Effect of vertex corrections in spin pumping into a two-dimensional electron gas

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We theoretically consider effect of the vertex correction in spin pumping from a ferromagnetic insulator (FI) into a two-dimensional electron gas (2DEG) in which the Rashba and Dresselhaus spin-orbit interactions coexist. We calculate the enhancement of Gilbert damping and the shift of the resonant frequency by solving the vertex functions. Although the vertex correction usually affects the results moderately, elastic spin flipping is strongly suppressed when the two spin-orbit interactions competes with each other.

I. INTRODUCTION

In the field of spintronics$^{[12]}$, spin pumping has been used as one of the powerful methods for spin injection into various materials for a long time since its discovery$^{[13]}$. Spin pumping has first been used for spin injection from a ferromagnetic metal into an adjacent normal metal (NM)$^{[14]}$, and subsequently, has also been realized in ferromagnetic insulator (FI)/NM junctions$^{[15]}$. Because spin injection is generally related to the loss of the magnetization in ferromagnets, it affects the Gilbert damping measured in ferromagnetic resonance (FMR) experiment$^{[16]}$. When we employ spin injection from the FI, the modulation of the Gilbert damping reflects properties on spin excitation in adjacent materials such as magnetic thin film$^{[17]}$, magnetic impurities on metal surface$^{[18]}$, and superconductor$^{[19]}$.

Recently, spin injection from ferromagnetic metals into two-dimensional electron gas (2DEG) or its inverse phenomenon has been attracting interest as a new direction of spintronics devices. Spin injection has been realized through the Rashba–Edelstein effect or its inverse effect in various materials such as surface Rashba systems$^{[20]}$, atomic layer materials$^{[21]}$, and transition oxides$^{[22]}$. In these experiments, the existence of the spin-orbit interactions in two-dimensional materials is essential. The Rashba spin-orbit interaction$^{[23]}$ causes an effective Zeeman field on conduction electrons whose direction varies on the Fermi surfaces, while anisotropic Fermi surfaces and additional spin-orbit interactions lead to more complex spin-texture near the Fermi surfaces. Two-dimensional systems with such a spin texture are expected to exhibit characteristic spin excitation, which can be detected in the modulation of the Gilbert damping in the FI/2DEG junctions.

In our previous work$^{[24]}$, we theoretically studied spin pumping into a 2DEG in semiconductor heterostructures with both Rashba and Dresselhaus spin-orbit interactions, which can be regarded as a prototype for 2DEG with a complex spin-texture near the Fermi surface (see Fig. 1(a)). In this study, we formulated the modulation of the Gilbert damping in the FI using the second-order perturbation with respect to interfacial coupling$^{[25,26]}$ and related it to the dynamic spin susceptibility of the 2DEG. We further calculated the spin susceptibility without taking the vertex correction into account and obtained characteristic features due to elastic spin flipping and magnon absorption. However, the vertex corrections frequently play an important role in theoretical description of spin transport$^{[27,28]}$.

In this study, we consider the same setting, i.e., a junction composed of an FI and a 2DEG as shown in Fig. 1(a) and discuss effect of the vertex correction. We theore-
icantly calculate the modulation of the Gilbert damping and the shift of the FMR frequency by solving the vertex function within the ladder approximation. Our study also provides a concrete formulation for spin pumping into general 2DEG with a complex spin-texture induced by spin-orbit interactions.

The rest of this work is organized as follows. In Sec. II, we briefly summarized our model for a FI/2DEG junction and describe a general formulation for the magnon self-energy following Ref. 26. In Sec. III, we formulate the vertex correction that corresponds to the self-energy in the Born approximation. We show the modulation of the vertex correction that corresponds to the self-energy in the Born approximation. Finally, we summarize our model for a FI/2DEG junction shown in Fig. 1 (a) and formulate the spin relaxation rate in a detail. Finally, we summarize our results in Sec. IV. In the following four appendices, we give a detailed calculation in Sec. III.

II. FORMULATION

In this section, we introduce a model for the FI/2DEG junction shown in Fig. I (a) and formulate the spin relaxation rate in a FMR experiment. Because we have already described details on the model and the general formulation in our previous paper 26, we summarize them briefly. For a detailed description, see Ref. 26.

A. Two-dimensional electron gas

We consider a 2DEG by the Hamiltonian

\[ H_{\text{kin}} = \sum_k \left( c_{k \uparrow}^\dagger c_{k \downarrow} + c_{k \downarrow}^\dagger c_{k \uparrow} \right) \hat{h}_k \left( c_{k \uparrow}^\dagger c_{k \downarrow}^\dagger \right), \]

where \( c_{k \sigma} \) is the annihilation operator of conduction electrons with wavenumber \( k = (k_x, k_y) \) and \( z \)-component of a spin, \( \sigma \) (\( \uparrow, \downarrow \)), \( \hat{I} \) is an identity matrix, and \( \sigma_a \) (\( a = x, y, z \)) are the Pauli matrices, and \( \xi_k = \hbar^2 k^2 / 2m - \mu \) is a kinetic energy measured from the chemical potential. The two kinds of the spin-orbit interactions are described in the form of the effective Zeeman field \( h_{\text{eff}} \) defined as

\[ h_{\text{eff}} = k_F (-\alpha \sin \varphi - \beta \cos \varphi, \alpha \cos \varphi + \beta \sin \varphi, 0), \]

where \( \alpha \) and \( \beta \) respectively denote the amplitudes of the Rashba- and Dresselhaus-type spin-orbit interactions and \( \varphi \) is the azimuth angle of the conduction electrons. We assume that \( k_F \alpha \) and \( k_F \beta \) are much smaller than the Fermi energy.

We also consider the impurity Hamiltonian

\[ H_{\text{imp}} = u \sum_{i \in \text{imp}} \sum_\sigma | \Psi_{\sigma}^\dagger (r_i) \Psi_{\sigma} (r_i) |, \]

where \( \Psi_{\sigma} (r) = A^{-1/2} \sum_k c_{k \sigma} e^{i k \cdot r} \), \( A \) is the junction area, \( u \) is the strength of the impurity potential, and \( r_i \) is the position of the impurity site.

A finite-temperature Green's function for the conduction electrons is defined as a 2 \( \times \) 2 matrix \( \hat{g}(k, \omega_n) \), whose elements are

\[ g_{\sigma \sigma'}(k, \omega_n) = \int_0^{\hbar \beta} d\tau \, e^{i \omega_n \tau} g_{\sigma \sigma'}(k, \tau), \]

where \( c_{\sigma \tau}(\tau) = e^{H_{\text{NM}} \tau / \hbar} c_{\sigma \tau} e^{-H_{\text{NM}} \tau / \hbar} \), \( H_{\text{NM}} = H_{\text{kin}} + H_{\text{imp}}, \omega_n = \pi (2n + 1) / \hbar \beta \) is the fermionic Matsubara frequency, and \( \beta \) is the inverse temperature. By employing the Born approximation, the finite-temperature Green's function is calculated as

\[ \hat{g}(k, \omega_n) = \left( i \hbar \omega_n - \xi_k + i \Gamma \text{sgn}(\omega_n) / 2 \right) \hat{I} - h_{\text{eff}} \cdot \sigma, \]

where \( E_{\pm k} = \xi_k \pm |h_{\text{eff}}(\varphi)| / 2 \) is the spin-dependent electron dispersion, \( \Gamma = 2 \pi n \hbar^2 D(\epsilon_F) \) is level broadening, and \( D(\epsilon_F) \) is the density of states per spin per unit volume.

B. Ferromagnetic insulator

We consider the quantum Heisenberg model for the FI. We employ the spin-wave approximation assuming that the temperature is much lower than the magnetic transition temperature and the magnitude of the localized spins, \( S_0 \), is large enough. The resultant Hamiltonian is given as

\[ H_{\text{FI}} = \sum_k \hbar \omega_k b_k^\dagger b_k, \]

where \( b_k \) is a magnon annihilation operator with wavenumber \( k \), \( \hbar \omega_k = D k^2 + \hbar \gamma h_{\text{dc}} \) is energy of a magnon, \( D \) is spin stiffness and \( h_{\text{dc}} \) is DC magnetic field applied externally. We note that the external DC magnetic field controls the direction of the ordered spins. When we introduce a new coordinate \( (x', y', z') \) fixed on the ordered spins by rotating the original coordinate \( (x, y, z) \) as shown in Fig. I (b), the magnon annihilation operator is related with the spin ladder as \( S_k^z = (2S_0)^{1/2} b_k \). The spin correlation function is defined as

\[ G(k, \omega_n) = \int_0^{\hbar \beta} d\tau \, e^{i \omega_n \tau} G(k, \tau), \]

where \( \omega_n = 2n \pi / \beta \) is the bosonic Matsubara frequency. Using the relation \( S_k^z = (2S_0)^{1/2} b_k \), the spin correlation function is calculated for the Hamiltonian as

\[ G(k, \omega_n) = \frac{2 S_0 / \hbar}{i \omega_n - \omega_k + i \alpha g |\omega_n|}, \]
where $\alpha_G$ is a phenomenological dimensionless parameter that describes the strength of the Gilbert damping in the bulk FI.

C. Effect of the FI/2DEG interface

The Hamiltonian for the coupling between the FI and 2DEG is given as

$$H_{\text{int}} = \sum_k (T_k S_k^+ s_k^- + T_k s_k^+ S_k^-),$$

(12)

where $T_k$ is an exchange interaction at the interface and $s_k^\pm$ are the spin ladder operators for conduction electrons and are obtained by the coordinate rotation as

$$s_k^\mp = \frac{1}{2} \sum_{\sigma, \sigma'} k' \sigma (\hat{\sigma}^\mp)_{\sigma \sigma'} C_{k' \sigma} x ko',$$

(13)

$$\hat{\sigma}^\mp = -\sin \theta \sigma_x + \cos \theta \sigma_y \pm i \sigma_z.$$  

(14)

Here, we have assumed a clean interface, for which the momentum of spin excitation is conserved.

We consider a second-order perturbation with respect to the interfacial exchange interaction $H_{\text{int}}$. Then, the spin correlation function is calculated as

$$G(k, i\omega_n) = \frac{1}{(G_0(k, i\omega_n))^{-1} - \Sigma(k, i\omega_n)},$$

(15)

$$\Sigma(k, i\omega_n) = -|T_k|^2 A \chi(k, i\omega_n),$$

(16)

where $\Sigma(k, i\omega_n)$ is the self-energy due to the interfacial exchange coupling and $\chi(k, i\omega_n)$ is the spin susceptibility for conduction electrons per area defined as

$$\chi(k, i\omega_n) = \int_0^{\pi} d\theta e^{i\omega_n \theta} \chi(k, \theta),$$

(17)

$$\chi(k, \theta) = -\frac{1}{\hbar A} (s_k^+(\theta) s_k^-(0)), $$

(18)

where $s_k^\mp(\theta) = e^{iH_{\text{NM}}/\hbar} s_k^\mp e^{-iH_{\text{NM}}/\hbar}$. Using the analytic continuation, the uniform component of the retarded spin correlation function is given as

$$G_R(0, \omega) = \frac{2S_0/\hbar}{\omega - (\omega_0 + \delta\omega_0) + i(\alpha_G + \delta\alpha_G) \omega},$$

(19)

$$\frac{\delta\omega_0}{\omega_0} \approx -\frac{2S_0|T_0|^2 A}{\hbar \omega_0} \text{Re} \chi_R(0, \omega_0),$$

(20)

$$\delta\alpha_G \approx \frac{2S_0|T_0|^2 A}{\hbar \omega_0} \text{Im} \chi_R(0, \omega_0),$$

(21)

where the superscript $R$ indicates retarded correlation function, $\omega_0 = \omega_{0G} - \omega$, $\delta\omega_0$ is a shift of the FMR frequency, and $\delta\alpha_G$ indicates the increase of the Gilbert damping. In Eqs. (20) and (21), we made approximation by replacing $\omega$ with the FMR frequency $\omega_0$ assuming that the FMR peak is sufficiently sharp ($\alpha + \delta\alpha \ll 1$). Thus both the FMR frequency shift and the modulation of the

$$\chi(0, i\omega_n) = \hat{g}(k, i\omega_n) + \hat{g}(k, i\omega_n) \hat{\Gamma}(k, i\omega_n),$$

(22)

FIG. 2. (a) Relationship between magnetic susceptibility and vertex corrections. (b) Feynman diagram of Bethe-Salpeter equation.

Gilbert damping are determined from the spin susceptibility of the conduction electrons, $\chi(0, \omega)$. In this work, we consider the vertex correction for the spin susceptibility, which has not been treated in our previous work.

III. VERTEX CORRECTIONS

We evaluate the spin susceptibility in the ladder approximation that obeys the Ward-Takahashi relation with the self-energy in the Born approximation. The Feynman diagrams for the corresponding spin susceptibility and the vertex function are shown in Fig. 2(a) and (b), respectively. Then, the spin susceptibility is written as

$$\chi(0, i\omega_n) = \frac{1}{4\beta A} \sum_{k, i\omega_m} \text{Tr} \left[ \hat{g}(k, i\omega_m) \hat{\Gamma}(k, i\omega_m, i\omega_n) \right].$$

(22)

The vertex function $\hat{\Gamma}(k, i\omega_m, i\omega_n)$ is a $2 \times 2$ matrix, whose components are determined by the Bethe-Salpeter equation shown in Fig. 2(b) as

$$\Gamma_{\sigma\sigma'}(k, i\omega_m, i\omega_n) = (\hat{\sigma}^\mp)^{\sigma\sigma'} \frac{u^2 N_0}{A} \sum_{q, \sigma_1, \sigma_2} g_{\sigma\sigma_1}(q, i\omega_m) g_{\sigma_2\sigma}(q, i\omega_m + i\omega_n),$$

(23)

Since the right-hand side of this equation is independent of $k$, the vertex function can simply be described as $\hat{\Gamma}(i\omega_m, i\omega_n)$. Using the Pauli matrices, the Bethe-Salpeter equation is rewritten in a simplified form as

$$\hat{\Gamma}(i\omega_m, i\omega_n) \equiv \hat{\sigma}^\mp + E\hat{I} + X\hat{\sigma}_x + Y\hat{\sigma}_y + Z\hat{\sigma}_z \equiv EI + X\hat{\sigma}_x + Y\hat{\sigma}_y + Z\hat{\sigma}_z.$$  

(24)
Green’s function for the conduction electrons can also be rewritten as
\[ \hat{g}(\mathbf{q}, i\omega_n) = \frac{\hat{A} + B\hat{\sigma}_x' + C\hat{\sigma}_y'}{D}, \]
\[ A(i\omega_n) = i\hbar\omega_n - \xi + \frac{i\Gamma}{2}\text{sgn}(\omega_m), \]
\[ B = -\hbar\text{eff} \cos(\varphi - \theta), \]
\[ C = -\hbar\text{eff} \sin(\varphi - \theta), \]
\[ D(i\omega_n) = \prod_{n=\pm}(i\hbar\omega_n - E_q^\nu + \frac{i\Gamma}{2}\text{sgn}(\omega_m)). \]

By substituting Eqs. (24) and (25) into Eq. (23), we obtain
\[ \left( \begin{array}{c} E' \\ X' \\ Y' \\ Z' \end{array} \right) = \left( \begin{array}{cccc} \Lambda_0 + \Lambda_1 & 0 & 0 & 0 \\ 0 & \Lambda_0 + \Lambda_2 & \Lambda_3 & 0 \\ 0 & \Lambda_3 & \Lambda_0 - \Lambda_2 & 0 \\ 0 & 0 & 0 & \Lambda_0 - \Lambda_1 \end{array} \right) \left( \begin{array}{c} E \\ X \\ Y \\ Z \end{array} \right). \]

where \( \Lambda_j(i\omega_n, i\omega_n) \) \( (j = 0, 1, 2, 3) \) is given as
\[ \Lambda_0(i\omega_n, i\omega_n) = \frac{u^2N_i}{A} \sum_q \frac{AA'}{DD'}, \]
\[ \Lambda_1(i\omega_n, i\omega_n) = \frac{u^2N_i}{A} \sum_q \frac{h^2_{\text{eff}}}{DD'}, \]
\[ \Lambda_2(i\omega_n, i\omega_n) = \frac{u^2N_i}{A} \sum_q \frac{h^2_{\text{eff}} \cos 2(\varphi - \theta)}{DD'}, \]
\[ \Lambda_3(i\omega_n, i\omega_n) = \frac{u^2N_i}{A} \sum_q \frac{h^2_{\text{eff}} \sin 2(\varphi - \theta)}{DD'}, \]

using the abbreviated symbols, \( A = A(i\omega_n), A' = A(i\omega_m + i\omega_n), D = D(i\omega_m), \) and \( D' = D(i\omega_m + i\omega_n). \) Here, we have used the fact that the contributions of the first-order terms of \( B \) and \( C \) becomes zero after replacing the sum with the integral with respect to \( k \) and performing the angular integration.

Using \( \hat{\sigma}^{x'} = \hat{\sigma}^y + i\hat{\sigma}^z \), we can solve Eqs. (24) and (30) as
\[ E = 0, \]
\[ X = \frac{\Lambda_3}{(1 - \Lambda_0)^2 - \Lambda_2^2 - \Lambda_3^2}, \]
\[ Y = \frac{1 - \Lambda_0 - \Lambda_2}{(1 - \Lambda_0)^2 - \Lambda_2^2 - \Lambda_3^2}, \]
\[ Z = \frac{1}{1 - \Lambda_0 + \Lambda_1}. \]

By replacing the sum with an integral as
\[ \frac{1}{\Lambda} \sum_k (\cdots) \simeq D(\varepsilon_F) \int_{-\infty}^{\infty} d\xi \int_0^{2\pi} \frac{d\varphi}{2\pi} (\cdots), \]
\[ \Lambda_j(i\omega_m, i\omega_n) = \theta(-\omega_m)\theta(\omega_m + \omega_n)\tilde{\Lambda}_j(i\omega_n), \]
\[ \tilde{\Lambda}_j(i\omega_n) = \frac{i\Gamma}{4} \int_0^{2\pi} \frac{d\varphi}{2\pi} \times \sum_{\nu'\nu} \frac{f_j(\nu, \nu', \varphi)}{i\hbar(\omega_n + (\nu - \nu')|h_{\text{eff}}(\varphi)| + i\Gamma),} \]
\[ \frac{2BCX}{2D^2} \]
\[ \chi(0, i\omega_n) = \frac{1}{4\beta A_k} \sum_{k, i\omega_m} \left[ \frac{2BCX}{2D^2} \right] \]
\[ + (AA' - B^2 + C^2)Y - i(AA' - B^2 - C^2)Z. \]

After the analytical continuation by \( i\omega_n \to \omega + i\delta \), the retarded spin susceptibility is obtained as
\[ \chi^R(0, \omega) = \frac{D(\varepsilon_F)\hbar\omega}{2i\Gamma} \left[ \frac{\tilde{\Lambda}_0^R(1 - \tilde{\Lambda}_0^R) - \tilde{\Lambda}_2^R(1 - \tilde{\Lambda}_2^R) + (\tilde{\Lambda}_4^R)^2}{(1 - \tilde{\Lambda}_0^R)^2 - (\tilde{\Lambda}_2^R)^2 - (\tilde{\Lambda}_4^R)^2} \right] \]
\[ + \frac{\tilde{\Lambda}_0^R - \tilde{\Lambda}_2^R}{1 - \tilde{\Lambda}_0^R + \tilde{\Lambda}_2^R} - D(\varepsilon_F), \]
\[ \Lambda_j^R = \Lambda_j(\omega) = \Lambda_j(i\omega_n \to \omega + i\delta) \]
\[ = \mathcal{I} \int \frac{d\varphi}{2\pi} \times \sum_{\nu'\nu} \frac{f_j(\varphi, \nu, \nu')}{\hbar(\omega_n + (\nu - \nu')|h_{\text{eff}}/\Delta_0 + i\Gamma/\Delta_0).} \]

Here, we have introduced a unit of energy, \( \Delta_0 = k_F\beta \), for convenience in making physical quantities dimensionless. Using Eqs. (20) and (21), we finally obtain the shift of the FMR frequency and the modulation of the Gilbert damping as
\[ \frac{\omega_0}{\omega_0} = -\alpha_{G,0} \text{Re} F(\omega_0), \]
\[ \delta\alpha_G = \alpha_{G,0} \text{Im} F(\omega_0), \]
\[ F(\omega) = \frac{\Delta_0}{2\pi i\Gamma} \left[ \frac{\tilde{\Lambda}_0^R(1 - \tilde{\Lambda}_0^R) - \tilde{\Lambda}_2^R(1 - \tilde{\Lambda}_2^R) + (\tilde{\Lambda}_4^R)^2}{(1 - \tilde{\Lambda}_0^R)^2 - (\tilde{\Lambda}_2^R)^2 - (\tilde{\Lambda}_4^R)^2} \right] \]
\[ + \frac{\tilde{\Lambda}_0^R - \tilde{\Lambda}_2^R}{1 - \tilde{\Lambda}_0^R + \tilde{\Lambda}_2^R} - \frac{\Delta_0}{\pi\hbar\omega_0}. \]
where $\alpha_{G,0} = 2\pi S_0 |T_0|^2 AD(\epsilon_F)/\Delta_0$ is a dimensionless constant that describes the coupling strength at the interface. This is the main result in this work.

The spin susceptibility without the vertex correction can be obtained by taking the first-order term with respect to $\Lambda$ as

$$\chi(0, \omega_n) \approx \frac{\hbar \omega D(\epsilon_F)}{2\Gamma} [\tilde{2} \tilde{\Lambda}^R - \tilde{\Lambda}_1^R - \tilde{\Lambda}_2^R] - D(\epsilon_F)$$

$$= \hbar \omega D(\epsilon_F) \int \frac{d\varphi}{2\pi} \left[ \frac{1}{\hbar \omega + i\Gamma 1 - \cos^2(\varphi - \theta)} \right]$$

$$+ \frac{1}{h \omega - 2h_{\text{eff}} + i\Gamma} \frac{1 + \cos^2(\varphi - \theta)}{4} - D(\epsilon_F).$$

The imaginary part of $\chi(0, \omega_n)$ reproduces the result of Ref. [26]. Using this expression, the shift of the FMR frequency and the modulation of the Gilbert damping considering no vertex correction is obtained as

$$\frac{\delta \omega_{NV}}{\omega_0} = -\alpha_{G,0} \text{Re} F_{\text{nv}}(\omega_0),$$

$$\delta \alpha_{G}^{\text{nv}} = \alpha_{G,0} \text{Im} F_{\text{nv}}(\omega_0),$$

$$F_{\text{nv}}(\omega) = \frac{\Delta_0}{2\pi \Gamma} \left[ \tilde{2} \tilde{\Lambda}^R \tilde{\Lambda}_1^R - \tilde{\Lambda}_2^R \right] - \frac{\Delta_0}{\pi h \omega_0}.$$ 

IV. MODULATION OF THE GILBERT DAMPING

A. Case of $\alpha/\beta = 0$

Let us first discuss the case of $\alpha/\beta = 0$, i.e., the case when only the Dresselhaus spin-orbit interaction exists. Fig. 3 (a) shows the effective Zeeman field $h_{\text{eff}}$ along the Fermi surface. Fig. 3 (b) and (c) shows the modulations of the Gilbert damping without and with the vertex correction, respectively. The horizontal axes in Fig. 3 (b) and (c) denotes the resonant frequency $\omega_0 = h \gamma h_{\text{dip}}$ in the FMR experiment. We note that the modulation of the Gilbert damping, $\delta \alpha_{G}$, does not depend on the orientation of the ordered spins in the FI, $\theta$. The four curves in Fig. 3 (b) and (c) correspond to $\Gamma/\Delta_0 = 0.1, 0.2, 0.5, 1.0$, respectively. The qualitative feature is common for the results without and with the vertex correction. The modulation of the Gilbert damping has two peaks at $\omega_0 = 0$ and $\omega_0 = 2\Delta_0$. The former corresponds to elastic spin-flip of conduction electrons induced by the transverse magnetic field via the exchange bias of the FI, while the latter is caused by magnon absorption accompanying spin excitation of the conduction electrons as discussed in Ref. [26]. We note that the peak position of the latter is given by the spin-splitting energy $2h_{\text{eff}}$, which is constant along the Fermi surface in the case of $\alpha/\beta = 0$. As $\Gamma$ increases, the widths of the two peaks in $\delta \alpha_{G}$ becomes larger for both the cases without and with the vertex correction.

B. Case of $\alpha/\beta = 1$

In the case of $\alpha/\beta = 1$, the effective Zeeman field $h_{\text{eff}}$ points in the direction of $(-1, 1)$ or $(1, -1)$ as shown in Fig. 3 (d). The amplitude of $h_{\text{eff}}$ depends on the orientation of wavenumber of the conduction electrons, $\varphi$, as

$$h_{\text{eff}}(\varphi) = k_F \beta \sqrt{2} |\cos \varphi + \sin \varphi|,$$  

and varies in the range of $0 \leq h_{\text{eff}} \leq 4\Delta_0$. We show the modulation of the Gilbert damping without and with the vertex correction in Fig. 3 (e) and (f), respectively. The five curves correspond to $\theta = -\pi/4, -\pi/8, 0, \pi/8, \pi/4$, respectively. Both $\delta \alpha_{G}$ and $\delta \alpha_{G}^{\text{nv}}$ show broad enhancement in the range of $0 \leq h_{\text{eff}} \leq 4\Delta_0$, which is caused by magnon absorption when the spin-splitting energy coincides with the resonance frequency ($h \omega_0 = 2h_{\text{eff}}$). This broad structure depends on the orientation of the magnetization of the FI, $\theta$ and takes a maximum (a minimum) when $\theta = -\pi/4 (\theta = \pi/4)$. Here, the most remarkable feature in Fig. 3 (e) and (f) is that the peak of $\delta \alpha_{G}$ at $\omega_0 = 0$ disappears when the vertex correction is taken into account.

C. Case of $\alpha/\beta = 3$

In the case of $\alpha/\beta = 3$, the orientation of the effective Zeeman field $h_{\text{eff}}$ varies along the Fermi surface as shown in Fig. 3 (g). We show the modulation of the Gilbert damping without and with the vertex correction for $\Gamma/\Delta = 0.5$ in Fig. 3 (e) and (f), respectively. In this case, the peak at $\omega_0 = 0$ appears again even for the result with the vertex correction, $\delta \alpha_{G}$. The broad structure in the range of $4\Delta_0 \leq h \omega_0 \leq 8\Delta_0$ corresponds to the magnon absorption process. The range of this structure is determined by the distribution of the amplitude of $h_{\text{eff}}$ along the Fermi surface. The peak structure at $\omega_0 = 0$ becomes clearly sharper when considering the vertex correction while the broad structure is slightly enhanced. Therefore, we conclude for this case that effect of the vertex correction is restricted to the moderate change of the peak widths.

V. SHIFT OF THE RESONANT FREQUENCY

Next, we discuss the shift of the resonant frequency. Before showing the result for it, we summarize the modulation of the Gilbert damping as contour plots in Fig. 3 (a), (b), and (c) for $\alpha/\beta = 0, 1,$ and 3, respectively. The feature of these plots is the same as Fig. 3 and has been discussed in the previous section. For $\alpha/\beta = 0$, the modulation of the Gilbert damping, $\delta \alpha_{G}$, does not depend on the angle of the magnetization, $\theta$ and has two peaks at $\omega_0 = 0, 2\Delta_0$. For $\alpha/\beta = 1$, the peak at $\omega_0 = 0$ disappears and the broad structure in the range...
FIG. 3. (Left panels) Spin textures at the Fermi surface. (Middle panels) $\delta\alpha_G$ with no vertex correction is shown as a function of the resonance frequency $\omega_0$. (Right panels) $\delta\alpha_G$ with the vertex correction. The spin-orbit interactions are taken as follows. (a), (b), (c): $\alpha/\beta = 0$. (d), (e), (f): $\alpha/\beta = 1$. (g), (h), (i): $\alpha/\beta = 3$. In the plots of (d)-(i), we have chosen $\Gamma/\Delta_0 = 0.5$. of $0 \leq \hbar\omega_0 \leq 4\Delta_0$ depends on the angle of the magnetization. For $\alpha/\beta = 3$, the peak at $\omega_0 = 0$ appears again and the broad structure in the range of $4\Delta_0 \leq \hbar\omega_0 \leq \Delta_0$ depends on the angle of the magnetization. For $\alpha/\beta > 0$, the peak at $\omega = 0$ has a maximum when $\theta = \pi/4, 5\pi/4$ if it exists while the broad structure at finite frequencies has a maximum when $\theta = 3\pi/4, 7\pi/4$.

We show the shift of the resonant frequency $\delta\omega_0/\omega_0$ with the vertex correction as contour plots in Fig. 3(d), (e), and (f) for $\alpha/\beta = 0, 1,$ and $3$, respectively. In this plot, no structure appears around $\omega_0$ while a characteristic sign change is observed at $\hbar\omega_0 = \max 2h_{\text{eff},\text{max}}$ where $h_{\text{eff},\text{max}}$ is a maximum value of $h_{\text{eff}}$ on the Fermi surface. For $\alpha/\beta > 0$, the structure at finite frequencies depend on the angle of the magnetization and $|\delta\omega_0/\omega_0|$ has a maximum at $\theta = 3\pi/4, 7\pi/4$ as for the modulation of the Gilbert damping. This feature comes from the magnon absorption process. Thus, the shift of the resonant frequency has the information on the finite-frequency structure. We note that $\delta\omega_0/\omega_0$ and $\delta\alpha_G$ are related to each other through the Kramers-Kronig conversion because they are determined by the real and imaginary parts of the retarded spin susceptibility for the conduction electrons.

VI. SUMMARY

We theoretically investigated spin pumping into a two-dimensional electron gas (2DEG) with a spin-textured effective Zeeman field caused by the Rashba- and Dresselhaus-type spin-orbit interactions. We expressed a change of the peak position and the linewidth in a ferromagnetic resonance (FMR) experiment that is induced by the 2DEG within a second-order perturbation with respect to the interfacial exchange coupling by taking the vertex correction into account. In our previous work,$^{26}$ we calculated the modulation of the linewidth without
considering the vertex correction and reported there are two kinds of processes that modulate the Gilbert damping, i.e., (a) an elastic process induced by the transverse field felt by conduction electrons and (b) magnon absorption/emission processes. The former is dominant when the FMR frequency is sufficiently low compared with the energy scale of the spin-orbit interaction, while the latter becomes important when the FMR frequency is comparable to the spin splitting energy by spin-orbit coupling in the 2DEG. In this work, we found that the vertex correction modifies the modulation of the Gilbert damping only moderately in general and does not change the features obtained in our previous paper. However, when the effective Zeeman field on conduction electrons points in the same direction on the Fermi surface, the elastic process is completely suppressed. We showed that this phenomenon can be understood by conservation of the total spin along the Zeeman field.

Our work provides a theoretical foundation for spin pumping into two-dimensional electrons with a spin-textured Zeeman field on the Fermi surface. Although we have treated a specific model for two-dimensional electron systems with both the Rashba- and Dresselhause spin-orbit interactions, our formulation and result will be helpful for description of spin pumping into general two-dimensional electron systems such as surface states and atomic layer compounds.

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