On the Runtime-Efficacy Trade-off of Anomaly Detection Techniques for Real-Time Streaming Data

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ABSTRACT

Ever growing volume and velocity of data coupled with decreasing attention span of end users underscore the critical need for real-time analytics. In this regard, anomaly detection plays a key role as an application as well as a means to verify data fidelity. Although the subject of anomaly detection has been researched for over 100 years in a multitude of disciplines such as, but not limited to, astronomy, statistics, manufacturing, econometrics, marketing, most of the existing techniques cannot be used as is on real-time data streams. Further, the lack of characterization of performance – both with respect to real-time and accuracy – on production data sets makes model selection very challenging.

To this end, we present an in-depth analysis, geared towards real-time streaming data, of anomaly detection techniques. Given the requirements with respect to real-time and accuracy, the analysis presented in this paper should serve as a guide for selection of the "best" anomaly detection technique. To the best of our knowledge, this is the first characterization of anomaly detection techniques proposed in very diverse set of fields, using production data sets corresponding to a wide set of application domains.

CCS CONCEPTS

- Computing methodologies → Machine learning algorithms;
- Computer systems organization → Real-time systems;

KEYWORDS

Stream Mining, Anomaly Detection, Time Series, Machine Learning, Pattern Mining, Clustering

1 INTRODUCTION

Advances in technology – such as, but not limited to, decreasing form factor, network improvements and the growth of applications, such as location-based services, virtual reality (VR) and augmented reality (AR) – combined with fashion to match personal styles has fueled the growth of Internet of Things (IoT). Example IoT devices include smart watches, smart glasses, heads-up displays (HUDs), health and fitness trackers, health monitors, wearable scanners and navigation devices, connected vehicles, drones et cetera. In a recent report [27], Cisco projected that, by 2021, there will be 929 M wearable devices globally, growing nearly 3× from 325 M in 2016 at a CAGR of 23%.

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In this section we define the terms used in the rest of the paper.

Definition 2.1. Point Anomalies: are data points which deviate so much from other data points so as to arouse suspicions that it was generated by a different mechanism [48].

Definition 2.2. Pattern Anomalies: Continuous set of data points that are collectively anomalous even though the individual points may or may not be point anomalies.

Definition 2.3. Change Detection: This corresponds to a permanent change in the structure of a time series, e.g., change in the mean level (Level Shift), change in the amplitude of seasonality (Seasonal Level Shift) or change in the noise amplitude (Variance Change).

Table 1: Classification of anomaly detection techniques. P: Parametric technique, N-P: Non-parametric technique, Pt: Point anomalies, Pa: Pattern anomalies, Inc.: Incremental technique, Robust: Robustness to noise, Recency: Ability to weigh observations by age, TG: Time Granularity of a data stream that can use the method, CFAR: Constant False Alarm Rate.

| Domain          | Technique                      | Summary                                                                 | P/N-P | Inc. | Robust | Recency | TG | CFAR |
|-----------------|--------------------------------|-------------------------------------------------------------------------|-------|------|--------|---------|----|------|
| Statistics      | Mu-Sigma [8, 73, 85, 92, 99, 106, 107] | Thresholds based on mean and standard deviation                          | ✓     | ✓    | ✓      | ✓       | 1sec |      |
|                 | Med-Mad [10]                     | Thresholds based on median and median absolute deviation                 | ✓     | ✓    | ✓      | ✓       | 1sec |      |
|                 | Generalized tSD [62, 83, 90, 102] | Uses Student t-distribution to calculate a max number of outliers         | ✓     | ✓    | ✓      | ✓       | 100msec |      |
|                 | E-M Estimator [7, 80, 89, 113]    | Measure of Spread with better Gaussian efficiency than MAD               | ✓     | ✓    | ✓      | ✓       | 100msec |      |
|                 | Huber M-Estimator [74, 80]        | Huber’s M-estimator                                                     | ✓     | ✓    | ✓      | ✓       | 100msec |      |
|                 | t-digest [36, 32, 63]            | Streaming percentile based detection                                      |✓     | ✓    | ✓      | ✓       | 100msec |      |
|                 | Adalyn Flite [75, 106]            | Adjusted whiskers for box plots                                          | ✓     | ✓    | ✓      | ✓       | 100msec |      |
| Time Series Analysis | STL [39, 46, 64, 102]            | Seasonality Decomposition                                               | N-P   | ✓    | ✓      | ✓       | 100msec |      |
|                 | SARIMA [79, 90, 91, 98, 96]       | Seasonal Auto Regression Moving Average (ARIMA)                         | ✓     | ✓    | ✓      | ✓       | 1sec  |      |
|                 | STL-ARMA [70, 79, 84, 94, 96]     | STL-ARMA on residual                                                    | ✓     | ✓    | ✓      | ✓       | 100msec |      |
|                 | STL-RobustRE [5, 97]              | ARIMA with Robust Kaiman                                                | ✓     | ✓    | ✓      | ✓       | 100msec |      |
|                 | SDAR [101, 108, 109]              | Sequential Discounting AR                                               | ✓     | ✓    | ✓      | ✓       | 100msec |      |
|                 | RobustOutliers [14, 33, 60, 75]   | Intervention Analysis with ARMA                                        | ✓     | ✓    | ✓      | ✓       | 10sec  |      |
|                 | THATS [59, 84, 97, 92, 73]        | Exponential Smoothing with Fourier terms for Seasonality                | ✓     | ✓    | ✓      | ✓       | 1sec  |      |
| Pattern Mining  | HOTSAK [63, 75]                   | Pattern Distance based on SAX                                           | N-P   | ✓    | ✓      | ✓       | 1sec  |      |
|                 | RRA [59, 91]                      | Rare Rule Anomaly based on Grammar Induction                            | N-P   | ✓    | ✓      | ✓       | 1sec  |      |
|                 | DensStream [22]                   | Online Density Micro-Clustering                                        | N-P   | ✓    | ✓      | ✓       | 20msec |      |
|                 | CluStree [84]                     | Hierarchical Micro-Clustering                                          | N-P   | ✓    | ✓      | ✓       | 100msec |      |
|                 | THFab [34, 46, 57, 103]           | Incremental Shared Density Based clustering                            | N-P   | ✓    | ✓      | ✓       | 10msec |      |
| Machine Learning| MBS-means [11, 103]               | Mini-batch clustering with k-means                                       | N-P   | ✓    | ✓      | ✓       | 10msec |      |
|                 | PICS [12, 55, 59, 73]             | Principal Components Analysis                                          | ✓     | ✓    | ✓      | ✓       | 1sec  |      |
|                 | RobustPCA [21, 105]               | Low Rank Approximation                                                  | ✓     | ✓    | ✓      | ✓       | 1sec  |      |
|                 | HForest [84, 11, 96]              | Isolation Forests                                                       | N-P   | ✓    | ✓      | ✓       | 100msec |      |
|                 | OneLayerSVM [12, 89]              | One Layer SVM                                                           | ✓     | ✓    | ✓      | ✓       | 1sec  |      |

We present a map of the accuracy-runtime trade-off for the anomaly detection techniques.

We present detailed insights into the performance – as measured by precision, recall and F1 score – of the anomaly detection techniques listed in Table 1. Specifically, we present a deep dive view into the behavior subject to the following:

- Trend and level shifts
- Change in variance
- Change in seasonal level
- Change in seasonality period

For the sake of brevity, we do not list the full names of the techniques. However, the readers can refer to the full list in Table 1 for more details.

Further, other characteristics of IoT devices such as, but not limited to, small storage, small power budgets, consumption, etc., limit the use of off-the-shelf anomaly detection techniques. Last but not least, constantly evolving nature of data streams in the wild call for support for continuous learning.

As overviewed in [3], anomaly detection has been researched in a wide variety of disciplines, for example, but not limited to, operations, computer vision, networking, marketing, and social media. Unfortunately, there does not exist a characterization of the performance of anomaly detection techniques – both with respect to real-timeliness and accuracy – on production data sets. This in turn makes model selection very challenging. To this end, in this paper, we present an in-depth analysis, geared towards real-time streaming data, of a large suite of anomaly detection techniques. In particular, the main contributions of the paper are as follows:

- Lack of support for constant false alarm rate
- Lack of scalability

We present a classification of over 20 (!) anomaly detection techniques across seven dimensions (refer to Table 1).

As a first, using over 25 (!) real-world data sets and real hardware, we present a detailed evaluation of the real-time performance of the anomaly detection techniques listed in Table 1. It is important to note that the evaluation was carried out in an unsupervised setting. In other words, irrespective of the availability of labels, a model was not trained a priori.

We present detailed insights into the performance – as measured by precision, recall and F1 score – of the anomaly detection techniques listed in Table 1. Specifically, we present a deep dive view into the behavior subject to the following:

- Trend and level shifts
- Change in variance
- Change in seasonal level
- Change in seasonality period

Given the requirements with respect to real-timeliness and accuracy, we believe that the analysis presented in this paper should serve as a guide for selection of the "best" anomaly detection technique.

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3 BACKGROUND

As mentioned earlier in Section 1, the subject of anomaly detection has been researched for over 100 years [3]. A detailed walkthrough of prior work is beyond the scope of this paper (the reader is referred to the books [7, 14, 48, 88] or surveys [17, 24, 26, 40, 45, 79, 114] written on the subject). In the section, we present a brief overview of the techniques listed in Table 1.

3.1 Statistics

In this subsection we briefly overview the common statistical techniques used for anomaly detection.

3.1.1 Parametric Approaches. One of the most commonly used rule to detect anomalies — popularly referred to as the $\mu \pm 3\cdot \sigma$ rule — whereby, observations that lie 3 or more deviations ($\sigma$) away from the mean ($\mu$) are classified as anomalies. The rule is based on the following two assumptions: (a) the underlying data distribution is normal and (b) the time series is stationary. In practice, production time series often do not satisfy the above, which results in false positives. Further, both $\mu$ and $\sigma$ are not robust against the presence of anomalies. To this end, several robust estimators have been proposed. Specifically, Huber M-estimator [54] is commonly used as a robust estimate of location, whereas median, $\tau$ estimator [111] and Median Absolute Deviation (MAD) are commonly used as robust estimates of scatter.

In the presence of heavy tails in the data, $t$-distribution [112] is often used as an alternative to the normal distribution. The Generalized Extreme Studentized Deviate (ESD) test [86] uses the $t$-distribution to detect outliers in a sample, by carrying out hypothesis tests iteratively. GESD requires an upper bound on the number of anomalies, which helps to contain the false alarm rate (FAR).

3.1.2 Non-parametric Approaches. It is routine to observe production data to exhibit, for example but not limited to, skewed and multi-modal distribution. For finding anomalies in such cases, several non-parametric approaches have been proposed over the years. For instance, $t$-digest [32] builds an empirical cumulative density function (CDF), using adaptive bin sizes, in a streaming fashion. Maximum bin size is determined based on the quantile of the value $\text{max}(1, 4\cdot N\cdot 6\cdot (1 - q))$, where $q$ is the quantile and $\delta$ is a compression factor that controls the space requirements. In a similar vein, adjusted Boxplots [104] have been proposed to identify anomalies in skewed distributions. For this, it uses a robust measure of the skew called medcouple [19].

3.2 Time Series Analysis

Observations in a data streams exhibit autocorrelation. Thus, prior to applying any anomaly detection technique, it is critical to weed out the autocorrelation. Auto Regressive Moving Average (ARMA) [18] models have been commonly used for analysis of stationary time series. ARMA models are formulated as State Space Models (SSM) [33], where one can employ Kalman Filters for model estimation and inference. Kalman filters (KF) [61] are first order Gaussian Markov Processes that provide fast and optimal inference for SSMs. KFs assume that the hidden and observed states are Gaussian processes. When that assumption fails, the estimates obtained via KFs can potentially be biased. Robust Kalman Filters (RobustKF) [99] treat the residual error as a statistical property of the process and down weight the impact of anomalies on the observed and hidden states.

Sequential Discounting AutoRegressive (SDAR) filters assign more weight to recent observations in order to adapt to non-stationary time series or change in dynamics of a system [108]. Further, a key feature of SDAR filters is that they update incrementally. A discount rate is specified to guide the rate of adaptation to changing dynamics.

3.3 Pattern Mining

Time series with irregular but self-similar patterns are difficult to model with parametric methods. Non-parametric data mining approaches that find anomalous patterns and/or subsequences have been proposed for such time series. SAX is a discretization technique that transforms a series from real valued domain to a string defined over a finite alphabet $\mathcal{F}$ of size $a$ [63]. It divides the real number scale into equal probabilistic bins based on the normal model and assigns a unique letter from $\mathcal{F}$ to every bin. Before discretization, SAX z-normalizes the time series to map it to a probabilistic scale. It then forms words from consecutive observations that fall into a sliding window. The time series can now be represented as a document of words. SAX employs a dimensionality reduction technique called Piecewise Aggregate Approximation (PAA) which chunks the time series into equal parts and computes the average for each part. The reduced series is then discretized for further processing.

The key advantage of SAX over other discretization heuristics [20, 38] is that the distance between two subsequences in SAX lower bounds the distance measure on the original series. This allows SAX to be used in distance based anomaly detection techniques. For example, HOTSAX [63] uses SAX to find the top-k discords.

Another method that leverages SAX is the Rare Rule Anomaly (RRA) technique [90]. RRA induces a context free grammar from the data. The grammar induction process compresses the input sequence by learning hierarchical grammar rules. The inability of compressing a subsequence is indicative of the Kolmogorov randomness of the sequence and hence, can be treated as being an anomaly. RRA uses the distance to the closest non-self match subsequence as the anomaly score.

3.4 Machine Learning

Machine Learning approaches such as clustering, random forests, and deep learning are very effective in modeling complex time
series patterns. Having said that, the training time is usually very high and many of these techniques are not incremental in nature. Thus, most of these techniques work in batch mode where training is performed periodically.

Isolation forests [71] is a tree based technique that randomly splits the data recursively in an attempt to isolate all observations into separate leaves. The number of splits needed to reach a data point from the root node is called the path length. Many such random split forests are constructed and the average path length to reach a point is used to compute the anomaly score. Anomalous observations are closer to the root node and hence have lower average path lengths.

One Label Support Vector Machines (SVMs) [89] are often used to construct a non-linear decision boundary around the normal instances, thereby isolating anomalous observation that lie away from the dense regions of the support vectors. A key advantage of this technique is that it can be used even with small number of data points, but are potentially slow to train.

**Clustering**

There are two main approaches to handle the stream clustering problem: Micro-Clustering (MC) and Mini-Batching (MB).

**Micro-Clustering**

A large number of techniques follow a 2-phase Micro-clustering approach [8] which has both an online as well as an offline component. The online component is a dimensionality reduction step that computes the summary statistics for observations very close to each other (called micro-clusters). The offline phase is a traditional clustering technique that ingests a set of MCs to output the final clustering, which can be used to identify anomalous MCs.

**DenStream** is a density-based streaming technique which uses the MC paradigm [22]. In the online phase, it creates two kinds of MCs: potential micro-clusters ($p_{MC}$) and anomalous micro-clusters ($o_{MC}$). Each cluster maintains a weight $w$ which is an exponential function of the age of the observations in the cluster. $w$ is updated periodically for all clusters to reflect the aging of the observations. If $w$ is above a threshold ($\alpha$), it is deemed as a core micro-cluster. If $w$ is more than $\beta_{st}$ (where $0<\beta<1$), the cluster is deemed as a $p_{MC}$, otherwise it is deemed a $o_{MC}$. When a new observation arrives, the technique looks for a $p_{MC}$ that can absorb it. If no $p_{MC}$ is found, it looks for an $o_{MC}$ that can absorb the observation. If no $o_{MC}$ is found, a new $o_{MC}$ is instantiated. Older clusters are periodically removed as their $w$ become smaller.

**DBStream** is an incremental clustering technique that decays MCs, akin to DenStream. It also keeps track of the shared density between MCs. During the offline phase, it leverages this shared density for a DBSCAN-style clustering to identify anomalous MCs.

Hierarchical clustering techniques such as Clustree use the MC paradigm to construct a hierarchical height balanced tree of MCs [64]. MC corresponding to an inner node is an aggregation of the clusters of all its children. A key advantage of these techniques is their being incremental; having said that, the data structures can grow balloon in size. For anomaly detection, the distance of a new observation from the closest leaf MC is used as the anomaly score.

**Mini-Batching**

The second approach for a data stream clustering entails batch clustering of a sample generated from the data stream. These samples are significantly larger than the micro-clusters and hence the name mini-batch. An example technique of this kind is the mini-batch $k$-means which uses cluster centroids from the previous clustering step to reduce the convergence time significantly [35].

3.5 Potpourri

Seasonality in time series is commonly observed in production data. Filtering of the seasonal component is critical for effective anomaly detection. In this regard, a key challenge is how to determine the seasonal period. For this, a widely used approach is to detect strong peaks in the auto-correlation function coupled with statistical tests for the strength of the seasonality.

Seasonal-Trend-Loess (STL) [28] is one the most commonly used techniques for removing the seasonal and trend components. STL uses LOESS [29] to smooth the seasonal and trend components. Presence of anomalies can potentially induce distortions in trend which in turn can result in false positives. To alleviate this, Robust STL iteratively estimates the weights of the residual -- the number of robustness iterations is an input manual parameter. The downside of this is that the robustness iterations slow down the run time performance. Vallis et al. [103] proposed the use of piecewise medians of the trend or quantile regression – this is significantly faster than using the robustness iterations. Although STL is effective when seasonality is fixed over time, moderate changes to seasonality can be handled by choosing a lower value for the “s.window” parameter in the STL implementation of R.

Seasonal ARMA (SARMA) models handle seasonality via the use of seasonal lag terms [33]. A key advantage with the use of SARMA models is the support for change in seasonality parameters over time. However, SARMA is not robust to anomalies. This can be addressed via the use of robust filters [25]. Note that the estimation of extra parameters increases the relative model estimation time considerably. SARMA models can also handle multiple seasonalities at the expense of complexity and runtime. Akin to SARMA, TBATS is an exponential smoothing model which handles seasonality using Fourier impulse terms [72].

Principal Components Analysis (PCA) is another common method used to extract seasonality. However, the use of PCA requires the data to be represented as a matrix. One way to achieve this is to fold a time series along its seasonal boundaries so as to build a rectangular matrix ($M$) where each row is a complete season. Note that PCA is not robust to anomalies as it uses the covariance matrix which is non-robust measure of scatter. To alleviate this, Candes et al. proposed Robust PCA (RPCA) [21], by decomposing the matrix $M$ into a low rank component($L$) and a sparse component($S$) by minimizing the nuclear-norm of $M$.

4 EXPERIMENTAL SETUP

In this section, we detail the data sets used for evaluation, the underlying methodology and walk through any transformation and/or tuning needed to ensure a fair comparative analysis.

4.1 Data Sets

Table 2 details the data sets used for evaluation. Note that the data sets belong to a diverse set of domains. The diversity of the data sets is reflected based on the following attributes:

- **Seasonality Period (SP):** Most of these time series exhibit seasonal behavior and the SP increases with TG. For instance, the minutely time series of operations data experience daily
### Table 2: Anomaly detection datasets. Len: Average length of the time series, WinSize: Window Size, PL: Pattern Length, Cnt: Number of Series, TG: Time Granularity, SP: Seasonal Period, SJ: Seasonal period Jitter, LS: Level Shift, VC: Variance Change, SLD: Seasonal Level Shift

| Domain | Description | Acronym | Len | WinSize | PL | Cnt | TG | Labels | SP | SJ | LS | VC | SLD | SLS |
|--------|-------------|---------|-----|---------|----|-----|----|--------|----|----|----|----|-----|-----|
| NAB [67] | NAB Advertising Click Rates | nab-ctr | 1200 | 1440 | 60 | 1 | min | Manual | ✓ | ✓ | ✓ | ✓ |
| NAB | NAB Tweet Volumes | nab-twt | 3000 | 2880 | 100 | 1 | sec | ✓ | 20000 | ✓ | ✓ | ✓ | ✓ |
| NAB Ambient Temperature | nab-iot01 | 7200, 22609 | 600 | 20 | 1 | sec | ✓ | 120000 | ✓ | ✓ | ✓ | ✓ | ✓ |
| YAD [66] | Real operations series | iot-fridge01 | 1800 | 720 | 10 | 1hr | Manual | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ |
| | Synthetic operations series | iot-fridge02 | 1800 | 720 | 10 | 1hr | Manual | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ |
| HFT [4] | Facebook Trades Dec. 2016 | fb-trd | 35758 | 10000 | 60 | 1 | sec | Manual | ✓ | ✓ | ✓ | ✓ |
| | Twitter Trades Dec. 2016 | twitter-trd | 17738 | 10000 | 60 | 1 | sec | Manual | ✓ | ✓ | ✓ | ✓ |
| | SPY Trades Dec. 2016 | spy-trd | 39274 | 10000 | 60 | 1 | sec | Manual | ✓ | ✓ | ✓ | ✓ |
| Ops. | Minutely Operations Data | ops | 124000 | 2000 | 60 | 1 | min | Manual | ✓ | ✓ | ✓ | ✓ |
| IoT [4] | Power Usage of Freezer | int-freezer01 | 43200 | 2382 | 1800 | 1 | sec | Manual | ✓ | ✓ | ✓ | ✓ |
| | Power Usage of Fridge | int-fridge01 | 43200 | 8190 | 1800 | 1 | sec | Manual | ✓ | ✓ | ✓ | ✓ |
| | Power Usage of Dishwasher | int-dishwasher01 | 43200 | 2000 | 1800 | 1 | sec | Manual | ✓ | ✓ | ✓ | ✓ |
| Health [62] | ECG Sleep Apnea [62] | health-apnea-ecg/a02 | 13000 | 2000 | 40 | 10 | minsec | ✓ | 100 | 90-110 | ✓ | ✓ | ✓ | ✓ |
| | ECG Premature Ventricular Contraction [63] | health-pvc-0066 | 12000 | 2000 | 40 | 4 | 4 sec | Manual | ✓ | ✓ | ✓ | ✓ |

**Figure 2:** Illustration of the characteristics of data sets from different domains

Health series have coarse TG but the seasonal periodicity are usually small (period=100 - 200).

- **Seasonal Jitter:** It refers to the presence of jitter in seasonal periods, and is an artifact of coarse TG, IoT and Health time series exhibit this property.

- **Non-stationary:** Time series exhibit one or more types of nonstationarities with respect to their level (amplitude) and variance.

Figure 2 illustrates characteristics of these time series. From the figure we note that most of the series are non-stationary and exhibit one of these types of nonstationarities:

- The Numenta Anomaly Benchmark (NAB) [67] and the Yahoo Anomaly Dataset (YAD) [66] – which are considered industry standard – provide labels. NAB itself is a collection of time series from multiple domains like Advertising Click Through Rates (nab-ctr), volume of tweets per hour (nab-twt), sensor data for temperature (nab-iot).

The NAB series used herein have hourly granularity. YAD is composed of three distinct data sets: yahoo-a1 comprises a set of operations time series with hourly granularity, whereas yahoo-a2 and yahoo-a3 are synthetic time series.

Anomalies detection in the context of high frequency trading (HFT) surfaces arbitrage opportunities and hence can have potentially large impact to the bottom line. In light of this, we included a month long time series of trades for the tickers FB, GOOG, NFLX and SPY. The time series, purchased from QuantQuote [4], are of secondarily granularity.

We collected 44 minutely time series of operations data from our production environment. The ECO dataset [15] comprises of secondarily time series of power usage. Given the increasing emphasis on the Runtime-Efficacy Trade-off of Anomaly Detection Techniques for Real-Time Streaming Data.
on healthcare apps owing to the use IoT devices in Healthcare domain, we included seven data sets from Physiobank [41].

4.2 Methodology
To emulate unbounded evolutionary data streams, we chose long time series and applied anomaly detection techniques for every new data point in a streaming fashion. Further, we limit the number of data points a technique can use to determine whether the most recent observation is anomalous. A common way of doing this is to define a maximum Window Size that can be used for training or modeling.

Window size is an important hyper-parameter which has a direct correlation with runtime and accuracy. Longer windows require more processing time, while shorter windows result in drop in accuracy. For a fair comparative analysis, we set an upper limit on the values of window size for different data sets, as listed in Table 2. The values were set based on the data set, e.g., for the minutely operations time series (seasonal period=1440), one would need at least 10 periods [68] to capture the variance in the seasonal patterns, giving a seasonal period of 14400. This is the maximum allowed value; techniques may choose a shorter seasonal period depending on their requirements. For instance, TBATS needs fewer number of periods to learn and hence, we used only 5 periods (WinSize=7200) in the experiments. Data sets such as YAD have fixed seasonal periods due to hourly TG, and hence, require much smaller window sizes to achieve maximum accuracy. IoT data sets have the longest window sizes due to the long seasonal periods.

Pattern Length (PL) is a hyper-parameter of all pattern mining techniques. The value of PL is dependent on the application and the type of anomalies. For example, IoT workloads require a PL of ≈ 30 minutes, whereas Health time series usually require a PL of only 40. A moving window equal to the PL is used to extract subsequences from the time series. Although pattern anomalies are typically not of fixed length in production data, most techniques require a fixed length to transform the series into subsequences. To alleviate this, post-processing can be used to string together multiple length anomalies [90].

In order to characterize the runtime performance of the technique listed in Table 1, we measured the runtime needed to process every new point in a time series. For every new data point in a time series, each technique was run 10 times and the median runtime was recorded. Across a single time series, the run-times for all the data points in the time series were averaged. Across multiple time series in a group, geometric mean of the run-times1 of the individual series is used to represent the runtime for the group. Given that some of the data sets listed in Table 2 have short time series (e.g., NAB and YAD), we replicate these series 10 times to increase their length. All run-times are reported in milliseconds (mssec).

4.2.1 Metrics. Accuracy for labeled data sets is calculated in terms of the correctly identified anomalies called True Positives (TP), falsely detected anomalies called False Positives (FP) and missed anomalies called False Negatives (FN). To measure accuracy of a single time series, we use the following three metrics:

- **Precision**: defined as the ratio of true positives (TP) to the total detected anomalies: \( Pr = \frac{TP}{TP+FP} \)
- **Recall**: defined as the ratio of true positives (TP) to the total labeled anomalies: \( Re = \frac{TP}{TP+FN} \)
- **F1-score**: defined as the weighted harmonic mean of Precision and Recall: \( F_1 = \frac{1 + \beta^2}{\frac{\beta^2}{Pr} + \frac{1}{Re}} \) where \( \beta \) is a constant that weights precision vs recall based on the application

In most applications, it is common to set \( \beta=1 \), giving equal weightage to precision and recall. But for healthcare, recall is sometimes more important than precision and hence \( \beta=2 \) is often used [69]. This is because false negatives can be catastrophic. To calculate accuracy across a group of time series, we report the micro-average \( F_1 \)-score [36], which calculates precision and recall across the whole group. The use of this is subject to time series in a group being similar to each other.

Most of our data sets are labeled with point anomalies. In light of this, we propose the following methodology to compute accuracy for detected patterns against labeled point anomalies. Let \( Y_1, Y_2, Y_3, ..., Y_p, Y_{p+1} \) denote a time series and the pattern \( Y_2-Y_p \) be detected as a pattern anomaly. Let \( Y_{p-1} \) be a true anomaly. A naive way to compute a TP is to have a pattern anomaly end at the true anomaly. In this case, \( Y_2-Y_p \) would be considered a FP. In contrast, TP can correspond to an instance where a true anomaly occurs anywhere inside the boundary of an anomalous pattern. Pattern anomalies are very often closely spaced due to the property of neighborhood similarity as described by Handschin and Mayne [63]. Given this, it is important to count each true anomaly only once if multiple overlapping pattern anomalies are detected. A post-processing pass can help weed out such overlapping subsequences.

Given that the methodology of calculating accuracy is so different for pattern techniques, we advise the reader to only compare accuracies of pattern techniques with other pattern techniques.

4.2.2 System Configuration. Table 3 details the hardware and software system configuration. Table 4 details the R and Python packages used. For HOTSSAX and RRA, we modified the implementation so as to make them amenable to a streaming data context.

| Architecture | Intel(R) Xeon(R) | Frequency | 2.40GHz |
|--------------|-----------------|-----------|---------|
| Num Cores    | 24              | Memory    | 128GB   |
| L1d cache    | 32K             | L1i cache | 32K     |
| L2 cache     | 256K            | L3 cache  | 15360K  |
| OS Version   | CentOS Linux release 7.1.1503 | R, Python | 3.2.4, 2.7 |

Table 3: System Configuration

| R            | MASS | rrcover | jmotif |
|--------------|------|---------|-------|
| stream, streamMOA |      |         |       |

| Python       | pykalman | scikit-learn | tidigest | statsmodels |
|--------------|----------|--------------|----------|-------------|

Table 4: Packages and Libraries

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1 This approach is an industry standard as evidenced by its use by SPEC [1] for over 20 years.
4.3 Hyper-parameters
In the interest of reproducibility, we detail the hyper-parameters used for the techniques listed in Table 1. In addition, we detail any transformation and/or tuning needed to ensure a fair comparative analysis.

4.3.1 Statistical techniques. For parametric statistical techniques such as mu-sigma, med-mad et cetera, we set the threshold to $3 \cdot \sigma$ or its equivalent robust estimate of scale.

A constant false alarm rate can be set for techniques such as t-digest and GESD. In case of the former, we set the threshold at 99.73th percentile which is equivalent to $3 \cdot \sigma$. In case of the latter, one can set an upper limit on the number of anomalies. Based on our experiments, we set this parameter to 0.02 for all, but the Health data sets. The parameter was set to 0.04 for the the Health data sets to improve recall at the expense of precision.

Model parameter estimates are computed at each new time step over the moving window. As these techniques do not handle seasonality or trend, we removed seasonality and trend using STL and evaluate these techniques on the residual of the STL. In light of this, i.e., statistical techniques are not applicable to the raw time series in the presence of seasonality or trend, the accuracy of these techniques should not be compared with ML and Pattern Mining based techniques.

4.3.2 Time series analysis techniques. Parametric time series analysis techniques such as TBATS, SARMA, STL-ARMA estimate model parameters and evaluate incoming data against the model. Retraining at every time step is often unnecessary as the temporal dynamics do not change at every point. In practice, it is common to retrain the model periodically and this retraining period is another hyper-parameter. This period depends on the application, but it should not be greater than the window size. We set the retraining period to be the same as the window size. We include the training runtime as part of the total runtime of a techniques so as to assess the total detection time for anomalies.

STL with default parameters assumes periodic series. To allow gradual seasonal level drift, we set the $stl$-periodic parameter equal to 21. For RobustSTL we use 4 robust iterations. SDAR is an incremental technique that requires a learning rate parameter ($\alpha$). Based on our experiments, we set $\alpha = 0.0001$. RobustKF is the robust kalman filter by Ting et al. [99] which requires parameters $\alpha$ and $\beta$ for the Gamma prior on the robustness weights. We set $\alpha = \beta = 1$.

We also evaluate techniques based on Intervention Analysis of time series implemented in the tsoutliers package of R. These techniques are significantly slower than most other techniques we evaluate in this work, e.g., for a series with 2k data points, it took over 5 minutes (!) for parameter estimation. Clearly, these techniques are non-viable for real-time streaming data.

4.3.3 Pattern mining and machine learning techniques. Most pattern techniques require pattern length (PL) as an input parameter. Table 5 lists the specific parameters and their respective values for each technique. HOTSAX and RRA are robust to presence of trend in a time series as they use symbolic approximation, but they do require the series to be studentized. All the other pattern techniques are not robust to presence of trend as they use the underlying real valued series directly. Thus, all the subsequences need to be mean adjusted (i.e., subtract the mean from all the data points) to avoid spurious anomalies due to changing trend. Scale normalization is not carried as change in scale is an anomaly itself.

Keogh et al. proposed a noise reduction technique wherein a subsequence is rejected if its variance lies below a threshold, $\epsilon$ [63]. Our experiments show that this preprocessing step is critical, from an accuracy standpoint, for all pattern mining techniques considerably. Therefore, we use it by default, with $\epsilon=0.01$. DenStream is the only MC technique that works without an explicit re-clustering step. The technique can classify outlier micro-clusters (oMC) as anomalous, but this leads to a higher false positive rate. Alternatively, one can take the distance of the points to the nearest pMC as the strength of the anomaly. This makes DenStream less sensitive to the MC radius $\epsilon$ as well. DenStream and DBStream are incremental techniques. Hence, they do not need an explicit window size parameter, having said that, they use a decay constant $\lambda$ to discount older samples. IForest and OneSVM are not incremental techniques and hence, need to be retrained for each time step. MBMeans is also not incremental, however, it uses cluster centroids from previous run to calculate the new clusters, which allows the clustering to converge faster.

5 ANALYSIS
In this section we present a deep dive analysis of the techniques listed in Table 1, using the data sets detailed in Table 2.

5.1 Handling Non-stationarity
In this subsection we walk the reader through how the different techniques handle the different sources of non-stationarity exhibited by the data. Most techniques assume that the underlying process is stationary or evolving gradually. However, in practice, this assumption does not hold true thereby resulting in a larger number of false positives immediately after a process change. Though detecting the change is itself important, the false positives adversely impact the efficacy in a material fashion.

- **Trend and Level Shifts (LS):** Statistical techniques are not robust to trend or level shifts. Consequently, their performance is dependent on the window size, which decides how fast these techniques adapt to a new level. Time series analysis techniques based on state space models (e.g., SARMA, TBATS) can identify level shifts and adapt to the new level without adding false positives. Figure 3a illustrates a financial time series, where SARMA and SDAR detect the level shift as an anomaly. med-mad can also detect the level shift but it surfaces many false positives right after the level shift. Pattern techniques mean-adjust the patterns. Hence, in the

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| Technique   | Parameters          | Description                        |
|-------------|---------------------|------------------------------------|
| HOTSAX      | paa-size=4, a-size=5 | PAA Size, Alphabet Size            |
| RRA         | paa-size=4, a-size=5 | PAA Size, Alphabet Size            |
| DenStream   | epsilon=0.9 lambda=0.01 | MC radius, Decay Constant          |
| DBStream    | r=0.5, Cm=5, shared-density=True | MC radius, minimum weight for MC    |
| ClusFree    | max-height=5, lambda=0.02 | Tree Height, Decay Constant        |
| DBScan      | eps=0.05            | Threshold used to re-cluster ClusTree |
| IForest     | n-estimators=50, contamin=0.05 | Number of Trees, Number of Outliers |
| OneSVM      | n=0.3, gamma=0.1    | Support vectors, kernel coeff       |
| MBMeans     | n-clusters=10, batch-size=200 | Number of Clusters, Batch Size     |

Table 5: Parameters for Pattern Mining and Machine Learning Techniques
presence of level shifts, they do not surface false positives as long as the pattern shapes do not change rapidly.

- **Variance Change (VC):** Figure 3b shows an operations series with a variance change. Iterative techniques such as **STL-SDAR** adapt faster to changing variance which allows them to limit the number of false positives. On the other hand, **STL-ARMA** and **SARMA** are periodically re-trained and oscillate around the true error variance.

- **Seasonal Drift (SD):** Gradually changing seasonal pattern is often observed in **Iot** and **ops** time series. **SARMA** adapts to such a drift with default parameters. **STL** adapts as well if the **periodic** parameter is set to false – this ensures that seasonality is not averaged out across seasons.

- **Seasonal Level Shift (SLS):** SLS is again exhibited predominantly in **Iot** and **ops** series as illustrated in Figure 3c. **TBATS** does not adapt to SLS or SD as it handles seasonality using Fourier terms and assumes that the amplitudes of seasonality do not change with time. **SARMA** handles SLS smoothly as it runs a Kalman-Filter on the seasonal lags and hence only detects anomalies when the shift happens. On the other hand, **STL** is not robust to SLS and may result in false positives as exemplified by Figure 3c. Pattern Techniques such as **DBStream** are very robust to SLS and can detect the pattern around the shift without any false positives.

- **Seasonal Jitter (SJ):** SJ is an artifact of fine-grain TG and is predominantly exhibited in **Iot** (**sec**) and **health** (**msec**) time series. Statistical and time series analysis techniques do not model this non-stationary behavior. As a consequence, in such cases, only pattern anomaly techniques can be used.

### 5.2 Runtime Analysis

In this section we present a characterization of the techniques listed in Table 1 with respect to their runtimes. In the figures referenced later in the section, the benchmarks are organized in an increasing order of seasonal period from left to right.

- **Statistical techniques:** From Figure 4a we note that **mu-sigma** and **t-digest** – recall that these are incremental too – are the fastest (<10 µsec) in this category. Robust techniques are at least an order of magnitude slower! This stem from the fact that these techniques solve an optimization problem. Although **GESD** is the slowest technique in this category, it let’s one set an upper bound on the number of anomalies, which in turn helps control the false alarm rate (CFAR).

- **Time series analysis techniques:** From Figure 4b we note that **STL** is the fastest technique (1-5 msec) in this category. Having said that, the runtime increases considerably as the seasonal period increases (left to right). **Robust STL** is an order of magnitude slower than **STL** even when the number of robust iterations is limited to 4. This can be ascribed to the fact that iterations with the robust weights in **STL** are significantly slower than the first one. **SARMA** and **TBATS** are significantly slower than most other techniques in this category. This is an artifact of the window length needed to fit these models being proportional to the seasonal period and thus, model parameters need to be estimated on a much larger window. On the other hand, a technique such as **STL-ARMA** applies **ARMA** on the residual of **STL** and therefore does not need to deal with seasonality, which allows for a much smaller training window. Runtimes for **TBATS**, **SARMA** and **RPCA** increase exponentially with an increase in seasonal period. Hence, for secondly time series, these methods become nonviable.

**SDAR** and **RobustKF** are fast incremental techniques that can execute in µsecs. However, these techniques cannot be applied to seasonal series as is. This limitation can be alleviated by applying **STL** as a preprocessing step. From Figure 4b, we note that **STL-SDAR** and **STL-RobustKF** are almost as fast as **STL**. Even though **STL-ARMA** trains on small training windows, note that it adds significant additional runtime to **STL**. This impact is not as prominent in the case of the **ops** and **fin** data sets - this is due to the fact that **STL** itself has
On the Runtime-Efficacy Trade-off of Anomaly Detection Techniques for Real-Time Streaming Data

(a) Statistical techniques
(b) Time series analysis techniques
(c) Pattern and machine learning techniques

Figure 4: Runtime characterization of anomaly detection techniques

long runtimes for these data sets. Although we note that PCA is a very fast technique, its accuracy is very low (this is discussed further in the next subsection). This is owing to the PCs being not robust to anomalies themselves.

Pattern and machine learning techniques: From Figure 4c we note that IForest and OneSVM have the worst runtime performance as they are not incremental and they need to be trained for every new data point. MB-Kmeans is relatively faster even though it performs clustering for every new point. This is because the clustering has fast convergence if the underlying model drift is gradual. Although the internal data structures HOTSAX and RRA can be generated incrementally, finding the farthest point using these structures accounts for majority of the runtime. Thus, the runtime for HOTSAX, RRA is not dependent on the window size; instead, it is a function of the variance in the data. Data sets such as fin, ops and iot exhibit high variance due to fine grain TG – this impacts the runtime of HOTSAX, RRA. med data set on the other hand has coarse TG and therefore have a low variance in terms of the shapes of patterns and hence, HOTSAX and RRA are significantly faster for them.

DBStream is the fastest micro-clustering technique across all data sets even though it does have an offline clustering component which is executed for every new point. This is because it maintains the shared density between MCs on-the-fly and then uses DBScan over these MCs to produce the final clustering. DenStream is slower than DBStream because the distance of a data point to all pMC’s needs to be computed to calculate the strength of the anomaly. Alternatively, one can tag all oMCs as anomalous which helps to reduce runtime but adversely impacts the FAR. From Figure 4c we note that ClusTree is the slowest of all the micro-clustering techniques.

5.3 Accuracy-Speed Tradeoff

Figure 5 charts the landscape of accuracy-speed tradeoff for the techniques listed in Table 1 across all data sets tabulated in Table 2. Table 6 details the Precision, Recall along with the F1-score. In this rest of this subsection, we present an in-depth analysis of the tradeoff from multiple standpoints.

Robustness and False Positives: Techniques such as mad surface a higher number of anomalies, which improves recall at the expense of precision as evident from Table 6. For best accuracy, we recommend to use robust techniques with CFAR such as GESD. From Table 6 we note that GESD outperforms most other statistical techniques. On the other hand, from Figure 6 we note that GESD has the highest runtime amongst the statistical techniques.

Model Building: Estimating model parameters in the presence of anomalies can potentially impact accuracy adversely if the technique is not robust. This is observed from Figure 5 wherein an extreme anomaly biases the model obtained
via SARMA thereby inducing false positives in the nab-ctr dataset. STL is susceptible to this as well. In contrast, Robust STL effectively down-weights the anomalies during model parameter estimation. From Figure 5 we also note that pattern mining techniques such as DenStream are more robust to anomalies, as they do not fit a parametric model.

**Anomaly Bursts:** It is not uncommon to observe bursts of anomalies in production data. In such a scenario, the accuracy of a technique is driven by how soon the technique adapts adapt to the new “normal”. If a burst is long enough, then most techniques do adapt but with different lag. CFAR techniques such as t-digest and GESD fair quite poorly against anomaly bursts. For instance, in the health data set, the anomalies happen in bursts and a CFAR system puts an upper bound on the number of anomalies, thereby missing many of them. Having said that, this can also be
Table 6: Precision, Recall and F1-scores for all the techniques. For each dataset, the most accurate technique is highlighted.

![Figure 7: Performance of t-digest anomaly bursts](image)

Figure 7: Performance of t-digest anomaly bursts

advantageous as exemplified by the nab-ctr data set wherein there are a few spaced out anomalies. Table 6 shows that t-digest improves both precision and recall. Figure 7 illustrates an operations time series that highlights why t-digest does not surface anomaly bursts.

**Unique Pattern Anomalies:** The performance of HOT SAX and RRA is abysmal on the yahoo-a2 and yahoo-a3 data sets. This is because these synthetic data sets comprise of many similar anomalies. Both HOT SAX and RRA are not robust to the presence of such similar anomalies as the anomaly score is based on the nearest neighbor distance. Figure 8 highlights how similar anomalies may be missed. In contrast, DenStream and DBStream are able to detect self-similar anomalies as they create micro-clusters of similar anomalies.

**Scale of distance measure:** Accuracy of an anomaly detection technique is a function of the distance measure used to differentiate normal vs. anomalous data points. For instance, let us consider IForest and DBStream (refer to Figure 9). The latter creates much better separation between normal and anomalous data points. This can be ascribed to anomaly score in IForest being the depth of the leaf in which the point resides, which is analogous to the log of the distance.

5.4 Model Selection

As mentioned earlier, the analysis presented in this paper should serve as a guide for selection of the "best" anomaly detection technique. In general, model selection is a function of the application domain and latency requirement. Table 7 enlists the various application domains, the attributes exhibited by the datasets in these domains and the "best" algorithms for a given latency requirement.
When latency requirement is in the range of <1 msec, the use of pattern and machine learning based anomaly detection techniques is impractical owing to their high computation requirements. Although techniques geared towards detecting point anomalies can be employed, the “best” technique is highly dependent on the attributes. For instance, STL-SDAR accurately detects anomalies for operations time series that exhibit non-stationarities such as LC, VS. On the other hand, in the case of minutely operations time series which typically tend to exhibit long Seasonal Periods (SP), STL-SDAR is not a bottleneck. It is more effective. It has been shown in prior work [63, 90] that techniques such as HOTSAX are effective in finding anomalies in ECG data in an offline setting. From Figure 6 we note that the detection runtime is over 100 msec, which is significantly larger than the TG for ECG series. DBStream on the other hand is much faster (<10msec) and can detect the same set of anomalies as HOTSAX, refer to Figure 10. Having said that, DBStream does surface more false positives than HOTSAX. A post processing step can help reduce the false positive rate. It should be noted that none of the aforementioned techniques satisfy the <1 msec latency requirement for the health time series.

Finally, when the latency requirements are >10msec, a wide range of anomaly techniques can be leveraged. In many cases, DBStream is still the most accurate technique. When an application has very few unique anomalies, HOTSAX usually is the most effective, as is the case with the datasets nab-twt and nab-iot.

Table 7: Best Techniques For Different Latency Requirements

| Application Domain | DataSets | Attributes | ≤1msec | 1-10msec | >10msec |
|--------------------|----------|------------|--------|----------|---------|
| Hourly Operations  | yahoo-xl,yahoo-xl,yahoo-xl | LS, SL, Noisy | STL-SDAR | DenStream | DenStream |
| Anomaly            | nba, nba | No Anomaly | DBStream | DenStream | DenStream |
| Hourly IoT         | nba, nba | Unique Anomalies | med-mad | DBStream | HOTSAX |
| Financial          | web | LS, VC | DBStream | DenStream | HOTSAX |
| Healthcare         | health | SL, LS | - | DBStream | DenStream |
| Minutely Operations| ops | SL, VS, SLO, Large DP | med-mad | DBStream | DenStream |

Figure 9: Anomaly Score Separation

(according to the accuracy-speed trade-offs discussed in the previous section).

Figure 10: HOTSAX and DBStream for health-qtdb/0606

6 CONCLUSION

In this paper, we first presented a classification of over 20 anomaly detection techniques across seven dimensions (refer to Table 1). Next, as a first, using over 25 real-world data sets and real hardware, we presented a detailed evaluation of these techniques with respect to their real-timeliness and performance – as measured by precision, recall and F1 score. We also presented a map of their accuracy-runtime trade-off. Our experiments demonstrate that the state-of-the-art anomaly detection techniques are applicable for data streams with 1 msec or higher granularity, highlighting the need for faster algorithms to support the use cases mentioned earlier in Section 1. Last but not least, given an application domain and latency requirement, based on empirical evaluation, we made recommendations for the “best” technique for anomaly detection.

As future work, we plan to extend our evaluation to other data sets such as live data streams as exemplified by Facebook Live, Twitter Periscope video applications and other live data streams on platforms such as Satori.

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