The 3-dimensional numerical simulation of artificially altitude-triggered negative lightning

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Abstract. A 3-dimensional numerical model for artificially altitude-triggered negative lightning is developed based on an analytic thunderstorm model and the Dielectric Breakdown Model (DBM). Two major parameters are concerned, they are the thundercloud electric field and the length of the nylon wire which isolates the triggering wire from the ground. A few groups of contrast numerical experiments are done to study their effects on the success rates of altitude-triggered lightning. It is found that the success rates of altitude-triggered lightning increase when the thundercloud electric field enhances or the length of the nylon wire increases. Another interesting phenomenon is that the upward positive leader is always initiated earlier than the downward negative leader in either case.

1. Introduction
Artificially altitude-triggered lightning is an efficient way to study some unknown physical processes related to the lightning flashes. It is initiated by a small rocket towing an ungrounded triggering wire aloft under a thunderstorm. When the triggering wire is introduced rapidly into a strong-field region, it might actually initiate a bi-directional stepped leader, i.e., the upward positive leader and the downward negative leader. The positive leader propagates towards the thundercloud and might initiate a negative lightning, while the negative leader is an efficient way to imitate the lightning connection process to a grounded structure.

Since numerical work complements the experimental studies, much effort has been done to develop the lightning model. A numerical model on the stepped leader path in the earth atmosphere was developed by Odim Mended to study the effect of the tropospheric electric conductivity on the lightning behaviour [1], and a self-consistent upward leader model was proposed by Marley Becerra to study the initiation and propagation of an upward connecting leader in the presence of a downward moving leader [2], and so on. Nevertheless, much work remains to be done. It is worthy to mention that P. Lalande once developed a model on the altitude-triggered lightning to study the lightning connection process [3]. In the model, the Charge Simulation Method (CSM) was applied to the computation of the electric field which was distorted by the introduction of the triggering wire, and the distribution of the induced charge on the surface of the triggering wire was assumed according to

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experience, so it might be a good work but seems to be not very scientific. Besides, there are few systemic reports about the growth rule of the bi-directional stepped leader.

In this paper, a 3D numerical model on the altitude-triggered lightning is proposed and it contains 4 major parts:

- A background electric field model based on an analytic thunderstorm model.
- A new method to calculate the distorted electric field based on the combination of the Laplace equation and the Successive Over Relaxation (SOR) method.
- A stepped leader model based on the Dielectric Breakdown Model which ensures the lightning path to have enough branches.
- A space resolution conversion model to save the calculation resource.

A study of the relationship between the success rates of altitude-triggered lightning and the two parameters, the background electric field intensity and the length of the nylon wire which will be introduced in the following text, is carried out based on the model. And finally, some growth rules of the bi-directional stepped leader are indicated.

2. Modelling

The experiment device of altitude-triggered lightning is shown in figure 1: the triggering wire is spooled to the end of a small rocket and introduced into the air. It has a constant total length of \( l \), and is isolated from the ground by a nylon wire with a length of \( H \). When the triggering wire has been trailed out over a sufficient length of \( l_t \), the bi-directional leader is initiated from both of its two ends and propagates towards the cloud and the ground respectively. For simplification, the speed of the rocket is assumed to be constant and the value is 100 m s\(^{-1}\).

2.1. Background electric field

An appropriate background electric field model is necessary to develop the numerical model of the altitude-triggered lightning. It should not only be able to express the main characteristics of the thundercloud electric field, but also not be too complex to calculate.
Williams, Uman and Rakov proposed the Three Layers Cylindrical Charged Pile Model (TLCCPM) to calculate the thundercloud electric field and it overcame the drawback of Dipole Model that the distribution of the charge was simplified too much. And then Amoruso and Lattarulo pointed out that when the charge density inside the thundercloud was symmetrical the TLCCPM could be simplified to the Charged Disks Model. It is convenient and reasonable to describe the thundercloud electric field with this model [4, 5]. And the final expression for the background electric field is obtained [6]

\[ \hat{E}(x) \approx \frac{\rho_N}{4\pi r_1} \int_{s_1} \overrightarrow{u_1} \frac{dS'}{r_1} + \frac{\rho_{P-L}}{4\pi r_2} \int_{s_2} \overrightarrow{u_2} \frac{dS'}{r_2} + \frac{\rho_{P-P}}{4\pi r_3} \int_{s_3} \overrightarrow{u_3} \frac{dS'}{r_3} \]  \tag{1}

It is an analytic expression, where \( \rho_N \), \( \rho_{P-L} \), \( \rho_{P-P} \) are the charge density of the main negative, lower positive, positive charge center respectively, and \( \overrightarrow{u_1}, \overrightarrow{u_2}, \overrightarrow{u_3} \) are the respective outward unit normal vectors of the interface. The surface integral in equation (1) is

\[ \int_{s} \frac{dS'}{r} = 2 \left[ -\left(1-e'\right) \frac{\pi h + r_e E(p) + a^2 - R^2}{r_e} K(p) + \frac{h^2}{r_e} \frac{a - R}{a + R} \Pi(m, p) \right] \]  \tag{2}

Where \( K, E, \Pi \) are the first, second, and third kind of complete elliptic integral respectively, \( a \) is the radius of the thundercloud, \( R \) is the horizontal distance to the center of the thundercloud, \( h \) is the height of the charged disk, \( r_e = \left(\left( (a + R)^2 + h^2 \right)^{1/2} \right) \), \( m = 2(aR)^{1/2} (a + R)^{-1} \), and \( p = 2(aR)^{1/2} r_e^{-1} \). \( e' \) is -1, 0 or 1 respectively when \( R \) is greater than, equal to or less than \( a \). Taking account of the Mirror Effect of the ground, the total electric field is obtained.

The space concerned is cubic with a size of 4 x 4 x 5 km. Integrating the E-field, the electric potentials of the whole space will be obtained. The potentials of points which are on the surface of the space will be the constant outer boundary conditions of the Laplace Equation.

2.2. The distorted electric field
The electric field is distorted when the triggering wire is introduced into the air. Thus the E-field around the triggering wire is enhanced and might exceed the initiating criterion of the stepped leader. Since both the distorted E-field and the charge distribution on the wire are unknown, it is not easy to calculate the distribution of the space potential in fact. As the relaxation velocity of charge is much faster than the velocity of the rocket, the triggering wire could be treated as an equipotential conductor at each time step. Thus we have one more unknown parameter, the potential of the triggering wire, besides the solutions to the Laplace equations. On the other hand, the net charge on the surface of the triggering wire must be zero, so a new equation related to the potential of the triggering wire is obtained.

\[- \int_{\partial \Omega_1} \frac{\partial \phi}{\partial n} = 0 \]

Where \( \partial \Omega_1 \) is the surface of the triggering wire. Adding this equation to the finite difference Laplace equations as an inner boundary condition, we obtain a definite problem

\[ \begin{align*}
\nabla^2 \phi &= 0 \\
- \int_{\partial \Omega_2} \frac{\partial \phi}{\partial n} &= 0 \\
\phi|_{\partial \Omega_3} &= \psi
\end{align*} \]  \tag{3}

Where \( \partial \Omega_2 \) is the outer boundary of the space.

The seven-point finite difference scheme of the Laplace equations [7] is
\[
\frac{\varphi_{i+1,j,k} - 2\varphi_{i,j,k} + \varphi_{i-1,j,k}}{\Delta x^2} + \frac{\varphi_{i,j+1,k} - 2\varphi_{i,j,k} + \varphi_{i,j-1,k}}{\Delta y^2} + \frac{\varphi_{i,j,k+1} - 2\varphi_{i,j,k} + \varphi_{i,j,k-1}}{\Delta z^2} = 0
\] (4)

Where \(\Delta x\), \(\Delta y\) and \(\Delta z\) are the respective resolution of \(x\), \(y\) and \(z\) axis. Applying the method of Successive Over Relaxation (SOR) to equation (4), we will obtain the iteration scheme of potential \(\varphi\)

\[
\varphi_{i,j,k}^{(t+1)} = \varphi_{i,j,k}^{(t)} + \frac{\omega}{2(\Delta x^2 + \Delta y^2 + \Delta z^2)} \left\{ \Delta x^2 \left( \varphi_{i+1,j,k}^{(t)} + \varphi_{i-1,j,k}^{(t)} \right) + \Delta y^2 \left( \varphi_{i,j+1,k}^{(t)} + \varphi_{i,j-1,k}^{(t)} \right) + \Delta z^2 \left( \varphi_{i,j,k+1}^{(t)} + \varphi_{i,j,k-1}^{(t)} \right) \right\}
\] (5)

Where the superscript \(t\) represents the iteration, and \(\omega\) is the relaxation factor

\[
\omega = \frac{6}{3 + [9 - (\cos \frac{\pi}{l-1} + \cos \frac{\pi}{m-1} + \cos \frac{\pi}{n-1})]^2}
\] (6)

Where \(l\), \(m\), and \(n\) are the respective number of lattices in the direction of \(x\), \(y\) and \(z\) axis, and it is evident that \(\omega \in [1, 2]\).

When the criterion is exceeded, the bi-directional leader will be initiated and propagate rapidly, the wire will be melted by the heat produced by the leader discharge. Both the channel of the wire and the developed leader will be treated as new constant boundary conditions for another iterative calculation.

### 2.3. The stepped leader model

The E-field can be determined when the distribution of the space potential is known. And the Dielectric Breakdown Model [8] is applied to calculate the probability of each possible direction for the next stepped leader

\[
P = \begin{cases} 
\left\lvert E_y - E_{\text{crit}} \right\rvert / F & E_y \geq E_{\text{crit}} \\
0 & E_y < E_{\text{crit}} 
\end{cases}
\] (7)

\[
F = \sum_{j(y_j \neq y_{\text{crit}})} \left\lvert E_y - E_{\text{crit}} \right\rvert
\]

Where \(E_y = (\varphi_y - \varphi) / \Delta l\) is the average electric field intensity between the developed and developing point, \(\Delta l\) is the distance of the two points, and \(E_{\text{crit}}\) is the threshold value. The Monte Carlo Method is used in the selection.

Considering the finite conductivity of the leader channel, we use \(E_m\) to describe the axial electric field intensity. Thus the potential of the newly chosen point becomes

\[
\varphi_i = \varphi_0 - E_m \sum \Delta l
\] (8)

Where \(\varphi_0\) is the potential of the initiating point, and \(\sum \Delta l\) is the length along the channel. Thus a new boundary condition is obtained.

### 2.4. The resolution conversion

In this paper, a resolution of 50 m is applied to the whole 3D space concerned, and it is enough to show the details of the stepped leader. But for some areas where the electric field is distorted by the triggering wire and has a greater gradient, it is necessary to convert the resolution into a finer one to ensure a precise solution. The 3-spline interpolation method is employed to do this conversion [9]. The procedure is shown in figure 2, and the finer resolution is 5 m.
2.5. The criterion of success rates
An altitude-triggered lightning numerical experiment succeeds when the upward positive leader reaches the cloud and the downward negative leader reaches the ground respectively. But not every numerical experiment will succeed, because the total length of the triggering wire is limited. The numerical experiment will fail if the bi-directional leader is not initiated before the wire runs out. And the longer the triggering wire is used, more difficult the numerical experiment to trigger a lightning. So the success rate of the altitude-triggered lightning experiment will be

\[ \text{Rate} = \frac{\alpha (l_0 - l_n)}{l_0} \]  

Where \( l_n \) is the length of the triggering wire, \((l_0 - l_n)\) is the remaining length, and \( \alpha \) is an adjustable parameter which will be determined by the numerical experiment.

3. Numerical experiment
Numerical experiments are conducted while the parameters are: \( \rho_p = \rho_n = 0.5 \text{nC} \cdot \text{m}^{-3} \), \( \rho_L = 0.1 \text{nC} \cdot \text{m}^{-3} \), the respective height \( h_p = 10 \text{ km} \), \( h_n = 1.5 \text{ km} \), \( h_L = 1.4 \text{ km} \), the radius of the thundercloud \( a = 2 \text{ km} \), the radius of the lower positive center \( a_n = 0.5 \text{ km} \), the total length of the triggering wire \( l_0 = 350 \text{ m} \), the length of the nylon wire \( H_n = 350 \text{ m} \), the axial electric field intensity of the channel \( E_n = 500 \text{ V} \cdot \text{m}^{-1} \), the threshold value of the upward positive leader \( E_{\text{up}} = 150 \text{ V} \cdot \text{m}^{-1} \), the threshold value of the downward negative leader \( E_{\text{crit}} = 180 \text{ V} \cdot \text{m}^{-1} \), the error threshold of the SOR method \( \delta = 0.1 \text{ V} \). The result of the simulation is shown in figure 3.

In this case, the electric field intensity \( E_0 \) at the center of the ground \((0, 0, 0)\) is about \(-15 \text{ kV} \cdot \text{m}^{-1}\). An upward positive leader is initiated from the top of the triggering wire and propagates towards the cloud when the length is 110 m. And about 40 steps later, a downward negative leader is developed from the bottom of the triggering wire and propagates towards the ground. Both of them have abundant branches which make it look like a real lightning. And according to the criterion, the success rate in this case is obtained

\[ \text{Rate} = \alpha (350 - 110)/350 = 68.6\% \]

If it is assumed that \( \alpha = 1 \), the success rate will be 68.6%.
In the following text two groups of contrast numerical experiments are conducted to study the effect of the thunderstorm electric field intensity $E_n$ and the length of the nylon wire $H_n$ on the success rates of altitude-triggered lightning. Other parameters are invariant unless mentioned.

3.1. The effect of the thunderstorm electric field intensity
To describe the problem quantitatively, the length of the nylon wire is assumed to be constant $H_0=350$ m in this case. And for convenience, the electric field intensity $E_n$ at the center of the ground is employed to express the variation of the electric field instead of the thundercloud charge density, where $E_n = E_0 / n$ and $E_0$ is $-15$ kVm$^{-1}$. The results of the contrast numerical experiments are shown in table 1, where $l_n$ is the length of the triggering wire.

| $n$ | $E_n$ (kVm$^{-1}$) | $l_n$ (m) | Rate  | $n$ | $E_n$ (kVm$^{-1}$) | $l_n$ (m) | Rate |
|-----|------------------|----------|-------|-----|------------------|----------|------|
| 2   | -7.500           | 130      | 62.9% | 10  | -1.500           | 290      | 17.1%|
| 4   | -3.750           | 165      | 52.9% | 12  | -1.250           | 335      | 4.2% |
| 6   | -2.500           | 215      | 38.6% | 14  | -1.071           | Failed   | 0    |
| 8   | -1.875           | 260      | 25.7% | 16  | -0.937           | Failed   | 0    |

Evidently, $l_n$ grows when the magnitude of the electric field intensity decreases, and the success rates decrease quickly at the same time. When $E_n$ is less than $-1.071$ kVm$^{-1}$, there seems no possibility to trigger a lightning in the case. The relation between $E_n$ and the success rates are shown in figure 4, where the success rates decrease in a parabola law with the variation of $E_n$. 

Figure 3. Diagram of an altitude-triggered lightning discharge and the resolution conversion.
The success rate

3.2. The effect of the length of the nylon wire

In this case the length of the nylon wire $H_n$ is variable, and $E_n$ is constant. We assume $E_n = -7.5 \text{ kV/m}$. Other parameters are invariant as mentioned above. The results of the contrast numerical experiments are shown in table 2, where $l_n$ is the length of the triggering wire.

| $H_n$ (m) | $l_n$ (m) | Rate | $H_n$ (m) | $l_n$ (m) | Rate |
|----------|-----------|------|-----------|-----------|------|
| 250      | 140       | 60.0%| 450       | 120       | 65.7%|
| 300      | 140       | 60.0%| 500       | 120       | 65.7%|
| 350      | 130       | 62.3%| 550       | 110       | 68.8%|
| 400      | 130       | 62.3%| 600       | 110       | 68.6%|

The success rates increase when the nylon wire gets longer. A probable reason is that the electric field intensity increases along the altitude [10, 11]. But the variation is not as evident as the thundercloud electric field intensity does. The relation between $H_n$ and the success rates are shown in figure 5, where the success rates form a ladder-type curve coursed by the discrete time step. A linear fitting has been done to the data, and it fits quit well.
The success rate $R$ is plotted against the length of the nylon wire $H_n (m)$ in Figure 5. The relation of $H_n$ and the success rates.

3.3. The bi-directional stepped leader
Another interesting phenomenon is found in all the simulations that the upward positive leader is always initiated earlier than the downward negative leader. And this phenomenon gets more obvious when the electric field intensity decreases. The initiating procedure is shown in figure 6.

The result is highly consistent with the experimental data proposed by P. Lalande [3]. Two probable reasons are responsible for the phenomenon. Firstly, the electric field intensity increases along the altitude, what has been mentioned in section 3.2. The top of the triggering wire where the upward positive leader is initiated has a higher electric field and it is easier to exceed the initiating criterion. Secondly, the threshold of the positive leader is lower than the negative leader [12]. It means that the positive leader is easier to be initiated.

4. Conclusion
A 3-dimensional numerical model on artificially altitude-triggered lightning has been developed in a new and more physical way. It combines the charge conservation equation with the Laplace equations and is solved with the application of the SOR method. A criterion is proposed to describe the success rates of artificially altitude-triggered lightning. Through groups of contrast numerical experiments, it is
found that the success rates of artificially altitude-triggered lightning increase when the thundercloud electric field enhances or the length of the nylon wire increases. And the former one is the main influence factor. Another interesting phenomenon is found that the upward positive leader is always initiated earlier than the downward negative leader in either case. A typical time interval is about 40 steps and it becomes more obvious when the electric field decreases.

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