Dimension-two gluon condensate from large-$N_c$ Regge models\textsuperscript{*}

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It is shown that in the large-$N_c$ limit radial Regge trajectories give rise in a natural way to the presence of the dimension-2 gluon condensate, $\langle A^2 \rangle$, in meson correlators. We match these models to QCD and provide estimates for $\langle A^2 \rangle$ in terms of other physical quantities. In particular, in the simplest strictly linear radial Regge model with equal residues $\langle A^2 \rangle$ is proportional to the pion decay constant squared. However, the linear model fails a consistency condition based on matching the short- and long-distance string tensions, nor reproduces the phenomenological values of the gluon condensates. On the contrary, in Regge models departing from strict linearity one may reproduce both the consistency condition and the signs of condensates. We demonstrate this in a simple explicit model.

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The dimension-two gluon condensate was originally proposed by Celentano and Shakin twenty years ago and appears as rather elusive gauge-invariant non-perturbative and non-local operator which generates the lowest $1/Q^2$ power corrections. In fact, in spite of increasing evidence provided by instanton model studies\textsuperscript{2}, phenomenological QCD sum rules re-analyses\textsuperscript{3}, phenomenological studies of the $\tau$ decay data\textsuperscript{4}, theoretical considerations\textsuperscript{3, 6, 7, 8}, quark-model calculations\textsuperscript{9, 10}, lattice simulations\textsuperscript{11, 12}, and their relevance in the confinement-deconfinement phase transition\textsuperscript{13}, the dynamical origin of $\langle A^2 \rangle$ is still unclear. For short and recent reviews see, e.g.,\textsuperscript{14, 15}.

In this paper we stress that within the large-$N_c$ expansion the $1/Q^2$ corrections appear naturally. The issue was discussed briefly by Afonin, Andrianov, Andrianov, and Espriu\textsuperscript{16} in the framework of a Regge model. Here we reexamine this point and demonstrate that the signs and magnitudes of both the dimension-2 and dimension-4 gluon condensates can be accommodated comfortably with reasonable values of the parameters of the hadronic spectra. The idea of comparing Regge models to OPE has been explored in a number of works\textsuperscript{17, 18, 19, 20, 21, 22}, however with the exception of Ref.\textsuperscript{22} the dimension-2 condensate has been systematically ignored. We note here a recent attempt to parameterize such a dimension-2 object within the holographic approach based on the AdS/CFT correspondence and the resulting Regge behavior\textsuperscript{23}.

Our method is based on the analysis of the vector and axial vector meson correlators and has three basic elements: Firstly, we use the operator product expansion (OPE) of QCD with the non-standard $1/Q^2$ power correction in the $V + A$ meson correlator (the contribution vanishes in the $V - A$ combination). Secondly, confinement is incorporated in terms of the radial Regge spectra which necessarily satisfy certain constraints at high energies, in particular, residues must asymptotically become constant and the string tensions in the vector and axial vector channels must be the same. Finally, the large number of colors is assumed, i.e. the correlators are saturated with sharp non-interacting meson states. Matching to OPE allows for enforcement of the two Weinberg sum rules and for the identification of the QCD condensates in terms of the parameters of the model of the hadronic spectra. A novel element of our analysis is a consistency condition based on matching the values of the short- and long-distance string tensions.

We begin with the basic formulas in order to fix the notation as well as derive formal constraints on the Regge models in the presence of the $\langle A^2 \rangle$ condensate. The OPE of the chirally even and odd combinations of the transverse parts of the vector and axial vector currents, $j^{\mu;0}_{VA} = \bar{\psi}\gamma^\mu(1, \gamma_5)\frac{Q^6}{2\pi}\psi$, gives in the strict chiral limit

\begin{align}
\Pi_{V,A}^T(Q^2) &= \frac{1}{4\pi^2} \left\{ -\frac{N_c}{3} \log \frac{Q^2}{\mu^2} - \frac{\alpha_S}{\pi} Q^2 + \frac{256\pi^2}{27} \frac{\alpha_S}{Q^6} \right\} \nonumber \\
&\quad - \frac{\alpha_S}{3} \frac{\lambda^2}{Q^4} + \frac{\alpha_S}{3} \frac{G^2}{Q^4} + \frac{268}{81} \frac{\alpha_S}{Q^6} \langle \bar{q}q \rangle^2 + \ldots, \\
\Pi_{V-A}^T(Q^2) &= -\frac{32\pi}{9} \frac{\alpha_S}{Q^6} \langle \bar{q}q \rangle^2 + \ldots,
\end{align}

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with the non-standard $1/Q^2$ term proposed in Ref. 24 present. According to Ref. 23 the dimension-2 coefficient $\lambda^2$ is interpreted as the tachyonic gluon mass, $m_g$, providing the short-distance string tension

$$\sigma_0 = -2\alpha_s \lambda^2 / N_c.$$  

(2)

It is also related to the dimension-2 gluon condensate, 

$$\langle 0 | T (\bar{c} c) | 0 \rangle = m_0^2 = - \frac{N_c}{4(4N_c - 3)} g^2 (A^2),$$

where we have inserted appropriate factors of $N_c$, such that $\alpha_s \lambda^2 \sim N_c$ and $m_g \sim 1$.

Now comes an important observation: The coefficient of $1/Q^2$ is proportional to $\sigma_0$, and on the other hand, as we show shortly, it involves the series over mesonic spectra which depend on the long-distance string tension $\sigma$. This allows for building a sum-rule or a consistency check, since $\lambda \sim \sigma_0$. The near equality is supported by $SU(2)$ lattice simulations 23, where only a 92% reduction of $\langle A^2 \rangle$ at the deconfinement transition point is found suggesting a similar factor between $\sigma$ and $\sigma_0$.

In the large-$N_c$ limit the vacuum sector of QCD becomes a theory of infinitely many non-interacting mesons and glueballs, hence the correlators may be saturated by infinitely many meson states. Thus one has, up to subtractions,

$$\Pi_V^T (Q^2) = \sum_{n=0}^{\infty} \frac{F_{V,n}^2}{M_{V,n}^2 + Q^2} + c.t.,$$

$$\Pi_A^V (Q^2) = \frac{f^2}{Q^2} + \sum_{n=0}^{\infty} \frac{F_{A,n}^2}{M_{A,n}^2 + Q^2} + c.t.,$$

(3)

where the first term in the axial-vector channel is the massless pion contribution with $f = 86$ MeV denoting the pion decay constant in the chiral limit. Next, we use the radial Regge spectra to saturate the vector and axial-vector channels,

$$M_{V,n}^2 = M_V^2 + a_V n, \quad M_{A,n}^2 = M_A^2 + a_A n, \quad n = 0, 1, \ldots.$$  

(4)

which is well fulfilled 26 in the experimentally explored region. The vector correlator satisfies the unsubtracted dispersion relation, hence

$$\Pi_V^T (Q^2) = \sum_{n=0}^{\infty} \left( \frac{F_{V,n}^2}{M_{V,n}^2 + a_V n + Q^2} - \frac{F_{V,n}^2}{M_{V,n}^2 + a_V n} \right).$$

(5)

The sum needs to reproduce the log $Q^2$ term of the OPE expansion 11, for which the asymptotic part of the spectrum matters. We note that this is the case if at large $n$ the behavior of the residues is $F_{V,n} \approx F_V$, i.e. there is no $n$-dependence. Similarly, $F_{A,n} \approx F_A$. If this is satisfied, then the sum 23 leads to the digamma function, which upon expansion produces the log $Q^2$ term with pure power-law corrections (see the following). At first sight this seems a rather surprising result, since the condition on residues means that all highly-excited radial states are coupled to the current with equal strength. However, this is to be expected. The spectral density is $\rho_V(s) = F_V^2 \sum_{n=0}^\infty \delta (s - M_V^2 + a n)$, which from the far Euclidean end looks as a continuum of constant strength in the dispersive integral, which obviously gives rise to the log $Q^2$. Any dependence of $F_{V,n}$ or $F_{A,n}$ on $n$ would spoil this behavior and damage the twist expansion. Thus the parton-hadron duality requests, for linear Regge spectra, asymptotically constant residues.

The chirally-odd combination of currents satisfies the unsubtracted dispersion relation, which implies

$$F_V^2 / a_V = F_A^2 / a_A.$$  

(6)

With this constraint we may now compute the series

$$\Pi_{V-A}^V (Q^2) \simeq - \frac{F_V^2}{a_V} \psi \left( \frac{M_V^2 + Q^2}{a_V} \right) + \frac{F_A^2}{a_A} \psi \left( \frac{M_A^2 + Q^2}{a_A} \right)$$

$$\frac{f^2}{Q^2} = \frac{F_V^2}{a_V} \log a_V + O(1/Q^2),$$

(7)

where $\psi(z) = \Gamma' (z) / \Gamma (z)$ is the digamma function, and compare to OPE 11. We also assume $a_V = a_A$, which together with 6 means that for asymptotic $n$

$$F_V = F_A = F, \quad a_V = a_A = a.$$  

(8)

Note that $a_V = a_A$ yields the same density of states in the $V$ and $A$ channels which complies to the “chiral symmetry restoration” in the spectra 20,27. Moreover, the independent fits of Ref. 23 give very close experimental values $\sqrt{\sigma_A} = 464$ MeV and $\sqrt{\sigma_V} = 470$ MeV, compatible to 5.

In the first model considered below we thus assume 5,8 for all $n$, which means strictly linear radial Regge trajectories with constant residues. The evaluation of $\Pi_{V-A}^V (Q^2)$ with conditions 5,8 gives

$$\Pi_{V-A}^V (Q^2) = \frac{F_V^2}{a} \left[ - \psi \left( \frac{M_V^2 + Q^2}{a} \right) + \psi \left( \frac{M_A^2 + Q^2}{a} \right) \right]$$

$$- \frac{f^2}{Q^2} \simeq \Pi_{V-A}^V (Q^2) = \left( \frac{F_V^2}{a} (M_A^2 - M_V^2) - f^2 \right) \frac{1}{Q^2}$$

$$+ \left( \frac{F_A^2}{2a} (M_A^2 - M_V^2)(a - M_A^2 - M_V^2) \right) \frac{1}{Q^4} + \ldots$$  

(9)

Matching to 11 yields the two Weinberg sum rules:

$$f^2 = \frac{F_V^2}{a} (M_A^2 - M_V^2),$$

(WSR I)

$$0 = (M_A^2 - M_V^2)(a - M_A^2 - M_V^2).$$

(WSR II)

As mentioned, the $V + A$ channel requires regularization. One may use the $\zeta$-function, however, a simpler and equivalent procedure is to carry the $d/dQ^2$ differentiation, compute the convergent sum, and then integrate back over $Q^2$. The result is

$$\Pi_{V+A}^V (Q^2) = \frac{F_V^2}{a} \left[ - \psi \left( \frac{M_V^2 + Q^2}{a} \right) - \psi \left( \frac{M_A^2 + Q^2}{a} \right) \right]$$
\[ + f^2 \frac{Q^2}{Q^2} + \text{const} \simeq -\frac{2F^2}{a} \log \frac{Q^2}{\mu^2} \]

\[ + \left( f^2 + F^2 - \frac{F^2}{a} (M_A^2 + M_V^2) \right) \frac{1}{Q^2} \]

\[ + \frac{F^2}{6a} \left( a^2 - 3a(M_A^2 + M_V^2) + 3(M_A^2 + M_V^2) \right) \frac{1}{Q^4} + \ldots \]

The integration constant has been absorbed in the scale \( \mu \). Matching of the coefficient of the log \( Q^2 \) to (11) gives

\[ a = 2\pi\sigma = 24\pi^2 F^2 / N_c, \]

where \( \sigma \) is the (long-distance) string tension. If we use \( F = 154\text{MeV} \) from the \( \rho \to 2\pi \) decay \cite{28}, we get \( \sqrt{\sigma} = 546\text{MeV} \). The lattice calculation of Ref. \cite{29} gives \( \sqrt{\sigma} = 420\text{MeV} \). We stress that the conditions (8,11) come solely from the asymptotic spectrum and are insensitive to the low-lying states. In addition, we may read off from (10) the dimension-2 and -4 condensates.

The Regge spectrum assumes confinement, \( \sigma \neq 0 \), as well as \( F \neq 0 \). The chiral symmetry might be a priori be broken or unbroken. However, since \( \sigma \simeq \sigma_0 \), also \( \sigma_0 \neq 0 \), and this this via \( \sigma \) implies spontaneous chiral symmetry breaking, \( f \neq 0 \). Thus, from WSR I \( M_A \neq M_V \) and from WSR II \( a = M_A^2 + M_V^2 \). In this case (10) give

\[ M_A^2 = M_V^2 + 24\pi^2 f^2 / N_c, \]

\[ a = M_A^2 + M_V^2 = 2M_V^2 + 24\pi^2 f^2 / N_c, \]

and

\[ -\frac{\alpha_s \lambda^2}{4\pi^3} = f^2, \]

\[ \frac{\alpha_s\langle G^2 \rangle}{12\pi} = \frac{M_A^4 - 4M_V^2 M_A^2 + M_V^4}{48\pi^2} = \frac{288\pi^4 f^4 / N_c - 24\pi^2 f^2 M_V^2 / N_c - M_V^4}{24\pi^2}. \]

The numerical value for the dimension-2 condensate is \( -\frac{\alpha_s \lambda^2}{4\pi^3} = 0.3\text{GeV}^2 \) as compared to the value of 0.12\text{GeV}^2 from Ref. \cite{30}. We note that Andreev \cite{22} also quotes the estimate \( -\frac{\alpha_s \lambda^2}{4\pi^3} = 0.3\text{GeV}^2 \). Eq. (10) gives \( \sqrt{\sigma_0} = 782\text{MeV} \), about twice as much as deduced from Eq. (11), thus the consistency check is violated badly. Also, the dimension-4 gluon condensate is negative for \( M_V \geq 0.46 \text{GeV} \). Actually it never, not even at very low values of \( M_V \), reaches the QCD sum-rules value of the condensate. The dimension-6 condensate in the model is zero in the \( V + A \) channel, while from OPE it should not be. All these problems show that the strictly linear radial Regge model with constant residues is too restrictive.

More parameters can be inserted in models of the spectra by treating separately the low-lying states, both their residue and position. Actually, this is physical. We know that there are departures from the linear Regge trajectories at low \( n \), also there is no reason why at low energies the couplings should be the same. In principle, these constants are measurable, thus in the large-\( N_c \) world the values of the OPE condensates can be expressed in terms of the parameters of the large-\( N_c \) spectra. This is very close to the approach of Ref. \cite{31,32} with the important extension of admitting the possibility of the dimension-2 condensate. For the purpose of illustration we consider the following simple modification of the previous model:

\[ M_{V,0} = m_\rho, \quad M_{V,n}^2 = M_V^2 + an, \quad n \geq 1, \]

\[ M_{A,n}^2 = M_A^2 + an, \quad n \geq 0. \]

In words, the lowest \( \rho \) mass is shifted, otherwise all is kept “universal”, including constant residues for all states. In the present case the Weinberg sum rules have the form (we set \( N_c = 3 \) from now on)

\[ M_A^2 = M_V^2 + 8\pi^2 f^2, \]

\[ a = 8\pi^2 F^2 = \frac{8\pi^2 f^2 (4\pi^2 f^2 + M_V^2)}{4\pi^2 f^2 - m_\rho^2 + M_V^2}. \]

When \( m_\rho = 0.77\text{GeV} \) is fixed, the model has only one free parameter left. We may take it to be \( M_V \), however, it is more convenient to express it through the string tension \( \sigma \), which is then treated as a free parameter. Thus

\[ M_V^2 = -\frac{16\pi^3 f^4 + 4\pi^2 f^2 - m_\rho^2 \sigma}{4f^2 \pi - \sigma}, \]

and the glueon condensates, obtained by matching to (11), are

\[ -\frac{\alpha_s \lambda^2}{4\pi^3} = \frac{16\pi^3 f^4 - \pi\sigma^2 + m_\rho^2 \sigma}{16f^2 \pi^3 - 4\pi^2 \sigma}, \]

\[ \frac{\alpha_s\langle G^2 \rangle}{12\pi} = \frac{2\pi^2 f^4 - \pi\sigma f^2}{3\sigma \left( \frac{m_\rho^2 \sigma}{\sigma - 4f^2 \pi^2} - 2\pi \right)} + \frac{\sigma^2}{12}. \]

Figure (11) shows the two gluon condensates plotted as functions of \( \sqrt{\sigma} \). The constant lines mark the “physical” values \( -\alpha_s \lambda^2 / (4\pi^3) = 0.003\text{GeV}^2 \) and \( \alpha_s\langle G^2 \rangle / (12\pi) = 0.001\text{GeV}^2 \). Remarkably, the window for which both condensates are simultaneously positive yields very acceptable values of \( \sigma \), cf. Ref. \cite{23}. Moreover, the consistency check \( \sigma = \sigma_0 \) is satisfied for \( \sqrt{\sigma} = 497\text{MeV} \). The magnitude of the condensates is in the ball park of the “physical” values. The value of \( M_V \) in the fiducial range is around 0.82 GeV – again a very reasonable value. Note the close value of \( M_V \) to \( m_\rho \), 0.82 GeV vs. 0.77 GeV. This shows sensitivity of the condensates even to small deviations from the linearity of the Regge trajectory. The experimental \( \rho \) states are at 770, 1450, 1700, 1900, and 2150 MeV, with the last two states not well established, while the model with \( \sigma = (0.47 \text{GeV})^2 \) gives 770, 1355, 1795, and 2147 MeV. In the \( a_1 \) channel the experiment shows states at 1260 and 1640 MeV, while we get somewhat lower 1015 and 1555 MeV.
Finally, we wish to point out that the $V - A$ channel is actually very well reproduced with the radial Regge models. As in recent study of Ref. [9], we apply the Das-Mathur-Okubo sum rule to evaluate the low-energy constant $\lambda_{10}$, and the Das-Guralnik-Mathur-Low-Yang sum rule to obtain the electromagnetic pion mass splitting. In the strictly linear Regge model with, for instance, $M_\sigma^2 = 2M_\rho^2$ and $M = \sqrt{24\pi^2/\sigma} = 764\text{MeV}$, we have $\sigma = 3M_\rho^2$, or $\sqrt{\sigma} = \sqrt{3/2\pi M} = 532\text{MeV}$, and $F = \sqrt{3f} = 150\text{MeV}$, rather reasonable results. Then $L_{10} = -N/(6\sqrt{3}) = -5.74 \times 10^{-3}(-5.5 \pm 0.7 \times 10^{-3})_{\exp}$ and $m_{\pi^+}^2 - m_{\sigma_0}^2 = (31.4\text{MeV})^2 (35.5\text{MeV})^2_{\exp}$.

In our second model with $\sigma = (0.48\text{GeV})^2$ the values are $L_{10} = -5.2 \times 10^{-3}$ and $m_{\pi^+}^2 - m_{\sigma_0}^2 = (34.4\text{MeV})^2$, in quite remarkable agreement with the data.

In conclusion, we note that the scheme of matching OPE to the radial Regge models produces, in a natural way, the $1/Q^2$ correction to the vector and axial vector correlators, which is attributed to the dimension-2 gluon condensate. Thus our explicit calculation illustrates the significance of confinement also for the short-distance expansion, as discussed in Ref. [3, 14]. More generally, OPE with the dimension-2 and all other condensates can be matched by radial Regge models, provided conditions [8, 11] are satisfied by the asymptotic spectra, and the parameters of the low-lying states are adjusted to reproduce the values of the condensates. In principle, these parameters are measurable, hence the information encoded in the low-lying states is the same as the information in the condensates and we could verify consistency. Yet the sensitivity of the values of the condensates to the parameters of the spectra, as seen by comparing the two explicit models considered in this paper, make such a study difficult at a more precise level.

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