Dark neutrino interactions make gravitational waves blue

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Abstract: New interactions of neutrinos can stop them from free streaming in the early Universe even after the weak decoupling epoch. This results in the enhancement of the primordial gravitational wave amplitude on small scales compared to the standard ΛCDM prediction. In this paper we calculate the effect of dark matter neutrino interactions in CMB tensor B-modes spectrum. We show that the effect of new neutrino interactions generates a scale or ℓ dependent imprint in the CMB B-modes power spectrum at ℓ ≳ 100. In the event that primordial B-modes are detected by future experiments, a departure from scale invariance, with a blue spectrum, may not necessarily mean failure of simple inflationary models but instead may be a sign of non-standard interactions of relativistic particles. New interactions of neutrinos also induce a phase shift in the CMB B-mode power spectrum which cannot be mimicked by simple modifications of the primordial tensor power spectrum. There is rich information hidden in the CMB B-modes spectrum beyond just the tensor to scalar ratio.
1 Introduction

Despite steep experimental challenges, detecting the signatures of primordial gravitational waves (PGW) in the cosmic microwave background (CMB) remains one of our top priorities. It is justifiably so since the PGW, produced during a very early phase of the universe, such as inflation, may carry signatures of physics of exceptionally high energy or the ultraviolet (UV) physics. The evolution of the PGW and their imprint on CMB has been studied in the standard ΛCDM cosmology for many decades [1–3]. A major breakthrough was the realization that they leave a particular polarization pattern (curl modes or the $B$-modes) in the CMB which cannot be created by the scalar modes in linear theory [4–6]. The CMB $B$-modes, therefore, provide a clean way of detecting the PGW using the CMB as a detector. The results from the BICEP experiment [7, 8], which originally claimed to have found hints of PGW, gave rise to tremendous excitement in the cosmology and high energy physics community. Since then, a significant amount of effort has gone in the direction of finding new physics that can also source tensor perturbations [9–17] and modify the CMB temperature and polarization anisotropies. Naturally, this over abundance of possible scenarios that can, in principle, give rise to the tensor modes awakens an intellectual inverse problem – if and when traces of $B$-modes are observed, how can we actually disentangle the exact nature of UV-physics which might have caused the signal?

The purpose of this paper is not to find another UV phenomenon that might source detectable $B$-modes. We rather point out a previously overlooked fact that even if one predicts the exact initial conditions for the $B$-modes, non-standard physics during the evolution through the radiation dominated era may alter the CMB $B$-modes spectrum non-trivially. This simply implies that the inverse problem of the unentangling UV becomes far more challenging than we usually anticipate. In other words, new physics before the cosmological recombination may leave a unique imprint in the primordial gravitational waves and the CMB $B$-modes. Therefore, CMB $B$-modes, if detected, may give us information about not only the very high energy phenomenon but also new physics operating at low energies. In this paper we construct one proof of principle that demonstrates how new physics before the cosmological recombination leaves characteristic imprints in the CMB $B$-modes, providing a new probe of physics beyond the standard model among the constituents of the universe and at the same time confusing our view of the initial PGW.

The physics of the evolution of the PGW is rather simple. These are frozen in or conserved outside the horizon [18], and redshift on subhorizon scales similar to any other radiation species with energy density decreasing with redshift $z$ (namely, $\rho_{GW} \propto (1 + z)^4$). The only non-trivial changes in the spectrum occur in the presence of free streaming radiation during the evolution. We define a particle species to be free streaming if its mean free path is greater than the horizon size. Gravitational waves entering the horizon source anisotropic stress in the free streaming species, which ultimately gets dissipated. The presence of the free streaming relativistic particles, therefore, simply results in the removal of energy from the gravitational waves which, in turn, get damped. In terms of the Boltzmann equations
for tensor perturbations, the effect of the free streaming relativistic particles shows up as an
inhomogeneous piece (source or sink), which co-evolves with PGW. The example we construct
in this work relies on the fact that the inhomogeneous piece changes anytime non-standard
interactions of these radiative species are introduced, which ultimately leaves its imprints on
the tensor modes.

To be concrete, we consider neutrinos and show how their interactions can suppress
their free streaming properties. Consequently, the $B$-mode component of the CMB escapes
neutrino damping. The CMB $B$-modes, therefore, get enhanced compared to the standard
$\Lambda$CDM solution obtained by neglecting all interactions of neutrinos.

Neutrino interactions are severely restricted when one considers the full symmetry structure
(gauged and global) of the Standard Model of particle physics. When new degrees of
freedom are included, however, non-trivial interactions can arise even in a gauge invariant and
flavor symmetric way. There is significant cosmological evidence that new degrees of freedom
beyond the Standard Model do exist, namely dark matter(DM) and dark energy [19–21]. Un-
fortunately, decades of WIMP (Weakly Interacting Massive Particles) searches (laboratory
experiments with atomic nuclei [22–25]), axion searches [26–30], and indirect searches (detection
experiments looking for the annihilation or decay of dark matter into the Standard Model
particles in $\gamma$-rays, X-rays as well as the radio part of the electromagnetic spectrum [31–33])
have failed to yield any definitive signal for dark matter and, therefore, failed to shine any
light on the dark matter interaction with Standard Model particles. Though Neutrino tele-
scopes [34–37] have put constraints on DM-Neutrino interaction looking for DM annihilating
to neutrinos, those bounds are highly model dependent(only applicable if DM is produced
via thermal freezeout mechanism). This vast lack of our knowledge regarding the dark mat-
ter (especially, its interactions with neutrinos) motivates us to go beyond simplistic scenarios
which the current experiments severely constrain and look for new probes of dark interactions
in more general settings.

In fact, there has been a renewed interest recently in the interactions of neutrinos with
dark matter, motivated by the steady accumulation of evidence for discrepancies between
simulations that use a cold, collisionless fluid for dark matter, and observations at scales
smaller than galaxy clusters [38, 39]. Introducing a relatively strong coupling of dark matter
to neutrinos and photon [40–45], with new radiative species or with itself [46–54] remains one
of the simplistic ways to affect structure formation on small scales. The interactions of the
dark matter with the standard model, with other dark particles, and with itself can influence
the cosmological observables such as CMB scalar modes and the large scale structure (LSS).
This work, on the other hand, implies that the imprints of these interactions can also be found
on CMB $B$-modes. Therefore, one can turn the argument around to use CMB $B$-modes as
independent probes of interaction between dark matter and neutrinos.

In this work, we use a fairly generic set-up to account for the dark matter neutrino
interaction and calculate its effects on the time evolution of tensor perturbations. However,
we emphasize that our formalism and results are very general and apply to any new physics
which can stop the neutrinos from free streaming including neutrino self interactions[55, 56].
The only important model dependence comes from how the free streaming properties of neutrinos evolve with the redshift. In this paper, we will consider two different redshift dependences motivated by particle physics. Our results apply to any new physics which has the same time dependence as far as neutrino free streaming is concerned and can be easily extended to models with different redshift dependence.

This paper is organised as follows: in section 2, we begin with the a model where we implement interactions between the dark matter and neutrinos, calculate the size of cross-section, and its dependence on the scale factor; in section 3, we derive the modifications to the Boltzmann equations for tensor modes; in section 4, we summarize results of our numerical studies; and finally we conclude in section 5. We use natural units with the speed of light, reduced Planck constant and Boltzmann constant $c = h = k_B = 1$.

2 Dark matter neutrino interactions as extensions of the Standard Model of particle physics

Building models where dark matter can interact with neutrinos sufficiently strongly so that there remain sizeable imprints on the cosmological observables is non-trivial. Note that we are interested in direct coupling between the dark matter and active neutrinos, which come from the lepton electroweak doublets. The effective interactions involving these neutrinos must, therefore, arise in a gauge invariant way. Further, as far as the neutrino sector is concerned, we confine ourselves entirely with the degrees of freedom of the Standard Model (SM) of particle physics (no right-handed neutrinos) and for the rest of the paper we take neutrinos to be massless. Summarizing the underlying assumptions:

- In our setup, the degrees of freedom of the SM are extended to include new multiplets which are inert under the SM gauge group.

- The effective interaction involving neutrinos must arise from electroweak invariant operators. On one hand, it paves the way for a straightforward UV completion, and on the other, gives the natural size of the strength of interaction.

- Neutrinos arise from SM lepton doublets and, as a result, inherit the $SU(3)_l$ flavor symmetry, even after electroweak symmetry breaking. Additionally, we assume the neutrinos to be massless.

- We will focus on the dark sector that additionally preserves at least a $U(1)_D$.

The lowest order gauge invariant polynomial in SM fields is given by $H^\dagger l$. This operator, is charged under global $U(1)_L \times SU(3)_L$. We attempt to preserve the full symmetry structure in the interaction, which implies that the polynomial with hidden sector fields must transform under the global symmetries above. The minimum solution is a dark-sector fermion with $-1$ lepton number and triplet under $SU(3)_L$. Even though this would be the simplistic scenario, it does not quite work – after electroweak symmetry breaking these operators become Dirac
masses for SM neutrinos and we know that these Dirac adjoints can only contribute a small fraction of dark matter density [57]. Additionally, this construction does not allow for a preserved $U(1)_D$. The minimum configuration, therefore, consists of two multiplets from the dark sector – one candidate for dark matter (namely, $\chi$) and an additional field (denoted here by $\psi$). We will assign flavor quantum number to $\psi$, whereas dark-charges to both $\chi$ and $\psi$. Since both $\psi$, and $\chi$ carry $U(1)_D$ charges, we additionally require $m_\chi \leq m_\psi$, for $\chi$ to be dark matter. Note that in both cases, either $\psi$ or $\chi$ is a fermion and, therefore, an additional spinor (the Dirac adjoint) needs to be introduced with appropriate charges that allows us to write Dirac mass term for the corresponding dark fermion.

With the above charge assignments, the minimal interaction turns out to be a 5-dimensional operator

$$\mathcal{L} \supset Y \frac{1}{\Lambda} \left( H^\dagger l \right) (\psi \chi) \Rightarrow \eta \delta_{ij} \nu_i \psi_j \chi \quad \text{where} \quad \eta = Y \frac{v}{\sqrt{2} \Lambda}. \quad (2.1)$$

Note that the flavor indices in $l$ and $\psi$ are contracted among each other and, therefore, $Y$ is simply a complex number. Also, we explicitly show flavor indices for neutrinos, to signify that neutrinos of all flavor interact with identical strength in our setup. It is important to note that the operator $H^\dagger l$, when expanded around the Higgs vacuum expectation value $v/\sqrt{2}$, yields couplings consisting of only neutrinos among all SM particles. This is particularly suitable for avoiding constraints from observables with charged leptons.

It is straightforward to UV complete the effective interaction shown in Eq. (2.1). We take the opportunity to provide one such example. Consider new vector-like fermions namely $N$ with quantum numbers $(3, -1, 0)$ under $SU(3)_l \times U(1)_L \times U(1)_D$, and its adjoint $N^c$. The global charges allow for the following interactions

$$\mathcal{L} \supset Y_N N \left( H^\dagger l \right) + Y_N^c N^c (\psi \chi) + M_N N N^c. \quad (2.2)$$

At scales below $M_N$, these fermions are integrated out and one recovers the effective interactions shown in Eq. (2.1) at tree level (see Fig. 1) with the identification $Y/\Lambda = 2 Y_N Y_N^c / M_N$. Before moving on, we emphasise that the above toy model in Eq. (2.2) presents only one way

|   | $SU(3)_l$ | $U(1)_L$ | $U(1)_D$ | spin | Case I | Case II |
|---|-----------|----------|----------|------|--------|---------|
| $(H^\dagger l)$ | 3         | 1        | 0        | 1/2  |        |         |
| $\chi$       | 1         | 0        | 1        | 1/2  | 0      |         |
| $\psi$       | 3         | -1       | -1       | 0    | 1/2    |         |

Table 1: Charge assignment of dark sector fields
**Figure 1:** Feynman diagrams to demonstrate the effective operator in Eq. (2.1) can be generated at tree level.

to UV complete Eq. (2.1) and the main purpose of Eq. (2.2) is to provide a proof of principle. None of the calculations shown below depend on the particularities of Eq. (2.2).

Interactions in Eq. (2.1) give rise to dark matter neutrino scattering in both cases (i.e., whether the dark matter $\chi$ is a fermion or a scalar). In Fig. 2, we show example Feynman diagrams that give rise to $\chi$-$\nu$ scattering. For massless neutrinos (which is an excellent approximation before recombination) of any flavor, the scattering matrix element between neutrinos of left helicity and dark matter (spin averaged) is simply given by

$$|M|^2 = \begin{cases} 
\frac{2|\eta|^4}{(-m^2\Delta + 2p_1 \cdot k_1)^2 + \gamma^2} \left[ k_1 \cdot p_1 k_2 \cdot p_2 \right], & \text{Case I} \\
\frac{2|\eta|^4}{(-m^2\Delta + 2p_1 \cdot k_1)^2 + \gamma^2} \left[ 2(k_1 \cdot p_1)^2 - (p_1 \cdot p_2)(m^2 + 2k_1 \cdot p_1) \right], & \text{Case II}
\end{cases}$$

where $$\Delta \equiv \frac{m^2_{\psi} - m^2_{\chi}}{m^2_{\chi}}$$

(2.3)
Here $p^\mu_{1,2}$ and $k^\mu_{1,2}$ are four-momentum of incoming and outgoing neutrino and $\chi$ respectively, $\gamma$ is the width of $\psi$. As clearly seen in Fig. 2, the flavor triplet $\psi$ play the role of mediator and the cross-section depends crucially on the mass-splitting parameter of the dark sector (namely, $\Delta$).

We are interested in the redshift range where the new interactions can stop the free streaming of neutrinos after the neutrino decoupling epoch ($z \lesssim 10^9$). Also we want to see the effects of these new interactions on the CMB B-modes generated at the time of recombination at $z \approx 1100$, and these are only sensitive to the evolution of primordial gravitational waves at $z \gtrsim 1000$. In this redshift range of interest, $10^3 \lesssim z \lesssim 10^9$, massless neutrinos is an excellent assumption and we make this assumption from now on. In this redshift range, we also assume that $\chi$ is non-relativistic and, in particular, that the neutrino temperature is below DM mass (i.e. $|k_1| \ll m_\chi$). On the other hand, throughout our redshift range of interest, neutrinos remain relativistic with $p^2 = 0$ and follow Fermi-Dirac statistics. After integration over the phase space, $E_\nu \equiv |p|$ dependence of the amplitude-squared will give a temperature dependent cross section for a thermal (Fermi-Dirac) distribution of neutrinos. We can write the general intensity averaged cross section needed in the Boltzmann evolution equations (see Appendix A), $\sigma_{\chi\nu}$, as

$$\sigma_{\chi\nu} = \sum_n \sigma^{(n)} \left( \frac{T_\nu}{1.95 \text{ K}} \right)^n,$$

with one of the terms typically dominating over the rest, where $T_\nu$ is the neutrino temperature. In $\Lambda$CDM cosmology, the neutrino temperature is given by $T_\nu = 1.95(1 + z) \text{ K}$. For $n \neq 0$, $\sigma^{(n)}$ is, therefore, the cross section extrapolated to $z = 0$ assuming massless neutrinos.

We arrive at distinct scenarios depending on which term in the denominator of Eq. 2.3 dominates. Clearly, if $(k_1 \cdot p_1)$ dominates, the cross-section should go as $T_\nu^2$.

**Limit 1**: $\Delta \ll 2 (k_1 \cdot p_1)/m_\chi^2$

Note that in our redshift range of interests $E_\nu \ll m_\chi$. Consequently, limit 1 necessarily implies $\Delta \ll 1$, i.e., highly degenerate masses in the dark sector. From a model building side, constructing a UV theory that naturally flows towards such degenerate spectrum is non-trivial. In this work, however, we remain agnostic of the origin of such a degeneracy and reserve any such speculations for future considerations. The scattering matrix element squared in limit 1 goes as,

$$|\mathcal{M}|^2 = \begin{cases} |\eta| \frac{i}{2} \{1 + \cdots\} & \text{Case I} \\ |\eta| \frac{i}{2} \{ (1 + \hat{p}_1 \cdot \hat{p}_2) + \cdots \} & \text{Case II} \end{cases}$$

The dots represents correction of $O\left(m_\chi^2 \Delta / (k_1 \cdot p_1)\right)$. The leading term in the amplitude-squared is constant as expected, which leads to a temperature independent cross-section.
\( \sigma^{(0)} \). Also we define \( \sigma^{(0)} \) to be the angularly averaged cross section in Case-II. The angular dependence is not important since we do not observe the neutrinos directly but only the average effect of their interactions on the gravitational waves. In particular, we find that

\[
\sigma_{\chi \nu} \approx \sigma^{(0)} \approx 10^{-13} \times \sigma_{\text{Th}} \times \left( \frac{\eta}{0.1} \right)^4 \left( \frac{m_\chi}{100 \text{ GeV}} \right)^{-2}
\]  

\[
(2.6)
\]

where \( \sigma_{\text{Th}} = 6.65 \times 10^{-25} \text{ cm}^2 \) is the Thomson scattering cross-section.

\textbf{Limit 2:} \( \Delta \gtrsim 2 \left( \frac{k_1 \cdot p_1}{m_\chi^2} \right) \)

The case with \( \Delta \gg 2 \left( \frac{k_1 \cdot p_1}{m_\chi^2} \right) \) is simple to understand. In fact, in all cases where \( \Delta \gtrsim 1 \), we end up with limit 2, simply because in our redshift range of interest \( k_1 \cdot p_1 \ll m_\chi^2 \). Unless special mechanisms are invoked UV models predict splittings of order masses themselves. Even if, \( \Delta \) is taken to be negligible, renormalization drives \( \Delta \) to order one values. One can then simply replace the denominator in Eq. (2.3) by \( m_\chi^4 \Delta^2 \), which does not have any dependence on \( E_\nu \). The leading term in the cross-section goes as \( T^2 \).

Even in the case where \( \Delta \approx 2 \left( \frac{k_1 \cdot p_1}{m_\chi^2} \right) \), so that the propagator falls on shell, the same temperature dependence follows. The only difference is that one simply replaces denominator by \( \gamma \), which in these limits is given as \( \gamma \sim (\eta^2/16\pi^2) m_\psi^2 \Delta^2 \). Of course, in order to write the expression of \( \gamma \), we assume that \( \psi \) can decay only via the interactions present in Eq. (2.1). Note, however, that both \( \psi \) and \( \chi \) carry conserved \( U(1)_D \), and \( \chi \) is the lightest particle with a non-zero \( U(1)_D \) number. Therefore, \( \chi \) should always be present in the decay product of \( \psi \) irrespective of hidden sector details. Further, because of phase space considerations, we still expect \( \gamma \) to vanish in the limit \( \Delta \to 0 \). The form of \( \gamma \) is, therefore, is quite general except for the numerical pre-factor which can change from model to model. In any case, the denominators still gets replaced by a term independent of \( E_\nu \).

Summarizing, in case \( \Delta \gtrsim 1 \) we obtain

\[
|M|^2 = \begin{cases} 
2|\eta|^4 \frac{p_1^2}{m_\chi^2 \Delta^2} \{ 1 + \cdots \} & \text{Case I} \\
2|\eta|^4 \frac{p_1^2}{m_\chi^2 \Delta^2} \{ (1 + \hat{p}_1 \cdot \hat{p}_2) + \cdots \} & \text{Case II}
\end{cases}
\]  

\[
(2.7)
\]

This ultimately yields a leading \( T_\nu^2 \) dependence in the cross-section

\[
\sigma_{\chi \nu} \approx \sigma^{(2)} \left( \frac{T_\nu}{1.95 \text{ K}} \right)^2 \simeq 10^{-39} \sigma_{\text{Th}} \left( \frac{T_\nu}{1.95 \text{ K}} \right)^2 \left( \frac{\eta}{0.1} \right)^4 \left( \frac{\Delta}{0.1} \right)^{-2} \left( \frac{m_\chi}{100 \text{ GeV}} \right)^{-4}
\]  

\[
\simeq 10^{-39} \sigma_{\text{Th}} a^{-2} \left( \frac{\eta}{0.1} \right)^4 \left( \frac{\Delta}{0.1} \right)^{-2} \left( \frac{m_\chi}{100 \text{ GeV}} \right)^{-4},
\]  

\[
(2.8)
\]

where \( a \) is the scale factor normalized so that \( a = 1 \) today. In the last line of Eq. (2.8) we have used the temperature dependence of massless neutrinos and \( T_\nu \propto 1/a \). In the case of resonance \( \Delta \approx 2 \left( \frac{k_1 \cdot p_1}{m_\chi^2} \right) \), the cross-section will go as \( \sigma_{\chi \nu} \sim \Delta^{-4} \). We note that even
when we start with $\Delta \ll 2(k_1 \cdot p_1)/m^2_\chi$ with constant cross section, neutrino temperature would decrease due to cosmological expansion and at some point we will transition to limit 2: $\Delta \gtrsim 2(k_1 \cdot p_1)/m^2_\chi$. In terms of dynamics, we will, therefore, transition from a constant cross section to a $T^2_\nu$ dependence at some point. This will not affect our results as long as the transition happens well after the neutrinos have decoupled from the dark matter but may lead to a qualitative modification of dynamics if the transition happens in the radiation dominated era when the neutrinos are still coupled to the dark matter. We will ignore this additional complication in this paper.

3 Boltzmann equations for Dark Matter - Neutrino Interaction

At linear order in perturbation theory, tensor and scalar modes evolve independently[58]. We will only consider tensor modes in this paper and comment on existing results on scalar modes in section 4. The non-relativistic particles like dark matter give negligible contribution to the anisotropic stress. We, therefore, do not need to consider the evolution of dark matter perturbations for tensor modes. Their only role, as far as tensor modes are concerned, is to provide scattering targets for neutrinos and stop these from free streaming. We can also ignore the energy exchange between the neutrinos and the dark matter because the entropy or heat capacity of neutrinos is much larger than that of dark matter. To be precise, we are in the regime where dark matter is non-relativistic and neutrino temperature $T_\nu \ll m_\chi$. The energy exchange between neutrinos and dark matter can make the dark matter temperature equal to the neutrino temperature. The energy-fraction that needs to be transferred from neutrinos to dark matter to accomplish this is of order $\Delta \rho_\nu/\rho_\nu \sim n_\chi k_B T_\nu/(n_\nu k_B T_\nu) = n_\chi/n_\nu \sim 10^{-9}$ for $m_\chi \sim$ GeV, where $n_\chi$ is the number density of interacting dark matter particles and $n_\nu$ is the number density of neutrinos. Note that this is the upper (saturation) limit to the energy that can be transferred from neutrinos to dark matter at high scattering rate. In reality, since the energy transfer in each elastic scattering between neutrinos and dark matter is of order $\sim T_\nu/m_\chi \ll 1$, the actual energy lost by the neutrinos would become smaller as the mean free path of neutrinos increases with the expansion of the Universe and scattering rate drops. This situation is similar to the energy lost by photons to baryons due to Compton scattering. In the photon-electron-baryon system also, because of large entropy in photons compared to baryons (baryon to photon ratio $\approx 6 \times 10^{-10}$), the energy lost by photons can be neglected for most applications except when considering small distortions of the CMB spectrum [59–62].

In the following derivation we will closely follow ref [63]. The Boltzmann equation for the neutrinos is given by

$$\frac{df_\nu(x, p, t)}{dt} = C[f_\nu(x, p, t)]$$

(3.1)

where $f_\nu$ is the neutrino distribution function, $C[f_\nu]$ is collision term arising from DM-neutrino interaction discussed in the previous section, $t$ is the proper time, $x$ is the comoving spatial coordinate and $p = p\hat{p}$ is the neutrino momentum measured by a comoving observer. We
define fluctuations of neutrino distribution $\delta f_\nu$ as

$$f_\nu(x, p, t) = \bar{f}_\nu + \delta f_\nu(x, p, t) \quad (3.2)$$

where $\bar{f}_\nu$ is the zeroth order Fermi-Dirac distribution function of the neutrinos. Instead of $\delta f_\nu$, it is convenient to work with dimensionless intensity perturbation,

$$J(x, \hat{p}, t) = \frac{N_\nu}{a^4 \bar{\rho}_\nu} \int_0^\infty \delta f_\nu(x, p, t) 4\pi p^3 dp \quad (3.3)$$

where $N_\nu$ is the effective number of neutrino species and $\bar{\rho}_\nu$ is the background energy density of neutrinos. The Boltzmann equation for tensor perturbation in terms of this new variable reads,

$$\frac{\partial J(x, \hat{p}, t)}{\partial t} + \hat{p}_i a(t) \frac{\partial J(x, \hat{p}, t)}{\partial x^i} + 2\hat{p}_i \hat{p}_j \frac{\partial}{\partial t} \left[ D_{ij}(x, t) \right] = \begin{cases} \ -n_\chi \sigma^{(0)}(0) \left[ J(x, \hat{p}, t) - \frac{1}{4\pi} \int d^2 \hat{p}' J(x, \hat{p}', t) \right] : T_\nu \text{ independent} \\ -n_\chi \sigma^{(2)}(2) \left[ J(x, \hat{p}, t) - \frac{1}{4\pi} \int d^2 \hat{p}' J(x, \hat{p}', t) \right] : T_\nu^2 \text{ dependent} \end{cases} \quad (3.4)$$

with $D_{ij}$ being the metric tensor perturbation. The detailed derivation for R.H.S of Eq 3.4 is given in Appendix A. In Fourier space, we decompose $J(x, \hat{p}, t)$ as

$$J(x, \hat{p}, t) = \sum_{\lambda = \pm 2} \int d^3 q e^{i q \cdot x} \beta(q, \lambda) \delta_{ij}(\hat{q}, \lambda) \hat{p}_i \hat{p}_j \Delta_T^\nu(q, \hat{p} \cdot \hat{q}, t) \quad (3.5)$$

Here $\beta(q, \lambda)$ is the stochastic initial condition with wave number $q$ and helicity $\lambda$, $\delta_{ij}(\hat{q}, \lambda)$ is the symmetric, divergence and traceless polarization tensor, and $\Delta_T^\nu(q, \hat{p} \cdot \hat{q}, t)$ is the neutrino tensor transfer function. One notable feature of this decomposition is that the second integral on R.H.S of (3.4) for both the cases vanishes. Defining $\cos \theta \equiv \hat{p} \cdot \hat{q}$ we expand the neutrino transfer function in multipole components,

$$\Delta_T^\nu(q, \cos \theta, t) = \sum_{l=0}^{\infty} \frac{1}{l!} (2l + 1) P_l(\cos \theta) \Delta_{\nu,l}^T(q, t) \quad (3.6)$$

Following the convention of [64],

$$\delta_T^\nu \equiv \Delta_{\nu,0}^T, \quad \theta_T^\nu \equiv \frac{3}{4} q \Delta_{\nu,1}^T, \quad \sigma_T^\nu \equiv \frac{1}{2} \Delta_{\nu,2}^T \quad (3.7)$$
and using conformal time $\eta$ defined by the relation $dt = ad\eta$, we write down the modified tensor equations for neutrinos:

\[
\frac{\partial \delta^T_\nu}{\partial \eta} = -\frac{4}{3} \theta^T_\nu - 2 \partial \frac{\mathcal{D}_q}{\partial \eta} - \dot{\mu} \delta^T_\nu
\]

(3.8)

\[
\frac{\partial \theta^T_\nu}{\partial \eta} = q^2 \left[ 4 \frac{\delta^T_\nu - \sigma^T_\nu}{4} \right] - \dot{\mu} \theta^T_\nu
\]

(3.9)

\[
\frac{\partial \sigma^T_\nu}{\partial \eta} = \frac{4}{15} q^T_\nu \left[ 3 \frac{\Delta^T_{\nu,3} - \dot{\mu} \sigma^T_\nu}{10} \right]
\]

(3.10)

\[
\frac{\partial \Delta^T_{\nu,l}}{\partial \eta} + q \left( \frac{2l + 1}{(2l + 1)} \left[ (l + 1) \Delta^T_{\nu,l+1} - l \Delta^T_{\nu,l-1} \right] \right) = -\dot{\mu} \Delta^T_{\nu,l} \quad \text{for } l > 2
\]

(3.11)

In the above equations, $\mathcal{D}_q$ is the Fourier transformation of tensor perturbation,

\[
\mathcal{D}_{ij}(x, t) = \sum_{\lambda = \pm 2} \int d^3q \, e^{i \mathbf{q} \cdot \mathbf{x}} \beta(\mathbf{q}, \lambda) (\hat{q}_{ij}) (\mathbf{q}, \lambda) \mathcal{D}_q(t)
\]

(3.12)

whose evolution is given by

\[
\frac{\partial^2}{\partial \eta^2} \mathcal{D}_q + 2aH \frac{\partial \mathcal{D}_q}{\partial \eta} + q^2 \mathcal{D}_q = 16\pi a^2 G \pi^T_q
\]

(3.13)

where $\pi^T_q$ is the total anisotropic stress sourced by photons and neutrinos and $H \equiv \frac{1}{a^2} \frac{da}{d\eta}$ is the Hubble rate. The (dominant) neutrino part of the anisotropic stress is given by,

\[
\pi^T_{\nu} q = 2\rho_\nu \left[ \frac{1}{15} \Delta^T_{\nu,0} + \frac{2}{15} \Delta^T_{\nu,2} + \frac{1}{35} \Delta^T_{\nu,4} \right]
\]

(3.14)

The last two equations are included for completeness, these remain unaffected from the no-interaction case. In Eqs (3.8-3.11), $\dot{\mu}$ is the differential optical depth of neutrinos. It is the inverse of comoving mean free-path of neutrino ($\lambda_\nu = 1/\dot{\mu}$) which is the collisional length scale of the problem. The model dependence of DM-neutrino interaction comes as a ratio of cross section over mass of DM. In general, we parametrize this interaction strength with respect to Thompson scattering and dark matter mass $m_\chi = 100$ GeV. The expression of $\dot{\mu}$ for the two different limits are given by

\[
\dot{\mu} \equiv \begin{cases} 
    a \rho_\chi \sigma^{(0)} \left( \frac{1}{m_\chi} \right) \equiv au^{(0)} \rho_\chi \left( \frac{\sigma_{\text{Th}}}{100 \text{GeV}} \right) & : T_\nu \text{ independent} \\
    a \rho_\chi \sigma^{(2)} \left( \frac{1}{m_\chi} \right) \left( \frac{1}{a} u^{(2)} \rho_\chi \left( \frac{\sigma_{\text{Th}}}{100 \text{GeV}} \right) \right) & : T^2_\nu \text{ dependent}
\end{cases}
\]

(3.15)

where we define

\[
u^{(0)} \equiv \left( \frac{\sigma^{(0)}}{\sigma_{\text{Th}}} \right) \left( \frac{100 \text{GeV}}{m_\chi} \right), \quad \nu^{(2)} \equiv \left( \frac{\sigma^{(2)}}{\sigma_{\text{Th}}} \right) \left( \frac{100 \text{GeV}}{m_\chi} \right)
\]

(3.16)

and $\rho_\chi$ is DM energy density.
Figure 3: We have plotted the strength of neutrino dark matter interaction, \( u^{(0)} \) (constant interaction) and \( u^{(2)} \) (temperature-dependent interaction) as a function of redshift such that at each redshift the comoving mean free path \( \lambda_\nu \) of neutrino is equal to comoving Hubble radius \( 1/aH \).

4 Results

We have implemented the above mentioned modifications in the Boltzmann equation solver CLASS (Cosmic Linear Anisotropy Solving System) [65]. We only need to modify the tensor neutrino evolution equations. The evolution equation for the tensor metric perturbation and initial conditions remain unchanged. We use the following flat \( \Lambda \)CDM cosmological parameters for all calculations [21]:

\[
\begin{align*}
    h &= 0.67 \quad \Omega_b = 0.049 \quad \Omega_{cdm} = 0.26 \quad N_{\text{eff}} = 3.04 \quad Y_{\text{He}} = 0.25 .
\end{align*}
\]  

(4.1)

with \( h = H_0/100 \) where \( H_0 \) is Hubble constant, \( \Omega_b \) and \( \Omega_{cdm} \) are the ratio of baryon and DM energy density to critical density respectively, \( N_{\text{eff}} \) is the effective number of neutrinos and \( Y_{\text{He}} \) is the helium nucleon fraction. The amplitude and shape of CMB B-modes also depends
on tensor to scalar ratio $r$ and tensor spectral index $n_T$, which is defined as

$$P_T(k) = A_{T*} \left( \frac{k}{k_*} \right)^{n_T}$$

where $P_T(k)$ is (dimensionless) primordial tensor power spectrum, $k_*$ is pivot scale, $A_{T*}$ is amplitude at pivot point.

Changing $n_T$ tilts the power spectrum about pivot point i.e, it increases power on one side of the pivot scale w.r.t the scale invariant spectrum ($n_T = 0$), while suppressing power on the other side. For example, setting $n_T > 0$ ($n_T < 0$) increase (decrease) power for $k > k_*$ and decrease (increase) power for $k < k_*$. The spectral index $n_T$ is conventionally fixed using the self consistency condition in terms of $r$ assuming slow roll inflation of single scalar field, $n_T \approx -r/8$. However, it can take different values in more complicated models of inflation. For the current bound on the tensor to scalar ratio, $r < 0.07$ [8], we have $n_T \approx -r/8 \sim 0$ for the fiducial $\Lambda$CDM model with no new neutrino interactions. We use $\Lambda$CDM hereafter to mean the standard cosmological evolution without any new physics,

$$\Lambda\text{CDM Limit } \equiv \dot{\mu} \rightarrow 0$$

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure.png}
\caption{Evolution of the mean free path of neutrinos for our two models is compared with the comoving Hubble radius. The evolution of comoving Hubble radius is shown in solid black line.}
\end{figure}
Before looking at the CMB power spectra, we can try to build intuition about what to expect. We should expect redshifting due to the expansion of the Universe after a mode enters the horizon giving $D_q \propto 1 + z$ from second term in Eq. 3.13. There is additional damping if the neutrinos are free streaming when a mode enters the horizon due to anisotropic stress $\pi_q^T$ in R.H.S of Eq. 3.13. This additional damping should be suppressed if the neutrinos are scattering with themselves or another particle. When the neutrinos have a short mean streaming distance or equivalently large differential optical depth $\dot{\mu}$, the neutrino tensor anisotropies are suppressed. This can be seen from eqns. 3.8-3.11. For very high $\dot{\mu}$, the solution is approximately given by

$$\Delta_{\nu,l}^T \sim e^{-\int \dot{\mu} d\eta}$$

Therefore,

$$\dot{\mu} \to \infty \quad \Rightarrow \quad \pi_q \to 0$$

from (3.14), and the gravitational wave (3.13) solution deep inside the horizon becomes[63],

$$\frac{\partial^2}{\partial \eta^2} D_q + 2aH \frac{\partial D_q}{\partial \eta} + q^2 D_q = 0 \quad \Rightarrow \quad D_q \propto \frac{1}{qa} \sin \left( q \int d\eta \right)$$

Figure 5: Evolution of the gravitational waves with three different wavelengths in ΛCDM cosmology and after including new neutrino interactions is compared.
In this limit gravitational waves just oscillate with their amplitude decreasing due to the cosmological redshifting.

A useful rough measure of the effect on a given mode $q$ is given by considering whether the mean free path of the neutrinos is smaller or larger than the horizon scale at the time when the mode enters the horizon. We would expect the modes which entered the horizon when the mean free path of neutrinos is smaller than the horizon size would be amplified with respect to the $\Lambda$CDM. The modes which enter the horizon when the mean free path of the neutrinos is already larger than the horizon size would be damped similar to the $\Lambda$CDM case. We plot the values of interaction parameters $u^{(0)}, u^{(2)}$ as a function of redshift $z$ when the comoving mean free path of neutrinos is equal to the comoving Hubble radius $1/(aH)$ in Fig. 3. The top axes in Fig. 3 shows the corresponding (approximately to a given value of $u^{(0)}, u^{(2)}$) smallest CMB multipole $\ell = q(z)(\eta_0 - \eta_*)$, that would be affected, where $\eta_0$ is the conformal time today, $\eta_*$ is the conformal time at recombination, $\eta_0 - \eta_*$ is the comoving distance to the last scattering surface and $q(z)$ is wavenumber of the mode entering the horizon at redshift $z$. For a given value of $u^{(0)}, u^{(2)}$, we can read off the multipole $\ell$ from the curves above which we should expect significant change in the CMB $B$-modes power spectrum. In Fig. 4, we compare the comoving mean free path of neutrinos with the comoving Hubble distance $1/(aH(z))$ for our two models of neutrino DM interaction.

We show in Fig. 5 the evolution of $D_q$ for different modes $q$. The curves labelled $\Lambda$CDM are the standard evolution including both the cosmological redshift as well as the effect of standard neutrino damping. If we turn on the neutrino interactions with a dark particle by making the interaction parameter non-zero, the neutrino anisotropic stress is suppressed. We, therefore, see that the gravitational waves are enhanced w.r.t. the $\Lambda$CDM limit for mode $q = 0.1 \text{ Mpc}^{-1}$ which enters the horizon when the neutrinos are still tightly coupled to the dark matter and have a short mean free path. This is seen more clearly in Fig. 6, where we have scaled out the cosmological redshift of the gravitational waves. We can see in Fig. 3 that, for $u^{(0)} = 0.1$, the neutrino decoupling happens around recombination. Therefore, for $q = 0.01 \text{ Mpc}^{-1}$, which enters the horizon just before recombination, the enhancement is less pronounced. The mode $q = 0.001 \text{ Mpc}^{-1}$ is still outside the horizon when the neutrinos decouple and remains unaffected. For the case when the neutrino scattering cross section is temperature dependent we show the evolution for $u^{(2)} = 10^{-9}$. For this value of $u^{(2)}$, we see from Fig. 3 that the neutrino decoupling happens a little earlier, around matter radiation equality, giving a smaller effect on the gravitational waves in Figs. 5 and 6, especially for $q = 0.01 \text{ Mpc}^{-1}$, which is just entering the horizon at this time. We also see that the new neutrino interactions induce a phase shift in Gravitational waves w.r.t. the $\Lambda$CDM case. This phase shift is a unique signature of neutrino interactions and cannot be mimicked by a change in the initial tensor power spectrum. We can see the effect of neutrino dark matter interactions more clearly on the neutrino tensor mode anisotropic stress plotted in Fig. 7. The development of the anisotropic stress is highly suppressed for $q = 0.1 \text{ Mpc}^{-1}$ mode. The suppression gets weaker as we go to smaller wavenumbers or larger scales.

The quadrupolar anisotropy in the CMB at the last scattering surface sourced by the
Figure 6: Zoom in version of Fig. 5. We have divided out the trivial redshifting of the gravitational due to the expansion of the Universe and zoomed in on the redshift range near recombination.

Gravitational waves creates polarization on Thomson scattering, imprints the signature of PGWs in the CMB polarization. CMB experiments observe the $Q$ and $U$ Stokes parameters as a function of direction in the sky (denoted here by $\hat{n}$), which depend on the choice of frame. We would like to construct observables which are independent of frame choice and can distinguish between scalar and tensor modes. Of particular interest is the combination $Q \pm iU$. On rotation by angle $\alpha$,

$$Q \pm iU \rightarrow e^{\mp 2i\alpha} (Q \pm iU)$$

and, therefore, form spin-2 fields. They can, therefore, be decomposed into spin-2 spherical harmonics, $\pm 2 Y_{\ell m}$ [5, 66],

$$(Q \pm iU) (\hat{n}) = \sum_{\ell m} a_{\pm 2, \ell m} \pm 2 Y_{\ell m} (\hat{n})$$

We can define orthogonal combinations of spin weighted spherical harmonic coefficients $a_{\pm 2, \ell m}$
Figure 7: Evolution of source term of Gravitational wave equation with three different wavelengths in ΛCDM cosmology and after including new neutrino interactions is compared.

which have a definite parity,

\[ a_{\ell m}^E = -\frac{1}{2} (a_{2,\ell m} + a_{-2,\ell m}) \]
\[ a_{\ell m}^B = -\frac{i}{2} (a_{2,\ell m} - a_{-2,\ell m}) \]

where \( a_{\ell m}^E \) is a scalar (even parity, same as the electric field) and \( a_{\ell m}^B \) is a pseudo-scalar (odd parity, same as the magnetic field). The E- and B-modes power spectra are defined by the ensemble average (for theoretical predictions) or average over the \( m \) modes (for observations),

\[ C_{\ell}^{EE} = \langle a_{\ell m}^E a_{\ell m}^{*E} \rangle = \frac{1}{2\ell + 1} \sum_m a_{\ell m}^E a_{\ell m}^{*E} \]
\[ C_{\ell}^{BB} = \langle a_{\ell m}^B a_{\ell m}^{*B} \rangle = \frac{1}{2\ell + 1} \sum_m a_{\ell m}^B a_{\ell m}^{*B} \]

The PGW create both E and B modes. The CMB B-modes power spectrum, however, vanishes at linear order for scalar perturbations (which are much larger than the tensor metric perturbations) and, therefore, provides a powerful observable for primordial gravitational
waves. The $E$-modes created by scalar mode quadrupole at the last scattering surface can be converted to $B$-modes through gravitational lensing by the large scale structure at lower redshifts. These lensing $B$-modes provide the biggest challenge to the detection of PGW signatures in the CMB [67–81]. We are interested in the effect of dark matter neutrino scattering on the unlensed $B$-modes power spectrum, $C_{BB}^{\ell}$. In the following part, we will refer to unlensed $B$-modes power spectrum as the $B$-modes power spectrum.

We show the effect of temperature independent DM-neutrino interaction on the CMB $B$-modes power spectrum in Fig. 8 (full spectrum) and Fig. 9 (fractional change w.r.t. $\Lambda$CDM) and in Fig. 10 for the $\sigma_{\chi\nu}\propto T_{\nu}^2$ case. We also give, for reference, the CMB $B$-modes power spectrum (labelled “no damping”) when we set the neutrino anisotropic stress (last term in Eq. 3.13) to zero. This curve represents the maximum change in the CMB $B$-modes we can expect in the extreme case when the neutrinos never decoupled and free streamed. We see that increasing the interaction strength causes less damping at high $l$, and the CMB power spectrum approaches the “no damping” curve as $u^{(0)}, u^{(2)} \to \infty$. The phase shift in gravitational waves (Fig. 6) due to new neutrino interactions results in the oscillatory features in the power spectrum difference on top of the damping effect. We also show for reference a blue initial tensor power spectrum with the tensor spectral index $n_T = 0.05$ but without any new neutrino interactions. We have chosen a pivot scale at $q = 0.002$ Mpc$^{-1}$ (pivot scale is irrelevant when $n_T \approx 0$), and tensor to scalar ratio is $r = 0.01$ for all curves.

A blue initial spectrum will also enhance the small scale power compared to the almost scale invariant $\Lambda$CDM case for which $n_T \approx 0$. The shape of the blue spectrum at $\ell \lesssim 100$ is different from the effect of neutrino non-damping and sufficiently high precision measurements of the CMB $B$-modes should be able to distinguish between the two. In particular, for a blue primordial spectrum with a constant spectral index $n_T > 0$, all $\ell$ modes are affected tilting the whole $B$-modes power spectrum. In the case of interacting neutrinos only the scales which are either entering the horizon or are already sub-horizon at the time of recombination are enhanced w.r.t. $\Lambda$CDM, while the super-horizon modes remain unaffected. We expect that the first detections of the CMB $B$-modes would be just above the noise in which case the effect of non-standard neutrino interactions may be mistaken to be a blue initial power spectrum.

Interactions of neutrinos with dark matter would also modify the scalar CMB and matter power spectra. This effect has been considered previously in [43, 82, 83]. For the scalar modes we can classify the changes broadly into two physical effects:

1. If new interactions stop the neutrinos from free streaming, the scalar perturbations in them do not decay away and contribute as a source to the scalar metric perturbations enhancing the scalar CMB and matter power spectra.

2. If the source of the short mean free path of neutrinos is their scattering with dark matter particles, neutrinos provide pressure to the coupled neutrino-dark matter fluid, preventing the growth of dark matter perturbations on small scales.

The two main physical effects, therefore, oppose each other for the scalar case. On the scales
below the horizon scales and above the neutrino mean free path, we have both effects. On the small scales, suppression of dark matter and acoustic oscillations of dark matter are the dominant effects, while on the large scales, neutrino clustering is the dominant effect. This means that there is a range of intermediate scales where both effects may be important and can partially cancel each other.

Scalar modes (CMB as well as LSS) are mostly sensitive to modifications in the dark matter power spectrum. The tensor modes, on the other hand, only care about what is happening to neutrinos. There is, therefore, a complimentarity between scalar and tensor modes. In particular, we can suppress the effect of neutrino pressure on the total matter power spectrum on all scales in a multi-component dark matter model with only a fraction of dark matter interacting with neutrinos while keeping the effect on the tensor modes unchanged. Thus, even though there are strong bounds when all of the dark matter interacts with neutrinos, e.g., Ref. [82] puts a bound of $u^{(0)} \lesssim 9 \times 10^{-5}$ and $u^{(2)} \lesssim 3 \times 10^{-14}$ on the neutrino-dark matter interactions from CMB and LSS data, they can be weakened significantly in a multi-
component dark matter model. In particular, matter power spectrum on small scales remains unaffected if only a small fraction of dark matter interacts with neutrinos and only the CMB constraints, which are sensitive to effects of neutrino free streaming on large scales, remain relevant in multicomponent dark matter models. Thus, for example, a multi-component dark matter model can easily escape the much stronger Lyman-α forest constraints [43].

For a fraction $f_i$ of the total dark matter interacting with neutrinos, we see from Eq. 3.15 that the tensor mode power spectrum depends only on the combinations $f_i u^{(0)}, f_i u^{(2)}$, and we can get the predictions for tensor modes in multi-component models trivially by re-labeling $u^{(0)} 	o f_i u^{(0)}, u^{(2)} 	o f_i u^{(2)}$ in all our plots. The scalar mode total dark matter perturbation $\delta_{DM}$ is given by

$$\delta_{DM} = f_i \delta_i + (1 - f_i) \delta_{CDM},$$

where $\delta_i$ is the perturbation in the interacting component of dark matter and $\delta_{CDM}$ is the perturbation in the non-interacting component. Only $\delta_i$ is affected by interactions with neutrinos, getting damped and exhibiting oscillations because of the neutrino pressure [43, 82, 83]. As an extreme example, if we take $f_i \to 0$ while keeping $f_i u^{(0)}$ or $f_i u^{(2)}$ constant, the effect on total dark matter perturbation vanishes while the effect on tensor modes remains unaffected. The only effect that remains on the scalar modes is the contribution of neutrinos to the metric perturbations on large scales.

For scalar modes, a multi-component dark matter model is studied in [84], where they consider neutrinos interacting with a fraction of total DM with $T^2$ dependent cross section. They parametrize the interaction by the parameter $Q$ which is related to our parameter $u^{(2)}$ as $Q = u^{(2)} \sigma_{TH}/100$ GeV. The constraints from Fig. 2 of that paper show that for $f_i \approx 10\%$ the constraints weaken considerably to $Q \lesssim 3 \times 10^{-40}$ cm$^2$ MeV$^{-1}$ or $f_i u^{(2)} \lesssim 5 \times 10^{-12}$. We note that this earlier paper uses WMAP (Wilkinson Microwave Anisotropy Probe) and SDSS (Sloan Digital Sky Survey) LRG (Luminous Red Galaxies) data. We do not expect this result to change significantly with Planck since Planck only adds small scale information which remains unaffected when $f_i \ll 1$. We will explore these possibilities in detail in an upcoming paper [85].

Comparing the power spectra for the temperature independent cross-section, Fig. 9, and temperature dependent cross section, Fig. 10, we see that, in the temperature dependent case the initial enhancement around $\ell = 200$ is much smaller before it follows a similar trend as the temperature independent case at higher $\ell$. Therefore, high precision measurements of the CMB B-modes can in principle also tell us about not just the amplitude of the neutrino interaction cross section but also its temperature dependence.

5 Summary and conclusion

Primordial gravitational waves are damped by the free streaming relativistic particles on horizon entry. In the standard ΛCDM cosmology, neutrinos free stream after they decouple from the baryonic plasma when the temperature of the Universe falls below $\sim 1$MeV. In
Figure 9: Fractional change of tensor $BB$ power spectrum with different strengths of DM-$\nu$ interaction compared to no interaction case for constant interaction strength. We also show for comparison a blue initial spectrum with $n_T = 0.05$. We have defined $\Delta C_{\ell}^{BB}/C_{\ell}^{BB} \equiv (C_{\ell}^{BB}-C_{\ell}^{BB})/C_{\ell}^{BB}$, where $C_{\ell}^{BB}$ stands for the modified $B$-modes power spectrum and $C_{\ell}^{BB}$ stands for $\Lambda$CDM $B$-modes power spectrum.

In particular, the modes to which the CMB is sensitive, $k \lesssim 0.1 \text{ Mpc}^{-1}$, enter the horizon much after neutrino decoupling and suffer damping due to neutrino free streaming [18, 86]. If neutrinos have new interactions, either with dark matter or with itself, which prevents the neutrinos from free streaming, PGW are damped. Compared to the standard cosmology, we then see an amplification of the PGW amplitude and consequently the CMB $B$-modes for scales which enters the horizon when the neutrinos are still coupled. For a neutrino scattering cross section that is either a constant or increasing function of temperature, we expect the neutrinos to be more tightly coupled at higher redshifts. The small scale modes, which enter the horizon earlier, are therefore, more strongly affected compared to the long wavelength modes. In other words, the spectrum of gravitational waves and, therefore, the CMB $B$-modes, become blue ($n_T > 0$) compared to standard $\Lambda$CDM.

We explore these consequences of non-standard (or dark) neutrino interactions within the context of a specific model of neutrino interacting with dark matter. We find that, as
far as consequences for cosmology are concerned, our model allows for either a neutrino dark matter elastic scattering cross section $\sigma$ that is independent of neutrino temperature or $\propto T_\nu^2$. These two cases exhaust almost all possibilities apart from a small portion of parameter space where we may have transition from a constant cross section to a $T_\nu^2$ dependent cross section.

Quantitatively, we find that there is enhancement of the CMB $B$-modes at $\ell \gtrsim 100$ if neutrinos are coupled to dark matter and not free streaming when the modes of interest enter the horizon. This corresponds to $u^{(0)} > 0.001$ for constant cross section case and $u^{(2)} \gtrsim 10^{-12}$ in the case of the cross section having the $T_\nu^2$ temperature dependence (where, $u^{(0)}$ and $u^{(2)}$ are dimensionless parameter characterising the ability of our new physics to stop the neutrino free streaming) assuming we are looking at $\ell < 1000$ modes in the CMB. In principle, if we could detect $\ell > 1000$ modes in CMB primordial $B$-modes power spectrum, we could constrain even smaller interaction strengths.

Since we enhance the small scale modes compared to the large scale modes, the effect of new neutrino interactions mimics a blue spectrum. A blue spectrum but with a constant spectral index $n_T$ has, however, a different CMB $B$-modes power spectrum shape compared
to the case of neutrino interactions and can in principle be distinguished if we measure the \( B \)-modes with high enough precision. To be precise, if we measure the B-modes power spectrum in only a few \( \ell \) bins, the upward trend in the power spectrum due to the new neutrino interactions may be indistinguishable from a blue spectrum. In addition, we can mimic the general increase in power on small scales due to new neutrino interactions if we allow a scale dependent primordial spectral index, i.e. \( n_T \) changing from \( \sim 0 \) to large positive values as we go from large scales to small scales. However, the phase shift in the primordial gravitational waves, which causes the oscillatory behaviour in difference power spectrum, will be hard to mimic by simple changes in the primordial power spectrum and provides a potential way of distinguishing these two effects. The oscillations are, however, only a fraction of the total change in the power spectrum and will require very precise measurement of the B-modes power spectrum with \( \ell \) resolution better than \( \delta \ell \sim 25 \). We, therefore, conclude that although in principle it is possible to distinguish between simple primordial tensor power spectra and new neutrino interactions, in practice it will be very challenging and there is potential degeneracy between the two. This point is very important, since in the event primordial \( B \)-modes are detected with a large enough amplitude, the tensor spectral index being almost scale invariant \( n_T \sim 0 \) (or slightly negative), is expected to be a diagnostic which can rule out a large class of inflationary models [87–89]. Non-standard neutrino interactions confuse our ability to test early Universe physics such as inflation.

The biggest challenge in using CMB \( B \)-modes as a probe of new physics in the neutrino/dark sector is that even though we have a considerably large effect (\( \gtrsim 10\% \) for \( u \gtrsim 10^{-3} \)), most of the effect of the dark neutrino interactions is confined to small scales, \( \ell > 100 \). On these scales, given current limits on the tensor to scalar ratio [8], the CMB lensing \( B \)-modes, arising from lensing of primordial E-modes by structure at low redshifts, dominate over the primordial \( B \)-modes. Our ability to detect the primordial \( B \)-modes at \( \ell < 100 \) will, therefore, depend crucially on how efficiently we can delens the CMB [67–81].

Finally we note that the new interactions between neutrino and dark matter would also affect the scalar modes and there are already constraints from the current CMB and large scale structure data [42–44, 82–84, 90]. The effect on scalar modes is a sum of two effects which are opposite in sign, the non-free streaming of neutrinos which enhances the matter perturbations and radiation pressure in the dark matter fluid which suppresses the matter perturbations. It may be possible to weaken these constraints by playing these two effects against each other within some region of parameter space while still getting large effect on the primordial CMB \( B \)-modes. We will explore these possibilities in a future publication [85].

Keeping in mind the strong focus of the CMB community on the detection of primordial \( B \)-modes with ground based, balloon based and space based experiments in the next decade, we have tried to explore the consequences of an eventual detection. We find that new physics in the neutrino-dark sector may influence the cosmological signal in a non-trivial way. In particular, in the event that the primordial \( B \)-modes are measured up to a few hundred \( \ell \), a deviation from a scale invariant spectrum could be interpreted either as the failure of the simplest inflationary models or new physics in the dark sector. These two can be disentangled.
if the CMB B-modes are detected with high S/N at \( \ell > 100 \).

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**A Collision terms of DM-neutrino interaction**

In this section, we derive the collision terms for neutrino Boltzmann equations in Section 3. The Boltzmann equations for tensor perturbation from Eqn 3.1 reads

\[
\frac{\partial \delta f_\nu(x, p, t)}{\partial t} + \frac{p_i}{a(t)p} \frac{\partial \delta f_\nu(x, p, t)}{\partial x^i} - \frac{\partial \tilde{f}_\nu(p)}{\partial p} \frac{p_i p_j}{2p} \frac{\partial}{\partial t} D_{ij}(x, t)
\]

\[
= \left\{ \begin{array}{l}
-\eta \rho_\nu \left[ \delta f_\nu(p\hat{p}) - \frac{1}{4\pi} \int d^2 \hat{p}' \delta f_\nu(p\hat{p}') \right] : \text{Limit 1} \\
-\eta \rho_\nu \frac{p_i}{m^2} \left[ \delta f_\nu(p\hat{p}) - \frac{1}{4\pi} \int d^2 \hat{p}' \delta f_\nu(p\hat{p}') \right] : \text{Limit 2}
\end{array} \right.
\]

(A.1)

In this limit, there is no momentum dependence in the collision term. Therefore, we can trivially carry out the momentum integration to arrive at,

\[
\frac{\partial J(x, \hat{p}, t)}{\partial t} + \frac{\hat{p}_i}{a(t)} \frac{\partial J(x, \hat{p}, t)}{\partial x^i} + 2\hat{p}_i \hat{p}_j \frac{\partial}{\partial t} [D_{ij}(x, t)] = -n\eta \rho_\nu \left[ J(x, \hat{p}, t) - \frac{1}{4\pi} \int d^2 \hat{p}' J(x, \hat{p}', t) \right]
\]

(A.3)

This, as anticipated, gives the \( T_\nu \)-independent collision term.
A.2 Limit 2: \((T^2_\nu \text{ dependent})\)

We can relate the fluctuations in the neutrino distribution function to the fluctuations of the neutrino temperature \(\Theta\) defined by

\[
f_\nu(x, p, \hat{p}, t) = \left[ \exp \left( \frac{p}{T_\nu(t) \left[ 1 + \Theta(x, \hat{p}, t) \right]} \right) + 1 \right]^{-1}
\]

(A.4)

by expanding \(f_\nu\) to first order in \(\Theta\),

\[
\delta f_\nu(x, p, \hat{p}, t) = -p \frac{\partial f_\nu}{\partial p} \Theta(x, \hat{p}, t),
\]

(A.5)

where \(\bar{f}_\nu = f_\nu(\Theta = 0)\) is the zeroth order Fermi-Dirac distribution for neutrino. The relation of this new variable \(\Theta(x, \hat{p}, t)\) with \(J(x, \hat{p}, t)\) is

\[
J(x, \hat{p}, t) = \frac{N_\nu}{a^4 \rho_\nu} \int_0^\infty \delta f_\nu(x, p, t) 4\pi p^3 dp
\]

(A.6)

\[
= \frac{N_\nu}{a^4 \rho_\nu} \int_0^\infty -p \frac{\partial \bar{f}_\nu}{\partial p} \Theta(x, \hat{p}, t) 4\pi p^3 dp
\]

(A.7)

\[= 4\Theta(x, \hat{p}, t)
\]

(A.8)

Integrating over momentum and using the relation above, we get

\[
\frac{\partial J(x, \hat{p}, t)}{\partial t} + \frac{\dot{\rho}_\nu}{a(t)} \frac{\partial J(x, \hat{p}, t)}{\partial x} + 2\dot{\rho}_\nu \frac{\partial}{\partial t}[D_{ij}(x, t)]
\]

(A.9)

\[
= -n_\chi \sigma \frac{1}{m_\chi^2} \left[ \frac{\int_0^\infty \delta f_\nu(p\hat{p}) 4\pi p^5 dp}{\int_0^\infty f_\nu 4\pi p^3 dp} - \frac{1}{4\pi} \int d^2 \hat{p}' \left\{ \Theta(\hat{p}) \int_0^\infty \delta f_\nu(p\hat{p}') 4\pi p^5 dp \right\} \right]
\]

(A.10)

\[
= -n_\chi \sigma \frac{1}{m_\chi^2} \left[ \frac{\int_0^\infty -p \frac{\partial \bar{f}_\nu}{\partial p} \Theta(\hat{p}) p^5 dp}{\int_0^\infty f_\nu 4\pi p^3 dp} - \frac{1}{4\pi} \int d^2 \hat{p}' \left\{ \int_0^\infty -p \frac{\partial \bar{f}_\nu}{\partial p} \Theta(\hat{p}') p^5 dp \right\} \right]
\]

(A.11)

\[
= -n_\chi \sigma \frac{6}{m_\chi^2} \left( \frac{310\pi^2 T^2_\nu}{147} \right) \left[ \Theta(\hat{p}) - \frac{1}{4\pi} \int d^2 \hat{p}' \Theta(\hat{p}') \right]
\]

(A.12)

\[
= -n_\chi \sigma \frac{6}{4m_\chi^2} \left( \frac{310\pi^2 T^2_\nu}{147} \right) \left[ J(x, \hat{p}, t) - \frac{1}{4\pi} \int d^2 \hat{p}' J(x, \hat{p}', t) \right]
\]

(A.13)

\[
= -n_\chi \sigma \left( \frac{T_\nu}{1.95 \text{ K}} \right)^2 \left[ J(x, \hat{p}, t) - \frac{1}{4\pi} \int d^2 \hat{p}' J(x, \hat{p}', t) \right].
\]

(A.14)

We have used the following integral in the second last line,

\[
\int_0^\infty \bar{f}_\nu p^5 dp = \frac{310\pi^2 T^2_\nu}{147}
\]

(A.15)

and defined

\[
\sigma^{(2)} \equiv \sigma \left( \frac{3}{2} \right) \left( \frac{310\pi^2}{147} \right) \left( \frac{T_{\nu,0}}{m_\chi} \right)^2
\]

(A.16)

where \(T_{\nu,0}\) is neutrino temperature today.
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