Acoustic black hole evaporation as diffusion phenomena in plasmas?

by

L.C. Garcia de Andrade

Departamento de Física Teórica – IF – Universidade do Estado do Rio de Janeiro-UERJ
Rua São Francisco Xavier, 524
Cep 20550-003, Maracanã, Rio de Janeiro, RJ, Brasil
Electronic mail address: garcia@dft.if.uerj.br

Abstract

Acoustic analogues of Kerr black hole in plasmas are considered, by taking for granted the existence of acoustic ion waves in plasmas. An effective black holes (BH) in curved Riemannian spacetime in a random walk plasmas is endowed with a naked singularity, when plasmas are in the lowest diffusion mode. The plasma particle diffusion is encoded in the effective metric. The diffusive solution has a horizon when the plasma flow reaches the sound velocity in the medium and a shock wave is obtained inside the slab. The sonic black hole curved Riemannian metric is also found in terms of particle number density in plasmas. The sonic BH singularity is found at the center of the plasma diffusive slab from the study of the Ricci curvature scalar for constant diffusion coefficient. It is suggested and shown that the Hawking temperature is proportional to the plasma Kelvin temperature through diffusion coefficient dependence to this temperature. Therefore Unruh sonic or dumb BH is shown to have a relation between Hawking and plasma diffusive temperatures. BH evaporation is analogous to the diffusive phenomena in plasmas, since in both cases Hawking temperature is inversely proportional to mass. It is shown that Hawking analogue temperature of a plasma torus is $T_H(torus) \approx 10^{-4} K$ which is much higher than the gravitational Hawking temperature of a one solar mass BH, $T_H^{GR} \approx 10^{-8} K$, but still very small for being detectable in plasma laboratories. PACS numbers: 02.40.Ky: Riemannian geometries
I Introduction

Relativistic and non-relativistic acoustic black holes geometries have been considered respectively by Bilic [1] and Unruh [2] and Visser [3]. Other types of effective artificial black holes [4] in other laboratory settings as optical and acoustic media as well as more recently plasma matter [5] have also been addressed in the literature. These acoustic black holes are effective analogue pseudo-Riemannian metrics given by the homogeneous wave equation from linearised Euler flows or Navier-Stokes equation [6]. According to Visser, the vortex plasma flow consider here though not irrotational can possesses a sonic BH solution. Artificial black holes have been considered with vorticity in the eikonal approximation where again homogeneous wave equation yields an acoustic BH. In this paper one shows that under the low diffusion limit one obtains the homogeneous wave equation from the inhomogeneous one obtained from plasma particle diffusion. Actually when one considers the limit of lowest diffusion mode a Riemann flat spacetime is obtained and since Riemannian curvature vanishes no BH is found at all. When diffusion increases a BH is found on a curved Riemannian effective plasma through the Unruh metric. Superfluid BHs have been found in other contexts [7]. A non-diagonal sonic metric is obtained in this effective plasma spacetime. Actually the Riemannian geometry of the diffusion processes in random walks has been treated previously by Molchanov [8]. Here the diffusion processes are shown to possibly be responsible for being the analogous to the BH evaporation, since in Schwarzschild and acoustic BHs the Hawking temperature are inversely proportional to the mass of the system. The presence of analogue BHs in plasmas is motivated by the acoustic ion waves which are common in non neutral plasmas. The paper is organised as follows: In section 2 the wave equation is deduced from particle diffusive equation by simply applying the partial time derivative in both sides of the equation. In section 3 a constant diffusion coefficient may lead to the presence of naked singularities in the case of lowest diffusion plasma modes in curved Riemannian effective spacetime. In section 4 Hawking and plasma temperatures are shown to be related through the diffusion coefficient dependence of Kelvin absolute temperature and collision frequency of plasma particles. Conclusions are presented in section 5.
II Acoustic BHs in effective diffusive plasma flows

In this section we present the brief results described above. The Riemannian acoustic effective geometry endowed with slow diffusion can be started with the Fick’s law [9]

\[ \vec{\Gamma} = n \vec{v} = -D \nabla n \] (II.1)

where \( D \) is the variable diffusion coefficient, and \( n \) is the plasma particle density. The diffusion coefficient depends only on \( x \) and \( t \) coordinates, such as \( D = D(t, x) \). Thus since the diffusion coefficient is not in principle constant \( \vec{v} \) is not irrotational, the plasma flow is a vortex flow and the vortex expression becomes

\[ \vec{\Omega} = \nabla \times \vec{v} = \nabla D \times \frac{1}{n} \nabla n \] (II.2)

From the equation of particle diffusion in plasmas

\[ \partial_t n + (\vec{v} \cdot \nabla) n = \nabla \cdot (D \nabla n) \] (II.3)

where one has to consider that the diffusion coefficient is in principle stationary. Actually below this would not be a problem since one would consider only constant diffusion coefficients. Equation (II.3) yields the wave equation format required to build the effective spacetime

\[ \partial_t (\partial_t + (\vec{v} \cdot \nabla) n) = \nabla \cdot [(D - \partial_t D) \nabla n - D \nabla (\partial_t n + (\vec{v} \cdot \nabla) n)] = -D \nabla \cdot (\frac{D}{n} [\nabla n]^2) \] (II.4)

which yields the wave equation in the format

\[ \Box n = -D \nabla \cdot (\frac{D}{n} [\nabla n]^2) \] (II.5)

where the LHS of this equation is given by

\[ \Box n = \frac{1}{\sqrt{-g}} \partial_\mu (\sqrt{-g} g^{\mu \nu} \partial_\nu n) \] (II.6)

where \( g^{\mu \nu} (\mu = 0, 1, 2, 3) \) is the effective sonic metric of Unruh if one of course drops the RHS of equation (II.6). But this can be easily done if one notes that that term depends on the squared diffusion coefficient \( D^2 \). So the Unruh sonic metric in this diffusive plasma would be reduced to

\[ ds^2 = (dx - (\frac{D}{n} \partial_x n + c) dt)(dx - (\frac{1}{n} D \partial_x n + c)) - dy - dz \] (II.7)
which represents the existence of sonic BHs in plasmas. Let us now consider a simple example which shows that this curved BHs sonic Riemannian metric, reduces to a Ricci-flat and Riemann-flat metric when one considers the solution of the diffusion equation is the lowest diffusion mode solution. To compute this example one considers the diffusion equation above in the approximation

\[
\frac{\partial^2 n}{\partial t^2} + \partial_t [\mathbf{v} \cdot \nabla n] = \partial_t [\nabla \cdot (D \nabla n)]
\]

(II.8)

where \( \Omega_1 \) is the vorticity fluctuation according to the rules the fields fluctuations [4]

\[
p = p_0 + \epsilon p_1
\]

(II.9)

\[
\psi = \psi_0 + \epsilon \psi_1
\]

(II.10)

\[
\rho = \rho_0 + \epsilon \rho_1
\]

(II.11)

\[
\vec{\Omega} = \vec{\Omega}_0 + \epsilon \vec{\Omega}_1
\]

(II.12)

Actually by noticing that diffusion equation is the generalised conservation equation for \( \rho = nm \) where \( m \) is the mass of particles in plasmas one obtains the following equation

\[
\partial_t \rho + \nabla \cdot (\rho \mathbf{v}) = 0
\]

(II.13)

which along with the barotropic equation of state

\[
p = p(\rho)
\]

(II.14)

where \( p \) is the pressure, the generalised Unruh effective ”general relativistic” equation as

\[
\partial_t [c^{-2}_{\text{sound}} \rho_0 (\partial_t \mathbf{v}_1 + \mathbf{v}_0 \cdot \nabla \psi_1)] = \nabla \cdot [\rho_0 \nabla \psi_1 - c^{-2}_{\text{sound}} \rho_0 \mathbf{v}_0 (\partial_t \psi_1 + \mathbf{v}_0 \cdot \nabla \psi_1)] + \partial_t \nabla^2 \psi_1 + \mathbf{v}_0 \cdot \nabla \nabla^2 \psi_1
\]

(II.15)

from the Unruh wave equation

\[
\Box \psi_1 = \partial_t \nabla \cdot \mathbf{v}_1 + (\mathbf{v}_0 \cdot \nabla) \nabla \cdot \mathbf{v}_1
\]

(II.16)

Thus when one takes the hypothesis that the perturbed flow \( \mathbf{v}_1 \), besides irrotational is incompressible

\[
\nabla \cdot \mathbf{v}_1 = 0
\]

(II.17)
This condition of incompressibility is distinct from the used by Visser one that it calls almost incompressible flow. Actually instead of the above condition one may use the fact that only the perturbed flow $v_1$ is irrotational and that this perturbation is on the vortex flow $v_0$ where the diffusion coefficient is non-constant. In this case in formula (II.7) $n$ shall be replaced by the unperturbed plasma number particle density must be substituted by $n_0$. One notes that the velocity at the horizon $g_{00} = 0$ is inward velocity and therefore the trapped surface can form. However in the next section one shall show that naked singularity can form when the diffusion coefficient is not constant.

### III  Naked singularities in effective plasma spacetime

R Penrose [10] has investigate the presence of naked singularities in general relativistic BHs. This is connected with a presence of a Cosmic censorship hypothesis which forbides the presence of naked singularities, which are essentially the coincidence of event horizons with the physical real general relativistic BH singularity. In this section it is shown that the presence of a naked singularity is granted in sonic diffusive plasmas BHs in the lowest diffusion mode where the diffusion coefficient $D$ is constant. The only possibility, though never been tested, is that naked singularities may exist in the case of highly ionised plasmas where the diffusion coefficient is nonconstant. A lowest diffusion mode is found as a solution like [9]

$$n = n_0 \cos\left(\frac{\pi x}{L}\right)e^{-\frac{t}{\tau}} \quad \text{(III.18)}$$

Thus the spacetime metric (II.7) becomes

$$ds^2 = (dx - \left(\frac{D(x)}{n}\partial_x n + c\right)dt)(dx - (-\frac{1}{n}D(x)\partial_x n + c)) - dy - dz \quad \text{(III.19)}$$

Thus the horizon $g_{00} = 0$ surface becomes

$$v_0 = D(x)\tan\left(\frac{\pi x}{L}\right) \quad \text{(III.20)}$$

which shows that at $x = 0$ which a true physical singularity coincides with the event horizon unphysical singularity. Now to confirm that this is not a flat spacetime and that the naked singularity is not a spurious one, one must compute the Riemann curvature of
metric (II.7)

\[ R_{0101} = R_{txtx} = -DAD'' + 2D'A' + DA'' \]  

(III.21)

where \( A(x) = \frac{d}{dx} \ln(n^{-1}) \) and the dash means derivative with respect to the x-coordinate. Note that in at \( x = 0 \) the Riemann curvature component in case of constant \( D \) is

\[ R_{0101} = R_{txtx}(t, x = 0, y, z) = -D^2 AA'' |_{x=0} \approx \frac{2\pi}{L} D^2 \frac{1}{\tan(\frac{\pi x}{L})} \approx \infty \]  

(III.22)

which shows that the Riemannian effective plasma spacetime is curved in the case of lowest diffusion mode and may possess a naked singularity unless the diffusion coefficient is constant. Confirmation of singular structure can be given by the Kretschmann scalar as used by Royston and Gass [11] in the case of optical effective BH or as done here and in Fischer and Visser [12] by computing the Ricci curvature scalar

\[ R = \frac{1}{A} A'' \]  

(III.23)

which is given by

\[ R \approx \frac{1}{\tan(\frac{\pi x}{L})} |_{x=0} \approx \infty \]  

(III.24)

at \( x = 0 \). Thus the sonic BH physical singularity is at the center of the plasma slab. Note that at the boundaries of the slabs the Ricci scalar is given by

\[ R \approx \frac{1}{\tan(\frac{\pi x}{L})} |_{x=L/2} = 0 \]  

(III.25)

at the boundaries of the slab given by \( x = \frac{L}{2} \) where \( L \) is the width of the retangular plasma slab. Note that Roystn and Gass [11] have shown from the computation of Kretschmann scalar

\[ R^2 = R_{\mu\nu\alpha\beta}R^{\mu\nu\alpha\beta} \]  

(III.26)

that the cosmic censorship hypothesis exists and no naked singularity is found which does not happens here.
IV Hawking and plasma temperatures in dumb holes

One of the main motivations for the investigation of acoustic analogues in effective spacetime, is the investigation of Hawking radiation and its relation to quantum field theory. Therefore it seems important to check for the presence of Hawking radiation in these plasmas acoustic BHs. Let us start by computing the Hawking temperature [3]

\[ T_{\text{Hawking}} = \frac{\hbar}{4\pi k_B} \left| \frac{\partial c_{\text{sound}}}{\partial x} \right|_c^2 = v^2 \] (IV.27)

Now let us apply this expression into the above plasma diffusive effective spacetime solution to yield

\[ T_{\text{Hawking}} = \frac{h D}{4\pi k_B} \left( \left| \frac{\partial^2 n(x)}{\partial x^2} \right| \right) \] (IV.28)

By recalling the relation between the diffusion coefficient and the plasma Kelvin temperature \( T_{\text{Plasma}} \) [9]

\[ D = \frac{k_B T_{\text{Plasma}}}{m\nu} \] (IV.29)

where the coefficient \( \nu \) is the collision frequency. Thus by substitution of (IV.29) into (IV.28) yields

\[ T_{\text{Hawking}} = \frac{h T_{\text{Plasma}}}{4\pi m\nu} \left( \left| \frac{\partial^2 n(x)}{\partial x^2} \right| \right) \] (IV.30)

From these last two expressions and noting that the gravitational general relativistic Hawking temperature in Schwarzschild static BH the Hawking temperature is inversely proportional to the mass \( M \) of the BH as \( T_H = \frac{\hbar}{4\pi GM} \). allows us to suggest that the diffusion processes in analogue plasma effective spacetime the diffusion phenomena would play the hole of the BH evaporation in general relativistic BHs. Therefore a relation of type \( T_{\text{Hawking}} \approx T_{\text{Plasma}} \) between Hawking and plasma temperatures is obtained. Let us now consider the computation of Hawking radiation on a more specific example of a plasma retangular slab with a localized source [9]. This can be given by the solution of steady diffusion equation with source

\[ \nabla^2 n(r) = -\frac{Q(r)}{D} = \frac{d^2 n(x)}{dx^2} \] (IV.31)

where \( Q(r) \) is the source term. Due to the localization or concentration of plasma source one obtains [9]

\[ \nabla^2 n(r) = -\frac{\delta(0)}{D} = \frac{d^2 n(x)}{dx^2} \] (IV.32)
which yields a Hawking temperature of

\[ T_{\text{Hawking}} = \frac{hT_{\text{plasma}}}{4\pi m\nu} \left( \frac{d}{dx} \left[ n n(x) \right] \right) \] (IV.33)

which for a large width L of the retangular plasma slab with the solution

\[ n = n_0 (1 - \frac{|x|}{L}) \] (IV.34)

yields the Hawking temperature relation

\[ T_{\text{Hawking}} \approx \frac{h}{4\pi k_B} \delta(0) \] (IV.35)

Of course this is not the only condition for the existence of Hawking radiation and further investigation is necessary before one can confirm the presence of this radiation in dumb holes analogue of GR Kerr BHs. Let us now estimate the Hawking radiation for the plane slab plasma geometry in (III.20), by assuming that the slab is thin and that the horizon is close to the physical singularity at \( x = 0 \), thus the diffusion coefficient is

\[ v_0 \approx D \left( \frac{\pi x_h \L}{L} \right) \] (IV.36)

where \( x_h \) is the horizon locus. Since at the horizon \( v_0 = c_{\text{sound}} \), where sound is almost trapped inside the plasma analogue BH, thus

\[ x_h = \frac{c_{\text{sound}} L}{\pi D} \] (IV.37)

since in the plasma slab one can take [9] \( L = \pi \)

\[ x_h = \frac{c_{\text{sound}}}{D} \] (IV.38)

which from the \( D = 4 \times 10^3 cm^2s^{-1} \) yields a horizon locus at

\[ x_h = 10^{-3} c_{\text{sound}} \] (IV.39)

From this expression one finally obtains the Hawking temperature as

\[ T_{\text{Hawking}} \approx \frac{h}{4\pi k_B} x_h \approx 10^{-11} c_{\text{sound}} \] (IV.40)

since the sound velocity in plasma \( c_{\text{sound}} = \left( \frac{\gamma K T_e}{M} \right)^{\frac{3}{2}} = 10^6 K T_e V \) and by taking this value of \( KT \approx 10 \), for a typical torus, yields \( T_H(\text{Plasma}) \approx 10^{-4} K \) which still much weaker than the astrophysical one solar mass BH Hawking temperature, where we take a Planck constant of \( h \approx 10^{-27} \text{erg.s} \). Here one considers that the adiabatic constant \( \gamma = 1 \).
V  Conclusions

Recently a deep connection between vortex flows and non-Riemannian artificial acoustic BHs and the by Garcia de Andrade [1]. In this report one shows that this sonic non-Riemannian BHs is not needed in the random walk plasmas, even in the presence of vorticity. A naked singularity is found in the lowest diffusion mode is obtained when the diffusion coefficient is non-constant. In this case the Riemannian curvature does not vanishes as when it is constant, and therefore our effective diffusive spacetime endowed with a naked singularity is found. Note that only large perturbations of the spherical geometry of GR BHs may induce a naked singularity, contrary to what happens here in the sonic black holes in plasmas in the lowest diffusion mode small perturbation in the flow. Hawking radiation and its presence in the dumb holes need further investigation as was done recently by Schützhold and Unruh [13] in the case of the electromagnetic wave-guides in dumb holes where quantum commutation relations have to match as well. The interesting point here is that in the plasma physics examples discussed the set-up for testing Hawking radiation already exists in plasma laboratories. When a plasma jet reaches a speed higher that the sound speed a shock wave is formed and this is exactly the condition analogous for the trapped sound in Unruh dumb holes in plasmas. A more detailed investigation of the random walk fluid motion and its connection with the presence of acoustic BHs in the flow and Hawking BH evaporation [14] shall appear elsewhere.

VI  Acknowledgements

Thanks are due to R. Kerr and S. Bergliaffa for helpful discussions on this letter. Thanks go to Universidade do Estado do Rio de Janeiro (UERJ) and CNPq for financial aid.
References

[1] B Bilic, Relativistic Acoustic Geometry, gr-qc/9908002v1. L C Garcia de Andrade, Phys Rev D 70, 6 64004-1. (2004). L C Garcia de Andrade, Phys Lett A (2005).

[2] G Volovik, Universe in a Helium droplet (2003) Oxford University Press.

[3] W Unruh, Phys Rev Letters 46 (1981) 1351. R P Kerr, Phys Rev Lett (1963).

[4] M Visser in Artificial Black Holes(2002) Ed M Novello, M Visser and G Volovik.

[5] M. Stone, Phys Rev E 62,1:1341.

[6] Uwe Fischer and M Visser, Phys Rev Lett (2003).

[7] M. Stone, Phys Rev E 64 (2000).

[8] S. Molchanov, Russian mathematical surveys.

[9] F Chen, Introduction to plasma physics.

[10] R Penrose, Rivista del Nuovo Cimento , numero speciale 1 (1969) 252.

[11] A Royston and R Gass, An optical analogue of a Black hole, (2002) gr-qc/0212122v1.

[12] U Fischer and M Visser, Europhys Lett (2003).

[13] R Schützhold and W Unruh, Hawking radiation in an electromagnetic wave-guide?, arxiv:quant-ph/0408145v1 (2004).

[14] R Schützhold and W Unruh, On the origin of the particles in black hole evaporation, 08041686v1 (2008).