Harmonic decomposition of three-particle azimuthal correlations at RHIC

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We present measurements of three-particle correlations for various harmonics in Au+Au collisions at energies ranging from $\sqrt{s_{NN}} = 7.7$ to 200 GeV using the STAR detector. The quantity $\langle \cos(m\phi_1 + n\phi_2 - (m+n)\phi_3) \rangle$ is evaluated as a function of $\sqrt{s_{NN}}$, collision centrality, transverse momentum, $p_T$, pseudo-rapidity difference, $\Delta\eta$, and harmonics $(m,n)$. These data provide detailed information on global event properties like the three-dimensional structure of the initial overlap region, the expansion dynamics of the matter produced in the collisions, and the transport properties of the medium. A strong dependence on $\Delta\eta$ is observed for most harmonic combinations consistent with breaking of longitudinal boost invariance. Data reveal changes with energy in the two-particle correlation functions relative to the second-harmonic event-plane and provide ways to constrain models of heavy-ion collisions over a wide range of collision energies.

I. INTRODUCTION

Heavy nuclei are collided at facilities like the Relativistic Heavy Ion Collider (RHIC) and the Large Hadron Collider (LHC) in order to study the emergent properties of matter with quarks and gluons as the dominant
degrees-of-freedom: a quark-gluon plasma (QGP) \[1-4\]. The QGP is a form of matter that existed in the early universe when its ambient temperature was more than 155 MeV or 200 thousand times hotter than the center of the sun \[3, 6\]. As temperatures drop, quarks and gluons no longer possess the energy necessary to overcome the confining forces of QCD and they become confined into color neutral hadrons and the QGP transitions smoothly and continuously into a gas of hadrons \[3\]. This transition occurred in the early universe at about one microsecond after the big bang. Heavy-ion collisions provide the only known method to recreate and study that phase transition in a laboratory setting.

To provide the clearest possible picture of this phase transition, a beam energy scan was carried out at RHIC with collision energies ranging from \[\sqrt{s_{NN}}=200\ \text{GeV}\] down to 7.7 GeV. Lowering the beam energy naturally reduces the initial temperature of the matter created in the collisions providing information on how the transport properties and equilibrium of the matter vary with temperature \[8\]. These heavy-ion collisions however create systems that are both very small and short-lived. The characteristic size of the collision region is the size of a nucleus or approximately \[10^{-14}\ \text{meters}\] across. This system expands in the longitudinal direction and eventually in the transverse direction so that the energy density drops quickly. Any quark gluon plasma that exists will only survive for on the order \[5 \times 10^{-23}\ \text{seconds}\]. Given the smallness of the system and its very brief lifetime, it is challenging to determine the nature of the matter left behind after the initial collisions. Physicists rely on indirect observations based on particles streaming from the collision region which are observed long after any QGP has ceased to exist. Correlations between these produced particles have provided insight into the early phases of the expansion as well as the characteristics of the matter undergoing the expansion \[9\]. The dependence of the correlations on the azimuthal angle between particles \[\Delta \phi = \phi_1 - \phi_2\] has proven to be particularly informative. Data have revealed that even when particle pairs are separated by large angles in the longitudinal direction (large \[\Delta \eta\]), they remain strongly correlated in the azimuthal direction. This correlation manifests itself as a prominent ridge-like structure in two-particle, \[\Delta \eta, \Delta \phi\], correlation functions \[10\]. The origin of this ridge has been traced to the initial geometry of the collision region where flux tubes are localized in the transverse direction but stretch over a long distance in the longitudinal direction \[11, 14\]. How well these structures from the initial geometry are translated into correlations between particles emitted from the collision region reveals information about the medium’s viscosity: the larger the viscosity, the more washed out the correlations will become \[13\].

To study these effects, it is convenient to examine the coefficients of a Fourier transform of the \[\Delta \phi\] dependence of the two-particle correlation functions \[10\]. These coefficients have been variously labeled as \[V_n\], \[a_n\], or \[\psi_n(2)\] where \(n\) is the harmonic. Although the latter is perhaps more cumbersome, we have maintained its usage owing to its connection to the original terminology used for two-particle cumulants which has been in use for more than a decade \[17\]. While \[\psi_n(2) = \langle \cos n(\Delta \phi) \rangle\] has been studied as a function of \[\sqrt{s_{NN}}\], centrality, harmonic \(n\), \(p_T\), and \(\Delta \phi\) \[18\], in this paper we extend this analysis from two-particle correlations to three-particle mixed harmonic correlations of the form \[\langle \cos (m \phi_1 + n \phi_2 - (m+n) \phi_3) \rangle\] \[19\] where \(m\) and \(n\) are positive integers.

Extending the analysis of azimuthal correlations from two to three particles provides several benefits. First, the three particle correlations provide greater sensitivity to the three-dimensional structure of the initial state by for example revealing information about the two-particle \[\Delta \eta - \Delta \phi\] correlations with respect to the reaction plane. Many models of heavy-ion collisions make the simplifying assumption that the initial geometry of the collision overlap does not vary with rapidity and that a boost invariant central rapidity plateau may be considered \[20\]. It is likely however that this assumption is broken by the asymmetric nature of the initial state and that precision comparisons between models and data will require a better understanding of the initial state fluctuations in all three dimensions \[21\]. Second, the new measurements can constrain models \[22, 23\]. When signals seen in two-particle correlations may be mocked up by multiple effects, three-particle correlations can break those ambiguities. This is important as models become more sophisticated by including for example bulk viscosity, shear viscosity, and their temperature dependence \[24\]. Also, three-particle correlations can reveal information about how two-particle correlations change as a function of their angle with respect to the reaction plane. When one of the harmonics \(m, n,\) or \(m+n\) is equal to two, that harmonic will be dominated by the preference of particles to flow in the direction of the reaction plane. This feature has been exploited to study charge separation relative to the reaction plane through measurements of the charge dependence of \[\langle \cos (\phi_1 + \phi_2 - 2 \phi_3) \rangle\] \[27, 28\]. The motivation for those measurements was to search for evidence of the chiral magnetic effect (CME) in heavy-ion collisions \[29-31\]. By extending the measurements to other harmonics we can ascertain more information about the nature of the correlations interpreted as evidence for CME. Finally, three-particle correlations reveal information about how various harmonics are correlated with each other. For example, Teaney and Yan \[22\] originally proposed the measurement of \[\langle \cos (\phi_1 + 2 \phi_2 - 3 \phi_3) \rangle\] because initial state models predict a strong correlation between the first, second and third harmonics of the spatial density distribution. That correlation can be traced to collision geometries where a nucleon from one nucleus fluctuates toward the edge of that nucleus and impinges on the oncoming nucleus. This leads to something similar to a \(p + A\) collision and a high density near the edge of the main collision region. That configuration increases the predicted \(v_3\) by a factor of 2-3 in noncentral collisions so that \(v_3\) deviates from the \[1/\sqrt{N_{\text{part}}}\] one would expect from ran-
dom fluctuations in the positions of the nucleons participating in the collision \cite{15, 16, 18}. That configuration should also be asymmetric in the forward and backward rapidity directions, again pointing to the importance of understanding the three-dimensional structure of the initial state. If the evidence proposed by Teaney and Yan is not confirmed, then one may question the validity of any model that predicts the centrality dependence of $v_2$ based on those initial condition models. In this paper we present measurements of $\langle \cos(m\phi_1 + n\phi_2 - (m+n)\phi_3) \rangle$ as a function of energy, centrality, $\Delta \eta$, $p_T$, and harmonics $m$ and $n$. Data confirm the predicted correlation between the first, second and third harmonics but the $\Delta \eta$ dependence points to the potential importance of including the three-dimensional structure of the initial state in the model calculations.

In the following, we first describe the experiment and the analysis (Sec. \textbf{II}). We then present the results in Sec. \textbf{III} including the $\Delta \eta$ dependence (Sec. \textbf{III A}), the centrality dependence (Sec. \textbf{III B}), the $p_T$ dependence (Sec. \textbf{III C}), and the beam energy dependence (Sec. \textbf{III D}). Conclusions are presented in Sec. \textbf{IV}. We present measurements of $v_2^2$ (\textbf{2}) for $n=1, 2, 4$, and $5$ in the appendix.

\section{II. EXPERIMENT AND ANALYSIS}

Our measurements make use of data collected from Au+Au collisions with the STAR detector at RHIC in the years 2004, 2010, 2011, 2012, and 2014. The charged particles used in this analysis are detected through their ionization energy loss in the STAR Time Projection Chamber \cite{32}. The transverse momentum $p_T$, $\eta$, and charge are determined from the trajectory of the track in STAR's solenoidal magnetic field. With the 0.5 Tesla field used during data taking, particles can be reliably tracked for $p_T > 0.2$ GeV/c. The efficiency for finding particles drops quickly as $p_T$ decreases below this value \cite{33}. Weights have been used to correct the three-particle correlation functions for the $p_T$-dependent efficiency and for imperfections in the detector acceptance. The quantity analyzed and reported is

$$C_{m,n,m+n} = \langle \cos(m\phi_1 + n\phi_2 - (m+n)\phi_3) \rangle = \left\langle \left( \frac{\sum_{i,j,k} w_i w_j w_k \cos(m\phi_i + n\phi_j + (m+n)\phi_k)}{\sum_{i,j,k} w_i w_j w_k} \right) \right\rangle$$

(1)

where $\langle \rangle$ represents an average over events and $\sum_{i,j,k}$ is a sum over unique particle triplets within an event. Each event is weighted by the number of unique triplets in that event. The weights $w_i w_j w_k$ are determined from the inverse of the $\phi$ distributions after they have been averaged over many events (which for a perfect detector should be flat) and by the $p_T$ dependent efficiency. The $w_{i,j,k}$ depend on the particles’ $p_T$, $\eta$, and charge and the collisions’ centrality and $z$-vertex location. The correction procedure

is verified by checking that the $\phi$ distributions are flat after the correction so that $\langle \cos(m\phi) \rangle$ and $\langle \sin(m\phi) \rangle$ are near zero. With these corrections, the data represent the $C_{m,n,m+n}$ that would be seen by a detector with perfect acceptance for particles with $p_T > 0.2$ GeV/c and $|\eta| < 1$. In practice, calculating all possible combinations of three particles individually would be computationally too costly to be practical, particularly for the larger data sets at 200 GeV. In that case we use algebra based on Q-vectors ($Q_n = \Sigma \exp(in\phi)$) to reduce the computational challenge \cite{33}. Differential measurements like the $\Delta \eta$ dependence of the correlations, however, require explicit calculations for at least two of the particles. Studying the $\Delta \eta$ dependence of the correlations also allows us to correct for the effect of track-merging on the correlations. Track-merging leads to a large anti-correlation between particle pairs that are close to each other in the detector. The effect becomes large in central collisions where the detector occupancy is largest. After weight corrections have been applied to correct for single particle acceptance effects, the effect of track-merging is the largest remaining correction. Data have been divided into standard centrality classes (0-5%, 5-10%, 10-20%, ..., 70-80%) based on the number of charged hadrons within $|\eta| < 0.5$ observed for a given event. In some figures, we will report the centrality in terms of the number of participating nucleons ($N_{part}$) estimated from a Monte Carlo Glauber calculations \cite{34, 35}.

The three-particle correlations presented in this paper are related to the low-resolution limit of the event-plane measurements that have been explored at the LHC \cite{36}. Practically this would be carried out by dividing $C_{m,n,m+n}$ by $\langle v_m v_n v_{m+n} \rangle$. Typically, however, $v_n$ is measured from a two-particle correlation function such as the two-particle cumulants $v_n = \sqrt{v_n^2 (\textbf{2})}$ or a similar measurement and the $v_n^2 (\textbf{2})$ are not positive-definite quantities. As such, $v_n^2 (\textbf{2})$ can, and often does, become imaginary. This is particularly true for the first harmonic and also at lower collision energies. For this reason we report the pure three-particle correlations which, in any case, do not suffer from the ambiguities related to the low- and high-resolution limits associated with reaction plane analyses \cite{19, 37} and are therefore easier to interpret theoretically.

\section{III. RESULTS}

In the following, we present the $\Delta \eta$ dependence of the three-particle correlations for several harmonic combinations corrected for track-merging. After removing the effects of track merging and Hanbury Brown and Twiss (HBT) correlations \cite{38}, we integrate over the $\Delta \eta$ dependence of the correlations and present the resulting integrated correlations as a function of centrality for the energies $\sqrt{s_{NN}}=200, 62.4, 39, 27, 19.6, 14.5, 11.5$, and 7.7 GeV. We also investigate the $p_T$ dependence of the correlations by plotting them as a function of the $p_T$ of
either the first or second particle used in the correlation. Finally, we study how the data depends on the beam energy.

A. $\Delta \eta$ Dependence

![Graph showing $\Delta \eta$ dependence of $C_{1,1.2}$ for Au+Au collisions.](image)

**FIG. 1.** (color online) The $\Delta \eta$ dependence of $C_{1,1.2}$ scaled by $N_{\text{part}}$ for 9 centrality intervals with the three most central classes shown in the top panels and the three most peripheral in the bottom. The $N_{\text{part}}$ values used for the corresponding centralities are 350.6, 298.6, 234.3, 167.6, 117.1, 78.3, 49.3, 28.2 and 15.7. In the panels on the left, $\Delta \eta$ is taken between particles 1 and 2 while on the right it is between particles 1 and 3 (which is identical to 2 and 3). Data are from 200 GeV Au+Au collisions and for charged hadrons with $p_T > 0.2$ GeV/c, $|\eta| < 1$.

Figure II shows the $\Delta \eta$ dependence of $C_{1,1.2}$ scaled by $N_{\text{part}}$ for charged hadrons with $p_T > 0.2$ GeV/c and $|\eta| < 1$. The scaling accounts for the natural dilution of correlations expected if the more central collisions can be treated as a linear superposition of nucleon-nucleon collisions. Results for nine different centrality intervals from 200 GeV Au+Au collisions are shown. We do not include the uncertainty on $N_{\text{part}}$ in the uncertainties in our figures. The left panels show the correlations as a function of the difference in $\eta$ between the first and second particle. Note that the subscripts in $C_{m,n,m+n}$ refer to the harmonic number while the subscripts for the $\eta$ refers to the particle number. The right panels show the same but as a function of the difference between particles 1 and 3. The $C_{1,1.2}$ correlation is similar to the correlation used in the search for the chiral magnetic effect except that we do not separate out the cases when particles 1 and 2 have like-sign charges vs unlike-sign charges as is done when looking for charge separation with respect to the reaction plane. These measurements can be approximately related to the reaction-plane based measurements by scaling the three-particle correlations by $1/\nu_2$.

We note that the difference in $C_{1,1.2}$ for different charge combinations is as large as the signal with $C_{1,1.2}$ being nearly zero for unlike-sign combinations of particle 1 and 2. This correlation may also be influenced by momentum conservation effects as well. It’s not clear however how those effects would be distributed with respect to $\Delta \eta$.

In the left panels of Fig. I we see a strong dependence for $C_{1,1.2}$ on $|\eta_1 - \eta_2|$. In central collisions, the data starts out negative at the smallest values of $|\eta_1 - \eta_2|$ but then begins to increase and becomes close to zero or even positive near $|\eta_1 - \eta_2| = 1.5$. At small $|\eta_1 - \eta_2|$, a narrow peak is seen in the correlation that is related to HBT. As we progress from central to peripheral collisions, the trends change with $C_{1,1.2}$ in peripheral collisions exhibiting a positive value at small $|\eta_1 - \eta_2|$, perhaps signaling the dominance of jets in the correlation function in these peripheral collisions.

The left panels share the same scales as the right panels making it clear that the dependence of $C_{1,1.2}$ on $|\eta_1 - \eta_3|$ is much weaker than the dependence on $|\eta_1 - \eta_2|$. This is expected since the $e^{-2i\phi_3}$ term in $C_{1,1.2} = (e^{i\phi_1}e^{i\phi_2}e^{-2i\phi_3})$ will be dominated by the global preference of particles to be emitted in the direction of the reaction plane. For all but the most central collisions, the almond shaped geometry of the collision overlap region is approximately invariant with rapidity. This is not likely the case for other harmonics.

Figure II shows $C_{1,2.3}$ scaled by $N_{\text{part}}^2$ as a function of $|\eta_1 - \eta_2|$ (left panels) and $|\eta_1 - \eta_3|$ (right panels). In this case, $C_{1,2.3}$ exhibits a stronger dependence on $|\eta_1 - \eta_3|$ than on $|\eta_1 - \eta_2|$. The variation with $|\eta_2 - \eta_3|$ is very similar to the variation with $|\eta_1 - \eta_2|$ and is omitted from the figures to improve legibility. Again, the $e^{2i\phi_3}$ component of $C_{1,2.3}$ is dominated by the reaction plane which is largely invariant within the $\eta$ range covered by these measurements so that $C_{1,2.3}$ depends very little on the $\eta_2$, $|\eta_1 - \eta_2|$, or $|\eta_2 - \eta_3|$. However, $C_{1,2.3}$ depends very strongly on $|\eta_1 - \eta_3|$. This dependence may arise from the longitudinal asymmetry inherent in the fluctuations that lead to predictions for large values of $C_{1,2.3}$ [24]. In models for the initial geometry, the correlations are induced between the first, second, and third harmonics of the eccentricity by cases where a nucleon fluctuates towards the edge of the nucleus [24]. If that occurs in the reaction plane direction and towards the other nucleus in the collision, then that nucleon can collide with many nucleons from the other nucleus. This geometry will cause the first and third harmonics to become correlated with
the second harmonic. Since the collision of one nucleon from one nucleus with many nucleons in the other nucleus is asymmetric along the rapidity axis, we argue that we can expect a strong dependence on $|\eta_1 - \eta_3|$. Models that assume the initial energy density is symmetric with rapidity (boost invariant) will likely fail to describe this behavior. One may also speculate that the variation with $|\eta_1 - \eta_3|$ could arise from sources like jets or resonances particularly if they interact with the medium so that they become correlated with the reaction plane. Making use of the full suite of measurements provided here will help delineate between these two scenarios.

In Fig. 3 we present the $|\eta_1 - \eta_2|$ and $|\eta_1 - \eta_3|$ dependence of $C_{2,2,4}$. This correlation is more strongly influenced by the reaction plane correlations and exhibits much larger values than either $C_{1,1,2}$ or $C_{1,2,3}$. The dependence on $|\eta_1 - \eta_2|$ and $|\eta_1 - \eta_3|$ are also weaker with $C_{2,2,4}$ in central and mid-central collisions showing little variation over the $|\eta_1 - \eta_2|$ range, consistent with a mostly $\eta$-independent reaction plane within the measured range. A larger variation is observed with $|\eta_1 - \eta_3|$ which in mid-central collisions amounts to an approximately 20% variation. We also note that in mid-central collisions, the change in value of $C_{2,2,4}$ over the range $0 < |\eta_1 - \eta_3| < 2$ is similar in magnitude to the change of $C_{1,1,2}$ over $0 < |\eta_1 - \eta_2| < 2$ and $C_{1,2,3}$ over $0 < |\eta_1 - \eta_3| < 2$.

In Fig. 4 we present the $|\eta_1 - \eta_2|$ and $|\eta_2 - \eta_3|$ dependence of $C_{2,3,5}$. Again, $C_{2,3,5}$ only exhibits a weak dependence on $|\eta_1 - \eta_2|$ but a stronger dependence on $|\eta_2 - \eta_3|$. In central and mid-central collisions, a strong short-range correlation at $|\eta_2 - \eta_3| < 0.4$ is apparent consistent with HBT and Coulomb correlations that vary with respect to the reaction plane. In addition to that peak, $C_{2,3,5}$ decreases as $|\eta_2 - \eta_3|$ increases. Although the relative variation of $C_{2,3,5}$ is similar to $C_{2,2,4}$, the absolute change is much smaller than for $C_{1,1,2}$, $C_{1,2,3}$, or $C_{2,2,4}$.

The combination of the various $C_{m,n,m+n}$ can help elucidate the nature of the three-particle correlations. If the $|\eta_1 - \eta_3|$ dependence of $C_{1,2,3}$ arises from correlations between particles from jets correlated with the reaction plane.
plane, we would expect the particles at small $\Delta \eta$ to predominantly come from the near-side jet (at $\Delta \phi \approx 0$) and particles at larger $\Delta \eta$ to come from the away-side jet (at $\Delta \phi \approx \pi$ radians). In that case, at small $\Delta \eta$, $C_{m,n,m+n}$ for all harmonics will have a positive contribution from the jets. The same is not true however for large $\Delta \eta$ where we would expect the correlations to be dominated by the away-side jet separated by $\pi$ radians. For this case at large $\Delta \eta$, $C_{1,1,2}$ and $C_{1,2,3}$ would receive negative contributions from the away side jet while $C_{2,2,4}$ and $C_{2,3,5}$ would both receive positive contributions. The trends observed across the variety of $C_{m,n,m+n}$ measurements are inconsistent with this simple picture with $C_{2,2,4}$ decreasing by nearly the same amount as $C_{1,2,3}$ as $\Delta \eta$ is increased. A more complicated picture of the effect of jets would therefore be required to account for the observed variations but we do not know of any specific model that has been shown to describe our data.

### B. Centrality Dependence

In Figs. 5 and 6 we show $C_{m,n,m+n}$ correlations scaled by $N_{\text{part}}^2$ with $(m,n) = (1,1), (1,2), (1,3), (2,2), (2,3), (2,4), (3,3)$, and $(3,4)$ for $\sqrt{s_{\text{NN}}} = 200, 62.4, 39, 27, 19.6, 14.5, 11.5,$ and $7.7$ GeV Au+Au collisions as a function of $N_{\text{part}}$. Data are for charged particles with $|\eta| < 1$ and $p_T > 0.2$ GeV/c. The correlation $C_{2,2,4}$, by far the largest of the measured correlations, has been scaled by a factor of 1/5. Otherwise, the scales on each of the three panels are kept the same for each energy to make it easier to compare the magnitudes of the different harmonic combinations.

At 200 GeV, $C_{1,1,2}$ is negative for all centralities except for the most peripheral where it is slightly positive but consistent with zero. $C_{1,2,3}$ is consistent with zero in peripheral collisions, positive in mid-central collisions but then becomes negative in central collisions. If the second and third harmonic event planes are uncorrelated, then $C_{1,2,3}$ should be zero. The $C_{1,2,3}$ correlation is non-zero deviating from that expectation. The magnitude is however much smaller than originally anticipated based on a linear hydrodynamic response to initial state geometry fluctuations [22]. Non-linear coupling between harmonics, where the fifth harmonic for example is dominated by a combination of the second and third harmonic, has been shown to be very important [23, 40]. In the case of $C_{1,2,3}$, the non-linear contribution has an opposite sign to the linear contribution and similar magnitude canceling out most of the expected strength of $C_{1,2,3}$. This suggests that $C_{1,2,3}$ is very sensitive to the nonlinear nature of the hydrodynamic model. $C_{1,3,4}$ is close to zero for all centralities indicating little or no correlation between the first, third, and fourth harmonics. The other $C_{m,n,m+n}$ correlations are positive for all centralities. When considering the comparison of this data to hydrodynamic models, it is important to also consider the strong $\Delta \eta$ dependence of the correlations as shown in the previous section.

The correlations involving a second harmonic are largest with $C_{2,2,4}$ being approximately 5 times larger in magnitude than the next largest correlator $C_{2,3,5}$. The correlations decrease quickly as harmonics are increased beyond $n=2$. The higher harmonic correlations $C_{3,3,6}$ and $C_{3,4,7}$ are both small but non-zero. The correlations $C_{1,1,2}, C_{1,2,3}, C_{2,2,4}, C_{2,3,5},$ and $C_{3,3,6}$ scaled by $N_{\text{part}}^2$ all exhibit extrema in mid central collisions where the initial overlap geometry is predominantly elliptical. We note that the centrality at which $N_{\text{part}}^2 C_{2,2,4}$ reaches a maximum is different than the centrality at which $N_{\text{part}}^2 C_{2,3,5}$ reaches a maximum.

As the collision energy is reduced, although the magnitude of the correlations becomes smaller, the centrality dependence and ordering of the different harmonics seems to remain mostly the same. The $C_{1,2,3}$ correlation

![Image of plots showing centrality dependence](image.png)
FIG. 5. (color online) The centrality dependence of the $C_{m,n,m+n}$ correlations scaled by $N_{\text{part}}^2$ for charged hadrons with $p_T > 0.2$ GeV/$c$ and $|\eta| < 1$ from 200, 62.4, 39, and 27 GeV Au+Au collisions for $(m, n) = (1, 1)$, $(1, 2)$, $(1, 3)$ (left) $(2, 2)$, $(2, 3)$, $(2, 4)$ (center) and $(3, 3)$, $(3, 4)$ (right). Systematic errors are shown as bands. All panels in the same row share the same scale but $C_{2,2,4}$ has been divided by a factor of 5 to fit on the panel. The labels in the top panels apply to all the panels in same column.

However, an exception. While at 200 GeV, $C_{1,2,3}$ is mostly positive, at 62.4 GeV it is consistent with zero or slightly negative and at lower energies it becomes more and more negative. We speculate that this behavior may be related to the increasing importance of momentum conservation as the number of particles produced in the collision decreases. No theoretical guidance exists however for the energy dependence of these correlations at energies below 200 GeV. This data should provide useful constraints for the models being developed to describe lower energy collisions associated with the energy scan program at RHIC.

Figure 6 shows the same correlations as Fig. 5 except for lower energy data sets: $\sqrt{s_{NN}} = 19.6, 14.5, 11.5,$ and 7.7 GeV. Trends similar to those seen in Fig. 5 are for the most part also exhibited in this figure. Although the
The statistical precision is poor for the lowest energy points, it appears that $C_{1,1,2}$ at 7.7 GeV is smaller in magnitude than at higher energies, becoming consistent with zero. This was also observed in the charge dependent measurements of $C_{1,1,2}$ [41]. A second phase of the RHIC beam energy scan planned for 2019 and 2020 will significantly increase the number of events available for analysis at these lower energies while expanding the $\eta$ acceptance from $|\eta| < 1$ to $|\eta| < 1.5$ [42] so that this intriguing observation can be further investigated. The increased acceptance will increase the number of three-particle combinations by approximately a factor of three and will make it possible to measure the $\Delta\eta$ dependence of the $C_{m,n,m+n}$ correlations to $|\Delta\eta| \approx 3$.

C. $p_T$ Dependence

If the three-particle correlations presented here are dominated by correlations between event planes, then one might expect that the $p_T$ dependence of the three-particle correlations will simply track the $p_T$ dependence.
The same quantity but for a different set of centrality for understanding the primary reason for introducing Eq. 2 is to provide a context of simplicity in this publication, we have scaled the $v$ by $N_{\text{part}}$, which is a powerful measurement to help tune those models. At intermediate $p_T$, $C_{1,2,3}$ is positive for central collisions but negative for peripheral collisions. At $p_T > 7$ GeV/$c$, $C_{1,2,3}$ is strongly negative, perhaps again, indicative of the contribution of back-to-back jets to the correlations. Such strong negative correlation seems to be absent in central collisions where $C_{1,2,3}$ appears to remain positive, although with large error bars. This is consistent with a scenario where di-jets have been quenched in central collisions. As with $C_{1,1,2}$, the $p_T$ trends for $C_{1,2,3}$ are very different in the relationship between $C_{m,n,m+n}$ and harmonic planes in Eq. 2 is not guaranteed to hold and is particularly likely to be broken for correlations involving the first harmonic where momentum conservation effects will likely play an important role or where a strong charge sign dependence has been observed [27, 28].

In Fig. 7 we show $N_{\text{part}}C_{1,1,2}/p_T$ as a function of the $p_T$ of particle one. The top panel shows the more central collisions while the bottom panel shows more peripheral collisions. In this and in the following figures related to the $p_T$ dependence, we sometimes exclude centrality bins and slightly shift the positions of the points along the $p_T$ axis to make the figures more readable. For more central collisions, $C_{1,1,2}/p_T$ is negative and slowly decreases in magnitude as $p_T$ increases. This indicates that $C_{1,1,2}$ is generally increasing with the $p_T$ of particle one but that for central collisions at high $p_T$, $C_{1,1,2}$ starts to saturate. For the more peripheral 30-40% and 40-50% collision however, $C_{1,1,2}$ appears to be linear in $p_T$ without an indication of saturation even up to $p_T \approx 10$ GeV/$c$. For the much more peripheral 60-70% and 70-80% centrality intervals, $C_{1,1,2}$ starts out at or above zero then becomes more and more negative as $p_T$ is increased. The trends in the most peripheral centrality intervals, particularly at high $p_T$, are consistent with being dominated by momentum conservation and jets. A pair of back-to-back particles aligned with the reaction plane will lead to a negative value for $C_{1,1,2}$. Although the data exhibit a smooth transition from the trends in more central collisions to the trends in more peripheral collisions, the trends are quite distinct and indicative of very different correlations in those different regions. In peripheral collisions, the correlations get stronger as $p_T$ is increased. In central collisions, the opposite is observed.

For the case of $C_{1,2,3}$ in Fig. 8 we show the $p_T$ dependence of both particle one (left panels) and particle two (right panels). The dependence of $C_{1,2,3}/p_T$ on $p_T$ is quite weak indicating that where $C_{1,2,3}$ is non-zero, it increases roughly linearly with $p_T$. The dependence of $C_{1,2,3}/p_T$ on $p_T$, however, exhibits several notable trends. First we note that for the 20-30% centrality interval, $C_{1,2,3}/p_T$ changes sign up to three times. In hydrodynamic models, the value of $C_{1,2,3}$ is very sensitive to the interplay between linear and non-linear effects and to viscous effects. The sign oscillations exhibited in the data may be a consequence of subtle changes in the relevant sizes of those effects. If this is the case, then this confirms that $C_{1,2,3}$ is positive for central collisions but negative for peripheral collisions. At $p_T > 7$ GeV/$c$, $C_{1,2,3}$ is strongly negative, perhaps again, indicative of the contribution of back-to-back jets to the correlations. Such strong negative correlation seems to be absent in central collisions where $C_{1,2,3}$ appears to remain positive, although with large error bars. This is consistent with a scenario where di-jets have been quenched in central collisions. As with $C_{1,1,2}$, the $p_T$ trends for $C_{1,2,3}$ are very different in the

![FIG. 7. (color online) Three-particle azimuthal correlations $C_{1,1,2}$ scaled by $N_{\text{part}}$ as a function of the first particles $p_T$ for 200 GeV Au+Au collisions for charged hadrons with $p_T > 0.2$ GeV/$c$ and $|\eta| < 1$. The top and bottom panels show the same quantity but for a different set of centrality intervals. Systematic errors are shown as solid lines enclosing the respective data points.](image-url)
Systematic errors are shown as solid lines enclosing the respective data points.

The competition of these two trends leads to a maximum at intermediate values of $\eta$ with model calculations that show in a parton cascade. We have tried to point out interesting features in the data for $p_T > 0.2$ GeV/c and $|\eta| < 1$. The top and bottom panels show the same quantity but for a different set of centrality intervals. Systematic errors are shown as solid lines enclosing the respective data points.

most peripheral and most central collisions.

The $C_{2,2,4}$ correlation is the largest of the $C_{m,n,m+n}$ correlations since it is strongly affected by the tendency of particles to preferentially line up with the reaction plane. In Fig. III we show $N_{part}^2 C_{2,2,4}/p_T$ as a function of $p_T$ for charged hadrons. This observation is consistent with model calculations that show in a parton cascade even up to $p_T \approx 5$ GeV/c there are a significant number of partons whose final momentum has been increased by interactions with the medium. The $p_T$ dependence of $C_{2,2,4}/p_T$ supports that picture as well.

In Fig. III we show the $p_T$ dependence of $N_{part}^2 C_{2,3,5}/p_T$ where $p_T$ is either the $p_T$ of particle one (left panels) or particle two (right panels). Again, the top panels show more central collisions and the bottom panels more peripheral. For $p_T < 5$, $C_{2,3,5}/p_T$ is mostly flat as a function of the $p_T$ of either particle one or particle two. Above that, the correlations seem to become smaller but with large statistical errors. One can discern a slight difference between the trends in the left and right panels: $C_{2,3,5}/p_T$ seems to decrease slightly as a function of $p_T$, while $C_{2,3,5}/p_T$ to increase slightly. This is likely related to the different $p_T$ dependences of $v_2$ and $v_3$ where $v_3$ has been found to saturate at lower $p_T$ while $v_3$ is still growing. In central collisions, it is even found that $v_3$ becomes larger than $v_2$ at intermediate $p_T$.
Intervals. Systematic errors are shown as solid lines enclosing the respective data points.

Trends are very different when comparing central collisions to peripheral collisions. We expect that while Figs. 5 and 6 show the centrality dependence of $C_{224}$ for a selection of collision energies. Both $C_{224}$ and $C_{235}$ remain positive for the centralities and energies shown with no apparent changes in the centrality trends. We note that although $C_{224}$ drops significantly from 200 down to 19.6 GeV, we observe little change with energy below 19.6 GeV. A similar lack of energy dependence between 7.7 and 19.6 GeV was also observed in recent measurements of $v_2^2$ ([2]) [18]. This is notable since one would naively expect either of these correlation measurements to continuously increase as the density of the collision region increases.

To better view the energy trends, in Fig. 12 we show $N_{part}C_{m,m+n}$ as a function of $p_T$ for three centrality intervals: 10-20%, 20-30%, and 30-40%. The $v_2$ values are based on a two-particle cumulant analysis as discussed in Appendix A. The scaling will be further discussed in the next paragraph. For all centrality intervals shown, $C_{1,1,2}/v_2$ is negative at the highest energy but the magnitude of the correlation decreases as the energy decreases and becomes consistent with zero, although with large errors, at 7.7 GeV. This behavior was also observed in the charge dependence of this correlator which has been studied to search for the charge separation predicted to be a consequence of the chiral magnetic effect [41]. As noted above, both $C_{2,2,4}$ and $C_{2,3,5}$ are positive for all energies. The energy dependence of $C_{1,2,3}/v_2$ is unique in that it is positive at 200 GeV but then drops below zero near 62.4 GeV and continues to become more negative at lower energies. In the following paragraph, we discuss the implications that this trend has for how two-particle correlations with respect to the reaction plane change with energy.

The correlations $C_{1,1,2}$, $C_{1,2,3}$, $C_{2,2,4}$, and $C_{2,3,5}$ presented in Fig. 12 have either $m = 2$, $n = 2$, or $m + n = 2$. When $v_2$ is large, as it is for the 10-20%, 20-30% and 30-40% centrality intervals, then $\langle \cos(1\phi_1 + 1\phi_2 - 2\phi_3)/v_2 \rangle$
FIG. 10. (color online) Three-particle azimuthal correlations $C_{2,3,5}$ scaled by $N_{\text{part}}^2/p_T$ as a function of $p_T$ where the $p_T$ is taken for either particle one (left panels) or particle two (right panels) for 200 GeV Au+Au collisions. Data are for charged hadrons with $p_T > 0.2$ GeV/$c$ and $|\eta| < 1$. The top and bottom panels show the same quantity but for a different set of centrality intervals. Systematic errors are shown as solid lines enclosing the respective data points.

FIG. 11. (color online) The centrality dependence of $C_{11,2}$ (left) and $C_{1,2,3}$ (right) scaled by $N_{\text{part}}^2$ for a selection of energies.
FIG. 12. (color online) The centrality dependence of $C_{2,2,4}$ (left) and $C_{2,3,5}$ (right) scaled by $N_{\text{part}}^2$ for a selection of energies.

FIG. 13. (color online) The $\sqrt{s_{NN}}$ dependence of $N_{\text{part}}C_{m,n,m+n}/v_2^2$ for $(m, n) = (1, 1)$ (top left), (1, 2) (top right), (2, 2) (bottom left) and (2, 3) (bottom right) for three selected centrality intervals. In the bottom right panel, the lowest energy points for the 20-30% and 30-40% centrality intervals, having large uncertainties, are omitted for clarity. Statistical uncertainties are shown as vertical error bars while the systematic errors are shown as shaded regions or bands.
The reaction plane deserves further theoretical investigation. Correlations including a second harmonic should then provide information about two-particle correlations with respect to the second harmonic reaction plane:

\[
\langle \cos(\phi_1 + 1\phi_3 - 2\phi_2) \rangle / v_2 \approx \langle \cos(1\phi_1' + 1\phi_3') \rangle, \\
\langle \cos(\phi_1 + 2\phi_3 - 3\phi_2) \rangle / v_2 \approx \langle \cos(1\phi_1' - 3\phi_2') \rangle, \\
\langle \cos(2\phi_1 + 2\phi_3 - 4\phi_2) \rangle / v_2 \approx \langle \cos(2\phi_1' - 4\phi_2') \rangle, \\
\langle \cos(2\phi_3 + 3\phi_1 - 5\phi_2) \rangle / v_2 \approx \langle \cos(3\phi_1' - 5\phi_2') \rangle, 
\]

where \(\phi' = \phi - \Psi_{RP}\). Since we are integrating over all particles in these correlations, the subscript label for the particles is arbitrary so we have reassigned them so that particle 3 is always associated with the second harmonic. For illustration, Table I shows values for \(C_{m,n,m+n}/v_2\) for specific values of \(\phi_1'\) and \(\phi_2'\). At 200 GeV, all measured correlations are positive except \(\cos(\phi_1' + \phi_2')\). This points to an enhanced probability for a pair of particles in one of two possible configurations: either \(\phi_1' \approx \pi/3\) and \(\phi_2' \approx 2\pi/3\) or \(\phi_1' \approx -\pi/3\) and \(\phi_2' \approx -2\pi/3\) (these correspond to the right-most column of Table I). This result is surprising since it implies a preference for both of the correlated particles to either lie in the upper hemisphere, or both in the lower hemisphere. We note however, that hydrodynamic models with fluctuating initial conditions correctly predict this trend \[45\] which could arise from increased density fluctuations at either the top or the bottom of the almond shaped overlap region. A high density fluctuation in the lower half of the almond zone naturally leads to particles moving upward and away from that density fluctuation so that they both end up in the upper hemisphere. This response was described in Ref. \[22\] and was illustrated as “Position B” in Fig. IV of that reference. For energies below 200 GeV, \(C_{1,2,3}\) changes sign so that \(\cos(\phi_1' + \phi_2')\) and \(\cos(1\phi_1' - 3\phi_2')\) are both negative while \(\cos(2\phi_1' - 4\phi_2')\) and \(\cos(3\phi_1' - 5\phi_2')\) are both positive. This condition does not match any of the scenarios in the table but it could indicate an increased preference for particle pairs with \(\phi_1' \approx 0\) and \(\phi_2' \approx \pi\). A preference for back-to-back particle pairs aligned with the reaction plane would be consistent with an increased importance for momentum conservation at lower energies. Momentum conservation naturally leads to a tendency for particles to be emitted with back-to-back azimuthal angles \[40\]. As the beam energy is decreased, the multiplicity decreases and we should expect the effects of momentum conservation to become more prominent (in the case that only two particles are emitted, they must be back-to-back). The implications of this change in the configuration of two-particle correlations with respect to the reaction plane deserves further theoretical investigation.

The discussion in the above paragraph illustrates how measurements of \(C_{m,n,m+n}\) reveal information about two-particle correlations with respect to the reaction plane and we pointed out two specific conclusions based on the \(p_T\)- and \(\Delta\eta\)-integrated measurements. The value of \(C_{1,2,3}\) changes sign as a function of centrality, \(\Delta\eta\) and \(p_T\) suggesting that further specific configurations may arise when triggering on a particular \(p_T\) or investigating particles separated by an \(\eta\)-gap. We have not examined the charge dependence of \(C_{m,n,m+n}\) but future work placing a like-sign or unlike-sign requirement on \(\phi_1'\) and \(\phi_2'\) may be useful for interpreting charge separation measurements and determining whether they should be taken as evidence for the chiral magnetic effect.

### Table I

| \((\phi_1', \phi_2') [\text{rad}]\) | \((0, 0)\) | \((0, \pi)\) | \((-\pi, 0)\) | \((-\pi, \pi)\) | \((-\pi, 2\pi)\) |
|---|---|---|---|---|---|
| \(C_{1,1,2}/v_2\) | +1 | -1 | -1 | +1 | -1 |
| \(C_{1,2,3}/v_2\) | +1 | -1 | -1 | +1 | +\(\frac{1}{2}\) |
| \(C_{2,2,4}/v_2\) | +1 | +1 | -1 | -1 | +1 |
| \(C_{2,3,5}/v_2\) | +1 | -1 | -1 | +1 | +\(\frac{1}{2}\) |

IV. CONCLUSIONS

We presented measurements of the energy, centrality, \(p_T\), and \(\Delta\eta\) dependence of three-particle azimuthal corrections \(C_{m,n,m+n}\) for a variety of combinations of \(m\) and \(n\). We find a strong dependence of \(C_{1,1,2}\) on \(|\eta_1 - \eta_2|\) and a strong dependence of \(C_{1,2,3}\) on \(|\eta_1 - \eta_3|\). Meanwhile, \(C_{2,2,4}\) and \(C_{2,3,5}\) exhibit a smaller but still appreciable dependence on \(|\eta_1 - \eta_3|\). This may indicate either the presence of short-range non-flow correlations or a rapidity dependence to the initial energy density signaling a breaking of longitudinal invariance. Simple pictures of non-flow however, appear to be inconsistent with the overall trends observed in the data. The integrated correlations with \(m = 1\) are generally negative or consistent with zero except for \(C_{1,2,3}\) which, at 200 GeV, is positive for mid-central collisions while it is negative for all centralities at all of the lower energies. Nonzero values for \(C_{1,2,3}\) imply correlations between the second and third harmonic event plane that are predicted from models of the initial overlap geometry. The \(p_T\) dependence of the correlations exhibits trends suggesting significant differences between the correlations in peripheral collis-
sions and more central collisions as well as differences for \( p_T > 5 \text{ GeV/c} \) and \( p_T < 5 \text{ GeV/c} \). The quantity \( C_{1,2,3} \) as a function of \( p_T \), changes sign as many as three times. While \( C_{1,2} \) is negative for higher energies, it becomes positive or consistent with zero at 7.7 GeV. By examining the energy dependence of \( C_{1,1,2}, C_{1,2,3}, C_{2,2,4} \), and \( C_{2,3,5} \), divided by \( v_2 \), we are able to infer that in mid-central collisions at 200 GeV, there is a preference for particle pairs to be emitted with angles relative to the reaction plane of either \( \phi_1 \approx \pi/3 \) and \( \phi_2 \approx 2\pi/3 \) or \( \phi_1 \approx -\pi/3 \) and \( \phi_2 \approx -2\pi/3 \). At 62.4 GeV and below, this appears to change due to a possible preference for back-to-back pairs (\( \phi_1 \approx 0 \) and \( \phi_2 \approx \pi \)) aligned with the reaction plane. These data will be useful for constraining hydrodynamic models. In order to facilitate such future data-model comparisons we also include the measurements of \( v_n^2 \{2\} \), \( n = 1, 2, 4, 5 \), over a wide range of energy, in the appendix of this paper. Measurements of the charge dependence of the correlations presented here, by revealing information about the preferred directions of correlated particles with respect to the reaction plane, should provide valuable insights into whether or not the charge separation observed in heavy-ion collisions is related to the chiral magnetic effect.

\section{SUMMARY}

The very first measurement of charge inclusive three-particle azimuthal correlations from the RHIC beam energy scan program, presented in this paper, can provide several new insights into the initial state and transport in heavy ion collisions. These observables go beyond conventional flow harmonics and provide the most efficient way of studying the correlation between harmonic amplitudes and their phases over a wide range of multiplicities. These observables are well defined and of general interest even when the azimuthal correlations are not dominated by hydrodynamic flow. The major finding of this analysis is the strong relative pseudorapidity (\( \Delta \eta \)) dependence between the particles associated with different harmonics, observed up to about two units (\( \Delta \eta \approx 2 \)) of separation. Non-flow based expectations such as fragmentation (flat in \( \Delta \eta \)) can not provide a simple explanation to such observations. If the observed correlations are dominated by flow, the current results strongly hint at a breaking of longitudinal invariance of the initial state geometry at RHIC. The comprehensive study of momentum and centrality dependence of three-particle correlations over a wide range of energy (7.7-200 GeV), presented here, will help reduce the large uncertainties in the transport parameters involved in hydrodynamic modeling of heavy ion collisions over a wide range of temperature and net-baryon densities. In addition, the charge inclusive three-particle correlations will provide baselines for the measurements of the chiral magnetic effect.

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\section{Appendix A: Two-particle Cumulants \( v_n^2 \{2\} \)}

In this appendix we present the measurements of \( v_n^2 \{2\} \) for \( n = 1, 2, 4 \) and 5. The second harmonic \( v_2^2 \{2\} \) was used to scale \( C_{m,n,m+n} \) in Fig. 13. Under the assumption that

\[
\langle \cos(m\phi_1 + n\phi_2 - (m + n)\phi_3) \rangle \approx \langle v_m v_n v_{m+n} \rangle \cos(m\Psi_m + n\Psi_{m+n} - (m + n)\Psi_{m+n})
\]

where \( \Psi_m \) is the participant plane angle for harmonic \( m \), one can convert the \( C_{m,n,m+n} \) correlations into reaction plane correlations in the low-resolution limit by dividing by \( \sqrt{v_m^2 \{2\} v_n^2 \{2\} v_{m+n}^2 \{2\}} \). The relationship of the \( C_{m,n,m+n} \) to \( v_m \) and \( \Psi_m \) assumes that non-flow correlations are minimal. Similar assumptions must also be made when using event-plane angles in the analysis. The analysis of \( v_2^2 \{2\} \) was performed in a similar manner to that of \( v_3^2 \{2\} \) presented in Ref. 13. The \( \Delta \eta \) dependence of \( \langle \cos(2(\phi_1 - \phi_2)) \rangle \) is analyzed for \( p_T > 0.2 \text{ GeV/c} \) and \( |\eta| < 1 \). Short-range correlations are parameterized with a narrow Gaussian peak centered at \( \Delta \eta = 0 \) and the remaining longer-range correlations are integrated (weighting by the number of pairs at each \( \Delta \eta \)) to obtain the \( \Delta \eta \)-integrated \( v_2^2 \{2\} \) results. The quantity labeled \( v_2 \) in Fig. 13 is \( \sqrt{v_2^2 \{2\}} \).

Figure 14 shows the results for \( v_2^2 \{2\} \) (left) and \( v_3^2 \{2\} \) (right) as a function of centrality for 200, 62.4, 39, 27, 19.6, 14.5, 11.5, and 7.7 GeV Au+Au collisions. The data are scaled by \( N_{\text{part}} \) and plotted verses \( N_{\text{part}} \) for convenience. At 200 GeV, \( v_2^2 \{2\} \) is positive for central collisions but becomes negative for \( N_{\text{part}} < 150 \). The negative values are expected from momentum conservation and present a conceptual challenge for dividing \( C_{m,n,m+n} \) by \( \sqrt{v_2^2 \{2\}} \). The values of \( v_2^2 \{2\} \) become more negative.
at lower energies. This is consistent again with momentum conservation effects which are expected to become stronger as multiplicity decreases. In the limit of a collision that produces only two particles, momentum conservation would require that \( v_1^2(2) = -1 \). The \( v_1^2(2) \) results follow a monotonic energy trend except for peripheral collisions at 19.6 GeV which appear to be elevated with respect to the trends. The right panel of Fig. 14 shows the results for \( N_{\text{part}} v_2^2(2) \) which remain positive for all energies and collision centralities. While it is unusual to scale \( v_2^2 \) by \( N_{\text{part}} \), we keep this format for consistency. The scaled results exhibit a strong peak for mid-central collisions due to the elliptic geometry of those collisions.

Figure 15 shows the data for \( N_{\text{part}} v_4^2(2) \) (left) and \( N_{\text{part}} v_5^2(2) \) (right) for a more limited energy range. Results for \( N_{\text{part}} v_4^2(2) \) are available in Ref. [18]. At the lower energies the relative uncertainties on these data become too large to be of use. This presents another challenge to recasting \( C_{m,n,m+n} \) in terms of reaction plane correlations because scaling by \( \sqrt{v_4^2(2)} \) or \( \sqrt{v_5^2(2)} \) leads to a large uncertainty on the resulting ratios.

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[1] J. C. Collins and M. J. Perry, “Supersdense Matter: Neutrons Or Asymptotically Free Quarks?,” Phys. Rev. Lett. 34, 1353 (1975).
[2] S. A. Chin, “Transition to Hot Quark Matter in Relativistic Heavy Ion Collision,” Phys. Lett. B 78, 552 (1978).
[3] J. I. Kapusta, “Quantum Chromodynamics at High Temperature,” Nucl. Phys. B 148, 461 (1979).
[4] R. Anishetty, P. Koehler and L. D. McLerran, “Central Collisions Between Heavy Nuclei at Extremely High Energies: The Fragmentation Region,” Phys. Rev. D 22, 2793 (1980).
[5] S. Borsanyi et al. [Wuppertal-Budapest Collaboration], “Is there still any \( T_c \) mystery in lattice QCD? Results with physical masses in the continuum limit III,” JHEP 1009, 073 (2010).
[6] T. Bhattacharya et al., “QCD Phase Transition with Chiral Quarks and Physical Quark Masses,” Phys. Rev. Lett. 113, 082001 (2014).
[7] Y. Aoki, G. Endrodi, Z. Fodor, S. D. Katz and K. K. Szabo, “The Order of the quantum chromodynamics transition predicted by the standard model of particle physics,” Nature 443, 675 (2006).
[8] M. M. Aggarwal et al. [STAR Collaboration], “An Experimental Exploration of the QCD Phase Diagram: The Search for the Critical Point and the Onset of Deconfinement,” arXiv:1007.2613 [nucl-ex].
[9] S. A. Voloshin, A. M. Poskanzer, R. Snellings, “Collective phenomena in non-central nuclear collisions,” arXiv:0809.2949 [nucl-ex]; P. Sorensen, “Elliptic Flow: A Study of Space-Momentum Correlations In Relativistic Nuclear Collisions,” arXiv:0905.0174 [nucl-ex]; H. G. Ritter and R. Stock, “Collective Flow of QCD Matter: a Historical Introduction,” J. Phys. G 41, 124002 (2014).
[10] J. Adams et al. [STAR Collaboration], “Minijet deformation and charge-independent angular correlations on momentum subspace (eta, phi) in Au-Au collisions at \( \sqrt{s_{NN}} = 130 \) GeV,” Phys. Rev. C 73, 064907 (2006); J. Adams et al. [STAR Collaboration], “Delta phi Delta eta Correlations in Central Au+Au Collisions at \( \sqrt{s_{NN}} = \)
non-trivial dependence and centrality dependence of $N_{\text{part}}v_4(2)$ (left) and $N_{\text{part}}v_5(2)$ (right) after short-range correlations, predominantly from quantum and Coulomb effects, have been subtracted. For more details see Ref. [18].

200 GeV,” Phys. Rev. C 75, 034901 (2007); A. Adare et al. [PHENIX Collaboration], “Dihadron azimuthal correlations in Au+Au collisions at $\sqrt{s_{NN}} = 200$ GeV,” Phys. Rev. C 78 (2008) 014901; B. I. Abelev et al. [STAR Collaboration], “Long range rapidity correlations and jet production in high energy nuclear collisions,” Phys. Rev. C 80, 064912 (2009); B. Alver et al. [PHOBOS Collaboration], “High transverse momentum triggered correlations over a large pseudorapidity acceptance in Au+Au collisions at $\sqrt{s_{NN}} = 200$ GeV,” Phys. Rev. Lett. 104, 062301 (2010); S. Chatrchyan et al. [CMS Collaboration], “Long-range and short-range dihadron angular correlations in central PbPb collisions at a nucleon-nucleon center of mass energy of 2.76 TeV,” JHEP 1107, 076 (2011); K. Aamodt et al. [ALICE Collaboration], “Harmonic decomposition of two-particle angular correlations in PbPb collisions at $\sqrt{s_{NN}} = 2.76$ TeV,” Phys. Lett. B 708, 249 (2012); G. Aad et al. [ATLAS Collaboration], “Measurement of the azimuthal anisotropy for charged particle production in $\sqrt{s_{NN}} = 2.76$ TeV lead-lead collisions with the ATLAS detector,” Phys. Rev. C 86, 014907 (2012).

[11] S. A. Voloshin, “Two particle rapidity, transverse momentum, and azimuthal correlations in relativistic nuclear collisions and transverse radial expansion,” Nucl. Phys. A 749, 287 (2005).

[12] A. P. Mishra, R. K. Mohapatra, P. S. Saumia, A. M. Srivastava, “Super-horizon fluctuations and acoustic oscillations in relativistic heavy-ion collisions,” Phys. Rev. C77, 064902 (2008).

[13] P. Sorensen, “Searching for Superhorizon Fluctuations in Heavy-Ion Collisions,” arXiv:0808.0503 [nucl-ex]; P. Sorensen, “Implications of space-momentum correlations and geometric fluctuations in heavy-ion collisions,” J. Phys. G 37, 094011 (2010).

[14] J. Takahashi et al., “Topology studies of hydrodynamics using two particle correlation analysis,” Phys. Rev. Lett. 103, 242301 (2009).

[15] P. Sorensen, B. Bolliet, A. Mocay, Y. Pandit and N. Pruthi, “The Rise and Fall of the Ridge in Heavy Ion Collisions,” Phys. Lett. B 705, 71 (2011).

[16] K. Aamodt et al. [ALICE Collaboration], “Higher harmonic anisotropic flow measurements of charged particles in Pb-Pb collisions at $\sqrt{s_{NN}}=2.76$ TeV,” Phys. Rev. Lett. 107, 032301 (2011), A. Adare et al. [PHENIX Collaboration], “Measurements of Higher-Order Flow Harmonics in Au+Au Collisions at $\sqrt{s_{NN}} = 200$ GeV,” Phys. Rev. Lett. 107, 252301 (2011), L. Adamczyk et al. [STAR Collaboration], “Third Harmonic Flow of Charged Particles in Au+Au Collisions at $\sqrt{s_{NN}} = 200$ GeV,” Phys. Rev. C 88, 014904 (2013).

[17] C. Adler et al. [STAR Collaboration], “Elliptic flow from two and four particle correlations in Au+Au collisions $\sqrt{s_{NN}} = 130$ GeV,” Phys. Rev. C 66, 034904 (2002).

[18] L. Adamczyk et al. [STAR Collaboration], “Beam Energy Dependence of the Third Harmonic of Azimuthal Correlations in Au+Au Collisions at RHIC,” Phys. Rev. Lett. 116, 112302 (2016).

[19] R. S. Bhalerao, J. Y. Ollitrault and S. Pal, “Event-plane correlators,” Phys. Rev. C 88, 024909 (2013).

[20] J. D. Bjorken, “Highly Relativistic Nucleus-Nucleus Collisions: The Central Rapidity Region,” Phys. Rev. D 27, 140 (1983).

[21] G. Denicol, A. Monnai and B. Schenke, “Moving forward to constrain the shear viscosity of QCD matter,” Phys. Rev. Lett. 116, 212301 (2016).

[22] D. Teaney and L. Yan, “Triangularity and Dipole Asymmetry in Heavy Ion Collisions,” Phys. Rev. C 83, 064904 (2011).

[23] Z. Qiu and U. Heinz, “Hydrodynamic event-plane correlations in Pb+Pb collisions at $\sqrt{s} = 2.76$ATeV,” Phys. Lett. B 717, 261 (2012).

[24] D. Teaney and L. Yan, “Event-plane correlations and hydrodynamic simulations of heavy ion collisions,” Phys. Rev. C 90, 024902 (2014).

[25] H. Niemi, K. J. Eskola and R. Paatelainen, “Event-by-event fluctuations in perturbative QCD + saturation + hydro model: pinning down QCD matter shear viscosity in ultrarelativistic heavy-ion collisions,”
[26] S. Ryu, J.-F. Paquet, C. Shen, G. S. Denicol, B. Schenke, S. Jeon and C. Gale, “Importance of the Bulk Viscosity of QCD in Ultrarelativistic Heavy-Ion Collisions,” Phys. Rev. Lett. 115, 132301 (2015).

[27] B. I. Abelev et al. [STAR Collaboration], “Azimuthal Charged-Particle Correlations and Possible Local Strong Parity Violation,” Phys. Rev. Lett. 103, 251601 (2009).

[28] B. I. Abelev et al. [STAR Collaboration], “Observation of charge-dependent azimuthal correlations and possible local strong parity violation in heavy ion collisions,” Phys. Rev. C 81, 054908 (2010).

[29] D. Kharzeev, R. D. Pisarski and M. H. G. Tytgat, “Possibility of spontaneous parity violation in hot QCD,” Phys. Rev. Lett. 81, 512 (1998).

[30] D. Kharzeev, “Parity violation in hot QCD: Why it can happen, and how to look for it,” Phys. Lett. B 633, 260 (2006).

[31] S. A. Voloshin, “Parity violation in hot QCD: How to detect it,” Phys. Rev. C 70, 057901 (2004).

[32] STAR Collaboration, C. Adler et al., Nucl. Instr. Meth. A 499, 624 (2003).

[33] A. Bilandzic, R. Snellings and S. Voloshin, “Flow analysis with cumulants: Direct calculations,” Phys. Rev. C 83, 044913 (2011); A. Bilandzic, C. H. Christensen, K. Gulbrandsen, A. Hansen and Y. Zhou, “Generic framework for anisotropic flow analyses with multiparticle azimuthal correlations,” Phys. Rev. C 89, 064904 (2014).

[34] B. I. Abelev et al. [STAR Collaboration], “Systematic Measurements of Identified Particle Spectra in pp, d+ Au and Au+Au Collisions from STAR,” Phys. Rev. C 79, 034909 (2009).

[35] M. L. Miller, K. Reygers, S. J. Sanders and P. Steinberg, “Glauber modeling in high energy nuclear collisions,” Ann. Rev. Nucl. Part. Sci. 57, 205 (2007).

[36] G. Aad et al. [ATLAS Collaboration], “Measurement of event-plane correlations in \( \sqrt{s_{NN}} = 2.76 \) TeV lead-lead collisions with the ATLAS detector,” Phys. Rev. C 90, 024905 (2014).

[37] M. Luzum and J. Y. Ollitrault, “Eliminating experimental bias in anisotropic-flow measurements of high-energy nuclear collisions,” Phys. Rev. C 87, 044907 (2013).

[38] M. A. Lisa, S. Pratt, R. Soltz and U. Wiedemann, “Femtoscopy in relativistic heavy ion collisions,” Ann. Rev. Nucl. Part. Sci. 55, 357 (2005).

[39] Q. Y. Shou, Y. G. Ma, P. Sorensen, A. H. Tang, F. Videbck and H. Wang, “Parameterization of Deformed Nuclei for Glauber Modeling in Relativistic Heavy Ion Collisions,” Phys. Lett. B 749, 215 (2015).

[40] D. Teaney and L. Yan, “Non linearities in the harmonic spectrum of heavy ion collisions with ideal and viscous hydrodynamics,” Phys. Rev. C 86, 044908 (2012).

[41] L. Adamczyk et al. [STAR Collaboration], “Beam-energy dependence of charge separation along the magnetic field in Au+Au collisions at RHIC,” Phys. Rev. Lett. 113, 052302 (2014).

[42] STAR Collaboration, “Proposal for STAR TPC Inner Sector Upgrade (iTPC)”, STAR Note 0619: https://drupal.star.bnl.gov/STAR/starnotes/public/sn0619

[43] K. H. Ackermann et al. [STAR Collaboration], “Elliptic flow in Au + Au collisions at \( \sqrt{s_{NN}} = 130 \) GeV,” Phys. Rev. Lett. 86, 402 (2001); J. Adams et al. [STAR Collaboration], “Particle type dependence of azimuthal anisotropy and nuclear modification of particle production in Au + Au collisions at \( \sqrt{s_{NN}} = 200 \) GeV,” Phys. Rev. Lett. 92, 052302 (2004); J. Adams et al. [STAR Collaboration], “Azimuthal anisotropy in Au + Au collisions at \( \sqrt{s_{NN}} = 200 \) GeV,” Phys. Rev. C 72, 014904 (2005).

[44] D. Molnar, “Novel mechanism of high-p_T production from an opaque quark-gluon plasma,” [arXiv:1701.06497 [nucl-ex]].

[45] L. Adamczyk et al. [STAR Collaboration], “Constraining the initial conditions and temperature dependent transport with three-particle correlations in Au+Au collisions,” [arXiv:1701.06497 [nucl-ex]].

[46] L. Adamczyk et al. [STAR Collaboration], “Fluctuations of charge separation perpendicular to the event plane and local parity violation in \( \sqrt{s_{NN}} = 200 \) GeV Au+Au collisions at the BNL Relativistic Heavy Ion Collider,” Phys. Rev. C 88, no. 6, 064911 (2013).