Neutron stars with hyperon cores: stellar radii and EOS near nuclear density

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1. Introduction

Hyperons, baryons containing at least one strange quark, are known for more than 50 years. They are frequently studied in terrestrial laboratories. Although unstable on Earth, it is expected however that they are stably present in the dense interiors of neutron stars (NSs). Recent measurements of 2 M⊙ pulsars represent a challenge for equations of state of NSs with hyperon cores. The difficulty of reaching a high mass is related to a significant softening of the EOS associated with the hyperonization of the matter. Only EOSs based on the relativistic mean field (RMF) models, after tuning of their Lagrangians, turned out to be able to produce NS with maximum allowable mass M\textsubscript{max} > 2 M⊙ and sizable hyperon cores (see Colucci & Sedrakian 2013 and references therein). These successful models with hyperonization of the NS matter (one of them being hereafter referred to as EOS.H) merit a careful inspection. We will focus here on two specific features of these models: NS radii and the EOS at pre-hyperon density; as we will show, both features are interrelated.

The EOS below nuclear density ρ\textsubscript{n} = 2.7 × 10\textsuperscript{14} g cm\textsuperscript{-3} (corresponding to the baryon number density n\textsubscript{b} = 0.16 fm\textsuperscript{-3}) is commonly believed to be rather well known (see Hebeler et al. 2013). However, to construct a complete family of NS models, up to the maximum allowable mass M\textsubscript{max}, one needs the EOS for up to ~ 5ρ\textsubscript{n}. Nuclear densities ρ\textsubscript{n} and n\textsubscript{b} are suitable units for densities in the NS core. In what follows we will use dimensionless (reduced) densities 7 ≡ ρ/ρ\textsubscript{n} and 13 ≡ n\textsubscript{b}/n\textsubscript{b}. Because of uncertainties in the theory of dense matter, the only chance to unveil the actual EOS of the degenerate matter at supra-nuclear density relies on the observations of NSs.

Mathematically, both M\textsubscript{max} and the radius of NS of (gravitational) mass M, R\textsubscript{M} ≡ R(M), are functionals of the EOS. We expect that NS matter at 0.5 < 7 < 2 (so called "outer core") is composed mostly of neutrons, with a few percent admixture of protons, electrons, and muons. At higher density hyperons or even quark gluon plasma might appear, forming a strangeness carrying NS-core. In the present paper we restrict ourselves to the NSs cores where quarks are confined into baryons - nucleons and hyperons. EOSs fulfilling M\textsubscript{max} > 2 M⊙ and allowing for the presence of hyperons in NSs cores form the set EOS.H.

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A very recent paper of Yamamoto et al. (2014) is based on different many-body theory. Their MPa model yields M\textsubscript{max} > 2 M⊙ for NSs with hyperon cores. As we show in Sect. 10, the properties of NS models for the MPa EOS are consistent with those obtained by our set of the RMF EOSs.
Independently of the uncertainties related to the structure of NSs cores, any theoretical EOS\(H\) has to be consistent with the semi-empirical parameters of nuclear matter at \(\rho \approx 1\). Moreover, the hyperon component should be consistent with semi-empirical estimates (potential wells, \(\Lambda\)-interaction) coming from hypernuclear physics. Basic features of EOS\(H\) resulting from various theoretical models and puzzles which remain to be resolved are briefly summarized in Sect. 2. Another constraint, related to the value of the pressure \(P\) at nuclear (saturation) density, is discussed in Sect. 3.

The impact of the uncertainty in the EOS on the mass vs. central density dependence for NS is described in Sect. 4. In what follows, we will prefer to use \(\rho_c\) instead of \(\rho\), because it is the former which characterizes the degree of packing of baryons at the NS center. As we show there, different segments (domains) of the EOS \(P(\rho)\) determine measurable global stellar parameters in different NS mass domains. The radius of a 1.4 \(M_\odot\) NS, \(R_{1.4}\), is mostly determined by EOS\((1 < \rho < 3)\), while the value of \(M_{\text{max}}\) is to a large extent determined by EOS\((4 < \rho < 7)\).

As we show in Sect. 5, the NS radius for 1.0 \(M_\odot < M < 1.6\, M_\odot\) for EOS\(H\) is larger than 13 km. This seems to be an unavoidable consequence of \(M_{\text{max}} > 2\, M_\odot\) condition, which implies a very high stiffness of the hyperon (i.e., purely nucleon) segment 1 < \(\rho\) \(\lesssim\) 2 - 3 of EOS\(H\).

Sect. 6 explores the difference in the values of NS masses at a given central pressure and connects it with the difference in pressure distribution within NS models. In Sect. 7, the causal-limit EOS is used to provide a bound on the NS radius, \(R(M)\). Sect. 8 is devoted to the effects of the NS rotation.

In Sect. 9 we discuss a possible meaning of large radii of NSs with hyperon cores in the context of recent measurements of radii of NSs. Our conclusions are formulated in Sect. 10.

Preliminary results of our work were presented at the Nuclear Physics in Astrophysics VI conference, Lisbon, Portugal, May 19-24, 2013, at EWASS 2013 Symposium "Extreme physics of neutron stars" at Turku, Finland, July 10-13, 2013, at EMMI Meeting, FIAS, Frankfurt, Germany, October 7-10, 2013, and at the conference "The structure and signals of neutron stars, from birth to death", Florence, Italy, March 24-28, 2014.

\section{Equation of state of neutron-star matter}

Except for a very outer layer, whose contribution to NS mass and radius can be neglected, the matter in NS interior is strongly degenerate, and can be approximated by that calculated at \(T = 0\) assuming the ground state composition (cold catalyzed matter, see, e.g., Haensel et al. (2007). For such EOS, pressure depends on the density only.

In general theory of relativity (GTR) the matter density is defined as \(\rho = \mathcal{E}/c^2\), where \(\mathcal{E}\) is the total energy density (including rest energy of particles). Baryon density \(n_b\) is defined as the baryon number (baryon charge) in a unit volume. Using elementary thermodynamics one obtains the relation between \(P\) and \(\rho\) from the calculated function \(\mathcal{E}(n_b)\),

\[
P(n_b) = n_b^4 \frac{d(\mathcal{E}/n_b)}{dn_b}, \quad \rho(n_b) = \mathcal{E}(n_b)/c^2 \implies P = P(\rho) .
\]

\section{EOS satisfying the semi-empirical nuclear-hypernuclear constraints and \(M_{\text{max}} > 2\, M_\odot\)}

The models of the EOS\(H\) set reproduce (within some tolerance) four semi-empirical values of parameters of nuclear matter at saturation: saturation density \(n_s\), energy per nucleon \(E_\text{n}\), symmetry energy \(S_\text{s}\) and incompressibility \(K_\text{s}\). (The semi-empirical value of a fifth parameter, the density slope of symmetry energy \(L_\text{s}\), is relatively poorly known (Hebeler et al. (2013)) and hence it is not imposed as a constraint.) Together with \(M_{\text{max}} > 2\, M_\odot\), this makes five constraints imposed on an EOS belonging to EOS\(H\).

Apart from the five constraints described above, a given EOS\(H\) has to reproduce additionally semi-empirical values of three potential wells of (zero momentum) hyperons in nuclear matter at \(n_s\): these are sixth, seventh, and eighth constraints on EOS\(H\). Moreover, for most of the EOS\(H\) models, a ninth condition, the fitting a semi-empirical estimate of the depth of potential well of \(A\) in \(\Lambda\)-matter, is also imposed. Summarizing, there are eight or nine constraints to be satisfied by an EOS\(H\). Constructing an EOS of the EOS\(H\) set is therefore associated with a strong tuning of the dense matter models. With exception of a very recent EOS\(H\) of Yamamoto et al. (2014), only the EOSs based on the non-linear relativistic mean-field (RMF) theories can satisfy these conditions.

Selecting a very specific type of approximation - the RMF one - is a first tuning of the dense matter model. Moreover, in order to satisfy \(M_{\text{max}} > 2\, M_\odot\), baryon fields are coupled not only to (standard) \(\sigma\), \(\omega\), and \(\rho\) meson fields. Namely, the hyperon fields are additionally coupled to the vector \(\phi\) meson field. The addition of \(\phi\) (and possibly also of a \(\sigma^*\) meson field, which provides a scalar coupling between hyperons only) is a second tuning. All EOSs from EOS\(H\) are able to satisfy \(M_{\text{max}} > 2\, M_\odot\) due to repulsion produced by the \(\phi\)-meson coupled to hyperons only. Moreover, in some cases, an amplification of the hyperon repulsion due to SU\(\{6\}\)-symmetry breaking in the vector-meson coupling to hyperons is introduced, which is a third tuning. Finally, some models have density-dependent coupling constants of baryons to meson fields - this allows for a fourth tuning. The models included in EOS\(H\) set are listed in Table 1 together with brief characteristics and references.

Among the eight models consistent with a 2 \(M_\odot\) NS in Sulaksono & Agrawal (2012), we selected four of them, three with SU\(\{6\}\) symmetry, the stiffest, the softest and an intermediate one, and one with SU\(\{6\}\) symmetry broken. Lopes & Menezes (2014) obtain similar results to Weissenborn et al. (2012). The models included in EOS\(H\) set are listed in Table 1 together with brief characteristics and references.
three of them listed in the upper part of Table 1. These EOSs produce "standard NS", are consistent with all semi-empirical constraints and yield \( M_{\text{max}} > 2.0 \, M_\odot \).

The EOS of the core is supplemented with an EOS of NS crust. We assume that the crust is composed of cold catalyzed matter. For the very outer layer with \( \rho < 10^8 \, \text{g cm}^{-3} \) we use classical BPS EOS \cite{Baym:1971}. The outer crust with \( \rho > 10^9 \, \text{g cm}^{-3} \) is described by the EOS of Haensel & Pichon \cite{1994}, while for the inner crust we apply the SLy EOS of Douchin & Haensel \cite{2001}. A smooth matching with an interpolation between the crust and core EOSs is applied to get a complete EOS of NS interior. We start our comparative study of EOSs based on the nucleon in-medium EOS where the crust and core EOSs are based on the same non-empirical constraints and yield

\[ 3.0 \leq \rho \leq 10^3 \, \text{g cm}^{-3} \]

Overpressure of EOS.H at nuclear density

\[ \rho > \rho_\text{max}(n_0) \]

\[ \text{EOS.H} \]

Relation \( n \sim n_0 \) for NS matter for the set of EOSs presented in Figs. 2 and 3. Dotted segments correspond to the central densities of NS models which are unstable with respect to radial oscillations. Relation \( \rho = \rho(n_0) \) deviates from linearity for \( n_0 > 0.3 \, \text{fm}^{-3} \). Nonlinearity grows with increasing \( n_0 \) and is EOS-dependent. For \( n_0 \approx 0.2 \, \text{fm}^{-3} \), the linear approximation \( \rho \approx \rho(n_0) \approx n_0 \rho_m \) (where \( \rho_m \) is neutron mass) is valid.

3. Overpressure of EOS.H at nuclear density

We start our comparative study of EOS.H and EOS.N_{\text{ref}} by calculating the pressure at nuclear density, \( P(N)(n_0) \) and \( P(H)(n_0) \) respectively. Results are collected in the Appendix Table 5. In the following \( P_{33} \) refers to the pressure \( P \) in the units of \( 10^{15} \, \text{dyn cm}^{-2} \). We notice a striking difference between EOS.H and EOS.N_{\text{ref}}. The values of \( P_{33}(N)(n_0) \) are concentrated within \( 3.3 \pm 0.3 \), while the values of \( P_{33}(H)(n_0) \) are significantly larger, within \( 8 \pm 2.5 \)

\[ n_0 \leq n_0 \approx n_0 \text{ can be reliably calculated using up-to-date many-body theory of nuclear matter. Their results are in remarkable agreement with the ones by Gandolfi et al. (2012) using an approach completely different from the one adopted by Hebeler et al. (2013). At such density, NS matter in beta equilibrium is expected to be somewhat softer than the pure neutron matter one. Hebeler et al. (2013) calculate the effect of the presence of an admixture of protons and electrons in beta equilibrium on the EOS combining the EOS of neutron matter and available semi-empirical information about nuclear symmetry energy and its density dependence (slope parameter \( \nu_L \)). Interpolating between the values in their Table 5, we conclude that Hebeler et al. (2013) provide the following constraint on the pressure of NS matter at \( n_0 \):

\[ 2.7 < P_{33}(n_0) < 4.4 \]

This constraint is satisfied by EOS.N_{\text{ref}}. On the contrary, it is badly violated by EOSs from EOS.H, which give \( P_{33}(n_0) \) significantly larger than the upper bound in Eq. 2. Before considering consequences of the "overpressure" of the nucleon (pre-hyperon) segment of EOS.H for NS radii, we discuss two different parametrizations of NS models.

4. Two densities and two parametrizations of neutron star models

When investigating the EOS of NS matter, we have to consider two distinct densities, \( \rho = \rho^2 \) and \( n_0 \) (sect. 2). While \( \rho \) is the relevant quantity for GTR calculations of the NS structure, it is \( n_0 \) that is associated with an average distance between baryons (treated as point-like objects), \( r_0 \approx n_0^{-1/3} \). Therefore, knowing \( n_0 \), we can compare an actual \( r_0 \) with the average distance between nucleons in nuclear matter at normal nuclear density, \( r_0 \approx n_0^{-1/3} \). At subnuclear densities, \( \rho \) of NS matter can be very well approximated by \( n_0 m_n \), where \( m_n \) is neutron rest mass. However, at supranuclear densities \( \rho \) grows non-linearly with \( n_0 \). This non-linear dependence is model-dependent, see Fig. 1, and determines actually the EOS, see Eq. 3.

In Fig. 2, we plot the relations between \( M \) and the central baryon density \( n_0 \) for non-rotating NS models. Several conclusions result from this figure. First, the central density in a 2 \( M_\odot \) NS is typically \( \rho_\text{c} \approx 4 \). At the star’s center \( r_0/r_\text{c} \approx 0.6 \). Second, the N segment of EOS.H corresponding to \( 1 < \rho < 2 \) is so much stiffer than a similar segment of the EOS. N_{\text{ref}} one, that \( M(H)(\rho_\text{c} = 2) \approx 2 M(N)(\rho_\text{c} = 2) \). In other words, in order to yield \( M_{\text{max}} > 2.0 \, M_\odot \) despite of the hyperon softening, the pre-hyperon (nucleon) segment 1 < \( \rho_\text{c} < 2 \) of EOS.H has to be very stiff.

\[ M(\rho_\text{c}) \] curves are shown in Fig. 3. One notices that \( \rho_\text{c} \) in 2 \( M_\odot \) stars can be as high as 6 - 7, significantly larger than the corresponding values of \( \rho_\text{c} \). For the \( M_{\text{max}} \) configurations the difference is even larger. However, let us remind that it is \( n_0 \) and not \( \rho_\text{c} \) that determines the mean inter-baryon distance at the center of the star.

5. Radii of neutron stars with hyperon cores

\( R \) is a measurable NS parameter and therefore large radii of NS with hyperon cores could be subject to an observational test. The \( M(R) \) lines for selected EOSs from EOS.H
Table 1. Equations of state. For EOS.N_ref (upper part) we selected three widely used EOS which produce standard values of NS parameters. Lower part of the table contains our EOS.H set. For further explanations see the text.

| EOS     | theory                                      | reference                      |
|---------|---------------------------------------------|--------------------------------|
| APR     | Variational, infinite chain summations      | Akmal et al. (1998)            |
| DH      | energy-density functional, Skyrme type      | Douchin & Haensel (2001)       |
| BSk20   | energy-density functional, Skyrme type      | Fantina et al. (2013)          |
| BM165   | RMF, constant couplings, SU(6)              | Bednarek et al. (2012)         |
| DS08    | RMF, constant couplings, SU(6)              | Dexheimer & Schramm (2008)     |
| GM1Z0   | RMF, constant couplings, SU(6) broken       | Weissenborn et al. (2012b)     |
| M.CQMCC | RMF, constant couplings, SU(3)              | Miyatsu et al. (2013)          |
| SA.BSR2 | RMF, constant couplings, SU(6)              | Sulaksono & Agrawal (2012)     |
| SA.TM1  | RMF, constant couplings, SU(6) broken       | Sulaksono & Agrawal (2012)     |
| G.TM1   | RMF, constant couplings, SU(6)              | Gusakov et al. (2014)          |
| M.TM1C  | RMF, constant couplings, SU(3)              | Miyatsu et al. (2013)          |
| SA.NL3  | RMF, constant couplings, SU(6)              | Sulaksono & Agrawal (2012)     |
| M.NL3B  | RMF, constant couplings, SU(6)              | Miyatsu et al. (2013)          |
| M.GM1C  | RMF, constant couplings, SU(3)              | Miyatsu et al. (2013)          |
| SA.GM1  | RMF, constant couplings, SU(6)              | Sulaksono & Agrawal (2012)     |
| UU1     | RMF, density-dependent couplings, SU(6)     | Uechi & Uechi (2009)           |
| UU2     | RMF, density-dependent couplings, SU(6)     | Uechi & Uechi (2009)           |

are plotted in Fig. 4. By construction, selected EOS.Hs include those producing an envelope of a complete H-bundle of $M(R)$ curves.

In the mass range $1 < M/M_\odot < 1.6$, the H-bundle is centered around $\sim 14.5$ km. There is a wide $> 1$ km gap between the H and N_ref bundles in this mass range. More specifically, in the considered mass range, we find a lower bound $R^{(H)} > 13$ km. In Fig. 4 we plot the points calculated for EOS.H in the $P_{33}(n_s) - R_{1.4}$ plane, where $R_{1.4} \equiv R(1.4 M_\odot)$. Large values of $R^{(H)}_{1.4}$ are correlated with a large $P(n_s)$ violating the upper bound of [Hebeler et al. (2013)]. Let us consider now the largest, up to date, measured pulsar mass, $2.0 M_\odot$, and EOS-dependent (theoretical) maximum allowable mass $M_{\text{max}}$. The EOS-dependent radius of $2.0 M_\odot$ star and the radius at $M_{\text{max}}$ are denoted by $R_{2.0}$ and $R_{M_{\text{max}}}$, respectively. Calculated points in the $R_{M_{\text{max}}} - R_{2.0}$ plane are shown in Fig. 6. Those for EOS.N_ref are tightly grouped (within less than 1 km) around $R_{2.0} = 11$ km. The points calculated for EOS.H are loosely distributed along the diagonal of the bounding box, with $R^{(H)}_{M_{\text{max}}}$ ranging within 10.5-12.5 km, and $R^{(H)}_{2.0}$ within 11.5-14 km.

6. Mass vs. central pressure for neutron stars with hyperon cores

The difference between $M - R$ relations for EOS.N_ref and EOS.H and in particular an $R$-gap for $1.0 < M/M_\odot < 1.6$ reflects a difference in the pressure distributions within NS models of a given $M$. This is visualized in Fig. 4 where we show $M(P_c)$ plots for EOS.N_ref and EOS.H families. For $M \lesssim 1.6 M_\odot$ we find that $M = M(P_c)$ is well approximated by

$$M/M_\odot \simeq A(P_{c,34})^\beta,$$

(3)
Fig. 2. (Color online) Gravitational mass $M$ vs. central baryon density $n_c$ for non-rotating NS models based on the sets EOS.H (blue lines - H) and EOS.N$_\text{ref}$ (black lines - N$_\text{ref}$). In N$_\text{ref}$: A - APR, B - BSk20, C - DH; in H: a - SA.BSR2, b - BM165, c - GM1Z0, d - UU1, e - G.TM1C. EOS labels from Table 1. Solid lines: stable NS configurations. Dotted lines: configurations unstable with respect to small radial perturbations. Vertical lines crossing the $M(n_c)$ curves indicate configurations with $n_c/n_0 = 2, 3, \ldots$. Hatched strip correspond to $M = 1.4 \pm 0.05$ M$_\odot$, and the observational constraints for J1614-2230 and J0348+0432 are marked in blue and magenta, respectively (1-$\sigma$ errors).

where $P_{34} = P/10^{34}$ erg cm$^{-3}$ and $\beta_H \approx 0.52$, while prefactor $A_N = 0.36$ for the N$_\text{ref}$-bundle is significantly smaller than $A_H = 0.52$ for the H-one.

There is a sizable gap between $M^{(N)}(P_c)$ and $M^{(H)}(P_c)$ bundles. The $M$-gap ranges from $\sim 0.2$ M$_\odot$ at $P_{c,34} = 3$ to $\sim 0.7$ M$_\odot$ at $P_{c,34} = 8$. The fit $M \propto P_c^\beta$ with $\beta = \text{const.}$ breaks down for the H-family for $M > 1.6$ M$_\odot$ due to the hyperon softening of the EOS. For $M \sim 2$ M$_\odot$ (not shown in Fig. 7) the gap between the N$_\text{ref}$ and H bundles disappears.

7. On the causal bounds on $R(M)$

The radius of a neutron star of given mass $M$, based on a causal EOS, cannot exceed a limit which is calculated by replacing this EOS above some "fiducial density" $n_*$ by the causal-limit (speed of sound = c) continuation. The causal-limit (CL) segment of the EOS.CL is:

$$P^{(\text{CL})} = P_* + c^2 (\rho - \rho_*)$$

for $n_b \geq n_*$, whereas $P_*$ and $\rho_*$ are calculated from the original EOS, $P_* = P(n_*)$, $\rho_* = \rho(n_*)$.

We choose $n_* = n_b$. Our choice is more conservative, for the reasons explained later in this section, than $1.8n_0$ used by Hebeler et al. (2013). We fix the model for the crust segment of all EOS, i.e., for $n_b < 0.5n_0$, we use the EOS described in the final fragment of Sect. 2.1.

In Fig. 8 we show $M(R)$ curves for selected models from EOS.N$_\text{ref}$ and EOS.H (DH and DS08 EOS, respectively), together with their causal-limit (CL) bounds. The CL-curves bifurcate from the actual $M(R)$ curves at $n_c = n_*$, corresponding to $M = M_*$. For a given $M > M_*$, the CL-bound is denoted by $R_{\text{max}}(M)$.

Consider first $\Pi_* = 1$. We get $M_*^{(N)} \approx 0.2$ M$_\odot$ and $M_*^{(H)} \approx 0.5$ M$_\odot$. As $P_*^{(H)}$ is significantly larger than $P_*^{(N)}$.

Fig. 3. (Color online) Same as in Fig. 2 but with $n_c$ replaced by $\rho_c$.

Fig. 4. (Color online) Gravitational mass $M$ vs. circumferential radius $R$ for non-rotating NS models. For the labels, see details in the caption of figure 2.
Fig. 5. (Color online) Pressure at nuclear density vs. NS radius for 1.4 $M_\odot$ for EOS.H and EOS.N$_{\text{ref}}$ (See table 1 for the labels). Grey horizontal strip - range of $P(n_0)$ determined by Hebeler et al. (2013). Circles correspond to EOS with the TM1, squares with NL3, pentagons with GM1 model for the nucleon sector, respectively. Triangles indicate EOS built with RMF models with density-dependent couplings.

Fig. 6. (Color online) NS radius for 2.0 $M_\odot$ vs. radius at maximum allowable mass. EOS labels from Table 1.

Fig. 7. (Color online) Dependence of $M$ on the central pressure, for non-rotating NS models, for $M < 1.6 M_\odot$. Dashed and dashed-dotted lines describe the $M/M_\odot = A(P_{34})^\beta$ fits for N and H families, respectively. For details, see the text.

Fig. 8. (Color online) Solid lines: mass-radius relations for an EOS.N(DH) and an EOS.H (DS08), respectively. Long-dash line: causal-limit upper bound on $R_M$ for these EOS, assuming $n_\ast = n_0$. Dotted line: similar upper bound on $R_M$, but with $n_\ast = 1.8 n_0$, as in Hebeler et al. (2013).

Fig. 9. (Color online) $M(R_{eq})$ for two selected EOS: N (DH) and H (DS08). $R_{eq}$ is defined in the text. Solid curves: non-rotating configurations. Dashed lines: configurations rotating rigidly at $f = 300$ and 600 Hz.

$R_{\text{max}}^{(H)}(M)$ is larger than $R_{\text{max}}^{(N)}(M)$, e.g. by about 1 km at 1.4 $M_\odot$. The difference decreases to 0.5 km at 2 $M_\odot$.

Consider now $n_\ast = 1.8$ chosen by Hebeler et al. (2013). We get then $M_\ast^{(N)} \approx 0.5 M_\odot$ and $M_\ast^{(H)} \approx 1.4 M_\odot$. At 1.4 $M_\odot$ the actual $R_\ast^{(H)}(M)$ is larger than $R_{\text{max}}^{(N)}(M)$ by about 1.4 km. The H configuration there violates the condition $P_c \gg P_{\text{(H)}}$ needed for a weak dependence of $R_{\text{max}}(M)$ on the EOS below $n_\ast$. Therefore, a CL-bound derived for EOS.N$_{\text{ref}}$ should not be applied to EOS.H stars. Even at 2 $M_\odot$ the CL-bound for H is larger by 1 km than the
8. Effect of neutron star rotation

For stationary rigidly rotating configurations, which are axially symmetric, we consider the circumferential equation for a given EOS. The rotational increase of the radius $R_{\text{eq}}$ is obtained from

$$R_{\text{eq}} = R_{\text{eq}}(f, M) - R(f = 0, M) \approx B(M) f^2 .$$

For the EOSs used in Fig. 9 the rotational increase of $R_{\text{eq}}$ is for EOS.H roughly twice as large as for EOS.N. For $M = 1.4 \, M_\odot$ we have $B_H = 3.1 \, \text{km/kHz}^2$ and $B_N = 1.6 \, \text{km/kHz}^2$.

For $1.4 \, M_\odot$ and at 600 Hz the value of $R_{\text{eq}}^{(H)}$ is by 10% larger than the static one. At a given $f$, $\Delta R_{\text{eq}}$ decreases with increasing $M$ and at 1.8 $M_\odot$ the rotational increase of $R_{\text{eq}}$ is 7%.

For NS masses between 1.2 $M_\odot$ and 1.8 $M_\odot$, the decrease of $B(M)$ with an increasing mass is approximately linear, from 2.0 km/kHz$^2$ to 1.2 km/kHz$^2$, and from 3.8 km/kHz$^2$ down to 2.2 km/kHz$^2$, for our selected EOS.N and EOS.H, respectively.

9. Observational determination of radii of neutron stars

The radius $R$ can be extracted from the analysis of X-ray spectra emitted by the NS atmosphere. Recent attempts are based on the modeling the X-ray emission from four different types of objects (see Potekhin 2014 for a review):

- isolated NSs (INSs);
- quiescent X-ray transients (QXTs) in Low-Mass X-Ray binaries, i.e. NSs in a binary system observed when the accretion of matter from their binary companion has stopped or is strongly reduced;
- bursting NSs (BNSs) i.e. NSs from which recurring and very powerful bursts, so-called photospheric radius expansion (PRE) bursts, are observed;
- rotation-powered radio millisecond pulsars (RP-MSPs).

The modelling of the X-ray emission and thus the radius determination strongly depend on the distance to the source, its magnetic field and the composition of its atmosphere. Note that even in the simplest case of non-rotating and non-magnetized NSs, due to the space-time curvature, the apparent radius $R_\infty$ that is constrained by the modeling actually depends on both the stellar radius and mass:

$$R_\infty = R/\sqrt{1 - 2GM/R^2} .$$

While the magnetic field and the chemical composition of the atmosphere of NSs are unknown and difficult to determine, QXTs have a low magnetic field, as a result of its decay due to the accretion of matter, and an atmosphere likely to be composed of light elements (H, possibly He; see Catuneanu et al. 2013, Heinke et al. 2014). In addition, QXTs in globular clusters are promising sources for the mass-radius determination since their distance are well-known. BNSs that also have low magnetic field and a light-element atmosphere are interesting sources, all the more if the distance to them is known. However the modeling of the PRE bursts is still subject to uncertainties and debates (see Ponti et al. 2014, Steiner et al. 2013). Finally, constraints can be derived from the modeling of the shape of the X-ray pulses observed from RP-MSPs, in particular if the mass is known from radio observations (Bogdanov 2013).

Conflicting constraints on the radius from the X-ray emission of QXTs, BNSs and RP-MSPs have been obtained by different groups. In what follows, we restrict ourselves to the most recent publications, indicated in Table 2. Figure 10 shows the constraints with 2-σ error bars derived in these papers for the radius $R_{1.4}$ of a NS with a mass of 1.4 $M_\odot$.

Among the five QXTs studied in Guillot et al. (2013), the radius of one of them, in the core of the globular cluster NGC 6397, is low compared to the four other sources: $R_\infty \sim 11 \, \text{km}$ compared to $R_\infty \sim 12.5 \, \text{km}$ according to their table 6 for Run 7. As a consequence they derive a constraint suggesting a small NS radius: $R = 9.1^{+1.1}_{-1.2} \, \text{km}$ (90% confidence level). However Heinke et al. (2014), using new and archival X-ray data, re-performed an analysis of the spectrum of the QXT in NGC 6397 and argue that an helium atmosphere is favoured instead an hydrogen one such as the one used in Guillot et al. (2013) and obtain a larger radius $R_\infty \sim 12.5 \, \text{km}$. Owing to this uncertainty on the atmosphere composition and radius of the NS in NGC 6397 we adopt the constraint derived in Guillot et al. (2013) not including it: $R = 10.7^{+1.1}_{-1.2} \, \text{km}$, labelled GS13m in Table 2.

For all sources in table 2 if no available, constraints on $R_{1.4}$ with a 2-σ error bars were derived by assuming a Gaussian distribution for the radius, so that the confrontation between the results is made easier. The constraint from Güver & Özel (2013) is extracted from their figure 3.

Formally, at 2-σ level, all constraints on $R_{1.4}$ but the one by Ponti et al. (2014) are consistent with one another and give the radius of $\sim 12.8 \, \text{km}$ for $M = 1.4 \, M_\odot$. All present EOS.H have a substantially larger radius $R_{1.4} \geq 13.51 \, \text{km}$ as indicated in table A and figure E and thus violate this upper bound. They are consequently consistent only with the lower bounds on $R_{1.4}$ obtained by Ponti et al. (2013) and Bogdanov (2013).

However note that, the determination of the radius of NSs is subject to many assumptions, uncertainties and systematics effects, (see e.g., Table 1 in Potekhin 2014). In addition, the inclusion of rotation strongly complicates the analysis of the collected X-ray spectra from both QXTs and BNSs which are likely to rotate at a frequency of few hundred Hz and is expected to affect the radius determination by $\sim 10\%$ according to Ponti et al. (2014).
The observational constraint that \( M_{\text{max}} > 2 M_\odot \) for NS with hyperon cores imposes a fine tuning of the dense matter model which has to be consistent at the same time with semi-empirical nuclear and hyper-nuclear parameters. With one very recent exception (Yamamoto et al. 2014), that will be discussed separately, only specific types of nonlinear RMF models are able to satisfy this constraint. The EOSs of NS matter for these models, forming the set EOS.H, have following features described below.

**Overpressure at \( n_0 \).** Pressures \( P^{(H)}(n_0) \) are significantly higher than the robust upper bound obtained by Hebeler et al. (2013). Introducing a density-dependence for the coupling constants in the RMF Lagrangian does not help in this respect.

**Large radii.** For \( 1.0 < M/M_\odot < 1.6 \) one gets \( 13 < R^{(H)}/\text{km} < 15 \). Such radii are consistent at 2-\( \sigma \) confidence level only with two lower bounds out of the five most recent constraints, derived by analyzing and modeling the X-ray emission from NS in quiescent LMXB (from the ones exhibiting photospheric expansion bursts) and from radio millisecond pulsars. Future simultaneous determination of \( M \) and \( R \) by analysing the X-ray spectra with a \( \sim 5\% \) precision thanks to the forthcoming NICER (Gendreau et al. 2012), Athena+ (Motch et al. 2013) and possibly LOFT (Feroci et al. 2013) missions, could either rule out hyperon cores in NS or keep the possibility of sizable hyperon cores still open.

Overpressure at \( n_0 \) and large radii are likely to be inter-related. To get \( M_{\text{max}} > 2 M_\odot \), an EOS.H should necessarily have unusually stiff pre-hyperon segment, and this results in large radii for \( M \lesssim 1.6 M_\odot \). However, in our opinion the upper bound on \( P(n_0) \) by Hebeler et al. (2013) is sufficiently robust to be respected. Therefore, we propose to include this constraint in the procedure of determination of the RMF Lagrangian coupling constants, in addition to the standard fitting four parameters \( E_\infty, n_0, K_\text{sym}, K_s \). In the weakest form, this constraint would read

\[
P^{(\text{PNM})}(n_0) \leq P^{(\text{Heb})}_{\text{max}}(n_0) = 5.4 \times 10^{33} \text{ dyn cm}^{-2},
\]

where \( P^{(\text{PNM})} \) is the pressure of pure neutron matter (PNM) calculated for a given RMF model, and \( P^{(\text{Heb})}_{\text{max}}(n_0) \) is the upper bound to this pressure obtained by Hebeler et al. (2013). We prefer to use constraint (6) instead of fitting the very uncertain \( L_s \) referring to weakly asymmetric nuclear matter. The possibility of imposing both the PNM constraint (6) and getting \( M_{\text{max}} > 2 M_\odot \) to narrow the EOS.H family of RMF models is now being studied by the authors.

A very recent model of EOS.H fulfilling \( M_{\text{max}} > 2 M_\odot \) (Yamamoto et al. 2014) deserves a separate discussion. This model does not rely on the RMF approximation but is calculated using the G-matrix theory. A crucial new element is a strong three- and four-baryon repulsion resulting from the multi-pomeron exchanges between baryons. The many-body theory is applied to a number of terrestrial nuclear and hyper-nuclear data which is sufficient to fit three sets of parameters of the models. The set MPa yields the stiffest hyperon NS-cores and is the only one satisfying \( M_{\text{max}} > 2 M_\odot \). The MPa curves in figures 8, 9, 11 of Yamamoto et al. (2014) indicate that the MPa EOS has similar basic features as those characteristic of the successful RMF models of our EOS.H set. Namely, \( R^{(\text{MPa})} \gtrsim 13.5 \text{ km for } 1.2 < M/M_\odot < 1.6 \) and \( M^{(\text{MPa})}(\tau = 2) \approx 1.3 M_\odot \), which is twice the value characteristic of our EOS.N_{\text{ref}} set. This indicates that the pre-hyperon segment of the MPa EOS is very stiff. Concluding, these features of the MPa EOS are coinciding with those of our EOS.H set, and should be subject to the same tests.

As far as observations are concerned, owing to the current large uncertainties on the radius determination that exist due to assumptions in the models and systematic effects, no stringent conclusion on the radius of a 1.4 \( M_\odot \) NS can be derived.

### Table 2. Radius determination: most recent publications.

| Abbreviations | Objects | References |
|---------------|---------|------------|
| PN14          | BNS: 4U 1608-522 | (a) |
| GS13          | QXT: M28, NGC 6397, M13, ω Cen, NGC 630 | (b) |
| GO13m         | BNS: SAX J1748.9-2021 | (c,d) |
| SL13          | BNS: 4U 1608-522, KS 1731-260, EXO 1745-248, 4U 1820-30 | (e) |
|               | QXT: M13, 47 Tuc, NGC 6397, ω Cen | (f) |
| B13           | RP-MSP: PSR J0437-4715 | (f,g) |

Fig. 10. Observational constraints on the radius \( R_{1.4} \) of a 1.4 \( M_\odot \) NS from the most recent publications. See table 2 for the labels and text for details. 2-\( \sigma \) error-bars are plotted.

### 10. Summary, discussion, and conclusions

The observational constraint that \( M_{\text{max}} > 2 M_\odot \) for NS with hyperon cores imposes a fine tuning of the dense matter model which has to be consistent at the same time with semi-empirical nuclear and hyper-nuclear parameters. With one very recent exception (Yamamoto et al. 2014), that will be discussed separately, only specific types of nonlinear RMF models are able to satisfy this constraint. The EOSs of NS matter for these models, forming the set EOS.H, have following features described below.

| Abbreviations | Objects | References |
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|               | QXT: M13, 47 Tuc, NGC 6397, ω Cen | (f) |
| B13           | RP-MSP: PSR J0437-4715 | (f,g) |
In our opinion, a robust observational upper bound on $R$ will become available only with advent of high-precision X-ray astronomy, like that promised by the NICER, Athena+ and LOFT projects. A simultaneous measurement of $M$ and $R$ within a few percent error is expected to be achieved, and then used in combination with a maximum measured pulsar mass (at present $2.01 \pm 0.04 \, M_{\odot}$) as a robust criterion in our quest for unveiling the structure of neutron star cores.

Acknowledgements. We thank V. Dexheimer, M. Oertel, A. Sulaksono, H. Uechi, and Y. Yamamoto for providing us with the EOS tables. We are grateful to H. Uechi and Y. Yamamoto for helpful comments concerning their EOS. Correspondence with N. Chamel and A. Fantina about the BSk EOSs was very helpful. We are grateful to J.M. Lattimer for his comments after a talk of one of the authors (PH) during the EMMI meeting at FIAS (Frankfurt, Germany, November 2013). One of the authors (MF) was supported by the French-Polish NCN grant no 2011/01/B/ST9/04838. We thank V. Dexheimer, M. Oertel, A. Sulaksono, H. Uechi, and Y. Yamamoto for providing us with the EOS tables. We are grateful to H. Uechi and Y. Yamamoto for helpful comments concerning their EOS. Correspondence with N. Chamel and A. Fantina about the BSk EOSs was very helpful. We are grateful to J.M. Lattimer for his comments after a talk of one of the authors (PH) during the EMMI meeting at FIAS (Frankfurt, Germany, November 2013). One of the authors (MF) was supported by the French-Polish NCN grant no 2011/01/B/ST9/04838. We thank V. Dexheimer, M. Oertel, A. Sulaksono, H. Uechi, and Y. Yamamoto for providing us with the EOS tables. We are grateful to H. Uechi and Y. Yamamoto for helpful comments concerning their EOS. Correspondence with N. Chamel and A. Fantina about the BSk EOSs was very helpful. We are grateful to J.M. Lattimer for his comments after a talk of one of the authors (PH) during the EMMI meeting at FIAS (Frankfurt, Germany, November 2013). One of the authors (MF) was supported by the French-Polish NCN grant no 2011/01/B/ST9/04838.

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Table A.1. Parameters of the EOS and of NS models based on them.

| EOS         | $P(n_0)$ ($10^{33}$ dyn cm$^{-2}$) | $\rho(n_0)$ ($10^{14}$ g cm$^{-3}$) | $R_{1.4}$ (km) | $R_{1.4}^{(\text{CL})}$ (km) | $L_s$ (MeV) | $R_{M_{\text{max}}}$ (km) | $M_{\text{max}}$ (M$_\odot$) |
|-------------|-----------------------------------|-------------------------------------|----------------|-----------------------------|-------------|--------------------------|-------------------------------|
| APR         | 3.05                              | 2.72                                | 15.01          | 11.34                       | 59          | 9.93                     | 2.19                          |
| BSk20       | 3.20                              | 2.72                                | 14.95          | 11.75                       | 37          | 10.18                    | 2.17                          |
| DH          | 3.60                              | 2.72                                | 15.03          | 11.73                       | 46          | 9.99                     | 2.05                          |
| BM165       | 6.45                              | 2.74                                | 15.46          | 13.59                       | 74          | 10.68                    | 2.03                          |
| DS08        | 7.58                              | 2.74                                | 15.52          | 13.91                       | 88          | 12.02                    | 2.05                          |
| GM1Z0       | 7.45                              | 2.72                                | 15.51          | 13.95                       | 94          | 12.05                    | 2.29                          |
| M.CQMCC     | 7.47                              | 2.73                                | 15.61          | 13.97                       | 91          | 12.12                    | 2.08                          |
| SA.BSR2     | 5.60                              | 2.70                                | 15.40          | 13.51                       | 62          | 11.65                    | 2.03                          |
| SA.TM1      | 9.58                              | 2.82                                | 16.35          | 14.86                       | 110         | 12.52                    | 2.10                          |
| G.TM1       | 8.78                              | 2.75                                | 15.91          | 14.51                       | 110         | 12.51                    | 2.06                          |
| M.TM1C      | 8.77                              | 2.74                                | 15.94          | 14.57                       | 111         | 12.61                    | 2.03                          |
| SA.NL3      | 8.91                              | 2.72                                | 16.14          | 15.02                       | 118         | 12.83                    | 2.32                          |
| M.NL3B      | 8.97                              | 2.74                                | 15.98          | 14.92                       | 118         | 13.18                    | 2.07                          |
| M.GM1C      | 7.45                              | 2.72                                | 15.61          | 14.06                       | 94          | 12.28                    | 2.14                          |
| SA.GM1      | 7.41                              | 2.71                                | 15.64          | 14.03                       | 94          | 11.98                    | 2.02                          |
| UU1         | 9.95                              | 2.72                                | 15.78          | 15.04                       | 117         | 11.97                    | 2.21                          |
| UU2         | 10.09                             | 2.73                                | 15.79          | 13.81                       | 117         | 10.98                    | 2.12                          |