A Magnetic Rotator Wind-Disk Model for Be Stars

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Abstract. We consider a Magnetic Rotator Wind-Disk (MRWD) model for the formation of Keplerian disks around Be stars. Material from low latitudes of the stellar surface flows along magnetic flux tubes and passes through a shock surface near the equatorial plane to form a pre-Keplerian disk region. Initially, the density in this region is small and the magnetic field helps to maintain super-Keplerian rotation speeds. After a fill-up time, the density of the disk is significantly larger and the magnetic force in this region becomes negligible compared with the centrifugal force. The material then moves outwards to form a quasi-steady Keplerian disk. During the fill-up stage, the meridional component $B_{m,*}$ of the magnetic field at the stellar surface must be larger than a minimum value $B_{m,*,\text{min}}$. The radial extent of the quasi-steady Keplerian region will be larger when $B_{m,*}$ is larger or when viscosity plays a role. In B-type stars, the values of $B_{m,*,\text{min}}$ are of order 1 G to 10 G. We find that a condition for the formation of shock-compressed disk regions is that in faster rotating stars, the wind speed must be correspondingly larger.

1. Disk Formation Processes

The MRWD (Magnetic Rotator Wind-Disk) model for the formation of Keplerian disks involves two processes. The first is a fill-up process in which material from a star flows along magnetic field lines and passes through a shock surface to form a pre-Keplerian disk region. The magnetic field supplies angular momentum to the wind and assists the flow of material towards the disk region. Initially, the density in the pre-Keplerian region is small and it increases significantly during this process.

The second process occurs after the magnetic force in the disk region becomes small compared to the centrifugal force. Then, super-Keplerian material in the pre-Keplerian region expands to form a quasi-steady Keplerian disk. If there is no viscosity or inflow into the disk region, angular momentum will be conserved. The time at which the second process starts depends on the rotational speed, magnetic field strength and rate of fill-up of the disk region. In some stars, sub-Keplerian wind material may continue to flow into the super-Keplerian disk region. Also, as the angular velocity of the expanding disk approaches a Keplerian distribution, magnetorotational instability (MRI) and viscosity can become important. When there is inflow or viscosity, the radial extent of the quasi-steady Keplerian disk region will be larger. In situations where the magnetic force in the disk is stronger than the centrifugal force at the end of the first process, the disk will continue to maintain its pre-Keplerian structure.
Figure 1 gives a schematic meridional cross-section of a pre-Keplerian disk during the fill-up stage. It shows the magnetic field lines from the stellar surface to the inner and outer end points $X_{\text{inn}}$ and $X_{\text{lim}}$, respectively, of the pre-Keplerian region. In a meridional plane, streamlines of flow from the star to the disk coincide with magnetic field lines. The limiting streamline from $P_{\text{lim}}$ to $X_{\text{lim}}$ separates streamlines that flow into the disk region from the open streamlines that flow away from the star and the disk. Let $X$ be a point on the equatorial plane in the pre-Keplerian region. Let $P$ be the point on the stellar surface that is connected to $X$ by a magnetic field line that loops back to a point $P'$ on the star. If $\omega$ is the angular velocity at $P$ and $P'$, the angular velocity at $X$ during the pre-Keplerian stage will also be equal to $\omega$ (Maheswaran 2003). The ratio of the angular velocity at $P$ to the critical angular velocity at the same point is $\alpha = \frac{\omega R^{3/2}}{GM(1-\Gamma)^{1/2}}$, where $M$ and $R$ are the stellar mass and radius, respectively. $\Gamma$ is the ratio of the continuum radiation force to gravity. For points on the equatorial plane we put $x = r/R$ where $(r, \theta, \phi)$ are spherical polar coordinates. Let $X_{\text{kep}}$ and $X_{\text{esc}}$, respectively, be the points in the pre-Keplerian region at which the rotational velocities are equal to the Keplerian and escape velocities. Then, $x_{\text{kep}} = \alpha^{-2/3}$ and $x_{\text{esc}} = 2^{1/3}\alpha^{-2/3}$.

When the meridional component $B_{m,*}$ of the magnetic field at the point $P_{\text{lim}}$ on the stellar surface is increased, the value of $x_{\text{lim}}$ also increases. The MRWD model requires that a super-Keplerian region be present in the pre-Keplerian disk. Hence, $X_{\text{lim}}$ must be further away from the star than $X_{\text{kep}}$, so that $x_{\text{lim}} > x_{\text{kep}}$. Thus, $B_{m,*}$ must be larger than a minimum value $B_{m,*,\text{min}}$, which is the meridional field strength when $X_{\text{lim}}$ coincides with $X_{\text{kep}}$. Figure 2 schematically shows the different possible locations of $X_{\text{inn}}$ and $X_{\text{lim}}$ in relation to $X_{\text{kep}}$ and $X_{\text{esc}}$. During the second process, material between $X_{\text{kep}}$ and $X_{\text{esc}}$ will move into Keplerian orbits. When there is no viscosity or inflow of wind material, the material beyond $X_{\text{esc}}$ will flow away. If viscosity or inflow becomes important, material in the region between $X_{\text{esc}}$ and $X_{\text{lim}}$ can become part of a quasi-steady Keplerian disk. A pre-Keplerian region having configuration 1 in Figure 2 will yield a quasi-steady Keplerian disk region with maximal radial extent. The optimal model corresponds to configuration 5, where $X_{\text{lim}}$ coincides with $X_{\text{esc}}$ and $X_{\text{inn}}$ coincides with $X_{\text{kep}}$ or is closer to the stellar surface than $X_{\text{kep}}$. This model requires the least magnetic field strength at the stellar surface.
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1. Magnetic Rotator Wind-Disk Model

A magnetic rotator wind-disk model for Be stars is described. The model is used to form a Keplerian disk with the largest radial extent in the absence of inflow or viscosity.

The Keplerian region formed when \( B_{m,\star} \) is only slightly larger than \( B_{m,\star,\text{min}} \) will be in the shape of a ring. For larger values of \( B_{m,\star} \), the radial extent of the Keplerian region will be larger. In an inviscid model without inflow, the Keplerian region has its largest radial extent when \( B_{m,\star} \) equals \( B_{m,\star,\text{opt}} \), which is the field strength required for the optimal model. For values of \( B_{m,\star} \) larger than \( B_{m,\star,\text{opt}} \), there is no increase in the radial extent of the quasi-steady Keplerian region as long as the material beyond \( X_{\text{esc}} \) moves away from the disk region.

When the rotation rate \( \alpha \) of the sector \( P_{\text{lim}}P_{\text{inn}} \) is approximately constant, the inner radius of a quasi-steady Keplerian disk is \( \alpha^{-2/3} R \). Let \( x_{\text{end}} \) be a point on the outer boundary of the quasi-steady Keplerian disk. When there is no viscosity or inflow, we have \( x_{\text{end}} = x_{\text{lim}}^{4/3} \alpha^2 \) when \( B_{m,\star,\text{min}} < B_{m,\star} < B_{m,\star,\text{opt}} \) and \( x_{\text{end}} = 2^{4/3} \alpha^{-2/3} \) when \( B_{m,\star} \geq B_{m,\star,\text{opt}} \). However, if there is flow of wind material into the disk region or if there is an onset of MRI during the second process and viscosity becomes significant, the value of \( x_{\text{end}} \) will be much larger because of a contribution from material between \( X_{\text{esc}} \) and \( X_{\text{lim}} \).

2. Magnetic Field Strengths and Disk Models

Let \( B_{m,\star,\text{circ}} \) be the lower bound derived by Maheswaran & Cassinelli (1988, 1992) for the meridional field to withstand the effects of meridional circulation near the stellar photosphere and let \( t_{\text{circ}} \) be the turnover time of this circulation. Let \( B_{m,\star,\text{mtd}} \) be the minimum surface magnetic field required in the MTD model of Cassinelli et al. (2002). For Be stars with realistic rotation rates we have...

Figure 2. A schematic picture of the different possible locations of the end points \( X_{\text{inn}}, X_{\text{lim}} \) of pre-Keplerian disks in relation to the critical points \( X_{\text{kep}}, X_{\text{esc}} \).
$B_{m, \text{min}} < B_{m, \text{circ}} < B_{m, \text{mtd}}$. Also, $t_{\text{fill}} \ll t_{\text{circ}}$, where $t_{\text{fill}}$ is the fill-up time of the pre-Keplerian disk. Table 1 gives the critical values of magnetic fields for the formation of disks in different stellar models.

Table 1. Critical values of $B_{m, \star}$ for MRWD and MTD type disks in different stellar models with specified rotation rates (Cassinelli et al 2002; Maheswaran 2003)

| Spectral Type | $\alpha$ | $B_{m, \star, \text{min}}$ (G) | $B_{m, \star, \text{opt}}$ (G) | $B_{m, \star, \text{circ}}$ (G) | $t_{\text{circ}}$ (yr) | $B_{m, \star, \text{mtd}}$ (G) |
|---------------|---------|-------------------------------|-----------------------------|-----------------------------|-----------------------|---------------------------|
| O3            | 0.6     | 437                           | 893                         | 139                         | 0.53                  | 22000                     |
| O6.5          | 0.6     | 101                           | 207                         | 175                         | 0.57                  | 5000                      |
| B0            | 0.6     | 33                            | 67                          | 131                         | 0.31                  | 1600                      |
| B2            | 0.5     | 7                             | 13                          | 25                          | 1.1                   | 335                       |
| B5            | 0.45    | 1.4                           | 2.5                         | 3.6                         | 6.7                   | 64                        |
| B9            | 0.4     | 0.8                           | 1.3                         | 0.5                         | 30.6                  | 30                        |

Here, we consider models that are appropriate for stars with different values of $B_{m, \star}$. Although the azimuthal component may also play a role, we do not include it in this discussion.

(a) If $B_{m, \star, \text{min}} < B_{m, \star} < B_{m, \star, \text{circ}}$, quasi-steady Keplerian disks satisfying the MRWD model can be formed. The surface magnetic field will be affected by meridional circulation near the photosphere. This can lead to changes in the disk over time scales of order $t_{\text{circ}}$.

(b) When $B_{m, \star, \text{circ}} < B_{m, \star} < B_{m, \star, \text{mtd}}$, the MRWD model is appropriate. The surface magnetic field will be able to withstand the effects of circulation and persist over time periods that are long compared with $t_{\text{circ}}$.

(c) If $B_{m, \star} > B_{m, \star, \text{mtd}}$, the disks will be of MTD type. If the magnetic field lines from the star thread the disk region and loop back to the star, like field lines of a dipole-type field, the angular velocity distribution in this disk will be the same as that in the region $P_{\text{inn}}P_{\text{lim}}$ of the stellar surface from which the field lines emerge. On the other hand, if the field lines from the star pass through the disk region and travel outwards to large distances from the star without looping back, the rotation law for the disk will be similar to the empirical formula specified in the MTD model of Cassinelli et al (2002).

3. Discussion

The requirement that the disk density should be positive gives the condition $v_n^2 > (B_{\text{disk}}^2 - B_{\text{wind}}^2) / 8\pi r_{\text{wind}}$ for disk formation (Maheswaran 2003). Here, all quantities are evaluated on the appropriate sides of a point $Q$ on $\Sigma_{\text{shock}}$ and $v_n$ is the normal component of the wind velocity. Thus, for given values of the equatorial rotation speed and surface magnetic field strength, the wind velocity must be larger than a critical value for a disk to be formed. Since
this critical value depends on the stellar rotation speed, a faster rotation rate does not ensure the formation of a shock-compressed disk region unless \( v_n \) is correspondingly larger.

The MRWD model does not require that the magnetic field be uniformly strong across the entire stellar surface. The field must have the required strength on the sector \( \theta_{\text{in}} \theta_{\text{lim}} \), whose spread in latitude is only a few degrees. The basic processes in this model do not depend on whether the field is axially symmetric. The results obtained for symmetrical systems will qualitatively apply to stars possessing magnetic fields with flux loops that thread the disk region.

Observational evidence (e.g., Donati et al. 2001, 2002; Neiner 2002, 2004) indicates that several stars of the type we consider for disk formation are oblique magnetic rotators with dipole fields. Preuss et al. (2004) find that disk-like structures can form along the magnetic equatorial planes of models with strong magnetic fields in which the Alfvén speed is faster than the rotation speed of the magnetosphere. Thus, when the MRWD model is applied to oblique rotators, a pre-Keplerian region can be formed along a plane or surface that is determined by gravity, centrifugal force and magnetic force acting on the disk material. During the second process, if the magnetic force in the disk becomes small, we expect that super-Keplerian material will move into Keplerian orbits in the rotational equatorial plane. In the case of oblique rotators with \( B_{m,*} > B_{m,* \text{mtd}} \), the results of Preuss et al imply that an MTD disk will be located in the magnetic equatorial plane rather than the rotational equatorial plane.

Because the magnetic force in the disk region of the MRWD model becomes small at the end of the fill-up stage, the one-armed spiral pattern of the Global Disk Oscillation model (e.g., Okazaki 1997) can be used to explain the V/R variability in disks of Be stars.

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**References**

- Cassinelli, J. P., Brown, J. C., Maheswaran, M., Miller, N. A., & Telfer, D. C. 2002, ApJ, 578, 951
- Donati, J.-F., Wade, G. A., Babel, J., Henrichs, H. f., de Jong, J. A., Harries, T. J. 2001, MNRAS, 326, 1265
- Donati, J.-F., Babel, J., Harries, T. J., Howarth, I. D., Petit, P., Semel, M. 2002, MNRAS, 333, 55
- Maheswaran, M., & Cassinelli, J. P. 1988, ApJ, 335, 931
- Maheswaran, M., & Cassinelli, J. P. 1992, ApJ, 386, 695
- Maheswaran, M. 2003, ApJ, 592, 1156
- Neiner, C. 2002, Ph.D. Thesis, University of Amsterdam
- Neiner, C. 2004, in ASP Conf. Ser., The Nature and Evolution of Disks Around Hot Stars, ed. R. Ignace & K. Gayley (San Francisco: ASP)
- Okazaki, A. T. 1997, A&A, 318, 548
- Preuss, O., Schüssler, M. Holzerwarth, V., & Solanki, S. K. 2004, A&A, 417, 987