Magnetoconductivity of quantum wires
with elastic and inelastic scattering

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Abstract

We use a Boltzmann equation to determine the magnetoconductivity of quantum wires. The presence of a confining potential in addition to the magnetic field removes the degeneracy of the Landau levels and allows one to associate a group velocity with each single-particle state. The distribution function describing the occupation of these single-particle states satisfies a Boltzmann equation, which may be solved exactly in the case of impurity scattering. In the case where the electrons scatter against both phonons and impurities we solve numerically - and in certain limits analytically - the integral equation for the distribution function, and determine the conductivity as a function of temperature and magnetic field. The magnetoconductivity exhibits a maximum at a temperature, which depends on the relative strength of the impurity and electron-phonon scattering, and shows oscillations when the Fermi energy or the magnetic field is varied.

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I. INTRODUCTION

The possibility of making quasi-one-dimensional quantum wires, due to advances in microfabrication technology, has greatly stimulated the interest in the transport properties of such low-dimensional systems. A number of recent papers have treated the magnetoconductivity of quantum wires using a variety of different theoretical approaches. The suppression of scattering between edge-states, due to the presence of the magnetic field, was discussed by several authors. The effects of impurity and boundary scattering on the Hall effect in quantum wires were treated by Akera and Ando, starting from a Boltzmann equation. The Kubo method has been used for considering the effect of the coupling to optical phonons and the influence of impurity scattering on the magnetotransport through quantum wires. Momentum- and energy-balance equations have been derived for a quasi-one-dimensional gas of electrons in a magnetic field. Other authors have used the Keldysh method to discuss magnetotransport in the presence of impurity scattering, while the formalism due to Landauer and Büttiker was employed in Ref. to treat the influence of disorder on the Hall effect in quantum wires.

Transport in strong magnetic fields has traditionally been treated within the Kubo formalism, since this allows one to take fully into account the quantization of the motion of an electron in a magnetic field. The resulting diagrammatic expansion of the conductivity requires the consideration of both self-energy effects and vertex corrections. It was demonstrated in Ref. that it is essential to include vertex corrections in order to determine the effect of impurity scattering on the magnetoconductivity of a quantum wire. The physical reason for this is that only back-scattering from the impurities contributes to the resistivity.

In the present paper we use a Boltzmann equation to determine the transport properties of quantum wires formed by additional confinement of the two-dimensional electron gas of e.g. a GaAs-GaAlAs heterojunction. The presence of a confining potential in addition to the magnetic field removes the degeneracy of the Landau levels and allows one to associate a group velocity with each single-particle state. The distribution function describing the occupation of these single-particle states satisfies a Boltzmann equation, which we solve exactly in the case of impurity scattering. The resulting magnetoconductivity agrees with that obtained in Ref. within the Kubo-approach. In the case of electron-phonon scattering we solve numerically - and in certain limits analytically - the integral equation for the distribution function. The conductivity is determined as a function of temperature and magnetic field in the case when both impurities and phonons contribute to the scattering of the electrons. Our paper extends previous work discussed above by giving a more complete account of the simultaneous influence of electron-impurity and electron-phonon scattering. By starting from the Boltzmann equation we include vertex corrections from the outset, thus simplifying the formal development in comparison to that involved in the use of the Kubo formalism.

Apart from its simplicity, the use of the Boltzmann equation has the additional advantage that it allows one to go beyond linear response theory in a straightforward manner. Although the present paper treats in detail only the regime of linear response, our approach may readily be generalized to take nonlinear effects into account.

Since conduction in a quantum wire involves current transport in one dimension, it is necessary to consider the role of localization. This was done in Ref. where it was shown...
that weak localization in quantum wires is destroyed in a magnetic field greater than a critical value $B_c$, where the critical magnetic field $B_c$ apart from numerical factors is given by $B_c \approx \frac{\hbar \ell}{e L_\phi W^2}$. Here $\ell$ is the elastic scattering length, $W$ the width of the quantum wire, while $L_\phi$ is the phase coherence length. For the parameters considered in Ref. 16 this yields a $B_c$ somewhat less than 0.1 T. In what follows we shall assume that the magnetic field is sufficiently strong that localization effects are always negligible. The transport properties of the quantum wires may then be obtained from a Boltzmann equation, in which the quantizing effect of the magnetic field is incorporated in the manner described below. We shall also disregard Coulomb-blockade effects which have been shown to be of importance in some systems.

The paper is organized as follows. In section 2 we introduce the single-particle states derived from the presence of a confining potential in addition to a homogeneous magnetic field. The Boltzmann equation and its solution is discussed in section 3. In section 4 we treat scattering due to impurities, while scattering from impurities and acoustical phonons forms the subject of section 5, where we consider both deformation-potential and piezoelectric coupling. Finally section 6 deals with the case where scattering from impurities and optical phonons is important.

II. THE SINGLE-PARTICLE ENERGY SPECTRUM

We consider an electron moving in the $xy$-plane under the influence of a constant magnetic field in the $z$-direction. In addition a parabolic confinement potential limits its motion in the $y$-direction whereby a wire is formed in the $x$-direction. The Hamiltonian for such an electron is

$$\hat{H} = \frac{1}{2m^*}(\vec{p} + e\vec{A})^2 + \frac{1}{2}K_y^2,$$

(1)

$m^*$ being the effective (band) mass of the electron. For simplicity we have neglected the Zeeman splitting, since we shall be mainly concerned with quantum wires in GaAs-based structures, where the Zeeman energy is a few percent of the cyclotron energy, $\hbar \omega_c$, to be defined below. The electron spin therefore only enters as a factor of 2 in the expression for the current density. In the $x$-direction we impose periodic boundary conditions, and for the vector-potential $\vec{A}$ we use the Landau-gauge, $\vec{A} = B(-y, 0, 0)$. Consequently the eigenfunctions of the Hamiltonian are products of plane waves and harmonic-oscillator functions involving Hermite polynomials $H_n$,

$$\Psi_{nk}(x, y) = \frac{1}{\sqrt{L}} e^{ikx} \frac{1}{\sqrt{\sqrt{\pi} 2^n n! \ell_h}}$$

$$\times H_n \left( \frac{y - y_k}{\ell_h} \right) \exp \left( -\frac{(y - y_k)^2}{2\ell_h^2} \right),$$

(2)

with $L$ being the length of the quantum wire. The center coordinate, $y_k$, and the characteristic length, $\ell_h$, of the harmonic oscillator is specified below. The wave function in the $z$-direction is taken to be a delta function.
In the case where $K$ is zero and only the magnetic field is present the natural system of units is based on the cyclotron frequency $\omega_c$ and the magnetic length $\ell_c$, given by

$$\omega_c = \frac{eB}{m^*}, \quad \ell_c^2 = \frac{\hbar}{eB}. \quad (3)$$

When $K$ is different from zero, we define the quantity $\gamma$, which describes the relative strength of the confinement potential, by

$$\gamma \equiv \frac{K}{m^* \omega_c^2}, \quad (4)$$

and the natural units are now based on the hybrid frequency, $\omega_h$, and the hybrid characteristic length, $\ell_h$, given by

$$\omega_h = (1 + \gamma)^{\frac{3}{2}} \omega_c, \quad \ell_h = (1 + \gamma)^{-\frac{3}{4}} \ell_c. \quad (5)$$

By inserting the function (2) in the Schrödinger-equation

$$\hat{H} \Psi_{nk} = \varepsilon_{nk} \Psi_{nk}, \quad (6)$$

one finds that (4) is satisfied, provided

$$\varepsilon_{nk} = \hbar \omega_h \left( n + \frac{1}{2} \right) + \frac{\hbar^2 k^2}{2m}, \quad n = 0, 1, 2, \ldots, \quad (7)$$

where $m$ is a renormalized mass defined by

$$m = \frac{1 + \gamma}{\gamma} m^*. \quad (8)$$

The center of the harmonic-oscillator wavefunctions is given by

$$y_k = \frac{\omega_h \hbar k}{m^* \omega_h^2} = (1 + \gamma)^{-\frac{3}{4}} k \ell_h^2. \quad (9)$$

The confinement potential $K y^2 / 2$ removes the degeneracy of the Landau-levels through the $k$-dependence exhibited in (4). Note that the dispersion relation is that of a free particle with a renormalized mass $m$. When the strength of the confinement potential goes to zero, the renormalized mass $m$ becomes infinite (corresponding to degenerate Landau-levels), while in the opposite limit $(\gamma \to \infty)$, the renormalized mass becomes equal to the effective mass $m^*$. The $k$-dependent energy levels are plotted in Fig. 1, where we have also indicated the position of the Fermi level corresponding to a definite electron density.

In the following we shall describe conduction through the quantum wire on the basis of the Boltzmann equation for the distribution function of the excitations with energy $\varepsilon_{nk}$. The group velocity $v$ of the excitations is seen to be

$$v_{nk} = \frac{1}{\hbar} \frac{\partial \varepsilon_{nk}}{\partial k} = \frac{h k}{m}. \quad (10)$$

If the confinement potential is not simply parabolic, the relation between $v_{nk}$ and $k$ becomes more complicated than Eq. (10), but the method described in the following is still applicable, provided one modifies the group velocity accordingly, and incorporates the new wavefunctions into the matrix elements appearing in the collision probability.
III. THE BOLTZMANN EQUATION

We consider the distribution function $f_{nk}$ for the excitations specified by the energy $\varepsilon_{nk}$, with group velocity $v_{nk} = \hbar k/m$. Unlike the usual semi-classical description of transport in a magnetic field, the magnetic field has here been taken into account from the very beginning, in defining the excitations with dispersion relation (7). The effect of an electric field $E$ in the $x$-direction is, however, included in the usual manner through the acceleration equation

$$\hbar \dot{k} = -eE.$$  \hspace{1cm} (11)

The Boltzmann equation is

$$\frac{\partial f_{nk}}{\partial t} + k \frac{\partial f_{nk}}{\partial k} = \left( \frac{\partial f_{nk}}{\partial t} \right)_{\text{coll}},$$  \hspace{1cm} (12)

where the right hand side of the equation contains the collision term. Since we are interested in the linear response to a static electric field, the time-derivative $\partial f / \partial t$ is equal to zero, while $f_{nk}$ in the second term on the left hand side may be replaced by the equilibrium function $f_{0nk}$. Using Eq. (11) the Boltzmann equation then becomes an inhomogeneous integral equation of the form

$$\frac{eE}{k_B T} v_{nk} f_{0nk} (1 - f_{0nk}) = \left( \frac{\partial f_{nk}}{\partial t} \right)_{\text{coll}},$$  \hspace{1cm} (13)

with the integral term on the right hand side - the collision term - to be specified below.

Before we consider specific scattering mechanisms, let us write down the conductivity $\sigma$ in terms of the distribution function. In the present work we restrict ourselves to the calculation of the diagonal conductivity element $\sigma_{xx} \equiv \sigma$. The current density $j$ is

$$j = \sigma E = -2e \sum_n \int_{-\infty}^{\infty} \frac{dk}{2\pi} v_{nk} f_{nk}.$$  \hspace{1cm} (14)

We shall introduce the deviation function $\psi_{nk}$ by the definition

$$f_{nk} \equiv f_{nk}^0 + f_{nk}^0 (1 - f_{nk}^0)\psi_{nk}.$$  \hspace{1cm} (15)

Then the current density becomes

$$j = -2e \sum_n \int_{-\infty}^{\infty} \frac{dk}{2\pi} v_{nk} f_{nk}^0 (1 - f_{nk}^0)\psi_{nk}.$$  \hspace{1cm} (16)

Since we shall be dealing with distribution functions that change sign upon $k \to -k$, we may restrict $k$ to positive values and work instead with the distribution function as a function of energy: $\psi_n(\varepsilon)$. In this notation $k$ is then a function of energy, and we denote the $k$-value which solves $\varepsilon = \varepsilon_{nk}$ by $k_n(\varepsilon)$. Furthermore, by introducing the function $\phi_n$ defined by

$$\phi_n \equiv \frac{k_B T}{eE} \psi_n,$$  \hspace{1cm} (17)
we may express the conductivity as

$$\sigma = \frac{2e^2}{h} 2 \sum_n \int_0^\infty d\varepsilon \left( -\frac{\partial f^0(\varepsilon)}{\partial \varepsilon} \right) \phi_n(\varepsilon)$$  \hspace{1cm} (18)$$

since \( dk = d\varepsilon/\hbar\). Note that \( \phi \) has dimension of length. The chemical potential entering the equilibrium distribution function \( f^0 \) will be assumed to be independent of temperature, equal to \( \varepsilon_F \), since the quantum wire under typical experimental conditions is in contact with a large reservoir of electrons.

The Boltzmann equation considered in this paper is an inhomogeneous integral equation of the form

$$X_{nk} = H\psi_{nk},$$  \hspace{1cm} (19)$$

where

$$X_{nk} = -\frac{e E}{k_B T} \nu_{nk} f^0_n (1 - f^0_n),$$  \hspace{1cm} (20)$$

while the integral operator \( H \) is defined by

$$\left( \frac{\partial f_{nk}}{\partial t} \right)_{\text{coll}} = -H\psi_{nk}$$  \hspace{1cm} (21)$$

with

$$H\psi_{nk} = \sum_{n'} \int_{-\infty}^{\infty} \frac{L}{2\pi} dk' K_{nn'}(k, k') (\psi_{nk} - \psi_{n'k'}).$$  \hspace{1cm} (22)$$

Here \( K \) is an integral kernel to be specified in later sections.

By introducing the scalar product \((A, B)\) through the definition

$$ (A, B) = \sum_n \int_{-\infty}^{\infty} \frac{dk}{2\pi} A_{nk} B_{nk} $$  \hspace{1cm} (23)$$

the conductivity may be written as

$$\sigma = \frac{2k_B T}{E^2} (X, \psi).$$  \hspace{1cm} (24)$$

In cases where we cannot solve explicitly for \( \psi \), it is useful to employ a variational principle, which yields a lower bound on \( \sigma \) by virtue of the Schwarz inequality

$$ (\psi, H\psi)(U, HU) \geq (U, H\psi)^2,$$  \hspace{1cm} (25)$$

where \( U \) is an arbitrary trial function. Since \( H\psi = X \), this gives a lower bound on the conductivity

$$\sigma \geq \frac{2k_B T}{E^2} \frac{(X, U)^2}{(U, HU)}.$$  \hspace{1cm} (26)$$
This lower bound will be used to determine the low-temperature conductivity when the scattering is due to acoustic phonons, both with and without impurities, and the resulting analytic expressions are compared to the results of the numerical solution of the integral equation.

We shall always write our calculated values of the conductivity, as limited by the various scattering mechanisms to be considered in subsequent sections, in the form

\[ \sigma = \frac{2e^2}{h} l, \]

where \( l \) has the dimension of a length. As may be expected, in the presence of impurity scattering \( l \) tends to a finite value as \( T \) tends to zero, while it increases exponentially in the absence of impurity scattering, when the electrons are scattered only by phonons.

**IV. IMPURITY SCATTERING**

In this section we consider the case of elastic impurity scattering treated recently in Ref. 9. Our aim is to show how the results of Ref. 9, which were obtained by use of the Kubo-formalism, are derived within the present framework. The collision integral is

\[ \left( \frac{\partial f_{nk}}{\partial t} \right)_{\text{coll}} = -\sum_{n'} \int_{-\infty}^{\infty} \frac{L}{2\pi} dk' w_{nn'}(k, k')(f_{nk} - f_{n'k'}), \]  

(28)

since the scattering is elastic. According to the Golden Rule we have

\[ w_{nn'}(k, k') = \frac{2\pi}{\hbar} \frac{1}{|\langle nk|V|n'k'\rangle|^2} \delta(\varepsilon_{nk} - \varepsilon_{n'k'}). \]  

(29)

The square of the matrix-element \( \langle nk|V|n'k'\rangle \) for scattering from impurities may be written as

\[ |\langle nk|V|n'k'\rangle|^2 = \int d^2r \int d^2r' V(r)V(r') \times \langle nk|r\rangle \langle r|n'k'\rangle \langle n'k'|r'\rangle \langle r'|nk\rangle. \]  

(30)

We shall perform an ensemble average over the distribution of impurities, corresponding to the replacement

\[ V(r)V(r') \rightarrow \langle V(r)V(r') \rangle = F(r, r'). \]  

(31)

When the impurity potential may be approximated by a delta-function in space, the function \( F \) is proportional to the delta-function \( \delta(r - r') \), corresponding to a “white-noise” model

\[ F(r, r') = A^2 n_{\text{imp}} \delta(r - r'). \]  

(32)

Here we have introduced the number of impurities per unit area \( n_{\text{imp}} \) together with the constant \( A \), which denotes the magnitude of the matrix-element for scattering from a single impurity. The dimension of \( A \) is that of an energy times an area.
In order to distinguish between states of equal energy but opposite sign of \( k \), we introduce the “branch-index” \( s = \pm 1 \).

\[
k_{ns}(\varepsilon) \equiv sk_n(\varepsilon) = s\sqrt{\frac{2m}{\hbar^2}(\varepsilon - (n + \frac{1}{2})\hbar\omega_0)},
\]

provided

\[
(n + \frac{1}{2})\hbar\omega_0 \leq \varepsilon.
\]

The density of states at the Fermi energy \( \varepsilon_F \) is inversely proportional to the \( s \)-independent Fermi velocity \( v_n(\varepsilon_F) = \frac{\hbar}{m}k_n(\varepsilon) \). Finally, since \( \phi_{ns} \) is odd in \( k \) and hence changes sign with \( s \), we may write \( \phi_{ns} = s\phi_n \). The problem then becomes that of solving the matrix equation

\[
s = \sum_{n's'} B_{ns}^{n's'} (s\phi_n - s'\phi_{n'}),
\]

and calculating the conductivity according to Eq. (18). The matrix \( B_{ns}^{n's'} \) is positive and symmetric with respect to interchange of \( ns \) with \( n's' \). Its elements are

\[
B_{ns}^{n's'} = \frac{1}{\hbar^2}A^2n_{imp} \frac{1}{v_nv_{n'}} \frac{1}{\ell_h}\sqrt{2\pi}F_{ns}^{n's'}.
\]

Here we have defined the following dimensionless quantities

\[
F_{ns}^{n's'} = \sqrt{2\pi} \ell_h \int_{-\infty}^{\infty} dy |u_n(y - y_{k_{ns}})|^2 |u_{n'}(y - y_{k_{n's'}})|^2.
\]

In Fig. 2 we show the result of solving the matrix equation (35) and calculating the conductivity according to Eq. (18) as a function of the Fermi energy \( \varepsilon_F \), for different choices of the parameter \( \gamma \) determining the strength of the confining potential. The pronounced structure is due to the opening of new “channels” for scattering each time the Fermi energy exceeds \( (n+1/2)\hbar\omega_0 \). At finite temperature this structure is somewhat smeared, as evidenced by the figure, but still clearly visible.

When \( \hbar\omega_0/2 \leq \varepsilon_F \leq 3\hbar\omega_0/2 \) only the lowest Landau-level is involved, corresponding to \( n = n' = 0 \) in Eq. (35). Then the solution becomes particularly simple, \( \phi_0 = 1/2B_{0+}^{0+} \). By means of the Sommerfeld-expansion we obtain the following low-temperature expression for the conductivity \( \sigma = 2e^2/\hbar B_{0+}^{0+} \),

\[
\sigma = \frac{2e^2}{\hbar} \frac{\hbar^2}{A^2n_{imp}} \frac{h^2k_F^2}{m^2} \exp \left( \frac{2k_F^2\ell_h^2/[1 + \gamma]}{\hbar\omega_0} \right)
\times \left[ 1 + \frac{8\pi^2}{3\gamma^2} \left( 1 + \frac{1 + \gamma}{k_F^2\ell_h^2} \right) \left( \frac{k_B T}{\hbar\omega_0} \right)^2 \right],
\]

where \( k_F = k_{0+}(\varepsilon_F) \) is the Fermi momentum. The conductivity increases with temperature because electrons are excited into states which have a smaller overlap with the states at the opposite branch. Consequently the electrons have a smaller back-scattering probability. The
result (38) is valid only to lowest order in \((k_B T/\bar{\hbar}\omega_h)^2\). In obtaining our numerical results we do not use the Sommerfeld-expansion, but calculate directly the integral over energy according to the expression (38).

As an illustration of the matrix-inversion involved in the solution of the Boltzmann equation we shall also give the expression for the zero-temperature conductivity in the case when \(3\bar{\hbar}\omega_h/2 \leq \varepsilon_F \leq 5\bar{\hbar}\omega_h/2\). By utilizing the symmetry \(\phi_{n+} = -\phi_{n-}\) the \(4 \times 4\) matrix-equation becomes a \(2 \times 2\) equation which may readily be solved for \(\phi_{0+}\) and \(\phi_{1+}\). The resulting conductivity \(\sigma = (2e^2/h)2(\phi_{0+} + \phi_{1+})\) is at \(T = 0\ K\) given by

\[
\sigma = \frac{2e^2}{h} \frac{2(2B_{0+}^{1+} + B_{1+}^{1-} + B_{0+}^{0-})}{2(B_{0+}^{1+}B_{0+}^{1-} + B_{0+}^{0+)B_{0+}^{1-}} + (B_{0+}^{1+} + B_{0+}^{0+})(B_{0+}^{0-} + B_{1+}^{1-})}, \tag{39}
\]

The result (39) yields the conductivity at zero temperature, when the Fermi energy \(\varepsilon_F\) satisfies the condition \(3\bar{\hbar}\omega_h/2 \leq \varepsilon_F \leq 5\bar{\hbar}\omega_h/2\), in agreement with Ref. 9. The generalization to more Landau-levels is straightforward. The \(B\)-matrix in Eq. (35) is in general a \(2(n_0 + 1) \times 2(n_0 + 1)\) matrix given by Eq. (36), where \(n_0\) is the quantum number associated with the highest occupied Landau-level at \(T = 0\ K\).

It is convenient to relate the conductivity calculated in the following two sections to the zero-temperature conductivity \(\sigma_{2D}\) associated with motion in two dimensions in the absence of a magnetic field and a confinement potential. The latter is given by

\[
\sigma_{2D} = \frac{e^2n_{2D}}{m^*} \tau_{imp} = en_{2D}\mu_{imp}, \tag{40}
\]

where \(n_{2D}\) is the electron sheet density and \(\mu_{imp}\) is the zero-field, zero-temperature mobility,

\[
\mu_{imp} = \frac{e\tau_{imp}}{m^*}; \tag{41}
\]

with

\[
\frac{1}{\tau_{imp}} = \frac{m^* A^2 n_{imp}}{\hbar^3}. \tag{42}
\]

We shall thus express our calculated conductivity in units of \((2e^2/h)l_{imp}\), where

\[
l_{imp} = \tau_{imp}\omega_c l_c \tag{43}
\]

is a characteristic length which depends on the impurity content as well as the magnetic field.

**V. ELECTRON-PHONON SCATTERING**

Next we turn to the consideration of inelastic processes. The present section treats the case where the electrons are scattered by acoustical phonons due to the combined deformation-potential and piezoelectric coupling, while the following section discusses the coupling of electrons to optical phonons.
Before specializing to a particular model let us write down the contribution to the collision operator from the scattering against phonons. The phonons are assumed to be in thermal equilibrium. Then the integral operator $H$ is given by the following expression

$$H\psi_{nk} = \frac{2\pi}{\hbar} \sum_{n'} \sum_{k'} \sum_{q} |g_q|^2 f^0(\varepsilon_{nk})(1 - f^0(\varepsilon_{n'k'}))\delta_{k,k'+q_x}(\psi_{nk} - \psi_{n'k'})$$

$$\times \{ \delta(\varepsilon_{n'k'} - \varepsilon_{nk} + \hbar \omega_q)(1 + N^0(\omega_q)) + \delta(\varepsilon_{n'k'} - \varepsilon_{nk} - \hbar \omega_q)N^0(\omega_q) \}.$$  (44)

Here $g_q$ is the electron-phonon coupling parameter, which will be specified below, while $N^0(\omega_q)$ is the equilibrium phonon distribution. For simplicity we have assumed that the phonon frequencies only depend on the magnitude of $q$. In Eq. (44) we have explicitly written the factor $\delta_{k,k'+q_x}$ discussed in the appendix. Utilizing the relationship

$$f(\varepsilon)(1 - f(\varepsilon \pm \hbar \omega_q)) = (f(\varepsilon) - f(\varepsilon \pm \hbar \omega_q))(1 + N^0(\pm \omega_q)),$$  (45)

the collision integral (44) can be written as

$$H\psi_{nk} = \frac{2\pi}{\hbar} \sum_{n'} \sum_{q} |g_q|^2 \left| \frac{f^0(\varepsilon_{nk}) - f^0(\varepsilon_{nk} + \sigma \hbar \omega_q)}{4 \sinh^2(\hbar \omega_q/2k_BT)} \right|$$

$$\times (\psi_{nk} - \psi_{n'k-q_x})\delta(\varepsilon_{n'k-q_x} - \varepsilon_{nk} - \sigma \hbar \omega_q),$$  (46)

with $\sigma$ assuming the two values $+1$ and $-1$ corresponding to phonon absorption and emission respectively.

Our calculations of $g_q$ are based on the standard electron-phonon interaction Hamiltonian, which disregarding umklapp processes takes the form

$$\hat{H}_{ep} = \sqrt{\frac{\hbar}{2V\rho}} \sum_{\lambda,q} M_\lambda(q)\hat{\rho}(q)(\hat{a}_q + \hat{a}^\dagger_{-q}),$$  (47)

where $\rho$ is the ion mass density, $V$ the normalization volume, $\hat{\rho}(q)$ the Fourier-component of the electron density operator, $\hat{a}_{\pm q}$ a phonon creation operator, and $\lambda$ the polarization index. The coupling function $M$ is given by

$$M_\lambda(q) = -V(q) \cdot \xi_\lambda,$$  (48)

$V(q)$ being the electron-ion potential and $\xi_\lambda$ a unit vector describing the polarization $\lambda$. For a detailed treatment of electron-phonon coupling in semiconductors see e.g. Ref. 19. The values of the GaAs parameters used in our calculations are listed in Table I.

A. Coupling to acoustical phonons

In GaAs-heterostructures at low temperatures the electron-phonon scattering is mainly due to the combined piezoelectric coupling and the deformation-potential coupling. Below we briefly sketch how these couplings are derived.

The deformation potential coupling only involves the longitudinal acoustical (LA) phonons. The coupling to the transverse acoustical (TA) phonons is suppressed by the square
of the ratio between the speed of sound and the speed of light. In the long-wavelength limit the coupling function $M^{df}$ for the deformation-potential coupling is written as

$$M^{df} = \Xi q,$$

(49)

where $\Xi$, known as the deformation potential, is the zero-wave-vector limit of $V(q)$.

The electron-ion potential $V$ for the piezoelectric coupling is found from the basic piezoelectric equations. For GaAs (zinc-blende structure) the only non-vanishing independent piezoelectric constant is $h_{14}$ (reduced notation), and the coupling function $M^{pz}_\lambda$ in this case becomes

$$M^{pz}_\lambda = i2e h_{14} (\hat{q}_x \hat{\xi}_{\lambda,x} + \hat{q}_y \hat{\xi}_{\lambda,y} + \hat{q}_z \hat{\xi}_{\lambda,z}),$$

(50)

where $\hat{q}_i = (q/q)_i$ and $\hat{\xi}_{\lambda,i} = (\xi_\lambda)_i$. In the piezoelectric case $\lambda$ is retained.

It is noted that $M^{df}$ is real while $M^{pz}$ is imaginary; thus to second order the two terms do not interfere, and the absolute square of the total coupling function $M^{ac}$ is given by

$$|M^{ac}|^2 = |M^{df}|^2 + |M^{pz}|^2.$$  

(51)

To obtain a more tractable form of the piezoelectric coupling we perform angular averages for the longitudinal and the (two) transverse modes separately and then add the terms. While this represents an approximation compared to retaining the full $q$-dependence of the coupling in the collision integral, we expect it to involve only minor quantitative differences. In adding the transverse and longitudinal contribution we must remember the different average sound velocities associated with each of the terms, originating in the factor $1/\sqrt{\omega_q} = 1/\sqrt{cq}$ in Eq. (47). Expressing the transverse sound velocity as $x$ times the longitudinal sound velocity we obtain the following angular average of the absolute square of the piezoelectric coupling function $M^{pz}$:

$$|M^{pz}|^2 = (eh_{14})^2 \left( \frac{12}{35} + \frac{116}{x35} \right) \equiv P,$$

(52)

where we have introduced the constant $P$, and where it is understood that the only sound velocity appearing in the following is the longitudinal one.

The electron-phonon coupling parameter $g_q$ introduced in Eq. (44) now becomes

$$|g^{ac}(q)|^2 = \frac{\h}{2\rho Vc} \frac{1}{q} (\Xi^2 q^2 + P) \langle nk |e^{iq\cdot r}|n'k'\rangle|^2.$$

(53)

The matrix element appearing in this expression is treated in detail in the appendix. In writing Eq. (53) we have taken the phonon frequency $\omega_q$ to be given by

$$\omega_q = cq.$$

(54)
B. Coupling to acoustical phonons in the low-temperature limit

In an analytical study of the low-temperature limit we use the variational principle discussed in section 3 above and choose a trial function given by

\[ U = U_{nk} = \text{sgn}(k). \tag{55} \]

Thus \( U \) is 1 on the branches corresponding to \( k \) being positive, while it is \(-1\) on the branches corresponding to \( k \) being negative. This choice will lead to an expression for the conductivity which in the case of a single occupied Landau-level and in the limit of low temperature agrees with a numerical solution of the integral equation. This suggests that the trial function is in fact the exact one under these conditions.

First we evaluate the scalar product \( (U, X) \) appearing in the general expression (26) for the lower bound on the conductivity. By changing the integration variable to the energy \( \varepsilon_{nk} \) we get

\[ (X, U) = \frac{eE}{\hbar \pi} \sum_n f^0(\varepsilon_{n0}), \tag{56} \]

where \( \varepsilon_{n0} \) is the \( k = 0 \) value of \( \varepsilon_{nk} \). Because of the symmetry of the integral operator \( H \) given in Eq. (46) the denominator occurring in Eq. (26) becomes

\[ (U, HU) = \sum_n \int_{-\infty}^{\infty} \frac{dk}{2\pi} \int_{-\infty}^{\infty} \frac{dk'}{2\pi} \sum_{n'} K_{nn'}(k, k') \times \frac{1}{2} [1 - \text{sgn}(k)\text{sgn}(k')]. \tag{57} \]

As discussed in the appendix \( k' = k - q_x \). The occurrence of the factor \([1 - \text{sgn}(k)\text{sgn}(k - q_x)]\) in the integrand of Eq. (57) implies that the summation over \( q_x \) becomes restricted by \( q_x > k \) for \( k > 0 \), while \( q_x < k \) for \( k < 0 \). Using the symmetry with respect to reversal of all momenta variables we may thus restrict the \( k \)-integral to the interval \( 0 \) to \( \infty \), which limits \( q_x \) to the region \( q_x > k \). If \( \theta \) denotes the angle between \( q \) and the \( x \)-axis, the integration over \( \theta \) is therefore restricted by

\[ \cos \theta > \frac{k}{q}, \quad 0 < q < \infty. \tag{58} \]

Thus we have to carry out three integrations (over \( q \), \( k \) and \( \cos \theta \)) as well as two sums (over \( n \) and \( n' \)).

Let us consider the simplest case, in which \( n = n' = 0 \). If the sound velocity is much less than the Fermi-velocity then the energy-conserving delta-functions are

\[ \delta(\varepsilon_{n'k'} - \varepsilon_{nk} \pm \hbar \omega_q) \simeq \delta\left(\frac{\hbar^2}{2m} (q^2 \cos^2 \theta - 2kq \cos \theta)\right). \tag{59} \]

The \( q \)-integral is now restricted to the interval \( q > 2k \), while \( k/q < \cos \theta < 1 \). This yields, after performing the \( \phi \)-integral as shown in the appendix,
\[(U, HU) = \frac{1}{4\pi^3\hbar^2 c \rho} \int_0^\infty \frac{d\varepsilon}{v(\varepsilon)^2} \int_{2k}^\infty dq \left( \Xi^2 q^2 + P \right) \]
\[\times I_{0,0}(q, 2k/q) \frac{|f^0(\varepsilon_{nk}) - f^0(\varepsilon_{nk} + \sigma \hbar \omega_q)|}{4 \sinh^2 (\hbar \omega_q / 2k_B T)}, \quad (60)\]

where \(I_{0,0}(q, 2k/q)\) is given by Eq. (A.5). We may carry out the final integration over energy by expanding the difference of the Fermi-functions in powers of \(\omega_q\). Since the contribution due to terms involving higher order derivatives of the Fermi function is seen to vanish by the use of partial integration, we obtain

\[(U, HU) = \frac{1}{2\pi^3\hbar^2 c \rho} \int_0^\infty \frac{d\varepsilon}{v(\varepsilon)} \left( -\frac{\partial f^0}{\partial \varepsilon} \right) \]
\[\times \int_{2k}^\infty dq \left( \Xi^2 q^2 + P \right) I_{0,0}(q, 2k/q) \hbar \omega_q \]
\[\frac{\hbar \omega_q}{4 \sinh^2 (\hbar \omega_q / 2k_B T)}. \quad (61)\]

The remaining steps are standard. We assume that the temperature is much less than the Fermi temperature, so that the integral over \(\varepsilon\) yields \(1/v_F^2\), while \(k\) may be replaced by the Fermi momentum \(k_F\). Furthermore, we assume that the temperature is small compared to the characteristic temperature \(\Theta\) defined by

\[k_B \Theta = \hbar k_F c. \quad (62)\]

Then we obtain

\[(U, HU) = \frac{8}{\pi^2} \frac{m^2 k_F k_B T}{\rho c \hbar^4} \left( \Xi^2 + \frac{P}{4k_F^2} \right) \]
\[\times e^{-2k_F^2 \ell_\sigma^2/(1+\gamma)} e^{-2\Theta/T}. \quad (63)\]

This results in the final conductivity expression

\[\sigma = \frac{2e^2}{h} l_\sigma, \quad (64)\]

where the length \(l_\sigma\) is given by

\[l_\sigma = \frac{\pi c \hbar^3 \rho k_F}{m^2 (4k_F^2 \Xi^2 + P)} e^{2k_F^2 \ell_\sigma^2/(1+\gamma)} e^{2\Theta/T}. \quad (65)\]

At low temperatures the conductivity thus increases exponentially, in agreement with the result of the numerical calculation (see Fig. 3). In the presence of impurities, as we shall see in the following subsection, the scattering against phonons yields a contribution to the inverse conductivity which is proportional to \(T^3\) rather than \(\exp(-2\Theta/T)\), provided the temperature is sufficiently low that the impurities dominate the scattering.
C. Scattering from acoustical phonons and impurities

When the impurity scattering dominates we may use the variational principle with a trial function which is proportional to the solution for impurity scattering alone. We shall consider the case where only the lowest Landau level, \( n = 0 \), is important. The trial function is thus chosen to be proportional to \( 1/B_0^- \) as given by Eq. (36),

\[
U(k) = k^2 \exp \left( \frac{2k^2l_0^2}{[1 + \gamma]} \right),
\]

(66)

Since we have chosen the exact solution to the impurity problem as our trial function, the calculated upper bound on the contribution from electron-phonon scattering, \( 1/\sigma_{ph} \), is exact, to lowest order in the magnitude of the electron-phonon coupling. At low temperatures we may make the approximation

\[
(U(k) - U(k'))^2 \approx \left( \frac{dU(k)}{dk} \right)^2 (k - k')^2,
\]

(67)

which is justified, since \( k - k' = q_x \) and the restriction to low temperatures implies that the phonon momentum is small. Furthermore, we may neglect the deformation-potential coupling since the piezoelectric coupling dominates for small \( q \) according to Eq. (53), and also set the matrix element appearing in Eq. (53) equal to unity. By inserting \( U \) in Eq. (26) and carrying out the integrals we obtain

\[
\sigma_{ph} = \frac{h}{2e^2} l_{ph}^{-1} \frac{3\zeta(3)}{8\pi} \frac{1 + \gamma}{\gamma \bar{e}_F^3} \left( 1 + \frac{4\bar{e}_F}{\gamma} \right)^2 \left( \frac{k_B T}{\hbar \omega_h} \right)^3,
\]

(68)

where \( \bar{e}_F = \varepsilon_F / \hbar \omega_h \), and

\[
l_{ph}^{-1} = \frac{Pm^*}{\rho e^2 \hbar^2}.
\]

(69)

This \( T^3 \)-behavior agrees well with our numerical calculations in the parameter range where it is expected to apply. In Figs. 3 and 4 we show the calculated conductivity, obtained by numerical solution of the integral equation, for samples with different amounts of impurities.

VI. SCATTERING FROM OPTICAL PHONONS

Next we investigate the combined effects of scattering by longitudinal optical phonons and impurities. The chief differences from the previous section involve the \( q \)-dependence of the electron-phonon matrix-element and the absence of dispersion in the phonon frequencies. The conductivity is obtained by solving the Boltzmann equation in the limit where the temperature is much less than \( \hbar \omega_0 / k_B \).
A. Coupling to optical phonons

We shall use the simple model where the phonon frequency is independent of momentum,

\[ \omega_q \simeq \omega_0. \]  

(70)

The electron-phonon matrix element is given by

\[ |g(nn', q)|^2 = \frac{1}{V} \frac{M_0^2}{q^2} M_n^{n'} (u), \]  

(71)

where \( V \) is the normalization volume and where

\[ M_0^2 = \frac{2}{\pi} \varepsilon \frac{e^2 h \omega_0}{\varepsilon_0} \left( \frac{1}{\kappa_\infty} - \frac{1}{\kappa_0} \right). \]  

(72)

The function \( M \) is given by

\[ M_n^{n'} (u) = \left| \langle nk | e^{i \mathbf{q} \cdot \mathbf{r}} | n'k' \rangle \right|^2, \]  

(73)

which is calculated in the appendix, where also \( u \) is defined.

The approximation in Eq. (70) allows us to integrate over the \( y \) and \( z \) components of the phonon momentum. Using a delta-function for the wave-function in the \( z \)-direction, we get

\[ \sum_{q_y q_z} |g(nn', q)|^2 = \frac{M_0^2}{(2\pi)^2 L} K_n^{n'} (q_x), \]  

(74)

where

\[ K_n^{n'} (q_x) = \int_{-\infty}^{\infty} \frac{dq_y}{\sqrt{q_x^2 + q_y^2}} M_n^{n'} (u(q_x, q_y)). \]  

(75)

B. Scattering from optical phonons and impurities

We now consider the case of impurity and optical phonon scattering. The Boltzmann equation in this case reads

\[ \frac{eE}{k_B T} v_{nk} f_0^{0} (1 - f_{nk}^{0}) = \left( \frac{\partial f_{nk}}{\partial t} \right)^{\text{imp}} + \left( \frac{\partial f_{nk}}{\partial t} \right)^{\text{coll}}. \]  

(76)

The collision integral for the phonon scattering given in Eq. (46) simplifies in the case of optical phonons to

\[ \left( \frac{\partial f_{nk}}{\partial t} \right)^{\text{coll}} = -\frac{M_0^2}{\hbar 2\pi L 4 \sinh^2 (\hbar \omega_0 / 2k_B T)} \sum_{n'q_x} \sum_{\sigma} K_n^{n'} (q_x) \left| f_{nk}^0 (\varepsilon_{nk}) - f_{nk}^0 (\varepsilon_{nk} + \sigma \hbar \omega_0) \right| \]  

\[ \times (\psi_{nk} - \psi_{n'k-q_x}) \delta(\varepsilon_{n'k-q_x} - \varepsilon_{nk} - \sigma \hbar \omega_0), \]  

(77)
with \( K_n'(q_x) \) given in Eq. (72). The argument of the energy-conserving delta-function is zero for

\[
q_x = k \left( 1 - s \sqrt{1 + \frac{\hbar \omega_h (n - n') + \sigma \hbar \omega_0}{\hbar^2 k^2/2m}} \right),
\]

(78)

for each value of \( nk \), provided the square root is real. The branch index \( s \) assumes the values +1 and -1 corresponding to forward and backward scattering. The Boltzmann equation now simplifies to

\[
1 = \sum_{n's'} B_{n+s'}^{n's'}(\varepsilon)(\phi_n(\varepsilon) - s'\phi_{n'}(\varepsilon)) + \sum_{n's'} C_{n+s'}^{n's'}(\varepsilon, \sigma)(\phi_n(\varepsilon) - s'\phi_{n'}(\varepsilon + \sigma\hbar\omega_0)),
\]

(79)

where the matrix \( B \) is given by Eq. (36). The optical-phonon scattering gives rise to the matrix \( C \), which is given by

\[
C_{n's'}^{n's'}(\varepsilon, \sigma) = \frac{1}{f^0(\varepsilon)[1 - f^0(\varepsilon)]} \frac{M_0^2}{\hbar^2(2\pi)^2} \frac{1}{4 \sinh^2(\hbar \omega_0/2k_B T)} \times \sum_{\sigma} \left| f^0(\varepsilon) - f^0(\varepsilon + \sigma\hbar\omega_0) \right| \frac{K_{n'}(sk_n(\varepsilon) - s'k_{n'}(\varepsilon + \sigma\hbar\omega_0))}{v_n(\varepsilon)v_{n'}(\varepsilon + \sigma\hbar\omega_0)}.
\]

(80)

In GaAs the typical optical phonon energies are about 36 meV. Furthermore, since the cyclotron energy \( \hbar \omega_c \) in GaAs is 1.5 meV \( B/T \), the typical situation will be that the energy spacing between the Landau levels as modified by the confinement potential, \( \hbar \omega_h \), is much smaller than the phonon energies, \( \hbar \omega_0 \gg \hbar \omega_h \). Since we are interested in temperatures that are small or comparable in magnitude to the energy spacing \( \hbar \omega_h \), we are therefore always in a situation where \( k_B T \) is much less than \( \hbar \omega_0 \). Under these circumstances it is possible to simplify the Boltzmann equation for electrical transport by considering it to be a coupled system of equations for the functions \( \phi(\varepsilon \pm n\hbar\omega_0) \) with \( n \) integer. Since the current is mainly determined by the distribution of electrons \( \phi(\varepsilon) \) within a thermal layer of thickness comparable to \( k_B T \), it is sufficiently accurate to neglect the contributions from \( \phi(\varepsilon \pm n\hbar\omega_0) \), when \( n \) is different from zero. The collision integral may therefore be approximated by the “scattering-out” term, corresponding to the neglect of vertex-corrections in the Kubo-approach.

In calculating the conductivity from Eq. (79) we thus neglect the “scattering-in” terms involving \( \phi_{n'}(\varepsilon + \sigma\hbar\omega_0) \), while retaining the full collision matrix for the impurities. We have then evaluated the solution of Eq. (79) numerically and inserted the solution in the conductivity formula (18). Results are shown in Fig. 5. The low-temperature behavior is dominated by the impurities, which give rise to an initial increase of the conductivity, in accordance with Eq. (88). The optical phonons come in at higher temperatures, yielding a maximum of the conductivity as a function of temperature. The position of the maximum is roughly proportional to the logarithm of the strength of the impurity scattering, since the phonon contribution depends exponentially on temperature. Note however that the acoustical phonons have reduced the conductivity for the high mobility cases rather strongly.
at the temperature range shown here, see Fig. 4. The dashed line represents the conductivity without optical phonon scattering.

In comparing the curves Figs. 3-5 we observe that the optical phonon scattering in clean systems tends to dominate the total scattering already at fairly low temperatures, around 50 K. This contrasts the situation in typical GaAs-based two-dimensional electron gases\textsuperscript{22}, where the optical phonon scattering begins to dominate around 100 K.

VII. CONCLUSION

We have shown that the magnetoconductivity of quantum wires may be discussed in a simple and unified fashion within the framework of a Boltzmann equation, by taking into account the influence of the magnetic field on the electron group-velocity and the matrix-elements governing the transition probability. By treating in detail the scattering from acoustical as well as optical phonons in the presence of impurity scattering, we have determined the temperature dependence of the magnetoconductivity for realistic choices of parameters in GaAs-based structures. In particular, we have found that the magnetoconductivity exhibits a maximum as a function of temperature, depending on the relative strength of the impurity and electron-phonon scattering. The calculated magnetoconductivity oscillates when the Fermi energy or the magnetic field is varied. Our detailed calculations show that the scattering against optical phonons in quantum wires is significant at temperatures somewhat smaller than the corresponding temperatures for the two-dimensional case.

In order to relate our theoretical results to experiment\textsuperscript{24–26} it is necessary to consider not only the diagonal conductivity element $\sigma_{xx}$, but also $\sigma_{xy}$. To compare with experiment one must therefore either determine $\sigma_{xy}$ theoretically or compare our calculated $\sigma_{xx}$ to simultaneous measurements of $\rho_{xx}$ and $\rho_{xy}$. Alternatively, our results could be compared to experiments that use a Corbino-type geometry. The effects predicted in this paper should be observable in quantum wires of sufficient purity, since otherwise the electron density may vary considerably due to fluctuations in the electrostatic potential from the donors.

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APPENDIX:

In this appendix we study the square of the matrix element $\langle nk | \exp( i \mathbf{q} \cdot \mathbf{r} ) | n' k' \rangle$. Using the single electron wave functions in Eq. (2) and performing the $x$- and $y$-integrals one obtains

$$|\langle nk | \exp( i \mathbf{q} \cdot \mathbf{r} ) | n' k' \rangle|^2 = \delta_{k,k'} \frac{1}{n_{\text{max}}!} \frac{n_{\text{min}}!}{n_{\text{max}}!} \left| \left[ L_{n_{\text{min}}} (u) \right] \right|^2 e^{-u},$$

(A1)
where \( n_{\text{min}} = \min(n, n') \) and \( n_{\text{max}} = \max(n, n') \); \( L_m^m(u) \) are the Laguerre polynomials while \( u \) is given by

\[
u = \frac{1}{1 + \gamma^2} \left( \frac{q_x \ell_h}{2} \right)^2 + \frac{1}{2} (q_y \ell_h)^2.
\]

(A2)

The Kronecker delta appearing in Eq. (A1), and which is written explicitly in the expression for the integral operator \( H \) in Eq. (44), is a consequence of the translational invariance along the \( x \)-direction.

In calculations involving acoustical phonons it is natural to use the polar coordinates \((q, \theta, \phi)\) with the \( x \)-axis as the polar axis so that \( q_x = q \cos \theta \), \( q_y = q \sin \theta \cos \phi \), and \( q_z = q \sin \theta \sin \phi \). According to Eqs. (44) and (53) the only \( \phi \)-dependence is through \( u \) defined above. It is therefore of interest to calculate the integral

\[
I_{n,n'}(q, \cos \theta) \equiv \int_0^{2\pi} d\phi \, u^{n-n'} \left[ L_{\min}^{n-n'} \right] e^{-u}.
\]

(A3)

From Eq. (A2) it follows that the \( \phi \)-dependence of \( u \) is of the form \( u = \beta + \alpha \cos^2 \phi \), so the integrand of \( I_{n,n'} \) therefore takes the form of a polynomial in \( \cos^2 \phi \) times \( \exp(-\alpha \cos^2 \phi) \). The integral over \( \phi \) may then be carried out explicitly in terms of the beta-function and Gauss’ hypergeometric function.

In the case where we consider only the lowest Landau level we just need to calculate \( I_{0,0} \). Using the results quoted above we find

\[
I_{0,0}(q, q_x/q) = 2\pi \exp \left( -\frac{1}{2} (q_x \ell_h)^2 \frac{1}{1 + \gamma} - \frac{\ell_h^2}{4} (q^2 - q_x^2) \right),
\]

\[
\times I_0 \left( \frac{1}{4} \ell_h^2 (q^2 - q_x^2) \right)
\]

(A4)

where \( I_0(x) \) is the \( 0^{th} \) Bessel function of imaginary argument.
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TABLES

TABLE I. The GaAs constants used in this paper. Unless otherwise indicated the values are taken from Ref. 27.

| Parameter                        | Symbol | Value            |
|----------------------------------|--------|------------------|
| ion mass density                 | $\rho$ | $5.3 \times 10^3$ kg m$^{-3}$ |
| longitudinal sound velocity      | $c$    | $5.2 \times 10^3$ m s$^{-1}$ |
| transverse sound velocity        | $x_c$  | $3.0 \times 10^3$ m s$^{-1}$ |
| sound velocity ratio             | $x$    | 0.58             |
| static dielectric constant       | $\kappa_0$ | 12.8             |
| high-frequency dielectric constant | $\kappa_\infty$ | 10.6             |
| piezoelectric constant, Eq. 52   | $h_{14}$ | $1.38 \times 10^9$ V m$^{-1}$ |
| piezoelectric coupling, Eq. 52   | $P$    | $5.4 \times 10^{-20}$ J$^2$ m$^{-2}$ |
| deformation potential, Eq. 52    | $\Xi$  | $2.2 \times 10^{-18}$ J |
| effective electron mass          | $m^*$  | 0.067 $m_0$      |
| optical phonon energy            | $\hbar\omega_0$ | 36 meV          |
FIGURES

FIG. 1. The two lowest $k$-dependent energy bands $\varepsilon_0k$ and $\varepsilon_1k$ of the quantum wire. The dashed line represents an arbitrary Fermi level above the bottom of the second band.

FIG. 2. The conductivity for a quantum wire with impurity scattering plotted versus the Fermi level for three different choices of the confinement potential corresponding to $\gamma = 1, 0.5,$ and $0.25$. The dashed line is the $T = 0$ result and the solid lines correspond to $k_B T = 0.05 \hbar \omega_h$. The magnetic field is $9$ T.

FIG. 3. Plots of the normalized conductivity versus temperature for a GaAs quantum wire at the magnetic field $B = 9$ T, the confinement parameter $\gamma = 0.5$, and the Fermi level $\varepsilon_F = 0.6 \hbar \omega_h$. Taking into account both impurity scattering, Eq. (35), and acoustical phonon scattering, Eqs. (46) and (53), the five dotted curves, $\sigma_1, \ldots, \sigma_5$, are numerical results for the zero-field, zero-temperature mobilities, $\mu_{\text{imp}} = 7.5, 75, 300, 3000,$ and $30000$ m$^2$/Vs respectively. The last two rather unrealistic high mobilities are considered to allow a study of the transition from impurity-dominated scattering to phonon-dominated scattering at temperatures low enough ($T < \Theta \simeq 4.0$ K – see Eq. (62)) for the approximative result Eq. (64) to apply. The two full curves, $\sigma_{a4}$ and $\sigma_{a5}$, are plots of the conductivity (rescaled to match $\sigma_4$ and $\sigma_5$) calculated from Eq. (64) where only the acoustical phonon scattering is present. The full curve $\sigma_{\text{imp}}$ is the case where only impurity scattering is present. The two full curves, $\sigma_{t4}$ and $\sigma_{t5}$, are the results of assuming that the inverse conductivities for each scattering mechanism considered separately may be added, $\sigma_{tj} = \sigma_{aj} \sigma_{\text{imp}} / (\sigma_{aj} + \sigma_{\text{imp}})$ ($j = 4, 5$), to approximate the exact numerical calculations of the total conductivity.

FIG. 4. Plots of the normalized conductivity versus temperature for a GaAs quantum wire at the magnetic field $B = 9$ T and the Fermi level $\varepsilon_F = 0.6 \hbar \omega_h$. The confinement parameter in panel (a) is $\gamma = 1.0$ and in (b) it is $\gamma = 0.5$. The dashed curve in each panel is the case where only impurity scattering is taken into account. The four full curves in each panel is the result of combining impurity and acoustical phonon scattering for each of the following values of the zero-field, zero-temperature mobility, $\mu_{\text{imp}} = 0.9, 9, 90, \text{ and } 900$ m$^2$/Vs.

FIG. 5. The conductivity with combined impurity and optical phonon scattering for a GaAs quantum wire. The normalized conductivity is plotted versus temperature for different choices of confinement potential strengths, Fermi energies and mobilities. The magnetic field is $B = 9$ T. The confinement parameter is $\gamma = 1$ for panel (a) and (c) and $\gamma = 0.5$ for (b) and (d); the Fermi level is $\varepsilon_F = 0.6 \hbar \omega_h$, i.e., close to the bottom of the first band, for (a) and (b), and panel (c) and (d) have $\varepsilon_F = 1.3 \hbar \omega_h$. The conductivity is shown for each of the following values of the zero-field, zero-temperature mobility, $\mu_{\text{imp}} = 0.9, 9, 90, \text{ and } 900$ m$^2$/Vs. The dashed curve in each panel is the case where only impurity scattering is taken into account.