Study on Hydromechanics with Particles and Bubbles Based on Finite Element Modeling

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Abstract. The hydromechanics with foams and particles has important scientific and commercial significance. The presence of solid particles will greatly change the behavior of the multiphase/multicomponent flow in the bubble, which is a scientific issue unexplored. So this article simulates the foam layer and platform edge and the hydrodynamic laws of particles along the interface and inside the film were studied. As a result, the explicit analytical expressions for the drag coefficient and torque coefficient of spherical particles attached to a flat stress-free interface between two immiscible fluids are constructed by using the finite element method based on computational fluid dynamics (CFD) under the condition of low Reynolds number. The hydrodynamic data of the translational sphere and the rotating sphere under different motion states are analyzed respectively, and the flow mechanics law of the foam bridging region is analyzed.

Keywords: Finite Element Modelling, Solid Particles, Hydromechanics, Bubbles

Introduction
The discharge of the interstitial phase and the rise of the bubbles will seriously affect the homogeneity of the granular foam [1-3]. The dry foam is formed at the fluid-fluid interface with topological changes, and the polyhedron is intersected by infinitely thin flake materials at the junction known as the platform boundary [4-6]. The dynamics of such bubble clusters are multifaceted and are related to the complex interactions between micro-scale fluid flows within thin slices and platform boundaries, and macro-scale movements of gases within bubbles, as illustrated by a groundbreaking study by Sbragaglia et al. [7-9]. The presence of solid particles, however, will greatly change the behavior of multiphase/multicomponent flow in the foam, and it is the scientific issue unexplored. In this paper, the finite element method based on computational fluid Dynamics (CFD) is used to study the fluid dynamics of particles along the interface and inside the thin film by simulating the boundary of the foam sheet and platform under the condition of low Reynolds number. It will provide the first theoretical prediction for the conditions (particle size, shape wettability and heterogeneity) under which the resulting foam can be stable to drainage. As capillary force is formed among particles to be contacted, the particles are surrounded by a thin liquid film in the direction parallel to the line connecting the particle's center of mass.
1. Finite Element Simulation Modeling

Typically, there are two types of bubbles: bubbles that are small in diameter and dispersed in a fluid. The other is the bubbles with large diameter and segmented by liquid film. In addition, bubbles and emulsion are distinguished from density. Thick foams and emulsions (polyhedral foams and emulsions) refer to bubbles that are polyhedra and the size of bubbles is larger than the thickness of liquid film. The bubble size of thin foam and emulsion is smaller than the thickness of liquid film and takes the shape of sphere. Liquid foam and emulsion have multi-scale characteristics and are characterized with highly organized structure inside, as shown in Figure 1.

![Figure 1. Internal characteristics of bubbles with different particle sizes](image)

Based on the translation and rotation motion of solid particles, it can be expressed as the sum of pure flat motion and pure rotation motion. Explicit analytical expressions of the drag coefficient and torque coefficient of spherical particles attached to a flat stress-free interface between two immiscible fluids are constructed respectively, and the drag coefficient and torque coefficient are obtained from these solutions.

In the translational case, the particle velocity (dimensionless) is equal to \( \hat{\mathbf{z}} \), and \( \hat{\mathbf{z}} \) is the unit vector in the Z direction. The normal component of this velocity is equal to the fluid velocity of the surface (\( \mathbf{v_s} \)), i.e., \( \mathbf{n} \cdot (\hat{\mathbf{z}} - \mathbf{v_s}) = 0 \), where \( \mathbf{n} \) is the local (outward) normal of the particle. \( \mathbf{t} \) is used to represent the unit tangent vector of particle surface, and \( \tau_s = \nabla \mathbf{v} - \nabla \mathbf{v}^\top \) refers to the (non-dimensional) viscosity stress tensor assessed on the surface, the tangential translation condition is formulated for the two orthogonal tangents on the surface, and the translation condition is:

\[
t \cdot (\hat{\mathbf{z}} - \mathbf{v_s}) = \frac{\lambda}{a} (\mathbf{n} \cdot \mathbf{\tau_s} \cdot \mathbf{t})
\]

For pure rotation, the (dimensionless) rotation velocity vector of the surface is \( \hat{\mathbf{x}} \), and \( \hat{\mathbf{x}} \) is the unit vector in the X direction, and the tangential velocity is \( \hat{\mathbf{x}} \times (\mathbf{x_p} - \mathbf{x_o}) \), where \( \mathbf{x_p} \) and \( \mathbf{x_o} \) are position vectors of the particle surface and center (scaled in \( a \)) respectively. The normal component of the surface fluid velocity is zero (\( \mathbf{n} \cdot \mathbf{V_s} = 0 \)), and the tangential sliding condition is

\[
t \cdot (\hat{\mathbf{x}} \times (\mathbf{x_p} - \mathbf{x_o}) - \mathbf{v_s}) = \frac{\lambda}{a} (\mathbf{n} \cdot \mathbf{\tau_s} \cdot \mathbf{t})
\]

Hydrodynamic flows produce traction and torques on the surfaces of translational or rotating particles. For realization of each flow, these forces and torques are given in the following (dimensionless form):

\[
F_p = \oint_{\mathbf{r}_p} (\mathbf{\sigma} \cdot \mathbf{n}) \, ds
\]

\[
\Gamma_p = \oint_{\mathbf{r}_p} (\mathbf{x_p} - \mathbf{x_o}) \times (\mathbf{\sigma} \cdot \mathbf{n}) \, ds.
\]
$\Gamma_p$ refers to the contacting part between colloid surface and the liquid, and $\sigma = -pI + \nabla \cdot \tau \cdot \nabla$ is the total stress tensor. According to the calculated traction and torque, the drag coefficient and torque coefficient can be calculated.

To perform translational motion along the z axis and rotate along the x axis for the solid spherocolloid with radius $a$ in the original point of rectangular coordinate system by using COMSOL Multiphysics numerical software. The size in the computation space of the rectangular box is $250a$ (in $z$) $\times$ $250a$ (in $x$), which $M$ changes between 1.2 to 35, indicating that the different film thickness can implement semi-infinite condition. The typical translational and rotational discretization around the sphere is shown below and compared.

The problem of calculating the drag and torque on a sphere moving through a static fluid is solved, which can be verified by the analysis data in the literature [8]. Maximum boundary grid element size (unit: $\mu m$) = maximum size $B$, the minimum boundary grid element size (unit: $\mu m$) = minimum size $B$, the grid size of maximum contact line (edge) (unit: $\mu m$) = maximum size $C$, minimum contact line (edge) grid element size, the unit of micron $C$ = minimum size $C$.

2. Discussion on Results and Analysis

2.1. Fluid Dynamics of the Translational Sphere

Translational particles of different penetration depth, the drag coefficient and torque coefficient of dimension are determined and the dimensionless drag coefficient increases with the decrease of the three phase tenticles of particles until the particles completely immersed and away from any wall, and asymptotic stokes resistance value (i.e., $6\pi$) is achieved, and the torque is still on the rise. The particle reaches its maximum value when it is half-immersed.

Figure 2 b corresponds to $\theta = \pi/8$, the sphere is endowed with strong hydrophilicity, so it is weakly kept in the interface only by capillary force, and Figure 2 c shows that capillary force increases as $\theta = 7\pi/8$ with strong hydrophobic particles, and particles in the structure is stably affected by the viscous force.

Figure 2. Velocity distribution of fluid around translational particles with different wettability (a: Neutral B: hydrophilic C: hydrophobic)

The resistance and torque of neutral wetting spherical particle is calculated, and the drag and torque is the function of thickness of foam sheet and bubbles on a solid substrate, and they show the opposite trend, which are close to the asymptotic value from either side. As additional shearing action was applied to the solid matrix, there was a sharp velocity gradient near the surface of the particles. See Figures 3 and 4.
Figure 3. a. Velocity distribution around particles in liquid film on solid matrix

b. Velocity distribution around particles in foam of finite thickness

Figure 4. Calculation of foam particle resistance and torque in free medium

2.2. Hydrodynamics of the Rotating Sphere

For the calculation of a rotating sphere, the component of the surface velocity of the particle is perpendicular to the interface around the contact line, which confirms the appearance of non-integrable singularities. In order to eliminate the singularity, the sliding coefficient was set to 100, the torque generated by rotation was the same as the drag generated by translation, and the maximum velocity gradient appeared at the corners of the sphere, as shown in Figure 5.

Figure 5. Velocity distribution of fluid around rotating particles with different wettability

a. Neutral b. hydrophobic c. hydrophilic
Figure 6. a. Functional relationship between drag coefficient and immersion depth  
b: Functional relationship between torque coefficient and immersion depth

As shown in Figure 6, when the immersion depth was 1.75, that is, when the particle was very hydrophilic but not fully immersed, the torque generated by the rotation was maximized, and two particle sizes (1µm and 10µm) were calculated. As the sphere enters the volume, the torque decreases to or exceeds a maximum and reaches an asymptotic value of 8π.

2.3. Hydrodynamics of the Bridging of the Foam Layer
When particles are more immersed in the liquid film until the film thickness is infinite big and the resistance value is close to the stokes resistance, the resistance of the translational particles used to bridge the interface between the two bubbles will increase

Figure 7. a. Translation particle b) Distribution of fluid velocity around the rotating particle

Figure 8. a. Functional relationship between drag coefficient and immersion depth  
b. Functional relationship between torque coefficient and immersion depth
The torque of a translational particle and the resistance are equal to zero. As the bridging particles or particles symmetrically placed in the liquid film will not independently rotate, so when the particle rotates, the torque of rotating sphere increases to the maximum, the particle size is equal to the thickness of lamella, this is because the thin liquid film on both sides of the particles produced the largest imbalance torque.

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