Tailoring Single and Multiphoton Probabilities of a Single Photon On-Demand Source

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(Dated: October 29, 2018)

As typically implemented, single photon sources cannot be made to produce single photons with high probability, while simultaneously suppressing the probability of yielding two or more photons. Because of this, single photon sources cannot really produce single photons on demand. We describe a multiplexed system that allows the probabilities of producing one and more photons to be adjusted independently, enabling a much better approximation of a source of single photons on demand.

PACS numbers:

The advent of photon-based quantum cryptography, communication and computation schemes is increasing the need for light sources that produce individual photons. It is of particular importance that these single photons be produced in as controlled a manner as possible, as unwanted additional photons can render quantum cryptographic links insecure and degrade quantum computation efficiencies.

Single photons (or more precisely, approximations thereof) are now commonly created via the process of parametric downconversion (PDC), although attenuated lasers and quantum-dots and other single quantum site sources are also used. Because PDC creates photons in pairs, the detection of one photon indicates, or “heralds”, the existence of its twin, a significant advantage over other methods (even aside from the potential to directly produce entangled states). In addition, because the PDC process is governed by the constraints of phase matching, it is possible to know the output trajectory, polarization, and wavelength of that heralded photon. While PDC has a long history as a single photon source and there have been many recent improvements, the scheme has a couple of problems. The conversion process is random, so while an output photon is heralded by its twin, there is no control or prior knowledge of when the heralding event will occur. In addition, there is a possibility of producing more than one pair at a time and because that probability increases nonlinearly with the one photon probability, one must operate at low one photon probabilities. So to be assured that more than one photon is not produced, one must operate where it is most likely that no photon is produced at all.

The faint laser scheme suffers the same difficulty as the PDC method, in possibly producing more than one photon at a time, with the added difficulty of not having any herald at all. Quantum-dot sources offer promise as a new way of definitively producing single photons, although it remains to be seen whether their output/collection efficiencies can be made to approach unity in practice, a requirement for a true on-demand source.

To surmount the problem of random production in PDC, one uses a pulsed laser to pump the nonlinear crystal (see for instance [8]). With a pulsed source, photon pairs can only be produced at certain times. Unfortunately the multiple photon emission problem remains; a high probability of producing a single photon pair during each pulse leads to an increased likelihood of producing more than one photon pair during each pulse, defeating the original goal of having a source of single photons. This problem occurs regardless of whether sources with Poisson or Bose statistics are used. Because of this, pulsed systems are usually operated with low probability of producing an output photon pair during a laser pulse (P1~0.1 to 0.3). Thus, while photons can only come during specific time windows, most of these time windows will contain no photons at all, illustrating the trade off between the two requirements of producing a photon on demand and being assured that there is, in fact, just one photon.

The improvement presented here allows these two competing requirements to be adjusted independently by decoupling P1 and P>1. We can then select both the desired likelihood of production of a photon pair and the desired suppression of multiple pair events. This is accomplished using an array of downconverters and detectors (Fig. 1a) pumped simultaneously by the same laser pulse. The laser power is chosen so each downconverter has some small probability of producing a photon pair, while the number of downconverters is chosen so there is a high likelihood of at least one pair being created somewhere in the array. The detector associated with each downconverter allows us to determine which of the downconverters has fired. This information is used to control an optical switching circuit directing the other photon of the pair onto the single output channel. This arrangement allows a much truer approximation of a single photon on-demand source than is possible with other methods.

A simple extension of this arrangement can produce a regularly spaced series of single photons. By gating up a series of these “single photon on demand setups” with additional optical switches and a series of optical delays, it is possible to produce an arbitrary length train of single photons. By producing a pulse train long enough to last until the next pump laser pulse arrives, one can create
a continuous train of single photon pulses synchronized with an external signal.

These basic concepts could be used to produce higher order photon number states as well. By using detectors with the capability of sensing the number of photons in a single pulse, the switching circuit could just as well direct the outputs of those converters that produced multiple photon pairs to the output channel. That would result in an output pulse train with each pulse containing the desired number of photons.

While the scheme just described is conceptually the simplest way of presenting the method, there are a number of modifications that can improve the efficiency, construction, and convenience of the system. The first of these is that the array of downconverters can be implemented with a single PDC crystal. This is possible because while phasematching requires a PDC photon pair to be constrained to a plane containing the pump beam, the azimuthal angle is not constrained. Thus, the PDC process produces light distributed azimuthally around the pump laser direction (for type I phasematching). So, each azimuthal plane can be thought of as a separate downconverter. Thus the multiple PDC setup is achieved by placing many detectors azimuthally in an annular pattern around the pump direction of a single downconverter and collecting at the correlated positions.

A second modification allows the array of detectors to be replaced by a single detector. By sending each of the potential herald photons to an increasing sequence of delays and then directing the delay outputs to a single detector, the timing of the detector pulse indicates which of the input downconverters has created a pair. Of course, several of the downconverters may produce a photon pair, but only the first photon herald received by the detector causes it to fire. Detector deadtime causes subsequent photons to be ignored. The timing of the detector pulse is used to select which correlated photon channel to direct to the output of the system.

A third implementation (Fig. 1b) even eliminates the output switching network circuit, while still maintaining a significant advantage over both the conventional PDC and faint laser photon sources. The output photons are simply collected with a single lens output port. This most basic implementation allows production of pulses with individual certainties of that pulse containing exactly one photon. In other words, this source provides single photons and a “certificate” tied to each output photon quantifying the likelihood that just one photon was emitted. Some of these pulses can achieve a significantly higher single photon certainty than is possible with the conventional single photon source setups. This more complete characterization of the emitted pulse and its tighter constraints on $P_{>1}$, will allow more efficient use of the emitted light. This reduces the need for overhead tasks such as privacy amplification.

The basic reason that this arrangement can produce single photons with lower probability of multiple photons is that the delay system provides extra information about the photons produced. For instance, in the cases where one of the longer delays happened to cause the detector to fire, we know that all the prior delays did not cause the detector to fire. If the detection quantum efficiency is high, it is very likely that there were no photons in those modes coupled to those shorter paths. Thus $P_{>1}$ is greatly reduced because it is just the multiphoton probability for only the last delay, rather than for all $N_D$ of them, which has a mean photon number of $\pi$ vs. $\pi/N_D$.

We now quantify the advantage of this last scheme in its simplest and most straightforward implementation. We will see this scheme results in production of photon pulses where each pulse has its own individualized single photon certification, and as expected, these certifications can be significantly better than the uniform result obtained from the conventional arrangement. To begin, we consider the standard PDC setup for producing heralded single photons. The trigger detector registers one photon of a pair and indicates the existence of the second photon exiting the correlated channel. The collection optics for that correlated channel are designed to collect, as close as can be approximated, just the photons correlated to those seen by the trigger detector. In this arrangement, both the trigger channel and output channel are set up to collect only one mode of the field $|1\rangle$. When the trigger detector fires, one photon has been received (assuming negligible dark counts), but we do not know if additional photons were also present as the considered detectors cannot distinguish a one-photon from a multi-
photon event. Given that the trigger detector has fired, the probability that there were \( n \) photons incident is:

\[
P^E_{\eta, n}(n) = \frac{(1 - (1 - \eta)^n) \times P_{\eta}(n)}{\sum_{k=1}^{\infty} (1 - (1 - \eta)^k) \times P_{\eta}(k)}. \tag{1}
\]

Note that \( 1 - (1 - \eta)^n \) is the probability of the detector firing for \( n \) photons incident and detector quantum efficiency, \( \eta \) defined as the probability of the detector firing when just one photon is incident. We use Bose statistics for the probability, \( P_{\eta}(n) \), of having \( n \) photons emitted into a single mode of the PDC light, given \( \eta \) \cite{5, 13}.

With this basis, we now describe a system with a number of delay lines of increasing length placed between the PDC crystal and the trigger detector. Each of these delay line channels intercepts a single, but separate, mode of the field. The output channel collection optic is also modified to include the extra modes correlated to those of the additional trigger modes. Each of these trigger paths has a chance to cause the trigger detector to fire, with the shortest path providing the first chance, and the next longer path providing the next chance, and so on. But once the trigger detector fires due to a photon in a particular path, it cannot fire due to subsequent photons in the longer delay paths. The timing of the trigger detector firing relative to the pump pulse tells which delay path caused the firing. Thus the result of a single pulse of the pump laser is that either no trigger was produced or a trigger was produced and we know which delay path produced it. This last piece of information allows us to make a better determination of the probability that the photon produced was a single photon. If the photon that causes the trigger to fire is one of the later delay paths, we will have a much lower likelihood of there being more than one photon. We now calculate this likelihood as a function of which delay path caused the firing.

For a system of \( N_D \) delay paths where the \( i \)th delay path caused the firing, the one photon probability is:

\[
P_{\eta, N_D}(i) = \left(1 - P^E_{\eta, \eta}\right)^{i-1} P^E_{\eta, \eta} (1) \left(P_{\eta}(0)\right)^{N_D-i}, \tag{2}
\]

where \( P^E_{\eta, \eta} \) is the probability that photons were incident if the detector did not fire. Then we use Eq. (2) with \( n=1 \) for a Bose distribution in each of the modes collected in Fig. 1b. Figure 2 shows the functional form of these probabilities, where each line of the fan shaped family of curves represents a system of \( N_D \) delays. Each point on a given line corresponds to the trigger firing at a particular \( i \)th delay out of a set of \( N_D \). The point’s value is the probability that this event indicates that exactly one photon pair exists in the system. It can be seen that for \( N_D \) of more than a few, and with high \( \eta \), we can have emission events with single photon probabilities around 90%, greatly exceeding the conventional arrangement results for the same \( \eta \) and \( \eta \). The conventional result is represented by the single \( N_D = 1 \) point (i.e. the standard PDC setup with one trigger path).

We have calculated two additional probabilities: the overall probability of producing a heralded single photon (Eq. (3)), and that same probability given that the trigger did fire (Eq. (4)). The first is obtained by taking the product of the probability of a particular delay event occurring and the probability of that event being due to a single photon and then summing over all possible types of events. The second probability is obtained by eliminating the case where the trigger did not fire and renormalizing. Via some algebra we obtain:

\[
P_1(\eta, \eta, N_D) = \eta \times \left(\frac{N_D}{\eta + N_D}\right)^{1+N_D} \tag{3}
\]

\[
P_1(\eta, \eta, N_D|\text{trigger}) = \frac{\eta \left(\frac{N_D}{\eta + N_D}\right)^{1+N_D}}{1 - \left(\frac{N_D}{\eta + N_D}\right)^{N_D}}. \tag{4}
\]

for a Bose distribution in each of the modes collected in Fig. 1b. These two results are also shown on Fig. 2. For these curves, the abscissa is \( N_D \) rather than delay \( i \). Note that the total single photon probability rises somewhat as \( N_D \rightarrow \infty \). This probability, calculated for a thermal source approaches the result that would be obtained for a Poisson distribution \( P_1(\eta, \eta, N_D) = \eta \times e^{-\eta} \), which is independent of \( N_D \). This independence for a set of Poisson subsystems is what one would expect, as a collection of Poisson subsystems taken together yield a result with Poisson statistics. The tending of the collection of many single mode Bose subsystems toward the Poisson result is also understandable in that the more independent subsystems that are included in the sum, the more the individual events in the system are independent of each other, which is the definition of Poisson statistics. The higher values of \( P_1(\eta, \eta, N_D|\text{trigger}) \) seen in Fig. 2 indicate the advantage of having heralded photons, rather than those from a faint laser. We also note that for Poisson instead of Bose distributions, the above analysis yields qualitatively the same shaped curves seen in Fig. 2, but all the \( P_{\eta, \eta, N_D}(i) \) probabilities are somewhat higher.

Analysis of Eq. (2) shows that \( \eta \) near one gives the best single photon certifications. This is because high \( \eta \) means that the system provides more complete information about what has happened, e.g. an instance of the trigger not firing means with high certainty that no photon was incident, while low \( \eta \) decreases our certainty of that. Highlighting the advantage of this method, Fig. 2 also shows that in almost all cases, the multiplexed heralded system presented here significantly surpasses the single photon probability of a faint laser described by the Poisson limit to the curve. This advantage still holds for \( \eta < 1 \), although the certifications are not as high.

Eq. (4) also shows that increasing \( \eta \) increases the spread of the certifications, while decreasing the maxi-
the setup would produce single photons with individual certifications that the photons produced are actually the desired single photons. Such a better-defined single photon source will allow for better use of quantum channel resources in a cryptographic system by reducing the need for overhead tasks such as privacy amplification, as well as having implications for the field of quantum computation. As photon counting becomes more convenient at telecom wavelengths we expect that integrated all solid state implementations of these schemes will be made even easier and we will have truly achieved the dream of a convenient source of single photons on demand. We are currently working on experimental implementations.

FIG. 2: The fan of curves labeled 1-8 are the probabilities of exactly one photon being produced given that the \( i^{th} \) delay, in a system of \( N_D \) delays, caused the trigger to fire for \( \eta, \overline{\eta}=1 \).

The lowest curve is the total probability of a system of \( N_D \) producing a single heralded photon per pump pulse with the Poisson limit shown just above. The dashed curve above the Poisson limit is the probability that the emitted light is a single photon given that the trigger did fire. (For these last 3 curves, the \( x \)-axis is \( N_D \) rather than triggered delay.)

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