Unfortunate Terminology

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Abstract. The phrase “negative squared rest mass” can sometimes be found in papers on neutrinos and frequently occurs in the tachyonic literature. Consequently, some authors say that “the rest mass of tachyons is imaginary”. Besides, the statement “photons have zero rest mass” is almost common. In terms of relativity, however, the state of rest cannot be reasonably defined for luxons and tachyons, and, therefore, today it does not make sense to speak of rest mass of such objects. It is shown here that the phrases “negative squared mass”, “imaginary mass”, and “photon’s zero mass” result from applying bradyonic dynamical relativistic relations to determine properties of luxons and tachyons; and that this erroneous procedure results from an unfortunate interpretation of kinematical relativistic relations. It is also shown that the use of proper relativistic relations, i.e. luxonic or tachyonic ones, gives a positive quantity having the squared mass dimension. Thus we obtain a nonzero real quantity having the mass dimension (called masslike quantity), which is positive for the photon and may (by intuition – should) be positive for other luxons and for tachyons.
From some papers on measurements of the neutrino mass one can learn that “the squared rest mass of neutrinos is negative”. In the tachyonic literature it is frequently stated that “the squared rest mass of tachyons is negative”, and, consequently, some authors conclude that “the rest mass of tachyons is imaginary”, though in the case of neutrinos I have not met such a heroic author. Besides, the statement “photons have zero rest mass” is almost common. These statements are said to be conclusions from relativity but this is not true.

In relativity the term “rest mass” does not make sense in the case of luxons and tachyons, since the state of rest can be reasonably defined for these objects neither within standard relativity nor in its consistent extensions. This is obvious in the luxonic case since, e.g., the Lorentz transformation is singular for speeds equal to \( c \). If we were to assume that any tachyon may be at rest, then three independent states of its rest would have to exist since observers who see our spacetime as that having three time dimensions and only one space dimension would then have to be admitted.

As regards the phrases “negative squared mass”, “imaginary mass”, and “photon’s zero mass”, we shall proceed step by step.

Consider the world line \( x^\mu (\sigma) \) of a pointlike object. Assume, for simplicity, that the object is free in flat spacetime endowed with the Lorentzian coordinates (i.e. \( x^\mu (\sigma) \) is straight), that \( \sigma \) is the normalized affine parameter of \( x^\mu (\sigma) \), and that the signature is, e.g., \( + + + - \). Note that in the metric form expressions, \( ds^2 = dx_\mu dx^\mu \), \( ds^2 \) is only a conventional symbol, and therefore it need not be the square of an infinitesimal real quantity. In the case under consideration

\[
ds^2 = dx^2 + dy^2 + dz^2 - c^2 dt^2,
\]

and for \( x^\mu (\sigma) \) we have

\[
ds^2 = -k (d\sigma)^2,
\]

where \( d\sigma \) is indeed an infinitesimal real quantity, and where the discrete dimensionless parameter \( k \) is as follows:
\[ k = 1 \text{ in the bradyonic (timelike, subluminal) case,} \]
\[ k = 0 \text{ in the luxonic (null, luminal) case, and} \]
\[ k = -1 \text{ in the tachyonic (spacelike, superluminal) case.} \]

(If the signature + − − − were chosen, then by Eq. (2) we would have \( k = -1 \) in the bradyonic case and \( k = 1 \) in the tachyonic case.) Dividing Eqs. (1) and (2) by \((d\sigma)^2\) we get

\[ -k = (u^x)^2 + (u^y)^2 + (u^z)^2 - (u^t)^2, \tag{3} \]

where \(u^\mu := dx^\mu /d\sigma\) is a four-velocity vector. The kinematical Eq. (3) concerns every type of world lines – timelike, null, and spacelike. The type is determined by \(k\).

Multiplying Eq. (3) by \(m^2 c^2\), where \(m\) has the mass dimension (we do not yet determine physical meanings of \(m\)), we get the well-known special relativistic formula for a four-momentum vector \(p^\mu\):

\[ -km^2 c^2 = (p^x)^2 + (p^y)^2 + (p^z)^2 - (p^t)^2 \equiv p^2 - c^{-2}E^2, \tag{4} \]

where

\[ p^\mu := mcu^\mu, \tag{5} \]

and where \((p^x)^2 + (p^y)^2 + (p^z)^2 \equiv p^2\) and \((p^t)^2 \equiv c^{-2}E^2\). If we had \(m = 0\), then by definition (5) we would have no four-momentum, i.e. no object on our world line (not speaking that the multiplication of equations by zero does not make sense). Thus

\[ m \neq 0. \tag{6} \]

If \(m\) were imaginary, then by definition (5) also the four-momentum components \(p^\mu\) would be imaginary, which would give us a new physics yet unknown. If we had real \(m < 0\), then by definition (5) we would have opposite senses of the four-vectors \(u^\mu\) and \(p^\mu\). Such a situation is yet unknown and today seems strange, though perhaps it will be considered in future. Anyway, we are entitled to put real \(m > 0\) for every type of the objects under consideration (Ockham’s principle!).
The unfortunate phrases have resulted from the fact that some authors have not taken into account the existence of three values of \( k \): \((1, 0, -1)\) and have applied the bradyonic variants of Eqs. (1)–(4) for luxons and tachyons. The use of proper values of \( k \) allows to avoid the difficulties.

In the bradyonic case, \( m \) is the rest mass of our object. In the luxonic case the physical meaning of \( m \) is not determined in general, though it is so for the photon for which \( m = c^{-2}E = c^{-2}h\nu > 0 \). Anyway, the dynamical luxonic relation \( p^2c^2 = E^2 \) does not result from the condition \( m = 0 \), which is false (inequality (6)), but it does result from the condition \( k = 0 \), i.e. it is determined at the kinematical level of Eqs. (1)–(3). In the tachyonic case we have yet no operational definition of \( m \) (for lack of rest), and therefore the term “masslike quantity” has been proposed. (The terms “pseudomass” or “quasimass” are shorter but semantically inferior.)

Additional remarks and amusing details can be found in Section 6 of my prior paper [1].

[1] J. K. Kowalczyński, Acta Physica Slovaca 50 (2000) 381, or hep-ph/9911441.