Possibile Presence and Properties of Multi Chiral Pair-Bands in Odd-Odd Nuclei with the Same Intrinsic Configuration

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Abstract

Applying a relatively simple particle-rotor model to odd-odd nuclei, possible presence of multi chiral pair-bands is looked for, where chiral pair-bands are defined not only by near-degeneracy of the levels of two bands but also by almost the same expectation values of squared components of three angular-momenta that define chirality. In the angular-momentum region where two pairs of chiral pair-bands are obtained the possible interband M1/E2 decay from the second-lowest chiral pair-bands to the lowest chiral pair-bands is studied, with the intention of finding how to experimentally identify the multi chiral pair-bands. It is found that up till almost band-head the intraband M1/E2 decay within the second chiral pair-bands is preferred rather than the interband M1/E2 decay to the lowest chiral pair-bands, though the decay possibility depends on the ratio of actual decay energies. It is also found that chiral pair-bands in our model and definition are hardly obtained for $\gamma$ values outside the range $25^\circ < \gamma < 35^\circ$, although either a near-degeneracy or a constant energy-difference of several hundreds keV between the two levels for a given angular-momentum $I$ in ”a pair bands” is sometimes obtained in some limited region of $I$. In the present model calculations the energy difference between chiral pair-bands is always one or two orders of magnitude smaller than a few hundreds keV, and no chiral pair-bands are obtained, which have an almost constant energy difference of the order of a few hundreds keV in a reasonable range of $I$.

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I. INTRODUCTION

The total Hamiltonian for the nuclear system is taken to be invariant under the exchange of the right- and left-handed geometry. Chirality in triaxial nuclei is characterized by the presence of three angular-momentum vectors which are noncoplaner and thereby make it possible to define chirality. Since possible triaxial even-even nuclei are generally expected to collectively rotate mainly about the intermediate axis (taken as the 2-axis in the present article) as is expected from irroational-flow-like moments of inertia, other two angular-momenta to define chirality must come from particle configurations. In odd-odd nuclei the simplest example is the angular momenta of odd neutron and odd proton, which prefer to pointing out the directions of the shortest and longest axes.

The occurrence of the chirality in a nuclear structure was considered theoretically in [1], and since then experimental spectra exhibiting two $\Delta I = 1$ rotational bands, which presumably have the same parity and an almost constant energy difference, have been reported in the region of the mass number $A \approx 130$ and 110 region. The usual interpretation is that in the $A \approx 130$ region a proton-particle and a neutron-hole in the $h_{11/2}$-shell play a role in producing the two angular-momenta to define chirality, while in the $A \approx 110$ region a proton-hole in the $g_{9/2}$-shell and a neutron-particle in the $h_{11/2}$-shell play the role. The energy difference between the observed levels with the same $I$ belonging to those observed pair-bands is typically several hundreds keV in the $A \approx 130$ region where more data are reported, though it is not clear whether or not the observed difference is really close to a constant in the relevant angular-momentum region. It is not easy to find the origin which gives rise to such an amount of constant energy difference when the chiral pair-bands are realized.

It has been theoretically known for years [2, 3] that in some limited range of $I$ multiple pair-bands, of which the two levels with a given $I$ are degenerate with very good accuracy, are obtained for $\gamma \sim 30^\circ$ and a given chiral-candidate configuration with one high-$j$ proton-particle and one high-$j$ neutron-hole. In the present article we use the conventional way of defining the triaxial parameter [4], $0^\circ \leq \gamma \leq 60^\circ$, which corresponds to the region $0^\circ \geq \gamma \geq -60^\circ$ in the Lund convention [5] employed conventionally in high-spin physics. A high-$j$ orbit in a given major shell has unique properties such as a unique parity and a large angular-momentum compared with other energetically close-lying one-particle orbits so that
the high-j one-particle wave-functions remain relatively pure under both deformation and rotation and, furthermore, the states containing high-j particles appear close to the yrast line in high angular-momenta. Therefore, it may be possible to observe higher-lying chiral pair-bands consisting of the same high-\( j_p \) quasiproton and high-\( j_n \) quasineutron configuration as that of the lowest chiral pair-bands. Indeed, the present study was prompted by the recent experimental finding of four (or five) very similar \( \Delta I = 1 \) bands with possibly the same parity in the odd-odd nucleus \(^{104}_{45}Rh\) \([6]\), which may well be interpreted to come from the same chiral-candidate configuration.

Pushing further the notion of chiral pair-bands, based on adiabatic and configuration-fixed constrained triaxial relativistic mean field approaches the presence of multiple chiral pair-bands for different deformation (\( \beta, \gamma = 22^\circ \sim 31^\circ \)) and different intrinsic configurations in a given nucleus \(^{106}Rh\) was theoretically suggested \([7]\), while in a recent publication \([8]\) observed data on \(^{133}_{58}Ce\) were interpreted in terms of two chiral doublet bands with positive and negative parity and different deformations (\( \beta = 0.20 \sim 0.23, \gamma = 11^\circ \sim 15^\circ \)). One may wonder whether it is possible to obtain chiral bands for a weak triaxial-deformation such as \( \gamma = 15^\circ \), as the most favourable triaxial deformation for realizing chirality is known to be \( \gamma \approx 30^\circ \).

In the present work a relatively simple particle-rotor model of odd-odd nuclei consisting of a triaxial collective rotor together with one high-\( j_p \) quasiproton and one high-\( j_n \) quasineutron is used to study the possible presence and properties of two pairs of chiral bands as well as the relative structure of three angular-momenta, which define chirality. We would identify chiral pair-bands, only when not only the near degeneracy of two \( \Delta I = 1 \) bands but also the very similarity of the expectation values of squared components of the three angular momenta in some finite region of \( I \gg 1 \). When the latter condition is fulfilled, the energies of the two bands as well as corresponding various intraband transitions are expected to be almost identical. In the case that the presence of two pairs of chiral bands is obtained, the decay properties of the higher chiral pair-bands to the lower chiral pair-bands are examined. First we study the case of a proton-particle and a neutron-hole in a given \( j(= h_{11/2}) \)-shell coupled to the collectively rotating core, since in this case a quantum-number was present \([9]\) for \( \gamma = 30^\circ \) so that the understanding of numerical results is transparent. Then, we proceed to the case of a proton-hole in the \( g_{9/2} \)-shell and a neutron-particle in the \( h_{11/2} \)-shell which may be applicable to the possible pair-bands in the \( A \approx 110 \) region.
In Sec.II main points of our model are briefly summarized, while numerical results and discussions are presented in Sec.III. Conclusions and discussions are given in Sec.IV.

II. MODEL

Our particle-rotor model Hamiltonian \[^4\] of odd-odd nuclei is written as

\[ H = H_{\text{rot}} + H_{\text{intr}}^{(p)} + H_{\text{intr}}^{(n)} \]  

where the first term on r.h.s. expresses the rotor Hamiltonian of the even-even core

\[ H_{\text{rot}} = \sum_{k=1}^{3} \frac{\hbar^2}{2\mathcal{I}_k} R_k^2 \]  

with the irrotationa-flow-like moments of inertia

\[ \mathcal{I}_k = \frac{4}{3}\mathcal{I}_0 \sin^2(\gamma + \frac{k\pi}{3}) \]  

The total angular-momentum \( I \) is a good quantum-number, while the core angular-momentum \( R \) is not. In the present work we take the case in which both odd quasiproton and odd quasineutron are in respective single-\( j \)-shells. The intrinsic Hamiltonian in the case of a single-\( j \)-shell is conveniently written as \[^{11, 12}\]

\[ H_{\text{intr}} = \sum_{\nu} (\epsilon_{\nu} - \lambda) a_{\nu}^\dagger a_{\nu} + \frac{\Delta}{2} \sum_{\mu,\nu} \delta(\bar{\mu}, \nu) (a_{\mu}^\dagger a_{\nu}^\dagger + a_{\nu} a_{\mu}) \]  

where \( \epsilon_{\nu} \) expresses one-particle energies for a single-particle with angular-momentum \( j \) moving in a general triaxially-deformed quadrupole potential

\[ V = \frac{\kappa}{j(j+1)} \left( \{3j_3^2 - j(j+1)\} \cos \gamma + \sqrt{3} (j_1^2 - j_2^2) \sin \gamma \right). \]  

In eq. \[^6\] \( \kappa \) is used as a convenient energy unit for a single-\( j \)-shell, and the value of \( \kappa \) is proportional to the quadrupole deformation parameter \( \beta \[^4\]. An appropriate value of \( \kappa \) may be something between 2 and 2.5 MeV, depending on nuclei \[^{11}\]. The quantities, \( \epsilon_{\nu}, \lambda, \Delta \) and \( j \), appearing in \( H_{\text{intr}}^{(p)} \) and \( H_{\text{intr}}^{(n)} \) of \[^{11}\] depend on protons or neutrons, respectively, while we take the values of \( \kappa \) and \( \gamma \) common to protons and neutrons.

Our particle-rotor Hamiltonian \[^{11}\] is numerically diagonalized in the space consisting of the rotor coupled with one-quasiproton and one-quasineutron which are obtained from the
BCS approximation \[4\]. The total number of basis states for spin $I$ is given by \((1/2) (I + 1/2) (2j_p + 1) (2j_n + 1)\).

For M1 transitions the operator

\[
(M1)_\mu = \sqrt{\frac{3}{4\pi}} \frac{e\hbar}{2mc} \left\{ (g_\ell - g_R) \ell_\mu + (g_\ell^{eff} - g_R) s_\mu \right\}
\]

is used.

When chiral geometry is realized, observed two states with $I$ in the chiral pair-bands may be written as \[13\]

\[
|I+\rangle = \frac{1}{\sqrt{2}} (|IL\rangle + |IR\rangle) \quad \text{and} \quad |I-\rangle = \frac{i}{\sqrt{2}} (|IL\rangle - |IR\rangle)
\]

where left- and right-handed geometry states are written as $|IL\rangle$ and $|IR\rangle$, respectively. For states with $I \gg 1$ it is expected that

\[
\langle IL | H | IR \rangle \approx 0, \quad \langle IL | M1 | IR \rangle \approx 0, \quad \text{and} \quad \langle IL | E2 | IR \rangle \approx 0
\]

As $I$ increases the matrix-elements in (9) rapidly decrease and may approach zero for the $I$-value at which chiral pair-bands are realized. If so, two levels with a given $I$ in the chiral pair-bands are almost degenerate, while mutually corresponding intraband M1 and E2 transitions in the chiral pair-bands are almost identical.

Those energies and electromagnetic transitions can be, in principle, measured experimentally, though the measurement of the absolute magnitude of the latter with good accuracy is at present not so easy. On the other hand, the occurrence of chiral pair-bands can be theoretically explored by examining the similarity of the three angular-momenta of the first band to that of the second band, which define chirality. Checking the similarity is more fundamental and strict than just exploring the degeneracy of levels for a given $I$.

The expectation values of squared components of three angular momenta that define chirality were previously calculated in Ref.\[10\]. We define

\[
R_i(I) \equiv \sqrt{\langle I | R_i^2 | I \rangle} = \sqrt{\langle I | (I_i - j_{pi} - j_{ni})^2 | I \rangle}
\]

\[
j_{pi}(I) \equiv \sqrt{\langle I | j_{pi}^2 | I \rangle}
\]

\[
j_{ni}(I) \equiv \sqrt{\langle I | j_{ni}^2 | I \rangle}
\]

where $i=1, 2$ and 3. However, the similarity (or difference) of those expectation values in the lowest two levels with a given $I$ was not really used to pin down the identification of chiral
pair-bands. We study the quantities in (10) because when chiral pair-bands are realized the quantities $R_i$, $j_{pi}$ and $j_{ni}$ must be very similar for the pair-bands for a given $I$. On the other hand, if those quantities are not very similar, say different by more than 10 percent, in the two bands, they are not regarded as chiral pair-bands, even if the two bands are nearly degenerate.

III. NUMERICAL RESULTS

We show numerical results, in which the parameters, $\lambda_p$, $\lambda_n$, $\Delta_p$, $\Delta_n$, $\gamma$ and $\Im_0$ are taken to be independent of $I$. This is partly because only in a limited region of $I$ so-called "chiral pair-bands" are expected to occur and partly because we want to keep our model as simple as possible. In reality, those parameters may depend on $I$ even within the limited range of $I$, and in a quantitative comparison with experimental data one may have to perform more elaborate calculations. However, we think that if chiral pair-bands will ever appear, a simple schematic model such as the present one should already indicate the basic elements.

When we obtain the angular-momentum region, in which two pairs of chiral bands are identified, we have calculated the interband M1 and E2 transitions between the first pair-bands and the second pair-bands. Then, it is found that the E2 transitions may not win against the M1 transitions for possible transition energies. Thus, only the calculated $B(M1)$ values are shown and discussed in the following.

In numerical calculations presented in this article we use $\Delta_p = \Delta_n = 0.1\kappa$, $(\Im_0/h^2)\kappa = 70$, $g_R = 0.4$ and $g_{s eff}^f = 0.7 g_{s free}$. In the following the numerical results for $\gamma = 30^\circ$, in which chiral pair-bands are most favorably produced, are mostly presented, except at the end of subsection B where an example of a constant energy difference between two bands is discussed.

A. Both protons and neutrons in the $h_{11/2}$-shell

In this subsection we consider the case that both protons and neutrons are in the $h_{11/2}$-shell. In particular, one quasiproton represents almost one particle in the $h_{11/2}$-shell while one quasineutron expresses almost one hole in the $h_{11/2}$-shell. This choice of Fermi levels together with $\gamma = 30^\circ$ is theoretically known to be most favorable for producing a pair
of chiral bands. Moreover, if the condition, $\lambda_p = -\lambda_n$, is fulfilled the quantum-number $A$ defined in Ref. [9] is a good quantum number of the system.

First, in the case of the parameters that the quantum number $A$ in Ref. [9] is valid, we give a schematic sketch of possible $M1(I \rightarrow I - 1)$ transitions between the two pairs of idealistic chiral pair-bands, $(f1, u1)$ and $(f2, u2)$. In Fig. 1 we show the allowed $M1(I \rightarrow I - 1)$ transitions from the second-lowest pair of chiral bands $(f2, u2)$ to the lowest pair $(f1, u1)$, which are denoted by dotted-line arrows, as well as the allowed $M1(I \rightarrow I - 1)$ transitions within respective pairs that are expressed by solid-line arrows. The quantum-number $A$ of each level is expressed by $\pm$ sign. In this example respective bands $(f1, u1, f2, u2)$ are defined so that $E2(I \rightarrow I - 2)$ transitions are always allowed within a given band, though there may be some band-crossings within a given pair-bands (for example along the yrast line) experimentally observed. It is noted that because the M1 operator contains a non-negligible isoscalar component weak M1 transitions are possible between levels with the same sign of the quantum-number $A$, but such weak M1 transitions are not drawn in Fig. 1. There are two kinds of M1-decay scheme between the first and the second chiral pair-bands. In Fig. 1a states belonging to the $f2$ band $M1(I \rightarrow I - 1)$ decay always to those in the $f1$ band, while states in the $u2$ band $M1(I \rightarrow I - 1)$ decay to those in the $u1$ band. In contrast, in the case of Fig. 1b the states in the $f2$ ($u2$) band $M1(I \rightarrow I - 1)$ decay to those in the $f1$ and $u1$ bands alternatively in $I$. Whether or not M1 transitions denoted by dotted lines in Fig. 1 can be observed depends on both the B(M1)-values and transition energies relative to those shown by solid arrows. Examples of numerical values are shown later.

We note that in Fig. 1 the feature of chiral pair-bands is represented by the energy degeneracy of the two bands with the quantum-number $A = \pm$ for a given $I$ level as well as the equality of the $B(M1)$- and $B(E2)$-values of mutually corresponding intraband transitions. In contrast, the presence of the quantum-number $A$ and the resulting selection rule in electromagnetic transitions come from the symmetry properties of the Hamiltonian with the present parameters and not from the realization of chiral pair-bands.

In Fig. 2a calculated energies of the lowest four bands are shown for $\gamma = 30^\circ$ and the proton Fermi level $\lambda_p$ (neutron Fermi level $\lambda_n$) placed on the lowest (highest) single-particle energies of the $h_{11/2}$-shell. These parameters, $\gamma$, $\lambda_p$ and $\lambda_n$ together with a moderate amount of pair-correlation are known to be most favorable for producing chiral pair-bands. As is well known, chiral pair-bands may appear at best only in the region of intermediate values
of $I$. In the following presentation of our numerical results the name, \(b_1, b_2, \ldots\) are given for the sequence of the levels counting simply from the energetically lowest to the higher levels for a given $I$, and not a sequence of levels which are connected by strong $E2(I \rightarrow I - 2)$ transitions such as shown in Fig. 1. This naming is chosen because in the practical presence of non-vanishing interactions between two bands it is not always trivial to define a band, of which levels are connected by large $B(E2; I \rightarrow I - 2)$ values, in contrast to the present case in which the quantum number $A$ can be used to define a sequence of levels in a band.

In Fig. 2b calculated values of $R_1$ and $R_2$ for the lowest-lying two bands are shown, while in Fig. 2c calculated values of $j_{pi}$ and $j_{ni}$ are plotted. Due to the construction of the model and the parameters used here, the values of $R_1$ are equal to those of $R_3$. From Fig. 2c it is seen that the quantities $j_{pi}$ and $j_{ni}$ are not very efficient for showing the character of chiral pair-bands, because both the variation in $I$ and the dependence on bands are relatively minor. From Figs. 2a, 2b and 2c one may safely state that the lowest two bands, $b_1$ and $b_2$, form chiral pair-bands in the region of $16 \leq I \leq 24$. Indeed it is amazing to see that in the limited region of $I$ the lowest two bands, $b_1$ and $b_2$, form an almost perfect chiral pair-bands. Examining calculated $R_i$-values of the third- and fourth-lowest bands shown in Fig. 2d, together with energies exhibited in Fig. 2a, we may say that the bands, $b_3$ and $b_4$, may form chiral pair-bands in the region of $18 \leq I \leq 24$.

In Fig. 3a examples of calculated $B(M1; I \rightarrow I - 1)$ values both between the bands $b_1$ and $b_2$ and between the bands $b_3$ and $b_4$ are shown in the angular-momentum region, where the character of respective pair-bands is confirmed above to be chiral. The parameter set used in this numerical example is the one, in which the quantum number $A$ defined in Ref. 9 should work exactly. Therefore, the zigzag behavior of $B(M1; I \rightarrow I - 1)$ values sketched qualitatively in Figs. 1a or 1b appears precisely, except at $I=20$ and 21 where a band-crossing occurs within both pairs of chiral bands, $[b_1$ and $b_2]$ and $[b_3$ and $b_4]$, though the band-crossings are somewhat difficult to be recognized in the scale of Fig. 2a. The occurrence of the band-crossing may be understood in terms of a very small difference of the effective moments of inertia between the lowest (third lowest) and the second lowest (fourth lowest) bands, which originates from a very small matrix-element $<IL|H|IR > \neq 0$.

In Fig. 3b we plot examples of calculated $B(M1; I \rightarrow I - 1)$ values of the M1 transitions from the second chiral pair-bands to the first ones. The M1 transitions are seen to be again controlled by the quantum number $A$. However, the point here is that the $B(M1)$-values in
Fig. 3b are at least about a factor 50 smaller than those of competing M1 transitions shown in Fig. 3a, due to the different structure of three vectors, $\vec{R}$, $\vec{j}_p$, $\vec{j}_n$, in the first and second chiral pair-bands. That means, when the second chiral pair-bands are formed in addition to the first ones, the decays of the former will occur within the pair-bands to the levels of the band-head and not to the first chiral pair-bands, if for a given initial state the ratio of the decay energy of the M1 transition shown in Fig. 3b to that shown in Fig. 3a is not larger than by a factor of about $(50)^{1/3} \approx 3.7$. Such a large ratio of the decay energies is hardly obtained from numerical examples of the present model such as those shown in Fig. 2a.

Though we have tried various numerical calculations varying the parameters, $\lambda_p$, $\lambda_n$, $\Delta_p$, $\Delta_n$, $\gamma$ and $\Im_0$, it is found that outside the region of $25^\circ < \gamma < 35^\circ$ chiral pair-bands defined in the present work are hardly obtained. It is also found that we could not obtain any chiral pair-bands, of which the energy difference is nearly a constant of the order of a few hundreds keV in a certain range of $I$.

**B. Protons in the $g_{9/2}$-shell and neutrons in the $h_{11/2}$-shell**

First we consider the case that for $\gamma = 30^\circ$ one quasiproton expresses almost one hole in the $g_{9/2}$-shell while one quasineutron represents almost one particle in the $h_{11/2}$-shell. This choice of parameters is again most favorable for producing a pair of chiral bands.

In Fig. 4a calculated energies of the lowest four bands are shown for $\lambda_p$ placed on the highest single-particle energy of the $g_{9/2}$-shell and for $\lambda_n$ on the lowest single-particle energy of the $h_{11/2}$-shell. The behavior of the degeneracy of calculated energies of the lowest pair bands, $b_1$ and $b_2$, and the second lowest pair bands, $b_3$ and $b_4$, looks very similar to that in Fig. 2a.

In Fig. 4b calculated values of $R_i$ of the lowest-lying two bands are shown. From Fig. 4a and 4b one may state that the lowest-lying two bands, $b_1$ and $b_2$, form chiral pair-bands in the region of $15 \leq I \leq 26$. Examining calculated $R_i$ values of the third- and fourth-lowest bands, $b_3$ and $b_4$, shown in Fig. 4c, together with energies exhibited in Fig. 4a, we may say that the bands, $b_3$ and $b_4$, form chiral pair-bands in the region of $17 \leq I \leq 22$. It is noted that the appearance of chiral pair-bands, both the lowest pair and the second lowest pair, in a certain range of $I$ is very similar to the case that both protons and neutrons are in the $h_{11/2}$-shell discussed in the previous subsection. We also note that in the present model
the character of chiral pair-bands becomes dubious before the calculated energy difference between two bands approaches 100 keV.

In Fig. 5a examples of calculated $B(M1; I \rightarrow I - 1)$ values within the bands $b_1$ and $b_2$ as well as within the bands $b_3$ and $b_4$ are plotted in the angular-momentum region, where the chiral character of respective pair-bands is identified. It is seen that a strong zigzag pattern of the $B(M1)$-values, which is reminiscent of Fig. 3a, appears especially for lower $I$-values. The zigzag pattern diminishes as $I$ increases.

In Fig. 5b calculated $B(M1; I \rightarrow I - 1)$ values of the M1 transitions from the second chiral pair-bands to the first ones are plotted. It is again seen that the B(M1)-values in Fig. 5b are at least about a factor 50 smaller that those in Fig. 5a. That means, when the second chiral pair-bands are formed in addition to the first ones, the M1 decays of the levels of the former to those of the latter will hardly occur except possibly around the bandhead, though the decay possibility depends on actual transition energies.

For reference, in Fig. 6a we show the energies of the lowest four bands calculated for the same parameters as those used in Fig. 4a except $\gamma = 20^\circ$. Examining Fig. 6a we see that the energy difference between the third and fourth bands are nearly constant in the region of $16 \leq I \leq 24$, while the difference between the lowest and second-lowest bands is monotonically increasing. In Fig. 6b calculated $R_i$ values for the bands $b_1$ and $b_2$ are plotted, of which relative values are also monotonically changing as a function of $I$. Calculated $R_i$-values for the bands $b_3$ and $b_4$ are shown in Fig. 6c, which are drastically and independently changing in contrast to the nearly constant energy-difference between the bands $b_3$ and $b_4$ shown in Fig. 6a. From Figs. 6a, 6b and 6c it is concluded that any two of the four bands, $b_1$, $b_2$, $b_3$ and $b_4$, hardly form chiral pair-bands.

**IV. CONCLUSIONS AND DISCUSSIONS**

Defining chiral pair-bands not only by the degeneracy of the two levels with a given $I$ but also by almost equal (within $\pm 20$ percent) expectation values of squared components of three angular-momenta that define chirality, we have explored the presence and properties of multi chiral pair-bands in odd-odd nuclei, using a particle-rotor model. The model is relatively simple, but we believe that it contains basic elements in physics so that if our model does not at all produce chiral pair-bands, then there will be presumably little hope
to obtain chiral pair-bands in more elaborate models.

With the parameters of the model that are most favorable for producing chiral bands it is amazing to see that two lowest $\Delta I = 1$ bands form an almost perfect chiral pair-bands in the range of $I$ varying the value by so much as 10 units. And, it is also possible to obtain the second chiral pair-bands in the range of $I$ varying the value at least by several units. In the region of $I$ the energy difference between the two $I$-levels of respective chiral pair-bands is one or two orders of magnitude smaller than a few hundreds keV, and mutually corresponding intraband $M1/E2$ transitions are nearly equal. On the other hand, interband $M1/E2$ transitions from the second lowest chiral pair-bands to the lowest ones are found to be too weak to compete with the intraband $M1/E2$ transitions within the second chiral pair-bands.

Chiral pair-bands, of which the energy difference of the two levels with a given $I$ is nearly constant and is the order of a few hundreds keV, have never been obtained in the present model. It is noted that the energy difference comes essentially from the matrix-element $\langle IL|H|IR \rangle \neq 0$, which will rapidly decrease as $I$ increases. If a pair of bands in odd-odd nuclei, of which the energy difference is a constant of a few hundreds keV as experimentally reported in the region of $A \sim 130$, are pinned down to be approximately chiral pair-bands, an important and interesting question is what is the origin of the energy difference. On the other hand, as shown in an example given at the end of section III, in the present model we sometimes obtain two bands, of which the energy difference is nearly a constant of the order of a few hundreds keV in a certain range of $I$. However, in such cases the examination of the components of three vectors in (10) indicates that those two bands are far from being a pair of chiral bands. Such two bands may happen to appear from the presence of many close-lying levels (or bands) in odd-odd nuclei.

Choosing the most appropriate Fermi levels for obtaining chiral pair-bands, namely that one quasiproton (quasineutron) expresses almost one hole in the high-$j_p$ (high-$j_n$) shell while one quasineutron (quasiproton) represents one particle in the high-$j_n$ (high-$j_p$) shell, we have looked for the possibility of obtaining a chiral pair-bands by varying $\gamma$ values. It is found that outside the region of $25^\circ < \gamma < 35^\circ$ chiral pair-bands following our definition have hardly been obtained.

On the other hand, we have tried numerical calculations by keeping $\gamma = 30^\circ$ while relaxing the condition that a set of one quasiproton and one quasineutron almost expresses a set of
one hole and one particle in respective high-$j$ shells. Then, the range of $I$ for the occurrence of chiral pair-bands becomes in general narrower even when the range can ever be obtained. For example, simulating somewhat the case of nuclei $^{104,106}$Rh, we place the proton Fermi level on the second-highest single-particle level of the $g_{9/2}$-shell and the neutron Fermi level on the second-lowest single-particle level of the $h_{11/2}$-shell. Then, it is found that relative values of energies and $R_i$ of four bands, $b_1$, $b_2$, $b_3$ and $b_4$, are monotonically changing as a function of $I$. When we take our definition of chiral pair-bands we may barely state that the bands, $b_1$ and $b_2$, form chiral pair-bands in the region of $14 \leq I \leq 17$, while the bands, $b_3$ and $b_4$, form a pair of chiral bands for $16 \leq I \leq 18$, though the quality of being chiral pair-bands is much poorer than that shown in Figs. 4a, 4b and 4c. For larger values of $I$ the difference of zigzag pattern (namely odd- and even-$I$ dependence) of $R_i$ values between bands $b_1$ and $b_2$ (and between bands $b_3$ and $b_4$) becomes so prominent that the interpretation of the two bands as a chiral pair does not work, though their energies are not so far away from each other.

An important question is: if "the pair" of $\Delta I = 1$ bands observed in odd-odd nuclei in the region of $A \approx 130$ and 110 region are not understood in terms of chiral pair-bands, we must find what makes the systematic occurrence of such "pair" bands.

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Figure captions

Figure 1: Schematic sketch of the selection rule expected for both interbands and intrabands $M_1(I \rightarrow I - 1)$ and $E_2(I \rightarrow I - 1)$ transitions between the two pairs of idealistic chiral bands, $(f_1, u_1)$ and $(f_2, u_2)$, using the model discussed in [9]. The band $u_1$ ($u_2$) is slightly shifted upward from the band $f_1$ ($f_2$). The quantum number $A$ of each level is denoted by $\pm$ sign. A band is arranged so that $E_2(I \rightarrow I - 2)$ transitions are always allowed within a given band. The transitions within respective pairs $(f_i, u_i)$ are expressed by solid-line arrows, while those from the second pair-bands to the lowest pair-bands are denoted by dotted-line arrows. There are two possible relations of the quantum number $A$ of the first chiral-pair to that of the second chiral-pair, which are shown in Figs. 1a and 1b. See the text for details.

Figure 2: (a) Calculated energies of the lowest four bands for $\gamma = 30^\circ$ and for the proton (neutron) Fermi level placed on the lowest (highest) single-particle energy of the $h_{11/2}$-shell. (b) Values of $R_1$ and $R_2$ calculated for the lowest two bands. (c) Values of $j_{pi}$ and $j_{ni}$ calculated for the lowest two bands. (d) Values of $R_1$ and $R_2$ calculated for the second-lowest two bands.

Figure 3: Parameters are the same as those used in Fig. 2. The B(M1) values are expressed in units of $(\frac{e\hbar}{2mc})^2$. (a) Examples of calculated $B(M1; I \rightarrow I - 1)$ values of the transitions within the first and second pair-bands in the angular-momentum region, where the character of respective chiral pair-bands is confirmed. (b) Examples of calculated $B(M1; I \rightarrow I - 1)$ values of the transitions from the second chiral pair-bands to the first ones.

Figure 4: (a) Calculated energies of the lowest four bands for $\gamma = 30^\circ$ and for the proton Fermi level placed on the highest single-particle energy of the $g_{9/2}$-shell while the neutron Fermi level on the lowest single-particle energy of the $h_{11/2}$-shell. (b) Values of $R_1$ and $R_2$ calculated for the lowest two bands. (c) Values of $R_1$ and $R_2$ calculated for the third and fourth bands.

Figure 5: Parameters are the same as those used in Fig. 4. The B(M1) values are expressed in units of $(\frac{e\hbar}{2mc})^2$. (a) Examples of $B(M1; I \rightarrow I - 1)$ values of the
transitions within the first and second pair-bands, respectively, which are calculated in the angular-momentum region, where the character of respective chiral pair-bands is confirmed. (b) Examples of calculated $B(M1; I \rightarrow I - 1)$ values of the transitions from the second chiral pair-bands to the first ones.

Figure 6: Parameters are the same as those used in Fig. 4 except $\gamma = 20^\circ$. (a) Calculated energies of the lowest four bands. (b) Values of $R_1$, $R_2$ and $R_3$ calculated for the lowest two bands. (c) Values of $R_1$, $R_2$ and $R_3$ calculated for the third and fourth lowest bands.