The weak-gravity bound in asymptotically safe gravity-gauge systems

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Abstract: The weak-gravity bound has been discovered in asymptotically safe gravity-matter systems, where it limits the maximum strength of gravitational fluctuations. In the present paper, we explore it for the first time in systems with more than one gauge field, to discover whether systems with 12 gauge fields (like the Standard Model) exhibit a weak-gravity bound and whether the gravitational fixed point evade it. Further, we test the robustness of the present and previous results on the weak-gravity bound by exploring their dependence on a gravitational gauge parameter. Finally, the existence of the weak-gravity bound also has important phenomenological consequences: it is key to a proposed mechanism that bounds the spacetime dimensionality from above to four or five dimensions. In this paper, we strengthen the evidence for this mechanism. Thus, the predictive power of the asymptotic safety paradigm could extend to parameters of the spacetime geometry, such that the four-dimensionality of our universe could be explained from first principles.
1 Introduction

How much can be learned about quantum gravity without direct observational access to Planck-scale phenomena? This question drives a significant part of research on the phenomenology of quantum gravity, see [1] for an extensive review. Within asymptotically safe quantum gravity (see [2–8] for reviews and [9] for a critical discussion of the current status), it has been found that the existence of Standard Model (SM) matter at low energies strongly constrains the properties of the regime at transplanckian energies, in the following ways:

First, the interaction structure of matter models at low energies constrains asymptotically safe gravity at high scales: to accommodate nonvanishing Yukawa couplings (giving rise to
fermion masses in the SM), the values of gravitational couplings at transplanckian energy scales are constrained [10–12]. Similarly, specific beyond SM (BSM) models impose their own constraints on asymptotically safe gravity, see, e.g., [13, 14], while in turn the consistent coupling to asymptotically safe gravity reduces the parameter space in many BSM settings, see, e.g., [15–23]. Second, the existence of certain matter degrees of freedom is already enough to constrain asymptotically safe gravity, without considerations of specific interaction structures at low energies: if asymptotically safe gravity is too strongly coupled, the strong gravitational fluctuations destroy fixed points in matter interactions [10, 24, 25]. The matter interactions in question are not relevant for low-energy phenomenology, because they are canonically higher-order interactions which are induced by gravity [10, 11, 24–29]. It is required that they lie at an interacting, i.e., asymptotically safe fixed point \(^1\) at high energies and thus one can conclude that asymptotically safe gravity is restricted to be sufficiently weakly interacting. Thus, asymptotic safety has to satisfy the weak-gravity bound (WGB)\(^2\).

This program of relating effective theories for matter to asymptotically safe gravity is in its aim partially similar to the swampland program in string theory [33], see [34, 35] for reviews, in that it determines which effective field theories could be ultraviolet completed by their coupling to quantum gravity. However, it goes beyond the swampland program in that within asymptotic safety, these considerations narrow down the gravitational parameter space very significantly.

Both types of constraints on the transplanckian regime give rise to boundaries in the microscopic gravitational parameter space. These have been determined, but are subject to systematic uncertainties. Therefore, it is critical to extend previous work to determine those boundaries with reduced systematic uncertainties, in order to decide whether or not asymptotically safe gravity is phenomenologically viable. In this paper, we make a significant step in that direction with a focus on gauge interactions. First, we extend the work in [25, 26] to a complete basis of interactions at a given order in fields and derivatives. Second, we consider settings with more than one gauge field and explore the dependence of the WGB on the number of vectors, similar to a corresponding recent study for scalar fields [27]. Third, we use gauge dependence as a proxy for the stability of results, i.e., investigate whether physical statements, such as the existence of a fixed point, are gauge independent, as they should \(^3\).

This paper is organized as follows. In Sec. 2 we discuss the constraints a UV-complete Abelian gauge sector imposes on the gravitational parameter space in more detail. Sec. 3 is dedicated to the study of the system consisting of one field, in Sec. 4, two gauge fields are considered. In sec. 5 we perform a study of \(N_V > 2\) gauge fields, also investigating the

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\(^1\)For symmetry reasons, which we also review in the present paper, a non-interacting, i.e., asymptotically free fixed point is unavailable for these interactions.

\(^2\)Though similar in name, weak gravity bound should not be confused with weak gravity conjecture [30]. For the relation between string theory, asymptotic safety and the weak gravity conjecture, see [31, 32].

\(^3\)Within asymptotically safe gravity, approximations can result in a gauge dependence even in physical quantities. In turn, gauge dependence allows to determine whether approximations are robust (when gauge dependence is low/absent) or insufficient (when gauge dependence becomes large).
large $N_V$ limit. Finally, in Sec. 6 we discuss the gauge dependence of the WGB as a test for the robustness of our results. Sec. 7 summarizes our work. In App. A we discuss the case when the WGB ceases to be a function in the space of couplings.

2 The Abelian gauge sector in asymptotically safe quantum gravity

We will briefly review the status of the Abelian gauge sector in asymptotically safe quantum gravity. We will first focus on the Abelian gauge coupling $g_Y$, and then discuss the role of higher-order, induced matter interactions.

2.1 The Abelian hypercharge

Quantum fluctuations of charged matter have a screening effect on the Abelian gauge coupling, resulting in a Landau pole in perturbation theory and a non-perturbative triviality problem [36–38]. The associated scale of new physics is transplanckian, suggesting that the new physics required to solve the triviality problem could be quantum gravity.

For asymptotically safe gravity, the leading-order terms in the beta function for the Abelian hypercharge coupling, including the gravitational contribution, read

$$
\beta_{g_Y} = -f_g g_Y + \frac{g_Y^3}{16\pi^2} \frac{41}{6} + \ldots ,
$$

(2.1)

where $f_g$ parameterizes the quantum gravitational contribution, which depends on the gravitational couplings, see, e.g., [39] for the explicit form.

Explicit computations using the functional Renormalization Group (FRG), cf. 3.1, yield $f_g > 0$ [25, 26, 39–43] (with a general argument that $f_g \geq 0$ in [44]), indicating an antiscreening effect of gravitational fluctuations $^4$. Thus, gravitational fluctuations change the scaling dimension of the gauge coupling near $g_Y = 0$, thereby solving the Landau pole/triviality problem.

Additionally, gravitational fluctuations induce a second, interacting fixed point $g_{Y^*} > 0$, at which the gauge coupling is irrelevant, which means that its value at all scales is predictable. Thus, its infrared (IR) value can be computed from first principles, see [39, 41]. Within approximations that cause significant systematic uncertainties, the calculated IR value is 35 % above the measured value. Accordingly, two universality classes are currently compatible with observations within the systematic uncertainties: First, from an asymptotically safe fixed point, the gauge coupling is predicted at all scales; second from an asymptotically free fixed point the Abelian gauge coupling can reach any IR-value within an interval bounded by the prediction from the interacting fixed point. Given that the current

$^4$Since the gravitational couplings are not marginal, the gravitational contribution $f_g$ is not universal, but is scheme-dependent. Studies using dimensional regularization within perturbation theory indicate that $f_g = 0$ [45–49], but neglect the contribution of higher-order couplings to the scale dependence of $g_Y$ [25], cf. Sec. 2.2. In addition, perturbative studies do not compute universal quantities; in contrast to the FRG setting, in which $f_g$, when evaluated on a gravitational fixed point, corresponds to the critical exponent of the free fixed point $g_{Y^*} = 0$, and is thus universal. In the following, we will choose a scheme where $f_g > 0$, and investigate scheme independent quantities, e.g., critical exponents or the existence of fixed points, in this scheme.
estimate for this upper bound on the gauge coupling is 35% above the measured value (with systematic uncertainties expected to be roughly of a similar size), asymptotically safe gravity appears to indeed solve the Abelian triviality problem.

2.2 Higher order, induced interactions

Under the impact of gravity, additional matter interactions besides the couplings of the Standard Model have to be scrutinized, because these additional interactions are generated by gravitational fluctuations. In particular, all matter interactions that are compatible with the symmetries of the kinetic term of a given matter field are expected to be induced by gravitational fluctuations which are nonvanishing at an asymptotically safe fixed point \cite{11}. This general argument has been explicitly confirmed for scalars \cite{24, 27, 28, 50, 51}, fermions \cite{52–54} and gauge fields \cite{25, 26} as well as mixed scalar-fermion systems \cite{10, 11}. For gauge fields, for instance the interaction $w_2 (F_{\mu\nu} F^{\mu\nu})^2$ is induced. Schematically, the scale dependence of $w_2$ at finite values of the dimensionless Newton coupling $G$ reads

$$\beta_{w_2} = w_2 C_1(G) + C_0(G) + w_2^2 C_2 + \ldots , \quad (2.2)$$

where explicit expressions for $C_0, C_1$ and $C_2$ can be found in \cite{25}, and the ancillary notebook\footnote{The term linear in $w_2$ contains the canonical contribution generated by the dimensional nature of the $F_{\mu\nu} F^{\mu\nu}$ operator, and quantum contributions, i.e., $C_1 = 4 + C_{1, q}$.}. Their key property is that $C_0 \to 0$ as $G \to 0$. Consequently, in the absence of gravitational fluctuations, there is a free fixed point\footnote{This applies to the setting of a pure gauge theory. In the presence of matter, matter loops also generate $w_2$, e.g., as part of the Euler-Heisenberg effective action. In contrast to the situation in gravity, the generation of $w_2$ by charged matter is an IR effect.} $w_{2*} = 0$. Conversely, in the presence of gravitational fluctuations, $C_0 \neq 0$, such that $w_2 = 0$ is no longer a fixed point. Instead, the free fixed point is shifted into an interacting, shifted Gaussian fixed point (sGFP). At sufficiently weak gravitational interactions, the scaling exponent at the shifted Gaussian fixed point is close to the Gaussian one. The fixed-point value is given by

$$w_{2*} = \frac{-C_1(G)}{2C_2} + \sqrt{\frac{C_1(G)^2}{4C_2^2} - \frac{C_0(G)}{C_2}} , \quad (2.3)$$

for $C_2 > 0$ \footnote{Actually, there are two fixed points differing by the sign in front of the square root. The sign of $C_2$ determines which one is the sGFP. In the following we assume that $C_2 > 0$, which we confirm by an explicit computation later on.}. Hence, in the presence of gravity the gauge sector is necessarily interacting at high energies \cite{11, 25, 26} and complete asymptotic freedom in the gauge sector cannot be achieved. Instead, if a fixed point exists, it necessarily features a set of nonvanishing interactions.

Additionally, it is not even clear whether the gauge sector actually is UV complete under the impact of gravity, as suggested by studies focusing on the gauge coupling $g_Y$ alone: The sGFP becomes complex (and thus no longer a viable fixed point) due to a fixed-point collision at

$$C_{0, \text{crit}}(G) = \frac{C_2^2(G)}{4C_2} . \quad (2.4)$$
Therefore, for $C_0 > C_{0, \text{crit}}$, there is no real fixed point for $w_2$ and the gauge sector is not UV-complete for $C_0 > C_{0, \text{crit}}$. The condition on $C_0$ translates into a bound on the gravitational couplings. This bound is known as the weak gravity bound (WGB) in the literature, and appears in different gravity-matter systems \cite{10, 11, 25–27} at roughly similar values of the gravitational interactions \cite{55}. It owes its name to the fact that gravity has to be sufficiently weak in order for the sGFP to be real. Thus, the WGB separates the region in the gravitational parameterspace, where a UV-completion of the matter sector is possible, from the excluded, strong-gravity region.

In summary, a UV-completion for the Abelian gauge sector requires that two conditions hold simultaneously, namely

$$f_g > 0 \quad \text{and} \quad C_0(G) \leq C_{0, \text{crit}}(G).$$  \hspace{1cm} (2.5)

The largest part of the gravitational parameter space satisfies both conditions in four dimensions, but no longer above five dimensions \cite{26}, providing a constraint on the dimensionality of asymptotically safe gravity with Standard Model matter.

In the following, we extend the studies in \cite{25, 26} in various directions. In particular, we will investigate:

- how a second gauge invariant interaction involving four gauge fields, given by $(F \tilde{F})^2$, impacts the WGB,
- how this second gauge invariant interaction impacts the presence and value of a critical spacetime dimensionality, discovered in \cite{26},
- which interaction structures are induced in a system containing $N_V$ gauge fields,
- how robust the WGB is under changes of the gauge choice, and number of gauge fields $N_V$.

3 One species of gauge fields

3.1 Setup

We use functional RG (FRG) techniques \cite{56–58} adapted to gravity \cite{59} to extract the scale dependence of the couplings and wavefunction renormalizations, see \cite{7, 60–67} for introductions and reviews. The FRG realizes the Wilsonian paradigm of integrating out quantum fluctuations in the path integral according to their momentum shell. This is implemented for the scale-dependent effective action $\Gamma_k$, which interpolates between the classical action $S$ and the full effective action $\Gamma$. The power and versatility of FRG techniques, see \cite{67} for a recent review from statistical physics to quantum gravity, comes from the functional differential equation for $\Gamma_k$. This flow equation encodes the change of $\Gamma_k$ in response to quantum fluctuations in the momentum shell between $k$ and $k - \delta k$. The flow equation reads

$$k \partial_k \Gamma_k = \frac{1}{2} \text{STr} \left[ (\Gamma_k^{(2)} + R_k)^{-1} k \partial_k R_k \right],$$  \hspace{1cm} (3.1)
where the matrix $\Gamma^{(2)}_k$ is the second functional derivative of $\Gamma_k$ with respect to the fields and also carries spacetime and internal indices. The super-trace $\text{STr}$ involves the summation over these spacetime and internal indices, as well as an integration over $d$-dimensional space. The beta-functions of couplings, which parameterize the different interaction monomials in $\Gamma_k$ can be extracted from the flow equation by projecting onto the corresponding field monomials. The flow equation Eq. (3.1) can thus be understood as a compact summary of all beta functions and anomalous dimensions of the theory.

The regulator $R_k$ suppresses modes of the field with momenta below $k$ in the scale-dependent effective action $\Gamma_k$. The introduction of the regulator $R_k$ guarantees UV and IR finiteness of the flow equation, and implements the momentum-shell wise integration of quantum fluctuations. To ensure this, the regulator function has to satisfy several conditions, but a certain freedom in its choice exists. We use this freedom to select a regulator which is proportional to the momentum-dependent part of $\Gamma^{(2)}_k$, i.e.,

$$R_k = \left. \Gamma^{(2)}_k \right|_{\Lambda=0} r_k \left( \frac{p^2}{k^2} \right),$$

(3.2)

with the shape-function $r_k$. This spectrally adjusted regulator ensures that no further symmetries are broken by the regulator, see [61, 68–70]. For the shape-function $r_k$, we choose a Litim-type cutoff [71]

$$r_k(p^2/k^2) = \left( \frac{k^2}{p^2} - 1 \right) \theta \left( 1 - \frac{p^2}{k^2} \right),$$

(3.3)

which gives rise to analytic expressions for the beta-functions.

Even though along the flow all field monomials compatible with the symmetries are induced, for practical calculations one has to truncate the effective action. We approximate the dynamics of the system by

$$\Gamma_k = \Gamma_k^{\text{EH}} + \Gamma_k^{U(1)},$$

(3.4)

with the Einstein-Hilbert action

$$\Gamma_k^{\text{EH}} = -\frac{1}{16\pi G_k} \int d^4x \sqrt{g} \left( R - 2\Lambda_k \right) + S_{gf,h},$$

(3.5)

where we have introduced the dimensionless versions of the Newton coupling $G$ and of the cosmological constant $\Lambda$, respectively. To include gravitational fluctuations, we apply the background-field method, and decompose the full metric into a background metric and a fluctuation field, according to

$$g_{\mu\nu} = \delta_{\mu\nu} + h_{\mu\nu}.$$  

(3.6)

We choose a flat background which suffices to extract all beta functions that we are interested in. To calculate the propagator for the fluctuation field, we introduce a gauge-fixing condition $F^\mu = 0$, with

$$F^\mu = \left( \delta^{\mu\nu} \bar{D}^\nu - \frac{1 + \beta h}{4} \delta^{\nu\lambda} \bar{D}^\mu \right) h_{\nu\lambda}.$$  

(3.7)
β_h is one of two gauge parameters, which we fix to \( \beta_h = 1 \) in the following, except for Sec. 6, where we keep it more general. The second, \( \alpha_h \), enters the gauge-fixing action

\[
S_{gf,h} = \frac{1}{32 \pi \alpha_h G k^{-2}} \int d^4 x \sqrt{g} F^\mu \bar{g}^{\mu \nu} F_{\nu}, \quad \alpha_h \to 0.
\]

(3.8)

The gauge fixing also introduces Fadeev-Popov ghosts. These might contribute to the beta functions for the Abelian gauge field indirectly, through induced ghost-matter interactions [50] which we neglect here.

For the Abelian gauge sector, we approximate the dynamics by the standard kinetic term, and the two independent and gauge invariant interactions at mass dimension eight for four gauge fields:

\[
\Gamma_{U(1)}^{(1)} = \frac{1}{4} \int d^4 x \sqrt{g} F_{\mu \nu} F^{\mu \nu} + S_{gf,A}
+ \frac{w_2 k^{-4}}{8} \int d^4 x \sqrt{\bar{g}} (F_{\mu \nu} F^{\mu \nu})^2 + \frac{v_2 k^{-4}}{8} \int d^4 x \sqrt{\bar{g}} (F_{\mu \nu} \tilde{F}^{\mu \nu})^2.
\]

(3.9)

\( F_{\mu \nu} = \partial_\mu A_\nu - \partial_\nu A_\mu \) is the field strength tensor, and \( \tilde{F}_{\mu \nu} = \frac{i}{2} \epsilon_{\mu \nu \rho \sigma} F_{\rho \sigma} \) its dual tensor, with the totally antisymmetric tensor \( \epsilon_{\mu \nu \rho \sigma} \). In the presence of a non-flat metric, \( \epsilon_{\mu \nu \rho \sigma} \) is related to its flat space counterpart \( \epsilon_{ijkl} \) by \( \sqrt{\bar{g}}^{-1} \). The interactions labeled by the dimensionless and scale-dependent couplings \( w_2 \) and \( v_2 \) are the only two independent and gauge invariant interactions up to this order in mass dimension \(^8\). The action \( \Gamma_{U(1)}^{(1)} \) defined in Eq. (3.9) actually contains the same terms as the weak-field expansion of the Euler-Heisenberg effective action [73]. The gauge-fixing action for the U(1) gauge field is implemented by

\[
S_{gf,A} = \frac{1}{2 \alpha_A} \int d^4 x \sqrt{\bar{g}} \left( \bar{D}^\mu A_\mu \right) \left( \bar{D}^\mu A_\mu \right), \quad \alpha_A \to 0,
\]

(3.10)

and the corresponding Fadeev-Popov ghosts decouple from all beta functions in the non-gravitational sector.

After expanding the action Eq. (3.5) to second order in metric fluctuations \( h_{\mu \nu} \), we perform a rescaling of the fields to bring the kinetic terms to canonical form:

\[
h_{\mu \nu} \to \sqrt{Z_h} 16 \pi k^{-2} G h_{\mu \nu}, \quad A_\mu \to \sqrt{Z_A} A_\mu.
\]

(3.11)

The wave-function renormalizations \( Z_h \) and \( Z_A \) for metric fluctuations and the gauge field absorb the scale-dependence of the respective field \(^9\). The wave-function renormalizations give rise to anomalous dimensions, defined as

\[
\eta_h = -\partial_t \ln Z_h, \quad \eta_A = -\partial_t \ln Z_A.
\]

(3.12)

\(^8\)We neglect the operators \( F_{\mu \nu} \Box F^{\mu \nu} \) and \( F_{\mu \nu} \Box^2 F^{\mu \nu} \) here, which would contribute to the momentum-dependent anomalous dimension of the gauge field, see [72].

\(^9\)In the presence of diffeomorphism symmetry breaking by the regulator and gauge fixing, there is more than one "avatar" of the Newton coupling. However, with our rescaling of the metric fluctuation we introduce one single gravitational coupling \( G \) for each of the distinct gravity-matter vertices, as well as a single graviton-mass parameter. These choices assume a near-perturbative nature of quantum gravity, where the modified Slavnov-Taylor identities, which relate different gravity-matter and pure-gravity vertices, are trivial. Evidence for the agreement of different gravity-matter vertices has been found in [54, 74–76], see also [77] for a comparison of different graviton mass parameters.
3.2 Results

We now evaluate the scale dependence of the matter interactions $w_2$ and $v_2$, and of $\eta_A$. Their diagrammatic representation can be found in [25, 26], and we employ the Mathematica packages xAct [78–82], as well as the FormTracer [83], for their evaluation. For the purpose of simplicity, we will discuss the choice $\beta_h = 1$ in the following. We provide the analytical expressions at $\Lambda = 0$ below, because these suffice to understand the key mechanisms at play. The full results can be found in the ancillary notebook. In the perturbative approximation, where the anomalous dimension coming from the regulator insertion is neglected, the scale-dependences are given by

\begin{align}
 f_g = \frac{\eta_A}{2} = \frac{G}{4\pi} - \frac{v_2}{24\pi^2} - \frac{w_2}{6\pi^2}, \\
 \beta_{w_2} = \left(4 - \frac{7G}{2\pi} + \frac{5v_2}{12\pi^2}\right)w_2 + \left(8G^2 - \frac{Gv_2}{\pi} + \frac{v_2^2}{6\pi^2}\right) + \frac{35}{24\pi^2}w_2^2, \\
 \beta_{v_2} = \left(4 - \frac{25G}{6\pi} + \frac{11w_2}{12\pi^2}\right)v_2 - \left(8G^2 + \frac{7Gw_2}{6\pi} + \frac{w_2^2}{24\pi^2}\right) + \frac{1}{8\pi^2}v_2^2.
\end{align}

We observe that, depending on the signs of $v_2$ and $w_2$, these contributions could counteract the gravitational antiscreening in Eq. (3.13). As it turns out $v_2^* \approx -w_2^*$ and $w_2^* < 0$, cf. Fig. 1, such that $f_g$ is antiscreening also in the presence of four-photon interactions.

We now explore whether or not the two Eqns. (3.14) and (3.15) give rise to a WGB. Because the system of equations is coupled, this is less straightforward than for a single beta function. We start with the observation that Eq. (3.14) gives rise to a WGB for small enough $v_2$. Conversely, Eq. (3.15) does not give rise to a WGB for small enough $w_2$, because the signs of the quadratic term $\sim v_2^2$ and the gravitational term $\sim G^2$ are opposite, cf. (2.3). However, at large enough $w_2$, the term $\sim w_2^2$ in Eq. (3.15) might change the situation. Similarly, at large enough $v_2$, the term $\sim Gv_2$ in Eq. (3.14) might prevent the WGB. In turn, the fixed-point values $w_2^*$ and $v_2^*$ grow in magnitude with $G$. Unless they grow faster than the contributions $\sim G$ in the beta functions, the conclusion holds that a WGB arises from Eq. (3.14), but no second bound from Eq. (3.15) independently.

A numerical study of fixed-point solutions confirms this conclusion: The left panel in Fig. 1 shows that the WGB is only shifted somewhat by the inclusion of $v_2$, but not altered qualitatively. Indeed, the fixed-point value for $v_2$ turns out to be positive, thus leading to a shift of the weak-gravity bound to larger $G$ (at constant $\Lambda$).

The signs of the fixed-point values $w_2^*$ and $v_2^*$ might tempt one to speculate about the stability-properties of the fixed point. We caution that for stability studies, the IR values of couplings are decisive, not the UV fixed-point values. Nevertheless, a negative sign for $w_2$ might indicate that in the UV, the effective potential $W(F^2) = F^2 + w_2(F^2)^2 + ...$ is not locally stable about the origin, $F^2 = 0$, but no reliable conclusion can be drawn without a study of higher-order terms. We caution further that the sign of the fixed-point values for $w_2$ and $v_2$ can be changed by a change of basis, see Sec. 3.3, emphasizing that the full effective potential $V(F^2, \bar{F}F)$ is required to infer the presence of non-trivial minima.
3.3 Critical dimensionality revisited

In the following we extend the study of [26] and investigate, how the second independent four-gauge-field interaction impacts the critical dimensionality for asymptotic safety. The critical dimensionality arises from a mechanism discovered in [26]: In dimensions $d > 4$, the Abelian gauge coupling is dimensionful, i.e., $\bar{g}_Y = (4 - d)/2$. This results in an additional contribution to the scale-dependence of the dimensionless counterpart of the gauge coupling $g_Y$, i.e.:

$$\beta_{g_Y} = \left(\frac{d - 4}{2} - f_g\right) g_Y + \mathcal{O}(g_Y^3).$$

(3.16)

In $d > 4$, this dimensional contribution competes with the anti-screening contribution from metric fluctuations, because both take the form of a scaling dimension. The overall scaling dimension has to be positive for a UV completion, see Sec. 2,

$$f_g(d) > \frac{d - 4}{2},$$

(3.17)

such that the Abelian gauge coupling is a relevant direction at the Gaussian fixed point. From Eq. (3.17), it follows that the gravitational contribution $f_g$ would have to increase with the dimensionality $d$ to induce a UV-completion for all values of $d$. However, $f_g$ actually shows the opposite behavior and decreases with the dimensionality (for fixed values of the gravitational couplings), see [26]. Thus, the required growth of $f_g(d)$ with $d$ requires increasing values of the gravitational couplings, i.e., a shift into the strong-gravity regime. However, the WGB for the induced four-gauge interactions excludes this regime from the

\[\text{Figure 1. Left panel: We show the WGB in the } G - \Lambda \text{ plane, which separates the allowed weak-gravity regime (white) from the excluded strong-gravity regime (grey). The green dotted line shows the WGB from previous work, where } v_2 \text{ was neglected; the blue dashed line shows the WGB under the inclusion of } v_2 \text{ that is consistently evaluated at its fixed-point value } v_2^* = v_2^*(G, \Lambda).\]

\[\text{Right panel: We show the real and imaginary parts of the two fixed-point values } w_2^* \text{ and } v_2^* \text{ as a function of } G \text{ for } \Lambda = 0. \text{ The WGB is visible as the point at which the imaginary parts depart from zero.}\]
viable parameter space. In summary, there are two competing effects: A sufficiently large $f_g$ requires an increasing strength of gravity with increasing $d$; avoiding the WGB imposes a bound on the strength of gravity that is present at all $d$. The competition of these two effects gives rise to a critical dimensionality, beyond which the Abelian gauge sector remains UV-incomplete. In a truncation only taking into account the $w_2(F^2)^2$ interaction, this critical dimensionality was found to be $d_c \approx 5.8$ [26].

We test how robust this result is by adding the second independent four-gauge-field interaction\(^\text{10}\). Because the dual field-strength tensor $\tilde{F}_{\mu \nu}$ in Eq. (3.9) is not a two-tensor away from $d = 4$ dimensions, we exploit that in Euclidean spaces\(^\text{11}\)

\[
F_{\mu \rho} F_{\nu} + \tilde{F}_{\mu \rho} \tilde{F}_{\nu} = \frac{1}{2} F_{\rho \sigma} F_{\mu \nu} g_0^{\mu \nu},
\]

\[
F_{\mu \rho} \tilde{F}_{\nu} = \tilde{F}_{\mu \rho} F_{\nu} = \frac{1}{4} F_{\sigma} \rho \sigma g_0^{\mu \nu},
\]

which holds in $d = 4$, see [84, 85] for the Lorentzian version, and from which it follows that

\[
(F_{\rho \sigma} \tilde{F}_{\rho \sigma})(F_{\mu \nu} \tilde{F}_{\mu \nu}) = -4 F_{\mu \rho} F_{\nu} F_{\rho \sigma} F_{\mu \nu} + 2(F_{\rho \sigma} \tilde{F}_{\rho \sigma})(F_{\mu \nu} \tilde{F}_{\mu \nu}),
\]

such that we can replace the $v_2(F\tilde{F})^2$ interaction in Eq. (3.9) with a $h_2 F^4$ interaction, because the difference between them is proportional to $w_2(F^2)^2$. In $d = 4$ dimensions, this is just a different choice of basis operators. The physical properties of the system, for example critical exponents, and the existence and location of a WGB are the same in both bases, as we have explicitly confirmed in our computation. In $d > 4$ dimensions, the $F^4$ basis is the appropriate one to work with. To extend the study of a single species of gauge field to $d > 4$, we generalize the matter part of the scale-dependent effective action in Eq. (3.9) to

\[
\Gamma_k^{(1)} = \frac{1}{4} \int d^d x \sqrt{g} F_{\mu \nu} F_{\mu \nu} + S_{g, A}
\]

\[
+ \frac{w_2 k^{-d}}{8} \int d^d x \sqrt{g} (F_{\mu \nu} F_{\mu \nu})^2 + \frac{h_2 k^{-d}}{8} \int d^d x \sqrt{g} (F_{\mu \nu} F_{\nu \rho} F_{\rho \sigma} F_{\sigma \mu}).
\]

With this action, we can investigate the WGB for the couplings $w_2$ and $h_2$, as well as the condition (3.17) for arbitrary dimensionality. For a given dimensionality, these two conditions give rise to a region in parameter space, where metric fluctuations are strong enough to render the Abelian gauge coupling UV-complete, but weak enough to avoid the WGB for induced interactions. The two-dimensional area of that region (spanned by $G \in [0, 1000]$ and $\Lambda \in [-1500, 0.5]$) is denoted $A(d)$ see [26] for details. In Fig. 2 we compare $A(d)$ in the $(F^2)^2$ truncation studied in [26] (green, dashed line) with the truncation defined in Eq. (3.21) (blue, solid line). In both cases, we normalize to the allowed region for

\(^{10}\)A study of the gauge-dependence of the existence and value of the critical dimensionality as an independent test of the robustness of the system can be found in [55], showing only a quantitative impact of the gauge-parameter $\beta_h$ on $d_c$ in the $w_2(F^2)^2$ truncation.

\(^{11}\)We thank Benjamin Knorr for pointing this out to us.
Figure 2. We show the area of the viable region in the gravitational parameter space for \( \Lambda \in [-1500, 0.5] \) and \( G \in [0, 1000] \), where the Abelian gauge sector could be UV-complete in the presence of gravitational fluctuations, as a function of the dimensionality \( d \). The green (dashed) line shows the area of the viable region \( A(d) \) in the pure \((F^2)^2\) truncation studied in [26]. The blue (solid) line shows the area of the viable region \( A(d) \) in the more complete truncation defined in Eq. (3.21). In both truncations, the viable region in the investigated parameter region shrinks to zero at \( d_c < 6 \).

\( d = 4 \). The inclusion of the second independent interaction \( h_2 \) clearly only has a subleading quantitative effect. The main result of [26], namely the existence of a critical dimension \( d_c < 6 \), above which the Abelian gauge sector remains UV-incomplete, even in the presence of quantum gravity, remains unchanged. In fact, the inclusion of \( h_2 \) reduces the critical dimensionality to \( d_c \approx 5.5 \), compared to \( d_c \approx 5.8 \) reported in [26].

We interpret this small impact of the second four-gauge-field interaction as an indication for the robustness of our study. Thus, four (and, with a significantly reduced parameter space also five) dimensions indeed appear to be special in asymptotically safe gravity-matter models.

4 Two species of gauge fields

We now extend the system and add a second Abelian gauge field. The motivation for such an extension is twofold: First, nature might contain more than a single Abelian gauge field, e.g., dark photons are considered in Standard-Model extensions [86]. Second, gravitational effects are independent of internal symmetries, because gravity is "blind" to those. Accordingly, our study of several Abelian gauge fields is also relevant for non-Abelian gauge groups, at least at small values of the non-Abelian gauge coupling, which remains asymptotically free in the presence of gravitational fluctuations [40, 42, 44], see also the corresponding discussion in [11].

We study a gauge field \( A_a^\mu \), where \( a \in (1, 2) \) is a species-index, with field strength tensor \( F^a_{\mu \nu} \). For a study of the gravity-induced interactions of these two gauge fields, a thorough understanding of their global symmetries is critical. These global symmetries are
always preserved by the RG flow, unless explicit symmetry violations are introduced by the regulator. Starting from the two kinetic terms, i.e., \( F_a^{\mu\nu} F^{\mu\nu, a} \), we identify the following global symmetry: The sum of the two kinetic terms is preserved under the global \( O(2) \) rotation

\[
\begin{pmatrix}
A_1^\mu \\
A_2^\mu
\end{pmatrix} \rightarrow \begin{pmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{pmatrix} \begin{pmatrix} A_1^\mu \\
A_2^\mu
\end{pmatrix}.
\]

For the special case of \( \theta = \pi \), this reduces to two independent \( \mathbb{Z}_2 \) symmetries, under which \( A_1^\mu \rightarrow -A_1^\mu \), or \( A_2^\mu \rightarrow -A_2^\mu \) respectively. These two \( \mathbb{Z}_2 \) symmetries are preserved even if the two gauge fields acquire different anomalous dimensions, i.e., if the rescaling (3.11) is performed for both gauge fields individually. If the anomalous dimensions are the same, then the full global \( O(2) \) symmetry is preserved by the flow.

It was recently investigated for scalar fields, whether the analogous global \( O(2) \) symmetry is preserved by the flow and a positive answer was found [27]. Together with an argument about the diagrammatic structure of the flow equation (spelled out in [87]), where no symmetry-breaking terms are generated if regulator, propagator and interaction vertices satisfy a symmetry, we do not expect that violations of the \( O(2) \) symmetry are induced by gravitational fluctuations. Nevertheless, we test this hypothesis explicitly, by working in an enlarged space of couplings, which only preserves the two \( \mathbb{Z}_2 \) symmetries.

The most general action for the gauge fields, which satisfies these symmetries, and including all linearly independent interaction up to dimension eight with four gauge fields is given by

\[
\Gamma_k^{U(1) \times U(1)} = \frac{1}{4} \int d^4x \sqrt{g} F_{\mu\nu}^{a} F^{\mu\nu, a} + S_{gf, A} \\
+ \frac{k^{-4}}{16} \int d^4x \sqrt{g} \left( w_2 \left[ F_{\mu\nu}^{a} F^{\mu\nu, a} \right]^2 + y_2 \left( F_{\mu\nu}^{a} F^{\mu\nu, b} \right) \left( F_{\rho\sigma}^{a} F^{\rho\sigma, b} \right) \\
+ x_2 \left( F_{\mu\nu}^{2} F^{\mu\nu, 2} \right)^2 + z_2 \left( F_{\mu\nu}^{1} F^{\mu\nu, 2} \right) \left( F_{\rho\sigma}^{1} F^{\rho\sigma, 2} \right) \right) \\
+ \frac{k^{-4}}{16} \int d^4x \sqrt{g} \left( v_2 \left[ F_{\mu\nu}^{a} \tilde{F}_{\mu\nu}^{a} \right]^2 + t_2 \left( F_{\mu\nu}^{a} \tilde{F}_{\mu\nu}^{b} \right) \left( F_{\rho\sigma}^{a} \tilde{F}_{\rho\sigma}^{b} \right) \\
+ u_2 \left[ F_{\mu\nu}^{2} \tilde{F}_{\mu\nu}^{2} \right]^2 + s_2 \left( F_{\mu\nu}^{1} \tilde{F}_{\mu\nu}^{2} \right) \left( F_{\rho\sigma}^{1} \tilde{F}_{\rho\sigma}^{2} \right) \right),
\]

where we have introduced the scale-dependent and dimensionless couplings \( x_2, y_2, z_2, u_2, t_2, \) and \( s_2 \), describing the self-interaction of the second gauge field, and the interaction between both fields. For these interactions, the \( O(2) \) symmetry of the kinetic term is only respected, if

\[
x_2 = 0, \quad u_2 = 0, \quad z_2 = 0, \quad s_2 = 0,
\]

and for equal wave-function renormalizations of the two gauge fields. Our explicit computations confirm that both anomalous dimensions are equal and

\[
\left( \beta_{x_2}, \beta_{y_2}, \beta_{u_2}, \beta_{s_2} \right) \big|_{x_2 = y_2 = u_2 = s_2 = 0} = (0, 0, 0, 0),
\]
indicating that the $O(2)$-symmetry breaking couplings $x_2$, $z_2$, $u_2$, and $s_2$ can be consistently set to zero. Hence, once restricted to the $O(2)$ theory-space, the non-$O(2)$-symmetric interactions $x_2$, $z_2$, $u_2$ and $s_2$ are not induced by gravitational fluctuations. Based on this finding, we will restrict the following analysis to the $O(2)$-symmetric theory-space. This reduces the number of induced couplings to $w_2$, $v_2$, $y_2$, and $t_2$, and a single wave-function renormalization $Z_A$. The beta functions for $\Lambda = 0$ (for the general expression see the ancillary notebook) are given by

$$\beta_{w_2} = \left( 4 - \frac{13G}{6\pi} + \frac{5t_2}{16\pi^2} + \frac{5v_2}{24\pi^2} + \frac{23y_2}{24\pi^2} \right) w_2 + \frac{55}{48\pi^2} w_2^2$$

$$+ G \left( \frac{v_2}{2\pi} - \frac{3t_2}{4\pi} - \frac{2v_2}{3\pi} \right) + \frac{5t_2^2}{192\pi^2} + \frac{t_2 v_2}{48\pi^2} + \frac{t_2 y_2}{12\pi^2} + \frac{v_2^2}{16\pi^2} + \frac{17y_2^2}{192\pi^2}.$$ 

$$\beta_{y_2} = \left( 4 - \frac{17G}{6\pi} + \frac{t_2}{6\pi^2} + \frac{5v_2}{24\pi^2} + \frac{41v_2}{48\pi^2} \right) y_2 + \frac{45}{64\pi^2} y_2^2$$

$$+ 16G^2 - G \left( \frac{t_2}{4\pi} + \frac{3v_2}{2\pi} + \frac{4w_2}{3\pi} \right) + \frac{5t_2^2}{64\pi^2} + \frac{7t_2 v_2}{48\pi^2} + \frac{v_2^2}{48\pi^2} + \frac{w_2^2}{48\pi^2},$$

$$\beta_{v_2} = \left( 4 - \frac{8G}{3\pi} + \frac{5t_2}{48\pi^2} + \frac{17w_2}{24\pi^2} + \frac{3y_2}{8\pi^2} \right) v_2 + \frac{1}{12\pi^2} v_2^2$$

$$+ G \left( \frac{w_2}{2\pi} - \frac{3t_2}{4\pi} - \frac{5y_2}{6\pi} \right) - \frac{v_2^2}{96\pi^2} + \frac{t_2 y_2}{96\pi^2} - \frac{w_2 y_2}{48\pi^2} - \frac{y_2^2}{96\pi^2},$$

$$\beta_{t_2} = \left( 4 - \frac{41G}{12\pi} + \frac{v_2}{16\pi^2} + \frac{17w_2}{24\pi^2} + \frac{25y_2}{48\pi^2} \right) t_2 + \frac{7}{64\pi^2} t_2^2$$

$$- 16G^2 - G \left( \frac{3v_2}{2\pi} + \frac{5w_2}{3\pi} + \frac{y_2}{3\pi} \right) - \frac{v_2^2}{48\pi^2} + \frac{v_2 y_2}{8\pi^2} - \frac{w_2^2}{48\pi^2} - \frac{w_2 y_2}{48\pi^2} - \frac{y_2^2}{64\pi^2}. \quad (4.5)$$

We first note the absence of pure gravitational inducing contributions to $\beta_{w_2}$ and $\beta_{v_2}$. This is an artefact of the chosen basis, and only occurs for $\Lambda = 0$, and does therefore not have any physical interpretation. Even at $\Lambda = 0$, the couplings $w_2$ and $v_2$ are indirectly induced by non-vanishing fixed-point values for $y_2$ and $t_2$, which in turn feature direct gravitational contributions to the inducing term also at $\Lambda = 0$. Away from $\Lambda = 0$, all these four couplings are induced by gravity, i.e., feature terms $\sim G^2$ in their beta functions. In the left panel of Fig. 3 we show the fixed-point structure of the two-species system for $\Lambda = -0.5$. We observe that the fixed-point values of $w_2$ and $v_2$ remain small compared to $y_2$ and $t_2$. This is a consequence of the small pure-gravitational inducing terms in their respective beta-functions, which cancel for $\Lambda = 0$.

As in the single-species case, the matter contributions to the scale dependences, which encode the back-reaction between both gauge fields, are suppressed by a factor of $1/\pi$ for gravity-matter contributions, and by a factor of $1/\pi^2$ for pure matter contributions, compared to the pure-gravitational contributions, see [11] for a detailed discussion. Therefore, as long as the fixed-point values at the sGFP remain small, the second gauge field will only result in small quantitative changes, compared to the single-species system.

To better quantify the impact of the second gauge field, we define the critical value of the Newton coupling $G_{\text{crit}}(\Lambda)$ as the line in the $G - \Lambda$ plane, where the sGFP becomes
fixed-point values
excluded
strong-
gravity
regime
allowed
weak-gravity regime
Re(w^2*)
Re(v^2*)
Re(y^2*)
Re(t^2*)
Im(w^2*)
Im(v^2*)
Im(y^2*)
Im(t^2*)

Figure 3. Left panel: We show the real and imaginary parts of the four fixed-point values of the $O(2)$-symmetric, two-species system described by Eq. (4.2) as a function of $G$ for $\Lambda = -0.5$. We show the imaginary parts of the sGFP, once the fixed-point values become complex. The WGB is visible as the point at which the imaginary parts depart from zero, marked by the vertical gray line. Left panel: We show the relative deviation of the critical value of the Newton coupling $G_{\text{crit}}$ between the single-gauge field system (see Eq. (3.9)), and the $O(2)$ symmetric system of two vector fields (see Eq. (4.2)). $G_{\text{crit}}$ is defined as the value for the Newton coupling, where the sGFP becomes complex, i.e., the value of the WGB as a function of $\Lambda$, see (4.6). The blue (solid) line shows the comparison of both systems for $\beta_h = 1$, indicating that the inclusion of the second gauge field only modifies the location of the WGB on the percent level. The green (dashed) and magenta (dotted) line show the same quantity for different choices of the gauge-fixing parameter $\beta_h$, see Sec. 6 for a discussion. We do not plot the magenta (dotted) line for $\Lambda \in (-0.16, 0.02)$ due to numerical artefacts, which we briefly discuss in App. A.

complex, i.e., where the condition (2.4) is satisfied,

$$G_{\text{crit}}(\Lambda) = G(\Lambda), \text{ such that } \min_{G(\Lambda)} \text{Im}[w^2, v^2, \ldots] \neq 0. \quad (4.6)$$

In the right panel of Fig. 3 we compare the values of $G_{\text{crit}}(\Lambda)$ of the single-species and two-species systems. We see that the second gauge field only changes the position of the WGB slightly, with the relative difference of $G_{\text{crit}}(\Lambda)$ at the level of few percent. This is in line with the expectation that for low numbers of species, the gravitational terms are dominant, due to the suppression by factors of $1/\pi$ of pure-matter contributions. This indicates that the two gauge fields are almost decoupled, leading to a very small impact of the second gauge field on the WGB. This observation is qualitatively similar to the WGB of induced scalar interactions, where only the inclusion of many scalar fields results in a significant change of $G_{\text{crit}}$, see [27].

5 $N_V$ species of vector fields

We extend the previous analysis and study a coupled system of $N_V$ Abelian gauge fields. Based on our findings in the previous section, where we have seen that gravitational fluc-
tuations respect the global $O(2)$ symmetry of the kinetic term, we will consider the generalization to a globally $O(N_V)$ symmetric system in the following. We stress that this symmetry is realized at vanishing gauge coupling, also if the $N_V$ gauge fields transform in a local, non-Abelian symmetry group. Given that asymptotically safe gravity fluctuations preserve the asymptotically free fixed point in non-Abelian gauge couplings (at least in studies based on truncations of the full dynamics [40, 42, 44]), the fixed-point structure of induced interactions of a system of $N_V = 8 + 3 + 1$ gauge fields with global $O(N_V)$ symmetry is likely relevant for the Standard Model, with its 8 gluons, 3 weak gauge bosons and one Abelian gauge field. Once the gauge couplings depart from their vanishing fixed-point value (or if the Abelian gauge coupling starts out at an interacting fixed point [39, 41]), additional interactions will be generated and the global $O(N_V)$ symmetry will be broken.

5.1 The weak-gravity bound for many vector fields

The most general $O(N_V)$ symmetric and gauge invariant effective action up to dimension 8 operators with four gauge fields is given by extending the range of the species index $a \in (1, \ldots, N_V)$ in Eq. (4.2). Thus, we consider the flowing action

$$
\Gamma_{U(1)^{N_V}}^{k} = \frac{1}{4} \int d^4 x \sqrt{g} F_{\mu \nu}^{a} F^{\mu \nu, a} + S_{gf, A} + \frac{k^{-8}}{8N_V} \int d^4 x \sqrt{g} \left( w_2 [F_{\mu \nu}^{a} F^{\mu \nu, a}]^2 + y_2 (F_{\mu \nu}^{a} F^{\mu \nu, b})(F_{\rho \sigma}^{a} F^{\rho \sigma, b}) \right) + \frac{k^{-8}}{8N_V} \int d^4 x \sqrt{g} \left( v_2 [\tilde{F}_{\mu \nu}^{a} \tilde{F}^{\mu \nu, a}]^2 + t_2 (F_{\mu \nu}^{a} \tilde{F}^{\mu \nu, b})(F_{\rho \sigma}^{a} \tilde{F}^{\rho \sigma, b}) \right),
$$

(5.1)

where the species indices $a$ and $b$ run from 1 to $N_V$. For the choice $N_V = 2$, we recover the $O(2)$-symmetric system studied in Sec. 4. The beta functions for $\Lambda = 0$ (for the general
expression see the ancillary notebook) and for general \( N_V \) are given by

\[
\beta_{w_2} = \left( 4 - \frac{13G}{6\pi} + \frac{5(N_V + 1)t_2}{24\pi^2 N_V} + \frac{5v_2}{12\pi^2 N_V} + \frac{(5N_V + 36)y_2}{24\pi^2 N_V} \right) w_2 + \frac{21N_V + 13}{24\pi^2 N_V} \frac{w_2^2}{24\pi^2 N_V} \\
+ G \left( -\frac{3t_2}{4\pi} + \frac{v_2}{2\pi} - \frac{2y_2}{3\pi} \right) \\
+ \frac{(N_V + 8)t_2^2}{192\pi^2 N_V} + \frac{t_2 v_2}{24\pi^2 N_V} + \frac{v_2^2}{8\pi^2 N_V} + \frac{(N_V + 14)t_2 y_2}{96\pi^2 N_V} + \frac{(N_V + 32)y_2^2}{192\pi^2 N_V} ,
\]

\[
\beta_{y_2} = \left( 4 - \frac{17G}{6\pi} + \frac{(7N_V + 18)t_2}{96\pi^2 N_V} + \frac{5v_2}{12\pi^2 N_V} + \frac{(12N_V + 17)y_2}{24\pi^2 N_V} \right) y_2 + \frac{23N_V + 224}{192\pi^2 N_V} \frac{y_2^2}{192\pi^2 N_V} \\
+ 8G^2 N_V + G \left( -\frac{t_2}{4\pi} + \frac{3v_2}{2\pi} - \frac{4w_2}{3\pi} \right) \\
+ \frac{(7N_V + 16)t_2^2}{192\pi^2 N_V} + \frac{7t_2 v_2}{24\pi^2 N_V} + \frac{v_2^2}{24\pi^2 N_V} + \frac{w_2^2}{24\pi^2 N_V} ,
\]

\[
\beta_{v_2} = \left( 4 - \frac{8G}{3\pi} + \frac{(2N_V + 1)t_2}{24\pi^2 N_V} + \frac{(6N_V + 5)w_2}{12\pi^2 N_V} + \frac{(N_V + 7)y_2}{12\pi^2 N_V} \right) v_2 + \frac{1}{6\pi^2 N_V} \frac{v_2^2}{6\pi^2 N_V} \\
+ \frac{8G^2 N_V + G \left( -\frac{3t_2}{4\pi} + \frac{w_2}{2\pi} - \frac{5y_2}{6\pi} \right) - \frac{(N_V + 2)t_2^2}{192\pi^2 N_V} - \frac{(N_V - 4)t_2 y_2}{96\pi^2 N_V} - \frac{w_2 y_2}{24\pi^2 N_V} - \frac{(N_V + 2)y_2^2}{192\pi^2 N_V} ,
\]

\[
\beta_{t_2} = \left( 4 - \frac{41G}{12\pi} + \frac{v_2}{8\pi^2 N_V} + \frac{(6N_V + 5)w_2}{12\pi^2 N_V} + \frac{5(3N_V + 14)y_2}{96\pi^2 N_V} \right) t_2 + \frac{5N_V + 4}{64\pi^2 N_V} \frac{t_2^2}{64\pi^2 N_V} \\
- 8G^2 N_V + G \left( -\frac{3t_2}{2\pi} + \frac{5w_2}{3\pi} - \frac{y_2}{3\pi} \right) \\
- \frac{v_2^2}{24\pi^2 N_V} + \frac{v_2 y_2}{4\pi^2 N_V} - \frac{w_2 y_2}{24\pi^2 N_V} - \frac{w_2 y_2}{24\pi^2 N_V} - \frac{(N_V + 4)y_2^2}{192\pi^2 N_V} ,
\]

(5.2)

from which we recover the beta-functions of the \( O(2) \) symmetric system (4.5) by choosing \( N_V = 2 \), and the beta functions of the single-species system (3.14) and (3.15) by choosing \( N_V = 1 \), rescaling \( w_2 \to w_2/2 \) and \( v_2 \to v_2/2 \), identifying \( y_2 = w_2/2 \) and \( t_2 = v_2/2 \), and adding \( \beta_{w_2} + \beta_{y_2} \) and \( \beta_{v_2} + \beta_{t_2} \), respectively. Contributions that are linear in one of the matter couplings are independent of \( N_V \), while contributions that are quadratic in matter couplings feature \( N_V \) independent parts, as well as parts that are suppressed by \( 1/N_V \). Finally, only the contributions that are independent of the matter couplings, i.e., the gravitational contribution to the inducing terms, are linear in \( N_V \). Thus, the WGB becomes stronger when increasing \( N_V \), because the absolute value of the coefficients \( C_0 \) in (2.3) increases linearly with \( N_V \), while the other coefficients remain constant to leading order. Hence, the condition (2.4) for the WGB is met at smaller values for the Newton couplings. Thus, at large enough \( N_V \), an infinitesimally small value of the \( G \) is sufficient to trigger a fixed-point collision in the matter sector, i.e., we expect that \( G_{\text{crit}} \to 0 \) as \( N_V \to \infty \).

In Fig. 4 we show the WGB for different values of \( N_V \). We see that the excluded region indeed grows monotonically with larger \( N_V \). Therefore, theories with more gauge fields will have less gravitational parameter space available where the they might be UV-complete.
Figure 4. We show the WGB for the $O(N \nu)$ symmetric system for different values of $N \nu$, labeled by the dashed (dotted, dashdotted, dashdotdotted) line. The respective gray shaded region indicates the strong-gravity regime, which is excluded due to the presence of UV-divergences in the matter sector. The magenta line and the markers indicate the gravitational fixed-point values in the background-field approximation as a function of $N \nu$, see Sec. 5.2.

5.2 Gravitational fixed-point values in relation to the weak-gravity bound

So far, we have treated $\Lambda$ and $G$ as free input parameters. This enabled us to develop an understanding of the gravitational parameter space beyond just the behavior of the matter couplings at the gravitational fixed point. In addition, it allows us to interpret our results in light of scenarios of effective asymptotic safety, see [31, 88, 89]. In these scenarios, an effective-field theoretic description emerges from a more fundamental theory at a cutoff scale above the Planck scale. Studying the WGB in the parameter space spanned by $\Lambda$ and $G$ constrains the initial conditions for $\Lambda$ and $G$ of this effective field theory.

Yet, it is of course crucial to supplement the analysis by the actual fixed-point values for $G$ and $\Lambda$ to discover whether or not asymptotic safety evades the weak-gravity bound and passes this critical viability test. The existence of a fixed point for pure gravity has been established in numerous works [70, 72, 74, 90–142] and under the inclusion of matter in [14, 28, 42, 54, 75, 76, 138, 143–155]. To search for fixed points, we use the beta-functions for $\Lambda$ and $G$ [156], supplemented with the matter contributions from [144, 147]. We find that the fixed-point values stay below the WGB for all values of $N \nu$ that we investigate explicitly. In particular, this means that systems with 12 gauge fields, like the SM, evade the corresponding weak-gravity bound.

Our numerical investigation stops at a finite, large $N \nu$, and we supplement it by an analytical large $N \nu$ study: To determine whether or not the WGB is still evaded for $N \nu \rightarrow \infty$, we evaluate the beta functions of the induced matter couplings (5.2) at the gravitational fixed-point values for $G$ and $\Lambda$. The resulting beta functions feature a well-defined $N \nu \rightarrow \infty$
limit, given by
\[
\beta w^2 = \left( \frac{5t^2}{24\pi^2} + \frac{5y^2}{24\pi^2} + 4 \right) w^2 + \frac{7}{8\pi^2}w_2^3 + \left( \frac{t^2}{192\pi^2} + \frac{t_2y^2}{96\pi^2} + \frac{y_2^2}{192\pi^2} \right),
\]
\[
\beta y^2 = \left( \frac{7t^2}{96\pi^2} + \frac{w_2^2}{2\pi^2} + 4 \right) y^2 + \frac{23}{192\pi^2}y_2^2 + \frac{7t^2}{192\pi^2},
\]
\[
\beta v^2 = \left( \frac{t^2}{12\pi^2} + \frac{w^2}{2\pi^2} + \frac{y^2}{12\pi^2} + 4 \right) v^2 + \left( -\frac{t^2}{192\pi^2} - \frac{t_2y^2}{96\pi^2} - \frac{y_2^2}{192\pi^2} \right),
\]
\[
\beta t^2 = \left( \frac{w_2^2}{2\pi^2} + \frac{5y_2^2}{32\pi^2} + 4 \right) t^2 + \frac{5}{64\pi^2}t_2^2 - \frac{y_2^2}{192\pi^2}. \tag{5.3}
\]

The existence of this limit indicates that $G_* \to 0$ faster than $1/\sqrt{N_V}$ as $N_V$ increases, cf. (5.2). Hence, gravity and matter become weakly coupled for large $N_V$ and decouple entirely in the $N_V \to \infty$ limit. Since gravity and matter are decoupled for $N_V \to \infty$, the self-interactions are no longer induced in this limit, but assume their Gaussian fixed-point values.

In summary, our studies indicate that asymptotically safe quantum gravity could be compatible with any number of $O(N_V)$-symmetric gauge fields. Therefore, within our truncation and the investigated mechanism, the interplay of gravity and matter does not give rise to any bound on the number of vector fields. This is in contrast to the scalar sector, where no number of scalar fields (in the absence of spinning matter) gives rise to a UV-complete scalar-gravity system [27].

6  Gauge dependence as a test of the robustness of the results

In our discussion so far, we have fixed the gauge parameter $\beta_h = 1$. In the following, we will investigate the robustness of the WGB under variations of $\beta_h \in [-\infty, 3]$ in the Landau limit, i.e., $\alpha_A \to 0$ and $\alpha_h \to 0$. The point $\beta_h = 3$ is an incomplete gauge fixing, thus results for values $\beta_h \lesssim 3$ become untrustworthy.

This serves as a test for the robustness of our truncation: physical information, such as the existence of a fixed point, and physical quantities, such as the critical exponents, should be independent of the gauge parameters, when $\Gamma_k$ is not truncated. Conversely, gauge dependence arises in physical quantities, when truncations are employed. When extending the truncation, these gauge dependences are expected to decrease, converging to gauge-independent physical quantities when $\Gamma_k$ is not truncated at all. Similarly, when a truncation already captures the physics of a system well, gauge dependences are expected to be mild. In this spirit, we explore the gauge dependence of our results as a test of their robustness.

In the following, we focus on the WGB for the different systems discussed in the previous sections, namely a single species, two species and $N_V$ species of gauge fields. For simplicity, we specify to $\Lambda = 0$, and compute the critical value $G_{\text{crit}}(\Lambda = 0)$ as a function of $\beta_h$, cf. Eq. (4.6).

By investigating $G_{\text{crit}}$ for the specific choice $\Lambda = 0$ as a function of $\beta_h$, we gain insight into the robustness of the WGB.
6.1 One species of gauge fields

We follow a similar strategy as in the previous sections, and first investigate the gauge-dependence of the WGB for two truncations, which contain either only $w_2$ or only $v_2$ and consider a third truncation which contains both couplings as a second step.

For the individual couplings, the relative sign between the coefficients $C_0$ and $C_2$ determines if a WGB is possible in principle, cf. Eq. (2.3). Since only pure gauge-field diagrams contribute to $C_2$, it is independent of $\beta_h$.

For a truncation with $w_2$ only, the gauge-dependent coefficient $C_{0,w_2}(G,v_2 = 0)$ reads

$$C_{0,w_2}(G,v_2 = 0) = \frac{4G^2}{3(-3 + \beta)} \left( 225 - 60\beta - 154\beta^2 + 100\beta^3 - 15\beta^4 \right),$$

which is positive for $\beta_h \gtrsim -1.03$, and negative for smaller $\beta_h$. Thus, for $\beta_h \lesssim -1.03$, there is no WGB in a truncation with only $w_2$. For larger $\beta_h$, a WGB exists, and the resulting $G_{\text{crit}}$ at $\Lambda = 0$ only quantitatively depends on $\beta_h$, see Fig. 5.

The situation is reversed when considering $v_2$ as the only interaction of the system, since

$$C_{0,v_2}(G,w_2 = 0) = -C_{0,w_2}(G,v_2 = 0).$$

As $C_{2,v_2} > 0$, cf. Eq. (3.15), a WGB for $v_2$ can only exist for $\beta_h \lesssim -1.03$, cf. Fig. 5.

We have seen in Sec. 3 that the effect of $v_2$ in $\beta_{w_2}$ and vice-versa is suppressed compared to the gravitational contributions. Therefore, we expect that the truncation containing both
$w_2$ and $v_2$ features a WGB that is quantitatively close to the sum of the two individual WGBs for all values of $\beta_h$. An explicit computation of $G_{\text{crit}}$ confirms this expectation, and $G_{\text{crit}}$ is finite and non-zero for all $\beta_h \in [\infty, 3)$, see Fig. 5. This result indicates the reliability of the previous results [25, 26] for the choice $\beta_h = 1$ that was employed in these works, but also highlights the importance of considering a full basis of induced operators at a given mass dimension, to reliably investigate the presence of a WGB.

In summary, the WGB of the full single gauge-field system features a WGB for all values of the gauge parameter $\beta_h < 3$. The position of the WGB in the gravitational parameter space only depends quantitatively on $\beta_h$. This indicates that indeed the physical information, namely that a strongly coupled regime of quantum gravity appears to be incompatible with a UV-complete gauge sector, is independent of the gauge choice.

### 6.2 Multiple species of gauge fields

For an estimate on the gauge-dependence of the $O(2)$-symmetric system discussed in Sec. 4, we compare the location of the WGB with the single-species system for various choices of $\beta_h$, see the right panel of Fig. 3. For each displayed gauge-choice, the difference of the WGB in the single-species and two-species systems is at the level of a few percent. An exception to this small deviation appears for $\Lambda \approx 0$, which is produced by a sign-flip in the beta-functions, giving rise to a dent in the WGB. We comment on this feature in App. A.

The overall small deviation of the WGB between the single-species and two-species systems indicates that the impact of the second gauge field on the WGB remains small, independent of the gauge choice and that the gauge dependence is overall not very large, except close to $\beta_h = -1$, where somewhat larger variations occur.

We study the gauge-dependence of the $O(N_V)$-symmetric system by computing the critical value $G_{\text{crit}}$ at $\Lambda = 0$. As in the single- and two-species systems, $G_{\text{crit}}$ depends on $\beta_h$ only quantitatively, and remains finite for all displayed values of $N_V$ and $\beta_h$, see Fig. 6. The strongest dependence on $\beta_h$ is found for $-5 \lesssim \beta_h < 3$, see the right panel of Fig. 6. This is a consequence of the incompleteness of the gauge fixing at $\beta_h = 3$. Since these gravitational contributions at $\Lambda = 0$ are functions in $(\beta_h - 3)^{-n}$, where $n \in \mathbb{Z}_+$, their variation with $\beta_h$ decreases for more negative $\beta_h$.

Furthermore, the overall gauge dependence decreases when increasing the number of gauge fields. This is because the contribution from the gauge fields is independent of $\beta_h$, and starts to become more important than the gravitational contribution, once $N_V$ is large enough. At these values of $N_V$, the WGB is only weakly dependent on $\beta_h$ \footnote{This is easier to see after rescaling the matter couplings with $N_V$. This rescaling is just a redefinition of the couplings and does not change the location of the WGB. In the rescaled version of the beta functions, the matter contributions is proportional to $N_V$, while the gravitational contribution is $N_V$-independent.}. Therefore, the qualitative feature that the inclusion of more gauge fields result in a stronger WGB, is independent of the gauge choice.
Figure 6. We show the WGB in a function of $N_V$ and $\beta_h$. $G^{\text{crit}}$ denotes the value of $G^*$ at which the sGFP becomes complex.

To the left: Dependence of the $G^{\text{crit}}(\Lambda = 0)$ on $N_V$ given selected values of $\beta_h$.

To the right: Dependence of the $G^{\text{crit}}(\Lambda = 0)$ on $\beta_h$ given selected values of $N_V$.

7 Conclusions and discussion

In this work, we have developed a deeper and more extensive understanding of the weak-gravity bound, discovered in [25] for gauge-gravity systems.

First, we have extended previous work [25, 26] by the second 4-photon-interaction, namely $(F\tilde{F})^2$. Our first key result shows the robustness of previous studies: Including the additional term changes the critical strength of gravity, i.e., the weak-gravity bound, only very slightly. This nontrivial result could even be interpreted as an indication for the onset of apparent convergence of the weak-gravity bound under extensions of the truncation of the dynamics.

Further we strengthen the evidence, first found in [26], that spacetime dimensionalities close to four are preferred by asymptotically safe gravity-matter systems. The mechanism is simple: To render the Abelian gauge coupling asymptotically free or safe, and solve its triviality problem, the strength of metric fluctuations must increase as a function of dimensionality beyond four. At the same time, the weak-gravity bound continues to exist in dimensionality beyond four, and prohibit gravitational fixed-point values to enter the regime where they are strong enough to solve the Abelian triviality problem.

We also consider systems with more than one gauge field for the first time. Here, we observe results similar to scalar-gravity systems [27]: First, the weak-gravity bound becomes stronger with increasing number of fields. Second, the increase is slow, i.e., the weak-gravity bound at 12 gauge fields (as in the SM) is still quantitatively close to the weak-gravity bound at one gauge field. Unlike for scalar-gravity systems, the gravitational fixed-point values lie below the weak-gravity bound for any number of gauge fields. The latter statement is subject to systematic uncertainties, both on the location of the weak gravity bound (which
is not very large, if the change under the inclusion of \((F\bar{F})^2\) provides a robust estimate) and on the location of the gravitational fixed-point values.

Our fourth key result shows the robustness of the weak-gravity bound: We use gauge dependence as a proxy for systematic uncertainties; because physical results (such as the existence of a bound, beyond which no asymptotically safe fixed point can exist) exhibit gauge dependence when truncations of the dynamics are used, the amount of gauge dependence quantifies the impact of terms beyond the truncation. We find that the weak-gravity bound is stable under variations of a gravitational gauge parameter. To obtain this result, the inclusion of the second type of four-photon interaction, \((F\bar{F})^2\), is crucial: for a range of values of the gauge parameter, the mechanism behind the weak-gravity bound is exhibited by \((F^2)^2\); for the neighboring range, it is exhibited by \((F\bar{F})^2\). Thus, our result shows not just the stability of the weak-gravity bound, but also the importance of including a full basis of interactions at the interaction-order of interest.

Our results are subject to systematic uncertainties from our choice of truncation, as discussed above. Further, we have used the Functional Renormalization Group in Euclidean spacetime to obtain our results. Therefore, one may well wonder about their applicability to Lorentzian spacetime. In this context, the weak-gravity bound could turn out to be an important piece of information: While an analytical continuation from Euclidean to Lorentzian signature is not possible in full quantum gravity, or even on a general background, it is (under certain conditions on the propagators, currently under investigation, e.g., in [132, 136, 142, 157]), possible around a perturbative, flat background. One might interpret the weak-gravity bound as an indication that asymptotically safe gravity dynamically prefers this more weakly-coupled regime, in which fluctuations about a flat background might be a good approximation to the dynamics of full quantum gravity. In turn, this would make an analytical continuation potentially feasible, and our results therefore relevant for Lorentzian signature.

Acknowledgments

We would like to thank Benjamin Knorr, Gustavo de Brito and Rafael Robson Lino dos Santos for insightful discussions. A. E. is supported by a research grant (29405) from VILLUM FONDEN and J. H. K. acknowledges the NAWA Iwanowska scholarship PPN/IWA/2019/1/00048. The research of M. S. has been supported by a scholarship of the German Academic Scholarship Foundation and by the Perimeter Institute for Theoretical Physics. Research at Perimeter Institute is supported in part by the Government of Canada through the Department of Innovation, Science and Economic Development and by the Province of Ontario through the Ministry of Colleges and Universities. M. S. and J. H. K. are grateful to CP3-Origins at the University of Southern Denmark for extended hospitality during various stages of this work.
A Cases in which the Weak Gravity Bound is not a function in the $G - \Lambda$ plane

The $\beta_h \approx 0$ case deserves special attention, since the WGB features a dent in the $G - \Lambda$ plane, see Fig. 7, which leads to a discontinuity in $G_{\text{crit}}$, cf. Fig 5. Here we discuss it for the representative case of the single species system for $\beta_h=0$. The WGB for $\beta_h = 0$ is not a function for $\Lambda \approx 0$, see the left panel of the Fig. 7. This is different for other gauge choices, where $G_{N,\text{crit}}(\Lambda)$ is a function, i.e., there is a single value $G_{N,\text{crit}}$ for a given value of $\Lambda$. This behavior is present only for the full truncation consisting of $w_2$ and $v_2$ and absent for truncations consisting of $w_2$ or $v_2$ only. In particular, for $\Lambda = 0$ there are two allowed intervals, $G < 2.05$ and $2.31 < G < 2.55$. This follows from the form of beta function for $w_2$. Let us consider the schematic form of $\beta_{w_2}$, which we repeat here for convenience

$$\beta_{w_2} = C_{2,w_2}(G, v_2, \Lambda = 0) w_2^2 + C_{1,w_2}(G, v_2, \Lambda = 0) w_2 + C_{0,w_2}(G, v_2, \Lambda = 0). \quad (A.1)$$

As long as the discriminant $\Delta = C_{1,w_2}^2 - 4C_{0,w_2}C_{2,w_2}$ is non-negative, the real shifted Gaussian FP exist. At $\Delta = 0$, there is collision of the fixed points. For $v_2 \equiv 0$ the $\text{Re}(C_{1,w_2}^2(v_2 = 0))$ decreases with $G$ (the blue dot-dashed line in right panel of Fig. 7), while $\text{Re}(4C_{0,w_2}C_{2,w_2}(v_2 = 0))$ increases (the red dotted line). As they cross at $G \approx 1.75$ the $\text{Re}(\Delta_{w_2})$ becomes negative and the fixed points are no-longer real. On the other hand, the contribution from $v_2$ to the $C_1$ have opposite sign to the one from $G$

$$\beta_{w_2} = \left(4 - \frac{155G}{54\pi} + \frac{v_2}{12\pi^2}\right) w_2 + \left(\frac{100G^2}{27} - \frac{Gv_2}{\pi} + \frac{v_2^2}{6\pi^2}\right) + \frac{35}{24\pi^2} w_2^2. \quad (A.2)$$

Since $v_{2,*} > 0$, the $\text{Re}(C_{1,w_2}^2)$ starts to grow at $G \approx 2.0$ (in the right panel of Fig. 7 depicted as the dashed orange line). Furthermore the contributions from $v_{2,*}$ to the $\text{Re}(C_{0,w_2})$ are such that $\text{Re}(C_{0,w_2}(v_2, \Lambda = 0))$, depicted as the cyan dashed line. This two effects combined results in $\text{Re}(\Delta_{w_2})$ (the purple line) being slightly positive in the interval $2.31 < G < 2.55$, resulting in a second allowed interval. This behavior is a reminder that one has to be cautious when studying systems with multiple couplings.

The presence of a second allowed interval for $G$ at fixed $\Lambda$ gives rise to a jump in $G_{\text{crit}}(\Lambda)$, cf. the left panel of Fig. 7. The exact value for $\Lambda$ where this jump happens depends on the gauge parameter $\beta_h$, and on the number of vector fields $N_V$. Therefore, the discussed property of the WGB leads to the jump in $G_{\text{crit}}(\beta_h)$ for the single-species system, cf. the blue (solid) line in Fig. 5. It also leads to the seemingly large difference between the WGB of the single-species and two-species system around $\Lambda = 0$, cf. the right panel of Fig. 3, since the jump in $G_{\text{crit}}$ occurs for slightly different values of $\Lambda$ in both systems.
Figure 7. In the left panel: Comparison of the WGB for $w_2$ and $w_2$ plus $v_2$ truncations for $\beta_h = 0$. For the full truncation there is a kink at $\Lambda \approx 0$ absent for $w_2$ truncation only. In the right panel: The behavior of coefficients of the coupled $w_2$ and $v_2$ system.

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