MaBµlS-2: high-precision microlensing modelling for the large-scale survey era

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ABSTRACT

Galactic microlensing datasets now comprise in excess of $10^4$ events, and with the advent of next generation microlensing surveys that may be undertaken with facilities such as the Vera Rubin Observatory (formerly LSST) and Nancy Grace Roman Space Telescope (formerly WFIRST), this number will increase significantly. So too will the fraction of events with measurable higher order information such as finite source effects and lens–source relative proper motion. Analysing such data requires a more sophisticated Galactic microlens modeling approach. We present a new second-generation Manchester–Besançon Microlensing Simulator (MaBµlS-2), which uses a version of the Besançon population synthesis Galactic model that provides good agreement with stellar kinematics observed by HST towards the bulge. MaBµlS-2 provides high-fidelity signal-to-noise limited maps of the microlensing optical depth, rate and average timescale towards a 400 deg$^2$ region of the Galactic bulge in several optical to near-infrared pass-bands. The maps take full account of the unresolved stellar background as well as limb-darkened source profiles. Comparing MaBµlS-2 to the efficiency-corrected OGLE-IV 8,000 event sample shows a much improved agreement over the previous version of MaBµlS, and succeeds in matching even small-scale structural features in the OGLE-IV event rate map. However, there remains evidence for a small under-prediction in the event rate per source and over-prediction in timescale. MaBµlS-2 is available online (www.mabuls.net) to provide on-the-fly maps for user supplied cuts in survey magnitude, event timescale and relative proper motion.

Key words: gravitational lensing: micro – methods: numerical – Galaxy: structure – Galaxy: kinematics and dynamics – planets and satellites: detection

1 INTRODUCTION

Gravitational microlensing is an important tool in exoplanet science, courtesy of its ability to detect exoplanets beyond the snowline (Batista 2018); an under-surveyed portion of the orbital parameter space that is important for testing planet formation theories (Qi et al. 2013).

Microlensing has also proven to be an important tool in the analysis of Galactic structure (Moniez 2010) due to its ability to detect faint and low-mass objects such as M-dwarf stars and brown dwarfs that are typically undetectable beyond a few kiloparsecs. Accurate Galactic microlensing models also offer the ability to provide important prior constraints on the modelling of individual lens systems.

Theoretical models of microlensing optical depth $\tau$, Einstein crossing timescale $\langle t_E \rangle$ or microlensing rate $\Gamma$ can be calibrated against efficiency-corrected observational data from ongoing large-scale microlensing surveys such as MOA-2 (Sumi 2010), OGLE-IV (Udalski et al. 2015) and KMTNet (Park et al. 2018). The dependency of $\tau$ on the density distribution of the Galaxy makes this a good probe of Galactic structure, allowing us to compare empirical results such as from Mróz et al. (2019) to simulation results. The microlensing rate $\Gamma$ probes not just the Galactic density distribution but also the lens and source kinematics and the lens mass function. It is therefore an important tool when informing future exoplanet microlensing surveys, such as the Nancy Grace Roman Space Telescope (hereafter NGRST, formerly WFIRST) (Bennett et al. 2018) and the Vera Rubin Observatory (formerly LSST) (Street et al. 2018), as regions of high $\Gamma$ will naturally yield more microlensing events and subsequently exoplanet detections. Finally, the Einstein
radius crossing time \( (\tau) \) is the characteristic timescale of a microlensing event and can be extracted directly from microlensing light curves. Measuring the timescale distribution from a statistically large sample of light curves can provide a good probe for estimating the mass distribution of lenses. Such data was used to calibrate a brown dwarf initial mass function (IMF) for microlensing simulations by Awiphan et al. (2016) as well as providing tentative evidence for possible populations of free-floating planets (FFPs) (Mróz et al. 2017).

Generating parameter maps from microlensing observations is difficult due to the requirement of a detection efficiency for events of a given timescale, which typically requires extensive Monte-Carlo simulations (Sumi et al. 2011). Survey sky coverage, sampling rate and lifetime are also major limiting factors when generating empirical microlensing maps. In the work by Sumi et al. (2013), maps with resolutions of 1 degree were generated using around 470 events from the MOA-II survey. More recently, the OGLE-IV survey (Mróz et al. 2019) used around 8000 events to generate microlensing maps with a resolution of 10 arcminutes. Both analyses also considered two subsets of their microlensing data, namely red-clump giant (RCG) sources, which are bright and easily resolved, and difference image analysis (DIA) sources, which include fainter strongly blended source stars that may only be visible close to the magnification peak. Each of these subsets provide separate, but strongly correlated, measures of \( \tau \).

The most detailed study to date comparing theoretical microlensing models and observational data was presented by Awiphan et al. (2016) using the Manchester–Besançon Microlensing Simulator (MaB\( lS\))^1. MaB\( lS\) used the Besançon Galactic populations synthesis model (Robin et al. 2012a) to produce detailed microlensing maps for different photometric bands. All of the required ingredients for microlensing rate calculations, including lens mass, kinematic and density distributions, as well as source magnitude, kinematic and density distributions, are supplied by the Besançon model, together with a fully calibrated 3D extinction model (Marshall et al. 2006). Comparison of MaB\( lS\) with MOA-II microlensing results reported evidence for a mass deficit in the Galactic bar as the model significantly under-predicted the observed optical depth. However Sumi & Penny (2016) subsequently found a systematic problem with the way in which source stars were counted by MOA-2 in constructing their observed maps. Correcting for this, Penny & Sumi found reasonable agreement between MOA-2 data and MaB\( lS\) predictions.

While our first generation MaB\( lS\) simulation provides a good level of agreement with MOA-2 data, the more recent OGLE-IV dataset provides a much greater challenge, involving a sample that is 17 times larger and allowing much higher fidelity observational maps. To provide a realistic comparison to such large datasets, we have developed a new generation of MaB\( lS\). MaB\( lS\)-2 incorporates several important improvements, including calculation for fixed signal-to-noise ratio, accounting explicitly for unresolved stellar backgrounds of the event signal-to-noise, and a more detailed treatment of finite source size effects that are important for low mass FFPs. There is some independent evidence for a nearby FFP counterpart populations (Liu et al. 2013), as well as individual candidates discovered through microlensing (OGLE Collaboration et al. 2019), so this will be a high priority area of study for future surveys such as NGRST. Whilst the previous version of MaB\( lS\) could calculate rates and optical depths for arbitrary magnitude and timescale cuts, MaB\( lS\)-2 additionally allows for arbitrary cuts in lens–source relative proper motion. This functionality makes it much more useful for providing model constraints on individual events where proper motion measurements are available.

The paper is structured as follows: Section 2 outlines the Galactic model used to simulate the microlensing sources and lenses used in this work, including the stellar kinematics and mass functions. Section 3 details the method behind generating the microlensing \( \tau \), \( (\tau) \) and \( \Gamma \) maps, including the equations used for calculating these parameters from a discreet stellar population and lists the first order effects introduced to the simulation in this work, such as finite source effects and background light contributions. The results and discussion of the simulation are shown in section 4, with microlensing parameter maps generated with different constraints, as well as a comparison with the OGLE-IV data compiled by Mróz et al. (2019). Conclusive remarks are given in section 5.

2 THE BESANÇON GALACTIC MODEL

The approach of using Galactic population synthesis simulations to model microlensing was first demonstrated by Kerins et al. (2009) using the Besançon Galactic Model (BGM) (Robin et al. 2003; Marshall et al. 2006; Robin et al. 2012a). This method has also been used to make detailed predictions for exoplanet microlensing yields for forthcoming space missions such as Euclid (Penny et al. 2013) and NGRST (Penny et al. 2019) using a more recent version of the BGM (variant BGM1106). Recently, a population synthesis based microlensing model has been developed to study microlensing due to black holes (Lam et al. 2020).

Penny et al. (2019) noted that the BGM1106 model does not provide a close match to the HST stellar kinematics study of Clarkson et al. (2008). Awiphan et al. (2016) presented a more recent version of the model (variant BGM1307) and compared it to a large ensemble of 470 events from the MOA-II survey (Sumi et al. 2013). Awiphan et al. (2016) concluded that there was a reasonable level of agreement once allowance was made for brown dwarfs that are missing from the BGM. However, the model appeared to significantly under-predict the observed rate from the Galactic bulge. Sumi & Penny (2016) subsequently showed that at least part of the cause of this discrepancy was due to under-counting of source stars within the MOA-2 analysis, and correcting for this resulted in much better agreement with the model. After investigation of a number of different BGM variants we have decided to use BGM1307 as the basis for MaB\( lS\)-2.

The BGM separates the Galaxy into four components; the thin disk, thick disk, bar and halo, each with their own stellar initial mass functions (IMFs), star formation rates (SFRs), kinematics and ages. Interstellar extinction is in-

1 http://www.mabuls.net/
cluded for UBVRJHK photometric bands using a 3D extinction model from Marshall et al. (2006). The solar position relative to the Galactic centre and plane in the BGM is taken to be $R_0 = 8$ kpc and $z_0 = 15$ pc, respectively.

### 2.1 The Thin Disk

The thin disk component is sub-divided into seven age groups, each with their own luminosity and effective temperature distributions. The mass density is modelled by the difference of two exponentials in cylindrical polar coordinates $(r, z)$; one representing the body of the disk with scale length $R_d = 2170$ pc and the other representing a central hole with scale length $R_h = 1330$ pc (Robin et al. 2012b). It is given by

$$
\rho(r, z) = \rho_0 \left\{ \exp \left(-\frac{1}{4} \frac{a^2}{R_d^2} \right) - \exp \left(-\frac{1}{4} \frac{a^2}{R_h^2} \right) \right\},
$$

where $\rho_0$ is the component normalisation density, $a^2 = r^2 + (\xi/\epsilon)^2$ and $\epsilon$ the axis ratio of the ellipsoid. The SFR is taken to be constant across the whole disk. The IMF is given by a broken power law, which is the same across all age groups

$$
\xi(M) \propto \begin{cases} 
M^{-1.6}, & M \leq 1M_\odot \\
M^{-3}, & M > 1M_\odot 
\end{cases}, \quad \text{with } 0.079M_\odot \leq M < 1M_\odot.
$$

Each age group of the thin disk has its own velocity dispersion, to mimic the secular evolution, and follows the relation from Gomez et al. (1997); these average to $\sigma_{UVW} = (30, 20, 13)$ km s$^{-1}$, with a maximum of $\sigma_{UVW} = (43, 28, 18)$ km s$^{-1}$ for the oldest component. The rotational velocity of the LSR is 240 km s$^{-1}$, with the rotation curve taken from Caldwell & Ostriker (1981). The peculiar motion of the sun relative to the LSR is assumed to be $v_0 = (11, 12, 7)$ km s$^{-1}$, following Schönrich et al. (2010).

### 2.2 The Thick Disk

The thick disk is an older and less dense component of the model. It is most prevalent at latitudes beyond the scope of the thin disk at $|b| > 9^\circ$ and has a modified exponential density law with scale height $h_z = 533.4$ pc, scale length $h_R = 2355.4$ pc and break distance $\xi = 658$ pc,

$$
\rho(r, z) = \rho_0 \exp \left( \frac{r_0 - r}{h_R} \right) \begin{cases} 
1, & z \leq \xi \\
\left( \frac{2h_z}{2h_z + r_0} \right) \exp \left( \frac{2h_z}{2h_z + r_0} \right) \left( \frac{\xi - z}{h_z} \right), & z > \xi 
\end{cases},
$$

where $\rho_0$ is the local stellar density with $(r_0, z_0)$ representing the coordinates of the sun. The star formation history is modelled by a single burst 10 Gyr ago (Robin et al. 2014). We use the Bergbusch & Vandenbreg (1992) luminosity function with an isochrone of 10 Gyr, a mean metallicity of -0.5 dex and alpha enhanced to simulate this population. The IMF of the thick disk is given by a single power law of the form

$$
\xi(M) \propto M^{-1.5}, \quad M > 0.154M_\odot.
$$

The velocity dispersion of the thick disk is taken to be $\sigma_{UVW} = (67, 51, 42)$ km s$^{-1}$ with a rotational velocity of 176 km s$^{-1}$.

### 2.3 The Bar

The bar population dominates Galactic densities at low latitudes, $|b| < 5^\circ$. The bar is represented by a Padova isochrone of 8 Gyr, with a solar metallicity. It is modelled as a triaxial ellipsoid with scale lengths $x_0 = 1.46$ kpc, $y_0 = 0.49$ kpc, $z_0 = 0.3$ kpc, with the major axis ($x$) offset from the sun-Galactic centre axis by 12.89° (Robin et al. 2012b). Its pitch and roll is set to be zero. Its density distribution is given in galactocentric cartesian coordinates as

$$
\rho(x, y, z) = \rho_0 \text{sech}^2 \left( -R_b(x, y, z) \right),
$$

where $R_b(x, y, z) = \sqrt{x^2 + y^2}$ within the cutoff radius $R_C = 3.43$ kpc,

$$
\sigma_{xy} = \sqrt{x^2 + y^2} \leq R_C
$$

The parameters $C_\parallel$ and $C_\perp$ control the ‘disky’ or ‘boxy’ shape of the ellipsoid; for this version of the BGM, a boxy bar shape was used, with $C_\parallel = 0.5$ and $C_\perp = 3.007$. The bar IMF was modelled with a broken power law,

$$
\xi(M) \propto \begin{cases} 
M^{-1.5}, & M \leq 0.15M_\odot \\
M^{-2.3}, & M > 0.15M_\odot 
\end{cases}, \quad \text{with } 0.15M_\odot \leq M < 0.7M_\odot.
$$

The bar has the largest velocity dispersion with $\sigma_{UVW} = (150, 115, 100)$ km s$^{-1}$. Its kinematics have been modelled by an N-body simulation, outlined in Gardner et al. (2014).

### 2.4 The Halo

The halo component mainly contributes at high latitudes and for magnitudes fainter than $m \sim 18$. Stars in the halo are generated with an age of 14 billion years and naturally have a low metallicity, with [Fe/H] = -1.78 (Robin et al. 2014). The density profile of the halo is given by a power law,

$$
\rho(r, z) = \rho_0 \left( \frac{r^2 + (z/\epsilon)^2}{\epsilon^2} \right)^{-1.695},
$$

where $\epsilon = 0.768$ is the axis ratio of the ellipsoid. The halo IMF follows a single power law,

$$
\xi(M) \propto M^{-1.5}, \quad M > 0.085M_\odot.
$$

The halo has a velocity dispersion of $\sigma_{UVW} = (131, 106, 85)$ km s$^{-1}$. 

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MNRA 000, 1-10 (2020)
Figure 1. The kinematics of BGM1307, in comparison to HST data (Clarkson et al. 2008). The top left shows the distributions of proper motion in Galactic latitude and longitude ($\mu_l, \mu_b$) for stars simulated by the BGM (small points) for $H < 25$, relative to the mean motion of the bar population. Bar stars are shown in red with disk stars shown in blue. A subset of these points are shown as circles; these represent ‘bar proxy’ and ‘disk proxy’ star samples, meant to replicate the selection of stars from Clarkson et al. (2008). The bright red and blue circles show the 1-sigma contours of the HST stars, while the dark red and blue represent the corresponding BGM contours. The HST blue/disk population has a central offset ($\mu_l, \mu_b$) = (3.24, −0.81) mas and width ($\sigma_l, \sigma_b$) = (2.2, 1.3) km s$^{-1}$ compared to the BGM values of ($\mu_l, \mu_b$) = (2.58, −0.61) mas and width ($\sigma_l, \sigma_b$) = (2.23, 1.28) km s$^{-1}$. The widths of the red/bar population from HST data is ($\sigma_l, \sigma_b$) = (3.0, 2.8) km s$^{-1}$ with a BGM value of ($\sigma_l, \sigma_b$) = (3.12, 2.2) mas. The top right and bottom left plots show the variation of $\mu_l$ and $\mu_b$ with distance from the observer. The dotted lines represent the Galactic centre at 8 kpc. In the bottom right is the colour-magnitude diagram of the simulated stars in the Sloan $r$ and $i$ filters, which were used to approximate the Hubble F606W and F814W filters.

Table 1. Summary of the dispersion velocities for each component in BGM1307, using the galactic (UVW) coordinate system. The numbers listed for the thin disk are averaged over each age group sub-division.

| Component | $\sigma_U$ km s$^{-1}$ | $\sigma_V$ km s$^{-1}$ | $\sigma_W$ km s$^{-1}$ |
|-----------|-----------------|-----------------|-----------------|
| Thin Disk | 30              | 20              | 13              |
| Thick Disk| 67              | 51              | 42              |
| Bar       | 150             | 115             | 100             |
| Halo      | 131             | 106             | 85              |

2.5 Comparison with HST kinematics

Figure 1 follows the kinematics comparison of BGM1106 by Penny et al. (2019), this time applied to BGM1307. The model predictions are compared to stellar kinematics derived from HST data presented by Clarkson et al. (2008). Figure 1 shows stellar proper motions along the Galactic longitude and latitude ($\mu_l$ and $\mu_b$) directions produced by BGM1307 together with 1-$\sigma$ contours for the Clarkson et al. (2008) study. The alignment of the 1$\sigma$ contours shows very good agreement between BGM1307 predicted and HST observed kinematics and is a clear improvement over BGM1106. The kinematics for BGM1307 are summarised in Table 1.

2.6 Low mass stars and brown dwarfs

Low mass M dwarfs are not included in some components of the BGM, as evidenced by some of the IMF mass limits discussed above. Brown dwarfs are also not included in the
shown in figure 2. Not all the parameters were used in this minimisation on MOA II timescales, but does the addition of brown dwarfs and still matches the data quite well. We ensure that we sample rarer bright stars sufficiently well. To this end, for each line of sight, four catalogues were generated, each covering a fixed range of apparent magnitudes shown in Table 2. The solid angles of each magnitude interval for each line of sight were calibrated to produce ~ 10,000 stars per catalogue resulting in 40,000 stars in total, per line of sight. Our microlensing calculations must therefore be appropriately re-scaled to account for these solid angle choices.

3 SIMULATION METHOD

3.1 Microlensing parameters

The simplest gravitational microlensing scenario is the point-source point-lens (PSPL or Paczynski) model (Paczynski 1986). In this case, both the lens and source are considered single objects with an infinitesimal angular size; this is accurate to describe most microlensing events observed to date. The magnification due to the lens as a function of time \( A(t) \) at a specific normalised angular impact parameter \( u(t) \), is given by

\[
A(t) = \frac{u(t)^2 + 2}{u(t)^2 + 4}.
\]

This IMF slope was acquired by performing a \( \chi^2 \) minimization over a range of slope values in the interval [-0.9, 1.0] in 0.1 increments, as shown in figure 2. The minimisation used 258 evenly spaced points in the OGLE timescale map by Mróz et al. (2019). A modified version of the MaBµlS algorithm, outlined in section 4.2, was used to ensure event selection criteria were as faithful as possible to the OGLE events. Although this work is comparing the MaBµlS-2 prediction of \( \theta_E \) to the data after fitting a brown dwarf mass slope to minimize the \( \chi^2 \) between the two, it is worthy of note that the optical depth is not significantly affected by the addition of brown dwarfs and still matches the data quite well (section 4.2). The previous version of MaBµlS also attempted this minimization on MOA II timescales, but does not achieve as successful a result as the new simulation, as shown in figure 2.

### Table 2. The range of apparent magnitude in each magnitude interval for simulated stars is shown. Low mass objects from section 2.5 only appear in interval 4 as they are assigned an apparent magnitude of 99. Microlensing sources are only considered from intervals 1 \( \rightarrow \) 3.

| Interval No. | Magnitude Range |
|--------------|-----------------|
| 1            | 0 \( \leq K < 15 \) |
| 2            | 15 \( \leq K < 20 \) |
| 3            | 20 \( \leq K < 24 \) |
| 4            | 24 \( \leq K < 99 \) |

This IMF slope was acquired by performing a \( \chi^2 \) minimization over a range of slope values in the interval [-0.9, 1.0] in 0.1 increments, as shown in figure 2. The minimisation used 258 evenly spaced points in the OGLE timescale map by Mróz et al. (2019). A modified version of the MaBµlS algorithm, outlined in section 4.2, was used to ensure event selection criteria were as faithful as possible to the OGLE events. Although this work is comparing the MaBµlS-2 prediction of \( \theta_E \) to the data after fitting a brown dwarf mass slope to minimize the \( \chi^2 \) between the two, it is worthy of note that the optical depth is not significantly affected by the addition of brown dwarfs and still matches the data quite well (section 4.2). The previous version of MaBµlS also attempted this minimization on MOA II timescales, but does not achieve as successful a result as the new simulation, as shown in figure 2.

\[
\zeta(M) \propto M^{-0.1}, \quad 0.001M_{\odot} < M < 0.079M_{\odot}. \quad (11)
\]

This magnification due to the lens as a function of time \( A(t) \) at a specific normalised angular impact parameter \( u(t) \), is given by

\[
A(t) = \frac{u(t)^2 + 2}{u(t)^2 + 4}. \quad (12)
\]

for a lens mass \( M \) at a distance \( D_l \) and a source at a distance \( D_s \). The microlensing optical depth \( \tau \)

\[
\tau = \frac{4\pi G}{c^2} \int_0^{D_l} \rho(D_l)D_l(D_s - D_l) dD_l. \quad (15)
\]

for a continuous lens mass-density distribution \( \rho(D_l) \). For a discrete catalogue of \( N_s \) source stars and \( N_l \) lens stars we can instead use (Kerins et al. 2009; Awiphan et al. 2016):

\[
\tau = \frac{\sum_{s}^{N_s} \sum_{l}^{N_l} D_l D_s \Theta_{\text{max}}^2 \frac{1}{\Omega_{\text{max}}}}{\sum_{s}^{N_s} (w_s^2) \frac{1}{\Omega_s}}. \quad (16)
\]

The variable \( \Theta_{\text{max}} \equiv \Theta(A_{\text{min}}) \) represents the largest impact parameter that permits a magnification \( A \geq A_{\text{min}} \). It is common to adopt an absolute threshold maximum impact parameter \( u_t \), where often \( u_t = 1 \) is adopted. We can incorporate this into the definition of \( \Theta_{\text{max}} \) by defining \( \Theta_{\text{max}} = \min[u(A_{\text{min}}, u_t)] \). The variable \( (w^2) \) is the 2nd moment of the source weight \( w \), which counts the effective number of sources; in the case
of a source resolved by the telescope at baseline, this will be equal to unity, however for DIA sources which are only visible during magnification, \( w \) can drop below unity, resulting in a smaller contribution to the total optical depth. In general, the \( \mu \)th moment of \( w \) is given by
\[
\langle w^\mu \rangle = \frac{\sum_{i} N_i w_i \mu_{\text{rel}} D^2_i \theta_E^2 \Omega_i}{\sum_{i} N_i \mu_{\text{rel}} D^2_i \theta_E^2 \Omega_i},
\]
where \( \mu_{\text{rel}} \) is the lens-source relative proper motion. One way to consider equation 16 is as a summation over the 'sensitivity regions' in the sky formed by circles of radius \( u_{\text{max}} \theta_E \), averaged over all possible sources, as a ratio to the total survey solid angle. The rate-weighted average Einstein radius crossing time \( \langle \tau \rangle \) for a line of sight is given by
\[
\langle \tau \rangle = \frac{\sum_{i} N_i \sum_{j, D_i>D_j} w_i \mu_{\text{rel}} D^2_j \theta_E^2 \Omega_j}{\sum_{i, j, D_i>D_j} N_i N_j \mu_{\text{rel}} D^2_i D^2_j \theta_E^2 \Omega_i \Omega_j}.
\]
Similarly, the rate-weighted average relative proper motion \( \langle \mu_{\text{rel}} \rangle \) for a line of sight is given by
\[
\langle \mu_{\text{rel}} \rangle = \frac{\sum_{i} N_i \sum_{j, D_i>D_j} w_i \mu_{\text{rel}} D^2_j \theta_E^2 \Omega_j}{\sum_{i, j, D_i>D_j} N_i N_j \mu_{\text{rel}} D^2_i D^2_j \theta_E^2 \Omega_i \Omega_j}.
\]
Finally, the microlensing rate per source star is simply given by
\[
\Gamma = \frac{2}{\pi} \langle \tau \rangle / \langle \mu_{\text{rel}} \rangle.
\]

### 3.2 Finite source effects

In cases where the angular size of the source star is comparable to its impact parameter with the lens, finite source effects may become evident in the light curve, as different parts of the source are magnified by appreciably different amounts. This typically results in a flattening of the light curve peak, as the singularity in equation 12 is avoided as \( u \to 0 \). This has an important consequence for the optical depth, as \( u_{\text{max}} = u(A_{\text{min}}) \) can no longer be inverted from equation 12, but requires a numerical solution. The treatment by Lee et al. (2009) was used to evaluate \( u_{\text{max}} \) as a function of both \( A_{\text{min}} \) and the angular source radius normalised to the Einstein radius, \( \rho \). A lookup table was generated with a resolution of 100 \( \times \) 100 \( u_{\text{max}} \) evaluations over the range 0.01 \( \leq \rho \leq 5 \) and \( A_{\text{base}} \leq A_{\text{min}} \leq 200 \), with \( A_{\text{base}} = 1.01695 \) (the PSPL magnification for \( u = 3 \)); bilinear interpolation was then used to extract a continuous value for use in the simulation.

For a uniform disk, there is a maximum possible magnification \( A_{\text{cut}} \) which a microlensing event can reach for a particular \( \rho \) given by
\[
A_{\text{cut}} = A(\rho, u = 0) = \sqrt{1 + \frac{4}{\rho^2}}.
\]
Consequently, simply using \( A_{\text{min}} \) for the vertical axis of the \( u_{\text{max}} \) distribution results in most of the grid being zero due to finite source effects preventing magnifications greater than \( A_{\text{cut}} \) from being achieved. To account for this, a magnification threshold proxy parameter \( \alpha \) was used, where
\[
\alpha = \frac{\ln(A_{\text{min}} - A_{\text{base}} + 1)}{\ln(A_{\text{cut}} - A_{\text{base}} + 1)},
\]
which scales the top of the distribution to \( A_{\text{cut}} \) and also more evenly distributes the information, resulting in a more slowly varying derivative, as evident from figure 3.

As \( \rho \to 1 \), stellar limb darkening becomes a significant factor. To deal with this, a linear limb darkening coefficient was introduced to the numerical calculation, which varies as a function of observation wavelength and effective temperature. UBVIJHK linear LDCs from Claret & Bloemen (2011) were used to construct curves in each wavelength band (excluding \( L \) band, which reused \( K \)-band data) as a function of effective temperature, which were then interpolated in real time during the simulation for each source star. To account for this in the \( u_{\text{max}} \) grid, a simple scaling of \( \alpha \) and \( \rho \) were performed before the bilinear interpolation to approximate the effects of limb darkening. This was necessary to reduce computation time, as no analytical version of equation 21 exists with limb-darkening considerations.

### 3.3 Background light contributions

The calculation of \( A_{\text{min}} \) is derived from the event selection criterion that at peak magnification, a signal-to-noise ratio \( S/N \geq 50 \) must be achieved. In the previous version of MaBILIS, calculations were based only on a source magnitude threshold cut, not a survey \( S/N \) cut. For our survey \( S/N \) cut we assume three component contributions: photons from the source \( N_{\text{src}} = t_{\text{exp}} A_{\text{min}} 10^{-0.4(zp_m-zp)} \), a uniform sky background \( N_{\text{sky}} = t_{\text{exp}} \Omega_{\text{psf}} 10^{-0.4(zp_m-zp)} \) and the light contribution from all other stars under the source’s PSF, \( N_{\text{BG}} = t_{\text{exp}} \Omega_{\text{psf}} \sum_{i} 10^{-0.4(zp_i-zp)} \). The combination of the zero-point magnitude \( m_{zp} \) and exposure time \( t_{\text{exp}} \) is fixed to provide 4% photometric precision for an assumed seeing of \( \Omega_{\text{psf}} = 1 \) arcsec at a telescope limiting magnitude \( m_{\text{lim}} \) (Ban et al. 2016). 4% photometric precision is therefore achieved for \( m_{m} = m_{zp} \) if \( t_{\text{exp}} = 625 \) s. More generally, the \( S/N \) at
Table 3. The sky brightness as a function of Johnson-Cousins filter.

| Filter | Sky Brightness / mag arcsec\(^{-2}\) |
|--------|--------------------------------------|
| U      | 22.28                                |
| B      | 22.64                                |
| V      | 21.61                                |
| R      | 20.87                                |
| I      | 19.71                                |
| J      | 16.50                                |
| H      | 14.40                                |
| K      | 13.00                                |

The particular value of $\mu_{sky}$ is band dependent. Ground-based values for Paranal Observatory from Patat (2003) are adopted. These values are shown in table 3. We take $t_{exp} = 625$ s and use the survey limiting magnitude for 4% photometric precision, $m_{lim}$, for $m_{zp}$.

3.4 Error calculation

The treatment of the parameter errors has been made more rigorous; previously, the error was estimated by distributing all sources into two bins and taking the difference of the two results as an approximation for the error. This ignored the variance due to the lenses, which were kept constant across both bins, as well as the inaccuracies of estimating the standard deviation with only two source bins. To account for both shortcomings, ten bins were used for both sources and lenses. The error for a particular source, $\sigma_{t}$, was calculated by distributing the lenses randomly into the ten lens bins and calculating the standard deviation. At this stage in the error propagation, only the numerator terms of equations 16 and 18 were calculated, as the numerators and normalisation terms must be summed up separately during map generation. After obtaining the parameter values $x_{i}$, error estimates $\sigma_{x}$ and normalisation $(w_{i})$ for a particular source, the ten source bins were then populated by looping through all sources. The sums over $x_{i}$, $\sigma_{x}$, and $(w_{i})$ in source bin $i$ are $x_{i}$, $\sigma_{x}$, and $(w_{i})$ respectively. The final error value on a parameter $x$ is

$$
\epsilon = \sqrt{\frac{\sum_{i}(x_{i}^{2} + \sigma_{x}^{2})(w_{i})}{\sum_{i}(w_{i})}} + \frac{\sum_{i} x_{i} (w_{i})}{\sum_{i} (w_{i})}.
$$

3.5 Map generation method

The simulation distributes the resulting parameters, normalisations and errors into 3-dimensional bins of source star magnitude $m_{s}$ (10 bins), average timescale $\langle t_{E} \rangle$ (10 bins) and average relative proper motion $(\mu_{rel})$ (5 bins). The bin edges for apparent magnitude were uniformly sampled between 12 and 23. For average timescale, the 10\(^{th}\) percentiles of the $I$-band timescale distribution multiplied by the timescale were found; the multiplication by timescale was implemented to account for the fact that event weights for optical depth are linearly proportional to their corresponding timescale. The bin edges for $\mu_{rel}$ were found by calculating the 20\(^{th}\) percentiles of the $I$-band distribution.

The output files contain the integrals of optical depth and timescale up to their corresponding upper bin edges; this was to eliminate the necessity of looping over all bins interior to a user-specified cut in each of the three dimensions, allowing the map generator to simply add and subtract the integral limits. To account for a user specified cut in any direction which does not lie on bin edges (as would be the case most of the time), 6D interpolation is performed using the up-to 64 integrals bounding the user’s selected parameter ranges; this is the product of 3D interpolation in the bin containing the upper bound and the bin containing the lower bound. The dimensionality of the interpolation method is automatically reduced if the user picks bounds on a bin edge.

This method of interpolation is more accurate than the previous version of MaBulS, which used 2-dimensional interpolation, using $m_{s}$ and $(\mu_{rel})$ cuts, over the parameter values, errors and normalisations, before integrating over these interpolants. Switching the order of operations to interpolating over the integrals prevents inaccuracies accumulated by the interpolation process, as integrals confined to bin edges are exact; as such, interpolating over them ensures that the result will vary smoothly between the various bounds and is also more reliable at preserving timescale ranges to be within the range specified.

4 RESULTS

Examples of microlensing maps are shown below. The parameters $\tau$, $(\langle t_{E} \rangle)$ and $\Gamma$ are simulated over the region $|b| \leq 10^\circ$ or $|l| \leq 10^\circ$, centred on the Galactic coordinates origin, $l, b = 0^\circ$. The microlensing event rate $\Gamma$ is shown as either the event rate per star or the event rate per square degree. In figures 5, 6 and 7, these maps are shown in $V, I$ and $K$ band respectively, with their corresponding errors. The parameter range is chosen to be the same as from Mróz et al. (2019), with $m_{s} < 21$ (in each band) and $(\mu_{rel}) < 300$ days. The full range of $(\mu_{rel})$ is used in those graphs. The increased effect of dust in the Galactic plane is evident at shorter wavelength, as well as the change in optical depth.

Later graphs show various cuts in $m_{s}$, $(\langle t_{E} \rangle)$ and $(\mu_{rel})$ in $\tau$ and $(\langle t_{E} \rangle)$. The effects of selecting different survey magnitude ranges on the optical depth and timescale are shown in figures 8 and 9. The effect of reduced survey sensitivity (brighter $m_{lim}$) is evident from the thicker dust bar; this is due to the dependence of source distance on apparent stellar brightness, as fewer lenses are likely to exist between the observer and a close star than for a more distant star. The effect on the timescale maps is also seen as a thinning of the bar as one selects fainter stars, however the lower bound of the timescale also drops as we select fainter stars. This is likely due to selecting stars with smaller Einstein radii, as
well as those with a larger velocity dispersion (such as those in the bar), which result in a larger \( \langle \mu_{\text{rel}} \rangle \) relative to \( \theta_{\text{E}} \).

The effects of selecting different timescale ranges for optical depth and timescale maps are shown in figures 10 and 11. Optical depth maps show a thickening of the dust bar for longer timescales and also a noticeable increase in noise due to the lower statistics. For timescale lower bounds of \( \langle t_E \rangle < 25 \) days, the timescale maps respect the bounds chosen; however, as one picks timescale cuts above this range, the minimum timescale observed on the map can dip significantly below the bounds chosen. This is due to an artifact of the linear interpolation method, which struggles to preserve parameter bounds when interpolating between bins with significantly different statistics and weights.

Finally, the cuts in relative proper motion are shown in figures 12 and 13. Choosing the lowest cuts in proper motion provided the highest optical depth values, which is consistent with the linear weighting of optical depth by timescale. The effect on the dust bar is subtle, as it is not as obviously thicker as the proper motion gets higher, although it does become more defined. The effect on timescale is prominent, as higher relative proper motion results naturally in a smaller timescale, which is evident from the range in values shown in figures. There is also evidence of the near side of the bar in the slowest proper motion cut, which is seen as a local increase in optical depth centred around \( l = 3^\circ \) with a width of \( \sim 5^\circ \), which is consistent with the orientation and scale lengths of the bar outlined in section 2.3. This asymmetry is mirrored by the average timescale, which is larger around the \( l = 357^\circ \) region (shown as \( l = -3^\circ \) in the figure). The primary contributor to this asymmetry is Bar-Disk lensing, which contributes 40% of the total optical depth along this line of sight.

4.1 The brown dwarf mass function

The discrepancy of the fitted brown dwarf mass function slopes between the previous work by Awiphan et al. (2016) and this work is due mainly to the difference in \( \langle t_E \rangle \) between the MOA-II results and OGLE-IV results, which is likely a result of differing event selection criteria. Figure 4 shows this discrepancy, with a noticeable excess in the OGLE-IV timescale map in most parts other than around \( 2 < l < 4 \), with an average excess of 3.16 days across the overlapping region (shown in the bottom of figure 4). As the mass function slope was fitted by sampling across the entire OGLE-IV timescale map, fewer simulated brown dwarfs were ultimately required to bring the timescale in accordance with the OGLE-IV results.

Comparing the optical depth between MOA-II and OGLE-IV shows more similarity between the resolved RCG exponential than the all-sources exponential from MOA-II results (Sumi & Penny 2016), as shown in figure 18. This suggests that strong blending could be the source of tension between the two brown dwarf slopes.

4.2 Comparison with OGLE IV data

The OGLE-IV parameter maps are shown in figure 14 with their associated error maps\(^2\). A modified version of MaB\(\frac{\text{ll}}{\text{BB}}\) was run which more closely matched the event selection criteria of the OGLE-IV survey, including an accurate treatment of the field cadences, with results shown for \( I \)-band in figure 15. The signal-to-noise criterion for OGLE-IV outlined in Mróz et al. (2019) was a time integrated one, as opposed to requiring a particular signal-to-noise threshold at peak magnification (equation 23). It required that the sum over all the flux difference at least \( 3\sigma \) above baseline, normalised to the scatter, was at least 32,

\[
\chi^2_{3+} = \sum_i \frac{F_i - F_{\text{base}}}{\sigma_i} \geq 32
\]  

The residuals (OGLE - model), normalised to their errors are shown in figure 16, with equivalent residuals shown for the older version of MaB\(\frac{\text{ll}}{\text{BB}}\) (brown dwarfs included) shown in figure 17. The error on the residual was calculated by finding the standard deviation of all nearby points within a donut shaped kernel, with an inner radius \( r_{\text{inner}} = 15' \) for optical depth, rate per source star and rate per square degree and \( r_{\text{inner}} = 30' \) for the average timescale and an outer radius \( r_{\text{outer}} = 3r_{\text{inner}} \). The inner radius prevents correlating the error estimate with the parameter value, while the outer

\(^2\) The OGLE-IV parameter map and microlensing event data is available at [http://ogle.astrouw.edu.pl/cgi-ogle/get_o4_tau.py](http://ogle.astrouw.edu.pl/cgi-ogle/get_o4_tau.py)

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**Figure 4.** Maps of \( \langle t_E \rangle \) for the MOA-II survey (top, Sumi et al. (2013)), OGLE-IV survey (middle, Mróz et al. (2019)) and the difference between them (bottom, OGLE - MOA). The mean difference in timescale is an excess of 3.16 days for the OGLE-IV data. For comparisons’ sake, the positive latitude portion of the OGLE-IV timescale map is not shown, as the MOA-II map only includes data in the negative latitude sky. The resolution of both maps was fixed at 0.25\(^\circ\).
radius prevents sampling points too far away from the target location to act as representative of the local region. The resulting distribution of residuals is an estimate of the accuracy of the model; in the ideal scenario, the distributions would be unit gaussians with a mean of zero. Offsets in the mean and standard deviation thus suggest inaccuracies in the model.

The residual maps suggest that the model is consistent with the data, with some spatial variations visible in the optical depth depth, with a notable over-prediction in the bar. This discrepancy is propagated to the rate per source star. The over-prediction in $\tau$ is similar to the original MaB$\nu$S residual map, which shows similar features. The new timescale residual map suggests strong agreement with the data, with a marked improvement over the original, which under-predicted the timescale above the Galactic plane, although this is to be expected given that the previous model was optimised for the smaller MOA II sample with different event selection. The microlensing rate per source star shows good agreement in the residual histogram, with spatial variation in the residual map consistent with the propagation of the optical depth. This is opposed to the old MaB$\nu$S, which shows a much stronger over-prediction of the rate per source star, with a non unit Gaussian distribution of residuals. In both models, the rate per square degree is over predicted near the bar, although this effect is much more dominant in the previous version, which over-predicts the rate across much of the field. The new model shows a uniform under-prediction of the rate per square degree outside the bar, although in the context of survey rate predictions, these are less interesting areas. The inconsistencies in the data with the rate per square degree could likely be due to inaccuracies with the IMFs used, which in turn influence the luminosity of the sources; changing the IMFs would not necessarily have a large effect on the optical depth or timescale, which depend on lens mass as $\sqrt{M}$, but luminosities would, with a stronger dependency on the order of $L(M) \propto M^{\alpha}$ with values of $\alpha$ in the range $[2,4]$. Alternatively, the discrepancy could also be caused by an insufficiently sophisticated source weighting which does not replicate the OGLE IV survey conditions.

5 CONCLUSION

A new microlensing model has been presented which develops significantly off of previous work. The inclusion of a formal finite source treatment, improved error calculations, background light contributions and lens-source relative proper motion cuts allows for improved microlensing optical depth, timescale and rate estimations. Calculating the distribution of normalised residuals for each microlensing parameter (optical depth, timescale, rate per source star and rate per square degree) showed improvements in all parameters over the old model, except for optical depth which remained the same. A notable discrepancy between the new model and the OGLE-IV results was seen in the rate per square degree, with a notable under prediction by the model outside of the bulge region. This discrepancy was the inverse of the old model, which displays a strong over prediction across the whole OGLE-IV rate map. Two possible solutions to the discrepancy are proposed, namely that the stellar IMFs (and by extension, luminosity functions) are inaccurate, leading to a smaller population of resolved stars, or that the source weighting used by the simulation is insufficiently faithful to the OGLE-IV survey, resulting in incorrect contributions from each source star. The brown dwarf mass function was fitted to the OGLE-IV data with a $\chi^2$ minimisation and the resulting mass function slope was found to be $+0.1$, contrasting significantly with previous work, suggesting a slope of $-0.4$, which was determined to be a result of the effect of differing event selection criteria between the OGLE-IV and MOA-II surveys on the resulting mean event timescale; as the OGLE-IV timescale map was on average higher than the MOA-II map, the necessity for short timescale brown dwarfs in the new model was lessened.

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Figure 5. V-band parameter maps are shown on the left column, with associated percent errors on the right column. Parameter ranges for all plots are $V < 21$, $\langle t_E \rangle < 300$ days and $\langle \mu_{\text{rel}} \rangle < 20$ mas year$^{-1}$. From top to bottom, the parameter maps are: the microlensing optical depth $\tau$, the average Einstein radius crossing time $\langle t_E \rangle$, the event rate per source star and the event rate per square degree, $\Gamma$. 
Figure 6. $I$-band parameter maps are shown on the left column, with associated percent errors on the right column. Parameter ranges for all plots are $I < 21$, $\langle t_E \rangle < 300$ days and $\langle \mu_{\text{rel}} \rangle < 20$ mas year$^{-1}$. From top to bottom, the parameter maps are: the microlensing optical depth $\tau$, the average Einstein radius crossing time $\langle t_E \rangle$, the event rate per source star and the event rate per square degree, $\Gamma$. 

D. Specht et al.
Figure 7. $K$-band parameter maps are shown on the left column, with associated percent errors on the right column. Parameter ranges for all plots are $K < 21$, $\langle n_b \rangle < 300$ days and $\langle \mu_{\text{rel}} \rangle < 20$ mas year$^{-1}$. From top to bottom, the parameter maps are: the microlensing optical depth $\tau$, the average Einstein radius crossing time $\langle t_E \rangle$, the event rate per source star and the event rate per square degree, $\Gamma$. 

MNRAS 000, 1–10 (2020)
Figure 8. The optical depth in \(I\)-band is shown over various survey magnitude ranges with associated error. From top to bottom, these ranges are \(I < 15\), \(15 \leq I < 18\), \(18 \leq I < 21\) and \(21 \leq I < 23\). The timescale and relative proper motion ranges are kept as \(\langle \eta \rangle < 300\) days and \(\langle \mu_{\alpha} \rangle < 20\) mas year\(^{-1}\). The thinning dust bar and rising optical depth is evident as the magnitude cut selects fainter stars.
Figure 9. The average timescale in $I$-band is shown over various survey magnitude ranges with associated error. From top to bottom, these ranges are $I < 15$, $15 \leq I < 18$, $18 \leq I < 21$ and $21 \leq I < 23$. The timescale and relative proper motion ranges are kept as $\langle t_E \rangle < 300$ days and $\langle \mu_{\text{rel}} \rangle < 20$ mas year$^{-1}$. The falling lower timescale bound is evident as the magnitude cut selects fainter stars.
Figure 10. The optical depth in $I$-band is shown over various timescale ranges with associated error. From top to bottom, these ranges are $\langle \tau \rangle < 20$ days, $20 \text{ days} \leq \langle \tau \rangle < 30$ days, $30 \text{ days} \leq \langle \tau \rangle < 40$ days and $40 \text{ days} \leq \langle \tau \rangle < 300$ days. The magnitude and relative proper motion ranges were kept constant at $I < 21$ and $\langle \mu_{\text{rel}} \rangle < 20$ mas year$^{-1}$. 

\[ \tau \] estimate
Figure 11. The average timescale in $I$-band is shown over various timescale ranges with associated error. From top to bottom, these ranges are $\langle t_E \rangle < 20$ days, $20$ days $\leq \langle t_E \rangle < 30$ days, $30$ days $\leq \langle t_E \rangle < 40$ days and $40$ days $\leq \langle t_E \rangle < 300$ days. The magnitude and relative proper motion ranges were kept constant at $I < 21$ and $\langle \mu_{\text{rel}} \rangle < 20$ mas year$^{-1}$. The interpolation method struggles to preserve the chosen timescale bounds as the range increases, as evident in the lower two rows.
Figure 12. The optical depth in $I$-band is shown over various relative proper motion ranges with associated error. From top to bottom, these ranges are $\langle \mu_\text{rel} \rangle < 4 \text{ mas year}^{-1}$, $4 \text{ mas year}^{-1} \leq \langle \mu_\text{rel} \rangle < 5 \text{ mas year}^{-1}$, $5 \text{ mas year}^{-1} \leq \langle \mu_\text{rel} \rangle < 6 \text{ mas year}^{-1}$, $6 \text{ mas year}^{-1} \leq \langle \mu_\text{rel} \rangle < 20 \text{ mas year}^{-1}$. The magnitude and average timescale ranges were kept constant at $I < 21$ and $\langle t_\text{E} \rangle < 300$ days. The nearside of the bar is evident in the lowest proper motion cut, where the optical depth is larger around $l = -3^\circ$. 
Figure 13. The average timescale in $I$-band is shown over various relative proper motion ranges with associated error. From top to bottom, these ranges are $\langle \mu_{\text{rel}} \rangle < 4$ mas year$^{-1}$, $4$ mas year$^{-1} \leq \langle \mu_{\text{rel}} \rangle < 5$ mas year$^{-1}$, $5$ mas year$^{-1} \leq \langle \mu_{\text{rel}} \rangle < 6$ mas year$^{-1}$, $6$ mas year$^{-1} \leq \langle \mu_{\text{rel}} \rangle < 20$ mas year$^{-1}$. The magnitude and average timescale ranges were kept constant at $I < 21$ and $\langle t_E \rangle < 300$ days.
Figure 14. Data compiled from the OGLE IV analysis between the years 2010 to 2017. The parameter ranges used are $14 < I < 21$, $(\langle t \rangle) < 300$ days. The corresponding error maps shown are the errors on the residual between the OGLE survey and the MaBplS-2 model, which for a perfect model would be an accurate representation of the error. Given that the model is imperfect, these errors are approximate.
Figure 15. A MaBµlS-2 simulation using the same event selection criteria outlined in Mróz et al. (2019), with a mask applied over the regions not applicable in the OGLE data. The parameter ranges used are $14 < I < 21$, $\langle t_E \rangle < 300$ days. The signal-to-noise selection criterion characterised by equations 23 and 24 was modified to match the time-integrated signal to noise from equation 26. The sharp contours in the timescale map are due to the differing OGLE field cadences, and the effect this has on the signal to noise criterion.
Figure 16. The residuals (OGLE - Model) for each of the parameter maps shown in figure 14. Residuals have been normalised to their variance. The left column shows the map of residuals, with the corresponding 1D histograms in the right column. Two Gaussians are plotted on each of the histograms. The solid black Gaussian is a unit Gaussian with $\mu = 0$ and $\sigma = 1$, representing the ideal scenario where the model matches the data. The dashed Gaussian is fitted to the distribution with the mean and standard deviation listed on the plot.
Figure 17. The residuals (OGLE - Model) for each of the parameter maps shown in figure 14, for the old version of MaBμlS (Awiphan et al. 2016). Residuals have been normalised to their variance. The left column shows the map of residuals, with the corresponding 1D histograms in the right column.
Figure 18. The comparison between the optical depth variations of MOA-II source stars (all sources and exclusively RCG sources) from Sumi & Penny (2016) and OGLE-IV source stars from Mróz et al. (2019) over the longitude range \(|l| < 5^\circ\) as well as the new MaB\(\mu\)lS2 result from the simulation. In black is the OGLE-IV data, binned in intervals of 0.5° in galactic latitude. The MOA-II all-sources and RCG-sources results are shown in blue and red respectively. MaB\(\mu\)lS-2 is shown in green. The lines show the optimal exponential fits for their respective colours, with equations shown at the top of the figure.