Linear magnetoresistance in the charge density wave state of quasi-two-dimensional rare-earth tritellurides

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We report measurements of the magnetoresistance in the charge density wave (CDW) state of rare-earth tritellurides, namely TbTe$_3$ and HoTe$_3$. The magnetic field dependence of magnetoresistance exhibits a temperature dependent crossover between a conventional quadratic law at high $T$ and low $B$ and an unusual linear dependence at low $T$ and high $B$. We present a quite general model to explain the linear magnetoresistance taking into account the strong scattering of quasiparticles on CDW fluctuations in the vicinity of “hot spots” of the Fermi surface (FS) where the FS reconstruction is the strongest.

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I. INTRODUCTION

Interaction between pairs of quasiparticles often leads to broken-symmetry ground states in solids. Typical examples are the formation of Cooper pairs in superconductors, or charge (CDW) and spin (SDW) density waves driven by electron-phonon and electron-electron interactions respectively. A CDW ground state is characterized by a spatial modulation $\sim \cos(Qx + \varphi)$ of the electron density and a periodic lattice distortion with the same $Q_{CDW} = 2k_F$ wave vector inducing opening of a gap, $\Delta$, in the electron spectrum. From one-dimensional (1D) weak coupling mean field theories, with $\Delta/E_F \ll 1$, the Peierls instability is driven by the electronic energy gain which originates mostly from the Fermi surface (FS) nesting with $Q = 2k_F$.

In the case of not complete nesting in quasi-one-dimensional (Q1D) compounds or in the case of quasi-two-dimensional (Q2D) or three-dimensional (3D) conductors the ground state in the CDW state is semimetallic because electron and hole pockets remain in the FS. Properties of these carriers can be modified by the CDW ordering. One of the methods to understand this possible modification is to study magnetoresistance.

In conventional metals, the Lorentz force caused by an applied magnetic field changes the electron trajectory and gives rise to a positive magnetoresistance (MR) which increases quadratically with the strength of the field. Only in a few cases the MR may grow linearly with the field (LMR). For the first time such type of behavior was observed by Kapitza in polycrystalline metals. It was shown that LMR is attributed to the presence of open Fermi surfaces. The quantum mechanism of LMR was proposed by Abrikosov. In his model LMR is realized basically in gapless semiconductors or semimetals with a linear energy spectrum and with a very small carrier concentration, so that only one Landau band participates in the conductivity. Parish and Littlewood considered a macroscopically disordered and strongly inhomogeneous semiconductor and showed that a classical mechanism will give LMR in this case. In Ref. it was shown that LMR may occur in weakly inhomogeneous systems, for fields where the cyclotron orbit period exceeds the scattering time.

From many published works one can also conclude that this unusual LMR may be a common feature of CDW systems. Indeed, LMR was observed in Q1D compounds exhibiting a CDW with incomplete FS nesting such as NbSe$_2$, and in PdTe$_3$. Effect of LMR was reported for Q2D compounds with a CDW: transition metal dichalcogenides 2H-NbSe$_2$; 2H-TaSe$_2$, 1T-TaSe$_2$, 1T- NbTe$_2$, monophosphonate tungsten bronzes (PO$_2$)$_3$(WO$_3$)$_2m$ for $m=4,6,7$; molybdenum purple bronze, K$_0.5$Mo$_3$O$_{11}$ and molybdenum oxides $\eta$-Mo$_3$O$_{11.5}$. In the present work we have studied galvanomagnetic properties in another type of Q2D compounds with a CDW, namely, rare-earth tritellurides. We have measured magnetoresistance in the temperatures range across the Peierls transition temperature and show that effectively LMR appears below this temperature.

Rare-earth tritellurides RT$_3$ ($R$ = Y, La, Ce, Nd, Sm, Gd, Th, Ho, Dy, Er, Tm) exhibit an incommensurate CDW through the whole $R$ series with a wave vector $Q_{CDW} = (0, 0, \sim 2/7c^*)$ with a Peierls transition temperature above 300 K for the light atoms (La, Ce, Nd). For the heavier $R$ (Dy, Ho, Er, Tm) a second CDW occurs with the wave vector $Q_{CDW2} = (\sim 2/7a^*, 0, 0)$. The superlattice peaks measured from X-ray diffraction are very sharp and indicate a long range 3D CDW order.

Below the Peierls transition, in all RT$_3$ compounds, the Fermi surface is partially gapped resulting in a metallic behavior at low temperature. The layered RT$_3$ compounds exhibit a large anisotropy between the resistivity along the $b$-axis and that in the $(a,c)$ plane, typically $\sim 10^2$ below $T_{CDW}$ and much higher at low temperature. Because the unidirectional character of the upper CDW, a conductivity anisotropy in the $(a,c)$ plane arises in the CDW state as was observed experimentally and explained theoretically in Ref. The effect of the upper CDW on the in-plane resistivity observed in experiments is very weak, no more than a few percent of the total resistance.

For our study we chose two compounds: TbTe$_3$ as a system with unidirectional CDW and HoTe$_3$ exhibiting a bidirectional CDW. In TbTe$_3$ the CDW ordering...
is observed well above room temperature ($T_{CDW1}=336$ K). In HoTe$_3$ the first and the second CDW transitions take place at $T_{CDW1} = 283$ K and $T_{CDW2} = 110$ K correspondingly.$^{19}$

II. EXPERIMENTAL

Single crystals of TbTe$_3$ and HoTe$_3$ were grown by a self-flux technique under purified argon atmosphere as described previously.$^{22}$ Thin single-crystal samples with a square shape and with a thickness less than 1 µm were prepared by micromechanical exfoliation of relatively thick crystals glued on a sapphire substrate. The untwinned character of selected crystals and the spatial arrangement of crystallographic axes were controlled with a square shape and with a thickness less than 1 µm. Room temperature resistivity of samples demonstrates nearly the same behavior: magnetic field, in Fig. 3 (a) and (b) we plot MR as a function of square $B$ for TbTe$_3$ (b) in a log-log plot. Both compounds demonstrate nearly the same behavior: magnetoresistance changes by more than four order of magnitude as temperature $T$ decreases from 300 K to 20 K. Simultaneously, the power-law field dependence of MR changes monotonically from quadratic (red straight line segment) at high $T$ and at low $B$ to linear (blue straight line segment) at low $T$ and high $B$. Note, that in the studied magnetic field range (up to 6.5 T) we never observed any deviation of MR from quadratic law at temperatures above the Peierls transition temperatures. The examples of MR($B$) dependencies measured at $T$ above $T_{CDW}$ (330 K and 290 K for TbTe$_3$ and HoTe$_3$ respectively) and below $T_{CDW}$ (40 K for both compounds) are shown in Fig. 2. Note, that at the same temperature $T = 40$ K, the linear $R_{xx}(B)$ is more pronounced for HoTe$_3$ in which two CDWs exist at this temperature.

To make this quadratic-to-linear MR crossover clearer, in Fig. 1 (a) and (b) we plot MR as a function of square magnetic field, $B^2$ for HoTe$_3$ and for TbTe$_3$ correspondingly. Solid black lines are quadratic dependencies which coincide with the experimental curves at low magnetic fields. At a certain magnetic field, $B^*$, experimental dependencies deviate from these lines. Temperature dependence of this characteristic field $B^*$ is shown in Fig. 4. As can be seen, $B^*$ increases rapidly or even diverges when $T$ approaches the CDW transition temperature.

III. EXPERIMENTAL RESULTS

The temperature variation of the field dependence of magnetoresistance, defined as $MR= |R_{xx}(B) - R_{xx}(0)|/R_{xx}(0)$, in the temperature range from 10 K up to the temperature well above $T_{CDW}$ is shown in Fig. 1 for HoTe$_3$ (a) and for TbTe$_3$ (b) in a log-log plot. Both compounds demonstrate nearly the same behavior: magnetoresistance changes by more than four order of magnitude as temperature $T$ decreases from 300 K to 20 K. Simultaneously, the power-law field dependence of MR changes monotonically from quadratic (red straight line segment) at high $T$ and at low $B$ to linear (blue straight line segment) at low $T$ and high $B$. Note, that in the studied magnetic field range (up to 6.5 T) we never observed any deviation of MR from quadratic law at temperatures above the Peierls transition temperatures. The examples of MR($B$) dependencies measured at $T$ above $T_{CDW}$ (330 K and 290 K for TbTe$_3$ and HoTe$_3$ respectively) and below $T_{CDW}$ (40 K for both compounds) are shown in Fig. 2. Note, that at the same temperature $T = 40$ K, the linear $R_{xx}(B)$ is more pronounced for HoTe$_3$ in which two CDWs exist at this temperature.

IV. THEORETICAL MODEL AND DISCUSSION

Thus, most of charge-density wave systems with imperfect nesting exhibit a linear magnetoresistance that is, probably, related to the CDW electronic structure. To propose a possible explanation of this linear MR we invoke a usually neglected scattering mechanism of quasi-particles on the fluctuations of the order parameter of charge density wave, which violate the space uniformity and lead to the momentum relaxation of quasi-particles. The scattering on CDW fluctuations is the strongest near the so-called ”hot spots” on the Fermi surface (FS). Somewhat similar mechanism of linear MR but above the CDW transition temperature was proposed in Refs. 28, 29. In Ref. 28, the scattering in the hot spots, with large momentum and low energy transfer, in-
Involves umklapp processes. In Ref. 29 it involves the scattering by soft phonons, appearing due to the proximity to Peierls instability. In our case these hot spots are the FS areas where the FS reconstruction due to CDW is the strongest. Usually the hot spots are the ends of the ungapped FS parts. The electron dispersion in such hot spots depends strongly on the CDW structure, and the electrons in such hot spots may be easily scattered by CDW fluctuations. Thus, in cuprate high-Tc superconductors such hot spots are the ends of Fermi arcs, but the FS reconstruction is driven by the pseudogap or antiferromagnetic ordering, rather than by CDW. In organic metals, e.g. α-(BEDT-TTF)$_2$KHg(SCN)$_4$, where the CDW leads to the FS reconstruction and changes its topology, such hot spots are the points of intersection of the original FS and the FS shifted by the CDW wave vector. In these hot spots the electron dispersion changes strongly, somewhat similar to the change of electron dispersion at the boundaries of the Brillouin zone in the weak-coupling approximation, where the energy gap is formed due to periodic potential, which is the CDW in our case. Since the periodic potential and the formed energy gap in the electron spectrum is of the order of CDW order parameter and much less that the Fermi energy, in high magnetic field the electron trajectories in these hot spots are subject to magnetic breakdown in addition to the direct scattering by CDW fluctuations. This leads to an additional indirect scattering mechanism of conductive electrons, which may be rather strong.

The electron scattering in the hot spots leads to the linear field dependence of the scattering rate and, hence, to the linear magnetoresistance. To show this linear field dependence of the electron mean-free time $\tau$ we assume that in each hot spot an electron is scattered with some probability $w_{hs} < 1$; the possible origin of this scattering is discussed later. If this hot-spot scattering is the main scattering mechanism of conducting electrons, the corresponding electron mean-free time $\tau_{hs} = t_{hs}/w_{hs}$, where $t_{hs}$ is the mean time between electron passing through these hot spots. This time $t_{hs}$ is determined by the FS details. In magnetic field $H$ electrons move in momentum space along the Fermi surface due to the Lorentz force, $dp/dt = (e/c)[v_\perp \times H]$, and periodically pass through such hot spots. Hence, the mean free time $\tau_{hs}$ is proportional to the length of the Fermi-surface between hot spots divided by magnetic field $H$ strength and by electron velocity in real space $v_\perp$:

$$\tau_{hs} = (e/cHw_{hs}) \int dl/v_\perp. \quad (1)$$

If the electron trajectory in magnetic field is closed, its motion is periodic given by the cyclotron (or Laron...
which gives for the resistivity tensor or holes from different Fermi-surface parts, the quadratic magnetoresistance appears, which saturates at high magnetic field. Thus, for the simplest isotropic model of only two types of carriers the calculation based on the kinetic equation gives (see Eq. 7.163 of Landau)

$$\frac{\Delta R_{xx}(H)}{R_{xx}(0)} = \frac{\sigma_1 \sigma_2 (\omega_c \tau_1 - \omega_c \tau_2)^2}{(\sigma_1 + \sigma_2)^2 + (\omega_c \tau_1 \sigma_1 + \omega_c \tau_2 \sigma_2)^2},$$

(4)

where the subscripts 1 and 2 denote the charge carriers of the first and second type respectively. The quadratic dependence here comes from $\omega_c$, in numerator, and the saturation at $\omega_c \tau \gtrsim 1$ comes from $\omega_c$ in denominator. Usually, the relaxation time depends on the speed $v_i$ of an individual charge carrier, so that one cannot describe the motion of the carriers in terms of a single drift velocity even for metals with a single electron band. Therefore, even in single-band metals in weak field one observes the quadratic magnetoresistance

$$\frac{\Delta R_{xx}(H)}{R_{xx}(0)} = \frac{R_{xx}(H) - R_{xx}(0)}{R_{xx}(0)} = \frac{\alpha (\omega_c \tau)^2}{1 + \beta (\omega_c \tau)^2},$$

(5)

where the coefficients $\alpha \sim \beta \sim 1$, which saturates in strong field when $\omega_c \tau \gtrsim 1$. This quadratic magnetoresistance in weak fields can be understood also in terms of the curvature of electron trajectories, which geometrically reduces the electron mean-free path $l_i = \tau v_i$ by the quantity $\sim l_i (l_i/r_L)^2$, where $r_L \gg l_i$ is the Larmor radius. Since $l_i \propto \tau$, this effect can be taken into account in Eq. (4) by the renormalization of the electron mean-free time $\tau (H)$ in weak magnetic field $H$ according to

$$\frac{\tau (0)}{\tau (H)} = 1 + \frac{\alpha (\omega_c \tau)^2}{1 + \beta (\omega_c \tau)^2},$$

(6)

which leads to Eq. (5).

If the electron scattering is dominated by the scattering in the hot spots, instead of Eq. (4) we have $\tau (H) \approx \tau_{hs} \propto 1/H$, and Eq. (3) gives $R_{xx} \propto H$. However, in real compounds the total scattering rate $\tau^{-1}$ is a sum of the contribution from various mechanisms, including those from the scattering by impurities $\tau_{i}^{-1}$ and by phonons $\tau_{ph}^{-1}$. The scattering by phonons at high temperature is somewhat similar to scattering by short-range impurities, as in both cases the momentum transfer during each scattering is large and comparable to the Fermi momentum. In rather weak magnetic field, when the Landau levels are not separated and the magnetic quantum oscillation can be neglected, the scattering rates $\tau_{i}^{-1}$ and $\tau_{ph}^{-1}$ depend on magnetic field according to Eq. (6). Then $\tau^{-1} \approx \tau_{i}^{-1} + \tau_{ph}^{-1} + \tau_{hs}^{-1}$, and the linear MR appears only in rather strong magnetic field, when $\tau_{hs}^{-1} > \tau_{i}^{-1} + \tau_{ph}^{-1} \equiv \tau_{i+ph}^{-1}$, i.e., when $\omega_c \tau_{i+ph} \gtrsim 2\pi/w_{hs}n_{hs}$, or when the quadratic magnetoresistance saturates. At high temperature, when the scattering rate by phonons $\tau_{ph}^{-1}$ becomes larger than $\tau_{hs}^{-1} \approx \omega_c w_{hs}n_{hs}/2\pi$, one should...
observe a usual quadratic MR. On contrary, at lower temperature and in higher field in the CDW state, when \( \tau_{hs} < \tau_{i+ph} \), the linear MR should be observed as a general phenomenon. This crossover is clearly seen on experimental data in Figs. 1-3 and, according to the theoretical model, the crossover field increases with temperature, as shown in Fig. 4.

Let us discuss the possible microscopic origin of the electron scattering in the hot spots in more detail. The mechanism of linear MR above the CDW transition temperature \( T_{CDW} \), proposed in Ref. 29, assumes a strong scattering by soft phonons with a wave vector close to the nesting vector \( Q_N \) due to the Peierls instability. Then the hot spots are those connected by the CDW wave vector, as shown in Fig. 5a. However, in our experiment the linear MR is observed much below \( T_{CDW} \). Then the expected hot spots are the ends of the ungapped FS parts, as shown in Fig. 5b. In these hot spots the FS reconstruction due to CDW is the strongest, and the electron dispersion depends strongly on the CDW order parameter. Therefore, electrons in such hot spots may be easily scattered by CDW fluctuations. Unfortunately, the available experimental data do not give detailed informations about the Fermi surface in RTe\(_3\) compounds in the CDW state: the ARPES data\(^{21}\) do not have sufficient resolution to determine even the FS topology in the CDW state, while the magnetic quantum oscillations in the CDW state are complicated by the second CDW transition in some RTe\(_3\) materials and by magnetic breakdown. Therefore, there are possibly other hot spots in the ungapped FS parts, and without detailed information about the FS in the CDW state we cannot predict the coefficient in the dependence \( \tau_{hs} \propto 1/H \).

There are two types of CDW fluctuations: amplitude and phase fluctuations. Both may arise, e.g., due to the CDW pinning by crystal defects or local inhomogeneities. The amplitude CDW defects may strongly scatter the conducting electrons, e.g., due to the inhomogeneous magnetic breakdown (MB)\(^{31}\) because the MB probability depends exponentially\(^{29,32}\) on the gap opened in the electron spectrum due to the CDW. At the ends of the gapped region the gap values decrease, as explicitly shown by ARPES data\(^{21}\), and the magnetic breakdown become possible even in low field. Moreover, the amplitude fluctuations of the CDW gap may lead to spatial variations of the boundary between gapped and ungapped FS parts. Therefore in Fig. 5b we place the hot spots at the ends of the gapped region. The MB amplitude depends strongly not only on the gap value, but also on the electron velocity and dispersion in the MB region\(^{32}\) which is also affected by the amplitude fluctuations of CDW. The inhomogeneous MB probability leads to the strong electron scattering in the MB regions, playing the role of hot spots. Note, that this strong scattering mechanism may be important not only for transverse but also for the longitudinal MR and may even lead to the phase inversion of magnetic quantum oscillations\(^{31}\).

The CDW phase fluctuations mean local variations of the CDW wave vector, which also change the electron dispersion in our hot spots and affect the MB probability in addition to the direct scattering in these hot spots by the CDW periodic potential with inhomogeneous wave vector. Hence, the CDW fluctuations may indeed lead to the strong electron scattering in the hot spots of the Fermi surface, and, consequently, to linear MR.

Our theoretical model is in many aspects similar to that in Ref. 29, but there are some important differences. First, the model in Ref. 29 is developed and applied only slightly above the CDW transition temperature \( T_{CDW} \), while our model can be applied much below \( T_{CDW} \). Second, the model in Ref. 29 is applied only to the unrecon-
structured FS, while we apply our model to the strongly
reconstructed FS. Third, in Ref. 22 the CDW fluctuations
have the wave-vector equal to the nesting vector, while in
our model any Fermi-surface reconstruction due to CDW
leads to scattering by CDW fluctuations, even if there
are no ungapped Fermi-surface points connected by the
CDW wave vector. Fourth, we propose a temperature-
driven crossover between the quadratic and linear mag-
etoresistance, which cannot be found in the model of
Ref. 22 where the CDW fluctuations are considered only
in the vicinity of transition temperature.

In conclusion we have shown that in the CDW
state of RTe$_3$ compounds there is a crossover from linear
magnetoresistance at low temperature to usual quadratic
magnetoresistance at higher temperature. We propose a
general explanation of this phenomenon as being related
to the electron scattering in the hot spots of Fermi surface
due to the spatial fluctuations or inhomogeneity of the
charge-density-wave order parameter.

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and the interlayer transfer integral $t_z$, in addition to quan-
tum oscillations the effective mean-free time $\tau$ acquires
a monotonic magnetic-field dependence. As a result, at
$\hbar \omega_c \gg t_z, h/\tau$ the square-root longitudinal interlayer mag-
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