{\eta'}^2 + i\epsilon} + (x \leftrightarrow \bar{x}) \right],$$  \hspace{1cm} (9)

$$F^{(g)}_{\eta' g^* g^*}(q_1^2, q_2^2, m_{\eta'}^2) = 4\pi \alpha_s(Q^2) \frac{C}{2} \int_0^1 dx \phi^{(g)}(x, Q)$$

$$\times \left[ \frac{1}{xq_1^2 + \bar{x}q_2^2 - (1 + \bar{x}x)m_{\eta'}^2 + i\epsilon} - (x \leftrightarrow \bar{x}) \right].$$  \hspace{1cm} (10)

Note that the gluonic wave-function of the $\eta'$-meson [3] satisfies the antisymmetry condition $\phi^{(g)}(x, Q) = -\phi^{(g)}(\bar{x}, Q)$ [18, 13, 20]. It implies that if the relative sign in the brackets of Eq. (10) were a “+” (as given in Eq. (6) in Ref. [17], and with which we differ) the gluonic contribution to the $\eta' g^* g^*$ form factor would vanish identically.

The quark and gluonic wave-functions contain both free ($B^{(q)}_n$, $B^{(g)}_n$) and constrained parameters, with the latter depending on the anomalous dimensions. The free parameters can be fitted from the experimental data, for example, from the $\eta' \gamma^* \gamma$ transition form factor. We take the following restrictions on the first correction to the leading order quark-antiquark wave-function: $|B^{(q)}_2| < 0.1$ and $|\rho^{(g)}_2 B^{(g)}_2| < 0.1$ in order to keep $\phi^{(q)}(x, Q)$ close to its
asymptotic value: $\phi_{\text{as}}(x) = 6x\bar{x}$, in agreement with the experimental data on the $\eta'\gamma^*\gamma$ transition form factor [13]. Taking into account the anomalous dimension-dependent quantity $\rho_2(g) \simeq -1/90$ from Ref. [16] we get\footnote{This differs considerably from the values used in [17].} $|B_2^{(g)}| < 9.0$. Below, we shall present the $\eta'g^*g^*$ vertex function for the maximum allowed values of the non-perturbative parameters, i.e., $|B_2^{(q)}| = 0.1$ and $|B_2^{(g)}| = 9.0$. We note that there is not much sensitivity to the variation of the parameter $B_2^{(q)}$ on the overall $\eta'g^*g^*$ vertex function. Hence, we fix this parameter to its maximum allowed value $|B_2^{(q)}| = 0.1$. However, there is considerable sensitivity to the variation of the parameter $B_2^{(g)}$. To show this we shall take $|B_2^{(g)}| = 9.0$ and $|B_2^{(g)}| = 3.0$. The resulting theoretical dispersion between the two cases ($|B_2^{(g)}| = 9.0$ vs. $|B_2^{(g)}| = 3.0$) can be seen in Fig. 3, where we show the $\eta'g^*g$ vertex function for an on-shell gluon ($q_2^2 = 0$) with the other having a time-like virtuality ($q_1^2 > 0$) in the Brodsky-Lepage approach. The various leading order and next-to-leading order components and the sum contributing to $F_{\eta'g^*g}(q_1^2, 0, m_{\eta'}^2)$ are displayed individually. The gluonic contribution (called NLG) for the maximum values of the free parameters is comparable to the leading quark contribution (called LQ), as shown in the upper plot in Fig. 3 increasing the $\eta'g^*g$ vertex by almost a factor 2 as compared to the case when only the quark content of the $\eta'$-meson is assumed. This may be considered as the maximum gluonic content of the $\eta'$-meson allowed by current data. Even in the more realistic case with $B_2^{(q)} = 0.1$ and $B_2^{(g)} = 3.0$, we see from the lower plot in Fig. 3 that the gluonic contribution is not small, and also in this case it enhances the value of the total $\eta'g^*g$ vertex function (the solid curve in Fig. 3) at the level of few tens percent.

IV. THE $\eta'g^*g^*$ VERTEX FUNCTION IN THE MHSA APPROACH

In the Brodsky-Lepage approach to form factors, the transverse momentum dependence of the partons in the mesons is neglected as the hard scattering (perturbative) approach is applicable only when the virtualities of the external particles are much larger than the typical value of the parton transverse momenta $k_{\perp i}$. Including the transverse momenta of the partons in the meson, the perturbative expansion of the transition form factor encounters large logarithms of the form $\ln(Q^2/k_{\perp i}^2)$, and it becomes mandatory to sum the multiple-
gluon emissions. The formalism for the soft and collinear gluon resummation was introduced by Collins and Soper \[28\] and by Collins, Soper and Sterman \[29\]. Such gluon emissions give rise to powers of double logarithms in each order of perturbation theory and their contribution exponentiates into the Sudakov function \[30\]. The Sudakov exponents are known both for the quark-antiquark case, from the Drell-Yan (DY) and the deep inelastic scattering (DIS) processes, and for gluons from the gluon fusion into $2\gamma$, gauge, or Higgs boson final states. This formalism is suitable for the description of the hadronic wavefunctions and the hadronic form factors, such as the electromagnetic and transition form factors of the pion \[12, 22, 23, 31, 32\], and has also been employed in calculating the $\eta'_g g^*$ vertex in Ref. \[16\].

In the modified Hard Scattering Approach (mHSA) \[12\], we take the $\eta'$-meson wavefunction in a form similar to the pion wave-function \[22, 23\]:

$$\hat{\Psi}^{(p)}(x, Q, b) = \frac{2\pi C}{\sqrt{2}N_c} \phi^{(p)}(x, Q) \exp \left[ -\frac{x\bar{x}b^2}{4a^2} \right] S^{(p)}(x, Q, b),$$  \(11\)

where $p = q$ for the quark and antiquark case, and $p = g$ for gluons; the constant $C$ is already defined in Eq. (3), $b$ is the separation between the $\eta'$-meson constituents in the transverse configuration space, with $b = |b|$ often called the impact parameter, and $\phi^{(q)}$ and $\phi^{(g)}$ have the form presented in Eqs. (4) and (5), respectively. The transverse size parameter $a$ can be determined from the average transverse momentum of the $\eta'$-meson. For the numerical analysis, the value $a^{-1} = 0.861$ GeV is used, following from the analysis of the form factor involving the $\pi$-meson \[22, 23\]. The soft-gluon emission from the quark, antiquark and gluons in the $\eta'$-meson can be taken into account by including the QCD Sudakov factors $S^{(p)}(x, Q, b)$, the details of which can be found in Ref. \[16\].

In the space-like region of the gluon virtualities, the quark and gluonic vertex functions are \[16\]:

$$F^{(q)}_{\eta'_g g^*}(Q, \omega, \eta) = -4\pi\alpha_s(Q^2) \frac{C}{N_c\Lambda^2} \int_0^1 dx \phi^{(q)}(x, Q)$$

$$\times \int_0^1 db_\Lambda b_\Lambda \exp \left[ -\frac{x\bar{x}b^2}{4a^2\Lambda^2} \right] S^{(q)}(x, Q, b) K_0^{(+)}(x, b_\Lambda Q_\Lambda),$$  \(12\)

$$F^{(g)}_{\eta'_g g^*}(Q, \omega, \eta) = -4\pi\alpha_s(Q^2) \frac{C}{\Lambda^2} \int_0^1 dx \phi^{(g)}(x, Q) \int_0^1 db_\Lambda b_\Lambda \exp \left[ -\frac{x\bar{x}b^2}{4a^2\Lambda^2} \right] S^{(g)}(x, Q, b)$$

$$\times \left[ |\eta| K_0^{(-)}(x, b_\Lambda Q_\Lambda) - (x - \bar{x})\omega K_0^{(+)}(x, b_\Lambda Q_\Lambda) \right].$$  \(13\)
Here,
\[ K_0^{(+)}(x, b\lambda Q_\Lambda) = \frac{1}{2} \left[ K_0 \left( b\lambda Q_\Lambda \lambda_\pm(x,\omega,\eta) \right) \pm K_0 \left( b\lambda Q_\Lambda \lambda_\pm(\bar{x},\omega,\eta) \right) \right], \quad (14) \]
where the various dimensionless parameters are defined as: \( b\lambda = b\Lambda, \) \( Q_\Lambda = Q/\Lambda, \) \( |\eta| = m_{\eta'}^2/Q^2, \omega = (q_1^2 - q_2^2)/Q^2, \) \( K_0(z) \) is the modified Bessel function, and
\[ \lambda_\pm^2(x,\omega,\eta) = \frac{1}{2} \left[ 1 + \omega(x - \bar{x}) \pm 2x\bar{x}|\eta| \right]. \quad (15) \]
The vertex function \( |F_{\eta'g^*g^*}(q_1^2, q_2^2, m_{\eta'}^2)| \) calculated in the space-like region is plotted in Fig. 4 as a function of \( q_1^2 \) for given values of \( q_2^2 \) for the Brodsky-Lepage case (upper figure) and in the mHSA formalism (lower figure). The function \( q^2 F_{\eta'g^*g^*}(q_1^2, q_2^2, m_{\eta'}^2) \) in the two approaches is shown in Fig. 5. We note the improved perturbative behaviour of the vertex functions for smaller virtualities in the mHSA formalism, while for larger virtualities the two approaches yield very similar results.

The transition from the space-like region of gluon virtualities to the time-like one for the \( \eta'g^*g^* \) vertex function is discussed in Ref. \[16\]. The final result for the quark and gluonic contributions in the time-like region is:
\[
F_{\eta'g^*g^*}^{(q)}(Q,\omega,\eta) = 4\pi\alpha_s(Q^2) \frac{i\pi C}{2N_c\lambda^2} \int_0^1 dx \phi(q)(x,Q) \\
\times \int_0^1 db\lambda b\lambda \exp \left[ -\frac{x\bar{x}}{4a^2\lambda^2} b\lambda^2 \right] S^{(q)}(x,Q,b) H_0^{(+)}(x,b\lambda Q_\Lambda), \quad (16)
\]
\[
F_{\eta'g^*g^*}^{(g)}(Q,\omega,\eta) = 4\pi\alpha_s(Q^2) \frac{i\pi C}{2\lambda^2} \int_0^1 dx \phi(g)(x,Q) \int_0^1 db\lambda b\lambda \exp \left[ -\frac{x\bar{x}}{4a^2\lambda^2} b\lambda^2 \right] S^{(g)}(x,Q,b) \\
\times \left[ \eta H_0^{(-)}(x,b\lambda Q_\Lambda) + (x - \bar{x})\omega H_0^{(+)}(x,b\lambda Q_\Lambda) \right]. \quad (17)
\]
where
\[ H_0^{(\pm)}(x,b\lambda Q_\Lambda) = \frac{1}{2} \left[ H_0^{(2)}(b\lambda Q_\Lambda \lambda_\pm(x,\omega,\eta)) \pm H_0^{(2)}(b\lambda Q_\Lambda \lambda_\pm(\bar{x},\omega,\eta)) \right]. \quad (18) \]

Here, \( H_0^{(2)}(z) \) is the second Hankel function \[33\]. It is interesting to note that due to the \( i\epsilon \) prescription of the propagators in the hard scattering part of the \( \eta'g^*g^* \) vertex function, an imaginary part is generated. This can be seen in Fig. 6 where the upper two figures show the real and imaginary parts of the vertex function \( F_{\eta'g^*g^*}(q_1^2, q_2^2, m_{\eta'}^2) \) as a function of \( q_1^2 \) for the indicated values of \( q_2^2 \), calculated in the mHSA approach. In the lower two figures, we compare the magnitude of the vertex function in this approach with the one in the Brodsky-Lepage approach. A comparison shows that there is a marked difference between the vertex
functions calculated in the Brodsky-Lepage and the mHSA approaches, pertaining to the
absence of the singularity at \( Q^2 = m^2_{\eta'} \) in the latter case.

V. INTERPOLATING EXPRESSIONS FOR THE \( \eta'g^*g^* \) VERTEX FUNCTIONS

For the applications in various decay and production processes it is useful to find an
approximate expression for the vertex function which is simple and can be used over a large
domain of the gluon virtualities. We recall that Brodsky and Lepage [34] presented an
approximate form for the \( \pi^-\gamma \) transition form factor which interpolates between the PCAC
value and the QCD prediction in the large \( Q^2 \) region. Subsequently, in Ref. [35], this form
was extended to the case of the \( \eta' - \gamma \) transition form factor. Very much along the same
lines, a similar expression can be written for the \( \eta'g^*g^* \) transition form factor [16]:

\[
F_{\eta'g^*g^*}(q^2, \omega) = 4\pi\alpha_s(Q^2) \frac{2\sqrt{3}f_\pi D(q^2, \omega)}{\sqrt{3}q^2 - 8\pi^2 f_\pi^2 D(q^2, \omega)},
\]

where the largest energy scale parameter \( Q^2 = q^2 \) for the time-like total gluon virtuality and
\( Q^2 = -q^2 \) for the space-like one. The function \( D(q^2, \omega) \) is defined as follows [16]:

\[
D(q^2, \omega) = f_0(\omega) + \frac{q^2}{Q^2} \left[ 16B_2^{(q)} \left( \frac{\alpha_s(Q^2)}{\alpha_s(\mu_0^2)} \right) \frac{g^2(\omega)}{4 \pi^2} \right] \frac{g_2(\omega)}{f_\pi^2 D(q^2, \omega)} + 5B_2^{(q)} \left( \frac{\alpha_s(Q^2)}{\alpha_s(\mu_0^2)} \right) \frac{g_2(\omega)}{f_\pi^2 D(q^2, \omega)},
\]

where \( \omega \) is the asymmetry parameter having values in the interval \( 0 \leq \omega \leq 1 \). The asymptotic functions for the quark-antiquark \( f_0(\omega) \) and gluon \( g_2(\omega) \) cases are derived in Ref. [16]
and obey the bounds \( 2/3 \leq f_0(\omega) \leq 1 \) and \( 0 \leq g_2(\omega) \leq 1/6 \). We remark that the difference
between the asymptotic behaviour of the vertex functions in the Brodsky-Lepage and mHSA
approaches is small [16].

The above expression reproduces both the anomaly value and the large \( Q^2 \) asymptotics
of the vertex functions:

\[
F_{\eta'g^*g^*}(Q^2, \omega) \big|_{Q^2 \to 0} = 4\pi\alpha_s(m^2_{\eta'}) \frac{\sqrt{3}}{4\pi^2 f_\pi}, \quad \text{(21)}
\]

\[
F_{\eta'g^*g^*}(Q^2, \omega) \big|_{Q^2 \to \infty} = 4\pi\alpha_s(Q^2) \frac{2f_\pi D(q^2, \omega)}{q^2}. \quad \text{(22)}
\]

Presented in Eq. (14), the interpolating function is a smooth function in the space-like region
of the gluon virtualities but has a pole at \( q^2 = 8\pi^2 f_\pi^2 D(q^2, \omega)/\sqrt{3} \) in the time-like region.
A similar behavior for the form factor in the time-like region was obtained by Kagan and Petrov [7] as the result of the evaluation of the triangle diagram.

The mHSA approach naturally removes the unphysical singularity from the vertex function in the time-like region of the gluons virtualities but the vertex function gets an imaginary part. In this case the real part of the approximate formula interpolates between the anomaly value and the large $Q^2$ asymptotics, while the imaginary part goes to zero as $Q^2 \to 0$. An approximate interpolating form in the time-like region of the gluon virtualities is given in Ref. [16] to which we refer for further details.

VI. SUMMARY

We have studied the $\eta'g^*g^*$ vertex function $F_{\eta'g^*g^*}(q_1^2, q_2^2, m_{\eta'}^2)$ in perturbation theory for the most general case when both gluons are virtual. The evolution equations involving the eigenfunctions of the quark-antiquark and gluonic components of the $\eta'$-meson wave function are solved, and the input parameters in the wave function are determined using data on the electromagnetic transition form factor of the $\eta'$-meson. It is shown that within the allowed variation of the parameters $B_{2}^{(q)}$ and $B_{2}^{(g)}$ of the $\eta'$-meson wave-function, the gluonic contribution is not small. For extremal values of the parameters allowed by data, $B_{2}^{(q)} = 0.1$ and $B_{2}^{(g)} = 9.0$, the gluonic correction is found to be comparable to the quark contribution. But, even for a lower value $B_{2}^{(q)} = 3.0$, the gluonic contribution is present at the level of few tens percent. Our work corrects and extends the existing results on the $\eta'g^*g$ vertex function, reported earlier in Ref. [17] for the case of one virtual gluon in the time-like region.

We find that the Brodsky-Lepage approach leads to the appearance of a singularity in the region of the $\eta'$-meson mass for time-like gluon virtualities. This reflects the observation made earlier in the literature that the Brodsky-Lepage approach to exclusive form factors is valid only in the asymptotic region. In conformity with this, we find that for the total gluon virtuality $|q^2| = |q_1^2 + q_2^2| \gg m_{\eta'}^2$, the $\eta'g^*g^*$ vertex function has the usual asymptotic behaviour: $F_{\eta'g^*g^*} \sim 1/q^2$, anticipated for the pseudoscalar meson transition form factors.

In the mHSA approach, where the transverse momentum dependence of the hard scattering amplitude as well as the transverse momentum distribution and the soft-gluon emission (the Sudakov factor) in the $\eta'$-meson wave-function are taken into account, the mentioned
singularity in the Brodsky-Lepage-approach at $q^2 = m^2_{\eta'}$ disappears. Also, the validity of the perturbative approach is extended to smaller virtualities in the mHSA formalism, though asymptotically the two approaches yield very similar results for the space-like gluon virtualities. In the time-like region, in the mHSA formalism, we find that due to the $i\varepsilon$ prescription of the propagators, the $\eta'g^*g^*$ vertex function obtains an additional phase factor in comparison with the space-like expression. Thus, in applications where the $\eta'g^*g^*$ vertex appears with off-shell gluons in the time-like region, this perturbative phase should be included.

An approximate expression for the $\eta'g^*g^*$ vertex function is presented for the case when both gluons are off mass shell and have space-like virtualities. This expression interpolates between the anomaly value of the $\eta'g^*g^*$ vertex and the asymptotic QCD prediction in the large $Q^2$ region. The results summarized here have obvious applications in rare $B$-meson decays $B \to \eta'K$ and $B \to \eta'X_s$, and in a number of other radiative and hard processes involving the vertex $\eta'g^*g^*$.

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FIG. 1: Lowest order quark-antiquark contribution to the $\eta'g^*g^*$ vertex.

FIG. 2: Lowest order gluonic contribution to the $\eta'g^*g^*$ vertex.

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FIG. 3: The $\eta'g^*g$ vertex $F_{\eta'g^*g}(q_1^2,0,m_\eta^2)$ as a function of $q_1^2$ with $B_2^{(q)} = 0.1$ and two values of $B_2^{(g)}$: $B_2^{(g)} = 9.0$ (upper plot) and $B_2^{(g)} = 3.0$ (lower plot) in the Brodsky-Lepage approach. The dashed curves are the leading (LQ), next-to-leading quark-antiquark (NLQ), and gluonic (NLG) components, and the solid curve is the sum.
FIG. 4: The $\eta'g^*g^*$ vertex function $|F_{\eta'g^*g^*}(q_1^2, q_2^2, m_{\eta'})|$ for space-like gluon virtualities in the Brodsky-Lepage approach (upper figure) and in the mHSA formalism (lower figure), with $B^{(g)}_2 = 0.1$ and $B^{(g)}_2 = 3.0$. The legends are as follows: $q_2^2 = 0$ (solid curve), $q_2^2 = -1$ GeV$^2$ (long-dashed curve), $q_2^2 = -5$ GeV$^2$ (medium-dashed curve), $q_2^2 = -10$ GeV$^2$ (short-dashed curve), and $q_2^2 = -25$ GeV$^2$ (dotted curve).
FIG. 5: The $\eta'g^*g^*$ vertex function $q^2F_{\eta'g^*g^*}(q_1^2,q_2^2,m_{\eta'}^2)$ for space-like gluon virtualities in the Brodsky-Lepage approach (upper figure) and in the mHSA formalism (lower figure), with $B_2^{(g)} = 0.1$ and $B_2^{(g)} = 3.0$. Legends are the same as in Fig. 4.
FIG. 6: The real and imaginary parts of the $\eta' g^* g^*$ vertex $F_{\eta' g^* g^*}(q_1^2, q_2^2, m_{\eta'}^2)$ in the mHSA formalism (top two figures), and its magnitude $|F_{\eta' g^* g^*}(q_1^2, q_2^2, m_{\eta'}^2)|$ (bottom left figure) with time-like gluon virtualities, $B_2^{(q)} = 0.1$ and $B_2^{(q)} = 3.0$. The bottom right hand figure shows the form factor in the Brodsky-Lepage approach. The legends are as follows: $q_2^2 = 0$ (solid curve), $q_2^2 = 1$ GeV$^2$ (long-dashed curve), $q_2^2 = 5$ GeV$^2$ (medium-dashed curve), $q_2^2 = 10$ GeV$^2$ (short-dashed curve), and $q_2^2 = 25$ GeV$^2$ (dotted curve).