Axino warm dark matter and $\Omega_b - \Omega_{DM}$ coincidence

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Abstract

We show that axinos, which are dominantly generated by the decay of the next-to-lightest supersymmetric particles produced from the leptonic $Q$-ball (L-ball), become warm dark matter suitable for the solution of the missing satellite problem and the cusp problem. In addition, $\Omega_b - \Omega_{DM}$ coincidence is naturally explained in this scenario.

PACS numbers: 95.35.+d, 12.60.Jv, 98.80.Cq, 04.65.+e
I. INTRODUCTION

Recent observations of cosmic microwave background anisotropies such as Wilkinson Microwave Anisotropy Probe measured the abundance of components of the Universe very precisely. However, their origins are still one of the major mysteries of cosmology and particle physics. The fact that the abundances of dark matter and baryon are of the same order may give us a great hint for their origins.

In the minimal supersymmetric standard model (MSSM), flat directions consist of squarks/sleptons and produce a non-zero baryon or lepton number through the Affleck-Dine (AD) mechanism [1]. Then, $Q$-balls, which are non-topological solitons [2], can be produced due to the instability and absorb almost all the produced baryon or lepton numbers [3]. In the gravity mediated supersymmetry breaking model, the mass per charge of a $Q$-ball is larger than that of a nucleon so that they are unstable against the decay into light fermions. Then, they can directly decay into baryons and the lightest supersymmetric particle (LSP). In case that the charge of the produced $Q$-balls is large enough, $Q$-balls can survive even after the freeze-out of weakly interacting massive particles (WIMPs). Thus, the reason why energy densities of dark matter and baryon are almost the same magnitude can be explained in this scenario [4, 5].

However, it was pointed out that LSPs are often overproduced by the decay of $Q$-balls if the LSP is the lightest neutralino in the MSSM [6], which gives the stringent conditions on the neutralino LSPs and AD fields. Only a few models free from this overproduction have been proposed [7,8]. Instead, the supergravity models in which the LSPs is a stable gravitino are investigated [9]. In this scenario, $Q$-balls decay into the next-to-lightest supersymmetric particle (NLSP) directly instead of the LSP gravitino. The LSP gravitinos are produced by the decay of NLSP and becomes dominant over other gravitinos produced by thermal processed [10,11] and by the decay of thermal produced NLSP [12,13] in case that $Q$-balls can survive the evaporation. Then, it is found that such a gravitino dark matter scenario is still viable if the late decay of NLSP does not spoil the success of Big Bang Nucleosynthesis (BBN). Another interesting possibility is that the LSP is an axino, which is the fermionic superpartner of an axion. Axinos are also produced by the decay of NLSPs produced from the $Q$-ball [14] as well as by thermal processes and by the decay of thermally produced NLSP [15,16,17,18,19].

Such gravitinos and axinos often become an ideal candidate for cold dark matter. The models of cold dark matter (CDM) and dark energy combined with inflation-based scale-invariant primordial density fluctuations have succeeded at explaining many properties of the observed universe, especially the large scale structure of the universe. However, going into the smaller scales, some observations on galactic and subgalactic ($\lesssim$ Mpc) seem to conflict with predictions by high-resolution N-body simulations as well as analytic calculations based on the standard CDM model. The first discrepancy is called the missing satellite problem [20]. The CDM-based models predict an order of magnitude higher number of halos than those actually observed within the Local Group. The other is called the cusp problem [21]. The CDM-based models also predict overly cuspy mass profile for the CDM halos compared to actual observations within the Local Group. In order to reconcile such discrepancies, several authors proposed modifications to the standard CDM-based model though the photoionization mechanism may overcome such difficulties [22]. One method is to reduce the small-scale power of primordial density fluctuations, which can be realized in a specific model of inflation [23]. Another is to change the properties of dark matter.
Spergel and Steinhardt introduced strong self-interaction among cold dark matter particles (collisional CDM), which enhances satellite destruction and suppress cusp formation [24]. The warm dark matter [25], which can have relatively large velocity dispersion at the epoch of the matter-radiation equality, can also reduce satellite production and cusp formation.

In this paper, we consider axinos dominantly generated by the decay of NLSPs produced from the leptonic $Q$-ball (L-ball). Such axinos become warm dark matter suitable for the solution of the missing satellite problem and the cusp problem. In addition, $\Omega_b - \Omega_{DM}$ coincidence is naturally explained through the Affleck-Dine mechanism and the subsequent L-ball formation in this scenario. In the next section, we discuss $\Omega_b - \Omega_{DM}$ coincidence based on the Affleck-Dine mechanism and the subsequent L-ball formation. In section III, we show that axinos in our scenario become warm dark matter suitable for the solution of the missing satellite problem and the cusp problem. In the final section, we give concluding remarks.

II. $\Omega_b - \Omega_{DM}$ COINCIDENCE FROM AFFLECK-DINE LEPTOGENESIS

We now discuss baryogenesis via Affleck-Dine leptogenesis and dark matter production from $Q$-ball decays within the framework of gravity mediated supersymmetry breaking.

A. Lepton asymmetry

The potential of the AD flat direction field is, in general, lifted by soft supersymmetric (SUSY) breaking terms and non-renormalizable terms [26, 27]. The full potential of the AD field is given by

$$V(\phi) = \left( m_\phi^2 \left[ 1 + K \ln \left( \frac{|\phi|^2}{\Lambda^2} \right) - c_1 H^2 \right] + \sum_{n=1}^{\infty} \frac{c_2 H + A m_{3/2}}{n M^{n-3}} + \text{H.c.} \right) + \lambda^2 |\phi|^{2n-2} \frac{m_3}{M^{2n-6}}.$$  

Here, $m_\phi$ is the soft SUSY breaking scalar mass for the AD field with radiative correction $K \ln |\phi|^2$. A flat direction dependent constant, $K$, takes values from $-0.01$ to $-0.1$ [28]. $\Lambda$ denotes a renormalization scale and $-c_1 H^2$ represents the negative mass squared induced by the SUSY breaking effect which comes from the energy density of the inflaton, with an order unity coefficient $c_1 > 0$ [27]. $\lambda$ is the coupling of a nonrenormalizable term and $M$ is some large scale acting as its cut-off. Terms proportional to $A$ and $c_2$ are the A-terms coming from the low energy SUSY breaking and the inflaton-induced SUSY breaking, respectively, where $m_{3/2}$ denotes the gravitino mass. Here, we omitted possible terms which may appear by thermal effects [29, 30]. These terms are negligible as long as we consider a sufficient low reheating temperature after inflation, as we will. Moreover, the model would face with “gravitino problem” [31], if the reheating temperature after inflation is so high that these thermal effect become effective, unless gravitino is LSP [8].

The charge number density for the AD field $n_q$ is given by $n_q = i q (\dot{\phi}^* \dot{\phi} - \dot{\phi}^* \dot{\phi})$ where $q$ is the baryonic (or leptonic) charge for the AD field. By use of the equation of motion of the AD field, the charge density can be rewritten as

$$n_q(t) \simeq \frac{1}{a(t)^3} \int^t dt' a(t')^3 \frac{2q \lambda m_{3/2}}{M^{n-3}} \text{Im}(A \phi^o),$$  

(2)
with $a(t)$ being the scale factor. When the AD field starts to oscillate around the origin, the charge number density is induced by the relative phase between A-terms. By taking into account $s = 4\pi^2 g_* T^3 / 90$, the charge to entropy ratio after reheating is estimated as

$$\frac{n_q}{s} = \frac{T_R n_q}{4 M_P^2 H^2} \bigg|_{t_{\text{os}}} \approx \frac{q |A| \lambda m_{3/2}^2}{2 M_P^2 H^3} T_R |\phi_{\text{os}}|^n \sin \delta. \quad (3)$$

Here, $M_P \approx 2.4 \times 10^{18}$ GeV is the reduced Planck mass, $t_{\text{os}}$ is the time of the start of the oscillation and $\sin \delta$ is the effective CP phase. In case that thermal corrections are ineffective, $H_{\text{os}} \approx m_\phi$, which yields

$$|\phi_{\text{os}}| \approx \left( \frac{m_\phi M^{n-3}}{-2m_\phi^2} \right)^{1/(n-2)} \quad (4)$$

From now on, as a concrete example, we consider a $LL\bar{e}$ direction of $n = 6$ as the AD field for our scenario. Since this is a pure leptonic direction, the lepton asymmetry generated by the Affleck-Dine mechanism can be estimated as

$$\frac{n_L}{s} \approx 1 \times 10^{-10} \frac{q |A| \sin \delta}{\lambda^{1/2}} \left( \frac{m_{3/2}}{100 \text{GeV}} \right)^{3/2} \left( \frac{10^3 \text{GeV}}{m_\phi} \right)^2 \left( \frac{T_R}{100 \text{GeV}} \right) \left( \frac{M}{M_P} \right)^{3/2} \quad (5)$$

B. Baryon asymmetry and LSP production from $Q$-balls

The produced lepton asymmetry is not directly released to thermal bath. Instead, $L$-balls are formed due to the instability and almost all produced lepton numbers are absorbed into $L$-balls [3].

First of all, we briefly summarize relevant properties of $Q$-balls in gravity mediated SUSY breaking models. The radius of a $Q$-ball, $R$, is estimated as $R^2 \sim 2/(|K|m_\phi^2)$ [5]. Numerical calculations provide a fitting formula for the $Q$-ball charge

$$Q \approx 3 \left( \frac{|\phi_{\text{os}}|}{m_\phi} \right) \times \left\{ \begin{array}{cl} \epsilon & \text{for } \epsilon \geq \epsilon_c \\ \epsilon_c & \text{for } \epsilon < \epsilon_c \end{array} \right\} \quad (6)$$

with

$$\epsilon \equiv \frac{n_L}{n_\phi} \bigg|_{t_{\text{os}}} \approx 2 q |A| \frac{m_{3/2}^2}{m_\phi^2} \sin \delta \quad (7)$$

where $\epsilon_c \approx 10^{-2}$ and $\bar{\beta} = 6 \times 10^{-3}$ [3]. The $Q$-ball charge can be evaluated as

$$Q \sim 2 \times 10^{20} \left( \frac{\epsilon}{4 \times 10^{-1}} \right) \left( \frac{1 \text{TeV}}{m_\phi} \right)^{3/2} \left( \frac{M^3}{\lambda M_P^3} \right)^{1/2} \quad (8)$$

where we assumed $\epsilon > \epsilon_c$ because it looks to be more natural than the other which can be realized only for an accidental small $\sin \delta$ . Furthermore, if $\epsilon < \epsilon_c$, additional “unnatural” parameters are required for our scenario, as we will show.

A part of the charge of a $Q$-ball can evaporate by the interaction with particles in the thermal bath. The evaporation of charge of $Q$-ball is done by the evaporation with the rate

$$\frac{d Q}{dt} = -4\pi R_Q D_{\text{ev}} n^{\text{eq}} \sim -4\pi R_Q^2 D_{\text{ev}} \mu_Q T^2, \quad (9)$$
with $D_{ev} \lesssim 1$ and by the diffusion with the rate

$$\Gamma_{\text{diff}} \equiv \frac{dQ}{dt} = -4\pi k R_Q D_{\text{diff}} n^\phi \simeq -4\pi k R_Q D_{\text{diff}} \mu_Q T^2, \quad (10)$$

where $\mu_Q$ is the chemical potential of $Q$-balls and the numerical constant $k$ is very close to unity so that we will drop it hereafter. $D_{\text{diff}} \approx a/T$ is a diffusion constant \[32\] and $a$ is a particle dependent coefficient given by \[33, 34\]

$$a \simeq \begin{cases} 
4 & \text{for squark} \\
6 & \text{for quark} \\
100 & \text{for left- handed (s)lepton} \\
380 & \text{for right- handed (s)lepton} 
\end{cases} \quad (11)$$

Here, we see that both the evaporation and the diffusion are efficient for low temperature, from Eqs. (9) and (10) with the relation between the cosmic time and the temperature:

$$\frac{dt}{dT} = \begin{cases} 
\frac{-8}{\pi} \sqrt{\frac{10}{g_*}} \frac{T_R^2 M_P}{T^3} & \text{for } T \gtrsim T_R \\
\frac{-3}{\pi} \sqrt{\frac{10}{g_*}} \frac{M_P}{T^2} & \text{for } T < T_R 
\end{cases} \quad (12)$$

Moreover, by comparing $\Gamma_{\text{diff}}$ and $\Gamma_{\text{evap}},$

$$\frac{\Gamma_{\text{diff}}}{\Gamma_{\text{evap}}} \simeq \left( \frac{m_\phi}{T} a \sqrt{\frac{|K|}{2}} \right) \left( \frac{1}{D_{ev}} \right), \quad (13)$$

we can find that for low temperature

$$T \lesssim a \sqrt{\frac{|K|}{2}} m_\phi \sim 10 m_\phi \left( \frac{a}{10^2} \right) \left( \frac{|K|}{10^{-2}} \right)^{1/2}, \quad (14)$$

the diffusion is more crucial for estimation of the evaporated charge from $Q$-ball. Equation (10) is rewritten as

$$\frac{dQ}{dT} \simeq -4\pi R_Q D_{\text{diff}} \mu_Q T^2 \left( \frac{dt}{dT} \right), \quad (15)$$

Integrating Eq. (15) from $m_\phi$, because the evaporation from $Q$-ball is suppressed by the Boltzmann factor, we can estimate the total evaporated charge as

$$\Delta Q \simeq 32k R_Q \sqrt{\frac{10}{g_*}} a \mu_Q \frac{T_R^2 M_P}{3 m_\phi^3}$$

$$\sim 3.2 \times 2.4 \sqrt{10} \times 10^{19} \left( \frac{1 \text{TeV}}{m_\phi} \right)^3 \left( \frac{T_R}{1 \text{TeV}} \right)^2 \left( \frac{a}{300} \right) \sqrt{\frac{200}{g_*}} \sqrt{\frac{0.01}{|K|}} \left( \frac{\mu_Q}{m_\phi} \right),$$

for $m_\phi \gtrsim T_R,$

$$\Delta Q \sim 3.6 \times 2.4 \sqrt{10} \times 10^{19} \left( \frac{1 \text{TeV}}{m_\phi} \right) \left( \frac{a}{300} \right) \sqrt{\frac{200}{g_*}} \sqrt{\frac{0.01}{|K|}} \left( \frac{\mu_Q}{m_\phi} \right) \quad (16)$$

$$\sim 3.6 \times 2.4 \sqrt{10} \times 10^{19} \left( \frac{1 \text{TeV}}{m_\phi} \right) \left( \frac{a}{300} \right) \sqrt{\frac{200}{g_*}} \sqrt{\frac{0.01}{|K|}} \left( \frac{\mu_Q}{m_\phi} \right) \quad (17)$$
for \( m_\phi < T_R \). By taking Eq. (8) into account, we obtain

\[
\frac{\Delta Q}{Q} \sim \frac{3}{2} \times 10^{-1} \left( \frac{4 \times 10^{-1}}{\epsilon} \right) \left( \frac{a}{300} \right) \left( \frac{m_\phi}{\text{1 TeV}} \right)^{1/2} \sqrt{\frac{\lambda M_p^3}{M^3}} \sqrt{\frac{200}{g_*}} \sqrt{\frac{0.01}{|K|}} \left( \frac{\mu_Q}{m_\phi} \right)
\]

(18)

for \( m_\phi < T_R \) \(^1\) and find that about 10% of Q-ball charge would be evaporated. Here, one can see why the case of \( \epsilon < \epsilon_c \) is irrelevant for us. If \( \epsilon < \epsilon_c \simeq 10^{-2} \), \( \epsilon \) is replaced with \( \epsilon_c \) in Eq. (18). Then these Q-balls cannot survive the evaporation unless the AD field mass is extremely small as \( m_\phi = \mathcal{O}(10) \text{ GeV} \) or \( (\lambda^{1/3} M_p/M)^{3/2} \ll 1 \).

The evaporated charges are released into the thermal bath so that a part of them is transformed into baryonic charges through the sphaleron effects \[35\]. Then, the resultant baryon asymmetry is given as

\[
\frac{n_b}{s} = \frac{8}{23} \times \frac{\Delta Q}{Q} \times \frac{n_L}{s} \\
\simeq \frac{8}{23} \Delta Q \times 10^{-30} \left( \frac{10^3 \text{GeV}}{m_\phi} \right)^{-1} \left( \frac{T_R}{100 \text{GeV}} \right) \\
\simeq 10^{-10} \left( \frac{a}{300} \right) \sqrt{\frac{200}{g_*}} \sqrt{\frac{0.01}{|K|}} \left( \frac{\mu_Q}{m_\phi} \right) \times \left\{ \left( \frac{T_R}{\text{1 TeV}} \right)^2 \left( \frac{\mu_Q}{m_\phi} \right) \right\}^2 \left( \frac{T_R}{\text{1 TeV}} \right)^3 \\
\text{for } m_\phi > T_R \quad \text{for } m_\phi < T_R.
\]

(19)

Interestingly, the baryon asymmetry does not depend on the effective CP phase sin \( \delta \) unlike usual Affleck-Dine baryogenesis, because the CP phase dependences in both lepton asymmetry \( n_L/s \) and the charge of Q-ball \( Q \) cancel each other. In addition, for \( m_\phi < T_R \), the baryon asymmetry basically depends on only one free parameter, the reheating temperature \( T_R \), because other parameters are not free but known in a sense.

When Q-balls decay, the supersymmetric particles are released from them. Since the Q-ball consists of scalar leptons, the number of the produced supersymmetric particles is given by

\[
Y_{\text{NLSP}} = N_Q \frac{n_L}{s} \\
= 2 \times 10^{-9} \left( \frac{N_Q}{1} \right) \left( \frac{\epsilon}{4 \times 10^{-1}} \right) \left( \frac{1 \text{ TeV}}{m_\phi} \right)^{1/2} \left( \frac{T_R}{\text{1 TeV}} \right)^2 \left( \frac{M^3}{\lambda M_p^3} \right)^{1/2},
\]

(20)

where \( N_Q \) is the number of produced NLSP particles per one leptonic charge.

Such produced NLSPs decay into axino LSP with a typical lifetime of \( \mathcal{O}(0.1 - 1) \) second. Thus, NLSPs produced by the Q-ball decay become a source of axino production. Of course, like gravitinos, axinos can also be produced by other processes such as thermal processes (TP), namely the scatterings and decays in the thermal bath, and non-thermal processes (NTP), say the late decay of NLSPs produced thermally. The relevant Boltzmann equations can be written as

\[
\dot{n}_{\text{NLSP}} + 3Hn_{\text{NLSP}} = -\langle \sigma v \rangle (n_{\text{NLSP}}^2 - n_{\text{eq,NLSP}}^2) + \gamma_{\text{Q-ball}} - \Gamma_{\text{NLSP}}n_{\text{NLSP}},
\]

(21)

\[
\dot{n}_\tilde{a} + 3Hn_\tilde{a} = \langle \sigma v(i + j \rightarrow \tilde{a} + ...) \rangle_{ij} n_in_j + \langle \sigma v(i \rightarrow \tilde{a} + ...) \rangle_i n_i + \Gamma_{\text{NLSP}}n_{\text{NLSP}},
\]

(22)

\(^1\) For \( m_\phi > T_R \), the result is of the same magnitude but with different dependence on \( m_\phi \) and \( T_R \).
where $\gamma_{Q\text{-ball}}$ denotes the contribution to NLSP production by $Q$-balls decay, $\langle \sigma v \rangle_{ij}$ and $\langle \sigma v \rangle_i$ are the scattering cross section and the decay rate for the thermal production of axinos, and $\Gamma_{\text{NLSP}}$ is the decay rate of the NLSP. The total NLSP abundance, before its decay, is given by

$$Y_{\text{NLSP}} = N_Q \frac{n_L}{s} + Y_{\text{TP}}^{\text{NLSP}},$$

where $N_Q n_L/s$ denotes the NLSP produced by $L$-ball decay and $Y_{\text{TP}}^{\text{NLSP}}$ is the abundance of NLSP produced thermally and given by

$$Y_{\text{TP}}^{\text{NLSP}} \simeq \left. H \right|_{T=m_{\text{NLSP}}} \frac{m_{\text{NLSP}}/T_f}{\langle \sigma v \rangle_{\text{ann}}}.$$ 

Here $\langle \sigma v \rangle_{\text{ann}}$ is the annihilation cross section and $T_f \sim m_{\text{NLSP}}/20$ is the freeze-out temperature. The resultant total axino abundance is expressed as

$$Y_{\tilde{a}} = Y_{\tilde{a}}^{\text{NTP}} + Y_{\tilde{a}}^{\text{TP}}.$$  

Here

$$Y_{\tilde{a}}^{\text{NTP}} = Y_{\text{NLSP}} = N_Q \frac{n_L}{s} + Y_{\text{TP}}^{\text{NLSP}}$$

is the nonthermally produced axino through the NLSP decay and $Y_{\tilde{a}}^{\text{TP}}$ denotes the axinos produced by thermal processes. For nonthermally produced axinos, while the NLSP abundance produced by $L$-ball decay is

$$N_Q \frac{n_L}{s} = 2 \times 10^{-9} \left( \frac{N_Q}{1} \right) \left( \frac{n_L/s}{2 \times 10^{-9}} \right),$$

the typical value of $Y_{\text{TP}}^{\text{NLSP}}$ is given by

$$Y_{\text{TP}}^{\text{NLSP}} \simeq 10^{-11} \left( \frac{100 \text{GeV}}{m_{\text{NLSP}}} \right) \left( \frac{10^{-10} \text{GeV}^{-2}}{\langle \sigma v \rangle_{\text{ann}}} \right).$$

Thus, nonthermal production of axinos due to the thermal relic NLSPs decay, $Y_{\text{NLSP}}^{\text{TP}}$, can be negligible compared to that from $Q$-ball produced NLSPs, $N_Q n_L/s$. On the other hand, axino production by thermal processes is dominated by scattering processes for the case that the reheating temperature is larger than the masses of neutralinos and gluinos. In this case, the abundance of such axinos is proportional to the reheating temperature $T_R$ and the inverse square of Peccei-Quinn (PQ) scale $f_a$ and given by

$$Y_{\tilde{a}}^{\text{TP}} \simeq 10^{-8} \left( \frac{T_R}{1 \text{TeV}} \right) \left( \frac{10^{11} \text{GeV}}{f_a/N} \right)^2,$$

where $N$ is the number of vacua and $N = 1(6)$ for the KSVZ (DFSZ) model. Thus, for $T_R \simeq 1 \text{ TeV}$, if $f_a/N \gtrsim \text{several} \times 10^{11} \text{ GeV}$, $Y_{\tilde{a}}^{\text{TP}}$ is subdominant compared with $Y_{\tilde{a}}^{\text{NTP}} \simeq N_Q n_L/s$.

If this is the case, the energy density of axino is given by $\rho_{\tilde{a}} = m_{\tilde{a}} n_{\text{NLSP}}$ due to the $R$-parity conservation. Recalling

$$\rho_{\text{DM}} \simeq 3.9 \times 10^{-10} \left( \frac{\Omega_{\text{DM}} h^2}{0.11} \right) \text{GeV},$$

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the density parameter of axinos is expressed as

$$\frac{\Omega_a h^2}{0.11} \approx \left( \frac{m_{\tilde{a}}}{0.2 \text{GeV}} \right) \left( \frac{n_L/s}{2 \times 10^{-9}} \right) \left( \frac{N_Q}{1} \right).$$  \hspace{1cm} (31)$$

Thus, axinos with the sub-GeV mass can be dark matter in our scenario. Now, one can see that the $\Omega_b$ and $\Omega_{DM}$ is related through the lepton asymmetry. In fact, from Eqs. (19) and (31), we obtain a relation between the abundances of dark matter and baryon asymmetry,

$$\frac{\Omega_b}{\Omega_{\tilde{a}}} \approx \frac{2}{11} \left( \frac{N_Q}{1} \right) \left( \frac{\Delta Q/Q}{1 \times 10^{-1}} \right) \left( \frac{0.2}{m_{\tilde{a}}/m_p} \right),$$  \hspace{1cm} (32)

where $m_p(\approx 1 \text{ GeV})$ is the mass of proton. One may find the similar relation in the case of baryonic $Q$-ball ($B$-ball) [14]. The difference between the case of $B$-ball and $L$-ball is that the required mass of LSP from $L$-ball can be an order of magnitude smaller than that in $B$-ball where the mass of LSP dark mater must be $\approx 1 \text{ GeV}$, mainly because a part of lepton asymmetry produced by the Affleck-Dine mechanism, that is, only evaporated charges $\Delta Q/Q$ are converted to baryon asymmetry so that the number density of NLSPs produced by the $L$-ball decay become larger for a fixed baryon asymmetry. As shown in the next section, such a difference of axino masses is crucial for solving the missing satellite problem and the cusp problem.

Equation (31) with $T_R \approx 1 \text{ TeV}$ to explain the observed baryon asymmetry yields a quite natural scale of the AD field mass

$$m_{\phi} \approx 1 \text{TeV} \left( \frac{\epsilon}{4 \times 10^{-1}} \right)^2 \left( \frac{M^3}{\lambda M_p^2} \right) \left( \frac{m_{\tilde{a}}}{0.2 \text{GeV}} \right)^2.$$  \hspace{1cm} (33)

As mentioned above, in our scenario, NLSP decays into axino at late time. Such late decay is potentially constrained by BBN. The lifetime of NLSP is given as

$$\tau_{\chi} \equiv \tau(\chi \rightarrow \tilde{a} + \gamma) = 0.33 \text{sec} \left( \frac{1}{C_{aYY}^2 Z_{11}^2} \right) \left( \frac{\alpha_{em}}{1/128} \right)^{-2} \left( \frac{f_a/N}{10^{11} \text{GeV}} \right)^2 \left( \frac{10^2 \text{GeV}}{m_{\chi}} \right)^3$$  \hspace{1cm} (34)

for the case that the lightest neutralino $\chi$ is NLSP in [18]. Here, $C_{aYY}$ is the axion model dependent coupling coefficient between axion multiplet and $U(1)_Y$ gauge field, $Z_{11}$ denotes the fraction of $b$-ino component in the lightest neutralino. According to Ref. [18], we can summerize the constraints as follows. First of all, for $\tau_{\chi} \leq 0.1 \text{ sec.}$, there is no constraint. The corresponding mass of the NLSP neutralino is

$$m_{\chi} = 320 \text{GeV} \left( \frac{0.1 \text{sec}}{\tau_{\chi}} \right)^{1/3} \left( \frac{1}{C_{aYY}^2 Z_{11}^2} \right)^{1/3} \left( \frac{f_a/N}{\sqrt{10} \times 10^{11} \text{GeV}} \right)^{2/3},$$  \hspace{1cm} (35)

from Eq. (34). Thus, if $m_{\chi} \gtrsim 320 \text{ GeV}$, this model is free from problems by the late decay of NLSP. This lower bound is a bit stringent than that in [18], because we need to take the PQ scale somewhat larger as we mentioned. On the other hand, for $0.1 \text{ sec.} \leq \tau_{\chi} < 1 \text{ sec.}$, the lower bound on axino mass exists and can be roughly expressed as

$$\frac{m_{\tilde{a}}}{0.1 \text{GeV}} \gtrsim -4 \left( \frac{m_{\chi}}{10^2 \text{GeV}} \right) \left( C_{aYY} Z_{11} \right)^{2/3} \left( \frac{f_a/N}{10^{11} \text{GeV}} \right)^{-2/3} + 6 \approx -4 \left( \frac{0.33 \text{sec.}}{\tau_{\chi}} \right)^{1/3} + 6,$$  \hspace{1cm} (36)
by reading Fig. 4 in Ref. [18]. In this case, axino must be heavier than a few hundred MeV. For $\tau_\chi \simeq 1$ sec., the corresponding mass of the NLSP neutralino and the lower bound of axino mass is given by

$$m_\chi \simeq 150\text{GeV},$$
$$m_\tilde{a} \gtrsim 320\text{MeV}.$$  \hspace{1cm} (37)

### III. A SOLUTION TO THE MISSING SATELLITE PROBLEM AND THE CUSP PROBLEM

An interesting consequence of such light axinos is their large velocity dispersion. Therefore, they can potentially solve the missing satellite problem and the cusp problem as stated in the introduction. For the scenario of dark matter particle produced by the late decay of long-lived particle, it is shown that the missing satellite problem and the cusp problem can be solved simultaneously if the lifetime of long-lived particle and the ratio of mass between dark matter particle and the mother particle satisfy the following relation [39]:

$$\left(\frac{6.3 \times 10^2 m_\tilde{a}}{m_\chi}\right)^2 \text{sec.} \lesssim \tau_\chi \lesssim \left(\frac{1.0 \times 10^3 m_\tilde{a}}{m_\chi}\right)^2 \text{sec.},$$  \hspace{1cm} (38)

where we identified dark matter with axino (LSP) and the long-lived particle with the lightest neutralino (NLSP), respectively.

Combining Eq. (34) with Eq. (38), we have the following relation between the masses of the axino (LSP) and the lightest neutralino (NLSP),

$$m_\chi \simeq (0.33 - 0.83) \left(\frac{m_\tilde{a}}{1\text{GeV}}\right)^{-2} \left(\frac{1}{C_{aYY}^2 Z_{11}^2}\right) \left(\frac{\alpha_{em}}{1/128}\right)^{-2} \left(\frac{f_a/N}{10^{11}\text{GeV}}\right)^2 \text{GeV}. \hspace{1cm} (39)$$

For $m_\tilde{a} \simeq 1$ GeV in the $B$-ball case [14], $m_\chi$ becomes a few GeV with its lifetime $\tau_\chi \gg 10^3$ second even if we take $f_a/N$ to be several times $10^{11}$ GeV, which is excluded. On the other hand, for $m_\tilde{a} = \mathcal{O}(0.1)$ GeV in the $L$-ball case of this paper, $m_\chi$ becomes $\mathcal{O}(100)$ GeV with its lifetime $\tau_\chi \lesssim 1$ second if we take $f_a/N$ to be several times $10^{11}$ GeV. Thus, we find that axinos in this scenario can solve the missing satellite problem and the cusp problem simultaneously for natural mass scales with $m_\tilde{a} = \mathcal{O}(0.1)$ GeV and $m_\chi = \mathcal{O}(100)$ GeV.

### IV. CONCLUDING REMARKS

In this paper, we show that Affleck-Dine leptogenesis can explain baryon asymmetry and dark matter abundance simultaneously and that $\Omega_b - \Omega_{DM}$ coincidence is explained for sub-GeV mass of the LSP axino. Though the basic idea is the same as Ref. [14], where $B$-balls are considered, the mass of LSP axino becomes an order of magnitude smaller in this scenario. On the other hand, the PQ scale is determined as $f_a/N = \text{a few} \times 10^{11}$ GeV, which will be tested in the future if PQ scale can be measured by e.g., the manner proposed in [40].

The other attractive point is that axinos considered in this paper can potentially solve the missing satellite problem and the cusp problem simultaneously because they are relatively light and have large velocity dispersion. We have shown that axions in our scenario can solve
both problems for natural mass scales with $m_a = \mathcal{O}(0.1) \text{GeV}$ and $m_{\text{NLSP}} = \mathcal{O}(100) \text{GeV}$. For simplicity, we concentrate on the case of the lightest neutralino NLSP with the mass to be $\mathcal{O}(100) \text{GeV}$. Such neutralinos are detectable in Large Hadron Collider. In addition, the corresponding lepton asymmetry is almost the maximal value under the assumption of $(M^3/\lambda M^3_P) \simeq 1$. Hence, if this model is the simultaneous answer to both $\Omega_b - \Omega_{\text{DM}}$ coincidence, the missing satellite problem, and the cusp problem, we would discover the NLSP neutralinos with the mass of $\mathcal{O}(100) \text{GeV}$ and their decays into photons (and negligible missing energy) with the lifetime of about $(0.1 - 1)$ second.

**Acknowledgments**

We thank M. Kawasaki and T. Takahashi for useful comments. We are grateful to John McDonald for pointing out an error in the power of a coefficient in an earlier version. O.S. would like to thank RESCEU at the University of Tokyo for the hospitality where this work was initiated. The work of O.S. is supported by the MEC project FPA 2004-02015 and the Comunidad de Madrid project HEPHACOS (No. P-ESP-00346). M.Y. is supported in part by JSPS Grant-in-Aid for Scientific Research No. 18740157 and the project of the Research Institute of Aoyama Gakuin University.

[1] I. Affleck and M. Dine, Nucl. Phys. B249, 361 (1985).
[2] S. Coleman, Nucl. Phys. B262, 263 (1985).
[3] S. Kasuya and M. Kawasaki, Phys. Rev. D 62, 023512 (2000).
[4] A. Kusenko and M. Shaposhnikov, Phys. Lett. B 418, 46 (1998).
[5] K. Enqvist and J. McDonald, Phys. Lett. B 425, 309 (1998); K. Enqvist and J. McDonald, Nucl. Phys. B 538, 321 (1999).
[6] K. Enqvist and J. McDonald, Phys. Lett. B 440, 59 (1998); Nucl. Phys. B570, 407 (2000).
[7] M. Fujii, K. Hamaguchi and T. Yanagida, Phys. Rev. D 64, 123526 (2001).
[8] M. Fujii and K. Hamaguchi, Phys. Lett. B 525, 143 (2002); Phys. Rev. D 66, 083501 (2002).
[9] O. Seto, Phys. Rev. D 73, 043509 (2006).
[10] M. Bolz, W. Buchmuller, and M. Plumacher, Phys. Lett. B 443, 209 (1998).
[11] For a recent analysis, see e.g., J. Pradler and F. D. Steffen, Phys. Rev. D 75, 023509 (2007); Phys. Lett. B 648, 224 (2007).
[12] J. L. Feng, A. Rajaraman, and F. Takayama, Phys. Rev. Lett. 91, 011302 (2003).
[13] J. L. Feng, S. Su, and F. Takayama, Phys. Rev. D 70, 075019 (2004).
[14] L. Roszkowski and O. Seto, Phys. Rev. Lett. 98, 161304 (2007).
[15] K. Rajagopal, M. S. Turner, and F. Wilczek, Nucl. Phys. B358, 447 (1991).
[16] J. E. Kim, A. Masiero, and D. V. Nanopoulos, Phys. Lett. 139B, 346 (1984).
[17] L. Covi, J. E. Kim, and L. Roszkowski, Phys. Rev. Lett. 82, 4180 (1999).
[18] L. Covi, H. B. Kim, J. E. Kim and L. Roszkowski, J. High Energy Phys. 0105, 033 (2001).
[19] A. Brandenburg and F. D. Steffen, JCAP 0408, 008 (2004).

Of course, NLSP might be another kind of particle, although we do not discuss here.
[20] A. A. Klypin, A. V. Kravtsov, O. Valenzuela, and F. Prada, Astrophys. J. 522, 82 (1999); B. Moore, et al., Astrophys. J. 524, L19 (1999); A. R. Zentner and J. S. Bullock, Astrophys. J. 598, 49 (2003).

[21] B. Moore, Nature (London) 370, 629 (1994); R. A. Flores and J. A. Primack, Astrophys. J. 427, L1 (1994); W. J. G. De Blok and S. S. McGaugh, Mon. Not. R. Astron. Soc. 290, 533 (1997); J. J. Binney and N. W. Evans, Mon. Not. R. Astron. Soc. 327, L27 (2001); A. R. Zentner and J. S. Bullock, Phys. Rev. D 66, 043003 (2002); J. D. Simon et al., Astrophys. J. 621, 757 (2005).

[22] J. S. Bullock, A. V. Kravtsov, and D. H. Weinberg, Astrophys. J. 539, 517 (2000).

[23] For example, see J. Yokoyama, Phys. Rev. D 62, 123509 (2000).

[24] D. N. Spergel and P. J. Steinhardt, Phys. Rev. Lett. 84, 3760 (2000).

[25] P. Colin, V. Avila-Reese, and O. Valenzuela, Astrophys. J. 542, 622 (2000); J. Hisano, K. Kohri and M. M. Nojiri, Phys. Lett. B 505, 169 (2001); W. B. Lin, D. H. Huang, X. Zhang and R. H. Brandenberger, Phys. Rev. Lett. 86, 954 (2001). P. Bode, J. P. Ostriker, and N. Turok, Astrophys. J. 556, 93 (2001); J. L. Feng, A. Rajaraman, and F. Takayama, Phys. Rev. Lett. 91, 011302 (2003); J. A. R. Cembranos, J. L. Feng, A. Rajaraman, and F. Takayama, Phys. Rev. Lett. 95, 181301 (2005); M. Kaplinghat, Phys. Rev. D 72, 063510 (2005).

[26] K. W. Ng, Nucl. Phys. B 321, 528 (1989).

[27] M. Dine, L. Randall and S. Thomas, Nucl. Phys. B 458, 291 (1996).

[28] K. Enqvist and J. McDonald, Phys. Lett. B 425, 309 (1998); K. Enqvist, A. Jokinen and J. McDonald, Phys. Lett. B 483, 191 (2000).

[29] R. Allahverdi, B. A. Campbell and J. R. Ellis, Nucl. Phys. B 579, 355 (2000).

[30] A. Anisimov and M. Dine, Nucl. Phys. B 619, 729 (2001).

[31] M. Y. Khlopov and A. D. Linde, Phys. Lett. B 138, 265 (1984); J. R. Ellis, J. E. Kim and D. V. Nanopoulos, Phys. Lett. B 145, 181 (1984).

[32] R. Banerjee and K. Jedamzik, Phys. Lett. B 484, 278 (2000).

[33] M. Joyce, T. Prokopec and N. Turok, Phys. Rev. D 53, 2930 (1996).

[34] H. Davoudiasl and E. Westphal, Phys. Lett. B 432, 128 (1998).

[35] V. A. Kuzmin, V. A. Rubakov, and M. E. Shaposhnikov, Phys. Lett. 155B, 36 (1985); S. Y. Khlebnikov and M. E. Shaposhnikov, Nucl. Phys. B308, 885 (1988); J. A. Harvey and M. S. Turner, Phys. Rev. D 42, 3344 (1990).

[36] J. E. Kim, Phys. Rev. Lett. 43, 103 (1979); M. A. Shifman, V. I. Vainshtein, and V. I. Zakharov, Nucl. Phys. B166, 493 (1980).

[37] M. Dine, W. Fischler, M. Sredicki, Phys. Lett. 104B, 199 (1981); A. R. Zhitnitsky, Yad. Fiz. 31, 497 (1980) [Sov. J. Nucl. Phys. 31, 260 (1980)].

[38] For the latest constraint on the breaking scale of the PQ symmetry, see M. Yamaguchi, M. Kawasaki, and J. Yokoyama, Phys. Rev. Lett. 82, 4578 (1999).

[39] J. Hisano, K. T. Inoue and T. Takahashi, Phys. Lett. B 643, 141 (2006).

[40] A. Brandenburg, L. Covi, K. Hamaguchi, L. Roszkowski and F. D. Steffen, Phys. Lett. B 617, 99 (2005).