Binary Black Hole Accretion Flows From a Misaligned Circumbinary Disk

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Abstract

We studied the basic properties of accretion flows onto binary supermassive black holes, including cases in which a circumbinary disk is misaligned with the binary orbital plane, by means of three-dimensional smoothed particle hydrodynamics simulations. We find that a circular binary system with a misaligned circumbinary disk normally produces a double-peaked mass-accretion-rate variation per binary orbit. This is because each black hole passes across the circumbinary disk plane and captures gas twice in one orbital period. Even in misaligned systems, however, a single peaked mass-accretion-rate variation per binary orbit is produced, if the orbital eccentricity is moderately large (e ≥ 0.3). The number of peaks in the mass accretion rates can be understood simply in terms of the orbital phase dependence of the distance between each binary black hole and its closest inner edge of the circumbinary disk. In the cases of eccentric binary black holes having different masses, the less massive black hole can get closer to the circumbinary disk than the massive one, thus tidally splitting gas from its inner edge, but the created gas flows are comparably captured by both black holes with a short time delay. As a consequence, the combined light curve shows periodic occurrence of double-peaked flares with a short interval. This may account for the observed light variations of OJ287.

Key words: accretion, accretion disks — binary black holes — black hole physics — Galaxies: nuclei

1. Introduction

Hierarchical structure formation scenario tells us that a galaxy grows by the merger of smaller galaxies. The recently confirmed correlation between the mass of supermassive black holes (SMBHs) and the mass or luminosities of the bulge of their host galaxies strongly support the idea that SMBHs have grown along with the growth of their host galaxies (Magorrian et al. 1998; Gebhardt et al. 2000; Ferrarese & Merritt 2000). These relationships suggest that each SMBH in the center of each galaxy should have evolved toward coalescence in a merged galaxy. If this is the case, a binary of SMBHs should be formed in a merged galactic nucleus before two black holes finally coalesce; yet, no binary SMBHs have clearly been identified so far, except for a dual active galactic nucleus (AGN)/SMBH at a large separation (>kpc) (Komossa et al. 2003) and its candidates (Comerford et al. 2011; McGurk et al. 2011; Fu et al. 2011; Comerford et al. 2012).

The merger of two SMBHs progresses via three stages (Begelman et al. 1980). First, each black hole sinks into a common center of a merged galactic nucleus by dynamical friction with the surrounding field stars and gas (Escala et al. 2005; Dotti et al. 2007). When the separation between the two black holes becomes as short as one parsec or so, the dynamical friction is no longer efficient, and a hard binary is formed (Mayer et al. 2007). The separation of such a hard binary is then reduced by some unknown mechanism. When the separation becomes as short as one parsec or less, finally, the binary black holes rapidly merge by emitting gravitational wave radiation to become a single SMBH. However, there has been many discussions on how the binary orbit decays in the second stage. One of predominant candidates is the interaction between the binary and the circumbinary disk (Ivanov et al. 1999; Gould & Rix 2000; Armitage & Natarajan 2002; Hayasaki 2009; Haiman et al. 2009; Cuadra et al. 2009; Lodato et al. 2009; Hayasaki et al. 2010; Nixon et al. 2011a; Kocsis et al. 2012a, 2012b).

It is an observational challenge to identify binary SMBHs on such a subparsec scale [see Komossa (2006) for a review]. Several ways have been proposed. Periodic optical and radio outbursts (e.g., OJ 287) (Sillanpää et al. 1988; Valtonen et al. 2011), wiggled patterns of the radio jet, indicating precessional motions on a parsec scale (Yokosawa & Inoue 1985; Lovanov & Roland 2005), an X-shaped morphology of radio lobes (Merritt & Ekers 2002), double-peaked and unusual broad emission line profiles in AGNs (Gaskell 1996; Bogdanović et al. 2009; Dotti et al. 2009; Boroson & Lauer 2009; Montuori et al. 2011, 2012; Eracleous et al. 2012), see also Popović (2012) for a recent review double compact cores with a flat radio spectrum (Rodríguez et al. 2006), orbital motion of the compact core with a periodic flux variation (Sudou et al. 2003; Iguchi & Sudou 2010), a spectroscopically resolved sub-parsec binary orbit by detecting periodic variations in the light and radial velocity curves (Bon et al. 2012), and so on.

With successive discoveries of binary black hole candidates and ongoing constructions of advanced gravitational-wave detectors, such as eLISA (Amaro-Seoane et al. 2013), much attention has been recently paid to electromagnetic signatures from binary SMBH systems, such as afterglows (Milosavljević & Phinney 2005; Tanaka & Menou 2010; Tanaka et al. 2010),
precursors (Chang et al. 2009; Bode et al. 2010; Hayasaki 2011; Farris et al. 2011, 2012; Bode et al. 2012; Baruteau et al. 2012), periodic emissions (Artyomowicz & Lubow 1996; Lehto & Valtonen 1996; Hayasaki et al. 2007, 2008; Bogdanović et al. 2008; MacFadyen & Milosavljević 2008; Roedig et al. 2011; Sesana et al. 2012; Shi et al. 2012; Noble et al. 2012; D’Orazio et al. 2012), persistent emission with unusual AGN spectra (Tanaka et al. 2012), a dual-jet structure (Palenzuela et al. 2010; Mösta et al. 2010), and a collimated electromagnetic emission enhanced by strongly amplified magnetic fields (Giacomazzo et al. 2012), in the context of a massive binary black hole coalescence. There are also several theoretical studies on the observational methodology to probe the presence of a gaseous disk around coalescing binary black holes through waveform analyses of the emitted gravitational waves (Kocsis et al. 2011; Yunes et al. 2011; Hayasaki et al. 2013).

In most of the previous studies, it has been assumed that the circumbinary disk is aligned with the binary orbital plane. However, the angular momentum vector of the circumbinary disk does not always coincide with that of the binary, because the orientation of a circumbinary disk is primarily due to the angular momentum distribution of the gas, and not significantly due to the angular momentum of the binary black holes. Therefore, the orientation of a circumbinary disk plane can be taken arbitrarily with respect to the binary orbital plane.

There are two stable states: a co-alignment state in which the circumbinary disk rotates in the prograde direction of the binary motion, and a counter-alignment state, in which the circumbinary disk rotates in the retrograde direction of the binary. That is, when the circumbinary disk is initially misaligned with the binary orbital plane, it transits to either stable state because of the tidal action working between them. If the misalignment angle is larger than \( \pi/2 \), the circumbinary disk makes a transition to the counter-alignment state (Nixon 2012). When it is smaller than \( \pi/2 \), conversely, the circumbinary disk transits to the co-alignment state. The transition timescale is comparable to the viscous timescale of the circumbinary disk (Nixon et al. 2011b), which is much longer than the binary orbital period. During the transition, the circumbinary disk is still misaligned with the binary orbital plane.

However, accretion flows onto binary SMBHs around such a misaligned circumbinary disk have been poorly investigated. The misalignment angle is expected to produce complex light variations that never appear otherwise. Especially, two periodic outbursts per orbital period may be obtained, since there are two epochs in one orbital period, when the distance between the binary and the inner edge of circumbinary disk is shortest. By contrast, the binary orbital eccentricity produces a single, steep peak per binary orbit in a coplanar system (Hayasaki et al. 2007; Roedig et al. 2011).

In this paper, we focus our attention on basic properties of accretion flows onto binary SMBHs, including cases involving a misaligned circumbinary disk. In the next section, we describe our models and calculation methods. The numerical results are presented in section 3. In section 4, we provide a simple model in order to understand the numerical results. The final sections are devoted to a summary and discussion.

## 2. Our Models and Calculation Methods

In this section, we first explain our models, and then describe our calculation methods.

### 2.1. Initial Settings

Figure 1 illustrates a schematic picture of the setting of our model; binary black holes orbiting each other are surrounded by a misaligned circumbinary disk. The binary is put on the \( x-y \) plane with its center of mass located at the origin. The masses of the primary and secondary black holes are represented by \( M_1 \) and \( M_2 \), respectively. We put a circumbinary disk around the origin. The unit vector of specific angular momentum of the circumbinary disk is expressed by (e.g., Pringle 1996)

\[
f_{\text{d}} = (\cos \gamma \sin \beta, \sin \gamma \sin \beta, \cos \beta),
\]

where \( \beta \) is the tilt angle between the circumbinary disk plane and the binary orbital plane, and \( \gamma \) is the azimuth of tilt. The position vector of each black hole is given by

\[
r_i = (r_i \cos \phi, r_i \sin \phi, 0) \quad [i = 1, 2],
\]

where

\[
r_i = \frac{\eta_i a(1 - e^2)}{1 + e \cos \phi}
\]

and \( \phi \) is the true anomaly (hereafter, we regard the true anomaly as the orbital phase) (e.g., Murray & Dermott 1999), \( \eta_1 \equiv q/(1 + q) \) and \( \eta_2 \equiv 1/(1 + q) \) with a binary mass ratio of \( q = M_2/M_1 \); \( a = a_1 + a_2 \) is the semi-major axis of the binary, where \( a_1 \equiv \eta_1 a \) and \( a_2 \equiv \eta_2 a \).

The circumbinary disk initially has a radially uniform density profile between the initial radius of the disk-inner edge, \( r_{\text{ini}} \), and a radius of \( r = r_{\text{ini}} + 0.05 a \) (i.e., the initial width of the circumbinary disk is set to be 0.05a). The material in the circumbinary disk rotates around the origin on the circular orbit with the Keplerian rotation velocity. The vertical density structure of the circumbinary disk is exponential; i.e., we assume a hydrostatic balance in the vertical direction, and that the disk temperature is set to be constant, \( T = 30700 \) K, in both space and time. Note that this corresponds to the typical central temperature of a standard disk at \( r = 2a \) around a single black hole with \( 10^6 M_\odot \) for a given mass input rate of \( M_{\text{ini}} = 1.0 M_\odot \text{yr}^{-1} \) (Kato et al. 2008). The initial mass of the circumbinary disk is \( 1.0 \times 10^{-2} M_\odot \).

In table 1, we summarize common model parameters for all models used in the present study: the total black hole mass \( (M_1 + M_2) \), semi-major axis of the binary orbit \( (a) \), initial mass of the circumbinary disk \( (M_{\text{disk}}) \), mass injection rate \( (M_{\text{inj}}) \), disk temperature \( T_d \), and Shakura–Sunyaev viscosity parameter, \( \alpha_{\text{SG}} \) (Shakura & Sunyaev 1973). The relation between \( \alpha_{\text{SG}} \) and the SPH (Smoothed Particle Hydrodynamics) artificial viscosity parameters are described in subsection 2.3. The binary orbital period, \( P_{\text{orb}} \), is estimated to be approximately 9.4 yr by using the Kepler’s third law. The mass of a SPH particle is set to be \( 1.0 \times 10^{-15} \) in units of the total black hole mass.
Fig. 1. Configuration of our model. There are two angles ($\beta, \gamma$) that specify the orientation of the circumbinary disk plane with respect to the binary orbital plane ($x$–$y$ plane).

Table 1. Common model parameters.

| Parameter                   | Value                                      |
|-----------------------------|--------------------------------------------|
| Total black hole mass       | $M_1 + M_2 = 10^8 M_\odot$                |
| Semi-major axis             | $a = 0.01$ pc                              |
| (Orbital period)            | $P_{\text{orb}} = 2\pi a^{3/2} / \sqrt{G(M_1 + M_2)} \approx 9.4$ yr |
| Initial disk mass           | $M_{\text{disk}} = 1.0 \times 10^{-4} M_\odot$ |
| mass injection rate         | $M_{\text{inj}} = 1.0 M_\odot$ yr$^{-1}$  |
| Disk temperature            | $T_d = 30700$ K                            |
| Disk viscosity              | $\alpha_{\text{SS}} = 0.1$                |

2.2. Boundary Conditions

Gas particles are added to the outer edge of the circumbinary disk from its outside in arbitrary angles at a constant rate of $M_{\text{inj}} = 1.0 M_\odot$ yr$^{-1}$. They have the Keplerian velocity in the azimuth direction at the injected radius, but no velocities in the radial and vertical directions. The temperature of the injected particles is 30700 K; i.e., the same with that of the disk. The mass injection may affect the dynamics in the early stage of disk evolution. However, we are interested in the late stage of disk evolution, a quasi-steady state, in which the disk mass is much larger than the mass injected per orbital period, so that the mass injection cannot affect the gas dynamics.

The inner edge of the circumbinary disk is determined by the balance between the tidal/resonant torque exerted by the binary black holes and the viscous torque of the circumbinary disk. We take the initial inner edge radius of the circumbinary disk to be $r_{\text{ini}} = 2.5 a$ for $e = 0.5$. In an equal-mass and circular binary, on the other hand, we take the initial inner edge radius as to be $r_{\text{ini}} = 1.68 a$, corresponding to the tidal truncation radius where the tidal torque of the binary equals to the viscous torque of the circumbinary disk (Papaloizou & Pringle 1977; see also table 1 of Artymowicz & Lubow 1994). We set the outer calculation boundary at $r = 6.0 a$, which is sufficiently far from the disk region, so that the outer boundary should not affect the flow dynamics in the binary SMBH system. The SPH particles passing outward across the outer calculation boundaries are removed from the simulation box.

The accretion radius depends on the mass of each black hole. The black holes are modeled by sink particles with a fixed accretion radius of $r_{\text{acc}} = 0.1 a$ or 0.05 $a$, depending on the black hole mass. Note that each accretion radius is two orders of magnitude larger than the Schwarzschild radius of each black hole. Numerically, we remove all of the particles that enter the region inside the accretion radius.

2.3. Numerical Method

The simulations described below were performed using the three-dimensional (3D) SPH code, which is based on a version originally developed by Benz (1990); Benz et al. (1990); Bate et al. (1995), and has been extensively used for various systems by many authors (e.g., Okazaki et al. 2002; Hayasaki et al. 2007; Okazaki et al. 2011; Takata et al. 2012). The SPH equations are composed of a mass-conservation equation, a momentum equation with the SPH standard artificial viscosity, and an isothermal equation of state in substitution for an energy equation. These equations with the standard cubic-spline kernel are integrated using a second-order Runge-Kutta-Fehlberg integrator with individual time steps for each particle and a variable smoothing length (Bate et al. 1995), which results in saving enormous computational time when a large range of dynamical timescales are involved.

The calculation methods and equations are the same as those adopted in Hayasaki et al. (2007). The momentum equation for gas particles moving under the binary potential is given by equating acceleration with the sum of the pressure gradient, viscous force, and gravitational force [e.g., see equations (3) and (4) of Hayasaki et al. (2007)], where the self-gravity of gas particles is neglected. We have also used the equation of state, in which a local pressure is proportional to a local density, where the proportional coefficient, square of the isothermal
Models B and C are eccentric binaries with equal black hole masses (Model B) and unequal black hole masses (Model C). Through this paper, we assign Model A2 as a fiducial model.

### 3. Accretion Flows from a Misaligned Circumbinary Disk to Binary Black Holes

In this section, we examine how the basic properties of accretion flows onto each black hole depend on $\beta$, $\gamma$, $e$, and $q$, based on 3D SPH simulations.

#### 3.1. Mass Accretion Rates onto Binary Black Holes

We first show in figure 2 the orbital phase dependence of the mass accretion rates for all models. Through out the present work, the mass accretion rate was calculated by counting the number of SPH particles entering the accretion radius of each black hole per unit time. Each mass accretion rate was folded on the orbital period over $40 \leq t \leq 60$. For eccentric binaries (Models B1–C3), the binary is at the periastron (or at the apastron) at phase 0.0 (0.5). For clarity, we vertically offset the mass accretion rates for Models A1 and B1 by +0.3, for Models A2, B2, and C1 by +0.2, and for Models A3, B3, and C2 by +0.1 with respect to those of Models A4, B4, and C3, respectively. The smoothing length is longer than the scale-height over the whole disk in all models, whereas it is smaller than the radial dimension of the circumbinary disk. This means that the vertical structure of the circumbinary disk is not resolved, while the radial disk structure is resolved.

Hayasaki et al. (2007) already studied the aligned disk case (i.e., $\beta = 0$) corresponding to Model A1. Since the mass continuously falls onto the binary from the circumbinary disk, there is no remarkable orbital-phase dependence in a circular orbit. As a result, no periodic variations are produced in neither the mass accretion rates onto the two black holes nor the luminosity of the accretion disks surrounding the two black holes. In other words, the accretion proceeds in a quasi-steady fashion, and, hence, no large light variations, but small amplitude, random fluctuations are observed. More details are shown in figures 2, 4, and 8 of Hayasaki et al. (2007). We should note, however, that even in the case of a circular binary, periodic variations may be produced by the formation of spiral patterns excited on the circumbinary disk (MacFadyen & Milosavljević 2008; D’Orazio et al. 2012). Such features never appear in our calculations with limited spatial dimension of the circumbinary disk and, so, will be explored in the future.

The situations differ markedly, if the circumbinary disk is misaligned with respect to the binary orbital plane. In Models A2 and A3, the circumbinary disks are tilted by $\beta = \pi/6$ and $\pi/4$, respectively, while $\gamma$ is kept the same (i.e., $\gamma = \pi/2$). Both models produce mass-accretion-rate variations with (more than) two peaks in one orbital period, as shown in the left panel of figure 2. This can be understood in terms of the geometrical effect. In these misaligned systems, two black holes stay away from the circumbinary disk plane most of the time, but twice per orbital period they cross the circumbinary disk plane. Thus, there are two chances in one orbital period that black holes approach the inner region of the circumbinary disk and strip gas from the circumbinary disk. This effect makes the mass accretion rates enhanced around

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**Table 2. Individual model parameters.**

| Model | $q \equiv M_2/M_1$ | $e$ | $r_{in}/a$ | $(\beta, \gamma)$ |
|-------|-------------------|----|----------|-----------------|
| A1    | 1.0               | 0.0| 1.68     | (0, 0)          |
| A2    | 1.0               | 0.0| 1.68     | ($\pi/6, \pi/2$) |
| A3    | 1.0               | 0.0| 1.68     | ($\pi/4, \pi/2$) |
| A4    | 1.0               | 0.0| 1.68     | ($\pi/6, 0$)    |
| B1    | 1.0               | 0.5| 2.50     | (0, 0)          |
| B2    | 1.0               | 0.5| 2.50     | ($\pi/6, \pi/2$) |
| B3    | 1.0               | 0.5| 2.50     | ($\pi/4, \pi/2$) |
| B4    | 1.0               | 0.5| 2.50     | ($\pi/6, 0$)    |
| C1    | 0.5               | 0.5| 2.50     | (0, 0)          |
| C2    | 0.5               | 0.5| 2.50     | ($\pi/6, \pi/2$) |
| C3    | 0.5               | 0.5| 2.50     | ($\pi/4, \pi/2$) |

sound speed that is proportional to temperature, is assumed to be constant in space and time. We therefore do not need to explicitly solve an energy equation.

The artificial viscosity commonly used in SPH consists of two terms: a term that is linear in the velocity differences between particles, which produces a shear and bulk viscosity, and a term that is quadratic in the velocity differences, which is needed to eliminate particle interpenetration in high Mach number shocks. The parameters $\alpha_{\text{SPH}}$ and $\beta_{\text{SPH}}$ control the linear and quadratic terms, respectively. In the simulations described in this paper, the artificial viscosity is adjusted so as to keep the Shakura–Sunyaev viscosity parameter $\alpha_{\text{SS}} = 0.1$ (Shakura & Sunyaev 1973), using the approximate relation $\alpha_{\text{SS}} = 0.1 \alpha_{\text{SPH}} h/H$ and $\beta_{\text{SPH}} = 0$ (see subsection 2.2 of Hayasaki et al. 2007), where $h$ and $H$ are the smoothing length of individual particles and the scale-height of the circumbinary disk, respectively.

Sink particles (i.e., black holes) are orbiting around each other, following the Kepler’s third law, because perturbations by the interaction with SPH particles are negligible. In all models described in the next subsection, total run time is 60 in the unit of $P_{\text{orb}}$, and the simulation time $t$ is normalized by $P_{\text{orb}}$ through this paper. The orbital phase is forwardly shifted by 0.03 at the end of a run ($t = 60$) so as to correct any cumulative numerical errors arising from the second-order accuracy of the time integration scheme in our code.

#### 2.4. Calculated Models

In the present study we are concerned with the observable quantities for various configurations of binary SMBHs with the circumbinary disk. Accordingly, we calculated eleven models, in total, by varying the binary mass ratio, binary orbital eccentricity, tilt angle, and azimuth of tilt. In table 2, we summarize the model parameters: from left to right, model number (first column), mass ratio of the secondary black hole to the primary one (second column), orbital eccentricity (third column), initial radius of the inner edge of the circumbinary disk in units of $a$ (fourth column), and tilt angle and azimuth of tilt $(\beta, \gamma)$ (fifth column, see figure 1 and equation (1) for definitions of $\beta$ and $\gamma$).

While Model A are an equal-mass and circular binary,

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Fig. 2. Orbital phase dependence of mass accretion rates for all models. Models A1–A4, Models B1–B4, and Models C1–C3 are shown in the left panel, the middle panel, and the right panel, respectively. The data is folded on the orbital period over $40 \leq t \leq 60$. Here, we redefine the orbital phase in order that the phase 0 corresponds to the epoch of the periastron passage (see subsection 2.3). The binary black holes are at the periastron (or at the apastron) at phase 0.0 (0.5) for eccentric binaries (Models B1–C3). For clarity, we vertically offset the mass accretion rates for Models A1 and B1 by $+0.3$, for Models A2, B2, and C1 by $+0.2$, and for Models A3, B3, and C2 by $+0.1$ with respect to those of Models A4, B4, and C3, respectively. In the right panel, the solid line and dashed (red) line represent the mass accretion rates onto the primary black hole and that of the secondary one, respectively. (Color online)

In Model B4, we assign $\phi = 0$, while keeping other parameters the same as those in Model B2. The resultant mass accretion rate has a single peak per binary orbit, and the variation amplitude is smaller than that of Model B2. It is interesting to note that the double-peak nature, which was observed in Models A2 and A3, is no longer noticeable in Models B2–B4. This is because the effect of orbital eccentricity is stronger than both the misalignment effect and the rotation effect in the azimuth direction, thus erasing the double-peak nature (see section 4). Note that it seems that the mass-accretion-rate variations have a double-peaked structure at $\phi = 0$ in Models B2 and B3, but such a structure could be a numerical artifact arising from the much larger accretion radius than the Schwarzschild radius.

We finally performed simulations of Models C1–C3 to see what variations in the mass accretion rates are produced by the binary with different masses. The results are shown for Models C1–C3 in the right panel, where the solid line and the dashed (red) line represent the mass accretion rates onto the primary and secondary black holes, respectively. Single peaked light variations are obtained for these models.1 The mass accretion rate onto the secondary black hole is slightly smaller than those onto the primary black hole. Remarkably, the peak phases slightly differ among the two black holes. This can be

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1 Model C1 has the same model parameters as those of Model 4 of Hayasaki et al. (2007). However, the resultant variations of the mass accretion rates slightly differ with each other, since the size of the accretion radius in the previous calculation was not small enough to properly calculate the actual mass accretion rate.
orbital period in Models B1 and C1. The most remarkable peaks is shorter than those in Models A2, A3, and A4. The resultant superposed light curve exhibits one double-peaked flare per binary orbit, and its interval between two small points and solid circles denote the positions of two black holes and the accretion radii surrounding them, respectively. The figure is shown in the inertial frame; i.e., both of the binary and the circumbinary disk are rotating around the center in the anti-clockwise direction. Note that the disk plane is tilted by $\beta = \pi/6$ from the binary orbital plane with $y = \pi/2$ in both models. The density levels of each panel can be seen in the color chart ($-3.5 \leq \log \Sigma \leq 1.5$) at the right side of lower panel (d). The dashed circle represents the initial inner-edge radius $r_{\text{in}} = 1.68 a$ for Model A2, while it is the $1:3$ outer Lindblad resonance radius $\sim 2.1 a$ for Model B2. Annotated in each panel are the major scale in units of $0.01 \text{pc}$, time in units of $P_{\text{orb}}$ and the number of SPH particles $N_{\text{SPH}}$, respectively. (Color online)

understood as follows. The gas in the inner edge of the circumbinary disk is tidally stripped at the apastron when the secondary black hole gets closest to the circumbinary disk. The gas freely infalls onto the primary black hole moving around the center of mass of the binary system, another part of the gas accretes onto the secondary black hole at the periastron passage. The resultant superposed light curve exhibits one double-peaked flare per binary orbit, and its interval between peaks is shorter than those in Models A2, A3, and A4.

We have also found other periodicities longer than the binary orbital period in Models B1 and C1. The most remarkable periodicity ($\sim 2 P_{\text{orb}}$) corresponds to the Keplerian period at the $1:3$ resonance radius, where the tidal torque by the binary potential is balanced by the viscous torque of the circumbinary disk. The physical origins of the other periods are not clear. They may be due to some kind of resonance within the circumbinary disk or artificial effects appearing because of very thin radial width ($0.05 a$) in our simulations. More extensive simulations, in which the width of the circumbinary disk is taken larger, are needed to identify their origins (e.g., see D’Orazi et al. 2012).

3.2. Accretion Flow Patterns

In order to visualize how gas accretion onto binary SMBHs occurs, we show in figure 3 sequences of surface density contours for Model A2 (upper panels) and Model B2 (lower panels) at four different phases; at the phases of $t = 59.0$, 59.25, 59.5, and 59.75 from the left to the right, respectively. These panels are illustrated in the inertial frame; both of the black holes and the circumbinary disk are rotating in the counter-clockwise direction. Note that the black holes are rotating more rapidly than the gas in the circumbinary disk according to the Kepler’s third law. The two small points represent the locations of the primary black hole and secondary black hole, respectively. The solid small circles surrounding them represent their accretion radii, which are set to be $r_{\text{acc}} = 0.1 a$ from the center of each black hole. The dashed circles in both models represent the inner edge of a circumbinary disk of $\sim 1.68 a$ for Model A2 and the $1:3$ outer Lindblad resonance radius $\sim 2.1 a$ for Model B2 (see Artymowicz & Lubow 1994), respectively. These density maps are projected to the circumbinary disk plane (denoted as the $x^* - y^*$ plane).

First, we describe the case of Model A2 (upper panels). Although the flow patterns shown in the upper four panels appear to be quite similar to those in Model A1 reported by Hayasaki et al. (2007), we notice some differences between them. At elapsed times of $t = 59.0$ and 59.5, as shown in panels (a) and (c), the innermost part of the circumbinary disk is most strongly distorted by the tidal force of each black hole, and therefore the surface density of the innermost part of the circumbinary disk is enhanced at the two positions of $\sim 3\pi/4$ and $\sim 7\pi/4$ from the $x^*$ axis, where cusp structures are observed. This is because each black hole is located within the circumbinary disk plane at those times. By contrast, at elapsed times of $t = 59.25$ and 59.75, as shown in panels (b) and (d), the cusp density structure disappears, though density

Fig. 3. Sequence of snapshots of the accretion flow from a circumbinary disk onto binary SMBHs in Model A2 (upper four panels) and Model B2 (lower four panels). The color contours of surface density are displayed in the chronological order from the left (a) to the right (d) for each model. The surface density was calculated by integrating density in the direction of the circumbinary disk plane. The two small points and solid circles denote the positions of two black holes and the accretion radii surrounding them, respectively. The figure is shown in the inertial frame; i.e., both of the binary and the circumbinary disk are rotating around the center in the anti-clockwise direction. Note that the disk plane is tilted by $\beta = \pi/6$ from the binary orbital plane with $y = \pi/2$ in both models. The density levels of each panel can be seen in the color chart ($-3.5 \leq \log \Sigma \leq 1.5$) at the right side of lower panel (d). The dashed circle represents the initial inner-edge radius $r_{\text{in}} = 1.68 a$ for Model A2, while it is the $1:3$ outer Lindblad resonance radius $\sim 2.1 a$ for Model B2. Annotated in each panel are the major scale in units of $0.01 \text{pc}$, time in units of $P_{\text{orb}}$ and the number of SPH particles $N_{\text{SPH}}$, respectively. (Color online)
enhancements are seen to some extent. This is because each black hole is most distant from the circumbinary disk plane. We can now understand that the time variations of the overall density distribution repeat every half orbital period, which is the period of the passage of the black holes across the circumbinary disk plane. This is responsible for the double-peaked variations in the mass accretion rates.

We now, examine the case of Model B2, which is illustrated in the lower four panels of figure 3. At the phase of $t = 59.25$, as shown in panel (b), the separation between the two black holes is increasing and, hence, the black holes are approaching the inner edge of the circumbinary disk. At $t = 59.5$ the binary separation reaches its maximum, and the distance to the circumbinary disk is at the minimum [see panel (c)]. The gas in the innermost part of the circumbinary disk is pulled out by the black hole from $t = 59.25$ to 59.5, thereby a tidal tail being formed and extending inward from the innermost part of the circumbinary disk. This tidal tail continuously grows, and at the phase of $t = 59.75$, as shown in panel (d), we can see a bridge connecting the two tidal tails extending from the opposite sides of the circumbinary disk. Gas is supplied to the black hole in a next moment, which can be clearly seen in panel (a). In fact, it takes about a half of binary period for gas to fall onto the black holes from the inner edge of the circumbinary disk. This is the reason why the mass accretion rate reaches its maximum value at around phase zero.

It should be noted that such a gas dynamical behavior is very similar to that in Model B1, which was reported previously (Hayasaki et al. 2007). In other words, the effects of the tilt angle, $\beta$, is not so appreciable for high eccentricity cases. We discuss why it is so in section 4.

3.3. Averaged Mass Accretion Rates and Circularization Radii

In table 3, we summarize the averaged mass-accretion rates and the averaged circularization radii around black holes, and the number of SPH particles at the end of each run for all models, where the circularization radius, $r_{c,i}$, is defined by $r_{c,i} = J_i^2/GM_i$, [$i = 1, 2$] and $J_i$ represents the specific angular momentum of the SPH particles which inside each accretion radius. Thus, their specific angular momentum is proportional to the 1/2 power of the circularization radius.

It is noted from table 3 that the same amount of mass injection is assumed for Models A1–A4, but nevertheless the averaged accretion rate is higher in Model A1 than in the others. These trends are also seen in Models B and C.

The averaged circularization radii are about two orders of magnitude larger than the Schwarzschild radius corresponding to each black hole mass. This suggests that the averaged circularization radius of infalling material indicates the size of an accretion disk formed around each black hole. We also note that the averaged circularization radii for the case of $\beta \neq 0$ is larger than those for the case of $\beta = 0$ in all models. This is because the distance between each black hole and its nearest inner edge of the circumbinary disk is longer in the misaligned system ($\beta \neq 0$) than that of the coplanar system ($\beta = 0$). The mass tidally stripped from the circumbinary disk does not directly accrete onto each black hole, but via the accretion disk around each black hole. We briefly discuss how the accretion disk evolve in section 5.

4. Simple Semi-Analytical Models

To understand in a simpler way how a variety of variation patterns in mass accretion rates arises, we construct a semi-analytical model. Since mass accretion occurs by the tidal stripping of gas from the inner edge of the circumbinary disk, and since the gravitational attraction force to the circumbinary disk is strongest when the distance between the black hole and its nearest inner edge of the circumbinary disk is at a minimum, we can guess the number of peaks per orbital period by calculating the minimum distance as a function of the orbital phase for parameter sets of all models.

The position vector of the inner edge of the circumbinary disk, $r_{in}$, can be expressed by

\[
r_{in} = \sqrt{\left( r_{in} \cos \theta \sin \gamma + \sin \theta \cos \gamma \cos \beta \right)^2 + \left( r_{in} \sin \theta \sin \gamma \cos \beta - \cos \theta \cos \gamma \right)^2 + \left( -r_{in} \sin \theta \sin \beta \right)^2},
\]

(4)

where the orbital phase $\phi$ and the azimuth angle $\theta$ are measured from $x$-axis and the descending node, respectively (see figure 4). From equations (2) and (4), we obtain the formula for the distance:

\[
d(\beta, \gamma, e, q; \phi) = |r_{in} - r_{i}| = \sqrt{4a^2 + r_i^2 - 4ar_i \sin \theta \cos (\phi - \gamma) \cos \beta - \cos \theta \sin (\phi - \gamma)},
\]

(5)

where the value of $\theta$ for a given $\phi$ is numerically chosen so as to give the minimum distance. Here, we assign $i = 2$, since the less-massive black hole can move on a larger extent, thereby getting closer to the circumbinary disk than the massive one.
Figure 5 shows the orbital phase dependence of the distance between the black hole and its nearest inner edge of circumbinary disk. There are four different parameters: orbital eccentricity \( (e) \), mass ratio \( (q) \), tilt angle \( (\beta) \), and azimuth of tilt \( (\gamma) \). The dependences of \( d(\beta, \gamma, e, q; \phi) \) normalized by the semi-major axis on these parameters are shown in panels (i)–(v). The fiducial parameters are those of Model A2; i.e., \((\beta, \gamma, e, q) = (\pi/6, \pi/2, 0.0, 1.0)\).

Panel (i) shows the dependence of the normalized distance, \( d(\beta, \gamma, e, q; \phi)/a \), for fixed values of \((q, e, \beta) = (1.0, 0.0, \pi/6)\). The solid line, dotted line, dashed line, and dash-dotted line represent \( d(\phi)/a \) for \( \gamma = \pi/2, 0, \pi/6, \) and \( \pi/4 \), respectively. We note that \( d(\phi)/a \) is the shortest twice per binary orbit, and that the phases of the minimum distance shift by varying the parameter \( \gamma \). Here, let us go back to see the difference between the mass accretion rate of Model A2 and that of Model A4. The phase difference between the solid line \((\gamma = \pi/2)\) and the dotted line \((\gamma = 0)\) is \( \pi/2 \). This supports our simulation results of Models A2 and A4.

Panel (ii) shows the \( \beta \)-dependence of the normalized distance for the fixed values of \((q, e, \gamma) = (1.0, 0.0, \pi/2)\). The solid line, dashed line, dash-dotted line, and dotted line represent \( d(\phi)/a \) for \( \beta = \pi/6, 0, \pi/4, \) and \( \pi/2 \), respectively. The normalized distance reaches its minimum value twice per binary orbital period, except for the case with \( \beta = 0 \), in which \( d(\phi)/a \) is constant. That is, the non-zero values of \( \beta \) are essential to produce two peaks per binary orbit in the mass-accretion-rate variations. This panel provides a reasonable explanation about why the results of Models A2–A4 are shown in the left panel of figure 2.

Panel (iii) shows the \( e \)-dependence of the normalized distance for the fixed values of \((q, \beta, \gamma) = (1.0, \pi/6, \pi/2)\). The solid line, dashed line, dash-dotted line, dotted line, and three-dotted line represent \( d(\phi)/a \) for \( e = 0, 0.1, 0.2, 0.3, \) and \( 0.4 \), respectively. The normalized distance has two round peaks for \( e = 0 \), but the larger \( e \), the deeper does become a hollow at around \( \phi = \pi \). Although we have mentioned that non-zero values of \( \beta \) give rise to the double peaked modulations in the mass accretion rates, this is not always the case when the orbital eccentricity is not zero. This will explain that the mass accretion rates of Models B2, B3, and B4 have a single peaked shape. We have also calculated the cases with \( \gamma = 0.0 \), reaching the same conclusion; i.e., single peaked variations are found for \( e \geq 0.3 \). This supports the result of Model B4.

Panel (iv) shows the \( q \)-dependence of the normalized distance between the secondary (less massive) black hole and its nearest inner edge of the circumbinary disk for the fixed values of \((e, \beta, \gamma) = (0, \pi/6, \pi/2)\). The solid line, dashed line, dash-dotted line, dotted line, and three-dotted line represent \( d(\phi)/a \) for \( q = 1.0, 0.1, 0.3, 0.5, \) and \( 0.7 \), respectively. The lower is the binary mass ratio, the shorter does \( d(\phi)/a \) become. Variations in the mass ratio do not change the number of peaks of the normalized distance. Obviously, the smaller is \( q \), the smaller does the mean distance become. This is because the semi-major axis of the secondary black hole \( r_2 \) in equation (3) increases as \( q \) decreases.

Panel (v) shows the same \( \gamma \)-dependence as those in panel (i), but for \( e = 0.1 \). This panel shows how the normalized distance changes with \( \gamma \) for the cases with an even smaller orbital eccentricity. The solid line, dotted line, dashed line, and dash-dotted line represent \( d(\phi)/a \) for \( \gamma = \pi/2, 0, \pi/6, \) and \( \pi/4 \), respectively. The solid line is the same as the dashed line of panel (iii). It is interesting to note that the double-peaked variation curve for the case of \( \gamma = 0 \) changes to a single-peaked one as \( \gamma \) increases.

### 5. Discussion

Periodic light variations are not generally observed from single SMBH systems. There is a report concerning
the presence of quasi-periodic oscillations (QPOs) in AGNs (Gierliński et al. 2008), but the QPOs do not mean a periodic occurrence of clear-cut flares, but rather gradual variations. Unlike single SMBH systems, periodic light variations can be seen in an eccentric binary black hole system because of periodically enhanced binary-disk interactions. In most previous studies, it was implicitly assumed that the binary orbital plane is aligned with the circumbinary disk plane. Since there are no strong reasons to believe that binary-disk alignment exists, it is natural to relax this assumption, and to examine what light variations are expected for a misaligned system. As shown in figure 2, a circular binary surrounded by a misaligned circumbinary disk exhibits a double-peak structure in the variations of the mass accretion rates, whereas an eccentric binary with a moderately large orbital eccentricity shows a single peak per orbit, even if the circumbinary disk is inclined from the binary orbital plane.

5.1. Inner Edge Radius of Misaligned Circumbinary Disks

The radius of the inner edge of the circumbinary disk is determined by the balance between the viscous torque of the prograde circumbinary disk and the tidal/resonant torque that acts on it (Artymowicz & Lubow 1994). In the case of moderate orbital eccentricity, the typical inner edge radius is at the 1 : 3 outer Lindblad resonance radius, which is estimated to be $\sim 2.1 a$. If the circumbinary disk is misaligned with respect to the binary plane, the resonant torque will be weaker than otherwise, which makes the inner edge smaller. In order to more precisely determine the size of the inner edge of the circumbinary disk, we need to investigate how the tidal/resonant torque acts on the misaligned circumbinary disk. This is also essential to calculate the circumbinary disk structure.

5.2. Triple Disk Model for OJ 287

A blazar, OJ287, exhibits quasi-periodic optical outbursts with a 12-year interval. In order to explain such a quasi-periodic nature, a binary black hole model was firstly proposed by (Sillanpää et al. 1988). Lehto and Valtonen (1996) subsequently proposed a modified model is which a less-massive black hole orbits around a more-massive one while undergoing relativistic precession and impacts twice per binary orbit on the accretion disk around the more-massive black hole. This model can explain the quasi-periodicity of outbursts with a double-peak structure at a few-year interval.
the circumbinary disk around an eccentric binary composed of black holes with different masses is misaligned by a relatively small tilt angle \(0 \lesssim \beta \lesssim \pi/6\).

5.3. Precessions of Misaligned Circumbinary Disks

The binary-disk interaction also gives rise to a precession of the circumbinary disk (MacFadyen & Milosavljević 2008; Hayasaki & Okazaki 2009; D’Orazio et al. 2012). With the assumption that \(r \gg a\) and \(M_2 < M_1\), the precession frequency of the misaligned circumbinary disk is given (e.g., Nixon et al. 2011b) as

\[
P_{\text{prec}} \approx \frac{4}{3} q \left(\frac{r}{a}\right)^{7/2} \frac{1}{\cos \beta} \left(-\pi/2 < \beta < \pi/2\right),
\]

where \(P_{\text{prec}}\) and \(P_{\text{orb}}\) are the precession period and binary orbital period, respectively. It is clear that \(P_{\text{prec}}\) is much longer than \(P_{\text{orb}}\), e.g., \(P_{\text{prec}} \sim 100 P_{\text{orb}}\) for \(q = 0.1\) and \(r = 2a\). This can produce a light variation in the circumbinary disk with a beat period of \(P_{\text{beat}}\), where it is expressed by \(1/P_{\text{beat}} = 1/P_{\text{orb}} - 1/P_{\text{prec}}\). From equation (6), we obtain the beat period normalized by the binary orbital period,

\[
P_{\text{beat}} = \left[1 - \frac{3}{4} \left(\frac{r}{a}\right)^{7/2} q \frac{1}{1 + q \cos \beta}\right]^{-1},
\]

which is slightly longer than the binary period, as long as \(r \gg a\). The beat period for the case of \(q = 0.1\) and \(r = 2a\) is, for example, approximately 1.01 \(P_{\text{orb}}\). In a subsequent paper, we will take up this topic in more detail.

5.4. Luminosity Variations

Another interesting topic is how the mass finally accretes onto each black hole via an accretion disk from the circumbinary disk. In subsection 3.3, we show that the infalling material is circularized around each black hole (the estimated circularization radii are given in Table 3). This leads to the formation of a triple disk system, which is composed of two accretion disks around black holes, and one circumbinary disk surrounding them (Hayasaki et al. 2008). However, it is poorly known what structure each accretion disk has and how it evolves. Once these two accretion disks are formed by gas supply from the circumbinary disk, they viscously evolve, and gas in the accretion disk finally accretes onto each black hole after the viscous timescale. Assuming that the accretion disk is the standard disk, the viscous timescale measured at the circularization radius is much longer than the binary orbital period. The precise shape of the light curves may be different from that of variations in the mass accretion rates. It is interesting to examine the basic properties of radiations emitted from each accretion disk.

6. Conclusions

We have carried out numerical simulations of accretion flows from a circumbinary disk that is inclined from the binary orbital plane, in order to examine to what extent the basic properties of the mass-accretion-rate variations may alter, compared with the coplanar cases. Our main conclusions are summarized as follows:

1. We find that the mass accretion rates exhibit a double peak per binary orbit in a circular binary system, when the circumbinary disk is misaligned with the binary orbital plane. This is because each black hole passes across the circumbinary disk plane twice per binary orbit, and then attracts the gas there. This double-peak nature of the mass accretion rates is also independent of the azimuth of the tilt. The tilt angle is one of the important orbital parameters to determine the variation patterns of radiations emitted from a binary black hole system.

2. The orbital eccentricity remains to be an important orbital parameter to produce single sharply peaked variations per binary orbit in the mass accretion rates, even in a misaligned circumbinary disk system. This is because each black hole is closest to the inner edge of the circumbinary disk once per binary orbit in most cases. The simple semi-analytic model (see section 4) predicts that this single peak nature is independent of both the tilt angle and the azimuth of tilt, as long as \(e \gtrsim 0.3\).

3. In the case of an eccentric binary composed of black holes with different masses, the less-massive black hole can get closer to the circumbinary disk than the massive one, thus tidally splitting gas from its inner edge, but the created gas flows are comparably captured by both black holes with a short time delay. The superposed accretion rates show periodic outbursts with an apparent double-peaked structure with a short interval.

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