Space-Time Medium Functions as a Perfect Antenna-Mixer-Amplifier Transceiver

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We show that a space-time-varying medium can function as a front-end transceiver, i.e., an antenna-mixer-amplifier. Such a unique functionality is endowed by space-time surface waves associated with complex space-time wave vectors in a subluminal space-time medium. The proposed structure introduces pure frequency up- and down-conversions and with very weak undesired time harmonics. In contrast to other recently proposed space-time mixers, a large frequency up-/down conversion ratio, associated with gain is achievable. Furthermore, as the structure does not operate based on progressive energy transition between the space-time modulation and the incident wave, it possesses a subwavelength thickness (metasurface). Such a multi-functional, highly efficient and compact medium is expected to find various applications in modern wireless telecommunication systems.

I. INTRODUCTION

Reception and transmission of electromagnetic waves is the essence of wireless telecommunications [1, 2]. Such a task requires various functions, such as power amplification, frequency conversion, and wave radiation. These functions have been conventionally performed by electronic components and electromagnetic structures, such as transistor-based amplifiers, semiconductor-based diode mixers, and resonance-based antennas. The ever increasing demand for versatile wireless telecommunication systems has led to a substantial need in multi-functional components and compact integrated circuits. In general, each natural medium may introduce a single function, e.g., a resonator may operate as an antenna at a specific frequency. To contrast this conventional approach, a single medium exhibiting several functions concurrently could lead to a disruptive evolution in telecommunication technology.

Recently, space-time-modulated media have attracted a surge of scientific interest thanks to their capability in multifunctional operations, e.g., mixer-duplexer-antenna [3], unidirectional beam splitters [4], nonreciprocal filters [5, 6], and signal coding metagratings [7]. In addition, a large number of versatile and high efficiency electromagnetic systems have been recently reported based on the unique properties of space-time modulation, including space-time metasurfaces for advanced wave engineering and extraordinary control over electromagnetic waves [8–25], nonreciprocal platforms [26–32], frequency converters [17, 33], and time-modulated antennas [34–39]. Such strong capability of space-time-modulated media is due to their unique interactions with the incident field [21, 28, 40–44].

This paper reveals that a space-time medium can function as a full transceiver front-end, that is, an antenna-mixer-amplifier-filter system. Specific related contributions of this study are as follows.

- We show that the space-time medium operates as an antenna per se. Such an interesting functionality of the space-time medium is endowed by space-time surface waves. Other recently proposed space-time antenna systems are formed by integration of the space-time-modulated medium with an antenna [3, 35, 45], and hence suffer from a number of drawbacks, i.e., requiring long structures, low efficiency and narrow-band operation.

- The proposed antenna-mixer-amplifier introduces large frequency up- and -down conversion ratios. This is very practical, because in real-scenario wireless telecommunication systems, a large frequency conversion is required, i.e., a frequency conversion from a microwave/millimeter-wave frequency to an intermediate frequency in receivers. In contrast, recently proposed space-time frequency converters suffer from very low frequency conversion ratios (up-/down-converted frequency is very close to the input frequency) [3, 17, 35, 45].

- Recently proposed space-time systems operate based on a progressive coupling between the incident wave and the space-time modulation [3, 33, 46–48], and hence a thick (compared to the wavelength) slab is required. In contrast, the proposed antenna-mixer-amplifier does not work based on progressive coupling between the incident wave and the space-time modulation and hence is formed by a thin (sub-wavelength) slab. As a result, the proposed structure is classified among metasurface categories and is compatible with the compactness requirements of modern wireless telecommunication systems.

- The proposed medium inherently operates as a band-pass filter, i.e., a pure frequency down- and up-conversion occurs so that the undesired time harmonics are highly suppressed. Such a property is governed by the subluminal operation of space-time modulation.
In contrast to conventional mixers that introduce a significant conversion loss, here a conversion gain may be achieved in the down-link or up-link. In addition, the radiation from the antenna-mixer-amplifier is very directive.

The paper is organized as follows. Section II presents the operation principle of the antenna-mixer-amplifier. Then, Sec. III investigates the theoretical aspects of the work, and provides an analysis of the wave propagation in the antenna-mixer-amplifier space-time medium. Next, Sec. IV presents the design procedure of the structure, and gives an approximate closed-form solution to calculate the thickness and receive/transmit power gain of the structure. Section V gives the time and frequency domain numerical simulation results for the designed antenna-mixer-amplifier. Finally, Sec. VI concludes the paper.

II. OPERATION PRINCIPLE

Consider the antenna-mixer-amplifier slab in Fig. 1(a), with the thickness of $d$ and a spatiotemporally-periodic electric permittivity as

$$\epsilon(z,t) = \epsilon_{av} + \epsilon_m \cos(\beta_m z - \omega_m t),$$  

with $\epsilon_{av}$ being the average electric permittivity of the background medium, and $\epsilon_m$ representing the modulation amplitude. In Eq. (1), $\omega_m$ denotes the temporal frequency of the modulation, and $\beta_m$ is the spatial frequency of the modulation represented by

$$\beta_m = \frac{\omega_m}{v_m} = \frac{\omega_m}{\Gamma v_b},$$  

where $v_m$ and $v_b$ are the phase velocity of the modulation and the background medium, respectively, and $\Gamma = v_m/v_b$ is the space-time velocity ratio [28].
The slab in Fig. 1(a) is obliquely illuminated by a y-polarized incident electric field under an angle of incidence of \( \theta_i \), as

\[
E_i(x, z, t) = \hat{y}E_0 e^{i(k_x x + k_z z - \omega_0 t)},
\]

where \( E_0 \) is the amplitude of the incident wave, and \( \omega_0 \) and \( k_0 = \sqrt{k_x^2 + k_z^2} = \sqrt{(k_0 \sin(\theta_i))^2 + (k_0 \cos(\theta_i))^2} \) are respectively temporal and spatial frequencies of the incident wave. Here, \( k_0 = \omega_0/c \), with \( c \) being the velocity of light in vacuum.

In the receiving state (down-link) in Fig. 1(a), strong transition between a space-wave, with temporal frequency \( \omega_0 \), and space-time waves with temporal and spatial frequencies of the space-time harmonics inside the structure are governed by the momentum conservation law, i.e.,

\[
\gamma_{z,n} = \beta_{z,0} + n \beta_m + i \alpha_{z,n},
\]

and the energy conservation law, i.e.,

\[
\omega_n = \omega_0 + n \omega_m.
\]

The scattering angles of the space-time harmonics in regions 1 and 3 (reflection to the top and transmission to the bottom of the medium, respectively) may be determined by the Helmholtz relations, i.e.,

\[
k_0^2 \sin^2(\theta_i) + k_{0,n}^2 \cos^2(\theta_{r,n}) = k_{0,n}^2 \quad \text{and} \quad k_0^2 \sin^2(\theta_i) + k_{0,n}^2 \cos^2(\theta_{t,n}) = k_{0,n}^2,
\]

with \( \theta_{r,n} \) and \( \theta_{t,n} \) being the reflection and transmission angles of the \( n^{\text{th}} \) space-time harmonics, reading [7]

\[
\theta_{r,n} = \theta_{i,n} = \sin^{-1} \left( \frac{\sin(\theta_i)}{1 + n \omega_m / \omega_0} \right),
\]

where \( k_{0,n} = \omega_n / c = (\omega_0 + n \omega_m) / c \). Therefore, given the common tangential wavenumber, \( k_z = k_0 \sin(\theta_i) \) in all the regions, the reflection and transmission angles for the \( n^{\text{th}} \) space-time harmonic are equal. Equation (5) shows that the harmonics in the \( n \)-interval \([\omega_0(\sin \theta_i - 1)/\omega_m, +\infty[\) are scattered at different angles ranging from 0 to 1/2 through \( \theta_i \) for \( n = 0 \). However, the space-time harmonics outside of this interval represent imaginary \( k_{z,n} \) and hence propagate as space-time surface waves along the two boundaries of the slab at \( z = 0 \) and \( z = d \).

The scattering angles of the space-time harmonics inside the space-time-modulated medium read

\[
\theta_{n} = \tan^{-1} \left( \frac{k_{z,n}}{\beta_{z,n}} \right) = \tan^{-1} \left( \frac{k_0 \sin(\theta_i)}{\beta_{z,0} + n \beta_m} \right).
\]

Figure 2 shows the structure of the antenna-mixer-amplifier space-time medium. We aim to design the structure such that the first lower space-time harmonic, \( n = -1 \), outside the medium scatters along the boundary of the medium, i.e., \( \theta_{t,-1} = \theta_{i,-1} = 90^\circ \). Hence, using Eq. (5), the incident angle reads

\[
\theta_i = \sin^{-1} \left( \frac{\omega_m}{\omega_0} \right).
\]

In addition, to achieving a strong transition to the \( n = -1 \) harmonic, the scattered \( n = -1 \) harmonic inside the medium should propagate in parallel to the two space-time surface waves on the two boundaries of the medium at \( z = 0 \) and \( z = d \), i.e., \( \theta_{n=-1} = 90^\circ \). Thus, using Eq. (6), we achieve

\[
\beta_{z,-1} = 0.
\]

As a result, the \( z \) component of the Wave vector inside the medium is purely imaginary, i.e., \( \gamma_{z,-1} = i \alpha_{z,-1} \), whereas the incident field wavenumber \( k_z \) is purely real. The space-time-modulated medium presents transition from the fundamental harmonic \( n = 0 \) to a large (theoretically infinite, \( -\infty < n < \infty \)) number of space-time harmonics. Such a transition is very strong for the luminal space-time modulation, where the space-time modulation velocity is close to the background phase velocity, i.e., \( v_m = v_b [28, 30] \).

To prevent generation of strong undesired time harmonics, here the space-time-modulated medium operates...
in the subluminal regime, where $0 < v_m < v_b$, i.e.,

$$0 < \Gamma_{\text{subluminal}} < \sqrt{\frac{\epsilon_r}{\epsilon_{av} + \epsilon_m}}.$$  \tag{9}

As a result, a pure and precise transition between the fundamental ($n = 0$) harmonic and the desired (here $n = -1$) space-time surface wave harmonic can occur.

### III. THEORETICAL INVESTIGATION

#### A. Analytical solution

Considering the TE$_0$ incident field in Eq. (3), the incident magnetic field reads

$$\mathbf{H}_i(x, z, t) = \frac{1}{\eta_1} \left[ \mathbf{k}_i \times \mathbf{E}_i^F(x, z, t) \right]$$

$$= -\hat{x}\cos(\theta_i) + \hat{z}\sin(\theta_i) \frac{E_0}{\eta_1} e^{i(k_x x + k_z z - \omega_0 t)},$$  \tag{10}

where $\eta_1 = \sqrt{\mu_0/\epsilon_0}$ is the characteristic impedance of free space. Since the space-time-modulated medium is periodic in space and time, the electric and magnetic fields in the slab may be decomposed to space-time Bloch-Floquet harmonics as

$$\mathbf{E}_m(x, z, t) = \hat{y} \sum_n E_n e^{i\gamma_{z,n} z} e^{i(k_x x - \omega_0 t)},$$  \tag{11a}

and

$$\mathbf{H}_m(x, z, t) = \frac{1}{k} \sum_n H_n [-\hat{x}\beta_{z,n} + \hat{z}k_x] e^{i\gamma_{z,n} z} e^{i(k_x x - \omega_0 t)}.$$  \tag{11b}

The unknown coefficients in Eqs. (11a) and (11b), $\gamma_{z,n} = \beta_{z,n} + i\alpha_{z,n}$, $E_n$ and $H_n$ shall be found through the following procedure. We first construct and solve the corresponding wave equation. Next, we fix the source boundary conditions at the edges of the slab, i.e. at $x = 0$ and $z = d$, for all the $(\omega_0, \gamma_{z,n})$ states in the dispersion diagram. This provides the unknown field coefficients $E_n$ and $H_n$ inside the slab, as well as the scattered (reflected and transmitted) fields outside the slab, i.e., $E_R$ and $E_T$.

The homogeneous wave equation reads

$$\nabla^2 \mathbf{E}_m(x, z, t) - \frac{1}{c^2} \frac{\partial^2 [\epsilon(z, t)\mathbf{E}_m(x, z, t)]}{\partial t^2} = 0.$$  \tag{12}

Inserting (11a) and (1) into (12), and using the Fourier series expansion for the permittivity, results in

$$E_n \left[ \frac{k_x^2 + (\beta_{z,n} + i\alpha_{z,n})^2}{(\omega_0 + n\omega_m)/c} \right] - \sum_{k=-\infty}^{\infty} \hat{\epsilon}_k E_{n-k} = 0.$$  \tag{13}

where $\hat{\epsilon}_k$ are the Fourier series coefficients of the permittivity. Equation (13) may be cast to the matrix form as

$$[K] \cdot [E_n] = 0,$$  \tag{14}

where $[K]$ is the $(2N+1) \times (2N+1)$ matrix with elements

$$K_{nn} = \left[ \frac{k_x^2 + (\beta_{z,n} + i\alpha_{z,n})^2}{(\omega_0 + n\omega_m)/c} \right] - \epsilon_0,$$  \tag{15}

where $2N + 1$ is the number of truncated harmonics. Then, the dispersion relation reads

$$\det \{[K]\} = 0.$$  \tag{16}

Once the dispersion diagram is formed, for a given incident temporal frequency $\omega_0$, the corresponding wave number, i.e., $\gamma_{z,n}$, inside the slab can be computed. Next, the unknown field amplitudes $E_n$ in the slab are found by solving (14) after determining the $E_0$ satisfying boundary conditions. Figure 3 shows a qualitative 3D dispersion diagram of a subluminal space-time-modulated medium, composed of an infinite periodic set of double-cones with apexes at $\beta_{z,0} = -n\beta_m$ and $k_x = 0$ and the slope $\Gamma$. As the slope $\Gamma$ increases and goes towards unity, the cones get closer to each other and start touching other cones(harmonics). As a result, stronger transition between harmonics occurs and yields transitions of the power from the fundamental harmonic to a large number of harmonics. However, this is not what we aim at. As we are interested in a pure transition to a desired harmonic $(n = -1)$, we require a subluminal space-time modulation defined in Eq. (9).

Figure 4 plots the analytical isofrequency dispersion diagram, computed using (16), for the subluminal regime where $\Gamma = 0.2$. We have chosen a specific $\theta_i$, where $n = -1$ harmonic is excited at a point corresponding to $\beta_{z,-1} = 0$ (i.e., $\beta_0/\beta_m = +1$). As a consequence, the $n = -1$ harmonic scatters along the $x$-direction, i.e., $\theta_{z,-1} = 90^\circ$. Although the real part of the $z$ component of the wavevector of the $n = -1$ harmonic is zero ($\beta_{z,-1} = 0$), its imaginary part is greater than zero, and hence $\gamma_{z,-1} = i\alpha_{z,-1}$.

Figure 4 shows that the transition between the forward $n = 0$ and $n = -1$ harmonics (highlighted with magenta arrows) occurs for both reception (down-link) and transmission (up-link). This may as well be verified by Eq. (6) as follows. For the down-link, the incident wave $(n = 0)$ with the incident angle of $\theta_i^{RX}$ and wavenumber of $\beta_{z,0}^{RX} = \beta_m$ makes a transition to the down-converted harmonic $(n = -1)$, which is attributed to $\theta_i^{TX} = 90^\circ$, $\beta_{z,-1}^{TX} = 0$ and $\epsilon_{z,-1}^{TX} = i\alpha_{z}^{RX}$. However, for the up-link (transmission state), the incident wave $(n = 0)$ with the incident angle of $\theta_i^{TX} = 90^\circ$ and wavenumber of $\beta_{z,0}^{TX} = 0$ makes a transition to the up-converted harmonic $(n = +1)$ with $\theta_{z,+1}^{TX} = \theta_i^{RX} = \theta_i^{TX}$, which is attributed to $\beta_{z,+1}^{TX} = \beta_m$ and $\gamma_{z,+1}^{TX} = \beta_m$. 
isofrequency \((\omega, \gamma_z)\) close to each other, where \(\Gamma \) is the subluminal regime, i.e., \(\text{Re} \epsilon = 0\) for \(\omega_0 \rightarrow 0\). It may be seen from this figure that for \(n = 0\) and \(n = -1\) harmonics are excited very close to each other, where \(n = 0\) is excited at an angle of scattering of \(\theta_0 = \theta_1 = 15^\circ\). However, the \(n = -1\) harmonic is intentionally excited at the angle of scattering of \(\theta = 90^\circ\).

Figure 5(b) plots the same isofrequency \(\beta_{z,n}(k_z)\) diagram as Figure 5(a) except for a greater modulation amplitude of \(\epsilon_m = 0.45\). As a result of non-equilibrium in the electric and magnetic permissivities of the medium, several electromagnetic badgaps appear at the intersections between space-time harmonics [30]. As a consequence, strong coupling between some of the harmonics has occurred, e.g. between the \(n = 0\) and \(n = -1\) harmonics.

Figure 5(c) plots the complex isofrequency dispersion diagram \(\gamma_{z,n}(k_z)\) for the medium in Fig. 5(b). This diagram is formed by two different curves, i.e., the real \(\beta_{z,n}(k_z)\) and the imaginary \(\alpha_{z,n}(k_z)\) parts of the wavenumber. For the sake of clarification, we have included only a few number of harmonics. This figure shows that the excited angle of incidence \(\theta_1 = 15^\circ\), the \(n = 0\) harmonic introduces a pure real wavenumber, i.e., \(\gamma_{z,0} = \beta_{z,0}\), whereas the \(n = -1\) harmonic acquires a pure imaginary wavenumber, that is, \(\gamma_{z,-1} = i\alpha_{z,-1}\). As a result, a perfect space-time transition from a pure propagating wave to a pure space-time surface wave is ensured.

Figure 5(d) plots the complex dispersion diagram, i.e., \(\gamma_{z,n}(\omega_0)\) for the medium in Fig. 5(b) and 5(c). This figure shows the strong transition between \(n = 0\) and \(n = -1\) harmonics from the \(k_z = 0.218\) (corresponding to \(\theta_1 = 15^\circ\) cut).

\[
E_0(z) = E_1 \cos \left( \frac{\epsilon_m k_0 k_{-1}}{4\gamma_{z,-1}} z \right), \tag{17}
\]

\[
E_{-1}(z) = i E_1 \frac{k_{-1}}{k_0} \sqrt{\frac{\gamma_{z,0}}{\gamma_{z,-1}}} \sin \left( \frac{\epsilon_m k_0 k_{-1}}{4\gamma_{z,-1}} z \right), \tag{18}
\]

where \(k_{-1} = \omega_{-1}/c^2\). The optimal transition from the incident field to the space-time surface wave occurs at \(z = d\), where

\[
\frac{d}{dz} E_{-1}(z)|_{z=d} = 0, \tag{19}
\]

yielding \(|\epsilon_m k_0 k_{-1} d/(4\sqrt{\gamma_{z,-1}\gamma_{z,0}})| = \pi/2\) which corresponds to

\[
d = 2\pi \frac{\sqrt{\gamma_{z,-1}\gamma_{z,0}}}{\epsilon_m k_0 k_{-1}}. \tag{20}
\]

The down-link transition power ratio reads

\[
G_d = \frac{|E_{-1}(z = d)|^2}{|E_1|^2} = \left| \frac{k_{-1}^2 \gamma_{z,0}}{k_0^2 \gamma_{z,-1}} \right| = \frac{\omega_{-1}^2 \beta_m}{\omega_{0}^2 \alpha_{z,-1}}, \tag{21}
\]

Therefore, for a set of modulation parameters, i.e., \(\omega_m, \alpha_{z,-1}\) and \(\Gamma\), a down-link power gain may be achieved. For the particular case, with the dispersion diagram in Figs. 5(c) and 5(d), since \(\alpha_{z,-1} << \beta_m\) (i.e., \(\alpha_{z,-1}/\beta_m = 0.03\) and \(\omega_{0}^2/\omega_{0}^2 = 0.071\)), the down-link power gain reads \(G_d = 3.75\) dB. This power gain emerges from the coupling of the space-time modulation power to the incident wave [4, 40, 49, 50].
The total down-link electric field inside the slab reads

\[ E(x, z, t) = E_i \cos \left( \frac{\epsilon_m k_0 k_{-1}}{4i(\alpha_{z,0}^2 - 1)z} \right) e^{-i(k_x x + \beta_{z,0} z - \omega_0 t)} + \]

\[ iE_i \frac{k_{-1}}{k_0} \frac{\beta_{z,0}}{\alpha_{z,0}} \sin \left( \frac{\epsilon_m k_0 k_{-1}}{4\sqrt{i(\alpha_{z,0}^2 - 1)\beta_{z,0}}} \right) e^{-i(k_x x + i\alpha_{z,0} z - \omega_1 t)} \quad (22) \]

2. Transmission state (up-link)

For the up-link, the matrix differential equations (S7) and (S9) with the initial conditions of \( E_{-1}(0) = E_i \) and \( E_0(0) = 0 \) yields

\[ E_{-1}(z) = E_i \cos \left( \frac{\epsilon_m k_0 k_{-1}}{4\sqrt{\alpha_{z,0}^2 - 1} z} \right) \quad (23) \]

\[ E_0(z) = iE_i \frac{k_0}{k_{-1}} \frac{1}{\alpha_{z,0}} \frac{\beta_{z,0}}{\gamma_{z,0}} \sin \left( \frac{\epsilon_m k_0 k_{-1}}{4\sqrt{\gamma_{z,0}^2 - 1} \beta_{z,0}} \right) \quad (24) \]

The up-link transition power ratio reads

\[ G_u = \frac{|E_0(z = d)|^2}{|E_i|^2} = \frac{|k_0^2 \gamma_{z,0} - 1|}{|k_{-1}^2 \beta_{z,0}|} = \frac{\omega_0^2 \alpha_{z,0} - 1}{\omega_{-1}^2 \beta_m} \quad (25) \]

which is the inverse of the down-link transition power ratio. This shows that the proposed antenna-mixer-amplifier introduces a power amplification only in one direction (here down-link), as shown in Fig. 1(c).

V. RESULTS

This section investigates the functionality and efficiency of the antenna-mixer-amplifier space-time slab using FDTD numerical simulation. We consider oblique incidence to the slab and compare the numerical results with the analytical solution provided in Secs. IIIA and IVB. Figure 6(a) depicts the structure of the antenna-mixer-amplifier space-time surface wave medium. A substrate integrated waveguide (SIW) is integrated with the space-time slab to efficiently launch and receive space-time surface waves. The slab assumes \( \omega_0 = 2\pi \times 2.2 \text{ GHz}, \epsilon_m = 0.45, \Gamma = 0.55, \) and \( d = 0.8\lambda_0. \)

A plane wave with temporal frequency \( \omega_0 = 2\pi \times 3 \text{ GHz} \) is propagating in the +z-direction under an angle of incidence of \( \theta_i = 15^\circ \), and impinges on the slab.

Figure 6(b) shows the time domain numerical simulation result for the receiving state (down-link). It may be seen from this figure that a pure transition from the incident space-wave at \( \omega_0 \) to the space-time surface wave, propagating along the x-direction, at frequency \( \omega_T = \omega_0 - \omega_m = 2\pi \times 0.8 \text{ GHz} \) occurs. Furthermore, it may be seen from Fig. 6(b) that the amplitude of the

![Diagram showing analytical isofrequency dispersion diagram with annotations](image-url)
FIG. 5. Analytical dispersion diagram of the sinusoidally space-time surface wave medium with the electric permittivity in (1) for subluminal regime, i.e., $\Gamma = 0.55$. (a) Isofrequency diagram, i.e., $\beta_z(n,k_x)$ at $\omega / \omega_m = 1.363$, and for $\epsilon_m \rightarrow 0$. (b) Same as (a) except for greater modulation amplitude $\epsilon_m = 0.45$. (c) Isofrequency diagram of the medium in (b), which includes the real and imaginary parts of the $\gamma_z(n,k_x)$, i.e., $\beta_z(n)$ and $\alpha_z(n)$, for $n = 0$ and $n = -1$. (d) Dispersion diagram, i.e., $\gamma_z(n,\omega_0)$ for the medium in (a) and (b) for the $k_x = 0.218$ (corresponding to $\theta_i = 15^\circ$) cut.

received wave is stronger than the amplitude of the incident wave. Figure 6(c) plots the frequency domain numerical simulation result. This plot clearly shows a pure and strong transition (frequency-conversion) from the incident wave to the down-converted space-time surface wave. The 3.5 dB power conversion gain is in agreement with the analytical result in Eq. (21). In addition, the amplitudes of the undesired harmonics are more than 33 dB lower than the amplitude of the down-converted harmonic at $\omega_{IF}$.

Figure 7(a) shows the time domain FDTD numerical simulation result for the transmission state (up-link). Here, a transition (up-conversion) from the space-time surface wave at $\omega_{IF}$ to the space-wave at $\omega_0 = \omega_{IF} + \omega_m$ occurs.

Figure 7(b) plots the frequency domain numerical simulation result for the transmission state (up-link). This plot clearly shows a pure and strong transition (frequency-conversion) from the incident wave to the down-converted space-time surface wave. The 3.52 dB power conversion loss is in agreement with the analytical result in Eq. (25). In addition, the amplitude of the un-
desired harmonics are more than 27 dB lower than the amplitude of the down-converted harmonic at $\omega_0$.

VI. CONCLUSIONS

We showed that a space-time-varying medium operates as an antenna-mixer-amplifier. Such a unique functionality is achieved by taking advantages of space-time surface waves associated with complex space-time wave vectors in a subluminal space-time medium. The proposed structure is endowed with pure frequency up- and down-conversions and very weak undesired time harmonics. In contrast to the recently proposed space-time mixers, a large frequency ratio between the incident wave frequency and the up-/down-converted wave frequency, with a down- or up-conversion gain, is achievable. Furthermore, as the structure does not operate based on progressive energy transition between the space-time modulation and the incident wave, it possesses a subwavelength thickness (metasurface). Such a multi-functional, highly efficient and compact medium represents a new class of integrated electronic-electromagnetic component, and is expected to find various applications in modern wireless telecommunication systems.
FIG. 7. FDTD simulation results of the antenna-mixer-amplifier space-time surface wave medium with $\omega_0/\omega_m = 1.363$, $\epsilon_m = 0.45$, $d = 0.8\lambda_0$, and $\theta_i = 15^\circ$. (a) and (b) Time-domain and frequency-domain responses for the up-link transition.

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SUPPLEMENTAL MATERIAL

The antenna-mixer-amplifier space-time medium is placed between \( z = 0 \) and \( z = d \), and represented by the space-time-varying permittivity of

\[
\epsilon_{eq}(z, t) = \epsilon_{av} + \epsilon_{m} \cos(\beta_{m} z - \omega_{m} t),
\]

and \( \mu = \mu_{0} \). The electric field inside the slab is defined based on the superposition of the \( m = 0 \) and \( m = -1 \) space-time harmonics fields, i.e.,

\[
E_{S}(x, z, t) = E_{0}(z)e^{-i(k_{x} x + \gamma_{z} z - \omega_{0} t)} + E_{-1}(z)e^{-i(k_{x} x + \gamma_{z-1} z - \omega_{-1} t)},
\]

where \( \omega_{-1} = \omega_{0} - \omega_{m} \). The corresponding homogeneous wave equation reads

\[
\frac{\partial^{2}E}{\partial x^{2}} + \frac{\partial^{2}E}{\partial z^{2}} = \frac{1}{c^{2}} \frac{\partial^{2}[\epsilon_{eq}(t, z)E]}{\partial t^{2}}.
\]

Inserting the electric field in (S2) into the wave equation in (S3) results in

\[
\left( \frac{\partial^{2}}{\partial x^{2}} + \frac{\partial^{2}}{\partial z^{2}} \right) \left[ E_{0}(z)e^{-i(k_{x} x + \gamma_{z} z - \omega_{0} t)} + E_{-1}(z)e^{-i(k_{x} x + \gamma_{z-1} z - \omega_{-1} t)} \right]
= \frac{1}{c^{2}} \frac{\partial^{2}}{\partial t^{2}} \left[ \epsilon_{av} + \frac{\epsilon_{m}}{2} e^{i(\beta_{m} z - \omega_{m} t)} + \frac{\epsilon_{m}}{2} e^{-i(\beta_{m} z - \omega_{m} t)} \right]
\times \left( E_{0}(z)e^{-i(k_{x} x + \gamma_{z} z - \omega_{0} t)} + E_{-1}(z)e^{-i(k_{x} x + \gamma_{z-1} z - \omega_{-1} t)} \right),
\]

and applying the space and time derivatives, while using a slowly varying envelope approximation (i.e., \( \partial^{2}E_{-1}(z)/\partial z^{2} = 0 \) and \( \partial^{2}E_{0}(z)/\partial z^{2} = 0 \)), yields

\[
\left[ (k_{z}^{2} + \gamma_{z,0}^{2}) E_{0}(z) - 2i\gamma_{z,0} \frac{\partial E_{0}(z)}{\partial z} \right] e^{-i(k_{x} x + \gamma_{z} z - \omega_{0} t)}
+ \left[ (k_{z}^{2} + \gamma_{z,-1}^{2}) E_{-1}(z) - 2i\gamma_{z,-1} \frac{\partial E_{-1}(z)}{\partial z} \right] e^{-i(k_{x} x + \gamma_{z-1} z - \omega_{-1} t)}
= \frac{1}{c^{2}} \left( \omega_{0}^{2}\epsilon_{av} + \frac{\epsilon_{m}}{2}\omega_{0} \omega_{m} \right) e^{i(\beta_{m} z - \omega_{m} t)}
+ \frac{\epsilon_{m}}{2} \omega_{0}^{2} e^{-i(\beta_{m} z - \omega_{m} t)}
\times \left( E_{0}(z)e^{-i(k_{x} x + \gamma_{z} z - \omega_{0} t)} + E_{-1}(z)e^{-i(k_{x} x + \gamma_{z-1} z - \omega_{-1} t)} \right).
\]

We then multiply both sides of Eq. (S5) with \( e^{i(k_{x} x + \gamma_{z} z - \omega_{0} t)} \), which gives

\[
\left[ (k_{z}^{2} + \gamma_{z,0}^{2}) E_{0}(z) - 2i\gamma_{z,0} \frac{\partial E_{0}(z)}{\partial z} \right]
+ \left[ (k_{z}^{2} + \gamma_{z,-1}^{2}) E_{-1}(z) - 2i\gamma_{z,-1} \frac{\partial E_{-1}(z)}{\partial z} \right] e^{i(\beta_{m} z - \omega_{m} t)}
= \frac{1}{c^{2}} \left( \omega_{0}^{2}\epsilon_{av} + \frac{\epsilon_{m}}{2}\omega_{0} \omega_{m} \right) e^{i(\beta_{m} z - \omega_{m} t)}
+ \frac{\epsilon_{m}}{2} \omega_{0}^{2} e^{-i(\beta_{m} z - \omega_{m} t)}
\times \left( E_{0}(z)e^{i(\beta_{m} z - \omega_{m} t)} + E_{-1}(z)e^{i(\beta_{m} z - \omega_{m} t)} \right),
\]

and, next, applying \( \int_{0}^{2\pi/m} dt \) to both sides of (S6) yields

\[
\frac{dE_{0}(z)}{dz} = i \epsilon_{m} k_{z}^{2} \epsilon_{av}\gamma_{z,0} E_{-1}(z).
\]

Following the same procedure, we next multiply both sides of (S6) with \( e^{-i(\beta_{m} z - \omega_{m} t)} \), and applying \( \int_{0}^{2\pi/m} dt \) in both sides of the resultant, which reduces to

\[
\left[ (k_{z}^{2} + \gamma_{z,-1}^{2}) E_{-1}(z) - 2i\gamma_{z,-1} \frac{dE_{-1}(z)}{dz} \right] = \frac{\omega_{-1}^{2}}{c^{2}} \left( \frac{\epsilon_{m}}{2} E_{0}(z) + \epsilon_{av} E_{-1}(z) \right),
\]
\[
\frac{dE_{-1}(z)}{dz} = i \frac{\epsilon_m k_{-1}^2}{4 \gamma_{z,-1}} E_0(z),
\]  

(S9)

where \( k_{-1} = \omega_{-1}/c^2 \). Solving the coupled equations (S7) and (S9) together, we achieve the field coefficients \( E_0(z) \) and \( E_{-1}(z) \) as given in Eqs.(17) and (18) for the reception state and in Eqs.(23) and (24) for the transmission state.