Polarised beampattern synthesis against array manifold mismatch

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Abstract
The problem of joint synthesis of spatial power pattern and mainlobe radiation polarisation using a vector antenna array is considered in the presence of array manifold errors. A general approach to the synthesis problem is to formulate the restriction on the power and radiation polarisation as a convex optimisation programming. However, for certain types of vector antennas like tripole and electromagnetic vector sensors, the optimal weights might be extremely large and the resultant beampattern could be very sensitive to the array manifold errors. An intuitive explanation on this phenomenon is given based on the subspace theory. A worst case performance optimisation technique-based scheme is presented to draw attention to the polarised beampattern synthesis against the large weights problem and array manifold errors. Numerical examples regarding different types of vector antennas used for one- and two-dimensional polarised beampattern synthesis are provided to validate the efficacy of the scheme.

1 | INTRODUCTION

The utilisation of polarisation information of the electromagnetic (EM) waves has shown many advantages in various areas when using multi-polarised vector antenna (VA) arrays. In radar system, it can greatly improve the capabilities of remote sensing and target detection, estimation, and tracking [1]. In wireless communications, it can significantly increase the communication capacity and interference rejection capability [2, 3].

In many of these applications, it is required that the radiated beampattern has a certain polarisation and spatial power pattern with desired sidelobe level (SLL), null positions, and mainlobe beam width [4–13]. The joint synthesis of polarisation and power pattern has received much attention over recent years. An iterative least square method is proposed in [4] to synthesise a shaped radiation beampattern with linear or circular polarisation. However, this method cannot ensure to find the optimal excitation weights. In [5], a convex formulation is provided for the polarised beampattern synthesis problem. The achieved beampattern has a specific polarisation towards the target direction whereas the sidelobe level is minimised. A convex optimisation scheme is developed in [6] to synthesise the radiation polarisation optimally over an angular region, given a pre-specified SLL bound.

In [7, 8], the author compares the tri-polarised antenna with a dual-polarised antenna in polarised beampattern synthesis, and shows that the sidelobe and cross-polarisation level of tri-polarised array can be significantly reduced in contrast with the dual-polarised array, especially in wide scanning angle. Concerning synthesis problem of fixed user-defined desired polarisation and shaped power pattern, [9, 10] define the realisable co-polarisation and cross-polarisation directions and exploit the semi-definite relaxation or convex optimisation schemes to control co-polarisation, cross-polarisation and total power in respective regions. In addition, the wideband polarised beampattern synthesis problem is investigated in [11], and a matched dual-polarised beampattern synthesis scheme is studied in [12] for polariometric radar measurements. More recently, compressive sensing-based methods are also proposed to simultaneously optimise the polarised beampattern as well as the antenna locations of the sparse array [13].

It is not considered in the aforementioned methods, the possible array manifold errors caused by imperfect array calibration (such as channel mismatch, VA misorientation, VA position perturbations and inter-VA mutual coupling) and the excitation errors caused by the impairments in the amplifiers or attenuators of the element feeding network [14–17]. The
presence of model errors would generally lead to seriously degraded polarised beampattern synthesis.

While, the robust issue has been well addressed in the conventional polarisation fixed beampattern synthesis work (without the ability of controlling radiation polarisation) using an array of uni-polarised antennas instead. Most methods are based on the worst case performance optimisation technique [18–23]. For example, a scheme is presented in [18] by optimising the worst case SLL. In [19], the antenna mutual coupling effect is taken into account with worse case optimisation. Based on the promising weight vector orthogonal decomposition algorithm, it is discussed in [20] that how the worst case upper boundaries of beampattern in the given sidelobe points can be precisely adjusted as the desired level.

The focus of this article is on the polarised beampattern synthesis against array manifold mismatch. First, it is demonstrated, that extremely large weights (or large dynamic range) are very likely to occur when the high dimensional VAs are utilised, which are very similar to the super-directivity or super-gain phenomenon in polarisation fixed beampattern synthesis [24, 25]. This would cause relatively low radiation efficiency and high sensitivity to the array manifold errors. A robust polarised beampattern synthesis scheme is then presented by invoking worst case performance optimisation also.

Throughout the article, the following notations and conventions are used.

- $\otimes$ Kronecker product
- $I_{n \times n}$ The $n \times n$ identity matrix
- $O_{m \times n}$ The $m \times n$ zero matrix
- $Re$ Real part of a complex number
- $(\cdot)^T$ Transpose of a vector or matrix
- $(\cdot)^H$ Hermitian transpose of a vector/matrix
- $(\cdot)^\dagger$ Orthogonal complement
- $\lfloor \cdot \rfloor$ Total angle size of an angular region
- $\lceil \cdot \rceil$ The smallest integer equal to or greater than the number in the bracket
- $\| \cdot \|$ The $L_2$ norm of a column/row vector
- $\angle(\cdot)$ Principle argument of a complex number
- $\propto$ Proportion to

2 | PROBLEM FORMULATION

Consider a multi-polarised array composed of $N$ identically oriented $P$-dimensional VAs (see Figure 1), and defined as

$$h(\phi) = [-\sin \phi, \cos \phi, 0]^T \quad (1)$$
$$v(\theta, \phi) = [\cos \theta \cos \phi, \cos \theta \sin \phi, -\sin \theta]^T \quad (2)$$
$$r(\theta, \phi) = [\sin \theta \cos \phi, \sin \theta \sin \phi, \cos \theta]^T \quad (3)$$

**FIGURE 1** Co-ordinate system and the schematic view of the array where $\phi \in [0, 2\pi)$ and $\theta \in [0, \pi/2]$ are the azimuth and elevation angle, respectively.

Let $w$ be the $PN \times 1$ complex valued weight vector (i.e., the excitation current vector) applied to the $PN$ array elements for polarized beampattern design, the synthesised electric field in the direction $r(\theta, \phi)$ can be written as follows

$$E_w(\theta, \phi) = \begin{bmatrix} E_{h,w}(\theta, \phi) \\ E_{v,w}(\theta, \phi) \end{bmatrix} = \begin{bmatrix} w_{h1}^T a_{h}(\theta, \phi) \\ w_{v1}^T a_{v}(\theta, \phi) \end{bmatrix} \quad (4)$$

where $E_{h,w}(\theta, \phi)$ and $E_{v,w}(\theta, \phi)$ are the horizontal and vertical components of the synthesised electric field along $h(\phi)$ and $v(\theta, \phi)$, respectively, while $a_{h}(\theta, \phi)$ and $a_{v}(\theta, \phi)$ are the horizontal and vertical polarisation steering vectors with

$$a_{h1}(\theta, \phi) = a(\theta, \phi) \otimes a_{iso-h1}(\theta, \phi) \quad (5)$$
$$a_{v}(\theta, \phi) = a(\theta, \phi) \otimes a_{iso-v}(\theta, \phi) \quad (6)$$

$$a_{iso-h1}(\theta, \phi) = J \begin{bmatrix} h^T(\phi), v^T(\theta, \phi) \end{bmatrix}^T \quad (7)$$
$$a_{iso-v}(\theta, \phi) = J \begin{bmatrix} v^T(\theta, \phi), -h^T(\phi) \end{bmatrix}^T \quad (8)$$

Here $j = \sqrt{-1}$, $p_{n} = [x_n, y_n, z_n]^T$ represents the position vector of the $n$-th VA, and $k(\theta, \phi) = 2\pi r(\theta, \phi)/\lambda$ is referred to as the wave-vector, with $\lambda$ being the wavelength of interest;

$J$ is an $P \times 6$ selection matrix. For instance, $J = I_{6 \times 6}$ corresponds to the six-dimensional VA with orthogonal dipoles and loops $\{e_x, e_y, e_z, b_x, b_y, b_z\}$ (also known as the EM vector sensor/EMVS); $J = [I_{3 \times 3}, O_{3 \times 3}]$ corresponds to the tripoles antenna $\{e_{iso-x}, e_{iso-y}\}$. Moreover, it is noted that $\|a_{h1}(\theta, \phi)\|$ and $\|a_{v}(\theta, \phi)\|$ may be angle dependent without the use of the six-dimensional VAs, in what follows, $a_{h1}(\theta, \phi)$ and $a_{v}(\theta, \phi)$ are both normalised with respect to

$$\max_{\theta, \phi} \max \left\{ \frac{\|a_{h1}(\theta, \phi)\|}{\|a_{v}(\theta, \phi)\|}, \frac{\|a_{v}(\theta, \phi)\|}{\|a_{h1}(\theta, \phi)\|} \right\} \quad (10)$$
The polarisation of the synthesised electric field can be characterised by the following polarisation vector:

\[
\xi_w(\theta, \phi) = \begin{bmatrix} \cos \gamma_{t,\phi} \\ \sin \gamma_{t,\phi} e^{j\eta_{t,\phi}} \end{bmatrix} \propto E_w(\theta, \phi) \tag{11}
\]

where \(\gamma_{t,\phi} \in [0, \pi/2]\) and \(\eta_{t,\phi} \in [-\pi, \pi]\) are the auxiliary polarisation angle and polarisation phase difference, respectively.

In practice, various beampattern shapes might be desired, while the most commonly employed shape has minimised the sidelobe power while maintaining the mainlobe power and polarisation state. More specifically, given a desired direction \((\theta_0, \phi_0)\), the magnitude of the electric field in that direction should be no less than \(E_0\), while the radiation polarisation over the mainlobe angular region \(\mathcal{P}\) is constrained to be \((\gamma_0, \eta_0)\), and the sidelobe level over the angular region \(\mathcal{S}\) is optimised to be minimum. Following [6], the optimisation problem here can be formulated as follows:

\[
\begin{align*}
\min_w & \quad \rho \\
\text{s.t.} & \quad \max_{(\theta, \phi) \in \mathcal{P}} |E_v,w(\theta, \phi) - \kappa_0 E_{1,v,w}(\theta, \phi)| \leq \tau \\
& \quad \max_{(\theta, \phi) \in \mathcal{S}} \|E_{1,w}(\theta, \phi), E_v,w(\theta, \phi)\| \leq \rho \\
& \quad \text{Re}\{E_{1,w}(\theta_0, \phi_0)\} \geq E_{0,1} \tag{12}
\end{align*}
\]

where \(\kappa_0 = \tan \gamma_0 e^{j\eta_0}\) controls the radiation polarisation in the mainlobe region, \(\tau\) tunes the degree of purity for the mainlobe radiation polarisation, \(\rho\) handles the spatial power constraint over the sidelobe, and

\[
E_{0,1} = E_0 \left(1 + |\kappa_0|^2\right)^{-1/2} \tag{13}
\]

represents the magnitude of the horizontal component of the electric field in the desired direction.

If pure vertical polarisation synthesis in the target direction is of interest \((\gamma_0 = \pi/2)\), the first constraint of (12) turns to be \(\max_{(\theta, \phi) \in \mathcal{P}} |E_{1}(\theta, \phi)| \leq \tau\), and (12) is still applicable in this scenario. Substituting (4) into (12),

\[
\begin{align*}
\min_w & \quad \rho \\
\text{s.t.} & \quad \max_{(\theta, \phi) \in \mathcal{P}} |w^{1H} a_v(\theta, \phi) - \kappa_0 w^{1H} a_t(\theta, \phi)| \leq \tau \\
& \quad \max_{(\theta, \phi) \in \mathcal{S}} \|w^{1H} a_t(\theta, \phi), w^{1H} a_v(\theta, \phi)\| \leq \rho \\
& \quad \text{Re}\{w^{1H} a_t(\theta_0, \phi_0)\} \geq E_{0,1H} \tag{14}
\end{align*}
\]

The optimisation problem (14) is convex and can be formulated as the SOCP problem and solved by using the convex optimisation toolbox CVX [26].

## 3 | ROBUSTNESS ANALYSES

In this section, the robustness of the polarised beampattern synthesis problem formulated in (12) and (14) is studied. A simulation example is given first to show the performance improvement of the polarised beampattern synthesis by increasing the VA dimensionality. However, the robustness of the synthesised beampattern against the array manifold mismatch would be degraded. Then an intuitive explanation is provided for this unfavourable phenomenon.

Four types of 12-VA uniform linear arrays (ULAs), composed of 1) crossed-dipole {\(e_x, e_y\)}, with \(J = [O_{2\times1}, I_{2\times2}, O_{2\times3}]\); 2) COLD {\(e_z\)}, with \(J = I_{2\times2} \otimes [0, 0, 1]\); 3) tripole; and 4) EMVS, respectively, were considered. The VAs are located along the \(y\)-axis and separated by \(\lambda/2\).

For simplicity, only one-dimensional beampattern in the \(x\)-plane was studied. Figure 2 shows the corresponding synthesised power patterns, where the mainlobe is designed towards \((90^\circ, 0^\circ)\), with unit power and right-handed circular radiation polarisation. The beamwidth is 16°, with \(\mathcal{P} = [-8^\circ, 8^\circ]\), and \(\mathcal{S} = [-90^\circ, -8^\circ] \cup [8^\circ, 90^\circ]\). The angular sample interval in the mainlobe and sidelobe regions is 0.5°, and \(\tau = 0.1\). The magnitudes of the corresponding optimal weight vectors of the four arrays are plotted in Figure 3.

It can be seen that the EMVS array exhibits the lowest SLL, followed by the tripole array, crossed-dipole array and COLD array which agrees with the analysis in [5]. However, the corresponding weight magnitudes of EMVS and tripole arrays are quite large as compared with those of the crossed-dipole and COLD arrays. Therefore, the EMVS or tripole array based polarised beampattern synthesis would be very sensitive to the practical array manifold errors.

Let

\[
\Theta_M = \left\{ (\theta_M, k, \phi_{M,k}), k = 1, 2, \ldots, K \right\} \tag{15}
\]

\[
\Theta_S = \left\{ (\theta_S, q, \phi_{S,q}), q = 1, 2, \ldots, Q \right\} \tag{16}
\]

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure2.png}
\caption{Power patterns of the crossed-dipole array, COLD array, tripole array and EMVS array}
\end{figure}
denote the collections of the angular samples in the mainlobe region and sidelobe region, respectively. The following denote the collections of the angular samples in the mainlobe region and sidelobe region, respectively. The following

\[ \mathbf{B}(\Theta_M, \Theta_S) = [\mathbf{A}_{HV}(\Theta_S), \mathbf{A}_{CR}(\Theta_M)] \]  

(17)

with

\[ \mathbf{A}_{HV}(\Theta_S) = [\mathbf{a}_{HV}(\theta_{S,1}, \phi_{S,1}), \cdots, \mathbf{a}_{HV}(\theta_{S,Q}, \phi_{S,Q})] \]  

(18)

\[ \mathbf{A}_{CR}(\Theta_M) = [\mathbf{a}_{CR}(\theta_{M,1}, \phi_{M,1}), \cdots, \mathbf{a}_{CR}(\theta_{M,K}, \phi_{M,K})] \]  

(19)

\[ \mathbf{a}_{HV}(\theta_{S,q}, \phi_{S,q}) = [\mathbf{a}_1(\theta_{S,q}, \phi_{S,q}), \mathbf{a}_V(\theta_{S,q}, \phi_{S,q})] \]  

(20)

\[ \mathbf{a}_{CR}(\theta_{M,k}, \phi_{M,k}) = -\kappa_0 \mathbf{a}_1(\theta_{M,k}, \phi_{M,k}) + \mathbf{a}_C(\theta_{M,k}, \phi_{M,k}) \]  

(21)

Here, \( \mathbf{a}_{CR}(\theta_{M,k}, \phi_{M,k}) \) is the array response to the cross-polarised component of the electric field with radiation polarisation \( (\gamma_0, \eta_0) \) towards \( (\theta_{M,k}, \phi_{M,k}) \). Then minimising the left-hand-side of the first two constraints in (14) approximately is

\[ \min_{\mathbf{w}} \| \mathbf{w}^H \mathbf{B}(\Theta_M, \Theta_S) \| \]  

(22)

Consider further the singular value decomposition (SVD) of \( \mathbf{B}(\Theta_M, \Theta_S) \) as follows

\[ \mathbf{B}(\Theta_M, \Theta_S) = [\mathbf{u}_1, \cdots, \mathbf{u}_{PN}] \Sigma_{PN} \mathbf{O}_{PN \times (2Q + K - PN)} [\mathbf{v}_1, \cdots, \mathbf{v}_{2Q + K}]^H \]

(23)

where \( \Sigma_{PN} = \text{diag}(\sigma_1, \sigma_2, \cdots, \sigma_{PN}) \), with \( \sigma_1 \geq \sigma_2 \geq \cdots \geq \sigma_{PN} \) be the singular values of \( \mathbf{B}(\Theta_M, \Theta_S) \). The subspace is defined

\[ \mathcal{B} = \text{span}\{\mathbf{B}(\Theta_M, \Theta_S)\} = \text{span}\{\mathbf{U}\} \]  

(24)

Then, the first two constraints in (14) or (22) can be interpreted as finding a weight vector \( \mathbf{w} \) such that \( \| \mathbf{P}_B \mathbf{w} \| \) is as small as possible, where \( \mathbf{P}_B \) denotes the operator of orthogonal projection onto \( \mathcal{B} \). Intuitively, if the dimension of the subspace \( \mathcal{B} \) is less than \( PN \), the orthogonal complement of \( \mathcal{B} \), denoted by \( \mathcal{B}^\perp \), can be constructed by collecting the singular vectors in \( \mathbf{U} \) that correspond to the zero singular values. Then, the weight vector \( \mathbf{w} \) is very likely to fall into the subspace \( \mathcal{B}^\perp \), and \( \| \mathbf{P}_B \mathbf{w} \| \) reduces to zero. Therefore, the dimension of \( \mathcal{B} \) is important in the synthesis problem.

As shown in [27], the number of degrees of freedom (DOFs) of the EM wave radiated by the transmit antenna array in the far field is related not only to the antenna number but also to the wave-vector aperture polarisation product (WAPP). This metric is analogous to the dimension of the space of time-limited and band-limited signals. More specifically, if the signal

**FIGURE 3** Magnitudes of the optimal weight vectors corresponding to the beampatterns of the four arrays.
in time domain is approximately limited to [−T/2, T/2] and in frequency domain its spectrum is limited to [−W, W], the dimension of the space of the signals with these two constraints is essentially 2WT [28].

In the context of array polarised beampattern synthesis, there are three signal domains of interest, that is, array domain, wave-vector domain (or angular domain) and polarisation domain. The value of WAPP, σWAPP, is determined by the array effective aperture (in array domain), total angle of radiated angular sector (in wave-vector domain) and the radiation polarisation in (polarisation domain). Following [3, 27, 29, 30], σWAPP can be evaluated by 2A [S] + A [P], or 2(|A [S]| +1) + (|A [P]| +1) as more applicable integer value. Here, A is the normalised array effective aperture.

For a linear VA array located along the γ-axis and radiating to the broadside (x > 0) in the x-γ plane, A equals to L/λ, where L is the physical length of the array. For a planar array on the y-x plane and radiating to the upper hemisphere (z > 0), A is approximately πR^2/λ^2, where R is the radius of the minimal circle which completely encloses the array. Suppose Ω (S or P) contains the angular sector(s) that the radiated EM wave covers, the angle [Ω] can be calculated as ∫Ω cos Φ dΦ dΩ for the linear and planar array, respectively. The factor of two in front of 2A [S] represents the two-fold DOFs involved in the horizontal and vertical polarisation diversity in the sidelobe region. On the other hand, the radiated field in the mainlobe has a fixed radiation polarisation and, hence no additional DOFs are involved.

Figure 4 shows the singular value distributions of B(θM, θS) for the four twelve-VA uniform linear arrays considered in the previous synthesis example. All the parameter settings remain the same. Then A = 5.5, [P] ≈ 0.28, [S] ≈ 1.72, and the resultant σWAPP is about 25. It can be seen that, for tripole and EMVS arrays, PN > σWAPP and σn approaches zero once n has exceeded σWAPP. On the other hand, P = 2 for crossed-dipole and COLD array, PN < σWAPP and σn does not reach zero when n = PN.

In summary, given a specific array aperture, mainlobe/sidelobe region and radiation polarisation constraints, WAPP can be used to characterise the dimension of the wave-vector domain. When PN is much larger than σWAPP (e.g., for the tripole and EMVS arrays, one can form

\[ U_1 = [u_1, u_2, \ldots, u_{\sigma_{\text{WAPP}}}] \]

\[ U_2 = [u_{\sigma_{\text{WAPP}}+1}, u_{\sigma_{\text{WAPP}}+2}, \ldots, u_{\text{PN}}] \]

Then B⊥ is almost the same as span{U2}, and most of the weight vector components of w will be involved in B⊥ that is orthogonal to span{U1}. Consider the mainlobe magnitude constraint in (14): Re{w^H aM(\theta_0, \phi_0)} ≥ E_{0,H}, where aM(\theta_0, \phi_0) can be approximated by a linear combination of some columns in U1; therefore w is nearly orthogonal to aM(\theta_0, \phi_0). Furthermore, note that

\[ |w^H aM(\theta_0, \phi_0)| \geq \text{Re}\{w^H aM(\theta_0, \phi_0)\} \]

Now,

\[ |w^H aM(\theta_0, \phi_0)| = \|w\| \cdot \|aM(\theta_0, \phi_0)\| \cos \theta \geq E_{0,H} \]

and

\[ \|w\| \geq \frac{E_{0,H}}{\|aM(\theta_0, \phi_0)\| \cos \theta} \]

**FIGURE 4** Singular value distributions of matrix B(θM, θS) for the crossed-dipole, COLD, tripole and EMVS array
where $\theta \in [0, \pi/2]$ is the angle between $\mathbf{w}$ and $\mathbf{a}_1(\theta_0, \phi_0)$, and it is close to $\pi/2$, based on the above discussion; $E_{0,H}$ is a relatively large number compared with $\rho$ and $\tau$; as a consequence, $\|\mathbf{w}\|$ turns out to be quite large.

On the other hand, when PNs less than or comparable with $\omega_{\text{WAPP}}$ (e.g. for crossed-dipole and COLD arrays), one could not find the orthogonal complement $\mathbf{B}^\perp$. Although $\mathbf{w}$ can be constructed by a linear combination of some singular vectors of $\mathbf{B}(\mathbf{H}_M, \mathbf{H}_A)$ that correspond to the small singular values, there may exist common components between $\mathbf{w}$ and $\mathbf{a}_1(\theta_0, \phi_0)$. Therefore, the angle between $\mathbf{w}$ and $\mathbf{a}_1(\theta_0, \phi_0)$ would not approach $\pi/2$ and, eventually, $\|\mathbf{w}\|$ would be of moderate magnitude.

It is interesting to note that, given the VA dimensionality $P$ and VA number $N$ in the multi-polarised array, changing the inter-VA spacing or mainlobe/sidelobe regions might result in different WAPP. The weight vector would have quite large magnitudes if $PN > \omega_{\text{WAPP}}$.

Heuristically, the metric of WAPP measures the number of DOF that can be effectively used in the beampattern synthesis problem. When some extra DOFs are provided in the array domain (i.e. $PN > \omega_{\text{WAPP}}$), some weights will be left free to vary, and this may result in extremely large magnitude when catering for the optimisation criteria. Although imposing more constraints on the polarisation/power of mainlobe/sidelobe seems to consume more DOFs, we find that, in most of the cases, this will make the optimisation problem unsolvable. Obviously, if we add a norm constraint on the weight vector (i.e., $\|\mathbf{w}\| < c$) to (14), the magnitude of $\mathbf{w}$ will be limited immediately. However, as shown in (29), this constraint is in conflict with the mainlobe magnitude constraint, and the norm bound constant $c$ is hard to determine. Moreover, the side effects of larger weight magnitude are mainly related to the array system errors. Concerning these issues, a robust method will be derived in the next section.

4 | PROPOSED METHOD

In this section, a robust polarised beampattern synthesis scheme is proposed to eliminate the large weight magnitude phenomenon in the polarised beampattern synthesis and alleviate the impact of the potential array manifold errors on the beampattern. Due to the array manifold errors caused by the VA position error, inter-VA mutual coupling, or channel mismatch, the actual polarimetric array manifold vectors $\tilde{\mathbf{a}}_1(\theta, \phi)$ and $\tilde{\mathbf{a}}_v(\theta, \phi)$ are assumed to belong to the following spherical uncertainty sets:

$$\mathcal{U}_1(\theta, \phi) = \left\{ \tilde{\mathbf{a}}_1(\theta, \phi) \mid \tilde{\mathbf{a}}_1(\theta, \phi) = \mathbf{a}_1(\theta, \phi) + \mathbf{e}_1(\theta, \phi) \right\}$$

$$\mathcal{U}_v(\theta, \phi) = \left\{ \tilde{\mathbf{a}}_v(\theta, \phi) \mid \tilde{\mathbf{a}}_v(\theta, \phi) = \mathbf{a}_v(\theta, \phi) + \mathbf{e}_v(\theta, \phi) \right\}$$

where $\mathbf{e}_1(\theta, \phi)$ and $\mathbf{e}_v(\theta, \phi)$ are the distortion vectors, and

$$\|\mathbf{e}_1(\theta, \phi)\| \leq \epsilon_1$$

$$\|\mathbf{e}_v(\theta, \phi)\| \leq \epsilon_v$$

Here $\tilde{\mathbf{a}}_1(\theta, \phi)$ and $\tilde{\mathbf{a}}_v(\theta, \phi)$ are modelled as

$$\tilde{\mathbf{a}}_1(\theta, \phi) = C_1 \mathbf{a}_1(\theta, \phi) = \mathbf{a}_1(\theta, \phi) + C_1^\perp \mathbf{a}_1(\theta, \phi)$$

where $C_1^\perp = C_1 - I$. Then

$$\|\mathbf{e}_1(\theta, \phi)\| \leq \|C_1^\perp\| \cdot \|\mathbf{a}_1(\theta, \phi)\|$$

in which, $\|C_1^\perp\| = \sqrt{\lambda_{\text{max}}(\{C_1^\perp \}^H C_1^\perp)}$ is the spectral matrix norm of $C_1^\perp$ with $\lambda_{\text{max}}(\cdot)$ denoting the operation of finding the largest eigenvalue, and max $\|C_1^\perp\|$ has been derived for various system errors in [14, 20]. It follows that

$$\epsilon_1 = \left( \max_{\theta, \phi \in \mathcal{D}^P} \|C_1^\perp\| \cdot \max_{\theta, \phi \in \mathcal{D}^P} \|\mathbf{a}_1(\theta, \phi)\| \right)$$

Similar result applies to $\epsilon_v$.

With the manifold uncertainty considered, the constraints imposed in (14) should be modified to guarantee that the radiation polarisation and the SLL of the array beampattern are maintained in the worst case scenario. That is, for any $\tilde{\mathbf{a}}_1(\theta, \phi) \in \mathcal{U}_1(\theta, \phi)$ and $\tilde{\mathbf{a}}_v(\theta, \phi) \in \mathcal{U}_v(\theta, \phi)$, we have:

$$\min_{\mathbf{w}} \rho_{\text{rob}}$$

s.t.

$$\max_{(\theta, \phi) \in \mathcal{D}^P} \max_{\tilde{\mathbf{a}}_1(\theta, \phi) \in \mathcal{U}_1(\theta, \phi)} \|\mathbf{w}^H \tilde{\mathbf{a}}_1(\theta, \phi) - \kappa_0 \mathbf{w}^H \tilde{\mathbf{a}}_1(\theta, \phi)\| \leq \tau_{\text{rob}}$$

$$\max_{(\theta, \phi) \in \mathcal{S}^P} \max_{\tilde{\mathbf{a}}_1(\theta, \phi) \in \mathcal{U}_1(\theta, \phi)} \|\mathbf{w}^H \tilde{\mathbf{a}}_1(\theta, \phi), \mathbf{w}^H \tilde{\mathbf{a}}_v(\theta, \phi)\| \leq \rho_{\text{rob}}$$

$$\min_{\tilde{\mathbf{a}}_1(\theta, \phi) \in \mathcal{U}_1(\theta, \phi)} \Re\left\{ \mathbf{w}^H \tilde{\mathbf{a}}_1(\theta, \phi) \right\} \geq E_{0,H}$$

By using the triangle and Cauchy-Schwarz inequalities, (notation $(\theta, \phi)$ is dropped for brevity in the following derivations)

$$\|\mathbf{w}^H \tilde{\mathbf{a}}_v - \kappa_0 \mathbf{w}^H \tilde{\mathbf{a}}_1\|$$

$$\leq \|\mathbf{w}^H \tilde{\mathbf{a}}_v - \kappa_0 \mathbf{w}^H \tilde{\mathbf{a}}_1\| + \|\mathbf{w}^H \tilde{\mathbf{a}}_v - \kappa_0 \mathbf{w}^H \tilde{\mathbf{a}}_1\|$$

$$\leq \|\mathbf{w}^H \tilde{\mathbf{a}}_v - \kappa_0 \mathbf{w}^H \tilde{\mathbf{a}}_1\| + \|\mathbf{w}^H \tilde{\mathbf{a}}_v - \kappa_0 \mathbf{w}^H \tilde{\mathbf{a}}_1\|$$

$$\leq \|\mathbf{w}^H \tilde{\mathbf{a}}_v - \kappa_0 \mathbf{w}^H \tilde{\mathbf{a}}_1\| + c_1 \|\mathbf{w}\|$$

$$\leq \|\mathbf{w}^H \tilde{\mathbf{a}}_v - \kappa_0 \mathbf{w}^H \tilde{\mathbf{a}}_1\| + c_1 \|\mathbf{w}\|$$
where $c_1 = \epsilon_V + |\kappa_0| \epsilon_{11}$. Moreover, the equalities hold when

$$e_{11} = -\epsilon_{11} \left( \frac{w}{\|w\|} \right) \hat{c} \left( \psi_{\phi_{\text{rob}}} \right)$$

(39)

$$e_{V} = \epsilon_{V} \left( \frac{w}{\|w\|} \right) \hat{c} \left( \psi_{\phi_{\text{rob}}} \right)$$

(40)

Similarly,

$$||[w_{\text{rob}}^H a_{11_i}, w_{\text{rob}}^H a_{1v}]|| = ||[w_{\text{rob}}^H a_{11}, w_{\text{rob}}^H a_{V}] + [w_{\text{rob}}^H e_{11}, w_{\text{rob}}^H e_{V}]||$$

$$\leq ||[w_{\text{rob}}^H a_{11}, w_{\text{rob}}^H a_{V}]||$$

$$+ ||[w_{\text{rob}}^H e_{11}, w_{\text{rob}}^H e_{V}]||$$

$$\leq ||[w_{\text{rob}}^H a_{11}, w_{\text{rob}}^H a_{V}]|| + c_2 \|w\|$$

(41)

with $c_2 = \sqrt{\epsilon_{11}^2 + \epsilon_{V}^2}$. When $e_{11}$ and $e_{V}$ are chosen to be

$$e_{11} = \epsilon_{11} \left( \frac{w}{\|w\|} \right) \hat{c} \left( \psi_{\phi_{\text{rob}}} \right)$$

(42)

$$e_{V} = \epsilon_{V} \left( \frac{w}{\|w\|} \right) \hat{c} \left( \psi_{\phi_{\text{rob}}} \right)$$

(43)

and under the condition that $|w_{\text{rob}}^H a_{11}^*| - |w_{\text{rob}}^H a_{V}|^{-1} = \epsilon_{11}/\epsilon_{V}$, the equalities in the last two expressions of (41) hold.

For the last inequality in (37), notice that

$$||w_{\text{rob}}^H a_{11}|| = ||w_{\text{rob}}^H a_{11} + w_{\text{rob}}^H e_{11}|| \geq ||w_{\text{rob}}^H a_{11}||$$

$$- ||w_{\text{rob}}^H e_{11}|| \geq ||w_{\text{rob}}^H a_{11}|| - \epsilon_{11} \|w\|$$

(44)

and the equalities hold by choosing

$$e_{11} = -\epsilon_{11} \left( \frac{w}{\|w\|} \right) \hat{c} \left( \psi_{\phi_{\text{rob}}} \right)$$

(45)

Consider the fact that arbitrary phase rotation of $w$ does not change the objective function,

$$\text{Re} \left( w_{\text{rob}}^H a_{11} \right) \geq \text{Re} \left( w_{\text{rob}}^H a_{11} \right) - \epsilon_{11} \|w\|$$

(46)

Based on (38), (41) and (46), the optimisation problem (37) can be reformulated as

$$\min_{\rho_{\text{rob}}} \rho_{\text{rob}}$$

s.t.

$$\max_{(\theta,\phi) \in P} \left| w_{\text{rob}}^H a_{11}(\theta, \phi) - \kappa_0 w_{\text{rob}}^H a_{11}(\theta, \phi) \right| + c_1 \|w\| \leq \tau_{\text{rob}}$$

$$\max_{(\theta,\phi) \in S} \left| w_{\text{rob}}^H a_{11}(\theta, \phi), w_{\text{rob}}^H a_{V}(\theta, \phi) \right| + c_2 \|w\| \leq \rho_{\text{rob}}$$

$$\text{Re} \left( w_{\text{rob}}^H a_{11}(\theta_0, \phi_0) \right) - \epsilon_{11} \|w\| \geq E_{0,11}$$

(47)

With the above modified constraints, a robust solution $w_{\text{rob}}$ to the polarised beampattern synthesis problem can be obtained by using the SOCP solver.

It is interesting to mention that the regularisation terms $c_1 \|w\|$ and $c_2 \|w\|$ added on the constraints in the optimisation procedure are also of great importance in mitigating the large magnitudes in the weight vector.

To ensure that the optimisation problem (47) has a feasible solution, a discussion is made on the lower bound of $\rho_{\text{rob}}$ and the parameter choice of $\tau_{\text{rob}}$. The notations $\tau$, $\rho$ and the optimised weight vector $w$ in the optimisation problem (14) are designated, respectively, as $\tau_{\text{nom}}$, $\rho_{\text{nom}}$ and $w_{\text{nom}}$ for the nominal case.

From the third constraint in (47),

$$e_{11} \|w_{\text{rob}}\| \leq \text{Re} \left( w_{\text{rob}}^H a_{11}(\theta_0, \phi_0) \right) - E_{0,11}$$

(48)

$$\leq ||w_{\text{rob}}^H a_{11}(\theta_0, \phi_0) - E_{0,11} || - E_{0,11}$$

Hence, a lower bound on $\|w_{\text{rob}}\|$ can be obtained as follows

$$\|w_{\text{rob}}\| \geq E_{0,11} / ||a_{11}(\theta_0, \phi_0) - c_1 e_{11} + \tau_{\text{nom}}$$

(49)

In order to achieve comparable polarisation purity in the mainlobe similar to that in (14), the parameter $\tau_{\text{rob}}$ in the first constraint of (47) should be no larger than

$$\frac{c_1 E_{0,11}}{||a_{11}(\theta_0, \phi_0)|| - c_1 e_{11} + \tau_{\text{nom}}}$$

(50)

As for the sidelobe level, let $\bar{\rho}_{\text{rob}}$ and $\bar{\rho}_{\text{nom}}$ be the optimal values of peak sidelobe level of the synthesis problem (47) and (14), respectively, that is

$$\bar{\rho}_{\text{rob}} = \max_{(\theta,\phi) \in S} \left| w_{\text{rob}}^H a_{11}(\theta, \phi), a_{V}(\theta, \phi) \right| + c_2 \|w_{\text{rob}}\|$$

(51)

$$\bar{\rho}_{\text{nom}} = \max_{(\theta,\phi) \in S} \left| w_{\text{nom}}^H a_{11}(\theta, \phi), a_{V}(\theta, \phi) \right|$$

(52)

When $\tau_{\text{rob}}$ is set as its upper bound in (50), the robust solution $w_{\text{rob}}$ is also feasible for the problem (14), but possibly not optimal:

$$\bar{\rho}_{\text{rob}} - \bar{\rho}_{\text{nom}} \geq c_2 \|w_{\text{rob}}\| \geq \frac{c_2 E_{0,11}}{||a_{11}(\theta_0, \phi_0)|| - c_1 e_{11} + \tau_{\text{nom}}}$$

(53)

Thus

$$\bar{\rho}_{\text{rob}} - \bar{\rho}_{\text{nom}} \geq c_2 \|w_{\text{rob}}\| \geq \frac{c_2 E_{0,11}}{||a_{11}(\theta_0, \phi_0)|| - c_1 e_{11} + \tau_{\text{nom}}}$$

(54)
This means that for the problem (47), we cannot achieve a lower value of $\tilde{\rho}_{\text{rob}}$ than

$$\tilde{\rho}_{\text{rob},0} = c_2 E_{0,1}/(\|a_{1}(\theta_0, \phi_0)\| - \epsilon_{H}) + \tilde{\rho}_{\text{nom}}$$

(55)

However, this value corresponds to the worst case, in practice, the actual peak sidelobe level

$$\max_{(\theta,\phi)\in S} \| w_{\text{rob}}^{H} [\tilde{a}_{H}(\theta, \phi), \tilde{a}_{V}(\theta, \phi)] \|$$

(56)

might be lower than $\tilde{\rho}_{\text{rob},0}$, when $\tilde{a}_{H}(\theta, \phi)$ and $\tilde{a}_{V}(\theta, \phi)$ involves less errors.

At the end of this section, the same simulation example, shown at the beginning of the above section, is repeated by using the proposed robust optimisation method, where the manifold error vectors $e_{H}(\theta, \phi)$ and $e_{V}(\theta, \phi)$ are assumed to be direction independent zero mean complex Gaussian random vectors, with

$$\epsilon_{H} = 0.04 \max_{\theta, \phi} \| a_{H}(\theta, \phi) \|$$

(57)

which means that 4% uncertainty is involved in the two polarimetric array manifold vectors. Figure 5 shows the magnitudes of the robust weight vectors, from which we can see that the largest excitation magnitudes of tripole array and EMVS array are similar to those of the crossed-dipole array and COLD array, and the dynamic ranges of tripole array and EMVS array excitations are much smaller than the results shown in Figure 3, which are calculated by using the original optimisation method.

5 | NUMERICAL RESULTS

In this section, numerical simulation results are provided to illustrate the performance of the presented polarised beampattern synthesis scheme against the array manifold mismatch. For comparison, the corresponding results of the original optimisation method (14) and polarimetric Dolph-Chebyshev (P-D-C) method introduced in [8] are also included in the figures. The weight vector of P-D-C can be calculated by
average polarisation deviations (APDs) in the mainlobe original and robust optimisation schemes, with and without polarisation synthesis deviations of the $P$ and synthesised polarisation states in direction azimuth only beampatterns and the corresponding mainlobe defined as follows:

$$
\mathbf{w}_{\text{P-D-C}} = E_0 \mathbf{w}_{\text{D-C}}(\theta_0, \phi_0) \otimes \mathbf{w}_{\text{iso}}(\theta_0, \phi_0, \gamma_0, \eta_0)
$$

where

$$
\mathbf{w}_{\text{iso}}(\theta_0, \phi_0, \gamma_0, \eta_0) = [\mathbf{a}_{\text{iso-H}}(\theta_0, \phi_0), \mathbf{a}_{\text{iso-V}}(\theta_0, \phi_0)] \mathbf{\xi}(\gamma_0, \eta_0)
$$

and $\mathbf{w}_{\text{D-C}}(\theta_0, \phi_0)$ is the Dolph-Chebyshev weighting vector when the mainlobe is steered to $\theta_0, \phi_0$ and the beamwidth coincides with the other two methods.

In the first example, the same crossed-dipole, tripole and EMVS ULAs shown in Section 3 are used for the polarised beampattern synthesis. The radiated beam is steered towards $-30^\circ$, with unit magnitude, $18^\circ$ beamwidth, and elliptical radiation polarisation $(45^\circ, 60^\circ)$. The WAPP of the simulation scenario is about 25. The manifold error vectors $\mathbf{e}_H(\theta, \phi)$ and $\mathbf{e}_V(\theta, \phi)$ are the same as those described in (57) and (58). The polarisation constraint parameters of the original and the robust optimisation methods are the following:

$$
\tau_{\text{nom}} = 0.05 \\
\tau_{\text{rob}} = c_1 E_{0.11} / (\| \mathbf{a}_{\text{HH}}(\theta_0, \phi_0) \| - c_{11}) + \tau_{\text{nom}}
$$

The metric used for measuring the mainlobe radiation polarisation purity is the polarisation synthesis deviation defined as follows:

$$
\Delta(\theta, \phi) = 2 \arccos( | \mathbf{\xi}_{\text{syn}}(\theta, \phi) \mathbf{\xi}_{\text{des}}(\theta, \phi) | )
$$

where $\mathbf{\xi}_{\text{des}}(\theta, \phi)$ and $\mathbf{\xi}_{\text{syn}}(\theta, \phi)$, respectively, are the desired and synthesised polarisation states in direction $(\theta, \phi)$.

Presented in Figures 6–11 are the one dimensional (1D) azimuth only beampatterns and the corresponding mainlobe polarisation synthesis deviations of the P-D-C scheme, the original and robust optimisation schemes, with and without the array manifold mismatch. The corresponding SLLs and the average polarisation deviations (APDs) in the mainlobe are listed in Table 1, where the APD is calculated by $\int_0^{2\pi} \Delta(\theta, \phi) d\theta d\phi$, and ‘F’ denotes failing to synthesise a feasible beampattern.

From these results, it can be seen that increasing the VA dimensionality helps to suppress the sidelobe and control the mainlobe polarisation, and has also been verified in [5]. Compared with the robust method and P-D-C, the original optimisation method provides the lowest SLL at the expense of a little polarisation purity loss in the mainlobe. However, when manifold errors exist, the original optimisation method fails to synthesise normal beampatterns with tripole array and EMVS array, because WAPP is much less than the dimensionality of these arrays, as analysed in Section 3, and the weighting magnitudes can be as large as $10^6$ in order to radiate a beampattern with just a unit power (0dB) in the mainlobe direction. When little uncertainties are involved in the manifold vectors, the error terms will have great effects on the original beampatterns, as seen from the Figure 10 and 11.

The robust optimisation method and P-D-C, in contrast, exhibit stable performance in all the cases, while the robust method can provide much lower SLL with comparable polarisation purity in the mainlobe. These results show the good performance of the proposed robust method in 1D polarised beampattern synthesis. The two dimensional (2D) azimuth and elevation polarised beampattern synthesis are considered. Since the problem interpretation for the 2D scenario is very similar to that of 1D scenario, in the following example, only the tripole array is focussed to show the corresponding results. Suppose $8 \times 8$ tripoles are uniformly located in the $xy$ plane and separated by $\lambda/2$. The mainlobe is steered towards $24^\circ$ with right-handed circular radiation polarisation. The other parameter settings follow those of the first simulation. Thus, $\mathcal{A} \approx 19.24$, $[\mathcal{P}] \approx 0.14$, $[\mathcal{S}] \approx 3.00$, and $\mathbf{w}_{\text{WAPP}}$ is about 122, which is much smaller than the tripole array dimensionality (192). Therefore, large weights might occur when using the original optimisation method.

The synthesis results of the P-D-C, original optimisation method and robust optimisation method are shown in
Figures 12–17. The subfigures on the top left are 3D view of synthesised power patterns, the subfigures on the top right are the superimposed cuts of the power patterns at $\phi \in \{0^\circ, 10^\circ, 20^\circ, \ldots, 170^\circ\}$, while the elevation angle varying from $-90^\circ$ to $90^\circ$, the subfigures on the bottom left are the 3D view of the polarisation distances, and the subfigures on the bottom right are the superimposed cuts of polarisation distances at the same azimuth angles selected from the above set. In the figures, In the figures, $u = \sin \theta \cos \phi$, and $v = \sin \theta \sin \phi$.

**FIGURE 7** Tripole array-based synthesis results in the absence of array manifold errors (a) power pattern, (b) polarisation deviation

**FIGURE 8** EMVS array-based synthesis results in the absence of array manifold errors (a) power pattern, (b) polarisation deviation

**FIGURE 9** Crossed-dipole array-based synthesis results in the presence of manifold errors (a) power pattern, (b) polarisation deviation
FIGURE 10  Tripole array-based synthesis results in the presence of array manifold errors (a) power pattern, (b) polarisation deviation

FIGURE 11  EMVS array-based synthesis results in the presence of manifold errors (a) power pattern, (b) polarisation deviation

| Array Type         | Methods | Peak SLL (dB) |  | APD (Deg.) |  |
|--------------------|---------|---------------|---|------------|---|
|                    |         | No errors     | With errors | No errors | With errors |
| Crossed-dipole     | P-D-C   | −14.4         | −13.9       | 2.7       | 3.2         |
|                    |         | −16.2         | −15.7       | 2.8       | 3.4         |
|                    |         | −16.1         | −15.5       | 1.4       | 2.0         |
|                    | Original opt. | −14.1         | −14.0       | 0.2       | 1.3         |
|                    |         | −18.5 F       | 6.9 F       |  |
|                    |         | −16.6 −15.9   | 2.0         | 2.4       |
| Tripole array      | P-D-C   | −14.2         | −11.1       | 0         | 1.0         |
|                    | Original opt. | −29.7 F       | 7.1 F       |  |
|                    |         | −16.8 −16.4   | 1.3         | 1.7       |

Abbreviation: EMVS, electromagnetic vector sensor; SLL, sidelobe level.
Similar conclusions can be drawn as those in the 1D beampattern synthesis. The original optimisation method is very sensitive to the array manifold errors, while the P-D-C and the proposed robust optimisation method are quite robust to these errors. Moreover, compared with P-D-C, a lower peak sidelobe level and smaller polarisation synthesis deviation can be achieved by the proposed robust method.

6 | CONCLUSION

In this article, the polarised beampattern synthesis problem has been considered in the presence of manifold errors, aiming at the sidelobe suppression and mainlobe radiation polarisation control with convex optimisation programing. It is observed that when the VA dimension is larger than 2, large weights are very likely to appear in the optimal excitation solution. This
would severely deteriorate the beampatterns even with very small array manifold errors. Associated with this are the increases of ohmic losses and reactive power compared with the radiated power, and hence, the antenna efficiency will be greatly reduced. Meanwhile, precise adjustment of the amplitudes and phases of the array elements is also very difficult. These problems greatly limit the usage of the original optimisation method in practice. The DOF of the EM wave in the wave-vector domain are analysed and an intuitive explanation on this large weight magnitude phenomenon based on the subspace theory is given. A robust optimisation method has also been proposed on the basis of worst-case performance optimisation paradigm. Numerical examples have validated the analyses and illustrated the effectiveness of the presented robust scheme.

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FIGURE 16 The synthesis results of the original optimisation method for the tripole planar array involving manifold errors

FIGURE 17 The synthesis results of the robust optimisation method for the tripole planar array involving manifold errors

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