One-Loop Renormalization of the Electroweak Sector with Lorentz Violation

Don Colladay and Patrick McDonald

New College of Florida
Sarasota, FL, 34243, U.S.A.

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The one-loop renormalizability of the electroweak sector of the Standard Model Extension with Lorentz violation is studied. Functional determinants are used to calculate the one-loop contributions of the higgs, gauge bosons and fermions to the one-loop effective action. The results are consistent with multiplicative renormalization of the SME coupling constants. Conventional Electroweak symmetry breaking is effectively unaltered relative to the standard case as the renormalized SME parameters are sufficient to absorb all infinite contributions.

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I. INTRODUCTION

The Standard Model Extension (SME) provides a framework within the context of conventional quantum field theory that allows for general Lorentz-breaking effects. The construction generates all possible couplings of standard model fields to constant background fields that serve as a source for spontaneous Lorentz breaking.

In a recent series of articles, various one-loop renormalization calculations have been performed. All of these have indicated that multiplicative renormalization still works, even when Lorentz symmetry is not intact. Explicit verification in the case of Lorentz-breaking theories is important since conventional techniques frequently make use of Lorentz invariance arguments to establish renormalizability. This paper extends these previously obtained results to the electroweak sector of the SME. More specifically, the functional determinant formalism is used to calculate radiative corrections to one-loop in the Higgs, fermion, and gauge boson sectors of the electroweak model. The renormalization is incorporated using multiplicative factors as in the conventional case. This renormalization procedure is formally carried out before SU(2) × U(1) breaking so that the symmetries can be utilized fully.

The investigation of the renormalizability properties of the SME was first started in [1, 2] where the authors studied one-loop radiative corrections for QED with Lorentz violation. In this prior work, the one-loop renormalizability of general Lorentz and CPT violating QED was established. The manuscript [1] includes an analysis of the explicit one-loop structure of Lorentz-violating QED and the resulting running of the couplings. The authors established that conventional multiplicative renormalization succeeds and they find that the beta functions indicate a variety of running behaviors, all controlled by the running of the charge. Portions of this analysis have been extended to allow for a curved-space background [2], while other analysis involved finite, but undetermined radiative corrections due to CPT violation [3, 4, 5, 6, 7, 8, 9, 10]. In addition, recent papers have addressed anomalies in the presence of Lorentz-violating terms [11, 12]. The main results indicate that the anomaly is present even in the absence of Lorentz symmetry and the fundamental nature of the anomaly is essentially the same as in the conventional case.

The Lorentz violating QED results of [1] were extended to non-abelian gauge theories including QCD in [13, 14] where the authors established that Yang-Mills theory is renormalizable at one-loop, provided the gauge group remains unbroken. The electroweak sector presents additional challenges, mainly due to the SU(2) × U(1) symmetry breaking and the parity violating fermion sectors. In addition, the Higgs participates as a scalar field that was not considered in previous work on the subject. The present paper focuses on the functional determinants that integrate out the Higgs and fermion sectors as the gauge sector has already been handled in sufficient detail in [14].

The current work should be viewed as part of an extensive, systematic investigation of Lorentz violation and its possible implications for Planck-scale physics [15, 16, 17, 18, 19, 20, 21, 22, 23]. Recent work involving Lorentz violation and cosmic microwave background fluctuations [24] suggest that the SME might play a useful role in cosmology. In addition to the above, the SME formalism has been extended to include gravity [25, 26, 27], where it has been suggested that Lorentz violation provides an alternative means of generating General Relativity [28, 29].

Some other work relevant to the current paper includes a study of deformed instantons in pure Yang-Mills theory with Lorentz Violation [30, 31], an analysis of the Casimir effect in the presence of Lorentz violation [32], an analysis of gauge invariant of Lorentz-violating QED at higher-orders [33], and possible effects due to nonpolynomial interactions [34]. Some investigations into possible Lorentz-violation induced from the ghost sector of scalar QED have also been performed [35]. Higher powers of spatial derivatives that violate Lorentz invariance have been used to argue improved behavior of renormalization for scalar and gauge theories [36, 37] at higher-order.
In addition, functional determinants have been used to compute finite corrections to CPT-violating gauge terms arising from fermion violation \[38\].

II. NOTATION AND CONVENTIONS

In this paper we adopt the conventions used in \[38\] to define standard model fields and Lorentz-violating couplings. The electroweak sector contains left- and right-handed lepton and quark multiplets denoted as

\[
L_A = \begin{pmatrix} \nu_A \\ l_A \end{pmatrix}_L, \quad R_A = (l_A)_R,
\]

\[
Q_A = \begin{pmatrix} u_A \\ d_A \end{pmatrix}_L, \quad U_A = (u_A)_R, \quad D_A = (d_A)_R,
\]

where

\[
\psi_L \equiv \frac{1}{2}(1 - \gamma_5) \psi, \quad \psi_R \equiv \frac{1}{2}(1 + \gamma_5) \psi,
\]

as usual, and where \(A = 1, 2, 3\) labels the flavor: \(l_A \equiv (e, \mu, \tau)\), \(\nu_A \equiv (\nu_e, \nu_\mu, \nu_\tau)\), \(u_A \equiv (u, c, t)\), \(d_A \equiv (d, s, b)\).

We denote the Higgs doublet by \(\phi\).

The lagrangian terms in the usual SU(2) × U(1) electroweak sector of the minimal standard model are

\[
\mathcal{L}_{\text{lepton}} = \frac{i}{2} i \bar{L}_A \gamma^\mu \overleftrightarrow{D}_\mu L_A + \frac{i}{2} i \bar{R}_A \gamma^\mu \overleftrightarrow{D}_\mu R_A + \lambda \frac{1}{3!}(\phi^\dagger \phi)^3 \equiv \mathcal{L}_{\text{Yukawa}}, \tag{3}
\]

\[
\mathcal{L}_{\text{quark}} = \frac{i}{2} i \overleftrightarrow{Q}_A \gamma^\mu \overleftrightarrow{D}_\mu Q_A + \frac{i}{2} i \overleftrightarrow{U}_A \gamma^\mu \overleftrightarrow{D}_\mu U_A + \lambda \frac{1}{3!}(\phi^\dagger \phi)^3 \equiv \mathcal{L}_{\text{Yukawa}}, \tag{4}
\]

\[
\mathcal{L}_{\text{Higgs}} = (D_f^\mu \phi)^\dagger D^\mu_f \phi + \mu^2 \phi^\dagger \phi - \frac{\lambda}{3!}(\phi^\dagger \phi)^2 \equiv \mathcal{L}_{\text{Yukawa}} + \mathcal{L}_{\text{Higgs}}, \tag{5}
\]

where \(\mathcal{L}_{\text{Higgs}}\) is the Higgs sector Lagrangian with \(SU(2)_L\) doublet \(\phi\) is written as

\[
\mathcal{L}_{\text{Higgs}} = (D_f^\mu \phi)^\dagger D^\mu_f \phi + \mu^2 \phi^\dagger \phi - \frac{\lambda}{3!}(\phi^\dagger \phi)^2. \tag{6}
\]

The Yukawa coupling \(G_L\) for the leptons may be diagonalized in the usual way before any \(SU(2) \times U(1)\) symmetry breaking occurs. The quark couplings \(G_U\) and \(G_D\) are more complicated due to the existence of right-handed up quarks. It is only possible to simultaneously diagonalize these couplings in a specific gauge, so they are left arbitrary at this point. Note that massive neutrinos can be incorporated easily by making the structure of the neutrino sector match that of the quark sector. This is not done here so that the two sectors may be contrasted more effectively.

The corresponding Lorentz-violating terms in the SME are given for each sector as they appear in the following sections of the paper.

III. HIGGS SECTOR CORRECTIONS

In this section we introduce the Lorentz-violating terms in the SME involving Higgs couplings. Recall, the one-loop effective action for a field theory can be written as a functional integral over fields \(\Psi\):

\[
\exp i\Gamma[\Psi] = \int \mathcal{D}\Psi e^{i\int d^4x \mathcal{L}[\Psi]}, \tag{8}
\]

The effective action is constructed by writing the underlying fields as the sum of classical background fields and fluctuating quantum fields. The effective action is given by a classical term perturbed by terms quadratic in the fluctuation. The quadratic term gives rise to a Gaussian integral, which in turn can be described by a functional determinant \[41\]. Using \(\mathcal{L}_{\text{cl.c.t.}} = \mathcal{L}_0 + \mathcal{L}_{\text{c.t.}}\) for the classical Lagrangian as a function of the background field where \(\mathcal{L}_{\text{c.t.}}\) is the counterterm Lagrangian, the expression generates terms of the form

\[
\exp i\Gamma[\Psi] = e^{i\int d^4x \mathcal{L}_{\text{c.t.}} \det(\Delta)^n}, \tag{9}
\]

where \(\Delta\) are operators which are given explicitly below, and \(n\) is an exponent that depends on the field type. To compute the above determinants, dimensional regularization is used. Each determinant is treated separately, beginning with the pure Yang-Mills gauge field contribution. The calculation is performed to first order in Lorentz violating parameters. As this is the case, the computations of the various terms decouple and the CPT-even and CPT-odd cases can be treated independently.

The conventional Higgs sector lagrangian with \(SU(2)_L\) doublet \(\phi\) is written as

\[
\mathcal{L}_{\text{Higgs}} = (D_f^\mu \phi)^\dagger D^\mu_f \phi + \mu^2 \phi^\dagger \phi - \frac{\lambda}{3!}(\phi^\dagger \phi)^2. \tag{10}
\]

The covariant derivative acting on \(\phi\) is

\[
D_\mu \phi = \left(\partial_\mu - i f^{\alpha \beta \gamma}_2 W_\mu^\alpha \sigma^\beta - i g^f_2 B_\mu\right) \phi. \tag{11}
\]

For notational simplicity, it is convenient to introduce the quantity

\[
A_\mu = f^{\alpha \beta \gamma}_2 W_\mu^\alpha \sigma^\beta + \frac{g^f_2}{2} B_\mu. \tag{12}
\]

The Lorentz-violating contributions split into CPT even and CPT odd terms

\[
\mathcal{L}^{\text{CPT-even}}_{\text{Higgs}} = \frac{1}{2} (k_\phi)^{\mu \nu} (D_\mu \phi)^\dagger D_\nu \phi + \text{h.c.} - \frac{1}{2} (k_\phi^{1 \mu} \phi^\dagger B_{\mu \nu} - \frac{1}{4} (k_\phi W)^{\mu \nu} \phi^\dagger W^{\mu \nu} \phi), \tag{13}
\]

and

\[
\mathcal{L}^{\text{CPT-odd}}_{\text{Higgs}} = i (k_\phi)^{\mu \phi} D_\mu \phi + \text{h.c.} \tag{14}
\]
We will write the Higgs fields as the sum of a classical background field (denoted with an underline) and a fluctuating quantum field:

$$\phi \rightarrow \phi + \phi'. \quad (15)$$

First, the one-loop contribution of the term $k_{\phi\phi}$ is calculated. The quadratic contribution to the lagrangian is

$$L_{\text{higgs}} = \phi^\dagger \left[ -D^2 + \mu^2 - \frac{1}{2} \left( (k_{\phi\phi})^{\mu\nu} D_\mu D_\nu + \text{h.c.} \right) \right] \phi \quad (16)$$

Integration over the Higgs fields yields logdet of the operator in the brackets. To facilitate calculation, $k_{\phi\phi} = k_{\phi\phi}^R + i k_{\phi\phi}^I$ is split into a symmetric real part and an antisymmetric imaginary piece. The kinetic, field-independent piece of the operator

$$P = -(\eta^{\mu\nu} + k_{\phi\phi}^{\mu\nu}) \partial_\mu \partial_\nu + \mu^2, \quad (17)$$

is factored out of the expression. The inverse of this operator is written using the Fourier expansion

$$P^{-1} = \int \frac{d^4p}{(2\pi)^4} e^{-ip(x-y)} \frac{1}{(\eta^{\mu\nu} + k_{\phi\phi}^{\mu\nu}) p_\mu p_\nu + \mu^2}. \quad (18)$$

Note that $\mu^2$ appears with opposite sign to the usual scalar field propagator mass term. This occurs because the renormalization is being performed before spontaneous breaking of $SU(2) \times U(1)$ is implemented. The operator expansion

$$\frac{1}{A + B} \simeq \frac{1}{A} - \frac{1}{A} \frac{B}{A} + \cdots, \quad (19)$$

is then used to expand the inverse kinetic operator for small $k_{\phi\phi}$. The result of a calculation involving the real component yields the Lorentz-violating contribution

$$\log \text{det} \left[ -D^2 - \frac{1}{2} (k_{\phi\phi}^R)^{\mu\nu} D_\mu D_\nu \right] = \frac{1}{6} \frac{i}{(4\pi)^2} \Gamma(2 - \frac{d}{2}) (k_{\phi\phi}^R)^{\mu\nu} Q^{\mu\nu} \quad (20)$$

where the gauge invariant operator $Q^{\mu\nu}$ is defined as

$$Q^{\mu\nu} = tr \int \frac{d^4k}{(2\pi)^4} (k^\mu k^\nu A^2 - 2k^\mu A^\nu k^A + k^2 A^\mu A^\nu). \quad (21)$$

Note that this term contributes radiative corrections to the anti-self-dual components of the Lorentz-violating gauge terms

$$\mathcal{L}_{\text{gauge}}^{\text{CPT-even}} = -\frac{1}{2} Tr (k_W^{\mu\nu} \omega^{\alpha\beta} W_\mu W_\alpha W_\beta) = -\frac{1}{4} (k_B^{\mu\nu} \omega^{\alpha\beta} B_\mu B_\alpha B_\beta. \quad (22)$$

A calculation using the antisymmetric component of $k_{\phi\phi}$ yields

$$\log \text{det} \left[ -D^2 - \frac{1}{2} (k_{\phi\phi}^I)^{\mu\nu} D_\mu D_\nu \right] = \frac{g'}{4} \int \frac{d^4p}{(2\pi)^4(p^2 + \mu^2)} (k_{\phi\phi}^I)^{\mu\nu} B^{\mu\nu} \quad (23)$$

a quadratically divergent linear field instability. This suggests that $k_{\phi\phi}^I$ should be set to zero in any sensible theory, although it is possible to arrange a cancellation as described next.

The remaining CPT-even Higgs corrections are the $k_{\phi W}$ and $k_{\phi B}$ couplings. The contribution of $k_{\phi W}$ is zero due to the Lie Algebra trace, but the term $k_{\phi B}$ contributes

$$\log \text{det} \left[ -D^2 - \frac{1}{2} (k_{\phi B})^{\mu\nu} B_{\mu\nu} \right] = \frac{1}{2} \int \frac{d^4p}{(2\pi)^4(p^2 + \mu^2)} (k_{\phi B})^{\mu\nu} B_{\mu\nu} \quad (24)$$

again introducing a linear instability. If the bare coefficients are chosen appropriately, it is possible to obtain a cancellation between $k_{\phi W}$ and $k_{\phi B} = 0$ at one loop. This choice simply corresponds to choosing the $k_{\phi B}$ term to cancel the contribution of $k_{\phi B}$, reducing the CPT-odd Higgs sector term.

The remaining CPT-odd Higgs sector term is the vector field $k_{\phi}$. It’s contribution to the divergent piece of the determinant is in fact zero as the various terms cancel out in the equations. In fact, the field redefinition

$$\phi \rightarrow e^{i k_{\phi} x} \phi, \quad (25)$$

of the Higgs field eliminates this term entirely from the theory to all orders, contributing only to the mass parameter for the Higgs at second order in $k_{\phi}$. In addition, this redefinition removes the nontrivial vacuum expectation value found for the $Z$ boson in [40]. This implies that the electroweak symmetry breaking is unaltered from the conventional case when the fields are appropriately defined.

### IV. FERMION SECTOR CORRECTIONS

The left- and right-handed fermion fields are treated differently with respect to the covariant derivative, therefore, they must be separated in the construction of the determinant. To accomplish this in the leptonic sector, the fermion fields in each generation are arranged into a six-component multiplet of the form

$$L_A = \left( \begin{array}{c} \nu_L \\ l_L \\ l_R \end{array} \right). \quad (26)$$

The Lorentz-invariant portion of the Lagrangian can be written in the form $\mathcal{L} = \overline{\mathcal{M}} L \mathcal{L}$ using the cross-generational matrix

$$M_{AB} = \left( \begin{array}{ccc} \frac{g}{\sqrt{2}} \overline{W}^3 - \frac{g}{\sqrt{2}} \overline{B} \\ \frac{g}{\sqrt{2}} \overline{W}^+ - \frac{g}{\sqrt{2}} \overline{B} \end{array} \right) \left( \begin{array}{c} (G_L)^{AB\phi} \\ (G_L)^{AB\phi^*} \end{array} \right) \quad (27)$$
Where the Feynman slash notation indicates contraction with one of the $2 \times 2$ representations $\sigma^\mu = (Id, \vec{\sigma})$, or $\bar{\sigma}^\mu = (Id, -\vec{\sigma})$ for the right- and left-handed fields respectively:

$$\beta = \sigma^\mu A_\mu, \quad \bar{\beta} = \bar{\sigma}^\mu A_\mu. \quad (28)$$

The Lorentz-violating terms can be easily included into the above matrix. In the $4 \times 4$ Chiral representation for $\gamma^\mu$, they take the form

$$\mathcal{L}^{\text{CPT-\text{even}}}^{\text{lepton}} = \frac{1}{2} i (cL)_{\mu\nu AB} T_A \gamma^\mu \overleftrightarrow{D^\nu} L_B + \frac{1}{2} i (cR)_{\mu\nu AB} \overline{R} A \gamma^\mu \overleftrightarrow{D^\nu} R_B \quad , \quad (29)$$

$$\mathcal{L}^{\text{CPT-\text{odd}}}^{\text{lepton}} = -(aL)_{\mu AB} \overline{L} \gamma^\mu L_B - (aR)_{\mu AB} \overline{R} A \gamma^\mu R_B \quad , \quad (30)$$

and the Yukawa terms are

$$\mathcal{L}^{\text{Yukawa}}^{\text{CPT-\text{even}}} = -\frac{1}{2} \left[ (H_L)_{\mu - \nu AB} \overline{L} \phi \sigma^{\mu \nu} R_B + \text{h.c.} \right] \quad . \quad (31)$$

The kinetic, field-independent portion of the operator can be written as

$$P_{AB} = \begin{pmatrix} (P_L)_{AB} & 0 & 0 \\ 0 & (P_L)_{AB} & 0 \\ 0 & 0 & (P_R)_{AB} \end{pmatrix} \quad , \quad (32)$$

where $(P_L)_{AB} = i(\partial^\delta A + (cL)_{\mu\nu AB} \sigma^\mu \delta^\nu)$, and $(P_R)_{AB} = i(\partial^\delta A + (cR)_{\mu\nu AB} \sigma^\mu \delta^\nu)$. The contribution of the $c$-terms to the logdet in this case yields

$$T_{\text{lep}} = T_{\text{lep}}^\text{B} + T_W$$

where

$$T_{\text{lep}}^\text{B} = -\frac{1}{3} \frac{i}{(4\pi)^2} g^2 \Gamma(2 - \frac{d}{2}) Tr(c_L + 2c_R)_{\mu\nu} Q_B^{\mu\nu} \quad , \quad (33)$$

and

$$T_W = -\frac{1}{3} \frac{i}{(4\pi)^2} g^2 \Gamma(2 - \frac{d}{2}) Tr(c_L)_{\mu\nu} Q_W^{\mu\nu} \quad , \quad (34)$$

where $Q_B^{\mu\nu}$ and $Q_W^{\mu\nu}$ are defined as in eq.(21) with $A$ replaced with $B$ or $W$ respectively. The trace includes a summation over generational indices that have been suppressed for notational simplicity. This contribution can be absorbed into the Lorentz-violating gauge $k_B$ and $k_W$ terms given in eq.(22). The Lorentz-violating Yukawa terms yield traces over sigma matrices that vanish, so they don’t contribute to one-loop radiative corrections. In addition, the CPT-odd leptonic terms can be eliminated using field redefinitions, so they don’t contribute either.

The quark sector may be handled similarly by defining the multiplet

$$L_A = \begin{pmatrix} u_L \\ d_L \\ u_R \\ d_R \end{pmatrix} \quad . \quad (35)$$

The calculation proceeds in exactly the same manner as in the leptonic sector with $M_{AB}$ replaced by the corresponding $4 \times 4$ matrix. As in the leptonic sector, the only contribution arises from the $c$ couplings

$$\mathcal{L}^{\text{CPT-\text{even}}}^{\text{quark}} = \frac{1}{2} i (cQ)_{\mu\nu AB} \overline{T} A \gamma^\mu \overleftrightarrow{D^\nu} Q_B + \frac{1}{2} i (cU)_{\mu\nu AB} \overline{U} A \gamma^\mu \overleftrightarrow{D^\nu} U_B + \frac{1}{2} i (cD)_{\mu\nu AB} \overline{D} A \gamma^\mu \overleftrightarrow{D^\nu} D_B \quad , \quad (36)$$

with the result for logdet of $T_q = T_q^\text{B} + T_W$ with $T_W$ the same as in the leptonic calculation of eq.(34) and

$$T_q^\text{B} = -\frac{1}{2} \frac{i}{(4\pi)^2} g^2 \Gamma(2 - \frac{d}{2}) Tr(cQ + 8cU + 2cD)_{\mu\nu} Q_B^{\mu\nu} \quad . \quad (37)$$

The difference in coupling factors are due to the standard hypercharge assignments of the quark fields relative to the leptonic fields.

V. ELECTROWEAK SYMMETRY BREAKING

Once the coupling constants have been renormalized, it is a simple matter to incorporate the electroweak symmetry breaking. Specifically, a term can be added to the Lagrangian of the form

$$\mathcal{L}_{SB} = s \cdot \phi(x) \quad , \quad (38)$$

where $s$ is a small, external source field that induces the breaking. The vacuum expectation value for the Higgs field is therefore fixed to point along the direction of $s$. The inverse of the renormalization factor used to rescale $\phi$ can be used to renormalize the external source to keep the symmetry breaking term finite. This procedure is the same as the procedure used to incorporate renormalization in the linear sigma model with symmetry breaking [42].

Once the renormalization factors are included in the standard SME parameters and they are rendered finite, it is a simple matter to perform an extremization of the static electroweak potential. This calculation has already been performed in [40] and will not be reproduced here. One additional feature worth mentioning is that the diagonalization of the mass matrices for the photon and $Z$ introduces quadratic photon-Z couplings proportional to the difference between the CPT-even gauge couplings $k_W$ and $k_B$. This may lead to interesting novel experimental effects and may place very stringent bounds on the difference between these two parameters.

VI. SUMMARY

Functional determinant techniques can be easily adapted to the electroweak sector by performing the renormalization prior to $SU(2) \times U(1)$ breaking. It is
found that conventional multiplicative renormalization factors suffice to renormalize the theory to one loop as with the other sectors of the SME. Electroweak breaking is then essentially the same as in the usual standard model, provided the proper field redefinition is implemented on the phase of the Higgs field.

This paper (together with references [13, 14]) exhausts the uses of functional determinants in computing one-loop renormalization effects in the SME. This serves as an important first step towards future work that will hopefully involve demonstrating full renormalization to all orders in perturbation theory for the SME. The one-loop renormalizability of the SME is promising as it is used effectively in the standard case to argue all-orders loop renormalizability of the SME is that parameters that are not bounded very tightly in the tree-level theory (such as tau couplings...) contribute to the one-loop effective action suppressed by the square of the appropriate coupling. This indicates that very stringent photon bounds (for example) may be used to estimate reasonable levels for other coupling constants in the SME that are so far unbound by direct experiment.

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