Renormalization Group Induced Neutrino Mass in Supersymmetry without R-parity

Enrico Nardi

Department of Particle Physics
Weizmann Institute of Science, Rehovot 76100, Israel

Abstract

We study supersymmetric models without $R$ parity and with universal soft supersymmetry breaking terms. We show that as a result of the renormalization group flow of the parameters, a misalignment between the directions in field space of the down-type Higgs vacuum expectation value $v_d$ and of the $\mu$ term is always generated. This misalignment induces a mixing between the neutrinos and the neutralinos, resulting in one massive neutrino. By means of a simple approximate analytical expression, we study the dependence on the different parameters that contribute to the misalignment and to $m_\nu$. In large part of the parameter space this effect dominates over the standard one-loop contributions to $m_\nu$; we estimate $1 \text{ MeV} \lesssim m_\nu \lesssim 1 \text{ GeV}$. Laboratory, cosmological and astrophysical constraints imply $m_\nu \lesssim 100 \text{ eV}$. To be phenomenologically viable, these models must be supplemented with some additional mechanism to ensure approximate alignment and to suppress $m_\nu$. 

E-mail address: ftnardi@wicc.weizmann.ac.il
I. INTRODUCTION

The field content of the Standard Model (SM) together with the requirement of $SU(2)_L \times U(1)_Y$ gauge invariance, implies that at the renormalizable level the most general Lagrangian possesses additional accidental $U(1)$ symmetries. The $U(1)$ generators correspond to Baryon ($B$) and Lepton flavor ($L_i$) charges. The conservation of $B$, $L_i$ and hence of total Lepton number ($L = \sum_i L_i$) naturally explains nucleon stability as well as the non observation of $L$ and $L_i$ violating transitions. This nice feature of the SM is lost in its Supersymmetric (SUSY) extensions: once the SM fields are promoted to superfields, additional gauge and Lorentz invariant terms are allowed, which violate $B$, $L_i$ and $L$ at the renormalizable level. To forbid these dangerous terms, a parity quantum number $R = (-1)^{3B+L+2S}$ ($S$ being the spin) is assigned to each component field, and invariance under $R$ transformation is imposed. Therefore, in SUSY frameworks $B$ and $L$ quantum numbers are assigned ad hoc to the superfields to reproduce the accidental symmetries of the SM, and ensure the absence of non observed transitions. However, $R$ parity is by no means the only symmetry which allows for building viable SUSY extensions of the SM [1]. From a phenomenological point of view, the first priority is to ensure the absence of operators leading to fast nucleon decay, and in this respect other discrete symmetries can be more effective than $R$. This is because $R$ parity only forbids dimension 4 $B$ and $L$ violating terms, but does not forbid dimension 5 operators which can still be dangerous, even when suppressed by factors as large as the Planck mass. Some interesting alternatives to $R$ parity exist, which forbid dimension 4 and 5 $B$ violating terms but do not imply the same for the $L$ non-conserving terms [1], and thus imply a rather different phenomenology from models with $R$-parity. Since a mild violation of $L$ can be phenomenologically tolerated, SUSY extensions of the SM with highly suppressed $B$ violation but without $R$ parity and without $L$ number, represent interesting alternatives to the Minimal Supersymmetric Standard Model (MSSM). Two new types of Lagrangian terms characterize this class of models: (i) renormalizable interactions responsible for $L$ and $L_i$ violating transitions; (ii) superrenormalizable terms which mix the three lepton doublets
with the down-type Higgs. These mixing terms are present because the four hypercharge
\( Y = -1 \) doublets transform in the same way under the full gauge group. They also imply that
after the gauge symmetry is spontaneously broken, all the color singlet fermion fields with the
same electric charge are mixed (left and right handed charged leptons mix with Higgsinos
and winos, neutrinos mix with neutral Higgsinos and with the zino). This situation can
appear phenomenologically untenable, however the mixing acquires a well defined physical
meaning only when a physical basis for the various fields is defined. We define the down-
type Higgs as the particular combination of the four \( Y = -1 \) doublets which acquires a non
vanishing vacuum expectation value. If all the superrenormalizable terms in the Lagrangian
are such that in this basis the remaining three combinations are decoupled from the Higgs,
we can still assign to the fields a lepton number which is violated only by the renormalizable
interactions \( (i) \) while, at lowest order, it is conserved by the mass terms and by the gauge
interactions. In the class of models where the soft terms responsible for SUSY breaking
are universal, the conditions required to realize this scenario seem to be satisfied. Since
minimal SUSY extensions of the SM generally belong to this class, most of the literature
on SUSY without \( R \) parity concentrated in studying the effects of the interaction terms
\( (i) \) \cite{2,3}, while less attention has been payed to the consequences of \( (ii) \) \cite{2,3}. However,
even in the minimal models, universality of the soft terms holds only at some high energy
scale where these terms are generated. The set of loop corrections induced by the terms
\( (i) \) imply deviations from universality for the low energy parameters, and this unavoidably
results in the appearance of the terms \( (ii) \) which therefore have to be always included in the
\( R \)-parity nonconserving superpotential. Moreover, since the scale where universality holds
can be as large as the Planck scale, deviation from universality at low energy can be relevant
and imply that the effects of the Renormalization Group (RG) induced superrenormalizable
terms cannot be neglected.

In Section II we first present a qualitative discussion, based on symmetry considerations,
of the mass spectrum for the color singlet fermions. We also review the conditions for the
alignment in field space between the down-type Higgs vacuum expectation value and the
μ term \[3,14\] which plays a crucial role in ensuring the suppression of neutrino masses. In Section III we derive a simple formula which parametrizes the RG induced misalignment, and we discuss the main dependence of this effect on the model parameters. The fermion mass spectrum is discussed quantitatively in Section IV, where the mass of the heaviest neutrino arising from the misalignment is estimated. Section V contains a brief review of the main laboratory, cosmological and astrophysical constraints on the neutrino mass, which can be translated into constraints on the relevant parameters responsible for the misalignment. Our results are summarized in Section VI.

In many aspects our analysis complements recent works that discuss the same effect \[32,34\]. Refs. \[32\] and \[34\] restrict their analysis to models in which \(L\) violation enters only through the bilinear terms (ii). The renormalizable interactions (i) arise from the Yukawa terms only after the fields are rotated to a basis where the lepton doublets are decoupled from the Higgs. Therefore the form of these terms is not general, but is determined by the corresponding Yukawa couplings with a proportionality factor accounting for the field rotation. Issues analogous to the ones studied here are also addressed in Section IV of \[33\], where some results corresponding to specific choices of the parameters are presented. A brief discussion of these effects is also given in Section VI of \[13\]. Most of the results presented in these studies are given in numerical or graphical forms, which render difficult to appreciate the details of the physics involved. In the absence of analytical results it also appears awkward the task of taking properly into account these effects in future studies of SUSY models without \(R\)-parity. In the present work we study the general \(R\)-parity violating case by including all the terms consistent with the SM gauge symmetry and with \(B\) conservation. In contrast to previous works, our approach is essentially analytical. We give a simple basis independent expression for the RG induced misalignment which highlights its physical meaning. We present an analytical formula for the neutrino mass that shows explicitly the main effects involved, and makes it easy to appreciate the various interrelations between the different parameters of the model. All our main results are summarized in a few simple expressions that can be easily used for investigating further this class of models.
II. ALIGNMENT

In this section we examine the qualitative features of the fermion mass spectrum which can be expected in SUSY models without $R$ parity. The SUSY extension of the SM contains eight color-singlets chiral multiplets, corresponding to the up-type Higgs field, three right-handed leptons, three left-handed leptons and the down-type Higgs doublets. Under the electroweak gauge group $SU(2)_L \times U(1)_Y$ their quantum number assignments are

$$\hat{H}_u \sim (2,1),$$  
$$\hat{\ell}_i \sim (1,2), \quad (i = 1, 2, 3)$$  
$$\hat{H}_\alpha \sim (2,-1), \quad (\alpha = 0,1,2,3).$$

(1)

Here $\hat{H}_\alpha$ denotes collectively the supermultiplets containing the down-type Higgs and left-handed lepton doublets, which in the MSSM are distinguished by different $R$-parity assignments. If $R$-parity is not imposed, the gauge interactions posses a global $SU(4)$ symmetry corresponding to rotations of the four $\hat{H}_\alpha$ superfields \[34\]. However, other terms are generally present which select some preferred directions in $SU(4)$ field space. The relevant terms that break the symmetry in the fermion sector are:

(a) The bilinear superpotential term

$$\mu_\alpha \hat{H}_\alpha \hat{H}_u,$$

(2)

provides a mass for the fermionic component of one combination of the $\hat{H}_\alpha$ doublets (the Higgsino). The symmetry is broken down to $SU(3)$ acting on the three combinations orthogonal to the Higgsino.

(b) The vacuum expectation values (vevs)

$$\langle H_\alpha \rangle = v_\alpha,$$

(3)

which contribute to the spontaneous breaking of the electroweak (EW) symmetry, induce a mixing between the neutral members of the $\hat{H}_\alpha$ fermion doublets and the
neutral gauginos, thus breaking the symmetry down to $SU(2)$. A second combination of the neutral members in $\tilde{H}_\alpha$ acquires a mass in this way. Since the vector $v_\alpha$ fixes a direction only for the neutral fields, no additional charged fermion becomes massive at this stage.

(c) Finally, the following trilinear terms in the superpotential break the symmetry completely:

$$\lambda_{\alpha\beta k} \tilde{H}_\alpha \tilde{H}_\beta \tilde{\ell}_k + \lambda'_{\alpha j k} \tilde{H}_\alpha \hat{Q}_j \tilde{d}_k. \tag{4}$$

Here $\hat{Q}_j$ and $\tilde{d}_k$ denote the quark doublet and down-quark singlet superfields, and $\lambda_{\alpha\beta k} = -\lambda_{\beta\alpha k}$ due to the antisymmetry in the $SU(2)$ indices. For the charged fields the breaking is induced at tree level by the $\lambda_{\alpha\beta k}$ couplings, which generate three new vectors in $SU(4)$ space corresponding to the mass terms $(m^\ell)_{\alpha k} = \lambda_{\alpha\beta k} v_\beta$, ($k = 1, 2, 3$). In the neutral sector the residual $SU(2)$ symmetry is broken only at the loop level, through quark-squark and lepton-slepton loop diagrams which generate the mass terms $[15,25–31]$

$$m^\nu_{\alpha\beta} \simeq \frac{3 \lambda'_{\alpha ij} \lambda'_{\beta kl} (m^d)_{ik} (\tilde{M}^2_{LR})_{jl}}{8\pi^2 \tilde{m}^2} + \frac{\lambda_{\alpha\gamma j} \lambda_{\beta\sigma k} (m^\ell)_{\gamma k} (\tilde{M}^2_{LR})_{j\sigma}}{8\pi^2 \tilde{m}^2}. \tag{5}$$

Here $m^d$ is the $d$–quark mass matrix which arises at tree level from the second term in (4), $\tilde{M}^2_{LR}$ is the left–right sector in the $\tilde{d}$-squark mass-squared matrix, $\tilde{M}^2_{LR}$ is the left–right sector in the mass-squared matrix for the charged $\tilde{\ell}_j-H_\alpha$ scalars, $\tilde{m}$ represents a slepton or squark mass, and the expression holds at leading order in $\tilde{M}^2_{LR}/\tilde{m}^2$.

The qualitative features of the fermion mass spectrum for the fields in (4) arising from the pattern (a)-(c) are the following:

1. Only one combination of the charged $\tilde{H}_\alpha$ acquires a large mass of order $\mu = (\mu_\alpha \mu_\alpha)^{1/2}$ (or of the order of the EW breaking scale) while the remaining three charged fermions get masses proportional to (arbitrarily small) Yukawa couplings.
2. Two neutral combinations of the $\tilde{H}_\alpha$ doublets acquire small masses only at the loop level (c), while other two get large tree level masses as a consequence of (a) and (b).
Since three neutral fermions (neutrinos) are known to be very light, (2) represents a major challenge for reconciling this scenario with our experimental knowledge of the fermion mass spectrum. The qualitative features of the predicted spectrum can be reconciled with the observations if some mechanism ensures that

\[ v_\alpha \propto \mu_\alpha. \]  

(6)

If this relation is satisfied, no new direction is singled out by the vevs \( v_\alpha \), \( SU(3) \) still remains a good symmetry after \((b)\) and three neutral fermions acquire their mass only through the loop effects \((c)\). As we will see below, in general \((6)\) cannot be enforced as an exact (low-energy) relation. However, since the tree level mass which is induced at stage \((b)\) is proportional to the amount of \( SU(3) \) breaking, an \textit{approximate} alignment between \( v_\alpha \) and \( \mu_\alpha \) can be sufficient to avoid conflicts with the limits on neutrino masses.

The conditions for \( v_\alpha \) and \( \mu_\alpha \) alignment were studied in [3,14]. The direction of \( v_\alpha \) is determined by the minimum equations for the scalar potential, which depend on the soft SUSY breaking terms

\[ B_\alpha H_\alpha H_u, \quad \tilde{m}^2_{H_\alpha H_\beta} H_\alpha H_\beta, \]  

(7)

and on \( \mu_\alpha \). Terms proportional to \( \lambda_{\alpha\beta j} \) and \( \lambda_{\alpha ij} \) as well as the soft SUSY breaking trilinear \( A \) terms which also carry \( SU(4) \) indices, always involve a charged field and hence at lowest order do not contribute to determine \( v_\alpha \). Relation \((6)\) holds if the following two conditions are satisfied \([14]\):

(A) \( \mu_\alpha \) is an eigenvector of \( \tilde{m}^2_{H_\alpha H_\beta} \): \( \tilde{m}^2_{H_\alpha H_\beta} \mu_\beta = \tilde{m}^2 \mu_\alpha \);

(B) \( B_\alpha \) is proportional to \( \mu_\alpha \): \( B_\alpha = B \mu_\alpha \).

To show this, let us rotate the \( H_\alpha \) fields to the basis \((H_\parallel, H_\perp)\) where \( H_\parallel = \mu_\alpha H_\alpha / \mu \) and \( H_\perp \) denotes the three combinations orthogonal to \( H_\parallel \). According to \((A)\), in this basis \( \tilde{m}^2_{H_\parallel H_\beta} = \tilde{m}^2 \delta_{\parallel \beta} \) \( (\beta = \parallel, \perp) \) while \((B)\) implies that, like \( \mu_\alpha \), also \( B_\alpha \) has the only non-vanishing component along \( H_\parallel \). Then the solution of the minimum equations corresponds to \( \langle H_\perp \rangle = 0 \). The vector \( v_\alpha \) is thus aligned with \( \mu_\alpha : v_d \equiv (v_\alpha v_\alpha)^{1/2} = \langle H_\parallel \rangle = \mu_\alpha v_\alpha / \mu \).
III. RENORMALIZATION GROUP INDUCED MISALIGNMENT

In models where SUSY breaking is induced by universal soft breaking terms, both conditions (A) and (B) of the previous section hold. However, these conditions are exactly satisfied only at the scale $\Lambda_U$ where the universal terms are induced. After defining $B_\alpha \equiv B_{\alpha\beta} \mu_\beta$, the universality conditions at $\Lambda_U$ read

$$\left( \tilde{m}_{\mu_{\alpha\beta}} \right)_{\Lambda_U} = \tilde{m}_U^2 \delta_{\alpha\beta}$$
$$\left( B_\alpha \right)_{\Lambda_U} = B_U \delta_{\alpha\beta} .$$

As a result of the RG running of the parameters, these relations become only approximate at low energy, and a misalignment between $\mu_\alpha$ and $v_\alpha$ is generated. The deviations from conditions (A) and (B) at a generic energy scale can be parametrized as

$$\left( \frac{1}{\tilde{m}_{\mu_{\alpha\beta}}} \tilde{m}_{\mu_{\alpha\beta}}^2 - \delta_{\alpha\beta} \right) \mu_\beta = \Delta_{\alpha\beta} \tilde{m}_\mu \mu_\beta$$
$$\left( \frac{1}{B} B_{\alpha\beta} - \delta_{\alpha\beta} \right) \mu_\beta = \Delta_{\alpha\beta} B \mu_\beta$$

where $\tilde{m}_2 \simeq (\det \tilde{m}_{\mu_{\alpha\beta}})^{1/4}$ and $B = (B_\alpha B_\alpha)^{1/2}$. To estimate the misalignment induced by the RG running, we write the result of the minimization of the (low energy) scalar potential as

$$v_\alpha = \frac{v_d}{\hat{\mu}} \left[ \delta_{\alpha\beta} + \Delta_{\alpha\beta} \right] \mu_\beta ,$$

where $\Delta_{\alpha\beta} = \Delta_{\alpha\beta} + \Delta_{\alpha\beta}$ accounts for the misalignment induced by violations of conditions (A) and (B), and the normalization factor $\hat{\mu} \simeq \mu$.

It is now convenient to introduce two unit vectors $e^\nu_\alpha$ and $e^\mu_\alpha$ with components $v_\alpha/v_d$ and $\mu_\alpha/\mu$. The misalignment can be quantified by means of an angle $\xi$ defined as

$$\sin \xi = \left| e^\mu \wedge e^\nu \right| .$$

By means of (10) we obtain

$$\sin^2 \xi = \frac{1}{2} \sum_{\alpha,\beta} \left[ \left( e^\mu_\alpha \Delta_{\alpha\gamma} - e^\mu_\beta \Delta_{\beta\gamma} \right) e^\mu_\gamma \right]^2$$
$$= e^\mu \cdot \Delta^2 \cdot e^\mu - \left( e^\mu \cdot \Delta \cdot e^\mu \right)^2 .$$
We note that all factors proportional to $\delta_{\alpha\beta}$ in $\Delta_{\alpha\beta}$ cancel in (12). Therefore, in computing $\sin \xi$ it is sufficient to retain only the terms which carry non trivial $SU(4)$ indices. In particular, the contributions proportional to the up-quarks Yukawa couplings and to the gauge couplings can be dropped off, and only terms involving the couplings in (4) and the corresponding trilinear soft SUSY breaking $A$ terms need to be kept. An approximate analytical expression for $\Delta_{\alpha\beta}$, obtained by assuming constant coefficients and by integrating the RG equations in one step, is derived in the Appendix. Proceeding in this way, it is possible to single out the main effects which generate misalignment and to keep track of the various interrelations among different parameters. Besides universality, in the following we will also assume that at $\Lambda_U$ the trilinear soft SUSY breaking $A$ terms are proportional to the corresponding couplings in (4). This assumption is made only for reasons of simplicity, since it allows factoring out the overall scale $A_U$ of the soft-breaking terms and this gives simpler expressions. However, the results of the analysis do not depend on this assumption, and it is straightforward to replace terms like $A_U \lambda_{\alpha\beta i}$ with the more general parameters $A_{\alpha\beta i}$. At the EW scale $\sim m_Z$, the RG induced misalignment matrix reads

$$\Delta_{\alpha\beta} = \frac{t_U}{8\pi^2} \left( 3 + \frac{A_U^2}{\tilde{m}_U^2} + \frac{A_U}{B_U} \right) \left( \lambda_{\alpha\gamma i} \lambda_{\beta\gamma i} + 3 \lambda'_{\alpha ij} \lambda'_{\beta ij} \right), \quad (13)$$

where $t_U = \log \frac{M_Z}{\Lambda_U}$, $A_U$, $B_U$ and $\tilde{m}_U^2$ are the soft SUSY breaking parameters at $\Lambda_U$, and only terms inducing $SU(4)$ rotations are displayed. In (13) the term proportional to $A_U/B_U$ comes from the running of $B_{\alpha\beta}$, while all the others originate from $\tilde{m}_{\alpha\beta}^2$. We learn the following:

(a) The RG induced $v_\alpha - \mu_\alpha$ misalignment originates mainly from the running of the the soft-breaking scalar masses. For $A_U \sim B_U \sim \tilde{m}_U$ this effect dominates by a factor of $\sim 4$. Only if $B_U \ll A_U \ll \tilde{m}_U$ the misalignment is dominantly induced by the evolution of the $B$ terms.

(b) Apart from fine tuned cancelations and as long as $L$ is a broken symmetry, there is no limit for the soft-breaking terms in which alignment can be recovered.
(c) If \( A_U \ll B_U, \tilde{m}_U \), the misalignment is independent of the initial values of the soft-breaking parameters.

(d) Since \( SU(4) \) rotations in the evolution of \( B_{\alpha\beta} \) are induced only by terms proportional to \( A_U \) (see the Appendix) if \( B_U = 0 \) the third term in the first parenthesis in (13) is unity. If \( A_U = B_U = 0 \) the \( B \) term does not contribute (at this order) to the misalignment.

From (12) we see that \( \sin \xi \) is a basis independent physical parameter. It is convenient to give its explicit expression in a specific basis. We define the down-type Higgs field \( H_d \) at the EW scale by the condition \( \langle H_d \rangle = v_d \) (that is \( H_d = e^v_\alpha H_\alpha \)) and we choose the basis \( \{ \hat{H}_d, \hat{L}_i \} \) where \( \hat{L}_i \) are three states orthogonal to \( \hat{H}_d \). In this basis \( e^v_\alpha = \delta_{0\alpha} \), while \( h^d_{ij} = \lambda_{\alpha jk} e^v_\alpha \) and \( h^{\ell k}_\beta = \lambda_{\alpha \beta k} e^v_\alpha \) (with \( h^\ell_{0k} = h_{0\alpha k} e^v_\alpha = 0 \) for the antisymmetry of the \( \lambda \) couplings) are the \( H_d \) Yukawa couplings to the fermions. After inserting (13) in (12) and using \( e^\mu_\alpha \approx (\delta_{0\beta} - \Delta_{0\beta}) e^v_\beta \approx \delta_{0\alpha} \) we obtain

\[
\sin^2 \xi \simeq \sum_i \Delta_{i0} \Delta_{i0} = \left( \frac{t_U}{8\pi^2} \right)^2 \left( 3 + \frac{A^2_U}{\tilde{m}_U^2} + \frac{A_U}{B_U} \right)^2 \sum_i \left( h^\ell_{jk} \lambda_{ijk} + 3 h^d_{jk} \lambda'_{ijk} \right)^2,
\]

where it is understood that \( \lambda_{ijk} \) and \( \lambda'_{ijk} \) are now the couplings (4) rotated to the \( \{ \hat{H}_d, \hat{L}_i \} \) basis. From (14) we learn the following:

(e) To generate misalignment is enough to have at least one of the \( L \) violating \( \lambda \) or \( \lambda' \) couplings (or one of the corresponding \( A \) terms, if the assumption of proportionality is dropped) non-vanishing.

(f) Assuming no particular suppression of the R-parity violating \( b \)-quark couplings \( \lambda'_{i33} \) with respect to the couplings involving the first two families, the dominant contribution to the misalignment is proportional to the \( b \)-quark Yukawa coupling \( h^d_{33} \).

\[\text{\footnote{For simplicity we do not distinguish between the couplings at the EW scale and at \( \Lambda_U \). For our approximate solutions of the RG equations the difference is formally of higher order.}}\]
(g) Since only the leptons and $d$-quarks couplings appear in (14), the misalignment depends strongly also on the value of $\tan \beta = v_u/v_d$ (where $v_u = \langle H_u \rangle$). For the leading contributions we obtain

$$\sin^2 \xi \simeq \left[ \frac{3t_U m_b}{8\pi^2 v} \right]^2 \left( 3 + \frac{A_U^2}{m_U^2} + \frac{A_U}{B_U} \right)^2 \left( \sum_i \lambda'_{i33} \lambda_{i33} \right) (1 + \tan^2 \beta), \quad (15)$$

where $v = (v_u^2 + v_d^2)^{1/2} \simeq 246$ GeV. If in addition we assume that the $L$ violating couplings are proportional to the corresponding Yukawa couplings (as is the case in models based on horizontal symmetries [14][15]) then $\sin^2 \xi \sim \tan^4 \beta$.

IV. FERMION MASS SPECTRUM

In this section we investigate the consequences of the RG induced misalignment on the fermion mass spectrum. Numerically, the factor in square brackets in (13) is at most of order 1% resulting in $\sin \xi \ll 1$ and approximate low energy alignment. As we will see, this implies that $L$ violation in the mass terms is small, and that a distinction between ‘leptons’ and charginos and neutralinos is still a meaningful one. We define the ‘right handed leptons’ as the mass eigenstates having as main components the three $SU(2)$ singlets $\bar{\ell}_i$. Their mass partners are the ‘left handed charged leptons’ which are dominantly combinations of just the $Y = -1$ doublets. Their neutral $SU(2)$ partners constitute the main components of the neutrinos, while the remaining mass eigenstates are the charginos and the neutralinos.

The mass matrix for the charged fermions $M_c$ is $5 \times 5$, with rows corresponding to $\{\tilde{W}^-, \tilde{H}_d^\alpha, \tilde{\nu}_L^i\}$, and columns to $\{\tilde{W}^+, \tilde{H}_u^+, \bar{\ell}_k^+\}$:

$$M_c = \begin{pmatrix} M_2 & \frac{\mu}{\sqrt{2}} v_u & 0_{1 \times 3} \\ \frac{\mu}{\sqrt{2}} v_{\alpha} & \mu_\alpha & \lambda_{\alpha \beta k} v_{\beta} \end{pmatrix}. \quad (16)$$

Here $0_{1 \times 3}$ denotes a zero $1 \times 3$ block and $M_2$ is the $SU(2)_L$ gauginos Majorana mass. In the $\{\tilde{W}^-, \tilde{H}_d, \tilde{\nu}_L^i\}$ basis (in which we denote particles and superparticles according to the usual convention) this becomes
\[ M'_c = \begin{pmatrix} M_2 & \sqrt{2} m_W \sin \beta & 0_{1 \times 3} \\ \sqrt{2} m_W \cos \beta & \mu \cos \xi & 0_{1 \times 3} \\ 0_{3 \times 1} & \mu (e^v \wedge e^\mu)_i & h_{ij}^i \nu_d \end{pmatrix}, \tag{17} \]

where \( m_W = g v / 2 \). As expected the charged lepton masses originate from the Yukawa couplings to \( H_d \), and their mixings with the charginos, induced by the \((3 2)\) block, is suppressed at least as \( \sin \xi \). (If \( \sin \xi \) is not too small, this mixing could still give rise to interesting processes like \( Z \rightarrow \tilde{W}^+ L_i^- \), \( L_i^+ \ell_j^- \), \( \ell_i^+ \ell_j^- \) \((i \neq j)\), etc.)

The full neutralino mass matrix is \( 7 \times 7 \). In the basis with rows and columns corresponding to \( \{ \tilde{B}, \tilde{W}_3, \tilde{H}_u^0, \tilde{H}_d^0 \} \) it reads

\[ M_n = \begin{pmatrix} M_1 & 0 & g \tan \theta_W & -g \tan \theta_W \\ 0 & M_2 & -g_2 v_i & g_2 v_i \\ g \tan \theta_W v_i & -g_2 v_i & 0 & \mu \alpha \\ -g \tan \theta_W v_i & g_2 v_i & -\mu \alpha & 0_{4 \times 4} \end{pmatrix}. \tag{18} \]

Here \( M_1 \) is the \( U(1)_Y \) gaugino mass and \( \theta_W \) is the weak mixing angle. In the \( \{ \tilde{H}_d, L_i^- \} \) basis \( v_\alpha = (v_d, 0, 0, 0) \) and \( \mu_\alpha \approx (\mu, -\Delta_{i0} \mu) \). \( M_n \) gives 5 massive states and two massless ones. Four massive states correspond to the neutralinos while the fifth one, a neutrino, corresponds to a combination of the neutral members in \( L_i \). The mass of the neutrino is given by

\[ m_\nu \approx \frac{\det' M_n}{\det' M_n |_{\xi = 0}}, \tag{19} \]

where \( \det' \) denotes the product of the nonvanishing eigenvalues of the respective mass matrices. We have

\[ \det' M_n = \mu^2 M_3 m_Z^2 \cos^2 \beta \sin^2 \xi, \]

\[ \det' M_n |_{\xi = 0} = \mu M_3 m_Z^2 \sin 2 \beta - \mu^2 M_1 M_2, \tag{20} \]

where \( m_Z = g v / (2 \cos \theta_W) \) and \( M_3 = M_1 \cos^2 \theta_W + M_2 \sin^2 \theta_W \) is the photino mass term. The final expression for the mass of the neutrino reads

\[ m_\nu \approx \mu \left[ \sin 2 \beta - \frac{\mu M_1 M_2}{M_3 m_Z^2} \right]^{-1} \left[ 3 t_U m_b / 8 \pi^2 v \right]^2 \left( 3 + \frac{A_U^2}{m_U^2} + \frac{A_U}{B_U} \right)^2 \left( \sum_i \lambda'_{i33} \lambda_{i33} \right). \tag{21} \]
From this expression we see that the numerical value of $m_\nu$ depends on several parameters, and this explains why it is not easy to derive any simple scaling behavior from a numerical study of these effects. However, the leading behaviors are as follows: if $\mu M_2/m_Z^2 \gg \sin 2\beta$ (which is more easily satisfied in the $\tan\beta \gg 1$ limit) then $m_\nu \propto m_Z^2/M_2$, practically independently of $\mu$ and $\tan\beta$. Therefore, as was noted in [32], $m_\nu$ vanishes in the limit of very large SUSY breaking scales. In the opposite limit, which is consistent only for moderate values of $\tan\beta$ (\lesssim 5) and for small values of $M_2$ and $\mu$, $m_\nu$ is approximatively proportional to $\mu \tan\beta$. It is interesting to note that in the first equation (20) $\cos^2\beta$ cancels against the explicit $1 + \tan^2\beta$ term in the misalignment parameter $\sin^2\xi$ (15), leaving only a mild dependence on $\tan\beta$ in the final result. However, if the $R$-parity violating couplings are proportional to the Yukawa couplings, an implicit $\tan^2\beta$ dependence from the misalignment is still present in the last parenthesis in (21). We also note that the first square bracket in (21) cannot approach zero, since is bounded by the lower limits on the neutralino masses. Banning possible fine tunings, for natural values of the parameters we obtain that $\mu$ divided by the first bracket yields a dimensionful factor $\sim 10^{−100}$ GeV. The square of the second bracket provides a suppressing factor in the range $10^{−3}−10^{−5}$ corresponding respectively to $\Lambda_U \sim m_{\text{Planck}}$ and $\Lambda_U \sim 10^5$ GeV, where the second value is typical of gauge mediated SUSY breaking scenarios [36–41]. Finally, the square of the term containing the soft breaking parameters yields approximatively a one order of magnitude enhancing factor.

As a result, in the absence of further suppression from the $\lambda'$ couplings, we would estimate $1$ MeV $\lesssim m_\nu \lesssim 1$ GeV.

Before concluding this section, it is interesting to compare the RG induced effects on $m_\nu$ with the standard contributions to the neutrino mass matrix from one-loop diagrams [15,25,31]. From (13) the corresponding leading term reads

\[ \text{\textsuperscript{†}} \]

\[ \text{The lower limit can be pushed down to 1 GeV in a phenomenologically viable scenario in which } \mu \text{ is very small resulting in two neutralinos not much heavier than a few } \text{GeV}. \]
\[ m_{ij}^\nu \simeq \frac{3}{8\pi^2} \frac{m_h^2}{m^2} (A - \mu \tan \beta) \lambda'_{i33} \lambda'_{j33}. \] (22)

The misalignment yields the dominant contribution to the mass of the heaviest neutrino as long as

\[ t_U \gtrsim \left[ \frac{8\pi^2}{3} F(m_{\text{soft}}) \right]^{1/2}, \] (23)

where the dimensionless function \( F \) depends in a complicated way on the various soft SUSY breaking parameters, as well as on \( \mu \) and \( \tan \beta \). For most values of \( F \) (23) is satisfied as long as \( \Lambda_U > 10^5 \text{ GeV} \), and thus in general the induced misalignment gives the leading effect. This implies that predictions for the neutrino masses in models without \( R \)-parity and with high energy alignment based solely on an estimate of the loop contributions [31], should be modified to include this effect.

On the other hand, \( F \) is maximal when \( A_U \ll \tilde{m}_U, B_U \) and in this limit we obtain \( F \lesssim (1/g^2) (\mu M_2/\tilde{m}^2) \tan \beta \). This situation is interesting since \( A_U = 0 \) can arise in gauge mediated SUSY breaking models [37]. In this case, for values of the relevant parameters such that \( F \sim (1/g^2) \) and for \( \Lambda_U \lesssim 10^6 \text{ GeV} \) the two effects yield contributions which can be comparable in magnitude. Finally, for small \( A_U \) and \( \tan \beta \) rather large \((\mu M_2 \tan \beta/\tilde{m}^2 \gtrsim 25)\) the one-loop contributions (22) dominate over the misalignment effects up to \( \Lambda_U = m_{\text{Planck}} \), and thus determine the mass of the heaviest neutrino.

V. PHENOMENOLOGICAL CONSTRAINTS

As we have seen, in SUSY models without \( R \)-parity and without Lepton number the induced \( v_\alpha - \mu_\alpha \) misalignment results in one massive neutrino which, in the absence of suppression of the \( L \) violating trilinear couplings, is naturally in the range \( 1 \text{ MeV} \lesssim m_\nu \lesssim 1 \text{ GeV} \). In this section we argue that laboratory and cosmological constraints imply that this window is excluded. In addition the massive neutrino of these models is very likely stable on a cosmological time scale, and thus the cosmological limit \( m_\nu \lesssim 100 \text{ eV} \) applies. As a conse-
quence, to render these models phenomenologically viable some mechanism to suppress the $L$-violating couplings in (21) down to $\sum_i \lambda'_{i33} \lambda'_{i33} \lesssim 10^{-4} - 10^{-7}$ is called for.

The flavor composition of the neutrino is determined by the relative rotation in the $\{\tilde{H}_d, L_i\}$ basis arising from the diagonalization of the submatrix in (18) containing $\Delta_{i0}$ (for the $\nu$’s) and of the Yukawa couplings matrix $h^\ell h^{\ell\dagger}$ (for the left-handed leptons). This can be studied only by specifying further the model. We will avoid doing this, and we will conservatively assume that our massive state is mainly $\nu_\tau$, so that the laboratory limit on the mass is $m_\nu < 23$ MeV [45]. With regards to the neutrino mixing angles, the discussion below is purely phenomenological and does not require any theoretical estimate.

Cosmological considerations of the age and the present energy density of the Universe provide constraints relating the mass and lifetime of the neutrino. For masses in the range 100 eV – a few MeV the constraint reads $m_\nu^2 \tau_\nu \lesssim 2 \times 10^8$ MeV$^2$ sec [16]. When charged particles are present in the final state, a stronger bound from the absence of distortions in the cosmic microwave background radiation (CMBR) applies, $\tau_\nu \lesssim 10^4$ sec. Detailed studies of the effects of a massive $\nu_\tau$ during the nucleosynthesis era imply an even stronger limit $\tau_\nu \lesssim 10^2$ sec, for masses in the range 0.5 MeV – 35 MeV and independently of the decay modes [47–52]. For visible decay modes (final states containing $\gamma$ or $e^\pm$) also a lower bound on $\tau_\nu$ exists, $\tau_\nu \gtrsim 10^8$ sec. This bound follows from the limits on the gamma-ray fluence around the time when the neutrinos from the Supernova 1987A were detected [53]. This set of constraints already suggests that $m_\nu \gtrsim 1$ MeV is very likely ruled out.

Such a suppression can be easily accommodated in models for fermion masses based on Abelian horizontal symmetries [12–44]. The required suppression for the case when the soft SUSY breaking terms are not universal and the misalignment arises as a tree level effect was studied in [14,15]. In contrast to that case which required horizontal charges for the $L_i$ doublets larger than $\sim 7$, in the present scenario a sufficiently small neutrino mass is obtained with the more natural values $Q_H(L_i) \sim 0 – 3$. 

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In order to avoid some (or all) of these constraints, the massive \( \nu \) should decay fast enough, and preferably into invisible final states. However, most likely the dominant \( \nu \) decay mode is \( \nu \rightarrow e^+e^-\nu_e \) which proceeds via \( W \)–mediated tree level diagrams. All other decay modes, as \( \nu \rightarrow 3\nu_\ell \) or \( \nu \rightarrow \gamma\nu_\ell \left( \ell = e, \mu \right) \) are flavor changing neutral current (FCNC) processes, and are more suppressed. In particular, the invisible decay mode into three neutrinos involves the FCNC \( Z\bar{\nu}\nu_\ell \) vertex, which is quadratic in the neutrino mixing with the isomultiplet neutralino \( \tilde{W}_3 \), and hence very small. For the leading decay mode the lifetime is

\[
\tau_\nu = \left( \frac{m_\mu}{m_\nu} \right)^5 \frac{\tau_\mu}{|U_{1\nu}|^2} \approx \frac{2.8 \times 10^4 \left(1 \text{MeV} \right)^5}{|U_{1\nu}|^2} \text{sec},
\]

where \( \tau_\mu \approx 2.2 \times 10^{-6} \) has been used. On the other hand, peak and kink searches in \( \pi, K \) and \( \beta \) decays yield stringent upper limits on \( |U_{1\nu}|^2 \). We have \( |U_{1\nu}|^2 < 5 \times 10^{-6} \left(1 \times 10^{-4} \right) \) for a mass of about 20 MeV (5 MeV), implying lifetimes in conflict with the nucleosynthesis (and CMBR) constraint. For smaller masses the laboratory limits on the mixing parameters are less stringent. However, below about 3.5 MeV (1.5 MeV) the constraint from nucleosynthesis (and CMBR) is not satisfied even for maximal mixing. For \( m_\nu < 1 \text{MeV} \) only FCNC decay channels are open, implying that also the weaker mass-lifetime constraint from the age of the Universe is not evaded. Therefore, we conclude that independently of the mass and mixing angles the \( \nu \) decay rate is not fast enough to evade all the constraints, and the limit for cosmologically stable neutrinos holds.

VI. CONCLUSIONS

In this paper we have presented an analysis of SUSY models without \( R \) parity and without Lepton number. We have shown that even when universality of the soft SUSY breaking terms is assumed, at low energy the vector \( v_\alpha \) of the vevs of the hypercharge \(-1\) doublets \( H_\alpha \) is not aligned with the vector \( \mu_\alpha \) of the generalized \( \mu \)-term \( \mu_\alpha \hat{H}_\alpha \hat{H}_u \). The misalignment is induced by the renormalization group flow of the parameters from the
scale where the soft SUSY breaking terms are generated, down to low energy. We have derived a simple analytical expression which describes the dependence of the misalignment on the relevant parameters. Our treatment is basis independent, and shows that this effect cannot be rotated away or neglected. In the basis where the fields are physical, the bilinear superpotential terms \( \mu_i \hat{L}_i \hat{H}_u \) which violate Lepton number by one unit, are always present. A major consequence of \( v_\alpha - \mu_\alpha \) misalignment is that one neutrino becomes massive, and the mass induced in this way is generally larger than the contributions from one-loop diagrams. We have estimated that in the absence of additional suppression \( 1 \text{ MeV} \lesssim m_\nu \lesssim 1 \text{ GeV} \). A brief analysis of various laboratory, cosmological and astrophysical constraints strongly suggests that this neutrino is cosmologically stable, and thus its mass must be below 100 eV. This bound can be translated into a constraint on the \( R \)-parity violating trilinear couplings \( \sum_i \lambda'_{i33} \lambda'_{i33} \lesssim 10^{-4} - 10^{-7} \). We conclude that, to be phenomenologically viable, SUSY models without \( R \)-parity must be supplemented with some mechanism (as for example a horizontal flavor symmetry) yielding a sufficient suppression of these couplings.

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**APPENDIX A:**

In this Appendix we compute the misalignment matrix

\[
\Delta_{\alpha\beta} = \Delta^B_{\alpha\beta} + \Delta_{\alpha\beta}
\]

induced by the RG evolution of the soft SUSY breaking parameters \( \tilde{m}_{H_\alpha H_\beta} \) and \( B_{\alpha\beta} \) from the high scale \( \Lambda_U \) down to the EW scale. The running is controlled by the RG equations

\[
\frac{d\tilde{m}_{H_\alpha H_\beta}^2}{dt} = \frac{\tilde{m}^2}{16\pi^2} G_{\alpha\beta} \tilde{m}^2,
\]
\[
\frac{dB_{\alpha\beta}}{dt} = \frac{A}{16\pi^2} G_{\alpha\beta}^B, \quad (A2)
\]

where we have factored out the overall scale \(\tilde{m}^2\) and \(A\) of the soft SUSY breaking masses and of the trilinear soft breaking terms. The boundary conditions at \(\Lambda_U\), where by assumption universality holds, are given in (8). We solve (A2) in first approximation, neglecting the scale dependence of the coefficients. This gives

\[
\Delta \tilde{m}^2_{\alpha\beta} \equiv \frac{1}{\tilde{m}^2_U} \tilde{m}^2_{H_{\alpha}H_{\beta}} - \delta_{\alpha\beta} \simeq t_U \frac{G_{\alpha\beta}}{16\pi^2} \Lambda_U, \quad (A3)
\]

\[
\Delta B_{\alpha\beta} \equiv \frac{1}{B_U} B_{\alpha\beta} - \delta_{\alpha\beta} \simeq A_U \frac{t_U}{B_U} \frac{G_{\alpha\beta}}{16\pi^2} \Lambda_U,
\]

where \(t_U = \log(M_Z/\Lambda_U)\). The SUSY RG equations including R-parity violation have been recently presented in a number of papers [33,55,56]. The equations for the soft SUSY breaking terms can be read off from [33]. For the running of \(\tilde{m}^2_{H_{\rho}H_{\sigma}}\) in (A2) we have

\[
\tilde{m}^2 G_{\alpha\beta}^\rho^\sigma = C^H_{\alpha\beta} \tilde{m}^2_{H_{\rho}H_{\sigma}} + C^\ell_{\alpha\beta} \tilde{m}^2_{\ell_{i}\ell_{j}} + C^Q_{\alpha\beta} \tilde{m}^2_{Q_{i}Q_{j}} + C^d_{\alpha\beta} \tilde{m}^2_{d_{i}d_{j}} + A^2 C^A_{\alpha\beta} + C^G \delta_{\alpha\beta} \quad (A4)
\]

where

\[
C^H_{\alpha\beta} = \lambda_{\alpha\gamma i} \lambda_{\beta\gamma i} \delta_{\alpha\sigma} + 2\lambda_{\alpha\sigma i} \lambda_{\beta\rho i} + 3(\lambda'_{\alpha ij} \lambda'_{\rho ij} \delta_{\beta\sigma} + \lambda'_{\beta ij} \lambda'_{\rho ij} \delta_{\alpha\sigma})
\]

\[
C^\ell_{\alpha\beta} = 2 \lambda_{\alpha\gamma i} \lambda_{\beta\gamma j}
\]

\[
C^Q_{\alpha\beta} = 6 \lambda'_{\alpha ki} \lambda'_{\beta kj}
\]

\[
C^d_{\alpha\beta} = 6 \lambda'_{\alpha ki} \lambda'_{\beta kj}
\]

\[
C^A_{\alpha\beta} = 2(\lambda_{\alpha\gamma i} \lambda_{\beta\gamma i} + 3\lambda'_{\alpha ij} \lambda'_{\beta ij})
\]

\[
C^G = -\sum_{\sigma=\text{all}} g_1^2 Y_{\sigma} \tilde{m}^2_{\sigma\sigma} - 2g_1^2 M_1^2 - 6g_2^2 M_2^2.
\]

(A5)

The running of the \(B\) term is determined by

\[
G_B^{\alpha\beta} = D_{\alpha\beta} + D \delta_{\alpha\beta} \quad (A6)
\]

with

\[
D_{\alpha\beta} = 2 \left(\lambda_{\alpha\gamma i} \lambda_{\beta\gamma i} + 3\lambda'_{\alpha ij} \lambda'_{\beta ij}\right)
\]

\[
D = 2 \left(3h_i^u h_i^u + g_1^2 M_1 + 3g_2^2 M_2\right), \quad (A7)
\]

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where $h^u_{ij}$ are the up-quark Yukawa couplings. Using the boundary conditions (8) together with $(\tilde{m}_{\ell_i,\ell_j})_{\Lambda_U} = (\tilde{m}_{\ell_i,\ell_j})_{\Lambda_U} = (\tilde{m}_{\ell_i,\ell_j})_{\Lambda_U} = \tilde{m}_U^2 \delta_{ij}$ we obtain

$$(G_{\alpha\beta})_{\Lambda_U} = \left(6 + 2 \frac{A_U^2}{m_U^2}\right) \left(\lambda_{\alpha\gamma_i} \lambda_{\beta\gamma_i} + 3 \lambda'_{\alpha i k} \lambda'_{\beta i k}\right) - \left[\sum_{\sigma=all} g_1^2 Y_{\sigma} + 2 g_1^2 \frac{M_U^2}{m_U^2} + 6 g_2^2 \frac{M_U^2}{m_U^2}\right] \delta_{\alpha\beta}$$

$$(G^B_{\alpha\beta})_{\Lambda_U} = 2 \left(\lambda_{\alpha\gamma_i} \lambda_{\beta\gamma_i} + 3 \lambda'_{\alpha i j} \lambda'_{\beta i j}\right) + 2 \left[3 h^u_{ij} h^u_{ij} + g_1^2 \frac{M_U}{A_U} + 3 g_2^2 \frac{M_U}{A_U}\right] \delta_{\alpha\beta}, \quad (A8)$$

where $M_U$ is the universal gaugino mass. The terms in square brackets which are proportional to $\delta_{\alpha\beta}$ do not generate misalignment, and for our purposes can be dropped off. Inserting the relevant terms of equations (A8) into (A3) gives the expression (13) for the misalignment matrix $\Delta_{\alpha\beta}$.

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5In models where SUSY breaking is communicated to the visible sector via gauge interactions, the SUSY breaking terms are flavor-symmetric but not universal. The corresponding modifications of equations (A8) are straightforward, and do not affect our conclusions.
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