Doping induced inhomogeneity in high-$T_c$ superconductors

Ivar Martin and Alexander V. Balatsky

Theoretical Division, Los Alamos National Laboratory, Los Alamos, NM 87545, USA

Abstract

Doping and disorder are inseparable in the superconducting cuprates. Assuming the simplest possible disordered doping, we construct a semiphenomenological model and analyze its experimental consequences. Among the affected experimental quantities are the ARPES spectra and thermodynamic properties. From our model we make a prediction for the width of the local superconducting gap distribution with the only experimentally unknown parameter being the superconducting correlation length. Thus, our model provides a direct way of determining the superconducting correlation length from a known experimental gap distribution.

PACS numbers: 74.20.-z, 74.25.Dw

1 Introduction

Ground state of all cuprate superconductors is a Mott insulator at half filling. To render this state a superconductor one has to dope it with carriers (holes for p-type superconductors such as LSCO and Bi2212, and electrons for n-type, e.g NdSCO). Below we will focus on hole doped superconductors, although discussion about intrinsic disorder is relevant for electron doped materials as well. In the low doping limit, dopants, such as Sr in LSCO and oxygen in Bi2212 are certainly randomly distributed across the sample without forming any regular array. The central point we argue for in this paper is that effect of carriers and intrinsic disorder produced by the very same doping are inseparable in underdoped and possibly overdoped cuprates.

* Expanded version of a talk presented at the ISS 2000, Oct 14-16, Tokyo, Japan
1 ivar@viking.lanl.gov, avb@lanl.gov
The microscopic inhomogeneity of the superconducting state state has im-
portant consequences for the average properties obtained in the bulk measure-
ments. Immediate conclusion we draw is that even with the perfectly grown
crystals doping will produce artificial impurities that disorder the system. The
affected properties include the average tunneling spectra, as the ones obtained
by ARPES and measured specific heat. Another implication is that since the
state is inhomogeneous, there should be no well defined superconducting tran-
sition temperature, much like in the conventional granular superconductors [1].
This “granularity” should also clearly enhance the electric and the thermal
transport above $T_c$.

2 Density of States and gap distribution in the presence of doping
induced disorder

To analyze the effects of doping-induced disorder, we use a minimal model
of randomly distributed dopants in the reservoir layer. The superconducting
order parameter can only change on the length scale of the superconducting
coherence length, $\xi$. Hence we can coarse-grain the Cu-O plane into patches
of the size $\xi$ and on these patches define the effective dopant concentration as

\[ n_\xi(r) = \frac{N_\xi(r)}{\xi^2}, \]  

(1)

where $N_\xi(r)$ is the number of dopants in the adjacent reservoir plane which
happen to be within a circle of radius $\xi$ around the point $r$ (Figure 1).

Fig. 1. The effective local doping at a particular point is defined through the random
number of dopants which happen to be in the $\xi$-vicinity of this point.

Assuming that the positions of the dopants are uncorrelated, which is expected
to hold particularly at low doping levels, the numbers $N_\xi$ obey the Poisson
distribution, with a probability density,

\[ P(N) = \frac{(\bar{n}\xi^2)^N}{N!} \exp(-\bar{n}\xi^2). \]  \hspace{1cm} (2)

Here, the average dopant density, \( \bar{n} \), is a function of the nominal doping fraction, \( \bar{x} = ab\bar{n} \), with \( a \) and \( b \) the in-plane lattice constant. An example of Poisson distribution is shown in Figure 2.

The main characteristic of the Poisson distribution is that its dispersion is equal to its average value. In the limit of large average, \( \bar{N} = \bar{n}\xi^2 \gg 1 \), the Poisson distribution reduces to a Gaussian. In this limit, the probability distribution of the effective local doping fractions becomes

\[ P(x) = \frac{\xi}{\sqrt{2\pi\bar{x}ab}} \exp\left[\frac{(x - \bar{x})^2}{2\bar{x}} \frac{\xi^2}{ab}\right]. \]  \hspace{1cm} (3)

The distribution of the local doping should lead to a corresponding distribution of the gap values over the regions of size \( \xi \). Assuming that the distributions of doping is sufficiently narrow, the value of the local gap as a function of the local doping can be linearized as

\[ \Delta(x) = \Delta_0 - \Delta_1 x. \]  \hspace{1cm} (4)

To the first order approximation, the parameters \( \Delta_0 \) and \( \Delta_1 \) can be extracted, e.g. from the ARPES data which determines the average gap values as a function of the average doping. Then, from Equations (3) and (4), the probability
distribution for the local superconducting gaps follows:

\[ P(\Delta) = \frac{\xi}{\Delta_1 \sqrt{2\pi\bar{x}ab}} \exp \left[ \frac{(\Delta - \Delta_1)^2 \xi^2}{2\bar{x}\Delta_1^2 ab} \right]. \]  

(5)

The probability distribution of the local values of the gap \( \Delta(x) \) is completely specified up to the unknown parameter \( \xi \). Hence, one of the applications of the local gap statistic, and in particular the distribution width is the determination of the doping-dependent correlation length, \( \xi \).

To illustrate the smoothing effect of the gap fluctuations on the bulk properties we compute the average density of states (DOS), as measured by photoemission. In a conventional \( d \)-wave BCS superconductor with the amplitude of the superconducting gap \( \Delta \), the density of state is given by

\[ N_\Delta(\omega) = \begin{cases} N_0(\omega/\Delta)K(\omega/\Delta), & \text{if } \omega < \Delta \\ N_0K(\Delta/\omega), & \text{if } \omega > \Delta \end{cases} \]  

(6)

Here, \( N_0 \) is the electronic DOS at the Fermi level in the normal state and \( K(x) \) is the complete elliptic integral of the first kind.

To determine the average DOS is the presence of the gap amplitude fluctuations, we compute the integral

\[ \bar{N}(\omega) = \int P(\Delta)N_\Delta(\omega)d\Delta. \]  

(7)

The average DOS, \( \bar{N}(\omega) \), clearly preserves the some rule \( \int d\omega \bar{N}(\omega) = 1 \) due to normalization of \( P(\Delta) \).

In Figure 3 we show the DOS curves for various widths of the gap distribution. In the “clean” case one indeed observes a divergence in DOS at the energy equal to the amplitude of the gap. For a finite-width gap distributions, the divergence is replaced by a hump near the average gap value. For wide gap distributions, such that the distribution width is comparable to the magnitude of the gap itself (case \( \sigma = 1 \) in Fig. 3), normal regions can be expected to appear in the sample. Such regions give a finite contribution to the DOS at the Fermi level. This contribution is similar in spirit to the well-known effect of non-magnetic impurities on the DOS in superconductors [2,3].

For the optimally-doped Bi2212, the observed width of the gap distribution is about 10% of the average[4]. As is clear from the Figure 3, such width corresponds to a rather smooth peak, much like the one observed in ARPES [5].
Fig. 3. The average BCS $d$-wave density of states (DOS) for different widths of the gap distribution, $\sigma$. Such spatial averaging may explain the smooth features commonly observed in the photoemission of optimally and underdoped cuprates. The finite DOS at the Fermi level for large $\sigma$ is caused by the zero-gap (normal) regions.

3 Acknowledgments

We would like to thank J.C. Davis and S.H. Pan for useful discussions. This work was supported by the U.S. DOE.

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