Robust control-based power quality optimization strategy for inverters

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Abstract—A dynamic compensation control strategy based on a residual observer combined with a gradient descent optimization algorithm is proposed to address the power quality problem of distributed AC networks. The robust control structure based on the residual observer is designed according to the theory of Plug and Play control framework. The proposed structure is effective in improving the dynamic performance of the system without changing the structure and parameters. A gradient descent optimization algorithm is used to optimize the parameters of the rectified control structure online. Based on the need for fast response capability and stable performance of distributed grid systems, the gradient descent algorithm is optimized using the model matching principle, which provides a theoretical basis for the selection of initial points in the optimization algorithm. A low-order derivation form of the robust control structure is given through a step-down process to further enhance the speed of optimization under this strategy. The analysis shows that the strategy effectively enhances the robustness of the distributed grid system in comparison with the conventional double closed-loop control. Modelling simulation results verify the effectiveness of the proposed control strategy.

1. INTRODUCTION

In a distributed grid system, the switching devices of power electronic converters will generate harmonics to pollute the bus voltage, the temporary rise and fall of the output bus voltage when the load and distributed power supply cut in and out, and the unbalanced load and single-phase power supply access will cause the bus voltage to be unbalanced in three phases. The above power quality problems will affect the stable operation of the whole system. Therefore, it is very important to enhance the robustness of the distributed grid system and to improve the dynamic and steady-state performance of the system.

The paper [1] discusses the coordinated control of reactive, unbalanced and harmonic power in distributed grids, using unbalanced sag controllers and harmonic sag controllers in order to share harmonic and unbalanced power proportionally between distributed generation units and to effectively improve the bus voltage power quality. The paper [2] addresses the power quality problems caused by
non-linearity and unbalanced load access in distributed grid systems, and introduces a 5th and 7th harmonic compensation control loop load negative sequence control loop on top of the traditional double closed loop to solve the three-phase unbalance and harmonic problems. In the paper [3], a control strategy of using the load current as a disturbance current and using the Disturbance Observer (DOB) as a feed-forward compensation of the voltage outer loop is proposed to solve the problem of output bus voltage fluctuation when the load is cut in and out, and a universal parameter design method for the Disturbance Observer is given. The design method of the disturbance observer with universal applicability is given. In [4], an observer-based voltage optimization control strategy is proposed for UPS systems, using feedback control and feedforward compensation to resolve the uncertainties caused by system errors and disturbances, effectively solving three typical power quality problems. In the paper [5], a voltage and current control strategy with internal mode control is proposed, and using small signal analysis, eigenvalue and sensitivity analysis is performed for islanded distributed grid systems, which further illustrates that internal mode control has good robustness and can effectively solve the power quality problems arising in distributed grid systems. In the paper [6], a robust voltage controller is transformed into a convex optimization problem solved for a linear objective function subject to a linear matrix inequality, which improves the dynamic response of the system and reduces the steady-state error of the system, using an islanded DC distributed grid system as the object.

In order to improve the robustness of distributed power grids against three typical power quality problems, a dynamic compensation control strategy based on a residual observer is proposed in this paper. Finally, the effectiveness of the strategy is verified by simulation.

2. ROBUST CONTROL STRUCTURE

2.1. Robust control framework design based on residual observer

For a true rational transfer function \( G(s) \) has a minimal state space realization of the form

\[
G(s) = \begin{bmatrix} A & B \\ C & D \end{bmatrix}
\]

Its left-right mutual mass decomposition (LCF and RCF) is

\[
G(s) = \hat{M}^{-1}(s) \hat{N}(s) = N(s)M^{-1}(s)
\]

In which \( \hat{M}(s) \) and \( \hat{N}(s) \) are left coprime over \( RH_\infty \), an \( M(s) \) and \( N(s) \) are right coprime over \( RH_\infty \). If \( \hat{X}(s) \) and \( \hat{Y}(s) \) are left coprime over \( RH_\infty \), \( X(s) \) and \( Y(s) \) are right coprime over \( RH_\infty \), they can be shown:

\[
\begin{bmatrix} \hat{N}(s) & \hat{M}(s) \\ \hat{X}(s) & \hat{Y}(s) \end{bmatrix} = \begin{bmatrix} I_n \\ I_r \end{bmatrix}
\]

\[
\begin{bmatrix} M(s) & \hat{Y}(s) \\ N(s) & \hat{X}(s) \end{bmatrix} = \begin{bmatrix} A + BF & B & L \\ F & I & o \\ C + DF & D & I \end{bmatrix}
\]

\[
\begin{bmatrix} X(s) & Y(s) \\ \hat{X}(s) & \hat{Y}(s) \end{bmatrix} = \begin{bmatrix} A - LC & -(B - LD) & -L \\ F & I & o \\ C & -D & I \end{bmatrix}
\]

where \( F \) and \( L \) are chosen such that \( A + BF \) and \( A - LC \) are both stable.

Consider a standard feedback control loop as shown in Figure 1.
Then the set of all proper controllers via Youla parameterization achieving internal stability can be parameterized by:

\[
K(s) = -\left(\hat{Y}(s) + M(s)Q(s)\right)\left(\hat{X}(s) - N(s)Q(s)\right)^{-1};
\]

\[
= -\left(X(s) - Q(s)\hat{N}(s)\right)^{-1}\left(Y(s) + Q(s)\hat{M}(s)\right)
\]

where, \(Q(s)\) is Youla parameterization matrix.

From the above theoretical derivation, it follows that for a true rational transfer function \(G(s)\), and an output control signal \(u_0\), if this control system is internally stable, then all internally stable controllers can be parameterized as:

\[
u(s) = u_0 + Q(s)r
\]

where \(Q(s)\) is a stable parameterization matrix and \(r\) is the residual vector.

As shown in Figure 2, the essence of this robust control theory is that the actual and estimated values of the output are processed in the residual observer and the resulting residual signal is fed back into the controller to achieve compensated control of the disturbance.

\[\text{Observer-based Residual generator}\]

**2.2. Robust controller modelling**

Based on the above theoretical proof, a robust control strategy for voltage inverters is proposed by combining the traditional inverter double closed-loop control strategy with a robust control architecture based on residual observers, whose architecture is shown in Figure 3. When the control performance of the controller fails to meet the standard conditions due to external disturbing effects, the original double closed-loop feedback controller can be redesigned without changing or redesigning, and only the parameter matrix \(Q(s)\) needs to be reconfigured.

The state space equations for the voltage and current control loop are as follows:

\[
\begin{bmatrix}
\dot{\theta}_{dq} \\
\dot{\phi}_{dq}
\end{bmatrix} =
A_{uc}
\begin{bmatrix}
\theta_{dq} \\
\phi_{dq}
\end{bmatrix} +
B_{uc}
\begin{bmatrix}
e_{dq} \\
e_{dlq}
\end{bmatrix}
\]

\[
\begin{bmatrix}v_{id}^* \\
v_{iq}^*
\end{bmatrix} =
C_{uc}
\begin{bmatrix}
\theta_{dq} \\
\phi_{dq}
\end{bmatrix} +
D_{uc}
\begin{bmatrix}
e_{dq} \\
e_{dlq}
\end{bmatrix}
\]

(8)
where \( A_{vc} \) is the 4X4 zero matrix and \( B_{vc} \) is the 4X4 unit matrix. \( C_{vc} \), \( D_{vc} \) are shown below.

\[
C_{vc} = \begin{bmatrix}
K_{pi} & 0 & 0 & -L_t \omega K_{iv} \\
0 & K_{ii} & L_t \omega K_{iv} & 0
\end{bmatrix},
\]

\[
D_{vc} = \begin{bmatrix}
K_{pi} & L_t \omega & L_t C_t \omega^2 & -L_t \omega K_{pv} \\
-L_t \omega & K_{pi} & L_t \omega K_{pv} & L_t C_t \omega^2
\end{bmatrix}
\]

The original PI controller (20) can be reformulated into a discrete state-space representation as shown:

\[
\begin{align*}
x_{vc,k+1} &= A_{vc} x_{vc,k} + B_{vc} e_k \\
u_{vc,k} &= C_{vc} x_{vc,k} + D_{vc} e_k
\end{align*}
\]

(9)

The measured inverter output voltage differs from the estimated value when there is load cut-off, high harmonics from non-linear loads and load unbalance disturbances occur, resulting in a residual signal. Therefore, a Luenberger observer can be established. Then the state-space representation of the observer based residual generator can be expressed:

\[
\begin{align*}
z_{k+1} &= A_z z_k + B_z u_k + L_z r_k \\
\hat{y}_k &= C_z z_k \\
r_k &= y_k - \hat{y}_k
\end{align*}
\]

(10)

In order to design the parameter matrix \( Q(s) \) and reduce dynamic complexity, we can represent it in a controlled canonical form [7], which has fewer design parameters and has the following state space representation.

\[
\begin{align*}
x_{r,k+1} &= A_z x_{r,k} + B_z r_k \\
u_{r,k} &= C_z x_{r,k} + D_z r_k
\end{align*}
\]

(11)
Where * is a non-zero element of the matrix $A$, and the state vector $x_{r,k}$ is selectable by the designer.

Combining the above discrete state space expressions (9), (10) and (11) and Figure 2 the state space expression with the reference signal $w_k$ and the residual signal $r_k$ as inputs and the error tracking signal $e_k$ and the control signal $u_k$ as outputs can be established as:

$$
\begin{align*}
A_r &= \begin{bmatrix}
0 & 1 & \cdots & 0 \\
0 & 0 & \ddots & \vdots \\
\vdots & \vdots & \ddots & 1 \\
* & * & \cdots & * & \cdots & * & \cdots & * & \cdots & * & \cdots & * & \cdots & * & \cdots & 1 \\
\end{bmatrix},
B_r &= \begin{bmatrix}
0 \\
\vdots \\
\vdots \\
0 \\
1 \\
0 \\
0 \\
\vdots \\
\vdots \\
0 \\
1 \\
\end{bmatrix},
C_r &= \begin{bmatrix}
c_{i1} & \cdots & c_{iQ} \\
\vdots & \ddots & \vdots \\
c_{i1} & \cdots & c_{iQ} \\
\end{bmatrix},
D_r &= \begin{bmatrix}
d_{i1} & \cdots & d_{iM} \\
\vdots & \ddots & \vdots \\
d_{i1} & \cdots & d_{iM} \\
\end{bmatrix},
\end{align*}
$$

$$J = \frac{1}{2N} \sum_{k=k_0}^{N+k_0-1} \left[ e_k^T W_{c_k} e_k + u_k^T W_{u_k} u_k \right]$$

$k_0$ and $N$ represent the starting point of the gradient descent optimization process and the time window of the whole optimization process respectively. $W_{e_k} \geq 0$ and $W_{u_k} \geq 0$ are weighting matrices of the tracking error and control signal, respectively.

The value of the parameter $\theta$ is optimized by the gradient descent algorithm. In order to successfully find the optimal value of $\theta$, the gradient algorithm is chosen to be designed in the steepest direction of descent, where the gradient function is:
The update rule for the value of parameter $\theta$ is

$$\theta_{j+1} = \theta_j - \Gamma_j \frac{\partial J}{\partial \theta_j}, \quad \theta_0 = 0$$

(15)

where $j$ is the number of iterations and $\Gamma_j$ is the iteration step.

The size of the iteration step can determine the convergence speed of the objective function, so a suitable iteration step can be designed according to the optimization theory to speed up the convergence speed. The Taylor expansion of the objective function yields.

$$\Gamma_j = \left[ \frac{\partial^2 J}{\partial \theta_j \partial \theta_j} \right]^{-1}$$

(16)

Where $\Gamma_j$ is the Hessian matrix derived from the objective function $J$.

According to equation (11), the $C_r$ and $D_r$ matrices contain unknown parameters $c$ and $d$, and the derivatives of the error tracking signal $e_k$ and the control signal $u_k$ with respect to $\theta$ for the $C_r$ and $D_r$ matrix row vectors (column vectors), respectively, can be found as:

$$\begin{bmatrix}
\frac{\partial z_{k+1}}{\partial \theta_r}
\frac{\partial x_{c,k+1}}{\partial \theta_r}
\end{bmatrix} = \begin{bmatrix}
A_r - B_r D_{vec} C_r & B_r C_{vec}
-B_{vec} C_r & A_{vec}
\end{bmatrix} \begin{bmatrix}
\frac{\partial z_k}{\partial \theta_r}
\frac{\partial x_{c,k}}{\partial \theta_r}
\end{bmatrix}
+ \begin{bmatrix}
B_r I_r
0
\end{bmatrix} x_{r,k}$$

\begin{align*}
\begin{bmatrix}
\frac{\partial e_k}{\partial \theta_r}
\frac{\partial u_k}{\partial \theta_r}
\end{bmatrix} &= \begin{bmatrix}
-C_r & 0
d_{vec} C_r & C_{vec}
\end{bmatrix} \begin{bmatrix}
\frac{\partial z_k}{\partial \theta_r}
\frac{\partial x_{c,k}}{\partial \theta_r}
\end{bmatrix}
+ \begin{bmatrix}
0
I_r
\end{bmatrix} x_{r,k}
\end{align*}

(17)

Where, $I_r = \frac{\partial C_r}{\partial \theta_r}$. 
Where, \( I_{di} = \frac{\partial D_i}{\partial \theta_d} \)

By bringing equations (17) and (18) into the gradient expression (15), a suitable parameterisation matrix \( Q(s) \) can be obtained by the gradient descent algorithm. In the gradient descent algorithm, the choice of initial points also has an impact on the optimization time and optimization speed, so the choice of initial points can be optimized according to robust model matching theory to obtain the appropriate parameterization matrix \( Q(s) \) more quickly.

2.4 Model matching problem solving

According to the dynamic compensation theory based on the residual generator in Fig. 3, the dynamic compensation structure is designed as follows:

\[
\begin{split}
\left[ \frac{\partial z_{d+1}}{\partial \theta_d} \right] &= \left[ A_z - B_z D_v C_z, B_z C_v C_z \right] \left[ \frac{\partial z_k}{\partial \theta_d} \right] \\
&\quad + \left[ B_z I_{d1} \right] r_k \\
\left[ \frac{\partial z_k}{\partial \theta_d} \right] &= \left[ -C_z, 0 \right] \left[ \frac{\partial z_k}{\partial \theta_d} \right] \\
&\quad + \left[ 0 \right] r_k
\end{split}
\]

where \( I_{di} = \frac{\partial D_i}{\partial \theta_d} \)

After the dynamic compensation structure in Fig. 4 is simplified and refined, and its compensation process is shown in Fig. 5.
Disturbance acts on the control object

Reverse compensation branch

The controller \( Q(s) \) is shown in:

\[
\min \| T_{y_d}(s) + G_p(s) Q(s) T_{y_d}(s) \|_\infty \tag{19}
\]

Through model-matching theory, the parameterization matrix \( Q(s) \) is:

\[
\begin{align*}
Q(1.1) &= -3(s + 1001)(s + 12.5) \\
Q(1.2) &= -700(s + 1001) \\
Q(2.1) &= 700(s + 1001) \\
Q(2.2) &= -3(s + 1001)(s + 12.5)
\end{align*}
\tag{20}
\]

It can be seen that the numerator order of the parameterized matrix controller \( Q(s) \) derived by the model matching method is higher than the denominator order. By introducing two non-dominated poles \( s = -1e^3 \) farther away from the imaginary axis, the denominator order can be made equal to the numerator order without affecting the performance of the original system, thus ensuring the physical realizability of \( Q(s) \).

\[
\begin{align*}
Q(1.1) &= \frac{-3(s + 1001)(s + 12.5)}{s^2 + 2000s + 1e^6} \\
Q(1.2) &= \frac{-700(s + 1001)}{s^2 + 2000s + 1e^6} \\
Q(2.1) &= \frac{700(s + 1001)}{s^2 + 2000s + 1e^6} \\
Q(2.2) &= \frac{-3(s + 1001)(s + 12.5)}{s^2 + 2000s + 1e^6}
\end{align*}
\tag{21}
\]

Using the reduced-order theory of controllers proposed in the paper [8], \( Q(s) \) can be simplified as:

\[
\begin{align*}
Q(1.1) &= -1.5 + \frac{580}{s + 500} \\
Q(1.2) &= -0.35 - \frac{175}{s + 500} \\
Q(2.1) &= 0.35 + \frac{175}{s + 500} \\
Q(2.2) &= -1.5 + \frac{580}{s + 500}
\end{align*}
\tag{22}
\]

The parameter matrix \( Q(s) \), obtained according to the model matching theory, is processed by order reduction, causing the problem of inaccurate controller model, but simplifying the amount of system operations. Equation (22) can also be used as the initial point of the gradient descent algorithm for the purpose of optimizing the gradient descent algorithm within the margin of error allowed.
3. EXPERIMENTAL VERIFICATION AND ANALYSIS
In order to verify the correctness and effectiveness of the control strategy mentioned in this paper. Taking a single inverter as an example, the inverter system is built in Matlab/Simulink to analyze the suppression of current disturbances caused by switching of different types of loads under the isolated operation state of distributed power supply.

### TABLE I. INVERTER PARAMETERS

| Parameters | Inverter value |
|------------|----------------|
| Vdc        | 700V           |
| Ts         | 5e-6s          |
| Lf         | 2mH            |
| Cf         | 1500μF         |
| rf         | 0.01           |
| Kpv        | 10             |
| kiv        | 100            |
| kpi        | 100            |
| kii        | 100            |

3.1. Power Load Switching Experiment
In the simulation environment of Matlab/Simulink, when the inverter performs load shedding, there are transient fluctuations in the output voltage. In order to simulate the effect of the robust control structure on the perturbation suppression of load throwing, the inverter is connected to a 20kW active load and a 20kVar reactive load for stable operation, and the output d-axis voltage of the inverter with robust control is observed by throwing a 20kW active load and a 20kVar reactive load once per second for comparison with different initial points selected. The results of the simulation experiments are shown in Figure 5.

From the comparison waveforms, it can be concluded that when the default starting point is 0, the voltage fluctuation of the final output d-axis voltage is ±0.6V when the simulation time is set to 15 seconds; when the starting point is obtained by the model matching method, the output voltage fluctuation is ±0.6V in 2 seconds when the simulation time is set to 15 seconds, and the final output voltage fluctuation is ±0.2V. Therefore, the simulation shows that the proposed optimization strategy not only effectively suppresses the voltage fluctuation of the load switching, but also speeds up the response time to return to the steady state value. Therefore, the simulation shows that the proposed optimization strategy not only effectively suppresses the voltage fluctuations arising from load switching, but also speeds up the response time of voltage recovery to the steady-state value and improves the robustness of the inverter.
3.2. Nonlinear load switching Experiment

In the Matlab/Simulink simulation environment, harmonics are injected into both the three-phase output voltage and the d-axis output voltage when the inverter is connected to a non-linear load. To simulate the disturbance suppression of the injected harmonic sources by the robust control structure, the output d-axis voltages and THD with and without robust control are observed and compared by connecting the inverter to the 5th, 7th and 11th harmonic sources.

![Fig. 7 Comparison of d-axis output voltage under three control strategies](image)

![Fig. 8 Comparison of THD under three control strategies](image)

A comparison of the above three-phase output voltage waveforms and THD shows that when the 5th, 7th and 11th harmonic sources are connected, the THD is 1.44% without the use of optimal control theory. With robust control, the THD drops to 0.94% when the initial point is selected as the default initial point and to 0.86% when the initial point is obtained by the model matching method with the simulation time set at 15 seconds; the comparison of the d-axis output voltage at 1.6s and 13s shows that the voltage disturbance suppression effect has improved. Therefore, it can be concluded from the simulated experimental waveforms that when the 5th, 7th and 11th harmonic sources are connected, the theory proposed in this paper effectively improves the system's immunity to the injected harmonics, enhances the robustness of the inverter and suppresses the impact of harmonic disturbances on the distributed grid.
4. CONCLUSION
The method proposed in this paper can effectively suppress voltage disturbances caused by load throwing and cutting, three-phase voltage unbalance caused by unbalanced loads and harmonic disturbances caused by non-linear loads, and improve the stability of inverter operation.

In the robust control structure based on the residual observer, a data-driven iterative optimization is carried out for the robust controller $Q(s)$ parameter, on the basis of which the robust model matching principle is introduced, basically solving the problem that this iterative optimization algorithm is limited by the initial point.

One of the future research works in this paper is how to improve the proposed optimization strategy to obtain better optimization results that can be achieved by optimizing the iterative step size.

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