The Einstein-Hilbert Action, Horizons and Connections With Thermodynamics

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Abstract

It is shown that the Einstein-Hilbert action can be constructed by minimizing free energy. The entropy used to determine the free energy is determined on the horizon of a black hole. Some further considerations with regard to generalizations of these ideas to other situations of physical importance are presented as well.

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1. Introduction.

For some time, it has been known that there are deep connections between the areas of gravity and thermodynamics [1-5]. One of the main reasons for this is that the principle of equivalence has led naturally to the idea that gravity is a manifestation of curved space-time [6-7]. Gravity then is of geometric origin and can be termed a geometric effect. In general relativity, the local inertial frames at the various events are patched together in the presence of gravity to generate a curved space-time. If the global space-time were flat, the different local inertial frames would fit together to form an extended inertial frame. The idea of gravity in general relativity as an aspect of inertia merges with the elimination of the absolute set of extended inertial frames. Since these distortions of space-time can be regarded as surfaces or types of membranes which are capable of being deformed, distorted or even wrapped up, it is frequently the case that information from one region is not accessible to observers in a different region. Sakharov thought of gravitation as a metric elasticity. This metric elasticity was based upon a microscopic structure analogous to molecular structure behind material elasticity. In this picture, any static space-time with horizon, space-time dynamics can be regarded as similar to the thermodynamic limit in solid state. It is frequently the case that information from one region is not accessible to observers in a different region. It is also known that there is a deep relationship between entropy and information, or accessibility of information. All told, it must be concluded that there must be many connections between gravity and thermodynamics, even just thinking at the classical level [8-9]. There has also been a lot of interest in looking at thermodynamic systems as extremal hypersurfaces [10].

If the equivalence principle leads to the formulation of a geometrical description of gravity, then there is the possibility that membranes will arise in viewing space-time under various circumstances. Regarding space-time as a membrane means they can act as one-way interfaces for the transfer of information, as discussed extensively by Padmanabhan [11-13]. Thus, a connection is established with thermodynamics, or a thermodynamic view, and a particular instance in which such a surface-like structure, or horizon, can physically appear is in the vicinity of a black hole, namely the event horizon. Moreover, its possible horizons play a similar role in other relativistic
situations as well. Thus the main objective here is to examine this relationship, as well as to discuss other physical situations in which similar ideas may play an important role. In the case of a black hole, the information content entangled across a horizon is proportional to the area of the horizon. This brings with it the idea that the fundamental constant characterizing gravity is the quantum of area $4A_P$, which can contain about one bit of information. The situation is similar to the case of bulk matter made of individual atoms. The conventional gravitational constant $G = A_P c^3 / \hbar$ will diverge if $\hbar \rightarrow 0$ when $A_P$ is kept constant. In a way similar to the fact that an atomic system does not exist in this limit, space-time and gravity are inherently quantum mechanical.

A new coordinate system can be constructed by first transforming to the local inertial frame and then using standard transformations between the inertial coordinates and Rindler coordinates. In this context, it can be postulated that the horizon in the local Rindler frame has an entropy per unit transverse area and then demand that any feature of gravity have this incorporated in it. Of course, the static situation in terms of a black hole space-time corresponds to an equilibrium state in the thermodynamic picture. Moreover, it will be shown that by minimizing the free energy, this is sufficient to lead to the correct Einstein-Hilbert action principle for gravity. By using the local Rindler frame and demanding that gravity must incorporate these thermodynamical aspects leads in a natural way to the action functional itself. The action is built up from its surface behaviour, which is to say that gravity is fundamentally holographic in nature. Finally, in addition to finding that Einstein’s equations are equivalent to the principle of minimization of the free energy, some speculation as to the relevance of these ideas to other systems in which the equivalence principle plays a role, such as accelerating systems or free fall, will be discussed at the end.

2. Gravitational Functional.

It will be important to have some information concerning the gravitational functional at the start. It is this functional which will result from thermodynamic considerations. Begin by considering some general properties of the gravitational functional. The conventional action principle
for general relativity is the Einstein-Hilbert action, which is given as \[ S_{EH} = \frac{1}{16\pi} \int R\sqrt{-g} \, d^4x, \] (2.1)

where \( R \) is the Ricci scalar curvature calculated using the components of the metric \( g_{ij} \). In fact, (2.1) can be reexpressed by making use of the following relation

\[ R\sqrt{-g} = \frac{1}{4}\sqrt{-g}M^{abcijk}g_{ab,c}g_{ij,k} - \partial_j P^j = \sqrt{-g}L_{quad} - \partial_j P^j. \] (2.2)

In (2.2), comma indicates partial differentiation with respect to the indicated coordinate. The factor needed to define \( L_{quad} \) is given by

\[ M^{abcijk} = g^{ck}(g^{ab}g^{ij} - g^{ai}g^{bj}) + 2g^{cj}(g^{ai}g^{bk} - g^{ki}g^{ba}), \] (2.3)

and \( P^j \) is given by

\[ P^j = \sqrt{-g} g_{ac,i}(g^{ac}g^{ji} - g^{ia}g^{cj}) \equiv \sqrt{-g}V^j. \] (2.4)

The equality between the first term on the left and last on the right in (2.2) is well known \[14\].

If the Lagrangian of the gravitational field is taken as the noninvariant quantity

\[ L_{quad} = g^{ab}(\Gamma^s_{ar} \Gamma^r_{bs} - \Gamma^s_{ab} \Gamma^r_{rs}), \] (2.5)

then \( \sqrt{-g}L_{quad} \) differs from \( \sqrt{-g}R \) by a divergence term of the form \( \partial(\sqrt{-g}V^j)/\partial x^j \). From (2.2), \( L_{quad} \) can be identified to have the form,

\[ L_{quad} = \frac{1}{4} M^{abcijk}g_{ab,c}g_{ij,k}. \]

Upon differentiating both sides of this with respect to \( g_{uv,w} \), the following result is obtained

\[ 4\frac{\partial L_{quad}}{\partial g_{uv,w}} = M^{uvwijk}g_{ij,k} + M^{abcuvw}g_{ab,c} \]

\[ = g^{wk}(g^{uv}g^{ij} - g^{ui}g^{vj})g_{ij,k} + 2g^{wj}(g^{ui}g^{vk} - g^{ki}g^{vu})g_{ij,k} \]

\[ + g^{kw}(g^{ij}g^{uv} - g^{iu}g^{jv})g_{ij,k} + 2g^{kv}(g^{iu}g^{jw} - g^{wu}g^{ji})g_{ij,k} \]

\[ = 4g_{ij,k}(g^{ij}g^{kw} - g^{wj}g^{ik}). \] (2.6)
The first important result that is obtained from (2.2) and (2.6) is a direct relationship between $P^j$ and Lagrangian $\mathcal{L}_{quad}$:

$$g_{ab} \frac{\partial \mathcal{L}_{quad}}{\partial g_{ab, q}} = g_{ij,k}(g^{ij}g^{qk} - g^{jk}g^{qj}) - V^q = \frac{1}{\sqrt{-g}} P^q. \tag{2.7}$$

By replacing $P^j$ in (2.2), this result shows that the scalar curvature can be put in the form,

$$R = \mathcal{L}_{quad} - \frac{1}{\sqrt{-g}} \partial_k [\sqrt{-g} g_{ij} \frac{\partial \mathcal{L}_{quad}}{\partial g_{ij,k}}]. \tag{2.8}$$

Choose a coordinate system in which the metric has the form,

$$ds^2 = g_{nn} (dx^n)^2 + g_{ij}^\perp dx^i dx^j, \tag{2.9}$$

and $n = 0, \cdots, 3$. For each choice of $n, i$ and $j$ run over the other three coordinates, assuming cross terms vanish. Then $P^n$ in this coordinate system can be worked out by using (2.4),

$$P^n = -\frac{1}{\sqrt{-g}} \partial_k (gg^{kn}) = -\frac{2}{\sqrt{g_{nn}}} \partial_n \sqrt{g^\perp}. \tag{2.10}$$

With $C$ a constant, the normal to the surface $x^n = C$ is given by $n^a = g_{mn}^{-1/2} \delta^a_n$, and the trace of the extrinsic curvature of the $x^n = C$ surface is given by

$$K = -\nabla_a n^a = -\frac{1}{\sqrt{g^\perp}} \frac{1}{\sqrt{g_{nn}}} \partial_n \sqrt{g^\perp}. \tag{2.11}$$

Consequently,

$$\int_V d^4x \partial_a P^a = \sum_{\partial V} 2 \int K \sqrt{g^\perp} d^3x. \tag{2.11}$$

The sum in (2.11) is over the boundary surfaces, so the total divergence term can be expressed as the sum over the integrals of the extrinsic curvatures on each boundary. Comparing (2.11) with (2.8), a dynamical interpretation of $K$ can be obtained

$$2K = n_c g_{ab} \frac{\partial \mathcal{L}_{quad}}{\partial g_{ab,c}} = n_c g_{ab} \Pi^{abc}. \tag{2.12}$$

The quantity $\Pi^{ab} = n_c \Pi^{abc}$ is the energy-momentum conjugate to $g_{ab}$ with respect to the surface defined by the normal $n_c$. 

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If the Lagrangian is taken to have the form $L(q_A, \partial_i q_A)$ and depends on a collection of dynamical variables $q_A$, where in turn $A$ can denote a collection of indices, a second Lagrangian can be obtained as follows

$$L_\pi = L - \partial_i [q_A \frac{\partial L}{\partial (\partial_i q_A)}] = L - \partial_i (q_A p^{ Ai}). \quad (2.13)$$

For gravity $q_A$ would be taken as $g_{ab}$ with $A$ denoting a pair of indices. Each of these Lagrangians will lead to the same equations of motion, provided that $q_A$ is fixed while varying $L$ and the $p^{ Ai}$ are fixed while varying $L_\pi$. In the context of gravity, Lagrangian $L$ corresponds to the quadratic Lagrangian while $L_\pi$ corresponds to the Einstein-Hilbert Lagrangian.

Consider any Lagrangian $L(q_A, \partial_i q_A)$ which contains dynamical variables $q_A$ such that the Euler-Lagrange function will have the form

$$E^A \equiv \frac{\partial L}{\partial q_A} - \partial_i [q_A \frac{\partial L}{\partial (\partial_i q_A)}]. \quad (2.14)$$

Contracting this with $q_A$, (2.14) can be put in the form,

$$q_A E^A = q_A \frac{\partial L}{\partial q_A} - \partial_i [q_A \frac{\partial L}{\partial (\partial_i q_A)}] + (\partial_i q_A) \frac{\partial L}{\partial (\partial_i q_A)}. \quad (2.15)$$

For example, suppose $L$ is a homogeneous function of degree $\mu$ in the $q_A$ and homogeneous of degree $\lambda$ in the $\partial_i q_A$. Then (2.15) reduces to

$$q_A E^A = (\lambda + \mu) L - \partial_i [q_A \frac{\partial L}{\partial (\partial_i q_A)}]. \quad (2.16)$$

In the case of gravity, $E^A$ works out to be

$$E^A = -(R^{ab} - \frac{1}{2} g^{ab} R) \sqrt{-g}. \quad (2.17)$$

Contracting (2.17) with $q_A = g_{ab}$, it turns into

$$q_A E^A = g_{ab}[-(R^{ab} - \frac{1}{2} g^{ab} R) \sqrt{-g}] = R \sqrt{-g}.$$

In this case, it can be verified that $\mu = -1$ and $\lambda = 2$, so (2.16) becomes identical to (2.8).

3. Entropy and Horizons.
To work entirely in the Lorentzian space-time, some semi-classical features must be adopted. First, the time integration must be restricted to a suitable finite range in defining the action. Second, there should be a suitable surface term in the action describing gravitational dynamics which acquires a contribution from the horizon. The horizon is the only surface or interface which is common to both the inside and outside regions. Quantum entanglement effects across a horizon can only appear as a surface term in the action. The equivalence principle then leads to the conclusion that the action functional describing gravity must contain certain boundary terms which can encode information which is equivalent to that present beyond the horizon. The idea here is to determine this surface term from general principles. This can be used to make a correspondence with entropy and out of the equations from thermodynamics, obtain the form of the full action for gravity.

In order to provide a local Lagrangian description, the sought after boundary term has to be expressible as an integral of a four-divergence. This allows the action functional to have the form,

$$S_{grav} = \int d^4x \sqrt{-g} L_{grav} = \int d^4x \sqrt{-g} (L_{bulk} + \nabla_i W^i) = S_{bulk} + S_{sur}.$$  \hspace{1cm} (3.1)

In (3.1) $L_{bulk}$ is quadratic in the first derivatives of the metric and of course

$$\nabla_i W^i = (-g)^{-1/2} \partial_i [(-g)^{1/2} W^i],$$  \hspace{1cm} (3.2)

irrespective of whether $W^i$ is a general four vector or not. It is required to determine $S_{sur}$ so that a connection with entropy can be established.

Let $(M, g_{ab})$ be a globally hyperbolic space-time, which can be foliated by Cauchy surfaces, $\Sigma_t$, parametrized by a global time function, $t$. Let $u^a$ be the unit normal vector field to the hypersurfaces $\Sigma_t$. The space-time metric $g_{ab}$ induces a three-dimensional metric $h_{ab}$ on each $\Sigma_t$, as discussed below. Let $t^a$ be a vector field on $M$ satisfying $t^a \nabla_a t = 1$, which is decomposed into its parts normal and tangential to $\Sigma_t$, by defining the lapse function, $N$, and the shift vector, $N^a$, with respect to $t^a$ by

$$N = -t^a n_a = (n^a \nabla_a t)^{-1}, \quad N_a = h_{ab} t^b.$$  

The lapse function and shift vector are not considered dynamical, since they merely prescribe how to move forward in time, a gauge transformation. The horizon for a class of observers arises in a
specific gauge and \( S_{\text{sur}} \) will in general depend on the gauge variables \( N, N_\alpha \). The lapse function \( N \) plays a more important role than the \( N_\alpha \), and so we set \( N_\alpha = 0 \) without loss of generality.

Next a \((1+3)\) foliation with the standard notion for the metric components \( g_{00} = -N^2, g_{0\alpha} = N_\alpha \). Let \( u^i = (N^{-1}, 0, 0, 0) \) be the four-velocity of observers corresponding to this foliation; that is, the normal to this foliation. Let \( a^i = u^j \nabla_j u^i \) be the related acceleration and \( K_{ij} = -\nabla_i u_j - u_i a_j \) be the extrinsic curvature of the foliation, with \( K \equiv K_i^i = -\nabla_i u^i \), hence \( K_{ij} u^i = K_{ij} u^j = 0 \) and \( K_{ij} \) is purely spatial.

Consider a series of hypersurfaces \( \Sigma \) with normals \( u^i \). The following differential geometric identity will be required [6]. Beginning with

\[
R_{ijka} u^a = (\nabla_i \nabla_j - \nabla_j \nabla_i) u_k,
\]

the following required identity is obtained,

\[
R_{ij}^i u^j = g^{ac} R_{aicj} u^i u^j = u^i \nabla_a \nabla_i u^a - u^i \nabla_j \nabla_a u^a
\]

\[
= \nabla_a (u^i \nabla_j u^a) - (\nabla_a u^i) (\nabla_i u^a) - \nabla_j (u^i \nabla_a u^a) + (\nabla_j u^i)^2
\]

\[
= \nabla_i (K u^i + a^i) - K_{ij} K^{ij} + K_i^i K_j^j,
\]

(3.3)

where \( K_{ij} = K_{ji} = -\nabla_i u_j - u_i a_j \) is the extrinsic curvature with \( K = K_i^i = -\nabla_i u^i \) and \( K_{ij} K^{ij} = (\nabla_i u^j)(\nabla_j u^i) \). In any space-time, there is the geometric identity (3.3). In static space-time with \( K_{ij} = 0 \), this reduces to

\[
\nabla_i a^i = R_{ij} u^i u^j.
\]

(3.4)

Thus all possible vector fields \( W^i \) which can be used in (3.1) may be accounted for. It must be made up of \( u^i, g_{ij} \) and \( \nabla_i \) acting only once, since the equations of motion must be no higher order than two. Given these conditions, there is only one vector field, \( u^i \) itself, and only three vectors

\( (u^i \nabla_j u^i, u^j \nabla_i u_j, u^i \nabla_j u_j) \)

which are linear in \( \nabla_i \). The first one is the acceleration \( a^i = u^j \nabla_j u^i \).

The second identically vanishes since \( u^j \) has unit norm and the third can be written as \( -u^i K \).

Consequently, \( W^i \) appearing in the surface term has to be a linear combination of the terms \( u^i, u^i K \) at lowest order. Hence \( S_{\text{sur}} \) must have the form

\[
S_{\text{sur}} = \int d^4 x \sqrt{-g} \nabla_i W^i = \int d^4 x \sqrt{-g} \nabla_i [\lambda_0 u^i + \lambda_1 K u^i + \lambda_2 a^i],
\]

(3.5)
where $\lambda_j$ are numerical constants.

Let the region of integration be a four-volume $V$ bounded by two space-like surfaces $\Sigma_1$, $\Sigma_2$ and two time-like surfaces $S$ and $S_1$. The space-like surfaces are constant time slices with normals $u^i$, and the time-like surfaces have normals $n^i$ such that $n_i u^i = 0$. The induced metric on the space-like surface $\Sigma$ is $h_{ij} = g_{ij} + u_i u_j$, while the induced metric on the time-like surface $S$ is $\gamma_{ij} = g_{ij} - n_i n_j$. These two surfaces intersect on a two-dimensional surface $Q$ with induced metric $\sigma_{ij} = h_{ij} - n_i n_j = g_{ij} - u_i u_j - n_i n_j$. In this instance, the first two terms in (3.5) contribute only on $\Sigma_1, \Sigma_2$ with $t$ constant, while the third term contributes on $S$, that is on a horizon. Consequently, on the horizon

$$S_{\text{sur}} = \lambda_2 \int d^4 x \sqrt{-g} \nabla_i a^i = \lambda_2 \int_S dt d^2 x N \sqrt{\sigma} (n_i a^i). \tag{3.6}$$

In any static space-time with horizon, the integration over $t$ becomes multiplication by $\beta = 2\pi/\kappa$, where $\kappa$ is the surface gravity of the horizon, since there is a periodicity in the Euclidean sector. As $S$ approaches the horizon, $N(a_i n^i)$ in the integrand tends to $-\kappa$, which is constant over the horizon. Thus on the horizon,

$$S_{\text{sur}} = -\lambda_2 \kappa \int_0^\beta dt \int d^2 x \sqrt{\sigma} = -2\pi \lambda_2 A_H, \tag{3.7}$$

where $A_H$ is the area of the horizon.

Treating the action as analogous to entropy, the information blocked by a horizon, and encoded in the surface term, must be proportional to the area of the horizon. Taking into consideration non-compact horizons like Rindler, the entropy or information content per unit area of the horizon is a constant related to $\lambda_2$. Writing $\lambda_2 = -1/8\pi A_P$, where $A_P$ is a fundamental constant with dimensions of area, the entropy associated with the horizon is

$$S_H = \frac{A_H}{4A_P}. \tag{3.8}$$

4. Einstein’s Equations Based on a Thermodynamic Argument.

The information content which is entangled across a horizon surface is proportional to the area of the horizon. Consequently, there is a fundamental constant characterizing gravity, the quantum
of area. Consider a four-dimensional region of space-time defined as follows: a three-dimensional spatial region is taken to be some compact volume $V$ with boundary $\partial V$. The time integration is taken over the interval $[0, \beta]$ because there is periodicity in Euclidean time.

From the discussion in the preceding section, we now define the entropy associated with the space-time region to be

$$S = \frac{1}{8\pi G} \int_V d^4x \sqrt{-g} \nabla_i a^i = \frac{\beta}{8\pi G} \int_{\partial V} d^2x \sqrt{\sigma} (N n_\mu a^\mu). \tag{4.1}$$

The integral is cast into the second form on account of the previous considerations. The time integration reduces to multiplication by $\beta$, and only the spatial components are non-zero, so the divergence is three-dimensional over $V$, which can be transformed into an integral over $\partial V$. If the boundary $\partial V$ is a horizon, the quantity $N n_\mu a^\mu$ will tend to a fixed quantity, namely, the constant surface gravity $\kappa$, so the integral reduces to an expression for area. Using $\beta \kappa = 2\pi$, it is found that $S = A/4G$, where $A$ is the area of the horizon. Similar considerations apply to each piece of any area element when it acts as a horizon for some Rindler observer.

The total energy $E$ in the region, acting as a source for gravitational acceleration, is given by the Tolman energy defined by

$$E = 2 \int_V d^3x \sqrt{\gamma} N (T_{ij} - \frac{1}{2} T g_{ij}) u^i u^j. \tag{4.2}$$

The covariant combination $2(T_{ij} - \frac{1}{2} T g_{ij}) u^i u^j$, which reduces to $(\rho + 3p)$ for an ideal fluid, is the correct source for gravitational acceleration. Note that both $S$ and $E$ depend on the congruence of timelike curves chosen to define them through $u^i$. The free energy of space-time must have direct geometrical meaning independent of the congruence of observers used to define the entropy $S$ and $E$. The energy in (4.2) is not just

$$U = \int_V d^3x \sqrt{\gamma} N (T_{ij} u^i u^j), \tag{4.3}$$

based on $\rho = T_{ij} u^i u^j$, but the integral of $(\rho + 3p)$. The free energy needs to be defined as $F \equiv U - T S$, since pressure, which is an independent thermodynamic variable, should not appear in the free energy. This gives,

$$\beta F = \beta U - S = -S + \beta \int_V d^3x \sqrt{\gamma} N (T_{ij} u^i u^j) = -S + \int_V d^4x \sqrt{-g} T_{ij} u^i u^j. \tag{4.4}$$
Using the expression for the entropy (4.1), \( R = -8\pi GT \) and (3.3)-(3.4), it is found that

\[
\beta F = -\frac{1}{8\pi G} \int_V d^4x \sqrt{-g} \nabla_i a^i + \int_V d^4x \sqrt{-g} T_{ij} u^i u^j
\]

\[
= \int_V d^4x \sqrt{-g} (-\frac{1}{8\pi G} \nabla_i a^i + T_{ij} u^i u^j) = \frac{1}{2} \int_V d^4x \sqrt{-g} T_{ij} g_{ij} u^i u^j = -\frac{1}{8\pi G} \int_V d^4x \sqrt{-g} R g_{ij} u^i u^j
\]

\[
= \frac{1}{8\pi G} \int_V d^4x \sqrt{-g} R. \quad (4.5)
\]

The equation (4.5) is just the Einstein-Hilbert action. The equations of motion are obtained by minimizing the action. They can be equivalently thought of as arising by minimizing the macroscopic free energy.

5. Horizons Applied to Other Physical Situations and Conclusions.

It is clear that the presence of horizons which appear so naturally in the case of black-holes leads to a link between entropy and the Einstein-Hilbert action. t’Hooft’s horizon algebra posits that ingoing and outcoming fields with respect to an event horizon are projected onto the horizon and determined there. An event horizon contains all field theoretic information of a black hole.

What can be said with regard to other systems when there is an acceleration involved, or when the equivalence principle can be applied. In particular, the state of motion of the measuring device can affect whether or not particles are observed to be present [5,15]. The question as to which set of modes furnishes the best description of a physical vacuum is not easy to answer. For example, a free-falling detector will not always register the same particle density as a non-inertial, accelerating detector. One of the main reasons for the vagueness of the particle concept is in its global or extended nature. The modes are defined on the whole of space-time, and of course, when there are particles in large enough numbers, there is the possibility that thermodynamic variables can be assigned to them, such as an entropy or free energy, as one would do with any gas. Similarly, the absence of particles signifies the absence of properties such as entropy as well. Perhaps the nature of horizons is to limit the extended nature when considering the particle concept. Another physical example is the system which contains a charge fixed with respect to a stationary observer which is passed by an observer who occupies an accelerating frame of reference. The accelerating
observer would presume the charge is accelerating and should appear to radiate. On the other hand, the stationary observer would observe the charge at rest and not radiating.

Perhaps the idea of the physics of horizons and the holographic principle has a greater generality which goes beyond the study of black holes where the idea of a horizon has a very prominent role. This is inevitable since a horizon appears automatically during formation due to the enormous concentration of mass. In fact, it is well known that boundaries appear in other related relativistic contexts as well. If a timelike curve $x^a(\tau)$ in the space-time is considered parametrized by the proper time $\tau$ of the clock moving along that curve, the union of past light cones $\{C(\tau), -\infty \leq \tau \leq \infty\}$ determines whether an observer on $x^a$ can obtain information from all events in the space-time or not. If there is a nontrivial boundary, there will be regions in the space-time from which this observer cannot receive signals, and a family of such curves is usually called a congruence. This could be used to define a type of horizon. Acceleration itself could be a determining factor much in the way mass determines a horizon in the black hole context. That is to say, perhaps it is possible for a sufficiently large acceleration to twist space-time locally in the vicinity of the accelerated mass that it can be said a horizon exists at least locally and has a physical impact. Much of this could depend on how far one would like to push the view that space-time can be regarded as a membrane which can be distorted, of course.

To see how this might be approached, let us return to the identity connecting the $\Gamma^2$ Lagrangian $\mathcal{L}_{bulk}$ and the Einstein-Hilbert Lagrangian $\mathcal{L}_{grav}$. This relation is a purely differential geometric identity, and can be written in general as

$$\mathcal{L}_{grav} = \mathcal{L}_{bulk} - \nabla_c [g_{ab} \frac{\partial \mathcal{L}_{bulk}}{\partial (\partial_c g_{ab})}]$$

$$\mathcal{L}_{bulk} = \mathcal{L}_{grav} - \nabla_c [\Gamma^j_{ab} \frac{\partial \mathcal{L}_{grav}}{\partial (\partial_j \Gamma^j_{ab})}]$$

These show that the really important degrees of freedom in gravity are the surface degrees of freedom. It is quite plausible that other situations can be described by equations similar to these.

References.

[1] J. D. Bekenstein, Phys. Rev. D7, 2333-2346, (1973).

[2] J. D. Bekenstein, Phys. Rev. D9, 3292-3300, (1974).
[3] S. W. Hawking, Commun. Math. Physics, 43, 199-220, (1975).
[4] S. A. Fulling, Phys. Rev. D7, 2850-2862, (1973).
[5] S. A. Fulling, Aspects of Quantum Field Theory in Curved Spacetime, Cambridge University Press, Cambridge, (1989).
[6] R. M. Wald, General Relativity, The University of Chicago Press, Chicago, (1984).
[7] W. Rindler, Relativity-Special, General and Cosmological, 2nd Ed., Oxford Univ. Press (2006).
[8] T. Padmanabhan, Physics Reports, 380, 235-320, (2003).
[9] T. Padmanabhan, Physics Reports, 406, 49-125, (2005).
[10] A. Vázquez, H. Quevedo, A. Sánchez, J. Geom. and Physics, 60, 1942-1949, (2010).
[11] T. Padmanabhan, Astro. and Space Science, 285, 407-417, (2003).
[12] T. Padmanabhan, Class. Quantum Grav., 19, 3551-3566, (2002).
[13] T. Padmanabhan, Class. Quantum Grav., 19, 5387-5408, (2002).
[14] M. Carmeli, Classical Fields, General Relativity and Gauge Theory, World Scientific, Singapore, (2001).
[15] N. D. Birrell and P. C. W. Davies, Quantum Fields in Curved Space, Cambridge University Press, (1982).