Biomechanical simulations with dynamic muscle paths on NURBS surfaces

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In order to model the behaviour of muscles that work around the joints in musculoskeletal simulations, information about the muscle paths are needed. Typically, musculotendon paths, their lengths, and their force directions are modeled as the shortest connection wrapping around obstacles representing bones and adjacent tissue. In this work, the shortest path problem is described via constrained variational dynamics and is applied to a biomechanical example with muscle paths on non-uniform rational B-spline (NURBS) surfaces representing the wrapping obstacles. The results are compared with muscle paths from a GI-continuous combination of geodesics \([2, 4, 5]\).

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1 Introduction

When simulating musculoskeletal motion with multibody systems representing bones and joints, the muscle paths are highly relevant for the muscle forces, moment arms and resulting body and joint loads. Typically, muscle paths cannot be adequately represented as straight lines because the anatomical structure of the human body forces the muscles to wrap around bones and adjacent tissue. Assuming that the muscles and tendons are always under tension, their path can be modelled as a locally length minimizing curve that wraps smoothly over adjacent obstacles \([2, 4, 5]\).

Due to the complex geometry of bones and tissue, the determination of muscle paths is very challenging. To model these wrapping obstacles in this work, the widely used and well known mathematical model of NURBS surfaces \([7]\) is used to create and display arbitrary surfaces.

2 Dynamic muscle paths on obstacle surfaces

Within this work, the muscle path is modeled as the shortest connection between two points on dynamic obstacle surfaces. Assuming that the muscle completely touches the surface, the Lagrangian is augmented by a scalar valued function of holonomic surface constraints \(\phi(\gamma) = 0 \in \mathbb{R}\) and a Lagrange multiplier \(\lambda \in \mathbb{R}\). Furthermore, the action is given by a scalar functional \(E[\gamma, \lambda] = \int_{s_0}^{s_3} \left( \frac{1}{2} \left( \gamma'(s) \right)^2 - \phi(\gamma) \cdot \lambda \right) \, ds\) of a curve \(\gamma \in C^2([s_0, s_3], \mathbb{R}^3)\) from start point \(\gamma(s_0)\) to the end point \(\gamma(s_3)\). The local minimizers of \(E\), that also locally minimize the length are the so called geodesics \([6]\). The variational principle \(\delta E = 0\) yields that for a stationary point \(\gamma\) of \(E\), the corresponding Euler-Lagrange equation (1) has to hold, where \(\Phi(\gamma) = \frac{\partial \phi(\gamma)}{\partial \gamma} \in \mathbb{R}^{1 \times 3}\) is the surface constraint Jacobian and \(\gamma'\) is the derivative of the geodesic curve with respect to \(s\). Given a nonsingular differentiable parametrization \(\gamma = F(\nu)\) in terms of surface coordinates \(\nu \in \mathbb{R}^2\), the Jacobian \(\frac{\partial F(\nu)}{\partial \nu} \in \mathbb{R}^{3 \times 2}\) can be used to project equation (1) into the tangent space of the manifold defined by the surface constraint. Thus, differential equation (2) is equivalent to equation (1) and the Jacobian \(\frac{\partial F(\nu)}{\partial \nu}\) plays the role of a null space matrix \([1, 3, 4]\). From the point of view of classical mechanics, the solution of equations (1) and (2) is the trajectories of a free particle on a constraint manifold. In fact, this means that the acceleration vector \(\frac{d}{ds} \frac{\partial \mathcal{L}(\gamma')}{\partial \gamma'}\) of the curve has no components in the normal direction of the surface and the motion is entirely determined by the surface curvature.

3 Musculoskeletal example

As a practical example, the lifting of the human arm from an outstretched initial configuration to a flexed elbow is examined. The multibody configuration \(q \in \mathbb{R}^{24}\) consist of two rigid bodies, which represent the upper and lower arm. For simplification, a revolute joint is used to model the elbow and the upper arm is fixed in space. Given a control net \(B_{ij}(q)\) with \(n \times m\) control points as function of the configuration \(q\), polynomial orders \(k\) and \(l\), the knot vectors \(u = [u_0, u_1, ..., u_{n+k+1}]\) and...
Fig. 1: **Left:** Muscle path of the biceps and the triceps around NURBS bone geometry approximation\(^2\). **Center:** Muscle path of the biceps and the triceps around multiple obstacles with G1-continuous transitions from [4]. **Right:** Length evolution of the biceps and triceps for two different wrapping surfaces.

\[ \mathbf{v} = [v_0, v_1, ..., v_{m+l+1}], \]  

a NURBS surface is given by the tensor product

\[ F(u, v) = \sum_{i=1}^{n} \sum_{j=1}^{m} B_{ij}(\mathbf{q}) N_{ij}(u,v) \]  

(3)

where \( N_{ij}(u,v) \) are the so-called rational basis functions, see [7]. By coupling the control network with the configuration, the NURBS surface moves together with the multibody system, forming a closed wrapping surface for all muscles. Fig. 1 shows the implemented exemplary wrapping surfaces for the biceps and the triceps. Moreover, the evolution of the muscle length is shown for two different wrapping formulations. On the left hand side, the bone geometry is approximated via a NURBS surface, while the muscles in the centre wrap around several cylinders and spheres with G1-continuous transitions, see [4]. The geometry of this example should be understood as a theoretical example and is not based on real anatomy data. However, the evolution of muscle lengths for the biceps and triceps shows qualitatively the same behaviour.

### 4 Conclusion

This work shows a muscle wrapping formulation on dynamic NURBS surfaces. The shortest connection between two points on an obstacle surface is derived using constrained variational dynamics. This allows to constrain the muscle path to arbitrarily complex obstacle surfaces, which can be represented by the widespread and well-known NURBS description. In the given example, the evolution of the muscle length and orientation shows the same qualitative behaviour as in widely used multi obstacle wrapping methods, see e.g. [4, 5]. The major difference is that the wrapping obstacle is modeled as a single closed surface and no calculation of G1-continuous transitions is necessary. Due to this, we expect that this formulation is well suited to be used in more complex musculoskeletal models, which is our next step.

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\(^2\) For the 3d bone model see https://www.thingiverse.com/thing:1543880.