**pp Solar Neutrinos at DARWIN**

André de Gouvêa,1 Emma McGinness,2,3 Ivan Martinez-Soler,1,4,5,6 and Yuber F. Perez-Gonzalez1,4,5,7

1Northwestern University, Department of Physics & Astronomy, 2145 Sheridan Road, Evanston, IL 60208, USA
2University of California Berkeley, Department of Physics, 366 Physics North, Berkeley, CA 94720, USA
3University of California Berkeley, Department of Astronomy, 501 Campbell Hall, Berkeley, CA 94720, USA
4Colegio de Física Fundamental e Interdisciplinaria de las Américas (COFI)
5254 Norzagary street, San Juan, Puerto Rico 00901
6Theoretical Physics Department, Fermilab, P.O. Box 500, Batavia, IL 60510, USA
7University of California Berkeley, Department of Physics, 366 Physics North, Berkeley, CA 94720, USA
8Department of Physics & Laboratory for Particle Physics and Cosmology, Harvard University, Cambridge, MA 02138, USA
9Institute for Particle Physics Phenomenology, Durham University, South Road, Durham, United Kingdom.

The DARWIN collaboration recently argued that DARWIN (DARk matter WImp search with liquid xenoN) can collect, via neutrino–electron scattering, a large, useful sample of solar pp-neutrinos, and measure their survival probability with sub-percent precision. We explore the physics potential of such a sample in more detail. We estimate that, with 300 ton-years of data, DARWIN can also measure, with the help of current solar neutrino data, the value of sin²θ13, with the potential to exclude sin²θ13 = 0 close to the three-sigma level. We explore in some detail how well DARWIN can constrain the existence of a new neutrino mass-eigenstate ν4 that is quasi-mass-degenerate with ν1 and find that DARWIN’s sensitivity supersedes that of all current and near-future searches for new, very light neutrinos. In particular, DARWIN can test the hypothesis that ν1 is a pseudo-Dirac fermion as long as the induced mass-squared difference is larger than 10⁻¹³ eV², one order of magnitude more sensitive than existing constraints. Throughout, we allowed for the hypotheses that DARWIN is filled with natural xenon or ¹³⁶Xe-depleted xenon.

**I. INTRODUCTION**

Multi-ton-scale, next-generation dark matter experiments are expected to collect significant statistics of atmospheric and solar neutrinos. The DARWIN collaboration recently argued that DARWIN (DARk matter WImp search with liquid xenoN) can collect a large, useful sample of solar pp-neutrinos, measured via elastic neutrino–electron scattering [1]. There, they argued that the survival probability of pp-neutrinos can be measured with sub-percent precision and that one can measure the Weinberg angle at low momentum transfers with 10% precision, independent from the values of the neutrino oscillation parameters. Here, we explore other neutrino-physics-related information one can obtain from a high-statistics, high-precision measurement of the pp-neutrino flux.

In a nutshell, pp-neutrinos are produced in the solar core via proton-proton fusion: \( p + p \rightarrow ^2\text{H} + e^+ + \nu \). The vast majority of neutrinos produced by the fusion cycle that powers our Sun are produced via proton-proton fusion. pp-neutrinos have the lowest energy among all solar neutrino “types” (other types include pep-neutrinos, ⁷Be-neutrinos, B-neutrinos, and CNO-neutrinos) and are characterized by a continuous spectrum that peaks around 300 keV and terminates around 420 keV. Theoretically, the pp-neutrino flux is known at better than the percent level [2] given that they are created early in the pp-fusion cycle – they are the first link in the chain – and their flux is highly correlated with the photon flux, measured with exquisite precision. For the sake of comparison, the flux of ⁷Be-neutrinos and B-neutrinos, which provide virtually all information on the particle-physics properties of solar neutrinos, can be computed at, approximately, the 6% and 12% level, respectively [3] [4]. The pp-neutrino flux has been directly measured, independent from the other flux-types, by the Borexino collaboration [5], with 10% precision.

A percent-level measurement of the pp-neutrino flux is expected to be sensitive to new-physics effects in neutrino physics that are also at the percent level. This includes, for example, effects from the so-called reactor angle θ13 – not new physics but very small for solar neutrinos – and the presence of new neutrino states or neutrino interactions. Furthermore, the fact that pp-neutrinos have energies that are significantly lower than those of the other solar neutrino types renders them especially well-suited to constrain (or discover) new, very long oscillation lengths associated to very small new neutrino mass-squared differences. These searches are expected to add significantly to our ability to test the hypothesis that the neutrinos are pseudo-Dirac fermions [6] [7] (for relevant recent discussions, see, for example, [9] [12]). Here, as far as new-physics hypotheses are concerned, we concentrate on the search for new, very light neutrino states.

In Sec. [1] we review the relevant features of the proposed DARWIN experiment and provide information on how we simulate and analyze DARWIN data on pp-neutrinos. In Sec. [11] we show that a percent-level measurement of the pp-neutrino flux allows for a “solar-neutrinos-only” measurement of sin²θ13. In Sec. [IV] we compute the sensitivity of
DARWIN to the hypothesis that there is a fourth neutrino with a mass \( m_4 \) that is quasi-degenerate with the mass of the first neutrino state, \( m_1 \) (in the Appendix, we discuss how this can be generalized). We concentrate on the region of parameter space where the new mass-squared difference is \( 10^{-13} \text{ eV}^2 \lesssim |m^2_4 - m_1^2| \lesssim 10^{-6} \text{ eV}^2 \). We add some concluding remarks in Sec. V.

II. DARWIN AS A LOW-ENERGY SOLAR NEUTRINO EXPERIMENT

DARWIN is projected to be a large – 40 tons fiducial volume – liquid xenon time-projection chamber, aimed at searching for weakly interacting massive particles (WIMP) in the GeV to TeV mass range \([13]\) via elastic WIMP-nucleon scattering. It will inevitably be exposed to a large flux of solar and atmospheric neutrinos and is large enough that solar-neutrino scattering events will occur at an observable rate.

According to \([1]\), DARWIN is expected to collect a sample of almost ten thousand \( pp \)-neutrinos per year via elastic neutrino-electron scattering:

\[
\nu_\alpha + e^- \rightarrow \nu_\alpha + e^-, \tag{II.1}
\]

where \( \alpha = e, \mu, \tau \) is the flavor of the incoming neutrino. The flavor of the outgoing neutrinos is, of course, never observed. For \( pp \)-neutrino energies, the cross section for \( \nu_\alpha e^- \)-scattering is around six times larger than that of \( \nu_\alpha e^- \)-scattering, \( \alpha = \tau, \mu \) and the differences between the cross sections for \( \nu_\alpha e^- \)-scattering and \( \nu_\alpha e^- \)-scattering are negligible. At leading order in the weak interactions, the differential cross section in the rest frame of the electron is

\[
\frac{d\sigma}{dT}(\nu_\alpha + e^- \rightarrow \nu_\alpha + e^-) = \frac{2G_F^2 m_e}{\pi} \left[ a_e^2 + b_e^2 \left( 1 - \frac{T}{E_\nu} \right)^2 - a_e b_e \frac{T}{E_\nu} \right], \tag{II.2}
\]

where \( T \) is the kinetic energy of the recoil electron, \( E_\nu \) is the incoming neutrino energy, \( m_e \) is the electron mass and \( G_F \) is the Fermi constant. The dimensionless couplings \( a_e, b_e \) are

\[
a_e = -\frac{1}{2} - \sin^2 \theta_W, \quad b_e = -\sin^2 \theta_W; \quad a_\alpha = \frac{1}{2} - \sin^2 \theta_W, \quad b_\alpha = -\sin^2 \theta_W, \tag{II.3}
\]

where \( \theta_W \) is the weak mixing angle. DARWIN measures the kinetic energy spectrum of the recoil electrons.

If filled with natural xenon, one expects a large number of electron-events in the energy range of interest from the double-beta decays of \(^{136}\text{Xe}\). These events are a powerful source of background for solar-neutrino studies and, according to \([1]\), obviate the study of solar neutrinos with energies higher than 1 MeV. They are a powerful nuisance for measurements of the \(^7\text{Be}\)-neutrino flux and have a significant but not decisive impact on the measurement of the \( pp \)-neutrinos (around a 30\% decrease in the precision with which the overall \( pp \)-neutrino flux can be measured \([1]\)).

The reason one can measure the \( pp \)-neutrino flux in spite of the \(^{136}\text{Xe}\) background is well known and the experiment can detect events over a large range of recoil-electron energies, effectively measuring it with excellent precision. There is the possibility of filling DARWIN with liquid xenon depleted of the double-beta-decaying \(^{136}\text{Xe}\) isotope. This would allow the study of higher energy solar neutrinos. Here we consider these two different scenarios, i.e., the \(^{136}\text{Xe}\)-depleted version of DARWIN and the one where the abundance of \(^{136}\text{Xe}\) agrees with natural expectations.

Other than the background from \(^{136}\text{Xe}\), for \( pp \)-neutrinos, the double electron capture decay of \(^{124}\text{Xe}\) leads to two narrow peaks at 37 keV and 10 keV \([1]\) and, at higher recoil energies, radioactive backgrounds from the detector components and the liquid volume supersedes the \( pp \)-neutrino events for recoil kinetic energies above 200 keV or so. When simulating DARWIN data, we restrict our sample to events with recoil kinetic energies below 220 keV and assume that, in this energy range, the only sources of background are those from \(^{136}\text{Xe}\) and \(^{124}\text{Xe}\). We simulate the backgrounds using the results published in \([1]\). When analyzing the simulated data, we marginalize over the normalization of the two \(^{124}\text{Xe}\) lines, which we treat as free parameters, and the normalization of the \(^{136}\text{Xe}\) recoil spectrum, which we assume is independently measured with 0.1\% precision. We assume the shape of the \(^{136}\text{Xe}\) recoil spectrum is known with infinite precision. For the \(^{136}\text{Xe}\)-depleted version of DARWIN, we assume the \(^{136}\text{Xe}\) background is 1\% of the background presented in \([1]\). We organize the simulated data into recoil-kinetic-energy bins with 10 keV width, consistent with the recoil-kinetic-energy resolution quoted in \([1]\), starting at 1 keV. We use a simple \( \chi^2 \)-test in order to address questions associated to the sensitivity of DARWIN to different parameters and in order to combine simulated DARWIN data with those from other experiments.
II. TESTING THE THREE-MASSIVE-NEUTRINOS PARADIGM

In the absence of more new physics, existing data reveals that the neutrino weak-interaction-eigenstates $\nu_\alpha$, $\alpha = e, \mu, \tau$, are linear combinations of the neutrino mass-eigenstates $\nu_i$ with mass $m_i$, $i = 1, 2, 3$:

$$\nu_\alpha = U_{\alpha i} \nu_i,$$

(III.1)

where the $U_{\alpha i}$, $\alpha = e, \mu, \tau$, $i = 1, 2, 3$, define the elements of a unitary matrix. Here, we are only interested in solar neutrinos so all accessible observables are sensitive to $|U_{ei}|^2$, $i = 1, 2, 3$. These, in turn, are parameterized with two mixing angles, $\theta_{12}$ and $\theta_{13}$. Following the parameterization of the Particle Data Group \[14\],

$$|U_{e2}|^2 = \sin^2 \theta_{12} \cos^2 \theta_{13}, \quad |U_{e3}|^2 = \sin^2 \theta_{13},$$

(III.2)

and unitarity uniquely determines the third matrix-element-squared: $|U_{e1}|^2 = 1 - |U_{e2}|^2 - |U_{e3}|^2$. Combined fits to the existing data reveal that the two independent mass-squared differences are $\Delta m^2_{21} \equiv m^2_2 - m^2_1 \sim 10^{-4}$ eV$^2$ and $|\Delta m^2_{31}| \equiv m^2_3 - m^2_1 \sim 10^{-3}$ eV$^2$. For more precise values see, for example, \[15\]. While $\Delta m^2_{21}$ is defined to be positive, the sign of $\Delta m^2_{31}$ is still unknown; for our purposes here, it turns out, this is irrelevant. The two mixing parameters of interest have been measured quite precisely. According to \[15\], at the one-sigma level,

$$\sin^2 \theta_{12} = 0.304^{+0.013}_{-0.012}, \quad \sin^2 \theta_{13} = 0.02221^{+0.00068}_{-0.00062}.$$  

(III.3)

The experiments that contribute most to these two measurements are qualitatively different. $\theta_{12}$ is best constrained by solar neutrino experiments – and is often referred to as the “solar angle” – while $\theta_{13}$ is best constrained by reactor antineutrino experiments – and is often referred to as the “reactor angle.”

We are interested in the solar pp-neutrinos. These have a continuous energy spectrum that peaks around 300 keV and terminates at around 420 keV. The matter-potential $V = \sqrt{2} G_N N_e$, where $G_F$ is the Fermi constant and $N_e$ is the electron number-density, inside the Sun is $V_\odot < 2 \times 10^{-5}$ (eV$^2$/MeV) so, for neutrino energies $E < 0.420$ MeV, $|\Delta m^2_{31}|/(2E), |\Delta m^2_{21}|/(2E) \gg V_\odot$. This, in turn, implies that, given what is known about the neutrino mass-squared differences, matter effects can be neglected. Including the fact that, for all practical purposes, solar neutrinos lose flavor coherence as they find their way from the Sun to the Earth, it is trivial to show that the $\nu_e$ survival probability is energy independent and given by

$$P_{ee} = |U_{e1}|^4 + |U_{e2}|^4 + |U_{e3}|^4.$$  

(III.4)

On the other hand, solar neutrino experiments cannot distinguish $\nu_\mu$ from $\nu_\tau$ – the neutrino energies are too small – but are potentially sensitive to the combination $P_{ea} = P_{e\mu} + P_{e\tau}$. In the three-massive-neutrinos paradigm

$$P_{ea} = 1 - P_{ee}.$$  

(III.5)

Given our current knowledge of mixing parameters, for pp-neutrinos, we can indirectly infer that $P_{ee} = 0.552 \pm 0.025$, naively combining the uncertainties in Eq. (III.3) in quadrature.

According to \[14\], after 20 ton-years of exposure, DARWIN can measure $P_{ee}$ with better than 1% accuracy. Assuming the three-massive-neutrinos paradigm, this can be converted into a measurement of the relevant mixing parameters. Fig. 1(top,left) depicts the allowed region of the $\sin^2 \theta_{12} \times \sin^2 \theta_{13}$ parameter space assuming DARWIN can measure $P_{ea}$ for pp-neutrinos at the 1% level, and assuming the best-fit value is $P_{ee} = 0.552$. There is very strong degeneracy between different values of $\sin^2 \theta_{12}$ and $\sin^2 \theta_{13}$, for obvious reasons. The degeneracies can be lifted by including constraints from other neutrino experiments.

It is interesting to investigate how well one can constrain neutrino-mixing parameters using only solar-neutrino data. In order to estimate that, we add to the hypothetical pp-neutrino measurement from DARWIN current information from $^8$B neutrinos, mostly from the Super-Kamiokande and SNO experiments, see \[16, 17\] and references therein. These provide the strongest constraints on $\sin^2 \theta_{12}$. Here, we address this in a simplified but accurate way \[18\], postulating that $^8$B experiments measure

$$\langle P_{ee}\rangle_{^8B, \text{average}} = (1 - |U_{e3}|^2) \left[0.9 |U_{e2}|^2 + 0.1 |U_{e1}|^2\right] + |U_{e3}|^4,$$

(III.6)

* See also [http://www.nu-fit.org](http://www.nu-fit.org)

† For example, at the center of the Sun, for neutrino energies less than 420 keV, the “mass equivalent” of $\sin^2 2 \theta_{12}$ differs from its vacuum counterpart by less than one percent.
FIG. 1: Top: One-, two- and three-sigma allowed regions of the $\sin^2 \theta_{12} \times \sin^2 \theta_{13}$ parameter space assuming DARWIN can measure $P_{ee} = 0.552$ at the one percent level, excluding (left) and including (right) external constraints on the neutrino-mixing parameters from other solar experiments. The tiny empty ellipse in the right-hand panel indicates the best-fit point. The open regions bound by dashed lines (left-hand panel) represent one-, two- and three-sigma results from current $^8$B neutrino experiments, as discussed in the text. Bottom: Solar-only $\chi^2$ as a function of $\sin^2 \theta_{13}$, marginalized over $\sin^2 \theta_{12}$, assuming 300-ton-years of simulated DARWIN data. The full line corresponds to the assumption of a depleted background, while the dashed line is obtained including the expected natural $^{136}$Xe background.

with 4% accuracy, consistent with the current uncertainty on $\sin^2 \theta_{12}$, mostly constrained by high-energy solar neutrino data. $(1 - |U_{e3}|^2) \times 0.9$ (or $(1 - |U_{e3}|^2) \times 0.1$) is the average probability that a $^8$B neutrino arrives at the surface of the Earth as a $\nu_2$ (or a $\nu_1$). When the $^8$B data is treated as outlined above, it translates into the open regions bound by dashed lines in Fig. 1(top, left). Strong matter effects lead to the boomerang-shaped allowed region of the parameter space and restrict the parameter space to values of $\sin^2 \theta_{12} \lesssim 0.5$. The results of the joint $pp-^8$B analysis are depicted in Fig. 1(top, right). All degeneracies present in the $pp$-neutrino data are lifted and one is constrained to small values of $\sin^2 \theta_{13}$ and $\sin^2 \theta_{12} < 0.5$. 
The combined $^8$B and DARWIN data can rule out $\sin^2 \theta_{13} = 0$ with some precision. This is important; it implies that a hypothetical DARWIN measurement of the $pp$-neutrino flux, combined with the current $^8$B solar neutrino data, can measure $\sin^2 \theta_{13}$ in a way that is independent from all non-solar measurements. The marginalized $\chi^2$ as a function of $\sin^2 \theta_{13}$ is depicted in Fig. 1(bottom), for 300 ton-years of simulated DARWIN data and the current $^8$B solar neutrino data. On average, if the $pp$-neutrino flux can be measured at the percent level, we expect to measure $\sin^2 \theta_{13}$ at the 35% level and rule out $\sin^2 \theta_{13} = 0$ at almost the three-sigma level. Here we consider the two scenarios outlined earlier, one with natural xenon (dashed line), the other with $^{136}$Xe-depleted xenon (solid line).

The precision on $\sin^2 \theta_{13}$ obtained above is not comparable to that of the current measurement of $\sin^2 \theta_{13}$, Eq. (III.3). However, these measurements are qualitatively different. The most precise measurements of $\sin^2 \theta_{13}$ come from reactor antineutrino experiments and a baseline of order 1 km [19–21]. The estimate discussed above is a “solar only” measurement, i.e., it exclusively makes use of measurements of neutrinos (and not antineutrinos) produced in the Sun. Current measurements of $\sin^2 \theta_{13}$ that make use of neutrinos (as opposed to antineutrinos), from T2K and NOvA, are much less precise (at the 50%, see [22, 23]).

Looking further into the future, the DUNE experiment, for example, is expected to independently measure the “neutrino-only” value of $\sin^2 \theta_{13}$ at the 20% level [24] (or worse, depending on the assumptions made in the analysis).

IV. BEYOND THE THREE-MASSIVE-NEUTRINOS PARADIGM

The fact that the $pp$-neutrino flux can be computed with great precision, combined with the sub-MeV $pp$-neutrino energies, allows a high-statistics measurement of the $pp$-neutrino flux to meaningfully search for phenomena beyond the thee-active-neutrinos paradigm. Here we concentrate on testing the hypothesis that the neutrinos produced in the Sun have a nonzero probability of behaving as “sterile neutrinos” $\nu_s$, characterized by their lack of participation in charged-current and neutral-current weak interactions.

We first discuss, in Sec. [IV.A] the case where the oscillation probabilities are energy-independent for the energies of interest, as in the case of the thee-active-neutrinos paradigm discussed in Sec. III. In particular, we test the hypothesis that $P_{ee} + P_{ea} = 1$ for $pp$-neutrinos. Then, in Sec. [IV.B] we compute DARWIN’s ability to constrain the hypothesis that there is a fourth neutrino $\nu_4$ and that its mass is quasi-degenerate with $m_1$.

A. Model-Independent Considerations

As discussed in Sec. [III] we are interested in the shape and normalization of the electron recoil-energy spectrum from neutrino–electron elastic scattering. The differential cross-section for $\nu_e$ and $\nu_\alpha$ scattering are different, both in normalization and shape and hence, in principle, one can obtain independent information on both $P_{ee}$ and $P_{ea}$.

We simulate and analyze 300 ton-years of DARWIN $pp$-data, as discussed in Sec. [III] and attempt to measure $P_{ee}$ and $P_{ea}$ independently. The results are depicted in Fig. 2(left) for both the natural xenon (dashed) and the $^{136}$Xe-depleted xenon (solid) hypotheses. Strong departures from $P_{ee} + P_{ea} = 1$ are allowed and the “natural” data are not capable of ruling out $P_{ea} = 0$ at the three-sigma confidence level. The “depleted” data can rule out $P_{ea} = 0$ at the five-sigma confidence level. For both scenarios, one can constrain the departure of $P_{ee} + P_{ea}$ from one, which we interpret as the oscillation probability into sterile neutrinos $P_{es} = 1 - P_{ee} - P_{ea}$. The colorful diagonal lines in Fig. 2(left) correspond to different constant values of $P_{es}$. Fig. 2(right) depicts $\chi^2$ as a function of $P_{es}$, marginalized over $P_{ee}$ and restricting $P_{ea}$ to non-negative values for both scenarios. If DARWIN data are consistent with the three-active-neutrinos paradigm, they will be capable of constraining $P_{es} < 0.35$ at the two-sigma confidence level even if DARWIN is filled with natural xenon.

B. Fourth-Neutrino Hypothesis

We explore in more detail the scenario where there is one extra neutrino mass-eigenstate $\nu_4$ with mass $m_4$. In this case, the interaction eigenstates are, including $\nu_4$, related to the four mass-eigenstates via a $4 \times 4$ unitary matrix $U_{\alpha s}$, $\alpha = e, \mu, \tau, s, i = 1, 2, 3, 4$. We will concentrate on the scenario where, among the four $U_{si}$, only $U_{s1}$ and $U_{s4}$ are potentially nonzero. In this case, we can parameterize the $|U_{si}|^2$ entries of the mixing matrix using three mixing

\[ U_{s1} = U_{s4} = U_{es} \]

\[ U_{s2} = \cos \theta_s \quad \text{and} \quad U_{s3} = \sin \theta_s \]

where $\theta_s$ is the mixing angle between $\nu_s$ and $\nu_4$. This parameterization is advantageous because it allows for a high-statistics measurement of $\sin^2 \theta_s$ and $\cos^2 \theta_s$ by measuring the electron recoil-energy spectrum, in a way that is independent from all non-solar measurements. The marginalized $\chi^2$ as a function of $\sin^2 \theta_s$ and $\cos^2 \theta_s$ is depicted in Fig. 3(left), for 300 ton-years of simulated DARWIN data and the current $^8$B solar neutrino data. On average, if the $pp$-neutrino flux can be measured at the percent level, we expect to measure $\sin^2 \theta_s$ at the 35% level and rule out $\sin^2 \theta_s = 0$ at almost the three-sigma level. Here we consider the two scenarios outlined earlier, one with natural xenon (dashed line), the other with $^{136}$Xe-depleted xenon (solid line).

The precision on $\sin^2 \theta_s$ obtained above is not comparable to that of the current measurement of $\sin^2 \theta_{13}$, Eq. (III.3). However, these measurements are qualitatively different. The most precise measurements of $\sin^2 \theta_s$ come from reactor antineutrino experiments and a baseline of order 1 km [19–21]. The estimate discussed above is a “solar only” measurement, i.e., it exclusively makes use of measurements of neutrinos (and not antineutrinos) produced in the Sun. Current measurements of $\sin^2 \theta_s$ that make use of neutrinos (as opposed to antineutrinos), from T2K and NOvA, are much less precise (at the 50%, see [22, 23]).

Looking further into the future, the DUNE experiment, for example, is expected to independently measure the “neutrino-only” value of $\sin^2 \theta_s$ at the 20% level [24] (or worse, depending on the assumptions made in the analysis).

\[ \chi^2 = \sum_{i=1}^{N} \frac{1}{2} \left( \frac{S_i - P_i}{\sigma_i} \right)^2 \]

where $S_i$ is the observed number of events, $P_i$ is the predicted number of events, and $\sigma_i$ is the statistical error on the predicted number of events. The minimum value of $\chi^2$ occurs when the observed and predicted number of events are equal. In this case, we obtain $\sin^2 \theta_s = 0.35$ at the three-sigma confidence level. Here we consider the two scenarios outlined earlier, one with natural xenon (dashed line), the other with $^{136}$Xe-depleted xenon (solid line).

However, these measurements are qualitatively different. The most precise measurements of $\sin^2 \theta_s$ come from reactor antineutrino experiments and a baseline of order 1 km [19–21]. The estimate discussed above is a “solar only” measurement, i.e., it exclusively makes use of measurements of neutrinos (and not antineutrinos) produced in the Sun. Current measurements of $\sin^2 \theta_s$ that make use of neutrinos (as opposed to antineutrinos), from T2K and NOvA, are much less precise (at the 50%, see [22, 23]).

Looking further into the future, the DUNE experiment, for example, is expected to independently measure the “neutrino-only” value of $\sin^2 \theta_s$ at the 20% level [24] (or worse, depending on the assumptions made in the analysis).
angle $\theta_{12}, \theta_{13}, \theta_{14}$. Eqs. (III.2) are still valid, along with

$$|U_{e1}|^2 = \cos^2 \theta_{12} \cos^2 \theta_{13} \cos^2 \theta_{14}, \quad |U_{e4}|^2 = \cos^2 \theta_{12} \cos^2 \theta_{13} \sin^2 \theta_{14}. \quad (IV.1)$$

It is easy to check that $\sum_{i=1}^{4} |U_{ei}|^2 = 1$. The non-zero “sterile” entries of the mixing matrix are

$$|U_{s1}|^2 = \sin^2 \theta_{14}, \quad |U_{s4}|^2 = \cos^2 \theta_{14}. \quad (IV.2)$$

Given the quasi-two-flavors nature of these solar neutrino oscillations, to be discussed momentarily, the entire physical parameter space is spanned by either fixing $\Delta m^2_{41} > 0$ and allowing $\sin^2 \theta_{14} \in [0,1]$ or allowing both signs for $\Delta m^2_{41}$ and restricting $\sin^2 \theta_{14} \in [0,0.5]$ in such a way that $\nu_4$ is always “mostly sterile.” Here, the former convention – to fix the sign of $\Delta m^2_{41} > 0$ – is most convenient. With this choice, when $\sin^2 \theta_{14} \in [0,0.5]$, the heaviest of the two quasi-degenerate states (i.e., $\nu_4$) is mostly sterile, when $\sin^2 \theta_{14} \in [0.5,1]$, the lightest among the two quasi-degenerate states (i.e., $\nu_1$) is mostly sterile. For historical reasons, we will refer to $\sin^2 \theta_{14} \in [0,0.5]$ as the light side of the parameter space and $\sin^2 \theta_{14} \in [0.5,1]$ as the dark side $^{29}$.

We are interested in the hypothesis that $\Delta m_{41}^2 \ll \Delta m_{31}^2$ and outside the reach of all current neutrino experiments. In this case, the current neutrino oscillation data constrain the oscillation parameters $\Delta m_{21}^2$, $\Delta m_{31}^2$, $\sin^2 \theta_{12}$, and $\sin^2 \theta_{13}$ exactly as in the three-massive-neutrinos paradigm. Furthermore, building on the discussion in Sec. III, it is easy to conclude that the oscillation probabilities of interest $P_{\alpha\nu}$, $\alpha = e, a, s$, are only functions of $\sin^2 \theta_{12}, \sin^2 \theta_{13}, \sin^2 \theta_{14}$, and $\Delta m_{41}^2$. Further taking advantage of the fact that $|\Delta m_{21}^2|/(2E), |\Delta m_{31}^2|/(2E) \gg V_0$, it is straightforward to compute

$$P_{ee} = |U_{e2}|^4 + |U_{e3}|^4 + (1 - |U_{e2}|^2 - |U_{e3}|^2)^2 P_{ee}^{2f} (\Delta m_{41}^2, \sin^2 \theta_{14}, V_{\text{eff}}), \quad (IV.3)$$

$$P_{ea} = (1 - |U_{e2}|^2 - |U_{e3}|^2)(1 - P_{ee}^{2f} (\Delta m_{41}^2, \sin^2 \theta_{14}, V_{\text{eff}})), \quad (IV.4)$$

$$P_{es} = 1 - P_{ee} - P_{ea}, \quad (IV.5)$$

where $P_{ee}^{2f}$ is the survival probability obtained in the scenario where there are only two flavors, $\nu_e^{2f}$ and $\nu_e^{2f}$, characterized by the mass-squared difference $\Delta m_{41}^2$ and the mixing angle $\theta_{14}$, defined via $\nu_e^{2f} = \cos \theta_{14} \nu_e + \sin \theta_{14} \nu_4$. Inside $P_{ee}^{2f}$, the matter potential is replaced by an effective matter potential $V_{\text{eff}}$. It takes into account the neutral-current contribution to the matter potential $V_{NC} = -\sqrt{2}/2G_F N_n$, where $N_n$ is the neutron number density in the medium.

FIG. 2: Left: One- and three-sigma allowed region of the $P_{ee} \times P_{es}$-plane, for 300 ton-years of simulated DARWIN data. The diagonal lines correspond to constant $P_{ee} = 1 - P_{ea} - P_{es}$ values. The burgundy line segment with positive slope corresponds to the values of $(P_{ee}, P_{es})$ accessible via Eqs. (IV.3). Right: Marginalized $\chi^2$ as a function of $P_{es}$. The full line correspond to the assumption of a depleted background, while the dashed line is obtained considering no cuts in the $^{136}$Xe background.
while the charged-current contribution is rescaled by \( (1 - |U_{e2}^2 - |U_{e3}|^2) = \cos^2 \theta_{13} \cos^2 \theta_{12} \):

\[
V^\text{eff} = \sqrt{2} G_F \left( N_c \cos^2 \theta_{13} \cos^2 \theta_{12} - \frac{1}{2} N_n \right).
\]

(IV.6)

In the sun, the position-dependency of the electron and neutron number densities are slightly different \[2, 5\]. In the sun’s core, \( N_n \) is around 50% of \( N_c \) and \( V^\text{eff} \) is slightly less than one half of the standard matter potential in the Sun.

Eq. (IV.3) allows us to estimate, in very general terms, the impact of the sterile neutrinos. For \( P^2_{ee} = 1 \), we recover the three-active-neutrinos result, \( P_{ee} = \cos^4 \theta_{12} \cos^2 \theta_{13} + \sin^4 \theta_{12} \cos^2 \theta_{13} + \sin^4 \theta_{13} \), Eq. (III.4). On the other hand, for \( P^2_{ee} = 0 \), \( P_{ee} = \sin^4 \theta_{12} \cos^2 \theta_{13} + \sin^4 \theta_{13} \) such that, given the current knowledge of oscillation parameters,

\[
P_{ee} \in [0.09, 0.55].
\]

(IV.7)

\( P_{ea} \) values, on the other hand, are allowed to be as small as zero and as large as 0.68.

In Sec. IV.A we discussed that, very generically, DARWIN can rule out \( P_{es} < 0.35 \) at the two-sigma level. The situation here is more constrained as \( P_{ee}, P_{ea}, P_{es} \) are not only required to add up to one but depend on the same oscillation parameters. We proceed to discuss the sensitivity of DARWIN to the new oscillation parameters \( \Delta m^2_{41}, \sin^2 \theta_{14} \) by taking advantage of the fact that the properties of \( P^2_{ee} \) are well known (see, for example, \[24\]).

For large-enough values of \( \Delta m^2_{41} \), \( P^2_{ee} \) is well approximated by averaged-out vacuum oscillations:

\[
P^2_{ee, \text{ave}} = 1 - \frac{1}{2} \sin^2 2\theta_{14}.
\]

(IV.8)

This occurs, keeping in mind we are interested in energies below 420 keV, for \( \Delta m^2_{41} \gtrsim 10^{-5} \text{ eV}^2 \), when the solar matter effects can be ignored. In this case,

\[
\begin{align*}
P_{ee} &= 0.55 - 0.23 \sin^2 2\theta_{14}, \\
P_{es} &= 0.34 \sin^2 2\theta_{14}, \\
P_{ea} &= 0.45 - 0.11 \sin^2 2\theta_{14}.
\end{align*}
\]

(IV.9)

Here, it is impossible to distinguish the light from the dark side of the parameter space since the oscillation probabilities are invariant under \( \sin^2 \theta_{14} \leftrightarrow 1 - \sin^2 \theta_{14} \). Varying \( \sin^2 2\theta_{14} \in [0, 1] \), Eqs. (IV.9) define a line segment in the \( P_{ee} \times P_{ea} \) plane, depicted in Fig. 2 (left) – burgundy line with positive slope – keeping in mind the segment extends to \( P_{ee} \) values below 0.4. Fig. 3 depicts \( \chi^2 \) as a function of \( \sin^2 \theta_{14} \) in the regime where Eqs. (IV.9) are a good approximation, for 300 ton-years of simulated DARWIN data, for both the natural (dashed) and \(^{130}Xe\)-depleted (solid) scenarios. In this analysis, and in the upcoming analyses discussed this subsection, we assume that \( \sin^2 \theta_{13} \) and \( \sin^2 \theta_{12} \) are known with infinite precision. This is, currently, a good approximation for \( \sin^2 \theta_{13} \) and will be a good approximation for \( \sin^2 \theta_{12} \) once data from the JUNO experiment is analyzed [27]. Similar results were recently presented and discussed in [28]. Where are assumptions agree, the estimated sensitivity also agrees.

For intermediate values of \( \Delta m^2_{41} \), \( P^2_{ee} \) is well described by the strong MSW effect in the adiabatic regime. In this case, for a range of energies,

\[
P^2_{ee, \text{adiabatic}} = \sin^2 \theta_{14}.
\]

(IV.10)

For \( pp \)-neutrinos, this occurs for, very roughly, \( \sin^2 \theta_{14} \gtrsim 10^{-3} \) and \( 10^{-9} \lesssim \Delta m^2_{41}/(\text{eV}^2) \lesssim 10^{-6} \). Under these conditions,

\[
\begin{align*}
P_{ee} &= 0.09 + 0.46 \sin^2 \theta_{14}, \\
P_{es} &= 0.68 - 0.68 \sin^2 \theta_{14}, \\
P_{ea} &= 0.27 + 0.22 \sin^2 \theta_{14}.
\end{align*}
\]

(IV.11)

Here, oscillation probabilities are very different in the light and dark sides. In particular, in the light side of the parameter space \( P_{ee} \) (\( P_{es} \)) is small (large) and increases (decreases) linearly with \( \sin^2 \theta_{14} \). If DARWIN data are consistent with three-active neutrinos, in this region of parameter space, small values of \( \sin^2 \theta_{14} \) will be excluded.

\[5\] There are relatively more neutrons in the center of the sun relative to its edges. This is due to the fact that most of the solar helium is concentrated in the core.
while large values of $\sin^2 \theta_{14}$ are allowed.

For small-enough values of $\Delta m^2_{41}$, $P^{2f}_{ee}$ is well described by the strong MSW effect in the very non-adiabatic regime and turns out to be well approximated by vacuum oscillations,

$$ P^{2f,\text{ave}}_{ee} = 1 - \sin^2 2\theta_{14} \sin^2 \left( \frac{\Delta m^2_{41} L}{4E} \right), \quad \text{(IV.12)} $$

This occurs, for pp-neutrinos, for $\Delta m^2_{41} \lesssim 10^{-9} \text{ eV}^2$. In this case,

$$ P_{ee} = 0.55 - 0.46 \sin^2 2\theta_{14} \sin^2 \left( \frac{\Delta m^2_{41} L}{4E} \right), $$

$$ P_{es} = 0.68 \sin^2 2\theta_{14} \sin^2 \left( \frac{\Delta m^2_{41} L}{4E} \right), \quad \text{(IV.13)} $$

$$ P_{ea} = 0.45 - 0.22 \sin^2 2\theta_{14} \sin^2 \left( \frac{\Delta m^2_{41} L}{4E} \right). $$

Here, again, it is impossible to distinguish the light from the dark side of the parameter space. Given the average Earth–Sun distance $L = 1.5 \times 10^{12} \text{ m}$, the oscillation phase is

$$ \frac{\Delta m^2_{41} L}{4E} = 4.8 \left( \frac{\Delta m^2_{41}}{10^{-11} \text{ eV}^2} \right) \left( \frac{400 \text{ keV}}{E} \right), \quad \text{(IV.14)} $$

so we expect the vacuum oscillations to average out for $\Delta m^2_{41} \gtrsim 10^{-11} \text{ eV}^2$. This means that, for $10^{-11} \lesssim \Delta m^2_{41}/(\text{eV}^2) \lesssim 10^{-9}$, the oscillation probabilities are well described by Eqs. (IV.9).

Fig. 3 depicts contours of constant $P_{ee}$ in the $\Delta m^2_{41} \times \sin^2 \theta_{14}$-plane for $E_{\nu} = 300 \text{ keV}$. The other parameters are fixed to their current best-fit values, Eq. (III.3). We assume the matter potential is spherically symmetric and drops exponentially, $V^{\text{eff}} \propto e^{-r/r_0}$. We fit information from the prediction of the B16-GS98 solar model $[29]$ and obtain $r_0^* = R_{\odot}/10.37$ where $R_{\odot} = 6.96 \times 10^{11} \text{ m}$ is the average radius of the Sun; see Fig. 4 for a comparison of the matter potential in the standard case (left) and in the scenario of interest here (right, labeled sterile neutrino).

Under these circumstances, $P^{2f}_{ee}$ can be computed exactly $[29]$. For simplified pedagogical discussions see, for example, $[26, 30]$. We assume all solar neutrinos are produced in the exact center of the Sun; we explicitly verified that the results we get are very similar to the results we would have obtained by integrating over the region where
FIG. 4: Solar matter potential for active (left) and sterile (right) neutrinos – the scenario of interest here – as function of the distance from the center in units of the Solar radius, from the B16-GS98 Solar Model. We also present our fitted exponential forms, where $r_0 = R_{\odot}/10.43$, and $r_0' = R_{\odot}/10.37$, in dashed lines.

$pp$-neutrinos are produced. The region where matter effects are strong and the adiabatic condition holds correspond to the vertical sides of the constant $P_{ee}$ regions that form quasi-triangles. The “return” to vacuum oscillations at low and high values of $\Delta m_{41}^2$ is highlighted by the vertical, dark lines, which correspond to constant values of the averaged-out vacuum oscillation probability. For a detailed discussion of the boundary between the adiabatic and non-adiabatic transition, including $L$ dependent effects, see [31].

FIG. 5: Contours of constant $P_{ee}$ in the $\Delta m_{41}^2 \times \sin^2 \theta_{14}$-plane for $E = 300$ keV. $\sin^2 \theta_{12}$, $\sin^2 \theta_{13}$ are fixed to their best-fit values, Eq. (III.3). The vertical lines correspond to constant values of the averaged-out vacuum oscillation probability.

We simulate 300 ton-years of DARWIN data consistent with the three-massive-neutrinos paradigm and assuming the true values of $\sin^2 \theta_{12}$ and $\sin^2 \theta_{13}$ are the ones in Eq. (III.3). We restrict our discussion to values of $\Delta m_{41}^2 < 10^{-6}$ eV$^2$. Larger values are constrained by measurements of higher-energy solar neutrinos; these constraints have been explored in [32, 33], along with a detailed discussion of the oscillation probabilities. The expressions we derive here are contained in the analyses of [32, 33] if one explores them in the appropriate regime.
Fig. 6 depicts the region of the $\tan^2 \theta \times \Delta m^2_{41}$-plane inside of which 300 ton-years of DARWIN data is sensitive, at the 90% confidence level, to the fourth neutrino for both the natural-xenon scenario (dashed line) and the depleted-136Xe scenario (solid). On a log-scale, the contour is symmetric relative to $\tan^2 \theta_{14} = 1$ when one cannot distinguish the light from the dark side of the parameter space [25], a feature one readily observes, as advertised, for small values of $\Delta m^2_{41}$. The impact of nontrivial matter effects is also readily observable. For larger values of $\Delta m^2_{41}$, the sensitivity to small mixing angles is expected to “shut-off” quickly – see Fig. 5 – and would return to values similar to those around $\Delta m^2_{41} \sim 10^{-16}$ eV$^2$, minus the tiny wiggles.

The low energies of the pp-neutrinos combined with the long Earth–Sun distance render DARWIN a specially powerful probe of the hypothesis that neutrinos are pseudo-Dirac fermions. This is the hypothesis that there are right-handed neutrinos coupled to the left-handed lepton doublets and the Higgs doublet via a tiny Yukawa coupling $y$ and that lepton number is only slightly violated. In these scenarios, each of the neutrino mass eigenstates is “split” into two quasi-degenerate Majorana fermions, each a 50–50 mixture of an active neutrino (from the lepton doublet) and a sterile neutrino (the right-handed neutrino). The mass splitting is small enough that, for most applications, the two quasi-degenerate state act as one Dirac fermion. Pseudo-Dirac neutrinos reveal themselves via active–sterile oscillations associated with very large mixing and very small mass-squared differences.

In the language introduced here, a pseudo-Dirac neutrino corresponds to $\sin^2 2\theta_{14} = 1$ (maximal mixing) and the small mass splitting leads to a nonzero $\Delta m^2_{41} = 4\epsilon m_1$ where $m_1 \pm \epsilon$ are the masses of the two quasi-degenerate states (here $\nu_1$ and $\nu_4$), $m_1$ is the Dirac mass, proportional to the neutrino Yukawa coupling and $\epsilon$ characterizes the strength of the lepton-number violating physics. Fig. 7 depicts $\chi^2$ as a function of $\Delta m^2_{41}$ for $\sin^2 2\theta_{14} = 1$ associated with 300 ton-years of simulated DARWIN data for both the natural xenon (dashed) and the 136Xe-depleted (solid) scenarios, assuming the data are consistent with no new neutrino states. Current solar neutrino data exclude $\Delta m^2_{41}$ values larger than $10^{-12}$ eV$^2$ [9, 10] so DARWIN can extend the sensitivity to $\Delta m^2_{41}$ and $\epsilon$ by an order of magnitude.

V. CONCLUDING REMARKS

Next-generation WIMP-dark-matter-search experiments will be exposed to a large-enough flux of solar neutrinos that neutrino-mediated events are unavoidable. The DARWIN collaboration recently argued that DARWIN can collect a large, useful sample of solar pp-neutrinos, detected via elastic neutrino–electron scattering [11], and measure the survival probability of pp-neutrinos with sub-percent precision. Here we explored the physics potential of such a sample in more detail, addressing other concrete neutrino-physics questions and exploring whether one can also extract information from a precise measurement of the shape of the differential pp-neutrino flux.
We estimate that, with 300 ton-years of data, DARWIN can not only measure the survival probability of \( pp \)-neutrinos with sub-percent precision but also determine, with the help of current solar neutrino data, the value of \( \sin^2 \theta_{13} \), with the potential to exclude \( \sin^2 \theta_{13} = 0 \) close to the three sigma level. Such a \( pp \)-neutrino sample would allow one to perform a “neutrinos-only” (and solar-neutrinos-only) measurement of \( \sin^2 \theta_{13} \) and \( \sin^2 \theta_{12} \). Such a measurement can be compared with, for example, reactor-based “antineutrinos-only” measurements of the same mixing parameters and allow for nontrivial tests of the CPT-theorem and other new physics scenarios.

DARWIN can also test the hypothesis that \( pp \)-neutrinos are oscillating into a combination of active and sterile neutrinos. We estimate that DARWIN data can exclude the hypothesis that the \( pp \)-neutrinos are “disappearing” in an energy independent way – assuming their data are consistent with the three-active-neutrinos paradigm – especially if the experiment manages to fill their detector with \( ^{136} \text{Xe} \)-depleted xenon.

We explored in some detail how well DARWIN can constrain the existence of a new neutrino mass-eigenstate \( \nu_4 \) (mass \( m_4 \)) that is quasi-degenerate and mixes with \( \nu_1 \), i.e., \( \Delta m^2_{41} \ll \Delta m^2_{21} \), \( U_{s1}, U_{s4} \neq 0 \), \( U_{s2} = U_{s3} = 0 \). Our estimated sensitivity is depicted in Fig. 6. It supersedes that of all current and near-future searches for new, very light neutrinos. In particular, DARWIN can test the hypothesis that \( \nu_1 \) is a pseudo-Dirac fermion as long as the induced mass-squared difference is larger than \( 10^{-13} \text{ eV}^2 \). This is one order of magnitude more sensitive than existing constraints [9, 10].

Throughout, we allowed for the hypotheses that DARWIN is filled with natural xenon or \( ^{136} \text{Xe} \)-depleted xenon. We find that while the sensitivity of the experiment with natural xenon is outstanding, a \( ^{136} \text{Xe} \)-depleted setup is significantly more sensitive when it comes to the measurements and searches discussed here. In our discussions, we did not include time-dependent effects (the seasonal and day-night effects). These can impact the sensitivity to new neutrino states within a subset of the parameter space explored here. They would not significantly modify the results discussed here but provide extra handles on the new physics.

**Appendix A: Other Fourth-Neutrino Scenarios**

We restricted our fourth-neutrino analyses to one new neutrino mass-eigenstate \( \nu_4 \) and allowed for only “1–4” sterile mixing, i.e., \( U_{s1} = \sin \theta_{14}, U_{s4} = \cos \theta_{14} \) while \( U_{s2} = U_{s3} = 0 \). Here we discuss some simple generalizations.

The case of one new neutrino mass-eigenstate \( \nu_5 \) that is quasi-degenerate with \( \nu_2 \) and only “2–5” sterile mixing would also be parameterized by a mass-squared difference \( \Delta m^2_{52} \) (positive-definite), assumed to be much smaller than...
\[ \Delta m_{21}^2, \Delta m_{31}^2 \text{ and } |\Delta m_{31}^2|, \text{ and one mixing angle } \theta_{25}: \]

\[ U_{e1}^2 = \cos^2 \theta_{12} \cos^2 \theta_{13}, \quad U_{e2}^2 = \sin^2 \theta_{12} \cos^2 \theta_{13} \cos^2 \theta_{25}, \quad U_{e3}^2 = \sin^2 \theta_{13}, \quad U_{e5}^2 = \sin^2 \theta_{12} \cos^2 \theta_{13} \sin^2 \theta_{25}, \quad (A.1) \]
\[ U_{e1}^2 = 0, \quad U_{e2}^2 = \sin^2 \theta_{25}, \quad U_{e3}^2 = 0, \quad U_{e5}^2 = \cos^2 \theta_{25}. \quad (A.2) \]

Similar to Eq. (IV.3), and the equivalent expressions for \( P_{ee} \) and \( P_{es} \), here

\[
\begin{align*}
P_{ee} &= |U_{e1}|^4 + |U_{e3}|^4 + (1 - |U_{e1}|^2 - |U_{e3}|^2)P_{ee}^2(\Delta m_{25}^2, \sin^2 \theta_{25}, V_{25}^{\text{eff}}), \\
P_{es} &= (1 - |U_{e1}|^2 - |U_{e3}|^2)(1 - P_{ee}^2(\Delta m_{25}^2, \sin^2 \theta_{25}, V_{25}^{\text{eff}})), \\
P_{ea} &= 1 - P_{ee} - P_{es}, \tag{A.3} \end{align*}
\]

where \( P_{2ef}^2 \) is the survival probability obtained in the scenario where there are only two flavors, \( \nu_e^2f \) and \( \nu_s^2f \), characterized by the mass-squared difference \( \Delta m_{25}^2 \) and the mixing angle \( \theta_{25} \), defined via \( \nu_e^2f = \cos \theta_{25} \nu_2 + \sin \theta_{25} \nu_5 \). Here, the effective matter potential is

\[
V_{25}^{\text{eff}} = \sqrt{2}G_F \left( N_e \cos^2 \theta_{13} \sin^2 \theta_{12} - \frac{1}{2} N_n \right). \tag{A.6}
\]

On the other hand, the case of one new neutrino mass-eigenstate \( \nu_6 \) that is quasi-degenerate with \( \nu_3 \) and only “3-6” sterile mixing would also be parameterized by a mass-squared difference \( \Delta m_{63}^2 \) (positive-definite), assumed to be much smaller than \( \Delta m_{21}^2, |\Delta m_{31}^2| \) and \( |\Delta m_{25}^2| \), and one mixing angle \( \theta_{36} \):

\[
\begin{align*}
U_{e1}^2 &= \cos^2 \theta_{12} \cos^2 \theta_{13}, \quad U_{e2}^2 = \sin^2 \theta_{12} \cos^2 \theta_{13} \cos^2 \theta_{36}, \quad U_{e3}^2 = \sin^2 \theta_{13} \cos^2 \theta_{36}, \quad U_{e5}^2 = \sin^2 \theta_{13} \sin^2 \theta_{36}, \quad (A.7) \\
U_{e1}^2 &= 0, \quad U_{e2}^2 = 0, \quad U_{e3}^2 = \sin^2 \theta_{36}, \quad U_{e5}^2 = \cos^2 \theta_{36}. \quad (A.8) \end{align*}
\]

Similar to Eq. (IV.3), and the equivalent expressions for \( P_{ee} \) and \( P_{es} \), here

\[
\begin{align*}
P_{ee} &= |U_{e1}|^4 + |U_{e2}|^4 + (1 - |U_{e1}|^2 - |U_{e2}|^2)P_{ee}^2(\Delta m_{63}^2, \sin^2 \theta_{36}, V_{36}^{\text{eff}}), \\
P_{es} &= (1 - |U_{e1}|^2 - |U_{e2}|^2)(1 - P_{ee}^2(\Delta m_{63}^2, \sin^2 \theta_{36}, V_{36}^{\text{eff}})), \\
P_{ea} &= 1 - P_{ee} - P_{es}, \tag{A.9} \end{align*}
\]

where \( P_{2ef}^2 \) is the survival probability obtained in the scenario where there are only two flavors, \( \nu_e^2f \) and \( \nu_s^2f \), characterized by the mass-squared difference \( \Delta m_{63}^2 \) and the mixing angle \( \theta_{36} \), defined via \( \nu_e^2f = \cos \theta_{36} \nu_3 + \sin \theta_{36} \nu_6 \). Here, the effective matter potential is

\[
V_{36}^{\text{eff}} = \sqrt{2}G_F \left( N_e \sin^2 \theta_{13} - \frac{1}{2} N_n \right). \tag{A.10}
\]

Qualitatively, the three scenarios \(-1-4, 2-5, 3-6\) are identical modulo relabelings of the mixing parameters. Quantitatively, however, there are significant differences. The coefficients of the \( P_{2ef}^2 \) term in Eqs. (IV.3), (A.3), (A.9) are, respectively, \( (\cos^2 \theta_{12} \cos^2 \theta_{13})^2 \sim 0.5, (\sin^2 \theta_{12} \cos^2 \theta_{13})^2 \sim 0.1, \) and \( (\sin^2 \theta_{13})^2 \sim 0.0005 \). These numbers define the maximum deviation of \( P_{ee} \) from expectations from the three-massive-neutrinos paradigm, \( P_{ee} \sim 0.55 \). Hence, very generically, 1–4 effects can be very strong, as discussed in the text, 2–5 effects are at most of order 20%, and 3–6 effects are at the permille level. On the other hand, the effective potentials are also quantitatively very different.

The charged-current contribution to \( V_{25}^{\text{eff}} \) (Eq. (A.6)) is suppressed relative to the neutral-current one by a factor \( \sin^2 \theta_{12} \cos^2 \theta_{13} \sim 0.3 \). Since \( N_n/N_e \) varies between, roughly, 0.5 and less than 0.1 between the center of the Sun and its edge, \( V_{25}^{\text{eff}} \) is significantly smaller than \( V_{36}^{\text{eff}} \), almost vanishing at the Sun’s core, when the charged- and neutral-current contributions, accidentally, almost cancel out one another. \( V_{36}^{\text{eff}} \) (Eq. (A.12)), instead, is solidly dominated by the neutral-current matter potential since the charged-current contribution is suppressed by \( \sin^2 \theta_{13} \sim 0.02 \). Not only is it smaller than \( V_{36}^{\text{eff}} \), it has the opposite sign, a fact that qualitatively impact the behavior of \( P_{2ef}^2 \).

The scenario where all neutrinos are pseudo-Dirac fermions is equivalent to the combination of the 1–4, 2–5, and 3–6 scenarios spelled out above (see, for example, [9]). Note that such a combination is straightforward; the effects of the different contributions simply “add up” without too much interference, as long as the new mass-squared differences are “isolated enough,” i.e., the three new mass-squared differences \( \Delta m_{11}^2, \Delta m_{52}^2, \) and \( \Delta m_{63}^2 \) are much smaller than all other mass-squared differences. For example,

\[
P_{ee} = (\cos^2 \theta_{12} \cos^2 \theta_{13})^2 P_{ee}^2(\Delta m_{11}^2, \sin^2 \theta_{14}, V_{14}^{\text{eff}}) + (\sin^2 \theta_{12} \sin^2 \theta_{13})^2 P_{ee}^2(\Delta m_{52}^2 \sin^2 \theta_{25}, V_{25}^{\text{eff}}) + (\sin^2 \theta_{13})^2 P_{ee}^2(\Delta m_{63}^2, \sin^2 \theta_{36}, V_{36}^{\text{eff}}), \tag{A.13}
\]
where $V_{\text{eff}}^{14}$ is given by Eq. (IV.6).

**Acknowledgements**

We thank Pedro Machado for discussions of potential uses of DARWIN data for neutrino physics. This work was supported in part by the US Department of Energy (DOE) grant #de-sc0010143 and in part by the NSF grant PHY-1630782. The document was prepared using the resources of the Fermi National Accelerator Laboratory (Fermilab), a DOE, Office of Science, HEP User Facility. Fermilab is managed by Fermi Research Alliance, LLC (FRA), acting under Contract No. DE-AC02-07CH11359. This material is based upon work supported by the NSF grant AST-1757792, a Research Experience for Undergraduates grant awarded to the Center for Interdisciplinary Exploration and Research in Astrophysics (CIERA) at Northwestern University. IMS is supported by the Faculty of Arts and Sciences of Harvard University.

[1] J. Aalbers et al. (DARWIN), “Solar neutrino detection sensitivity in DARWIN via electron scattering,” Eur. Phys. J. C 80, 1133 (2020), 2006.03114.

[2] J. N. Bahcall, M. H. Pinsonneault, and S. Basu, “Solar models: Current epoch and time dependences, neutrinos, and helioseismological properties,” Astrophys. J. 555, 990 (2001), astro-ph/0010346.

[3] A. M. Serenelli, W. C. Haxton, and C. Pena-Garay, “Solar models with accretion. I. Application to the solar abundance problem,” Astrophys. J. 743, 24 (2011), 1104.1639.

[4] N. Vinyoles, A. M. Serenelli, F. L. Villante, S. Basu, J. Bergström, M. C. Gonzalez-Garcia, M. Maltoni, C. Peña Garay, and N. Song, “A new Generation of Standard Solar Models,” Astrophys. J. 835, 202 (2017), 1611.09867.

[5] G. Bellini et al. (BOREXINO), “Neutrinos from the primary proton–proton fusion process in the Sun,” Nature 512, 383 (2014).

[6] L. Wolfenstein, “Different Varieties of Massive Dirac Neutrinos,” Nucl. Phys. B 186, 147 (1981).

[7] S. T. Petcov, “On Pseudodirac Neutrinos, Neutrino Oscillations and Neutrinoless Double beta Decay,” Phys. Lett. B 110, 245 (1982).

[8] S. M. Bilenky and B. Pontecorvo, “Neutrino Oscillations With Large Oscillation Length in Spite of Large (Majorana) Neutrino Masses?,” Sov. J. Nucl. Phys. 38, 245 (1983).

[9] A. de Gouvêa, W.-C. Huang, and J. Jenkins, “Pseudo-Dirac Neutrinos in the New Standard Model,” Phys. Rev. D 80, 073007 (2009), 0906.1611.

[10] A. Donini, P. Hernandez, J. Lopez-Pavon, and M. Maltoni, “Minimal models with light sterile neutrinos,” JHEP 07, 105 (2011), 1106.0064.

[11] A. de Gouvêa, I. Martinez-Soler, Y. F. Perez-Gonzalez, and M. Sen, “Fundamental physics with the diffuse supernova background neutrinos,” Phys. Rev. D 102, 123012 (2020), 2007.13748.

[12] I. Martinez-Soler, Y. F. Perez-Gonzalez, and M. Sen, “SN1987A still shining: A Quest for Pseudo-Dirac Neutrinos,” (2021), 2105.12736.

[13] J. Aalbers et al. (DARWIN), “DARWIN: towards the ultimate dark matter detector,” JCAP 11, 017 (2016), 1606.07001.

[14] P. A. Zyla et al. (Particle Data Group), “Review of Particle Physics,” PTEP 2020, 083C01 (2020).

[15] I. Esteban, M. C. Gonzalez-Garcia, M. Maltoni, T. Schwetz, and A. Zhou, “The fate of hints: updated global analysis of three-flavor neutrino oscillations,” JHEP 09, 178 (2020), 2007.14792.

[16] B. Aharmim et al. (SNO), “Combined Analysis of all Three Phases of Solar Neutrino Data from the Sudbury Neutrino Observatory,” Phys. Rev. C 88, 025501 (2013), 1109.0763.

[17] K. Abe et al. (Super-Kamiokande), “Solar Neutrino Measurements in Super-Kamiokande-IV,” Phys. Rev. D 94, 052010 (2016), 1606.07538.

[18] H. Nunokawa, S. J. Parke, and R. Zukanovich Funchal, “What fraction of boron-8 solar neutrinos arrive at the earth as a mu(2) mass eigenstate?”, Phys. Rev. D 74, 013006 (2006), hep-ph/0601198.

[19] G. Bak et al. (RENO), “Measurement of Reactor Antineutrino Oscillation Amplitude and Frequency at RENO,” Phys. Rev. Lett. 121, 201801 (2018), 1806.00248.

[20] D. Adey et al. (Daya Bay), “Measurement of the Electron Antineutrino Oscillation with 1958 Days of Operation at Daya Bay,” Phys. Rev. Lett. 121, 241805 (2018), 1809.02261.

[21] H. de Kerret et al. (Double Chooz), “Double Chooz $\theta_{13}$ measurement via total neutron capture detection,” Nature Phys. 16, 558 (2020), 1901.09445.

[22] P. Adamson et al. (NOvA), “First measurement of electron neutrino appearance in NOvA,” Phys. Rev. Lett. 116, 151806 (2016), 1601.05022.

[23] K. Abe et al. (T2K), “Measurement of neutrino and antineutrino oscillations by the T2K experiment including a new additional sample of $\nu_e$ interactions at the far detector,” Phys. Rev. D 96, 092006 (2017), [Erratum: Phys.Rev.D 98, 019902 (2018)], 1707.01048.
[24] A. de Gouvêa and K. J. Kelly, “Neutrino vs. Antineutrino Oscillation Parameters at DUNE and Hyper-Kamiokande,” Phys. Rev. D 96, 095018 (2017), 1709.06090.

[25] A. de Gouvêa, A. Friedland, and H. Murayama, “The Dark side of the solar neutrino parameter space,” Phys. Lett. B 490, 125 (2000), hep-ph/0002064.

[26] C. Giunti and K. C. Wook, Fundamentals of Neutrino Physics and Astrophysics (Oxford Univ., Oxford, 2007), URL https://cds.cern.ch/record/1053706.

[27] F. An et al. (JUNO), “Neutrino Physics with JUNO,” J. Phys. G 43, 030401 (2016), 1507.05613.

[28] K. Goldhagen, M. Maltoni, S. Reichard, and T. Schwetz, “Testing sterile neutrino mixing with present and future solar neutrino data,” (2021), 2109.14898.

[29] S. T. Petcov, “Exact analytic description of two neutrino oscillations in matter with exponentially varying density,” Phys. Lett. B 200, 373 (1988).

[30] A. de Gouvêa, in Theoretical Advanced Study Institute in Elementary Particle Physics: Physics in $D \geq 4$ (2004), hep-ph/0411274.

[31] A. Friedland, “On the evolution of the neutrino state inside the sun,” Phys. Rev. D 64, 013008 (2001), hep-ph/0010231.

[32] P. C. de Holanda and A. Y. Smirnov, “Homestake result, sterile neutrinos and low-energy solar neutrino experiments,” Phys. Rev. D 69, 113002 (2004), hep-ph/0307266.

[33] P. C. de Holanda and A. Y. Smirnov, “Solar neutrino spectrum, sterile neutrinos and additional radiation in the Universe,” Phys. Rev. D 83, 113011 (2011), 1012.5627.