EVOLUTIONARY EFFECTS OF IRRADIATION IN CATACLYSMIC VARIABLES

P. MCCORMICK \(^1\) AND J. FRANK \(^2\)
Department of Physics and Astronomy, Louisiana State University, Baton Rouge, LA 70803-4001

Received 1997 October 3; accepted 1998 February 2

ABSTRACT

The orbital evolution of cataclysmic variables in which the companion is illuminated by a fraction of the accretion luminosity consists of irradiation-driven limit cycles on thermal timescales, superimposed on secular evolution toward shorter periods due to systemic angular momentum losses. We show that positive orbital period derivatives during bright phases are a natural consequence of the expansion of the companion during high mass transfer phases in the limit cycle. The irradiation instability may be enhanced by consequential angular momentum losses \(J_{\text{CAML}}\) accompanying the limit cycle. We investigate the secular evolution of cataclysmic binaries under the combined effects of irradiation and \(J_{\text{CAML}}\) and show that faster-than-secular orbital period excursions of either sign may occur. We discuss whether the mass transfer fluctuations that occur during these cycles can account for the observed dispersion in disk luminosities or estimated accretion rates at a given orbital period. If indeed irradiation-driven and CAML-assisted mass transfer fluctuations on timescales faster than secular occur, as discussed in this paper, then we may be able to predict the relative abundances of the different types of cataclysmic variable at a given orbital period. For example this mechanism may explain the relative paucity of dwarf novae with respect to nova-like variables with periods between 3 and 4 hr.

Subject headings: accretion, accretion disks — binaries: close — instabilities — novae, cataclysmic variables — radiative transfer

1. INTRODUCTION

A cataclysmic variable (CV) is a semidetached binary star with an orbital period of a few hours (\(\leq 0.5\) days) containing an accreting white dwarf and a companion or mass donor star, which in most cases is a main-sequence hydrogen-burning star (see Warner 1995 for a comprehensive review of CVs; also see Frank, King, & Raine 1992b for an introduction to accretion physics). In such a binary, the donor star transfers mass to its compact companion via Roche lobe overflow. For this mass transfer to be dynamically and thermally stable, the companion must be less massive than the white dwarf and systemic angular momentum losses are required. Binary evolution, in the standard CV evolution scenario (see, e.g., King 1988; Warner 1995), is determined by the interplay between mass transfer, which tends to expand the orbit, and systemic angular momentum losses, which tend to shrink the orbit. Examples of systemic angular momentum losses include gravitational quadrupole radiation, as predicted from general relativity, and magnetic braking, due to mass loss from the companion along the star’s magnetic field lines, which produces a braking torque. The balance between systemic angular momentum losses and the tendency of the binary to expand usually results in a stable secular mass transfer rate whose magnitude is determined by the rate at which angular momentum is lost from the binary. The secular effect of these losses is to shrink the orbit and drive the binary to ever shorter periods until, late in the evolution, the companion becomes partially degenerate and begins to expand. Thus the shortest orbital period for a CV with a hydrogen-rich companion is \(P_{\text{min}} \approx 75\) minutes (Paczynski & Sienkiewicz 1981; Rappaport, Joss, & Webbink 1982), in agreement with the observed period distribution (see, e.g., Kolb 1996). Most CVs are expected to be in the phase of secular orbital contraction with the possible exception of some old systems—the TOADs—which may have evolved beyond the period minimum and are faint except when they undergo large-amplitude outbursts (Howell & Szkody 1990; Howell, Szkody, & Cannizzo 1995; Howell, Rappaport, & Politano 1997).

An obvious and important feature of the observed orbital period distribution of CVs known as the “period gap” is caused by the relative paucity of systems with periods between 2 and 3 hr. Currently, the best theoretical explanation for this feature is the interrupted magnetic braking picture (Spruit & Ritter 1983; Rappaport, Verbunt, & Joss 1983). According to this picture, magnetic braking either ceases or is drastically reduced near \(P_{\text{orb}} \approx 3\) hr as the companion becomes fully convective and the angular momentum loss rate drops suddenly by a factor of \(\sim 10\). The natural tendency of the binary to expand under mass transfer then briefly wins, the system detaches, and mass transfer halts. Since the companion has been driven out of thermal equilibrium by the mass loss sustained above the gap, the star returns to its thermal equilibrium radius and must wait until gravitational radiation grinds the orbit down to \(P_{\text{orb}} \approx 2\) hr when contact is reestablished and the binary becomes visible again as a CV.

The energy released by accretion in a disk or accretion column is radiated away and a fraction of it illuminates the side of the companion facing the white dwarf. The irradiation of the companion can influence the above-mentioned secular balance between angular momentum losses and mass transfer and under certain conditions can cause limit cycles. The stability of mass transfer in binaries with irradiated companions and the nature of the resulting limit cycles has been the subject of a number of recent papers (King 1995; King et al. 1995, 1996, 1997; Ritter, Zhang, & Kolb 1998; Hameury & Ritter 1997). Earlier studies of the reac-
tion of the companion to irradiation assumed that the illumination was either spherically symmetrical and steady (Podsiadlowski 1991; Harpaz & Rappaport 1991; Frank, King, & Lasota 1992a) or adopted other approximations and treated closely related situations (Gontikakis & Hameury 1993; Hameury et al. 1993; Harpaz & Rappaport 1995).

The dispersion in estimated mass transfer rates \( M_z \) at a given orbital period (Patterson 1984) or, similarly, the dispersion in estimates of accretion-disk absolute magnitudes at a given binary orbital period (Warner 1987; Warner 1995, see Fig. 9.8; Sproats, Howell, & Mason 1996) is unlikely to be the result of comparable dispersion in systemic angular momentum loss rates. If cataclysmic variables were driven at such diverse rates at the same orbital period over secular timescales, the predicted period minimum and the position and width of the period gap would be inconsistent with the observed period distribution (Verbunt 1984; Hameury, King, & Lasota 1989). Instead it has been proposed that the dispersion in \( M_z \) is the result of cyclical evolution on timescales too long to be observed directly but too short to affect the secular evolution significantly (Warner 1987; King 1995). The irradiation-driven limit cycles discussed above may well provide a physical mechanism for such cyclical evolution as proposed by King (1995) and King et al. (1996). In subsequent papers, it was shown that pure irradiation cycles are limited to orbital periods \( \gtrsim 6 \text{ hr} \) because the thermal inertia of the convective envelope of the companion increases rapidly toward lower masses (King et al. 1996; Ritter et al. 1998). King et al. (1996) noted that a positive feedback between thermal limit cycles and angular momentum losses (e.g., by coupled fluctuations in the rate of magnetic braking) would extend the range of the instability to lower masses and shorter orbital periods.

This paper explores the above scenario in detail using numerical simulations. The effects of cyclical changes in the mass transfer rate on the observable orbital parameters of compact binaries are investigated under various assumptions including both irradiation and coupled fluctuations in angular momentum losses. Increasing the systemic losses in proportion to the mass transfer during a thermal cycle is effectively a form of consequential angular momentum loss (CAML; Webbink 1985) and affects the overall stability of the binary (King & Kolb 1995).

2. BIPOLYTROPIC CODE

In a bipolytropic binary evolution code, the structure of the donor star is represented by a composite polytropic model. Much of the following draws heavily on work done by Chandrasekhar (1939), Rappaport et al. (1982), Rappaport et al. (1983), and Kolb & Ritter (1992). In the polytropic model, pressure \( P \) is taken to be a function of only density \( \rho \):

\[
\rho = \rho_1 \theta_1^{\alpha_1}, \quad P = K_1 \rho_1^{\alpha_1 + 1} \theta_1^{-1} \theta_1^{\alpha_1 + 1}
\]

where \( K \) is constant. The equation of state is represented as

\[
P = \frac{k \rho v T D}{m \mu},
\]

where \( k \) is the Boltzmann's constant, \( m \) is the atomic mass unit, \( \mu \) is the mean molecular weight, and \( D \) is an electron degeneracy factor (ratio of total gas pressure to ideal gas pressure). The bipolytropic model divides the lower main-sequence star (stars with masses less than 1.2 \( M_\odot \)) into an inner radiative region ("1" or "core") with a polytropic index of 1.5–4, surrounded by a convective region ("2" or "envelope") with polytropic index 1.5. The boundary of these two regions is determined by a parameter \( Q \), which is the ratio of the radiative region's radius to the total stellar radius, so that \( R = 0 \) is a fully convective star and \( Q = 1 \) is a fully radiative star. The free parameters \( n_1 \) (the polytropic index in the radiative region) and \( h_1 \) (the entropy jump between the top of the radiative region and the surface of the star) are adjusted to match theoretical and observed lower main sequence radii and luminosities closely. We write the densities and pressure as

\[
\rho_1 = \rho_1 \theta_1^{\alpha_1}, \quad P_1 = K_1 \rho_1^{\alpha_1 + 1} \theta_1^{-1} \theta_1^{\alpha_1 + 1}
\]

in the radiative region, and

\[
\rho_2 = \rho_2 \theta_2^{\alpha_2}, \quad P_2 = K_2 \rho_2^{\alpha_2 + 1} \theta_2^{-1} \theta_2^{\alpha_2 + 1}
\]

in the convective region. Here "c" denotes quantities evaluated at the center of the star, and "i" denotes quantities evaluated at the core/envelope interface, and \( \theta \) is the dimensionless temperature obtained from the Lane-Emden equation,

\[
1 \frac{d}{\xi} \left( \frac{\xi^2 d\theta}{d\xi} \right) = -\theta^j, \quad j = 1, 2
\]

We solve the Lane-Emden equation for \( \theta \) as a function of \( \xi \) (dimensionless radius), with \( n_1 = 1.5, 2.0, \ldots 4.0 \) in region 1 and \( n_2 = 1.5 \) in region 2. Solutions are evaluated at the core/envelope interface, approaching from region 2 and from region 1, as well as at the stellar surface. These solutions are labeled \( 1, 1i, 1s, 1z \) respectively. A table of \( \theta_{1i}, \xi_{1s}, \theta_{1s}, \xi_{1z}, \), and \( (-\xi_1^2 \theta_{1s})^{-1/3} \) (where primes denote radial derivatives, is computed for \( \approx 50 \) \( Q \) values. The above quantities allow one to calculate the pressure and density at the interface, the mass of the radiative region, the total mass of the star, and the polytropic constant in the convective region, respectively. Our bipolytrope code includes some modest improvements in the treatment of the degeneracy, which are important for low masses but is otherwise very similar to the version described in more detail in Kolb & Ritter (1992).

In the evolutionary calculations presented here, the stellar radius, binary separation and Roche lobe radius all change on timescales comparable to the thermal timescale of the convective envelope (King et al. 1996, 1997). Therefore, the stationary mass transfer equation does not apply, and we must calculate the mass transfer using (Ritter 1988; Kolb & Ritter 1990)

\[
M_2 = M_0 \exp \left( \frac{R_2 - R_1}{H_P} \right)
\]

where \( H_P = \epsilon R_2 \) is the pressure scale height, and we take \( M_0 = -10^{-8} M_\odot \text{ yr}^{-1} \). The exact value of this constant does not matter as we are not interested in the precise depth of contact which yields the stationary mass transfer. In order to reduce trivial numerical instabilities (which can be overcome by smaller timesteps), we adopt \( \epsilon = 10^{-7} \) rather than \( \epsilon = 10^{-4} \), which is more appropriate for near-main-sequence companions. However, we have done some calculations with \( \epsilon = 10^{-4} \) and find qualitatively similar results. Including the smaller \( \epsilon = 10^{-4} \) results in a higher frequency
of mass transfer oscillations and a slightly higher peak mass transfer rate. In every timestep, we calculate the mass transfer, remove that mass from the secondary, adjust the binary according to the angular momentum and mass losses experienced, and then move to the next period and iterate.

Using our bipolytrope code, we are able to reproduce all of the standard evolution of CVs. This includes the existence of a minimum period and a period gap between approximately 2 and 3 hr, mass transfer rates ∼1–2 orders of magnitude larger above the gap than below, and a “flag” when contact is reached at the lower limit of the period gap. A simple example of a set of standard evolutions coming into contact at different periods and converging to a common evolutionary path is shown in Figure 1 (see also Stehle, Ritter, & Kolb 1996).

3. CYCLIC EVOLUTION WITH IRRADIATION

After a brief discussion of previous related work, we present results of our simulations for cyclic evolution driven by irradiation with and without taking into account mass loss from the primary during nova explosions.

Podsiadlowski (1991) showed that in the spherically symmetric irradiation case with a flux of $F_{\text{irr}} \approx 10^{11} - 10^{12}$ ergs cm$^{-2}$ s$^{-1}$, a lower main-sequence $(M_2 \lesssim 0.8 ~ M_\odot)$ star’s radius will swell significantly. The expansion occurs on the thermal timescale and eventually the star will become fully radiative. The irradiated surface acts as a thermal blanket causing the star to lose energy in a less efficient manner. A low-mass star under these conditions may expand to a few times its original equilibrium radius as a result of the star trying to store the blocked energy in its gravitational and internal energy. In the completely spherical case, this is the only place the excess energy can be diverted. In the non-spherically symmetric case, which is more likely to reflect the true nature of the irradiation in binaries, the excess energy can be transported efficiently by convection and released through the unirradiated side. This has the effect of diminishing the expansion of the star. However, even when a fraction of the star’s surface is illuminated, the resultant radial expansion of a few percent is still much larger than the atmospheric scale height and may cause mass transfer cycles of large amplitude. In our quantitative treatment of this effect, we adopt the simple model introduced by Ritter et al. (1998; hereafter RZK) in which a one-zone approximation is used for the superadiabatic layers. In the one-zone model, convection is assumed to be adiabatic up to the base of the superadiabatic zone $(V = V_{\text{ad}})$. In the superadiabatic zone, convection is an ineffective method of energy transport, so the simplifying assumption is made that energy is transferred via radiation only $(V = V_{\text{rad}})$. Irradiation reduces the temperature gradient $V$ by raising the photospheric temperature and thus the rate at which energy can be lost through the superadiabatic zone. In this way, the superadiabatic zone acts as a valve that is open when there is no irradiation and restricts the amount of energy that can be released as the amount of irradiation is increased.

In order to describe the effective irradiation region, we assume that the radiation is coming from a point source. The irradiation flux at the stellar surface is

$$F_{\text{irr}}(\theta) = \frac{\eta}{4\pi} \frac{G M_2 M_1^2}{R_i a^2} h(\theta).$$

In the equation above, $\eta$ is a dimensionless parameter describing the efficiency of the X-ray irradiation, $a$ is the binary separation, $R_i$ is the radius of the primary, and $h(\theta)$ is a geometric factor defined as

$$h(\theta) = \frac{\cos \theta - f_2}{(1 - 2f_2 \cos \theta + f_2^2)^{3/2}},$$

where $f_2$ is the ratio of the secondary’s radius to the orbital separation $(R_2/a)$. We can define a dimensionless irradiating flux as the ratio of impinging flux over unirradiated stellar flux,

$$x(\theta) = \frac{F_{\text{irr}}(\theta)}{F_0} = \frac{F_{\text{irr}}(\theta)}{F_0},$$

and a dimensionless stellar flux as

$$G[x(\theta)] = \frac{F}{F_0} = \frac{T_4^4}{T_0^4} - x(\theta),$$

the ratio of the emergent stellar flux from an irradiated star to that of the unirradiated star.

In this model, the effective luminosity of the secondary, taking into account the effects of external irradiation, is

$$L = 4\pi R_2^2 \sigma T_0^4 \left[ \frac{3}{2} [1 + f_2(q)] + \frac{1}{2} \left( \int_{0}^{\theta_{\text{max}}} G[x(\theta)] \sin \theta \, d\theta \right) \right],$$

(11)

where the sum of the bracketed terms gives the fractional effective surface area through which the stellar luminosity escapes. The function $G(x)$ contains all the information on how the superadiabatic layers adjust and how effectively they block the internal luminosity. In this paper, we present evolutions obtained mostly with the one-zone model described above, but we also discuss and compare with results of calculations done with more detailed treatments of the blocking (Hameury & Ritter 1997) in which full stellar models are used to produce a table of $G(x)$.

In our first set of irradiated models, shown in Figure 2, we include only the effects of systemic angular momentum losses, i.e., gravitational radiation and magnetic braking, ignoring possible angular momentum losses during nova explosions. The mass transfer rate is plotted as a function of
As illustrated in Figure 2, RZK included systemic and angular momentum losses due to mass loss from the primary assuming all mass accreted was lost in nova explosions, carrying away the specific angular momentum of the primary. We assume for our calculations that mass is transferred in a nonconservative manner. That is, the mass of the compact object remains constant while angular momentum is carried away at a rate

$$\frac{J}{M} = \beta_1 M_2^2 \frac{M_2}{M_1^2},$$

(12)

where $\beta_1$ is an adjustable dimensionless parameter measuring the specific angular momentum carried away in units of the specific angular momentum of the white dwarf. Clearly this angular momentum loss is a form of CAML. In a “real” CV, things are a bit more complex: between nova outbursts, mass is conserved, but the mass accumulated on the primary is lost on timescales generally much shorter than those characteristic of the cycle. Thus only in an average sense do orbital parameters evolve following equation (12). A more sophisticated calculation akin to the “hibernation scenario” (Livio & Shara 1987; Kovetz, Prialnik, & Shara 1988), in which nova explosions and the effects of the irradiation by the hot white dwarf are modeled as well, is beyond our present scope.

A sequence of evolutions with different $\eta$ values and $\beta_1 = 1.0$, that is, assuming that all the mass loss from the system leaves with the specific angular momentum of the primary, are presented in Figure 3. These cases are analogous to the cases presented in RZK in which they assume similar angular momentum losses due to mass leaving the system from the primary. In addition, we find that, although a minimum threshold is necessary in order to produce cycles at all, as the magnitude of $\eta$ increases, the mass range in which oscillations occur decreases.

In the next section, we discuss the effects of simultaneously allowing two forms of consequential angular momentum losses: angular momentum losses due to mass loss from the primary (e.g., nova explosions) and fluctuations of $J$ coupled to irradiation-driven cycles. These results indicate that angular momentum loss due to mass loss from the primary or coupled CAML cycles can extend the irradiation-driven instability to lower periods and shift its onset to smaller $\eta$ values.

4. COUPLED CONSEQUENTIAL ANGULAR MOMENTUM LOSSES

Angular momentum losses due to mass leaving the primary have some effect in enhancing mass transfer cycles as shown in the previous section. However, since it is an order of magnitude smaller than magnetic braking, it is incapable of extending the mass transfer cycle very much. It is possible to get losses of the form of equation (12) to drive cycles at all orbital periods by assuming a much weaker magnetic braking; this however will cause the size of the period gap to shrink to unacceptably small values. Thus there is a need for another mass transfer–dependent angular momentum loss in the system.

As suggested in King et al. (1996), we introduce an additional angular momentum loss mechanism that is pro-

![Graph 1](image1)

![Graph 2](image2)

**Fig. 2.**—Set of evolutions with increasing irradiation efficiency $\eta$ values from top to bottom. The mass transfer rate is shown here as a function of companion mass. The discontinuity occurs when the companion becomes fully convective and the binary detaches and enters the period gap. Limit cycles below the period gap occur in the last $\eta = 0.2$ model. All the models have some mass transfer oscillations but they become significant only with $\eta \geq 0.05$. It is assumed that $M_1 = \text{const.}$ and no angular momentum is lost with the mass leaving the system (e.g., in nova explosions).

**Fig. 3.**—Same as Fig. 2, but with mass loss carrying away the specific angular momentum of the primary, $\beta_1 = 1.0$ (see text for definitions). Mass transfer oscillations below the period gap occur in the last $\eta = 0.2$ model. All cases shown have some significant mass transfer oscillations.
portionate to the instantaneous mass transfer rate and is thus a type of CAML. Enhanced magnetic braking or some other mass transfer–coupled angular momentum loss could be responsible for these fluctuations. We show that, from a combination of irradiation and $J_{\text{cy}}/J \propto M_2$, mass transfer cycles can be extended to all masses.

In previous work, King et al. (1996) already noted that the main reason the irradiation-driven instability dies, as the secondary mass is reduced in the course of mass transfer, is that the thermal timescale of the convective envelope becomes too long. They also pointed out that oscillations in angular momentum losses directly coupled to the mass transfer cycle could extend the occurrence of limit cycles to lower companion masses. They proposed a simple form of coupling that could be shown analytically to produce always an instability if the binary as a whole was pushed closer to dynamical instability. However, their analysis was limited to the onset of the instability without investigating the form of the cycles nor their evolutionary effects. To determine if the resultant cyclic evolution can be applied to CVs or other systems, detailed simulations of such cycles, allowing us to study their amplitude and time dependence and their sensitivity to input parameters, are necessary.

We follow King et al. (1996) and formally allow for coupled variations in $J$, writing the variations in the form

$$-\frac{\ddot{J}}{J} = \frac{j(R_2, \dot{M}_2)}{t_j},$$

(13)

where $t_j$ is the secular mean angular momentum loss timescale and $j$ is a dimensionless function. To investigate the main effects of allowing for variations of the angular momentum losses coupled to the transfer rate, King et al. (1996) considered the simplest possible case, in which these increase linearly with the instantaneous $M_2$, i.e.,

$$j = 1 + \frac{M_2}{\dot{M}_2}, \quad l > 0,$$

(14)

where $[\dot{M}_2]_{ad} = -M_2/(D_2 t_j)$ is the adiabatic transfer rate and $D_2 = (C_2 - C_R)/2$ is the standard adiabatic stability denominator (see § 7). In equation (14) above, the first term represents systemic losses while the second term gives the fluctuating angular momentum losses coupled to the cycle $J_{\text{cy}}/J = lM_2/[\dot{M}_2 t_j]$. Formally, this is an example of a consequential angular momentum loss (CAML; see Webbink 1985; King & Kolb 1995): $J/J$ is the sum of an $M_2$-independent “systemic” component, represented by $1/t_j$, and the “consequential” components

$$\frac{J_{\text{CAML}}}{J} = \frac{J_{\text{cy}}}{J} + \frac{J_{\text{ad}}}{J} = lD_2 M_2 + \beta_1 \frac{M_2^2}{(\dot{M}_2)} M_2.$$  

(15)

The stability of CAML is discussed in King & Kolb (1995) in general terms. King et al. (1996) show that stability against dynamical-timescale mass transfer requires $D = D_2(1 - l) > 0$, i.e., $l < 1$ when $\beta_1 = 0$. The case with arbitrary $\beta$ is discussed in § 7 below.

A sequence of evolutions with different values of the irradiation efficiency $\eta$ is shown in Figure 4, where we adopt $l = 0.9$. In order to isolate the effects of the CAML oscillations postulated by King et al. (1996), we excluded angular momentum losses due to nova explosions by taking $\beta_1 = 0$ in this example. Cycles appear both above and below the gap but there is an intermediate range of masses or, correspondingly, orbital periods in which mass transfer is stable. Wherever cycles occur, the mass transferred per cycle increases with $\eta$, but the total mass range over which mass transfer is unstable diminishes. The combined effects of CAML from the primary and coupled oscillations are illustrated by a sequence of models with $l = 0.25$ and $\beta_1 = 1.0$ shown in Figure 5. Note that a reduction in $l$ is necessary to get qualitatively the same results as in a case with $\beta_1 = 0$ since both these effects increase the coupling between CAML and mass transfer cycles. There are

\[ \text{FIG. 4.} \text{— Same as Figs. 2 and 3, with systemic angular momentum oscillations coupled to the limit cycle with } l = 0.9. \text{ In order to isolate the effects of such cycles, no CAML due to mass loss from the primary } \beta_1 = 0.0 \text{ was included in this case. Mass transfer oscillations below the period gap occur all models. All the models have some significant mass transfer oscillations. The amplitude of oscillations increases with larger } \eta \text{ values but the overall duration decreases.} \]

\[ \text{FIG. 5.} \text{— Same as Figs. 2, 3, and 4, with systemic angular momentum oscillations coupled to the limit cycle with } l = 0.25. \text{ CAML due to mass loss from the primary was assumed to leave the system with the specific angular momentum of the primary, i.e., } \beta_1 = 1.0. \text{ Mass transfer oscillations below the period gap occur in the last } \eta = 0.2 \text{ model. All the models have some significant mass transfer oscillations. The amplitude of oscillations increases with larger } \eta \text{ values but later decreases for larger } \eta \text{ values.} \]
however some significant differences between angular momentum losses due to nova explosions \((\dot{J}_{\mathrm{ne}}/J)\) and cyclic angular momentum losses \((\dot{J}_{\mathrm{cyc}}/J)\) as parameterized by \(l\). Losses through novae have a more dominant effect on higher mass systems than on lower mass systems since they are \(\propto M_2^2/M\) for a constant \(M_2/M_2\). In contrast, \(\dot{J}_{\mathrm{cyc}}/J\) remains relatively constant with a constant \(M_2^2/M_2\). \(\dot{J}_{\mathrm{cyc}}/J\) will decrease somewhat because of a shrinking \(\zeta_2\) as the secondary becomes more convective but will eventually increase as \(\zeta_2\) becomes more negative. Therefore \(\dot{J}_{\mathrm{ne}}/J\) makes higher mass systems more unstable but has a diminishing effect as the secondary mass decreases. Since pure irradiation-driven mass transfer cycles are easier to produce at higher masses, \(\beta_1\) does not contribute much to extending the duration of cycles and only amplifies the initial peak mass transfer rates. Another case with \(l = 0.95, \beta_1 = 0,\) and \(\eta = 0.1\), is shown in Figure 6, now plotted as a function of the binary orbital period. An example of CV evolution calculated using the \(G(x)\) tables obtained by Hameury & Ritter (1997) from full stellar models is shown on Figure 7. These examples demonstrate that a suitable adjustment of \(l\) or the combined effects of \(l\) and \(\beta_1\) can produce cycles at all orbital periods. However, the peak mass transfer rates are uncomfortably high, and the period gap is wider than in the other cases. This is a natural corollary of the fact that CAML as given by equation (15) can only enhance angular momentum losses from the system and drive transfer rates higher than secular.

The above discussion suggests that the coupling introduced in equation (14) could be modified to read

\[
j = 1 + l \left( \frac{M_2^2 - [M_2^2]_{\mathrm{sec}}}{[M_2^2]_{\ad}} \right), \quad l > 0,
\]

where \([M_2^2]_{\sec}\) is the secular equilibrium rate under irradiation (King et al. 1996). With this modification, there is no enhanced CAML if the system transfers mass at the secular rate and if the transfer is stable. However, if there is an irradiation-driven limit cycle, then coupled CAML occurs, and the net effect is to enhance the cycle without changing the mass transfer averaged over secular timescales. An example of evolutions obtained with this form of coupling is shown in Figure 8. The peak mass transfer during cycles and the width of the period gap are now in line with observational limits. The same evolution calculated with an atmospheric scale height 10 times smaller, \(\epsilon = 10^{-4}\), is shown for comparison in Figure 9. The peak mass transfers in this case are no more than a factor of 2 higher than in the calculation with \(\epsilon = 10^{-3}\) at the same orbital period. The mass and period changes during individual cycles are smaller as expected, but the overall evolution remains qualitatively unchanged.

5. ANATOMY OF A CYCLE

In order understand physically the nature of the cycles, it is interesting to analyze what happens during a typical cycle.
in a purely irradiation driven case and in a case where CAML is present, and to compare the results. As we shall see below, pure irradiation cycles are driven by the thermal expansion of the secondary, while in CAML assisted cycles at higher secondary masses, one is enhancing angular momentum losses to the extent that it is the contraction of the Roche lobe that dominates. As the companion mass decreases, the thermal expansion of the companion again dominates, but the net result is that irradiation-driven cycles now occur at shorter orbital periods.

Let us examine first the effects of changes in the radius of the secondary and how it feeds back into changes in the systemic orbital angular momentum losses. The systemic orbital angular momentum losses $J_{sys}$ respond to changes in $R_2$ since the magnetic braking angular momentum losses $J_{MB}/J \propto M_2 R_2^2/a^5$. We refer to Figure 10, which shows results obtained for a pure irradiation-driven cycle to illustrate the discussion given below. Examining the bottom panel of Figure 10, we see that the rate of change of magnetic braking goes through six distinct stages corresponding to distinct behavior identified by numeric labels. These stages are correlated to rates of change in $R_2$, $M_2$, and $a$ (shown in the middle panel of Fig. 10). Initially, before mass transfer starts, as the star approaches Roche lobe contact, the companion is near equilibrium and its mass and radius are virtually constant. Hence increasing $J_{MB}/J \propto a^5$, rapidly as $a$ decreases (stage 1). Once contact is reached, the secondary begins to expand because of irradiation, and $a$ is still decreasing, thus driving an even stronger angular momentum loss (stage 2). As the mass transfer rate increases, $\dot{a}$ changes sign and the orbital separation begins expanding at an increasing rate. Even though the radius of the secondary is still expanding during this phase, the combination of enhanced mass transfer and orbital expansion rapidly reduces the braking (stage 3). After the peak rate of orbital separation and mass transfer is reached, the rate of decrease in braking slows down as the secondary is still expanding, but this stage is relatively short lived (stage 4). At some point, the effects of irradiation saturate and the companion starts to shrink toward its new equilibrium radius. Thus, initially, the magnetic braking rate decreases as $J_{MB}/J \propto R^4$, (stage 5) since on this timescale both $a$ and

---

**Fig. 10.**—Plot of one mass transfer cycle. In the top graph, $R_2$ (dotted line) and $R_e$ (dashed line) have been plotted as functions of time. The middle graph shows $\dot{a}/a$ (dashed line), $\dot{a}/a$ (dotted line), and $M_2/M_2$ (solid line) as functions of time. In the bottom graph, $d[\ln(J/J)]/dt$ has been plotted over the same time interval. This model was calculated with no CAML with an irradiation efficiency of $\eta = 0.1$.

**Fig. 11.**—Detailed plot of one CAML-assisted mass transfer cycle selected from the evolution shown on Fig. 9, at a period above 7 hr. In the top graph, $R_2$ (dotted line) and $R_e$ (dashed line) have been plotted as functions of time. In the middle graph, $R_2/R_2$ (dotted line), $\dot{a}/a$ (dashed line), and $M_2/M_2$ (solid line) have been plotted as functions of time. In the bottom graph, $d[\ln(J/J)]/dt$ has been plotted over the same time interval.
$M_2$ are constant. As the secondary adjusts thermally, the magnetic braking rate returns (stage 6) again to the detached behavior $\dot{J}_{MB}/J \propto a^{-5}$ (stage 1). During the nearly detached phase, the system is slowly driven back into contact and the cycle repeats.

We also present results for two individual cycles selected from the run with CAML-assisted cycles shown in its entirety on Figure 9. The cycle shown on Figure 11 occurs early in the evolution when $M_2 \approx 0.9 \, M_\odot$. This evolution was calculated with $\epsilon = 10^{-4}$ and therefore, during contact, $R_2$ and $R_1$ follow each other more closely than in the case shown in Figure 10. It is clear from the top panel of Figure 11 that the enhanced braking during high mass transfer leads to a contraction of the binary, the Roche lobe, and the companion. In the early stages of contact, as the mass transfer increases, the secondary expands in response to irradiation but eventually the effect of CAML dominates the evolution and results in a rapid overall contraction. As the companion is trying simultaneously to adjust thermally to the incident irradiation, a high peak of mass transfer results. Later, in the same evolution, once the companion’s mass has decreased to $M_2 \approx 0.4 \, M_\odot$, the thermal expansion of the secondary dominates during a typical cycle as shown in Figure 12. The reasons for this behavior are twofold: (1) the specific angular momentum of the primary is reduced, and (2) the mass transfer rate is lower. As a consequence, both CAML terms in equation (15) are smaller than before and the mechanical tendency for the system to expand during high mass transfer episodes wins. These effects translate in accompanying period variations, which are discussed in § 6 below.

6. PERIOD VARIATIONS

The balance between the systemic angular momentum losses and the mechanical tendency for the orbit to expand during mass transfer yields in the standard evolution a stable mass transfer rate that changes only secularly. However, in evolution under irradiation, this balance is temporarily affected by the cycles resulting in orbital period changes of either sign. Generally one expects that during the bright phases of the cycle the orbit expands on timescales of about a few times $10^6$–$10^7$ yr as the companion expands. On the other hand, during the faint quasi-detached phase of the cycle, the system contracts undetected and the overall effect is a secular evolution toward shorter periods. The simulations shown on Figure 13 illustrate that this is indeed the case for irradiation-driven cycles without CAML, which can occur only at longer orbital periods. This is potentially interesting because it would predict a systematic effect, supposing one could disentangle these intermediate timescale variations from other short term variations: bright systems should statistically show a large and positive $P$. 

![Fig. 12.—Detailed plot of another CAML-assisted mass transfer cycle selected from the evolution shown on Fig. 9, at a period below 4 hr. The same conventions as in Figs. 10 and 11 were used here.](image)

![Fig. 13.—Upper panel: Plot of $M_2$ as a function of the orbital period $P_{orb}$. Lower panel: Plot of the orbital period derivative $\dot{P}_{orb}$ as a function of orbital period. An irradiation efficiency of $\eta = 0.1$ is assumed in this model. No CAML is included in this simulation. We show only a range of periods between 6 and 8 hr in order to emphasize the structure of the oscillation. The full calculation is shown in Fig. 2 ($\eta = 0.1$ panel).](image)
However, when CAML is included, the calculations show that enhanced magnetic braking can actually dominate and cause orbital contraction at longer periods. In that case, if CAML remains at a sufficient level, cycles can occur at all orbital periods with the bright phase of the cycle yielding $P < 0$ at long periods and $P > 0$ just above and below the gap (see Figure 14). This behavior is easily understood analytically if one considers the logarithmic derivative of the orbital period including all mass and angular momentum losses present,

$$\frac{\dot{P}}{P} = 3 \left( \frac{d}{dt} \right)_{\text{sys}} + \frac{\dot{M}_2}{M_2} \left[ -3 + (1 - \alpha) \frac{q}{1 + q} + 3\alpha q + 3\beta q \times (q + 1) + 3D_1 \right],$$

where $\dot{M}_1 = -\alpha \dot{M}_2$ has been assumed. The last two terms are the contributions due to losses associated with CAML (mass loss from the primary and coupled fluctuations). Without these terms, the contribution to $\dot{P}$, which is proportional to the mass-loss rate, is always positive. Thus in the pure irradiation case in which mass loss is enhanced, large positive orbital derivatives occur during bright phases. In CAML-assisted cycles, the increase in mass loss during irradiation-driven expansion of the secondary also increases $\dot{J}_{\text{CAML}}$, to the point where angular momentum losses dominate over the mechanical tendency of the system to expand upon mass exchange. Eventually as $M_2$ decreases, the specific angular momentum carried away by mass loss decreases and the expansion tendency during high mass transfer dominates. The expansion becomes more pronounced as $M_2$ is reduced, occurring at smaller peak mass transfer rates and for longer durations.

The above suggests that further studies may lead to a statistical observational test of the picture. If there was a certain period range over which the period variations during bright phases had a definite sign, one could in principle deduce the relative importance of thermal and CAML effects, for example. It is, however, important to note that period variations occur in a variety of timescales. Over the longest secular timescale, $\geq 10^6$ yr, the period should decrease as long as the companion is nondegenerate. The period variations induced by cycles occur at intermediate timescales, $P/\dot{P} \geq 10^6$ yr and should be easier to observe than the secular trend. However, other "short-term" effects, presumably caused by stellar processes such as magnetic cycles, starspots, etc., with timescales $(P/\dot{P} \geq 10-100)$ yr, have been observed. Just as cycles can dominate the secular trend, these short-term effects may make it difficult to detect changes due to thermal and CAML cycles. Nevertheless, it is worth pursuing this test since one could learn much about processes governing the evolution of close binaries if shorter term orbital effects could be successfully filtered out.

### 7. Stability Considerations

In order to analyze the stability of mass transfer in semidetached compact binaries under the influence of irradiation and mass loss from the primary and CAML cycles, it is necessary to examine how the radius of the mass donor star, $R_1$, changes in comparison to the size of the Roche lobe, $R_L$. When $R_2$ expands faster than $R_L$, we get unstable mass transfer, whereas if $R_2$ expands slightly more slowly than $R_L$, mass transfer can greatly decrease or even stop altogether. It is only in the case where $R_2$ and $R_L$ expand at the same rate that stable mass transfer occurs. Changes in $R_2$ can be expressed as (Ritter 1995)

$$\frac{\dot{R}_2}{R_2} = \xi_s \frac{\dot{M}_2}{M_2} + \left( \frac{\partial \ln R_2}{\partial t} \right)_{\text{nuc}} + \left( \frac{\partial \ln R_2}{\partial t} \right)_{\text{th}} - \xi_{\text{irr}} \frac{\dot{M}_2}{M_2},$$

where the adiabatic response to mass loss is described by the adiabatic mass-radius exponent

$$\xi_s = \left( \frac{\partial \ln R_2}{\partial \ln M_2} \right)_{\text{ad}},$$

the second and third terms describe the effects of nuclear evolution and thermal relaxation, respectively, and the rate of expansion due to irradiation has been written in analogy with the adiabatic expansion, using an "irradiation mass-radius exponent," defined by

$$\xi_{\text{irr}} = -M_2 \frac{\partial}{\partial M_2} \left( \frac{\partial \ln R_2}{\partial t} \right)_{\text{th}}.$$

Similarly, changes in the Roche lobe radius can be expressed as

$$\frac{\dot{R}_L}{R_L} = \xi_R \frac{M_2}{M_2} + 2 \left( \frac{\partial \ln J}{\partial t} \right)_{M_2=0} - \xi_{\text{CAML}} \frac{\dot{M}_2}{M_2},$$

where the term with $\xi_R$ describes changes due to redistribution of mass and angular momentum in the system, $(J/J)_{M_2=0}$ is the rate of systemic angular momentum losses, and the term containing

$$\xi_{\text{CAML}} = -2M_2 \frac{\partial}{\partial M_2} \frac{\partial \ln J}{\partial t}$$

gives the rate of change due to CAML. By equating equation (18) with equation (21) and solving for $\dot{M}_2$ for stationary mass transfer, we get

$$-\dot{M}_2 = \frac{M_2}{\xi_s - \xi_R - \xi_{\text{irr}} + \xi_{\text{CAML}}} \times \left[ \left( \frac{\partial \ln R_2}{\partial t} \right)_{\text{nuc}} + \left( \frac{\partial \ln R_2}{\partial t} \right)_{\text{th}} - 2 \left( \frac{\partial \ln J}{\partial t} \right)_{M_2=0} \right],$$

which leads to the criterion for adiabatic stability,

$$D_{\text{ad}} = \xi_s - \xi_R - \xi_{\text{irr}} + \xi_{\text{CAML}} > 0.$$
For a system driven only by systemic angular momentum losses, in the absence of irradiation and CAML, equation (24) yields the standard result \( -\dot{M}_{2\alpha} = D_\alpha \frac{1}{J}(J/J)_M = 0 \), which is adiabatically stable when the standard adiabatic denominator \( D_\alpha = (\zeta_\alpha - \zeta_\alpha)/2 \) is positive. If we assume that consequential angular momentum loss (CAML) is proportional to the instantaneous mass-loss rate and angular momentum losses due to mass leaving the primary can be parameterized by \( \beta_1 \), we can write these angular momentum losses as in equation (15):

\[
\frac{J}{J_{\text{CAML}}} = D_\alpha \frac{1}{\dot{M}_2} \left( \frac{\beta_1 M_2^2 M_2}{(M M_1)} \right)_2 ,
\]

which, using the definition in equation (22), gives

\[
\zeta_{\text{CAML}} = -2D_\alpha \frac{1}{2} \beta_1 M_2^2/(M M_1) .
\]

Adopting Paczyński’s approximation for the Roche lobe radius, \( R_L/a = f(q) \),

\[
\frac{R_L}{a} = \frac{2}{3^{1/3}} \left( \frac{q}{1 + q} \right)^{1/3} = 0.462 \left( \frac{M_2^2}{M} \right)^{1/3} , \quad q \approx 0.8 ,
\]

and the definitions \( q = M_2/M_1 \) and \( \dot{M}_1 = -\alpha \dot{M}_2 \), a general expression for \( \zeta_R \) is obtained:

\[
\zeta_R = -\frac{5}{3} + \frac{2}{3} (1 - \alpha) \frac{q}{1 + q} + 2qz .
\]

Therefore, for conservative mass transfer (\( \alpha = 1 \), no mass is lost from the binary),

\[
\zeta_R = -\frac{5}{3} + 2M_2/M_1 ,
\]

whereas for nonconservative mass transfer (\( \alpha = 0 \), all matter transferred is eventually lost),

\[
\zeta_R = -\frac{5}{3} + 2M_2/3M .
\]

The use of Paczyński’s approximate formula rather than Eggleton’s more exact formula (see, e.g., Frank et al. 1992b) for stability considerations will give approximate stability boundaries but is convenient for simplicity. Furthermore, for the mass range that we are considering, this approximation is accurate enough for most numerical work.

During the cyclic mass transfer phases, mass transfer has become adiabatically unstable because of the irradiation expansion term of \( R_2 \). The bipolytrope code provides an adiabatic expansion term that for higher masses agrees in sign with the full stellar model but is somewhat smaller in magnitude. This results in stars with mass \( \gtrsim 0.7 M_\odot \) being slightly more unstable than a full stellar model would predict. The agreement for stars with mass \( \lesssim 0.7 M_\odot \) is good and improves as \( M_2 \) decreases. Now that we have expressions for \( \zeta_R \) and \( \zeta_{\text{CAML}} \), and \( \zeta_i \) is obtained from our bipolytrophic code, an explicit expression for \( \zeta_{\text{irr}} \) is all we need to apply the stability criterion given by equation (24).

From equation (20), we have

\[
\zeta_{\text{irr}} = -M_2 \frac{\partial}{\partial L} \left( \frac{\partial \ln R_2}{\partial t} \right) \frac{dL}{dM_2} .
\]

Computing \( \frac{\partial^2 \ln R_2/\partial t \partial L}{\partial L} \) in equation (31) from the bipolytrope model (see, e.g., Kolb & Ritter 1992), we obtain

\[
\frac{\partial}{\partial L} \left( \frac{\partial \ln R_2}{\partial t} \right)_{\text{th}} = -\frac{R_2}{GM_2^2} \mathcal{F}(Q, n_i) ,
\]
where $\mathcal{F}(Q, n_i)$ is a numerical function calculated by the bipolytropic code. This leads to a generalized stability criterion similar to the $\lambda$ criterion given by RZK (eq. [59]) and to equation (5) from Hameury & Ritter (1997):

$$\lambda = 2(\zeta_s - \zeta_R + \zeta_{\text{CAML}}) \frac{\tau_{\text{KH}}}{\tau_{\text{a}}^2} \mathcal{F}^{-1}(Q, n_i)$$

$$> \frac{1}{2\pi} \int_0^{\theta_{\text{max}}} x(\theta)g[x(\theta)] \sin \theta \, d\theta = I_{p\omega}, \quad (33)$$

where, following notation introduced in RZK, we define $g[x(\theta)] = -dG/dx$, $\tau_a = M_*/M_2$ is the mass transfer timescale, and $\tau_{\text{KH}}$ is the Kelvin-Helmholtz timescale. In RZK, the stability criterion is derived from one of several starting points: any one of their equations (25), (40), or (52) can be used, taking into account the definition of $\Lambda$, but the result does not include the effects of any CAML. Angular momentum losses through the primary are incorporated into the definition of $\zeta_{\text{CAML}}$ in RZK, whereas we include the effect explicitly in the definition of $\zeta_{\text{CAML}}$. In RZK, the stability criterion—in the absence of CAML—can be written as $\zeta_s - \zeta_R > \zeta_{\text{irr}}$, where $\zeta_{\text{irr}}$ can be expressed as work proportional to the right-hand side of equation (5) in Hameury & Ritter (1997). The criterion is written in this way in order to group in $\Lambda$ terms that are related to the stellar and binary structure on the left-hand side, while the terms on the right-hand side depend on the modeling of the irradiation.

We present two examples to illustrate the application of the stability criterion. In each case we plot $\lambda$ and $I_{p\omega}$ versus $M_*$ calculated along a fictitious evolution in which irradiation is present and affects the stationary mass transfer but cycles are suppressed. This is necessary since the stability criterion is based on a linear analysis of small departures from the stationary mass transfer (the “fixed points” of the equations; see King et al. 1996 for details). In the regions in which $I_{p\omega}$ is greater than $\Lambda$, we anticipate that the system will be unstable and experience mass transfer cycles. The bottom panel in each graph shows the complete nonlinear evolution with cycles generally occurring where $I_{p\omega} > \Lambda$. The case shown on Figure 15 starts unstable, and the instability weakens around 0.7 $M_\odot$ but strengthens around 0.6 $M_\odot$ and finally stabilizes above the gap. The mass transfer below the gap is predicted to be stable but a few oscillations occur before the mass transfer settles down to its stationary value. The case shown on Figure 16 is unstable throughout according to the top panel and the nonlinear simulation shown below confirms this. Therefore the linear criterion works quite well but the remaining differences indicate that ultimately a nonlinear simulation is necessary since large departures from the stationary points occur during cycles.

The stability criterion (eq. [24]) shows that any form of CAML with $\zeta_{\text{CAML}} < 0$ reduces the adiabatic denominator $D_{ad}$ and makes mass transfer more unstable. The simulations presented in this paper do indeed show that $\beta_I$ and $l$ nonzero and positive make mass transfer more unstable, as expected from equation (26), thus extending the range of the irradiation-driven cycles.

8. DISCUSSION AND CONCLUSIONS

The evolutionary calculations presented in this paper incorporate the effects of irradiation of the companion and of consequential angular momentum losses (CAML). These simulations confirm previous analytic and numerical work on pure irradiation-driven cycles, explore secular evolution with CAML-assisted cycles, examine in detail what happens during a cycle, analyze orbital period variations, and extend the discussion of stability to the more general case including CAML.

Our results show that mass transfer cycles can occur at all orbital periods when CAML is strong enough. Therefore irradiation-driven cycles assisted by CAML may account for the dispersion observed in estimates of the mass transfer for CVs at a given orbital period without blurring significantly the period gap. The main weakness of the model in its present form is the absence of a physical mechanism that would enable one to predict the strength of the coupling between CAML fluctuations and mass transfer and to ascertain its dependence on system parameters. It is conceivable that the rate of magnetic braking for example could be affected by irradiation, by varying the underlying wind mass-loss rate, or by altering the state of ionization of the wind, but so far, no working model exists for these effects. In the meantime, one can assume a certain form for the coupling and follow the secular evolution, predicting the properties of the resulting cycles and comparing these with observations. For example, the correlations predicted between the sign of the orbital period derivative over intermediate timescales, the orbital period itself, and the relative importance of CAML may serve as a test of the picture.

Another test may be provided by nonmagnetic CVs with accretion disks if one adopts the premise that their outburst properties are the combined result of the irradiation instability determining the accretion rate over intermediate timescales and the disk instability determining the short-term behavior (see, e.g., Cannizzo 1993, 1998 for reviews). Above a critical accretion rate, the disk will be hotter than the hydrogen ionization temperature throughout and thus will be stuck in the hot state (Smak 1983). Estimates for this critical rate depend on the assumed size and shape of the disk and on the effects of irradiation on the disk temperature and vertical structure (van Paradijs 1996). In any case, the disk instability may be simply assumed not to occur above some critical mass transfer rate $M_{\text{crit}}$, which is itself subject to some uncertainty. This approach is adequate if one is interested only in the overall outburst behavior (e.g., the presence/absence of dwarf nova outbursts) rather than in the detailed shape of individual outbursts.

A well-known example of the insight one may gain from studies of the period distribution is provided by the relative paucity of dwarf novae relative to nova-like variables at orbital periods just above the gap, in the range 3–4 hr (Shafter, Wheeler, & Cannizzo 1983; Shafter 1992). The simulations presented here suggest that irradiation-driven cycles assisted by a certain amount of coupled CAML could produce this effect in one of two ways:

1. If a moderate CAML is assumed, then mass transfer oscillations will damp out at periods around 4 hr. Depending on the efficiency of the irradiation assumed (how effective the blocking is), the mass transfer rate between the periods above the gap of 3–4 hr may be above the critical mass transfer rates for dwarf novae and may be observed as nova-like systems.

2. If a stronger CAML is assumed, cycles do occur at periods of 3–4 hr, but most systems still spend most of the time they are visible accreting at rates above the critical
uncertainties mentioned above, we postpone a detailed coupling CAML and irradiation-driven cycles. Given the enable us to place constraints on the possible mechanism(s) over the observed orbital period range. Such a study should

An important caveat to this scenario is that the gap width is very sensitive to the mass-loss timescale at the upper edge of the gap. The width of the period gap depends on the ratio of the mass-loss timescale to the thermal timescale of the secondary star. This led us to explore a coupling (eq. [16]) different in form from the original suggestion of King et al. (1996), which yields relatively lower mass transfer rates at the upper edge of the gap and yet produces cycles at all orbital periods. A population-synthesis study incorporating the cyclic evolution described here will be carried out during future investigations in order to determine if the predicted observational properties of such an ensemble of CVs agree with existing observational limits. A particularly interesting result would be a distribution of CV subtypes like most of the time.

Finally, on a more speculative note, we note that in some of the CAML-assisted evolutions, it is possible to get mass transfer rates about few times $10^{-7} M_\odot$ yr$^{-1}$ for 10$^{5-6}$ yr. Perhaps it is possible to get mass transfer rates in this range extending down to orbital periods of 3–4 hr by adjusting the CAML losses adequately. It is tempting to identify such systems with the short period CV-type supersoft X-ray sources (e.g., RX J0439.8–6809 in the LMC and 1E 0035.4–7230 in the SMC), which do not conform to the generally accepted model in which the companion is more massive than the white dwarf (Kahabka & van den Heuvel 1997).

We are grateful to Andrew King, Uli Kolb, Hans Ritter, and Jean-Marie Hameury for helpful discussions at various stages of this work. This work was partially supported by NSF grant AST 90-20855 and NASA grant NAG 5-3082 to LSU. J. F. acknowledges the kind hospitality of the STScI, where part of this work was done.

REFERENCES
Cannizzo, J. K. 1993, in Accretion Disks in Compact Stellar Systems, ed. J. C. Wheeler (Singapore: World), 6
— 1998, in ASP Conf. Proc. 137, Wild Stars in the Old West: Proc. 13th North American Workshop on Cataclysmic Variables and Related Objects, ed. S. B. Howell, E. Kuulkers, & C. Woodward (San Francisco: ASP), in press
Chandrasekhar, S. 1939, An Introduction to the Study of Stellar Structure (Chicago: Univ. Chicago Press)
Frank, J., King, A. R., & Lasota, J.-P. 1992a, ApJ, 385, L45
Frank, J., King, A. R., & Raine, D. J. 1992b, Accretion Power in Astrophysics (2d ed.; Cambridge: Cambridge Univ. Press)
Gontikakis, C., & Hameury, J.-M. 1993, A&A, 271, 118
Hameury, J.-M., King, A. R., Lasota, J.-P. 1989, MNRAS, 237, 39
Hameury, J.-M., King, A. R., Lasota, J.-P., & Raison, F. 1993, A&A, 277, 81
Hameury, J.-M., & Ritter, H. 1997, A&AS, 123, 273
Harpaz, A., & Rappaport, S. 1991, ApJ, 383, 739
— 1995, A&A, 294, L49
Howell, S. B., Rappaport, S., & Politano, M. 1997, MNRAS, 287, 929
Howell, S. B., & Szkody, P. 1990, ApJ, 356, 623
Howell, S. B., Szkody, P., & Cannizzo, J. K. 1995, ApJ, 439, 337
Kahabka, P., & van den Heuvel, E. P. J. 1997, ARA&A, 35, 69
King, A. R. 1988, QJRAS, 29, 1
— 1995, in Cataclysmic Variables, ed. A. Bianchini, M. Della Valle, & M. Orio (Dordrecht: Kluwer), 523
King, A. R., Frank, J., Kolb, U., & Ritter, H. 1995, ApJ, 444, L37
— 1996, ApJ, 467, 761
— 1997, ApJ, 482, 919
King, A. R., & Kolb, U. 1995, ApJ, 439, 330

Kolb, U. 1996, in Cataclysmic Variables and Related Objects, ed. A. Evans & J. H. Wood (Dordrecht: Kluwer), 433
Kolb, U., & Ritter, H. 1990, A&A, 236, 385
— 1992, A&A, 254, 213
Kovetz, A., Prialnik, D., & Shara, M. M. 1987, ApJ, 325, 828
Livio, M., & Shara, M. M. 1987, ApJ, 319, 282
Paczynski, B., & Sienkiewicz, R. 1981, ApJ, 248, L27
Patterson, J. 1984, ApJS, 54, 443
Podsiadlowski, P. 1991, Nature, 350, 136
Rappaport, S., Joss P. C., & Webbink, R. F. 1982, ApJ, 254, 616
Rappaport, S., Verbunt, F., & Joss P. C. 1983, ApJ, 275, 713
Ritter, H. 1988, A&A, 202, 93
— 1995, in Evolutionary Processes in Binary Stars, NATO ASI Ser. C, Vol. 477, ed. R. Wijers, M. Davies, & C. Tout (Dordrecht: Kluwer), 223
Ritter, H., Zhang, Z.-Y., & Kolb U. 1998, A&A, submitted (RZK)
Shafter, A. W. 1992, ApJ, 394, 268
Shafter, A. W., Wheeler, J. C., & Cannizzo, J. K. 1986, ApJ, 305, 261
Smak, J. 1985, ApJ, 272, 324
Spruit, H. C., & Ritter, H. 1983, A&A, 124, 267
Steiehe, R., Ritter, H., & Kolb, U. 1996, MNRAS, 279, 581
van Paradijs, J. 1996, ApJ, 464, L139
Verbunt, F. 1984, MNRAS, 209, 227
Warner, B. 1987, MNRAS, 227, 23
— 1995, Cataclysmic Variable Stars (Cambridge: Cambridge Univ. Press)
Webbink, R. F. 1985, in Interacting Binary Stars, ed. J. Pringle & R. A. Wade (Cambridge: Cambridge Univ. Press), 39