$\alpha_S$ from LEP

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Abstract. Recent results on measurements of the strong coupling $\alpha_S$ from LEP are reported. These include analyses of the 4-jet rate using the Durham or Cambridge algorithm, of hadronic $Z^0$ decays with hard final state photon radiation, of scaling violations of the fragmentation function, of the longitudinal cross section, of the $Z^0$ lineshape and of hadronic $\tau$ lepton decays.

1. Introduction
Determinations of the strong coupling constant $\alpha_S$ using data from the LEP experiments ALEPH, DELPHI, L3 and OPAL are among the most precise and least ambiguous measurements [1]. The initial state is purely leptonic avoiding initial and final state interference, the data samples are generally large and the experiments have large acceptances, good resolutions and low backgrounds resulting in small experimental corrections. When the re-analysed data from the PETRA experiment JADE are included a centre-of-mass (cms) energy range of more than an order of magnitude is covered by many sets of measurements.

The LEP program ran from 1989 to 2000 and covered cms energies from 89 to 209 GeV. The samples of hadronic events in $e^+e^-$ annihilation contain $O(10^5-6)$ events near $m_{Z^0}$ (LEP 1), $O(10^{2-3})$ events at higher energies (LEP 2) and $O(10^{3-4})$ events below $m_{Z^0}$ from JADE between 14 and 44 GeV cms energy.

2. 4-jet rate
In the analyses [2] [3] [4] [5] hadronic final states are clustered using the Durham algorithm, which defines $y_{ij} = 2 \min^2(E_i, E_j)(1 - \cos \theta_{ij})/E_{\text{vis}}^2$, as the phase space distance between two particles $i, j$ with energies $E_i$ and $E_j$ with $E_{\text{vis}} = \sum_k E_k$. The pair with the smallest $y_{ij}$ is combined by adding the four-vectors. The procedure is repeated until all $y_{ij} < y_{\text{cut}}$. The fraction of four-jet events $R_4(y_{\text{cut}})$ is studied as a function of $y_{\text{cut}}$. DELPHI uses the Cambridge variant of the Durham algorithm. Since a four-jet final state corresponds to at least four partons in QCD the prediction is $O(\alpha_S^2)$ in leading order (LO). The relative error $\Delta \alpha_S/\alpha_S = \Delta R_4/(2R_4)$, i.e. a precise measurement of $\alpha_S$ is possible if experimental uncertainties can be kept under control. The data, corrected for experimental effects, are compared with NLO calculations combined with resummed next-to-leading logarithm (NLLA) terms (DELPHI use NLO only) including hadronisation corrections derived from simulation. The results are summarised in figure 1. The average value $\alpha_S(m_{Z^0}) = 0.1172 \pm 0.0010(\text{exp.}) \pm 0.0016(\text{had.}) \pm 0.0014(\text{theo.})$ was found assuming partially correlated experimental uncertainties while hadronisation and theory uncertainties were evaluated by repeating the procedure with simultaneously changed input values. Since the dependency of $\alpha_S$ on the renormalisation scale (RS) in the theory is at a
minimum in the analyses the conventional estimation of the theoretical uncertainty by variation of the RS may yield an underestimate [4, 5].

Figure 1. Results for $\alpha_S(m_{Z^0})$ from the four-jet rate $R_4$ [2, 3, 4, 5]. The dashed vertical line and shaded band indicate the average value with errors shown on the figure.

3. Radiative $Z^0$ decays
Hadronic decays of the $Z^0$ with energetic and isolated photons can be used to study QCD at lower effective cms energies $\sqrt{s}$ [6, 7] assuming that the effects of wide-angle and hard photon radiation and of QCD processes can be factorised. The recent OPAL analysis [6] shows a successful comparison of event shape distributions using simulated $Z^0$ decays with hard and isolated photons and using events generated at lower cms energy. The background from events with isolated neutral hadrons misidentified as photons has to be calculated from data since the rates of isolated neutral hadrons in the simulations do not agree with measurements. The events are binned according to the photon energy corresponding to $24 \leq \sqrt{s} \leq 78$ GeV. The event shape analysis uses NLO+NLLA QCD calculations including hadronisation corrections predicted by simulations; the fitted theory is found to be consistent with the data. Within the uncertainties of the analysis assuming factorisation of hard and isolated photon radiation and QCD radiation leads to a good description of the data.

4. Fragmentation
In studies of fragmentation in $e^+e^-$ annihilation to hadrons production of charged particles is measured. The distributions of $x = 2p/\sqrt{s}$, where $p$ is the particle momentum, are independent of $\sqrt{s}$ in the static quark-parton model. Dependence on $\sqrt{s}$ is referred to as scaling violation and can be described by NLO QCD calculations. In [1] the results from ALEPH, DELPHI and OPAL are averaged giving $\alpha_S(m_{Z^0}) = 0.119 \pm 0.009$. The distribution of angles between charged particles and the beam direction allows to extract the fraction of events where the intermediate
gauge boson appears with longitudinal polarisation, the so-called longitudinal cross section $\sigma_L$. Since longitudinal polarisation is only possible with an additional gluon in the final state $\sigma_L$ can be used to measure $\alpha_S$. In [1] measurements of $\sigma_L$ by ALEPH, DELPHI and OPAL are averaged with the result $\sigma_L/\sigma_{tot.} = 0.056 \pm 0.002$. This yields $\alpha_S(m_{Z^0}) = 0.117 \pm 0.008$ using a NLO QCD calculation. The average for $\alpha_S(m_{Z^0})$ from fragmentation analyses in $e^+e^-$ annihilation from [1] is $\alpha_S(m_{Z^0}) = 0.118 \pm 0.004(\text{exp.}) \pm 0.001(\text{had.}) \pm 0.007(\text{theo.})$.

5. $Z^0$ lineshape and hadronic $\tau$ decays
In [1] the results of the $Z^0$ lineshape measurements [8] were used to extract measurements of $\alpha_S(m_{Z^0})$ from four sensitive observables: the hadronic width $\Gamma_h$, the ratio of hadronic and leptonic widths $R_Z$, and the total peak cross sections for hadron and lepton production $\sigma_h$ and $\sigma_L$. The dependence on the RS [9] and the Higgs mass are studied. The result based on NNLO QCD calculations is $\alpha_S(m_{Z^0}) = 0.1189 \pm 0.0027(\text{exp.}) \pm 0.0015(\text{theo.})$.

The analysis of the hadronic branching ratio $R_\tau$ of the $\tau$ lepton in [10] uses NNLO QCD calculations. Contributions from non-perturbative processes are estimated using the spectral functions $dR_\tau/ds$ for final states with even or odd numbers of pions and turn out to be consistent with zero for $R_\tau$. The result is $\alpha_S(m_{Z^0}) = 0.1221 \pm 0.0006(\text{exp.}) \pm 0.0004(\text{had.}) \pm 0.0019(\text{theo.})$. A similar analysis [11] uses partially calculated NNLO terms in addition and is consistent.

In both cases the theoretical uncertainty is determined by varying the renormalisation scale between 0.5 and 2.0 of the central value such that the theory errors can be compared directly with other results.

6. Summary
The most reliable measurements of $\alpha_S$ are those based on NNLO calculations and with small hadronisation uncertainties and thus small model dependence. The average of the results from section 5 with a related measurement using the total cross section for hadron production in $e^+e^-$ annihilation at low $\sqrt{s}$ from [1] is $\alpha_S(m_{Z^0}) = 0.1211 \pm 0.0010(\text{exp.}) \pm 0.0018(\text{theo.})$. All other measurements discussed here as well as recent determinations of world average values [12, 13, 14] are consistent with this value. The results are summarised in figure 2 adapted from [1].

Further progress can be expected from the use of NNLO calculations in the analysis of event shape observables [15, 16] and from improved analysis of fragmentation data with NNLO results [17].

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