Limitation of electron mobility from hyperfine interaction in ultra-clean quantum wells and topological insulators

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The study of electron transport and scattering processes limiting electron mobility in high-quality semiconductor structures is central to solid-state electronics. Here, we uncover an unavoidable source of electron scattering which is caused by fluctuations of nuclear spins. We calculate the momentum relaxation time of electrons in quantum wells governed by the hyperfine interaction between electrons and nuclei and show that this time drastically depends on the spatial correlation of nuclear spins. Moreover, the scattering processes accompanied by a spin flip are a source of the backscattering of Dirac fermions at conducting surfaces of topological insulators.

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I. INTRODUCTION

The invention of modulation-doped semiconductor structures [1] and the subsequent progress in semiconductor technology have led to the fabrication of ultrahigh-mobility two-dimensional electron systems [2, 3] and discovery of novel exciting quantum phenomena such as the fractional quantum Hall effect [4] and microwave-induced resistance oscillations [3]. Much effort is focused now on the search for new technological approaches and the optimization of quantum well (QW) design to reduce structure disorder and increase electron mobility. This raises the question of the fundamental limitation of electron mobility that could be achieved in defect-free QW structures with ideal interfaces, see Ref. [5] for a recent discussion.

Here, we analyze a source of electron scattering stemming from hyperfine interaction between electron spins and spins of nuclei constituting the crystal lattice. This scattering mechanism is unavoidable in III-V compounds since all stable and long-lived isotopes of anions (N, P, As, Sb) and cations (B, Al, Ga, In) possess non-zero nuclear spins. Hyperfine interaction in semiconductors has been extensively studied in the context of coupled electron and nuclear spin dynamics in bulk semiconductors [7], quantum dots (QDs) [8, 9] and QWs [10, 11], and also in the spin-dependent electron transport along edge channels of a two-dimensional electron gas [12, 13] or through QDs (spin-blockade effect) [20, 21], but not in bulk charge transport measurements. Here, we calculate the electron mobility limited by electron-nuclear hyperfine interaction in QWs for various spin configurations of the electron and nuclear subsystems including the case of unpolarized electrons and nuclei and the dynamic nuclear polarization (DNP). It is shown that the effect of electron-nuclear interaction on the electron mobility drastically depends on the spatial correlation of nuclear spins. Generally, both spin-conserving and spin-flip processes contribute to the electron scattering. The quadrupole splitting of the nuclear spin levels in strained QWs or the Zeeman splitting of the electron and nuclear levels in a magnetic field can suppress the spin-flip electron scattering by nuclear spin fluctuations at low temperatures. A uniform nuclear polarization achieved, e.g., by DNP, suppresses in turn the spin-conserving scattering processes. Spin-flip scattering is an unavoidable source of the backscattering of two-dimensional Dirac fermions emerging at conducting surfaces of topological insulators.

II. SCATTERING BY UNCORRELATED NUCLEAR SPINS

The effective Hamiltonian of hyperfine interaction between the conduction-band electrons and nuclei can be presented in the form [7]

$$ V = \sum_{\alpha,n} A_{\alpha} v_0 S \cdot I_{\alpha,n} \delta(r - R_{\alpha,n}) ,$$

(1)

where $\alpha$ is the index of nucleus species, $n$ enumerates nuclei of certain species, $A_{\alpha}$ are the constants of interaction, $v_0$ is the volume of the primitive cell, $S$ and $I_{\alpha,n}$ are the electron and nucleus spin operators, respectively, and $R_{\alpha,n} = (\rho_{\alpha,n}, z_{\alpha,n})$ are the positions of the nuclei [24].

We assume first that nuclei are unpolarized on average and their spin states are uncorrelated with each other. Then, the scattering of an electron by different nuclei occurs independently and the total probability of the scattering from the initial state $(k, s)$ to the final state $(k', s')$ is given by the golden rule rate

$$ W_{k',s',k,s} = \frac{2\pi}{\hbar} \sum_{\alpha,n} \sum_{j,j'} |M_{s,s'}^{(\alpha,n)}|^2 p_{\alpha j}$$

$$ \times \delta (\varepsilon_k + \varepsilon_s + \varepsilon_{\alpha j} - \varepsilon_{k'} - \varepsilon_{s'} - \varepsilon_{\alpha j'}) ,$$

(2)

where $k$ and $k'$ are the wave vectors in the QW plane, $s, s' = \pm 1/2$ are the electron spin projections, $j$ and $j'$ are the positions of the nuclei, $S_{\alpha,j}$ are the components of the nuclear spin operator on the $\alpha, j$ nucleus, $I_{\alpha,n}$ is the total nuclear spin operator on the $\alpha, n$ nucleus, and $p_{\alpha j}$ is the nuclear polarization of the $\alpha, j$ nucleus.
are the initial and final nuclear spin projections, $M^{(n,n)}_{s',j',s,j}$ is the matrix element of scattering at the potential given in Eq. (1), $p_{o,j}$ is the nuclear spin distribution function, $\varepsilon_k = \hbar^2 k^2/(2m^*)$ is the electron kinetic energy, $m^*$ is the effective mass, $\varepsilon_s$ is the energy related to electron spin, e.g., in an external magnetic field, and $\varepsilon_{o,j}$ are the nuclear energies. The hyperfine interaction is weak and short-range as compared to the de Broglie wavelength of electrons. It allows us to study the scattering in the first Born approximation and neglect screening.

The squared modulus of the scattering matrix element has the form

$$|M^{(n,n)}_{s',j',s,j}|^2 = A_n^2 v_{0}^2 |\langle s' j'| S \cdot I_{a,n} | s j \rangle|^2 \psi^4(z_{a,n}), \quad (3)$$

where $\psi(z)$ is the function of electron size quantization in the QW. The operator $S \cdot I$ can be rewritten in the form

$$S \cdot I = S_x I_x + (S_y I_+ - S_z I_-)/2, \quad (4)$$

where $S_\pm = S_x \pm i S_y$, $I_\pm = I_x \pm i I_y$, and $S_\beta$ and $I_\beta$ ($\beta = x,y,z$) are the Cartesian components, which yields

$$\langle s' j'| S \cdot I | s j \rangle = s j \delta_{s',s} \delta_{j',j} + \frac{1}{2} \sqrt{(I - j)(I + j + 1)} \delta_{s',s-1} \delta_{j',j+1} + \frac{1}{2} \sqrt{(I - j + 1)(I + j)} \delta_{s',s+1} \delta_{j',j-1}. \quad (5)$$

The electron relaxation time $\tau_p$ determining the mobility is expressed via the probability of scattering (2). To derive the expression for $\tau_p$, we follow Boltzmann’s approach and write down the collision integral, which plays the role of the “friction force” in Boltzmann’s equation,

$$\text{St} f_{k s} = \sum_{k' s'} \left[ f_{k' s'}(1-f_{k s}) W_{k, k' s'} - f_{k s} (1-f_{k' s'}) W_{k, k' s'} f_{k, k' s'}, \right] \quad (6)$$

where $f_{k s}$ is the electron distribution function. In the presence of a weak driving electric field, the distribution function has the form $f_{k s} = f_s^{(0)}(\varepsilon_k) + \delta f_{k s}$, where $f_s^{(0)}$ is the equilibrium function and $\delta f_{k s}$ is a small anisotropic correction. To first order in $\delta f_{k s}$, and for $W_{k, k' s'}$ independent of the directions of the wave vectors $k$ and $k'$, which is valid for short-range scattering by nuclei, Eq. (6) yields

$$\text{St} f_{k s} = -\frac{\delta f_{k s}}{\tau_p}, \quad (7)$$

where

$$\tau_p^{-1} = \sum_{k' s'} \left[ f_s^{(0)}(\varepsilon_{k'}) W_{k s, k' s'} + [1-f_s^{(0)}(\varepsilon_{k'})] W_{k, k' s'} f_{k s} \right], \quad (8)$$

Finally, for the scattering probability given by Eq. (2), homogeneously distributed nuclei and degenerate electron gas, we obtain

$$\tau_p^{-1} = \frac{m^*}{\hbar^3} \int \psi^4(z) dz \sum_{\alpha} A_n^2 v_{0}^2 N_{\alpha} \sum_{j' s'} |\langle s' j' | S \cdot I_{a,n} | s j \rangle|^2 \times \left[ p_{o,j'} + (p_{o,j'} - p_{o,j}) f_s^{(0)}(E_F + \varepsilon_{o,j} - \varepsilon_{s'} - \varepsilon_{o,j'}) \right] \times \delta(E_F + \varepsilon_{o,j} - \varepsilon_{s'} - \varepsilon_{o,j'}), \quad (9)$$

where $N_{\alpha}$ are the densities of nuclei of certain species, $E_F$ is the Fermi energy, and $\theta(z)$ is the Heaviside step function. We note that, for rectangular QWs with infinitely high barriers, $\int \psi^4(z) dz = 3/(2d)$ with $d$ being the QW width. Also note that, in general, $\tau_p$ can depend on the electron spin $s$ unless the nuclear-spin distribution is symmetric, $p_{o,j} = p_{o,j'}$. The time reversal symmetry imposing the condition $p_{o,j} = p_{o,j'}$ can be broken by an external magnetic field or DNP (studied below), spontaneously (see discussions on the possibility of nuclear self-polarization in Refs. [22]), or, in principle, by driving an electric current through the quantum well. In the latter case, the electric current induces a spin polarization of electrons due to spin-orbit coupling which results, in turn, in a build up of DNP [22]. The emerging electron and nuclear spin polarizations are both proportional to the electric current and, therefore, do not affect the linear electron transport. Non-linear effects are beyond the scope of this paper. Below, we analyze Eq. (9) for some particular cases which can be of interest.

(i) All spin states of nuclei are degenerate and equally populated, $\varepsilon_{o,j} = 0$, $p_{o,j} = 1/(2I_{a,n} + 1)$, where $I_{a,n}$ are the nuclear spins; electrons are unpolarized, $\varepsilon_s = 0$. In this case, both spin-conserving and spin-flip scattering processes are allowed. Taking into account that

$$\sum_j |\langle s j | S \cdot I | s j \rangle|^2 = I(I + 1)(2I + 1)/12,$$

$$\sum_j |\langle s \pm 1, j \mp 1 | S \cdot I | s, j \rangle|^2 = I(I + 1)(2I + 1)/6,$$

we obtain

$$\tau_p^{-1} = \frac{m^*}{\hbar^3} \int \psi^4(z) dz \sum_{\alpha} A_n^2 v_{0}^2 N_{\alpha} I_{a,n} (I_{a,n} + 1). \quad (10)$$

We note that spin-flip scattering processes in the context of electron and nuclear spin relaxation were theoretically considered in Ref. [12].

The degeneracy of nuclear spin levels may be lifted due to the quadrupole interaction of the nuclear spins with the strain-induced gradient of the crystal field in lattice-mismatched structures [3]. In a simple axial model relevant for (001)-oriented QWs, the quadrupole interaction determining the splitting and order of the spin levels is proportional to $I_{1}^2$, i.e., levels with different $j^2$ have different energies. The sign of the quadrupole splitting is opposite for axial tension and compression. Typical values of the quadrupole splitting in III-V heterostructures are of the order of $1 \sim 10$ meV which corresponds to $10^{-5} \sim 10^{-4}$ K at the temperature scale [3, 27]. Therefore, non-equal thermal population of the nuclear spin levels at sub-mK temperatures can occur and affect the electron-nuclear interaction.

(ii) Nuclear spin levels are split by strain in such a way that the ground levels are characterized by the highest spin projections $j = \pm I_{a,n}$, $I_{a,n} > 1/2$; electrons are
unpolarized, $\varepsilon_s = 0$. Temperature is lower than the
nuclear quadrupole splitting and, therefore, only ground
levels contribute to scattering, $p_{\alpha j} = 1/2$ if $j = \pm I_\alpha$
and $p_{\alpha j} = 0$ otherwise. Only spin-conserving scattering
processes can occur and the corresponding electron relaxation time is given by

$$\tau_p^{-1} = \frac{m^*}{4\hbar^3} \int \psi^4(z)dz \sum_\alpha A^2_{\alpha} v_0^2 N_\alpha \bar{T}_\alpha^2.$$ (11)

(iii) Nuclear spin levels are split by strain in such a
way that the ground levels are characterized by the
lowest spin projections $j = \pm 1/2$, $I_\alpha$ is half-integer; electrons
are unpolarized, $\varepsilon_s = 0$. Temperature is lower than the
quadrupole splitting energy and only ground levels con-
tribute to scattering, $p_{\alpha,\pm 1/2} = 1/2$. In this case, both
spin-flip and spin-conserving processes contribute to scat-
tering and the electron relaxation time has the form

$$\tau_p^{-1} = \frac{m^*}{8\hbar^3} \int \psi^4(z)dz \sum_\alpha A^2_{\alpha} v_0^2 N_\alpha [1/2 + (I_\alpha + 1/2)^2].$$ (12)

(iv) All spin states of nuclei are degenerate and equally
populated, $\varepsilon_{\alpha j} = 0$, $p_{\alpha j} = 1/(2I_\alpha + 1)$; electrons
are completely spin polarized by an external magnetic field
(thermal nuclear polarization is small and neglected). In
this particular case, only spin-conserving processes can occur and Eq. (9) yields

$$\tau_p^{-1} = \frac{m^*}{12\hbar^3} \int \psi^4(z)dz \sum_\alpha A^2_{\alpha} v_0^2 N_\alpha I_\alpha (I_\alpha + 1).$$ (13)

In III-V semiconductor structures, nuclear spins can be
efficiently polarized by DNP [7, 8]. In the case of uniform
nuclear polarization, the average nuclear field should be
excluded from the Hamiltonian of electron-nuclear in-
teraction causing the electron scattering because it does
not introduce any disorder which breaks the translational
symmetry of the crystal lattice. Accordingly, the opera-
tors $I_\alpha$ in Eqs. (9) and (10) should be replaced by $\vec{I}_\alpha$
where the vectors $\vec{I}_\alpha$ are given by

$$\vec{I}_\alpha = \sum_\beta A_\alpha N_\beta \vec{I}_\beta$$ \quad \text{(16)}

the index $\beta$ runs over the nuclei of anions or cations if $\alpha$
stands for a nucleus of an anion or a cation, respectively,
and $I_\alpha$ is the average nuclear spin of a certain isotope.
In the simple case of an isotopically pure crystal, where
all anions and cations are of certain isotopes, $\vec{I}_\alpha = \vec{I}_0$
We assume that all the vectors $\vec{I}_\alpha$ point along the same
axis $z$ and choose $z$ as the spin quantization axis. Then,
one obtains $\vec{I}_\alpha = j_\alpha = \sum j p_{\alpha j}$ and the scattering marix elements

$$\langle s^j | S \cdot (I_\alpha - \vec{I}_\alpha) | s^j \rangle = s (j - \vec{I}_\alpha) \delta_{s^j s^j} \delta_{j_j^j}$$
$$+ \frac{1}{2} \sqrt{(I_\alpha - j)(I_\alpha + j + 1)} \delta_{s^j, s^j} \delta_{j^j, j^j} + \frac{1}{2} \sqrt{(I_\alpha - j + 1)(I_\alpha + j)} \delta_{s^j, s^j} \delta_{j^j, j^j}.$$ (15)

Now we discuss the electron relaxation time in the presence
of DNP.

(v) The nuclei are spin polarized; electrons of both spin
states are present at the Fermi level. The splitting of
nuclear levels is lower than temperature so that both
spin-conserving and spin-flip processes can contribute to scat-
tering. In such conditions, the momentum relaxation time
becomes spin dependent and is given by

$$\tau_{p,s}^{-1} = \frac{m^*}{4\hbar^3} \int \psi^4(z)dz \sum_\alpha A^2_{\alpha} v_0^2 N_\alpha \left[I_\alpha^2 + \bar{T}_\alpha^2 - 2j_\alpha (I_\alpha + s)\right].$$ (16)

(vi) The nuclei are spin polarized; electrons are com-
pletely spin polarized by the nuclear (Overhauser) effec-
tive magnetic field or an external magnetic field. Then,
only spin-conserving scattering occur and we obtain the relaxation time

$$\tau_p^{-1} = \frac{m^*}{4\hbar^3} \int \psi^4(z)dz \sum_\alpha A^2_{\alpha} v_0^2 N_\alpha \left[I_\alpha^2 - 2j_\alpha \bar{T}_\alpha + \bar{T}_\alpha^2\right],$$ (17)

where $\bar{T}_\alpha = \sum_j j^2 p_{\alpha j}$. For isotopically purified crys-
tals, the expression in the square brackets is reduced to $I_\alpha^2 - j^2_\alpha$. This quantity is zero for fully polarized nuclei
when both $\bar{T}_\alpha$ and $j^2_\alpha$ are equal to $I_\alpha^2$. The absence of spin-conserving electron scattering by the fully polarized
nuclei, when the crystal translational symmetry is re-
stored, is in accordance with the free motion of Bloch
electrons in a periodic potential. In isotopically mixed
QW structures, the scattering may occur even in the case
of fully polarized nuclei due to the disorder caused by a
difference in the nuclear spins $I_\alpha$ and/or the interaction
constants $A_\alpha$ of anions (or cations). The corresponding relaxation time is given by

$$\tau_p^{-1} = \frac{m^*}{4\hbar^3} \int \psi^4(z)dz \sum_\alpha A^2_{\alpha} v_0^2 N_\alpha (I_\alpha - \bar{I}_\alpha)^2.$$ (18)

Now we estimate the electron relaxation time governed
by the scattering from unpolarized nuclei following
Eq. (11). The estimation for a 10-nm-wide QW
grown from GaAs, where $m^* \approx 0.067m_0$ with $m_0$
being the free electron mass, $I_{Ga} = 3/2$, $I_{As} = 3/2$,
and $v_0^2 \sum \alpha A^2_{\alpha} N_\alpha \approx 0.2 \times 10^{-24}$ meV^2 cm^3
(Ref. 28), gives $\tau_p \approx 10^{-5}$ s. This corresponds to the mobility
$\mu \approx 4 \times 10^{11}$ cm^2/Vs that is still a few orders of
magnitude higher than the mobility achieved to date [2].
The relaxation time governed by the hyperfine interaction
is much shorter in structures made of atoms with large
nuclear spins (e.g., $I_{In} = 9/2$) or heavy atoms, where the
interaction constants are larger.

The probabilities of spin-conserving and spin-flip pro-
cesses are comparable. Therefore, electron scattering by
unpolarized nuclei makes a contribution to electron spin
relaxation with the time $\tau_s$ comparable to the momentum
relaxation time $\tau_p$ calculated above. This spin re-
laxation mechanism can be important if other mecha-
nisms are suppressed, e.g., in (110)-oriented QWs where
the D’yakonov-Perel’ spin relaxation mechanisms is suppressed for the out-of-plane spin component \( \mathbf{k} \) and the spin lifetime up to 0.5 \( \mu s \) has been recently determined \cite{32}.

### III. SCATTERING OF DIRAC FERMIONS

Spin-flip processes contribute to the backscattering of two-dimensional Dirac fermions emerging at surfaces of 3D topological insulators (TIs). In such materials, the strong spin-orbit interaction giving rise to topologically protected surface states locks the carrier spin and momentum \( \mathbf{k} \). Therefore, elastic scattering between the states with the opposite momenta is forbidden in the presence of time reversal symmetry. Interaction with nuclear spins breaks the time reversal symmetry in the subsystem of the Dirac fermions and enables the backscattering. Electron-nuclear interaction leading to backscattering between one-dimensional helical edge states of 2D TIs was studied in Refs. \cite{18} and \cite{19}. Here, we calculate the probability of scattering for two-dimensional Dirac fermions on surfaces of 3D TIs and provide estimates for \( \text{HgTe and Bi}_2\text{Se}_3 \).

\( \text{HgTe} \) is a gapless semiconductor with the inverted band structure which becomes a 3D TI if strained and the strain opens a gap in the otherwise four-fold degenerate \( \Gamma_6 \) states \cite{33,35}. Within the 6-band \( \mathbf{k}-\mathbf{p} \) theory, relevant for narrow-band materials, the topological surface states are described by the wave functions

\[
\psi_{\mathbf{k}}(\mathbf{r}, z) = \sum_{m=\pm 1/2} \psi_{\Gamma_6,m}(z)|\Gamma_6, m\rangle + \sum_{m=\pm 1/2, \pm 3/2} \psi_{\Gamma_8,m}(z)|\Gamma_8, m\rangle \exp(i \mathbf{k} \cdot \mathbf{r}),
\]

where \( \psi_{\Gamma_6,m}(z) \) and \( \psi_{\Gamma_8,m}(z) \) are the envelope functions in the direction normal to the surface, \( |\Gamma_6, m\rangle \) and \( |\Gamma_8, m\rangle \) are the basis Bloch amplitudes of the \( s \)-type \( \Gamma_6 \) and \( p \)-type \( \Gamma_8 \) states, respectively, and \( \mathbf{k} \) is the wave vector in the surface plane. For strained \( \text{HgTe} \) films, the wave function \cite{19} contains considerable contribution (~20\%) of the \( \Gamma_6 \) states \cite{35}. Since the hyperfine interaction for \( s \)-type Bloch amplitudes given by the Fermi contact term is much stronger than that for \( p \)-type Bloch amplitudes \cite{8,30}, a good estimation is that the electron-nuclear interaction for the Dirac fermions at \( \text{HgTe} \) surface is of contact type and determined by the contribution of the \( \Gamma_6 \) states.

The electron spin of the surface states in the axial approximation lies in the surface plane and points perpendicular to the wave vector. Accordingly, the spinor composed of the functions \( \psi_{\Gamma_6, \pm 1/2}(z) \) can be presented in the form

\[
\begin{bmatrix}
\psi_{\Gamma_6, +1/2}(z) \\
\psi_{\Gamma_6, -1/2}(z)
\end{bmatrix} = \frac{1}{\sqrt{2}} \begin{bmatrix}
1 \\
i \exp(i \varphi_{\mathbf{k}})
\end{bmatrix} \psi_{\Gamma_6}(z),
\]

where \( \varphi_{\mathbf{k}} \) is the polar angle of the vector \( \mathbf{k} \). Following Eqs. \( (11) \) and \( (15) \) one can readily calculate the matrix element of scattering. For the scattering from the initial state \( \mathbf{k} \) to the final state \( \mathbf{k}' \) the squared modulus of the scattering matrix element has the form

\[
|M_{\mathbf{k}, \mathbf{k}'}, (\alpha, \mathbf{n})|^2 = \frac{1}{16} A_{\alpha}^2 (v_0^2) |\psi_{\Gamma_6}(z_{\alpha, \mathbf{n}})|^4 \left[ 4 \sin^2(\theta/2) j^2 \delta_{j,j'} + (I_\alpha - j + 1)(I_\alpha + j) \delta_{j,j+1} + (I_\alpha - j)(I_\alpha + j + 1) \delta_{j,j-1} \right],
\]

where \( \theta = \varphi_{\mathbf{k}'} - \varphi_{\mathbf{k}} \) is the angle of scattering.

The differential probability of elastic scattering of a test particle by the angle \( \theta \) in 2D systems is given by

\[
dw(\theta) = \frac{k}{2\pi v^2} \sum_{\alpha,n,j,j'} |M_{\mathbf{k}, \mathbf{k}', (\alpha, \mathbf{n})}|^2 p_{\alpha,j} d\theta,
\]

where \( v = (1/\hbar)\partial E/\partial k \) is the velocity which is independent of the energy for linearly dispersive Dirac fermions.

We assume that all spin states of the nuclei are degenerate and equally populated, \( p_{\alpha,j} = 1/(2I_\alpha + 1) \). Then, summing up over the nuclei, we obtain

\[
dw(\theta) = \frac{k}{24\pi v^2} \left[ 1 + \sin^2(\theta/2) \right] d\theta \times \int |\psi_{\Gamma_6}(z)|^4 dz \sum_{\alpha} A_{\alpha}^2 (v_0^2) N_{\alpha} I_\alpha (2I_\alpha + 1).
\]

Equation \((22)\) describes the scattering of two-dimensional Dirac fermions by nuclear spin fluctuations. It shows that the backscattering (\( \theta = \pi \)) determined by spin-flip processes is twice as efficient as the forward scattering (\( \theta = 0 \)) determined by spin-conserving processes.

In natural \( \text{HgTe} \), about 30\% of \( \text{Hg} \) nuclei and about 8\% of \( \text{Te} \) nuclei possess non-zero spins \cite{19}. To the best of our knowledge, the hyperfine interaction constants have not been measured yet. Considering the fraction of nuclei with non-zero spins, the typical parameters of surface states in \( \text{HgTe} \) films \cite{35}: \( k = 2 \times 10^6 \, \text{cm}^{-1} \), \( v = 0.5 \times 10^8 \, \text{cm/s} \), the characteristic length of the surface-state localization \( d = 10 \, \text{nm} \), and the \( \Gamma_6 \) band partition 0.2, we estimate that the probability of scattering by nuclear spin fluctuations is two-three orders of magnitude lower than that in \( \text{GaAs quantum wells} \).

Other prominent examples of 3D TIs are binary and ternary compounds of \( \text{Bi} \) with \( \text{Se} \) and \( \text{Te} \) \cite{33}. Natural \( \text{Bi} \) consists of the only isotope \( ^{209}\text{Bi} \) with the nuclear spin \( 9/2 \). Recent measurements of nuclear magnetic resonance in \( n \)-type \( \text{Bi}_2\text{Se}_3 \) crystals have revealed very strong contact interaction between electrons and \( ^{209}\text{Bi} \) nuclei (in spite of the fact that the Bloch amplitude is mostly of \( p \)-type) \cite{37}. The contact hyperfine interaction constant is found to be comparable and even exceed those for the conduction-band electrons in \( \text{GaAs} \). These results suggest that the scattering of Dirac fermions at the \( \text{Bi}_2\text{Se}_3 \) surface by nuclear spin fluctuations can be as efficient as that for electrons in \( \text{GaAs} \)-based structures.
IV. SCATTERING BY FLUCTUATIONS OF MACROSCOPIC NUCLEAR POLARIZATION

The electron-nuclear interaction is drastically enhanced if the nuclear spins are spatially correlated and polarized at a macroscopic scale. Such a nuclear spin polarization inhomogeneous in the QW plane can be created, e.g., via DNP by optical grating technique \[28-40\]. In the case of macroscopic nuclear polarization, the interaction can be described as the Zeeman term

\[ V(\rho) = g\mu_B S \cdot B_n(\rho) \]  

with the effective nuclear (Overhauser) magnetic field

\[ B_n(\rho) = \sum_{\alpha} (A_{\alpha} \psi_0 N_{\alpha}/g\mu_B) \int I_{\alpha}(\rho, z) \psi^2(z)dz \]  

which varies in the QW plane at a scale much larger than the crystal lattice constant. Here, \( \mu_B \) is the Bohr magneton, \( g \) is the effective electron \( g \)-factor, and the overline denotes quantum mechanical averaging over the ensemble of nuclear wave functions.

The effective magnetic field Eq. \[23\] produces a spin-dependent electron potential which causes the scattering. For the effective field \( B_n(\rho) \) oriented along a certain axis, e.g., the growth direction, the momentum relaxation time assumes the form

\[ \tau_p^{-1} = \frac{m^*(g\mu_B)^2}{8\pi^3 \hbar^3} \int_{-\pi}^{\pi} \langle B_n(\rho) \cdot B_n(\rho') \rangle_{\mathbf{q}} 2k_F \sin \theta/2 (1-\cos \theta) d\theta, \]  

where \( \langle B_n(\rho) \cdot B_n(\rho') \rangle_{\mathbf{q}} \) is the Fourier image of the spatial correlation function \( \langle B_n(\rho) \cdot B_n(\rho') \rangle \), \( k_F \) is the Fermi wave vector, and \( \theta \) is the angle of scattering.

To estimate the momentum relaxation time we assume that the nuclear polarization is randomly distributed in the QW plane with the characteristic correlation length \( l \) and zero mean value. Then, for GaAs-based QWs with the nuclear polarization \( F_n \sim 1 \), \( v_0 \sum_{\alpha} A_{\alpha} N_{\alpha} \approx 0.1 \text{ meV} \) (Ref. \[28\]), the Fermi wave vector \( k_F = 10^6 \text{ cm}^{-1} \), and the correlation length \( l \sim 1/k_F \), one obtains \( \tau_p \sim 10^{-9} \text{ s} \). Such \( \tau_p \) is comparable to the momentum relaxation time in high-mobility structures. It indicates that strong spatially inhomogeneous spin polarization of nuclei can considerably affect the electron transport in quantum wells and can be probed by electrical measurements. Spatially oscillating nuclear polarization created by optical grating technique may cause the Bragg diffraction of electrons and has even stronger impact on the electron transport.

To summarize, we have calculated the limitation of electron mobility in III-V quantum wells from the unavoidable source of disorder originating from the fluctuations of nuclear spins. We have analyzed various spin configurations of the electron and nuclear subsystems and shown that the electron mobility determined by the electron-nuclear hyperfine interaction drastically depends on the spatial correlation of nuclear spins. While the electron mobility limited by the hyperfine interaction with uncorrelated nuclear spins is still few orders of magnitude higher than that achieved in high-mobility quantum wells, nuclear spins that are spatially correlated and polarized at a macroscopic scale can considerably affect the electron transport in modern high-mobility quantum wells.

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