Kalb-Ramond field interactions in a braneworld scenario

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Electromagnetic and (linearized) gravitational interactions of the Kalb-Ramond (KR) field, derived from an underlying ten dimensional heterotic string in the zero slope limit, are studied in a five dimensional background Randall-Sundrum I spacetime with standard model fields confined to the visible brane having negative tension. The warp factor responsible for generating the gauge hierarchy in the Higgs sector is seen to appear inverted in the KR field couplings, when reduced to four dimensions. This leads to dramatically enhanced rotation, far beyond observational bounds, of the polarization plane of electromagnetic and gravitational waves, when scattered by a homogeneous KR background. Possible reasons for the conflict between the theory and observation are discussed.

I. INTRODUCTION

The massless antisymmetric tensor field, also known as the Kalb-Ramond (KR) field, is generic to any closed string spectrum. It is also not a degree freedom of the standard low energy theory of fundamental particle interactions consisting of QCD, the electroweak theory and general relativity. Any observational effect involving the KR field, obtained using standard fields as probes, is then a window into the otherwise inaccessible world of very high energy physics supposedly predicted by string/M-theories. On the contrary, non-observation of a large predicted effect is equally likely to illuminate the grey area where stringy considerations meet the real world.

The weakly coupled heterotic string [1] is known to be consistent and to have $N = 1$ supersymmetry, provided the KR 3-form field strength is augmented by addition of $(E_8 \otimes E_8)$ Yang-Mills Chern-Simons 3-form and local Lorentz Chern-Simons 3-form. This augmentation induces electromagnetic and gravitational interactions of the KR field which lead to potentially interesting physical effects showing up in the Maxwell and Einstein equations, when the theory is compactified to four dimensions. The electromagnetic effect mainly comprise a rotation of the polarization plane of electromagnetic waves from large redshift sources, upon scattering from a homogeneous KR background [2]-[3]. This rotation is independent of the wavelength of the electromagnetic wave and cannot be explained by Faraday effect where the plane of polarization of the electromagnetic wave rotates depending quadratically on the wavelength while passing through galactic/ intergalactic magnetized plasma. The magnitude of the effect is sensitive to the dimensional compactification of the underlying theory. For toroidal compactification (as well as for the Calabi-Yau compactification) of the theory (in the zero slope limit), the predicted rotation is proportional to the appropriate KR field strength component (scaled by the inverse scale factor in a Friedmann universe), so that bounds on the observed rotation translate into a stringent upper bound on the size of the KR field strength component. However, compactifications of type IIB string theory with p-form fluxes lead to warped spacetimes [6] which lead to rather extraordinary couplings of the KR field, as we discuss below.

In contrast to the Maxwell field, the gravitational couplings of the KR field have not been studied substantively in the literature, except in relation to parity violating KR field couplings [5]. This paper addresses this lacuna in the literature. The main finding in this regard is that gravitational waves exhibit rotation of their plane of polarization through an angle that is once again proportional to (a power of) of the KR field strength component. If one uses the bounds on the KR field strength obtained from non-observation of the cosmic optical activity, the effect for similar activity for gravitational waves remains small.

The situation changes dramatically in a Randall-Sundrum background of type I [7], i.e., with two three-branes embedded in an exponentially warped five dimensional anti-deSitter (AdS) spacetime. In this scenario, the standard model fields are confined to the 3-brane but gravity can propagate in bulk. Since gravity mode is a part of the closed string spectra, it is not unreasonable to include the closed string modes like the KR field and dilaton among the $5 - d$ fields. For the case in hand, the dilaton is not significant as it couples to the Maxwell Lagrangian and the kinetic term of the KR field and cannot affect the optical/gravitational activity induced by the KR- field in any major way. Henceforth, the dilaton will be freezeed to its vacuum expectation value. If we assume that (a) the heterotic string admits of compactifications to such a spacetime (which is then compactified to four dimensions a la ref. [7]) and (b) the augmented electromagnetic and gravitational couplings of the KR field survive such compactification, then the couplings undergo an anti-warping, i.e., they are enhanced exponentially, leading to extraordinarily large rotation of

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the plane of polarization. Such large-angle rotations of course contradict observational bounds on these quantities, and hence point to an area of tension between the set of assumptions made and the real world. While the exponential enhancement in case of cosmic optical activity was first derived [8] through an analysis involving the five dimensional action, in this paper we provide a more direct approach to this where the interplay between the equations of motion and the Bianchi identities becomes manifest.

The plan of the paper is as follows: In section 2, we briefly review our original derivation of the Cosmic Optical Activity and extend it to the case of gravitational waves from distant sources. In section 3, the anti-warped coupling of the KR field to electromagnetism is obtained by manipulating the field equations and Bianchi identities in the warped spacetime and effecting the compactification to four dimensions. This is repeated in section 4 for gravitational waves. We conclude in section 5 with an attempt to analyze possible reasons for the conundrum.

II. PHYSICAL EFFECTS OF KR INTERACTIONS

The KR field is characterized by a 2-form potential $B$ which has a 3-form field strength $H = dB$; the field strength is invariant under the KR gauge transformation $\delta_{\Lambda}B = d\Lambda$, where $\Lambda$ is a one-form gauge parameter. The free KR action is given (in $D$ dimensional spacetime) as

$$S_H = \int_{M_D} H \wedge ^*H ,$$

where $^*H$ is the Hodge-dual of the field strength $H$. Varying this action w.r.t. $B$ yields the KR field equation

$$d^*H = 0$$

which has the immediate local solution

$$^*H = dV,$$

where $V$ is a $D-4$ form. Substituting this in the KR Bianchi identity

$$dH = 0$$

one obtains for the field $V$

$$d^*dV = 0 .$$

Ten dimensional heterotic string theory (where $B$ occurs in the massless spectrum of the free string) reduces in the zero slope (infinite tension) limit to ten dimensional $N=1$ supergravity coupled to $N=1 E_8 \otimes E_8$ super-Yang-Mills theory. The requirement of ten dimensional supersymmetry requires that the KR field strength $H$ be augmented as [1]

$$H = dB - M_D^{1-D/2} \Omega_{YM} ,$$

where

$$\Omega_{YM} = tr(A \wedge dA + \frac{2}{3}g A \wedge A \wedge A)$$

is the Yang-Mills Chern-Simons 3-form with $A$ the gauge connection 1-form and $M_D$ is the Planck mass in $D$-dimensional spacetime. The theory in this form still suffers from the presence of mixed anomalies at the quantum level; consistency is restored upon a further augmentation [1]

$$H = dB - M_D^{1-D/2} (\Omega_{YM} - \Omega_{L}),$$

where $\Omega_{L}$ is the gravitational Chern-Simons 3-form obtained by replacing the Yang-Mills gauge connection $A$ by the spin connection 1-form $\omega$ in (7), and the trace is taken over the local Lorentz indices.

The augmentation in eq. (8) has the following consequences:
In order that $H$ remain gauge invariant under both Yang-Mills gauge transformations and under local Lorentz transformations, $B$ must now transform non-trivially under both gauge transformations. This is a surprise as the field $B$ is neutral and also has no magnetic moment. We consider the gauge transformation of the Yang-Mills field $A$, given by

$$\delta_{YM}A = d\Sigma + [A, \Sigma], \quad (9)$$

where, $\Sigma$ is a matrix of infinitesimal parameters. The Chern-Simons term varies as

$$\delta_{YM}\Omega_{YM} = tr(d\Sigma \wedge dA) \quad (10)$$

Thus, to achieve gauge invariance for the $H$ field, the transformation law for $B$ should include the 2-form in (10) so that under Yang-Mills gauge transformation

$$\delta_{YM}B = M_{YM}^{1-D/2} tr(d\Sigma \wedge dA) \quad (11)$$

Also, the gravitational field in the vielbein formalism can be treated very similarly to the Yang-Mills field. Specifically the Yang-Mills potential $A$ is analogous to the spin connection 1-form $\omega_{AB}$, where $A,B$ are Lorentz indices. Under an infinitesimal Lorentz transformation with parameters given by an $SO(D-1,1)$ matrix $\Theta$, the transformation of $\omega$ is

$$\delta_{L}\omega = d\Theta + [\omega, \Theta], \quad (12)$$

The Lorentz Chern-Simons term varies as

$$\delta_{L}\Omega_{L} = tr(d\Theta \wedge d\omega) \quad (13)$$

Similar to the argument above, transformation law for $B$ should include the 2-form in (13) so that under Lorentz transformation

$$\delta_{L}B = -M_{L}^{1-D/2} tr(d\Theta \wedge d\omega) \quad (14)$$

Retaining the form of the KR action (1), it follows that the KR field equation does not change. Therefore, $^*H$ still has the local solution (3). However, the KR Bianchi identity certainly changes, leading to

$$d^*dV = M_{D}^{1-D/2} tr(F \wedge F - R \wedge R), \quad (15)$$

where $F(R)$ is the Yang-Mills (spacetime) curvature 2-form.

The Yang-Mills and Einstein equations change non-trivially. We shall consider these below in special situations viz., the Maxwell part of the gauge interaction and linearized gravity.

A. Cosmic optical activity

In this subsection we confine ourselves to the electromagnetic interactions of the KR field after toroidally compactifying the heterotic string in the zero slope limit to four dimensional Minkowski spacetime. The relevant four dimensional field equations are

$$\partial_{\mu}H_{\mu\nu\rho} = 0 \quad (16)$$

$$\partial_{\mu}F_{\mu\nu} = M_{P}^{-1} H_{\mu\nu\rho} F_{\rho\eta}$$

The corresponding Bianchi identities are

$$\Box \Phi_H = M_{P}^{-1} F_{\mu\nu} * F_{\mu\nu}$$

$$\partial_{\mu}F_{\mu\nu} = 0 \quad (17)$$

where, in four dimensions, the (pseudo)scalar $V = \Phi_H$ and $M_P$ is the Planck mass in 4d-spacetime.

Rather than solving these equations simultaneously, we introduce another simplifying assumption: the ‘axion’ field $\phi_H$ is homogeneous and provides a background with which the Maxwell field interacts. We restrict our attention to
lowest order in the inverse Planck mass $M_P$, so that terms on the RHS of the axion field equation (17) are ignored to a first approximation. Consequently, $\Phi_H \equiv d\Phi_H/dt = f_0$ where $f_0$ is a constant of dimensionality of $(\text{mass})^2$. Under these conditions, the Maxwell equations can be combined to yield the inhomogeneous wave equation for the magnetic field $B$

$$\square B = -\frac{2f_0}{M_P} \nabla \times B.$$ (18)

With the ansatz for a plane wave travelling in the $z$-direction, $B(x,t) = B_0(t) \exp ikz$, we obtain, for the left and the right circular polarization states $B_{0\pm} \equiv B_{0x} \pm iB_{0y}$,

$$\frac{d^2 B_{0\pm}}{dt^2} + (k^2 \mp \frac{2f_0k}{M_P}) B_{0\pm} = 0.$$ (19)

Similarly, we can obtain the wave equation for the left and right circularly polarization states for the electric field which has exactly the same form as that of magnetic field. We concentrate on the equation for magnetic field as the conclusions will be same for that of electric field. Thus, the right and left circular polarization states have different angular frequencies (dispersion)

$$\omega^2 \pm = k^2 \mp \frac{2k f_0}{M_P}$$ (20)

so that over a time interval $\Delta t$, the plane of polarization undergoes a rotation (for large $k$)

$$\Delta \Psi_{op} \equiv |\omega_+ - \omega_-| \Delta t \simeq \frac{2 f_0}{M_P} \Delta t.$$ (21)

In a radiation or matter dominated Friedmann universe, the formula for the angle of rotation changes slightly [3]

$$\Delta \Psi_{op} \equiv |\omega_+ - \omega_-| \Delta t \simeq \frac{2 f_0}{a(t) M_P} \Delta t,$$ (22)

where, $a(t)$ is the scale factor and $\Delta t$ is now to be taken as the look-back time. This means that $\Delta \Psi = \Delta \Psi(z)$, where $z$ is the red-shift, and increases with red-shift. This rotation also differs from the better-understood Faraday rotation in that it is achromatic in the limit of high frequencies, [12]. Observationally, even for large redshift sources, the angle of rotation is less than a degree, which imposes the restriction on the dimensionless quantity $f_0/M_P^2 < 10^{-20}$, [13], [14].

**B. Cosmic gravitational activity**

To find the gravitational analogue of the optical activity discussed above, we first note that the augmentation of $H$ in (8) implies that the $tr\mathbf{R} \wedge \mathbf{R}$ term contributes an additional term to the Einstein equation over and above the energy-momentum tensor of the KR field. Formally,

$$G_{\mu\nu} = \frac{8\pi}{M_P^2} T_{\mu\nu} + \frac{16\pi}{M_P^2} \frac{1}{\sqrt{-g}} \frac{\delta}{\delta g^{\mu\nu}} \int d^4x' \sqrt{-g(x')} \, \Phi_H(x') \, R_{\rho\lambda\sigma\eta}(x') \ast R^\rho\lambda\sigma\eta(x'),$$ (23)

where,

$$T_{\mu\nu} = H_{(\mu||\rho} \, H_{\nu)}^{\tau\rho} - \frac{1}{6} g_{\mu\nu} H^2.$$ (24)

Since our focus is on gravitational waves, it is adequate to consider the Einstein equation in a linearized approximation. To this effect, we decompose the metric $g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$ with the fluctuation $h_{\mu\nu}$ being considered small so that one need only retain terms of $O(h)$ in the Einstein equation. We further impose on the fluctuations $h_{\mu\nu}$ the Lorenz gauge $h_{\mu\nu,\nu} = \frac{1}{2} h_{\mu,\nu}$. In this gauge, the linearized Einstein equation becomes

$$-\square h_{\mu\nu} = \frac{16\pi}{M_P^2} T_{\mu\nu} - \frac{128\pi}{M_P^2} \epsilon_{(\mu|}^{\sigma\alpha\beta} \left[ \Phi_{H,\lambda\sigma} \left( h_{\beta\nu,\alpha}^\lambda + h_{\beta,\alpha\nu}^\lambda \right) + \Phi_{H,\alpha} \square h_{\beta\nu,\sigma} \right]$$ (25)
In analogy with the last subsection, we regard the axion field $\Phi_H$ as a homogeneous background satisfying eq. (17) and consider its effect on a plane gravitational wave. We restrict to the lowest inverse power of the Planck mass for which a nontrivial effect is obtained, and as such, ignore terms on the RHS of the axion field equation.

$$\Box \Phi_H = M_P^{-1} R^\mu_{\nu\lambda\sigma} * R_{\mu\nu\lambda\sigma}$$  \hspace{1cm} (26)$$

Now, the field equations are (before gauge choice) invariant under general coordinate transformation. As we have chosen the Lorenz gauge, this invariance is broken and not all components of $h_{\mu\nu}$ are independent. In fact, the only physical degrees of freedom of the spin 2 field are contained in $h_{ij}$, for which we choose a plane wave ansatz travelling in the z-direction,

$$h_{ij} = \varepsilon_{ij}(t) \exp(-ikz).$$  \hspace{1cm} (27)$$

The Latin indices above correspond to spatial directions. The other components of $h_{\mu\nu}$ can be gauged away, so that their field equation need not be considered. The only non-vanishing polarization components can be chosen to be $\varepsilon_{11} = -\varepsilon_{22}, \varepsilon_{12} = \varepsilon_{21}$; from these the circular polarization components can be constructed as in the Maxwell case: $\varepsilon_\pm \equiv \varepsilon_{11} \pm i\varepsilon_{12}$. Further, we write the energy momentum tensor in eq. (24) in terms of $\Phi_H$ using eq. (3).

Then, under the approximation of homogeneous axion field, these polarization components satisfy the inhomogeneous differential equation

$$\left[ \frac{d^2}{dt^2} + k^2 + F_\pm \right] \varepsilon_\pm = -F_\pm,$$  \hspace{1cm} (28)$$

where,

$$F_\pm \equiv \frac{8\pi f_0^2}{M_P^2 (1 \pm 128\pi kf_0/M_P^3)}.$$  \hspace{1cm} (29)$$

The difference between (83) and the analogous equation (19) is that the former has a forcing term absent in the latter; this forcing term is dependent on the wave number $k$ and controlled by the constant $f_0$ which characterizes the strength of the KR field coupling.

Even though we are interested in large $k$, we would still remain within the Planckian regime $k < M_P$ so that the quantity $16\pi kf_0/M_P^3 << 1$ and can serve as an expansion parameter, leading to

$$\left[ \frac{d^2}{dt^2} + k^2 + 8\pi f_0^2/M_P^2 \mp 1024\pi^2 kf_0^3/M_P^5 \right] \varepsilon_\pm \simeq -8\pi f_0^2 (1 \mp 16\pi kf_0/M_P^3)/M_P^5.$$  \hspace{1cm} (30)$$

We can now read off the dispersion relation

$$\omega_\pm^2 = k^2 + 4\pi f_0^2/M_P^2 \mp 1024\pi^2 kf_0^3/M_P^5,$$  \hspace{1cm} (31)$$

whence the group velocity

$$v_{g\pm} = \frac{d\omega_\pm}{dk} = 1 + O(k^{-2}),$$  \hspace{1cm} (32)$$

and the phase velocity

$$v_{p\pm} = \frac{\omega_\pm}{k} = 1 \mp 512\pi^2 f_0^3/M_P^5 k.$$  \hspace{1cm} (33)$$

Thus for large $k$ the violation of Lorentz invariance can be ignored. As in the electromagnetic case, the rotation of the polarization plane for gravitational waves is given by

$$\Delta \Psi_{grav} \simeq 1024\pi^2 f_0^3/M_P^5 \Delta t.$$  \hspace{1cm} (34)$$

Admittedly, the effect is immeasurably tiny; with the limits on $f_0$ given in the previous subsection, it is $O(10^{-30})$. However, in contrast to the optical case, the tensor perturbations characterizing the gravitational wave do not get randomized, so the effect is in principle observable.
III. KR-MAXWELL INTERACTIONS IN AN RSI BRANEWORLD

Using eq. (15), and retaining only the coupling to the Maxwell field, the KR Bianchi identity in a five dimensional RS1 background [7]

\[ ds^2 = e^{-2\sigma(y)} \eta_{\alpha\beta} \, dx^\alpha dx^\beta + dy^2, \]  

is given by

\[ \Box_{RS} V^M = M_5^{-3/2} * F^{MNP} \, F_{NP}, \]  

where, \( \Box_{RS} \) is the covariant d’Alembertian on five dimensional warped RS1 spacetime, \( M,N,P = 0,1,\ldots,4 \), \( \sigma(y) = k|y| \), \( F_{NP} \equiv \partial[N A_P] \) and \( * F^{MNP} \) is the tensor derived using the antisymmetric tensor in RS1 spacetime. Since the Maxwell field is supposed to be confined to the (visible) brane, \( A_4 = 0 \), and also \( A_\mu \) are independent of the compact coordinate \( y \), so that \( F_{4\mu} = 0 \). Recalling that the five-vector \( V^M = (V^\mu, \Phi_H) \), the axion field \( \Phi_H(x,y) \) satisfies

\[ \Box_{RS} \Phi_H = M_5^{-3/2} * F^{\mu\nu} F_{\mu\nu} \, \delta(y - a). \]  

The corresponding vector equation is

\[ \Box_{RS} V^\mu = 0 \]  

Using the metric (35), eq. (37) can be rewritten

\[ e^{2\sigma(y)} \left[ \Box \Phi_H + e^{2\sigma(y)} \partial_y \{ e^{-4\sigma(y)} \partial_y \Phi_H \} \right] = M_5^{-3/2} g^{\lambda\mu} g^{\sigma\nu} \tilde{F}_{\lambda\sigma} F_{\mu\nu} \, \delta(y - a), \]  

where, \( \Box \) is the d’Alembertian on four dimensional Minkowski space, and \( y \in [0, a] \) is the coordinate on the fifth dimension. Also, we have made use of the transformation properties of complete antisymmetric tensor from the RS spacetime to that of the Minkowski spacetime. The \( F^{\mu\nu} \) tensor is that as observed in Minkowski spacetime.

The compactification to four dimensions now proceeds through the ansatz

\[ \Phi_H(x, y) = \sum_n \Phi^{(n)}_H(x) \, \chi_n(y), \]  

where, the mode functions \( \chi_n(y) \) satisfy the eigenvalue equation (for each value of \( n \))

\[ \frac{d}{dy} \left[ e^{-4\sigma(y)} \frac{d\chi_n(y)}{dy} \right] = m_n^2 e^{-2\sigma(y)} \chi_n(y). \]  

So that the equation (39) reduces to

\[ \sum_n e^{-2\sigma(y)} \left[ \Box \Phi^{(n)}_H(x) + m_n^2 \Phi^{(n)}_H(x) \right] \chi_n(y) = M_5^{-3/2} \tilde{F}^{\mu\nu} F_{\mu\nu} \, \delta(y - a), \]  

Our interest here is in the zero-mode \( \chi_0(y) \) corresponding to \( m_0 = 0 \). It is easy to verify that the second order differential operator in eq. (41) is self-adjoint in the domain \([0, a]\) provided

\[ \chi_0(0) = \frac{1}{\alpha} \frac{d\chi_0(y)}{dy} \bigg|_{y=0}, \]  

\[ \chi_0(a) = \frac{1}{\beta} \frac{d\chi_0(y)}{dy} \bigg|_{y=a}. \]  

where \( \alpha, \beta \) are real constants having value \( 4k \). The eigenfunctions \( \chi_n(y) \) satisfy the orthonormality condition

\[ \int_0^a e^{-2\sigma(y)} \chi_m(y) \chi_n(y) \, dy = \delta_{mn}. \]  

Choosing an ansatz for the equation (41), \( \chi_0(\theta) = C_1 \exp 4\sigma(y) + C_2 \), the boundary condition (43) implies \( C_2 = 0 \). Using (44), we obtain the normalized solution \( \chi_0(y) = \sqrt{\frac{k}{6k}} \exp k(4y - 3a) \). Substituting this solution in equation
(42), and using the orthonormality of the eigenfunctions in (44), one obtains the four-dimensional equation of motion for the zero mode KR axion field
\[ \square \Phi_H^{(0)}(x) = \frac{\exp \sigma(a)}{M_P} F^\mu_\nu \tilde{F}_\mu_\nu , \] 
where, \( M_P = M_3^3/k^\frac{3}{2} \sim (M_3^3/k)^\frac{1}{2} \) is the Planck mass in \( 4-d \) spacetime. Notice that the warp factor in the RS metric (35) now appears inverted, implying an exponentially large enhancement in the KR-Maxwell coupling strength [15].

The mode solution for the field \( V^\mu(x,y) \) can be calculated similarly. The field \( V^\mu(x,y) \) when expanded in the Kaluza-Klein modes
\[ V^\mu(x,y) = \sum_n V^{(n)}_\mu(x) \zeta_n(y) , \] 
leads to the mode equation (for each mode)
\[ \frac{d}{dy} \left( e^{-2\sigma(y)} \frac{d}{dy} \zeta_n(y) \right) = m_n^2 \zeta_n(y) . \] 
The orthonormality condition for \( \zeta(y) \) is
\[ \int_0^a \zeta_m(y) \zeta_n(y) dy = \delta_{mn} \] 
It is easy to verify using (47) and (48) that the zero mode is a constant on the visible brane.

Analysis similar to (36) leads to the Maxwell equation on the visible brane: one starts with the equation
\[ \partial_\mu F^{\mu_\nu} = M_5^{-3/2} \tilde{F}^{\nu_\lambda} \left[ \partial_\lambda \Phi_H - \partial_\rho V_\lambda \right] . \] 
Expanding the dual KR vector field in modes and using its dynamics, the zero mode of \( V_\lambda \) can be shown to decouple, yielding
\[ \partial_\mu F^{\mu_\nu} = \sqrt{6} \frac{\exp \sigma(a)}{M_P} \tilde{F}^{\nu_\sigma} \partial_\sigma \Phi_H^{(0)}(x) . \] 
Once again, we observe the exponentially enhanced coupling strength seen in the effective four-dimensional field equation for the axion (45). This enhancement in the effective interaction strength between the KR and Maxwell fields was first seen in ref. [8] using a compactification analysis of the effective action. Here, we have preferred to re-derive it using direct compactification of the field equations and Bianchi identities in the bulk, underlining thereby the crucial role played by the latter which are not directly derived from the action. In this manner, the result of ref. [8] is placed on a firmer footing.

The exponentially large coupling translates immediately into an exponential increase in the size of the angle through which the polarization plane of an electromagnetic wave, interacting with a homogeneous sea of KR axions, rotates [8].
\[ \Delta \Psi_{op} \equiv |\omega_+ - \omega_-| \Delta t \simeq \frac{2\sqrt{6} f_0 e^{ka}}{M_P} \Delta t . \] 
Now, we had denoted the strength of the homogeneous axion field in flat spacetime as \( f_0 \). In RS spacetime, this will convert to \( f_0 = f_0 e^{ka} \) on the negative tension brane. So, essentially, the strength of the axion field gets enhanced by an anti-warping factor \( e^{ka} \). Thus, the rotation of the plane of polarization is
\[ \Delta \Psi_{op} \equiv |\omega_+ - \omega_-| \Delta t \simeq \frac{2\sqrt{6} f_0}{M_P} \Delta t . \] 
The cosmic optical activity thus is far in excess of anything that is, or will be, ever observed from cosmologically distant galaxies, and is therefore physically untenable. One may argue, as already mentioned, that the incompatibility
between theory and observation so far is actually itself rather unlikely from a cosmological standpoint, since vector perturbations of the metric (e.g., electromagnetic waves) would certainly thermalize like the CMBR on time scales shorter than those under consideration. Note however that the same enhancement shows up in a parity-violating modification leading to exponentially large $B$-type parity-violating polarization anisotropy of the CMBR itself [10], which is certainly within the realm of observability. As we show below, the same is true for tensor (i.e., gravitational) perturbations, where thermalization is not effective in washing out long wavelength effects. An exponential coupling to KR fields would lead to an unobservably large effect on the polarization of gravitational waves, far beyond what one expects to see in gravitational wave experiments like LIGO and LISA.

IV. KR-Gravitational Interactions in an RSI BraneWorld

The five dimensional RS metric [7], given by eq. (35) is subjected to linear perturbations

$$ds^2 = dy^2 + (e^{-2\sigma(y)} \eta_{\mu\nu} + h_{\mu\nu}) \, dx^\mu dx^\nu \quad (53)$$

i.e. $g_{MN} = \delta g_{MN} + h_{MN}$. These perturbations further obey the RS gauge conditions [7],

$$h_{55} = h_{\mu 5} = 0$$
$$h_{\mu \nu , \nu} = 0,$$
$$h_{\mu \nu} = 0 . \quad (54)$$

In what follows, we retain terms up to $O(h)$ in the field equations.

To obtain a degree of freedom count, observe that the totality of 10 gauge conditions in (54) leave 5 independent components of the spin 2 graviton tensor in five dimensional spacetime. These degrees of freedom are further split into 4 dimensional graviton (2 polarizations), a 4 dimensional spin 1 photon (2 polarizations) and a spin zero scalar, also in four dimensions. In the general case of $n$ extra dimensions, the number of degrees of freedom of graviton follows from the irreducible tensor representations of the isometry group as $\frac{1}{2}(n + 1)(n + 2)$. The massive modes of the 5 dimensional gravitons are represented by the massive modes of all these fields on the brane. Since we will be concerned with the tensor perturbations, only the $h_{ij}$ is of interest. The standard 4 dimensional graviton corresponds to the massless mode of $h_{ij}$.

We look for the solution, to $O(h)$, of the five dimensional Einstein equation

$$(5)G_{MN} \equiv (5)R_{MN} - \frac{1}{2} (5)g_{MN} (5)R = -\Lambda (5)g_{MN} + (5)\kappa T_{MN} \quad (55)$$

where, $\Lambda = -6k^2$ is the cosmological constant of the $AdS_5$ spacetime, $(5)\kappa = 8\pi G_5$, $G_5 = 1/M_5^3$ is the five dimensional Newton’s constant and $T_{MN}$ is the energy-momentum tensor which is the sum of energy-momentum of the KR field and the term obtained from the variation of the Chern-Simons- Kalb-Ramond field i.e.

$$T_{MN} = t_{MN} + \frac{1}{M_5^2} \frac{1}{\sqrt{-g}} \frac{\delta}{\delta g^{MN}} \int d^5x' \sqrt{-g(x')} \, V_P(x') \, R_{QRST} (x') \ast R^{QRST} (x') \quad (56)$$

where,

$$t_{MN} = H_{(M|TR} H_{N)}^{TR} - \frac{1}{6} g_{MN} H^2 \quad (57)$$

The aim here is to obtain the leading order coupling of the induced four dimensional graviton fluctuations on the visible brane, with the Kalb-Ramond field, in order to ascertain the effect of the latter on the polarization of (to be) observed gravitational waves. In this venture, we are guided by the earlier work of ref. [11], with the input of the appropriate energy momentum tensor involving the antisymmetric tensor fields.

Proceeding as in [11], the linearized Einstein equation in RS gauge reduces to

$$\left( e^{2\sigma(y)} \square^{(4)} + \partial_y^2 - 4k^2 + 4k(\delta(y) + \delta(y - a)) \right) h_{\mu\nu} = -2\kappa \Sigma_{\mu\nu} \delta(y - a) \quad (58)$$

where,

$$\Sigma_{\mu\nu} = \left( T_{\mu\nu} - \frac{1}{3} \gamma_{\mu\nu} T \right) + 2\kappa^{-1} \xi^g_{\mu\nu} . \quad (59)$$
Here, the delta functions in (58) will enforce the discontinuities. The combination in (59), includes the “bending” of the wall $\xi^5$, and will play the role of the source term in the RS gauge. Note that $\gamma_{\mu\nu} = e^{-2\sigma(y)}\eta_{\mu\nu}$ is the background spatial metric.

Define the five dimensional retarded Green’s function, which satisfies

$$\left[e^{2\sigma(y)} \Box^{(4)} + \partial_y^2 - 4k^2 + 4k(\delta(y) + \delta(y - a))\right] G_R(x, x') = \delta^{(5)}(x - x').$$

The formal solution of (58) is then given by

$$h_{\mu\nu}(x) = -2\kappa \int d^5x' G_R(x, x') \Sigma_{\mu\nu}(x') \delta(y - a),$$

where integration is taken over the corresponding surface for which $h_{\mu\nu}$ is to be determined. Since $h^{\mu\nu}$ must vanish, we must impose $\Sigma_{\mu\nu} = 0$, which implies the “equation of motion” for $\xi^5$.

$$\Box^{(4)} \xi^5 = \frac{\kappa}{6} T$$

With this choice of $\xi^5$, it is easy to check that $h_{\mu\nu}$ given by eq.(61) satisfies the harmonic condition $h_{\mu\nu,\nu} = 0$. The metric perturbation on the brane can be decomposed into pieces induced by standard model fields on the brane and the deformation of the brane itself, as [11]

$$h_{\mu\nu} = h_{\mu\nu}^{(m)} + 2k \gamma_{\mu\nu} \xi^5,$$

where

$$h_{\mu\nu}^{(m)} = -2\kappa \int d^4x' G_R(x, x') \left(T_{\mu\nu} - \frac{1}{3} \gamma_{\mu\nu} T\right)(x'),$$

$$h^{(\xi)} = -4 \int d^4x' G_R(x, x') \xi^5(x').$$

Here in (64), $h_{\mu\nu}^{(m)}$ corresponds to the metric fluctuation induced by matter fields and $h^{(\xi)}$ in (65) corresponds to that induced by brane deformation.

Now, let us look into the solution of eq. (60). To dimensionally reduce to four dimensions, the five dimensional graviton field is expanded in Kaluza-Klein modes as

$$h_{\mu\nu}(x, y) = \sum_{p=0}^{\infty} h_{\mu\nu}^{(p)}(x) \psi^p(y)$$

Defining $z \equiv \text{sgn}(y) \left(e^{\sigma(y)} - 1\right)/k$, $\psi_p(z) \equiv \psi_p(y)e^{\sigma(y)/2}$, $h_{\mu\nu}(x, z) \equiv h_{\mu\nu}(x, y)e^{\sigma(y)/2}$, the Kaluza Klein modes of the graviton field can be expressed as solutions to an effective one dimensional Schrödinger problem

$$\left[\partial_z^2 - V(z)\right] \psi_p(z) = m^2 \psi_p(z),$$

with a potential

$$V(z) = \frac{15k^2}{4(k|z| + 1)^2} - 3k\delta(z) + \frac{3k}{1 + kz_c}\delta(z - z_c).$$

Our interest is in the zero mode which corresponds to the solution $\psi_0(z) = N_0(k|z| + 1)^{-3/2}$, where, $N_0$ is a normalization constant. It is easily seen that that this mode satisfies boundary conditions required for the self adjointness of the differential operator in eq.(67)

$$\partial_z \psi(0) = -\frac{3k}{2} \psi(0),$$

$$\partial_z \psi(z_c) = -\frac{3k}{2(kz_c + 1)} \psi(z_c).$$
where, \( z_c \equiv (e^{\sigma(a)} - 1)/k \). The modes satisfy the orthogonality condition

\[
\int_{-a}^{a} e^{2\sigma(y)} \psi_p(y) \psi_q(y) dy = \delta_{pq}
\]

which fixes the value of \( \psi_0(y) = \sqrt{k} e^{-2\sigma(y)}/(1 - e^{-2\sigma(a)}) \).

Notice that if we want to have a four dimensional Lagrangian for \( h_{\mu\nu}(x,y) \) with a canonical kinetic term and with masses of the \( h_{\mu\nu}(x,y) \) field coming from the Kaluza-Klein mode, we must have \( \text{dim}[h] = 3/2 \), since \( h_{\mu\nu}(x,y) \) is a 5 dimensional field. Also further since \( \psi \) is of dimension 1/2, \( h_{\mu\nu}(x) \) is of dimension 1 as expected. But, since \( \eta_{\mu\nu} \) is dimensionless, \( h_{\mu\nu}(x,y) \) must be dimensionless. We choose to make the modes of \( h_{\mu\nu}(x,y) \) dimensionless by scaling \( h_{\mu\nu}(x,y) \) by \( M_P \). Essentially, we scale each \( h_{\mu\nu}(x,y) \) by \( M_P^2 = M_P \ k^{\pm} \).

Thus, in terms of the redefined fields, we have

\[
\psi_0(y) = e^{-2\sigma(y)}/(1 - e^{-2\sigma(a)})
\]

The Green's function is then constructed from the complete set of eigenfunctions

\[
G_R(x, x') = -\int \frac{d^4p}{(2\pi)^4} e^{i p \cdot (x - x')} \left[ \frac{e^{2\sigma(y)} e^{2\sigma(y')}}{p^2 - (\omega + i\epsilon)^2} \frac{1}{1 - e^{-2\sigma(a)}} \right] + \int_0^\infty dm \frac{u_m(y) u_m(y')}{m^2 + p^2 - (\omega + i\epsilon)^2} \right]
\]

where the first term corresponds to the zero mode and the rest corresponds to the continuum of KK modes \( u_m(y) = \sqrt{m/2k} \{ J_1(m/k)Y_2(m/ka) - Y_1(m/k)J_2(m/ka) \}/\sqrt{J_1(m/k)^2 + Y_1(m/k)^2} \).

The Green's function for the zero mode contribution for the negative and positive tension brane are given respectively as

\[
G_R(x, x') = \frac{e^{4\sigma(a)} \delta^{(4)}(x - x')}{\Box^{(4)}(1 - e^{-2\sigma(a)})}
\]

\[
G_R(x, x') = \frac{\delta^{(4)}(x - x')}{\Box^{(4)}(1 - e^{-2\sigma(a)})}
\]

We find that in the zero mode approximation the gravitational field on each of the branes satisfies [11]

\[
\left(e^{2\sigma(y)} \Box^{(4)} h_{\mu\nu}\right)^{(\pm)} = -\sum_{\alpha = \pm} 16\pi G^{(\alpha)} T_{\mu\nu} - \frac{1}{3} \gamma_{\mu\nu} T \left(\alpha\right) \pm \frac{16\pi G^{(\pm)} \sinh(ak)}{3} e^{\pm a k} \gamma_{\mu\nu} T^{(\pm)},
\]

where the plus and minus refer to quantities on the brane with positive and negative tension respectively. Here, we have introduced

\[
G^{(\pm)} = \frac{G_5 e^{\pm a k}}{2 \sinh(a k)}
\]

which plays the role of Newton’s constant in a Brans-Dicke parameterization [11].

Let us now look at the KR - gravitational interaction as observed on the visible brane. The equation (15) becomes,

\[
\Box_{RS} V^M = M_{5}^{-3/2} R^{MNPS} R_{NPQS} ,
\]

The axion coupling is

\[
\Box_{RS} \Phi_H = M_5^{-3/2} \left[ R^\mu_{\nu\lambda\sigma} R^\nu_{\mu\lambda\sigma} + 2e^{\mu\nu\lambda\sigma} R_{\lambda\sigma} \delta^{(4)} R_{\mu\nu}\right] |_{y=a}.
\]

For the zero modes of the fields, at the negative tension brane, we obtain

\[
\Box^{(0)} \Phi_H(x) = \frac{\exp(\sigma(a))}{\sqrt{6} M_P} h_{\mu\nu}(x) \frac{\gamma_{\mu\nu}}{\gamma_{\alpha\beta}} [-h_{\alpha}^\beta(x), \gamma_{\sigma}^\alpha - h_{\alpha}^\lambda(x), \gamma_{\sigma}^\alpha - h_{\alpha}^\lambda(x), \gamma_{\sigma}^\alpha e^{\mu\nu\lambda\sigma} ,
\]

For the vector field, the corresponding equation yields

\[
\Box^{(0)} V^\mu(x) = 4\sqrt{\frac{2k}{M_P}} h_{\nu\beta}(x) \gamma_{\mu}^\alpha [-h_{\alpha}^\beta(x), \gamma_{\sigma}^\alpha - h_{\alpha}^\lambda(x), \gamma_{\sigma}^\alpha - h_{\alpha}^\lambda(x), \gamma_{\sigma}^\alpha e^{\mu\nu\lambda\sigma} ,
\]
Note that while the axion coupling to the graviton is enhanced by an exponential anti-warping factor, the vector field coupling shows no such enhancement. Now, let us return to the equation of motion for the graviton and consider the case where, \(ka\) is large, so that \(1 - e^{-2ka} \approx 1\). Then from (74), on the negative tension brane the equation reduces to[16]

\[-\Box^{(4)} h_{\mu\nu} = \frac{8\pi e^{-2\sigma(a)}}{3} T_{\mu\nu},\]  

(80)

Let us now look at the energy-momentum tensor in eq. (80). Under a linear approximation, we get

\[T_{\mu\nu} = t_{\mu\nu} - \frac{8\sqrt{6}ke^{4\sigma(a)}}{M_S^2} \epsilon_{[\mu|}^{\sigma|\alpha|\beta|} \left[ \Phi_H(x, \lambda \sigma (\beta_{[\nu]}(x), \alpha ++ h^{\lambda}_{\beta}(x), \sigma) + \Phi_H(x, \lambda \square h_{\beta[\nu]}(x), \sigma) \right] - \frac{8k\sqrt{2}e^{4\sigma(a)}}{M_S^2} \epsilon_{[\mu|}^{\sigma|\alpha|\beta|} \left[ V_\alpha(x, \sigma h_{\rho\beta}(x), \rho_{[\nu]} + V_\alpha(x, \sigma^\rho h_{\rho\beta}(x), \sigma) \right],\]  

(81)

In the above equation, we have used the mode solutions of the fields on the brane. The energy momentum tensor \(t_{\mu\nu}\) can be calculated from (57) by using (3). Then, under the assumption of a homogeneous axion background \(\Phi_H^0(x) = \Phi_H^0(t)\) such that \(d\Phi_H^0(x)/dt = f_0\) and homogeneous vector field, restricting to the lowest order in Planck mass and \(O(h)\) for which a nontrivial effect is observed and hence ignoring terms in the R.H.S. of eq. (78) and eq. (79), the equation of motion for the graviton turns out to be (with the relevant terms in view of above approximation)

\[\Box^{(4)} h_{\mu\nu} = \frac{8}{M_P^2} (\partial_\sigma \Phi_H(x) \partial_\eta \Phi_H(x) \eta^{\sigma\eta}) (\eta_{\mu\nu} + h_{\mu\nu}(x)) + \frac{4}{3M_P^2} (\partial_\sigma V_\alpha(x) \partial_\eta V_\beta(x) \eta^{\sigma\alpha} \eta^{\eta\beta}) (\eta_{\mu\nu} + h_{\mu\nu}(x)) + \frac{8\sqrt{6}ke^{3\sigma(a)}}{M_P^2} \epsilon_{[\mu|}^{\sigma|\alpha|\beta|} \left[ \Phi_H(x, \lambda \sigma (\beta_{[\nu]}(x), \alpha ++ h^{\lambda}_{\beta}(x), \sigma) + \Phi_H(x, \lambda \square h_{\beta[\nu]}(x), \sigma) \right] + \frac{8\sqrt{k}e^{3\sigma(a)}}{3M_P^2} \epsilon_{[\mu|}^{\sigma|\alpha|\beta|} \left[ V_\alpha(x, \sigma h_{\rho\beta}(x), \rho_{[\nu]} + V_\alpha(x, \sigma^\rho h_{\rho\beta}(x), \sigma) \right],\]  

(82)

Proceeding as in the section III, the circularly polarized components lead to the equation[17]

\[\left[ \frac{d^2}{dt^2} + p^2 + F_\pm \right] \epsilon_\pm = - F_\pm ,\]  

(83)

where,

\[F_\pm \equiv \frac{8\pi f_0^2}{M_P^2} \left( 1 \pm 64\sqrt{6}\pi p f_0 e^{3\sigma(a)}/M_P^2 \right),\]  

(84)

and

\[h_{ij} = \epsilon_{ij}(t) \exp - ipz .\]  

(85)

Even though we are interested in large \(p\), we would still remain within the Planckian regime \(p < M_P\) so that the quantity \(64\sqrt{6}\pi f_0 e^{3\sigma(a)}/M_P^2 \ll 1\) and can serve as an expansion parameter, leading to

\[\left[ \frac{d^2}{dt^2} + p^2 + 8\pi f_0^2/M_P^2 \neq 512\sqrt{6}\pi^2 p f_0^3 e^{3\sigma(a)}/M_P^5 \right] \epsilon_\pm \simeq - 8\pi f_0^2 (1 \pm 64\sqrt{6}\pi p f_0 e^{3\sigma(a)}/M_P^3)/M_P^2 .\]  

(86)

We can now read off the dispersion relation

\[\omega_\pm^2 = p^2 + 8\pi f_0^2/M_P^2 \neq 512\sqrt{6}\pi^2 p f_0^3 e^{3\sigma(a)}/M_P^5 .\]  

(87)

Therefrom, we calculate the group velocity

\[v_{g\pm} = \frac{d\omega_\pm}{dp} = 1 + O(p^{-2}) ,\]  

(88)
and the phase velocity

\[ v_{pk} = \frac{\omega_p}{p} = 1 \pm 256 \sqrt{6} \pi^2 p f_0^{3/3} e^{3\sigma(a)}/M_P^5. \]  

(89)

Thus, for large \( p \), the violation of Lorentz invariance can be ignored. As in the electromagnetic case, the rotation of the polarization plane for gravitational waves in RS spacetime is given by

\[ \Delta\Psi_{\text{grav}} \approx 512 \sqrt{6} \pi^2 f_0^{3/3} e^{3\sigma(a)} M_P^5 \Delta t = 512 \sqrt{6} \pi^2 f_0^{3/3} M_P^5 \Delta t. \]  

(90)

So, the rotation of the plane of polarization for the gravitational wave is enhanced by the anti-warping factor of \( \exp 3\sigma(a) \). Even with very conservative limits for \( f_0 \), the effect becomes far larger than what anyone ever expects to observe.

\section*{V. DISCUSSION}

The rotation of the plane of polarization of gravitational waves upon interaction with a homogeneous Kalb-Ramond field background, is a new aspect of Kalb-Ramond physics which has hardly been explored. The exponential enhancement of the effect in an RS1 scenario is quite analogous to the corresponding effect of KR fields on electromagnetic waves from cosmologically distant sources. However, the huge disparity between our theoretical range of values for the angle of rotation of the plane of polarization, both for electromagnetic and gravitational waves, and observational constraints already available, points to serious flaws within the theoretical framework used. The issue is then, precisely \textit{where} are these flaws? The basic assumptions made in the work have already been mentioned in an earlier part of the paper, but for completeness we recapitulate them

1. Ten dimensional heterotic string theory admits the warped five dimensional (RS1) spacetime as a consistent compactification. Note that ‘RS1’ for us means that the visible brane possesses negative tension, as is crucial to produce a naturally light Higgs scalar.

2. The Kalb-Ramond field couplings to Maxwell fields and linear metric fluctuations in this warped spacetime is the straightforward dimensional reduction of the couplings in ten dimensions in quantum heterotic string theory.

3. The compactification preserves the gauge invariance of the five dimensional Hodge-dual Kalb-Ramond vector field of which the axion is the fourth component.

If any of these assumptions are incorrect, our results no longer stand. On the other hand, if none is actually incorrect, our result points to a fundamental tension between heterotic string theory and the warping of spacetime introduced by [7] in their attempt to resolve naturalness and gauge hierarchy issues in the standard strong-electroweak theory. Put somewhat differently, reconciling the results we have obtained with observational constraints requires fine tuning of the constant \( f_0 \) to the same extent as one is forced to do within the standard electroweak theory to obtain a naturally light Higgs boson. Such a high degree of fine tuning is an extremely unsavoury feature of a scheme meant to do away with the fine tuning problem in the standard model.

One may mention that the RS1 braneworld scenario, with the visible brane having negative tension, has already been shown to lead to inconsistencies with observational constraints [11]. The spin two fluctuations on the visible brane lead to a Brans-Dicke type of gravity with a Brans-Dicke parameter lying far beyond the range restricted by observational constraints based on solar system tests. Some have tried to argue away such inconsistencies by conjecturing that the effective Brans-Dicke scalar will, on embedding in some sort of a string theory, turn out to be a moduli field which may acquire a potential, and therefore no longer produce the offending effect. While this is a distinct possibility, the crucial task of showing that the vacuum which stabilizes the Brans-Dicke moduli field also has the standard model fields with the correct spectrum, remains.

Why cannot a similar moduli stabilization work for the KR axion? Consistent with our assumptions, the KR axion is the fourth component of a five-vector gauge field whose masslessness is protected by a dual version of Kalb-Ramond gauge invariance. Stabilizing a one-off scalar moduli may be easier than one which is a component of a massless gauge field, since one now needs a Higgs mechanism to be simulated by appropriate fluxes on the brane. Additionally, the problem of demonstrating that the same Higgs vacuum yields the standard model spectrum remains. Nothing so far is known about this latter vacuum. Until such time as these problems are resolved, the fundamental inconsistencies discerned above cannot be argued away.
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[11] J. Garriga and T. Tanaka, 2000, Phys. Rev. Lett., 84, 2778.
[12] We have made an assumption here which is shared with Faraday rotation: the existence of a coherent electromagnetic wave over a time $\Delta t$. Cosmologically, any vector perturbation tends to thermalize with time scales typically smaller than $\Delta t$. We thank T. Padmanabhan for pointing this out to us.
[13] In regard to astrophysical observations of optical activity, it appears that there is definite evidence that the rotation of the plane of polarization travelling over cosmologically large distance is not entirely attributable to Faraday rotation due to magnetic fields present in the galactic plasma [4]. It is therefore not unlikely that the axion field will endow observable effect in CMB.
[14] A variant of the above KR-Maxwell interaction with parity violating effects, not obviously grounded in string/M-theory, has been shown to lead to parity violating correlations between temperature and polarization anisotropy in the Cosmic Microwave Background [9].
[15] It is easy to verify that the remaining components of the 5-dimensional dual KR gauge field $V^M$ decouple from the Maxwell field on the brane and need no longer be considered.
[16] One could also start from eq. (74) but since the result will not change under such a consideration, we prefer to get the result more simply.
[17] We are interested only in the axion coupling. Also the KR vector coupling does not lead to the rotation of polarization plane of the gravitational wave. We will ignore the KR vector-graviton coupling here.