Analytic formula for the dynamics around inflation end and implications on primordial gravitational waves

Asuka Ito, Jiro Soda, and Masahide Yamaguchi

1 Department of Physics, Kobe University, Kobe 657-8501, Japan
2 Department of Physics, Tokyo Institute of Technology, Tokyo 152-8551, Japan

We argue that primordial gravitational waves have a spectral break and its information is quite useful for exploring the early universe. Indeed, such a spectral break can be a fingerprint of the end of inflation, and the amplitude and the frequency at the break can tell us the energy scale of inflation and the reheating temperature simultaneously. In order to investigate the spectral break, we give an analytic formula for evolution of the Hubble parameter around the end of inflation where the slow roll approximation breaks down. We also evaluate the spectrum of primordial gravitational waves around the break point semi-analytically using the analytic formula for the inflation dynamics.

I. INTRODUCTION

Inflation, the accelerated expansion in the early universe, generates primordial density and tensor perturbations as well as solving the flatness and the horizon problems [1-3]. The experiments of cosmic microwave background (CMB) anisotropies have directly observed primordial density perturbations and their amplitudes were shown to be $\sim 10^{-5}$ on large scales [4, 5], which yield important information on the dynamics of inflation, that is, $\delta \rho/\rho \sim H^2/\dot{\phi}$ [6, 7]. Unfortunately, this quantity depends on the velocity of an inflaton as well as on the Hubble parameter during inflation, which determines the energy scale of inflation. Thus, the detection of primordial tensor perturbations, whose amplitudes are solely given by the Hubble parameter [8, 9], is pressing. The primordial tensor perturbations (gravitational waves) can be probed in two ways. One way is to observe them through the B-mode polarizations of the CMB indirectly. The other way is to directly detect them through, for example, interferometer experiments. In fact, recent observations of the gravitational waves sourced by binary blackholes through the LIGO/VIRGO collaborations open gravitational wave astronomy [12]. There are several ongoing and planned experiments to directly detect the primordial gravitational waves.

The primordial gravitational waves produced during inflation have fruitful information on the dynamics of inflation. Since they have almost scale invariant spectrum on horizon exit in general, their amplitudes give us the energy scale of inflation. While they are frozen and their amplitudes are kept on superhorizon scales, they behave as damped oscillations on subhorizon scales. Thus, the (would-be) observed spectrum of primordial gravitational waves depend on cosmic history much. That is, even if they were exactly scale invariant when they exited the horizon, the (would-be) observed one can be scale-dependent. By use of this kind of scale dependence, one can probe the change of the number of relativistic degrees of freedom [13, 14], the reheating temperature of the universe [15, 16], for example.

An important feature in the spectrum of primordial gravitational waves is a break due to the end of inflation. Indeed, we will show that the frequency and the amplitude at the break tells us the energy scale of inflation and the reheating temperature simultaneously. So far, very rough analytic estimates or detailed numerical calculations have been done to clarify the spectral shape of the (would-be) observed primordial gravitational waves around the break point. However, in order to properly incorporate the history of reheating in the analysis of the spectrum, it is desired to have a more accurate analytic formula. For example, Ema et al. recently pointed out that, even after inflation end, primordial gravitational waves might be produced as a result of inflaton annihilation into graviton pairs [18]. Thus, in order to smoothly connect the spectral shape at the frequency which exits the horizon just at the end of inflation, we need the detailed information around this frequency. For this purpose, we will derive an analytic formula to approximate the dynamics of inflation around its end, which also yields a semi-analytic formula of the spectral of the primordial gravitational waves around the frequency which exits the horizon at the end of inflation. It should be noted that, typically, the frequency of the spectral break is around $10^7$ Hz (see Eq. (2)). In this frequency range, new gravitational wave detectors are being proposed and developed intensively [19, 20]. Therefore, our study is also important for the future gravitational wave experiments.

The paper is organized as follows. In Sec.II after giving the basic expression of the amplitude of primordial gravitational waves generated during inflation, we explain how to determine the Hubble parameter and the reheating temperature from the information of primordial gravitational waves on the break scale. In Sec.III an analytic formula for evolution of the Hubble parameter around the end of inflation is derived. In Sec.IV we first discuss the spectral index of the spectrum at the end of inflation. An implication to reconstruct the inflaton potential from the observation of the spectral index with our formula is given. We also semi-analytically give
an example of the spectrum around the break point at present. Final section is devoted to conclusion.

II. THE SPECTRAL BREAK

Inflation predicts a nearly scale invariant spectrum of primordial gravitational waves [10, 11]. More explicitly, the dimensionless power spectrum is given by

$$P_k(k) = \frac{2}{\pi^2} \frac{H^2}{M_{Pl}^2} k = aH,$$

where $H$ is the Hubble parameter, $M_{Pl}$ is the reduced Planck mass, and $a$ is the scale factor. Here, we evaluated the power spectrum at the horizon crossing, $k = aH$. The amplitude of the power spectrum is determined by $H$ and nearly scale invariant because $H$ is almost constant during inflation. Therefore, if we observe the scale invariant spectrum of primordial gravitational waves, we see the energy scale of inflation through the parameter $H$. Indeed, upper bounds on the energy scale of inflation at the pivot scale is given by the observation of cosmic microwave background [5].

A feature in the primordial gravitational wave spectrum produced during inflation is the break around the end of inflation. Above the break frequency, production of primordial gravitational waves should be exponentially suppressed though Ema et al. recently pointed out that, even above such break frequency, the production of primordial gravitational waves might happen as a result of inflaton annihilation into graviton pairs [18]. Even in this case, this frequency still represents the break of the spectral shape and can become a fingerprint. The detection of the break of the spectrum would be a smoking gun proving the existence of inflation. Also, it would tells us the energy scale of inflation because the break frequency is determined by the Hubble parameter at the end of inflation:

$$f_* = \frac{H_{end}}{2\pi},$$

where $f_*$ and $H_{end}$ are the break frequency and the Hubble parameter at the end of inflation. After inflation, this cutoff frequency is red-shifted due to the expansion of the universe to the frequency $f_{break}$:

$$f_{break} = \frac{a_{end}}{a_0} f_*,$$

where $a_{end}$ and $a_0$ are the scale factor at the end of inflation and today, respectively.

In general, an inflaton field begins to oscillate around the bottom of the potential after inflation and particle production occurs [24, 25]. In this reheating phase, the evolution of the universe mimics that of matter dominated phase approximately [26]. As a result of (light) particle production during the reheating, radiation dominated phase follows. Therefore, the Hubble parameter evolves during the reheating as

$$\frac{H_{reh}}{H_{end}} = \left(\frac{a_{reh}}{a_{end}}\right)^{-3/2},$$

where $a_{reh}$ is the scale factor at the end of the reheating phase and thus at the beginning of the radiation dominated phase. The Hubble parameter at the end of the reheating phase can be parameterized by the reheating temperature, $T_{reh}$:

$$H_{reh}^2 = \frac{\pi^2 g(T_{reh}) T_{reh}^4}{90 M_{Pl}^2},$$

where $g(T)$ stands for effective degrees of freedom for the energy density at a temperature $T$ [26, 27]. From Eqs. (4) and (5), one can estimate the expansion rate $a_{end}/a_{reh}$ by using parameters, $H_{end}$, $T_{reh}$ and $g(T_{reh})$.

In order to estimate the ratio $a_{reh}/a_0$, we use the fact that the entropy $S$ conserves in an adiabatic universe:

$$S \propto g_s(T) a(T)^3 T^3 = \text{const.},$$

$g_s(T)$ represents effective degrees of freedom for the entropy at a temperature $T$ [26, 27].

Using Eqs. (2)-(5), we eventually obtain

$$f_{break} \simeq 1.4 \times 10^7 \text{ Hz} \left(\frac{H_{end}}{10^{-5} M_{Pl}}\right)^{1/3} \left(\frac{T_{reh}}{10^{13} \text{ GeV}}\right)^{1/3}.$$

Here we used the values for each parameter: $g_s(T_{reh}) = g(T_{reh})$, $g_s(T_0) \simeq 3.36$, $T_0 \simeq 2.73K$. Suppose that we observe gravitational waves at the break frequency. Then, because the amplitude also depends on $H_{end}$ and $T_{reh}$, it means we can determine not only the energy scale of inflation but also the reheating temperature simultaneously.

Finally, we mention that the maximum value of the break frequency is given by the case of the instantaneous reheating, i.e., $H_{end} = T_{reh}$, that is

$$f_{break} \simeq 1.0 \times 10^8 \text{ Hz} \left(\frac{200}{g(T_{reh})}\right)^{1/12} \left(\frac{H_{end}}{10^{-5} M_{Pl}}\right)^{1/2}.$$

III. HUBBLE PARAMETER AROUND THE BREAK

As mentioned in the section [11] the spectrum of primordial gravitational waves is determined by the Hubble parameter. However, gravitational waves around the break frequency [8] was produced just before the end of inflation where the slow roll approximation gets worse. Therefore, we need a prescription away from the slow roll approximation to discuss the spectrum of primordial gravitational waves around the break frequency. In this section we give an analytic expression for the Hubble parameter near the end of inflation.
Let us consider an inflationary universe which can be characterized by the scale factor $a(t)$ defined by

$$ds^2 = -dt^2 + a^2(t)dx^2 .$$  \hspace{1cm} (9)$$

Exponential expansion of the universe is driven by an inflaton $\phi(t)$ whose potential is $V(\phi)$. Then, one can obtain Einstein equations:

$$H^2 = \frac{1}{3M_{pl}^2} \left[ \frac{1}{2} \dot{\phi}^2 + V \right] ,$$ \hspace{1cm} (10)

$$\dot{H} = -\frac{1}{2M_{pl}^2} \phi^2 ,$$ \hspace{1cm} (11)

where a dot represents derivative with respect to $t$ and $H = \dot{a}/a$ is the Hubble parameter. It is more convenient to express the equations by taking $\phi$ as a time variable [28][29]. Dividing Eq. (11) by $\phi$ yields

$$H'(\phi) = -\frac{1}{2M_{pl}^2} \phi ,$$ \hspace{1cm} (12)

where a dash stands for derivative with respect to $\phi$. Note that we assume $\dot{\phi} > 0$ ($H' < 0$) throughout our discussion. Using Eq. (12) to eliminate $\phi$ in Eq. (10), one obtains

$$H^{(n)} = \frac{1}{2H(1)} \left[ -\sum_{r=1}^{n-2} \binom{n-1}{r} H^{(r+1)} H^{(n-r)} + \frac{3}{2M_{pl}^2} \sum_{r=0}^{n-1} \binom{n-1}{r} H^{(r)} H^{(n-r)} - \frac{1}{2M_{pl}^2} V^{(n)} \right] ,$$ \hspace{1cm} (15)

for $n \geq 3$ and $\binom{n}{r} = \frac{n!}{r!(n-r)!}$.

We now define the break point $\phi_0$ by the equation

$$\epsilon(\phi_0) \equiv 2M_{pl}^2 \frac{H'^2}{H^2} = 1 .$$ \hspace{1cm} (16)

On the break point, the Hubble parameter and the potential are linked,

$$H(\phi_0) = \sqrt{\frac{V(\phi_0)}{2M_{pl}^2}} .$$ \hspace{1cm} (17)

Then, we can expand the Hubble parameter:

$$H(\phi) = \sum_{n=0}^{\infty} \frac{1}{n!} H^{(n)}(\phi_0) (\phi - \phi_0)^n$$

$$= \sum_{n=0}^{\infty} \frac{1}{n!} H^{(n)}(\phi_0, \cdots, V^{(n-1)}(\phi_0))$$

$$\times (\phi - \phi_0)^n .$$ \hspace{1cm} (18)

We used Eqs. (15) and (17) to get the second equality. Eq. (18) shows that the evolution of the Hubble parameter, which is needed to calculate the spectrum of primordial gravitational waves, can be described by the inflaton potential and its derivatives, which are evaluated at $\phi_0$. In Fig. 1, we plot Eq. (18) (red line) at second order, $n = 2$, from the end of inflation with the exact numerical value (black dotted line) and results from the slow roll approximation [30] (blue, green, orange, pink lines). We see that our formula (18) to the second order expansion ($n = 2$) shows good agreement with the numerical one between, $\phi = -2M_{pl} \sim -M_{pl}$, where the slow roll approximation gets worse. It should be mentioned that the slow roll approximation seems to break down near the end of inflation, namely higher order corrections do not make the result better. Therefore, Eq. (18) is complementary to the slow roll approximation [30].

1. In [29], the Padé approximant was used and a better result was obtained compared with simple slow roll approximations. However, even in that case, the deviation is unavoidable around the end of inflation.
FIG. 1. The Hubble parameters near the end of inflation in the case of $V(\phi) \propto \phi^2$ are depicted. $\phi/M_{pl} = -1$ corresponds to the end of inflation where a slow roll parameter becomes unity. The red line represents Eq. (18) with $n = 2$ and expanded from the end of inflation, i.e., $\phi_0/M_{pl} = -1$. The black dotted line is a numerical solution of Eq. (13). The blue, green, orange, pink lines shows approximated solutions of 1st, 2nd, 3rd and 4th slow roll approximations, respectively [30].

IV. PRIMORDIAL GRAVITATIONAL WAVES AROUND THE BREAK

In the previous section, we gave an analytic expression (18) to describe the evolution of the Hubble parameter near the end of inflation. It can be used to derive the spectrum of primordial gravitational waves, which is expected to be given by Eq. (1). In this section, we first explicitly calculate the spectral index of the spectrum at the end of inflation. Next, we give a spectrum around the spectral break at present.

A. Spectral index

We define the spectral index $n_T$ at the end of inflation by

$$n_T = \frac{d \ln (P_h(k))}{d \ln k} = \frac{d \ln (P_h(k))}{d \ln a} \times \frac{d \ln a}{d \ln k}. \tag{19}$$

Together with Eq. (1), we have

$$d \ln (P_h(k)) = 2 \ln H \frac{d \ln a}{d \ln a} = -\frac{4M_{pl}^2H'^2}{H^2}. \tag{20}$$

On the other hand,

$$\frac{d \ln a}{d \ln k} = \left( \frac{d \ln k}{d \ln a} \right)^{-1} = \left( 1 + \frac{d \ln H}{d \ln a} \right)^{-1} = \left( 1 - \frac{2M_{pl}^2H'^2}{H^2} \right)^{-1}. \tag{21}$$

Note that we used the relation $k = aH$. Therefore, from Eqs. (19)-(21), we obtain

$$n_T = -\frac{4M_{pl}^2H'^2(H,V)}{H^2 - 2M_{pl}^2H'^2(H,V)}, \tag{22}$$

where we explicitly indicated that $H'$ is a function of $H$ and $V$ through Eq. (13).

Observing that the scale factor can be expressed by an integral,

$$\ln a = \int -\frac{H}{2M_{pl}^2H'(H,V)} d\phi, \tag{23}$$

one can rewrite the wave number $k$ (or the frequency $f$) as

$$k = \exp \left( \int -\frac{H}{2M_{pl}^2H'(H,V)} d\phi \right) H \left( = 2\pi f \right). \tag{24}$$

We see that Eqs. (22) and (24) are expressed by $H$ and $V$. Replacing $H$ in the equations by the analytic expression (18), we can calculate the spectral index against $k$ semi-analytically. In Fig. 2, we depicted the spectral index near the end of inflation in the case of $V(\phi) \propto \phi^2$ is depicted. The plot runs from $\phi/M_{pl} = -2$ to $\phi/M_{pl} = -1$ (inflation end), where Eq. (18) shows good agreement with a numerical solution and the slow roll approximation breaks down. (see Fig. 1) The horizontal axis is the frequency normalized by the frequency at the end of inflation $f_*$. The numerical result is depicted by the black dotted line. The red line represents Eq. (18) with $n = 2$ and expanded from the end of inflation. The blue and green lines are for $n = 4$ and $n = 6$, respectively.
breaks down (see Fig. 1). Note that the spectral index diverges at the end of inflation because the denominator of Eq. (22) is zero at there. This clearly shows the existence of the break point. Fig. 2 shows that our analytic expression (22) gets closer to the numerical solution by including higher order corrections. Therefore, one can use it with truncation at an appropriate order depending on the accuracy required.

An advantage of using the analytic expression is that one can extract the important information from the formula. For example, if one expands $n_T$ in powers of $(\phi - \phi_0)$, we have

$$n_T = \left( \frac{\sqrt{2}/M_{pl} - V'(\phi_0)/V(\phi_0)}{(\phi - \phi_0)^{-1}} \right)^{-1} + \frac{4}{M^2_{pl}} \left( \frac{V(\phi_0)}{V'(\phi_0)} \right)^2 + \frac{9}{2 M^2_{pl}} \left( \frac{V(\phi_0)}{V'(\phi_0)} \right)^2 + \frac{1}{2} \left( \frac{V''(\phi_0)}{V'(\phi_0)} \right)^2 + \cdots .$$

(25)

We note that the denominators, $(\sqrt{2}/M_{pl} - V'(\phi_0)/V(\phi_0))$, are not zero and, furthermore, not necessarily small because the slow roll approximations is broken at $\phi_0$. This analytic expression can be used as a fitting function for data analysis. Since the fitting function includes information of the inflaton potential at the break point, it is useful to discriminate inflation models. Indeed, for example, the sign of $V''(\phi_0)$ is different from a model to a model, so that determining the sign of $V''(\phi_0)$ from observations is important to distinguish inflation models. Therefore, our analytic expression is useful.

**B. Spectrum at present**

In this subsection, we give an example of the primordial gravitational wave spectrum around the spectral break at present. The dimensionless power spectrum around the end of inflation $P_h(k)$ is given by Eq. (1). After inflation, in the super horizon regime, $k < aH$, the power spectrum keeps the initial amplitude. When a mode $k$ reenter the Hubble horizon, namely in the sub horizon regime $k > aH$, the power spectrum starts to decay proportional to $a^{-2}$. It means the spectrum at present depends on the background evolution of the universe through the scale factor.

It would be useful to define the energy density parameter instead of the power spectrum and evolve it to the present time:

$$\frac{d\ln \Omega_{GW}^0}{d\ln k} = \frac{k^2 P_h(k)}{12a_0^2 H_0^2} \times \left( \frac{a_{\text{reenter}}(k)}{a_0} \right)^2 .$$

(26)

where $H_0 = 70\, \text{km/s/Mpc}$ is the Hubble parameter at present and $a_{\text{reenter}}$ is the scale factor when a mode reenters the Hubble horizon, i.e., $k = a_{\text{reenter}}H$.

We now assume that, as was done in Sec. I, a reheating phase where the inflaton oscillates around the bottom of its potential follows after inflation. In the phase, the Hubble parameter evolves as $H \propto a^{-3/2}$ and then $a_{\text{reenter}}(k) \propto k^{-2}$. Therefore, the energy density parameter would proportional to $k^{-2}$. More explicitly, the energy density parameter for the modes which reenter during the inflaton oscillation is

$$\frac{d\ln \Omega_{GW}^0}{d\ln f} = \frac{n_f^2 P_h(f)}{3H_0^2} \left( \frac{f}{f_{\text{break}}} \right)^{-2} \times \left( \frac{g_*(T_0) T_0^3}{g_*(T_{\text{reh}}) T_{\text{reh}}^3} \right)^{2/3} \left( \frac{H_{\text{reh}}}{H_{\text{end}}} \right)^{4/3} .$$

(27)

Note that $H_{\text{reh}}$ is related to $T_{\text{reh}}$ by Eq. (18) and $f_{\text{break}}$ is determined by $T_{\text{reh}}$ and $H_{\text{end}}$ in Eq. (17). Therefore,

\begin{itemize}
  \item We have not considered the effect of anisotropic stress of neutrinos here. It would give an overall suppression factor to Eq. (27).
\end{itemize}
for a set of free parameters $T_{\text{reh}}, H_{\text{end}}$, one can calculate Eq. (27) immediately with the semi-analytic formula we investigated in the previous sections.

Let us give an example of the spectrum in the case of $V(\phi) \propto \phi^2$ where we set $T_{\text{reh}} = 10^{13}$ GeV and $H_{\text{end}} = 10^{-5} M_{\text{Pl}}$. The result is depicted in Fig. 3. We see that our semi-analytic method (green line) well fits the numerical result (black dot line) around the spectral break where the slow roll approximation (blue line) gets worse. Note that we used the 1st order slow roll approximation in Fig. 3 because higher order collections make the result worse around the spectral break (see Fig. 1). Moreover, Fig. 3 shows that our formula can be connected to the slow roll approximation at a crossing point. Therefore, our formulation with the slow roll approximation enables us to complete the spectrum in full frequency range in analytic ways.

V. CONCLUSION

We studied a spectral break of primordial gravitational waves as a fingerprint of the end of inflation. Indeed, the amplitude and the frequency on the break can tell us the energy scale of inflation and the reheating temperature simultaneously. This spectral break scale may exist around $10^7$ Hz (see Eq. (7)) and thus the theoretical investigation is crucial for new gravitational wave detectors developed intensively [19–23].

We gave an analytic formula for describing the evolution of the Hubble parameter near the end of inflation in Sec. III. Because the slow roll approximation breaks down near the end of inflation, our analytic formulation is useful and complementary to it. In Sec. IV we semi-analytically investigated the spectral index of the spectrum at the end of inflation. We showed how to extract important information of the inflaton potential from the observation of the spectral index. Furthermore, we presented an example of the spectrum around the break point at present in Fig. 3 for specific values of parameters in Eq. (27).

It should be mentioned that probing the reheating temperature with primordial gravitational waves was also proposed in [15–17]. However, the paper focused on a bend of the spectrum due to the transition of background spacetime from the reheating to the radiation dominant phase. Then, the possibility to observe the bend with gravitational wave interferometers was discussed. This situation is realized when the reheating temperature is low enough. In the present paper, we intend to observe the spectral break due to the end of inflation and then higher reheating temperature is favorable compared with the above case.

ACKNOWLEDGMENTS

A.I. and J.S. was supported by JSPS KAKENHI Grant Numbers JP17H02894, JP17K18778. M.Y. is supported in part by JSPS Grant-in-Aid for Scientific Research Numbers 18K18764, Mitsubishi Foundation, and JSPS Bilateral Open Partnership Joint Research Projects.

[1] A. A. Starobinsky, Adv. Ser. Astrophys. Cosmol. 3, 130 (1987).
[2] K. Sato, Mon. Not. Roy. Astron. Soc. 195, 467 (1981).
[3] A. H. Guth, Adv. Ser. Astrophys. Cosmol. 3, 139 (1987).
[4] G. F. Smoot et al. (COBE), Astrophys. J. Lett. 396, L1 (1992).
[5] Y. Akrami et al. (Planck), (2018), arXiv:1807.06211 [astro-ph.CO].
[6] V. F. Mukhanov and G. V. Chibisov, JETP Lett. 33, 532 (1981).
[7] S. Hawking, Phys. Lett. B 115, 295 (1982).
[8] A. A. Starobinsky, Phys. Lett. B 117, 175 (1982).
[9] A. H. Guth and S. Pi, Phys. Rev. Lett. 49, 1110 (1982).
[10] L. Grishchuk, Sov. Phys. JETP 40, 409 (1975).
[11] A. A. Starobinsky, JETP Lett. 30, 682 (1979).
[12] B. Abbott et al. (LIGO Scientific, Virgo), Phys. Rev. Lett. 116, 061102 (2016), arXiv:1602.03837 [gr-qc].
[13] Y. Watanabe and E. Komatsu, Phys. Rev. D 73, 123515 (2006), arXiv:astro-ph/0604176.
[14] S. Kuroyanagi, T. Chiba, and N. Sugiyama, Phys. Rev. D 79, 103501 (2009), arXiv:0804.3249 [astro-ph].
[15] K. Nakayama, S. Saito, Y. Suwa, and J. Yokoyama, Phys. Rev. D 77, 124001 (2008), arXiv:0802.2452 [hep-ph].
[16] K. Nakayama, S. Saito, Y. Suwa, and J. Yokoyama, JCAP 06, 020 (2008), arXiv:0804.1827 [astro-ph].
[17] S. Kuroyanagi, K. Nakayama, and J. Yokoyama, PTEP 2015, 013E02 (2015), arXiv:1410.6618 [astro-ph.CO].
[18] Y. Ema, R. Jinno, and K. Nakayama, (2020), 10.3204/PUBDB-2020-02704, arXiv:2006.09972 [astro-ph.CO].
[19] V. Domcke and C. Garcia-Cely, (2020), arXiv:2006.01161 [astro-ph.CO].
[20] A. Ito and J. Soda, Eur. Phys. J. C 80, 545 (2020), arXiv:2004.04646 [gr-qc].
[21] A. Ito, T. Ikeda, K. Miuchi, and J. Soda, Eur. Phys. J. C 80, 179 (2020), arXiv:1903.04843 [gr-qc].
[22] F. Li, N. Yang, Z. Fang, J. Baker, Robert M.L., G. V. Stephenson, and H. Wen, Phys. Rev. D 80, 064013 (2009), arXiv:0909.4118 [gr-qc].
[23] F. Li, J. Baker, Robert M.L., Z. Fang, G. V. Stephenson, and Z. Chen, Eur. Phys. J. C 56, 407 (2008), arXiv:0806.1989 [gr-qc].
[24] L. Abbott, E. Farhi, and M. B. Wise, Physics Letters B 117, 29 (1982).
[25] A. Dolgov and A. Linde, Physics Letters B 116, 329 (1982).
[26] E. W. Kolb and M. S. Turner, The Early Universe, Vol. 69 (1990).
[27] M. Maggiore, Phys. Rept. 331, 283 (2000), arXiv:gr-
[28] D. Salopek and J. Bond, Phys. Rev. D 42, 3936 (1990).
[29] J. Lidsey, Phys. Lett. B 273, 42 (1991).

[30] A. R. Liddle, P. Parsons, and J. D. Barrow, Phys. Rev. D 50, 7222 (1994), arXiv:astro-ph/9408015.