Intermediate Mass Black Hole Formation in compact Young Massive Star Clusters

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ABSTRACT

Young dense massive star clusters are a promising environment for the formation of intermediate mass black holes (IMBHs) through collisions. We present a set of 80 simulations carried out with Nbody6++GPU of 10 initial conditions for compact \( \sim 7 \times 10^4 \) solar mass star clusters with half-mass radii \( R_h \lesssim 1 \) pc, central densities \( \rho_{\text{core}} \gtrsim 10^5 M_\odot \text{pc}^{-3} \), and resolved stellar populations with 10% primordial binaries. Very massive stars (VMSs) with masses up to \( \sim 400 M_\odot \) grow rapidly by binary exchange and three-body scattering events with main sequence stars in hard binaries. Assuming that in VMS - stellar BH collisions all stellar material is accreted onto the BH, IMBHs with masses up to \( \sim 350 M_\odot \) can form on timescales of \( \lesssim 15 \) Myr. This process was qualitatively predicted from Monte Carlo MOCCA simulations. Despite the stochastic nature of the process - typically not more than 3/8 cluster realisations show IMBH formation - we find indications for higher formation efficiencies in more compact clusters. Assuming a lower accretion fraction of 0.5 for VMS - BH collisions, IMBHs can also form. The process might not work for accretion fractions as low as 0.1. After formation, the IMBHs can experience occasional mergers with stellar mass BHs in intermediate mass-ratio inspiral events (IMRIs) on a 100 Myr timescale. Realised with more than \( 10^5 \) stars, 10% binaries, the assumed stellar evolution model with all relevant evolution processes included and 300 Myr simulation time, our large suite of simulations indicates that IMBHs of several hundred solar masses might form rapidly in massive star clusters right after their birth while they are still compact.

Key words:

gravitational waves ⋆ methods: numerical ⋆ stars: black holes ⋆ stars: dynamics ⋆ stars: mass-loss ⋆ galaxies: star clusters: general.

1 INTRODUCTION

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black holes (BH) with masses from 5 $M_\odot$ up to about 60 $M_\odot$. A large number of X-ray and optical observations provide solid evidence for the existence of stellar BHs (Webster & Murdin 1972; Remillard & McClintock 2006; Casares & Jonker 2014). Their presence is further confirmed by the recent discovery of gravitational waves generated by BH mergers (Abbott et al. 2016, 2017; Lange & LIGO-Virgo Collaboration 2018). It is also well established that waves generated by BH mergers (Abbott et al. 2016, 2017; Lange et al. 2018) are further confirmed by the recent discovery of gravitational evidence for the existence of BHs bridging the mass range between stellar and supermassive black holes. These intermediate-mass black holes (IMBHs) could originate from stellar BHs and might be the seeds for SMBHs. Finding them and understanding their formation mechanism is crucial for a full understanding of the BH population in the Universe. There are three theoretical paths for IMBH formation leading to SMBHs discussed in the literature (see reviews Volonteri 2010; Koliopanos 2017, and citations therein).

In the first scenario IMBHs form through direct collapse of dense gas at high redshifts. This scenario predicts the formation of IMBHs of $10^4$ to $10^6$ $M_\odot$ (Bagel & 2006; Agarwal et al. 2012; Luo et al. 2020). A second possibility is that IMBHs are the remnants of first generation (PopIII) stars. These stars formed from zero metallicity gas and are expected to collapse into IMBHs more massive than 100 $M_\odot$ (Madau & Rees 2001; Ryu et al. 2016). A third family of models assumes that IMBHs are generated in dense stellar environments through runaway collisions. Several studies have demonstrated that IMBHs can form through dynamical interactions in dense stellar systems. Those studies include analytical approaches (Bagelman & Rees 1978; Stone et al. 2017) as well as N-body simulations (Portegies Zwart & McMillan 2002; Portegies Zwart et al. 2004; Mapelli 2016; DiCarlo et al. 2020), and Monte Carlo simulations (Freitag et al. 2006; Gürkan et al. 2006; Giersz et al. 2015). In particular, the effect of tidal capture of stars by BHs has been discussed in Patruno et al. (2006) for massive star clusters and Stone et al. (2017) for nuclear star clusters.

Several IMBH candidates have been discovered in our galaxy and others nearby. For example, IMBHs have been proposed to explain the nature of ultra-luminous X-ray emitters (ULXs). ULXs are extra-galactic and off-center X-ray sources that could be generated by BHs of intermediate mass which accrete gas isotropically below the Eddington rate (Colbert & Mushotzky 1999). Most ULXs observed are, however, more likely generated by smaller objects such as magnetized neutron stars and stellar BHs with super-Eddington accretion (Feng & Soria 2011; Gladstone 2013; Roberts et al. 2016; Kaaret et al. 2017; King & Lasota 2020). This is also confirmed by dynamical evidence (Liu et al. 2013) as well as X-ray pulsations, which indicate the presence of neutron stars (Fürst et al. 2016). Nevertheless, there exists a group of hyper-luminous X-ray sources (HLXs) that might be laborious to explain by super-Eddington accretion since their luminosity exceeds $10^{41}$ erg s$^{-1}$. Probably the best IMBH candidate known so far is HLX-1. This HLX is believed to host an IMBH because it has an X-ray luminosity of $1.1 \times 10^{42}$ erg s$^{-1}$ (Farrell et al. 2009). This luminosity would imply a mass of 500 $M_\odot$ even assuming an accretion rate ten times larger than the Eddington limit (Farrell et al. 2009). The mass estimates of the BH associated with HLX-1 is estimated between $3.0 \times 10^4$ and $3.0 \times 10^5$ $M_\odot$ (see review Mezcua et al. 2017, and references therein).

Other notable sources in the same category are NGC 5252 and NGC 2276-3C, which have been estimated to host IMBHs of $\sim 10^4 M_\odot$ (Kim et al. 2020) and $\sim 5 \times 10^4 M_\odot$ (Mezcua et al. 2013, 2015), respectively. Another HLX is M82 X-1. This X-ray source is associated with a young massive star cluster (MGG-11) in the starburst galaxy M82 (Matsumoto & Tsuru 1999; Kaaret et al. 2001). First observations have suggested that it is generated by a BH with a mass of $200 - 5000 ~M_\odot$ (Kaaret et al. 2001; Matsumoto et al. 2001; Strohmayer & Mushotzky 2003; Patruno et al. 2006). A more recent analysis indicates the presence of an IMBH, estimating its mass to $\sim 400 ~M_\odot$ (Pasham et al. 2014). However, M82 X-1 could still be a stellar BH with super-Eddington accretion (Brightman et al. 2016).

Globular clusters have been popular targets for the search of IMBHs. Due to their high central density, and possibly even higher density at formation (Lahen et al. 2019), they provide a promising environment for the formation of IMBHs through runaway core-collapse and collision. Many studies have attempted to detect IMBHs in globular clusters through their accretion signatures. However, so far G1 in M31 is the only globular cluster detected in X-rays (Pooley & Rappaport 2006; Kong 2007). This signal might be generated by an IMBH with a mass of $2 \times 10^4 M_\odot$ (Gebhardt et al. 2002, 2005). A recent observational study reports the lack of IMBHs accretion signatures in 19 globular clusters located in the early-type galaxy NGC 3115 (Wrobel & Nyland 2020). Here it is important to note that globular clusters contain little gas, which might explain the absence of X-ray and radio signals. Observations based on kinematic measurements might suggest the presence of IMBHs in globular clusters such as M15 (Bahcall & Wolf 1976; Peterson et al. 1989), and Centauri (Noyola et al. 2008, 2010), NGC 1904 and NGC 6266 (Lützgendorf et al. 2013). However, these observations could also be explained by a central concentration of compact objects (Baumgardt et al. 2003; van den Bosch et al. 2006; Baumgardt et al. 2020). Measurements of pulsar accelerations indicate that the globular cluster Tucanae 47 might host an IMBH of $2300^{+1500}_{-850} M_\odot$ (Kiziltan et al. 2017). Another study suggests that current observations of pulsar accelerations are insufficient to confirm the presence of an IMBH in Tucanae 47, they can only be used to estimate an upper limit on its mass (Abbate et al. 2019).

Dwarf galaxies are very promising systems for IMBH searches. NGC 4395 seems to be one of the most plausible candidates for an active and central IMBH. Observations of the central stellar velocity dispersion reveal a value of 30 km/s (Filippenko & Ho 2003), suggesting an IMBH mass of $\sim 10^5 M_\odot$. Further measurements based on the broad profile of the H$\beta$ line, from X-ray variability (Filippenko & Ho 2003), reverberation mapping (Peterson et al. 2006; Edzi et al. 2012), and integral field kinematics (den Brok et al. 2015) indicate a mass in the range between $10^4$ to $10^5 M_\odot$. Also our Galaxy might host an IMBH in the vicinity of the galactic center as suggested by recent high-resolution molecular line observations that indicates the presence of a $\sim 10^4 M_\odot$ candidate BH in the central region of the Milky Way (Takekawa et al. 2020).

In this paper, we study IMBH formation paths in young concentrated star clusters with numerical simulations. The simulations incorporate detailed models for single and binary stellar evolution as well as mass loss due to stellar winds. In Section 2, we describe the N-body code used to simulate star cluster evolution and focus on the description of the adopted stellar evolution and collision models. In Section 3, we describe the initial conditions for our models. The results of our simulations are discussed in Section 4. In Section 5 we compare these results with previous studies. In the final section we summarise the main points of this paper.

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1 The lower and upper mass boundaries depend on the stellar evolution model, i.e. details of stellar wind mass loss and supernova explosions.
2 THE METHOD

To investigate the possible formation of IMBHs in massive star clusters we generated initial conditions for 80 isolated systems using MCLUSTER (Küpper et al. 2011). The systems were set up with two different half-mass radii and various central concentrations. Each initial condition is evolved for a few hundred million years employing NBODY6++GPU (Wang et al. 2015; Wang et al. 2016), a direct N-body simulation code designed to follow the dynamical and stellar evolution of individual stars and binaries.

2.1 NBODY6++GPU

NBODY6++GPU is a high-precision direct N-body simulation code based on the earlier N-body codes NBODY1-6 (Aarseth 1999) and NBODY6++ (Spurzem 1999). It uses for time integration Taylor series up to $4^{th}$ order, due to the Hermite scheme it can be based on two time points only. This together with the hierarchically blocked variable time step scheme allows an efficient parallelization of the code for massively parallel supercomputers (since NBODY6++); gravitational forces between particles are offloaded to graphics processing units (GPUs), used for high-performance general purpose computing (NBODY6++GPU, Wang et al. (2015)). The parallelisation is achieved via MPI and OpenMP on the top level, distributing work within a group of particles due for time integration, and efficient parallel use of GPU cores at the base level (every MPI process using a GPU), for computing the gravitational forces between particles. The GPU implementation in NBODY6++GPU provides a significant performance improvement, especially for the long-range (regular) gravitational forces (see Nitadori & Aarseth 2012; Wang et al. 2015; Wang et al. 2016).

The code accurately computes the evolution of binaries, multiples and close encounters between them and single stars and between multiple systems, using the Kustaanheimo-Stiefel (KS, Kustaanheimo & Stiefel 1965) regularisation with the classical chain algorithm by Mikkola & Aarseth (1998). It also follows single and binary stellar evolution based on the SSE and BSE recipes by Hurley (see Sect. 2.2), including also rapid tidal circularization for binaries with small pericenters and tidal captures according to the prescription given in Mardling & Aarseth (2001), which is based on the previous work of Press & Teukolsky (1977); Lee & Ostriker (1986); Mardling (1996). The integrator fully resolves orbits and dynamical evolution of binaries, even during phases of mass loss or when one of the two stars undergoes a supernova explosion. The binary orbit is adjusted to the corresponding loss of mass, energy and angular momentum with appropriate time stepping; in case of a supernova explosion it is always ensuring that the remnant and its companion leave the explosion with the corrected orbital positions and velocities.

In the implementation used for this study we compute the gravitational wave energy loss of hard binaries according to the orbit averaged approximation of Peters & Mathews (1963), using the average change of energy and angular momentum per orbit from their work. At each KS integration time-step, which is much smaller than the orbital time, we apply a corresponding fractional loss of energy and angular momentum. This allows for the proper representation of the evolution of gravitational radiation driven shrinking and circularization of the orbit, until the time scale of orbit shrinking becomes comparable to the orbital time. When this happens, the time to final coalescence is very short and we assume coalescence.

The publicly available code NBODY6++GPU has been significantly upgraded in three respects:

(i) For collisions between a compact remnant and a main sequence star or red giant a free parameter $f_c$ is introduced, which describes the mass loss from the system in the process. The previous NBODY6 versions used only $f_c = 1$, i.e. no mass loss in the process (see Eq. (1) and (2) above). Routine involved: coal.f.

(ii) Simultaneous treatment of classical tidal interactions (Roche lobe overflow) and Post-Newtonian orbit-averaged orbit shrinking due to gravitational wave emission has been made possible. Both are treated technically in a similar way, and can now be switched on together. Routines and parameters involved: ksint.f, kstide.f, tides3.f, KZ(27).

(iii) Strongly bound binaries of two compact objects, which are subject to Post-Newtonian relativistic energy loss are prevented from unperturbed two-body integration. Routine involved: unpert.f.

2.2 Stellar Evolution

The simulations in this paper were performed with the same stellar evolution models as the DRAGON simulations presented in Wang et al. (2016). Stellar evolution is implemented using analytical fits to the models of (Eggleton et al. 1989, 1990) developed by (Hurley et al. 2000b) and (Hurley et al. 2000a) for single stars (SSE) and by (Hurley et al. 2002) for binary stars (BSE). A few updates were included for strong kicks at neutron star birth (Hobbs et al. 2005) and for fallback and more massive black hole formation of massive stars at low metallicities (Belczynski et al. 2002).

The code is able to follow the main properties of single stars (such as radius, mass, luminosity, and core mass) from the zero-age main-sequence to the remnant stage. This also includes mass loss due to stellar winds for a wide range of masses and metallicities. As long as the orbit of a binary star is wide enough, the evolution of each star is assumed not to be affected by its companion and just the single star tracks are used. However, if one of the two stars is losing mass by a stellar wind, the companion has the chance to accrete material and deviate from its standard evolution. For close enough orbits either star might fill its Roche-lobe leading to mass transfer. For these cases, the code computes the accretion rate as a function of the masses, the radii, the stellar types, and the separation of the donor and the accretor, ensuring that it never exceeds 100 times the Eddington limit. If the matter ejected by a star is not entirely absorbed by its companion it might accumulate in a common envelope around the two stars. All the above effects: mass transfer, Roche phase, and common envelope evolution have significant consequences for the orbit and stellar properties of the binary and are included in the simulations based on the models of Tout et al. (1997).

In dense environments, where stellar collision rates are high, runaway collisions (Lee 1987; Shapiro 1987; Quinlan & Shapiro 1989, 1990), can generate stars above the maximum IMF mass of 100 $M_{\odot}$ (Portegies Zwart & McMillan 2002; Portegies Zwart et al. 2004; Gürkan et al. 2004; Mapelli 2016; DiCarlo et al. 2020; Wang et al. 2020) for which we use the term ”very massive stars” (VMS). To track the evolution of VMSs, in the absence of observational constraints, we extrapolate our stellar evolution model to stars with

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2 We study clusters in isolation to investigate internal dynamical effects without possible external influences.

3 Link to repository: http://silkroad.bao.ac.cn/repos/Nbody6++GPU-Aug2020/
Table 1. Model parameters of the cluster simulations: $r_c$: initial core radius; $\rho_0$: initial central density; $W_0$: central potential parameter for the King density profile (King 1966); $R_0$: half mass radius; $\sigma$: dispersion velocity; $t_{rh}$: half mass relaxation time computed using eq. 3 with $\gamma = 0.02$; $t_s$: segregation time scale for 100 $M_\odot$ objects computed using eq. 4; $f_c$: fraction of mass absorbed by a compact object during a direct collision with a star; # IMBH: Number of BHs with masses $>100$ $M_\odot$ formed out of 8 realisations; $M_{\text{IMBH}}$: IMBH masses; $t_{\text{form}}$: IMBHs formation times; $M_f/M_1$: the total stellar mass accreted by the IMBH divided by its final mass.

| Model Name | $r_c$ pc | $\rho_0$ $M_\odot$/pc$^3$ | $W_0$ | $R_0$ pc | $\sigma$ km/s | $t_{rh}$ Myr | $t_s$ Myr | $f_c$ | # IMBH | $M_{\text{IMBH}}$ $M_\odot$ | $t_{\text{form}}$ Myr | $M_f/M_1$ |
|------------|---------|----------------------|------|--------|------------|-----------|---------|------|--------|----------------|-----------|--------|
| R06W9F01   | 0.04    | $3.0 \times 10^2$    | 9    | 0.6    | 15         | 56        | 1.4     | 0.1  | 0/8    | /             | /         | /      |
| R06W9P05   | 0.04    | $3.0 \times 10^2$    | 9    | 0.6    | 15         | 56        | 1.4     | 0.5  | 2/8    | 138,110       | /         | /      |
| R06W6      | 0.19    | $1.1 \times 10^2$    | 9    | 0.6    | 15         | 56        | 1.4     | 1.0  | 4/8    | 307,151,138,122 | 8.6, 83.9, 6.4, 8.4 | 86%, 72%, 80% |
| R06W7      | 0.13    | $5.0 \times 10^1$    | 7    | 0.6    | 15         | 56        | 1.4     | 1.0  | 2/8    | 148,147       | 22.9, 8.2 | 76%, 87% |
| R06W8      | 0.06    | $3.0 \times 10^0$    | 8    | 0.6    | 15         | 56        | 1.4     | 1.0  | 3/8    | 336,171,110   | 6.6, 12.3, 8.1 | 98%, 87%, 77% |
| R06W9      | 0.04    | $3.0 \times 10^2$    | 9    | 0.6    | 15         | 56        | 1.4     | 1.0  | 3/8    | 355,349,120   | 8.57, 8.19, 16.3 | 81%, 89%, 73% |
| R1W7       | 0.2     | $1.0 \times 10^2$    | 5    | 1.0    | 12         | 120       | 3.0     | 1.0  | 0/8    | /             | /         | /      |
| R1W8       | 0.11    | $4.0 \times 10^2$    | 8    | 1.0    | 12         | 120       | 3.0     | 1.0  | 0/8    | 239           | 16.4      | 92%    |
| R1W9       | 0.05    | $3.0 \times 10^2$    | 9    | 1.0    | 12         | 120       | 3.0     | 1.0  | 1/8    | 233           | 133,110   | 13.2, 7.8 | 83%, 83% |
| R1W10      | 0.03    | $1.8 \times 10^2$    | 10   | 1.0    | 12         | 120       | 3.0     | 1.0  | 2/8    | 133,110       | 13.2, 7.8 | 83%, 83% |

2.3 Collisions

The outcome of a collision depends on the relative velocity, the relative sizes and the internal structure of the two colliding objects. In general, full 3D radiation magneto-hydrodynamical simulations are required to robustly determine the properties of the final object (such as mass, size, and internal structure). In the absence of results covering the full parameter space NBODY6++GPU adopts a simplified treatment. If a collision does not involve red giants the two objects are merged when the radii of two colliding objects overlap and the merger of the two masses is assumed to be instantaneous.

However, VMSs are affected by strong stellar wind mass loss and lose a significant fraction of their mass during their lifetime. In our stellar evolution framework, it is therefore impossible to form massive BHs from direct stellar collapse. In fact, with the adopted stellar evolution recipes, even an isolated star with a mass of 500 $M_\odot$ generates a BH of only about 30 $M_\odot$. In the absence of well-established theories for VMS evolution, we therefore assume a conservative model $^4$. It is worth mentioning that theories of stellar evolution predict that a low metallicity star more massive than 260 $M_\odot$ collapses directly into a BH without significant mass loss in supernova explosions (see Woosley & Heger 2015, and citations therein). However, it is important to take into account the complex stellar structure acquired by VMSs during merger events. A detailed computation of the evolution of collision products shows that, VMSs formed through runaway collisions and tidal capture, are dominated by mass loss from stellar winds that drastically reduce their final remnant mass (Giebbeck et al. 2009).

Despite using a stellar evolution model where stellar winds strongly affects the most massive stars and direct collapse into IMBHs is not possible we can still form BHs more massive than 100 $M_\odot$ through BH-BH and BH-star collisions. We will show, in the next sections, that BH-VMS collisions provide the main channel for the formation of IMBHs. In other words, our results show that black holes with a mass above 100 $M_\odot$ form when a stellar black hole merges with a VMS in agreement with previous works (Giersz et al. 2015; Mapelli 2016). Runaway tidal captures (Stone et al. 2017) can also produce IMBHs.

If $M_1$ and $M_2$ are the masses of two stars or two compact objects (black holes, neutron stars, with dwarf, etc.), the final object will have a mass $M_f$ equal to:

$$M_f = M_1 + M_2.$$  

(1)

Otherwise, if $M_1$ is the mass of a compact object and $M_2$ is the mass of a star, the final mass of the compact object, $M_f$, is given by:

$$M_f = M_1 + f_c \times M_2,$$

(2)

where $0 < f_c < 1$ represents the amount of stellar material falling back and accreted onto the black hole during the coalescence. The value of $f_c$ has only been investigated in the simulation literature for some specific cases (Law-Smith et al. 2019; Dai et al. 2018; Metzger & Stone 2016; Shiokawa et al. 2015). In the absence of solid results for all mass ratios we treat our recipes and black stars, the core mass coincides with the mass of the objects itself.

4 Our recipes do not include pair-instability supernovae and pulsation pair instability.

5 For main sequence stars and black holes, the core mass coincides with the mass of the objects itself.
composed of the appropriate remnants. Otherwise, the two cores are destined to spiral into each other. The final mass depends on their relative density. If they are of different compactness, equation 2 applies, otherwise the code uses 1.

When two stars of similar compactness merge, we assume that they coalesce and mix completely. As a consequence the final star can be rejuvenated since the core of the final object absorbs new fuel. This phenomenon is well established as it explains the peculiar evolution of blue stragglers as the product of binary evolution and direct stellar collisions (Smith & Tombleson 2015; Davies et al. 2004; Davies 2015). This rejuvenation procedure also applies to MS stars, even if they are assumed to have no core. For more details see section 2.6.6 of Hurley et al. (2002).

When two black holes (or neutron stars) form hard binaries, gravitational waves emission is computed with the Peters & Mathews formulae (Peters & Mathews 1963). They interact with the surrounding stars and the initial conditions are then changed according to the amount of gravitational wave energy emitted.

3 INITIAL CONDITIONS

We created initial conditions for 10 isolated star cluster models following a King density profile (King 1966) with two different half-mass radii and five different central densities varying the dimensionless central potential parameter \( 6 \leq W_0 \leq 10 \). For a fixed value of the half-mass radius, the central density increases with increasing \( W_0 \) (see Fig. 1). All simulated clusters were initialized with a very low metallicity of \( Z = 0.0002 \) and \( N = 1.1 \times 10^5 \) stars sampled from a Kroupa IMF (Kroupa 2001) as zero-age main-sequence stars in a mass range of 0.08 \( M_\odot \) to 100 \( M_\odot \) (see fig. 2). No primordial mass segregation was included. We assumed a primordial binary fraction of 10% (10,000 binaries) with a uniform semi-major axis distribution on a logarithmic scale from 0.001 AU to 100 AU, a uniform distribution of mass ratios, and a thermal distribution of eccentricities; With this binary distribution about 30% of the primordial binaries are weak and they dissolve in the cluster at the beginning of the simulation. For each initial condition parameter set we created 8 realisations with different random number seeds. In Tab. 1 we list the initial conditions parameters of the 10 models including the number of realizations that lead to the formation of IMBHs and their respective masses. All simulations were run for more than 300 Myr up to 500 Myr.

The clusters studied in this work are initialized with a low metallicity value (about two orders of magnitude lower than the solar metallicity), they are rather compact with high initial central densities and small half-mass radii. For this reason, they do not resemble the typical properties of young massive star clusters (YMSCs) in the observable range. The latter have higher, and closer to solar, metallicity and are typically less compact; observed YMSCs with masses similar to our models have virial radii typically in the range from 3 to 30 pc (Portegies Zwart et al. 2010). Even considering the expansion driven by stellar and dynamical evolution our less compact systems would have a virial radius from 2 to 30 times smaller than most of the observed massive clusters.

However, it is still possible to find a few compact YMSCs in the local Universe. An example is Westerlund 1, which has a virial radius of \( \sim 1.7 \) pc (our \( R_0 = 0.6 \) pc model at the same age has a virial radius of about 1.15 pc) and a mass \( \sim 6 \times 10^5 M_\odot \) (Mengel & Tacconi-Garman 2007; Portegies Zwart et al. 2010). Another exception is MGG-11 in M82. This cluster has a half-light radius of about 1.2 pc and a mass of about \( 3.50 \times 10^5 M_\odot \) (McCrady et al. 2003).

Our models start with velocity dispersions \( \sigma \) between 15 km/s and 12 km/s (see Tab. 1) which drop to values between 6.1 km/s and 5.3 km/s after 300 Myr as shown in Tab. 2. We also show the half mass radius, the relaxation time, the number of particles and fraction of total mass left in the cluster after 300 Myr. It is interesting to notice that R06W9, which formed the most massive IMBHs, after 300 Myr has a dispersion velocity and half mass radius very similar to the least compact model R1W7 (see table 2), which never formed a BH more massive than 100 \( M_\odot \). The velocity dispersions are shown in Tab. 2 are approximately in the same range of the observed values of the globular clusters in our Galaxy (Baumgardt & Hilker 2018). However, our clusters are too small to resemble the initial conditions of present-day globular clusters and estimating the particle number of our clusters at late times using equation 22 in

\[ W_0 \]
Figure 2. Left panel: stellar mass function (the mass function is computed including all stellar types apart from BHs, neutron stars and white dwarfs) at 0, 5, 10, 15, 30 and 60 Myr. Right panel: compact objects mass function (the mass function includes only BHs, neutron stars and white dwarfs) at 0.0, 5, 10, 15, 30 and 60 Myr.

Figure 3. Peak masses of the very massive stars (VMS) formed by collisions (red stars) and black holes (black circles) for all simulations with $f_c = 1$ sorted by initial core density (see Tab. 1). The left (right) panel shows models with a half-mass radius of $r_h = 1$ pc ($r_h = 0.6$ pc). Each model has 8 realizations (plotted with random offset). $W_0$ does not seem to have a strong impact on the final black hole mass. A larger cluster half-mass radius apparently makes black holes mass growth less likely (left panel).

Gieles et al. (2011), our systems might not survive for 10 Gyr even if we assume no external tidal forces.

We have chosen these initial conditions mainly to investigate the formations of IMBHs through dynamical interactions. Nevertheless, our theoretical models might approximate the properties of clusters formed at a high redshift, which may be located around any galaxy in the LIGO/Virgo sensitivity volume ($\sim 1$ Gpc$^3$).
4 RESULTS

The dynamical evolution of a star cluster is dominated by two-body relaxation on time scales longer than the relaxation time (Spitzer 1987):

\[ t_{\text{rel}} = \frac{0.138N^{1/2}}{\ln \Lambda} \left( \frac{R_h^3}{GM} \right)^{1/2}. \]  

(3)

Here, \( N \) is the number of stars in the cluster, \( R_h \) is the half mass radius and \( \tilde{m} \) is the average star mass of the cluster and the argument of the Coulomb logarithm is \( \Lambda = \gamma N \). Numerical experiments indicate a value for the parameter \( \gamma = 0.11 \) for single-mass systems (Giersz & Heggie 1994) and \( \gamma = 0.02 \) for multi-mass stellar systems (Giersz & Heggie 1996). We have used Eq. 3 to estimate half-mass relaxation times of 56 Myr and 120 Myr for systems with 0.6 pc and 1 pc half-mass radii, respectively (see Tab. 1).

The internal structure of star clusters evolves into a dense hot core and an extended halo. Since bound self-gravitating systems have negative heat capacity (Lynden-Bell & Wood 1968; Lynden-Bell 1999), the center will keep releasing energy to the outer part and contracts in a core collapse. In an isolated equal-mass system, this happens on the order of \( 15t_{\text{rel}} \) (Cohn 1980). In clusters with a broad mass spectrum, low mass stars gain kinetic energy when interacting with massive stars; the former tend to expand their orbits, while the latter tend to lose kinetic energy and segregate to the central part of the cluster. In this case, core collapse is driven by the amassing of heavy stars in the core that typically occur in a time scale of the order of the segregation time (Spitzer & Hart 1971; Portegies Zwart et al. 2004):

\[ t_s = \frac{\tilde{m}}{M_{\text{max}}} \frac{0.138N}{\ln (0.11M/M_{\text{max}})} \left( \frac{R_h^3}{GM} \right)^{1/2}. \]  

(4)

where \( M_{\text{max}} \) is the mass of the most massive object in the cluster and \( M \) is the total mass of the cluster. Consequently multi-mass clusters undergo core collapse in much shorter time than single-mass systems. According to equation 4 our \( R_h = 0.6 \) pc and \( R_h = 1.0 \) pc models are expected to experience mass segregation at \( t_s \approx 1.4 \) and 3 Myr respectively (see Tab. 1).

Core collapse lead to dramatic growth in the central density which in turn triggers violent few-body interactions between single stars and binaries, either primordial or dynamically formed. By means of this interaction binary stars release energy in the core and balance the loss of energy from the centre preventing the core to collapse further (see Heggie & Hut 2003, and references therein).

We show in Fig. 3 that the most massive stars formed in each simulation (red crosses) consistently have masses much higher than the initial limit of 100 M\(_{\odot}\). These system have formed by mergers of lower mass main sequence stars. As described in section 2.2 our stellar evolution recipes do not allow for the formation of a stellar black hole more massive than 30 M\(_{\odot}\). Nevertheless, Fig. 3 demonstrates that several cluster systems generate BHs more massive than 100 M\(_{\odot}\). In our framework, the only possible way to grow such a massive object is through dynamical interactions (see Sec. 4.1).

The likelihood to form a very massive star by mergers or tidal captures seems to be correlated with the compactness of the cluster. Only 6 star clusters with \( R_h = 1.0 \) pc form stars with masses above 200 M\(_{\odot}\), while clusters with \( R_h = 0.6 \) pc form 15 stars more massive than 200 M\(_{\odot}\) (see Fig. 3). Those stars play an important role in the production of IMBHs as their collisions with stellar BH are the main IMBH formation channel. Such a formation path for IMBHs has been predicted by Monte-Carlo models (Giersz et al. 2015) and is discussed in more detail below.

4.1 Intermediate Mass Black Hole Formation

Figure 4 illustrates how IMBHs of a few hundred solar masses are typically generated in our simulations. The formation consists of three main steps. First, a sequence of binary stellar collisions, triggered by triple interactions and hyperbolic collisions generates a VMS which can live up to 10 Myr due to mixing rejuvenation (see section 2.2). Second, in a merger with a stellar mass BH, a great part of the mass of the VMS is absorbed by the BH. In a third step, the IMBH can grow in mass by collisions with other stellar BHs. Our results indicate that, for our models, the dominant process for the formation of IMBHs is a collision between a VMS and a stellar mass BH. This type of collisions can lead to the formation of IMBHs of up to 350 M\(_{\odot}\) within the first 10 Myr of cluster evolution. Our simulations also indicate that after all massive stars disappear from the cluster, the IMBH can still grow moderately in mass by merging with other stellar mass black holes and other types of stars.

Our most compact models register about 300 collisions within 300 Myr, 40% of which happen in the first 15 Myr. Most of these collisions are triggered by three-body scattering events between a hard binary and a third particle. These interactions have the overall effect of increasing the binding energy of the binaries and they can also raise their eccentricity (Heggie 1975; Nash & Monaghan 1978; Hut & Bahcall 1983; McMillan & Hut 1996). In general, when the distance of closest approach of the third object is comparable with the semi-major axis of the hard binary, the net effect of the interaction is to harden the binary (Heggie 1975; Nash & Monaghan 1978; Hut & Bahcall 1983; McMillan & Hut 1996), On the other hand, when the pericenter of the intruder is considerably larger than the size of the binary, the interaction is approximately adiabatic, and there is no exchange of energy between the binary and the intruder. However, the binary and the third object can form a hierarchical triple, which excites the eccentricity of the inner binary to values close to unity (Lidov 1962; Kozai 1962) and induces the two components of the binary to merge. Figure 6 illustrates the dynamical process leading to the collision of two massive stars with 170 and 80 M\(_{\odot}\), respectively. The 170 M\(_{\odot}\) star approaches a binary (0.1 – 80 M\(_{\odot}\)) in a hyperbolic orbit (left panel); during the interaction, the intruder forms a hard binary with the 80 M\(_{\odot}\) star, while the lightest component escapes in a wider orbit (panel 2). Due to the perturbation of the third object the eccentricity of the new 170 – 80 M\(_{\odot}\) binary increases from 0.002 to 0.45 and the two massive stars crash into each other. Figure 6 also shows that the colliding objects do not necessarily need to be in a primordial binary. Exchanges are very frequent in triple interactions. Typically, encounters leading to exchange increase the mass of the components of a hard binary because the lowest mass is most likely to escape (see Heggie & Hut 2003, and references therein).

In summary, when a single object interacts with hard binaries, the binaries tend to harden and become more massive, and they also gain angular momentum and eccentricity. For all these reasons triple interactions drive the chain of star-star collisions leading to the formation of VMSs. They are also the main process that triggering BHs-VMSs mergers (see Fig. 7).

Here it is important to mention that gravitational kicks are not implemented in our simulations. Theoretically, an IMBH could be ejected from the cluster by gravitational wave recoil after a collision with one of the stellar mass BHs left in the system. However, except for the simulation illustrated in Fig. 5, we expect the clusters to have good chances of retaining their IMBHs as the IMBH - BH
mass ratio is large. Therefore the gravitational velocity kick is likely to be smaller than the local escape velocity according to Campanelli et al. (2007); Baker et al. (2008); Kulte et al. (2015); Morawski et al. (2018); Zivancev et al. (2020). The absence of gravitational recoil velocity does not influence the series of collisions and interactions that lead to the VMS-BH mergers. The BHs involved in these collisions are the remnant of massive stars and they did not merge with any other BH. This can be clearly seen from fig. 4 and fig. 10. Both figures report all the collisions experienced by each object that participates in the collision chain. Therefore if a BH appears in the plot only once it implies that the object never experiences any collision before.

The IMBHs in our simulations are mainly growing through two channels:

- Accretion of stellar material through binary collisions with stars and, to a lesser extent, through hyperbolic collisions with stars.
- Collision with low mass BHs.

BHs can also become more massive through multiple mass transfer events that do not lead to coalescence. However, the mass absorbed in those events is negligible. Moreover, BHs could also grow through tidal captures. However, according to our simulations, tidal captures do not seem to be relevant for the IMBHs final mass. Tidal captures are very rare (10-15 events per run) and only a small fractions of these events lead to collisions.

Collisions with stellar BHs typically add little (~ 10–30%, see Tab. 1) to the total IMBH mass. BH-BH collisions are not very likely to occur. This type of binary requires very small semi-major axes, or very high eccentricities to enter in the post-Newtonian regime, experience gravitational wave energy loss, and eventually merge. This requires the BH binary to undergo several strong triple interactions (or binary-binary interactions). For this reason, as we can see in Fig. 4, and later also in Fig. 10, that most of the material accreted onto a massive BH originates from stars. We form in total 17 BHs with a masses above 100 $M_\odot$ (see Tab. 1). Despite being unlikely, one of the IMBHs grows its mass almost entirely by swallowing other black holes (as shown in Fig. 5). The formation process occurred in about ~ 90 Myr and it involves a chain of low mass BH mergers with masses of 17 : 28, 25 : 45, 68 : 70 $M_\odot$. According to Fig. 3 the less concentrated model R06W0 and the more dense model R06W9 have comparable probability to produce IMBHs and VMSs despite the marked difference in the initial central density. Fig. 8 displays the time evolution of the core radius (plot on the top) and the core density (plot on the button) for two simulations with $W_0 = 9$ and $W_0 = 6$. The two systems experience different initial evolution: the core of the cluster with higher central density expand from the beginning of the simulation due to primordial binaries interactions, contrarily clusters with $W_0 = 6$ undergo core collapse. (this is also confirmed by the evolution of the Lagrangian radii shown in Fig. 12 ). Consequently, the initial concentration discrepancy between the two models rapidly decreases. This fact might explain why clusters with $W_0 = 6$ and $W_0 = 9$ lead
to comparable results although the initial central densities of the two models differ of more then two orders of magnitude.

4.2 Collision Fraction $f_c$

So far we have assumed an accretion fraction of $f_c = 1$ i.e. in star - BH collisions all stellar material is accreted onto the BH. Under these circumstances IMBHs regularly form in the simulations. However, the accretion fraction is highly uncertain and we have simulated R06W9 with lower fractions of $f_c = 0.5$ and $f_c = 0.1$. The results indicate that the accretion fraction can significantly change IMBH growth (see Fig. 9). In fact, while the stars still grow in mass by collisions, no IMBH is formed in the simulation with $f_c = 0.1$, similar to Fig. 4. However, here the black hole reaches a mass of about 140 $M_\odot$ mostly through collisions (horizontal dashed lines) with other BHs. This is the only case, out of 80 simulations analysed, where a BH above 100 $M_\odot$ built up its mass mostly through collisions with other BHs.

4.3 Cluster Evolution

In this section, we highlight the evolution of three different simulations resulting in very different peak BH masses, which we label, for simplicity, with $S_1$, $S_2$ and $S_3$. $S_1$ and $S_2$ are two different realizations of the model R06W6. The latter creates an IMBH of 140 $M_\odot$ (the evolution path of this BH is shown in Fig. 5). The most massive BH in $S_1$ only reaches 60 $M_\odot$. $S_3$ is one realization of the model R06W9 generating an IMBH, with a mass of about 350 $M_\odot$ (see evolution path in Fig. 4). The left, center and right panels in Fig. 12 refer to $S_1$, $S_2$ and $S_3$ respectively. Each panel consist of three plots. The plots at the top display the time evolution of the radii enclosing 3%, 5%, 10%, 30%, 50% and 70% of the cumulative stellar mass (Lagrangian radii). The middle plots illustrate the evolution of the average stellar mass within 3%, 5%, 10% and 30% Lagrangian radii. The plots at the bottom show the mass (and the type) of the most massive object in each cluster as a function of time and its distance from the cluster center.

The evaluation of these three systems highlights the complex interplay between stellar evolution and dynamical interactions. The former has a strong impact on the early phase, while the latter plays a major role in driving the long-term change. Stellar evolution mass loss triggers a strong expansion on early cluster evolution (Applegate 1986; Chernoff & Shapiro 1987; Chernoff & Weinberg 1990; Fukushige & Heggie 1995) Our simulations confirm these results. The 30%, 50%, 70% Lagrangian radii indicate a strong monotonically cluster inflation in the first ~ 10 Myr as a consequence of mass loss due to supernovae explosion and stellar winds followed by a moderate expansion driven by primordial binaries and further mass loss due to stellar/binary evolution.

Since our three simulations are multi-mass systems they undergo mass segregation in about ~ 1.4 Myr (see table 1). As a consequence of that, the average stellar mass in the inner part of the three clusters strongly increases in the first ~ 2 Myr (middle panels of Fig. 12). $S_1$ and $S_2$ also register an initial core collapse connected with mass segregation (rapid decrease for 3%, 5%, and 10% Lagrangian radii) in agreement with previous numerical work (Portegies Zwart & McMillan 2002; Portegies Zwart et al. 2004; Güruz et al. 2004; McMillan 2008). On the other hand, $S_3$ does not show any indication of core collapse. The system is already so dense that binary interactions prevent the collapse and force the core to expand (the innermost Lagrangian radii are in constant expansion since the beginning of the simulation. See top-right plot of Fig. 12).

Subsequently, the death of the most massive stars leads to a mass average drop in the inner part of the cluster for both $S_1$ and $S_2$ (see red line in central plots). The average mass then continues to oscillate, but while in $S_1$ it presents a steady decline, $S_2$ raises again due to a BH-BH merger after about 15 Myr. After that, it declines at a constant rate due to stellar evolution mass loss. The initial drop is less evident in $S_3$ also because the presence in the center of the IMBH\cite{imbh} compensates for the absence of the massive stars.

Massive objects located in the core of the cluster experience frequent strong interactions. This effect is reflected in the strong oscillations of the mass average in the core (see central plots) as well as the strong variations in the position of the IMBHs (see green lines in the button plots). In $S_3$ the IMBH oscillates in position between 0.01 and 0.5 pc with an average position of about 0.1 pc. Similarly the BH in $S_1$ moves around 0.1 pc with slightly stronger oscillations due to its lower mass. On the other hand, the BH in $S_2$ experiences

\begin{figure}
\centering
\includegraphics[width=0.5\textwidth]{figure5.png}
\caption{Formation and evolution of an IMBH in a simulation with $W = 6$, $R_0 = 0.6$ pc and $f_c = 0.1$, similar to Fig. 4. However, here the black hole reaches a mass of about 140 $M_\odot$ mostly through collisions (horizontal dashed lines) with other BHs. This is the only case, out of 80 simulations analysed, where a BH above 100 $M_\odot$ built up its mass mostly through collisions with other BHs.}
\end{figure}
Figure 6. Sketch of the triple interaction that induces the collision of two massive stars of 170 and 80 $M_\odot$. Left panel: the 170 $M_\odot$ approaches a binary in a hyperbolic orbit. Central panel: the exchange between the lightest component of the binary and the intruder of 170 $M_\odot$. Right panel: the perturbed 170-80 $M_\odot$ binary increase its eccentricity, the two components are about to merge.

Figure 7. Sketch of the last five triple interactions experienced by a BH-VMS binary before the collision. In the last interactions, the BH-VMS binary forms a hierarchical triple with a stellar BH. Consequently the binary increases its eccentricity to a value close to unity that leads the BH to merge with the VMS.
strong radial change few millions years before its coalescence with an other BH at 84 Myr. These heavy oscillations are generated by the strong interactions that triggered the last BH-BH collision.

4.4 Comparison with Observations

Very massive stars, formed through collisions between lower mass stars, appear in almost all the realizations of our ten models. If the VMSs formation mechanism proposed in this and other works (Portegies Zwart & McMillan 2002; Portegies Zwart et al. 2004; Gürkan et al. 2004; Mapelli 2016; DiCarlo et al. 2020; Wang et al. 2020) is correct they might be observed inside or close to dense star clusters. Early studies of the Arches cluster indicated an upper star mass limit of 150 M_☉ (Figer 2005). However, more recent observations indicate the existence of stars greatly exceeding this limit, suggesting the presence of VMSs up to 300 M_☉ in the vicinity of young massive star clusters (Crowther et al. 2010). Another study claimed the discovery of a VMS of initial mass in the range between 90–250 M_☉ in the central cluster of the region W49 (Wu et al. 2014). These stars might be generated via runaway collisions as indicated by the outcome of our and previous studied discussed in section 5. However, gas accretion could be an equally valid mechanism for the formation of VMSs (Krumholz 2015).

It has been shown that the massive BH that power the hyper-luminous source associated with MGG-11 might have a dynamical origin. The high density and compactness of this cluster allow for the formation of a few thousand solar masses star via collisional runaway, which might directly collapse into an IMBH (Portegies Zwart et al. 2004). Also, our results suggest that IMBHs could be the origin of HLXs associated with dense star clusters. It is not rare for our simulated IMBHs to accrete mass and merge with stars, as shown in the top and central plots of Fig. 10 (both plots show the IMBH merging with a red giant at 10 and 21 Myr). However, even if our results show that IMBHs do receive mass from other stars, the mass transfer events are very often interrupted by strong interactions and therefore they do not last more than 1 Myr. In other words the IMBHs tend to spend only a small fraction of their time accreting material from their companion. This, in addition with the fact that IMBHs have a non negligible probability to be ejected from the cluster after a BH-IMBH collision (Arca-Sedda et al. 2020; Mapelli et al. 2020), might explain the absence of IMBHs accretion signature in may star clusters and most of globular clusters (Wrobel et al. 2015; Wrobel & Nyland 2020).

Our simulations reveal that IMBHs, right after formation, tend to bind with a low mass black hole in a BH-BH binary. The binary, located a the center of the cluster, experiences constant gravitational interaction with other objects, and it merges in an interval of time
Collision

Main-Sequence

Black Hole

Figure 4. A figure showing a comparison of models with different initial conditions and decreasing $f_c$. The time of IMBH formation is indicated by a black circle in the two top panels. Here, we show a different realization of model R06W9 than in Fig. 5.

Figure 10. Comparison of three models with identical initial conditions and decreasing $f_c$ of 1.0, 0.5, and 0.1 from top to bottom. For $f_c = 0.1$, no IMBH forms. The time of IMBH formation is indicated by a black circle in the two top panels. Here, we show a different realization of model R06W9 than in Fig. 4.

Figure 11. Comparison between 5% Lagrangian radii (see subsection 4.3 for the definition of Lagrangian radii) for a simulation with $f_c = 1.0$ (blue) and a simulation with $f_c = 0.1$ (orange). Observing the orange line, we can notice a fast expansion at about 5 Myr triggered by mass-loss. This dilatation is generated by a BH-VMS collision event that occurred at 4.98 Myr. During the collision, 90% of the total mass of the VMS is assumed to be ejected instantly from the cluster. For simulations with $f_c = 1.0$, these types of expansions do not occur because the mass of stars is fully retained in the cluster when BH-star collisions occur.

between $\sim 10 - 100$ Myr generating gravitational wave signal. The results also show that hierarchical black hole mergers $^{13}$ could be observed in dense stellar systems, as shown in Fig. 5 and 10 (top panel), however as pointed out in recent studies (Mapelli et al. 2020; Arca-Sedda et al. 2020) these events might be suppressed by the gravitational wave kicks. The latter were not included in our simulations although we expect them to generate a large recoil velocity especially if the two colliding BHs have comparable masses (Campanelli et al. 2007; Baker et al. 2008; Kulier et al. 2015; Morawski et al. 2018; Zivancev et al. 2020).

BH-IMBH coalescence, as well as the inspiral phase, will be detected by the next generation of gravitational wave detectors. Events that involve an IMBH with mass $< 200$ $M_\odot$ should be detected by LIGO while signal generated by IMBHs with masses $< 2000$ $M_\odot$ should be observable with the Einstein Telescope (Arca-Sedda et al. 2020).

5 COMPARISON WITH PREVIOUS WORK

Various groups have predicted runaway merger scenario for the formation of VMSs and IMBHs in dense stellar environments. For example, direct N-body simulations, carried out by Portegies Zwart et al. (2004), indicate the formation of a few thousand solar masses stars produced by multiple stellar mergers. The simulated star clusters, containing 128 $M_\odot$ stars, were evolved for 12 Myr using Starlab (Portegies Zwart et al. 2001) and NBODY4 (Aarseth 1999). The

$^{13}$ Multiple mergers of BHs that form more massive ones.
stellar evolution prescription adopted was based on Hurley et al. (2000b) for stars with masses \(< 50 \, \text{M}_\odot\) while more massive stars follow the evolution track given by Stothers & Chin (1997) and Ishii et al. (1999). With these models stars more massive than 260 \, \text{M}_\odot\) collapse directly into IMBHs without losing mass in supernova explosions.

A similar mechanism is observed in the 30 simulations presented by Mapelli (2016) where VMSs reach about 500 \, \text{M}_\odot\) through runaway collisions. Each of these simulations is initialized with 10^5 stars following a King density profile with W_0 = 9 and R_h = 1.0 pc. The clusters were evolved for 17 Myr using Starlab. Due to the different stellar evolution model adopted for massive stars, these simulations generate IMBHs of few hundred solar masses through direct collapse of VMSs (the VMSs at low metallicity lose a relatively small fraction of their masses).

The analysis of the 6000 simulations of lower mass clusters pre-
sent in DiCarlo et al. (2020) shows that BHs of about 300 M_☉ can form through dynamical interaction and collisions. The clusters, evolved using NBODY6++GPU, adopted the MOBSE stellar evolution (Mapelli et al. 2017; Giacobbo et al. 2018). This prescription is based on Hurley et al. (2000b); Hurley et al. (2002) and it includes new prescriptions for massive stars reducing the mass loss in supernovae explosion and stellar winds. The systems were initialized with an initial mass in the range between 10^3 and 3 × 10^5 M_☉. The initial central densities and initial half-mass radii were computed as a function of the initial mass of the cluster.

Simulations evolved with Monte Carlo codes reveal similar outcomes. Freitag et al. (2006) computed over 100 models, varying the cluster size, particle number, and central concentration. They systematically changed the number of stars between 10^5 and 10^6 represented by a maximum of 9 × 10^6 particles. Their result show that 20 % of the clusters with an initial central potential parameter W_0 > 8 form a VMS with a mass > 400 M_☉. Other simulations carried out using Monte Carlo models show that multiple VMSs can form within the same cluster (Gürkan et al. 2006).

The analysis of 2000 of simulations (Leigh et al. 2013; Giersz et al. 2015), evolved using the MOCCA (MOnte Carlo simulator) code, reveals the formation of IMBHs in dense stellar environment (Giersz et al. 2015). The outcome of these simulations indicates that about 20% of the simulated clusters generate a BH with a mass larger than 100 M_☉. These BHs have formed through collisions between a VMS and a stellar BH. The formation path is very similar to the one indicated by our N-body simulations (see Figs. 10 and 4). However, in general, the IMBHs produced in MOCCA simulations tend to be systematically more massive as the runaway main-sequence star collisions lead to more massive VMSs in MOCCA than in N-body. In fact, in MOCCA simulations the stellar evolution time-step is performed at the end of the relaxation time-step (that is about 10 Myr). As a consequence of that, the masses of main-sequence stars, the mass segregation, and central density are larger in the MOCCA simulations than in N-body simulations leading to a larger interaction rate.

The analytical work carried on by Stone et al. (2017) shows how stellar mass BHs in nuclear star clusters can grow into IMBH through runaway tidal captures of low mass stars. As stated in their work, runaway tidal captures can be triggered only in massive and compact clusters with a velocity dispersions σ > 40 km/s. According to their criteria, none of our models would have made IMBHs. However, Stone et al. (2017) do not include massive stars and primordial binaries in their study, which are the key elements for the IMBH formation mechanism proposed in our work.

As we have shown in this section the debate whether and how IMBH form in dense star clusters is ongoing for at least two decades. Recent MOCCA Monte Carlo simulations have provided the so far strongest evidence for IMBH formation.

6 SUMMARY AND DISCUSSION

We have provided evidence for the formation of intermediate mass black holes (IMBH) through collisions of massive stars, formation, and evolution of binaries including black holes. Debated for decades and recently underpinned by a large set of Monte Carlo (MOCCA) simulations our direct N-body models are the largest and longest simulations supporting this idea of IMBH formation in dense star clusters, made possible by the use of the massively parallel GPU accelerated code NBODY6++GPU and the use of suitable supercomputers in Germany and China.

We ran and analyzed 80 N-body simulations of compact young massive star clusters with different central densities (central potential parameters W_0 = 6, 7, 8, 9, 10) and sizes (half mass radii R_h = 0.6, 1.0 pc). The simulated clusters were evolved for at least 300 Myr 14. All our models lead to the collisional formation of at least one star above 100 M_☉ (the upper initial mass function limit) and several simulations create stars with masses higher than ~ 400 M_☉ within the first ~ 10 Myr of cluster evolution. Most of the collisions were triggered by triple interactions between hard binaries and single objects. With the stellar evolution model assumed for this study, isolated massive stars cannot collapse directly into IMBHs (BHs with masses > 100 M_☉). Even stars with ~ 500 M_☉ lose most of their mass through stellar winds and collapse into a BH of about 30 M_☉.

However, a sizable fraction (about 20%) of our simulations result in the formation of IMBHs by means of direct collisions between stellar-mass BHs and massive stars as already observed in MOCCA simulations (Giersz et al. 2015). This process is more likely in compact clusters as they form more massive stars and it takes less time for the BHs to sink into the center. Nevertheless, if only a small fraction of the stellar mass is accreted in a collision with a BH (e.g. a collision fraction of f_c = 0.1) the above process becomes unlikely for the formation of IMBHs in compact ~ 7 × 10^5 M_☉ clusters investigated in this study.

After its formation, the IMBH can still grow moderately colliding with other low mass BHs. Here it is important to mention that kicks from gravitational radiation consequent to BH-BH mergers have not been implemented in our code. As a consequence of that, we might have overestimated the probability for the clusters to retain the IMBH because, during a IMBH-BH collision, the recoil velocity might exceed the escape velocity (Campanelli et al. 2007; Baker et al. 2008; Kulier et al. 2013; Morawski et al. 2018; Zivancev et al. 2020).

During the growth process of all simulations with IMBHs, with

14 The simulations that formed an IMBH of about 350 M_☉ were evolved for 500 Myr.
one exception shown in Fig. 5, there are no BH - BH mergers\textsuperscript{15} before the formation of the IMBH in a VMS - stellar BH collision. Therefore the inclusion of gravitational kicks will not change this result. After the IMBH has formed it occasionally collides with a stellar mass BH in an intermediate mass-ratio inspiral (IMRIs) event, which has the potential to kick the IMBH out of the cluster. If common, such a process might explain the missing observational evidence for IMBHs in present day globular clusters. IMBHs might have formed in many GCs early on and, once lost, float around in the galaxies.

Adding gravitational wave kicks and spins will be possible in the future using the approximate models of Baker et al. (2008) for the kick velocity (magnitude and direction) and a new model of how initial BH spins depend on mass and metallicity by Belczynski et al. (2017). Morawski et al. (2018) have analyzed large samples of BH mergers from MOCCA simulations, and show that the BH retention fraction in the cluster varies between 20% and 100% depending on evolutionary time and parameters of the cluster. Brem et al. (2013) have included full Post-Newtonian dynamics in their N-body simulation and reproduced results of Rezzolla et al. (2008), who fitted fully relativistic models. This could also be used to derive recoil velocities, in the way done by Gerosa et al. (2018), the latter again using fully relativistic modeling.

Our results show that the models R06W6 and R06W9, despite the difference in the initial central density, have a comparable probability to form an IMBH. The different evolution of the inner part of the clusters in these two models seems to mitigate the impact of the initial central density on the probability to form an IMBH: in very concentrated systems, the high central density forces the clusters to expand because of early energy generation by primordial binaries; on the other hand, less dense clusters undergo core collapse. Therefore, already at the very beginning of the simulation, the initial difference in central concentration between the models is reduced.

Our results indicate that compact star clusters can rapidly generate an IMBH of few hundred solar masses in about 5 – 15 Myr. Assuming the scenario that nuclear star clusters are generated by globular clusters that spiral toward the nucleus (Tremaine et al. 1975), the IMBHs, if present in the clusters, can further grow in mass colliding with each other and swallowing smaller objects in the center of the nuclear cluster (Arca-Sedda & Gualandris 2018; Arca-Sedda & Capuzzo-Dolcetta 2019; Askar et al. 2020) leading to the formation of a SMBH. This scenario is investigated by Stone et al. (2017) adopting an analytical approach. They show how low mass BHs located in dense nuclear star cluster could rapidly grow in mass via runaway tidal captures, transforming the cluster into a SMBH.

With order $10^5$ particles our models are currently the best available (in the sense of modeling all processes directly for the simulated particle number, without any scaling or averaging). However, young massive clusters in our galaxy and massive extragalactic clusters (also: nuclear star clusters) can be much more massive with particle numbers of up to $10^8$ or more. NBODY6++GPU has been used for the million-body DRAGON simulations (Wang et al. 2015; Wang et al. 2016) and for million body simulation of a nuclear star cluster (Panamarev et al. 2019). But in the first case, the central density was much lower than in this paper, so the DRAGON simulations are not prone to IMBH formation, and in the second case still, some scaling had to be used, because $10^8$ particles are not enough to model nuclear star clusters. In the next ongoing studies, we are running dense star cluster models with a million bodies, and more in the future. It is feasible because only a shorter simulation time is needed (a few hundred Myr versus 12 Gyr for DRAGON). This will help us to get a better understanding of the statistics of the presence of IMBHs, not only in our Galaxy but also out to distant regions relevant for LIGO/Virgo and space-based gravitational wave detections.

In future models we plan to improve the modeling of the external tidal forces on the cluster, reflecting its true orbit around the host galaxy; also stellar evolution has been significantly updated recently,\textsuperscript{16} and is used for our ongoing and future simulations, see for information e.g. Banerjee et al. (2019).

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\textsuperscript{15} In general, BH-BH collisions events are rare. This type of binaries must undergo several strong close interactions to enter the post-Newtonian regime where gravitational radiations can lead to a rapid coalescence. Because of these interactions, the binary is often ejected from the cluster before the merger occurs.

\textsuperscript{16} The most updated version of NBODY6++GPU (partly inspired by LIGO data) contains some recent stellar evolution updates (see Belczynski et al. 2008; Banerjee 2020) These updates were made after the completion of the simulations presented in this work.
