Anomalous low frequency dissipation processes in metal springs

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Abstract. The dissipation processes of leaf springs used in seismic isolation chains of Gravitational Wave detectors have been studied. A low frequency phase transition from viscous-like to fractal-like dissipation, controlled by Self Organized Criticality of dislocations, was observed. The new understandings suggest different best practices for the operations of the seismic isolation chains of the second generation of Gravitational Wave observatories and require new techniques and materials for the third generation.

1. Introduction.
Measurement on Maraging steel blades used in seismic attenuators of Gravitational Wave interferometers [1-5], coupled with an old prediction on self organization of dislocations, lead to novel understanding of low frequency losses in metals, with unpredictable, avalanche dominated, loss mechanisms and mechanical noise.

It is commonly accepted that dislocation motion inside metal grains (crystals) plays an important role in dissipation [6,7].

The dislocation model of dissipation (considering the motion of many independent dislocations) leads to a description of viscous-like dissipation, or a loss angle one, which can be easily mathematically treated (damped oscillator) and lead to predictable results [8, 9].

Neither is completely satisfactory, especially when dealing with equilibrium point hysteresis. As a result dissipation studies are full of exceptions from the rule, and ad hoc explanations [10], often with no basis on microscopic behavior understanding.

Mostly the models do not explain hysteresis [11] (which should not be confused with plasticity, in which the material deforms permanently) and low frequency instabilities (sudden collapses [12], random walk [13], excess losses, history dependent mechanical noise [14]).

To get some deeper understanding, one needs better understanding of the collective processes at the microscopic level.

Dislocations are ubiquitous and isotropic in polycrystalline materials, a large number of dislocations are required by the process of rapid freezing of a liquid into a solid, due to the material’s thermal expansion coefficient, to reconnect separate parts of growth of a crystal during the rather chaotic freezing process. Crystalline defects (not all line dislocations) appearing as often as every 200 lattice spacings [8, 15], are required to generate the electron scattering necessary to explain the measured metal resistivity in pure metals. Many of these defects are dislocations of various kind.
Line dislocations take different forms. One of the simpler and most common ones is the edge dislocation, that appears when there is a missing half plane of atoms in the crystal. For simplicity we limit the reasoning to edge dislocations, although screw and other kinds of dislocations can participate as well. While we picture line dislocation as straight, they have no reason to be straight, neither at birth, nor as a result of movements; line dislocations form arches and bends.

Dislocations are deviations from the crystal, which is the state of minimal energy. Therefore dislocations are energetically expensive (annealing dislocations in a material is an exothermal process).

Forcing two dislocations to overlap would cause additional energy expense. Therefore line dislocation generally display short range repulsion force (except for combinations that may lead to annihilation processes).

Atoms in a metal lattice are tied by covalent bonds, therefore missing atomic bonds along a dislocation can switch to the neighboring atoms at virtually no energetic cost. If there is an external stress applied to the material, say a compression gradient, there is an energetic advantage for the dislocation to move along that gradient. Although quantum and thermal fluctuations may cause bond switching in either direction, the net result is always in the direction of lower energy states and therefore of energy dissipation.

Additionally, dislocations, being the expression of local deformations of the crystal, carry both strain and stress, which means that the slow accumulation of dislocations in response to a stress gradient results in a material deformation and appearance of internal forces, either permanent (if grain cleavage, creep [16,17], acoustic emission [18], or other irreversible processes occur) or recoverable by simply reversing the force (hysteresis).

Although the individual bond switching in a plane happens almost instantaneously, a sequence of switches must repeat in a “zipper”-fashion along the dislocation line for actual dislocation movement, which takes considerable time. This makes that dislocations respond sluggishly to applied forces, thus leading to dissipation.

Figure 1: Schematics of a dislocation constrained between two anchoring points (diamonds), responding to alternating stress (the arrows). The dislocation arches until the energy gain from drifting along the stress equals to the energy necessary to stretch it longer. In the left and centre panel the dislocation is further constrained by entanglement (circle) with another dislocation. In the right panel the other dislocation is removed and the dislocation movement covers much more surface, thus dissipating more energy; it also takes longer time to move farther. Entangled dislocations lead to lower dissipation losses.

The zipping propagates along the dislocation length, as long as the pressure gradient offers energetic advantage. A zipping event moves a dislocation by an atomic spacing; it also allows the start
of additional zipping events on the same dislocation. A sequence of many zipping processes is needed to allow a dislocation to glide over any appreciable distance. If a dislocation is anchored in one or more points it will deform into an arching profile until the energy cost of stretching the dislocation length exceeds the advantage of following the stress gradient (figure 1.). Dislocations in an anchoring-defect-poor grain respond to a varying stress gradient with longer drifts, resulting in longer arches, which absorb more energy from the external excitation.

If the stress field is cyclic (as in a macroscopic oscillation or a vibration) the dislocation’s motion is also cyclic, but because dislocation movement happens with a delay with respect to the applied force, it will dissipate energy. The result is either a viscous or a loss angle dissipation regime if the oscillation period is shorter or longer than the average arch response time.

These well-known behaviors hypothesize independent movement of dislocations.

The system becomes much more interesting when dynamic and static interactions between dislocations are considered, resulting in non-linear, unpredictable, and sometimes even preposterous effects.

When a large number of dislocations respond to a stress gradient, they tend to pile up against fixed defects. Because they repel and cannot cross, they entangle to form a chaotic and meta-stable lattice permeating the ordered crystal lattice. Because dislocations carry stress and strain, the dislocation lattice contributes to the mechanical properties of the sample with its own stiffness and its own internal stresses and strains. When piled up and entangled, dislocations are blocked at many points along their length, and can respond to oscillating stresses only with short arches, which being short respond more rapidly, thus dissipating little energy.

If a sufficiently large number of dislocations is introduced (work hardening) the dislocation lattice can even become permanently entangled, and the material becomes harder and more elastic.

If there are only few dislocations and blocking defects (as in the annealed state of a pure metal grain), dislocations can glide under stress across the entire crystal volume, and pile up at the edge of it, until the accumulated stress exceeds the material ultimate strength and grain cleavage ensues, with acoustic emission [19], crystal cleavage, and creep; this also happens if the applied stress is sufficient to allow dislocations to jump blocking defects [16,17].

In the intermediate case, a sufficient number of dislocations moving under a pressure gradient can easily entangle. If one waits for a time sufficient to allow all dislocations to settle in a tangled pileup, the material can be expected to become slightly more elastic (slightly larger Young’s modulus) and less lossy (entangled dislocations either cannot move or move less in response to variable stress gradients and therefore dissipate less energy).

The lattice of entangled dislocations is not stable though. Reversing the stress field in absence of irreversible events (like crystal cleavage for example) eventually results in disentanglement. Disentanglement is not immediate, though, and can require non-negligible stress reversal. If disentanglement is induced the excess stiffness contributed by the chaotic lattice disappears, and the disentangled dislocations become mobilized in the form of avalanches, and available for larger dissipation.

Avalanching and its effects are non local. The impact of a whipping dislocation can activate a nearby one. Similarly the stress field of a dislocation avalanche can trigger the motion of other avalanches at longer range. The entire sample can eventually be interested by spreading avalanches and temporarily change its physical properties.

A regime where dislocation can pile up and get entangled is described by the statistics of systems controlled by Self Organized Criticality [20,21] (SOC). Such a behavior for dislocations was foreseen by [22-24] and observed in our laboratory. When in SOC controlled regime, the character of material internal dissipation changes dramatically.

In a SOC-controlled regime entangled dislocations act collectively, i.e. they their entanglement is stable below a critical “slope” and above that they move in avalanches of unpredictable amplitudes and timing that return the system below critical slope but in a different equilibrium state. The dislocation under varying stresses act like sand in a box that is tilted until the sand’s critical slope is
reached and the sand starts flowing. The avalanching process is scale-free, in the sense that any event size can happen, between a single dislocation shifting over a small distance, and the dislocation lattice of the entire macroscopic sample rearranging dramatically.

Therefore one cannot expect anymore a smooth reaction from the spring in response to movements sufficiently slow to allow time for entanglement.

If the macroscopic movements are sufficiently fast, many smaller avalanches will happen at the same time, and merge, masking the effects of individual larger avalanches happening at low frequency, and an apparently smooth behavior is restored.

If an oscillatory motion faster than the avalanche growth time is applied, no avalanche have time to fully grow before the stress field reverses direction and aborts it. Because the avalanches have exponential growth, i.e. the bulk of action of an avalanche happens at its very end, the energy dissipated by the avalanching process should drop off dramatically for oscillations with period shorter than the avalanching characteristic time. Note that, because the dislocation avalanches can spread over the entire elastic sample, this characteristic time depends on the macroscopic dimensions of the object, as well as the material characteristics and its stress conditions.

2. Expected unexpected behaviours in GAS springs and other low frequency oscillators

Although quantitative predictions would require unyielding simulations, extending from the microscopic to the macroscopic level, several qualitative predictions can be made from the theory of SOC of dislocations. Many of these “predictions” were made with hindsight, taking advantage of many puzzling experimental observations accumulated over years by one of the authors and his collaborators. The theory of SOC of dislocations can also explain many unexplained or ill-explained effects observed in other experiments and precision instruments.

The rest of this section is a listing of some “predictions”, while the next section lists the corresponding observations, either from our own measurements, or from effects or behaviors reported in literature.

2.1. Geometric Anti Springs (GAS) are mechanical oscillators that can be tuned to very low frequency [25,26] by progressively neutralizing their restoring forces by means of radial blade compression and then by means of ElectroMagnetic Anti Springs (EMAS) [27]. At sufficiently low frequency the stable crystalline lattice elasticity can be completely neutralized and the oscillator may remain stable only because of the metastable elasticity contributed by the entangled dislocation lattice. In such a state, if dislocation movement is triggered (by an external excitation of by a continuing slowly changing stress, like a thermal drift), the component of the restoring force provided by the entangled dislocations can temporarily disappear and the oscillator can run-off from its equilibrium position. In absence of slow stress variations, like thermal drifts for GAS springs and tilt drifts for IPs, the metastable equilibrium may last indefinitely even with excessively small restoring forces.

2.2. A similar effect should also be observed in Inverted Pendulum (IP), when their restoring force is reduced by means of progressive top loading they should fall in arbitrary direction [11,28].

The run-off events in both GAS and IP should be reversible by simply returning the system to an equilibrium point and holding them there for the time necessary for dislocation entanglement.

2.3. The initial phase of the run-off, when only a small number of individual and localized avalanches is involved, the acceleration of a run-off should to be “bumpy”, reflecting the effect of individual, non predictable, avalanches.

The same should be observed in ringdowns of oscillators close to their instability limit.

2.4. A sequence of small excitations combined with a slowly changing stress can induce many small avalanches before the critical slope is reached. While a slow thermal drift can reach the critical slope,
and result in a catastrophic run-off in a quiet environment, in noisy conditions it can produce a series of bumpy motion and less likely run-off.

2.5. If slow motion of the oscillator is induced, with period longer than avalanching characteristic times, dislocation avalanches that are mobilized by the motion are expected to re-entangle in a different configuration. When the force inducing the motion is removed the oscillator’s equilibrium point must be changed by an amount that depends, on the relative strength of the residual “true” and stable restoring force from the crystal lattice, and the changeable contribution provided by the entangled dislocation lattice. The hysteresis is expected to reach dramatic levels when the oscillator is tuned close to the instability (i.e. the residual restoring forces are dominated by the entangled dislocation contribution).

Multi stability can be observed in these conditions if forces of different amplitudes and directions are applied.

The dislocation landscape can be altered by a sequence of fast stresses, even orthogonal to the oscillator motion, thus inducing random changes of equilibrium point.

2.6. The amount of hysteresis should depend on the past motion or stress history, if the applied force causes the oscillator to go in directions just explored, the dislocations are already entangled in the correct location and no further hysteresis should be observed.

2.7. An oscillator tuned to high frequency with period shorter than the characteristic avalanche growth time should show much lower dissipation than when tuned with longer periods, thus affecting for example the Quality factor of the system.

2.8. Forced oscillations of a metal flexure cause dislocation disentanglement, which can be expected to grow with larger induced motion amplitudes. The thus disentangled dislocations should then result in a reduction of the fraction of Young’s modulus contributed by the entangled dislocation lattice, and in an enhanced viscous-like dissipation.

2.9. When an oscillator is tuned or moves to a frequency sufficiently low to allow sizable entanglement, and is subject to excitations sufficient to generate dislocation avalanches, it cannot be expected to respond like a harmonic oscillator, and its transfer function must be affected by 1/f response characteristic of avalanching systems.

2.10. Precision instruments based on mechanical oscillators (like tiltmeters, seismometers, torsional pendulums and accelerometers) should present low noise performance only after a sufficient dislocation settling time, and only in absence of slow excitations, like thermal drifts and tilts, or large excursions or stresses even in orthogonal vibrations, that can drive parts of the system to its “critical” slope and generate dislocation avalanches. These instruments cannot be expected to produce low noise operation during and after large excitations like earthquakes.

3. Observational behaviour and results
For lack of time only some of our experimental observations were illustrated in the conference presentation, and for lack of space even fewer of them are reported here, please refer to [29] and to the forthcoming longer paper [30] for more detail.

3.1. GAS springs were observed to spontaneously run-off either up or down (figure 2. Left panel). In particularly quiet conditions, under vacuum and by night, the HAM SAS system remained stable at 10 mHz tuning until an external perturbation was applied [31].
Figure 2: Example of GAS filter run-offs. In the left panel the resonant frequency of the filter is tuned to lower and lower frequency, until it destabilizes and runs off to the end stops. Note that run-off in an ideal spring should occur only at frequency equal to zero, while run-off is observed for frequencies as high as 200 mHz. The run-off of the 65 kg mass happens indifferently up or down (ruling out creep or material failing effects). In the right panel a run-off in a filter tuned at fixed resonant frequency, happens after several hours of night operation. Note, in the zoomed insert, the very slow start of the event, with an irregular and bumpy motion signalling the effects of individual avalanches, followed by a progressive acceleration.

3.2. IP were similarly observed to fall in arbitrary direction, sometimes after waiting times as long as many hours. In both IP and GAS filters the equilibrium can be easily restored.

3.3. Runoff and very slow frequency oscillations are indeed bumpy and of unpredictable shape (figure 2. Insert of right panel).

3.4. GAS filters were observed to show jumping equilibrium point and less run-off during the noisy day time and run-off more easily after long, quiet periods during the nights.

3.5. Hysteresis was observed to grow very rapidly in GAS filters below 0.2 Hz, much more than what could be expected by the lower stiffness (Figure 3.). Slow forces can drive the system to almost arbitrary equilibrium points [32] over a wide range of positions. Bad-weather-induced perturbations were observed to induce a slow equilibrium point random walk in the Virgo Superattenuators Inverted Pendula [13].

3.6. While excitations of alternate sign show alternate hysteresis of approximately the same amplitude, repeated excitations of the same sign produce no additional hysteresis (Figure 3.).

3.7. The Quality factor of GAS filters was observed to increase dramatically above a threshold of 0.2 Hz [29,30]. Below 0.2 Hz it behaves quadratically, as expected from time independent (completely developed avalanches) dissipation processes.
Figure 3: Measurement of hysteresis at 0.21 and 0.15 Hz GAS filter tune, responding to the same half-sine excitation. The restoring force differs by less than a factor of two, while the measured hysteresis grows by more than an order of magnitude, marking the phase transition between viscous and avalanche dominated dissipation. Note how the leading edge of the response in the left panel has the same curvature of the sine, while in the right panel the leading edge has opposite curvature as if the system was about to run-off.

Figure 4: Left panel, frequency deficit as function of oscillation amplitude (19% in frequency for oscillation amplitude of 4 mm, 34% in elastic constant k). The dots were evaluated along ringdown measurements, the bigger squares in swept sine measurements. Right panel, change of damping constant as function of amplitude, the dissipation is more than doubled for amplitude oscillations of 4 mm. Both measurements taken at 0.21 Hz (without EMAS).

3.8. The oscillation frequency of GAS filters is lower than the frequency measured for vanishing amplitude by a small amount (several percent) roughly linear with the oscillation amplitude, while the dissipation was observed to more than double with a growth rate compatible with the square root of the oscillation amplitude (Figure 4.). These effects were observed both in our measurements of swept sine excitation and ringdowns of GAS springs.

3.9. The 1/f transfer function was observed by [33,34] in a GAS filter operated at 100 mHz. It remains to observe if, at lower excitation level, the transfer function resume its ordinary 1/f2 slope as would be expected for oscillations small enough not to generate critical slope and not to trigger avalanches.
3.10. Development of at least three generations of mechanical tiltmeters ended in disappointing performance [35,36], thousands of times the level that could be extrapolated from higher frequency behavior, growing at least with 1/f at the low frequency side. Additionally, it is well known that high precision seismometers need to settle for a “thermalization” time of many hours, which is much longer than the thermalization time constants that one could expect from the heat capacitance and conductivity of their structure. Careful choice of the flexure material and its treatments can minimize this effect but never fully eliminate it. Torsion pendula reach their thermal noise performance only after settling time of hours, to be compared with the wire thermalization time of seconds or less [37]. In view of our understandings, the cause of the settling times is more likely connected to the settling of dislocations in their flexures and along the wires. Similarly it is well known that seismometers subject to thermal drifts are more noisy. Finally, attempts to use low noise, co-located seismometers to measure the actual displacement generated by an earthquake through integration of their signal, are reported to give inconsistent values and even directions.

4. Conclusions

Hysteresis effects and excess low frequency noise were a nagging and long studied issue in the development of the SAS systems for Virgo, LIGO, TAMA and for the HAM chambers for Ad-LIGO. After more than 10 years of studying, these effects were linked to the theory of SOC of dislocation. While work is still ongoing, better practices are slowly emerging from the new understandings, which include different engineering, different materials, and sometimes simply different procedures.

Most of the measurements leading to these results were performed with precipitation wire Maraging steel, but hysteretic behavior foreseen by SOC was observed also in work hardened piano wire [38] and other metals (Copper Beryllium, Nispan-C, et c.). SOC effects are thought to be ubiquitous in low frequency behavior of metal flexures.

During the dislocation rearrangement leading to hysteresis, springs and flexures can be expected to be noisy. Rearrangement of dislocations may take substantial time, even hours of settling time after perturbation in high sensitivity seismometers or tilt pendulum. To reach quiet spring behavior, the dislocation rearrangement in the flexures must lead to a “flat” dislocation landscape. Dislocation landscape smoothing happens naturally (a SOC controlled system always settles below its “critical” slope). The corresponding instrument settling time can likely be reduced, and the flattening of the dislocation landscape can be improved, and driven well below the critical slope, by inducing gentle oscillations around the true equilibrium point and slowing tapering them down to zero amplitude.

Fluctuation dissipation estimations of thermal noise are valid only in absence of avalanching, and therefore they are valid only when the dislocation landscape is well below its critical “slope”, i.e. after a sufficiently long settling time, in absence of external mechanical perturbations and of any other slow stress variations. All sources susceptible to alter the dislocation landscape slope must be avoided (which include thermal drifts, slowly changing forces, physical tilt variations, vibrations and occasionally stronger excitations of flexures, even in orthogonal degrees of freedom, and any other stimuli that we may not have identified yet) if minimal noise conditions are desired. While any sustained external perturbation (like seismic noise, thermal drifts and other slow moving forces) will produce avalanches, it is possible that, at the lower stages of the GW detectors seismic attenuation chains, the residual seismic and mechanical excitation may be insufficient to induce avalanching, and that the full design attenuation power and thermal noise performance may be recovered. Thermal stabilization of a small fraction of a degree should be required. An example of behavior to be avoided is to compensate for earth’s tides by pushing on the mirror itself. The slow flexure bending close to the test mass would generate avalanching noise especially at the extremities of the tidal motion range. Tidal compensation should be applied as far upstream as possible.

We also suggest that any oscillator expected to operate at low noise should be operated around its “true” equilibrium point, i.e. the one determined by the elasticity of the crystalline structure alone, in absence of the contribution from the entangled dislocations lattice. Tests have shown that this state can be reached through induced oscillations of slowly decreasing amplitudes [32]. If the system does
not have sufficient Q-factor to do it by simple ringdown, these oscillations must be forced [30]. Old practices based on hysteresis effects, like finding the working tilt of a suspended mirror by over-tilting it by hand and then slowly releasing it, were effective but inductive of a metastable equilibrium determined by large dislocation pile-up and likely source of noisy conditions. These practices should be avoided, the required tilt of the mirror should be determined by moving weights up the chain, after a suitable dislocation landscape relaxation routine.

Materials free of dislocations (like glasses) or with frozen dislocations (like ceramics) can be expected to be free of SOC noise. Such SOC noise free materials will be needed for the seismic isolation of the third generation, low frequency Gravitational Wave detectors.

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