Chaotic fluctuation of temperature on environmental interface exchanging energy by visible and infrared radiation, convection and conduction

D.T. Mihailović *
Faculty of Agriculture, University of Novi Sad, Try D. Obradovića 8, 21 000 Novi Sad, Serbia

D. Kapor
Department of Physics, Faculty of Sciences, University of Novi Sad, Try D. Obradovića 4, 21 000 Novi Sad, Serbia

M. Budincević
Department of Mathematics, Faculty of Sciences, University of Novi Sad, Try D. Obradovića 4, 21 000 Novi Sad, Serbia
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The concept of environmental interface is defined and analyzed from the point of view of the possible source of non-standard behavior. The energy balance equation is written for the interface where all kinds of energy transfer occur. It is shown that under certain conditions, the discrete version of the equation for the temperature time rate turns in to the well-known logistic equation and the conditions for chaotic behaviour are studied. They are determined by the Lyapunov exponent. The realistic situation when the coefficients of the equation vary with time, is studied for the Earth-environment general system.

I. INTRODUCTION

The concept of "balance", either global or local in any given context, it is undoubtedly the cornerstone in the increasing number of environmental problems. The question is: Why are the environmental problems in the focus now? One particular answer can be found in a hierarchy of the main scientific problems in this century. According to the physicists, the world scientific community, will be occupied in this century among others, by the environmental problems that are primarily expressed through the problem of climate changes [1, 2] as well as many other linked problems that comprises a wide range of time and spatial scales. This is the first time in the history of science that the environmental problems take the place at the research front of the sciences. The question, why it is happening now and why it will go on happening in the future, could be answered by the well known fact that in the scientific as well as in other worlds the main "dramatic event" takes place at the interface between either two media or two states [3]. The field of environmental sciences is abundant with various interfaces and it is the right place for application of new fundamental approaches leading towards better understanding of environmental phenomena. We define the environmental interface as an interface between the two, either abiotic or biotic environments each, which are in a relative motion exchanging energy through biophysical and chemical processes and fluctuating temporally and spatially regardless of its space and time scale. In our opinion, this definition broadly covers the unavoidable multidisciplinary approach in environmental sciences and also includes the traditional approaches in sciences that are dealing with the environmental space less complex than any one met in reality. The wealth and complexity of processes at this interface determine that the scientists, as it often seems, are more interested in a possibility of non-linear "dislocations" and surprises in the behaviour of the environment than in a smooth extrapolation of current trends and a use of the approaches close to the linear physics [3 - 6]. To overcome the current situation we have to do the following: (a) establish a way in approaching the non-linear physics and the non-linearity in describing the phenomena in physics as well as environmental sciences, and (b) solve or, at least, understand the problem of predictability. These two problems are problems par excellence of the methodology. Their successful solving will help us avoid the current problems in mathematical, physical, biological and chemical interpretation of the nature. In the next section, we write down the equation for the energy balance at the environmental interface. We show that under certain conditions, its discrete version becomes well-known logistic equation leading to the chaotic behaviour of the temperature. The conditions for such behaviour are investigated and particular examples studied. The analysis is based on the Lyapunov exponent.

* e-mail: guto@polj.ns.ac.yu
II. ENERGY BALANCE EQUATION FOR ENVIRONMENTAL INTERFACE

We see the outside world, i.e. the world of phenomena (ambience) from the observer’s perspective (its inner world). In the ambience there are systems of different levels of complexity and their environments. System in the ambience is a collection of precepts while whatever lies outside, like the component of a set, constitutes the environment [7,8]. The “fate” of science lies in the fact that it is focused on the system [8]. Furthermore, to be able to anticipate something, we describe the system by the states (determined by observations) while the environment is characterized through its effects on the system. The environmental interface as a complex system is a suitable area for occurrence of the irregularities in temporal variation of some physical or biological quantities describing their interaction. For example, such interface can be placed between: human or animal bodies and surrounding air, aquatic species and water and air around them, natural or artificially built surfaces (vegetation, ice, snow, barren soil, water, urban communities) and atmosphere [4-6,9,10], etc. The environmental interface of different media was recently considered for different purposes [3-6,11-13]. In these systems visible radiation provides almost all of the energy received on the environmental interface. Some of the radiant energy is reflected back to space. The interface also radiates, in the thermal waveband, some of the energy received from the sun. The quantity of the radiant energy remaining on the environmental interface is the net radiation (the net radiation energy available on the surface when all inward and outward streams of radiation have been considered as seen in Fig. 1 where all kinds of energy are expressed in terms of the flux density), which drives certain physical processes important to us. The energy balance equation may be written as

\[ c_i \frac{dT_i}{dt} = R - H - E - S \]  \hspace{1cm} (2.1)

where \( c_i \) is the environmental interface soil heat capacity per unit area, \( T_i \) is the environmental interface temperature (EI), \( t \) is the time, \( H \) and \( E \) are the sensible and the latent heat, respectively, transferred by convection, and \( S \) is the heat transferred by conduction into deeper layers of underlying matter.

**Figure 1**

[Schematic diagram of terms included in the energy balance equation for environmental interface: (a) net radiation (\( R \)) that includes (1) visible radiation, (2) infrared radiation of underlying matter and (3) infrared counter radiation of the gas; (b) sensible(\( H \)) and latent heat (\( E \)) transferred by convection and (c) heat (\( S \)) transferred by conduction.]

The sensible heat is calculated as \( C_H(T_i - T_a) \) where \( C_H \) is the sensible heat transfer coefficient and \( T_a(t) \) is the gas temperature given as the upper boundary condition. The heat transferred into underlying soil material is calculated as \( C_D(T_i - T_d) \) where \( C_D \) is the heat conduction coefficient while \( T_d(t) \) is the temperature of deeper layer of underlying matter that is given as the lower boundary condition. Following Bhumralkar [14] the net radiation term in Eq. (1) can be represented as \( C_R(T_i - T_a) \) where \( C_R \) is the radiation coefficient. According to [15], for small differences of \( T_a \) and \( T_d \), the expression for the latent heat can be written in the form \( C_L f(T_a) \left[ b(T_i - T_a) + b^2(T_i - T_a)^2 \right]/2 \). Here \( C_L \) is the latent heat transfer coefficient, \( f(T_a) \) is the gas vapor pressure at saturation and \( b \) is a constant characteristic for a particular gas. Calculation of time dependent coefficients, and can be found in [16]. After collecting all terms in Eq. (1) we get

\[ \frac{dT_i}{dt} = A_1(T_i - T_a) - A_2(T_i - T_a)^2 - A_3(T_i - T_d) \]  \hspace{1cm} (2.2)

where \( A_1 = [C_R - C_H - C_L b f(T_a)]/c_i, \ A_2 = b^2 f(T_a)/(2c_i), \ \text{and} \ A_3 = C_D/c_i \). This is a non-linear Riccati type differential equation that practically always has to be solved numerically, i.e.,

\[ DT_i = F_n \]  \hspace{1cm} (2.3)

(3) where \( D \) is the finite difference operator defined as \( DT_i = (T_{i,n+1} - T_{i,n})/Dt \), \( n \) the time level, \( Dt \) is the time step and \( F_n \) is the r.h.s. of Eq. (2.2) defined at the \( n \)th time level.

III. NUMERICAL RESULTS ON CHAOS IN TEMPERATURE ON ENVIRONMENTAL INTERFACE

We consider Eq. (2.2) with the lower boundary condition, at some time interval, given in the form

\[ T_d = T_a - (c_i/C_D) \frac{dT_a}{dt} \]  \hspace{1cm} (3.1)
expressing slow temperature changes in both the environment and underlying material. Then equation becomes

$$\frac{d\xi}{dt} = A_0\xi - A_2\xi^2$$

(3.2)

where $\xi = T_i - T_a$, and $A_0 = A_1 - C_D/c_i$. Substituting the time derivative by the finite difference operator $D$ in this equation and after some transformations, we obtain

$$\Gamma_{n+1} = A_1^p\Gamma_n(1 - \Gamma_n)$$

(3.3)

where $\Gamma = (A_1^p/A_2^p)\xi$, $A_1^p = 1 + A_0Dt$ and $A_2^p = A_2Dt$. This equation has the same form as the well known logistic difference equation $\Gamma_{n+1} = \beta\Gamma_n(1 - \Gamma_n)$ where $\beta$ is a constant (see, e.g. [17], among others). Conditions for occurrence of deterministic chaos for logistic mapping given by this equation are: (a) $0 \leq \Gamma \leq 1$ and (b) $3.57 \leq \beta \leq 4$. Here we have to bear in mind that $A_1^p$ depends on discrete "time" $n$. However, chaos is still expected, although we can give no quantitative prediction of the parameter regime. We analyze Eq. (3.3) in the following way. With $Dt_p = 1/A_0$ we indicate the scaling time range of energy exchange at the environmental interface including coefficients, that express all kind of energy reaching and departing the environmental interface. For any chosen time interval, for solving Eq. (3.3), there always exists $Dt_{p,f} = Min[\{Dt_p(c_i,C_{IR},C_{HR},C_{HL})\}]$ when energy at the environmental interface is exchanged in the fastest way by radiation, convection and conduction. If we define dimensionless time $\tau = Dt/Dt_{p,f}$, then Eq. (3.3) becomes

$$\Gamma_{n+1} = (1 + \tau)\Gamma_n(1 - \Gamma_n).$$

(3.4)

Regarding to order of magnitude of parameters included in coefficients $A_1^p$ and $A_2^p$, the condition (a) is always satisfied. From (b) we get the interval $(\beta = 1 + \tau)$ where the solution is chaotic, i.e., $2.57 \leq \tau \leq 3$. We analyze now the occurrence of the chaos in solution of Eq. (3.4). Since conditions (a) and (b) are necessary but not sufficient for identification of the chaos, we shall calculate values of the Lyapunov exponent for situations when these two conditions are satisfied. Here we define the Lyapunov exponent $\lambda_L$ for the case of Eq. (3.4), which has a single degree of freedom $\Gamma$ which depends on discrete "time" $j$ following the form [18]

$$\lambda_L = lim_{n \to \infty} -\frac{1}{n}\sum_{j=1}^{n} ln|P'(\Gamma_j)|$$

(3.5)

where $P(\Gamma) = (1 + \tau)\Gamma(1 - \Gamma)$.

To examine how changes in $A_0$ and $A_2$ determine the irregularities in behaviour of the temperature at the environmental interface obtained from the energy balance equation, we perform numerical experiment. We solve Eq. (3.2) for different values of $A_0$ and $A_2$, varying them in a broad range of energy exchange coefficients. After that we calculate the corresponding Lyapunov exponent $\lambda_L$ as a function of $\tau$. Figure 2 depicts dependence of Lyapunov exponent $\lambda_L$ on $\tau$. It takes mostly positive values indicating chaotic fluctuations of $\Gamma$. However, inside of the chaotic interval there are a lot opened periodical "windows" where $\lambda_L < 0$.

**Figure 2**

Dependence of Lyapunov exponent $\lambda_L$ on the dimensionless time $\tau$.

Now, we give a specific example of the above analysis applied to the interface of the Earth and its environment. It is now accepted that the Earth is a complex system which consists of the biota and their environment. These two elements of the system are closely coupled: the biota regulate the environment (e.g., climate on planetary scale) and, in turn, the environment restricts the evolution of the biota and dictates what type of life can exist. Changes in one part will influence the other, being opposed by negative feedback or enhanced by positive feedback, and this may lead to oscillation or chaos in the system. In the system considered, the values of the dimensionless time $\tau$ for long term atmospheric integration are in the interval $1 \leq \tau \leq 3$. Corresponding bifurcation map given in Fig. 3 depicts the regions with the chaotic fluctuations of temperature at the environmental interface biota-surrounding air.

**Figure 3**

Bifurcation diagram of $\Gamma$ as a function of dimensionless time $\tau$ in the long term atmospheric integration.
IV. CONCLUSION

The main aim of the report was to indicate the possibility of chaotic behaviour of the temperature at the interface of two media where the energy is exchanged by all three known mechanisms. The conditions for such phenomenon are discussed. Once the dimensionless time is introduced, one can study the problems on rather different scales in the same manner (scaling approach).

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1 H.Rodhe, R.J.Charlson and T.L.Anderson, Clim. Change 44, 409 (2000)
2 R.Pielke, Clim.Change 52, 1 (2002)
3 D.T.Mihailović, Plenary Talk in Ecological problems of our days -from global to local scale: Vulnerability and adaptation, 30 November - 1 December 2006, Keszthely (Hungary). (Electronic source: CD and Internet: http://www.georgikon.pate.hu/tanszekek
4 V.Chuprin, W.Mihajlovic, AGE, 28, 111(2006)
5 D.S.Niyogi, S.Raman, in: Mesoscale Atmospheric Dispersion, Ed. Z. Boybeyi, WIT Publications, Southampton, UK, Advances in Air Pollution, Vol 9, 1-51. (2001)
6 N.Nikolov, W.Massman and A.Schoettle, Ecol. Modelling, 80, 205 (1995)
7 T.H.Sivertsen, Phys.Chem.Earth 30 35
8 R.Rosen, Life Itself: A Comprehensive Inquiry into the Nature, Origin, and Fabrication of Life (Columbia University Press. New York, 1991) pp. 285.
9 K.Bentley and C.Clack, Morphological plasticity: Environmentally driven morphogenesis. M. Capcarrere et al. (Eds.): ECAL 2005, LNAI 3630, pp. 118-127, 2005.
10 M.R.Carter, Agricul.Ecosys.and Environ., 83, 3 (2001)
11 J.A.Glazier and F.Graner, Physical Review E 47, 2128 (1993)
12 L.Shulenburger, Y-C.Lai, T.Yalcinkaya, R.D.Holt, Physics Letters A 260, 156 (1999)
13 M. L. Martins, G. Ceotto, S. G. Alves, C. C. B. Bufon, J. M. Silva, and F. F. Laranjeira, Physical Review E 62, 7024 (2000)
14 C.M.Bhumralkar, J.Appl.Meteor., 14, 1246 (1975)
15 D.T.Mihailović, D.V.Kapor, B.Lalić and I.Arsenić in Abstracts of the 26th General Assembly of European Geophysical Society, March 20-25. 2001 Nice, France
16 J.L.Monteith, M.Unsworth, Principles of Environmental Physics, (Second Edition, Elsevir, Amsterdam, 1990) p. 304.
17 M.R.May, Nature, 261, 459 (1976)
18 T.S.Parker and L.O. Chua, Practical numerical algorithms for chaotic systems. (Springer-Verlag, New York, Berlin, Heidelberg, 1989) p. 348
