Abstract

In this article, we take the point of view that the bottomed \((0^+, 1^+)\) mesons \(B_{s0}\) and \(B_{s1}\) are the conventional \(b\bar{s}\) meson, and calculate the strong coupling constants \(g_{B_{s0}BK}\) and \(g_{B_{s1}B^*K}\) with the light-cone QCD sum rules. The numerical values of strong coupling constants \(g_{B_{s1}B^*K}\) and \(g_{B_{s0}BK}\) are very large, and support the hadronic dressing mechanism. Just like the scalar mesons \(f_0(980), a_0(980), D_{s0}\) and axial-vector meson \(D_{s1}\), the \((0^+, 1^+)\) bottomed mesons \(B_{s0}\) and \(B_{s1}\) may have small \(b\bar{s}\) kernels of the typical \(b\bar{s}\) meson size, the strong couplings to the hadronic channels (or the virtual mesons loops) may result in smaller masses than the conventional \(b\bar{s}\) mesons in the potential quark models, and enrich the pure \(b\bar{s}\) states with other components.

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Key Words: Bottomed mesons, light-cone QCD sum rules

1 Introduction

Recently, the CDF Collaboration reports the first observation of two narrow resonances consistent with the orbitally excited \(P\)-wave \(B_s\) mesons using 1 fb\(^{-1}\) of \(p\bar{p}\) collisions at \(\sqrt{s} = 1.96\) TeV collected with the CDF II detector at the Fermilab Tevatron \[1\]. The masses of the two states are \(M(B_{s1}) = (5829.4 \pm 0.7)\) MeV and \(M(B_{s2}^*) = (5839.7 \pm 0.7)\) MeV, and they can be assigned as the \(J^P = (1^+, 2^+)\) states in the heavy quark effective theory \[2\]. The D0 Collaboration reports the direct observation of the excited \(P\)-wave state \(B_{s2}^*\) in fully reconstructed decays to \(B^+K^-\), the mass of the \(B_{s2}^*\) meson is measured to be \((5839.6 \pm 1.1 \pm 0.7)\) MeV \[3\]. While the \(B_s\) states with spin-parity \(J^P = (0^+, 1^+)\) are still lack experimental evidence.

The masses of the \(B_s\) mesons with \((0^+, 1^+)\) have been estimated with the potential quark models, heavy quark effective theory and lattice QCD \[4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16\], the values are different from each other. In our previous work \[17\], we study the masses of the bottomed \((0^+, 1^+)\) mesons with the QCD sum rules, and observe that the central values are below the corresponding \(BK\) and \(B^*K\) thresholds respectively. The strong decays \(B_{s0} \rightarrow BK\) and \(B_{s1} \rightarrow B^*K\) are kinematically forbidden, the \(P\)-wave heavy mesons \(B_{s0}\) and \(B_{s1}\) can decay through the isospin violation precesses \(B_{s0} \rightarrow B_s\eta \rightarrow B_s\pi^0\) and \(B_{s1} \rightarrow B_s^*\eta \rightarrow B_s^*\pi^0\), respectively \[18\]. The \(\eta\) and \(\pi^0\) transition matrix is very small according to Dashen’s theorem.

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The bottomed mesons $B_{s0}$ and $B_{s1}$ may have interesting feature, just like their charmed cousins $D_{s0}$ and $D_{s1}$, have small $b\bar{s}$ kernels of the typical $b\bar{s}$ mesons size, strong couplings to the virtual intermediate hadronic states (or the virtual mesons loops) may result in smaller masses than the conventional $b\bar{s}$ mesons in the potential quark models, enrich the pure $b\bar{s}$ states with other components [14, 20, 21, 22, 23, 24].

In previous works, the mesons $f_0(980)$, $a_0(980)$, $D_{s0}$ and $D_{s1}$ are taken as the conventional $q\bar{q}$ and $c\bar{s}$ states respectively, and the values of the strong coupling constants $g_{f_0KK}$, $g_{a_0KK}$, $g_{D_{s0}DK}$ and $g_{D_{s1}D^*K}$ are calculated with the light-cone QCD sum rules [22, 23, 24, 25, 26]. The large values of the strong coupling constants support the hadronic dressing mechanism.

In this article, we take the bottomed mesons $B_{s0}$ and $B_{s1}$ as the conventional $b\bar{s}$ states, and calculate the values of the strong coupling constants $g_{B_{s0}BK}$ and $g_{B_{s1}B^*K}$ with the light-cone QCD sum rules, and study the possibility of the hadronic dressing mechanism in the bottomed channels.

The light-cone QCD sum rules approach carries out the operator product expansion near the light-cone $x^2 \approx 0$ instead of the short distance $x \approx 0$ while the non-perturbative matrix elements are parameterized by the light-cone distribution amplitudes (which classified according to their twists) instead of the vacuum condensates [27, 28, 29, 30, 31, 32]. The non-perturbative parameters in the light-cone distribution amplitudes are calculated by the conventional QCD sum rules and the values are universal [33, 34, 35, 36].

The article is arranged as: in Section 2, we derive the strong coupling constants $g_{B_{s0}BK}$ and $g_{B_{s1}B^*K}$ with the light-cone QCD sum rules; in Section 3, the numerical result and discussion; and in Section 4, conclusion.

\section{Strong coupling constants $g_{B_{s1}B^*K}$ and $g_{B_{s0}BK}$ with light-cone QCD sum rules}

In the following, we write down the definition for the strong coupling constants $g_{B_{s0}BK}$ and $g_{B_{s1}B^*K}$,

\begin{align}
\langle B_{s1} | B^* K \rangle &= -ig_{B_{s1}B^*K} \eta^* \cdot \epsilon = -iM_A \hat{g}_{B_{s1}B^*K} \eta^* \cdot \epsilon , \\
\langle B_{s0} | B K \rangle &= g_{B_{s0}BK} = M_S \hat{g}_{B_{s0}BK} ,
\end{align}

where the $\epsilon_\alpha$ and $\eta_\alpha$ are the polarization vectors of the mesons $B^*$ and $B_{s1}$ respectively. The masses $M_S$ and $M_A$ can serve as an energy scale, we factorize the masses from the corresponding strong coupling constants $g_{B_{s0}BK}$ and $g_{B_{s1}B^*K}$ respectively.

We study the strong coupling constants with the two-point correlation functions
\[ \Pi_{\mu\nu}(p, q) \text{ and } \Pi_\mu(p, q) \text{ respectively,} \]

\[ \Pi_{\mu\nu}(p, q) = i \int d^4x e^{-iq \cdot x} \langle 0 | T \{ J_\mu^V(0) J_\nu^{A+}(x) \} | K(p) \rangle, \]  

\[ \Pi_\mu(p, q) = i \int d^4x e^{-iq \cdot x} \langle 0 | T \{ J_\mu^S(0) J_\nu^{S+}(x) \} | K(p) \rangle, \]  

\[ J_\mu^V(x) = \bar{u}(x) \gamma_\mu b(x), \]

\[ J_\mu^A(x) = \bar{s}(x) \gamma_\mu \gamma_5 b(x), \]

\[ J_\mu^S(x) = \bar{s}(x) b(x), \]  

where the currents \( J_\mu^V(x), J_\mu^A(x), J_\mu^S(x) \) and \( J_\mu^S(x) \) interpolate the bottomed mesons \( B^*, B_{s1}, B \) and \( B_{s0} \), respectively, the external \( K \) meson has four momentum \( p_\mu \) with \( p^2 = m_K^2 \). The correlation functions \( \Pi_{\mu\nu}(p, q) \) and \( \Pi_\mu(p, q) \) can be decomposed as

\[ \Pi_{\mu\nu}(p, q) = i \Pi_A(p, q) g_{\mu\nu} + \Pi_A1(p, q) (p_\mu q_\nu + p_\nu q_\mu) + \cdots, \]

\[ \Pi_\mu(p, q) = i \Pi_S(p, q) q_\mu + \Pi_{S1}(p, q) p_\mu + \cdots \]  

due to the Lorentz invariance.

According to the basic assumption of current-hadron duality in the QCD sum rules approach \cite{33, 34, 35, 36}, we can insert a complete series of intermediate states with the same quantum numbers as the current operators \( J_\mu^V(x), J_\mu^A(x), J_\mu^S(x) \) and \( J_\mu^S(x) \) into the correlation functions \( \Pi_{\mu\nu}(p, q) \) and \( \Pi_\mu(p, q) \) to obtain the hadronic representation. After isolating the ground state contributions from the pole terms of the mesons \( B^*, B_{s1}, B \) and \( B_{s0} \), we get the following results,

\[ \Pi_{\mu\nu} = \frac{\langle 0 | J_\mu^V(0) | B^*(q + p) \rangle \langle B^* | B_{s1} K \rangle \langle B_{s1}(q) | J_\nu^{A+}(0) | 0 \rangle}{[M_{B^*}^2 - (q + p)^2] [M_A^2 - q^2]} + \cdots \]

\[ = - \frac{ig_{B_{s1} B^* K} f_{B^*} f_A M_B M_A}{[M_{B^*}^2 - (q + p)^2] [M_A^2 - q^2]} g_{\mu\nu} + \cdots, \]  

\[ \Pi_\mu = \frac{\langle 0 | J_\mu^S(0) | B(q + p) \rangle \langle B | B_{s0} K \rangle \langle B_{s0}(q) | J_\nu^{S+}(0) | 0 \rangle}{[M_{B}^2 - (q + p)^2] [M_{s0}^2 - q^2]} + \cdots \]

\[ = \frac{ig_{B_{s0} B K} f_B f_S M_S}{[M_B^2 - (q + p)^2] [M_{s0}^2 - q^2]} (p + q)_\mu + \cdots, \]

where the following definitions for the weak decay constants have been used,

\[ \langle 0 | J_\mu^V(0) | B^*(p) \rangle = f_{B^*} M_{B^*} \epsilon_\mu, \]

\[ \langle 0 | J_\mu^A(0) | B_{s1}(p) \rangle = f_A M_A \eta_\mu, \]

\[ \langle 0 | J_\mu^S(0) | B(p) \rangle = i f_{B_{s0}} \rho_\mu, \]

\[ \langle 0 | J_\mu^S(0) | B_{s0}(p) \rangle = f_S M_S. \]  

\[ (8) \]
In the following, we briefly outline the operator product expansion for the correlation functions $\Pi_{\mu\nu}(p,q)$ and $\Pi_{\mu}(p,q)$ in perturbative QCD theory. The calculations are performed at the large space-like momentum regions $(q+p)^2 \ll 0$ and $q^2 \ll 0$, which correspond to the small light-cone distance $x^2 \approx 0$ required by the validity of the operator product expansion approach. We write down the propagator of a massive quark in the external gluon field in the Fock-Schwing er gauge firstly [37],

\[
\langle 0| T \{ q_i(x_1) \bar{q}_j(x_2) \} | 0 \rangle = i \int \frac{d^4k}{(2\pi)^4} e^{-ik(x_1-x_2)} \delta^{ij} - \int_0^1 dv g_s G_{ij}^{\mu\nu} (v x_1 + (1-v) x_2) \frac{1}{2} \frac{k+m}{(k^2-m^2)^2} \sigma_{\mu\nu} - \frac{1}{k^2-m^2} v(x_1-x_2)_{\mu} \gamma_{\nu} \right \}. \tag{9}
\]

Substituting the above $b$ quark propagator and the corresponding $K$ meson light-cone distribution amplitudes into the correlation functions $\Pi_{\mu\nu}(p,q)$ and $\Pi_{\mu}(p,q)$, and completing the integrals over the variables $x$ and $k$, finally we obtain the analytical results, which are given explicitly in the appendix.

In calculation, the two-particle and three-particle $K$ meson light-cone distribution amplitudes have been used [38, 39, 40, 41], the explicit expressions are given in the appendix. The parameters in the light-cone distribution amplitudes are scale dependent and are estimated with the QCD sum rules [38, 39, 40, 41]. In this article, the energy scale $\mu$ is chosen to be $\mu = 1\text{GeV}$, to be more precise, one can choose $\mu = \sqrt{M_B^2 - m_b^2} \approx 2.4\text{GeV}$.

After straightforward calculations, we obtain the final expressions of the double Borel transformed correlation functions $\Pi_A(M_A^2, M_B^2)$ and $\Pi_S(M_A^2, M_B^2)$ at the level of quark-gluon degrees of freedom. The masses of the bottomed mesons are $M_A = 5.72\text{GeV}$, $M_S = 5.70\text{GeV}$, $M_{B^*} = 5.33\text{GeV}$ and $M_B = 5.28\text{GeV},$ \n
\[
\frac{M_A^2}{M_A^2 + M_{B^*}^2} \approx \frac{M_S^2}{M_S^2 + M_B^2} \approx 0.54, \tag{10}
\]

there exists an overlapping working window for the two Borel parameters $M_A^2$ and $M_B^2$, it’s convenient to take the value $M_A^2 = M_B^2$. We introduce the threshold parameter $s_0$ and make the simple replacement,

\[
e^{-\frac{m_b^2 + u_0(1-u_0)m_b^2}{M_b^2}} \rightarrow e^{-\frac{m_b^2 + u_0(1-u_0)m_b^2}{M_b^2}} - e^{-\frac{s_0}{M_b^2}}
\]

to subtract the contributions from the high resonances and continuum states [37], finally we obtain the sum rules for the strong coupling constants $g_{B_{s0}BK}$ and $g_{B_{s1}B^*K}$.
\[ g_{B_{s0}B K} = \frac{1}{f_B f_S M_S} \exp\left(\frac{M_S^2}{M_1^2} + \frac{M_B^2}{M_2^2}\right) \left\{ \exp\left(-\frac{\Xi}{M^2}\right) - \exp\left(-\frac{s_0^2}{M^2}\right) \right\} \]

\[ + \frac{f_K m_K^2 M^2}{m_u + m_s} \left[ \varphi_p(u_0) - \frac{d\varphi_\sigma(u_0)}{6du_0} \right] + \exp\left(-\frac{\Xi}{M^2}\right) \left[ -m_b f_K m_K^2 \int_0^{u_0} dt B(t) \right. \]

\[ + f_3 K m_K^2 \int_0^{u_0} d\alpha_s \int_{u_0}^{1-\alpha_s} d\alpha_g \varphi_3 K(1 - \alpha_s - \alpha_g, \alpha_g, \alpha_s) \frac{2(\alpha_s + \alpha_g - u_0) - 3\alpha_g}{\alpha_g^2} \]

\[ - \frac{2m_b f_K m_K^4}{M^2} \int_{1-u_0}^1 d\alpha_g \frac{1 - u_0}{\alpha_g^2} \int_0^{\alpha_g} d\beta \int_0^{1-\beta} d\alpha \Phi(1 - \alpha - \beta, \beta, \alpha) \]

\[ + \frac{2m_b f_K m_K^4}{M^2} \left( \int_0^{1-u_0} d\alpha_g \int_{u_0}^{u_0} d\alpha_s \int_0^{\alpha_s} d\alpha + \int_{1-u_0}^1 d\alpha_g \int_{u_0}^{1-\alpha_g} d\alpha_s \int_0^{\alpha_s} d\alpha \right) \]

\[ \frac{\Phi(1 - \alpha - \alpha_g, \alpha_g, \alpha)}{\alpha_g} \right\}, \quad (11) \]
\[ g_{B_1B^*K} = \frac{1}{f_{B^*} f_A M_{B^*} M_A} \exp \left( \frac{M_A^2}{M_{1}^2} + \frac{M_{B^*}^2}{M_{2}^2} \right) \left\{ \exp \left( -\frac{\Xi}{M^2} \right) - \exp \left( -\frac{s_A^0}{M^2} \right) \right\} \\
\times f_K \left[ \frac{m_b m_{s}^2 M^2}{m_u + m_s} \varphi_B (u_0) + \frac{m_{s}^2 K (M^2 + m_b^2)}{8} \frac{d}{du_0} A(u_0) - \frac{M^4}{2} \frac{d}{du_0} \phi_K (u_0) \right] \\
- \exp \left( -\frac{\Xi}{M^2} \right) \left[ f_K m_{b}^2 m_K^2 \int_{0}^{u_0} dt B(t) \right] \\
+ m_{K}^2 \int_{0}^{u_0} d\alpha_s \int_{u_0 - \alpha_s}^{1 - \alpha_s} d\alpha_g (u_0 f_K m_{K}^2 \Phi + f_{3K} m_{\Phi} \varphi_{3K}) (1 - \alpha_s - \alpha_g, \alpha_s, \alpha_g) \\
+ f_K m_{K}^2 M^2 \frac{d}{du_0} \int_{0}^{u_0} d\alpha_s \int_{u_0 - \alpha_s}^{1 - \alpha_s} d\alpha_g A_{\parallel} (1 - \alpha_s - \alpha_g, \alpha_s, \alpha_g) \frac{\alpha_s + \alpha_g - u_0}{2\alpha_g} \\
- f_K m_{K}^2 M^2 \frac{d}{du_0} \int_{0}^{u_0} d\alpha_s \int_{u_0 - \alpha_s}^{1 - \alpha_s} d\alpha_g A_{\parallel} (1 - \alpha_s - \alpha_g, \alpha_s, \alpha_g) \frac{\alpha_s + \alpha_g - u_0}{2\alpha_g} \\
+ f_K m_{K}^4 \left( \int_{0}^{1 - u_0} d\alpha_g \int_{u_0 - \alpha_g}^{u_0} d\alpha_s \int_{0}^{\alpha_s} d\alpha + \int_{1 - u_0}^{1} d\alpha_g \int_{u_0 - \alpha_g}^{1 - \alpha_g} d\alpha_s \int_{0}^{\alpha_s} d\alpha \right) \\
\left[ \frac{1}{\alpha_g} \left( \Phi + 3 \frac{2 m_{b}^2}{M^2} \frac{\alpha_s + \alpha_g - u_0}{\alpha_g^2} (A_{\perp} + A_{\parallel}) \right) \right] (1 - \alpha - \alpha_g, \alpha, \alpha_g) \\
\frac{d}{du_0} \int_{1 - u_0}^{1} d\alpha_g \int_{u_0 - \alpha_g}^{u_0} d\alpha_s \int_{0}^{\alpha_s} d\alpha + \int_{1 - u_0}^{1} d\alpha_g \int_{u_0 - \alpha_g}^{1 - \alpha_g} d\alpha_s \int_{0}^{\alpha_s} d\alpha \right) \\
\frac{\alpha_g}{\alpha_g} \left( 4 - \frac{2 m_{b}^2}{M^2} \right) \\
+ \frac{4 m_{b}^2 (1 - u_0)^2}{\alpha_g^2} (A_{\parallel} + A_{\perp}) (1 - \alpha - \beta, \alpha, \beta) \\
+ f_K m_{K}^4 \frac{d}{du_0} \int_{1 - u_0}^{1} d\alpha_g \int_{u_0 - \alpha_g}^{u_0} d\alpha_s \int_{0}^{\alpha_s} d\alpha \Phi (1 - \alpha - \beta, \alpha, \beta) \frac{u_0 (1 - u_0)}{\alpha_g^2} \right) \right\}, \quad (12)\]

where

\[ \Xi = m_b^2 + u_0 (1 - u_0) m_K^2, \]
\[ u_0 = \frac{M_1^2}{M_1^2 + M_2^2}, \]
\[ M^2 = \frac{M_1^2 M_2^2}{M_1^2 + M_2^2}. \]

The term proportional to the $M^4 \frac{d}{du_0} \phi_K (u_0)$ in Eq.(12) depends heavily on the asymmetric coefficient $a_1 (\mu)$ of the twist-2 light-cone distribution amplitude $\phi_K (u)$ in the
limit \(u_0 = \frac{1}{2}\) (see also the sum rules for the strong coupling constant \(g_{D_1 D^* K}\) in Ref. [23]), if we take the value \(a_1(\mu) = 0.06 \pm 0.03\) [38 39 40 41], no stable sum rules can be obtained, the value of the \(g_{B_1 B^* K}\) changes significantly with the variation of the Borel parameter \(M^2\). In this article, we take the assumption that the \(u\) and \(s\) quarks have symmetric momentum distributions and neglect the coefficient \(a_1(\mu)\).

In the heavy quark limit \(m_b \to \infty\),

\[
\begin{align*}
  s_0^0 & \rightarrow m_b^2 + 2m_b \omega_0^0, \\
  s_A^0 & \rightarrow m_b^2 + 2m_b \omega_A^0, \\
  M_1^2 & \rightarrow 2m_b T_1, \\
  M_2^2 & \rightarrow 2m_b T_2, \\
  M^2 & \rightarrow 2m_b T, \\
  M_S & \rightarrow m_b + \Lambda_1, \\
  M_A & \rightarrow m_b + \Lambda_1, \\
  M_B & \rightarrow m_b + \Lambda_0, \\
  M_{B^*} & \rightarrow m_b + \Lambda_0, 
\end{align*}
\]

the two sum rules in Eqs.(11-12) are reduced to the following form,

\[
\begin{align}
g_{B_{s0} BK} &= \frac{1}{f_{B_{s0}} f_s} \exp \left( \frac{\Lambda_1}{T_1} + \frac{\Lambda_0}{T_2} \right) \left\{ \left[ 1 - \exp \left( -\frac{\omega_0^0}{T} \right) \right] \left[ 2 f_K m_K^2 T \frac{m_u + m_s}{m_u} \int_{u_0}^0 dt B(t) \right] - \int_{u_0}^0 dt B(t) \right\}, \\
g_{B_{s1} B^* K} &= \frac{1}{f_{B_{s1}} f_A} \exp \left( \frac{\Lambda_1}{T_1} + \frac{\Lambda_0}{T_2} \right) \left\{ \left[ 1 - \exp \left( -\frac{\omega_A^0}{T} \right) \right] \left[ f_K m_K^2 T \frac{m_u + m_s}{m_u} \int_{u_0}^0 dt \phi(u_0) \right] - \frac{f_K m_K^2}{m_u} \int_{u_0}^0 dt B(t) \right\},
\end{align}
\]

where the decay constants take the behavior \(f_A = \frac{C_1}{\sqrt{M_A}}, \ f_S = \frac{C_1}{\sqrt{M_S}}, \ f_B = \frac{C_2}{\sqrt{M_B}}, \ f_{B^*} = \frac{C_2}{\sqrt{M_{B^*}}}\) (according to the definition in Eq.(8)), the \(C_i\) are some constants.

### 3 Numerical result and discussion

The input parameters are taken as \(m_s = (140 \pm 10)\text{MeV}, \ m_u = (5.6 \pm 1.6)\text{MeV}, \ m_b = (4.7 \pm 0.1)\text{GeV}, \ \lambda_3 = 1.6 \pm 0.4, \ f_{3K} = (0.45 \pm 0.15) \times 10^{-2}\text{GeV}^2, \ \omega_3 = -1.2 \pm 0.7, \ \eta_3 = 0.6 \pm 0.2, \ \omega_4 = 0.2 \pm 0.1, \ a_2 = 0.25 \pm 0.15\) [32 38 39 40 41], \(f_K = 0.160\text{GeV}, \ m_K = 0.498\text{GeV}, \ M_B = 5.279\text{GeV}, \ M_{B^*} = 5.325\text{GeV}\) [42], \(M_S = (5.70 \pm 0.11)\text{GeV}, \ M_A = (5.72 \pm 0.09)\text{GeV}, \ f_s = f_A = (0.24 \pm 0.02)\text{GeV}\) [17],
Figure 1: The strong coupling constants $g_{B_s^1 B^* K}(A)$ and $g_{B_s^0 BK}(B)$ with the Borel parameter $M^2$.

\[ f_{B^*} = f_B = (0.17 \pm 0.02) \text{GeV} \quad [32, 43, 44], \quad s_S^0 = (37 \pm 1)\text{GeV}^2, \quad s_A^0 = (38 \pm 1)\text{GeV}^2 \]
\[ \Lambda_0 = \frac{M_B + 3M_{B^*}}{4} - m_b = (0.6 \pm 0.1)\text{GeV}, \quad \Lambda_1 = \frac{M_S + 3M_A}{4} - m_b = (1.0 \pm 0.1)\text{GeV}, \]
\[ \omega_S^0 = (1.6 \pm 0.1)\text{GeV} \quad \text{and} \quad \omega_A^0 = (1.6 \pm 0.1)\text{GeV}. \]

The Borel parameters are chosen as $M^2 = (5 - 7)\text{GeV}^2$, in this region, the values of the strong coupling constants $g_{B_s^1 B^* K}$ and $g_{B_s^0 BK}$ are rather stable, which are shown in Fig.1. In the heavy quark limit, the Borel parameters are chosen as $T = (0.7 - 1.5)\text{GeV}$, in this region, the values of the strong coupling constants $g_{B_s^1 B^* K}$ and $g_{B_s^0 BK}$ are rather stable, which are shown in Fig.2.

In the limit of large Borel parameter $M^2$, the strong coupling constants $g_{B_s^1 B^* K}$ and $g_{B_s^0 BK}$ take up the following behaviors,

\[
\begin{align*}
g_{B_s^0 BK} & \propto \frac{M^2 \varphi_p(u_0)}{f_B f_S}, \\
g_{B_s^1 B^* K} & \propto \frac{m_b M^2 \varphi_p(u_0)}{f_{B^*} f_A}.
\end{align*}
\] (17)

It is not unexpected, the contributions from the two-particle twist-3 light-cone distribution amplitude $\varphi_p(u)$ are greatly enhanced by the large Borel parameter $M^2$, (large) uncertainties of the relevant parameters presented in above equations have significant impact on the numerical results. The contributions from the two-particle twist-2, twist-3 and twist-4 light-cone distribution amplitudes $\phi_K(u_0)$, $\varphi_\sigma(u_0)$ and $A(u_0)$ are zero due to symmetry property.

Taking into account all the uncertainties of the input parameters, finally we
obtain the numerical values of the strong coupling constants

\[
\begin{align*}
g_{B_1B^*K} &= (18.1 \pm 6.1) \text{GeV}, \\
g_{B_0B^*K} &= (20.0 \pm 7.4) \text{GeV}, \\
\hat{g}_{B_1B^*K} &= 3.2 \pm 1.1, \\
\hat{g}_{B_0B^*K} &= 3.5 \pm 1.3
\end{align*}
\]  

(18)

from Eqs.(11-12) and

\[
g_{B_1B^*K} = g_{B_0B^*K} = (19.6 \pm 5.7) \text{GeV}
\]

(19)

from Eqs.(15-16). The uncertainties are large, about 30%. The contributions from three-particle light-cone distribution amplitudes vanish in the heavy quark limit, the uncertainties are reduced slightly, as the dominating contributions come from the two-particle twist-3 light-cone distribution amplitude \(\varphi_p(u)\).

The large values of the strong coupling constants \(g_{B_1B^*K}\) and \(g_{B_0B^*K}\) obviously support the hadronic dressing mechanism [45, 46, 47], the scalar meson \(B_{s0}(D_{s0})\) and axial-vector meson \(B_{s1}(D_{s1})\) can be taken as having small scalar and axial-vector \(b\bar{s}\) (\(c\bar{s}\)) kernels of typical meson size with large virtual \(S\)-wave \(BK(DK)\) and \(B^*K(D^*K)\) cloud respectively.

In Refs.[48, 49], the authors analyze the unitarized two-meson scattering amplitudes from the heavy-light chiral lagrangian, and observe that the scalar mesons \(D_{s0}\) and \(B_{s0}\), and axial-vector mesons \(D_{s1}\) and \(B_{s1}\) appear as the bound state poles with the strong coupling constants \(g_{D_{s0}DK} = 10.203\text{GeV}, g_{D_{s1}D^*K} = 10.762\text{GeV},\)
Table 1: Theoretical estimations of the strong coupling constants from different models, where * stands for the strong coupling constants in the heavy quark limit.

|                | $g_{B_s^1 B^* K}$ (GeV) | $g_{B_s^0 B K}$ (GeV) | $g_{D_s^1 D^* K}$ (GeV) | $g_{D_s^0 D K}$ (GeV) |
|----------------|-------------------------|-----------------------|-------------------------|-----------------------|
| Ref. [22, 23] | 10.5 ± 3.5              | 9.3 ± 2.1             |                         |                       |
| Ref. [48, 49] | 23.572                  | 23.442                | 10.762                  | 10.203                |
| This work     | 18.1 ± 6.1              | 20.0 ± 7.4            |                         |                       |
| This work*    | 19.6 ± 5.7              | 19.6 ± 5.7            |                         |                       |

$g_{B_s^1 B^* K} = 23.572$ GeV and $g_{B_s^0 B K} = 23.442$ GeV. Our numerical results for the strong coupling constants are certainly reasonable and can make robust predictions. However, we take the point of view that the mesons $D_s^0$, $B_s^0$, $D_s^1$ and $B_s^1$ be bound states in the sense that they appear below the corresponding $D K$, $B K$, $D^* K$ and $B^* K$ thresholds respectively, their constituents may be the bare $c\bar{s}$ and $b\bar{s}$ states, the virtual $D K$, $B K$, $D^* K$ and $B^* K$ pairs and their mixing, rather than the $D K$, $B K$, $D^* K$ and $B^* K$ bound states.

In Ref. [50], the authors calculate the strong coupling constants $g_{D_s^0 D_s^0 \eta}$ and $g_{D_s^1 D_s^1 \eta}$ with the light-cone QCD sum rules, then take into account $\eta - \pi^0$ mixing and calculate their pionic decay widths. The bottomed mesons $B_s^0$ and $B_s^1$ can decay through the same isospin violation mechanism, $B_s^0 \rightarrow B_s \eta \rightarrow B_s \pi^0$ and $B_s^1 \rightarrow B_s^* \eta \rightarrow B_s^* \pi^0$. We study the strong coupling constants $g_{B_s^0 B_s^0 \eta}$ and $g_{B_s^1 B_s^1 \eta}$ with the light-cone QCD sum rules and make predictions for the corresponding small decay widths [18].

4 Conclusion

In this article, we take the point of view that the bottomed mesons $B_s^0$ and $B_s^1$ are the conventional $b\bar{s}$ mesons and calculate the strong coupling constants $g_{B_s^0 B K}$ and $g_{B_s^1 B^* K}$ with the light-cone QCD sum rules. The numerical results are compatible with the existing estimations, the large values support the hadronic dressing mechanism. Just like the scalar mesons $f_0(980)$, $a_0(980)$, $D_s^0$ and axial-vector meson $D_s^1$, the bottomed mesons $B_s^0$ and $B_s^1$ may have small $b\bar{s}$ kernels of typical $b\bar{s}$ meson size. The strong couplings to virtual intermediate hadronic states (or the virtual mesons loops) can result in smaller masses than the conventional $0^+$ and $1^+$ mesons in the potential quark models, enrich the pure $b\bar{s}$ states with other components.

Appendix

The analytical expressions of the $\Pi_S(p, q)$ and $\Pi_A(p, q)$ at the level of the quark-gluon degrees of freedom,
\[
\Pi_S = \frac{f_K m_K^2}{m_u + m_s} \int_0^1 du \frac{\varphi_p(u)}{\Delta} - m_b f_K m_K^2 \int_0^1 du \int_0^a dt \frac{B(t)}{\Delta^2} \\
+ \frac{1}{6} \frac{f_K m_K^2}{m_u + m_s} \int_0^1 du \varphi_s(u) \frac{d}{du} \frac{1}{\Delta} \\
+ f_3 m_K^2 \int_0^1 dv \int_0^1 d\alpha_g \int_0^{1-\alpha_g} d\alpha_s \varphi_3 (\alpha_u, \alpha_g, \alpha_s) \frac{2v - 3}{\Delta^2} |_{u=(1-v)\alpha_g + \alpha_s} \\
- 4m_b f_K m_K^4 \int_0^1 dv \int_0^1 d\alpha_g \int_0^{1-\alpha_g} d\beta \int_0^{1-\beta} d\alpha \frac{\Phi(1 - \alpha - \beta, \beta, \alpha)}{\Delta^3} |_{u=(1-v)\alpha_g} \\
+ 4m_b f_K m_K^4 \int_0^1 dv \int_0^1 d\alpha_g \int_0^{1-\alpha_g} d\alpha_s \int_0^{\alpha_s} d\alpha \frac{\Phi(1 - \alpha - \alpha_g, \alpha_g, \alpha)}{\Delta^3} |_{u=(1-v)\alpha_g + \alpha_s} ,
\]

\[
\Pi_A = \frac{f_K m_b m_K^2}{m_u + m_s} \int_0^1 du \frac{\varphi_p(u)}{\Delta} + f_K m_K^2 \int_0^1 du \int_0^a dt \frac{B(t)}{\Delta^2} \\
+ \frac{f_K}{2} \int_0^1 du \left\{ \phi_K(u) \frac{d}{du} \log \Delta + \frac{A(u) m_K^2}{4} \frac{d}{du} \left[ \frac{1}{\Delta} + m_b^2 \right] \right\} \\
+ m_K^2 \int_0^1 dv \int_0^1 d\alpha_g \int_0^{1-\alpha_g} d\alpha_s \\
\frac{[f_K m_b^2 u \Phi + f_3 m_K^2 \varphi_3 (1 - \alpha_s - \alpha_g, \alpha_g, \alpha_s)]}{\Delta^2} |_{u=\alpha_s + (1-v)\alpha_g} \\
- \frac{f_K m_b^2}{2} \int_0^1 dv \int_0^1 d\alpha_g \int_0^{1-\alpha_g} d\alpha_s [(1 - 2v) A_{||} - V_{||}] (1 - \alpha_s - \alpha_g, \alpha_s, \alpha_g) \\
\frac{d}{du} \frac{1}{\Delta} |_{u=\alpha_s + (1-v)\alpha_g} \\
+ f_K m_K^4 \int_0^1 dv \int_0^1 d\alpha_g \int_0^{1-\alpha_g} d\alpha_s \int_0^{\alpha_s} d\alpha \Phi(1 - \alpha - \alpha_g, \alpha_g) \\
\left\{ \frac{4}{\Delta^2} - \frac{4 m_b^2}{\Delta^3} + u \frac{d}{du} \frac{1}{\Delta^2} \right\} |_{u=\alpha_s + (1-v)\alpha_g} \\
- f_K m_K^4 \int_0^1 dv \int_0^1 d\alpha_g \int_0^{1-\alpha_g} d\beta \int_0^{1-\beta} d\alpha \Phi(1 - \alpha - \beta, \alpha, \beta) \\
\left\{ \frac{4}{\Delta^2} - \frac{4 m_b^2}{\Delta^3} + u \frac{d}{du} \frac{1}{\Delta^2} \right\} |_{u=1-v\alpha_g} \\
+ 8 f_K m_b^2 m_K^4 \int_0^1 dv \int_0^1 d\alpha_g \int_0^{1-\alpha_g} d\alpha_s \int_0^{\alpha_s} d\alpha [(A_{||} + A_{||}) (1 - \alpha - \alpha_g, \alpha, \alpha_g) \\
\frac{1}{\Delta^3} |_{u=\alpha_3 + (1-v)\alpha_g} \\
- 8 f_K m_b^2 m_K^4 \int_0^1 dv v^2 \int_0^1 d\alpha_g \int_0^{1-\alpha_g} d\beta \int_0^{1-\beta} d\alpha [(A_{||} + A_{||}) (1 - \alpha - \beta, \alpha, \beta) \\
\frac{1}{\Delta^3} |_{u=1-v\alpha_g} ,
\]

(20)
where

\[
\Delta = m_b^2 - (q + up)^2, \\
\Phi = A\parallel + A\perp - V\parallel - V\perp.
\]

The light-cone distribution amplitudes of the \( K \) meson are defined by

\[
\langle 0|\bar{u}(0)\gamma_\mu\gamma_5s(x)|K(p)\rangle = if_{Kp}\int_0^1 du e^{-iu p \cdot x} \left\{ \phi_K(u) + \frac{m_K^2x^2}{16}A(u) \right\} \\
+ f_Km_K^2\frac{ixp}{2p}\int_0^1 du e^{-iu p \cdot x} B(u),
\]

\[
\langle 0|\bar{u}(0)i\gamma_5s(x)|K(p)\rangle = \frac{f_Km_K^2}{m_s + m_u}\int_0^1 du e^{-iu p \cdot x} \varphi_p(u),
\]

\[
\langle 0|\bar{u}(0)\sigma_{\mu\nu}\gamma_5s(x)|K(p)\rangle = i(p_\mu x_\nu - p_\nu x_\mu)\int_0^1 du e^{-iu p \cdot x} \varphi_\sigma(u),
\]

\[
\langle 0|\bar{u}(0)\sigma_{\alpha\beta}\gamma_5g_s G_{\mu\nu}(vx)s(x)|K(p)\rangle = f_{3K}\left\{ (p_\mu p_\alpha g_{\nu\beta} - p_\nu p_\alpha g_{\mu\beta}) - (p_\mu p_\beta g_{\nu\alpha} - p_\nu p_\beta g_{\mu\alpha}) \right\} \int D\alpha_i\varphi_{3K}(\alpha_i) e^{-ip x(\alpha_i + voa)},
\]

\[
\langle 0|\bar{u}(0)\gamma_\mu\gamma_5g_s G_{\alpha\beta}(vx)s(x)|K(p)\rangle = f_Km_K^2p_\mu p_{\alpha\beta} - p_{\beta\alpha} \frac{p \cdot x}{p} \int D\alpha_i A_\parallel(\alpha_i) e^{-ip x(\alpha_i + voa)} + f_Km_K^2(p_\beta g_{\alpha\mu} - p_\alpha g_{\beta\mu}) \int D\alpha_i A_\perp(\alpha_i) e^{-ip x(\alpha_i + voa)},
\]

\[
\langle 0|\bar{u}(0)\gamma_\mu g_s \tilde{G}_{\alpha\beta}(vx)s(x)|K(p)\rangle = f_Km_K^2p_\mu p_{\alpha\beta} - p_{\beta\alpha} \frac{p \cdot x}{p} \int D\alpha_i V_\parallel(\alpha_i) e^{-ip x(\alpha_i + voa)} + f_Km_K^2(p_\beta g_{\alpha\mu} - p_\alpha g_{\beta\mu}) \int D\alpha_i V_\perp(\alpha_i) e^{-ip x(\alpha_i + voa)},
\]

where the operator \( \tilde{G}_{\alpha\beta} \) is the dual of the \( G_{\alpha\beta} \), \( \tilde{G}_{\alpha\beta} = \frac{1}{2}\epsilon_{\alpha\beta\mu\nu}G^{\mu\nu} \) and \( D\alpha_i \) is defined as \( D\alpha_i = d\alpha_1 d\alpha_2 d\alpha_3 \delta(1 - \alpha_1 - \alpha_2 - \alpha_3) \). The light-cone distribution amplitudes are
parameterized as

\[
\begin{align*}
\phi_K(u, \mu) &= 6u(1 - u) \left\{1 + a_1 C_1^3 (2u - 1) + a_2 C_2^3 (2u - 1)\right\}, \\
\varphi_p(u, \mu) &= 1 + \left\{30 \eta_3 - \frac{5}{2} \rho^2\right\} C_2^3 (2u - 1) \\
&\quad + \left\{-3 \eta_3 \omega_3 - \frac{27}{20} \rho^2 - \frac{81}{10} \rho^2 a_2\right\} C_4^3 (2u - 1), \\
\varphi_\sigma(u, \mu) &= 6u(1 - u) \left\{1 + \left[5 \eta_3 - \frac{1}{2} \eta_3 \omega_3 - \frac{7}{20} \rho^2 - \frac{3}{5} \rho^2 a_2\right] C_2^3 (2u - 1)\right\}, \\
\varphi_{3K}(\alpha_i, \mu) &= 360 \alpha_u \alpha_s \alpha_g^2 \left\{1 + \lambda_3 (\alpha_u - \alpha_s) + \omega_3 \frac{1}{2} (7 \alpha_g - 3)\right\}, \\
V_{\parallel}(\alpha_i, \mu) &= 120 \alpha_u \alpha_s \alpha_g (v_{00} + v_{10} (3 \alpha_g - 1)), \\
A_{\parallel}(\alpha_i, \mu) &= 120 \alpha_u \alpha_s \alpha_g a_{10} (\alpha_s - \alpha_u), \\
V_{\perp}(\alpha_i, \mu) &= -30 \alpha_g^2 \left\{h_{00} (1 - \alpha_g) + h_{01} [\alpha_g (1 - \alpha_g) - 6 \alpha_u \alpha_s]\right. \\
&\quad + \left. h_{10} \left[\alpha_g (1 - \alpha_g) - \frac{3}{2} (\alpha_u^2 + \alpha_s^2)\right]\right\}, \\
A_{\perp}(\alpha_i, \mu) &= 30 \alpha_g^2 (\alpha_u - \alpha_s) \left\{h_{00} + h_{01} \alpha_g + \frac{1}{2} h_{10} (5 \alpha_g - 3)\right\}, \\
A(u, \mu) &= 6u(1 - u) \left\{\frac{16}{15} + \frac{24}{35} a_2 + 20 \eta_3 + \frac{20}{9} \eta_4\right. \\
&\quad + \left. \left[-\frac{1}{15} + \frac{1}{16} - \frac{7}{27} \eta_3 \omega_3 - \frac{10}{27} \eta_4\right] C_2^3 (2u - 1)\right\} \\
&\quad + \left[-\frac{11}{210} a_2 - \frac{4}{135} \eta_3 \omega_3\right] C_4^3 (2u - 1) \left\{2u^3 (10 - 15u + 6u^2) \log u + 2u^3 (10 - 15u + 6u^2) \log \bar{u}\right. \\
&\quad \left. + u\bar{u} (2 + 13u\bar{u})\right\}, \\
g_K(u, \mu) &= 1 + g_2 C_2^3 (2u - 1) + g_4 C_4^3 (2u - 1), \\
B(u, \mu) &= g_K(u, \mu) - \phi_K(u, \mu),
\end{align*}
\]
where

\[
\begin{align*}
    h_{00} &= v_{00} = -\frac{\eta_4}{3}, \\
    a_{10} &= \frac{21}{8} \eta_4 \omega_4 - \frac{9}{20} a_2, \\
    v_{10} &= \frac{21}{8} \eta_4 \omega_4, \\
    h_{01} &= \frac{7}{4} \eta_4 \omega_4 - \frac{3}{20} a_2, \\
    h_{10} &= \frac{7}{2} \eta_4 \omega_4 + \frac{3}{20} a_2, \\
    g_2 &= 1 + \frac{18}{7} a_2 + 60 \eta_3 + \frac{20}{3} \eta_4, \\
    g_4 &= -\frac{9}{28} a_2 - 6 \eta_3 \omega_3,
\end{align*}
\]

(24)

here \(C_2^2, C_4^2\) and \(C_2^3\) are Gegenbauer polynomials, \(\eta_3 = \frac{f_K}{f_K} \frac{m_a + m_s}{m_K}\) and \(\rho^2 = \frac{(m_a + m_s)^2}{m_K^2}\) [27, 28, 29, 30, 38, 39, 40, 41].

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**References**

[1] T. Aaltonen, et al, arXiv:0710.4199.
[2] M. Neubert, Phys. Rept. **245** (1994) 259.
[3] V. Abazov, et al, arXiv:0711.0319.
[4] D. Ebert, V. O. Galkin and R. N. Faustov, Phys. Rev. **D57** (1998) 5663.
[5] S. Godfrey and R. Kokoski, Phys. Rev. **D43** (1991) 1679.
[6] W. A. Bardeen, E. J. Eichten and C. T. Hill, Phys. Rev. **D68** (2003) 054024.
[7] P. Colangelo, F. De Fazio and R. Ferrandes, Nucl. Phys. Proc. Suppl. **163** (2007) 177.
[8] A. M. Green, et al, Phys. Rev. **D69** (2004) 094505.
[9] M. Di Pierro and E. Eichten, Phys. Rev. **D64** (2001) 114004.
[10] J. Vijande, A. Valcarce and F. Fernandez, arXiv:0711.2359.
[11] M. A. Nowak, M. Rho and I. Zahed, Acta. Phys. Polon. B35 (2004) 2377.
[12] I. W. Lee, T. Lee, D. P. Min and B. Y. Park, Eur. Phys. J. C49 (2007) 737.
[13] I. W. Lee and T. Lee, Phys. Rev. D76 (2007) 014017.
[14] A. M. Badalian, Yu. A. Simonov and M. A. Trusov, arXiv:0712.3943.
[15] T. Matsuki, K. Mawatari, T. Morii and K. Sudoh, Phys. Lett. B606 (2005) 329.
[16] T. Matsuki, T. Morii and K. Sudoh, Prog. Theor. Phys. 117 (2007) 1077.
[17] Z. G. Wang, arXiv:0712.0118.
[18] Z. G. Wang, arXiv:0801.1932.
[19] R. F. Dashen, Phys. Rev. 183 (1969) 1245.
[20] E. S. Swanson, Phys. Rept. 429 (2006) 243; and references therein.
[21] P. Colangelo, F. De Fazio and R. Ferrandes, Mod. Phys. Lett. A19 (2004) 2083; and references therein.
[22] Z. G. Wang and S. L. Wan, Phys. Rev. D73 (2006) 094020.
[23] Z. G. Wang, J. Phys. G34 (2007) 753.
[24] Z. G. Wang and S. L. Wan, Phys. Rev. D74 (2006) 014017.
[25] P. Colangelo and F. D. Fazio, Phys. Lett. B559 (2003) 49.
[26] Z. G. Wang, W. M. Yang and S. L. Wan, Eur. Phys. J. C37 (2004) 223.
[27] I. I. Balitsky, V. M. Braun and A. V. Kolesnichenko, Nucl. Phys. B312 (1989) 509.
[28] V. L. Chernyak and I. R. Zhitnitsky, Nucl. Phys. B345 (1990) 137.
[29] V. L. Chernyak and A. R. Zhitnitsky, Phys. Rept. 112 (1984) 173.
[30] V. M. Braun and I. E. Filyanov, Z. Phys. C44 (1989) 157.
[31] V. M. Braun and I. E. Filyanov, Z. Phys. C48 (1990) 239.
[32] P. Colangelo and A. Khodjamirian, hep-ph/0010175.
[33] M. A. Shifman, A. I. Vainshtein and V. I. Zakharov, Nucl. Phys. B147 (1979) 385.
[34] M. A. Shifman, A. I. Vainshtein and V. I. Zakharov, Nucl. Phys. B147 (1979) 448.

[35] L. J. Reinders, H. Rubinstein and S. Yazaki, Phys. Rept. 127 (1985) 1.

[36] S. Narison, QCD Spectral Sum Rules, World Scientific Lecture Notes in Physics 26 (1989) 1.

[37] V. M. Belyaev, V. M. Braun, A. Khodjamirian and R. Rückl, Phys. Rev. D51 (1995) 6177.

[38] P. Ball, JHEP 9901 (1999) 010.

[39] P. Ball and R. Zwicky, Phys. Lett. B633 (2006) 289.

[40] P. Ball and R. Zwicky, JHEP 0602 (2006) 034.

[41] P. Ball, V. M. Braun and A. Lenz, JHEP 0605 (2006) 004.

[42] W.-M. Yao, et al, J. Phys. G33 (2006) 1.

[43] Z. G. Wang, W. M. Yang and S. L. Wan, Nucl. Phys. A744 (2004) 156.

[44] J. M. Verde-Velasco, arXiv:0710.1790; and references therein.

[45] N. A. Tornqvist, Z. Phys. C68 (1995) 647.

[46] E. van Beveren and G. Rupp, Phys. Rev. Lett. 91 (2003) 012003.

[47] Yu. A. Simonov and J. A. Tjon, Phys. Rev. D70 (2004) 114013.

[48] F. K. Guo, P. N. Shen, H. C. Chiang and R. G. Ping, Phys. Lett. B641 (2006) 278.

[49] F. K. Guo, P. N. Shen and H. C. Chiang, Phys. Lett. B647 (2007) 133.

[50] W. Wei, P. Z. Huang and S. L. Zhu, Phys. Rev. D73 (2006) 034004.