High intensity laser hybrid guiding for electron acceleration

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Abstract. Controlled guiding of laser pulses at relativistic intensities in plasmas over distances exceeding the diffraction length is a crucial requirement of a Laser Plasma Accelerator Stage (LPAS) for achieving high quality electron beams. A hybrid guiding scheme is proposed using a dielectric capillary tube, inside which a plasma channel is generated. A high intensity laser ($I = 4.3 \times 10^{18}$ W/cm$^2$) focused at the entrance of this structure can be efficiently guided by the combined refraction of this channel and reflection at capillary walls. The efficiency of this guiding scheme as a LPAS has been investigated through numerical simulations of a 10 pC electrons beam injected at 150 MeV and accelerated up to 1.9 GeV inside a plasma channel of density $1.5 \times 10^{17}$ e.cm$^{-3}$ and of 15 cm length.

1. Introduction

Plasma-based accelerators [1] are promising candidates for application of relativistic electron beams due to their capability of sustaining an accelerating gradient of several GeV/m. The configuration of such accelerators is constituted of an injector, based either on laser plasma interaction at high intensity or on Radio-Frequency fields, connected to one or several laser-plasma acceleration stage (LPAS) providing an energy gain of several GeV. In this paper we present the design of such an LPAS operating in the quasi-linear regime. We propose a specific design which allows to reduce the total plasma energy of the LPAS, with the main objective of facilitating operation at high repetition rates.

For a LPAS operating in the quasi-linear regime, one of the most severe issue is to get an efficient guiding of the high intensity laser over distances much larger than the diffraction one. For a Gaussian beam this length is given by the Rayleigh length $Z_R = \pi w_0^2 / \lambda$, where $w_0$ is the laser waist in vacuum at the focal plane and $\lambda$ the laser central wavelength. For the Eupraxia design of an LPAS in the quasi-linear regime [2], one gets $Z_R \sim 1$ cm, while the accelerating field is between 10 and 20 GeV/m. It means that a few GeV LPAS should be at least 10 times longer than the diffraction length $Z_R$, requiring to introduce a guiding scheme. To this end, presently the most effective guiding scheme, demonstrated both theoretically and experimentally [3], is through a quasi-parabolic plasma channel (PPC). In a PPC the radial profile of the plasma electron density $n_e (r)$ inside the LPAS has a parabolic form:

$$n_e (r) = n_{e0} \left[ 1 + \frac{\Delta n_e}{n_{e0}} \frac{r^2}{w_0^2} \right] \equiv n_{e0} \left[ 1 + \frac{r^2}{r_{ch}^2} \right],$$

(1.1)

where $n_{e0}$ is the on-axis electron density, $r$ is the distance to this axis, $\Delta n_e$ the channel depth and $r_{ch} = w_0 \sqrt{n_{e0} / \Delta n_e}$ the radius of the plasma channel at which the density is increased by a factor of 2. Guiding by a PPC occurs through the radial variation of the optical refractive index $\delta$, given by $\delta = (1 - \omega_p^2 / (\beta_\omega \omega_\delta)^2) / \sqrt{1 - \omega_p^2 / (\beta_\omega \omega_\delta)^2}$, where $\omega_p$ and $\omega_\delta$ being the angular frequency of the laser and of the plasma respectively, while $\beta_\omega$ is the average Lorentz factor of a plasma electron oscillating in the laser field. $\omega_p$ is given by $\omega_p = \left( \frac{e}{m_e} \right) \sqrt{\omega_0^2 + m_e^2 v^2}$, $e$ and $m_e$ being the charge and mass of an electron and $\omega_0$ the vacuum permittivity, while $\beta_\omega = (1 + a^2 / 2)^{1/2}$, $a$ being the amplitude of the reduced vector potential of
the laser beam. For $\lambda = 0.8$ $\mu$m relevant to Ti-Sa based laser technology, and linear polarization, we have $a = 0.68\sqrt{I_0}$, $I_{18}$ being the laser intensity in unit of $10^{18}$ W/cm$^2$.

One of the most interesting property of a PPC is that, at low intensity $a \ll 1$, a laser beam having an initially Gaussian radial profile, keeps a radial Gaussian profile during its propagation. Moreover, for specific conditions, the waist of the laser beam can remain constant. As a consequence, the longitudinal and radial profiles of the electric field of the plasma wave remain unchanged. In particular, an electron bunch injected at the optimum phase, can stay in an optimized zone where it is both longitudinally accelerated and radially focused, staying close to the laser axis of propagation. At higher laser intensities $a \sim 1$ self-focusing can play a non-negligible role through $\vec{p}_e$. Nevertheless, numerical simulations [2] have shown that a PPC can lead to an efficient guiding of a high intensity laser with $a \sim 1.2$ in configurations close to the quasi-linear regime.

Experimentally, the generation of a plasma channel over large distances has been demonstrated using a high power discharge inside a capillary tube. In such a scheme, the discharge current is initiated at the capillary walls, leading to some evaporation of the capillary material. In order to reduce the pollution of the evaporated material on the plasma properties close to the laser axis during electron acceleration, the radius $R_{cap}$ of the capillary tube has to be rather large. Moreover, for lasers at the PW level, the mitigation of laser induced damages leads also to a large value for the ratio $\chi = R_{cap}/w_L$. As detailed in the next section a large value of $\chi$ prevents optimization of radial gradient close to the laser axis. In particular, in the experiment of ref [3] a sophisticated additional heating procedure was needed in order to increase this gradient. In the present paper we consider a lower laser power guided inside a capillary with a $\chi$ value of only two. As shown in the next section such a small value has the advantage of strongly reducing the amount of energy required to generate the plasma channel and also to damp high radial spatial frequencies that can be generated at the capillary entrance. The efficiency of such a configuration for a LPAS is examined in section 3, for an injected electrons bunch having similar parameters values as in the Eupraxia project [2].

2. Guiding in a capillary tube of small radius

Here we consider the guiding in a capillary tube of a high intensity laser having a bi-Gaussian shape and a peak power slightly lower than the critical power for relativistic self-focusing $P_c\approx 17n_e^2/n_0$, $n_e$ being the critical density equal to $1.74\times10^{21}$ cm$^{-3}$ for $\lambda = 0.8$ $\mu$m. The capillary is made of glass and has optically smooth inner wall surface. In the presence of a uniform low density plasma inside the tube, the high intensity laser can be guided over large distances by reflections at the inner walls [4]. Such guiding over 100 $Z_a$ has been experimentally demonstrated [5]. Multi-GeV acceleration of an externally injected electron bunch has also been demonstrated by numerical study [6], while this study also shows that to attenuate the negative influence of interferences between the E.M. modes of the tube it is necessary to consider high values of the laser waist leading to laser power in excess of the PW level.

In order to get a more efficient guiding at lower values of the laser waist we consider a PPC inside the capillary tube. The radial profile of the electron density is given by equation (1.1), with $\Delta n_c$ satisfying the best-matched guiding condition for a low intensity Gaussian laser beam [1]: $\Delta n_c = \Delta n$, where $\Delta n_c = 1/(\pi r_w^2)$ is called the critical channel depth, $r_w = e^2/(4\pi n_0 m_e c^2)$ being the classical electron radius. The best-matched value of the channel radius is then given by

$$r_w = r_{BM} = w_L\sqrt{n_0}/\Delta n = Z_a\sqrt{n_0}/n_e$$

(1.2)

For a given value of $\lambda$, $r_{BM}$ increases as the square of the laser waist and as the square root of the density. From equations (1.1) and (1.2) one can deduce the evolution with $\chi$ of the ratio $\eta = n_e (R_{cap})/n_0$, which is reported in figure 1a for $w_L = 50.5$ $\mu$m and in figure 1b for $R_{cap} = 100$ $\mu$m.
We see in these figures that $\eta$ is quickly increasing with $\chi$ at $Z_k = 1 \text{ cm}$ ($w_L = 50.5 \text{ µm}$) and even more for a fixed capillary radius. Therefore, the average density of energy of the plasma electrons significantly increases with $\chi$. Taking into account the additional $R_{cap}$ dependence for the energy per capillary length one can conclude that the minimum of energy deposited inside the tube in order to generate the plasma channel is obtained by reducing simultaneously $R_{cap}$ and $\chi$. Lower deposited energy will is desirable to operate at higher rates. Moreover, this minimization of deposited energy inside the capillary tube is accompanied by a reduction of the energy fluence at inner capillary walls.

One important limitation on the minimum value of $\chi$ comes from the possible laser-induced damage (LID) at the entrance surface of the capillary. For a laser having a FWHM duration of 45 fs and a wavelength of 0.8 µm, the intensity threshold for LID of glass is around $10^{13} \text{ W.cm}^{-2}$ [7]. Taking a maximum intensity on axis of $I_n = 4.3 \left( a = \sqrt{2} \right)$, the condition for maintaining the intensity below the intensity threshold for LID at a plane entrance surface which is perpendicular to the laser axis is $\chi \geq \chi_{LID} = 2.5$. Our calculations also show that for $\chi \geq \chi_{LID}$, reflections on the inner capillary walls during the laser propagation have a negligible role, therefore one retrieves in this case the limit of a plasma channel of infinite radius. For $\chi < \chi_{LID}$, a conical adapter at the capillary entrance can be used. In order to reduce the length of the cone one can introduce a more general shape for the capillary radius $R_{cap}(z) = f_{LID}(z)$ over a limited distance $z \leq d_{LID}$ ($z = 0$ is at the capillary entrance). $f_{LID}(z)$ is determined by setting the intensity at the capillary wall at a value lower than the intensity threshold for LID. An example of such a shape is shown in figure 2, for a laser having $a = \sqrt{2}$, a FWHM duration of 45 fs and a waist corresponding to $Z_k = 1 \text{ cm}$, focused in a capillary tube having $R_{cap} = 100 \text{ µm}$, which leads to $\chi \approx 2$. The laser focal plane is set 1.5 mm inside the capillary tube. The same laser configuration will be used in the following calculations. We can observe in figure 2 that the LID have some effect only over a longitudinal distance less than 1.5 mm. We note that a shape similar to the one represented in figure 2 can be generated directly by the high intensity laser beam interaction with a cylindrical capillary tube.
Figure 2. Cross section of the capillary hole close to the entrance of the capillary tube (z=0) as determined by \( f_{\text{LID}}(z) \), setting the maximum intensity at the capillary inner wall at 5x10^{12} \text{W.cm}^{-2}.

The length of \( d_{\text{LID}} \) increases quickly when \( \chi \) is reduced below two. Therefore, in our laser conditions \( \chi = 2 \) appears as the threshold value for getting a limited contribution of LID, allowing to operate a large number of shots with the same capillary tube.

Diffraction and reflection of the laser beam wings at the dielectric surface close to the capillary entrance generate inside the capillary tube high orders E.M. propagating modes. In the presence of a plasma channel, these modes provide, close to the laser axis, spatial fluctuations of the plasma wave field at high frequencies, in particular concerning the radial field. For an LPAS, these fluctuations lead to significant emittance growth of the injected electron bunch, and even to ejection of electrons from the accelerating zone. However, during propagation in vacuum, the amplitude of each E.M. propagating modes evolves as \( \exp(-z/\ell_{a.m}) \), where \( \ell_{a.m} \) is the absorption length for the mode \( m \), which scales as \( \ell_{a.m} \propto 1/m^2 \) for \( m \gg 1 \) [4]. Therefore, in our configuration, the high order modes can be damped over a few mm of propagation.

The longitudinal profile of the electron density on axis inside the capillary tube that we have chosen is reported in figure 3. \( n_o \) is zero along the first 35 mm of the capillary tube, then it has a linear increase during 1 cm up to its plateau value. At 150 mm, the density is linearly decreasing up to the capillary exit at 170 mm. The plateau value of \( n_o \) is fixed at 1.5x10^{17} \text{e.cm}^{-3}.

In order to generate the accelerating field of the LPAS, the high intensity laser beam is focused 1.5 mm inside the capillary tube. The plasma is a PPC satisfying the best matched condition of equation (1.2) at the plateau value of \( n_o \). The propagation of the high intensity beam has been determined using the quasi-static PIC code Wake-EP [6], which is well adapted to the considered configuration. The evolution of the maximum laser intensity on axis is reported in figure 3. During the first 40 mm of propagation, the laser starts to diverge then is focused to the axis by reflexion at the capillary inner wall.
During this propagation, fast oscillations appearing close to the capillary entrance are damped. Because there is no plasma up to \( z = 35 \) mm, these fast oscillations of the laser will not affect an electron beam injected behind the laser. The laser is then guided by the PPC during the whole length of the density plateau, then during the decrease of density, the laser beam diverges again up to the capillary exit. At the plateau density, the maximum of laser intensity has an oscillation around its average value due to relativistic self-focusing, these oscillations remain of moderate amplitude (\(-10\%\) indicating that propagation stays in the quasi-linear regime.

3. Acceleration of an injected relativistic electrons beam

The efficiency of the configuration shown in figure 3 for accelerating an injected electron bunch was also studied with the WakeEP code. The injected electron bunch has an average energy of 150 MeV, with a rms dispersion of 0.5 % and a normalized phase emittance \( \epsilon_{n,x} \) of 0.5 µm. Its focal plane is set 37 mm inside the capillary tube, with a radial dimension determined by the emittance matching conditions \[ 2 \mu \text{m}. \]

\[ T \text{ of } 0.5 \mu \text{m}^{-1}, \]

of the plasma channel is introduced in order to reduce the focusing angle of the bunch, to get a radial size much shorter than \( R_{op} \) at the capillary entrance. Similarly, the 20 mm linear decrease in density at the capillary exit allows to reduce significantly the final bunch divergence.

We have first determined the bunch electrons slice properties at the capillary exit in term of their injection phase \( \xi_B = z - ct \) relative to the laser beam without considering beam loading effects. We have reported in figure 4 the obtained results, as calculated by the Wake-EP code, for the slice average energy \( E_{sl} \), the ratio of accelerated electrons \( \eta_{trap} \), the slice relative dispersion in energy \( \delta E/E \) of and the increase in the slice normalized phase emittance \( \epsilon_{n,x,sl} / \epsilon_{n,x,sl,0} \). These slice properties have been calculated with a bin size for \( \xi_B \) of 0.5 µm.

In figure 4, we observe the expected decrease of energy with \( \xi_B \). However, due to a change of sign of the radial field during propagation, electrons too close to the maximum of the accelerating field becomes ejected from the plasma wave. The decrease of the average energy is directly connected to an increase of the dispersion in energy. A part of this increase is due to the finite value of the initial dispersion, while a second contribution comes from the fact that \( \partial^2 E / \partial \xi_B^2 > 0 \) in the accelerating zone, \( E \) being the longitudinal electric field, close to the laser axis, of the plasma wave. We also observe an emittance increase of about 10 %, which has not a clear dependence on \( \xi_B \).

Finally, we have introduced beam loading effect by injecting a 10 pC bunch at the average phase corresponding to the maximum of energy gain without losing electrons, which correspond to \( \xi_B = -10 \) µm in figure 4. Two bunch durations have been considered: a rms length \( \sigma_z \) of either 1 or 2 µm. The obtained results are summarized in table 1.
Table 1. Properties of the bunch electrons at the exit of the capillary tube for two initial bunch durations. The slice properties are calculated for a bin size of 0.2 µm

|            | \( E \) | \( \delta E / E \) | \( (\delta E / E)_x \) | \( \varepsilon_{\alpha x} \) | \( (\varepsilon_{\alpha x})_x \) | \( \sigma_{\alpha x} \) |
|------------|---------|-------------------|----------------------|-----------------|----------------------|------------------|
| \( \sigma_x = 1 \mu m \) | 1.88    | 0.82              | 0.28                 | 0.63            | 0.62                 | 190              |
| \( \sigma_x = 2 \mu m \) | 1.87    | 1.41              | 0.27                 | 0.58            | 0.58                 | 192              |

Looking at results of table 1 and the ones reported in figure 4, we observe that the main consequence of charge effects is to increase slightly the final emittance. The distribution in energy (not shown) is also significantly affected by the bunch charge and duration.

4. Conclusion
We have analyzed a configuration of a laser plasma accelerator stage in which the guiding of the high intensity laser is achieved by a plasma channel having a parabolic radial profile inside a capillary tube having optically smooth inner surface. We show that using a small ratio between the capillary radius and the waist of the high intensity laser can lead to a strong reduction in the energy required to generate the plasma channel. This opens the way to new mechanisms for generating this channel, in particular without the need to fully ionize the plasma. We also show that using dielectric tube with a small radius allow to benefit from the filtering properties of such capillary yielding a more stable acceleration over tens of cm.

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