Electromagnetically coupled processes – An error estimator for electromagnetic computational models

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Abstract. Reliable modelling of electromagnetic processing technologies largely depends on the accuracy of electromagnetic computations. This work will thus focus on the development of error estimators for evaluating the accuracy of finite element electromagnetic computations dedicated to improving induction heating process modelling accuracy. Application to induction heating and magnetic stirring cases will prove the efficiency of the approach. Error estimation results will be used later on for carrying out anisotropic adaptive remeshing.

1. Introduction
Electromagnetically coupled processes are increasingly used in the manufacturing industry. Induction heating processes for instance are commonly used for initial bulk heating or surface heat treatment. The computational modelling tools for designing these processes require dealing with electromagnetic/heat transfer couplings, as well as metallurgical evolution and mechanical distortions for residual stress prediction. However, one of the problems with the finite element tools used for modelling these processes is to make sure that these tools can be used in a reliable way. Several estimators have been proposed in literature for eddy current problems, but most of them deal with harmonic formulations ([1]). We aim here at introducing an error estimator suited for time-dependent formulations of the electromagnetic problem in fully immersed FE approach.

As electromagnetically coupled processes often involve non-linear magnetic effects, we have developed a model ([2]) which takes into account these effects using a full-time integration scheme. We shall introduce here an error estimator suited to our model. We have then tested it in an induction heating case as well as a magnetic stirring case, and carry out an error analysis by studying convergence as a function of varying mesh size.
2. The computational model

The computational model is based on a coupling between the electromagnetic model and additional physical models – such as heat transfer, solid mechanics, ….

2.1. The electromagnetic model

The electromagnetic model is classically based on the Maxwell equations (1) completed by the electromagnetic constitutive laws (2).

\[
\begin{align*}
\nabla \times \mathbf{H} &= \mathbf{j} \\
\nabla \times \mathbf{E} &= -\frac{\partial \mathbf{B}}{\partial t} \\
\n\nabla \cdot \mathbf{B} &= 0
\end{align*}
\]

(1)

\[
\begin{align*}
\mathbf{B} &= \mu(|\mathbf{H}|, T) \mathbf{H} \\
\mathbf{j} &= \sigma(T) \mathbf{E}
\end{align*}
\]

(2)

2.2. The numerical approximation for the electromagnetic problem

The previous equation system is solved using an \((A, \Phi)\) formulation.

\[
\begin{align*}
\sigma \frac{\partial \mathbf{A}}{\partial t} + \nabla \left( \frac{1}{\mu} \nabla \times \mathbf{A} \right) &= -\sigma \nabla \Phi \\
\nabla \left( \sigma \frac{\partial \mathbf{A}}{\partial t} + \nabla \Phi \right) &= 0
\end{align*}
\]

(3)

Space discretization is based on a global finite element discretization approach where the coil, the workpiece as well as the air are meshed with P1 edge-based finite elements ([3]).

This means that the magnetic potential vector field \( \mathbf{A} \) is approximated as \( \mathbf{A}^{P1} \) by constant values on each edge of the mesh – while the electric potential is approximated as \( \Phi^{P1} \) by its values at the mesh nodes.

2.3. Time discretization and coupling with other physical problems involved in EPM processes

For time discretization, we have chosen to use a full-time integration scheme for the electromagnetic problem instead of a harmonic scheme – in order to deal with non-linear magnetic materials. In a coupled Multiphysics problem as this one, we typically have two specific time scales: the first one for the electromagnetic problem - typically a fraction of the current period \( 10^{-2} \) down to \( 10^{-6} \) s. – while the second is for the thermomechanical problem - typically 1 s. Coupling with the heat transfer equation is carried out here by estimating an average dissipated Joule heat source term and using it over several thermomechanical time steps.

Coupling with the other physical problems involved is carried out by computing the average Joule heat source term due to eddy currents and of instantaneous Lorentz forces (4)

\[
\begin{align*}
P_{\text{eddy}} &= \frac{1}{T} \int_0^T \sigma \left\| \frac{\partial \mathbf{A}}{\partial t} \right\|^2 \\
\mathbf{F}_{\text{Lorentz}} &= \sigma \frac{\partial \mathbf{A}}{\partial t} \times (\nabla \times \mathbf{A})
\end{align*}
\]

(4)
3. Error estimators for the electromagnetic problem

Error estimation for electromagnetically coupled processes arises from the multi-physical coupled problems. It exists various error sources. It could be solving the electromagnetic problem, from the heat transfer model, in case of metallurgical couplings, error from the metallurgy model, in case of mechanical couplings, error from the solid mechanics model.

This paper is focused on the error that occurs when solving only the electromagnetic model. There are two main kinds of error estimators: a priori estimators – based on extrapolation of solutions obtained with mesh refinement or interpolation increase - or a posteriori estimators – based on postprocessing of numerical solutions. We shall focus here on this second category.

There are three main kinds of a posteriori estimators: residual-based estimators, equilibrated estimators and recovery-based estimators. We present here an estimator based on this last approach.

The main idea behind recovery-based estimators is to build a higher interpolation order field compared to the one calculated with the finite element approximation. This approach for a posteriori estimators was introduced by Ainsworth and Craig[4].

In the electromagnetic model, we obtain an elementwise approximated solution of the magnetic potential vector \(\mathbf{A}^p\) in order to obtain a more accurate representation of its first-order curl-derivative – namely \(H^p\).

The strategy we have thus developed is based on the following stages:
- Computation of the \(A^p\) solution
- Derivation of the \(H^p\) magnetic field: \(\text{curl}(A^p)\)
- Recovery of an \(H^p\) magnetic field
- Computation of the \((H^p - H^0)\) difference on the mesh

Two strategies can be carried out for carrying out the recovery operation: a first one based on Super Convergent Patch Recovery (SPR method) and another based on Recovery by Galerkin-based method. This last method proves to be more robust in terms of numerical convergence and efficient in terms of CPU time. Hence, the approach for the computation of the recovered solution in this work is based on a Galerkin minimization method.

The \(H^0\) magnetic field is derived using the curl of the \(A^p\) vector magnetic potential field; the values for the \(H^0\) field are computed and then stored at the Gauss integration points. Then, using the recovery method the \(H^p\) field is computed on the edges starting from the \(H^0\) field and the edge element shape functions. This minimization stage leads to solving a finite element system with a mass matrix and a load vector, as follows

\[
\langle \psi_j, \psi_i \rangle \{H^p\} = \langle \psi_i, H^0(X_{\text{GaussPts}}) \rangle
\]  

(5)

To compute the \(H^p\) field values on the integration points, an interpolation of the \(H^p\) field on the edges is carry out, as follows

\[
H^p(X_{\text{GaussPts}}) = \sum_{e=1}^{\text{nbedges}} H^p_e \psi_e(X_{\text{GaussPts}})
\]  

(6)

Finally, the estimator is computed as follows

\[
\varepsilon = \|H^p - H^0\|
\]  

(7)
4. Applications
This methodology has been applied to several cases involving electromagnetic couplings. These cases have been carried out within the framework of the commercial software FORGE® & THERCAST®. The estimators show a good convergence rate with the mesh size. We present here an application to an induction heating case modelled with FORGE®, as well as a magnetic stirring case modelled with THERCAST®.

4.1. An induction heating case
The case investigated here is the TEAM 3 (TEAM (Testing Electromagnetic Analysis Methods) represents an open international working group aiming to compare electromagnetic analysis computer codes) problem; it consists of a conducting ladder, having two holes with a coil above carrying a sinusoidal current.

![Figure 1-a. The case](image1)
![Figure 1-b. The associated mesh](image2)

Figure 1-a displays the geometry of both the part to be heated as well as the inductor. Meshes for the part and inductor are displayed in Figure 1-b. It should be noted that these meshes are embedded in a global mesh modelling the surrounding air.

Figure 2 displays the magnetic field distribution after FE calculation on the part.

![Figure 2. Magnetic field distribution on the part to be heated.](image3)

The next stage is to use the finite element error estimator introduced previously in order to get a first estimate of error values. Moreover, we know from finite element theory that numerical errors should decrease when the average mesh size is reduced. We have thus carried out a mesh sensitivity analysis in order to check the convergence of our estimator. Meshes based on a decreasing mesh size are displayed in Figure 3a.
Figure 3b presents the errors distributions obtained with the estimator. As one can see, larger errors can be observed in the coarser mesh, as well as a continuous decrease as the mesh gets finer. It can thus be clearly seen that the finite element computational error vanishes with decreasing mesh size, which is coherent with finite element theory.

4.2. A magnetic stirring case

The benchmark used to test the estimator on the electromagnetic stirring process is based on the results proposed by Musaeva [5]. The case consists in a laboratory scale stirring application of Galinstan melt in a Plexiglas mould. Figure 4 displays the geometry, as well as the associated mesh; thanks to the axial symmetry of the geometries, a reduced 15° angular section has been simulated.
As for the previous case, a sensitivity analysis has been carried out to analyze the behavior of our error estimator as the mesh gets refined. Therefore, we have carried out computations on an increasingly refined sequence of meshes.

Figure 5a shows the increasingly refined meshes, while Figure 5b displays the errors obtained at a selected time step with the estimator. Here again, the errors keep decreasing as the mesh gets refined.

Figure 6 provides additional information by providing the error distribution for a selected mesh (mesh size = 2 mm) at different time steps of the simulation (the selected time steps are plotted on the sine wave curve).

The results show how the error distribution changes over space and time in conjunction with the travelling magnetic force field.
5. Conclusion

We have introduced in this paper a recovery-based a posteriori estimator. This estimator has been tested on the solutions provided by our computational model – which is based on a fully immersed finite element method approach in conjunction with a full-time integration of the Maxwell equations. The recovered solution is computed using a Galerkin-based approach.

This estimator has been tested on two electromagnetically coupled processes – an induction heating case, as well as a magnetic stirring case. The results on the induction heating case show a satisfactory behaviour; it enables to localise in space the areas with the larger errors and shows how these errors converge as the mesh gets refined. Moreover, the results for the magnetic stirring case show how the estimator manages to dynamically follow the magnetic field at every time step and thus enable the detection of the areas most affected by the error.

This error estimator will now be used in conjunction with an automatic adaptive remeshing algorithm in order to perform computations for which the errors comply with a specific tolerance defined by the end-user.

6. References

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