Dynamic Control of Soft Manipulators to Perform Real-World Tasks

Oliver Fischer\textsuperscript{1}, Yasunori Toshimitsu\textsuperscript{1,2}, Amirhossein Kazemipour\textsuperscript{1,3}, Robert K. Katzschmann\textsuperscript{1}

Dynamic motions are a key feature of robotic arms, enabling them to perform tasks quickly and efficiently. Soft continuum manipulators do not currently consider dynamic parameters when operating in task space. This shortcoming makes existing soft robots slow and limits their ability to deal with external forces, especially during object manipulation. We address this issue by using dynamic operational space control. Our control approach takes into account the dynamic parameters of the 3D continuum arm and introduces new models that enable multi-segment soft manipulators to operate smoothly in task space. Advanced control methods, previously afforded only to rigid robots, are now adapted to soft robots; for example, potential field avoidance was previously only shown for rigid robots and is now extended to soft robots. Using our approach, a soft manipulator can now achieve a variety of tasks that were previously not possible: we evaluate the manipulator’s performance in closed-loop controlled experiments such as pick-and-place, obstacle avoidance, throwing objects using an attached soft gripper, and deliberately applying forces to a

\textsuperscript{1}ETH Zurich, Switzerland
\textsuperscript{2}University of Tokyo, Japan
\textsuperscript{3}The Sapienza University of Rome, Italy
surface by drawing with a grasped piece of chalk. Besides the newly enabled skills, our approach improves tracking accuracy by 59% and increases speed by a factor of 19.3 compared to state of the art for task space control. With these newfound abilities, soft robots can start to challenge rigid robots in the field of manipulation. Our inherently safe and compliant soft robot moves the future of robotic manipulation towards a cageless setup where humans and robots work in parallel.
Introduction

Fig. 1: A soft manipulator performs various dynamic tasks. (A) The manipulator draws with a grasped piece of chalk by moving from left to right. Motion snapshots are visually overlaid and spaced in time with two second intervals. (B) The manipulator throws an object in a controlled manner. The motion from right and to left is depicted with snapshots at intervals of 130 milliseconds. (C) The manipulator avoids multiple obstacles while moving from right to left. Motion snapshots are spaced with 2 second intervals.

When robots work in real-life situations, they must be able to quickly and precisely interact with their environment to avoid any harm and allow for effective operations. Modeling the contact interaction behavior between the robot and the objects is a promising approach to dexterously interact with objects and manipulate them (1). Another approach is to move towards the target object based on a desired grasping pose, which will be generated from a data-driven model (2). In these practical model-based techniques, it is essential to have a model-based controller for the arm that can accurately control its pose in task space. Traditionally, rigid robotic arms are used as the platform for these tasks.

An emerging alternative to conventional rigid robotics is the field of soft robotics (3, 4). Soft robots adopt the principle of physical artificial intelligence to achieve an inherently compliant robotic behavior and embodied (5, 6). In soft robotics, robots are fabricated mainly from functional soft materials (7–10) to achieve inherently adaptive and intelligent characteristics. The inherent compliance of soft robots enables various behaviors that were previously difficult
for conventional, rigid-linked robots to achieve, such as navigating through tight environments or gentle gripping (11–13). Continuum robots (14–16), one of the precursors to soft robots, have shown improved adaptability and safety when moving from rigid to soft actuator materials and support structures. However, it is more challenging to model soft robots’ behavior due to the continuous properties and infinite dimensionality of their state space. Therefore, various solutions have been proposed to make model-based control feasible for soft robot (17–25).

In this work, we present the first example of a physical implementation of dynamic task space control on a physical soft continuum manipulator that can operate in a three-dimensional space and demonstrate its capabilities in various real-world tasks, such as drawing on a whiteboard, avoiding obstacles, and applying force. We introduce new actuation and deformation modeling elements to address the real-world limitations of previous models. Our new approach allows us to show new skills previously not possible with soft manipulators, such as throwing, pick-and-place and obstacle avoidance. Furthermore, our approach improves the operational space tracking accuracy by 59% and increases the operational speed by a factor of 19.3 compared to previous methods. We implement an obstacle avoidance method based on potential fields (26), which has previously been used on conventional rigid robotics (27), but has never before been done with soft manipulators. The newfound capabilities of soft manipulators are examined in multiple experiments, of which a brief overview is shown in Fig.1.

This work aims to create a soft modeling and control framework that benefits from the research on both rigid and soft robotics. We combine the inherently advantageous properties of soft robots with the extensive literature on advanced control methods for rigid robots. The experimental results presented here demonstrate new abilities indicating that soft robots can begin to practically challenge rigid robots in the field of object manipulation and real-world task interaction. We see this work as an important step towards a future where robotic manipulation occurs in a safe and cageless setting with humans and robots working together.
Background

Soft robots have recently been increasing in popularity due to their unique properties (3, 28, 29). Soft robots, lacking electronics and metals, can operate in environments where metal components are impractical, such as in magnetic resonance imaging machines (30). Furthermore, soft robots’ inherent compliance is useful in several ways (31). The compliance makes soft robots safe, as they will deform adaptively if they collide with another object (18, 32). This feature makes soft robots particularly interesting for working side-by-side with humans. The continuous bending of the actuators also means they are able to take on unique shapes, which may be of use when operating within constrained environments (33–37). Their continuous nature also gives them a significantly large nullspace, a feature which has only been utilized so far in rigid robotics (38).

With regard to modeling and control, however, soft robotic manipulators currently fall short of rigid robotic manipulators. Dynamic control on a physical soft manipulator has, to the best of our knowledge, only been shown within configuration space. There have been several approaches to construct a dynamic model for soft continuum manipulators, akin to the manipulator equation (39). Godage et al. (40) derived the dynamic model of a continuum manipulator through integral Lagrangian formulation with the assumption of continuous mass distribution along the arm. However, the closed-form expressions of dynamic terms become complicated as the number of segments increases, making the model unpractical for using it in real-time control. In order to simplify the dynamic model, the mass distribution of each segment can be approximated into a single lumped mass. Under this assumption, Falkenhahn et al. derived the dynamic model for a continuously bending manipulator (Festo Bionic Handling Assistant) where the lumped masses are assumed to be concentrated in the tips of the soft continuum sections (17). With the addition of a dynamic model, they showed acceleration-level control methods in joint space. In a later work, Falkenhahn et al. explicitly considered valve dynamics
to achieve higher model accuracy (41). Curvature space methods can also be combined with inverse kinematic approaches to control real-world coordinates (42, 43). This combination was shown by Gong et al. (44), who used an underwater continuum arm to grab samples.

Alternatively, it is possible to compute the dynamic parameters of a continuously bending soft body by using an augmented rigid body model to approximate its kinematic and dynamic characteristics (18, 21). This model supplements the piecewise constant curvature (PCC) model by adding a rigid link model and mass points. Dynamic parameters can then be obtained using methods previously designed for rigid link models. This model has been implemented on a planar manipulator (18, 19) and on a 3D manipulator (21). In Della Santina et al. (45), it was shown that the actuation space must have at least the same dimensions as the operational space, and dynamic operational space control was demonstrated in a simulated soft continuum arm model. A proof of concept is shown in Mustaza et al. (46), which uses a dynamic model to actuate a single continuum segment in task space. Kapadia et al. also show task space for planar continuum manipulators (47–49). However, to the best of our knowledge, model-based dynamic task space control has never been shown on a physical three-dimensional soft manipulator.

Previous works that have applied task-space control to physical arms have employed a quasi-static assumption using a locally approximated Jacobian, which is derived either from a model (20, 50) or learned (51–54), to perform tip-follower actuation using local kinematics. This means that these works have not considered dynamic parameters such as gravity and inertia. Instead, they use local kinematics at every step to incrementally move toward the target. This strategy doesn’t allow for quick movements and skillful force application, as the actuation steps must be kept small to prevent the robot from oscillating.

In comparison, rigid manipulators are simpler in their structural composition and their rigid body dynamics can be accurately described (55). Indeed, accurately identified models and extensive control theory allow rigid manipulators to quickly move through their task space (56).
Among these control schemes, operational space control formulates the dynamics in an operational space, and allows for accurate force application and dynamic motion while fulfilling the task given to the controller in the operational space (57, 58). The extension of operational space control to soft robots with their complex structural composition has yet to be shown. This would allow soft robots to move quickly through their task space while performing tasks, similar as to with rigid robots.

**Contributions**

To the best of our knowledge, we describe and implement the first dynamic task space controller on a multi-segment soft manipulator operating in a three-dimensional space. In this way, we enable soft manipulators to perform new velocity-, acceleration-, and force-based tasks, such as throwing, force-compensation, force-application, and even drawing. These tasks were previously not possible due to the quasi-static approach that is often used, which is prone to oscillation, runs very slowly, and does not consider forces. Dynamic control allows soft manipulators to perform the same tasks as rigid manipulators, while the manipulator benefits from the properties that are limited to soft manipulators such as compliance, safety, and an infinitesimal body. During the simulation-to-real transfer, we identify the shortcomings of previous modeling approaches and create additional model parameters that assist in smooth control. Our new model elements reduce the task space error by 59% for an operational space controller. We present a soft manipulator that is able to avoid obstacles without a trajectory planner thanks to accelerations caused by potential fields, and demonstrate a pick-and-place task new to soft robotics. Moreover, our dynamic controller can not only throw objects in a controlled manner, but also skillfully apply force using our manipulator’s end effector and use this to draw.
Results

Simulation to Real World Transfer

Fig. 2: Model errors before and after new model elements were added. These errors directly impact absolute accuracy of control. (A) The offset between the desired polar angle of actuation and the measured polar angle of actuation. (B) A visual representation of the angular error. (C) The radial stiffness profile, which was measured during static gravity and stiffness compensation. (D) A visual representation of the stiffness error.
A precise model is a core part of making a robot controllable. A real-world soft manipulator suffers from unmodeled fabrication imperfections which are not found in simulation. To correct for this, we introduce a phase adjustment element and a magnitude adjustment element, which will be explained in more detail in the section Dynamic Model and its Adjustments. These elements serve the purpose of reducing directional errors resulting from model anisotropy. The effects of the adjustment elements are vital to understanding all of the experiments and are therefore presented first.

We performed static feedforward experiments to characterize the robot’s observed behavior and we compared it with the behavior of an unmodified model. We gave the manipulator pressure inputs of equal magnitude while varying actuation direction from 0° to 360°, and we measured the error between the desired polar angle and the measured polar angle. The experiment was then performed again using our phase adjustment element. The results are plotted in Fig. 2-A. The different offsets are demonstrated in Fig. 2-B, with the blue point representing the desired tip location. The mean absolute error between the desired and measured polar angle for the uncorrected model was 7±6°, and the peak absolute error was 23±1°. The corrected model showed a mean absolute error of 1±1° and a peak absolute error of 5±1°. The error of the corrected model was largest in regions where the uncorrected model’s error was changing quickly. This indicated localized concentrations of instability.

We then performed experiments to determine the effect of our magnitude adjustment element on the accuracy of the model. The manipulator was commanded to compensate for gravity and stiffness at a static polar radius and height with a polar angle varying from 0° to 360°, for which the expected polar radius and the measured polar radius were compared. The experiment was conducted with a static assumption to eliminate possible errors that could have occurred as a result of the inertia, damping, and coriolis effects. The experiment was performed once without any model additions and once with the magnitude adjustment element. The results are
plotted in Fig. 2C. Without model additions, the mean absolute error between the desired and measured radial magnitude was 1.8±1.3cm, and the peak absolute error was 4.7±0.1cm. When the magnitude adjustment element was used, the mean absolute error was 0.3±0.3cm, and the peak absolute error was 1.8±0.1cm.

**Pick-and-Place**

![Image of manipulator](image)

**Fig. 3:** Soft manipulator picking up and dropping a grape in a controlled manner. The grape’s position was assumed to be known. (A) Manipulator traversing the workspace from picking position to dropping position, and dropping the grape in a cup. Motion frames are overlaid with 2.5 second intervals. (B) Manipulator trajectories during the task. (C) Manipulator coordinates plotted against their references.

We examined a core behavior of robotic manipulators in the form of a pick-and-place task, something previously never shown with soft manipulators. The task was performed 7 times to examine repeatability. The manipulator was given a trajectory to a set point, at which a grape was located. The manipulator was then commanded to grab the grape, and given a second
trajectory to a drop-off point over a small container. Finally, the manipulator was made to drop the grape into the container. The trajectories are plotted in Fig.3-B,C. Average tip error during the trials was 1.8cm±0.6cm. This is equivalent to 6.7%±2.2% of total arm length. The task was performed successfully in 6 out of 7 trials, with one failure where the gripper lost grip of the grape while moving towards the container.

**Throwing**

The manipulator was commanded to traverse its task space as fast as possible and to release a gripped object along the way, leading to a throwing behavior. Different objects with varying weights were thrown. We performed the experiment twice for each object: once with our model additions and once with state of the art modeling (45).

When our additions to the model were included, the manipulator threw the objects and stabilized fully. When using the unmodified model, the manipulator was still able to throw objects, but it was unable to stabilize due to a mismatch between the calculated dynamic parameters and the real-world performance. With a quasi-static controller, throwing was not possible due to severe limitation in speed.

**Drawing**

The manipulator was commanded to a position close to a blackboard set up, which was placed inside the task space. A soft gripper was mounted to the manipulator. The gripper grasped a piece of pink chalk, which is shown in Fig.5-B. When it reached the blackboard’s surface, we input a tip force in the direction of the blackboard to the tip. Then, using the controller, we made the manipulator move in a straight line along the blackboard while applying a force in order to draw a line. The end effector followed the trajectory of a line, as is shown in Fig.5-C. The manipulator was able to track the line smoothly. The mean absolute vertical error was 0.34cm.
Fig. 4: The soft manipulator’s stabilization behavior while throwing various objects. All graphs displayed the first 5 seconds of manipulator behavior upon starting the throw. (A) Manipulator throwing tape, shown in the top left corner. The images are spaced in 65ms intervals. (B,C) Trajectories taken by the manipulator when throwing tape weighing 11g using our model and state of the art modeling, respectively. (D,E) Trajectories taken when throwing a gluestick weighing 24g. (F,G) Trajectories taken when throwing electric tape weighing 40g.

The error appeared mostly beneath the desired trajectory, which may result from the shape of the arm’s task space.
Obstacle Avoidance

We placed obstacles in the manipulator’s task space and commanded it to traverse between two task space points. The location of the object was obtained using a motion capture system. We performed this experiment multiple times both with and without active potential fields. Fig. 6-B shows the path taken by the manipulator when it traverses the obstacle course with the potential fields activated. The objects are also marked in the graph. The potential fields pushed the manipulator away from the objects before it could collide with them, leading to a collision-free trajectory. Fig. 6-A demonstrates the manipulator’s trajectory when the potential fields were deactivated. The manipulator traversed the obstacle course without explicit consideration of the objects, leading to collisions. These are marked by stars. On the return path, the manipulator
Fig. 6: A soft manipulator avoiding objects using potential fields. Objects are marked with black circles. (A) The manipulator’s trajectory when potential fields were disabled. Images A1-A6 are all spaced with 1 second intervals. The manipulator collides with the objects multiple times. (B) The manipulator’s trajectory when potential fields were enabled. Images B1-B6 are spaced with 1 second intervals. The manipulator is repelled by the objects and avoids collision. got stuck on one of the objects after a collision and was unable to continue.

**Circular Tracking**

A robot’s ability to respond quickly and accurately to given trajectories is essential in robotic manipulation. In this experiment, we evaluated our controller’s ability to follow a task space trajectory in the shape of a circle.

We gave the manipulator a planar circular trajectory to follow within task space. The radius of the circular trajectory was set at 15cm. A new reference position, velocity, and acceleration
Fig. 7: Comparison of circular tracking with different tip loads. (A) Different controllers performing a circle with 24g tip load, plotted against time. The quasi-static controller requires significantly more time to complete the circle. (B-E) Comparison of dynamic controllers with and without our model additions. The tip weights are 24g, 36g, 54g, 78g, respectively.

were given to the manipulator every 0.1s. The full circle was completed in 8s, corresponding to an average tip speed of 10cm/s. The experiment was completed with four different loads: 24g (gripper only); 36g (gripper and brush); 54g (gripper and glue stick); and 78g (gripper and tape). The experiment was conducted twice for every weight: once with model additions and once without. The movements and paths are shown in Fig.[7] When the model additions were disabled, the controller incorrectly compensated for stiffness and actuated in an offset direction, which interfered greatly with the control performance.
We also compare the operational space controller to a quasi-static controller. The quasi-
static controller uses local kinematics to add a pressure differential to the current pressure. This
controller iteratively approaches the desired target position. A comparison of the performance
of these controllers for a tip weight of 24g is shown in Fig. 7-A. While following a circular tra-
jectory, both the average error and the completion time were measured. The dynamic controller
with model additions was able to complete a circle in 8s with an average error of 2.9-5.4cm,
depending on weight. The dynamic controller without our additions completed the circle in
the same time, with an average error of 7.7-10.7cm. The quasi-static controller took between
154s and 301s, depending on weight, but the average error was reduced to 2.0-2.1cm. This is
a result of the reference trajectory only advancing once error was reduced to sufficient margin.
Therefore, when comparing our dynamic approach to the previous quasi-static approach, error
is increased by 45-170% during dynamic motion while the speed is increased by a factor of of
19.3-37.6.

Applying Force

The formulation of our manipulator’s operational space control enables us to command specific
tip forces. A manipulator’s ability to apply tip force is essential for almost every manipulation
task in which interaction with the environment is required. In this experiment, we evaluated
the accuracy of the force application using our controller, both with and without our model
additions.

We commanded the robot to compensate for gravitational and stiffness torques, making it
stay in a desired position. Using a string, the tip of the manipulator was attached to a load
cell with 0.001N precision. The string was loose before force was applied. The controller then
received a desired tip force input in a single direction, causing the manipulator to pull against the
load cell. The desired forces ranged from 0.5N to 2.5N, with increments of 0.5N. We performed
the experiment twice: once with our model additions and once without. The measured force linearly correlated with the desired force in both experiments. The mean absolute error between the desired and measured force was 0.06±0.03N when we applied forces in the magnitude of 0.5N to 2.5N with model additions. Without model additions, the mean absolute error in the range of 0.5 to 2.5N was 0.28±0.22N. Therefore, the model additions reduced the error by 79%.

**Discussion**

**Conclusion**

The results shown in this work considerably advance the state of soft manipulators. We controlled a soft manipulator on an acceleration level in a task space and were able to apply tip forces. Our manipulator was able to complete a circle a 19.3 to 37.6 times faster than the previous approach for task space control, and it did so with reduced oscillations. While the error during this trajectory was slightly higher than in the quasi-static method, the benefits of increasing speed justify the increase in error, as the robot is able to perform a variety of new velocity- and acceleration-based tasks. The demonstration of pick-and-place in particular shows an advancement towards equalizing the playing field between rigid and soft manipulators. It is also of note that the dynamic method has no theoretical speed limitation and that the shown speeds are simply an example. Therefore, we propose improving dynamic models of soft manipulators in the future rather than focusing on quasi-static methods. Existing approaches to achieving task space control for soft manipulators have limitations in the categories of speed and force application due to a lack of consideration for dynamic parameters. The use of local tip kinematics means that these manipulators cannot apply force, as the required torque cannot be estimated without a dynamic full-body model. Moreover, the previous quasi-static assumption limits a manipulator’s movements, and makes velocity and acceleration impossible to control.

We have shown that, with a dynamic model, rapid and accurate motions and force applica-
tion are possible for soft manipulators in a task space. Thus, our approach has greatly improved control of physical soft manipulators.

Furthermore, we can use rigid robot control algorithms to solve soft manipulator problems. Control methods that have already been researched for rigid robotic manipulators can be implemented with few changes in a constant curvature model. Our model additions ensure that these algorithms are executed seamlessly. We have shown this by implementing potential field avoidance, which was originally developed for rigid manipulators. We were also able to apply tip forces precisely, allowing us to use the manipulator to draw a line.

Soft manipulators’ inherent safety, compliance, and high degrees of freedom make them more interesting than rigid manipulators. These properties allow soft manipulators to achieve certain types of behavior that are impossible or inconvenient for rigid manipulators. Therefore, this work lays the groundwork for an exciting future within the field of robotic manipulation.

Limitations and Future Work

While we have demonstrated that dynamic control for soft manipulators is very useful, the methods do still suffer from some limitations. For example, our potential field approach for obstacle avoidance was investigated only for segments tips. The effect of the continuum body on the potential field approach was therefore not considered. Potential fields may also introduce instability at their activation boundary, which is highly dependent on parameter tuning.

The piecewise constant curvature modeling approach is also limited. While it acts as a decent approximation of the robot’s curvature properties, the actual robot does not display a constant curvature. Instead, it bends in a nonlinear curved fashion. Additionally, an assumption is made that neither actuation nor stiffness vary with curvature intensity, which is also an approximation. The real robot is nonlinear with regards to curvature as a result of variation of cross section during actuation.
Our model correction approach could also be improved. We add a heuristically determined error correction term to our actuation and stiffness matrices, as the exact source of the errors is unknown. These error correction terms work around unknown modeling errors and require system identification of every manipulator we wish to control.

Finally, we rely heavily on a motion capture system to gather both the robot configuration and the location of objects within task space. This is limiting due to the large amount of setup and cost required for a motion capture system. It also makes the robot impractical to be used outside of a laboratory setting.

We propose the following solutions to the aforementioned problems. Continuum body interaction with potential fields could be investigated in a first step by placing additional control points along the continuum sections, in a similar way to how control points are placed along rigid links in rigid robotics. Smooth transitory phases could be introduced to the activation boundary in the form of activation functions to ease the transitory behavior.

Another way to possibly overcome the limitations in terms of modeling would be to replace the extra modeling layers with a model that accounts for unknown parameters, which cause the offset between the model and reality. First, the source of error would need to be correctly identified. This could be the anisotropic stiffness or differences in the wall thickness and silicone casting quality. Moreover, supplementing the model with a finite element model of the manipulator could lead to a better state estimation, as the elastic body would be more precisely modeled.

The limitation of PCC modeling’s inaccurate forward kinematics has already been investigated for soft robotics with new methods such as Cosserat rods (59–63). However, non-constant curvature approaches with Cosserat are difficult to model from a dynamic perspective. In future works, improved kinematic modeling approaches could be used in conjunction with a dynamic model (obtained from either an Augmented Rigid Arm or a Lagrangian) to combine the dynamic
properties of our arm with better kinematic modeling approaches. The nonlinear actuation and stiffness behavior could be modeled in order to improve model accuracy beyond what is currently possible. This could lead to an arm that is possible to control even in a model predictive manner.

The dependency on motion capture systems may be addressed using cheaper sensor options such as embedded bend sensors (64) or inertial measurement units (65) to estimate the state of the manipulator. Objects within the task space could be found using a comparatively cheap depth sensing camera and previously developed object detection methods (66) to enable an integrated system without dependency on bulky and expensive motion capture setups.

**Materials and Methods**

**System Architecture**

The flow diagram of the full system is shown in Fig. [8] The soft manipulator used in the experiments is the *Soft Proprioceptive Arm (SoPrA)* manipulator that was introduced by Toshimitsu et al. (64). The SoPrA manipulator is cast in silicone, and it is fiber-reinforced to prevent bubbling. The arm’s length is 0.27m and it weighs 299g in total. The gripper weighs 24g and is included in the arm’s total weight. The manipulator has two continuum segments. Each segment contains 3 actuation chambers. A soft silicone gripper is attached to the tip of the manipulator. Silicone tubing runs through the arm to actuate the chambers and gripper. Spherical reflective markers are mounted at the base of the arm and around the tip of each actuated segment. Task-space objects are also marked with reflective markers. The motion capture system consists of 8 infrared cameras, all of which are mounted around the manipulator and connected to a laptop that runs motion capture software [Miqus M3, Qualisys AB]. The motion capture data is used to estimate the manipulator’s curvature and velocity as well as the locations of the objects. The potential fields are derived from the

\[ \text{potential fields} = f(\text{curvature, velocity, locations}) \]
Fig. 8: The signal flow of the complete system. The router acts as an intermediary between the controller on the laptop and the physical system. Motion capture detects the manipulator pose, which is used to determine dynamic parameters such as gravity and inertia. The pose and parameters are used with the operational space formulation to generate a pressure signal for the manipulator. The pressure signal is then sent to the valve array, which internally regulates pressure to the desired value.
objects’ locations, and the resulting manipulator acceleration is computed. The controller then uses the location, velocity, and desired acceleration to calculate the desired pressure, which is then output to the valve system. The valve system is able to output up to 2 bars of pressure, but for this work we never exceeded 600mbar to avoid ruptures in the manipulator. The desired pressures are output to 7 channels (3 for each of the two segments and 1 for the gripper) and regulated with an internal proportional-integral-derivative controller in the valve system.

**Dynamic Model and its Adjustments**

The manipulator is split into \( N_{\text{seg}} \) controllable segments (\( N_{\text{seg}} = 2 \) for the experiments). According to the piecewise constant curvature approach (67), the curvature of a single segment is constant. Traditionally, PCC segments are described by the variables \( \phi \) and \( \theta \), which describe the angle in plane of bending and curvature, respectively. We opt for the method used by Toshimitsu et al. (64), which uses the following parameters: \( \theta_x = \theta \cos(\phi) \) and \( \theta_y = \theta \sin(\phi) \). These parameters describe the curvature in the \( x \) and \( y \) direction, respectively. They were chosen for easier understanding and identification of the dynamic equation matrices. Using our coordinates, we can describe the dynamic model of the soft manipulator:

\[
A p_{xy} + J^T f_{\text{ext}} = B(q) \ddot{q} + c(q, \dot{q}) + g(q) + K q + D \dot{q}
\]

where \( A \) maps the pressure \( p_{xy} \) to generalized torques, \( J \) is the Jacobian, and \( f_{\text{ext}} \) is the force at the tip of the manipulator. \( q \) is the curvature-space parameterization of the manipulator and has a vector size of \( 2N_{\text{seg}} \). The corresponding values for each segment are \( \theta_x \) and \( \theta_y \). The dynamic parameters \( B(q) \), \( c(q, \dot{q}) \), and \( g(q) \) correspond to the inertia matrix, the coriolis/centripetal torque vector, and the gravity torque vector, respectively.

To calculate the dynamic parameters \( B(q) \) and \( c(q, \dot{q}) \) for the PCC model, we create an augmented rigid body model like the one that was originally proposed by Della Santina et al. (18).
We extended this model to the 3rd dimension in a previous work (21). This augmented rigid body model consists of 5 joints per PCC section. The configuration of the joints is computed from the PCC coordinates that are used in Toshimitsu et al. (64). By representing each PCC element as a rigid body model, we are able to use the standard robotics library Drake (68) to compute the dynamic parameters of the rigid body model in rigid body coordinates $\xi$. These coordinates are then transformed back into configuration space of the PCC model $q$ by using the relation between $q$ and the augmented rigid body’s joint angles $\xi$. This gives us the dynamic parameters in PCC coordinate parameterization, which can be used for control.

Based on a first-order approximation, we assumed that the bending stiffness of the soft manipulator $K$ was constant, regardless of bending state in the previously proposed model (64). However, the bending magnitude varied by as much as 33.6% depending on the angle of bending. This likely results from both the cross-sectional shape of the robot, which differs from that of the model due to manufacturing limitations, and the first-order approximation used in Toshimitsu et al. (64), which does not hold up at higher degrees of bending. Additionally, fabrication errors and tapering cause the stiffness to vary between the chambers. This introduces a disparity between the model and reality: the torque inputs are of equal magnitude, but a different direction of actuation causes different magnitudes of deformation, despite the model predicting an equal deformation.

To compensate for this, we introduce the magnitude adjustment element into the stiffness matrix $K$. We define the magnitude adjustment element as a function of the angle of bending plane, $\phi$. To identify the variant parameter, we begin by assuming an invariant stiffness and identify this invariant term with a constant stiffness fit to quasi-static poses. Then, the arm is actuated with desired torques of equal magnitude for all angles in a rotating motion. Due to the aforementioned stiffness differences, the amount of bending deformation varies according to the angle of the bending plane $\phi$. The radial intensity can then be fitted to a function $f(\phi(q))$. It
was heuristically determined that fitting a 3rd degree polynomial to each 120° sweep covered by the three pneumatic chambers would provide a satisfactory characterization of the radial error. We pre-multiply the stiffness matrix $K$ with the inverse of the radial magnitude function $\frac{1}{f(\phi(q))}$ to obtain the angle-dependant stiffness matrix $K(\phi(q))$.

There is also an error in the desired plane of bending $\phi_{des}$ and in the one observed, $\phi_{meas}$. This error is corrected with a phase adjustment element. Again, we determine the error function, which can be expressed as $g(\phi_{des}) = \phi_{des} - \phi_{meas}$, by inputting feedforward pressures for $\phi \in [0, 360)$. Similar to the magnitude adjustment element, this error can be described with a 3rd degree polynomial with regard to the angle $\phi$. We compensate for this error by adding an extra layer during actuation: by offsetting the input’s direction with the angular error $\Delta \phi$, we can obtain an actuation signal that will produce the desired angle. We extract the intended direction of actuation from $p_{xy}$ and then rotate $p_{xy}$ by the angle corresponding to the error function $g(\phi)$. This way, the pressure’s magnitude remains untouched. This transformation can be expressed as a rotational matrix $R(-g(\phi(p_{xy})))$, which we directly include in $A_{xy}$. This results in an actuation matrix that varies in behavior dependent on configuration: $A_{xy}(p_{xy}) = R(-g(\phi(p_{xy})))A_{xy}$.

**Controller Design**

In this section, we detail how we have applied a force-based operational space controller to the soft robotic arm so that it can take advantage of its continuous body to fulfill tasks.

The command given to our operational space control law is a proportional-derivative controller in the form of

$$\ddot{x}_{ref} = k_p(x_{des} - x) + k_d(\dot{x}_{des} - \dot{x}) + \ddot{x}_{des} + \ddot{x}_{pot} \quad (2)$$

where $x$ is the tip position in Cartesian coordinates, $x_{des}$ is the desired tip position, and $k_p$ and $k_d$ are gain terms for the position and velocity, respectively. $\ddot{x}_{pot}$ is the acceleration caused
by potential fields. Additionally, the gains are made to saturate after reaching a set magnitude to prevent control inputs which could lead to unstable behavior.

The potential field acceleration used for avoiding obstacles, $\ddot{x}_{pot}$ is obtained as follows:

$$
\ddot{x}_{pot} = \begin{cases} 
k_{pot} \frac{1}{|\rho| - r_0} \frac{\rho}{|\rho|} & \text{if } |\rho| < r_m \\
0 & \text{otherwise}
\end{cases}
$$

(3)

where $k_{pot}$ is the gain of the potential field, $\rho$ is the Cartesian vector from the center of the potential field to the tip of the manipulator. $r_m$ is the cutoff radius of the potential field, after which it no longer affects the manipulator, and $r_0$ is the radius of the object that causes the potential field.

Objects are approximated as perfect spheres. As the tip of the manipulator approaches the object, the potential field acceleration $\ddot{x}_{pot}$, which is normal to the object, increases as the manipulator approaches the object. This leads to the tip of the manipulator being deflected away from the object. Similar to control gains, $\ddot{x}_{pot}$ saturates to prevent unstable behavior.

Having now obtained the reference acceleration $\ddot{x}_{ref}$, we perform a sequence of transformations: acceleration to a force in operational space, force to generalized torque, and then generalized torque to pressure. We thereby make use of the operational space formulation (57,58):

$$
f_{ref} = B_{op} \ddot{x}_{ref} + f_{supp}
$$

$$
\tau_{ref} = J^T f_{ref} + g(q) + c(q, \dot{q}) + K(q)q + D\dot{q} + (I - J^T J^T) \tau_{null}
$$

(4)

$$
p = A(q)^+ \tau_{ref}
$$

where $B_{op}$ is the operational space inertia matrix ($(JB^{-1}J^T)^{-1}$) and $f_{supp}$ is a vector that contains supplementary forces such as the desired tip force. $(I - J^T J^T) \tau_{null}$ is the nullspace term, which utilizes redundant degrees of freedom to perform secondary tasks. In our work, we use the nullspace input $\tau_{null} = -\alpha q - \beta \dot{q}$, which pushes the manipulator to the straightest possible configuration and dampens oscillations. $\alpha$ and $\beta$ are the straightness and damping gains, respectively.
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Author Contributions

O.F., Y.T. and R.K.K. conceptualized the outline for this work. O.F. added model elements and identified parameters, as well as performing experiments. O.F. and Y.T. developed the control software for the manipulator and manufactured the manipulators used for experiments. O.F., A.K. and Y.T. performed system identification for the manipulator. O.F., Y.T., A.K. and R.K.K. wrote the paper. R.K.K. oversaw the project as a whole and directed the research.

Competing Interests

The authors declare that they have no competing financial interests.

Data and materials availability

Contact O.F. for source code and other materials.
Supplementary Materials

Videos

A video is included showcasing the manipulator performing the experiments discussed in this paper. Commentary is provided and discussion points are overlaid. A second video with all experimental data is also included. This video shows raw experimental data without commentary.

Movie 1: Overview of experiments performed in this work. Commentary is provided. Pick-and-place, tracking, throwing, drawing and obstacle avoidance are shown.

Movie S1: Raw video of experiments performed in this work. More examples of tracking, throwing and obstacle avoidance are shown.

Force Application

The setup and graphs for our force application experiment are pictured in Fig. S1. The manipulator compensates for gravity in Fig. S1-A and pulls against the load cell in Fig. S1-B. The measured force is plotted against the desired force in Fig. S1-C,D. With enabled model additions, the relation appears linear and shows little error. Without our model additions, the accuracy is lowered and the relation between desired and measured is no longer as expected.
**Fig. S1**: Force experiment setup and graphs. (A) Manipulator compensating for gravity, string is not under tension. (B) Manipulator pulling against the load cell, string is under tension. (C) Measured force plotted against desired force when model corrections are used. (D) Measured force plotted against desired force when model corrections are not used.

**Throwing**

Fig. S2 shows the trajectory during a throw with enabled model additions. The line color changes based on speed measured during the interval. Maximum speed is reached approximately three quarters into the throw. The manipulator tip moves at 1 m/s in this interval. With model additions, it is able to stabilize after reaching this velocity.
Fig. S 2: Manipulator speed during a throw. (A) Manipulator performing a throw. (B) Speed of manipulator tip during a throw. Speed is plotted according to the color bar on the right, where the unit for speed is m/s.

Tracking

Table 1 shows quantitative data of the trajectory tracking experiments performed. Model Add. refers to our model additions.

Manipulator Task Space

The task space of the manipulator used in this work is shown in Fig. S3, which was obtained with a Monte Carlo analysis. The arm was given random pressure inputs to follow, and tip position was logged. The task space resembles a half-moon, extending out to a maximum radius of 0.16m. The height varies between -0.27cm and -0.21cm. Half of the random points lie between the heights -0.27cm and -0.25cm, after which the amount of points decay inversely with regards to height.
Table 1: Comparison of tracking error, completion time, and stability for different model approaches and weights. Model Add. refers to our modeling improvements described here.

| Weight | Model Approach | Model Add. | Completion Time | Stability | Average Error |
|--------|----------------|------------|-----------------|-----------|---------------|
| 24g    | Quasi-Static   | -          | 154s            | Low       | 2.0cm         |
| 24g    | Dynamic        | No         | 8s              | Medium    | 7.7cm         |
| 24g    | Dynamic        | Yes (Ours) | 8s              | High      | 2.9cm         |
| 36g    | Quasi-Static   | -          | 217s            | Low       | 2.0cm         |
| 36g    | Dynamic        | No         | 8s              | Medium    | 10.4cm        |
| 36g    | Dynamic        | Yes (Ours) | 8s              | High      | 3.5cm         |
| 54g    | Quasi-Static   | -          | 264s            | Low       | 2.0cm         |
| 54g    | Dynamic        | No         | 8s              | Low       | 9.7cm         |
| 54g    | Dynamic        | Yes (Ours) | 8s              | High      | 4.2cm         |
| 78g    | Quasi-Static   | -          | 301s            | Low       | 2.0cm         |
| 78g    | Dynamic        | No         | 8s              | Low       | 10.7cm        |
| 78g    | Dynamic        | Yes (Ours) | 8s              | High      | 5.4cm         |

Controller and Model Parameters

Parameters used in this work are listed in Table 2. Vectors start at the value used for the topmost segment.
Table 2: Parameters used within the frame of this work. Listed are physical parameters from which dynamic parameters are calculated, controller gains and potential field parameters.

| Model                  |                  |
|------------------------|------------------|
| Lengths                | {0.135, 0.135} m|
| Masses                 | {0.180, 0.105} kg|
| Shear Moduli           | {40686, 59116} Pa|
| Drag Coefficients      | {28000, 8000}    |

| Controller             |                  |
|------------------------|------------------|
| $k_p$                  | 170              |
| $k_d$                  | 5.5              |
| $\alpha$              | 0.001            |
| $\beta$               | 0.0001           |
| Threshold              | 0.05 m           |

| Potential Fields       |                  |
|------------------------|------------------|
| $k_{pot}$              | 0.11 m           |
| Object Radius          | 0.045 m          |
| Cutoff                 | 0.5 m            |
| Threshold              | 15 m/s$^2$       |
Fig. S 3: Monte carlo task space analysis of the soft robotic arm.

Physical Design and Modeling of the Manipulator

Fig. S 4: Model Aspects of the manipulator. (A) Design of the SoPrA soft manipulator. Modular silicone chambers are connected with resin pieces. (B) Augmented Rigid Arm joint configuration, using 3 rotational joints and 2 prismatic joints. (C) Two links of the piecewise constant curvature model. Images adapted from (64).
A brief overview of the manipulator’s physical design is shown in Fig. S4-A. Each segment consists of 3 identical segments, all of which are cast in silicone *Dragonskin 10* and wrapped in fiber. The center of the manipulator contains air tunnels to allow pressures to reach segments further down. An optional flex sensor can be added to the core of the segment for extra sensor readouts. Multiple segments are connected using resin connectors which redirect the pneumatic air tunnels to the segment below. The configuration of the joints of the Augmented Rigid Arm model is shown in Fig. S4-B. The rigid robotic links enable the mass point on the middle of the segment to be aligned approximately correctly, while positioning the end effector in the correct position. Fig. S4-C shows an explanation for the piecewise constant curvature (PCC) modeling approach. Two PCC segments are shown, therefore modeling the curvature of two manipulator segments. The angles used in the figure are traditional PCC coordinates and vary slightly from the coordinates used in this work.

**System Identification**

Multiple characterizing functions were created to autonomously determine the optimal parameters of the arm. Their workings and usage are described here.

**calcK(int segment, int directions, int vertical)**

This function fits a new $K$ value for segment `segment`, with `segment 0` being the uppermost. To do this, the segment will actuate in `directions` equispaced polar directions. For each direction, the segment actuates at `vertical` equispaced pressures between 0 and the maximum allowed pressure, for a total of `directions`*`vertical` data points. The function waits 10 seconds after each actuation to ensure quasi-static condition. It then gathers curvature, pressure and gravity vectors and moves on to the next position. After completing all positions, a least squares fit is performed as following:
\[ p_{rest} = p - A^{-1}g(q) \]
\[ p_K = A^{-1}(K_{previous})q \]
\[ k_{scalar} = p^+_K p_{rest} \]
\[ K_{new} = k_{scalar} K_{previous} \]

**logRadialPressureDist(int segment, std::string filename)**

This function identifies the proposed *Model Additions*. Again, the parameter `segment` identifies the segment, although in practice 0 was always used as the bottom segment’s errors are negligible when compared with the top segment. The `filename` parameter identifies the name of the file the result should be output to. The function feeds forward pressures at an equal magnitude, with the polar angle varying by 1° per step. The function waits one second between new pressure inputs to achieve quasi-static behavior. The reason this function sleeps for less time than the `calcK` function is because of the small input differences, quasi-static conditions are possible also with higher frequency of input. At every step, the pressure, expected angle as well as measured angle are noted.

After all data points have been gathered, the function performs three least-squares fits to third degree polynomials describing angular error, one every 120°.

\[ g = \phi_{des} - \phi_{meas} \]

\[
\begin{pmatrix}
1 & 1 & 1 & 1 \\
8 & 4 & 2 & 1 \\
27 & 9 & 3 & 1 \\
\vdots & \vdots & \vdots & \vdots \\
\phi^3 & \phi^2 & \phi & 1
\end{pmatrix}^+ \quad g = \begin{pmatrix}
\alpha_3 \\
\alpha_2 \\
\alpha_1 \\
\alpha_0
\end{pmatrix}
\]
Likewise, for the radial profile, the same movement is performed again after applying the angular corrections. The radial profile is used as an approximation of the nonlinear behavior of $K$. This time, the radius ($r = \sqrt{x^2 + y^2}$) is logged. Again, we fit to 3 polynomials of 3rd degree, each covering a $120^\circ$ sweep. These polynomials are considered the error function, with which we then norm $K(\phi(q))$ by using the initial value as desired.
Motion Capture Setup

**Fig. S 5:** Motion Capture System Setup. Left: Camera Localization in *Qualisys Track Manager*. Right: Physical Camera Locations.

The motion capture setup used 9 cameras, of which 8 were IR cameras and 1 was a normal RGB camera. The configuration of the cameras is shown in Fig. S5. Two cameras are mounted in the rear bottom of the motion capture setup. The other 7 are mounted at various points in the front of the motion capture setup, pointing inward. The frames being captured are also shown in Fig. S5. The red frame is the static base mount of the manipulator. The green frame is the tip of the first segment, and the blue frame is the tip of the second segment. The frames are aligned such that the x axis points out of the motion capture setup and the z axis points downward. The frame transformations needed for this are a rotation of 90° around y followed by a rotation of 180° around z. The settings within the motion capture software (*Qualisys Track Manager*) were a capture rate of 100Hz with 84μs exposure time and a 90% marker threshold.
**Manufacturing of Manipulator**

Manufacturing of the used arms was performed by following the principles shown in (64). The individual chambers were cast using *Dragonskin 10* using a core mold and a surrounding chamber, as seen in Fig. S6-A. The chambers were cured for a minimum of 2 hours at 50°C. The chambers were then removed from their mold with the assistance of isopropanol. The cores remained inside the chambers after this step. Fiber was then wrapped around each individual chamber in a crossing pattern. This is illustrated in Fig. S6-B. The chambers were then cast together, again using *Dragonskin 10*. Two chambers were placed inside 3D-printed alignment pieces and a core was cast between them. Channels for pressure were created in the core using sticks, as seen in Fig. S6-C. Then, the third chamber was placed on top of the core into the alignment pieces and cast in place. The three-chamber segment was then coated with *Dragonskin 10* and wrapped in a mold, shown in S6-D. The coating ensured that the fiber wrapping does not slip, as slipping would risk the silicone expanding plastically and damaging the manipulator.

After the segment was finished, the cores were removed and plugs were cast into their place to seal off the chambers, as seen in Fig. S6-E. Resin pieces were printed with the *Formlabs Form 3* and cast to have an outside layer of *Dragonskin 30*. These resin pieces have channels to connect the pressure channels of upper segments correctly to the bottom segments. They can be seen in Fig. S6-F. Finally, segments and resin pieces are assembled into a total manipulator, as seen in Fig. S6-G. The resin pieces’ inlets and outlets are connected to the silicone segments and cast in *Dragonskin 10* to hold the manipulator together.

The silicone for all parts was treated with *SloJo* and vacuumed until no bubbles were present. All molds were printed using PLA and sprayed with mold release prior to casting.
Controller Loop Code

The control loop is made to run at a constant frequency with a `Rate` function. The control loop runs in its own thread. The core implementation of the control loop is as follows:

```cpp
cc->get_state(state); //update state vector with latest mocap frame
stm->updateState(state); //update soft trunk model with state vector

if ((x_ref - x).norm() > threshold)
    ddx_des = ddx_ref + kp*(x_ref - x).normalized()*threshold + kd*(dx_ref - dx);
else
    ddx_des = ddx_ref + kp*(x_ref - x) + kd*(dx_ref - dx);

for (int i=0; i<potfields.size(); i++)
    potfields[i].set_position(get_object(i)); //update potfield position
    ddx_des += potfields[i].get_ddx(); //add ddx from potfield

for (int i=0; i<singularity_order(J); i++)
    handle_singularity(J,i);

B_op = (J*stm->B.inverse()*J.transpose()).inverse();
J_inv = stm->B.inverse()*J.transpose()*B_op;
f = B_op*ddx_des;
f(2) += loadAttached + 0.24*gripperAttached; //add z direction tip forces

tau_null = -beta*state.dq -alpha*state.q;
tau_ref = J.transpose()*f + (MatrixXd::Identity(q_size, q_size)
    - J.transpose()*J_inv.transpose())*tau_null;
p = stm->pseudo2real(stm->A_pseudo.inverse()*tau_ref
    + gravitycompensate(state));
actuate(p);
```

where `cc` is the state estimator, `stm` is the model containing dynamic parameters, `pseudo2real` is a function which maps `x,y` pressures to three chambers and `gravitycompensate` is a vector containing all dynamic compensation terms.

Various parameters can be changed from outside of the control thread. Here is a header file containing these.
**Runtime Performance**

The greatest bottleneck of the runtime performance is by far the call that updates the Augmented Rigid Arm model and calculates new dynamic parameters based on the rigid equivalent’s pose. The usage of *Drake* (68) within this function to calculate the dynamic parameters of the Augmented Rigid Arm equivalent takes on average $0.014\pm0.01$, with times of up to $0.061s$ observed when executing on our test setup’s laptop. Optimizing or replacing this function is therefore critical to enabling the controller to run smoothly at higher frequencies.
Fig. S 6: Manufacturing process of the SoPrA manipulator. (A) Molds used for a single chamber. (B) Wrapping a chamber with fiber. (C) Casting chambers together to form a segment. (D) Coating a segment with silicone to prevent fiber slippage. (E) Plugging the chambers. (F) Curing a resin connector piece. (G) Assembling the full manipulator.