The Color Dipole Picture and the ratio of $R(W^2, Q^2) = \frac{\sigma_L}{\sigma_T}$

Masaaki Kuroda$^a$ and Dieter Schildknecht$^b$

$a$ Institute of Physics, Meiji Gakuin University
Yokohama 244-8539, Japan

$b$ Fakultät für Physik, Universität Bielefeld
D-33501 Bielefeld, Germany
and
Max-Planck Institute für Physik (Werner-Heisenberg-Institut),
Föhringer Ring 6, D-80805, München, Germany

Abstract

The transverse size of $q\bar{q}$ fluctuations of the longitudinal photon is reduced relative to the transverse size of $q\bar{q}$ fluctuations of the transverse photon. This implies $R(W^2, Q^2) = 0.375$ or, equivalently, $F_L/F_2 = 0.27$ at $x \ll 0.1$ and $Q^2$ sufficiently large, while $R(W^2, Q^2) = 0.5$, if this effect is not taken into account. Forthcoming experimental data from HERA will allow to test this prediction.

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2email: kurodam@law.meijigakuin.ac.jp; Dieter.Schildknecht@physik.uni-bielefeld.de
In the present paper, we present our prediction for the ratio of the longitudinal to the transverse photoabsorption cross section, \( R(W^2, Q^2) \equiv \sigma_{\gamma L}(W^2, Q^2) / \sigma_{\gamma T}(W^2, Q^2) \), in the diffraction region of low values of the Bjorken variable \( x \approx Q^2/W^2 \ll 1 \). The prediction is based on a careful reconsideration of our previous formulation [1] of the color-dipole picture (CDP) [2].

At low values of \( x \ll 1 \), in terms of the imaginary part of the virtual forward Compton-scattering amplitude, deep inelastic scattering (DIS) proceeds via forward scattering of (timelike) quark-antiquark, \( q \bar{q} \), fluctuations of the virtual spacelike photon on the proton. In its interaction with the proton, a \( q \bar{q} \) fluctuation acts as a color dipole. A massive \( q \bar{q} \) fluctuation is identical to the \( (q \bar{q})^J=1 \) vector state originating from a timelike photon in \( e^+e^- \) annihilation at an \( e^+e^- \) energy equal to the mass, \( M_{q\bar{q}} \), of the \( q\bar{q} \) state.

A well-known life-time argument [3] allows one to put upper bounds on the magnitude of the scaling variable, \( x \approx Q^2/W^2 \), and on the magnitude of the contributing \( q\bar{q} \) masses, \( M_{q\bar{q}} \). Validity of the color-dipole-picture (CDP) requires the life-time of a \( q\bar{q} \) fluctuation

\[
L = \frac{W^2}{Q^2 + M_{q\bar{q}}^2 M_p^2} \equiv L_0 \frac{1}{M_p}
\]

(1)

to be large compared with the scale set by the proton mass, \( M_p \).

Requiring

\[
L_0 = \frac{1}{x + \frac{M_{q\bar{q}}^2}{W^2}} \gg 1
\]

(2)

implies

i) small \( x \), e.g. \( x \ll 0.1 \), as well as

ii) a restriction on the masses of the \( q\bar{q} \) states that actively contribute to the scattering process at a given center-of-mass energy \( W \),

\[
\frac{M_{q\bar{q}}^2}{W^2} \ll 0.1.
\]

(3)

Specifying the restriction (3) to

\[
\frac{M_{q\bar{q}}^2}{W^2} = 0.01,
\]

(4)

for e.g. a center-of-mass energy of \( W = 225 \) GeV, we obtain \( M_{q\bar{q}} = 22.5 \) GeV. The upper bound on the actively contributing dipole states must coincide with the upper end of the mass spectrum for copious diffractive production in \( \gamma^* \)-proton scattering at any given energy. Indeed, the upper end of the diffractive mass spectrum at \( W = 225 \) GeV at HERA [4] roughly coincides with the simple estimate based on the crude limit adopted in (4). In most
applications [5] of the CDP, the upper bound on the q̅q mass is ignored. Such an approximation is valid under kinematic restrictions on $Q^2$ and $x$, compare ref. [6]. A suitable upper bound extends [6] the kinematic region of validity of the CDP.

With respect to the ensuing discussions, it will be useful to start [7] from the transition of a timelike photon, $\gamma^*$, for definiteness assumed to originate from an $e^+e^-$-annihilation process, to a q̅q pair. The mass of the q̅q pair, $M_{qq}$, is identical to the $\gamma^*$ energy in the q̅q rest frame. A Lorentz boost leads from the q̅q rest frame to the proton rest frame. The direction of the Lorentz boost coincides with the direction of the $\gamma^*$ three-momentum in the proton rest frame. The relation between the q̅q rest frame and the proton rest frame is assumed to be such that the $\gamma^*$ center-of-mass energy is much larger than the q̅q mass, $W \gg M_{qq}$.

In terms of the transverse momentum, $\vec{k}_\perp$, of the (massless) quark and antiquark with respect to the photon direction, the mass $M_{q^\perp}$ is given by

$$M_{\bar{q}q}^2 = \frac{\vec{k}_\perp^2}{z(1-z)} \equiv \vec{k}_\perp^2.$$  \hspace{1cm} (5)

Here, $0 \leq z \leq 1$, denotes the usually employed variable that is related to the q̅q rest-frame angle between the $\gamma^*p$-axis and the three-momentum of the quark,

$$\sin^2 \vartheta = 4z(1-z).$$  \hspace{1cm} (6)

The electromagnetic $q\bar{q}$ current determines the coupling strength of the time-like photon to the $q\bar{q}$ pair of mass $M_{qq}$. The squares of the longitudinal and the transverse component of the electromagnetic $q\bar{q}$ current with respect to the $\gamma^*p$ axis are given by [7]

$$\sum_{\lambda=-\lambda'=-1} |j^\lambda_{L,X}|^2 = 8M_{q\bar{q}}^2 z(1-z)$$  \hspace{1cm} (7)

and

$$\sum_{\lambda=-\lambda'=-1} |j^\lambda_{T,X}(\pm)|^2 = \sum_{\lambda=-\lambda'=-1} j^\lambda_{T,X}(\mp)|^2 = 2M_{q\bar{q}}^2 (1 - 2z(1-z)).$$  \hspace{1cm} (8)

Here, $\lambda = -\lambda' = \pm 1$ refers to twice the helicity of the massless quark and the antiquark, and $j^\lambda_{T,X}(\pm)$ and $j^\lambda_{T,X}(\mp)$ refer to positive and negative helicity, respectively, of the transversely polarized photon. Note that $z(1-z)$ in (7) and (8) may be replaced by the production angle in the q̅q rest frame according to (6).

The q̅q state of mass $M_{qq}$ originating from the coupling to the photon consists of a quark and an antiquark of opposite helicity forming a spin 1 (vector) state. In the limit of very high energy, $W \gg M_{qq}$, the longitudinal momenta of the quark and antiquark in the proton rest frame, in good
approximation, become equal in magnitude, independently of the value of 
\(0 \leq z \leq 1\); the \(q\bar{q}\) vector state, as far as the longitudinal momenta of the quark and antiquark are concerned, does not contain any memory on the value of \(0 \leq z \leq 1\) it originated from. The interaction cross section of the \(q\bar{q}\) pair with the proton, as far as the longitudinal quark and antiquark momenta are concerned is independent of \(z\), and independent on whether the \(q\bar{q}\) state originates from a longitudinal or a transverse photon. The magnitude of the longitudinal momentum components in this high-energy limit only enters via a dependence of the \(q\bar{q}\)-proton interaction on the energy, \(W\).

The situation is different with respect to the transverse momentum of the quark and antiquark. The difference in the transverse momenta for different values of \(z\) at fixed mass, \(M_{q\bar{q}}\),

\[
\vec{k}^2_\perp = z(1-z)M^2_{q\bar{q}}
\]  

is independent of the value of \(W\), and it remains the same specifically also in the high-energy limit of \(W \gg M_{q\bar{q}}\) under consideration in the present context. The normalized distributions of the quark (antiquark) transverse momentum resulting from the coupling strengths in (7) and (8), for longitudinal and transverse photons,

\[
f_L(z) \equiv \frac{z(1-z)}{\int dz \ z(1-z)} = 6z(1-z),
\]

and

\[
f_T(z) \equiv \frac{1-2z(1-z)}{\int dz \ (1-2z(1-z))} = \frac{3}{2}(1-2z(1-z)),
\]

imply different average transverse momenta squared of the quark (antiquark) originating from longitudinal and transverse photons,

\[
\langle \vec{k}^2_\perp \rangle_{L,T} = M^2_{q\bar{q}} \int_0^1 dz \ z(1-z) f_{L,T}(z).
\]

Explicitly one obtains from (12),

\[
\langle \vec{k}^2_\perp \rangle_L = \frac{4}{20} M^2_{q\bar{q}}.
\]

and

\[
\langle \vec{k}^2_\perp \rangle_T = \frac{3}{20} M^2_{q\bar{q}}.
\]

and from (13) and (14),

\[
\rho = \frac{\langle \vec{k}^2_\perp \rangle_L}{\langle \vec{k}^2_\perp \rangle_T} = \frac{4}{3}.
\]

The result (15) is qualitatively expected, since a non-vanishing transition of a longitudinal photon, \(\gamma^*_L\), to a \(q\bar{q}\) pair according to (7) requires \(z \neq 0, 1\), or
equivalently, a non-vanishing rest-frame production angle \( \theta \), in distinction from the transverse case \( \langle \bar{q}q \rangle \), where \( z = 0, 1 \) is by no means excluded.

From the uncertainty relation, the ratio of the effective transverse sizes, \( \langle \bar{q}^2 \rangle_{L,T} \), for longitudinal and transverse photons according to (15) is given by

\[
\frac{\langle \bar{r}^2 \rangle_L}{\langle \bar{r}^2 \rangle_T} = \frac{1}{\rho} = \frac{3}{4}.
\]

(16)

Longitudinal photons, \( \gamma_L^* \), produce “small-size” pairs, while transverse photons, \( \gamma_T^* \), produce “large-size” pairs. The ratio of the average sizes at any fixed \( q\bar{q} \) mass is given by (16).

We summarize: as far as the longitudinal quark momenta are concerned, their approximate equality in the high-energy limit of \( W \gg M_{q\bar{q}} \) implies that they only affect the \( q\bar{q} \)-proton interaction via a dependence on \( W \) that is independent on whether the \( q\bar{q} \) state originates from a longitudinal or a transverse photon. In contrast, the difference in the average quark (anti- quark) transverse momenta (13) and (14) for longitudinal and transverse photons will affect the \( q\bar{q} \)-proton interaction via the transverse size of the \( q\bar{q} \) state according to (16).

We take the different interaction size into account by introducing a proportionality factor, \( \rho \), connecting the \( q\bar{q} \)-proton interactions induced by longitudinal, \( (q\bar{q})^j_{L=1} \), and transverse, \( (q\bar{q})^j_{T=1} \), quark-antiquark states of mass \( M_{q\bar{q}} \),

\[
\sigma_{(q\bar{q})^j_{L=1},p}(M_{q\bar{q}}^2, W^2) = \rho \sigma_{(q\bar{q})^j_{T=1},p}(M_{q\bar{q}}^2, W^2).
\]

(17)

The value of \( 1/\rho \) from (16) suppresses “small-size” longitudinal versus “large-size” transverse hadronic \( q\bar{q} \) cross sections. Note that \( \sigma_{(q\bar{q})^j_{L=1}} \equiv \frac{1}{2} \sigma_{(q\bar{q})^j_{L=1}}^\ast \), i.e. \( \rho = 1 \), instead of (16), is identical to helicity independence for \( (q\bar{q}) \) vector-state scattering. We stress that the suppression of small-size longitudinal versus large-size transverse \( (q\bar{q})^j_{L=1} \)-scattering cross sections is independent of the mass, \( M_{q\bar{q}} \), of the \( (q\bar{q})^j_{L=1} \) state under consideration. As a consequence, when passing to \( q\bar{q} \) fluctuations of spacelike photons, the suppression effect enters as a normalization factor, as in (17), while being irrelevant for the \( Q^2 \) dependence.

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\( ^3 \)The \( q\bar{q} \) state originating from a longitudinally polarized photon, \( \gamma_L^* \), only differs in the average (transverse) momentum-squared, \( \langle \bar{k}_L^2 \rangle_L \), from the \( q\bar{q} \) state originating from a transversely polarized photon, \( \gamma_T^* \). Accordingly, the (transverse) size-squared associated with \( \gamma_T^* \), namely \( \langle \bar{r}_T^2 \rangle_T \), is obtained from the (transverse) size-squared associated with \( \gamma_L^* \), namely \( \langle \bar{r}_L^2 \rangle_L = A/\langle \bar{k}_L^2 \rangle_L \), where \( A \leq \frac{1}{4} \), by the replacement \( \langle \bar{r}_L^2 \rangle_L \rightarrow \langle \bar{r}_T^2 \rangle_T \), i.e. \( \langle \bar{r}_T^2 \rangle_T = A/\langle \bar{r}_L^2 \rangle_T \) without change of \( A \) under this replacement. Relation (16) follows immediately.

\( ^4 \)In the talk at DIS2008 given by one of us (D.S.), helicity independence was erroneously presented as necessary. The written version of the contribution to DIS2008\[9\] agrees with the results of the present paper, see also ref. [10], where helicity independence is introduced as a hypothesis.
This is at variance with the frequently presented discussion, e.g. ref. [3], where transverse-size effects are associated with the $Q^2$ dependence of the longitudinal versus the transverse photoabsorption cross section.

The transition from the scattering of a $q\bar{q}$ pair of mass $M_{q\bar{q}}$ to the scattering of (timelike) $q\bar{q}$ fluctuations of a virtual spacelike photon may be described in transverse position space in terms of the so-called photon wave function $\hat{r}_{\perp}$. In terms of the variable $r_{\perp}'$, related to the transverse quark-antiquark distance in (16) via

$$r_{\perp}' = r_{\perp} \sqrt{z(1 - z)},$$

the longitudinal and the transverse photoabsorption cross section is given by

$$\sigma_{\gamma L,T\bar{p}}(W^2, Q^2) = \int \frac{dz d^2r_{\perp}'}{z(1 - z)} |\psi_{L,T}(r_{\perp}' Q, z(1 - z))|^2 \sigma_{(q\bar{q})L,T\bar{p}}(r_{\perp}', W^2) = \frac{6\alpha}{2\pi^2} Q^2 \Sigma_q Q_q^2 \left\{ \begin{array}{c}
4 \int dz z(1 - z) d^2r_{\perp} K_0^2(r_{\perp}' Q, z(1 - z), W^2), \\
\int dz (1 - 2z(1 - z)) d^2r_{\perp} K_1^2(r_{\perp}' Q, z(1 - z), W^2),
\end{array} \right\} (19)$$

In (19), $\Sigma_q Q_q^2$ denotes the sum over the quark charges (with $3\Sigma Q^2_q \equiv R_{e^+e^-}$), and $K_0(r_{\perp}' Q)$ and $K_1(r_{\perp}' Q)$ are modified Bessel functions. The essential point in (19), in distinction from the usually employed [5] form of the dipole picture, is the factorization of the $z(1 - z)$ dependence identical in form to the (squares of the) longitudinal and transverse electromagnetic-current components in (17) and (8). Indeed, (19) follows from the formulation of the CDP in terms of the interquark transverse separation, $r_{\perp}$

$$\sigma_{\gamma L,T\bar{p}}(W^2, Q^2) = \frac{6\alpha}{2\pi^2} Q^2 \Sigma_q Q_q^2 \left\{ \begin{array}{c}
4 \int dz z^2(1 - z)^2 d^2r_{\perp} K_0^2(\sqrt{z(1 - z)} Q, \sigma_{(q\bar{q})\bar{p}}(r_{\perp}, z(1 - z), W^2)), \\
\int dz (1 - 2z(1 - z)) z(1 - z) d^2r_{\perp} K_1^2(\sqrt{z(1 - z)} \sigma_{(q\bar{q})\bar{p}}(r_{\perp}, z(1 - z), W^2)),
\end{array} \right\} (20)$$

by requiring a $z(1 - z)$ dependence identical in form to the one in (17) and (8),

$$\int dz z(1 - z) \sigma_{(q\bar{q})\bar{p}}(r_{\perp}' Q, z(1 - z), W^2) = \int dz z(1 - z) \sigma_{(q\bar{q})\bar{p}}(r_{\perp}' Q, z(1 - z), W^2), (21)$$

and

$$\int dz (1 - 2z(1 - z)) \sigma_{(q\bar{q})\bar{p}}(r_{\perp}' Q, z(1 - z), W^2) = \int dz (1 - 2z(1 - z)) \sigma_{(q\bar{q})\bar{p}}(r_{\perp}' Q, z(1 - z), W^2). (22)$$

\footnote{Compare [11] for a careful examination of the usual formulation of the CDP.}
Substitution of \( \vec{r}_\perp \) in terms of \( \vec{r}_\perp' \) in (20) and substitution of (21) and (22) immediately imply (19).

The factorization implies the discrimination between dipole cross sections for longitudinally and transversely polarized \((q\bar{q})^{j=1}\) states in (19). The form (19) of the color-dipole picture (CDP) will be referred to as the \( r'_\perp \) representation.

Carrying out the integration over \( z \) in (19), we have

\[
\sigma_{\gamma_{L,T}^p}(W^2, Q^2) = \frac{2\alpha_R e^+ e^-}{3\pi^2} Q^2 \int d^2r_\perp' K_{0,1}(r_\perp' Q) \sigma_{(q\bar{q})}^{l=1, p}(r_\perp', W^2).
\]  

(23)

In (19) and (23), we now introduce the \( q\bar{q} \)-size effect (17) that becomes

\[
\sigma_{(q\bar{q})}^{l=1, p}(r_\perp', W^2) = \rho \sigma_{(q\bar{q})}^{l=1, p}(r_\perp', W).
\]

(24)

To incorporate the coupling of the \( q\bar{q} \) color dipole to two gluons, the \( r'_\perp \) representation must be supplemented by

\[
\sigma_{(q\bar{q})}^{l=1, p}(r_\perp', W^2) = \rho \sigma_{(q\bar{q})}^{l=1, p}(r_\perp', W^2) = \rho \int d^2l_\perp' \sigma_{(q\bar{q})}^{l=1, p}(l_\perp'^2, W^2)(1 - e^{-i\vec{l}_\perp' \cdot \vec{r}_\perp'}),
\]

(25)

where (24) was incorporated. In (25), \( l_\perp' \) is related to the transverse momentum of the gluon, \( \vec{l}_\perp' \), absorbed by the quark or antiquark by

\[
\vec{l}_\perp' = \frac{\vec{l}_\perp}{\sqrt{(1 - z)}}.
\]

(26)

In the \( r'^2_\perp \rightarrow 0 \) limit, (25) may be approximated by

\[
\sigma_{(q\bar{q})}^{l=1, p}(r_\perp', W^2) = \rho \sigma_{(q\bar{q})}^{l=1, p}(r_\perp', W^2) \approx \rho r'^2_\perp \int d^2l_\perp'^2 \sigma_{(q\bar{q})}^{l=1, p}(l_\perp'^2, W^2).
\]

(27)

The generic structure of two-gluon couplings to the \( q\bar{q} \)-pair implies “color transparency” [2]: vanishing of the color-dipole interaction for vanishing interquark distance. Due to the strong decrease of the modified Bessel functions \( K_0(r_\perp' Q) \) and \( K_1(r_\perp' Q) \) in (23) for large values of their argument, \( r_\perp' Q \), the large-\( Q^2 \) behavior of \( \sigma_{\gamma_{L,T}^p}(W^2, Q^2) \) in (23) can be obtained by substitution of the small-\( r'_\perp \) approximation (27).

In passing, we quote the explicit ansatz [1] consistent with (27) that was previously used in a (successful) representation of the experimental data \(^6\).

\[
\sigma_{(q\bar{q})}^{l=1, p}(r_\perp', W^2) = \rho \sigma_{(q\bar{q})}^{l=1, p}(r_\perp', W^2) = \rho \sigma_{(q\bar{q})}^{(\infty)}(W^2)(1 - J_0(r_\perp' \Lambda_{sat}^2(W^2)));
\]

(28)

\(^6\)Actually \( \rho = 1 \), i.e. helicity independence was used in the description of the experimental data for \( \sigma_{\gamma^p}(W^2, Q^2) \). The difference of \( \rho = 1 \) and \( \rho = 4/3 \) from (15) is mainly relevant for \( R(W^2, Q^2) \).
with a power-law ansatz for the “saturation scale” \( \Lambda_{\text{sat}}^2(W^2) \),

\[
\Lambda_{\text{sat}}^2(W^2) = \frac{\pi}{\sigma^{(\infty)}(W^2)} \int d\ell_\perp^2 \ell_\perp^2 \bar{\sigma}_{(q\bar{q})L}^{1-p}(\ell_\perp^2, W^2). \tag{29}
\]

The hadronic cross section, \( \sigma^{(\infty)}(W^2) \), is approximately constant. The ansatz \( \Lambda_{\text{sat}}^2(W^2) \) is mentioned in order to explicitly display the role of \( \rho \) as a factor that determines the relative magnitude of the total hadronic cross sections for transversely relative to longitudinally polarized \( (q\bar{q})^J=1 \) states; the hadronic cross section, \( \sigma^{(\infty)}(W^2) \), for scattering of \( (q\bar{q})^J=1 \) states of mass \( M_{q\bar{q}} \) is effectively replaced by \( \rho \sigma^{(\infty)}(W^2) \) when passing from longitudinally polarized to transversely polarized \( (q\bar{q})^J=1 \) states, compare the discussion following (17).

A unique consequence on the ratio \( R(W^2, Q^2) \) of the longitudinal to the transverse photoabsorption cross section at large values of \( Q^2 \) follows immediately by substituting the \( r'_{\perp} \rightarrow 0 \) approximation (27) into the \( r'_{\perp} \)-representation of the CDP (23). Indeed, in the large-\( Q^2 \) limit, the dependence on the details of the \( (q\bar{q})p \) interaction, compare (28) as an example, cancels in \( R(W^2, Q^2) \), and, for \( Q^2 \) sufficiently large, we have

\[
R(W^2, Q^2) = \frac{\sigma_{\gamma p}^{(\infty)}(W^2, Q^2)}{\rho \sigma_{\gamma p}^{(\infty)}(W^2, Q^2)} = \frac{1}{\rho} \frac{\int d^2 r'_\perp r'_\perp^2 K_0^2(r'_\perp Q)}{\int d^2 r'_\perp K_1^2(r'_\perp Q)} = \frac{1}{2 \rho} = \frac{3}{8} = 0.375. \tag{30}
\]

Equivalently, in terms of the structure functions,

\[
\frac{F_L(W^2, Q^2)}{F_T(W^2, Q^2)} = \frac{1}{1 + 2 \rho} = \frac{3}{11} \approx 0.27. \tag{31}
\]

In (30) and (31), the equality [12]

\[
\int_0^\infty dy \: y^3 K_0^2(y) = \frac{1}{2} \int_0^\infty dy \: y^3 K_1^2(y) \tag{32}
\]

was used, and the value of \( \rho = 4/3 \) from (15) was inserted. We note that helicity independence, \( \rho = 1 \), leads to \( R(W^2, Q^2) = 0.5 \) and \( F_L/F_T = 1/3 \). This case of \( \rho = 1 \), using the ansatz (28), in ref. [13] was evaluated and discussed in detail, including the transition to \( Q^2 \rightarrow 0 \), and compared with the H1 analysis of the longitudinal structure function available at the time [14] that was based on certain theoretical input assumptions. Compare fig.1. Replacing \( \rho = 1 \) by \( \rho = \frac{4}{3} \) decreases the theoretical predictions for \( \sigma_{\gamma p}^{(\infty)}(W^2, Q^2) \) in fig.1 by the factor of \( \frac{9}{11} \approx 0.82 \), improving agreement with the data.

We stress that the predictions (30) and (31) only rely on the CDP in the \( r'_{\perp} \) representation given by (19), (23) combined with color transparency (27) and the \( q\bar{q} \)-transverse-size effect incorporated into the proportionality (24). The predictions are independent of any specific ansatz for the dipole-cross section. We note that the predictions (30) and (31) are most reliable for
$5 GeV^2 \lesssim Q^2 \lesssim 100 GeV^2$. They become less reliable, when $Q^2$ increases to $Q^2 \gg 100 GeV^2$, since in this case the CDP has to be refined by a restriction on the actively contributing $q\bar{q}$ masses, $M_{q\bar{q}}$ [6]. For a value of $F_2 \cong 1.2$, that is typical for the $Q^2$ and $W$ values where final separation data will become available, according to (31), we find $F_L \simeq 0.32$.

The parameter $\rho$ from (24), making use of the first equality in (31), can be determined from measurements of DIS at different electron-proton center-of-mass energies, $\sqrt{s}$, for fixed values of $x$ and $Q^2$. The reduced cross section of DIS is given by

$$\sigma_r(x, y, Q^2) = F_2(x, Q^2) \left(1 - \frac{y^2}{1 + (1 - y)^2} \frac{1}{1 + 2\rho}\right),$$

(33)

where $y = Q^2/\sqrt{s}$. The slope of a straight-line fit of $\sigma_r(x, y, Q^2)$ as a function of $0 \leq y^2 / (1 + (1 - y)^2) \leq 1$ determines $\rho$. A value of

$$\rho = 1$$

(34)

corresponds to helicity independence, i.e. equality of the forward scattering amplitudes of $(q\bar{q})_{h=1}^J$ fluctuations of the photon on the proton for helicities $h = 0$, $h = +1$ and $h = -1$. A deviation from $\rho = 1$ rules out helicity independence. Longitudinally polarized $(q\bar{q})^{J=1}$ states have a reduced transverse size relative to transversely polarized $(q\bar{q})^{J=1}$ states. This implies a deviation from $\rho = 1$ of predictable magnitude, compare (16). The theoretically preferred value of $\rho$, accordingly, is

$$\frac{1}{\rho} = \frac{3}{4}.$$  

(35)

The measurement of $\rho$ provides insight into the dynamics of the scattering of the $q\bar{q}$ fluctuations of the photon on the proton and a strong constraint on the CDP.

An interesting upper bound on $R(W^2, Q^2)$ was recently derived in the framework of the CDP. The bound is given by [11]

$$R(W^2, Q^2) \leq 0.37248,$$

(36)

or, in terms of $\rho$ with $R(W^2, Q^2) = 1/2\rho$ from (30), $\rho > 1.34235$. The approximate numerical coincidence of our prediction (30) with the upper bound (36) is accidental.

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7 Preliminary results [15] from HERA were presented at DIS 2008, 7-11 April 2008, University College London.
8 Compare e.g. ref. [16].
9 In refs. [17] and [10], the interpretation of $\rho$ in terms of a proportionality of sea quark and gluon distributions led to a certain theoretical preference of (34) versus (35) in contrast to our present point of view. This needs some further investigation.
The upper bound (36) is obtained from (20) by adopting the frequently employed approximation of \( z(1 - z) \) independence of the ansatz for the color-dipole cross section

\[
\sigma_{(q\bar{q})p}(r_\perp, z(1 - z), W^2) \equiv \sigma_{(q\bar{q})p}(r_\perp, W^2) \tag{37}
\]

In this case of (37), the integration over \( z \) may be carried out in (20), and the longitudinal and transverse cross sections become integrals over \( r_\perp \)-dependent probability densities multiplied by the dipole cross section (37). The bound (36) follows from the maximum of the ratio of the longitudinal-to-transverse probability densities as a function of \( r_\perp \). If the assumption (37) is dropped, the derivation of the bound fails.

The bound (36) on \( R(W^2, Q^2) \) means that models for the dipole cross section that imply violations of the bound must necessarily contain a dependence on the configuration variable \( z(1 - z) \). An example is provided by our ansatz (28) with \( \rho = 1 \) and \( R(W^2, Q^2) = 0.5 \). Experimental values of \( R(W^2, Q^2) \) below the bound (36) neither require nor rule out a \( z(1 - z) \) dependence of the dipole cross sections. Indeed, the theoretical restriction (3) on the contributing \( q\bar{q} \) masses even requires the approximation (37) to break down at a certain level of accuracy in dependence on the range of the kinematical variables \( Q^2 \) and \( W^2 \).

The \( r_\perp' \) representation rests on an explicit factorization of the photon-production cross section in terms of a three-step process: \( \gamma^*(q\bar{q}) \) coupling, \( (q\bar{q}) \) propagation and \( (q\bar{q}) \) scattering. The factorization is intimately related to the underlying notion of a \( q\bar{q} \) fluctuation of the photon interacting with the proton. It even appears as an unavoidable consequence of this picture. The transverse size of the \( (q\bar{q})^{J=1} \) state emerging from the photon being large for transverse relative to longitudinal polarization, we predict \( R(W^2, Q^2) = 3/8 = 0.375 \) or, equivalently, \( F_L(W^2, Q^2)/F_2(W^2, Q^2) \approx 0.27 \).

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Figure caption
Fig.1: The longitudinal photoabsorption cross section $\sigma_{\gamma p}(W^2,Q^2) \equiv \sigma_L$ from ref. [13] as a function of the scaling variable $\eta \equiv (Q^2 + m_0^2)/\Lambda_{sat}^2(W^2)$ compared with HERA data available in 2001 and based on an H1 analysis [13] with theoretical QCD input assumptions rather than on a longitudinal-transverse separation measurement.
\[ \eta = \frac{(Q^2 + m_0^2)}{\Lambda^2(W^2)} \]

H1 data

GVD/CDP:
- \( W^2 = 90000 \text{ GeV}^2 \)
- \( W^2 = 20000 \text{ GeV}^2 \)
- \( W^2 = 900 \text{ GeV}^2 \)