An Extensive Analysis of Oscillations in Arbitrary Harbors with Reflecting, Absorbing and Transmitting Boundaries – Case study of the Port of Indiana

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Abstract: The harbor resonance numerical model can be applied to wave response in the harbor, groups of linear and nonlinear waves with high crest pose hazards to ships, inducing harbor resonance and over-topping of structures. The model, a two-dimensional linear, inviscid, dispersive, hybrid finite element to simulate harbor resonance with conservation of energy approach, is applicable in an offshore or coastal harbor of arbitrary shape. The model incorporates the effects of variable bathymetry, bottom friction, variable, full or partial absorbing boundaries as well as wave transmission through permeable breakwaters. The accuracy and efficiency of the present model will be verified by comparing with those of experimental and field data of the Port of Indiana, where wave incidences of ship damage, broken mooring lines and barge sinking, particularly in the grain dock area. It is important to study the behavior of incident wave periods and the associated maximum harbor resonant response at key points within the harbor. The model results were proved to be consistent with observations which lead to the conclusion that the model is well suited for use in harbors of irregular geometry with transmissivity boundaries. The model presented herein is a realistic method for solving arbitrary harbor resonance problems.

1. Introduction
An important aspect in the planning and design of a new harbor or modification of an existing harbor is the investigation of the response characteristics of the harbor basin to the incident wave system. Experience has shown that a harbor is dynamically similar to a mechanical system which responses selectively to a forcing function which is frequency dependent. When the wave length of the incident wave is on the same order of magnitude of the characteristic dimension of the harbor basin, the wave amplitude in the harbor basin can be larger than the amplitude of the incident wave. This phenomenon usually is referred to as “Harbor Resonance”, “Seiche”, or “Harbor Surging”. Large oscillations of the water surface inside the harbor could potentially damage ships, mooring systems, and marine structures. Therefore, it is important to estimate the significance of any induced harbor resonance to avoid or minimize its occurrence.

Numerous studies have been conducted on various aspects of the harbor resonance problem. Ippen and Goda [1] studied the wave induced oscillation in harbor and proposed the solution for a rectangular harbor connected to the open sea, Lee [2] developed a theory for wave-induced
oscillations in constant depth harbor of arbitrary geometry, by applying Weber’s solution of the Helmholtz equation. Chen and Mei [3, 4] developed a linear inviscid, nondispersive, hybrid finite element model based on a variation principle for treating arbitrary platform geometries. Berkhoff [5] developed the mild-slope equation to compute the combined refraction-diffraction, Houston [6] used a model based on the mild-slope equation to study the interaction of tsunamis with the Haeaiian Islands. Tsay, et. al. [7, 8] and Hamidi, et. al. [9] applied the mild-slope equation to effects of topographical variation and energy dissipation. Ganaba, et. al. [10] and Woo, et. al. [11, 12] developed finite element methods for boundary layer modelling with application to dissipative harbor resonance problems. The first order boundary condition has previously been used in finite element models, but the higher order boundary conditions to account for the effect of partial reflecting boundaries have been formulated herein for linear wave waters for the first time. Researchers took advantage of rapid growth of computation capacity of computer to further develop tools analysing long waves [13, 14] and nonlinear waves [15]. Analytical investigation results also showed that the boundary condition greatly affects the harbor resonance. Many research programs focused on the harbor depths from constant slope [16], Cosine squared bottom [17], exponential bottom [18], to parabolic bottom [19], geometric configuration [20], and the singular boundary method (SBM) [21], (ISBM) [22]. These previous numerical model studies have contributed significantly to the understanding of harbor resonance, but have not included wave energy dissipation with bottom friction, boundary absorption, and wave transmission through permeable breakwaters. Therefore, the need to develop realistic and fully interactive models for the prediction of wave conditions inside a basin is imperative.

Chang and Wang [23] developed a harbor resonance numerical model that is a conservation of energy approach is applied to develop a linear, inviscid, dispersive, hybrid finite element resonance model for calculating free surface wave response in an adjoining coastal basin of arbitrary shape. The governing model equation is based on the mild-slope wave equation, including wave refraction, diffraction, and reflection, and the effects of variable bathymetry, bottom friction, full or partial absorbing boundaries, and wave transmission through permeable breakwaters. The functional can be easily extended to include bottom friction as well as absorption of energy along boundaries of the area of calculation. The accuracy and efficiency of the present model will be verified by comparing numerical results with experimental and field data of the Port of Indiana. The Port of Indiana (Burns Waterway Harbor) is located at the south end of Lake Michigan (see Figure 1). The port of Indiana layout, shown in Figure 2 was constructed in 1968 as an international commercial and industrial harbor. The harbor consists of an attached, L-shaped rubble-mound north breakwater protecting an outer harbor, and two arms (west and east) formed by reclaimed land behind rip-rap revetments and steel bulkheads. All of the vertical bulkheads are aligned north-south with the exception of a rectangular grain dock, projecting from and centred on the north wharf [24, 25].

The Indiana Port Commission reports incidences of ship damage, broken mooring lines and barge sinking’s, particularly in the grain dock area. Also, along the harborside of the north breakwater many stones have been displaced resulting in serious breakwater damage. It is speculated that excessive transmission of wave energy through the breakwater, producing a large wave response, is responsible for these events. Thus, it is important to study the behaviour of incident wave periods and the associated maximum harbor resonant response at key points within the harbor. Field data were collected by the U.S. Army Corps of Engineers for wave conditions in the harbor, under the Monitoring of Completed Coastal Projects (MCCP) program (McGehee) [26]. Under this program, wave gages were placed outside of and behind the harbor breakwaters from December 1985 to June 1988, and in front of the grain dock from January to June, 1987. Available data from this program were used to verify the harbor resonance model by comparing them with numerical results.

2. Theoretical of numerical model
A linear gravity wave on the water surface is assumed to satisfy condition that the fluid is homogeneous, inviscid, incompressible, and that fluid motion is irrotational. The governing model
equation is based on the mild-slope wave equation [5], including wave refraction, diffraction, and reflection.

For an arbitrary-shape harbor, the domain is divided into two regions (Figure 3): Region A is the limit of the harbor, Region R is the infinite open-sea [1]. Mathematically, the boundary value problem for the arbitrary-shape harbor can be expressed as follows:

Figure 1. The Port of Indiana (Burns Harbor) Location (located at the south end of Lake Michigan).

Figure 2. The Port of Indiana (Burns Harbor) Layout.

Figure 3. Configuration of coastline harbor with permeable breakwater.
From the conservation of wave energy approach, we can derive a functional which includes the effects of variable depth, bottom friction, variable full or partial absorbing boundaries, and wave transmission through permeable breakwaters. The functional can be expressed as:

$$F(\Phi) = \int_{A} \left[ C_{g}(\nabla \Phi)^2 - \frac{C_{g}}{C} \omega^2 \Phi^2 \right] dx dy + \int_{\partial A} \left[ \frac{1}{2} C_{g} \Phi \frac{\partial \Phi^s}{\partial n_{A}} d s - \int_{\partial A} C_{g} \Phi_{A} \frac{\partial \Phi^s}{\partial n_{A}} d s \right.$$

$$\left. - \int_{\partial A} C_{g} \Phi_{A} \frac{\partial \Phi^s}{\partial n_{A}} d s + \int_{\partial A} C_{g} \Phi_{A} \frac{\partial \Phi^s}{\partial n_{A}} d s - \int_{\partial A} \frac{1}{2} C_{g} \omega \alpha \Phi^s d s \right]$$

$$- \int_{A} \left[ \frac{4}{3} \sum_{m=1}^{\infty} \kappa_{b} (\Phi)^2 \right] dx dy + \int_{\partial A} C_{g} \kappa_{b} \Phi^s d s$$

(7)

Terms $I_1$ to $I_8$ can be solved by applying the hybrid finite element method (Chen and Mei 1975) [2, 3]. Using this approach the functional, Eq. (7), can be evaluated as follows:

(1) First term (energy in region A):

$$I_1 = \int_{A} \left[ C_{g}(\nabla \Phi)^2 - \frac{C_{g}}{C} \omega^2 \Phi^2 \right] dx dy = \frac{1}{2} \left( \Phi^s \right)_{A}^T \left( K^s_{A} \right)_{M \times M} \left( \Phi^s \right)_{M \times 1}$$

(8)

With

$$[K^s_{1 \times M}]_{M \times M} = \frac{1}{4A} \sum_{n=1}^{\infty} \sum_{m=1}^{\infty} [C_{g} W_{n}]$$

(9)

Where $W_{n}$ is the weighting function, and $k(t, \{1, 2, ..., N\}) = \sum_{n=1}^{N} \sum_{m=1}^{\infty} C_{g} \omega^2 W_{n}$.

(2) Second term (energy flux along $\partial A$):

$$I_2 = \int_{\partial A} C_{g} \Phi \frac{\partial \Phi^s}{\partial n_{A}} d s = \frac{1}{2} \left[ \mu^s \right]_{1 \times M}^T \left( K^s_{1 \times M} \right)_{M \times 1}$$

(10)

With

$$\left[ \mu \right]_{1 \times M} = \left\{ \alpha_0, \alpha_1, ..., \alpha_\infty \right\}$$

(11)

$$[K^s_{1 \times M}] = \frac{1}{2} \pi \kappa_{b} [C_{g}] \left[ H_0, H_1, H_1', ..., H_\infty, H_\infty' \right]$$

(12)

(3) Third term (energy flux along $\partial A$):

$$I_3 = \int_{\partial A} \frac{\partial \Phi^s}{\partial n_{A}} d s = \frac{1}{2} \left[ \phi \right]_{1 \times M}^T \left( K^s_{1 \times M} \right)_{M \times 1}$$

(13)

With
(14) Fourth term (energy flux along $\partial A$):

$$
I_4 = - \int_{\partial A} CC_g \Phi \frac{d \Phi}{d n} d s = \left\{ Q_1^{\partial A} \right\}_{1 \times M} \Phi_{1 \times 1}
$$

With

$$
\left\{ Q_1^{\partial A} \right\}_{1 \times M} = \sum_{i=1}^{P} \frac{1}{2} kCC_g \left( \frac{\partial \Phi}{\partial n} \right)_{i \times 1}
$$

(15) Fifth term (energy flux along $\partial A$):

$$
I_5 = \int_{\partial A} CC_g \Phi \frac{d \Phi}{d n} d s = \left\{ Q_5^{\partial A} \right\}_{1 \times M} \Phi_{1 \times 1}
$$

With

$$
\left\{ Q_5^{\partial A} \right\}_{1 \times M} = - 2 nkCC_g \left( \frac{\partial \Phi}{\partial n} \right)_{i \times 1}
$$

(16) Sixth term (boundary absorption energy flux along $\partial B$):

$$
I_6 = - \int_{\partial B} CC_g \Phi \frac{d \Phi}{d n} d s = \frac{1}{2} \left\{ \Phi \right\}_{1 \times K} \left[ K_B \right]_{K \times 1} \left( \Phi \right)_{1 \times 1}
$$

With

$$
\left\{ K_B \right\}_{K \times 1} = \sum_{i=1}^{L} \frac{1}{2} L_B \left[ (\alpha \omega C_g)_{1} + (\alpha \omega C_g)_{2} \right]
$$

(17) Seventh term (bottom friction energy flux along $\partial A$):

$$
I_7 = - \int_{\partial A} \left[ \frac{1}{2} k_B \left( \frac{\partial \Phi}{\partial n} \right) \left( \left( \frac{\partial \Phi}{\partial n} \right) \right) \right] d x d y = \frac{1}{2} \left\{ \Phi^T \right\}_{1 \times M} \left[ K_T \right]_{M \times M} \left( \Phi^T \right)_{1 \times 1}
$$

(18) Eighth term (energy transmitting through porous structures along $\partial C$):

$$
I_8 = \int_{\partial C} \left[ \frac{1}{2} k_T \left( \frac{\partial \Phi}{\partial n} \right) \left( \left( \frac{\partial \Phi}{\partial n} \right) \right) \right] d x d y = \frac{1}{2} \left\{ \Phi^T \right\}_{1 \times M} \left[ K_T \right]_{M \times M} \left( \Phi^T \right)_{1 \times 1}
$$

(19) Where $L$ is the total number of line elements along the inner boundary $\partial B$, $L_B$ is the length of each segment, and $\alpha$ is a boundary absorption coefficient, dependent on empirical and laboratory data.

(20) Where $L$ is the total number of line elements along the porous structures (breakwater), and $\beta$ is an empirical bottom friction coefficient.

(21) Where $L$ is the total number of line elements along the porous structures (breakwater), and $\beta$ is an empirical bottom friction coefficient.

Finally, assembling the local stiffness matrix into a global stiffness matrix, and taking the stationary functional results in a functional expressed as:

$$
\text{[K]}_{M \times M} \{ \Phi \} = \{ Q \}_{N \times 1}
$$

Where the total stiffness matrix [K] is usually large and banded.
\[
[K] = \begin{bmatrix}
[K_1] \\
[K_2] \\
[K_3]
\end{bmatrix}
\]  
with \( [K_1] = [K_1 + K_B + K_F + K_T] \)  
\[
(26)
\]
\[
\text{And the total load vector}
\]
\[
\{Q\}^T = \{0\}^T, \{Q_1\}^T, \{0\}^T, \{Q_2\}^T
\]  
\[
(27)
\]
This set of \( N \) linear algebraic equations for \( N \) unknowns can be solved by the decomposition LDU algorithm method. The total velocity potential at all nodal points and unknown coefficients can also be determined.

3. The Port of Indiana numerical simulation model

3.1. Configuration

The present numerical model has been configured for the Port of Indiana. For numerical computations the entire harbor is divided into 374 nodes and 610 triangular shaped elements connecting there nodal points (Figure 4). Numerical analysis was conducted for four specific locations within the Port of Indiana. The locations are shown in Figure 4 as: Location A, Behind Breakwater; Location B, Grain Dock; Location C, West Basin End; Location D, East Basin End. The characteristic length “a”, which is 1150 ft. (near the harbor entrance in Figure 5) is used for normalization of the wave number “\( K_a \)”. The harbor model grid system is used for computing harbor response for a wave period range of 5.8 to 2013.4 seconds (0.1 \( < K_a \leq 35 \)), where \( K \) is the wave number defined as \( 2\pi/L \), the ratio of the wave amplitude inside the harbor to the wave amplitude outside the harbor. The maximum amplification factor defines the maximum wave inside the harbor regardless of its location. Parameters used in the model are: (1) Water depth: from 16.5 ft. to 40 ft. (actual field data, U.S. Army Corps of Engineers, 1989), (2) Bottom friction coefficient = 0.3 (sand, silt), (3) Permeable breakwater transmission coefficient = 0.5, and (4) Boundary absorption coefficient: from 0.05 to 0.4 (as shown in Figure 5).

3.2. Numerical results

3.2.1. Response of harbor with different incident wave conditions
To investigate the effect of wave response inside the Port of Indiana due to incident wave direction, four different incident wave angles ($\theta$) were selected (Figure 4): (1) $\theta = 40^\circ$, along the coastline straight to the entrance; (2) $\theta = 90^\circ$, from N33°33'E straight into the harbor; (3) $\theta = 140^\circ$, from N15°W to the harbor; (4) $\theta = 180^\circ$, from N55°W to the harbor; Figure 6 shows the maximum response curves for 4 different incident waves (Ka from 0 to 15), as incident wave angle increases (moves east to west), the maximum wave response increases. Conversely, Figure 7 shows the maximum response curves for 4 different incident waves (Ka from 15 to 35), when incident wave angle increases, the maximum wave response decreases. The incident wave angle $\theta = 90^\circ$, shows the highest response (Ka from 24 to 25). These results are reasonable because the maximum response peaks usually occur at the harbor mouth. As the incident wave angle increases, more incident long wave energy is reflected from the north facing shore side rubble mound breakwater. The interaction of the incident and reflected wave energy produce an excessive response at the entrance. For the short period wave, when the incident wave angle increases (becomes more westerly), the north breakwater shadow is increased, and wave response decreases.

![Figure 6](image1.png)

**Figure 6.** Comparison of the maximum response curves for 4 different incident waves (Ka=0–15).

![Figure 7](image2.png)

**Figure 7.** Comparison of the maximum response curves for 4 different incident waves (Ka=15–35).

### 3.2.2. Response of harbor with variable absorption coefficient

In order to investigate the effect of energy dissipation resulting from variable absorption characteristics of the north breakwater, nine different absorption coefficients were used: $\alpha = 0.0, 0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7,$ and $0.8$. All other boundary absorption coefficients in the harbor are held constant as shown in Figure 5, the transmission coefficient ($k_T$) = 0.5, bottom friction ($f_w$) = 0.3, and incident wave direction is from the north. Figure 8 and Figure 9 show a comparison of the amplification factor (R) and north breakwater absorption coefficient ($\alpha$), for six large resonant modes, at a position behind the north breakwater (location A) and at the grain dock (location) respectively. It can be seen that, as anticipated, as the boundary absorption coefficient increases, the amplification factor decreases. However, the curves show a steep fall within the range of $\alpha = 0.0$ to 0.4, thus showing the relative sensitivity of the wave response characteristics to breakwater material selection. It is important to note that the amplification factor (R) at the grain dock is substantially higher than behind the north breakwater regardless of the boundary absorption coefficient.
3.2.3. **Response of harbor with variable transmission coefficient**

In order to examine the effect of wave transmission through the north breakwater on the harbor oscillation behaviour, ten different transmission coefficients were investigated: \( k_T = 0.0, 0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, \) and 0.9. All incident waves are from the north direction, bottom friction \( (f_b) = 0.3, \) and all boundary absorption coefficients are as shown in Figure 5; Figure 10, and Figure 11 show a comparison of the amplification factor \( (R) \) and the north breakwater transmission coefficient \( (k_T) \), for six large resonant modes, behind the north breakwater (location A) and at the grain dock (location). These results seem reasonable and point to the fact that as the transmission coefficient increases beyond \( k_T = 0.5, \) the amplification factor increases rapidly. For long period waves, as the value of \( k_T \) varies from 0.5 to 0.8, the amplification factors are very sensitive. For short period waves (i.e. \( K_a = 23.4, T= 8.6 \) sec.), the amplification curve remains flat, thus showing that energy from these waves is poorly transmitted through the breakwater. In general, long period wave response at the grain dock was significantly larger than behind the north breakwater.

4. **Comparison of field data and numerical results**

4.1. **Previous investigation for Burns Harbor**
In 1967, Jackson used a large scale, two-dimensional physical model to evaluate breakwater stability and wave transmission. Experimentation conducted in a flume measured waves in front of and behind rubble breakwaters. Results showed a predicted wave transmission coefficient of 24 percent for an incident height of 13 ft. at 11 seconds, and 32 percent for the maximum incident height 18 ft. at 11 seconds. These trials considered monochromatic incident waves only, they did not fully consider incident diffraction from the harbor entrance, and neglected reflected wave effect from the entire harbor geometry. In addition to this laboratory experiment, field data has also been collected at Burns Harbor. The source for most of the field data is the Monitoring of Completed Coastal Projects (MCCP) program. Data was acquired by self-contained no directional wave gages at three locations (Figure 2); on the lake side of the north breakwater and in the harbor behind the breakwater, during the period from December 1985 through June 1988, and in front of the grain loading dock from January to June, 1987. The U.S. Army Coastal Engineering Research Center (CERC) (McGehee 1991), present a number of these field data on wave height variation at the measuring sites. Figure 12 shows wave heights in front of the north breakwater vs. wave heights behind the breakwater for both the 1967 model data and for prototype data measured in 1987. Figure 13 shows a similar plot for a location in front of the grain dock, during the same time period. A majority of wave heights are below one foot inside the harbor when the wave height is below 6 feet outside the harbor. When the lakeside wave heights are greater than 6 feet, the harbor side wave heights are 25 percent to 35 percent that of the incident wave heights. The maximum measured significant wave height inside the harbor (grain dock) is 9.5 feet, which is over 70 percent of the incident wave height (13 feet). Also, wave heights at the grain dock are generally higher than waves behind the breakwater for the same offshore incident wave height.

4.2. Comparison of field data and numerical results
Based on the limited available field measurements, a comparison was carried out using measured wave heights from outside and inside Burns Harbor and numerical model results from this study. Histograms were compiled of the amplification factor (R) versus the probability of occurrence (distribution frequency, F). The amplification factor (R) is defined as the ratio of wave height inside the harbor to wave height outside the harbor. Figure 14 shows a comparison of the wave response frequency distribution of the amplification factor (R) for the position behind the north breakwater (location A), using field data (Figure 12) and the present numerical results for a given Ka range of 15 to 25 (prototype wave period from 13.4 to 8 seconds). Figure 15 shows a similar comparison for the grain dock (location B) using the same wave period range (Figure 13). These histograms show that the numerical results agree well with the field data in Figure 14 and Figure 15. Location A has a large number of wave amplification occurrences in the 0.0 to 0.3 range of R (i.e. 0.0 to 30 percent of incident wave heights). In the range of 0.4 to 0.6, the numerical result is larger than the field data, but not significantly so. This difference may exist because of the chosen value of Ka. Similarly, at the grain dock (location B) the majority of wave amplification factors (R) are in the 0.0 to 0.3 range. In the range of 0.5 to 0.7, the numerically predicted results are again slightly larger than the field data. Once again, this difference could exist because of the chosen value of Ka. On February 8, 1987 the Port of Indiana experienced a 100 year storm event. During that storm the maximum measured significant wave height at the grain dock was 9.5 feet with a wave period of 11 second, which is over 70 percent of the incident wave height of 13 feet. Comparison to Figure 12 and Figure 13 for a numerically predicted prototype wave period of 11 seconds, from the north direction, shows the amplification (R) is 0.82 (82 percent). This indicates that the numerically predicted results are in good agreement with the field data.
5. Presentation and discussion of result

5.1. Peak harbor resonance

In order to provide a better visualization of the response characteristics inside Burns Harbor, three dimensional plots were generated of the harbor resonance response. From Figure 16 to Figure 19 show three dimensional plots of wave resonance response within Burns Harbor for systematically increasing wave periods. These three-dimensional plots are useful for comparison with aerial and storm damage photographic records. Comparison of aerial photographs and the three-dimensional graphs of numerical simulation (Figure 16 and Figure 17), showed that when short period incident waves approach the harbor, the west and east slip basins are very stable and calm on the water surface. For long period waves (the prototype wave period (T) larger than 11 seconds), wave response is quite sensitive with large wave responses occurring from the harbor entrance along the north breakwater and north wharf toward the west slip basin, yet the east slip basin wave response is quite small. Post storm photographs showing apparent wave damage along the north breakwater agree well with the numerically predicted wave distributions. Most notable is the large scale damage observed in the northwest corner of the outer breakwater which corresponds directly with high R values shown in Figure 18 and Figure 19. Overall, these qualitative comparisons tend to support the predictive results from the numerical model simulations.
5.2. Discussion of results
Based on the preceding numerical analysis, it was found that for short period waves (prototype wave period (T) less than 9 seconds) with any incident direction (θ), boundary absorption (a) and transmission coefficient (k_T), that an overall low wave response is predicted in the harbor. The west and east slip basins were found to remain very stable and calm under these conditions. For long period waves (prototype wave period (T) 11 seconds and larger), the above parameters affected wave response significantly. Even when the incident angle increased (moved away from harbor entrance), if the incident period matched the harbor basin frequency, an excessive wave response was predicted.

For example, at the grain dock, when the incident wave from N8^o 40'E, and the prototype wave period was about 14 seconds, the amplification (R) approached 1.3. In the west basin, almost all cases showed a small wave response, except when the incident wave was from N33^o 30'E, and the prototype wave period was about 12 seconds. In this case, the amplification factor (R) approached 1.4. This is because the incident wave passes directly through the entrance and proceeds toward the west basin, thus a high wave response occurs in that basin. The east basin is not influenced by incident waves passing through the entrance due to its nearly orthogonal alignment to the entrance. Hence, the east basin almost always remains calm and stable. The analysis of Burns Harbor using the numerical model developed in this study showed the harbor to have a maximum resonant frequency (K_a) in the range of 14 to 18. Thus, the prototype maximum resonant wave period (T) ranges from 14.4 to 11.2 seconds, and the maximum corresponding amplification factor (R) is usually greater than one. In the case of the incident wave period being within the resonant range, and wave height larger than the north breakwater height (+12 ft.), even more wave energy will be transmitted through the breakwater. This will produce an excessive harbor wave resonance response. A wind generated wave model (McGehee) [26], was used to determine that an 11+ second wave in southern Lake Michigan could be created by a 40 knot wind with a duration of only 10 hours, blowing from due north. Since strong low pressure systems passing across Lake Michigan develop 40 knot winds with substantial duration, the maximum resonant condition predicted by the model are likely to occur within Burns Harbor.
6. Conclusions
A realistic harbor resonance model has been developed based on the conservation of wave energy principle. This approach was used to develop a two-dimensional linear, inviscid, hybrid finite element model for calculating the free surface response characteristics inside a harbor, which is subjected to excitation by incident waves from outside the harbor. Wave induced oscillation in the harbor is influenced by the following parameters: the geometry of the harbor, boundary absorption within the harbor, and wave transmission through the breakwater. Results showed that the effects of bottom friction and variable bathymetry were found to intensify as a function of depth. This result is in agreement with accepted theory regarding the behaviour of waves in transition water and indicates that as basin depth decreases, the need for accurate bottom friction coefficients and water depths increases. The degree of sensitivity of this model to boundary absorption coefficient values is significant. Specifically, this parameter becomes increasingly more important as the percentage of engineered boundaries (low absorption coefficients) increases. Preliminary measurement and analysis indicate that model results are in good agreement with observed localized long-wave and shore-wave events. The model developed in this study is more realistic and a better approximation for the active harbor resonance problem. However, in order to improve model response prediction results, more experimental work is needed to better determine values for absorption and transmission coefficients.

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