ICI and PAPR Enhancement in MIMO-OFDM System Using RNS Coding

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Abstract— The Inter-Carrier-Interference (ICI) is considered a drawback in the utilization of Multiple-Input-Multiple-Output Orthogonal Frequency Division Multiplexing (MIMO-OFDM) systems, due to the sensitivity of the OFDM towards frequency offsets which lead to loss of orthogonality, presence of signal interference and degrading the overall system performance. In this paper Residue Numbers as a coding scheme is impeded in MIMO-OFDM systems, where the ICI levels is measured and evaluated with respect to the conventional ICI mitigation techniques as pulse shaping, windowing and self-cancellation techniques implemented in MIMO-OFDM system. The Carrier-to-Interference Ratio (CIR), Bit-Error-Rate (BER) and the Complementary Cumulative Distribution Function (CCDF) for MIMO-OFDM system with Residue Number System (RNS) coding are analyzed and evaluated. The results demonstrated a performance enhancement in the transmission model with RNS implementation.

Keywords— BER, CIR, ICI, Mitigation techniques, MIMO-OFDM system, RNS.

I. INTRODUCTION

In MIMO systems, the information data at both sides of the communication link is combined through the usage of various Space–Time Block Coding (STBC) algorithms to achieve either higher transmission data rates or enhanced system BER performance for the same data rate [1]. The OFDM as a multi-carrier modulation scheme had shown its ability to provide high transmission rates, because it has several unique features like robustness to multipath fading overcoming Inter-Symbol-Interference (ISI), high spectral efficiency, inviolability to impulse interference, overcoming time dispersion issues, flexibility and simple equalization over wireless channels.

For MIMO-OFDM communication systems [2], the orthogonality seen in OFDM technique is lost within the subcarriers due to the sensitivity of OFDM to frequency offset induced from the Doppler shift between the transmitter and the receiver. This results in an ICI between the transmitted symbols that degrade the overall performance [3].

Different ICI cancellation techniques are currently available like time-domain windowing, pulse shaping and frequency equalization, which reduce the ICI levels and thus improve the BER performance of MIMO-OFDM systems. These techniques are still costly and high complex either on the transmitter or receiver side.

This paper propose an efficient ICI cancellation scheme based on utilization of Residue coding; where the system is analyzed and compared to current mitigation techniques.

In section 2, the paper provides basic background on Residue system, and section 3 provides analysis of the ICI. Section 4 summarizes existing mitigation techniques, present the relation between frequency offset and number of subcarriers and give a theoretical rational about the RNS coding effect on ICI reduction. Section 5 describes the proposed MIMO-RNS-OFDM communication system. In section 6 the simulation results are provided to measure the system performance and finally in section 7, the conclusion has been given.

II. BASIC BACKGROUND

A. Residue Number System Review

The RNS represents large integers by set of smaller ones, and has two unique features; a carry-free arithmetic that enable to perform parallel mathematical operations related to the individual residue symbols, and no weight-information are carried between carriers which prevent error propagation [4].

RNS is defined by selecting v positive pair-wise relatively prime primes mi (i= 1, 2, 3 … v), such that any integer N, describing a message, is represented by the sequence (r1, r2, …,rv) in the range 0<N<M1 in a unique matter, where;

\[ r_i = N \mod m_i \]  \hspace{1cm} (1)

Where;

r_i least positive remainder when N divided by modulus m_i

\[ M_i = \prod m_i \] \text{ is symbols’ dynamic range.}  \hspace{1cm} (2)

Then use the Mixed Radix Conversion (MRC) method [5], to recover symbols, where for a given set of pair-wise relatively prime moduli \{m_1, m_2, ..., m_n\} and a residue state \{r_1, r_2, ..., r_n\} of a number X, that number can be uniquely represented in mixed-radix form as seen in next:

\[ X = \{z_1, z_2, ..., z_n\} \] \hspace{1cm} (3)

And;

\[ X = z_1 + z_2m_1 + ... + z_nm_n, 0 \leq z_i \leq r_i \] \hspace{1cm} (4)

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Where; \( z_i \) is represented as function of the moduli and residue representations as seen in table (1);

| Parameter | Representation |
|-----------|----------------|
| \( z_1 \) | \( r_1 \) |
| \( z_2 \) | \( m_1 z_1 \) |
| \( z_3 \) | \( (m_2 m_1)^{-1} m_3 (r_3 - z_2 m_1 + z_1) \) |
| \( Z_n \) | \( (m_n \ldots m_2 m_1)^{-1} m_n (r_n - z_{n-1} m_{n-2} \ldots z_2 m_1 + z_1) \) |

**B. Redundant Residue Number System**

The RNS moduli utilized for error detection and correction through implementation of additional RNS moduli as redundancy symbols; being named the Redundant Residue Number System (RRNS).

In this configuration each redundant moduli selected to be greater than any of the other chosen moduli set and don’t play any role in determining the system dynamic range. So, an RRNS is obtained by appending an additional \( u / g_237 v \) number of moduli \( m_{v+1}; m_{v+2}; \ldots; m_u \), where \( m_{v+j} \geq \max\{m_1; m_2; \ldots; m_v\} \) is referred to as a redundant moduli, to the previously introduced RNS. This is in order to form an RRNS of \( u \) positive, pairwise relative prime moduli. [6]

For the correction of the error, using the MRC method, a test on each of the information moduli with the two redundant moduli is performed. Through the test we are able to identify and correct the bit which generated the error [7].

**III. ANALYSIS OF INTER-CARRIER-INTERFERENCE**

In MIMO-OFDM systems, the loss of orthogonally between subcarriers, increases the ICI between sub-carriers and leads to degradation for the system performance. This is due to the Doppler shift induced from the relative motion between the transmitter and receiver sensitivity and caused a frequency offset between sub-carriers, and would result in a reduced signal amplitude and ICI as presented in Fig. 1.

The frequency offset \( \epsilon \) is modeled as shown in Fig. 2, where the received signal represented as;

\[
Y(n) = x(n) e^{j\frac{2\pi n \epsilon}{N}} + w(n)
\]

**IV. RNS AS ICI MITIGATION TECHNIQUE**

The accurate frequency and time synchronization are essential factors for OFDM technique. It is sensitive towards frequency offset factors as attenuation and rotation of subcarriers. For that reason the orthogonality is lost between subcarriers, yielding to constraint named inter-carrier interference (ICI), which degrades the efficiency of the system.

In this section a summary for the research conducted to mitigate the ICI errors, as well as the theoretical behind ability of RNS as a coding scheme to reduce the ICI errors in the communication system is provided as seen next.

**A. ICI Current Mitigation Schemes**

A lot of researchers [8, 9] have proposed numerous ICI mitigation techniques to resolve this problem which is categorized as; self-cancellation, frequency-domain equalization, time–windowing, and Pulse shaping techniques [10 – 14].

These techniques are employed as well for the reduction of the Peak-Average-Power Ratio (PAPR) through the reduction of side lobes in each carrier, permitting higher power to be transmitted to for a constant peak power, and making enhancement in the overall signal to noise ratio (SNR) at the receiver.

**B. The Frequency Offset and Number of Sub-carriers**

The impact of Carrier Frequency Offset (CFO) on the degradation of the SNR in OFDM systems [15], as seen in equation (8);

\[
D_{\text{req}} = \frac{10}{3\ln 10} (\pi \Delta f T)^2 \frac{E_b}{N_0}
\]

Where,

The SNR = \( \frac{E_b}{N_0} \) and \( \Delta f \), \( T \), \( E_b \), \( N_0 \) are the frequency offset, symbol duration, energy per bit and the noise respectively.
Thus, for $N$ sub-carriers, equation (8) would yield to equation (9):

$$\text{Losses (dB)} = \frac{D_{\text{req}}}{N} = \frac{10}{3\ln 10} (\pi c)^2 \cdot 20 \log N \quad (9)$$

In Fig. 3 the impact of the sampling offset on the degradation of SNR is provided, where it is seen a degradation in the OFDM system when increasing the number of subcarriers.

![Fig.3. Effect of Number of Sub-carriers on SNR](image)

**C. RNS Coding as ICI Reduction Approach**

The OFDM system use a dedicated sub-carrier for each data symbol, so for $N$-data samples it is required an $N$ sub-carrier frequencies used in the IFFT/FFT stage at the transmitter/receiver side. The Residue Number System on the other hand use only $v$ sub-carriers, each with $N$-sample data. These $v$ sub-carriers are equivalent to the selected number of RNS moduli.

Moreover, previously in sub-section B, it was stated that the system degradation due to frequency offset which generates ICI is function of the number of used sub-carriers in the transceiver system, such that when the number of subcarriers are duplicated the ICI increased by 3 dB.

Consequently, as selected number of RNS moduli is always less than the $N$-samples, the conversion to RNS is able to mitigate the effect of ICI seen in OFDM system.

**V. SYSTEM MODEL**

The proposed MIMO-RNS-OFDM system is shown in Fig. 4 is initialized with a binary data random source, converted to residue system. The resultant packet is modulated, coded through the Space-Time Block Coding (STBC) encoder, passed to a Serial-To-Parallel (S/P) converter for parallel transmission, passed through an IFFT block, and finally transmitted through the antenna. At the receiver side the communication blocks are the reverse of the transmitter.

![Fig.4. MIMO-OFDM System model](image)

The above system shown in Fig. 4; is evaluated by measuring the Carrier-Interference-Ratio (CIR) given in equation (10), and the Bit Error Rate (BER) of the signal shown in equation (11), respectively.

$$\text{CIR} = \frac{\sum_{l=0}^{L-1} |s(l)|^2}{\Sigma_{l=0}^{L-1} |S(l-k)|^2} \quad (10)$$

Where;
- $s(l)$ Complex coefficient for ICI components in the receiving signal.

And; the probability of error for M-PSK modulated transmission [16] is given by:

$$P_{\text{ERR}} = \gamma \sum_{k=1}^{\min(2, \frac{M}{2})} Q \left( \sqrt{2\sigma x} \sin \left( \frac{(2k-1)\pi}{M} \right) \right) \quad (11)$$

$$\gamma = \max(\log_2 M, 2) \quad (12)$$

Where;
- $M$; The constellation size
- $\sigma$; The SNR per symbol
- $x$; Is a chi-square distributed random variable

**VI. SIMULATION RESULTS**

The results obtained from the MATLAB simulations are discussed, where various analysis had been performed on MIMO-RNS-OFDM system to measure its resilience towards ICI in comparison to current MIMO-OFDM systems.

In this simulation, 1000 symbols are 512-QAM modulated and transmitted over a MIMO-OFDM communication system using RNS coding technique with redundant moduli’s (17, 13, 11, 7, 5, 3), were (11, 7, 5, 3) are the information moduli’s and the set (13, 17) are the redundant moduli’s.

**A. BER vs. SNR for various offset MIMO-RNS-OFDM**

The performance of communication system using 32QAM in the presence of frequency offset between the transmitter and the receiver over a Rician fading channel is seen in Fig. 5.

![Fig.5. Effect of frequency offset on System Performance](image)

In Fig. 5, it is shown the degradation in performance when increasing the frequency offset. Thus, when the offset is small, the system has a lower BER (better).
B. ICI Measurements for MIMO-RNS-OFDM System

For a pre-defined SNR value (80), the transmission signal error is plotted versus the frequency offset as seen in Fig. 6 for OFDM system with and without RNS moduli’s (13, 11, 7, 5, 3) as coding scheme;

![Fig. 6: Error for MIMO-RNS-OFDM System](image)

Where; in Fig. 6 an absolute 25 dB improvement when using the RNS scheme, which is better than the achieved improvement using ICI cancelation scheme indicated in section (4.A). The RNS coding enhancement is attributed to the usage of less number of sub-carriers compared to OFDM system, and even enhanced with low offset values due to the inherent properties of RNS that does not allow the transmission of error between different moduli’s.

In addition, it is seen as the frequency offset increase this would increase the error due to the increasingly loss of orthogonality between inter-carriers.

C. ICI Measurements for MIMO-RRNS-OFDM System

Using RNS technique with redundant moduli’s (17, 13, 11, 7, 5, 3), where (11, 7, 5, 3) are the information moduli’s and the set (13, 17) are the redundant moduli’s, and measuring ICI for the system comparing its value with the communication system without any redundant modus as seen in Fig. 7;

![Fig 7: Error for MIMO-RRNS-OFDM Systems](image)

Here; in Fig 7 the improvement is more than 30 dB, which is better than ICI cancellation scheme and RNS coding scheme seen in sections (4.A) and (6.B) respectively.

Moreover, the system exhibit similar performance as that shown when using RNS as a coding scheme only, as seen in section (6.B).

D. Effect of RNS moduli selection on ICI performance

Increasing the order of RNS moduli set and measures the system performance to see the effect of the selection of the RNS on ICI reduction.

![Fig 8: ICI vs. RNS moduli set](image)

From Fig 8, it is noted that each time the amplitude of the RNS set increased this would increase the ICI error, and thus the increased signal amplitude would directly increase the interference between sub-carriers.

Now; in the comings sub-sections E, F, G, and H; various mitigation schemes are implemented and analyzed in the MIMO-RNS-OFDM communication system to study and evaluate its performance in combination with Residue coding technique.

E. MIMO-RNS-OFDM with “Frequency equalization”

A frequency domain equalizer is used in the receiver, and the system performance is evaluated as seen in Fig. 9.

![Fig. 9: MIMO-OFDM RNS system with/without Equalizer](image)

Where; at SNR = 10, BER for the communication system with error correction is $2\times10^{-3}$ while it reaches $6\times10^{-3}$ for the system without error correction.
F. MIMO-RNS-OFDM with “Self-Cancellation” Scheme

Using data conjugate technique in self-cancellation scheme, where the system performance is evaluated as seen in Fig. 10 over a Rayleigh fading channel.

![Fig. 10: MIMO-OFDM RNS system self-cancellation](image)

Where; at SNR = 10, BER for the communication system with error correction is $4 \times 10^{-3}$ while it reaches $7 \times 10^{-3}$ for the system without error correction.

G. MIMO-RNS-OFDM with “Pulse Shaping” Scheme

A raised cosine pulse shaping added to the system, and evaluated through the coming simulations.

1) PAPR Measurement:

Perform recurrent measurement to evaluate the PAPR of the communication system with and without pulse shaping mitigation scheme as seen in Fig. 11.

![Fig 11.a: With mitigation scheme](image)

![Fig 11.b: Without mitigation scheme](image)

From Fig. 11.a and 11.b, it is shown that reduction in PAPR seen over different wireless communication systems when using pulse shaping mitigation scheme in comparison to that without the mitigation scheme.

2) ICI and BER Measurement:

Then measuring the ICI error and overall BER performance for the system with and without pulse shaping mitigation is measured, as seen in Fig 12 and 13.

![Fig 12.a: With mitigation scheme](image)

![Fig 12.b: Without mitigation scheme](image)

From the above Fig. 12, it is shown that using the mitigation scheme as in Fig 12.a the ICI reduction using RNS coding is around 40 dB while without the mitigation scheme as seen in Fig 12.b the reduction is only 30 dB. The improved features seen in Fig 12.a is attributed to the use of pulse shaping scheme as a mitigation technique in the communication system.

And from Fig. 13 the BER performance for the system with and without mitigation scheme is measured, it is shown that BER performance in Fig.13.a is better than that seen in Fig 13.b. Where, at SNR = 10 dB, the BER for the RRNS communication system with mitigation scheme seen in Fig13.a is $10^{-4}$ while for the same system without mitigation scheme as seen in Fig 13.b is $10^{-3}$.

This result is coherent with that obtained in Fig. 12 indicating the decrease of ICI when implementing mitigation scheme.

H. MIMO-RNS-OFDM vs. ICI Reduction Techniques

In this subsection a comparisons of various ICI cancelation schemes that are implemented within the MIMO-RNS-OFDM system are studied and analyzed as seen in Fig. 14, to determine the best choice of ICI mitigation techniques that is suitable of RNS coding scheme.
ICI self cancelation
Raised Cosine Pulse Shaping
Equalization Scheme

Fig 14.a: ICI for MIMO-RNS-OFDM with varies mitigation schemes

BER performance of MIMO-RNS-OFDM 32-QAM

With various ICI techniques

Without
cancellation
ICI self cancellation
Equalization Scheme
Raised Cosine Pulse Shaping

Fig 14.b: BER for MIMO-RNS-OFDM with varies mitigation schemes

From the above Fig 14, windowing/pulse shaping provides the best performance with respect to the other schemes. The rationale behind this is that conventional technique for ICI Reduction like time domain equalization, self-cancelation, does not properly reduce ICI at the receiver side as through these techniques, ICI reduces only band limited channel which is not the main source of ICI. Where, the main source of ICI is due to frequency mismatch between transmitter and receiver that is corrected and reduced through pulse shaping mitigation method.

VII. CONCLUSION

In this paper, a review for MIMO-OFDM system performance using ICI self-cancelation, pulse shaping, windowing mitigation techniques had been provided and discussed.

An RNS coding insertion in MIMO-OFDM communication system has been proposed, and evaluated with respect to both CIR and BER performance. The usage of residue system had showed its advantage in improving the communication system features through decreasing the ICI and improving the BER performance.

The MIMO-OFDM with RNS coding scheme further enhanced through the insertion of ICI mitigation technique in the communication system, where the pulse shaping mitigation scheme had proven its enhanced performance with the residue system over the equalization scheme; through the recorded improvement in the BER, PAPR and ICI parameters.

REFERENCES

[1] Biradar, R. (2015, July). Study and Analysis of 2x2 MIMO Systems for Different Modulation Techniques Using MATLAB. Retrieved September 10, 2017, from http://www.techrepublic.com/resource-library/whitpapers/study-and-analysis-of-2x2-mimo-systems-for-different-modulation-techniques-using-matlab/

[2] Ghassan M. T. Abdalla “Orthogonal Frequency Division Multiplexing, Theory and Challenges” UoKEJ Vol. 1 Issue 2 pp. 1-8 (October 2011)

[3] T. Wang, J. G. Proakis, and J. R. Zeidler, “Performance Degradation of OFDM Systems Due to Doppler Spreading,” IEEE Transactions on Wireless Communications, vol. 5, pp. 1422-1432, June 2006.

[4] M. Roshanzadeh, A. Ghaffari and S. Saqaeeyan, Using Residue Number Systems for Improving QoS and Error Detection & Correction in Wireless Sensor Networks, Communication Software and Networks (ICCSN), May 2011 IEEE 3rd International Conference on Page: 1-5.

[5] Pallab Maji, June 2011. Application of Residue Arithmetic in Communication and Signal Processing. Master of Science. NATIONAL INSTITUTE OF TECHNOLOGY, Rourkela, Orissa-769008, India

[6] Angel James, Ameenudeen Pe, Jilu James, Minny George, “Multiple error correction using non-binary Redundant Residue Number System”, India Conference (INDICON) 2015, 2015, ISSN 2325-9418.

[7] Salifu Abdul-Mumin1, Kazeem Alagbe Gbolagade, “An Improved Redundant Residue Number System Based Error Detection and Correction Scheme for the Moduli Set”. Advances in Wireless Communications and Networks journal, 2016; 2(1): 11-14.

[8] Jasdeep Singh, Komal Arora, Inter carrier interference removal in MIMO-OFDM system. IOSR Journal of Electronics and Communication Engineering (IOSR-JECE) e-ISSN: 2278-2834, p-ISSN: 2278-8735.Volume 9, Issue 2, Ver. VII (Mar - Apr. 2014), PP 87-91.

[9] V. Kumbasar and O. Kucur, “ICI Reduction in OFDM Systems by using Improved Sinc Power Pulse,” Digital Signal Processing, Vol.17, Issue 6, pp.997-1006, Nov. 2007.

[10] Mrityunjaya Hatagundi1, Ramakrishna Joshi “ ICI Cancellation using Self Cancellation Method”. International Journal of Advanced Research in Computer and Communication Engineering ISO 3297:2007 Certified Vol.5, Issue 12, December 2016, ISSN (Online) 2278-1021.

[11] Yin-Ray Huang1, Carson C “Frequency Domain Equalization for OFDM Systems with Insufficient Guard Interval using null subcarriers”.17th European Signal Processing Conference (EUSIPCO 2009) Glasgow, Scotland, August 24-28, 2009

[12] R. Kumar, S. Malavizhi “Time Domain Equalization Technique for Intercarrier Carrier Interference Suppression in OFDM System.”Information Technology Journal, ANSINET, Vol 7(1), PP.149-154,1-jan-2008.

[13] M. Palaniyelvan, Sheila Anand “PAPR and ICI Reduction in OFDM Systems using Modified Raised Cosine Power Pulse Shape”. European Journal of Scientific Research Vol.72 No.4 (2012), pp. 618-627.

[14] Kusumarsi Malikanti “Analysis of OFDM System by Using Pulse Shaping Filters for DSP Applications”. SSRG International Journal of Electronics and Communication Engineering—(ICRTESTM-2017) Special Issue- April 2017. ISSN: 2348 – 8549.

[15] Abdul Gani Abshir, M. M. Abdullahi, “A Comparative Study of Carrier Frequency Offset (CFO) Estimation Techniques for OFDM Systems”. IOSR Journal of Electronics and Communication Engineering ISSN: 2278-2834. Volume 9, Issue 4, (Jul - Aug. 2014), PP 01-06.

[16] Yong Soo Cho, Jackwon Kin “MIMO-OFDM Wireless Communication with MATLAB”. Text Book, John Wiley and Sons Ltd- Year 2010, Print-ISBN: 978-0-470-82561-7.