Quantum Zeno effect and parametric resonance in mesoscopic physics

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As a realization of the quantum Zeno effect, we consider electron tunneling between two quantum dots with one of the dots coupled to a quantum point contact detector. The coupling leads to decoherence and to the suppression of tunneling. When the detector is driven with an ac voltage, a parametric resonance occurs which strongly counteracts decoherence. We propose a novel experiment with which it is possible to observe both the quantum Zeno effect and the parametric resonance in electric transport.

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The interaction of a quantum system with a macroscopic measurement device is known\textsuperscript{1} to generate decoherence. The frequent repetition of a decohering measurement leads to a striking phenomenon known as the quantum Zeno effect\textsuperscript{2}: The suppression of transitions between quantum states. The standard example is a two–level system with a tunneling transition between the two levels. For small times, the probability to tunnel out of level system with a tunneling transition between the two quantum states. The standard example is a two–level system with a tunneling transition between the two levels. For small times, the probability to tunnel out of one of the two levels is \( \sim t^2 \). With a device that projects the system onto that same level, \( N \) repeated measurements yield the reduced probability \( \sim N(t/N)^2 \). The suppression of tunneling in bound systems has its parallel in systems with unstable states. Here, the quantum Zeno effect predicts the suppression of decay and the enhancement of the lifetime of unstable states.

Despite considerable theoretical work on the quantum Zeno effect there is only little experimental proof for it. An experimental test using an induced hyperfine transition of Be ions\textsuperscript{3} has been reported. Experiments on optical transitions\textsuperscript{4} or atomic Bragg scattering\textsuperscript{5} have been proposed. However, the observation of the quantum Zeno effect in some of these experiments is hampered by other sources of decoherence not related to the measurement. Further experimental evidence for the existence of the quantum Zeno effect is clearly desirable. Gurvitz\textsuperscript{6} first pointed out that the quantum Zeno effect may be observed in semiconductor microstructures. He studied theoretically the tunneling of an electric charge between two weakly coupled quantum dots. A quantum point contact (QPC) located in the vicinity of one of the dots served as a non–invasive detector for the charge on that dot. As expected from the quantum Zeno effect, Gurvitz found that the coupling to the QPC suppresses the tunneling oscillations between the two dots.

In this Letter we use a microscopic theory to study the quantum Zeno effect in coupled quantum dots. We propose an arrangement which is within reach of present–day experimental techniques, and through which the quantum Zeno effect can be investigated. Our work is motivated by the first experimental demonstration of complementarity and controlled dephasing in a which–path semiconductor device by Buks et al.\textsuperscript{7}. The observation of interference in an Aharonov–Bohm interferometer with a quantum dot (QD) embedded in one of its arms showed that electrons pass coherently through the device and the QD. A QPC measuring the electric charge in the QD led to dephasing. Several theoretical explanations of the experimental results have been given\textsuperscript{8}. Our work goes considerably beyond that of Ref.\textsuperscript{9}. Indeed, we identify several novel aspects of the quantum Zeno effect in coupled quantum dots. First, we show that the application of an AC voltage with frequency \( \omega \) across the QPC leads to parametric resonance and to a strong reduction of decoherence. The resonance occurs when \( \omega \) equals twice the frequency \( \omega_0 \) of the internal charge oscillations in the double–dot system. Second, the power spectrum of the QPC displays a peak which is a clear signal for the quantum Zeno effect. Third, we propose a new transport experiment with the two quantum dots in a parallel circuit and each dot coupled to external leads. With current flowing into the first dot, we calculate the branching ratio of the current transmitted through the second dot and that through the first dot. Measurements with the QPC–detector necessarily induce an energy exchange between the dots and the QPC. Therefore, the coupling to the QPC induces a correction to the branching ratio which has both an elastic and an inelastic contribution. This correction is proportional to the decoherence rate. A measurement of the branching ratio thus provides a direct signature of dephasing, and of the quantum Zeno effect in the two coupled quantum dots.

We deal first with a simple system: Two coupled quantum dots without coupling to external leads and occupied by a single (excess) electron. We consider only one energy level in each dot and assume that both levels are degenerate. Let \( E_0 \) denote the energy of each level and \( \Omega_0/2 \) with \( \Omega_0 = \hbar \omega_0 \) the coupling matrix element between the two dots. The lower dot interacts with a QPC, which continuously measures the electron position. The QPC is modeled as a single–channel device which is symmetric with respect to the lower dot. The relation of this simple model to a realistic dot system is briefly discussed later. In order to derive a master equation for the density matrix \( \rho_{\text{dot}} \) of the two quantum dots, we calculate the to-
tal density matrix of dots plus QPC and then trace out the QPC-variables. The total density matrix is obtained from a scattering approach which generalizes a method first introduced in Ref. 3. The scattering matrix $S_{\text{QPC}}$ through the QPC has dimension two and depends on the location of the electron. If the electron is on the lower [upper] dot, we write $S$ in the form $S_{\text{QPC}} = \exp(i\theta_1a_x)$

$$[S_{\text{QPC}} = \exp(i\theta_a a_x)],$$

respectively. Here, $\tau$ is a Pauli spin matrix. In both cases we have suppressed a global phase, and we have used time–reversal symmetry and the above–mentioned symmetry of the QPC. Both cases can be combined in writing the two–particle scattering matrix as $S_{\sigma \sigma'} = \delta_{\sigma \sigma'}[\delta_{\sigma a} e^{i\theta a} + \delta_{\sigma u} e^{i\theta u} a_x]$, where $\sigma$ and $\sigma'$ both stand for either quantum dot labeled $l$ and $u$, respectively. With $\rho^{(0)} = \rho_{\text{dot}} \rho_{QPC}$ the density matrix of the total system prior to the passage of an electron through the QPC, the density matrix after scattering through the QPC is given by $\rho = S\rho^{(0)} S^\dagger$. The reduced density matrix of the two dots is obtained by tracing over the QPC-variables and given by

$$\rho_{\text{dot}} = \text{Tr}_{\text{QPC}} \rho = \rho_{\text{dot}}^{(0)} - \frac{1}{2} (\Delta \theta)^2 \left[ \begin{array}{cc} \rho_{\text{dot},12}^{(0)} & \rho_{\text{dot},11}^{(0)} \\ \rho_{\text{dot},21}^{(0)} & \rho_{\text{dot},22}^{(0)} \end{array} \right].$$

We have used $\Delta \theta = \theta_u - \theta_l$. Assuming that the coupling between the lower dot and the QPC is weak, we have expanded $\cos(\Delta \theta) \approx 1 - (\Delta \theta)^2/2$. The term $(\Delta \theta)^2$ can be expressed in terms of the transmission coefficients $T_l, T_u$ [4] so that $(\Delta \theta)^2 = (\Delta T)^2/[4T(1 - T)]$ where $\Delta T = T_u - T_l$ and $T = (T_u + T_l)/2$. We simplify the notation by writing the last term in Eq. (1) as $- (\Delta \theta)^2 \Sigma/2$. We note that with $\mu$ the voltage drop across the QPC, the time between two scattering events in the QPC is given by $\Delta t = h/(2\mu)$. During this time, the dynamics of the two–dot system is governed by the tunneling Hamiltonian $\Omega_0 \sigma_x/2$. Combining this fact with Eq. (1), we obtain the master equation

$$\frac{d \rho_{\text{dot}}}{dt} = -\frac{(\Delta T)^2}{8T(1 - T)} \frac{e\mu}{\pi \hbar} \Sigma - i\frac{\Omega_0}{\hbar} \sigma_x \rho_{\text{dot}}.$$  

A master equation for the double–dot system has previously been derived by Gurvitz [5] using a different method. We note that the factor $1 - T$ is missing in Gurvitz’ result.

Since $\rho_{\text{dot}} = \rho_{\text{dot}}^{(0)}$ and $\text{Tr} \rho = 1$, we can parameterize $\rho_{\text{dot}}$ by the two quantities $a = 1/2 - \rho_{\text{dot},11}, b = \rho_{\text{dot},12}$. Substitution into Eq. (2) yields the equations of motion $(da)/(dt) = \omega_0 b a$ and

$$\frac{d^2 a}{dt^2} + \frac{(\Delta T)^2}{8T(1 - T)} \frac{e\mu}{\pi \hbar} \frac{da}{dt} + \omega_0^2 a = 0.$$ 

For a time–independent voltage drop $\mu = \mu_0$ both $a$ and $b$ display damped oscillations $\sim \exp[-\kappa t] \cos \omega_0 t$ with the damping constant $\kappa = (\Delta \theta)^2 e\mu/(4\pi \hbar)$. The exponential suppression of oscillations in $a$ clearly demonstrates the quantum Zeno effect. The off–diagonal elements of $\rho$ vanish for large times in agreement with the standard picture of decoherence, and the density matrix reduces to the random statistical ensemble $\rho_{\text{dot}} \rightarrow \text{diag}(1/2, 1/2)$. The charge oscillations in the double–dot system modulate the current in the QPC and, hence, modify its power spectrum. They cause a peak with FWHM $2\kappa$ centered at the shifted frequency $\omega_0 \sqrt{1 - \kappa^2/\omega_0^2}$. This peak will be present in addition to standard shot noise. Both location and width of this peak are clear signatures of the quantum Zeno effect. The experimental investigation of these features would amount to a time–resolved study of the quantum–mechanical measurement process and would be of considerable interest.

Interesting new physical aspects arise if $\mu$ has an AC–component, $\mu(t) = \mu_0 - \mu_1 \sin \omega t$ where $\mu_0, \mu_1 \geq 0$ and $\mu_1 \leq \mu_0$. According to Eq. (2) this corresponds to a harmonic oscillator with an oscillatory damping constant. Using a simple ansatz for $a(t)$ and neglecting terms of order $O(\mu^2)$, one is led to an equation of the Mathieu type which is known [12] to display parametric resonance close to the frequencies $\omega = 2\omega_0/n$ where $n$ is a positive integer. Parametric resonance is most pronounced for $\omega \approx 2\omega_0$. The damping near the resonance is strongly reduced, $\kappa = \frac{\pi}{2(4\omega_0^2)} \Omega_0^2 [\mu_0 - \frac{1}{2}\mu_1]$. The resulting time evolution of $a$ near resonance is illustrated in Fig. 1 and compared with the case where $\mu$ is time–independent. The resonance condition $\omega = 2\omega_0$ is interpreted as follows: The position of the electron is not measured when $\mu$ is close to zero. The electron uses this time to tunnel from one dot to the other.

![Fig. 1. Parametric resonance in the double-dot system coupled to a QPC-detector. The upper part shows the oscillations of the diagonal elements of the density matrix for constant voltage (dashed curve) and for a time-dependent voltage as shown in the lower part (full curve). Damping of the oscillations is reduced by a factor two.](image)

We now ask: How does the quantum Zeno effect manifest itself in a transport experiment? We consider the arrangement shown in Fig. 2 with the two dots in parallel and each dot coupled to two external leads. For simplicity we consider only one transverse channel in each lead labeled $c$. The quantum dots are assumed to be in the
resonant tunneling regime close to a Coulomb blockade resonance so that it is sufficient to consider only a single level in each dot. Both levels are assumed to have the same energy $E_0$ and width $\Gamma$. The QPC detector is described in terms of plane waves with energy $\epsilon_k$ and mean density $\rho_F$ which are scattered from a spatially local potential with Fourier components $U_{kk'}$. To model the capacitive coupling of the QPC with the lower dot, we write the Hamiltonian for the QPC plus interaction as

$$H' = \sum_k \epsilon_k b_k^\dagger b_k + \sum_{k, k'} (U_{kk'} + V_{kk'}) d_{k}^\dagger d_{k'} b_{k'}^\dagger b_{k'}.$$ (4)

Here $b_k^\dagger$ and $d_{\sigma}^\dagger$, $\sigma = l, u$ denote the creation operators for the QPC states and for the states on either QD, respectively. We note that the interaction vanishes for an electron on the upper dot (see Fig. 2).

$$\begin{array}{ccc}
T_u & u & T_u \\
R_L & \Omega_0 & T_1 \\
V & & \\
\end{array}$$

FIG. 2. Upper part: double–dot system coupled to a quantum point contact. Lower part: the table shows the change of transmission and reflection coefficients through the double–dot system.

To find the transmission through the system, we calculate the two–particle scattering amplitude $S_{\sigma c, \sigma' c'; kk'}$ for the joint transition between channels $\sigma c$ and $\sigma' c'$ in the leads and the states $k$ and $k'$ in the detector from the Lippmann–Schwinger equation. This yields

$$\begin{array}{l}
\Delta T_u = -4C + C \\
\Delta R_L = 4C + (1 + 2T^2)/(2\Omega_0^2) C \\
\Delta T_L = -4\epsilon_0 C /\Omega_0^2 + (1 + 2T^2)/(2\Omega_0^2) C \\
C = (\epsilon_0 p_{\sigma} /\Omega_0^2) / (2(1 - T) \Omega_0^2) \\
\end{array}$$

and the connected Green function $G_{kk'}^{\text{conn}}(\Omega) = G_0^{-1}(\Omega) - \frac{4}{\Gamma^2 + \Omega^2} \frac{-V}{\Gamma_U F_{U+V}} \frac{\delta_{\epsilon_k, \epsilon_{k'}} + \Omega}{(2(\Omega + \Gamma)^2 - \Omega_0^2)} \times \frac{\Omega_0^2}{\Omega_0^2 - i\Omega_0 \Gamma}.$ (7)

Here, $F_U = 1 + 2\pi iU \rho_F$. The $\Omega$–dependent prefactor effectively limits inelastic processes to an interval of width $\Gamma$ around $E_0 + \epsilon_k$. The energy exchange is essential to ensure the unitarity of the $S$–matrix. Moreover, it allows for a position measurement of the dot electron without violating the Heisenberg uncertainty relation.

To calculate the single–particle transmission and reflection coefficients through the double–dot system we trace over the degrees of freedom of the QPC. We emphasize that we add the two–particle scattering probabilities and not the two–particle scattering amplitudes since the paths of electrons going through the QPC can be observed in principle. In the case $V = 0$ all scattering processes are elastic and the transmission
and reflection coefficients at resonance $E = E_0 + \epsilon_k$ are $T^{(0)}_u = \Omega_0^2 T^2 (\Gamma^2 + \Omega_0^2)^{-2}$, $T^{(0)}_l = \Gamma^4 (\Gamma^2 + \Omega_0^2)^{-2}$, and $R^{(0)}_l = \Omega_0^4 (\Gamma^2 + \Omega_0^2)^{-2}$. The branching ratio is $T^{(0)}_u / T^{(0)}_l = \Omega_0^2 / \Gamma^2$. We note that these branching ratios have no oscillatory dependence on $\Omega_0 / \Gamma$ as one might expect when naively applying the idea of time-dependent oscillations between the two quantum dots to a transport problem.

Corrections to the transmission coefficients due to the interaction $V$ are calculated in a weak–coupling expansion to second order in $V$. This is the appropriate limit realized experimentally [8]. In this limit the application of a drain source voltage $\mu$ across the QPC is equivalent to the simultaneous scattering of $2e\mu \rho_F$ particles in different longitudinal QPC modes. The total effect of these particles is obtained by multiplying the result for one QPC–particle with the number of longitudinal modes.

The corrections to the transmission and reflection coefficients due to $V$ are collected in Fig. 2 and arise both from coherent (elastic) and incoherent (inelastic) scattering. When the quantum dots are embedded in an Aharonov–Bohm interferometer only the coherent part of transport contributes to the flux–dependent current oscillations. However, in the setup depicted in Fig. 2, one measures the total current. The results show that measurements with the QPC detector have a twofold effect: (i) They suppress tunneling from the feeding lead into the lower dot and (ii) they suppress tunneling from the lower into the upper dot. Observation (i) follows from the increase in reflection, and (ii) from the decrease of the branching ratio

$$T_u = \frac{\Omega_0^2}{\Gamma^2} \left[ 1 - \frac{e\mu}{\pi \Gamma} \frac{(\Delta T)^2}{4 T (1 - T)} \right].$$  
(8)

Both effects (i), (ii) have an obvious interpretation as manifestations of the quantum Zeno effect. We note that the coefficients given in Tab. 1 depend on our choice for the scattering amplitudes $\gamma_{sc}$ at the junctions in the circuit of Fig. 2. However, one universally finds an increase of reflection and a decrease of the branching ratio. Moreover, the coefficient given in square brackets in Eq. (8) is independent of the $\gamma_{sc}$’s. We add that the second term in the square bracket is up to a factor $\Gamma/(4\hbar)$ the damping constant found for the isolated double–dot system. The appearance of the damping constant in the branching ratio shows that the parametric resonance discussed above for isolated dots can also be observed in a transport experiment.

An interesting special case of the S–matrix (8) is obtained for $\Omega_0 = 0$ when the two dots are completely isolated from each other. In this limit we can study how the interaction with the QPC reduces the transmission through a single dot. This corresponds to the reduction of the Aharonov–Bohm contrast in the experiment of Ref. [8]. For the modulus of the elastic transmission amplitude at resonance, we find

$$|t_{el}| = \sqrt{T_{el}} = 1 - \frac{e\mu \rho_F}{\pi \Gamma} \frac{(\Delta T)^2}{2 T (1 - T)}$$  
(9)

which is up to a factor of 1/2 in agreement with previous calculations [11]. The missing 1/2 is recovered [14] if one includes the Fermi sea in the QPC.

How are these results modified by the Fermi sea in the leads? Eqs. (3,7) show that a measurement with the QPC necessarily involves an energy transfer between the QPC and the dots. Our result (8) applies provided this transfer is not restricted by phase space. In this case, we predict dephasing due to the QPC even for zero temperature. This case is realized if the applied drain source voltages are much larger than the resonance widths as in the experiment of Ref. [8]. In the opposite regime of small drain source voltages the Fermi surfaces block all inelastic processes. In this case, we find [14] that dephasing at $T = 0$ is completely suppressed, in agreement with a recent general theorem [13].

In summary, we have investigated the quantum Zeno effect for a system of two quantum dots coupled to each other and to a QPC, and either isolated from the rest of the world or connected to it by leads. In the first case, the frequency spectrum of the QPC displays clear signatures of the quantum Zeno effect. In the second case, the ratio of the transmission coefficients carries similar information.

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