A Note on the $J/\psi$ Strong Couplings

A. Deandrea
Institut de Physique Nucléaire, Université de Lyon I
4 rue E. Fermi, F-69622 Villeurbanne Cedex, France

G. Nardulli
Dipartimento di Fisica, Università di Bari and INFN Bari,
via Amendola 173, I-70126 Bari, Italia

A. D. Polosa
CERN - Theory Division
CH-1211 Geneva 23, Switzerland

Abstract

In this note we present an evaluation of the couplings $JD^{(*)}D^{(*)}$ and $JD^{(*)}D^{(*)}\pi$ in the Constituent Quark Model. These couplings are a crucial ingredient in the calculation of cross sections for the processes $\pi J/\psi \rightarrow D^{(*)}D^{(*)}$, an important background for the $J/\psi$ suppression signal in quark-gluon plasma.
A NOTE ON THE $J/\psi$ STRONG COUPLINGS

A. Deandrea, G. Nardulli and A.D. Polosa

Abstract
In this note we present an evaluation of the couplings $JD^{(*)}D^{(*)}$ and $JD^{(*)}D^{(*)}\pi$ in the Constituent Quark Model. These couplings are a crucial ingredient in the calculation of cross sections for the processes $\pi J/\psi \to D^{(*)}\bar{D}^{(*)}$, an important background for the $J/\psi$ suppression signal in quark-gluon plasma.

This note is a preliminary report on a study of absorption effects of the $J/\psi$ resonance due to its interaction with the hot hadronic medium formed in relativistic heavy ion scattering. We will give the full analysis elsewhere [11]; here we limit the presentation to the study of the strong couplings of $J/\psi$, low mass charmed mesons and pions. In the calculation of the relevant cross sections one encounters tree-level diagrams such as those depicted in Fig. 1. Previous studies of these effects can be found in [2]-[5]. Besides the $g(D\bar{D}^*\pi)$ couplings, for which both theoretical [7],[8] and experimental [9] results are available, in Fig. 1 the $JD^{(*)}D^{(*)}$ and $JD^{(*)}D^{(*)}\pi$ couplings appear. They have been estimated by different methods, that are, in our opinion, unsatisfactory. For example the use of the $SU(4)$ symmetry puts on the same footing the heavy quark $c$ and the light quarks, which is at odds with the results obtained within the Heavy Quark Effective Theory (HQET), where the opposite approximation $m_c \gg \Lambda_{QCD}$ is used (for a short review of HQET see [10]). Similarly, the rather common approach based on the Vector Meson Dominance (VMD) should be considered critically, given the large extrapolation $p^2 = 0 \to m_{J/\psi}^2$ that is involved. A different evaluation, based on QCD Sum Rules can be found in [10] and presents the typical theoretical uncertainties of this method. In this note we will use another approach, based on the Constituent Quark Model (CQM), which is a quark-meson model taking into account explicitly the HQET symmetries (for more details on the CQM see [11]).

The CQM model has turned out to be particularly suitable for the study of exclusive heavy meson decays. Since its Lagrangian contains the Feynman rules for vertices formed by a heavy meson, a heavy quark and a light quark, transition amplitudes are computable via simple (constituent) quark loop diagrams where mesons enter as external legs. The model is relativistic and incorporates, besides the heavy quark symmetries (for more details on the CQM see [11]).

In the CQM the evaluation of the loop diagram depicted on the l.h.s. of the VMD equation in Fig. 2 amounts to the calculation of the Isgur-Wise function, which can be found in [11]. The result is

$$\xi(\omega) = Z_H \left[ \frac{2}{1+\omega} I_5(\Delta_H) + \left( m + \frac{2\Delta_H}{1+\omega} I_5(\Delta_H,\Delta_H,\omega) \right) \right],$$

where the $I_i$ integrals are listed in the appendix. This result arises from the calculation of the following loop integral (for the $JDD$ process):

$$m_DZ_H \frac{iN_c}{16\pi^4} \int d^4\ell \frac{\text{Tr}[(\gamma \cdot \ell + m)\gamma_5(1 + \gamma \cdot \ell')\gamma_{\mu}(1 + \gamma \cdot \ell')\gamma_5]}{4(\ell^2 - m^2)(\ell \cdot \ell + \Delta_H)(\ell' \cdot \ell' - \Delta_H)},$$

where

$$\frac{1 + \gamma \cdot v}{2} \frac{1}{v \cdot k}$$

is the heavy quark propagator of the HQET, $v$ and $v'$ are the 4-velocities of the two heavy quarks; they are assumed equal, in the infinite quark mass limit, to the hadron $D^{(*)}$ velocities. On the other hand $\omega = v \cdot v'$. 

$$\omega_0 = \frac{(\Delta_H)^2}{4m_D^2}$$

is the heavy quark propagator of the HQET, $v$ and $v'$ are the 4-velocities of the two heavy quarks; they are assumed equal, in the infinite quark mass limit, to the hadron $D^{(*)}$ velocities. On the other hand $\omega = v \cdot v'$. 

$$\omega_0 = \frac{(\Delta_H)^2}{4m_D^2}$$
Let us also introduce $k$, the meson residual momentum, defined by $p^\mu_D = m_c v^\mu + k^\mu$; it enters the calculation through the parameter $\Delta_H = v \cdot k$ which is equal to the mass difference $m_D - m_c$. Its numerical value is in the range $0.3 - 0.5$ GeV \[\text{[1]}\]. If we consider a $D^*$ meson instead of a $D$, a factor $-\gamma_5$ must be substituted by $\gamma \cdot \epsilon$, $\epsilon$ being the polarization of $D^*$. The constant $Z_H$ arises from the coupling of $D^{(*)}$ mesons to their constituent quarks (more precisely the coupling constant is $\sqrt{Z_H m_D}$); $Z_H$ is computed and tabulated in \[\text{[1]}\].

We note that the Isgur-Wise function obeys the normalization condition $\xi(1) = 1$, arising from the flavor symmetry of the HQET. This is the Luke’s theorem, whose ancestor for the light flavors is the Ademollo-Gatto theorem \[\text{[12]}\]. The explicit definition of the Isgur-Wise form factor is:

$$\langle H(v')|\bar{c}\gamma_\mu c|H(v)\rangle = -\xi(\omega) \text{Tr} (\bar{H}\gamma_\mu H).$$  \hspace{1cm} (4)

Here $H$ is the multiplet containing both the $D$ and the $D^*$ mesons \[\text{[8]}\):

$$H = \frac{1 + \gamma \cdot v}{2} (-P_5 \gamma_5 + \gamma \cdot P),$$  \hspace{1cm} (5)

and $P_5$, $P^\mu$ are annihilation operators for the charmed mesons. As an example, for the transition between two pseudoscalar mesons $D$ one finds:

$$\langle D(v')|\bar{c}\gamma_\mu c|D(v)\rangle = m_D \xi(\omega) (v + v')_\mu.$$  \hspace{1cm} (6)

One can compute in the CQM the Isgur-Wise function for any value of $\omega$ and not only in the region $\omega > 1$, which is experimentally accessible via the semileptonic $B \rightarrow D^{(*)}$ decays. Since

$$\omega = \frac{p_1^2 + p_2^2 - p^2}{2 \sqrt{p_1^2 p_2^2}},$$  \hspace{1cm} (7)

($p_1$, $p_2$ = momenta of the two $D$ resonances), differently from the naive use of VMD, by our method we can have a control on the dependence on $p^2$ (and also on the off-shell behavior in the variables $p_1^2$, $p_2^2$).

Let us now consider the r.h.s. of the equation depicted in Fig. 2. For the coupling of $J/\psi$ to the current we use the matrix element

$$\langle 0|\bar{c}\gamma_\mu c|J(q, \eta)\rangle = f_J m_{J/\psi} e^\mu$$  \hspace{1cm} (8)

with $f_J = 0.405 \pm 0.014$ GeV. As to the strong couplings $JD^{(*)}D^{(*)}$, the model in Fig. 2 gives the following effective lagrangians

$$\mathcal{L}_{JDD} = ig_{JDD} \left( \bar{D}^{\alpha \nu} D^\nu D^\alpha \right),$$

$$\mathcal{L}_{JDD^*} = ig_{JDD^*} e^{\nu \alpha \beta} J_\nu \bar{D}^\alpha D^\beta,$$

$$\mathcal{L}_{JDD^*D^*} = ig_{JDD^*D^*} \left[ \bar{D}^{\mu \nu} (\partial_\mu D_\nu^*) J^{\nu} - D^{\mu \nu} (\partial_\mu \bar{D}_\nu^*) J^{\nu} \right. - \left. \left( \bar{D}^{\mu \nu} \partial_\nu D_\mu^* \right) J^{\nu} \right].$$  \hspace{1cm} (9)

As a consequence of the spin symmetry of the HQET we find:

$$g_{JDD^*D^*} = g_{JDD},$$

$$g_{JDD^*} = \frac{g_{JDD}}{m_D},$$  \hspace{1cm} (10)

while the VMD ansatz gives:

$$g_{JDD}(p_1^2, p_2^2) = \frac{m_{J/\psi}^2 - p^2}{f_J m_{J/\psi}} \xi(\omega).$$  \hspace{1cm} (11)
Since \( g_{JDD} \) has no zeros, eq. (11) shows that \( \xi \) has a pole at \( p^2 = m_{J/\psi}^2 \), which is what one expects on the basis of dispersion relations arguments. The CQM evaluation of \( \xi \) does show a strong peak for \( p^2 \approx (2m_c)^2 \), even though, due to \( \mathcal{O} \left( \frac{1}{m_c} \right) \) effects, the location of the singularity is not exactly at \( p^2 = m_{J/\psi}^2 \). This is shown in Fig. 3 where we plot \( g_{JDD}(p_1^2, p_2^2, s) \) for on shell \( D \) mesons, as a function of \( p^2 \) (the plot is obtained for \( \Delta_H = 0.4 \text{ GeV} \) and \( Z_H = 2.36 \text{ GeV}^{-1} \)). For \( p^2 \) in the range \((0,4) \text{ GeV}^2\), \( g_{JDD} \) is almost flat, with a value
\[
 g_{JDD} = 8.0 \pm 0.5 .
\] (12)

For larger values of \( p^2 \) the method is unreliable due to the above-mentioned incomplete cancellation between the kinematical zero and the pole (the distorted shape around the \( J/\psi \) pole suggests that the contribution of the nearby \( \psi(2S) \) pole could also be relevant). Therefore, we extrapolate the smooth behavior of \( g_{JDD} \) in the small \( p^2 \) region up to \( p^2 = m_{J/\psi}^2 \) and assume the validity of the result (12) also for on-shell \( J/\psi \) mesons. On the other hand in the \( p_1^2, p_2^2 \) variables we find a smooth behavior, compatible with that produced by a smooth form factor. Let us finally observe that the result (12) agrees with the outcome of the QCD sum rule analysis of \cite{13}; the smooth behavior of the form factor found in \cite{10} agrees with our result. This is not surprising, as the QCD sum rules calculation involves a perturbative part and a non perturbative contribution which is however suppressed; the perturbative term has its counterpart in CQM in the loop calculation of Fig. 2 and the overall normalization should agree as a consequence of the Luke’s theorem.

Let us now consider the \( J^D(s) D^*(s) \pi \) couplings. As discussed in \cite{[13]}, but see also \cite{8}, the leading contributions to the current matrix element \( \langle H'(v')\pi|\bar{c}\gamma^\mu c|H(v)\rangle \) in the soft pion limit (SPL) are the pole diagrams. The technical reason is that, in the SPL, the reducing action of a pion derivative in the matrix element is compensated in the polar diagrams by the effect of the denominator that vanishes in the combined limit \( q_\pi \rightarrow 0, m_c \rightarrow \infty \). Since the effect of the pole diagrams is explicitly taken account in Fig. 1, we should not include any further contribution. In any event, for the sake of a numerical comparison, let us consider the coupling \( g_{JDD\pi} \); it can be obtained by a VMD ansatz similar to Fig. 2, but now the l.h.s is modified by the insertion of a soft pion on the light quark line (with a coupling \( q_\pi^2/f_\pi\gamma_5\gamma_5 \)). We call \( \xi^\pi(\omega) \) the analogous form factor in the soft pion limit and we find:
\[
 \xi^\pi(\omega) = Z_H \left[ \frac{4m + 2\Delta_H}{1 + \omega} I_4(\Delta_H) - \left( m^2 + \frac{2\Delta_H^2 + 4m\Delta_H}{1 + \omega} \right) \frac{\partial I_5(\Delta_H, \Delta_H, \omega)}{\partial m^2} \right] .
\] (13)

(the integral \( I_4 \) is given in the appendix). On the other hand from the VMD ansatz of Fig. 2 we get
\[
 \mathcal{L}_{JDD\pi} = ig_{JDD\pi} \epsilon_{\mu\nu\alpha\beta} J_\mu \partial_\nu D\partial_\alpha \bar{D}\partial_\beta \pi
\] (14)
and
\[
 g_{JDD\pi}(p_1^2, p_2^2, p^2) = \frac{(m_j^2 - p^2)\xi^\pi(\omega)}{f_\pi f_D m_j}.
\] (15)

In Fig. 4 we plot our result for the \( g_{JDD\pi} \) coupling with on shell \( D \) mesons. By the same arguments used to determine \( g_{JDD} \) in Fig. 3 we get, with all mesons on the mass-shell,
\[
 g_{JDD\pi} = 125 \pm 15 \text{ GeV}^{-3} .
\] (16)

Let us now compare this result with the effective \( JDD\pi \) coupling obtained by a polar diagram with an intermediate \( D^* \) state. We get in this case
\[
 g_{JDD\pi}^{\text{polar}} \approx \frac{g_{JDD} \cdot g_{D^* D\pi}}{2q_\pi \cdot p_D} .
\] (17)
All the calculations presented in this note are valid in the SPL, therefore one should consider pion momenta not larger than a few hundred MeV. Using \[9\] the result \(g_{D^*D^\pi} = 2m_D/f_{\pi D}\), with \(g = 0.59 \pm 0.01 \pm 0.07\), we get therefore \(g_{J\pi DD^\pi}^{\text{polar}} \approx 393,196,98 \text{ GeV}^{-3}\) for \(|q_\pi|\) respectively equal to 50, 100, 200 MeV. This analysis shows that, within the region of validity of the model, in spite of the rather large value of the coupling \(13\), the diagrams containing this coupling are in general suppressed. Similar conclusions are reached considering \(D^*\) mesons instead of \(D\) mesons.

Let us finally discuss the kinematical limits of our approach. To allow the production of a \(D^{(*)}D^{(*)}\) pair, as shown in Fig. 1, we must extend the region of validity of the model beyond the SPL, since the threshold for the charmed meson pair is \(|q_\pi| = 700 - 1000 \text{ MeV}\). The CQM, as the other models existing in the literature, is a chiral model and this puts limits on the pion momenta. Therefore one has to include a form factor enhancing the small pion momenta region, for example

\[
f(|q_\pi|) = \frac{1}{1 + \frac{|q_\pi|}{m_\chi}}.
\]

A similar form factor is considered in \([3]\), with a different motivation. Here we introduce it to ensure the validity of our approach (in this sense the cross sections we can compute by this model should be considered as a lower bound). Since the main effect of \((13)\) should be that of reducing contributions from pion momenta larger than a few hundred MeV, a reasonable estimate for \(m_\chi\) is in the range 400-600 MeV. This choice implies that the direct couplings of Fig. 1 (diagrams 1c, 2d and 3e) should not dominate the final result since their contribution is larger where the form factor is more effective.

**Appendix**

We list the expressions used to numerically compute the integrals \(I_i\) quoted in the text. The ultraviolet cutoff \(\Lambda\), the infrared cutoff \(\mu\) and the light constituent mass \(m\) are fixed in the model \([11]\) to be \(\Lambda = 1.25 \text{ GeV}, \mu = 0.3 \text{ GeV}\) and \(m = 0.3 \text{ GeV}\).

\[
I_3(\Delta) = -\frac{iN_c}{16\pi^4} \int_{1/\Lambda^2}^{\text{reg}} d^4k \frac{d^4k}{(k^2 - m^2)(v \cdot k + \Delta + i\epsilon)}
\]

\[
= \frac{N_c}{16\pi^3/2} \int_{1/\Lambda^2}^{1/\mu^2} ds \frac{d^4k}{s^{3/2}} e^{-s(m^2 - \Delta^2)} (1 + \text{erf}(\Delta \sqrt{s}))
\]

\[
I_4(\Delta) = -\frac{iN_c}{16\pi^4} \int_{1/\Lambda^2}^{\text{reg}} d^4k \frac{d^4k}{(k^2 - m^2)(v \cdot k + \Delta + i\epsilon)}
\]

\[
= \frac{N_c}{16\pi^3/2} \int_{1/\Lambda^2}^{1/\mu^2} ds \frac{d^4k}{s^{1/2}} e^{-s(m^2 - \Delta^2)} [1 + \text{erf}(\Delta \sqrt{s})]
\]

\[
I_5(\Delta_1, \Delta_2, \omega) = \frac{iN_c}{16\pi^4} \int_{1/\Lambda^2}^{\text{reg}} d^4k \frac{d^4k}{(k^2 - m^2)(v \cdot k + \Delta_1 + i\epsilon)(v' \cdot k + \Delta_2 + i\epsilon)}
\]

\[
= \int_0^1 dx \frac{1}{1 + 2x^2(1 - \omega) + 2x(\omega - 1)} \times
\]

\[
\left[ \frac{6}{16\pi^3/2} \int_{1/\Lambda^2}^{1/\mu^2} ds \frac{d^4k}{s^{3/2}} e^{-s(m^2 - \sigma^2)} s^{-1/2} (1 + \text{erf}(\sigma \sqrt{s})) + \frac{6}{16\pi^2} \int_{1/\Lambda^2}^{1/\mu^2} ds e^{-s\sigma^2} s^{-1} \right],
\]

where

\[
\sigma(x, \Delta_1, \Delta_2, \omega) = \frac{\Delta_1 (1 - x) + \Delta_2 x}{\sqrt{1 + 2 (\omega - 1) x + 2 (1 - \omega) x^2}}.
\]
Fig. 1: Feynman diagrams for $J/\psi$ absorption by the pion. (1) $J/\psi \pi \rightarrow D \bar{D}$, (2) $J/\psi \pi \rightarrow D D^*$ and $J/\psi \pi \rightarrow D^* D^*$. 

References

[1] A. Deandrea, G. Nardulli and A.D. Polosa, in preparation.
[2] Z.W. Lin and C.M. Ko, Phys. Rev. C 62, 034903 (2000) [arXiv:nucl-th/9912046];
[3] K. L. Haglin and C. Gale, Phys. Rev. C 63, 065201 (2001) [arXiv:nucl-th/0010017].
[4] Y. Oh, T. Song and S.H. Lee, Phys. Rev. C 63, 034901 (2001) [arXiv:nucl-th/0010064].
[5] J. w. Qiu, J. P. Vary and X. f. Zhang, Phys. Rev. Lett. 88, 232301 (2002) [arXiv:hep-ph/9809442].
[6] H. Fujii, arXiv:hep-ph/0209197.
[7] P. Colangelo, F. De Fazio and G. Nardulli, Phys. Lett. B 334 (1994) 175 [arXiv:hep-ph/9406320];
    P. Colangelo, G. Nardulli, A. Deandrea, N. Di Bartolomeo, R. Gatto and F. Feruglio, Phys. Lett. B
    339, 151 (1994) [arXiv:hep-ph/9406295].
[8] R. Casalbuoni, A. Deandrea, N. Di Bartolomeo, R. Gatto, F. Feruglio and G. Nardulli, Phys. Rept.
    281, 145 (1997) [arXiv:hep-ph/9605342].
[9] S. Ahmed et al. [CLEO Collaboration], Phys. Rev. Lett. 87, 251801 (2001) [arXiv:hep-ex/0108013].
[10] R. D. Matheus, F. S. Navarra, M. Nielsen and R. Rodrigues da Silva, Phys. Lett. B 541, 265 (2002)
    [arXiv:hep-ph/0206198]; F. O. Duraes, S. H. Lee, F. S. Navarra and M. Nielsen, arXiv:nucl-
    th/0210075.
[11] A. Deandrea, N. Di Bartolomeo, R. Gatto, G. Nardulli and A.D. Polosa, Phys. Rev. D 58, 034004
    (1998) [arXiv:hep-ph/9802308]; A.D. Polosa, Riv. Nuovo Cim. 23N11, 1 (2000) [arXiv:hep-
    ph/0004183].
[12] M. Ademollo and R. Gatto, Phys. Rev. Lett. 13, 264 (1964); M. E. Luke, Phys. Lett. B 252, 447
    (1990).
[13] A. F. Falk, H. Georgi, B. Grinstein and M. B. Wise, Nucl. Phys. B 343, 1 (1990).
Fig. 2: The Vector Meson Dominance equation giving the coupling of $J/\psi$ with $D, D^*$ in terms of the Isgur-Wise function $\xi$. The function $\xi$ on the l.h.s. is computed by a diagram with a quark loop. The coupling of each $D^{(*)}$ meson to quarks is given by $\sqrt{Z_H} m_D$.

Fig. 3: The $p^2$ dependence of $g = g_{JDD}(m_D^2, m_D^2, p^2)$, showing the almost complete cancellation between the pole of the Isgur-Wise function and the kinematical zero. Units are GeV$^2$ for $p^2$.

Fig. 4: The $p^2$ dependence of $g^\pi = g_{JDD\pi}(m_D^2, m_D^2, p^2)$; as in Fig. 3 there is an almost complete cancellation between the pole of the form factor and the kinematical zero. Units are GeV$^2$ for $p^2$ and GeV$^{-3}$ for $g_{JDD\pi}$.