Charmonium Hybrid Production in Exclusive B Meson Decays

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Recent data on charmonium production in B-meson decays suggest that charmonium hybrid mesons with mass \( \sim 4 \) GeV may be produced in \( B \)-decay via \( c\bar{c} \) colour octet operators. Some of these states are likely to be narrow with clean signatures to \( J/\psi \pi^+\pi^- \) and \( J/\psi \gamma \) final states. Experimental signatures and search strategies for existing \( B \)-factories are described.

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I. INTRODUCTION

The existence of gluonic excitations in the hadron spectrum is one of the most important unanswered questions in hadron physics \(^1\). Hybrid mesons form one such class which consists of a \( q\bar{q} \) with an excited gluonic degree of freedom. Although there is mounting evidence for hybrids consisting of light quarks they still await confirmation \(^2\). Recent observations of charmonium states in exclusive \( B \)-meson decays \(^3\) \(^4\) \(^5\) \(^6\) \(^7\) \(^8\) \(^9\) \(^10\) indicate that charmonium is produced significantly in the color octet. \(^8\) leading us to argue that charmonium hybrids (\( \psi_c \)) \(^11\) should also be produced in \( B \)-meson decay \(^12\) \(^13\). The unambiguous discovery of such a state would herald an important breakthrough in hadronic physics, and indeed, in our understanding of Quantum Chromodynamics, the theory of the strong interactions. It would also provide important input to refine models of hadron structure. In this letter we examine the production of charmonium hybrids and how one might observe these new types of mesons in \( B \)-decays. While various elements of our arguments have appeared elsewhere, by putting all the ingredients together, a more complete picture of charmonium hybrid production and detection emerges that, we hope, will encourage experimenters to pursue the necessary analysis. In some cases we can only resort to what are at best order of magnitude estimates of various processes. However, we expect that they will provide useful guidance for the initial exploration of this new physics frontier. Our goal is to point out that it may be possible to discover charmonium hybrids in \( B \) decay and to suggest likely signatures to do so. We review their spectroscopy, argue for their production in \( B \)-decays, and suggest experimental strategies for detecting charmonium hybrid mesons.

II. HYBRID CHARMONIUM SPECTROSCOPY

Lattice gauge theory and hadron models predict a rich spectroscopy of charmonium hybrid mesons \(^11\) \(^14\) \(^15\) \(^16\) \(^17\) \(^18\) \(^19\) \(^20\) \(^21\). For example, the flux tube model predicts \( 8 \) low lying hybrid states in the \( 4 \) to \( 4.2 \) GeV mass region with \( J^{PC} = 0^{\pm\mp} \), \( 1^{\mp\mp} \), \( 2^{\mp\mp} \), and \( 1^{\pm\pm} \). Of these states the \( 0^{+-} \), \( 1^{-+} \), and \( 2^{+-} \) have exotic quantum numbers; quantum numbers not consistent with the constituent quark model. The flux-tube model predicts \( M(\psi_c) \approx 4 - 4.2 \) GeV \(^14\) \(^17\); lattice QCD predictions for the \( J^{PC} = 1^{-+} \) state range from \( 4.04 \) GeV to \( 4.4 \) GeV \(^17\) \(^18\) with a recent quenched lattice QCD calculation \(^20\) finding \( M(1^{-+}) = 4.428 \pm 0.041 \) GeV. These results have the \( 1^{-+} \) lying in the vicinity of the \( D^{*+}D \) threshold of \( 4.287 \) GeV. There is the tantalising possibility that the \( 1^{-+} \) could lie below \( D^{*+}D \) threshold and therefore be relatively narrow.

III. HYBRID PRODUCTION

Recent developments in both theory and experiment lead us to expect that charmonium hybrids will be produced in \( B \) decays. The partial widths for \( B \rightarrow c\bar{c} + X \), with \( c\bar{c} \) representing specific final states such as \( J/\psi, \psi', \chi_{c0}, \chi_{c1}, \chi_{c2}, \chi_{c3}, D_2^+, D_2 \) etc., have been calculated in the
NRQCD formalism which factorizes the decay mechanism into short (hard) and nonperturbative (soft) contributions. The hard contributions are fairly well understood but the soft contributions, included as colour singlet and colour octet matrix elements, have model dependent uncertainties. Insofar as hybrid $c\bar{c}$ wavefunctions have a non-trivial colour representation they can be produced via a colour octet intermediate state.

Over the last decade there has been great theoretical progress in the understanding of exclusive $B$ decay to hidden charm final states. Central to this is the recognition of the importance of the colour octet contributions to these decays. Although the colour octet terms are higher order in the velocity expansion for the soft contributions, the Wilson coefficients for the colour octet subprocess are significantly larger than that of the colour singlet subprocess in the hard contributions to the decay. The net result is that the colour octet components play an important role in these decays. This has been dramatically confirmed by the observation by the CLEO, Babar, and Belle collaborations of the decays $b \to \chi_{c0}$ and $b \to \chi_{c2}$ which proceed via colour octet and have been measured to have comparable branching fractions to the decays $b \to \psi + X$ and $b \to \chi_{c1}$ which have sizeable colour singlet components. Calculations of the BR's to the $1D(\bar{c}c)$ states, which are higher order in the NRQCD expansion but have a large colour octet contribution, also predict BR's roughly comparable to those of other charmonium states.

The NRQCD approach assumes a Fock space expansion of the state which gives a velocity scaling of the various wavefunction terms contributing to the soft process. For conventional mesons the colour singlet term is the leading term in the expansion and the colour octet term is higher order in $v$. In contrast, in many models describing hybrid states with constituent glue, such as the bag model and constituent gluon models, the colour octet $c\bar{c}$ configuration is the leading term. In the flux tube model the factorization of the $Q\bar{Q}$ colour is not so clear and the main uncertainty is estimating the hadronic matrix elements. Chiladze et al. estimated the octet matrix element by explicitly calculating the wavefunction in an approximation to the Isgur-Paton flux tube model while Close estimated the S-wave matrix element by rescaling the P-wave octet matrix element by $1/v$. Chiladze et al. estimated the branching ratio $BR[B \to \psi_g(0^{++}) + X] \sim 10^{-3}$ for $M \sim 4$ GeV (though recent quenched lattice calculations suggest $M(0^{++}) = 4.70 \pm 0.17$ GeV, and hence will be inaccessible). Note that the decay to the $0^{++}$ hybrid is suppressed by a spin factor of $1/3$ while hybrid states with higher spin have larger statistical factors leading to larger branching ratios. Close et al. estimate a similar branching ratio to $1^{-+}$ and argued that if $M_g < 4.7$ GeV, the total branching ratio to $\psi_g$ for all $J^{PC}$ could be $BR[\psi_g(0^{++}) + X] \sim O(1\%)$. Thus, using two different approaches for estimating $BR[B \to \psi_g + X]$ both Chiladze et al. and Close et al. obtain similar results. Both calculations estimate BR's of $O(0.1 - 1\%)$ which are comparable to the BR's for conventional $c\bar{c}$ states. We conclude that charmonium hybrids should be expected to be produced with roughly the same branching fractions as conventional charmonium states.

IV. DECAYS

There are three important decay modes for charmonium hybrids: (i) the Zweig allowed fall-apart mode $\psi_g \to D^{(*)}D^{(*)}$; (ii) the cascade to conventional $c\bar{c}$ states, of the type $\psi_g \to (c\bar{c})(gg) \to (c\bar{c}) + \text{(light hadrons)}$ and $\psi_g \to (c\bar{c}) + \gamma$; (iii) decays to light hadrons via intermediate gluons, $\psi_g \to (ng)$, analogous to $J/\psi \to \text{light hadrons}$ and $\eta_c \to \text{light hadrons}$. Each mode plays a unique role. $\psi_g$ hybrids with exotic $J^{PC}$ quantum numbers offer the most unambiguous signal since they do not mix with conventional quarkonia.

(i) Decays to $D^{(*)}D^{(*)}$: In addition to $J^{PC}$ selection rules (for example, $2^{-}\to 2^{-}$ decay to $D\bar{D}$ is forbidden by parity and the exotic hybrid $\psi_g(0^{++})$ decays to $D^{(*)}D^{(*)}$ final states are forbidden by $P$ and/or $C$ conservation) a general feature of most models of hybrid meson decay is that decays to two mesons with the same spatial wave function are suppressed. Therefore, decays to $D\bar{D}$ should not occur, but small differences in wavefunctions could lead to small but finite widths to $D\bar{D}^{*}$. Seeing $D\bar{D}^{*}$ but not $DD^{*}$ or $D^{*}D^{*}$ would be a striking signal for a hybrid meson. The dominant coupling of charmonium hybrids is to excited states, in particular $D^{(*)}(L = 0)$ and $D^{*}(L = 1)$ states for which the threshold is $\sim 4.3$ GeV. This is at the kinematic limit for most mass predictions so that decays into the preferred $D^{(*)}D^{*}$ states are expected to be significantly suppressed if not outright kinematically forbidden. Estimates for the various $\psi_g$ decay widths and branching ratios are given in Table I. In columns 1-4 a hybrid mass of 4.1 GeV is assumed, which is below $DD^{*}$ threshold, while column 5 and 6 employs a mass of 4.4 GeV, above $DD^{*}$ threshold. This enables us to gauge the model dependence of the results and the effect of opening up the $DD^{*}$ channel on the total width. The original Isgur-Kokoski Paton flux tube model predicts partial widths of $\sim 1-20$ MeV, depending on the $J^{PC}$ of the hybrid while a refined version of this model predicts smaller partial widths of 0.3-1.5 MeV. These widths are quite narrow for charmonia of such high mass. If the hybrid masses are above $D^{*}$ threshold then the total widths increase to $4-40$ MeV for 4.4 GeV charmonium hybrids which are still relatively narrow. The challenge is to identify decay modes that can be reconstructed by experiment.

(ii) Decays to $(c\bar{c}) + \text{(light hadrons)}$: The $\psi_g \to (c\bar{c}) + \text{(light hadrons)}$ mode offers the cleanest signature for $\psi_g$ observation if its branching ratio is large enough. In ad-
dition, a small total width also offers the possibility that the radiative branching ratios into $J/\psi$, $\eta_c$, $\chi_{cJ}$, and $h_c$ could be significant and offer a clean signal for the detection of these states.

For masses below $DD^*$ threshold the cascade decays $\psi_g \rightarrow (\psi, \eta_c, \ldots) + (gg)$ and annihilation decays $\psi_g(C = +) \rightarrow (gg) \rightarrow$ light hadrons will dominate. If the masses of exotic $J^{PC}$ states are above $DD^*$ threshold their widths are also expected to be relatively narrow for states of such high mass, in which case cascades to conventional $cc$ states transitions of the type $\psi_g \rightarrow (\psi, \psi') +$ (light hadrons) should have significant branching ratios making them important signals to look for in $\psi_g$ searches. The hadronic transition rates to conventional charmonia from either $C = +$ or $C = -$ are similar because both are the same order in $\alpha_s$ and the charge conjugation of the conventional $cc$ daughter should be the same as that of the $cc$ hybrid parent since two gluons ($C = +$) are emitted in the lowest order process.

A transition between two quarkonium states proceeds via the emission of gluons by the heavy quark and the subsequent conversion of the gluons into light hadrons. The emission of the gluons is typically treated as a multiple expansion of the colour gauge field to estimate rates for hadronic transitions between $QQ$ states. Kuang and Yan estimated the matrix elements between quarkonium states by inserting intermediate states with the string in its vibrationally excited lowest mode, i.e. hybrid states. Thus, the matrix elements for hadronic transitions between conventional quarkonia are related to hybrid-conventional quarkonium hadronic transitions. The widths $(cc) \rightarrow \pi\pi J/\psi$ are typically $\mathcal{O}(10-100)$ keV while $(cc) \rightarrow \eta J/\psi$ are typically $\mathcal{O}(10)$ keV. BES has recently measured $\Gamma(\psi(3770) \rightarrow J/\psi\pi^+\pi^-) = (139 \pm 61 \pm 41)$ keV which is a $D_{11} \rightarrow S_{11}$ transition. It seems reasonable to assume that the partial widths for the decays $\psi_g(1^{+}) \rightarrow \eta_c + \pi \pi$ and $\psi_g(0^{+}, 2^{+}) \rightarrow J/\psi \pi + (\pi \pi, \eta, \eta')$ will be similar in magnitude, of $\mathcal{O}(10-100)$ keV. Clearly this is not a rigorous result but it does offer a reasonable order of magnitude estimate.

While there are no calculations for radiative transitions involving charmonium hybrids there are estimates of radiative transitions involving hybrids with light quarks. Both calculations found that the $E1$ transitions between hybrid and conventional states to be comparable in magnitude to transitions between conventional mesons. While neither calculation can be applied directly to $cc$ one might take this to suggest that the partial widths for $\psi_g(1^{+}) \rightarrow \gamma + (J/\psi, h_c)$ and $\psi_g(0^{+}, 2^{+}) \rightarrow \gamma + (\eta_c, \chi_{cJ})$ are the same order of magnitude as transitions between conventional charmonium states. However, it is not at all clear if this extrapolation to charmonium hybrids is correct as in the flux-tube model Close and Dudek showed that the $\Delta S = 0$ $E1$ transitions to hybrids only occur for charged particles, and hence would vanish for $cc$. The $\Delta S = 1$ $M1$ transitions can occur, but are non-leading and less well defined. Estimates for their widths are $\mathcal{O}(1-100)$ keV. Clearly, given our general lack of understanding of radiative transitions involving hybrids, the measurement of these transitions, $\psi_g \rightarrow (cc)\gamma$, has important implications for model builders.

(iii) Decays to light hadrons: Decays of the type $\psi_g \rightarrow$ light hadrons offer the interesting possibility of producing light exotic mesons. Estimates of annihilation widths to light hadrons will be order of magnitude guesses at best due to uncertainties in wavefunction effects and QCD corrections. We estimate the annihilation widths $\Gamma(\psi_g(C = +) \rightarrow$ light hadrons) and $\Gamma(c\bar{c}(C = +) \rightarrow$ light hadrons) by comparing them to $\Gamma(\psi(\gamma) \rightarrow$ light hadrons) and $\Gamma(\eta(\gamma) \rightarrow$ light hadrons). The light hadron production rate from $\psi_g(C = +)$ decays is suppressed by one power of $\alpha_s$ with respect to $\psi_g(C = +)$ decays. This very naive assumption gives $\Gamma(\psi_g(C = +) \rightarrow$ light hadrons) $\sim \mathcal{O}(100)$ keV and $\Gamma(c\bar{c}(C = +) \rightarrow$ light hadrons) $\sim \mathcal{O}(10)$ MeV. These widths could be smaller because the $q\bar{q}$ pair in hybrids is expected to be separated by a distance of order $1/A_{QCD}$ resulting in a smaller annihilation rate than the $S$-wave $\psi'$ and $\eta_c$ states.

V. SIGNATURES

The decays discussed above lead to a number of possible signals: $\psi_g \rightarrow D^{(*)}(s)D^{(*)\, (s)}$, $\psi_g(0^{+}, 2^{+}) \rightarrow J/\psi + (\pi^{+}\pi^{-}, \eta, \eta')$, $\psi_g(1^{+}) \rightarrow \eta_c + (\pi^{+}\pi^{-}, \eta, \eta')$, $\psi_g \rightarrow (cc)\gamma$, and $\psi_g \rightarrow$ light hadrons. Of the possible decay modes, $\psi_g \rightarrow J/\psi \pi^{+}\pi^{-}$, $\psi_g \rightarrow J/\psi \eta$, and $\psi_g \rightarrow (cc)\gamma$ give distinctive and easily reconstructed signals. In the former case the subsequent decay, $J/\psi \rightarrow e^{+}e^{-}$ and $\mu^{+}\mu^{-}$ offers a clean tag for the event.

A good place to search for hybrids in $\psi_g \rightarrow J/\psi\pi^{+}\pi^{-}$ is to look for peaks in the invariant mass distributions $M(e^{+}e^{-}\pi^{+}\pi^{-}) - M(e^{+}e^{-})$. Babar observed a strong signal for the $\psi'$ such in a distribution from the decay chain $B \rightarrow \psi(2S) + X \rightarrow J/\psi(e^{+}e^{-})\pi^{+}\pi^{-}$. Babar’s efficiency for the $\psi(e^{+}e^{-})\pi^{+}\pi^{-}$ final state is about 20%. With $2.3 \times 10^{7} BB$ pairs from an integrated luminosity of 20.3 fb$^{-1}$ Babar observed $\approx 972 \psi(2S) \rightarrow J/\psi\pi^{+}\pi^{-}$ events. Both the Babar and Belle collaborations have collected over 100 fb$^{-1}$ of integrated luminosity and each expects to collect 400 fb$^{-1}$ over the next few years.

Both the $0^{+}$ and $2^{+}$ should decay via the $\psi_g \rightarrow J/\psi\pi\pi$ cascade. Although the lattice predictions for the $0^{+}$ and $2^{+}$ masses are above $DD^*$ threshold there is still considerable uncertainty in these values and the flux tube model predicts masses approximately 4.1 GeV. We therefore consider both cases, where the $\psi_g$ lies both below and above $DD^*$ threshold. For the low mass scenario, combining our estimates of $B(B \rightarrow \psi_g + X) \approx 10^{-3}$ and $B(\psi_g(2^{+}) \rightarrow J/\psi\pi^{+}\pi^{-}) \approx 0.2$ (the 4.1 GeV PSS case in Table I) with the PDG value of $B(\psi \rightarrow \ell^{+}\ell^{-}) = 11.81\%$ and the Babar detection efficiency we esti-
mate that for 100 fb\(^{-1}\) of integrated luminosity each experiment should observe roughly 50 events. If the 2\(^{+-}\) lies above the DD\(^{**}\) threshold the BR for 2\(^{+-}\) → J/ψππ decreases significantly to 2.6 × 10\(^{-2}\) (the 4.4 GeV PSS case in Table I) lowering the expected number to about 6 events. Similarly, for the 0\(^{+-}\) hybrid we estimate roughly 1200 events if it lies below threshold but only 5 events once the DD\(^{**}\) decay modes open up.

The 1\(^{-+}\) state is expected to be the lightest exotic c\(\bar{c}\) hybrid and therefore the most likely to lie below DD\(^{**}\) threshold. However, in this case the cascade goes to η\(_c\)ππ, a more difficult final state to reconstruct. We use our estimate of B(B → ψ\(_g\) + X) ≃ 10\(^{-3}\) and the estimate given in Table I for the 4.1 GeV PSS case of B(ψ\(_g\)(1\(^{-+}\) → η\(_c\)π\(^+\)π\(^-\)) ≃ 9 × 10\(^{-3}\). The Babar collaboration studied the decay B → η\(_c\)K by observing the η\(_c\) in KKπ and KKK final states. Combining the PDG values for the BR’s to these final states with the Babar detection efficiencies of roughly 15\% and 11\% respectively we estimate that for 100 fb\(^{-1}\) each experiment should observe roughly 10 events. If the 1\(^{-+}\) lies above the DD\(^{**}\) threshold, the BR for 1\(^{-+}\) → π\(^+\)π\(^-\)η\(_c\) decreases to 3 × 10\(^{-3}\) lowering the expected number to about 3 events.

The radiative transition, ψ\(_g\)(1\(^{-+}\) → γJ/ψ, also has a distinct signal if it has a significant branching ratio. If we take the conservative value of Γ(ψ\(_g\)(1\(^{-+}\) → γJ/ψ) ≃ 1 keV, the BR’s for this transition would be rather small. On the other hand, a monochromatic photon offers a clean tag with a high efficiency. One could look for peaks in M(μ\(^+\)μ\(^-\)γ) − M(μ\(^+\)μ\(^-\)). Babar observed χ\(_{c1}\) and χ\(_{c2}\) this way obtaining ≃ 394 χ\(_{c1}\)’s and ≃ 1100 χ\(_{c2}\)’s with a 20.3 fb\(^{-1}\) data sample and an efficiency of about 20\% for the J/ψγ final state. So although the rate may be too small to observe, given the potential payoff, it is probably worth the effort to perform this search. We also note that both Babar and Belle should be able to see the 3\(D_2\) state via radiative transitions so that even if they do not discover a charmonium hybrid they will almost certainly add to our knowledge of quarkonium spectroscopy.

We note that Babar has measured BR’s for B → (c\(\bar{c}\))\(^+\)h at the 10\(^{-6}\) level with 61.6 × 10\(^6\) BB pairs (B(B\(^{+}\) → χ\(_{c0}\)K\(^+\), χ\(_{c0}\) → π\(^+\)π\(^-\)) = (1.46 ± 0.35 ± 0.12) × 10\(^{-6}\)) demonstrating the accessibility to these levels of combined BR’s.

Experiments might also look for charmonium hybrids in invariant mass distributions of light hadrons. For example, Belle observed the χ\(_{c0}\) by looking at the invariant mass distributions from the decays χ\(_{c0}\) → π\(^+\)π\(^-\) and χ\(_{c0}\) → K\(^+\)K\(^-\). They found efficiencies of 21\% for χ\(_{c0}\) → π\(^+\)π\(^-\) and 12.9\% for χ\(_{c0}\) → K\(^+\)K\(^-\), obtaining ~ 16 events in the former case and ~ 9 in the latter.

The decay to charmed mesons also needs to be stud-
ied. Because there are more particles in the final state it will be more difficult to reconstruct the charmonium hybrid. On the other hand, with sufficient statistics these channels will be important for measuring the \( \psi_g \) quantum numbers and distinguishing their properties from conventional \( cc \) states.

The final consideration in charmonium hybrid searches is distinguishing the signal from background. The largest backgrounds in these final states will be via charmonium states with similar masses and fairly narrow total widths. The only charmonium states with these properties are the missing \( cc \) \( 3D_2 \) and \( 1D_2 \) states whose masses are predicted to be \( \sim 3.8 \) GeV. They lie below \( DD^* \) and are forbidden to decay into \( DD^* \) because of parity conservation. So they are expected to have narrow widths of 300-400 keV and should be easily tagged through the dominant E1 transitions into the \( \chi_{cJ} \) states. These states are expected to be produced with branching ratios of \( O(1\%) \) with \( B(\psi(2S)) \to \pi \pi J/\psi \sim 0.12 \). The successes of QCD motivated quark models for these states give reasonably reliable predictions for their masses so it should be possible to distinguish them from the charmonium hybrid on this basis. In any case, their discovery would be interesting in their own right. For exclusive processes such as \( B \to \psi_g + K^{(*)} \) the \( K^{(*)} \) would have a definite momentum in the \( B \) rest frame. Careful study of \( K^{(*)} \) momentum spectra is another tool that could be used to separate the signal from other sources, and seek excess of low momentum \( K^{(*)} \) recoiling against the \( \psi_g \sim 4 - 4.5 \) GeV.

VI. CONCLUSIONS

Establishing the existence of mesons with explicit gluonic degrees of freedom is one of the most important challenges in strong interaction physics. As demonstrated by the discovery of the \( \eta_c(2S) \) in \( B \) decay, \( B \) decays offer a promising approach to discovering charmonium hybrid mesons. In this letter we have described a strategy to search for these states. While there is no question that our estimates for the various partial widths are crude, the essential point is that these states are expected to be relatively narrow and that distinctive final states are likely to have observable branching ratios. Given how much we can learn by finding these states we strongly advocate that some effort be devoted to their searches.

Note added: After the completion of this paper the Belle Collaboration published the observation of a new charmonium state in exclusive \( B^\pm \to K^{\pm} \pi^+ \pi^- J/\psi \) decays with mass \( 3871.8 \pm 0.7 \) (stat) \( \pm 0.4 \) (syst) MeV which is most likely the \( 3D_2 \) state. However, if this state were found to have natural spin parity, \( 0^+, 1^- \) etc, then a dynamical suppression, such as expected for hybrids, would be called for.

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