Quantum Population of Magnons via Spin Shot Noise

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Recent measurements in current-driven spin valves demonstrate a magnon population that deviates from semiclassical predictions. We posit that the origin of this deviation is spin shot noise. On this basis, our theory predicts that the magnon population asymmetrically increases in biased junctions irrespective of the current direction. At low temperatures, the number of magnons linearly increases with the bias, but at different rates for opposite current directions. Quantum effects control the magnon population even at higher temperatures. Our results are in semiquantitative agreement with recent experiments and are in contradiction to semiclassical theories of spin-transfer torque.

Spin-transfer torque (STT), which is the transfer of spin angular momentum from spin-polarized currents to localized magnetic moments, is a cornerstone in spintronics.[1–5] The low power consumption and scalable architecture open a promising path for STT-based devices in future data storage and information-processing technologies. For example, STT facilitates state-of-the-art non-volatile random-access memory[5–7] and spin-transfer nanooscillators[8].

STT affects the magnetization dynamics via (an) a (anti)damping-like torque. The torque is proportional to the current. In the simplest manifestation, the magnetization dissipation is enhanced or reduced and is captured by an effective Gilbert damping constant. At finite temperatures, there are additional torques that fluctuate. In a semiclassical picture, the fluctuation is modeled by stochastic temperature-dependent random magnetic fields. These fields obey the dissipation-fluctuation theorem; the two-point autocorrelation function is proportional to the effective Gilbert damping parameter. Thus, the (anti)damping-like torque also results in a (enhancement) reduction of the magnetization fluctuations[9, 10].

Recently, Zholud et al. measured an anomalous behavior in the magnon population in a current-driven spin valve[11]. Measurements at low temperatures suppress thermal fluctuations, but there were observations of an enhanced magnon density with a different origin. These measurements cannot be explained by semiclassical STT models[1, 2]. Rather, Ref. [11] suggests that quantum effects are essential. Thus far, there have only been a few theoretical works describing STT in a quantum picture[11, 12].

In this Letter, we investigate the effects of a particular type of quantum noise on STT phenomena. Electronic shot noise is due to the discrete nature of electric charge[13, 14]. Similarly, the discrete nature of itinerant electron spin in units of $\hbar/2$ causes spin shot noise[15]. We demonstrate that spin shot noise can explain the recent experimental observations. At low temperatures and when there is a current flowing through the spin valve, spin shot noise dominates and affects the magnon distribution. These bias-driven quantum fluctuations exert additional torques on the magnetization dynamics that do not obey the fluctuation-dissipation theorem. As a result of the competition between the fluctuations and the dissipation (through the Gilbert damping), the magnons are driven out of equilibrium. Consequently, our theory predicts that the magnon population in spin valves depends on the bias voltage.

Our main finding is a microscopic expression for the magnon population in spin-valves,

$$\langle N_m \rangle \simeq \frac{1}{1 - \frac{I}{I_c}} \left[ f_{BE}(\Omega, T) + \frac{1}{2} + \Xi^{(sh)}(U, T) \right], \quad (1)$$

where $I < I_c$ is the applied current, $I_c$ is the critical current for ferromagnetic instability, $f_{BE}$ is the Bose-Einstein distribution function at temperature $T$ and magnon frequency $\Omega$, and $U$ is the applied bias voltage. In Eq. (1), the prefactor is due to the semiclassical STT. The first term within the bracket is the contribution
thermal magnons. The second term arises from vacuum fluctuations. Our main contribution is the third term that originates from quantum spin shot noise. We will demonstrate that $\Xi^{(sh)}(U, T)$ is an even function of the bias voltage $U$.

As an illustration, in Fig. 2 we plot the magnon population as a function of bias voltage for different temperatures. The curve is in semiquantitative agreement with recent experimental results [11]. Moreover, we can quantify the various contributions in terms of microscopic parameters that can be evaluated by first-principles band-structure calculations. We encourage detailed ab initio evaluations of these parameters to further explore the quantitative consistency between our approach and measurements.

To explain our approach, we first review how Zholud et al. [11] measures the magnon population of Eq. (1). The experimental observation is the magneto-resistance of a spin-valve nanopillar, shown in Fig. 2, as a function of current. In spin valves, the magneto-resistance depends on the relative angle $\theta$ between the magnetization directions in the free layer, FM2, and the polarizer layer, FM1 [11]. The resistance varies as $R(\theta) = R_0 + \Delta R(1-\cos \theta)/2$, where $\Delta R$ is the magneto-resistance. Meanwhile, the number of available magnons is $\langle N_m \rangle = (M_s V/\mu_B)(1 - \cos \theta)$, where $M_s$ is the saturation magnetization, $V$ is the volume of the ferromagnet, and $\mu_B$ is the Bohr magneton. Thus, there is a linear relation between the giant magneto-resistance and the magnon population: $\langle N_m \rangle = (M_s V/\mu_B)(R(\theta) - R(0))/\Delta R$. Therefore, measuring the magneto-resistance reflects the magnon number density. Ref. [11] finds an asymmetric enhancement of the magnon population as a function of current, which is in contradiction to semiclassical STT theory.

To model the aforementioned experiment, we consider the spin-valve structure depicted in Fig. 2. This system consists of a hard ferromagnetic layer, FM1; a normal metal spacer, N; and a free ferromagnetic layer, FM2, attached to two left and right reservoirs, as shown in Fig. 2. We will derive Eq. (1) and discuss its consequences. At finite temperature and when there are spin currents, the stochastic LLG describes the dynamics of the magnetization direction $m$ in the free layer,

$$\dot{m} = -\gamma m \times [H_{\text{eff}} + h(t)] + \alpha m \times \dot{m} + \beta m \times (m \times \dot{p}),$$

where $\gamma$ is the effective gyromagnetic ratio; $H_{\text{eff}} = -(M_s V)^{-1} \delta F/\delta m$ is the effective magnetic field, which is a functional derivative of the thermodynamic free energy $F$ with respect to the magnetization; $h(t)$ is the stochastic magnetic field arising from various sources of fluctuations; $\alpha$ is the effective Gilbert damping parameter, $\beta = \gamma p I/(M_s V e)$ is the STT parameter, with $p$ and $\dot{p}$ parameterize the spin-current polarization density and its spin direction, respectively; and $e$ is the electron charge.

There are two contributions to the stochastic field $h$. First, there are intrinsic contributions related to the intrinsic Gilbert damping. According to the fluctuation-dissipation theorem, a stochastic and uncorrelated thermal field describes these effects. Our focus is on the second, extrinsic, contributions to the random field. These fields relate to the fluctuations of the spin transfer torque, the difference between the spin currents to the left and the right of the free magnetic layer. The spin-transfer torque fluctuations consist of equilibrium thermal fluctuations arising from the spin-pumping-induced enhancement of the Gilbert damping, and bias-driven quantum spin shot noise fluctuations.

We will now compute the stochastic field due to the spin-transfer torque fluctuations. On both sides of the soft ferromagnet (FM2), we define currents with respect to the flow towards the ferromagnet. The rate of change of the magnetization direction, the STT, is then $-\gamma I_{\sigma,\text{abs}}(t)/(M_s V)$, where the absorbed spin current is $I_{\sigma,\text{abs}}(t) = \sum_{A \in \{N, R\}} I_{\sigma, A}(t)$. Since the spin-transfer torque is transverse to the magnetization, the associated stochastic magnetic field appearing in the LLG equation is given by $h(t) = -(M_s V)^{-1} m \times \delta I_{\sigma,\text{abs}}(t)$, where $\delta I_{\sigma,\text{abs}}(t)$ is the deviation of the absorbed current from its average value. The extrinsic fluctuating fields vanish on average $\langle h_i(t) \rangle = 0$, and the transverse components

![FIG. 2: (a) A spin-valve structure attached to left (L) and right (R) reservoirs with chemical potentials $\mu_L$ and $\mu_R$, respectively. FM1 is a hard ferromagnet, FM2 is a soft ferromagnet, and N is a normal metal. (b) A two-terminal scattering representation of the spin-valve structure in (a) with two spin-dependent scattering matrices $S^{(1)}$ and $S^{(2)}$. $\hat{a}_A$ and $\hat{b}_A$ are fermion annihilation operators representing incoming and outgoing electrons, respectively, in the left lead ($A = L$), middle of the normal metal ($A = N$), and the right lead ($A = R$).]
are correlated as \[ \langle h_i(t) h_j(t') \rangle = \frac{1}{M^2 \nu^2} \sum_{A,B \in \{N,R\}} C_{ji,AB}(t-t'), \]

\[ \langle h_i(t) h_i(t') \rangle = \frac{1}{M^2 \nu^2} \sum_{A,B \in \{N,R\}} C_{jj,AB}(t-t'), \]

(3)

where \( i \neq j = x, y \), \( C_{ij,AB}(t-t') = \langle \delta \mathbf{I}_{\mathbf{r},A}(t) \delta \mathbf{I}_{\mathbf{r},B}(t') \rangle \)

is the spin-current correlation function and \( \delta \mathbf{I}_{\mathbf{r},A} \) is the deviation of the vector component \( i \in \{ x, y \} \) of the spin current in the lead \( A \), \( \mathbf{I}_{\mathbf{r},A}(t) = I_{\pi,A}(t) - \langle I_{\pi,A}(t) \rangle \).

The spin current is \( \mathbf{I}_{\pi,A} = -\langle h/2e \rangle \sum_{\sigma, \alpha} \sigma^{\alpha \beta} \mathbf{I}_{\alpha}^{\beta} \),

where \( \sigma \) is the vector of Pauli matrices. In a scattering formalism, the components of the spin current tensor are

\[ \mathbf{I}_{\alpha}^{\beta}(t) = \frac{e}{\hbar} \left( \mathbf{b}_{A,\alpha}(t) \mathbf{a}_{A,\beta}(t) - \mathbf{a}_{A,\alpha}(t) \mathbf{b}_{A,\beta}(t) \right), \]

(4)

where \( \mathbf{a}_{A,\alpha} \) and \( \mathbf{b}_{A,\alpha} \) are vector operators containing all transverse transport channels, which annihilate electrons with spin \( \alpha \) in lead \( A \) that move toward and away from the ferromagnets, respectively. For simplicity, in expressing the formulas, we drop all channel indices, but contributions from all channels are taken into account in all of our calculations and results.

Since our system consists of two ferromagnets, we need to explicitly compute the currents to the left and right of the free layer, FM2. To this end, we must relate the scattering properties in the subsystems. The outgoing and incoming modes are, in the absence of spin-flip processes, related via

\[ \begin{pmatrix} \mathbf{b}_{La} \\ \mathbf{b}_{Na} \end{pmatrix} = \begin{pmatrix} s_{LA}^{(1)} & s_{LA}^{(2)} \\ s_{NA}^{(1)} & s_{NA}^{(2)} \end{pmatrix} \begin{pmatrix} \mathbf{a}_{La} \\ \mathbf{a}_{Na} \end{pmatrix}, \]

(5)

\[ \begin{pmatrix} \mathbf{a}_{Na} \\ \mathbf{b}_{Ra} \end{pmatrix} = \begin{pmatrix} s_{NA}^{(2)} & s_{NA}^{(1)} \\ s_{RA}^{(2)} & s_{RA}^{(1)} \end{pmatrix} \begin{pmatrix} \mathbf{b}_{Na} \\ \mathbf{a}_{Ra} \end{pmatrix}. \]

(6)

The diagonal and off-diagonal elements of the scattering matrices represent the reflection and the transmission coefficients, respectively. A scattering matrix that relates the annihilation operators associated with incoming and outgoing waves from the left and right leads is

\[ \begin{pmatrix} \mathbf{b}_{La} \\ \mathbf{b}_{Ra} \end{pmatrix} = \begin{pmatrix} s_{RLA}^{(1)} & s_{RLA}^{(2)} \\ s_{RRA}^{(1)} & s_{RRA}^{(2)} \end{pmatrix} \begin{pmatrix} \mathbf{a}_{La} \\ \mathbf{a}_{Ra} \end{pmatrix}. \]

(7)

The scattering matrix of the total system \( S \) is related to the scattering matrices of FM1, \( S^{(1)} \), and FM2, \( S^{(2)} \), via

\[ \begin{align*}
\mathcal{D} &= |1 - s_{N,\alpha}^{(1)} s_{N,\alpha}^{(2)}|^{-1}.
\end{align*} \]

In order to compute the spin currents, we must express the currents in Eq. \( 3 \) as functions of the properties of the incoming modes in the left and right lead only, \( \hat{a}_{L} \) and \( \hat{a}_{R} \). Using Eqs. \( 3 \)-\( 5 \), we can rewrite the operators in the normal spacer as a linear combination of the left and right leads as

\[ \hat{a}_{N,\alpha} = K_{\alpha}^{L} \hat{a}_{La} + K_{\alpha}^{R} \hat{a}_{Ra}, \]

(9a)

\[ \hat{b}_{N,\alpha} = K_{\alpha}^{L} \hat{b}_{La} + K_{\alpha}^{R} \hat{b}_{Ra}, \]

(9b)

where

\[ \begin{align*}
K_{\alpha}^{L} &= s_{N,\alpha}^{(1)} s_{N,\alpha}^{(2)} D, \\
K_{\alpha}^{R} &= s_{N,\alpha}^{(2)} s_{N,\alpha}^{(1)} D, \\
K_{\alpha}^{R} &= s_{N,\alpha}^{(1)} s_{N,\alpha}^{(2)} D, \\
K_{\alpha}^{R} &= s_{N,\alpha}^{(2)} s_{N,\alpha}^{(1)} D.
\end{align*} \]

(10a)

(10b)

(10c)

(10d)

Using fermionic statistics and preforming straightforward calculations, we can find the total correlator of the transverse components of the stochastic magnetic field as a function of the scattering matrix elements. In the simplest limit, considering that the frequency of the spin-current noise is the lowest energy scale in the system, we can approximate \( C_{ij,AB}(t-t') \) \( \simeq C_{ij,AB}(\omega = 0) \delta(t-t') \), where

\[ C_{ij,AB}(\omega = 0) = \frac{\hbar}{8\pi} \sum_{\alpha, \in \{ \uparrow, \downarrow \} \alpha, \beta} \sigma^{\alpha \beta} \sigma^{\beta \alpha} \]

\[ \times \int d\varepsilon \left[ (G_{1}(\varepsilon) + G_{2}(\varepsilon)) \delta_{\mathbf{A}} \delta_{\mathbf{B}} + G_{3}(\varepsilon) \delta_{\mathbf{A}} \delta_{\mathbf{B}} \right], \]

(11)

where

\[ \begin{align*}
G_{1} &= \text{Tr} \left[ 2\delta_{\mathbf{A}} - s_{\mathbf{A},\beta}(\varepsilon)s_{\mathbf{A},\alpha}(\varepsilon) - s_{\mathbf{B},\beta}(\varepsilon)s_{\mathbf{B},\alpha}(\varepsilon) \right] \\
&\times f_{\mathbf{A}}(\varepsilon)(1 - f_{\mathbf{A}}(\varepsilon)), \\
G_{2} &= \sum_{C,D \in \{ L,R \}} \text{Tr} \left[ s_{\mathbf{A},\alpha}^{\dagger} s_{\mathbf{A},\beta}^{\dagger} s_{\mathbf{B},\alpha}^{\dagger} s_{\mathbf{B},\beta}^{\dagger} \right] \\
&\times f_{\mathbf{C}}(\varepsilon)(1 - f_{\mathbf{D}}(\varepsilon)), \\
G_{3} &= \sum_{C,D \in \{ L,R \}} \sum_{i,j \in \{ a,b \}} \text{Tr} \left[ (K_{\mathbf{C},\alpha}^{\dagger})^{i} (K_{\mathbf{D},\beta}^{\dagger})^{j} K_{\mathbf{D},\alpha}^{i} K_{\mathbf{C},\beta}^{j} \right] \\
&\times f_{\mathbf{C}}(\varepsilon)(1 - f_{\mathbf{D}}(\varepsilon)).
\end{align*} \]

(12a)

(12b)

(12c)

In Eqs. \( 12 \), the trace is a sum over the transverse channels and \( f_{\mathbf{A}}(\varepsilon) \) is the Fermi-Dirac distribution at energy \( \varepsilon \) in lead \( A \) with chemical potential \( \mu_{A} \).

The correlator of Eq. \( 11 \) can be further simplified. Typically, the thermal energy \( k_{B}T \), and the applied bias potential \( eU \equiv \mu_{L} - \mu_{R} \), are much smaller than the Fermi energy of reservoirs \( \varepsilon \). Just like many other cases of electron transport in metals, the scattering matrix elements can then be evaluated at the Fermi level. In this limit,
we find that the total correlator of spin currents, Eq. (11) can be decomposed into two parts. The first part is the thermal contribution that vanishes at zero temperatures, and the second part is bias dependent and finite even at zero temperature. Finally, by using Eqs. (11) and (12), the extrinsic stochastic thermal spin-current noise and spin shot noise correlators become

\[ \langle h_i^{(th)}(t) h_j^{(th)}(t') \rangle = \xi^{(th)}(\delta_{ij} \delta(t - t')), \quad (13) \]

\[ \langle h_i^{(sh)}(t) h_j^{(sh)}(t') \rangle = \xi^{(sh)}(\delta_{ij} \delta(t - t')), \quad (14) \]

with the following correlator amplitudes:

\[ \xi^{(th)}(\omega = 0) = \frac{4 \pi \alpha_{sp}}{\gamma M_a V} k_B T, \quad (15) \]

\[ \xi^{(sh)}(\omega = 0) = \frac{k_B T \sigma[W_{LR}(\varepsilon_F)]}{4 \pi M_a^2 V^2} \left( \frac{eU}{\tanh(\frac{eU}{2k_B T})} - 2k_B T \right). \quad (16) \]

To derive the above results, we have used the following integrals: \( \int dz (f_L - f_R)^2 = 2k_B T + eU \coth(eU/2k_B T) \) and \( \int dz f_L f_R (1 - f_L f_R) = k_B T. \) \( \alpha_{sp} \) is a Gilbert-type damping parameter arising from the spin pumping that depends on the spin-mixing conductance \( \alpha_{sm} \). \( \sigma[W_{LR}(\varepsilon_F)] \) is a function of the spin-dependent scattering matrices, see Eqs. (12) whose quantity depends on the details of the spin-valve structure and can be calculated using first principle calculations which is a topic beyond the scope of this Letter. In a naive approximation, we can estimate \( \sigma[W_{LR}] \sim N \delta \varepsilon/\varepsilon_F \), where \( \delta \varepsilon \) is the exchange splitting in the free ferromagnetic layer \( \delta \varepsilon \).

The extrinsic thermal spin-current noise of Eq. (15) is proportional to the spin-pumping-induced enhancement of the damping parameter \( \alpha_{sp} \) and obeys the fluctuation-dissipation theorem. This term is analogous to the intrinsic contribution of the thermal noise arising from the intrinsic Gilbert damping \( \alpha_0 \). Thus, we can take the latter contribution into account by replacing \( \alpha_{sp} \rightarrow \alpha = \alpha_{sp} + \alpha_0 \), in Eq. (15). At finite frequencies, we could rewrite the total thermal noise correlator in the frequency domain as

\[ \langle h_i^{(th)}(\omega) h_j^{(th)}(\omega') \rangle = \frac{2 \pi \alpha_0 \xi}{\gamma M_a V \tanh(\frac{\hbar \omega}{2k_B T})}. \quad (17a) \]

\[ \xi^{(th)}(\omega) = \frac{2 \pi \alpha_0 \xi}{\gamma M_a V \tanh(\frac{\hbar \omega}{2k_B T})}. \quad (17b) \]

Now we discuss the shot noise correlator of Eq. (16). When the applied bias potential is larger than the thermal energy, \( k_B T \ll |eU| \ll \varepsilon_F \), the shot noise amplitude is \( \xi^{(sh)} \propto |eU| \), whereas in the opposite limit, \( |eU| \ll k_B T \ll \varepsilon_F \), we obtain \( \xi^{(sh)} \propto e^2 U^2 / (6k_B T) \).

We, finally, calculate the magnon density in the presence of the spin shot noise as well as thermal stochastic noise, Eqs. (13), (16) and (17), for a uniaxial and collinear ferromagnetic layer. The total free energy of the free ferromagnetic layer is written as \( F = \int dr (-K (m \cdot \dot{z})^2 / 2 - \mu_B m \cdot B), \) where \( K > 0 \) is the anisotropy energy density and \( B = B \hat{z} \) is the external magnetic field along the \( z \)-direction. We expand the unit vector along the magnetization in terms of the transverse excitations \( \delta m \), as \( m = \sqrt{1 - \delta m^2} \hat{z} + \delta m \), with \( \dot{z} \cdot \delta m = 0 \). The number of magnons is proportional to the small deviation of the magnetization along the equilibrium \( z \)-direction, \( \langle N_m \rangle = (M_s V / 4 \mu_B) \langle \delta m^2 \rangle \). Linearizing the LLG equation, Eq. (2), in the presence of spin currents with a polarization in the \( z \)-direction results in an effective equation of motion for magnons

\[ i(1 + \alpha) \dot{\psi}(t) - (\Omega + i \beta) \psi(t) = \gamma \dot{h}(t), \quad (18) \]

where \( \dot{h}(t) = \delta m_x(t) - i \delta m_y(t), \) \( \dot{h}(t) = h_x(t) - i h_y(t), \) and \( \Omega = \gamma (K + \mu_B B) / (M_s V) \) is the ferromagnetic resonance frequency. Through a Fourier transformation, the solution of Eq. (18) becomes

\[ \psi(\omega) = \frac{\gamma \dot{h}(\omega)}{\omega - \Omega + i(\alpha \omega - \beta)}. \quad (19) \]

To obtain Eq. (11), we compute the population of magnons in the limit of small damping and bias voltage as

\[ \langle N_m \rangle = \frac{M_s V}{4 \mu_B} \int \frac{d\omega}{2 \pi} \frac{d\omega'}{2 \pi} \langle \psi(\omega) \psi^*(\omega') \rangle 
\approx \frac{1}{1 - \beta^2 / \gamma^2} \left[ \left( f_{BE}(\Omega, T) + \frac{1}{2} \right) + \frac{\gamma M_s V}{4 \pi h \alpha \Omega} \xi^{(sh)} \right]. \quad (20) \]

The total magnon number density has two contributions. The first term in Eq. (20) is the contribution of thermal magnons, that obey the Bose-Einstein statistics, including zero-temperature quantum fluctuations; and the second term is the contribution of the spin shot noise, which has a purely quantum mechanical nature and is finite even at zero temperature.

Figure 1 shows the magnon population number of Eq. (20) as a function of the charge current for different temperatures. We consider a ferromagnetic free layer with a ferromagnetic resonance of \( \Omega = 10 \) GHz and an effective Gilbert damping of \( \alpha = 0.01 \). There is a zero-bias singularity in magnon number at zero temperature, see Eq. (20), that is rapidly broadened by increasing the temperature, see Fig. 1. This broadening is not due to the contribution of thermal magnons but is rather the contribution of spin shot noise at finite temperature. The piecewise and asymmetric dependence of the magnon population to the bias current survives even at higher temperatures. In Fig. 1 we also compare the magnon populations with and without the contribution from the quantum shot noise. In the absence of the quantum shot noise (dotted lines), depending on the direction of the applied bias voltage, the magnon fluctuations increase due to the antidamping-like STT or decrease because of the damping-like STT. The quantum spin shot noise, on the
other hand, leads to an increase in the magnon number irrespective of the current direction.

In summary, in addition to the semiclassical picture of STT [1, 2], there is an important and so far neglected quantum effect arising from the spin shot noise contribution. This effect originates from the discrete nature of itinerant electron spins. At low temperatures, the resulting quantum fluctuations strongly affect the magnon population in spin valves. The result is in good agreement with the recent observation of a piecewise-linear dependence of the magnon population on the applied current measured by Zhould et al.

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Note added—During the completion of the work, we became aware of another paper [19] that attributes the quantum STT [11] to the spin fluctuations of small magnetic junctions.

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