Simple grading model for financial markets

Chu Thuy Anh, Nguyen Tri Lan and Nguyen Ai Viet

Institute of Physics, Vietnam Academy of Science and Technology, 10 Dao Tan, Ba Dinh, Hanoi, Vietnam

E-mail: ctanh@iop.vast.ac.vn

Abstract. A simple way to estimate and grade a financial market by comparison the evolution process and the shape of distribution functions was proposed. In normal working state of financial market, the shape of distribution functions have one-peak form and change from Boltzmann-like to Gaussian-like distributions, while in risk moment might have two-peak form. The grad of financial markets was characterized by overlap area of initial and final distribution functions, and for risk degree by the separation between two shoulders of distribution function. The meaning of Levi tails of distribution and laws of general entropy and information was discussed.

1. Introduction

The transition from an initial distribution, such as Boltzmann distribution appearing in short period data set of econophysics study, to the final Gaussian distribution is a universal phenomenon in several observations in nature, social subjects included. In econophysics, investigation using returns distributions is very common [6, 7, 8], in which Gaussian distribution seemed to be the most used approximation for close-to-stable state [1, 2] of the market. The most observed for disordered state of the market obviously follows Boltzmann distribution. The transition process from disordered state to close-to-stable state of the market has received much attention in theoretical study and also in data mining study. In this framework, a new concept will be introduced to clarify some parts of this process.

There are several parameters affect on the transition from exponential distribution to normal distribution of market returns. In a series of works on econophysical study, few models, dynamic and non-dynamic, were proposed [3, 5][4] to explain the natural insights of this transition, considering an economy system as a basin in contact with environment. The most common input data for these investigation is return index, which can be measure as an internal value of an economy system. Various external values affect on this basin and its transformation process. However, one can not measure all external values, transformation process can not be observed and described in details.

In this work, we will use initial and final distribution without considering the details of transformation process, then will introduce a new concept to describe the changes on the market state. By investigating and comparing initial and final states of markets, information contained in returns distributions can be read out by several ways, reflecting the behaviours that characterize states of market. For a simple consideration, a new measurement tool will be introduced, using returns data to indicate in which state the financial market is, stable or critical ones.
2. Graphical study of overlap
Under several affections such as background noises [4] or dynamic process [3], an initial returns distribution evaluates to some final one. In the transition, the information dissipation from the initial to the final one has been done. However, the information remained in the transition reflects many properties of system and of process that happened along with the transition. In this presentation a simple method based on the geometrical overlap of graphics between initial and final distributions is proposed to estimate and to grade the system under consideration.

As it was mentioned in previous works [3], initial return distribution $P(x)$ of a short term data set is often approximated by Boltzmann form

$$P_B(x) = \frac{1}{2} \lambda e^{-|x|\lambda}, \quad (1)$$

where $\lambda$ is rate parameter, which is strongly associated with the stock or index liquidity, i.e. the rate parameter is implicitly depended on so-called market temperature. In general, this initial state could be considered as a chaotic state of the market containing a large set of in-time information and causing various scenarios that would happen in next time steps.

Observation of the same data after several internal evolutions, theoretical model and empirical data show that returns rearranged and tends be to Gaussian distribution with variance $\sigma$ and $\mu$ the mean value of $x$

$$P_G(x) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{(x-\mu)^2}{2\sigma^2}}, \quad (2)$$

In that close-to-stable state, market already follows some deterministic trajectory and market figures are dramatically reduced. Information contained in returns distribution is reduced and more ordered in some aspects. Information transition can be read via examining initial and final returns distributions. It’s ordinary to use distribution overlap to study similarity or difference between two given distributions. However, in this work, the overlap is proposed to be a new tool for market crisis measurement reflecting insights of transition process.

For a big enough data set, distributions are approximatively symmetric, so for the theoretical and analytical study, one can use the continuous version of distributions and the investigation is realized on the positive half axis, $x > 0$ only. Obviously, the equation

![Figure 1. Overlap area of initial and final distribution observed on $x > 0$.](image-url)
\[ P_B(x) = P_G(x), \]  

has two different solutions, which are

\[ x_{\pm} = \mu + \lambda \sigma^2 \pm \sqrt{2\lambda \mu \sigma^2 + \lambda^2 \sigma^4 - 2\sigma^2 \log\left(\frac{\sqrt{\pi} \lambda \sigma}{2}\right)} . \]  

Then overlap area of two distributions is determined by expression

\[ O_{i,f} = \frac{1}{2} \left( e^{-\lambda x_-} - e^{-\lambda x_+} \right) + \frac{1}{2} \left( \text{Erf} \left( \frac{\mu}{\sqrt{2} \sigma} \right) + \text{Erf} \left( \frac{x_- - \mu}{\sqrt{2} \sigma} \right) + \text{Erfc} \left( \frac{x_+ - \mu}{\sqrt{2} \sigma} \right) \right) . \]  

It is well known that the overlap area of initial and final distribution contains appearance possibilities of both initial and final states. It could be used to measure the stability of the market.

![Figure 2. The change of overlap area in \( \sigma \).](image)

An Information Degree denoted by \( I \) is introduced to describe market states.

\[ I = \frac{O_{i,f}}{100} . \]  

The newly defined information degree might vary from 0 to 100. The 0 degree corresponds to a complete chaotic state, 100 degree corresponds to almost stable state. It’s opposite to thermal degree where higher degrees correspond to more disordered states.

In other hand, for a study on the real market data, the discrete version would be employed. In this context, the overlap is numerically determined by following sum

\[ O_{i,f} = \frac{1}{2} \Delta x \sum_l \left[ \theta (P_{f,l} - P_{i,l}) P_{i,l} + \theta (P_{i,l} - P_{f,l}) P_{f,l} \right] . \]  

The above expression will be used in our future extensive study on real market data.
3. Conclusion
A simple way to estimate and to grade a financial market by comparison the evolution process and the shape of distribution functions was proposed. The investigation within the short and long period data sets is performed graphically. A new index, Information Degree, has been introduced to determine in which state the market is. Information Degree used in this paper is different from other statistical view on fat tail distribution, where most information is got and analysed for tails part only, but as shown, it could be considered as another measurement tool which is easier to use by economists, especially who work on financial markets. As Information Degree is of inverse order to thermal degree, we wish to make a connection between the two in order to simplify the tool and to bring more clear physical meaning to our newly proposed approach. Further consideration will take place to define more practical quantities for real financial markets.

References
[1] Victor M. Yakovenko A. Christian Silva, Richard E. Prange. Exponential distribution of financial returns at mesoscopic time lags: a new stylized fact. *Physica A*, (344), 2004.
[2] V. M. Yakovenko A. Dr. Probability distribution of returns in the heston model with stochastic volatility. *Quant. Finance* 2, (443), 2002.
[3] D. H. Lien Chu T. A and N. A. Viet. Simple model for market returns distribution. *Communications in Physics*, 23(2), 2013.
[4] N. T. Lan Chu T. A and N. A. Viet. Boltzmann-gaussian transition under specific noise effect. *J. Phys.: Conf. Ser.*, (537), 2014.
[5] N. T. Lan Chu T. A, D. H. Lien and N. A. Viet. Study of Hanoi and Hochiminh Stock Exchange by Econophysics Methods. *Communications in Physics*, 24(3S2), 2014.
[6] Stauffer D. Phys. rev. e 60 (1999) 6519. *Physica A*, (336):1–5, 2004.
[7] H. Salunkay K. Matia, M. Pal and H.E. Stanley. *Europhys. Lett.*, 66(909), 2004.
[8] V. M. Yakovenko. Econophysics, statistical mechanics approach to. In *Encyclopedia of Complexity and System Science*. Springer, 2007.