Band-Gap Nonlinearity in Lead Chalcogenide (PbQ, Q = Te, Se, S) Alloys

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Band-Gap Nonlinearity in Lead Chalcogenide (PbQ, Q = Te, Se, S) Alloys

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ABSTRACT: Narrow band-gap lead chalcogenides have been developed for several optical and electronic applications. However, band-gap energies of the ternary and quaternary alloys have received little attention compared with the parent binary phases. Here, we have fabricated single-phase ternary (PbTe)_{1-x}(PbSe)_{x} and quaternary (PbTe)_{0.9-y}(PbSe)_{y}(PbS), and (PbTe)_{0.65-x}(PbSe)_{x}(PbS) alloys and shown that although lattice parameters follow Vegard’s law as a function of composition, the band-gap energies exhibit a substantial bowing effect. The ternary (PbTe)_{1-x}(PbSe)_{x} system features a smaller bowing parameter predominantly due to the difference in electronegativity between Se and Te, whereas the larger bowing parameters in quaternary alloys are generated from a larger crystal lattice mismatch and larger miscibility gap. These findings can lead to further advances in tuning the band-gap and lattice parameters for optical and electronic applications of lead chalcogenides.

INTRODUCTION

Lead chalcogenides (PbQ, Q = Te, Se, S) are unique compared with other semiconductors due to their interesting electronic and transport properties, such as narrow band gaps, low resistivities, large carrier mobilities, and positive temperature coefficients, showing an increase in the energy of band gap with temperature. These optical and electronic properties have led to the development of lead chalcogenides for several applications, including infrared lasers and detectors, thermophotovoltaics, infrared optoelectronic devices, photovoltaics, and thermoelectrics.

The optical and thermoelectric properties of lead chalcogenides have been shown to vary significantly with alloying. Despite significant recognition of band engineering in lead chalcogenide alloys, the band-gap energies of ternary and quaternary lead chalcogenides have received little attention in contrast to the parent binary phases of PbTe, PbSe, and PbS. Conventional semiconductor physics wisdom suggests that “band-gap bowing” is a common effect in semiconductor alloy systems and that it is likely present in lead chalcogenide alloys as well. Regardless, many authors have overlooked this and assumed that lead chalcogenide alloy band gaps change linearly with composition, in a similar way to their lattice parameters. This work aims to provide a better understanding of the band gap and bowing parameters for a wide range of lead chalcogenide alloys, to provide fundamental information required to tune the energy gap for optoelectronic and electronic applications.

In this work, we have fabricated intrinsic polycrystalline single-phase bulk ternary and quaternary lead chalcogenide alloys and measured their room-temperature energy gaps. The ternary system of (PbTe)_{1-x}(PbSe)_{x} was selected to determine the effect of PbSe content on the band-gap energy of PbTe. Whereas, quaternary samples of (PbTe)_{0.9-y}(PbSe)_{y}(PbS) and (PbTe)_{0.65-x}(PbSe)_{x}(PbS) were specially fabricated to elucidate the effects of PbS alloying on the band gap of ternary solid solution alloys of (PbTe)_{0.5}(PbSe)_{0.1} and (PbTe)_{0.65}(PbSe)_{0.35} respectively.

We show that although the lattice parameter variation of alloys with composition follows Vegard’s law, the energy gaps exhibit nonlinearity. The measured room-temperature band gaps, as a function of composition, consistently differ from any linear projection and show a deviation, with a parabolic function (bowing). Bowing parameters have been determined for ternary and quaternary systems, and factors influencing the degree of bowing are described in detail.

RESULTS AND DISCUSSION

Extrinsically doped semiconductors show high charge-carrier concentrations that increase the measured optical band-gap energy values due to the Burstein–Moss shift. Therefore, all samples in the current study were undoped. The room-temperature electrical resistivity of sintered samples was measured to assure negligible effect of intrinsic doping. Extrinsically doped n-type and p-type lead chalcogenides show room-temperature resistivity values below 1 mΩ cm, whereas the current study samples show resistivities between 6 and 90 mΩ cm. Nevertheless, the optical band gaps measured for n-type PbTe samples show a noticeable increase in band-
Figure 1. Powder XRD patterns of (a) ternary \((\text{PbTe})_{1-x}(\text{PbSe})_x\) \((x = 0, 0.10, 0.15, 0.35, 0.50, 0.70, 0.85, \text{and} \ 1.0)\), (b) quaternary \((\text{PbTe})_{0.9-y}(\text{PbSe})_{0.1-y}(\text{PbS})_y\) \((y = 0, 0.05, 0.08, \text{and} \ 0.9)\), and (c) quaternary \((\text{PbTe})_{0.65-z}(\text{PbSe})_{0.35-z}(\text{PbS})_z\) \((z = 0, 0.05, 0.10, 0.15, 0.20, \text{and} \ 0.65)\) alloys. Lattice parameters of (d) ternary \((\text{PbTe})_{1-x}(\text{PbSe})_x\) \((x = 0, 0.10, 0.15, 0.35, 0.50, 0.70, 0.85, \text{and} \ 1.0)\), (e) quaternary \((\text{PbTe})_{0.9-y}(\text{PbSe})_{0.1-y}(\text{PbS})_y\) \((y = 0, 0.05, 0.08, \text{and} \ 0.9)\), and (f) quaternary \((\text{PbTe})_{0.65-z}(\text{PbSe})_{0.35-z}(\text{PbS})_z\) \((z = 0, 0.05, 0.10, 0.15, 0.20, \text{and} \ 0.65)\) alloys as a function of composition.

Figure 2. (a) Normalized Kubelka–Munk transformations for the ternary \((\text{PbTe})_{1-x}(\text{PbSe})_x\) \((x = 0, 0.10, 0.15, 0.35, 0.50, 0.70, 0.85, \text{and} \ 1.0)\) system from raw data obtained using diffuse reflectance infrared Fourier transform spectroscopy (DRIFTS). (b) Normalized Kubelka–Munk function for the ternary \((\text{PbTe})_{1-x}(\text{PbSe})_x\) \((x = 0, 0.10, 0.15, 0.35, 0.50, 0.70, 0.85, \text{and} \ 1.0)\) system for absorption coefficient fit along with a few linear extrapolations to obtain the direct band gap by the Tauc method.
The solubility of PbS in PbTe results in the precipitation of PbS within the PbTe structure. All patterns show the single-phase NaCl-type face-centered cubic structure. Rietveld refinement was employed to accurately determine lattice parameters of all samples by extrapolating from high angle diffraction peaks. The results show that the lattice parameters of the alloys change linearly with composition and follow Vegard’s Law, indicating solid solution phases for all systems.

**Optical Band-Gap Energy.** The room-temperature absorption coefficient was calculated using the Kubelka-Munk function ($\alpha/S$), where $\alpha$ is the absorption and $S$ the scattering coefficient, respectively. Normalized spectra are shown in Figure 2a for the ternary system ($PbTe_{0.9} (PbSe)_{0.1}$, $x = 0$, 0.10, 0.15, 0.35, 0.50, 0.70, 0.85, and 1.0) system. Optical band-gap energies are determined by applying the Tauc method, where ($h\omega$)$^2$ is extrapolated to zero as a function of $\alpha/S$.

**Figure 3.** Normalized Kubelka–Munk transformations for quaternary alloys of (a) $PbTe_{0.9} (PbSe)_{0.1} (PbS)_y$ ($y = 0, 0.05, 0.08, 0.10, 0.15, 0.20, 0.35, 0.50, 0.70, 0.85$, and 1.0) for application of the Tauc method for determining direct band gaps.

**Figure 4.** Direct band-gap energies for (a) the ternary system $PbTe_{0.9} (PbSe)_{0.1}$, ($x = 0, 0.10, 0.15, 0.35, 0.50, 0.70, 0.85$, and 1.0) system as a function of the PbSe content, compared to those of films grown by the molecular beam epitaxy method; (b) the quaternary system $PbTe_{0.9} (PbSe)_{0.1} (PbS)_y$ system and (c) the $PbTe_{0.65} (PbSe)_{0.35} (PbS)_z$ system as functions of PbS concentration.

Band-gap energies of $PbTe_{0.9} (PbSe)_{0.1} (PbS)_y$ ($y = 0, 0.05, 0.08, 0.10, 0.15, 0.20, 0.35, 0.50, 0.70, 0.85$, and 1.0) samples were calculated from the normalized Kubelka–Munk graph shown in Figure 3a,b, respectively. The shift in energy of the band gaps to higher values with PbS content is evident in the quaternary samples compared to that in the ternary system. The band-gap energy of PbS concentrations. The lattice parameter of the PbTe ($a = 6.46$ Å) phase is larger than that of PbSe ($a = 6.13$ Å) and PbS ($a = 6.93$ Å). A Hallbauer et al. (2003) study shows that the lattice parameters of the alloys change linearly with composition and follow Vegard’s Law, indicating solid solution phases for all systems.

**Crystal Structure.** X-ray diffraction (XRD) patterns of $PbTe_{1-x}(PbSe)_x$ ($x = 0, 0.10, 0.15, 0.35, 0.50, 0.70, 0.85$, and 1.0), $PbTe_{0.9} (PbSe)_{0.1} (PbS)_y$ ($y = 0, 0.05, 0.08, 0.10, 0.15, 0.20, 0.35, 0.50, 0.70, 0.85$, and 0.9), and $PbTe_{0.65} (PbSe)_{0.35} (PbS)_z$ ($z = 0, 0.05, 0.10, 0.15, 0.20, 0.35, 0.50, 0.70, 0.85$, and 0.65) samples are shown in Figure 1a–c, respectively. All patterns show the single-phase NaCl-type face-centered cubic structure. Figure 2 shows that diffraction peaks are progressively shifted to a higher angle by increasing PbSe and PbS concentrations. The lattice parameter of the PbTe ($a = 6.46$ Å) phase is larger than that of PbSe ($a = 6.13$ Å) and PbS ($a = 6.93$ Å). Rietveld refinement was employed to accurately determine lattice parameters of all samples by extrapolating from high angle diffraction peaks. The results show that the lattice parameters of the alloys change linearly with composition and follow Vegard’s Law, indicating solid solution phases for all systems.

The room-temperature absorption coefficient was calculated using the Kubelka-Munk function ($\alpha/S$), where $\alpha$ is the absorption and $S$ the scattering coefficient, respectively. Normalized spectra are shown in Figure 2a for the ternary system ($PbTe_{0.9} (PbSe)_{0.1}$, $x = 0$, 0.10, 0.15, 0.35, 0.50, 0.70, 0.85, and 1.0) system. Optical band-gap energies are determined by applying the Tauc method, where ($h\omega$)$^2$ is extrapolated to zero as a function of $\alpha/S$.

Band-gap energies of $PbTe_{0.9} (PbSe)_{0.1} (PbS)_y$ ($y = 0, 0.05, 0.08, 0.10, 0.15, 0.20, 0.35, 0.50, 0.70, 0.85$, and 1.0) samples were calculated from the normalized Kubelka-Munk graph shown in Figure 3a,b, respectively. The shift in energy of the band gaps to higher values with PbS content is evident in the quaternary samples compared to that in the ternary system. The band-gap energy of PbS concentrations. The lattice parameter of the PbTe ($a = 6.46$ Å) phase is larger than that of PbSe ($a = 6.13$ Å) and PbS ($a = 6.93$ Å). A Hallbauer et al. (2003) study shows that the lattice parameters of the alloys change linearly with composition and follow Vegard’s Law, indicating solid solution phases for all systems. **Crystal Structure.** X-ray diffraction (XRD) patterns of $PbTe_{1-x}(PbSe)_x$ ($x = 0, 0.10, 0.15, 0.35, 0.50, 0.70, 0.85$, and 1.0), $PbTe_{0.9} (PbSe)_{0.1} (PbS)_y$ ($y = 0, 0.05, 0.08, 0.10, 0.15, 0.20, 0.35, 0.50, 0.70, 0.85$, and 0.9), and $PbTe_{0.65} (PbSe)_{0.35} (PbS)_z$ ($z = 0, 0.05, 0.10, 0.15, 0.20, 0.35, 0.50, 0.70, 0.85$, and 0.65) samples are shown in Figure 1a–c, respectively. All patterns show the single-phase NaCl-type face-centered cubic structure. Figure 2 shows that diffraction peaks are progressively shifted to a higher angle by increasing PbSe and PbS concentrations. The lattice parameter of the PbTe ($a = 6.46$ Å) phase is larger than that of PbSe ($a = 6.13$ Å) and PbS ($a = 6.93$ Å). Rietveld refinement was employed to accurately determine lattice parameters of all samples by extrapolating from high angle diffraction peaks. The results show that the lattice parameters of the alloys change linearly with composition and follow Vegard’s Law, indicating solid solution phases for all systems. **Optical Band-Gap Energy.** The room-temperature absorption coefficient was calculated using the Kubelka-Munk function ($\alpha/S$), where $\alpha$ is the absorption and $S$ the scattering coefficient, respectively. Normalized spectra are shown in Figure 2a for the ternary system ($PbTe_{0.9} (PbSe)_{0.1}$, $x = 0$, 0.10, 0.15, 0.35, 0.50, 0.70, 0.85, and 1.0) system. Optical band-gap energies are determined by applying the Tauc method, where ($h\omega$)$^2$ is extrapolated to zero as a function of $\alpha/S$.
PbS (0.41 eV) is much higher than that of PbSe (0.27 eV) and PbTe (0.29 eV) and hence there is a large difference in the energy gap between the PbS-free samples (PbTe)\(_{0.93}\) (PbSe)\(_{0.35}\) and the PbTe-free samples (PbSe)\(_{0.41}\) (PbSe)\(_{0.59}\) and (PbSe)\(_{0.53}\) (PbSe)\(_{0.47}\), resulting in a larger energy range for the measurements to spread across.

Figure 4a illustrates nonlinear variation of the optical band gap of ternary (PbTe)\(_{1−x}\) (PbSe)\(_{x}\) alloys with composition. Although the differences in the measured band gaps from linear assumption values are small, the adopted measurement technique DRIFTS has been shown to be reliable in quantifying changes in the band gap, with high resolution of ±0.004 eV.\(^{30}\) It is evident that the band gaps of (PbTe)\(_{1−x}\) (PbSe)\(_{x}\) alloys vary in a parabolic contour with composition rather than in a linear relationship. This deviation is due to a phenomenon known as the bowing effect. This effect has been previously observed in (PbTe)\(_{1−x}\) (PbSe)\(_{x}\) alloys grown on the (111) BaF\(_2\) substrate by the molecular beam epitaxy method\(^6\) (Figure 4a). Although both studies show a bowing effect, the measured energy gaps are different. This might have originated from fabrication methods. The molecular beam epitaxy technique might have introduced point defects in the structure, resulting in higher carrier concentration, which is shown to increase the optical band gap due to the Burstein–Moss effect.\(^{12}\)

Figure 4b,c shows the calculated linear assumption and measured variation of the direct band-gap energy with respect to the PbS content for quaternary systems of (PbTe)\(_{0.99-0.65}\) (PbSe)\(_{0.65}\) (PbSe)\(_{0.35}\) (PbSe)\(_{0.1}\), and (PbTe)\(_{0.35-0.1}\) (PbSe)\(_{0.65}\) (PbSe)\(_{0.35}\) (PbSe)\(_{0.1}\). The calculated curve is a linear prediction of the band gap, using the band-gap energy measured for pure binary phases of PbTe, PbSe, and PbS. The linear assumption lines in Figure 4b,c are the predicted variation in the energy of the band gap with composition between the ternary alloys of PbS-free (PbTe)\(_{0.93}\) (PbSe)\(_{0.1}\) and (PbTe)\(_{0.65}\) (PbSe)\(_{0.35}\) and PbTe-free samples of (PbSe)\(_{0.41}\) (PbSe)\(_{0.59}\) and (PbSe)\(_{0.53}\) (PbSe)\(_{0.47}\). The curves in Figure 4b,c fit the parabolic equation for the measured energies of the band gaps. The experimental data in Figure 4c deviate slightly from the linear relationship, although the calculated curves for (PbTe)\(_{0.99-0.65}\) (PbSe)\(_{0.65}\) (PbSe)\(_{0.1}\), compounds show band gaps much smaller than with the linear assumption and experimental data due to the smaller band-gap values obtained for binary PbSe, PbTe, and PbS. It is worth noting that experimental band-gap energies measured for these binary compounds are equal to values extracted from the literature (Table 1). The composition gaps in the experimental data in Figure 4a,b exhibit a phase separation region, wherein the PbS-rich phase precipitates within the PbTe-rich matrix.

The nonlinear nature of the variation of the optical band gap with composition is not uncommon and has been observed in a number of classical semiconductor systems, such as Si–Ge,\(^{10,23}\) InGaAs, AlGaAs, AlInN,\(^{18}\) and CdSe–CdTe.\(^{13}\) The bowing parameter in semiconductors is described by a parabolic polynomial\(^{12}\)

\[
E_{g,AB} = xE_{g,A} + \frac{(1-x)}{b(1-x)}
\]

where \(E_{g,A}\) and \(E_{g,B}\) are intrinsic energy gaps of the initial semiconductors A and B and x is the phase ratio of A that is used to calculate the energy gap of the alloy (\(E_{g,AB}\)). The degree to which the curve deviates from the linear fit is described by the bowing parameter b.\(^{12}\)

Bowing parameters calculated for ternary and quaternary systems are summarized in Table 2. The negative sign of bowing parameters is due to the inverted nature of the parabola for these systems.\(^{35}\) It is worth noting that the bowing parameter for the (PbTe)\(_{0.99-0.65}\) (PbSe)\(_{0.65}\) (PbSe)\(_{0.1}\) system has been obtained by fitting limited experimental data, which can contribute to a larger error value. Nevertheless, the absolute value of the bowing parameter (b) for the quaternary (PbTe)\(_{0.65}\) (PbSe)\(_{0.35}\) (PbSe)\(_{0.1}\) system (−0.02352 eV) is much smaller than that for the quaternary (PbTe)\(_{0.99}\) (PbSe)\(_{0.65}\) (PbSe)\(_{0.1}\) system (−0.02183 eV), larger than that (−0.052 eV) for the ternary (PbTe)\(_{1−x}\) (PbSe)\(_{x}\), system, and similar to that of the ternary (PbSe)\(_{1−x}\) (PbSe)\(_{x}\) system (−0.190 eV).\(^{12}\) There are several factors that affect the bowing parameter of solid solution materials, including ionicity and crystal structure mismatch, solubility, electron potential, and electronegativity.\(^{8,12,33,36}\)

### Table 2. Bowing Parameter (b) Calculated for Band-Gap Energy of Ternary and Quaternary Systems

| System                        | Bowing Parameter (eV) |
|-------------------------------|-----------------------|
| (PbTe)\(_{1−x}\) (PbSe)\(_{x}\)| (−0.052 ± 0.011)      |
| (PbTe)\(_{0.99}\) (PbSe)\(_{0.65}\) (PbSe)\(_{0.1}\)| (−0.2183 ± 0.01561)  |
| (PbTe)\(_{0.65}\) (PbSe)\(_{0.35}\) (PbSe)\(_{0.1}\)| (−0.08326 ± 0.02352) |
| (PbSe)\(_{1−x}\) (PbSe)\(_{x}\)| (−0.190 ± 0.045)      |

### Atomic Size and Ionicity

The atomic size and ionicity mismatch of the atoms have been shown to be the main cause for bowing of band gaps in solid solution semiconductor alloys.\(^{20,32,34}\) In systems such as (Zn, Mg, Be)O\(_3\),\(^{34}\) BaTiO\(_3\)–CaTiO\(_3\),\(^{35}\) BaTiO\(_3\)–BaZrO\(_3\), SrTiO\(_3\)–BaZrO\(_3\),\(^{36}\) and Mg\(_{3}Zn_{1−x}\)Se,\(^{20}\) there are large variations in the atomic size and ionicity of participant atoms, which are believed to cause the band-gap bowing.

Lattice parameters of PbTe (6.50 Å) and PbSe (6.12 Å) are very similar but larger than those of PbS (5.94 Å). Concurrently, Se ions (1.98 Å) are slightly smaller than Te ions (2.21 Å), and both are larger than S ions (1.84 Å). All have valence 2+. Therefore, it is unlikely that atomic size and ionicity play a significant role in the observed bowing of the band gap in ternary (PbTe)\(_{1−x}\) (PbSe)\(_{x}\) and (PbSe)\(_{1−x}\) (PbSe)\(_{x}\) systems. On the other hand, replacing Te with S atoms in quaternary systems creates a slightly larger lattice mismatch that, in turn, increases the bowing parameter.\(^{36}\)

### Electronegativity

The electronegativity of individual anions bonded to cations has also been identified as a major factor determining the degree to which band-gap bowing occurs in alloys.\(^{33}\) The electronegativity effect is used to measure an atom’s ability to draw electrons from surrounding atoms toward it. It is determined by the atomic number and how far the outer electrons lie from the inner nucleus. The Pauling scale is the...
most frequently used method to compare electronegativities, in which fluorine is given the highest value of 4.0, and this value decreases down through all elements to cesium and francium at 0.7.\(^{31}\)

Electronegative values of Te, Se, and S atoms are 2.10, 2.55, and 2.58, respectively, according to the Pauling scale. The band-gap bowing in the ternary (PbTe)\(_{1-x}\)(PbS)\(_x\) system might be attributed to the larger electronegativity of Se ions compared to that of Te ions (2.55 > 2.10). It is unlikely, however, that replacing Te ions with S ions with slightly larger electronegativity (2.58 > 2.55) in quaternary systems could cause bowing in the energy gaps.

Valence Electron Potential. Similar to electronegativity, conflicting valence electron potentials of individual atoms in a specific system have also been identified to play a significant role in the band-gap bowing of alloys. This valence electron potential is a quantitative indication of the reactivity of individual atoms and can be calculated using the equation\(^{37}\)

\[
-eV = \frac{k_n}{r}
\]

(2)

where \(-eV\) is the valence electron potential, \(n\) is the number of valence electrons, \(r\) is the ionic radius (Å), and \(k\) is the proportionality factor used to convert angstrom (Å) into centimeter (cm) and transform the force of valence electrons into electron volt (eV).

The valence electron potential of S (160 eV) is higher than that of Se (120 eV) and is much higher than that of Te (59 eV), suggesting an increase in the core potential fluctuation and increased deviation from the symmetric alloy behavior.\(^{35}\)

Therefore, one would expect that quaternary systems show a larger bowing parameter than that of ternary compounds.

Solubility. The solubility of two compounds is directly linked to the magnitude of bowing in the band gaps of their alloys. Solubility is a direct reflection of atomic size, ionicity, and electronegativity.\(^{38}\) Alloys that contain large regions where phase separation occurs tend to have larger bowing parameters than those of alloys with minimal or no miscibility gaps.\(^{33,35}\)

It is evident from the pseudobinary phase diagram of PbSe–PbTe that the miscibility gap only occurs at temperatures below 170 K for (PbTe)\(_{1-x}\)(PbSe)\(_x\) alloys.\(^{39}\) Hence, the solubility factor has minimal influence on the band-gap bowing of the (PbTe)\(_{1-x}\)(PbSe)\(_x\) system, although it might result in bowing of band gap for (PbTe)\(_{0.9}\)(PbSe)\(_{0.1}\)(PbS)\(_y\) and (PbTe)\(_{0.65}\)(PbSe)\(_{0.35}\)(PbS)\(_z\) systems, as their alloys have a sizeable miscibility gap.\(^{29}\)

The size of the miscibility gap for the (PbTe)\(_{0.65}\)(PbSe)\(_{0.35}\)(PbS)\(_z\) system is much smaller than that of the (PbTe)\(_{0.9}\)(PbS)\(_{0.1}\) system.\(^{29}\) Therefore, one would expect to observe a larger bowing parameter for the (PbTe)\(_{0.9}\)(PbSe)\(_{0.1}\)(PbS)\(_y\) system. Both quaternary systems show larger bowing parameters than those of ternary (PbTe)\(_{1-x}\)(PbSe)\(_x\) alloys, with complete solubility over the whole composition range.

The solubility, atomic size, and ionicity factors suggest minimal band-gap bowing for ternary (PbTe)\(_{1-x}\)(PbSe)\(_x\) and (PbSe)\(_{1-x}\)(PbS)\(_x\) systems, whereas, the electron potential and electronegativity predict bowing in the energy of band gaps in the ternary (PbTe)\(_{1-x}\)(PbSe)\(_x\) system. It is worth noting that the band gap for ternary (PbSe)\(_{1-x}\)(PbS)\(_x\) alloys was determined for nanocrystals, suggesting that the bowing is influenced by quantum confinement and changes of the quantum dot size.\(^{32}\) A linear relationship of band-gap energy with composition is expected for bulk (PbSe)\(_{1-x}\)(PbS)\(_x\) alloys.

On the other hand, the atomic size, electronegativity, and solubility that result in spinodal decomposition (below 1050 K) in the PbTe–PbS system appear to be likely factors behind the band-gap bowing of quaternary systems.

A potential reason for the reduced bowing parameter in the (PbTe)\(_{0.65}\)(PbSe)\(_{0.35}\)(PbS)\(_y\) system compared to that of the (PbTe)\(_{0.9}\)(PbSe)\(_{0.1}\)(PbS)\(_y\) system is the smaller miscibility gap that exists between the (PbTe)\(_{0.65}\)(PbSe)\(_{0.35}\) and (PbSe)\(_{0.35}\)(PbS)\(_y\) phases in the (PbTe)\(_{0.65}\)(PbSe)\(_{0.35}\)(PbS)\(_z\) system compared to that between (PbTe)\(_{0.9}\)(PbSe)\(_{0.1}\) and (PbSe)\(_{0.1}\)(PbS)\(_0\) in the (PbTe)\(_{0.9}\)(PbSe)\(_{0.1}\)(PbS)\(_0\) system.

## CONCLUSIONS

The unique physical properties of Pb chalcogenides have led to their development for several optical and electronic applications, wherein the tuned band-gap energy can tailor electronic transport properties. Herein, intrinsic polycrystalline samples of ternary and quaternary Pb chalcogenide alloys were fabricated and their optical band-gap energies and lattice parameters were measured as a function of composition. Although lattice parameters of these alloys follow Vegard’s equation, the band-gap energies exhibit nonlinear relationships with compositions. We have determined the bowing parameters for these systems.

The ternary (PbTe)\(_{1-x}\)(PbSe)\(_x\) system featured a small amount of band-gap bowing, predominantly due to the difference in electronegativity between Se and Te. Bowing parameters for the quaternary (PbTe)\(_{0.9}\)(PbSe)\(_{0.1}\)(PbS)\(_y\) system are larger than those for (PbTe)\(_{0.65}\)(PbSe)\(_{0.35}\)(PbS)\(_z\) due to the larger crystal lattice mismatch and miscibility gap. This opens up promising perspectives for tuning the band-gap energy and lattice parameters of Pb chalcogenide alloys for optical and electronic applications.

## METHODS

Sample Fabrication. Intrinsic polycrystalline samples of PbS, PbSe, and PbTe were prepared by mixing stoichiometric ratios of high purity Pb (99.999%), Se (99.999%), and dried S (99.99%) in vacuum-sealed quartz ampoules. These were reacted at 1373 K to produce high purity PbS, PbSe, and PbTe.

The final polycrystalline samples of ternary and quaternary compounds of (PbTe)\(_{1-x}\)(PbSe)\(_x\) (\(x = 0, 0.15, 0.35, 0.50, 0.70, 0.85\), and 1), (PbTe)\(_{0.9}\)(PbSe)\(_{0.1}\)(PbS)\(_y\) (\(y = 0, 0.05, 0.08, \) and 1), and (PbTe)\(_{0.65}\)(PbSe)\(_{0.35}\)(PbS)\(_z\) (\(z = 0, 0.05, 0.10, 0.15, 0.20, \) and 1) were synthesized by mixing stoichiometric quantities of high purity PbS, PbSe, Pb, and Te. The mixtures were sealed in carbon-coated quartz tubes under vacuum, reacted at 1373 K for 10 h, and then cooled to room temperature in the furnace. The resulting ingots were hand-ground to powder in an agate mortar and used to measure the energy of band gaps.

XRD Analysis. The crystallographic structure of samples was characterized by XRD using a GBC Scientific X-ray diffractometer with Cu Kα radiation (\(λ = 1.544\) Å, 40 kV, 30 mA). To measure phase ratios and calculate lattice parameters, XRD patterns were refined using the Rietveld analysis.

Infrared Optical Properties. Room-temperature diffuse reflectance spectra of finely ground powders were recorded using a Shimadzu IRPrestige-21 Fourier Transform Infrared (FTIR) Spectrophotometer equipped with diffuse reflectance. The spectra were monitored in the mid-IR region (6000–400 cm\(^{-1}\)), which were then converted to electron volts (eV) using...
Planck’s law. Absorption data ($\alpha/S$) were calculated from reflectance data via the Kubelka–Munk transformation. Optical energy gaps were extracted using the Tauc method. 

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**Notes**

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