MOND: time for a change of mind?
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1. What is MOND?

MOND is a quarter-century old paradigm of dynamics propounded as replacement for Newtonian and relativistic dynamics. It greatly departs from these venerated theories when accelerations in a system under study are very small: much smaller than those encountered on earth or in the solar system, but quite typical of galaxies and other galactic systems. I shall limit the discussion to non-relativistic MOND, applicable for most problems now under study in galactic astronomy. MOND introduces into physics a new constant $a_0$, with the dimensions of acceleration, which marks the boundary of validity of classical dynamics, and also features prominently in the phenomenology of low acceleration systems. In this sense, the role of $a_0$ is similar to that of the speed of light, $c$, in the context of the relativistic departure from classical physics, or the role of the Planck constant, $\hbar$, in the quantum context. It appears in the equations of dynamics of any theory that is anchored in the MOND paradigm, but for systems with accelerations that are much larger than $a_0$, such a theory reduces to the standard, Newtonian theory. This can be achieved, formally, by simply substituting everywhere in the MOND equations $a_0 = 0$, which should reduce them to the equations of standard physics (just as taking an infinite speed of light, or $\hbar = 0$, reduces relativistic results, or quantum results, to the corresponding classical limit).

But how does the theory look like and what does it imply for accelerations that are smaller than $a_0$? The most appealing way to describe MOND for very low accelerations is to point to a symmetry principle: in this limit the theory becomes invariant to simultaneous scaling of times and distances: Take any physically permissible configuration--such as a planetary system, or a galaxy of stars--and increase all distances in the system by a certain factor, and also increase all times by the same factor. The symmetry implies that we then obtain another physically permissible configuration. For example, in a planetary system that has very low accelerations and obeys the MOND dynamics, a result of the symmetry is that the length of the planetary year increases in proportion to the orbital radius, and the orbital velocities are then independent of the orbital radius. In contrast, standard dynamics do not have this symmetry: if we increase the radius of the earth's orbit around the sun by a factor of four, say, the orbital time will increase by a factor of eight, not by a factor four as would be required by such a symmetry.

More generally, the symmetry dictates how the orbital (centripetal) acceleration, $a$, of a body on a circular orbit of radius $R$ around a central mass, $M$, depends on these parameters in the very low acceleration limit: Since $a$ can depend only on $R$, $M$, and on the constants $G$ and $a_0$, the symmetry and dimensional considerations tell us that $a$ must be proportional to $\sqrt{MGa_0}/R$, and we define the value of $a_0$ once and for all so as to have equality $a = \sqrt{MGa_0}/R$. This dependence is in sharp contrast with the Newtonian relation $a = MG/R^2$. The main differences to note are: (i) the MOND acceleration decreases in inverse proportion to $R$, not to $R^2$, and (ii) it is no longer proportional to the attracting mass, but to its square root. Note also that MOND generically gives higher accelerations than Newtonian dynamics, for a given mass.

Any theory based on the MOND paradigm should interpolate between the classical and the MOND limits and should thus cover the full range of phenomena from very low to very high accelerations. For example, if we have a planetary system with high accelerations for the inner planets and accelerations much smaller than $a_0$ for the outer ones, the theory should describe the system in full, accounting for both limits...
correctly. In such a theory the dependence of the acceleration on mass and radius will be of the form \(a \mu(a/a_0) = MG/R^2\), such that at small radii, where \(a\) is much larger than \(a_0\), \(\mu\) is very nearly unity, so the Newtonian expression is valid, and \(a_0\) disappears. At large radii, where \(a\) is small compared with \(a_0\), we have to have \(\mu(x) = x\) to good accuracy, and the MOND relation obtains. Figure 1 shows the schematic dependence of \(a\) on \(M\) and \(R\), compared with the Newtonian prediction, for a range of masses representing various astronomical objects.

The function \(\mu(x)\) encapsulates the transition from the classical to the modified regime. Such functions appear commonly in other instances of modifications of classical physics. For example, Planck’s black-body function describes the spectrum of the black body with full classical-quantum coverage. It reduces to the classically predicted spectrum at low frequencies (where the Planck constant does not appear). The Lorentz factor, which appears in various expressions of special relativistic kinematics and dynamics, is another famous interpolating function between the classical and relativistic regime.

2. Why MOND? The mass discrepancy and the appearance of dark matter

Why consider such a major modification to Newtonian dynamics and General Relativity when it is common knowledge that these two are so very successful in accounting for non-quantum phenomena? The truth of the matter is, however, that these theories fail miserably in accounting for the observed dynamics of most galactic and cosmological systems—such as galaxies, binary galaxies, small groups of galaxies, and rich galaxy clusters. These theories can be saved only if new, ad hoc, dominant ingredients of matter-energy are introduced into the universe; these ingredients are known as “dark matter” (DM) and “dark energy” (DE). Standard dynamics fail to explain the observed motions in such systems if the only mass present is what we directly observe (stars, cool and hot gas, etc.). The velocities in these systems are so high that, with only the observed mass to arrest them gravitationally, the systems would just fly apart in a relatively short time. The amount of extra mass needed to keep such systems from dispersing varies from object to object, and from place to place within an object, but it is routinely found to be several times to tens of times larger than the mass observed directly. The dark matter paradigm then assumes that whatever extra mass is needed is there in the form of “dark matter”, of some yet unspecified nature.

Consider just one example of such measurements leading to a mass discrepancy, arguably the most reliable and clear-cut to date. A disc galaxy, such as is shown in Fig. 2 is made of gas and stars in a rather thin, flat disc; both components circle the galaxy on nearly circular orbits with a velocity \(V\) that depends on the radius, \(R\), of the orbit. The examination of the galaxy rests on two kinds of measurements: First, we measure the dependence of \(V\) on \(R\) to give the so-called rotation curve, \(V(R)\), as is depicted in Fig. 2. Second, we measure the distribution of mass we see in the galaxy in the form of stars and gas. Given the mass distribution, one calculates the gravitational field of the galaxy using one’s favorite theory of gravity (e.g., Newtonian gravity or MOND). From this one calculates the expected orbital velocities based on the law of inertia by which we equate the centripetal acceleration \(V^2(R)/R\) to the gravitational force per unit mass. It is a fact established for all disc galaxies studied to date that the Newtonian prediction of the rotational speeds fall short of the measured ones. In some galaxies the discrepancy appears only beyond a certain radius, and then increases with radius; in many other cases the discrepancy is present at all radii. For the rotation curve shown in Fig. 2 the discrepancy exhibited by the predicted Newtonian velocities is of a factor 2.25 in the outer parts, corresponding to a factor of five discrepancy in mass. So, in this case a dark matter mass four times as large as that of normal matter is needed. Similar analyses lead to similar mass discrepancies in all astronomical systems from the scale of galaxies and up.
Fig. 1.— The MOND centrifugal acceleration of a particle on a circular orbit around a mass $M$, as a function of orbital radius (in heavy lines), for a star of one solar mass ($1M_\odot$)(red), globular cluster of mass $10^5 M_\odot$ (blue), a galaxy of mass $3 \times 10^{10} M_\odot$ (green), and a galaxy cluster of mass $3 \times 10^{13} M_\odot$ (magenta). The Newtonian accelerations are shown as dashed lines. Departure of MOND from Newtonian dynamics occurs at different radii for different central masses, but always at the same value of the acceleration, $a_0$, below which we are in the MOND regime, and above which we are in the Newtonian regime.

Fig. 2.— A spiral (disc) galaxy.
Such mass discrepancies are deduced not only by following the motion of massive objects bound to the system such as stars and gas. A powerful method that complements such studies involves the measurement of the bending of light rays coming from far objects in the background, as they pass near the object under study—so called gravitational lensing. The results of such measurements are consistent with those of bound particles measurements in regions where the two coincide. The method also enables us to measure the discrepancy at very large radii, where the first method is not applicable for lack of probing objects.

The need for DM arises also in the context of cosmology, in particular in connection with the way galactic systems have formed out of the uniform, primordial soup. It is thought that the universe, which has been expanding since the initial “big bang”, contained in the early stages a rather uniform, hot mixture of known matter (protons, electrons, photons, some nuclei, etc.) and DM in roughly a one to five proportion. As the universe expanded and cooled, the small seed non-uniformities in the mass distribution have gradually increased in magnitude, due to gravitational self attraction, a process that eventually led to the formation of galactic systems. Normal matter was initially hot and ionized (charged) and so submissive to the black body radiation ambiance then present in abundance. In such a state, the normal matter could not efficiently collapse to enhance the strength of the mass agglomerates, because the domineering radiation does not collapse. However, at a certain stage, matter cooled enough and formed neutral atoms. Its non-uniformities—now free of the restraints of the radiation—could then continue to grow effectively.

Alas, without auxiliaries, standard dynamics tells us that such non-uniformities of normal matter did not have enough time to grow into the structures we see today in the time since it became neutral. Dark matter comes to the rescue because, being neutral, it could have collapsed freely even before the neutralization. When normal matter neutralized it already found itself in the presence of better developed clumps of DM, which then pulled the normal matter into them, hastening the collapse. Thus begins a complex process of continued collapse, interaction and mergers between clumps, dissipation of the normal-matter gas, its forming stars, and its expulsion from galaxies by various processes, etc.. This, still ongoing, process has led to the matter agglomerates that we see today as galaxies, galaxy groups, clusters, and super-clusters. These agglomerates are composed of an extended invisible “halo” of DM, at the center of which sits the normal matter, which is visible as radiation emitting and absorbing gas, and as shining (and dead) stars that formed later. All this is what the DM paradigm would have us believe. (Dark energy will be discussed later.)

The DM paradigm can make hardly any prediction without further specification of the nature of the DM. As a general concept DM is only a filler, whose presence is assume where needed. However, there is a class of candidates for the DM substance that is now favored, called cold DM (CDM). Attention on CDM has converged after several other candidates have been ruled out and discarded over the years, such as neutrinos, and massive, dead, stellar objects. This particular choice does lend itself to certain predictions, and from now on I shall refer to this option.

There are, at present, two types of particles that are considered the leading candidates for the CDM constituents: supersymmetric counterpart of ordinary particles is the one, the so called “axion” is the other. Both are hypothetical particles whose existence is rooted in different elementary particle theories. They are of very different nature, so the ways to detect them are very different, and the ways they have appeared in the cosmos, in the first place, are very different. However, their effects in cosmology are similar, as they simply act as inert, hardly interacting, relatively heavy particles.

Neither of the two has been produced in the laboratory, or is even known to exist on other secure grounds, let alone to have been caught in the act of playing the role of DM (even if such particles do exist, they need not be the DM particles). One of the declared goals of the LHC accelerator in CERN is to produce
and detect the supersymmetric partners in the debris of high-energy subatomic collisions. If these are indeed discovered it will tell us at least that a candidate exist. But, to establish it as a DM particle we need to detect these particles directly as dark matter: The solar system, and with it earth, is presumed to be bathed in the sea of dark matter particles engulfing our galaxy, as it does all galaxies; these could, in principle, be detected directly. There is a number of underground experiments afoot, trying for some years now to do just that.

Having taken all that in, it has to be realized that the deduction of large mass discrepancies is based on a combination of the law of gravity (the way gravity depends on mass and distance), and on the law of inertia (describing how motion responds to an applied force). These are the central pillars of Newtonian dynamics, and hence also of General Relativity, which rests on Newtonian dynamics. They work very well in the laboratory and the solar system; but, can they also be applied in the realm of the galaxies?

Enters MOND: it is possible to device a new theory of dynamic—incorporating standard dynamics at high accelerations, but forgoing it at low ones—that explains almost all aspects of the mass discrepancies in galactic systems with no need to invoke dark matter. This is what MOND claims to achieve.

And the “dark energy”? MOND does not obviate it; in fact, as we shall see below, MOND may be part and parcel of a universe governed by “dark energy”.

3. What does MOND predict and how does it perform?

One can construct various detailed theories that incorporate the basic MOND tenets listed above. For example, Newtonian gravity is known to be governed by the Poisson equation, by which the gravitational potential is determined from the mass distribution (the same equation governs the electrostatic field). This equation has been generalized to incorporate the MOND tenets. The resulting MOND theory is as complete a theory as Newtonian dynamics, and has been the basis for many analytic and computational studies. A recent example of numerical simulations based on this theory of the complex, interacting binary galaxy system known as The Antennae, by Tiret and Combes from the Paris Observatory, is shown in Fig.4.

There are other theories that one can write for the nonrelativistic regime. In addition, we would like to construct a MOND theory that is compatible with the principles of relativity. There has been progress made in this direction, notably the relativistic MOND formulation call TeVeS proposed by Jacob Bekenstein from the Hebrew University, but it is felt that even this is not the ultimate theory.

Luckily, there are many predictions of MOND that follow directly from the basic premises and do not require a specific theory. These may be viewed as the MOND analogues and extensions of Kepler’s laws for planetary motions. Kepler discovered his laws as purely phenomenological regularities in the motions of the planets in our solar system, without understanding their origin. These laws then served Newton in deriving his general theory of dynamics, which not only reproduced Kepler’s laws, but have greatly extended them. For example, Newton’s theory also predicted the motion of unbound bodies on hyperbolic orbits, and of planetary motions in other systems, around other central stars. More generally, it predicted the behavior of arbitrary systems governed by gravity.

A MOND theory, likewise, predicts the general behavior of arbitrary systems held by gravity. As in the Newtonian case, such predictions are usually deduced from involved calculations (for instance, solving the motions of the planets in our solar system in Newtonian dynamics, with all the interactions between planets taken into account, is a formidable, yet incomplete task). But, in MOND too, one can extract a set
Fig. 3.— The measured rotation curve of the galaxy NGC1560 shown by the data points. The predicted Newtonian curve based on the measured mass distribution is shown in blue. It shows a velocity disparity of a factor 2.25 at the last measured point, which corresponds to a factor-of-five mass discrepancy. The MOND prediction is shown in green. The best fit with a dark matter halo of the type predicted by CDM simulations is shown in red. It has two free parameters over which the fit is optimized: the mass and scale of the halo (no such freedom exists for MOND). Courtesy of Stacy McGaugh.

Fig. 4.— Simulation of closely interacting galaxy pair known as The Antennae with MOND (right) compared to the observations from Hibbard et al. 2001 (left). In the observations, the gas is represented in blue and the stars in green. In the simulation the gas is in blue and the stars are in yellow/red. Considering that many of the details of the history of the collision are not known and cannot be incorporated in the simulation, the agreement is quite remarkable. From Tiret and Combes 2008.
of general regularities that can be directly applied to galactic systems without further calculations. Here are some examples:

1. As we saw already, MOND predicts that orbital velocities on circular orbits around a concentrated mass become independent of the orbital radius, for large radii (where the acceleration becomes smaller than $a_0$). For disc galaxies, this means that the rotational velocity should become constant with radius at large radii, as indeed it does (see Fig. 5).

2. We also saw that the constant asymptotic rotational velocity in a galaxy should be proportional to the fourth root of the galaxy’s mass. This is also in very good agreement with observations.

3. As summarized in Fig 1, the mass discrepancy in galaxies should appear at a different radii for different galaxies, but always at the same value of the centrifugal acceleration $V^2/R = a_0$. Galaxies for which the acceleration is smaller than $a_0$ at all radii, should show a discrepancy everywhere. All this is amply born out by the observations.

4. MOND also predicts that an inflated, spheroid-like halo of dark matter, as is predicted by the cold dark matter paradigm, should not suffice to explain all the facets of the mass discrepancy in disc galaxy: an additional flat, disc-like component, with predictable properties, should be necessary.

5. MOND predicts that the discrepant acceleration in galactic systems can never much exceed $a_0$.

Quite a few more such predicted laws are known (e.g., ones pertaining to elliptical galaxies and other such systems). They all conform well with the data.

Above and beyond such Kepler-like laws, the flagship of MOND phenomenology is the full prediction of the exact rotation curves of individual disc galaxies: MOND does this for each and every galaxy, given only the observed distribution of the normal mass in the galaxy. Over a hundred galaxies have been analyzed in this way to date, with generally, very good success. We already saw one example shown in detail in Fig. 3; some more are shown in Fig. 6.

4. MOND vs. CDM

The fact that the initial motivation for DM and MOND is similar, and that they aim to account for similar phenomena, may give the impression that they are similar paradigms that can be tested by similar criteria. This is anything but true: MOND is much more predictive, and lends itself to falsification to a far greater degree then the CDM paradigm, to say nothing of DM in general. MOND makes definite predictions on dynamics for individual objects based on just the observed mass distribution. For CDM, predictions of this kind are impossible, with very few exceptions, as they would have to hinge on understanding of the interrelations between normal and dark matter in a given system, which, in turn, would result from complex formation and evolution histories that cannot be known for a given object. This is further complicated by the fact that normal matter and DM follow very different evolution paths.

CDM is capable of making some predictions on general properties of the bare CDM halos themselves, if we neglect the effects of normal matter on the CDM: Using high power computer simulations, a uniform distribution of CDM, seeded with some initial irregularities, can be evolved to obtain some statistical properties of CDM halos at the present time. This includes population statistics, such as the distribution of total masses of these halos, and general intrinsic properties of the halos such as the form of the density distribution in them. However CDM is almost completely dumb on the expected properties of the normal
Fig. 5.— The MOND rotation curves for several galaxies, shown as the solid lines going through the data points (the other lines show Newtonian curves calculated for different mass components such as stars and gas).
matter inside these halos, and on interrelations between CDM and normal matter; why? Normal matter is far more active and interactive than the purported CDM; it undergoes more complex and erratic influences: it can emit and absorb radiation, it is subject to energy losses by dissipation, it interacts with magnetic fields, it can be ejected from galaxies by stellar explosions, it can form stars, etc., etc., all of which the CDM does not partake. It is enough to glance at galaxies to see that the normal matter in then have very different characteristics from those attributed to CDM, even though they are presumed to have started as a well mixed soup: In many galaxies the normal matter forms a thin rotating disc, while the DM is thought to constitute a spherical or elliptical halo that is hardly rotating. The DM halo is much more extended then the normal matter in galaxies. The ratio of DM to normal matter in galaxies is much larger than the cosmic ratio with which they presumably started. All this tells us quite cogently that the two components should not be well correlated, and that their mutual relations are unpredictable in the CDM paradigm. Even the most basic fact one would like to be able to predict: the total amount of CDM to be expected around a given visible galaxy, cannot be estimated, to say nothing of finer details, such as the exact distribution of DM around a given, visible galaxy.

MOND, on the other hand, is capable of predicting all this (or rather the equivalent, since it repudiates DM).

Take, as an example, the rotation curve shown in Fig. 3; it demonstrates clearly the differences between the predictive power and success of MOND and CDM on galactic scales. The figure shows the measured curve together with the MOND curve, which is predicted with hardly any freedom. MOND does well in predicting the general shape of the rotation curve, and even reproduces the peculiar feature consisting of a “dip”. In contrast, the CDM curve, which is also shown, is not a prediction, as it involves also a dominant contribution from a DM halo with a priori unknown properties. Numerical simulations yield CDM halos that have a certain form of the density distribution characterized by two structural parameters: the mass of the halo and its characteristic radius (as well as some unpredictable degree of departure from sphericity; but, in rotation curve fits the halo is usually taken as spherical). The observed velocity curve is then fitted with such a halo optimizing the fit with a necessary best choice of mass and size for the halo. This allows great freedom in fitting to the data, as any quantity of DM can be assumed with impunity. And yet, even with all its freedom, the best CDM fit in Fig. 3 is rather inferior. It misses the general trend, and totally fails to reproduce the observed “dip”. (The dip results from a feature in the mass distribution in the gas disc, and the putative spherical halo “erases” the effect.)

On rare occasions the CDM paradigm does predict the amount of dark matter that should accompany a normal matter galaxy. A case in point involves the dwarf galaxies that form out of the debris of high-velocity collisions between cut-and-dried disc galaxies. The colliding galaxies come with their purported halos of CDM and collide violently, ejecting out some of their mass (both normal and DM) and continuing on their speedy way. The ejected debris typically form semi-ordered, elongated structures of gas, such as rings or arcs, coming from the colliding gas discs. An example of such a ring-like debris is shown in Fig. 6. The interacting galaxies shown in Fig. 4 are believed to undergo a milder, lower-speed collision and will eventually merge; they too eject “antennae” of gas and stars.) From the gas ashes of the debris, little phoenix galaxies form in the course of time. In the system shown in Fig. 6 believed to have resulted from a collision some half a billion years ago, a few such little phoenixes have been identified and studied in detail. The CDM, which supposedly come from the spheroidal halos of the colliding galaxies, is spread all around with hardly any of it accompanying the gas, and hardly any finding its way into the phoenix dwarfs. This assertion is strongly supported by numerous simulations (the right panel in Fig. 6 shows the gas structure resulting from such a simulation of the system on the left). In a unique instance of predictability on normal-to-dark-matter
relation, the CDM paradigm thus predicts no, or very small, mass discrepancies in these phoenixes. And yet, analysis of the three accessible phoenixes in the system found all three to show mass discrepancies of about a factor three, in conflict with the prediction of the CDM paradigm.

And what does MOND predict for these? These dwarfs are measured to rotate with centripetal acceleration a few times smaller than $a_0$, and MOND thus predicts substantial mass discrepancies in them, just at levels and distributions that match those found.

The DM paradigm is known to face a number of other severe difficulties in explaining the observed properties of galaxies, which I shall not discuss here, and which DM advocates are hard at work to explain away.

5. Deeper significance?

MOND already boasts two main achievements: it obviates the need for dark matter in galactic systems, and it predicts, explains, and organizes a large body of galactic dynamics data. But, MOND’s most far reaching ramifications may yet be in store. This expectation is based on the following observation: The MOND acceleration constant, $a_0$, appears in quite a few predictions of MOND regarding dynamics of galaxies (just as the Planck constant appears in the description of disparate quantum phenomena, such as the black body spectrum, atomic spectra, superconductivity, the quantum Hall effect, etc.). Comparing these predictions with the observations, we can determine its value in several independent ways. The value obtained is the same for all methods, within the uncertainties, and is $a_0 \approx 1.2 \times 10^{-8} \text{cm s}^{-2}$. This value is tantalizingly close to acceleration constants that typify our universe as a whole. One such cosmic acceleration constant, call it $a_H$, is gotten by multiplying the speed of light by the expansion rate of the universe, known as the Hubble constant. (To envisage this numerical proximity, note that a body starting from rest and accelerating with acceleration $a_0$ will approach the speed of light in the life time of the universe since the big bang.) Another constant, call it $a_\Lambda$, is derived from the recently observed acceleration of the expansion rate of the universe. This accelerated expansion is difficult to explain in the context of standard physics (General Relativity) if only matter of conventional properties is present (normal or dark matter). The majority view is that the acceleration is due to the dominant presence in the cosmos of an entity known as “dark energy”, whose gravity acts, unlike that of conventional matter, to accelerate the expansion; it is required to make up about 75 percent of the mass-energy content of the cosmos today. In General Relativity, gravity and geometry of space-time are one and the same. The gravity effect of the “dark energy” also means that our space-time is approximately a “spherical” one with a radius that is related to the acceleration rate. It is a great mystery that for some reason the values of the seemingly unrelated constants $a_H$ and $a_\Lambda$ are today nearly equal ($a_H$ varies over time, while $a_\Lambda$ could be a veritable constant). It is yet another mystery that $a_0$, which apparently derives from completely unrelated phenomena, is also numerically close to these two.

Is this a mere coincidence? Personally I view it as a strong hint that MOND, which pertains to objects that are very small on cosmological scales is, nonetheless, strongly related to the global state of the cosmos. Exactly how? we do not yet know for sure, but there are ideas in this vein. The hope is that when the connection is understood, MOND, and the value of $a_0$ will follow from this understanding. The situation might be schematically similar to the dynamics of moving bodies near the earth surface, which was found to be governed by some acceleration constant $g$, Galilei’s free-fall acceleration. As long as we only perform experiment near the earth’s surface we appear to have a theory with some acceleration constant. But once we understand gravity better, we see that this is only an approximate description, and that the constant can
Fig. 6.— Left: The gas ring ejected from the high speed collision of the galaxy NGC5291 with another galaxy (present position outside of frame). The forming phoenix dwarfs NGC5291N,S,SW are marked. Right: Numerical simulation of the collision and its outcome. Curtesy of Frederic Bournaud.
be derived from properties of the earth, such as its mass and radius.

One interesting consequence of the above numerical proximity of $a_0$ to cosmic accelerations is that there cannot exist in nature systems with both relativistic gravity and deep-MOND accelerations (such as a black hole with low accelerations): such a systems would have to be much larger than the visible universe.

The MOND-cosmology connection is further supported by the identification of symmetries that are common to the deep MOND limit, and a dark-energy dominated space-time.

6. Open questions

MOND is a paradigm still under construction. In the nonrelativistic regime we have a working, full fledged theory, as described above, but we are not sure that it is the ultimate one, since there are other possibilities. We also have a working relativistic formulation of MOND called TeVeS. This replaces Einstein's field equations for calculating the geometry of space-time, or the gravitational field, in a given system. TeVeS is already a great advance, because it enables us to calculate the effects of gravitation on light rays (gravitational lensing), and the cosmological evolution of the universe, which are beyond the capabilities of nonrelativistic theories. However, TeVeS too seems to have its limitations, and the quest for a relativistic formulation of MOND continues. In any event, theories such as TeVeS do not embody the above mentioned connection of MOND with cosmology; one may then wish for a more fundamental theory that does.

On the phenomenological front MOND is also not all roses. For example, the need for dark matter in the cosmological setting has not yet been convincingly explained, although there have been suggestions on how this too may be replaced by a MOND effect. Also, MOND does not completely explain away the need for dark matter in galaxy clusters: MOND does reduce substantially the amount of DM needed, but there is a remaining discrepancy between the required and the directly observed masses in clusters. We, MOND advocates, attribute this to normal matter that could easily have hidden in clusters, perhaps in the form of massive neutrinos, perhaps as extinguished stars, or in the form of cool, dense gas clouds. Note that, in any event, even the normal matter believed to exist in the universe is still largely unaccounted for, and the amount still needed in clusters makes up only a very small fraction of the normal matter that is still missing.

All in all, these are exciting times for those involved in MOND research, with a tailwind of continuing successes, and the attraction of remaining challenges.

7. Further reading:

- Adam Frank: The Einstein Dilemma, “Discover” Magazine, August 2006 (http://discovermagazine.com/2006/aug/cover/?searchterm=milgrom)
- Marcus Chown: Dark Matter is Dead, “BBC Sky at Night” Magazine, September 2007 (http://www.skyatnightmagazine.com/viewIssue.asp?id=837)
- The MOND pages (http://www.astro.umd.edu/~ssm/mond/index.html). Various accounts of MOND with a link to an extensive list of papers on MOND
- M. Milgrom: Does Dark Matter Really Exist? Scientific American, August 2002 (appeared in Hebrew as well).