Can Reputation Ensure Efficiency in the Structured Finance Market?

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In Elamin (2013), the credit rating agency (CRA) cannot credibly fully reveal its information about the quality of a rated structured finance project, when ratings are unverifiable. Can the fear of losing its reputation discipline the CRA? In this paper, there is incomplete information about the type of the CRA. With some probability, it can be a truthful type, always fully revealing its information. At every period, the (updated) probability that the CRA is of the truthful type is its reputation. With only two project types and when the CRA's reputation is high enough, an informationally-efficient equilibrium, where investors are fully informed, exists. With more than two project types, no matter how high the CRA’s patience level or its reputation, there is no informationally-efficient equilibrium. The many-project-types case is clearly the relevant case; therefore, I conclude that the fear of losing a reputation is not enough of a deterrent in the structured finance market.

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1 Introduction

The recent financial crisis exposed many problems with the ratings of structured finance products. Criticizing ratings of such products became a common practice, both in the academic literature and the public sphere. Congress conducted hearings questioning credit rating agencies behavior in the crisis. Numerous articles lambasted them for the findings. In the academic literature, among many others, Benmelech et al (2009) discussed the “Credit Rating Crisis”. Coval et al (2009) discussed the economics behind CDOs and showed problems with the Credit Rating Agency’s (CRA’s) models. After the crisis, the CRAs themselves admit that they face a deep crisis, and they acknowledge the need to work on restoring the market’s faith and confidence in their ratings’ credibility.

Before the last crisis, the CRAs insisted adamantly that their business is built on market participants’ trust and confidence in their ratings. The much-repeated claim is that CRAs can not afford not to do a good job rating products because if they do their reputation will collapse. Reputation, as the story goes, is the main and only capital in the credit rating business. If the CRA’s reputation collapses, the ratings then have no credibility and market participants would ignore them. This would deal a fatal blow to the credit rating business. In general, the CRAs were thought to mitigate information asymmetries in financial markets. They independently certified information about issuances, revealing relevant information to the public about rated projects’ probabilities of default. Firms in need of financing used the CRAs as a signaling device. Good firms asked to be rated, providing investors with independent certification of the quality of their projects. But since the financial crisis revealed an across-the-board failure in rating a whole class of assets—structured finance assets–confidence in the CRAs’ ability to rate structured finance products has been severely damaged. Elamin (2013) identified a key feature of structured finance products: their ratings are effectively unverifiable relative to bond ratings. It showed that because structured finance ratings are not verifiable, there is no equilibrium where they fully reveal their information. This paper builds on the unverifiable ratings model of Elamin
(2013) and evaluates whether the CRA’s fear to lose its reputation is potent enough to deter it from misrating structured finance products.

2 Results and Relationship with the Literature

Elamin (2013) considered an infinitely repeated game between a sequence of short-run firms and short-run investors and a long-run CRA. The firms are cash constrained and need to borrow from the investors to finance their projects. Projects could be either good or bad. They only differ in their probability of default and hence are ex-post undistinguishable. Projects pay zero in default and the same amount when they do not default. A new project is selected every period for the firm playing in that period. The firm decides whether to access the CRA to signal the type of project it has. The CRA is perfectly informed of the type of project the firm has when accessed, and makes a public announcement (rating) about the project type. Investors Bertrand compete after seeing the move of that period’s firm and the CRA’s announcement if there is any. Elamin (2013) showed that in this infinitely repeated game, no matter how high the CRAs discount factor is, there is no infomationally-efficient equilibrium, where the investors are fully informed of the project type they face every period on the equilibrium path. Therefore, the CRA can not fully fulfill its role in the structured finance market.

This paper starts from where Elamin (2013) ends and builds on it. Can the CRAs concern for its reputation discipline its ratings in the structured finance market? In general, reputation is thought of in two ways. First, reputation is synonymous with an infinitely repeated game. The fact that interactions are repeated means that a player should care not only about his payoff today but also about his payoff from tomorrow on. The mere existence of continuation payoffs in infinitely repeated games is considered as a representation of reputation. Think of this in the following sense, if a player behaves properly today then the players will reward him from tomorrow on. Otherwise, he is punished from tomorrow. When
punishment starts it is considered that the player lost his reputation. The second concept of reputation is represented as incomplete information about the type of the player. Reputation is represented as incomplete information about the CRAs type. With some probability, the CRA might be a truthful CRA that has a dominant strategy to always be truthful.

The infinitely repeated game concept of reputation was tackled already by Elamin (2013), it was shown there that it does not make the CRA fully fulfill its role in the structured finance market. In this paper, I will consider reputation as incomplete information about the type of the CRA. Reputation here is considered as incomplete information about the type of the CRA. With some probability, the CRA could be a truthful type that always reveals truthfully. I build on the infinitely repeated game of Elamin (2013) with unverifiable ratings. Nature selects a type for the CRA in the beginning of time and only informs the CRA of it. At every period, the (updated) probability that the CRA is the truthful type is considered to be its reputation. The CRA is fully informed of the project type when accessed. Its role is to mitigate the information asymmetry between the investors and the firms.

Will the CRA fully fulfills its role in the game, mitigating the information asymmetry in the structured finance market? To start the discussion, let us define an informationally-efficient equilibrium. In an informationally-efficient equilibrium the investors are fully informed of the type of project they face in every period along the equilibrium path and the CRA’s role is fully fulfilled. In the model with only two types of projects, I derive a cut-off reputation level and show that if the CRA’s initial reputation is above that cutoff then there is an informationally-efficient equilibrium. If the CRA’s initial reputation is below that cutoff level then, independent of how high the discount factor is, there is no informationally-efficient equilibrium.

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1But what realistic concerns of market participants justify using reputation as incomplete information about the type of the CRA? In general, market participants are not sure about the exact mechanics that govern the CRA’s internal structure. A lot of safeguards has been put in place by the CRAs to deal with possible conflicts of interests inherent in the rating business. Market participants might not be fully confident of the potency of these safeguards. Moreover market participants might be under the impression that if the CRA does not do the right thing, that would be considered as fraud. Intentionally falsifying or misrepresenting information might be punished by the courts. These considerations among others create incomplete information about the type of the CRA in the mind of market participants leading it to have a reputation as modeled here.
efficient equilibrium. This existence result seems to be encouraging at a first glance. I next test the limits of this existence result in the two project types case. Is this existence result an anomaly? Or is it a particular instance of a more general result? The result fails in the two directions I extended the model. In the first extension, there are still only two project types but the firms in the game are informed of the CRA’s type. The second extension is where the CRA is the only player informed of its type but there are more than two projects. In both these extensions, no matter how high the CRA’s patience level or its initial reputation are, there is no informationally-efficient equilibrium. Hence I conclude that the existence result is indeed an anomaly. Reputation does not ensure efficiency in the infinitely repeated game with unverifiable ratings.

The rest of the paper is organized as follows: section 2 presents the infinitely repeated game with reputation, section 3 presents the case of two projects but with firms informed of the CRA’s type, section 4 presents the case of more than two projects and section 5 concludes.

3 Infinitely Repeated Game with Reputation

Consider an infinitely repeated game between short-run investors, short-run firms and a long-run CRA with reputation.\(^2\) Time is discrete and infinite. A sequence of cash constrained firms need investors funding for a project. The firm’s project is either good or bad. Firms know their project type, and could ask the CRA to rate it, signaling the project type to

\(^2\)Investors are short-run players to prevent collusion in the infinitely repeated game. To give an idea, assume there are two investors who are long-lived and consider the following scheme: investor 1 gives an offer that wipes out the firm 1 at period 1, while investor 2 gives an unacceptable offer that gives negative profits. In period 2, investor 2 gives an offer that wipes out firm 2 and investor 1 gives an unacceptable offer. Investors competing to provide funding to firms do not collude. So I model investors as short-run players to prevent collusion. On the other hand, modeling firms with projects as short-run firms is in line with the structured finance market modeled here. Structured finance securities’ payment depends only on cash flows specific to the particular pool of loans that back these securities. Issuers sell the pool of loans to a special purpose vehicle constructed for the purpose of dealing with that pool. An issuer then receives a cash payment for its sale of loans. The special purpose vehicle itself issues the structured finance securities backed by the pool. The original issuer is not liable if the special purpose vehicle he constructed defaults. This quick background motivates viewing firms as short-run players (or as one-shot projects).
investors. There is incomplete information about the type of CRA. It could be a truthful type that, when accessed, always reveals its information truthfully or a strategic type that maximizes its discounted payoffs. At the beginning of time, Nature selects the CRA’s type, and informs only the CRA of it. Let \(0 < \alpha_1 < 1\) be the prior probability that the CRA is of the truthful type. Every period, the investors and the firm playing in that period see the public history that has transpired up to then, and update the probability that they face a truthful CRA. As is standard in the game theory literature, this updated probability is the CRA’s reputation.

### 3.1 Stage Game and Setup

Every period \(t\) has a short-run firm, firm \(t\), and two short-run investors \(t1\) and \(t2\), who play in period \(t\) only. Every firm \(t\), is born with a new project. Projects yield \(\bar{R}\) when they pay and zero in default and differ only in their probability of default. Projects are ex-post indistinguishable: it is not possible to perfectly distinguish between the two types ex post. The probability of default is selected iid across time from \(\{p_H, p_L\}\) with prior \(\eta\) on the low quality project \(p_L\)

Firm \(t\) knows its project type and decides whether to access the CRA (A) or not (NA). At period \(t\), if accessed, the CRA sees the project’s default probability and makes a public announcement from \(\{H, L\}\). Investors \(t1\) and \(t2\), each with 1 unit of a good, simultaneously make offers \((R_{ti}, b_{ti})\), where \(R_{ti} \in \mathbb{R}\) is the required rate of return by investor \(ti\) and \(0 \leq b_{ti} \leq 1\) is the issuance size demanded by investor \(ti\). Investors \(t1\) and \(t2\) move in the following three possible contingencies (occurring in period \(t\)): firm \(t\) did not access the CRA, it accessed the CRA and \(H\) was announced, or it accessed the CRA and \(L\) was announced. In all contingencies, firm \(t\) then observes the offers \(\{(R_{t1}, b_{t1}), (R_{t2}, b_{t2})\}\) and decides which offer to pick. If the offers give the firm the same level of profit, each is picked with probability \(\frac{1}{2}\).

\(0 < p_H < p_L < 1\) and \(p_L\) is the low quality project.

Denote the access action by \(a_t\) and let \(a_t = 0\) if firm \(t\) did not access the CRA at time \(t\), and \(a_t = 1\) if it decides to access.
If the CRA was not accessed at time $t$ and $b$ is the issuance size of the picked offer, $b$ is invested in the project. When accessed, only the amount $(1 - \epsilon)b$ is invested in the project, and the amount $\epsilon b$ is paid to the CRA upfront. Then, the time $t$ project either defaults or pays. And the game in period $t$ ends. Firm $t$ exits the game with its profits. Investors $t1$ and $t2$ consume and exit. We start period $t + 1$ with the same CRA, different firm with a new project (firm $t + 1$) and two new investors.

3.2 Payoffs

Investors are identical and have log utility. An unpicked investor always consumes his endowment and gets zero utility. If the CRA was not accessed at time $t$, the picked investor $ti$ consumes $c_{ti}$ when the project selected at time $t$ defaults, and $c_{ti} + R_{ti}b_{ti}$ when the project pays. If the CRA was accessed, the picked investor $ti$ consumes $1 - b_{ti}$ when the project defaults, and $c_{ti} + R_{ti}(1 - \epsilon)b_{ti}$ when the project pays where $c_{ti} = 1 - b_{ti}$. Assume the selected offer in period $t$ is $(R_{ti}, b_{ti})$. Firm $t$ earns zero profit when the project defaults (limited liability) and if the project pays it earns: $(\bar{R} - R_{ti})b_{ti}$, if the CRA was not accessed in period $t$ and $(1 - \epsilon)(\bar{R} - R_{ti})b_{ti}$, if the CRA was accessed in period $t$. The truthful CRA has a dominant strategy to act truthfully. The payoff of the strategic CRA with discount factor $0 < \delta < 1$ is the discounted sum of its one period payoff. Let the selected issuance size in period $t$ be $b_t$ and let $a_t$ be the indicator function of the access decision by firm $t$ as explained before. The strategic CRA evaluates the sequence $\{b_t\}_{t=1}^{\infty}$ in the following way:

$$(1 - \delta) \sum_{t=1}^{\infty} a_t \delta^{t-1} \epsilon b_t$$

A detailed timeline of the events can be found in Appendix B.3

3.3 Equilibrium Characterization

I use the sequential equilibrium concept. A detailed analysis is contained in Appendix B.2

7
The following two conditions on the parameters of the game make it interesting.

**Condition 1.** \( p_L < \frac{(1-\epsilon)\bar{R}-1}{(1-\epsilon)\bar{R}} \)

**Condition 1** states that even when the project is thought to be bad and after adjusting for the fee, the project’s expected payment is better than just saving the endowment. Let \( q = \eta p_L + (1-\eta) p_H \) be the posterior on default under the prior.

**Condition 2.** \((1-\epsilon)\bar{R} - \frac{((1-\epsilon)\bar{R}-1)^{q}}{(1-q)^{1-\eta}q^{\eta}} > (\bar{R} - \frac{(\bar{R}-1)^{p_L}}{(1-p_L)^{1-p_L}p_L})\)

**Condition 2** states that the profit when the firm accesses the CRA and investors retain their priors is strictly higher than when the firm plays no access and investors believe it to be bad. Hence, the fee is small enough that firms prefer to access and pool, than not access and be considered bad. In particular, the condition implies that \( \epsilon < 1 \).

Lemmas 1, 2 and 3 listed here are from Elamin (2010a). These Lemmas detail the investors’ optimal strategies given their beliefs. The only difference between this environment and the environment of Elamin (2010a) is that the beliefs themselves may be different. But optimal strategies given the beliefs are still the same.

**Lemma 1.** Under **Condition 1** and **Condition 2**, and given \( \mu \),

1. An optimal response of the investors given beliefs in the contingency following access and announcement is a vector \(((R_1, b_1), (R_2, b_2))\) for the two investors s.t. for \( i \in \{1, 2\} \), \((R_i, b_i)\) solves the following problem (Problem P1):

\[
(1-\epsilon)\max_{R_i, b_i}(\bar{R} - R_i)b_i \text{ s.t.} \\
0 \leq b_i \leq 1 \\
q\log(1 - b_i) + (1 - q)\log(1 + (R_i(1 - \epsilon) - 1)b_i) = 0
\]

where:

\[
q = \mu(A, L)p_L + (1 - \mu(A, L))p_H \text{ if there was access and } L \text{ was announced.} \\
q = \mu(A, H)p_L + (1 - \mu(A, H))p_H \text{ if there was access and } H \text{ was announced.}
\]
2. An optimal response of the investors given beliefs in the contingency following no access is a vector \(((R_1, b_1), (R_2, b_2))\) for the two investors s.t. for \(i \in \{1, 2\}\), \((R_i, b_i)\) solves the following problem (Problem P2):

\[
\max_{R_i, b_i} (\bar{R} - R_i) b_i \quad \text{s.t.}
\]

\[
0 \leq b_i \leq 1
\]

\[
q \log(1 - b_i) + (1 - q) \log(1 + (R_i - 1)b_i) = 0
\]

where:

\[
q = \mu(NA)p_L + (1 - \mu(NA))p_H
\]

**Lemma 2.** Under **Condition 1** and **Condition 2**, there exists a solution \((R, b)\) that solves P1. Moreover the solution is unique and is characterized by the following two equations:

\[
b = 1 - \left[\frac{q}{(1-q)(1-\epsilon)\bar{R}-1}\right]^{1-q}
\]

\[
R = \frac{1}{q}\left[(1-q)\bar{R} - \frac{1}{(1-\epsilon)}\right]^{1-b}
\]

**Lemma 3.** Under **Condition 1** and **Condition 2**, there exist a solution \((R, b)\) that solves P2. Moreover, the solution is unique and is characterized by the following two equations:

\[
b = 1 - \left[\frac{q}{(1-q)(R-1)}\right]^{1-q}
\]

\[
R = \frac{1}{q}\left[(1-q)\bar{R} - 1\right]^{1-b}
\]

### 3.4 Informationally-Efficient Equilibrium

The CRA is fully informed of the project type, when accessed by a firm. Its role is to mitigate the information asymmetry between firms and investors. The notion of efficiency we consider reflects this role. Intuitively speaking, in an informationally-efficient equilibrium, the investors are always informed of the type of project they face, and there is no information asymmetry. Hence in an informationally-efficient equilibrium the CRA fully fulfills its role revealing all the information it has about the projects it rates when accessed. A more formal definition follows now.
**Definition 1.** A sequential equilibrium is informationally-efficient if on the equilibrium path the investors are correct in their beliefs about the type of project they face.

An informationally-efficient equilibrium exists in this game, when there are only two kinds of projects if reputation is high enough. Define a cutoff reputation value $\alpha^*$ at which the bad firm is indifferent between access and no access when the strategic CRA says it is the good firm after it accesses and the investors believe the CRA, and the bad firm is known to be the bad firm when it does not access. The bad firm faces a tradeoff between accessing, paying the CRAs fee, and considered to be the good firm, and between not accessing, saving the fee, and considered to be the bad firm. $\alpha^*$ is the level of CRA reputation at which the bad firm is indifferent between these two alternatives. If the CRA’s reputation is above this cutoff, the bad firm would not access.

Let $\alpha^*$ be the $0 < \alpha < 1$ that solves the following equation:

$$
\alpha \left[ (1-\epsilon)^{\bar{R}} - \frac{((1-\epsilon)^{\bar{R}} - 1)^{plL}}{(1-pL)^{1-plL}pL^{plL}} \right] + (1-\alpha) \left[ (1-\epsilon)^{\bar{R}} - \frac{((1-\epsilon)^{\bar{R}} - 1)^{phL}}{(1-pH)^{1-phH}pH^{phH}} \right] = \bar{R} - \frac{((\bar{R} - 1)^{plL})}{(1-pL)^{1-plL}pL^{plL}}
$$

I note that under **Condition 1** and **Condition 2**, a solution $\alpha^*$ to the above equation

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5I clarify two possible confusions. The first concerns what is meant by beliefs being correct. Beliefs are correct in equilibrium if they are updated according to the definition of the equilibrium used. Here beliefs are always correct in the sense that they adhere to the sequential equilibrium definition used. They are computed by Bayes rule on the equilibrium path, and are Bayes rule limits off the equilibrium path. What is meant by correct in Definition 2 is that the investors believe they face the bad project in the periods where nature selects a bad project. And they believe they face the good project when nature selects a good project. The best way to clarify the other misconception concerning the equilibrium path is by defining a slightly different concept which I call on the equilibrium path in period $t$. Play is on the equilibrium path in period $t$ if firm $t$ plays according to its equilibrium strategy. Play is on the equilibrium path in the infinitely repeated game if it is on the equilibrium path every period. In an informationally-efficient equilibrium, in any period $t$ the beliefs of the time $t$ investors should be correct when play is on the equilibrium path in period $t$. Let us see what an informationally-efficient equilibrium requires of the beliefs of investors after a deviation. Assume that at period 1 the bad firm was not supposed to access the CRA but it did. In an informationally-efficient equilibrium, the CRA does not have to mitigate the information asymmetry in that particular period and the investors might not have correct beliefs in that particular period. But in any period following that first period, if the firms play according to their equilibrium strategies, then the CRA should mitigate the information asymmetry and the investors should have correct beliefs. What is important to understand here is that following a deviation in period $t$, the CRA expects all firms playing afterwards to stick to their equilibrium strategy, and hence in any informationally-efficient equilibrium the CRA’s continuation payoff will be fixed after every possible contingency. The relevance of this subtle point should be clear after reading the proofs of Propositions 4, 2, and 3.
exists by the intermediate value theorem. Moreover the solution $\alpha^*$ is unique because of monotonicity in $\alpha$ of the LHS of the equation.

**Proposition 1.** Under [Condition 1] and [Condition 2], independent of the discount factor $0 \leq \delta < 1$, if the prior reputation is greater than the reputation cutoff ($\alpha_1 \geq \alpha^*$) then there is an informationally-efficient equilibrium. If $\alpha_1 < \alpha^*$ then there is no informationally-efficient equilibrium.

Proposition 1 showed that there is an informationally-efficient equilibrium when there are only two types of projects and when the reputation level is high enough. The intuition is very simple. Assume that the reputation of the CRA is almost 1. Bad firms would not want to access, and good firms would want to access. Access by the bad firms would almost surely reveal they are the bad firms. The worst that could happen if they do not access is that they are considered to be the bad firms. But even then they save the fee. The good firms on the other hand would want to access because that almost surely signals they have the good project. When the good firms access and the bad firms do not, the reputation level stays constant in the game.⁶

This seems like a promising result, but we will see in the remaining sections that this existence result is not robust to two very natural extensions. First, is it robust to changes in the information structure of the game? Second, does the result extend to when the number of projects increase to more than two? Proposition 2 answers the first question in the negative, and Proposition 3 answers the second question in the negative. The result seems to be an anomaly stemming from the fact that we only have two project types, and in this case, the access/no access decision itself is enough to separate the types in the game.

⁶Note that firms get higher profits when they are thought to be the good firms than the bad firms.
4 Firms Informed of CRA’s Type

This section motivates the first robustness question and checks if the existence result obtained in Proposition 1 extends to the case where the firms are informed of the type of the CRA.

The CRAs operate under what is known as the issuer-pays business model. Issuers of the securities themselves pay the CRA raising some doubts about their clout with the CRA. Moreover the process of rating a security is a give and take process between the issuer of the security and the CRA. There is a high level of interaction between the firms who have the projects and the CRA. This interaction endows the firm with a deeper knowledge of the CRA’s inner workings. I adjust the information structure to account for these close interactions, the firms now know the CRA’s true type. They know that the CRA is the strategic type when it is. They know the CRA is the truthful type when it is. It is only the investors who have the incomplete information about the type of the CRA because they are far removed from direct dealing with the CRA. Investors see the CRA as a black box which issues ratings. Will the existence result of Proposition 1 hold in this environment? I will skip making the adjustments to the strategies of the firms and to the equilibrium concept because these adjustments are straightforward.

Proposition 2. Under Condition 1 and Condition 2, in the infinitely repeated game with reputation if the firms are informed of the true type of the CRA, then for every $\delta : 0 \leq \delta < 1$ and for every $\alpha : 0 \leq \alpha_1 < 1$ there is no informationally-efficient equilibrium.

Proof. .

Assume that there is an informationally-efficient equilibrium. The arguments in the proof of Proposition 1 show that the case to consider is where the good firm accesses and is revealed to be the good firm, and the bad firm does not access. The same profitable deviation as before still holds. Assume that Nature selected the strategic type, and the CRA has threatened the bad firm by revealing its type when it is the strategic type with high enough probability to
convince it not to access. No matter what the strategic CRA says it will do, it will always say the project is good when it is presented with the contingency of access by the bad firm. Hence the profitable deviation is that the bad firm accesses the CRA, the CRA says the project is good, and the investors think the project is good. The equilibrium unravels and our Proposition is proved.

Proposition 2 shows that if firms are informed of the type of the CRA, there is no informationally-efficient equilibrium and the CRA can not fulfill its role and mitigate the information asymmetry.

5 More than Two Project Types

This section checks if the existence of an informationally-efficient equilibrium when reputation is high enough in Proposition 1 extends to the case of more than two projects. Obviously in reality there are more than two project types, and I refrain from motivating the obvious. Here, just like before changing the information structure for our first robustness exercise, only the CRA knows its type. The investors and firms have to use the public history to update their probability that the CRA is the truthful type.

This paragraph tries to make the necessary changes to Condition 2 and quickly sets up the new game. Assume there are \( N \) projects with \( N \) different probabilities of default \( p_1 < p_2 < \ldots < p_N \). Let \( N \geq 3 \). Also assume the prior on each project is \( \eta_n > 0 \) where \( \sum_{n=1}^{N} \eta_n = 1 \). A time \( t \) public history becomes an element \( h_t^p \in \left[ \{NA, (A,1), (A,2), \ldots, (A,N)\} \times \mathbb{R} \times [0,1]^2 \times \{1,2\} \times \{D,P\} \right]^{t-1} \). Firm \( t \)'s strategy becomes \( \sigma_{Firm\ t} : H_t^p \times \{p_1, \ldots, p_N\} \to \triangle\{A,NA\} \) and the CRA's strategy becomes \( \sigma_{CRA} : \zeta_0 \times H^p \times A \times \{p_1, \ldots, p_N\} \to \triangle\{1, \ldots, N\} \). Investor \( ti \), sees the public history from the past and picks his offer given the \( N+1 \) contingencies he might find himself in today. Time \( t \) investor \( i \)'s strategy is \( \sigma_{ti} : H_t^p \times \{NA,(A,1), \ldots,(A,N)\} \to \mathbb{R} \times [0,1] \).

The condition we will have now in this environment will allow the before-worst firm to
get higher profits when it accesses the CRA and its true type is revealed than not access and be considered the worst firm.

**Condition 3.** \((1 - \epsilon)\bar{R} - \frac{((1-\epsilon)\bar{R} - 1)^{p_{N-1}}}{(1-p_{N-1})^{1-p_{N-1}}p_{N-1}^{p_{N-1}}} \geq \bar{R} - \frac{(\bar{R} - 1)^{p_{N}}}{(1-p_{N})^{1-p_{N}}p_{N}^{p_{N}}}\)

**Condition 3** is in line with our informationally-efficient equilibrium definition. If this condition does not hold there is no hope of getting an informationally-efficient equilibrium since the before-worst firm would never access and pay the fee to reveal its type. The worst firm never accesses to reveal its type since it prefers to always not access and save the fee. But then the worst and the before-worst firm pool together and no informationally-efficient equilibrium exists. I also note here that if this condition holds for the before-worst firm, then it holds for every firm that has a better project. Hence when **Condition 3** holds, every firm other than the worst firm prefers to access and pay the fee when their true type is revealed than not access and save the fee but be considered the worst firm.

To prepare for the proof of **Proposition 3** for \(n \neq N\) let \(b_n^*\) be the optimal issuance size investors are willing to buy after access when they know that they face the project with default \(p_n\). From **Lemma 2** we know that \(b_n^* = 1 - \left[\frac{p_n}{(1-\epsilon)\bar{R} - 1(1-p_n)^{p_{N-1}}}\right]^{1-p_{N-1}}\). I remind the reader that \(b_1^* > b_2^* > \ldots > b_{N-1}^*\). The optimal issuance size increases when the investors know that they face a less risky project.

**Proposition 3.** Under **Condition 1** and **Condition 3** in the infinitely repeated game with reputation if the number of projects is more than two then independent of the discount factor \(\delta : 0 \leq \delta < 1\) and the initial reputation level \(\alpha_1 : 0 \leq \alpha_1 < 1\) there is no informationally-efficient equilibrium.

**Proof.**

We first note that a bad firm will never access if its true type is revealed. No access is always a profitable deviation. When it does not access, the worst the investors could believe about the firm is that it is the bad firm. But even then the bad firm saves the fee. So in an informationally-efficient equilibrium the bad firm will not access the CRA. Now note
that an informationally-efficient equilibrium separates between the types on the equilibrium path. Hence in an informationally-efficient equilibrium all the firms with types better than the worst will access and the CRA will reveal their type truthfully.

We note now that everything in the paragraph above has got to happen in every time period along the equilibrium path. Now fix the first period of an informationally-efficient equilibrium and let us look at the continuation payoffs from period 2 on. In every contingency along the equilibrium path every firm better than the worst will access and its type will be revealed to the investors. Hence the continuation payment of the CRA after every contingency that happens on the equilibrium path is constant and in particular is equal to 

$$\sum_{n=1}^{N-1} \eta_n \epsilon b_n^{*} 1 - \delta.$$ 

We now know that the continuation from period 2 onwards is fixed at 

$$\sum_{n=1}^{N-1} \eta_n \epsilon b_n^{*} 1 - \delta$$

and that all the firms except the worst firm will access and their true type will be revealed. Consider the following profitable deviation for the CRA, after access by a $p_2$ firm, it announces it is a $p_1$ firm. The investor would assume we are still on the equilibrium path and would act accordingly increasing the CRA’s current payment ($\epsilon b_1^{*} > \epsilon b_2^{*}$), and the future payment is still fixed by the equilibrium’s continuation payment ($\sum_{n=1}^{N-1} \eta_n \epsilon b_n^{*} 1 - \delta$). This profitable deviation concludes our simple proof.

\[\square\]

6 Conclusion

In this paper I have shown that reputation is not potent enough for the CRA to mitigate information asymmetry in the structured finance market. In the model with reputation, we expected that the fear to lose this reputation will discipline the CRA and help it fulfill its role fully.

What we found is that in the model with non-verifiable ratings, if there are only two types of projects and only the CRA is informed of its type, then there is a cutoff reputation level such that if the initial reputation is above the cutoff then an informationally-
efficient equilibrium exists. If the initial reputation is below the cutoff then there is no informationally-efficient equilibrium. Although at a first glance the result is encouraging, but we fail to extend the result in two possible directions. The first direction is a change of the information structure of the game. If the firms are informed of the type of the CRA, then there is no informationally-efficient equilibrium. The second direction is that even when only the CRA is informed of its type, if there are more than two project types there is no informationally-efficient equilibrium. It does seem that the existence result we got in the two project types hinges on the fact that when reputation is high, the bad firms do not want to access. The fact that there is the access and no access choice then separates the types. The bad firms do not access, and the good firms access. The separation here is not really happening because of what the CRA itself does.

When the firms know the true type of the CRA, the intuition above fails. The bad firms are not afraid to access the CRA any more when it is the strategic type. Hence there is no informationally-efficient equilibrium. And when there are more than two types of projects, the access and no access decision is not enough any more to separate the types. The actions of the CRA itself are needed to help separate between the types. The CRA can not reveal truthfully when it is the strategic type, and there is no informationally-efficient equilibrium. If investors believe the CRA will always reveal truthfully, then the CRA prefers to deviate today to make the highest gains in the current payment. This increases its current payment without having any effect on the continuation payoff which is fixed in any informationally-efficient equilibrium.

Therefore because of the lack of robustness of our existence result and because the world is composed of more than two types of projects, I conclude that reputation is not strong enough to ensure efficiency in the structured finance market.
A Proofs

Proof of Proposition 1.

Proof. Let us prove the first part of the proposition first. Assume $\alpha_1 \geq \alpha^*$ and $0 \leq \delta < 1$. The following is an informationally-efficient equilibrium. Good firms access, and bad firms do not. The strategic CRA always says $L$ (good) after access by $p_L$ firm and by $p_H$ firm. Beliefs of investors after they see no access is that it is the bad firm, after they see access and $L$ that it is the good firm, and after they see access and $H$ that it is the bad firm. Reputation level stays constant at $\alpha_1$ on the equilibrium path, and goes to 1 if the investors see access and $H$ and stays at 1 forever. This proves the first part of the Proposition.

Now to prove the second part, assume there is an informationally-efficient equilibrium and $\alpha_1 < \alpha^*$. There are three cases to consider. First, at any time period the bad firm accesses and it is revealed to be the bad firm and the good firm does not access. That would never arise because the bad firm would profitably deviate to no access. Second, at any time period both types of firms access and the CRA truthfully reveals the types. This case would never arise either. The bad firm would prefer to deviate to no access. The worst that could happen when the bad firm does not access is that it is thought to be the bad firm. But even then, it saves the fee. These two cases show that the bad firm would never access when its true type is revealed after access.

To prepare for the third case we need to understand the possible CRA’s continuation payoffs in an informationally-efficient equilibrium. It might be useful at this point to review the comments after Definition 2. The two cases discussed before show that the bad firm would never access in an informationally-efficient equilibrium. Hence to separate the types in an informationally-efficient equilibrium, the good firm has to accesses and the CRA reveals the true type of the project. Notice that because of the definition of an informationally-efficient equilibrium, the good firm has to access every period after every possible contingency.
This means that in any informationally-efficient equilibrium the continuation payoff of the CRA on the equilibrium path is fixed no matter what happens today. At any period, the continuation payoff of an informationally-efficient equilibrium is determined by the expected payments coming from access by good firms from that period on. Hence let $b^*_L$ be the optimal issuance size investors are willing to buy after access when they know that they face the project with default $p_L$ (the good project). From Lemma 2 we know that $b^*_L = 1 - \left[ \frac{p_L}{(1-\epsilon)\bar{R} - 1} \right]^{1-p_L}$. The continuation payoff after every possible contingency in an informationally-efficient equilibrium is fixed at $\eta \epsilon b^*_L$. 

Now we are ready to present the third case, bad firms do not access and good firm access. The strategic CRA says $L$ (good) after access by good firm, and threatens the bad firm with enough punishment to force it not to access. But no matter what the strategic CRA says it will do after access by $p_H$ (bad) firm, consider the following profitable deviation: the bad firm accesses and the CRA says $L$ (good). The unsuspecting investors think they are on the equilibrium path where only good firms access. After access and an $L$ announcement by the CRA, the investors believe the project is good giving offers accordingly. The reputation level stays the same, and the CRA and the bad firm both profit from the (wrong) belief of the investor that the project is good. The bad firm gets higher profits because it is thought to be the good firm (by Condition 2 and the monotonicity of profits in beliefs). The CRA’s continuation payoff is fixed from tomorrow on at $\frac{\eta \epsilon b^*_L}{1-\delta}$ no matter what it says today. With this deviation it gets $\epsilon b^*_L$, which is the highest payment it could get in a period, and the continuation is constant. Hence the total payment is definitely higher than saying bad, getting the fee for a bad project today and getting the same continuation payoff from tomorrow on. The proposed equilibrium unravels. This concludes our proof. \qed
B Histories, Strategies, and Sequential Equilibrium

B.1 Histories and Strategies

I now define the relevant histories of the infinitely repeated game necessary to define the strategies of the players. At any time period $t$, a player’s strategy might possibly depend on anything he knows at that period of what has transpired in the past, be it private (like the true type of the project for a CRA that was accessed in some previous period) or public (like the decision to access the CRA or not, or if the project defaulted or paid). But here I impose the standard restriction that the players use the past in one way only: the players only use the public events from the past in their strategies. The players can still make their strategy today depend on what they see privately today, but it can depend on the past only through what is publicly known. There is an exception to this of course and that is that the CRA knows what type it is when it moves. At any time period, the CRA knows the move of nature on its type and uses the public history from the past and what it sees privately today when it picks a strategy. Hence, I will first define the public history, and then for each player add to it what he observes privately today and for the CRA its type. Note here that for simplicity, I will not mention the firm $t$’s second move because as we saw in the Lemmas in Elamin (2010a) in any equilibrium the firm will not move again, and these nodes will be off-path. Tracking behavior in these nodes will be merely cumbersome with no real benefit. The Lemmas derived in Elamin (2010a) still apply to this environment with almost no change in the formulation or the proofs. The only difference is that the beliefs themselves might be different, but behavior given beliefs is still the same. These Lemmas are listed in this paper as Lemmas 1, 2, and 3.

B.1.1 Public History

At any time period $t$, the entries recorded in the public history are the following elements: if the CRA was accessed or not by firm $t$, the public announcement if accessed, the offers of
the investors, the chosen investor, and the realization of the project at the end of the period.

A time $t$ public history is an element:

$$h^p_t \in \left[\{NA, (A, H), (A, L)\} \times \mathbb{R}^2 \times [0, 1]^2 \times \{1, 2\} \times \{D, P\}\right]^{t-1}$$

where:

- NA stands for CRA not accessed.
- $\{(A, H), (A, L)\}$ denotes the decision to access the CRA and the subsequent announcement.
- $\mathbb{R}^2 \times [0, 1]^2$ denotes the set of possible required rates and possible issuance sizes of investors $t1$ and $t2$.
- $\{1, 2\}$ denotes the set of possible chosen investor in each period.
- $\{D, P\}$ denotes the set of what could happen to the project. It either defaults (D) or pays (P).

Let $h^p_1 = \emptyset$, $H^p_t$ be the space of all possible time $t$ public histories $h^p_t$ and $H^p = \bigcup_t H^p_t$.

### B.1.2 Strategies

Firm $t$, sees the public history from the past and the true project type at period $t$ and randomizes on access and not access. Firm $t$’s strategy is a function $\sigma_{Firm \ t} : H^p_t \times \{p_H, p_L\} \to \Delta\{A, NA\}$. The CRA sees a private history in the time periods it was accessed. The CRA, knowing its type, only uses the public history from the past for its strategy. It specifies what it will do after access by a $p_H$ firm and a $p_L$ firm. The CRA’s strategy is $\sigma_{CRA} : \zeta_0 \times H^p \times A \times \{p_H, p_L\} \to \Delta\{H, L\}$. Investor $ti$, sees the public history from the past and picks his offer given the three contingencies he finds himself in. Time $t$ investor $i$’s strategy is $\sigma_{ti} : H^p_t \times \{NA, (A, H), (A, L)\} \to \mathbb{R} \times [0, 1]$. Investor $ti$ picks his required rate of return and the issuance size given what he knows up to then.
B.2 Sequential Equilibrium

I now define the sequential equilibrium concept I will use. I note here that there are two “kinds” of incomplete information in this game: transient and permanent. The incomplete information about the type of project selected for investment at each period is transient in nature. A new project is selected every period. So there is no incomplete information carrying over from one period to the other concerning project types. The incomplete information about the type of the CRA playing in the game on the other hand is more permanent in nature. This incomplete information may carry over from one period to the other. Only the CRA is informed of its type. The firms and investors have to use the public information from the past, and whatever information they see in the period they play in to update their belief about the CRA being the truthful type. This posterior belief is the reputation of the CRA.

**Definition 2.** A sequential equilibrium (SE) is a tuple \((\sigma, \mu, \alpha)\):

1. \(\sigma = \{\sigma_{\text{Firm } t}\}_{t=1}^{\infty}, \sigma_{\text{CRA}}, \{\sigma_{t1}\}_{t=1}^{\infty}, \{\sigma_{t2}\}_{t=1}^{\infty}\)
   - \(\sigma_{\text{Firm } t}\) is the strategy of the time \(t\) firm.
   - \(\sigma_{\text{CRA}} = \{\sigma_{\text{CRA } t}\}_{t=0}^{\infty}\) is the strategy of the CRA in the infinitely repeated game, a collection of time \(t\) strategy components.
   - \(\sigma_{ti}\) is the strategy of investor \(ti\).

2. \(\mu = \{\mu_{t}\}_{t=1}^{\infty}\) where:
   \[\mu_{t} = (\mu(H_{t}^{P} \times NA), \mu(H_{t}^{P} \times (A, L)), \mu(H_{t}^{P} \times (A, H)))\]
   At each period \(t\) given the public history, \(\mu_{t}\) is a collection of three numbers between 0 and 1. These probabilities denote the beliefs of the investors in each of their information sets about the probability of facing a \(p_{L}\) project in each possible contingency.

3. \(\alpha = \{\alpha_{t}(H_{t}^{P})\}_{t=1}^{\infty}\) where:
   \(\alpha_{t}(H_{t}^{P})\) is the updated probability that the CRA is of the truthful type that firm \(t\) and
investors $t_1$ and $t_2$ start the period with.

Satisfying the following requirements:

i. Given $\mu$, $\alpha$, $\{\sigma_{Firm} t\}_{t=1}^{\infty}$, $\{\sigma_{t1}\}_{t=1}^{\infty}$, and $\{\sigma_{t2}\}_{t=1}^{\infty}$:

   $\sigma_{CRA}$ is optimal at every node the CRA moves on.

ii. Given $\mu_t$, $\alpha_t(H_p^p)$ and $H_p^p$,

   a. $\sigma_{ti}$ is optimal given $\sigma_{tj}$ with $i, j \in \{1, 2\}$ and $j \neq i$.

   b. $\sigma_{Firm t}$ is optimal given $\{\sigma_{CRA t}, \sigma_{t1}, \sigma_{t2}\}$.

iii. Given $\alpha_t(H_p^p)$ and $H_p^p$, $\mu_t$ and $\alpha_{t+1}(H_p^p)$ are derived from the perturbed time $t$ strategies of the players as Bayes rule limits of (totally mixed) perturbed strategies converging to $(\sigma_{Firm t}, \sigma_{CRA t})$

   where $\sigma_{CRA t}$ is the time $t$ component of the CRA’s strategy:

   $\sigma_{CRA t} : \zeta_0 \times H_1^p \times A \times \{p_H, p_L\} \to \Delta\{H, L\}$ consistent with $\sigma_{CRA}$.

iv. If $\alpha_T(H_p^p) = 0$ or $\alpha_T(H_p^p) = 1$ for some $T$, then $\alpha_t = \alpha_T \forall t \geq T$.

B.3 Timeline

To make visualizing the game easier a detailed chronological timeline follows.

1. At the beginning of time, Nature picks a type for the CRA and only informs the CRA of it. The CRA is either a strategic type $\zeta_0$ or a truthful type $\zeta_T$.

2. Then at every period $t$, Nature moves and picks a time $t$ project from the set of possible default probabilities $\{p_H, p_L\}$ with prior $\eta$ on $p_L$ where $0 < p_H < p_L < 1$ and $p_L$ is the low quality project.

3. Firm $t$ sees the move of nature and then decides to access (A) the CRA or not access (NA).
4. If accessed by firm \( t \), the CRA sees the move of nature on project type in that period and makes a public announcement from the set \( \{H, L\} \).

5. Two investors \( t_1 \) and \( t_2 \) each with an endowment of one unit of a good, simultaneously make offers \((R_{ti}, b_{ti})\) in the following three possible contingencies: firm \( t \) did not access the CRA, it accessed the CRA and \( H \) was announced, or it accessed the CRA and \( L \) was announced. Note that \( R_{ti} \) is the required rate of return required by investor \( ti \), a real number in \( \mathbb{R} \) and \( b_{ti} \) is the issuance size \( ti \) is willing to buy where \( 0 \leq b_{ti} \leq 1 \).

6. In all contingencies, firm \( t \) observes the offers \( \{(R_{t1}, b_{t2}), (R_{t2}, b_{t2})\} \) set by the investors playing in period \( t \). If the offers give the firm the same profits then Nature moves again picking each offer with probability \( \frac{1}{2} \). If the offers give the firm different profits, then the firm decides which offer to pick.

7. Let \( b_{t} \) be the issuance size of the picked offer. If the firm has decided to access the CRA then the amount \((1 - \epsilon)b_{t}\) is invested in the project, and the amount \( \epsilon b_{t} \) is paid to the CRA upfront. If the CRA was not accessed then \( b_{t} \) is invested in the project.

8. The time \( t \) project realizes as a public outcome, it either defaults or pays. Investors \( t_1 \) and \( t_2 \) consume and exit the game.

9. The time \( t \) game ends. And we start period \( t + 1 \) with two new investors, a new firm with a new project and the same long-run CRA.
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