Active nematics are intrinsically phase-separated

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Two-dimensional nonequilibrium nematic steady states, as found in agitated granular-rod mono-
layers or films of orientable amoeboid cells, were predicted [Europhys. Lett. 62 (2003) 196] to have
giant number fluctuations, with standard deviation proportional to the mean. We show numerically
that the steady state of such systems is macroscopically phase-separated, yet dominated by fluctu-
ations, as in the Das-Barma model [PRL 85 (2000) 1602]. We suggest experimental tests of our
findings in granular and living-cell systems.

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The ordering or “flocking” \[^2\] of self-propelled particles obeys laws strikingly different from those
governing thermal equilibrium systems of the same spatial symmetry. Even in two dimensions, the ve-
clocities of particles in such flocks show true long-range order \[^2\], despite the spontaneous breaking
of continuous rotational invariance. Density fluctuations in the ordered phase are anomalously large \[^3\],
and the onset of the ordered phase is discontinuous \[^4\]. The ultimate origin of these nonequilibrium phe-
nomena is that the order parameter is not simply an orientation but a macroscopic velocity. It is thus in-
triguing that even the nematic phase of a collection of self-driven particles, which is apolar and hence has
zero macroscopic velocity, shows \[^4\] giant number fluctuations \[^5\], as a result of the manner in which
orientational fluctuations drive mass currents. This Letter takes a closer look at these fluctuations and
shows that they offer a physical realisation of the remarkable nonequilibrium phenomenon known as
fluctuation-dominated phase separation \[^6\], hitherto a theoretical curiosity.

Before presenting our results, we make precise the term active nematic. An active particle extracts en-
ergy from sources in the ambient medium or an internal fuel tank, dissipates it by the cyclical motion
of an internal “motor” coordinate, and moves as a consequence. For the anisotropic particles that con-
cern us here, the direction of motion is determined predominantly by the orientation. Our definition
ecompasses self-propelled organisms, living cells, molecular motors, and macroscopic rods on a verti-
cally vibrated substrate (where the tilt of the rod

serves as the motor coordinate). An active nematic
is a collection of such particles with axes on average spontaneously aligned in a direction \[^b\], with invari-
ance under \[^b\] \rightarrow -\[^b\]. We know of two realisations of
active nematics: collections of living amoeboid cells
\[^10\] and granular-rod monolayers \[^11\],[^12].

We study active nematics in a simple numerical
model described in detail below. Our results confirm
(see Fig. 1) the giant number fluctuations (standard
deviation \(\propto\) mean) \[^6\] predicted by the linearised
analysis of \[^5\], but are far richer: (i) A statisti-
cally uniform initial distribution of particles, on a
well-ordered nematic background, undergoes a deli-
cate “fluctuation-dominated” \[^5\] phase separation,
where the system explores many statistically sim-
ilar segregated configurations. (ii) The equal-time
two-point density correlator \(C(r)\), Fig. 3 shows a
collapse when plotted as a function of \(r/L(t)\), where
\(L(t)\) is the location of the first zero-crossing of \(C(r)\),
with a cusp at small \(r/L(t)\) signalling a departure
from Porod’s Law, i.e., the absence of sharp inter-

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\begin{figure}
\centering
\includegraphics[width=\textwidth]{fig1}
\caption{Number standard deviation \(\Delta_N\) scales roughly as the mean \(N\), for system sizes \(L = 128, 256\).}
\end{figure}
faces. (iii) We confirm that the large density inhomogeneities are indeed best thought of as phase separation, by showing (1) that the saturation value of \( L(t) \) is proportional to the linear size of the system and (2) that the phase-separation order parameter – the time-averaged first spatial Fourier component of the particle density – approaches a nonzero value in the limit of large system size (Fig. 2). (iv) \( L(t) \) grows clearly faster than the \( t^{1/3} \) for normal conserved-order-parameter coarsening (see inset to Fig. 2), and is consistent with \( t^{1/2} \) as expected by analogy to \( \mathbb{R} \). Below we show how these results were obtained, discuss them in detail, explain the analogy to \( \mathbb{R} \), and suggest experimental tests for these striking phenomena.

We begin with some background information. An apolar, uniaxial, compressible nematic liquid crystal is described by a director and density fields \( \hat{n}(r) \) and \( c(r) \), with fluctuations \( \delta n(r) \) and \( \delta c(r) \) about their uniform mean values \( \bar{n}_0 \) and \( c_0 \). Let us review first what happens at thermal equilibrium. The system is then governed by an extended Frank free-energy \( F[\hat{n}, c] = (1/2) \int d^3r |K(\nabla \hat{n})|^2 + A(\delta c)^2/c_0 + C_1 \hat{n} \cdot \nabla c \nabla \cdot \hat{n} + C_2 \hat{n} \times \nabla c \cdot \nabla \times \hat{n} \), where \( K \) is an elastic tensor, \( A \) the compressional modulus at constant orientation, and \( C_{1,2} \) couple orientation and density in the simplest symmetry-allowed fashion. Equipartition applied to \( F \) implies that the static structure factor \( S_q \equiv \int d^3r \exp(-i q \cdot r) \delta n(0) \delta n(r) / c_0 \) is finite for \( q \to 0 \); i.e., the mean \( N \) and standard deviation \( \Delta_N \) of the number of particles obey \( \Delta_N \propto \sqrt{N} \) at equilibrium even when \( C_1, C_2 \neq 0 \).

An active nematic is a steady state away from thermal equilibrium, in which not only dynamic correlators but equal-time quantities like \( S_q \) or \( \Delta_N \) as well must be inferred from equations of motion for \( \hat{n} \) and \( c \). As shown in \( \mathbb{R} \), the equation of motion for \( \hat{n} \) is qualitatively the same as for equilibrium nematics. The feature \( \mathbb{R} \) that distinguishes active nematics crucially from their equilibrium counterparts is that the current \( j \) in the continuity equation \( \partial_t c = -\nabla \cdot j \) for the density has a contribution \( \propto \nabla \cdot c(\hat{n}) \). This term, which is ruled out at thermal equilibrium, has a simple, physically appealing origin: spatial variation in the director field \( \hat{n} \) defines a curve; the normal to this curve defines a local vectorial asymmetry; for a driven system, such an asymmetry implies a current \( \mathbb{R} \). If \( \hat{n} = (\cos \theta, \sin \theta) \) then inhomogeneities in \( \theta \) give a curvature-induced current \( j = (j_x, j_z) \propto (\partial \theta/\partial z, \partial \theta/\partial x) \) (see Fig. 3), analogous to \( \mathbb{R} \) where particles slide with velocity \( \propto \nabla h \) on a fluctuating interface with height field \( h \), except that our current is not a gradient. The Since nematic order is a spontaneous breaking of rotation-invariance, large fluctuations in \( \theta \) at long wavelengths are expected to be present in abundance, and to decay slowly, in any nematic, equilibrium or otherwise. We showed in the previous paragraph that the effect of these broken-symmetry modes on the density field was benign in an equilibrium nematic. In an active nematic, however, the same orientational fluctuations, because of the curvature-induced current we just mentioned, will affect the density fluctuations substantially. A linearised small-fluctuations analysis \( \mathbb{R} \) showed that they lead to giant fluctuations in the number of particles: \( \Delta_N / \sqrt{N} \propto N^{1/d} \) in \( d \) dimensions, i.e., \( \Delta_N \propto N \) for \( d = 2 \).

Such large fluctuations prompt the suspicion that an analysis beyond Gaussian fluctuations would reveal that the system is in fact phase-separated, as in \( \mathbb{R} \). There are two issues here: (i) whether the nonequilibrium coupling mentioned above inevitably arises in an active nematic; and (ii) whether it leads to phase separation. Ref. \( \mathbb{R} \) effectively answers the first question in the affirmative; we focus on the second.

We find it convenient to separate the density and orientation degrees of freedom, and employ a discrete model of lattice-gas particles coupled to an angle field, incorporating explicitly the the nonequilibrium curvature-induced current \( j \propto (\partial \theta, \partial \theta, \partial \theta) \) mentioned above, via a suitable choice of particle-hopping rates. We consider a two-dimensional lattice with angles \( \theta_i \in [0, \pi] \) and noninteracting lattice-gas occupancy variables \( n_i = 0, 1 \) at each site \( i \). The angles evolve by Metropolis Monte Carlo updates governed by the Lebwohl-Lasher \( \mathbb{R} \) hamilto-
Fig. 3: Director variation from the bottom to the top of the picture gives rise to a current in a transverse direction.

Fig. 4: Anisotropic clustering of particles, 20000 time steps, system size 100, 50% occupancy.

The saturation length is numerically small compared to $L$, probably because of the poorly-defined clusters (Fig. 4) of fluctuation-dominated phase separation. That we work with hard-core particles, and on timescales on which the macroscopic variation of the mean nematic orientation is very small, also probably contributes. The exponent of 1/2 is because the particles aggregate not by diffusion plus short-range capture, but rather, by analogy with \cite{1N}, because the broken-symmetry mode of the nematic order sweeps the particles over large distances via curvature-induced drift. A nematic fluctuation on a scale $\ell$ collects particles in a time of order $\ell$. For two such domains to coalesce requires the nematic director field on that scale to turn over, which is a time $\sim \ell^2$ where $z = 2$ is the dynamic exponent of transverse fluctuations of the nematic director.

At long times a steady state is reached, and $C(r/L(t \to \infty))$ shows a cusp at small argument ($C(x) \propto x^a$, $a \approx 0.33$), which can be seen in Fig. 4 as well, signalling the absence of sharp interfaces between regions rich and poor in particles, and a power-law distribution of cluster sizes. For steady state in the largest system, we also measured the standard deviation $\Delta_N$ in the number of particles in an observation containing $N$ particles on average. The plot of $\Delta_N$ vs $N$, Fig. 1 shows precisely the linear dependence predicted by \cite{1}. Faced with these results, we ask: is this phase separation or a single phase with large fluctuations? This is answered by measuring the magnitude of the time-averaged lowest spatial Fourier-component $Q(1,1)$ of the density, shown in Fig. 4. Although the data are not conclusive, the flattening of the semilog plot as a function of system size rules out an exponential decay to zero. Together with the proportionality of the
coarsening length to the system size, and the nature of the mechanism, this suggests strongly that active nematics offer the most natural physical realisation of macroscopic fluctuation-dominated phase separation. As in [8], we find that the time-series of configuration to another, spending very little time in non-phase-separated states. Lastly, the velocity autocorrelation of tagged particles at low (15%) concentration agrees qualitatively with the 1/t tail (plot not shown) predicted by [3] over the range in which a given particle moves unimpeded by others.

What experiments can test these results? The best would be agitated layers of granular rods, for which nematic phases have been reported [12]. Although many features of these systems can be rationalized in terms of equilibrium hard-rod theories [10], some properties such as global circulation and swirls [11] are clearly very nonequilibrium. These systems as well as the living melanocyte nematic of Gruler et al. [10] remain the most promising candidates for experimental tests of the rich range of results made here and in [3]. The confirmation of giant number fluctuations in the numerical experiments of [8] is encouraging in this regard.

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![FIG. 6: Time-averaged fourier component of the density profile vs. wave vector \( k \) for wave vector direction (1,1), for system sizes \( L = 16, 32, 64, 128 \); flattening in semilog plot at largest size suggests nonzero value for \( L \to \infty \). Inset: Time series of \( Q \) for (1,1) direction.

\[ Q(1,1) \text{ shows enormous fluctuations, (Fig. 6 inset), with \"crashes\" during which other nearby fourier-components gain weight. Thus, as in [8], the system lurches from one macroscopically phase-separated } ]

\[ \text{configuration to another, spending very little time in non-phase-separated states. Lastly, the velocity autocorrelation of tagged particles at low (15\%) concentration agrees qualitatively with the } 1/t \text{ tail (plot not shown) predicted by [3] over the range in which a given particle moves unimpeded by others.} \]

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