Dark energy model selection with current and future data

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ABSTRACT

The main goal of the next generation of weak lensing probes is to constrain cosmological parameters by measuring the mass distribution and geometry of the low redshift Universe and thus to test the concordance model of cosmology. A future all-sky tomographic cosmic shear survey with design properties similar to Euclid has the potential to provide the statistical accuracy required to distinguish between different dark energy models. In order to assess the model selection capability of such a probe, we consider the dark energy equation-of-state parameter \( w_0 \). We forecast the Bayes factor of future observations, in the light of current information from Planck, by computing the predictive posterior odds distribution. We find that Euclid is unlikely to overturn current model selection results, and that the future data are likely to be compatible with a cosmological constant model. This result holds for a wide range of priors.

Key words: cosmology: dark energy – weak gravitational lensing – methods: statistical

1 INTRODUCTION

There is no shortage of data in modern cosmology, and information from various experiments has allowed us to measure the parameters in our cosmological model with increasing precision. These data include cosmic microwave background measurements (e.g. Hinshaw et al. 2013, WMAP; Planck Collaboration 2013, Planck), supernovae compilations (e.g. Goldhaber 2009, SCP), large scale structure maps (e.g. Ahn et al. 2014, SDSS), weak lensing observations (e.g. Parker et al. 2007, Schrabback et al. 2010). The next generation of experiments (e.g. Amendola et al. 2012, Euclid; Blake et al. 2004, SKA) should provide even better precision.

The \( \Lambda \) cold dark matter (\( \Lambda \)CDM) concordance model can fit current astrophysical data with only 6 parameters describing the mass-energy content of the Universe (baryons, cold dark matter and dark energy) and the initial conditions. However, this is not a statement on whether the model is correct. It merely implies that deviations from \( \Lambda \)CDM are too small compared to the current observational uncertainties to be inferred from cosmological data alone. One obvious example is the addition of hot dark matter to the \( \Lambda \)CDM model, i.e. parameters for the physical neutrino density and the number of massive neutrino species (see e.g. Abazajian et al. 2011, Audren et al. 2013, Basse et al. 2013). Although massive neutrinos are not required by current cosmological observations, neutrino oscillation observations have shown that neutrinos have a non-zero mass.

The fundamental questions facing cosmologists are not simply about parameter estimation, but about the possibility of new physics, therefore about model selection. In addition to estimating the values of the parameters in the model, this involves decisions on which parameters to include or exclude. In some cases, the inclusion of parameters is possible only by invoking new physical models.

This is the case with the dark energy problem (Albrecht et al. 2006, Peacock et al. 2006). There is firm observational evidence suggesting that the Universe entered a recent stage of accelerated expansion. The physical mechanism driving this expansion rate remains unclear and there exist several potential models. In the framework of General Relativity applied to a homogeneous and isotropic universe, the acceleration could be produced either by an additional term in the gravitational field equations or by a new isotropic comoving perfect fluid with negative pressure, called dark energy (see e.g. Peebles & Ratra 2003, Johri & Rath 2007, Polarski 2013).

The main science goal of the next generation of cosmological probes is to test the concordance model of cosmology. In the case of dark energy, the objective is to measure the expansion history of the Universe and the growth of structure (see Weinberg et al. 2013 for a comprehensive review). In this work we will consider weak cosmic shear from a future all-sky survey similar to the European Space Agency mission Euclid, due for launch in 2020 (see Amendola et al. 2012). The main scientific objective of Euclid is to understand the origin of the accelerated expansion of the Universe by probing the nature of dark energy. It could potentially test for departures from the current concordance model (see e.g. Heavens, Kitching & Verde 2007, Zhao et al. 2012, Jain & Khoury 2013).
In a previous paper (Debono 2014) we studied the ability of Euclid to distinguish between dark energy models. This time we go a step further by addressing the following question: based on current data, what model will we select using future data from Euclid? What is the probability that Euclid could favour ΛCDM?

Cosmologists are faced with the task of constructing valid physical models based on incomplete information. In this relation between data and theory, Bayesian inference provides a quantitative framework for plausible conclusions (see e.g. Robert, Chopin & Rousseau 2009; Hobson et al. 2011; Jenkins & Peacock 2011 for a discussion) and can be understood as operating on three levels:

(i) Parameter inference (estimation): we assume that a model $M$ is true, and we select a prior for the parameters $p(\theta | M)$.

(ii) Model selection or comparison: there are several possible models $M_i$. We find the relative plausibility of each in the light of the data $D$.

(iii) Model averaging: there is no clear evidence for a best model. We find the inference on the parameters which accounts for the model uncertainty.

The dark energy question is a model comparison or model averaging problem. In the present work, we will confine ourselves to model selection.

In order to produce an accelerated expansion at the present epoch, the dark energy equation-of-state parameter should satisfy the conservative bound $w_{DE} = \frac{p_{DE}}{\rho_{DE}} < -0.5$. Observations suggest a lower value, close to $-1$. If the data are compatible with this value, then in model selection terms it means they are compatible with ΛCDM. However, ΛCDM is not merely a special case of some more general model where $w_{DE} = -1$. It contains smaller number of free parameters, and if it fits the data, it is favoured by the Occam’s razor effect because it is more predictive. In the cosmological context, the question is whether there is evidence that we need to expand our cosmological model beyond ΛCDM to fit this data (see e.g. Liddle 2004; March et al. 2011).

Consider a result from an experiment quoted in terms of a mean value $\mu$, and a confidence interval $\sigma$. This is a parameter estimation result. In the frequentist interpretation, a confidence interval of 68.3 per cent means that if we were to repeat the experiment many times and obtain a $\sigma$ distribution for the mean, the true value would lies inside the intervals thus obtained 68.3 per cent of the time. This is not the same as saying that the probability of $\mu$ lying within a given interval is 68.3 per cent. The latter is a statement on model selection, and it only follows if we use Bayesian techniques.

In simple terms, how do we know if a given accuracy on a certain mean value enough to falsify a model? If it falsifies a model, what does it verify? It is therefore clear that model selection calculations must include information on the alternatives under consideration. We cannot reject a hypothesis unless an alternative hypothesis is available that fits the facts better.

In the context of the dark energy problem, it means that a claim such as ‘ΛCDM is false’ is not enough. We need an alternative model, for we would know at least the number of free parameters and their allowed ranges, before the data come along. This is the model prior. Intuitively, we know that the prior affects the model selection outcome. We know $w_{DE}$ to be within range of values around $-1$, but for a cosmological constant, the prior width is zero. Statistically, we will measure $w_{DE}$ to be a different value each time, so we take some average. It is the average that is compatible with theory that we understand to be the value of the cosmological constant.

The question is therefore to know which range of measured values leads us to choose ΛCDM, and which values lead us to discard it. In terms of model selection, we need to quantify the degree of compatibility with ΛCDM of a measured value of $w_{DE}$ that is $x\sigma$ away from $-1$.

One important point to note is that in constructing models, we are seeking to find that model which will best predict future data. We can always include all possible parameters and obtain a perfect fit to the current data, but we also want our model to be predictive. Thus, the model that explains the past data best may not be most predictive model. Bayesian evidence quantifies this trade-off between goodness of fit and predictivity or model simplicity.

We are interested in forecasting the result of a future model comparison, by predicting the distribution of future data. In this work, we assess the potential of Euclid to address model comparison questions, based on current information. We derive a predictive distribution for the dark energy equation-of-state parameter for a cosmic shear survey with the Euclid probe using the predictive posterior odds distribution (POD) method developed by Trotta (2007b). We also study the dependence of our results on the prior width.

This paper is organized as follows. In Section 2 we describe the Bayesian framework. The POD method is described in Section 3. Our cosmological and weak lensing formalism for Euclid, together with the current and future data are described in Section 4. We apply the POD to Euclid in Section 5 and present our conclusions in Section 6.
vector of free parameters \( \psi \) is a restricted submodel of \( M_1 \), which contains the parameters \( \psi \) and \( \omega \). In \( M_0 \), the additional parameters are fixed at \( \omega = \omega_* \). In the \( \Lambda \)CDM model, the dark energy equation-of-state parameter \( \omega_{DE} \) is fixed at \(-1\). We assume separable priors i.e.

\[
p(\omega, \psi | M_1) = p(\omega | M_1)p(\psi | M_0). \tag{4}
\]

Then the Bayes factor can be written as the ratio of the marginalized posterior over the prior marginal density of \( \omega \) under the extended model \( M_1 \), evaluated at the value \( \omega = \omega_* \):

\[
B_{01} = \frac{p(\omega|d, M_1)}{p(\omega| M_1)} \bigg|_{\omega = \omega_*}, \tag{5}
\]

which is the Savage-Dickey density ratio or SDDR \citep{Dickey1971}. The SDDR expresses the Bayes factor as an amount of information brought by the data. It is therefore a good tool for model selection in cosmology.

We use the Jeffreys scale to interpret the logarithm of the Bayes factor in terms of the strength of evidence. We adopt a slightly more conservative version of the convention used by \cite{Jeffreys1961} and \cite{Trotta2007b}. This is shown in Table 1.

| \( \ln B_{01} \) | Probability | Evidence |
|-----------------|-------------|----------|
| > 0             | < 0.5       | Negative |
| -2.5 to 0       | 0.5 to 0.923| Positive |
| -5 to -2.5      | 0.923 to 0.993| Moderate |
| < -5            | > 0.993     | Strong   |

Table 1. Jeffreys’s scale for the strength of evidence when comparing two models \( M_0 \) (restricted) against \( M_1 \) (extended), interpreted here as the evidence for the extended model. The probability is the posterior probability of the favoured model, assuming non-committal priors on the two models, and assuming that the two models fill all the model space. Negative evidence for the extended model is equivalent to evidence for the simpler model. Note that the labels attached to the Jeffreys scale are empirical, and their interpretation depends to a large extent on the problem being modelled. An experiment for which \( |\ln B_{01}| < 1 \) is usually deemed inconclusive.

3 THE PPOD

In a companion paper \citep{Debono2014}, we examined the ability of \textit{Euclid} cosmic shear measurements to distinguish between different dark energy models. Our current knowledge is included in the calculations in two ways: through our choice of fiducial model, and through the prior ranges. Implicit in the former is the assumption that the future maximum likelihoods for the cosmological parameters common to all models under consideration will be roughly the same as the current likelihoods. However, we do not include information on the position of the current maximum likelihood of the extra parameters (in this case, the dark energy equation-of-state parameters). In other words, we do not take into account the present posterior distribution.

One way of including this information is through the predictive posterior odds distribution (PPOD) developed by \cite{Trotta2007b}. This extends the idea of posterior odds forecasting introduced by \cite{Trotta2007a} and also \cite{Pahud2006, Pahud2007}, which is based on the concept of predictive probability. This uses present knowledge and uncertainty to predict what a future measurement will find, with corresponding probability. The predictive probability is therefore the future likelihood weighted by the present posterior.

The PPOD is a hybrid technique, combining a Fisher matrix analysis \citep{Fisher1935, Fisher1936} with the SDDR. It gives us the probability distribution for the model comparison result of a future measurement. It is conditional on our present knowledge, and gives us the probability distribution for the Bayes factor of a future observation. In other words, it allows us to quantify the probability with which a future experiment will be able to confirm or reject the null hypothesis.

The PPOD showed its usefulness in predicting the outcome of the \textit{Planck} experiment \citep{Trotta2007b} found that \textit{Planck} had over 99 per cent probability of obtaining model selection result favouring a scale-dependent primordial power spectrum, with only a small probability that it would find evidence in favour of a scale-invariant spectrum. This result was confirmed by actual data a few years later, when \textit{Planck} temperature anisotropy measurements combined with the WMAP large-angle polarization found a value of \( n_s = 0.96 \pm 0.0073 \), ruling out scale invariance at over 5\( \sigma \) \citep{PlanckCollaboration2013}.

In this paper, we apply the PPOD technique to the dark energy equation-of-state parameter, comparing the evidence for \( \Lambda \)CDM against a dynamical dark energy model \( w \)CDM. From the current posterior we can produce a PPOD for the \textit{Euclid} satellite. In this section we review the formalism of the PPOD.

The predictive distribution for future data \( D \) is

\[
p(D|d) = \sum_{i=0}^{1} p(D|d, M_i)p(M_i|d) = \sum_{i=0}^{1} p(M_i|d) \int p(D|\theta, M_i)p(\theta|d, M_i) d\theta, \tag{6}
\]

where \( d \) is the current data, and the sum runs over the 2 models we are considering. In the equation above, \( p(D|\theta, M_i) \) is the predicted likelihood for future data, assuming \( \theta \) is the correct value for cosmological parameters under model \( M_i \). We obtain a Gaussian approximation to the future likelihood by performing a Fisher matrix analysis assuming \( \theta \) as a fiducial model. This gives us a forecast of the parameter covariance matrix \( C \) for future data \( D \).

The PPOD for the future Bayes factor \( B_{01} \), conditional on current data \( d \) is then

\[
p(B_{01}|d) = \int p(B_{01}, D|d) dD
\]

\[
= \int p(B_{01}|D, d)p(D|d) dD
\]

\[
= \delta(D - B_{01}(D))p(D|d) dD, \tag{7}
\]

where \( \delta \) is the Dirac delta function.

Let us consider the case of nested models, with \( \theta = (\psi, \omega) \) as defined previously. It is reasonable to assume that the current and future likelihoods for the data considered in this paper are both Gaussian. For future data, this assumption is implicit in our use of Fisher matrix analysis to forecast the future covariance matrix \( C \). We make the further assumption that the covariance matrix does not depend on the fiducial values chosen for the common parameters \( \psi \). Then we can marginalize over the common parameters, and compare a 1-dimensional model \( M_1 \) with a model \( M_0 \) with no free parameters.
The priors on the extra parameter are taken to be Gaussian, centred on 0, with a prior width equal to unity. The current likelihood is also assumed to be Gaussian, centred on $\omega = \mu$ of width $\sigma$. The Gaussian mean and width are expressed in units of the prior width and are therefore dimensionless. Likewise, the predicted likelihood is assumed to have a Gaussian distribution, with mean $\omega = \nu$ and constant standard deviation $\tau$. The latter is the forecast error $\tau = \sqrt{C_{11}}$ obtained from a Fisher matrix calculation. It is assumed to be independent of $\omega$, which is a reasonable assumption, since the marginalised errors are very stable to a change in the fiducial values of the model parameters over the region of interest (see Debono2014 for the variation of the dark energy figure-of-merit in the $w_0 - \omega$ parameter space).

The predicting distribution can then be expressed analytically as

$$p(D|d) \propto \frac{p(M_0)}{\tau \sigma} \exp \left( -\frac{1}{2} \frac{\nu^2 \sigma^2 + \mu^2 \tau^2}{\tau^2 \sigma^2} \right) + \frac{p(M_1)}{\sqrt{\tau^2 + \sigma^2 + \tau^2 \sigma^2}} \exp \left( -\frac{1}{2} \frac{(\nu - \mu)^2 + \sigma^2 \nu^2 + \tau^2 \mu^2}{\tau^2 + \sigma^2 + \tau^2 \sigma^2} \right)$$

(8)

where the normalising constants for the probability distribution are left out. This gives the probability of obtaining a value $\omega = \nu$ from a future measurement as a function of the future mean $\nu$ conditional on the present data $d$.

Note that in the equation above, $p(M_0) = p(M_1) = \frac{1}{2}$. At present, there is no evidence which justifies assigning a higher probability to a particular model. This is a statement on our prior knowledge, which is based on the accumulation of information from a multitude of experiments (see Brewer & Francis2009). In this paper, we justifiy assigning equal probabilities to each model because we are testing two at a time.

The PPOD is obtained by applying the SDDR using the relation between $\nu$ and the future model selection outcome $B_{01}(D)$:

$$\nu^2 = \tau^2 (1 + \tau^2) \left( \ln \frac{1 + \tau^2}{2 \pi \tau^2} - 2 \ln B_{01} \right)$$

(9)

This relation only holds for a Gaussian prior and a posterior distribution that is accurately described by a Gaussian. The latter assumption will most likely not hold in the tails of the distribution, where $|\nu - \omega|/\tau \gg 1$. In other words, this is the region where the mean value of the extra parameter in the extended model is many sigmas away from its fixed value in the restricted model. In this case, parameter estimation should be enough to provide evidence against $M_0$, even though we might not be able to calculate a precise value for the expected odds.

4 FORECASTS FOR EUCLID

We apply the PPOD technique to assess the potential of the Euclid mission in terms of model selection, taking into account the information from current data. Our current information is taken from Planck results, while we forecast our future cosmic shear data from Euclid.

4.1 The future data

We forecast the errors for future Euclid cosmic shear data using the Fisher matrix technique. The restricted fiducial cosmological model used for our forecast contains parameters describing baryonic matter, cold dark matter (CDM), massive neutrinos (or hot dark matter – HDM) and dark energy. We drop the requirement for flat spatial geometry by including a dark energy density parameter $\Omega_{DE}$ together with the total matter density $\Omega_m$.

We choose fiducial parameter values based on the Planck 2013 best-fit values (Planck Collaboration2013a), with the exception of the scalar spectral index, which we set to 1:

(i) Total matter density: $\Omega_m = 0.31$ (which includes baryonic matter, HDM and CDM).
(ii) Baryonic matter density: $\Omega_b = 0.048$.
(iii) Neutrinos (HDM): $m_\nu = 0.25$ eV (total mass); $N_\nu = 3$ (number of massive neutrino species).
(iv) Dark energy density: $\Omega_{DE} = 0.69$.
(v) Hubble parameter: $h = 0.67(100 \, \text{km} \, \text{s}^{-1} \, \text{Mpc}^{-1})$ and
(vi) Primordial power spectrum parameters: $\sigma_8 = 0.82$ (amplitude); $n_s = 1$ (scalar spectral index); $\alpha = 0$ (its running).

We assume a total of three neutrino species, with degenerate masses for the most massive eigenstates. The temperature of the relativistic neutrinos is assumed to be equal to $(4/11)^{1/3}$ of the photon temperature. We model $N_\nu$, the number of massive neutrino species, by a continuous variable.

CMB anisotropy observations from the Planck probe suggest caution in employing an overly simple parametrisation of the primordial power spectrum (Planck Collaboration2013a,b). For this reason, we allow for possible departures from a scale-invariant primordial power spectrum.

For simplicity, we shall refer to this fiducial model as ACDM. Note that our work implicitly assumes that future best-fit values for the restricted model will not deviate significantly from the current ones. This assumption must be used carefully (see Starkman, Trotta & Vaudrevange2010). We use the numerical Boltzmann code CAMB (Lewis, Challinor & Lasenby2000) to calculate the linear matter power spectrum. This includes the contribution of baryonic matter, cold dark matter, dark energy and massive neutrino oscillations. We use the Smith et al.2003 HALOFIT fitting formula to calculate the non-linear power spectrum, with the modification by Bird, Viel & Haehnelt2012. The power spectrum is normalised using $\sigma_8$, the root mean square amplitude of the density contrast inside an $8 \, h^{-1} \text{Mpc}$ sphere.

To calculate future errors, we use forecasts for an all-sky tomographic weak lensing survey similar to Euclid (Amendola et al.2012; Laureijs et al.2011), using the method described in Debono (2014).

We follow the power spectrum tomography formalism by Hu & Jain2004, with the background lensed galaxies divided into 10 redshift bins. Cross-correlations of shears are carried out within and between bins. The 3D power spectrum is projected onto a 2D lensing correlation function using the Limber (1953) equation:

$$C_{ij} = \int dz \frac{H(z)}{D_A} W_i(z) W_j(z) P(k = \ell/D_A, z),$$

(10)

where $i, j$ denote different redshift bins. The weighting function $W_i(z)$ is defined by the lensing efficiency:

$$W_i(z) = \frac{3}{2} \Omega_m H_0 \frac{H_0 \Delta_D(z)}{a} \int_z^{\infty} dz' \frac{D_L(z')}{D_{OS}(z')} P(z'),$$

(11)
Table 2. fiducial parameters for the Euclid-type all-sky weak lensing survey used for our future data.

| Survey property | Requirements | Goals |
|-----------------|--------------|-------|
| $A_\mu$/arcmin | 15 000 | 20 000 |
| $z_{\text{median}}$ | 0.9 | 0.9 |
| $n_\ell$/arcmin$^2$ | 30 | 40 |
| $\sigma_\ell$(z)/(1 + z) | 0.05 | 0.03 |
| $\sigma_\ell$ | 0.25 | 0.25 |

where the angular diameter distance to the lens is $D_{\text{OL}}$, the distance to the source is $D_{\text{OS}}$, and the distance between the source and the lens is $D_{\text{LS}}$ (see [Hu & Jain 2004]). Our multipole range is $10 < \ell < 5000$. We assume the probability distribution function for the galaxies:

$$P(z) = z^a \exp \left[ -\left( \frac{z}{z_0} \right)^b \right],$$

where $a = 2$ and $b = 1.5$, and $z_0$ is determined by the median redshift of the survey $z_{\text{median}}$.

We calculate the measurement errors based on two configurations of the Euclid-type survey, referred to as the ‘requirements’ and ‘goals’ in the Euclid Definition Study Report [Laureijs et al. 2011]. The experiment is defined by the following parameters: the survey area $A_\mu$, median redshift of the density distribution of galaxies $z_{\text{median}}$, the observed number density of galaxies $n_\ell$, the photometric redshift errors $\sigma_\ell(z)$ and the intrinsic noise in the observed ellipticity of galaxies $\sigma_\ell$, such that $\sigma_\ell^2 = \sigma_\ell^2 + \gamma^2$, where $\sigma_\ell$ is the variance in the shear per galaxy. These parameters are shown in Table 2.

The Fisher matrix for the shear power spectrum is given by [Hu & Jain 2004]

$$F_{\alpha\beta} = f_{\text{sky}} \sum_\ell \frac{(2\ell + 1)\Delta \ell}{2} \text{Tr} \left[ D_{\alpha\beta} \tilde{G}^{-1}_\ell D_{\alpha\beta} \tilde{G}^{-1}_\ell \right],$$

(13)

where the sum is over bands of multipole $\ell$ of width $\Delta \ell$. $\text{Tr}$ is the trace, and $f_{\text{sky}}$ is the fraction of sky covered by the survey. We assume the likelihood to have a Gaussian distribution, with zero mean. From the Fisher matrix we calculate the covariance matrix, which gives us the error forecasts on the parameters in our model.

4.2 Dark energy parametrization

This paper examines the question of whether dark energy is $\Lambda$ or whether there evidence for dynamical dark energy. Specifically, we ask how well the future Euclid probe will be able to answer this in the light of the current model selection outcome.

Here we have a case of a restricted model $\Lambda$CDM nested within an extended model, which we call $w$CDM. We consider an extension of $\Lambda$CDM by adding two dark energy parameters: the equation-of-state parameter at the present epoch $w_0$ and its variation $\Delta w_0$. The dynamical dark energy equation-of-state parameter, $w = p/\rho$, is expressed as function of redshift and is parametrized by a first-order Taylor expansion in the scale factor $a$ (Chevallier & Polarski 2001; Linder 2003):

$$w(a) = w_0 + (1 - a)\Delta w_0,$$

where $a = (1 + z)^{-1}$. This parametrization is motivated by the quintessence model, in which dark energy is some minimally coupled scalar field, slowly rolling down its potential such that it can have negative pressure. Scalar field models typically have a time-varying $w \geq -1$, and constant $w \neq -1$ models are poorly motivated, which is why we include the parameter $\Delta w_0$. In this study, however, we will focus on the value of $w_0$.

We include dark energy perturbations in all our calculations by using the parametrized post-Friedmann framework (Hu & Sawicki 2007, Hu 2008) as implemented in CAMB (Fang, Hu & Lewis 2008; Fang et al. 2008).

4.3 The current data

As our current posterior, we use the results from four Planck data sets used by the Planck Collaboration (2013a) to estimate the values of cosmological parameters. In these parameter estimation calculations, the Planck temperature power spectrum is combined with a WMAP polarization low-multipole likelihood and with four other data sets, as detailed below:

(i) Planck+WP+BAO: Planck and WMAP, combined with baryon acoustic oscillation measurements;
(ii) Planck+WP+Union 2.1: Planck and WMAP, combined with an updated Union2.1 supernova compilation by Suzuki et al. (2012);
(iii) Planck+WP+SNLS: Planck and WMAP, combined with the Supernova Legacy Survey compilation by Conley et al. (2011); and
(iv) Planck+WP+H0: Planck and WMAP, combined with the Riess et al. (2011) $H_0$ measurements.

We use a Gaussian approximation for the current likelihoods of the dark energy equation-of-state parameter. The values for the mean $w_0$ and width $\sigma$ for the likelihood from each dataset are given in Table 3. From each current likelihood, using a Gaussian prior centred on $w_0$ with width $\Delta \omega$, we can calculate the Bayesian evidence using [Trotta 2007a]

$$\ln B_{w0}(\beta, \lambda) = \frac{1}{2} \ln(1 + \beta^{-2}) - \frac{\lambda^2}{2(1 + \beta^2)},$$

(15)

where $\lambda = |\mu - \omega_0|/\sigma$ and $\beta = \sigma/\Delta \omega$. We choose the prior width to be 0.5, and we shift and rescale the parameters so the Gaussian likelihood is centred on zero and parameters are dimensionless. Thus, $\lambda$ is the discrepancy between the mean value of $w_0$ and $w_0 = -1$ expressed in number of sigmas. The quantity $\beta$ is the factor by which the prior accessible space is reduced by the data. It is evident from equation 15 that Bayesian evidence is a function of both the data and the prior.

The Bayesian evidence for $\Lambda$CDM against a dynamical dark energy model $w$CDM with $-1.5 < w_0 < -0.5$ is given in the fourth column of Table 3. Our model selection results qualitatively confirm general conclusions of the Planck parameter estimation results (Planck Collaboration 2013a), where a wider prior range is used. In the Planck paper, the BAO and Union2.1 data sets were found to be compatible with a cosmological constant, SNLS data weakly favour the phantom domain, while $H_0$ data are in tension with $w = -1$.

From our results we conclude that most of the current Planck data favour a cosmological constant. Next we turn to the question of whether future data from the Euclid probe can overturn these results.
Table 3. Model selection results with four current data sets including Planck, assuming a Gaussian prior centred on \( w_0 = -1 \) with a prior width of 0.5. Most of the data favour \( \Lambda \)CDM. A combination of Planck, WP and \( H_0 \) data shows positive evidence for dynamical dark energy. Note that this calculation uses a rather restrictive prior. This model of dynamical dark energy would have an equation-of-state parameter in the range \(-1.5 < w_0 < -0.5\).

| Current data               | \( w_0 \) | \( \sigma \) | \( \ln B_{01} \) | Evidence            |
|----------------------------|-----------|-------------|-----------------|---------------------|
| Planck+WP+BAO              | -1.13     | 0.120       | 0.900           | Positive for \( \Lambda \)CDM |
| Planck+WP+Union2.1         | -1.09     | 0.085       | 1.241           | Positive for \( \Lambda \)CDM |
| Planck+WP+SNLS             | -1.13     | 0.065       | 0.082           | Inconclusive to weak for \( \Lambda \)CDM |
| Planck+WP\( H_0 \)        | -1.24     | 0.090       | -1.713          | Positive for \( \omega \)CDM |

5 THE PPOD APPLIED TO THE DARK ENERGY QUESTION

We produce a PPOD forecast for Euclid following the method described in Section 3. The physical question we study is the choice of dark energy model. We therefore focus on the dark energy equation-of-state parameter \( w_0 \), comparing a cosmological constant model (\( \Lambda \)CDM) with \( w_0 = -1 \) against a dynamical dark energy model (\( \omega \)CDM) with a Gaussian prior of width \( \Delta w_0 = 0.5 \).

We calculate the PPOD for two configurations of the Euclid survey, based on the present knowledge from the 4 data sets described earlier. The constant future and current errors, \( \tau \) and \( \sigma \) respectively, are expressed in units of the prior width \( \Delta w_0 = 0.5 \). Thus the current errors are \( \tau = 0.0551/\Delta w_0 = 0.1102 \) and \( \sigma = 0.0382/\Delta w_0 = 0.0764 \) for the requirement and goal survey, respectively. Likewise, we express the current mean \( \mu \) and future mean \( \nu \) in units of the prior width.

The predictive distribution for Euclid using current knowledge from Planck is shown in Figure 1. For the Planck+BAO+supernova data, the peak of the distribution is located at \( w_0 = -1 \). This is a consequence of the fact that the errors around the current mean with these data sets are too large to exclude \( w_0 = -1 \). For the Planck+WP\( H_0 \) data set, the most probable models are located around \( w_0 = -1.24 \).

The PPOD results obtained from the predictive distribution are shown in Table 4. The main result is that Euclid has a low probability of finding high-odds evidence [i.e. \( p(\ln B < -5) \)] for \( \omega \)CDM for the Planck data sets using BAO and supernova data. If the current mean is given by the Planck+WP\( H_0 \) data set, then this probability increases to more than 50 per cent. This is consistent with the model selection results for Planck given in Table 3. It means that Euclid is not likely to overturn the current model selection results for this choice of prior.

There are two points to note about the PPOD results given here. Firstly, the Gaussian approximation used in the PPOD breaks down in the tails of the distribution. Secondly, the intervals for \( \ln B \) used in the Jeffreys scale are arbitrary. Furthermore, the interpretation given to each region has an empirical origin in betting odds and depends to some extent on the nature of the model selection question (see Kass & Raftery [1995], Efron & Gous [2001], Nesseris & García-Bellido [2013]). For these reasons, the results given here should be interpreted as a rough guide to the model selection outcome. A more general result can be obtained at the expense of computational speed by dropping the assumption of Gaussianity of the current and future likelihoods and sampling from both using Markov chain Monte Carlo techniques.

Figure 1. The predictive data distribution for Euclid cosmic shear, conditional on current knowledge. The probability distribution (normalized to the peak) is that of future measurements of the dark energy equation-of-state parameter \( w_0 \). The peaks centred on \( w_0 = -1 \) correspond to \( \Lambda \)CDM. We use Euclid ‘requirement’ and ‘goal’ survey parameters in the top and bottom panel, respectively. In each panel, we plot \( p(D/d) \) for four current data sets: Planck+WP+BAO (black), Planck+WP+Union 2.1 (red), Planck+WP+SNLS (blue), and Planck+WP\( H_0 \) (green).
were testing an extended model with large departures from it is evident that the required accuracy needs to be higher than if we
We have applied the predictive posterior odds distribution tech-
6 CONCLUSIONS
Euclid

5.1 The dependence on the prior
The dependence of model selection conclusions on the prior range is an important aspect of modern cosmology (see e.g. Kunz, Trotta & Parkinson 2006). The prior width determines the strength of the Occam’s razor effect, since a larger prior favours the simpler model. While the prior range should be large enough to contain most of the likelihood volume, an arbitrarily large prior can result in an arbitrarily small evidence for the extended model. For a discussion on the dependence of evidence on the choice of prior see e.g. Kunz, Trotta & Parkinson (2006), Trotta (2007a) and Brewer & Francis (2009). In Section 5 we used a fixed prior width of 0.5. We now examine the impact of a change of prior on our PPPOD results.

In Figure 2 we show the dependence on the prior width of the probabilities for Euclid to obtain different levels of evidence for a dynamical dark energy model wCDM against a cosmological constant model ΛCDM. As current data, we use the Planck+WP+BAO data set. Our results hold for a wide range of priors. We note that the probability of evidence for ΛCDM approaches 75 per cent while the probability of strong evidence for wCDM falls below 10 per cent as the prior is widened beyond Δw0 = 1. The prior would have to be narrowed to less than 0.2 for the model selection conclusion to be reversed, namely, for the probability of strong evidence for wCDM to be greater than the evidence for ΛCDM. For any reasonable choice of prior, and for both Euclid survey configurations, there is at most 25 probability of strong evidence for wCDM.

These results show the important role of the prior in the dark energy question. The narrower the prior, the greater the precision on w0 that is required to provide evidence for the extended model. The prior width defines the model predictivity space. When we seek some significant evidence for ΛCDM we are in fact finding the space of models that can be significantly disfavoured with respect to w0 = −1 at a given accuracy. This point is highlighted in Trotta (2006). For an extended model with small departures from ΛCDM, it is evident that the required accuracy needs to be higher than if we were testing an extended model with large departures from ΛCDM.

These conclusions qualitatively support the results in Debono (2014), in which we find that Euclid cosmic shear data forecasts return an undecided Bayesian evidence result if the true values of w0 and wCDM are close to their ΛCDM fixed values of −1 and 0. Furthermore, the present work shows that ΛCDM is still well-supported by the forecasts if we include current information, since the inclusion of the extra parameter wCDM is not required by Bayesian evidence, even if the alternative model has a relatively narrow prior range (Δw0 = 0.5). As we widen the prior range, the probability of evidence for ΛCDM becomes overwhelming.

Improving the parameter precision by going from a requirement to a goal survey configuration increases the probability of evidence in favour of ΛCDM. This result holds for all prior ranges considered in this paper. This highlights the essential role of parameter precision in deciding model selection questions.

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REFERENCES

Abazajian K. N. et al., 2011, Astroparticle Physics, 35, 177
Ahn C. P. et al., 2014, ApJS, 211, 17
Albrecht A. et al., 2006, eprint (arXiv:0609591)
Amendola L. et al., 2012, eprint (arXiv:1206.1225)
Audren B., Lesgourgues J., Bird S., Haehnelt M. G., Viel M., 2013, J. Cosmol. Astropart. Phys., 1, 26

Table 4. Probability of future model selection results for the two survey configurations of the Euclid mission described in the test, using cosmic shear data, conditional on present knowledge from 4 Planck data sets. The probability that Euclid will favour ΛCDM is shown in the last column. The third to fifth columns give the probability that Euclid will provide strong, moderate and positive evidence for wCDM, respectively.

| Future data       | Current data     | p(ln B < −5) | p(−5 < ln B < −2.5) | p(−2.5 < ln B < 0) | p(ln B > 0) |
|-------------------|------------------|--------------|---------------------|-------------------|------------|
| Euclid requirement survey |
| Planck+WP+BAO     | 0.150            | 0.160        | 0.337               | 0.353             |
| Planck+WP+Union2.1| 0.108            | 0.156        | 0.356               | 0.380             |
| Planck+WP+SNLS    | 0.252            | 0.225        | 0.278               | 0.245             |
| Planck+WP+H0      | 0.531            | 0.200        | 0.160               | 0.109             |
| Euclid goal survey |
| Planck+WP+BAO     | 0.163            | 0.135        | 0.309               | 0.393             |
| Planck+WP+Union2.1| 0.125            | 0.137        | 0.323               | 0.414             |
| Planck+WP+SNLS    | 0.286            | 0.185        | 0.253               | 0.276             |
| Planck+WP+H0      | 0.621            | 0.119        | 0.132               | 0.128             |

Dark energy: w0 = −1 vs −1.5 ≤ w0 ≤ −0.5 (Gaussian)
Figure 2. The PPOD dependence on the prior width of $w_0$, using Planck+WP+BAO as the current data, and forecasts for Euclid with a requirement survey configuration. The vertical dotted line shows the prior width of 0.5 used to calculate $P(D|d)$ in the previous figure. The lines show the future probability of evidence for $\omega$CDM according to the Jeffreys scale for Bayesian evidence: positive (magenta dots), moderate (cyan dashes), strong (thick red line) and negative (thick black line). Negative evidence for the extended model is equivalent to evidence for the restricted model $\Lambda$CDM. In order to obtain a larger probability for moderate evidence for $\omega$CDM one would have to use a prior width smaller than about 0.4. With a prior width smaller than about 0.2, when the evidence for $\Lambda$CDM drops sharply, the bulk of the data will provide only positive evidence for $\omega$CDM, falling short of strong or even moderate evidence. In Jeffreys's terminology, the evidence is in the inconclusive regime. The probability of strong evidence for $\omega$CDM with this prior width is below 20 per cent.
Planck Collaboration, 2013a, A&A, preprint (arXiv:1303.5076)
Planck Collaboration, 2013b, eprint (arXiv:1303.5082)
Polarski D., 2013, in American Institute of Physics Conference Series, Vol. 1514, American Institute of Physics Conference Series, Dabrowski M. P., Balcerzak A., Denkiewicz T., eds., pp. 111–117
Riess A. G. et al., 2011, ApJ, 730, 119
Robert C. P., Chopin N., Rousseau J., 2009, Statistical Science, 24, 141
Schrabback T. et al., 2010, A&A, 516, A63
Smail I., Ellis R. S., Fitchett M. J., 1994, MNRAS, 270, 245
Smith R. E. et al., 2003, MNRAS, 341, 1311
Starkman G. D., Trotta R., Vaudrevange P. M., 2008, eprint (arXiv:0811.2415)
Starkman G. D., Trotta R., Vaudrevange P. M., 2010, MNRAS, 401, L15
Suzuki N. et al., 2012, ApJ, 746, 85
Trotta R., 2006, eprint (arXiv: 0607496)
Trotta R., 2007a, MNRAS, 378, 72
Trotta R., 2007b, MNRAS, 378, 819
Weinberg D. H., Mortonson M. J., Eisenstein D. J., Hirata C., Riess A. G., Rozo E., 2013, Phys. Rep., 530, 87
Zhao G.-B., Crittenden R. G., Pogosian L., Zhang X., 2012, Phys. Rev. Lett., 109, 171301