Optimal design of a pair of vibration suppression devices for a multi-storey building

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Summary
This paper investigates the use of two two-terminal vibration suppression devices in a building and assesses the performance benefits over those achieved using a single device. The inerter-combined configurations for a multi-storey building structure are considered. The inerter is a two-terminal device, with the property that the applied force is proportional to the relative acceleration across its terminals. In this paper, a five-storey building model with two absorbers of the same kind subjected to base excitation is studied, where one is located between ground and the first floor and the other is between the first and second floors of the building. Three passive suppression layouts, two dampers, two tuned inerter dampers, and two tuned viscous mass dampers are considered. The optimal configurations for minimising the maximum interstorey drifts of the building are obtained with respect to the inerter's size and the damping boundary. The corresponding parameter values are also presented. For the sake of comparison, the single device mounted between the ground and first floor is also considered. Finally, with specific inerterance and damping values, the frequency response is provided to show the potential advantage of the proposed optimal configurations. It is demonstrated that the optimal configurations with a pair of devices are more effective than the optimal single device with equal total inerterance and the same total damping boundary. The approach demonstrated in this paper is applicable to the investigation of using more than two devices for multistorey buildings.

KEYWORDS
a pair of absorbers, base excitation, Inerter, structural control, vibration suppression

1 | INTRODUCTION

Human safety and comfort are two main concerns for dynamic design of building structures. The tuned mass damper (TMD) proposed by Frahm1 has been widely accepted as an effective passive control device. Den-Hartog2 proposed and others3-5 have refined the tuning method to choose the damping ratio of the TMD to maximise energy dissipation. Considering the motion of the structure as well as that of the TMD, Krenk6 characterised the damping properties and identified the optimal damping of TMD. In most real applications, only a single TMD is used, and it is always installed near the top of the building, see for example the analysis in other studies.7,8 To investigate the performance of multiple TMDs, Iwanami and Seto9 proposed dual tuned mass dampers (2TMD) for suppressing the vibration of a single degree of freedom.
Deploying multiple tuned mass dampers (MTMDs) in a structure has also been studied in other studies,\textsuperscript{10-15} which concluded that MTMDs can result in a better performance than a single TMD with the same total mass. In a study,\textsuperscript{16} Rana and Soong investigated the control of multiple structural modes with MTMD located between the different floors in a MDOF building. Xiang and Nishitani\textsuperscript{17} proposed the TMD floor system that takes advantages of both the floor isolation system and TMD. Using the multimode approach, it has been shown that a structure with several TMD floors can perform better than one with a single TMD floor. For multiple dampers, Takewaki\textsuperscript{18} proposed an efficient and systematic procedure for finding the optimal placement of the dampers. The inerter is a two-terminal device, introduced by Smith\textsuperscript{19} in the early 2000s, which has the property that the force generated by it is proportional to the relative acceleration across its terminals. The inerter completes the force-current analogy between mechanical and electrical networks with the inerter corresponding to the capacitor, the spring corresponding to the inductor, and the damper corresponding to the resistor. The performance advantages of various mechanical systems incorporating inerterss have been identified in numerous studies.\textsuperscript{20-29} The application of inerter-based suppression systems in buildings has also been analysed by many researchers as the inerter can provide a high inertance with a much lower mass due to internal gearing. In 2010, Wang et al.\textsuperscript{30} proposed several simple inerter-combined absorber layouts that have been shown to be effective in reducing vibration of one DOF and two DOF building models. A new inerter-based device, the tuned viscous mass damper (TVMD), was presented by Ikago et al.\textsuperscript{31} as a vibration suppression device for a SDOF system. In 2013, Lazar et al.\textsuperscript{42} proposed a tuned inerter damper (TID) by substituting the mass of the widely used TMD with an inerter. The study showed that the performance with a TID mounted between the structure and the ground can be better than that with a TMD mounted at the top of the structure. A tuned mass damper inerter consisting of an inerter mounted in series with a TMD has been proposed in Marian and Giaralis.\textsuperscript{32} In Zhang et al.,\textsuperscript{33} the effect of the size of inerter and the stiffness of the brace that links the device across floors on the optimal inerter-based device layout were considered. They reported three optimal layouts across the inertance-brace stiffness parametric space: the D (damper), the IPD (inerter in parallel with damper), and the TID. It was also noted that beyond a certain value of inertance, a suppression device becomes suboptimal regardless of which layout is used. All these applications focused on using one inerter-based control device in a building structure. In addition, the effect of the inertial mass damper for cables has also been studied in previous studies.\textsuperscript{34,35}

In Ikago et al.,\textsuperscript{36} TVMDs were mounted within every storey in a multistorey building, and Takewaki et al.\textsuperscript{37} investigated the earthquake response reduction with inerterlike devices known as inertial dampers used between several floors of a building. Also in 2016, Giaralis and Marian\textsuperscript{38} studied the benefits of the tuned mass damper inerters over the traditional TMDs in a linear building structure. These studies are based on a specific predetermined layout for the suppression device. As a first step towards multiple inerter-based device investigation, considering two devices makes it easier to interpret the obtained results and understand the influence of using more than one inerter-based device for vibration suppression in a building model. In addition, it is preferable to use fewer devices from cost and maintenance points of view. In this paper, multiple layouts that have all been shown to be effective as a single device will be compared for the case where a pair of devices is used. In considering their effectiveness, both performance and device size are discussed. Many cost functions could be selected for the optimisation; herein, the inter-storey drifts are selected as the performance index. Three candidate layouts—two dampers, TVMDs, or TIDs—are considered as the suppression devices, with one device located between the ground and first floor and the other mounted between the first and second floors of the building. In order to scale the device in the same way as is done by selecting the mass ratio for a TMD, the total inertance of the layouts is selected before optimisation. Considering the implementation and potential cost implications, the upper limit of the total damping value is also fixed during optimisation. By optimising the objective function, the optimum results and the corresponding parameter values of different layouts can be obtained with respect to the inerter’s size and the damping boundary. Other performance measures and constraints can also be adopted for optimisation, for example, the weight-normalised device forces. However, because the focus of this work is on the optimum vibration absorber methodology, which can be adopted for any performance constraint consideration, we do not pursue these other possible metrics/constraints in this work.

This paper is arranged as follows. In Section 2, we introduce an idealised five-storey building model and the optimisation procedure used in the paper. The candidate suppression device layouts are also proposed. In Section 3, optimisation without restricting the damping value is carried out to show the limitation of fixed-sized-inerter method. The optimisation results with respect to the inerter’s size and the damping boundary are obtained in Section 4. Conclusions are drawn in Section 5.
2 | A BUILDING MODEL WITH CANDIDATE VIBRATION SUPPRESSION LAYOUTS AND OPTIMISATION PROCEDURE

In this section, a five storey building model with two absorbers is introduced, where one device is located in the bottom storey and the other one is mounted between the first and second floors as shown in Figure 1. The objective function and the optimisation approach are also provided. Three candidate layouts with two vibration suppression devices of the same kind are considered. Note, we define layouts as the topological arrangements formed by specific connections of springs, dampers, and inerters, whereas the configurations mean the layouts with element values specified.

2.1 | A multistorey building model with general suppression devices

In the present paper, we consider a five-storey building model with equivalent floor mass $m_s$ and equivalent inter-storey elasticity $k_s$ shown in Figure 1, in which the structural damping is taken to be zero because its value is typically small compared with the control device. The locations of the two absorbers are taken to be between the ground and the first and the first and the second floors, defined as Device1 and Device2, respectively. The control system can be assumed to be a passive mechanical admittance $Y(s) = F(s)/v(s)$,\(^{20,40}\) where $F(s)$ is the force exerted by the control device in the Laplace domain, and $v(s)$ is the relative velocity between the two terminals, also in the Laplace domain. $Y_1(s)$ and $Y_2(s)$ shown in Figure 1 are the general admittances of the two absorbers. The parameters of the five storey building model are fixed as $m_s = 1000$ kg and $k_s = 1500$ kN/m. These numerical values were selected for convenience while retaining realistic natural frequencies and noting that the parameters scale linearly.

Defining the variable matrix

$$x = \begin{bmatrix} x_1 & x_2 & x_3 & x_4 & x_5 \end{bmatrix}^T,$$

where $x_i (i = 1, \ldots, 5)$ is the displacement of the $i$th floor relative to the ground. For the building model of Figure 1, excited by a ground acceleration $a_g$, we can obtain its equation of motion in matrix form, in the Laplace domain as

$$M s^2 X + K X + \delta K X = -M E A_g,$$  \hspace{1cm} (1)

where $M = m_s I$ is the $5 \times 5$ mass matrix of the structure and where $I$ is the identity matrix; $E$ is a $5 \times 1$ vector of ones; $X$ and $A_g$ represent the vectors of relative floor displacements and ground acceleration in the Laplace domain and

$$K = \begin{bmatrix} 2k_s & -k_s & 0 & 0 & 0 \\ -k_s & 2k_s & -k_s & 0 & 0 \\ 0 & -k_s & 2k_s & -k_s & 0 \\ 0 & 0 & -k_s & 2k_s & -k_s \\ 0 & 0 & 0 & -k_s & k_s \end{bmatrix},$$

FIGURE 1  Schematic representation of an idealised building and lower floor suppression device
2.2 Optimisation procedure and candidate vibration suppression layouts

Many design criteria for a vibration absorber are proposed in Warburton, such as limiting the absolute displacement or the absolute acceleration. We consider the interstorey drift of each floor as the performance index in this paper, as potential damage of a building structure is strongly correlated with the values of interstorey drifts. Thus, the objective for the optimisation of device parameters in this study is to reduce the maximum magnitude of the frequency response functions of the interstorey drifts among all the floors, with the objective function defined as

\[
J_\infty = \max_i \{\max_\omega |T_{A_g \rightarrow Z_i}(j\omega)|\},
\]

where \(Z_i = X_i - X_{i-1}\) and \(X_0 \equiv 0\). \(T_{A_g \rightarrow Z_i}\) denotes the transfer function from ground acceleration \(A_g\) to interstorey drift \(Z_i\), and \(\max |T_{A_g \rightarrow Z_i}(j\omega)|\) represents the maximum magnitude of \(T_{A_g \rightarrow Z_i}\) across all frequencies. We note that most real-world ground accelerations are focused on low frequencies under 10 Hz; although the infinity norm can be focused on high frequencies, for the building model considered in this paper, the maximum interstorey drift always occurs at the first fundamental frequency, around 1.7 Hz, which can be seen from, for example, Figures 16 and 20 in Section 5. The objective function taken here is the same as that proposed by Xiang and Nishitani. The optimisation problem herein can be formulated as to make objective function \(J_\infty\) as small as possible. The \texttt{patternsearch} and \texttt{fminsearch} functions in MATLAB are used in this paper to find the optimum value of \(J_\infty\). Although the effectiveness of these optimisation algorithms depends on a proper choice of initial values, but these algorithms are faster than global optimisation methodologies that need no initial values (e.g., generic algorithms). To avoid getting stuck at local minima, 10 sets of random initial values, uniform in appropriate ranges, are used for each optimisation, and the result with minimum objective function \(J_\infty\) is used.

Figure 2 illustrates the four kinds of passive suppression devices: (a) a traditional damper (D), (b) a TVMD, (c) a TID, and (d) an inerter with a parallel connected damper (IPD, a special case of TVMD). The mechanical admittance \(Y(s)\) for these devices (D, TVMD, TID, and IPD) can be respectively calculated as

\[
Y(s) = c, \quad Y(s) = \frac{k(bs + c)}{bs^2 + cs + k}, \quad Y(s) = \frac{bs(cs + k)}{bs^2 + cs + k}, \quad Y(s) = bs + c.
\]

In Zhang et al., using a fixed-sized-inerter admittance—four configurations, the damper, the IPD, the TID, and the dual tuned inerter dampers (TTID; shown in Figure 6b in Zhang et al.)—were found to be the optimal structures for reducing the vibration for different regions making up the inerter–brace stiffness parametric space. Also it was found that the TTID was a suboptimal structure because, for the region where it outperformed the other layouts, the resulting cost function could be matched or bettered by reducing either the inerter’s size or the damping value. Hence, in this paper, candidate layouts two dampers (2Ds), dual tuned mass viscous inerter dampers (2TVMD), and dual tuned mass dampers (2TID) are considered; for example, 2Ds mean one damper at the bottom and the other damper mounted between the first and second floors of the building model. For the sake of comparison, the performance of each single device shown in Figure 2a–c located in the bottom storey will be analysed as well. The admittance functions \(Y(s)\) obtained in (3) will be used to derive the objective function \(J_\infty\) for optimisation.

FIGURE 2 Four kinds of suppression layouts: (a) the damper (D), (b) the tuned viscous mass damper, (c) the tuned inerter damper and (d) the inerter with a parallel connected damper
LIMITATION OF FIXED-SIZED-INERTER OPTIMISATION

A fixed-sized-inerter optimisation is carried out, where the inertance of the layouts is selected before optimisation. The range of the inerter’s size considered in this paper is $b \in (100 \text{ kg}, 3000 \text{ kg})$, that is, $b/m_s \in (0.1, 3)$. For the single device mounted at the bottom, the optimum results of the damper, the TVMD, and the TID have been shown in Figure 3a, respectively. It can be seen that the TVMD provides the best performance over the whole range of inerter’s size, slightly better than that of a damper. Comparing with the TID, the TVMD can have up to 93.1% performance improvement when $b = 100$ kg, and the improvement percentage decreases with the increasing inerter’s size, falling to 26.7% when $b = 3000$ kg.

Considering a pair of vibration suppression devices, the selected inertance is split between the two inerters, with the ratio of split being one of the optimisation parameters. Figure 3b shows the optimal values of the objective function $J_\infty$ with the layouts 2D, 2TVMD, and 2TID. We observe that the 2TVMD provide the best performance over the whole range of inerter values considered, with up to 94.3% and 13.1% performance improvement comparing with the 2TID and 2D layouts, respectively. From Figure 3a,b, it can be noticed that a pair of devices can give significantly smaller values of the objective function $J_\infty$. The percentage improvement with two absorbers has been shown in Figure 3c, comparing with the corresponding single device. It suggests that the 2TVMD perform much better than a single TVMD in the whole range of the inerter’s size, with around 62% improvement. For the 2TID layout, comparing with the single TID, the improvement is greatest at small inertance and decreases until $b \approx 600$ kg and over the range $b \in (1000 \text{ kg}, 3000 \text{ kg})$, the improvement percentage is always around 12%. The 2D layout can result in 62.4% smaller value of $J_\infty$, comparing with a single damper.

Forces exerted by the devices are also of interest for the optimal design of the vibration suppression devices. Figure 4 shows the force $F$ for all the absorber layouts with the optimum parameter values for each value of $b$, where $F$ is defined as

$$F = \max_w |T_{A_g \rightarrow F_d}(j\omega)|,$$

which is the maximum magnitude of the transfer function from ground acceleration $A_g$ to the device force $F_d$. From Figure 4a it can be noticed that although the TVMD and the damper can result in better performance than the TID, the exerted forces of these three devices are in the same level. For the multiple devices considered, similar conclusions can be drawn from Figure 4b.

3. For the single device, the optimum results of the damper, the TVMD, and the TID have been shown in Figure 3a, respectively. It can be seen that the TVMD provides the best performance over the whole range of inerter’s size, slightly better than that of a damper. Comparing with the TID, the TVMD can have up to 93.1% performance improvement when $b = 100$ kg, and the improvement percentage decreases with the increasing inerter’s size, falling to 26.7% when $b = 3000$ kg.

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**FIGURE 3** The optimisation results for minimising $J_\infty$ over all device parameters: (a) a single device, (b) a pair of devices and (c) the percentage improvement of using two devices over a single one for the layouts D (green dash dotted), TVMD (blue dashed) and TID (red). D, damper; TID, tuned inerter damper; TVMD, tuned viscous mass damper.

**FIGURE 4** Device forces for (a) a single device with the TID (red), the D (green dash dotted) and the TVMD (blue dashed), (b) a pair of devices with the 2TID (red), the 2TVMD (blue dashed) and the 2D (green dash dotted), where thick line represents the force of Device1 and the thin one is that of Device2. 2D, two dampers; 2TID, dual tuned inerter dampers; 2TVMD, dual tuned viscous mass dampers.
The optimal damping values, nondimensionalised by a damping value $c_{10} = 76.4 \text{kN} \cdot \text{s} / \text{m}$, for a single and a pair of devices are given in Figure 5a and b, respectively. The damping value $c_{10}$ is calculated by

$$c_{10} = \frac{10\% \times 2 \times \sqrt{m_{m1} k_{m1}}}{\phi_{1,1}}$$

where $m_{m1}$ and $k_{m1}$ are the modal mass and modal stiffness of the first vibration mode of the building model; $\phi_{1,1}$ is the first component of the first mode shape. Assuming that only the contribution of the first vibration mode is significant, $c_{10}$ represents the damping coefficient that provides a 10% damping ratio of the first mode if a pure damper is installed within the bottom storey. It should be noted that, for a pair of devices, the damping values for Device1 are represented by thick lines whereas those for Device2 are plotted in thin lines. From Figure 5a, it can be seen that the damping value required for the TID is much smaller than that of both the TVMD and the damper, of which the optimal damping values are almost twice the value of $c_{10}$. Figure 5b suggests that although the devices (2TVMD and 2D) can achieve a smaller value of the objective function, their damping values are significantly larger than that of the 2TID layout. Comparing with the single device, the damping values of the 2TID are similar to that of the TID whereas the 2TID can achieve a better performance. It is known that damping devices have realistic range of effective damping values, related with fluid and valve designs. In order to make meaningful and fair comparisons among the proposed candidate layouts, the damping limit on a single or a pair of devices will need to be implemented. In this work, we select $c_{10}$ as the maximum damping limit, which is in line with that used by other researchers; for example, to illustrate the performance benefit of the TVMD, Ikago compared it with a traditional damper using the same damping ratio for a SDOF model, and the maximum damping ratio considered is 10%. In subsequent optimisation, we impose a bound on the sum of the damping coefficients added to the main structure. Similar work on restricting damping elements, especially the sum of the damping coefficient has been conducted by many researchers for identifying optimal suppression devices in building structures, for example, in previous studies.

### OPTIMISATION RESULTS WITH DAMPING RESTRICTION

This section reports the optimisation results when restricting the upper bound on the damping values required. The total inertance $b$ is selected before optimisation, and the total damping values should not exceed the proposed damping up-boundary, denoted as $c_u$. For the candidate layouts 2TID and 2TVMD, the parameters for optimisation are inertance ratio $u_b$ with $u_b = b_1/b$, the damping values $c_1$ and $c_2$ satisfying the condition $c_1 + c_2 \leq c_u$ and the stiffnesses $k_1$ and $k_2$. The subscripts 1 and 2 relate to components in Device1 and Device2, respectively. As $c_{10}$ is considered as the maximum upper bound for the total damping size added to the main structure, to show performance and optimal configuration trends when applying different damping constraints $c_u$ ($c_u \leq c_{10}$), three example damping upper limits are analysed:

$c_u = c_{10}/16$, $c_u = c_{10}/5$, and $c_u = c_{10}$.

For the damping upper bound $c_u = c_{10}/16$, the optimum values of $J_\infty$ using a single device and a pair of devices with three different devices shown in Figure 2 are given in Figure 6a and b, respectively. A comparison of the optimisation results between using a single device and a pair of devices is also given in terms of the percentage improvement in cost function using two devices of a certain layout over the use of one device of the same layout (see Figure 6c). It can be seen from Figure 6a that, for the building model with a single device mounted at the bottom, the TID can provide the best performance over the whole range of inerter’s size significantly better than both the damper and the TVMD. Figure 6b shows that for two devices, the 2TVMD and the 2TID have different optimal inertance range, namely $b \in (100 \text{ kg}, 700 \text{ kg})$.
and \( b \in (700 \text{ kg}, 3000 \text{ kg}) \), respectively. The percentage improvement in \( J_\infty \) of the pair of devices is presented in Figure 6c; it can be seen that the 2D has the same performance with the single damper (over the whole inertance range). The 2TVMD gives a much smaller value of \( J_\infty \) comparing with a single TVMD, and the relative improvement increases as the value of \( b \) increasing. Comparing with that of the TID, the smallest percentage improvement of the 2TID is 5.5\%, which occurs when \( b = 700 \text{ kg} \). The maximum forces of the single device and a pair of devices are shown in Figure 7a and b, respectively. It can be seen from Figure 7a that the damper provides the smallest force, whereas results in the worst suppression performance. The force of the TVMD increases rapidly with a larger value of \( b \), whereas the TID has similar level of force as the damper. For the multiple devices, the exerted forces are all in the same level, as noted from Figure 7b.

Figure 8 shows the optimal element values for the case \( c_u = c_{10}/16 \), but only showing where a particular suppression layout is optimal, with short vertical lines to note the transition points between layouts. For example, values for neither D nor 2D are shown in Figure 8 as they are not the optimal layout choices for any value of \( b \) in \((100 \text{ kg}, 3000 \text{ kg})\). From Figure 8a, it can be seen that the damping value of the single device TID in the range of \( b \in (100 \text{ kg}, 1500 \text{ kg}) \) is smaller than the damping boundary \( c_{10}/16 \), which means, in this range, increasing the upper bound on the damping value will not result in improved TID performance. Figure 8c gives the inertance ratio \( u_b \) of the pair of devices, and from it, we note that the value of \( u_b \) for the 2TVMD is approximately zero when \( b \in (100 \text{ kg}, 700 \text{ kg}) \). Also, over this range, the reciprocal value of the corresponding stiffness \( k_1 \) for the 2TVMD is almost zero. This suggests that for the 2TVMD layout, the lower device can be simplified to a damper in its whole optimal range of inerter’s size, and we denote this structure as DTVMD—a damper located at the bottom and a TVMD mounted between the first and second floors. Also note that the total optimal damping value of the 2TID is smaller than \( c_{10}/16 \) when \( b \) is less than 1900 kg.

The results in Figure 9, suggest that for the case \( c_u = c_{10}/5 \), the optimal range of the inerter’s size for the TVMD and 2TVMD becomes larger as more damping is available. The optimal values of the cost function \( J_\infty \) with the TVMD and 2TVMD are much smaller than that for the case \( c_u = c_{10}/16 \). For the TID and 2TID layouts, comparing with the case \( c_u = c_{10}/16 \), the optimal results are the same in the range of \( b \in [100 \text{ kg}, 1500 \text{ kg}] \) and \( b \in (100 \text{ kg}, 1900 \text{ kg}) \), respectively. For a higher inertance, both the TID and 2TID can provide better performance. For the single device mounted at the bottom, it can be seen from Figure 9a, the optimal configuration is the TVMD when \( b \in (100 \text{ kg}, 300 \text{ kg}) \) and the TID over the remaining range. And from Figure 9b, the optimal configuration for a pair of devices is 2TVMD and 2TID in the range of \( b \in (100 \text{ kg}, 1700 \text{ kg}) \) and \( b \in (1700 \text{ kg}, 3000 \text{ kg}) \), respectively. The performance improvement has also been
The corresponding optimal element values for $c_u = c_{10}/16$: for a single device (a) $c$, (b) $k$; for a pair of devices (c) the inertance ratio $u_b$, (d) $c_1$ (bold) and $c_2$ (thin), (e) $k_1$ (bold) and $k_2$ (thin) with the layouts TVMD (blue dashed), TID (red). 2D, two dampers; 2TID, dual tuned inerter dampers; 2TVMD, dual tuned mass viscous mass dampers.

The optimisation results for $c_u = c_{10}/5$: (a) a single device, (b) a pair of devices, (c) the percentage improvement of using two devices over a single one for the layouts D (green dash dotted), TVMD (blue dashed) and TID (red). 2D, two dampers; 2TID, dual tuned inerter dampers; 2TVMD, dual tuned mass viscous mass dampers.

Device forces for $c_u = c_{10}/5$: (a) a single device with the TID (red), the D (green dash dotted) and the TVMD (blue dashed), (b) a pair of devices with the 2TID (red), the 2TVMD (blue dashed) and the 2D (green dash dotted), where thick line represents the force of Device1 and the thin one is that of Device2. 2D, two dampers; 2TID, dual tuned inerter dampers; 2TVMD, dual tuned mass viscous mass dampers.

shown in Figure 9c, where 2D have the same performance with a damper. The 2TID achieves up to 52.1% improvement over the TID, whereas for the 2TVMD, the improvement over the TVMD increases as the inertance increases, with a 81% improvement when $b = 3000$ kg. Figure 10 suggests that the optimum suppression configurations in their corresponding optimal range of the inerter’s size exert a similar level of control forces. In addition, for the 2TID device, Device1 provides
larger force than Device2, whereas in the optimal range of the 2TVMD, the TVMD located at the bottom exerts smaller force comparing with that located between the first and second floors.

The damper and spring values for the optimal TVMD and TID are shown in Figure 11a,b. The optimal damping values of the TID is smaller than $c_{10}/5$ in the whole range of the inerter’s size. For the TVMD, the reciprocal value of the optimal stiffness of the TVMD is almost zero when $b$ equals 100 kg, which means the TVMD can be reduced to a IPD, shown in Figure 2d, at this inerance. For the case where a pair of devices is used, Figure 11c,e shows that the 2TVMD can be simplified to the DTVMD when $b \in (100 \text{ kg}, 1700 \text{ kg})$. From Figure 11d, we notice that total damping values of the 2TID are smaller than the damping boundary $c_{10}/5$ in the whole range of the inerter’s size, indicating that any damping boundary with larger value will not result in a better performance with the 2TID layout, this can be demonstrated in the following case $c_u = c_{10}$.

With the damping upper boundary set to $c_{10}$, Figure 12a shows that the TVMD outperforms the others when $b \in (100 \text{ kg}, 2900 \text{ kg})$ and $b \in (2900 \text{ kg}, 3000 \text{ kg})$, respectively. It can be noted from Figure 12b that the 2TVMD provides the best performance in the whole range of the inerter’s size, with up to 83.9% and 46.4% improvement comparing with 2TID and 2D, respectively. From Figure 9b and Figure 12b, it can be seen that comparing with the previous case $c_u = c_{10}/5$, the value of $J_\infty$ of 2TID is the same and that of 2TVMD and 2D is much smaller for the case $c_u = 10$. The comparison between a single device and a pair of devices with the damping restriction $c_u = c_{10}$ has been shown in Figure 12c. From it, we can

![FIGURE 11](image1.png)

**FIGURE 11** The corresponding optimal element values for $c_u = c_{10}/5$: for a single device (a) $c$, (b) $k$; for a pair of devices (c) the inerance ratio $u_b$, (d) $c_1$ (bold) and $c_2$ (thin), (e) $k_1$ (bold) and $k_2$ (thin) for a pair of devices with the layouts TVMD (blue dashed), TID (red). 2D, two dampers; 2TID, dual tuned inerter dampers; 2TVMD, dual tuned viscous mass dampers

![FIGURE 12](image2.png)

**FIGURE 12** The optimisation results for $c_u = c_{10}$: (a) a single device, (b) a pair of devices, (c) the percentage improvement with the configurations D (green dash dotted), TVMD (blue dashed), TID (red). 2D, two dampers; 2TID, dual tuned inerter dampers; 2TVMD, dual tuned viscous mass dampers
obtain that different from the previous cases, the improvement of 2D is about 2% comparing with the single damper. The 2TVMD still performs better than the TVMD, with 17.2%–52.9% improvement.

The device forces under this damping restriction are shown in Figure 13, also suggesting that all the devices result in similar levels of control force and the TVMD has much smaller force when the inerter’s size is large, comparing with that for previous two damping limit cases. It can also be noted that different from the case that \( c_u = c_{10}/5 \), the TVMD located at the bottom has larger control force than the one mounted between the first and second floors in most range of inerter’s sizes. The optimal stiffness for the TVMD is shown in Figure 14b and appears to be extremely large in the range of \( b \in (100 \text{ kg}, 1600 \text{ kg}) \), hence, the TVMD can again be simplified to the IPD. The inertance ratio and the optimal stiffness of the 2TVMD in Figure 14c,e suggest that the 2TVMD can be reduced to the DTVMD in the whole range of the inerter’s size.

Overall, we note that limiting the dampers’ parameter has much less effect on the TID than the other layouts, as the TID typically requires much less damping whereas the TVMD and D appear to rely heavily on significant damping to achieve optimal performance. In addition, from Figures 7, 10, and 13, it can be noted that for a single damper with a higher damping, the exerted force becomes larger while the objective function becomes smaller. This shows a trade-off between the lower value of the objective function \( J_\infty \) and lower control force \( F \) acting on the main structure. For the TID and TVMD, this trade off does not hold; for example, with a same inerter’s size \( b = 500 \text{ kg} \), obtained from Figures 9 and 4, the force of TVMD is \( F = 18 \text{ kg} \) with the cost function \( J_\infty = 0.16 \text{ s}^2 \) when \( c_u = c_{10}/16 \), and based on Figures 9 and 10, when \( c_u = c_{10}/5 \), we can obtain that \( F = 12 \text{ kg} \) and \( J_\infty = 0.07 \text{ s}^2 \). We can note that with the TID and the TVMD, a lower

**FIGURE 13** Device forces for \( c_u = c_{10} \): (a) a single device with the TID (red), the D (green dash dotted) and the TVMD (blue dashed), (b) a pair of devices with the 2TID (red), the 2TVMD (blue dashed) and the 2D (green dash dotted), where thick line represents the force of Device1 and the thin one is that of Device2. 2D, two dampers; 2TID, dual tuned inerter dampers; 2TVMD, dual tuned viscous mass dampers

**FIGURE 14** The corresponding optimal element values for \( c_u = c_{10} \): for single device (a) \( c \); (b) \( k \); for a pair of device (c) the inertance ratio \( u_b \), (d) \( c_1 \) (bold) and \( c_2 \) (thin), (e) \( k_1 \) (bold) and \( k_2 \) (thin) with the layouts TVMD (blue dashed), TID (red). 2D, two dampers; 2TID, dual tuned inerter dampers; 2TVMD, dual tuned viscous mass dampers
control force $F$ can result in a lower value of $J_\infty$. This is because for the TID and the TVMD, one additional degree of freedom is added to the primary structure, and the performance resulted from such device is not only influenced by the acting force but also related to the movement of the added degree.

5 | OVERALL BENEFICIAL LAYOUTS

From the analysis above, it can be concluded that the optimised layout is affected by the amount of inertance available and the upper bound of damping selected. With two devices, 2TVMD and 2TID always outperform the single devices, the TVMD and TID, respectively. In contrast, the optimisation result of 2D is better than that of the single damper only when the damping boundary is large. In addition, the TVMD and 2TVMD can be reduced to the IPD and DTVMD over some range of the inerter’s size for some upper damping limits.

Based on the conclusions obtained for the three specific damping boundaries $c_u = c_{10}/16$, $c_u = c_{10}/5$, and $c_u = c_{10}$ with a single device and a pair of devices, we optimise the objective function with these proposed layouts for many damping restrictions in the range of $(0, c_{10})$. The optimal regions with respect to the $b$ value and the damping boundary for a single device and a pair of devices are shown in Figure 15a,b together with the optimisation results for the cost function. This will provide a guidance for selecting the appropriate configurations given a certain boundary for inertance, damping values, and performance requirements. In some regions of the inerter’s size and the damping boundary, the TVMD and 2TVMD are simplified to the IPD and DTVMD, respectively. It can be seen that using a pair of devices can provide a much better performance than the single device of equivalent inertance and damping boundary. From Figure 15(a), we notice that for some selected inerter’s size, the optimisation result of the TID does not change when the damping boundary is higher than a specific value (see the vertical contour lines), this is because increasing the value of $c_u$ does not result in a better performance. As a result, if limiting the value of the dampers’ parameter is important, the TID is a preferable layout. Furthermore, it can be seen that for reducing the interstorey drifts of a building model, both a single damper and a 2D do not provide superior performance comparing with the proposed inerter-based devices. The results in Figure 15b, suggest that the DTVMD has a very large optimal region, where a bigger $b$ and bigger $c_u$ can always give a smaller value of $J_\infty$. It can be also noted that the sensitivity of the DTVMD improves with the value of $b$ and the damping boundary $c_u$ increasing.

Finally, two illustrative numerical examples are conducted for demonstrating the results obtained in Figure 15, for selecting the optimal design of the suppression devices with given values of inerter $b$ and the damping up-boundary $c_u$. The actual values a designer might use will depend on a range of functions including practical acceptable device sizes, where the inerter’s size is dependent on the physical inerter prototypes, and the damping values are also needed to be constrained in a reasonable range, related with the fluid and valve design shown in Makris and Constantinou.\(^{43}\) In terms of physical realisation, we take the inerter as an example. With a ball–screw inerter, the inertance is achieved by

$$b = \left(\frac{2\pi}{P}\right)^2 J,$$

where $P$ is the pitch of of the ball-screw assembly, and $J$ is the moment of inertia of the flywheel. It can be seen that by selecting smaller pitch values, the achieved inertance can be far greater than the actual mass of the device. For example, if $P = 5\text{mm}$, to achieve a predetermined value of the inertance $b$, the flywheel moment can be obtained as $J = 6.3 \times 10^{-7} \text{kgm}^2$. Subsequently, based on this obtained moment, the dimensions the flywheel can be calculated, with which

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure15}
\caption{The optimal layouts and the optimisation results of $J_\infty$ for (a) a single device and (b) a pair of devices. 2TID, dual tuned inerter dampers; DTVMD, a damper located at the bottom and a tuned viscous mass damper mounted between the first and second floors; IPD, an inerter with a parallel connected damper; TVMD, tuned viscous mass dampers.}
\end{figure}
the ball–screw inerter can be designed and manufactured. For example, this could be achieved for \( b = 2500 \) kg using a 30-mm-thick 90-mm-diameter solid steel dish. Together with this ball–screw inerter, the optimum absorbers can then be assembled using the viscous damper and the coil spring. This approach has been adopted for realising the TVMD, and the manufactured prototype has been applied to a 14-storey building in Sendai, Japan.\(^{47}\)

We first consider an example that \( b = 750 \) kg and \( c_u = 30.75 \) kN\(s/m\). Based on the Figure 15, it can be seen that the DTVMD is the optimal design for providing the smallest value of \( J_\infty \). To show the superiority of this device, three other devices, the D, the TVMD, and the TID, with the same inertance \( b = 750 \) kg and the same damping up-boundary \( c_u = 30.75 \) kN\(s/m\) are analysed for a sake of comparison, with which, the optimisation results are summarised in Table 1.

It can be seen that comparing with the damper, the TID, and the TVMD, the optimal device DTVMD can respectively provide 51\%, 38\%, and 47\% performance improvement. The control forces \( F \) have also been shown in Table 1, suggesting that similar force levels are required for these four configurations. It can also be noted that for the single device, the TID results in a value of the objective function \( J_\infty \) as 0.029, which is in line with that obtained from Figure 15a. Also note that at this point in \( b, c_u \) space, the TID provides better performance than the TVMD (14.7\% improvement), as expected from Figure 15a. Using the optimum parameter values obtained in Table 1, the corresponding interstorey drift frequency response of the building model is shown in Figure 16, from which, it can also be seen that the DTVMD provides the best performance.

To further verify the seismic performance of these four configurations, the five-storey building model with these devices is studied with respect to two earthquake base excitations, the Japan Meteorological Agency N-S Tohoku earthquake of March 11, 2011, with the duration as 180s and the Japan Meteorological Agency N-S Kobe earthquake of January 17, 1995, with duration as 50s. These two earthquakes are shown in Figures 17a and 18a, respectively. With both the earthquakes, Figures 17b and 18b illustrate the time response of the interstorey drift between the first floor and the ground. It can be seen that all the devices D, TID, TVMD, and DTVMD suppress the vibration, and among these, the DTVMD provides the best performance.

| Configurations | \( J_\infty \), \( s^2 \) | \( F \), kN\(s^2/m \) | Optimum parameter values, kN\(s/m\), kN/m |
|----------------|---------------------|----------------|-----------------------------------------|
| \( D \)        | 0.037               | 12.5           | \( c = 30.75 \)                        |
| \( TID \)      | 0.029               | 12.7           | \( c = 1.66, k = 89.03 \)              |
| \( TVMD \)     | 0.034               | 12.4           | \( c = 30.75, k = 1689.6 \)            |
| \( DTVMD \)    | 0.018 \( F_1 = 6.21, F_2 = 7.79 \) | \( c_1 = 29.27, c_2 = 1.48, k = 95.1 \) |

Abbreviations: D, damper; DTVMD, a damper located at the bottom and a TVMD mounted between the first and second floors; TID, tuned inerter damper; TVMD, tuned viscous mass damper.

**FIGURE 16** Magnitudes of frequency response functions from ground accelerations \( A_g \) to interstorey drifts \( Z_i \) with (a) the damper (D), (b) the tuned viscous mass damper, (c) the tuned inerter damper, and (d) the DTVMD when \( b = 750 \) kg and \( c_u = 30.75 \) kN\(s/m\). DTVMD, a damper located at the bottom and a tuned viscous mass damper mounted between the first and second floors.
For the Tohoku earthquake: (a) Ground acceleration time-history and (b) first floor inter-storey drift time history with the damper (green dashed), tuned inerter damper (red), tuned viscous mass damper (black dotted), DTVMD (blue) for $b = 750$ kg and $c_u = 30.75$ kNs/m. DTVMD, a damper located at the bottom and a tuned viscous mass damper mounted between the first and second floors.

For the Kobe earthquake: (a) Ground acceleration time-history and (b) first floor inter-storey drift time history with the damper (green dashed), tuned inerter damper (red), tuned viscous mass damper (black dotted), DTVMD (blue) for $b = 750$ kg and $c_u = 30.75$ kNs/m. DTVMD, a damper located at the bottom and a tuned viscous mass damper mounted between the first and second floors.

| Configurations | Tohoku earthquake | Kobe earthquake |
|----------------|-------------------|-----------------|
|                | $F_{1\text{max}}$ (kN) | $F_{1\text{rms}}$ (kN) | $F_{2\text{max}}$ (kN) | $F_{2\text{rms}}$ (kN) | $F_{1\text{max}}$ (kN) | $F_{1\text{rms}}$ (kN) | $F_{2\text{max}}$ (kN) | $F_{2\text{rms}}$ (kN) |
| D              | 9.78              | 1.65            |                  |                  | 16.1             | 2.94            |                  |                  |
| TID            | 10.8              | 1.98            |                  |                  | 16.9             | 3.23            |                  |                  |
| TVMD           | 10.4              | 1.75            |                  |                  | 17.3             | 3.05            |                  |                  |
| DTVMD          | 7.22              | 8.30            | 1.16            | 1.31            | 14.8             | 11.5            | 2.41            | 2.33            |

Abbreviations: D, damper; DTVMD, a damper located at the bottom and a TVMD mounted between the first and second floors; TID, tuned inerter damper; TVMD, tuned viscous mass damper.

The second example is to identify the optimal absorber with the constraints that the required inertance and damping constraint are $b = 2500$ kg and $c_u = 20$ kNs/m. From Figure 15a, the approximate optimum control configuration of the performance over the whole time range, which is significantly better than the other devices. The TID, D, and the TVMD have very similar performance under these two earthquake excitations, as expected from Table 1 and Figure 16. The peak forces $F_{\text{max}}$ and the root mean square value of the exerted forces ($F_{\text{rms}}$) with these four configurations are summarised in Table 2 for the Tohoku and Kobe earthquakes, respectively. Note that these values are taken over the whole duration of the considered earthquakes, that is, 180s for Tohoku earthquake and 50s for Kobe earthquake. It can be noted that under the Kobe earthquake, higher control forces are required comparing with the Tohoku earthquake and subjected to both these two earthquakes, all the four devices, the D, TID, TVMD, and DTVMD, exert similar force levels, as expected from Table 1. Also note that the maximum force exerted by the optimum device, the DTVMD, is 8.3kN and 14.8kN when the structure is subjected to the Tohoku and Kobe earthquakes, respectively. These are 16.6% and 30% of the structure weight, 49kN. However, when comparing with the traditional damper with the same amount of damping, 30.75kNs/m, the DTVMD results in not only the better performance but also the smaller force. Using these four configurations in a building model with 3% structural damping, the maximum drift between each two connected stories during the earthquake are shown in Figure 19 for both the Tohoku and the Kobe earthquakes. Comparing with the response for this building model with no device, these figures confirm that the proposed vibration suppression devices reduce the interstorey drifts for all the floors, and the DTVMD provides the best performance.
single device is the TID. Figure 15b suggests that the DTVMD and the 2TID provide similar performance at this $b - c_u$ point. The optimum results for these three devices, the TID, the 2TID, and the DTVMD are summarised in Table 3, where a single damper is also provided for comparison. The optimum result shows that the 2TID provides 11.8% performance improvement comparing with a single TID and results in same value of $J_{\infty}$ as the DTVMD.

The drift frequency response of the building model with the four configurations are shown in Figure 20, suggesting the similar conclusions obtained in Table 3. Table 4 summarises the forces exerted by the three inerter-based devices,

![Figure 19](image1)

**Figure 19** Maximum inter-storey drift for the building model with 3% structural damping ratio controlled by no device, DTVMD, TVMD, IPD and TID for $b = 750$ kg and $c_u = 30.75$ kN.s/m against (a) Tohoku earthquake, (b) Kobe earthquake. DTVMD, a damper located at the bottom and a tuned viscous mass damper mounted between the first and second floors; IPD, an inerter with a parallel connected damper; TID, dual tuned inerter dampers; TVMD, tuned viscous mass dampers.

![Figure 20](image2)

**Figure 20** Magnitudes of frequency response functions from ground accelerations $A_g$ to inter-storey drifts $Z_i$ with (a) the damper (D), (b) the tuned inerter damper, (c) the dual tuned inerter damper and (d) the DTVMD when $b = 2500$ kg and $c_u = 20$ kN.s/m. DTVMD, a damper located at the bottom and a tuned viscous mass damper mounted between the first and second floors.

**Table 3** Optimisation results with the D, TID, 2TID and DTVMD when $b = 2500$ kg and $c_u = 20$ kN.s/m

| Configurations | $J_{\infty}$, $s^2$ | $F$, kN.s$^2$/m | Optimum parameter values, kN.s/m, kN/m |
|----------------|---------------------|-----------------|---------------------------------------|
| D              | 0.056               | 12.4            | $c = 20$                              |
| TID            | 0.017               | 12.5            | $c = 11.14, k = 277.6$                |
| 2TID           | 0.015, $F_1 = 7.74$, $F_2 = 7.86$ | $u = 0.57, c = 3.62$ |
| DTVMD          | 0.015, $F_1 = 1.82$, $F_2 = 12.3$ | $k_1 = 139.8, c_1 = 3.16, k_2 = 132.9$ |

Abbreviations: 2TID, dual tuned inerter damper; D, damper; DTVMD, a damper located at the bottom and a tuned viscous mass damper mounted between the first and second floors; TID, tuned inerter damper.
TABLE 4  The peak value $F_{\text{max}}$ and the root mean square value $F_{\text{rms}}$ of the exerted forces with the four devices subjected to Tohoku and Kobe earthquakes when $b = 2500$ kg and $c_u = 20$ kNs/m.

| Configurations | Tohoku earthquake | Kobe earthquake |
|----------------|-------------------|-----------------|
|                | $F_{1\text{max}}$ (kN) | $F_{2\text{max}}$ (kN) | $F_{1\text{rms}}$ (kN) | $F_{2\text{rms}}$ (kN) | $F_{1\text{max}}$ (kN) | $F_{2\text{max}}$ (kN) | $F_{1\text{rms}}$ (kN) | $F_{2\text{rms}}$ (kN) |
| D             | 7.93              | 1.38            | 11.6           | 2.25              | 17.1              | 2.45            | 28.6           | 5.06               |
| TID           | 11.2              | 8.47            | 15.1           | 3.47              | 3.01              | 15.1            | 5.47           | 29.2               |
| 2TID          | 11.1              | 2.45            | 1.38           | 5.47              | 11.2              | 5.47            | 15.3           | 3.40               |
| DTVMD         | 3.01              | 15.1            | 0.40           | 2.37              | 11.2              | 8.47            | 1.51           | 2.37               |

Abbreviations: 2TID, dual tuned inerter damper; D, damper; DTVMD, a damper located at the bottom and a tuned viscous mass damper mounted between the first and second floors; TID, tuned inerter damper.

FIGURE 21  Maximum inter-storey drift for the building model with 3% structural damping ratio controlled by no device, DTVMD, 2TID, D and TID for $b = 2500$ kg and $c_u = 20$kNs/m against (a) Tohoku earthquake, (b) Kobe earthquake. 2TID, dual tuned inerter damper; D, damper; DTVMD, a damper located at the bottom and a tuned viscous mass damper mounted between the first and second floors; TID, tuned inerter damper.

together with the traditional damper for the case that $b = 2500$ kg, $c_u = 20$kNs/m. It can be seen that the DTVMD exerts the maximum force $F_{2\text{max}} = 29.2$kN for the Kobe earthquake, approximately 60% of the structure weight, which is a significant large force acting on the structure. The 2TID device provides similar seismic performance (see Table 3 and Figure 20) but results in much smaller maximum forces. Hence, the 2TID device is considered as the optimum vibration suppression device. The maximum interstorey drifts for a building model with 3% structural damping ratio subjected to the two earthquakes are also obtained in Figure 21, demonstrating the effectiveness of the optimum device, the 2TID.

6 | CONCLUSIONS

The main focus of this paper is to identify the potential benefits of using two inerter-based suppression devices over the use of a single device. Three layouts mounted between the ground and the first and the first and second floors of a building model are taken into consideration. For the sake of comparison, a single device located between the ground and the first floor is also considered. Results from fixed-sized-inerter optimisations are presented to demonstrate that the value of damper(s) in the devices can be very large in some cases. Hence, to form a fair comparison, and also to take limitations for physical implementation into consideration, analysis with damping constraints are conducted. Using the maximum magnitude across all the interstorey drifts and across the whole frequency as the cost function, an optimisation procedure is carried out for fixed inerter and with a limit on the maximum damping imposed. As a result, the optimum layouts and the optimal results for a single and a pair of suppression devices have been obtained with respect to the different inerter’s sizes and different up-boundaries of the damping value limits, respectively. It has been shown that comparing with the single suppression device, a pair of devices can always provide better performance with the same inerter and damping constraints. Furthermore, the TID requires much less damping to result in the optimal performance compared to the TVMD or the damper layouts, and so becomes the optimal configuration over a wide range of acceptable inerter as the constraint on the damping is lowered. Finally, the frequency responses were presented to show the validity of the proposed optimal configurations. The approach taken in this work is applicable for investigation of the case where multiple devices are used.
ACKNOWLEDGEMENTS

The authors would like to acknowledge the support of the Engineering and Physical Sciences Research Council (EPSRC), the University of Bristol, and the China Scholarship Council: Simon Neild is supported by an EPSRC fellowship EP/K005375/1, Sara Ying Zhang is supported by a University of Bristol studentship and the China Scholarship Council. Jason Zheng Jiang is supported by an EPSRC grant EP/P103456/1.

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**How to cite this article:** Zhang SY, Neild S, Jiang JZ. Optimal design of a pair of vibration suppression devices for a multi-storey building. *Struct Control Health Monit*. 2020;27:e2498. [https://doi.org/10.1002/stc.2498](https://doi.org/10.1002/stc.2498)