Entropy-parametric control of stochastic system

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Abstract. The paper contains material about controlled methods of stochastic systems. The article considers the features of establishing control over a stochastic system, which are found when using parametric and informational criteria. It is proposed to use the potential of the Renyi entropy when building control over a complex multilevel system. The possibility of combining entropy and parametric criteria to increase the stability of control over the system is discussed.

1. Introduction

In modern technology, the dynamic system is used to describe and control physical objects. Formally, a dynamical system from a mathematical point of view is a system of differential equations for modeling changes in the physical processes of an observed object, designed to analyze the evolutionary changes of an object over time.

The observer judges the state of the dynamic system by tracking parameters. Despite the data on the observed properties of the system, the state of most complex dynamical systems remains uncertain for the observer due to reasons such as approximation errors in mathematical modeling of the internal structure, as well as due to the influence of random noise or random changes in the environment.

Due to random changes in the state of complex systems and systems with a probabilistic change in at least one parameter, probabilistic methods for analyzing the dynamic state should be used to make decisions on system control. If at least one of the observed parameters of an object is specified by a probabilistic dependence, then the mathematical models of objects are referring to stochastic systems.

The task of controlling a stochastic system is to estimate the parameters of the probability distribution of the observed properties, on the basis of which a control action is formed for a purposeful change in its state.

It is well known that stochastic systems have been an active research area for the last few decades. There has been a great number of research results from which it is possible to allocate two basic approaches that differing in the analysis of the statistical and information properties of the system.

2. Methods of stochastic systems control

The first approach is to control the parameters of the statistical distribution of random variables derived from the object. One of the main tasks of the control system consists in the processing of data received from the stochastic system. This task is implemented by minimizing the control criterion. Traditionally, the control parameters "Euclidean measure" is used. This measure is based on standard deviation of the
sample data. When calculating a measure, the sampled values of the observed quantity are independent components of the \(N\)-dimensional random sample. For calculating the measure, the formula is given

\[
D(x) = \frac{1}{N-1} \sum_{i=1}^{N} \left( x_i - m_x \right)^2
\]

(1)

Where \(m_x\) is mathematical expectation that was found for the sample values \(x_i\).

Minimizing of variances or standard deviations are widespread as necessary conditions for solving of optimal control problem [1, 2]. These conditions are given by

\[
D \rightarrow \min,
\sigma \rightarrow \min.
\]

(2)

More generally, this approach was implemented in algorithms of stochastic systems that are based on the calculation of statistical moments for distributions of random data. Due to this approach, it is possible to control the state of the stochastic system by a transformation of the density distribution of the output variable [3].

Another approach of the stochastic systems control is to analyze information’s that are contained in the output. For this purpose, dynamic systems algorithms are based on the entropy control of the distributions data in the output [4, 5, 6]. In this approach, the level of uncertainty of sampling data on the based Shannon’s entropy \(H(x)\) of the discrete source is estimated. The level of uncertainty is equal to the mathematical expectation of the logarithm of the unit related to the conditional probability of events \(x\).

\[
H(x) = M \left( \ln \frac{1}{p(x)} \right)
\]

(3)

When analyzing the data, it is convenient to estimate the uncertainty given in the units of the observed values. For this reason, the entropy potential as an estimate of the uncertainty interval for the observed parameter of a complex system has recently been used. This is obtained by potentiating the \(H(x)\)-entropy. The expression for calculating the entropy potential is given as

\[
\Delta_e = 0.5 \cdot e^{H(x)}
\]

(4)

In [7], it was proposed to use minimization of the entropy potential as an optimality condition in the control of a stochastic system

\[
\Delta_e \rightarrow \min
\]

(5)

The entropy potential is proportional to the standard deviations. For calculation of entropy potential, the formula is given

\[
\Delta_e = K_e \cdot \sigma_x
\]

(6)

Where \(K_e\) is an entropy coefficient.

It can be seen from Equation (6) that there are two possibilities to organization the control of a dynamic system. The first option is possible if the coefficient of entropy is known from observations or obtained by the methods of mathematical analysis.

The first option of dynamic system control is possible if the coefficient of entropy is known from observations or it was obtained by the methods of mathematical analysis. In this case, the entropy potential reduces to scaling standard deviation, that limits to use of the properties of the distribution of information for the control of the object.
Another option of dynamic system control is implemented by minimizing of the entropy potential. Wherein it is necessary to take into consideration that standard deviation and coefficient of entropy may change at the same time at transformation of distribution shape of controlled parameter. The main disadvantage of this method of control is self-transition of a dynamic system in steady unfavorable condition with low values of entropy potential. Such a transition is possible due to a change in the internal structure of the dynamic system [8]. The effect of changing the behavior of a complex object is often found in modern robotics.

In the modern literature, when building control systems [9] and building neural networks [10], the information potential is used, which is set as a scalar value of the logarithm argument of the differential entropy of Renyi.

So in the work [9], the information potential was used to track the $e(i)$ error of the desired output signal of the system as $r(i)$ by the values of the measurable output vector of $y(i)$. The formula for determining the error is given as

$$e(i) = r(i) - y(i).$$

The $γ(e)$ density probability distribution of the random error is given using the expansion in real symmetric kernel functions with the specifications presented in [11]. The formula for calculating the error probability density distribution is given

$$γ(e) = \frac{1}{N} \sum_{i=1}^{N} K_\alpha(e_i - e).$$

Where $K_\alpha(e - e_i)$ is real symmetric kernel function, $e_m$ is mean error, $N$ is number of independent readout of a target vector of $y(i)$.

For the random error, the known measure of the Renyi entropy of $α$-order is given as follows:

$$H_\alpha(e) = \frac{1}{\alpha - 1} \log \left( \int (γ(e))^\alpha \cdot de \right).$$

The formula is used to determine the information potential of a random error on the base of the Renyi entropy of $α$-order that is given as

$$V_{\alpha_\mu}(e) = \frac{1}{N^\mu} \sum_{i=1}^{N} \left[ \sum_{i=1}^{N} K_\alpha(e_i - e_i) \right].$$

The disadvantage of the information potential is that for the first-order Renyi entropy, the information potential is identically equal to one. For this reason, in control systems and neural networks, the information potential is used for the second-order Renyi entropy specified using the expression [12].

Units of Rainier information potential coincide with the units of density distribution and it is inversely proportional to the units of the observed value. To determine the second-order Renyi entropy from a known potential, the formula is used, that is given by

$$H_2(x) = -\log(V_{\alpha_2}(x)).$$

From expression (11) we see that the Rainier entropy decreases monotonically with an increase in the information potential. Consequently, with minimizing entropy, the information potential increases to its maximum value, which is inconvenient for constructing a control algorithm. Therefore, in a number of well-known works, the stochastic system control algorithm is built on the basis of the inverse index proportional to the information potential.

3. **Entropy-parametric control**

An effective way to control the state of the stochastic system is to change the distribution shape of its output values. It is possible to achieve control over a stochastic dynamic system by ensuring the position
of the system in the region of optimal States. This is achieved by converting the distribution shapes for
the parameters of the controlled system so that the $\Delta K_H$ entropy coefficient variation and the $\Delta \kappa$ anti-
kurtosis variation of this distribution tends to zero [8], that is given by

$$
\Delta K_H \to 0,
\Delta \kappa \to 0.
$$

The simultaneous provision of conditions (12) is possible by using parameter uncertainty, including
both as information and statistical uncertainty of the output. In this case, the entropy-parametric potential
should be used as an optimality criterion for a stochastic dynamical system with random external
influences, since it takes into account the properties of entropy for a number of values in a single
measurement cycle and reduces the influence of the entropy potential during the transition to a low-
entropy state due to the square of the standard deviation.

The formula of the entropy-parametric potential is given by

$$
\Delta_{ep} = \sqrt{\left(\Delta r \right)^2 + \left(k_{p_{anti}} \cdot \sigma\right)^2}.
$$

Under these conditions (12), for the stochastic dynamical system the construction of optimal control
is reduced to the minimizing condition of entropy-parametric potential

$$
\Delta_{ep} \to \min.
$$

In the area of optimal control, the entropy-parametric potential is characterized by the following
properties. Entropy-parametric potential is finite, real, and differentiable with respect to the parameter
of output quantity, monotonically increasing with the deviation of the expectation from its optimum
value. In the steady state of the system, entropy-parametric potential has a minimum value with the
known coefficient of entropy and anti-kurtosis of the output parameter. Change in condition is cause a
change in the distribution of output parameter of system that is reflected in the increase in entropy-
parametric potential.

The example of entropy parametrical control of stochastic system is shown on a figure 1 where
numbers 1 and 2 are accordingly the Gauss and Lapses distributions, numbers 3 and 4 are Weibull–
Gnedenko distributions with entropy factors that are equal 1,628 and 1,599 accordingly, number 5 is the
change of the object condition in space of average quadratic disorder and entropy potential, number 6 is
a direction of transition of system in an optimum condition 7 at preservation of the distribution data
shape.

![Figure 1. Space of entropy potential and root-mean-square.](image-url)
On a figure 1 arrow with number 6 illustrates the transition of stochastic system in an optimum condition at performance of a condition of minimization of entropy–parametric potential. The curve with number 5 illustrates real moving of system at control maintenance.

The application of entropy-parametrical control has allowed to dilate functionality of system cardio diagnostics at build-up of the plan of stochastic monitoring of electrophysiological performances of heart [13].

4. Conclusions
Due to the intense external influences can yield dynamic system of the boundary of the optimal state and its transition to a low-entropy state. Return the system to an optimum state occurs through a series of stable intermediate states in which the system can be a long time, increasing the time of one cycle. In this case, a forced return of the system to the area of optimal control is necessary that occurs on the basis of the analysis of the entropy-parametric criterion. Thus, the optimal control of the dynamic system is provided through the preservation of form output parameter distribution.

In conclusion, it should be noted that the entropy-parametric approach provides stable control of a dynamic system due to new features. First, it allows us to differentiate the field of sustainable inefficient and not productive states with respect to the area of the optimum state by comparing entropy-parametric test with the new value for the field of optimal control. Second, by minimizing the entropy-parametric potential you maintain control over the stochastic system when it picked up as in the low-entropy state, as and in the intermediate states of the system during its return to an optimal state by continuously adjusting the form of distribution. Third, the definition of entropy-parametric test directly on a set of measurements of the output parameter provides a return of a dynamic system from unfavorable conditions in the optimal state.

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