Superconducting cascade electron refrigerator

M. Camarasa-Gómez,1 A. Di Marco,2 F. W. J. Hekking,2 C. B. Winkelmann,3,4 H. Courtois,3,4 and F. Giazotto1

1)NEST, Instituto Nanoscienze-CNR and Scuola Normale Superiore, I-56127 Pisa, Italy
2)LPMMC-CNRS, Université Joseph Fourier, 25 Avenue des Martyrs, 38042 Grenoble, France
3)Univ. Grenoble Alpes, Institut Néel, F-38042 Grenoble, France
4)CNRS, Institut Néel, F-38042 Grenoble, France

(Dated: 21 April 2014)

The design and operation of an electronic cooler based on a combination of superconducting tunnel junctions is described. The cascade extraction of hot-quasiparticles, which stems from the energy gaps of two different superconductors, allows for a normal metal to be cooled down to about 100 mK starting from a bath temperature of 0.5 K. We discuss the practical implementation, potential performance and limitations of such a device.

Electronic heat transport at the mesoscopic scale has been in the spotlight during the last few years.1 In particular, efforts have been made to develop different types of solid-state electronic refrigerators based on tunnel junctions between a normal metal and superconductors.2 Since the first observation of electronic cooling,3 different kinds of devices have been studied such as SINIS and S2IS1IS2, where S1 and S2 are different superconductors, N is a normal metal and I stands for a tunnel barrier. Symmetric structures avoid the use of any other contact than the cooling junctions, while the cooling power, being an even function of the voltage, is doubled. In every case, electronic cooling is obtained by applying a voltage bias related to the gaps of the superconductors S1,2. This allows the extraction of hot quasiparticles from N to S1 or from S1 to S2 in a SINIS or a S2IS1IS2 structure, respectively, so that as a whole cooling occurs in the normal metal or the low-gap superconductor. Such devices are of wide interest for cooling microscopic5 as well as macroscopic objects.6

In the SINIS case, the cooling power is maximum at a temperature around Tc/3, where Tc is the superconducting critical temperature. By exploiting aluminum (Al) as superconducting material with a critical temperature of about 1 K, this optimum occurs at a bath temperature of about 300 mK, and electronic cooling down to below 100 mK of a, for instance copper (Cu), island can be routinely achieved.2 The cooling of a superconductor by quasiparticle tunneling in a S2IS1IS2 has also been demonstrated using aluminum-oxide-titanium junctions.7 Operation over a wider temperature range calls for the use of alternative superconducting materials and/or new architectures. For instance, a SIS’IS nanorefrigerator based on vanadium (V) with a critical temperature of about ~ 4 K was used to efficiently cool down electrons in an Al island from 1 K to 0.4 K.4

In this Letter, we theoretically discuss the feasibility and performance of a multistage superconducting refrigerator, hereafter called cascade cooler. By using suitable materials and device parameters, we show that it is possible to cool down a normal metal with improved performance with respect to more conventional SINIS refrigerators.

We consider an electron cooler based on tunnel junctions arranged in a symmetric configuration, i.e. S2IS1INIS1IS2, as displayed in Fig. 1(a). The structure includes two superconductors S1 and S2 with respective energy gaps Δ1,2 so that Δ1 < Δ2. R1 and R2 denote the normal-state resistances of the individual S1IN and S2IS1 junctions, respectively. The present structure actually consists of a SINIS micro-cooler to which one superconducting tunnel contact has been added at each end. In the following, the cascade cooler S2 electrodes are voltage-biased at a voltage ±V, so that the inner superconducting islands (S1) reach a voltage ±V1. Here, we also assume that inelastic electron-electron interaction drives each individual part of the the system into a quasi-equilibrium regime. Therefore, the electron populations in N and S1 can be respectively described by a Fermi-Dirac energy distribution function at temperatures T_N and T1, which can largely differ.

![Fig. 1.](image-url)
from the bath temperature $T_{\text{bath}}$. The outer superconductor $S_2$ is considered at thermal equilibrium with the phonon bath so that $T_{S_2} = T_{\text{bath}}$.

We first discuss the behavior of each individual junction in the cascade cooler. The charge current $I_{N_1}$ and the heat current $\dot{Q}_{N_1}$ flowing from $N$ to $S_1$ through a NIS$_1$ junction under voltage bias $V_1$ are given by

$$ I_{N_1} = \frac{1}{eR_1} \int_{-\infty}^{\infty} dE n_1(E - eV_1) [f_N(E) - f_i(E - eV_1)], \hspace{1cm} (1) $$

$$ \dot{Q}_{N_1} = \frac{1}{e^2 R_1} \int_{-\infty}^{\infty} dE E n_1(E - eV_1) [f_N(E) - f_i(E - eV_1)]. \hspace{1cm} (2) $$

Here, $f_{1,2,N}$ is the quasiparticle energy distribution function in $S_1$, $S_2$ or $N$, respectively, and $n_{1,2}$ denote the dimensionless BCS density of states of $S_{1,2}$ smeared by the Dynes parameter $\gamma_{1,2}$.

In a NIS$_1$ junction, a non-zero Dynes parameter for $S_1$ induces heating in $N$, so that cooling vanishes at an electron temperature $T_N \approx T_2, 5(\Delta_2)^{1/3}$. In practice, the Dynes parameter ranges from $10^{-2}\Delta_{1,2}$ to $10^{-7}\Delta_{1,2}$. Figure 1(b) shows the voltage bias dependence of the charge and heat currents in a NIS$_1$ junction. At a sub-gap bias, the heat current $\dot{Q}_{N1}$ is positive, meaning heat removal from $N$ into $S_1$. At low temperature $k_BT_N < \Delta_1$, the maximum cooling power is obtained at a voltage $V_1 \approx \Delta_1 - 0.66k_BT_N$. At this optimum value, the corresponding charge current reads

$$ I_{N1,\text{opt}} \approx 0.48 \frac{\sqrt{k_BT_N\Delta_1}}{eR_1}. \hspace{1cm} (3) $$

As each tunneling event removes an energy of about $k_BT$, the related heat current $\dot{Q}_{N1}$ is about $I_{N1,\text{opt}}k_BT/e$. For $eV_1 > \Delta_1$, the N electrode is heated with a power $-\dot{Q}_{N1}$ close to $IV_1/2$. In every case, the superconductor receives a heat $-\dot{Q}_{N1} = IV_1 + \dot{Q}_{N1} > 0$.

In a S$_1$S$_2$ junction biased with a voltage $V_2$, the charge current $I_{12}$ and heat current $\dot{Q}_{12}$ flowing from $S_1$ to $S_2$ are given by

$$ I_{12} = \frac{1}{eR_2} \int_{-\infty}^{\infty} dE n_1(E)n_2(E - eV_2) \times [f_2(E - eV_2) - f_i(E)], \hspace{1cm} (4) $$

$$ \dot{Q}_{12} = \frac{1}{e^2 R_2} \int_{-\infty}^{\infty} dE E n_1(E)n_2(E - eV_2) \times [f_2(E) - f_2(E - eV_2)]. \hspace{1cm} (5) $$

Figure 1(c) shows the voltage bias dependence of the charge and heat current in S$_1$S$_2$ case. We note the sharp maximum of thermal and charge currents occurring at a voltage bias $V_2$ equal to $(\Delta_2 - \Delta_1)/e$. This peak shows up only at non-zero temperatures and corresponds to electrons occupying states above the gap in $S_1$ tunneling to empty states below the gap in $S_2$. Both the charge and the heat current at the peak are strongly affected by the temperature and the Dynes parameter. In particular, we have calculated the charge current to be

$$ I_{12,\text{opt}} = \frac{-\sqrt{\Delta_1\Delta_2}}{eR_2} \exp\left[ -\frac{\Delta_1}{k_BT_1}\right] \ln \left( \sqrt{\gamma_1} + \sqrt{\gamma_2} \right). \hspace{1cm} (6) $$

The thermal balance in $N$ reads

$$ 2\dot{Q}_{N1} + P_{e-ph} = 0, \hspace{1cm} (7) $$

the factor 2 coming from the presence of two symmetric cooling NIS junctions. On the other hand, the thermal balance in each $S_1$ reads

$$ \dot{Q}_{12} + \dot{Q}_{IN} + P^S_{e-ph} = 0, \hspace{1cm} (8) $$

where we have taken into account the heat $-\dot{Q}_{IN} > 0$ deposited by the $S_1$I$N$ junction into the superconductor 1. The behavior of the cascade cooler is governed by the above three non-linear integral equations. It depends strongly on different parameters such as the dimensionless Dynes parameters.
An aluminum island cooled in this way can reach 10 K to the optimum. Here, the parameters are determined by the resistance ratio \( R_1 / R_2 \) of 100, close to the optimum (see below). From Fig. 2, the electronic cooling of the N island (full lines) is more efficient in the cascade system, which performs well up to 0.7 K whereas the SINIS refrigeration (dashed lines) is little efficient. At a bath temperature of 1 K, the SIN stage is inefficient, while the SIS stage operates well. The capability of the cascade refrigeration scheme is illustrated by the large quasiparticle cooling obtained in S1 at every bath temperature below 1 K (dotted lines).

Still in the case of a V-Al-Cu device, Figure 3 displays the minimum achieved electronic temperature in N (\( T_N \)) [panel (a)] and the voltage drops \( V_{1,\text{opt}} \) and \( V_{2,\text{opt}} \) [panel (b)] across the two S1IN and S2IS1 junctions at the minimum temperature \( T_N \) versus the junctions’ resistance ratio \( R_1 / R_2 \). A bath temperature \( T_{bath} \) of 0.5 K and a fixed resistance \( R_1 \) of 500 \( \Omega \) is considered here. At large \( R_1 / R_2 \) value, the S1IN junctions dominate and the optimum cooling is obtained at a voltage drop \( V_1 \) close to the expected value \( (\Delta_1 - k_B T_N) / e \). At small \( R_1 / R_2 \) value, it is the S2IS1 junctions that dominate, and the optimum cooling is obtained at \( V_2 \) close to the expectation \( (\Delta_2 - \Delta_1) / e \). Overall, the best performance is obtained in the region where the two kinds of junctions can operate close to the optimum. Here, the parameters are \( \gamma_{1,2} = 10^{-5} \) and \( \gamma_N = 10^{-2} \) \( \mu \)m\(^2\). We have used the well-accepted material-specific values \( \Sigma_{Al} = 0.2 \times 10^{6} \text{Wm}^{-3}\text{K}^{-5} \) and \( \Sigma_{Cu} = 2 \times 10^{6} \text{Wm}^{-3}\text{K}^{-5} \). In this case, we achieve a good and somewhat constant performance for a resistance ratio between 10 and 200. This order of magnitude is consistent with the factor \( \exp((\Delta_1 / k_B T_1)) \) between the currents \( I_{N1,\text{opt}} \) and \( I_{12,\text{opt}} \) at an identical junction resistance \( R_1 \). The relatively large span of this region stems from the existence of the singularity in the electric current as a function of the bias voltage. This rectifies any imbalance that might occur in the structure, similarly to what happens for an asymmetric pair of NIS junction in series. At higher bath temperature, the window for optimal resistance ratio gets narrower, and is slightly shifted towards lower values.

Let us now discuss practical issues in a cascade cooler’s design. As stated above, the performance of the cascade cooler configuration strongly depends on the value of the ratio \( R_1 / R_2 \). Due to the smaller value of the current \( I_1 \) through a S1IS2 junction compared to the current \( I_{S1} \) through a S1IN junction of comparable normal-state conductance, the resistance \( R_2 \) has to be made significantly smaller than \( R_1 \) in order to get an efficient cascade cooler. Optimal values of the \( R_1 / R_2 \) ratio for bath temperatures and material configurations of experimental interest therefore lie in the range \( 15 - 150 \), while depending strongly on subtle parameters like the Dynes parameters of \( S_{1,2} \). From the fabrication point of view, it might be difficult to tune the \( R_1 / R_2 \) ratio at its optimum with a good degree of precision. This leads to the practical necessity of tuning the voltage \( V_1 \) independently from the main bias voltage \( V \). One possible solution to this problem is to tunnel-couple to each \( S_1 \) electrode an additional superconductor \( S_2 \), as shown in Fig. 1(a). Biasing with a second positive (negative) voltage \( U \) these two tunnel junctions would enable to add (subtract) some current in the S1IS2 junctions compared to the S1IN ones. The S1INIS1 current can then be tuned from zero to the double of its value at zero bias \( U \). The latter limitation comes from the fact that the voltage \( U \) needs to be always sub-gap in order to prevent any extra heating of the \( S_1 \) electrode.

For practical sample fabrication issues, one would preferably use the same tunnel barrier characteristics (in particular...
shows the results for the electron temperature in a suspended metal geometry would improve. In the present design, the outer superconductor S\textsubscript{1} plays this role, with an increased efficiency thanks to its density of states singularity at the gap edge. An improved cooling, because it also reduces the available heat current in the superconducting electrodes. The optimum resistance ratios were adjusted so that $\Delta_{N_1}/\Delta_{N_2} \approx 0.5$ K as a function of the dimensionless bias voltage $eV/\Delta_{Al}$, in different cases: Al-Cu one-stage cooler (orange), V-Al-Cu (dotted red) with identical volumes for N and S\textsubscript{1}, V-Al-Cu (blue) and Nb-Al-Cu (green) with volumes adapted to the resistances’ ratio so that $V_1/V_N = R_1/R_2$. The ratio $R_1/R_2$ is set at the optimal value in every case: 100 (V-Al-Cu), 30 (Nb-Al-Cu), 80 (V-Al-Cu, adapted volumes’ ratio) respectively. We take $\Delta_{N_1} = 1407 \mu eV$, $R_1 = 1 \Omega$, and the other parameters are identical to Fig. 2, including $V_N = 10^{-2}\mu m^2$.

Another crucial issue for the present cascade electronic cooler resides in a proper quasiparticle thermalization in the intermediate superconductor S\textsubscript{1}. It is well known that superconducting-based electronic refrigerators generally suffer from poor evacuation of highly-energetic quasiparticles in the superconducting electrodes. To this end, quasiparticle traps of various kinds have been envisaged in order to allow their evacuation into nearby-connected normal metal layers. In the present design, the outer superconductor S\textsubscript{2} actually plays this role, with an increased efficiency thanks to its density of states singularity at the gap edge. An incomplete quasiparticle energy relaxation in the superconductor S\textsubscript{1} should actually not hinder the cooling in the low-gap superconductor S\textsubscript{1} compared to the present quasi-equilibrium calculations. The cascade cooler appears as rather immune against poor electronic equilibration in S\textsubscript{1}. Finally, the outer superconducting electrodes S\textsubscript{2} can be efficiently thermalized through quasiparticles traps, just as it is done in the case of conventional superconducting refrigerators.

In conclusion, we have discussed a novel kind of electronic cooler based on hybrid superconducting tunnel junctions. A cascade geometry allows to cool a first superconducting stage, which is used as a local thermal bath in a second stage. The correct operation of the device strongly depends on the matching between the resistances of the two kinds of tunnel junctions. The resulting constraint can be easily implemented in a practical device, using of a set of two additional tunnel junctions. Decoupling of local phonon population from the thermal bath would improve performances compared to the situation considered here.

We acknowledge the Marie Curie Initial Training Action Q-NET no. 264034 and the EU Capacities MICROKELVIN project no. 228464 for partial financial support. We thank M. A. Pascal for her contribution at the very beginning of this project, A. Ronzani for help in numerics, and J. P. Pekola for fruitful discussions.

ACKNOWLEDGMENTS

1F. Giazotto, T. T. Heikkilä, A. Luukanen, A. M. Savin, and J. P. Pekola, Rev. Mod. Phys. 78, 217 (2006).
2J. T. Muhonen, M. Meschke, and J. P. Pekola, Rep. Prog. Phys. 75, 046501 (2012).
3M. Nahum, T. M. Eiles and J. M. Martinis, Appl. Phys. Lett. 65, 3123 (1994).
4O. Quanta, P. Spathis, F. Beltram, and F. Giazotto, Appl. Phys. Lett. 98, 032501 (2011).
5K. J. Claram, N. A. Miller, A. Williams, S. T. Ruggiero, G. C. Hilton, L. R. Vale, J. A. Beall, K. D. Irwin, and J. N. Ullom, Appl. Phys. Lett. 86, 173508 (2005).
6P. J. Lowell, G. C. O’Neil, J. M. Underwood, and J. N. Ullom, Appl. Phys. Lett. 102, 082601 (2013).
7A. J. Manninen, J. K. Suoknuuti, M. M. Leivo, and J. P. Pekola, Appl. Phys. Lett. 74, 3020 (1999).
8R. C. Dynes, J. P. Garno, G. B. Hertel, T. P. Orlando, Phys. Rev. Lett. 53, 2437 (1984).
9B. Frank and W. Krech, Phys. Lett. A 235, 281 (1997).
10F. Giazotto and J. P. Pekola, J. Appl. Phys. 97, 023908 (2005).
11J. P. Pekola, V. F. Maisi, S. Kafanov, N. Chekurov, A. Kemppinen, Yu. A. Pashkin, O.-P. Saira, M. Mottönen, and J. S. Tsai, Phys. Rev. Lett. 105, 026803 (2010).
12F. C. Wellstood, C. Urbina, and J. Clarke, Phys. Rev. B 49, 5942 (1994).
13A. V. Timofeev, C. Pascual Garcia, N. B. Kopnin, A. M. Savin, M. Meschke, F. Giazotto, and J. P. Pekola, Phys. Rev. Lett. 102, 017003 (2009).
14L. M. A. Pascal, A. Fay, C. B. Winkelmann, and H. Courtois, Phys. Rev. B 88, 100502 (2013).
15H. Q. Nguyen, L. M. A. Pascal, Z. H. Peng, O. Buissen, B. Gilles, C. B. Winkelmann, and H. Courtois, Appl. Phys. Lett. 100, 252602 (2012).
16J. P. Pekola, A. J. Manninen, M. M. Leivo, K. Arutyunov, J. K. Suoknuuti, T. I. Suppula, B. Collaudin, Physica B 290, 485 (2000).
17S. Rajauria, H. Courtois and B. Pannetier, Phys. Rev. B 80 214521 (2009), and references therein.
18G. C. O’Neil, P. J. Lowell, J. M. Underwood, and J. N. Ullom, Phys. Rev. B 85, 134504 (2012).
19H. Q. Nguyen, T. Aref, V. J. Kauppila, M. Meschke, C. B. Winkelmann, H. Courtois, and J. P. Pekola, New J. of Phys. 15, 085013 (2013).