Noncommutative Theory in Light of Neutrino Oscillation

Shao-Xia Chen and Zhao-Yu Yang

Institute of High Energy Physics, Chinese Academy of Sciences
P.O.Box 918(4), 100039 Beijing, China

Abstract

Solar neutrino problem and atmospheric neutrino anomaly which are both long-standing issues studied intensively by physicists in the past several decades, are reckoned to be able to be solved simultaneously in the framework of the assumption of the neutrino oscillation. For the presence of the Lorentz invariance in the Standard Model, the massless neutrino can’t have flavor mixing and oscillation. However, we exploit the $q$-deformed noncommutative theory to derive a general modified dispersion relation, which implies some violation of the Lorentz invariance. Then it is found that the application of the $q$-deformed dispersion relation to the neutrino oscillation can provide a sound explanation for the current data from the reactor and long baseline experiments.

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The current knowledge concerning high energy neutrinos mainly comes from solar, atmospheric and long baseline neutrino experiments, which all present some inconsistency with the electro-weak Standard Model. In the Super-Kamiokande (S-K) experiment [1, 2], the results of directly detecting $\nu_e e$ scattering via the observation of the Cherenkov light in the large water Cherenkov detector, and the detection in the SNO [3]-[5] using a heavy water Cherenkov detector have significantly confirmed

1ruxanna@mail.ihep.ac.cn
2yangzhy@mail.ihep.ac.cn
that the number of solar neutrinos detected in these experiments is less than that predicted by the theories of the standard solar model. Some experiments detecting atmospheric neutrino, including several water Cerenkov experiments (Kamiokande [6], S-K and IMB [7]) together with the iron calorimeter Soudan 2 experiment [8], observing atmospheric neutrinos over energies varying from sub-GeV to tens of GeV have shown obvious discrepancy between the expected and measured ratio of the numbers of muon and electron events \( R_{\mu/e} \). As one of the new generation of long baseline neutrino oscillation searches, KamLAND [9] has published its first results recently, which give strong evidence for the disappearance of neutrinos travelling from a power reactor to a far detector. Meanwhile, the results of long baseline K2K experiment [10] imply a reduction of \( \nu_\mu \) flux together with a distortion of the energy spectrum. Thanks to all the experiments above, it is now convincing that the long-standing solar neutrino \( \nu_e \) deficit and the atmospheric neutrino \( \nu_\mu \) anomaly are both from neutrino oscillations between the flavor eigenstates.

Neutrino oscillations may arise, in general, when some terms are added to the neutrino sector of the Standard Model Lagrangian in the way generating the discordance of the eigenstates of the total Hamiltonian with the neutrino flavor eigenstates. In addition to the general scenario of neutrino oscillations [11]-[15] from the mismatch between the diagonalization of the charged lepton mass matrix and that of the neutrino mass matrix in an arbitrary flavor basis, there have appeared considerable papers [16]-[19] discussing the neutrino oscillation in the context of the premise that the oscillations may be the effects of the disagreement of the flavor eigenstates with the velocity eigenstates which are defined as the energy eigenstates of the massless neutrino and much has already been achieved. In the perspective of the violation of the Lorentz invariance several kinds of outcomes have appeared in the literature: \( \lambda \propto E^0 \) [20, 21], \( \lambda \propto E^{-1} \) [22, 23], \( \lambda \propto E^{-2} \) [16, 24, 25] and \( \lambda \propto E^{-3} \) [26], here \( \lambda \) is the neutrino oscillation length which will be defined in the subsequent discussion and \( E \) is the mean value of observed neutrino energy.

In this Letter, we will start from a modified dispersion relation induced by the
$q$-deformed noncommutative theory which also indicates the violation of the Lorentz invariance, to investigate the oscillations between the different flavor eigenstates of the massless neutrinos.

The $q$-deformed noncommutative theory, as one of the significant application of the quantum group [27]-[31] to the modern physics, plays a useful role in the study of the high energy physical phenomena. In the $q$-deformed noncommutative theory physical quantity $[x]$ is defined in terms of a parameter $q$ (taken to be real for simplicity):

$$[x] = \frac{q^x - q^{-x}}{q - q^{-1}}.$$  

The parameter $q$ measures the degree of the deviation of the considered system from the usual commutative case. Now that neutrino oscillation is a physical phenomenon related to the GeV scale energy, it is natural to make use of the $q$-deformed noncommutative theory to investigate it. The dispersion relation for a fermion in the $q$-deformed case is of the form [32, 33]:

$$E = \sqrt{m^2 + p^2} + \sqrt{m^2 + p^2 (4m^2 + 4p^2 - \omega^2)} \frac{(q - 1)^2}{24\omega^2}.$$

(1)

For the purpose of subsequent discussion, we derive the approximated form of Eq. (1) for massless neutrinos:

$$E = p \left( 1 + \frac{(q - 1)^2 p^2}{6\omega^2} \right) = p(1 + \alpha p^2),$$

(2)

here the notation $\alpha=\frac{(q - 1)^2}{6\omega^2}$ has been introduced and characterizes the Lorentz invariance violation in $q$-deformed noncommutative theory.

As argued in [20, 34], the Lorentz invariance violation would indicate that neutrinos may differ in their maximal attainable velocities, which disagree with the light velocity $c$ in vacua. In this scenario, neutrino oscillations can occur if the neutrino flavor eigenstates decided by the weak interactions are coherent superposition of the neutrino velocity eigenstates. In the case of two neutrino flavors, the flavor eigenstates can be expressed in terms of the velocity eigenstates:

$$\nu_\mu = \nu_1 \cos \theta + \nu_2 \sin \theta, \quad \nu_e = \nu_2 \cos \theta - \nu_1 \sin \theta.$$  

(3)
here $\theta$ is the mixing angle between different flavor eigenstates. The probability that neutrino oscillates from $\alpha$ flavor to $\beta$ flavor is:

$$P_{\alpha \rightarrow \beta} = \sin^2 2\theta \sin^2 \left(\frac{\pi L}{\lambda}\right),$$

where $L$ is the cosmological distance travelled by neutrinos between the emission and the detection, and the oscillation length $\lambda$ is defined as $\lambda = \frac{2\pi}{E_1 - E_2}$. The mixing angle $\theta$ only determines the amplitude of the oscillation but not affect the oscillation length. It should be noted that, although here we consider merely the simplified case of two neutrino flavors, however, in the case of more than two flavors, Eq. (4) will take a more complicated form but not contain more intrinsical physical contents, so we will concentrate only on the two flavors case.

During their evolution, neutrinos propagate as a linear superposition of their velocity eigenstates whose energy eigenvalues are $E_a$ ($a = 1, 2$). Exploiting Eq. (2), one obtains that the energy difference is

$$E_1 - E_2 = \delta \alpha E^3,$$

where $\delta \alpha = \alpha_1 - \alpha_2$. Combining the definition of the oscillation length $\lambda$, (5) and (4), one can acquire

$$\lambda = \frac{2\pi}{\delta \alpha E^3}, \quad P_{\alpha \rightarrow \beta} = \sin^2 2\theta \sin^2 \left(\frac{LE^3 \delta \alpha}{2}\right).$$

It is obvious that the oscillation length $\lambda$ is of the form $\lambda \propto E^{-3}$, which is identical with that appeared in the literature [26]. For a ulterior discussion, we can plot several curves in Fig.1 describing the dependence of the neutrino oscillation length $\lambda$ on $\delta \alpha$ for different energies. From the curves we find that only a tiny deviation of $q$ from 1 can unravel the existent data, which means that in the phenomenon of neutrino oscillation, at least on the current energy scale of detection, the degree of noncommutativity is very small. Of course, we must note that here we only take into account the neutrino oscillation from the disagreement of the neutrino flavor eigenstates with velocity eigenstates and omit the conventional oscillation mechanism in which neutrino oscillation comes from
the difference between neutrino mass eigenstates and flavor eigenstates. However, it is evident that in the presence of neutrino mass the requisite deviation of $q$ from 1 will be smaller than that for massless neutrinos. In general, if neutrinos have mass, they may have simultaneous velocity and mass oscillations. Then exploiting the argument in [20, 34], one can further discuss the situation in the presence of two oscillations.

**Experimental Data**

| EXP. | STATUS       | $\langle E \rangle$(GeV) | L(km) |
|------|--------------|--------------------------|-------|
| CHORUS | closed 1997  | 26                       | 0.85  |
| NOMAD  | closed 1999  | 24                       | 0.94  |
| SK     | operating    | 1.3                      | $10^{-4}$ |
| K2K    | operating    | 1.3                      | 250   |
| SNO    | operating    | 0.008                    | $10^{8}$ |
| MINOS  | starting 2003| 15                       | 730   |
| CNGS   | starting 2005| 17                       | 732   |

*Table 1.* Shown for each experiment are its operation status, mean value of observed neutrino energy and typical neutrino flight distance $L$. The table is quoted from the paper [16].

To summarize, we exhibit in this Letter the relation between the neutrino oscillation length $\lambda$ and the Lorentz invariance violation parameter $\delta \alpha$, which can be verified by the existent and future neutrino oscillation experiments. Especially, when the data from long baseline experiments and neutrino factories come to be available, Lorentz invariance violation in neutrino oscillation can be probed to new and significant levels. Once the oscillation length are measured, $\delta \alpha$ will be derived as additional contribution apart from the observations of ultrahigh energy cosmic rays [33] to determine the $q$-deformed noncommutativity parameter.

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Fig 1. Oscillation length for different energies
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