Dark Matter Induced Brownian Motion

Ting Cheng,1† Reinard Primulando,2 ‡ and Martin Spinrath1 ‡

1 Department of Physics, National Tsing Hua University, Hsinchu 30013, Taiwan
2 Center for Theoretical Physics, Department of Physics, Parahyangan Catholic University, Jl. Ciumbuleuit 94, Bandung 40141, Indonesia

We discuss a novel approach for directional, light dark matter searches inspired by the high precision position measurements achieved in gravitational wave detectors. If dark matter interacts with ordinary matter, movable masses are subject to an effect similar to Brownian motion induced by the scattering with dark matter particles which exhibits certain characteristics and could be observed. We provides estimates for the sensitivity of a hypothetical experiment looking for that motion. Interestingly, if successful, our approach would allow to constrain the local distribution of dark matter momentum.

INTRODUCTION

Cosmological and astrophysical data provides overwhelming evidence for dark matter (DM). Unfortunately, this data does not tell us anything definite about the nature of DM itself.

To solve this riddle, there has been tremendous experimental effort in the last decades focussing, in particular, on theoretically well-motivated weakly interacting TeV scale DM, so called Weakly Interacting Massive Particles (WIMPs). These efforts remain until today without any conclusive evidence for a discovery. For that reason, recently other potential mass regions for DM are more seriously considered and new ideas are developed to test them experimentally, see, e.g. the working group report [1]. In this paper we follow this line and discuss a novel experimental approach for light DM. As we will see our method would work, in principle, to very small masses. However, under more realistic assumptions, its sensitivity lies in the range above a few MeV/c^2.

Our proposal is motivated by the great achievements in laser interferometry for gravitational wave detectors, but as we will see later LIGO and other current earth-bound gravitational wave detectors are not well suited for our method.

We are not the first to propose to use gravitational wave detectors as DM detectors, see for instance [2,7]. Nevertheless, our approach is very different from theirs. They usually focus on very light DM with masses well below 1 keV/c^2, where DM behaves more like a classical field with a very long wave-length. In our method the particle nature of DM is essential. In fact, it is loosely inspired by the work of one of the authors [8]. Interestingly, results of the KWISP detector were presented which is looking for dark energy particles with an opto-mechanical setup [9] somewhat similar to our proposal.

The idea of this paper is based on the picture that any movable target with mass M_T, is actually in a bath of DM. If DM has some interaction with ordinary matter, that will induce some random motion of the target, which is conceptually similar to Brownian motion. We call this here Dark Brownian Motion (DBM) which can in principle be observed using high precision laser interferometers. It is experimentally easier to study the DBM if the target is constrained to a certain volume, e.g., if it is a pendulum. The concrete system is not relevant here, but we assume that the motion of the target in absence of any external forces can be understood well. In the following section, we will discuss the details of our method.

THE DETECTION METHOD

Since the earth has a relative velocity to the DM bath we expect that the DBM has a preferred direction. This directional dependence is well-known for directional dark matter detection with more conventional detectors [10]. In particular, we adopt here the formalism for the DM recoil momentum spectrum as it was discussed, for instance, in [11]. To keep the discussion simple, we will assume that the DM interaction with ordinary matter is fully elastic. The total event rate, \( R \), is given by

\[
R = (Z + N)^2 \sigma_{\text{DM} - N} \frac{M_T}{M_{\text{mol}}} \frac{\rho_{\text{DM}}}{m_{\text{DM}}} \langle \vec{v} \rangle_{\text{DM}},
\]

where \( Z + N \) is the atomic number, \( \sigma_{\text{DM} - N} \) is the average cross section of DM with a target nucleon, \( M_T \) the target mass, \( M_{\text{mol}} \) is the molar mass of the target, \( \rho_{\text{DM}} = 0.3 \text{ GeV/cm}^3 \) [12] is our assumed local DM energy density, \( \langle \vec{v} \rangle_{\text{DM}} = \int d^3v_{\text{DM}} v_{\text{DM}} f(\langle \vec{v} \rangle_{\text{DM}}) \) is the average local DM velocity.

We assume here that DM couples dominantly to nucleons in an isospin conserving manner, and throughout the paper we assume \(^{12}\text{C}\) as target material motivated by the use of graphene as reflective material [13] and tunable oscillators [14]. Actually, our suggested method does not necessarily rely on laser interferometers. Any monitoring of the targets with high precision can be employed.

For the DM velocity distribution we assume the Standard Halo Model

\[
f_{\text{gal}}(v) = \begin{cases} 
\frac{N}{(2\pi\sigma_z)^{3/2}} \exp\left(-\frac{|v|^2}{2\sigma_z^2}\right) & \text{if } |v| < v_{\text{esc}}, \\
0 & \text{if } |v| > v_{\text{esc}},
\end{cases}
\]

where \( v_{\text{esc}} \) is the escape velocity of the target.
where $v_{\text{esc}} = 550 \text{ km/s}$, $\sigma_v = 220 \text{ km/s}$ \cite{12} and for the translation between the galactic rest frame and the local laboratory frame we used the formulas in Appendix A of \cite{12}.

It is not clear, that the SHM is a good description of the DM velocity distribution. In a recent work \cite{17}, for instance, it was argued that especially for light DM one would expect a significant shift towards larger velocities opening up direct detection constraints to a lower mass region. A higher DM velocity would make our setup more sensitive as well. Nevertheless, to make our results more easily comparable to other studies, we will follow the conventional SHM assumption.

Under these assumptions and the formalism of \cite{11} we can simulate the distribution of recoil momenta, which allows us to define an asymmetry parameter

$$A = \frac{N_+ - N_-}{N_+ + N_-} = p_+ - p_- ,$$

where $N_{\pm}$ is the number of events of the DM hitting the detector with positive/negative target recoil momentum $q_R$ with $|q_R| > q_{\text{min}}$ along the axis we are probing with a laser interferometer. The parameter $q_{\text{min}}$ is a cutoff parameter which we will discuss later in more detail. We have also introduced here the likelihoods $p_{\pm} = N_{\pm} / (N_+ + N_-)$.

To determine $A$ we simulated for each parameter point one million events to get the distribution of the recoil momenta, from which we can derive $p_{\pm}$ easily. We estimate the statistical uncertainty for $A$ using ordinary error propagation with the $1\sigma$ uncertainty of the number of events given by $\sigma_{\pm} = \sqrt{N_{\pm}}$, and find

$$\sigma_A^2 = \frac{4}{R \Delta t} (p_+^2 p_- + p_-^2 p_+)$$

with $\Delta t$ as the length of data taking, which we have fixed for the rest of the paper to be 10 minutes.

At this point we want to sketch how the counting of events could be performed. We mentioned already, that we assume to have a mathematical model for the experiment which predicts the position and velocity of the target mass in the absence of forces. Suppose now the data is taken in time bins $t_1, t_2, \ldots$ and the target is hit by DM at some time between $t_h$ and $t_{h+1}$. We also assume that the time resolution of the detector can resolve the DM hits. The target position and velocity at $t_{h+1}$ will then differ from its predicted values given by the values measured at the previous time bins, and from the differences we can reconstruct the recoil momentum of the DM hit. For a harmonic oscillator, this can be done straight-forwardly using a Runge-Kutta algorithm as we convinced ourselves. In fact, for the asymmetry we only really need to reconstruct the sign of the recoil momenta, which is comparatively easy.

In Fig. 1 we plot the time dependence of the asymmetry for an imaginary experiment located in Hsinchu within 24 hours on June 1st, 2020. This is in fact an advantage of our experiment, since due to its directional information, we could measure the daily modulation easily instead of just the annual modulation. For the error bars we have assumed that we take data for ten minutes with a target mass of $10^{-3} \text{ g}$ and $\sigma_{\text{DM} - N} = 10^{-31} \text{ cm}^2$. In the rest of the paper our hypothetical experiment is aligned along the east-west direction and we take data at 10pm local Hsinchu time. Then $A \approx 0.37$, $\bar{v}_{\text{DM}} \approx 341 \text{ km/s}$ and the relative velocity of the lab with respect to the DM halo along this direction is $v_{\text{lab, EW}} \approx -183 \text{ km/s}$. A positive $A$ here means that there are more hits expected with a recoil momentum in the eastern direction.

Looking at Fig. 1 one might also consider the north-south direction since the asymmetry there at 4pm is more extreme than in the east-west direction. Nevertheless, we prefer here the latter since the daily modulation effect in that direction is much more pronounced. Of course, it would be better to study both directions simultaneously although this is certainly experimentally even more challenging.

One of the major reasons to consider $A$ as the relevant observable, and not just the total number of events or the measured momentum distribution itself, is its characteristic of a more pronounced time dependence, which can be a powerful discrimination from backgrounds. Moreover, it is more robust against uncertainties of the recoil momentum measurement after applying a momentum cutoff as will be introduced later.
The estimated $2 \sigma$ sensitivity of our experimental setup for a total target mass of $10^{-3}$ g. The black line is for an idealized experiment, the blue line includes a symmetric Gaussian background noise and the red line includes beyond the noise a minimal momentum cutoff, $q_{\text{min}} = 10^{-23} \text{ kg m/s}$. We also show the exclusion bound of the CRESST 2017 surface run [17] as a purple dotted line. The blue diamond corresponds to the blue diamond in Fig. 2 for more details, see main text.

**ESTIMATED SENSITIVITY**

In Fig. 2 we show an estimate for the sensitivity. We begin our discussion with an idealized experiment with infinitely precise momentum measurement and no background events. For a total target mass of $M_T = 10^{-3}$ g we find the black line as $2 \sigma$ exclusion level, i.e. $(A)/\sigma_A = 2$. The dependence of this line can be easily understood. For smaller masses, the DM flux, and hence the event rate, increases while the shape of the recoil momentum distribution remains the same, i.e.

$$
\frac{(A)^2}{\sigma_A} = R \Delta t \frac{(p_+ - p_-)^2}{4(p_+^2 + p_-^2 + p_+).}
$$

(5)

In this formula the only explicit dependence on the DM mass and the DM-nucleon cross section is in $R$, cf. Eq. (1).

In reality though there will be experimental uncertainties which alter the sensitivity. To illustrate their potential impact we introduce two sources of uncertainty.

The first is a generic background giving a recoil momentum of the test masses. That could be seismic noise, nearby traffic, radioactivity, etc. As already mentioned many of these backgrounds could be easily discriminated from a potential DM signal due to the different characteristic time-dependences of the asymmetries. For simple illustrative purposes we model them in terms of a Gaussian distribution of recoil momenta centered around zero, i.e. $(A)_{\text{bkg only}} = 0$. Furthermore, we set the width of the background distribution to be $\sigma_{\text{bkg}} = 3 \times 10^{-23} \text{ kg m/s}$ and assume an average background event rate $R_{\text{bkg}} = 0.1 \text{ Hz}$. A symmetric background will generally speaking reduce the asymmetry and hence lower the sensitivity. This can also be seen in Fig. 2 where we draw the estimated sensitivity as a blue line for a $2 \sigma$ deviation from the background hypothesis, i.e. $(A)/\sigma_A + \sigma_{\text{bkg}} = 4$. Note that $\langle A \rangle$ and $\sigma_A$ include both of the DM signal and the background.

The second major deviation from an ideal experiment is the uncertainty of the position and hence the recoil momentum measurement. Considering a harmonic oscillator with an eigenfrequency $\omega_0$ and a position resolution $d_{\text{min}}$ as a toy model, this implies a minimal momentum resolution of the order of $q_{\text{min}} = M_T \frac{\omega_0}{d_{\text{min}}}$. To improve the sensitivity one could imagine to split the detector mass into smaller detector cells, which would lower $q_{\text{min}}$ linearly with the number of cells. For definiteness we assume $\omega_0 = 100 \text{ Hz}$ and $d_{\text{min}} = 10^{-19} \text{ m}$, which is inspired by LIGO’s capabilities [18]. Therefore we assume a momentum cutoff $q_{\text{min}} = 10^{-23} \text{ kg m/s}$. Nevertheless, LIGO with its 40 kg mirrors has a much larger momentum cutoff and cannot resolve the momentum recoil of an individual light DM hit.

The uncertainty in the momentum measurement affects recoil momenta larger than $q_{\text{min}}$ as well, but in the asymmetry $A$ only the sign of the momentum enters which we know well, after we discarded all events with recoil momenta smaller than $q_{\text{min}}$.

A momentum cutoff $q_{\text{min}} > 0$ on one hand reduces the number of events in the signal region and thus potentially lowers the significance. On the other hand, it can enhance $A$ and the two effects are competing with each other.

In Fig. 2 we show the final estimated sensitivity curve including background noise and the momentum resolution cutoff as the red line. First of all, we can see in Fig. 2 that the momentum cutoff makes it impossible for our hypothetical experiment to discover DM masses with less than a few MeV/c$^2$ (the vertical dashed line). This will be clear from the kinematical arguments which we discuss later. But we can also see, that for the mass window between about 15 to 500 MeV/c$^2$ the momentum cutoff increases the theoretical sensitivity compared to the background only case which is due to the increase in the asymmetry.

From Fig. 2 it is also clear, that the $q_{\text{min}}$ plays the equivalent role of the energy threshold in conventional DM detectors. In our case though, this depends only indirectly on the chosen target material, instead it depends dominantly on the resolution of the interferometer and the target mass neglecting potential excitation modes of the target material.

To give an impression on how our sensitivity compares to experimental results, we also show the exclusion line from the CRESST 2017 surface run [17] which is relevant.
FIG. 3. The dependence of the asymmetry $A$ on a lower/upper bound for the recoil momentum $q_{\min}/q_{\max}$ in orange/green for a benchmark point. The dashed lines are for an idealized experiment without any background noise. As soon as $q_{\min}$ is larger than the maximal DM recoil momentum there are no events left and the line ends at the red cross. The solid lines are for an experiment with some assumed background noise. The straight, dotted lines indicate the value of $A$ when no cuts are applied without background (upper one) and with background (lower one). The blue diamond corresponds to the blue diamond in Fig. 2. For more explanations, see main text.

for DM masses above 140 MeV/c$^2$, and in that region, it is better than our suggestion. Cosmology also provides a bound over the whole shown DM mass region, but it is rather weak. Ref. [19], for instance, finds that the bound over the whole shown DM mass region, but it is better than our suggestion. Cosmology also provides bounds over the whole displayed mass region should be less than a few times $10^{-7}$ cm$^2$/GeV. Ref. [19], for instance, finds that the bound over the whole shown DM mass region is rather weak.

It is instructive to have a closer look at the momentum cutoff $q_{\min}$. We can easily understand from the most extremal case that the asymmetry can increase for an increasing cutoff. Let $v_{\text{lab}}$ be the lab velocity with respect to the galactic halo in the direction we are probing. For $|v_{\text{esc}} - v_{\text{lab}}| < q_{\min}/(2 m_{\text{DM}}) < |v_{\text{esc}} + v_{\text{lab}}|$ the asymmetry is extremal, $|A| = 1$, due to simple kinematics. This leads to another characteristic feature of a DM signal. Increasing $q_{\min}$ the modulus of the DM induced asymmetry continuously increases until no events are left for an idealized experiment. This is the dashed orange line in Fig. 2 which ends at $q_{\min} = 5 \times 10^{-23}$ kg m/s. Again this would be affected if we add a background. Assuming the same background as we have used in Fig. 2 we see that the asymmetry raises first before it goes to zero, cf. the solid, orange line in Fig. 3. It is plausible to assume that the background distribution is broader than a DM signal, and for a large $q_{\min}$ the signal-to-noise ratio decreases so that the asymmetry goes to zero.

Actually, this could be exploited to our advantage by introducing an upper cutoff $q_{\max}$ and only include events in the asymmetry $A$ with a recoil momentum $|q_R| < q_{\max}$. This could help reducing background from neutrinos or cosmic rays, for instance. In the SHM, or any other halo model, there is a prediction for the maximum recoil momentum possible given the DM mass and we could discard all events with momenta higher than that to increase the signal-to-noise ratio. We have shown the impact of such a cutoff as well in Fig. 3 for an idealized experiment (dashed, green line) and the case including a background (solid, green line).

A set of benchmark cutoffs could hence be useful to understand the data better, and once a potential DM signal is identified from the shape of the recoil momentum distribution, the DM mass scale and halo model could be easily constrained.

SUMMARY AND CONCLUSIONS

In this paper we have suggested a novel way to search for light DM which is based on the fact that freely movable test masses could be subject to an effect very similar to Brownian motion. This motion has a characteristic, time- and direction-dependent asymmetry of the recoil momenta due to the relative motion of the earth to the DM halo. This asymmetry and, in particular, its daily modulation could be observed in an experiment similar to gravitational wave detectors, which measure the position of a target object to very high precision. Nevertheless, current gravitational wave detectors like LIGO are not suited for our proposal since their mirror masses imply a momentum cutoff well above our expectations for light DM. Assuming another experiment with a lighter target mass instead but a position resolution similar to LIGO one could test DM masses down to the few MeV/c$^2$ region. For our example, we find that for a DM mass of 20 MeV/c$^2$ we could test a DM-nucleon cross section down to $6.7 \times 10^{-23}$ cm$^2$ with only 10 minutes of data taking. This could even improve taking into account that light DM can be boosted by cosmic rays [10].

Another way to improve the setup is to look at several lighter test masses simultaneously, lowering the recoil momentum threshold and hence increasing the sensitivity towards lighter DM masses. The sensitivity could also be improved by optimizing it with a maximum and/or minimum momentum cutoff. Interestingly, once DM would be found in this way the measured time- and direction-dependent momentum distributions can be used to constrain the DM mass and halo model.

Acknowledgements — We would like to thank Yue-Lin Sming Tsai for useful comments on the manuscript. TC and MS are supported by the Ministry of Science and Technology (MOST) of Taiwan under grant number MOST 107-2112-M-007-031-MY3.
[1] J. Alexander et al. (2016), arXiv:1608.08632 [hep-ph].
[2] Y. V. Stadnik and V. V. Flambaum, Phys. Rev. Lett. 114, 161301 (2015).
[3] Y. V. Stadnik and V. V. Flambaum, Phys. Rev. A 93, 063630 (2016).
[4] S. Jung and C. S. Shin, Phys. Rev. Lett. 122, 041103 (2019), arXiv:1712.01396 [astro-ph.CO].
[5] A. Pierce, K. Riles, and Y. Zhao, Phys. Rev. Lett. 121, 061102 (2018), arXiv:1801.10161 [hep-ph].
[6] S. Morisaki and T. Suyama, (2018), arXiv:1811.05000 [hep-ph].
[7] H. Grote and Y. V. Stadnik, (2019), arXiv:1906.06193 [astro-ph.IM].
[8] V. Domcke and M. Spinrath, JCAP 1706, 055 (2017), arXiv:1703.08629 [astro-ph.CO].
[9] S. Arguedas Cuendis et al. (CAST), (2019), arXiv:1906.01084 [hep-ex].
[10] D. N. Spergel, Phys. Rev. D37, 1353 (1988).
[11] P. Gondolo, Phys. Rev. D66, 103513 (2002), arXiv:hep-ph/0209110 [hep-ph].
[12] M. C. Smith et al., Mon. Not. Roy. Astron. Soc. 379, 755 (2007), arXiv:astro-ph/0611671 [astro-ph].
[13] E. Carrasco, M. Tamagnone, and J. Perruisseau-Carrier, Applied Physics Letters 102, 104103 (2013), https://doi.org/10.1063/1.4795787.
[14] C. Chen, S. Lee, V. V. Deshpande, G.-H. Lee, M. Lekas, K. Shepard, and J. Hone, Nature Nanotechnology 8, 923 EP (2013).
[15] F. Mayet et al., Phys. Rept. 627, 1 (2016), arXiv:1602.03781 [astro-ph.CO].
[16] T. Bringmann and M. Pospelov, Phys. Rev. Lett. 122, 171801 (2019), arXiv:1810.10543 [hep-ph].
[17] G. Angloher et al. (CRESST), Eur. Phys. J. C77, 637 (2017), arXiv:1707.06749 [astro-ph.CO].
[18] B. P. Abbott et al. (LIGO Scientific, Virgo), Phys. Rev. Lett. 116, 131103 (2016), arXiv:1602.03838 [gr-qc].
[19] W. L. Xu, C. Dvorkin, and A. Chael, Phys. Rev. D97, 103530 (2018), arXiv:1802.06788 [astro-ph.CO].