Reliability Analysis of Technical Means of Transport

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Abstract: The importance of system reliability within military logistics should be considered in terms of the ability to ensure the readiness of all available resources, e.g., means of transport, which are necessary during the realization of operational tasks. A special role is played by technical security, which enables the performance of all the specific tasks by the realization of the process supporting the subsystem in the area of providing the necessary assemblies, subassemblies and spare parts. The objective of the work was to define reliability in relation to technical means of transport and to illustrate an original solution leading to the determination of the expected fitness time of the available vehicle fleet, using the example of a selected military unit. The GNU Octave software—designed to conduct, among other things, advanced numerical computations—was used for the study. The daily operational mileage for a selected group of means of transport and the moments of failures were recorded during the tests, for the period from 31 December 2013 until 30 June 2015. The conducted analysis enabled the determination of the fundamental reliability indicators. The presented model has been supported with numerical examples, along with the interpretation of the obtained results.

Keywords: vehicle; operation; reliability; mathematical model

1. Introduction

The basic components of a logistics system’s potential include the technical measures enabling the storage, displacement and handling of cargo units, integrated computer networks and decision-making services managing the physical movement of goods. The individual elements of a logistics system, along with the methods of their utilization and operation, form an infrastructure of logistic processes [1–4].

Due to the structure of the technical objects, the following infrastructural elements can be distinguished:
- Linear, understood as a grid of specified strips of land, intended for parking or traffic of means of transport;
- Point, comprising objects intended for the stationary handling of cargo and means of transport;
- Informative, consisting of media, telecommunication lines and measures ensuring data transfer [5–9].

A logistics system shall also ensure the obtaining of a required level and quality of logistics support and be characterized by appropriate efficiency. The reliability of technical objects, in technical terms, shall be understood as a set of properties determining the readiness of an object and impacting such readiness:
- Operability—the ability of an object to maintain or restore, under specific operating conditions, a state enabling the execution of required functions;
- Indestructibility—the ability of an object to function properly and be uninterrupted by failure under specific conditions within a given time interval [10];

- Ensuring the necessary operating measures [11,12].

The literature often equates reliability, in terms of meaning (and as a measure), to operational readiness. If a given object is composed of multiple subassemblies, then it is important to determine not only its reliability characteristics but also the impact of the reliability of individual subassemblies on the reliability of this specific object. The theory of reliability defines a system as an organized set of objects intended for the execution of specific tasks. The method of system element interconnections, which determines the impact of system failure depending on the failure of individual elements, is called the reliability structure of a system [13–15].

The objective of this research paper is to define reliability in relation to technical means of transport and to propose an original solution leading to the determination of the expected fitness time of the available vehicle fleet, using the example of a selected military unit (MU).

The literature of the subject is often related to the issue of transport system reliability, whereas less focus is placed on the reliability analysis of means of transport, which are the components of the entire system. Therefore, this publication focuses on issues associated with the reliability analysis covering technical means of transport and the determination of further research directions.

The most frequently estimated measures of transport system efficiency are:

- Operational readiness or availability;
- Safety;
- Reliability [16,17].

A transport process implemented by a transport system is complex and involves a set of activities that enable the execution of a specific transport task. Figure 1 shows the components of a transport system.

![Figure 1. Components of a transport process](image)

The reliability of a transport system depends on both its structure and the method of management at the engineering and operation stages. As a result, the analysis of a transport system reliability process shall take into account the possible occurrence of issues associated, among others, with:

- Possible vehicle unreliability;
- Unreliable infrastructure;
- Lack of information exchange;
- Erroneous operator decisions;
- Random events.
The indicated problems may adversely impact the efficiency of the entire transport process. Transport process reliability can be understood as the ability of a system to operate correctly over a planned time period. According to [18], the concept of transport system reliability can be applied to express the probability of a transport system satisfying the passenger demand for a transport service, without undesired events resulting from vehicle and infrastructure failure or the participation of other means of transport. The research paper [19] describes changes within the technologies for monitoring transport services and increasing their reliability in the era of advancing electronic communication and available localization technologies. In paper [20], the archived data from the automatic systems of localization are used to obtain information related to possible sources of vehicle unreliability. In the studies, the actual data from the CTM SpA public transport company from Cagliari in Italy were applied. The collected numerical data included 30 bus lines servicing approximately 35,500,000 passengers per year. A thorough analysis was conducted based on the archived data from the automatic systems of vehicle localization, whereas a reliability test concerned the prevention strategies. During the reliability analysis of the transport system, particular attention should be focused on the anomalies, which are frequently eliminated or neglected. In paper [21], the influence of the anomalies in the archived data from the automatic systems of localization is described in order to estimate both the regularity and the punctuality of transport services.

From a passenger’s perspective, the reliability of a transport system can be perceived in terms of delay (longer travel times). The source literature contains numerous reliability models for travel time in passenger transport, where the travel time reliability is studied, among others, in terms of measuring its duration variability [22–26], whereas the authors of [27] analyzed the travel time reliability based on a triangular distribution. An innovative approach to the problem of travel time reliability is presented in paper [28], in which two measurements of reliability are introduced, namely, PPR—the percentage of passengers receiving a regular service and PPP—the percentage of passengers receiving a punctual service. The proposed factors are of key importance while defining the capabilities of transport companies aiming to improve passenger service performance.

Article [29] presents a public transport service reliability analysis using the graph theory. Paper [30] describes a thorough reliability analysis of a transport network, based on multimodal transport systems. In relation to the reliability of means of transport, the issue was analyzed in paper [31], in which the author focused on describing the reliability characteristics of means of transport within urban areas. Article [32] suggests a procedure for estimating the reliability measures for means of transport in relation to a selected route of municipal mass transit. The model, developed using the PT VISION VISSIOM (ver. 5.4) app, points out that the analysis covered only a single, 10.33 km long trolleybus route. It was emphasized that the conducted studies were based on a simulation model developed in the course of the previous research. As a result, in order to carry out a more thorough analysis, it would be necessary to extend the research area into a municipal transport system and take all public means of transport into account.

The authors of this paper believe that the reliability of technical means of transport shall be understood as a foreseeable ability to perform a task that involves the correct use of the functions available within a specified time period, while simultaneously taking into account the actual operating conditions.

The reliability of technical means of transport that provide services for the military is of crucial importance from the perspective of conducting operational tasks at various command levels. This is a consequence of the fact that the basic tasks of military logistics, in technical terms, are to ensure correct operation and maintain an appropriate level of military equipment readiness. A high reliability of technical means of transport is required both in terms of carrying out tasks and ensuring the safety of soldiers performing such tasks. Monitoring the vehicle technical condition in terms of the frequency of conducted preventive replacement of their sub-assemblies, assemblies and parts significantly increases the reliability of the entire transport system.
The ability to predict future phenomena and determine the expected time before failure, i.e., expected fitness time, are of particular importance. It is possible to utilize numerical integration methods for this purpose, which greatly facilitate the calculations. Therefore, this publication proposes a novel solution in the form of source code developed in the Notepad++ text editor, designed to determine the reliability measure, which is to be the expected fitness time of the studied fleet of vehicles. After calling a file name in the GNU Octave command window, the software displays a message prompting the user to enter a reliability function formula along with the defining integration limits. The study compares the selected numerical integration methods and indicates their advantages and disadvantages.

2. Methodology

This section describes the theoretical grounds for developing a mathematical model. The functional characteristics of reliability, considered in terms of probability and described in Section 2.1, are related to continuous random variables and may be also applied during mathematical transformations in order to determine specific functions analytically. If the considered objects are unrepairable, it is necessary to know relevant probability distributions in order to determine a value of a specified reliability factor. Then, a value of this factor should be determined based on the analytical methods of a probability theory. From a practical perspective, only the values of reliability factors in a certain population and statistical methods for the estimation of these values may be referred to. Functions characterizing a random time of object fitness, described in Section 2.1, are simply interpreted statistically in the case when the probability of events is replaced with their occurrence frequency. There can be distinguished two groups of methods for the statistical presumption of reliability:

- Non-parametric methods, in which the probabilistic characteristic of a random variable is deduced directly;
- Parametric methods, in which the probabilistic characteristics of random variables are defined indirectly through determining the probabilistic characteristics of specific parameters from an experiment.

A correlation of parametric and non-parametric methods, for a statistical test of object reliability, is presented in the form of a chart (Figure 2).

![Figure 2. Flowchart of the correlation of parametric and non-parametric methods.](image-url)
If it is required to estimate the functional characteristics of reliability based on the empirical data, obtained during either observations or experimental tests, the object reliability may be characterized through a set of numerical data randomly selected from a given population (set) of objects. Therefore, it is crucial to appropriately estimate the parameters of the distribution of the considered variable as well as to determine the empirical characteristics, both numerical and functional, described in Section 3.

2.1. Theoretical Grounds for Developing a Mathematical Model

Theoretically, each object can be considered as a system in the course of tests. There is no objective division of objects into systems and elements. The developed formal model for this object constitutes grounds for determining whether a given object, the reliability of which is to be tested, will be treated as a system. In the case of reliability, the objective is to study the relationships between the system element reliability and the entire system reliability [10].

A mathematical model of a non-renewable technical object that describes its reliability, understood as the ability to execute tasks under specific conditions and within a known time interval, is a non-negative and constant random variable \( T \) [33–39]. The basic measure of the reliability of an object \( R(t) \) within a time interval \([0, t]\) is the object probability described by the following formula in Equation (1):

\[
R(t) = P(T \geq t) \quad \text{for } t \geq 0
\]  

The reliability function of an object \( R(t) \) for each \( t \geq 0 \) has a value equal to the probability of an event involving object failure-free operation at least until \( t \), which is the probability of an object being in a state of fitness until \( t \).

The function, which for each established \( t \geq 0 \) adopts the value of the probability of an event that the object at moment \( t \) is damaged, is referred to as an unreliability function, described by the following formula in Equation (2):

\[
F(t) = P(T < t) = 1 - R(t)
\]  

If the reliability function is absolutely continuous, then:

\[
R(t) = \int_{t}^{\infty} f(u)\,du \quad \text{for } t \geq 0
\]  

Function \( f \) satisfying the condition in Equation (3) is called a probability density. At all continuity points, the probability density can be expressed as a derivative, using the following relationship in Equation (4):

\[
f(t) = \frac{d}{dt}[F(t)] = -\frac{d}{dt}[R(t)]
\]  

If the probability density and reliability are known, then it is also possible to determine the failure rate with Equation (5):

\[
\lambda(t) = -\frac{d}{dt} \ln R(t) = \frac{f(t)}{R(t)}
\]  

Using the Taylor’s formula, the relationships in Equations (6) and (7) can be noted as:

\[
f(t) \approx \frac{R(t) - R(t + \Delta t)}{\Delta t}
\]  

\[
\lambda(t) \approx \frac{R(t) - R(t + \Delta t)}{R(t) \Delta t}
\]
The formula in Equation (6) indicates an interpretation of the probability density, which is understood as a decline in reliability over a short time interval of \( \Delta t \), whereas the relationship in Equation (7) means that the failure rate is a relative decline in the reliability of an object per unit of time.

A measure of the depletion of an object task execution reserve is a leading function or cumulative failure rate described by the formula:

\[
\Lambda(t) = -\ln[R(t)] = \int_0^t \lambda(u)du \text{ for } t \geq 0
\]  

(8)

Each of the presented functions characterizing the object fitness time can be expressed by any different one of them [40–43].

If the reliability function \( R(t) \) is known, the expected operating time before failure is calculated from the relationship in Equation (9):

\[
ET = \int_0^\infty R(t)dt
\]  

(9)

In relation to non-renewable objects, the expected value of the random variable \( T \) is also often called an expected fitness time [43], and its measure can be calculated using numerical integration methods. If an integrand is not an elementary function, the analytical determination of the integral can be very difficult or even impossible. Numerical formulas for the integration of functions with single independent variables are called quadratures [44].

The following designations were introduced to precisely explain the applied numerical formula:

- a—lower integration limit;
- b—upper integration limit;
- \( t_i \)—equidistant quadrature nodes, such as \( t_i = a + \frac{(b-a)}{n^*}i, i = 0, \ldots, n^*; \)
- \( h \)—length of the smallest subdivision (applies to compound quadratures).

The rectangle rule involves approximating an area limited by a function graph by rectangles with a base equal to the integration step length and a height equal to the function within the interval determined by the integration step.

The rectangle quadrature based on the node \( t_0 = (a+b)/2 \) can be expressed as:

\[
\int_a^b R(t)dt \approx (b-a)R\left(\frac{a+b}{2}\right)
\]  

(10)

The trapezoidal rule consists of approximating the area limited by a function graph by rectangular trapezoids with a height equal to the length of the integration step and bases with lengths corresponding to the function values at nodal points on the interval boundary [45].

The trapezoidal quadrature based on nodes \( t_0 = a \) and \( t_1 = b \) shall be calculated using the following relationship:

\[
\int_a^b R(t)dt \approx \frac{(b-a)}{2}(R(a) + R(b))
\]  

(11)

The Simpson’s rule involves approximating an area limited by a function graph by a square function spread over the value of a function integrated at the integration interval central points, for which the sides are the values of a function integrated at boundary points.

Simpson’s quadrature based on nodes \( t_0 = a, t_1 = b, \) and \( t_2 = (a+b)/2 \) can be determined based on the formula:
\[
\int_{a}^{b} R(t) \, dt \approx \frac{b-a}{6} \left( R(a) + 4R \left( \frac{a+b}{2} \right) + R(b) \right)
\]

(12)

Calculation accuracy is often improved by applying a division of the integration interval in question in an \(n^*\) number of sub-intervals and determining a compound trapezoidal quadrature and compound Simpson’s quadrature, which are described by the relationships in Equations (13) and (14):

\[
\int_{a}^{b} R(t) \, dt \approx h \left( \frac{R(a) + R(b)}{2} + \sum_{i=1}^{n^*-1} R(a + ih) \right)
\]

where \( h = \frac{(b-a)}{n^*} \)

(13)

\[
\int_{a}^{b} R(t) \, dt \approx \frac{h}{3} \left( \frac{R(a) + R(b)}{2} + \sum_{i=1}^{n^*-1} R(a + ih) + 2 \sum_{i=0}^{n^*-1} R(a + ih + \frac{h}{2}) \right)
\]

(14)

Table 1 describes the advantages and disadvantages for each of the applied rules.

| Quadrature (Rule)     | Disadvantages                              | Advantages                          |
|-----------------------|--------------------------------------------|-------------------------------------|
| Rectangular           | Very low accuracy resulting from node selection; high-error method | Calculation simplicity               |
| Trapezoidal           | Accuracy depends on node selection         | Calculation simplicity; higher accuracy than rectangular rule; need for fewer calculations in order to obtain better result accuracy |
| Simpson’s             | Complicated calculation formula based on three nodes | Higher accuracy than rectangular and trapezoidal rules |
| Compound trapezoidal  | Sensitive to a number of nodes; complex calculation formula | Rapid convergence rate; possibility to complete the calculation process after reaching a set accuracy |
| Compound Simpson’s    | Sensitive to a number of nodes; extended numerical formula | Very rapid convergence rate; possibility to complete calculations after obtaining a set accuracy; method accuracy increases with an increase in the number of nodes |

3. Mathematical Model—Implementation

The characteristics resulting from a group randomly selected from a given object population are called empirical characteristics. The determination of empirical characteristics, both functional and numerical, can be presented in the case of observing the fitness time to failure of \(n\) studied objects. The following designations were adopted for the development of the vehicle reliability model:

(a) The number of studied vehicles:

\[
n = n(t) + m(t),
\]

where:

\(n(t)\)—the number of fit for the use of vehicles until moment \(t\);

\(m(t)\)—the number of unserviceable vehicles until moment \(t\);

(b) The reliability empirical function:

\[
\bar{R}(t) = \frac{n(t)}{n} = \frac{n - m(t)}{n} = 1 - \frac{m(t)}{n},
\]

(16)
The unreliability empirical function:
\[
Q(t) = 1 - R(t) = \frac{n - n(t)}{n} = 1 - \frac{n(t)}{n},
\]  
(17)

The failure probability density empirical function:
\[
f(t) = \frac{n(t) - n(t + \Delta t)}{n\Delta t} = \frac{R(t) - R(t + \Delta t)}{\Delta t},
\]  
(18)

The failure rate empirical function:
\[
\lambda(t) = \frac{n(t) - n(t + \Delta t)}{n(t)\Delta t} = \frac{R(t) - R(t + \Delta t)}{R(t)\Delta t},
\]  
(19)

The empirical leading function:
\[
\Lambda(t) = \sum_i \lambda(t_i)\Delta t.
\]  
(20)

Glivenko’s theorem indicates that at each continuity point of a distribution function of a random variable describing the fitness time within a population, in which the sample was randomly selected, for each \(\varepsilon > 0\), the following boundary property occurs:
\[
\lim_{n \to \infty} P\left\{ \sup_t |Q(t) - Q(t)| < \varepsilon \right\} = 1.
\]  
(21)

It justifies applying the empirical distribution function to evaluate the reliability of the objects selected from a given population [43].

The knowledge of the failure rate function’s waveform enables the obtaining of important information in the fields of:

- Properly equipping the objects with tools and spare parts;
- Determining the spare part production output;
- Determining the usefulness of applying the preventive part or assembly replacements;
- Determining the object operating periods;
- Determining the optimum object durability;
- Planning the recovery of scarce elements or assemblies from repaired and scrapped objects [42].

4. Numerical Example—Real Case Study

The potential of a MU (Military Unit) machinery park consists of 11 vehicles necessary to execute specific tasks. The operational process analysis involved a group of average-capacity, high-mobility means of transport with a permissible gross vehicle mass below 7000 kg. The main purpose of the studied group of vehicles was transporting people, transporting cargo and using the chassis for installing special devices, depending on the performed tasks. The research involved recording daily operational mileage in the period from 31 December 2013 to 30 June 2015 (18 months) and the date of the failure. Unfortunately, due to the lack of access to repair sheets, a simplified assumption that the vehicles in question were not renewable was adopted. The figures used for the calculations were taken from military operational documents such as departure orders and technical service sheets. The task involves analyzing the reliability of technical means of transport and determining the expected fitness time of an MU fleet. Table 2 lists the figures concerning fit and unfit technical objects over a defined operating time.
Table 2. List of fit and unfit vehicles from 31 December 2013 to 30 June 2015.

| Period                  | \( n(t) \) |
|-------------------------|-------------|
| 31 December 2013        | 11          |
| 1 January 2014–31 March 2014 | 8 3        |
| 1 April 2014–30 June 2014 | 8          |
| 1 July 2014–30 September 2014 | 6 5      |
| 1 October 2014–31 December 2014 | 5 6    |
| 1 January 2015–31 March 2015 | 5 6        |
| 1 April 2015–30 June 2015 | 4 7        |

A vehicle reliability empirical function for subsequent periods was calculated based on the relationship in Equation (16), and its waveform is depicted in Figure 3.

The reliability function graph indicates that there was no change in the number of fit vehicles over the period in question on two occasions. The geometrical interpretation of the integral and formula in Equation (16) indicates that the expected fitness time is equal to the area limited by the reliability function and coordinate system axes.

There are currently more and more computer programs and packages available, such as MATLAB, C++, Pascal or BASIC, that can be used for numerical integration. However, it should be noted that they require the user to be proficient in their operation and have a thorough knowledge of the available functions. An alternative to commercial software can be the freeware environment called Octave, which enables the application of the programming techniques and loops as well as the generation of new user functions or the conducting of complex numerical computations. By applying numerical integration, the values of individual simple and complex quadratures were calculated, and the results are shown in Figure 4.

Figure 3. Reliability empirical function waveform.

GNU Octave is a relatively little, popular programming environment, and thus, a script for calculating the values of individual quadratures (expected fitness times for a studied vehicle group) was developed for the purposes of this study. Figure 5 illustrates the source code of the software using the Notepad++ text editor.
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It should be noted that in order to develop a function, the text file needs to be saved in the working directory under the same name as the function executed in the GNU Octave command window. The integration limits in the attached script are designated with the letters $s$ and $t$.

Table 3 shows the results of applying the individual quadratures depending on the adopted reliability empirical function.

| Function/Method | Rectangle | Trapezoidal | Simpson’s | Compound Trapezoidal | Compound Simpson’s | R2  |
|-----------------|-----------|-------------|-----------|----------------------|---------------------|-----|
| $R_1(t)$        | 9.9522    | 9.9522      | 9.9522    | 9.9522               | 9.9522              | 0.85|
| $R_2(t)$        | 8.8902    | 10.348      | 9.3762    | 9.3763               | 9.3762              | 0.91|
| $R_3(t)$        | 9.4440    | 10.669      | 9.8524    | 9.8491               | 9.8490              | 0.93|
Where:

\[
\bar{R}_1(t) = -0.0337t + 0.8562, \quad (22)
\]
\[
\bar{R}_2(t) = -0.0002t^2 + 0.0064t^2 - 0.0897t + 0.9286, \quad (23)
\]
\[
\bar{R}_3(t) = 0.8685e^{-0.056t}. \quad (24)
\]

The lowest values of the match factor $R^2$ were obtained by approximation using the linear function. An approximation of the reliability empirical function using a polynomial of the first degree subsequently led to the determination of the same expected fitness time value for each of the applied quadratures. A third-degree polynomial was used to obtain a higher match factor than the one determined using a linear function. However, it should be noted that the highest $R^2$ factor of 93% was recorded when approximating the reliability empirical function using an exponential function.

Another method for the determination of quadrature values is using the ”quad” feature provided in the Octave 3.4.3 software. The computation results for individual approximations of the reliability empirical function are shown in Figure 6.

**Figure 6.** Result of applying the “quad” feature.

Numerical integration using the “quad” feature enabled the obtaining of results corresponding to a complex Simpson’s quadrature. Figure 7 shows the expected fitness time calculated using mathematical software.

**Figure 7.** Expected fitness time.

The expected fitness time value when $t \to \infty$ is 15.509 months. Figure 8 illustrates the computation results.

It should be noted that, depending on the integration interval length, the values obtained using complex quadratures differ to a very small extent, whereas a significant increase in the integration interval division leads to the convergence of the obtained results.

Another reliability characteristic is a failure rate function that depends on the physical properties, imposed requirements and operating conditions of the objects in question. It can be a function that
is constant over time, monotonically decreasing or monotonically increasing and can have a single extreme (maximum or minimum) or several extreme points. The presence of extreme points results from the fact the number of unfit vehicles in the second quarter of 2014 and the first quarter of 2015 did not change. The results of the conducted calculations are shown in Figure 9.

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Figure 8. Complex quadrature values depending on the integration interval division.

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Figure 9. Failure rate function waveform.

The cumulative failure rate graph indicates that an increase in the number of unfit vehicles was recorded over time, which may be caused by the ongoing adaptation and wear processes. The so-called premature failures, which result from the improper performance of the objects or the application of incorrect technology, are the most common during the adaptation period. When the impact of the adaptation period process is negligible, fatigue and wear of the objects are reflected in a growing function of the cumulative failure rate, which is shown in Figure 10.
The subject of the research included a group of medium-capacity, high-mobility military vehicles, with a permissible gross vehicle mass below 7000 kg. The main purpose of the analyzed means of transport was carrying people and equipment and, depending on the needs, the ability to adapt the chassis for the installation of special devices. Over the course of the 18-month studies, using departure orders and technical servicing sheets, the operational mileage of each vehicle and the dates of the failure were recorded on a daily basis. Based on the collected data, it was concluded that the number of roadworthy (fit) vehicles over the period in question changed on two occasions. The highest alignment factor of 93% was obtained through the approximation of the empirical reliability function using an exponential function. It should be noted that, depending on the adopted integration interval length, the expected fitness time values calculated using the compound trapezoidal rule and compound Simpson’s rule were slightly different. The expected fitness time for the studied group of vehicles was approximately 15 months. The results of tests covering medium-capacity, high-mobility military vehicles have their specific characteristics and have not yet been the subject of other publications.

This study focused on determining the reliability characteristics, both functional and numerical. One of the numerical characteristics is the expected fitness time, the value of which can be calculated using numerical integration methods. The article presents a novel solution in the form of a script, the implementation of which in GNU Octave leads to the determination of the expected fitness time for a studied group of vehicles. It also discusses the theoretical grounds for the structure of the mathematical model and explains the purposefulness of applying the numerical formulas for the integration of functions with single independent variables, so-called quadratures. Each of the applied methods was compared regarding the indicated advantages and disadvantages resulting from their application.

The objective of this article was to define the reliability of technical means of transport, using the example of a selected MU, and to determine the expected fitness time, using the GNU Octave and Notepad++ software. The source literature contains numerous studies on the reliability of a transport system, with much less elaboration related to the issues associated with the reliability of means of transport, which are, undoubtedly, components of the entire system. Ensuring an appropriate level of military equipment readiness and proper operating conditions for vehicles enables the execution of tasks at individual command levels. Every vehicle failure can lead to growing operating costs resulting from the need for diagnostics and repairs. It should be noted that the operating conditions of means of transport significantly influence their technical condition, durability and reliability. This can be reflected in the decreasing operating efficiency of the vehicles and can impose a higher frequency of maintenance work.

5. Conclusions

The objective of this article was to define the reliability of technical means of transport, using the example of a selected MU, and to determine the expected fitness time, using the GNU Octave and Notepad++ software. The source literature contains numerous studies on the reliability of a transport system, with much less elaboration related to the issues associated with the reliability of means of transport, which are, undoubtedly, components of the entire system. Ensuring an appropriate level of military equipment readiness and proper operating conditions for vehicles enables the execution of tasks at individual command levels. Every vehicle failure can lead to growing operating costs resulting from the need for diagnostics and repairs. It should be noted that the operating conditions of means of transport significantly influence their technical condition, durability and reliability. This can be reflected in the decreasing operating efficiency of the vehicles and can impose a higher frequency of maintenance work.

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The subject of the research included a group of medium-capacity, high-mobility military vehicles, with a permissible gross vehicle mass below 7000 kg. The main purpose of the analyzed means of transport was carrying people and equipment and, depending on the needs, the ability to adapt the chassis for the installation of special devices. Over the course of the 18-month studies, using departure orders and technical servicing sheets, the operational mileage of each vehicle and the dates of the failure were recorded on a daily basis. Based on the collected data, it was concluded that the number of roadworthy (fit) vehicles over the period in question changed on two occasions. The highest alignment factor of 93% was obtained through the approximation of the empirical reliability function using an exponential function. It should be noted that, depending on the adopted integration interval length, the expected fitness time values calculated using the compound trapezoidal rule and compound Simpson’s rule were slightly different. The expected fitness time for the studied group of vehicles was approximately 15 months. The results of tests covering medium-capacity, high-mobility military vehicles have their specific characteristics and have not yet been the subject of other publications.

Given the above, the authors were not able to compare them with the study results presented in other
publications. The determination of the expected fitness time is significantly simplified by the possibility of applying the “quad” feature provided in the GNU Octave software, which consequently led to obtaining a result corresponding to the assumed Simpson’s quadrature. The conducted calculations showed that the GNU Octave software is an effective tool when applied for the determination of the expected fitness time, depending on the adopted reliability function. The implementation of the proposed solution reduces the calculation time and enables a user to determine the expected fitness time based on the individually specified criteria.

The failure rate function—which enables the obtaining of information regarding, among other things, the purposefulness of applying preventive subassembly, assembly and spare part replacements—is important. While analyzing the waveform of a failure rate empirical function, it should be noted that the cause of the double occurrence of extreme points, resulting from the fact that the numbers of unfit vehicles in 2014 Q2 and 2015 Q1 were similar, did not change. Furthermore, the growing waveform of the cumulative failure rate empirical function means that the number of unfit vehicles grew over years, which might have resulted from the adaptation processes in the initial operation phase or wear resulting from the progressing corrosion processes.

Expanding the reliability analysis with the Monte Carlo method will be a reasonable direction for further research. It is one of the numerical integration methods, enabling the modeling of the processes, the course of which depends on random factors. This involves entering information regarding the integration interval in question and the number of points to be generated, in order to calculate the integral value. The accuracy of the Monte Carlo method depends on the applied pseudo-random number generator. The biggest advantage resulting from the application of this method is the fact that the increasing computing power of computers enables the solving of increasingly complex and complicated problems. However, it should be noted that the results depend on the quality of the used pseudo-random number generator.

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**References**

1. Niziński, S.; Żurek, J.; Ligier, K. *Logistics for Engineers*; Transport and Communication Publisher: Warsaw, Poland, 2011; p. 229.
2. Barlow, R.E.; Proschan, F. *Mathematical Theory of Reliability, Classics in Applied Mathematics*; Society for Industrial and Applied Mathematics: Philadelphia, PA, USA, 1965; p. 12.
3. Zieja, M.; Ziółkowski, J.; Oszczypała, M. Comparative Analysis of Available Options for Satisfying Transport Needs Including Costs. In Proceedings of the 23rd International Scientific Conference on Transport Means, Palanga, Lithuania, 2–4 October 2019; pp. 1433–1438.
4. Ziółkowski, J.; Legas, A. Problem of Modelling Road Transport. *J. Konbin 2019*, 49, 159–193. [CrossRef]
5. Cygan, Z. *Fundamentals of Economics of Motor Transport in the Army*; Ministry of National Defence: Warsaw, Poland, 1978; p. 75.
6. Jacyna, M. *Selected Problems of Modeling Transport. Systems*; Warsaw University of Technology Publishing House: Warsaw, Poland, 2009; p. 143.
7. Migdalski, J. *Reliability Engineering: Handbook*; ZETOM: Warsaw, Poland, 1992; Volume 2, p. 658.
8. Nowakowski, T. *Reliability of Logistic Systems*; Wroclaw University of Science and Technology Publishing House: Wroclaw, Poland, 2011; p. 102.
9. Lefebvre, M. *Basic Probability Theory with Applications*; Springer: New York, NY, USA, 2009; p. 103.
10. O’Connor, P.; Kleyner, A. *Practical Reliability Engineering*, 5th ed.; John Wiley & Sons, Ltd.: Chichester, UK, 2012; p. 19.
11. Pham, H. *Handbook of Reliability Engineering*; Springer-Verlag: London, UK, 2003; p. 651.
12. PN-93/N-50191: Polish Standard. *Electricity Glossary. Reliability*; Quality of Service, Polish Committee for Standardization: Warsaw, Poland, 1994.

13. Migdalski, J.; Bartoszewicz, J.; Bobrowski, D.; Ciechanowicz, K.; Dwiłirski, L.; Jaźwiński, J.; Kalinowska, H.; Kiliński, A. Probabilistic Methods in Reliability. In *Reliability Handbook: Mathematical Basics*; Migdalski, J., Ed.; WEMA: Warsaw, Poland, 1982; p. 68.

14. Żurek, J.; Ziętkowski, J.; Borucka, A. A method for Determination of Combat Vehicles Availability by Means of Statistic and Econometric Analysis. In Proceedings of the 27th European Safety and Reliability Conference, Portoroz, Slovenia, 18–22 June 2017; pp. 2925–2933.

15. Karpinski, J.; Firkowicz, S. *Preventive Maintenance Policies of Technical Objects*; National Scientific Publishers: Warsaw, Poland, 1981; p. 85.

16. Kutz, M. (Ed.) *Handbook of Transportation Engineering*; McGraw-Hill Companies: New York, NY, USA, 2013; pp. 25–41.

17. Młyńczak, M.; Nowakowski, T.; Restel, F.; Werbińska-Wojciechowska, S. Problems of Reliability Analysis of Passenger Transportation Process. In Proceedings of the European Safety and Reliability Conference, Balkema, Leiden, 2–4 October 2011; pp. 1433–1438.

18. Fricke, J.D.; Whitford, R.K. *Fundamentals of Transportation Engineering. A Multimodal Systems Approach*; Pearson Education, Inc.: Upper Saddle River, NJ, USA, 2004; pp. 243–276.

19. Levinson, H.S. The Reliability of Transit Service: An Historical Perspective. *J. Urban. Technol.* 2005, 12, 99–118. [CrossRef]

20. Barabino, B.; Di Francesco, M.; Mozzi, S. An Offline Framework for the Diagnosis of Time Reliability by Automatic Vehicle Location Data. *IEEE Trans. Intell. Transp. Syst.* 2016, 18, 583–594. [CrossRef]

21. Barabino, B.; Di Francesco, M.; Mozzi, S. Time Reliability Measures in bus Transport Services from the Accurate use of Automatic Vehicle Location raw Data. *Qual. Reliab. Eng. Int.* 2017, 33, 969–978. [CrossRef]

22. Pulugurtha, S.S.; Imran, M.S. Modeling Basic Freeway Section Level-of-Service Based on Travel Time and Reliability. *Case Stud. Transp. Policy 2020*, 8, 127–134. [CrossRef]

23. Zhao, L.; Guan, H.; Zhang, X.; Zhao, P.; Wang, P. Study on Travel Time Reliability Considering Route Travel Time Boundary. *J. South. China Univ. Technol.* 2019, 47, 127–135.

24. Tu, Q.; Cheng, L.; Sun, C.; Tang, F.; Li, M. Reliability-based Network Equilibrium Model with Truncated Stochastic Travel Time. *J. Southeast. Univ.* 2020, 50, 175–181.

25. Zheng, L.; Hensher, D.A.; Rose, J.M. Willingness to Pay for Travel Time Reliability in Passenger Transport: A Review and some New Empirical Evidence. *Transp. Res. Part. E Logist. Transp. Rev.* 2010, 46, 384–403.

26. Chakrabarti, S. The Demand for Reliable Transit Service: New Evidence Using Stop Level Data from the Los Angeles Metro Bus System. *J. Transp. Geogr.* 2015, 48, 154–164. [CrossRef]

27. Nam, D.; Park, D.; Khamkongkhun, A. Estimation of Value of Travel Time Reliability. *J. Adv. Transp.* 2005, 39, 39–61. [CrossRef]

28. Prashker, J.N. Direct Analysis of the Perceived Importance of Attributes of Reliability of Travel Modes in Urban Travel. *Transportation 1979*, 8, 329–346. [CrossRef]

29. Bell, M.G.H.; Schmocker, J.-D. Network Reliability: Topological Effects and the Importance of Information. In Proceedings of the 3rd International Conference on Traffic and Transportation Studies, Guilin, China, 23–25 July 2002; pp. 453–460. [CrossRef]

30. D’Este, G.M.; Taylor, M.A.P. Network Vulnerability: An Approach to Reliability Analysis at the Level of National Strategic Transport Networks. In Proceedings of the 1st International Symposium on Transport Network Reliability, Kyoto, Japan, 31 July–1 August 2001; pp. 23–44.

31. Prashker, J.N. Direct Analysis of the Perceived Importance of Attributes of Reliability of Travel Modes in Urban Travel. *Transportation 1979*, 8, 329–346. [CrossRef]

32. Yatskiw, I.; Pticina, I.; Savrasovs, M. Urban Public Transport System’s Reliability Estimation Using Microscopic Simulation. *Transp. Telem. 2012*, 13, 219–228. [CrossRef]

33. Bazaraa, M.S.; Jarvis, J.J.; Sherali, H.D. *Linear Programming and Network Flows*, 4th ed.; John Wiley & Sons Inc.: New York, NY, USA, 2010; p. 748.

34. Kececioglu, D.B. *Reliability Engineering Handbook*; DEStech Publications: Lancaster, PA, USA, 2002; Volume 1, p. 62.
35. Macha, E.; Niesłony, A. Reliability of Mechatronic Systems. Academic Handbook; Opole University of Technology Publishing House: Opole, Poland, 2010; pp. 27–45.

36. Rausand, M.; Høyland, A. System Reliability Theory, Models, Statistical Methods and Applications, 2nd ed.; John Wiley & Sons Inc.: New Jersey, NJ, USA, 2004; p. 148.

37. Szkutnik-Rogoż, J.; Ziolkowski, J. Determine Transportation Costs with Using Octave 3.4.3. In Research Approach in Logistics Processes and Transport Systems; Warsaw University of Technology Publishing House: Warsaw, Poland, 2016; pp. 531–542.

38. Woropay, M.; Landowski, B.; Żurek, J. Operational Availability of the Executive Subsystem in the Transport System within Serial Changing Maintenance Stages. Mach. Exploit. Issues 2004, 39, 87–100.

39. Birolini, A. Reliability Engineering Theory and Practice, 3rd ed.; Springer: Berlin/Heidelberg, Germany, 1999; p. 298.

40. Młynarski, S. Problems of Definition of Reliability in Machines and Vehicles Operating. J. Mach. Constr. Maint. 2003, 2, 165–174.

41. Korzan, B. Reliability Theory Elements; Military University of Technology: Warsaw, Poland, 1986; p. 21.

42. Niziński, S.; Żołtowski, B. Information Management Systems for the Operation of Technical Object; MARKAR-B.Z: Bydgoszcz, Poland, 2001; p. 146.

43. Bobrowski, D. Models and Mathematical Methods of Reliability Theory in Examples and Assignments; Scientific and Technical Publishers: Warsaw, Poland, 1985; p. 188.

44. Kosma, Z. Numerical Methods for Engineering Applications; Radom University of Technology: Radom, Poland, 1999; p. 263.

45. Gil, A.; Segura, J.; Temme, N.M. Numerical Evaluation of Airy-Type Integrals Arising in Uniform Asymptotic Analysis. J. Comput. Appl. Math. 2020, 371, 112717. [CrossRef]

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