Abstract—Existing electricity market designs assume risk neutrality and lack risk-hedging instruments, which leads to suboptimal market outcomes and reduces the overall market efficiency. This paper enables risk-trading in the chance-constrained stochastic electricity market by introducing Arrow-Debreu Securities (ADS) and derives a risk-averse market-clearing model with risk trading. To enable risk trading, the probability space of underlying uncertainty is discretized in a finite number of outcomes, which makes it possible to design practical risk contracts and to produce energy, balancing reserve and risk prices. Notably, although risk contracts are discrete, the model preserves the continuity of chance constraints. The case study illustrates the usefulness of the proposed risk-averse chance-constrained electricity market with risk trading.

I. INTRODUCTION

Uncertain renewable energy sources (RES) challenge the efficiency of existing wholesale electricity markets, which still lack risk-hedging financial instruments, [1]. As a result, electricity markets are incomplete with respect to uncertainty and risk, i.e., it does not provide market participants with a mechanism to secure their positions relative to all probable future states of the system. In [2–5], we developed a chance-constrained stochastic electricity market design, which internalizes the RES uncertainty and produces uncertainty-aware electricity prices that support welfare efficiency, revenue adequacy and cost recovery. However, this market design is risk-neutral and ignores risk perceptions (preferences) of market participants, which leads to suboptimal market outcomes.

Although common in the fields of stochastic optimization [6] and finance [7–9], the notion of risk aversion has only recently gained attention in power system operations and electricity markets. For example, Sopasakis et al. [10] and Hans et al. [11] developed risk-averse control strategies for decentralized generation resources, and Kazempour et al. [12] explored the effects of risk-averse electricity producers in a two-stage market equilibrium. However, while hedging market outcomes against risk using the conditional value-at-risk (CVaR), [10–12] do not allow for risk trading. On the other hand, Ralph and Smeers [13], [14] demonstrated in a layout reminding wholesale electricity markets that risk trading using Arrow-Debreu Securities (ADS) results in a risk-complete market design and the resulting risk-aware prices can be related to risk-neutral market outcomes. Motivated by [13], [14], Philpott et al. [15] extended the ADS trading to a multi-stage scenario-based stochastic market and showed the existence of a risk-averse competitive equilibrium, if all market participants are endowed with a coherent risk measure. Gérard et al. [16] applied the result from [15] to a two-stage stochastic electricity market and showed that a risk-averse equilibrium is not unique. In line with [15], [16], Cory-Wright and Zakeri [17] demonstrated that different risk perceptions of market participants may encourage them to act strategically, thus causing suboptimal market outcomes, which can be eliminated if the electricity market is completed with risk trading.

Departing from scenario-based stochastic programming used in market designs in [12], [13], [16], [18–20], this paper explores risk trading in the chance-constrained electricity market proposed in [2–5] by introducing ADS. Since the chance-constrained approach does not require pre-defined scenarios, the paper shows that infinite-dimensional ADS instruments can be discretized to design practical risk contracts for a given set of uncertain outcomes, thus making the market risk complete. Finally, this paper derives a risk-averse equilibrium and analyzes risk-averse market outcomes.

II. CHANCE-CONSTRAINED ELECTRICITY MARKET

Consider a chance-constrained electricity market as in [2–4]. Let $\mathcal{N}$, $\mathcal{G}$, $\mathcal{U}$ be sets of nodes, conventional generators, and RES. The market operator solves the following optimization:

\[
\min_{p_{G,i}, \alpha_i} \mathbb{F}_0 \left[ \sum_{i \in \mathcal{G}} c_i(p_{G,i}(\omega)) \right] \quad (1a)
\]

s.t. $p_{U,i}(\omega) = p_{U,i} + \omega_i$ $\forall i \in \mathcal{U}$ $(1b)$

$p_{G,i}(\omega) = p_{G,i} - \alpha_i^T \omega$ $\forall i \in \mathcal{G}$ $(1c)$

($\delta_i^+)$ : $\mathbb{P} [p_{G,i}(\omega) \leq P_{G,i}] \geq 1 - \epsilon_g$ $\forall i \in \mathcal{G}$ $(1d)$

($\delta_i^-$) : $\mathbb{P} [p_{G,i}(\omega) \geq P_{G,i}] \leq 1 - \epsilon_g$ $\forall i \in \mathcal{G}$ $(1e)$

($\theta$) : $\mathbb{P} [F(p_{G,i}(\omega), p_{U,i}(\omega), p_{D,i}) \in \mathcal{F}] \geq 1 - \epsilon_f$ $(1f)$

($\lambda_i$) : $p_{G,i} + p_{U,i} + p_{D,i} = p_{D,i}$ $\forall i \in \mathcal{N}$ $(1g)$

($\chi_u$) : $\sum_{i \in \mathcal{G}} \alpha_i = 1$ $\forall u \in \mathcal{U}$, $(1h)$

where (1a) minimizes the system operating cost evaluated by measure $\mathbb{F}_0$ (e.g., expectation, if $\mathbb{F}_0 = \mathbb{E}$) over the random vector of RES forecast errors $\omega = [\omega_i, i \in \mathcal{U}]$ and given the cost function of each generator $c_i(p_{G,i})$. Eq. (1b) models the uncertain RES power output $p_{U,i}(\omega_i)$ at node $i$ as the RES forecast $p_{U,i}$ plus the RES forecast error $\omega_i$. Eq. (1c) defines the power output of conventional generators under uncertainty $p_{G,i}(\omega)$ using an affine control policy, where $p_{G,i}$ and $\alpha_i = [0 \leq \alpha_{i,u} \leq 1, u \in \mathcal{U}]$ are decisions for the scheduled power output and the vector of participation factors for balancing reserve of generator $i$. Note that $\alpha_{i,u}$ denotes the participation factor of generator $i$ in response to the RES forecast error at node $u \in \mathcal{U}$. Chance constraints (1d) and (1e) ensure the power output of conventional generator $i$ under uncertainty does not exceed the upper or lower limits $P_{G,i}$ and $P_{G,i}$, with a probability of $1 - \epsilon_g$. Similarly, (1f) ensures that DC power flows computed using function $F(p_{G,i}(\omega), p_{U,i}(\omega), p_{D,i})$, which maps net nodal injections in power flows, are contained in a convex set of feasible power...
flows give by $\mathcal{F}$ with a probability of $1 - \epsilon_f$. Finally, $[13]$ is the nodal power balance constraint given the nodal demand and power flow injections $p_{D,i}$ and $p_i(F)$. Eq. (1h) ensures balancing reserve sufficiency to mitigate $\omega$. Greek letters in parentheses in $[14]$–[18] denote dual multipliers.

A. Deterministic Reformulation

Assuming that $c_i(p_{G,i})$, is quadratic:

$$c_i(p_{G,i}(\omega)) = c_{0i} + c_{1i}p_{G,i}(\omega) + c_{2i},$$

where $c_{2i}$, $c_{1i}$, $c_{0i}$ are cost coefficients, and using $\mathbb{F}_i \equiv \mathbb{E}_i$ and $\omega \sim \mathcal{N}(0, \Sigma)$, where $\Sigma$ is the covariance matrix of $\omega$, the optimization in (1) has a tractable convex (second-order conic) reformulation:

$$\begin{align*}
\min_{p_{G,i}(\omega)} & \quad \sum_{i \in G} c_i(g_i) + c_{2i} \left\| \alpha_i^T \Sigma^{1/2} \right\|^2_2 \\
\text{s.t.} & \quad (\zeta_i) : \quad s_{p_{G,i}} \geq \left\| \alpha_i^T \Sigma^{1/2} \right\|^2_2 \\
& \quad (\delta_i^+) : \quad p_{G,i} + z_{1-\epsilon_f} s_{p_{G,i}} \leq \mathbb{F}_i, \quad \forall i \in G \\
& \quad (\delta_i^-) : \quad -p_{G,i} + z_{1-\epsilon_f} s_{p_{G,i}} \leq -\mathbb{F}_i, \quad \forall i \in G \\
& \quad (\theta) : \quad F_{\epsilon_f}(p_{G,i}, p_{U,i}, p_{D,i}, \alpha) \leq 0 \\
& \quad (\lambda) : \quad p_{G,i} + p_{U,i} + p_i(F_{\epsilon_f}) = p_{D,i}, \quad \forall i \in \mathcal{N}, \\
& \quad (\chi_u) : \quad \sum_{i \in G} \alpha_{i,u} = 1, \quad \forall u \in \mathcal{U},
\end{align*}$$

where $z_{1-\epsilon_f} = \Phi^{-1}(1 - \epsilon_f)$ is the quantile function of the standard normal distribution and $s_{p_{G,i}}$ is an auxiliary decision variable modeling the standard deviation of $p_{G,i}(\omega)$. (Note that less restrictive distribution assumptions could be invoked on $\omega$, see [3].) Function $F_{\epsilon_f}()$ in (3c) maps the decision variables, parameters, statistical characteristics of $\omega$ and security threshold $\epsilon_f$ into a vector of security adjusted power flows, i.e. power flows with security margins so that enforcing (3c) is equivalent to enforcing the original chance constraint in (1f).

B. Equilibrium Formulation

The optimization problem in (1) and (3) represents a risk-neutral market operator and has been proven to yield energy and balancing reserve prices $\lambda_i$ and $\chi_u$, which solve the following equilibrium, [2]–[4]:

$$\begin{align*}
\left\{ \begin{array}{l}
\max_{p_{G,i},\alpha_i} \quad \lambda_i p_{G,i} + \chi^T \alpha_i - \mathbb{E}[c_i(p_{G,i}(\omega))] \\
\text{s.t.} \quad (3b) - (3d)
\end{array} \right.
\end{align*}$$

$$\begin{align*}
0 \leq p_{G,i} + p_{U,i} + p_i(F) - p_{D,i} \leq \lambda_i \geq 0, \quad \forall i \in \mathcal{G} \\
0 \leq \sum_{i \in \mathcal{G}} \alpha_{i,u} - 1 \leq \chi_u \geq 0, \quad \forall u \in \mathcal{U} \\
0 \leq -F_{\epsilon_f}(p_{G,i}, p_{U,i}, p_{D,i}, \alpha) \leq 0
\end{align*}$$

where (4a) is a profit maximization solved by each producer and (4b)–(4d) are the market-clearing conditions. As shown in [2]–[4], $\lambda_i$ and $\chi_u$ can be interpreted as equilibrium energy and reserve prices.

III. Risk-Averse Chance-Constrained Electricity Market

The optimization in (4a) solved by each producer is risk neutral because it assumes average (expected) outcomes of random $\omega$. In practice, however, producers are likely to hedge against the risk of uncertain costs based on their risk perception. This section considers risk-averse profit maximizing producers endowed with a risk measure $\mathbb{F}_i$.

A. Coherent Measures of Risk

Intuitively, a risk measure evaluates an uncertain outcome $Z$ in terms of an equivalent deterministic outcome $\mathbb{F}[Z]$ so that a producer endowed with risk measure $\mathbb{F}$ is indifferent between accepting uncertain $Z$ or its certainty equivalent $\mathbb{F}[Z]$. Additionally, a risk measure is called coherent if, [7], [9]:

(i) $\mathbb{F}[c] = c$, i.e. the certainty equivalent of a deterministic constant $c \in \mathbb{R}$ is equal to the constant,

(ii) $\mathbb{F}[cZ] = c \mathbb{F}[Z]$, i.e. an uncertain outcome $Z$ scaled by some positive constant $c > 0$ is equal to the scaled certainty equivalent,

(iii) $\mathbb{F}(1 - c)Z + cY \leq (1 - c)\mathbb{F}[Z] + c\mathbb{F}[Y]$ for $c \in [0,1]$, i.e. the risk measure is convex, and

(iv) $\mathbb{F}[Z] \leq \mathbb{F}[Y]$ if $Z \preceq Y$, i.e. the risk measure is monotone.

The expectation operator $\mathbb{E}$ used to obtain (3a) fulfills (i)–(iv) and is, therefore, a coherent measure of risk, [9]. However, $\mathbb{E}$ neglects all information on the volatility of the outcome leading to its common interpretation as a risk-neutral measure.

Any coherent risk measure can be expressed as, [9], [22]:

$$\mathbb{F}[Z] = \sup_{P \in \mathcal{D}} \mathbb{E}_P[Z]$$

where $\mathcal{D}$ denotes the risk set (risk envelope) of $\mathbb{F}$, i.e. a compact and convex set of probability measures, and $\mathbb{E}_P$ is the expectation over the probability measure $P$. Risk set $\mathcal{D}$ uniquely defines $\mathbb{F}$ and can be structured such that $\sup_{P \in \mathcal{D}} \mathbb{E}_P[Z]$ is identical to specific risk measures, e.g. CVaR, [9].

Remark 1. Defining a risk measure in terms of a worst-case probability distribution as in (5) is structurally identical to distributionally robust optimization that can be applied to chance constraints [14]–[18], see e.g. [5]. This work, however, focuses on the evaluation of the objective function.

B. Risk-Averse Profit Maximization

To derive a risk-averse modification of (3), we define a risk set using a moment ambiguity set, [23], which generally yields tractable convex optimization problems. For example, it can be implemented by restricting all distributions in a given set to share the same moments, [24], or to have their moments in a given closed set, [25]. Accordingly, the risk set of each producer $i$ can be defined as:

$$\mathcal{D}_i = \{ P(\omega) \in \mathcal{P} \mid \mathbb{E}_P[\omega] = 0, \text{Var}_P[\omega] \in S_i \}$$

where $\mathcal{P}$ is the set of probability distributions and $S_i = \{ \Sigma_1, \ldots, \Sigma_K \}$ is the set of $K$ covariance matrices $(\Sigma_1, \ldots, \Sigma_K)$, where $\Sigma$ is the same for all producers.

The notable feature of risk set $\mathcal{D}_i$ is that it represents a finite set of continuous distributions as opposed to discrete polyhedral probability measures in [15], [16], which capture a given set of pre-described scenarios. Hence, using (5) and (6) yields:

$$\begin{align*}
\min_{p_{G,i},\alpha_i} & \quad \sup_{P \in \mathcal{D}_i} \mathbb{E}_P[c_i(p_{G,i}(\omega))] \\
= & \quad \min_{p_{G,i},\alpha_i} c_i(p_{G,i}) + \sup_{k=1,\ldots,K} c_{2i} \left\| \alpha_i^T \Sigma_k^{1/2} \right\|^2_2
\end{align*}$$

Although $\mathcal{D}_i$ as defined in (6) is non-convex, solving (7)
is equivalent to the following problem with convex polyhedral set \( \tilde{S}_i = \text{conv}(S_i) \), (26 Section 6.4.2):

\[
\min_{p_{G,i}, \alpha_i} \quad c_i(p_{G,i}) + \sup_{\Sigma_k \in \tilde{S}_i} c_{2i} \| \alpha_i^T \Sigma_k^{1/2} \|_2^2
\]

and we can define:

\[
\mathcal{D}_i = \{ \mathbb{P}(\omega) \in \mathcal{P} \mid \mathbb{E}_\mathcal{P}[\omega] = 0, \text{Var}_\mathcal{P}[\omega] \in \tilde{S}_i \}
\]

as the convex counterpart of \( \mathcal{D}_i \), which yields the following coherent risk measure:

\[
\mathbb{F}_i(c_i(p_{G,i}, \omega)) = \sup_{\mathbb{P} \in \mathcal{D}_i} \mathbb{E}_\mathcal{P}[c_i(p_{G,i}, \omega)].
\]

Using the epigraph form of (8), each producer now solves the following risk-average profit maximization problem:

\[
\max_{p_{G,i}, \alpha_i} \quad \lambda_i p_{G,i} + \chi^T \alpha_i - t_i \tag{11a}
\]

s.t. (3b)–(3d) and (11c)

\[
(\eta_{i,k}) : \quad t_i \geq c_i(p_{G,i}) + c_{2i} \| \alpha_i^T \Sigma_k^{1/2} \|_2^2 \quad \forall \Sigma_k \in \tilde{S}_i.
\]

Hence, the risk-average version of (3) is:

\[
\min_{p_{G,i}, \alpha_i} \quad (c_i(p_{G,i}) + t_i) \tag{12a}
\]

s.t. (3b)–(3d) and (12c)

\[
(\eta_{i,k}) : \quad t_i \geq c_{2i} \| \alpha_i^T \Sigma_k^{1/2} \|_2^2 \quad \forall \Sigma_k \in \tilde{S}_i, \forall i.
\]

Remark 2. Unlike in (3a), the risk-average profit maximization in (11) allows different producers to have different perceptions of the system uncertainty, which can be modeled as different risk attitudes drawn from producer-specific set \( \mathcal{D}_i \).

IV. RISK TRADING IN THE CHANCE-CONSTRAINED ELECTRICITY MARKET

If producer \( i \) is endowed with coherent risk measure \( \mathbb{F}_i \) given by risk set \( \mathcal{D}_i \) and seeks to maximize its risk-adjusted profit as in (11), it will lead to suboptimal market outcomes because (12) is incomplete with respect to risk. This section describes an approach to complete the chance-constrained market with respect to risk by introducing ADS trading.

A. Continuous Risk Trading

ADS as introduced in (27) is a common security contract that depends on the outcome of an uncertain asset, which in the case of the chance-constrained electricity market in (12) is the RES forecast error given by \( \omega \). Specifically, a buyer of the contract pays price \( \mu(\omega) \) to receive a payment of 1 for a pre-defined realization of \( \omega \). Hence, if producer \( i \) seeks to receive a payment of \( a_i(\omega) \) for a realization of \( \omega \), it must pay in advance:

\[
\pi_{a_i} = \int \mu(\omega)a_i(\omega)d\omega \tag{13}
\]

where \( \Omega \) is the space of all possible outcomes of \( \omega \). If \( a_i(\omega) \leq 0 \), then producer \( i \) sells ADS (i.e. provides security to the system) and receives the payment of \( \pi_{a_i} \leq 0 \). Otherwise, if \( a_i(\omega) \geq 0 \), producer \( i \) purchases ADS and pays \( \pi_{a_i} \geq 0 \). Further, the market trading ADS must ensure that the amount of securities purchased and sold match, i.e. risk trading must be revenue adequate:

\[
\sum_{i \in \mathcal{G}} a_i(\omega) = 0 \quad \forall \omega \in \Omega \tag{14}
\]

Given the risk trading model in (13) and (14), each profit-maximizing producer can be modeled as follows:

\[
\max_{p_{G,i}, \alpha_i} \quad \lambda_i p_{G,i} + \chi^T \alpha_i - t_i - \pi_{a_i} \tag{15a}
\]

s.t. (3b)–(3d) and (15c)

\[
(\eta_{i,k}) : \quad t_i \geq \mathbb{E}_\mathcal{P}_k[c_i(p_{G,i}(\omega))] - \mathbb{E}_\mathcal{P}_k[a_i(\omega)], \quad \forall \mathbb{P}_k \in \mathcal{D}_i.
\]

where \( \pi_{a_i} \) reflects the additional cost or revenue due to risk trading, as given in (13), and \( \mathbb{E}_\mathcal{P}_k[a_i(\omega)] \) in (15c) is the expected ADS cost or revenue over probability measure \( \mathbb{P}_k \).

Given (13) and (14), extending the risk-averse market-clearing in (12) with risk trading yields:

\[
\min_{p_{G,i}, \alpha_i, \omega} \quad \sum_{i \in \mathcal{G}} (c_i(p_{G,i}) + t_i) \tag{16a}
\]

s.t. (3b)–(3d) and (15c)

\[
(\mu(\omega)) : \quad \sum_{i \in \mathcal{G}} a_i(\omega) = 0 \quad \forall \omega \in \Omega, \tag{16c}
\]

where (16c) enforces the market-clearing condition in (14), yielding dual multiplier \( \mu(\omega) \). Using (16) and under the assumption that set \( \mathcal{F} \) is sufficiently large to accommodate injections \( p_G(w), p_U(w), p_D \) without causing network congestion\(^1\) (i.e. energy prices are uniform \( \lambda = \lambda_i \)), we prove:

**Proposition 1.** Let \( \lambda^*, \chi^*, \) and \( \mu^*(\omega) \) be equilibrium energy, balancing, and risk prices, respectively, so that \( \{\lambda^*, \chi^*_i, \mu^*(\omega) : \mathcal{G}, \forall i \in \mathcal{G} \} \) solves (16). Then \( \mu^*(\omega) \) can be interpreted as a probability measure that solves the risk-neutral profit maximization equivalent of the risk-averse profit maximization with ADS trading.

**Proof.** The market-clearing problem in (16) is convex as long as \( a_i(\omega) \) is convex in \( \omega \). Therefore, KKT conditions can be invoked. The Lagrangian function of the profit maximization of each producer in (15) can be written as:

\[
\mathcal{L}_i = \lambda p_{G,i} + \chi^T \alpha_i - t_i - \pi_{a_i} - \sum_{k=1}^K \eta_{i,k}(\mathbb{E}_\mathcal{P}_k[c_i(p_{G,i}(\omega))] - a_i(\omega)) - t_i
\]

\[
- \zeta_i(\| \alpha_i^T \Sigma_k^{1/2} \|_2^2 - s_{G,i})
\]

Hence, the resulting optimality conditions for \( t_i \) and \( a_i(\omega) \) are

\[
\frac{\partial \mathcal{L}_i}{\partial t_i} = -1 + \sum_{k=1}^K \eta_{i,k} = 0 \quad \Rightarrow \quad \sum_{k=1}^K \eta_{i,k} = 1 \tag{18}
\]

\[
\frac{\partial \mathcal{L}_i}{\partial a_i(\omega)} = -\mu(\omega) + \sum_{k=1}^K \eta_{i,k} f(\omega, \sigma_k) = 0 \tag{19}
\]

where \( f(\omega, \Sigma_k) \) denotes the probability density function of a multivariate, zero-mean normal distribution with covariance matrix \( \Sigma_k \). Note that for the derivation of (19) we used the

\(^1\)This assumption simplifies derivations. Subsequent results can be generalized for the congested case by pricing transmission assets, (28).
following two standard functional derivatives, \([29]\):
\[
\frac{\partial \pi_{a_i}}{a_i(\omega)} \equiv \frac{\partial}{a_i(\omega)} \int_\Omega \mu(\omega) a_i(\omega) d\omega = \mu(\omega),
\]
\[
\frac{\partial}{a_i(\omega)} \mathbb{E}_{P_k}[a_i(\omega)] = \frac{\partial}{a_i(\omega)} \int_\Omega a_i(\omega) f(\omega, \Sigma_k) d\omega = f(\omega, \Sigma_k).
\]
(20)
(21)

Conditions \((18)\) and \((19)\) lead to two relevant observations:

(O1) Dual multiplier \(\mu(\omega)\) in \((16c)\) is a probability measure as it is the weighted average of \(K\) probability density functions with zero means and covariance matrices \(\Sigma_1, ..., \Sigma_K\). In other words, random \(Z(\omega) \sim \mu(\omega)\) has the expected value of \(\mathbb{E}_\mu[Z(\omega)] = 0\) and the variance of \(\text{Var}_\mu[Z(\omega)] = \sum_{k=1}^K \eta_{i,k} \Sigma_k\).

(O2) Since \(S_i\) is a convex set, condition \((18)\) ensures that \(\sum_{k=1}^K \eta_{i,k} \Sigma_k \in S_i\) and thus \(\mu(\omega) \in \mathbb{D}_i\).

The set of optimal decisions \(\{\lambda^*; \chi^*_i; \mu^*(\omega); p^*_{G,i}; \forall i \in G; \alpha^*_i, \chi^*_i, \mu^*(\omega); p^*_{G,i}; \forall i \in G\}\) maximize \(L_i\) given in \((17)\). Note that the fifth term in \((17)\) can be recast using observation O1 above as:
\[
\sum_{k=1}^K \eta_{i,k} \mathbb{E}_{P_k}[c_i(p_{G,i}(\omega)) - a_i(\omega)]
\]
\[
= \sum_{k=1}^K \eta_{i,k} \int_\Omega [c_i(p_{G,i}(\omega)) - a_i(\omega)] f(\omega, \Sigma_k) d\omega
\]
\[
= \int_\Omega [c_i(p_{G,i}(\omega)) - a_i(\omega)] \sum_{k=1}^K \eta_{i,k} f(\omega, \Sigma_k) d\omega
\]
\[
= \int_\Omega [c_i(p_{G,i}(\omega)) - a_i(\omega)] \mu(\omega) d\omega
\]
\[
= \mathbb{E}_\mu[c_i(p_{G,i}(\omega))] - \pi_{a_i}.
\]
(22)

Substituting \((22)\) in \((17)\) leads to:
\[
L_i = p_{G,i} + \chi_i - \mathbb{E}_\mu[c_i(p_{G,i}(\omega))] - y_\delta,
\]
(23)
where \(y_\delta\) denotes the last three terms in \((17)\). Hence, \((23)\) is a risk-neutral equivalent, evaluated with respect to probability measure \(\mu(\omega)\), of the risk-averse profit of producer \(i\) participating in risk trading with ADS.

Given Proposition \([1]\) the optimization of individual producers in \((15)\) is related to the risk-averse chance-constrained electricity market with ADS trading in \((16)\).

**Proposition 2.** Let \(\lambda^*, \chi^*_i, \mu^*(\omega)\) be equilibrium energy, balancing, and risk prices so that \(\{\lambda^*; \chi^*_i; \mu^*(\omega); p^*_{G,i}; \forall i \in G; \alpha^*_i, \chi^*_i, \mu^*(\omega); p^*_{G,i}; \forall i \in G\}\) solves problem \((16)\). Assuming that risk sets \(\mathbb{D}_i, i \in G\) are non-disjoint, i.e. \(\bigcap_{i \in G} \mathbb{D}_i \neq \emptyset\), then these prices and allocations solve the risk-averse chance-constrained market with risk trading with \(\mathbb{D}_0 = \bigcap_{i \in G} \mathbb{D}_i\) and worst case probability measure \(\mu(\omega)\).

**Proof.** Given the optimal solution for each producer, it follows from the complementary slackness of \((15c)\):
\[
\eta_{i,k} \mathbb{E}_{P_k}[c_i(p_{G,i}(\omega)) - a_i(\omega)] - t_i = 0.
\]
(24)
Using \((14)\), \((22)\) and \((24)\), we write:
\[
\sum_{i \in G} t_i = \sum_{i \in G} \sum_{k=1}^K \eta_{i,k} \left(\mathbb{E}_{P_k}[c_i(p_{G,i}(\omega)) - a_i(\omega)]\right)
\]
\[
= \mathbb{E}_\mu \left[\sum_{i \in G} c_i(p_{G,i}(\omega))\right],
\]
(25)
Also, since \((15c)\) is a convex epigraph, it follows:
\[
t_i = \max_{\mu(\omega) \in \mathbb{D}_i} \mathbb{E}_{\mu}[c_i(p_{G,i}(\omega)) - a_i(\omega)]
\]
(26)
Given \((26)\), term \(\sum_{i \in G} t_i\) in \((25)\) can also be written as:
\[
\sum_{i \in G} t_i = \max_{\mu(\omega) \in \mathbb{D}_i} \mathbb{E}_{\mu}[c_i(p_{G,i}(\omega)) - a_i(\omega)]
\]
\[
\geq \max_{\mu(\omega) \in \bigcap_{i \in G} \mathbb{D}_i} \mathbb{E}_{\mu}[\sum_{i \in G} c_i(p_{G,i}(\omega)) - a_i(\omega)]
\]
\[
\geq \max_{\mu(\omega) \in \bigcap_{i \in G} \mathbb{D}_i} \mathbb{E}_{\mu}[\sum_{i \in G} c_i(p_{G,i}(\omega))]
\]
(27)
where transition \((\Xi)\) is due to the replacement of individual risk sets \(\mathbb{D}_i\) with the intersection of all risk sets \(\mathbb{D}_0 = \bigcap_{i \in G} \mathbb{D}_i\) and transition \((\Theta)\) is due to the market-clearing ADS condition in \((14)\). Since \(\mu(\omega) \in \mathbb{D}_i, i \in G\) and \(\mathbb{D}_0 \neq \emptyset\), due to observation O2 above, \((25)\) and \((27)\) yield
\[
\mathbb{E}_{\mu}[\sum_{i \in G} c_i(p_{G,i}(\omega))] = \max_{\mu(\omega) \in \mathbb{D}_0} \mathbb{E}_{\mu}[\sum_{i \in G} c_i(p_{G,i}(\omega))],
\]
(28)
showing that \(\mu(\omega)\) is the worst-case probability measure for the risk-averse chance-constrained market with risk trading. \(\square\)

**Remark 3.** Propositions \([1]\) and \([2]\) hold if \((16)\) has binding constraints in \((15)\) and can be proven analogously.

**B. Discrete Risk Trading**

Recall that Section \([4, A]\) defines ADS contracts as continuous over \(\omega\), which leads to an infinite-dimensional problem in \((16)\). On the other hand, a more practical and computationally tractable approach would be to discretize the probability space of \(\omega\) and consider contracts for discrete outcomes. Hence, consider the system-wide (aggregated) RES forecast error given as \(\Omega = \{e^\omega\} \omega = 0\) and variance \(\text{Var}_{P_k}[e] = \sum_{\omega \in \Omega} e(\omega) e(\omega)\) for vector of ones of appropriate dimensions. The probability space of \(\Omega\) can then be divided into \(W\) events \(w = 1, ..., W\), where each event is a closed interval given by \(\mathcal{W}_w = [l_w, u_w]\) so that \(\bigcup_{w=1}^W \mathcal{W}_w = \Omega\). These intervals are sequential such that \(l_w = -\infty, u_w = \infty\) and \(u_w = l_{w+1}\), \(w = 1, ..., W, w \neq 1, W - 1\). Using this discretization, the probability of each discrete outcome is defined from the underlying probability density function as:
\[
P_w(\sigma_k) := P_k[\mathcal{W}_w = \{\mathcal{W}_w \in \mathcal{W}_w \cap (\Omega \cap \Omega \geq l_w)\}]
\]
\[
= \int_{l_w}^{u_w} f(x, \sigma_k) dx
\]
(29)
and can be pre-computed for all \(w = 1, ..., W\) and \(k = 1, ..., K\). Using the discrete space notation, \((11)\) recasts as:
\[
\pi_{a_i} = \sum_{w=1}^W \mu(a_{w,i}, w)
\]
(30)
where \(a_{w,i} \in \Omega\). Next, using \((29)\), the expected cost or payment \(a_{w,i}\) under \(P_k\) can be computed as:
\[
\mathbb{E}_{P_k}[a_{w,i}(\omega)] \equiv \sum_{w=1}^W a_{w,i} P_w(\sigma_k).
\]
(31)
Finally, using (30) and (31) and the optimality condition for \( a_{i,w} \), the discrete-space equivalent of \( \mu(\omega) \) is computed as:

\[
\mu_w = \sum_{k=1}^{K} \eta_k P_w(\sigma_k) = \sum_{k=1}^{K} \eta_k \int_{x=0}^{x_w} f(x, \sigma_{i,k}) dx
\]

\[
= \int_{x=0}^{x_w} \sum_{k=1}^{K} \eta_k f(x, \sigma_{i,k}) dx, \quad \forall i
\]

where \( \sigma_{i,k} = \left\| e^T \Sigma_{i,k}^{1/2} \right\|_2 \) with \( \Sigma_k \in S_i \). Hence, due to (32), \( \mu_w \) retains the interpretation of \( \mu(\omega) \) from observation O1 of Proposition 1. Indeed, a random variable with probability density function \( \sum_{k=1}^{K} \eta_k f(x, \sigma_{i,k}) \) has variance \( \left\| e^T (\sum_{k=1}^{K} \eta_k \Sigma_k)_{1/2} \right\|_2 = e^T (\sum_{k=1}^{K} \eta_k \Sigma_k) e \), it follows that \( \text{Var}_r(\sigma) = e^T (\sum_{k=1}^{K} \eta_k \Sigma_k) e. \)

Using this result and (29)–(32), a discrete modification of the risk-averse chance-constrained electricity market (33) is computed as:

\[
\min_{p_{G,i}, \alpha_{i,w}, t_i} \sum_{i \in G} (c_{i}(p_{G,i}) + t_i)
\]

s.t. (3b)–(3g) (33b)

\[
(\eta_{i,k}) : \quad t_i \geq c_{2i} \left\| \alpha_i \Sigma_{i}^{1/2} \right\|_2^2 + \sum_{w=1}^{W} a_{i,w} P_w \left\| e^T \Sigma_{i,k}^{1/2} \right\|_2, \quad \forall \Sigma_k \in S_i, \forall i
\]

(33c)

\[
(\mu_w) : \quad \sum_{i \in G} a_{i,w} = 0, \quad \forall w = 1, ..., W.
\]

Since the discrete representation of ADS contracts in (33) is a special case of the infinite-dimensional representation in (16), the results of Propositions 1 and 2 hold for (33).

C. Price Analysis with Risk Trading

Using the risk-averse chance-constrained electricity market with discrete risk trading in (33), this section analyzes resulting energy, balancing reserve and risk prices as follows:

**Proposition 3.** Consider the risk-averse chance-constrained market with risk trading in (33). Let \( \lambda_i, \chi_u \) and \( \mu_w \) be the dual multipliers of the active power balance (31), the reserve sufficiency constraint (3g) and the ADS market-clearing constraint (33d). Then \( \mu_w \) is given by (32) and \( \lambda_i, \chi_u \) are:

\[
\lambda_i = 2c_{2i} \alpha_{i}^T \left\| \Sigma_i \right\|_2 + c_{1i} + (\delta^+_i - \delta^-_i) + y_{pg,i}(\theta)
\]

\[
\chi_u = \frac{1}{|G|} \sum_{i \in G} \left( 2c_{2i} \alpha_{i}^T \left\| \Sigma_i + z_{1-e} \delta^+_i \right\|_2 + s_{G,i}(\theta) \right),
\]

where \( y_{pg,i}(\theta) := \theta^T \frac{\partial f_i}{\partial a_{i,w}}, \quad y_{pg,i}(\theta) := \theta^T \frac{\partial f_i}{\partial a_{i,w}}, \quad \Sigma_i := \left\{ \Sigma_k \mid \Sigma_k \in S_i \right\}, \delta_i := \delta^+_i - \delta^-_i, \quad [X]_u \) is the vector of elements in the \( u \)-th column of matrix \( X \), and \( s_{G,i} = \left\| \alpha_{i}^T \Sigma_i^{1/2} \right\|_2 \), i.e. the standard deviation of \( p_{c,i}(\omega) \).

**Proof.** Let \( L \) be the Lagrangian function of (33), its first-order optimality conditions for \( p_{G,i}, \alpha_{i,w}, s_{G,i}, \) and \( a_{i,w} \) are:

\[
\frac{\partial L}{\partial a_{i,w}} = \sum_{k=1}^{K} \eta_k P_w(\sigma_k) - \mu_w = 0
\]

\[
\frac{\partial L}{\partial p_{G,i}} = 2c_{2i} \alpha_{i}^T \left\| \Sigma_i \right\|_2 + c_{1i} + (\delta^+_i - \delta^-_i)
\]

\[
+ y_{pg,i}(\theta) - \lambda_i = 0, \quad \forall i \in G
\]

\[
\frac{\partial L}{\partial \alpha_{i,w}} = 2c_{2i} \alpha_{i}^T [\Sigma_i]_{11} + \chi_u \frac{\alpha_{i}^T [\Sigma_i]_{11}}{\left\| \alpha_{i}^T \Sigma_i^{1/2} \right\|_2}
\]

\[
+ y_{pg,i}(\theta) - \chi_u = 0, \quad \forall i \in G
\]

Expressions (32) and (34) follow immediately from (36a) and (36d), respectively. Expressing \( \chi_u \) from (36d) and summing over all \( i \in G \) in (36e) yields (36).

Notably, energy prices in (34) are driven by cost coefficients of \( c_{i}(\cdot) \) and do not explicitly depend on random \( \omega \), risk set \( D_i \) and tolerance to chance constraint violations \( \epsilon_u \). On the other hand, the balancing reserve price in (34) depends on \( \omega \) (via parameter \( \Sigma \)), \( D_i \) (via parameter \( \Sigma_{k} \)) and \( \epsilon_u \). Finally, risk prices in (32) depends on the degree of discretization \( W \), which affects interval limits \( l_w \) and \( u_w \), and individual risk perception given by set \( D_i \) (via parameter \( \sigma_{i,k} \)).

**V. CASE STUDY**

The case study compares the risk-averse chance-constrained electricity market without risk trading (NO-RT case) and with risk trading (RT case) in (12) and (33), respectively. The case study includes five conventional producers with parameters reported in Table I and five undispatchable stochastic RES producers. The total system demand is \( \sum_{i \in G} p_{D,i} = 100 \text{ MW} \), while each RES producer is forecasted to produce \( p_{U,i} = 5 \text{ MW} \) with the standard deviation of 20% of the forecast value (1 MW). Each producer has its own set \( S_k = \{ \Sigma_k \}, k = 1, ..., 10 \}, \) which was generated such that (i) each producer has an unique risk perception and (ii) all producers share at least one \( \Sigma_k \), which is the “true” covariance of \( \omega \) in our case. We discretize the probability space of \( \omega \) in eight intervals using breakpoints \([-0.2, -0.1, -0.05, 0, 0.05, 0.1, 0.2]\) as illustrated in Fig. 1(a). The code and input data is available in [30].

Relative to the NO-RT case, the RT case reduces the operating cost by 0.2%. Notably, the energy cost (4,656.50 $) and energy prices (62.09 $) are the same in both cases and the balancing reserve cost reduces by 11% (from 6.17 $ to 5.39 $) due to the ADS trading. Since the system-wide ADS trading is revenue-neutral, i.e. \( \sum_{i \in G} \eta_u = 0 \) as per (14), ADS contracts do not contribute to the operating cost. On the other hand, the introduction of ADS trading changes the balancing reserve provision and its prices, as shown in Table I, which is influenced by different risk beliefs of market participants.

**TABLE I: Parameters of conventional generators**

| Prod. | 1 | 2 | 3 | 4 | 5 |
|-------|---|---|---|---|---|
| \( c_{1i} \) [$/(MW)]\) | 10 | 7 | 7 | 15 | 17 |
| \( c_{2i} \) [$/(MW h)]\) | 1.07 | 0.7 | 1.5 | 1.7 | 1.9 |
| \( P_{G,i} \) [MW] | 30 | 10 | 10 | 25 | 25 |
TABLE II: Power Outputs and Balancing Participation Factors

| i   | μ_i,u in the RT case | μ_i,u in the NO-RT case |
|-----|---------------------|-------------------------|
| 1   | 26.04               | 0.09                    |
| 2   | 10.00               | 0.41                    |
| 3   | 10.00               | 0.31                    |
| 4   | 15.70               | 0.01                    |
| 5   | 13.36               | 0.18                    |

| X_u | 1.24 | 1.54 | 0.72 | 0.69 | 1.31 | 0.91 | 1.47 | 1.31 | 1.34 | 1.11 |

Fig. 1: Risk trading results in the RT case: (a) itemizes the event probabilities \( P_w(\sigma_{i,k}) \) drawn from all individual risk sets (shown in thin gray lines) relative to the “true” distribution (dashed green line) and the ADS prices \( \mu_w \) (solid red line); (b) ADS trades, where negative (purple) values indicate a producer selling ADS and positive (orange) values indicate a producer buying ADS.

the RES producer \( u = 1 \) than in the RT case. In other words, when producer 5 can generate additional revenue from ADS selling, it is incentivized to procure more balancing reserve for the RES producer \( u = 1 \). This risk-averse perception also affects the ADS prices in Fig. 1(b) given by dual \( \mu_w \) of the ADS market-clearing constraint \( \text{[14]} \) for each event. As shown in Fig. 1(a), the values of risk prices \( \mu_w \) in Fig. 1(b), match the “true” event probabilities. That is, \( \mu_w \) is indeed a probability measure, as in Proposition \( \text{[1]} \) and captures the risk perception at the intersection of all risk sets \( \mathcal{D}_i \), as in Proposition \( \text{[2]} \).

VI. CONCLUSION

This paper has developed a risk-averse modification of the chance-constrained electricity market proposed in \( \text{[3]}-\text{[5]} \) by completing it with ADS-based risk trading. By discretizing the outcome space of the system uncertainty, we formulated practical ADS contracts that lead to a computationally tractable market-clearing optimization with risk trading. This optimization reduces the system operating cost relative to the case with no risk trading and produces energy, balancing reserve and risk prices. In particular, both qualitative and quantitative analyses indicate that system uncertainty and risk parameters do not explicitly affect the energy prices, but explicitly contribute to the formation of the balancing reserve and risk prices.

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