Learning to Remember, Forget, and Ignore using Attention Control in Memory

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Abstract

Typical neural networks with external memory do not effectively separate capacity for episodic and working memory as is required for reasoning in humans. Applying knowledge gained from psychological studies, we designed a new model called Differentiable Working Memory (DWM) in order to specifically emulate human working memory. As it shows the same functional characteristics as working memory, it robustly learns psychology inspired tasks and converges faster than comparable state-of-the-art models. Moreover, the DWM model successfully generalizes to sequences two orders of magnitude longer than the ones used in training. Our in-depth analysis shows that the behavior of DWM is interpretable and that it learns to have fine control over memory, allowing it to retain, ignore or forget information based on its relevance.

1 Introduction

Keeping information in mind after it is no longer present in the environment is critical for all higher cognitive behaviors. Working memory (WM) is the term used for this ability, which is distinct from the storage of vast amount of information in long-term memory [Bad03, Obe09]. The two main distinguishing characteristics of WM are the limited capacity (3-5 items) [CKB+05] and temporary retention (secs-minutes). Hence, WM is not a storage per se, but a mental workspace utilized during planning, reasoning and solving problems. Most psychologists differentiate WM from “short-term” memory because it can involve the manipulation of information rather than being a passive storage [Cow17]. Along the same lines, Engle et al. [ETLC99] argued that WM is all about the capacity for controlled, sustained attention in the face of interference or distraction. Attention-control is a fundamental component of the WM system and probably the main limiting factor for capacity [CE94, EK04]. Consequently, the inability to effectively parallel process two-attention demanding tasks limits our multitasking performance severely.

Over the past several decades psychologists have developed tests to measure the individual differences in WM capacity and better understand the underlying mechanisms. These tests have been carefully crafted to focus on the specific aspects of WM such as task-driven attention control, interference and capacity limits [OL17]. The best known and successfully applied

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class of tasks for measuring WM capacity is the “complex span” paradigm. The challenge presented by complex span tasks is recalling the list of items, despite being distracted by the processing task. Studies show that individuals with high WM capacity are less likely to store irrelevant distractors [VMM05] and they are better at retaining task-relevant information [MRH13]. Developing task-driven strategies for cognitive control are essential for the effective use of WM.

The power of maintaining information over time has also been recognized by the AI community. Starting with the basic recurrent neural network architectures [Elm90, HT86] followed by the introduction of gating mechanisms [HS97], the research has recently moved onto more complex architectures with memories [GWD14, JM15, WCB15, GWR+16, SBB+16, GCB17]. These models are typically applied to tasks (e.g. associative recall, bAbI QA [WBCM15]) that require a complex mixture of long-term memory (episodic and semantic) and working memory. In the human brain, these kinds of memory systems are distinct: working memory is instantiated in multiple interconnected areas with the prefrontal cortex playing a major role [CK16], whereas for episodic memory the hippocampus is the critical structure [FAE02]. Studying these mechanisms separately is necessary to disentangle the contributions of each memory system and develop a detailed understanding of human intelligence.

In this work, we introduce a Differentiable Working Memory (DWM) model that more closely mimics the functional behavior of human WM. We also present a battery of tasks adopted from the cognitive psychology literature that allow us to elucidate the working memory behavior of a neural network directly. In contrast to prior work on neural nets with external memory, our starting point is the functional attributes of WM and the tasks that primarily tests those.

The main contributions of this work are:

- A differentiable WM model that is easy to train and accurately generalizes to sequences two orders of magnitude longer than the training data.
- A new attention control mechanism in memory, which can deal with interference.
- Algorithmic working memory tasks adapted from the cognitive psychology.
- An examination of strategies developed for handling memory scarcity.

## 2 Methods

### 2.1 Psychometric tasks for working memory

Over last several decades cognitive psychologists have developed many tasks to measure the performance of human WM (See Fig. 1 for examples). These tasks are mainly sequential and typically divided into verbal and visuospatial domains. One of the fundamental goals of these psychometric tests is to measure individual differences and correlate to performance in reasoning and fluid intelligence. In this regard, WM capacity, retention and the ability to switch between tasks are the key predictors.

Given the large number of WM tasks in the psychology literature and various categorizations by different researchers, we built a taxonomy of tasks (Fig. 2) and carefully selected tasks that seem to be the most representative for a given category. First order categorization is based on the number and complexity of tasks. For simple tasks, the presence of data manipulation is the next level sub-category. Serial recall is a prime example for a simple task without manipulation. Other WM tasks may require the manipulation of the memory content, which could be divided
in spatial and temporal domains. The complex WM tasks involve multiple sequential inputs or sub-tasks but not necessarily implies multi-tasking. We follow the framework developed by Clapp and Gazzeley to distinguish the sources of goal interference, i.e., Distraction (to-be-ignored) and Interruption (i.e., multi-tasking). For example, Operation Span (Fig. 1c) is a dual task because the subjects must attend and process the summation ( Interruption ) even though they do not need to recall the results afterwards. In Reading Span (1980 version by [DC80]) subjects read sentences and need to recall the last word of each one.

2.2 Differentiable Working Memory (DWM)

Inspired by human working memory, we designed a Differentiable Working Memory model with the appropriate functional characteristics. We illustrate its operation on Fig. 3. As a neural network with external memory, the DWM has three main components: a controller, an external memory and an interface between the two [Zar16, ZMJF16]. The controller generates parameters that allow the interface to pay attention to specific locations in memory and then read and write to them. The procedure is sketched in Algorithm 1. The detailed equations for steps 4–6 can be found in the Supplemental Information.

**Attention control.** For our model to act like human working memory, it must have the same characteristics, the most important of which is that working memory is accessed and written to sequentially [STH18]. One condition is that it has to use circular convolution shifting enabling it to shift attention over memory. For that purpose we use a mechanism similar to the one used by Neural Turing Machine (NTM) [GWD14]:

$$w_t = \text{convolution}(w^0_t, s_t)$$

(1)

where $w^0_t$ and $w_t$ are the vectors of attention weights over cells in memory at time $t$ before and after shifting, and $s_t$ is a shift vector outputted by the controller. The other condition that sequentiality suggests is that the read and write operations should jointly share a single attention so that all access happens in sequential order. As is standard with the linear shifting in memory

![Figure 1: Exemplary tasks for testing the performance of human working memory](image-url)
Algorithm 1 Operation of the Differentiable Working Memory

1: Initialize the hidden state $h_0$ and memory array $M_0$
2: Initialize the attention vector $w_0$ and bookmark vectors \( \{B^i_0: i = 1, 2, \ldots, N_B\} \)
3: for \( t \in \{1, 2, \ldots, T\} \) do
4: \hspace{1em} Read from the memory: \( r_{t-1} \leftarrow M_{t-1} w_{t-1} \)
5: \hspace{1em} Compute a new controller hidden state and parameters: \( h_t, P_t \leftarrow \phi(x_t, r_{t-1}, h_{t-1}) \)
6: \hspace{1em} Write and erase from memory: \( M_t \leftarrow \text{edit}(w_{t-1}, P_t, M_{t-1}) \)
7: \hspace{1em} Update attention and bookmarks: \( w_t, \{B^i_t\} = \text{attn\_control}(w_{t-1}, \{B^i_{t-1}\}, P_t) \)

enhanced neural networks, we also apply a weight sharpening step as detailed in Graves et al. [GWD14].

Working memory also involves some limited model of when information was attended to in the past [STH18]. This characteristic suggests that human working memory could contain some limited internal record of its past attention. In DWM, this is accomplished by storing bookmarks of system’s attention at previous time steps. This is recorded in $N_B$ bookmark vectors \( \{B^i_t: i = 1, 2, \ldots, N_B\} \) at time $t$. At each time step, the DWM must decide whether to remember its

Figure 2: Taxonomy of working memory tasks

Figure 3: Illustration of the operation of the DWM Model
previous attention $w_{t-1}$ by recording it in a bookmark, as:

$$B_t^i = g_t^i w_{t-1} + (1 - g_t^i)B_{t-1}^i,$$  \hfill (2)

where the gating parameter $g_t^i$ is emitted by the controller. Additionally, the DWM keeps one bookmark fixed to the initial attention at time $t = 0$ so the model maintains a fixed reference frame. As discussed below, we found even when limiting the bookmark memory to only two bookmarks that we could still solve all tasks.

The DWM must also decide before moving sequentially whether it wishes to return to a previous bookmark. For this purpose we once again use a gating mechanism, this time in a slightly more sophisticated form:

$$w_t^q = \delta_t^0 w_{t-1} + \sum_{i=1}^{N_B} \delta_t^i B_{t-1}^i,$$  \hfill (3)

where $\delta_t^i, i = 0 \ldots N_B$ are gating parameters emitted by the controller. These gating parameters are scalars, normalized using a softmax function.

The DWM attention control incorporates the presented mechanisms using the combination of equations (3),(2),(1) in order.

2.3 Formal description of tasks

In order to investigate the capabilities of working memory (WM) such as retention, forgetting and ignoring we introduce a battery of tasks presented in Table 1. These tasks are motivated by, and have a direct correspondence to the categorization shown in Fig. 2, but modified so that they are agnostic to audio/visual processing. In addition to the classical psychometric tasks, we introduce additional tasks that will test the effectiveness of attention control in memory (Ignore, Forget and Scratch Pad).

The input to every WM task is a time-indexed stream of items. At a higher level, we view the input as a concatenation of various subsequences that represent different functional units of processing. Additionally we use a constant-sized set of special items (called command markers) to both mark the beginning of a subsequence as well as indicate its functional type. Important note is that the system does not know a priori what kind of operation is associated with a given type of marker and must learn that from data. We ignored markers in Table 1 to keep the description simple. Also, note that such markers are also commonly employed in the psychometric tests (e.g. see [MKo8]).

For all Simple tasks, there is only one type of subsequence, and the output will be reproduced from the memory with or without manipulation. The $|$ sign indicates the delay between input and output of the primary subsequence(s). In our notation, $x^\circ$ represents doing a circular shift of the bit representation of element $x$ by half the number of bits. The Complex tasks may involve a secondary set of subsequences, which may or may not require immediate output as indicated in the Forget and Operation Span tasks.

3 Experimental results

We evaluated the performance of DWM on the proposed tasks and compared it to two models: LSTM (Long Short-Term Memory) [HS97], considered as a classical baseline for sequential problems, and DNC (Differentiable Neural Computer) [GWR+16] being the state-of-the-art memory
| Task               | (I)input/(O)output sequences | Notes                                                                 |
|--------------------|------------------------------|----------------------------------------------------------------------|
| Serial Recall I:   | \( x_1x_2 \ldots x_n \) | Store the input and recall it in the same order                      |
| O:                 | \( \_\_\_ \ldots \_ x_1x_2 \ldots x_n \) |
| Reverse Recall I:  | \( x_1x_2 \ldots x_n \) | Store the input and recall it in the reversed order                 |
| O:                 | \( \_\_\_ \ldots \_ x_nx_{n-1} \ldots x_1 \) |
| Rotate Shape I:    | \( x_1x_2 \ldots x_n \) | Rotate each element of sequence                                      |
| O:                 | \( \_\_\_ \ldots \_ x_i^\circ \_ \ldots \_ x_n^\circ \) |
| Reading Span I:    | \( x_1 \ldots x_k \) | Recall \( z_i \) being the last element of \( x_i \)               |
| O:                 | \( \_\_\_ \ldots \_ z_1 \ldots z_k \) |
| Forget I:          | \( x_1y_1 \ldots x_ky_k \) | Recall every \( y_i \) immediately, recall all \( x_i \) in the same order |
| O:                 | \( \_\_\_ \ldots \_ y_1 \ldots \_ y_k \ldots \_ x_1 \ldots x_k \) |
| Operation Span I:  | \( x_1y_1 \ldots x_ky_k \) | Rotate \( y_i \) immediately, recall all \( x_i \) in the same order |
| O:                 | \( \_\_\_ \ldots \_ y_1^\circ \ldots \_ y_k^\circ \ldots \_ x_1 \ldots x_k \) |
| Scratch Pad I:     | \( x_1 \ldots x_k \) | Return only the last \( x_k \)                                       |
| O:                 | \( \_\_\_ \ldots \_ x_k \) |
| Ignore I:          | \( x_1y_1 \ldots x_ky_k \) | Ignore \( y_i \), recall \( x_i \)                                  |
| O:                 | \( \_\_\_ \ldots \_ x_1 \ldots x_k \) |

Table 1: The Working Memory Tasks used in our experiments
augmented neural model. In our implementation we used the Pytorch framework [PGC+17]. During training we used the Adam (Adaptive Momentum) optimizer [KB14] and (average) binary cross-entropy as the loss function. We apply early stopping based on validation loss ($10^{-4}$). Additionally we terminate training when the number of training episodes reach 100,000, where a single episode involves processing a batch of sequences. The size of batch was a hyper-parameter that was tuned for each model along with training rate using validation loss. The exact parameters describing each tasks can be found in the Supplemental Information.

It is well-known that neural networks augmented by external memory learn algorithms to process tasks rather than memorizing data [GWD14, GWR+16]. Therefore, to determine the robustness of our models we train them on sequences of lengths of order 10 and then validate and test them on sequences of size 100 and 1000, respectively. As seen in Table 2, the DWM, the LSTM and the DNC are all capable of learning each of the tasks for sequences of order 10. To train on these tasks, the DWM required on average the fewest number of sequences with only Forget and Operation Span requiring more than 10000 batches of size 16 to converge. The DNC required the most number of sequences with only Serial Recall, Rotate Shape and Ignore converging with fewer than 20000 batches of size 16 and the rest requiring between 20000 to 90000 batches.

Only the DWM was able to successfully generalize to sequences of size 1000 even with only 1066 trainable parameters. The DNC with its 4,792 trainable parameters generalized to sequences of length 100 just on Serial Recall, Scratch Pad and Rotate Shape and did not fully generalize on sequences of length 1000 for any of the tasks. Despite that the LSTM possessed over 5 million trainable parameters it was only able to generalize to sequences of length 100 on Serial Recall.

The DWM performance on these tasks indicates that it is actually acting as a form of artificial working memory. Human working memory’s operation is straightforward compared to episodic memory and therefore it is more effective at solving the relatively simple working memory tasks. We hypothesize that the DNC, which was explicitly designed to mimic an episodic memory [GWR+16], did not learn these simple working memory tasks as effectively because the complexity required for true episodic remembrance interfered with learning tasks that do not require episodic memory.

3.1 Analysis of strategies for solving tasks

In contrast to long-term memory, working memory has an extremely small capacity. Therefore, dealing with interference (e.g. distractions) is a major challenge for memory capacity and attention control. As mentioned earlier, ignoring distractions without encoding them in the memory is arguably the best strategy to minimize memory consumption. On the other hand, for a complex task with an interruption (i.e. multi-tasking), the secondary task cannot be ignored and may require extensive memory usage. In this case, the best strategy might be to forget (e.g. erase or overwrite) the secondary information as soon as possible in order to maintain sufficient memory capacity for the main task. Ignoring and forgetting may be good strategies for the efficient use of capacity but they also complicate the attention control and addressing during writing in the memory.

During the training and testing of all of the tasks reported in Table 2, we provided sufficient memory size, so that the system could store all the encoded input items in the memory (if it has chosen to). However, limitation of the memory size can force the system to develop more memory efficient strategies, thus we decided to investigate that issue further.
### Task Summary

| Task          | Best Train Accuracy | Validation Accuracy | Test Accuracy |
|---------------|---------------------|---------------------|---------------|
|               | Seq. Size 10 [%]    | Seq. Size 100 [%]   | Seq. Size 1000 [%] |
|               | LSTM    | DNC    | DWM    | LSTM    | DNC    | DWM    | LSTM    | DNC    | DWM    |
| Serial Recall | 100     | 100    | 100    | 100     | 100    | 100    | 50.20   | 64.64  | 100    |
| Reverse Recall| 100     | 100    | 100    | 52.96   | 50.62  | 100    | 50.38   | 50.15  | 99.76  |
| Rotate Shape  | 100     | 100    | 100    | 52.17   | 53.36  | 100    | 50.44   | 49.04  | 91.88  |
| Reading Span  | 100     | 100    | 100    | 50.00   | 53.90  | 98.92  | 50.45   | 49.94  | 94.11  |
| Forget        | 100     | 100    | 100    | 58.16   | 79.22  | 99.95  | 51.26   | 53.61  | 99.64  |
| Operation Span| 100     | 100    | 100    | 71.28   | 100    | 100    | 70.02   | 74.97  | 100    |
| Scratch Pad   | 100     | 100    | 100    | 56.13   | 69.32  | 100    | 50.89   | 49.99  | 90.05  |
| Ignore        | 100     | 100    | 100    | 100     | 100    | 100    | 50.89   | 49.99  | 90.05  |

Table 2: Summary of experimental results. The first column is the average of the best accuracy achieved during training for each run. The second column is the average of the best validation accuracy for each run. The third column is the average of the test accuracy for the model parameters with the best validation accuracy. For the DNC and LSTM, the validation loss did not reach the threshold for some tasks (i.e. training was stopped at 100,000 episodes), in which cases we decided to include the best scores of single best (but diverged) models.

### Strategies for the Scratch Pad task

The goal of the Scratch Pad task is to recall only the last input subsequence. Given the DWM mechanisms, we expect two possible strategies for the model to learn in order to solve this task.

The "Expand" strategy exploits the fact that memory can be used in a similar way to a circular buffer, storing each consecutive subsequence one after the other in the memory. In this case the model should write each subsequence, then place the dynamic bookmark (i.e. the one that does not have fixed attention) at the start of the given subsequence, and then update the bookmark position to the beginning of the next subsequence. Finally, when the model receives a command marker indicating it needs to recall, it should recall the attention associated with that dynamic bookmark and then retrieve consecutive items one by one using circular convolution.

The "Overwrite" strategy for the Scratch Pad relies on the fact that when a new subsequence appears, the elements from the previous one can be discarded. The model could exploit this by learning to recall attention stored in the static bookmark (pointing at address 0) every time it processes a command marker denoting the next subsequence, which will result in overwriting the previous subsequences until the system is told to recall. This strategy is clearly more memory efficient, as the system reuses the same addresses and overwrites the memory repeatedly.

To our (initial) surprise, the model always developed the "Overwrite" strategy, irrespective of the memory size (i.e. as long as the memory size was sufficient to fit all the encoded items of a single subsequence). A typical example run of an early training episode is presented in Fig. 4. Please note that memory addresses 1 and 2 remain unchanged and the model stores consecutive items of subsequences $x_1$ to $x_5$ in the same addresses 3-7. After analyzing several runs, we hypothesize that overwriting was simpler to learn for this task because: a) both for storing and recalling the command markers, the model had to learn exactly the same behavior: recalling the attention stored in the static bookmark, b) for every other input item (data and dummy) it had to shift by one address location with the circular convolution. As a result, it could converge rapidly...
by disregarding the control (update, recalling) of the dynamic bookmark (in the last training episodes the dynamic bookmark was typically "following" the current attention, despite it wasn’t recalled at all).

3.1.2 Strategies for the Ignore task

The main goal of the Ignore task is to test the retention capabilities of the system in the presence of distractors. For this task the input consists of two types of subsequences $x$ and $y$, where the system is supposed to ignore all $y_i$ and at the end recall $x_i$ one by one in the order of their appearance. The task can be solved with two strategies which we call "Overwrite" and "Skip".

The "Overwrite" strategy involves overwriting of the distractors, as we have already observed

Figure 4: "Overwrite" strategy developed by DWM for Scratch Pad (episode 969). Memory plot contains a snapshot of the memory content from the last iteration, whereas the other ones present concatenation of states from consecutive iterations (evolution in time).

Figure 5: Strategies developed by DWM for solving the Ignore task (two different training runs)
The difference is, however, that in this case the model must learn to use the dynamic bookmark for that purpose. Our experiments with sufficient memory have shown that the system can learn this strategy. An example plot from one of the final training episodes (Fig. 5a) shows that the dynamic bookmark retains its attention while processing items from \( y_1 \) and \( y_2 \). As soon as the command marker indicating \( x \) appears, the model jumps back its attention to the dynamic bookmark and starts to overwrite. Finally, when the recall marker appears, it recalls the attention stored in the static bookmark.

The "Skip" strategy to solve this task would be to ignore elements within the \( y \) subsequences and skip writing these into the memory. Our experiments with limited memory have shown that the model could also learn this strategy. Example plots from the final episode from one of the training runs is presented in Fig. 5b. Please notice that in this case the model has learned to keep its attention focused on a single address for all items of \( y_i \) and shift attention only for items belonging to \( x_i \).

That behavior of the model that mastered the "Skip" strategy seems to be more difficult from the operational point of view. In the "Overwrite" strategy the system develops a reactive behavior, i.e. it always performs convolutional shift except for the rare cases when it hits the command marker – at that point it has to retrieve attention from one or the other bookmarks. In the "Skip" strategy the command markers for \( x \) and \( y \) activate one of two distinct operation modes that will executed for the whole subsequence until hitting the next marker, i.e. for \( x \) attention is supposed to move to the next address, whereas for \( y \) it is supposed to stay at the same position. The only way to perform this is that the controller must learn how to carry the information about the current operation mode from one iteration to another in its hidden state, which is more difficult to learn.

We performed several experiments to support that hypothesis. In Fig. 6 we present two episodes from one of the training runs when the operation of the system seems to be evolving from one strategy to the other. At the early stages of the training (Fig. 6a) we can observe that the shift of attention with the circular convolution is active for both types of input subsequences, whereas the dynamic bookmark already learned how to follow attention for \( x \) and freeze for \( y \). As learning to shift attention is crucial for learning both storing and recall, model has to first master that. However, once achieved, it seems to switch to different operation mode. Obviously, learning
two modes is simpler for dynamic bookmark, as it possesses much simpler gating mechanism and cannot shift its attention. As the training progresses (Fig. 6b) the model finally learns to freeze its attention when processing \(y\) subsequences. Developing that strategy is harder, however it more efficiently utilizes memory.

4 Summary

We decided to study human working memory in order to improve artificial learning models. Our analysis allowed us to distill a novel attention control mechanism (bookmarks), which is quantitatively more robust and more data-efficient than previously published work (i.e. DNC, LSTM) based on the tasks tested in this study. Moreover, our model showed generalization to sequences two orders of magnitude longer than the training regime. We also studied the behavior of our model during learning and found out that for complex tasks, it has developed efficient strategies to control attention and use its memory resources.

In our future work we plan to use our working memory model as one component of more complex system and combine it with long-term memory and expand the capabilities to solve tasks that require both working and episodic memory.

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Appendix A  Details of operation of DWM

A.1  Read from the memory

Differentiable models with addressable external memory read from the memory $M$ with $N_M$ addresses through with soft attention using the formula (e.g. \cite{WCB15,GCCB16}):

$$r_t = M_t w_t,$$

where $r_t$ is a vector read from memory.

A.2  Memory update

There are several schemes for memory update, e.g. using least recently used access \cite{SBB16,GCCB16}. In DWM we decided to use the simple erase–add scheme derived from NTM \cite{GWD14}:

$$M_t = M_{t-1} \odot (E - w_t \otimes e_t) + w_t \otimes a_t$$

where $E$ is a matrix of all ones, $e_t$ and $a_t$ are vectors of content to be erased and added to memory, respectively. $e_t$, $a_t$ and $w_t$ are emitted by interface mechanisms directed by a controller network.
A.3 Controller

Controller processes inputs in order to produce outputs and interface parameters. In DWM we use a single-layer recurrent neural network controller:

$$h_t = \sigma(W_h[x_t, h_{t-1}, r_{t-1}])$$  \hspace{1cm} (6)

where $x_t$ denotes the current input and $h_{t-1}$ and $r_{t-1}$ are the hidden state and vector read from memory in the previous time step, respectively. To prevent the controller from acting as a separate working memory, the hidden state size is chosen to be smaller than that of a single input vector (in all of our experiments it was set to 5). The output logits, $y_t$ and interface vector $P_t$ are produced similarly as:

$$y_t = W_y[x_t, h_{t-1}, r_{t-1}]$$  \hspace{1cm} (7)

$$P_t = W_P[x_t, h_{t-1}, r_{t-1}]$$  \hspace{1cm} (8)

$W_h$, $W_y$ and $W_P$ are the only trainable parameters of our DWM model. The interface vector $P_t$ contains all of the parameters that control reading, writing, and the attention mechanisms. Denoting the unprocessed parameters from the interface with a hat, the full list of parameters is as follows:

- The write vector $a_t \in \mathbb{R}^{N_M}$
- The erase vector $e_t = \sigma(\hat{e}_t) \in [0,1]^{N_M}$
- The shift vector $s_t = \text{softmax}(\text{softplus}(\hat{s})) \in [0,1]^3$
- The bookmark update gates $g^i_t = \sigma(\hat{g}^i_t) \in [0,1]^{N_B-1}$
- The attention update gate $\delta^i_t = \text{softmax}(\hat{\delta}^i_t) \in [0,1]^{N_B+1}$
- The sharpening parameter $\gamma = 1 + \text{softplus}(\hat{\gamma}) \in [1,\infty]$ 

Appendix B Definition of input sequences

The input to every WM task is a time-indexed stream of items (Fig. 7). At a higher level, we view the input as a concatenation of various subsequences that represent different functional units of processing. For all "Simple" tasks, there is only one type of subsequence, and the output will be reproduced from the memory with or without manipulation. The "Complex" tasks may involve a secondary set of subsequences, which may or may not require immediate output as indicated in the "Forget" and "Operation Span" tasks.

In the actual encoding of the input, we use a constant-sized set of special items (called Command Markers, please refer to Fig. 7a) to both mark the beginning of a subsequence as well as indicate its functional type. Important note is that the system does not know a priori what kind of operation is associated with a given type of marker and must learn that from data. We ignored markers in Table 1 to keep the description simple.

Each subsequence is either real data or dummy (Fig. 7b). The dummy subsequences represent elements in the input processing where a suitable target of the same length needs to be output. In Table 1 we denote this by the $\omega$ symbol. Introduction of dummies enables delays in the input processing for capturing memory retention and other aspects of WM tasks.
Throughout the experiments, we fixed the input item size to be 8 bits supplemented by additional 2–4 control bits, depending on the task. In order to ensure that the sequences returned by our data generators for training, validation and testing are distinctive, we followed the conditions for length and number of subsequences presented in Table 3. For the memory based models, the memory size was chosen dynamically for each episode to equal the (common) length of the input sequences within the batch.

### Appendix C  Experiments

#### C.1 Number of parameters per model

During our experiments with LSTM model we have used stacked LSTM with 3 layers and 512 hidden units in each of them. DNC model used an single-layer LSTM controller with 20 hidden units. Our DWM model used RNN with sigmoid activation function and 5 hidden units. In Table 4 we report number of trainable parameters for each of our models.

#### C.2 Number of runs and converged models

For each model and task pair we performed 10 runs. In Table 5 we report number of models that converged, i.e. the validation loss went below the 1e-4 threshold.
### Table 4: Number of parameters of the used models

| Models | LSTM | DNC | DWM |
|--------|------|-----|-----|
| Number of model parameters | 5,279,752 | 4,792 | 1,066 |
| Learning Rate | $5 \cdot 10^{-3}$ | $5 \cdot 10^{-5}$ | $1 \cdot 10^{-2}$ |
| Optimizer | Adam | Adam | Adam |

### Table 5: Success criterion: validation loss < $10^{-4}$

| Task         | Number of Successful Runs |
|--------------|---------------------------|
|              | LSTM | DNC | DWM |
| Serial Recall| 0/10 | 10/10 | 10/10 |
| Reverse Recall| 0/10 | 0/10 | 7/10 |
| Rotate Shape | 0/10 | 9/10 | 10/10 |
| Reading Span | 0/10 | 0/10 | 8/10 |
| Forget       | 0/10 | 0/10 | 5/10 |
| Operation Span| 0/10 | 0/10 | 3/10 |
| Scratch Pad  | 0/10 | 1/10 | 9/10 |
| Ignore       | 0/10 | 0/10 | 8/10 |

**C.3 Convergence plots**

The following plots present convergence of the best models for each of the tasks, expressed in training loss.
Figure 8: Training Loss of the best models on the Serial Recall task

Figure 9: Training Loss of the best models on the Rotate Shape task
Problem: Reverse Recall

Training Loss DWM
Training Loss DNC
Training Loss LSTM

Figure 10: Training Loss of the best models on the Reverse Recall task

Problem: Operation Span

Training Loss DWM
Training Loss DNC
Training Loss LSTM

Figure 11: Training Loss of the best models on Operation Span task
Problem: Scratch Pad

Figure 12: Training Loss of the best models on Scratch Pad task

Problem: Reading Span

Figure 13: Training Loss of the best models on Reading Span task
Figure 14: Training Loss of the best models on Ignore task

Figure 15: Training Loss of the best models on Forget task