Exact shape of the lowest Landau level in a double–layer system and a superlattice with uncorrelated disorder

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Abstract

We extend Wegner’s exact solution for the 2D density of states at the lowest Landau level with a short–range disorder to the cases of a double–layer system and a superlattice. For the double–layer system, an analytical expression for the density of states, illustrating the interplay between the tunnel splitting of Landau levels and the disorder–induced broadening, is obtained. For the superlattice, we derive an integral equation, the eigenvalue of which determines the exact density of states. By solving this equation numerically, we trace the disappearance of the miniband with increasing disorder.

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The shape of the Landau levels (LL) in a 2D system in the presence of a disorder was the subject of intensive study during the last two decades.\textsuperscript{[1]–[19]} The complexity of the problem arises from the fact that in the absence of the disorder the energy spectrum is discrete. As a result, the self–energy of an electron appears to be real in any finite order of the perturbation theory. Therefore, obtaining a finite width of the LL requires summation of the entire diagram expansion. It was demonstrated\textsuperscript{[10], [16]} that such a summation is possible when the number of the LL is large. The simplifications, arising in this limit, are different in the case of a short–range and a smooth disorder. In the former case only a subsequence of diagrams without self–intersections contributes to the self–energy, or, in other words, the self–consistent Born approximation\textsuperscript{[1], [3]} becomes asymptotically exact.\textsuperscript{[11]} The shape of the LL in this case is close to semielliptical. For a smooth disorder, with correlation radius larger than the magnetic length, all diagrams are of the same order of magnitude, but in this case magnetic phases, caused by self–intersections of impurity lines, become small. The origin of these phases lies in an uncertainty in the position of the center of the Larmour orbit. Having the phases dropped, the entire perturbation series can be summed up with the help of the Ward identity, resulting in the Gaussian shape of the LL.\textsuperscript{[16]}

For low LL numbers and short–range disorder, the magnetic phases in diagrams are of the order of unity. A small parameter appears in the problem only if the energy \(\varepsilon\) (measured from the lowest LL) is much larger than the LL width \(\Gamma\), making possible a calculation of the density of states (DOS) in the tails of LL. Such calculations were carried out in the framework of the instanton approach\textsuperscript{[4], [5], [8], [11], [12], [14], [19]} and the tails were shown to be Gaussian. In the domain \(\varepsilon \sim \Gamma\) the problem has no small parameter and no simplifications are possible. However, for the lowest LL, the exact DOS was found by Wegner\textsuperscript{[6]} for an arbitrary ratio \(\varepsilon/\Gamma\). Wegner has shown that the diagrammatic expansion of the disorder–averaged Green function, \(G(\varepsilon)\), can be mapped onto that of the zero–dimensional complex \(\varphi^4\)–model with the partition function \(Z_0^{(1)}\) given by a simple integral

\[
Z_0^{(1)}(\varepsilon, \Gamma) = \int d\varphi^* d\varphi \exp \left[ i\varphi^* \varphi - \frac{\Gamma^2}{4}(\varphi^* \varphi)^2 \right].
\]  

(1)
The crucial observation made by Wegner was that the number of diagrams for the disordered system, which are mapped onto a single graph of the $\varphi^4$-model, equals (up to an overall factor) the inverse value of the diagram itself. The electron Green function is then given by

\[ G = -\frac{1}{2\pi l^2} \frac{\partial \ln Z_0^{(1)}}{\partial \varepsilon}, \]  

where $l$ is the magnetic length. Wegner has proved that coefficients in front of $\Gamma^n$ in each side of this equation coincide. Having a closed expression for $G(\varepsilon)$, Wegner obtained the following formula for the DOS in the lowest LL

\[ g(\varepsilon) = \frac{1}{2\pi^2 l^2} \frac{2}{\sqrt{\pi}} \frac{e^{\varepsilon^2/l^2}}{1 + \left(\frac{2}{\sqrt{\pi}} \int_0^{\varepsilon/\Gamma} dx e^{x^2}\right)^2}. \]  

The magnetic field dependence of the width $\Gamma$ is $\Gamma \propto \sqrt{B}$. More precisely, for the correlator of the random potential $V(r)$ of the form $\langle V(r)V(r') \rangle = \gamma \delta(r-r')$, one has $\Gamma = (\gamma/2\pi l^2)^{1/2}$. An alternative derivation of Wegner’s result was given by Brézin, Gross and Itzykson in the framework of functional–integral approach.

Consider now a system consisting of two parallel two–dimensional layers. In the absence of a disorder and magnetic field, a tunnel coupling between the layers would cause a splitting of size quantization levels by an amount of $2t$, $t$ being the tunnel integral. In a perpendicular magnetic field, the spectrum of the system represents two staircases of LL shifted in energy by $2t$. Assume that the field is strong, so that the cyclotron energy is much larger than $t$. If a disorder is present in the layers, the resulting shape of two adjacent LL’s would depend on the ratio $\Gamma/t$. If this ratio is large, then the tunnel coupling does not play any role, so that the DOS is twice the DOS in an individual layer, which is given by Eq. (3). In the opposite case, $t \gg \Gamma$, the peaks in the DOS, corresponding to the symmetric and antisymmetric combinations of size–quantization wave functions, are broadened independently. The centers of the peaks are distanced by $2t$ and their shape is described by Eq. (3) with the width $\Gamma/\sqrt{2}$. The factor $1/\sqrt{2}$ appears because the effective random potential for symmetric (antisymmetric) state is $[V_1(r) \pm V_2(r)]/2$, where $V_1(r)$ and $V_2(r)$ are the random potentials in the layers. If $\langle V_1(r)V_2(r') \rangle = 0$, the correlator for each effective potential appears to be
twice as small as that for an individual layer.

In the case $\Gamma \sim t$, calculation of the DOS in a double–layer system seems to pose even harder problem than for a single layer, since here the DOS represents a two–parametric function, $g_{dl}(\varepsilon/\Gamma, t/\Gamma)$, with both arguments of the order of unity. Nevertheless, as we demonstrate below, for the lowest LL the exact DOS can be obtained in a closed form by generalizing Wegner’s approach. Moreover, such a generalization can be carried out for an arbitrary number of layers, and, in particular, we consider the case when the number of layers is infinite (superlattice). In the absence of a disorder, each LL in a superlattice gives rise to a miniband of a width $4t$. Gradual switching on a disorder first smears out the singularities in DOS at the edges of the miniband and then, as $\Gamma$ exceeds $t$, transforms the DOS into a single peak, corresponding to an individual layer. We derive an integral equation, the eigenvalue of which determines the DOS in a superlattice, and trace this transformation by solving it numerically.

Consider first the double–layer system. The Hamiltonian has the form

$$\hat{H} = \sum_i \int d\mathbf{r} \left[ \left( \hat{\mathbf{P}} - \frac{e}{c} \mathbf{A} \right) \psi_i \right]^2 + \psi_i^* \mathbf{V}_i \psi_i + t \int d\mathbf{r} (\psi_1^* \psi_2 + \psi_2^* \psi_1)$$  \hspace{1cm} (4)

where $\mathbf{A} = (-By/2, Bx/2)$ is the vector-potential in the symmetric gauge measured from the origin in both layers. It will be convenient to include the last term in (4) into the definition of the free Hamiltonian. Then, after projecting onto the lowest LL, the free Green function represents a $2 \times 2$ matrix

$$\hat{G}^0(\mathbf{r}, \mathbf{r}') = \frac{\hat{Q}}{2\pi l^2} \exp \left[ -\frac{(\mathbf{r} - \mathbf{r}')^2}{4l^2} + \frac{i}{2l^2} (\mathbf{r} \times \mathbf{r}') \right]$$  \hspace{1cm} (5)

with

$$\hat{Q} = (\varepsilon - \hat{t})^{-1}, \quad \hat{t} = \begin{pmatrix} 0 & t \\ t & 0 \end{pmatrix}. \hspace{1cm} (6)$$

The perturbation expansion of the Green function, averaged over random potentials $V_1$ and $V_2$, has the same diagrammatic representation as for a single layer. Some of the first diagrams are shown in Fig. 1(a) and (b). The solid lines correspond to $\hat{G}^0$ and the dashed
lines correspond to the correlator of the random potential. In contrast to the single-layer case, solid lines carry indices, reflecting the fact that electron can tunnel from one layer to another between two successive scattering acts. Since the scattering retains an electron in the same layer, the indices at the ends of each dashed line are the same. Let us introduce projecting operators $\hat{\tau}_i$ as

$$
\hat{\tau}_1 = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}, \quad \hat{\tau}_2 = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}.
$$

(7)

Then the expressions corresponding to diagrams (a) and (b) can be written as

$$
\hat{G}^{(1)} = F^{(1)} \Gamma^2 \sum_i (\hat{Q} \hat{\tau}_i \hat{Q} \hat{\tau}_i \hat{Q}),
$$

$$
\hat{G}^{(2)} = F^{(2)} \Gamma^4 \sum_{ij} (\hat{Q} \hat{\tau}_i \hat{Q} \hat{\tau}_j \hat{Q} \hat{\tau}_i \hat{Q} \hat{\tau}_j \hat{Q}),
$$

(8)

where $F^{(1)}$ and $F^{(2)}$ are spatial integrals. Similarly, in any nth order diagram the spatial integrals are separated out as factors in front of products of matrices $\hat{Q}$ and $\hat{\tau}_i$, which are responsible for the energy dependence. Important is that coefficients $F^{(n)}$ are exactly the same as those for a single-layer.

The mapping is carried out following Wegner’s prescription: one identifies pairs of points in a diagram connected by dashed lines, and one gets a graph with four lines entering each vertex [see Fig. 1(c) and (d)]. In doing so, one obtains, in general, a set of diagrams yielding the same graph. It is clear, however, that since assigning indices to the lines does not alter in any way the topology of diagrams or graphs, the number of diagrams in a set is the same for both single- and double-layer cases. Moreover, one observes that the contractions of matrices $\hat{\tau}_i$ precisely follow the identification of points described above [as it can be seen, e.g., in Fig. 1(b) and (d)], so that all the diagrams in such a set are equal. The fundamental relation, established by Wegner, is that for each diagram in the set one has $F^{(n)} = 1/Ns$, where $N$ is the number of diagrams in the set and $1/s$ is the symmetry factor of the graph ($s$ is the number of permutations leaving graph invariant). The latter factor is also unchanged by assigning indices to the graph. For example, the graph (d) is invariant under permutation
of upper and lower lines so its symmetry factor is 1/2 in both cases. Thus, the contribution of the set, being proportional to $\mathcal{N} F^{(n)}$, is $\mathcal{N}$-independent, and the problem again reduces to the zero-dimensional field theory. The remaining question is whether matrix products of type (8) can be generated in the perturbation expansion of some generalized $\varphi^4$-model. Our main observation is that the model with the partition function

$$Z_0^{(2)} = \int d\Phi^* d\Phi \exp \left[ i\Phi^* \hat{Q}^{-1} \Phi - \frac{\Gamma^2}{4} \sum_i (\Phi^* \tau_i \Phi)^2 \right],$$

accomplishes this task. Here $\hat{Q}$ and $\hat{\tau}_i$ are matrices defined by (6), and $\Phi$ is a two-component complex field: $\Phi = (\varphi_1, \varphi_2)$. Indeed, the $n$th order term in the expansion of exponent (9) in terms of $\Gamma^2$ represents a product of $2n$ matrices $\hat{\tau}_i$ (with all pairwise contractions) separated by $2n$ products of the form $\Phi \Phi^*$. Then the gaussian integral over $\Phi$ inserts the “Green function” $\hat{Q} = -i \langle \Phi \Phi^* \rangle$ in place of each pair of fields $\Phi$ and $\Phi^*$, with all possible contractions between them yielding all the $n$th order graphs with appropriate symmetry factors.

Having the mapping established, the DOS in the double-layer system can be calculated directly from (9). It is also instructive to rewrite $Z_0^{(2)}$ in a different form. First, we decouple the quartic term in the exponent of (9),

$$iS = \sum_{j=1,2} \left[ i\varepsilon \varphi_j^* \varphi_j - \frac{\Gamma^2}{4} (\varphi_j^* \varphi_j)^2 \right] - it (\varphi_1^* \varphi_2 + \varphi_2^* \varphi_1),$$

with the help of gaussian integral over a pair of auxiliary variables. Performing the remaining integral over $\varphi_1$ we then obtain

$$Z_0^{(2)} = \left( \frac{i\pi}{\pi \Gamma^2} \right)^2 \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \frac{d\lambda_1 d\lambda_2}{(\varepsilon + \lambda_1)(\varepsilon + \lambda_2) - t^2} \exp \left( -\frac{\lambda_1^2}{\Gamma^2} - \frac{\lambda_2^2}{\Gamma^2} \right).$$

From the form (11), the both limiting cases of large and small $t$ are evident. For small $t$, the partition function factorizes, $Z_0^{(2)} = \left( Z_0^{(1)} \right)^2$, yielding twice the DOS (9). For $t \gg \Gamma$ the characteristic values of $\lambda_1$, $\lambda_2$ in (11), being of the order of $\Gamma$, are much smaller than $t$. This allows to neglect the product $\lambda_1 \lambda_2$ in the denominator; $Z_0^{(2)}$ is not small only if $(\varepsilon - t) \sim \Gamma$ or $(\varepsilon + t) \sim \Gamma$. In both cases one should introduce new variables $\mu_1 = \lambda_1 + \lambda_2$.
and $\mu_2 = \lambda_1 - \lambda_2$. Then the integration over $\mu_2$ would contribute a factor $\sqrt{2\pi \Gamma}$, and the integral over $\mu_1$ would reproduce Wegner’s result with the width $\Gamma/\sqrt{2}$, as discussed above. The evolution of DOS between two limits, calculated from (12) with $I_{Z,0}$, is shown in Fig. 2.

Let us now turn to a superlattice. The partition function (13) can be straightforwardly generalized to a multilayer system, and for $n$ layers with nearest-neighbor tunneling it takes the form

$$Z_{\varepsilon,\Gamma}^{(n)}(\varepsilon, \Gamma) = \int \prod_{i=1}^{n} d\varphi_i^* d\varphi_i \exp \left[ i \varepsilon \sum_{j=1}^{n} \varphi_j^* \varphi_j - it \sum_{j=1}^{n-1} (\varphi_j^* \varphi_{j+1} + \varphi_{j+1}^* \varphi_j) - \frac{\Gamma^2}{4} \sum_{j=1}^{n} (\varphi_j^* \varphi_j)^2 \right].$$

We are interested in the asymptotic behavior of $Z_{\varepsilon,\Gamma}^{(n)}(\varepsilon, \Gamma)$ as $n \to \infty$. For this purpose we employ a method similar to the transfer–matrix method in the theory of 1D spin chains. Note that the expression (12) for $Z_{\varepsilon,\Gamma}^{(n)}$ can be rewritten as

$$Z_{\varepsilon,\Gamma}^{(n)}(\varepsilon, \Gamma) = \int d\varphi^* d\varphi \exp \left[ i \varepsilon \varphi^* \varphi - \frac{\Gamma^2}{4} (\varphi^* \varphi)^2 \right] I_n(\varphi^*, \varphi),$$

where $I_1 = 1$ and the functions $I_n(\varphi^*, \varphi)$ satisfy the following recurrence relation

$$I_{n+1}(\varphi^*, \varphi) = \hat{T}_{\varepsilon,\Gamma} I_n = \int d\varphi^* d\varphi \exp \left[ i \varepsilon \varphi_1^* \varphi_1 - \frac{\Gamma^2}{4} (\varphi_1^* \varphi_1)^2 - it(\varphi^* \varphi_1 + \varphi_1^* \varphi) \right] I_n(\varphi_1^*, \varphi_1).$$

Consider now the eigenvalues, $\lambda^{(k)}(\varepsilon, \Gamma)$, and eigenfunctions, $\Omega^{(k)}_{\varepsilon,\Gamma}(\varphi^*, \varphi)$, of the operator $\hat{T}_{\varepsilon,\Gamma}$: $\hat{T}_{\varepsilon,\Gamma} \Omega^{(k)}_{\varepsilon,\Gamma} = \lambda^{(k)} \Omega^{(k)}_{\varepsilon,\Gamma}$. Assume that $\lambda^{(0)}$ has the maximal absolute value. Then in the limit $n \to \infty$, $Z_{\varepsilon,\Gamma}^{(n)}$ will behave as $(\lambda^{(0)})^n$. Hence, the DOS per layer in a superlattice can be expressed through $\lambda^{(0)}(\varepsilon, \Gamma)$ in the following way

$$g_{sl}(\varepsilon, \Gamma) = \frac{1}{2\pi^2 \Gamma} \text{Im} \lim_{n \to \infty} \frac{1}{n} \frac{\partial \ln Z_{\varepsilon,\Gamma}^{(n)}}{\partial \varepsilon} = \frac{1}{2\pi^2 \Gamma} \text{Im} \frac{\partial \ln \lambda^{(0)}(\varepsilon, \Gamma)}{\partial \varepsilon}.$$ 

Thus, we have reduced the calculation of DOS to the solution of an integral equation

$$\lambda^{(0)}(\varepsilon, \Gamma) \Omega^{(0)}_{\varepsilon,\Gamma}(\varphi^*, \varphi) = \int d\varphi_1^* d\varphi_1 \exp \left[ i \varepsilon \varphi_1^* \varphi_1 - \frac{\Gamma^2}{4} (\varphi_1^* \varphi_1)^2 - it(\varphi^* \varphi_1 + \varphi_1^* \varphi) \right] \Omega^{(0)}_{\varepsilon,\Gamma}(\varphi_1, \varphi_1).$$

Consider first the case of a weak disorder, $\Gamma \to 0$. One can check that eigenfunctions of $\hat{T}_{\varepsilon,0}$ in this case have the form

$$\Omega^{(p,m)}_{\varepsilon,0} = e^{ima - (\varepsilon + \sqrt{4t^2 - \varepsilon^2})R^2/2} \left( R^2 \sqrt{4t^2 - \varepsilon^2} \right)^{m/2} L_p^m \left( R^2 \sqrt{4t^2 - \varepsilon^2} \right).$$

(17)
where $R$ and $\alpha$ are, respectively, the absolute value and the phase of $\varphi$, and $L^m_p(x)$ is the Laguerre polynomial. The corresponding eigenvalues, $\lambda^{(p,m)}(\varepsilon, 0)$, are equal to

$$\lambda^{(p,m)}(\varepsilon, 0) = \frac{\pi i}{t} \left( \frac{2it}{i\varepsilon - \sqrt{4t^2 - \varepsilon^2}} \right)^{2p+m+1},$$  

where for $|\varepsilon| > 2t$ the square root is defined as $i^{-1} \text{sgn}(\varepsilon) \sqrt{\varepsilon^2 - 4t^2}$. Outside the interval $|\varepsilon| < 2t$, the phases of eigenvalues (18) have no energy dependence, supporting the obvious observation that the DOS is zero outside the miniband. Within the miniband, all $\lambda^{(p,m)}(\varepsilon, 0)$ have the same absolute value. This is a manifestation of the fact that for a large but finite number of layers the DOS in the absence of disorder represents a set of delta–peaks. However, with arbitrary weak disorder present, only the eigenvalue $\lambda^{(0,0)}(\varepsilon, 0)$ will survive in the limit $n \to \infty$, yielding the familiar result

$$g_{sl}(\varepsilon, 0) = \frac{1}{2\pi^2l^2} \text{Im} \frac{\partial \ln \lambda^{(0,0)}(\varepsilon, 0)}{\partial \varepsilon} = \frac{1}{2\pi^2l^2 \sqrt{4t^2 - \varepsilon^2}}$$

Assume now that the disorder is finite but $\Gamma \ll t$. It is convenient to formally rewrite Eq. (16) in the following form

$$\hat{T}_{\varepsilon, \Gamma} \Omega^{(0)}_{\varepsilon, \Gamma} = \hat{T}_{E, 0} \Omega^{(0)}_{E, 0} = \frac{1}{\sqrt{\pi \Gamma}} \int_{-\infty}^{\infty} dE \exp \left[ -\frac{(E - \varepsilon)^2}{\Gamma^2} \right] \lambda^{(0,0)}(E, 0) \Omega^{(0)}_{E, 0}.$$  

For small $\Gamma$, only the energies $E$ close to $\varepsilon$ contribute to the integral (20). This suggests to start the iteration procedure by substituting, as a zero approximation, the $m = p = 0$ eigenfunction of $\hat{T}_{E, 0}$, $\Omega^{(0,0)}_{E, 0} = \exp \left[ -\left( iE + \sqrt{4t^2 - E^2} \right) R^2 / 2 \right]$, into the right–hand side.

This generates the first approximation for the function $\Omega^{(0)}_{\varepsilon, \Gamma}$

$$\tilde{\Omega}^{(0)}_{\varepsilon, \Gamma} = \frac{1}{\sqrt{\pi \Gamma} \lambda^{(0)}(\varepsilon, \Gamma)} \int_{-\infty}^{\infty} dE \exp \left[ -\frac{(E - \varepsilon)^2}{\Gamma^2} \right] \lambda^{(0,0)}(E, 0) \Omega^{(0,0)}_{E, 0}.$$  

Substituting this function back into (20), we obtain

$$\hat{T}_{\varepsilon, \Gamma} \tilde{\Omega}^{(0)}_{\varepsilon, \Gamma} = \frac{1}{\pi \Gamma^2 \lambda^{(0)}(\varepsilon, \Gamma)} \int_{-\infty}^{\infty} dE \int_{-\infty}^{\infty} dE' \exp \left[ -\frac{(E - \varepsilon)^2}{\Gamma^2} - \frac{(E' - \varepsilon)^2}{\Gamma^2} \right] \lambda^{(0,0)}(E', 0) \tilde{T}_{E, 0} \Omega^{(0,0)}_{E', 0}.$$  

Note now, that $\Omega^{(0,0)}_{E', 0}$ as a function of $E'$ changes significantly on the scale $E' \sim t$. On the other hand, exponential factors in (22) enforce the difference between $E$ and $E'$ be of the
order of $\Gamma$. This allows to replace $\Omega_{E',0}^{(0)}$ by $\Omega_{E,0}^{(0)}$ under the integral. Then we immediately observe that the right-hand side takes the form

$$\tilde{\lambda}^{(0)}(\varepsilon, \Gamma) \tilde{\Omega}_{\varepsilon,1}^{(0)}$$

with

$$\tilde{\lambda}^{(0)}(\varepsilon, \Gamma) = \frac{2\pi}{\sqrt{\pi}} \frac{dE}{\sqrt{4t^2 - E^2 - iE}} \exp \left[ -\frac{(E - \varepsilon)^2}{\Gamma^2} \right].$$

(23)

In other words, for small $\Gamma$ the function $\tilde{\Omega}_{\varepsilon,1}^{(0)}$ satisfies Eq. (20), yielding the eigenvalue (23).

In principle, to assess the region of large $\Gamma$ one should keep iterating Eq. (20). However, as we have established numerically, the function $\tilde{\Omega}_{\varepsilon,1}^{(0)}$ is already a very good approximation for $\Omega^{(0)}$ and $\tilde{\lambda}^{(0)}$ is a very good approximation for $\lambda^{(0)}$ not only for small, but for arbitrary ratio $\Gamma/t$. The reason for this is the following. As $\Gamma/t$ increases, one should reproduce Wegner’s result, which corresponds to $t = 0$ and, hence, $\Omega^{(0)} = const$ in Eq. (16). On the other hand, it is easy to see that $\tilde{\Omega}^{(0)}$ turns to constant as $t \to 0$, and that in this limit $\tilde{\lambda}^{(0)}$ turns to $Z_0^{(1)}$. Thus, Eq. (23) is exact in both limits. The numerical results for the DOS, obtained by substituting $\tilde{\lambda}^{(0)}$ into (15), are shown in Fig. 3. We see that the miniband is completely destroyed as $\Gamma/t$ exceeds 1.4.

Note in conclusion, that a decade ago there was a significant interest in the study of transport phenomena in multilayer systems in a strong magnetic field (see, e.g., Refs. 21 and 22). In the recent publications this interest was renewed. The question of interest is how a transition from a purely 2D to three-dimensional behavior of the conductivity occurs with increasing $t$. As was shown in Ref. 24, the structure of electronic states in a multilayer system can be efficiently tuned by tilting magnetic field. In this case the role of the parallel component of $B$ reduces to the suppression of the interlayer tunneling.
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FIGURES

FIG. 1. First (a) and second (b) order diagrams for the Green function mapped on graphs (c) and (d), respectively.

FIG. 2. The DOS per layer for a double-layer system in units of $g_1 = (2\pi l^2)^{-1} \Gamma^{-1}$ for values of $t/\Gamma=0.0$ (highest curve), 0.4, 0.6, 0.8, 1.0, 1.2, 1.4, 1.6, 1.8, 2.0, 2.2, 2.4, 2.6, 2.8, and 3.0.

FIG. 3. The DOS per layer for a superlattice in units of $g_2 = (2\pi l^2)^{-1}(2t)^{-1}$ for values of $\Gamma/2t=0.1$ (highest curve), 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9, 1.0, 1.2, 1.4, 1.6, 1.8, and 2.0.
Fig. 2
Fig. 3