Dispersive CQED interactions between matter qubits and bright squeezed light

Fang-Yu Hong and Shi-Jie Xiong
National Laboratory of Solid State Microstructures and Department of Physics, Nanjing University, Nanjing 210093, China
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Dispersive interactions of matter qubits with bright squeezed light in a high-Q cavity is studied. Numerical simulation shows that higher fidelity of operations to obtain a certain phase shift of the pulse through the dispersive light-matter interaction may be reached using bright squeezed light than that using bright coherent light.

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The dispersive interaction of an intensive optical pulse with a single atom in a high-Q cavity has been explored by many experiments in cavity quantum electrodynamics (CQED) [1]. Such interactions are essential for non-destructive measurement of atoms [2], quantum optic computers [3], and quantum communication [4]. Ladd et al [5] have studied the interaction between an intense, off-resonant coherent optical pulse and a single atom in a high-Q cavity. In this paper, the cavity-based dispersive interaction of bright squeezed light with a three-level atom has been discussed. Numerical simulation shows that to achieve a certain detectable phase shift of the bright pulse, higher fidelity of operation may be obtained using squeezed pulses than that using coherent pulses.

The basic matter qubit in a cavity is formed by the two lower states of a three-state Λ-system, as shown in Fig.1. The two metastable qubit states are denoted by |0⟩ and |1⟩. Coherence transitions (rotations) between these two states are assumed to be possible through methods, such as stimulated adiabatic Raman transitions [6] or spin-resonance techniques [7]. In this article, we focus on optical transitions between |1⟩ and an excited state |e⟩. We assume that our light is completely incoherent, and some combination of the two. One example of such a system may be found in a semiconductor donor-bound impurity, where the qubit states are provided by electron Zeeman sublevels and |e⟩ is provided by the lowest bound-exciton state. In this paper, the matter qubit is always referred to as an atom although it may be a semiconductor impurity or quantum dot comprised of many atoms. Particularly, we assume the state of the qubit is in the state (|0⟩ + |1⟩)/√2. The probe pulse is sufficiently detuned from the transition between |1⟩ and the exited state to guaranty a strictly weak dispersive light-matter interaction.

When large number photons are introduced into a cavity, a numerical approach is required. For very large photon numbers, a full-quantum analysis may be computationally intensive; an appropriate approximation is the semi-classical optical Bloch equation approach. We presume that ωp = ω0, that is, the center frequency ωp of the pulse is on-resonance with the cavity (and both are offset from the atomic transition by ω0). To keep track of the atomic dynamics, we define several 'partial' characteristic functions,

\[ \chi^p(\eta, t) = \text{Tr}(\hat{J}^e \hat{\rho}^i_a, \hat{\eta}, \hat{a}^\dagger a_{\eta t}) \hat{\rho}(t), \]  

where \( \hat{\rho}(t) \) is the density operator of the light-matter system, states \( |j⟩ \) and \( |k⟩ \) are atomic states and the trace is over the optical field. Assuming a rotating reference frame rotating at the center frequency of the optical pulse, for a narrow-band pulse, the master equations in a fully quantum setting in which any quantum state of light is allowed have the form [7]:

\[
\begin{align}
\dot{\chi}^{eε}(\eta, t) &= ig \left( S(t) \left( \frac{\eta}{2} + \frac{\partial}{\partial \eta} \right) \chi^{εε}(\eta, t) + S^*(t) \left( \frac{\eta^*}{2} + \frac{\partial}{\partial \eta} \right) \chi^{εε}(\eta, t) \right) - 2\Gamma \chi^{εε}(\eta, t), \\
\dot{\chi}^{11}(\eta, t) &= ig \left( S(t) \left( \frac{\eta}{2} - \frac{\partial}{\partial \eta} \right) \chi^{εε}(\eta, t) + S^*(t) \left( \frac{\eta^*}{2} - \frac{\partial}{\partial \eta} \right) \chi^{εε}(\eta, t) \right) + 2\Gamma \chi^{εε}(\eta, t), \\
\dot{\chi}^{ε1}(\eta, t) &= igS(t) \left( \frac{\eta}{2} + \frac{\partial}{\partial \eta} \right) \chi^{εε}(\eta, t) + \left( \frac{\eta^*}{2} + \frac{\partial}{\partial \eta} \right) \chi^{εε}(\eta, t) + (i\Omega - \Gamma) \chi^{εε}(\eta, t), \\
\dot{\chi}^{00}(\eta, t) &= igS(t) \left( \frac{\eta}{2} + \frac{\partial}{\partial \eta} \right) \chi^{εε}(\eta, t) + (i\Omega - i\Delta - \Gamma) \chi^{εε}(\eta, t), \\
\dot{\chi}^{10}(\eta, t) &= igS(t) \left( \frac{\eta^*}{2} - \frac{\partial}{\partial \eta} \right) \chi^{εε}(\eta, t) - i\Delta \chi^{10}(\eta, t), \\
\dot{\chi}^{00}(\eta, t) &= 0,
\end{align}
\]
where $g$ is the atom-cavity coupling factor, $S(t)$ is related to a cavity-waveguide coupling factor $\kappa$, cavity decay parameter $\gamma$ which imply that any optical power in the cavity leaks out of the cavity as $e^{-\gamma t}$, and the input pulse shape $S_{in}(t)$ coupling into the cavity as follows [7],

$$S(t) = 2 \sqrt{\frac{S_{in}(t)}{\gamma}}$$

In Eq. (3), $2\Gamma$ is the total decay rate of the atom in the cavity, including the influence of the Purcell effect:

$$2\Gamma = \frac{1 + P(\omega_p)}{\tau_r} + \frac{1}{\tau_{nr}},$$

where $\tau_r$ and $\tau_{nr}$ describes spontaneous emission and non-radiative decay, respectively, and $P(\omega)$ is the Purcell factor

$$P(\omega) = \frac{\tau_r g^2}{\omega^2 + \gamma^2/4}.$$  

In Eq. (4), $\Omega$ is the atomic detuning from the cavity, including the ac-Stark shift,

$$\Omega = \omega_p \left[ 1 + \frac{P(\omega_p)}{\gamma \tau_r} \right].$$

Numerical solutions of this system of equations at large number photons are computationally intensive, so we use the semiclassical approximation. The assumptions underpinning the semiclassical approximation are that the quantum state of the pulse during the light-matter interaction is always a squeezed state, and that it always remains unentangled from state $|\epsilon\rangle$. (Similar assumptions were used in [7].) Then the density operator has the form

$$\hat{\rho}(t) = |\tilde{\rho}(t)\rangle_{gg} \otimes [\rho^{ss}(t)\sigma^s \sigma^s + |\rho^{11}(0) - \rho^{11}(t)|\sigma^- \sigma^- + \rho^{11}(t)\sigma^+ \sigma^-] + |\tilde{\rho}(t)\rangle_{gg} (\beta \otimes \rho^{00}(t)|0\rangle\langle 0| + \rho^{01}(t)|1\rangle\langle 0| + |\beta\rangle_{gg} (\tilde{\rho}(t) \otimes [\rho^{ss}(0)|\epsilon\rangle\langle \epsilon| + |\rho^{01}(0)|0\rangle\langle 1| + |\beta\rangle_{gg} (\beta \otimes \rho^{00}(0)|0\rangle\langle 0|),$$

where $\sigma^+ = |\epsilon\rangle\langle 1|$, $\sigma^- = |1\rangle\langle \epsilon|$, and $|\beta\rangle_{gg}$ is a two-photon coherence state defined as

$$|\beta\rangle_{gg} = \Xi(\epsilon)D(\beta)(0),$$

where

$$D(\beta) = \exp(\beta a^\dagger - \beta^* a),$$

and

$$\Xi(\epsilon) = \exp\left(\frac{1}{2} \epsilon e^{\phi} - \frac{1}{2} \epsilon e^{\phi^2}\right)$$

are the displacement operator and the unitary squeezed operator respectively, $a^\dagger$ is the creation operator of photons and $\epsilon = re^{2i\phi}$, $r$ is the squeeze factor. Squeezed states $|\alpha, \epsilon\rangle$ defined by

$$|\alpha, \epsilon\rangle = D(\alpha)\Xi(\epsilon)|0\rangle,$$

are equivalent to two-photon coherence states $|\beta\rangle_{gg}$ [10]:

$$|\alpha, \epsilon\rangle = |\beta\rangle_{gg},$$

where

$$\beta = \mu \alpha + \nu \alpha^*,$$

with $\mu = \cosh r$ and $\nu = e^{2i\phi} \sinh r$.

Substituting this density operator into equations [11-12], focusing on $\eta = 0$, using the following formulas [10]

$$\Xi(\epsilon) a \Xi(\epsilon) = \mu a - \nu a^*,$$

we arrive at the optical Bloch equations:

$$\rho^{ss} = i g [S^{*}(t)\tilde{\delta}(t)\rho^{ss}(t) - S(t)\tilde{\delta}(t)\rho^{ss}(t)] - 2\Gamma \rho^{ss},$$

$$\rho^{11} = \rho^{11} - [i\Delta - \Omega + F + c(t)]\rho^{11},$$

$$\rho^{00} = -[i\Delta + \Omega + F + c(t)]\rho^{00},$$

where

$$c(t) = \frac{\partial}{\partial t} \ln(\beta^2(t)) + 2 \frac{\partial}{\partial t} \left(\beta^2 + \beta^*\hat{\beta}(t)\right),$$

with

$$\hat{\beta}(t) = \mu \tilde{\beta}(t) + \nu \tilde{\beta}(t),$$

$\Delta$ is the energy separation of states $|0\rangle$ and $|1\rangle$, and

$$\tilde{\delta}(t) = \mu \tilde{\beta}(t) - \nu \tilde{\beta}(t).$$

If $\alpha$ experiences a phase shift of $\phi$, i.e., $a(t) = ae^{i\phi}$, according to the following formula [12]:

$$\langle \Delta N(t)\rangle^2 = \langle \Delta N(0)\rangle^2 = 2 \sinh^2 r \cos^2 r,$$

and

where

$$\frac{1}{2} \epsilon e^{\phi} + e^{2i\phi} \sin^2(\theta - \frac{\phi}{2}).$$

FIG. 1: Schematic of the dispersive interaction between a three-level atom and a bright squeezed pulses in a high-Q cavity.
ment by internal loss. Those two equations show a phase advance and dephasing from the light.

They can be simplified by defining

\[ \alpha - \kappa \gamma = \sigma, \]  
\[ \sigma = 3 \text{ ns}, \]  
\[ \Omega/2\pi = 100 \text{ GHz}. \]

FIG. 2: Numerical simulations of the weak dispersive interaction between squeezed lights and a matter qubit in a cavity. The parameters are \( \alpha = 100, \ r = 1, \ g/2\pi = 0.17 \text{ GHz}, \ \Gamma/2\pi = 1 \text{ MHz}, \ \kappa = \gamma = 0.2 \times 2\pi \text{ GHz}, \ \sigma = 3 \text{ ns}, \ \text{and} \ \Omega/2\pi = 100 \text{ GHz}. \)

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FIG. 3: The phase shift \( \theta \) of \( \alpha(t) \) and fidelity \( F \) of the matter qubit after dispersive light-matter interaction versus \( \Gamma \). The parameters are \( \alpha = 100, \ r = 1, \ g/2\pi = 0.17 \text{ GHz}, \ \kappa = \gamma = 0.2 \times 2\pi \text{ GHz}, \ \sigma = 3 \text{ ns}, \ \text{and} \ \Omega/2\pi = 100 \text{ GHz}. \)

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FIG. 4: The phase shift \( \theta \), the distinguishability \( d \), and the fidelity \( F \) of the matter qubit after dispersive light-matter interaction versus \( \alpha \). The parameters are \( r = 1, \ g/2\pi = 0.17 \text{ GHz}, \ \kappa = \gamma = 0.2 \times 2\pi \text{ GHz}, \ \sigma = 3 \text{ ns}, \ \text{and} \ \Omega/2\pi = 100 \text{ GHz}. \ \Gamma/2\pi = 10 \text{ MHz}. \)

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FIG. 5: The phase shift \( \theta \) of \( \alpha(t) \) and the fidelity \( F \) of the matter qubit after dispersive light-matter interaction versus coupling \( g \). The parameters are \( \alpha = 100, \ r = 1, \ \Gamma/2\pi = 1\text{MHz}, \ \kappa = \gamma = 0.2 \times 2\pi \text{ GHz}, \ \sigma = 3 \text{ ns}, \ \text{and} \ \Omega/2\pi = 100 \text{ GHz}. \)

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FIG. 6: The phase shift \( \theta \) of \( \alpha(t) \) and the fidelity \( F \) of the matter qubit after dispersive light-matter interaction versus squeezing factor \( r \). The parameters are \( \alpha = 100, \ g/2\pi = (\cosh^2(1) + \sinh^2(1))/(\cosh^2(r) + \sinh^2(r)) \times 0.17 \text{ GHz}, \ \Gamma/2\pi = 10\text{MHz}, \ \kappa = \gamma = 0.2 \times 2\pi \text{ GHz}, \ \sigma = 3 \text{ ns}, \ \text{and} \ \Omega/2\pi = 100 \text{ GHz}. \)

where \( N = a^\dagger a \) and \( \langle \cdot \rangle \) denotes the expectation value for squeezed state \( |\alpha, \epsilon\rangle \), \( \epsilon \) should have a corresponding phase shift of \( 2\theta \). For simplicity, hereafter, we assume \( \alpha \) and \( \epsilon \) are real.

We first discuss the approximate solution of Eq. (16), then numerically solve them. Eqs. (16a, 16b) are related to the fidelity of dispersive light-matter interaction, which is degraded by internal loss. Those two equations show a phase advancement by \( \Delta \), and both a phase and loss from \( c(t) \), which corresponds to the phase advance and dephasing from the light. They can be simplified by define

\[ \Sigma^0(t) = \langle \beta | \hat{\beta}(t) | e^{i\Lambda} r^{0}(t), \]  
\[ \Sigma^1(t) = \langle \beta | \hat{\beta}(t) | e^{i\Lambda} r^{1}(t), \]  

\[ (21a) \]  
\[ (21b) \]
FIG. 7: The phase shift $\tilde{\alpha}(t)$ and the fidelity $F$ of the matter qubit after dispersive light-matter interaction versus log$_{10}(\sigma/3)$ where $\sigma$ is in units of ns. The parameters are $\alpha = 100$, $r = 1$, $g/2\pi = 0.17$ GHz, $\Gamma/2\pi = 10$ MHz, $\kappa = \gamma = 0.2 \times 2\pi$ GHz, and $\Omega/2\pi = 100$ GHz.

![Graph](image)

FIG. 8: The fidelity of the matter qubit after dispersive light-matter interaction $F_\alpha$ using squeezed light and $F_\epsilon$ using coherent light versus phase shift $-\theta$ of $\tilde{\alpha}(t)$. The parameters are $\alpha = 100$, $\Gamma/2\pi = 10$ MHz, $\kappa = \gamma = 0.2 \times 2\pi$ GHz, $\Omega/2\pi = 100$ GHz, $r = 1$ (a), and $r = 0$ (b).

![Graph](image)

which obey

$$\dot{\Sigma}^{0}(t) = \Sigma^{10}(t) = -ig\tilde{\alpha}(t)S(t)\Sigma^{0}(t) + (i\Omega - \Gamma)\Sigma^{0}(t),$$

$$\dot{\Sigma}^{10}(t) = -ig\tilde{\alpha}^*(t)\Sigma^{0}(t).$$

From Eq. (16a), (16c), we obtain

$$|\tilde{\alpha}(t)|^2 = |\alpha|^2 - \int_0^t \rho^{ee}(\tau)\frac{1 + \mu^2 + |\nu|^2}{2\rho^{11}(0) - 2\rho^{ee}(t)}d\tau' - 2\Gamma \int_0^t \rho^{ee}(\tau)\frac{1 + \mu^2 + |\nu|^2}{2\rho^{11}(0) - 2\rho^{ee}(t)}d\tau'.$$

Since $\rho^{ee}(t) \ll \rho^{11}(0)$, which we will see in the following numerical simulation, and $\rho^{ee}(t) \to 0$ as $t \to \infty$, we arrive at

$$|\tilde{\alpha}(\infty)|^2 = |\alpha|^2 - 2\Gamma \int_0^\infty \rho^{ee}(\tau')\frac{1 + \mu^2 + |\nu|^2}{2\rho^{11}(0)}d\tau'.$$

All optical losses arise finally from atomic decay. Other optical losses from the cavity independent from atomic decay are already incorporated into the definition of $S(t)$. We may approximately solve Eq. (16a)-(16c) for the phase shift and optical loss in the limit of a narrow-band, far detuned pulse. This approximation may be obtained by assuming $g\tilde{S}(t)\tilde{\alpha}(t)$ is constant in time, with value $g\tilde{S}(t)\alpha$, and solving them for $\rho^{ee}$ and $\rho^{11}$. (A similar approach was adopted by [7] and [11].)

Using Laplace transforms, only the zero-valued pole of $\rho^{ee}$ and $\rho^{11}$ being important, we may arrive at the approximation solutions

$$\rho^{ee} \to \frac{\rho^{11}(0)g^2|\tilde{S}(t)|\alpha^2}{\Gamma^2 + \Omega^2 + 2g^2|\tilde{S}||\alpha|^2},$$

$$\rho^{11} \to \frac{\rho^{11}(0)g\tilde{S}(t)\alpha}{\Gamma^2 + \Omega^2 + 2g^2|\tilde{S}||\alpha|^2}(\Omega - i\Gamma).$$

Presuming this solution for $\rho^{11}$ is maintained as $S(t)$ varies in time, integrating Eq. (16a), we find

$$\tilde{\alpha}(t) = \alpha \left[1 - ig^2 \int_0^t \rho^{ee}(\tau)\frac{(\Omega - i\Gamma)(1 + \mu^2 + |\nu|^2)}{2(\Gamma^2 + \Omega^2 + 2g^2|\tilde{S}||\alpha|^2)}d\tau'\right].$$

In a similar way, we can approximate solution of Eq. (22a), (22b),

$$\Sigma^{0} \to \Sigma^{0}(0)\frac{ig\tilde{\alpha}(t)S(t)}{2(i\Omega - \Gamma)},$$

$$\Sigma^{10} \to \Sigma^{10}(0)\exp\left(-g^2 \int_0^\infty \frac{\alpha + \tilde{\alpha}(t)|\tilde{S}(t)|^2}{2(\Gamma - i\Omega)}d\tau'\right).$$

From Eqs. (27), (29), we may find that to achieve a certain phase shift of $\alpha$, the larger the squeeze factor $r$, the smaller the magnitude of $g$, thus the higher the fidelity of the matter qubit after dispersive light-matter interactions. The total magnitude of the damping to the desired coherence is

$$|\rho^{10}(t)| = e^{\beta - \tilde{\beta}(t)^2/2} |\Sigma^{10}(t)|$$

For the calculations presented here, we assume this interaction is used for entanglement distribution, in which case $\rho^{10} = 1/2$. Then the final fidelity of our qubit may be written

$$F = \frac{1}{2}(1 + 2|\rho^{10}(t)|).$$

Now we discuss the numerical solution of equations (16a), (16c), (22a), (22b). All the following simulations assume that $S_m$ takes a Gaussian shape, $S_m = \sqrt{\frac{\pi}{\lambda^2}} \exp(-\frac{x^2}{\lambda^2})$, and $\kappa = \gamma$, thus, from Eq. (3), we have

$$S(t) = 2\sqrt{\frac{\sqrt{2}}{\gamma/\sqrt{\pi}r}} \exp(-\frac{t^2}{\sigma^2}).$$

We also assume $\epsilon = re^{\pi s}$ and the initial state of the matter qubit is $|0\rangle + |1\rangle/2$. The parameters are assumed to be $\Omega/2\pi = 100$ GHz, $\kappa/2\pi = \gamma/2\pi = 0.2$ GHz, $g/2\pi = 0.17$ GHz, which are typical for $^{31}P$ [5], and $\alpha = 10$, $r = 0$ for coherent state, $|\alpha\rangle$, we have (1) $F_\epsilon = 0.99999724, \theta = -5.77896,$
\( \langle t \rangle - |\alpha| = -5.8 \times 10^{-7} \) for \( \Gamma/2\pi = 1 \) MHz, and
(2) \( F_i = 1.00000014, \theta = -5.77896, \langle t \rangle - |\alpha| = 2.7 \times 10^{-13} \) for
\( \Gamma/2\pi = 0 \). Figure 4 shows the dependence of the phase shift
we have fidelity. The fidelity of the matter qubit of this operation is
ignorable, and so does the absorption of photons in the interaction.
pling factor magnitude of \( \langle t \rangle \) of \( \tilde{\alpha} \) shows that the change in the magnitude of
\( \langle t \rangle \), as the atomic decay \( \Gamma \) increase in three orders from \( 10^{-3} \)
to 1. Figure 2 shows that the magnitude of the phase shift \( -\theta \) decrease very slight, so does the fidelity
\( F \), as the atomic decay \( \Gamma \) increase in three orders from \( 10^{-3} \)
to 1. Figure 4 shows the dependence of the phase shift \( \theta \), the
distinguishability \( d \equiv \langle t \rangle \sin \theta \), and the fidelity \( F \) on the magnitude of \( \alpha \). With the increasing of the magnitude of
coupling factor \( g \), the fidelity \( F \) decreases at first, then increases
again (see Fig. 5). This shows that the decoherence factor
\( \Gamma \) plays less role when \( g \) become larger than about \( 0.22 \times 2\pi \)
GHz.

If we increase the squeeze factor \( r \) while keeping \( g(\cosh^2 r + \sinh^2 r) \) constant, the fidelity \( F \) and the magnitude of phase
shift \( \theta \) both increase (see Fig. 6). The phase \( \theta \) and the fidelity
\( F \) are dependent on the length of the pulse \( \sigma \) (see Fig. 7). The results of Fig. 8 tell us that we can obtain higher fidelity
of the dispersive interaction using squeezed pulses to get a
certain phase shift of \( \langle t \rangle \) than that using coherent ones. Those
characters show that this scheme may have good adaptability
to wide range different systems.

In conclusion, this paper has discussed the dispersive in-
teraction of bright squeezed light with an three-level atom
in a high-Q cavity. Numerical simulation shows that (1) the
lower decoherence of the atom arising from the interaction
with the light will available, the larger the squeeze factor
of the squeezed pulse is, (2) compared with that using bright co-
herent light, higher fidelity of the atom qubit can be realized
using bright squeezed light.

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