A Sliding mode control method for trajectory tracking control of wheeled mobile robot

Lu Yang¹ and Shenghui Pan²,³

¹ School of Electrical and Information Engineering, Guangxi University of Science and Technology, Liuzhou, china, 545006;
² School of Electrical and Information Engineering, Guangxi University of Science and Technology, Liuzhou, china, 545006;
³ E-mail: 40352670@qq.com

Abstract. This paper presents a trajectory tracking control scheme for a three wheeled mobile robot using sliding mode control method system. The indoor mobile robot system is subject to parameter perturbation and external interference during trajectory tracking control, and the problem of slow tracking speed and large tracking error will occur. In order to improve the tracking precision, a sliding mode control algorithm based on new approach law is proposed. Firstly, the motion control model of mobile robot is established, and a sliding mode trajectory tracking controller is designed. Then adopting MATLAB to track the linear and circular trajectories, the tracking error converges to zero around 3s. Experimental results show that sliding mode control method effectively reduces sliding mode control input chattering, and the convergence speed and tracking accuracy improve. Finally, the effectiveness of the control method is thus verified in practice.

1. Introduction

Mobile robot is a comprehensive system integrating environmental perception, dynamic planning and decision-making, behavioral control and execution, etc. [1]. Trajectory tracking means that mobile robot through feedback control, making mobile robot follow a preset expectation trajectory from any initial position [2]. Due to the constraint of robot's nonlinear control system, mobile robot existed in the uncertainty and complexity of indoor environment. When robot is large in quality or moving at high speed, the control system is influenced on parameter disturbance, as well as the system stability is poorly. How to solve and weaken the system input chattering in the tracking control process of wheel mobile robot. The main problem will further research.

At present, many literatures have studied the trajectory tracking problem of wheeled mobile robot and proposed different trajectory tracking algorithms. The terminal sliding mode control (TSMC) method involves non-smooth sliding surfaces. While on the sliding surface, the error states converge to the origin in finite time thus ensuring finite-time tracking in [3]. A global sliding mode control for linear velocity is proposed on [4], which bring the position error to zero. Unlike in [4], a sliding mode variable structure controller with continuous state feedback is designed to realize the error tracking control of non-complete mobile robot in [5]. In this regard, a trajectory tracking control using by sliding mode technique, and the stability of designed sliding mode dynamics is analysis and accessibility of the sliding mode is guaranteed in [6,7]. Another work in [8] proposed that a dynamic inverse control method of PID control is used to ensure that the tracking error is convergent to zero.
Whereas, the linear sliding mode controller designed and applied to actual wheeled mobile robot in [9]. Combining the sliding mode controller and Lyapunov function, a fast terminal sliding mode controller is proposed in [10] to ensure asymptotic convergence in the kinematics error system of mobile robot. Similar work can be found in [11,12]. A novel sliding mode controller is given at torques level such that both position and orientation tracking errors convergence to zero within finite time.

The contributions of this paper are summarized as follows: 1) Because of the traditional sliding mode speed reaching law is discontinuous, which causes that the process of trajectory tracking in switching surface produces large chattering. To solve this problem, a new type of control law is proposed such that the state trajectory converges continuously to the switching surface. 2) Considering mobile robots are non-complete systems, therefore, it introduces indoor environment constraints, the effectiveness of controller is illustrated by simulation to track linear and circular trajectory.

2. Kinematic model of wheeled mobile robot
Wheeled mobile robot is the movement mechanism that all wheel contact with the ground. It automatically performs the work. Omnidirectional mobile robot consists of three omnidirectional wheels, as shown in figure 1, including two driving wheels and a universal wheel, each wheel forming a 120° angle with mounted in a radial symmetry. Three wheel are exactly the same as size and quality, which are powered by the same DC motor. To describe the movement along a flat, a robot mobile platform build as shown in figure 2. Figure 2 have world coordinate system and robot coordinate system, the angle \( \theta \) is between the robot coordinate system and world coordinate system. To facilitate kinematic analysis, the mobile platform needs to simplify. As shown in figure 3, the angle between the driving wheels is \( \varphi \), \( \varphi = 120^\circ \). The horizontal distance which from the robot centers to each wheel center is \( L \), and \( v_1, v_2, v_3 \) is omnidirectional speed.

![Figure 1. Wheeled Mobile Robot. Figure 2. Mobile robot model. Figure 3. A simplified motion model.](image)

Supposing that the velocity component of robot coordinate system is \( v, v, w \), the world coordinate system's velocity component is \( V, V, w \), \( w \) is angular velocity of robot rotation. The relationship between world coordinate system \((V, V, w)^T\) and robot coordinate system \((v, v, w)^T\) can be expressed as follows:

\[
\begin{bmatrix}
    v \\
    v \\
    w
\end{bmatrix} =
\begin{bmatrix}
    \cos(\theta) & \sin(\theta) & 0 \\
    -\sin(\theta) & \cos(\theta) & 0 \\
    0 & 0 & 1
\end{bmatrix}
\begin{bmatrix}
    V \\
    V \\
    w
\end{bmatrix}
\]

(1)

According to the plane movement decomposition and synthesis relationship, speeds decomposition carrying out toward a fixed wheel in the positive direction. From figure 3 concluded that the relationship between the driving wheel speed \((v_1, v_2, v_3)^T\) and the robot coordinate system can be described as the following equation:
\[
\begin{bmatrix}
v_x \\
v_y \\
v_z \\
w
\end{bmatrix} =
\begin{bmatrix}
0 & 1 & L & 0 \\
-\cos 60 & \sin 60 & L & v_r \\
-\cos 60 & -\sin 60 & L & w
\end{bmatrix}
\]

(2)

From (1) and (2) concluded the relationship between the robot's speed in the world coordinate system and driving wheel speed, when establishing a relative coordinate system, to simplify the calculation formula, \( \theta = 0 \), the robot kinematics model is

\[
\begin{bmatrix}
v_x \\
v_y \\
w
\end{bmatrix} =
\begin{bmatrix}
0 & 1 & \sqrt{3}L & v_r \\
-\frac{1}{2} & \frac{\sqrt{3}}{2} & L & v_e \\
-\frac{1}{2} & -\frac{\sqrt{3}}{2} & L & w
\end{bmatrix}
\]

(3)

The kinematics models of wheeled mobile robots focus on two-dimensional planes. In this paper, the coordinate transformation method is used to construct the pose error model. Three wheeled mobile robot takes as a specific research object. Assuming that mobile robot does not slide relative to the ground when it moves, the robot's motion model is simplified to two driving wheels. So, the pose error of non-complete wheeled mobile robot on the flat is shown in figure 4.

Figure 4. Simplified position of mobile robot. The position of wheeled mobile robot can be presented by the body axis mid-point \( E \) and heading angle \( \theta \), then, the posture coordinate vector of mobile robot is described as \( P = (x, y, \theta) \). The robot moves from \( E \) point to point \( F \) by time \( t \), the robot's position is \((x, y)\). It is the angle \( \theta \) between the moving direction of robot and the axis \( X \). When mobile robot moves to the point \( F \), the angle \( \theta \) is between robot forward direction and the axis \( X \), \( \theta = \theta + \theta \).

The robot's control input can be described as \( q = (v, w)^T \), \( v \) is robot's linear speed and \( w \) is angular speed. The motion equation of mobile robot is established:

\[
\begin{bmatrix}
x' \\
y' \\
\theta'
\end{bmatrix} =
\begin{bmatrix}
cos \theta & 0 \\
\sin \theta & 0 \\
0 & 1
\end{bmatrix}
\begin{bmatrix}
x' \\
y' \\
\theta'
\end{bmatrix} +
\begin{bmatrix}
0 \\
0 \\
q
\end{bmatrix}
\]

(4)

Supposing the robot's speed \( q = (v, w)^T \) and reference position \( P_r = (x, y, \theta)^T \) is tracked. Wheeled mobile robots move from pose \( P = (x, y, \theta)^T \) to \( P_r = (x, y, \theta)^T \), and get a new coordinate system \( Xe-Ye \). The position of mobile robot is \( P = (x, y, \theta)^T \) in \( Xe-Ye \). The angle between the new coordinate system \( Xe-Ye \) and the coordinate system \( X-Y \) is \( \theta \). According to the coordinate formula change, the error equation to describe the position of mobile robot is obtained in [9, 10, 11]

\[
P =
\begin{bmatrix}
x \\
y \\
\theta
\end{bmatrix} =
\begin{bmatrix}
\cos \theta & \sin \theta & 0 \\
-\sin \theta & \cos \theta & 0 \\
0 & 0 & 1
\end{bmatrix}
\begin{bmatrix}
x - x_r \\
y - y_r \\
\theta - \theta_r
\end{bmatrix}
\]

(5)

From above we can get the differential equation of pose error:
\[ \dot{P} = \begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\theta} \end{bmatrix} = \begin{bmatrix} y, w - v + v \cos \theta \\ -x, w + v \sin \theta \\ w - w \end{bmatrix} \] (6)

Achieving robot trajectory track control is to find a suitable input \( q = (v, w) \), the system made \( P = (x, y, \theta) \) bounded without initial error requirements, and \( \lim_{t \to \infty} (x, y, \theta) = 0 \).

3. Sliding mode trajectory tracking control

3.1. Switch function design

In this paper, the inversion function is used to design the sliding mode controller's switching function [2], when \( x = 0 \), the Lyapunov function was chosen as:

\[ V = \frac{1}{2} y^2 \] (7)

When \( \theta = -\arctan(v, y) \), then we can get the following result:

\[ \dot{V} = -y, x, w - v, y, \sin(\arctan(v, y)) \] (8)

According to this lemma, any \( x \in R, \) and \( |s| < \infty, \) if \( \phi(x) = x, \sin(\arctan(x)) \geq 0 \), and only \( x = 0 \) then "=" set up [2], we can get \( v, y, \sin(\arctan(v, y)) \geq 0 \) (only then "=" is established), which is \( V \leq 0 \). It can be concluded: As soon as \( x, \) converges to zero and \( \theta \) converges to \(-\arctan(v, y)\), the state of system \( y, \) and \( \theta \), converges to zero. According to the conclusion, the design of switching function is:

\[ s = \begin{bmatrix} s_1 \\ s_2 \end{bmatrix} = \begin{bmatrix} x \\ \theta + \arctan(v, y) \end{bmatrix} \] (9)

Therefore, we can design sliding mode controller. When \( s_1 \to 0, s_2 \to 0 \), achieving \( x \), convergence to zero and \(-\arctan(v, y)\), then achieving \( x \to 0 \) and \( \theta \to 0 \), the trajectory tracking control of mobile robot is further realized.

3.2. Design of sliding mode controller

Early work in [8], the Switching function can be designed as:

\[ s = \begin{bmatrix} s_1 \\ s_2 \end{bmatrix} = \begin{bmatrix} \theta + a\theta + b\theta^2 \\ x - y \end{bmatrix} \] (10)

The next step is designing a sliding controller, which make \( s_1 \to 0, s_2 \to 0 \) to achieve 0, and \( x \to y \), the reaching law can be defined as:

\[ \dot{s} = -ks \left| s \right| \cdot \text{sgn}(s) \] (11)

\( k > 0, a > 0 \). Substituting (10) into (11), it can be derived as:

\[ q = \begin{bmatrix} v \\ w \end{bmatrix} = \begin{bmatrix} w, y, + v, + w, x, + k|x| \left| s \right| \cdot \text{sgn}(s) \\ w, + a\theta, + b\theta^2 \end{bmatrix} \] (12)

This paper is not same as the control law in [8]. Due to the control law exist in discontinuous control items \( k \cdot \text{sgn}s \), and it generate chatter on the switching surface \( s = 0 \). Thus, considering the continuity of saturation function, so that state trajectory converges to the switching plane. System buffeting is suppressed to a large extent. Saturation function is used to replace the discontinuous
control term \( k_{\text{sgn}} s \) of control law for weaken chattering. A new sliding mode control approach law construct in

\[
\dot{s}_i = - k_i \frac{s_i}{|x_i| + \delta_i} \tag{13}
\]

Where \( i = 1, 2 \) and \( \delta_i \) is a positive decimal.

By setting \( T = \arctan(v, y) \), then \( \frac{\partial T}{\partial x} = \frac{y}{1 + (v, y)} \), \( \frac{\partial T}{\partial y} = \frac{v}{1 + (v, y)} \), Eq.(13) becomes:

\[
\dot{s} = \begin{bmatrix} \dot{s}_1 \\ \dot{s}_2 \end{bmatrix} = \begin{bmatrix} -k_1 \frac{s_1}{|x_1| + \delta_1} \\ -k_2 \frac{s_2}{|x_2| + \delta_2} \end{bmatrix} = \begin{bmatrix} \dot{x} \\ \dot{y} \end{bmatrix} = \begin{bmatrix} \dot{x} + \frac{\partial I}{\partial x} \dot{v} + \frac{\partial I}{\partial y} \dot{y} \\ \dot{y} + \frac{\partial I}{\partial x} \dot{v} + \frac{\partial I}{\partial y} \dot{y} \end{bmatrix} = \begin{bmatrix} \dot{x} + \frac{\partial I}{\partial x} (x - w - v \cos \theta) \\ \dot{y} + \frac{\partial I}{\partial y} (w - v \sin \theta) \end{bmatrix} \tag{14}
\]

By sorting out, we obtain the trajectory tracking control laws as follows

\[
q = \begin{bmatrix} v \\ w \end{bmatrix} = \begin{bmatrix} y + v \cos \theta + k_1 \frac{s_1}{|x_1| + \delta_1} \\ w + \frac{\partial I}{\partial x} \dot{v} + \frac{\partial I}{\partial y} (v \sin \theta) + k_2 \frac{s_2}{|x_2| + \delta_2} \end{bmatrix} \tag{15}
\]

According to the results of literature [8], it adopts sliding mode control law as in equation (12). The tracking error converges to 0 in the 6s, nevertheless, this paper uses news control law, and tracking error converges to 0 about 3s. Therefore, the trajectory tracking time of sliding mode control method in this paper is shorter. The practical application effect is better.

4. Simulation results and analysis

4.1. Linear trajectory tracking

Mobile robot initial posture is \([1 \quad 0 \quad \pi/2]^{\top}\) in experiment, the actual speed is \( v = 0.4 \) m/s, \( w = 0.4 \) rad/s, the initial pose of mobile robot is \([1 \quad 0 \quad \pi/3]^{\top}\), reference speed is \( v_r = 0.2 \) m/s, \( w_r = 0 \). \( k_1 = 3 \), \( k_2 = 3 \), \( k_3 = 9 \), \( k_4 = 3 \) is selected control law parameters, the simulation results of linear trajectory tracking are shown in the following figure (refer to figure 5, 6, 7).

![Figure 5. Linear tracking curve.](image-url)

![Figure 6. Control law v, w curve.](image-url)
4.2. Circular trajectory tracking

Mobile robot initial posture is \([1.2 -0.3 2\pi/3]\) in experiment, the actual speed is \(v = 0.3 \text{ m/s}\), \(w = 0.4 \text{ rad/s}\), the desired speed is \(v_c = 0.2 \text{ m/s}\), \(w_c = 0.2 \text{ rad/s}\), the radius of mobile robot is \(r = v/6\). Mobile robot's reference pose is \(P = (x, y, \theta)^T\), \(x = r \cos(\theta)\), \(y = r \sin(\theta)\), \(\theta = \pi/2 + 0.1w\). The article selects the control parameters \(k_1 = 1.5, k_2 = 1.5, k_4 = 25, k_5 = 5\), the simulation results of mobile robot tracking \(R=1\) circular trajectory are shown in the following figures 8-10.

**Figure 7.** Tracking pose error. Figure 5 to 7 presents that mobile robot adjusts its direction in limited time, when \(t = 2.2s\), the actual position of mobile robot is \([1.2206 0.3817 1.0469]\), and desired position is \([1.2200 0.3811 1.0472]\). Then, position coordinate tracking is realized through forward motion, thus actual and desired trajectory is basically overlapped. We observe that linear speed increases linearly within specified time, until 2.6s, the control input signal \(v = 0.1999 \text{ m/s}\) and \(w = -0.062 \text{ rad/s}\) approaches reference speed. To the end, state error is basically zero.

**Figure 8** Linear tracking curve.

**Figure 9** Control law \(v, w\) curve.

**Figure 10** Tracking pose error. As shown in figure, 8 to 10, the linear and angular velocity curves oscillate and fluctuate. Mobile robot system is about \(2.4s\), its actual pose and expected pose is \([0.8999 0.4631 2.069]\), \([0.8870 0.4619 2.050]\). The direction of movement is adjusting in a limited time, it can smoothly track the circular trajectory, which making actual trajectory gradually converge desired trajectory, and pose error \(x, y, \theta\) quickly converge to zero about 2.5s. Mobile robot system control input signal as \(v = 0.2003 \text{ m/s}\), \(w = 0.1952 \text{ rad/s}\) in \(t = 3s\). Finally, speed converges to the reference speed over time.
5. Conclusions
This paper addressed the problem of tracking control around a desired position, taking into account the kinematic model for mobile robots. A new trajectory tracking control method for mobile robot based on sliding mode control theory is proposed. A new sliding mode control can make the system states converge to zero in finite time. The convergent characteristic of new sliding mode control is superior to that of the normal sliding mode control. Experiment test that the proposed controller exhibits good tracking performance. Simulation results show that the control method effectively not only restrains the chattering phenomenon, but also achieve the rapidity and accuracy of trajectory tracking. Besides, the validity of this method is verified.

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