Yaw stability control of automated guided vehicle under the condition of centroid variation

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Received: 25 February 2021 / Accepted: 1 December 2021 / Published online: 15 December 2021
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Abstract
The centroid of an automated guided vehicle (AGV) changes due to the irregular position and uneven weight of the cargo on the load platform, which affects the completion of the handling task between stations in intelligent factories. This paper presents a hierarchical control strategy to improve yaw stability considering centroid variation. Firstly, the vehicle body and hub motor models are established based on dynamics. Secondly a hierarchical controller is designed by using the method of extension theory, model predictive control and sliding mode control. Then based on CarSim and Simulink, the low-speed step co-simulation condition of the AGV is carried out. Compared to the uncontrolled condition, the maximum deviation of the yaw rate is reduced from 0.58 to 0.52 rad/s, and the difference with the theoretical value is reduced from 16 to 4%; the maximum deviation of the centroid sideslip angle is reduced from −0.84 rad to −0.77 rad, and the difference with the theoretical value is reduced from 12 to 3%. Finally, a four-wheel drive and four-wheel steering AGV are manufactured to carry out inter station steering experiments in simulated factory environment on different road adhesion coefficients. The difference between simulation and experiment is less than 5%. The results show that the designed controller is effective, and the research can provide theoretical and experimental basis for the low-speed steering control stability of AGV.

Keywords Four-wheel steering · Extension theory · Model predictive control · Sliding mode control · Centroid variation

1 Introduction
Distributed drive electric vehicles can effectively alleviate the environmental protection and safety problems in the development of automobile industry, and become a popular carrier for many scholars at home and abroad to research vehicle stability control [1, 2]. It has the advantages of short transmission chain, high transmission efficiency and compact structure, which provides more possibilities for the overall intelligent by wire control and dynamics control of the chassis.

Four-wheel drive and four-wheel steering is one of the main forms of distributed drive. Compared to traditional vehicles, it optimizes the torque distribution of each driving wheel, makes full use of road adhesion, changes the steering direction of front and rear wheels at different speeds, so as to improves vehicle safety and handling stability [3]. The vehicle steering stability has a certain impact on the safety of the vehicle. The control system of four-wheel drive and four-wheel steering electric vehicle is more integrated and more complex than the traditional vehicle. Therefore, scholars at home and abroad have done a lot of research on
the yaw stability of four-wheel steering vehicle and obtained some achievements. The evaluation indexes of yaw stability mainly include yaw rate and centroid sideslip angle. In the literature [4, 5], the extension theory, sliding mode control integrated control algorithm and LQR control algorithm are used to control the yaw rate and centroid sideslip angle at the same time. The two kinds of control weights are changed in real time according to the actual road conditions. The solved compensation torque is constrained and then distributed to four wheels motors. The experiment results show that the response speed of vehicle parameters and driving stability are improved. Real time tracking of parameters in ideal state can effectively improve vehicle handling stability and yaw stability. In the literature [6–12], corresponding reference model are established for the problem to be solved, which are used to tracked the theoretical values. The controller parameters are dynamically adjusted according to different steering states to reduce the difference between the actual and the theoretical values.

The above yaw stability research is mainly through optimizing the control algorithm to strengthen the control of yaw rate and centroid sideslip angle, so as to improve the yaw stability. Another research hot topic of yaw stability control system is control strategy. In many conditions, a single control strategy can not achieve the desired goal, so the application of integrated control can further improve the vehicle yaw stability. In the literature [13–16], a control strategy integrating direct yaw control (DYC) with different controls is adopted to make up for the deficiency of tire linear control and improve the handling performance in nonlinear range, so as to reduce the demand of yaw moment and improve vehicle yaw stability. In the literature [17], the integrated control strategy of DYC and steering angle adaptive compensation is adopted. Through the distribution of braking torque and the correction of steering angle, the difference between the centroid sideslip angle and zero is reduced. Literature [18] innovatively combines grey predictive control theory, extension theory and fuzzy control theory to realize “leading control”, so as to preprocesses the yaw rate and centroid sideslip angle. The results show that it can effectively guarantee the yaw stability of the vehicle and reduce energy consumption. Literature [19–21] proposed to build a four-wheel steering model based on Ackerman’s angle theorem. When the four-wheel steering torque is the same, the steering stability can be improved by changing the wheel angle distribution.

The main object of the above research is the traditional car with constant centroid, and the application scenarios are obstacle free and wide road. It does not take into account the limited movement space and strict speed requirements of the factory environment. The cargo’s irregular position and uneven weight lead to the variate of the AGV centroid. It is easy to produce the problem of understeer and oversteer, which affects the yaw stability, so it is necessary to research this working condition. In order to solve this problem, a control method is proposed to improve the yaw stability of low-speed, which combines extension theory, model predictive control theory, fuzzy control theory and sliding mode control. According to the vehicle stability, the yaw moment of the four driving wheels is dynamically compensated, so as to improve the yaw stability of the vehicle.

The rest of this paper is arranged as follows. In the second chapter, the dynamic model of the AGV is established. In the third chapter, the yaw stability controller is designed. In the fourth chapter, the controller is simulated and analyzed. In the fifth chapter, the algorithm verification experiment is carried out. Finally, the conclusion is given in the sixth chapter.

2 Dynamic model

2.1 3-DOF model of vehicle

The stability of the AGV is mainly determined by the lateral motion and yaw motion, so the vehicle model only needs to consider the longitudinal motion, lateral motion and yaw motion. The dynamics model is simplified to reduce the calculation of the control algorithm without affecting the characterized vehicle dynamics process. In vehicle dynamics modeling, the following idealized assumptions should be made. Firstly, it is assumed that the AGV runs on a flat road and ignores the vertical motion; secondly, it only considers the pure cornering characteristics of the tire; then it ignores the coupling of transverse and longitudinal tire forces; next, it does not consider the lateral load transfer of the tire; finally, it ignores the influence of lateral and longitudinal aerodynamics on the yaw characteristics of the AGV.

Based on the above assumptions, the yaw dynamic model can be obtained, as shown in Fig. 1. O is the
instantaneous center; \( CG \) is the centroid; \( L \) is the distance from the front axle to the rear axle; \( M_2 \) is the yaw moment; \( \beta \) is the centroid sideslip angle.

According to the force balance and moment balance, the motion equation of 3-DOF is obtained as follows:

**Equation of longitudinal motion**

\[
m(v_x - v_y) = \sum F_x + \sum F_y \\
\begin{aligned}
&= F_{xfl} \cos \delta_{fl} + F_{xfr} \cos \delta_{fr} + F_{xrr} \cos \delta_{rr} \\
&\quad + F_{yfl} \cos \delta_{fl} \sin \delta_{fl} + F_{yfr} \cos \delta_{fr} \sin \delta_{fr} \\
&\quad - F_{yrr} \sin \delta_{rr} - F_{yfr} \sin \delta_{fr} 
\end{aligned} \quad (1)
\]

where, \( m \) is the vehicle mass; \( v_x \) is the longitudinal acceleration; \( v_y \) is the lateral velocity; \( \gamma \) is the yaw rate; \( F_{xfl}, F_{xfr}, F_{xrr} \) and \( F_{yfl}, F_{yfr}, F_{yrr} \) are the longitudinal force and lateral force of the tire, respectively; \( \delta_{fl} \) and \( \delta_{fr} \) are the left and right steering angles of the front wheels, respectively; \( \delta_{rl} \) and \( \delta_{rr} \) are the left and right steering angles of the rear wheels, respectively.

**Equation of lateral motion**

\[
m(v_y + v_x) = F_{yfr} \sin \delta_{fr} + F_{yrr} \sin \delta_{rr} \\
\begin{aligned}
&+ F_{xfr} \cos \delta_{fr} + F_{xrr} \cos \delta_{rr} \sin \delta_{fr} \\
&- F_{xfr} \sin \delta_{fr} - F_{xrr} \sin \delta_{rr} 
\end{aligned} \quad (2)
\]

where \( v_x \) is the lateral acceleration and \( v_y \) is the longitudinal velocity of the centroid in the vehicle body coordinate system.

**Equation of yaw motion**

\[
I_\gamma = I_f (F_{xfr} \sin \delta_{fr} + F_{xrr} \cos \delta_{fr} \sin \delta_{fl} + F_{yfr} \sin \delta_{fr} + F_{yrr} \sin \delta_{fr}) \\
\begin{aligned}
&+ I_r (F_{xfr} \sin \delta_{fr} - F_{xrr} \cos \delta_{fr} \sin \delta_{fr} + F_{yfr} \sin \delta_{fr} + F_{yrr} \cos \delta_{fr}) \\
&+ d_1 (F_{xfr} \cos \delta_{fr} - F_{yfr} \sin \delta_{fr} \sin \delta_{fr} + F_{xrr} \cos \delta_{fr} + F_{yrr} \sin \delta_{fr}) \\
&+ d_2 (F_{xfr} \cos \delta_{fr} - F_{yfr} \sin \delta_{fr} \sin \delta_{fr} - F_{xrr} \cos \delta_{fr} - F_{yrr} \sin \delta_{fr}) 
\end{aligned} \quad (3)
\]

where \( I_\gamma \) is the moment of inertia around the \( z \)-axis; \( \gamma \) is the yaw angular acceleration; \( L_f \) and \( L_r \) are the distances from the centroid to the front and rear axles, respectively; \( d_1 \) and \( d_2 \) are the distances from the left wheel and the right wheel to the equivalent wheel, respectively.

### 2.2 Hub motor model

In this paper, the hub motor adopts brushless DC motor, and the research focuses on the yaw stability control strategy of four-wheel steering vehicle. The model of hub motor needs the following specifications. Firstly, the eddy current loss and hysteresis loss of the motor are ignored; secondly, the saturation of the motor core is ignored; then, the air gap magnetic field distribution is approximately a trapezoidal wave with a flat top angle of 120 degrees; next, the cogging effect is ignored, and the armature conductor is evenly distributed on the armature surface; finally, the freewheeling diode and power transistor are regarded as ideal components (Fig. 2).

The voltage equation of hub motor is as follows:

\[
\begin{bmatrix}
\frac{d}{dt} i_A \\
\frac{d}{dt} i_B \\
\frac{d}{dt} i_C
\end{bmatrix} =
\begin{bmatrix}
R & 0 & 0 \\
0 & R & 0 \\
0 & 0 & R
\end{bmatrix}
\begin{bmatrix}
i_A \\
i_B \\
i_C
\end{bmatrix}
+ \begin{bmatrix}
0 & 0 & 0 \\
0 & L-M & 0 \\
0 & 0 & L-M
\end{bmatrix}
\begin{bmatrix}
e_A \\
e_B \\
e_C
\end{bmatrix}
\]

(4)

\[
P_e = e_A i_A + e_B i_B + e_C i_C
\]

(5)

\[
P_e = T_e \cdot \Omega
\]

(6)

It can be obtained from Eqs. (5) and (6):

\[
T_e = \frac{e_A i_A + e_B i_B + e_C i_C}{\Omega}
\]

(7)

The motion equation of the motor is:

\[
T_e - T_f = J \frac{d^2 \Omega}{dt^2} + B \omega
\]

(8)
where $T_e$ is the electromagnetic torque; $\Omega$ is the mechanical angular velocity of the motor; $T_l$ is the load torque; $J$ is the rotor moment of inertia; $B_v$ is the viscous friction coefficient; $u_{a}$, $u_{b}$, and $u_{c}$ are the $ABC$ three-phase winding voltage; $i_{a}$, $i_{b}$, and $i_{c}$ are the $ABC$ three-phase current; $e_A$, $e_B$ and $e_C$ are the $ABC$ three-phase back electromotive force; $L$ is the phase winding self-inductance; $M$ is the phase winding mutual inductance.

3 Design of yaw stability controller

The vehicle control strategy is shown in Fig. 3. This paper adopts hierarchical control, which is divided into two layers. The upper controller is used to solve the additional yaw moment required by the yaw stability control, which is composed of yaw rate controller, centroid sideslip angle controller, extension joint controller and sliding mode controller; the lower controller is the driving torque distribution controller, which distributes the additional yaw moment calculated by the upper controller to four driving wheels after constraint.

The target vehicle speed and steering angle are input into the dynamic model to solve the actual values of yaw rate $\gamma$ and centroid sideslip angle $\beta$. The theoretical values of yaw rate $\gamma^*$ and centroid sideslip angle $\beta^*$ are solved by MPC controller. The upper controller inputs $\gamma^*$, $\beta^*$, $\gamma$, $\beta$, the yaw rate controller, the centroid sideslip angle controller and the extension joint controller outputs the yaw moment $\Delta M_{\gamma}$, $\Delta M_{\beta}$, $\xi_1\Delta M_{\gamma} + \xi_2\Delta M_{\beta}$, where $\xi_1$ and $\xi_2$ are the weights of the yaw rate controller and the centroid sideslip angle controller, respectively. Then, the additional yaw moment $\Delta M_z$ is obtained by sliding mode control. The control domain module in the lower controller receives the inputs of $\Delta M_z$ and $\gamma^*$, $\beta^*$, $\gamma$, $\beta$ from the upper controller, judges the control domain according to the vehicle state, and allocates $T_{di}(i=fl, fr, rl, rr)$ to the four hub motors through constraint conditions.

3.1 Ideal reference model

When AGV steering, we hope that the centroid sideslip angle can be as small as possible, so the ideal centroid sideslip angle is zero. The linear 2-DOF vehicle model can effectively reflect
the linear relationship between yaw rate, centroid sideslip angle and steering angle, and can be used as an ideal reference model. The State space equation of AGV can be expressed as:

\[
\begin{align*}
\dot{x}(t) &= A_x x(t) + B_x u(t) \\
y_c(t) &= C_x x(t)
\end{align*}
\]  

(9)

where

\[
\begin{align*}
x(t) &= \begin{bmatrix} \beta^* \cr \gamma^* \end{bmatrix}, & u(t) &= \begin{bmatrix} \delta_f \cr \delta_r \end{bmatrix}, \\
A_c &= \begin{bmatrix} \frac{(2k_f+2k_r)}{mv} - \frac{(2L_f k_f + 2L_r k_r)}{mv^2} \\
&\frac{1}{L_f} \left(2L_f k_f - 2L_r k_r \right) \frac{1}{L_f v_f} \left(2L_f^2 k_f + 2L_r^2 k_r \right) \end{bmatrix}, \\
B_c &= \begin{bmatrix} -\frac{k_f}{mv} - \frac{k_r}{mv} \\
-\frac{k_f}{L_f} - \frac{k_r}{L_r} \end{bmatrix}, & C_c &= \begin{bmatrix} 1 & 0 \\
0 & 1 \end{bmatrix}
\end{align*}
\]

where \(k_f\) is the front wheel lateral deflection stiffness; \(k_r\) is the rear wheel lateral deflection stiffness.

Under the condition of low-frequency input of the wheel angle, the response of the yaw rate to the wheel angle can be simplified as a first-order lag link, namely

\[
\frac{\gamma^*(s)}{\delta(s)} = \frac{G_{aw}}{1 + \tau_{aw}s}
\]

(10)

where, \(\tau_{aw}\) is the time constant of inertia link; \(G_{aw}\) is the steady-state gain of yaw rate to wheel angle of ideal vehicle model.

\[
G_{aw} = \frac{1}{1 + K v_c L}
\]

(11)

where \(K\) is the steering characteristic stability factor of the vehicle.

The response of the centroid sideslip angle to the wheel angle should always be zero, namely:

\[
\frac{\beta^*(s)}{\delta(s)} = 0
\]

(12)

Substituting Eq. (10), (12) into the Eq. (9) can be obtained:

\[
\begin{bmatrix} \dot{\beta}^* \\
\dot{\gamma}^* \end{bmatrix} = \begin{bmatrix} 0 & 0 \\
-1/\tau_{aw} & 0 \end{bmatrix} \begin{bmatrix} \beta^* \\
\gamma^* \end{bmatrix} + \begin{bmatrix} 0 \\
G_{aw}/\tau_{aw} \end{bmatrix} u(t)
\]

(13)

Considering that the different adhesion coefficient of the road will limit the tire force, and to avoid large lateral acceleration that exceeds tire cornering capability, the yaw rate is constrained as:

\[
\gamma^* = \begin{cases} \frac{v_c \delta_f}{L + kmv^2} & |\gamma^*| \leq 0.85 \frac{\mu g v_c}{v_c} \\
0.85 \frac{\mu g v_c}{v_c} \text{sgn} \left( \frac{v_c \delta_f}{L + kmv^2} \right) & |\gamma^*| > 0.85 \frac{\mu g v_c}{v_c} \end{cases}
\]

(14)

where \(\mu\) is the road adhesion coefficient.

### 3.2 Model predictive control

Model predictive control (MPC) is a feedback control strategy, according to the established model to predict the future state of the system. The difference between the predicted output and the actual output can be reduced by the rolling optimization method. The optimization problem can be continuously updated to achieve the optimal control. The MPC control flow chart is shown in Fig. 4.

In Fig. 4. At time k, prediction model solves the ideal yaw rate \(\gamma_k^*\) and ideal centroid sideslip angle \(\beta_k^*\), INS...
measures the actual value of yaw rate $\gamma_k$ and the estimate value of centroid sideslip angle $\beta_k$, enter them into the feedback correction module to obtain the difference $\Delta \gamma_k = |\gamma_k - \gamma_k^*|$, $\Delta \beta_k = |\beta_k - \beta_k^*|$. Based on the stability objective function $J_{stb}$ and related constraints for $\Delta \gamma_k$, $\Delta \beta_k$ to optimization. Obtain the ideal yaw rate $\gamma_{k+1}^*$ and the ideal centroid sideslip angle $\beta_{k+1}^*$ at $k + 1$ time and act on the controlled object. Repeat the above process until the end of the current sampling period. Assuming that $p$ sampling periods are taken, the ideal yaw rate $\gamma_{k+p}^*$ and the ideal centroid sideslip angle $\beta_{k+p}^*$ obtained at $k + p$ time after $p$ closed-loop control are solved. This is the optimal yaw rate input and the centroid sideslip angle input to solve the instability state of the AGV within $p$ sampling times. Namely, the optimal solution of MPC controller in $p$ sampling periods.

The accuracy of prediction model determines the control effect of MPC controller. With the improvement of accuracy, the complexity of mathematical model and the increase of calculation, the real-time performance decreases and the control system loses its function. Considering that MPC has the characteristics of feedback correction and has certain robustness. Therefore, the prediction model in this paper selects the ideal reference model established in 3.1 above, discretizes Eq. (9), and takes the sampling time $T_s$ as 0.05 s:

$$\begin{aligned}
X(k+1) &= Ax(k) + Bu(k) \\
Y_c(k) &= C_s x(k)
\end{aligned} \quad (15)$$

In order to meet the solution requirements of MPC, Eq. (15) is rewritten into incremental form to obtain:

$$\begin{aligned}
\Delta x(k+1) &= A \Delta x(k) + B \Delta u(k) \\
y_c(k) &= C_s \Delta x(k) + y_c(k - 1)
\end{aligned} \quad (16)$$

where

$$A = e^{A T_s}, \quad \Delta x(k) = x(k) - x(k - 1), \quad \Delta u(k) = u(k) - u(k - 1), \quad B = \int_0^{T_s} e^{A \tau} d\tau \cdot B_c.$$ According to the current AGV state information $x(k)$, $y(k)$, the state variable values in the next $p$ sampling time region can be predicted, namely:

$$\begin{aligned}
\Delta x(k + p | k) &= A^p \Delta x(k) + A^{p-1} B \Delta u(k) + A^{p-2} B \Delta u(k + 1) + \cdots + A B \Delta u(k + p - 1) \\
= A^p A \Delta x(k) + A^{p-1} B \Delta u(k) + A^{p-2} B \Delta u(k + 1) + \cdots + A B \Delta u(k + p - 1)
\end{aligned} \quad (17)$$

$$Y_p(k+1 | k) = S_x \Delta x(k) + \Gamma y_c(k) + S_u \Delta U(k) \quad (18)$$

where

$$Y_p(k+1 | k) = \begin{bmatrix} y_c(k+1 | k) \\
y_c(k+2 | k) \\
\vdots \\
y_c(k+p | k) \end{bmatrix}, \quad \Delta U(k) = \begin{bmatrix} \Delta u(k) \\
\Delta u(k+1) \\
\vdots \\
\Delta u(k+m-1) \end{bmatrix}.$$

The MPC online finite time region rolling optimization module needs to be based on a certain objective function. This paper research the instability caused by the change of the centroid position of the AGV, so the stability objective function is used for optimization, namely:

$$J_{stb} = \left\| \tau = (Y_p(k+1 | k) - R(k+1)) \right\|^2 + \left\| \tau u \Delta U(k) \right\|^2 \quad (19)$$

where $\tau$ is a weighting coefficient matrix related to vehicle state deviation, $\tau u$ is the weighting coefficient matrix related to the change rate of control quantity, $R(k+1)$ is the output state of 2-DOF model at the next moment. Where the first term indicates that the vehicle output follows the expected output, and the second term indicates that the control quantity should not be too large, so as to affect the stability of the AGV.

In order to ensure the optimal control performance of the AGV after instability, the following constraints need to be imposed on the input according to the actual situation.

$$\begin{aligned}
\begin{cases}
u_{\min}(k+i) \leq u(k+i) \leq u_{\max}(k+i), & i = 0, 1, \ldots, m-1 \\
\Delta u_{\min}(k+i) \leq \Delta u(k+i) \leq \Delta u_{\max}(k+i), & i = 0, 1, \ldots, m-1 \\
x_{\min}(k+i) \leq x(k+i) \leq x_{\max}(k+i), & i = 0, 1, \ldots, p \\
x_{\min}(k+i) \leq \dot{x}(k+i) \leq \dot{x}_{\max}(k+i), & i = 0, 1, \ldots, p \\
\end{cases}
\end{aligned} \quad (20)$$
3.3 Design of upper controller

According to the extension theory, the AGV can be divided into three states when steering at low speed: stable region, single control region and joint control region. In the stable region, the AGV runs smoothly without control; in the single control region, the AGV gradually loses stability or tends to lose stability, and the yaw rate controller starts to work; in the joint control region, the AGV has become instability. The yaw rate controller and the centroid sideslip angle controller work at the same time, which the control weight is determined by the extension theory. The division of control region is shown in Fig. 5.

Firstly, the fuzzy controller of yaw rate is designed. When the AGV transports cargoes in the factory, the randomness of cargoes position and weight causes the vehicle’s yaw rate to be too large or too small, which leads to the instability of the vehicle body when steering. The controller uses the combination of MPC and fuzzy control to modify the yaw rate. The membership functions of fuzzy subsets are shown in Figs. 6 and 7. The fuzzy control rules of yaw rate controller are shown in Table 1.

In Figs. 6 and 7, the membership functions of yaw rate controller adopt triangle function. The input are the difference \( \Delta \gamma \) between the ideal yaw rate and the predicted yaw rate and the change rate of the difference \( \Delta \gamma \). The output is the additional yaw moment \( \Delta M_\gamma \). The horizontal coordinate is the discourse region, and the longitudinal coordinate is the degree of membership. According to the system model, the input region is \([-6, 6]\), the output region is \([-3, 3]\), and the fuzzy sets are all \([\text{NB, NM, NS, ZO, PS, PM, PB}]\).

Taking the yaw rate as the control parameter, the 49 fuzzy inference rules in Table 1 are designed by repeatedly adjusting the simulation experiment. Input the data in Table 1 into Matlab, use “Mamdani” type reasoning, area center of gravity method to de-fuzzify, and get the additional yaw moment response diagram of yaw rate controller, as shown in Fig. 8.

In Fig. 8, the x-axis and y-axis are the \( \Delta \gamma \) and the \( \Delta \gamma \). The z-axis is the \( \Delta M_\gamma \). It can be seen visually that the output surface of the \( \Delta M_\gamma \) in the discourse region varies with the operating conditions.

Secondly, the fuzzy controller of centroid sideslip angle is designed. When the AGV transports cargoes between two stations in the factory, the randomness of the cargoes position makes the AGV produce understeer or oversteer in the process of low-speed steering, and deviate from the expected path. The controller uses the combination of MPC and fuzzy control to modify the centroid sideslip angle. The membership function of fuzzy subsets are shown in Figs. 9 and 10. The fuzzy control rules of centroid sideslip angle controller are shown in Table 2.

In the Figs. 9 and 10, the membership functions of centroid sideslip angle controller adopt double S-type function. The input is the difference \( \Delta \beta \) between the ideal and predicted centroid sideslip angles and the change rate of the difference \( \Delta \beta \). The output is the additional yaw moment \( \Delta M_\beta \). The horizontal coordinate is the discourse region, and the longitudinal coordinate is the degree of membership. According to the system model, the input region is \([-6, 6]\), the output region is \([-3, 3]\), and the fuzzy sets are all \([\text{NB, NM, NS, ZO, PS, PM, PB}]\).

Taking the centroid sideslip angle as the control parameter, 49 fuzzy inference rules in Table 2 are designed through repeated adjustment of simulation experiment. Input the data in Table 2 into Matlab, use “min–max” type reasoning and center of gravity method to get the additional yaw moment response diagram of yaw rate controller, as shown in Fig. 11.

![Fig. 5 Control region division](image)

![Fig. 6 Membership function of input fuzzy subsets](image)

![Fig. 7 Membership function of output fuzzy subsets](image)
In Fig. 11, the x-axis and y-axis are the $\Delta \beta$ and $\Delta \dot{\beta}$. The z-axis is the $\Delta M_{\beta}$. It can be seen visually that the output surface of the $\Delta M_{\beta}$ in the discourse region varies with the operating conditions.

Then, in order to realize extension joint control, it is necessary to determine the size of extension set, that is, the stability boundary of AGV. According to the following steps, the extension joint controller is established.

### Table 1 Fuzzy control rules of yaw rate controller

| $\Delta \gamma$ | $\Delta M_{\gamma}$ |
|-----------------|---------------------|
|                | NB | NM | NS | ZO | PS | PM | PB |
| NB | PB | PB | PM | PM | PS | ZO | ZO |
| NM | PB | PB | PM | PM | PS | ZO | NS |
| NS | PM | PM | PM | PS | ZO | NS | NS |
| ZO | PM | PM | PS | ZO | NS | NM | NM |
| PS | PS | PS | ZO | NS | NS | NM | NM |
| PM | PS | ZO | NM | NM | NM | NB | NB |
| PB | ZO | ZO | NM | NM | NB | NB | NB |

(1) The control parameter is selected. Yaw rate and centroid sideslip angle are the main performance indexes to evaluate vehicle driving stability. The yaw rate represents the vehicle stability when steering, and the centroid sideslip angle shows the deviation of the actual running track from the expected path. In this paper, the $\Delta \gamma$ and $\beta$ are selected as the control variables. The vehicle driving state is divided into stability region, single control region and joint control region.

(2) The extension set is divided. Select the $\beta^*$ as the horizontal coordinate, the $\Delta \gamma$ as the longitudinal coordinate. Among them, the longitudinal coordinate is divided by the tolerance band method: Stable region $|\Delta \gamma| < |\xi_1\gamma^*|$; Single control region $|\xi_1\gamma^*| \leq |\Delta \gamma| \leq |\xi_2\gamma^*|$; Joint control region $|\Delta \gamma| > |\xi_2\gamma^*|$. Where, $\xi_1$ and $\xi_2$ are constants, taking 0.05 and 0.15, respectively. The stable region of the centroid sideslip angle is determined according to whether the yaw rate gain is in the linear interval. Firstly, when no control is applied, the wheel angle input is gradually increased at different vehicle speeds. Secondly, the relationship between the extreme value of the angle in the linear region and the vehicle speed is solved. Finally, this relation is input into the 2-DOF vehicle model, and obtain the stable boundary $\beta_i$. When $\beta \in (-\beta_i, \beta_i)$, the AGV is in a stable state and does not need control. Through a large number of simulations, when the centroid sideslip angle is
When $\beta \in (-\beta_2, -\beta_1) \cup (\beta_1, \beta_2)$, the AGV has the tendency of instability, but the centroid sideslip angle is still small. The vehicle body direction deviates little from the driving direction or does not deviate. The vehicle yaw stability control can be realized by yaw rate controller alone. When the vehicle is in an instability state, the limit value of the centroid sideslip angle $|\beta| = |\arctan(0.02 \mu g)|$, where $\mu$ is the adhesion coefficient of pavement. Set the value of centroid sideslip angle at this time to $\beta_3$. When $\beta \in (-\beta_3, -\beta_2) \cup (\beta_2, \beta_3)$, the AGV is in an instability state, it is necessary to control the yaw rate and the centroid sideslip angle at the same time in order to make the vehicle tend to be stable.

In summary, the stable region is $0 < |\Delta \gamma| < \Delta \gamma_1$ and $0 < |\beta| < \beta_1$.

### Table 2: Fuzzy control rules of centroid sideslip angle controller

| $\Delta \beta$ | NB | NM | NS | ZO | PS | PM | PB |
|---------------|----|----|----|----|----|----|----|
| $\Delta \beta$ | NB | NB | NM | NM | NS | ZO | ZO |
| $\Delta \beta$ | NM | NB | NM | NS | NS | ZO | ZO |
| $\Delta \beta$ | NS | NB | NM | NS | NS | ZO | PS |
| $\Delta \beta$ | ZO | NM | NS | NS | ZO | PS | PM |
| $\Delta \beta$ | PS | NM | NA | ZO | PS | PM | PB |
| $\Delta \beta$ | PM | ZO | ZO | PS | PM | PB | PB |
| $\Delta \beta$ | PB | ZO | ZO | PS | PM | PB | PB |

### Fig. 11: Additional yaw moment response diagram of centroid side-slip angle controller

1.45$\beta_1$, it corresponds to the instability boundary $\beta_2$. When $\beta \in (-\beta_2, -\beta_1) \cup (\beta_1, \beta_2)$, the AGV has the tendency of instability, but the centroid sideslip angle is still small. The vehicle body direction deviates little from the driving direction or does not deviate. The vehicle yaw stability control can be realized by yaw rate controller alone. When the vehicle is in an instability state, the limit value of the centroid sideslip angle $|\beta| = |\arctan(0.02 \mu g)|$, where $\mu$ is the adhesion coefficient of pavement. Set the value of centroid sideslip angle at this time to $\beta_3$. When $\beta \in (-\beta_3, -\beta_2) \cup (\beta_2, \beta_3)$, the AGV is in an instability state, it is necessary to control the yaw rate and the centroid sideslip angle at the same time in order to make the vehicle tend to be stable.

In summary, the stable region is $0 < |\Delta \gamma| < \Delta \gamma_1$ and $0 < |\beta| < \beta_1$.

### Fig. 12: One-dimensional extension set

Single control region $\begin{cases} \Delta \gamma_1 < |\Delta \gamma| < \Delta \gamma_2 \\ \beta_1 < |\beta| < \beta_2 \end{cases}$

Joint control region $|\Delta \gamma| > \Delta \gamma_2$ and $\beta_2 < |\beta| < \beta_3$

(3) The correlation function is constructed. The origin $O (0, 0)$ is the optimal point in the extension set. Take a point $Q$ in the single control region, connect $Q$ and $O$ and extend it. The line $OQ$ intersects the boundaries of the stable region and the joint control region at points $Q_1, Q_2, Q_3, Q_4$. As shown in Fig. 5, it can be seen that the line segment $OQ$ is the shortest distance that $Q$ is close to the optimal point. The two-dimensional extension set is transformed into a one-dimensional extension set to calculate the extension distance. The one-dimensional extension set is shown in Fig. 12.

Let the set of stable regions be $X_w$, the set of single control regions be $X_d$. The extension distance from point $Q$ to the stable region is $\rho (Q, X_w)$, the extension distance from point $Q$ to the single control region is $\rho (Q, X_d)$. Depending on the position of the $Q$ point, the extension distance from the point to the control region is different. Taking the extension distance from point $Q$ to single control region as an example.

$$
\rho(Q, X_d) = \begin{cases} 
|QO_2|, Q \in \langle -\infty, Q_2 \rangle \\
-|QO_2|, Q \in \langle Q_2, 0 \rangle \\
|QO_2|, Q \in \langle 0, Q_3 \rangle \\
|QO_2|, Q \in \langle Q_3, +\infty \rangle
\end{cases}
$$  

(21)
The correlation function is

\[ K(S) = \frac{\rho(Q X_d)}{D(Q X_d X_w)} \]

where \( D(Q X_d X_w) = \rho(Q X_d) - \rho(Q X_w) \).

(4) The weight of joint control is determined. When it is in the stable region, the vehicle runs smoothly and does not need extra control. At this time, \( \xi_1 = 0, \xi_2 = 0 \). When it is in the single control region, the vehicle gradually loses stability, and only need to apply the yaw rate control to make the car run smoothly. At this time, \( \xi_1 = 1, \xi_2 = 0 \). When it is in the joint control region, the vehicle is already instability, and the yaw rate control and centroid sideslip angle control need to be applied at the same time. At this time, \( \xi_1 = 1 k(s)/100 \), and \( \xi_2 = 1 — 1 k(s)/100 \).

Next, the sliding mode controller is designed. Sliding mode control is mainly to select the sliding mode surface. Different switching surfaces have different control effects. This paper researches the simultaneous control of the yaw rate and the centroid sideslip angle, so the following switching surface is constructed:

\[ s = \xi_1 (\dot{\gamma} - \gamma_d) + \xi_2 (\dot{\beta} - \beta_d) \]  

This control method can not only track the ideal yaw rate quickly, but also keep the centroid sideslip angle not far from the theoretical value, ensuring the yaw stability of the vehicle when steering.

Select the index approach rate:

\[ s = -k_1 \cdot \text{sgn}(s) - k_2 \cdot s \]  

where \( \text{sgn}() \) is the sign function, \( k_1 \) reflects the approach speed of the system state to the sliding mode surface \( s = 0 \) under sliding mode control, \( k_2 \) reflects the convergence speed of the movement toward the equilibrium point after the system state reaches the sliding surface.

In sliding mode control, the discontinuity of the sign function will cause chattering in the control system. In this paper, the saturation function \( \text{sat}(s/c) \) is used to replace the sign function, namely:

\[ s = -k_1 \cdot \text{sat}(s/c) - k_2 \cdot s \]  

where \( c \) is the normal number of the boundary layer thickness around the sliding surface \( s = 0 \).

Finally, the additional yaw moment is calculated. The derivation of Eq. (23) leads to the conclusion that:

\[ \dot{s} = \xi_1 (\dot{\gamma} - \gamma_d) + \xi_2 (\dot{\beta} - \beta_d) \]  

The additional moment of body stability is

\[ \Delta M_c = L_f (F_{xfl} \sin \delta_{fl} + F_{yfl} \sin \delta_{fr}) + L_r (F_{xrl} \sin \delta_{rl} + F_{yrl} \sin \delta_{rr}) - d_1 (F_{xfl} \cos \delta_{fl} + F_{yfl} \cos \delta_{fr}) + d_2 (F_{xrl} \cos \delta_{rl} + F_{yrl} \cos \delta_{rr}) \]  

Equation (3) can be transformed into

\[ \dot{\gamma} = \frac{1}{I_z} \left[ \frac{\Delta M_c + L_f (F_{xfl} \cos \delta_{fl} + F_{yfl} \cos \delta_{fr})}{L_f (F_{yfl} \cos \delta_{fl} - F_{yfl} \cos \delta_{fr}) + d_1 (F_{yfl} \sin \delta_{fl} - F_{yfl} \sin \delta_{fr}) + d_2 (F_{yfl} \sin \delta_{fl} - F_{yfl} \sin \delta_{fr})} \right] - \dot{\gamma}_d \]  

Substituting Eqs. (28) into (26), get

\[ \dot{s} = \xi_1 (\dot{\gamma} - \gamma_d) + \xi_2 (\dot{\beta} - \beta_d) \]  

Combined with exponential approach rate, additional torque can be obtained

\[ \Delta M_c = \left[ L_f (F_{xfl} \cos \delta_{fl} + F_{yfl} \cos \delta_{fr}) + L_r (F_{xrl} \cos \delta_{rl} - F_{yrl} \cos \delta_{rr}) + d_1 (F_{yfl} \sin \delta_{fl} - F_{yfl} \sin \delta_{fr}) + d_2 (F_{yfl} \sin \delta_{fl} - F_{yfl} \sin \delta_{fr}) \right] \frac{1}{I_z} \left[ \begin{array}{c} -\frac{k_1 \cdot \text{sat}(s/c)}{\xi_1} - k_2 s + \xi_1 \dot{\gamma}_d \left[ -\xi_2 (\dot{\beta} - \beta_d) \right] \end{array} \right] \]  

3.4 Design of lower controller

The role of the lower controller is to convert the additional yaw moment solved by the upper controller into driving force and transmit it to the drive wheel.

The balance equation between the driving force of each wheel and the additional yaw moment is as follows

\[ \begin{align*}
M_f &= L_f (F_{xfl} \sin \delta_{fl} + F_{yfl} \sin \delta_{fr}) - d_1 F_{xfl} \cos \delta_{fl} + d_2 F_{yfl} \cos \delta_{fr} \\
M_r &= L_r (F_{xrl} \sin \delta_{rl} + F_{yrl} \sin \delta_{rr}) - d_1 F_{xrl} \cos \delta_{rl} + d_2 F_{yrl} \cos \delta_{rr}
\end{align*} \]
\[ M_f = iM_z, M_r = (1 - i)M_z \]

\[ \begin{align*}
F_{xfr} + F_{xfl} &= \frac{T_d}{r} \\
F_{xrl} + F_{xrr} &= (1 - j) \frac{T_d}{r}
\end{align*} \]

where \( M_f \) and \( M_r \) are the additional torque of the front and rear wheels, respectively; \( i \) is the additional torque adjustment coefficient; \( j \) is the dynamic adjustment coefficient; \( T_d \) is the driving torque of the whole vehicle; \( r \) is the tire radius.

According to the above constraints and the additional yaw moment, the longitudinal forces of the four wheels can be calculated:

\[ \begin{align*}
F_{xfr} &= \frac{jT_d}{r} \left( L_f \sin \delta_f + d_1 \cos \delta_f \right) - iM_z \\
F_{xfl} &= \frac{jT_d}{r} \left( d_2 \cos \delta_f - L_f \sin \delta_f \right) + iM_z \\
F_{xrl} &= \frac{(1 - j)T_d}{2r} \left( 1 - i \right) M_z \\
F_{xrr} &= \frac{(1 - j)T_d}{2r} \left( 1 - i \right) M_z
\end{align*} \]

The obtained longitudinal force is transformed into driving torque:

\[ T_i = F_{xfr} + I \dot{\omega}_r \]

where \( I \) is the moment of inertia of the wheels; \( F_{xfr} \) is the four-wheel drive force.

The calculated four-wheel drive torque is used as the input of CarSim vehicle model. The co-simulation platform was built by Carsim and Simulink to realize the research on the yaw stability of AGV under low-speed steering conditions.

### 4 Simulation results and analysis

To verify the effectiveness of the control method, this paper conducts a co-simulation based on Carsim and Matlab/Simulink to examine the control effect of yaw stability under low-speed steering conditions. The vehicle dynamics model and ideal reference model are established in Carsim, and the hub motor and controller models are established in Simulink. The step condition, which is prone to instability under low-speed steering conditions, is selected for simulation analysis. The parameters of the whole vehicle are shown in Table 3.

| Name                        | Value |
|------------------------------|-------|
| vehicle mass (m/kg)          | 128   |
| Vehicle curb quality (m/kg)  | 148   |
| Distance from centroid to front axle \( L_f \) | 0.32  |
| Distance from centroid to rear axle \( L_r \) | 0.37  |
| Track width (d/m)            | 0.59  |
| Wheel rolling radius (r/m)   | 0.10  |
| Yaw moment of inertia \( I_y \)/(kg·m²) | 70    |
| Tire moment of inertia \( I \)/(kg·m²) | 0.42  |

In low-speed steering driving condition, the longitudinal speed is 25 km/h, the adhesion coefficient of the ground is 0.7. After the AGV runs at a constant speed for 3 s, wheel steering angle is affected by a 20 degrees (about 0.349 rad) step signal, resulting in the vehicle instability. The controller corrects the yaw rate and the centroid sideslip angle to restore the stability of the vehicle, and the whole working condition lasts for 20 s. The simulation results of low-speed step are shown in Fig. 13.

It can be seen from Fig. 13. In the first 3 s of normal driving, the AGV is in the stable region. \( \gamma \) and \( \beta \) can track the changes of theoretical values well. After 3 s, the vehicle starts to steer. Affected by the step signal, \( \gamma \) and \( \beta \) begin to deviate from the theoretical value and gradually increase, reaching the maximum value around 5 s. The maximum yaw rate is 0.58 rad/s, which exceeds the theoretical value by 16%. There is a hysteresis delay of 0.4 s at the maximum value. The maximum value of the centroid sideslip angle is -0.84 rad, which is 12% beyond the theoretical value. There is a 0.3 s lag delay at the maximum value, and fluctuations appear after the maximum value. It indicates that the AGV has a tendency to lose stability or has already lost stability, and its stability performance is not well. After adding hierarchical control, \( \gamma \) and \( \beta \) can closely track the theoretical value. The difference of \( \gamma \), \( \beta \) and the theoretical value are maintained at 4% and 3%, respectively. The hysteresis delay is small and negligible, which improves the low-speed operating conditions of the AGV Yaw stability.

### 5 Algorithm verification experiment

#### 5.1 Development of prototype vehicle

The experiment object is self-designed and manufactured four-wheel drive four-wheel steering AGV. The topology diagram is shown in Fig. 14. The vehicle parameters and motor parameters are shown in Table 4. The AGV is equipped with a DSPF28335 as the lower computer, and the
It uses the hub motor as the driving source and the stepping motor as the steering source. The load platform uses the push rod motor as the power source to realize up and down movement, which is convenient for loading and unloading cargoes.

During the experiment, the vehicle's position and attitude information, such as the vehicle's two-dimensional position, vehicle speed, yaw rate, and centroid sideslip angle, are determined by the high-precision INS inertial navigation system. The real-time parameters of the hub motor, such as the motor speed and torque, are measured by the motor encoder and torque sensor, respectively. The signal is collected in real time by the DSP single-chip microcomputer and fed back to the upper computer. The upper computer issues control instructions to the single-chip microcomputer through serial communication, and the single-chip directly controls the corresponding equipment according to the command.

### 5.2 Experimental results and analysis

The operating environment of the AGV research in this paper is the low-speed yaw stability research between two workstations in the factory. For safety reasons, the road with an adhesion coefficient of 0.7 is selected for this experiment, and the average vehicle speed is maintained at 25 km/h, the station layout and vehicle operating condition route are shown in Figs. 15 and 16. The two chairs in the picture represent the loading area and the unloading area, respectively. The AGV reach to the first chair for loading, then drives out of the loading area and turns into another chair for unloading. The black line is the driving path of the AGV. In order to verify the effectiveness of the control algorithm under different adhesion coefficients, the pavement was sprinkled to make the pavement adhesion coefficient approximately 0.4, and the above experiment was repeated.

| Parameter                                    | Value          |
|----------------------------------------------|----------------|
| Overall dimensions of the vehicle (long/wide/high)/mm | 890/690/560    |
| wheelbase/mm                                  | 690            |
| Tire diameter/mm                              | 200            |
| Track width/mm                                | 590            |
| vehicle mass/kg                               | 128            |
| Braking type                                  | Disc electric brake |
| Rated output power/W                          | 50–300         |
| Rated speed/RPM                               | 400            |
| Rated torque/(N·m)                            | 5              |
| Rated voltage/VDC                             | 24–48          |
The AGV enters the experiment site at a constant speed to reach the first chair for loading cargoes. After 3 s, it starts to steer. After the 20 s, it ends steering and arrives at another chair for unloading. The experimental results are shown in Figs. 17 and 18.

It can be seen from Fig. 17 that after loading cargoes, the irregular placement of the cargo leads to change the AGV centroid. The yaw rate and centroid sideslip angle start responding to changes. When the steering starts in 3 s, the yaw stability begins to fluctuate and the controller starts to work. 4 s to 7 s, the response amplitude of yaw rate is large, and the response amplitude of centroid sideslip angle is small. The AGV is in the state of single control region, and the yaw rate controller works independently. 7 s to 12 s, the response amplitude of centroid sideslip angle increases. The AGV is in the state of joint control region, and the yaw rate controller works together with the centroid sideslip angle controller. 12 s to 17 s, the response amplitude of centroid sideslip angle decreases and fluctuates up and down around the theoretical value. The AGV recovers to the single control region state, the centroid sideslip angle controller stops working. 17 s to 20 s, the response values of yaw rate and centroid sideslip angle tend to the theoretical value and change gently. The AGV gradually and smoothly enters the stability region state. At this time the yaw rate controller stops working. It can be seen from Fig. 18 that the fluctuation amplitude of the yaw rate and the centroid sideslip angle under the low-adhesion road is greater than that on the high-adhesion road. 0 s to 5 s, the response values of yaw rate and centroid sideslip angle tend to the theoretical value, and there is no obvious change. The AGV is in the stable region state and does not need control. 5 s to 12 s, the response amplitude of yaw rate and centroid sideslip angle increases. The AGV is in the state of joint control region, and the yaw rate controller works together with the centroid sideslip angle controller. For 12 s to 17 s, the response amplitude of centroid sideslip angle decreases. The AGV is in the single control region state, the centroid sideslip angle controller stops working. 17 s to 20 s, the response values of yaw rate and centroid sideslip angle fluctuate up and down around the theoretical value. The AGV enters the stability region state, the yaw rate controller stops working.

The experiments show that the actual values of yaw rate and centroid sideslip angle can closely track the theoretical values at different road adhesion coefficients. The deviation from the theoretical value at the peak of the actual value is kept within 5%. The input of the controller improves the yaw stability of the AGV at low-speed steering and also proves the effectiveness of the control strategy.

![Station layout](image1)

**Fig. 15** Station layout

![Experimental trajectory](image2)

**Fig. 16** Experimental trajectory

![Graphs](image3)

(a) Response of $\gamma$ under low-speed steering test

(b) Response of $\beta$ under low-speed steering test

**Fig. 17** Low-speed steering test response on high-adhesion road
6 Conclusion

(1) This paper takes the four-wheel drive and four-wheel steering AGV as the research object. Establish a 3-DOF dynamical model of the vehicle body and a hub dynamical motor model to provide a theoretical basis for building the model in Simulink.

(2) Aiming at the problem of the handling task completion of the AGV under the condition of centroid variation, a control strategy combining MPC, sliding mode control and extension theory is designed. The upper layer is the additional yaw moment solution layer, which calculates the compensation moment when the AGV is steering. The lower layer is the driving force distribution layer, which reasonably distributes the compensation torque calculated by the upper controller to the four wheels to ensure the yaw stability control of the AGV at low speed.

(3) Establish a co-simulation platform based on Carsim and Matlab/Simulink for experiment. The vehicle model and simulation conditions are established in the Carsim, and the hub motor and controller models are established in Simulink. In the uncontrolled state, the yaw rate and the centroid sideslip angle exceed the theoretical value of the reference model by 16% and 12%, respectively. After adding hierarchical control, the yaw rate and centroid sideslip angle can closely track the theoretical value, and the difference remains within 4%. Experiment results show that the established joint control strategy is reasonable under low-speed steering conditions and can improve the yaw stability of the AGV.

(4) Carry out real-vehicle experiments on the control strategy, and design a four-wheel drive and four-wheel steering AGV. Simulation the handling task between two stations in the factory on different road adhesion coefficients. The experiments show that the yaw rate and the centroid sideslip angle fluctuate within 5% of the theoretical value. The designed control strategy can reduce the delay, speed up the response, and improve the yaw stability of the AGV. The experiment results are basically consistent with the simulation results, verifying the correctness of the algorithm.

Acknowledgements Thanks are due to Dr. Ni for assistance with the experiments and to Dr. Zhao for valuable discussion.

Author contributions The author’ contributions are as follows: Wei Liu was in charge of the whole trial; Qingjie Zhang wrote the manuscript; Yidong Wan, Yue Yu, Ping Liu, Jun Guo assisted with sampling and laboratory analyses.

Funding This work was supported by National Natural Science Foundation of China (Grant No. 51405419), Natural Science Foundation of the Jiangsu Higher Education Institutions of China (Grant No. 18KJB460029) and Yancheng Institute of Technology Training Program of Innovation and Entrepreneurship for Postgraduates (Grant No. SJCX21_X2009).

Data availability The datasets supporting the conclusions of this article are included within the article.

Declarations

Conflict of interest The authors declare no competing financial interests.

Consent to participate Authors agree to the authorship order.

Consent to publish All authors have read and agreed to the published version of the manuscript.

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