GALAXY LUMINOSITY FUNCTION AND TULLY–FISHER RELATION: RECONCILED THROUGH ROTATION-CURVE STUDIES

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\textbf{ABSTRACT}

The relation between galaxy luminosity \( L \) and halo virial velocity \( v_{\text{vir}} \) required to fit the galaxy luminosity function differs from the observed Tully–Fisher relation between \( L \) and disk speed \( v_{\text{rot}} \). Because of this, the problem of reproducing the galaxy luminosity function and the Tully–Fisher relation simultaneously has plagued semianalytic models since their inception. Here we study the relation between \( v_{\text{rot}} \) and \( v_{\text{vir}} \) by fitting observational average rotation curves of disk galaxies binned in luminosity. We show that the \( v_{\text{rot}} - v_{\text{vir}} \) relation that we obtain in this way can fully account for this seeming inconsistency. Therefore, the reconciliation of the luminosity function with the Tully–Fisher relation rests on the complex dependence of \( v_{\text{rot}} \) on \( v_{\text{vir}} \), which arises because the ratio of stellar mass to dark matter mass is a strong function of halo mass.

\textbf{Key words:} galaxies: halos – galaxies: kinematics and dynamics – galaxies: luminosity function, mass function – galaxies: spiral

\textbf{Online-only material:} color figures

1. INTRODUCTION

In the standard $\Lambda$CDM cosmology, $\sim 84\%$ of the mass of the universe is composed of dark matter (DM), which dominates gravitational evolution on large scales. Galaxies are baryonic condensations at the centers of DM halos formed by the gravitational instability of primordial density fluctuations. The main goal for studies of galaxy formation in a cosmological context is to explain the properties of galaxies (luminosities, spectra, sizes, and morphologies) in terms of the growth histories of their host halos.

Historically, the main observational constraints on the link between luminous galaxies and the underlying DM distribution come from the galaxy luminosity function (LF) and from the Tully–Fisher (TF) relation $L \propto v_{\text{rot}}^n$ that links the luminosity $L$ of a disk galaxy and its reference speed $v_{\text{rot}}$ ($n \sim 3–4$ depending on the band. Figure 1 shows the $I$-band TF relation of Yegorova & Salucci (2007). Today, there are further constraints from weak-lensing data (Mandelbaum et al. 2006; Leauthaud et al. 2010; Reyes et al. 2012), velocity dispersion profiles (Martinsson et al. 2013), and kinematics of satellite galaxies (Conroy et al. 2007; More et al. 2011). The latter probe the gravitational potential of spiral galaxies at large radii. Yegorova et al. (2011) find that their results are consistent with those from rotation-curve (RC) data in a statistical sense.

The importance of the LF and the TF relation for constraining the relation between galaxy luminosity and halo mass is readily explained. Let us start from the LF. If a volume of the universe contains $n$ halos of mass $M_h$, and if each halo of mass $M_h$ hosts a galaxy of luminosity $L$, then the volume must contain $n$ galaxies with luminosity $L$. More generally, the number of galaxies brighter than $L$ in a given band must be equal to the number of halos more massive than $M_h$ if $L$ is a growing function of $M_h$—a general prediction of galaxy formation models and simulations in bands where luminosity traces stellar mass. The LF of galaxies is observed and the $\Lambda$CDM model makes strong predictions for the mass function of DM halos. Hence, there is a well-defined $L - M_h$ relation that cosmological models must satisfy to fit the galaxy LF. It is straightforward to convert this relation into a relation between $L$ and $v_{\text{vir}}$ by using the fitting formula of Bryan & Norman (1998) for the virial overdensity contrast.

The black dashed curve in Figure 1 shows the $L - v_{\text{vir}}$ relation that we obtain when we apply this method to the LF of Papastergis et al. (2012) and to the halo mass function for a $\Lambda$CDM cosmology with $h_0 = 0.73$, $\Omega_m = 0.24$, $\Omega_{\Lambda} = 0.76$, and $\sigma_8 = 0.76$. The $I$-band LF of Papastergis et al. (2012) is constructed by computing the Kron–Cousin $I$-band magnitude of each individual galaxy from its Sloan Digital Sky Survey (SDSS) $r$- and $i$-band magnitudes. We have verified that this LF is consistent with the SDSS DR6 $i$-band LF of Montero-Dorta & Prada (2009) when we convert the latter to $I$ band by using mean $I - i$ colors in Fukugita et al. (1995). The halo mass function comes from a cosmological $N$-body simulation that was run by the Horizon Project (http://www.projet-horizon.fr) and is the same that we used in Papastergis et al. (2012), to whom we refer for all details concerning the abundance-matching (AM) procedure.

Let us now consider the TF relation. The disk rotation speed $v_{\text{rot}}(r)$ measures the total mass $M$ within radius $r$, which is dominated by DM for large values of $r$. Since the RCs of disk galaxies are nearly flat at large radii, the TF relation implies a relation between $L$ and $v_{\text{vir}}$. The trouble is that this relation differs from the one that we find from the LF.

Figure 1 illustrates the problem by comparing the $L - v_{\text{vir}}$ relation that we derive from AM (black dashed line) with the $I$-band TF relation of Yegorova & Salucci (2007, black symbols). This discrepancy cannot be attributed to variations in mass-to-light ratio because the entire analysis has been done using the...
Kron–Cousin $I$-band luminosity both for the LF and the TF relation. This is a strong point of our work and the reason why Figure 1 is totally independent of any assumption on the stellar mass-to-light ratio (on the issue of mass-to-light ratios, see also Dwarf spheroidal galaxy kinematics and spiral galaxy scaling laws por Portinari & Salucci 2010).

The discrepancy shown in Figure 1 has been known for 20 yr and is independent of the AM method because any model that matches the galaxy LF ends up with an $L_I^\prime$–$v_{vir}$ relation that we obtain by matching the Kron–Cousin $I$-band LF of Papastergis et al. (2012) and the halo mass function from a cosmological simulation with $h_0 = 0.73$, $\Omega_m = 0.24$, $\Omega_\Lambda = 0.76$, and $\sigma_8 = 0.76$.

Figure 1. Black symbols: the TF relation for the spiral-galaxy sample of Yegorova & Salucci (2007). $M_I$ is the absolute magnitude in the $I$ band of Kron–Cousin, $v_{opt}$ is the speed at the optical radius (equal to 3.2 exponential radii). Red symbols with error bars: four dwarf galaxies from Salucci et al. (2012) inserted to extend the TF relation to $M_I \sim -13$. Black dashed curve: the $L_I^\prime$–$v_{vir}$ relation that we obtain by matching the Kron–Cousin $I$-band LF of Papastergis et al. (2012) and the halo mass function from a cosmological simulation with $h_0 = 0.73$, $\Omega_m = 0.24$, $\Omega_\Lambda = 0.76$, and $\sigma_8 = 0.76$.

(A color version of this figure is available in the online journal.)

Our article follows the same general philosophy of these two previous studies, but the method is completely different because first we determine $v_{rot}/v_{vir}$ as a function of $L_I$ from the modeling of disk RCs and then we use this result to convert the $L_I^\prime$–$v_{vir}$ relation from AM into a relation between $L_I$ and $v_{rot}$, which can be compared with the TF relation. The originality of the article is that, following Salucci et al. (2007), we compute the $v_{rot}/v_{vir}$ with a method that is completely independent of the galaxy LF and we find that this relation is precisely the one we need to reconcile the TF relation with the galaxy LF.

The structure of the article is as follows. In Section 2, we present our RC analysis and our results for the $v_{rot}$–$v_{vir}$ relation. In Section 3, we combine the results of the previous section with those from AM to compute a TF that will be found to be in agreement with the observations, and we discuss the significance of this final result.

2. THE RELATION BETWEEN $v_{opt}$ AND $v_{vir}$

Our method to compute $v_{vir}$ is conceptually quite simple. We fit the RCs of spiral galaxies by assuming that they are made of two components: a baryonic disk and a DM halo. The best-fit DM halo profile is extrapolated out to the radius $r_{vir}$, where the mean density equals the critical density of the universe times the virial overdensity contrast computed with the formula by Bryan & Norman (1998). The virial velocity $v_{vir}$ is equal to the circular velocity at this radius. In practice, to apply this method, we need to specify three things: a model for the density distribution of the baryonic disk, a model for the density distribution of the DM halos, and the RCs on which we intend to do the analysis. Here, we analyze these three elements one by one.

2.1. The Baryonic Disk Model

We model the baryonic disk with a single exponential profile. This model contains two parameters: the total disk mass $M_d$ (stars plus gas) and the disk exponential radius $r_d$. We do not treat $r_d$ as a free parameter of the fit. Instead, we require it to be equal to the exponential radius of the $I$-band surface-brightness profile. This is equivalent to assuming that the gas and the stellar disk have the same scale length. In fact, the gas distribution is usually more extended, but this has almost no
effect on our results for the following reason. Small galaxies are completely DM-dominated. An error on the spatial extension of the baryonic component will have little effect on the best-fit parameters for the dominant DM component (this point is shown quantitatively in Persic et al. 1996). In massive galaxies, the baryonic component is dynamically important, but these galaxies have low gas fractions. Therefore, the gas component has a negligible effect on the size of the baryonic disk.

2.2. The DM Halo Model

Cosmological simulations of dissipationless hierarchical clustering in a cold DM universe find that the density distribution of DM halos are described by the Navarro–Frenk–White (Navarro et al. 1997) profile:

$$\rho(r) = \frac{\rho_0}{\left( \frac{r}{r_0} \right)^{2}} \left( 1 + \frac{r}{r_0} \right)^{-3},$$

(1)

with concentration

$$c(M_{\text{vir}}) = \frac{r_{\text{vir}}}{r_0} = 9.6 \left( \frac{M_{\text{vir}}}{10^{12} h^{-1} M_\odot} \right)^{-0.075}$$

(2)

for halos at $z \simeq 0$ (Klypin et al. 2011). This profile provides a poor fit to the RCs of spiral galaxies in the inner regions (Flores & Primack 1994; Moore 1994, but also see Swaters et al. 2003). The Burkert (1995) profile:

$$\rho(r) = \frac{\rho_0}{\left( 1 + \frac{r}{r_0} \right) \left( 1 + \frac{r}{r_0}^{2} \right)},$$

(3)

gives a much better fit to the observed RCs (Figure 2), though with concentration-parameter values that are $\sim 1.4–1.5$ times larger than suggested by Equation (2). As this is a phenomenological paper, the fact that the Burkert model fits the observation is a good enough reason for using it independently of any theoretical argument. However, some readers may wonder whether it is self-consistent to use the Burkert profile (Equation (3)) alongside the halo mass function from a simulation that assumes a cold DM cosmology. To address this objection, we remark that our choice is justified for two reasons. First, Equations (1) and (3) are almost identical at $r \gtrsim r_0$ and differ only at small radii, while here we are interested in the DM density distribution at large radii. Recent hydrodynamical simulations have shown that the difference at small radii may be understood as an effect of supernova feedback, which was not considered in pure N-body simulations (Governato et al. 2010; Pontzen & Governato 2012; Brook et al. 2012; Teyssier et al. 2013). Second, the concentration difference can be explained as being due to adiabatic contraction. We have verified this point quantitatively by comparing the DM density profile in a simulation by Geen et al. (2013) in the cases with pure DM and with baryon cooling.

2.3. The RCs

Our analysis is based on a sample of 967 spiral galaxies for which there are high-quality Hz data and I-band photometry (Persic & Salucci 1995; Mathewson et al. 1992). The sample is split into 11 luminosity bins and each luminosity bin is analyzed separately. Instead of analyzing the RC of each galaxy individually and then computing an average $v_{\text{rot}}/v_{\text{vir}}$, we directly analyze the co-added RCs of Persic et al. (1996, hereafter PSS), obtained by stacking the RCs of the galaxies in each bin. Following PSS, the co-added RCs are computed by averaging the values of $v_{\text{rot}}$ in bins of $r/r_{\text{vir}}$, where $r_{\text{crit}} = 3.2 r_d$ ($r_d$ is the exponential radius of the I-band surface-brightness profile). The points with error bars in Figure 2 show the co-added RCs in the 11 I-band magnitude bins.

2.4. The Fit

In each magnitude bin, we fit the co-added RC with a disk plus halo model (Figure 2). We do the fit at $0.5 < r/r_{\text{vir}} < 2$ to give more weight to the outer regions. The result is generally quite good (compare the solid lines and the points with error bars).

The disk contribution is a strong function of luminosity. Faint galaxies ($M_I \gtrsim 19$) are entirely DM-dominated. Their RCs rise steeply out to $r = 2 r_{\text{crit}}$. Bright spirals ($M_I < -22$) are disk dominated out to very large radii. Their RCs peak at $0.5 < r/r_{\text{vir}} < 1$ and decrease at larger radii.

Let $v_{\text{opt}}$ be the mean rotation speed at the optical radius and let $v_{\text{vir}}$ be the virial velocity obtained for the best-fit halo parameter by extrapolating the halo contribution out the virial radius (the radius within which the mean density equals the critical density of the universe times the critical overdensity contrast computed with the fitting formula of Bryan & Norman 1998). By computing $v_{\text{opt}}$ and $v_{\text{vir}}$ for each of our magnitude bins, we obtain the $v_{\text{opt}}$–$v_{\text{vir}}$ relation that is shown by the red solid curve in Figure 3.

Figure 3 shows that $v_{\text{opt}}$ differs from $v_{\text{vir}}$. The difference is by a factor of $\sim 1.3$ but its precise value varies with $v_{\text{vir}}$ and has a maximum of $v_{\text{opt}}/v_{\text{vir}} \sim 1.5$ for $v_{\text{vir}} \sim 100 \text{ km s}^{-1}$. This result is consistent with the one from weak lensing by Reyes et al. (2012) when we correct for the difference in our definition of $v_{\text{crit}}$. Dutton et al. (2010) find a curve $v_{\text{rot}}/v_{\text{vir}}$ versus $v_{\text{vir}}$ with the same shape and the maximum in the same position but their values of $v_{\text{rot}}/v_{\text{vir}}$ are systematically lower (their maximum value for $v_{\text{rot}}/v_{\text{vir}}$ is closer to 1.1).

In Figure 3, we have also compared our results to those obtained with the AM method by Papastergis et al. (2011, solid blue curve; the same approach has also been explored by Trujillo-Gomez et al. 2011). Papastergis et al. (2011) have considered the galaxy velocity function computed by using the H I rotation speed for disk galaxies and $\sigma \sqrt{2}$ for elliptical galaxies where $\sigma$ is the stellar velocity dispersion at 1/8 effective radii and they have matched this velocity function to the virial velocity function of DM halos. Given that the two methods are totally unrelated, the broad agreement of the red curve and the blue curve is quite significant. Furthermore, a small deviation is not unexpected, since rotation speeds measured from the width of the H I line are generally slightly larger than $v_{\text{rot}}$ (Dutton et al. 2010).

Finally, we note that in Tonini et al. (2006), we had computed the spin parameter of spiral galaxies by using PSS’s universal RC (automatically in agreement with the TF relation) in conjunction with the disk–halo mass relation from AM (see also Shankar et al. 2006). Our article shows that the approach of these studies was well grounded.

3. LF AND TF RELATION: RECONCILED AT LAST

In Section 2, we have used our analysis of disk RCs to determine the relation between $v_{\text{rot}}$ and $v_{\text{vir}}$ (the red curve in Figure 3). Now, we use this relation to transform the $M_I$–$v_{\text{vir}}$ relation from AM (the dashed curve in Figure 1) into a relation
Figure 2. Points with error bars: the co-added RCs in 11 $I$-band magnitude bins for the 967 spiral galaxies in PSS’s sample. Curves: each co-added RC is fitted with a disk (dotted curve) and a halo (dashed curve) contribution. Their sum is shown by the solid curve. Faint spirals are DM-dominated at all radii. Bright spirals are disk-dominated at all radii.

between $M_I$ and $v_{\text{opt}}$. This relation is shown by the red curve in Figure 4 and is in excellent agreement with the TF data points (black symbols; they are the same in Figures 1 and 4). Our conclusion is that we do not encounter any problem at reproducing the LF and the TF relation simultaneously when the dependence of $v_{\text{opt}}$ on $v_{\text{vir}}$ is properly accounted for.

Our result is established in the magnitude range of $-22 < M_I < -18$. LF data (from which we derive the dashed curve in Figure 1) exist down to $M_I \sim -14$ but we still lack a statistically significant dwarf galaxy sample with high-quality RCs that may allow us to extend our analysis at $M_I > -18$, although progress in this direction is being made (e.g., the 30 galaxy sample by Martinsson et al. 2013). Even with these uncertainties, there is evidence that the TF relation may bend at low luminosities (e.g., Trujillo-Gomez et al. 2011 and our lowest-luminosity dwarf galaxy data point).

The quality of the agreement at $-22 < M_I < -18$ is also linked to the consistency of our procedure. The $v_{\text{opt}}/v_{\text{vir}}$ that we use to pass from $v_{\text{vir}}$ to $v_{\text{opt}}$ is measured from the same RCs from which we extract the TF data points. Furthermore, our choice to work with $I$-band luminosity throughout the article, both for the LF and TF relation, avoids introducing the uncertainty of stellar mass-to-light ratios.

At high luminosities, the red curve shows a hint of flattening, which is not seen in TF data points, but this happens because the AM relation (black dashes) is computed from the total LF, which is dominated by early-type galaxies at high luminosity, while TF relation is shown for spiral galaxies only. The implication
The dependence of $v_{\text{opt}}/v_{\text{vir}}$ on $v_{\text{vir}}$ is of paramount importance to explain how it is possible to reconcile TF relation (a single power law) with the LF, which has a break at the characteristic luminosity $L_*$. This dependence arises from the strong trend in $M_*/M_{\text{halo}}$ with halo mass. This can be seen both through the disk/halo decomposition of the RCs (PSS) and through the results of AM, stellar kinematics, and weak-lensing studies (see Figure 3).
Papastergis et al. 2012, where we also compare the results of many different authors).

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