Time Delay Estimation via Correlation in the Non-Synchronous Sensor Network by the Internet of Things

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Abstract. This paper is concerned with the elimination of time delay among data from different channels sampled by the sensor network from the Internet of things (IoT), especially the wireless sensor network (WSN) in Structural Health Monitoring (SHM). In this paper, a technique using correlation has been proposed and moving window plays an important role in this algorithm. Obtaining the correlation coefficient at each point, use interpolation function to fit a curve so that the sub-sampling -interval part of time delay can be calculated, which is enlightened by sub-pixel deformation algorithm from Digital Image Correlation (DIC). The length of each segmentation and the choice of interpolation function will affect the computational efficiency and the quality of result, so they should be carefully confirmed. Finally, a numerical experiment has been conducted to verify this method and analysis the error of this method, and gives out a satisfactory consequence.

Key words: Non-synchronous sensing, correlation, sensor network, sub-sampling -interval

1. Introduction
In the Structural Health Monitoring and vibration experiment (like shaking table experiment), a lot of sensors will be applied to measure different kinds of data. However, ideal synchronized sensing is unprocurable [1]. The time difference of each channel (this difference will be called time delay in the following discussion) should be determinate and adjust the signal, otherwise it will make a tremendous difference to the result of modal analysis [1, 2]. Time delay comes from several sources [1, 3, 4]. Time delay [5] could be divided into 4 parts: (a) clock drift because of the physical environment; (b) ping because of the data transmission speed; (c) inherited error because of the accumulation of the error; (d) error in time transmission in GPS. These time delay caused by hardware, and the optimization of hardware is out of discussion, yet time delay can be eliminated through algorithm.

To eliminate time delay among these channels, many techniques have been developed in both hardware [5-7] and software [8-13]. In the paper about software, correlation, as a common software method, has been expected a lot.

For most sampling device, they gather discrete-time signal, and there are many other kinds of correlation function for discrete-time signal. In DIC (Digital Image Correlation), four types of correlation criteria commonly used, CC criteria, SAD criteria, SSD criteria and PSSD criteria. Three kind of correlation criteria, CC, SSD and PSSD criteria, often used according to [14], and they are equivalent.

However, using correlation cannot calculate the sub-sampling -interval part of time delay [4], and
time delay is impossible to be just several time steps so that the accuracy of this method is only the reciprocal of sampling frequency. Meanwhile, this method only used to calculate the time delay between transmitting and reflecting signals. Furthermore, due to the randomness and frequency shift of the sampling, different time points might have inconsistent time delay, and can only give a general time delay but not the time delay at each time point.

In this paper, in order to calculate the time delay at each time point, at each specific time point in a channel, other certain number of data before and after this time point will be regarded as a group, which can be used to calculate cross-correlation coefficient between this group and another respective group in its channel. Moving window is the basic idea of this paper.

2. Methodology

In a SHM system or a vibration experiment, assume that \( N \) sensors have been set up in a structure. The signals recorded by these channels can be written as:

\[
x = \begin{bmatrix} x_1 & x_2 & \cdots & x_N \end{bmatrix}^T
\]  

(2.1)

Where \( x_1, x_2, \ldots, x_N \) can be displacement, velocity, acceleration data or even temperature data recoded by the sensors network. However, for time delay estimation via correlation analysis, the value of correlation coefficient affects the result, sufficiently. That is to say, for example, temperature data and displacement data have low relationship which will make out a terrible consequence compared to the relativity between displacement data and velocity data. In the following discussion, two rows of signals will be considered to estimate the time between these two channels, and this method can be simply generalized to N-dimension signal groups.

Assume two rows of acceleration measured from a SHM system or a vibration experiment, \( x_1 = x_1[n] \) and \( x_2 = x_2[n] \). Firstly, calculate the cross-correlation sequence between \( x_1[n] \) and \( x_2[n] \), so that the initial time delay \( D \) can be estimated by seeking the peak in the cross-correlation sequence. Figure 1 shows an example using cross-correlation method to find out the general time delay of two rows of signal. In this example, one row of acceleration data had been moved backward 0.04s, deliberately. In the right figure, it shows that the initial time delay \( D \) is 0.04s, which proves that using cross-correlation can find out the delay time.

![Cross-Correlation between Two Rows of Signal](image)

**Figure 1.** Cross-Correlation between Two Rows of Signal

However, this value \( D \) has some problems which have been stated in the Introduction. Therefore, in this paper, an approach based on segmentation is useful to determinate \( D \) at each specific time point. The process is shown below (this process will be called PROCESS in the later discussion): (1) Regard that one row of data (which will be called row A below) was sampled at standard time point and another row should be modified (which will be called row B below). Otherwise, these two row would better be same length; (2) Choose a time point which one is interested with, and make sure that before
and after this time point, there are still enough sampling data; (3) Calculate the cross-correlation sequence between these two rows of data to determinate an initial time delay \( D \); (4) Move row B forward or backward \( D \) time points, this new row will be called row C below; (5) Move row C \( k \) time steps forward and calculate a cross-correlation coefficient and record it; (6) Move backward 1 time steps, calculate a cross-correlation coefficient and record it; (7) Repeat step (6) until row C is \( k \) time steps backward row A. (8) Use interpolation function to fit \( (21^2 + 21^3) \) time points and cross-correlation coefficients, determinate the peak of the interpolation function by derivative. According to the peak, the sub-sampling -interval part of time delay is \( d \), \( (D+d) \) is the total time delay.

### 2.1. The Length of Time Series

To determinate the time delay at a specific time point, a corresponding time sequence length should be guaranteed to calculate the correlation coefficient. If two channels measure same value, such as the acceleration in a same floor (almost same), Ianniello [15] had made a good estimation below:

\[
\text{Var}(D) = \frac{4\pi^2 b^2}{L} \left[ \frac{S/N}{1 + 2S/N} \right]^{\frac{1}{4}}
\]

Where \( L \) is the length of time series. Other details can refer to [15]. That is to say, the larger \( L \) is, the larger time delay cross-correlation can determinate. However, a more common case is that these two channels measure different values, and their relation coefficient also will affect the result a lot. The (2.2) may give out a well estimation, but the calculation of correlation coefficient costs a large amount of time. On the other hand, if the cross-correlation coefficient is too low such as 0.15 or even lower, the result will be unreliable. In this case, even if \( L \) is long enough to satisfy (2.2), the estimation of \( D \) is also unreliable, and also it will be time-consuming if \( L \) is too long. Hence, the length of \( L \) is better at [150,350], and the reliable internal of \( D \) can be regarded as \( (1/5 \leq 1/15) L \).

### 2.2. Correlation Coefficient

Usually, the general cross-correlation coefficient can be calculated as below:

\[
z(\lambda) = \frac{1}{T} \int_0^T x_1(t) x_2(t + \lambda) dt
\]

For discrete-time signal in this paper, SSD criteria will be used, and the formula is below:

\[
C_{\text{SSD}} = \sum \left( \frac{\bar{f}_i}{\sqrt{\sum f_i^2}} - \frac{\bar{x}_i}{\sqrt{\sum x_i^2}} \right)
\]

### 2.3. Accuracy Target

In this paper, the purpose of estimation of time delay is the reduction of the error in modal analysis. The core question is; how small the time delay should be so that the affect to modal analysis can be negligible. Obviously, after the estimation of time delay and moving each data in row B to the nearest time point in row A (and called new row B as row D), in row D, each data can be regarded that they are sampled near standard time point. The word “near” needs experiment to determinate.

Actually, if the interval of sampling is \( \Delta T \), even 0.1\( \Delta T \) error in row D has little affect to the result of parameter identification. Table 1 and Table 2 show an example of modal analysis, Table 1 is the result under standard sampling and Table 2 is under 0.1\( \Delta T \) error, respectively.
Table 1. Result under Standard Sampling

| Frequency | Model Shape |
|-----------|-------------|
| 1st Order | 2.2461      |
| 1st Order | -0.2843     |
| 1st Order | -0.5458     |
| 1st Order | -0.7632     |
| 1st Order | -0.9189     |
| 1st Order | -1.0000     |
| 2nd Order | 6.5430      |
| 2nd Order | -0.7660     |
| 2nd Order | -1.0000     |
| 2nd Order | -0.5408     |
| 2nd Order | 0.2957      |
| 2nd Order | 0.9334      |
| 3rd Order | 10.1563     |
| 3rd Order | -1.0000     |
| 3rd Order | -0.2740     |
| 3rd Order | 0.9346      |
| 3rd Order | 0.5559      |
| 3rd Order | -0.7613     |

Table 2. Result under 0.1ΔT Sampling Error

| Frequency | Model Shape |
|-----------|-------------|
| 1st Order | 2.2461      |
| 1st Order | -0.2843     |
| 1st Order | -0.5458     |
| 1st Order | -0.7632     |
| 1st Order | -0.9189     |
| 1st Order | -1.0000     |
| 2nd Order | 6.5430      |
| 2nd Order | -0.7660     |
| 2nd Order | -1.0000     |
| 2nd Order | -0.5408     |
| 2nd Order | 0.2957      |
| 2nd Order | 0.9334      |
| 3rd Order | 10.1563     |
| 3rd Order | -1.0000     |
| 3rd Order | -0.2740     |
| 3rd Order | 0.9423      |
| 3rd Order | 0.5606      |
| 3rd Order | -0.7674     |

From Table 1 and Table 2, there is no error in natural frequency, which is very stable, while some small errors can be found in model shape. In this paper, Frobenius-norm had been used to describe the difference between the results in Table 1 and Table 2. It is only 0.38%. That is to say, accuracy target can be regarded as 10% sampling interval.

2.4. Interpolation Function
In [15], linear interpolation had been used to determinate the non-integer part of time delay. The expression is shown below:

\[
\frac{\hat{D}}{\Delta T} = i_p + \frac{1}{2} \frac{z(i_{p+1})}{z(i_p)} - \frac{z(i_{p-1})}{z(i_{p+1})}
\] (2.5)

Obviously, linear interpolation will lose lots of information from other points nearby. Therefore, in this paper, spline interpolation has been used to determinate the non-integer part of time delay. The order of interpolation function can be regarded as the same as the reliable internal of \( D \) in Section 3.1.

3. Numerical Experiments
In this paper, two rows of signal have been created manually to verify PROCESS. They are the sum of finite sine wave function. The general expression of manual signals in this paper are as:

To simulate damping, an exponential decay factor depend on time also should be applied to the manual signals. In (4.1), they should have same natural frequency \( \omega_i \) but different amplitude \( A_i \) and \( B_i \). Their phase, \( \varphi_i \) and \( \varphi_i' \) can be same or different (but in this paper, they all had been taken as 0). Their orders, \( n \) and \( m \), would better be the same.

In the following numerical experiment, the sampling frequency for \( x_1 \) is standard (1Hz), while for \( x_2 \) is a random walk around 1 second, and the sampling interval obeys uniform distribution \( U(0.9,1.1) \). Total time points are 4000. The sampling frequency in this paper is 1Hz, so that it will be convenient to see the percentage of the error relative to sampling interval.

3.1. The Accuracy of Identification
Firstly, the accuracy of PROCESS should be determinate. The method is to use same signal with on time delay and to use different signal with on time delay.
In Fig 2, there is no error in the result, because they are same signal. While in Fig 3, even this two rows of signal are synchronous, this method has systematic error, and this error depends on interpolation function and cross-correlation coefficient.

3.2. Examples of Identification
In this Section, 4 cases have been created to verify PROCESS. These 4 cases are shown below:
Case 1: $x_2$ has a time shift (3 seconds). Result should be a horizontal line.
Case 2: $x_2$ has a frequency shift (0.999). Result should be a skew line.
Case 3: $x_2$ has a sine wave shift. Result should be a sine wave shape.
Case 4: $x_2$ has random walk shift.
Table 3 shows the result of identification and errors in these 4 cases.

|                      | Case 1 | Case 2 | Case 3 | Case 4 |
|----------------------|--------|--------|--------|--------|
| Maximal Error        | 0.0180 | 0.0170 | 0.1422 | 0.2896 |
| Average Error        | 0.0083 | 0.0069 | 0.0601 | 0.0734 |

4. Conclusion and Comments

An estimation of time delay via cross-correlation has been proposed in this paper. A series of numerical experiments have been conducted to verify PROCESS. Obviously, PROCESS can determine the time delay of each sampled point, and the quality of results depends on the cross-correlation coefficient between signals. For the accuracy, even two rows of signals have no time delay, this method still has systematic error, and it depends on interpolation function and cross-correlation coefficient. In other words, iteration can improve the result but still have a limitation, and need further study. And as it has been illustrating in Section 2.1, the determination of the length of time series still need further study. Finally, in this paper, Pearson correlation coefficient has been used. However, other linear correlation coefficient (like Spearman and Kendall correlation coefficient) will not reduce the systematic error, and it is common that data do not obey linearly dependent coefficient, so the estimation of time delay in this paper will fail. Copula might be a new way to solve this problem, but the choice of correlation function still needs further study.

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