Optimal Capacitor Placement in a Radial Distribution System using Plant Growth Simulation Algorithm

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Abstract—This paper presents a new and efficient approach for capacitor placement in radial distribution systems that determine the optimal locations and size of capacitor with an objective of improving the voltage profile and reduction of power loss. The solution methodology has two parts: in part one the loss sensitivity factors are used to select the candidate locations for the capacitor placement and in part two a new algorithm that employs Plant growth Simulation Algorithm (PGSA) is used to estimate the optimal size of capacitors at the optimal buses determined in part one. The main advantage of the proposed method is that it does not require any external control parameters. The other advantage is that it handles the objective function and the constraints separately, avoiding the trouble to determine the barrier factors. The proposed method is applied to 9, 34, and 85-bus radial distribution systems. The solutions obtained by the proposed method are compared with other methods. The proposed method has outperformed the other methods in terms of the quality of solution.

Keywords—Distribution systems, Capacitor placement, loss reduction, Loss sensitivity factors, PGSA.

I. INTRODUCTION

The loss minimization in distribution systems has assumed greater significance recently since the trend towards distribution automation will require the most efficient operating scenario for economic viability variations. Studies have indicated that as much as 13% of total power generated is wasted in the form of losses at the distribution level [1]. To reduce these losses, shunt capacitor banks are installed on distribution primary feeders. The advantages with the addition of shunt capacitors banks are to improve the power factor, feeder voltage profile, Power loss reduction and increases available capacity of feeders. Therefore it is important to find optimal location and sizes of capacitors in the system to achieve the above mentioned objectives.

Since, the optimal capacitor placement is a complicated combinatorial optimization problem, many different optimization techniques and algorithms have been proposed in the past. Schmill [2] developed a basic theory of optimal capacitor placement. He presented his well known 2/3 rule for the placement of one capacitor assuming a uniform load and a uniform distribution feeder. Duran et al [3] considered the capacitor sizes as discrete variables and employed dynamic programming to solve the problem. Grainger and Lee [4] developed a nonlinear programming based method in which capacitor location and capacity were expressed as continuous variables. Grainger et al [5] formulated the capacitor placement and voltage regulators problem and proposed decoupled solution methodology for general distribution system. Baran and Wu [6, 7] presented a method with mixed integer programming. Sundharajan and Pahwa [8] proposed the genetic algorithm approach to determine the optimal placement of capacitors based on the mechanism of natural selection. In most of the methods mentioned above, the capacitors are often assumed as continuous variables. However, the commercially available capacitors are discrete. Selecting integer capacitor sizes closest to the optimal values found by the continuous variable approach may not guarantee an optimal solution [16]. Therefore the optimal capacitor placement should be viewed as an integer-programming problem, and discrete capacitors are considered in this paper. As a result, the possible solutions will become a very large number even for a medium-sized distribution system and makes the solution searching process become a heavy burden.

In this paper, Capacitor Placement and Sizing is done by Loss Sensitivity Factors and Plant Growth Simulation Algorithm (PGSA) respectively. The loss sensitivity factor is able to predict which bus will have the biggest loss reduction when a capacitor is placed. Therefore, these sensitive buses can serve as candidate locations for the capacitor placement.

PGSA is used for estimation of required level of shunt capacitive compensation to improve the voltage profile of the system. The proposed method is tested on 9, 34 and 85 bus radial distribution systems and results are very promising.

The advantages with the Plant Growth Simulation algorithm (PGSA) is that it treats the objective function and constraints separately, which averts the trouble to determine the barrier factors and makes the increase/decrease of constraints convenient, and that it does not need any external parameters such as crossover rate, mutation rate, etc. It adopts a guiding search direction that changes dynamically as the change of the objective function.

The remaining part of the paper is organized as follows: Section II gives the problem formulation; Section III sensitivity analysis and loss factors; Sections IV gives brief description of the plant growth simulation algorithm; Section...
V develops the test results and Section VI gives conclusions.

II. PROBLEM FORMULATION

The objective of capacitor placement in the distribution system is to minimize the annual cost of the system, subjected to certain operating constraints and load pattern. For simplicity, the operation and maintenance cost of the capacitor placed in the distribution system is not taken into consideration. The three-phase system is considered as balanced and loads are assumed as time invariant.

Mathematically, the objective function of the problem is described as:

$$\min f = \min(COST)$$

where COST is the objective function which includes the cost of power loss and the capacitor placement.

The voltage magnitude at each bus must be maintained within its limits and is expressed as:

$$V_{\text{min}} \leq V_i \leq V_{\text{max}}$$

where $V_i$ is the voltage magnitude of bus $i$, $V_{\text{min}}$ and $V_{\text{max}}$ are bus minimum and maximum voltage limits, respectively.

The power flows are computed by the following set of simplified recursive equations derived from the single-line diagram depicted in Fig. 1.

$$P_{i+1} = P_i - P_{Li,i+1} - R_{i,i+1} \cdot \frac{(P_i^2 + Q_i^2)}{V_i^2}$$

$$Q_{i+1} = Q_i - Q_{Li,i+1} - X_{i,i+1} \cdot \frac{(P_i^2 + Q_i^2)}{V_i^2}$$

$$|V_{i+1}|^2 = |V_i|^2 - 2(R_{i,i+1}P_i + X_{i,i+1}Q_i) + (R_{i,i+1}^2 + X_{i,i+1}^2) \cdot \frac{(P_i^2 + Q_i^2)}{|V_i|^2}$$

where $P_i$ and $Q_i$ are the real and reactive powers flowing out of bus $i$, and $P_{Li}$ and $Q_{Li}$ are the real and reactive load powers at bus $i$. The resistance and reactance of the line section between buses $i$ and $i+1$ are denoted by $R_{i,i+1}$ and $X_{i,i+1}$, respectively.

The power loss of the line section connecting buses $i$ and $i+1$ may be computed as

$$P_{\text{Loss}}(i,i+1) = R_{i,i+1} \cdot \frac{(P_i^2 + Q_i^2)}{|V_i|^2}$$

The total power loss of the feeder, $P_{T,\text{Loss}}$, may then be determined by summing up the losses of all line sections of the feeder, which is given as

$$P_{T,\text{Loss}} = \sum_{i=0}^{n-1} P_{\text{Loss}}(i,i+1)$$

Considering the practical capacitors, there exists a finite number of standard sizes which are integer multiples of the smallest size $Q_0^C$. Besides, the cost per kVar varies from one size to another.

In general, capacitors of larger size have lower unit prices. The available capacitor size is usually limited to

$$Q_{\text{max}} = LQ_0^C$$

where $L$ is an integer. Therefore, for each installation location, there are $L$ capacitor sizes $\{Q_0^C, 2Q_0^C, 3Q_0^C, \ldots, LQ_0^C\}$ available. Given the annual installation cost for each compensated bus, the total cost due to capacitor placement and power loss change is written as

$$\text{COST} = K_p P_{T,\text{Loss}} + \sum_{i=1}^{n} (K_{cf} + K_i Q_i^C)$$

where $n$ is number of candidate locations for capacitor placement, $K_p$ is the equivalent annual cost per unit of power loss in $$/kW\cdot\text{year}$; $K_{cf}$ is the fixed cost for the capacitor placement. The constant $K_i$ is the annual capacitor installation cost, and, $i = 1, 2, \ldots, n$ are the indices of the buses selected for compensation. The bus reactive compensation power is limited to

$$Q_i^C \leq \sum_{i=1}^{n} Q_{Li}$$

where $Q_i^C$ and $Q_{Li}$ are the reactive power compensated at bus $i$ and the reactive load power at bus $i$, respectively.

III. SENSITIVITY ANALYSIS AND LOSS SENSITIVITY FACTORS

The candidate nodes for the placement of capacitors are
determined using the loss sensitivity factors. The estimation of these candidate nodes basically helps in reduction of the search space for the optimization procedure.

Consider a distribution line with an impedance $R+jX$ and a load of $P_{\text{eff}}+jQ_{\text{eff}}$ connected between ‘$p$’ and ‘$q$’ buses as given below.

\[
\begin{array}{c}
P^{p} \\
\downarrow \quad \downarrow \quad \downarrow \\
R+jX \\
\downarrow \quad \downarrow \quad \downarrow \\
\text{k}^{\text{th}}\text{-Line} \\
\downarrow \quad \downarrow \quad \downarrow \\
P_{\text{eff}}+jQ_{\text{eff}} \quad \quad \quad q
\end{array}
\]

Active power loss in the $k^{\text{th}}$ line is given by,

\[
[P_{\text{lineloss}}[q] = \frac{(P_{\text{eff}}[q]+Q_{\text{eff}}[q])R[k]}{(V[q])^2}]
\]

Similarly the reactive power loss in the $k^{\text{th}}$ line is given by

\[
[Q_{\text{lineloss}}[q] = \frac{(P_{\text{eff}}[q]+Q_{\text{eff}}[q])X[k]}{(V[q])^2}]
\]

Now, both the Loss Sensitivity Factors can be obtained as shown below:

\[
\partial P_{\text{lineloss}}/\partial Q_{\text{eff}} = \frac{2*Q_{\text{eff}}[q]*R[k]}{(V[q])^2}
\]

\[
\partial Q_{\text{lineloss}}/\partial Q_{\text{eff}} = \frac{2*Q_{\text{eff}}[q]*X[k]}{(V[q])^2}
\]

Candidate Node Selection using Loss Sensitivity Factors:

The Loss Sensitivity Factors ($\partial P_{\text{lineloss}}/\partial Q_{\text{eff}}$) are calculated from the base case load flows and the values are arranged in descending order for all the lines of the given system. A vector bus position ‘bpos[i]’ is used to store the respective ‘end’ buses of the lines arranged in descending order of the values ($\partial P_{\text{lineloss}}/\partial Q_{\text{eff}}$). The descending order of ($\partial P_{\text{lineloss}}/\partial Q_{\text{eff}}$) elements of “bpos[i]” vector will decide the sequence in which the buses are to be considered for compensation. This sequence is purely governed by the ($\partial P_{\text{lineloss}}/\partial Q_{\text{eff}}$) and hence the proposed ‘Loss Sensitive Coefficient’ factors become very powerful and useful in capacitor allocation or Placement. At these buses of ‘bpos[i]’ vector, normalized voltage magnitudes are calculated by considering the base case voltage magnitudes given by (norm[i]=V[i]/0.95). Now for the buses whose norm[i] value is less than 1.01 are considered as the candidate buses requiring the Capacitor Placement. These candidate buses are stored in ‘rank bus’ vector. It is worth note that the ‘Loss Sensitivity factors’ decide the sequence in which buses are to be considered for compensation placement and the ‘norm[i]’ decides whether the buses needs $Q$-Compensation or not. If the voltage at a bus in the sequence list is healthy (i.e., norm[i]>1.01) such bus needs no compensation and that bus will not be listed in the ‘rank bus’ vector. The ‘rank bus’ vector offers the information about the possible potential or candidate buses for capacitor placement. The sizing of Capacitors at buses listed in the ‘rank bus’ vector is done by using Plant Growth Simulation Algorithm.

IV. PLANT GROWTH SIMULATION ALGORITHM

The plant growth simulation algorithm [15] is based on the plant growth process, where a plant grows a trunk from its root; some branches will grow from the nodes on the trunk; and then some new branches will grow from the nodes on the branches. Such process is repeated, until a plant is formed. Based on an analogy with the plant growth process, an algorithm can be specified where the system to be optimized first “grows” beginning at the root of a plant and then “grows” branches continually until the optimal solution is found.

By simulating the growth process of plant phototropism, a probability model is established. In the model, a function $g(Y)$ is introduced for describing the environment of the node $Y$ on a plant. The smaller the value of $g(Y)$, the better the environment of the node for growing a new branch. The outline of the model is as follows: A plant grows a trunk $M$, from its root $B_{\text{r}}$. Assuming there are $k$ nodes $B_{M1}, B_{M2}, B_{M3}, ...., B_{MK}$ that have better environment than the root on the trunk $M$, which means the function $g(Y)$ of the nodes and satisfy $g(B_{M1}) < g(B_{M2})$ then morphactin concentrations $C_{M1}, C_{M2}, ...., C_{MK}$ of nodes $B_{M1}, B_{M2}, B_{M3}, ...., B_{MK}$ are calculated using

\[
C_{Mi} = \frac{g(B_{Mi}) - g(B_{M1})}{\Delta_i} \quad (i = 1,2,.......,k)
\]

\[
\Delta_i = \sum_{i=1}^{k} (g(B_{Mi}) - g(B_{M1}))
\]

Fig. 2 Morphactin concentration state space

The significance of equation (15) is that the morphactin concentration of a node is not only dependent on its environmental information but also depends on the
environmental information of the other nodes in the plant, which really describes the relationship between the morphactin concentration and the environment. From (15), we can derive $\sum_{i=1}^{k} C_{Mi} = 1$, which means that the morphactin concentrations $C_{M1}, C_{M2}, \ldots, C_{MK}$ of nodes $B_{M1}, B_{M2}, B_{M3}, \ldots, B_{MK}$ form a state space shown in Fig. 2. Selecting a random number $\beta$ in the interval $[0, 1]$, $\beta$ is like ball thrown to the interval $[0, 1]$ and will drop into one of $C_{M1}, C_{M2}, \ldots, C_{MK}$ in Fig. 2, then the corresponding node that is called the preferential growth node will take priority of growing a new branch in the next step. In other words, $B_{M1}$ will take priority of growing a new branch if the selected $\beta$ satisfies

$$0 \leq \beta \leq \sum_{i=1}^{q} C_{Mi}(T = 1)$$

or

$$\sum_{i=1}^{T-1} C_{Mi} < \beta \leq \sum_{i=1}^{T} C_{Mi}(T = 2, 3, \ldots, k).$$

For example, if random number $\beta$ drops into $C_{M2}$, which means $\sum_{i=1}^{T-1} C_{Mi} < \beta \leq \sum_{i=1}^{2} C_{Mi}$, then the node $B_{M2}$ will grow a new branch $m$. Assuming there are $q$ nodes $B_{m1}, B_{m2}, B_{m3}, \ldots, B_{mq}$ which have a better environment than the root $B_{m0}$ on the branch $m$, and their corresponding morphactin concentrations are $C_{m1}, C_{m2}, \ldots, C_{mq}$. Now, not only the morphactin concentrations of the nodes on branch $m$ need to be calculated, but also the morphactin concentrations of the nodes except $B_{M2}$ (the morphactin concentration of the node $B_{M2}$ becomes zero after growing the branch $m$) on trunk $M$ need to be recalculated after growing the branch $m$. The calculation can be done using (16), which is gained from (15) by adding the related terms of the nodes on branch $m$ and abandoning the related terms of the node $B_{M2}$.

$$C_{Mi} = \frac{g(B_{m}) - g(B_{Mi})}{\Delta_1 + \Delta_2} \quad (i = 1, 3, \ldots, k)$$

$$C_{mj} = \frac{g(B_{m}) - g(B_{mj})}{\Delta_1 + \Delta_2} \quad (j = 1, 2, \ldots, q)$$

(16)

where

$$\Delta_1 = \sum_{i=1, i \neq 2}^{k} (g(B_{m}) - g(B_{Mi}))$$

$$\Delta_2 = \sum_{j=1}^{q} (g(B_{m}) - g(B_{mj})).$$

We can also derive $\sum_{i=1}^{k} C_{Mi} + \sum_{j=1}^{q} C_{mj} = 1$ from (16). Now, the morphactin concentrations of the nodes (except $B_{M2}$) on trunk $M$ and branch $m$ will form a new state space (the shape is the same as Fig. 2, only the nodes are more than that in Fig. 2). A new preferential growth node, on which a new branch will grow in the next step, can be gained in a similar way as $B_{M2}$.

Such process is repeated until there is no new branch to grow, and then a plant is formed.

From the viewpoint of optimal mathematics, the nodes on a plant can express the possible solutions; $g(Y)$ can express the objective function; the length of the trunk and the branch can express the search domain of possible solutions; the root of a plant can express the initial solution; the preferential growth node corresponds to the basic point of the next searching process. In this way, the growth process of plant phototropism can be applied to solve the problem of integer programming.

A complete algorithm for the proposed method of capacitor placement is given below:

1. input the system data such as line and load details of the distribution system, constraints limits etc.;
2. form the search domain by giving the range of capacitor ratings (kVAr ratings) available which corresponds to the length of the trunk and the branch of a plant;
3. give the initial solution $X_0$ ($X_0$ is vector) which corresponds to the root of a plant, and calculate the initial value objective function (power loss);
4. let the initial value of the basic point $X^*$, which corresponds to the initial preferential growth node of a plant, and the initial value of optimization $X^{best}$ equal to $X_0$, and let $F^{best}$ that is used to save the objective function value of the best solution $X^{best}$ be equal to $f(X_0)$, namely, $X^* = X^{best} = X_0$ and $F^{best} = f(X_0)$;
5. identify the candidate buses for capacitor placement using Loss Sensitivity Factors;
6. initialize iteration count, $i=1$;
7. for $j=n$ to $m$ (with step size 1), where $m$ is the minimum available size and $n$ is maximum available size;
8. search for new feasible solutions: place kVAr at sensitive nodes in a sequence starting from basic point $X^*=[X_1^*, X_2^*, \ldots, X_n^*]$, where $X^*$ corresponds to the initial kVAR;
9. for each solution $X^*$ in step 8, calculate the nodes voltages of the buses;
10. if the node voltage constraints is satisfied go to step 10; otherwise abandon the possible solution $X^*$ and goto step 12;
11. calculate powerloss $f(X^*)$ for each solution of $X^*$ in step 8 and compare with $f(X_0)$. Save the feasible solutions if $f(X^*)$ less than $f(X_0)$; otherwise goto step 12;
12. if $i \geq N_{max}$ go to step 16; otherwise goto step 14;
13. calculate the probabilities $C_1, C_2, \ldots, C_k$ of feasible solutions $X_1, X_2, \ldots, X_k$, by using equation (15), which corresponds to determining the morphatin concentration of the nodes of a plant;
14. calculate the accumulating probabilities $\Sigma C_1, \Sigma C_2, \ldots, \Sigma C_k$ of the solutions $X_1, X_2, \ldots, X_k$. Select a random number
Input data (line and load data and constraint limits)

Initialize $X_0$ and compute $f(X_0)$ Assign $X_b = X_0$

Sensitivity analysis for identifying potential locations and initialize $i = 1$

For $j = m$ to $n$ (capacitor min. and max. Size)
Place capacitors at sensitivity nodes and search for new feasible solutions

Node voltage computation

Voltages violating the constraints?

Yes

Compute $f(X')$ for all possible solutions of $[X']$

For $j = i + 1$

No

No

Save possible feasible solution set

Abandon possible solution $[X']$

Probabilities of all feasible solutions

New basic point $X^*$ for next iteration

Stop

Fig. 3 Flow chart for proposed method

V. TEST RESULTS

The proposed method has been programmed using MATLAB and run on a Pentium IV, 3-GHz personal computer with 0.99 GB RAM. The effectiveness of the proposed method for loss reduction by capacitor placement is tested on 9 bus, 34 bus and 85 bus radial distribution systems. The results obtained in these methods are explained in the following sections.

A. 9 - Bus system

The first test case for the proposed method is a 10-bus, single feeder, radial distribution system [16] shown in Fig. 4. This system has zero laterals. The rated line voltage of the system is 23 kV. The details of the feeder and the load characteristics are given in Table 1.

![Fig. 4 A 9-Section feeder](image)

TABLE I. LOAD AND LINE DATA OF 9-BUS SYSTEM

| Line No. | From Bus, i | To Bus, i+1 | $R_{i,i+1}$ (Ω) | $X_{i,i+1}$ (Ω) | $P_i$ (kW) | $Q_i$ (kVAR) |
|----------|-------------|-------------|----------------|----------------|------------|-------------|
| 1        | 0           | 1           | 0.1233         | 0.4127         | 1840       | 460         |
| 2        | 1           | 2           | 0.0140         | 0.6057         | 980        | 340         |
| 3        | 2           | 3           | 0.7463         | 1.2050         | 1790       | 446         |
| 4        | 3           | 4           | 0.6984         | 0.6084         | 1598       | 1840        |
| 5        | 4           | 5           | 1.9831         | 1.7276         | 1610       | 600         |
| 6        | 5           | 6           | 0.9053         | 0.7886         | 780        | 110         |
| 7        | 6           | 7           | 2.0552         | 1.1640         | 1150       | 60          |
| 8        | 7           | 8           | 4.7953         | 2.7160         | 980        | 130         |
| 9        | 8           | 9           | 5.3434         | 3.0264         | 1640       | 200         |

For this test feeder, $K_P$ is selected is selected to be 168 $$/kW-year$ [16]. Commercially available capacitors sizes with $$/kVAR are used in the analysis. Table 2 shows the example of such data.

TABLE II AVAILABLE THREE PHASE CAPACITOR SIZES AND COSTS

| Size (kVAR) | Cost ($) |
|------------|----------|
| 150        | 750      |
| 300        | 975      |
| 450        | 1140     |
| 600        | 1320     |
| 900        | 1650     |
| 1200       | 2040     |

Only fixed capacitors are used in the analysis and the marginal cost of capacitors ($K_i$) [18] given in Table 3 are used to compute the total annual cost. The fixed cost of the capacitor, $K_f$, is selected as $1000 [20]$ with a life expectancy of ten years (the maintenance and running costs are neglected). The substation voltage (bus 1) is considered as 1.0 p.u. The limit of voltage magnitude is taken between 0.90 ~ 1.10 p.u. The method of sensitive analysis is used to select the candidate installation locations of the capacitors to reduce the search space. The buses are ordered according to their sensitivity value ($\frac{f(X^*)}{\partial Q_{eff}}$) (i.e., bus 6, 5, 9, 10, 8 and 7). Top four buses are selected as optimal candidate locations and then amount of kVAR to be injected in the selected buses is optimized by PGSA.
Using this method, the capacitors of rating 1200, 1200, 200, 407 kVAR are placed at the optimal candidate locations 6, 5, 9, and 10 respectively. The initial power loss is 783.77 kW and it is reduced to 694.93 kW after capacitor placement using the proposed method. The results of the proposed method are shown in Table 4. Table 4 also shows the comparison of results with Fuzzy reasoning [19] and Particle Swarm Optimization (PSO) [21]. The minimum and maximum voltages before capacitor placement are 0.8375 p.u (bus 10) and 0.9929 p.u (bus 2) and these are improved to 0.901 p.u (bus 10) and 0.9991 p.u (bus 2) after capacitors placement.

From Table 4, it is observed that the power loss obtained with the proposed method is less than the Fuzzy reasoning [19] and PSO [21]. The optimal candidate locations are the same with all methods but the total kVAr injected by the proposed method is less than the other two. The selection of the allowable consecutive iterative number \( N_{\text{max}} \) depends highly on the solved problem. The \( N_{\text{max}} \) value is tried from 2 to 25. All of the results converge to the same optimal solution with \( N_{\text{max}} \) greater than 4. The convergence characteristics of power loss of the PGSA in this test system are shown in fig. 5. It can be observed that the number of iterations taken is only four on this computer. The CPU time needed is only 0.6 seconds.

### Table III Possible sizes of capacitors and sizes in $$/kVAr

| \( Q^c_j \) | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
|---|---|---|---|---|---|---|---|
| S/kVAR | 150 | 300 | 450 | 600 | 750 | 900 | 1050 |
| \( Q^c_j \) | 1200 | 1350 | 1500 | 1650 | 1800 | 1950 | 2100 |
| S/kVAR | 0.170 | 0.207 | 0.201 | 0.193 | 0.187 | 0.211 | 0.176 |
| \( Q^c_j \) | 2250 | 2400 | 2550 | 2700 | 2850 | 3000 | 3150 |
| S/kVAR | 0.197 | 0.170 | 0.189 | 0.187 | 0.183 | 0.180 | 0.195 |
| \( Q^c_j \) | 3300 | 3450 | 3600 | 3750 | 3900 | 4050 | -- |
| S/kVAR | 0.174 | 0.188 | 0.170 | 0.183 | 0.182 | 0.179 | -- |

### Table IV Simulation results of 9-bus system

| Items | Un-compensated | Compensated | Fuzzy reasoning [19] | PSO [21] | Proposed |
|---|---|---|---|---|---|
| Total losses (kW) | 783.77 | 704.883 | 696.21 | 694.93 |
| Loss reduction (%) | --- | 10.065 | 11.17 | 11.33 |
| Optimal locations and size in kVAR | --- | 4 1050 6 1174 6 1200 | 5 1050 5 1182 5 1200 | 6 1950 9 264 9 200 | 10 900 10 566 10 407 |
| Total kVAR | 4950 | 3186 | 3007 |
| Annual Cost ($/year) | 131,674 | 119,420 | 118,582 | 118,340 |
| Net Savings ($/year) | --- | 12,255 | 13,091 | 13,334 |
| %Saving | --- | 9.31 | 9.94 | 10.13 |

From Table 4, it is observed that the power loss obtained with the proposed method is less than the Fuzzy reasoning [19] and PSO [21]. The optimal candidate locations are the same with all methods but the total kVAR injected by the proposed method is less than the other two. The selection of the allowable consecutive iterative number \( N_{\text{max}} \) depends highly on the solved problem. The \( N_{\text{max}} \) value is tried from 2 to 25. All of the results converge to the same optimal solution with \( N_{\text{max}} \) greater than 4. The convergence characteristics of power loss of the PGSA in this test system are shown in fig. 5. It can be observed that the number of iterations taken is only four on this computer. The CPU time needed is only 0.6 seconds.

### B. 34 - Bus system

The second test case for the proposed method is a 34-bus radial distribution system [22]. This system has a main feeder and four laterals (sub-feeders). The single line diagram is shown in fig. 6. The line and load data of the feeders are taken from the reference [22]. The rated line voltage of the system is 11 kV.

![Fig. 6 34 - bus distribution network configuration](image-url)

Similar to test case 1, the sensitive analysis method is used to select the candidate installation locations of the capacitor to reduce the search space. The buses are ordered according to their sensitivity value as {19, 22, 20, 21, 23, 24, 25, 26, and 27}. Top three buses are selected as optimal candidate locations and the amount of kVAr injected are 1200, 639, and 27 kVAR respectively. The constants \( K_p \), \( K_c^t \), and \( K_{cf} \) are same as in test case 1. The power loss before and after capacitor placement are 221.67 and 161.07 kW. The minimum and maximum voltages before capacitor placement are 0.9417 p.u. and 1.000 p.u after capacitor placement. All of the results converge to the same optimal solution with \( N_{\text{max}} \) greater than 8. The results of the proposed method are compared with the results of PSO method [21] and Heuristic based method [22] and is shown in Table 5. The implementation shows that the sub-feeder connected to node 6 of main feeder only need the compensation. From the results shown in Table 5, it is observed that the optimal candidate installation locations are three for the proposed and PSO methods [21], but it is four for the Heuristic method [22].
power loss and net annual savings obtained with proposed method are less than PSO methods and Heuristic based. The CPU time needed by the proposed method is 11 sec.

### Table V Simulation results of 34-bus system

| Items                  | Un-compensated | Heuristic based [19] | PSO [21] | Proposed |
|------------------------|----------------|----------------------|----------|----------|
| Total losses (kW)      | 221.67         | 168.47               | 168.8    | 161.07   |
| Loss reduction (%)     | ---            | 23.999               | 23.850   | 27.337   |
| Optimal locations and Size in kVAR | --- | 26 1400 19 781 | 19 1200 | 11 750 22 803 | 22 639 |
|                        | 17 300         | 20 479               | 20 200   | 4 250    | --- -- | --- |
| Total kVAR             | --             | 2700                 | 2063     | 2039     |
| Annual Cost ($/year)   | 37,241         | 33,182               | 29,936   | 28,484   |
| Net Saving ($/year)    | --             | 4,089                | 7,306    | 8,756    |

C. 85-Bus system

The third test case is 85-bus radial distribution system which is same as in Das et al [23]. The line data and feeder characteristics are taken from reference [23]. Based on sensitivity analysis buses 8, 58, and 7 are selected as optimal candidate locations for the capacitor placement. Using proposed PGSA, the amount of kVAR injected are 1200, 908, and 200 kVAR at the above buses. The constants in the objective function are same as test case 1. The N_max value tried from 2 to 80 and all of the results converge to the same optimal solution with N_max greater than 8. The minimum and maximum voltages before compensation are 0.8877 and 0.9952 p.u and are improved to 0.96571 and 0.9991 p.u respectively. The power loss power loss before and after compensation and net savings are shown in Table 6.

### Table VI Simulation results of 85-bus system

| Items                  | Un-compensated | PSO [21] | Proposed |
|------------------------|----------------|----------|----------|
| Total losses (kW)      | 315.714        | 163.32   | 161.4    |
| Loss reduction (%)     | ---            | 48.27    | 48.88    |
| Optimal locations and Size in kVAR | --- | 8 796 8 1200 | 8 453 58 908 |
|                        | 58 453         | 58 908   | 7 324    | 7 200    |
|                        | 27 300         | 27 901   | 27 901   | 27 901   |
| Total kVAR             | 2473           | 2473     | 2308     |
| Annual Cost ($/year)   | 53,040         | 29,051   | 28,585   |
| Net Saving ($/year)    | ---            | 23,990   | 24,455   |

The Table 6 also shows the comparison of results with the PSO method [21]. From the results it is observed that the power loss after compensation and net annual savings are almost the same, but optimal candidate locations with the proposed method is less than the PSO method. The CPU time needed by the proposed method is 20.4 sec.

VI. CONCLUSION

A new and efficient approach that employs loss sensitivity factors and PGSA for capacitor placement in the distribution system has been proposed. The loss sensitivity factors are used to determine the candidate locations of the buses required for compensation. The PGSA is used to estimate the required level of shunt capacitive compensation at the optimal candidate locations to enhance the voltage profile the system and reduce the active power loss. The simulation results based on 9, 34, 85-bus systems have produced the best solutions that have been found using a number of approaches available in the literature. The advantages of the proposed method are: 1) it handles the objective function and the constraints separately, avoiding the trouble to determine the barrier factors; 2) the proposed approach does not require any external parameters; 3) the proposed approach has a guiding search direction that continuously changes as the change of the objective function. This method places the capacitors at less number of locations with optimum size and offers much net annual saving in initial investment.

REFERENCES

[1] Y. H. Song, G. S. Wang, A. T. Johns and P.Y. Wang, “Distribution network reconfiguration for loss reduction using Fuzzy controlled evolutionary programming,” IEEE Trans. Gener., Trans., Distri., Vol. 144, No.4, July 1997.
[2] J. V. Schmill, “Optimum Size and Location of Shunt Capacitors on Distribution Feeders,” IEEE Transactions on Power Apparatus and Systems, vol. 84, pp. 825-832, September 1965.
[3] H. Dura “Optimum Number Size of Shunt Capacitors in Radial Distribution Feeders: A Dynamic Programming Approach,” IEEE Trans. Power Apparatus and Systems, Vol. 87, pp. 1769-1774, Sep 1968.
[4] J. J. Grainger and S. H. Lee, “Optimum Size and Location of Shunt Capacitors for Reduction of Losses on Distribution Feeders,” IEEE Trans. on Power Apparatus and Systems, Vol. 100, No. 3, pp. 1105-1118, March 1981.
[5] J.J. Grainger and S. Civanlar, “Volt/var control on Distribution systems with lateral branches using shunt capacitors as Voltage regulators-part I, II and III,” IEEE Trans. Power Apparatus and systems, vol. 104, No. 11, pp. 3278-3297, Nov. 1985.
[6] M. E Baran and F. F. Wu, “Optimal Sizing of Capacitors Placed on a Radial Distribution System”, IEEE Trans. Power Delivery, vol. No.1, pp. 1105-1117, Jan. 1989.
[7] M. E. Baran and F. F. Wu, “Optimal Capacitor Placement on radial distribution system,” IEEE Trans. Power Delivery, vol. 4, No.1, pp. 725-734, Jan. 1989.
[8] Sundharajan and A. Pahwa, “Optimal selection of capacitors for radial distribution systems using genetic algorithm,” IEEE Trans. Power Systems, vol. 9, No.3, pp.1499-1507, Aug. 1994.
[9] H. N. Ng, M. M. A. Salama and A. Y. Chikhani , “Capacitor Allocation by Approximate Reasoning: Fuzzy Capacitor Placement,” IEEE Trans. Power Delivery, vol. 15, no.1, pp. 393-398, Jan. 2000.
[10] H.C.Chin, “Optimal Shunt Capacitor Allocation by Fuzzy Dynamic Programming,” Electric Power Systems Research, pp.133-139, Nov. 1995.
[11] N. I. Santos, O. T. Tan, “Neural- Net Based Real- Time Control of Capacitors Installed on Distribution Systems,” IEEE Trans. Power Delivery, vol. PAS-5, No.1, pp. 266-272, Jan. 1990.
[12] M. Kaplan, "Optimization of Number, Location, Size, Control Type and Control Setting Shunt Capacitors on Radial Distribution Feeders," IEEE Trans. on Power Apparatus and System, Vol.103, No.9, pp. 2659-63, Sep 84.
[13] Chun Wang and Hao Zhong Cheng, “Reactive power optimization by plant growth simulation algorithm,” IEEE Trans. on Power Systems, Vol.23, No.1, pp. 119-126, Feb. 2008.
[14] Ji-Ping Chiong, Chuang-Fu Chang and Chung-Tzong Su, “Capacitor placement in large scale distribution system using variable scaling hybrid differential evolution,” Electric Power and Energy Systems, vol. 28, pp.739-745, 2006.
[15] Chun Wang, H. Z. Cheng and L. Z. Yao, “Optimization of network reconfiguration in large distribution systems using plant growth simulation algorithm,” DRPT 2008 Conference, Nanjing, China, pp. 771-774, 6-9, April 2008.
[16] Baghzouz, Y. and Ertem, S., “Shunt capacitor sizing for radial distribution feeders with distorted substation voltages,” *IEEE Trans Power Delivery*, Vol. 5, pp. 650–57, 1990.

[17] J. B. Bunch, R. D. Miller, and J. E. Wheeler, “Distribution system integrated voltage and reactive power control,” *IEEE Trans. Power Apparatus and Systems*, vol. 101, no. 2, pp. 284–289, Feb. 1982.

[18] S. F. Mekhame et al., “New heuristic strategies for reactive power compensation of radial distribution feeders,” *IEEE Trans Power Delivery*, Vol. 17, No. 4, pp. 1128–1135, October 2002.

[19] Su, C. T and Tsai, C. C, “A new fuzzy reasoning approach to optimum capacitor allocation for primary distribution systems,” Proc IEEE on Industrial Technology Conf. 1996; pp. 237–41.

[20] Das, D., “Reactive power compensation for radial distribution networks using genetic algorithms,” *Electric Power and Energy Systems*, vol. 24, pp. 573-581, 2002.

[21] Prakash K. and Sydulu M., “Particle swarm optimization based capacitor placement on radial distribution systems,” *IEEE Power Engineering Society general meeting* 2007, pp. 1-5.

[22] M. Chis, M. M. A. Salama and S. Jayaram, “Capacitor Placement in distribution system using heuristic search strategies,” *IEEE Proc-Gener, Transm, Distrib*, vol. 144, No.3, pp. 225-230, May 1997.

[23] Das et al., “Simple and efficient method for load flow solution of radial distribution network,” *Electric Power and Energy Systems*, vol. 17, No.5, pp. 335-346, 1995.

[24] H. D. Chiang, J. C. Wang, O. Cockings, and H. D. Shin, “Optimal capacitor placements in distribution systems: Part I & Part II,” *IEEE Trans. Power Delivery*, vol. 5, pp. 634–649, Apr. 1990.

[25] M. Jaeger and P. H. De Reffye, “Basic concepts of computer simulation of plant growth,” *Journal of Bioscience (India)*, Vol. 17, No. 3, pp. 275-291, September 1992.

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