I. INTRODUCTION

Since it was first commercialized in 2006 by Amazon’s EC2, cloud-based computing service has become more popular in recent years in information technology (IT) industry because of its performance, accessibility, low cost and many other advantages of computing activities including business support [1-4]. Cloud computing is an approach in which the users or clients can save their data to an off-site storage system that belongs to a third party and can access the pool of computing resources as well as the computing power of their own in a network environment. It also provides a dynamic, shared and flexible resources from remote data centers to the users, and supports service request where the Internet can develop resources to them.

The cloud computing is based on mainly three functional units:

1. Cloud service provider: A cloud service provider (CSP) is a company that offers network services, infrastructure, or business applications in the cloud. It has a significant storage capacity to preserve the user’s data and high computation power, as well as provides the security of stored application. The large benefit of using a CSP is due to its efficiency, accessibility and low cost of computing activities. Furthermore, the individuals and companies, instead of building their own infrastructure to support internal services and applications, can also purchase the service from the CSP which provides the same to many customers from a shared infrastructure. There are, however, three main services in the cloud by CSPs, which can be discussed as follows:

   - Software as a service (SaaS): It is the top priority model in which a service provider (third-party) hosts the software or application on the cloud infrastructure and makes them available to users over the Internet. However, the users can not manage or control the underlying cloud infrastructure, network, servers, operating systems or even individual application capabilities except some limited user-specific application configuration settings. It also helps the users save cost by renting the application instead of licensing of the traditional packages from cloud service.

   - Platform as a service (PaaS): A middle or second category of the cloud service model that provides a platform or software environment allowing users to design, develop, run, and manage their applications without worrying about the complexity of building and maintaining the cloud infrastructure.

   - Infrastructure as a service (IaaS): It is an instant computing infrastructure, provisioned and managed over the Internet. It helps users avoid the expense and complexity of buying and managing their own physical servers and other data-center infrastructure. However, the users can purchase, install, configure and manage their own software-operating systems, middle-ware and applications.

2. Owner/Third party: It is an organization which stores large data files in the cloud and relies on it for data maintenance and computation.

3. User/Client: It is usually registered with the owner and it uses the data of owner stored in the
Data security is one of the main concerns of cloud computing system as it encompasses many technologies such as network, database, operating system, memory management etc. So, the service providers must ensure that the users preserve their data (i.e., no data loss and data theft) in storage section without any serious risk. In this context, various security issues and challenges have been addressed in the literature (see, e.g., Refs. 5–8). Some architectural security issues are also there which are changing due to various architectural designs over cloud computing. Users primarily want two kinds of securities:

- High-end security which provides automated maintenance and support, low cost for all Internet-capable user devices, and individual security settings regardless of operating system and user device.

- Data security in which all the data is encrypted and sent via the secure system, i.e., no one can intercept data and security is everywhere even when one uses other devices.

On the other hand, the trust issues and challenges (such as trust in efficiency and trust in belief) are also matters of concern in the cloud computing. Note that trust is a significant indicator for service selection and recommendation, and it is still an emerging topic in the cloud computing. For a comprehensive and systematic review of trust evaluation, readers are suggested to Ref. 3. Though we are not considering the theory of trust evaluation, however, we will be proposing a cloud computing model from a different point of view which is, in fact, chaotic in nature [9–11]. This model will be useful for users to share or store data in the cloud through chaotic processes more securely. We will also analyze and discuss various security issues and challenges, and propose a scheme for the analysis of data loss, and to show that the probability of data loss can be negligibly small.

Thus, a dynamical model can be constructed for cloud computing with SaaS for distribution of data, their storage and security. The main issue is that one can store and share data or files in the chaotic regime without any risk. In this context, several theoretical attempts have been made from different points of view [12–13], however, as of now no comprehensive discrete or continuous dynamical model is available in the literature. Since for a confidential data storage, cloud computing model can be owner independent, the user section requires a dynamical model which exhibit chaos. In the proposed algorithm, to be discussed shortly, we will see that how at different iterations or discrete times a file is stored randomly for each and every user and securely in the chaotic regime so that no one can access the files of the others. In the proposed model we also use some scaling parameters for the flexible storage and the independent users, and fix them in such a way that the model exhibits chaos for a system of two or more users and a single owner.

In this work, we propose a discrete dynamical model where a cloud storage operating system provides a set of network-based interfaces i.e., file management and storage which are accessible from various devices of the users. Here, a user can access, save and transfer their files from anywhere (using web, Internet etc.) in a chaotic regime securely and with low cost and less time. The model exhibits periodic, quasiperiodic and chaotic states for certain ranges of parameter values. Though there are different considerations for cloud computing, we will consider only on a private multi-user single-owner model and it can also be used for multi-owner model as well. In this process the total data communication system is shown to be fully secured, i.e., the main problem of security issues may be reduced. In our model we construct a discrete system in which an owner or third party buys a storage capacity from the cloud storage company and distributes to the users in such a way that the storage capacity depends on the authentication of the users. In this situation the owner can provide a well distribution of the storage as per demand of the users in a chaotic regime. This service is provided by the owner only, no user and also no cloud storage company can access without the permission of the owner. We also propose a random replication scheme to ensure that the probability of data loss can be highly reduced compared to the existing scheme in the literature [12].

II. PROPOSED MODEL FOR CLOUD COMPUTING

Mathematical modeling and analysis of cloud computing may be important due to understanding of the interdependencies of demand of the users and capacity of a owner involved. A cloud storage system requires a distributed file system that allows many users to have access to data and supports operations (e.g., create, delete, modify, read and write) on that data, and different file system can have different scale size. In our model, we choose $n$ users, namely $U_1$, $U_2$, ..., $U_n$ and one owner. We assume that $v_m$ is the maximum storage capacity of the owner. Also, let $x_i(t)$ be the $l$-th demand of the user $U_i$ with a scaling parameter $\xi_i$, and $v_c(t)$ the storage capacity of the owner with the scaling parameter $\alpha$ at the $l$-th state or time such that the maximum capacity is $v_m$. Though the variables $x_i(t)$ are independent, however, each of them depends on the storage capacity $v_c(t)$. Here, the owner with storage capacity $v_c(t)$ is flexible with respect to the user’s demand $x_i(t)$. In the cloud computing system, an owner buys a maximum storage capacity $v_m$ from a cloud service provider as per users’ flexible demands, and requires a dynamical model for distribution and share of data through a random process in which a chaotic regime exists where the users’ files may be encrypted by the public key RSA algorithm and their passwords may be secured by the Hash function. Thus, we can construct a
discrete model which will describe sharing of data, distribution of storage from owner’s section to every user \(U_i\) as per their demand \(x_i^{(l)}\) at any \(l\)-th state or \(l\)-th discrete time.

In the process of cloud computing a storage device is connected with the owner and initially a storage size is distributed with the capacity variable \(v_c^{(l)}\) to the users as per their demands. Since \(\alpha\) is the scaling parameter for the capacity variable \(v_c^{(l)}\), the maximum storage capacity \(v_m\) of the owner is such that \(\alpha v_c^{(l)} \leq v_m\), where \(0 < \alpha \leq 1\). Also, the sum of the storage distributions to the \(n\) users at any stage \(l\) is always less than the maximum capacity of the owner \(v_m\), i.e.,

\[
0 < \sum_{i=1}^{n} (-1)^i \xi_i x_i^{(l)} \leq \alpha v_c^{(l)} \leq v_m,
\]

where \(\sum_{i=1}^{n} (-1)^i \xi_i \leq 1\) and \(\xi_i \geq 0 \forall i\). The minus sign in Eq. (1) is introduced to measure the demand or storage of an user relative to the other. Next, we measure the changes in the proposed system at any state \(l\). In the distribution of storage from an owner to the users, the capacity variable at \((l+1)\)-th state, \(v_c^{(l+1)}\) must be equal to the capacity in the \(l\)-th state (with the unit scale \(\alpha\)) minus the storage already allocated as per the demands of the users (with the unit scale \(\xi_i\)). Thus, the capacity in the owner’s section in the \((l+1)\)-th state is given by

\[
v_c^{(l+1)} = \alpha v_c^{(l)} - \sum_{i=1}^{n} (-1)^i \xi_i x_i^{(l)},
\]

where \(l = 0, 1, 2, \ldots\) On the other hand, the demands of an user also varies, in general, from \(l\)-th state to \((l+1)\)-th state. Since at any state \(l\), \(v_c^{(l)}\) denotes the percentage capacity of \(v_m\), the demand of an user \(U_i\) at \((l+1)\)-th state depends on the storage availed by the other users \(U_j\) \((j \neq i)\) and the storage of the user \(U_i\) in the \(l\)-th state. Thus, we have for \(l = 0, 1, 2, \ldots\) and \(i = 1, 2, \ldots, n\)

\[
x_i^{(l+1)} = (1) \xi_i x_i^{(l)} v_c^{(l)} - \sum_{j=1, j \neq i}^{n} (-1)^j \xi_j x_j^{(l)},
\]

At \(l = 0\), we have certain initial values of the capacity and the demand variables, i.e., \(v_c^{(0)}\) and \(x_i^{(0)}\). We assume that initially with the capacity \(v_c^{(0)}\), the company or owner uniformly distributes to all the users the amount of storage \(x^{(0)}\). As an illustration, we analyze our model for one owner and two users. Thus, Eqs. (2) and (3) reduce to

\[
\begin{align*}
    v_c^{(l+1)} &= \alpha v_c^{(l)} + \xi_1 x_1^{(l)} - \xi_2 x_2^{(l)}, \\
x_1^{(l+1)} &= -\xi_1 x_1^{(l)} v_c^{(l)} - \xi_2 x_2^{(l)}, \\
x_2^{(l+1)} &= \xi_1 x_1^{(l)} + \xi_2 x_2^{(l)} v_c^{(l)},
\end{align*}
\]

where \(l = 0, 1, 2, \ldots; x_i, \xi_i \geq 0\) for \(i = 1, 2\) and \(0 < \alpha \leq 1\).

A schematic diagram for a cloud computing model [panel (a)] and its description [panel (b)] as in Sec. II are shown in Fig. 11. The model can be implemented with the following steps.

1. Assume the initial storage capacity of the owner as \(v_c^{(0)}\) at \(l = 0\) and distribute it uniformly to the users with demands \(x_1^{(0)}, x_2^{(0)}, \ldots, x_n^{(0)}\).
2. Consider the proposed dynamical model with the initial conditions \(x_i^{(0)}, v_c^{(0)}\) for \(i = 1, 2, \ldots, n\).
3. After a certain stage or discrete time \(l\), distribute the storage from the owner’s section to users as per the flexible storage variable \(x_i^{(l)}\) and the capacity variable \(v_c^{(l)}\). At the next iteration, i.e., in the \((l+1)\)-th state, the corresponding variables are different from \(x_i^{(l)}\) and \(v_c^{(l)}\), and they are \(x_i^{(l+1)}\) and \(v_c^{(l+1)}\) respectively.
4. The owner notes the distributions of storage to the users at each stage \(l\) and makes secure communication between them. Any user does not get any kind of information of the other user.

III. DYNAMICAL PROPERTIES OF THE PROPOSED MODEL

In this section, we study the dynamical properties of the proposed model (4) for a single owner or company and two users. First, we perform the linear stability analysis of the model about the fixed points, and second, we show that the chaos may be established by the analysis of Lyapunov exponent spectra and the bifurcation diagram.

A. Stability analysis of the proposed model

We study the stability of the fixed points of the iterated map (4) with certain parameter values. The fixed points of Eq. (4) are obtained as \((0, 0, 0)\) and \((1, -\alpha/2\xi_1, \alpha/2\xi_2)\), where \(\alpha, \xi_1, \xi_2 \geq 0\). Then the eigenvalues of the Jacobian matrix, given by,

\[
J = \begin{bmatrix}
    \alpha & \xi_1 & -\xi_2 \\
    0 & 0 & -\xi_2 \\
    0 & \xi_1 & 0
\end{bmatrix}
\]

corresponding to the fixed point \((0, 0, 0)\), are \(\lambda_1 = \alpha\) and \(\lambda_2 = \pm \sqrt{-\xi_1 \xi_2}\), i.e., \(\lambda_1 > 0\) and \(\lambda_2\) is purely imaginary. This means that the equilibrium is unstable and there is a possibility of Hopf bifurcation. From the bifurcation diagram (cf. Fig. 5), we will see that a critical point indeed exists at which the system looses its stability leading to chaos. The latter is established by the analysis of Lyapunov exponent spectra [cf. panels (b) of Figs. 3 to 4].
Next, we study the stability of the fixed point 
\( (1, -\alpha/2\xi_1, \alpha/2\xi_2) \). To this end, we apply the linear transformations as follows:
\[
\begin{align*}
\hat{v}_c^{(l)} &\leftarrow (v_c^{(l)} - 1), \\
\hat{x}_1^{(l)} &\leftarrow (x_1^{(l)} + \alpha/2\xi_1^{(l)}), \\
\hat{x}_2^{(l)} &\leftarrow (x_2^{(l)} - \alpha/2\xi_2^{(l)}).
\end{align*}
\]
Thus, Eq. (4) reduces to the following linearized form
\[
\begin{align*}
\hat{v}_c^{(l+1)} &= \alpha\hat{v}_c^{(l)} + \xi_1\hat{x}_1^{(l)} - \xi_2\hat{x}_2^{(l)}, \\
\hat{x}_1^{(l+1)} &= -\xi_1\left(\hat{x}_1^{(l)} - \frac{\alpha}{2\xi_1}\hat{v}_c^{(l)}\right) - \xi_2\hat{x}_2^{(l)}, \\
\hat{x}_2^{(l+1)} &= \xi_1\hat{x}_1^{(l)} + \xi_2\left(\hat{x}_2^{(l)} + \frac{\alpha}{2\xi_2}\hat{v}_c^{(l)}\right).
\end{align*}
\]

The corresponding Jacobian matrix about the point \( O(0, 0, 0) \) becomes
\[
J^* = \begin{bmatrix}
\alpha & \xi_1 & -\xi_2 \\
\alpha/2 & -\xi_1 & -\xi_2 \\
\alpha/2 & \xi_1 & \xi_2
\end{bmatrix}.
\]

The eigenvalues corresponding to the matrix \( J^* \) are given by
\[
\begin{vmatrix}
\alpha - \lambda & -\xi_1 & -\xi_2 \\
\alpha/2 & \xi_1 - \lambda & -\xi_2 \\
\alpha/2 & -\xi_1 & \xi_2 - \lambda
\end{vmatrix} = 0.
\]
This gives
\[
\lambda^3 + P\lambda^2 + Q\lambda + R = 0,
\]
where the coefficients are
\[
P = -(\alpha - \xi_1 + \xi_2), \quad Q = \frac{3}{2}\alpha(\xi_2 - \xi_1), \quad R = 2\alpha\xi_1\xi_2.
\]
Now, if $P$, $Q$, $R > 0$ and $PQ > R$, then by the Routh’s stability criterion all the roots of Eq. (10) have negative real parts implying that the linearized system is stable. Thus, for the linear stability of the system (4) about the fixed point $(1, -\alpha/2\xi_1, \alpha/2\xi_2)$, we must have $0 < \alpha < \xi_2 - \xi_1 \leq 1$. Otherwise, if at least one of $P$, $Q$ and $R$ is negative (which holds when $0 < \xi_1 - \xi_2 < \alpha$) and $\Re \lambda > 0$, the system is said to be unstable (cf. Figs. 2 to 4). Furthermore, in order to have a Hopf bifurcation of the system indeed exhibits a stable structure (periodic limit cycle) for $\alpha = 0.96$ and $\xi_2 - \xi_1 = 0.98$ [see panel (a)]. This means that when $\alpha \approx 1$, the whole system is running through the owner’s capacity and the users demands periodically. It happens only when the owner has almost utilized the capacity and the users are filled with their demands, i.e., no demand after a certain stage or time remains. The periodic structure is justified with the corresponding Lyapunov exponents which are all negative as expected [see panel (b)].

**Calculation of Lyapunov exponents:** In order to calculate the Lyapunov exponents of the discrete dynamical system (4), we redefine the variables as $X^{(l)} = (v_c^{(l)}, x_1^{(l)}, x_2^{(l)})$ and the initial condition as $X^{(0)} = (v_c^{(0)}, x_1^{(0)}, x_2^{(0)})$. Then Eq. (4) can be rewritten as

$$X^{(l+1)} = B^{(l)}X^{(l)},$$

where

$$B^{(l)} = \begin{bmatrix} \alpha & \xi_1 & -\xi_2 \\ 0 & -\xi_1 v_c^{(l)} & -\xi_2 \\ 0 & \xi_1 & \xi_2 v_c^{(l)} \end{bmatrix}. \quad (13)$$

Next, to calculate the Jacobian $J^{(l)}$ of the above equation for determining the eigenvalues, we differentiate the right-hand side of Eq. (4) with respect to the variables $X^{(l)}$. Thus, we write

$$J^{(l+1)} = A^{(l)}J^{(l)}.$$  \quad (14)

where $A^{(l)}$ is given by

$$A^{(l)} = \begin{bmatrix} \alpha & \xi_1 & -\xi_2 \\ -\xi_1 x_1^{(l)} & -\xi_1 v_c^{(l)} & -\xi_2 \\ \xi_2 x_2^{(l)} & \xi_1 & \xi_2 v_c^{(l)} \end{bmatrix}. \quad (15)$$

Furthermore, for $l = 0$, $J^{(0)}$ is the identity matrix and $A^{(0)}$ is evaluated with the initial condition $X^{(0)}$. So, the solution of the Eq. (14) is given by

$$\Lambda = \lim_{l \to \infty} \frac{1}{2l} \log \left( J^{(l)} [J^{(l)}]^T \right).$$

where the Lyapunov exponents are obtained as the eigenvalues $\lambda_i$ of the matrix $\Lambda$ for $i = 1, 2, 3$. Next, substitution of the initial condition $X^{(0)} = (0.01, 0.01, -0.01)$ and different parameter values of $v_c$, $x_1$ and $x_2$ gives different eigenvalues.

In order to verify the results with the linear stability analysis as above, we numerically solve Eq. (4) by the fixed point iteration scheme within the ranges of values of the parameters $\alpha$, $\xi_1$ and $\xi_2$ where the system exhibits stable and unstable fixed points. We consider the initial condition as $v_c = 0.01$, $x_1 = 0.01$ and $x_2 = -0.01$. The results are displayed in Figs. 2 to 4. Figure 2 clearly shows that when the condition $0 < \alpha < \xi_2 - \xi_1$ is satisfied for the stability of the fixed point $(1, -\alpha/2\xi_1, \alpha/2\xi_2)$, the system indeed exhibits a stable structure (periodic limit cycle) for $\alpha = 0.96$ and $\xi_2 - \xi_1 = 0.98$ [see panel (a)].

![FIG. 2: The periodic limit cycle [panel (a)] and the corresponding Lyapunov exponents (all are negative) [panel (b)] are shown for $0 < \alpha \equiv 0.96 < \xi_2 - \xi_1 \equiv 0.98 < 1$.](image)
a transition from torus structure to chaos occurs. The phase-space structures [panels (a)] are well justified with the Lyapunov exponents [panels (b)] as shown in Figs. 3 and 4.

![Fig. 3](image)

**FIG. 3**: The torus [panel (a)] and the corresponding Lyapunov exponents (one is close to zero and other two are negative) [panel (b)] are shown for $0 < \xi_1 - \xi_2 \equiv 0.6 < \alpha \sim 0.9$; $\xi_1 = 1.4$, $\xi_2 = 0.8$.

**Bifurcation diagram**: Figure 4 shows the bifurcation diagram for the capacity variable $v_c$ with respect to the scaling parameters $\alpha$ [left panel (a)] and $\xi$ [right panel (b)]. We find that for some fixed values of $\xi_1$ and $\xi_2$, as the value of $\alpha$ increases in $0 < \alpha \lesssim 1$, the system approaches from chaotic state to a periodic state with a series of period-halving bifurcations [panel (a)]. This is a consequence of Fig. 2 where the system’s periodicity is shown for a value of $\alpha$ close to the unity. However, an opposite trend occurs when a fixed value of $\alpha = 0.5$ is considered and one of $\xi_1$ and $\xi_2$ varies [see panel (b)]. In this case, a series of period-doubling bifurcations leads the system from order to chaos. Thus, whether the system exhibits periodicity or chaos that depends on the scaling parameters $\alpha$, $\xi_1$ or $\xi_2$.

![Fig. 4](image)

**FIG. 4**: Transition from torus to chaos [Panel (a)] and the corresponding Lyapunov exponents (two are positive and other one is negative) [Panel (b)] are shown for $0 < \xi_1 - \xi_2 \equiv 0.05 < \alpha \sim 0.6$.

IV. ANALYSIS OF DATA SECURITY AND DATA LOSS

Data security [5][6] and data loss [12][13] are two main challenging issues in cloud computing systems. Though, a number of authors have considered these issues in their different works, however, we review the security issues, especially, loss of user identity and password, brute force attack, unauthorized server, data privacy etc. to justify how our proposed model fits to resolve these security issues. Furthermore, we propose a random replication scheme which ensures that the probability of data loss is highly reducible compared to the existing scheme in the literature [12]. There are two main issues in the cloud computing system, these are

- A model of organization has to be built up with a company (owner) and users, and distribution of storage must be in a proper way and secured.
- Once the storage is distributed properly, one must be ensured that the data loss or data leakage is minimum.
The first issue is resolved by proposing a chaotic dynamical model which we have already done in Sec. II. However, we analyze and discuss about the second issue, i.e., the data security and data loss in Secs. IV A and IV B.

### A. Analysis of data security

We start by considering an example that a financial company provides a data storage system to each of its users as per their demands in a certain interval of discrete time. The details are demonstrated in Table I. The model (4) gives an information about how much data an user can store in the time interval. We assume that the company or owner uses the model for data storage as SaaS. In the following, we define different terms that are related to computation of storage in the user section in a given interval of time or at a particular state.

- **Time interval for demand**: In the model (4), we consider the iteration $l$ as some particular time.

- **Data in use**: In a cloud computing system, the full storage in the owner’s section cannot be used for data storage of users, some part of it may be used for some other purpose, e.g., software development. This means that all the cloud computing systems go on by an excess of storage. Some scaling parameters are also used to calculate the storage/demand available in the owner/user section. These are namely, $\alpha$ for the owner and $\xi_i$ for the $i$-th user.

- **Initial storage**: There are some initial storage for distribution of data in the owner section, i.e., $\alpha c_v^{(0)}$ and in the user section, i.e., $\xi_i x_i^{(0)}$ for the $i$-th user.

Using the model (4), one can calculate how much data is required by the user’s section and how much data can be distributed by the owner section to the users in a particular interval of time (cf. Table I).

The cloud computing model (4) is such that it exhibits chaos in the distribution of storage between the owner and users. So, the data storage (in the cloud) to the owner section may be random which ensures that the database system allocates data randomly as per users’ demands. Thus, the proposed model is capable of preventing external threats like data stealing and it secures data privacy. Next, we discuss about some other issues, namely user authentication, data security and other external threats.

1. **User authentication**: One of the most security concerns in cloud computing is the user’s authentication to the owner. Here, the process of verifying the user’s identity to the owner consists of two steps:

   - **User identification**: This step presents an identification for each and every user, and the user section carefully preserves its identity (ID) and password which are provided by the owner. These ID and password must be kept secretly and known to the user section only. Otherwise, the data can be accessed by some other one or data can be stolen.

   - **Verification by the owner section**: A verification of ID and password is made by the owner which is necessary to allocate data to each user distinctively. To verify the ID and password, owner section uses a secure hash algorithm (SHA). When an user logs into the system with its own ID and password, the owner section applies the SHA and matches with the user’s ID and Password. If it matches then user can access, otherwise it will be denied by the owner section. Also, each user is connected with the owner section using a unique media access control (MAC) address as well.

2. **Data Privacy and security**: Data privacy, also called the information privacy, is an important aspect of information technology (IT) that deals with the ability of an organization or individual to determine what data in a computing system can be shared with the third parties. The main issue is the trust on the user and also on the owner. So, in this manner owner should develop the user’s data privacy. The users and owner all are connected through a network, and users require to create nodes to save data as per the storage distribution by the owner. So, in each node the data must be securely encrypted and stored. In this way, one can use AES-256 encryption scheme (4) for data encryption and decryption (18, 19) by which the user can know the private key to access their files when it is needed. Also as the process of data transfer occurs in the cloud with superior encryption procedure, the data is not corrupted or tampered in any way.

   On the other hand, keeping all cryptographic algorithms up to date is another factor when the information needs to be protected by encryption. In this case, users must use advanced cryptographic algorithms that are up-to-date which deny faulty algorithms of old cryptographic process. This certainly prevents the information leakage for the owner and users. If the owner is not aware of or concerned about these changes, the security risk will continue to increase, while attackers are looking for specific vulnerabilities dealing with unsuitable cryptographic algorithms.

3. **External threats**: An owner and users must be aware of the external threats that an attacker exploits the vulnerabilities in services provided to an user. External threats can be characterized by attacks that occur outside the user’s domain. External threats include brute force attack, hacking of data, hacking of user ID and password etc. The attacks occur when a hostile user deploys a proxy application in between an user and owner without them knowing, and the attacker intercepts personal information such as user ID and password.

   In order to avoid brute force attack in our proposed model we use AES-256 encryption scheme. The encryption occurs in two ways—one by the owner section in which
the data is stored in the cloud and other by the user section where the data is stored at the physical level. In Sec. IV.B, we have analyzed the process of data loss and how it can be minimized in the cloud computing system. For this purpose we construct nodes for data storage where each device in the user section contains 50% of the total data that are stored in nodes in such a way that a single block of nodes does not contain full data information, implying that the data is safe, i.e., an attacker can not recover any information without collection of full data. Again in Sec. IV.B, we have discussed about chunks where the data storage is divided into different parts. So, to recover a data from nodes while the data is transferred, an attacker has to recover all the chunks. However, in the process of creating chunks of files, users always use the initial condition for the capacity variable and it is almost impossible for an attacker to choose the initial condition. Furthermore, since the data is in transit by parts it is also impossible to get all the parts of the storage nodes. Thus, this approach, not only safeguards data where it is stored, but also helps assure users that data is secured while in transit.

4. Unauthorized server: As the data needs to be transmitted over a network to the owner’s cloud, there are numerous means through which an attacker can easily get into the Internet based network and act as an user to the owner’s data, thus resulting into the loss of data. To prevent the loss of data, a well developed authorized certification may be used. Here, Certificate Authorities (CAs) issue each certificate, which is a credential for the online world, to only one specific domain or server. The cloud server first sends the identification information to the owner when it connects, then sends the owner a copy of its certificate. The owner verifies the certificate and then sends a message to the server and the server sends back a digitally signed acknowledgement to start an encrypted session, enabling encrypted data transfer between the owner and the user. Moreover, the data and keywords are stored on the cloud in encrypted form.

### B. Analysis of data loss

Replication scheme has been widely used as a means to achieve high availability and to avoid failure of data in distributed cloud storage systems [14–16, 20, 21]. However, in most of the existing schemes, e.g., random replication scheme, copyset replication scheme, HDFS random placement policy, the probability of data loss may not be reduced. Here, we propose a different replication scheme which is resilient, fault-tolerant and high-efficient global replication algorithm (RFH) for distributed cloud storage systems. Before proposing a scheme for the calculation of the probability of data loss, we note that in order to maintain a low-cost of data storage for users, owners may choose a pseudo-randomness system of partitioning their storage as per the demand of users. These partitions are random in storage and may be called as nodes, i.e., the l-th node for the i-th user may be defined as $N_{il}^{(l)} = \xi x_i^{(l)}$.

It is to be mentioned that a cloud storage system may have some limitations in physical (IaaS) as well as software (SaaS) levels, and so it may not provide an effective performance in between the owner and users without a well-designed storage allocation policy. In fact, there is always an unpredictable demand of storage in the owner’s section by the users owing to customer services. As the demand rate of an user in any interval of time or stage becomes higher and hence the storage in the owner’s section, the chance of machine failure or device crush increases. Thus, proper design of a replication scheme is most important to save the storage nodes or data of users.

The traffic load or cluster-wide power outage is one of

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**TABLE I:** An example of a cloud computing system with one owner and two users. The symbols $\alpha$, $(\xi_1, \xi_2)$ denote the scaling parameters corresponding to the capacity and demand variables $v_c^{(l)}$ and $x_i^{(l)}$ respectively.

| Parameters | Initial storage $(\alpha v_c^{(0)})$ of owner | Initial storage $(\xi x_i^{(0)})$ of an user for $i = 1, 2$ | Storage distribution at $l$-th stage by the owner $(\alpha v_c^{(l)})$ | Storage usage at $l$-th stage by an user $(\xi x_i^{(l)})$ |
|------------|-----------------------------------------------|--------------------------------------------------|-------------------------------------------------|--------------------------------------------------|
| $\alpha$   | $\xi_1$                                      | $\xi_2$                                         | $l = 1$                                         | $l = 1$                                          |
| 0.6        | 1.25                                         | 1.28                                            | 480 Mb                                          | 12.29 Mb                                         |
|            |                                               |                                                 | 13 Mb                                           |                                                 |
|           |                                               |                                                 | 103 Mb                                          |                                                 |
|           |                                               |                                                 | 102.76 Mb                                       |                                                 |
|           |                                               |                                                 | 1.16 Gb                                         |                                                 |
|           |                                               |                                                 | 1.16 Gb                                         |                                                 |
|           |                                               |                                                 | 5.03 Gb                                         |                                                 |
|           |                                               |                                                 | 4.02 Gb                                         |                                                 |
|           |                                               |                                                 | 123.5 Mb                                        |                                                 |

| $l = 10$   | 369 Mb                                        | 0.1 Gb                                          |                                                 |                                                 |
| $l = 20$   | 3.5 Gb                                        |                                                 |                                                 |                                                 |
| $l = 200$  | 10.45 Gb                                      |                                                 |                                                 |                                                 |
| $l = 365$  | 7.07 Gb                                        |                                                 |                                                 |                                                 |
the main concerns for data loss in a cloud storage system unless any replication of nodes for data storage is done. The replication of nodes can restore the data again and repair the affected nodes. Sometimes the power outage can kill a data in storage nodes. Even after when the power is restored, the nodes can not be recovered. However, in our proposed scheme, the replication of nodes gives much lower probability of losing data from the affected nodes than the existing schemes [12]. Before providing a replication scheme we review some main issues of data loss and how our model is useful for avoiding data loss as follows:

1. Traffic load in cloud computing system

Network traffic control is used to control the load of data traffic in which the data is transferred from users to owner and vice versa in a given interval of time or at any stage (say, l-th iteration) of the computing system. In our proposed model [1], since the allocations of data for every users are given as per their demand rates, the possibility of traffic load is low. However, there are some other traffic issues, namely the consumption of data bandwidth during working hours, bandwidth competition to purchase spectrum etc. which can affect the networking system. This issue can be resolved by creating chunk files in the storage nodes of data traffic due to the demands of users. If consumption of data bandwidth occurs in the user section in any node, then it is required to partitioning the allocated storage in the respective nodes which reduces the traffic problem. Below we show how the chunk files in the nodes may be created.

The demand rate of an i-th user at the l-th stage is given by \( q^{(l)} = \xi x^{(l)}_{i} v_{c}^{(l)} \), where \( l = 1, 2, ... \). Since the forward demand rate creates some traffic, the demand of an user at l-th stage can be written as the difference of the demands during the traffic at l’-th stage and that at the l’-th stage, i.e., \( x^{(l)} = x^{(l')} - x^{(l’)} \) with \( l' \leq l \leq l'' \), so that the storage in the l-th node of an i-th user is \( \xi x^{(l)} \). Next, to calculate the number of chunk files to be created in the node \( N^{(l)} \) in the interval \( l' \leq l \leq l'' \), where each l may not be equally spaced, we note that the size of the storage in the l-th node of an i-th user is \( c^{(l)} = \xi x^{(l)} \) where \( l' \leq l \leq l'' \). So, at \( l = 0 \), \( c^{(l)} \) represents the initial chunk size for a node. The chunks in the next stages are calculated as \( c^{(l+1)} = c^{(l)} - c^{(l)} v_{c}^{(l)} \), so that \( c^{(l)} \) represents the total number of chunk files in the node \( N^{(l)} \) for an i-th user.

2. Cluster-wide power outage

The data loss in cloud computing may be due to the cluster-wide power outage. Here, a cluster means a collection of interconnected different machines (devices or computers) which store all the chunk files that contain the total users’ data in a node. The cluster-wide power outage causes the failures of the machines in the cluster or crush of the devices where the database is being stored. In a cloud computing system there is a connection between the physical storage in a device and the virtual storage in the cloud. In our model the user section has nodes in physical level and uploads its data to the owner section’s cloud storage. If in a cluster all the machines fail, there is a possibility of nodes failure at physical level, however, the data may be recovered from the cloud storage. Note that the data may also be lost or crushed in the cloud due to some other reasons, e.g., virus attack. To resolve these issues we propose a design for preventing data loss as follows:

Having calculated nodes for each user and the chunk files for each node, we redistribute the chunk files into one primary and two secondary nodes in such a way that in the first replica 100% of the chunk files remain in the primary node and in the second replica 50% of the chunk files are in one secondary node and rest 50% in the other. Here, each node is stored in the owner section (cloud storage) as well as in the users’ section (physical device). Let us now consider n number of nodes, say \{1, 2, 3, ..., n\}. The distribution of nodes is given in Table II. The distribution of nodes in the owner section is such that \( N_{i} = \{P_{i}S^{1}_{i}, P_{i}S^{2}_{i+1}, S^{1}_{i}S^{2}_{i+2}\} \) for \( i = 1, 2, 3, ..., n \) and \( S^{1}_{n+1} = S^{1}_{1}, \ S^{2}_{n+2} = S^{1}_{2} \), i.e., the process is cyclic to fill the node blocks properly to prevent data loss. Here, \( P_{i} \) and \( S_{i} \) stand for the primary and secondary nodes. Similarly, in the user section to fill the node blocks we take the secondary data values in such a way that \( N_{i} = \{S^{1}_{i}, S^{1}_{i+1}S^{2}_{i+2}\} \) and \( S^{1}_{n+1} = S^{1}_{1}, \ S^{2}_{n+2} = S^{1}_{2} \). Thus, the number of blocks in the owner’s and user’s sections is each n.

3. Probability of data loss

We note that the data is replicated three times one in primary nodes and two in secondary nodes. If a machine or device can store 50% of the total chunk files in the nodes then for each block in the owner section we require four machines to store the data and in the user section we require three machines. To calculate the probability of data loss, we assume that the probability of machine failure is \( p \).

Since the number of blocks is twice the number of nodes, i.e., \( 2n \), and two blocks in the user section and owner section have three plus four, i.e., seven machines, there are \( 7n \) machines for \( n \) nodes in two racks (one is for owner and other is for the user). We calculate the probability of data loss for \( f \) number of machine failures randomly in both the racks where \( n \) represents the number of nodes, \( b = (2n) \) the number of total blocks, \( 7n \) the number of machines in a cluster, and \( p \) the probability of machine failure.

We mention that the above replication scheme is valid.
for $n \geq 3$ and in order to have data loss there must be
failure of at least three machines at a time. In what fol-
lows, we construct a generating polynomial from Table I
and obtain the coefficients of this polynomial to calcu-
late the probability of data loss. The generating polynomi-
al is given by

$$P(x) = (1 + a_1 x + a_2 x^2 + a_3 x^3 + a_4 x^4 + a_5 x^5)^n, \quad (17)$$

where $a_1 = \binom{7}{0} = 7$, $a_2 = \binom{7}{1} = 7$, $a_3 = \binom{7}{2} - 1 = 34$, $a_4 = \binom{7}{3} - 1 = 30$, and $a_5 = \binom{7}{4} - \binom{7}{3} = 12$. Here, the coefficient of $x^f$ is the number of ways of choosing $f$
machines out of $7n$ machines, i.e., in $(\binom{7}{f})$ ways with the
condition as given in Table I. The probability of no data
loss for failure of $f$ machines is given by

$$P_n = \begin{cases} (\text{coefficient of } x^f \text{ in } P(x)) \big/ \binom{7}{f} & \text{if } f \leq 5n \\ 0 & \text{otherwise.} \end{cases} \quad (18)$$

So, the probability of data loss for failure of $f$ machines
is $(1 - P_n) \times P_f$, where $P_f$ denotes the probability of $f$
machine failures, i.e., $P_f = (\binom{7}{f}) p^f (1-p)^{7n-f}$ since there
is no data loss for $f = 1, 2$. Note that the coefficient of $x^f$
in $P(x)$ for $f \leq 5n$ can be obtained by considering all the
possible combinations of $k_1$, $k_2$, $k_3$, $k_4$, $k_5$ which satisfy
$k_1 + 2k_2 + 3k_3 + 4k_4 + 5k_5 = f$. So, the coefficient of $x^f$
in $P(x)$ is $\sum_{(k_1, k_2, k_3, k_4, k_5)} \binom{7}{k_1} a_1^{k_1} a_2^{k_2} a_3^{k_3} a_4^{k_4} a_5^{k_5}$. Thus, if the probability of a machine failure is $p$ then the
probability of data loss is given by $P = \sum_{f=3}^{5n} \binom{7}{f} p^f (1-p)^{7n-f}$

Next, we compute the probabilities of data loss for some sample values of $n$ as given in Table II. Here, we
assume that different clusters of machines have different
number of nodes $n$. For example, we consider that 70,
140, 280, 560, 700, 980 and 1400 numbers of machines
have 10, 20, 40, 80, 100, 140 and 200 nodes respectively.
We also assume that the probability of machine failure is
$p = 0.01$.

Figure [6] shows the graphs for the probability of data
loss using our proposed scheme [panel (a)] and as in Ref.
[12] [panel (b)]. It is found that in the random machine
failures with $p = 0.01$, the probability of data loss in our
scheme is highly reduced compared to that in Ref. [12].

| Primary (P) | Secondary-1 ($S^1$) | Secondary-2 ($S^2$) | Nodes in the owner section | Nodes in the user section |
|-------------|---------------------|---------------------|---------------------------|---------------------------|
| $P_1, P_2, \ldots, P_n$ | $S^1_1, S^1_2, \ldots, S^1_n$ | $S^2_1, S^2_2, \ldots, S^2_n$ | {($P_1, S^1_1, S^1_2 \cup \ldots, S^1_n$), ($P_2, S^1_1, S^1_2 \cup \ldots, S^1_n$), \ldots, ($P_n, S^1_1, S^1_2 \cup \ldots, S^1_n$)} | {($S^1_1, S^2_1, S^1_2$), ($S^1_1, S^2_2, S^1_2$), \ldots, ($S^1_n, S^2_1, S^1_2$)} |

TABLE II: Replication scheme is shown for user and owner sections with primary (P’s) and secondary ($S^1$’s) nodes.

V. CONCLUSION

We have proposed a discrete dynamical model for cloud
computing and management of storage between a third
party or company and users. Though the model is
applicable for an arbitrary number of users, however, for
simplicity, we have analyzed it for one owner and two
users. The basic dynamical properties of the model are
studied. It is found that the model exhibits chaos for
certain ranges of parameter values. A framework for dis-
tribution of storage of data and its implementation with
users and the owner is also given. Some issues of data
security are analyzed and discussed. Furthermore, we have
proposed a random replication scheme and calculated the
probability of data loss. It is found that the probability of
data loss is highly reduced compared to that using the
existing scheme in the literature [12].

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### TABLE III: The probability of data loss is calculated using the random replication scheme as in Sec. [IV B 2] for different choices of nodes and machines or devices. The latter are assumed to failure randomly.

| Number of nodes ($n$) | Number of machines ($7n$) | Failure policy | Probability of data loss ($\times 10^{-4}$) |
|-----------------------|---------------------------|----------------|--------------------------------------|
| 10                    | 70                        | Random         | 0.12120                              |
| 20                    | 140                       | Random         | 0.2220                               |
| 40                    | 280                       | Random         | 0.42419                              |
| 80                    | 560                       | Random         | 0.82817                              |
| 100                   | 700                       | Random         | 1.0301                               |
| 140                   | 980                       | Random         | 1.4341                               |
| 200                   | 1400                      | Random         | 2.04                                 |

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FIG. 5: Bifurcation diagram for the capacity variable $v_c$ with respect to the scaling parameters $\alpha$ and $\xi_1$. Panels (a) and (b) are corresponding to the parameters ($\xi_1 = 1.28$, $\xi_2 = 1.23$) and ($\xi_2 = 1.28$, $\alpha = 0.5$) respectively.

FIG. 6: Probability of data loss is shown using our replication scheme [left panel (a)] and compared with that in Ref. [12] [right panel (b)]. It is seen that the probability of data loss in our proposed schemes is significantly reduced compared to that in Ref. [12].