Glauber gluon effects in soft collinear factorization

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Abstract

Effects of Glauber gluons, which cause the elastic scattering process between different jets, are studied in the frame of soft-collinear effective theory. Glauber modes are added into the action before being integrated out, which is helpful in studies on the Glauber couplings of collinear and soft particles. It is proved that the final state interactions cancel out for processes inclusive enough. So are interactions with the light cone coordinates $x^+$ and $x^-$ greater than those of the hard collision. The eikonalization of Glauber couplings of active particles and absorption of these couplings into soft and collinear Wilson lines are discussed, which is related to the loop level definitions of Glauber gluons here. The active-spectator coherence is proved to be harmless to the inclusive summation of spectator-spectator and spectator-soft Glauber exchanges. Based on this result, spectator-spectator and spectator-soft Glauber exchanges cancel out in the processes considered here. Graphic aspects of the cancellation are also discussed to explain relations between the graphic and operator level cancellation of Glauber gluons.

Keywords: soft-collinear effective theory, QCD factorization, Glauber gluons

1. Introduction

Soft-collinear factorization is crucial in connecting perturbative QCD calculations with experimental data of high energy hadron processes. Such factorization is proved in many typical processes in the frame of perturbative QCD, (see, for example, [1–8]). Compared to these results, the factorization in soft-collinear effective theory (SCET) [9–12] provides us with a new perspective to understand the QCD factorization, which seems more intuitive. Collinear, soft and ultrasoft particles are described by effective modes in SCET. Especially, the hard collisions between collinear particles are described by various effective operators in SCET. In the original SCET Lagrangian, ultrasoft gluons decouple from collinear fields after a unitary transformation and collinear-soft couplings are absorbed into soft Wilson lines. Thus the soft-collinear factorization holds at the Lagrangian level in the original SCET.

Despite great advantages, the original SCET did not deal with Glauber gluons properly. Glauber gluons, which take space-like momenta, are responsible for elastic scattering processes between different jets. Results in [13] displayed how Glauber gluons break the QCD factorization in processes involving two initial hadrons like the Drell–Yan process. For Drell-Yan process, leading pinch singularities in the Glauber region cancel out according to the unitarity [2–4]. One may then deform the integral path of loop momenta to avoid the Glauber region. After the deformation, couplings between Glauber gluons and collinear particles eikonalize. That is, Glauber gluons behave like soft or collinear gluons after the deformation. In summary, the soft-collinear factorization of the Drell–Yan process is not violated by Glauber gluons [2–4]. However, the factorization can be violated by Glauber gluons for processes that are not inclusive enough (see, for example, [14–18]). In these processes, cancellation of leading pinch singularities in the Glauber region is hindered by Glauber couplings of the detected final particles.

Although leading pinch singular singularities in the Glauber region may cancel out, the Glauber gluon effects are visible in processes in which the factorization works. Since the contour deformation to avoid the Glauber region relies on explicit processes, whether the collinear and soft Wilson lines appearing in parton (distribution and fragmentation) functions and soft factors are past-pointing or future-pointing is process dependent [19]. In other words, Glauber gluons affect the directions of various Wilson lines in the factorization. Such effects are more obvious...
in transverse momentum-dependent objects like the Sivers function [20–22], although they are not concerned here.

Glauber gluons in SCET are more subtle. In [23–27], Glauber gluon fields are added into the SCET Lagrangian to describe jets in dense QCD matter. The effective theory is termed SCET$_{G}$. Glauber gluons in SCET$_{G}$ behave like a QCD function in transverse momentum-dependent objects like the Sivers function.

These effective operators may violate the factorization theorem in SCET as they cause coherence between different jets. These approximations are helpful to describe the Glauber couplings in loop integrals. Let us consider the coupling between a gluon $q$ and a plus-collinear particle $k$. If $q$ is soft or ultrasoft or minus-collinear, the coupling between $q$ and $k$ eikonalize at a leading power of $\lambda$. Hence we can define the non-eikonalized part of the coupling as Gau- bner coupling of $k$ even if $q$ locates in the non-Glauber region without affecting the leading power results. On the other hand, the eikonalized part of the coupling can be absorbed into soft or ultrasoft or collinear Wilson lines even if $q$ locates in the Glauber region. Thus we always define the non-eikonalized part as Glauber coupling of $k$ and absorb the eikonalized part into soft or ultrasoft or collinear Wilson lines. For a Glauber gluon $q$ exchanged between plus-collinear and minus-collinear particles, one has

$$\frac{-i}{p^2 + i\varepsilon} \simeq \frac{-i}{(l_i)^2 + i\varepsilon}.$$  \hspace{1cm} (1)

One can also neglect $l^i(l^-)$ in couplings between $l$ and plus-collinear (minus-collinear) particles. These approximations are helpful to describe the Glauber couplings in loop integrals. Let us consider the coupling between a gluon $q$ and a plus-collinear particle $k$ further. If $q$ is soft or minus-collinear, the coupling between $q$ and $k$ eikonalize. The eikonalized part of the coupling can be absorbed into collinear or soft Wilson lines and does not affect the factorization even if $q$ locates in the Glauber region. The non-eikonalized part should be power suppressed in the minus-collinear or soft (ultrasoft) region of $q$. Hence the rapidity divergences in this definition can be controlled through the regulator presented in [30, 32, 33], which reads

$$\omega^2 |2q^-|^{\gamma_1} \propto |q^+ - q^-|^{-\gamma}.$$  \hspace{1cm} (2)

We find it convenient to introduce Glauber gluon fields into the SCET action before integrating them out. This is helpful to determine the power counting for couplings involving Glauber gluons. Especially, it helps us to see the origins of leading power effective operators in [30]. We should mention that the power counting for various modes may depend on explicit gauge conditions in perturbative calculations as shown in [28–30]. Compared to the covariant gauge in [28–30], it is more convenient to work in the Feynman gauge for issues considered here. There are super-leading powers in practical diagrams in the Feynman gauge. However, such superleading powers cancel out in physical observable according to the Ward identity [6, 7]. This is confirmed by the power counting result presented in this

Figure 1. Examples of ladder diagrams with Glauber gluons exchanged (a) between active particles and spectators; (b) between active particles.

Figure 2. Elastic scattering diagrams between spectators, where the dot lines represent Glauber gluons.
paper. Thus superleading powers in the Feynman gauge do not disturb us.

The subtraction of eikonalized couplings from the definition of Glauber gluon modes is important in the treatment of elastic scattering processes. The eikonalized part of couplings involving collinear particles should be viewed as soft and collinear and ultrasoft interactions of these collinear particles even if there are gluons with Glauber momenta. Especially, the Glauber couplings of active particles eikonalize and these couplings should be absorbed into the definition of collinear or soft gluons. Besides, the subtraction does not affect spectator-spectator type Glauber gluons in the dimensional regularization scheme after the approximation (1) as dimensionless integrals vanish in the scheme.

The paper is organized as follows. In section 2, we add Glauber gluons into the SCET action. The power counting for couplings involving Glauber gluons is also presented in this section. In section 3, we discuss how to understand the cancellation of ladder-like Glauber exchanges between spectators presented in [30]. These discussions can be viewed as operator level interpretation of such cancellation. The cancellation of final state interactions for processes inclusive enough is also proved in this section, which is crucial in our general discussions. In section 4, we consider the Glauber couplings of active particles. We prove the eikonalization of these couplings and explain why these couplings are equivalent to zero bins of soft and collinear couplings of active particles. The absorption of spectator-active Glauber exchanges (including non-ladder-like cases) into collinear Wilson lines and active-soft and active-active Glauber exchanges (including non-ladder-like cases) into collinear Wilson lines is the direct result of discussions in this section. In section 5, we prove the cross section level cancellation of spectator-spectator Glauber exchanges for processes inclusive enough. We exclude the influence of spectator-active coherence by proving that the coherence should occur before spectator-spectator Glauber exchanges. Hence the summation over final spectators without affecting spectator-active coherence is enough for the cancellation of these Glauber exchanges. After the cancellation, Glauber modes can be absorbed into collinear and soft Wilson lines which are graph independent. In section 6, we explain how our operator scheme is related to the graphic cancellation of spectator-spectator Glauber exchanges in [2–4]. Our conclusions and some discussions are presented in section 7. In appendix A, we discuss the evolution operator of the effective theory. In appendix B, we discuss how to define spectators and collinear particles. The reparameterization invariance of the effective theory is discussed in appendix C.

2. Glauber gluons in SCET

In this section, we introduce Glauber gluon fields into the SCET action and study Glauber interactions. These discussions are helpful for studies of Glauber effects in hadronic processes at leading power. Glauber gluons take space-like momenta and cause elastic scattering between collinear particles. For example, we consider a gluon with momentum scales as

\[(p^+, p^-, p_3) \sim Q(\lambda^2, \lambda^2, \lambda),\]  \hspace{1cm} (3)

where \(Q\) represents a hard energy scale and \(\lambda \ll 1\). The gluon is space-like as \(p^2 - Q^2 \lambda^2 < 0\). Exchanging of such gluons causes elastic scattering between particles collinear to the plus and minus directions. We discuss the power counting for couplings between Glauber gluons and other particles in this section. Results in this section are compatible with those in [30] and make our following discussions more clear.

For a light-like vector \(n^\mu = \frac{1}{\sqrt{2}}(1, \bar{n}, n_3)\), the power counting for relevant modes is presented in table 1 [9, 10, 30].

To obtain the power counting for Glauber gluon fields, we consider the Glauber propagators in the Feynman gauge,

\[
\int d^4x e^{ikx} T < A_\mu(x) A_\nu^\dagger(0) > = \frac{-i\epsilon^{\mu\nu\rho\sigma}}{k^2}. \hspace{1cm} (5)
\]

The Glauber momentum scale as \((n \cdot k, \bar{n} \cdot k, k_{nl}) \sim Q(\lambda^2, \lambda^2, \lambda)\) and the integral volume scale as \(\int d^4x \sim Q^{-4}\lambda^{-b}\). Thus the power counting for Glauber gluon fields reads \(A_\mu(x) \sim \lambda^2 + \lambda^2\). For future convenience, we present here the power counting for fermions with momenta scales as \((n \cdot k, n \cdot k, k_{nl}) \sim Q(\lambda^2, \lambda^2, \lambda)\), which reads \(\lambda^{2+b}\).

2.1. Power counting for couplings between Glauber gluons and other particles

In this section, we consider Glauber interactions. We first consider couplings between Glauber gluons and ultrasoft gluons. According to table 1, the power counting for ultrasoft particles and Glauber gluons reads \(\lambda^2, \lambda^2\) and \(\lambda^{2+b}\). The integral volumes of these couplings scale as \(\int d^4x \sim Q^{-4}\lambda^{-b}\). In these couplings, there are at least two Glauber gluons and one ultrasoft gluon. The power counting for the combination of these fields reads \(\lambda^{4+b}\). There is an additional gluon field or momentum operator in these couplings according to Lorentz covariance. The power counting for the additional gluon field reads \(\lambda^{2+b}\). That of the momentum operator reads \(\lambda^2\). Thus the infrared power counting for these couplings reads \(\lambda^2\), where

\[r \geq 1 + 4 + b + (-4 - b) = 1. \hspace{1cm} (6)\]

That is to say, the infrared power counting for these couplings reads \(\lambda^2\) or higher. For the couplings between Glauber gluons and ultrasoft fermions, the situation is similar. The integral volumes scale as \(\int d^4x \sim Q^{-4}\lambda^{-b}\). In these couplings, there are at least one ultrasoft fermion and one Glauber gluon and one fermion with momentum scales as \((n \cdot k, n \cdot k, k_{nl}) \sim Q(\lambda^2, \lambda^2, \lambda)\), which reads \(\lambda^{2+b}\).

In [9, 10], the authors work in the covariant gauge,

\[
\int d^4x e^{ikx} T < A_\mu(x) A_\nu^\dagger(0) > = \frac{-i}{k^2} \left\{ g^{\mu\nu} - (1 - \xi \frac{k^\rho k^\sigma}{k^2}) \right\}. \hspace{1cm} (4)
\]

where \(\xi\) is the gauge parameter. The power counting for the field \(A_\mu\) reads \((n \cdot A_{\mu}, \bar{n} \cdot A_{\mu}, k_{nl}) \sim (\lambda^2, \lambda, 1)\). It is required that \(1 - \xi\) is not too small to get this result. While working in the Feynman gauge, there are superleading power terms involving \(\vec{n} \cdot A_{\mu}\) in SCET Lagrangian. This does not disturb us as \(\vec{n} \cdot A_{\mu}\) and \(\bar{n} \cdot A_{\mu}\) appear in pairs in practical diagrams. In fact, if there is a collinear gluon polarized as \(n \cdot A_{\mu}\), then its other end should polarize as \(n \cdot A_{\mu}\) in the Feynman gauge according to the \(g^{\mu\nu}\) tensor in its propagator. The power counting for such a pair reads \(\lambda^4\) according to table 1, which is equivalent to that of the covariant gauge in [9, 10].

3
The power counting for these fields reads $\lambda^3$, $\lambda^{1+\frac{3}{2}}$ and $\lambda^{\frac{3}{2}}$ respectively. Momenta of these fields are quite small and there are no energy scales that may produce the minus power of $\lambda$. Thus the infrared power counting for these couplings reads $\lambda^3$ or higher.

For the couplings between Glauber gluons without other type particles, the integral volume scales as $\int d^4x \sim Q^{-4}\lambda^{-2-b}$. There are at least three Glauber gluons and the power counting for the combination reads $\lambda^{1+\frac{3}{2}}$. There is an additional gluon field ($\lambda^{1+\frac{3}{2}}$) or momentum operator ($\lambda$ or $\lambda^2$) in these couplings. Thus the power counting for these couplings reads $\lambda^3$ or higher.

For the couplings between Glauber gluon fields $A_{4G}$ and $A_{4G'}$, which involve soft gluons, the integral volume scales as $\int d^4x \sim Q^{-4}\lambda^{-2}$. There are at least two Glauber gluons ($\lambda^{1+\frac{3}{2}}$) and one soft gluon ($\lambda$) in these couplings. There is an additional gluon field ($\lambda^{1+\frac{3}{2}}$) or momentum operator ($\lambda$ or $\lambda^2$) in these couplings. Thus the power counting for these couplings reads $\lambda^3$ or higher.

For the couplings between Glauber gluon fields $A_{4G}$ and soft gluons without Glauber gluon fields $A_{4G'}$, the integral volume scales as $\int d^4x \sim Q^{-4}\lambda^{-4}$. In these couplings, there are at least two soft gluons and one Glauber gluon. The power counting for the combination reads $\lambda^{1+\frac{3}{2}}$. There is an additional gluon field ($\lambda^{1+\frac{3}{2}}$) or momentum operator ($\lambda$ or $\lambda^2$) in these couplings according to Lorentz invariance. Thus the infrared power counting for these couplings reads $\lambda^3$ or higher.

For the couplings between Glauber gluons and soft fermions, the integral volume scales as $\int d^4x \sim Q^{-4}\lambda^{-4}$. In these couplings, there are at least two soft fermions and one Glauber gluon. The power counting for the combination reads $\lambda^{1+\frac{3}{2}}$. Momenta of these fields are quite small and there are no energy scales that may produce the minus power of $\lambda$. Thus the infrared power counting for these couplings reads $\lambda^3$ or higher.

For the couplings between Glauber gluon fields $A_{4G}$ and $n$-collinear fermions, the integral volume scales as $\int d^4x \sim Q^{-4}\lambda^{-4}$. In these couplings, there are at least two collinear fermions and one Glauber gluon. The power counting for the combination reads $\lambda^{1+\frac{3}{2}}$. Thus the infrared power counting for these couplings reads $\lambda^3$ or higher.

For the couplings between Glauber gluon fields $A_{4G}$ and $n$-collinear gluons, the integral volume scales as $\int d^4x \sim Q^{-4}\lambda^{-4}$. In these couplings, there are at least two collinear gluons and one Glauber gluon. The infrared power counting for the combination reads $\lambda^{1+\frac{3}{2}}$. There is an additional gluon field ($\lambda^{1+\frac{3}{2}}$) or momentum operator ($\lambda^3$, $\lambda$ or $\lambda^2$) in these couplings. Thus the power counting for these couplings reads $\lambda^3$ or higher.

For the couplings between Glauber gluon fields $A_{4G}$ and $\bar{n}$-collinear fermions, the integral volume scales as $\int d^4x \sim Q^{-4}\lambda^{-2-b}$. In these couplings, there is at least one collinear fermion, one Glauber gluon and one fermion with momentum scales as $(n \cdot k, n \cdot \bar{n}, k_{\perp}) \sim Q(1, \lambda^3, \lambda)$. The power counting for the combination reads $\lambda^{1+\frac{3}{2}}$. Thus the infrared power counting for these couplings reads $\lambda^3$ or higher.

For the couplings between Glauber gluon fields $A_{4G}$ and $\bar{n}$-collinear gluons, the integral volume scales as $\int d^4x \sim Q^{-4}\lambda^{-2-b}$. In these couplings, there is at least one collinear gluon, one Glauber gluon and one gluon with momentum scales as $(n \cdot k, n \cdot \bar{n}, k_{\perp}) \sim Q(1, \lambda^3, \lambda)$. The power counting for the combination of these fields reads $\lambda^{3+\frac{3}{2}}$ or higher. There is an additional gluon field or momentum operator in these couplings. The power counting for these objects reads $\lambda^3$ or higher. Thus the power counting for these couplings reads $\lambda^3$ or higher.

Our results in this section are presented in table 2.

### 2.2. Leading power action including Glauber gluons

We consider the leading power SCET action with Glauber gluon fields in this section. There are two kinds of SCET Lagrangian in literature, SCETI and SCETII, which are suitable for studies on different objects. We do not distinguish them here. For simplicity, we neglect the couplings involving ultrasoft particles and the couplings between Glauber gluons without soft gluons at first.

We start from a Glauber gluon $A_{4G}$ which couples to $n$-collinear particles. The power counting for such coupling reads $\lambda^{-1}$ according to the results in table 2. The other end of the Glauber gluon may couple to ultrasoft particles, Glauber gluons, soft particles or particles collinear to other directions. If the Glauber gluon couple to particles collinear to other directions at that end, then the power counting for that coupling reads $\lambda^{-1}$. The final power counting for couplings at two ends of the Glauber gluon reads $\lambda^{-4}$.

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4 There may be additional powers of $\lambda$ in practical diagrams even if these diagrams involve only leading order couplings as shown in [30]. This does not disturb us here as we are only concerned with the power counting for effective iterations in this paper.
If the other end of the Glauber gluon involves soft particles, then the power counting for that coupling reads $\lambda^2$ or $\lambda^3$. For the former case, the final power counting for couplings at two ends of the Glauber gluon reads $\lambda^{b-1} \lambda^0$. For the latter case, in which $b = 1$, the other end of $A_{nG}$ involves a Glauber gluon of the type $A_{nG}$. If the other end of the $A_{nG}$ couple to particles collinear to $b^0$, then the final power counting for these couplings reads $\lambda^{b+1} \lambda^0 = \lambda^b$. If the other end of $A_{nG}$ couples to soft particles, then we can repeat the procedure and get the same result. In conclusion, the final power counting for these couplings reads $\lambda^0$ if there are couplings between Glauber gluons and soft particles.

If a Glauber gluon is exchanged between soft particles, then the power counting for that coupling reads $\lambda^1$. We notice that the minus powers of $\lambda$ can only be produced by couplings between Glauber gluons and collinear particles. Thus the combination of couplings involving Glauber gluons does not produce minus powers of $\lambda$. Thus couplings involving ultrasoft particles and couplings between Glauber gluons and collinear particles are involved.

We then consider couplings involving ultrasoft particles and couplings between Glauber gluons without soft gluons. The power counting for the couplings between Glauber gluons and ultrasoft particles reads $\lambda^1$ or $\lambda^{3/2}$. The power counting for couplings between Glauber gluons without soft gluons reads $\lambda^2$. According to the above discussions, the combination of other couplings involving Glauber gluons does not produce minus powers of $\lambda$. Thus couplings involving ultrasoft particles and couplings between Glauber gluons without soft gluons are power suppressed.

In summary, (1) Glauber gluons are exchanged between collinear particles and soft particles or between collinear particles at leading power; (2) there may be intermediate couplings between Glauber gluons and soft particles in these exchanges at leading power; (3) couplings between Glauber gluons and ultrasoft particles are power suppressed; (4) couplings between Glauber gluons without soft gluons are power suppressed. This is compatible with the results in [30].

The leading power effective action can then be written as

$$I_{\text{eff}} = \sum_n I_n^{G} + I_n^{A} + I_{nAS} + \sum_n I_{nG}$$  \hspace{1cm} (7)

$\text{5}$ One should not be confused with the possible fractional power in this coupling. There are other powers of $\lambda$ in diagrams involving this coupling. For example, one considers the coupling between three Glauber gluons of the type $A_{nG}$. Two of them connect to a fermion collinear to $n^\nu$ and one of them collinear to $b^\nu$. At leading power, two of them are of the type $\bar{n} \cdot A_{nG}$ and one of them is of the type $\bar{n} \cdot A_{nG}$ in the coupling between these three Glauber gluons. According to Lorentz covariance, there is a momentum term of the type $n \cdot b \sim \lambda^2$ instead of the type $k_i \sim \lambda$, which is produced by the coupling

$\text{6}$ The perpendicular components $A_{nG \perp}$ decouple from other fields at the leading power of $\lambda$. We simply disregard them here.

$\text{7}$ We should mention that there is overlap between $A_{nG}^\nu$ and $A_{nG}^{\perp}$, which should be subtracted from the effective action. The subtraction is not displayed here for simplicity.

$$I_n^{G} = \int d^4x \xi_n \{in \cdot D + gn \cdot (A_n + A_{nG})$$

$$+ (\mathcal{P}_1 + gA_n^\nu) W_1 \mathcal{W}_1 (\mathcal{P}_1 + gA_n^\nu) [\xi_n$$

$$+ \int d^4x \frac{-1}{2g} tr \{[iD_{nG}^\nu + gA_n^\nu, iD_{nG}^\nu + gA_n^\nu] \}^2$$

$$+ \int d^4x 2 tr \{c_n[iD_{nG}, [iD_{nG}^\nu + gA_n^\nu, c_n]] \}$$

$$+ \int d^4x tr \{[iD_{nG} + A_n^\nu + gn \cdot A_{nG} b^\nu]$$

$$\times [iD_{nG}, A_{nG} + gn \cdot A_{nG} b^\nu] \} \}$$  \hspace{1cm} (8)

$$I_n^{A} = \int d^4x \bar{\psi}_A^\nu (\mathcal{P} + gA_{nG}) \psi_A$$

$$- \frac{1}{2} \int d^4x tr \{G_{\mu\nu}^A G_{\mu\nu}^A \}$$

$$+ \int d^4x 2 tr \{c_n[i\mathcal{P}, [i\mathcal{P}^\nu + gA_n^\mu$$

$$+ g\sum_n \bar{n} \cdot A_{nG} h^{\mu\nu}, c_n]] \}$$

$$+ \int d^4x tr \{[\mathcal{P}_{nG}, A_n^\mu + g\sum_n \bar{n} \cdot A_{nG} h^{\mu\nu}]$$

$$\times [\mathcal{P}_{nG}, A_{nG} + g\sum_n \bar{n} \cdot A_{nG} h^{\mu\nu}] \}$$  \hspace{1cm} (9)

$$I_{nAS} = \int d^4x \bar{\psi}_{nAS} \mathcal{P} \psi_{nAS} - \int d^4x \frac{1}{2} \{G_{\mu\nu} \}$

$$+ \int d^4x 2 tr \{c_n[i\mathcal{P}, [i\mathcal{P}^\nu + gA_n^\mu, c_n]] \}$$

$$+ \int d^4x tr \{[\bar{\partial} \cdot A_{nAS}] \}$$  \hspace{1cm} (10)

$$I_{nG} = \int d^4x tr \{[\mathcal{P} \bar{n} \cdot A_{nG}]([\mathcal{P} \bar{n} \cdot A_{nG}]$$

$$- \frac{1}{4} \int d^4x tr \{[\bar{n} \cdot \mathcal{P} n \cdot A_{nG}]([\bar{n} \cdot \mathcal{P} n \cdot A_{nG}] \}$$  \hspace{1cm} (11)

where $(\xi_n, A_{nG}, c_n)$ are collinear fields and $(\psi_A, A_{nAS})$ are soft fields and $(\psi_{nAS}, A_{nAS})$ are ultrasoft fields and

$$W_\nu(x) = P \exp (ig \int_{-\infty}^{0} ds \bar{n} \cdot A_{\nu}(x + s\bar{n})$$

$$D^\mu = g\gamma^\mu - igA_{\mu}^A$$

$$\mathcal{P}^\nu(\phi_{q_1} \cdots \phi_{q_n} \phi_{p_1} \cdots \phi_{p_n})$$

$$=(p_1^\mu + \cdots + p_n^\mu - q_1^\mu - \cdots - q_m^\mu)(\phi_{q_1} \cdots \phi_{q_n} \phi_{p_1} \cdots \phi_{p_n})$$  \hspace{1cm} (12)

$$iD_{\mu}^\nu = n^\mu \bar{n} \cdot \mathcal{P} + \mathcal{P}_{\mu}^\nu + \bar{n} \cdot in \cdot D$$

$$iG_{\mu\nu} = \frac{1}{g} [D_{\mu}^\nu + gA_{\mu}^\nu + g\sum_n \bar{n} \cdot A_{nG} h^{\mu\nu}$$

$$+ g\sum_n \bar{n} \cdot A_{nG} h^{\mu\nu}]$$  \hspace{1cm} (13)

$$G_{\mu}\nu = \frac{1}{g} [D^\mu, D_{\nu}]$$

$$\mathcal{D}_{nG}^\nu = \mathcal{D}_{nG} - \bar{n} \cdot in \cdot (A_{nG}).$$  \hspace{1cm} (14)

Some power suppressed terms are added into the action to maintain the BRST covariance as discussed in appendix A. We see that diagrams involving $A_{nG}(k)$ rely on $n \cdot k$ through various propagators and vertexes involving $A_{nG}(k)$ are independent of $n \cdot k$ at leading power. This is crucial in our following discussions.
3. Elastic scattering effects in hadron collisions

In this section, we consider the kinematic effects of Glauber gluons and prove the cancellation of final state interactions in inclusive processes. These discussions are independent of the details of Glauber couplings.

The processes considered here can be written as,

\[ H_1(P) + H_2(\bar{P}) \rightarrow l' + l^-(q) + X \]  \\
\[ or \  \\
H_1(P) + H_2(\bar{P}) \rightarrow J_3(P_3) + J_4(P_4) + X \]

with \( P_3 + P_4 = q \), where \( H_1 \) and \( H_2 \) represent the initial hadrons with momenta \( P \) and \( \bar{P} \) and \( l'^-l^- (q) \) represents a lepton pair with momentum \( q \) and \( J_3 \) and \( J_4 \) represent the detected final jets with momenta \( P_3 \) and \( P_4 \) and \( X \) represents any other states. We work in the center of mass frame of initial hadrons. At leading power of \( \Lambda_{QCD}/(q^2)^{1/2} \), \( P \) and \( \bar{P} \) are light-like. Without loss of generality, we assume that \( P^\mu \) is plus-collinear and \( \bar{P}^\mu \) is minus-collinear.

The hard subprocess is described by a hard vertex, which is denoted as \( J(x) \). For example, the hard electromagnetic vertex in SCETII takes the form [30],

\[ J = \xi_\alpha W_{\alpha} S^\alpha \Gamma S^\beta W^\beta \xi_\beta, \]

where \( W_{\alpha} \) and \( W_{\beta} \) are collinear Wilson lines and \( S_{\alpha} \) and \( S_{\beta} \) are soft Wilson lines.

We do not consider quantities dependent on \( q \) in this paper. Thus one can integrate out \( q \) in the following discussion.

3.1. Elastic scattering effects without interactions between spectators and active particles

In this section, we start from spectator–spectator interactions without the active-spectator coherence\(^8\). We show an operator level explanation for the spectator–spectator cancellations discussed in [30]. Our explanation based on the unitarity is quite general and helpful to understand what happens in inclusive processes. Given that the active-spectator coherence is neglected, such cancelation can be easily extended to non-Glauber interactions.

Let us start from the diagrams shown in figure 2. These diagrams can be written as,

\[ S(P, \bar{P}, p, \bar{p}) \equiv \int d^4x_1 \int d^4x_2 \int d^4x_3 \int d^4x_4 \]
\[ \times \int d^4x_5 \delta^{(4)}(P + \bar{P} - p - \bar{p} - z) \]
\[ \times \langle PP\{O^I(\chi_1)\bar{O}^J(\chi_2)J^I(z)U^\dagger(\infty, -\infty)\}|pp\rangle \]
\[ \times \langle pp|T\{O(\chi_3)\bar{O}(\chi_4)J(0)U(\infty, -\infty)\}|PP\rangle, \]

where \( T \) and \( \bar{T} \) represent the time order and anti-time order operators and \( J(x) \) represents the hard vertexes that annihilate active particles and \( O(x)\bar{O}(x) \) represents the vertex that produces active particles and spectators from \( H_1(H_2) \) and \( U(t_1, t_2) \) represents the time evolution operator of the effective theory in the interaction picture.

If we neglect the active-spectator coherence, then the effective action can be written as

\[ I = I_{act}(\psi_{act}, A^\mu_{act}) + I_{sp}(\psi_{sp}, A^\mu_{sp}), \]

where \( I_{act} \) is the active part of \( I_{eff} \) and \( I_{sp} \) is the spectator part of \( I_{eff} \).\(^9\) The interactions between spectators and active particles have been dropped in the above decomposition. We have

\[ U(t_1, t_2) = U_{act}(t_1, t_2)U_{sp}(t_1, t_2) = U_{sp}(t_1, t_2)U_{act}(t_1, t_2), \]

\[ J_{act}(\psi_{act}, A^\mu_{act}) + J_{sp}(\psi_{sp}, A^\mu_{sp}). \]

\[ \begin{array}{ccc}
\text{Couplings} & \text{Fields} & \text{Power counting} \\
\hline
\text{Glauber gluons and ultra-soft gluons} & (A_{soft}, A_{soft}) & \lambda^2 \text{ or higher} \\
\text{Glauber gluons and ultra-soft fermions} & (A_{soft}, A_{soft}) & \lambda^2 \text{ or higher} \\
\text{Glauber gluons} & (A_{soft}) & \lambda^2 \text{ or higher} \\
\text{Glauber gluons and soft gluons} & (A_{soft}, A_{soft}) & \lambda \text{ or higher} \\
\text{Glauber gluons and soft fermions} & (A_{soft}, A_{soft}) & \lambda^2 \text{ or higher} \\
\text{Gluons and collinear ferrions} & (A_{soft}, S) & \lambda^2 \text{ or higher} \\
\text{Gluons and collinear gluons} & (A_{soft}, A_{soft}) & \lambda^2 \text{ or higher} \\
\text{Gluons and collinear fermions} & (A_{soft}, S) & \lambda^2 \text{ or higher} \\
\hline
\end{array} \]

Table 2. The infrared power counting for couplings involving Glauber gluons, where \( \tilde{\eta} = \frac{1}{2\pi}(1, -\tilde{Q}). \)
where $U_{ip}(t_1, t_2)$ and $U_{sp}(t_1, t_2)$ represent the time evolution operators corresponding to $I_{ic}$ and $I_{sp}$.

We now consider the Wick contractions of the fields in (21). Fields in $U_{sp}$ do not contract with those in the current $J$ as $J$ is functional of active particle fields. We notice that the energy of spectators flows out of the vertexes $O$ and $\bar{O}$. Hence the couplings involving spectators all occur after the production of them. Especially, the interactions corresponding to $U_{sp}$ should occur after the vertexes $O(\bar{O})$. As a result, we can drop the contractions between $O(\bar{O})$ and $U_{sp}$ once the time coordinates of fields in $U_{sp}$ are smaller than those of $O(\bar{O})$. We have

$$
\langle PP| T \{ O' (x_1) \bar{O}' (x_2) J' (z) U' (\infty, -\infty) \}| pp \rangle = \langle PP| T \{ O' (x_1) \bar{O}' (x_2) J' (z) U_{ic} (\infty, -\infty) \} U_{sp} (\infty, -\infty) \rangle \langle pp | U_{sp} (\infty, -\infty) \rangle T \{ O (x_1) \bar{O} (x_2) J (0) U_{ic} \times (\infty, -\infty) \} | PP \rangle.
$$

(24)

(25)

We then consider the evolution of the final state $| pp \rangle$ under $U_{sp} (\infty, -\infty)$. Elastic scattering processes exchange the transverse momenta and colors and angular momenta between collinear particles. On the other hand, the total momentum and color and angular momentum of these particles are invariant in the elastic processes. Thus all possible pairs $| pp \rangle$ with fixed total momenta and colors and spins form an invariant subspace of $U_{sp}$ if we neglect inelastic scattering between spectators. That is,

$$
\sum \int d^2 \Delta p_p U_{sp} (\infty, -\infty) \langle pp | pp \rangle \langle pp | U_{sp} (\infty, -\infty) U_{sp} (\infty, -\infty) \rangle = \sum \int d^2 \Delta p_p \langle pp | pp \rangle \langle pp | pp \rangle,
$$

(26)

where the summation is made over all possible color and angular momentum distributions of the pair $| pp \rangle$ with fixed total color and angular momentum. $\Delta p_p$ is defined as

$$
\Delta p_p \equiv p_p - \bar{p}_p.
$$

(27)

We then have,

$$
\sum \int d^2 \Delta p_p S (P, \bar{P}, p, \bar{p}) = \sum \int d^2 \Delta p_p \int d^4 x_1 \int d^4 x_2 \int d^4 x_3 \int d^4 x_4 \\
\times \int d^2 z e^{i p \cdot z} \langle PP| T \{ O' (x_1) \bar{O}' (x_2) J' (z) U_{ic} (\infty, -\infty) \} | pp \rangle \langle pp | T \{ O (x_1) \bar{O} (x_2) J (0) U_{ic} \times (\infty, -\infty) \} \rangle \langle pp | \rangle.
$$

(28)

Thus the elastic scattering effects between spectators cancel out in processes inclusive enough once we neglect the active-spectator coherence. Such cancellation is independent of the details of elastic interactions between spectators.

If there are inelastic interactions involving spectators like those shown in figure 3, then the summation over all possible soft and ultrasoft final states is necessary. That is,

$$
\sum_{pp...} U_{ip} (\infty, -\infty) \langle pp... | pp... \rangle U_{ip} (\infty, -\infty) = \sum_{pp...} \langle pp... | pp... \rangle.
$$

(29)

where the summation is made over all possible final particles with fixed total momentum and color and angular momentum. Thus the spectator–spectator interactions cancel out in processes inclusive enough given that the active-spectator coherence have been dropped as in figure 3.

3.2. Cancellation of final state interactions

In this section, we take into account the active-spectator coherence and prove the cancellation of final state interactions in the process is inclusive enough. Such general cancellation originates from the unitarity.

For diagrams with the active-spectator coherence, like those shown in figure 4 and their conjugations, the situation is more complicated. Let us consider the quantity,

$$
\mathcal{H}(P, \bar{P}, q^+, q^-) \equiv \sum \int \frac{d^4 q}{(2\pi)^2} \int \frac{d^4 x_1}{2} \int \frac{d^4 x_2}{2} \\
\times \int \frac{d^4 x_3}{2} \int \frac{d^4 x_4}{2} \int \frac{d^2 e^{i q \cdot z}}{2} \langle PP| T \{ O' (x_1) \bar{O}' (x_2) J' (z) U' (\infty, -\infty) \} | H(q) X \rangle \langle H(q) X| T \{ O (x_1) \bar{O} (x_2) J (0) U (\infty, -\infty) \} \rangle \langle PP | \rangle.
$$

(30)

where $H(q)$ represents the detected lepton or jet pair with total momenta $q$ and the summation is made over all possible final states. Generally, there is active-spectator coherence in $U(t_1, t_2)$ and the factorization (23) does not simply work.

Fortunately, the contractions between $O(\bar{O})$ and other fields occur only if the time coordinates of fields in $O(x)$ are smaller than those of other fields. Otherwise, the contraction does not contribute to $\mathcal{H}$. We can write $\mathcal{H}$ as

$$
\mathcal{H}(P, \bar{P}, q^+, q^-) \equiv \sum \int \frac{d^4 q}{(2\pi)^2} \int \frac{d^4 x_1}{2} \int \frac{d^4 x_2}{2} \\
\times \int \frac{d^4 x_3}{2} \int \frac{d^4 x_4}{2} \int \frac{d^2 e^{i q \cdot z}}{2} \langle PP| T \{ O' (x_1) \bar{O}' (x_2) J' (z) U' (\infty, -\infty) \} | H(q) X \rangle \langle H(q) X| T \{ J (0) U (\infty, -\infty) \} \rangle \langle PP | \rangle.
$$

(31)

where the order between operators $O$ and $\bar{O}$ does not affect the result.
We notice that
\[
T(J(x)U(\infty, -\infty)) = U(\infty, x^0)J(x)U(x^0, -\infty) = U(\infty, \max\{x^0, 0\})U(\max\{x^0, 0\}, x^0)J(x)U(x^0, -\infty),
\]
and have
\[
\mathcal{H}(P, \bar{P}, q^+, q^-) = \sum_x \int \frac{d^2q}{(2\pi)^2}U^+(\infty, -\max\{x^0, 0\})\mathcal{H}(q)X
\times \langle H(q)X|U(\infty, \max\{x^0, 0\}) \rangle \times U^+(\infty, -\max\{x^0, 0\})U(\infty, \max\{x^0, 0\})
= \sum_x \int \frac{d^2q}{(2\pi)^2}\mathcal{H}(q)X\frac{\partial}{\partial x^0}\mathcal{O}(x_0)\mathcal{O}(x_0)|PP\rangle.
\]

The summation in the above quantity is made over all possible final states. The completeness of the final states $X^{10}$ and the unitarity of $U(t_1, t_2)$ hint that
\[
\sum_x \int \frac{d^2q}{(2\pi)^2}U^+(\infty, -\max\{x^0, 0\})\mathcal{H}(q)X
\times \langle H(q)X|U(\infty, \max\{x^0, 0\}) \rangle U^+(\infty, -\max\{x^0, 0\})U(\infty, \max\{x^0, 0\})
= \sum_x \int \frac{d^2q}{(2\pi)^2}\mathcal{H}(q)X\frac{\partial}{\partial x^0}\mathcal{O}(x_0)\mathcal{O}(x_0)|PP\rangle.
\]

We have,
\[
\mathcal{H}(P, \bar{P}, q^+, q^-) = \sum_x \int \frac{d^2q}{(2\pi)^2}U^+(\infty, -\max\{x^0, 0\})\mathcal{H}(q)X
\times \langle H(q)X|U(\infty, \max\{x^0, 0\}) \rangle \times U^+(\infty, -\max\{x^0, 0\})U(\infty, \max\{x^0, 0\})
= \sum_x \int \frac{d^2q}{(2\pi)^2}\mathcal{H}(q)X\frac{\partial}{\partial x^0}\mathcal{O}(x_0)\mathcal{O}(x_0)|PP\rangle.
\]

Hence final state interactions cancel out in $\mathcal{H}(P, \bar{P}, q^+, q^-)$. This is compatible with the results in [2–4].

Especially, the effects of couplings between Glauber gluons and final active particles cancel out in $\mathcal{H}$ as these couplings occur after the hard collision.

10 $X$ represents all possible other states. Hence $H(q)X$ represents all possible states that contain the detected lepton or jet pair. $H(q)X$ forms the invariant subspace of QCD evolution unless one considers more hard processes after the hard collision (so that the jet pair may vanish). Effects of such hard processes are power suppressed in $\mathcal{H}(P, \bar{P}, q^+, q^-)$ according to the physical picture of pinch singular surfaces [6, 34].

4. Glauber gluons coupling to active particles

In this section, we consider the Glauber couplings of active particles. They are absorbed into collinear and soft Wilson lines for ladder diagram cases [30]. We extend the conclusion to the general situation in this section. Explicitly we see that (1) the active-soft Glauber exchanges can be absorbed into soft Wilson lines in $\mathcal{H}(P, \bar{P}, q^+, q^-)$; (2) the active–active Glauber exchanges can be absorbed into soft Wilson lines in $\mathcal{H}(P, \bar{P}, q^+, q^-)$; (3) the active-spectator Glauber exchanges can be absorbed into collinear Wilson lines in $\mathcal{H}(P, \bar{P}, q^+, q^-)$. We should emphasize that these results are the case even if there is active-spectator coherence.

Glauber gluons look like special collinear or soft gluons. For example, the Glauber gluons $A_{np}^{\mu}$ can be viewed as the collinear gluons $A_{np}^{\mu}$ with $\vec{n} \cdot p = 0$ or the soft gluons $A_{np}^{\mu}$ with $n \cdot q = 0$. As we always integral over all loop momenta regions in practical diagrams, distinguishing different modes in the loop integrals is quite technical. In perturbative QCD, gluons coupling to active particles are not pinched in the Glauber region [2–4]. As a result, one can deform the integral contour to avoid the Glauber region of these gluons. In SCET, one needs systematic subtraction schemes of which the details are beyond the scope of this paper. We emphasize that whatever the subtraction scheme is, the eikonal approximation is important while dealing with the couplings between collinear and other particles.

The key point is the eikonalization of couplings between Glauber gluons and active particles. As a sequence, Glauber gluons behave like soft or collinear gluons while coupling to active particles. This provides us with the possibility to absorb these Glauber into the definition of soft or collinear gluons. Even if one is not concerned with the definition of different modes, absorption of these Glauber gluons into soft or collinear Wilson lines is the direct result of the eikonalization.

We do not consider the Glauber couplings of final particles as they cancel out in $\mathcal{H}(P, \bar{P}, q^+, q^-)$ (35). Without loss of generality, we consider the Glauber couplings of plus-collinear particles in this section. Explicitly, we consider the couplings between some Glauber gluons $l_1, \ldots, l_n$ and a plus-collinear active particle $k$ in the following sections.
4.1. Eikonal approximation in couplings between collinear particles and soft (ultrasoft) gluons

In this section, we briefly explain the eikonal approximation in soft and ultrastiff couplings of collinear particles.\textsuperscript{11} Let us consider the coupling between a plus-collinear particle $p$ and a soft or ultrastiff gluon $q$. The power counting for $p$ and $q$ reads

\[(p^\perp, p^\perp, p_\perp^\perp) \sim Q(1, \lambda^2, \lambda), \quad q^\mu \sim Q\lambda (\text{for soft gluons})\]

\[\text{or } q^\mu \sim Q\lambda^2 (\text{for ultrastiff gluons}).\]

We have

\[(p \pm q)^2 = p^2 \pm 2p \cdot q + q^2 \approx p^2 \pm 2p^\perp q^\perp,\]

for ultrastiff particles and

\[(p \pm q)^2 = p^2 \pm 2p \cdot q + q^2 \approx \pm 2p^\perp q^\perp,\]

for soft particles. The gluon field $A^\nu(q)$ should contract with some vectors. In the limit $\lambda \to 0$, $p$ behaves like a one-dimensional particle and there is only one direction relevant to $p$, that is, the plus direction. Hence $A^\nu(q)$ should contract with the plus direction in this limit. In other words, $A^\nu(q)$ should contract with vectors collinear to the plus direction at the leading power of $\lambda$. This conforms with the leading power action (2.2). In summary, we can make the approximation

\[(p \pm q)^2 \approx p^2 \pm 2p^\perp q^\perp \quad A_{\mu}^\perp \equiv (A^+, A^-, A^\perp) \approx (A^+, 0, 0),\]

in ultrastiff-collinear couplings and

\[(p \pm q)^2 \approx \pm 2p^\perp q^\perp \quad A^\mu \equiv (A^+, A^-, A^\perp) \approx (A^+, 0, 0),\]

in soft-collinear couplings. The approximations (39) and (40) are termed eikonal approximations in the literature.

It is interesting to consider the coordinate space version of the eikonal approximation,

\[A_{\mu}^\perp (x) \equiv (A^+ (x), A^- (x), A^\perp (x)) \approx (A^+ (x^+, 0, 0), 0, 0),\]

\[A_{\mu}^\mu (x) \equiv (A^\perp (x), A^\perp (x), A^\perp (x)) \approx (A^\perp (x^+, 0, 0), 0, 0).\]

We have\textsuperscript{12}

\[
\partial^\mu - ig(A^+_\perp(x^+, 0, 0), 0, 0) - ig(A^\perp_\perp(x^+, 0, 0), 0, 0) + \epsilon = P \exp \left( ig \int_{-\infty}^{x} ds(A^+_\perp + A^\perp_\perp)(x^+ + s, 0, 0) \right) \\
\times (\partial^\nu - \epsilon) P \exp \left( ig \int_{0}^{\infty} ds(A^+_\perp + A^\perp_\perp)(x^+ + s, 0, 0) \right).
\]

That is to say, the absorption of soft and ultrastiff gluons into light-like Wilson lines is the direct result of the eikonal approximation.\textsuperscript{13}

The approximation (40) also works in couplings between plus-collinear particles and minus-collinear gluons. Although the eikonal approximations (39) and (40) seem rather diagrammatic, it is necessary in loop level definition of effective modes considered here.\textsuperscript{14} It is also the origin of various Wilson lines in SCET [11]. Hence discussions on the approximations are necessary to examine whether Glauber interactions can be absorbed into collinear and soft Wilson lines or not.

However, the Glauber couplings of collinear particles are more subtle. Taking the coupling between a plus-collinear particle $p$ and a Glauber gluon $l$ as an example, the power counting for $p$ and $l$ reads

\[(p^\perp, p^\perp, p_\perp^\perp) \sim Q(1, \lambda^2, \lambda),\]

\[(l^\perp, l^\perp, l^\perp) \sim Q(\lambda^2, \lambda^2, \lambda)(b = 1, 2).\]

We have

\[2p^\perp l^\perp \sim 2p^\perp l^\perp \sim l^2\]

and the eikonal approximation does not simply work here.

4.2. Eikonalization of couplings between active particles and Glauber gluons in $\mathcal{H}(P, \bar{P}, q^+, q^-)$

In this section, we consider the Glauber couplings of active particles in $\mathcal{H}(P, \bar{P}, q^+, q^-)$ and prove the eikonalization of these couplings. We should emphasize that such eikonalization is independent of the other ends of the Glauber gluons. Hence we consider couplings between Glauber gluons and an arbitrary active particle with the other ends of the Glauber gluons discussed in sections 4.3–4.5.

The distinction between spectators and active particles is discussed in appendix B. According to results in appendix B, the plus-collinear (minus-collinear) active particles are defined as: (1) plus-collinear (minus-collinear) particles coupling to the hard vertex directly; (2) plus-collinear (minus-collinear) particles for which there is a path made up of plus-collinear (minus-collinear) particles and the plus momenta (minus momenta) of them flow into the hard vertex through the path. We discuss these two cases one by one.

\textsuperscript{11} Couplings between collinear particles and soft(ultrasoft) fermions are power suppressed [9, 10].

\textsuperscript{12} According to the formula $\delta(x - y) = \int_{-\infty}^{\infty} du e^{i(x-y)u}$, one has $(\partial_x + \epsilon)\delta(x - y) = \delta(x - y)$ and $(\partial_x - \epsilon)\delta(y - x) = -\delta(y - x)$.

\textsuperscript{13} We do not distinguish the Wilson lines of soft and ultrastiff gluons here as it is irrelevant to the main result in this section.

\textsuperscript{14} In fact, the eikonalization approximation is crucial for the higher order definition of spectator and active in [30].
We start from active particles which participate in the hard collision directly. Without loss of generality, we consider Glauber couplings of a plus-collinear active particle \( k \)

\[
|k^+| \gg |k_-| \gg |k^-|.
\]

(46)

Let us consider couplings between \( k \) and some Glauber gluons \( l_1, \ldots, l_n \) as shown in figure 5.

At the leading power of \( \lambda \) and \( \eta \), we have

\[
\text{Figure 5} = \int \frac{dl_1^-}{2\pi} \ldots \int \frac{dl_n^-}{2\pi} \int \frac{d^0-2k_±_1}{(2\pi)^D-2} \ldots \int \frac{d^0-2k_±_n}{(2\pi)^D-2} \times \frac{1}{k^- + l_i^- + \frac{(k_i^+ + \lambda_i^+ + \cdots + k_n^+ + \lambda_n^+)^2}{2\pi} + i\varepsilon} \times \frac{1}{k^- + l_i^- + \cdots + l_n^- + \frac{(k_i^+ + \lambda_i^+ + \cdots + k_n^+ + \lambda_n^+)^2}{2\pi} + i\varepsilon} \times \cdots \times \frac{1}{l_1^± + i\varepsilon} \ldots \frac{1}{l_n^± + i\varepsilon} \times \mathcal{F}(l_1, \ldots, l_n) \times \text{terms independent of } l_j^- \text{ and } l_j^±(1 \leq j \leq n),
\]

where \( D = 4 - 2\varepsilon \) and \( \mathcal{F}(l_1, \ldots, l_n) \) represents the terms on the other ends of these Glauber gluons.

\[
\text{One can easily verify that}
\]

\[
\int \frac{dl_1^-}{2\pi} \ldots \int \frac{dl_n^-}{2\pi} \left( \frac{1}{k^- + l_i^- + \frac{q_1^2 + \lambda_1^2}{2\pi} + i\varepsilon} \right) \ldots \left( \frac{1}{k^- + l_i^- + \frac{q_n^2 + \lambda_n^2}{2\pi} + i\varepsilon} \right) \times \frac{1}{l_1^± + i\varepsilon} \ldots \frac{1}{l_n^± + i\varepsilon} \times \mathcal{F}(l_1, \ldots, l_n) \times \text{terms independent of } l_j^- \text{ and } l_j^±(1 \leq j \leq n).
\]

(48)

According to similar discussions as those for (49), we see that the couplings between the Glauber gluons \( l_1, \ldots, l_n \) and the active particle \( k' \) eikonalize.

For the Glauber couplings of minus-collinear active particles, we have similar results. In conclusion, Glauber couplings of active particles eikonalize at the leading power of \( \lambda \) and \( \eta \) in \( \mathcal{H}(P, \vec{P}, q^+, q^-) \).

4.3. Glauber gluons exchanged between active and soft particles

We consider active-soft Glauber exchanges in this section. These Glauber gluons can be absorbed into soft Wilson lines according to discussions here.
Without loss of generality, we consider the Glauber gluons exchanged between plus-collinear active particles and soft particles. To specify our discussions, let us consider the Glauber gluons of exchanged between a plus-collinear active particle $k$ and a soft particle $k_s$.

The couplings between $k$ and Glauber gluons read (49)

$$
\int \frac{d^4l_i}{(2\pi)^D} \cdots \int \frac{d^4l_n}{(2\pi)^D} \frac{1}{k^- + l_i^- + \cdots + l_n^- + \frac{(l_i)^2}{2\kappa^2} + i\varepsilon} \cdots
$$

$$
\times \frac{1}{k^- + l_1^- + \cdots + l_n^- + \frac{(l_1)^2}{2\kappa^2} + i\varepsilon}
$$

$$
\times \frac{1}{l_1^\perp + i\varepsilon} \cdots \frac{1}{l_n^\perp + i\varepsilon}
$$

$$(\text{terms independent of } l_i)(1 \leq j \leq n)
$$

$$(\text{terms on the other ends of } l_1, \ldots, l_n)
$$

(51)

at leading power of $\lambda$ and $\eta$, where $D = 4 - 2\varepsilon$. We compare the result with the case that $k$ couple to soft gluons $q_1, \ldots, q_n$. After the eikonal approximation, the couplings between $q_1, \ldots, q_n$ and $k$ read

$$
\int \frac{d^4q_1}{(2\pi)^D} \cdots \int \frac{d^4q_n}{(2\pi)^D} \frac{1}{k^- + q_1^- + \cdots + q_n^- + \frac{(q_1)^2}{2\kappa^2} + i\varepsilon} \cdots
$$

$$
\times \frac{1}{k^- + q_1^- + \cdots + q_n^- + \frac{(q_1)^2}{2\kappa^2} + i\varepsilon}
$$

$$
\times \frac{1}{q_1^\perp + i\varepsilon} \cdots \frac{1}{q_n^\perp + i\varepsilon}
$$

$$(\text{terms independent of } q_j)(1 \leq j \leq n)
$$

$$(\text{terms on the other ends of } q_1, \ldots, q_n)
$$

(52)

4.4. Glauber gluons exchanged between spectators and active particles

We consider active-spectator Glauber exchanges in this section. These Glauber gluons are absorbed into collinear Wilson lines according to discussions here.

Without loss of generality, we consider the Glauber gluons exchanged between a minus-collinear active particle $k$ and a plus-collinear active particle $k(k^+ > 0$ as plus momenta flow from plus-collinear particles to the hard vertex). While coupling to spectators, these Glauber gluons behave like the collinear gluons $A_{\mu,\rho}$ with $\rho^- = 0$. On the other hand, couplings between Glauber and ultrasoft gluons are power suppressed according to the power counting results in table 2. Hence, we can use the eikonal approximation in couplings between Glauber and ultrasoft gluons without affecting the leading power results. So are couplings between these Glauber gluons and other Glauber gluons. In other words, these Glauber gluons behave like collinear gluons $A_{\mu,\rho}$ with $\rho^- = 0$ while coupling to spectators and ultrasoft and Glauber gluons.

On the other end, these Glauber gluons couple to plus-collinear active particles. According to the results of section 4.2, such couplings eikonalize at the leading power of $\lambda$ and $\eta$. The couplings between these Glauber gluons and $k$
The Wilson lines should be past pointing as $4.5$. Glauber gluons exchanged between active particles (read Commun. Theor. Phys.) collinear gluons. We denote the momenta of these collinear hand, one may consider the couplings between exchanges can be absorbed into collinear Wilson lines.\footnote{The Wilson lines should be past pointing as final interactions cancel out in $H(P, P, q^+, q^-)$ as discussed in section 3.2}

at leading power of $\lambda$ and $\eta$, where $D = 4 - 2 \varepsilon$. On the other hand, one may consider the couplings between $k$ and minus-collinear gluons. We denote the momenta of these collinear gluons as $k_1, \ldots, k_n$. The couplings between these collinear gluons and $k$ can be written as

\[ \int \frac{d^D k_1}{(2\pi)^D} \cdots \int \frac{d^D k_n}{(2\pi)^D} \frac{1}{k^+ - k_1^- + \frac{(k_1^+)^2}{2k^+} + i\varepsilon} \cdots \]

\[ \times \frac{1}{k^- + l_1^- + \cdots + l_n^- + \frac{(l_n^-)^2}{2k^+} + i\varepsilon} \times \text{(terms independent of $l_j$) (1} \leq j \leq n) \times \text{(terms on the other ends of $l_1, \ldots, l_n$)} \] 

(53)

at leading power of $\lambda$ and $\eta$, where we have made use of the eikonal approximation in couplings between $k$ and $k_1, \ldots, k_n$. We see that the couplings between $k$ and $l_1, \ldots, l_n$ behave like those between $k$ and $k_1, \ldots, k_n$.

In summary, Glauber gluons exchanged between $\bar{K}$ and $k$ behave like collinear gluons $A^\rho$ with $\rho^- = 0$ on both ends. The propagators of these Glauber gluons behave like those of the collinear gluons $A^\rho$ with $\rho^- = 0$ too. Hence the Glauber gluons exchanged between $\bar{K}$ and $k$ can be absorbed into minus-collinear Wilson lines by extending the collinear region to include the Glauber region in the loop integrals.\footnote{The Wilson lines should be past pointing as final interactions cancel out in $H(P, P, q^+, q^-)$ as discussed in section 3.2}

For general active-spectator Glauber exchanges, we have similar results. In conclusion, active-spectator Glauber exchanges can be absorbed into collinear Wilson lines.

4.5. Glauber gluons exchanged between active particles

We consider active–active Glauber exchanges in this section. They can be absorbed into soft Wilson lines for ladder diagrams \cite{30}. In this section, we extend the result to the general situation.

Without loss of generality, we consider the Glauber gluons$(l_1, l_n)$ exchanged between a plus-collinear active particle $k(k^+ > 0$ as plus momenta flow from plus-collinear particles to the hard vertex) and a minus-collinear active particle $\bar{k}(\bar{k}^+ > 0$ as minus momenta flow from minus-collinear particles to the hard vertex). Glauber couplings of active particles eikonalize according to the result in section 4.2. In other words, we can use the eikonal approximation on both ends of these Glauber gluons. The couplings between $k$ and these Glauber gluons can be written as (49)

\[ \int \frac{d^D l_1}{(2\pi)^D} \cdots \int \frac{d^D l_n}{(2\pi)^D} \frac{1}{k^- + l_1^- + \frac{(l_1^-)^2}{2k^+} + i\varepsilon} \cdots \]

\[ \times \frac{1}{k^- + l_1^- + \cdots + l_n^- + \frac{(l_n^-)^2}{2k^+} + i\varepsilon} \times \text{(terms independent of $l_j$) (1} \leq j \leq n) \times \text{(terms on the other ends of $l_1, \ldots, l_n$)} \] 

(55)

at leading power of $\lambda$ and $\eta$, where $D = 4 - 2 \varepsilon$. One may compare the result with the cases that $k$ couple to minus-collinear gluons $\bar{p}_1, \ldots, \bar{p}_n$ or soft gluons $q_1, \ldots, q_m$. After the eikonal approximation, the couplings between $\bar{p}_1, \ldots, \bar{p}_n$ and $k$ read

\[ \int \frac{d^D \bar{p}_1}{(2\pi)^D} \cdots \int \frac{d^D \bar{p}_n}{(2\pi)^D} \frac{1}{k^- + \bar{p}_1^- + \frac{(\bar{p}_1^-)^2}{2k^+} + i\varepsilon} \cdots \]

\[ \times \frac{1}{k^- + \bar{p}_1^- + \cdots + \bar{p}_n^- + \frac{(\bar{p}_n^-)^2}{2k^+} + i\varepsilon} \times \text{(terms independent of $\bar{p}_j$) (1} \leq j \leq n) \times \text{(terms on the other ends of $\bar{p}_1, \ldots, \bar{p}_n$)} \] 

(56)

and the couplings between $q_1, \ldots, q_n$ and $k$ read

\[ \int \frac{d^D q_1}{(2\pi)^D} \cdots \int \frac{d^D q_n}{(2\pi)^D} \frac{1}{k^- + q_1^- + \frac{(q_1^-)^2}{2k^+} + i\varepsilon} \cdots \]

\[ \times \frac{1}{k^- + q_1^- + \cdots + q_n^- + \frac{(q_n^-)^2}{2k^+} + i\varepsilon} \times \text{(terms independent of $q_j$) (1} \leq j \leq n) \times \text{(terms on the other ends of $q_1, \ldots, q_n$)} \] 

(57)
at leading power of $\lambda$ and $\eta$. Comparing (55) with (56) and (57), we see that the Glauber gluons behave like collinear gluons $A_{a\bar{b}}^{\mu}$ and soft gluons $A_{f}^{\mu}$ while coupling to $k$.

For the couplings between $t_{1}, \ldots, t_{n}$ and $\bar{k}$, we have a similar result and the Glauber gluons behave like collinear gluons $A_{a\bar{b}}^{\mu}$ and soft gluons $A_{f}^{\mu}$ in these couplings.

According to the above discussions, we see that Glauber gluons exchanged between $k$ and $\bar{k}$ behave like soft gluons on both ends. In addition, the propagators of $t_{1}, \ldots, t_{n}$ behave like those of $q_{1}, \ldots, q_{n}$ in the special momenta region $|k_{i}| \ll |k_{j}|$, $|q_{i}| \ll |q_{j}|$.

For general active–active Glauber exchanges, we have similar results. Hence active–active Glauber exchanges can be absorbed into soft Wilson lines by extending the soft region to include the Glauber region in loop integrals.\textsuperscript{18}

5. Cancellation of spectator–spectator and spectator-soft Glauber exchanges in $\mathcal{H}(P, \bar{P}, q^{+}, q^{-})$

In this section, we prove the cancellation of the spectator–spectator and the spectator-soft Glauber exchanges in $\mathcal{H}(P, \bar{P}, q^{+}, q^{-})$. Calculations in [30] show the cancellation of the spectator–spectator type Glauber exchanges in ladder diagrams. According to our discussions in section 3.1, ladder diagrams of Glauber gluons exchanged between spectators in $\mathcal{H}$ can be understood as perturbative series of the object

$$
\sum_{X} \langle p_{1}^{1} p_{1}^{t} \ldots |U^{t}(\infty, -\infty)|X \rangle U(\infty, -\infty)|p_{1}^{1} p_{1}^{t} \ldots \rangle \\
= \langle p_{1}^{1} p_{1}^{t} \ldots |U^{t}(\infty, -\infty)|p_{1}^{1} p_{1}^{t} \ldots \rangle
$$

(58)

where $|p_{1}^{1} p_{1}^{t} \ldots \rangle$ and $|p_{1}^{1} p_{1}^{t} \ldots \rangle$ represent initial spectators and $|X\rangle$ represents possible final states and $U(t_{2}, t_{1})$ represents the time evolution operator corresponding to the spectator–spectator Glauber exchanges. We have made use of the unitarity of the time evolution operator in the above equation. In fact, the spectator–spectator type Glauber ladder diagrams can be viewed as the possible evolution of spectators. While neglecting active-spectator coherence, the evolutions of active particles and spectators are independent of each other according to discussions in section 3.1. In this case, the summation over all possible spectator states is not obstructed by the coherence and the cancellation of the ladder diagrams is the direct result of the unitarity of the time evolution of spectators. That is, cancellation occurs between interactions in the evolution and the conjugation of the evolution of spectators.

In general cases, the active-spectator coherence may obstruct the summation over all possible final spectators. For example, one may consider the diagram shown in figure 6.

The gluon exchanged between spectators and active particles should not be collinear in the first two diagrams. Otherwise the two diagrams do not contribute to the process considered here. In other words, the summation over Glauber interactions of spectators is hampered by the active-spectator coherence in figure 6. One should deal with the active-spectator coherence carefully to get the cancellation of spectator-spectator and spectator-soft Glauber exchanges.

If the Glauber interactions occur after the spectator-active and the soft-active interactions like the first diagram in figure 7 then the summation over all states after the spectator-active and the soft-active interactions is inclusive enough for the spectator–spectator and the spectator-soft Glauber exchanges.

The cancellation (58) can be extended into this case even if there are spectator-active and soft-active interactions. It seems important for us to exclude spectator-active and soft-active interactions after spectator–spectator and spectator-soft Glauber exchanges like those in figure 6. However, instated of the time evolution of collinear and soft states, we find it convenient to consider the evolution of these states along a nearly light-like direction. This is displayed explicitly in the following sections.

According to the discussions in section 4, Glauber gluons coupling to active particles should be absorbed into zero bins of collinear or soft gluons. Hence we view Glauber interactions of active particles as collinear or soft interactions of these particles. While referring to Glauber interactions, we always mean the spectator–spectator and the spectator-soft Glauber exchanges in the following texts.

5.1. $\vec{n}_{+}$ $x$-evolution in $\mathcal{H}(P, \bar{P}, q^{+}, q^{-})$

It is convenient to consider the $x^{+}$ (or $x^{-}$) evolution instead of the time evolution of states in $\mathcal{H}(P, \bar{P}, q^{+}, q^{-})$. Such evolution is crucial in proofs of the factorization theorem of Drell–Yan process in perturbative QCD [4]. In this section, we consider a time-like evolution that is approximately equivalent to $x^{+}$ (or $x^{-}$) evolution.

Let us bring in a time-like direction $\vec{n}_{+}$ and a space-like direction $\vec{n}_{\perp}$ at first

$$
\vec{n}_{+}^{t} \equiv (\vec{n}_{+}^{t+}, \vec{n}_{+}^{t-}, \vec{n}_{+}^{t_{0}}) \equiv \frac{1}{\sqrt{2}} \left( \frac{1}{\sqrt{\lambda \omega}}, \sqrt{\lambda \omega}, 0 \right)
$$

(59)

where $\omega$ is a positive constant of order 1. We have

$$
\vec{n}_{+}^{2} = 1, \quad \frac{n_{+}^{z}}{n_{+}^{+}} = \lambda \omega
$$

$$
\vec{n}_{+}^{2} = -1, \quad \frac{n_{+}^{z}}{n_{+}^{+}} = -\lambda \omega.
$$

(60)

That is to say $\vec{n}_{+}^{t} (\vec{n}_{\perp})$ is time-like (space-like) and nearly collinear to the plus direction.
One can choose a reference system so that
\[ \tilde{\tau}^\mu_\tau (\tilde{\tau}^0_\tau, \tilde{\tau}^1_\tau) \rightarrow (1, \vec{0}) \] (61)
in the new reference system. According to the skills in section 3.2 one has
\[
\sum_X \int d^4x_1 \int d^4x_2 \int d^4x_3 \int d^4x_4 \int d^4ze^{i\eta z} \\
\times \langle H_2 | T \{ O(x_1) \tilde{O}(x_2) J(z) U^+(\infty, -\infty) \} | H(q) X \rangle \\
\times \langle H(q) X | T \{ \tilde{O}(x_4) \tilde{O}(x_3) J(0) U(\infty, -\infty) \} | H_2 \rangle \\
= \sum_X \int d^4x_1 \int d^4x_2 \int d^4x_3 \int d^4x_4 \int d^4ze^{i\eta z} \\
\times \langle H_2 | O(x_1) \tilde{O}(x_2) J(z) U^+ \rangle \\
\times \langle \max \{ \tilde{\tau}^1_\tau, z, 0 \}, -\infty) | H(q) X \rangle \\
\times \langle H(q) X | T \{ J(0) U(\max \{ \tilde{\tau}^1_\tau, z, 0 \}, -\infty) \} | \rangle \\
\times \langle x_3 | \tilde{O}(x_4) | H H_2 \rangle
\] (62)
in the new reference system.

According to the Lorentz invariance of QCD, one has
\[
\mathcal{H}(P, \tilde{P}, q^+, q^-) \\
\equiv \int \frac{d^4q}{(2\pi)^4} \sum_X \int d^4x_1 \int d^4x_2 \int d^4x_3 \int d^4x_4 \int d^4ze^{i\eta z} \\
\times \langle PP | \tilde{O}(x_1) \tilde{O}(x_2) J(z) U^+ \rangle \\
\times \langle \max \{ \tilde{\tau}^1_\tau, z, 0 \}, -\infty) | H(q) X \rangle \\
\times \langle H(q) X | T \{ J(0) U(\max \{ \tilde{\tau}^1_\tau, z, 0 \}, -\infty) \} | \rangle \\
\times \langle x_3 | \tilde{O}(x_4) | PP \rangle
\] (63)
in the center of mass frame of initial hadrons, where \( U^+ \) represents the evolution operator along the \( \tilde{\tau}^1_\tau \times \) direction. That is to say, interactions with the coordinates \( \tilde{\tau}^1_\tau \times \) greater than that of the hard collision cancel out in \( \mathcal{H}(P, \tilde{P}, q^+, q^-) \).\(^{20}\)

5.2. Couplings involving Glauber gluons \( A_{+G} \) in \( \mathcal{H}(P, \tilde{P}, q^+, q^-) \)

In this section, we consider the Glauber gluons \( A_{+G} \) exchanged between plus-collinear and other particles. We prove the cancellation of the Glauber couplings at vertexes of which the coordinates \( \tilde{\tau}^1_\tau \times \) are smaller than those of some vertexes free from \( A_{+G} \) (except for the hard vertex).

To distinguish vertexes involving \( A_{+G} \) from others, we denote the coordinates of couplings between \( A_{+G} \) and plus-collinear particles as \( y_i \) and those between \( A_{+G} \) and minus-collinear and soft particles as \( y'_i \) and those free from \( A_{+G} \) as \( z_i \). Without loss of generality, we consider a Glauber gluon \( l_i \) exchanged between the two vertexes \( y_i \) and \( y'_i \) can be neglected at the vertex \( y_i \).\(^{21}\)

The propagator of \( l_i \) is independent of \( l_i^+ \) and \( l_i^- \) at the leading power of \( \lambda \) and \( \eta \). As a result, \( \mathcal{H}(P, \tilde{P}, q^+, q^-) \) relies on \( l_i^+ \) and \( l_i^- \) only through the term
\[
\exp(-i l_i^+ y_i^- + i l_i^- y_i'^-) \tag{64}
\]
and the integrals over \( l_i^+ \) and \( l_i^- \) read
\[
\int \frac{dl_i^-}{2\pi} \frac{dl_i^+}{2\pi} \exp(-i l_i^+ y_i^- + i l_i^- y_i'^-) = \delta(y_i^-) \delta(y_i'^-) \tag{65}
\]
The vertex \( y_i \) and the hard vertex are connected through plus-collinear particles. For a plus-collinear particle with momenta \( \xi P + k \xi \sim O(1), \xi \ll Q \lambda \) for plus-collinear external lines, the propagator reads
\[
\int \frac{dk}{(2\pi)^4} \frac{N(P)}{2k^+ k^- + (k_i)^2 + i\varepsilon} e^{-i(k_i - x_2)} \propto \delta(x_1^+ - x_2^-), \tag{66}
\]
where \( N(P) \) represents possible numerators in the propagators.
We have
\[
y_i^- = 0, \quad \delta(y_i^+) = \frac{\lambda \omega}{2} \delta(\tilde{\tau}^1_\tau \cdot y_i) \tag{67}
\]
on the left side of the final cut and
\[
y_i'^- = z^-, \quad \delta(y_i'^+) \approx \delta(y_i'^- - z^+) = \frac{\lambda \omega}{2} \delta(\tilde{\tau}^1_\tau \cdot y_i - \tilde{\tau}^1_\tau \cdot z) \tag{68}
\]
on the right side of the final cut, where we have made use of the fact
\[
z^+ \sim z^- \sim 1/Q. \tag{69}
\]
For the vertex \( y_i'^+ \), we have
\[
\delta(y_i'^+) \approx \frac{1}{\sqrt{2\lambda \omega}} \delta(\tilde{\tau}^1_\tau \cdot y_i') \tag{70}
\]
\(^{21}\) \( l_i \ll O, \quad (Q\lambda) \ll P^+ \sim O(0) \quad \text{and} \quad \tilde{l}_i \sim O \quad (Q\lambda \ll P^- \sim O(Q), \quad \text{where} \ P^+ \text{represents the plus momenta of plus-collinear particles and} \ P^- \text{and} \ \tilde{l}_i \text{represent the minus momenta of soft and minus-collinear particles.} \)
on the left side of the final cut and
\[ \delta(y_i^+) \simeq \delta(y_i^+ - z_i) = \frac{1}{\sqrt{2\lambda\omega}} \delta(\vec{n}_+ \cdot y_i^+ - \vec{n}_+ \cdot y_1) \] (71)
onumber
on the right side of the final cut. Hence
\[ \vec{n}_+ \cdot z_i < 0 \Rightarrow \vec{n}_+ \cdot z_i \leq \vec{n}_+ \cdot y_1, \quad \vec{n}_+ \cdot z_i \leq \vec{n}_+ \cdot y_i^+ \] (72)
onumber
on the left side of the final cut and
\[ \vec{n}_+ \cdot z_i \leq \vec{n}_+ \cdot z \Rightarrow \vec{n}_+ \cdot z_i \leq \vec{n}_+ \cdot y_1, \quad \vec{n}_+ \cdot z_i \leq \vec{n}_+ \cdot y_i^+ \] (73)
onumber
on the right side of the final cut. That is to say
\[ \vec{n}_+ \cdot z_i \leq \vec{n}_+ \cdot y_1, \quad \vec{n}_+ \cdot z_i \leq \vec{n}_+ \cdot y_i^+ \] (74)
onumber
on both sides of the final cut.
For other vertices involving the Glauber gluons \( A_{G} \), we repeat the calculations and get similar results. We conclude that we divide the Glauber gluons \( A_{G} \) should couple to collinear or soft particles at vertexes with coordinates \( \vec{n}_+ \cdot y_i \) greater than those of vertexes free from \( A_{G} \)(except for the hard vertex), otherwise effects of the Glauber gluons \( A_{G} \) cancel out at leading power of \( \lambda \) and \( \eta \).

5.3. Cancellation of Glauber gluon \( A_{G} \)
In this section, we prove the cancellation of the Glauber gluons \( A_{G} \), which are exchanged between plus-collinear and minus-collinear spectators or between plus-collinear spectators and soft particles. According to the results of section 5.2, the Glauber gluons \( A_{G} \) should couple to collinear or soft particles at vertexes with the coordinates \( \vec{n}_+ \cdot x \) greater than those of vertexes free from \( A_{G} \)(except for the hard subprocess). After the absorption of Glauber gluons coupling to active particles into collinear and soft Wilson lines, we have
\[ \mathcal{H}(P, P, q^+, q^-) = \sum_{m_0, m_1} \int \frac{d^2q_z}{(2\pi)^2} \frac{d^4q_z}{(2\pi)^4} \int d^4z e^{iq_z} \langle PP| T \mathcal{O}^+|S_+, S_-, W_+, W_-\rangle(x)|X\rangle \times \langle X| T \mathcal{O}(S_+, S_-, W_+, W_-\rangle(0)|PP\rangle \otimes h(z), \] (75)
where \( h(z) \) represents contributions of the hard scattering subprocess and \( \mathcal{O}(S_+, S_-, W_+, W_-) \) represents the Wilson line structure of the hard vertex and
\begin{align*}
W_0(x) &= P \exp(i \int_{-\infty}^{0} ds A_+^{-1}(0, x^- + s, 0) \\
S_0(x) &= P \exp(i \int_{-\infty}^{0} ds A_-(x^+ + s, 0, 0) \\
W_0(x) &= P \exp(i \int_{-\infty}^{0} ds A_+^{-1}(x^- + s, 0, 0) \\
S_0(x) &= P \exp(i \int_{-\infty}^{0} ds A_+^{-1}(x^- + s, 0, 0). \end{align*}
(76)

The \( \vec{n}_+ \cdot x \)-evolution of these Wilson lines is determined by the operator
\[ U_{SCET}(\vec{n}_+ \cdot x_1, \vec{n}_+ \cdot x_2), \] (77)
where \( U_{SCET} \) represents the \( n \cdot x \) evolution operator related to the action (7) and
\[ A^n(\chi) = U_{SCET}(\vec{n}_+ \cdot x, 0) A^n(0, \vec{n}_+ \cdot x, x) U_{SCET}^{-1}(\vec{n}_+ \cdot x, 0) \] (78)
with \( A^n = A_n^+, A_n^-, A_n^\mu \).

According to the results of section 5.2, \( A_{G} \) couple to spectators and soft particles at vertexes with the coordinates \( n \cdot x \) greater than those of vertexes free from \( A_{G} \) in the matrix elements
\[ \langle X| T \mathcal{O}(S_+, S_-, W_+, W_-\rangle(0)|PP\rangle \] (79)
and
\[ \langle PP| T \mathcal{O}(S_+, S_-, W_+, W_-\rangle(x)|X\rangle. \] (80)

We make the perturbative expansion about interactions free from \( A_{G} \) and have
\[ \mathcal{H}(P, P, q^+, q^-) = \sum_{m_0, m_1} \int \frac{d^2q_z}{(2\pi)^2} \frac{d^4q_z}{(2\pi)^4} \int d^4z e^{iq_z} \times \langle PP| T \mathcal{O}^+_{m_0}(s) U_{G}^{-1}(\vec{n}_+ \cdot z, s)|X\rangle \times \langle X| U_{G}(0, s_2) \mathcal{O}_{m_0}(s_2)|PP\rangle \otimes h(z), \] (81)
where \( \mathcal{O}^+_{m_0} \) and \( \mathcal{O}_{m_0} \) represent the \( m_0 \)th and \( m_0 \)th perturbative order interactions free from \( A_{G} \) and \( U_{G} \) represents the \( n \cdot x \) evolution operator corresponding to the spectator–spectator and spectator-soft Glauber exchanges.\(^{22}\)

Considering that
\[ \vec{n}_+ \cdot x \simeq 0, \quad \vec{n}_- \cdot z \] (82)
for interactions involving the Glauber gluons \( A_{G} \)\(^{23}\) as discussed in section 5.2, we have
\[ U_{G}(0, s_2) \simeq U_{G}(\infty, -\infty) \] (83)
on the left side of the final cut and
\[ U_{G}^{-1}(\vec{n}_+ \cdot z, s) \simeq U_{G}^{-1}(\infty, -\infty) \] (84)
\(^{22}\) There could be spectator–spectator ladder diagrams with soft gluons connecting different ladders induced by soft-Glauber couplings in the effective action (7). We show some examples of these diagrams in figure 8. Effects of these diagrams should be absorbed into \( U_{G} \) once they do not bring in active-spectator coherence with the coordinates \( \vec{n}_+ \cdot x \) greater than those of the hard collision. Otherwise the diagrams can be simply neglected according to discussions in section 5.2.
\(^{23}\) While considering the next leading power results, one has \( \vec{n}_+ \cdot x \sim O(\lambda) \) and \( \vec{n}_- \cdot x = 0, \vec{n}_- \cdot z \simeq \vec{n}_+ \cdot z. \)
and other Glauber gluons diagrams with the Glauber gluons from the unitarity of the spectator-soft type Glauber exchanges of $A$.

In this section, we consider the Glauber gluons $A_+$ at leading power of $\omega$ on the right side of the final cut. Hence

$$\mathcal{H}(P, \bar{P}, q^+, q^-)$$

$$\approx \sum_{m_{m_1} m_{m_2}} \int \frac{d^2 q_{\perp}}{(2\pi)^2} \sum_{X} \int d^2 z_1 \int_{-\infty}^{\infty} dz_1 \int_{-\infty}^{0} dz_2 e^{i q \cdot z_2}$$

$$\times \langle PP|\mathcal{O}^b_{m_1}(s_1)U_{+G}^{+}\{\infty, -\infty\}|X\rangle$$

$$\times \langle X|U_{+G}(\infty, -\infty)\mathcal{O}_{m_2}(s_2)|PP\rangle \otimes h(z)$$

$$- \sum_{m_{m_1} m_{m_2}} \int \frac{d^2 q_{\perp}}{(2\pi)^2} \sum_{X} \int d^2 z_1 \int_{-\infty}^{\infty} dz_1 \int_{-\infty}^{0} dz_2 e^{i q \cdot z_2}$$

$$\times \langle PP|\mathcal{O}^b_{m_1}(s_1)|X\rangle \langle X|\mathcal{O}_{m_2}(s_2)|PP\rangle \otimes h(z)$$

$$= \sum_{m_{m_1} m_{m_2}} \int \frac{d^2 q_{\perp}}{(2\pi)^2} \sum_{X} \int d^2 z_1 \int_{-\infty}^{\infty} dz_1 \int_{-\infty}^{0} dz_2 e^{i q \cdot z_2}$$

$$\times \langle PP|\mathcal{O}^b_{m_1}(s_1)|X\rangle \langle X|\mathcal{O}_{m_2}(s_2)|PP\rangle \otimes h(z)$$

(85)

at leading power of $\lambda$ and $\eta$. That is, the spectator–spectator and spectator-soft type Glauber exchanges of $A_{+G}$ cancel out in $\mathcal{H}(P, \bar{P}, q^+, q^-)$. We emphasize that the cancellation originates from the unitarity of the $\hat{n}_{-} \cdot x$ evolution operator induced by interactions between spectators and soft particles. Hence, such cancellation is independent of the details of these interactions.

5.4. Cancellation of Glauber gluons $A_{-G}$

In this section, we consider the Glauber gluons $A_{-G}$ and outline the proof of cancellation of $A_{-G}$.

According to the results of sections 4 and 5.3, contributions of $A_{+G}$ can be absorbed into those of collinear gluons $A_{+G}$ and soft gluons $A_{s}$. Hence we can drop the Glauber gluons $A_{-G}$ in calculations of $\mathcal{H}(P, \bar{P}, q^+, q^-)$. After this, we consider the $\hat{n}_{-} \cdot x$-evolution, where the direction $\hat{n}_{-} \cdot x$ is defined as

$$\hat{n}_{-} \cdot x \equiv (\hat{n}_{-}^+, \hat{n}_{-}^-, \hat{n}_{-}^0) \equiv \frac{1}{\sqrt{2}} (\sqrt{\lambda \omega}, \frac{1}{\sqrt{\lambda \omega}}, 0),$$

(86)

with $\omega$ a positive constant of order 1. Attributing to proofs similar to those in section 5.1, interactions with the coordinates $\hat{n}_{-} \cdot x$ greater than those of the hard collision cancel out in $\mathcal{H}(P, \bar{P}, q^+, q^-)$. We have

$$\mathcal{H}(P, \bar{P}, q^+, q^-)$$

$$\equiv \int \frac{d^2 q_{\perp}}{(2\pi)^2} \sum_{X} \int d^2 x_1 \int d^2 x_2 \int d^2 x_3 \int d^2 x_4 \int d^2 z_3 e^{i q \cdot z_3}$$

$$\times \langle PP|\mathcal{O}^b_{m_1}(s_1)\mathcal{O}^b_{m_2}(s_2)|PP\rangle \otimes h(z)$$

$$\times \langle H(q)X|X|U_{+G}(\max\{\hat{n}_{-} \cdot z, 0\}, -\infty)\rangle$$

$$\times \langle (x_3)\mathcal{O}(x_4)|PP\rangle$$

(87)

in the center of mass system of initial hadrons, where $U_{+G}$ represents the evolution operator along the $\hat{n}_{-} \cdot x$ direction with the Glauber gluons $A_{+G}$ removed from the effective action.

We then repeat the discussions in section 5.2 and see that $A_{-G}$ should couple to collinear or soft particles at vertexes of which the coordinates $\hat{n}_{-} \cdot x$ are smaller than those of vertexes free from $A_{-G}$ (except for the hard vertexes). Otherwise, $A_{-G}$ cancels out at the leading power of $\lambda$ and $\eta$.

According to proofs similar to those in section 5.3, we conclude that spectator–spectator and spectator-soft type Glauber gluons (both $A_{+G}$ and $A_{-G}$) cancel out in $\mathcal{H}(P, \bar{P}, q^+, q^-)$.

In conclusion, spectator–spectator and spectator-soft Glauber exchanges cancel out in $\mathcal{H}(P, \bar{P}, q^+, q^-)$ and other Glauber gluons are equivalent to zero bins of collinear and soft gluons according to discussions in sections 4 and 5. We should mention that such cancellation works at cross section level instead of amplitude level and eikonalization of collinear-Glauber.
couplings should be viewed as a cross section instead of Lagrangian level result even if one is concerned with inclusive processes. Therefore effective theories like the original SCET, in which Glauber modes are absent or absorbed into collinear and soft Wilson lines, only work at cross section level for processes inclusive enough. In other words, the collinear-Glauber couplings do not eikonalize in the Lagrangian for general cases. Let us start from couplings involving plus-collinear particles and Glauber gluons here, \( \bar{\eta}_{+} \cdot x \)-like, they are approximately light-like (with corrections of order \( \lambda \)) in the center of mass frame of initial hadrons. Hence it is not surprising that the \( \bar{\eta}_{+} \cdot x \)-evolution (\( \bar{\eta}_{+} \cdot x \)-evolution) of collinear and soft particles is approximately equivalent to the light-cone evolution of them.

### 6.1. \( \bar{\eta}_{+} \times x \)-evolution of plus-collinear particles

Let us start from couplings involving plus-collinear particles at an arbitrary point \( y \).

We denote the momenta of particles at the vertex \( y \), \( q_{\ell}(\ell = 1, \ldots) \), which are defined as flow into the vertex \( y \). Interactions with the coordinates \( \bar{\eta}_{+} \cdot x \) greater than those of the hard collision cancel out in \( \mathcal{H}(P, \bar{P}, q^{+}, q^{-}) \). Hence

\[
\bar{\eta}_{+} \cdot y \leq \max(\bar{\eta}_{+} \cdot z, 0),
\]

where \( z \) and \( 0 \) are the coordinates of the hard collisions. At the vertex \( y \), one has

\[
\int d^{4}y \exp\left(-i\sum_{\ell}q_{\ell} \cdot y\right)\Theta\left(\max(\bar{\eta}_{+} \cdot z, 0) - \bar{\eta}_{+} \cdot y\right) = (2\pi)^{3}\delta(\bar{\eta}_{+} \cdot \sum_{\ell}j_{\ell})\delta^{(2)}(J_{i+}) \times i\exp\left(\sum_{\ell}q_{\ell} \cdot l_{\ell} \max(\bar{\eta}_{+} \cdot z, 0)\right) \sum_{\bar{n}_{+} \cdot l_{\ell} + i\varepsilon},
\]

where the term

\[
\exp\left(\sum_{\ell}q_{\ell} \cdot l_{\ell} \max(\bar{\eta}_{+} \cdot z, 0)\right),
\]

contributes to the momenta conservation \( \delta \)-function of the hard subprocess and can be dropped here.

Without loss of generality, we define \( l_{\ell} \) as the momentum of a plus-collinear internal line connecting to \( y \) and have

\[
l_{\ell}^{+} = -\sum_{i=1}^{\ell}l_{i}^{+} + \frac{1}{N}\sum_{j=1}^{N}l_{j}^{+}.
\]

Considering that

\[
l_{\ell}^{+} \sim O(Q^{2})
\]

for plus-collinear particles and Glauber gluons \( A_{i,G}^{\ell} \), we have

\[
l_{\ell}^{+} \approx -\sum_{i=1}^{\ell}l_{i}^{+} \Rightarrow \delta(\bar{n}_{+} \cdot \sum_{i}l_{i}) \approx \sqrt{\frac{2}{\lambda}}\delta(\sum_{i}l_{i}^{+})
\]

\[
\frac{i\delta(\bar{n}_{+} \cdot \sum_{i}l_{i})}{\sum_{i}n_{i}^{+} \cdot l_{i} + i\varepsilon} \approx \frac{i\delta(\sum_{i}l_{i}^{+})}{\sum_{i}n_{i}^{+} \cdot l_{i} + i\varepsilon}.
\]

That is to say, the couplings involving a plus-collinear particle \( l_{\ell} \) (except for hard interactions) can be calculated through the Feynman diagram skill except that one should make the substitution

\[
(2\pi)^{3}\delta^{(4)}(l_{\ell}^{+} + \ldots) \rightarrow (2\pi)^{3}\delta(l_{\ell}^{+} + \ldots)\delta^{(2)}(l_{i}^{+} + \ldots)
\]

\[
\times \frac{1}{l_{i}^{+} + \ldots + i\varepsilon},
\]

for the \( \delta \)-function of momenta conservation.

We then integrate out the minus momenta of plus-collinear particles \( k \). \( \mathcal{H} \) relies on \( l^{+} \) through the propagator

\[
N(l^{+}) \sim \frac{1}{2l^{+}l^{-} + (l_{\perp})^{2} + i\varepsilon},
\]

and the two vertexes

\[
\frac{1}{-l^{-} + q_{i}^{-} + \ldots + i\varepsilon}, \quad \frac{1}{l^{-} + q_{J}^{-} + \ldots + i\varepsilon},
\]

where \( q_{i} \) and \( q_{J} \) represent the momenta of Glauber and soft (ultrasoft) gluons. We take \( l^{+} > 0 \) as plus momenta of plus-collinear particle flow from the initial particle \( P \) to the hard vertex or final cut. We can then integrate out \( l^{+} \) by taking the residue of poles located in the upper half plane. After this

\[24\] Couplings between collinear and soft (ultrasoft) particles eikonalize. Hence the coordinates \( x \) of plus-collinear particles remain unchanged in these couplings. Be compatible with our discussions for couplings between Glauber gluons here, \( \bar{\eta}_{+} \cdot x \) and \( x^{+} \)-order of plus-collinear states are equivalent to each other even if one considers soft (ultrasoft) interactions of these states.
operation, we have
\[
\frac{1}{-i^+ + \cdots + q_i^+ + \cdots + i\varepsilon} \frac{1}{\mathcal{N}(\bar{l}')} \times \frac{1}{2 i^{2-} + (\bar{l}_-)^2 + i\varepsilon}.
\]

We repeat this procedure and get terms shaped like
\[
\frac{1}{P^- + \sum q_i^- + \sum \frac{d_{ij} \cdot \bar{l}_j}{2k_i} + \sum \frac{d_{ij} \cdot \bar{l}_j}{2k_i} - i\varepsilon},
\]
on the left side of the final cut and
\[
\frac{1}{P^- + \sum q_i^- + \sum \frac{d_{ij} \cdot \bar{l}_j}{2k_i} + \sum \frac{d_{ij} \cdot \bar{l}_j}{2k_i} + i\varepsilon},
\]
on the right side of the final cut, where \(q_i(q_j)\) represents the momenta of Glauber and soft(ultrasoft) gluons and \(l_i(l_j)\) represents the momenta of plus-collinear internal lines and \(k_i(k_j)\) represents the momenta of on-shell plus-collinear particles. The summation of \(l_i(l_j)\) is made over states with the coordinates \(\vec{n}_+\cdot x\) between two given vertices. The summation of \(q_i(q_j)\) and \(k_i(k_j)\) is made over states connecting to vertices with the coordinates \(\vec{n}_+\cdot x\) smaller than those of \(l_i(l_j)\). We should mention that \(l_i(l_j)\) may rely on transverse momenta of external lines and Glauber and soft(ultrasoft) gluons through the \(\delta\)-function of transverse momenta conservation.

Compared to the light-cone perturbative series in [4], terms of the type (99) and (100) correspond to contributions of states with the coordinates \(x^+\) (or the coordinates \(\vec{n}_+\cdot x\)) smaller than that of the hard collision. Interactions with the coordinates \(\vec{n}_+\cdot x\) greater than that of the hard collision cancel out as demonstrated in section 5.1.

6.2. \(\vec{n}_+\cdot x\) evolution of minus-collinear and soft particles

We then consider couplings free from plus-collinear particles. Considering that
\[
l_i^\mu \sim O(Q\lambda), \quad l_i^\mu \sim O(Q\lambda^2)
\]
for soft and ultrasoft particles and
\[
l_i^\mu \lesssim O(Q\lambda)
\]
for Glauber gluons, we have\(^{25}\)
\[
\frac{i\delta(\vec{n}_+\cdot \sum l_j)}{\sum \vec{n}_+\cdot l_j + i\varepsilon} \approx \frac{i\delta(\sum l_j)}{\sum l_j + i\varepsilon}.
\]
Hence we should make the substitution
\[
(2\pi)^4 \delta(\sum l_j^\mu) \rightarrow (2\pi)^3 \delta(\sum l_j^\mu) \delta^{(2)}(\sum l_j),
\]
in vertexes free from plus-collinear particles.

We then integral out the plus momenta of minus-collinear particles. For an arbitrary internal line \(\bar{l}'\), \(\mathcal{H}\) relies on \(\bar{l}^-\) through the propagator
\[
\frac{N(k)}{2 i^{2+} \bar{l}^- + (\bar{l}_-)^2 + i\varepsilon},
\]
and the two vertexes
\[
\frac{1}{-\bar{l}^- + \cdots + q_i^- + \cdots + i\varepsilon}, \quad \bar{l}^+ + \cdots + q_j^+ + \cdots + i\varepsilon;
\]
where \(q_i(q_j)\) represent the momenta of Glauber gluons and ultrasoft particles, which are defined as flow into \(\bar{l}'\). We have dropped terms independent of \(\bar{l}'\). We choose the direction of \(\bar{l}'\) so that \(\bar{l}' > 0\). We then integrate out \(\bar{l}'\) by taking the residues of poles in the upper half plane. After this operation, we can make the substitution
\[
\bar{l}' \rightarrow q_i^- + \cdots + i\varepsilon
\]
in remaining terms. That is
\[
\frac{1}{-\bar{l}^- + \cdots + q_i^- + \cdots + i\varepsilon} \frac{1}{\bar{l}^+ + \cdots + q_j^+ + \cdots + i\varepsilon} \times \frac{N(l)}{2 i^{2+} \bar{l}^- + (\bar{l}_-)^2 + i\varepsilon} \times \frac{1}{q_i^- + \cdots + \frac{d_{ij} \cdot l_j}{2k_i} + i\varepsilon}
\]
We repeat the procedure and get terms shaped like
\[
\frac{1}{P^- + \sum q_i^- + \sum \frac{d_{ij} \cdot \bar{l}_j}{2k_i} + \sum \frac{d_{ij} \cdot \bar{l}_j}{2k_i} + i\varepsilon},
\]
on the left side of the final cut and
\[
\frac{1}{P^- + \sum q_i^- + \sum \frac{d_{ij} \cdot \bar{l}_j}{2k_i} + \sum \frac{d_{ij} \cdot \bar{l}_j}{2k_i} - i\varepsilon},
\]
on the right side of the final cut, where \(q_i(q_j)\) represents the momenta of Glauber and soft(ultrasoft) gluons and \(l_i(l_j)\) represents the momenta of minus-collinear particles. The summation of \(l_i(l_j)\) is made over states with the coordinates \(\vec{n}_+\cdot x\) between two given vertices. The summation of \(q_i(q_j)\) is made over states connecting to vertices with the coordinates \(\vec{n}_+\cdot x\) smaller than those of \(l_i(l_j)\).

\(^{25}\)Couplings between soft and ultrasoft particles are power suppressed and can be neglected. So are couplings between Glauber gluons and ultrasoft particles and those between Glauber gluons.
Terms of the type (99) and (100) correspond to contributions of states with the coordinates x as \( \vec{n}_+ \cdot x = \frac{1}{2\sqrt{2\omega}}(1 + O(\lambda)) \) smaller than that of the hard collision in \( x \)-ordered perturbation theory. Minus-collinear states with the coordinates \( \vec{n}_+ \cdot x > 1 \) greater than that of the hard collision cancel out in \( \mathcal{H}(P, \vec{P}, q^+, q^-) \).

6.3. Glauber graphs in \( \mathcal{H}(P, \vec{P}, q^+, q^-) \)

According to the discussions in section 5.2, the Glauber gluons \( A_{1G} \) should couple to other particles at vertexes with the coordinates \( \vec{n}_+ \cdot x > 1 \) greater than those free from \( A_{1G} \) (except for the hard vertex). The result is helpful for us to exclude graphs in which the summation over all spectator–spectator and spectator-soft Glauber exchanges is hampered by the active-spectator coherence.

Let us start from an example shown in figure 9, in which there are Glauber couplings of a plus-collinear particle before its non-Glauber couplings (not the hard vertex). According to the substitution rule (94), we have

\[
\text{Figure 9} = \int \frac{dq^+}{2\pi} \int \frac{dp_1^-}{2\pi} \int \frac{dp_2^-}{2\pi} \int \frac{dq^-}{2\pi} \int \frac{dk^-}{2\pi} \times \frac{2q^+ + (q_2^-)^2 + i\varepsilon}{2(P^+ + p^+ + q^-)k^+ + (P_+ - p_+ - q_+ + l_+)^2 + i\varepsilon} \times \frac{1}{(P^+ + p^+ - q^+)^2 + 2p_+ + p_+ - q_+ + l_+)^2 + i\varepsilon} \times \frac{1}{(P^+ + q^+)^2 + 2p_+ + p_+ - q_+ + l_+)^2 + i\varepsilon} \times \frac{1}{(P^+ - k^- - p_2^- + 1 + e + q^-) + (q_2^- + p_+ + q_+ - l_+)^2 + i\varepsilon} \times \frac{1}{(P_1^- - q^- + \frac{1}{2p_2^-} + i\varepsilon} \times \text{(terms independent of } l^- \text{ and } p_j^-) \times (j = 1, 2).
\]

After the cancelation of interactions with the coordinates \( \vec{n}_+ \cdot x > 1 \) greater than that of the hard collision. We integrate out \( q^- \) and \( k^- \) and \( p_j^- (j = 1, 2) \) and have

\[
\text{Figure 9} = \int \frac{dq^+}{2\pi} \int \frac{dp_1^-}{2\pi} \int \frac{dp_2^-}{2\pi} \int \frac{dq^-}{2\pi} \int \frac{dk^-}{2\pi} \times \frac{\theta(q^+)}{2(P^+ + p^+ + q^-)k^+ + (P_+ - p_+ - q_+ + l_+)^2 + i\varepsilon} \times \frac{1}{(P^+ + p^+ - q^+)^2 + 2p_+ + p_+ - q_+ + l_+)^2 + i\varepsilon} \times \frac{1}{(P^+ + q^+)^2 + 2p_+ + p_+ - q_+ + l_+)^2 + i\varepsilon} \times \frac{1}{(P^+ - k^- - p_2^- + 1 + e + q^-) + (q_2^- + p_+ + q_+ - l_+)^2 + i\varepsilon} \times \frac{1}{(P_1^- - q^- + \frac{1}{2p_2^-} + i\varepsilon} \times \text{(terms independent of } l^- \text{)}.
\]

We see that all poles of \( l^- \) locate in the lower half plane and have

\[ \text{Figure 9} = 0. \quad (113) \]

For general diagrams, one may consider the flow of the plus momenta of plus-collinear particles. According to the discussions in section 6.1, the plus momenta should flow from these collinear particles to the hard vertex or the final cut. We consider the couplings involving these collinear particles through the order of the flow\(^{26}\). We can define the momenta of these particles so that the plus momenta of these particles are positive. We then integrate out the minus momenta of these collinear particles by taking the residues of poles of the vertex from which the plus momenta of the collinear particles flow out of. If we meet Glauber couplings before non-Glauber couplings in some diagrams then there are some Glauber gluons of which all poles of the minus momenta (no less than two) locate in the upper half plane as shown in the example figure 9 and the formula (112)\(^{27}\). Hence these diagrams vanish after the cancellation of interactions with the coordinates \( \vec{n}_+ \cdot x > 1 \) greater than that of the hard collision.

For Glauber couplings of minus-collinear and soft particles, one may consider the flow of the minus momenta of these particles. According to the discussions in section 6.2, we can consider these couplings through the order of the flow and meet the hard vertex or the final cut finally. We define the momenta of these particles so that the minus momenta of these particles are positive. Be similar to the Glauber couplings of plus-collinear particles, if we meet Glauber couplings before non-Glauber couplings in some diagrams then all poles of the minus momenta (no less than two) of some Glauber gluons locate in the upper half plane. These diagrams do not contribute to \( \mathcal{H}(P, \vec{P}, q^+, q^-) \).

In summary, the active-spectator coherence with the coordinates \( \vec{n}_+ \cdot x > 1 \) greater than those of Glauber interactions cancel out in \( \mathcal{H}(P, \vec{P}, q^+, q^-) \). As a result, the summation over final states in \( \mathcal{H}(P, \vec{P}, q^+, q^-) \) is inclusive enough for spectator–spectator and spectator-soft Glauber interactions involving \( A_{1G} \) even if there are detected final states. This is crucial in the cancellation of these Glauber interactions.

6.4. Cancellation of spectator–spectator and spectator-soft Glauber subgraphs

After excluding the effects of the active-spectator coherence, the Glauber subgraphs involving \( A_{1G} \) factorize from other parts of \( \mathcal{H}(P, \vec{P}, q^+, q^-) \). We can then consider the subgraphs separately.

\(^{26}\) According to the physical picture in [6, 34], such order should be definite on pinch singular surfaces.

\(^{27}\) These poles originate from the propagators of plus collinear particles coupling to Glauber gluons and vertexes which plus momenta of these particles flow into.
An example of cancellation of spectator-soft and spectator–s spectator Glauber exchange involving \( A_{+G} \) is shown in figure 10.

The cancellation in figure 10 is the direct result of the optical theorem if initial particles are on-shell. Considering that the momenta square of these particles is of order \( Q^2 \lambda^2 \), the graph in figure 10 should vanish at the leading power of \( \lambda \).

To confirm the cancellation of spectator–spectator and spectator-soft Glauber gluons no matter the initial spectators of the Glauber subprocess are on-shell or not, let us first consider an example shown in figure 11.

We have

\[
\text{Figure 11} = \int \frac{dl^+}{2\pi} \int \frac{dl^-}{2\pi} \int \frac{d\hat{P}^+}{2\pi} \int \frac{d\hat{P}^-}{2\pi} \frac{dp^+}{2\pi} \int \frac{dp^-}{2\pi} \int \frac{dp^+}{2\pi} \int \frac{dp^-}{2\pi} \int \frac{dp^+}{2\pi}
\]

\[
\times P^+ - k^- - p^- + i\varepsilon \hat{P}^+ - k^+ - p^+ + i\varepsilon
\]

\[
\times \left( i\delta \left( p^+ + \frac{q_\perp^2}{2p^+} \right) - i\delta \left( p^+ + \frac{q_\perp^2}{2p^+} \right) \right) i
\]

\[
\left( p^+ + \frac{q_\perp^2}{2p^+} + i\varepsilon \hat{P}^+ + \frac{q_\perp^2}{2p^+} + i\varepsilon \hat{P}^+ - l^+ - p^+ + i\varepsilon \right)
\]

\[
\times \left( \text{terms independent of } p^- \text{ and } p^+ \text{ and } l^- \text{ and } p^+ \text{ and } l^+ \right).
\]

We then integrate out \( p^- \) and \( p^+ \) and \( p^+ \) and \( p^+ \) and have

\[
\text{Figure 11} = 0.
\]

That is, the Glauber exchanges cancel out in figure 11 no matter whether the initial spectators are on-shell or not.

For general cases, one may consider Glauber couplings according to the \( \hat{\eta} \cdot x \)-order. We take the substitution (95) in couplings involving plus-collinear particles. After the substitution, the minus momenta of plus collinear particles are independent of each other. We then integrate out the minus momenta of Glauber gluons coupling to plus-collinear particles by taking residues of the Glauber vertexes. These vertexes look like

\[
\frac{-i}{l_i \cdots + i\varepsilon}.
\]

After these integrals, the Glauber exchange subprocess relies on the minus momenta of initial plus-collinear spectators only through their propagators or wave function. In other words, the whole process (including the Glauber and non-Glauber subprocess) relies on the minus momenta of the spectators only through the vertex at which the spectators are produced and propagators or wave functions of the spectators.

We then integrate out the minus momenta of the spectators by taking the poles of the propagators (if the spectators are off-shell) or using the on-shell condition (if the spectators are on-shell). Obviously, these two results are equivalent to each other. For Glauber couplings of other particles, we have similar results. That is, the cancellation of spectator–spectator and spectator-soft Glauber gluons is irrelevant to the off-shellness of initial particles of the Glauber subprocess at the leading power of \( \lambda \) and \( \eta \).

Generally, one may consider the \( \hat{\eta} \cdot x \)-version of operators and soft particles. Such evolution is unitary as \( \hat{\eta} \cdot \mu \) is time-like. As a result, one has a \( \hat{\eta} \cdot x \)-version of the optical theorem

\[
T_{\eta}^\dagger T_{\eta} = -i(T_{\eta}^\dagger - T_{\eta}^\dagger),
\]

where \( T_{\eta} \) represents the \( \hat{\eta} \cdot x \)-version of the scattering matrix. According to the relation (118), Glauber exchange with the coordinates \( \hat{\eta} \cdot x \) greater than those of the active–spectator coherence cancels out at the leading order of \( \lambda \)

The optical theorem (118) is the direct result of the unitarity of the \( \hat{\eta} \cdot x \)-evolution operator \( U_{+G}(\infty, -\infty) \) in (85).

The cancellation in figure 10 is a special case of (118). That is

\[
U_{+G}(\infty, -\infty) U_{+G}(\infty, -\infty) = 1
\]

\[
\Rightarrow T_{\eta}^\dagger T_{\eta} = -i(T_{\eta}^\dagger - T_{\eta}^\dagger).
\]

Hence the graphical cancellation of spectator–spectator and spectator-soft Glauber exchange involving \( A_{+G} \) based on the relation (118) is equivalent to our operator level method in section 5.3.

After the cancellation of the Glauber gluons \( A_{+G} \), one may consider an effective theory free from \( A_{+G} \). Calculations of \( \mathcal{H}(P, \hat{P}, q^+, q^-) \) in this effective theory are equivalent to

\[28 \text{ For example, one may check the } p^- \text{ terms in (115).} \]
those in the theory (7). One can then consider the $\sim n \cdot x$-evolution and repeat the procedure of $A + G$ to see the cancellation of $A - G$. The cancellation originates from the $\sim n \cdot x$ version of the optical theorem, which is equivalent to our operator level method.

7. Conclusions and discussions

We discuss the Glauber gluon effects in hadron collisions in this paper. It is proved that final state interactions cancel out in processes inclusive enough. For a time-like evolution (like $\sim n \cdot x$-evolution in section 5.1) which is approximately equivalent to a light-like evolution, we have a similar result. That is, interactions with the coordinates $\sim n \cdot x$ greater than that of the hard collision cancel out in processes inclusive enough. After the cancellation, the active-spectator coherence no longer disturbs us and we prove the cancellation of the spectator-spectator and spectator-soft Glauber exchanges for processes inclusive enough. We also present the proof of eikonalization of active-spectator and active-active and active-soft type Glauber exchanges. According to our discussions, these Glauber gluons should be viewed as zero-bins of collinear or soft gluons and absorbed into directions of collinear and soft Wilson lines of the hard vertex. The graphic cancellation of the spectator-spectator and spectator-soft Glauber exchanges are also discussed here to show how such graphic cancellation is related to operator level skills in this paper.

Exactly, the cancellation of interactions with the coordinates $\sim n \cdot x$ greater than that of the hard collision is equivalent to the cancellation of final states interactions as $\sim n$ is time-like. On the other hand, $\sim n$ is approximately light-like and the $\sim n \cdot x$-evolution of collinear and soft particles is approximately equivalent to the light cone evolution of these particles at the mass center frame of initial hadrons as discussed in sections 6.1 and 6.2. This explains why proofs based on the time-like evolution here give the same conclusion as those based on the light cone evolution [4].

According to our discussions, the eikonal approximation is crucial in the definition of collinear and soft modes at loop level. At the tree level, collinear and soft modes are characterized by their momenta. Considering that one usually runs over all momenta regions in practical loop integrals, the subtraction scheme to avoid double counting in loop level definition of different modes is necessary for the definition. Although the details of such a subtraction scheme are not concerned here, higher order definitions of different modes depending on the manner in which they couple to lower order modes. Especially, the eikonalized parts of couplings between collinear and other modes should be absorbed into collinear-collinear or soft-collinear or ultrasoft-collinear couplings. According to these definitions, active-spectator and active-soft exchanged gluons should be absorbed into the definition of loop level collinear and soft gluons.

Spectator-active interactions with the coordinates $\sim n \cdot x(\sim n \cdot x)$ greater than those of Glauber exchanges should be treated carefully as they may obstruct the
summation of spectator-soft and spectator-spectator Glauber exchanges. Fortunately, these interactions cancel out for processes considered here as discussed in section 5.2. Intuitively, spectator-soft and spectator-spectator exchange of Glauber gluons $A_{xG}$ and $A_{yG}$ should occur at the vertexes with the coordinates $x^+$ and $x^-$ equivalent to those of the hard collision according to the classical trajectories of collinear particles and locality of the Glauber propagators in the $x^+$ and $x^-$ directions. Therefore one should not be surprised to see such a cancellation.

Considering that Glauber couplings may change the transverse momenta of final particles, the summation over these final states, especially the integration over transverse momenta of these final states, is necessary for the cancellation of Glauber exchanges. While considered observable is not inclusive enough, the final states may not form the invariant subspace of the evolution induced by Glauber couplings and the Glauber cancellations induced by the unitarity of the evolution may break up. That is to say, our proofs of Glauber cancellations induced by the unitarity of the time evolution operator;

"see such a cancellation."

Appendix A. Time evolution operator of the effective theory

We discuss the time evolution operator of the effective theory here. Generally speaking, the time evolution operator in the interaction picture reads

$$U(t_1, t_2) = e^{iHt_2}e^{-iH(t_2-t_1)}e^{-iHt_1},$$

where $H_0$ and $H$ are the free and full Hamiltonian of the theory. However, the Hamiltonian formulation of non-Abelian gauge theory is quite nontrivial (see e.g. [35, 36]), not to mention the difficulties caused by Glauber gluon fields in (7), which should be viewed as constraints. According to the effective action (7), one has

$$(\mathcal{P}_n)^2 \cdot A_{\mu G},$$

$$= g(\bar{\xi}_8, t^\mu \nu \xi_8) + 2gtr(\bar{\xi}_8[\mathcal{D}_n, [t^\mu, c_n]]) + \frac{2}{g} \text{tr} [\mathcal{D}_n^\mu + gA_{\mu, n} \cdot \mathcal{D}_n + gn \cdot A_n][\mathcal{D}_n^\mu + gn \cdot A_n, t^\mu]$$

$$+ g(\bar{\psi}_n, t^\mu \nu \psi_n) + 2gtr(\bar{\psi}_n[n \cdot \mathcal{P}, [t^\mu, c_n]]) + \frac{2}{g} \text{tr} [(\mathcal{P}_n^\mu + gA_{\mu, n} + g\sum_n' = n \mathcal{R}_n \cdot A_{\nu G} n' \cdot n') + gn \cdot A_n + g\sum_n' = n \mathcal{R}_n \cdot A_{\nu G} n' \cdot n']$$

$$\times [\mathcal{P}_n + gA_{\mu, n} + g\sum_n' = n \mathcal{R}_n \cdot A_{\nu G} n' \cdot n']$$

$$\times (\mathcal{P}_n)^2 \cdot A_{\mu G},$$

$$= g(\bar{\xi}_8, t^\mu \nu \xi_8) + 2gtr(\bar{\xi}_8[\mathcal{D}_n, [t^\mu, c_n]]) + \frac{2}{g} \text{tr} [\mathcal{D}_n^\mu + gA_{\mu, n} \cdot \mathcal{D}_n + gn \cdot A_n]$$

$$\times [\mathcal{D}_n^\mu + gA_{\mu, n}, t^\mu].$$

where collinear and soft momenta are not displayed explicitly for simplicity.

Solving the constraints corresponding to Glauber gluon fields is equivalent to integrating out Glauber gluon fields in

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the effective theory. Let us consider the solution of $n \cdot A_{n,G}$

$$
\begin{align*}
    n \cdot A_{n,G}^{\mu} &= \frac{g}{(P_2)^2} \partial_{t_a}^\mu \xi_{n_{A}} + \frac{2g}{(P_2)^2} \text{tr} \{ \delta_n [ \partial_t \partial_n, [t_a, c_{n}]] \} \\
    + \frac{1}{(P_2)^2} \frac{2}{g} \text{tr} \{ [iD_n^\mu + gA^\mu_n, \partial_t \partial_n + n \cdot A_n] \} \\
    \times [iD_n^\mu + gA_n^\mu, t_a^\mu] \\
    + \frac{g}{(P_2)^2} (\bar{\psi}_n \partial_t \partial_n \psi_n) + \frac{2g}{(P_2)^2} \text{tr} \{ \delta_n [ \partial_t \partial_n, [t_a, c_{n}]] \} \\
    + \frac{1}{(P_2)^2} \frac{2}{g} \text{tr} \{ [P_n^\mu + gA_n^\mu + g\sum_{n'} = n \bar{n}^i \cdot A_{n,G} n_{1,i'}^\mu, n \cdot P_n] \\
    \times [P_n^\mu + gA_n^\mu + g\sum_{n'} = n \bar{n}^i \cdot A_{n,G} n_{1,i'}^\mu, t_a^\mu] \}. 
\end{align*}
$$

(A3)

We substitute the solution into couplings between $n$-collinear quarks and the Glauber gluons $A_{n,G}$ and have

$$
\begin{align*}
    g \bar{\psi}_n n \cdot A_{n,G} \bar{\psi}_n &= g \bar{\psi}_n t_a^\mu \xi_{n_{A}} + \bar{\psi}_n t_a^\mu \xi_{n_{A}} \\
    \times \frac{2g^2}{(P_2)^2} \text{tr} \{ \delta_n [ \partial_t \partial_n, [t_a, c_{n}]] \} \\
    + \frac{g^2}{(P_2)^2} \text{tr} \{ [iD_n^\mu + gA^\mu_n, \partial_t \partial_n + n \cdot A_n] \} \\
    + \frac{g}{(P_2)^2} (\bar{\psi}_n \partial_t \partial_n \psi_n) + \frac{2g}{(P_2)^2} \text{tr} \{ \delta_n [ \partial_t \partial_n, [t_a, c_{n}]] \} \\
    + \frac{g^2}{(P_2)^2} \text{tr} \{ [P_n^\mu + gA_n^\mu + g\sum_{n'} = n \bar{n}^i \cdot A_{n,G} n_{1,i'}^\mu, n \cdot P_n] \\
    \times [P_n^\mu + gA_n^\mu + g\sum_{n'} = n \bar{n}^i \cdot A_{n,G} n_{1,i'}^\mu, t_a^\mu] \}. 
\end{align*}
$$

(A3)

For the Glauber gluon terms in equation (A4), one may obtain their contribution order by order and see elastic processes induced by the exchange of more sequential Glauber gluons. We show some examples of such elastic scattering in figure A2.

At leading of power $\lambda$, figure A2 reads

$$
\begin{align*}
    \text{Figure A2}(a) &= \sum_{\langle p', p \rangle} \frac{\bar{u}(p')\gamma^\mu u(p)\bar{u}(p)\gamma^\mu u(p)}{(p' - p)^2 (p' - p)} \\
    \times \bar{n}(p')t^\mu u(p)\bar{u}(p)\gamma^\mu u(p) \\
    \times (p' - p) - 2n \cdot (p' - p) \\
    \text{Figure A2}(b) &= -\sum_{\langle p', p \rangle} \frac{ig^4}{(p' - p)^2 (p' - p)} \\
    \times \bar{n}(p')t^\mu u(p)\bar{u}(p)\gamma^\mu u(p) \\
    \times (f^{abc} f^{cde} (n^\mu n^\nu - n^\mu n^\nu) + f^{ace} f^{cbe} (g^{\mu \nu} - n^\mu n^\nu) \\
    + f^{ade} f^{bce} (g^{\mu \nu} - n^\mu n^\nu)). 
\end{align*}
$$

(A6)

One can verify that equations (A5) and (A6) are equivalent to the full QCD results at the leading power of $\lambda$.

For couplings between Glauber and $n$-collinear gluons, we have similar results. Although Glauber gluons are not integrated out here, one should keep in mind that they are constraints and do not correspond to any quantum states.

Instead of the Hamiltonian formula, we would like to discuss the time evolution operator in the frame of the path integral. Let us start from the time evolution operator of an ordinary gauge theory. According to the relation between the Hamiltonian formulation and path integral method, one has

$$
\begin{align*}
    \langle B_{\text{phy}} | U(t_2, t_1) | A_{\text{phy}} \rangle \\
    &= (B_{\text{phy}} | e^{iH(t_2-t_1)} | A_{\text{phy}} \rangle \\
    &= \mathcal{C} \int [D\psi_2][D\bar{\psi}_2][D\bar{A}_{\mu}][DA_{\mu}] [DC] [DC_2] \\
    \times [D\bar{\psi}_1][D\bar{A}_{\mu}][DA_{\mu}] [DC] [DC_1] \langle B_{\text{phy}} | \psi_2, \bar{\psi}_2, A_{\mu}^2 \rangle \\
    \times (\psi_1, \bar{\psi}_1, A_{\mu}^2 | A_{\text{phy}} \rangle \\
    \times \exp (-iE_0(\psi_2, \bar{\psi}_2, A_{\mu}^2, c_{\lambda}, c_{\lambda}) (t_2, 0)) \\
    \times \exp (ig(\bar{A}_{\mu}, c_{\lambda}, c_{\lambda}) (t_2, t_1)) \\
    \times \exp (i\bar{G}(\gamma, \bar{\psi}_1, A_{\mu}, c_{\lambda}, c_{\lambda}) (t_2, 0)) \\
    \times \exp (i\bar{G}_0(\psi_1, \bar{\psi}_1, A_{\mu}, c_{\lambda}, c_{\lambda}) (t_2, 0)) 
\end{align*}
$$

(A7)

for arbitrary physical states $| A_{\text{phy}} \rangle$ and $| B_{\text{phy}} \rangle$, where $\mathcal{C}$ is a constant independent of the fields and the states $| A_{\text{phy}} \rangle$ and $| B_{\text{phy}} \rangle$. The matrix element (A7) can be extended to arbitrary
Figure A1. Examples of tree level elastics scattering processes induced by a Glauber exchange. Figure (a) shows the scattering between $n$-collinear and $\bar{n}$-collinear quarks. Figure (b) shows the scattering between $n$-collinear quarks and transverse polarized $\bar{n}$-collinear gluons. Figure (c) shows the scattering between $n$-collinear and soft quarks. Figure (d) shows the scattering between $n$-collinear quarks and soft gluons.

Figure A2. Examples of tree level elastics scattering processes induced by two sequential Glauber gluons, where $q_s$ and $q'_s$ represent soft gluons.
states (including non-physical states)\(^{30}\)

\[
\langle B | U(t_2, t_1) | A \rangle \\
= \mathcal{C} \int [D\psi^c_2][D\tilde{\psi}_2][DA^n_{AD}][DC_2][DC_2] \\
\times [D\psi_2][D\tilde{\psi}_2][DA^n_{AD}][DC_2][DC_2] \\
\times [D\psi^c_2][D\tilde{\psi}_2][DA^n_{AD}][DC_2][DC_2] \langle B | \psi_2, \tilde{\psi}_2, A^c_2, c_2, \tilde{c}_2 \rangle \\
\times \langle \psi_1, \tilde{\psi}_1, A^c_1, c_1, \tilde{c}_1 | A \rangle \\
\times \exp \left( -iU^{(0)}(c_{2}, \tilde{c}_{2}, A_{2}^{cu}, c_{1}, \tilde{c}_{1}, c_{1}, \tilde{c}_{1}) \right) (t_2, 0) \\
\times \exp \left( iU^{(0)}(\xi_1, \tilde{\xi}_1, A_{1}^{cu}, c_{1}, \tilde{c}_{1}, c_{1}, \tilde{c}_{1}, c_{1}, \tilde{c}_{1}) \right) (t_2, 0) \\
\times \langle B_2^{(2)}, A_2^{(2)}, c_2^{(2)}, \tilde{c}_2^{(2)} | A \rangle \\
\times \langle \xi_1^{(1)}, \tilde{\xi}_1^{(1)}, A_{1}^{cu(1)}, c_{1}, \tilde{c}_{1}, c_{1}, \tilde{c}_{1}, c_{1} | A \rangle,
\] (A8)

The matrix element (A8) can be viewed as the Lagrangian formula of the time evolution operator.

For the effective theory, one has a similar Lagrangian formula of the time evolution operator:

\[
\langle B | U_{SCEG}^{(0)}(t_2, t_1) | A \rangle \\
= \mathcal{C} \int [D\xi_2^{(2)}] ... [D\tilde{\xi}_2^{(2)}] ... [DA_2^{(2u)}] ... \\
\times [D\xi_2^{(2)}] ... [D\tilde{\xi}_2^{(2)}] ... [DA_2^{(2u)}] ... \\
\times [D\xi_2^{(2)}] ... [D\tilde{\xi}_2^{(2)}] ... [DA_2^{(2u)}] ... \\
\times \exp \left( -iU_0^{(0)}(c_{2}, \tilde{c}_{2}, A_{2}^{cu}, c_{1}, \tilde{c}_{1}, c_{1}, \tilde{c}_{1}) \right) (t_2, 0) \\
\times \exp \left( iU_0^{(0)}(\xi_1, \tilde{\xi}_1, A_{1}^{cu}, c_{1}, \tilde{c}_{1}, c_{1}, \tilde{c}_{1}, c_{1}, \tilde{c}_{1}) \right) (t_2, 0) \\
\times \langle B_2^{(2)}, A_2^{(2)}, c_2^{(2)}, \tilde{c}_2^{(2)} | A \rangle \\
\times \langle \xi_1^{(1)}, \tilde{\xi}_1^{(1)}, A_{1}^{cu(1)}, c_{1}, \tilde{c}_{1}, c_{1}, \tilde{c}_{1}, c_{1} | A \rangle,
\] (A9)

where \(U_0^{(0)}\) represents the free part of the effective action (7). While considering hard collision processes, the relevant time evolution operator reads:

\[
\langle B | U_{SCEG}^{(0)}(t_2, t_1) | A \rangle \\
= \mathcal{C} \int [D\xi_2^{(2)}] ... [D\tilde{\xi}_2^{(2)}] ... [DA_2^{(2u)}] ... \\
\times [D\xi_2^{(2)}] ... [D\tilde{\xi}_2^{(2)}] ... [DA_2^{(2u)}] ... \\
\times [D\xi_2^{(2)}] ... [D\tilde{\xi}_2^{(2)}] ... [DA_2^{(2u)}] ... \\
\times \exp \left( -iU_0^{(0)}(c_{2}, \tilde{c}_{2}, A_{2}^{cu}, c_{1}, \tilde{c}_{1}, c_{1}, \tilde{c}_{1}) \right) (t_2, 0) \\
\times \exp \left( iU_0^{(0)}(\xi_1, \tilde{\xi}_1, A_{1}^{cu}, c_{1}, \tilde{c}_{1}, c_{1}, \tilde{c}_{1}, c_{1}, \tilde{c}_{1}) \right) (t_2, 0) \\
\times \langle B_2^{(2)}, A_2^{(2)}, c_2^{(2)}, \tilde{c}_2^{(2)} | A \rangle \\
\times \langle \xi_1^{(1)}, \tilde{\xi}_1^{(1)}, A_{1}^{cu(1)}, c_{1}, \tilde{c}_{1}, c_{1}, \tilde{c}_{1}, c_{1} | A \rangle,
\] (A10)

\(^{30}\) According to the discussions in \(\text{[37]}\), physical states should be annihilated by the BRST-charge.
and the collinear BRST transformation

\[
\delta \xi_{\mu} = 0, \quad \delta A_{\mu}^a = 0, \quad \delta c_{\mu} = 0 (n^\mu = n^0)
\]

\[
\delta A_{\mu}^\ast A_{\mu}^a = 0, \quad \delta c_{\mu} = 0, \quad \delta A_{\mu}^a = 0
\]

\[
\delta c_{\mu} = 0, \quad \delta \xi_{\mu} = \epsilon_c c_{\mu}, \quad \delta A_{\mu}^a = [D_{\mu\ast}^a - iA_{\mu\ast}^a, \epsilon c] \quad \delta c_{\mu} = \frac{g}{2} f^{abc} c_{\ast}^a c_{\ast}^b c_{\ast}^c, \quad \delta c_{\mu} = -\epsilon (D_{\mu}^a, A_{\mu\ast})
\]

\[
= - i \frac{g}{2} \hat{\partial} \cdot \delta \mathcal{P} n \cdot A_{\ast G} \]

\]

\[
\text{(A15)}
\]

where we have made use of the fact
\[
n \cdot \delta \mathcal{P} n \cdot A_{\ast G} = 0, \quad [\hat{\partial} \cdot D_{\ast}, n \cdot A_{\ast}] = 0. \quad \text{(A16)}
\]

Besides the effective action (7), the hard vertex (20) is also invariant under the BRST transformations (A13)–(A15) given that \( c_{\lambda}(\infty) = c_{\lambda}(\infty) = c_{\lambda}(\infty) = 0 \). In fact, once the global invariant hard vertex takes the form
\[
J_h = J_h(S_n W_i^a \tilde{c}_{\nu}, S_n W_i^a A_{\ast G}^b W_i^c S_n, \tilde{c}_{\nu} W_i^c S_n, \ldots)
\]

then \( J_h \) is invariant under the BRST transformations (A13)–(A15) in the case \( c_{\lambda}(\infty) = c_{\lambda}(\infty) = c_{\lambda}(\infty) = 0 \).

One may worry about the mixture between Glauber gluons and other modes. Fortunately, the cancellation of nonphysical polarized collinear and soft gluons is not affected by such a mixture. In fact, we have
\[
(P_{\mu} + i \delta_{\mu})(A_{\mu}^\ast + \sum_n n \cdot A_{\ast G} n^\mu) = (P_{\mu} + i \delta_{\mu})(1 + O(\lambda^{1/2}))
\]

\[
= (P_{\mu} + i \delta_{\mu})(A_{\mu}^\ast + \sum_n \tilde{n} \cdot A_{\ast G} n^\mu) = (P_{\mu} + i \delta_{\mu})A_{\mu}^\ast (1 + O(\lambda^{1/2})) \quad \text{(A18)}
\]

\[
(P_{\mu} + i \delta_{\mu})(A_{\mu}^\ast + \sum_n \tilde{n} \cdot A_{\ast G} n^\mu) = (P_{\mu} + i \delta_{\mu})(A_{\mu}^\ast + \sum_n \tilde{n} \cdot A_{\ast G} n^\mu)
\]

\[
= (P_{\mu} + i \delta_{\mu})(A_{\mu}^\ast + \sum_n \tilde{n} \cdot A_{\ast G} n^\mu)
\]

\[
= (P_{\mu} + i \delta_{\mu})(A_{\mu}^\ast + \sum_n \tilde{n} \cdot A_{\ast G} n^\mu)
\]

\[
= (P_{\mu} + i \delta_{\mu})(A_{\mu}^\ast + \sum_n \tilde{n} \cdot A_{\ast G} n^\mu)
\]

and the mixture between Glauber and nonphysical polarized gluons vanishes at the leading power of \( \lambda^{1/2} \), where
\[
\mathcal{D}^\mu = P_{\mu}, \quad \partial^\mu = \partial_{\mu} \quad \text{A20}
\]

Hence the mixture is power suppressed unless the contraction between nonphysical polarized gluons and superleading vertices occurs in the process. The superleading interactions (order \( \lambda^{1/2} \)) originate from couplings between the Glauber components \( n \cdot A_{\ast G} \) with \( b = 1 \) and \( n \)-collinear particles. Hence such superleading vertices can contract with the Glauber components \( n \cdot A_{\ast G} \) not \( n \cdot A_{\ast G} \). According to the power counting results of nonphysical polarized gluons in (A18) and (A19), such contraction (order \( \lambda^{2b-1} \) or higher) vanishes at the leading power of \( \lambda^0 \).

\[\dagger\] The power counting for couplings between soft and Glauber gluons reads \( \lambda^{-1/2} \) or higher. If there is additional suppression of order \( \lambda^{3/2} \) or higher, then the mixture between soft and Glauber gluons affects the cancellation of nonphysical soft gluons at order \( \lambda^0 \).

\[\ddagger\] The power counting for couplings between soft and Glauber gluons reads \( \lambda^{-1/2} \) or higher. If there is additional suppression of order \( \lambda^{3/2} \) or higher, then the mixture between soft and Glauber gluons affects the cancellation of nonphysical soft gluons at order \( \lambda^0 \).

\[\ddagger\ddagger\] At the leading power of \( \lambda^0 \), soft and ultrasoft particles and Glauber gluons are free from the hard subprocess.

\[\ddagger\ddagger\ddagger\] Obviously, \( \mathcal{D}^\mu(k_i) > 0 \) for \( 1 \leq j \leq n \) are the reflex of plus momenta, which contradicts classical trajectories of plus-collinear particles, is absent on the leading pinch singular surfaces.

\[\ddagger\ddagger\ddagger\ddagger\] \( k_j \) may couple to other non-Glauber particles (except the same). We define the momenta of these particles as variables independent of Glauber gluons coupling to \( k_j \).
changing the leading power result. This is compatible with discussions in section 4.2.

It is worth noting that the above definition distinguishes from one’s intuitive insight. For example, let us consider the particle \( p_1 \) in figure B2.

At first glance, \( p_1 \) should be defined as spectators. However, \( p_1 \) and \( p_2 \) are defined as active particles if \((p_1, p_2, q)\) are plus-collinear and \( P_1(q) = P_2(p_1 - p_2) > 0 \) according to the above discussions since there is a path \( p_1 \to p_2 \to q \to \text{hard vertex} \) through which \( p_1^\perp \) and \( p_2^\perp \) flow into the hard vertex. In this case, we choose \((p_1, l, p_3)\) as independent variables and figure B2 relies on \( l^\perp \) through the terms

\[
\frac{1}{p_1^\perp + l^\perp + \frac{(p_1 + l)^2}{2P(p_1)} + i\varepsilon} \times \frac{1}{p_n^\perp + l^\perp + \frac{m_p^2}{2P(p_n)} + \frac{m_l^2}{2P(p_n - p_1)} + i\varepsilon}.
\]

Hence \( l^\perp \) is not pinched in the Glauber region at the leading power of \( \lambda \). This is compatible with discussions in section 4.2.

For a plus-collinear particle \( p_1 \) free from the hard vertex, if there are no paths through which \( p_1^\perp \) flow into the hard vertex then \( p_1 \) flows into the final cuts. There should be a path \( p_1 \to \cdots p_n \to \text{final cuts} \). The propagator of \( p_j(1 \leq j \leq n - 1) \) reads

\[
N(j) = \frac{1}{p_1^\perp + l^\perp + \cdots + l_{j-1}^\perp + \cdots + \frac{(p_1 + l + \cdots + l_{j-1} + \cdots)^2}{2P(p_j)} + i\varepsilon},
\]

and the final cut on \( p_n \) reads

\[
\delta(p_1^\perp + l^\perp + \cdots + l_{j-1}^\perp + \cdots + \frac{(p_1 + l + \cdots + l_{j-1} + \cdots)^2}{2P(p_n)} - \frac{m_p^2}{2P(p_n)} - \frac{m_l^2}{2P(p_n - p_1)} + i\varepsilon)
\]

\[
- \frac{1}{p_1^\perp + l^\perp + \cdots + l_{j-1}^\perp + \cdots + \frac{m_p^2}{2P(p_n)} + \frac{m_l^2}{2P(p_n - p_1)} + i\varepsilon}.
\]

For minus-collinear particles, we have a similar definition.

In summary, we define plus-collinear(minus-collinear) active particles as:

\[1\text{ (the plus-collinear(minus-collinear)}
\]

The two cases may overlap.
particles coupling to particles collinear to other directions or hard modes directly; (2) the plus-collinear
(minus-collinear) particles of which the plus (minus) momenta flow into the hard vertex through a path made up of collinear particles. Other plus-collinear (minus-collinear) particles are defined as spectators. The definition of active particles is compatible with discussions in section 4.2. And further, we can deal with Glauber exchanges between spectators and other particles according to skills in sections 4.1 and 5.

Appendix C. Reparameterization invariance of the effective theory

We consider the reparameterization invariance in this appendix. For simplicity, we neglect ghost fields in this appendix. In original SCET, the reparameterization invariance of the n-collinear sector arises from two types of ambiguity [38, 39]:

1. The subscript p of the collinear fields ξ_{n,p} and A_{n,p} is arbitrary by order Q^2
2. Transformations on η_{n} and η_{n}

\[
\begin{align*}
& (a) \quad n_{\mu} \rightarrow n_{\mu} + \Delta^{\perp}_{\mu} \quad \text{and} \\
& (b) \quad n_{\mu} \rightarrow n_{\mu} \quad \text{and} \\
& (c) \quad \eta_{\mu} \rightarrow (1 + \alpha) \eta_{\mu} \quad \text{and} \\
& \eta_{\mu} \rightarrow (1 - \alpha) \eta_{\mu}
\end{align*}
\]

(C1)

should not change physical results, where the infinitesimal parameters satisfy the condition \( n \cdot \Delta^{\perp} = n \cdot \eta = 0 \).

The type-1 transformations leave \( n^{\mu} \) and \( \eta^{\mu} \) invariant and affect the subscripts of collinear fields. Keep in mind, the momenta of these particles are independent of the manner we decompose them. Hence the operators rely on \( \mathcal{P}^{\mu} \) and \( i \partial^{\mu} \) through the combination \( \mathcal{P}^{\mu} \) and \( i \partial^{\mu} \) are invariant under these transformations. At the leading power of \( \lambda \),

\[
\mathcal{P}^{\mu} + i \partial^{\mu} \simeq n \cdot \mathcal{P}^{\mu} + m \cdot \partial^{\mu} \quad \text{(C2)}
\]

for n-collinear particles and Glauber gluons and

\[
\mathcal{P}^{\mu} + i \partial^{\mu} \simeq \mathcal{P}^{\mu} \quad \text{(C3)}
\]

for soft particles and

\[
\mathcal{P}^{\mu} + i \partial^{\mu} = i \partial^{\mu} \quad \text{(C4)}
\]

for ultrasoft particles. Therefore the type-1 transformations leave the action (7) invariant at the leading power of \( \lambda \).

The type-2 transformations are more subtle. The transformations of relevant operators under the reparameterization-2 are exhibited in table C1 [38, 39]38. Where

\[
W_{\text{eff}} = \sum_{\text{perms}} \left( -\frac{g}{\bar{n} \cdot \mathcal{P}} \bar{n} \cdot (A_{n,p} + A_{nG}) \right)
\]

(C5)

at leading power of \( \lambda \). Considering the Glauber exchange causes \( n - \bar{n} \) and \( n - \bar{n} \)-soft correlations, we also present the transformation properties of \( \bar{n} \)-sector and soft sector in table C1.

One may verify that the transformations in table C1 leave the effective action (7) invariant at the leading power of \( \lambda \). Instead of explicit calculations, we would like to show the reason for the invariance. Let us start from the decomposition of the fields

\[
\psi(x) = \sum_{\mu} \psi_{\mu}(x) + \psi_{\text{us}}(x) + \psi_{\text{off}}(x)
\]

\[
A^{\mu} = \sum_{\mu} (A^{\mu}_{n} + A^{\mu}_{nG})(x) + A^{\mu}_{\text{us}}(x) + A^{\mu}_{\text{off}}(x),
\]

(C6)

where \((\psi, A^{\mu})\) represent fields in the basic theory and \((\psi_{\text{us}}, \psi_{\text{off}})\) represent the modes in the effective theory and \((\psi_{\text{us}}, A^{\mu}_{\text{off}})\) represent the modes we integrate out. As discussed in section 2.2, the subtraction to remove the overlap between \( A_{nG} \) and \( A_{nG} \) is not displayed here for simplicity. Although the reparameterization transformations may change fields in the effective theory, the decomposition (C6) is invariant under the transformations. Obviously, reparameterization transformations do not mix the \((\psi_{\text{us}}, A^{\mu}_{\text{off}})\) and \((\psi_{\text{us}}, A^{\mu}_{\text{off}})\). We simply drop the modes \((\psi_{\text{us}}, A^{\mu}_{\text{off}})\) from the basic theory and obtain the tree-level matching effective action,

\[
I_{\text{eff}} = \sum_{\mu} \bar{\psi}_{\mu}(i\mathcal{D}^{\mu}_{nG})\psi_{\mu} + \int d^{4}x \frac{1}{2g^{2}} \text{tr}
\]

\[
\times \left[ [i\mathcal{D}^{\mu}_{nG} + gA^{\mu}_{nG}, i\mathcal{D}^{\nu}_{nG} + gA^{\nu}_{nG}] \right]^{2}
\]

\[
+ \text{ghost terms}
\]

\[
+ I_{\mu}^{G} + I_{\mu} + \sum_{\mu} I_{\mu G}
\]

(C7)

which is invariant under the transformations.

We then consider the decomposition

\[
\psi_{\text{n}}(x) = \bar{\psi}(x) \quad \text{and} \quad \psi_{\text{n}}(x) \equiv \frac{g}{2} \bar{\psi}(x),
\]

(C9)

which is reparameterization invariant. We have

\[
\bar{\psi}_{\text{n}}(i\mathcal{D}^{\mu}_{nG})\psi_{\text{n}}
\]

\[
= \bar{\xi}_{\text{n}}(i\mathcal{D}^{\mu}_{nG})\xi_{\text{n}} + \bar{\xi}_{\text{n}}(i\mathcal{D}^{\mu}_{nG})\xi_{\text{n}}
\]

\[
+ \bar{\xi}_{\text{n}}(i\mathcal{D}^{\mu}_{nG})\xi_{\text{n}} + \bar{\xi}_{\text{n}}(i\mathcal{D}^{\mu}_{nG})\xi_{\text{n}}
\]

\[
= \bar{\xi}_{\text{n}}(i\mathcal{D}^{\mu}_{nG})\xi_{\text{n}} + \bar{\xi}_{\text{n}}(i\mathcal{D}^{\mu}_{nG})\xi_{\text{n}} + \bar{\xi}_{\text{n}}(i\mathcal{D}^{\mu}_{nG})\xi_{\text{n}}
\]

\[
+ \bar{\xi}_{\text{n}}(i\mathcal{D}^{\mu}_{nG})\xi_{\text{n}}
\]

(C10)
We integrate out $\xi_{\lambda}$ and have
\[
\psi_n = \left(1 + \frac{1}{\bar{n} \cdot D_{AG}} \bar{\nu}_{AG}(\bar{\mu}) \bar{\xi}_{n}\right) = \left(1 + W_{AG} \frac{1}{P} W_{AG}^{\dagger} \bar{\nu}_{AG}(\bar{\mu}) \bar{\xi}_{n}(1 + O(\lambda))\right). \tag{C11}
\]
Substituting the expression of $\psi_n(\lambda)$ into the effective action, we get results equivalent to those in (7). That is to say, the effective action (7) is reparameterization invariant at the leading power of $\lambda$.

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\[\text{Table C1. Transformations of relevant operators under the type-2 reparameterization.}\]

| Type 2a | Type 2b | Type 2c |
| --- | --- | --- |
| $n_\mu \to n_\mu + \Delta^\mu_\lambda$ | $n_\mu \to n_\mu$ | $n_\mu \to (1 + \alpha)n_\mu$ |
| $\bar{n}_\mu \to \bar{n}_\mu$ | $\bar{n}_\mu \to \bar{n}_\mu + \xi^\mu_\lambda$ | $\bar{n}_\mu \to (1 - \alpha)\bar{n}_\mu$ |
| $n \cdot D_{AG} \to n \cdot D_{AG} + \Delta \cdot D_{AG}$ | $n \cdot D_{AG} \to n \cdot D_{AG} + \xi \cdot D_{AG}$ | $n \cdot D_{AG} \to (1 + \alpha)n \cdot D_{AG}$ |
| $D_{AG} \to D_{AG} - \Delta \cdot D_{AG}$ | $D_{AG} \to D_{AG} - \xi \cdot D_{AG}$ | $D_{AG} \to D_{AG} - (1 - \alpha)n \cdot D_{AG}$ |
| $n \cdot D_{AG} \to n \cdot D_{AG} + \Delta \cdot D_{AG}$ | $n \cdot D_{AG} \to n \cdot D_{AG} + \xi \cdot D_{AG}$ | $n \cdot D_{AG} \to (1 + \alpha)n \cdot D_{AG}$ |
| $\xi_{s} \to (1 + \Delta \cdot \bar{\nu}_{AG}) \xi_{s}$ | $\xi_{s} \to (1 + \xi \cdot \bar{\nu}_{AG}) \xi_{s}$ | $\xi_{s} \to \xi_{s}$ |
| $W_{AG} \to W_{AG}$ | $W_{AG} \to (1 - \frac{1}{n \cdot D_{AG}} \Delta \cdot D_{AG}) W_{AG}$ | $W_{AG} \to W_{AG}$ |
| $\psi_n \to \psi_n$ | $\psi_n \to \psi_n$ | $\psi_n \to \psi_n$ |
| $A^\mu \to A^\mu$ | $A^\mu \to A^\mu$ | $A^\mu \to A^\mu$ |
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