Nonlocal advantage of quantum coherence in high-dimensional states

Ming-Liang Hu\textsuperscript{1,2,*} and Heng Fan\textsuperscript{3,4,†}

\textsuperscript{1}School of Science, Xi’an University of Posts and Telecommunications, Xi’an 710121, China
\textsuperscript{2}Institute of Physics, Chinese Academy of Sciences, Beijing 100190, China
\textsuperscript{3}School of Physical Sciences, University of Chinese Academy of Sciences, Beijing 100190, China
\textsuperscript{4}Collaborative Innovation Center of Quantum Matter, Beijing 100190, China

By local measurements on party A of a system AB and classical communication between its two parties, one can achieve a nonlocal advantage of quantum coherence (NAQC) on party B. For the $I_1$ norm of coherence and the relative entropy of coherence, we generalized the framework of NAQC for two qubits and derived the criteria which capture NAQC in the $(d \times d)$-dimensional states when $d$ is a power of a prime. We also presented a new framework for formulating NAQC, and showed through explicit examples its capacity on capturing the NAQC states. Moreover, we proved that any bipartite state with NAQC is quantum entangled.

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I. INTRODUCTION

Coherence is a basic notion in quantum theory \cite{1}. It stems from the superpositions of a set of basis states, and intuitively, a state is said to be coherent provided that there are nonzero elements in the nondiagonal position of its density matrix representation. Coherence is thought to be fundamental and crucial as it not only can deepen our understanding about the essence of quantum theory, but also can be used as a physical resource for developing many fascinating information processing tasks which outperform their classical counterparts \cite{2,3}.

While being widely accepted as a feature unique from classical physics, it has only recently been suggested to characterize coherence in a quantitative way. Baumgratz \textit{et al.} formulated the framework for defining a faithful coherence measure, and proposed the $I_1$ norm of coherence and relative entropy of coherence \cite{4}. Inspired by this, a number of other coherence measures have also been proposed \cite{5–11}. This sets the stage for a quantitative study of coherence, with much progress being achieved in recent years. Some notable ones include their interpretation from an operational perspective \cite{12,13}, the cohering and decohering power of a channel \cite{15–17}, the frozen phenomenon of coherence in noisy environments \cite{18,19}, and how coherent states work for improving efficiency of certain quantum information processing tasks \cite{20,21–23}. There are also works concentrating on the pivotal role of coherence in capturing the wave nature of a system \cite{23,24} and the maximum coherence in the optimal basis \cite{25,26}.

Quantum coherence could also be linked to quantum correlations, although they were defined in different scenarios (the former is for the single-partite states, while the latter is for the bipartite and multipartite states) and capture different aspects of the quantumness of a system. The essence for this intimate connection may be their origin of superposition principle, thus it is quite natural to ask how can one resource be converted to the other \cite{22}. In fact, inspired by the recent progresses on the resource theory of coherence, the interplay between coherence and quantum correlations has attracted people’s increasing interest, e.g., the relations between coherence and entanglement \cite{27} or between coherence and discord-like quantum correlations \cite{28,31} have already been established.

Apart from the aforementioned progresses, the interplay of coherence and quantum correlations can also be identified by scrutinizing the steered local coherence at one part of a bipartite system, while some works have been accomplished in this direction \cite{31–34}. In particular, Mondal \textit{et al.} proposed to think about steerability of the local coherence through a game between two players, Alice and Bob \cite{33}. By concentrating on the two-qubit system $AB$, they showed that by local operations on party A and classical communication between the two parties, the conditional states of $B$ can achieve a nonlocal advantage of quantum coherence (NAQC). It is then natural to quest whether a general bipartite state can also achieve the NAQC. We consider in this paper such a problem. We first generalize the framework for two qubits to high-dimensional states, then present a new framework for formulating the NAQC. We will also show that for any bipartite state that can achieve a NAQC, one is sure that it is quantum entangled.

II. MEASURES OF COHERENCE

First, we recall how to quantify coherence in a state. In general, the starting point for such a quantification is the identification of incoherent states and incoherent operations. Within the framework established by Baumgratz \textit{et al.} \cite{4}, a state is said to be incoherent if it is diagonal in the given reference basis $|\psi\rangle$, and the incoherent operations are those which map the incoherent states into incoherent states. This is in direct analogy to the resource theory of entanglement \cite{5} and quantum discord \cite{36}. Starting from this framework, Baumgratz \textit{et al.} \cite{4} introduced the defining conditions for a faithful coherence measure, and proposed to define measures of coherence as a distance to the closest incoherent states. They also defined the $I_1$ norm of coherence and relative entropy of coherence, which are given by

$$C_I(\rho) = \sum_{i \neq j} |\langle i | \rho | j \rangle|^2, \quad C_{re}(\rho) = S(\rho_{\text{diag}}) - S(\rho),$$

(1)
where \(S(\rho)\) is the von Neumann entropy of \(\rho\), and \(\rho_{\text{diag}}\) is the diagonal part of \(\rho\). A relation between \(C_l(\rho)\) and \(C_{se}(\rho)\) was established in Ref. \([37]\). Moreover, \(C_{se}(\rho)\) equals to the optimal rate of the distilled maximally coherent states by incoherent operations in the asymptotic limit of many copies of \(\rho\) \([9]\), and this endows it with an actual meaning.

Quantum coherence in a state \(\rho\) can also be quantified from other perspectives. In fact, most of the recently proposed measures were based on the framework of Baumgratz et al. \([4]\), with however the different distance measures of states being adopted. Of course, there are also coherence measures which were defined by relaxing the defining conditions or by redefining the free operations. See Refs. \([2, 3]\) for a review of these coherence measures.

### III. NAQC in High-Dimensional States

For two-qubit states, the criteria for capturing NAQC were established in Ref. \([33]\). Here, we derive the criterion for capturing NAQC in the \((d \times d)\)-dimensional states, with \(d\) being any power of a prime. Our starting point is the complementarity relation of coherence under mutually unbiased bases \([38]\). It states that for single-partite state \(\rho\) of dimension \(d\), we have

\[
\sum_{j=1}^{d+1} C^A_i(\rho) \leq C^m, \tag{2}
\]

where \(C^A_i(\rho)\) denotes any faithful coherence measure defined in the reference basis spanned by the eigenvectors of \(A_j\), and \(C^m\) is a state-independent upper bound that cannot be exceed by any \(\rho\). Moreover, \(\{A_j\}\) denotes the set of mutually unbiased observables, and by saying two observables are mutually unbiased, we mean that the bases comprising their eigenvectors are mutually unbiased \([39, 40]\).

Now, we extend the framework of NAQC for the two-qubit states \([33]\) to a more general scenario. Without loss of generality, we suppose Alice and Bob share a \((d \times d)\)-dimensional state \(\rho_{AB}\). Before the game commences, they agree on the set of measurements \(\{A_i\}\). Alice then carries out one of the measurements chosen at random and informs Bob of her choice \(A_i\) and outcomes \(a\). Bob’s task is to measure the coherence of his conditional states in the basis spanned by the eigenvectors of all possible \(A_j\) other than \(j \neq i\). After Alice performing all the possible measurements \(\{A_i\}\) with equal probability, Bob’s coherence averaged over all of his possible conditional states and all of his allowable bases is given by

\[
C^m(\rho_{AB}) = \frac{1}{d} \sum_{i,j} p(a|A_i) C^A_i(\rho_{B|A_i}^A), \tag{3}
\]

where \(p(a|A_i)\) is the probability for Alice’s measurement outcome \(a\) when measuring \(A_i\), and \(\rho_{B|A_i}^A\) is the conditional state of \(B\) (see Appendix A).

Based on the protocol stated above, one can establish a general criterion for capturing NAQC in \(\rho_{AB}\). It reads

\[
C^m(\rho_{AB}) > C^m, \tag{4}
\]

which is an immediate result of Eq. \((2)\) as the bound \(C^m\) is not achievable for any single-partite state of dimension \(d\). In the following, we will say \(\rho_{AB}\) an NAQC state if it obeys Eq. \((4)\).

Using the criterion \((4)\), one can prove that there are no separable states that can achieve the NAQC. This is because for any \(\rho_{\text{sep}} = \sum_k q_k |a_k\rangle \langle a_k|\), the conditional state of Bob is given by (see Appendix A)

\[
\rho_{B|A_i} = \frac{\sum_k q_k p_k(a|A_i) \rho_{B|A_i}^A}{p(a|A_i)}, \tag{5}
\]

where \(p_k(a|A_i) = \langle \phi_k | \rho_{A_i}^A | \phi_k \rangle\), and \(p(a|A_i) = \sum_k q_k p_k(a|A_i)\). Then by using the convexity of the coherence measure \(C\), one can obtain

\[
C^m(\rho_{\text{sep}}) = \frac{1}{d} \sum_{i,j} p(a|A_i) C^A_i(\rho_{B|A_i}^A) = \frac{1}{d} \sum_{k,l} q_k p_k(a|A_i) C^A_i(\rho_{B|A_i}^A) \leq C^m, \tag{6}
\]

where the second equality is due to \(\sum_k p_k(a|A_i) = 1\) (\(\forall i, k\)), and the last inequality is because \(\rho_{B|A_i}^A\) may not be the optimal state for saturating Eq. \((2)\). This completes the proof.

As for a bipartite entangled state \(\rho_{AB}\), it is also possible that \(C^m(\rho_{AB}) < C^m\), while Eq. \((6)\) implies that all the \(\rho_{AB}\) with NAQC are entangled, one may recognize what the NAQC captures as a kind of quantum correlation in \(\rho_{AB}\) which is stronger than quantum entanglement.

Similar to the two-qubit case, the bound \(C^m\) obtained with different coherence measures may be different. The violation of any one of them by Bob’s conditional states implies the existence of NAQC in \(\rho_{AB}\). That is, Eq. \((4)\) provides a sufficient coherence steering criterion. To be explicit, we consider in the following two faithful coherence measures, i.e., the \(l_1\) norm of coherence and the relative entropy of coherence.

#### A. \(l_1\) norm of coherence

We denote by \(C^A_{l_1}(\rho)\) for the \(l_1\) norm of coherence defined in the basis spanned by eigenvectors of \(A_j\). Then for any single-partite state \(\rho\) of dimension \(d\), we always have \([38]\)

\[
C^A_{l_1}(\rho) \leq \sqrt{d(d-1)} \sqrt{\left(P(\rho) - P(A_j|\rho)\right)}, \tag{7}
\]

where \(P(\rho) = \text{tr} \rho^2\), \(P(A_j|\rho) = \sum_i \langle a_j | \rho | a_i \rangle^2\), and \(\{|a_j\}_{i=1}^d\) is the eigenvectors of \(A_j\). By combining this equation with the mean inequality (i.e., the arithmetic mean of a list of nonnegative real numbers is not larger than the quadratic mean of the same list), one can obtain

\[
\sum_{j=1}^{d+1} C^A_{l_1}(\rho) \leq \sqrt{d(d^2-1)} \left[ (d+1) P(\rho) - \sum_{j=1}^{d+1} P(A_j|\rho) \right] = \sqrt{d(d^2-1)} \left[ d P(\rho) - 1 \right]. \tag{8}
\]
where the equality is due to $\sum_{j=1}^{d+1} P(A_i | \rho) = 1 + P(\rho)$ when $d$ is a power of a prime \[41\]. By further choosing $P(\rho) = 1$, one can obtain a strongest state-independent bound as

$$\sum_{j=1}^{d+1} C_{l_1}^{A_i}(\rho) \leq (d - 1) \sqrt{d(d + 1)} := C_{l_1}^m,$$  \(9\)

and it reduces to that of Ref. \[33\] when $d = 2$. So if one considers the $l_1$ norm of coherence, $C_{l_1}^{A_i}(\rho_{AB}) > C_{l_1}^m$ is a signature of NAQC existing in the state $\rho_{AB}$.

### B. Relative entropy of coherence

We first prove a lemma concerning the relation between the von Neumann entropy $S(\rho)$ and purity $P(\rho)$ for the general d-dimensional state $\rho$. By denoting $\{\lambda_j\}$ for the eigenvalues of $\rho$, $S(\rho)$ and $P(\rho)$ can be written explicitly as

$$S(\rho) = - \sum_j \lambda_j \log_2 \lambda_j, \quad P(\rho) = \sum_j \lambda_j^2,$$  \(10\)

then by using the inequality $-\log_2 x \geq (1 - x)/\ln 2 (\forall x > 0)$, one can show that

$$S(\rho) + P(\rho) \geq \sum_j \lambda_j \frac{1 - \lambda_j}{\ln 2} + \sum_j \lambda_j^2$$

$$= \frac{1}{\ln 2} \left[1 + (\ln 2 - 1) \sum_j \lambda_j^2\right] \geq 1.$$  \(11\)

By combining the above inequality with the complementarity relation for the relative entropy of coherence \[38\], one can obtain

$$\sum_{j=1}^{d+1} C_{re}^{A_i}(\rho) \leq (d + 1)[\log_2 d + P(\rho) - 1] - \frac{(d - 1) \log_2(d - 1)}{d(d - 2)}[dP(\rho) - 1],$$  \(12\)

where $C_{re}^{A_i}(\rho)$ denotes the relative entropy of coherence in the basis spanned by the eigenvectors of $A_i$. By further choosing $P(\rho) = 1$, one can obtain the strongest state-independent upper bound as

$$\sum_{j=1}^{d+1} C_{re}^{A_i}(\rho) \leq (d + 1) \log_2 d - \frac{(d - 1) \log_2(d - 1)}{d(d - 2)} := C_{re}^m,$$  \(13\)

and for the special case $d = 2$, we have $C_{re}^m = 3 - \log_2 e/2$, but this bound can be further sharpened to $3H(1/2 + \sqrt{3}/6)$ \[38\]. So if one uses the relative entropy of coherence, $C_{re}^m(\rho) > C_{re}^m$ captures the NAQC in $\rho_{AB}$.

### IV. NEW FRAMEWORK OF NAQC

In this section, we present a new framework for formulating the NAQC. Different from that of Sec. \[11\] in which Bob’s chosen basis may be spanned by any $A_j$ of the set $\{A_j\}_{j \neq 3}$ when Alice executed one round of the measurements and announced her choice $A_j$ and outcomes $a \in \{1, \ldots, d\}$. In this new framework Bob measures the coherence of his conditional states in the presupposed basis spanned by the eigenvectors of $A_{\alpha_l}$, where $\{\alpha_l\}$ is one of the possible permutations of the elements of $[l]$. That is, there should be a one-to-one correspondence between $l$ and $\alpha_l$. This is illustrated in Fig. 1.

Within this new framework, Bob’s coherence averaged over all his possible conditional states due to Alice’s different measurements can be written as

$$C_{na}^{\alpha_l}(\rho_{AB}) = \sum_{i,j} p(a_i | A_j) C_{A_i}(\rho_{B_iC}),$$  \(14\)

and it applies to any faithful coherence measure $C$. Moreover, for the special case of $d = 2$, $C_{na}^{\alpha_l}(\rho_{AB})$ can also be obtained analytically (see Appendix \[13\]). As for all the single-partite states $\rho$ of dimension $d$, we have $\sum_{i} C_{i}(\rho) \leq C_{max}$, the criterion for achieving NAQC becomes $C_{na}^{\alpha_l}(\rho_{AB}) > C_{max}$. It holds for any possible $\{\alpha_l\}$, so one can optimize $C_{na}^{\alpha_l}(\rho_{AB})$ over all the possible $\{\alpha_l\}$ and define

$$C_{na}(\rho_{AB}) = \max_{\alpha_l} C_{na}^{\alpha_l}(\rho_{AB}),$$  \(15\)

and an optimized criterion is obtained as

$$\tilde{C}_{na}(\rho_{AB}) > C_{max}.$$  \(16\)

This criterion could capture a wider regime of NAQC states than that of Eq. \[4\] for certain cases. As an explicit example, we consider the $(d \times d)$-dimensional isotropic state \[42\]

$$\rho_{I} = \frac{1 - x}{d^2 - 1} I_D + \frac{d^2 x - 1}{d^2 - 1} |\Phi\rangle \langle \Phi|, \quad x \in [0, 1],$$  \(17\)

where $|\Phi\rangle = \sum_{n} |nn\rangle / \sqrt{d}$, and $|\langle n|\rangle$ denotes the computational basis on $C^d$. By adopting the $l_1$ norm and the relative entropy as measures of coherence, we calculated $C_{na}(\rho_{I})$ and $\tilde{C}_{na}(\rho_{I})$. For $d = 2$, we found that Eqs. \[4\] and \[16\] capture the same set of $\rho_{I}$ that achieve NAQC. For $d = 3$ and 5, the corresponding results were showed in Fig. 2 from which one can see that Eq. \[16\] captures a wider regime of $\rho_{I}$ with NAQC than that of Eq. \[4\]. In particular, when one uses the $l_1$ norm of coherence, Eq. \[4\] cannot capture the NAQC in $\rho_{I}$ at all.

For the separable state $\rho_{sep}$, in a similar manner to proving Eq. \[6\], one can see that

$$C_{na}^{\alpha_l}(\rho_{sep}) \leq \sum_{\alpha_i, a} q(a_i | A_j) C_{A_i}(\rho_{B_iC}^a) \leq C_{na},$$  \(18\)
from which one can further obtain
\[ \tilde{C}^{na}(\rho_{\text{sep}}) < C^m, \]
thus Bob cannot achieve the NAQC for all the separable states even in the new framework. This implies that all the \((d \times d)\)-dimensional states \(\rho_{AB}\) which can achieve the NAQC at a part of the system are quantum entangled.

Experimentally, the NAQC of \(\rho_{AB}\) can be estimated by the conditional tomography on \(B\), and this is easier than the full state tomography. So Eq. (19) can also be used for witnessing entanglement. That is, whenever we observed \(\tilde{C}^{na}(\rho_{AB}) > C^m\), we are sure that \(\rho_{AB}\) is entangled. Furthermore, as the NAQC may be enhanced by performing local unitary operation \(U_A \otimes U_B\) on \(AB\) and the entanglement is a locally unitary invariant, we are sure that \(\rho_{AB}\) is entangled provided
\[ \max_{(U_A \otimes U_B)} \tilde{C}^{na}(U_A \otimes U_B \rho_{AB} U_A^{\dagger} \otimes U_B^{\dagger}) > C^m. \]  
Of course, one can also maximize \(C^{na}(\rho_{AB})\) over all local unitaries \((U_A \otimes U_B)\), but the witnessed region may be different.

V. SUMMARY AND DISCUSSION

In summary, we have derived the criteria which capture the NAQC in a bipartite state. We first generalized the framework of Mondal et al. \[33\] to \((d \times d)\)-dimensional states with \(d\) being any power of a prime, then derived the explicit criteria by considering the \(l_1\) norm of coherence and the relative entropy of coherence. We also presented a new framework for formulating NAQC which can capture a wider regime of NAQC states than that of the original one. Within both the two frameworks, we showed that any state with NAQC is entangled, so one can recognize what the NAQC captures as a kind of quantum correlation which is stronger than entanglement. We hope these results may lead to a better understanding of the interrelations between coherence and quantum correlations.

We remark that there are other faithful coherence measures \[2, 3\]. Their complementarity relations under mutually unbiased bases and the criteria for achieving NAQC could be expected in future research. Moreover, the present criteria apply to the case of \(d\) being a prime power, and they capture different sets of the NAQC states when one uses different measures of coherence. A criterion which applies to bipartite states of arbitrary dimensions still remains to be explored, while seeking a measure-independent criterion is also of practical significance as the NAQC is hoped to serve as a resource for quantum information processing.

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Appendix A: The mutually unbiased bases

When the dimension \(d\) of a system is a prime number, there are \(d + 1\) mutually unbiased bases. By denoting \(|\phi^0_m\rangle\) the \(m\)th vector \((m = 0, \ldots, d - 1)\) in the \(l\)th basis, we have \[39\]
\[ |\phi^0_m\rangle = \sum_{n=0}^{d-1} \delta_{mn} |n\rangle, \quad |\phi^d_m\rangle = \frac{1}{\sqrt{d}} \sum_{n=0}^{d-1} e^{i \frac{2\pi mn}{d}} |n\rangle, \]
and
\[ |\phi^l_m\rangle = \frac{1}{\sqrt{d}} \sum_{n=0}^{d-1} e^{i \frac{2\pi (mn+l)\alpha^2}{d}} |n\rangle, \quad \text{for} \quad l = 1, \ldots, d - 1, \]
while for \(d\) being a prime power, there are also \(d + 1\) mutually unbiased bases which have been constructed in Ref. \[40\].

Based on these mutually unbiased bases, one can obtain the ensemble of Bob’s conditional states as \(\{p(a|A_i), \rho_{BA_i}\}\), where the postmeasurement state of \(B\) is
\[ \rho_{BA_i} = \frac{\langle \phi^a_i | \rho_{AB} | \phi^a_i \rangle}{p(a|A_i)}, \]  
and \(p(a|A_i)\) is the probability for Alice’s outcome \(a\) when she measures \(A_i\). It can be written as
\[ p(a|A_i) = \text{tr}(|\phi^a_i \rho_{AB} |\phi^a_i \rangle). \]

If the bipartite state is separable, i.e., \(\rho_{\text{sep}} = \sum_k q_k \rho_A^k \otimes \rho_B^k\), one can obtain
\[ \rho_{BA_i} = \frac{\sum_k q_k \langle \phi^a_i | \rho_A^k | \phi^a_i \rangle \rho_B^k}{\sum_k q_k \text{tr}(|\phi^a_i \rho_A^k |\phi^a_i \rangle)} = \frac{\sum_k q_k p_k(a|A_i) \rho_B^k}{\sum_k q_k p_k(a|A_i)} \]
where we have defined \(p_k(a|A_i) = \langle \phi^a_i | \rho_A^k | \phi^a_i \rangle\).
Appendix B: Solution of Eq. (15) for two-qubit states

By denoting \( \vec{r} \) and \( \vec{s} \) the local Bloch vectors, and \( \vec{\sigma} \) the vector of Pauli operators, one can decompose a general two-qubit state \( \rho_{AB} \) as follows

\[
\rho_{AB} = \frac{1}{4} \left( I_4 + \vec{r} \cdot \vec{\sigma} \otimes I_2 + I_2 \otimes \vec{s} \cdot \vec{\sigma} + \sum_{ij} t_{ij} (\sigma_i \otimes \sigma_j) \right),
\]

where \( r_i = \text{tr}(\rho (I_2 \otimes I_2)) \), \( s_i = \text{tr}(I_2 \otimes \rho \sigma_i) \), and \( t_{ij} = \text{tr}(\rho \sigma_i \otimes \sigma_j) \) \((i, j = 1, 2, 3)\).

Based on this decomposition, the probability of Alice’s outcome \( a \) when she measures \( A_i \) can be obtained as

\[
p(a|A_i) = 1 + \frac{(-1)^a r_i}{2}.
\]

while the \( l_1 \) norm of coherence for the conditional state \( \rho_{BA}^{A_i} \) is given by

\[
C_{l_1}^{A_i}(\rho_{BA}^{A_i}) = \frac{\sqrt{\sum_{j=0}^{3}(s_j + (-1)^a t_{ij})^2}}{1 + (-1)^a r_j}.
\]

and the relative entropy of coherence for \( \rho_{BA}^{A_i} \) is given by

\[
C_{re}^{A_i}(\rho_{BA}^{A_i}) = H(\beta_{ia}) - H(\lambda_{ia}),
\]

where \( H(\cdot) \) is the binary Shannon entropy function, and

\[
\beta_{ia} = \frac{1}{2} + \frac{s_a + (-1)^a t_{ia}}{2[1 + (-1)^a r_i]}, \quad \lambda_{ia} = \frac{1}{2} + \frac{\sum_j [s_j + (-1)^a t_{ij}]^2}{2[1 + (-1)^a r_i]}.
\]

Finally, by substituting Eqs. (B3) or (B4) into Eq. (14), we obtain \( C_{it}^{\alpha_i}(\rho_{AB}) \) for a given \( \{\alpha_i\} \), and by optimizing over all possible \( \{\alpha_i\} \), one can further obtain \( C_{it}^{\alpha}(\rho_{AB}) \).

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