Broad-band emission properties of central engine powered supernova ejecta interacting with a circumstellar medium

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ABSTRACT

We investigate broad-band emission from supernova ejecta powered by a relativistic wind from a central compact object. A recent two-dimensional hydrodynamic simulation studying the dynamical evolution of supernova ejecta with a central energy source has revealed that outermost layers of the ejecta are accelerated to mildly relativistic velocities because of the breakout of a hot bubble driven by the energy injection. The outermost layers decelerate as they sweep a circumstellar medium surrounding the ejecta, leading to the formation of the forward and reverse shocks propagating in the circumstellar medium and the ejecta. While the ejecta continue to release the internal energy as thermal emission from the photosphere, the energy dissipation at the forward and reverse shock fronts gives rise to non-thermal emission. We calculate light curves and spectral energy distributions of thermal and non-thermal emission from central engine powered supernova ejecta embedded in a steady stellar wind with typical mass loss rates for massive stars. The light curves are compared with currently available radio and X-ray observations of hydrogen-poor superluminous supernovae, as well as the two well-studied broad-lined Ic supernovae, 1998bw and 2009bb, which exhibit bright radio emission indicating central engine activities. We point out that upper limits on radio luminosities of nearby superluminous supernovae may indicate the injected energy is mainly converted to thermal radiation rather than creating mildly relativistic flows owing to photon diffusion time scales comparable to the injection time scale.

Key words: supernova: general – gamma-ray burst: general – shock waves – radiation mechanisms: non-thermal

1 INTRODUCTION

Modern unbiased transient surveys have revolutionized our understanding of various explosive phenomena in the Universe. One of the remarkable results is the discovery of a special class of supernovae (SNe) characterized by their high luminosities (10–100 times higher than those of normal core-collapse SNe), which are now called superluminous supernovae (SLSNe; see Gal-Yam 2012 for a review). Although their volumetric rate is extremely small (< 0.1% of normal core-collapse SNe) (e.g. Quimby et al. 2013; McCrum et al. 2015; Pras et al. 2017), they could be detectable at high-z galaxies thanks to their extreme luminosities (Tanaka et al. 2012, 2013; Inserra & Smartt 2014). SLSNe are classified into a couple of subcategories based on the presence or absence of hydrogen features in their spectra. SLSNe without any hydrogen feature are called hydrogen-poor or type-I SLSNe (hereafter SLSNe-I) and suggested to be explosions of massive stars without hydrogen and helium envelopes (e.g. Quimby et al. 2007; Barbary et al. 2009; Pastorello et al. 2010; Quimby et al. 2011; Chomiuk et al. 2011). Observations of individual SLSNe-I and their host galaxies suggest that SLSNe-I are likely produced by massive stars born in dwarf galaxies with low metallicities and high specific star formation rates (Neill et al. 2011; Lunnan et al. 2014; Leloudas et al. 2015a; Thöne et al. 2015; Angus et al. 2016; Chen et al. 2017b; Perley et al. 2016; Schulze et al. 2018).

Despite intensive observational and theoretical studies on SLSNe-I, the energy source of their bright emission is still poorly understood. Currently, three different scenarios have been proposed, (1) core-collapse SNe interacting with massive hydrogen-poor circumstellar matter (Chevalier & Irwin 2011; Ginzburg & Balberg 2012; Moriya et al. 2013), (2) pair-instability SNe (Barkat et al. 1967; Rakavy & Shaviv

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The traditional and widely used way to assess the existing scenarios of the energy source is to examine whether light curves of SLSNe can successfully be explained in the framework of these scenarios. Supernova light curves are well explained by diffusion of thermal photons in freely expanding spherical ejecta (Arnett 1980, 1982, 1996). Therefore, the timescale of the luminosity evolution can be a key to constraining properties of exploding stars and the energy source. One-zone light curve models with multiple energy supplies, i.e., radioactive decay, CSM interaction, and central engine, have been formulated (e.g., Chatzopoulos et al. 2012) and applied for observed light curves of SLSNe and other extraordinary SNe (Inserra et al. 2013; Chatzopoulos et al. 2013; Nicholl et al. 2015b; Wang et al. 2015a,b, 2016, 2017; Nicholl et al. 2017c). Among the three scenarios, the pair-instability SN scenario requires extremely large nickel and ejecta masses, indicating slow light curve evolution. This is in tension with some SLSNe showing rapid evolution (Nicholl et al. 2013). However, distinguishing these scenarios solely from IR-optical photometric observations of SLSNe is generally difficult because of a number of adjustable parameters in theoretical light curve models.

Spectroscopic observations have also been conducted and revealed that early spectra of SLSNe-I exhibit a blue continuum and absorption features by highly-ionized oxygen (Quimby et al. 2007; Pastorello et al. 2010; Quimby et al. 2011; Howell et al. 2013). Numerical investigations on the spectral formation in SLSNe-I have also been attempted (Dessart et al. 2012; Mazzali et al. 2016). Spectra of SLSNe-I at various epochs are available particularly for well-observed nearby events, such as SN 2007bi, PTF09cnd, LSQ14an, and SN 2015bn (e.g., Gal-Yam et al. 2009; Pastorello et al. 2010; Quimby et al. 2011; Nicholl et al. 2013; McClure et al. 2014; Nicholl et al. 2015a; Chen et al. 2015; Leloudas et al. 2015b; Inserra et al. 2017; Chen et al. 2017a). Liu et al. (2017) compared spectra of SLSNe-I and stripped-envelope CCSNe in a systematic way by using the Markov Chain Monte Carlo (MCMC)-based spectral fitting method developed by Liu et al. (2016) and Modjaz et al. (2016). They found that the average photospheric velocity of SLSNe-I implied by FeII absorption lines (~ 15000 km s\(^{-1}\)) around 10 days after the peak) is higher than normal type Ic SNe (~ 7000 km s\(^{-1}\)) and similar to type Ic SNe characterized by broad absorption features (SNe Ic-BL). Recent observations of the SLSN-I 2015bn at z = 0.1136 also showed that the nebular spectra at later epochs were remarkably similar to those of SNe Ic-BL (Nicholl et al. 2016a; Jerkstrand et al. 2017). These findings may indicate a link between the two extraordinary classes of SNe, SLSNe-I and SNe Ic-BL. However, theoretical understanding of characteristic spectral features associated with different scenarios has still been limited, which makes it difficult by optical spectra to distinguish different scenarios.

Furthermore, an SLSN-like bump is found in the afterglow light curve of the ultra-long gamma-ray burst (GRB) 111209A and named SN 2011kl (Greiner et al. 2015; Kann et al. 2016). These observations may further support the scenario that SLSNe-I and SNe Ic-BL (or associated GRBs) are powered by the same engine. Actually, a fast rotating magnetized neutron star, which is the most popular power source for the central engine scenario of SLSNe-I, have also been considered as a potential central engine for GRBs (Usov 1992; Wheeler et al. 2000; Thompson et al. 2004; Bucciantini et al. 2008, 2009; Metzger et al. 2011). More recently, roles of a millisecond magnetized neutron star newly born in supernova ejecta are also paid a great attention in the context of fast radio bursts (FRBs). The recently realized localization of the repeating FRB 121102 and the associated persistent radio source have stimulated intense discussion on its progenitor and emission mechanism (Chatterjee et al. 2017; Marcote et al. 2017; Tendulkar et al. 2017). The similarity of the host galaxy of the FRB and those of SLSNe may indicate the possible FRB-SLSN or FRB-SLSN-GRB association (Tendulkar et al. 2017; Metzger et al. 2017; Nicholl et al. 2017d).

Another potential way to distinguishing energy sources of SLSNe-I is to identify emission signatures across a wide energy range from radio to gamma-rays. Radio waves and high energy photons from young CCSNe are usually attributed to emission from non-thermal electrons produced by blast waves driven by supernova ejecta. Such non-thermal emission is also naturally expected for SLSNe-I. Multi-wavelength observations of SLSNe-I have been conducted particularly for nearby events, such as SN 2015bn (Nicholl et al. 2016b), Gaia16apd (Coppejans et al. 2018), and SN 2017egm (also known as Gaia17biu) (Bose et al. 2018; Nicholl et al. 2017b). Systematic searches for X-ray emission from SLSNe-I have been carried out and compiled by Levan et al. (2013) and Margutti et al. (2017). Currently, possible detections of X-ray sources whose locations are consistent with SCP 06F6 (Gänsecke et al. 2009) and PTF12dam (Margutti et al. 2017) have been reported. However, the origin of the X-ray emission is still debated. Some SLSNe-I, e.g., Gaia 16apd, exhibit a significant UV excess, which should also be a key to revealing the energy source (Yan et al. 2017; Nicholl et al. 2017a; Kangas et al. 2017; Tolstov et al. 2017). Furthermore, a systematic search for gamma-ray emission associated with SLSNe has been conducted by Renault-Tinacci et al. (2018), although they obtained only an upper limit for the gamma-ray luminosity by assuming a photon spectrum of v\(^{-2}\).

If an SLSNe-I is powered by a relativistic wind from a fast-rotating magnetized neutron star in an analogy to Galactic pulsar wind nebulae, electron-positron pairs would be copiously produced in the downstream of the shock wave terminating the wind. These high energy particles with non-thermal energy spectra can serve as an ionizing photon source for the supernova ejecta surrounding the neutron star. Thus, the presence of a nascent neutron star at the centre of the expanding supernova ejecta could be probed by the ionization structures of the ejecta and/or radio, X-ray, and gamma-ray emission (Kotera et al. 2013; Metzger et al. 2014; Murase et al. 2015, 2016; Kashiwama et al. 2016). Metzger et al. (2017) considered radio emission associated with SLSNe in the context of the FRB-SLSNe connection and pointed out that the quiescent radio source found in the host galaxy of FRB 121102 could be an SLSN remnant having produced a magnetar. They considered radio emission from the pulsar wind nebula and the forward shock driven by the supernova ejecta. More recently, Omand et al. (2018) present similar
2 DYNAMICAL EVOLUTION AND THERMAL EMISSION OF SUPERNOVA EJECTA

In this section, we describe our model for the dynamical evolution of supernova ejecta powered by a central engine.

2.1 Energy injection from the central engine

Our previous simulation (Suzuki & Maeda 2017) is based on the central-engine scenario for SLSNe (Kasen & Bildsten 2010) and has assumed energy injection at a constant rate around the centre. On the other hand, the most popular model for central-engine powered supernovae adopts millisecond magnetar spin down as the primary power source. The energy injection rate is usually assumed to be proportional to \((1 + t/t_{\text{sd}})^{-s}\), where \(t_{\text{sd}}\) is the spin down time of the magnetar and \(s\) is an exponent (hereafter, \(s = 2\)). Therefore, while the energy injection rate is constant well before the characteristic spin down time \(t_{\text{sd}}\), it decays in a power-law fashion at \(t \gg t_{\text{sd}}\). In order to incorporate the multi-dimensional picture revealed by the numerical simulation into calculations of thermal and non-thermal emission powered by the central engine, we assume that the structure of the ejecta has been fixed after the total energy of the ejecta reaches \(E_{\text{ej}} = 10^{52} \text{ erg}\) at \(t = t_{\text{sd}}\). The radial density and velocity profiles of the ejecta are assumed to be those derived by Suzuki & Maeda (2017), which are reviewed in the next subsection. The normalization of the spin down energy deposition rate \(L_{\text{sd}}\) is determined so that the deposited energy reaches \(E_{\text{ej}}\) at \(t = t_{\text{sd}}\). Thus, the spin down rate is expressed as follows,

\[
L_{\text{sd}}(t) = \frac{2E_{\text{ej}}}{t_{\text{sd}}}(1 + t/t_{\text{sd}})^{-2}
\]  

With this normalization, the total deposited energy yields \(2E_{\text{ej}}\). The energy deposited at \(t > t_{\text{sd}}\) serves as a power source for thermal emission from the ejecta. Although a fraction of the energy may be used to accelerate the ejecta, it is smaller than the total energy of the ejecta and thus unlikely to significantly affect the subsequent dynamical evolution of the ejecta. This treatment makes the dynamical model not fully self-consistent. Nevertheless, important aspects of the dynamical evolution of the ejecta are certainly captured.

In this work, we are interested in thermal and non-thermal emission from the ejecta having experienced the hot bubble breakout. We start the calculation of the emission at the initial time \(t_i = t_{\text{sd}}\). The spin down time, which is now equal to the initial time of the calculation, is a free parameter specifying the timescale of the central energy injection.

2.2 Supernova ejecta with central energy injection

We review the dynamical evolution of supernova ejecta powered by the central energy injection at a constant rate and how the subsequent energy redistribution throughout the ejecta shapes the density and velocity structure of the ejecta.

2.2.1 Powering supernova ejecta

Suzuki & Maeda (2017) have considered the dynamical evolution of supernova ejecta powered by a relativistic wind injected from a central engine at a constant energy injection rate. The ejecta are assumed to be expanding in a homologous way, i.e., the radial velocity \(v\) of a layer is its radius divided by the elapsed time, \(v = r/t\). A widely used broken power-law model (Chevalier & Soker 1989), where the density is proportional to the radial velocity, \(\rho \propto v^{-6}\) for inner ejecta and \(\rho \propto v^{-m}\) for outer ejecta, is employed for
the density profile of the supernova ejecta. The inner density gradient should be shallow, $\delta < 3$, so that the mass of the ejecta should not diverge at the centre. On the other hand, the outer density gradient is usually assumed to be steep, with a typical index of $m \sim 10$. The numerical simulation adopted $\delta = 1$ and $m = 10$.

The important parameters characterizing the dynamical evolution of the ejecta are the original kinetic energy of the ejecta $E_{\text{sn}}$ and the energy injection rate $E_{\text{in}}$. These two parameters give the characteristic timescale $t_c = E_{\text{in}}/E_{\text{sn}}$, at which the injected energy reaches the original kinetic energy. The dynamical evolution can be scaled by this critical timescale. In other words, we can apply the following scenario for different energy injection rates by rescaling the time $t$.

The numerical simulation revealed that the dynamical evolution of supernova ejecta with an embedded relativistic wind can be divided into the following three stages (see the schematic representation in Figure 1): (1) The relativistic wind injected around the centre forms a quasi-spherical, geocentrically thin shell composed of the shocked wind and ejecta (quasi-spherical stage). In this stage, the shocked gas forms a quasi-spherical hot bubble well confined by the ram pressure of the ejecta. The dynamical evolution of the shell in this stage is described by a self-similar solution and the radius of the shell evolves as $r^\alpha$, where $\alpha = (6 - \delta)/(5 - \delta)$ (Chevalier 1982; Jun 1998; Chevalier 2005). (2) When the forward shock propagating in the ejecta reaches a layer above which the density gradient is steep, the ram pressure of the ejecta no longer confines the hot bubble. As a result, the steep density gradient efficiently accelerates the forward shock and the whole ejecta are gradually overwhelmed by the shocked gas (hot bubble breakout). This transition happens at $t_{\text{br}} = t_{\text{br}}(r)$, when the total amount of the energy injected from the central engine exceeds a threshold value $\rho_0 E_{\text{in}}$. The factor $t_{\text{br}}$ depends on the structure of the ejecta. We assume $t_{\text{br}} = 5$ (Suzuki & Maeda 2017; see also Blondin & Chevalier 2017). (3) After the emergence of the forward shock from the outermost layer of the ejecta, the energy of the ejecta is gradually redistributed and the ejecta approach the homologous expansion stage. The density structure in this stage is well represented by a power-law function of the radial velocity with an exponent $-6$ (Suzuki & Maeda 2017).

### 2.2.2 Homologous expansion of supernova ejecta

We focus on the evolution of the supernova ejecta after the power-law density structure is realized ($t > t_{\text{br}}$). The velocity distribution is again represented by

$$ v(t, r) = \begin{cases} \frac{r}{t} & \text{for } r \leq v_{\text{max}}t, \\ 0 & \text{for } v_{\text{max}}t < r, \end{cases} $$

with a maximum velocity $v_{\text{max}}$. We assume that the density profile of the freely expanding ejecta is described by a power-law function of the four-velocity with an exponent $-n$,

$$ \rho(t, r) = \begin{cases} \rho_0 \left( \frac{r}{t} \right)^{-3} \left( \frac{v}{v_{\text{max}}v_{\text{in}}} \right)^{-n} & \text{for } v_{\text{min}}t \leq r \leq v_{\text{max}}t, \\ 0 & \text{otherwise}, \end{cases} $$

where the Lorentz factor is given by

$$ \Gamma = \frac{1}{\sqrt{1 - (v/c)^2}}. $$

In this study, we use a fixed value for the maximum four-velocity, $v_{\text{max}} = c$, following our previous simulation (Suzuki & Maeda 2017). The outermost layer travelling at the maximum velocity has been transparent to optical photons at the time of creation. Thus, optical emission would not be affected by the adopted maximum velocity. Furthermore, the layer will soon be swept by the reverse shock, making the subsequent dynamical evolution and non-thermal emission from the shocked gas insensitive to the assumed value (Suzuki et al. 2017). Our previous study showed that the angle-averaged density structure of the supernova ejecta is well represented by a power-law profile with an exponent $n = 6$ (see, Section 2.2.1). We use $n = 6$ as our fiducial value and examine how different values affect the non-thermal emission. The normalization constant $\rho_0$ and the minimum velocity $v_{\text{min}}$ are determined for a given set of the ejecta mass and energy, $M_\text{ej}$ and $E_\text{ej}$, as follows,

$$ M_\text{ej} = 4\pi \rho_0 (c t_i)^3 \int_{v_{\text{min}}}^{v_{\text{max}}} \Gamma \left( \frac{v}{\Gamma_{\text{max}}v_{\text{max}}} \right)^{-n} v^2 dv, $$

and

$$ E_\text{ej} = 4\pi \rho_0 c^5 t_i^3 \int_{v_{\text{min}}}^{v_{\text{max}}} \Gamma(\Gamma - 1) \left( \frac{v}{\Gamma_{\text{max}}v_{\text{max}}} \right)^{-n} v^2 dv. $$

We assume $M_\text{ej} = 10 M_\odot$ and $E_\text{ej} = 10^{52}$ erg in order to imitate the freely expanding ejecta realized in our previous numerical simulation (Suzuki & Maeda 2017).

### 2.3 Photospheric emission

The supernova ejecta powered by the central energy injection give rise to bright thermal emission (Kasen & Bildsten 2010). We consider thermal photons diffusing out from the ejecta after $t = t_i$. The photospheric radius $R_\text{ph}$ at time $t$ can be calculated in the following way. The optical depth for a ray radially extending from a given radius $r$ to the outermost radius of the ejecta is calculated by

$$ \tau(r, t) = \int_r^{\Gamma_{\text{max}}} \kappa \rho(t, r') dr', $$

where $\kappa$ is the opacity for thermal photons and set to be $\kappa = 0.1 \text{ cm}^2 \text{ g}^{-1}$. Here we have ignored the motion of the ejecta while the ray is travelling. In addition, the outermost layers of the ejecta would be swept by the reverse shock and thus the density structure is modified. We also ignore the modification of the density structure for simplicity. The photospheric radius at $t$ is determined so that the optical depth is equal to unity, $\tau(R_\text{ph} t) = 1$. We particularly denote the photospheric radius at $t = t_i$ by $R_i$.

We calculate the photospheric emission from the ejecta being powered by the continuous energy injection at the centre. We basically use the Arnett’s solution for photon diffusion in freely expanding spherical ejecta (Arnett 1980, 1982). The bolometric luminosity of the photospheric emission from
the ejecta with energy input $L_{in}(t)$ is given by

$$L_{ph}(t) = \frac{2}{t_d} e^{-t(t+t_Rh)/t_d^2} \int_{t_d}^{t+t_Rh} e^{t(t+t_Rh)/t_d^2} \tilde{\rho}(t') \left( \frac{h}{t_d} + \frac{t'}{t_d} \right) dt',$$

where the timescales $t_0$, $t_h$, and $t_d$ are given by

$$t_0 = \frac{\kappa M_{ej}}{\beta c R_i},$$

$$t_h = \frac{R_i}{v(R_i)},$$

and

$$t_d = \sqrt{2 t_0 \beta c R_i}.$$  \hfill (8)

(Chatzopoulos et al. 2012; Inserra et al. 2013). Here $E_{bh,0}$ is the initial thermal energy and $v(R_i)$ is the radial velocity at the photosphere $r = R_i$, both given at $t = t_0$. The initial thermal energy can be obtained from the dynamical model. The thermal energy of the ejecta in the quasi-spherical stage almost linearly increases with time (Suzuki & Maeda 2017). The value at the end of the increase is given by

$$E_{bh} = \frac{2 - \gamma}{1 + 3\alpha(\gamma - 1)} E_{ej},$$

where $\gamma = 4/3$ is the adiabatic index. The non-dimensional constant $\beta$ depending on the density structure is set to be a commonly used value $\beta = 13.8$ (Arnett 1980, 1982).

We assume the energy injection at the rate given by Equation (1), where the spin down time $t_d$ is one of our input parameters, the effects of which are to be examined in this paper. The energy input into the ejecta is given by

$$L_{in}(t) = L_{sd}(t) (1 - e^{-t/T}),$$

where the last factor takes into account the leakage of the injected energy from the ejecta as gamma-rays (Wang et al. 2015a). We calculate the optical depth by the following integration,

$$\tau_\gamma = \kappa_\gamma \int_{\rho_{\min}}^{\rho_{\max}} \rho(t, r) dr,$$

which evolves as $\tau_\gamma \propto t^{-2}$.

The gamma-ray opacity $\kappa_\gamma$ should depend on the frequency of gamma-rays and the effective mass may possibly be dependent on the three-dimensional density distribution of the supernova ejecta. Therefore, the value is highly uncertain. Theoretical calculations by Kotera et al. (2013), who assumed simplified spherical ejecta, suggest that the gamma-ray opacity could be of the order of $\sim 0.1$ cm$^2$ g$^{-1}$ for photons with $\nu \gamma \approx 100$ keV because of Compton scattering and $\sim 0.01$ cm$^2$ g$^{-1}$ for photons with $\nu \gamma > 10$ MeV because of pair production. We should note that the effective opacity may be lower than these values when we take into account patchy density structure. Recently, several authors have incorporated the gamma-ray leakage effect into their light curve fitting models and tried to constrain the gamma-ray opacity. Liu et al. (2017) fitted light curves of 19 SLSNe-I by their light curve model and analyzed the results by an MCMC approach. They reported that the best-fit value of the gamma-ray opacity ranges from $\kappa_\gamma \approx 0.01$ cm$^2$ g$^{-1}$ to $\kappa_\gamma \approx 0.82$ cm$^2$ g$^{-1}$, Nicholl et al. (2017c) systematically studied multi-colour light curves of 38 SLSNe-I. For example, their analysis on SN 2015bn inferred $\kappa_\gamma \approx 0.01$ cm$^2$ g$^{-1}$. Other SLSNe with well-covered late-time evolutions also showed small values, indicating significant gamma-ray leakage especially at later epochs. Keeping in mind that the value should be treated with caution, we adopt a constant value of $\kappa_\gamma = 0.01$ cm$^2$ g$^{-1}$.

Finally, we determine the temperature of the photospheric emission. Determining the colour temperature of the photospheric emission requires sophisticated treatments of radiative transfer, the ionization states of different layers of the ejecta, and numerous line opacities contributing to the thermal balance of the ejecta. For simplicity, we assume that the spectrum of the emission is well represented by a Planck function. Thus, we estimate the effective temperature $T_{eff}$ of
the photospheric emission from the photospheric radius and the bolometric luminosity given above,
\[ L_{ph} = 4\pi R_{ph}^2 \sigma_{SB} T_{ph}^4, \]
where \( \sigma_{SB} \) is the Stefan-Boltzmann constant.

## 2.4 Ejecta-CSM interaction

Suzuki et al. (2017) considered the hydrodynamical interaction between spherical supernova ejecta travelling at mildly relativistic speeds and a steady wind with a mass-loss rate \( \dot{M} \) and a wind velocity \( v_w \),
\[ \dot{\rho}_{\text{csm}} = \frac{\dot{M}}{4\pi v_w r^2} \equiv A r^{-2} \]
where \( A \) is a free parameter specifying the CSM density. We introduce the non-dimensional parameter \( A_\star = A/(5 \times 10^{11} \text{ g cm}^{-1}) \). In this normalization, \( A_\star = 1 \) corresponds to a mass-loss rate of \( \dot{M} = 10^{-5} \, M_\odot \, \text{yr}^{-1} \) for a wind velocity of \( 10^7 \, \text{km s}^{-1} \). In the following, we assume CSM density parameters up to \( A_\star = 10 \), with which the CSM is still transparent for electron scattering. Thus we can safely assume that the ejecta-CSM interaction does not give rise to optically thick thermal radiation significantly contributing to the optical brightness of SNe. We use this semi-analytic model to describe the evolution of the forward and reverse shocks developed as a result of the collision of the ejecta with the CSM.

For a given set of the parameters, \( M_{\text{ej}}, E_{\text{ej}}, \) and \( n \), the density and radial velocity profiles of the ejecta are specified. Under the assumption that the ejecta start interacting with the surrounding gas at \( t = t_0 \), the semi-analytic model is used to calculate the shock radius, the rate of the energy dissipation via shock, and the swept mass, as a function of time for both the forward and reverse shocks. The rate of the energy dissipation at the shock front and the mass swept by the shock are used to specify the number and the average energy dissipation at the shock front, and the swept mass, as a function of time for both the forward and reverse shocks.

Figure 2 shows an example of the semi-analytic calculation with \( n_d = 10^8 \, \text{s} \). The temporal evolution of the shock radius, the shock velocity, the post-shock pressure, and the internal energy dissipation rate are plotted for the forward and reverse shocks. After the beginning of the calculation at \( t = t_0 (= 10^8 \, \text{s}) \), the forward and reverse shock radii steadily increase with time. The difference in the forward and reverse shock radii is much smaller than the shock radius, indicating that the shocked region can be described as a geometrically thin shell. The shock velocities decrease to \( \sim 0.2c \) by the end of the calculation at \( t = 10^8 \, \text{s} \). The post-shock pressure at the forward and reverse shock fronts are similar because of the pressure balance across the contact discontinuity separating the shocked ejecta and CSM.

### 3 NON-THERMAL EMISSION MODEL

In this section, we describe our non-thermal emission model. We treat the thin shocked region under a one-box approximation and calculate the temporal evolution of the isotropic momentum distribution, \( dN/d\epsilon \), of electrons uniformly distributed in the shocked region. Non-thermal electrons with a power-law momentum distribution are injected through the forward and reverse shocks and then they lose their energies via radiative and adiabatic cooling. We consider synchrotron and inverse Compton emission as radiative cooling processes. From the temporal evolution of the electron distribution, we calculate the spectrum of the non-thermal emission from the shocked gas.

#### 3.1 Electron injection at the shock front

The shock dissipation at the forward and reverse shock fronts creates non-thermal electrons. Our treatment of the electron injection is similar to studies on non-thermal emission from CCSNe in the literature (e.g., Chevalier 1998; Chevalier & Fransson 2006; see Chevalier & Fransson 2016 for a recent review). We introduce two free parameters, \( \epsilon_e \) and \( \epsilon_B \), representing the efficiencies of the non-thermal electron acceleration and magnetic field generation at the shock front. These values should ideally be self-consistently determined by microscopic plasma processes responsible for the energy equipartition in the shock downstream. However, how exactly electrons are accelerated and magnetic fields are amplified are still debated even with state-of-the-art numerical computations based on first principle approaches (e.g., Spitkovsky 2008). Thus, we fix their values to be constant.

The energy injected into the shocked region per unit

![Figure 2. Temporal evolutions of the shock radius (top panel), shock velocity (middle panel), and the energy dissipation rate (bottom panel) calculated by the one-zone model. In each panel, the solid and dashed lines correspond to the quantities at the forward and reverse shock fronts, respectively. The parameters specifying the ejecta and CSM models are assumed to be \( M_{\text{ej}} = 10 \, M_\odot, E_{\text{ej}} = 10^{52} \, \text{erg}, n = 6, \) and \( A_\star = 1.0 \).](image-url)
time is obtained by the semi-analytic model described in Section 2. We denote the energy injection rates at the forward and reverse shocks by $E_{\text{fs}}$ and $E_{\text{rs}}$. For a given set of the post-shock density $\rho$ and the internal energy density $u_{\text{int}}$, the internal energy density $u_{\text{ele}}$ and the number density $n_{\text{ele}}$ of the injected non-thermal electrons are

$$u_{\text{ele}} = e_{\text{f}} u_{\text{int}},$$

and

$$n_{\text{ele}} = \frac{Z \rho}{\Lambda n_{\text{ia}}},$$

where $A$ and $Z$ are the mass and atomic numbers of ions predominantly composing the ejecta and $n_{\text{ia}}$ is the atomic mass unit. We assume $Z/A = 0.5$ in the following. The average energy of the injected non-thermal electrons is obtained as follows,

$$\bar{\gamma} m_{\text{e}} c^2 = \frac{n_{\text{ele}}}{u_{\text{ele}}} = \frac{e_{\text{f}} \Lambda n_{\text{ia}} u_{\text{int}}}{Z \rho},$$

where $m_{\text{e}}$ is the electron mass. We assume that electrons are spontaneously accelerated by the shock passage and then obey the following simple power-law momentum distribution,

$$\left( \frac{dN}{dp_e} \right)_{\text{in}} = \left\{ \begin{array}{ll}
0 & \text{for } p_{\text{min}} \leq p_e \leq p_{\text{in}}, \\
K (p_e/p_{\text{in}})^{-p} & \text{for } p_{\text{in}} \leq p_e \leq p_{\text{max}},
\end{array} \right. \quad (20)$$

where $p_e$ is the electron momentum and $p_{\text{min}}$ and $p_{\text{max}}$ are the minimum and maximum values. The minimum and maximum momenta are set to be $p_{\text{min}} = 10^{-3} m_{\text{e}} c$ and $p_{\text{max}} = 10^5 m_{\text{e}} c$. We assume a power-law index of $p = 3$, which is commonly employed to account for radio observations of stripped-envelope CCSNe (e.g., Chevalier & Fransson 2006). Therefore, electrons at the minimum injection energy $c (m_{\text{e}}^2 c^2 + p_{\text{min}}^2)^{1/2}$ carry a considerable fraction of the internal energy of electrons. The normalization constant $K$ and the injection momentum $p_{\text{in}}$ characterize the momentum distribution of the injected electrons are determined by equating the mass and energy injection rates and the following two integrals,

$$\int_{p_{\text{in}}}^{p_{\text{max}}} \left( \frac{dN}{dp_e} \right)_{\text{in}} dp_e = \frac{E}{\bar{\gamma} m_{\text{e}} c^2}, \quad (21)$$

and

$$\int_{p_{\text{in}}}^{p_{\text{max}}} c (m_{\text{e}}^2 c^2 + p_e^2)^{1/2} \left( \frac{dN}{dp_e} \right)_{\text{in}} dp_e = \dot{E}, \quad (22)$$

with $\dot{E} = \dot{E}_{\text{fs}}$ or $\dot{E}_{\text{rs}}$.

### 3.2 Electron momentum distribution

The electrons injected at the shock front experience synchrotron, inverse Compton, and adiabatic cooling. The governing equation describing the temporal evolution of the electron momentum distribution is written as follows,

$$\frac{\partial}{\partial t} \left( \frac{dN}{dp_e} \right) = \frac{\partial}{\partial p_e} \left[ \left( \rho_{\text{syn}} + \rho_{\text{ic}} + \rho_{\text{ad}} \right) \frac{dN}{dp_e} \right] + \frac{dN}{dp_e} \left( \frac{\partial}{\partial t} \right)_{\text{in}}, \quad (23)$$

where $\rho_{\text{syn}}$, $\rho_{\text{ic}}$, and $\rho_{\text{ad}}$ are the momentum loss rates by the three cooling processes. The synchrotron and inverse Compton momentum loss rates are related to the corresponding energy loss rates, $E_{\text{syn}}$ and $E_{\text{ic}}$, as follows,

$$\rho_{\text{syn}, \text{ic}} = \frac{\sqrt{m_{\text{e}}^2 c^2 + p_e^2}}{p_e} E_{\text{syn}, \text{ic}}. \quad (24)$$

The synchrotron and inverse Compton energy loss rates are given by

$$\dot{E}_{\text{syn}} = \frac{4}{3} \sigma_T c u_{\text{b}} \beta_e^2 \gamma_e^2,$$

and

$$\dot{E}_{\text{ic}} = \frac{4}{3} \sigma_T c u_{\text{rad}} \beta_e^2 \gamma_e^2,$$

(e.g. Rybicki & Lightman 1979) where $\sigma_T$ is the Thomson cross section. The energy loss rates are proportional to the energy densities, $u_{\text{b}}$ and $u_{\text{rad}}$, of the magnetic field and the seed photons, which are described later. The electron velocity $\beta_e$ and the Lorentz factor $\gamma_e$ are expressed in terms of the corresponding electron momentum $p_e$ as follows,

$$\beta_e = \frac{p_e}{\sqrt{m_{\text{e}}^2 c^2 + p_e^2}} \quad (27)$$

and

$$\gamma_e = \sqrt{1 + p_e^2/(m_{\text{e}}^2 c^2)} \quad (28)$$

As we will see below, the photospheric emission serves as a dominant seed photon source for inverse Compton cooling. Thus, the seed photon temperature is of the order of 1 eV. On the other hand, the injection momentum of electrons is typically ~ 30$m_{\text{e}} c$ (see Section 4.2). Therefore, the energy of most seed photons in the rest frame of non-thermal electrons is much smaller than the electron rest energy $m_{\text{e}} c^2$, allowing us to neglect several processes reducing the efficiency of inverse Compton cooling, such as the Klein-Nishina suppression and the electron recoil effect.

The electrons in the shell can also cool according to the expansion of the shell. The adiabatic momentum loss rate is given by

$$\dot{\rho}_{\text{ad}} = \frac{p_e V}{3 V^2}, \quad (29)$$

where $V$ and $\dot{V}$ are the volume of the shell and its expansion rate. The temporal evolutions of these quantities are also obtained from the semi-analytic model.

The governing equation (23) is numerically solved by a simple upwind scheme with first-order implicit time integration. In the following calculations, the distributions of non-thermal electrons accelerated at the forward and reverse shock fronts are separately treated.

### 3.3 Synchrotron spectrum

The shock dissipation generates random magnetic field via some magnetohydrodynamics and/or plasma collective effects. In a similar way to the energy density of electrons, we use a parameter $e_{\text{f}}$ describing the fraction of the magnetic field energy density to the dissipated shock energy. Thus, using the downstream internal energy densities $u_{\text{fs}}$ and $u_{\text{rs}}$ for the forward and reverse shocks, the corresponding magnetic energy densities are $u_{\text{B,fs}} = e_{\text{f}} u_{\text{fs}}$ and $u_{\text{B,rs}} = e_{\text{f}} u_{\text{rs}}$. These magnetic energy densities are used to evaluate the
synchrotron energy loss rate, Equation (25). The magnetic field strengths are given by
\[ B_{\text{fs}} = (8\pi u_{B,\text{fs}})^{1/2} = (8\pi u_{B,\zeta}u_{\text{fs}})^{1/2}, \]
and
\[ B_{\text{rs}} = (8\pi u_{B,\text{rs}})^{1/2} = (8\pi u_{B,\zeta}u_{\text{rs}})^{1/2}. \]

For a given electron momentum distribution and a magnetic field strength, the synchrotron emissivity per unit frequency is calculated by the following formula,
\[ j_{\nu,\text{syn}} = \frac{1}{4\pi V} \int P_{\nu,\text{syn}}(\gamma_e) \frac{dN}{dp_e} dp_e. \]
The synchrotron power per unit frequency \( P_{\nu,\text{syn}}(\gamma_e) \) as a function of electron Lorentz factor \( \gamma_e \) and frequency \( \nu \) is described in Appendix A1. At low frequencies, synchrotron emission suffers from absorption by its inverse process. The synchrotron self-absorption coefficient is given by
\[ \alpha_{\nu,\text{syn}} = \frac{c^2}{8\pi V^2} \int \frac{\partial}{\partial p_e} \left[ \frac{P_e \gamma_e P_{\nu,\text{syn}}(\gamma_e)}{p_e} \right] \frac{1}{dN}{dp_e}. \]
Using these quantities and assuming that the emitting region is geometrically thin, the synchrotron intensity is obtained as follows,
\[ I_{\nu,\text{syn}}(\nu) = \frac{j_{\nu,\text{syn}}}{\alpha_{\nu,\text{syn}}}(1 - e^{-\tau_{\nu,\text{syn}}}), \]
(e.g. Rybicki & Lightman 1979), where \( \tau_{\nu,\text{syn}} \) is the corresponding optical depth.

### 3.4 Inverse Compton spectrum

We consider the photospheric emission from the ejecta as the dominant source of seed photons for inverse Compton emission. The radiation energy density corresponding to the photospheric luminosity \( L_{\text{ph}} \) is
\[ u_{\text{rad}} = \frac{E_{\text{ph}}}{4\pi c^2 R_{\text{sh}}^2}, \]
at the shell \( r = R_{\text{sh}} \), which is used to evaluate the inverse Compton energy loss rate, Equation (26). We assume that the photospheric emission is well represented by a blackbody spectrum with the colour temperature identical with \( T_{\text{eff}} \). Therefore, the photon spectrum is given by
\[ I_{\nu,\text{ph}}(\nu) = \frac{2\nu u_{\text{rad}}}{c^2 \alpha_c T_{\text{eff}}^4} e^{\nu/k_B T_{\text{eff}}} - 1, \]
\[ I_{\nu,\text{ph}} = \frac{2\nu u_{\text{rad}}}{c^2 \alpha_c T_{\text{eff}}^4} e^{\nu/k_B T_{\text{eff}}} - 1, \]
where \( \alpha_c \) and \( k_B \) are the radiation constant and the Boltzmann constant.

We also consider synchrotron emission as the other source of seed photons. We obtain the total intensity of seed photons by adding those of the photospheric emission and the synchrotron emission. \( I_{\text{seed}}(\nu) = I_{\text{ph}}(\nu) + I_{\text{syn}}(\nu) \). The spectrum \( I_{\nu}(\nu) \) of the inverse Compton emission is calculated by convolving the seed photon spectrum, the electron distribution, and the redistribution function of Compton scattering \( \Delta G \) as follows,
\[ I_{\nu}(\nu) = \int_{\nu_{\min}}^{\nu_{\max}} \Delta G_{\text{seed}}(\nu') \frac{dN}{dp_e} d\nu' dp_e. \]

### 4 RESULTS

In this section, we show light curves and spectra calculated by the method described above. In all the calculations below, the microphysics parameters are assumed to be \( p = 3 \), \( \epsilon_e = 0.1 \), and \( \epsilon_B = 0.02 \). We note that the electron spectral index \( p = 3 \) is widely used for stripped-envelope CCSNe (e.g., Chevalier & Fransson 2000). The parameter \( \epsilon_B \) of the order of 0.1 is also used in radio light curve modellings of highly energetic SNe including relativistic SNe (e.g., Soderberg et al. 2010; Barniol Duran et al. 2015; Nakasha et al. 2015), while the values of \( \epsilon_B \) show a variety depending on the radio brightness (e.g., Santana et al. 2014, for GRB afterglows).

We have determined the value of \( \epsilon_B \) so that the radio light curves of highly energetic SNe are reproduced by our fiducial model (see below).

The ejecta mass and energy are also fixed to be \( M_{\text{ej}} = 10 M_\odot \) and \( E_{\text{ej}} = 10^{52} \) erg, while we examine how light curves at different frequencies depend on the other parameters, \( t_{\text{sh}} \), \( n \), and \( A_\star \). Hereafter, the model with \( t_{\text{sh}} = 10^8 \) s, \( n = 6 \), and \( A_\star = 1.0 \) is called the fiducial model.

#### 4.1 Photospheric emission

First, we show theoretical light curves for photospheric emission from the ejecta powered by the central energy injection. In particular, we focus on the dependence of the light curves on the assumed spin down time.

Figure 3 shows the temporal evolutions of the photospheric luminosity, the effective temperature at the photosphere, and the photospheric radius. The duration and the peak luminosity of the bolometric light curve become shorter and less luminous for shorter spin down times. Therefore models with shorter \( t_{\text{sh}} \) result in small radiated energies. In these models, the energy injection is terminated at early stages of the evolution of the ejecta. The timescale of the energy injection is only a small fraction of the timescale at which the ejecta becomes transparent to thermal photons. Therefore, the injected energy suffers from significant adiabatic loss before escaping into the interstellar space as radiation, leading to low radiative efficiencies. In other words, the ratio of the spin down time \( t_{\text{sh}} \) to the diffusion time \( t_d \) plays a critical role in determining bolometric light curves of central engine-powered SNe (Kasen & Bildsten 2010; Metzger et al. 2015; Nicholl et al. 2015, 2015b, 2017c). The model with \( t_{\text{sh}} = 10^8 \) s can reproduce the timescale and the peak bolometric luminosity of SLSNe-I. On the other hand, the model with \( t_{\text{sh}} = 10^8 \) s exhibit a shorter timescale and a lower peak luminosity than SLSNe-I. Such models may be relevant to SNe associated with GRBs. The over-luminous SN 2011kl associated with the ultra-long GRB 111209A exhibited a fast evolving light curve with a timescale of 10–20 days and a peak luminosity of \( \sim 3 \times 10^{45} \) erg s\(^{-1}\). The timescale is similar to that of the model with \( t_{\text{sh}} = 10^8 \) s, while the peak luminosity is smaller by a factor of \( \sim 6 \). The discrepancy could be resolved by adjusting the free parameters or considering jet-like energy injection rather than a quasi-spherical wind.

In our model, the photospheric radius at time \( t \) is solely determined by the density structure of the ejecta. Thus, the temporal evolutions of \( R_{\text{sh}} \) for different models are exactly same as shown in the bottom panel of Figure 3. The entire ejecta become transparent at \( t \approx 190 \) days, after which
corresponding to radial rays with different directions could differ from each other. This indicates that more thorough investigations including effects of three-dimensional ejecta structure and sophisticated radiative transfer in the ejecta are required.

4.2 Electron momentum distribution

Figure 4 shows the electron momentum distributions at \( t - t_i = 10, 20, 50, \) and 100 days. The plotted distributions are multiplied by \( p^2 \rho E dN/dp_e \) so that the spectrum of the injected electrons appears to be flat. In this model, electrons accelerated at the forward shock front are more abundant and have higher average energy than those at the reverse shock, which reflects the large energy dissipation rate at the forward shock (see Figure 2). These electrons behind the forward shock thus predominantly contribute to the non-thermal emission. The distributions are divided into two segments separated by a peak. The peak momentum corresponds to the minimum injected momentum. As Figure 2 shows, the shock velocity is \( v_{sh} \approx 0.3-0.4c \) at several 10 days. The kinetic energy density of the flow is roughly given by \( \rho u^2_{sh} \), and a considerable fraction of this energy is supposed to dissipate at the shock front, \( u_{sh} \propto \rho u^2_{sh} \). Therefore, the average Lorentz factor of electrons is roughly estimated from Equation (19),

\[
\dot{\gamma} = \frac{A_{mol} v^2_{sh}}{Z m_e c^2} \approx 33 \left( \frac{t_e}{0.1} \right) \left( \frac{v_{sh}/c}{0.3} \right)^2 \left( \frac{Z/A}{0.5} \right)^{-1}
\]

for the forward shock, which agrees with the peak momenta shown in Figure 4. The lower peak momenta for the reverse shock are due to small shock velocities relative to the unshocked ejecta velocities.

At higher energies, the distributions are well represented by a power-law function with an index \(-(p + 1)\), \( dN/dp_e \propto p_e^{-(p+1)} \). This indicates that the fast cooling regime (Sari et al. 1998; Sari & Esin 2001) is realized at earlier epochs. This is because thermal photons abundantly produced by the photospheric emission can efficiently cool non-thermal electrons via inverse Compton scattering. At later epochs, e.g., the distribution at \( t - t_i = 100 \) days, a relatively flat distribution at \( p_e/(m_e c) = 20-100 \) indicates that injected electrons with lower energies remain uncooled because of the declining photospheric luminosity. At lower energies than the peak, on the other hand, the electron momentum distribution shows a hard spectrum, which is composed of electrons having lost most of their energies.

4.3 Radio light curve

Figure 5 shows the radio light curves at different frequencies, 1.4, 4.8, and 8.5 GHz, calculated by our fiducial model with \( t_{dd} = 10^6 \) s, \( n = 6 \), and \( A_4 = 1 \). Because of the hot bubble breakout and the subsequent acceleration of the outermost layers of the ejecta, central engine powered SNe give rise to bright radio emission especially at early epochs. The radio light curves of our fiducial model suggest that the radio luminosity exhibits a peak around \( \sim 5-10 \) days for \( v \approx 5-10 \) GHz, while the peak at \( v \approx 1 \) GHz appears around \( 50-100 \) days. These features are worth comparing with radio-loud
SNe. In Figure 5, we plot the radio light curves of SNe 1998bw and 2009bb for comparison. SN 1998bw was a widely known SN Ic-BL associated with GRB 980425 (Kulkarni et al. 1998; Galama et al. 1998). SN 2009bb was the SN Ic-BL whose properties were remarkably similar to GRB-associated SNe, but lacking any signature of gamma-ray emission (Soderberg et al. 2010). As shown in Figure 5, their radio luminosities were similar to each other. For SN 1998bw, the peak of the light curve was successfully observed at 1.4, 4.8 and 8.5 GHz thanks to early observations triggered by the gamma-ray detection. The peak was earlier at higher frequencies as is the case for radio emission from normal CCSNe interacting with their CSM (e.g. Chevalier & Fransson 2016). For SN 2009bb, the peaks at higher frequencies, ν > 4.8 and 8.5 Hz, were probably missed, while the peak at 1.4 GHz was successfully observed. The decline rates of the luminosities per unit frequency after the peak are similar for both events. For SN 2009bb, the presence of an ultra-relativistic jet is unlikely because of the absence of emission indicating off-axis jet. The radio emission is explained by trans-relativistic supernova ejecta (Soderberg et al. 2010, see also Nakauchi et al. 2015).

We also plot the upper limits obtained by radio observations of SN 2015bn (Nicholl et al. 2016b), Gaia16apd (Coppejans et al. 2018), and 2017egm (Bose et al. 2018) in Figure 5. We should note that the frequency bands for SN 2015bn (7.4 GHz), Gaia16apd (6.6 GHz), and 2017egm (1.5 and 10 GHz) are slightly different from the theoretical light curve and SNe 1998bw and 2009bb (1.5 and 8.5 GHz). However, the spectral energy distributions of the synchrotron emission (see Figure 8) suggest that the radio luminosities at the corresponding frequency bands are similar to the theoretical light curve shown in Figure 5 within a factor of a few.

In Figure 6, we show how the radio light curves depend on the free parameters, the spin down time $t_{\text{sd}}$, the power-law exponent $n$ of the density profile, and the CSM density $\rho_{\text{cs}}$. We first focus on the effect of the spin down time. As is seen in the left column of Figure 6, the models with longer $t_{\text{sd}}$ exhibit bright radio emission in early epochs but are less luminous at later epochs than those with shorter $t_{\text{sd}}$. The power-law exponent more significantly affects the radio light curve than the spin down time, since it determines how much fraction of the kinetic energy is distributed in the outermost layers interacting with the CSM. For shallower density slopes (smaller $n$), more energy is available in the outermost layer to produce non-thermal electrons, giving rise to brighter synchrotron emission. This trend of brighter radio luminosities for shallower density slopes is seen in the middle column of Figure 6. The increase in the CSM density makes the emission brighter because a dense CSM can efficiently dissipate the kinetic energy of the ejecta.

The radio light curves of our fiducial model in Figure 5 show good agreement with SNe 1998bw and 2009bb. The radio luminosities at 8.5, 4.8, 1.4 GHz show their peaks around $t \approx 5$, 7, and 50 days. After the peak, the model luminosity
to SN 2017egm, steep density gradients and/or small CSM densities are required to reconcile the disagreement. We will further discuss theoretical interpretations of the current radio constraints in Section 5.

4.4 X-ray light curve

Figure 7 shows the X-ray light curves of the inverse Compton emission. The curves in the panels of Figure 7 present the temporal evolution of $\nu L_\nu$ at $h\nu = 0.3$, 1.0, and 10 keV for models with $t_{\text{fd}} = 10^6$, $10^7$, and $10^8$ s. The other parameters are fixed as $n = 6$ and $A_\star = 1.0$. The luminosity of the inverse Compton emission from the ejecta can reach $\nu L_\nu \sim 10^{41-42}$ erg s$^{-1}$, which is brighter than most of normal CCSNe. The luminous X-ray emission is owing to the presence of the mildly relativistic ejecta and the optical photons abundantly provided by the photospheric emission. We plot the currently available upper limits for SLSNe-I (including SN 2015bn) and 0.3–10 keV X-ray luminosities of SCP 06F6 and PTF12dam (we used the data compiled by Margutti et al. (2017)). The luminous X-ray emission associated with SCP 06F6 have generated a lot of discussion on its origin (Gänsicke et al. 2009; Levan et al. 2013; Metzger et al. 2014). However, the X-ray luminosity is well above most of the upper limits obtained for SLSNe-I so far (Figure 7), leading to the consensus that such luminous X-ray emission is not common among SLSNe-I. Margutti et al. (2017) found an X-ray source at the location of PTF12dam. However, as they mention in their paper, the X-ray source can also be explained by X-ray emission associated with the star-forming activity in the host galaxy with a relatively high star formation rate $\sim 5M_\odot$ yr$^{-1}$. Therefore, further observations are required to see whether the X-ray emission is certainly associated with PTF12dam or not. In Figure 7, we also plot the X-ray light curves of the SNe Ic-BL 1998bw (Pian et al. 2000; Kouveliotou et al. 2004) and 2009bb (Soderberg et al. 2010) for comparison.

We first focus on our fiducial model in the top left panel of Figure 7. The theoretical light curve is below most of the upper limits placed for the other SLSNe, suggesting that deeper observations are needed to further constrain the central engine scenario for SLSNe-I. The X-ray luminosity of PTF12dam agrees with the theoretical value, but it may have to be treated as an upper limit because of the reason described above. The theoretical light curves exhibit their peaks around 10-30 days. Since the luminosity of inverse Compton emission is proportional to the product of the seed photon energy density and the energy of non-thermal electrons, the light curve is determined by the convolution of the bolometric light curve of the photospheric emission and the steadily declining energy dissipation rate at the shock front (see Figure 2). Thus, the peak in the X-ray light curve slightly precedes the optical maximum.

The theoretical light curves of the other models exhibit similar X-ray luminosities but earlier peaks for shorter spin down times. This is because the optical maximum shifts earlier for shorter $t_{\text{fd}}$ as we have described in Section 4.1. For models with shorter $t_{\text{fd}}$, the light curve exhibits a plateau rather than a peak and then the luminosity declines. This feature is similar to the X-ray light curve of SN 1998bw, although the observed light curve shows a longer flat part. Although only a single data point is available, SN 2009bb

![Figure 5](image-url). Radio light curves of the non-thermal emission model for 1.4 (top), 4.8 (middle), and 8.5 (bottom) Hz. The solid lines show the radio light curve calculated by the fiducial model with $t_{\text{fd}} = 10^6$ s, $n = 6$ and $A_\star = 1.0$. The microscopic free parameters are set to be $\kappa = 3.0$, $c_\phi = 0.1$, and $c_6 = 0.02$. For comparison, light curves of radio-loud SNe Ic-BL, 1998bw (blue square) and 2009bb (red circle) at the corresponding frequency, are also plotted. The star marks with arrows represent upper limits obtained by radio observations for two SLSNe-I. The green star in the bottom panel represents the upper limit for SLSN-I 2015bn at 7.4 GHz, the orange stars in the middle panel represent the upper limits for Gaia16apd at 6.6 GHz, and the magenta stars in the top and bottom panels are those for SLSN-I 2017egm at 1.5 and 10 GHz.
Figure 6. Dependence of the radio light curves on the spin down time $t_{sd}$ (left column), the power-law exponent $n$ of the density profile $n$ (middle column), and the CSM density $A_\star$ (right column). In the left panels, we compare the models with $t_{sd} = 10^6$ (solid), $10^5$ (dashed), $10^4$ (dotted), and $10^3$ (dash-dotted) s. The models with $n = 5$ (dash-dotted), 6 (solid), 7 (dashed), and 8 (dotted) are shown in the middle panels. The right panels represent the models with $A_\star = 1.0$ (dash-dotted), 1.0 (solid), 0.1 (dashed), and 0.01 (dotted). The other free parameters are set to the same one as the fiducial model in Figure 5.

also show similar X-ray luminosity, which agrees with the declining theoretical light curve at $\sim$30 days. One caveat on this comparison is that the corresponding theoretical bolometric luminosities of the photospheric emission (Figure 3) are brighter than those of SN 1998bw and SN 2009bb. This discrepancy indicates that we should explore appropriate parameters satisfying both optical and X-ray observational constraints and/or an improved treatment of the photospheric emission with multi-colour radiation transfer and other sources of seed photons would be required. We leave such improvements to future work.

As in the case of radio light curves, increasing the CSM density $A_\star$ makes the X-ray emission more luminous. Since the theoretical X-ray light curve of the fiducial model with $A_\star = 1.0$ in the upper left panel of Figure 7 roughly matches the X-ray flux of PTF12dam, the CSM density much larger than this value would predict too bright X-ray emission. This can place an upper limit on the CSM density by treating the X-ray flux as an upper limit. The adopted value $A_\star = 1.0$ corresponds to a steady wind at a mass-loss rate of $\dot{M} = 10^{-5} M_\odot$ yr$^{-1}$ for a wind velocity $10^3$ km s$^{-1}$. Therefore, mass-loss rates much larger than this value is unlikely. Margutti et al. (2017) have already constrained the CSM density by using the X-ray upper limit and reached a similar conclusion, $\dot{M} < 2 \times 10^{-5} M_\odot$ yr$^{-1}$.

4.5 Broad-band spectral energy distribution

Finally, we present spectral energy distributions at several epochs in Figure 8. The spectral energy distributions at different epochs look similar to each other, and qualitatively similar to those of normal CCSNe interacting with their CSM. Each distribution is composed of the synchrotron, photospheric, and inverse Compton components, whose peaks are located around $10^9$–$10^{10}$, $10^{15}$, and $10^{18}$ Hz. The peak in the radio energy range divides the synchrotron component into optically thick (lower frequencies) and thin (higher frequencies) regimes. The peak shifts toward lower frequencies with time because the non-thermal electrons in the shocked region gradually become transparent to radio waves with lower frequencies. This temporal shift of the radio peak frequency creates the peak in the radio light curve described in Section 4.3. The flux of the optically thick synchrotron emission follows a power-law function of $\nu$, $L_\nu \propto \nu^{5/2}$ (e.g. Rybicki & Lightman 1979). On the other hand, the spectral slope of the optically thin synchrotron emission depends on the power-law exponent $p$ of the electron energy spectrum. As we have described in Section 4.2, the fast cooling regime can apply, $L_\nu \propto \nu^{-p/2} = \nu^{-1.5}$.

Since the inverse Compton spectrum is the convolution of the energy spectra of non-thermal electrons and seed photons, the peak frequency of the inverse Compton component is determined by the peak in the electron momentum distribution and the effective temperature of the photospheric emission. As we have described in Section 4.2, the injection energy of non-thermal electrons is $\dot{\gamma} \approx 30$–40. The inverse
Compton scattering by electrons with this Lorentz factor increases the photon energy by a factor $\gamma^2$.

$$\nu_{\text{ic}} = \frac{3 \gamma^2 k_B T_{\text{eff}}}{\hbar} = 10^{16} \text{Hz} \left( \frac{\gamma}{40} \right)^2 \left( T_{\text{eff}} / 10^4 \text{K} \right).$$  \hspace{1cm} (39)

which explains the X-ray peak in the spectral energy distribution. The spectrum at frequencies higher than the X-ray peak frequency is well represented by a power-law distribution, whose exponent depends on the electron energy spectrum. The spectral index is same as the optically thin synchrotron emission, $L_\nu \propto \nu^{-p/2} = \nu^{-1.5}$.

5 DISCUSSION AND CONCLUSIONS

In this study, we have calculated broad-band emission from supernova ejecta powered by a central engine based on the picture revealed by the recent two-dimensional special relativistic hydrodynamic simulation (Suzuki & Maeda 2017). In the hydrodynamic simulation, the outermost layers of the ejecta are efficiently accelerated owing to a hot gas emerging from the central region of the ejecta and thus the maximum velocity of the ejecta can be mildly relativistic. While the ejecta emit thermal photons by radiative diffusion, the outermost layers colliding with a CSM create the forward and reverse shocks propagating in the CSM and ejecta, respectively. We model the photospheric emission from the ejecta by using the Arnett-type one-zone model for photon diffusion throughout the ejecta and at the same time...
we determine the photospheric radius and the effective temperature from the ejecta model. Furthermore, we calculated non-thermal emission from the shocked gas by using the semi-analytic model for the propagation of the forward and reverse shocks (Suzuki et al. 2017).

We found that non-thermal electrons produced in the shocked region can give rise to bright radio and X-ray emission via synchrotron and inverse Compton processes. When we adopt commonly assumed values for microphysics parameters, $\epsilon_e$, $\epsilon_B$, and $\rho$, and the CSM density corresponding to a steady mass-loss rate of $M = 10^{-3} M_{\odot}$ yr$^{-1}$ and a constant wind velocity of $10^3$ km s$^{-1}$, the theoretical radio light curves of central engine powered SNe well agree with those of radio-loud SNe Ic-BL, such as SNe 1998bw and 2009bb, consistent with the idea that they also harbour a central engine.

5.1 Radio and X-ray emission from SLSNe-I as a probe of a central engine

Our results suggest that SLSNe-I can also give rise to radio synchrotron emission with similar fluxes to radio-loud SNe Ic-BL. This could also be explained naturally if the SLSNe-I and SNe Ic-BL would be certainly linked to each other as suggested by similarities in spectra of SNe Ic-BL and SLSNe-I (Pastorello et al. 2010; Jerkstrand et al. 2017; Nicholl et al. 2016a; Liu et al. 2017). As shown in Figure 6, the radio brightness of the central-engine powered supernova ejecta highly depends on the density profile of the ejecta and the CSM density. Our previous hydrodynamics simulation (Suzuki & Maeda 2017) suggests that if an SN harbours a sufficiently energetic central engine to produce the hot bubble breakout, the impact of the blowout would create an ejecta component travelling at relativistic speeds. This situation corresponds to models with shallow density gradient and dense CSM densities. By observations conducted earlier (at several 10 days). On the other hand, for the recently discovered SLSN-I 2017egm by Gaia16apd, tighter upper limits are available. These upper limits rule out most of the models with shallow density gradients and dense CSM densities.

X-ray emission can also assess the presence of relativistic ejecta. The predicted X-ray fluxes are much lower than most of the currently available upper limits for SLSNe-I. The theoretical models cannot explain the X-ray luminosity of SCP 06F6, indicating a different origin for the unusually high X-ray luminosity.
bright X-ray emission. Our fiducial model agrees with the X-ray luminosity of PTF12dam, although it requires further observations to confirm the association of the X-ray source with PTF12dam. Furthermore, models with short spin down times well explain the X-ray emission from the SNe Ic-BL, 1998bw and 2009bb. Although X-ray observations of SLSNe-I are currently not so constraining as radio observations, future X-ray observations can also be used as a powerful tool for investigating the outermost ejecta structure of SLSNe-I and other extraordinary SNe.

We consider two possibilities to interpret the radio non-detections. First, some SLSNe may not experience the hot bubble breakout because only a small amount of additional energy is injected from the central engine. Although the kinetic energy of the supernova explosion preceding the central energy injection is not known, if we assume a typical kinetic energy of $10^{51}$ erg, the additional energy required to produce the hot bubble breakout ranges from a few $10^{51}$ to $10^{52}$ erg, depending on the density structure of the supernova ejecta (Suzuki & Maeda 2017; Blondin & Chevalier 2017). The light curve fitting of multi-colour optical data of SN 2017egm (Nicholl et al. 2017b) infers a relatively small kinetic energy, $1-2\times10^{51}$ erg, for this nearby event. The kinetic energy of Gaia16apd is estimated to be $3.69_{-0.59}^{+1.38}\times10^{51}$ erg (Nicholl et al. 2017c). The estimated kinetic energy of the ejecta also depend on the density structure of the freely expanding ejecta assumed in the light curve fitting model. Although there are uncertainties in the critical energy for the hot bubble breakout and the kinetic energy of the supernova ejecta, the injected energy may be smaller than the critical energy and thus relativistic ejecta may not be produced. Nicholl et al. (2017c) also performed light curve fitting to other SNSNe-I. According to their results, the ejecta mass and the kinetic energy of SLSNe-I are distributed in the range of $2.2-12.9\ M_\odot$ and $(1.9-9.8)\times10^{51}$ erg, respectively. Thus, the ejecta mass and energy assumed in our model are close to the upper end of the distributions. On the other hand, the spin down time $t_{sd} = 10^6 \text{ s}$ employed in our fiducial model is typical among the SLSNe. This may suggest the possibility that not all SLSNe-I experience the hot bubble breakout with bright radio emission.

The other possibility is that the density slope of the outermost layer of SLSNe ejecta is not so shallow as predicted by the hydrodynamic simulation by Suzuki & Maeda (2017). Although the hydrodynamic simulation suggests the presence of the mildly relativistic ejecta, it is still unclear whether the relativistic component is realized when correctly taking into account coupling between gas and radiation. In the hydrodynamics simulation without radiative transfer, gas and radiation are assumed to be strongly coupled, which enables the efficient acceleration of the outermost layers by radiation pressure. However, in reality, gas and radiation may be coupled only weakly in the outermost layers. In other words, radiation in the outermost layer may simply escape into the surrounding space rather than accelerating gas in the layer, leading to smaller kinetic energy and maximum velocity of the ejecta in the homologous expansion stage. This may be especially true for SLSNe-I, because they require spin down timescales comparable to the diffusion time of thermal photons in the ejecta.

5.2 SN explosions with a central engine

From the results of our broad-band light curve modelling combined with the dynamical evolution of SNe with central energy sources revealed by Suzuki & Maeda (2017), we can speculate the following scenario for SNe with central energy sources.

The most important factor is the total amount of the injected energy. For an injected energy exceeding a critical value depending on the original ejecta structure, the ejecta are significantly affected by the energy injection. Even when the additional energy is deposited as thermal energy, a quasi-spherical relativistic wind would soon be created around the centre and start pushing the ejecta. The hot bubble breakout and the associated energy redistribution throughout the ejecta potentially produce mildly relativistic ejecta with a shallow density gradient.

The presence of relativistic ejecta depends on the timescale of the energy injection compared with the diffusion timescale of the ejecta. For central energy injection with much shorter duration than the diffusion timescale, the injected energy would predominantly be converted to the kinetic energy of the ejecta via adiabatic expansion rather than escaping as thermal photons. This case likely produces supernova ejecta with a relatively large kinetic energy, which may be observed as SNe Ic-BL. On the other hand, for energy injection timescales comparable to or longer than the diffusion timescale, the injected energy can easily escape into interstellar space as thermal photons, giving rise to bright thermal emission. This may correspond to SLSNe-I. In terms of their radio and X-ray properties, SLSNe-I produced in such a way can be divided into two classes. One is the population harbouring sufficiently energetic central engine to produce the hot bubble breakout and thus they are radio-loud. The other is the population whose central engine can give rise to bright optical emission but is not accompanied by relativistic ejecta.

5.3 Other remarks

Finally, we mention the following two remarks on the broad-band light curve modelling. We should note that the expected radio and X-ray luminosities highly depend on the CSM density and the density profile of the supernova ejecta. The circumstellar environments of SLSNe-I are poorly known. They may explode in relatively clean environments, making non-thermal emission weak. The radio and X-ray light curve modelling of SLSNe-I significantly suffer from these uncertainties.

Another potential caveat is that the non-thermal emission could also arise from the wind nebula of the nascent neutron star. Recent theoretical modelings of non-thermal emission from the wind nebula embedded in spherical supernova ejecta suggest that the non-thermal emission start leaking the dense supernova ejecta after ~100 days (Kotera et al. 2013; Metzger et al. 2014; Murase et al. 2015; Kashiya et al. 2016; Omand et al. 2018). Although how early X-ray and radio emission starts leaking depends on the multi-dimensional density structure of the supernova ejecta, non-thermal emission from the wind nebula would basically be preceded by that from the shock interaction at the ejecta-CSM interface. Therefore, radio and X-ray detections at
early epochs likely indicate non-thermal emission from the blast wave driven by the supernova ejecta.

APPENDIX A: RADIATIVE PROCESSES

In this section, we summarize several formulae for radiative processes used in our non-thermal emission model.

A1 Synchrotron emission

Synchrotron radiation power per unit frequency by a single electron with a Lorentz factor $\gamma_e$ is given by

$$P_{\nu,\text{syn}}(\gamma_e) = \frac{\sqrt{3}e^3B}{3mc^2} \sin \theta_p F(\nu/\nu_{\text{syn}}),$$  \hspace{1cm} (A1)

with the synchrotron frequency $\nu_{\text{syn}}$ being

$$\nu_{\text{syn}} = \frac{3\sqrt{2}eB}{2mc^2} \sin \theta_p,$$  \hspace{1cm} (A2)

where $e$ is the elementary charge and the pitch angle $\theta_p$ specifies the angle between the electron orbit and the magnetic field line. In this paper, we set the pitch angle to be

$$\sin \theta_p = \frac{\sqrt{2}}{3},$$  \hspace{1cm} (A3)

in order for the power integrated with respect to the solid angle matches the synchrotron energy loss rate, Equation (25). The frequency dependence of the synchrotron power per unit frequency is determined by the function $F(x)$, which is given by the following form,

$$F(x) = \int_0^\infty K_0(y) y dy,$$  \hspace{1cm} (A4)

where $K_0(y)$ is the modified Bessel function of the second kind with an order 5/3. This function can be numerically evaluated in a straightforward way.

A2 Inverse Compton emission

The interaction between an electron and a photon has long been considered. The redistribution function of Compton scattering gives the distribution of an outgoing photon in the energy space for an incoming photon and is calculated by carefully integrating the differential cross section of the process (e.g. Jones 1968; Pomerantz 1973; Kershaw et al. 1986; Coppi & Blandford 1990).

In the same way as previous work (e.g., Vurm & Poutanen 2009), we obtain the redistribution function fully taking account relativistic effects by integrating the covariant form of the different cross section (or the invariant scattering amplitude). The result is described as follows,

$$\Delta G_i(E_e, E'_e, E_{\gamma}) = \sum_{i=0}^{4} \left[ G_i(w_+, E_e, E'_e, E_{\gamma}) - G_i(w_-, E_e, E'_e, E_{\gamma}) \right]$$

$$+ G_5(w_+, E_e, E'_e, E_{\gamma}) + G_5(w_-, E_e, E'_e, E_{\gamma}),$$  \hspace{1cm} (A5)

where $E_e$ is the energy of the incoming electron and $E'_e$ and $E_{\gamma}$ are the energies of incoming and scattered photons. The functions $G_i$ ($i = 0$–5) are given by

$$G_0(w_+, E_e, E'_e, E_{\gamma}) = -\frac{2}{E'_e E_{\gamma}} \left( (E'_e - E_{\gamma})^2 + \frac{4E_e E'_e}{1 + w^2} \right)^{1/2},$$  \hspace{1cm} (A6)

$$G_1(w_+, E_e, E'_e, E_{\gamma}) = -\frac{2m_e^2c^4}{E'_e E_{\gamma}} \sqrt{E_e^2 + m_e^2c^4 w^2},$$  \hspace{1cm} (A7)

$$G_2(w_+, E_e, E'_e, E_{\gamma}) = -\frac{2\sqrt{E_e^2 + m_e^2c^4 w^2}}{(E_e^2 - m_e^2c^4)(1 + w^2)},$$  \hspace{1cm} (A8)

$$G_3(w_+, E_e, E'_e, E_{\gamma}) = \frac{2m_e^2c^4}{(E_e^2 - m_e^2c^4)^3/2} \left[ 1 + \frac{2E_e^2 - m_e^2c^4}{E'_e E_{\gamma}} \right]$$

$$\times \text{Arctanh} \left( \frac{E'_e + m_e^2c^2 w^2}{E_e^2 - m_e^2c^4} \right),$$  \hspace{1cm} (A9)

and

$$G_5(w_+, E_e, E'_e, E_{\gamma}) = \frac{m_e^2c^4(E_e + E_e')}{E'_e E_{\gamma}} \frac{E_e}{\sqrt{E'_e^2 + m_e^2c^4 w^2}}$$  \hspace{1cm} (A10)

The quantity $w$ in these functions is related to the cosine of the scattering angle $\mu$,

$$w^2 = \frac{1 + \mu}{1 - \mu},$$  \hspace{1cm} (A11)

and the values of the scattering angle $\mu$ corresponding to $w_+$ and $w_-$ are obtained by solving the following quartic equation,

$$E'_e E_{\gamma}^2 (1 - \mu)^2 + 2E_e E'_e - E_e E'_e E_e + E_e - E_{\gamma} - m_e^2c^4 (1 - \mu)$$

$$+ m_e^2c^4 (E_e - E_e')^2 = 0.$$  \hspace{1cm} (A12)

The larger and smaller values of $w$ corresponding to the solutions are denoted by $w_+$ and $w_-$, respectively.

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ACKNOWLEDGEMENTS

We thank the anonymous referee for his or her constructive comments. This research was supported by MEXT as a Priority Issue on the post-K computer (Elucidation of the Fundamental Laws and Evolution of the Universe) and the Joint Institute for Computational Fundamental Science (JICFuS). K.M. acknowledges support by the Japan Society for the Promotion of Science (JSPS) KAKENHI grant 17H02864.

This paper has been typeset from a \TeX/La\TeX file prepared by the author.