A TimeStamp based Multi-version STM Protocol that satisfies Opacity and Multi-Version Permissiveness *

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Abstract

Software Transactional Memory Systems (STM) are a promising alternative to lock based systems for concurrency control in shared memory systems. In multiversion STM systems, each write on a transaction object produces a new version of that object. The advantage obtained by storing multiple versions is that one can ensure that read operations do not fail. Opacity is a commonly used correctness criterion for STM systems. Multi-Version permissive STM system never aborts a read-only transaction. Although many multi-version STM systems have been proposed, to the best of our knowledge none of them have been formally proved to satisfy opacity. In this paper we present a time-stamp based multiversion STM system that satisfies opacity and mv-permissiveness. We formally prove the correctness of the proposed STM system. We also present garbage collection procedure which deletes unwanted versions of the transaction objects and formally prove it correctness.

1 Introduction

In recent years, Software Transactional Memory systems (STM) [9], [18] have garnered significant interest as an elegant alternative for addressing concurrency issues in memory. STM systems take optimistic approach. Multiple transactions are allowed to execute concurrently. On completion, each transaction is validated and if any inconsistency is observed it is aborted. Otherwise it is allowed to commit.

An important requirement of STM systems is to precisely identify the criterion as to when a transaction should be aborted/committed. Commonly accepted correctness criterion for STM systems is Opacity proposed by Guerraoui, and Kapalka [7]. Opacity requires all the transactions including aborted to appear to execute sequentially in an order that agrees with the order of non-overlapping transactions. Opacity unlike traditional serializability [13] ensures that even aborted transactions read consistent values.

With the increase in concurrency, more transactions may conflict and abort, especially in presence many long-running transactions which can have a very bad impact on performance [2]. Perelman et al [14] observe that read-only transactions play a significant role in various types of applications. But long read-only transactions could be aborted multiple times in many of the current STM systems [10][5]. In fact Perelman et al [14] show that many STM systems waste 80% their time in aborts due to read-only transaction.

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It was observed that by storing multiple versions of each object, multi-version STMs can ensure that read-only transactions do not abort. Maintaining multiple versions was first successfully used in databases. Since then, many STM systems have been developed that store multiple version of objects \cite{17, 16, 14, 15}. However storing multiple versions poses a difficulty of deleting unwanted object versions. It is necessary to regularly delete unused versions which otherwise could use a lot of memory.

An important goal of STM system is to ensure that a transaction is not aborted when it does not violate correctness requirement. Many STM system however spuriously abort transactions \cite{11} even not required. A permissive STM \cite{6} does not abort a transaction unless committing of it violates consistency requirements. A multi-version permissive or mv-permissive STM system \cite{15} never aborts a read-only transaction; it aborts an update transaction (i.e transaction that also writes) when it conflicts with other update transactions.

Although many of the multi-version STM systems proposed in literature satisfy mv-permissiveness, no STM system to our knowledge has been formally of them are proved to satisfy opacity. In this paper, we propose a simple multi-version timestamp ordering STM system. We formally prove that our algorithm satisfies opacity and mv-permissiveness. To delete unwanted versions, we also give an algorithm for garbage collection and prove its correctness.

Roadmap. The paper is organized as follows. We describe our system model in Section 2. In Section 3 we formally define the graph characterization for implementing the opacity. In Section 4 we describe the working principle of MVTO protocol and its algorithm. In Section 5 we are collecting the garbage. Finally we conclude in Section 6.

2 System Model and Preliminaries

The notions and definitions described in this section follow the definitions of \cite{11}. We assume a system of \(n\) processes, \(p_1, \ldots, p_n\) that access a collection of objects via atomic transactions. The processes are provided with four transactional operations: the \(\text{write}(x, v)\) operation that updates object \(x\) with value \(v\), the \(\text{read}(x)\) operation that returns a value read in \(x\), \(\text{tryC}(\cdot)\) that tries to commit the transaction and returns \(\text{commit} (c \text{ for short})\) or \(\text{abort} (a \text{ for short})\), and \(\text{tryA}(\cdot)\) that aborts the transaction and returns \(\text{A}\). The objects accessed by the read and write operations are called as t-objects. For the sake of simplicity, we assume that the values written by all the transactions are unique.

Operations \(\text{write}, \text{read}\) and \(\text{tryC}()\) may return \(a\), in which case we say that the operations forcefully abort. Otherwise, we say that the operation has successfully executed. Each operation is equipped with a unique transaction identifier. A transaction \(T_i\) starts with the first operation and completes when any of its operations returns \(a\) or \(c\). Abort and commit operations are called terminal operations.

For a transaction \(T_k\), we denote all its read operations as \(Rset(T_k)\) and write operations \(Wset(T_k)\). Collectively, we denote all the operations of a transaction \(T_i\) as \(\text{evts}(T_k)\).

Histories. A history is a sequence of events, i.e., a sequence of invocations and responses of transactional operations. The collection of events is denoted as \(\text{evts}(H)\). For simplicity, we only consider sequential histories here: the invocation of each transactional operation is immediately followed by a matching response. Therefore, we treat each transactional operation as one atomic event, and let \(<_H\) denote the total order on the transactional operations incurred by \(H\). With this assumption the only relevant events of a transaction \(T_k\) are of the types: \(r_k(x, v), r_k(x, A), w_k(x, v), w_k(x, v, A), \text{tryC}_k(C)\) (or \(c_k\) for short), \(\text{tryC}_k(A), \text{tryA}_k(A)\) (or \(a_k\) for short). We identify a history \(H\) as tuple \((\text{evts}(H), <_H)\).

Let \(H|T\) denote the history consisting of events of \(T\) in \(H\), and \(H|p_i\) denote the history consisting of events of \(p_i\) in \(H\). We only consider well-formed histories here, i.e., (1) each \(H|T\) consists of a read-only prefix (consisting of read operations only), followed by a write-only part (consisting of write operations...
For a history of transactions in aborted transactions in with 0. The set of transactions that appear in transactions, where no new transaction begins before the last transaction completes (commits or aborts).

We assume that every history has an initial committed transaction \( T_0 \) that initializes all the data-objects with 0. The set of transactions that appear in \( H \) is denoted by \( \text{txns}(H) \). The set of committed (resp., aborted) transactions in \( H \) is denoted by \( \text{committed}(H) \) (resp., \( \text{aborted}(H) \)). The set of incomplete (or live) transactions in \( H \) is denoted by \( \text{incomplete}(H) \) (\( \text{incomplete}(H) = \text{txns}(H) - \text{committed}(H) - \text{aborted}(H) \)).

For a history \( H \), we construct the completion of \( H \), denoted \( \overline{H} \), by inserting \( a_k \) immediately after the last event of every transaction \( T_k \in \text{incomplete}(H) \).

Figure 1 shows a pictorial representation of a history \( H_1 \) where transactions \( T_1 \), \( T_2 \), \( T_3 \), and \( T_4 \) have a relationship in \( H \). The history \( H \) is valid if all its successful read operations are legal. Thus from these definitions we get that if \( H \) is legal then it is also valid.

Transaction orders. For two transactions \( T_k, T_m \in \text{txns}(H) \), we say that \( T_k \) precedes \( T_m \) in the real-time order of \( H \), denote \( T_k \prec_{RT} T_m \), if \( T_k \) is complete in \( H \) and the last event of \( T_k \) precedes the first event of \( T_m \) in \( H \). If neither \( T_k \prec_{RT} T_m \) nor \( T_m \prec_{RT} T_k \), then \( T_k \) and \( T_m \) overlap in \( H \). A history \( H \) is \( t \)-sequential if there are no overlapping transactions in \( H \), i.e., every two transactions are related by the real-time order.

Valid and legal histories. Let \( H \) be a history and \( r_k(x, v) \) be a successful read operation (i.e \( v \neq A \)) in \( H \). Then \( r_k(x, v) \), is said to be valid if there is a transaction \( T_j \) in \( H \) that commits before \( r_k \) and \( w_j(x, v) \) is in \( \text{evts}(T_j) \). Formally, \( \langle r_k(x, v) \rangle \) is valid \( \Rightarrow \exists T_j : (c_j \prec H r_k(x, v)) \land (w_j(x, v) \in \text{evts}(T_j)) \land (v \neq A) \). We say that \( T_k \) and \( T_j \) have a reads-from relation in \( H \). The history \( H \) is valid if all its successful read operations are valid.

We define \( r_k(x, v) \)'s lastWrite as the latest commit event \( c_i \) such that \( c_i \) precedes \( r_k(x, v) \) in \( H \) and \( x \in Wset(T_i) \) \( (T_i \) can also be \( T_0) \). A successful read operation \( r_k(x, v) \) (i.e \( v \neq A \)), is said to be legal if transaction \( T_i \) (which contains \( r_k \)'s lastWrite) also writes \( v \) onto \( x \). Formally, \( \langle r_k(x, v) \rangle \) is legal \( \Rightarrow \langle H \text{lastWrite}(r_k(x, v)) = c_i \rangle \land (w_i(x, v) \in \text{evts}(T_i)) \land (v \neq A) \). The history \( H \) is legal if all its successful read operations are legal. Thus from these definitions we get that if \( H \) is legal then it is also valid.

Opacity. We say that two histories \( H \) and \( H' \) are equivalent if they have the same set of events. Now a history \( H \) is said to be opaque [7, 8] if \( H \) is valid and there exists a \( t \)-sequential legal history \( S \) such that (1) \( S \) is equivalent to \( \overline{H} \) and (2) \( S \) respects \( \prec_{RT} \), i.e \( \prec_H \subseteq \prec_S \). By requiring \( S \) being equivalent to \( \overline{H} \), opacity treats all the incomplete transactions as aborted.

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*This restriction brings no loss of generality [12].*
3 Graph characterization of Opacity

To prove that a STM system satisfies opacity, it is useful to consider graph characterization of histories. The graph characterization described in this section is based on the characterization by Bernstein and Goodman [3] and is slightly different from the characterisation of Gueraroui and Kapalka [7, 8].

Consider a history $H$ which consists of multiple version for each t-object. Like [3, 7, 8], we use the notion of version order. Given $H$ and a t-object $x$, we define a version order for $x$ as any (nonreflexive) total order on all the versions of $x$ ever written by committed transactions in $H$. It must be noted that the version order may or may not be same as the actual order in which the version of $x$ are generated in $H$. A version order of $H$, denoted as $⟨≺⟩_H$ is the union of the version orders of all the t-objects in $H$. Using the notation that a committed transaction $T_i$ writing to $x$ creates a version $x_i$, a possible version order for $H$ of $H$ is: $⟨x_0 ≺ x_1⟩, ⟨y_0 ≺ y_2 ≺ y_3⟩, ⟨z_0 ≺ z_1 ≺ z_3⟩$.

We define the graph characterisation based on a given version order. Consider a history $H$ and a version order $≺$. Then a graph denoted as $OPG(H, ≪)$ (opacity graph) can be defined. There is a vertex for each transaction $T_i$ in $H$. The edges of the graph are of three kinds and are defined as follows:

1. $rt$(real-time) edges: If $T_i$ commits before $T_j$ starts in $H$, then there is an edge from $v_i$ to $v_j$. This set of edges are referred to as $rt(H)$. 

2. $rf$(reads-from) edges: If $T_j$ reads $x$ from $T_i$ in $H$, then there is an edge from $v_i$ to $v_j$. Note that in order for this to happen, $T_i$ must have committed before $T_j$ and $c_i ≺_H r_j(x)$. This set of edges are referred to as $rf(H)$. 

3. $mv$(multiversion) edges: The mv edges capture the multiversion relations and is based on the version order. Consider a successful read operation $r_k(x, v)$ and the write operation $w_j(x, v)$ belonging to transaction $T_j$. Here, $r_k(x, v)$ reads $x$ from $w_j(x, v)$ (it must be noted $T_j$ is a committed transaction and $c_j ≺_H r_k$). Consider a committed transaction $T_i$ which writes to $x$, $w_i(x, u)$ where $u ≠ v$. Thus the versions created $x_i, x_j$ are related by $≺$. Then, if $x_i ≺ x_j$ we add an edge from $v_i$ to $v_j$. Otherwise ($x_j ≺ x_i$), we add an edge from $v_k$ to $v_i$. This set of edges are referred to as $mv(H, ≪)$.

Using this construction, the $OPG(H_1, ≪_H_1)$ for history $H_1$ and $≺_H_1$ is given above is shown in Figure [1].

Given a history $H$ and a version order $≺$, consider the graph $OPG(Π, ≪)$. While considering the $rt$ edges in this graph, we only consider the real-time relation of $H$ and not $Π$. It can be seen that $r^RT_H ≲ r^RT_Π$ but with this assumption, $rt(H) = rt(Π)$. Hence, we get the following property,

Property 1 The graphs $OPG(H, ≪)$ and $OPG(Π, ≪)$ are the same for any history $H$ and $≺$.

Now we show the correctness of our graph characterization using the following lemmas and theorem.

Definition 2 For a t-sequential history $S$, we define a version order $≪_S$ as follows: For two version $x_i, x_j$ created by committed transactions $T_i, T_j$ in $S$, $⟨x_i ≺_S x_j ⇔ T_i ≺_S T_j⟩$.

Now, consider the following lemmas,

Lemma 3 Consider a legal t-sequential history $S$. Then the graph $OPG(S, ≪_S)$ is acyclic.
Proof: We numerically order all the transactions in $S$ by their real-time order by using a function $ord$. For two transactions $T_i, T_j$, we define $ord(T_i) < ord(T_j) \iff T_i <_S T_j$. Let us analyse the edges of $OPG(S, \ll_S)$ one by one:

- **rt edges**: It can be seen that all the rt edges go from a lower ord transaction to a higher ord transaction.

- **rf edges**: If $T_j$ reads $x$ from $T_i$ in $S$ then $T_i$ is a committed transaction with $ord(T_i) < ord(T_j)$. Thus, all the rf edges from a lower ord transaction to a higher ord transaction.

- **mv edges**: Consider a successful read operation $r_k(x,v)$ and a committed transaction $T_i$ writing $u$ to $x$ where $u \neq v$. Let $c_j$ be $r_k(x,v)$’s lastWrite. Thus, $w_j(x,v) \in \text{evts}(T_j)$. Thus, we have that $ord(T_j) < ord(T_k)$. Now there are two cases w.r.t $T_i$: (1) Suppose $ord(T_i) < ord(T_j)$. We now have that $T_i \ll T_j$. In this case, the mv edge is from $T_i$ to $T_j$. (2) Suppose $ord(T_i) < ord(T_j)$ which implies that $T_j \ll T_i$. Since $S$ is legal, we get that $ord(T_k) < ord(T_i)$. This case also implies that there is an edge from $ord(T_k)$ to $ord(T_i)$. Hence, in this case as well the mv edges go from a transaction with lower ord to a transaction with higher ord.

Thus, in all the three cases the edges go from a lower ord transaction to higher ord transaction. This implies that the the graph is acyclic.

Using these lemmas, we prove the following theorem.

**Theorem 5** A valid history $H$ is opaque iff there exists a version order $\ll_H$ such that $OPG(H, \ll)$ is acyclic.
Proof: (if part): Here we have a a version order \( \leq_H \) such that \( G_H = OPG(H, \leq) \) is acyclic. Now we have to show that \( H \) is opaque. Since the \( G_H \) is acyclic, a topological sort can be obtained on all the vertices of \( G_H \). Using the topological sort, we can generate a t-sequential history \( S \). It can be seen that \( S \) is equivalent to \( \overline{H} \). Since \( S \) is obtained by a topological sort on \( G_H \) which maintains the real-time edges of \( H \), it can be seen that \( S \) respects the rt order of \( H \), i.e \( \lessdot^R_H \subseteq \lessdot^S \).

Similarly, since \( G_H \) maintains reads-from order of \( H \), it can be seen that if \( T_j \) reads \( x \) from \( T_i \) in \( H \) then \( T_i \) terminates before \( r_i(x) \) and \( T_j \) in \( S \). Thus, \( S \) is valid. Now it remains to be shown that \( S \) is legal. We prove this using contradiction. Assume that \( \not\in H \) then there is a successful read operation \( r_k(x,v) \) such that its lastWrite in \( S \) is \( c_i \) and \( T_i \) writes value \( u(\not= v) \) to \( x \), i.e \( w_i(x,u) \in evts(T_i) \). Further, we also have that there is a transaction \( T_j \) that writes \( v \) to \( x \), i.e \( w_j(x,v) \in evts(T_j) \). Since \( S \) is valid, as shown above, we have that \( T_j \lessdot^R_ST_i \lessdot^S_Tk \).

Now in \( \leq_H \), if \( x_i \not\leq_H x_j \) then there is an edge from \( T_i \) to \( T_j \) in \( G_H \). Otherwise \( (x_j \not\leq_H x_i) \), there is an edge from \( T_k \) to \( T_i \). Thus in either case \( T_i \) can not be in between \( T_j \) and \( T_k \) in \( S \) contradicting our assumption. This shows that \( S \) is legal.

(Only if part): Here we are given that \( H \) is opaque and we have to show that there exists a version order \( \leq \) such that \( G_H = OPG(H, \leq) (= OPG(\overline{H}, \leq)) \). Property \([1]\) is acyclic. Since \( H \) is opaque there exists a legal t-sequential history \( S \) equivalent to \( \overline{H} \) such that it respects real-time order of \( H \). Now, we define a version order for \( S, \lessdot_S \) as in Definition \([2]\). Since the \( S \) is equivalent to \( \overline{H} \), \( \lessdot_S \) is applicable to \( \overline{H} \) as well. From Lemma \([3]\) we get that \( G_S = OPG(S, \lessdot_S) \) is acyclic. Now consider \( G_H = OPG(\overline{H}, \lessdot_S) \). The vertices of \( G_H \) are the same as \( G_S \). Coming to the edges,

- rt edges: We have that \( S \) respects real-time order of \( H \), i.e \( \lessdot^R_H \subseteq \lessdot^S \). Hence, all the rt edges of \( H \) are a subset of subset of \( S \).
- rf edges: Since \( \overline{H} \) and \( S \) are equivalent, the reads-from relation of \( \overline{H} \) and \( S \) are the same. Hence, the rf edges are the same in \( G_H \) and \( G_S \).
- mv edges: Since the version-order and the operations of the \( H \) and \( S \) are the same, from Lemma \([4]\) it can be seen that \( \overline{H} \) and \( S \) have the same mv edges as well.

Thus, the graph \( G_H \) is a subgraph of \( G_S \). Since we already know that \( G_S \) is acyclic from Lemma \([3]\) we get that \( G_H \) is also acyclic. \( \square \)

4 Multiversion Timestamp Ordering (MVTO) Algorithm

We describe a timestamp based algorithm for multi-version STM systems, multiversion timestamp ordering (MVTO) algorithm. We then prove that our algorithm satisfies opacity \([8,7]\) using the graph characterization developed in the previous section.

4.1 The working principle

In our algorithm, each transaction, \( T_i \) is assigned a unique timestamp, \( i \), when it is initially invoked by a thread. We denote \( i \) to be the id as well as the timestamp of the transaction \( T_i \). Intuitively, the timestamp tells the “time” at which the transaction began. It is a monotonically increasing number assigned to each transaction and is numerically greater than the timestamps of all the transactions invoked so far. All read and write operations carry the timestamp of the transaction that issued it. When an update transaction \( T_i \)
commits, the algorithm creates new version of all the t-objects it writes to. All these versions have the timestamp $i$.

Now we describe the main idea behind read, write and tryC operations executed by a transaction $T_i$. These ideas are based on the read and write steps for timestamp algorithm developed for databases by Bernstein and Goodman [3]:

1. **read rule:** $T_i$ on invoking $r_i(x)$ reads the value $v$, where $v$ is the value written by a transaction $T_j$ that commits before $r_i(x)$ and $j$ is the largest timestamp $\leq i$.

2. **write rule:** $T_i$ writes into local memory.

3. **commit rule:** $T_i$ on invoking tryC operation checks for each t-object $x$, in its Wset:
   
   (a) If a transaction $T_k$ has read $x$ from $T_j$, i.e. $r_k(x,v) \in evts(T_k)$ and $w_j(x,v) \in evts(T_j)$ and $j < i < k$, then tryC$_i$ returns abort,
   
   (b) otherwise, the transaction is allowed to commit.

### 4.2 Data Structures and Pseudocode

The algorithm maintains the following data structures. For each transaction $T_i$:

- $T_i.RS$(read set): It is a list of data tuples (d.tuples) of the form $\langle x, v \rangle$, where $x$ is the t-object and $v$ is the value read from the transaction $T_i$.

- $T_i.WS$(write set): It is a list of (d.tuples) of the form $\langle x, v \rangle$, where $x$ is the t-object to which transaction $T_i$ writes the value $v$.

For each transaction object (t.object) $x$:

- $x.vl$(version list): It is a list consisting of version tuples (v.tuple) of the form $\langle ts, v, rl \rangle$ where $ts$ is the timestamp of a committed transaction that writes the value $v$ to $x$. The list $rl$ is the read list consisting of a set of transactions that have read the value $v$ (described below). Informally the version list consists of all the committed transaction that have ever written to this t-object and the set of corresponding transactions that have read the value $v$ on $x$.

- $rl$(read list): This list contains all the read transaction tuples (rt.tuples) of the form $\langle j \rangle$. The read list $rl$ is stored in each tuple of the version list described above.

Figure 3 illustrates the how the version list and read list are managed.

In addition, the algorithm maintains two global data-structures:

- $tCounter$: This counter is used to generate the ids/timestamp for a newly invoked transaction. This is incremented everytime a new transaction is invoked.

- $liveList$: This list keeps track of all the transactions that are currently incomplete or live. When a transaction begins, its id is added to this list. When it terminates (by abort or commit), the id is deleted from this list.
The STM system consists of the following operations/functions. These are executed whenever a transaction begins, reads, write or tries to commit:

initialize() : This operation initializes the STM system. It is assumed that the STM system knows all the t-objects ever accessed. All these t-objects are initialized with value 0 by the initial transaction $T_0$ in this operation. A version tuple $(0, 0, \text{nil})$ is inserted into all the version list of all the t-objects.

begin_tran() : A thread invokes a transaction by executing this operation. It returns an unique transaction identifier which is also its timestamp. The id is used in all other operations exported by the STM system. The id is further stored in the liveList.

read$_i$(x) : To read any t-object by transaction $i$, this operation is invoked. First, the t-object $x$ is locked. Then the version list of $x$ is searched to identify the correct version tuple (i.e the version created by a writing transaction). From the version-list, the tuple with the largest timestamp less than $i$, say $(j, v)$ is identified. Then the value $v$ written by transaction $j$, is returned.

find_lts(i, x) : This function is invoked by read$_i$(x) and finds the tuple $(j, v, rl)$ having the largest timestamp $j$ value smaller than $i$ (lts).

write$_i$(x, v) : Here write is performed in the local memory. This operation appends the data tuple $(x, v)$ into the WS of transaction $T_i$.

tryC$_i$() : This operation is invoked when a transaction $T_i$ has completed all its operations and wants to commit. This operation first checks whether $T_i$ is read only or not. If it is read only transaction then it returns commit. Otherwise, for each t-object $x$ (accessed in a predefined order) in $T_i$’s write set, the following check is performed: if timestamp of $T_i$, $i$ between the timestamps of the $T_j$ and $T_k$, where transaction $T_k$ reads $x$ from transaction $T_j$, i.e $j < i < k$, then the transaction $T_i$ is aborted.

If this check succeeds for all the t-objects written by $T_i$, then the version tuples are appended to the version lists and the transaction $T_i$ is committed. Before returning either commit or abort, the transaction id $i$ is removed from liveList.

The system orders all the t-objects ever accessed as $x_1, x_2, ..., x_n$ by any transaction (assuming that the system accesses a total of $n$ t-objects). In this operation, each transaction locks and access t-objects in the increasing order which ensures that the system does not deadlock.

cHECK versions(i, x) : This function checks the version list of $x$. For all version tuples $(j, v, rl)$ in $x.vl$ and for all transactions $T_k$ in $rl$, it checks if the timestamp of $T_i$ is between the timestamp of the $T_j$ and $T_k$, i.e $j < i < k$. If so, it returns true else false.
Algorithm 1 STM `initialize()`: Invoked at the start of the STM system. Initializes all the t-objects used by the STM System

1: for all x used by the STM System do
2: /* T₀ is initializing x */
3: add ⟨0, 0, nil⟩ to x.vl;
4: end for;

Algorithm 2 STM `begin_tran()`: Invoked by a thread to being a new transaction Tᵢ

1: lock liveList;
2: // Store the latest value of tCounter in i.
3: i = tCounter;
4: tCounter = tCounter + 1;
5: add i to liveList;
6: unlock liveList;
7: return i;

Algorithm 3 STM `readᵢ(x)`: A Transaction Tᵢ reads t-object x

1: lock x;
2: // From x.vl, identify the right version_tuple.
3: ⟨j, v, rl⟩ = find_lts(i, x);
4: Append i into rl; // Add i into j’s rl.
5: unlock x;
6: return (v); // v is the value returned

Algorithm 4 `find_lts(i, x)`: Finds the tuple ⟨j, v, rl⟩ created by the transaction Tⱼ with the largest timestamp smaller than i

1: // Initialize closest_tuple
2: closest_tuple = ⟨0, 0, nil⟩;
3: for all ⟨k, v, rl⟩ ∈ x.vl do
4: if (k < i) and (closest_tuple.ts < k) then
5: closest_tuple = ⟨k, v, rl⟩;
6: end if;
7: end for;
8: return (closest_tuple);

Algorithm 5 STM `writeᵢ(x, v)`: A Transaction Tᵢ writes into local memory

1: Append the d_tuple(x, v) to Tᵢ.WS.
2: return ok;
Algorithm 6 STM tryC(): Returns ok on commit else return Abort

1: if (\(T_i.ws == NULL\)) then
2:   removeId(i);
3:   return ok; // A read-only transaction.
4: end if;
5: for all \(d_tuple(x, v)\) in \(T_i.ws\) do
6:   /* Lock the t-objects in a predefined order to avoid deadlocks */
7:   Lock x;
8:   if (check_versions\((i, x)\) == false) then
9:     unlock all the variables locked so far;
10:    removeId(i);
11:   return Abort;
12: end if;
13: end for;
14: /* Successfully checked for all the write variables and not yet aborted. So the new write versions can be
15: inserted. */
16: for all \(d_tuples(x, v)\) in \(T_i.ws\) do
17:   insert \(v_tuple(i, v, nil)\) into \(x.vl\) in the increasing order;
18: end for;
19: unlock all the variables;
20: removeId(i);
21: return ok;

Algorithm 7 check_versions\((i, x)\): Checks the version list; it returns True or false

1: for all \(v_tuples(j, v, rl)\) in \(x.vl\) do
2:   for all \(T_k\) in \(rl\) do
3:     /* \(T_k\) has already read the version created by \(T_j\) */
4:     if \((j < i < k)\) then
5:       return false;
6:     end if;
7:   end for;
8: end for;
9: return true;

Algorithm 8 removeId\((i)\): Removes transaction id \(i\) from the liveList

1: lock liveList;
2: remove \(i\) from liveList;
3: unlock liveList;
4.3 Proof of MVTO protocol

In this sub-section, we will prove that our implementation satisfies opacity. Consider the history \( H \) generated by MVTO algorithm. Recall that only the begin_tran, read, and tryC operations access shared memory. Hence, we call such operations memory operations.

Note that \( H \) is not necessarily sequential: the transactional operations can execute in overlapping manner. To reason about correctness we have to prove \( H \) is opaque. Since we defined opacity for histories which are sequential, we order all the overlapping operations in \( H \) to get an equivalent sequential history. We then show that this resulting sequential history satisfies operation.

We order overlapping memory operations of \( H \) as follows: (1) two overlapping begin_tran operations based on the order in which they obtain lock over tCounter; (2) two read operations accessing the same t-object \( x \) by their order of obtaining lock over \( x \); (3) a read \( r_i(x) \) and a tryC\(_j\), of a transaction \( T_j \) which has written to \( x \), are similarly ordered by their order of obtaining lock over \( x \); (4) begin_tran and a tryC operations are ordered by their order of obtaining locks over liveList; (5) similarly, two tryC operations based on the order in which they obtain lock over liveList.

Combining the real-time order of events with above mentioned order, we obtain a partial order which we denote as lockOrder\(_H\). (It is a partial order since it does not order overlapping read operations on different t-objects or an overlapping read and a tryC which do not access any common t-object).

In order for \( H \) to be sequential, all its operations must be ordered. Let \( \alpha \) be a total order or linearization of operations of \( H \) such that when this order is applied to \( H \), it is sequential. We denote the resulting history as \( \alpha = \text{linearize}(H, \alpha) \). We now argue about the validity of histories generated by the algorithm.

**Lemma 6** Consider a history \( H \) generated by the algorithm. Let \( \alpha \) be a linearization of \( H \) which respects lockOrder\(_H\), i.e. lockOrder\(_H\) \( \subseteq \alpha \). Then \( \alpha = \text{linearize}(H, \alpha) \) is valid.

**Proof**: Consider a successful read operation \( r_i(x) \) that returns value \( v \). The read function first obtains lock on t-object \( x \) (Algorithm read, Line 1). Thus the value \( v \) returned by the read function must have already been stored in \( x \)’s version list by a transaction, say \( T_j \) when it successfully returned ok from its tryC operation (if \( T_j \neq T_0 \)). For this to have occurred, \( T_j \) must have successfully locked and released \( x \) prior to \( T_i \)’s locking operation. Thus from the definition of lockOrder\(_H\), we get that \( \text{tryC}_j(\text{ok}) \) occurs before \( r_i(x, v) \) which also holds in \( \alpha \).

If \( T_j \) is \( T_0 \), then by our assumption we have that \( T_j \) committed before the start of any operation in \( H \). Hence, this automatically implies that in both cases \( \alpha \) is valid.

It can be seen that for proving correctness, any linearization of a history \( H \) is sufficient as long as the linearization respects lockOrder\(_H\). The following lemma formalizes this intuition.

**Lemma 7** Consider a history \( H \). Let \( \alpha \) and \( \beta \) be two linearizations of \( H \) such that both of them respect lockOrder\(_H\), i.e. lockOrder\(_H\) \( \subseteq \alpha \) and lockOrder\(_H\) \( \subseteq \beta \). Then, (1) \( \alpha = \text{linearize}(H, \alpha) \) is opaque iff \( \beta = \text{linearize}(H, \beta) \).

**Proof**: From Lemma 6, we get that both \( \alpha \) and \( \beta \) are valid histories. Now let us consider each case

**If**: Assume that \( \alpha \) is opaque. Then, we get that there exists a legal t-sequential history \( S \) that is equivalent to \( \alpha \). From the definition of \( \beta \), we get that \( \beta = \text{linearize}(H, \beta) \) as well. We also have that, \( \alpha \) and \( \beta \) are valid. Hence, \( S \) is equivalent to \( \beta \) as well. Thus \( \beta \) is opaque as well.

**Only if**: This proof comes from symmetry since \( \alpha \) and \( \beta \) are not distinguishable.
This lemma shows that, given a history $H$, it is enough to consider one sequential history $H^α$ that respects $lockOrder_H$ for proving correctness. If this history is opaque, then any other sequential history that respects $lockOrder_H$ is also opaque.

Consider a history $H$ generated by MVTO algorithm. We then generate a sequential history that respects $lockOrder_H$. For simplicity, we denote the resulting sequential history as $H_{to}$. Let $T_i$ be a committed transaction in $H_{to}$ that writes to $x$ (i.e. it creates a new version of $x$).

To prove the correctness, we now introduce some more notations. We define $H_{to}.stl(T_i, x)$ as a committed transaction $T_j$ such that $T_j$ has the smallest timestamp greater than $T_i$ in $H_{to}$ that writes to $x$ in $H_{to}$. Similarly, we define $H_{to}.lts(T_i, x)$ as a committed transaction $T_k$ such that $T_k$ has the largest timestamp smaller than $T_i$ that writes to $x$ in $H_{to}$. Using these notations, we describe the following properties and lemmas on $H_{to}$.

**Property 8** Every transaction $T_i$ is assigned an unique numeric timestamp $i$.

**Property 9** If a transaction $T_i$ begins after another transaction $T_j$ then $j < i$.

**Property 10** If a transaction $T_k$ reads $x$ from (a committed transaction) $T_j$ then $T_j$ is a committed transaction with $j$ being the largest timestamp smaller than $k$. Formally, $T_j = H_{to}.lts(x, T_k)$.

**Lemma 11** Suppose a transaction $T_k$ reads $x$ from (a committed transaction) $T_j$ in $H_{to}$, i.e. $\{w_j(x, v), r_k(x, v)\} \in evts(H_{to})$. Let $T_i$ be a committed transaction that writes to $x$, i.e. $w_i(x, u) \in evts(T_i)$. Then, the timestamp of $T_i$ is either less than $T_j$’s timestamp or greater than $T_k$’s timestamp, i.e. $i < j \oplus k < i$ (where $\oplus$ is xor operator).

**Proof:** We will prove this by contradiction. Assume that $i < j \oplus k < i$ is not true. This implies that, $j < i < k$. But from the implementation of read and tryC functions, we get that either transaction $T_i$ is aborted or $T_k$ reads $x$ from $T_i$ in $H$. Since neither of them are true, we get that $j < i < k$ is not possible. Hence, $i < j \oplus k < i$.

To show that $H_{to}$ satisfies opacity, we use the graph characterization developed in Section 3. For the graph characterization, we use the version order defined using timestamps. Consider two committed transactions $T_i, T_j$ such that $i < j$. Suppose both the transactions write to t-object $x$. Then the versions created are ordered as: $x_i \ll x_j$. We denote this version order on all the t-objects created as $\ll_{to}$. Now consider the opacity graph of $H_{to}$ with version order as defined by $\ll_{to}$. $G_{to} = OPG(H_{to}, \ll_{to})$. In the following lemmas, we will prove that $G_{to}$ is acyclic.

**Lemma 12** All the edges in $G_{to} = OPG(H_{to}, \ll_{to})$ are in timestamp order, i.e. if there is an edge from $T_j$ to $T_i$ then the $j < i$.

**Proof:** To prove this, let us analyze the edges one by one,

- rt edges: If there is a rt edge from $T_j$ to $T_i$, then $T_i$ terminated before $T_j$ started. This implies that $T_j$ started before $T_i$. Hence, from Property 9 and 10, we get that $j < i$.

- rf edges: This follows directly from Property 10.

- mv edges: The mv edges relate a committed transaction $T_i$ writing to a t-object $x$, $w_i(x, u)$; a successful read operation $r_k(x, v)$ belonging to a transaction $T_k$ reading $x$ written by a committed transaction
$T_j$, $w_j(x,v)$ and transaction $T_j$. Transactions $T_j, T_i$ create new versions $x_i, x_j$ respectively. According to $\ll_{to}$, if $x_i \ll_{to} x_j$, then there is an edge from $T_i$ to $T_j$. From the definition of $\ll_{to}$ this automatically implies that $i < j$.

On the other hand, if $x_j \ll_{to} x_i$ then there is an edge from $T_k$ to $T_i$. Thus in this case, we get that $j < i$. Combining this with Lemma 11, we get that $k < i$.

Thus in all the cases we have shown that if there is an edge from $T_j$ to $T_i$ then the $j < i$.

**Theorem 13** *The history $H_{to}$ is opaque.*

**Proof:** From the definition of $H_{to}$ and Lemma 6 we get that $H_{to}$ is valid. We show that $G_{to} = OPG(H_{to}, \ll_{to})$ is acyclic. We prove this by contradiction. Assume that $G_{to}$ contains a cycle of the form, $T_{c1} \rightarrow T_{c2} \rightarrow \ldots T_{cm} \rightarrow T_{c1}$. From Lemma 12 we get that, $c_1 < c_2 < \ldots < c_m < c_1$ which implies that $c_1 < c_1$. Hence, a contradiction. This implies that $G_{to}$ is acyclic. Thus from Theorem 5 we get that $H_{to}$ is opaque.

Now, it is left to show that our algorithm is live, i.e., under certain conditions, every operation eventually completes. We have to show that the transactions do not deadlock. The is so because all the transactions lock all the t-objects in a predefined order. As discussed earlier, the STM system the orders all t-objects. We denote this order as accessOrder and denote it as $\prec_{ao}$. Thus $x_1 \prec_{ao} x_2 \prec_{ao} \ldots \prec_{ao} x_n$. In addition to t-objects, the transactions also access the shared variable liveList. Thus we add liveList to this order: $x_n \prec_{ao} liveList$. We refer to the combined set of t-objects $x_1, \ldots, x_n$ and liveList as shared objects.

From accessOrder, we get the following property

**Property 14** Suppose transaction $T_i$ accesses shared objects $p$ and $q$ in $H$. Iff $p$ is ordered before $q$ in accessOrder, then lock($p$) by transaction $T_i$ occurs before lock($q$). Formally, $(p \prec_{ao} q) \iff (\text{lock}(p) <_{H} \text{lock}(q))$.

**Theorem 15** *Assuming that no transaction fails and all the locks are starvation-free, every operation of MVTO algorithm eventually returns.*

**Proof:** In our algorithm, a transaction $T_k$ executing some operation will not return only if the operation or a sub-function that is invoked by the operation is stuck waiting on a lock. This is possible only when a set of transactions, denoted as $D$ (which includes $T_k$) are deadlocked. Let $SO$ be the set of all shared objects locked by transactions in the $D$. Let $s$ be a shared object in the set $SO$ that is ranked highest according to accessOrder and locked by a transaction in $T_i$ in $D$. Since $T_i$ is deadlocked, it must be waiting to access a shared object, say $s' \in SO$ locked by a transaction $T_j$ (otherwise $T_i$ cannot be involved in the deadlocked). From Property 14 we get that $s \prec_{ao} s'$. But this contradicts our choice of $s$. Hence, a deadlock is not possible.

Finally, we prove that our algorithm satisfies mv-permissive.

**Theorem 16** *MVTO algorithm is mv-permissive.*

**Proof:** According to the algorithm a read operation never returns abort. Hence, when a read-only transaction executes tryC operation, it always ok. Thus, a read-only transaction never aborts. Further, an update transaction aborts only if another update transaction has already committed a previous version. This shows that MVTO algorithm is mv-permissive.
5  Garbage Collection

As one can see with multi-version STMs, multiple versions are created. But storing multiple versions can unnecessarily waste memory. Hence, it is important to perform garbage collection by deleting unwanted versions of t-objects. Some of the earlier STM systems solve this problem by maintaining a fixed number of versions for each t-object [4]. We on the other hand, do not restrict the number of versions. The STM system will detect versions that will never again be used (i.e. have become garbage) and delete them. The garbage collection routine will be invoked whenever the number of versions of a t-object has become greater than a predefined threshold. The threshold can be decided dynamically by the application invoking the STM system based on the current memory requirements.

The STM system will delete a version of a t-object \( x \) created by transaction \( T_i \) when the following conditions are satisfied:

1. At least one another version of \( x \) has been created by \( T_k \) and \( i < k \);
2. Any transaction \( T_j \) such that \( T_i < T_j < T_k \) has terminated (either committed or aborted).

To capture these conditions, we modify the data structure maintained. For each t-object \( x \), we augment the version tuples stored in its version list by adding another entry \( nts \). Thus each \( v_{\text{tuple}} \) is: \( (ts, v, r_l, nts) \). The entry \( nts \) (next timestamp) denotes a committed transaction with the smallest timestamp larger than \( ts \) that has created a version of \( x \). If there is no such transaction then \( nts \) is \( \text{nil} \). With this modification, we make the following changes to functions discussed in the previous section:

\( \text{initialize}() \): In the version tuple created for \( T_0 \) on every t-object \( x \), the \( nts \) entry is also initialized to \( \text{nil} \). Thus, the Line 3 is replaced with: add \( (0, 0, \text{nil}, \text{nil}) \) to \( x.vl \);

\( \text{read}(x), \text{find}(lts, i, x) \): The changes in both these functions are trivial. Whenever the version tuple is referred to, the \( nts \) entry is also assumed to be present. There field is not directly used in these functions.

\( \text{tryC}(i) \): We replace Line 16 of \( \text{tryC} \) with the function \( \text{ins}_{\text{tuple}}(x, i, v, \text{nil}) \). The description of this function is given below.

\( \text{ins}_{\text{tuple}}(x, i, v, \text{nil}) \): This function inserts the new value written by transaction \( i \) into \( x \)'s version list. First it creates a version tuple, \( \text{cur}_{\text{tuple}} \), with timestamp \( i \) and value \( v \). It then identifies the previous version tuple created by a transaction that has the largest id (timestamp) smaller than \( i \) (Line 4). Then it copies the \( nts \) entry of the \( \text{prev}_{\text{tuple}} \) into \( \text{cur}_{\text{tuple}} \). The \( nts \) entry of \( \text{prev}_{\text{tuple}} \) is stored as \( i \). This way, the \( nts \) entry for every version tuple is kept updated. Then garbage collection routine is invoked if the total number of \( v_{\text{tuples}} \) of \( x \) is greater than a predefined threshold.

Having described the changes necessary to keep \( nts \) entry updated, we will next describe the steps to perform garbage collection, \( gc() \).

\( gc(x) \): On being invoked, this function locks \( \text{liveList} \). It checks the version lists of the currently considered t-object \( x \). For each tuple in \( x \)'s version list, denoted as \( \text{cur}_{\text{tuple}} \), first \( \text{cur}_{\text{tuple}}.nts \) is checked. If it \( \text{nil} \), then the next tuple is checked. Otherwise, for each transaction with timestamp \( j \) in the range \( \text{cur}_{\text{tuple}}.ts \) to \( \text{cur}_{\text{tuple}}.nts \) is checked. If some transaction \( T_j \) is in \( \text{liveList} \), then the algorithm decides that \( \text{cur}_{\text{tuple}} \) is not yet garbage. It then checks the next tuple is checked. If no such transaction \( T_j \) in this range is in \( \text{liveList} \), then the algorithm decides that \( \text{cur}_{\text{tuple}} \) has become garbage. Hence, it deletes this tuple. Then, it returns the control. But before returning this function does not unlock \( \text{liveList} \) as \( \text{liveList} \) will again be locked in \( \text{tryC} \).
Figure 4: Data Structures for Garbage Collection

Figure 4 illustrates the idea of garbage collection. Here, the version tuple created by transaction $T_5$ has already been deleted. The version tuple created by transaction $T_{10}$ will be deleted when all the transactions between 10 and 17 have terminated (either aborted or committed).

**Algorithm 9** $\text{ins\_tuple}(x, i, v, \text{nil})$: Inserts the version tuple for $(i, v)$ created by the transaction $T_i$ into the version list of $x$

1: // Initialize $\text{cur\_tuple}$
2: $\text{cur\_tuple} = \langle i, v, \text{nil, nil} \rangle$;
3: /* Finds the tuple with the largest timestamp smaller than $i$ */
4: $\text{prev\_tuple} = \text{find\_lts}(i, x)$;
5: // $\text{prev\_tuple}$ is of the form $\langle ts, v', rl, nts \rangle$
6: $\text{cur\_tuple.nts} = \text{prev\_tuple.nts}$;
7: $\text{prev\_tuple.nts} = i$;
8: insert $\text{cur\_tuple}$ into $x.vl$ in the increasing order of timestamps;
9: /* $|x.vl|$ denotes number of versions of $x$ created and threshold is a predefined value. */
10: if $(|x.vl| > \text{threshold})$ then
11: $\text{gc}(x)$;
12: end if

### 5.1 Proof of Garbage Collection

Consider a history $H$ generated by the MVTO algorithm with garbage collection. As discussed Section 4, $H$ is not sequential. To prove the correctness, we order the overlapping operations to obtain a sequential history. Similar to Section 4, we use a total order that respects $\text{lockOrder}_H$ to order the overlapping operations. Although the $\text{tryC}$ function is modified due to invocation of garbage collection functions, $\text{ins\_tuple}$ and $\text{gc}$, it does not modify the lockOrder.

Thus Lemma[5] and Lemma[7] is applicable to $H$. Hence, we consider any total order that respects $\text{lockOrder}_H$ for ordering the overlapping operations of $H$. We denote the resulting sequential history as $H_{gc}$.

To prove of our garbage collection scheme, we now introduce some more notations. We denote $H_{gc}.vlist_{index}(ts, x)$, as the $v$ tuple in $x.vl$ created by transaction $T_{ts}$ in $H_{gc}$. If no such $v$ tuple exists then it is nil. We have the following useful lemmas on garbage collection. In these lemmas, we use the notations defined in SubSection 4.3.
Algorithm 10 STM gc(x): Unused version of a t-object x will deleted from x.vl

1: lock liveList;
2: // t-object x is already locked
3: for all (cur_tuple ∈ x.vl) do
4:   if (cur_tuple.nts == nil) then
5:     /* If nts is nil, check the next tuple in the version list */
6:       continue;
7:     end if
8:     j = cur_tuple.ts + 1;
9:   // Check for all ids j in the range j < nts
10:  while (j < cur_tuple.nts) do
11:     if (j ∈ liveList) then
12:       break;
13:     end if
14:  end while
15:  /* If all the tuples with timestamp j, such that i < j < nts have terminated then cur_tuple can be deleted*/
16:     delete cur_tuple;
17:  end for
18: /* liveList is not unlocked when this function returns */

Lemma 17 Consider any history $H_{gc}$ generated by the algorithm with garbage collection. Let $H_0$ be a prefix of $H_{gc}$. For every live transaction $T_l$ in incomplete($H_p$) and for every t-object x, we have: (a) Let $T_j = H_p.lts(T_l, x)$. Then $H_p.vlist.index(T_j, x) = vt_j ≠ nil$. (b) Let $vt_j.nts = T_k$. Then, $T_k = H_p.stl(T_l, x)$

Proof: We prove this using induction on number of version tuples, $count$, created for t-object x.

Base case, $count = 0$: When the STM system is initialized, the first version is created by $T_0$. Consider a prefix of $H_{gc}$, denoted as $H_0$, which has only this version of $x$ created by $T_0$. Thus in $H_0$ no transaction has yet committed. Since no transaction has yet executed tryC in $H_0$, the gc function would not have been executed. So the version tuple created by $T_0$ would not have been deleted. Let $T_l$ be a live transaction in $H_0$. So we get that $T_0 = H_0.lts(T_l, x), H_0.vlist.index(T_0, x) = vt_0 ≠ nil$. We also have that $vt_0.nts = nil$ and $H_{gc.stl}(T_l, x) = nil$. So this proves the base case.

Induction case, $count = m + 1$: In this case, we have to prove the lemma after $m + 1$ version tuples have been created assuming that it is true when $m$ tuples were present. Consider a prefix of $H_{gc}$, denoted as $H_m$, in which $m$ versions are created (note that the number of versions of $x$ present in $H_m$ could be less than $m$ since some tuples could have been deleted by gc() function). Let $T_l$ be a live transaction in $H_m$ that executes tryC to generate the next version of $x$. From induction hypothesis, we have that $T_j = H_m.lts(T_l, x), H_m.vlist.index(T_j, x) = vt_j ≠ nil$ and $vt_j.nts = T_k = H_m.stl(T_l, x)$.

Consider another prefix of $H_{gc}$, $H_{m+1}$ in which $T_l$ committed and created $m + 1$st version of $x$. As observed earlier, the lemma is true in $H_m$. When $T_l$ commits, the only live transactions that are affected are those transactions whose timestamps are between $j$ and $k$. Thus if we prove that it is true for all these live transactions, then the lemma is true for all live transactions in $H_{m+1}$. Consider two live transactions $T_{l1}$ and $T_{l2}$ such that $j < l1 < l < l2 < k$. From the tryC operation of $T_l$, we get that $T_j = H_m.lts(T_{l1}, x) = H_{m+1}.lts(T_{l1}, x)$ and $H_m.vlist.index(T_j, x) = H_{m+1}.vlist.index(T_j, x) = vt_j ≠ nil$. We also have
that $T_k = H_m.stl(T_{i2}, x) = H_{m+1}.stl(T_{i2}, x)$.

Since $T_i$ is committed in $T_{m+1}$ a new tuple of $x$ is created. Thus we get that $H_{m+1}.lts(T_{i2}, x) = T_i$ and $H_{m+1}.vlist\_index(T_{i1}, x) = vt_i \neq nil$. This proves the induction case.

This lemma intuitively states that for any live transaction $T_i$, its lts transaction for t-object $x$, $T_j$ is not deleted by gc function. It also states that for all version tuples, the nts entry is correctly maintained. Using this lemma, we next prove that that Property [10] and Lemma [11] are true even with garbage collection.

**Lemma 18** The history $H_{gc}$ generated by MVTO with garbage collection satisfies read rule: If a transaction $T_k$ reads $x$ from (a committed transaction) $T_j$ in $H_{gc}$ then $T_j$ is a committed transaction with $j$ being the largest timestamp smaller than $k$. Formally, $T_j = H_{gc}.lts(T_k, x)$.

**Proof:** Consider a history $H'$ (a prefix of $H_{gc}$) in which the read operation of $T_k$ is the last operation to execute in $H'$. Thus $T_k$ is a live transaction in $H'$. Let $T_j = H'.lts(T_k, x)$. From Lemma [17](a), we get that $T_k$’s lts transaction is correctly maintained by the algorithm (with garbage collection). Hence, $H'.vlist\_index(T_j, x) = vt_j \neq nil$ (which implies that the tuple $vt_j$ has not yet been deleted). Hence, $T_k$ reads $x$ from $T_j$.

**Lemma 19** The history $H_{gc}$ generated by MVTO with garbage collection satisfies write rule: Suppose a transaction $T_k$ reads $x$ from (a committed transaction) $T_j$ in $H_{gc}$, i.e. $\{w_j(x, v), r_k(x, v)\} \in \text{evts}(H_{gc})$. Let $T_i$ be a committed transaction that writes to $x$, i.e. $w_i(x, u) \in \text{evts}(T_i)$. Then, the timestamp of $T_i$ is either less than $T_j$’s timestamp or greater than $T_k$’s timestamp, i.e. $i < j \oplus k < i$.

**Proof:** Consider a sequential prefix of a $H_{gc}$, say $H'$, in which transaction $T_i$ has not yet executed tryC operation but $T_j$ has read from $T_j$. Thus $T_i$ is a live transaction in $H'$. From Lemma [18] we have that $T_j = H'.lts(T_k, x)$. Suppose by contradiction, $j < i < k$. This implies that $T_j = H'.lts(T_i, x)$ as well. Thus, $H'.vlist\_index(T_j, x) = vt_j \neq nil$ implying that $vt_j$ is not yet deleted in $H'$. The read function stores $T_k$ in $vt_j$. When $T_i$ executes tryC operation, the algorithm detects $j < i < k$ and aborts $T_i$ which contradicts our assumption.

Thus, we have that $i < j \oplus k < i$.

Since the read-rule and the write-rules are maintained, we get that Lemma [12] is true as well. Hence, Theorem [13] automatically follows. Thus the history generated by the algorithm with garbage collection is opaque as well.

6 Conclusion

There are many applications that require long running read-only transactions. Many STM systems can cause such transactions to abort. Multi-version STM system ensure that a read-only transactions does not need to abort by maintaining multiple versions. Two important properties that should be considered while building a STM system are: correctness which normally is opacity and progress condition which for multiversion systems is mv-permissiveness. Although several multi-version STM systems have been proposed to the best of our knowledge none of them have been proved formally satisfy opacity.

In this paper we presented a timestamp based multiversion STM system that satisfies opacity and mv-permissiveness. We also presented an algorithm for garbage collection that deletes version that will never be used. We have formally proved the correctness of our algorithm including garbage collection.

As a part of future work, we would like to implement this algorithm and test its performance on various benchmarks. Recently, Attiya and Hillel [1] proposed a single-version STM system that is mv-permissive.
Their system uses Compare and Swap (CAS) primitives in addition to lock. As a part of the implementation, we would like to compare the performance of our algorithm with theirs to see how much benefit do multiple versions offer.

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