Abstract—A dual-band compact-size ring antenna array is integrated with a proposed angle of arrival (AOA) algorithm to estimate directions of arrival sources. The impact of mutual coupling on the direction estimation accuracy is investigated based on the computed coupling matrices for both spectrum frequency bands. The mathematical model of direction of arrival (DOA) is illustrated and an appropriate decoupling method is integrated to eliminate the mutual coupling effects. A MATLAB code is developed to simulate various experimental scenarios. The results before and after compensation of coupling matrix are compared and discussed for several test examples.

Keywords—Dual-band Antenna array; angle of arrival; mutual coupling; tracking systems decoupling method.

I. INTRODUCTION

Direction finding (DF) is one of the most important fields in the array signal processing that has acquired significant attention from researchers and engineers [1-3]. The need for DF comes from the requirements of tracking and locating signal sources in many civilian and military applications [4, 5]. Examples of these applications are radar systems, medical and rescue services. These services mainly depend on the precision of the position estimation technique. The localization of radiating sources at spatially separated sensors has a considerable importance for these applications. For example in wireless communication systems, when DOA of the desired signal and interference signals are estimated properly, an adaptive beamforming technique can be applied to emphasize the gain towards the desired signal suppressing the noise and interference signals [6].

The size of an antenna array is crucial factor in several applications and it should be reduced as small as possible. Typically, localization and tracking systems are desirable when an efficient angle of arrival technique is integrated with small compact omnidirectional antenna arrays. One of the possible approaches that can be used to minimize the size of antenna array is by decreasing the separation distance between antenna array elements. However, the mutual coupling between these elements will increase and this in turn will influence on the estimation accuracy of localization systems negatively. From the electromagnetic point of view, the mutual coupling between the array radiators invalidates the initial calibration and this causes a significant deterioration in the performance of the array signal processing algorithms[7].

It is worthy to mention that the behavior of mutual coupling of an antenna array in the transmitting mode is different from that receiving end significantly [8, 9]. In order to estimate and/or track the directions of sources, firstly, it needs to consider the effects of mutual coupling. Secondly, an appropriate decoupling method has to apply to the covariance matrix in order to compensate the mutual effect. The effect of mutual coupling is often analyzed by computing the mutual impedances between the antenna sensors: the values of these impedances are complex and depend on position and type of antenna array mainly [10]. Many researches and studies have been achieved to minimize and compensate of the effects of mutual coupling by developing and proposing several technique [11-13]. In [13], the authors tested and investigated the performance of the AOA method by using uniform circular array (UCA) consists of 4 monopoles placed on a square metal plate. The error improved approximately by fifty percent after compensation mutual coupling of AOA technique. A compact low profile antenna array has been proposed in [14] to work at resonant frequency 400MHz. This array was used in the receiving end to estimate the DOAs by Multiple Signal Classification (MUSIC) algorithm [15]. The estimation accuracy was improved considerably after compensation the mutual coupling. Therefore, it is substantial to preprocess the covariance matrix before use it in the estimation process.

In this work, this antenna array in [14] has modified to work at dual bands in receiving mode, and then a new efficient AOA algorithm is applied. The proposed AOA approach is less complexity compared to MUSIC method. The rest of the paper is organized as follows: Section II gives the geometry and principal working of the proposed antenna array. The mathematical model and necessary conditions of DOA with circular array are presented section III. The mathematical model of the proposed method is presented in Section IV. Simulation results, discussion appear in Section V. Finally, Section VI summarizes the results and sets out conclusions.

II. ANTENNA ARRAY MODELLING

Although linear array is simple array and easy to implementation but it is not suitable to scenarios where 360 degrees of coverage or both elevation and azimuth for specific coverage area are required. In these situations, the natural choice is a uniform circular array. The spiral antenna that proposed in [14] is adopted here to work on dual bands 400 MHz and 868 MHz. Figure 1 shows the geometry of a circular
array in the x-y plane, where \( r \) is the array radius. The radius of the ring array is calculated based on half wavelength (\( \lambda_1 \)) distances between the 8 elements at 868 MHz as demonstrated by \( d_1 \) shown in Figure 1 and given by:

\[
\lambda_1 = \frac{c}{f_{c_1}} = \frac{3 \times 10^8}{868 \times 10^6} = 34.56 \text{ cm}
\]

\[
d_1 = \frac{\lambda_1}{2} = 17.28 \text{ cm}
\]

\[
r = 22.58 \text{ cm}
\]

Based on the above results the separated distances between the four antenna elements operated at 400 MHz (i.e., \( d_2 = 0.42 \lambda_2 \) at 400 MHz) is

\[
d_2 = \sqrt{r^2 + r^2} = \sqrt{2} \times r = 31.93 \text{ cm}
\]

It should be noted all 8 antenna elements will be used to higher frequency band 868 MHz, whereas four elements will be used for lower frequency band 400 MHz.

The antenna element is adopted from \([14]\), in which it is found appropriate to operate on dual spectrum bandwidth at 400 MHz and 868 MHz. The computed \( S_{11} \) for both bands are shown in Figure 2. The mutual coupling matrices for both bands are computed according to \([16]\) as shown in Tables 1 and 2. The coupling scattering parameters were quite reasonable for which the normalized coupling matrices between the elements for both bands.

### Table 1: Normalized receiving mutual impedances for an 8-element spiral array at 868 MHz.

| Z(8x8) | 1 | -0.55 + /0.19 | 0.044 - /0.45 | 0.023 - /0.91 | -0.42 - /0.019 | -0.16 + /0.44 | -0.12 + /0.13 | -0.13 - /0.24 |
|---|---|---|---|---|---|---|---|---|
| -0.55 + /0.19 | 1 | -0.53 + /0.13 | -0.14 + /0.15 | 0.13 - /0.02 | -0.12 + /0.03 | -0.12 - /0.24 | 0.01 + /0.23 |
| 0.044 - /0.45 | -0.53 + /0.13 | 1 | 0.25 - /0.26 | -0.12 + /0.12 | 0.43 - /0.091 | -0.32 + /0.03 | -0.16 + /0.44 |
| 0.023 - /0.91 | -0.14 + /0.15 | 0.25 - /0.26 | 1 | -0.11 + /0.05 | 0.044 - /0.45 | 0.023 - /0.91 | -0.42 - /0.019 |
| -0.42 - /0.019 | 0.13 - /0.02 | -0.12 + /0.12 | -0.11 + /0.05 | 1 | -0.56 + /0.29 | 0.13 + /0.11 | 0.13 - /0.91 |
| -0.16 + /0.44 | -0.12 + /0.03 | 0.43 - /0.091 | 0.044 - /0.45 | -0.56 + /0.29 | 1 | 0.17 + /0.22 | -0.2 - /0.19 |
| -0.12 + /0.34 | -0.12 + /0.03 | 0.32 + /0.03 | 0.023 - /0.91 | 0.13 + /0.11 | 0.17 + /0.22 | 1 | -0.42 + /0.11 |
| -0.49 + /0.17 | 0.01 + /0.23 | -0.16 + /0.44 | -0.42 - /0.019 | 0.13 - /0.91 | -0.2 - /0.19 | -0.42 + /0.11 | 1 |

### Table 2: Normalized receiving mutual impedances for a 4-element spiral array at 400 MHz.

| Z(4x4) | 1 | -0.013 - /0.22 | -0.02 + /0.091 | -0.201 - /0.202 |
|---|---|---|---|---|
| -0.013 - /0.22 | 1 | -0.013 - /0.22 | -0.02 + /0.091 |
| -0.02 + /0.091 | -0.013 - /0.22 | 1 | -0.013 - /0.22 |
| -0.201 - /0.202 | -0.02 + /0.091 | -0.013 - /0.22 | 1 |

### III. Mathematical Model of Angle of Arrival Methods

Consider there are \( i \) sources incident on the antenna array at an elevation angle (\( \theta \)) and an azimuth angle (\( \phi \)). The time sample of the received signal, \( X(t) \), can be defined by:

\[
X(t) = A(\theta, \phi) S(t) + N(t)
\]

(2)

\( S(t) \) is a sample of the complex vector of \( D \) incident signals:

\[
S(t) = [s_1(t), s_2(t), \ldots, s_D(t)]^T
\]

(3)

\( N(t) \) is an array of Additive White Gaussian Noise (AWGN) for each channel:

\[
N(t) = [n_1(t), n_2(t), \ldots, n_M(t)]
\]

(4)

\( A(\theta, \phi) \) is the \( M \times D \) matrix of steering vectors:

\[
A(\theta, \phi) = [a(\theta_1, \phi_1), a(\theta_2, \phi_2), \ldots, a(\theta_D, \phi_D)]
\]

(5)

The unit vector that includes the direction of \( \theta \) and \( \phi \) and can be defined as follows:
\[ v_{DA} = \cos \theta \sin \phi \hat{a}_x + \sin \theta \sin \phi \hat{a}_y + \cos \theta \hat{a}_z \]  

(6)

Where \( \hat{a}_x, \hat{a}_y \) and \( \hat{a}_z \) are the units vectors for Cartesian coordinates.

![Fig 3. The geometry of circular array located on x-y plane with M-elements.](image)

With a ring array located on x-y plane and phase angle \( \phi_i \), the unit vector from the origin point to the \( i \)th element is given as follows:

\[ v_i = \cos \phi_i \hat{a}_x + \sin \phi_i \hat{a}_y, \quad i = 1, 2, ..., M. \]  

(7)

where \( \phi_i \) is the angular location of each element in the ring array and can be expressed as follows:

\[ \phi_i = \frac{2\pi}{M} (i - 1), \quad i = 1, 2, ..., M. \]  

(8)

The angle \( \gamma_i \) can be obtained from the dot product between unit vectors \( v_i \) and \( v_{DA} \) as described below:

\[ \gamma_i = \cos^{-1} \left( \frac{v_i \cdot v_{DA}}{\|v_i\| \|v_{DA}\|} \right) \]  

(9)

where \( \|v_i\| \) is norm of \( v_i \) and equal to \( \|v_i\| = \sqrt{(\cos \phi_i)^2 + (\sin \phi_i)^2} = 1 \), while \( \|v_{DA}\| \) is norm of \( v_{DA} \) and is given as,

\[ \|v_{DA}\| = \sqrt{(\cos \theta \sin \phi)^2 + (\sin \theta \sin \phi)^2 + (\cos \theta)^2} = 1. \]

For \( \phi \) and \( \gamma \), are vectors with dimension \((1 \times M)\) and they given below:

\[ \Phi = [\phi_1, \phi_2, ..., \phi_M] \]  

(9)

\[ \gamma = [\gamma_1, \gamma_2, ..., \gamma_M] \]  

(10)

In order to compute the phase difference, the time difference of arrival of \( S(t) \) at reference element and each element needs to be calculated. The wave front time delay is calculated using the difference in distance \( \Delta t \) that is given by:

\[ \Delta t = r \cos \gamma = r \cos(\cos^{-1}(\sin \theta \cos(\phi - \phi_i))) = \sin \theta \cos(\phi - \phi_i) \]  

(11)

where \( r \) can be computed as follows:

\[ r = \frac{d}{2 \sin \left( \frac{2\pi}{M} \right)} \]  

(12)

where \( d \) is the separation between adjacent elements.

The phase difference \( \psi_i \) can be expressed in the following formula:

\[ \psi_i = \beta \cdot \Delta t \]  

(13)

Then, the circular array steering vector can be defined as follows:

\[ a(\theta, \phi) = \left[ e^{-j \frac{2\pi}{M} \sin \theta \cos(\phi - \phi_1)} e^{-j \frac{2\pi}{M} \sin \theta \cos(\phi - \phi_2)} \vdots \right] \]  

(14)

The array covariance matrix \( R_{xx} \) can be expressed in the following form \([17]\):

\[ R_{xx} = E[XX^H(t)] \]

(15)

where \( R_{xx} \) is the \((D \times D)\) source signal correlation matrix \( R_{xx} = E[S(t)S(t)^H] \), \( \sigma_n^2 \) is the noise variance and \( I_M \) refers to \( M \times M \) identity matrix. In practice the actual covariance matrix \( R_{xx} \) is not available and therefore we need to estimate it from the received data over \( N \) samples as follows:

\[ \tilde{R}_{xx} \approx \frac{1}{N} \sum_{k=1}^{N} X(t)X^H(t) \]  

(16)

After the covariance matrix is obtained, DOA estimation can be performed using a suitable angle of arrival algorithm.

IV THE PROPOSED AOA METHOD

After we obtained the covariance matrix from previous section, the AOA method is required to find the direction of incoming signals. The proposed algorithm depends on the orthogonality between the partial noise subspace (PNS) and antenna array steering vector to produce the spatial spectra of received array signals. The proposed algorithm depends on the orthogonality between the partial noise subspace (PNS) and antenna array steering vector to produce the spatial spectra of received array signals. The singular value decomposition of \( R_{xx} \) can be expressed as:

\[ R_{xx} = E \Sigma \Sigma^H = [E_N \ E_S] \begin{bmatrix} \Sigma_N & 0 \\ 0 & \Sigma_S \end{bmatrix} \begin{bmatrix} E_N^H \\ E_S^H \end{bmatrix} \]

(17)

where \( E_N \) and \( E_S \) refer to the noise and signal subspace respectively. While \( \Sigma_N = \{\lambda_1, \lambda_2, ..., \lambda_{M-D}\} \) \( \Sigma_S = \{\lambda_1, \lambda_2, ..., \lambda_D\} \). If the eigenvalues are sorted in ascending way (i.e. from smallest to largest) with corresponding eigenvectors:
where \( \lambda_1 < \lambda_2 < \lambda_3 \leq \cdots < \lambda_M \), and the eigenvectors matrix can be defined as:
\[
E = [e_1, e_2, \ldots, e_M]
\]  
(19)
where \( e_i \) is the eigenvector column with size \((1 \times M)\). As the eigenvalues are sorted from smallest to largest, then noise subspaces matrix (i.e. \( E_N \)) is given as:
\[
E_N = [e_1, e_2, \ldots, e_M-D]
\]  
(20)
Where \( E_N \) is noise subspace with size \((M) \times (M-D)\). Typically number of received signals (D) is less than \((M/2)\), therefore we suggest to use only the eigenvectors that corresponding to the least D eigenvalues as described below,
\[
Q = [e_1, e_2, \ldots, e_D]
\]  
(21)
This means we used matrix with dimension \((M \times D)\) instead of using matrix with size \((M) \times (M-D)\). Either if the number of arrival signals is equal or greater than \((M/2)\), then \( Q = E_N \).

Then spatial power spectrum of the partial noise subspace method can be expressed as follows:
\[
P_{PNS}(\theta) = \frac{1}{||a(\theta, \theta)^H Q||^2}
\]  
(22)

IV. COMPUTED RESULTS AND DISCUSSION

Two experiments have been achieved to verify the theoretical claims of proposed system. In the first experiment, a UCA with \( M = 4 \) elements and a half wave spacing between elements is set. Two plane waves \( (D) \) corrupted with AWGN are assumed incident on the antenna array, the carrier frequency \( f_{c1} = 400 \) MHz and the angular range is \([0^\circ - 360^\circ]\). The other simulation parameters are number of snapshots taken \( N = 100 \) and signal to noise ration \( \text{SNR} = 10 \) dB. The effect of mutual coupling matrix has been inserted inside the covariance matrix. The directions of the received signals have been represented with red line, the performance estimation before and after covariance matrix compensation are represented by black and blue curves respectively. The simulation results of the first experiment is presented in Figure 4. As can be seen from this graph, without using decoupling method, the performance estimation of the proposed method gives significant error and in Figure 4.b one of the signals direction is missed. However, after apply decoupling method the estimation resolution is improved significantly and there are sharp peaks towards the AOAs. In the second experiment, a UCA with \( M=8 \) and three signals are assumed impinging on this array, the carrier frequency \( f_{c2} = 868 \) MHz and \( d_2 = 0.42\lambda_2 \). The other simulations parameters are set as same the first experiment. The performance estimation of PNS is shown in Figure 5. Without compensate the mutual coupling, the PNS does not give indication in the some directions of arrival signals and there is deviation from the correct target as can be seen from this figures. In contrast, when a decoupling method is used reliability and estimation precision are improved considerably, where an accurate and sharp peaks are generated in the direction of the received signals.

![Fig 4. The performance estimation of PNS method with M=4, f_c = 400 MHz and two arrival signals.](image-url)
and the European Union’s Horizon 2020 research and innovation programme under grant agreement H2020-MSCA-ITN-2016 SECRET-722424.

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V. CONCLUSIONS

A small low profile spiral dual-band compact-size has been proposed in this paper. Eight elements ring antenna array has been designed to work at GSM network with resonant frequency 868 MHz. The same geometry was developed to work on 400 MHz band by switching off four elements. The coupling matrices for both bands were computed to take into account the effect of mutual coupling. Then, a new-efficient AOA method has been applied to estimate the directions of multiple arrival signals. An efficient decoupling method has been utilized to remove the mutual coupling effects from the covariance matrix. It has been observed that estimation accuracy and reliability improved significantly after compensation mutual coupling.

Acknowledgment

This work was supported partially by Yorkshire Innovation Fund, IETG Ltd. Contract, Research Development Project (RDP) and the European Union’s Horizon 2020 research and