Comments on differential cross section of $\phi$-meson
photoproduction at threshold

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Abstract

We show that the differential cross section $d\sigma/dt$ of $\gamma p \to \phi p$ reaction at the threshold is finite
and its value is crucial to the mechanism of the $\phi$ meson photoproduction and for the models of
$\phi N$ interaction.

PACS numbers: 13.88.+e, 13.60.Le, 14.20.Gk, 25.20.Lj
Now it becomes clear that the \( \phi \)-meson photoproduction at low energies \( E_\gamma \approx 2 - 3 \) GeV plays important role in understanding the non-perturbative Pomeron-exchange dynamics and the nature of \( \phi N \) interaction. It was expected that in the diffractive region the dominant contribution comes from the Pomeron exchange, since trajectories associated with conventional meson exchanges are suppressed by the OZI-rule \[1\]. The exception is the finite contribution of the pseudoscalar \( \pi, \eta \)-meson-exchange channel, but its properties are quite well understood \[2\]. Therefore, the low-energy \( \phi \)-meson photoproduction may be used for studying the additional (exotic) processes. Candidates are the Regge trajectories associated with a scalar and tensor mesons containing a large amount of strangeness \[3, 4\], glueball exchange \[1\] or other channels with \[3, 4, 7\] or without \[8\] suggestions of the hidden strangeness in the nucleon.

One possible indication of manifestation of the exotic channels is non-monotonic behavior of the differential cross section \( d\sigma/dt \) of \( \gamma p \to \phi p \) reaction, reported recently by the LEPS collaboration \[9\]. The data show a bump structure around \( E_\gamma \approx 2 \) GeV, which disagrees with monotonic behavior predicted by the conventional (Pomeron-exchange) model. Another peculiarity of the LEPS’s data is the tendency of \( d\sigma/dt \) at forward photoproduction angle \( (\theta \approx 0) \) to be finite when the photon energy \( E_\gamma \) approaches to the threshold value \( E_{\text{thr}} \approx 1.574 \) GeV. This is in contradiction with relatively old \[1, 10\] and recent \[11\] expectations \( \frac{d\sigma}{dt} = 0 \) at \( \theta = 0 \) and \( E_\gamma \approx E_{\text{thr}} \), based on a relation that near the threshold \( \frac{d\sigma}{dt} \) behaves as \( q^2_\phi/k^2_\gamma \) where \( k_\gamma \) and \( q_\phi \) are the momenta of the incoming photon and the outgoing \( \phi \) meson in center of mass, respectively. The aim of present communication is to concentrate on this particular aspects of the experimental data. We intend to show (i) absence of so called ”threshold factor” \( q^2_\phi/k^2_\gamma \) in differential cross section and (ii) to stress that \( d\sigma/dt \) at \( E_\gamma \approx E_{\text{thr}} \) is sensitive to the dynamics of \( \phi N \) interaction and is crucial for the modern QCD inspired models.

A. The threshold factor

The differential cross section of \( \gamma p \to \phi p \) reaction is related to the invariant amplitude as

\[
\frac{d\sigma_{\gamma p \to \phi p}}{dt} = \frac{1}{64\pi s k^2_\gamma} |T_{\gamma p \to \phi p}|^2 ,
\]

(1)
where $s$ is the total energy and averaging and summing over the spin projections in the initial and the final states are assumed. The arguments lead to appearance of the threshold factor $q_{\phi}^2/k_{\gamma}^2$ are shown in Ref. [11]. First, using the current-field identity (vector dominance model) one can express the invariant amplitude of the $\phi$ meson photoproduction through the amplitudes of the $Vp \to \phi p$ transitions ($V = \rho, \omega, \phi$)

$$T_{\gamma p \to \phi p} = \sum_{V} \frac{e}{2\gamma_{V}} T_{V p \to \phi p}, \quad (2)$$

where $\gamma_{\rho} \div \gamma_{\omega} \div \gamma_{\phi} \simeq 2.5 \div 8.5 \div 6.7$ are defined from the $V \to e^+ e^-$ decay. Keeping only the diagonal transition $\phi p \to \phi p$, one can express the cross section of $\gamma p \to \phi p$ reaction through the invariant amplitude of the elastic $\phi p \to \phi p$ scattering

$$\frac{d\sigma_{\gamma p \to \phi p}}{dt} = \frac{\alpha}{\gamma_{\phi}^2} \frac{1}{64\kappa_{\gamma}^2} |T_{\phi p \to \phi p}|^2. \quad (3)$$

The next step is evaluating $T_{\phi p \to \phi p}(\theta = 0)$. In [11] it is made by using the optical theorem

$$\text{Im} T_{\phi p \to \phi p}(\theta = 0) = -2q_{\phi} \sqrt{\sigma_{\phi p}^{\text{tot}}}, \quad (4)$$

where $\sigma_{\phi p}^{\text{tot}}$ is the total cross section of $\phi p$ interaction (for convenience, we use the same sign convention as in [11]). The consequence of Eq. (4) is a disappearance of $\text{Im} T_{\phi p \to \phi p}(\theta = 0)$ at $q_{\phi} \to 0$. The final result reads

$$\frac{d\sigma_{\gamma p \to \phi p}}{dt}(\theta = 0) = \frac{\alpha}{16\gamma_{\phi}^2} \frac{q_{\phi}^2}{k_{\gamma}^2} \left[1 + r^2\right] \sigma_{\phi p}^{\text{tot}}^2, \quad (5)$$

where $r = \text{Re} T_{\phi p} / \text{Im} T_{\phi p}$. Assuming $r$ to be a constant, one can get the threshold factor $q_{\phi}^2/k_{\gamma}^2$ in explicit form. But the weak point of such consideration is just assuming that $r$ is constant at $q_{\phi} \to 0$. The real part of invariant amplitude $T_{\phi p}$ is related to the $\phi p$ scattering length, that can not vanish at $q_{\phi} \to 0$ and therefore,

$$r^2(q_{\phi} \to 0) \sim \frac{1}{q_{\phi}^2}. \quad (6)$$

This leads to cancelation of $q_{\phi}^2$ dependence and eliminating the ”threshold factor” in Eq. (5).

**B. Threshold behavior of the differential cross section**

For more consistent analysis of the threshold behavior we express the differential cross section of $\gamma p \to \phi p$ reaction in Eq. (3) via differential cross section of $\phi p \to \phi p$ elastic
scattering

\[
\frac{d\sigma}{dt}^{\gamma p \rightarrow \phi p} = \frac{\alpha \pi^2}{\gamma_p^2 k_p^2} \frac{d\sigma}{d\Omega}^{\phi p \rightarrow \phi p}.
\] (7)

At small \( q_\phi \), the differential cross section \( d\sigma^{\phi p \rightarrow \phi p} / d\Omega \) becomes isotropic and it can be expressed through the spin averaged \( \phi p \) scattering length \( a_{\phi p} \)

\[
\frac{d\sigma}{d\Omega}^{\phi p \rightarrow \phi p} = a_{\phi p}^2.
\] (8)

This leads to the following estimation

\[
\frac{d\sigma}{dt}^{\gamma p \rightarrow \phi p}_{\text{threshold}} = \frac{\alpha \pi^2}{\gamma_p^2 k_p^2} a_{\phi p}^2.
\] (9)

One can see that at the threshold the cross section of \( \phi \) meson photoproduction is finite and its value is defined by the \( \phi p \) scattering length.

1. **direct estimations**

The direct estimation of the \( \phi p \) scattering length on the base of QCD sum rules was done by Koike and Hayashigaki \[12\]. They got \( a_{\phi p} \simeq -0.15 \text{ fm} \) which results in

\[
\frac{d\sigma}{dt}^{\gamma p \rightarrow \phi p}_{\text{thr}} \simeq 0.63 \text{ \mu b/GeV}^2.
\] (10)

This value is in qualitative agreement with the experimental indication \[9\].

One can estimate \( a_{\phi N} \) using the \( \phi N \) potential approaches. Thus for example, Gao, Lee and Marinov suggested to use the QCD van der Waals attractive \( \phi N \) potential \[14\] for analysis of \( \phi \)-nucleus bound states. This potential reads

\[
V_{\phi N} = -A \exp(-\mu r)/r,
\] (11)

where \( A = 1.25 \) and \( \mu = 0.6 \text{ GeV} \). The corresponding scattering length \( a_{\phi p} \simeq 2.37 \text{ fm} \), found by direct solution of the Schrödinger equation, leads to large cross section \( d\sigma/dt \simeq 1.6 \times 10^2 \text{ \mu b/GeV}^2 \). It is more than two order of magnitude greater than the experimental hint and provides a problem for this potential model. Thus, in order to get the scattering length \( a_{\phi p} \simeq \pm 0.15 \text{ fm} \) (and correspondingly, the cross section \( d\sigma/dt \) close to the experiment) one has to choose \( A = 2.56 \) or \( 0.226 \) for the positive (strong attraction) or negative (weak attraction) \( a_{\phi p} \), respectively. At \( A \simeq 2.75 \), the elastic scattering disappears (\( a_{\phi p} = 0 \)) and we get some kind of Ramsauer effect \[13\]. In principle, such analysis may be used for other potentials as well.
2. SU(3) symmetry considerations

Estimation of the upper bound of $|a_{\phi p}|$ may be done on assumption that the amplitudes of the $\phi p$ and $\omega p$ scattering are dominated by the scalar $\sigma$ meson exchange. Then the SU(3) symmetry gives relation

$$a_{\phi p} = \xi a_{\omega p}$$

(12)

where $\xi \equiv -\tan\Delta \theta_V$ ($\Delta \theta_V \simeq 3.7^0$ is the deviation of the $\phi - \omega$ mixing angle from the ideal mixing [15]). More complicated processes as $s$ channel exchange with intermediate nucleon or nucleon resonances, or box diagrams with $\omega(\phi)\pi\rho$ vertices would give terms proportional to $\xi^2$ and generally speaking, violate Eq. (12). But for crude estimation of order of magnitude of $a_{\phi p}$ one can utilize Eq. (12) using $a_{\omega p}$ as an input.

Thus, QCD sum rule analysis of Koike and Hayashigaki [12] results in $a_{\omega p} = -0.41$ fm. The coupled channel unitary approach of Lutz, Wolf and Friman [16] leads to $a_{\omega p} = (-0.44 \pm 0.20)$ fm. An effective Lagrangian approach based on the chiral symmetry developed by Klingl, Waas and Weise [17] results in $a_{\omega p} = (1.6 \pm 0.3)$ fm. The corresponding $\phi$ meson photoproduction cross sections for these scattering lengths, denoted with subscripts 2, 3 and 4, respectively, read

$$\frac{d\sigma_{\gamma p \rightarrow \phi p}^{\text{thr}}}{dt}^{[2]} = 2.0 \times 10^{-2} \text{mb/GeV}^2,$$

(13)

$$\frac{d\sigma_{\gamma p \rightarrow \phi p}^{\text{thr}}}{dt}^{[3]} = 2.7 \times 10^{-2} \text{mb/GeV}^2,$$

(14)

$$\frac{d\sigma_{\gamma p \rightarrow \phi p}^{\text{thr}}}{dt}^{[4]} = 3.1 \times 10^{-1} \text{mb/GeV}^2.$$  

(15)

Fig. 1 shows predictions of Eqs. (10), (13) - (15) by the enumerated symbols “plus”. Experimental data at $\theta = 0$ are taken from Refs. [9, 18]. The predictions of Eqs. (10) and (15) seems to be more preferable. The difference between Eqs. (13) and (14) and data can indicate small $\omega p$ scattering length or necessity to introduce large OZI-rule evading factor in Eq. (12) which can be related to the finite hidden strangeness in the nucleon. For example, analysis of $\phi$ meson photoproduction at large angles in Refs. [2, 19] favors for the large OZI-rule evading factor $x_{\text{OZI}} \simeq 3 - 4$. Such value results in increasing the threshold predictions based on $a_{\omega p}$ by almost of order of magnitude. Employing this evading factor seems to be consistent with predictions [2] and [3] and make a problem for that of [4].
FIG. 1: Differential cross section of $\gamma p \rightarrow \phi p$ reaction at $\theta = 0$ as function of the photon energy. The enumerated symbols "plus" correspond to the threshold predictions, given in Eqs. (10), (13) - (15). Experimental data are taken from Refs. [9, 18].

C. non-diagonal transitions

In principle, the non-diagonal transitions in Eq. (2) may also contribute near the threshold. Such example is the $\phi$ meson photoproduction with $\pi(\eta)$ meson exchange, shown in Fig. 2, which is associated with $\rho \rightarrow \phi$ transition.

\[ T_{\gamma p \rightarrow \phi p} = -i \frac{eg_{NN\pi}g_{\gamma\phi\pi}}{M_\phi(t - m_\pi^2)} \epsilon^{\mu \nu \alpha \beta} \epsilon_\mu(\gamma) \epsilon_\nu(\phi) k_{\gamma\alpha} q_{\beta\beta} [\bar{u}_{p'} \gamma_5 u_p] F(t), \]  \hspace{1cm} (16)

where $g_{\gamma\phi\pi}$ has a sense of $g_{\rho\phi\pi}/2g_{\rho\pi}$ and is taking from $\phi \rightarrow \gamma \pi$ decay ($g_{\gamma\phi\pi} \simeq 0.14$ [15]), $g_{NN\pi} \simeq 13.3$ and $F(t)$ is a product of the form factors in $\gamma\phi\pi$ and $NN\pi$ coupling vertices. This amplitude leads to the following estimate

\[ \frac{d\sigma_{\gamma p \rightarrow \phi p}^\text{(threshold)}}{dt} = \frac{\alpha g_{NN\pi}^2 g_{\gamma\phi\pi}^2 F^2(t_{\text{thr}})}{64 E_{\text{thr}} M_N^2 M_\phi^2} \frac{\left| t_{\text{thr}} \right|(M_\phi^2 - t_{\text{thr}})^2}{(t_{\text{thr}} - m_\pi^2)^2}, \]  \hspace{1cm} (17)
where $E_{\text{thr}} = (2M_N M_\phi + M_\phi^2) / 2M_N \simeq 1.574$ GeV and $t_{\text{thr}} = -M_N M_\phi^2 / (M_N + M_\phi) \simeq -0.5$ GeV$^2$. Taking $F(t) = ((\Lambda^2 - m_\pi^2) / (t - m_\pi^2))^2$ with $\Lambda \simeq 0.6 - 0.7$ GeV, one can find

$$\frac{d\sigma}{dt} \overset{\text{threshold}}{\simeq} (0.8 - 1.6) \times 10^{-2} \mu b / \text{GeV}^2.$$

(18)

Coherent sum of the $\pi$ and $\eta$ meson exchange results in $d\sigma_{\gamma p \to \phi p (\pi + \eta)} / dt \simeq (0.3 - 0.6) \times 10^{-1} \mu b / \text{GeV}^2$. So again, $d\sigma_{\gamma p \to \phi p} / dt$ is finite and its magnitude is in the range of uncertainty of other estimations. However, being smaller than the experimental indication it allows contribution of exotic channels discussed in literature, such as scalar/glueball exchange, direct knockout of hidden $\bar{s}s$ pairs and so on.

Finally we notice that the differential $d\sigma_{\gamma p \to \phi p} / d\Omega$ and the total $\sigma_{\gamma p \to \phi p}$ cross sections have the obvious kinematical phase space factor $q_\phi / k_\gamma$. For example, for the diagonal transition we get

$$\frac{d\sigma}{d\Omega} \overset{\text{threshold}}{\simeq} \frac{q_\phi \alpha \pi}{k_\gamma} a_\phi^2, \quad \sigma_{\gamma p \to \phi p} \overset{\text{threshold}}{=} \frac{4q_\phi \alpha \pi^2}{k_\gamma} a_\phi^2 .$$

(19)

If one accepts the threshold behavior of $d\sigma / dt$ as in Eq. (5) with a constant $r$, then the cross sections $d\sigma / d\Omega$ and $\sigma_{\gamma p \to \phi p}$ will decrease near threshold as $(q_\phi / k_\gamma)^3$ which seems to be rather strong.

In summary, we analyzed the differential cross section $d\sigma / dt$ of $\gamma p \to \phi p$ reaction at the threshold and have shown that it is finite and its value is crucial for the QCD inspired models of $\phi N$ interaction and for the mechanism of the $\phi$ meson photoproduction.

We thank E.L. Bratkovskaya, W. Cassing, H. Ejiri and B. Kämpfer for useful discussions.

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