STUDY OF A DYNAMIC COOPERATIVE TRADING QUEUE ROUTING CONTROL SCHEME FOR FREEWAYS AND FACILITIES WITH PARALLEL QUEUES

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1 Abstract

This article explores the coalitional stability of a new cooperative control policy for freeways and parallel queuing facilities with multiple servers. Based on predicted future delays per queue or lane, a VOT-heterogeneous population of agents can agree to switch lanes or queues and transfer payments to each other in order to minimize the total cost of the incoming platoon. The strategic interaction is captured by an n-level Stackelberg model with coalitions, while the cooperative structure is formulated as a partition function game (PFG). The stability concept explored is the strong-core for PFGs which we found appropriate given the nature of the problem. This concept ensures that the efficient allocation is individually rational and coalitionally stable. We analyze this control mechanism for two settings: a static vertical queue and a dynamic horizontal queue. For the former, we first characterize the properties of the underlying cooperative game. Our simulation results suggest that the setting is always strong-core stable. For the latter, we propose a new relaxation program for the strong-core concept. Our simulation results on a freeway bottleneck with constant outflow using Newell’s car-following model show the imputations to be generally strong-core stable and the coalitional instabilities to remain small with regard to users’ costs.

Keywords: lane-changing, stability, core, dynamic queue routing, parallel queues, connected vehicles
2 Introduction

Connected vehicle environments bring new opportunities in the operation of traffic infrastructures. Until now, vehicles traveling on a link selfishly self-organized themselves into a service order which corresponds on average to a First-Come First-Served (FCFS) order, without any mutual exchange of urgency information. For instance, lane changes naturally violate the FCFS service order through a gap acceptance distributed mechanism, with very limited or no precise future delay information. New connectivity can increase the efficiency of lane changes by providing better future delay estimation and by incorporating a new variable: travellers’ Value Of Time (VOT).

If it was possible for users to be informed of the downstream traffic conditions for each lane, i.e. the downstream delay on each lane at a particular timestep, and if they could communicate their VOT values to each other, travelers could decide which lane changes are the most beneficial for everybody. Thus, agents can violate the initial order by creating coalitions which give incentives to agents in front to choose longer queues in exchange of a side payment. This would lead to a different level of service for each lane or queue, the less congested lanes or queues then becoming faster than the more congested ones and preferable for the higher-VOT travellers who may be willing to pay. The concept presented in this article is applicable to both facilities with parallel queues (multiple parallel servers) i.e. access gates at ports, traffic intersections, and bottlenecks in a freeway section. We will use the terms “queue” and “lane” indistinctively, as the essential level of performance of each lane is captured by the queue associated with it, for the conceptual purposes of this paper.

This article presents a new dynamic queue routing control scheme which violates FCFS and outperforms it in efficiency while being core-stable. The policy is now outlined. Agents can choose which queue or lane they want to switch to. Naturally, vehicles in front get to choose earlier. Agents are assumed to be having perfect knowledge of the delay per lane (or queue) as well as the delay increases due to nearby vehicles’ lane changes. Agents can communicate their values of time to any other vehicle they want to interact with. Agents can form coalitions and exchange payments among them to improve their utility. Our mechanism implements the most efficient allocation and ensures that all agents and any subset of agents present in the system are better off by participating and cooperating with the outcome solution. This is ensured by employing the concept of the core, the pillar of cooperative game theory. Broadly, the core is a feasible set defined by constraints which define the stability of coalitions based on their worth. We hasten to add here that such exchanges may not legally allowed in traffic systems; however, we assume that demonstrated social/system efficiency can lead to regulatory changes in future.

Contrary to most common applications in cooperative game theory, traffic operations present externalities. This means that the worth of a particular coalition depends on what other coalitions do. This brings us to the domain of partition function games (PFG), a superset of the more commonly used characteristic function games (CFG), which are not complex enough to express externalities. Equivalent stability concepts to the core are defined for partition function games. In particular, we are going to use the strong core [7], which we believe is small enough to be meaningful and apparently non-empty for the simplified version of the current application. A fundamental result in [16] establishes an equivalence between strategic games and partition games. This paper will actually relate both approaches, since
the strategic-cooperative interaction is modeled as the optimization of the union of n-level Stackelberg games with coalitions.

We claim the following contributions in this article. First, we present this novel operational scheme for parallel queues and freeway management. Second, we are the first to use and solve a multiple-discrete-strategies n-level Stackelberg game with coalitions. Third, we found that the problem appears to be always strong-core stable for the vertical queue case, and that it is generally stable for the horizontal queue case as well. Finally, we propose a new relaxation for the strong-core concept found in [7] and generalize it to the dynamic domain.

The article is organized as follows: section 2 presents the meaningful literature from microeconomics on queue games and the utilized strategic structures, section 3 presents a static version on the cooperative queuing problem, modeling the queue routing as a parallel static vertical queue, section 4 presents a dynamic version of the problem, modeling the queue routing as parallel horizontal queues, and section 5 presents the conclusions and further research.

3 Literature review

Microeconomics literature has extensively explored the stability, fairness and truthfulness of priority queues [10] for single and parallel queues [11]. In priority queues, an unordered set of agents with heterogeneous values of time occupies positions on a line valued with linear delay. The efficient queue ordering is the one which places the agents sorted by decreasing value of time. However, queues in transportation systems involve agents with physical dimension and not all queue orderings are possible due to agent obstruction.

[5] studied pricing and incentives for a multiserver queuing facility. He analyzes both social welfare versus operator’s revenue but does not enforce budget balancedness nor cooperation of any kind. His results are based on steady state and reach standard marginal pricing conclusions in efficiency maximization.

[22] examines stationary equilibria in a GI/M/1 queue policy in which users agree over a common service toll, which is later redistributed among participants. However, it does not examine coalitional stability between agents nor applies a deterministic analysis on individual vehicles as the present article does. Other authors have tackled similar queue problem on the demand side [6].

The cooperative interaction between agents for this problem is represented by partition function games. PFG are normally classified by the externality, either positive or negative, that a two coalitions forming creates on a third one. [14] explores the role of convexity on efficiency and core stability. [1] explores PFG with either positive of negative externalities. Similarly to characteristic function games, although less studied, several stability concepts have been developed for this more expressive concept [15]. These concepts being generally too large when non-empty [8, 9] propose the gamma-core and the strong-core [7].

The strategic interaction between travelers has an inherent arrival ordering. The most adequate equilibrium concept is that of (multilevel) Stackelberg equilibrium. Much has been said about two and three level variants of this concept. Little has been studied on the more general multilevel case. [2] explore coalition formation in level Stackelberg games for linear
resource problems.

On a related application, [18] proposes a queue jumping mechanism for general purpose freeway operations, in which vehicles coming from upstream can pay queued vehicles for being overtaken. Stability in the problem emanates from envy-freeness, naturally found in position environments [21].

4 Parallel vertical queues: Static problem

We have a section of road which has a bottleneck downstream of constant outflow, this bottleneck can either be highway congestion, a multiserver queue from a port terminal or border crossing point or a saturated intersection. The section downstream has \( m \) lanes, and the section upstream has \( l \) lanes. \( m \) may be larger, equal or smaller than \( l \). From each lane \( l \in L \), a subset \( M_l \subseteq M \) is accessible. There is a set \( N \) of vehicles approaching the queue from the back. Each lane \( l \in L \) has \( N_l \subseteq N \) vehicles. Each downstream lane \( m \) has a queue \( Q_m \geq 0 \) built up. Without loss of generality, we assume that \( Q_{m'} \leq Q_m \), \( \forall m' > m \). These queues can represent actual queues of stopped vehicles or congested traffic in the link transmission model fashion. For the analysis in this section, the facility dispatches one vehicle per unit of time per queue.

We decompose the ordered set of agents \( N \setminus \{i\} \) in two sets. Let \( A(i) = \{k \in N \mid k < i\} \) be the set of predecessors of agent \( i \) and \( F(i) = \{k \in N \mid k > i\} \) the set of followers of \( i \). Let \( A(m, i) \) be the set of predecessors of \( i \) which choose lane \( m \). Let \( j_m = |A(m, i)|, \forall m \in M \). This defines a lane choice set \( \sigma : N \rightarrow M^N \). Contrary to priority queues, in our model, a traveler cannot advance a predecessor unless he joins a queue which is shorter than the queue that predecessor has joined.

The delay for agent \( i \in N \) joining lane \( m \) given lane choice set \( \sigma(N) \) is \( d_i(Q, \sigma(A(i))) = (Q_m + j_m - 1) \) and the valuation experienced by agent \( i \), \( v_i(Q, \sigma(A(i))) = -\theta_i d_i(Q, \sigma(A(i))) \), where \( \theta_i \) represents the value of delay in monetary units per unit of time. This variable will also be called the type of agent \( i \). Agent \( i \) is charged a price \( p_i \) for bearing the delay \( d_i(Q, \sigma(A(i))) \). Finally, his utility is \( u_i = v_i(Q, \sigma(A(i))) - p_i \).

Vehicles upstream belong to different lanes and are ordered based on their proximity to the downstream boundary. Again without loss of generality, we disregard the lanes \( l \) where vehicles are located, which is equivalent to assuming that \( M_l = M \). If there is no communication between agents, vehicles will join the downstream bottleneck on a First-Come First-Served (FCFS) basis, each vehicle selecting the shortest queue. It is easy to see that if the initial arrival order is not monotonically decreasing on types, the resulting queue ordering will be inefficient, if we view efficiency in a utilitarian social welfare sense. However, if vehicles were to cooperate with each other, that is, forming coalitions to alter this initial ordering, a more efficient ordering for everyone would be achieved. This cooperation would be on terms of multilateral agreements on which lane, every vehicle of the coalition would choose. This defines a multilevel Stackelberg game with coalitions. Of course, such coalition-forming would require communications and decision-making of the kind human drivers in traffic may not accomplish, but apps representing them can accomplish, and the technology for it certainly exists already.

[3, 2] explored some sufficient properties for stability for this kind of games, but on a
Figure 1: General problem configuration

linear resource allocation environment. We apply a similar recursion to our problem, but this time for pure strategies in extensive game. This recursion will give us the value generated by each coalition, given a particular coalition structure (partition of \( N \)). The recursion starts from the last vehicle and goes up. At each level, the user \( i \) attempts to optimize the sum of the valuation of users who are behind it, \( j \in N \mid j > i \) and which belong to the particular coalition \( i \) belongs to. The back recursion to solve the n-level Stackelberg problem with coalitions is, given a partition \( P = \{S_1, ..., S_j \} \mid \cap_j S_j = \emptyset \):

\[
V_{S(i)|P}(h) = \max_{k \in K_i(h)} \left\{ v_i(h, k) + V_{S(i)|P}(h \cup k) \right\} \quad \forall i = n - 1, ..., 1, \ h \in H_i \tag{1}
\]

\[
V_{S(n)|P}(h) = \max_{k \in K_n(h)} \left\{ v_n(h, k) \right\} \tag{2}
\]

Where \( S(i) \) is the coalition where \( i \) belongs.

\( \square \) shows that for every user \( i \), for every past history \( h \in H_i \) up to user \( i \), the agent selects the action \( k \) belonging to the set of available actions given \( h, K_i(h) \), which maximizes the sum of two terms. The optimal value \( V_{S(i)}^*(h \cup k) \) which emanates from the previous step \( i + 1 \) and, \( v_i(h, k) \), the valuation of user \( i \) from choosing action \( k \), given history \( h \).
Proposition 1. The computational complexity of the recursion for the vertical queue case is $O(n(n + l)^l)$:

Counting the number of nodes at the final level is equivalent to finding the number of $l$-combinations with repetitions:

$$\binom{l}{n} = \binom{l + n - 1}{n} = \prod_{i=1}^{l} \frac{(n+i)^l}{i-1} < (n + l)^l$$

Since there are $n$ levels, the final complexity is $O(n(n + l)^l)$.

The recursion above needs to be solved for every partition $P \in \mathcal{P}$ to obtain all coalition values. The interaction between all the $n$-level Stackelberg games with coalitions will be modeled as a cooperative game. Cooperative games are complete-information games in which users are allowed to form coalitions to improve their payoffs. In the absence of coalitional externalities, cooperative games can be represented in a Characteristic Function Form (CFF), which defines a Characteristic Function Game (CFG). However, in the current problem there are externalities, which translates into coalitional payoffs being dependent on which other coalitions are formed. Thus, we will represent the game on a partition function form (PFF) which defines a Partition Function Game (PFG). [16] shows that any strategic game can be represented as a partition function game. Thus, we will translate the former strategic games as a single PFG.

Let $P = \{S_1, \ldots, S_k\} \in \mathcal{P}$ be a partition of $N$ such that $S_i \cap S_j = \emptyset \forall i \neq j$. We define the partition function $v : 2^N \times \mathcal{P} \rightarrow \mathbb{R}$. That is, $v(S, P)$ represents the value of coalition $S$ when the partition $P$ is formed. $v(S, P)$ is in fact the sum of all the valuations $v_i \forall i \in S, \forall S \in P$, coming from the optimal order resulting from the cooperation between agents belonging to $S$ when the partition formed is $P$. The pair $< N, v >$ defines a partition function game. There is a coalition of special interest, the grand coalition $S_G = N$, which is composed by all members of the participant set.
A fundamental question in cooperative game theory is if this total cooperation will eventually occur. This is desirable when the grand coalition is the most efficient coalition. When the partition function game arises from a strategic game, the grand coalition is always efficient since the set of strategies of the grand coalition game includes all the strategies available in the other subgames. In fact, the grand coalition payoff is the shortest path on the graph defined by the recursion \([1]\) when \(P = N\). Conversely, a coalition can unanimously dissolve itself into singletons, since every member can still choose the same set of strategies. This is desirable when the grand coalition is the most efficient coalition. When the externalities are negative if the value of \(C\) increases the value of a third coalition \(C\), for any complementary partition \(\rho\). Conversely, the externalities are negative if the value of \(C\) is decreased. It is easy to see that this game has positive externalities.

Let \(\rho \in P(N \setminus (S \cup T \cup C))\). Suppose all agents forming coalition \(C\) precede those of coalitions \(S\) and \(T\). Then, the merging of \(S\) and \(T\) has no influence on \(v(C, \{C, S \cup T, \rho\}) \forall \rho \in P(N \setminus (S \cup T \cup C))\) and the externality is zero. If agents forming \(C\) go after \(S\) and \(T\), the externality can only be positive or zero since \(S \cup T\) either causes some agents to join longer queues or keep the positions if no improvement is possible, reducing the cost of the members of \(C\), which manage to advance some positions or none. However, if members of \(C\) are between those of \(S, T\) or \(S \cup T\), negative externalities may occur. We provide an example next:

**Example 1.** The static problem with instance \(N = \{13, 2, 14, 41\}, Q = \{4, 1\}\) has negative externalities. Let \(S = \{1, 4\}, T = \{3\}, C = \{2\}\). Then, by solving the game with the recursion algorithm, we reach the case of \(0 = v(C|\{S \cup T, C\}) < v(C|S, T, C) = 2\).

A useful property some PFG’s exhibit is superadditivity, which is defined next:

\[
\forall S, T \subseteq N \mid S \cap T = \emptyset, \forall \rho \in N \setminus (S \cup T), v(S \cup T; \{S \cup T\} \cup \rho) \geq v(S; \{S, T\} \cup \rho) + v(T; \{S, T\} \cup \rho)
\]

Superadditive means that if two coalitions \(S\) and \(T\) merge, their total payoff is larger than when unmerged. For the sake of exposition, the following counterexample shows that this game is not superadditive:

**Example 2.** \(N = \{1, 9, 5, 33\}\) and \(h = 3\). If \(\rho = \{1, 3\}\), \(S = \{2\}\), \(T = \{4\}\), then \(v(S \cup T, \{S \cup T, \rho\}) = 15\), \(v(S, \{S, T\} \cup \rho) = 9\), \(v(T, \{S, T\} \cup \rho) = 33\).

PFG’s stability concepts and characterizations generally focus on games which have either positive or negative externalities \([1]\) or exhibit superadditivity or convexity \([3]\). This is not the case of our problem. We found an exception in the literature, which is the strong-core for PFG \([7]\), which is defined next:
Definition 2. Strong-core for PFGs [7]

\[(x_1, \ldots, x_n) \in \mathbb{R}^n \mid \forall P \in \mathcal{P}, P = \{S_1, \ldots, S_p\} \neq [N], \exists S_i \in P, |S_i| > 1 \mid \sum_{j \in S_i} x_j \geq v(S_i, P) \text{ and if } P = [N], x_i \geq v(i; [N]) \forall i \in N\]

The definition above states that for every partition which contains non-singleton coalitions, exists at least one coalition of those coalitions which is worse off than in the strong-core imputation \((x_1, \ldots, x_n)\). Imputation is a term used in game theory to denote the utility agents obtain from a coalitional agreement. Moreover, every agent in the all-singleton partition is worse off. Contrary to other core specifications found in the literature such as the \(\alpha, \beta, \gamma, \delta\)-cores [15], the strong-core does not assume any coalition structure for the complementary partition when a given coalition forms. The solution concept is particularly useful for the treatment of the status-quo partition, which, in our case, corresponds to the FCFS queue allocation. If an imputation vector belongs to the strong-core, then all the singleton coalitions belonging to the coalition made of just singletons are better off by merging into the grand coalition. Now the question that remains is to prove non-emptiness of the strong-core.

Partial superadditivity is weaker than superadditive, which the game does not satisfy. Partial superadditivity is trivially satisfied for games with 3 or 4 players whenever the grand coalition is efficient.

Conjecture 1. The strong-core for the static queuing game as vertical queue is non-empty.

We leave the result as a conjecture since it was not possible to prove it. After having run a large number of simulations, we did not find any counterexample, either. The values employed for the simulation study are: \(n \sim \text{Unif}(1, \bar{n}), \bar{n} \in [2, 7], m \in [1, 4], \theta \sim \log(n \mu = 2.16, \sigma = 0.7), Q_m \sim \text{Unif}(1, 4), Q_i \sim \text{Unif}(1, Q_{i+1}) \forall i < m\). The semi-empirical distribution used to draw individual Valuation of Delay Savings is developed in [17].

Standard proof methods, such as direct proof used in operations research games for Minimum Spanning Tree Games and Shortest Path games [4], did not prove successful. Neither did other approaches such as reduction to market games. Moreover, the game did not prove to be convex, which is a sufficient condition for non-emptiness of strong-core.

Instead, we are going to evaluate the inclusion of two imputations which are generalizations of the Shapley value for partition function games. These imputations satisfy the four properties of the original Shapley value: efficiency, symmetry, additivity and null player. The two imputations are presented next:
Definition 3. [13, 12] Externality-free value:

$$\phi_i^{\text{free}}(v) = \sum_{S \subseteq N} \zeta_i^S v(S, \{S\} \cup \{\{j \mid j \in N \setminus S\}) \forall i \in N$$

(4)

Where:

$$\zeta_i^S = \begin{cases} 
\frac{(|S|-1)(|N|-|S|)!}{|N|!} & \text{if } i \in S \\
-\frac{|S|(|N|-|S|-1)!}{|N|!} & \text{if } i \notin S
\end{cases}$$

(5)

The $\zeta_i^S$ values arise from the reordering of the marginal increment $v(S \cup \{i\}) - v(S) \forall S \subseteq N \setminus \{i\}$ expression, often found in the Shapley value definition, in terms of all the partitions $v(S) \forall S \subseteq N$.

This value represents that an agent leaving the grand coalition always creates a new coalition, that is, a singleton. This value is inline with the $\gamma$-core present in the strong core existence theorem and we expect imputations from this value to generally belong to the strong-core. The second imputation to test is:

Definition 4. [19] McQuillin value:

$$\phi_i^{\text{McQ}}(v) = \sum_{S \subseteq N} \zeta_i^S v(S, \{S,N\setminus S\}) \forall i \in N$$

This value entails that an agent always chooses an existing coalition.

The following MILP will be used to test the feasibility of $\phi^{\text{free}}(v)$ and $\phi^{\text{McQ}}(v)$ in the strong-core. We add the following objective function and modify the group rationality condition for the coalitions which have non-singleton partitions $P \in \bar{P} = P \setminus \{[N] \cup \{N\}\}$. We call this relaxation the strong-core for PFG’s, in line with the $\epsilon$-core for characteristic function form games [20].

$$\min \epsilon$$

s.t.

$$\sum_{i \in S} x_i \geq v(S_j, P) - \epsilon - M z_{jp} \forall S_j \in \bar{P} \subseteq P, \forall P \in \bar{P}$$

(6)

$$x_i \geq v(\{i\}, [N]) - \epsilon \forall i \in N$$

(7)

$$\sum_{i \in N} x_i = v(N, \{N\})$$

(8)

$$\sum_{S_j \in \bar{P}} z_{jp} \leq |\bar{P}| - 1 \forall \bar{P} \subseteq P \in \bar{P}$$

(9)

$$\epsilon \geq 0, z_{jp} \in \{0,1\} \forall S_j \in \bar{P} \subseteq P, \forall P \in \bar{P}$$

(10)

Where $\bar{P} \cup \bar{P} = P \mid \bar{P} \cap \bar{P} \neq \emptyset$ are the collections of non-singleton coalitions $\bar{P}$ and singleton coalitions $\bar{P}$ of every partition $P$. Essentially, the program consists of the relaxed
group rationality constraints (6), the individually rational constraints (7), the grand coalition efficiency (8). The $\epsilon$ variable is the minimal slack for the most constraint coalition $S_j \in \hat{P} \subseteq P$ necessary to make the problem feasible. The binary terms $z_{jp}$ present in (6) and (9) enforce that at least one non-singleton coalition $S_j \in \hat{P} \subseteq P$ for every $P \in \hat{P}$ to be group rational.

The settings of this simulation are the same ones than for strong-core non-emptiness evaluation. For each of these settings, 250 experiments are run. The next tables show the percent of instances where the imputation was found in the strong-core:

| $\phi^{tree}$ : $\pi \backslash M$ | 2     | 3     | 4     |
|-----------------------------------|-------|-------|-------|
| 2                                | 100.0 | 100.0 | 100.0 |
| 3                                | 100.0 | 97.2  | 98.0  |
| 4                                | 96.0  | 85.2  | 88.4  |
| 5                                | 87.6  | 75.6  | 72.4  |
| 6                                | 84.0  | 67.6  | 64.0  |
| 7                                | 74.4  | 52.8  | 61.6  |

| $\phi^{McQ}$ : $\pi \backslash M$ | 2     | 3     | 4     |
|-----------------------------------|-------|-------|-------|
| 2                                | 100.0 | 100.0 | 100.0 |
| 3                                | 100.0 | 97.2  | 98.0  |
| 4                                | 96.0  | 85.2  | 88.4  |
| 5                                | 88.0  | 75.6  | 72.4  |
| 6                                | 83.6  | 67.6  | 64.0  |
| 7                                | 74.4  | 53.2  | 62.0  |

Table 1: Percent of experiments whose imputations are in the strong-core.

We observe that both imputations provide very similar percentages of inclusion into the strong-core, the only differences being due to numerical errors from the solution algorithm. As expected, the larger the vehicle set and the larger the number of lanes, the more the instances of not belonging to the strong core. This result seems to be weaker when increasing the number of lanes than when increasing the maximum platoon size. Both imputations having identical percents suggest that a certain degree of symmetry exists in the violation of the strong-core in the coalition formation process, for both all-singleton blocking coalitions and all-maximum-cardinality complementary blocking coalitions. This can provide some insights for the non-emptiness proof of this vertical queue model.

5 Parallel horizontal queues: Dynamic problem

Queues in transportation systems have a dynamic nature: they continuously receive arrivals and dispatch departures. Moreover, these queues are “horizontal”, that is, queue length has a physical dimension. Modeling such dynamic process as a succession of static queuing problems may lead to strong inter-temporal inefficiencies, coalitional stability and violation of individual rationality. One of the solutions, employed in [18] is to set up a reserve price which prevents low value vehicles from having too much present savings in detriment of further high value vehicles. Another alternative would be to add a terminal cost at the leaves of the static problem tree, however this would require a transfer of payment from former agents to new agents as well as knowledge of further arrivals and their values, which would difficult the dynamic budget balancedness and the efficiency of the system. In this particular problem, we will stick to a control solution which is always budget balanced and efficient, aiming an eventual V2V decentralized implementation.

We model again a link with $M$ lanes, but this time with length $\lambda$. Vehicles enter the link upstream at a constant speed $v_a$. There is a bottleneck downstream with a constant outflow
which corresponds to a headway $h_q$, spacing $s_q$ and speed $v_q < v_a$. Every time an event happens, vehicle communicate to each other their value of time and positions, including the vehicles at the back of the queue. An imputation which satisfies the minimal $\epsilon$-strong core is found and vehicles are assigned their new lanes. To minimize excessive perturbations due to the lane changes, only vehicles whose incorporation to the back of the queue is imminent will execute the lane change. The rest of vehicles will only execute the lane change once their incorporation becomes imminent. Naturally, the target lane can later change if there are further stability optimizations being executed due to new events happening.

Vehicles participate in lane-changing optimizations only as long as their incorporation to the back of the queue is not imminent. Once a vehicle is queued, they do not participate in other cooperative lane changes and are supposed to stay in the queue. The exchange optimization is run at every instant when there is a significant event. We define an event as the arrival of new vehicle to the link or imminent proximity of a moving vehicle to the back of the queue, or any external unpredictable event which could be detected by any of the vehicles. Events happen at time instants $t$, called epochs.

With this in mind, and using Newell’s simplified car-following model, vehicle’s $i$ predicted delay at epoch $t$ is the maximum quantity of two situations: arriving to the bottleneck downstream undelayed at free flow speed or being queued behind its predecessor:

$$d_t^i(S, P) = \max\{d_{\text{dep}}^i(S, P) + \frac{s_q}{v_q} a_i, a_i\}$$

Since vehicles may participate in multiple optimizations, the utility specification is composed of the predicted total cost at epoch $t$ since its arrival to the system minus the price charged at the optimization executed during epoch $t$ minus the accumulated price charged to vehicle $i$ until $t$. $i$’s imputation from being in coalition $S$ and partition $P$ is:

$$x_t^i(S, P) = v_t^i(S, P) - p_t^i(S, P) - \pi_{i-1}^t \forall i \in I_t, \forall S \subseteq P, \forall P \in \hat{P}$$

$$v_t^i(S, P) = -c_t^i(S, P) = -\theta d_t^i(S, P)$$

With the imputations and valuations being defined, the dynamic $\epsilon$-strong core program is defined by replacing $\forall i \in N^t$, $x_t^i$ and $v_t^i$ into equations (7 - 9). Naturally, the $\epsilon$ term in equations (10) and in the objective function will be replaced at each program $t$ by the corresponding $\epsilon_t$.

Parallel horizontal queues cannot benefit from the polynomial structure employed in the previous section. Vehicles’ costs do not depend this time only on the queue length in front of them, but on the dispatching times of the vehicles’ downstream. Since the departure times depend on the actual sequence of queued vehicles, a particular state is now defined by the sequence of vehicles in front of them and therefore the whole queuing tree needs to be explored. This increases the computational complexity of the problem. For this reason, we will limit the number of participant agents of every optimization to six. Any additional vehicles upstream will stay outside of the exchange and continue advancing through the link in a FCFS basis. The model with the full exponential tree is general enough to include jockeying strategies. That is, agents are allowed to switch queues after having joined a queue. We rule out this possibility in this paper for simplicity in exposition.
Proposition 2. The computational complexity of the recursion for the horizontal queue case is $O(l^n)$:

For the horizontal queue case, all the tree histories need to be explored. This defines a $l$-ary tree with $n$ levels. The last level has $O(l^n)$ nodes.

The exchange of information in positions also serves to define which lane changes are possible and which ones are obstructed. In our current formulation, if some lane change is not possible at a particular epoch $t$, the cost of it equivalent branch is set to $\infty$ and that strategy does not get explored. However, this is not implemented here.

We explore next the core stability of the dynamic problem as horizontal queues. We run 6 1-hour long simulations for each of the scenarios defined by: $\lambda = 200 \text{ m, } L \in \{2, 3\}, q_{in} \in \{360, 540, 720\} \text{ veh/h/lane, } q_{out} = 900 \text{ veh/h/lane, } \theta \sim logn(\mu = 2.16, \sigma = 0.7)$. Arrivals arise from a binomial distribution for implementation easiness, but the simulation is still event-based: both time and distance are continuous. The simulation is coded in MATLAB and the MILP programs are solved with Gurobi 6.0.5.

The next table (left) shows the percent of optimizations which are contained in the strong-score. We observe that for equal vehicle inflow, increasing the number of lanes increases core stability. This can be explained by the incoming platoon gets split into more lanes and their queues and interactions are smaller. Furthermore, increasing the incoming flow seems to increase instability, mostly due to a natural increase of complexity in the strategic interaction between agents. The table on the right displays the average ratio between $\epsilon$ and the average vehicle cost per optimization. This ratio is useful to compare the magnitude of the slack term, i.e. amount of utility that has to be transferred to a blocking coalition, with regard to the average cost of the participating agent. In line with the previous analysis, for equal inflow, a larger number of lanes decreases the magnitude of the instability. However, increasing the inflow seems to decrease the ratio, probably due to a larger increase in average vehicle cost.

| $q_{in}(\text{veh/h}) \setminus M$ | 2 | 3 |
|----------------------------------|---|---|
| 360                             | 91.5 | 99.6 |
| 540                             | 88.5 | 95.1 |
| 720                             | 78.7 | 93.2 |

| $q_{in}(\text{veh/h}) \setminus M$ | 2 | 3 |
|----------------------------------|---|---|
| 360                             | 0.29 | 0.17 |
| 540                             | 0.11 | 0.16 |
| 720                             | 0.04 | 0.02 |

Table 2: (left) % of strong-core stable optimizations. (right) ratio $\epsilon$/av.cost

6 Conclusions and further research

This article presented a new collaborative control mechanism for freeways and parallel queue facilities. Under this control scheme, agents observe predicted future delays per lane (or queue) and are allowed to collaborate to change lanes such that the total travel cost of their platoon is minimized. High VOT vehicles can pay low VOT vehicles to switch to a more congested lane while they can stay in the same lane or switch to another lane with less vehicles in front. The underlying cooperative principle is the strong-core for partition function games. While aimed as a decentralized, distributed control, the present paper
assumes a centralized optimization to evaluate the economic efficiency and stability without excessive technicalities.

The control policy has been first explored as a simpler vertical queue model. In this case, the strategic interaction between users structure forms a tree-like structure which is of polynomial-time complexity. While not proved, simulation results suggest that the problem may be strong-core stable. In addition, we have tested two generalizations of the Shapley value for partition function games which we found to generally be strong-core stable.

In the next section, we modeled the control policy as a dynamic horizontal queue. We have observed that the policy is generally strong-core stable except for situations when there are sharp increases in the incoming platoon size. However, it is computationally intensive. Further distributed optimization techniques should be used to make it applicable for an eventual real-world implementation. Alternatively, designing an approximation algorithm for the strong-core optimization would serve.

As further research we point out the following lines. We believe that developing a formal non-emptiness proof for the vertical case would represent a strong result in cooperative game theory. Also, from a theoretical point of view, exploring the dynamic vertical queue case would be interesting in the sense that dynamic applications in partition function games have never been explored outside of coalition formation in static settings. Concerning the dynamic horizontal queue, we will further explore unstable instances to better determine the source of instability and better understand the problem. Eventually, modeling such policy with a commercial microsimulation software, would provide insights on the efficiency increases of a real world situation, as well as including more realism and efficiency losses due to lane changing obstruction.

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