Thoughts on CP Violation

R. D. Peccei

Department of Physics and Astronomy, UCLA, Los Angeles, CA 90095-1547

Abstract

The necessary complex structures needed for CP violation are not easy to generate and, indeed, CP can be a symmetry in higher dimensional theories. In 4-dimensions CP violation argues for the existence of a scalar sector and, in its simplest manifestation, leads to the CKM model. Further CP violating phases, from an extended scalar sector, are constrained by the requirement of having no FCNC. Although new CP violating phases are expected in SUSY extensions of the SM, the good agreement of data with the CKM model only provides bounds on these phases. Baryogenesis via leptogenesis requires the presence of CP violating phases in the neutrino sector. However, the CP phases likely to be eventually measured experimentally are not necessarily directly connected to the ones which drive the leptogenesis.

1 Violating CP Is Not Easy

For CP (or T) to be violated one must have complex structures in the theory. This is easy to understand since under CP, effectively, operators are replaced by their Hermitian adjoints. For instance, for the charged $SU(2)$ gauge field $W_{\mu}^\pm$ under CP

$$W_{\mu}^{\pm}(x,t) \rightarrow \eta(\mu)W_{\mu}^{\pm}(-x,t),$$

(1)

where $\eta(0) = -1$ and $\eta(i) = +1$. Thus, schematically, under CP an operator $O(x,t)$ is replaced by:

$$O(x,t) \rightarrow O^\dagger(-x,t).$$

(2)

Because Lagrangians must be Hermitian, a term in the Lagrangian containing the operator $O$ has the structure

$$\mathcal{L} = aO + a^*O^\dagger,$$

(3)

where $a$ is a $c$-number. It is clear from the above that $\mathcal{L}$ is CP-invariant if $a^* = a$. So, without some complexity there will be no CP violation.

Having some complex structures in the theory, however, by itself may not suffice to give CP violation. A well known example is the two-generation Standard Model (SM) in which the phases in the complex Yukawa couplings can always be rotated away. There are also cases in which CP-violating terms appear in the theory without an apparent complex structure. The
most well known example is the famous $\theta F^{\mu \nu} \tilde{F}_{\mu \nu}$ term of QCD which gives rises to the strong CP problem. However, this term can be traced to the complex superposition of states which enters in the proper gauge theory vacuum. A more challenging problem, to which I return below, is why this allowed CP-violating term is apparently absent to a high degree of accuracy.

Although we observe that CP is violated experimentally, in certain contexts it is much more natural for CP to be conserved. In fact, as noted by Dine, Leigh, and MacIntire and Choi, Kaplan and Nelson, based on an original observation of Strominger and Witten in 10-dimensional heterotic string theory CP can be identified with the product of a Lorentz transformation in the 10-dimensional spacetime times a gauge transformation. In this theory fermions and gauge fields are in the adjoint representation of $E_8$, which is real, and CP acts as an inversion in a 6-dimensional compact space of the 10-dimensional theory. Thus, if our 4-dimensional world should originate from such a theory, the observed CP-violating effects have their origin as a result of the $10d \to 4d$ compactification. In these kinds of theories, in principle, one may be able to compute the resulting 4-dimensional CP-violating phases from the underlying geometry. I will return to this interesting possibility at the end of this paper.

Even in four dimensions, violating CP is not easy. In particular, a theory involving only fermions and gauge fields is CP-conserving, up to $\theta$-terms. Such theories have real coupling constants $g_i$, since the corresponding gauge fields $A_\mu^i$ transform according to the adjoint representation of the respective groups. The topological nature of the non-Abelian gauge theory vacuum, in the Standard Model allows for the presence of two CP-violating $\theta$-terms:

$$\mathcal{L} = \theta_W \frac{\alpha_2}{8\pi} W^\mu_a \tilde{W}_{a\mu} + \theta_S \frac{\alpha_3}{8\pi} F^\mu_{a\nu} \tilde{F}_{a\mu\nu}. \quad (4)$$

However, because the electroweak theory is chiral, the $\theta_W$ term can be rotated away. Furthermore, as mentioned above, the $\theta_S$ term is severely restricted, since no evidence has been found yet for a neutron electric dipole moment. The present bound on this moment [$\text{edm} < 6.3 \times 10^{-26}\text{ ecm}$] requires $\theta_S < 10^{-10}$. This is the strong CP problem.

The strong CP problem is still unresolved with four possibilities being bruited about:

i) It could be that $\theta_S$ accidentally happens to be small, just like other parameters in the SM—like the ratio $m_e/m_t$. However, this is hardly satisfactory as an explanation!

ii) The strong interactions also have a chiral symmetry, connected to the vanishing of the $u$-quark mass, which effectively allow $\theta_S$ to be rotated away. However, a careful current algebra analysis of the hadronic spectrum appear to argue against this possibility.

iii) The SM Lagrangian is augmented by an additional chiral global symmetry $U(1)_{\text{PQ}}$ which forces $\theta_S \to 0$ dynamically. In this case the parameter $\theta_S$ is effectively replaced by a dynamical axion field [$\theta_S \to a(x)/f_a$]. However, axions have not been seen, and the scale $f_a$ of $U(1)_{\text{PQ}}$ breaking is severely limited. Nevertheless, axions remain a tantalizing and beautiful candidate for the Universe’s dark matter.

iv) CP is a spontaneously broken symmetry and $\theta_S$ is a calculably small parameter. Although models exist where this is realized in general these models run into difficulties either with cosmology (see below) or cannot reproduce the structure of the observed CP violation at low energy.

At any rate, whatever the reason is for $\theta_S < 10^{-10}$, it is clear that this term by itself cannot be at the origin of the observed CP violation in K and B physics. These CP-violating phenomena are connected to flavor-changing transitions, while the $\theta_S F \tilde{F}$ term is flavor diagonal.
To account for the observed CP-violating phenomena, if there are no elementary scalar fields, it is necessary to imagine the formation of CP-violating fermion condensates. These, most likely, would need to involve some other fermion fields (techni-fermions) rather than the ordinary quarks and leptons. However, the formation of complex CP-violating condensates $\langle \bar{T}T \rangle \sim e^{i \delta_{TC} \Lambda_{TC}}$, with $\Lambda_{TC} \sim G_F^{-1/2}$, is very problematic. The origin of this problem was pointed out long ago by Zeldovich, Kobzarev and Okun. They showed that, if CP is a spontaneously broken symmetry, domains of different CP will form in the Universe, separated by walls whose slow dissipation with temperature is a cosmological catastrophe. Indeed, the energy density in the domain walls only decreases linearly with temperature, $\rho \sim \sigma^2 T$. If $\sigma$ is of order $\sigma \sim G_F^{-3/2}$ this energy density greatly exceeds the closure density of the Universe and the model makes no sense. One can countenance spontaneous violation of CP only if the scale where this violation occurs is above the scale where inflation takes place, since then one can inflate the domains away. However, this cannot happen in models where the fermion condensates must also serve to break $SU(2) \times U(1)$.

2 CP Violation and the Scalar Sector

In view of the above discussion, it seems very natural to assume that the experimentally observed CP violation is due to the presence of a scalar sector in the theory. Indeed, personally I think that the existence of CP violation at low energy is as compelling evidence for a Higgs field as are the precision electroweak tests which suggest the presence of a light Higgs boson, $M_H < 204$ GeV at 95% C. L. In fact, as discussed in detail by Buras at this Rencontre, all data in both K and B decays are perfectly consistent with the CKM paradigm where all the observed CP violation originates from the complex Yukawa couplings of the Higgs field with the quarks. As is well known, with three generations of quarks, this model has just one physical phase $\delta_{\text{CKM}}$. All data appear consistent with this phase, in the standard convention, being rather large: $\delta_{\text{CKM}} \sim (59 \pm 13)^\circ$.

It is clearly very important to look for deviations from the CKM paradigm, but both data and theory at this moment do not allow us to make any such pronouncement. I will return to this important point later on. However, first I want to discuss theoretically whether it might be possible to identify sources of possible flavor conserving CP-violating effects coming from the pure Higgs sector itself. As we shall see, these effects are not easy to find if one takes into account the structure of what we know about the weak interactions!

The SM, in which only one Higgs doublet is introduced, is very special. In this case, the required Hermiticity of the Higgs potential makes all parameters in the potential real:

$$V = \mu^2 \phi^\dagger \phi + \lambda (\phi^\dagger \phi)^2 . \quad (5)$$

Thus, there are no additional CP-violating phases in the SM besides $\delta_{\text{CKM}}$. However, if there is more than one Higgs doublet, the Higgs potential in general can have some possible CP-violating phases. But, even in this case, there are constraints. It is useful to illustrate the nature of these constraints in the case of having two Higgs doublets

$$\chi = \begin{pmatrix} \chi^+ \\ \chi^0 \end{pmatrix} ; \quad \phi = \begin{pmatrix} \phi^+ \\ \phi^- \end{pmatrix} . \quad (6)$$

The most general Higgs potential in the 2-doublet model can be written as the sum of three terms, reflecting specific additional symmetries besides $SU(2) \times U(1)$. To understand this
structure, it is useful to recall that under the weak hypercharge $U(1)$ the Higgs doublets in Eq. (6) transform as
\[ \chi \rightarrow e^{i\xi/2} \chi; \quad \phi \rightarrow e^{-i\xi/2} \phi. \] (7)

There is an additional Abelian symmetry, $U(1)_{\text{PQ}}$, which one can contemplate for these fields, which rotates them in the same way:
\[ \chi \rightarrow e^{i\alpha} \chi; \quad \phi \rightarrow e^{i\alpha} \phi. \] (8)

Such a symmetry allows for a chiral transformation of the quarks, thereby setting $\theta_S \rightarrow 0$ dynamically. Finally, one can also consider possible discrete symmetries for the $\chi$ and $\phi$ fields. In particular, to avoid flavor changing neutral currents (FCNC) one can consider a discrete symmetry $D$ which allows $\phi$ only to couple to $u_R$ and $\chi$ only to couple to $d_R$. Under $D$:
\[ \chi \rightarrow -\chi; \quad \phi \rightarrow \phi; \quad d_R \rightarrow -d_R; \quad u_R \rightarrow u_R. \] (9)

The full Higgs potential is the sum of three terms: $V = V_1 + V_2 + V_3$. The first term, $V_1$, is $SU(2) \times U(1) \times U(1)_{\text{PQ}} \times D$ invariant; $V_2$ is $SU(2) \times U(1) \times D$ invariant; and $V_3$ is only invariant under the electroweak group. In detail, one has:
\[ V_1 = \mu_1^2 \chi^\dagger \chi + \mu_2^2 \phi^\dagger \phi + \lambda_1 (\chi^\dagger \chi)^2 + \lambda_2 (\phi^\dagger \phi)^2 + \lambda_3 (\phi^\dagger \chi)(\chi^\dagger \phi) + \lambda_4 (\chi^\dagger \chi)(\phi^\dagger \phi) \] (10)
\[ V_2 = \lambda_5 e^{i\delta_5} (\chi^T C \phi)^2 + \lambda_6 e^{-i\delta_6} (\chi^T C \phi)^2 \] (11)
\[ V_3 = \mu_{12}^2 e^{i\delta_{12}} (\chi^T C \phi) + \mu_{12}^2 e^{-i\delta_{12}} (\chi^T C \phi)^\dagger + \left[ \lambda_6 e^{i\delta_6} (\chi^T C \phi) + \lambda_6 e^{-i\delta_6} (\chi^T C \phi)^\dagger \right] \chi^\dagger \chi + \left[ \lambda_7 e^{i\delta_7} (\chi^T C \phi) + \lambda_7 e^{-i\delta_7} (\chi^T C \phi)^\dagger \right] \phi^\dagger \phi. \] (12)

One sees therefore, that if one asks that $V$ be just $SU(1) \times U(1)$ invariant, the full Higgs potential contains 4 phases: $\delta_5$, $\delta_{12}$, $\delta_6$, and $\delta_7$. If, on the other hand, one asks that $V$ be also $U(1)_{\text{PQ}}$ invariant (so that $V = V_1$) all of the possible Higgs sector CP violating phases disappear.

If only the discrete symmetry $D$ is present (so that $V = V_1 + V_2$), one additional Higgs sector phase, $\delta_5$, appears in the potential. However, as Branco, Lavoura and Silva note in their nice book on CP violation, this phase gives no physical CP-violating effects. The phase $\delta_5$ in this case is correlated directly with the phase of the Higgs VEVs $\theta$:
\[ \langle \chi^0 \rangle = v_\chi; \quad \langle \phi^0 \rangle = v_\phi e^{i\theta}. \] (13)

Minimization of the potential $V = V_1 + V_2$ requires that $\sin(\delta_5 + 2\theta) = 0$. It is easy to check that all CP-violating phenomena, like for example the coupling of the axial Higgs field $A$ to $H^+ H^-$, are proportional to the phase combination $\delta_5 + 2\theta$, which vanishes:
\[ g_{AH^+H^-} \sim \sin(\delta_5 + 2\theta) = 0 \] (14)

Thus, remarkably, even in the case of having 2 Higgs doublets, the requirement that there be no FCNC (i.e. that $D$ be a good symmetry) prevents the appearance of any other CP-violating phases, besides the CKM phase $\delta_{\text{CKM}}$.

There are a number of corollaries to this result. For instance, in axion models where $U(1)_{\text{PQ}}$ is broken at a scale $f_a \gg v \sim G_F^{-1/2}$, no additional CP-violation ensues in the Higgs

*Here $C$ is a charge conjugation matrix.
sector. In such models the spontaneous breaking of $U(1)_{\text{PQ}}$ is effected by a complex singlet field $\sigma$, with VEV $\langle \sigma \rangle \simeq f_a$. The $U(1)_{\text{PQ}}$ invariant potential

$$V_a = \kappa e^{i\delta_a} \sigma^2 (\chi^T C \phi) + \kappa e^{-i\delta_a} [\sigma^2 (\chi^T C \phi)]^\dagger,$$

(15)

with $\kappa f_a^2 \equiv \mu_a^2 \sim v^2$, generates an additional complex term to the Higgs potential, beyond $V_1$. However, also in this case, the phase $\delta_a$ (just like $\delta_5$ in the previous discussion) is locked to the phase $\theta$ of the doublet Higgs VEV, and no physical CP-violating effects ensue.

It is, of course, possible to get Higgs sector CP-violating effects by complicating the theory further. The simplest case involves introducing an additional real singlet field $\eta$, which is also odd under $D$ ($\eta \rightarrow -\eta$) The total potential $V$ now is $V = V_1 + V_2 + V_4$, where $V_4$ contains an additional phase $\delta_4$:

$$V_4 = \kappa' (\eta^2 - f^2)^2 + \mu_4^2 e^{i\delta_4} \eta (\chi^T C \phi) + \mu_4^2 e^{-i\delta_4} \eta (\chi^T C \phi)^\dagger.$$

(16)

Because $\eta$ acquires a VEV, $\langle \eta \rangle = f$, the potential now has three phases: $\delta_4$; $\delta_5$; and $\theta$ the phase associated with the $\phi$ VEV. There is now enough freedom in the theory so that one linear combination of these phases gives rise to physical CP-violating effects. However having $\langle \eta \rangle \neq 0$ breaks the $D$-discrete symmetry spontaneously, and domain walls will ensue in the early Universe. So, it is not clear this model makes any sense cosmologically!

In general, however, if one introduces a sufficiently complicated Higgs sector it is possible eventually to have some non-trivial CP-violating phases. A good example is Weinberg’s 3 Higgs doublet model in which there are CP-violating phases associated to the coupling of the charged Higgs, $H^\pm$, to leptons and quarks. Such models can give rise to new observable phenomena, like the transverse muon polarization in $K^+ \rightarrow \mu^+ \pi^0 \nu_\mu$ decays. One finds

$$\langle P_\mu \rangle \sim \frac{M_K^2}{M_H^2} \text{Im}(g_{H\mu\nu} g_{Hd\bar{s}}^*).$$

(17)

This effect is interesting since, in principle, it can be larger than the induced polarization from final state interactions in $K^+ \rightarrow \mu^+ \pi^0 \nu_\mu$ ($\langle P_\mu \rangle_{\text{FSI}} \sim 10^{-6}$) and is an effect which is not present in the CKM model.

## 3 Supersymmetry and CP Violation

It is difficult to take CP-violating phenomena produced by multi Higgs models seriously, since there is no particular physical motivation for these models. In this respect, supersymmetric (SUSY) extensions of the SM have a much better pedigree. Unfortunately, CP violation in these models is largely a function of the assumed pattern of the SUSY-breaking terms! So these models, at this stage, are not predictive. In fact, as we shall see, there are difficulties in SUSY models both with CP violation and flavor conservation, which need careful control.

As is well known, the supersymmetric extension of the SM (SUSY SM) naturally requires the appearance of 2 Higgs doublets $\chi$ and $\phi$. The pure Higgs potential in the SUSY SM does not contain either the $V_2$ or $V_3$ terms, so that $V = V_1$, with the various parameters in $V_1$ taking particular values (related to the $SU(2) \times U(1)$ coupling constants). Without SUSY breaking, however, not only does the potential $V$ conserve CP but, in addition, it does not break $SU(2) \times U(1)$, since $\mu_1 = \mu_2 = 0$! As a result the introduction of soft SUSY breaking is
a necessity to render these models realistic, not only by producing a splitting of particles and sparticles in mass, but also to permit the breakdown of the electroweak theory.

In the simplest scheme of SUSY breaking, this breaking is mediated by gravitational strength forces and is assumed to be flavor blind. Even in this very restricted context, CP-violating phases appear in four different places:

i) In a complex gluino mass term: \( m_{1/2} \lambda_i \lambda_i \)

ii) In the complex coefficient \( A \) multiplying the Yukawa interactions in the scalar sector:
\[
A \Gamma_u \tilde{Q}_L \tilde{u}_R + A \Gamma_d \tilde{Q}_L \tilde{d}_R + \text{h.c.}
\]

iii) In the complex coefficient \( B \) multiplying bilinear scalar terms:
\[
B \mu (\chi^T C \phi) + \text{h.c.}
\]

iv) In the complex coefficient \( \mu \) characterizing the SUSY conserving Higgsino mass term:
\[
\mu (\tilde{\chi}^T C \tilde{\phi})
\]

One can check, however, that only 2 out of these possible 4 phases give rise to physical effects. For instance, the term \( B \mu \), is equivalent to \( \mu^2 e^{i \delta_{12}} \) in the notation of Eq. (12). However, as we remarked earlier, this phase by itself does not give rise to physical CP-violating effects.

Nevertheless, as the simple example above demonstrates, in general once SUSY breaking is introduced it is quite easy to get CP violation in the theory. The difficulty really is the opposite, not in generating CP-violating interactions but keeping the SUSY-breaking CP-violating contributions from violating experimental constraints! There are, in fact, two general types of constraints in SUSY extensions of the SM that one must be careful to satisfy. The first of these deals specifically with CP violating phenomena which preserve flavor, like the neutron electric dipole moment. Already nearly 20 years ago, Dugan, Grinstein, and Hall remarked that the neutron edm put powerful constraints on the CP-breaking phases appearing in SUSY extensions of the SM. They found, typically, that the neutron edm was given by the formula
\[
d_n \sim \left[ 300 \left[ \frac{100 \text{ GeV}}{\tilde{m}} \right]^2 \sin \phi_{A,B} \right] 6.3 \times 10^{-26} \text{ ecm.}
\]

Here \( \tilde{m} \) is a typical spartner mass and \( \phi_{A,B} \) are the two independent CP-violating phases entering in the simplest SUSY breaking structure described above. To satisfy the present experimental bound on \( d_n \), one sees that these phases have to be of \( O(10^{-3}) \) for spartner masses \( \tilde{m} \sim 100 \text{ GeV} \).

The second constraint on the flavor and CP structure of the SUSY breaking terms arises from the appearance of FCNC interactions due to SUSY matter entering at the loop level. These loop contributions, unless appropriately controlled, can lead to very large flavor changing effects, in complete contradiction with experiment. This whole subject has been analyzed in great detail by many authors and it would take me too far afield to review it here. For our more restricted purposes, connected with CP violation, a recent paper of Dine, Kramer, Nir, and Shadmi summarizes the relevant constraints very effectively.

The resulting structure of the SUSY CP violating effects in the flavor sector depends, in general, on how one assumes that the SUSY induced flavor violating effects are controlled. Three mechanisms are effective:

i) FCNC processes are suppressed through the imposition of near universality of the squark masses—\( \Delta \tilde{m}^2 \ll \tilde{m}^2 \)
ii) FCNC processes can also be suppressed by dynamical alignment of the squark-quark couplings to the gluinos—\(g_{gij} \sim \delta_{ij}\)

iii) Alternatively all loop processes can be suppressed by having heavy squarks and gluinos—\(\tilde{m} \gg \text{TeV}\).

It turns out that\(^2\) in general, only in the case of weak alignment one finds measurable SUSY breaking induced CP-violating effects. Even then, the results are rather model dependent. For example, about a year ago, Masiero, Piai, and Vives\(^2\) constructed a model in which \(\delta_{\text{CKM}} \equiv 0\). However, in their model CP violation due to SUSY breaking effects gave \(\epsilon \sim 10^{-3}\) (reflecting a phase in the squark-quark-gluino mixing). This model also gave a reasonable estimate for \(\epsilon'/\epsilon \sim 10^{-3}\), but predicted a small CP asymmetry for \(B \rightarrow \psi K_S\). This latter prediction, given the present data, makes this model not viable.

Because all the experimental data on CP violation is in excellent agreement with the CKM model, at the moment for the SUSY SM all that one has are constraints on squark mass splittings and on mixing in the gluino couplings. The typical parameters bounded are the quantities\(^6\)

\[
\Delta_{ij} = \left( \frac{\tilde{m}_{ij}^2 - \tilde{m}_{ii}^2}{\tilde{m}_{ii}^2} \right) g_{gij} \tag{19}
\]

for squarks associated with quarks of different helicities (L or R). Typical results of a recent analysis\(^3\) allow values in the B-sector (for either helicity) as large as

\[
\begin{align*}
\text{Re } \Delta_{13} &\approx 2 \times 10^{-2} \\
\text{Im } \Delta_{13} &\approx 10^{-2},
\end{align*}
\tag{20}
\]

without giving noticeable effects in the data.

### 4 What Should We Look For?

It is clearly of fundamental importance to pin down the unitarity triangle, to check whether or not all flavor changing CP violation effects originate solely from the single CKM phase \(\delta_{\text{CKM}}\). As remarked by Buras\(^7\) at this meeting, in the context of the CKM model, measurements of the (CP-conserving) sides of the unitarity triangle are as important as the measurement of an explicit CP-violating angle. In a more general context, however, what is crucial is to ascertain whether there are any additional CP-violating phases, besides \(\delta_{\text{CKM}}\).

In this respect, perhaps the most clear signals of CP-violation occur in extended Higgs models, where CP violating effects manifest themselves by the appearance of Higgs bosons with mixed CP properties. As an example, it is well known that the three neutral scalars \(h, H\) and \(A\) in the SUSY SM have well defined CP properties, with the first two fields being \(0^+\) objects and \(A\) being a \(0^-\) excitation. This, however, may no longer hold at loop order, since CP-violating loop effects involving stops and sbottoms can mix \(A\) with \(h\) and \(H\).\(^8\)

In general, therefore, it is important to look for the presence of possible CP-violating couplings of Higgs bosons. CP violation, for example, allows the lightest Higgs boson \(h\) to couple to two photons both through an \(F^2\) and an \(\tilde{F}\) term:

\[
\mathcal{L} = \frac{\alpha}{\pi} [aF^{\mu \nu}F_{\mu \nu} + bF^{\mu \nu}\tilde{F}_{\mu \nu}]h. \tag{21}
\]
In practice, however, it is going to be quite difficult to measure possible CP-violating coefficients (like the ratio $b/a$ above). At the LHC, typically, one is sensitive to CP-odd mixing at the level of about 30%\textsuperscript{32}. In this respect, an NLC would be much more effective at detecting Higgs sector CP violation, with the sensitivity to CP-odd mixings being pushed to about the 4% level.

It is natural to ask if there are other areas, besides the Higgs sector, where one should look for hints regarding CP violation. In this context, it is important to note that we know from the existence of a baryon asymmetry in the Universe that there must be other CP-violating phases besides $\delta_{\text{CKM}}$, since the SM does not seem to be able to produce such an asymmetry.\textsuperscript{33} It is barely possible that this asymmetry might be produced at the electroweak phase transition in the SUSY SM. However, the parameter space for this to happen is extremely restricted. Basically, as discussed extensively by Quiros and collaborators\textsuperscript{34} it is crucial to have a sufficiently strong first order transition to prevent the baryon asymmetry produced at the electroweak phase transition from being erased. For this to obtain in the SUSY SM one needs a very light stop ($\tilde{m}_t \sim 140$ GeV) and the lightest Higgs boson must have a mass near the edge of discovery ($m_h \sim 115$ GeV). So, although probably not totally ruled out yet, this possibility seems highly unlikely to me.

A much more intriguing possibility, and one which I believe is much more likely, is that the CP phase responsible for the baryon asymmetry in the Universe is actually connected to CP-violating phases in the neutrino sector. The scenario for this to happen is well known and was first outlined more than 15 years ago by Fukugita and Yanagida.\textsuperscript{35} What these authors observed was that, in theories with heavy Majorana neutrinos, the out of equilibrium decay of these heavy neutrinos (with masses $M \geq 10^{10}$ GeV) can establish a lepton asymmetry in the early Universe at temperatures of order $T \sim M$. Such a lepton asymmetry would naturally give a baryon asymmetry, through the Kuzmin Rubakov Shaposhnikov (KRS) mechanism.\textsuperscript{36} What KRS showed is that B+L-violating interactions are expected to be in equilibrium for a large temperature range ($100$ GeV $\leq T \leq 10^{12}$ GeV). Thus, as the Universe cools below $T \sim M$ these processes serve to erase any created (B+L)-asymmetry, and effectively serve to transmute any lepton asymmetry into a corresponding baryon asymmetry.

Although this is an attractive scenario for generating the Universe’s baryon asymmetry, as discussed by Hambye\textsuperscript{37} and Hernandez\textsuperscript{38} at this meeting it is, in general, difficult to relate directly the CP-violating phases at the root of the original lepton asymmetry with possible low energy CP-violating phenomena in the neutrino sector. Nevertheless, it is interesting to examine how one can conceivably extract from experiment information on CP violation of neutrinos.

Evidence for CP violation in the neutrino sector can come from two places: neutrino oscillations and neutrinoless double beta decay. These two processes are, in some sense, complementary\textsuperscript{39} since they are sensitive to different CP-violating phases. Measuring the difference in oscillations between neutrinos and antineutrinos gives direct evidence for CP violation, while the rate for neutrinoless double beta decay provides more indirect information on CP violation. As Blondel\textsuperscript{40} emphasised at this meeting, in either case the experimental challenges are enormous!

Neutrino (and antineutrino) oscillations can give information on $\delta_{\text{CKM}}^{\ell}$, the leptonic equivalent of the CKM CP violating phase. Neutrinoless double beta decay, on the other hand, is most sensitive to a CP violating phase $\varphi_{\text{M}}$ which enter only if neutrinos are Majorana particles. Let me briefly examine these two processes in turn.

For oscillation experiments to successfully detect CP-violating effects in the neutrino sector the angle $\theta_{13}$ in the leptonic mixing matrix must be near the CHOOZ bound $[\sin^2 2\theta_{13} < 0.1]$.\textsuperscript{41} In fact, it is easy to check that, as this angle tends to zero, the difference between the oscillation
probabilities of neutrinos and antineutrinos disappears. For example, for $\nu_\mu$ to $\nu_e$ transitions, one finds

$$P[\nu_\mu \rightarrow \nu_e] = a + b \sin 2\theta_{13} \sin \delta^\ell_{\text{CKM}},$$
$$P[\bar{\nu}_\mu \rightarrow \bar{\nu}_e] = a - b \sin 2\theta_{13} \sin \delta^\ell_{\text{CKM}}. \tag{22}$$

Thus, the accuracy with which one can determine $\sin \delta^\ell_{\text{CKM}}$ is directly related to the magnitude of $\theta_{13}$. For neutrinoless double beta decay, the situation is different, since one can detect the presence of a CP-violating Majorana phase $\varphi_M$ even in the limit when $\theta_{13}$ vanishes. Indeed, neglecting $\theta_{13}$ altogether yields for the effective neutrino mass $M_{ee}$ measured in this process a very simple formula. For the case of a normal hierarchy where $m_3 \gg m_2 \simeq m_1$, with $m_2 \simeq m_1 = m_e$ the mass of the neutrino measurable, in principle, in Tritium beta decay, one has

$$M_{ee} = m_e \left| \cos^2 \theta_{12} + e^{i\varphi_M} \sin^2 \theta_{12} \right|. \tag{23}$$

It is an open question whether even with the observation of neutrinoless double beta decay Eq. (23) will allow an extraction of $\varphi_M$, given the large theoretical error which accompanies the extraction of $M_{ee}$ from experiment.\cite{38,42}

Unfortunately, as alluded to earlier, neither $\varphi_M$ or $\delta^\ell_{\text{CKM}}$ are directly related to the CP-violating phases that control leptogenesis, except in very particular circumstances. The baryon asymmetry is directly related to the lepton asymmetry produced at temperatures of order $T \sim M$. In turn, the lepton asymmetry is proportional to the CP asymmetry in the decay of the heavy neutrino. One has, for the SM\cite{43}

$$\eta_B = \frac{8}{15} \eta_L = -\frac{8}{15} \left[ \frac{\kappa}{g^*} \right] \epsilon. \tag{24}$$

Here $\kappa$ is a washout factor connected with how the heavy neutrino decays go out of equilibrium\cite{33}, while $g^* \sim 100$ is the number of degrees of freedom at temperatures of $O(T \sim M)$. The CP asymmetry factor $\epsilon$ in Eq. (24) depends in detail on the phase structure of the heavy neutrino Yukawa coupling $h_\nu$ coupling the leptonic doublet $L_L$ to $N_R$ and the Higgs doublet. The phases, $\delta^\ell_{\text{CKM}}$ and $\varphi_M$, on the other hand, also depend on the phase structure of the electron Yukawa coupling $h_\ell$ coupling the leptonic doublet $L_L$ to $\ell_R$ and the Higgs doublet.

## 5 Concluding Remarks

Obviously, at this stage, the most important task still in front of us is to get additional experimental information on CP violation. Fortunately, the prospects for doing so are very good, since there are many experimental efforts precisely targeted in this direction. These range from experiments presently underway at the B factories at SLAC and KEK, with the BaBar and Belle detectors, to dedicated collider B-experiments planned for the Tevatron and the LHC. The list of experiments does not end here. Powerful information can come also from searches for $K_L \rightarrow \pi^0 \nu \bar{\nu}$\cite{34} (and its charged counterpart $K^+ \rightarrow \pi^+ \nu \bar{\nu}$, already seen by the Brookhaven experiment E787\cite{43}), and new experiments to detect the neutron and electron electric dipole moment with an order of magnitude or more precision.\cite{36} Finally, searches for CP violation in

---

\footnote{One should not forget the important role that CDF and $D^0$ can play in the near future at the Tevatron to elucidate aspects of B physics not accessible to the B factories.}
the neutrino sector are expected to be undertaken as the new generation of high flux neutrino experiments get underway. 40

One of the principal goals of this experimental program is to understand if the simple CKM paradigm explains all CP violating phenomena in the hadronic sector, or if there are other CP-violating phases. Of equal importance is to see if there is any signal of CP violation in the leptonic sector—something one is led to expect by analogy to the hadronic sector.

On the theoretical side, in my view, it is important to take seriously some of the hints we have from experiment regarding CP violation. Recall, from our discussion, that CP is a good symmetry in 10 dimensions, but is broken in our $d = 4$ world. Furthermore, even in four dimensions it is difficult to break CP. There is, apparently, no $\theta F \bar{F}$ term and there is at least indirect evidence that CP violation originates from the presence of scalars. For the SM this leads to the CKM paradigm. However, even with a more complicated scalar sector than that is the SM, the fact that there are no FCNC argues for the absence of terms which could introduce more CP phases. In SUSY theories, the problem is not how to generate CP violation but rather how to organize the SUSY breaking so as not to generate too large CP-violating effects.

To all of these restrictions, one must add the fact that the observed CP violating phenomena have quite different magnitudes [$\epsilon \sim 10^{-3}, \epsilon' / \epsilon \sim 10^{-3}; a_{B \to \psi K_S} \sim 1$]. However, these disparate measurements all appear to be explained by the same (large) CP-violating phase $\delta_{\text{CKM}}$. This leads one to speculate, as was done by Abel \(^6\) in this meeting, that perhaps all CP violation originates from the presence of a single geometrical phase \(^{17}\) associated with the compactification from $d = 10$ to $d = 4$. If I had to make a guess, I would imagine that this Ur-CP phase $\delta_0$ is related to some topological invariant of the associated geometry. An appealing formula is simply

$$\delta_0 = \frac{2\pi}{N_g}$$

with $N_g$ being the number of generations. \(^{19}\) It is amusing to note that the CKM phase is near $60^\circ$, in the usual convention.\(^ {20}\)

**Acknowledgements**

I am grateful to Xinmin Zhang and Tao Huang for discussions on some of the material presented here. This work has been supported in part by the Department of Energy under Contract No. FG03-91ER40662, Task C.

**References**

[1] R. D. Peccei in *Broken Symmetries*, eds L. Mathelitsch and W. Plessas (Springer Verlag, Berlin, 1999).
[2] R. Jackiw and C. Rebbi, *Phys. Rev. Lett.* 37, 172 (1976); C. G. Callan, R. Dashen and D. Gross, *Phys. Lett. B* 63, 334 (1976).
[3] M. Dine, R. G. Leigh and D. A. MacIntire, *Phys. Rev. Lett.* 69, 2030 (1992).
[4] K. Choi, D. B. Kaplan and A. E. Nelson, *Nucl. Phys. B* 391, 515 (1993).
[5] A. Strominger and E. Witten, *Comm. Math. Phys.* 101, 341 (1985).
[6] S. A. Abel, these Proceedings

\(^*\)This formula gives no CP-violation in the case of two generations. This is not necessarily the case if neutrinos are Majorana particles, although the presence of such Majorana phases is not mandated.
[7] N. V. Krasnikov, V. A. Rubakov and V. F. Tokarev, J. Phys A 12, L343 (1979); see also A. A. Anselm and A. A. Johansen, Nucl. Phys. B 412, 553 (1994).
[8] See, for example, Particle Data Group: K. Hagiwara et al, Phys. Rev. D 66, 010001 (2002).
[9] V. Baluni, Phys. Rev. D 19, 2227 (1979); R. Crewther et al, Phys. Lett. B 88, 123 (1979); Phys. Lett. B 91, 487 (1980) (E).
[10] For a recent review see, for example, R. D. Peccei, Nucl. Phys. B 72 (Proc. Suppl.) 3 (1999).
[11] H. Leutwyler, Phys. Lett. B 374, 163 (1996).
[12] R. D. Peccei and H. R. Quinn, Phys. Rev. Lett. 38, 1440 (1977); Phys. Rev. D 16, 1791 (1977).
[13] P. Sikivie, these Proceedings.
[14] A. E. Nelson, Phys. Lett. B 136, 387 (1983); Phys. Lett. B 143, 165 (1984); S. M. Barr, Phys. Rev. D 30, 1805 (1984).
[15] R. D. Peccei in 1990 International Workshop on Strong Coupling Gauge theories and Beyond, eds. T. Muta and K. Yamawaki (World Scientific, Singapore, 1991).
[16] Y. B. Zeldovitch, I. B. Kobzarev and L. Okun, Phys. Lett. B 50, 340 (1974).
[17] A. Buras, these Proceedings.
[18] N. Cabibbo, Phys. Rev. Lett. 12, 531 (1963); M. Kobayashi and T. Maskawa, Prog. Theo. Phys. 49, 652 (1973).
[19] K. Kleinhecht, these Proceedings.
[20] S. L. Glashow and S. Weinberg, Phys. Rev. D 15, 1958 (1977)
[21] G. C. Branco, L. Lavoura and J. P. Silva, CP Violation (Oxford University Press, Oxford, 1999).
[22] J. Kim Phys. Rev. Lett. 43, 103 (1979); M. A. Shifman, A. I. Vainshtein and V. I Zacharrov, Nucl. Phys. B 166, 493 (1980); A. Zhitnitski, Sov. Jour. Nucl. Phys. 31, 260 (1980); M. Dine, W. Fischler and M. Srednicki, Phys. Lett. B 104, 199 (1981).
[23] S. Weinberg, Phys. Rev. Lett. 37, 657 (1976).
[24] R. Garisto and G. Kane, Phys. Rev. D 44, 2038 (1991); G. Belanger and C. O. Geng, Phys. Rev. D 44, 2789 (1991).
[25] A. Zhitnitski, Sov. Jour. Nucl. Phys. 31, 529 (1980).
[26] For a review see, for example, A. Brignole, L. E. Ibanez and C. Munoz, in Perspectives in Supersymmetry, ed. G. L. Kane (World Scientific, Singapore, 1998).
[27] M. Dugan, B. Grinstein and L. Hall, Nucl. Phys. B 255, 413 (1985).
[28] M. Dine, E. Kramer, Y. Nir and Y. Shadmi, Phys. Rev. D 63, 116005 (2001).
[29] A. Masiero, M. Piai and O. Vives, Phys. Rev. D 64, 055008 (2001).
[30] D. Becirevic et al, hep-ph/0112303.
[31] A. Pilaftsis, Phys. Lett. B 435, 88 (1998).
[32] J. Conway et al, hep-ph/0203206.
[33] K. Rummukainen et al, Nucl. Phys. B 532, 283 (1998).
[34] For a discussion see, for example, M. Quiros, Nucl. Phys. B 101 (Proc. Suppl.) 401 (2001).
[35] M. Fukugita and T. Yanagida, Phys. Lett. B 174, 45 (1986).
[36] V. A. Kuzmin, V. A. Rubakov and M. E. Shaposhnikov, Phys. Lett. B 155, 36 (1985).
[37] T. Hambye, these Proceedings.
[38] P. Hernandez, these Proceedings.
[39] V. Barger et al, hep-ph/0205290.
[40] A. Blondel, these Proceedings.
[41] M. Apollonio et al, Phys. Lett. B 466, 415 (1999).
[42] S. Petcov, these Proceedings.
[43] Z. Berezhiani, these Proceedings.
[44] R. Tschirhart, these Proceedings.
[45] D. Bryman, these Proceedings.
[46] E. Hinds, these Proceedings.
[47] For a recent attempt in this direction, see S. A. Abel and A. W. Owen. [hep-ph/0205031]