Experimental investigation on the surface potential decays of dielectric materials with $q$-exponential function

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Abstract. We have studied the surface potential decays (SPD) of a variety of dielectric materials, and found that the SPD data of the samples with high charge retentions are well fitted by Tsallis $q$-exponential functions.

1. Introduction
In many different fields we have seen Tallis $q$-exponential distributions, which is equivalent to the Zipf-Mandelbrot distributions in the case with the parameter $q$ is larger than unity. For example, the distributions of family names [1], urban agglomeration [2], itineration of the internet over nonequilibrium stationary states [3], fluxes of cosmic ray [4], and football goals [5].

In this article, we show that the surface potential decay (SPD) profiles of various dielectric material powders are well fitted by Tsallis $q$-exponential functions. SPD measurement is a conventional experimental technique [6] for studying charge carrier transport and charge retention in dielectric (or insulating) materials. The SPD technique has been widely applied many kinds of dielectric materials concerning the industrial fields such as electro-photography or electrostatic powder coating or insulators for electric cable. A characteristic property of the SPD curves of dielectric materials with high electrical charge retention is an exponential behavior in the early stage and power law behavior in final stage as will be shown in figure 3 in section 2. Because of this property, the SPD profiles of dielectric materials with high electrical charge retention have not been well fitted by a single analytical function yet. To the best of our knowledge, no such reports are available.

The organization of the paper is as follows. In section 2, we explain the samples and the experimental procedure for SPD measurements. Then Tsallis $q$-exponential function and fitting procedure are explained. Section 3 shows the results and discussion. Remarkably the most of the SPD profiles are well fitted by Tsallis $q$-exponential functions. Finally, the paper is concluded in section 4.
2. Experimental and fitting procedures

2.1. Samples

Table 1 shows the dielectric materials used in this experiment, and their mean diameters. Polyester is used as a raw material for a PET bottle and fiber clothes. Epoxy is used by engineer plastic. Acrylic is used as a substitute of glass. Nylon is a kind of polyamide, and used as raw materials for clothes. Polyvinyl chloride powder, polyester powder, epoxy powder, and acrylic powder are used for electrostatic powder coating. Ferrite carriers is used as carrier particles in electro-photography (or Xerography). Those samples are high electrical charge retention. Nylon powder is used for powder coating.

| Sample name            | Mean diameter (µm) | Uses                  |
|------------------------|--------------------|-----------------------|
| Ferrite carriers       | 55                 | electro-photography   |
| Polyvinyl chloride powder | 200            | electrostatic powder coating |
| Polyester powder       | 27                 | electrostatic powder coating |
| Epoxy powder           | 20                 | electrostatic powder coating |
| Acrylic powder         | 10                 | electrostatic powder coating |
| Nylon powder           | 15                 | powder coating        |

2.2. Experimental procedure

Figure 1 show our experimental set up for DC corona-charging of samples. A sample is filled in the Aluminium container, which is 3 mm in depth and 30 mm in the diameter as shown figure 1. The sample container and grid mesh are electrically grounded. The sample is charged by DC corona discharging. For positive charging +5kV is applied to the tip electrode, and the grid potential is set to about +500 V. For negative charging the polarity of the applied voltages are reversed. The distance between the grid mesh and the surface of the sample is set to 5 mm. The sample is exposed to the DC corona discharge for about 1 min.

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**Figure 1.** Corona electrification experimental setup

**Figure 2.** Measurement experimental setup
After the corona charging the sample is immediately set to the under the measurement probe as shown in figure 2, and the surface potential is measured with an electro static voltmeter (TREK-344). The measurement probe (TREK MODEL 600B-8) is located in 3 mm from the sample surface.

2.3. Fitting procedure

A characteristic property of the SPD curves of dielectric materials with high electrical charge retention is an exponential behavior in the early stage and power law behavior in final stage as shown in figure 3, which is a typical SPD profile of ferrite carriers. Because of this property, the SPD profiles of dielectric materials with high electrical charge retention have not been well fitted by a single analytical function yet. An analytic function which has such property is Tsallis $q$-exponential function [7, 8], which is defined by

$$\exp_q(x) \equiv [1 + (1 - q)x]^{\frac{1}{1-q}},$$  \hspace{1cm} (1)

for a real parameter $q > 1$. In the limit of $q \rightarrow 1$, the $q$-exponential function reduces to $\exp(x)$. Since this $q$-exponential function behaves as

$$\exp_q(x) \sim \exp(x),$$  \hspace{1cm} (2)

for a small value of $|x|$, and

$$\exp_q(x) \sim |x|^{\frac{1}{q-1}}$$  \hspace{1cm} (3)

for a large value of $|x|$, we use this analytic function in order to fit the SPD data of the samples, especially with high charge retention. The measured SPD data is therefore fitted by using the following function

$$U(t) = U(0) \exp_q \left( -\frac{t}{\tau} \right),$$  \hspace{1cm} (4)

where $q$ and $\tau$ are fitting parameters, $U(t)$ is a surface potential at time $t$, $U(0)$ is an initial surface potential just starting measurement $t = 0$. 

![Figure 3. A typical SPD profile of ferrite carriers.](image)
In order to judge whether the fitting is well or not, we use the sum of the squares of the residuals (divided by the number) $R^2$ defined by

$$R^2 = \frac{1}{N} \sum_{i=1}^{N} \{V(t_i) - U(t_i)\}^2 .$$  \hspace{1cm} (5)

Here $N$ is the number of the measured data, $V(t_i)$ and $U(t_i)$ are a measured data and the value of the fitting function at $t = t_i$, respectively. The smaller $R^2$ the better the fitting.

The $q$-logarithmic function is defined by

$$\ln_q(x) = \frac{x^{1-q} - 1}{1 - q},$$  \hspace{1cm} (6)

which is the inverse function of $\exp_q(x)$. By taking $q$-logarithmic function of the both sides of equation (4), we have

$$\ln_q \left[ \frac{U(t)}{U(0)} \right] = -\frac{t}{\tau} .$$  \hspace{1cm} (7)

Then, the $q$-logarithmic functions of the normalized SPD data obey a straight line if the data is well fitted by the $q$-exponential function. Such a plot is called the $q$-logarithmic plot, which is as useful as the standard logarithmic plot.

3. Results and discussion

The powder samples for electrostatic coating and ferrite carrier have high charge retention characteristics, and consequently the SPD of those samples are relatively slow. Whereas nylon powder have low charge retention characteristics, and consequently the SPD of those samples are relatively fast.

3.1. Powders for electrostatic powder coating

Figures 4-7 are results of the SPD profiles of polyvinyl chloride powder, acrylic powder, epoxy powder, and polyester powder, respectively. Each SPD curve is plotted with the surface potential normalized by initial surface potential $V(0)$ against the total measurement time.

**Figure 4.** SPD profile of polyvinyl chloride powder. Relative humidity is 47% and temperature is 25.9°C. The total measurement time is 7000s. Parameter $q$ is 3.59, parameter $\tau$ is 5.53 and $R^2$ is $2.85 \times 10^{-5}$

**Figure 5.** SPD profile of acrylic powder. Relative humidity is 60% and 25.5°C. The total measurement time is 7000s. Parameter $q$ is 76.2, parameter $\tau$ is 0.52 and $R^2$ is $1.34 \times 10^{-5}$
Figure 6. SPD profile of epoxy powder. Relative humidity is 61% and temperature is 25.4°C. The total measurement time is 7000s. Parameter $q$ is 29.7, parameter $\tau$ is $16.5 \times 10^2$ and $R^2$ is $1.29 \times 10^{-6}$.

Figure 7. SPD profile of polyester powder. Relative humidity is 56% and temperature is 24.5°C. The total measurement time is 7000s. Parameter $q$ is 26.8, parameter $\tau$ is $26.0 \times 10^{-6}$ and $R^2$ is $4.09 \times 10^{-6}$.

The normalized surface potential $V(t)/V(0)$ soon decays to a certain value in the early stage, and then slow decay later for the result of SPD profile of polyvinyl chloride powder and that of acrylic powder. These properties are reflected in a small value of the fitting parameter $\tau$, which is a similar quantity as a time constant in the standard exponential decays. However $\tau$ in the $q$-exponential decay depends on $q$. As obvious from equation (7), at $t = \tau$ the value of the normalized surface potential decreases by the factor of

$$\exp_q(-1) = q^{1-q}. \quad (8)$$

3.2. Ferrite carriers for electro-photography

Figure 8 is a result of the SPD profile (dot) of ferrite carriers. Initial surface potential $V(0)$ is -496.8 V and the total measurement time is 6000 s.

Figure 8. SPD profile of ferrite carrier. Relative humidity is 57% and temperature is 27.8°C.

The SPD data is fitted (Figure 8) by using the $q$-exponential function 4, and plotted with solid line in figure 8. Remarkably the SPD data is well fitted by the $q$-exponential function with $q = 10.65$, $\tau = 144.2$, and $R^2 = 5.58 \times 10^{-6}$. This is more clear when the same data is expressed in the $q$-logarithmic plot (figure 9). As can be seen from the equation (7), SPD data in the
$q$-logarithmic plot becomes a straight line if the $q$-exponential fitting is well. Indeed this is so as shown in figure 9.

We also checked a possibility of a logarithmic decay, because the shape of logarithmic decay curve looks like a $q$-exponential curve. We have tried to fit the same SPD data shown in figure 8 with the following fitting function

$$U(t) = \frac{U(0)}{1 + \gamma \ln (1 + \frac{t}{\tau})},$$

where $\gamma$ and $\tau$ are fitting parameters. The broken line in figure 10 is the fitted result for the same SPD data in figure 8 with the logarithmic decay curve (9). The best fitted parameters are $\gamma = 0.1654$, $\tau = 28.62$, and $R^2 = 2.1 \times 10^{-4}$. As is clear from figure 10, the SPD data of ferrite carrier is not fitted with the logarithmic decay curve (9).

In this way the dielectric materials used for ferrite carrier and electrostatic powder coating are well fitted by the $q$-exponential function (4) with $q \neq 1$. However, as we will show in the next subsection, the results of nylon powder is different from those of ferrite carrier and electrostatic powder coating.

3.3. Nylon powder for powder coating

Figure 11 is a result of the nylon powders. Evidently $V(t)$ approaches zero as time $t$ passes

**Figure 10.** Logarithmic decay of SPD data in figure 8.

**Figure 11.** SPD profile of nylon powder. Relative humidity is 52% and temperature is 22.3°C.

**Figure 12.** It is fitted by stretched exponential of the same SPD data in figure 11.
due to the low charge retention characteristics of nylon powders. Consequently the SPD data of nylon powder is not fitted by $q$-exponential function. Instead we confirmed that a stretched exponential function (10) in the following form

$$U(t) = U(0) \exp\left\{-\left(\frac{t}{\tau}\right)^\beta\right\}, \quad (10)$$

is fitted with this data with the fitting parameter $\beta = 0.42, \tau = 198$, and $R^2 = 2.83 \times 10^{-5}$. Note that the sum of the square of residuals ($R^2 = 2.83 \times 10^{-5}$) of the stretched exponential fitting (10) is smaller than that ($R^2 = 2.63 \times 10^{-4}$) of the $q$-exponential fitting (4). Nylon has good electrification, but an increase of moisture content in nylon decreases its electrical insulation performance. A possible reason is therefore that the electrical insulation performance of nylon decreases due to relatively high humidity, then the electric charges may easily escape from the surface of the nylon powders. Consequently the surface potential $V(0)$ soon decays to zero.

4. Concluding remarks

We have studied the surface potential decay of various dielectric material powders: polyvinyl chloride powder; acrylic powder; epoxy powder; polyester powder; ferrite carriers; and nylon powder. Remarkably the most of the SPD profiles are well fitted by Tsallis $q$-exponential function for the various different samples except nylon powder. A physical reason for having these $q$-exponential behaviors might be related to some memory effect in the dielectric sample.

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