Identification of the mechanical behaviour of low density hyperelastic polymeric foams from full-field measurements

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Abstract. This paper presents a methodology aimed at the identification of the mechanical behaviour of hyperelastic low density polymeric foams using digital image correlation and the Virtual Fields Method. First, a simple bending/shear test enables the identification of the elastic parameters at low strain levels. Then, a uniaxial compression test is performed to identify the parameters of an Ogden-type law (hyperelastic). The strain fields exhibit strong localization caused by the elastic collapse of the cells clearly indicating that an approach based on global force/displacement curves will fail to provide intrinsic material parameters (structural effect). This is still very much a preliminary study but it clearly shows the potential of the approach for this type of materials.

1. Introduction
Low density polymeric foams are widely used in applications that require good low energy absorption capabilities, such as in packaging for instance. However, the determination of their mechanical properties is made difficult because of their hyperelastic behaviour and strong localization effects caused by the local collapse of cells in compression. For the present material, the latter is due to elastic buckling of the cells in compression, with total recovery of the initial cell shape after unloading, resulting in the hyperelastic behaviour. Trying to address such a complex mechanical behaviour with just a global stress/strain curve is extremely difficult. It is therefore essential to bring in more experimental information and optical full-field deformation measurements provide a great opportunity to improve the identification procedures for this type of materials.

The scientific literature provides still only few attempts at using full-field measurements on such foams. Some recent studies can be found but the measurements are often only used as qualitative observations of localization [1,2] or at best to validate finite element simulations [3]. It is also interesting to note that the progress in X-ray tomography has enabled several researchers to digitize the 3D structure of foam specimens. This can be used to build up a fully realistic 3D finite element model [4,5] but more recently, first attempts at producing deformation maps have been released, either using 2D correlation [6] or volume image correlation [7]. However, to the best knowledge of the present authors, full-field measurements on foams have never been used to identify a mechanical behaviour model through an inverse methodology.

The objective of the present paper is to provide some initial ideas and results on how full-field measurements can be coupled to an inverse identification procedure to identify mechanical constitutive parameters of such low density polymeric foams. 2D digital image correlation will enable
to measure the in-plane displacement field and the Virtual Fields Method (VFM, [8]) will be used to
identify the material parameters.

2. Elastic stiffnesses at low strains
The first case study performed here consists in determining the initial Young’s modulus and Poisson’s ratio when the foam undergoes very low elastic strains. One of the difficulties in this case is that if this is performed on a usual universal testing machine, the load will be very small and very inaccurate values of the elastic stiffnesses might be obtained (unless a specific machine with adapted load transducer is used). The use of full-field measurements associated to the Virtual Fields Method enables to perform very simple tests without having to worry about the actual stress field in the specimen.

Here, the test is a very simple bending test illustrated on Fig. 1. It consists in clamping a rectangular bar of foam in a manual clamp and loading through small weights positioned at a certain distance from the clamp along the upper surface of the beam. This provides a mixed stress state comprising longitudinal bending stresses and in-plane shear.

![Global view of the set up](image1.png)  
![Focus on the test specimen with field of view](image2.png)

**Fig. 1 – Bending test on foam specimen**

The material tested here is low density grey polyurethane packaging foam (27 kg.m$^{-3}$). The reason for the choice of this foam is that it has been used to produce auxetic (or negative Poisson’s ratio) foams [9]. The typical cell size is about 200 μm. A 1.3 Mpixels PCO Sensicam CCD camera was used to record the images of the specimen. A 60 mm focal length lens provided a pixel size of 55 μm. No surface preparation was performed, the natural texture of the foam providing sufficient contrast (though not ideal) for image correlation provided that lateral lighting was used. Images of the field of view represented in Fig. 1 were taken before and after the application of the load and a digital correlation software (Correli, [10]) was used to derive the two components of the in-plane displacement field. A correlation window of 16 by 16 pixels was used together with a shift of 8 pixels, providing an array of 154 x 104 non independent measurements points (because of the 8 pixels shift). In order to calculate the strains, the displacement maps were smoothed using a basis of piecewise functions (triangular 3-noded bilinear finite elements). The nodal displacements were obtained through least-square fitting of the data and the strains derived from the shape functions (for more details, see [11]).

The displacement fields are given on Fig. 2. The noise is rather large because the natural contrast of the foam is far from ideal. Nevertheless, the strain fields (Fig. 3), obtained for an average mesh size of 20 displacement points, clearly show the bending ($\varepsilon_{xx}$) and shear ($\varepsilon_{xy}$) strains coming respectively from the bending moment and the shear force.
The identification was performed using the Virtual Fields Method. This technique enables the identification of the elastic parameters from the strain fields in a direct way. An important issue concerns the choice of the virtual fields. They have a very significant impact on the quality of the identification procedure. Here, optimized piecewise virtual fields have been used. They are defined on the same mesh as that used for the displacement smoothing and an adequate procedure enables to reach the maximum likelihood solution (ie, the solution which is the less sensitive to noise). The details of this procedure can be found in [11]. This procedure is available in the Camfit open source software available on www.camfit.fr.

The results of the identification are reported in Table 1. On can see that the results are independent from the mesh size used, which is a good sign of stability and therefore, of quality of the identification. One can also see that the coefficient of variation is higher for Poisson’s ratio than for Young’s modulus, which is not surprising since the latter has a greater influence on the actual strain field than former. The present authors do not have a reference value for Young’s modulus of this foam but Poisson’s ratio compares well with values found in [9]. They report a Poisson’s ratio of 0.25 for a compressive strain of 20%. They also show that this coefficient decreases sharply with the
compressive strain. The present value of 0.33 found here is therefore coherent with their results. The simplicity of the present procedure should be underlined, the use of full-field measurements enabling a less rigorous control of the boundary conditions, the results being insensitive to the actual clamping conditions of the specimen as long as the strain distribution through the thickness remains constant (2D problem). The technique can therefore be used to obtain elastic parameter values for soft materials in a fast and inexpensive way, no tensile testing machine being necessary.

| Mesh size | E (kPa) | ν   |
|-----------|--------|-----|
| 12        | 398    | 0.335|
| 14        | 413    | 0.326|
| 16        | 415    | 0.346|
| 18        | 408    | 0.353|
| 20        | 417    | 0.372|
| Mean      | 404    | 0.333|
| Coef. Var. (%) | 3.9  | 10.7 |

Tab. 1 – Identified elastic constants for a 0.29 N load (30 grams).

3. Hyperelastic law

One of the main challenges in foam characterization concerns the identification of the hyperelastic behaviour at large strains in the presence of strong strain localization. This paragraph presents a first validation of this approach.

As a first attempt, a simple uniaxial compressive test of a rectangular foam specimen was performed. Fig. 4 shows the experimental setup and the specimen’s dimensions.

![Uniaxial compression test (dimensions in mm).](image)

The measurements have been performed with the same parameters as in section 2, except that chalk has been rubbed on the surface to hide the shiny spots and balance out the grey level histogram. Forty images have been recorded throughout the test. The stretches have been obtained through the same finite element smoothing procedure as that described in section 2, with a mesh size of 10 displacement points. It should be noted here that the correlation is performed incrementally, the reference image being updated at each load step. This enables to reach very high deformation levels. The compression strain $\frac{\partial u_y}{\partial Y}$ is plotted on Fig. 5 at different moments of the global load-displacement history (where Y is the vertical coordinate in the undeformed configuration and $u_y$ is the vertical displacement in the deformed configuration). One can see that the global load-displacement curve exhibits a long plateau where the force is constant after a first short linear stage. Then, the load takes up again. The plateau is
caused by the elastic buckling of the cells. This happens row by row of cells until the whole of the cells have undergone this phenomenon. Then, the stiffness increases sharply because of the deformation of the cell material (compacted stage). It is interesting to note that during the initial linear part and the final compacted stage, the strain field is reasonably uniform whereas on the plateau, the gradual elastic collapse of horizontal rows of cells produces a very heterogeneous strain field (as also noted in [1]). It is also interesting to note that the collapse of the cells starts at the loading surfaces (top and bottom), which is consistent with observations reported in [2].

The idea of this paper is to use the strain fields at each stage of the load history to identify a constitutive behaviour law. For this purpose, a simple two parameters Ogden law has been selected. The main hypothesis here is that the stress is uniaxial (the other stress components have been neglected with respect to \(\sigma_{yy}\)). Also, it was assumed that Poisson’s ratio is close to zero. This is consistent with the findings of [9] where it can be seen that Poisson’s ratio rapidly decreases to zero when the compressive strain increases. It was also shown in [12] that the effect of Poisson’s ratio on the identified behaviour of such a foam was not very important. In any case, the assumption of zero Poisson’s ratio enables to write that the Cauchy stress component \(\sigma_{yy}\) (deformed configuration) is equal to the Piola-Kirchhoff one \(\Pi_{yy}\) (reference configuration). The Ogden law writes [12]:

\[
\sigma_{yy} = \frac{1}{\lambda_y} \mu [\lambda_y^\alpha - 1]
\]  

(1)

where \(\lambda_y\) is the stretch in the vertical direction (supposed to be a principal direction of the stretch tensor) and \(\mu, \alpha\) are the two parameters to identify.

The Virtual Fields Method in the case of non linear constitutive behaviour consists in minimizing the difference between the virtual work of internal and external forces (the latter being obtained from the load cell and the former calculated from the strain fields and constitutive parameters). Details can be found in [13] for the case of elasto-plasticity.

Using a uniform compression virtual field, the cost function can be written as:

\[
\Phi(\mu, \alpha) = \sum_{i=1}^{n} \left( F^{(i)} L_0 - e \int \Pi^{(i)}_{yy}(\mu, \alpha) dxdy \right)^2
\]

(2)

where \(L_0\) is the initial height of the specimen and \(e\) its thickness (or depth, see Fig. 4), \(F^{(i)}\) is the load at load step \(i\) (\(n\) steps, here \(n=40\)) and \(\Pi^{(i)}_{yy}\) the Piola-Kirchhoff stress at load step \(i\). The integral in the above equation can be approximated by a discrete sum at all the measurement points. Substituting the stress from Eq. 1, the cost function becomes:

\[
\Phi(\mu, \alpha) = \sum_{i=1}^{n} \left( F^{(i)} - \frac{eb}{pq} \sum_{p,q} \frac{1}{\lambda_y^{(i)} (p, q)} \mu \lambda_y^{(i)\alpha} (p, q) - 1 \right)^2
\]

(3)

where \(p\) and \(q\) are the number of measurement points along the \(x\) and \(y\) directions and \(b\) is the specimen width. The left hand-side term in the squared difference represents the load measured by the load cell and the right hand-side term is the resultant load recalculated from the measured strains and the constitutive parameters.

This cost function was represented graphically in Fig. 6 for a large range of values of the Ogden parameters. A minimum is clearly visible around \([\mu, \alpha]=[7000 \text{ Pa}, 300]\), though above 100 for \(\alpha\), this parameter has little influence on the cost function. On the right hand-side of Fig 6 is a plot of the two terms in the cost function of Eq. 3. It can be seen that the Ogden model fits reasonably well but that the flat plateau is not so well described. This is not surprising since the mechanism of gradual elastic collapse of cell rows will produce a constant force that is not a feature of Ogden’s model which is not specific to foams. It would be possible to add up terms in the model but there might be uniqueness problems. It is clear that there is still a lot of work to be done in this area, particularly to identify the parameters on a more complex test with several stress components involved in the specimen’s response.
4. Evolution of Poisson’s ratio with compressive strain of an auxetic foam

The last part of this paper is dedicated to the measurement of Poisson’s ratio of an auxetic foam (negative Poisson’s ratio). The foam was produced in the group of Dr Fabrizio Scarpa at the University of Bristol from a precursor similar to the foam used in the previous sections. The specimen is an 85 mm long 20 by 20 mm square-base cylinder. It was tested in compression using a manual clamp (see Fig. 7). No force was measured as only Poisson’s ratio was sought here. 24 load steps were performed (corresponding to the deformation between the unloaded state in Fig. 7(a) to the maximum compression in Fig. 7(d)). The strain fields were obtained from the displacements by global polynomial fitting. 3rd degree polynomial were selected because there was no evidence of localization here as opposed to what happened in the specimen from section 3. Poisson’s ratio was calculated from the use of the virtual fields method with the following virtual field (with axes defined in Fig. 8 and $L$ the total length of the gauge length):

\[
\begin{align*}
\varepsilon_{xx}^* &= 0 \\
\varepsilon_{yy}^* &= -x(x - L) \\
\varepsilon_{xy}^* &= (-x + L / 2)y
\end{align*}
\] (4)

This virtual field is a virtual transverse compression field as represented in Fig. 8. The resulting equation using the virtual fields method [14] is:
\[
Q_{xx}\left(-\int x(x-L)\varepsilon_{xy}dV + \int (-x+L/2)\varepsilon_{xy}dV\right) + Q_{yy}\left(-\int x(x-L)\varepsilon_{xx}dV - \int (-x+L/2)\varepsilon_{xy}dV\right) = 0
\]

where \(Q_{xx}\) and \(Q_{yy}\) are the two isotropic stiffness components. Therefore, Poisson’s ratio is directly obtained by:

\[
\nu = -\frac{-\int x(x-L)\varepsilon_{xy}dV + \int (-x+L/2)\varepsilon_{xy}dV}{-\int x(x-L)\varepsilon_{xx}dV - \int (-x+L/2)\varepsilon_{xy}dV}
\]

where the integrals can be approximated by discrete sums over the array of measurement points.

Fig. 7 – Compression test on an auxetic foam specimen.

Fig. 8 – Identified Poisson’s ratio as a function of compressive strain.
Fig. 8 shows the identified values of Poisson’s ratio as a function of the global average compressive strain. The results are consistent with that in [9] where Poisson’s ratio decreases sharply from about -0.3 to near zero.

5. Conclusion
This paper has presented an overview of the possibilities of full-field deformation measurements to identify the constitutive behavior of low density polymeric open cell foams. Elastic and hyperelastic behaviour has been considered. An application on an auxetic foam specimen was also shown. There is great potential in the future to enhance the modelling of the mechanical behaviour of such materials thanks to full-field measurements that can tackle both strong localization and very flexible test specimens.

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