Stochastic Investigation of Natural Frequency for Functionally Graded Plates

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Abstract. This paper presents the stochastic natural frequency analysis of functionally graded plates by applying artificial neural network (ANN) approach. Latin hypercube sampling is utilised to train the ANN model. The proposed algorithm for stochastic natural frequency analysis of FGM plates is validated and verified with original finite element method and Monte Carlo simulation (MCS). The combined stochastic variation of input parameters such as, elastic modulus, shear modulus, Poisson ratio, and mass density are considered. Power law is applied to distribute the material properties across the thickness. The present ANN model reduces the sample size and computationally found efficient as compared to conventional Monte Carlo simulation.

1. Introduction
Functionally graded materials (FGM) belong to the category of advanced materials, which contain two different materials i.e. metal and ceramic. The mechanical, chemical and thermal properties of FGM are totally different from the individual parent materials [1]. Traditional methods cannot be used to dissolve two or more alloying elements due to thermal equilibrium limit. FGM is an advances of composite, in which material properties are varying continuously throughout the depth [2]. Figure 1 shows the geometric view of FGM cantilever plate, in which top of the plate is ceramic rich, whereas bottom is metal rich. Composites have some limitations i.e. it fails under extreme working condition due to delamination of layer, but in FGM there is no layer, so it can be used for long working condition [3]. Many researchers worked on free vibration analysis by using different theories, and models. The ANN model is developed for analysis of stochastic natural frequency analysis of laminated composite plates [4]. A combined application of FEM and ANN are utilised to found the effect of laminating stacking sequence of composite on ballistic limit [5]. First order deformation theory is applied to static and free vibration analysis of FG carbon nanotube skew plate with considering different skew angle, aspect ratio, thickness to width ratio, and boundary conditions [6]. Mori–Tanaka scheme, Power law, and third-order deformation theory are used to investigate the effect of loading condition, volume-fraction exponent, skew angle, and aspect ratio on deflection, natural frequency, and critical buckling load of FG skew plate [7]. Finite element method is applied to determine the natural frequency and mode shape of FGM at different boundary conditions, and different value of power law index [8]. 3D-shell model and discrete double directors shell model (DDDSM) are developed to analysis of free vibration of FGM plates [9]. State-space based differential quadrature method (SSDQM) is used to analysis of natural frequency of FGM plate. ANN is also applied for comparative study for different boundary conditions, thickness of the annular plate, and
material property graded index [10]. In past, various studies on stochastic analysis of composite have been conducted [11-16]. The present study is computationally investigated on stochastic natural frequencies of the FGM plates by using ANN approach.

Figure 1. Geometry of FGM cantilever plate

2. Mathematical Formulation

In Cartesian coordinate system (x, y, z) as shown in Figure 1, assume (x, y) is the mid plane of the reference plane. The displacement can be expressed as

\[
\begin{align*}
\mathbf{u}(x, y, z) &= u_0(x, y) - z\theta_x(x, y)u(x, y) \\
\mathbf{v}(x, y, z) &= z_0(x, y) - z\theta_y(x, y)v(x, y) \\
\mathbf{w}(x, y, z) &= w_0(x, y) = w(x, y)
\end{align*}
\]

Where \( u, v, \) and \( w \) are the displacement components in the x, y and z direction respectively. \( u_0, v_0 \) and \( w_0 \) are the displacement at the mid plane and \( \theta_x, \theta_y \) are the rotations of cross sections along the x and y axes. Using Hamilton’s principles, the dynamic equilibrium equation employing Lagrangian can be represented as

\[
L_f = T - U - W
\]

Where \( T, U \) and \( W \) denotes total kinetic energy, total strain energy and total potential of the applied load, respectively. The stochastic dynamic equilibrium equation considering free vibration can be expressed as

\[
[M(\dot{\omega})][\ddot{\delta}] + [K(\dot{\omega})][\dot{\delta}] = 0
\]

Where \( (\dot{\omega}) \) denotes the degree of stochasticity and \( \{\delta\} \) is global displacement vector and \( \{F\} \) is Global vector of externally applied load, \([M(\dot{\omega})]= \) Global mass matrix, \([K(\dot{\omega})]= \) Global stiffness matrix.

If ‘\( M' \) denotes a function of the material properties can be expressed as,

\[
M = M_0 + M_1 T^{-1} + 1 + M_2 T + M_3 T^2 + M_4 T^3
\]

Here, \( M_0, M_1, M_2, M_3, M_4 \) are the coefficients of temperature \( T \) in Kelvin. The effective material properties (\( M \)) are obtained using the power law distribution,
\[ M(t) = M_m + \left( M_c - M_m \right) \left[ \frac{z}{t} + \frac{1}{2} \right]^p \]  \quad (7)

Where ‘c’ and ‘m’ designates the equivalent values at the external surface (ceramic rich) and internal surface (metal rich) of functionally graded plates. Here t and p denote the thickness and power law exponent or material property graded index, respectively and, \( z = \frac{t}{2} \).

3. Stochastic Approach using ANN Model

In the present study, the most sensitive input parameters, namely elastic modulus, shear modulus, Poisson ratio and mass density are considered for combined variation cases. Hence, in this paper, the material properties of functionally graded plate are considered as input parameters for stochastic analysis of natural frequencies. A combined variations in input parameters(s) can be considered as follows:

\[
g\{E_1(\overline{\theta}), G_{12}(\overline{\theta}), \mu(\overline{\theta}), \rho(\overline{\theta})\} = \{\Phi_1(E_1(1), E_1(2), \ldots E_1(i)), \Phi_2(G_{12}(1), G_{12}(2), \ldots G_{12}(i)), \Phi_3(\mu(1), \mu(2), \ldots \mu(i)), \Phi_4(\rho(1), \rho(2), \ldots \rho(i))\} \quad (8)
\]

Where \( E_1, G_{12}, \mu, \\text{and} \rho \) are longitudinal elastic modulus, longitudinal shear modulus, Poisson’s ratio, and mass density respectively. Weights, bias, and an activation function are the main unit of the ANN. Figure 2 shows the basic elements of ANN, where input is given to each neuron, and weight function is multiplied with each input. The bias \( B_i \) is a nonzero value, which is added with the summation of inputs and corresponding weight, given as

\[ R_i = \sum_{j=1}^{n} W_{ij} Y_j + B_i \]  \quad (9)

The summation \( R_i \) is transformed using activation function also called transfer function, \( Z_i \) represents a value called the unit’s “activation”

\[ Z_i = f(R_i) \]  \quad (10)

4. Results and Discussion

In this paper, probability density function (PDF) plot is considered as the benchmark of the bottom line results. First three natural frequencies are considered to validate the results proposed by ANN model with respect to the direct Monto Carlo simulation. A comparative study between the results obtained from ANN model and MCS is carried out. Probability density function (PDF) for first three natural frequencies (rad/s) due to combined variation of variables, elastic modulus, shear modulus, Poisson’s ratio, and mass density for ANN model and MCS model is shown in Figure 2. ANN model shows better results with higher number of sample size as shown in the figure 2, in which sample size \( N=1024 \) gives results almost same to MCS. Scatter plot for first three natural frequencies (rad/s) for ANN model with respect to original finite element model considering sample size \( N=1,024 \) for combined variation of elastic modulus, shear modulus, Poisson’s ratio, and mass density is shown in Figure 3. It is evident from Figure 3, the results obtained from the ANN model are close to the MCS, which gives almost same range of natural frequency for fundamental natural frequency.
Figure 2. Probability density function for first three natural frequencies (rad/s) due to combined variation of variables for ANN with different sample size and MCS.

Figure 3. Scatter plot of first three natural frequencies (rad/s) for ANN model with respect to original finite element model considering $N_{\text{samp}} = 1024$ for combined variation of elastic modulus, shear modulus, Poisson’s ratio, and mass density.
Figure 4. Probability density function for first three natural frequencies (rad/s) due to combined variation of combined variables, elastic modulus, shear modulus, Poisson’s ratio, and mass density at different percentage of variation (P).

Figure 4 shows the probability density function for first three natural frequency due to combined variation at different percentage of variation (P) i.e. 10%, 20%, and 30%. It is clear from the Figure 4, with increase in the percentage of variation (P), stochasticity of all three natural frequencies is also increases. For all natural frequencies, stochasticity in output quantity of interest (QoI) is minimum, when percentage of variation is 10% as compared to 20% and 30%.
5. Conclusions
The novelty of the present study includes the incorporation of artificial neural network in conjunction to stochastic finite element algorithm for FGM plates. Both individual and combined variations of input variables are demonstrated. From the analyses presented in this article it is found that as the percentage of variation of input parameters increases, the sparsity of the stochastic output natural frequencies also increases while no notable variation of stochastic mean value of respective natural frequencies is identified. The computational time and cost is reduced by using the present ANN approach compared to traditional Monte Carlo simulation method. This ANN based approach can be extended to deal with more complex systems in future.

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