Data-Consistent Local Superresolution for Medical Imaging

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Abstract
In this work we propose a new paradigm of iterative model-based reconstruction algorithms for providing real-time solution for zooming-in and refining a region of interest in medical and clinical tomographic (such as CT/MRI/PET, etc) images. This algorithmic framework is tailor for a clinical need in medical imaging practice, that after a reconstruction of the full tomographic image, the clinician may believe that some critical parts of the image are not clear enough, and may wish to see clearer these regions-of-interest. A naive approach (which is highly not recommended) would be performing the global reconstruction of a higher resolution image, which has two major limitations: firstly, it is computationally inefficient, and secondly, the image regularization is still applied globally which may over-smooth some local regions. Furthermore if one wish to fine-tune the regularization parameter for local parts, it would be computationally infeasible in practice for the case of using global reconstruction. Our new iterative approaches for such tasks are based on jointly utilizing the measurement information, efficient upsampling/downsampling across image spaces, and locally adjusted image prior for efficient and high-quality post-processing. The numerical results in low-dose X-ray CT image local zoom-in demonstrate the effectiveness of our approach.

1. Introduction
Medical tomographic imaging is one of the pillar area in healthcare. Unlike other application scenarios in general computational imaging, medical imaging practitioners pay significant more attentions and cares in critical areas, aka, regions-of-interest (ROI) (Bubba et al., 2016). Very often, observing the abnormal (for example, a tumor or a tiny crack of some bone) in a patient is much more critical than reconstructing a overall clean image (Antun et al., 2020). The traditional and current state-of-the-art model-based iterative reconstruction algorithms, even the plug-and-play/regularization-by-denoising schemes (Venkatakrishnan et al., 2013; Romano et al., 2017; Reehorst and Schniter, 2018; Kamilov et al., 2017; Tang and Davies, 2020) with either kernel-based (Dabov et al., 2007; Buades et al., 2005; Milanfar, 2012; Tachella et al., 2021) or deep-learning based image priors (Zhang et al., 2017; Chen and Pock, 2017; Jin et al., 2017), are all tailored for global regularization and reconstruction of images and do not have a mechanism to focus on improving the reconstruction quality of local parts.

In this work, we propose a practical solution for superresolution and refine locally critical parts of tomographic images reconstructed from measurements given by medical imaging systems such as CT, MRI and PET, etc. Such imaging systems can be generally expressed as:

\[ b = Ax^\dagger + w, \]

where \( x^\dagger \in \mathbb{R}^d \) denotes the ground truth image (vectorized), and \( A \in \mathbb{R}^{n \times d} \) denotes the forward measurement operator, \( w \in \mathbb{R}^n \) the measurement noise (could be data-dependent), while \( b \in \mathbb{R}^n \) denotes the measurement data.
A classical way to obtain a reasonably good estimate of $x^\dagger$ is to solve a composite optimization problem:

$$x^\star \in \arg \min_{x \in \mathbb{R}^d} f(b, Ax) + \lambda g(x),$$

(2)

where data fidelity term $f(b, Ax)$ is typically a convex function (one example would be the least-squares $\|b - Ax\|_2^2$), while $g(x)$ being a regularization term, for example the total-variation (TV) semi-norm, and $\lambda$ denotes the regularization parameter controlling the strength of the regularization effect (Chambolle and Pock, 2016).

The regularization, both classical variational-regularization and more advanced learning-based regularization (Kamilov et al., 2017), is applied to the high-dimensional image space globally for overall good reconstruction quality. In the process of iterative reconstruction, the imaging quality of critical small regions of the image can often be compromised. These critical areas, such as infested parts and tumors, are usually patient-dependent, hence cannot be known a-priori. It would be desirable if we could perform a fast refinement and superresolution postprocessing step on critical parts pinpointed by clinicians on the reconstructed image by solving (2), utilizing a locally adjusted regularization. In this work, we propose a class of iterative reconstruction algorithms for this type of applications.

Alternatively, one may consider to reconstruct an extra-high resolution image instead. However, this would introduce a significant amount of extra computation, and meanwhile it still does not address the limitation of global regularization and would make the inverse problem more ill-posed. In clinical practice, it is highly-desirable to provide medical doctors a path of images with a grid of different regularization parameters and let the doctors decide which one of the images is the most likely to be the closest to the truth. In this situation, a global reconstruction of a sequence of images would be very computationally costly compare to our local reconstruction approach.

2. Data-Consistent Local Zoom-in

In this section we present our algorithmic framework for achieving efficient data-consistent local zoom-in for medical images.

2.1 Algorithmic Framework

Ideally, a successful algorithm for such a local zoom-in and refinement should take into account of the measurement data, whole reconstructed image from the first-stage, and locally adjusted image prior. Let $x_z$ denote the part of $x^\star$ which the clinicians wish to zoom-in and refine, and $x_o$ to be the complement of $x_z$, $A_z$ to be the block of $A$ associated with the coordinates of $x_z$, and $A_o$ being the complement of $A_z$, while $v \in \mathbb{R}^m$ to be the variable for the desired high-resolution image block. Denote

$$b_z = b - A_o x_o,$$

(3)

and $\mathcal{D}(\cdot)$ being some downsampling operator, and suppose we use least-squares data-fit, we write our program as:

$$x^* = \arg \min_{v \in \mathbb{R}^m} \|b_z - A_z \mathcal{D}(v)\|_2^2 + g_\lambda(v),$$

(4)

Then a proximal-gradient-like method for approximately solving (4) can be written as:

$$x_{k+1} = \mathcal{P}[x_k - \eta \cdot \mathcal{U}(A_z^T A_z \mathcal{D}(x_k) - A_z^T b_z)],$$

(5)
where $\mathcal{P}(\cdot) := \text{prox}_{\lambda \mathcal{P}}(\cdot)$, $\eta$ being the step-size, $\mathcal{U}(\cdot)$ being some up-sampling operator. Note that we do not need to restrict the regularization only to hand-crafted ones like TV, we could also consider advanced implicit regularization utilizing denoisers like BM3D and learning-based denoising-CNNs. Meanwhile both the upsampling and downsampling operator can be off-the-shelf algorithms, and can possibly be trained. In light of this, we present our generic algorithmic framework for data-consistent local zoom-in as:

**Local Zoom-in with Fast Gradients (LZFG)** — Initialize $x_0 \in \mathbb{R}^m$, $y_0 \in \mathbb{R}^m$, $a_0 = 1$

For $k = 0, 1, 2, ..., K$

\[
\begin{align*}
  x_{k+1} &= \mathcal{P}_{\theta_1}[y_k - \eta \cdot \mathcal{U}_{\theta_2}(A^T_z A_z D_{\theta_3}(y_k) - A^T_z b_z)] \\
  a_{k+1} &= (1 + \sqrt{1 + 4a_k^2})/2; \\
  y_{k+1} &= x_{k+1} + \frac{a_k^{-1}}{a_{k+1}} (x_{k+1} - x_k)
\end{align*}
\]

where $\theta_1, \theta_2, \theta_3$ are (possibly-trainable) parameters\(^1\) for denoiser $\mathcal{P}(\cdot)$, up-sampler $\mathcal{U}(\cdot)$ and down-sampler $D(\cdot)$, respectively. Our algorithm here can be view as an extension of the FISTA algorithm (Beck and Teboulle, 2009), and the plug-and-play framework (Venkatakrishnan et al., 2013; Kamilov et al., 2017; Reehorst and Schniter, 2018), with additional up and down sampling elements. For the momentum step of LZFG, we can alternatively use the choice of (Chambolle and Dossal, 2015) which reads:

\[
y_{k+1} = x_{k+1} + \frac{k - 1}{k + 3} (x_{k+1} - x_k), \quad (6)
\]

which has a provable convergence on iterates for convex problems.

To further reduce the per-iteration computational cost, we can further utilize the stochastic gradient schemes (Robbins and Monro, 1951; Xiao and Zhang, 2014; Allen-Zhu, 2017; Tang et al., 2017; Chambolle et al., 2018; Sun et al., 2019) to further accelerate our LZFG algorithm for several medical imaging applications such as CT and PET (Tang et al., 2020). The simplest case is below when we use vanilla minibatch stochastic gradient estimator:

**Local Zoom-in with Stochastic Gradients (LZSG)** — Initialize $x_0 \in \mathbb{R}^m$, $y_0 \in \mathbb{R}^m$

For $k = 0, 1, 2, ..., K$

\[
\begin{align*}
  x_{k+1} &= \mathcal{P}_{\theta_1}[y_k - \eta_k \cdot \mathcal{U}_{\theta_2}((S_k A_z)^T S_k A_z D_{\theta_3}(y_k) - (S_k A_z)^T S_k b_z)] \\
  y_{k+1} &= x_{k+1} + \frac{k - 1}{k + 3} (x_{k+1} - x_k)
\end{align*}
\]

This LZSG algorithm here uses a stochastic gradient estimator without variance-reduction (Johnson and Zhang, 2013), and hence typically we choose the step-size to be shrinking across iterations: $\eta_k = O\left(\frac{1}{\sqrt{k}}\right)$ to ensure convergence. We can also use the SVRG/SAGA-type (Johnson and Zhang, 2013; Defazio et al., 2014; Allen-Zhu, 2017; Tang et al., 2018) of gradient-estimators which are empowered with variance-reduction schemes, and for such cases we do not need the reducing step-sizes but instead a constant step-size\(^2\).

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\(^1\) Alternatively, we may also jointly optimize these parameters via alternating minimization (Bolte et al., 2014; Pock and Sabach, 2016; Driggs et al., 2021).

\(^2\) A side application of our LZFG/LZSG schemes would be the acceleration of global reconstruction of high-resolution images, where we choose the zoom-in region to be the whole global image.
2.2 Practical implementation and complexity of the block forward and adjoint operators

In practical programming of the reconstruction algorithms for CT and PET, we usually do not have direct access to the block operator $A_z$ but the full forward operator $A$. This issue can be fixed easily and efficiently for CT and PET reconstruction – in practice we just need to construct a zero-image of size $d$ and put the vector $D_{b_{z_k}}(y_k)$ at location of the region-of-interest (ROI), and then apply $A$ to this sparse image $v_s$ (since ROI is usually small), which is efficient. Since for CT/PET, the forward operator $A$ is a radon-transform which is sparse, $Av_s$ is also a sparse vector (also due to the fact that in each view only a small fraction of measurement can hit the ROI), hence the computation of $A^T Av_s$ is efficient – $O(nm)$ complexity instead of $O(ndq)$ where $q$ is the upscaling factor. Meanwhile the block backprojection term $A_z^T b_z$ in the gradient can also be easily precomputed in the same way (compute $A^T b_z$ and then take the block corresponds to the ROI). After computing the gradient, we just need to take the partial gradient regarding the ROI and perform the update.

3. Numerical Experiments

In this section we present some preliminary results for the proof-of-concept. We consider an example of a low-dose fan-beam CT imaging system where we have reconstructed a head image and a chest image, with TV regularization. The measurements we simulate are corrupted with Poisson noise:

$$b \sim \text{Poisson}(I_0 e^{-Ax})$$

and we take the logarithmic for the linearization of the measurements. We choose a low measurement energy with $I_0 = 2 \times 10^{13}$, the forward operator $A \in \mathbb{R}^{92160 \times 65536}$. We run all the experiments here in MATLAB R2018b.

For the first-stage reconstruction, we solve a TV-regularized least-squares optimization problem, and we grid search for the best global regularization parameter for this measurement system which gives the best mean squares error towards the global ground-truth.

As we can observe, the reconstructed image has a reasonable quality, but could oversmooth some of the local areas which may need to be refined and zoomed in. This observation suggest that although the regularization can be optimized to provide the best PSNR globally, classical iterative methods indeed have such a limitation for not taking care of local reconstruction performance. In this example we seek to zoom-in 4 times larger a 50 by 50 block of a 256 by 256 first-stage image. Hence the global iterative superresolution of the whole image will be 25 times more expensive than our approach in this setting. Direct zoom-in on this block would often have poor performance since it does not re-utilize the measurement data. We then test two instants of our LZFG framework. For the first one we use TV regularization while for the second one we use BM3D denoiser (Dabov et al., 2007). We use the MATLAB imresize function as the up-sampling and down-sampling operators, with the default bicubic interpolation.

We present in Figure 2, 3, 4, and 5 four examples of local zoom-in. We can observe from the numerical results that our method can indeed recover the details of the local blocks with high-quality. The direct zoom-in from the first-stage reconstruction fails to recover local details missed from the first-stage reconstruction since it does not utilize the measurement data, demonstrating the importance of data-consistency in local zoom-in of medical images. We present the convergence curves of LZFG with TV and BM3D in Figure 1 reporting the PSNR results towards the zoomed-in ground-truth image. From the PSNR result we can observe that our methods provide significantly improved reconstruction accuracy (5 to 10 dB better) compared to the naive approach.
Figure 1: Quantitative results for the 4 local zoom-in experiments on CT images. The first plot corresponds to the first example presented in Fig.2, and the second plot in the first row corresponds to the second example presented in Fig.3, and so on. From the PSNR results we can observe that our proposed data-consistent local zoom-in method LZFG achieves significantly improved reconstruction accuracy compared to the naive approach which directly zoom-in the first-stage reconstruction without utilizing the measurement data.

4. Conclusion

In this work, we propose a new paradigm of iterative model-based reconstruction algorithms tailored for the practical need of clinicians, to provide a real-time solution for local superresolution and refining regions of interests of medical images, which may be unclear with missing details due to oversmoothing from the classical global reconstruction. Our methods LZFG/LZSG build upon the proximal/PnP gradient methods (Beck and Teboulle, 2009; Chambolle and Dossal, 2015; Kamilov et al., 2017; Sun et al., 2019), with upsampling and downsampling operators allowing the algorithms to translate between the high-resolution and low-resolution image spaces, and approximate the gradient in high-resolution image space using the low-resolution one. The numerical results demonstrate that our methods provide much better local reconstruction with improved imaging on details, compared to the naive zoom-in which does not re-utilize the measurement data.

In clinical practice, we would advise medical imaging practitioners to predefine a range of regularization parameters (or parameters of the denoiser), and provide the clinicians a number of refined local images reconstructed under a grid of parameters, and let the clinicians to decide which image to chose. Computing a regularization path with our scheme on local regions of interests (pin-pointed by clinicians after checking the first-stage reconstruction) will be much more efficient (orders of magnitudes) than the brute-force approach which compute a regularization path of global reconstructions.
Figure 2: Local zoom-in results for low-dose fan-beam CT image (example 1)

Figure 3: Local zoom-in results for low-dose fan-beam CT image (example 2)
Figure 4: Local zoom-in results for low-dose fan-beam CT image (example 3)

Figure 5: Local zoom-in results for low-dose fan-beam CT image (example 4)
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