Abstract  Rolling bearing and squeeze film damper will introduce structural nonlinearity into the dynamic model of aeroengine. Rubbing will occur due to the clearance reduction design of the engine. The coupling of structural nonlinearity and fault nonlinearity will make the engine present rich vibration responses. This paper aims to analyze the nonlinear vibration behavior of the whole aeroengine including rolling bearing and squeeze film damper under rubbing fault. Firstly, the dynamic model of a turboshaft engine with nonlinear support and rubbing fault is established; The rolling bearing force, the oil film force and the rubbing force are introduced into a dual-rotor–casing model with six support points. Secondly, the linear part of the model is verified by the dynamic characteristics of the three-dimensional finite element model. Finally, the varying compliance vibration, the damping effect and the bifurcation mechanism are analyzed in detail in which the bearing clearance, speed ratio and rubbing stiffness are considered. Results show that the rubbing fault in the nonlinear support case will excite more significant varying compliance vibration in the low-speed region and expand the rotating speed range of the chaotic region in the high-speed region compared with that in the linear support case.

Keywords  Whole aeroengine model · Nonlinear vibration analysis · Rolling bearing · Squeeze film damper · Rubbing · Bifurcation

1 Introduction

The rotor-bearing-casing system is the core structure of a whole aeroengine including multi-nonlinearities. To reduce the vibration level during operation, the supporting structures are generally composed of rolling bearing and squeeze film damper (SFD). The complex relationships among these supports lead to the strong nonlinearity of the whole system. As the clearance between blade and casing becomes smaller and smaller for higher efficiency, the rub impact of blade–casing will be easily caused by the nonlinear supports, which will exhibit more complex nonlinearity [1]. These nonlinear factors will have a very distinct influence on the motion form and stability of
the aeroengine. According to the actual need in practice, accurate prediction of the nonlinear dynamic response is the key issue to solving structural design problems. Therefore, it is necessary to investigate the nonlinear dynamic response of the rotor-bearing-casing system considering the multi-nonlinearities including nonlinear bearing force, nonlinear oil film force and nonlinear rubbing force.

A three-dimensional (3D) finite element model is the model that can best reflect the real structure of an aeroengine. These models created by a fine mesh are widely used for dynamic characteristics [2, 4], stochastic response [3] and vibration energy transmission [5]. Although the finite element model with 3D solid elements is adapted to represent the structural characteristics of aeroengine, the nonlinear factors of rolling bearing, SFD and rubbing in these 3D whole aeroengine models have rarely been considered. The reason is that when these nonlinearities are considered in these finite element models, the computational cost for the dynamic characteristic and response analysis will be extremely large, which limits the application of this kind of modeling technique.

For that reason, many scholars have carried out abundant research considering the nonlinearities of bearing and oil film using the beam models, because the beam models are easier to be established and improve computational efficiency. In some models, only the nonlinear bearing forces were considered [6–11]. In other models, only nonlinear oil film forces were considered [12–19]. However, the supporting structure of a real aeroengine is a combination of SFD and rolling bearing. Due to the internal interaction of the composite supports, complex nonlinear supporting forces are generated, and the system exhibits abundant nonlinear phenomena. Thus, more and more scholars consider both nonlinear forces simultaneously in system modeling. The non-synchronous responses [20], the nonlinear dynamic response under bearing inner race defect [21] and the stability at different locations [22] were studied for single rotor systems, respectively. More abundant nonlinear dynamic response research and experimental verification were carried out for a double-rotor system with dual-frequency excitation [23]. However, the nonlinear dynamic characteristics of the whole aeroengine cannot be truly reflected only through the research of the rotor system. The flexibility of the casing is another important factor that must be considered. Furthermore, based on the combined supporting structure, some researchers studied the nonlinear dynamics of the whole aeroengine model including the casing [24–30], to more precisely describe the nonlinear dynamic behavior of the structure.

In addition to the nonlinearity of support, scholars were also constantly studying the complex vibration problems caused by fault nonlinearity. The nonlinear vibrations of the aeroengine structure may lead to various kinds of vibration faults, e.g., the blade–casing rub [31]. More serious nonlinearity will be caused by the blade–casing rub, which has been studied by many researchers. In a considerable part of rubbing studies [32–42], nonlinear bearings were often linearized as linear spring for the convenience of model simplification. However, in the real situation, the supporting structure of an aeroengine is composed of rolling bearings and SFDs, and there is a complex displacement relationship between the supporting parts. Anisotropy is the inherent characteristic of the support structures [23]. The expression of linear spring and damping is not capable of describing the complex nonlinear displacement relationship in the supporting structure. Besides, the complex nonlinear vibration caused by the supporting structure is also the source of many faults of the aeroengine, such as rubbing. So, the over-simplified linear supports cannot accurately predict the response of the whole structure. The supporting structure must be considered as nonlinear in the analysis of the whole aeroengine with rub fault.

As the research moves along, the nonlinearity of the support was gradually considered in the study of rubbing. The vibration under the coupling effect of nonlinear support and rubbing was revealed by numerical and experimental methods. In the early years, many scholars studied the nonlinear dynamics of simple single rotor systems by combining the nonlinear oil film force and rubbing force [43–48]. Although the bifurcation and chaos mechanisms caused by nonlinear support and rubbing can be revealed, these models are unable to include the dual-frequency excitation in more complex double-rotor systems. Later, the mechanism of nonlinear support and rubbing under dual-frequency excitation attracted more attention. Wang et al. [49] and Jin et al. [50] adopted a more complex double-rotor-bearing system to conduct nonlinear rubbing dynamics research. More complex frequency combinations were observed, which explained the complexity of the dynamic
behavior of the dual-rotor system. In addition to the dual-rotor system, the casing is another important structure in a complete aeroengine. The path of force transmission between the rotor and the aircraft is rotor-support-bearing-house-strut-casing-mounting joint-aircraft. Therefore, it is of engineering significance to study the rubbing vibration of the whole machine in a complete rotor-bearing-casing system. In recent years, in consideration of only the nonlinearity of the rolling bearing, the rubbing mechanism of a rotorbearing-casing system was revealed by Chen [51, 52] and Yang et al. [53]. Although rolling bearings or SFDs are considered in these rub studies, the actual supporting structure is a combination of them. The nonlinear bearing forces and the nonlinear oil film forces coexist in an aeroengine system. Under the combined action of the dynamic characteristics of oil film and the time-varying contact of the rolling body, the rubbing fault of a whole aeroengine needs to be more comprehensively studied.

The above studies on rubbing faults can be divided into two categories according to whether the casing was considered. Emphasis was put on the single rotor or dual-rotor without casing in references [32–36, 38–40, 42–46, 48–50]. Considering the flexibility of the casing, researchers paid attention to the nonlinearity of the rotor-bearing-casing [37, 41, 51–53]. The nonlinear bearing force and the nonlinear oil film force were not considered simultaneously in these rub researches. The bearing force, nonlinear oil film force and nonlinear rubbing force are only considered simultaneously in a rotor system [54] and a rotor-ball bearing-stator coupling system with a single rotor [24]. Most of the real aeroengine structures contain dual-rotor structures, and the rub impact mechanism under double-frequency excitation is more complex [17]. Overall, the mechanism of the nonlinear vibration of the whole dual-rotor aeroengine system under multi-nonlinearities, including the nonlinear Hertz contact force from rolling bearing, the nonlinear oil film force from SFD and the nonlinear rubbing force, should be further studied to make the analysis closer to the real status of a whole aeroengine, that is, the inspiration of this study.

This research work aims to establish a dual-rotor rolling bearing-SFD-casing system for studying the mechanism of the nonlinear vibration of whole dual-rotor aeroengine system under multi-nonlinearities including the nonlinear Hertz contact force from rolling bearing, the nonlinear oil film force from SFD and the nonlinear rubbing force. The following paper is organized as follows. In Sect. 2, a dual-rotor whole engine model is established according to a real turboshaft engine. The nonlinear rolling bearing force, the nonlinear film force of the SFD and blade-casing rub are all considered. In Sect. 3, the dynamic characteristics of the linear part of the engine’s model are verified by that of the corresponding complete 3D finite element model. In the fourth section, the VC vibration of the rolling bearing, the damping effect of the extruded oil film damper and the bifurcation phenomenon caused by rubbing under linear/nonlinear supports are all studied. An in-depth parametric analysis involving bearing clearance, oil film thickness, rotating speed ratio and rubbing stiffness was performed.

2 Methodology

2.1 Simplification and finite element modeling of the aeroengine

A dual-rotor turboshaft aeroengine is shown in Fig. 1. The gas generator rotor is a high-pressure rotor, which is supported by two bearings (bearing3 and bearing4). The power turbine rotor is a low-pressure rotor, which is supported by four bearings (bearing1, bearing2, bearing5 and bearing6). The front housing is shared by bearing1 to bearing3, while the back housing is shared by bearing4 to bearing6. The load is transferred between the housing and casing through an integrated strut. Oil film exists between the outer ring of each bearing and the housing. The existence of the SFD can reduce the vibration of the rotor.

The real aeroengine structure is very complex as shown in Fig. 1. The complete 3D finite element model based on the real structure is usually not suitable for nonlinear vibration analysis because of its high computational cost. Therefore, a simplified finite element model which can reflect the dynamic characteristics of the structure is often adopted in this case [25]. The principles of simplification are as follows:

1. The rotating shafts are considered as beam elements which include shear deformations, gyroscopic moments and inertia.
2. The disks are assumed as mass elements without considering deformation.
3. The casing and housing are considered as beam elements without gyroscopic moments.
4. The connections between casing and foundation, casing and housing as well as the squirrel cages are considered as linear springs.
5. The forces between the bearing outer ring and the rotor are considered as nonlinear bearing forces.
6. The forces between the bearing outer ring and the bearing housing are considered as nonlinear oil film forces.

According to the simplification principles and the structure of the real turboshaft engine, a simplified finite element model as shown in Fig. 2 can be established. The finite element model consists of 30 elements and 41 nodes. The casing, rotor and housing are all modeled by beam elements. The outer ring of the bearing and blade disk is modeled by discrete mass. Linear springs and dampers are used to model mounting joints and struts. Both the nonlinear bearing force and oil film force are adopted to model different kinds of contact load between rotor and housing. The rubbing force model is introduced at the rubbing point. All the components are coupled by linear springs and nonlinear forces, and a nonlinear dynamic model of the whole aeroengine is formulated.

![Fig. 1 Sketch of a dual-rotor turboshaft aeroengine with six supports](image1)

![Fig. 2 Finite element model of the whole aeroengine](image2)
2.2 Rolling bearing force model

The cross section of a typical rolling bearing is shown in Fig. 3. The rolling bearing is composed of inner ring, outer ring, rolling element and cage. According to the Hertz contact theory, contact deformation between the rolling element and raceway will produce a nonlinear restoring force. For ball bearing, the bearing force can be expressed as [55],

\[
F_{bx} = \sum_{j=1}^{N_b} C_b (x \cos \beta_j + y \sin \beta_j - \gamma_0)^{3/2} H(x \cos \beta_j + y \sin \beta_j - \gamma_0) \cos \beta_j
\]

\[
F_{by} = \sum_{j=1}^{N_b} C_b (x \cos \beta_j + y \sin \beta_j - \gamma_0)^{3/2} H(x \cos \beta_j + y \sin \beta_j - \gamma_0) \sin \beta_j
\]

(1)

For roller bearing, the bearing force can be expressed as [56],

\[
F_{bx} = \sum_{j=1}^{N_b} C_b (x \cos \beta_j + y \sin \beta_j - \gamma_0)^{10/9} H(x \cos \beta_j + y \sin \beta_j - \gamma_0) \cos \beta_j
\]

\[
F_{by} = \sum_{j=1}^{N_b} C_b (x \cos \beta_j + y \sin \beta_j - \gamma_0)^{10/9} H(x \cos \beta_j + y \sin \beta_j - \gamma_0) \sin \beta_j
\]

(2)

where \( N_b \) is the number of rolling elements, \( C_b \) is the Hertz contact stiffness, and \( x \) and \( y \) represent the relative displacements between the rotor and outer ring in \( x \)- and \( y \)-directions, respectively,

\[
x = x_{\text{rotor}} - x_{\text{outering}}
\]

\[
y = y_{\text{rotor}} - y_{\text{outering}}
\]

\( H(\cdot) \) is the Heaviside function, \( \gamma_0 \) is the radial clearance of bearing, \( \beta_j \) is the angle of \( j \)-th rolling element, and

\[
\beta_j = \omega_{\text{cage}} \times t + 2\pi/N_b(j - 1), j = 1, 2, \ldots, N_b
\]

\[
\omega_{\text{cage}} = \frac{\omega R_i}{R_0 + R_i}
\]

\( \omega_{\text{cage}} \) is the rotating speed of the cage, \( \omega \) is the rotating speed of the rotor, \( R_i \) is the radius of the inner ring, and \( R_0 \) is the radius of the outer ring.

2.3 Oil film force model

The structure of SFD is shown in Fig. 4. A certain clearance is reserved between the outer ring of the bearing and the housing. The rotation of the outer ring of the bearing is restricted by the squirrel cage which is elastic support for the rotor. Lubricating oil fills the gap between the outer ring of the bearing and the housing. The viscous damping of the lubricating oil plays an important role in reducing the vibration of the rotor. According to the assumption of short bearing, the nonlinear oil film force in \( x \)- and \( y \)-directions is [57]

\[
F_{oix} = -\frac{\mu R L^3}{C^2 \sqrt{X_i^2 + Y_i^2}} [X_i (I_1 \dot{e} + I_2 \dot{e}' - I_3 \phi') - Y_i (I_2 \dot{e} + I_3 \phi')]
\]

\[
F_{oiy} = -\frac{\mu R L^3}{C^2 \sqrt{X_i^2 + Y_i^2}} [Y_i (I_1 \dot{e} + I_2 \dot{e}') + X_i (I_2 \dot{e} + I_3 \phi')]
\]

(6)

where \( \mu \) is the viscosity of lubricating oil, \( R \) is the radius of damper, \( L \) is the length of damper, \( C \) is the clearance of SFD, \( X_i = x_i/C \), \( Y_i = y_i/C \), and \( x_i \) and \( y_i \) are the relative displacements of the outer ring of the bearing and the housing in \( x \)- and \( y \)-directions, respectively.

\[
x_i = x_{\text{outering}} - x_{\text{housing}}
\]

\[
y_i = y_{\text{outering}} - y_{\text{housing}}
\]

\( \varepsilon \) is eccentricity,

\[
\varepsilon = \sqrt{X_i^2 + Y_i^2}
\]

\[
\varepsilon' = \frac{x_i \dot{x}_i + y_i \dot{y}_i}{C \sqrt{X_i^2 + Y_i^2}}
\]

(8)

\( \phi \) is the angle of journal precession and \( \phi' \) is its first derivative, and
\[ \theta_{bi} = \frac{2\pi i}{N} + \omega t \] (12)

The normal rubbing force can be expressed as

\[ F_{iNx} = -F_{iNy} \cos(\theta_{bi}) \]
\[ F_{iNy} = -F_{iNy} \sin(\theta_{bi}) \] (13)

where

\[ F_{iNy} = \frac{1}{2} k_c \{ |d_{bi} - d_{ci} - \delta(\theta_{bi})| + |d_{bi} - d_{ci} - \delta(\theta_{bi})| \} \] (14)

\[ \delta(\theta_{bi}) = \begin{cases} \delta, & |\theta_{bi} - \theta| > \beta \\ \delta - A \left[ 0.5 + 0.5 \cos \left( \frac{\pi (\theta_{bi} - \theta)}{\beta} \right) \right], & |\theta_{bi} - \theta| \leq \beta \end{cases} \] (15)

The tangential rubbing force can be expressed as

\[ F_{iTx} = F_{iT} \sin(\theta_{bi}) \]
\[ F_{ITy} = -F_{IT} \cos(\theta_{bi}) \] (16)

where

\[ F_{iT} = fF_{iNy} \] (17)

\( f \) is the coefficient of friction.

Consequently, at time \( t \), the rubbing force acting on the rotor can be expressed as the combined rubbing force of \( N \) blades [52, 58].

\[ F_{iN} = \sum_{i=1}^{N} (F_{iNx} + F_{iTx}) \]
\[ F_{iN} = \sum_{i=1}^{N} (F_{iNy} + F_{ITy}) \] (18)

\[ \phi = \arctan \left( \frac{y_i}{x_i} \right) \]
\[ \phi' = \frac{x_i y_i' - y_i x_i'}{x_i^2 + y_i^2} \] (9)

\[ I_1, I_2 \text{ and } I_3 \text{ are the Sommerfeld integrals, and } \theta_1 \text{ is the} \]

starting point of the oil film positive pressure zone,

\[ I_1 = \int_{\theta_1}^{\theta_1 + \pi} \frac{\cos^2 \theta}{(1 + \varepsilon \cos \theta)^3} d\theta \]
\[ I_2 = \int_{\theta_1}^{\theta_1 + \pi} \frac{\sin \theta \cos \theta}{(1 + \varepsilon \cos \theta)^3} d\theta \]
\[ I_3 = \int_{\theta_1}^{\theta_1 + \pi} \frac{\sin^2 \theta}{(1 + \varepsilon \cos \theta)^3} d\theta \]
\[ \theta_1 = \arctan \left( -\frac{\varepsilon'}{\varepsilon} \right) \] (10)

2.4 Rubbing force model

A typical blade–casing rubbing model is shown in Fig. 5. By setting local deformation at one position of the casing, the single-point rubbing of the casing is realized. Assuming that the rotating speed is \( \omega \), the number of blades on the disk is \( N \), the initial clearance between the rotor and stator is \( d \), \( k_c \) is the stiffness of casing, and \( A \) is the deformation of the casing at angle \( \theta \). The rubbing region is in the angle range within \( \pm \beta \). The radial displacement of the blade and casing can be expressed as

\[ d_{bi} = x_{blade} \cos(\theta_{bi}) + y_{blade} \sin(\theta_{bi}) \]
\[ d_{ci} = x_{casing} \cos(\theta_{bi}) + y_{casing} \sin(\theta_{bi}) \] (11)

where \( \theta_{bi} \) is the angle between the \( i \)-th blade and \( x \)-axis at time \( t \),
2.5 Governing equation of the coupling system

Rotor1, rotor2, housing and casing are all modeled by beam elements that have 4 degrees of freedom (DOFS) for each node. The outer ring of the bearing is modeled by mass points containing 2 DOFS.

The governing equation of rotor1 can be expressed as

\[
M_{r1} \ddot{u}_{r1} + (C_{r1} - \omega_1 G_{r1}) \dot{u}_{r1} + K_{r1} u_{r1} = F_{r1}
\]  

(19)

\[
F_{r1} = [-F_{b1}, \ldots, -F_{a1}, \ldots, -F_{rub1}, \ldots, F_{gr1}]
\]  

(20)

where \(u_{r1i} = [x_{r1i}, y_{r1i}, \theta_{x1i}, \theta_{y1i}]^T\) is the displacement for the \(i\)th node of rotor1, and \(\omega_1\) is the rotating speed of rotor1. \(M_{r1}, C_{r1}, G_{r1}\) and \(K_{r1}\) are the mass matrix, damping matrix, gyroscopic matrix and stiffness matrix, respectively. Rayleigh damping is assumed. The generalized external force \(F_{r1}\) is composed of the bearing force \(F_{b1}\), the unbalanced force \(F_{a1}\), the rubbing force \(F_{rub1}\) and the force of gravity \(F_{gr1}\). The unbalanced force on the \(i\)th node of rotor1 is \(F_{u1i} = [m_1 \epsilon_1 \omega_2 \cos \omega_1 t, m_1 \epsilon_1 \omega_2 \sin \omega_1 t]^T\).

The governing equation of rotor2 can be expressed as

\[
M_{r2} \ddot{u}_{r2} + (C_{r2} - \omega_2 G_{r2}) \dot{u}_{r2} + K_{r2} u_{r2} = F_{r2}
\]  

(21)

\[
F_{r2} = [-F_{b2}, \ldots, -F_{a2}, \ldots, -F_{rub2}, \ldots, F_{gr2}]
\]  

(22)

where \(u_{r2i} = [x_{r2i}, y_{r2i}, \theta_{x2i}, \theta_{y2i}]^T\) is the displacement for the \(i\)th node of rotor2, and \(\omega_2\) is the rotating speed of rotor2. \(M_{r2}, C_{r2}, G_{r2}\) and \(K_{r2}\) are the mass matrix, damping matrix, gyroscopic matrix and stiffness matrix, respectively. Rayleigh damping is applied. The generalized external force \(F_{r2}\) is composed of the bearing force \(F_{b2}\), the unbalanced force \(F_{a2}\), the rubbing force \(F_{rub2}\) and the force of gravity \(F_{gr2}\). The unbalanced force on the \(i\)th node of rotor2 is \(F_{u2i} = [m_2 \epsilon_2 \omega_2 \cos \omega_2 t, m_2 \epsilon_2 \omega_2 \sin \omega_2 t]^T\).

The governing equation of casing can be expressed as

\[
M_{ch} \ddot{u}_c + C_{ch} \dot{u}_c + K_{ch} u_c = F_c
\]  

(23)

\[
F_c = [\ldots, F_{ch}, \ldots, F_{rub}, \ldots, F_{gc}]
\]  

(24)

where the displacement for the \(i\)th node of the casing is \(u_{ci} = [x_{ci}, y_{ci}, \theta_{xci}, \theta_{yci}]^T\), and \(M_{ch}, C_{ch}\) and \(K_{ch}\) are the mass matrix, damping matrix and stiffness matrix, respectively. Rayleigh damping is applied. The generalized external force \(F_c\) is composed of the elastic force \(F_{ch}\) between the housing and the casing, the rubbing force \(F_{rub}\) and the force of gravity \(F_{gc}\).

The governing equation of housing can be expressed as

\[
M_{ho} \ddot{u}_h + C_{ho} \dot{u}_h + K_{ho} u_h = F_h
\]  

(25)

\[
F_h = [\ldots, -F_{ch}, \ldots, -F_{ho}, \ldots, F_{oil}, \ldots, F_{gh}]
\]  

(26)

where \(u_{hi} = [x_{hi}, y_{hi}, \theta_{xhi}, \theta_{yhi}]^T\) is the displacement for the \(i\)th node of housing, and \(M_{ho}, C_{ho}\) and \(K_{ho}\) are the mass matrix, damping matrix and stiffness matrix, respectively. Rayleigh damping is applied. The generalized external force \(F_h\) is composed of the elastic force \(F_{ch}\) between the housing and the casing, the elastic force \(F_{ho}\) between the housing and the outer ring, the oil film force \(F_{oil}\) and the force of gravity \(F_{gh}\), where \(F_{ho} = K_{ho} u_{hi} - K_{ho} u_{ho}\), the displacement for the \(i\)th node of the outer ring is \(u_{oi} = [x_{oi}, y_{oi}, \theta_{xoi}, \theta_{yoi}]^T\), and \(K_{ho}\) is the stiffness of squirrel cage.
The governing equation of the outer ring of bearing can be expressed as

\[ M_o \ddot{u}_o = F_o \]  \hspace{1cm} (27)

\[ F_o = [\ldots, F_{ho}, \ldots, F_b, \ldots, -F_{oil}, \ldots, F_{go}] \]  \hspace{1cm} (28)

where \( M_o \) is the mass matrix. The generalized external force \( F_o \) is composed of the elastic force \( F_{ho} \) between the housing and the outer ring, the bearing force \( F_b \), the oil film force \( F_{oil} \) and the force of gravity \( F_{go} \).

The governing equation of the whole system can be expressed as

\[
\begin{bmatrix}
M_{r1} & M_{r2} \\
M_{r2} & M_r \\
C_{r1} - \omega_1 G_{r1} & C_{r2} - \omega_2 G_{r2} \\
C_{r2} - \omega_2 G_{r2} & C_c \\
K_{r1} & K_{r2} \\
K_{r2} & K_c + K_r - K_s \\
-K_s & K_h + K_m + K_i - K_m \\
-K_m & K_m
\end{bmatrix}
\begin{bmatrix}
\dot{u}_{r1} \\
\dot{u}_{r2} \\
\dot{u}_c \\
\dot{u}_h \\
\ddot{u}_o
\end{bmatrix}
= \begin{bmatrix}
F_{r1} \\
F_{r2} \\
F_c \\
F_h \\
F_o
\end{bmatrix}
\]  \hspace{1cm} (29)

Equation (29) is solved by the Newmark-\( \beta \) method, and the time step is set as \( 10^{-6} \) s in this study.

3 Validation

As shown in Fig. 2, the connections in the whole aeroengine model contain both linear and nonlinear parts. The mount, the strut and the squirrel cage are linear parts, while the rolling bearing and the SFDs are nonlinear parts. To verify the accuracy of the linear part of the model, a comparative study on the dynamic characteristics of the complete 3D finite element model will be conducted in this section. The structural dimensions and material parameters for rotors and stators are, respectively, shown in Tables 1 and 2. A 3D finite element model of a whole aeroengine as shown in Fig. 6 is established in the commercial finite element software. All the nonlinearities in SFDs and rolling bearings are ignored, and these connections are replaced by linear springs. According to the supporting form in Fig. 1, the first fulcrum and fifth fulcrum are assumed as rigid supports with stiffness coefficients equal to \( 10 \times 10^7 \) N/m. The other four fulcrum points are regarded as elastic supports with stiffness coefficients equal to \( 2.5 \times 10^7 \) N/m. The results of the first three vibration modes calculated from the proposed model are compared with those obtained by the 3D finite element model in Table 3 and Fig. 7. The errors in the modal frequencies are 5.2%, 4.1% and 3.3%, respectively. The accuracy of the linear part of the model is verified by the coincidence of mode shape and frequency. Based on the verified linear part of the model, the nonlinear behavior of the whole aeroengine will be carried out by adding the nonlinear bearing force, oil film force and rubbing force on the verified linear part of the model.

4 Results and discussion

4.1 Dynamic response of the whole engine without rubbing

The nonlinear response of the whole machine without rubbing is studied in this section. SFDs are installed on Nos. 2, 3, 4 and 6 bearings. The initial elastic supporting stiffness is \( 2.5 \times 10^7 \) N/m. Nos. 1 and 5 bearings are set as rigid supports with initial stiffnesses equal to \( 10 \times 10^7 \) N/m. The rotating speed ratio is 1.5. The unbalance on node 5 is \( 1 \times 10^{-5} \) kg m and on node 14 is \( 2 \times 10^{-5} \) kg m.

4.1.1 VC vibration of rolling bearing

When the rolling body passes through the loading zone periodically, it will cause varying stiffness excitation. When the frequency of the parametric excitation is close to the natural frequencies of the rotor system under certain parameter conditions and load conditions, it may cause resonance, which is regarded as VC vibration. The VC vibrational frequency is \( B_n \) times the rotating frequency. \( B_n \) is a number related to the radius of the inner and outer ring and the number of rolling bodies, which can be expressed by,

\[ B_n = \frac{R_i}{R_0 + R_i} \times N_b \]  \hspace{1cm} (30)
$B_{n1}$ and $B_{n2}$ have values equal to the VC frequency multiplied by the rotating frequency for rotor1 and rotor2, respectively. $f_{vc1}$ and $f_{vc2}$ are the VC frequencies of rotor1 and rotor2, respectively. According to the bearing parameters, $B_{n1}$ is 7.2 for rotor1 and $B_{n2}$ is 6.75 for rotor2 as shown in Table 4. VC frequency at low rotating speed is shown in Fig. 8 and Fig. 9 for rotor1 and rotor2, respectively. In this case, the VC frequency and its octave are easy to be observed. Fukata et al. [55] found that the rotating frequency of the ball and its harmonic are the main frequency component when the rotating speed is far away from the critical speed, which is consistent with the results in this paper.

### Table 1  The structural dimensions and material parameters for rotors

| Parameter                           | The value between the $i$th node and the $j$th node |
|-------------------------------------|-----------------------------------------------------|
| Length of rotor1 (m)                | $L_{12}$ $L_{23}~L_{34}~L_{45}~L_{56}~L_{67}$     |
|                                     | 0.1 0.065 0.05 0.1 0.11 0.095                      |
| Length of rotor2 (m)                | $L_{89}~L_{910}~L_{1011}~L_{1112}~L_{1213}~L_{1314}~L_{1415}~L_{1516}$ |
|                                     | 0.1 0.09 0.19 0.25 0.265 0.04 0.035 0.04          |
| Outer diameter of rotor1 (m)        | $D_{12}~D_{23}~D_{34}~D_{45}~D_{56}~D_{67}$      |
|                                     | 0.05 0.074 0.1 0.1 0.1 0.05                       |
| Outer diameter of rotor2 (m)        | $D_{89}~D_{910}~D_{1011}~D_{1112}~D_{1213}~D_{1314}~D_{1415}~D_{1516}$ |
|                                     | 0.06 0.045 0.03 0.03 0.03 0.045 0.045 0.045       |
| Inner diameter of rotor1 (m)        | $d_{12}~d_{23}~d_{34}~d_{45}~d_{56}~d_{67}$     |
|                                     | 0.04 0.064 0.09 0.09 0.09 0.038                   |
| Inner diameter of rotor2 (m)        | $d_{89}~d_{910}~d_{1011}~d_{1112}~d_{1213}~d_{1314}~d_{1415}~d_{1516}$ |
|                                     | 0.05 0.03 0.02 0.02 0.02 0.04 0.04 0.04           |
| Elastic modulus of rotor1 (GPa)     | $E_{12}~E_{23}~E_{34}~E_{45}~E_{56}~E_{67}$     |
|                                     | 196 121 121 121 121 196                          |
| Elastic modulus of rotor2 (GPa)     | $E_{89}~E_{910}~E_{1011}~E_{1112}~E_{1213}~E_{1314}~E_{1415}~E_{1516}$ |
|                                     | 204 204 204 204 204 204 204 204                   |
| Density of rotor1 (kg/m³)           | $\rho_{12}~\rho_{23}~\rho_{34}~\rho_{45}~\rho_{56}~\rho_{67}$ |
|                                     | 7800 4480 4480 4480 7800 7800                     |
| Density of rotor2 (kg/m³)           | $\rho_{89}~\rho_{910}~\rho_{1011}~\rho_{1112}~\rho_{1213}~\rho_{1314}~\rho_{1415}~\rho_{1516}$ |
|                                     | 8240 8240 8240 8240 8240 8240 8240 8240           |
| Mass of disk (kg)                   | $m_{2}~m_{3}~m_{4}~m_{5}~m_{6}~m_{14}~m_{16}$   |
|                                     | 1.008 0.393 0.177 3.253 3.395 2.859 4.345        |
| Diametric inertial moment (10⁻⁶ kg m²) | $J_{dd2}~J_{dd3}~J_{dd4}~J_{dd5}~J_{dd6}~J_{dd14}~J_{dd16}$ |
|                                     | 1.05 0.438 0.145 7.438 5.763 9.15 13.95          |
| Polar inertial moment (10⁻⁶ kg m²)  | $J_{pd2}~J_{pd3}~J_{pd4}~J_{pd5}~J_{pd6}~J_{pd14}~J_{pd16}$ |
|                                     | 1.945 0.856 0.289 14.21 11.13 17.49 25.13        |
| Poisson’s ratio                     | 0.3                                                 |

4.1.2 Effect of the rotating speed on bifurcation

Keeping the unbalances unchanged, the effect of rotating speed on bifurcation characteristics is studied with parameters in Table 4 in this section. Nodes 5 and 14 are selected as observation points for rotor1 and rotor2, respectively. Two sets of support stiffness are adopted as shown in Table 5. The rotating speed of rotor1 and rotor2 varies in [6300–42300] rpm and [4200–28200] rpm, respectively. Bifurcation diagrams of rotor1 and rotor2 are shown in Figs. 10 and 11, respectively.

As shown in Fig. 10a, with higher support stiffness and smaller bearing clearance, the dynamic response of rotor1 varies with the rotating speed as follows. When $\omega_1 < 14,400$ rpm, rotor1 is in the motion of
period 1; when $14,400 \text{ rpm} < \omega_1 < 25,200 \text{ rpm}$, period-doubling appears, and with the increase in rotating speed, the system motion is quasiperiodic; and when $\omega_1 > 25,200 \text{ rpm}$, through inverse bifurcation, rotor1 gets out of quasiperiodic and returns to the motion of period 1.
When the bearing clearance increases under the same supporting stiffness as used in Fig. 10a, the bifurcation of rotor1 is shown in Fig. 10b, and the nonlinear components of the system become more apparent. When $\omega_1 < 7200$ rpm, VC vibration becomes more obvious as the clearance of rolling bearing increases, and the motion is quasiperiodic. When $7200$ rpm $< \omega_1 < 9450$ rpm, with the increase of $\omega_1$, the rotating frequency gradually becomes the main component, and the motion is periodic. When $9450$ rpm $< \omega_1 < 26550$ rpm, the motion of rotor1 enters chaos through period-doubling bifurcation and then gets out of chaos by an explosive bifurcation. When $\omega_1 > 26,550$ rpm, the motion is quasiperiodic. It can be seen that a larger bearing clearance will bring serious nonlinearity and strongly increases the instability of the system.

Figure 10c represents the bifurcation at a lower support stiffness in comparison with Fig. 10a. Rotor1 takes a series of motions as $\{P1 \rightarrow P3 \rightarrow P1\}$. Compared with Fig. 10a, the motion form of period 1 occupies a longer rotating speed range which indicates that the lower support stiffness will result in better stability.

As shown in Fig. 11a, with higher support stiffness and smaller bearing clearance, the dynamic response of rotor2 varies with the rotating speed as follows: When $\omega_2 < 15,000$ rpm, the rotor2 enters quasiperiodic through period-doubling bifurcation. With the increase of $\omega_2$, the motion is periodic until $27,300$ rpm. When $\omega_2 > 27,300$ rpm, quasiperiodic appears through period-doubling.

When the bearing clearance increases under the same supporting stiffness as in Fig. 11a, the bifurcation of rotor2 is shown in Fig. 11b, and the nonlinear components of the system become more apparent. The motion of Rotor2 enters chaos through an explosive bifurcation when $\omega_2 < 7200$ rpm. Then, the motion is mainly in chaos. It can be seen that a larger bearing clearance will bring serious nonlinearity and make the motion form of the system very unstable.

Figure 11c represents the bifurcation at a lower support stiffness in comparison with Fig. 11a. When $\omega_2 < 15,000$ rpm, the rotor2 enters quasiperiodic through period-doubling bifurcation. When $15,000$ rpm $< \omega_2 < 20,400$ rpm, the motion is period1. When $\omega_2 > 20,400$ rpm, period-doubling appears, with the increase in rotating speed, the system gets into quasiperiodic finally. Compared with Fig. 11a, the motion form of period 1 occupies a longer rotating speed range which indicates that the lower support stiffness can make better stability.

### 4.1.3 Damping effect of the SFD

The function of the SFD is to reduce vibration. The design of the SFD should be conducted to achieve a favorable damping effect in the design stage of the whole machine structure. It can be seen from Eq. (6) that the main parameters affecting the oil film force are the clearance of SFD $C$, the viscosity of lubricating oil $\mu$, damper length $L$ and damper radius $R$. By keeping the structural parameters and bearing parameters unchanged, the influence of SFD parameters on the damping effect is studied in this section. The steady response of node 4 on rotor1 and node 14 on rotor2 is selected to observe the effect of vibration reduction.

As shown in Figs. 10a and 11a, the first critical speed of rotor1 is 10,800 rpm, and the first critical speed of rotor2 is 10,200 rpm. Steady responses of rotor1 and rotor2 at critical speeds are shown in Figs. 12 and 13, respectively. The vibration reduction efficiency of rotor1 and rotor2 is shown in Tables 6 and 7, respectively. The initial parameters of SFD are set as $\mu = 3.8 \times 10^{-3}$ Pa s, $L = 15$ mm and $R = 40$ mm. The steady responses of rotor1 and rotor2 with different $C$ are shown in Figs. 12a and 13a, respectively. Because $C$ is inversely proportional to the oil film force, with the decrease of $C$, the vibration reduction effect becomes more remarkable. When the thickness of the oil film is less than 0.1 mm, the vibration amplitude is greatly reduced. When $C = 0.05$ mm, the vibration reduction efficiency of rotor1 is 90.7%, and the vibration reduction efficiency of rotor2 is 87.1%. However, the decrease in oil film thickness will increase the difficulty of processing and manufacturing. Therefore, $C = 0.1$ mm is chosen in the following study. The influence of lubricating oil

| Mode order | Proposed model | 3D model | Errors (%) |
|------------|---------------|----------|------------|
| 1          | 140.73        | 148.38   | 5.2        |
| 2          | 210.79        | 219.53   | 4.1        |
| 3          | 270.12        | 261.47   | 3.3        |

Table 3 Comparisons of first three order frequencies between the proposed model and the 3D model (Hz)
viscosity on the damping effect is further studied which is shown in Figs. 12b and 13b. As it can be seen, a higher viscosity will increase the damping effect. When $\mu = 1.14 \times 10^{-2}$ Pa s, the vibration reduction efficiency of rotor1 is 81.2%, and the vibration reduction efficiency of rotor2 is 69.8%. Compared
with mechanical manufacturing, an optimized lubricating oil viscosity is not difficult to achieve. Therefore, \( l = 1.14 \times 10^{-2} \) Pa s is chosen, and the influence of different oil film lengths on vibration reduction is further studied in Figs. 12c and 13c. The oil film length \( L \) is a key parameter for the oil film force. Numerical results show that a larger oil film length can achieve a better damping effect. When \( L = 20 \) mm, the vibration reduction efficiency of rotor1 is 89.8\%, and the vibration reduction efficiency of rotor2 is 86.1\%. Therefore, \( L = 20 \) mm is chosen, and the influence of different oil film radii on vibration reduction is further studied in Figs. 12d and 13d. It is noted that the effect of oil film radius on vibration reduction is not significant after the design of the previous three parameters. Finally, with the optimized SFD parameters which are \( C = 0.1 \) mm, \( \mu = 1.14 \times 10^{-2} \) Pa s, \( L = 20 \) mm, \( R = 50 \) mm, the final vibration reduction efficiency of rotor1 is 91.4\%, and the final vibration reduction efficiency of rotor2 is 87.3\%, respectively.

4.2 Dynamic response of the whole engine with rubbing

To study the nonlinear effect and evolution mechanism of single point blade–casing rubbing on the whole engine system, the nonlinear dynamic responses under linear and nonlinear supporting conditions are studied, respectively, in this section. The number of blades is \( N = 20 \), the coefficient of friction is \( f = 0.3 \), and the initial clearance \( \delta \) is \( 10^{-6} \) m. The deformation of the casing at \( \theta = 180^\circ \) is \( A = 5 \times 10^{-6} \) m, and the deformation ranges in \( \beta = 5^\circ \). The unbalance on node 5 is \( 1 \times 10^{-5} \) kg m and on node 14 is \( 2 \times 10^{-5} \) kg m.
4.2.1 Bifurcation analysis with linear support

In this section, the connection between the rotor and the housing is simplified as a linear spring without considering bearing and SFD, and the support stiffness in Group2 of Table 5 is adopted. The bearing clearance

Table 5  Support stiffness at six supports (× 10^7 N/m)

| No. | 1    | 2    | 3    | 4    | 5    | 6    |
|-----|------|------|------|------|------|------|
| Group1 | 10   | 2.5  | 2.5  | 2.5  | 10   | 2.5  |
| Group2 | 5    | 1.5  | 1.5  | 1.0  | 5    | 1.5  |

4.2.1 Bifurcation analysis with linear support

In this section, the connection between the rotor and the housing is simplified as a linear spring without considering bearing and SFD, and the support stiffness in Group2 of Table 5 is adopted. The bearing clearance

Fig. 10  Bifurcation diagram of rotor1  a with support stiffness in Group1, γ0 = 1 μm, b with support stiffness in Group1, γ0 = 10 μm, c with support stiffness in Group2, γ0 = 1 μm
\( \gamma_0 = 1 \mu m \), and the rotating speed ratio \( \lambda = 1.5 \). Under
the linearized supporting condition, the rubbing stiffness \( k_c \) is the main parameter to be considered. The
bifurcation diagrams of rotor1 and rotor2 with the
change of rotating speed are shown in Figs. 14 and 15.

As shown in Fig. 14a, when \( \omega_1 < 12,000 \) rpm, the
motion is quasiperiodic mainly because the rubbing
frequency plays a major role when the rotating
frequency is lower. When \( 12,000 \) rpm \( < \omega_1 \)< 27,600 rpm, the motion is period 1. When
\( \omega_1 > 27,600 \) rpm, rotor1 enters chaos through Nei-
mark–Sacker bifurcation. Figure 14b shows bifurca-
tion diagrams with lower rubbing stiffness. By
comparing the bifurcation diagram with higher rub-
bishing stiffness, it can be found that the amplitude at the
resonant peak significantly increases, and the reso-
nance frequency shifts to the left. In the case of lower
rubbing stiffness, more periodic motion and less
chaotic motion occur. The lower rubbing stiffness

\[ fig:11 \text{ Bifurcation diagram of rotor2 a with support stiffness in Group1, } \gamma_0 = 1 \mu m, \text{ b with support stiffness in Group1, } \gamma_0 = 10 \mu m, \text{ c with support stiffness in Group2, } \gamma_0 = 1 \mu m \]

\[ fig:12 \text{ Steady response of node 5 on 10,800 rpm} \]
Fig. 13  Steady response of node 14 on 10,200 rpm

![Graphs showing steady response of node 14 on 10,200 rpm.](image)

can reduce the quasiperiodic and chaotic characteristics of rotor1.

As shown in Fig. 15a, when \( \omega_2 < 6000 \) rpm, the motion is periodic mainly because the rubbing frequency plays a major role when the rotating frequency is lower. When 6000 rpm < \( \omega_2 < 11,400 \) rpm, period-doubling appears, and rotor2 enters quasiperiodic through period-doubling bifurcation. With \( \omega_2 \) increasing, rotor2 gets out of quasiperiodic through inverse doubling bifurcation when 11,400 rpm < \( \omega_2 < 18,600 \) rpm, and then, periodic motion and chaotic motion appear alternately when \( \omega_2 > 18,600 \) rpm. Through explosive chaos, rotor2 finally enters the chaos motion.

### Table 6  Vibration reduction efficiency of rotor1

| C (mm) | Efficiency (%) | \( \mu \) (Pa·s) | Efficiency (%) | L (mm) | Efficiency (%) | R (mm) | Efficiency (%) |
|--------|----------------|-----------------|----------------|--------|----------------|--------|----------------|
| 0.2    | 5.9            | 0.0038          | 56.2           | 10     | 49.7           | 40     | 89.8           |
| 0.1    | 56.1           | 0.0076          | 77.3           | 15     | 81.2           | 45     | 90.7           |
| 0.05   | 90.7           | 0.0114          | 81.2           | 20     | 89.8           | 50     | 91.4           |

### Table 7  Vibration reduction efficiency of rotor2

| C (mm) | Efficiency (%) | \( \mu \) (Pa·s) | Efficiency (%) | L (mm) | Efficiency (%) | R (mm) | Efficiency (%) |
|--------|----------------|-----------------|----------------|--------|----------------|--------|----------------|
| 0.2    | 0.06           | 0.0038          | 39.9           | 10     | 35.2           | 40     | 84.8           |
| 0.1    | 38.8           | 0.0076          | 61.2           | 15     | 71.4           | 45     | 86.2           |
| 0.05   | 87.1           | 0.0114          | 69.8           | 20     | 86.1           | 50     | 87.3           |
Figure 15b shows a bifurcation diagram with lower rubbing stiffness. By comparing the bifurcation diagram with higher rubbing stiffness, it can be found that the amplitude at the resonant peak significantly increases, and the resonance frequency shifts to the left. In the case of lower rubbing stiffness, more periodic motion occurs. Rotor2 with rigid and elastic supports is more stable than rotor1 with full elastic supports.

4.2.2 Bifurcation analysis with nonlinear support

In real aeroengines, the supporting conditions are nonlinear, and the nonlinear mechanism in rubbing fault cannot be correctly reflected based on the linearization hypothesis. In this section, the rubbing of the blade–casing under the nonlinear supporting conditions with the rolling bearing and the SFD in Table 4 is comprehensively considered. The rubbing stiffness \( k_c = 1 \times 10^9 \text{ N/m} \). The rotating speed ratio \( \lambda \) and bearing clearance \( \gamma_0 \) are the two parameters to be considered. The bifurcation diagrams of rotor1 and rotor2 with the change of rotating speed are shown in Figs. 16 and 17, respectively.

As shown in Fig. 16a, when \( \omega_1 < 9900 \text{ rpm} \), the motion of rotor1 is quasiperiodic mainly because VC frequency plays a major role when the rotating frequency is lower. When 9900 rpm < \( \omega_1 < 21,000 \text{ rpm} \), the VC vibration is weak, and the motion is period 1. When \( \omega_1 > 21,000 \text{ rpm} \), rotor1 enters quasiperiodic through Neimark–Sacker bifurcation. Rotor1 gets into chaos finally through explosive chaos. It can be seen in Fig. 16b, when the rotating speed ratio is 1, the motion of rotor1 does not change much, except that the quasiperiodic interval caused by VC vibration extends to 10,800 rpm.

By comparing the bifurcation diagram without rubbing in Fig. 10c, it can be found from Fig. 16a that rubbing force significantly reduces the amplitude at the resonant peak, and the resonance frequency shifts to the right, which indicates that rubbing introduces the characteristics of a hard spring.
Compared with the linear support case in Fig. 14a, the nonlinear support case in Fig. 16a has the following changes: (1) The response at the resonant peak decreases; (2) the quasiperiodic motion is more obvious in the low-speed region; and (3) chaos occurs earlier in the high-speed region.

Figure 16c and d shows bifurcation diagrams with higher bearing clearance at the above two rotating speed ratios, respectively. It can be found that significant quasiperiodic and chaotic motions occur, which indicates that higher bearing clearance will increase the instability of the system under rubbing fault. The influence of the rotating speed ratio is not obvious.

As shown in Fig. 17a, b, when \( \omega_2 < 24,600 \) rpm, the motion of rotor2 is periodic and quasiperiodic. When \( \omega_2 > 24,600 \) rpm, the motion of rotor2 enters period 1. When \( \omega_2 > 27,900 \) rpm in Fig. 17b, the motion is mainly quasiperiodic. As shown in Fig. 17c, d, when the bearing clearance is higher, the motion of the system becomes more unstable, and the motion is mainly periodic and chaotic.

By comparing the bifurcation diagram without rubbing in Fig. 11c, it can be found from Fig. 17a that the resonance frequency shifts to the right, and rubbing force increases the amplitude at the resonant peak which presents different characteristics with rotor1. This phenomenon indicates that under all elastic support conditions, the rubbing force will increase the additional stiffness, while under the conditions of rigid support and elastic support, the influence of the rubbing force on the additional stiffness is not obvious.

Compared with the linear support case in Fig. 15a and the nonlinear support case as shown in Fig. 17a, the motions of rotor2 show similar characteristics which are mainly periodic and quasiperiodic in the low rotating speed region. However, in the high rotating speed region, the motion in Fig. 17a is more stable than that in Fig. 15a.

5 Conclusions

A dual-rotor whole aeroengine model with rolling bearing and SFD is established. The accuracy of the
The linear part of the model is first verified by its corresponding 3D finite element model formulated. The effectiveness of the nonlinear support is verified by the VC vibration frequency and the damping effect of the SFD. The nonlinear dynamic behavior of the whole aeroengine with rubbing fault under the nonlinear support is investigated in detail. The following conclusions can be drawn:

1. VC vibration is easy to occur in the low rotating speed region, especially in the case with larger bearing clearance and rubbing fault, and has a significant impact on the stability of the system.
2. It can be seen from the bifurcation analysis that when rubbing is not occur in the whole aeroengine, a more stable system motion form can be obtained by using lower bearing stiffness and lower bearing clearance.
3. A small oil film thickness can provide a very good effect on vibration reduction, but the design of SFDs can focus on the lubricating oil viscosity and oil film length after the oil film thickness has been determined to satisfy the convenience of processing and manufacturing. Oil film radius has a relatively weak impact on the vibration reduction, and it can be selected as the bearing outer diameter or the bearing housing inner diameter in the simulation model.
4. The motion of period, quasiperiodic and chaos are observed in the case of rubbing fault with both linear supports and nonlinear supports. Period-doubling bifurcation and explosive bifurcation are the routes to chaos.
5. The rubbing fault in the nonlinear support case will excite more significant VC vibration in the low-speed region and expand the rotating speed range of the chaotic region in the high-speed region compared with that in the linear support case.
6. When the rubbing fault occurs, the speed ratio has little influence on the stability of the system. However, a smaller rubbing stiffness and a smaller bearing clearance will be beneficial to achieve better system stability.
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Code availability The raw/processed code required to reproduce these findings cannot be shared at this time as the code also forms part of an ongoing study.

Declarations

Conflict of interest The authors declare that they have no conflict of interest.

Availability of data The data that support the findings of this study are available from the corresponding author, Qingguo Fei, upon reasonable request.

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