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New insights into electron spin dynamics in the presence of correlated noise

S Spezia, D Persano Adorno, N Pizzolato and B Spagnolo

Dipartimento di Fisica, Group of Interdisciplinary Physics, Università di Palermo and CNISM, Viale delle Scienze, Edificio 18, I-90128 Palermo, Italy
E-mail: stefano.spezia@unipa.it and dominique.persanoadorno@unipa.it

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Abstract

The changes in the spin depolarization length in zinc-blende semiconductors when an external component of correlated noise is added to a static driving electric field are analyzed for different values of field strength, noise amplitude and correlation time. Electron dynamics is simulated by a Monte Carlo procedure which takes into account all the possible scattering phenomena of the hot electrons in the medium and includes the evolution of spin polarization. Spin depolarization is studied by examining the decay of the initial spin polarization of the conduction electrons through the D’yakonov–Perel process, the only relevant relaxation mechanism in III–V crystals. Our results show that, for electric field amplitudes lower than the Gunn field, the dephasing length shortens with increasing noise intensity. Moreover, a nonmonotonic behavior of spin depolarization length with the noise correlation time is found, characterized by a maximum variation for values of noise correlation time comparable with the dephasing time. Instead, in high field conditions, we find that, critically depending on the noise correlation time, external fluctuations can positively affect the relaxation length. The influence of the inclusion of the electron–electron scattering mechanism is also shown and discussed.

(The possible utilization of electron spin as an information carrier in electronic devices is an engaging challenge for future spin-based electronics (spintronics). In particular, in semiconductor spin-based devices, information is encoded in the electron spin state, transferred by the conduction electrons, and finally detected [1–3]. However, a disadvantage of the use of spin degree of freedom is the fact that spin polarization is a quantity that is not preserved, as each initial non-equilibrium magnetization decays both in time and distance during the transport. Therefore, the spin depolarization length could not be long enough to permit a reliable manipulation, control and detection of information.

In spintronic devices, the information stored in a system of polarized electron spins can be transferred attached to mobile carriers by applying an external electric field [1–6]. Because of increasing miniaturization, to avoid too intense electric fields, which could lead the system to exhibit a strongly nonlinear physical behavior, applied voltages are very low. Since low voltages are more subject to background noise, it becomes essential to understand the influence of fluctuations of the electric field on the spin depolarization process.

The presence of noise is generally considered a disturbance, especially when determining the efficiency of a device, since strong fluctuations can affect its performance. The existence of fluctuations, for example, can limit the lifetime of the information stored in a memory cell, interfere with the opening (or closing) of random logic gates and cause the enlargement of the distribution of arrival times of signals in transmission lines. In quantum computation, a fundamental problem is the destruction of entangled states of qubits by interaction with the environment. This event is characterized by loss of coherence, which is not suitable for the design of quantum computers [7].
In the last decade, however, increasing interest has been directed toward possible constructive aspects of noise in the dynamical response of nonlinear systems [8–10]. The effect of an external source of noise on electron transport in GaAs crystals in the presence of static and/or periodic electric fields has been studied [11–14]. Furthermore, theoretical works which discuss the way to improve the ultra-fast magnetization dynamics of magnetic spin systems by including random fields have been recently published [15–17]. Nevertheless, to the best of our knowledge, an investigation of the role of noise in the electron spin dynamics in semiconductors is still missing.

In this paper, we investigate the spin relaxation process in low-doped n-type GaAs crystals driven by a randomly fluctuating electric field. The electron transport is simulated by a Monte Carlo procedure which takes into account all the possible scattering phenomena of hot electrons in the medium, and includes the evolution of the spin polarization vector. The effects caused by the addition of an external source of correlated noise are investigated by analyzing the change of the spin depolarization length with respect to the value obtained in the absence of noise. Our findings show that the presence of a random contribution can affect the value of the decoherence length, and that the variation is maximum for values of the noise correlation time comparable with the characteristic time of the spin relaxation process. Moreover, noise-induced effects are slightly reduced by the inclusion of the electron–electron Coulomb interaction.

The spin–orbit interaction couples the spin of conduction electrons to the electron momentum, which is randomized by scattering with impurities and phonons. The spin–orbit coupling gives rise to spin precession, while momentum scattering makes this precession randomly fluctuating, both in magnitude and orientation [18].

For electrons that are delocalized and under a nondegenerate regime, the D’yakonov–Perel (DP) mechanism [19, 20] is the only relevant relaxation process in n-type III–V semiconductors [3, 21, 22]. In a semiclassical formalism, the term of the single electron Hamiltonian, which accounts for the spin–orbit interaction, can be written as

\[ H_{SO} = \frac{\hbar}{2} \sigma \cdot \mathbf{\Omega}. \]  

This represents the energy of electron spins precessing around an effective magnetic field \( \mathbf{B} = h\mathbf{\Omega}/\mu_B g \) with angular frequency \( \mathbf{\Omega} \), which depends on the orientation of the electron momentum vector with respect to the crystal axes. Near the bottom of each valley, the precession vector can be written as

\[ \mathbf{\Omega}_\Gamma = \frac{\beta_{\Gamma}}{\hbar} \left[ k_x (k_y^2 - k_z^2) \hat{x} + k_y (k_z^2 - k_x^2) \hat{y} + k_z (k_x^2 - k_y^2) \hat{z} \right] \]  

in the \( \Gamma \)-valley [23],

\[ \mathbf{\Omega}_L = \frac{\beta_L}{\sqrt{3}} \left[ (k_x - k_z) \hat{x} + (k_z - k_y) \hat{y} + (k_y - k_x) \hat{z} \right] \]  

in the \( L \)-valleys located along the [111] direction in the crystallographic axes, and

\[ \mathbf{\Omega}_X = \frac{\beta_X}{2} [-k_y \hat{y} + k_z \hat{z}] \]  

in the X-valleys located along the \([100]\) direction [24]. In equations (2)–(4), \( k_i (i = x, y, z) \) are the components of the electron wavevector. \( \beta_{\Gamma}, \beta_L, \) and \( \beta_X \) are the spin–orbit coupling coefficients. Here, we assume \( \beta_L = 0.26 \text{ eV } \text{Å}^{-1} \times 2/\hbar \) and \( \beta_X = 0.059 \text{ eV } \text{Å}^{-1} \times 2/\hbar \), as recently theoretically estimated [25], while \( \beta_{\Gamma} \) is calculated as in [26].

Since the quantum-mechanical description of the electron spin evolution is equivalent to that of a classical momentum \( \mathbf{S} \) experiencing the magnetic field \( \mathbf{B} \), we describe the spin dynamics by the classical equation of precession motion

\[ \frac{d\mathbf{S}}{dt} = \mathbf{\Omega} \times \mathbf{S}. \]

The D’yakonov–Perel mechanism acts between two scattering events and reorients the direction of the precession axis and of the effective magnetic field \( \mathbf{B} \) in a random and trajectory-dependent way. This effect leads the spin precession frequencies \( \mathbf{\Omega} \) and their directions to vary from place to place within the electron spin ensemble. This spatial variation is called inhomogeneous broadening [27].

When a semiconductor is in contact with a spin polarization source at \( x = 0 \), in the absence of any magnetic field \( (\mathbf{B} = 0) \) and with a static electric field directed along the \( \hat{x} \)-direction \( (\mathbf{F} = \mathbf{F}_x) \), the drift–diffusion model predicts an exponential decay of the spin polarization \( S_i(x) = S_0 \exp(-x/L) \), in which \( L \) is the electric-field-dependent spin depolarization length [30]. The electron spin dephasing length can be seen as the distance that the electrons move with average drift velocity \( v_d \) within the spin lifetime \( \tau \) [30].

\[ L = v_d \tau = \mu F \tau = \frac{qF_p \tau}{m^*}, \]  

where \( \mu = q \tau_p / m^* \) is the electron mobility, \( q \) the elementary charge, \( \tau_p \) the momentum relaxation time and \( m^* \) the electron effective mass.

The inhomogeneous broadening, quantified by the average squared precession frequency \( \langle |\mathbf{\Omega}(\mathbf{k})|^2 \rangle \), together with the correlation time of the random angular diffusion of spin vector \( \tau_c \) are the relevant variables in the D’yakonov–Perel formula

\[ \tau^{-1} = \langle |\mathbf{\Omega}(\mathbf{k})|^2 \rangle \tau_c. \]  

Since, in our case, we include the electron–electron interaction mechanism, one needs to distinguish between the momentum relaxation time \( \tau_p \) and the momentum redistribution time \( \tau_p' \), which is practically equal to \( \tau_c \). This distinction is necessary because, although electron–electron scattering contributes to momentum redistribution, it does not directly lead to momentum relaxation [31, 32].

By using equations (6) and (7), the spin depolarization length can be expressed in the form

\[ L = \frac{eF}{m^* \langle |\mathbf{\Omega}(\mathbf{k})|^2 \rangle} \frac{1}{\tau_p'}. \]

In our simulations the semiconductor bulk is driven by a fluctuating electric field

\[ F(t) = F_0 + \eta(t) \]
where $F_0$ is the amplitude of the deterministic part and $\eta(t)$ is a random term, modeled by a stochastic process. Here, $\eta(t)$ is modeled as an Ornstein–Uhlenbeck (OU) process, which obeys the stochastic differential equation [33]

$$\frac{d\eta(t)}{dt} = -\frac{\eta(t)}{\tau_D} + \sqrt{\frac{2D}{\tau_D}} \xi(t)$$

(10)

where $\tau_D$ and $D$ are, respectively, the correlation time and the intensity of the noise described by the OU process with autocorrelation function $\langle \eta(t)\eta(t') \rangle = D \exp(-|t-t'|/\tau_D)$. $\xi(t)$ is a Gaussian white noise with zero mean $\langle \xi(t) \rangle = 0$ and autocorrelation function $\langle \xi(t)\xi(t') \rangle = \delta(t-t')$. Within the framework of the Ito calculus, the general solution of equation (10) leads to the following expression for the stochastic evolution of the amplitude of the electric field:

$$F(t) = F_0 + \eta(0)e^{-t/\tau_D} + \sqrt{\frac{2D}{\tau_D}} \int_0^t e^{-\frac{t-t'}{\tau_D}} dW(t'),$$

(11)

where the initial condition is $\eta(0) = 0$, and $W(t)$ is the Wiener process [33].

In a practical system, $\eta(t)$ could be generated by a $RC$ circuit driven by a source of Gaussian white noise, with correlation time $\tau_D = (RC)^{-1}$ (see equation (10)). The Gaussian white noise can be generated either by the Zener breakdown phenomenon in a diode or in a reversely polarized base–collector junction of a bipolar junction transistor, by amplifying the thermal noise in a resistor [34]. The correlation time $\tau_D$ is tunable by using a diode (varicap) with a voltage-dependent variable capacitance; the noise intensity $D$ can be chosen, for example, by suitably amplifying the noise produced through the Zener stochastic process.

The Monte Carlo code used here follows the procedure described in [28]. The spin polarization vector has been incorporated into the algorithm as an additional quantity and calculated for each free carrier. Furthermore, the electron–electron scattering is included, as extensively described in [29]. All simulations were performed in a GaAs crystal with a free electron concentration equal to $10^{13}$ cm$^{-3}$ and lattice temperature $T_L$ of 77 K. The temporal step is $10 \text{ fs}$ and an ensemble of $5 \times 10^4$ electrons is used to collect spin statistics. Moreover, we assume that all donors are ionized and that the free electron concentration is equal to the doping concentration. All physical quantities of interest are calculated after a transient time of typically $10^4 \text{ fs}$ time steps, long enough to achieve the steady-state transport regime. The spin relaxation simulation starts with all electrons of the ensemble initially polarized ($\langle S_z \rangle = 1$) along the $\hat{x}$-axis of the crystal, at the injection plane ($x_0 = 0$) [5, 3]. The spin lifetime $\tau$ and the spin depolarization length $L$ are calculated by extracting, respectively, the time and the distance from the injection plane of the center of mass of the electron ensemble, corresponding to a reduction of the initial spin polarization by a factor $1/e$.

In figure 1, we show the electron spin average polarization $\langle S_x \rangle$ as a function of the distance traveled by the center of mass of the electron cloud from the injection point, in the presence of a fluctuating field characterized by a deterministic component with amplitude $F_0$ and a random component with standard deviation $D^{1/2}$, for different values of noise correlation time $\tau_D$: $10^{-3}\tau_D$, $10^{-4}\tau_D$, $10^2\tau_D$, $10^3\tau_D$, and in the absence of noise. (a) $F_0 = 1 \text{ kV cm}^{-1}$, $D^{1/2} = 0.6 \text{ kV cm}^{-1}$ and (b) $F_0 = 6 \text{ kV cm}^{-1}$, $D^{1/2} = 3.6 \text{ kV cm}^{-1}$.

![Figure 1. Spin polarization ($S_x$) as a function of the distance from the injection point ($x = 0$) obtained by applying a fluctuating field characterized by a deterministic component $F_0$ and a random component with standard deviation $D^{1/2}$, for different values of noise correlation time $\tau_D$: $10^{-3}\tau_D$, $10^{-4}\tau_D$, $10^2\tau_D$, $10^3\tau_D$, and in the absence of noise. (a) $F_0 = 1 \text{ kV cm}^{-1}$, $D^{1/2} = 0.6 \text{ kV cm}^{-1}$ and (b) $F_0 = 6 \text{ kV cm}^{-1}$, $D^{1/2} = 3.6 \text{ kV cm}^{-1}$.](image)
In panel (a) of figure 2, we show the ratio between the spin depolarization length $L$ in the presence of noise and $L_0$, obtained in the absence of noise, as a function of the ratio between the noise correlation time $\tau_D$ and $\tau_0$, at different values of noise intensity $D$. For these parameter values, $L_0 = 32.6 \, \mu m$ and $\tau_0 = 0.16 \, ns$. The addition of a source of correlated fluctuations, characterized by $10^{-2} \tau_0 < \tau_D < \tau_0$, reduces the values of the spin depolarization length $L$ by up to 25%. In particular, $L/L_0$ is a nonmonotonic function of $\tau_D/\tau_0$ which exhibits a minimum for $\tau_D/\tau_0 \approx 0.1$. For both $\tau_D < \tau_0$ and $\tau_D > \tau_0$, the values of $L$ coincide with those of $L_0$. The presence of the minimum, which becomes deeper with increasing noise amplitude, can be explained by analyzing the temporal evolution of the quantities related to the electron transport and to the spin relaxation process. In panels (b) and (c) of figure 2, we report the electric field amplitude $F(t)$ and the squared precession frequency $|\Omega(k)|^2$, respectively, as a function of time $t$. For very low values of $\tau_D/\tau_0$, $|\Omega(k)|^2$ symmetrically fluctuates around its average value, corresponding to that obtained in the absence of noise. By increasing the value of $\tau_D$, the effective electric field felt by electrons, within a time window comparable to the spin relaxation time, becomes very different from the value $F_0$ (panel (b)). As a consequence, the temporal evolution of $|\Omega(k)|^2$ shows an evident asymmetry in the same temporal window (panel (c)). Because of the proportionality between the electron momentum $k_z$ and the electric field $F(t)$, equation (2) leads to a quadratic relation between $F(t)$ and $|\Omega(k)|^2$ on the $k_z^2$ term and at fourth power on the other two terms. Hence, the values of $F$ greater than $F_0$ give rise to values of $|\Omega(k)|^2$ much greater than those obtained for $F < F_0$. So, in accordance with equation (7), the asymmetry of $|\Omega(k)|^2$ is responsible for the observed reduction in spin lifetime. By further increasing the value of $\tau_D$, the random fluctuating term $\eta(t)$ of the electric field tends to its initial value $\eta(0) = 0$ (see equation (10)), and $F(t) \rightarrow F_0$. Therefore, the behavior of system becomes quasi-deterministic and the spin dephasing length $L$ approaches its deterministic value $L_0$.

In figure 3(a), we show the ratio $L/L_0$ as a function of $\tau_D/\tau_0$ for $F_0 = 6 \, kV \, cm^{-1}$, at different values of noise intensity $D$. For these parameter values, $L_0 = 279 \, nm$ and $\tau_0 = 1.13 \, ps$. In the presence of a driving electric field greater than the necessary static field to allow the electrons to move toward the upper energy valleys, the Gunn field $E_G = 3.25 \, kV \, cm^{-1}$, we find a positive effect of the field fluctuations. In fact, despite the error bars being large, our findings show that the addition of correlated fluctuations, characterized by $10^{-1} \tau_0 < \tau_D < \tau_0$, can increase the value of the spin depolarization length $L$ up to 20% of $L_0$. This effect is maximum for $\tau_D/\tau_0 \approx 1$. For the reasons discussed above, even in the high field case, both for very high and

Figure 2. (a) Ratio between the spin depolarization length $L$ in the presence of noise and $L_0$, obtained in the absence of noise, as a function of the ratio between the noise correlation time $\tau_D$ and the spin relaxation time $\tau_0$, at different values of noise intensity $D$. (b) Electric field amplitude $F(t)$ and (c) squared precession frequency $|\Omega(k)|^2$ as a function of time $t$. $F_0 = 1 \, kV \, cm^{-1}$. 

Figure 3. (a) Ratio between the spin depolarization length $L$ in the presence of noise and $L_0$, obtained in the absence of noise, as a function of the ratio between the noise correlation time $\tau_D$ and the spin relaxation time $\tau_0$, at different values of noise intensity $D$. (b) Electric field amplitude $F(t)$ and (c) squared precession frequency $|\Omega(k)|^2$ as a function of time $t$. $F_0 = 1 \, kV \, cm^{-1}$. 

In figure 2(a), we show the ratio between the spin depolarization length $L$ in the presence of noise and $L_0$, obtained in the absence of noise, as a function of the ratio between the noise correlation time $\tau_D$ and $\tau_0$, at different values of noise intensity $D$. Because of the

Figure 2. (a) Ratio between the spin depolarization length $L$ in the presence of noise and $L_0$, obtained in the absence of noise, as a function of the ratio between the noise correlation time $\tau_D$ and the spin relaxation time $\tau_0$, at different values of noise intensity $D$. (b) Electric field amplitude $F(t)$ and (c) squared precession frequency $|\Omega(k)|^2$ as a function of time $t$. $F_0 = 1 \, kV \, cm^{-1}$. 

In figure 2(a), we show the ratio between the spin depolarization length $L$ in the presence of noise and $L_0$, obtained in the absence of noise, as a function of the ratio between the noise correlation time $\tau_D$ and $\tau_0$, at different values of noise intensity $D$. Because of the
very low values of noise correlation time τ₀, the value of L approaches L₀. The presence of a positive effect of noise can be ascribed to the reduction of the electron occupation percentage in L-valleys, shown in panel (b) of figure 3. This finding, which can be considered as a further example of noise enhanced stability (NES) [13, 35, 36], leads a greater number of electrons to experience a spin–orbit coupling in the L-valleys at least of one order of magnitude weaker than that present in the L-valleys [26], causing a decrease in the efficacy of the D’yakonov–Perel dephasing mechanism.

Although for many decades it has been believed that the electron–electron (e–e) Coulomb scattering does not contribute to the spin relaxation/dephasing process, in the presence of inhomogeneous broadening in spin precession, each kind of scattering, including spin-conserving scattering, can cause irreversible spin dephasing. Moreover, the Coulomb interaction is fundamental to obtain the correct distribution of the electrons in k-space [3, 32]. Recently, it has been shown that the inclusion of e–e scattering leads to a strong increase in spin relaxation time in bulk semiconductors, in a wide range of electric field amplitudes, lattice temperatures and doping densities [29].

In order to quantify the effect of the Coulomb interaction on the spin depolarization process in the presence of external noise, we show a comparison between the results obtained with the e–e scattering mechanism and without it, analyzing the behavior of L/L₀ as a function of the ratio between the noise standard deviation D¹/² and the deterministic value of the driving field F₀ (see figure 4). We show the comparison at the values of noise correlation time corresponding with the maximum of the noise effect observed by neglecting the Coulomb interaction. Each panel of figure 4 shows the ‘ee-points’, obtained through full calculations including the e–e scattering mechanism, and the ‘no ee-points’, calculated without e–e interaction. In panel (a) τ₀ = 0.1τ₀ and F₀ = 1 kV cm⁻¹; in panel (b) τ₀ = τ₀ and F₀ = 6 kV cm⁻¹.

Figure 3. (a) Ratio between the spin depolarization length L in the presence of noise and L₀, obtained in the absence of noise, as a function of the ratio between the noise correlation time τ₀ and the spin relaxation time in the absence of noise τ₀ and (b) the electron occupation percentage in L-valleys η₀ as a function of τ₀/τ₀, at different values of noise intensity D. F₀ = 6 kV cm⁻¹.

Figure 4. Ratio between the spin depolarization length L in the presence of noise and L₀, obtained in the absence of noise, as a function of the ratio between the noise amplitude D¹/² and F₀. The ee-points are obtained through full calculations including the e–e scattering mechanism; the no ee-points are calculated without it. (a) τ₀ = 0.1τ₀ and F₀ = 1 kV cm⁻¹, (b) τ₀ = τ₀ and F₀ = 6 kV cm⁻¹.
slight noise-induced positive effect on spin relaxation length is found. In this case, the addition of a source of correlated fluctuations, having correlation time comparable with the spin lifetime, enhances the value of the spin depolarization length $L$ of only about 10–15%.

In this work, for the first time, we have investigated the influence of noise on the electron spin relaxation process in lightly n-doped GaAs semiconductor bulk by also including the e–e interaction. The findings show that a fluctuating electric field, obtained by adding a correlated source of noise to a static field, can modify the spin depolarization length. For electric fields lower than the Gunn field and values of the noise correlation time $\tau_D \sim \tau_0$, the spin lifetime obtained in the absence of noise, a reduction of the spin depolarization length up to 15% has been observed that is strongly dependent on both the strength of the applied electric field, obtained by adding a correlated source of noise to a static field, can modify the spin depolarization length.

To conclude, our preliminary results show that the presence of fluctuations in applied voltages changes the maintenance of long spin depolarization lengths in a way that is strongly dependent on both the strength of the applied electric field and the noise correlation time. Further studies are needed to learn more about the relationship between the semiconductor characteristic time scales, the noise correlation time and the e–e interaction, in order to find the most favorable conditions for the manipulation of electron spins.

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