Optical rogue waves in the generalized inhomogeneous higher-order nonlinear Schrödinger equation with modulating coefficients

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Abstract

Higher-order dispersive and nonlinear effects (alias the perturbation terms) such as third-order dispersion, self-steepening, and the self-frequency shift play important roles in the study of ultra-short optical pulse propagation. We consider optical rogue wave solutions and interactions for the generalized higher-order nonlinear Schrödinger (NLS) equation with space- and time-modulated parameters. An appropriate transformation is presented to reduce the generalized higher-order NLS equation to an integrable Hirota equation with constant coefficients. This transformation allows us to relate a certain class of exact solutions of the generalized higher-order NLS equation to the variety of solutions of the integrable Hirota equation. In particular, we illustrate the approach in terms of the two lowest-order rational solutions of the Hirota equation as seeding functions, to generate rogue wave solutions localized in time that have complicated evolution in space, with or without the differential gain or loss term. We simply analyze the physical mechanisms of the obtained optical rogue waves on the basis of these constraints. Finally, the stability of the obtained rogue wave solutions is addressed numerically. The obtained rogue wave solutions may raise the possibility of related experiments and potential applications in nonlinear optics and other fields of nonlinear science, such as Bose–Einstein condensates and ocean waves.

Keywords: higher-order effects, the ultra-short optical pulse, generalized higher-order NLS equation, optical rogue waves, Hirota equation

(Some figures may appear in colour only in the online journal)

1. Introduction

The laser beam intensity-dependent refractive index of the optical Kerr effect \(n(I) = n_0 + n_2 I\) can generate a nonlinear Schrödinger (NLS) equation

\[
\frac{\partial \psi}{\partial z} + \frac{\beta}{2} \frac{\partial^2 \psi}{\partial t^2} + g|\psi|^2 \psi = 0
\]

\(1\)

describing the propagation of light within an optical fiber or a uniform nonlinear medium [1–4], where \(n_0\) denotes the linear refractive index, \(n_2\) stands for a Kerr-type nonlinearity of the waveguide amplifier, \(I\) is the laser beam intensity, \(\beta\) denotes the group velocity dispersion (GVD), and \(g\) is the Kerr nonlinearity (or self-phase modulation). Equation (1) is also called the Gross–Pitaevskii equation without an external potential in Bose–Einstein condensates [5–8] if one replaces \(z\)
by $t$ and $r$ by $x$. Equation (1) has been shown to be completely integrable [9] and to admit optical solitons by balancing the GVD $\beta$ and Kerr nonlinearity $g$ (the self-focusing interaction and defocusing interaction corresponding to bright and dark solitons, respectively) [1, 9, 10]. For a varying second-order GVD $\beta \rightarrow \beta(z)$ and self-phase modulation (SPM) $g \rightarrow g(z)$, equation (1) describes the propagation of optical pulses in a self-similar manner in a nonlinear waveguide [11–13], and is shown to have novel (self-similar) optical solitons [14–16], including some free functions of space $z$, which generate complicated wave propagations. The nonlinear self-similar wave implies that its profile remains unchanged and its amplitude and width simply scale with time or propagation distance (see, e.g., [16–21]). If one exchanges the variables $z$ and $t$, then this kind of novel soliton is called a nonautonomous soliton (see, e.g., [24]). Other generalized models of equation (1) with varying parameters have been studied, such as the varying external potential [17, 22–25], the varying gain or loss term [17, 25, 26], and three-dimensional cases [26–28], and shown to admit exact solutions and complicated wave propagation.

When the optical pulses become shorter (e.g., 100 fs [1]), higher-order dispersive and nonlinear effects, such as third-order dispersion (TOD), self-steepening (SS), and the self-frequency shift (SFS) (alias the perturbation terms) arising from the stimulated Raman scattering become significant in the study of ultra-short optical pulse propagation (see, e.g., [1]). With the above-mentioned aim, Kodama and Hasegawa [29, 30] presented the higher-order NLS equation

$$i\frac{\partial \psi}{\partial z} + \frac{\beta^2}{2}\frac{\partial^2 \psi}{\partial t^2} + g|\psi|^2\psi = i\epsilon \left[ a_1 \frac{\partial^3 \psi}{\partial t^3} + a_2 \frac{\partial((|\psi|^2)\psi)}{\partial t} + a_3 \frac{\partial|\psi|^2}{\partial t} \right] + i\Gamma\psi,$$

(2)

reduced from the Maxwell equation $\nabla \times \nabla \times \mathbf{E} = -\frac{1}{c^2}\frac{\partial^2 \mathbf{D}}{\partial t^2}$, with $c$ being the speed of light and $\mathbf{D} = \epsilon \ast \mathbf{E}$ [30], where $\mathbf{E}$ is the electric field intensity, $\epsilon$ is a real-valued parameter, and the real-valued parameters $\beta$, $g$, $a_1$, $a_2$, $a_3$, and $\Gamma$ are related to GVD, SPM, SS, SFS, and the gain or loss terms, respectively. Equation (2) contains many types of integrable model, such as the Hirota equation [31], Sasa–Satsuma equation [32], KN-type derivative NLS equation [33], and CLL-type derivative NLS equation [34]. Equation (2) has been found to admit many types of solutions (see, e.g., [35–38]). For an ultra-short pulse in a self-similar matter, equation (2) with varying coefficients has been shown to support exact solutions for some constraints of coefficients (see, e.g., [39–43]). Experimental observations have been recently performed for optical solitons and pulse compression in $\sim$1 mm-long photonic crystal waveguides [44] based on the higher-order NLS model [45].

Recently, the NLS equation and its extensions have received much attention due to the interesting phenomenon of rogue waves, which are localized both in space and time and depict a unique event that appears from nowhere and disappears without a trace [46]. Rogue waves are also known as freak waves, monster waves, killer waves, giant waves or extreme waves. In particular, optical rogue waves have been experimentally verified in nonlinear optics and play an important role in supercontinuum generation [47, 48] on the basis of the generalized NLS equation with $N = 3–6$

$$i\frac{\partial A}{\partial z} + \sum_{n=2}^{N} \frac{i^n\beta_n}{n!} \frac{\partial^n A}{\partial t^n} = -g \left[ |A|^2 A + \frac{i}{\omega_0} \frac{\partial(|\psi|^2\psi)}{\partial t} - T_R A \frac{\partial|\psi|^2}{\partial t} \right],$$

where $A(z, t)$ denotes the field, $\beta_n$ are real values characterizing the fiber dispersion, $g$ is the nonlinear coefficient of the fiber, $\omega_0$ is the central carrier frequency of the field, and $T_R$ is a parameter that characterizes the delayed nonlinear response of silica fiber.

Moreover, optical rogue waves have been theoretically studied in telecommunication data streams [49]. Apart from optical rogue waves, rogue wave phenomena also appear in the ocean [50–53], Bose–Einstein condensates [54, 55], and even finance [56, 57]. Rogue wave solutions have analytically been found for many types of nonlinear physical models, such as nonlinear models with constant coefficients (e.g., the NLS equation [58, 59], higher-order NLS equations [60, 61], Hirota equations [62]), discrete models (e.g., discrete Ablowitz–Ladik and Hirota equations [63, 64], the generalized Ablowitz–Ladik–Hirota lattice with varying coefficients [65]), nonlinear models with varying coefficients (e.g., the NLS equation with varying coefficients [25], three-dimensional NLS equations with varying coefficients [55]), and coupled NLS equations [57, 66, 67]. Recently, controllable rogue waves of equation (2) with all coefficients depending only on space $z$ have been considered [68], but one more condition was missed to support the obtained results, that is to say, the sum of the parameters related to self-steepening and the self-frequency shift should be zero ($a_2 + a_3 = 0$).

Equation (2) with coefficients depending only on space $z$ has been studied before (see, e.g., [39–43, 68]). A natural problem is what waves space/time-modulated GVD and SPM terms can cause equation (2) to be excited. Moreover, could other terms (e.g., the additional external potential [17, 22–25], the differential gain or loss parameter [69], and the group velocity [1, 45, 70]) excite equation (2) to generate new phenomena? Based on these motivations related to some physical phenomena, in this paper we study the generalized model of equation (2) with space/time-modulated GVD, SPM, and gain or loss terms, space-modulated TOD, SS and SFS, and more terms such as a space/time-modulated external potential, linear group velocity, and differential gain or loss term [1, 69]

$$i\frac{\partial \psi}{\partial z} = \beta(z, t) \frac{\partial^2 \psi}{\partial t^2} + [V(z, t) + i\gamma(z, t)] \psi + g(z, t)|\psi|^2\psi + i \left[ \alpha_1(z) \frac{\partial^3 \psi}{\partial t^3} + \alpha_2(z) \frac{\partial((|\psi|^2)\psi)}{\partial t} + \alpha_3(z) \frac{\partial|\psi|^2}{\partial t} \right]$$

$$+ [\mu(z) + i\sigma(z, t)] \frac{\partial \psi}{\partial t},$$

(3)
where \( z \) is the normalized propagation distance along the optical fiber, \( t \) is the retarded time, \( \psi \equiv \psi(z,t) \) denotes the slowly varying envelope amplitude of the electric field measured in units of the square root of the power at position \( z \) in the optical fiber and at time \( t \), \( g(z,t) \), \( \alpha_1(z) \), \( \alpha_2(z) \), and \( \alpha_3(z) \) are all real-valued functions of the listed variables and stand for GVD, SPM, TOD, SS, and SFS arising from stimulated Raman scattering, respectively [1]. \( V(z,t) \) and \( \gamma(z,t) \) are the external potential and gain or loss distribution, respectively, \( \mu(z) \) denotes the differential gain or loss parameter [69], and \( \sigma(z,t) \) is related inversely to the group velocity of the modes (a walk-off effect) [1, 45, 70]. Equation (3) is associated with a variational principle or loss parameter [69], and

\[
\frac{\delta L}{\delta \psi^*} = 0 \quad \text{with the Lagrangian density}
\]

\[
\begin{align*}
\mathcal{L} &= i(\psi_\psi^* - \psi_\chi^*) - 2\beta(z,t)|\psi|^2 + g(z,t)|\psi|^4 \\
&+ 2[V(z,t) + i\gamma(z,t)]|\psi|^2 + \alpha_1(z)(\psi_\psi^* - \psi_\chi^*) + (\mu + i\sigma)(z,t)(\psi_\psi^*) - \psi_\chi^* \psi_\chi^* \\
&\quad + [\alpha_2(z)(|\psi|^2) + \alpha_3(z)(|\psi|^4)]|\psi|^2,
\end{align*}
\]

where \( \psi^* \) stands for the complex conjugate of the electric field \( \psi \), and the subscript denotes the partial derivative with respect to the variables \( z,t \). Equation (3) describes many types of nonlinear model, such as the NLS equation with varying coefficients [14–16], the derivative NLS equation with varying coefficients [33, 34], the Hirota equation with varying coefficients, the Sasa–Satsuma equation with varying coefficients [39–43]), and the higher-order NLS equation without three-photon nonlinear absorption [45].

The rest of this paper is organized as follows. In section 2, we present a proper transformation reducing equation (3) to the Hirota equation and determine the similarity variables and constraints satisfied by the external potential, GVD, SPM, TOD, SS, SFS, and gain or loss terms in equation (3). Section 3 mainly focuses on two types of time-localized rogue wave solutions for some chosen parameters. Moreover, we analyze the obtained first-order optical rogue wave solutions of equation (3) by using numerical simulations. Finally, these results and discussions are summarized in the conclusion.

2. Symmetry reductions and solutions

2.1. Symmetry reductions

In general, equation (3) is not integrable since these varying coefficients strongly affect the wave propagation of optical pulses in a self-similar manner. In order to study exact analytical solutions of equation (3), we need to look for some integrability conditions using methods such as the symmetric reduction approach or Painlevé analysis (see, e.g., [9, 71, 72] and references therein). Here we consider the symmetry reductions of equation (3). Equation (3) describes many different types of symmetry reduction by employing the Lie group transformation method (see, e.g., [71, 72]) to equation (3), which means that these similarity reductions can be obtained using the third-order propagation \( \text{Pr}^3(\chi) \) of the vector field (also called the infinitesimal generator)

\[
\chi = \frac{\partial}{\partial t} + Z \frac{\partial}{\partial z} + \theta_\beta \frac{\partial}{\partial \beta} + \theta_\gamma \frac{\partial}{\partial \gamma} + \theta_\mu \frac{\partial}{\partial \mu} + \theta_\sigma \frac{\partial}{\partial \sigma}
\]

acting on equation (3), i.e., \( \text{Pr}^3(\chi)F|_{\theta=0} = 0 \), where \( \psi(z,t) = \psi_R(z,t) + \psi_i(z,t) \) with \( \psi_R, \psi_i \in \mathbb{R} \), the variables \( T, Z, \theta_\beta \), and \( \theta_\gamma \) are all unknown functions of \( z,t, \psi_R, \psi_i \) to be determined, and

\[
F = \{-i\beta_0 + \beta_1 t^2 + (V + iy) + g|\psi|^2 + i[\alpha_1 t^2 + \alpha_2 |\psi|^2] + (\mu + i\sigma)|\psi|^4 + \alpha_3 G(|\psi|^2)\psi_\psi^* + \psi_\chi^* \psi_\chi^* \}, \quad \text{Pr}^3(\chi) = \chi + \theta_\beta \frac{\partial}{\partial \beta} + \theta_\gamma \frac{\partial}{\partial \gamma} + \theta_\mu \frac{\partial}{\partial \mu} + \theta_\sigma \frac{\partial}{\partial \sigma}.
\]

The similarity variables and transformations can be found by solving the characteristic equation

\[
\frac{\partial \psi}{\partial R} = \frac{\partial \psi}{\partial z} = \frac{\partial \psi}{\partial \sigma} = \frac{\partial \psi}{\partial \theta}
\]

connecting the solutions of equation (3) with those of the following Hirota equation with constant coefficients [31]

\[
\frac{i}{\partial R} \frac{\partial \psi}{\partial R} = - \frac{\partial^2 \psi}{\partial \tau^2} + G|\psi|^2\psi + 2\sqrt{2}In\left(\frac{\partial^2 \psi}{\partial \tau^2} + 3|\psi|^4\frac{\partial \psi}{\partial \tau}\right),
\]

where the physical field \( \Psi(\eta, \tau) \) is a function of the variables \( \tau(z,t) \) and \( \eta(z) \), which are the new temporal and spatial coordinates, respectively, and \( G, \nu \) are both real-valued constants. Since the main goal of this paper focuses on rogue waves of equation (3), we choose \( G < 0 \) (e.g., \( G = -1 \)) in equation (7), which corresponds to the attractive case (or self-focusing interactions in nonlinear optics [1]) and attractive interactions (negative scattering lengths) in the BEC theory [5–8].

Notice that the higher-order equation (7) with \( \nu \neq 0 \) differs from the NLS equation (1), which admits two basic transformations, i.e., the scaling and gauge transformations, leaving itself invariant (see, e.g., [73]). We find equation (7) has the following proposition:

**Proposition.** Equation (7) possesses two modified basic transformations leaving itself invariant, i.e., the following scaling-parameter transformation

\[
\eta \rightarrow \alpha^2 \eta, \quad \tau \rightarrow \alpha \tau, \quad \Psi \rightarrow \Psi/\alpha, \quad \nu \rightarrow \alpha \nu,
\]

leaving itself invariant for any SPM parameter \( G \neq 0 \), where \( \alpha \) is a real-valued constant, and the ‘gauge’ transformation

\[
\eta \rightarrow k^3 \eta, \quad \tau \rightarrow k \tau + \lambda \eta, \quad \Psi \rightarrow k^2 \psi e^{i(p\tau + q\eta)},
\]

leaving equation (7) invariant only for the self-focusing SPM parameter \( G = -1 \), which just makes equation (7) possess rogue wave solutions [62], where \( k = 1 + 6\sqrt{2}p, \lambda = -kp(1 + k), q = -p^2(2\sqrt{2}p + 1) \) with \( p \) being a real-valued constant.
Based on the above-mentioned method, the substitution of equation (6) into (3) with $\Psi(\eta, \tau)$ satisfying equation (7) yields the following system of partial differential equations

$$\begin{align*}
\alpha_1 \tau \tau_0 &= 0, \quad \alpha_2 + \alpha_3 = 0, \quad (10a) \\
\beta \tau_0 - 3\alpha_1 (\tau \psi + \psi \psi) + \mu \tau &= 0, \quad (10b) \\
\eta_1 + \chi_1^2 (\beta - 3\alpha_1 \psi) &= 0, \quad (10c) \\
\sigma_1 + 2\beta \tau \psi_0 + \alpha_1 (\tau_0 - 3 \chi_1^2 \psi_0) - \tau_0 &= 0, \quad (10d) \\
\rho_1 - \rho [\beta \psi_0 - 3\alpha_1 \psi \psi_0 + \mu \psi + \gamma] &= 0, \quad (10e) \\
V + \psi_0 - \beta \chi_1^2 + \alpha_1 (\chi_1^2 - \psi_0) - \sigma \psi_0 &= 0, \quad (10f) \\
\alpha_1 \chi_1^3 - 2\sqrt{2} \nu \eta_1 &= 0, \quad (10g) \\
\rho^2 (g - \alpha_2 \psi_0) - \sqrt{2} \nu g &= 0, \quad (10h) \\
\alpha_2 \rho^2 \tau_0 - 6\sqrt{2} \nu \eta_1 &= 0. \quad (10l)
\end{align*}$$

Generally speaking, the equations in system (10) may not be compatible with each other, however, one can find suitable constraints for these coefficients $\beta(z, t), g(z, t), V(z, t), \gamma(z, t), \alpha_1(t), \mu(z), \sigma(z, t)$ such that system (10) is compatible. This requirement leads us to the following procedure.

First of all, we solve equations (10a)–(10l) to obtain the similarity variables $\tau(z, t), \eta(z)$, and the phase $\psi(z, t)$ subject to the GVD parameter $\beta(z, t)$ and the differential gain or loss term $\mu(z)$.

Secondly, it follows from equations (10a)–(10l) that we can determine the amplitudes $\rho(z)$, the external potential $V(z, t)$, SPM $g(z, t)$, GVD $\beta(z, t)$, TOD $\alpha_1(z)$, $\sigma(z, t)$ SS $\alpha_2(z)$ and SFS $\alpha_3(z)$ in terms of the obtained variables $\eta(z), \tau(z, t), \psi(z, t)$.

Finally, we may establish a ‘bridge’ (also called a Lie–Bäcklund transformation) between exact solutions of equation (3) and those of the completely integrable Hirota equation (7). The latter admits an infinite number of solutions, thereby giving us an approach to find physically relevant solutions of equation (3). Here we consider only optical rogue wave solutions of equation (3) based on the rogue waves of the Hirota equation [62].

2.2. Determining similarity variables and controlled coefficients

For the considered equation (3) in the presence of TOD, i.e., $\alpha_1(z) \neq 0$, it follows from equation (10a) that the new temporal variable $\tau(z, t)$ should be of the form

$$\tau = \tau_1(z) t + \tau_0(z),$$

where $\tau_1(z)$ and $\tau_0(z)$ are functions of $z$. The substitution of equation (11) into equation (10b) yields

$$3\alpha_1 \psi_0 - \mu \tau_1 = 0,$$

which leads to two cases, since $\tau_1 \neq 0$ (i.e., $\tau_1(z) \neq 0$ is required, otherwise the field $\Psi(\eta, \tau)$ is only a function of $z$):

(I) $\mu(z) \equiv 0, \quad \psi(z, t) = \psi_1(z) t + \psi_0(z)$;

(II) $\mu(z) \neq 0, \quad \psi(z, t) = \psi_2(z) t^2 + \psi_1(z) t + \psi_0(z)$ with $\mu(z) = 6\alpha_1(z) \psi_2(z)$.

Based on the above-mentioned two cases, we have the following solutions of system (10):

Case I. In the absence of the differential gain or loss term $\mu(z)$, i.e., $\mu(z) \equiv 0$.

$$\tau(z, t) = \tau_1(z) t + \tau_0(z), \quad \psi(z, t) = \psi_1(z) t + \psi_0(z),$$

$$\eta(z) = \sqrt{\frac{\gamma}{4\nu}} \int_0^z \alpha_1(s) \tau_1^2(s) ds,$$

$$\rho(z) = \rho_0 \exp \left[ \int_0^z \gamma(s) ds \right],$$

$$\beta(z) = \alpha_1(z) \left[ 3\psi_1(z) - \frac{\tau_1(z)}{2\sqrt{2} \nu} \right],$$

$$g(z) = \frac{\alpha_1(z) \tau_1^2(z)}{\rho^2(z)} \left[ 3\psi_1(z) + \frac{G \tau_1(z)}{2\sqrt{2} \nu} \right],$$

$$\sigma(z, t) = \frac{\tau_1(z) t + \tau_0(z)}{\tau_1(z)} + \alpha_1(z) \psi_1(z) \left[ \frac{\tau_1(z)}{2\sqrt{2} \nu} - 3\psi_1(z) \right],$$

$$V(z, t) = \left[ \psi_1(z) \frac{\tau_1(z)}{\tau_1(z)} - \psi_1(z) \right] t + \psi_1(z) \frac{\tau_0(z)}{\tau_1(z)},$$

$$+ \alpha_1(z) \psi_1(z) \left( \frac{\tau_1(z)}{2\sqrt{2} \nu} - \psi_1(z) \right) - \psi_1(z),$$

$$\alpha_3(z) = -\alpha_2(z) = -3\alpha_1(z) \tau_1^2(z) \rho^{-2}(z),$$

where the dot over the variables denotes the space derivative, $\tau_1(z) \neq 0$ is the inverse of the widths of the pulse, $-\tau_0(z)/\tau_1(z)$ is the center of the pulse, $\phi_1(z)(j = 1, 2)$ denote the frequency shift and the phase-front curvature, respectively, $\tau_1(z)(j = 0, 1), \phi_1(z)(j = 1, 2)$, $\gamma(z)$ and $\alpha_1(z)$ are free differentiable functions of space $z$, and $\rho_0$ is a constant.

It is easy to see that in the absence of the differential gain or loss term $\mu \equiv 0$, the GVD parameter $\beta(z, t)$, SPM parameter $g(z, t)$ and gain or loss term $\gamma(z, t)$ are only functions of space $z$. It follows from equations (13b) to (13d) that the gain or loss terms $\gamma(z)$ can be used to manipulate the amplitude $\rho(z)$. SS parameter $\alpha_2(z)$, and SPM parameter $g(z)$. The TOD parameter $\alpha_1(z)$ is used to control the variable $\eta(z)$, the GVD parameter $\beta(z)$, SFS parameter $\alpha_3(z)$, SPM parameter $g(z)$, $\sigma(z)$, and potential $V(z, t)$.

Notice that the coefficients of the first degree term in $\tau(z, t)$ and the phase $\psi(z, t)$ differ from ones in which they must be constants [68], since we consider more two terms in equation (3), i.e., the group velocity term $\sigma(z, t)$ and the external potential $V(z, t)$. The varying parameters $\tau_1(z)$ and $\phi_1(z)$ will excite complicated structures which may be useful to control the propagation of optical ultra-short pulses (see, e.g., [17, 22–25] for the similar waves).

To clearly understand these constraints on the coefficients in equation (3) we will present the following special case. If we require that all the coefficients in equation (3) depend on only space, i.e., the external potential $V(z, t)$ and the group velocity $\sigma(z, t)$ depend on only space $z$, then it follows from equations (13e) and (13f) that we have the conditions

$$\tau_1(z) = 0, \quad \psi_1(z) = 0,$$
i.e., $\tau_1(z)$ and $\varphi_1(z)$ should be real constants:

$$\tau_1(z) = \sqrt{2}C_1, \quad \varphi_1(z) = C_2,$$

with $C_{1,2}$ being constants. In this case, system (13) takes the simple form

$$\tau(z, t) = C_1 t + \tau_0(z), \quad \varphi(z, t) = C_2 t + \varphi_0(z), \quad (16a)$$

$$\eta(z) = \frac{C_1^3}{v} \int_0^z \alpha_1(s) \, ds, \quad \beta(z) = \left(\frac{6\mu C_2 - C_1}{2\nu}\right) \alpha_1(z), \quad (16b)$$

$$g(z) = \frac{C_1^2(6\nu C_2 - C_1 G) \alpha_1(z)}{\rho^2(z)}, \quad (16c)$$

$$\sigma(z) = \frac{C_2(C_1 - 3\nu C_2)}{v} \alpha_1(z) + \frac{\tau_0(z)}{\sqrt{2}C_1}, \quad (16d)$$

$$V(z) = \frac{C_1^2(C_1 - 2\nu C_2)}{2\nu} \alpha_1(z) + \frac{C_2 \tau_0(z)}{\sqrt{2}C_1} \alpha_1(z), \quad (16e)$$

and $\rho(z)$ is given by equation (13b), where $\alpha_1(z)$, $\gamma(z)$, $\tau_0(z)$, $\varphi_0(z)$ are free functions of space, and $C_1$, $C_2$, $v$, $\rho_0$ are free constants.

Notice that these free parameters can modulate the coefficients $\beta(z)$, $g(z)$, $\sigma(z)$, $V(z)$, $\alpha_2,3(z)$ in equation (3), which enlarges the scope for these coefficients in studying models (see, e.g., [35, 38, 44, 45]) such that more wave phenomena may be generated by modulating these coefficients.

Case II. In the presence of the differential gain or loss term $\mu(z)$, i.e., $\mu(z) \neq 0$.

$$\tau = \tau_1(z)t + \tau_0(z), \quad \eta = \sqrt{\frac{2}{\nu}} \int_0^z \alpha_1(s) \tau_1^3(s) \, ds, \quad (17a)$$

$$\varphi = \frac{\mu(z)}{6\alpha_1(z)} \tau_1^2 + \varphi_1(z)t + \varphi_0(z), \quad (17b)$$

$$\gamma = -\frac{\mu^2(z)}{3\alpha_1(z)} t + \gamma_0(z), \quad (17c)$$

$$\rho = \rho_0 \exp \left\{ \int_0^z \left( \frac{\mu(s)}{2\nu} \left( \varphi'_1(s) - \frac{\tau_1(s)}{6\sqrt{2}} \right) + \gamma_0(s) \right) \, ds \right\}, \quad (17d)$$

$$\beta = \mu(z)t + \alpha_1(z) \left[ \frac{3\varphi_1(z) - \tau_1(z)}{2\sqrt{2}v} \right], \quad (17e)$$

$$g = \frac{\tau_1^2(z)}{\rho^2(z)} \left[ \mu(z)t + 3\alpha_1(z) \varphi_1(z) + \frac{G\alpha_1(z) \tau_1(z)}{2\sqrt{2}v} \right], \quad (17f)$$

$$\sigma = \frac{\mu^2(z)}{3\alpha_1(z)} \tau_1^2 + \sigma_1(z) t + \sigma_0(z), \quad (17g)$$

$$V = -\frac{\mu^3(z)}{27\alpha_1(z)} \tau_1^3 + \varphi_2(z) \tau_1^3 + \gamma_1(z)t + \gamma_0(z), \quad (17h)$$

$$\alpha_3(z) = -\alpha_2(z) = -\frac{3\alpha_1(z) \tau_1^2(z)}{\rho^2(z)}, \quad (17i)$$

where $\varphi_2(j = 0, 1)$, $\gamma_1(j = 0, 1)$, $\varphi_0(z)$ and $\alpha_1(z)$ are arbitrary differentiable functions of space $z$, and $\rho_0$ is a constant. We have introduced these functions in equations (17g) and (17h) given by

$$\sigma_1(z) = \frac{\mu(z)}{6v} \left[ \frac{3\sqrt{2}}{\tau_1(z)} - 12\varphi_1(z) \right] + \frac{\tau_1(z)}{\tau_1(z)}, \quad (18a)$$

$$\sigma_0 = \frac{\mu(z)}{\sqrt{2}v} \tau_1(z) - \frac{3\sqrt{2}}{2\nu} \varphi_1(z) + \frac{\tau_0(z)}{\tau_1(z)}, \quad (18b)$$

$$v_2(z) = \frac{\mu^2(z)}{3\alpha_1(z)} \left[ \frac{\tau_1(z)}{6\sqrt{2}v} - \varphi_1(z) \right] - \left[ \frac{\mu(z)}{6\alpha_1(z)} \right]^2 \tau_1(z), \quad (19)$$

$$\alpha_0 = \frac{\mu(z) \tau_1(z)}{3\alpha_1(z) \tau_1(z)}, \quad (20)$$

$$\alpha_0 = \frac{\mu(z) \varphi_1(z)}{3\alpha_1(z) \tau_1(z)} \left[ \frac{\tau_1(z)}{3\sqrt{2}v} - \varphi_1(z) \right] - \frac{\mu(z)}{6\alpha_1(z)} \tau_1(z), \quad (21)$$

It is easy to see that in the presence of the differential gain or loss term $\mu(z)$ $\neq 0$, the GVD parameter $\beta(z, t)$, SPM parameter $g(z, t)$ and gain or loss term $\gamma(z, t)$ are all functions of both $z$ and $t$, which differ from the usually considered higher-order NLS equation with varying coefficients. The differential gain or loss term $\mu(z)$ can be used to modulate the phase, gain or loss term $\gamma(z, t)$, the amplitude $\rho(z)$, the GVD parameter $\beta(z, t)$, the SS parameter $\alpha_2(z)$, SPM parameter $g(z, t)$, $\sigma(z, t)$ and potential $V(z, t)$. The phase $\varphi(z, t)$ is a second-degree polynomial in $t$ with coefficients being functions of $z$, which is similar to that in the solutions of the NLS equation with varying coefficients, but the external potential $V(z, t)$ is a third-degree polynomial in $t$, with coefficients being functions of $z$, which differs from that in the solutions of the NLS equation with varying coefficients [25].

Notice that the solutions in Case I can be found directly from the solutions in Case II with $\mu \equiv 0$, but we here list Case I in order to clearly point out that GVD $\beta(z)$, SPM $g(z, t)$, and the gain or loss $\gamma(z, t)$ depend only on space $z$, and $\sigma(z, t)$ and the potential $V(z, t)$ are linear functions of time with coefficients being functions of space in the absence of the differential gain or loss term $\mu$, whereas the GVD $\beta(z, t)$, SPM $g(z, t)$, and gain or loss $\gamma(z, t)$ are linear functions of time with coefficients being functions of space and $\sigma(z, t)$ and the potential $V(z, t)$ are second-degree and third-degree functions of time with coefficients being functions of space in the presence of the differential gain or loss term $\mu(z)$.

Thus these chosen differentiable functions in the amplitude $\rho(z)$, GVD $\beta(z, t)$, SPM $g(z, t)$, external potential $V(z, t)$, the gain or loss $\gamma(z, t)$, TOD $\alpha_1(z)$, SS $\alpha_2(z)$, SFS $\alpha_3(z)$, the phases $\varphi(z, t)$, and new variables $(\eta(z), \tau(z), t)$ can excite a wide range of nonlinear wave structures of equation (3), such as periodic wave equations, multi-soliton solutions, and even rogue wave solutions.

Note that from the analytical conditions given by (13) and (17), the TOD parameter $\alpha_1(z)$ influences all the system.
parameters, such as GVD $\beta(z, t)$, SPM $g(z, t)$, the external potential $V(z, t)$, the gain or loss $\gamma(z, t)$, SS $\alpha_2(z)$, SFS $\alpha_3(z)$ and the form factors of solutions, such as the amplitude $\rho(z)$ and the phase $\psi(z, t)$. The solutions found can exist only under certain conditions and the system parameter functions cannot all be chosen independently. Thus, similar to the corresponding discussions in [14, 74, 75], we can choose the equation parameters suitably to investigate the dynamic behavior of the solutions of equation (3). The choice of periodic function for the system parameters leads to alternating regions of positive/negative values, which are required for the eventual stability of solutions [14, 74, 75]. In section 3, we will discuss the dynamical behavior of optical rogue wave solutions in some periodic dispersive systems.

3. Optical rogue wave solutions

Here we only consider optical rogue wave solutions of equation (3). With the aim, we need to know rogue wave solutions of equation (7) with $G = -1$, which have been found [62] in terms of the Darboux transformation

$$
\Psi_{n+1} = \Psi_n - \frac{4i\eta_{n+1}p_{n+1}^*}{|p_{n+1}|^2 + |q_{n+1}|^2} \quad (n = 0, 1, 2, \ldots)
$$

(22)

with the proper initial (‘seed’) solution $\Psi_0 = e^{i\eta}$, where $p_{n+1}^*$ denotes the complex conjugate of $p_{n+1}$. The characteristic functions ($p_n, q_n$) and the solution $\Psi_n$ of equation (7) satisfy the simplified Lax pair in which the characteristic is chosen as $\lambda = i$ [62]

$$
\begin{pmatrix}
    p_{n+1} \\
    q_{n+1}
\end{pmatrix}
= \begin{pmatrix}
    -\frac{1}{\sqrt{2}}i\frac{1}{\sqrt{2}} & i\Psi_n^* \\
    i & -\frac{1}{\sqrt{2}}i\Psi_n
\end{pmatrix}
\begin{pmatrix}
    p_n \\
    q_n
\end{pmatrix},
$$

(23a)

$$
\begin{pmatrix}
    p_{n+1} \\
    q_{n+1}
\end{pmatrix}
= \begin{pmatrix}
    M_{11} & M_{12} \\
    M_{21} & -M_{11} \end{pmatrix}
\begin{pmatrix}
    p_n \\
    q_n
\end{pmatrix},
$$

(23b)

where the components $M_{ij}$ are given by

$$
M_{11} = \sqrt{2}v(\Psi^*\Psi_r - \Psi\Psi_r^*) - (2 + i/2)|\Psi|^2 - 4v - i,
$$

$$
M_{12} = 2i\sqrt{2}v\Psi^*_r = -\Psi_r^*(4i\nu - 1) + \Psi_r(2i\nu|\Psi|^2 + 4i\nu - 1),
$$

$$
M_{21} = 2i\sqrt{2}v\Psi + \Psi^*(4i\nu - 1) + \Psi(2i\nu|\Psi|^2 + 4i\nu - 1),
$$

3.1. First-order optical rogue wave solutions

Here we consider the lowest-order rational solutions of equation (7) [62], which serve as prototypes of rogue waves. As a result, we obtain the first-order self-similar (non-stationary) rogue wave solutions of equation (3) in the form

$$
\psi_1(z, t) = \rho(z) \left(1 - \frac{4 + 8i\eta(z)}{1 + [\sqrt{2}\tau(z, t) + 12v\eta(z)]^2 + 4\eta^2(z)} \right)
\times \exp[i\eta(z) + i\psi(z, t)]
$$

(24)

on the basis of the obtained similarity reduction transformation (6), where the new variables $\tau(z, t)$ and $\eta(z)$, the amplitude $\rho(z)$, and the phase $\psi(z, t)$ are given by systems (13) and (17). For Case I, $\tau_1(z), \psi_1(z) (j = 0, 1), \alpha_1(z), \gamma(z)$ are used to modulate the non-stationary rogue wave solutions (24) and coefficients of equation (3); whereas for Case II, $\tau_2(z), \psi_2(z) (j = 0, 1), \alpha_2(z), \mu(z), \gamma(z)$ are used to control the non-stationary rogue wave solutions (24) and coefficients of equation (3).

Here we simply analyze the physical mechanisms of rogue waves of equation (3) on the basis of the constraints in section 2. Concerning the constraints of Case I given by system (13), the TOD term $\alpha_1(z)$ is not equal to zero such that the SS $\alpha_2(z)$, and SFS parameter $\alpha_3(z)$ should not be zero. The GVD $\beta(z)$ and SPM $g(z)$ must not be zero except for the case $\tau_1(z) = 6\sqrt{2}v\psi_1(z)$. Similarly, the group velocity $\nu$ and external potential $V$ can be chosen to be zero or not. In brief, in this case given by system (13), the TOD, SS and SFS terms are key to control equation (3) to generate rogue wave solutions. Concerning the constraints of Case II given by system (17), the coefficients in equation (3) are not all zero since $\mu(z) \neq 0$. All these non-zero terms are modulated to make equation (3) admit a rogue wave solution (24) with the phase $\psi(z, t)$ given by equation (17b) being similar to that of the NLS equation with varying coefficients [25].

We must make sure that TOD $\alpha_1(z)$, SS $\alpha_2(z)$, SFS $\alpha_3(z)$, GVD $\beta(z)$ for Case I (or the coefficient of GVD $\beta(z)$ in time for Case II), SPM $g(z)$ for Case I (or the coefficient of SPM $g(z, t)$ in time for Case II), $\gamma(z)$ for Case I (or the coefficient of $\gamma(z, t)$ in time for Case II), the coefficients of $\sigma(z, t)$ in time, and the coefficients of the external potential $V(z, t)$ in time in system (3) are bounded for realistic cases.

For illustrative purposes, we choose these free parameters in Case I in the form

$$
\begin{align*}
    \rho_0 & = 1.0, \quad \nu = 0.6, \\
    \tau_1(z) & = \text{dn}(z, k_1), \quad \tau_0(z) = \text{cn}(z, k_2), \\
    \alpha_1(z) & = c_1\text{dn}(z, k_3), \quad \gamma(z) = c_2\text{sn}(z, k_4)\text{dn}(z, k_4), \\
    \psi_1(z) & = \text{sn}(z, k_5), \quad \psi_0(z) = \text{cn}(z, k_6),
\end{align*}
$$

(25)

where $\text{sn}$, $\text{cn}$, and $\text{dn}$ stand for the respective Jacobi elliptic functions, and $k_j \in (0, 1) (j = 1, 2–6)$ are their moduli, and $c_j (j = 1, 2)$ are real-valued constants.

Figure 1 depicts the profiles of GVD $\beta(z)$, SS $\alpha_2(z)$, and SPM $g(z)$ given by system (13) versus space $z$ and the profiles of $\sigma(z, t)$, $V(z, t)$, $\psi(z, t)$ given by system (13) for
the parameters given by equation (25). The evolution of the intensity distributions \( |\psi_1(z, t)|^2 \) of the rogue wavefields given by equation (24) is illustrated in figure 2. We can see that the solutions are localized in time and keep localization infinitely in space, and are generated from the varying coefficients and differ from the usual rogue wave solutions (see, e.g., [46, 56–58, 62]). The solutions may be useful for experimentalists, who can modulate these coefficients to generate different rogue wave phenomena in nonlinear optics.

On the other hand, if we choose the free parameters in another form

\[
\begin{align*}
\tau_1(z) &= 1 + 0.1 \sin(z), \\
\tau_0(z) &= \cos(z), \\
\alpha_1(z) &= 0.2 + 0.1 \sin(z), \\
\nu &= 0.1,
\end{align*}
\]

and \( \rho_0, \gamma(z), \) and \( \psi_j(z) \) \((j = 0, 1)\) are the same as those given by equation (25), then the evolution of the intensity distribution of the rogue wave solutions (24) will be changed. Figure 3 displays the profile of the rogue wave solution (24). The solutions are localized both in time and almost in space, thus almost revealing the usual rogue wave features (see, e.g., [46, 62]).

For Case II, we choose different gain or loss terms

\[
\begin{align*}
\mu(z) &= \mu_0 \text{cn}(z, k_7) \text{dn}(z, k_7), \\
\gamma_0(z) &= c_3 \text{sn}(z, k_8) \text{dn}(z, k_8),
\end{align*}
\]

where \( c_j \) \((j = 3, 4)\) are real-valued constants.

Figure 4 depicts the profiles of \( \mu(z), \) GVD \( \beta(z, t) \), SS \( \alpha_2(z), \) SPM \( g(z, t), \) \( \sigma(z, t), \) \( V(z, t), \) and \( \psi(z, t) \) given by system (17) for the parameters given by equations (27) and (25). The evolution of intensity distributions \( (|\psi_1(z, t)|^2) \) of the rogue wavefields given by equation (24) with parameters given by equations (27) and (25) is illustrated in figure 5.
equation (27) with \( P \) by \( \psi \) self-similar rogue wave solutions of equation (3) in the form higher-order Hirota equation (7) \([62]\) is used in the obtained equation (3). When a second-order rogue wave solution of the here we consider the interaction of two rogue waves of 3.2. Second-order optical rogue wave solutions

Here we consider the interaction of two rogue waves of equation (3). When a second-order rogue wave solution of the higher-order Hirota equation (7) \([62]\) is used in the obtained similarity transformation (6), we can obtain the second-order self-similar rogue wave solutions of equation (3) in the form

\[
\psi_2(z, t) = \rho(z) \left[ 1 + \frac{P(\eta, \tau) + i\eta Q(\eta, \tau)}{H(\eta, \tau)} \right] e^{i(\eta \psi(z, t))},
\]

where these functions \( P(\eta, \tau), Q(\eta, \tau) \) and \( H(\eta, \tau) \) are given by

\[
P = 48\tau^4 + 1152\sqrt{2}\nu\eta\tau^3 + 144\tau^2[4\eta^2(36\nu^2 + 1) + 1] + 576\sqrt{2}\nu\eta[12\eta^2(12\nu^2 + 1) + 7] - 36 + 192\eta^4[216\nu^2(6\nu^2 + 1) + 5] + 864\eta^2(44\nu^2 + 1),
\]

\[
Q = 96\tau^4 + 2304\sqrt{2}\nu\eta\tau^3 + 96\tau^2[4\eta^2(108\nu^2 + 1) - 3] + 1152\sqrt{2}\nu\eta[4\eta^2(36\nu^2 + 1) + 1] - 360 + 384\eta^4(36\nu^2 + 1)^2 + 192\eta^2(108\nu^2 + 1),
\]

\[
H = 8\tau^6 + 288\sqrt{2}\nu\eta\tau^5 + 12\tau^4[4\eta^2(180\nu^2 + 1) + 1] + 96\sqrt{2}\nu\eta\tau^3[12\eta^2(60\nu^2 + 1) - 1] + 6\eta^2[16\eta^4(216\nu^2 + 1) + 1] - 24\eta^2(60\nu^2 + 1) + 9 + 72\sqrt{2}\nu\eta[16\eta^4(36\nu^2 + 1)^2 - 8\eta^2(108\nu^2 - 1) + 17] + 64\eta^6(36\nu^2 + 1)^3 - 432\eta^2(52\nu^2 + 1)(12\nu^2 - 1) + 36\nu^2(556\nu^2 + 11) + 9,
\]

in which the new variables \( \tau(z, t) \) and \( \eta(z) \), the amplitude \( \rho(z) \), and the phase \( \psi(z, t) \) are given by systems (13) or (17), and \( \nu \) is a real-valued constant. These parameters \( \alpha_1, \alpha_2, \mu, \tau_1, \tau_0, \gamma, \phi_1, \phi_0, \) and \( \nu \) can be used to control the wave propagation of second-order self-similar rogue wave solutions (28) and the coefficients of equation (3).

Similar to section 3.1, we choose three kinds of parameters given by equations (25)–(27) to study the wave propagation of the second-order self-similar rogue wave solutions (28). Figures 6–8 illustrate the amplitude distributions of the interactions of the second-order rogue wave solution (28) in \((z, t)\)-space for the chosen parameters. It is easy to find that the amplitude propagation of the physical field in this case is more complicated. Figure 6 shows that the rogue wave solution is localized in time and not localized in space \( z \), since the modulated parameters are periodic functions of space \( z \). In the other case, figure 7 illustrates that the rogue wave solution is localized both in space \( z \) and in time, however the complicated structure differs.
from the usual rogue wave features, in particular one near the origin. For the presence of the differential gain or loss term $\mu(z) \neq 0$, figure 8 illustrates that the self-similar rogue wave solution is localized both in space $z$ and in time. This may be useful to generate novel experimental results. Moreover, there may exist other integrable conditions making equation (3) generate other types of self-similar optical rogue wave solutions.

Last, we analyze the stability of some obtained analytical solutions, that is, how they evolve with distance when they are disturbed from their analytically given forms. We perform direct numerical simulations for equation (3) in some cases with initial fields coming from solution (24). Two examples of such behavior are displayed in figure 9, which essentially presents a numerical rerun of figures 2 and 3. Our preliminary results indicate no collapse—instead stable propagation is observed, except for some oscillations in the wing of waves. Figure 9(a) shows the stable propagation of solution (24)
in the system (25). Figure 9(b) exhibits the stable wave shape of solution (24) in the system (26). The comparison of figures 9(a) and (b) shows that solution (24) in the system (25) is more stable than solution (24) in the system (26).

4. Conclusions

In conclusion, we have systematically presented an appropriate transformation reducing the generalized higher-order nonlinear Schrödinger equation with varying coefficients to the Hirota equation with constant coefficients under some modulated coefficients. This self-similarity transformation and some constraints on the coefficients allowed us to find certain classes of exact self-similar solutions of the generalized HONLS equation (3) in terms of the integrable Hirota equation (7). In this paper we illustrated the approach on the basis of the two lowest-order rogue wave solutions of the Hirota equation as seeding solutions to study rogue wave solutions of equation (3), which have complicated structures localized in time. For some chosen functions, we studied the wave propagation of rogue wave solutions. Moreover, we studied the stability of the obtained first-order rogue wave solutions.

The used technique can be extended to investigate self-similar rogue waves of coupled higher-order NLS equations with varying coefficients, such as the external potentials, GVD, SPM, SS, SFS, and the gain or loss terms. The obtained self-similar rogue wave solutions may raise the possibility of related experiments and potential applications in a self-similar manner in nonlinear optics and other related fields of nonlinear science.

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