Aggregation of landslide occurrence probability in spatially variable soil

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ABSTRACT

This paper presents a probabilistic slope stability analysis approach that formulates the slope failure event as a series of mutually exclusive and collectively exhaustive events using conditional probability and utilizes Monte Carlo simulation (MCS) to determine the occurrence probability for each of the mutually exclusive and collectively exhaustive events in a progressive manner. The probabilities of each event are aggregated to represent the overall slope failure probability $p_f$. The $p_f$ values obtained from the proposed approach are shown to agree well with those $p_f$ values that have been obtained by searching a large number of potential slip surfaces for the minimum factor of safety in each MCS sample. The computational efficiency, however, is shown to improve by, at least, an order of magnitude. In addition, the approach identifies the key failure modes (i.e., those slip surfaces that have significant effects on $p_f$) which can be readily used in the mitigation of landslide risk.

Keywords: probabilistic slope stability analysis, risk aggregation, conditional probability, Monte Carlo simulation

1 INTRODUCTION

It is well-recognized in slope stability analysis that a slope may fail along an unlimited number of potential slip surfaces, although evaluating failure probability, $p_f$, along all potential slip surfaces is considered a mathematically formidable task (e.g., El-Ramly et al. 2002, Wang et al. 2011). The $p_f$ is, therefore, frequently assessed only along one or a few pre-selected slip surfaces without proper consideration of the slip surface variation (e.g., Tang et al. 1976; Low et al. 1998; Hassan and Wolff 1999; Xue and Gavin 2007). There are a few exceptions. For example, Wang et al. (2011) used MCS to study the effect of the slip surface variation on $p_f$ and explicitly searching a large number of potential slip surfaces for the critical slip surface in each MCS sample. When the spatial variability of soil properties is considered, the critical slip surface was found to vary spatially, and such variation was shown to affect substantially the $p_f$. In their approach, however, the searching of the critical slip surfaces requires considerable time in each MCS sample. Because at least tens of thousands of MCS samples are needed in a MCS run, their approach is inevitably time consuming and computationally expensive. Therefore, an alternative approach is needed that is able to properly account for the slip surface variation with significantly reduced computational efforts. This paper presents a novel approach that properly selects a limited number of slip surfaces, evaluates failure probability along each of the selected slip surfaces, and aggregates these failure probabilities to represent the overall $p_f$ of the slope.

2 ESTIMATE OF SLOPE FAILAURE PROBABILITY USING RISK AGGREGATION AND CONDITIONAL PROBABILITY

In slope stability analysis, slope failure is defined as sliding along any potential slip surface or the factor of safety, $FS$, along any potential slip surface is less than unity. Let $N$ denote the number of slip surfaces considered in the slope stability analysis, $S_i$ denote the ith slip surface, $i = 1, 2, ..., N$, and $FS_0$ denote the $FS$ along the slip surface $S_i$. Then, the event of slope failure can be formulated as a series of mutually exclusive and collectively exhaustive events using conditional probability.

2.1 Conditional probability representation

Using conditional probability, the event of slope failure can be represented by a union of a two mutually exclusive and collectively exhaustive events as (Li et al. 2014):

$$\text{Failure} = E(S_i) \cup E(\text{Failure}|\overline{S_i})$$

where $E(S_i)$ denotes an event of slope failure along the slip surface $S_i$, $\overline{S_i}$ stands for no slope failure along $S_i$, and $E(\text{Failure}|\overline{S_i})$ denotes an event that, given no slope
failure along $S_i$, failure occurs along any of the remaining slip surfaces (i.e., any slip surface except $S_i$). Subsequently, $E(\text{Failure}|S_i)$ can be further represented by a union two mutually exclusive and collectively exhaustive events and expressed in a similar manner as:

\[
E(\text{Failure}|S_i) = E(S_i|S_i) U E(S_i|\overline{S_i})
\]

(2)

Similar formulations are extended for all $N$ slip surfaces, and the event of slope failure is expressed as:

\[
\text{Failure} = E(S_i) U E(S_j|S_i) U E(S_k|S_i) U \ldots U E(S_N|S_i)
\]

(3)

Because the events on the right hand side of Eq. 3 are mutually exclusive and collectively exhaustive, the slope failure probability, $P_f$, can be expressed as:

\[
P_f = P(E(S_i)) + P(E(S_j|S_i)) + P(E(S_k|S_i)) + \ldots + P(E(S_N|S_i))
\]

(4)

where $P(\text{Event})$ denotes probability of the event.

The $P_f$ in Eq. 4 can be evaluated conveniently using MCS. The MCS samples are grouped into corresponding sample subsets in a progressive manner, and $P(\text{Event})$ in Eq. 4 are evaluated one by one. Consider, for example, a MCS run with $n$ number of samples. These $n$ sets of input values are used to calculate the $FS$ along $S_i$, leading to a collection of failure samples for slip surface $S_i$. Subsequently, these failure samples for $S_i$ are excluded from the original MCS samples, and the remaining samples are used again to calculate the $FS$ along $S_i$, leading to a collection of additional failure samples for slip surface $S_i$. Then, these additional failure samples for $S_i$ are excluded from the original MCS samples, and the remaining samples are used again to calculate the $FS$ along $S_i$, leading to a collection of additional failure samples for slip surface $S_i$. This process is performed repeatedly until all N slip surfaces are evaluated.

Recent studies (e.g., Zhang et al. 2011, Li et al. 2013&2014) have shown that, although there are a large number of potential slip surfaces, the $FS$ for many of these slip surfaces are somehow correlated. When the $FS$s for a group of slip surfaces are highly correlated (i.e., the correlation coefficient $\rho$ is larger than a threshold value $\rho_0$), the limit state functions for this group of slip surfaces are nearly parallel in the uncorrelated reduced uncertainty space (Ang and Tang 1984; Zhang et al. 2011), and the failure domain of the slip surface with the minimum reliability index, $\beta$, includes the failure domains of the other slip surfaces. Therefore, the overall failure probability for the whole group of slip surfaces can be properly represented by the failure probability along the slip surface with the minimum $\beta$ value. This allows the number of slip surfaces evaluated to be substantially reduced to a limited number of properly selected slip surfaces. Li et al. (2013&2014) have shown that the number of slip surfaces selected is generally less than 10 to 20.

2.2 Risk aggregation approach

Slope failure probability problem can be considered as a system failure probability problem (e.g., Oka and Wu 1990; Chowdhury and Xu 1995; Low et al. 2011; Zhang et al. 2011, Li et al. 2013&2014). A slope may fail along a large number of potential slip surfaces, and each potential slip surface can be treated as a failure mode of the system. The overall failure probability $p_f$ of the slope is the system failure probability of interest, and it can be considered as an aggregation of the probabilities of failure along a limited number of properly selected slip surfaces (see Subsection 2.1).

The risk aggregation approach is performed together with MCS and uses the conditional samples to develop failure sample subsets in a progressive manner (Li et al. 2014). In MCS, $n$ sets of MCS random samples are firstly generated. Then, a large number $N$ of potential slip surfaces is randomly generated to uniformly cover the entire failure domain of the slope, and they collectively form a potential slip surface library (PSSL). Reliability indices of all these $N$ slip surfaces are then calculated. The slip surface with the minimum $\beta$ value in PSSL is selected as a slip surface (referred to as a representative slip surface) for calculation of $p_f$ in the risk aggregation (Li et al. 2014). Subsequently, the $n$ number of MCS samples are used to calculate the $FS$ along this first representative slip surface, leading to identification of $n_1$ number of failure samples along the first representative slip surface and a failure sample subset $U_1$ and $P(E(S_i))$ in Eq.4. Then, the correlation coefficients between the FS of this representative slip surface and that of the remaining slip surfaces in the PSSL are calculated. If a correlation coefficient is larger than a threshold value $\rho_0$, the corresponding slip surface is excluded from the PSSL. This generates a new PSSL with a reduced number of slip surfaces, and the slip surface with the minimum $\beta$ value in this new PSSL is selected as another representative slip surface. The remaining MCS sample (i.e., except those that have been included in the previous failure sample subset $U_1$) are used to calculate the $FS$ along this new representative slip surface, leading to identification of $n_2$ number of additional failure samples along this new representative slip surface and $P(E(S_i|\overline{S_i}))$ in Eq. 4. A new failure sample subset $U_2$ is defined that contains totally $n_1+n_2$ failure samples (i.e., $U_2$ includes $U_1$). Subsequently, the correlation coefficients between the $FS$ of this new representative slip surface and that of the remaining slip surfaces in the PSSL are calculated. If a correlation coefficient is larger than $\rho_0$, the corresponding slip surface is excluded from the PSSL. The process above is repeated until the PSSL contains no slip surface. Finally, the slope failure probability $p_f$
is calculated when all terms in the right hand side of Eq. 4 are determined from MCS.

3 ILLUSTRATIVE EXAMPLE

The risk aggregation approach is illustrated using a cohesive soil slope example, as shown in Fig. 1. The cohesive soil slope has a height $H = 10m$ and slope angle of $45^{\circ}$, corresponding to an inclination ratio of 1:1. The cohesive soil is underlain by a firm stratum at 12m below top of the slope. Short-term shear strength of the cohesive soil is characterized by undrained shear strength $S_u$, and the saturated unit weight of soil is $\gamma_{sat}$.

Short-term stability of the slope is assessed using Ordinary Method of Slices under undrained condition (e.g., Duncan and Wright 2005). The FS for a given slip surface is defined as the ratio of resisting moment to the overturning moment, and the slip surface is assumed to be a circular arc centered at coordinate $(x_c, y_c)$ and with radius $r$. The soil mass above the slip surface is divided into a number of vertical slices, each of which has a weight $W_i$, circular slip segment length $\Delta l_i$, undrained shear strength $S_u$ along the slip segment, and an angle $\alpha_l$ between the base of the slice and the horizontal. The FS for a given slip surface is then given by

$$FS = \frac{\sum S_u \Delta l_i}{\sum W_i \sin \alpha_l}$$

Eq. 5 is used repeatedly for FS along different circular slip surfaces with various combinations of $(x_c, y_c)$ and $r$.

Table 1 summarizes the values and probability distributions of input variables in this study. The $\gamma_{sat}$ is taken as a deterministic value of 20 kN/m$^3$, and the $S_u$ is modeled by a one-dimensional random field spatially varying along the vertical direction. The value of $S_u$ at the same depth is assumed to be fully correlated. The spatial variability in vertical direction is modeled by a homogeneous lognormal random field with an exponentially decaying correlation structure and a scale of fluctuation $\lambda$. In this example, the $\lambda$ value varies from 1m to 1000m for consideration of different spatial correlations. As shown in Fig. 1, the 12-m-thick cohesive soil layer is divided into 24 0.5-m-thick sub-layers, and $S_u$ at each sub-layer is represented by an entry in a $S_u$ vector with a length of 24. As shown in Table 1, the mean and standard deviation of $S_u$ are approximately equal to 40kPa and 8kPa (i.e., 0.2 coefficient of variation, COV), respectively. As a reference, the nominal value of FS that corresponds to the case where all $S_u$ values equal to their mean values of 40kPa is equal to 1.17.

Li et al. (2013&2014) have developed an in-house Fortran-based software package for probabilistic slope stability analysis using MCS and risk aggregation. The software package is used in the illustrative example.

4 RISK AGGREGATION RESULTS

30 risk aggregation runs with MCS are performed to illustrate the variation of $p_F$ with spatial variability (i.e., $\lambda$). The $\lambda$ value varies from 1m to 1000m. Note that $S_u$ at two different points are considered highly correlated when the distance between these two points is smaller than $\lambda$. Each run has a MCS sample number of 10000 (i.e., $n = 10000$). A threshold value $\rho_0=0.9$ is adopted herein when comparing the correlation coefficient of two slip surfaces and updating the PSSL. For validating the results, a separate set of MCS runs with explicitly searching of critical slip surface in each MCS sample (i.e., $\rho_0=1.0$) is also performed. A number $N=10000$ of potential slip surfaces are generated to cover all potential failure domain in Fig. 1.

Fig. 2 summarizes a variation of slope failure probability, $p_F$, as a function of the normalized scale of fluctuation, $\lambda/H$. As $\lambda/H$ increases from 0.1 to 2 (or $\lambda$ increases from 1 to 20 m for $H = 10m$), the value of $p_F$ increases significantly from about 6% to about 24%. When $\lambda/H > 2$, the effect of $\lambda$ on $p_F$ begins to diminish, and $p_F$ varies slightly as $\lambda/H$ further increases. Fig. 2 also includes the results that critical slip surface is searched explicitly in each MCS sample (i.e., $\rho_0=1.0$) using triangles. These two sets of results agree well with each other. The $p_F$ values based on the risk aggregation in this study are in good agreement with those $p_F$ values that have been obtained by searching a large number of potential slip surfaces for the minimum FS in each MCS sample.

Fig. 3 plots the computational time consumed for two different sets of analyses shown in Fig. 2. The analysis is conducted using a desktop computer with a CPU@ 3.40 GHz and 16.0 GB RAM. The computational time for risk aggregation approach with $\rho_0=0.9$ is about 100 sec, which at least one order of magnitude smaller than the time needed for explicit search of critical slip surface in each MCS sample (i.e.,
ρ₀=1.0). The computational efficiency has been improved substantially. Both computational efficiency and accuracy of the risk aggregation approach rely on a proper selection of ρ₀. Some guides on how to select a proper ρ₀ value are referred to Li et al. (2014).

5 IDENTIFICATION OF REPRESENTATIVE SLIP SURFACES

Five of the 30 aggregation runs are selected to further illustrate how the pᵣ is aggregated from representative slip surfaces, including those at the λ values of 1m, 5m, 10m, 20m, and 1000m, respectively. Fig. 4 plots the number of failure samples, nᵢ, for the ith representative slip surfaces and the total number of failure samples, nᵣ, at these five λ values. When λ=1000 m as shown in Fig. 4(e), the 1st representative slip surface accounts for 2408 out of 2413 failure samples obtained from MCS. The contribution from the first representative slip surface is more than 99.9%, and the key failure mode is the 1st representative slip. As the λ value decreases, the number of representative slip surfaces identified increases, and the pᵣ is attributed to more than one slip surface. This is consistent with findings of previous studies (e.g., Wang et al. 2011) that when the spatial variability is considered (i.e., a small λ value), the critical slip surface varies spatially
practical and efficient means to incorporate such slip surface variation in probabilistic slope stability analysis. In addition, the multiple representative slip surfaces identified from risk aggregation can be used as key failure scenarios when mitigation measures of the slope under analysis is planned or designed.

6 CONCLUSIONS

This paper presented a risk aggregation approach for probabilistic slope stability analysis that properly selects a limited number of slip surfaces, evaluates failure probability along each of the selected slip surfaces, and aggregates these failure probabilities to represent the overall slope failure probability $p_F$. The slope failure event is represented by a series of mutually exclusive and collectively exhaustive events using conditional probability. The proposed risk aggregation approach is performed together with MCS and uses the conditional samples to develop failure sample subsets in a progressive manner. The approach was illustrated using a cohesivé slope example. The $p_F$ values obtained from the risk aggregation approach were shown to agree well with those $p_F$ values that have been obtained by searching a large number of potential slip surfaces for the minimum FS in each MCS sample. The computational time, however, was shown to reduce by at least an order of magnitude. The risk aggregation approach provides a practical and efficient means to incorporate slip surface variation into probabilistic slope stability analysis when spatial variability of soil properties is considered in the analysis. In addition, the multiple representative slip surfaces identified from risk aggregation can be used as key failure scenarios when mitigation measures of the slope under analysis is planned or designed.

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