THz-driven zero-slippage IFEL scheme for phase space manipulation

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Abstract

We describe an inverse free electron laser (IFEL) interaction driven by a near single-cycle THz pulse that is group velocity-matched to an electron bunch inside a waveguide, allowing for a sustained interaction in a magnetic undulator. We discuss the application of this guided-THz IFEL technique for compression of a relativistic electron bunch and synchronization with the external laser pulse used to generate the THz pulse via optical rectification, as well as a laser-driven THz streaking diagnostic with the potential for femtosecond scale temporal resolution. Initial measurements of the THz waveform via an electro-optic sampling based technique confirm the predicted reduction of the group velocity, using a curved parallel plate waveguide, as a function of the varying aperture size of the guide. We also present the design of a proof-of-principle experiment based on the bunch parameters available at the UCLA PEGASUS laboratory. With a 10 MV m⁻¹ THz peak field, our simulation model predicts compression of a 6 MeV 100 fs electron beam by nearly an order of magnitude and a significant reduction of its initial timing jitter.

Introduction

The unique properties of radiation in the THz frequency regime have made it an invaluable tool for imaging and spectroscopy [1]. Until recently, the progress of research on further THz applications in the accelerator community has been limited by the attainable peak field, but with improved generation techniques, such as pulse-front tilted optical rectification [2, 3], partitioned nonlinear organic crystals [4], and photoconductive terahertz optoelectronics [5], THz sources have become an attractive tool for accelerator science. In particular, for beam phase space manipulation, the THz frequency range offers a unique middle ground between the high accelerating gradients of laser frequencies and the broad phase-acceptance window of radio frequency (RF) waves. For example, the strong energy chirp imparted to an electron bunch in an X-band structure, like the one used for phase space linearization at LCLS [6], could be accomplished by a THz field that is over fifty times smaller because of the higher frequency. At the same time, where laser coupling in a typical free electron laser (FEL) results in a train of microbunches, a THz-driven FEL interaction would capture and compress the entire beam in a single ponderomotive bucket.

To couple the THz wave to the longitudinal beam momentum, one can induce a longitudinal component in the field or a transverse component in the electron trajectory. Given excitation of the appropriate mode, a dielectric or corrugated waveguide structure can provide a longitudinal field for direct coupling to the beam, as demonstrated in [7] for low energy electrons. In an inverse free electron laser (IFEL), a series of alternating-polarity dipole magnet pairs (an undulator) is used to produce transverse oscillations in the beam trajectory to enable ponderomotive coupling with a transverse THz field. While the peak field available from today’s THz sources continues to improve, it remains the limiting factor for most THz accelerator applications. The IFEL...
coupling scheme has unique advantages which allow it to harness the peak field of a single cycle pulse more effectively, as we will see below, and is the basis for our THz-based beam manipulation technique.

For a thorough discussion of this THz-driven IFEL scheme, including simulations, we have chosen parameters suitable for a proof-of-concept experiment at the UCLA PEGASUS laboratory to demonstrate the compression of a low-charge electron bunch. A single Ti:Sapph laser source will be used to generate an intense, nearly single-cycle THz wave through optical rectification while simultaneously driving a 1.6 cell S-band photocathode gun. The ponderomotive force produced by the undulator and high-gradient THz fields induces a strong energy chirp on the electron bunch if the beam velocity is synchronous with the ponderomotive wave phase velocity and the phase of the ponderomotive bucket is centered on the arrival time phase of the bunch. When this happens, the induced energy chirp can be converted into longitudinal compression after a drift section. To sustain the IFEL-type interaction over the length of the undulator, the group velocity of the THz pulse will be matched to the propagation of the beam using a curved parallel plate waveguide (CPPWG) to produce a ‘zero-slippage’ resonance condition. This same set up is applicable also for reducing bunch jitter. When the phase of the ponderomotive bucket is centered on the average arrival time of the electron bunches, the induced energy chirp works to accelerate late bunches and decelerate early bunches, reducing the time-of-arrival jitter of the electron bunches relative to the laser pulse used to generate them. This effect is relevant for all applications involving laser–electron interactions which have tight time tolerances such as pump-probe techniques [8], inverse Compton scattering sources [9] and external injection into advanced accelerators [10].

This THz compressor setup could easily be re-purposed to perform a different THz-driven beam phase space manipulation. With the excitation of an odd symmetric waveguide mode it will be possible to use the THz field to streak the beam, that is, to impart to the electrons a transverse kick correlated with their longitudinal coordinate. After a drift section, the transverse kick maps the longitudinal profile of the beam to the transverse plane, which can then be imaged using a standard fluorescent screen. The use of an odd mode interaction within an undulator was first proposed by Zholents and Zolotorev [11] and recently discussed by Andonian et al [12]. Preliminary experimental results have confirmed the interaction using a CO2 laser. The wavelength of the CO2 laser is generally small relative to the length of the electron beam, resulting in a snake-like transverse streak pattern along the length of the beam that requires an additional orthogonal kick from an RF deflector to discern the series of streaked sections. The longer wavelength of THz radiation would drastically increase the acceptance of an individual transverse streak, long enough to capture the entire bunch in one section when its length is shorter than half a wavelength.

In principle, the scheme discussed above could find application in the design of a THz-based accelerator. Nevertheless, the efficiency of THz generation is still relatively low (on the order of a percent in the nonlinear downconversion process from the laser) and further hindered by low (also below a few percent) wall-plug efficiency of short pulse laser systems. Multiple groups are working towards optimizing the optical rectification process in order to increase the THz wall-plug efficiency by a few orders of magnitude and promote THz as a viable source for particle accelerators [4, 13, 14]. On the other hand, state-of-the-art laser-driven THz sources already have a lot to offer to accelerator science wherever efficiency is not a concern, as with beam phase space manipulations such as longitudinal compression, energy dechirping, and transverse streaking that are considered in the present article.

This paper is organized as follows. We begin with a discussion of the advantages of the IFEL ponderomotive coupling and introduce the ‘zero-slippage’ interaction. We derive analytical predictions for the energy and momentum modulations required for beam compression and high-streaking-speed THz-based temporal diagnostics and discuss their implementation using select modes in a CPPWG. We then show experimental results demonstrating the dispersive properties of the CPPWG, showing the feasibility of the group-velocity matching of the THz pulse with the electron beam. An electro-optic sampling (EOS) technique is used to characterize the dispersed radiation pulse profile. We finally discuss a proof-of-principle experiment to demonstrate this novel scheme and present simulation results of the zero-slippage guided-THz IFEL interaction using the beam parameters at PEGASUS laboratory.

**Zero-slippage IFEL interaction**

The main advantage of transverse ponderomotive coupling using an undulator magnetic field (commonly referred to as FEL or IFEL coupling) with respect to longitudinal ‘slow-wave’ (Cherenkov Smith–Purcell) coupling in a dielectric or corrugated waveguide structure is related to the transverse acceptance of the interaction region. This has been discussed in detail in the extensive comparison of laser-to-beam coupling schemes compiled in [15] which we briefly review here to highlight the result. In order to enable efficient energy transfer in an IFEL interaction between the electrons and a wave of frequency $\omega$ and wavenumber $k_z$, the phase synchronisation condition
\[ \frac{\omega}{c \beta_z} = k_z + k_u \]  

must be satisfied, where \( \beta_z \) is the normalized longitudinal component of the electron velocity in the undulator, \( k_z \) is the wavenumber describing the periodicity of the undulator, and for free-space propagation \( k_z = \omega/c \). For longitudinal coupling, the dielectric or the structure corrugation period are chosen so that there exists one waveguide mode that satisfies the phase synchronism condition \[ \frac{v}{v_{g1}} = k_z. \]

For both IFEL and longitudinal coupling schemes, the periodic magnetostatic field or slow radiation mode decays, in terms of the distance, \( x \), away from the magnetic field source or slow wave structure wall, like \( e^{-k_T x} \). The decay constant, \( k_T \), is determined by the geometry of the periodic field source or slow wave structure and by the interaction frequency \( c/\lambda \). Comparing the cases of transverse coupling in an undulator and longitudinal coupling with the TM mode in a slow-wave guiding structure we have

\[
\begin{align*}
  k_L = \begin{cases} 
    k_{\omega} & \text{transverse coupling} \\
    (k_z^2 - \omega^2 \gamma^2)^{1/2} & \text{longitudinal coupling}.
  \end{cases}
\end{align*}
\]

For the case of transverse ponderomotive coupling in an undulator, \( k_L \) must be equivalent to the undulator wavenumber \( k_i \) to satisfy the magnetostatic Laplace equation. In the case of longitudinal coupling with a TM mode, \( k_L \) should satisfy the Helmholtz equation. We can define an interaction range, \( 1/2k_L \), in which there is strong coupling for beam-wave interaction. Under the condition of phase synchronism (1), this interaction range can be expressed in terms of the beam Lorentz factor and normalization by the wavelength for both schemes:

\[
\frac{1}{2k_L} = \lambda \frac{1}{4\pi} \left\{ \begin{array}{ll}
  \frac{\beta_z}{1 - \beta_z} & \text{transverse coupling} \\
  \beta \gamma & \text{longitudinal coupling}.
\end{array} \right.
\]

In the non-relativistic limit, the interaction ranges have equivalent magnitudes for both manipulation techniques, but as one approaches the highly relativistic limit the interaction range is proportional to \( 2 \gamma^2 \) for the transverse coupling technique and to \( \gamma \) for the longitudinal technique. This is an important point when the wavelength of the accelerator is shrunk by two or three orders of magnitude from conventional radiofrequency to the THz range. From this analysis, we find that, for relativistic beams, the cross-section of the beam accelerated by the THz wave can be significantly larger in the undulator coupling case.

These considerations assumed dispersion-free vacuum propagation of the radiation for the undulator coupling, but yield equivalent results if one chooses to use a waveguide TE mode for the radiation in the undulator coupling. Note also that the TE modes used for transverse coupling typically suffer from less attenuation within the guiding structure than the TM modes in a dielectric or corrugated waveguide structure [15, 16]. For these reasons the use of a wiggler structure interaction scheme (IFEL) is advantageous over a slow wave structure interaction for the purpose of bunching relativistic beams. A representation of the guided-THz IFEL set-up is shown in figure 1, in which the vertical THz field, propagating in the TE\(_{01}\) mode, couples to the vertical oscillations of the beam trajectory.

The IFEL resonance condition for free-space interaction assumes that the radiation slips ahead of the particles by an integer number of wavelengths for every undulator period to maintain phase synchronism along the undulator. The near single-cycle THz pulse produced by optical rectification precludes the use of this slippage mechanism to keep phase synchronism, because the entire radiation pulse may slip away from the electron bunch before the end of the interaction length. Therefore, in order to sustain an interaction between a near single-cycle THz pulse and electron beam in the undulator, one must operate in the ‘zero-slippage’ condition, in which a waveguide is used to match the THz group velocity to the average longitudinal speed of the electron bunch [17, 18].

Group and phase velocity synchronism occur simultaneously when the dispersion curve of a waveguide mode, \( k = k_{\omega q}(\omega) \) with mode index \( q \), and the displaced dispersion relation of the e-beam modulation wave, \( k_z = \omega/c \beta_z \), intersect tangentially at the THz frequency, \( \omega_{0b} \), as shown in figure 2. The group velocity synchronism condition, stated as

\[
\omega = \frac{d\omega}{dk_z} \bigg|_{\omega_{0b}} = c \beta_z,
\]

is satisfied for frequencies at which the curves in figure 2 have the same slope, while the phase synchronism condition is satisfied for frequencies at which the curves intersect. For the case of tangential intersection, group and phase velocity synchronism are achieved at \( \omega = \omega_{0b} \), and nearly so in a wide range of frequencies around \( \omega_{0b} \). Simultaneous solution of both conditions, (1) and (4), can be found explicitly when the dispersion equation is expressed in terms of the waveguide cutoff frequency, \( \omega_{c0} \).
Substitution into equation (1) produces a quadratic equation with two solutions, corresponding to two intersections of the beam line with the mode dispersion relation curve of figure 2. In conventional FEL configurations, there are always two intersections, and one usually operates at the higher frequency intersection point. For the tangential intersection point satisfying conditions (1) and (4), the two solutions of the quadratic equation must be degenerate. This results in

\[ \omega_0 = \gamma_z \omega_{eq} = \frac{\gamma Z}{\beta_z c k_{in}} \] (6)

where

\[ \gamma_z = (1 - \beta_z^2)^{-\frac{1}{2}} = \frac{\gamma}{\sqrt{1 + K^2/2}} \] (7)

Here \( \beta_z = v_z/c \) is the average axial velocity of the beam, and \( K = \frac{\omega_{in}}{k_{in}} \) is the undulator parameter, where \( B_u \) is the magnetic field amplitude of a periodic linear undulator. Note that in the relativistic limit,

\[ \omega_0 = \gamma_z^2 c k_u \] (8)

is exactly half the frequency of the FEL synchronism frequency in free space propagation.

To summarize, in order to maximize the efficiency of the energy exchange between the near single-cycle THz wave and the electron beam, it will be necessary to choose the undulator and waveguide parameters to meet the ‘zero-slippage’ condition (satisfying both equations (1) and (4)) which will keep the beam in contact with the maximum available driving field for the entire length of the interaction.
THz pulse description

As the broad spectrum, near single-cycle THz pulse undergoes dispersion within the guide, the amplitude envelope may become less sharply peaked, relaxing this velocity-matching condition. This effect needs to be carefully considered when looking at the frequency-dependent radiation pulse evolution in the waveguide.

To be more quantitative, we start by assuming the incident field to be well matched to the waveguide so that only a single mode $q$ is excited. For a smooth waveguide, such as the parallel plates with which we will be working, we may assume the transverse profile is dispersionless and write the THz field propagating in mode $q$ as $\mathcal{E}_{q,1}(x, y) \Psi(z, t)$. We can then write the time dependent term, $\Psi(z, t)$, of the electric field as a Fourier integral in order to explicitly account for the frequency-dependent evolution of the spectral components.

$$\Psi(z, t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} c_q(z, \omega) e^{ik_q(\omega)z - i\omega t} d\omega.$$  

(9)

The $c_q(z, \omega)$ coefficients are determined by the Fourier transform of the field at the entrance, $z = 0$. Using the expanded expression for the propagation wavenumber around the peak frequency of our source,

$$k_q(\omega) = k_q(\omega_0) + \frac{dk_q}{d\omega}|_{\omega = \omega_0} (\omega - \omega_0) + \Delta k_q(\omega, \omega_0),$$  

(10)

where $\frac{dk_q}{d\omega}|_{\omega = \omega_0}$ is defined to be the inverse of the group velocity, $1/g_q$, we can gain insight into the evolution of the pulse envelope. $\Delta k_q(\omega, \omega_0)$ represents terms of second order and higher in $\omega - \omega_0$. Inserting this expansion into equation (9) and factoring the $\omega$-independent terms out of the integrand, we have

$$\Psi(z, t) = e^{ik_q(\omega_0)z - i\omega t} \frac{1}{2\pi} \int_{-\infty}^{\infty} c_q(0, \omega) e^{-i(\omega - \omega_0)(t - \frac{z}{g_q}) + i\Delta k_q(\omega, \omega_0)z} d\omega.$$  

(11)

We can interpret the form of this description of the THz pulse in terms of a sinusoidal carrier wave that accumulates phase $k_q(\omega_0)z$, propagates with constant phase velocity $\omega_0/k_q(\omega_0)$, and is modulated by a time-dependent envelope, which we will now refer to as $\Psi_{\text{env}}(t - \frac{z}{g_q}, t)$. The full expression for the field is

$$E(\mathbf{r}, t) = \text{Re}[\mathcal{E}_{q,1}(x, y) \Psi(z, t)] = \text{Re}[\mathcal{E}_{q}(x, y)\Psi_{\text{env}}(t - \frac{z}{g_q}, t)e^{ik_q(\omega_0)z - i\omega_0 t + i\phi_q}].$$  

(12)

Distortionless propagation

The initial envelope of the radiation pulse propagates nearly at the group velocity, but it may get distorted along the way if the higher order phase expansion terms in (10), beyond the linear term, are non-negligible. The envelope function becomes time-dependent when the higher order terms are negligible, indicating that the pulse propagates without distortion.

For a guide of length $L$, we can estimate a range of frequencies in which the higher orders of the phase expansion can be neglected by enforcing the condition $|\Delta k_q(\omega, \omega_0)L| \ll \pi$. Under the assumption that only the second order term in the Taylor expansion makes a significant contribution, the range of frequencies that satisfies the condition is

$$|\omega - \omega_0| \ll \frac{2\pi}{DL},$$  

(13)

where $D$ is the curvature parameter, $D = \frac{dk_q}{d\omega}|_{\omega_0}$. For the waveguide used to produce the tunable group-velocity results presented below, the frequency limit for distortionless propagation is 0.27 THz from the peak frequency. The spectrum of the THz pulse produced at UCLA includes a wider range of frequencies, as shown in figure 3(b), requiring numerical integration to track the dispersion of the higher order terms in the time-dependent envelope description.

For frequencies within the range defined by equation (13), the waveguide-moderated beam manipulation offers not only the enhanced efficiency of a ‘zero-slippage’ interaction, but also the potential for distortionless propagation. Under these conditions, the available chirping gradient for beam manipulation is not attenuated by dispersive pulse broadening, thereby extending the interaction length over which strong coupling is possible.

An alternative visualization of the zero slippage interaction can be obtained by a numerical integration of equation (11) and examining the distribution of the zero crossings of the THz waveform as it evolves inside the waveguide. In figure 4(b), the relative temporal positions, $\Delta t$, of each zero crossing within the THz pulse profile are plotted as a function of longitudinal position as the waveform propagates within the waveguide. $\Delta t$ is calculated in the rest frame of a 6 MeV bunch, so that the trajectory of the bunch, indicated by the pink line, appears stationary in time, while the zero crossings slip past. This plot gives a visual description of what it means
to satisfy the phase resonance condition and the ‘zero-slippage’ condition. The intersections between the trajectory of the electron bunch and the zero crossings of the pulse occur at distances given by the undulator period length, so that the relative phase between the electron oscillation and the field stays constant. The green markers show the time stamp of the peak field magnitude; an electron trajectory that follows of the peak field is group velocity matched.

**Phase space manipulation**

**Bunch compression.** The energy modulation imparted by a guided-THz IFEL interaction can be derived from the work–energy equation

\[
\frac{d\gamma}{dt} = -\frac{e}{mc^2} E(\mathbf{r}_e(t), t) \cdot \mathbf{v}_e(t).
\] (14)
Given the sinusoidal magnetostatic field of a linear undulator in the \( x \)-direction, the resulting electron velocity is

\[
\mathbf{v}_{\gamma}(z) = \text{Re} \left[ -\frac{K_e}{\gamma} e^{-ik_\perp z} \right] \hat{y}, \tag{15}
\]

Coupling between this transverse motion and a vertical THz field produces an energy change

\[
\Delta \gamma(t) = \frac{eK}{2\gamma mc} \text{Re} \left[ \mathcal{E}_{\perp} \int_{t_0}^{t} \Psi(z(t'), t') e^{ik_\perp z(t')} dt' \right]. \tag{16}
\]

That can be used for bunch compression, with the additional effect of improving timing synchronization to the external laser.

**Streak Diagnostic.** To calculate the transverse deflection that can be induced by this IFEL-type interaction, we begin from the force equation

\[
\frac{dp_y}{dt} = -e (\mathbf{E}_y(x, z, y) + v_{y0} \hat{y} \times \mathbf{B}_z(x, z, y) + v_\perp \times \hat{e}_z \mathbf{B}_y(x, z, y)). \tag{17}
\]

For the case of deflection within the plane of the wiggling electron trajectory, \( y-z \), the resulting expression is

\[
\frac{dp_y}{dt} = -e\alpha_q \text{Re} [\mathcal{E}_{\perp}(x, y) \Psi(z, t)], \tag{18}
\]

where

\[
\alpha_q = \begin{cases} 
1 - \beta_0 \frac{q}{k_0}, & \text{TE mode} \\
1 - \beta_0 \frac{k_0}{q}, & \text{TM mode} 
\end{cases} \tag{19}
\]

and the magnetic field profile in the waveguide is related to the electric field profile by

\[
\hat{e}_z \times \mathbf{B}_q = \frac{-\mathcal{E}_{\perp}}{\epsilon} \left\{ \begin{array}{ll}
k_{aq}/k_0, & \text{TE mode} \\
k_0/k_{aq}, & \text{TM mode}. \end{array} \right. \tag{20}
\]

The velocity matching condition requires \( \beta_z \approx \beta_y \), and the group velocity in the waveguide is given by \( \frac{k_0}{k_q} \), as follows from the waveguide property \( \gamma \approx \gamma_{ph} = c^2 \). Equation (19) then evaluates to

\[
\alpha_q = \begin{cases} 
\frac{1}{\gamma_z^2}, & \text{TE mode} \\
0, & \text{TM mode}. \end{cases} \tag{21}
\]

We see that, when sustaining a ‘zero-slippage’ interaction for deflection in the \( y \)-direction, a TE mode is required for the mode-dependent parameter \( \alpha_q \) to be non-zero.

For the antisymmetric mode profile necessary for transverse deflection in the \( y \)-direction, we can approximate the profile near the axis as

\[
\mathcal{E}_q(x, \chi) \approx \hat{e}_z \frac{\partial \mathcal{E}_{\perp}}{\partial y} \mid_{y=0} \chi(t) = -\hat{e}_z \mathcal{E}_{\perp} \frac{iKe^{-ik_\perp z}}{\beta_0 \gamma k_0}. \tag{22}
\]

Taking only the phase-synchronous term, the change in \( p_y \) becomes

\[
\frac{dp_y}{dt} = \frac{e\alpha_k}{2\beta_0 \gamma k_0} \text{Re} [i\mathcal{E}_{\perp} \Psi(z, t) e^{ik_\perp z}]. \tag{23}
\]

The resulting angular deflection is

\[
\Delta \Theta_y(t) = \frac{p_y(z(t), t)}{mc\gamma \beta_0} = \frac{e\alpha_k}{2\beta_0^2 \gamma^2 mc k_0} \text{Re} \left[ i\mathcal{E}_{\perp} \int_{t_0}^{t} \Psi(z(t'), t') e^{ik_\perp z(t')} dt' \right]. \tag{24}
\]

We can calculate the deflection in the plane perpendicular to the wiggling electron trajectory from the longitudinal coupling in the third term of equation (17). In this case, the angular deflection

\[
\Delta \Theta_y(t) = \frac{eK}{2\beta_0^2 \gamma^2 m} \text{Re} \left[ \mathcal{E}_{\perp} \int_{t_0}^{t} \Psi(z(t'), t') e^{ik_\perp z(t')} dt' \right] \tag{25}
\]

is optimized for an on-axis \( B \)-field.

As we see in equations (16), (24) and (25), the THz mode profile in the vicinity of the beam determines the type of phase space modulation achieved by the IFEL. Using the distortionless propagation condition to simplify calculations, we proceed with analytical predictions for these phase space modulations. Given the description of
the field in equation (12), the time-dependent integral appearing in equations (16), (24) and (25) becomes
\[ \int_{t_0}^{t_f} \Psi(z(t'), t') e^{i k z(t')} dt' = \int_{t_0}^{t_f} f_{\text{env}}(t' - z(t')/v_E) e^{i(k_0 t' + k_0 t - \omega_0 t') dt'}. \] (26)
Substituting \( z = v_z (t' - t_0) \) for an electron entering the wiggler on-axis with velocity \( v_z \) at initial time \( t_0 \) and assuming the velocity matching condition, \( v_z = v_g \), we have
\[ \int_{t_0}^{t_f} \Psi(z(t'), t') e^{i k z(t')} dt' = f_{\text{env}}(t_0) e^{-i \omega_0 t_0} \int_0^{L/t_0} e^{i(k_0 t + k_0 t - \omega_0 t') dt'}. \] (27)
after propagation a distance \( L \) at time \( t_f \). The beam modulations described by equations (16), (24) and (25) are then given by
\[ \Delta \gamma(t_0) = \frac{eKL \epsilon E_{\text{env}}(t_0)}{2 \gamma mc} \sin \frac{\theta t_0}{2} \cos \left( \omega_0 t_0 - \phi_0 + \frac{\theta t_0}{2} \right), \] (28)
\[ \Delta \Theta_x(t_0) = \frac{eKL \epsilon^* E_{\text{env}}^*(t_0)}{2 \beta^2 \gamma^2 mc^2 k_0} \sin \frac{\theta t_0}{2} \sin \left( \omega_0 t_0 - \phi_0 + \frac{\theta t_0}{2} \right), \] (29)
\[ \Delta \Theta_y(t_0) = \frac{eKL \beta E_{\text{env}}(t_0)}{2 \beta^2 \gamma mc} \sin \frac{\theta t_0}{2} \cos \left( \omega_0 t_0 - \phi_0 + \frac{\theta t_0}{2} \right), \] (30)
respectively, where \( \theta = \frac{\omega_0}{v_E} - (k_0 + k_u) \). In each case, we can see that the desired energy chirp or angular spread is maximized when \( \theta = 0 \), which is the condition for phase synchronism. For compression and streaking, particles should be injected at phase \( \omega_0 t_0 - \phi_0 \) corresponding to a zero-crossing in the waveform so that the interaction would impart an energy or transverse momentum chirp on the beam distribution.

**THz waveguide**

Although higher frequencies can offer larger acceleration gradients, the size of the guiding structures necessary for a waveguide-controlled interaction becomes prohibitively small for co-propagation of electrons and laser. The length scales necessary for a THz guiding structure are large enough to permit alignment of the electron beam without clipping. For reasonably low charge, wakefield effects in the guiding structure are negligible. The guided propagation has the added benefit of preserving the field intensity over the length of the guide rather than operating in the diffraction limited regime where high intensity would be particularly difficult to maintain for THz frequencies.

We have selected the CPPWG, cross section shown in figure 5 (a), to control the propagation of the THz pulse. This choice was motivated by the investigation of guiding structures for a THz FEL oscillator in [16] and [20]. The parallel plate structure has small ohmic losses relative to more conventional waveguides and offers the unique advantage of variable plate spacing. This dynamic control of the waveguide parameters allows for tuning of the guide’s dispersive properties to optimize the group velocity of the THz pulse for our application. The gap between plates results in some attenuation due to diffraction, but this effect is mitigated by adding a curvature to the plates which focuses the THz within the waveguide channel. Attenuation due to diffraction diminishes with increasing frequency, as shown in figure 5 (b). This feature results in a decrease in total attenuation with increasing frequency, unlike conventional waveguides. While decreasing the plate spacing reduces the group velocity to match the propagating beam, it also increases the cut-off frequency within the waveguide, see
figure 5(c), and increases the rate of pulse broadening within the guide. The utility of the THz pulse is eventually limited by the dispersive pulse broadening that reduces the available field gradient for chirping the beam as the pulse propagates down the length of the guide.

The THz mode profile in the CPPWG is determined by the dispersion relation giving the transverse wavenumber, \( k_{mn} \), and longitudinal wavenumber, \( k_z \), at frequency, \( k_0 \omega \),

\[
k_{mn} = \frac{1}{b} \left( n\pi + (2m + 1)\tan^{-1} \left( \frac{b}{\sqrt{2Rb - b^2}} \right) \right),
\]

\[
k_z = \sqrt{k_0^2 - k_{mn}^2}.
\]

(31)

The form of the longitudinal field, \( \Phi_{mn} \), corresponding to \( H_z \) for TE modes and \( E_z \) for TM modes, is given in terms of the Hermite polynomials, \( H_m \), as follows [21]

\[
\Phi_{mn} = \frac{e^{-\frac{1}{2}x^2}}{\sqrt{\alpha_{mn}(y)}} \begin{pmatrix} 2\beta_{mn}x \end{pmatrix}_{\sqrt{\alpha_{mn}(y)}} \left( \cos \left( \frac{\beta_{mn}y}{\sqrt{\alpha_{mn}(y)}} \right) \right) \left( \sin \left( \frac{\beta_{mn}y}{\sqrt{\alpha_{mn}(y)}} \right) \right) \times \left( k_{mn}y + \frac{2\beta_{mn}xy^2}{k_{mn}\alpha_{mn}(y)} - \left( m + \frac{1}{2} \right)\tan^{-1} \left( \frac{2\beta_{mn}y}{k_{mn}} \right) \right) e^{\pm ik_z z},
\]

(32)

where

\[
\alpha_{mn}(y) = 1 + \frac{\beta_{mn}^2 y^2}{k_{mn}^2},
\]

\[
\beta_{mn} = \sqrt{\frac{k_{mn}}{\sqrt{2Rb - b^2}}},
\]

(33)

and \( R \) and \( b \) are the plate curvature and spacing of the CPPWG. The transverse \( E \)-field components are then given by

\[
E_x = -\frac{i}{k_{mn}^2} \left( k_z \frac{\partial E_z}{\partial x} + \omega \mu_0 \frac{\partial H_x}{\partial y} \right),
\]

\[
E_y = -\frac{i}{k_{mn}^2} \left( k_z \frac{\partial E_z}{\partial y} - \omega \mu_0 \frac{\partial H_x}{\partial x} \right).
\]

(34)

### CPPWG Modes

The \( TE_{01} \) mode of the CPPWG is optimal for ponderomotive coupling to the beam to produce an energy chirp for bunch compression. The transverse profile, shown in figure 1, has a peak field on-axis with polarization parallel to the long axis of the CPPWG cross-section. This orientation allows maximum clearance for the parallel wiggle of the electron trajectory within the guide. When the \( E \)-field is linearly polarized in the direction parallel to the plate surfaces, direct coupling of a THz pulse with a Gaussian profile excites the \( TE_{01} \) mode of the CPPWG with 80% efficiency.

Selection of the optimal CPPWG mode for streaking depends on the coupling efficiency and the corresponding deflection parameter calculated from equations (29) and (30). Our primary candidates are the \( TE_{11} \) mode, with the antisymmetric mode profile necessary for deflection in the \( y \)-direction, see figure 6(a), and the \( TE_{20} \) mode with a longitudinal \( B \)-field, peaked on-axis, for deflection in the \( x \)-direction, see figure 6(c).

In the \( CO_2 \) streaking experiments [12] performed by Andonian et al, two mirror halves shifted by 1/4 of the wavelength produced a phase shift in the field profile in order to excite the desired mode. Using a similar method we will be able to generate the correct phase shift in a limited spectra region around the central peak frequency of our THz source, constraining our capacity for effective coupling of the entire THz spectrum into the \( TE_{11} \) or \( TE_{20} \) mode. However, due to the limited bandwidth of the interaction, the spectral components of the radiation which are more strongly coupled to the particles will be propagating in the waveguide in the desired mode configuration, as the shift at the edge of the interaction bandwidth (0.6 – 1 THz) is only 30° off from the desired 180°. With this modification of the coupling scheme, the experimental hardware from the compression project could be readily adapted for the THz streaking application.

A comparison of transverse coupling with the \( TE_{11} \) mode and longitudinal coupling with the \( TE_{20} \) mode, using the undulator parameters fixed by the compression experiment, can be found in table 1. For each mode, the coupling efficiency was calculated over a range of incoming transverse profiles that could be produced at the peak THz frequency through a combination of mirror offset and non-reflective coating. For these calculations we used the parameters from a proof-of-principle experiment which will be discussed in a later section of the
Given these parameters, other modes, like the TE02 mode, may provide greater angular deflection as predicted by equations (29) and (30), but the lack of suitable coupling schemes makes them unpractical to implement.

Assuming no position-angle correspondence in the initial beam distribution, the effective timing resolution is defined, for deflection in the x, or y, direction, as [12]

$$\Delta t = \frac{\epsilon_{n,x,y}}{\gamma \sigma_{x,y} \Delta \phi_{x,y,\text{max}} c k_x},$$

where $\epsilon_{n,x,y}$ is the normalized emittance and $\sigma_{x,y}$ is the beam size.

Despite the low field magnitude from our THz source, we can achieve high temporal resolution, because it scales with the wavelength of the modulating field. The resolution of our THz streaking method would be comparable to current femtosecond-scale timing diagnostics like the X-band deflector, but with the added advantage of timing synchronization to an external laser pulse. Further improvements, such as an increase in THz peak field or an extension of the IFEL interaction length, could push this THz streaking diagnostic to sub-femtosecond resolution.

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**Table 1. THz streaking parameters.**

|                  | TE11 mode | TE10 mode |
|------------------|-----------|-----------|
| Coupling efficiency | 0.278    | 0.638     |
| Resonant plate-spacing | 2.65 mm | 1.95 mm |
| Resonant energy   | 7.703 MeV | 6.225 MeV |
| Angular deflection| 3.3 mrad | 7.9 mrad |
| Effective angular deflection | 0.9 mrad | 5.0 mrad |
| Timing resolution | 30 fs    | 7 fs      |

(for $\epsilon_{n,x} = 0.1 \text{ mm mrad}$, $\sigma_y = 45 \mu\text{m}$)

**Table 2. Simulation Parameters.**

| Parameter                   | Value           |
|-----------------------------|-----------------|
| Bunch energy                | 6 MeV           |
| Bunch charge                | 1 pC            |
| Energy spread               | $3 \times 10^{-4}$ |
| Bunch length                | 100 fs rms      |
| Undulator period            | 3 cm            |
| Undulator parameter, K      | 1.12            |
| # of undulator periods      | 8               |
| CPPWG spacing, b            | 2.06 mm         |
| Plate curvature radius      | 2 mm            |
| Peak frequency              | 0.84 THz        |
| Peak THz field              | 10 MV m^-1      |
Guided THz propagation

In this section we discuss our preliminary experiments aimed at demonstrating the control of the group velocity and of the dispersive properties of the CPPWG structure. We use optical rectification in stoichiometric lithium niobate (sLN) to generate picosecond scale THz pulses from a Ti:Sapph laser centered at 800 nm with 30 nm bandwidth. To enhance the conversion efficiency from IR to THz, the intensity front of the laser pulse is tilted using a diffraction grating and imaged onto the sLN with a focusing lens [2, 3], producing a peak field of up to 4.6 MV m⁻¹ after focusing with a pair of off-axis parabolic mirrors. With the addition of a cooling chamber to improve the conversion efficiency at the sLN crystal and optimized focusing, we can expect the peak field to increase by at least a factor of two [22].

Measurements of the temporal field profile are conducted with EOS in a ZnTe crystal of thickness $d = 0.5$ mm. Within this nonlinear optical crystal, a THz induced rotation of the fast and slow axes changes the relative intensity between the vertical and horizontal polarization components of an IR probe pulse. A balanced detection scheme, shown in figure 7, utilizes a quarter waveplate and Wollaston prism to separate these components onto two photodiodes. The original THz field is then calculated in terms of the intensity difference [23].

The IR probe is aligned to the THz focus using a pyroelectric detector to ensure spatial overlap within the ZnTe. The CPPWG plates are then aligned to the IR probe line, to ensure coupling of the focused THz pulse to the $TE_{01}$ mode of the CPPWG waveguide. For measurements of the THz profile after the waveguide, the IR probe travels within the guide, before interacting with the THz in the ZnTe crystal placed at the exit. The IR probe pulse is undistorted in the guide, because its wavelength is much smaller than the waveguide dimensions. The IR probe pulse alignment was chosen to sample the peak on-axis field, corresponding to the region in which the IFEL interaction with the oscillating beam will occur. For the simulation parameters listed in table 2, the

**Figure 7.** Diagram showing the path of the THz pulse as it is collimated then focused into the CPPWG and the path of the IR for the EOS measurement. The planned electron trajectory is shown in pink.
maximum amplitude of the oscillating electron bunch trajectory is 457 μm. At this distance, the field magnitude of the TE₀₁ mode profile has decreased by only 6.5% from the on-axis maximum.

In figure 8, we show measurements of the THz longitudinal pulse profile after a 15 cm CPPWG for three different plate spacings. The plates, made from aluminum with a 3 mm radius of curvature and 4.25 mm thickness, were manufactured at UCLA using conventional machining techniques. The simulated pulse profile for each plate spacing, shown as the red dashed line in figure 8, was calculated from an initial pulse waveform determined by EOS measurements at the entrance to the guide, see figure 3(a), with initial field magnitude scaled to reflect imperfect coupling and neglecting losses due to diffraction within the guide. For the 1.5 mm plate spacing, there is good agreement between the measured and predicted waveforms, as we expect for a pulse that is not severely affected by diffraction losses. For wider plate spacings, the predicted peak field magnitude is larger than the 1.5 mm case, because there is less pulse dispersion in the guide, but in the experiment, the increased attenuation due to diffraction results in a lower field magnitude.

Using EOS measurements of the THz pulse profile at the entrance and exit of the CPPWG, a direct estimate of the group velocity of the pulse within the waveguide can be obtained from the IR probe delay between the peaks of the initial and final pulse envelopes. The envelope of the pulse is extracted from the raw data by factoring out the sinusoidal carrier wave determined by a least-squares fit in the region of the peak field. Experimental results for the group velocity determined using this method are shown in figure 9, along with the corresponding estimates found via application of this method to the simulated pulse propagation. The
theoretical group velocity at the peak frequency of the pulse, shown by the black dashed line in figure 9, provides a reasonable estimate for the expected group velocity for the case of 2.5 and 3.5 mm plate spacings. For the 1.5 mm plate spacing, both measurement and simulation deviate from the group velocity at the peak frequency. This behavior may be explained by the highly nonlinear dispersion at low frequencies which causes a frequency chirp that reduces the contribution of the low frequency components to the pulse envelope peak. The speed of the envelope peak is then determined by the high frequency components, with their correspondingly high group velocities. For larger plate spacings, the region of quasi-linear dispersion extends to lower frequencies, making the effect less significant.

Proof-of-principle experiment and simulations

We now proceed to discuss the design of a proof-of-principle experiment to demonstrate the application of the zero-slippage ponderomotive interaction to compress and synchronize a relativistic electron beam from an RF photoinjector. A layout of the guided-THz IFEL compressor as it would appear on the PEGASUS beamline is shown in figure 10. The waveguide parameters chosen for simulation of the guided-IFEL interaction correspond to a theoretical group velocity of $\beta_g = 0.994$ at the measured peak frequency of the PEGASUS THz source, within the range of the experimental results from our tunable CPPWG presented above. The simulated electron bunch parameters are well within the range that can be routinely generated at PEGASUS laboratory [24, 25]. With these constraints, the undulator parameters have been calculated from equation (1), the resonance condition modified for interaction within a waveguide, such that the average longitudinal beam velocity matches the THz group velocity. The undulator and waveguide parameters used for simulation of the guided-IFEL interaction are shown in table 2. In figure 11, we show the co-propagation of the simulated THz pulse and electron beam for this resonant interaction.

Ponderomotive coupling with a near single-cycle THz pulse cannot be well-described by the single frequency approximation used by most FEL simulation codes because of the broad spectral content of the THz pulse. We are currently developing a multi-frequency simulation code, similar to MUFFIN [26], which will track the evolution of the THz pulse, described by a sum of frequency components as in equation (9). The calculation
does not rely on the slowly varying envelope approximation or period averaging and includes the waveguide-
induced dispersion of the THz pulse.

For very low beam charges one can neglect the beam current term in the radiation field evolution (frozen
field approximation). Preliminary results from our multi-frequency code are shown in figure 12, using the
design parameters in table 2. For these results, the THz pulse was modeled by a Gaussian frequency spectrum
peaked at 0.84 THz with a FWHM of 0.75 THz and simulated by 101 spectral points. The peak
field was $-10 \text{MV m}^{-1}$, based on predictions for the cooled sLN conversion efficiency of the PEGASUS optical rectification
set-up. The sample electron beam with rms bunch length 100 fs, shown in figure 12, is compressed to 12 fs.

For optimum compression, the electron beam should be injected in the interaction at the zero crossing of the
THz waveform, so that it will receive a nearly linear energy chirp. In practice, the relative phase of the electron
bunches will be distributed throughout the ponderomotive bucket because of the inherent timing jitter at the
injection. Within the bucket, all electrons with negative phases will increase in energy, while all electrons with
positive phases will decrease in energy. In figure 13, we show a ‘late’ (red) and ‘early’ (green) bunch in the
ponderomotive bucket before and after phase space rotation in the IFEL. With the increase in energy, the ‘late’

![Figure 11. Snapshots of the simulated THz pulse propagation within the waveguide, after 0.5λ_c, 1λ_c, 3.5λ_c, 4λ_c, 7.5λ_c, and 8λ_c, with electron bunch position shown in red. As the system is evolved in terms of the longitudinal position coordinate, the time marked on the x-axis indicates the time required for the corresponding point on the THz waveform to cross the current longitudinal coordinate. The plot window tracks with propagation at the speed of light, therefore the THz pulse and the electron bunch appear to slip further back with each snapshot as they propagate at 0.994c. Note that at the half period intervals the sign of the THz field gradient flips.](image)

![Figure 12. Simulation results showing the longitudinal phase space distribution (a) before the undulator, (b) at the exit of the undulator, and (c) after a 1.03 m drift period.](image)
electrons will travel faster, catching up towards the equilibrium phase over a drift section. Similarly, ‘early’ electrons will travel slower, slipping back towards the equilibrium phase over a drift section. If the phase space distribution of a bunch is far from the resonant phase, it will fall out of the longitudinal acceptance of the compressor and the beam will acquire energy spread without decreasing the bunch length.

Fluctuations of the input energy from the resonant condition would also displace the bunch from optimal positioning within the ponderomotive bucket. Phase space rotation of a bunch with a small energy deviation would result in only slightly less effective bunching and a slight increase in the mean energy offset. In practice however, the arrival time of the electrons would also be affected by the electron beam energy. Using the same initial bunch distribution shown in figure 12 but adding an initial energy offset, and the corresponding accrued timing jitter, we can look at the comparative compression and timing jitter over a range of mean energies, as shown in figure 14. Even with a relatively large energy deviation of 0.2% and timing error of 100 fs, interaction in the THz-driven IFEL achieves a compression ratio of better than 3 and cuts the timing jitter by more than half.

The compression results discussed above are limited primarily by the THz peak field and available space on the PEGASUS beamline. Obviously, increasing the THz field would increase the accelerating gradient of the IFEL interaction and therefore the magnitude of the chirp. Direct improvement of THz generation techniques is an active area of research. In [13], it is predicted that the maximum field attainable via optical rectification in LiNbO$_3$ may be on the order of 100 MV cm$^{-1}$, in the range of 0.3 – 1.5 THz. Even larger THz fields, on the order of 1 GV m$^{-1}$, have been generated using optical rectification in a partitioned nonlinear organic crystal assembly [4]. Difference frequency generation in periodically poled LiNbO$_3$ [27] has also been proposed for obtaining very high peak field.
The higher frequencies characteristic of these techniques do not pose particular challenges, other than the requirement to adjust electron beam energy, undulator parameters and waveguide size to satisfy phase and group velocity matching. At low beam energies, phase matching with the TE$_{01}$ mode can be satisfied for higher THz frequencies without significant changes to the experimental parameters. At higher beam energies, the phase matching condition can be satisfied for the same waveguide parameters and peak undulator field by using longer undulator periods. Note that in the difference frequency generation technique proposed in [27], longer pulses might be available so that the need for zero-slippage interaction is relaxed and a more conventional ponderomotive coupling scheme might be employed.

Conclusion

As the development of THz sources pushes towards higher peak fields, the accelerator community eagerly looks to the potential of THz technology. To efficiently harness a near single-cycle THz field for beam manipulation, the problem of extending the interaction region must be solved. Group velocity matching between a THz pulse and an electron bunch can allow for a sustained ‘zero-slippage’ IFEL interaction. We intend to demonstrate this guided-THz IFEL technique at the PEGASUS laboratory to compress an electron bunch and reduce its time-of-arrival jitter with respect to the laser pulse generating the THz.

Towards this end, we have achieved tunable control of the group velocity of a THz pulse inside a CPPWG. Initial predictions of the 1D multifrequency simulation code that we are developing show bunch compression of up to an order of magnitude. The ‘zero-slippage’ interaction that we utilize for compression can be extended to a transverse deflection mechanism, like the one discussed in [12], without the need for an additional RF deflecting cavity.

Ground-breaking research on structural dynamics at an atomic level has already been accomplished using femtosecond electron diffraction [28]. With sub-fs electron beams it may be possible to delve even deeper, studying electronic motion within atoms [29]. Advances in THz technology are already underway to address this need [30], both for producing and characterizing these beams. With our guided-THz IFEL technique, we may bring beams with energies of a few MeV, capable of single-shot diffraction [31], into this sub-fs regime. The THz streaking technique based on ponderomotive coupling in a guided-THz IFEL could provide unprecedented resolution for longitudinal bunch diagnostics. Looking forward, an IFEL interaction driven by the guided-THz seed could prove to be a powerful tool for amplification of the THz source.

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