Effect of rotation on thermal convection in horizontal plane layer subject to circular vibration

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Abstract. The stability of quasi-equilibrium of a horizontal liquid plane layer with isothermal boundaries of different temperatures, subject to circular vibrations in the horizontal plane, is experimentally studied. The rotation of the cavity around the vertical axis is set independently. In order to exclude from consideration the destabilizing effect of the gravity field, the layer is heated from the top. In the absence of rotation, the circular vibrations lead to the threshold excitation of vibrational thermal convection in the layer of nonisothermal liquid, in spite of its stable stratified in the gravity field. At given vibration amplitude and temperature difference at the layer boundaries, the threshold is determined by sharp increase in heat transfer with a monotonic increase in the vibration frequency. The size of the convective spatial structures is determined by the layer thickness. In the case of high-frequency vibrations, convection is determined by dimensionless parameters: gravitational Rayleigh number $Ra$ and vibrational parameter $R_v$. The thresholds of vibrational convection excitation on the plane of these parameters in the case of circular vibrations coincide with the theoretically predicted stability boundary for linear vibrations. It is shown that rotation has a stabilizing effect on vibroconvective stability, similar to the case of gravitational convection. The threshold value of the vibration parameter $R_v$ grows with an increase in the dimensionless rotation velocity $\omega_{\text{rot}}$. Under conditions of the performed experiment, the structure of convective cells and the wave number of structures in the supercritical region do not change in comparison with $\omega_{\text{rot}} = 0$.

1. Introduction
The important place in the study of hydrodynamic systems belong to the problem of thermal convection in rotating cavities, which takes place in many processes occurring in the atmosphere, ocean, mantle of the Earth [1]. The classical problem is a thermal convection in a horizontal layer of liquid heated from below under rotation [2]. In this consideration, the Coriolis force and centrifugal force are added to the existing balance between gravitational buoyancy and viscous drag. The governing parameters for this thermal convection problem are the gravitational Rayleigh number $Ra = g \beta \Theta h^3 / \nu \chi$, centrifugal Rayleigh number $Ra_c = \Omega^2 R \beta \Theta h^3 / \nu \chi$ and the dimensionless rotation velocity, which characterizes the ratio of the Coriolis
force to the viscosity force, represented in the form \(\omega_{rot} = \Omega_{rot} h^2 / \nu\). Here \(g\) is the gravity acceleration, \(\Omega_{rot} = 2\pi f_{rot}\) – the angular velocity of the cavity rotation, \(\Theta\), \(h\) and \(R\) – characteristic temperature difference, cavity size (layer thickness) and distance to the axis of rotation, \(\beta\), \(\chi\) and \(\nu\) – coefficients of volumetric expansion, thermal conductivity and kinematic viscosity of the fluid. A detailed study of the case of gravitational convection in the cavity rotating around a vertical axis is done in [2]. The threshold of gravitational convection occurrence is determined in the plane of parameters \(\omega_{rot}, Ra\). It is shown that the stabilizing effect of Coriolis force on natural thermal convection stability grows with \(\omega_{rot}\), the wave number of cellular convective structures also increases with \(\omega_{rot}\).

The convective motion of non-isothermal fluid is also possible under the action of an oscillating force. The averaged motion excited by oscillations of non-isothermal fluid is called “thermal vibrational convection” [3,4]. The vibrational convection develops independently of the static field and can be applicable to experiments under reduced gravity or under weightlessness conditions. The averaged convection is determined by the vibrational parameter \(R_v = \left( b \Omega_{vib} \delta \Theta h \right)^2 / 2 \nu \chi\), where \(b\) is the vibration amplitude, and \(\Omega_{vib} = 2\pi f_{vib}\) – radian frequency of vibration. A theoretical description of vibrational convection excited by a rotating inertial field generated by circular translational vibrations of the cavity is analogous to the case of linear translational vibrations.

A similar phenomenon of averaged vibrational convection as in the case of rotating force field, takes place in the cavity rotating around a horizontal axis, the averaged convection excited in a vertical plane layer rotating around the horizontal axis was experimentally studied. The stability threshold, as expected, increases with an increase in the dimensionless rotation velocity. The averaged convection, which develops in non-isothermal fluid in a cavity rotating around the horizontal axis as a result of oscillations excited by gravity, is determined by a modified vibrational parameter \(R_y = \left( g \beta \delta \Theta h \right)^2 / 2 \nu \chi \Omega_{rot}^2\) [5]. Several works (see [5]) are devoted to experimental study of this type of “vibrational” convection in cavities of different shape. It is worth mentioning that in case of cavity rotation in gravity field, the rotation frequency is equal to the frequency of force oscillations in the cavity frame. Thus, the problem of influence of rotation of arbitrary rate on the vibrational convection and heat and mass transfer needs the experimental study.

The object of this study is to investigate the effect of rotation on the threshold of vibrational convection excitation in a horizontal plane layer subject to circular vibrations in the horizontal plane (Fig.1a). A rotating inertial force field under conditions of a given (independently) rotation of the cavity itself is capable to excite the vibrational convection in a non-isothermal fluid. To exclude the destabilizing effect of the gravitational convection mechanism, the layer is heated from above. Thus, we investigate the effect of rotation on the vibrational thermal convection in case of strong stabilizing effect of the gravity field.

2. Experimental technique

Vibrational thermal convection in a horizontal plane layer with isothermal boundaries of different temperatures \(T_1\) and \(T_2\) rotating around a vertical axis is studied experimentally. The layer is formed by two plane heat exchangers (Fig.1a), between which a gasket with a cylindrical thermal isolating lateral boundary is installed. The gasket thickness defines the thickness of the layer, which equals to \(h = 0.32\) cm, the diameter of lateral boundary \(d = 13.8\) cm.
In order to maintain a constant temperature at the layer boundaries, the fluid of definite temperature flows through the heat exchangers. The temperatures of the layer boundaries were measured using resistance thermometers with an accuracy $0.1 \, ^\circ C$. The temperature difference at the boundaries of the layer was determined as $\Theta = T_1 - T_2$ and changed from $\Theta = 0 \, ^\circ C$ to $\Theta = 35 \, ^\circ C$, where $\Delta T = T_2 - T_1$ is the temperature drop on the heat flux sensor. A thermocouple, which is arranged on the boundary of the fluid layer and in the cold heat exchanger, registers the heat flux through the layer.

The rotation of the cuvette is set by a stepper motor; the rotation velocity varies within $0.01 - 4.00 \, \text{rps}$. Rotation velocity instability does not exceed $0.01 \, \text{rps}$. The layer performs translational vibrations along a circular path with amplitude $b$ and frequency $\Omega_{\text{vib}}$. Amplitude and frequency vary in the range $b = 0 - 5.0 \, \text{cm}$, $f_{\text{vib}} = \Omega_{\text{vib}} / 2\pi = 0 - 8 \, \text{Hz}$. The relative error of the amplitude and frequency measurement does not exceed $1\%$ and $0.1\%$, respectively.

The fluid is stably stratified in the gravity field – the layer is heated from above ($T_1 > T_2$). The stabilizing effect of the gravity field is determined by the Rayleigh gravitational number, which has a negative value. To excite the vibrational convection, the parameter $R_v$ has to exceed the $Ra$, i.e. $R_v / |Ra| > 1$. To achieve the maximum vibrational effect, experiments were carried out in relatively thin layers at an amplitude $b = 5 \, \text{cm}$, close to maximum possible for the vibrator used, 95% ethanol solution was selected as the working fluid.

The experimental technique was as follows. At a given temperature difference $\Theta$ and rotation velocity $\Omega_{\text{rot}}$ the frequency of circular vibrations $\Omega_{\text{vib}}$ grows step by step at certain amplitude $b$. In this case, the heat transfer through the layer was measured and the structure of convective flows was studied using the photo-registration (Fig.1b). For this purpose, a thermochromic film which color varies depending on the liquid temperature was pasted on the lower (cold) boundary of the liquid layer. The temperature of the upper boundary of the layer $T_1$ was set by a transparent heat exchanger through which photo-registration of convective flows was carried out.

3. Experimental results. analysis
The temperatures of the layer boundaries and the temperature drop $\Delta T$ at the heat flux sensor were measured at gradual increase of the vibration frequency at given values of the temperatures $T_1$ and $T_2$. 

Figure 1. Statement of the problem (a) and fragment $\frac{1}{4}$ parts of the layer (top view) with convective structures in supercritical region (b)
vibration amplitude and rotation velocity (Fig. 2). The threshold of vibrational convection excitation manifests itself by a sharp increase in heat transfer. The Nusselt number is introduced as a characteristic of heat transfer \( \frac{\Delta T}{\Delta T_0} \), defined as the ratio of the heat flux through the layer to the heat flux in the absence of convection, at given \( \Theta \). In the absence of convection \( Nu = 1 \).

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Nu = \frac{\Delta T}{\Delta T_0},
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\( \frac{\Delta T}{\Delta T_0} \), defined as the ratio of the heat flux through the layer to the heat flux in the absence of convection, at given \( \Theta \). In the absence of convection \( Nu = 1 \).

Figure 2. Heat transfer \( Nu \) (a) and vibrational parameter \( R_v \) (b) versus the dimensionless vibrational frequency \( \omega_{\text{vib}} \) (at different values of rotation velocity and close values of gravitational Rayleigh number \( Ra \)).

The important limiting case of thermal vibrational convection generated by a circular vibration of the cavity in the absence of rotation was studied in [6]. It was found that for high-frequency circular oscillations of the cavity in the horizontal plane, convection is excited in a threshold manner at a certain critical dimensionless vibrational frequency \( \omega_{\text{vib}} = \Omega_{\text{vib}} h^2 / \nu \) (Fig. 2a, symbols 1) in the form of elongated rolls, divided along the length into relatively short segments. The vibrational convective structures in the absence of rotation \( \omega_{\text{rot}} = 0 \) are similar to those shown in Fig. 1b for rotating layer at \( \omega_{\text{rot}} = 50 \). With increasing the vibration intensity the wavenumber of convective cells practically does not change.

It is found that with an increase in the rotation rate (Fig. 2a), the threshold vibrational convection excitation, at definite negative Rayleigh number, occurs at larger value \( \omega_{\text{vib}} \). The structure of convective flows in the supercritical region of parameters is shown in Figure 1b. The increase of the rotation velocity \( f_{\text{rot}} \) (increase it in subsequent experiments) leads to an increase in the threshold value of the vibrational parameter \( R_v \) (Fig. 2b). One can see the kink in the curves, associated with a threshold change in the temperature difference on the layer boundaries due to increase in the temperature drop on the heat flux sensor with excitation of convection.
In the problem of the averaged thermal convection in a rotating horizontal plane layer under vibration, two mechanisms are simultaneously involved, gravitational and vibrational, determined by the gravitational Rayleigh number $Ra = g \beta \Theta h^3 / \nu \chi$ (which has a negative value) and vibrational parameter $R_v^2 = (b \Omega_{rot} \beta \Theta h)/(2 \nu \chi)$. Comparison of the threshold curve under rotation with the threshold in the absence of rotation is presented on the plane of these dimensionless parameters in Fig. 3a. The solid line presents the theoretical value for the stability boundary [3] in the absence of rotation. The line passes through two reference points: $Ra = 2129$ (at $Ra = 0$) and $Ra = 1708$ (at $Ra = 0$).

The dark symbols show the course of the experiment and the change of the parameter $Ra$ with increasing $\omega_{rot}$, in case of $\omega_{rot} = 50$ (symbols 2 in Figures 2a,b). Upon reaching the vibrational convection threshold, the value of $Ra$ decreases slightly – one can see the bending of the curve to the right in consequence of temperature differences fall associated with the growth of heat flux. With an increase of the rotation velocity, the vibrational convection threshold shifts to higher values of the parameter $R_v$. The graph shows a series of experiments at a fixed rotation velocity $f_{rot} = 0.75$ rps ($\omega_{rot} = 50$) and $f_{rot} = 1.30$ rps ($\omega_{rot} = 85$). Similar to the case, $\omega_{rot} = 0$, the threshold curves at a fixed dimensionless rotation velocity $\omega_{rot} = \text{const}$, are to abut the well-known threshold value of the parameter $Ra$ at $R_v = 0$ (the case of vibrational convection in a rotating plane layer heated from below [2]). Dashed lines in Figure 3 present an expected threshold curves at a given values of dimensionless rotation velocity $\omega_{rot} = 50$ and $\omega_{rot} = 85$.

In contrast to the averaged convection excited by circular translational vibrations [6], the Coriolis force plays an important role in the problem under consideration. As is shown in Figure 3, with increasing the dimensionless rotational velocity the dimensionless thresholds shifts to higher values of the parameter $R_v$. 

**Figure 3.** Threshold curves of vibrational convection excitation on the plane of control parameters $Ra, R_v$ (a) and the dependence of the threshold value of the vibrational parameter on the dimensionless rotation velocity $\omega_{rot}$ (b).
rotation stabilizes the threshold of the vibrational convection. It was found that the Coriolis force does not affect the flow structure in the investigated domain $\omega_{\text{rot}} < 100$. The wavenumber of convective structures with increasing the dimensionless rotation velocity remains practically unchanged.

4. Conclusion
Vibrational thermal convection in a horizontal plane layer with isothermal boundaries of different temperatures rotating around the vertical axis is experimentally investigated. It is shown that convection is excited by high-frequency circular oscillations of the cavity in the horizontal plane with an increase in the vibration frequency. The fluid is stably stratified in the gravity field - the layer is heated from above. The thresholds of vibrational thermal convection excitation under rotation are compared with the case of the rotation absence. It was found that in the rotating layer the vibroconvective structures take the form of regularly spaced cells, but the wave-number do not change compared to the nonrotating case in the investigated domain of dimensionless rotation velocity $\omega_{\text{rot}} < 100$. With the increase in the dimensionless velocity, the critical value of the vibrational parameter (at a certain Rayleigh number) increases. Thus, the action of rotation on the vibroconvective mechanism is similar to the action of rotation on natural convection in a horizontal plane layer.

Acknowledgments
This work was supported by the Russian Science Foundation (project 18-71-10053)

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