The existing bounds from CDF on the masses of the fourth generation quarks, $t'$ and $b'$, are reexamined. The bound of 256 GeV on the $t'$ mass assumes that the primary decay of the $t'$ is into $q+W$, which is not the case for a substantial region of parameter space. The bound of 268 GeV on the $b'$ mass assumes that the branching ratio for $b' \rightarrow b+Z$ is very large, which is not only not true for much of parameter space, but is never true for $b'$ masses above 255 GeV. In addition, it is assumed that the heavy quarks decay within the silicon vertex detector, and for small mixing angles this will not be the case. The experimental bounds, including all of these effects, are found as a function of the other heavy quark mass and the mixing angle.

The question of whether or not there exist quarks and leptons beyond the known three generations has generated a number of theoretical and experimental investigations\cite{1, 2}. Although direct experimental constraints did not (and do not) rule out a heavy fourth generation, until recently electroweak precision data appeared to disfavour its existence. In addition, there is a strong prejudice against quarks and leptons beyond the third generation which is usually paraphrased by the question: Why is the fourth neutrino so much heavier ($m_{\nu_4} > M_Z/2$) than the other three? Of course, one can very well imagine several scenarios in which this can “easily” be accomplished since much is yet to be learned about neutrino masses. In the end, it will be the verdict of experiments which will be the determining factor.

There is still the question: Why should one bother with a fourth generation? There might be several answers to that question. First, it is possible that a heavy fourth generation might help in bringing the mystery of family replication. We still have no satisfactory explanation for the question: Why is the fourth neutrino so much heavier than the third generation which is usually paraphrased by the strong prejudice against quarks and leptons beyond the three. We still have no satisfactory explanation for the mystery of family replication.

Recent reexaminations\cite{4, 5} of electroweak precision data led to the conclusion that the possible existence of a fourth generation is not only allowed but its mass range is correlated in an interesting way with that of the Higgs boson in the minimal Standard Model (SM). In\cite{4}, the masses of the fourth generation quarks ($t'$ and $b'$) are taken to be bounded from below at around 258 GeV. This lower bound was taken from CDF, who bounded the $t'$ mass$^6$ and the $b'$ mass$^7$. However, CDF made a number of assumptions. For instance, it was assumed that $B(b' \rightarrow b+Z) = 100\%$ which yields a lower bound $m_{b'} > 268$ GeV. As we will show below, this assumption is not only unjustified, but is in fact false for $m_{b'} > 255$ GeV. As a result, the CDF bound weakens considerably\cite{8}. Furthermore, we show that there exist unexplored regions in the mass- mixing angle parameter space which could prove very useful to future searches.

Most of the present mass bounds on the fourth generation quarks involved searches done within about 1 cm from the beam pipe, inside the silicon vertex detector of CDF. The customary focus is on the decay of $t'$ and $b'$ into quarks of the first three generations, in particular into the $b$ quark. As noted above, for example, the most recent bound on the $b'$ mass by CDF was obtained by searching for the decay mode $b' \rightarrow b+Z$. Similarly, the $t'$ search focused on the decay mode $t' \rightarrow q+W$.

Let us first look at the $t'$ decay. One can consider two regions: (I) $m_{t'} \leq m_W$, and (II) $m_{t'} > m_W$. For (I), it is obvious that the main decay mode will be $t' \rightarrow q+W$. For (II), whether or not $t' \rightarrow q+W$ dominates over $t' \rightarrow b'+W$ depends on the $b'$ mass and the mixing angle $\theta_{b't'}$. In this region, we calculate the branching ratio for $t' \rightarrow q+W$ as a function of the parameters. If it is lower than 60\%, the CDF bound will not apply (the choice of 60\% is based on viewing the graph in Ref.\cite{6}, and is somewhat arbitrary without a more detailed analysis—since the region shown is on a logarithmic scale, changing that number will not noticeably affect the result). If the $b'$ mass is between $m_{b'} - M_W$ and $m_{t'}$, then the three body decay predominates; whereas if it is lower than $m_{b'} - M_W$, the two-body decay will dominate. The decay rate is given in Refs.\cite{6, 10} and repeated (in this specific context) in equation [52] of Ref.\cite{1}.

In addition, even if the $t' \rightarrow q+W$ decay does domi-
nate, CDF assumed that it decayed within approximately 1 centimeter from the beam pipe. For very small mixing angles, this will not be the case. Of course, for extremely small mixing angles, such that the decay length is greater than about 3 meters, the $t'$ will appear to be a stable particle and can be ruled out (above some mass) by stable quark searches.

All of these results are plotted in Fig. 1, where we plot the bound on the $t'$ mass in the $m_{t'} - \sin^2 \theta_{bt'}$ plane. There are three distinct regions. The shaded region above and to the right of the curve with $m_{t'} = 256$ GeV represents the CDF lower bound on $m_{t'}$. In this region, the CDF bound applies. Below the shaded region, the curves correspond to the new lower bound on $m_{t'}$ from CDF based on the requirement that the $t' \rightarrow q + W$ decay is dominant. These curves all end to the left at $\sin^2 \theta_{bt'} \sim 6 \times 10^{-15}$. This corresponds to a decay length of approximately 1 cm. (Let us recall that present searches are sensitive to decays which occur very close to the beam pipe to a distance of about 1 cm.) To the left of those curves lies an unexplored window situated between $\sin^2 \theta_{bt'} \sim 6 \times 10^{-15}$ and $\sin^2 \theta_{bt'} \sim 2 \times 10^{-17}$, corresponding to a distance of roughly 1 cm out to 3 m. The far-left of the plot represents the constraint coming from the search\cite{11} for a stable $t'$ (at distances greater than approximately 3 m).

For the bounds on the $b'$ mass, CDF assumed that the branching ratio $B(b' \rightarrow b + Z)$ was 100 percent. In Fig. 2, we plot the branching ratio $B(b' \rightarrow b + Z)$ as a function of $m_{b'}$ for various $M_t$. Here we use the results of \cite{12} where the assumption $\sin \theta_{bt'} = -\sin \theta_{bt'} = x$ was made resulting in a GIM cancellation when $m_{t'} \sim m_t$. Furthermore, as in \cite{12}, we will assume that $|\sin \theta_{bt'}| < x^2$ so that the decay of $b'$ into “lighter” quarks will be mainly into the $t$ quark. Note that this assumption may not be justified, and if it is false the branching ratio will be even lower, weakening the CDF bound even further. The decay into $t$ is three-body for $m_{b'} < m_t + m_W$ and two-body otherwise. This is to be compared with $b' \rightarrow b + Z$. This analysis has been performed in \cite{12} (Table I) and \cite{11} (Fig. 14).

The results are shown in Fig. 2. It can be seen that $B(b' \rightarrow b + Z) < 100\%$ for a wide range of $b'$ mass above 200 GeV. Note that the bound of 268 GeV on $m_{b'}$ assuming $B(b' \rightarrow b + Z) = 100\%$ does not hold. As $m_{b'}$ gets above 200 GeV, the decay mode $b' \rightarrow t + W^*$ begins to be comparable with the mode $b' \rightarrow b + Z$ and starts to dominate for $m_{b'} \geq 250$ GeV \cite{12}. In particular, for $m_{b'} > 255$ GeV, the decay $b' \rightarrow t + W$ will be into real

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{fig1.png}
\caption{Bound on the $t'$ mass in the $m_{t'} - \sin^2 \theta_{bt'}$ plane. The shaded region corresponds to the CDF lower bound of 256 GeV.}
\end{figure}

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{fig2.png}
\caption{BR($b' \rightarrow b + Z$) as a function of $M_{b'}$ for various $M_t$.}
\end{figure}
particles, and thus this decay will always dominate. The CDF bound can thus never exceed 255 GeV. Using Fig. 2, we estimate the acceptance for $b' \to b + Z$ as had been done by CDF [2]. The results are then used as inputs into our analysis of the bounds on the $b'$ mass.

The $b'$ decay is treated in a similar fashion. Again, we subdivide the decay into two regions: (I) $m_{b'} \leq m_{t'}$, and (II) $m_{b'} > m_{t'}$. Different curves in the $m_{t'} - \sin^2 \theta_{bt'}$ plane correspond to different values of $m_{b'}$ for which $b'$ does not decay into $t'$. This is shown in Fig. 3. Here, we take into account the value of $B(b' \to b + Z)$ (denoted by $\beta$ in [2]) as obtained from Fig. 2 for a given $m_{t'}$ and $m_{b'}$. We then use this number to obtain the acceptance as given by [2] which is scaled by a factor $1 - (1 - \beta)^2$. Different values of $m_{b'}$ for different curves in Fig. 3 reflect this acceptance constraint. We also show an “unexplored region” similar to that shown in Fig. 1 for decays occurring between 1 cm and 3 m, as well as the region where $b'$ is “stable”. This unexplored region is not vertical, as in Fig. 1, since the rate for $b' \to b + Z$ is very sensitive to the $t'$ mass.

In summary, we reexamined the experimental bounds on the masses of the fourth generation quarks: the $t'$ and $b'$ quarks. We divide the search into three distance regions as measured from the center of the beam pipe: 1) $d = 0$ cm to $\sim 1$ cm, 2) $d \sim 1$ cm to 3 m and 3) $d > 3$ m. The first region is one where most searches at the Tevatron have been performed. We have computed the lower bounds on the $t'$ and $b'$ masses under the requirement that $t'$ and $b'$ decay primarily into quarks of the first three generations as shown in Fig. 1 and Fig. 3. For $b'$, we found that the CDF lower bound on its mass can never exceed 255 GeV, contrary to an earlier claim of 268 GeV which had made use of the assumption $B(b' \to b + Z) = 100\%$ and which is not correct when the $b'$ mass exceeds 200 GeV. For $t'$, bounds are shown, starting with the CDF bound 256 GeV. Region (3) (greater than 3 m) is bounded by searches for stable quarks. Region (2) (between 1 cm and 3 m) is unexplored and corresponds to a range of mixing angle $\sin^2 \theta_{bt'} \sim 6 \times 10^{-15}$ and $\sin^2 \theta_{bt'} \sim 2 \times 10^{-17}$. Such a small mixing angle might seem unlikely, but it could arise very naturally in a $3 + 1$ scenario. For example, if one simply had a $Z_2$ symmetry in which the fourth generation fields were odd and all other fields were even, then the mixing angle would vanish. However, discrete symmetries will generally be broken by Planck mass effects, which can lead to $\sin^2 \theta_{bt'}$ of $M_W/M_{Pl} \sim 10^{-17}$. Thus, such a small mixing angle could be natural, and we urge our experimental colleagues to explore this region. If the fourth generation quarks are indeed found in this region, it would shed light on the question of family replication.

Acknowledgments

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