A small displacement torsor (SDT) -based haptic rendering model for assembly of bolted joints

Yu Yong-Peng¹*¹, Xu De-Jian¹

¹School of Mechanical and Automotive Engineering, South China University of Technology, Guangzhou, P.R. China, 510640.
* Corresponding author. Email: 201820100437@mail.scut.edu.cn

Abstract. Haptic interaction technology is increasingly used in virtual assembly training, assembly path planning, teleoperation assembly and other fields. Obviously, the key issue of haptic interaction technology is the realistic haptic rendering models. This paper proposes a haptic rendering model based on small displacement torsor (SDT) for bolt assembly. For the first time, geometrical errors of parts are considered in the haptic rendering model of bolt assembly, which can improve the fidelity. The comparative experiment proves that the haptic rendering model proposed in this paper is more familiar to the actual assembly than the existing model.

1. Introduction
Haptic interaction technology is widely used in virtual assembly, product design and simulation, robot teleoperation and other fields [1]. Traditional assembly training requires a lot of cost in manual training and parts production. Fortunately, with the development of haptic interaction technology, virtual assembly training system with force feedback can achieve better training results with a lower cost [2]. Simultaneously, in robot teleoperation field, in some remote, high-risk, harsh, and difficult-to-reach environments, engineers can operate the controller locally to control the remote robot to complete the assembly of mechanical parts [3]. Further, the precise assembly haptic rendering can improve the reliability and safety of the system, helping the operators control accurately and improve the assembly quality.

Bolted joints are widely applied in the connection between mechanical parts due to the ease of assembly and disassembly. The assembly torque of the bolt assembly directly affects the pre-tightening force of the bolt, thus affecting the performance and safety of the machine. Hence, accurate haptic rendering is of great significance to improve the effect of virtual assembly training and the assembly quality of teleoperation assembly. Some scholars have carried out research on assembly haptic modelling. Liu et al. [4] decomposed the assembly process into two basic types of manipulations, i.e., axis-alignment assembly and face-mating assembly, and proposed different assembly haptic models by adding constraints. Q. H. Wang et al. [5] studied three fits of clearance, transition and interference for shaft-bushing assembly, and established the assembly haptic model. Wei Jiang et al. [6] proposed a virtual assembly haptic model integrated with physics engine, which can achieve more stable and realistic haptic rendering.

However, the contacts of the mating surfaces in the assemblies above are relatively simple, but the contact between bolted threads is more complicated, so the above assembly haptic models are not property to apply to bolt assembly. There are fewer haptic models for bolt assembly. Jing-Rong Li et al. [7] proposed a staged assembly haptic model for bolted joints. However, this approach did not consider
the influence of the geometrical errors of the parts. In fact, there are some differences in assembly force with different tolerance grades. Hence, existing methods cannot provide realistic and accurate haptic rendering for bolt assembly. Aiming at the shortcomings of the existing bolt assembly haptic model, this paper takes into account the influence of geometrical errors, using SDT to describe the tolerance of the parts, and proposes a bolt assembly haptic model based on SDT.

This paper proposes an SDT-based haptic rendering model of bolted joints. The rest of the paper is organized as follows. Section 2 introduces the SDT-based tolerance modelling. Then the force rendering model is proposed in section 3. While section 4 presents the case studies to evaluate the proposed force rendering model. And the last section, section 5 concludes the work.

2. The small displacement torsor (SDT) -based tolerance modelling

The relative error between two features of the parts can be represented by a set of SDT. For bolt assembly, the error that affects the assembly force is mainly the relative error between the contact surfaces of the nut and the connected part, as shown in Fig.4. In order to analyse the error, it is supposed to create a dimensional chain between different elements. Specifically, the feature of a part is represented by two numbers, namely (a, b), the first number represents the part code, and the second number represents the feature code on this part [8]. The representations of the features are shown in Fig.1. The paths of the dimensional chain transfer from feature (1,1) to feature (3,2) and from feature (1,1) to feature (2,2) are shown in Fig.2 respectively. The relative SDT between the features are represented as \( r_{1,2/1,1}, r_{3,1/1,2}, r_{3,2/3,1}, r_{2,1/1,1}, r_{2,2/2,1} \) respectively.

For bolt assembly, during assembly process, plane (1,1) and plane (2,1) are pressed against each other, hence the relative SDT \( r_{2,3/1,1} = 0 \); The threads of the bolt and the nut mesh with each other, and the centreline (1,2) and centreline (3,1) are aligned, hence the relative SDT \( r_{3,1/2,1} = 0 \). For the remaining relative SDTs, for convenience, \( r_1, r_2, r_3 \) are used to represent \( r_{1,2/1,1}, r_{2,3/1,1} \) and \( r_{3,1/2,1} \) respectively. Then the SDT of feature (3,2) relative to feature (2,2) \( r_\nu \) is represented as \( R_1 r_1 + R_3 r_3 - R_5 r_5 \), where, \( R_1, R_2, R_3 \) represent the projection matrix of \( r_1, r_2, r_3 \) in the basic coordinate system respectively. During the assembly process, the movement of the nut can be decomposed into a rotation movement around Z axis and a translation movement along Z axis. The bolt and the connected parts do not rotate, therefore, \( R_1 = R_3 = E \). However, the nut rotates relative to the connected part, the projection of \( r_2 \) in the basic coordinate system changes with the rotation angle \( \alpha \) of the nut. The projection matrix is represented as:

\[
R_2(\alpha) = \begin{bmatrix}
\cos \alpha & \sin \alpha & 0 \\
\sin \alpha & \cos \alpha & 0 \\
0 & 0 & 1
\end{bmatrix}
\]

Hence, the relative SDT \( r_\nu \) changing with \( \alpha \) is represented as \( r_\nu(\alpha) \):

\[
r_\nu(\alpha) = r_1 + R_2(\alpha) r_3 - r_5
\]

According to SDT theory, the specific expressions of \( r_1, r_2, r_3 \) are:

\[
\begin{align*}
r_1 &= \begin{bmatrix}
\rho_{11} & \rho_{12} & 0 \\
\rho_{12} & \rho_{13} & 0 \\
0 & 0 & \epsilon_{13}
\end{bmatrix} \\
r_2 &= \begin{bmatrix}
\rho_{21} & 0 & 0 \\
0 & \rho_{23} & 0 \\
0 & 0 & \epsilon_{23}
\end{bmatrix} \\
r_3 &= \begin{bmatrix}
0 & 0 & 0 \\
0 & 0 & 0 \\
0 & 0 & \epsilon_{33}
\end{bmatrix}
\end{align*}
\]

Substituting the specific SDT parameters of \( r_1, r_2, r_3 \) into the expression, \( r_\nu(\alpha) \) is represented as:
3. The haptic rendering model for assembling bolted joints

According to the different stress properties of bolted joints, the assembly process is divided into three stages: navigation stage, transition stage and linearity stage, as shown in Fig.3. Navigation stage refers to the stage before the nut contacts with the connected part, obviously, there is no tension in the bolt in this stage. Transition stage refers to the stage when the nut and the connected part are in contact but are not fully compressed, the tension in the bolt is generated in this stage. Linearity stage refers to the stage when the nut and the connected part are fully compressed, the tension in the bolt changes linearly with the rotation angle of the nut. The completion angles of the three stages are represented as $\alpha_1'$, $\alpha_2'$ and $\alpha_3'$ respectively.

3.1. Assembly torque in navigation stage

The assembly angle of the nut in this stage is within the range of $0 < \alpha < \alpha_1'$. The nut does not contact the connected part, and the assembly torque comes from the frictional resistance between the bolt and the nut thread. The frictional resistance between threads can be equivalent to the "slope-slider model" [7].

The inclination angle of the slope is equal to the lead angle $\sigma$, the gravity of the slider is equal to the gravity of the nut $G$. According to the calculation formula of sliding friction, assembly torque $T_1(\alpha)$ in navigation stage is represented as:

$$T_1(\alpha) = G \times (\sin \sigma - \mu \times \cos \sigma) \times \frac{d}{2}$$

Where, $\mu$ is the friction coefficient between threads, $d_c$ is the contact diameter of the threads.
3.2. Assembly torque in transition stage

The assembly angle of the nut in this stage is within the range of $\alpha_1 < \alpha < \alpha_2$. The nut is in contact with the connected part, and the bolt is tensioned, hence tensile stress is generated. When the assembly angle of the nut is $\alpha$, as shown in Fig.4, the rotation angle of the nut around the Y axis $\theta(\alpha)$ is represented as:

$$\theta(\alpha) = \frac{l_z}{d_n} = \frac{(\alpha - \alpha_1)P}{\pi d_n}$$  \hspace{1cm} (6)

Where, $d_n$ is the contact diameter of the nut and the connected part.

Extra torque $T_E(\alpha)$ caused by $\theta(\alpha)$ is represented as [9]:

$$T_E(\alpha) = \frac{\theta(\alpha) \mu}{\cos \beta \left( \frac{1}{K_T} + \frac{1}{K_B} + \frac{1}{K_P} \right)}$$  \hspace{1cm} (7)

Where, $\beta$ is the thread angle, $K_T$, $K_B$ and $K_P$ are the deformation stiffnesses of the nut, bolt and the connected parts respectively.

In the process of assembly, as shown in Fig.5, the nut and the connected part gradually fit. The boundary of the pressing area and the gap area is called the critical point, which is represented by $y_0$.

When assembly angle of the nut $\alpha = \alpha_1$, then $y_0 = d_3 / 3$; When $\alpha = \alpha_2$, then $y_0 = -d_3 / 3$. When $\alpha$ is within the range of $(\alpha_1, \alpha_2)$, $y_0(\alpha)$ can be calculated with linear interpolation function:

$$y_0(\alpha) = \frac{d_3}{2} - d_3 \left( \frac{\alpha - \alpha_1}{\alpha_2 - \alpha_1} \right)$$  \hspace{1cm} (8)

The distribution density function $q$ of the contact pressure of the nut and the connected part on the Y axis is represented as:

$$q = k_x y - k_x y_0$$  \hspace{1cm} (9)

Axial tension $F_{A2}$ can be calculated by integral:

$$F_{A2} = \int_{-y_0}^{y_0} k_x y - k_x y_0 dy = k_x \left( y_0 - r_1 \right)^2 = \frac{6M (\alpha_2)}{d_3} \left( \frac{\alpha - \alpha_1}{\alpha_2 - \alpha_1} \right)^2$$  \hspace{1cm} (10)

According to the MOTOSH equation [10], the assembly torque in transition stage is represented as:

$$T_a(\alpha) = T_E(\alpha) \times \left( \frac{P}{2\pi} + \frac{\mu r_1}{\cos \beta} + \mu r_2 \right) + T_E(\alpha)$$  \hspace{1cm} (11)

Where, $\mu$ is the friction coefficient of the nut and the connected part, $r_n$ is the contact radium of the nut and the connected part.

Fig.5 Change of tightness critical point during tightening
3.3. Assembly torque in linearity stage

The assembly angle of the nut in this stage is within the range of \( \alpha_l < \alpha < \alpha_u \). The nut is fully attached to the connected part, and the axial force in the bolt has a linear relationship with the assembly angle of the nut [11]. The gap between the nut and the connecting piece has been filled, hence the bending moment in the bolt no longer increases. Therefore, the extra torque caused by the bending moment is always the extra torque at the critical angle \( \alpha_l \), which is represented as:

\[
T_e = E_r (\alpha_l)
\]  

(12)

When the assembly angle of the nut is \( \alpha \), the axial tension in bolt \( F_{a3}(\alpha) \) is represented as:

\[
F_{a3}(\alpha) = F_{a3}(\alpha_l) + \frac{P(\alpha - \alpha_l)}{2\pi \left( \frac{1}{k_r} + \frac{1}{k_u} + \frac{1}{k_p} \right)}
\]

(13)

Where, \( k_r \), \( k_u \) and \( k_p \) are the stiffness coefficients of the nut, bolt and the connected parts respectively.

According to the MOTOSH equation, the assembly torque in linearity stage is represented as:

\[
T_a = F_{a3}(\alpha) \times \left( \frac{P_m}{2\pi \cos \beta} + \mu_r r + \mu_u r_u \right) + T_e (\alpha_l)
\]

(14)

Where, the meanings of the parameters are the same as that in formula (5) and (12).

4. Case study

Three sets of experiments are designed: Real Experiment (RE), Virtual Experiment 1 (VE1) and Virtual Experiment 2 (VE2).

To be comparative, the design parameters of the parts in the three sets of experiments are identical. The size of the bolt is M5, according to GB/T 3103.1, GB/T 197-2003, GB/T 1184-1996 and GB/T 3934-2003, geometric error parameters with tolerance grade IT6 are selected. In addition, the friction coefficient between the threads, the friction coefficient between the nut and the connected part are both 0.15, further, the assembly angular velocity is 0.05 rad/s (the linear velocity is 10 mm/s).

In RE, an assembly setup is designed to measure the actual assembly resistance (Fig.6). As designed, the robot takes the nut as the centre and makes a circular motion with an angular velocity of 0.05 rad/s. While at the same time, a force sensor installed on the arm records the assembly resistance along X and Y direction. The robot tightening force arm is designed to be 0.2 m. Three groups of different bolt parts with tolerance grade IT6 are measured, and the average values are taken.

In VE1, the force rendering approach in literature [7] is used. After setting the frictional parameters, the nut is tightened at the same speed.

In VE2, the force rendering approach proposed in this paper is applied, and the nut is tightened at the same speed.

The assembly torque recorded in RE, VE1 and VE2 are shown in Fig.7.
In order to quantitatively analyse the difference between assembly torque data, mean square error (MSE) is used for analysis. The MSE of VE1 and VE2 relative to RE are 0.2753 and 0.0361 respectively. It shows that the torque data in the VE1 and RE are quite different, while the torque data in VE2 and RE are similar. This is because the assembly torque model in VE1 is too ideal and does not take into account the influence of the geometric errors of the parts. On the one hand, the angle of transition stage is only 8° in VE1, while that angle in RE is approximately 25°; On the other hand, in the linearity stage, the assembly curve in VE1 is ideally linear, while in RE, its slope changes with the assembly angle of the nut. In addition, the completed assembly angle in VE1 is 146.09°, while the angle in RE is 135.13°. Obviously, the difference is too large. In actual assembly, the deviation of the assembly angle may lead to substandard of the pre-tightening force.

On the contrary, VE2 applies the assembly torque model proposed in this paper and takes into account the geometric errors between parts. In different stages, the torque model is corrected according to the geometric errors of the parts, and the final torque curve is similar with the actual assembly torque curve. Furthermore, the completed assembly angle in VE2 is 136.09°, which is 0.96° different from that in RE. But it is acceptable because this angle deviation is difficult to detect for operators. In addition, the tightening angle deviation will not have a significant impact on the bolted pre-tightening force in actual assembly. Therefore, in terms of assembly torque and completion angle, the force model proposed in this paper is more familiar to the real situation.

5. Conclusion
This paper presents an SDT-based haptic rendering model of bolted joints. Firstly, SDT theory is applied to represent the geometrical errors of the parts. And then by analysing the stress of parts during assembly, the haptic rendering model is proposed. To verify the proposed approach, a comparative study is designed, where the proposed approach is compared with the real assembly process and the existing haptic rendering approach. The results have shown that assembly torque proposed is generally similar to that with the real assembly torque. It certainly outweighs the other approach in terms of the accuracy of the force value and completed assembly angle during assembly.

References
[1] Seth A, Vance J M, Oliver J H (2011). Virtual reality for assembly methods prototyping: a review. Virtual reality, 15(1): 5-20. Another reference
[2] Wang QH, Wu SC, Liu JW, Li JR (2018). Design of a 6-DOF force device for virtual assembly (FDVA-6) of mechanical parts. Mech Based Des Struct Mech, 46 (5):567-577.
[3] Ni, D., Song, A., & Li, H. (2017). Haptic assisted teleoperation based on virtual fixture and dynamic modelling. Sensors & Materials, 29(9(2)): 1367-1381.
[4] Liu M, Wang D, Zhang Y (2010). A novel haptic rendering algorithm for stable and precise 6-dof virtual assembly. In: Proceedings of the Asme World Conference on Innovative Virtual Reality: 251-257
[5] QingHui Wang, Zhong-Dong, Huang et al. (2018). A force rendering model for virtual assembly of mechanical parts with clearance fits. Assembly Automation, 38(2): 173-181.

[6] Jiang W, Zheng JJ, Zhou HJ, Zhang BK (2016) A new constraint-based virtual environment for haptic assembly training. Adv Eng Softw, 98:58-68.

[7] Li J R, Liu J W, Wang Q H, et al. (2018). A staged haptic rendering approach for virtual assembly of bolted joints in mechanical assembly. The International Journal of Advanced Manufacturing Technology, 96(1): 161-171.

[8] Peng HePing, Jiang Xiangqian, Liu Xiaojun (2008), Concurrent optimal allocation of design and process tolerances for mechanical assemblies with interrelated dimension chains. International Journal of Production Research, 46(24): 6963-6979.

[9] Chen, D., Ma, Y., et al. (2019). Tightening behavior of bolted joint with non-parallel bearing surface. International Journal of Mechanical Sciences, 153-154: 240-253.

[10] Motosh N. (1976) Development of design charts for bolts preloaded up to the plastic range, 98(9): 849-851.

[11] Fukuoka T, Takaki T (2018). Mechanical Behaviors of Bolted Joint during Tightening Using Torque Control. Transactions of the Japan Society of Mechanical Engineers A, 63(2):1083-1088.