Electrical conductivity of Hot and Dense QCD matter at RHIC BES energies: A Color String Percolation Approach

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Recently, transport coefficients viz. shear viscosity, electrical conductivity of strongly interacting matter produced in heavy-ion collisions have drawn considerable interest. We study the normalised electrical conductivity ($\sigma_{el}/T$) of hot QCD matter as a function of temperature (T) at Relativistic Heavy-Ion Collider (RHIC) Beam Energy Scan (BES) energies within the Color String Percolation Model (CSPM). We also study the temperature dependence of shear viscosity to entropy density and the quantum fluctuations of the color fields eE $\approx \pi/2 \approx 10^{21}$ V/cm and eB $\approx m_{e}^2 \approx 10^{18}$ G [15].

The values of the electric and magnetic fields at RHIC are eE $\approx 10^{21}$ V/cm and eB $\approx m_{e}^2 \approx 10^{18}$ G [15]. Such a large electrical field influences the medium, which depends on the electrical conductivity of the medium.

I. INTRODUCTION

Ultra-relativistic heavy-ion collision programs at Relativistic Heavy-Ion Collider (RHIC) and Large Hadron Collider (LHC) produce a strongly interacting matter known as Quark-Gluon Plasma (QGP) [1]. Various experimental studies have been done in order to characterise the properties and behaviour of matter at extreme conditions of temperature and energy densities. The transport properties are very important to understand the evolution of the strongly interacting matter produced in heavy-ion collisions. These are mainly the theoretical inputs to the hydrodynamical calculations and affect various observables such as elliptic flow, transverse momentum spectra of particles created in heavy-ion collisions [2–4]. A very small shear viscosity to entropy density ratio explains the elliptic flow of identified hadrons produced at RHIC and LHC energies [5]. Various methods are used to estimate the shear viscosity ($\eta$) such as Kubo formalism [6], effective models [7–13] etc.

Electrical conductivity ($\sigma_{el}$) is another key transport coefficient in order to understand the behaviour and properties of strongly interacting matter. This plays an important role in the hydrodynamic evolution of the matter produced in heavy-ion collisions where charge relaxation takes place. In ref. [14], the electrical conductivity is extracted from charge dependent flow parameters from asymmetric heavy ion collisions. Experimentally, it has been observed that very strong electric and magnetic fields are created in the early stages (1-2 fm/c) of non-central heavy ion collisions at RHIC and LHC [14, 15].

The experimental measurement of electrical conductivity ($\sigma_{el}$) of the matter produced in heavy-ion collisions is not possible. Its information can be extracted from flow parameters measured in heavy-ion collision experiments [14]. Recently, various theoretical approaches have been used to study the electrical conductivity [17–31]. $\sigma_{el}$ is also related to the soft dilepton production rate [32] and the magnetic field diffusion in the medium [33, 34].

Color String Percolation Model is a QCD inspired model [35–39], which can be used as an alternative approach to Color Glass Condensate (CGC). In CSPM, the color flux tubes are stretched between the colliding partons in terms of the color field. The strings produce $q\bar{q}$ pair in finite space filled similarly as in the Schwinger mechanism of pair creation in a constant electric field covering all the space [40]. With the growing energy and the number of nucleons of participating nuclei, the number of strings grows. Color strings may be viewed as small discs in the transverse space filled with the color field created by colliding partons. The number of strings grows and starts to overlap and interact to form clusters as the energy and size of the colliding nuclei grows. After a critical string density reached, a macroscopic cluster appears that marks the percolation phase transition which spans the transverse nuclear interaction area. 2D percolation is a non-thermal second order phase transition. In CSPM, the Schwinger barrier penetration mechanism for particle production, the fluctuations in the associated string tension and the quantum fluctuations of the color fields make it possible to define a temperature. Consequently the particle spectrum is produced with a thermal distribution. When the initial density of interacting colored

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strings ($\xi$) exceeds the 2D percolation threshold ($\xi_e$) i.e. $\xi > \xi_e$, a macroscopic cluster appears, which defines the onset of color deconfinement. The critical density of percolation is related to the effective critical temperature and thus percolation may be a possible way to achieve deconfinement in ultrarelativistic heavy-ion collisions [41] and high multiplicity pp collisions [42, 43]. It is observed that, CSPM can be successfully used to describe the initial stages in high energy heavy-ion collisions [40]. Recently, we have performed collision centrality, energies and species dependent study of the deconfinement phase transition at RHIC Beam Energy Scan (BES) energies using color string percolation model [44]. We have also studied various thermodynamical and transport properties at RHIC BES energies in this approach [45].

In this work, for the first time we give the formulation of $\sigma_{cl}$ in the color string percolation approach. The paper is organised as: In section II, we give the detailed formulation for calculation of electrical conductivity in CSPM and present results and discussions in section III. Finally, we present summary and conclusions in section IV.

II. ELECTRICAL CONDUCTIVITY

In this section, we develop the formulation for evaluating the electrical conductivity of strongly interacting matter using the color string percolation approach. We start with few basic equations of CSPM. The percolation density parameter, $\xi$ for central Au+Au collisions at RHIC energies is calculated by using the parameterisation of pp collisions at $\sqrt{s} = 200$ GeV as discussed below. In CSPM one obtains:

$$\frac{dN_{ch}}{dp_T^2} = \frac{a}{(p_0 + \mu_T)^\alpha},$$

where, $a$ is the normalisation factor and $p_0$, $\alpha$ are fitting parameters given as, $p_0 = 1.982$ and $\alpha = 12.877$ [46]. Due to the low string overlap probability in pp collisions the fit parameters are then used to evaluate the interactions of the strings in Au+Au collisions as,

$$p_0 \rightarrow p_0 \left(\frac{\langle n_S_1/S_n \rangle_{Au+Au}}{\langle n_S_1/S_n \rangle_{pp}}\right)^{1/4}.$$  \hspace{1cm} (2)

Here, $S_n$ corresponds to the area occupied by $n$ overlapping strings. Now,

$$\frac{\langle n_S_1/S_n \rangle}{S_1} = \frac{1}{F_1^2(\xi)},$$  \hspace{1cm} (3)

where, $F(\xi)$ is the color suppression factor, which is given as,

$$F(\xi) = \sqrt{\frac{1 - e^{-\xi}}{\xi}}.$$  \hspace{1cm} (4)

To calculate the electrical conductivity of strongly interacting matter, which is one of the most important transport properties of QCD matter, we proceed as follows. The mean free path, which describes the relaxation of the system far from equilibrium can be written in terms of number density and cross-section as,

$$\lambda_{mfp} = \frac{1}{n\sigma_{tr}},$$  \hspace{1cm} (5)

where $n$ is the number density of an ideal gas of quarks and gluons and $\sigma_{tr}$ is the transport cross-section. In CSPM the number density is given by the effective number of sources per unit volume

$$n = \frac{N_{sources}}{S_nL}.$$  \hspace{1cm} (6)

Here, $L$ is the longitudinal extension of the string $\sim 1$ fm. The area occupied by the strings is given by the relation $(1 - e^{-\xi})S_n$. Thus, the effective number of sources is given by the total area occupied by the strings divided by the area of an effective string, $S_1 F(\xi)$ as shown below,

$$N_{sources} = \frac{(1 - e^{-\xi})S_n}{S_1 F(\xi)},$$  \hspace{1cm} (7)

In general, $N_{sources}$ is smaller than the number of single strings. $N_{sources}$ equals to the number of strings $N_s$ in the limit of $\xi = 0$. So,

$$n = \frac{(1 - e^{-\xi})}{S_1 F(\xi)L}.$$  \hspace{1cm} (8)

Now, using eqs. 5 and 8, we get,

$$\lambda_{mfp} = \frac{L}{(1 - e^{-\xi})},$$  \hspace{1cm} (9)

where $\sigma_{tr}$, the transverse area of the effective strings equals to $S_1 F(\xi)$.

Now we derive the formula for electrical conductivity. For this, we use Anderson-Witting model, in which the Boltzmann transport equation is given as [47],

$$p^\mu \partial_\mu f_k + q F^{\alpha\beta} \partial f_k/p^\beta = -p^\mu u_{\mu} \tau (f_k - f_{eq,k}),$$  \hspace{1cm} (10)

where $f_k$ is the equilibrium distribution function of $k^{th}$ species. $\tau$ is the mean time between collisions and $u_{\mu}$
FIG. 1: (colour online) $\sigma_{el}/T$ versus $T$ plot. Solid line is the result obtained in CSPM and blue triangles are PHSD results [26]. The blue and red dotted lines are the results of BAMPS [48] and NCH models [29], respectively. The blue circles are kinetic theory calculations [49]. The horizontal line is the result obtained for conformal supersymmetric (SYM) Yang-Mills Plasma [50].

$T$ is the fluid four velocity in the local rest frame. Eq. 10 provides a straightforward calculation of the quark distribution after applying the electric field. The gluon distribution function remains thermal and not altered by electric field. Here, we assume that there are as many quarks (charge $q$) as anti-quarks (charge $-q$) and uncharged gluons in the system. $F^{\alpha\beta}$ is the electromagnetic field strength tensor given by [48],

$$F^{\mu\nu} = u^{\nu} E^{\mu} - u^{\mu} E^{\nu} - B^{\mu\nu}. \quad (11)$$

Since we study the effect of electric field, the magnetic field is set to zero in the calculations. The electric current density of the $k^{th}$ species in the $x$-direction is given as,

$$j_{k}^{x} = q_{k} \int \frac{d^{3}p}{(2\pi)^{3}} p^{x} f_{k} = g_{k}\frac{8}{3} T^{2} e^{\frac{-M}{T}}. \quad (12)$$

According to Ohm’s law, $j_{k}^{x} = \sigma_{el} E^{x}$. Using eq. 12 and relation $n_{k} = g_{k} T^{3}/\pi^{2}$, electrical conductivity in the assumption of very small electric field and no cross effects between heat and electrical conductivity in the relaxation time approximation is given by,

$$\sigma_{el} = \sum_{k=1}^{M} g_{k}^{2} n_{k} \lambda_{mfp}. \quad (13)$$

Putting eq. 9 in eq. 13 and considering the density of only up flavour ($u$)-quark and antiquark in the calculation, we get the expression for $\sigma_{el}$ as,

$$\sigma_{el} = \frac{2}{3} \times 3T e^{\frac{2}{3} T} n_{q}(T) \frac{1}{(1-e^{-T})}. \quad (14)$$

Here, the pre-factor $2/3$ reflects the flavour averaged fractional quark charge squared ($\sum_{f} q_{f}^{2}$) and $n_{q}$ denotes the total density of quarks or antiquarks.

III. RESULTS AND DISCUSSIONS

In this section, we discuss the results obtained in CSPM along with that obtained in various approaches. In fig. 1, we show $\sigma_{el}/T$ as a function of temperature. The solid line shows the results of CSPM calculated using eq. 14, while blue triangles are IQCD [25, 51] estimations. We also show the results of various theoretical calculations. The blue dashed line is the result of microscopic transport model BAMPS [48], in which the relativistic $(3+1)$-dimensional Boltzmann equation is solved numerically to extract the electric conductivity for a dilute gas of massless and classical particles de-
scribed by the relativistic Boltzmann equation. BAMPS results show a slower increase of $\sigma_{el}/T$ with temperature and are above the IQCD results. A non-conformal holographic model [29] is used to estimate the electrical conductivity of the strongly coupled QGP, which is shown by the red dashed line and explains the IQCD data qualitatively. Kinetic theory [49] is also used to calculate electrical conductivity of hadron gas whose results are shown by blue circles in the figure, which shows a decrease of $\sigma_{el}/T$ with temperature. Parton-Hadron-String Dynamics (PHSD) model results [26] are also shown by the black triangles in the figure for both the phases- hadron gas and quark-gluon plasma. $\sigma_{el}$ in PHSD are quite higher than the IQCD data and decreases for HG and increases for QGP with temperature. The electrical conductivity for conformal Yang-Mills plasma [50] is also shown by the horizontal line in the figure. We find that CSPM results are in close agreement with the results obtained in PHSD and show a similar behaviour as observed in IQCD.

Figure 2, shows the variation of $\eta/s$ as a function of $T/T_c$. Here, $T_c$ is the critical temperature which is different in different model calculation. The black solid line is the CSPM result and the broken lines are quasiparticle model results [52]. Here, the dotted line is the result for anisotropic case while the dash-dotted is for isotropic case. The symbols are results of IQCD with (2+1)- dynamical flavours [53–55]. In CSPM, $\eta/s$ first decreases and after reaching a minimum value, it starts increasing with temperature. Thus, it forms a dip which occurs at $T/T_c = 1$. The quasiparticle model results [52] show a similar behaviour but the dip does not occur at critical temperature in this case. We notice that CSPM results stay little higher and vary more rapidly with temperature than the results obtained in the quasiparticle model.

Recently, the ratio $(\eta/s)/(\sigma_{el}/T)$ has gained a considerable interest in heavy-ion phenomenology [28]. QGP is expected as a good conductor due to the presence of deconfined color charges. A small value of $\eta/s$ suggests large scattering rates which can damp the conductivity. Since, we know that $\eta/s$ is affected by the gluon-gluon and quark-quark scatterings while $\sigma_{el}$ is only affected by the quark-quark scatterings. Thus, the ratio between them is important to quantify the contributions from quarks and gluons in various temperature regions. In this work, we have studied this ratio as a function of temperature using CSPM. In figure 3, we show the ratio of $\eta/s$ and $\sigma_{el}/T$ versus $T/T_c$. It is observed that, this ratio initially decreases rapidly as the temperature increases and tend to saturate at higher $T$. We have also shown the results obtained for the isotropic and anisotropic QGP using a quasi-particle model [52]. We find that CSPM results decrease rapidly in comparison to the quasiparticle model results. CSPM results are also confronted with the interpolated lattice QCD data [28] and explains the data satisfactorily. The dotted horizontal line is the Ads/CFT calculation [28] for strongly coupled system.

IV. SUMMARY AND OUTLOOK

In summary, we have developed a method to calculate the electric conductivity of strongly interacting matter using color string percolation approach. We use basically the well-known Drude formula for the estimation of electrical conductivity, which can be obtained after solving the Boltzmann transport equation in relaxation time approximation assuming very small electric fields and no cross effects between heat and electrical conductivity. We see that the CSPM results for the conductivity increase with increasing temperature in a similar fashion as shown by IQCD data but quantitatively both the results are very different. The CSPM results lie well above the IQCD results for all the temperatures except around $T = 0.1$–$0.15$ GeV. The temperature dependence of $\sigma_{el}/T$ is very similar to the results from PHSD and there is a good agreement between these two models. We have shown $\eta/s$ as a function of $T/T_c$ and compared our results with quasiparticle model results for isotropic and anisotropic case, and IQCD data. A similar behaviour is found for CSPM results as shown in IQCD data and quasiparticle model predictions. But, our results lie above the results obtained from other models. We have also plotted the ratio, $(\eta/s)/(\sigma_{el}/T)$ as a function of $T$, which decreases rapidly at lower temperature and seems to saturate at very high $T$. We have confronted CSPM results with the results obtained in quasiparticle model for isotropic and anisotropic QGP medium and IQCD predictions. The results obtained in CSPM lie below to the quasiparticle model calculations particularly at higher temperatures.
and explain the IQCD data within errorbars.

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