Topological spin-current in non-centrosymmetric superconductors

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We study the spin transport properties of the non-centrosymmetric superconductor with time-reversal-symmetry where spin-triplet $(p_x \pm ip_y)$-wave and spin-singlet $s$-wave pair potential can mix each other. We show that when the amplitude of $(p_x \pm ip_y)$-wave pair potential is larger than that of $s$-wave one, the superconducting state belongs to the topologically nontrivial class analogous to the quantum spin Hall system, and the resulting helical edge modes as Andreev bound states are topologically protected. We find that the incident angle dependent spin polarized current flows through the interface due to the presence of the helical edge modes. With a weak magnetic field, also the angle-integrated current is strongly spin polarized.

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The topological properties of the electronic states have been attracting intensive interests in condensed matter physics. Especially, it was highlighted by the discovery of the quantum Hall system (QHS) showing the accurate quantization of the Hall conductance \( \sigma_H \) which is related to the topological integer \( [1, 2] \).

Recently, the concept of the QHS has been generalized to the time-reversal (T) symmetric system, i.e., the quantum spin Hall system (QSHS) \([3, 4, 5]\). QSHS could be experimentally demonstrated for the quantum well system by the measurement of the charge conductance \( \sigma \) of HgTe system through the interface due to the presence of the helical edge modes.

In this Letter, we study the spin transport properties of the non-centrosymmetric (NCS) superconductor \([11]\) with T-symmetry, where $(p_x \pm ip_y)$-wave and spin-singlet $s$-wave pair potential can mix each other. We show that when the amplitude of $(p_x \pm ip_y)$-wave pair potential is larger than that of $s$-wave one, the superconducting state belongs to the topologically nontrivial class analogous to the quantum spin Hall system, and the resulting helical edge modes as Andreev bound states are topologically protected. We study Andreev reflection \([13]\) at low energy, which is determined mostly by the helical edge modes, and find the incident angle dependent spin polarized current flowing through the interface. When the magnetic field is applied, even the angle-integrated current is spin polarized.

We start with the Hamiltonian of NCS superconductor

\[
\hat{H} = \begin{pmatrix}
\hat{H}(k)
& \hat{\Delta}(k)

-\hat{\Delta}^*(k)
& -\hat{H}^*(k)
\end{pmatrix}
\]

with \( \hat{H}(k) = \xi_k + V(k) \cdot \hat{\sigma}, \ V(k) = \lambda(\hat{x}k_y - \hat{y}k_x), \ \xi_k = \hbar^2k^2/(2m) - \mu \).

Here, \( \mu, m, \sigma \) and \( \lambda \) denote chemical potential, effective mass, Pauli matrices and coupling constant of Rashba spin-orbit interaction, respectively \([12]\). The pair potential \( \hat{\Delta}(k) \) is given by

\[
\hat{\Delta}(k) = [d(k) \cdot \hat{\sigma}]\hat{\sigma}_y + i\psi(k)\hat{\sigma}_y.
\]

We choose \( (p_x \pm ip_y) \)-wave pair for spin-triplet component with \( d(k) = \Delta_p(\hat{x}k_y - \hat{y}k_x)/|k| \) and \( s \)-wave one with \( \psi(k) = \Delta_s \) with \( \Delta_p \geq 0 \) and \( \Delta_s \geq 0 \). The superconducting gaps \( \Delta_1 = \Delta_p + \Delta_s \) and \( \Delta_2 = |\Delta_p - \Delta_s| \) open for the two spin-splitted band, respectively, in the homogeneous state \([16]\).
However, as seen below, surface states are crucially influenced by the relative magnitude between $\Delta_p$ and $\Delta_s$. Let us consider wave function including ABS localized at the surface. Consider a two-dimensional semi-infinite superconductor on $x > 0$ where the surface is located at $x = 0$. The corresponding wave function is given by

$$
\Psi_S(x) = \exp(ik_y y)[c_1\psi_1 \exp(iq_{1x}x) + c_2\psi_2 \exp(-iq_{1x}x) + c_3\psi_3 \exp(iq_{2x}x) + c_4\psi_4 \exp(-iq_{2x}x)],
$$

with $k_{1(2)x} = k_{1(2)x}^+ = k_{1(2)x}^- = k_{1(2)x}$ for $|k_y| \leq k_{1(2)}$ and $k_{1(2)x} = -k_{1(2)x}^+ = k_{1(2)x}$ for $|k_y| > k_{1(2)}$. Here, $k_1$ and $k_2$ are Fermi momentum of the small and large magnitude of Fermi surface given by $-m\lambda/h^2 + \sqrt{(m\lambda/h^2)^2 + 2m\mu/h^2}$ and $m\lambda/h^2 + \sqrt{(m\lambda/h^2)^2 + 2m\mu/h^2}$, respectively. $k_{1(2)x}$ denotes the x component of the Fermi momentum $k_{1(2)}$ with $k_{1(2)x} = \sqrt{k_{1(2)}^2 - k_{x}^2}$. The wave functions are given by $T\psi_1 = (u_1, -i\alpha_1^{-1}u_1, i\alpha_1^{-1}u_1, 1)$, $T\psi_2 = (u_2, i\alpha_2^{-1}u_2, i\alpha_2^{-1}u_2, 1)$, $T\psi_3 = (u_2, i\alpha_2^{-1}u_2, i\alpha_2^{-1}u_2, 1)$, and $T\psi_4 = (u_2, i\alpha_2^{-1}u_2, i\alpha_2^{-1}u_2, 1)$, with $\gamma = \text{sgn}(\Delta_1 - \Delta_2)$. In the above, $u_{1(2)}$ and $v_{1(2)}$ are given as

$$
\frac{1}{2}\left[1 + \sqrt{\frac{E^2 - \Delta_1^2(2)}{E}}\right], \quad \text{and} \quad \frac{1}{2}\left[1 - \sqrt{\frac{E^2 - \Delta_1^2(2)}{E}}\right].
$$

Here we have introduced $\alpha_1 = (k_{1x}^+ - ik_y)/k_1$, $\alpha_2 = (k_{2x}^+ - ik_y)/k_2$, $\alpha_1 = (-k_{1x}^+ - ik_y)/k_1$, and $\alpha_2 = (-k_{2x}^+ - ik_y)/k_2$. $E$ is the quasiparticle energy measured from the Fermi energy.

By postulating $\Psi_S(x) = 0$ at $x = 0$, we can determine the ABS. The bound state condition can be expressed by

$$
\sqrt{(\Delta_1^2 - E^2)(\Delta_2^2 - E^2)} = \frac{1 - \zeta}{1 + \zeta}E^2 + \gamma\Delta_1\Delta_2,
$$

with $\zeta \leq 1$, $\cos \phi_1 = k_{1x}/k_1$ and $\cos \phi_2 = k_{2x}/k_2$. The critical angle $\phi_C$ is defined as $\sin^{-1}(k_1/k_2)$. For $\lambda = 0$, eq. 3 reproduces the previous result [16]. As seen from eq. 3, the ABS including zero energy state is only possible for $|\phi_2| \leq \phi_C$ and $\gamma = 1$, i.e., $\Delta_p > \Delta_s$. The present ABS is just the edge state, where the localized quasiparticle can move along the edge. The energy level of the edge state depends crucially on the direction of the motion of the quasiparticle. The inner gap edge modes are absent for large magnitude of $k_y$, i.e., $\phi_2$. The parameter regime where the edge modes survive is reduced with the increase of the magnitude of $\lambda$. However, as far as we concentrate on the perpendicular injection, the edge modes survive as the mid gap ABS [18, 19] irrespective of the strength of $\lambda$. If we focus on the low energy limit, ABS can be written as

$$
E = \pm \Delta_p(1 - \frac{\Delta_s^2}{\Delta_p^2})k_1 + k_2, k_y,
$$

with $\Delta_p < \Delta_p$ for any $\lambda$ with small magnitude of $k_y$. For $\Delta_s \geq \Delta_p$, the present ABS vanishes since the value of the right side of eq. (3) becomes negative due to the negative sign of $\gamma$ for $|E| < \Delta_1$ and $|E| < \Delta_2$. It should be remarked that the present ABS do not break the time reversal symmetry, since the edge current carried by each Kramers doublet flows in the opposite direction. Thus they can be regarded as helical edge modes, where two modes are connected to each other by time reversal operation.

Now we give an argument why the superconducting state with $\Delta_p > \Delta_s$ has the ABS from the viewpoint of $Z_2$ (topological) class [14]. We commence with the pure $(p_x \pm ip_y)$-wave state without the spin-orbit interaction $\lambda$. Spin Chern number [5] for the Bogoliubov-de Gennes (BdG) Hamiltonian is 2. Turning on $\lambda$ adiabatically, which leaves the T-symmetry intact and keeps the gap open, one can arrive at the BdG Hamiltonian of interest. Upon this adiabatic change of $\lambda$, the number of the helical edge mode pairs does not change. Then we increase the magnitude of $\Delta_s$ from zero. As far as $\Delta_p > \Delta_s$ is satisfied, the number of helical edge modes does not change, since it is a topological number. However, if $\Delta_s$ exceeds $\Delta_p$, the helical mode disappears. In this regime, the topological nature of superconducting state belongs to pure s-wave state without $\lambda$. It is remarkable, just at $\Delta_s = \Delta_p$, one of the energy gap of the quasiparticle in the bulk closes, where a quantum phase transition occurs.

Now we turn to the spin transport property governed by the ABS in the NCS superconductors [20]. First, we point out that the spin Hall effect, i.e., the appearance of the spin Hall voltage perpendicular to the superconducting current is suppressed by the compressive nature of the superconducting state by the factor of $(k_F\lambda_{m})^{-2}$ ($k_F$: Fermi momentum, $\lambda_m$: penetration depth) [21]. Instead, we will show below that the spin transport through the junction between the ballistic normal metal at $x < 0$ and NCS superconductor, i.e., (N/SC) junction, can be enhanced by the Doppler effect at the Andreev reflection. We assume an insulating barrier at $x = 0$ expressed by a delta-function potential $U(\delta(x))$. The wave function for spin $\sigma$ in the normal metal $\Psi_N(x)$ is given by

$$
\Psi_N(x) = \exp(ik_{Fy}y)[\psi_{i\sigma} + \sum_{\rho = 1, 1} a_{\sigma, \rho}\psi_{i\rho}]\exp(ik_{Fx}x) + \sum_{\rho = 1, 1} b_{\sigma, \rho}\psi_{i\rho} \exp(-ik_{Fx}x)]
$$

with $T\psi_{i\sigma} = T\psi_{i\rho} = (1, 0, 0, 0)$, $T\psi_{i\sigma} = T\psi_{i\rho} = (0, 1, 0, 0)$, $T\psi_{i\sigma} = (0, 0, 1, 0)$, and $T\psi_{i\sigma} = (0, 0, 0, 1)$. The corresponding $\Psi_S(x)$ is given by eq. (2). The coefficients
where \( \phi \) denotes the injection angle measured from the normal to the interface. First we consider pure \((p_x \pm ip_y)\)-wave state. In Fig. 1, the angle resolved spin conductance is plotted as a function of injection angle \( \phi \) and bias voltage \( V = eV \). Note here that the \( k_y \) is related to \( \phi \) as \( k_y = Fk \sin \phi \). It is remarkable that spin conductance has a non zero value although the NCS superconductor does not break time reversal symmetry. \( f_S(\phi) \) has a peak when the angle \( \phi \) is given by \( E \) in the energy dispersion of ABS. With this condition, the spin-dependent Andreev reflection occurs to result in the spin current. Besides this property, we can show that \( f_S(\phi) = -f_S(-\phi) \) is satisfied. By changing the sign of \( eV \), \( f_S(\phi) \) changes sign as seen in Fig. 1(a). Next, we look at the case where \( s\)-wave component coexists. We can calculate spin current similar to the pure \((p_x \pm ip_y)\)-wave case. For \( \Delta_s, \Delta_y \) where helical edge modes exist, \( f_S(\phi) \) has a sharp peak and \( f_S(\phi) = -f_S(-\phi) \) is satisfied [see Fig. 1(b)]. These features are similar to those of pure \((p_x \pm ip_y)\)-wave case. On the other hand, for \( \Delta_s > \Delta_y \), where the helical edge modes are absent, sharp peaks of \( f_S(\phi) \) as shown in Fig. 1 are absent.

We have checked that there is negligible quantitative change by taking \( \lambda = 0 \) limit compared to Fig. 1, e.g., less than 0.5% change of the peak height. In this limit, for pure \((p_x \pm ip_y)\)-wave state, \( f_S(\phi) \) is given simply as follows

\[
-8\sigma_0^2(1 - \sigma_N)\sin 2\phi \sin 2\varphi \cos \phi \left| \left( \sin^2 \phi - \sin^2 \varphi \right) + \sigma_N \exp(-2\varphi) \left( \sigma_N - 2 \right) + 2 \cos 2\phi \right|^2,
\]

for \( |E| < \Delta_y \) and \( f_S(\phi) = 0 \) for \( |E| > \Delta_y \) with \( \sin \varphi = E/\Delta_y \). Transparency of the interface \( \sigma_N \) is given by 

\[
4\cos^2\varphi/(4\cos^2\varphi + Z^2)
\]

with a dimensionless constant \( Z = 2mU/h^2k_F \). The magnitude of \( f_S(\phi) \) is largely enhanced at \( E = \pm \Delta_y \sin \phi \) corresponding to the energy dispersion of ABS. The origin of nonzero \( f_S(\phi) \) even without \( \lambda \) is due to the spin-dependent ABS. We have checked that even if we take into account the spatial dependence of the \((p_x \pm ip_y)\)-wave pair potential explicitly, the resulting \( f_S(\phi) \) does not qualitatively change [20].

Summarizing these features, we can conclude that the presence of the helical edge modes in NCS superconductor is the origin of the large angle resolved spin current through N/NCS superconductor junctions. However, the magnitude of the angle averaged normalized spin conductance becomes zero since \( f_S(\phi) = -f_S(-\phi) \) is satisfied.

![FIG. 1: (Color online) Angle resolved spin conductance for

\[ Z = 5. \] a: \( eV = 0.1\Delta_y \), b: \( eV = -0.1\Delta_y \), and c: \( eV = 0.6\Delta_y \)

with \( \lambda k_F = 0.1\mu \). (a) pure \((p_x \pm ip_y)\)-wave case with \( \Delta_s = 0. \) (b) \( \Delta_s = 0.3\Delta_y \).]

Magnetic field offers an opportunity to observe the spin current in a much more accessible way, where \( T \)-symmetry is broken by the shielding current at the interface. Here we consider the angle averaged normalized spin conductance \( \sigma_S \) and charge conductance \( \sigma_C \) as a function of magnetic field which are given by

\[
\sigma_S = \frac{\int_{-\pi/2}^{\pi/2} f_S(\phi) d\phi}{\int_{-\pi/2}^{\pi/2} f_{NC}(\phi) d\phi}, \quad \sigma_C = \frac{\int_{-\pi/2}^{\pi/2} f_C(\phi) d\phi}{\int_{-\pi/2}^{\pi/2} f_{NC}(\phi) d\phi},
\]

where \( f_{NC}(\phi) \) denotes the angle resolved charge conductance in the normal state with \( \Delta_p = \Delta_s = 0 \). Now we consider the magnetic field \( H \) applied perpendicular to the two-dimensional plane, which induces a shielding current along the N/NCS superconductor interface. When the penetration depth of the NCS superconductor is much longer than coherence length, the vector potential can be approximated as \( A(r) = (0, A_y(0), 0) \) with \( A_y(0) = -\lambda_m H \exp(-r/\lambda_m) \) with the penetration depth \( \lambda_m \). Here we consider the situation where the quantization of the Landau level can be neglected. Then quasiclassical approximation becomes available. The applied magnetic field shifts the quasiparticle energy \( E \) in wave function of \( \Psi_S(x) \) to \( E - H\Delta_p \sin \phi/H_0 \) with \( H_0 = h/(2e^2\xi \lambda_m) \) and \( \xi = h^2k_F/(\pi m\Delta_p) \). For typical values of \( \xi \approx 10 \text{nm}, \lambda_m \approx 100 \text{nm} \), the magnitude of \( H_0 \) is of the order of 0.2Tesla. Here the order of the energy of Doppler shift is given by \( H\Delta_p/H_0 \). Since the Zeeman energy is given by \( \mu_B H \), the order of the energy of Doppler shift is \( k_F \lambda_m \) much larger than that of Zeeman energy. Thus, we can neglect the Zeeman effect in
the present analysis. This is in sharp contrast to QSHS where the Zeeman energy is the main effect of $H$, which opens the gap in the helical edge modes and modulates the transport properties \[6\]. The enhanced spin current due to the Doppler shift is specific to superconducting state not realized in QSHS.

As shown in Fig. 1, to discuss topological nature of the helical edge modes, it is sufficient to consider pure $(p_x \pm i p_y)$-wave case. In the following, we choose $(p_x \pm i p_y)$-wave case. In Fig. 2, the spin conductance $\sigma_S$ and charge conductance $\sigma_C$ normalized by the charge conductance in the normal state are plotted. It should be noted that $\sigma_S$ becomes nonzero in the presence of the magnetic field $H$ (see curves b, c, and d), since $f_S(\phi)$ is no more odd function of $\phi$ due to the imbalance of the helical edge modes. For $\lambda = 0$ limit, the corresponding helical edge modes are given by $E = \Delta_p (1 + H/H_0) \sin \phi$ and $E = -\Delta_p (1 - H/H_0) \sin \phi$. As seen from the curves b and c, the sign of $\sigma_S$ is reversed by changing the direction of the applied magnetic field. On the other hand, the resulting charge conductance has different features. For $H = 0$, the resulting line shape of $\sigma_C$ is the same as that of chiral $\alpha$-wave superconductor (see curve a of right panel) \[16, 17, 24\]. As seen from curves b and c of right panel, $\sigma_C$ does not change with the change of the direction of the magnetic field $H$. Finally, we show in Fig. 3 the zero-voltage $\sigma_S$ and $\sigma_C$. $\sigma_S$ is nearly linearly proportional to $H$. Note that with a small magnetic field $H \approx 0.4H_0 \sim 100$Oe, $\sigma_S$ is already of the order of 1. Meanwhile, $\sigma_C$ is almost independent of $H$.

In conclusion, we have studied the spin transport property of non-centrosymmetric (NCS) superconductor from the viewpoint of topology and Andreev bound state (ABS). We have found the incident angle dependent spin polarized current flowing through the interface. When the weak magnetic field is applied, even the angle-integrated current is largely spin polarized. As the analogy to quantum spin Hall system (QSHS), the ABS in NCS superconductor corresponds to the helical edge modes in QSHS. The Andreev reflection via helical edge modes produces the enhanced spin current specific to NCS superconductor.

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