Hall Drag and Magnetodrag in Graphene

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Massless Dirac fermions in graphene at charge neutrality form a strongly interacting system in which both charged and neutral (energy) modes play an important role. These modes are essentially decoupled in the absence of a magnetic field, but become strongly coupled when a field is applied. We show that these ideas explain the recently observed giant magnetodrag, arising in classically weak fields when electron density is tuned near charge neutrality. We predict strong Hall drag in this regime, which is in stark departure from the weak coupling regime, where theory predicts the absence of Hall drag. Energy-driven magnetodrag and Hall drag arise in a wide temperature range and at weak magnetic fields, and feature an unusually strong dependence on field and carrier density.

Graphene near charge neutrality (CN) hosts an intriguing electron-hole system with unique properties\cite{1,10}. Our understanding of the behavior at CN would greatly benefit from introducing ways to couple the novel neutral modes predicted at CN to charge modes which can be easily probed in transport measurements. There is a long history of employing magnetic field for such a purpose, since transport in charge-neutral plasmas is ultra-sensitive to the presence of external magnetic fields\cite{11}.

A new interesting system in which magnetotransport at CN can be probed are atomically thin graphene double layer G/hBN/G structures\cite{12,13}. Strong Coulomb coupling between adjacent layers in these systems results in strong Coulomb drag, arising when current applied in one (active) layer induces a voltage in the adjacent (passive) layer\cite{13,22}. Recent measurements\cite{13} revealed drag resistance that peaks near CN and has dramatic magnetic field dependence, with the peak value increasing by more than an order of magnitude (and changing sign) upon application of a relatively weak $B$ field. Strong magnetic field dependence of drag has been observed previously in other double layer two-dimensional electron gas (2DEG) heterostructures\cite{23,24}, however these experiments were carried out in the quantum Hall regime, whereas the anomalous magnetodrag found in Ref.\cite{13} occurs at classically weak fields $B \lesssim 1$ T.

Here we explain this puzzling behavior in terms of an energy-driven drag mechanism which involves coupled energy and charge transport\cite{20,22} (see Fig.\ref{fig1}). Energy transport plays a key role because of fast vertical energy transfer due to interlayer Coulomb coupling in G/hBN/G systems\cite{20} and relatively slow electron-lattice cooling\cite{28,29}. As a result, current applied in one layer can create a spatial temperature gradient for electrons in both layers, giving rise to thermoelectric drag voltage. The effect peaks at CN, since thermoelectric response is large close to CN\cite{8,11} and diminishes as $1/E_F$ upon doping away from CN\cite{30,31}. Drag arising from this mechanism depends on thermoelectric response and, unlike the conventional momentum drag mechanism, it is insensitive to the electron-electron interaction strength.

Another interesting effect that can be probed in these systems is that of Hall drag. It has long been argued that, at weak coupling, no Hall voltage can arise in the passive layer in the presence of current in the active layer\cite{26,27}. This is because transferred momentum is parallel to velocity, allowing only a longitudinal “back-current” to develop in the passive layer. As we shall see, a very different behavior arises at strong coupling, owing to the long-range energy currents leading to electron-lattice temperature imbalance. Close to CN, the magnitude of the cross-
couplings between charge and energy currents becomes large, producing a finite Hall drag, \( V_H = R_{\text{drag}}^{\parallel} J_{\parallel} \).

As we will see, energy currents result in Hall and magnetodrag resistances, \( R_{\text{H}}^{\parallel} \) and \( R_{\text{drag}}^{\parallel} \), that are large and peak near CN, see Fig.1 and Fig.2. These large values arise even for classically weak fields \( B \sim 0.1 \text{T} \), exceeding by two orders of magnitude the values found previously in GaAs/AlGaAs 2DEG heterostructures \( [23,27] \) at similar fields. The mechanism based on coupled energy and charge transport predicts large and negative drag at CN that matches recent experiments (see Fig.1d). Our mechanism naturally leads to Hall drag because vertical energy transfer between layers does not discriminate between longitudinal and transverse heat currents since temperature profile is a scalar field. This stands in contrast to conventional momentum driven drag, where momentum transfer is parallel to the applied current \( [26,27] \).

The electric and thermal variables may depend on the transverse coordinate \( y \), see below.

Boundary conditions for a Hall bar require electric current being tangential to the side boundaries, \( y = \pm W/2 \), and zero temperature imbalance at the ends, \( x = \pm L/2 \), reflecting that the current and voltage contacts act as ideal heat sinks. The electric current parallel to the boundaries \( y = \pm W/2 \) gives rise to the Ettingshausen heat current that may have a component transverse to the Hall bar. The divergence of this heat current, appearing on the right hand side of Eq. (2), acts as an effective boundary delta function source in the heat transport equations. Boundary conditions can profoundly influence the symmetry of the resultant drag resistivity, see below.

We consider the case of a spatially uniform \( Q \) in both layers. The ideal heat sinks at \( x = \pm L/2 \) mean that no temperature imbalance develops in the \( x \)-direction (except for some “fringing” heat currents near the contacts which give a contribution small in \( W/L \ll 1 \), which we will ignore in the following discussion). Since no temperature gradients are sustained in the \( x \)-direction for from the ends, we can reduce our problem Eq.(2) to a quasi-1D problem with temperature profiles that only depend on the \( y \)-coordinate. As a result, the only heat source arises from the Ettingshausen effect \( Q^{\perp j} = (Q^{\perp xy} j) \tilde{y} \).

To describe transport in the presence of such a source, we will expand temperature variables in both layers in a suitable orthonormal set of functions. Here it will be convenient to use eigenstates of the operator \( \partial_y^2 \) with zero Neumann boundary conditions at \( y = \pm W/2 \), given by

\[
\begin{align*}
\delta T_{1,2}(y) &= \sum_{q_n} \delta T_{1,2}(q_n) \sin q_n y, \quad q_n = \frac{2\pi(n + \frac{1}{2})}{W}.
\end{align*}
\]

For each \( n \) we obtain a pair of algebraic equations

\[
\begin{align*}
q_n^2 \kappa_{1,2} \delta T_{1,2} + a(\delta T_{1} - \delta T_{2}) + \lambda \delta T_{1} &= 0, \\
q_n^2 \kappa_{1,2} \delta T_{1,2} + a(\delta T_{2} - \delta T_{1}) + \lambda \delta T_{2} &= 0
\end{align*}
\]

where \( \kappa_{1,2} = \kappa_{xx}^{(1,2)} \) and \( F_n = 2A(-1)^n Q^{\perp xy} j \). Solving Eq. (3), we find the temperature profile in layer 2:

\[
\delta T_{2}(y) = \sum_{n \geq 0} \frac{aF_n}{L_1(q_n)L_2(q_n) - a^2 v_n(y),}
\]

where \( L_i(q_n) = \kappa_i q_n^2 + a + \lambda \) (\( i = 1,2 \)). Since electron-lattice cooling is very slow \( [28,29] \), with the corresponding cooling length values in excess of few microns, we will suppress \( \lambda \) in what follows. Because the boundaries
\[
\Delta T = 4A^2 \sum_{n \geq 0} aQ_{xy}^{(1)} L_1 L_2 - a^2 = 8 W \kappa \sum_{n \geq 0} \frac{Q_{xy}^{(1)}}{q_n^2 (1 + \xi^2 q_n^2)},
\]

where we defined \( \kappa = \kappa_1 + \kappa_2 \) and a length scale \( \xi = \sqrt{\kappa_1 \kappa_2 / \alpha \kappa} \). We evaluate the sum using the identity \( \sum_{n=0}^{\infty} \frac{1}{(n+\frac{1}{2})^2 + \xi^2} = \frac{\pi^2}{2\xi^2} \left( 1 - \frac{\tanh(\xi)}{\xi} \right) \) to obtain

\[
\Delta T = \frac{W Q_{xy}^{(1)}}{\kappa} j G(\xi), \quad G(\xi) = 1 - \frac{2e}{W} \tanh\left( \frac{W}{2e} \right).
\]

Connecting \( \Delta T \) with the drag voltage, and in particular determining its sign, requires taking full account of Onsager reciprocity. This analysis is presented below.

In the same way that the applied charge current, \( j \), in layer 1 causes a heat current (Peltier/Ettingshausen), a temperature imbalance in layer 2, \( \Delta T \), can sustain voltage drops across the sample (Thermopower/Nernst). These two effects are related by Onsager reciprocity constraints. The cross couplings in the coupled energy and charge transport equations arise from

\[
\left( \begin{array}{c} -j \\ \kappa \end{array} \right) = \left( \begin{array}{cc} e L_{11}/T & e L_{12}/T \\ L_{21}/T & L_{22}/T \end{array} \right) \left( \begin{array}{c} \nabla \mu \\ \nabla T \end{array} \right)
\]

where \( L \) are 2 \( \times \) 2 matrices and \( e \) is the carrier charge.

In this notation, the electrical conductivity equals

\[
\sigma = c^2 L_{11} / T, \quad \text{and thermal conductivity is} \quad \kappa = L_{22} / T^2.
\]

Comparing to the heat current due to an applied charge current, \( e j \), we identify \( L_{12} = -\sigma L_{21} \).

Onsager reciprocity demands that the cross-couplings obey \( L_{12}(B) = (L_{21}^T)^{-1}(B) \) where \( B \) is the applied magnetic field (note the transposed matrix). In an isotropic system the off-diagonal components of \( L \) obey \( L^{(T)}(B) = L^{(y)}(-B) \). As a result, Onsager reciprocity reduces to

\[
L_{12}(B) = L_{21}(B)
\]

in an isotropic system. Applying Eq. (9) to Eq. (8) in an open circuit, we find \( \mathbf{E} = -e^{-1} \nabla \mu = T^{-1} L_{11}^L Q L_{11} \nabla T \). For an isotropic system \( Q = Q_{xx} = iQ_{xy} \), \( L = L_{xx} = iL_{xy} \), so that \( [Q, L] = 0 \), which gives Eq. (1).

Several different regimes arise depending on the relation between the interlayer cooling rate and the bar width \( W \). Using Eq. (3) and Eq. (1) we obtain

\[
\left( \begin{array}{c} V_H \\ V_H \end{array} \right) = \left( \begin{array}{cc} R_{H}^{\text{drag}} & -R_{H}^{\text{drag}} \\ -R_{H}^{\text{drag}} & R_{H}^{\text{drag}} \end{array} \right) \left( \begin{array}{c} I_L \\ 0 \end{array} \right),
\]

giving the magnetodrag and Hall drag resistance values

\[
R_{H}^{\text{drag}} = -G(\xi) Q_{xy}^{(1)} Q_{yy}^{(2)}, \quad R_{H}^{\text{drag}} = -\frac{L G(\xi)}{W T \kappa} Q_{xy}^{(1)} Q_{yy}^{(2)},
\]

where we used \( Q_{xx} = Q_{yy} \) and \( Q_{xy} = -Q_{yx} \) for an isotropic system.

For a narrow bar (or, slow cooling), we have \( \xi / W \gg 1 \) and \( G \rightarrow 0 \), giving vanishingly small \( R_{H}^{\text{drag}} \). For a wide bar (or, fast cooling) we have \( G \rightarrow 1 \) so that \( R_{H}^{\text{drag}} \) saturates to a universal value independent of the interlayer cooling rate. For typical device parameters, we estimate \( \xi \approx 40 \text{nm} \) at \( T = 300 \text{K} \) [20]. Since \( L, W \) are a few microns for typical graphene devices, we expect them to be firmly in the \( G = 1 \) regime, with the Hall drag and magnetodrag attaining universal values independent of the electron-electron interaction strength.

To describe the density and \( B \) field dependence, we use a simple model for \( Q \). Measurements indicate [8, 9] that thermopower and the Nernst effect in graphene are well described by the Mott formula [33], giving

\[
Q = \frac{\pi^2}{3e} k_B^2 T^2 \frac{\partial [\mu^{-1}]}{\partial \mu}, \quad \mu = \left( \frac{\rho_{\parallel}}{-\rho_{\perp}} \right),
\]

with \( \rho \) the resistivity, \( e < 0 \) the electron charge, and \( \mu \) the chemical potential. We use a simple phenomenological model [34] relevant for classically weak \( B \) fields:

\[
\rho_{\parallel} = \rho_0 \frac{\mu}{\sqrt{1 + n^2/n_0^2}} \text{ } \rho_{\perp} = -\frac{B n}{e(n^2 + n_0^2)},
\]

where \( \rho_0 \) is the resistivity peak value at the Dirac point, \( n \) is the carrier density, and parameter \( n_0 \) accounts for broadening of the Dirac point due to disorder. We account for disorder broadening of the density of states, \( dn/d\mu = (n^2 + n_0^2)^{1/4} \sqrt{2/(\pi h^2 v_F^2)} \),
From Eqs. (11), (12), (13) and the Wiedemann-Franz relation for $\kappa$, we obtain $\rho^{\text{drag}}_\parallel = (W/L) \kappa^2 \rho_\parallel$ and $R^{\text{drag}}_H$ (see Fig.1c and Fig.2c, respectively). In that, we used the parameter values $n_0 = 10^{11}$ cm$^{-2}$, $\rho_0 = \frac{h}{e^2}$, and a representative temperature, $T = 150$ K. These values match device characteristics (disorder broadening, $n_0$, and peak resistivity, $\rho_0$) described in Ref.[13]. As a sanity check, we plot the components of $Q$ (in Fig.2a) which show the behavior near CN matching thermopower and Nernst effects measured in graphene.[8, 9]

Analyzing magnetodrag, we find that $\rho^{\text{drag}}_\parallel$ peaks at dual CN, taking on large and negative values (Fig.1c). Magnetodrag peak exhibits a steep $B$ dependence, $\rho^{\text{drag}}_\parallel, \text{peak} \propto -B^2$, bearing a striking resemblance to measurements reproduced in Fig.1d. In particular, our model explains the negative sign of the measured magnetodrag.

Hall drag is large and sign-changing (see Fig.2a), taking on values consistent with measurements. Interestingly, the map in Fig.2c indicates that the sign of $R^{\text{drag}}_H$ is controlled solely by carrier density in layer 2, breaking the $n_1 \leftrightarrow n_2$ symmetry between layers. This behavior does not contradict Onsager reciprocity, it arises as a consequence of the asymmetric boundary conditions for the Hall bar: free boundary at $y = \pm W/2$ and ideal heat sinks at the ends, $\delta T(x = \pm L/2) = 0$. This allows for finite temperature gradients to be sustained across the bar but not along the bar, see Fig.2d.

For other geometries, the temperature gradient can be obtained by balancing the heat flux due to thermal conductivity against the net heat flux in the two layers, $(\kappa_1 + \kappa_2) \nabla T = D Q^{(1)} j_1$ (see Eq.(24) of Ref.[22]). The quantity $D$ can in principle be obtained by solving heat transport equations. Adopting the same approach as above, we find a magneto and Hall-drag resistivity

$$\rho^{\text{drag}} = \frac{1}{T R} Q^{(2)} D Q^{(1)}, \quad E_2 = \rho^{\text{drag}} j_1. \quad (14)$$

For isotropic heat flow, $D = 1$. In this case, since $Q^{(1)}$ and $Q^{(2)}$ commute, the resulting drag is layer-symmetric, $n_1 \leftrightarrow n_2$.[22] In particular, Hall drag for $D = 1$ vanishes on the diagonal $n_1 = -n_2$. In contrast, for anisotropic heat flow, such as that discussed above, we expect a generic tensor $D \neq 1$ and thus no layer symmetry.

We wish to clarify, in connection to recent measurements,[23] that layer symmetry $n_1 \leftrightarrow n_2$ implies a swap of current and voltage contacts. Layer symmetry, which implies $D = 1$ in Eq.(14), will therefore only hold for Hall bars equipped with wide voltage contacts, for which the contact and the bar widths are comparable. This is indeed the case for the cross-shaped devices used in Ref.[13]. However it is not the case for a Hall bar with noninvasive voltage probes which are much narrower than the bar width, as assumed in our analysis above. Noninvasive probes, which have little effect on temperature distribution in the electron system, translate into $D \neq 1$ and no layer symmetry.

In summary, magnetic field has dramatic effect on drag at CN because it induces strong coupling between neutral and charge modes, which are completely decoupled in the absence of magnetic field in a uniform system. Field-induced mode coupling leads to giant drag that dwarfs the conventional momentum drag contribution as well as a remnant drag due to spatial inhomogeneity.[20] Our estimates indicate that these two contributions are orders of magnitude smaller than the predicted magnetodrag, which also has an opposite sign. The giant magnetodrag and Hall drag values attained at classically weak magnetic fields, along with the unique density dependence and sign, make these effects easy to identify in experiments. The predicted magnetodrag is in good agreement with findings in Ref.[13]. Magnetic field, coupled with drag measurements at CN, provides a unique tool for probing the neutral modes in graphene.

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[1] J. Gonzales, F. Guinea, and M.A.H. Vozmediano, Nucl. Phys. B424, 595 (1994); Phys. Rev. B 59, R2474 (1999).
[2] D. E. Sheehy and J. Schmalian, Phys. Rev. Lett. 99, 226803 (2007).
[3] D. T. Son, Phys. Rev. Lett. 75, 235423 (2007).
[4] O. Vafek, Phys. Rev. Lett. 98, 216401 (2007).
[5] A. B. Kashuba, Phys. Rev. B 78, 085415 (2008).
[6] L. Fritz, J. Schmalian, M. Mueller, S. Sachdev, Phys. Rev. B 78, 085416 (2008).
[7] M. Mueller, L. Fritz, S. Sachdev, Phys. Rev. B 78, 115406 (2008).
[8] Y. M. Zuev, W. Chang, and P. Kim, Phys. Rev. Lett. 102, 096807 (2009).
[9] P. Wei, W. Bao, Y. Pu, C. N. Lau, and J. Shi, Phys. Rev. Lett. 102, 166806 (2009).
[10] J. G. Checkelsky and N. P. Ong, Phys. Rev. B 80, 081143(R) (2009).
[11] L. P. Pitaevskii, E.M. Lifshitz, Physical Kinetics (Pergamon, Oxford, 1981)
[12] L. Britnell, et al. Science 335, 947 (2012).
[13] R. V. Gorbachev, et al. Nature Physics 8, 896 (2012).
[14] S. Kim, I. Jo, J. Nah, Z. Yao, S. K. Banerjee, and E. Tutuc, Phys. Rev. B 83, 161401 (2011).
[15] W.-K. Tse and S. Das Sarma, Phys. Rev. B 75, 045333 (2007).
[16] B. N. Narozhny, Phys. Rev. B 76, 153409 (2007).
[17] E. H. Hwang, R. Sensarma, and S. Das Sarma , Phys. Rev. B 84, 245441 (2011).
[18] N. M. R. Peres, J. M. B. Lopes dos Santos, and A. H. Castro Neto, Europhys. Lett. 95, 18001 (2011).
[19] M. I. Katsnelson, Phys. Rev. B 84, 041407 (2011).
[20] J. C. W. Song and L. S. Levitov, Phys. Rev. Lett. 109, 236602 (2012).
[21] B. N. Narozhny, M. Titov, I. V. Gornyi, and P. M. Ostrovsky, Phys. Rev. B 85, 195421 (2012).
[22] J. C. W. Song, D. A. Abanin, L. S. Levitov, Nano Lett.,
10.1021/nl401475u (2013).
[23] N. K. Patel, E. H. Linfield, K. M. Brown, M. Pepper, D. A. Ritchie, and G. A. C. Jones, Semicond. Sci. Technol., 12, 309 (1997).
[24] H. Rubel, A. Fischer, W. Dietsche, K. von Klitzing, and K. Eberl, Phys. Rev. Lett. 78 1763 (1997).
[25] M. P. Lilly, J. P. Eisenstein, L. N. Pfeiffer, and K. W. West, Phys. Rev. Lett., 80 1714 (1998).
[26] A. Kamenev and Y. Oreg, Phys. Rev. B, 52 7516 (1995).
[27] M. C. Bonsager, K. Flensberg, B. Y.-K. Hu, and A.-P. Jauho, Phys. Rev. Lett., 77 1366 (1996). The authors note that Hall drag vanished for systems with inversion symmetry and when the carriers in the active layer can be described by a drifted Fermi-Dirac distribution.
[28] M. W. Graham, S-F. Shi, D. C. Ralph, J. W. Park, P. L. McEuen, Nature Physics, 9 103 (2013)
[29] A. C. Betz, S. H. Jhang, E. Pallecchi, R. Ferreira, G. Féve, J-M. Berroir, and B. Placais, Nature Physics, 9 109 (2013).
[30] J. M. Ziman, Principles of the Theory of Solids, Cambridge University Press (1979).
[31] E. H. Hwang, E. Rossi, and S. Das Sarma Phys. Rev. B 80, 235415 (2009).
[32] H. B. Callen, Phys. Rev., 73 1349 (1948).
[33] M. Jonson and S. M. Girvin, Phys. Rev. B, 29 1939 (1984).
[34] D. A. Abanin, et. al., Science, 332 328 (2011) and accompanying online supplement.
[35] A. K. Geim, private communication