New Developments in Open-String Theories

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ABSTRACT

The study of string models including both unoriented closed strings and open strings presents a number of new features when compared to the standard case of models of oriented closed strings only. We review some basic features of the construction of these models, describing in particular how gauge symmetry breaking can be achieved in this case. We also review some peculiar properties of the Green-Schwarz anomaly cancellation mechanism that present themselves in lower-dimensional open-string models.

Talk presented at the Tenth National General Relativity Conference
Bardonecchia, ITALY, September 1992
The study of models of oriented closed strings has been the object of most of the research carried out in String Theory over the last few years\textsuperscript{[1]} [2] [3]. Two key concepts have emerged. The first one is the (super)conformal invariance\textsuperscript{[1]} of the world-sheet theory describing the degrees of freedom of the string, a remnant, after gauge fixing, of the local invariances of the models. The second is modular invariance\textsuperscript{[4]}, a subtle global constraint deeply linked to the very nature of oriented closed strings. The meaning of this constraint may be simply appreciated by referring to the torus displayed in fig. 1. In sharp contrast with the case of particles, when oriented closed strings propagate there is generally no unique notion of “time” direction on the world sheet. The ambiguity, however, is harmless, as long as the models are endowed with a corresponding symmetry. The symmetry is precisely modular invariance, and for the torus one finds the infinite group $SL(2, Z)$ generated by the two transformations

$$\tau \rightarrow \tau + 1 \quad \text{and} \quad \tau \rightarrow -\frac{1}{\tau}.$$  \hspace{1cm} (1)

The torus amplitude is very important, since it contains a lot of information about the spectrum of the models. Thus, the modular invariance of the torus amplitude is a basic criterion to identify consistent models of oriented closed strings. The role of modular transformations in higher-genus amplitudes is also quite interesting. In particular, the invariance at genus two may be linked to the relation between spin and statistics\textsuperscript{[3]}.

The extension of these results to models including both unoriented closed strings and open strings (referred to, for brevity, as open-string models) has proved harder than was naively expected. There are actually two reasons for this. First of all, the propagation of unoriented and/or open strings reduces in part (or, at times, completely) the ambiguity in the choice of time direction on the world sheet. As a result, the role of modular transformations in these models need be reconsidered. Moreover, the end points of open strings are special, and they may be associated to the internal (Chan-Paton) symmetry\textsuperscript{[5]} of the models.
Referring to fig. 2, let us consider the propagation on an annulus of open strings with end charges possibly of two different kinds. This phenomenon corresponds to choosing a “vertical” time direction on the world sheet. On the other hand, if a “horizontal” time direction is chosen, for instance by performing a modular transformation on the previous setting, the picture changes dramatically: now one sees closed strings propagating along a tube with two boundaries at its ends. The lesson is therefore that, though quite important, modular transformations are not an invariance of these models. Rather, they link different sectors of the same model. In particular, open strings need closed strings for their consistency, since their propagation is, in some respect, the propagation of closed strings. This crucial fact was pointed out long ago by Lovelace [6]. What should we learn from these considerations? First of all, the structure of models of oriented closed strings only is simpler than that of open-string models. Thus, reverting the original observation of Lovelace, it would appear quite convenient to use models of oriented closed strings as the starting point in the process of model building for open-string theories. The suitable models should allow, in particular, for proper reflections at the ends of the tube that, as such, mix left-moving and right-moving waves. Thus, the natural condition is to require a symmetry in the spectrum under the interchange of these two kinds of waves. We shall call “left-right symmetric” the closed-string models in this class [7].

As to model building, it is natural to begin by associating to the simplest known (left-right symmetric) models of oriented closed strings, the closed bosonic string in $D = 26$ and the type-IIb superstring in $D = 10$, corresponding (classes of) open-string models, that we shall call their “descendants”. The $D = 10$ case is well known, of course, the open-string model being the $SO(32)$ type-I superstring of Green and Schwarz [8]. The novelty of this viewpoint is the connection between this model and the type-IIb superstring. In a similar fashion, in $D = 26$ one encounters a peculiar bosonic open-string model, with gauge group $SO(8192)$, first described in ref. [9]. Clearly, the issue is how to associate to a generic (left-right symmetric) closed-string model a (class of) open-string models, and in this respect it is quite
illuminating to take a critical look at the relation in these simple cases.

Referring to figure 3, let us note that, whereas in the bosonic closed string
the genus-one contribution to the vacuum energy is fully described by the torus
amplitude, in the $SO(8192)$ model there are three additional contributions. These
may be associated to the other three surfaces of vanishing Euler number that
contain holes and/or cross-caps, the Klein bottle, the annulus and the Möbius
strip. These surfaces are quite different: the first one has no boundaries and is not
orientable, the second one has two boundaries and is orientable, and the third one
has one boundary as is, again, not orientable. Their role is also quite distinct: the
first surface completes the symmetrization of the closed-string spectrum under the
interchange of left and right modes, and its contribution is

$$K = \frac{1}{2} \int_0^\infty \frac{d\tau}{\tau^{14}} \frac{1}{\eta(2i\tau)^{24}} \quad .$$

(2)

On the other hand, the annulus and the Möbius strip describe the open-string
sector, properly symmetrized under the interchange of the charges at the ends of
the open string. Their contributions are

$$A = \frac{N^2}{2} \int_0^\infty \frac{d\tau}{\tau^{14}} \frac{1}{\eta(i\tau/2)^{24}} \quad ,$$

(3)

and

$$M = -\frac{N}{2} \int_0^\infty \frac{d\tau}{\tau^{14}} \frac{1}{\eta(i\tau/2 + 1/2)^{24}} \quad .$$

(4)

All this is somewhat reminiscent of the familiar structure that one encounters
when passing from a closed-string model to a more complicated one via an orbifold
construction [2], whatever the geometrical interpretation of the latter. Namely,
one starts by restricting the original spectrum by a suitable projection, and then
adds new (twisted) sectors with rather peculiar features. In the closed-string case,
when starting from a (target-space) torus, the additional strings are closed only on the (target-space) orbifold but not on its covering torus. On the other hand, in this open-string example, the open strings are closed only on the double covers of the relevant Riemann surfaces. In this respect, open-string models should be regarded as “parameter-space orbifolds” of their “parent” closed-string models [7]. In models of oriented closed strings, modular invariance demands that one adds the twisted sector to the spectrum. The issue is then what fixes the (twisted) open-string sector of open-string models.

The answer to this crucial question may be described quite neatly by referring to fig. 4. There the three additional surfaces of fig. 3 are described in terms of a choice of time direction that corresponds to the propagation of closed strings. With due care, this choice has the virtue of making the three contributions of Klein bottle, annulus and Möbius strip comparable. Still, there is a crucial difference between these and the torus amplitude. Whereas the modular invariance of the torus amplitude allows one to restrict the integration region, while leaving out the “ultraviolet” line $\text{Im}(\tau) = 0$, the lack of modular invariance in the last three surfaces makes their Teichmüller spaces identical to their moduli spaces. All coincide with (translates of) the imaginary axis, and thus all include the “ultraviolet” point $\text{Im}(\tau) = 0$. The ultraviolet singularity may now be dealt with by considering suitable combinations of the contributions from the three surfaces, to be written in the vacuum channel. In this respect, one is actually cancelling an infrared divergence that arises in the limit of very long tubes. The idea, following ref. [8], is to express all amplitudes in terms of $q = \exp(-2\pi\tau)$, where $\tau$ denotes the “time” measured on the double covers along the tube, and to arrive at a “principal-part” prescription for the resulting pole part at $q = 0$. In the bosonic string one finds

$$
\tilde{Z}_{tot} = - \frac{N}{2} \int_0^1 \frac{dq}{2\pi q} \frac{1}{\eta(-q)^{24}} + \frac{N^2 + 2^{26}}{4 \cdot 2^{13}} \int_0^1 \frac{dq}{2\pi q} \frac{1}{\eta(q)^{24}},
$$

(5)
and for $N = 2^{13}$, corresponding to the $SO(8192)$ model [9],

$$\tilde{Z}_{tot} = \frac{2^{13}}{2} PP \int_{-1}^{1} \frac{dq}{2\pi q} \frac{1}{\eta(q)^{24}} . \quad (6)$$

This choice disposes of the massless tadpole, while the additional singularity introduced by the tachyon is not regulated.

In general, the “tadpole conditions” fix (part of) the dimensionalities of the allowed open-string gauge groups. These appear in a multiplicative fashion in the annulus and Möbius amplitudes, being linked to the degeneracy present in the definition of the boundaries. Upon factorization, the resulting conditions may then be related to suitable expectation values on the two “genus-one-half” surfaces, the disk and the projective plane [10] (fig. 5). For instance, in the bosonic model one finds for the total pole part

$$Z_{s.p.} = 24 \frac{(N - 2^{13})^2}{2^{13}} \int_{0}^{1} \frac{dq}{2\pi q} , \quad (7)$$

and from this one may deduce, by factorization, the total “genus-one-half” tadpole [10]

$$\Gamma = 2^8 \sqrt{3\pi} \left| \frac{N}{2^{13}} - 1 \right| . \quad (8)$$

More refined considerations [11] show that, in models with chiral fermions, some sectors contribute unphysical modes that may flow along the tubes. The tadpoles of these unphysical modes signal the presence of gauge and gravitational anomalies.

Let us illustrate the key features of the construction in the simplest of all possible new settings, by working out the open-string descendants of the left-right symmetric models in ten dimensions. To this end, we begin by recalling the defi-
nition of the theta constant with characteristics $\alpha$ and $\beta$,

$$\theta^{[\alpha \beta]} = \sum_{n=0}^{\infty} e^{i\pi n(n+\alpha/2)^2 + 2\pi i(n+\alpha/2)\beta/2}.$$  \hspace{1cm} (9)

Starting from this expression, one may define the level-one $SO(2n)$ characters,

$$O_{2n} = \frac{1}{2} \eta^n (\theta^n [0] + \theta^n [1/2]),$$
$$V_{2n} = \frac{1}{2} \eta^n (\theta^n [0] - \theta^n [1/2]),$$
$$S_{2n} = \frac{1}{2} \eta^n (\theta^n [1/2] + i^n \theta^n [1/2]),$$
$$C_{2n} = \frac{1}{2} \eta^n (\theta^n [1/2] - i^n \theta^n [1/2]),$$ \hspace{1cm} (10)

where $\eta$ is the Dedekind function. These four characters correspond to the four conjugacy classes of $SO(8)$ representations, and owe their names to the lowest-dimensional representations in these classes. These, in their turn, characterize the lowest-mass states in the corresponding sectors. Thus, $O_8$ starts with a scalar (a tachyon), while $V_8$, $S_8$ and $C_8$ start with massless states, a vector and two conjugate spinors, respectively. On this basis of characters, the two generating transformations of eq. (1) are represented by the two matrices

$$S = \frac{1}{2} \begin{pmatrix}
  1 & 1 & 1 & 1 \\
  1 & 1 & -1 & -1 \\
  1 & -1 & 1 & -1 \\
  1 & -1 & -1 & 1
\end{pmatrix} \hspace{1cm} \text{and} \hspace{1cm} T = \exp(-\frac{2i\pi}{3}) \begin{pmatrix}
  1 & 0 & 0 & 0 \\
  0 & -1 & 0 & 0 \\
  0 & 0 & -1 & 0 \\
  0 & 0 & 0 & -1
\end{pmatrix}. \hspace{1cm} (11)$$

In this notation, and leaving aside the contributions from the transverse bosons and from the measure over the moduli, the partition functions of the ten-dimensional
type-II models of ref.\cite{12} are
\begin{align}
T_{IIA} &= (V_8 - S_8)(\bar{V}_8 - \bar{C}_8) \\
T_{IIB} &= |V_8 - S_8|^2 \\
T_{0A} &= |O_8|^2 + |V_8|^2 + S_8 \bar{C}_8 + C_8 \bar{S}_8 \\
T_{0B} &= |O_8|^2 + |V_8|^2 + |S_8|^2 + |C_8|^2 .
\end{align}
\tag{12}

The descendant of the type-IIb theory is the type-I \( SO(32) \) model of Green and Schwarz. On the other hand, the type-IIa model is not left-right symmetric, and thus is not suitable for our construction. We are thus left with the remaining two non-supersymmetric models. Though tachyonic, they are an amusing testing ground for these ideas, since their open-string descendants contain a number of different sectors. In the following, we construct their descendants simultaneously, in order to elicit the differences between the two cases.

The first step is writing the Klein bottle amplitude in the direct channel. This contribution is meant to complete the symmetrization of the \( NS - NS \) sector of the closed string and the antisymmetrization of the \( R - R \) sector of the closed string under the interchange of left and right modes\footnote{As for the “spin-statistics” signs of the first of refs. \cite{3}, the need to antisymmetrize the \( R - R \) sector may be traced to the behavior of the gravitino determinant at genus two\cite{42}.}. One finds
\begin{align}
K_{0A} &= \frac{1}{2} (O_8 + V_8) \\
K_{0B} &= \frac{1}{2} (O_8 + V_8 - S_8 - C_8) ,
\end{align}
\tag{13}

where the argument is \( 2i\tau \), with \( \tau \) the “proper time” of the closed string. In the vacuum channel these expressions become
\begin{align}
\tilde{K}_{0A} &= \frac{2^5}{2} (O_8 + V_8) \\
\tilde{K}_{0B} &= \frac{2^6}{2} V_8 .
\end{align}
\tag{14}

The powers of two are introduced by the measure over the moduli, once these amplitudes are expressed in terms of the Teichmüller parameters of their double
covers, the natural choice if one is to compare contributions from all three additional surfaces.

The second, more difficult, step, is the contraction \[^{14,15}\] of the annulus amplitude. The starting point is, again, the closed-string spectrum, or rather the portion allowed in the tube by the holes at its ends. The rule for the allowed sectors is quite simple in this case: they are precisely the ones that fuse into the identity of the fusion algebra \[^{\dagger}\] with their anti-holomorphic partners in the closed-string GSO projection. This is precisely the condition that boundaries respect the transverse Lorentz group, $SO(8)$, a local symmetry of these models. In lower dimensions, one may relax this condition to define models where boundaries respect only part of the symmetries of the “parent” closed string. For instance, one may extend the familiar toroidal construction of ref.\[^{[16]}\]. The additional freedom is then related to marginal deformations of the conformal theory that draw their origin directly from the open-string sector\[^{[17]}\]. Returning to our $D = 10$ models, it is simple to see that, since all $SO(8)$ representations are self-conjugate, in the 0A models only $O_8$ and $V_8$ can flow in the vacuum channel, while in the 0B models all four characters can flow (figure 6).

The next, crucial observation, is that one still has the freedom to choose the reflection coefficients for the various sectors. Thus, one may write

$$
\tilde{A}_{0A} = \frac{2^{-5}}{2} ( (n_B + n_F)^2 V_8 + (n_B - n_F)^2 O_8 )
$$

(15)

and

$$
\tilde{A}_{0B} = \frac{2^{-6}}{2} \left( (n_o + n_v + n_s + n_c)^2 V_8 + (n_o + n_v - n_s - n_c)^2 O_8 + (n_o - n_v + n_s - n_c)^2 S_8 + (n_o - n_v - n_s + n_c)^2 C_8 \right) ,
$$

(16)

where the argument is $i\tau$, the Teichmüller parameter of the double covers. It should be appreciated that the coefficients in these amplitudes are perfect squares.

\[^{\dagger}\] In this case $V_8$, due to the Minkowski signature of the ten-dimensional space-time! \[^{[3]}\]
This reflects the fact that the individual sectors of the spectrum flow independently along the tube, while undergoing two reflections at its ends. In the direct (open-string) channel, these amplitudes become

\[ A_{0A} = \frac{n_B^2 + n_F^2}{2} (O_8 + V_8) - n_B n_F (S_8 + C_8) \]  (17)

and

\[ A_{0B} = \frac{n_o^2 + n_v^2 + n_s^2 + n_c^2}{2} V_8 + (n_o n_v + n_s n_c) O_8 - \\
( n_o n_c + n_v n_s) S_8 - (n_o n_s + n_v n_c) C_8 \]  (18)

where the argument is \( \frac{i\tau}{2} \), with \( \tau \) the "proper time" of open strings.

The final step is the construction of the Möbius amplitude. The guidance comes, again, from the transverse channel. The basic observation is that the Möbius strip may be represented as a tube with a hole and a cross-cap at its ends, and as such it may accommodate all the characters common to both \( \tilde{A} \) and \( \tilde{K} \). Moreover, apart from signs, the corresponding reflection coefficients are geometric means of those in \( \tilde{A} \) and \( \tilde{K} \). Thus†

\[ \tilde{M}_{0A} = - \left( (n_B + n_F) \dot{V}_8 + (n_B - n_F) \dot{O}_8 \right) \]  (19)

and

\[ \tilde{M}_{0B} = (n_o + n_v + n_s + n_c) \dot{V}_8 \]  (20)

where the argument is \( (i\tau + 1/2) \), the Teichmüller parameter of the double covers. The modular transformation to the direct channel is then effected by the matrix

\[ P = T^\frac{1}{2} S T^2 S T^\frac{1}{2} \]  (21)

† As in refs. [14] and [15], we are using a real basis of Möbius characters.
and the resulting projections of the open-string spectrum are

\[ M_{0A} = -\frac{1}{2} \left( (n_B + n_F) \hat{V}_8 + (n_B - n_F) \hat{O}_8 \right) \]  \hspace{1cm} (22)

and

\[ M_{0B} = -\frac{1}{2} (n_o + n_v + n_s + n_c) \hat{V}_8 , \]  \hspace{1cm} (23)

where the argument is \((i\tau/2+1/2)\), with \(\tau\) the “proper time” of open strings. These expressions are indeed the proper symmetrizations of those in eqs. (17) and (18).

The last step is then imposing the tadpole conditions. In the \(0A\) models there is only one tadpole condition, coming from the \(V_8\) sector, and the result is

\[ n_B + n_F = 32 \]  \hspace{1cm} (24)

On the other hand, in the \(0B\) models there are three tadpole conditions, coming from the sectors \(V_8, S_8\) and \(C_8\), that lead to

\[ n_o + n_v + n_s + n_c = 64 \]  \hspace{1cm} (25)

\[ n_o = n_v \quad \text{and} \quad n_s = n_c \]  \hspace{1cm} (26)

The presence of the additional conditions in the \(OB\) case is particularly gratifying, since these open-string models are chiral, and eqs. (26) are sufficient to ensure the cancellation of all anomalies. Strictly speaking, by imposing tadpole conditions one is eliminating the irreducible part of the anomaly polynomial. The antisymmetric tensors in the models then dispose of the non-irreducible traces, via a (generalized) Green-Schwarz mechanism\(^{[18]}\). In order to appreciate these remarks, let us first pause briefly to describe the meaning of these expressions.
The first observation is that the annulus amplitude is a second-degree polynomial in the Chan-Paton multiplicities. This is precisely as it should be, since open strings carry a pair of charges at their ends. Referring for simplicity to the 0A model, we see that if \( n_B = 32 \) there is a single open-string sector, with matter content corresponding to \( (O_8 + V_8) \), and all open strings carry a pair of identical charges. Thus, they all flow in the Möbius strip, and the gauge group is \( SO(32) \).

In agreement with ref. [5], one is filling up complete matrices, apart from the (anti)symmetrizations allowed by the “twist” symmetry. If, on the other hand, \( n_B \) is not equal to 32, there are additional open strings, with matter content corresponding to \( (S_8 + C_8) \) and, at their ends, two charges of different types, with multiplicities \( n_B \) and \( n_F \) respectively. All this has a nice interpretation, described in fig. 7: upon symmetry breaking, the “missing” Chan-Paton polarizations are simply moved to new sectors of the spectrum. If this is done compatibly with the fusion rules, the factorization constraints are still satisfied. This brings us to another interesting issue, that has to do with the fusion rules themselves. Namely, the very form of eqs. (17) and (18) is tailored to the fusion rules [14][15]. The basic suggestion came to us from an interesting paper of Cardy [19] on the annulus amplitude in rational conformal field theory. There, starting from the correspondence between types of boundaries and bulk sectors present in this case, the author relates the matter content flowing through the annulus with boundaries of types \( i \) and \( j \) to the fusion-rule coefficient \( N_{ij}^k \). Amusingly, if one extends Cardy’s expression writing

\[
A = \frac{1}{2} \sum_{ijk} N_{ij}^k n^i n^j \chi_k ,
\]

the Verlinde formula [20] implies that the coefficients of the transverse-channel amplitude are perfect squares. Indeed

\[
\tilde{A} \sim \sum_{jm} \left( \frac{S_{jm} \chi_m}{\sqrt{S_0}} \right)^2 \chi_m ,
\]

where \( S \) denotes the matrix that implements on the characters of the conformal
theory the transformation $\tau \to -1/\tau$. In lower-dimensional models, this choice does not exhaust all possibilities, and indeed different ones are available, even in rational models, at the minor price of altering the definition of the Möbius characters [14][15]. The basic idea is that one has the freedom to redefine the eigenvalues under “twist” of various Virasoro primaries within a generalized character, while preserving well-defined local currents on the double cover of the Möbius strip. In ref. [15], the resulting modifications were ascribed to “discrete” Wilson lines running along the boundaries of the annulus. Amusingly, in toroidal models the different choices of “discrete” Wilson lines are particular points of continuous lines of deformations associated with internal gauge fields, and the name is thus properly justified.

One may wonder whether the construction based on eq. (27) is limited to the case of simple models built out of free fermions and bosons on the world sheet or, rather, whether it may be extended to models with a more complicated fusion algebra. As a first step in this direction, in ref. [21] we have shown that, starting from the discrete series of Belavin, Polyakov and Zamolodchikov, one may derive “open-string descendants” whose disk amplitudes enjoy proper factorization in the presence of a (global) internal Chan-Paton symmetry. In other words, one may construct Veneziano-type analogues of the usual (Shapiro-Virasoro-like) amplitudes introduced in the third of refs. [1]. It would be nice to extend this work to the other discrete series and to $WZW$ models.

An important issue is whether the massless spectrum has some peculiar features that suffice to distinguish open-superstring models from heterotic models. In ten dimensions the answer is no, but this should be regarded as an accident. Indeed, in this case supersymmetry determines the low-energy theory completely up to two derivatives, and identifies it in both cases with $N = 1$ supergravity coupled to $N = 1$ supersymmetric Yang-Mills. The next interesting case is $D = 6$, and indeed in ref. [15] we constructed a class of supersymmetric open-string models in six dimensions that stands out by its peculiarities. Following the suggestion of ref. [7], these models were derived starting from $K_3$ compactifications of the type-
IIb superstring and performing the parameter-space orbifold construction. They show the novel feature (compared to the heterotic case) of containing a net number of self-dual tensors $B_{\mu\nu}$ in their spectra. Technically, these fields originate from incomplete projections of the $R-R$ sector of the “parent” closed string, and thus are certainly not present in the heterotic string. Still, they play a very important role, since they contribute to cancelling the irreducible part of the anomaly polynomial.

Referring to fig. 8, let us first confine our attention to the gauge anomaly in ten dimensions. The simplest relevant diagrams contain six external vectors and are of three types. In the planar diagrams all six vectors are emitted from one boundary of the annulus. In this case there is a potential singularity in the limit of very long tubes that is not regulated by any momentum flow. A similar pathology is also present in the non-orientable amplitudes, where all six vectors are emitted from the single boundary of the M"{o}bius strip. These two kinds of amplitudes determine the total irreducible contribution to the anomaly polynomial, proportional in this case to $\text{Tr} F^6$. The tadpole condition removes this contribution if the gauge group is $SO(32)$. On the other hand, the non-planar diagram is not singular by itself. It involves the emission of four vectors from one boundary and two vectors from the other boundary of the annulus, and therefore is regulated by the momentum flow along the tube. Still, from the viewpoint of the low-energy theory, this diagram is the most important one, since it is the site of the Green-Schwarz mechanism, whereby the $R-R$ antisymmetric tensor disposes of the residual anomaly. We should add that the vacuum diagrams contain detailed information on the cancellation mechanism for the irreducible part of the gravitational anomaly, proportional to $\text{Tr} R^6$. In this case the relevant contribution comes from the limiting behavior of the amplitude when all six emission vertices coalesce, and thus one may relate the phenomenon, once more, to the tadpole conditions. Since, by construction, they result in second-degree polynomials with coincident roots, one finds again that $SO(32)$ does the job. Indeed, the limiting behavior of the six-vector amplitudes is proportional to $(N-32)$, whereas the total contribution of the vacuum amplitudes is proportional to $(N-32)^2$. All this should be compared to the corre-
sponding mechanism in closed-string theories, where modular invariance removes
the ultraviolet singularity altogether, and is thus responsible for the whole anomaly
cancellation.

In six dimensions, the anomaly polynomial contains irreducible terms proportional to $\text{Tr}F^4$ and $\text{Tr}R^4$ that are disposed of by the tadpole conditions. They
are associated, respectively, with gauge and gravitational anomalies. The six-
dimensional models of ref. [15] contain a number of different sectors, and in par-
ticular a number of tadpole sectors. Since each of the tadpole sectors contains an
antisymmetric tensor*, one may wonder whether they might all play a role in the
cancellation mechanism. Indeed, the analysis carried out in ref. [18] revealed a
novel feature: in this case the Green-Schwarz mechanism does take a generalized
form! From a technical viewpoint, one finds that, after imposing the tadpole con-
ditions, the residual anomaly polynomial is a quadratic form built out of $\text{Tr}F^2$ and
$\text{Tr}R^2$ that, in sharp contrast with the usual case, does not factorize. Rather, the
quadratic form may be diagonalized and contains a number of nonzero eigenvalues.
Interestingly, this number is precisely equal to the number of antisymmetric ten-
sors in the models, that may thus dispose of the anomaly by acting in a combined
fashion. The low-energy effective supergravity should thus contain generalized cou-
plings between antisymmetric tensors and combinations of Chern-Simons forms for
the various simple factors of the gauge group, weighted by the matrix that imple-
ments on the characters the transformation $\tau \rightarrow -1/\tau$. The role of this matrix is
precisely as expected, since in the non-planar case the contributions to the anomaly
polynomial are weighted by the same factors as the contributions to the tadpole
graphs. This should indeed be the case, since the same Feynman rules determine
the GSO-type contributions in both cases. Thus, one may predict the canonical
form of the residual anomaly polynomial! On the other hand, the gravitational
Chern-Simons form couples only to the antisymmetric tensor in the supergravity
multiplet. One may also write a supersymmetric set of field equations for the low-

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* One of these is antiself-dual and belongs to the supergravity multiplet, while the remaining
ones are self-dual and belong to the matter multiplets.
energy field theory that incorporate these generalized Yang-Mills couplings [18]. Following common practice\cite{22}, these equations have been constructed only to lowest order in the spinors, but previous experience\cite{23} suggests that their completion to all orders should entail only difficulties of a technical nature.

In conclusion, we have seen how, in going from ten to six dimensions, open-string models display an enticing complexity. Of course, many major questions remain unanswered. First of all, we have omitted any direct reference to chiral four-dimensional models. As far as we know, open-string models appear to be plagued with a common disease: all chiral four-dimensional models they give rise to involve small gauge groups, typically products of $U(2)$ factors. Whereas in some cases this finding may be related to the need to introduce (quantized) background values of the $NS-NS$ antisymmetric tensors [17], the argument is not exhaustive, and a detailed analysis is called for. An additional problem is the geometrical formulation of these models. For instance, a basic question has to do with the residual antisymmetric tensors: how can one understand the incomplete projections of $R-R$ sectors directly in terms of the type-I superstring? Is the “parameter-space orbifold” more than an technical artifice? Hopefully, we shall be able to report on some of these issues at a future Meeting of this Society.

**Acknowledgments**

We are grateful to M. Bianchi, with whom several of the results reviewed in this talk were derived, during a long and enjoyable collaboration. A.S. would like to thank the Organizers for their kind invitation, while apologizing for his sudden inability to attend the Meeting.

**Figure Captions**

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Figure 1
The torus amplitude

Figure 2
The annulus amplitude

Figure 3
Genus-one vacuum amplitudes in open-string theories

Figure 4
Vacuum channels

Figure 5
Tadpole conditions

Figure 6 Allowed sectors in the annulus vacuum channel

Figure 7
Chan-Paton symmetry breaking

Figure 8
The Green-Schwarz mechanism
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