Coupling Warm Rain With an Eddy Diffusivity/Mass Flux Parameterization: 1. Model Description and Validation

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Abstract A new version of the stochastic multiplume Jet Propulsion Laboratory Eddy-Diffusivity/Mass-Flux (JPL-EDMF) parameterization which consistently couples the simplified Khairoutdinov and Kogan (2000), https://doi.org/10.1175/1520-0493(2000)128<0229:ANCPPI>2.0.CO;2, warm phase cloud microphysical parameterization with the parameterization of cloud macrophysical and subgrid scale dynamical processes is described. The new parameterization combines the EDMF approach with an assumed shape of a joint probability density function of thermodynamic and kinematic variables which provide the basis for the computation of all parameterized processes. As far as we are aware this is the first attempt to consistently couple all of these parameterized processes in the EDMF framework. This paper is part one of a two paper series. Here, the JPL-EDMF parameterization is described and benchmark simulations of precipitating stratocumulus and cumulus convection are performed in a single-column-model framework. The parameterization results compare favorably to the reference large-eddy-simulation results. In the second part (Smalley et al., 2022, https://doi.org/10.1029/2021MS002729) the JPL-EDMF parameterization is validated for a wide range of observation-based scenarios covering the continuous transition from subtropical stratocumulus to cumulus convection derived from global reanalysis, and parameterization uncertainties are studied in detail.

Plain Language Summary Parameterizations are components of atmospheric models that represent the impact of unresolved processes on the resolved scale flow and are crucial for shaping simulated dynamics and thermodynamics. In many atmospheric models, multiple parameterizations are used in parallel and each of them represents subgrid processes only in part, or only for a certain atmospheric regime. Recent parameterization developments focus on unified approaches which represent key unresolved processes in a mathematically consistent framework that allows to capture important interactions among unresolved processes, which is hard to achieve with modularized approaches. We describe a new unified parameterization which represents subgrid-scale dynamical, cloud and rain processes, thus removing key shortcomings of traditional parameterizations. In this new parameterization, the subgrid-scale dynamical processes are represented by the stochastic multiplume Eddy-Diffusivity/Mass-Flux framework from our previous works. Cloud and rain processes are represented by well-known parameterizations developed for high-resolution models. The key features of the new parameterization are a set of consistent assumptions about subgrid variability which is used to evaluate grid mean rates of these three processes, and representation of interactions between them. Our new parameterization is tested against benchmark simulations of marine stratocumulus and cumulus convection and the results compare favorably to the reference large-eddy-simulation results.

1. Introduction

The finite resolutions of weather and climate models fundamentally constrain the spatial and temporal scales of processes that can be resolved by these models. Atmospheric dynamical and thermodynamical processes are highly non-linear and interactive on a wide range of spatial and temporal scales. Therefore, the impact of unresolved processes on the resolved scale flow must be parameterized (e.g., Pielke, 2002). The parameterization problem would be greatly simplified if the subgrid scale joint-probability-density-function (JPDF) of the model's thermodynamic and kinematic variables were known because most of the parameterized quantities could be directly linked to this JPDF. In many atmospheric models, multiple parameterizations are used in parallel to represent either specific parts of the atmosphere or aspects of an unresolved processes (e.g., Donner et al., 2011; ECMWF, 2020; Molod et al., 2015; Skamarock et al., 2020). The underlying and sometimes implicit assumptions
about the subgrid JPDF used by different parameterizations within the same model often differ, which complicates the consistent simulation of unresolved processes that span across multiple parameterizations.

In recent years, systematic attempts to replace the modularized structure of atmospheric parameterizations with more unified representations of unresolved processes have been proposed. An example of such a unified approach is the Cloud Layers Unified By Binormals (CLUBB) parameterization (Golaz et al., 2002a, 2002b), which is designed around explicitly modeled subgrid scale JPDFs of key thermodynamic and kinematic variables. In the CLUBB framework, this JPDF is represented by a mixture of two normal distributions, one often interpreted as representing convective updrafts and the other the non-convective environment. To characterize the CLUBB’s JPDF, the key second order and some third order moments are prognosed. Different approaches that couple cloud microphysical (MP) parameterizations with CLUBB have been developed. They all assume that the MP parameterization represents relevant processes for the small homogeneous volume of air and are “upscaled” to a resolved scale either by the analytic integration of local MP rates over the CLUBB’s JPDF (Griffin & Larson, 2013, 2016a; Larson & Griffin, 2013), by a quadrature approximation of that integral (Chowdhary et al., 2015), or as a weighted average of MP rates computed from Monte-Carlo sampled thermodynamic properties from the CLUBB’s JPDF (Larson & Schanen, 2013; Larson et al., 2005).

Another family of unified parameterizations, which is the focus of this work, is based on the Eddy-Diffusivity/Mass-Flux (EDMF) approach. EDMF parameterizations were initially developed to represent subgrid motions in dry boundary layers (e.g., Han et al., 2016; Siebesma & Teixeira, 2000; Siebesma et al., 2007; Teixeira & Siebesma, 2000; Witek et al., 2011a, 2011b), and later extended to include shallow (non-precipitating) convection (e.g., Angeline, 2005; Han & Bretherton, 2019; Neggers, 2009; Neggers et al., 2009; Pergaud et al., 2009; Rio & Hourdin, 2008; Soares et al., 2004; Suselj et al., 2012, 2013, 2014, 2019a, 2020, 2021; Tan et al., 2018) and deep convection (Suselj et al., 2019b). The key idea behind the EDMF approach is to separate the subgrid dynamics into two distinct parts: convective plumes/updrafts and the non-convective environment. The updrafts describe non-local transport and are modeled with the mass flux parameterization, and the environment is characterized by local turbulence represented with the eddy-diffusivity parameterization.

In operational models, the EDMF parameterization often represents only the subgrid scale dynamical processes while the cloud macro- and microphysical processes are modeled with separate parameterizations and the subgrid scale coupling between cloud and dynamical processes is either neglected or represented in a very crude manner (e.g., Han & Bretherton, 2019; Hourdin et al., 2020; Olson et al., 2019). While the multiplume stochastic EDMF parameterization developed at the Jet Propulsion Laboratory-Eddy-Diffusivity/Mass-Flux (JPL-EDMF) has been shown to improve results of global atmospheric models (Suselj et al., 2014, 2021), its interaction with cloud processes was always constrained by the existing cloud parameterizations in those models. An important question that arises from that experience is how to develop a unified EDMF parameterization that includes the representation of cloud processes in a consistent way. Here, we address this very question by developing a new version of the JPL-EDMF parameterization based on our previous work (Suselj et al., 2012, 2013, 2019a, 2019b) but which also includes the representation and coupling with warm phase cloud MP processes described by Khaireoutdinov and Kogan (2000) (KK MP parameterization) in a physically sound manner. The KK parameterization was originally developed for models that explicitly resolve larger turbulent eddies and is here upscaled to the JPL-EDMF grid scale accounting for the subgrid scale variability of thermodynamic properties simulated by the JPL-EDMF parameterization. The coupling between cloud processes and JPL-EDMF’s subgrid dynamics is twofold: first, interactions with individual updrafts are explicitly represented; second, the interactions of the environment are represented through modification of mean environmental thermodynamic properties as well as higher order moments modeled by prognostic equations for turbulent kinetic energy (TKE) and saturation excess variance. As far as we are aware this is the first work that consistently couples the multiplume EDMF and cloud MP parameterizations and represents the interaction between cloud and subgrid dynamical processes in physically sound manner.

This paper is part one of a two paper series. In the first part, we describe a new version of the JPL-EDMF parameterization and in particular the details of the KK MP implementation together with the coupling between the subgrid dynamical and cloud processes (Section 2 and Appendices). We evaluate the new version of the JPL-EDMF parameterization implemented in a single-column-model (SCM) for two archetypal cases, namely drizzling marine stratocumulus and precipitating cumulus, against large-eddy-simulation (LES) results (Section 3). The stratocumulus case is based on the observations during the second research flight of the second
Dynamics and Chemistry of Marine Stratocumulus (DYCOMS) field study, follows Ackerman et al. (2009) LES setup and we refer to it as the DYCOMS case. The precipitating cumulus case is based on the Rain in Cumulus over the Ocean (RICO) field study over the north-western Atlantic Ocean, follows the van Zanten et al. (2011) LES setup and we refer to it as the RICO case.

In the second part (Smalley et al., 2022), we validate the JPL-EDMF parameterization against satellite observations for several hundred cases over the North Eastern Pacific Ocean in the Smalley et al. (2019) SCM framework. The primary goal of the second part is the identification of parameterized physical processes to which the JPL-EDMF parameterization is most sensitive. The results of these studies will guide future developments of the JPL-EDMF parameterization.

2. JPL-EDMF Model Description

The key idea of the JPL-EDMF parameterization is to divide the domain of the host model (in our case the SCM) into two types of subgrid elements: multiple surface-driven convective updrafts and the non-convective environment. For each of the two element types a different shape of JPDF of thermodynamic and kinematic variables is assumed and the rates of all subgrid processes are derived consistently with these JPDFs. The formulation of all cloud processes rates is simplified so that their subgrid variability is only a function of saturation excess ($s$; defined below) and rain water mixing ratio ($q_r$). Therefore, to evaluate the cloud processes rate for a certain subgrid element, the corresponding marginal JPDF of $s$ and $q_r$, denoted by $p(s, q_r)$, is needed.

Next, we explain the assumptions that define these marginal JPDFs. In the JPL-EDMF, the horizontal fractional area of individual updraft is small, which leads us to assume a uniform distribution of all variables within the horizontal plane of the updraft. The distribution of thermodynamic and dynamic properties for the convective part is thus represented as the sum of the uniform distributions where each of the uniform distributions corresponds to a single updraft. The non-convective environment accounts for a large horizontal fractional area and therefore the assumption of uniform distribution would be unreasonable. Instead, as detailed in Section 2.2, we assume that for each horizontal plane the environmental $p(s, q_r)$ follows a joint normal-delta-lognormal distribution, so that the marginal distribution of $s$ is normal and of $q_r$ is delta-lognormal. The delta-lognormal distribution is a mixture of the delta distribution at zero $q_r$ and the lognormal distribution, and they represent the non-precipitating and precipitating parts of the environmental area, respectively. We want to emphasize that the parameterization of subgrid scale dynamical processes is based on consistent JPDFs; on the assumption of a uniform distribution of variables within each updraft and normal marginal JPDFs of kinematic and thermodynamic properties (except for $q_r$) in the environment (see Suselj et al., 2019a, 2019b and Appendices for detail).

Following a conventional approach (e.g., Cotton et al., 2010), condensed (liquid) water is split into rain and cloud contributions. We assume that the velocity of the smaller cloud droplets instantaneously adjust to that of the surrounding airflow and therefore the cloud water is passively transported with surrounding air. The larger rain droplets are associated with their own velocity vectors and their mass-weighted averaged value is assumed to be valid for the rain water. To represent cloud macrophysical processes we assume that saturation adjustment of cloud water is a fast process that efficiently removes any sub- or supersaturation. MP processes, which include autoconversion, accretion and evaporation, are represented with a simplified KK parameterization. Because moist processes can significantly impact the subgrid scale dynamics, their rates as well as rain related quantities are computed separately for the non-convective environment and each of the convective updrafts, and their individual interactions with subgrid dynamics are considered. In contrast to many atmospheric models where convective and stratiform MP processes are based on different parameterization assumptions (e.g., Donner et al., 2011; Molod et al., 2015), here we use the same parameterization for both components as it improves consistency of the parameterization.

Figure 1 shows key components in the JPL-EDMF parameterization and the shapes of the marginal PDF $p(s)$ (insert in panel a) and JPDF $p(s, q_r)$ (panel b) in the cloud layer, as indicated in panel (a). The multiple surface forced convective updrafts (Figure 1 shows one precipitating and one non-precipitating updraft) are associated with Dirac delta functions in both marginal JPDFs. The updrafts interact with the non-convective environment through lateral entrainment and detrainment. The non-convective environment is associated with locally driven turbulence and can be partially covered by clouds and partially cloud free, represented by the positive and negative
parts of the saturation excess distributions, respectively. In panel (b), the non-precipitating and precipitating part of the environment are represented with a gray and the blue area of the environment.

The JPL-EDMF parameterization is implemented in a SCM (Suselj et al., 2019b), which solves prognostic equations for the grid-mean values of moist conserved thermodynamic variables (total water mixing ratio, $q_t = q_v + q_c$ and liquid-water potential temperature, $\theta_l = \theta - \frac{L_v}{c_p \Pi} q_c$; here $q_v$ and $q_c$ are the water vapor and cloud water mixing ratios, $\theta$ is the potential temperature, $L_v$ and $c_p$ are the latent heat of water and specific heat for dry air at constant pressure, and $\Pi$ is the Exner function), horizontal components of wind speed ($u$ and $v$), TKE and saturation excess variance ($\sigma_s^2$). With the help of the EDMF decomposition (Appendix B), we compute the TKE and $\sigma_s^2$ for the non-convective environment which are used to characterize the relevant statistical moments of the marginal JPDF $p(s, q_r)$ for the environment. The SCM prognostic equations are written in the anelastic framework and are given in Appendix C. Below we first summarize the general formulation of moist processes and then describe how they are applied for both the non-convective environment and convective updrafts (Sections 2.2 and 2.3, respectively).

### 2.1. Formulation of Cloud Macro- and Microphysical Processes

#### 2.1.1. Drop Size Distribution and Terminal Velocity of Rain Water

We assume that a certain value of rain water mixing ratio ($q_r$) is associated with the observationally based drop size distribution (DSD) from Abel and Boutle (2012):

$$N(D) = x_1 \lambda x_2 e^{-\lambda D}$$

(1)

where $N(D)$ is the DSD, which is a function of slope parameter $\lambda$ and constants $x_1$ and $x_2$ (given in Table A1 in Appendix A) and $D$ is the rain drop diameter.

Assuming the mass-diameter relationship for individual droplet ($m = a_r D^6$; where $m$ is the mass of the droplet, and $a_r$ and $b_r$ are constants assuming spherical drops and are given in Table A1) the slope parameter $\lambda$ can be expressed as a function of $q_r$ following the derivation of Grabowski (1998):

$$\lambda = \left[ \frac{a_r x_1 \Gamma(b_r + 1)}{\rho \eta} \right]^{1/(1+b_r-s_2)}$$

(2)
Figure 2. Terminal velocity of individual drops as a function of their diameter from Grabowski (1998) (blue line), from Equation 3 (red line), from Gunn and Kinzer (1949) observations (black circles for \(D > 100 \mu m\)) and from Stokes law (black diamonds for \(D < 100 \mu m\)).

The key advantage of the DSD from Abel and Boutle (2012) is that it is fully characterized by rain water mixing ratio and thus no additional knowledge of rain drop number concentration is required, unlike the more traditional two-parameter gamma or exponential distributions (e.g., Morrison & Gettelman, 2008; Morrison et al., 2005). The DSD from Equation 1 forms the basis of our parameterization of cloud MP processes that depend on the rain drop sizes, such as terminal velocity and evaporation efficiency of rain water. To parameterize the rates of these processes, we compute their corresponding rain drop mass-averaged rates and then apply these averaged rates to the whole DSD.

To characterize the mass-averaged terminal velocity of rain, we first determine the relation between terminal velocity and rain drop diameter across the range of realistic drop sizes. We fit a piecewise cubic polynomial to the Gunn and Kinzer (1949) measurements for larger drops (i.e., with diameters exceeding 100 \(\mu m\)) and to the Stokes-law relation for smaller drops (e.g., Cotton et al., 2010):

\[
w_c(D) = -\sum_{i=1}^{3} c_i D^i
\]  

(3)

where \(w_c\) is the velocity of individual drops, and fitting constants \(c_i\) (for \(i = 1 \ldots 3\)) are given in Table A1.

Figure 2 shows the magnitudes of terminal velocity \(w_c\) from Equation 3 and from Grabowski (1998), who uses a simpler exponential relation, and compares them to Gunn and Kinzer (1949) observations and the Stokes-law relation. Compared to Grabowski (1998), the terminal velocity formulation from Equation 3 shows a stronger dependence on rain drop diameter and it agrees well with the Gunn and Kinzer (1949) observations for larger drops (\(D > 100 \mu m\)) as well as the Stokes law relation for smaller drops (\(D \leq 100 \mu m\)). We find that the cubic spline relation from Equation 4 represents terminal velocity regimes well across the range of Reynolds numbers, which is not captured by the exponential relation from Grabowski (1998).

The mass-averaged terminal velocity is obtained as in Grabowski (1998) by integrating mass-weighted terminal velocity-diameter relation (Equation 3) over the DSD. Since the assumed DSD is fully determined by rain water mixing ratio \(q_r\), the resulting mass-averaged terminal velocity, \(w_r\), can be written as a function of \(q_r\):

\[
w_r = -\sum_{i=1}^{3} \alpha_{w,i} q_r^{b_{w,i}}
\]  

(4)

with

\[
\alpha_{w,i} = c_r \frac{\Gamma(b_r + i + 1)}{\Gamma(b_r + i)} \left(\frac{\rho}{a_r x_1 \Gamma(b_r + 1)}\right)^{b_{w,i}}
\]  

(5)

and

where \(\rho\) is air density and \(\Gamma\) represents the gamma function. Our slope parameter expression in Equation 2 differs from Grabowski (1998) because we assume a different functional form of DSD.

Abel and Boutle (2012) derived parameters \(x_1\) and \(x_2\) from in-situ DSD observations for a number of precipitating cloud types. Here, the value of \(x_1\) is taken directly from their work, while \(x_2\) is treated as a tunable parameter. Our final \(x_2\) values for the updrafts are smaller than for the non-convective environment, which means that for a given \(q_r\) there are more larger-size rain drops in the convective updrafts compared to the non-convective environment. Even though our \(x_2\) values are different than those suggested in Abel and Boutle (2012), they largely fall within their observational spread. In particular, their observed DSD for cumulus clouds, which is presumably dominated by active convective updrafts, is associated with a higher probability of large drops compared to the observations of stratocumulus and stratus clouds. The latter ones are more likely to be dominated by non-convective rain formation.
\[ \beta_{i,j} = \frac{i}{1 + b_i - x_2} \]  

(6)

The formulation in Equation 4 does not include any dependence of terminal velocity on air density, which might be important for clouds developing at higher altitudes (Foote & Du Toit, 1969). This formulation also differs from Grabowski (1998) due to different assumptions about both the DSD and relation between terminal velocity and diameter of rain drops.

### 2.1.2. Cloud Macro- and Microphysical Processes

Next, we describe the parameterization of cloud macro- and microphysical processes, and their application to the non-convective environment and convective updrafts is detailed in Sections 2.2 and 2.3, respectively. First, we define saturation excess or deficit with respect to the cloud water (e.g., Cheinet & Teixeira, 2003):

\[ s \equiv q_i - q_c(T, p) \]  

(7)

where \( q_i \) is the saturation mixing ratio for water at temperature \( T \) and pressure \( p \). We assume that saturation adjustment of cloud water is a fast process, therefore it eliminates any sub- or supersaturation nearly instantaneously. Therefore, the cloud cover (CC) and cloud water mixing ratio \( (q_c) \) can be written as:

\[ CC = H(s) \]  

(8)

\[ q_c = sH(s) \]  

(9)

where \( H \) is the Heaviside step function defined as: \( H = \{ 1 \text{ if } x > 0; 0 \text{ if } x \leq 0 \} \). Note that saturation adjustment is only assumed for small cloud water drops, while evaporation of rain drops is much slower process and its parameterization is described below.

The rain parameterization accounts for three key MP processes (e.g., Cotton et al., 2010): (a) autoconversion of cloud drops into rain drops, (b) accretion of cloud drops by rain, and (c) evaporation of rain. The rates of these three processes essentially follow the KK formulation:

\[ S_{ac} = a_{ac}s^{ac}H(s) \]  

(10)

\[ S_{ac} = a_{ac}q_i^{ac}H(s)H(q_i) \]  

(11)

\[ S_{ev} = -a_{ev}q_i^{ev}(-s)^{ev}H(-s)H(q_i) \]  

(12)

where \( S_{ac}, S_{ac}, \) and \( S_{ev} \) represent the autoconversion, accretion and evaporation rates, respectively (i.e., the rates of conversion of total water to rain water or vice versa, such that they are positive when rain water is formed). The constants for autoconversion and accretion rates in Equations 10 and 11 are taken from the KK LES study and are given in Table A1. We are aware that these constants were derived for a stratiform case, and we expect them to be more suitable for parameterization of rates for the non-convective environment than for convective updrafts. Kogan (2013) repeated the KK analysis for convective simulations, which we would expect to be better suited for convective updrafts, and arrived at very different values for these constants. However, the JPL-EDMF parameterization results discussed in this work are quite insensitive to whether constants from KK or Kogan (2013) are used (not shown) and therefore we decided to use the former. Because these MP constants are associated with significant uncertainty, in the second part of this work (Smalley et al., 2022) we investigate the impact of their uncertainty in the context of uncertainty of other parameterized processes in the JPL-EDMF parameterization.

The equation for rain evaporation rate (Equation 12) is derived by integrating the diffusional evaporation rate for the rain drops (e.g., Equation 15 in Khairoutdinov & Kogan, 2000) over the prescribed rain DSD. This allows us to derive the values of \( a_{ev}, b_{ev}, \) and \( c_{ev} \) which are:

\[ a_{ev} = \frac{2\pi \rho G x_1}{\rho q_i} \left( \frac{\rho}{a_c x_1 \Gamma(b + 1)} \right)^{b_{ev}} \]  

(13)

\[ b_{ev} = \frac{2 - x_2}{1 + b_r - x_2} \]  

(14)
where \( F \) is a constant and \( G \) is a function of temperature, and both are taken from Grabowski (1988, 1998) and given in Table A1.

The tendencies of moist conserved thermodynamic variables and rain mixing ratio resulting from MP processes are written as

\[
\frac{\partial q_i}{\partial t} \bigg|_{\text{micro}} = \frac{\partial q_i}{\partial t} \bigg|_{\text{micro}} = S
\]

and

\[
\frac{\partial \theta_l}{\partial t} \bigg|_{\text{micro}} = \frac{L_v}{c_p \Pi} S
\]

where the subscript \( \text{micro} \) indicates MP origin. For the remainder of this paper, \( S \) will denote the sum of all MP process rates \( (S = S_{\text{auto}} + S_{\text{ac}} + S_{\text{ev}}) \), and \( S \) any of the three MP rates \( (S = \{S_{\text{auto}}, S_{\text{ac}}, S_{\text{ev}}\}) \). Autoconversion and accretion contribute to the tendencies of \( \theta_l \) because the rain water mixing ratio is not included in definition of the \( \theta_l \) (and therefore the latent heating from conversion of cloud water to rain water must be taken into account), and evaporation represents cooling due to latent heat consumption.

### 2.1.3. Prognostic Equation for Saturation Excess Variance \( (\sigma_s^2) \)

One unique aspect of our parameterization is that it solves the prognostic equation for grid mean saturation excess variance \( (\sigma_s^2) \), from which its value for the non-convective environment is derived \( (\tilde{\sigma}_s^2) \). Here and in what follows the symbol tilde over a variable indicates that we are referring to the value of that variable in the non-convective environment. The \( \tilde{\sigma}_s^2 \) compactly combines the second order moments of moist conserved thermodynamic variables that are relevant for cloud and rain processes.

In the context of macrophysical parameterization, different methods have been proposed to estimate \( \sigma_s^2 \). The diagnostic approach of Bechtold et al. (1992) is used in some models (e.g., Cheinet & Teixeira, 2003; Suselj et al., 2012). This approach was developed for non-precipitating boundary layer clouds, and assumes the balance between the turbulent production and dissipation of the second order moments of moist conserved thermodynamic variables. While computationally efficient, the diagnostic approach might not be sufficiently accurate for transient cases and especially for the cases when precipitation is present. For examples, the studies by Khairoutdinov and Randall (2002) and Griffin and Larson (2016b) show that for precipitating cases the terms neglected by Bechtold et al. (1992) might be important for shaping the \( \sigma_s^2 \) profile. As detailed in Appendix C, the prognostic \( \sigma_s^2 \) equation which is derived from a linear combination of three prognostic equations for the second order moments of moist thermodynamic variables includes all relevant processes and is:

\[
\frac{\partial \sigma_s^2}{\partial t} \bigg|_{\text{tend}} = -\bar{u} \cdot \nabla \sigma_s^2 \bigg|_{\text{resolved adv}} - \frac{\partial}{\partial z} \bar{w}' \sigma_s' \bigg|_{\text{subgrid adv}} + 2 \left( a_i \bar{w}' q'_i - b_i \bar{w}' \theta'_l \right) \left( b_i \frac{\partial \theta_l}{\partial z} - a_i \frac{\partial \theta_l}{\partial z} \right) \bigg|_{\text{turb prod}} - 2 \left( a_i + b_i \frac{L_v}{c_p \Pi} \right) s^2 \sigma_s' \bigg|_{\text{micro}} - \varepsilon a_i \sigma_s' \bigg|_{\text{turb diss}}
\]

where in Equation 18 and in what follows the symbols with overbars represent grid-mean values and the primes denote perturbations around them. Saturation parameters \( a_i \) and \( b_i \) are given in Appendix C, \( \bar{u} \) represents the three-dimensional velocity vector, \( w \) is its vertical component and \( \sigma_s^2 \equiv s^2 \) is the saturation excess variance. The terms in Equation 18 represent tendency (tend), resolved advection (resolved adv), subgrid-scale advection (subgrid adv, where only the vertical component is retained), turbulent production (turb prod), MP contribution (micro) and turbulent dissipation (turb diss). In the JPL-EDMF parameterization, turbulent production is usually
a source term of \( \sigma_r^2 \), and turbulent dissipation and MP contribution are always sinks. The latter is a sink because in our MP model saturation excess is always positively correlated with the MP source terms (i.e., \( sS > 0 \)).

As detailed in Appendix C, the third-order moment in the turbulent advection term \((\overline{uw^2})^3\) is parameterized with the down-gradient approximation, the turbulent fluxes of moist conserved variables \((\overline{uw^2q_r^2} \) and \( \overline{w\theta_r^2} \)) are computed with the JPL-EDMF approach (Equation B4) and the turbulent dissipation with the standard approach (Equation D5). The grid-mean covariance of the saturation excess and the MP source term \((sS)\) are obtained as contributions from the non-convective environment and from the updrafts following Equation C12. To do so, the product between saturation excess and MP source terms in the non-convective environment (i.e., \( s\tilde{S} \) in Equation C12) is needed which is part of MP process evaluation (see Section 2.2).

Instead of prognosing \( \sigma_r \), we could in principle solve prognostic equations for all three second order moments of moist conserved thermodynamic variables (i.e., \( \theta_r^2 \), \( q_r^2 \), and \( \theta_r^2 q_r^2 \)) and diagnose the \( \sigma_r^2 \) from those three. However, since the rates of moist processes calculated by the JPL-EDMF depend on their combined effects that are encapsulated by saturation excess \( s \), it is computationally more efficient to solve a single \( \sigma_s^2 \) equation instead of three prognostic equations for the second order moments.

2.2. Nonconvective Environment

2.2.1. Joint Probability Density Function of Key Thermodynamic Variables

As we explain in detail below, to evaluate the mean rates of moist processes for the non-convective environment, we need to know the mean values of moist conserved thermodynamic variables over that part of the domain along with the marginal JPDF of saturation excess and rain water mixing ratio \( \tilde{p}(s,q_r) \), where as before the symbol tilde indicates that JPDF is evaluated for the non-convective environment. The value of moist conserved thermodynamic properties for the non-convective environment is diagnosed as a residual between the grid-mean and updraft properties using the EDMF decomposition (Equation B2) and the \( \tilde{p}(s,q_r) \) is described below.

To proceed, we first describe the one-dimensional PDFs of saturation excess and rain water mixing ratio for the non-convective environment and then their joint distribution. The PDF of saturation excess is assumed to be normal: \( \tilde{p}(s) = p_N(s) \) (where \( p_N \) denotes a normal distribution; Equation F1). The normal distribution of \( s \) is an approximation that results from assuming normal distributions of the JPL-EDMF moist conserved thermodynamic variables and linearizing \( s \) around its mean value. This normal PDF is defined by two parameters: mean environmental value of saturation excess, \( \tilde{s} \), and its variance \( \tilde{\sigma}_s^2 \). The mean is obtained from the mean most conserved thermodynamic variables and cloud macrophysical parameterization (described in the next subsection). The variance is derived from its prognosed grid-mean value \((\sigma_s^2)\) and from the saturation excess values in the updrafts using the EDMF decomposition for the second order moments (Equation B3).

Following Griffin and Larson (2016a), the marginal PDF of rain water mixing ratio \((q_r)\) in the environment is assumed to be the superposition of rain-dominated and rain-free regions that are described, respectively, by lognormal and delta PDFs:

\[
\tilde{p}(q_r) = a_r p_{\log N}(q_r) + (1 - a_r) \delta(q_r)
\]  

where \( p_{\log N} \) is the lognormal distribution (Equation F2), and \( \delta(q_r) \) represents the Dirac delta distribution, which is non-zero only for \( q_r = 0 \). The fractional area of the rain-dominated part of the non-convective environment is \( a_r \) and of the rain-free part is \( 1 - a_r \). We choose the lognormal PDF for the rain-dominated part because it can represent large skewness, which is in qualitative agreement with observations (e.g., Boutle et al., 2014). The lognormal distribution is characterized with two parameters that are derived from the mean values of rain water mixing ratio over the rain-dominated environment \((\bar{q}_r = \tilde{q}_r/a_r)\) and its variance \( \tilde{\sigma}_{\bar{q}_r}^2 \), where the latter one is parameterized following Larson and Griffin (2013) as:

\[
\tilde{\sigma}_{\bar{q}_r}^2 = d_{\bar{q}_r} \bar{q}_r^2
\]  

where \( d_{\bar{q}_r} \) is a constant defined in Table A1.
To specify the two-dimensional JPDF \( \tilde{p}(s, q_r) \) we assume that for any location on the horizontal plane of the environment there is no correlation between the saturation excess and the presence of rain. This practical assumption is likely true far below cloud base and less so within and below the clouds because the rain tends to be formed in the moister part of the environmental area. With this simplifying assumption, \( \tilde{p}(s, q_r) \) can be written as a sum of two mixture distributions:

\[
\tilde{p}(s, q_r) = a_1 p_{N, \text{log }N}([s q_r]) + (1 - a_1) p_N(s) \delta(q_r) \tag{21}
\]

where \( p_{N, \text{log }N} \) is the two-dimensional hybrid normal-lognormal PDF (defined by Equation F7), where one dimension is defined by the saturation excess and the other one by the rain water mixing ratio, and the second term in Equation 21 represents the normal distribution of saturation excess in the rain-free environment. To characterize this distribution, the covariance between the saturation excess and the rain water mixing ratio for the precipitating part of the environment, \( \tilde{p}_{\delta, s} \), is also needed, which in this parameterization is kept constant (see Table A1) following observational evidence from Lebsock et al. (2013).

Recent observation-based studies (Lebo et al., 2015; Witte et al., 2019) argue that \( \tilde{p}_{\delta, s} \) and \( \tilde{\sigma}_s^2 \) should both be functions of horizontal grid size of the host model and of the weather regime. Modifications of these parameters are left for future investigations.

The mean cloud MP rates (i.e., for accretion, autoconversion, and evaporation) in the environment are computed by integrating the local rates over the environmental JPDF:

\[
\tilde{S} = \int_{-\infty}^{\infty} ds \int_{0}^{\infty} dq_r \tilde{p}(s, q_r) s S(q_r, s) \tag{22}
\]

As discussed in Section 2.1.3 the MP processes can modify the \( \sigma_s^2 \), as described by its prognostic equation (Equation 18). In the \( \sigma_s^2 \) prognostic equation, the value of \( \tilde{S} \) is needed, which is calculated as:

\[
\tilde{\sigma}_s^2 = \int_{-\infty}^{\infty} ds \int_{0}^{\infty} dq_r \tilde{p}(s, q_r) s S(q_r, s) \tag{23}
\]

### 2.2.2. Cloud Properties

The parameterization of cloud macrophysical processes based on the normal PDF of saturation excess was first introduced by Sommeria and Deardorff (1977) and Mellor (1977) and has since been used in other works (e.g., Bechtold et al., 1995). Since our assumption about the PDF of saturation excess is the same as these works, we essentially use their expressions for the mean cloud cover and cloud water content in the non-convective environment:

\[
\overline{CC} = \frac{1}{2} \left( 1 + \text{erf} \left( \frac{\tilde{Q}_l}{\sqrt{2}} \right) \right) \tag{24}
\]

\[
\tilde{q}_c = \int_{0}^{\infty} \tilde{p}(s) ds = \tilde{\sigma}_s \left( \overline{CC} \cdot \tilde{Q}_l + \frac{\exp(-\tilde{Q}_l/2)}{\sqrt{2\pi}} \right) \tag{25}
\]

where \( \tilde{Q}_l \equiv \hat{s}/\tilde{\sigma}_s \) is the inverse of the normalized variance of saturation excess. The mean value of saturation excess \( \hat{s} \) is computed from the mean thermodynamic variables for the environment: \( \hat{s} = \tilde{q}_c - q_r (\tilde{T}, \tilde{p}) \), where we take \( \tilde{p} \approx \tilde{\bar{p}} \), and temperature \( \tilde{T} \) can be computed as \( \tilde{T} = \tilde{\bar{p}} \tilde{\bar{\Pi}} + L_r/c_p \tilde{\bar{q}}_c \). The value of \( \tilde{\bar{q}}_c \) is an implicit function of \( \hat{s} \), therefore the solutions for \( \overline{CC}, \tilde{q}_c \), and \( \tilde{T} \) are obtained iteratively using the linear expansion from Cheinet and Teixeira (2003) as a first guess. In Suselj et al. (2019b), we used similar approach to represent grid-mean cloudiness in the JPL-EDMF model.
2.2.3. Autoconversion

Because the autoconversion rate does not directly depend on \( q_r \), its mean value over the environment can be expressed as a single integral in Equation 22 over the marginal PDF of \( s \):

\[
\bar{S}_{\text{auto}} = a_{\text{auto}} \int_0^\infty p_N(s)s^{c_{\text{auto}}-2}ds
\]

\[
= a_{\text{auto}} \frac{2}{\sqrt{\pi}} \frac{\sigma_{\text{auto}}^{c_{\text{auto}}-1}}{\ddot{\sigma}} \left[ \frac{1}{\sqrt{2\pi}} \Gamma\left(\frac{c_{\text{auto}}}{2} + 1\right) F_1\left(\frac{1 - c_{\text{auto}}}{2}, 2 - \frac{Q_s^2}{2}\right) \right]
\]

where we took advantage of the fact that the PDF of saturation excess in the environment follows normal distribution, \( p_N(s) \), and \( F_1 \) is the Kummer confluent hypergeometric function (Appendix F). We used Equation F10 to derive the final expression in Equation 26.

To understand if the saturation excess variability can be neglected when evaluating the mean autoconversion rate for the non-convective environment, we first express the autoconversion source term in the limit of \( \ddot{\sigma} \to 0 \) (with the help of Equation F11) as:

\[
\bar{S}_{\text{auto,0}} \equiv \bar{S}_{\text{auto}, \ddot{\sigma} \to 0} = a_{\text{auto}} \ddot{s}^{c_{\text{auto}}} H(\ddot{s})
\]

which is the autoconversion rate expression that can be used if the saturation excess variability is neglected.

The upper panel of Figure 3 compares the values of autoconversion rates \( \bar{S}_{\text{auto}} \) and \( \bar{S}_{\text{auto,0}} \) from Equations 26 and 27, respectively, and their ratio \( E_{\text{auto}}^d \equiv \bar{S}_{\text{auto,0}} / \bar{S}_{\text{auto}} \) (in units of percent). The ratio \( E_{\text{auto}}^d \) can be interpreted as autoconversion “enhancement factor” due to subgrid variability (Boutle et al., 2014; Lebsock et al., 2013). We plot its inverse to avoid infinite values of \( E_{\text{auto}} \) when \( \ddot{s} < 0 \). In agreement with previous studies (e.g., Boutle et al., 2014; Lebsock et al., 2013) the autoconversion rate \( \bar{S}_{\text{auto,0}} \) always underestimates \( \bar{S}_{\text{auto}} \), and in relative terms this underestimation increases when both saturation excess and its standard deviation increase. In order to illustrate how much the \( \bar{S}_{\text{auto,0}} \) would underestimate \( \bar{S}_{\text{auto}} \) for a typical stratocumulus case, on top of \( E_{\text{auto}}^d \) we show the grid-mean values of \( s \) and \( \ddot{\sigma} \) for the cloudy part of the DYCOMS stratocumulus from the LES ensemble (Ackerman et al., 2009). These results show that the underestimation almost always exceeds 10% and often 40% or more, with the extreme values reaching 90%. These results illustrate the importance of considering subgrid variability of saturation excess in order to reliably estimate autoconversion rate in the atmospheric models.

The value of the product of saturation excess and the autoconversion rate, averaged over the environment \( \left( \ddot{s} \bar{S}_{\text{auto}} \right) \), which is used in the \( \ddot{\sigma}_s^2 \) equation is obtained with the help of Equation F10 and is:

\[
\ddot{s} \bar{S}_{\text{auto}} = a_{\text{auto}} \int_0^\infty p_N(s)s^{c_{\text{auto}}+1}ds
\]

\[
= a_{\text{auto}} \frac{2}{\sqrt{\pi}} \frac{\sigma_{\text{auto}}^{c_{\text{auto}}}}{\ddot{\sigma}} \left[ \frac{1}{\sqrt{2\pi}} \Gamma\left(\frac{c_{\text{auto}}}{2} + 1\right) F_1\left(\frac{1 - c_{\text{auto}}}{2}, 2 - \frac{Q_s^2}{2}\right) \right]
\]

2.2.4. Accretion

The accretion rate, averaged over the environment is expressed as:

\[
\ddot{S}_{\text{acc}} = a_s a_{\text{acc}} \int_0^\infty ds \int_{s}^{\infty} dq_{\text{res}} p_{N,N}(s, q_{\text{res}}) e^{q_{\text{res}}} \]

In Equation 29, we first take advantage of the fact that the source term integrated over the rain-free part of the environment (i.e., over the second term on the right hand side of Equation 21) is zero. Next, we transform the variable \( q_{\text{res}} \) to \( q_{\text{res}} = \log(q_{\text{res}}) \), where the basic PDF transformation law is used (i.e., \( p(s, q_{\text{res}})dq_{\text{res}} = p(s, q_{\text{res}})dq_{\text{res}} \); e.g., Press et al. (1992)). With this transformation, the JPDF of \( s \) and \( q_{\text{res}} \), denoted by \( p(s, q_{\text{res}}) \), is bivariate normal, and is denoted by \( p_{N,N} \).
With the help of Equation F12, we first evaluate the inner integral in Equation 29:

\[
I_1(b_a) \equiv \int_{b_a}^\infty P_{N,N}(s,q_{r\tau}) e^{\sigma_{acr} q_{r\tau}} dq_{r\tau} = \frac{1}{\sqrt{2\pi\sigma_s}} \exp\left(-\frac{\left(s - \tilde{s}\right)^2}{2\sigma_s^2} + b_a \tilde{q}_{r\tau} \sigma_{acr} \tilde{s} - \tilde{s} + b_a \tilde{q}_{r\tau} + \frac{b_a^2 \sigma_{acr}^2 (1 - \rho_{acr}^2)}{2}\right)
\]  

(30)

where \(\sigma_{acr} = q_{r\tau}^{1/2}\), \(\tilde{q}_{r\tau}\) and \(\tilde{\sigma}_{acr}\) can be expressed in terms of \(\tilde{q}_{r\tau}\) and \(\tilde{s}\), following Equations F3 and F4 and \(\rho_{acr}\) is the linear correlation between \(q_{r\tau}\) and \(s\) can be expressed in terms of the linear correlation between \(q_{acr}\) and \(s\) following Equation F9. The final solution of Equation 29 is obtained with the help of Equation F13 and is:

\[
\tilde{S}_{ac} = \frac{a_{ac} \sigma_{acr}^2}{\sqrt{2\pi}} \exp\left[\frac{b_a^2 \sigma_{acr}^2}{2} + b_a \tilde{q}_{r\tau} - \frac{\Lambda^2}{4}\right] \Gamma(\epsilon_{ac} + 1) D_{1-\epsilon_{acr}} (-\Lambda)
\]

(31)

Figure 3. Left panels: microphysical rates for the non-convective environment as a function of \(\tilde{s}\) and \(\tilde{\sigma}\) (\(\tilde{S}_{\text{au}}, \tilde{S}_{\text{au,0}}\), \(\tilde{S}_{\text{ac}}, \tilde{S}_{\text{ac,0}}\), \(\tilde{S}_{\text{ev}}, \tilde{S}_{\text{ev,0}}\); colors and red lines) and the corresponding rates obtained by neglecting the subgrid variability of \(s\) and \(q_{rr}\) (\(\tilde{S}_{\text{au}0}, \tilde{S}_{\text{au,0}}\), \(\tilde{S}_{\text{ac}0}, \tilde{S}_{\text{ac,0}}\), \(\tilde{S}_{\text{ev}0}, \tilde{S}_{\text{ev,0}}\); black lines). Right panels: the inverse of enhancement factors \(E_{\text{au}}^{-1}\), \(E_{\text{ac}}^{-1}\), and \(E_{\text{ev}}^{-1}\). Upper panel shows the results for autoconversion rate with typical cloud drop number concentration \(N_c\) (\(N_c = 50 \text{ cm}^{-3}\)), middle for accretion and lower for evaporation with \(\tilde{q}_{r\tau} = 0.1 \text{ g kg}^{-1}\) and \(a_r = 1\).
where we defined \( \Lambda \equiv b_{ac} \tilde{p}_{q_{\text{mix}}} \tilde{\sigma}_{q_{\text{mix}}} + \bar{Q}_r \) and \( D_{ac}(x) \) is the parabolic cylinder function with parameters \( \lambda \) and \( x \) (see Appendix F).

To understand the role of the subgrid variability of \( s \) and \( q_r \) for accretion rate, with the help of Equation F14 we compute its mean rate over the non-convective environment for \( \tilde{\sigma}_{q_{\text{mix}}} \rightarrow 0 \) and \( \tilde{s} \rightarrow 0 \):

\[
\tilde{S}_{ac,0} \equiv \tilde{S}_{ac} (\tilde{\sigma}_{q_{\text{mix}}} \rightarrow 0, \tilde{s} \rightarrow 0) = \max [b_{ac} \tilde{p}_{q_{\text{mix}}} 3^{\tilde{s}_{\text{env}}}, 0] = a_{ac} \tilde{p}_{q_{\text{mix}}} \tilde{s}_{\text{env}} H(\tilde{s})
\]  

(32)

Note that the solution \( \tilde{S}_{ac,0} \) is not consistent with the JPL-EDMF model assumptions because \( \tilde{\sigma}_{q_{\text{mix}}} \) and \( \tilde{q}_r \) are related following Equation 20.

The middle row of Figure 3 compares the solutions of Equations 31 and 32 and the inverse of enhancement factor, \( E_{ac}^{-1} \equiv \tilde{S}_{ac,0} / \tilde{S}_{ac} \), for a chosen value of \( \tilde{q}_r \) and for \( a_r = 1 \), as a function of \( \tilde{s} \) and \( \tilde{s}_r \). In the middle right panel on Figure 3, we show the values of \( E_{ac}^{-1} \) computed from the grid-mean values of the DYCOMS LES ensemble. Similar to autoconversion, the relative underestimation of accretion rate by \( \tilde{S}_{ac,0} \) increases with increasing both saturation excess and its variance. For the DYCOMS case, the \( \tilde{S}_{ac,0} \) underestimates the accretion rate by more than 20% and often times more than 50%. As for the autoconversion, these results indicate the importance of considering subgrid variability of saturation excess and rain water mixing ratio when evaluating accretion rate.

The value of the product of saturation excess and the accretion rate averaged over the environment is computed as:

\[
\tilde{S}_{ac} = a_{ac} \tilde{p}_{q_{\text{mix}}} \int_0^\infty ds \tilde{s}_r^{a_r+1} \int_0^\infty dq_r \tilde{p}_{N,N} (s, q_r) e^{b_{ac} q_r} \\
= a_{ac} \tilde{p}_{q_{\text{mix}}} \tilde{s}_{\text{env}} \frac{2}{\sqrt{2\pi}} \exp \left\{ \frac{b_{ac}^2 \tilde{\sigma}_{q_{\text{mix}}}^2}{2} + b_{ac} \tilde{q}_{\text{mix}} - \frac{\Lambda^2}{4} \right\} \Gamma(c_{ev} + 2) D_{ac}(c_{ev} + 2)(-\Lambda)
\]  

(33)

2.2.5. Evaporation

The functional form of evaporation rate is essentially the same as for accretion, therefore derivation of its mean rate for the non-convective environment follows that for accretion. The factor \( a_{ev} \) in the evaporation rate equation (Equation 13) is a function of thermodynamic variables (i.e., through both \( q_r \) and \( G \)) but we neglect its variability by computing its value for the environmental mean thermodynamic variables. Simple scaling analysis shows that for the evaporation source term, the variability of \( s \) and \( q_r \) from Equation 12 dominates the variability of \( a_{ev} \). With this simplification, the evaporation rate averaged over the environment can be expressed as:

\[
\tilde{S}_{ev} = -a_{ev} \tilde{p}_{q_{\text{mix}}} \int_0^\infty ds (-s)^{a_r} \int_0^\infty dq_r \tilde{p}_{N,N} (s, q_r) \exp (b_{ev} q_r) \\
= -a_{ev} \tilde{p}_{q_{\text{mix}}} \tilde{s}_{\text{ev}} \frac{2}{\sqrt{2\pi}} \exp \left\{ \frac{b_{ev}^2 \tilde{\sigma}_{q_{\text{mix}}}^2}{2} + b_{ev} \tilde{q}_{\text{mix}} - \frac{Y^2}{4} \right\} \Gamma(c_{ev} + 1) D_{ac}(c_{ev} + 1)(-\Lambda)
\]  

(34)

where we define: \( Y \equiv b_{ev} \tilde{p}_{q_{\text{mix}}} \tilde{\sigma}_{q_{\text{mix}}} - \bar{Q}_r \).

Neglecting the variability of rain water mixing ratio and saturation excess, the evaporation rate can be expressed as:

\[
\tilde{S}_{ev,0} \equiv \tilde{S}_{ev,\tilde{s}_r \rightarrow 0, \tilde{q}_r \rightarrow 0} = -H(-\tilde{s}) a_{ev} \tilde{q}_{\text{mix}} \tilde{s}_{\text{ev}} \tilde{s}_{\text{ev}}
\]  

(35)

The lower panel of Figure 3 shows the values of evaporation rates \( \tilde{S}_{ev} \) and \( \tilde{S}_{ev,0} \) as well as the corresponding inverse of the enhancement factor \( E_{ev}^{-1} \equiv \tilde{S}_{ev,0} / \tilde{S}_{ev} \) for a fixed value of \( \tilde{q}_r = 0.1 \) g kg\(^{-1}\) and \( a_r = 1 \). In the plot for the inverse of the enhancement factor, the grid mean values from the subcloud layer from the LES ensemble for the DYCOMS case are plotted. As for the accretion rate, the \( \tilde{S}_{ev,0} \) underestimates the magnitude of \( \tilde{S}_{ev} \) when the mean subsaturation and \( \tilde{s}_r \) increase. For the DYCOMS case, over much of the subcloud layer the relative difference between \( \tilde{S}_{ev} \) and \( \tilde{S}_{ev,0} \) is less than 20%. These results indicate that for the typical stratocumulus case, the subgrid scale variability might be less important for evaporation rate compared to autoconversion and accretion rates.

The value of the product of saturation excess and the evaporation rate averaged over the environment is computed as:
\[ \overline{S}_{ev} = \frac{a_{ev} \theta_{ev}^{2/n+1}}{\sqrt{2\pi}} \exp \left[ \frac{b_{ev} \theta_{ev}^2}{2} + b_{ev} \bar{q}_{rn} - \frac{Y^2}{4} \right] \Gamma (c_{ev} + 2) D_{n(c_{ev}+2)} (-Y) \] (36)

2.2.6. Rain Water-Mixing Ratio and Rain-Dominated Area

Below we derive equations for rain-water mixing ratio in the non-convective environment, \( \bar{q}_r \), and for rain-dominated part of the non-convective environment, \( a_r \). To do so, we first simplify the prognostic \( q_r \) equation (Equation C13) by neglecting the tendency and horizontal advection terms of \( q_r \) to obtain:

\[ \frac{\partial}{\partial z} (\rho (w + w_r) q_r) = \rho S \] (37)

where \( \rho \) is air density and \( w_r \) is the terminal velocity of rain water. Next, we perform Reynolds averaging and the EDMF decomposition (Equation B2) of Equation 37 to get

\[ \langle w + w_r \rangle q_r = a_r \left( \bar{w} q_r + \bar{w_r} \bar{q}_r \right) + \sum_n a_n (w_{r,n} + w_n) q_{r,n}, \]

where index \( n = 1 \ldots N \) refers to individual convective updrafts. We neglect the vertical advection of \( q_r \) by mean environmental flow (i.e., \( \bar{w} \bar{q}_r \approx 0 \)) and assume that density \( \rho \) is a function of height only, which leads to the final equation:

\[ \frac{\partial}{\partial z} (\rho a_r \bar{w} \bar{q}_r) = - \sum_n \frac{\partial}{\partial z} (\rho a_n (w_{r,n} + w_n) q_{r,n}) + \rho \bar{S} \] (38)

At this point, the interpretation of the terms in Equation 38 might not be obvious. Once we describe the equations for rain water mixing ratio and for mass conservation in the updrafts (see Equation 47 and steady-state version of Equation E11, respectively), Equation 38 can be rewritten in a physically more intuitive way as:

\[ \frac{\partial}{\partial z} (\rho a_r \bar{w} \bar{q}_r) = \rho \left[ a_r \bar{S} + \sum_n \left( q_{r,n} (D_n + \bar{D}_n) - E_n \bar{q}_r \right) \right] \] (39)

Equation 39 relates the divergence of rain rate with the MP source terms in the environment, the detrainment of rain water from all updrafts (here \( D_n \) and \( \bar{D}_n \) represent detrainment rates, and index \( n = 1 \ldots N \) denotes individual updraft) and the entrainment of the environmental rain water into the updrafts (\( E_n \) represents the entrainment rate). Except for the terms pertaining to the convective updrafts, Morrison and Gettelman (2008) use similar equation for \( q_r \).

To solve Equation 39, we need to find the relationship between the, \( \bar{w} \bar{q}_r \), and the mean rain water mixing ratio over the precipitating environmental area, \( \bar{q}_r \). To do so, we integrate the product of \( q_r \) and terminal velocity or rain (which is a function of \( q_r \); Equation 4) over the marginal JPDF of \( q_r \) for the non-convective environment:

\[ \bar{w} \bar{q}_r = \int_0^\infty \rho (q_r) u_r q_r dq_r \]

\[ = -a_r \int_0^\infty p_n(q_{rn}) \left[ \sum_j a_{uj} \exp \left( (\beta_{uj} + 1) q_{rn} \right) \right] dq_{rn} \]

\[ = - \sum_{j=1}^3 a_r a_{uj} \theta_{ur}^{2/n+1} \left( 1 + \frac{\sigma_{uj}^2}{\theta_{ur}^2} \right)^{-1/2} \] (40)

where we considered the assumed delta-lognormal distribution of \( q_r \) for the non-convective environment (Equation 21) and used the basic law of probabilities as well as Equation F15 to evaluate the integral after second equality sign.

A simple parameterization is used to describe the fractional area of rain-covered, non-convective environment \( a_r \). Except for the situations described below, we assume that vertical divergence of rain rate (which is usually dominated by evaporation of precipitation) results partially from the decrease of its mean intensity and partially from the decrease of the rain-dominated area \( a_r \). Here, we simply assume linear relation between vertical divergence of \( \frac{1}{a_r} \frac{\partial a_r}{\partial z} = a_r^{-1} \frac{1}{u_r \bar{q}_r} \frac{\partial u_r \bar{q}_r}{\partial z} \) (41)
where \( \alpha_r \) is a non-dimensional parameter given in Table A1. Equation 41 essentially represents power-law relation between the area \( a_r \) and the magnitude of \( \bar{\omega} \cdot \bar{q}_r \). Future work might be needed to refine this parameterization, even though in Smalley et al. (2022) we show that this parameterization contributes only a minor role the overall uncertainty of the JPL-EDMF results.

Whenever clouds in the environment or precipitating updrafts are present, the \( a_r \) is limited by:

\[
\alpha_r = \max \left( \alpha_r, C \bar{C}, \sum_n \alpha_{n,p} \right)
\]

where \( \sum_n \alpha_{n,p} \) represents the fractional area of the precipitating updrafts. Equation 42 assumes that if stratiform clouds are present, the cloudy area is always associated with some amount of precipitation. The last term in Equation 42 limits the minimum precipitating area to the area of precipitating updrafts, because precipitating updrafts detrain rain water into the environment (Equation 39). Here we simply assume that the area of detrained environment equals to the area of the corresponding updrafts. To compute \( \alpha_r \) Equation 41 is integrated from the top of the cloud layer to the surface and at each model level, it is modified following Equation 42 if necessary.

Because the rain rate and rain covered area are functions of MP source terms and the source terms are a function of \( \bar{q}_r \) the corresponding equations are solved in an iterative manner, similarly to Morrison and Gettelman (2008).

### 2.3. Mass-Flux Parameterization for Convective Updrafts

The convective part of the domain is represented with a steady-state laterally entraining multiplume stochastic parameterization. Multiple convective plumes/updrafts are initialized at the surface layer and each updraft ascends and terminates at the height where its vertical velocity vanishes. The fractional area of an individual updraft is assumed to be constant from the surface to its termination height. Thermodynamic and kinematic properties are unique to each updraft because each updraft is associated with its own surface conditions and (stochastic) profile of entrainment rate.

The surface conditions for the updrafts are taken from the tail of the assumed joint-normal distribution of vertical velocity, water vapor mixing ratio and virtual potential temperature, discretized along equidistant vertical velocity bins into \( N \) updrafts \((N = 20; \text{see Suselj et al. } (2019a), \text{for details}). \) We assume that updrafts cumulatively account for the surface vertical velocity between one and three times its standard deviation. The cumulative updraft surface area, therefore equals to the integral of the normal vertical velocity distribution between the two limits and is approximately 15.7%.

The equations for the moist conserved variables, three-dimensional velocity components as well as the rain water mixing ratio in the \( n \)th updraft follow Equations E13 and E14 and are similar to Suselj et al. (2013, 2019a, 2019b, 2020):

\[
\frac{\partial q_{r,n}}{\partial z} = \epsilon_a (\bar{q}_r - q_{r,n}) - \frac{S_n}{w_n}
\]

\[
\frac{\partial \theta_{r,n}}{\partial z} = \epsilon_a (\bar{\theta}_i - \theta_{r,n}) + \frac{L_e}{c_q \Pi} \frac{S_n}{w_n}
\]

\[
\frac{\partial u_{n}}{\partial z} = \frac{\epsilon_a}{3} (\bar{u}_i - u_{r,n})
\]

\[
\frac{1}{2} \frac{\partial u_{n}^2}{\partial z} = a_n B_n - b_n \epsilon_a u_{r,n}^2
\]

\[
\frac{\partial q_{r,n}}{\partial z} = \epsilon_a (\bar{q}_r - q_{r,n}) - \frac{1}{\rho w_n^2} \frac{\partial}{\partial z} \left( \rho u_{r,n} \bar{q}_{r,n} \right) + \frac{S_n}{w_n} - q_{r,n} \delta_n
\]
where \( e \) represent the entrainment rate (see below), \( a \) and \( b \) are vertical-velocity constants (given in Table A1), \( B = g(\theta_r/\theta_e - 1) \) is updraft buoyancy, where \( \theta = \theta(1 + 0.61 q_r - q_e) \) is virtual potential temperature. In Equation 45, \( i = \{0, 1\} \) where \( u \) and \( v \) are the two horizontal components of the velocity vector (\( u \) and \( v \)).

The symbol \( \delta \) in Equation 47 represents the detrainment of the rain water into the environment. This detrainment might occur because rain water can be associated with its own horizontal momentum relative to the surrounding air in the updraft. While there might be different processes that control this detrainment rate, here we assume that it is dominated by the vertical shear of horizontal velocity. The key idea behind this parameterization is that the shear is responsible for slanting of updrafts, while the falling rain is largely unaffected by this shear. The horizontal momentum of the rain relative to the air in the updrafts is therefore proportional to the magnitude of wind shear and the detrainment rate is parameterized as:

\[
\delta = \frac{1}{w_{f}} \left[ \left( \frac{\partial u}{\partial z} \right)^2 + \left( \frac{\partial v}{\partial z} \right)^2 \right]^{1/2} \tag{48}
\]

where velocity scale \( w_{f} \) is constant and is given in Table A1.

Because we assume that each individual updraft is associated with a uniform distribution of thermodynamic properties, the macrophysical properties as well as the rates of MP processes within individual updrafts are computed with Equations 8–12, where the \( s \) and \( q \) are replaced with the values for that particular updraft.

As in our previous works (Suselj et al., 2013, 2019b, 2020), the lateral entrainment rate over the depth \( \Delta z \) is assumed to be a result of a superposition of discrete stochastic entrainment events, where the number of events is assumed to follow Poisson distribution and where each event entrains a fixed fraction of environmental air denoted by \( \epsilon_{w} \). For an updraft that travels the depth of a model layer \( \Delta z \) the entrainment rate is:

\[
\epsilon_{w}(\Delta z) = \frac{s_{f} \epsilon_{0}}{\Delta z} P \left( \frac{\Delta z}{s_{f} L_{e}} \right) \tag{49}
\]

where the symbol \( P(x) \) represents a random number following a Poisson distribution with parameter \( x \) and \( s_{f} \) is the factor representing intermittency of the entrainment rate (Suselj et al., 2020). The entrainment length \( L_{e} \) is parameterized as:

\[
L_{e} = \frac{(z_{top}/z_{0})^{p_{e}}}{\phi_{e}} \tag{50}
\]

where \( z_{0} = 1 \) m and parameters \( \phi_{e}, p_{e}, s_{f} \) and \( \epsilon_{w} \) are given in Table A1.

Updraft equations (Equations 43–47) are challenging to solve because they are highly coupled, impacted by MP processes and because the boundary conditions for all variables but \( q_{r} \) are defined at the surface (see Suselj et al., 2019a), while for the \( q_{r} \) they are defined at the top of the updraft (i.e., the highest level where the updrafts reach, where \( q_{r} = 0 \)). Therefore, the set of the updraft equations represent a two point boundary value problem (e.g., Press et al., 1992). We solve updraft equations by iterating between two computational steps, A and B. In step A we integrate Equations 43–46 from the surface to the termination height of the updraft and the rates of the MP processes are taken from step B. First time when step A is computed, the rates of MP processes are taken to be zero. In step B, Equation 47 is integrated from the top of the updrafts to the surface and the rates of the MP sources are evaluated. The steps A and B are repeated until updraft properties converge.

3. Results

Two precipitating boundary layer cases are simulated with the JPL-EDMF parameterization implemented in a SCM and the results are compared against the reference LES simulations. The first case represents a precipitating version of coastal marine stratocumulus clouds and is based on measurements off the coast of California during the second research flight of the second DYCOMS field study (Stevens et al., 2003). As the simulation reaches quasi steady-state conditions relatively quickly, the case is simulated for 6 hr. The setup and the reference LES simulation results are both taken from Ackerman et al. (2009). Because in the JPL-EDMF parameterization the sedimentation of cloud water is not considered, the reference results consist of the 14 LES simulations that do not
model this process. The second case represents a slowly deepening cumulus dominated convection layer observed during the RICO field campaign in the trades over the Western Atlantic (Rauber et al., 2007). The convective evolution for the RICO case is simulated for 24 hr. The setup as well as the reference results, which consist of 13 LES simulations, are taken from van Zanten et al. (2011). For this comparison we consider the LES simulations which include the parameterization of MP processes. For both cases, all LES ensemble members are initialized and forced in exactly the same way and represent solutions of different models. We compute the LES ensemble mean and the interquartile range (IQR), which we take to represent the most likely value and the uncertainty range, respectively. In addition to the ensemble LES results just described, reference profiles of σ² were obtained from a single simulation with UCLA-LES (Stevens et al., 1999, 2005).

The LES models that contributed to the ensemble simulations used in this work differ in the numerical methods used for solving LES thermodynamical equations, parameterization of subgrid and MP processes, and computation of interactions between LES-resolved and parameterized processes. As detailed in Ackerman et al. (2009) and van Zanten et al. (2011), MP parameterizations in LESs range in complexity from single moment schemes (i.e., similar to the one used in the JPL-EDMF parameterization) to bin schemes. It is important to note that MP parameterizations are applied and coupled to the thermodynamical processes in very different manners in the LESs and the JPL-EDMF parameterization. In the LESs, convective structures and turbulent motions are resolved to a large degree and the MP process rates are evaluated and coupled to resolved motions, while in the JPL-EDMF MP processes are coupled to parameterized convection and turbulence.

The JPL-EDMF parameterization in the SCM model is run with a time step of 20 s and a vertical grid spacing of 20 m, except for a vertical resolution sensitivity study (Section 3.3) in which the vertical grid spacing is varied from 20 to 200 m. The vertical domain extends from the surface to 4,000 m, which is deep enough to capture moist convection for the two cases.

3.1. DYCOMS Case

As described above, the DYCOMS case setup follows Ackerman et al. (2009) from which the reference LES data is taken. At the start of simulation, the planetary boundary layer (PBL) is well mixed from the surface to the height of approximately 795 m, where it is topped by a strong inversion. The surface latent and sensible heat fluxes as well as the geostrophic wind speed components and vertically constant divergence are prescribed and are all constant with time. The short-wave radiative flux is set to zero, as the case represents the nocturnal boundary layer. In the clear sky above the PBL, the long-wave radiative flux is prescribed so that its cooling exactly balances the subsidence warming. For the cloud layer, it is computed interactively and is a function of cloud liquid water. In the JPL-EDMF parameterization, the cloud droplet concentration N_c is set to 55 cm⁻³. This value agrees with aircraft observations and is also used in those reference LES simulations that do not use bin microphysics (Ackerman et al., 2009). Because both the LES and JPL-EDMF simulations reach a quasi-steady state within a few hours (not shown), all shown JPL-EDMF vertical profiles are averaged across the last simulation hour (i.e., between 5 and 6 hr) and where possible they are compared against the ensemble of LES simulation results for that time period.

Figure 4 compares the profiles of grid-mean moist conserved variables (θ_l and q_t) and their turbulent fluxes from the JPL-EDMF simulation to the reference LES results. The JPL-EDMF simulation well reproduces the well-mixed profiles of both moist conserved variables and a sharp inversion at a height of around 850 m (Figures 4a and 4b). The only significant difference between the two profiles is the inability of the JPL-EDMF simulation to represent the sharp gradient of the total water mixing ratio at the surface, which indicates overly strong vertical mixing in the surface layer. The profiles of moist conserved variables from JPL-EDMF and LES compare well because JPL-EDMF closely matches the LES profiles of vertical turbulent fluxes (Figures 4c and 4d). One of the most notable differences is that the magnitude of the negative θ_l flux at the cloud base is larger for the JPL-EDMF simulation. In the JPL-EDMF parameterization, the turbulent fluxes are a sum of three terms (eddy-diffusivity, updraft and environmental mass-fluxes; see Equation B4). The eddy-diffusivity part accounts for the majority of both turbulent fluxes throughout the PBL. The mass-flux contribution is significant just above the surface and below the cloud top, where its contribution to cloud top entrainment of dry and warm air into the PBL is substantial. The environmental mass-flux is small throughout the PBL. For the stratocumulus cases, in the models of Suselj et al. (2013) and Wu et al. (2020), a qualitatively similar contribution of the eddy-diffusivity
and updraft mass-fluxes to the total turbulent fluxes is found. In particular, in both of these models the turbulent
fluxes are dominated by the eddy-diffusivity component while the updraft mass-flux component is significant
near the surface and just below the inversion.

Figure 5 compares the profiles of cloud macro- and microphysical properties which include cloud cover, cloud
water mixing ratio, rain rate, rain water mixing ratio, and the rates of MP processes. All plotted profiles are split
into the environmental and the updraft contributions (corresponding to the two terms in Equation B2). Exceptions
are the rates of MP processes for which the LES reference is not available. For all evaluated cloud macro- and
microphysical properties, the JPL-EDMF simulation results agree reasonably well with the reference LES results.
Some differences between the two simulations are discussed below. While the JPL-EDMF cloud cover reaches
100% in the upper cloud layer, which is in agreement with the LES simulations, the JPL-EDMF cloud cover in
the lower part of the cloud layer is underestimated compared to the LES (Figure 5a). The contribution of updrafts
to both the cloud cover and cloud water mixing ratio is small. In the JPL-EDMF, the total fractional area of moist
updrafts reaches about 2.5% just above the cloud base and it stays nearly constant throughout the cloud layer.
Because the surface fractional area of all updrafts is 15.7% and the fractional area of each individual updraft is constant from the surface until its termination height (see Section 2.3), this result indicates that most of the updrafts terminate before condensing, but those that do condense tend to survive all the way to the cloud top. In the JPL-EDMF simulation, the rain rate in the non-convective environment reaches a maximum value of around...
10 W m\(^{-2}\) in the lower cloud layer in agreement with LES results and sharply decreases below the cloud base due to evaporation (Figures 5c and 5e). The JPL-EDMF rain water mixing ratio in the cloud layer is somewhat smaller than in the LES simulations (Figure 5d). This together with good agreement between the LES and JPL-EDMF rain rates indicates stronger average sedimentation velocity for the JPL-EDMF parameterization. At this point it is unclear whether the difference in the average sedimentation velocity is related to the differences of rain drop sedimentation velocity, the assumptions related to DSDs, or the subgrid-distribution of the rain water mixing ratio. In the JPL-EDMF, the formation of rain in the cloud layer is dominated by autoconversion, while the contribution from accretion is smaller (Figure 5e). The evaporation rate, which is already active in the cloud-free area of the lower cloud layer, is strongest just below cloud base and generally decreases toward the surface.

Figure 6 compares the time series of cloud liquid water path (LWP), surface precipitation rate and cloud top entrainment rate \(E\), which is computed following Ackerman et al. (2009) as a balance between the boundary layer growth and the large scale subsidence velocity at the top of the boundary layer:

\[
E = \frac{dh_t}{dt} - \bar{w}(h_t)
\]

where \(h_t\) is the top of the boundary layer, characterized by the 8 g kg\(^{-1}\) isosurface of total water mixing ratio. The JPL-EDMF LWP agrees well with the reference LES results and reaches a value of around 150 g m\(^{-2}\) at the start of the simulation, and then gradually decreases to around 100 g m\(^{-2}\) by the end of the simulation (Figure 6a). Consistent with the results on Figure 5b, the JPL-EDMF LWP is dominated by contribution from the non-convective environment. Both the JPL-EDMF and LES surface precipitation rates peak within the first two simulation hours and then decreases; by the end of the simulation they reach a value of less than 1 W m\(^{-2}\) (Figure 6b). In the JPL-EDMF, the peak in surface precipitation occurs earlier in the simulation and by the end of the simulation the surface precipitation is on the smaller range of the LES results. This discrepancy in timing of the maximum precipitation rate is likely due to a much longer turbulence spinup time for LES. While the early JPL-EDMF precipitation peak is dominated by rain in the non-convective environment, at the end of the simulation it is dominated by rain in convective updrafts. The cloud top entrainment rate \(E\) from the JPL-EDMF agrees reasonably well with LES although it is somewhat larger compared to the LES ensemble mean (Figure 6c). As noted by Ackerman et al. (2009), the entrainment rate is about twice the subsidence velocity at the top of the boundary layer which indicates gradual growth of the PBL.

Figure 7 shows the vertical profiles of TKE and of the terms in the TKE budget equation (Equation C5), which include advection (the sum of the resolved and subgrid contributions), shear production, buoyancy production and dissipation. Unlike for the mean thermodynamic fields, the JPL-EDMF budget terms of the second order
moments (such as TKE) are highly parameterized. Therefore, we expect that it will be harder for the JPL-EDMF to reproduce these LES moments. Nevertheless, the profile of TKE as well as all of its budget terms from the two simulations are reasonably close. Some differences between the two simulations are discussed below. The JPL-EDMF TKE is somewhat lower compared to that from the LES throughout most of the PBL (Figure 7a).

Figure 7. Dynamics and Chemistry of Marine Stratocumulus case. Profiles of turbulent kinetic energy (TKE) (a), and budget terms in the TKE equation (Equation C5), which include advection (the sum of the resolved and subgrid contributions; panel (b)), shear production (c), buoyancy production (d), and dissipation (e) averaged across the last simulation hour. Black lines and gray areas are for the ensemble mean and the interquartile range from the large-eddy-simulation (LES) results and the red lines for the Jet Propulsion Laboratory Eddy-Diffusivity/Mass-Flux (JPL-EDMF).
except for the shear production away from the PBL top, all terms in the TKE budget equation significantly contribute to the shape of the TKE profile. In the lower part of the PBL (below 400 m), the JPL-EDMF advection of TKE overestimates its LES counterpart, which is compensated by a somewhat overestimated magnitude of TKE dissipation rate (Figures 7b and 7e). It appears that this overestimation of the advection is due to the excessive subgrid contribution (not shown). In the upper cloud layer, the underestimated TKE seem to be related to the excessive dissipation rate for this layer. In the JPL-EDMF, the dissipation rate is parameterized as a function of TKE (Equation D4). For the near steady-state TKE profile in the RICO case, the dissipation rate largely adjusts to the sum of the other terms. However, the fact that the JPL-EDMF dissipation rate agrees with the LES at smaller TKE implies that the JPL-EDMF parameterized dissipation rate is excessive.

Figure 8a compares the profiles of the JPL-EDMF and UCLA-LES saturation excess variance ($\sigma^2_l$). We use the results from the UCLA-LES because the $\sigma^2_l$ is not an available output from Ackerman et al.’s (2009) LES database. Figure 8b shows the profiles of terms in the JPL-EDMF budget $\sigma^2_l$ equation (Equation 18). The $\sigma^2_l$ shows small values in the subcloud layer and lower cloud layer, increase through the cloud layer and peak just below the cloud top. Qualitatively, these profiles agree with the profiles of variances of moist thermodynamic variables for the stratocumulus case from Stevens et al. (2005). The $\sigma^2_l$ profile from the JPL-EDMF simulation well follows the LES data, although its values tend to be lower in the cloud layer, which is similar to the above-discussed result for TKE and is likely the result of the same underlying reasons. Figure 8b shows that for the DYCOMS case, the turbulent dissipation and production are approximately in balance which indicates that the diagnostic model of Bechtold et al. (1995) could reproduce these results. The contribution from advection and the MP source terms to the $\sigma^2_l$ are generally small.

### 3.2. RICO Case

The RICO case is based on composite measurements over the trades in the western Atlantic and the JPL-EDMF setup follows van Zanten et al. (2011), from which the LES reference data are taken. The constant sea surface temperature is prescribed and surface fluxes are computed interactively from the evolving near-surface thermodynamic properties. The profiles of geostrophic wind speed components, subsidence and radiative heating as well as the horizontal advection of heat and moisture are also prescribed and are constant with time. The simulation starts with a 740 m deep subcloud mixed boundary layer topped by a conditionally unstable cloud layer.

Figure 9 compares the JPL-EDMF and LES profiles of the moist conserved variables, $\theta_l$ and $q_c$, and their corresponding turbulent fluxes averaged across the last simulation hour. The JPL-EDMF reproduces all of these LES profiles reasonably well, while some differences between the two simulations are discussed next. The JPL-EDMF
$q_t$ in the lower part of the PBL (below 500 m) is somewhat too well-mixed, a feature found also for the DYCOMS case, and shows a kink around the cloud base (between the heights of around 500 and 700 m) which is not present in the LES results (Figure 9b). This kink of $q_t$ corresponds to the local minimum of the turbulent flux of $q_t$ around that height, which is a feature of the JPL-EDMF parameterization and has been discussed in Suselj et al. (2019a) in the context of a shallow non-precipitating convection case. In the JPL-EDMF simulation, the upper cloud layer (between the heights of around 1,000 and 2,500 m) is marginally moister and colder compared to the LES mean values and the cloud layer is somewhat deeper which can be seen in all of the four profiles. Figures 9c and 9d shows that in the subcloud layer (below around 500 m) all three contributions to the turbulent fluxes are significant. In the cloud layer (above the height of around 500 m) the updraft mass-flux component dominates turbulent fluxes of both moist conserved variables. The contributions of the three JPL-EDMF components to the grid-mean fluxes of moist conserved variables is similar to partitioning discussed in Suselj et al. (2019a, 2019b). The vertical profiles of turbulent fluxes from the JPL-EDMF parameterization are somewhat noisier compared to the LES mean profiles. This noise is due to representation of the JPL-EDMF convection with a relatively small number of stochastic updrafts and decreases with the increased number of updrafts (not shown; see Suselj et al. (2019a), for discussion).

Figure 9. Rain in Cumulus over the Ocean case. Profiles of mean liquid-water potential temperature (a), total-water mixing ratio (b) and their corresponding subgrid fluxes (c and d) averaged across the last simulation hour. The black line and the gray area represent the LES ensemble mean and interquartile range. The red line is for JPL-EDMF grid-mean values. In panels (c and d) the green line represents eddy-diffusivity, blue updraft and orange mass-flux contribution to the total fluxes.
Figure 10 shows the profiles of cloud macro- and microphysical properties averaged across the last simulation hour. The shown profiles include cloud cover, cloud water mixing ratio, rain rate and the rates of the three MP processes. The LES cloud cover profile shows two peaks, one just above the cloud base (at a height of around 600 m) and another one with smaller magnitude near cloud top at a height of around 2,000 m (Figure 10a). Cloud cover from the JPL-EDMF simulation reasonably well matches the one from the LES but it slightly underestimates LES counterpart in most of the cloud layer, where it is exclusively due to updraft contribution. In the upper cloud layer, both the JPL-EDMF updrafts and non-convective environment contribute to the total cloud cover. In particular, the cloud cover from the JPL-EDMF non-convective environment overestimates the LES values. This overestimation is likely due to the differences in the mean thermodynamic variables between the two simulations as shown in Figures 9a and 9b. The JPL-EDMF profile of cloud water mixing ratio, which is dominated by updraft contribution, reasonably well reproduces LES results.

Figure 10. Rain in Cumulus over the Ocean case. Profiles of cloud cover (a), liquid water (b), rain rate (c), and (d) tendency of total water from to the three microphysical processes averaged across the last simulation hour. In panels (a–c), the black lines and gray areas are for the ensemble mean and the interquartile range from large-eddy-simulation (LES) results, the green lines for the environmental contributions and the blue lines for the contribution from updrafts (first and the second terms on the right hand side of Equation B2, respectively), and the red lines represent the sum of the two contributions. In panel (d), the tendency is split in its contributions from autoconversion (green), accretion (orange), and evaporation (cyan), and the red line represents the sum of the three terms.
The LES rain rate profile on Figure 10c is associated with significant uncertainty and thus only loosely constrains the JPL-EDMF simulation results. The ensemble mean LES rain rate peaks in the middle of the cloud layer and then decreases toward the surface which supports the hypothesis that significant rain evaporation occurs already within the cloud layer. Compared to the LES ensemble mean results, the JPL-EDMF rain rate in the upper cloud layer is underestimated while it is well represented for the lower cloud and subcloud layers. Part of the reason for the upper cloud layer underestimation is that the profile of JPL-EDMF moist updraft area underestimates its LES counterparts (not shown) in a similar manner to results shown in Suselj et al. (2019b). Throughout the JPL-EDMF cloud layer a significant fraction of precipitation falls within the non-convective environment. This precipitation is not produced in that part of the domain, instead it is detrained from convective updrafts. Figure 5d shows that unlike for the DYCOMS case, the JPL-EDMF MP source terms are dominated by accretion which peaks in the middle of the cloud layer and evaporation, which is active throughout the cloud layer and peaks in the lower cloud layer. Even though the precipitation rate in our previous version of the JPL-EDMF model described in Suselj et al. (2019b) better agrees with the mean LES results, we believe that the current version of the JPL-EDMF model is based on a more physically sound representation of cloud processes.

Figure 11 shows time series of cloud base and cloud top and cumulative rain rates at the surface, around cloud base (at 500 m) and in the upper cloud layer (at 1,500 m). The JPL-EDMF results well represent deepening of the cloud layer and lowering of the cloud base throughout the simulation (Figure 11a). Cloud extent from the two simulations differ significantly during the first 2 hr when the LES cloud top shows abrupt jumps which are not seen in the JPL-EDMF simulation. As for the DYCOMS case, this discrepancy is likely due to different turbulence spinup time for the two types of the models. At the cloud base and the surface, the JPL-EDMF cumulative rain rate agrees well with the LES mean values (Figures 11b and 11c). Consistent with the results on Figure 10c, the JPL-EDMF rain rate in the upper cloud layer is underestimated compared to the ensemble mean LES values (Figure 11d).

Figure 12 shows the profiles of TKE and the budget terms in its prognostic equation (Equation C5). The LES profile of TKE is reasonably well reproduced by the JPL-EDMF parameterization. Except for the lowest 100 m and in the upper cloud layer, the TKE in LES and JPL-EDMF is dominated by the balance of buoyancy production and turbulent dissipation (Figures 12b–12e). The JPL-EDMF buoyancy production term well represents that from the LES except above the height of around 2,000 m, where it becomes negative, a feature not found for the LES simulations. In the JPL-EDMF simulation the buoyancy production term is computed as a sum of the three EDMF terms (Equation B4) and the negative values in the upper cloud layer are dominated by contribution from convective updrafts overshooting neutral buoyancy (not shown).
Figure 12. Rain in Cumulus over the Ocean case. Profiles of turbulent kinetic energy (TKE) (a), and budget terms in the TKE equation (Equation C5), which include advection (the sum of the resolved and subgrid contributions; panel (b)), shear production (c), buoyancy production (d), and dissipation (e) averaged across the last simulation hour. Black lines and gray areas are for the ensemble mean and the interquartile range from the large-eddy-simulation (LES) results and the red lines for the Jet Propulsion Laboratory Eddy-Diffusivity/Mass-Flux (JPL-EDMF).

Figure 13a compares the profiles of the JPL-EDMF and UCLA-LES saturation excess variance ($\sigma_A^2$), and Figure 13b the profiles of terms in the JPL-EDMF budget $\sigma_A^2$ equation (Equation 18). The $\sigma_A^2$ from LES shows two peaks, one near the cloud base and one near the cloud top, both of which are reasonably well represented by the JPL-EDMF parameterization. Throughout the whole cloud and boundary layer, the $\sigma_A^2$ from the JPL-EDMF
somewhat underestimates that of the LES, a feature found also for the DYCOMS case. Figure 13b shows that in addition to the turbulent production and dissipation, advection plays an important role in the $\sigma_t^2$ budget. This is particularly noticeable for the upper cloud layer.

3.3. Sensitivity of the Results to the Vertical Resolution of the Model

For the DYCOMS and the RICO cases, Figure 14 shows the sensitivity of the JPL-EDMF simulated maximum cloud cover and surface rain rate to the vertical grid spacing $\Delta z$ of the SCM model in which the JPL-EDMF is implemented. We choose to show the selected two simulation results as they tend to show the highest sensitivity to the vertical grid spacing.

For the DYCOMS case, both the cloud cover and rain rate are fairly constant as long as the grid spacing is less than about 40 m, and then they sharply decrease with the increase of the grid spacing above that value. Because the DYCOMS case is characterized with a strong inversion at the top of the PBL, it is unsurprising that relatively high vertical resolution of the SCM is required to properly simulate the thermodynamic properties in the upper part of the PBL, as well as the associated cloud and rain processes that occur in that part of the atmosphere. For the RICO case the maximum cloud cover does not seem to be overly sensitive to $\Delta z$ only as long as it is below 40 m. With increase of $\Delta z$ above 40 m, the surface precipitation sharply decrease. This result indicates that even for the cumulus dominated boundary layers which are not associated with strong inversion relatively fine vertical resolution of the host model is needed to properly simulate rain processes.

Figure 14. Jet Propulsion Laboratory Eddy-Diffusivity/Mass-Flux maximum cloud cover (a) and surface rain rate (b) for the RICO and Dynamics and Chemistry of Marine Stratocumulus (DYCOMS) cases averaged over the last simulation hour, as a function of vertical grid spacing $\Delta z$ of the single-column-model. Blue line for the DYCOMS and red line for the RICO. For ease of presentation, the RICO cloud cover in panel (a) and the DYCOMS surface rain rate in panel (b) are multiplied by 10.
4. Summary and Conclusions

In this work, we describe a new version of the JPL-EDMF parameterization which represents subgrid scale dynamical, cloud macro- and microphysical processes in a unified and physically consistent manner and accounts for tight coupling between them. The new parameterization combines two approaches, the EDMF framework and an assumed shape of subgrid JPDF, which together form the basis for evaluation of all parameterized process rates. As far as we are aware, this is the first attempt to couple cloud macro- and microphysical processes in the EDMF framework in a physically consistent manner.

The underlying idea of the EDMF parameterization is to divide the domain of the host model into two types of elements: multiple surface-driven convective updrafts and the non-convective environment, each associated with a different shape of assumed JPDF for the model variables. Within a horizontal plane of individual updrafts this JPDF is taken to be uniform. The convective part of the domain is thus represented by the sum of uniform model variable distributions where each such distribution corresponds to a single updraft. For the non-convective environment, the variables within a horizontal plane are assumed to follow a joint normal distribution. The exception is the rain-water mixing ratio, which is assumed to follow a delta-lognormal distribution as a mixture of delta and lognormal distributions. These two mixture components represent the rain-free and rain-dominated parts of the environment, respectively. For each of the EDMF element types, the mean rates of cloud macro- and microphysical processes are computed by analytical integration of the local rates over the relevant marginal JPDFs. The subgrid dynamical processes are also represented consistently with these JPDFs. The local rates of MP processes for both the non-convective environment and updrafts are modeled by a simplified Khairoutdinov and Kogan (2000) parameterization with Abel and Boutle (2012) rain drop number concentration. For the updrafts, the tight coupling of dynamical and cloud processes is modeled by simultaneously solving equations for updraft thermodynamic and cloud macro- and microphysical processes. To do so, we use an iterative numerical method. In the JPL-EDMF parameterization, prognostic equations for TKE and saturation excess variance are solved. These equations both encapsulate strong coupling of the cloud and subgrid dynamical processes for the non-convective environment.

We use the JPL-EDMF parameterization implemented in the SCM to simulate two representative cases of subtropical marine precipitating stratocumulus and cumulus convection: the DYCOMS case (Ackerman et al., 2009) and the RICO case (van Zanten et al., 2011), respectively. The JPL-EDMF simulation results are compared against the reference LES ensemble. For the DYCOMS case, the JPL-EDMF well represents the well-mixed characteristics of boundary layer (Figure 4) and its quasi-steady behavior. The cloud and rain properties, such as cloud cover, cloud liquid and rain water profiles, are also reasonably well represented (Figure 5). As this case is characterized by relatively shallow clouds, the precipitation formation in the JPL-EDMF parameterization is dominated by autoconversion process while contribution from accretion is minor. Surface precipitation is only a small fraction of its cloud base value due to strong subcloud layer evaporation. Even though the total cloud liquid water is dominated by non-convective environment contribution, surface precipitation occurs primarily in the convective updrafts (Figure 6), which points to the different rates of precipitation evaporation for the updrafts and for the non-convective environment. The JPL-EDMF parameterization well represents the evolution of moist convective layers for the RICO case (Figure 11a) as well as the profiles of thermodynamic variables and cloud properties (Figures 9 and 10). For the RICO case, the convective cloud layer is much deeper than in the DYCOMS case, and therefore the rain formation is dominated by accretion rather than autoconversion. In the JPL-EDMF, the rain formation region is shallower than in the LES, but the rain rate profile from the JPL-EDMF generally falls within the IQR of that from the LES. For both the DYCOMS and the RICO cases, we show that JPL-EDMF prognosed second order moments, which include TKE and $\sigma^2$, are in reasonably good agreement with the LES results. In Section 3.3 we show that the vertical resolution of the SCM has to be relatively high for a reasonable representation of cloud and rain processes.

The question remains how well the JPL-EDMF parameterization represents the wider variety of subtropical moist convective cases and how sensitive the simulation results are to parameterization of different processes. This is answered in the second part of this study (Smalley et al., 2022) in which we evaluate the performance of the JPL-EDMF parameterization for several hundred observation-based scenarios covering the continuous transition from subtropical stratocumulus to cumulus convection derived from global reanalysis in the context of Smalley et al.’s (2019) SCM framework. An important goal of this second investigation is to identify physical processes that are responsible for the greatest amount of uncertainty in the JPL-EDMF results with the aim of guiding future development of the JPL-EDMF parameterization.
Appendix A: List of Constants

Table A1 lists all constants used in the JPL-EDMF model.

| Constant | Short description |
|----------|--------------------|
|          | Properties of rain drops |
| $x_1 = 0.22, x_2 = 1.9$ | DSD for convective updrafts (Equation 1) |
| $x_1 = 0.22, x_3 = 2.1$ | DSD for non-convective environment (Equation 1) |
| $a_r = 3$, $b_r = 3$ | Mass-diameter relationship |
| $c_{r,1} = 4.583 \text{ s}^{-1}$, $c_{r,2} = -7.527 \cdot 10^3 \text{ m}^{-1} \text{s}^{-1}$, $c_{r,3} = 4.022 \cdot 10^3 \text{ m}^{-2} \text{s}^{-1}$ | Mass-averaged terminal velocity relationship (Equation 3) |
|          | Microphysical source terms (constants assume SI units, except when described otherwise) |
| $a_{au} = \alpha_{au} N_c - \beta_{au}$ | Autoconversion rate ($N_c$ is prescribed cloud-droplet concentration in cm$^{-3}$; Equation 10) |
| $c_{au} = 2.47$ | Accretion rate (Equation 11) |
| $\alpha_{ac} = 67$, $b_{ac} = c_{ac} = 1.15$ | Evaporation rate ($e_s$ represents saturation water pressure; Equations 12–15) |
| $\beta_e = 2$, $F = 0.78$ | Correlation between saturation excess and rain water mixing ratio |
| $G = 10^{-7} (2.2 T / e_s(T) + 220 / T) ^{-1}$ | Relation between rain water mixing ration and its variance (Equation 20) |
|          | Rain dominated area fraction (Equation 41) |
| $a_{w} = 0.57$, $b_{w} = 0.5$ | Vertical velocity (Equation 46) |
| $c_{v} = 0.16$ | TKE (Equation D4) |
| $c_{ss} = 1.2$ | (Equation D5) |
|          | Diffusive and dissipation length-scale formulation |
| $N_0 = 6 \cdot 10^{-3} \text{ s}^{-1}$, $\alpha_t = 0.8$, $p_t = 1$ | Stability function for diffusive time scale (Equation D2) |
| $a_{diff} = 3$ | Diffusive length scale (Equation D3) |
| $a_{diss} = 1$ | Dissipative length scale (Equation D7) |
|          | Turbulent dissipation |
| $c_{t} = 0.16$ | $\sigma_t^2$ (Equation D5) |
| $c_{ss} = 1.2$ | |

Appendix B: Statistical Moments of Subgrid Variables in the JPL-EDMF Parameterization

At the heart of the Jet Propulsion Laboratory Eddy-Diffusivity/Mass-Flux (JPL-EDMF) parameterization is separation of host model domain into two subgrid element types, multiple convective updrafts and the non-convective environment. Here, we describe expressions for zeroth, first and second moments of subgrid distributions. By definition, the sum of horizontal fractional areas of all subgrid elements equals unity (Suselj et al., 2019a):

$$a_e + \sum_{i=1}^{N} a_i = 1$$  (B1)
where \( a \) represent fractional area and indices \( e \) and \( n \) are for the non-convective environment and for the updrafts, respectively.

The grid mean value of any variable \( \varphi \) can be computed as an area-weighted average of the corresponding values from each of the subgrid elements:

\[
\overline{\varphi} = a_n \varphi_n + \sum_{n=1}^{N} a_n \varphi_n
\]  

(B2)

As in the rest of this work, the overbar and the tilde over the symbol denote the subgrid and environmental mean of the corresponding variables, respectively.

Following an assumption of uniform distribution of model variables within individual updrafts, the covariance between two variables \( \varphi \) and \( \psi \) can be without any other approximations written as (e.g., Chinita et al., 2018; Suselj et al., 2019a, 2019b):

\[
\overline{\varphi'\psi'} = a_n \overline{\varphi'} \overline{\psi'} + a_e (\overline{\varphi} - \overline{\varphi}) (\overline{\psi} - \overline{\psi}) + \sum_{n=1}^{N} a_n (\varphi_n - \overline{\varphi}) (\psi_n - \overline{\psi})
\]  

(B3)

where \( \overline{\varphi'\psi'} \) represents the covariance of two variables in the non-convective environment.

Computation of the subgrid vertical fluxes is an important aspect of EDMF model and their expression can be derived from Equation B3:

\[
\overline{u'\varphi'} = a_n \overline{u'\varphi'} + a_e (\overline{u} - \overline{\varphi}) (\overline{\varphi} - \overline{\varphi}) + \sum_{n=1}^{N} a_n (u_n - \overline{\varphi}) (\varphi_n - \overline{\varphi})
\]  

(B4)

where \( u \) is the vertical velocity and \( \varphi \) any of the EDMF prognostic variables. As described in Appendix D, the term \( \overline{u'\varphi'} \) is parameterized with the eddy-diffusivity model and therefore we refer to it as the eddy-diffusivity component (Eddy-diff). The other terms in Equation B4 represent environmental mass-flux (Environment MF) and updraft mass-flux (Updraft MF) contributions to the total turbulent flux (Suselj et al., 2019a). The mass-flux terms are evaluated from the grid-mean and updraft properties once the updraft properties are computed with the mass-flux model (Section 2.3).

Appendix C: Single-Column-Model Equations

The single-column-model (SCM) prognostic equations for grid-mean values of the moist conserved thermodynamic variables (liquid water potential temperature, \( \theta_l \) and total water mixing ratio, \( q_t \)), horizontal wind speed components (\( u \) and \( v \)), and turbulent kinetic energy (TKE) (\( \varepsilon \)) can be written as:

\[
\frac{\partial \theta_l}{\partial t} = -\overline{u} \cdot \nabla \theta_l - \frac{1}{\rho} \frac{\partial}{\partial z} \left( \rho u'\theta'_l \right) + \frac{\partial \tilde{\theta}_l}{\partial t} \bigg|_{\text{micro}} + \frac{\partial \tilde{\theta}_l}{\partial t} \bigg|_{\text{rad}}
\]  

(C1)

\[
\frac{\partial q_t}{\partial t} = -\overline{u} \cdot \nabla q_t - \frac{1}{\rho} \frac{\partial}{\partial z} \left( \rho u'q'_t \right) + \frac{\partial \tilde{q}_t}{\partial t} \bigg|_{\text{micro}}
\]  

(C2)

\[
\frac{\partial u}{\partial t} = -\overline{u} \cdot \nabla \overline{u} - \frac{1}{\rho} \frac{\partial}{\partial z} \left( \rho u'u' \right) - f (\overline{v} - \overline{v}_s)
\]  

(C3)

\[
\frac{\partial v}{\partial t} = -\overline{u} \cdot \nabla \overline{v} - \frac{1}{\rho} \frac{\partial}{\partial z} \left( \rho u'v' \right) + f (\overline{u} - \overline{u}_s)
\]  

(C4)

\[
\frac{\partial \varepsilon}{\partial t} = -\overline{u} \cdot \nabla \varepsilon - \frac{1}{\rho} \frac{\partial}{\partial z} \left( \rho u' \varepsilon' \right) - \overline{u'} u' \frac{\partial \tilde{\varepsilon}}{\partial z} - \frac{\partial \tilde{\varepsilon}}{\partial \tilde{\varepsilon}} - \frac{\partial \varepsilon}{\partial \tilde{\varepsilon}}
\]  

(C5)
where $\mathbf{u} = \{u, v, w\}$ is the three-dimensional velocity vector, $z$ is the vertical direction, $\rho$ is density of the dry air, $u_q$ and $v_q$ are the horizontal geostrophic wind components and $f$ is the Coriolis parameter. In Equations C1–C5, overbar denotes the grid-mean average, prime deviation from it and subscripts micro and rad indicate the source of tendencies (from cloud microphysical [MP] and radiative processes, respectively). The thermodynamic variables $\theta_i$ and $q_i$ are conserved with respect to condensation, therefore cloud microphysical processes do not contribute to their source terms. In Equations C1–C4, the horizontal components of the subgrid-turbulent fluxes are neglected as is typical for atmospheric models of low horizontal resolution. The terms in Equation C5 are labeled because we refer to them in the figures and represent tendency (tend), resolved and subgrid-scale advection (resolved adv and subgrid adv, respectively), shear production (shear prod), buoyancy production or dissipation (buoy prod) and turbulent dissipation of TKE (diss).

Additionally, the SCM solves the prognostic equation for saturation excess variance ($\sigma_s^2$). Because this equation might be less well known, we briefly outline its derivation and describe necessary parameterizations. The starting point is the definition of saturation excess $s$ (Equation 7), which is linearized around the grid-mean value of moist conserved thermodynamic variables following Bechtold et al. (1995):

$$s = a_s q_i' - b_s \theta_i' + c_s$$  \hspace{1cm} (C6)

where saturation parameters $a_s = \left(1 + \frac{\rho_l}{\rho_v} \frac{\partial \theta_l}{\partial \theta_i} \right)^{-1}$ and $b_s = a_s \Pi \frac{\partial \theta_l}{\partial \theta_i} \frac{\partial q}{\partial \theta_i}$ are assumed to be only weakly dependent on liquid water temperature $T_i$ (which is defined as $T_i \equiv \Pi \theta$), and are evaluated at its grid-mean value, and $c_s = a_s \left(\bar{q}_i - q_i \left(\bar{T}_i, \rho \right)\right)$ represents the saturation excess with respect to the grid-mean moist conserved thermodynamic variables. The $\sigma_s^2$, which is defined as $\sigma_s^2 \equiv \bar{s}^2$ can be derived from Equation C6 as:

$$\sigma_s^2 = a_s^2 q_i'^2 + b_s^2 \theta_i'^2 - 2 a_s b_s q_i' \theta_i'$$  \hspace{1cm} (C7)

We combine Equation C7 with prognostic equations for second order moments of moist conserved variables. The latter ones are obtained as the difference between the full and Reynolds-averaged equations multiplied with the deviations (e.g., Pielke, 2002)

$$\frac{\partial \bar{q}_i'}{\partial t} = \bar{u}_i \cdot \nabla \bar{q}_i' - \frac{1}{\rho} \nabla \left( \rho \bar{u}' \bar{q}_i' \right) - 2 \bar{q}_i' \nabla \bar{q}_i' - 2 \bar{q}_i' \bar{S}' - \varepsilon_{q_i q_i}$$  \hspace{1cm} (C8)

$$\frac{\partial \bar{\theta}_i'}{\partial t} = -\bar{u}_i \cdot \nabla \bar{\theta}_i' - \frac{1}{\rho} \nabla \left( \rho \bar{u}' \bar{\theta}_i' \right) - 2 \bar{\theta}_i' \nabla \bar{\theta}_i' - 2 \bar{\theta}_i' \bar{S}' - \varepsilon_{\theta_i \theta_i}$$  \hspace{1cm} (C9)

$$\frac{\partial \bar{q}_i' \bar{\theta}_i'}{\partial t} = -\bar{u}_i \nabla \left( \bar{q}_i' \bar{\theta}_i' \right) - \frac{1}{\rho} \nabla \left( \rho \bar{u}' \bar{q}_i' \bar{\theta}_i' \right) - \bar{q}_i' \bar{u}' \nabla \bar{q}_i' - \bar{\theta}_i' \bar{u}' \nabla \bar{\theta}_i' - \bar{\theta}' \bar{S}' + \frac{L}{\Pi c_p} q_i' \bar{S}' - \varepsilon_{q_i \theta_i}$$  \hspace{1cm} (C10)

to obtain the following prognostic equation for $\sigma_s^2$:

$$\frac{\partial \bar{s}^2}{\partial t} = -\bar{u} \cdot \nabla \bar{s}^2 - \frac{\partial}{\partial z} \bar{u}' s'^2 + 2 \left( a_s u_i' q_i' - b_s u_i' \theta_i' \right) \left( b_s \frac{\partial \bar{q}_i'}{\partial z} - a_s \frac{\partial \bar{\theta}_i'}{\partial z} \right) - 2 \left( a_s + b_s \frac{L}{c_p \Pi} \right) \bar{s}^2 \varepsilon_{q_i q_i}$$  \hspace{1cm} (C11)

where the physical interpretation of terms in Equation C11 is given in Section 2.1.

To solve for prognostic $\sigma_s^2$ equation we need to parameterize the following terms. The covariance $\bar{s}^2 \bar{S}'$ is first computed separately from each of the EDMF components and the grid-mean value follows Equation B3:

$$\bar{s}^2 \bar{S}' = a_s \left( \bar{s}^2 - \bar{S} \right) + a_s \left( \bar{s} - \bar{S} \right) \left( \bar{S} - \bar{S} \right) + \sum_{n=1}^N g_n \left( s_n - \bar{S} \right) \left( S_n - \bar{S} \right)$$  \hspace{1cm} (C12)

where the grid-mean values $\bar{s}$ and $\bar{S}$ are obtained as the area-weighted means from all components (Equation B2) and we used the expression for decomposition of central moments to derive the first term on the right hand side.
of Equation C12. The value of $\tilde{s}\tilde{S}$ is evaluated by analytical integration of the product of saturation excess and MP source term over the assumed JPDF $\bar{p}(s,q)$ as explained in Section 2.2. The term representing the turbulent transport of $\sigma^2$ is modeled with down-gradient approximation: $u^2\bar{s}'^2 = -K \frac{\partial^2}{\partial z^2}$, where $K$ is the eddy-diffusivity coefficient and the dissipation of $\sigma^2$ is modeled with the usual approach (Equation D5).

With the help of Equation B3, we can express the value of $\sigma^2$, from $\sigma^2_{r}$, which characterizes an important statistical moment of $\bar{p}(s,q)$. For numerical reasons, we set the minimum values of $\bar{s}'^2$ and $\tilde{s}^2$ to $10^{-8}$.

The SCM solves the rain water mixing ratio ($q_r$) equation separately for the non-convective environment and updrafts, as detailed in Section 2. Here we give the basic equation for $q_r$, which is (e.g., Grabowski, 1998):

$$\frac{\partial q_r}{\partial t} + \frac{1}{\rho} \nabla \left( \rho (\bar{u} + \bar{u}_r) q_r \right) = S$$

(C13)

where $\bar{u}_r$ is the velocity of the rain water with respect to the surrounding air. We assume that vertical component as explained in Section 10.1029/2021MS002736.

Appendix D: Parameterization of Turbulent Fluxes in the Non-Convective Environment and Dissipation Rates of TKE and Saturation Excess Variance

In the Eddy-Diffusivity/Mass-Flux (EDMF) framework, the turbulent fluxes in the non-convective environment are approximated with the downgradient eddy-diffusivity approach $\overline{w'q'} \approx -K \frac{\partial^2}{\partial z^2}$ where $K = l \sqrt{e_c}$ is the eddy diffusivity coefficient and is a function of turbulent mixing length scale ($l$) and turbulent kinetic energy (TKE) in the non-convective environment ($e_c$). The latter one is diagnosed from the prognosed values of grid-mean TKE and updraft properties following the EDMF decomposition of second order moments (Equation B3).

One key part of the parameterization is the definition of the mixing length scale $l$, which here is a combination of the surface length scale (which itself is a product of von Karman constant $k = 0.4$ and height from the surface $z$) and the free tropospheric length scale modeled as $\tau \sqrt{e_c}$ (Teixeira & Cheinet, 2004):

$$l = \tau \sqrt{e_c} + \left( k z - \tau \sqrt{e_c} \right) e^{z_{sp}/z_{sp}}$$

(D1)

where $z_{sp} = 0.1 z_{sp}$ is the depth of the surface layer and $z_{sp}$ is the depth of the planetary boundary layer. The formulation in Equation D1 has also been proposed by Teixeira and Cheinet (2004).

The time scale $\tau$ is a combination of the time scale of the neutral atmosphere ($\tau_0$) with stability correction when the modified Brunt-Vaisala frequency ($N_f$) exceeds a threshold value of $N_0$ so that $\tau$ decreases with further increase of $N_f$:

$$\tau = \begin{cases} \tau_0 & \text{if } N_f < N_0 \\ \tau_0 \exp \left[ -\left( \frac{N_f - N_0}{N_0 a_t} \right)^{p_r} \right] & \text{if } N_f \geq N_0 \end{cases}$$

(D2)

where $N_f \equiv g/\theta_0 \sqrt{\partial \theta_0/\partial z}$ is equivalent to Brunt-Vaisala frequency except that in its definition the potential temperature is replaced by the liquid-water potential temperature $\theta_l \equiv \theta_i (1 + 0.61 q_r - q_s - q_r)$ as liquid-water potential temperature is adiabatically conserved variable for moist reversible processes (Grenier & Bretherton, 2001). The constants $N_0$, $a_t$, and $p_r$ are given in Table A1. Decreasing time scale $\tau$, and thus length scale $l$, with stability has been inspired by Deardorff (1976).

The time scale for the neutral atmosphere ($\tau_0$) is defined as the ratio of the depth of the dry layer ($z_{dry}$) and near-surface velocity scale, which is a combination of the Deardorff surface convective velocity scale ($w_d$) and surface friction velocity ($u_f$):

$$\tau_0 = \frac{1}{a_{dry}} \sqrt{\frac{z_{dry}}{w_d^2 + u_f^2}}$$

(D3)
with constant $a_{diff}$ given in Table A1.

The dissipation rate of TKE and $\sigma_z^2$ in Equations C5 and 18 are parameterized following traditional approach (e.g., Mellor & Yamada, 1974) which is also a simplified version of parameterization described by Golaz et al. (2002a):

$$\epsilon_z = \frac{c_ic_\varepsilon}{\tau_z} \quad (D4)$$

$$\epsilon_{\sigma_z^2} = \frac{c_s\sigma_z^2}{\tau_z} \quad (D5)$$

where the constants $c_i$ and $c_s$ are given in Table A1, and $\tau_z = \sqrt{c/l_z}$ is dissipation time scale where $l_z$ is the dissipation length scale and is defined as a combination of surface and free tropospheric length scale in a similar manner to mixing length scale (Equation D1):

$$l_z = l_{z,0} + (kz - l_{z,0})e^{-k/z_s} \quad (D6)$$

where the free tropospheric length scale follows definition from Mellor and Yamada (1982):

$$l_{z,0} = \frac{0.12}{a_{diss}} \int_0^\infty z\sqrt{edz} \int_0^\infty \sqrt{edz} \quad (D7)$$

where the constant $a_{diss}$ is also given in Table A1.

**Appendix E: Derivation of Equation Governing Updraft Properties**

Because the Eddy-Diffusivity/Mass-Flux (EDMF) model includes new equation for updraft rain water mixing ratio, $q_r$, we derive generic equation governing any kinematic or thermodynamic updraft variable following Siebesma (1998). Equation for $q_r$ is unique as rain-drops are associated with their own velocity relative to the mean flow in which they are embedded. The starting point is a general conservation law for kinematic or thermodynamic variable $\phi$:

$$\rho \frac{d\phi}{dt} + \nabla \left[ \rho \phi (\vec{u} + \vec{u}_r) \right] = \rho S_\phi \quad (E1)$$

where $\phi = \{ \theta, q_r, q_e, u, v, w \}$, $\vec{u}$ and $\vec{u}_r$ are velocity of the air flow, and relative velocity associated with the variable $\phi$ (and is non-zero only for $\phi = q_r$), and $S_\phi$ is the sum of all source terms corresponding to variable $\phi$.

First, we integrate Equation E1 over a volume $V$, which defines the nth updraft between the fixed heights $z$ and $z + \delta z$. With the help of the Leibnitz rule and the divergence theorem, we can rewrite the integrated updraft equation as:

$$\frac{d}{dt} \iiint_{V_n} \rho \phi \, dV + \iint_{\partial V_n} \rho \phi (\vec{u} + \vec{u}_r - \vec{v}_h) \cdot \vec{n} \, dA = \iiint_{V_n} \rho S_\phi \, dV \quad (E2)$$

where $\partial V_n$ represents the surface of the boundary that encompass the updraft between the heights $z$ and $z + \delta z$, $\vec{v}_h$ and $\vec{n}$ are the velocity vector of that boundary, and a unit vector perpendicular to that boundary, respectively. To derive Equation E2 we used the approximation $\partial \rho / \partial t = 0$.

We approximate each term in Equation E2 and assume that updraft can be represented with a vertically oriented cylinder with base area $B$, volume $V_n = B \times \delta z$, and side area $A_z$:

$$\frac{d}{dt} \iiint_{V_n} \rho \phi \, dV = \frac{\partial}{\partial t} (V_n \phi_n) \quad (E3)$$

$$\iint_{\partial V_n} \rho \phi (\vec{u} + \vec{u}_r - \vec{v}_h) \cdot \vec{n} \, dA = \rho B_n (w + w_{z,0}) \phi_n \frac{\delta z + z}{z} + \iint_{A_z} \rho \phi (\vec{u} + \vec{u}_r - \vec{v}_h) \cdot \vec{n} \, dA \quad (E4)$$
\[ \iiint_{V_n} \rho S_{\varphi} \, dV = V_n \langle \rho S_{\varphi} \rangle \]  

(E5)

where \( \langle \varphi \rangle \) denotes spatially averaged value of variable \( \varphi \) over the volume \( V_n \). \( w_{v_r} \) and \( w_{v_z} \) are the vertical velocity components of vectors \( \overline{u} \) and \( \overline{v}_r \), respectively. Consistent with the EDMF approximation we assume that the updraft properties are horizontally homogeneous and that the integral is performed over the fixed heights \( z \) and \( z + \delta z \) (therefore the location of the base area of updraft cylinder stays constant).

Next, we define entrainment and detrainment rates as the integrals of inflow and outflow of the mass through the side area \( A_n \) in agreement with Siebesma (1998):

\[ E_n \equiv -\frac{1}{V_0 \rho} \int_{A_n(\overline{u} - \overline{v}_b) \, \delta z = 0} \rho (\overline{u} - \overline{v}_b) \cdot \overline{n} \, dA \]  

(E6)

and

\[ D_n \equiv \frac{1}{V_0 \rho} \int_{A_n(\overline{u} - \overline{v}_b) \, \delta z = 0} \rho (\overline{u} - \overline{v}_b) \cdot \overline{n} \, dA \]  

(E7)

An inflow or outflow of mass through the side area \( A_n \) with the velocity \( \overline{u}_r \) is defined as:

\[ \dot{\overline{D}}_n \equiv \frac{1}{V_0 \rho} \int \rho \overline{u}_r \cdot \overline{n} \, dA \]  

(E8)

In Equation E6–E8, \( V_0 = \overline{B}_i \delta z \) is the volume of the air in which the updraft is embedded and \( B_i \) is the horizontal area of that volume. In our model, the value of \( \dot{D}_n \) is non-zero only for rain water, as this is the only variable associated with nonzero \( \overline{u}_r \).

We define the fractional updraft area as: \( a_n \equiv \overline{B}/B_{v_r} \). Using upstream approximation and assuming that Equation E8 primarily represents detrainment of rain water from the updraft, the second term on the r.h.s. in Equation E4 is expressed as:

\[ \int_{A_n} \rho \varphi (\overline{u} + \overline{u}_r - \overline{v}_b) \cdot \overline{n} \, dA = B_i \delta z \left( \overline{D}_n + \dot{\overline{D}}_n \right) \varphi_n - E_n \varphi_e \]  

(E9)

where \( \varphi_e \) represents the \( \varphi \) in the environment in which the updraft is embedded.

Expressions in Equation E3–E9 are used in Equation E2, which is then divided by \( V_0 \) and we let \( \delta z \to 0 \) to obtain the final result:

\[ \frac{1}{\rho} \frac{\partial}{\partial t} (\rho a_n \varphi_n) + \frac{1}{\rho} \frac{\partial}{\partial z} \left[ \rho a_n (w_v + w_{v-r}) \varphi_n \right] + (\dot{D}_n + \dot{\overline{D}}_n) \varphi_n - E_n \varphi_e = a_n S_{\varphi, x} \]  

(E10)

The mass-conservation equation can be recovered from Equation E10 by setting \( \varphi = 1 \), and \( S_{\varphi, r}, u_{v,r} \) and \( \dot{\overline{D}}_n \) to zero:

\[ \frac{1}{\rho} \frac{\partial}{\partial t} (\rho a_n) + \frac{1}{\rho} \frac{\partial}{\partial z} (\rho a_n w_v) + D_n - E_n = 0 \]  

(E11)

Equations E10 and E11 are combined to eliminate the detrainment rate \( D_n \) and the final result is:

\[ \frac{\partial \varphi_n}{\partial t} + w_v \frac{\partial \varphi_n}{\partial z} + \frac{1}{\rho a_n} \frac{\partial}{\partial z} \left[ \rho a_n (w_{v-r} \varphi_n) + w_{v-r} (\varphi_n - \varphi_e) + w_v \varphi_e \right] = S_n \]  

(E12)

where we defined \( \epsilon_e \equiv E_n/(w_v a_n) \) and \( \delta_n \equiv \dot{\overline{D}}_n/(w_v a_n) \).

Next, we neglect the tendency of \( \varphi_{er} \), use the assumption of height constant updraft area \( a_n \), and following Kurowski et al. (2019) we approximate \( \varphi_e \) with \( \overline{\varphi} \). The final updraft equation is:
\[
\nu_a \frac{\partial \varphi_a}{\partial z} + \frac{1}{\rho} \frac{\partial}{\partial z} (\rho \mathbf{u}_a \varphi_a) + \mathbf{u}_a \mathbf{e}_n (\varphi_a - \varphi) + \omega_a \varphi_a \hat{\delta}_a = S_a
\]  
(E13)

If the property \( \varphi_a \) is transported with the flow of the surrounding air (i.e., \( \mathbf{u}_a \) and \( \hat{\delta}_a = 0 \)), Equation E13 simplifies to:

\[
\frac{\partial \varphi_a}{\partial z} = \varepsilon_a (\varphi - \varphi_a) + \frac{S_a}{\nu_a}
\]  
(E14)

**Appendix F: Useful Formulations**

In this appendix, we provide some useful formulations for computation of micro- and macro-physical rates for the non-convective environment. First, we define the formulations of probability-density-function (PDF) and two-dimensional Joint Propulsion Laboratory Eddy-Diffusivity/Mass-Fluxes (JPDFs) that defines the JPDF of saturation excess and rain rate in the non-convective environment. We provide the integrals of functions used to derive environmental mean values of rates of macro- and microphysical processes.

Normal PDFs of variable \( x \), with the mean value \( \mu_x \) and variance \( \sigma_x^2 \) is defined as:

\[
p_{\text{N}}(x, \mu_x, \sigma_x^2) = \frac{1}{\sqrt{2\pi\sigma}} \exp \left( -\frac{1}{2} \left( \frac{x - \mu_x}{\sigma_x} \right)^2 \right)
\]  
(F1)

Similarly, the variable \( y \) is distributed lognormally if it follows the distribution:

\[
p_{\text{log}}(y, \mu_y, \sigma_y^2) = \frac{1}{\sqrt{2\pi y \sigma_y}} \exp \left( -\frac{1}{2} \left( \frac{\log y - \mu_y}{\sigma_y} \right)^2 \right)
\]  
(F2)

where \( \mu_y \) and \( \sigma_y^2 \) are the parameters of the distributions (and are equal to the mean and the variance of variable \( \log y \)) and which can be expressed in terms of the first two moments of variable \( y \) as (Garvey, 2000; Appendix B):

\[
\mu_y = \frac{1}{2} \log \left( \frac{\mu_y^4}{\mu_y^2 + \sigma_y^2} \right)
\]  
(F3)

\[
\sigma_y = \log \left( \frac{\mu_y^2 + \sigma_y^2}{\sigma_y} \right)
\]  
(F4)

where \( \mu_y \) and \( \sigma_y^2 \) are the mean value and variance of variable \( y \), respectively.

The bivariate joint normal distribution of variables \( x \) and \( y \) is:

\[
p_{x,y}(x, y) = \frac{1}{2\pi\sigma_x\sigma_y\sqrt{1-\rho^2}} \exp \left( -\frac{1}{2} \frac{1}{1-\rho^2} \Lambda_1 \right)
\]  
(F5)

with:

\[
\Lambda_1 = \left( \frac{x - \mu_x}{\sigma_x} \right)^2 + \left( \frac{y - \mu_y}{\sigma_y} \right)^2 - 2\rho \frac{x - \mu_x}{\sigma_x} \frac{y - \mu_y}{\sigma_y}
\]  
(F6)

where correlation coefficient between \( x \) and \( y \) is defined as \( \rho = \frac{\sigma_{x,y}}{\sigma_x \sigma_y} \). The parameters of the distribution on the l.h.s. of Equation F5 are written in the vector form and represent the mean values and covariance matrix of the two variables.

The bivariate normal-lognormal PDF \( p_{x,y,\text{log}} \) is defined (e.g., Chen, 2002) as:
\[
\begin{align*}
P_{N, \log}(x, y; \mu_x, \sigma_x, \mu_y, \sigma_y) &= \frac{1}{2\pi\sigma_x\sigma_y \sqrt{1-\hat{\rho}^2}} \exp \left( -\frac{1}{2 (1-\hat{\rho}^2)} \Lambda_2 \right) \\
\Lambda_2 &= \left( \frac{x - \mu_x}{\sigma_x} \right)^2 + \left( \frac{\log y - \hat{\mu}_y}{\hat{\sigma}_y} \right)^2 - 2\hat{\rho} \frac{x - \mu_x}{\sigma_x} \frac{\log y - \hat{\mu}_y}{\hat{\sigma}_y} \\
\end{align*}
\]

with
\[
\hat{\rho} = \frac{\hat{\sigma}_{xy}}{\sigma_x\hat{\sigma}_y} = \rho \sqrt{\exp \left( \hat{\sigma}_y^2 \right) - 1}
\]

where \(\rho\) is the correlation coefficient between variables \(x\) and \(y\).

Definite integral of the \(c\)th moment of the normal distribution \(p_N(x, \mu, \sigma^2)\) is:
\[
\int_{-\infty}^{\infty} x^c p_N(x, \mu, \sigma^2) \, dx = 2^{c-1} \sigma^{c-1} \sqrt{\pi} \left[ \sqrt{2\mu \Gamma \left( \frac{c}{2} + 1 \right)} \, F_1 \left( 1 - \frac{c}{2}, \frac{3}{2}, \frac{1}{2} \left( \frac{\mu}{\sigma} \right)^2 \right) + \sigma \Gamma \left( \frac{c}{2} + 1 \right) \, F_1 \left( \frac{c}{2}, \frac{1}{2}, \frac{1}{2} \left( \frac{\mu}{\sigma} \right)^2 \right) \right]
\]

where \(F_1\) is Kummer confluent hypergeometric function.

For arguments \(b - a > 0\) the asymptotic value for Kummer function is:
\[
\lim_{z \to \infty} F_1(a, b, z) = \Gamma(b) \left( \frac{-z}{} \right)^a \Gamma(b - a)
\]

The following two integrals, needed to evaluate value of accretion and evaporation in the non-convective environment, can be found in Gradshteyn and Ryzhik (1996):
\[
\int_{-\infty}^{\infty} \exp \left( -p^2 x^2 + qx \right) \, dx = \exp \left( \frac{q^2}{4p^2} \right) \frac{\sqrt{\pi}}{|p|}
\]
\[
\int_{0}^{\infty} x^{a-1} \exp \left( -\beta x^2 - \gamma x \right) \, dx = (2\beta)^{-a/2} \Gamma(a) \exp \left( \frac{\gamma^2}{8\beta} \right) D_{-a} \left( \frac{\gamma}{\sqrt{2}\beta} \right)
\]

Where \(D_{a}(z)\) is a parabolic cylinder function. In the limit when its argument approaches negative infinity, it can be simplified:
\[
\lim_{x \to -\infty} D_{-a}(z) = \frac{\sqrt{2\pi}}{\Gamma(a)} x^{a-1} \exp \left( x^2/4 \right)
\]

Finally, the following integral is used to estimate the flux of the rain water in the non-convective environment:
\[
\int_{-\infty}^{\infty} \rho_N(x, \mu_x, \sigma_x) e^{\alpha x} \, dx = \exp \left( a\mu_x + \frac{a^2\sigma_x^2}{2} \right)
\]

**Data Availability Statement**

The RICO and the Dynamics and Chemistry of Marine Stratocumulus model forcings and results may be found in van Zanten et al. (2011) and Ackerman et al. (2009), respectively. The Jet Propulsion Laboratory Eddy-Diffusivity/Mass-Flux results can be accessed on Zenodo (https://doi.org/10.5281/zenodo.5760085).
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