Light front thermal field theory at finite temperature and density

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We investigate quark matter at finite temperature and finite chemical potential as an example for a relativistic many-particle quantum system. Special relativity is realized through the front form that allows for a Hamiltonian formulation of a statistical operator. Utilizing our previous results we generalize the present formulation of a relativistic thermal field theory to include a finite chemical potential. As an application we use the Nambu-Jona-Lasinio model to investigate the gap equation and chiral restoration.

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I. INTRODUCTION

The front form [1] is widely recognized as an alternative form of a relativistic theory. It has been successfully applied in particular in the context of quantum chromo dynamics (QCD), e.g., to describe deep inelastic scattering or to model the structure of hadrons, see e.g. Ref. 2. The front form provides a rigorous relativistic many-body theory that has been applied, e.g., to investigate nuclear matter [3, 4]. Therefore it is quite natural to ask for the merits of light-front quantization in the context of quantum statistics to describe many-particle systems of finite temperature T and finite chemical potential µ. There is a need to do so, in particular, to model the evolution of the early universe, the dynamics of relativistic heavy ion collisions, multi-particle correlations in such a collision, relativistic Coulomb plasma, among others.

The difficulty to generalize the statistical operator to the light-front is related to the longitudinal mode \( k^+ = k^0 + k^3 \). Whereas temperature for the transverse modes related to \( k_\perp = (k^1, k^2) \) can easily be defined, the longitudinal modes, can only be related to the instant form by a limiting process (infinite momentum frame). Therefore the longitudinal modes appear in a proper definition of the statistical operator on the light-front. As a consequence the Fermi function (here given for a grand canonical ensemble) explicitly depends on the light-front energy \( k_\text{on}^- \),

\[
k_\text{on}^- = \frac{k_\perp^2 + m^2}{k^+}
\]  

(1)

and the light-front “mass” \( k^+ \), viz. [5]

\[
f(k^+, k_\perp) = \left[ \exp \left\{ \frac{1}{T} \left( \frac{1}{2} k_\text{on}^- + \frac{1}{2} k^+ - \mu \right) \right\} + 1 \right]^{-1}.
\]  

(2)

This function is consistent with the instant form and has already been used to investigate the formation of three-quark clusters (nucleons) in the hot and dense phase of nuclear matter close to the QCD phase transition [3, 6].

Recently, Brodsky [7, 8] suggested a partition function for a canonical ensemble (\( \mu = 0 \)) that would lead to a different Fermi function (naive generalization). It has been explicitly shown that the naive generalization has serious problems, because the Fermi function leads to divergent integrals [8], which is not the case for the distribution function given in Eq. (2) due to the presence of the \( k^+ \) term. Presently, there is a renewed interest and discussion on how to formulate thermal field theory on the light-front [10, 11, 12, 13]. So far, none of these generalizations consider finite chemical potentials that are relevant for QCD. Utilizing our previous results we generalize the present formulation of thermal field theory on the light-front to include a finite chemical potential.

As an example, we use the Nambu-Jona-Lasinio (NJL) model [14, 15] that is a powerful tool to investigate the non-perturbative region of QCD as it exhibits spontaneous breaking of chiral symmetry and the appearance of Goldstone bosons in a transparent way. The region of finite chemical potential and temperature can be investigated in relativistic heavy ion collisions of the newly constructed (RHIC) and upcoming machines (LHC, GSI). It is particularly interesting since the phase structure is expected to be complex [16, 17, 18, 19]. It might be also relevant, e.g. for the physics of compact stars. Lattice QCD studies of this region (\( \mu \neq 0 \)) are far more limited than for \( \mu = 0 \) [20], although some progress has been achieved recently, see e.g. [21] and refs. therein.

In Sec. IV we use our previous results to generalize the present formalism of thermal field theory on the light-front to finite chemical potentials. In Sec. IV we briefly introduce the NJL model on the light-front to give an example. This will be done along the lines of Ref. [22] and in Sec. IV we will present numerical results for the
quark mass at finite temperature and densities (chiral restoration).

II. LIGHT-FRONT STATISTICAL PHYSICS

Here we present more details of our previous results, and provide the derivation of the distribution function for the grand canonical ensemble. See the Appendix A for notation. The four-momentum operator \( P^\mu \) on the light-front is given by

\[
P^\mu = \int dS_+ T^{+\mu}(x),
\]

where \( T^{\mu\nu}(x) \) denotes the energy momentum tensor defined through the Lagrangian of the system and \( S_\mu \) is the quantization surface, see Eq. (A7). Hence the Hamiltonian is given by

\[
P^- = \int dS_+ T^{-+}(x).
\]

To investigate a grand canonical ensemble it is necessary to define the number operator. The number operator on the light-front is given by

\[
N = \int dS_+ J^+(x),
\]

where \( J^\nu(x) \) is the conserved current. These are the necessary ingredients to generalize the covariant calculations at finite temperature \(^{23, 24, 25}\) to the light-front. The grand canonical partition operator on the light-front is given by

\[
Z_G = \exp \left\{ \int dS_+ [ -\beta_\nu T^{+\nu}(x) + \alpha J^+(x)] \right\},
\]

where \( \alpha = \mu/T \), with the chemical potential \( \mu \). The velocity of the medium is

\[
\beta_\nu = \frac{1}{T} u_\nu
\]

with the normalized time-like vector \( u_\nu u^\nu = 1 \) \(^{23}\). We choose

\[
u^\nu = (u^-, u^+ = \bar{u}^\perp) = (1, 1, 0, 0),
\]

hence \( u^+ - u^- = 2u^3 = 0 \), i.e. the medium is at rest. The grand partition operator becomes

\[
Z_G = e^{-K/T}, \quad K = \frac{1}{2} (P^- + P^+) - \mu N
\]

with \( P^\pm \) and \( N \) defined in Eqs. (3) and (5). To further evaluate Eq. (9), the operators will be used in Fock space representation. For an ideal gas of Fermions we use the solution of the free Dirac field (conventions of \(^{26}\))

\[
\Psi_\alpha(x) = \sum_\lambda \int \frac{dk^+d^2k_\perp}{\sqrt{2k^+}} \left( b_\lambda(k) u_\alpha(k) e^{-ik_\perp x} + d_\lambda(k) v_\alpha(k) e^{ik_\perp x} \right)
\]

with the four-vector \( k_{\perp a} = (k_{\perp a}, k) \), the abbreviation \( k = (k^+, k_\perp) \), and the Fock operators

\[
\{ b_\lambda(k), b^\dagger_\mu(\vec{r}) \} = \delta(k^+ - p^+) \delta(\vec{k}_\perp - \vec{p}_\perp) \delta_{\lambda\sigma}, \quad \{ d_\lambda(k), d^\dagger_\nu(\vec{r}) \} = \delta(k^+ - p^+) \delta(\vec{k}_\perp - \vec{p}_\perp) \delta_{\lambda\sigma}.
\]

The Hamiltonian takes the form

\[
P^- = \sum_\lambda \int dk^+ d^2k_\perp k_{\perp a} \left( b^\dagger_\lambda(k) b_\lambda(k) + d^\dagger_\lambda(k) d_\lambda(k) \right)
\]

and for the plus component of the Dirac current,

\[
N = \sum_\lambda \int dk^+ d^2k_\perp \left( b^\dagger_\lambda(k) b_\lambda(k) - d^\dagger_\lambda(k) d_\lambda(k) \right).
\]

Inserting all the above into Eq. (10) gives

\[
K = \sum_\lambda \int dk^+ d^2k_\perp \left( \kappa^+ b^\dagger_\lambda(k) b_\lambda(k) + \kappa^- d^\dagger_\lambda(k) d_\lambda(k) \right),
\]

\[
\kappa^\pm = \frac{1}{2} (k^- + k^+) \mp \mu.
\]

The density operator for a grand canonical ensemble \(^{26}\) in equilibrium follows

\[
\rho_G = (\text{Tr} e^{-K/T})^{-1} e^{-K/T}.
\]

The light-cone time-ordered Green function for fermions is

\[
G_{\alpha\beta}(x-y) = \theta(x^+ - y^+) \mathcal{G}_{\alpha\beta}^{\geq}(x-y) + \theta(y^+ - x^+) \mathcal{G}_{\alpha\beta}^{\leq}(x-y),
\]

where

\[
\mathcal{G}_{\alpha\beta}^{\geq}(x-y) = i \langle \Psi_\alpha(x) | \bar{\Psi}_\beta(y) \rangle,
\]

\[
\mathcal{G}_{\alpha\beta}^{\leq}(x-y) = -i \langle \bar{\Psi}_\beta(y) | \Psi_\alpha(x) \rangle,
\]

We note here that the light-cone time-ordered Green function differs from the Feynman propagator \( S_F \) in the front form by a contact term (which is not the case in the instant form). Evaluating Eq. (14) for the vacuum, i.e. \( \langle \ldots \rangle = \langle 0 | \ldots | 0 \rangle \) (isolated case), the Green function is...
\[ iG(x - y) = \int \frac{d^4k}{(2\pi)^4} e^{-ik \cdot (x - y)} \left( \frac{\gamma k_0 + m}{2k^- - \frac{1}{2}k_0^\perp + i\varepsilon} \frac{\theta(k^+)}{2k^+} + \frac{\gamma k_0 + m}{2k^- - \frac{1}{2}k_0^\perp - i\varepsilon} \frac{\theta(-k^+)}{2k^+} \right) \]

where the Feynman propagator is given by

\[ S_F(k) = \frac{\gamma k + m}{k^2 - m^2 + i\varepsilon}. \]  

Eq. (20) coincides with the light-front propagator given previously in Ref. [28]. Using the independent components \( \Lambda^\perp \Psi \) instead of \( \Psi(x) \) in the definition of the chronological Green function Eqs. (15) and (16), leads to a different propagator suggested, e.g., in Ref. [9, 29].

To evaluate the ensemble average \( \langle \ldots \rangle = \text{Tr}(\rho \ldots) \) of Eqs. (15) and (16) (in-medium case), we utilize the imaginary time formalism [20, 27]. We rotate the light-front time of the Green function to imaginary values, i.e. \( u \rightarrow ix \). The anti-periodic boundary condition of the imaginary time Green function holds on the light-front as well. Hence the \( k^- \)-integral is replaced by a sum of light-front Matsubara frequencies \( \omega_n \) according to [3],

\[ \frac{1}{2}k^- \rightarrow i\omega_n - \frac{1}{2}k^+ + \mu \equiv \frac{1}{2}k_n^-, \]

where \( \omega_n = \pi \lambda T, \lambda = 2n + 1 \) for fermions [\( \lambda = 2n \) for bosons]. For noninteracting Dirac fields the imaginary time Green function becomes

\[ G(k_n^-, \vec{k}) = \frac{\gamma k_0 + m}{2k_n^- - \frac{1}{2}k_0^\perp + i\varepsilon} \frac{\theta(k^+)}{2k^+} (1 - f^+(\vec{k})) + \frac{\gamma k_0 + m}{2k_n^- - \frac{1}{2}k_0^\perp - i\varepsilon} \frac{\theta(k^+)}{2k^+} f^+(\vec{k}) \]

\[ + \frac{\gamma k_0 + m}{\frac{1}{2}k_n^- - \frac{1}{2}k_0^\perp + i\varepsilon} \frac{\theta(-k^-)}{2k^-} f^-(\vec{k}) + \frac{\gamma k_0 + m}{\frac{1}{2}k_n^- - \frac{1}{2}k_0^\perp - i\varepsilon} \frac{\theta(-k^-)}{2k^-} (1 - f^-(\vec{k})). \]  

For a grand canonical ensemble the Fermi distribution functions of particles \( f^+ \equiv f \) and antiparticles \( f^- \) are given by

\[ f^\pm(k^+, \vec{k}_\perp) = \left[ e^{\frac{\varepsilon}{T}} + 1 \right]^{-1} \]

\[ = \left[ \exp \left( \frac{1}{T} \left( \frac{1}{2}k_n^- + \frac{1}{2}k^+ \mp \mu \right) \right) + 1 \right]^{-1} \]

and \( k_n^- \) of Eq. (1). The (particle) fermionic distribution function on the light-front has been given in Eq. (2). The propagator for this case \( (f^- = 0) \) has been given previously and used to investigate the stability and dissociation of a relativistic three-quark system in hot and dense quark matter [3, 4]. Note that for the canonical ensemble, \( \mu = 0 \), this result coincides with the one given more recently in [3] (up to different conventions in the metric).

For equilibrium the imaginary time formalism and the real time formalism are linked by the spectral function [20, 27].

III. NJL MODEL ON THE LIGHT-FRONT

The Nambu-Jona-Lasinio (NJL) originally suggested in [14, 15] has been reviewed in Ref. [31] as a model of quantum chromodynamics (QCD), where also a generalization to finite temperature and finite chemical potential has been discussed. Most of the applications of the NJL model have utilized the instant form quantization. On the other hand light-front quantization of quantum field theories has emerged as a promising method for solving problems in the strong coupling regime. Examples and in particular the light-front formulation of QCD, are reviewed in [2]. Based on a Hamiltonian the light-front approach is particularly suited to treat bound states (correlations), the most relevant consisting of \( q \bar{q} \) and \( qqq \) valence quarks. The formation of bound states is in particular interesting in the vicinity of the quark-hadron phase transition. The front form is useful in the context of quantum statistics, since the Fock space representation allows a consistent formulation of thermodynamic properties, as demonstrated in the previous Section. Since the NJL model, on one hand, is a powerful tool to investigate the non-perturbative region of QCD and, on the other hand, is comparatively transparent and sim-
ple, we use it here as an example to tackle the following questions: How are hadrons formed/dissociated? Is the formation/disassociation transition at the same place in the phase diagram as the chiral phase transition? Approaches with a closer connection to QCD than the NJL model, based, e.g., on the method outlined in Ref. 2, have to follow. These may be of use as either complementary or supplementary approaches to lattice QCD (e.g. at large chemical potentials $\mu$, where lattice QCD is still not applicable).

The NJL Lagrangian is given by

$$\mathcal{L} = \bar{\psi}(i\gamma\partial - m_0)\psi + G ((\bar{\psi}\psi)^2 + (\bar{\psi}\gamma_5\tau\psi)^2).$$  \hspace{1cm} (25)

In mean field approximation the gap equation is

$$m = m_0 - 2G(\bar{\psi}\psi) = m_0 + 2iG\lambda \int \frac{d^4k}{(2\pi)^4} \text{Tr} S_F(k),$$  \hspace{1cm} (26)

where $\lambda = N_f N_c$ in Hartree and $\lambda = N_f N_c + \frac{1}{2}$ in Hartree-Fock approximation, $N_c$ ($N_f$) is the number of colors (flavors).

A. Isolated case

On the light-front the realization of chiral symmetry breaking is a subtle issue. Unlike the instant form, for a given momentum $(k^+, \vec{k}_\perp)$, $k^+ \neq 0$, the dispersion relation on the light-front is unambiguously determined. This is usually denoted as the simple vacuum structure on the light-front. A subtlety is related to the zero modes, $k^+ = 0$, which contain the information on symmetry breaking. For a recent review on the realization of chiral symmetry breaking on the light-front see, e.g., Ref. 31. Here we basically follow Ref. 22.

After performing the $k^-$ integration the gap equation becomes ($x > 0$)

$$m - \tilde{m}_0 = 2\tilde{G}\lambda \int \frac{dx d^2 k_\perp}{2x(2\pi)^3} 4m,$$  \hspace{1cm} (27)

where in addition $m_0 \to \tilde{m}_0$ and $G \to \tilde{G}$ have been renormalized. The parameters $m_0, G$ and $\tilde{m}_0, \tilde{G}$ are related via $1/N_c$ expansion (see appendix B of 22). The explicit relation is given below for the regularization schemes discussed here. Note that due to the trace either $G(k)$ or $S_F(k)$ of Eq. (20) might be used, since $\text{Tr} \gamma^+ = 0$. The integral of Eq. (27) clearly diverges and one has to introduce a regularization scheme. Several regularization schemes have been in use. A common scheme has been suggested by Lepage and Brodsky 36 (LB)

$$\tilde{k}_\perp^2 < \Lambda_{LB}^2 x(1 - x) - m^2,$$  \hspace{1cm} (28)

which implies $x_1 \leq x \leq x_2$, where

$$x_{1,2} = \frac{1}{2} \left( 1 \mp \sqrt{1 - 4m^2/\Lambda_{LB}^2} \right).$$  \hspace{1cm} (29)

The equivalence of the light front LB scheme to the instant form three momentum (3M) scheme, where (20) is integrated over $k^0$ and $k^2 < \Lambda_{3M}^2$ has been shown in Ref. 22 utilizing the Sawicki transformation 37. To compare results note that for the mentioned regularization schemes the following identifications hold

$$\tilde{m}_{0, LB} = m_{0, 3M},$$  \hspace{1cm} (30)

$$\tilde{G}_{LB} = G_{3M},$$  \hspace{1cm} (31)

$$\Lambda_{LB}^2 = 4(\Lambda_{3M}^2 + m^2).$$  \hspace{1cm} (32)

Numerical values are given in the next section. The calculation of the pion mass $m_{\pi}$, the pion decay constant $f_{\pi}$, and the condensate value follows 22.

B. In-medium case

Here we are concerned in an application of the formalism to chiral restoration in hot and dense matter. The statistical approach shown in the previous section for finite temperature $T$ and chemical potential $\mu$ will be used. Because of the medium, the propagator to be used in the gap equation (26) is given in Eq. (28). The gap equation becomes

$$m(T, \mu) = \tilde{m}_0 + 2\tilde{G}\lambda \int \frac{dk^+d^2 k_\perp}{2k^+ (2\pi)^3} 4m(T, \mu) (1 - f^+(k^+, \vec{k}_\perp) - f^-(k^+, \vec{k}_\perp)).$$  \hspace{1cm} (33)

The quark mass changes with the parameters $T$ and $\mu$. Note that being a single particle distribution the Fermi functions $f^\pm$ that appear in the gap equation depend on $k^+$, rather than $x = k^+/P^+$. Therefore the LB regularization scheme cannot be used without modification that will be shown below. To regularize Eq. (33) we require instead

$$k_{on}^- + k^+ < 2\Omega.$$  \hspace{1cm} (34)
As a consequence \( k_1^+ < k^+ < k_2^+ \) and
\[
\hat{\vec{k}}_{\perp}^2 < 2\Omega k^+ - (k^+)^2 - m^2, \tag{35}
\]
\[
k_{1,2}^+ = \Omega \mp \sqrt{\Omega^2 - m^2}. \tag{36}
\]
To connect this \( \Omega \) scheme with the LB and the 3M schemes we use the Sawicki transformation \[37\].

Starting from Eq. \[38\] with the regularization conditions \[39\] and \[40\] we substitute \( k^+ \) by \( x \) and replace the regularization conditions accordingly. If we choose
\[
\Lambda_{\text{LB}} = 2\Omega, \tag{39}
\]
we obtain the LB conditions \[41\] and \[42\]. The resulting regularized gap equation in the LB scheme is

\[
m(T, \mu) = \bar{m}_{0,\text{LB}} + 2\tilde{G}_{\text{LB}}\lambda \int_{\text{LB}} \frac{dxd\vec{k}_{\perp}}{2x(2\pi)^3} 4m(T, \mu) \left( 1 - f_{\text{LB}}^+(x, \vec{k}_{\perp}) - f_{\text{LB}}^-(x, \vec{k}_{\perp}) \right). \tag{40}
\]

\[
f_{\text{LB}}^\pm(x, \vec{k}_{\perp}) = \left[ \exp \left\{ \frac{1}{T} \left( \frac{k^2 + m^2}{2x\Lambda_{\text{LB}}} + \frac{x\Lambda_{\text{LB}}}{2} \mp \mu \right) \right\} + 1 \right]^{-1}, \tag{41}
\]

\[
m(T, \mu) = m_{0,\text{3M}} + 2G_{\text{3M}}\lambda \int_{\text{3M}} \frac{d^3k}{2\omega_k(2\pi)^3} 4m(T, \mu) \left( 1 - f^+(\omega_k) - f^-(\omega_k) \right), \tag{43}
\]

\[
f^\pm(\omega_k) = \left[ \exp \left\{ \frac{1}{T} \left( \omega_k \mp \mu \right) \right\} + 1 \right]^{-1}, \tag{44}
\]

and using \[45\] and \[46\] the regularisation condition follows
\[
k^2 < \Omega^2 - m^2 \equiv \Lambda_{\text{3M}}^2. \tag{45}
\]

This regularized version \[47\] is analytically the same equation as given previously in Ref. \[51\]. Therefore we have shown explicitly that also in medium the \( \Omega \) regularized light front gap equation leads to the same results as the 3M regularized instant form gap equation. We remark here also that, because of the \( T \) and \( \mu \) dependence of the mass, the regulators \( \Omega \) and \( \Lambda_{\text{LB}} \) need to depend parametrically on temperature and chemical potential to keep the equivalence with the 3M scheme.

### IV. RESULTS

The model parameters are adjusted to the isolated system. We use the Hartree approximation, i.e. \( \lambda = N_c N_f = 6 \). Parameter values shown in Table I are chosen to reproduce the pion mass \( m_\pi = 140 \text{ MeV} \), the decay constant \( f_\pi = 93 \text{ MeV} \), and to give a constituent quark mass of \( m = 336 \text{ MeV} \). The difference between the bare mass \( \bar{m}_0 \) and the constituent mass is due to the finite condensate, which is \( \langle \bar{u}\bar{d} \rangle^{1/3} = -247 \text{ MeV} \) for the parameters given in Table I. The parameters are reasonably close to the cases used in the review Ref. \[51\].

In hot and dense quark matter the surrounding medium leads to a change of the constituent quark mass due to the quasiparticle nature of the quark. The constituent mass as solution of Eq. \[47\] is plotted in Fig. I as a function of temperature and chemical potential. The fall-off is related to chiral symmetry restoration, which would be complete for \( m_0 = 0 \). It is related to the QCD phase transition. For \( T \lesssim 60 \text{ MeV} \) the phase transition is first order, which is reflected by the steep change of the constituent mass. To keep close contact with the 3M results we have chosen for the \( \Omega \) in-medium regulator mass \( \Omega^2(T, \mu) = \Lambda_{\text{3M}}^2 + m^2(T, \mu) \) with \( \Lambda_{\text{3M}} = 630 \text{ MeV} \) fixed.

| Table I: Parameters of the NJL model used in this analysis. | \( \bar{G} \) | \( \bar{m}_0 \) | \( \Omega \) |
|---|---|---|
| \( 10^{-6} \text{MeV} \) | \( \text{MeV} \) | \( \text{MeV} \) |
| 5.51 | 5.67 | 714 |
FIG. 1: Effective quark mass as a function of temperature and chemical potential. The fall-off is related to the vanishing condensate ⟨uu⟩, which shows the onset of chiral symmetry restoration. Critical temperature at μ = 0 is $T_c \approx 190$ MeV.

FIG. 2: Chiral phase transition as defined in [38]. The lower part is the chiral broken phase, whereas the upper part reflects the restored phase.

We define the phase transition to occur at a temperature at which $m(T, \mu)$ is half of the isolated constituent quark mass [38]. Because of the above mentioned equivalence we do not expect any qualitative difference to the results of the instant form analysis provided, e.g. by Refs. [31, 39]. The phase diagram is shown in Fig. 2. The line indicates the phase transition between the chiral broken and the unbroken phase.

V. CONCLUSION

We have given a relativistic formulation of thermal field theory utilizing the light front form. The proper partition operator (and the statistical operator) have been given for the grand canonical ensemble. The special case of a canonical ensemble is given for μ = 0. The resulting Fermi function depends on transverse and also on the $k^+$ momentum components. The $k^+$ components emerge in a natural way in a covariant approach and are also essential to fulfill the light front analog of the Thouless criterion [40] for the appearance of Cooper poles (color superconductivity) [41]. As an application we have revisited the NJL low energy model of QCD. Because of the zero range interaction a regularization is necessary. The equivalence of instant form 3M regularization and light front Ω regularization utilized here for the gap equation holds for a $T$ and μ dependent cut-off $\Omega^2(T, \mu) = \Lambda_{3M}^2 + m^2(T, \mu)$. Hence we reproduce the phenomenology of the the NJL model, in particular the gap-equation and the chiral phase transition. The NJL model constitutes a nontrivial example of the equivalence between instant form and front form quantization. We argue that the light front formulation of thermal field theory is a very useful and powerful alternative tool to make closer connection to QCD, in particular for the light front formulation of QCD, e.g. along the lines of [2]. The region of interest is that of finite μ, where lattice QCD just begins to become available for small μ [21]. Further merits of the light front approach are obvious when considering multi-quark correlations as has been already outlined in [3, 5].

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APPENDIX A: LIGHT-FRONT NOTATION

The contravariant components of a four vector in light-front coordinates may be written as

$$a^\mu = (a^-, a^+, a^1, a^2) \equiv (a^-, a^+, \vec{a}_\perp) \equiv (a^-, \vec{x}) \quad (A1)$$

If we define $a^\pm \equiv a^0 \pm a^3$, the standard Lorentz scalar product, can be recovered by defining the following metric tensor

$$g_{\mu\nu} = \begin{pmatrix} 0 & 1/2 & 0 & 0 \\ 1/2 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}, \quad g^{\mu\nu} = \begin{pmatrix} 0 & 2 & 0 & 0 \\ 2 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix} \quad (A2)$$

i.e.

$$a_\mu b^\nu = g_{\mu\nu} a^\mu b^\nu = \frac{1}{2} (a^+ b^- + a^- b^+) - \vec{a}_\perp \vec{b}_\perp. \quad (A3)$$
Hence the covariant components of the four vector are given by
\[ a_\mu = (a_-, a_+, a_1, a_2) = (\frac{1}{2}a^+, \frac{1}{2}a^-, a_\perp) \equiv (a_-, a). \]  
\text{(A4)}

Note
\[ a_- = \frac{1}{2}a^+, \quad a_+ = \frac{1}{2}a^- . \]  
\text{(A5)}

The four-dimensional volume element is
\[ d^4x = dx^0 dx^1 dx^2 dx^3 = \frac{1}{2} dx^- d^2x_\perp \]  
\text{(A6)}

The three-dimensional volume element on the light-like surface \( S_\nu \) characterized by the light-like vector \( \omega_\nu \) in the plane with \( \omega_\nu x^\nu = x^+ = \text{const} \), is
\[ d^4x \big|_{S_\nu} = \frac{1}{2} dx^- d^2x_\perp \equiv dS_+. \]  
\text{(A7)}

The scalar product of the light-like surface element \( dS_\nu \) with a vector \( a^\nu \) is then
\[ dS_\nu a^\nu = \frac{1}{2} dx^- d^2x_\perp a^+. \]  
\text{(A8)}

From the \( \gamma \)-matrices \( \gamma^\mu = (\gamma^+, \gamma^-, \gamma_\perp) \) it is possible to define Hermitian projection operators
\[ A^\pm = \frac{1}{2} = \gamma^\mp \gamma^\pm . \]  
\text{(A9)}

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