Detection Rates for Kaluza-Klein Dark Matter

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We consider the lightest Kaluza-Klein particle at N=1 mode (LKP) of universal extra dimension to be the candidate for Dark Matter and predict the detection rates for such particles for Germanium and NaI detectors. We have also calculated the nature of annual modulation for the signals in these two types of detectors for LKP Dark Matter. The rates with different values of speed of solar system in the Galactic rest frame are also evaluated.

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I. INTRODUCTION

There are strong indirect evidence (gravitational) from various observations like velocity curves of spiral galaxies, gravitational lensing etc., in favour of the existence of enormous amount of invisible, nonluminous matter or Dark Matter in the universe. This Dark Matter constitutes more than 90% of the matter in the universe. Although the constituents of Dark Matter still remain a mystery, the indirect evidence suggests that large part of Dark Matter should be of non-Baryonic in nature. They are stable, heavy, non relativistic (Cold Dark Matter or CDM) and are weakly interacting. Therefore they are often known as Weakly Interacting Massive Particles or WIMPs.

Particle physics has suggested a number of options for the candidates of these non baryonic Cold Dark Matter. Currently the most popular candidate is a lightest supersymmetric particle (LSP) neutralino ($\chi$) which of course is not a Standard Model particle. In Minimal Supersymmetric Standard Model or MSSM, the LSP is a neutralino ($\chi$). Neutralino is a Majorana fermion and it is a superposition of supersymmetric partners of neutral U(1) and SU(2) gauge bosons and neutral Higgs bosons. The conservation of R-parity ensures that the LSP is a stable particle. In literature there are a lot of work where neutralino in MSSM and supergravity models is consdered as a dark matter candidate [1,2,3,4,5]. In a recent work neutralino LSP as a Dark Matter candidate from minimal Anomaly Mediated Supersymmetric Breaking (mAMSB) model has been addressed [6].

In the present work we consider a bosonic particle, unlike the fermionic SUSY particle to be the candidate for Dark Matter. This particle is a lightest Kaluza-Klein (KK) particle or LKP in universal extra dimension (UED) [8]. The Kaluza-Klein theory [8] which is a 5-dimensional theory with 4 space-time dimensions and 1 extra space dimension, the extra dimension (5th dimension) is assumed to be compact. The fifth dimension has the topology of a circle with the compactification radius of the order of Planck scale. The topology of this 5-Dimensional space-time is $R^4 \times S^1$ and the fifth coordinate $y$ is periodic with $0 \leq my \leq 2\pi$, $m$ being inverse of the radius of the circle. This periodicity of the extra dimension enables one to obtain an infinite tower of fields by making Fourier expansion in the fifth coordinate. This tower is known as Kaluza-Klein tower. The Universal Extra Dimension model is one where all Standard Model particles can propagate in extra dimensions of size $R \sim \text{TeV}^{-1}$. In UED therefore every standard model particle has a KK partner in KK tower. In the present case UED scenario is considered with only one extra dimension. The parity conservation of KK particles ensures that the lightest KK particle or LKP is stable and thus can be a candidate for cold Dark Matter.

The direct detection of WIMP Dark Matter by a terrestrial detector uses the principle of elastic scattering off detector nuclei. But this is a difficult task as the WIMP-nucleus interaction is very feeble. The energy deposited by a WIMP of mass few GeV to 1 TeV on a detector nucleus is not more than 100 keV. Hence for direct detection of WIMP the detector has to be of low energy threshold and of low background.

Presently, there are several experiments engaged in direct detection of Cold Dark Matter. The direct WIMP search by DAMA/NaI collaboration [9] with $\sim 100$ kg NaI(TI) set up at Gran Sasso in Italy looks for annual modulation signature of WIMP. Due to the earth motion around the sun, the earth bound detector will experience a larger WIMP flux in the month of June when the earth’s rotational the velocity adds up to the velocity of solar system in Galaxy and minimum in December when...
these two velocities are antiparallel. The DAMA collaboration claimed to have detected this annual modulation of WIMP through their direct WIMP detection experiments. Their analysis suggests possible presence of Dark Matter with mass around 50 GeV. The Cryogenic Dark Matter Search or CDMS detector employs low temperature Ge and Si as detector materials to detect WIMP’s via their elastic scattering off these nuclei [12]. This is housed in a 10.6 m tunnel (~16 m.w.e) at Stanford Underground Facility beneath the University of Stanford. Although their direct search results are compatible with some regions of 3-σ allowed regions for DAMA analysis, it excludes DAMA results if standard WIMP interaction and a standard Dark Matter halo is assumed. The EDELWEISS Dark Matter search experiment which also uses cryogenic Ge detector at Frejus tunnel, 4800 m.w.e under French-Italian Alps observed no nuclear recoils in the fiducial volume [13]. The lower bound of recoil energy in this experiment was 20 keV. The Heidelberg Dark Matter Search (HDMS) uses their inner detector, highly pure $^{73}$Ge crystals [13] and with a very low energy threshold. They have recently made available their 26.5 kg day analysis. The recent low threshold experiment GENIUS (GErmenium in liquid Nitrogen Underground Setup) [14] at Gran Sasso tunnel in Italy has started its operation. Although a project for $\beta\beta$-decay search, due to its very low threshold (and expected to be reduced further) GENIUS is a potential detector for WIMP direct detection experiments and for detection of low energy solar neutrinos like pp-neutrinos or $^7$Be neutrinos. In GENIUS experiment highly pure $^{76}$Ge is used as detector material. For Dark Matter search 100 kg. of detector material is suspended in a tank of liquid nitrogen. In Fig. 3 of Ref. [14] the projected limit for WIMP mass $m_x$ and scalar cross-section for this detector is given. In the present work we have calculated the prediction for rates for the case two types of detector material namely $^{76}$Ge and NaI.

The theoretical predictions of rates has been addressed by several authors [15,16,17]. Although in Ref. [15] rates for detectors with various detecting material is given. In Refs. [16,17], the emphasis was mainly to analyse DAMA/NaI data.

The paper is organised as follows. In section 2 we give the theory for calculation of detection rates for Ge and NaI detectors. The actual calculation of differential rates for $^{76}$Ge detector and NaI detector for various choices of WIMP mass and other parameters are discussed in section 3. In Section 4 we give some concluding remarks.

II. THEORY

Differential detection rate of WIMPs per unit detector mass can be written as

$$\frac{dR}{d|q|^2} = N_T \Phi \frac{d\sigma}{d|q|^2} \int f(v)dv \quad (2.1)$$

where $N_T$ denotes the number of target nuclei per unit mass of the detector, $\Phi$ being the WIMP flux and $v$ is WIMP velocity in the reference frame of earth and $f(v)$ is the distribution of this velocity. The integration is over all possible kinematic configurations in the scattering process. In the above, $|q|$ is the momentum transferred to the nucleus in WIMP-nucleus scattering. Nuclear recoil energy $E_R$ is

$$E_R = \frac{|q|^2}{2m_{\text{nuc}}}$$

$$m_{\text{red}} = \frac{m_x m_{\text{nuc}}}{m_x + m_{\text{nuc}}} \quad (2.3)$$

where $\theta$ is the scattering angle in WIMP-nucleus centre of momentum frame, $m_{\text{nuc}}$ is the nuclear mass and $m_x$ is the WIMP mass.

Now expressing $\Phi$ in terms of local WIMP density $\rho_x$, WIMP velocity $v$ and WIMP mass $m_x$ and writing $|q|^2$ in terms of nuclear recoil energy $E_R$ with noting that $N_T = 1/m_{\text{nuc}}$, Eq. (1) takes the form

$$\frac{dR}{dE_R} = \frac{\rho_x}{m_x} 2 \frac{d\sigma}{d|q|^2} \int_{v_{\text{min}}}^{\infty} vf(v)dv \quad (2.4)$$

The WIMP-nucleus (or WIMP-nucleon) scattering cross-section has two parts, namely spin-independent or scalar cross-section and spin dependent cross-section. Here we make the assumption that scalar cross section dominates over the spin dependent cross-section. Following Ref. [18] the WIMP-nucleus differential cross-section for the scalar interaction can be written as

$$\frac{d\sigma}{d|q|^2} = \frac{\sigma_{\text{scalar}}}{4m_{\text{nuc}}^2 v_{\text{min}}^2} F^2(E_R) \quad (2.5)$$

In the above $\sigma_{\text{scalar}}$ is WIMP-nucleus scalar cross-section and $F(E_R)$ is nuclear form factor given by [19,20]

$$F(E_R) = \left[ \frac{3j_1(qR_1)}{qR_1} \right] \exp \left( \frac{q^2 x^2}{2} \right) \quad (2.6)$$

$$R_1 = (r^2 - 5x^2)^{1/2}$$

$$r = 1.2 A^{1/3}$$

where thickness parameter of the nuclear surface is given by $s \approx 1$ fm, $A$ is the mass number of the nucleus and $j_1(qR_1)$ is the spherical Bessel function of index 1.
For the distribution \( f(v_{\text{gal}}) \) of WIMP velocity \( v_{\text{gal}} \) with respect to Galactic rest frame, a Maxwellian form is considered here. The velocity \( v \) (and \( f(v) \)) with respect to earth rest frame can then be obtained by making the transformation

\[
v = v_{\text{gal}} - v_{\oplus} \tag{2.7}
\]

where \( v_{\oplus} \) is the velocity of earth with respect to Galactic rest frame and is given by

\[
v_{\oplus} = v_0 + v_{\text{orb}} \cos \gamma \cos \left( \frac{2\pi(t - t_0)}{T} \right) \tag{2.8}
\]

In Eq. (2.8), \( T = 1 \) year time period of earth motion around the sun, \( t_0 = 2^{nd} \) June, \( v_{\text{orb}} \) is earth orbital speed and \( \gamma \simeq 60^\circ \) is the angle subtended by earth orbital plane at Galactic plane. The speed of solar system \( v_\odot \) in the Galactic rest frame is given by,

\[
v_\odot = v_0 + v_{\text{pec}} \tag{2.9}
\]

where \( v_0 \) is the circular velocity of the Local System at the position of Solar System and \( v_{\text{pec}} \) is speed of Solar System with respect to the Local System. The latter is also called peculiar velocity and its value is 12 km/sec. Although the physical range of \( v_0 \) is given by \([21,22] \) 170 km/sec \( \leq v_0 \leq 270 \) km/sec (90\% C.L.), in the present work we consider the central value of \( v_0 = 220 \) km/sec. Here we mention that Eq. (2.8) is the origin of annual modulation of WIMP signal reported by DAMA/NaI experiment [14].

Defining a dimensionless quantity \( T(E_R) \) as,

\[
T(E_R) = \frac{\sqrt{\pi}}{2v_0} \int_{v_{\text{min}}}^{\infty} \frac{f(v)}{v} dv \tag{2.10}
\]

and noting that \( T(E_R) \) can be expressed as [18]

\[
T(E_R) = \frac{\sqrt{\pi}}{4v_{\oplus}} v_0 \left[ \text{erf} \left( \frac{v_{\text{min}} + v_{\oplus}}{v_0} \right) - \text{erf} \left( \frac{v_{\text{min}} - v_{\oplus}}{v_0} \right) \right] \tag{2.11}
\]

we obtain from Eqs. (2.4) and (2.5)

\[
\frac{dR}{dE_R} = \frac{\sigma_{\text{scalar}} \xi \rho_0}{4v_{\oplus} m_\chi m_p^2} F^2(E_R) \left[ \text{erf} \left( \frac{v_{\text{min}} + v_{\oplus}}{v_0} \right) - \text{erf} \left( \frac{v_{\text{min}} - v_{\oplus}}{v_0} \right) \right] \tag{2.12}
\]

In the above, \( \rho_0 \), is the total local Dark Matter density generally taken to be 0.3 GeV/cm\(^3\) and \( \xi \equiv \rho_\chi/\rho_0 \). The above expression for differential rate is for a monoatomic detector like Ge but it can be easily extended for a diatomic detector like NaI as we will see later.

In the present case, the lightest KK state or LKP, \( B^1 \), in simplest UED is considered to be the candidate for WIMP Dark Matter [17]. The spin independent cross-section for scattering of \( B^1 \) with mass \( m_{B^1} \) off nucleon or nucleus is given by [17]

\[
\sigma_{\text{scalar}} = \frac{m_N^2}{4\pi(m_{B^1} + m_N)^2} [Zf_p + (A - Z)f_n]^2 \tag{2.13}
\]

\[
f_p = \sum_{\text{all } q} \frac{B_q + \gamma_q}{m_q} m_p f_{T_q}^p
\]

\[
f_n = \sum_{\text{all } q} \frac{B_q + \gamma_q}{m_q} m_n f_{T_q}^n
\]

\[
\beta_q = \frac{m_q}{\cos^2 \theta_W} \left[ Y^2 \frac{m_{B^1}^2 + m_{q_L}^2}{(m_{B^1}^2 - m_{q_L}^2)^2} + (L \rightarrow R) \right]
\]

\[
\gamma_q = \frac{m_q}{\cos^2 \theta_W} \left[ 1 - 2 \frac{\cos^2 \theta_W}{m_{B^1}^2} \right] \tag{2.14}
\]

In Eq. (2.13), \( m_X \) is the nuclear or nucleon mass, \( q \) denotes the quarks, \( Y \) is the hypercharge and \( m_{h} \) is Higgs mass. For the mass \( m_{qL} \), we follow Ref. [17] and define a degeneracy parameter \( d = (m_{qL} - m_{B^1})/m_{B^1} \) and then assign different values of \( d \) for the actual calculation of WIMP detection rates. Needless to mention here that in the present calculation \( m_\chi \equiv m_{B^1} \).

The measured response of the detector by the scattering of WIMP off detector nucleus is in fact a fraction of the actual recoil energy. Thus, the actual recoil energy \( E_R \) is quenched by a factor \( q_{nX} \) (different for different nucleus \( X \)) and we should express differential rate in Eq. (2.12) in terms of \( E = q_{nX} E_R \). For Ge detector the expected energy spectrum per energy bin can be expressed as

\[
\frac{\Delta R}{\Delta E(E)} = \int_{E/q_{nX}}^{(E + \Delta E)/q_{nX}} dR_{Ge}(E_R) \frac{dE_R}{dE} \tag{2.15}
\]

and for a diatomic detector NaI, the above expression takes the form

\[
\frac{\Delta R}{\Delta E(E)} = a_{Na} \int_{E/q_{Na}}^{(E + \Delta E)/q_{Na}} dR_{Na}(E_R) \frac{dE_R}{dE} + a_{I} \int_{E/q_{I}}^{(E + \Delta E)/q_{I}} dR_{I}(E_R) \frac{dE_R}{dE} \tag{2.15}
\]

where \( a_{Na} \) and \( a_{I} \) are the mass fractions of Na and I respectively in a NaI detector and are given by (see Table 2)

\[
a_{Na} = \frac{m_{Na}}{m_{Na} + m_{I}} = 0.153 \quad a_{I} = \frac{m_{I}}{m_{Na} + m_{I}} = 0.847
\]

In this work theoretical prediction of the differential rate is calculated with \( \Delta E = 1 \) keV.
III. CALCULATION OF WIMP DETECTION RATES FOR GERMANIUM AND NAI DETECTORS

In order to calculate the theoretical predictions for the variation of detected signals with $E$, we use Eqs. (2.12 - 2.15) along with Eqs. (2.3, 2.6-2.9). The scalar cross-section for the direct detection of LKP Dark Matter is estimated using Eq. (2.13). For $f_{x}^{\pi}$ ($x = p$ or $n$), we adopt the central values of these quantities given in [23]. Thus, $f_{x}^{p} = 0.020, f_{x}^{p} = 0.026, f_{x}^{n} = 0.118, f_{x}^{n} = 0.014, f_{x}^{n} = 0.036, f_{x}^{n} = 0.118$ and $f_{x, i}^{n} = 2(1-f_{x}^{n}-f_{x}^{n}-f_{x}^{n})/27$. We have adopted four different values for degeneracy parameter $d = (m_{q} - m_{B})/(m_{B})$ and they are $d = 0.05, 0.2, 0.3$, and $0.4$.

For the actual calculations, we need to know among other things the value of $\xi$, the fraction of local WIMP density $\rho_{w}$ with respect to the total dark matter density $\rho_{0}$. This is usually estimated by calculating the relic WIMP density, $\Omega_{w}h^{2}$ where $\Omega_{w} = \rho_{w}/\rho_{c}$ ($\rho_{c}$ is the critical density of the universe) and $h$ is related to the value of Hubble parameter at the present epoch. This in turn depends on WIMP-WIMP annihilation cross-section (e.g. in [13]). In this first calculation of WIMP detection rates with a bosonic Kaluza-Klein particle considered as a candidate for WIMP, we adopt the following procedure for the estimation of $\xi$. DAMA/Nal direct Dark Matter detection experiment has furnished their results of annual modulation signal signature of WIMP in the form of allowed contours in the parameter space of $m_{\chi}$ and $\xi\sigma_{scalar}^{p}$ [10]. Recently started GENIUS experiment (uses pure $^{76}$Ge as detector material) [14] has predicted the variation of $\sigma_{scalar}^{p}$ for different values of WIMP mass $m_{\chi}$. Here $\sigma_{scalar}^{p}$ is WIMP scalar cross section on proton normalised to square of the nucleon number ($A^{2}$). Now, $\xi$ is estimated by first calculating $\sigma_{scalar}^{p}$ from Eq. (2.10) for different $m_{\chi}$’s (for different fixed values of $d$). From the values of $\xi\sigma_{scalar}^{p}$, predicted by the experiments (as the ones discussed above) for the same $m_{\chi}$, one can estimate the value of $\xi$ for different $m_{\chi}$ (and also for different $d$) using the ratio $\xi = \frac{(\xi\sigma_{scalar}^{p})_{Exp.}}{\sigma_{scalar}^{p}}$. In order to calculate $\sigma_{scalar}^{p}$ for KK Dark Matter from Eq. (2.10), we make the approximation $f_{p} = f_{n}$. Now replacing $m_{N}$ by $m_{p}$, the mass of the proton, in Eq. (2.10), and dividing the expression by $A^{2}$ gives $\sigma_{scalar}^{p}$ for LKP Dark Matter. We have calculated the variation of $\sigma_{scalar}^{p}$ with $m_{\chi}$ for KK Dark Matter (LKP) for four different values of $d$ ($d = 0.05, 0.1, 0.2, 0.4$). The results are plotted in Fig. 1. Also plotted in Fig. 1 are the $3\sigma$ C.L. DAMA/Nal allowed region in $\xi\sigma_{scalar}^{p}$ - $m_{\chi}$ plane for $v_{0} = 220$ km/sec (Fig. 4b in Ref. [10]) and the estimated limit of the variation $\sigma_{scalar}^{p}$ with $m_{\chi}$ from GENIUS detector with 100 kg. of natural $^{76}$Ge (Fig. 3 of Ref. [14]). Here we adopt the following prescription for estimation of $\xi$. If the ratio $\xi < 1$ then we take the actual calculated value of $\xi$ from Fig. 1. Otherwise if $\xi \geq 1$ then we simply take $\xi = 1$.

With this prescription, we see from Fig. 1, that if we use DAMA/Nal plot for the estimation of $\xi$ then for most of the cases $\xi = 1$. As this is an unlikely scenario, we consider instead the GENIUS (100 kg natural Ge) plot and estimate $\xi$ (using the ratio mentioned above) for different values of $m_{\chi}$ with different fixed values of $d$. The values of $d$ are chosen from the range given in Ref. [14]. We have taken four different values of degeneracy parameter $d$ ($d = 0.05, 0.1, 0.2, 0.4$) and for each value of $d$ we have taken five values of $m_{\chi}$ ($m_{hi} = 30$ GeV, 50 GeV, 80 GeV, 100 GeV, 150 GeV) for estimation of the ratio $\xi$. The results are furnished in a tabulated form in Table 1.

![Table 1: Estimated values of $\xi$, the ratio of local WIMP density and total local Dark Matter density for different WIMP mass $m_{\chi}$ and degeneracy parameter $d$ (see text).](image)

From Table 1 we see that $\xi$ increases with increase of the value of $d$ as also the mass $m_{\chi}$. This feature is reflective of the fact that $\sigma_{scalar}^{p}$ decreases with degeneracy $d$ and $m_{\chi}$ for the range of values chosen for present study.

Having obtained $\xi$ from Table 1, we now proceed to calculate the WIMP detection rates for a Germanium detector that uses $^{76}$Ge as detector material. These rates are computed using Eqs. (2.4 - 2.14) in case of four degeneracy parameters mentioned earlier ($d = 0.05, 0.1, 0.2, 0.4$) and for five values of $m_{\chi}$’s 30 GeV, 50 GeV, 80 GeV, 100 GeV and 100 GeV for each value of $d$ (see Table 1). The WIMP - nucleus scalar cross sections are evaluated by replacing $m_{N}$ in Eq. (2.13) by mass of the nucleus $m_{nuc}$ and use the values of $f_{p}$ and $f_{n}$ as given above. The nuclear mass $m_{nuc}$ is calculated using the relation $m_{nuc} = (Zm_{p} + (A-Z)m_{n}) + \Delta$, where $m_{p}$ and $m_{n}$ are respectively the mass of the proton and neutron. The mass excess is denoted as $\Delta$ (electron masses are neglected). Table 2 gives the values of $\Delta$ (in MeV) and calculated values of $m_{nuc}$ for $^{76}$Ge, $^{23}$Na, $^{127}$I nuclei.
We first calculate differential rates $\frac{dR}{dE}$ (Eq. 2.12) for a wide range of values of recoil energy $E_R$ with $v_0 = 220$ km/sec. The value of $t$ (day number) in Eq. (2.8) is taken to be $t_0$, i.e. the calculations are for 2nd June. Then we use Eq. (2.14) for predicting the observable rate $\frac{dR}{dt}$ (Eq. 14) in the units of kg$^{-1}$ day$^{-1}$ keV$^{-1}$.

Table 2: Mass excess $\Delta$ (in MeV) and calculated nuclear mass $m_{nuc}$ for three nuclei used for WIMP detection

| Z  | Mass Excess $\Delta$ (MeV) | Nuclear Mass $m_{nuc}$ (amu) |
|----|---------------------------|-----------------------------|
| 11 | 0.3                       | 11.29                       |
| 53 | 0.5                       | 53.29                       |
| 76 | 0.8                       | 76.29                       |

The results are shown in Fig. 2a to Fig. 2d. Each of the graphs (a - d) in Fig. 2 is for rates for five different values of $m_\chi$ with a fixed value of degeneracy $d$ (Table 1). For the sake of clarity of the plots, the upper limit of $\frac{dE}{dt}$ in all the four graphs in Fig. 2 is truncated at 0.2 kg$^{-1}$ day$^{-1}$ keV$^{-1}$. Therefore in Fig. 3 we plot the same (rate vs $E$) for the case of $m_\chi = 30$ GeV and for $d = 0.05$. The same for other values of $d$ are almost identical. A number of features of the nature and behaviour of the plots are apparent from Figs. 2 and 3 and they are the results of complicated dependence of rates on various factors, like variations of scalar cross sections with $m_\chi$ and $d$, the factor $\xi$, the nuclear recoil energy etc. Let us make the following observations in Figs 2 and 3.

1. Firstly we fix a value of $d$, say 0.05 (Fig 1a), the peak value of $\frac{dR}{dE}$ falls with increase of $m_\chi$. This is mainly due to $1/(m_\chi m_{\text{red}}^2)$ behaviour of rate equation and also due to decrease of $\sigma_{\text{scalar}}$ with $m_\chi$. From Fig 1a, one can also see that the rate corresponding to lower $m_\chi$ falls off early.

2. This feature has to do with the nature of variation of the expression $f_{\text{err}} = \text{erf} \left( \frac{m_{\text{nuc}} - m_{\text{nuc}}}{\Delta E} \right) - \text{erf} \left( \frac{m_{\text{nuc}} - m_{\text{nuc}}}{\Delta E} \right)$ in Eq. (2.12) with recoil energy $E_R$. We have actually checked that the plot for $E_R$ vs $f_{\text{err}}$ always remain lower for lower $m_\chi$ for a fixed value of $d$. It is also observed from Fig. 2a - 2b that the plots of $m_\chi = 30, 50, 80$, and 100 GeV’s for $d = 0.05$ (Fig. 2a) almost identical to those with $d = 0.1$ (Fig. 2b) but the plots differs for 150 GeV for these two values of $d$. This feature has its origin in the way $\xi$ is calculated. The relative decrease of $\sigma_{\text{scalar}}$ for increase in $d$ for a particular $m_\chi$ is compensated by the relative increase in the value of $\xi$ (see Fig. 1 and discussion above regarding estimation of $\xi$). For the case where this is not satisfied we find a difference in the nature of variation of rate with energy. Thus the plot corresponding to $m_\chi = 150$ GeV in Fig. 2b (where the value of $\xi$ is taken to be 1) is different from the corresponding plot in Fig. 2a. This therefore explains the deviations of a particular plot of a specific graph (a or b or c or d) in Fig. 2 from the corresponding plot of other three graphs.

From Fig. 2 we also observe that the differential rate is very small for WIMP masses greater than 100 GeV while for lower WIMP mass it is not that small. As for example for $m_\chi = 50$ GeV, the differential rate at $E = 4.5$ keV is $\approx 0.12$, whereas the same for a 150 GeV WIMP varies from 0.082 for $d = 0.05$ to $3.3 \times 10^{-3}$ for $d = 0.4$. More so, for this $m_\chi (= 50$ GeV) we obtain a $d$ independent rate. Also from Fig 1 and above discussions, it can be said that for WIMP mass upto about 60 GeV, we may obtain $d$ independent rate.

With the same values $\xi$, $m_\chi$ and $d$ as given in Table 1, we have also estimated the differential rate for a NaI detector. This detector is a diatomic detector with detecting material consists of $^{223}$Na and $^{127}$I nuclei. In this case we have calculated first $\frac{dR}{dE_R}$ from Eq. (2.12) for $^{23}$Na and $^{127}$I separately and then use Eq. (2.15) for the computation of $\frac{dR}{dE}$ for NaI detector. The value of $v_0$ is kept at 220 km/sec. The results are shown in four graphs of Fig. 4 (Fig. 4a - Fig. 4b) for four different values of $d$.

In order to investigate the annual variation of WIMP detection rates we calculate the variation of total rate in one year. For this purpose a representative value of degeneracy parameter $d = 0.2$ and WIMP mass $m_\chi = 50$ GeV is considered. The ratio $\xi$ is read from Table 1. In doing this, we vary $t$ in Eq. (2.8) from 1 to 365 and for each value of $t$ (i.e. for each day from January 1) and calculate the total rate by integrating the differential rate. In Fig. 5 we plot the results for the case of a $^{76}$Ge detector. Fig. 5 shows the sinusoidal behaviour of WIMP detection rate with respect to different days in a year. The prediction for maximum detection is in June and the minimum is in December, as expected. We repeated the calculation for NaI detector with similar results and they are shown in Fig. 6.

Although all the investigations above are for a fixed value of $v_0 = 220$ km/sec, we repeat the calculations for five representative values of $v_0$ in the range 170 km/sec $\leq v_o \leq 270$ km/sec. We calculate the variation of differential rates with $E$ (in keV) for the five values $v_0 = 170$ km/sec, $200$ km/sec, $230$ km/sec 250 km/sec and 270 km/sec. with the value of $t$ in Eq. (2.8) fixed at $t_0$ (as earlier). The results for Ge detector and NaI detector are plotted in Fig. 7 and Fig. 8 respectively.

IV. CONCLUSION

We have considered lowest state Kaluza Klein particle (LKP) in universal extra dimension to be a candidate for cold dark matter. Unlike the supersymmetric particle, neutralino, which is a fermion, this Kaluza Klein particle is bosonic. We have predicted differential rates for detection of WIMP signals for Ge and NaI detectors with KK particle as Dark Matter candidate. The prediction is made by first estimating the value of $\xi$ – the ratio of local Dark Matter density to total local Dark Matter density and for different values of a degeneracy parameter $d$ (related to particle in universal extra dimension) and different values of WIMP mass. Due to diurnal motion of earth around the sun, the WIMP signals detected by earth bound detectors suffer an annual modulation with a maximum when the WIMP velocity with respect to the earth is parallel to the earth’s rotational velocity and a
minimum when they are antiparallel. We have estimated the nature of annual modulation signal for WIMPs for the two types of detectors mentioned above. Lastly, we predicted the differential rate for five different values of $v_0$ within its 90% C.L. range to show the former’s variation with the circular velocity of the Local System. The new detectors like GENIUS, and running detectors like DAMA alongside with various other Dark Matter search programs of increased sensitivity, can verify the possibility of LKP to be a candidate for Dark Matter.

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Figure Captions

Fig. 1 Variation of $\sigma_p^\text{scalar}$ with $m_\chi$ for LKP Dark Matter obtained from Eq. (2.13) for $d = 0.05, 0.1, 0.2$ and 0.4 (see text). Also shown are 3$\sigma$ C.L. region given by DAMA/NaI direct detection results for WIMP and projected limit for GENIUS detector with 100 kg. Germanium.

Fig. 2 Prediction of WIMP detection rates at $^{76}$Ge detector for five different WIMP masses namely 30 GeV, 50 GeV, 80 GeV, 100 GeV and 150 GeV for different fixed values of $d$. Figs. 2a, 2b, 2c and 2d correspond to $d = 0.05, 0.1, 0.2$ and 0.4 respectively.

Fig. 3 $^{76}$Ge detector.

Fig. 4 Same as Fig. 2 but for NaI detector.

Fig. 5 Annual modulation of total WIMP detection rate per kg per day for $^{76}$Ge detector.

Fig. 6 Same as Fig. 5 but for NaI detector.

Fig. 7 WIMP detection rate prediction (per kg per day per keV) for different values of $v_0$ (see text) for $^{76}$Ge detector.

Fig. 8 Same as Fig. 7 but for NaI detector.
Fig. 1

DAMA Contour

GENIUS

d=0.05
d=0.1
d=0.2
d=0.4

\sigma_{\text{scalar}}^{p} \text{ or } \xi \sigma_{\text{scalar}}^{p} (in pb)

m_\chi (GeV)
Fig. 2
Fig. 3

$m_\chi = 30 \text{ GeV}$

d = 0.05
Fig. 4
Fig. 5

$\frac{m_\chi}{\chi} = 50 \text{ GeV}$

$^{76}\text{Ge}$

Total Rate (kg$^{-1}$ day$^{-1}$)

Days in a year
Fig. 6

Total Rate ($\text{kg}^{-1}\text{day}^{-1}$)

Days in a year

$m_\chi = 50 \text{ GeV}$

NaI
Fig. 7
\[ \Delta R/\Delta E \text{ (kg}^{-1} \text{ day}^{-1} \text{ keV}^{-1}) \]

Fig. 8