Phase diagram for rotating compact stars with two high density phases

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For the classification of rotating compact stars with two high density phases a phase diagram in the angular velocity (Ω) - baryon number (N) plane is investigated. The dividing line $N_{\text{crit}}(\Omega)$ between configurations with one and two phases is correlated to a local maximum of the moment of inertia and can thus be subject to experimental verification by observation of the rotational behavior of accreting compact stars. Another characteristic line, which also can be measured is the transition line to black holes that of the maximum mass configurations. The positions and the shape of these lines are sensitive to changes in the equation of state (EoS) of stellar matter. A comparison of the regional structure of phase diagrams is performed for polytropic and relativistic mean field type EoS and correlations between the topology of the transition lines and the properties of two-phase EoS are obtained. Different scenarios of compact star evolution are discussed as trajectories in the phase diagram. It is shown that a population gap in the $\Omega - N$ plane for accreting compact stars would signal a high density phase transition and could be observed in the distribution of so called Z sources of quasi periodic oscillations in low-mass X-ray binaries.

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At present, the existence of exotic phases of matter at high densities is under experimental investigation in ultrarelativistic heavy-ion collisions \[1\] the most prominent being the deconfined phase of QCD \[2\]. While the diagnostics of a phase transition in experiments with heavy-ion beams faces the problems of strong nonequilibrium and finite size, the dense matter in a compact star forms a macroscopic system in thermal and chemical equilibrium for which effects signalling a phase transition shall be most pronounced.

Signals of high density phase transitions have been suggested in the form of characteristic changes of observables such as the surface temperature \[3,4\], brightness \[5\], pulse timing \[6\] and rotational mode instabilities \[7\] during the evolution of the compact object. In particular the pulse timing signal has attracted much interest since it is due to changes in the kinematics of rotation. Thus it could be used not only to detect the occurrence, but also to determine the size of the high density matter core from the magnitude of the braking index deviation from the magnetic dipole value \[8\]. Besides of the isolated pulsars, one can consider also the accreting compact stars in low-mass X-ray binaries (LMXBs) as objects from which we can expect signals of a high density phase transition in their interior \[9,10\]. The observation of quasiperiodic brightness oscillations (QPOs) \[11\] for some LMXBs has lead to very stringent constraints for masses and radii \[12\] which according to \[13\] could even favour strange quark matter interiors over hadronic ones for these objects. Due to the mass accretion flow these systems are candidates for the formation of the most massive compact stars from which we expect to observe signals of the transition to either quark core stars, to a third family of stars \[14\] or to black holes.

In this work we introduce a classification of rotating compact stars in the plane of their angular frequency $\Omega$ and mass (baryon number $N$) which we will call phase diagram. In this diagram, configurations with high density matter cores are separated from conventional ones by a critical phase transition line.

Since the phase diagram of rotating compact objects seems to be a more general approach for investigations of phase transition effects in the interior of the star we assume that the deconfinement transition could be a particular case besides of other possibilities for phase transitions like pion or kaon condensation as discussed, e.g. in \[15,16\]. Therefore, our aim is to suggest it as a heuristic tool for obtaining constraints on the EoS at high densities from the rotational behaviour of compact stars. We will provide criteria under which a particular astrophysical scenario with spin evolution could be qualified to signal of high density phase transition.

The true EoS that describes the interior of compact stars is largely unknown. This results from the inability to verify experimentally the different theories that describe the strong interactions between baryons and the many-body theories of dense matter at densities larger than about twice the nuclear density \[17\].
Since our focus is on the elucidation of qualitative features of signals from the high density phase transition in the pulsar timing we will use a generic form of an equation of state (EoS) with such a transition. We use the polytropic type equation of state \( P_i = K^0_i \frac{n_i}{n_0} \left( \frac{n}{n_0} \right)^\gamma_i \); \( \epsilon_i = \frac{P_i}{\Gamma_i - 1} + m n \), where \( n \) is the baryon number density, \( P_i \) - the pressure, and \( \epsilon_i \) - energy density for both the low \((i = L)\) and the high \((i = H)\) density phases, respectively. \( K^0_i = K_i(n_0) \) is the value of the incompressibility \( K_i(n) = 9 \frac{dP}{dn} \) at the saturation density \( n_0 = 0.17/fm^3 \); \( \Gamma_i = d\ln(P_i)/d\ln\epsilon_i \) the adiabatic index and \( m \) is the nucleon mass. The phase transition between the lower and higher density phases is made by the Maxwell construction \([19]\) and compared to a relativistic mean field model consisting of a linear Walecka plus dynamical quark model EoS \([21,22]\) with a Gibbs construction \([23,24]\), see Fig.1.

The quark matter part of this EoS is obtained from a dynamical confining approach \([21]\) in the generalization to three flavors \([22]\), where the strange flavor remains confined at the deconfinement transition for the light and appears only at densities for which stars are close to the gravitational instability. The difference to most of the models for quark deconfinement in neutron star matter is that the ambiguity in the choice of the bag constant for the quark matter phase can be removed by a derivation of this quantity \([23,24]\) within the dynamical confining approach \([21,22]\).

With the EoS models discussed above we have performed calculations of rotating compact star configurations assuming stationary, rigid rotation. For our treatment of rotation within general relativity we employ a perturbation expansion following Refs. \([26]\). For a detailed discussion of the method and its application we refer to \([8]\). The results of our calculations of rotating compact star configurations can be classified in the plane of angular velocity \( \Omega \) and baryon number \( N \) which we call phase diagram.

In Fig. 2 we display the phase diagrams for the rotating star configurations, which correspond to the three model EoS of Fig. 1. These phase diagrams have four regions: (i) the region above the maximum frequency \( \Omega > \Omega_{\text{max}}(N) \) where no stationary rotating configurations are found, (ii) the region of black holes \( N > N_{\text{crit}}(\Omega) \), and the region of stable compact stars which is subdivided by the critical line \( N_{\text{crit}}(\Omega) \) into (iii) the region of hybrid stars for \( N > N_{\text{crit}}(\Omega) \) where configurations contain a core with a second, high density phase and (iv) the region of mono-phase stars without such a core.

From the comparison of the regional structure of these three different phase diagrams in Fig. 3 with the corresponding EoS in Fig. 1 we arrive at the main result of...
this paper that there are the following correlations between the topology of the lines \( N_{\text{max}}(\Omega) \) and \( N_{\text{crit}}(\Omega) \) and the properties of two-phase EoS:

- The hardness of the high density EoS determines the maximum mass of the star, which is given by the line \( N_{\text{max}}(\Omega) \). Therefore \( N_{\text{max}}(0) \) is proportional to the parameter \( K_H(n_H) \), where \( n_H \) is the density of the transition to high density phase.

- The onset of the phase transition line \( N_{\text{crit}}(0) \) depends on the density \( n_H \) and \( K_L(n_L) \) where \( n_L \) is the density of the transition to the low density phase.

- The curvature of the lines \( N_{\text{max}}(\Omega) \) and \( N_{\text{crit}}(\Omega) \) is proportional to the compressibility of the high and low density phases, respectively.

Therefore, a verification of the existence of the critical lines \( N_{\text{crit}}(\Omega) \) and \( N_{\text{max}}(\Omega) \) by observation of the rotational behavior of compact objects would constrain the parameters of the EoS for neutron star matter.

The determination of \( N_{\text{max}}(\Omega) \), the border between compact stars and black holes, is a traditional issue which has recently gained new impetus due to the interpretation of recent LMXB data. A new aspect characterizing the configurations with a phase transition is the critical line \( N_{\text{crit}}(\Omega) \) which can be measurable by changes in the rotational dynamics since it is correlated with a local maximum of the moment of inertia \( I(N,\Omega) \), the key quantity governing the rotational evolution via

\[
\dot{\Omega} = \frac{K(N,\Omega)}{I(N,\Omega)} \Omega \frac{\partial I(N,\Omega)}{\partial N}.
\]

In Fig. 3 we show for the example of the RMF EoS with a deconfinement phase transition from hadronic to quark matter how the structure of the lines of constant moment of inertia correlated with the critical lines in the phase diagram.

In Eq. (1) \( K = K_{\text{int}} + K_{\text{ext}} \) is the net torque acting on the star due to internal and external forces. The internal torque is given by \( K_{\text{int}}(N,\Omega) = -\Omega \dot{N} \frac{\partial I(N,\Omega)}{\partial N} \Omega \), the external one can be subdivided into an accretion and a radiation term \( K_{\text{ext}} = K_{\text{acc}} + K_{\text{rad}} \). The first one is due to all processes which change the baryon number, \( K_{\text{acc}} = \dot{N} dJ/dN \) and the second one contains all processes which do not. For the example of magnetic dipole and/or gravitational wave radiation it can be described by a power law \( K_{\text{rad}} = \beta \Omega^n \), see [29].

To prove that the appearance or disappearance of a high density phase during the rotational evolution of the star could entail observational consequences for the angular velocity we consider three main representatives different classes of trajectories which could cross the critical line on phase diagram. These classes of tracks are: (a) spindown of isolated (non-accreting, \( \dot{N} = 0 \)) pulsars due to magnetic dipole radiation [30], (b) spin up in accreting systems with weak magnetic field [6] (\( N \approx \text{const} \), vertical tracks) and (c) accretion either with strong magnetic field [4] or for accreting binaries emitting gravitational waves [28], for which \( \Omega \approx \text{const} \) (horizontal tracks).

In the case of (a) the spindown \( K = K_{\text{rad}} \) or (b) spinup \( K = K_{\text{acc}} \) evolutions (in both cases \( K_{\text{int}} \ll K_{\text{ext}} \)) the objects can undergo a phase transition if the baryon number lies within the interval \( N_{\text{crit}}(\Omega = 0) < N < N_c \), where \( N_c \) is the end point of the critical line \( N_{\text{crit}}(\Omega) \).

If the core of compact star is soft enough (as in case (SH-S)) the critical line \( N_{\text{crit}}(\Omega) \) crosses \( N_{\text{max}}(\Omega) \) at some \( \Omega_c(N_c) < \Omega_K(N_c) \). This means, that massive monophase configurations with total baryon number \( N > N_c \) rotating with angular velocities in the interval \( \Omega_c < \Omega < \Omega_K \) should encounter a transition to a black hole during the spin down evolution.

In cases, when the core EoS is harder (H-S-H and SH-H), the region of hybrid stars is a band which separates monophase configurations from black holes, see upper two panels of Fig. 3.

As it has been shown in [30] for the vertical tracks (a) and (b) in the phase diagram, the braking index \( n(\Omega) \) changes its value from \( n(\Omega) > 3 \) in the region (iii) to \( n(\Omega) < 3 \) in (iv). This is the braking index signal for a deconfinement transition introduced in Ref. [29].

The third evolutionary track is accretion with strong magnetic fields [4] and/or gravitational wave emission (horizontal tracks) [28]. For this case the \( \Omega \) first decreases as long as the moment of inertia monotonously increases.
with $N$. When passing the critical line $N_{\text{crit}}(\Omega)$ for the phase transition, the moment of inertia starts decreasing and the internal torque term $K_{\text{int}}$ changes sign. This leads to a narrow dip for $\dot{\Omega}(N)$ in the vicinity of this line. As a result, the phase diagram gets overpopulated for $N \gtrsim N_{\text{crit}}(\Omega)$ and depopulated for $N \gtrsim N_{\text{crit}}(\Omega)$ up to the second maximum of $I(N, \Omega)$ close to the black hole line $N_{\text{max}}(\Omega)$. This population gap marks the region of hybrid stars in the phase diagram and is a measurable. Moreover, as we have seen population clustering of compact stars at the phase transition line could be a signal for the occurrence of stars with high density matter cores and a measure for obtaining constraints on the EoS at high densities. In contrast to this scenario, in the case without a phase transition, the moment of inertia could at best survive before the transition to the black hole region and consequently $\dot{\Omega}$ would also saturate. This would entail a smooth population of the phase diagram without a pronounced structure [11].

As a strategy for the investigation of high density phase transitions in compact stars we suggest to perform a systematic observation of LMXBs for which the discovery of strong and remarkably coherent high-frequency QPOs with the Rossi X-ray Timing Explorer has provided new information about the masses and rotation frequencies of the central compact object [12]. If, e.g., the recently discussed period clustering for Atoll- and Z-sources [20] will correspond to objects in a narrow region of masses well below the maximum mass limit, this could be interpreted as a signal for the high density phase transition to be associated with a fragment of the critical line in the phase diagram for rotating compact stars [21].

In the present work we have shown that the existence and the shape of the suggested population gap for LMXBs in the phase diagram will signal the occurrence of a phase transition in the QCD EoS and constrains its properties at high densities.

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