Directly measuring mode purity of single component in superposed optical vortices

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Abstract: Mode purity is an important reference for the quality of the optical vortex. In this work, we propose a self-interference method to directly measure single component in superposed optical vortices based on phase-shifting technology. This method has excellent flexibility and robustness, which can be applied to a variety of occasions and harsh conditions. Careful alignment and optimized error analysis allow us to generate and measure single component with mode purity as high as 99.997%. © 2020 Optical Society of America

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1. INTRODUCTION

An optical vortex (OV) with the spiral wavefront is a kind of structured light which carries an orbit angular momentum (OAM) per photon of \( mh \), where \( m \) is topological charge (TC) and \( h \) denotes reduce Planck constant. In 1989, Couillet et al. proposed a kind of optical field bearing some analogy with the superfluid[1]. In 1992, Allen et al. showed that photons can carry OAM[2], which has aroused widespread concern among researchers [3-18]. Since then, OVs have been utilized in a plethora of applications in the field of optical micro-manipulation[19, 20], plasmonics[21, 22], and rotation speed measurement via the optical rotation Doppler effect[23-29]. The superposed OVs are often used here instead of a single component. Before practical application, we need to measure the mode purity of the superposed OVs. Since mode purity is an important reference for the quality of the OV, which determines the performance of OV in various applications. In rotating Doppler effect[30, 31], low mode purity means frequency spectrum expansion, which greatly increases the difficulty of signal. In the field of micro-manipulation, high mode purity is more conducive to the use of superposed OVs to achieve static capture of particles[32].

There are many methods to achieve measuring mode purity of superposed OVs[33-35]. But the most versatile approach is to resolve a field into a coherent sum of modes, each with a particular amplitude weighting and phase: a so-called modal decomposition[36]. However, we need to use an SLM to scan multiple holograms one by one to complete the measurement. Is there a way to complete the scanning process with a computer to improve measurement efficiency? The answer is yes. Recent work showed that the phase-shifting holography in an interferometer can be used to measure the mode purity of OVs[37]. This work required a separate reference path that increased the complexity of the setup and introduced additional phase noise. In 2019, Andersen et al. modified the work mentioned above to achieve the same goal with only one path[38]. However, these two works can only measure the mode purity of single OV, and they can't do anything about superposed OVs.

In this work, we propose a self-interference method to directly measure single component in superposed optical vortices based on phase-shifting technology. Since we use part of the OVs to be measured as the reference light, though use phase-shifting technology, no additional reference beams are required for interference anymore, which has not been reported before.
2. THEORY AND METHODS

A PHASE-SHIFTING THEORY OF SYMMETRY SUPERPOSED OVS

The self-interference method requires the measurement of four intensities $I(x, y; \phi_i)$ of superposed OVs. $\phi_i$ denotes the shifting-phase of one single component compared with another. For simplicity without loss of generality, we only consider the situation that the superposed OVs only have two symmetry components.

May the two components of optical vortices be

$$E_1 = A_1 \exp(i \phi) \exp(i \phi_1)$$
$$E_2 = A_2 \exp(-i \phi)$$

(1)

where $\phi = 0, \pi / 2, \pi, 3\pi / 2$. We can get four intensity distributions of superposed OVs:

$$I(x, y; \phi) = \begin{cases} I(x, y) + I'(x, y) \cos(2\phi) & I(x, y; \pi / 2) = I(x, y) - I'(x, y) \sin(2\phi) \newline I(x, y; \pi) = I(x, y) - I'(x, y) \cos(2\phi) & I(x, y; 3\pi / 2) = I(x, y) + I'(x, y) \sin(2\phi) \end{cases}$$

(2)

where $I'(x, y) = A_1^2 + A_2^2$, $I'(x, y) = 2A_1 A_2$. Consequently, we can calculate the phase $\phi$:

$$\phi = \frac{1}{2} \tan^{-1} \left( \frac{I(x, y; \pi/2) - I(x, y; \pi/2)}{I(x, y) - I(x, y; \pi)} \right)$$

(3)

Although the intensity of the two components does not affect the final result, according to interference contrast:

$$\gamma = \frac{I_{\text{max}} - I_{\text{min}}}{I_{\text{max}} + I_{\text{min}}}$$

(4)

we set the two equals, in order to get the best interference pattern. Where

$$I_{\text{max}} = A_1 + A_2; I_{\text{min}} = A_1 - A_2$$

(5)

B PHASE-SHIFTING THEORY OF ASYMMETRY SUPERPOSED OVS

We further promote the results. If the two components carry different OAM, that is

$$E_1 = A_1 \exp(i \phi_1) \exp(i \phi)$$
$$E_2 = A_2 \exp(i \phi_2)$$

(6)

where $\theta$ denotes azimuth angle. The result of phase-shifting is $\phi = \phi_1 - \phi_2$. If the TCs of superposed OVs are $m_1$ and $m_2$, we can get the final results are

$$\phi_1 = \frac{m_1}{m_1 - m_2}; \phi_2 = \frac{m_2}{m_1 - m_2}$$

(7)

C GENERATION OF PHASE-SHIFTING SUPERPOSED OVS

In this part, we briefly introduce the process of generating phase-shifting superposed OVs. To obtain four intensities of phase-shifting superposed OVs, four corresponding holograms are required. Each hologram is encoded with the sum of the phase-shifting superposed OVs and the blazed grating, as shown in Fig 1. The blazed grating is to prevent part of the unmodulated light from mixing into the required superposed OVs, diffracting the required light beam to the first order, and leave the unmodulated light in the zero order. This is due to the gaps in the SLM liquid crystal arrangement. The amplitude modulation is also added to the actual operation employed with the technique shown in ref.[39, 40]. Although using an SLM for complex amplitude modulation sacrifices phase depth, it allows us to radially modulate the incident light field to generate an eigenmode of OVs instead of hypergeometric mode[41]. Load the encoded holograms onto the SLM and we can get phase-shifting superposed OVs.
Fig 1 Schematic representation of generating phase-shifting superposed OVs. The TCs of superposed OVs here are ±1. (a) The phase of a blazed grating. (b) The phase of one component in superposed OVs. (c1-c4) Phases of another component of superposed OVs after phase-shifting. (d1-d4) Encoded holograms. The amplitude modulation is also added to the actual operation. (e1-e4) Experimental intensities of phase-shifting superposed OVs generated by (d1-d4). The color bar 1 indicates the phases range. The color bar 2 indicates the range of intensities.

D CALCULATING THE PHASE OF SINGLE COMPONENT IN SUPERPOSED OVS

Theoretically speaking, after obtaining four intensity distribution results, the desired phase can be deduced inversely. However, in the actual calculation, due to the introduction of trigonometric functions, the desired phase is wrapped in $2\pi$. We cannot get the desired result by simply dividing by 2, as shown in Eqs.(3). Consequently, the phase unwrapping technology is required[42]. Firstly, we need to trim the phase to achieve maximum efficiency of phase unwrapping, as depicted in Fig 2(b) and Fig 2(c). Finally, we can get the unwrapping phase by employed the phase unwrapping technology shown in ref.[42], as depicted in Fig 2(e1) and Fig 2(e2). We can directly perform phase unwrapping, or perform windowed Fourier transform[43] (WFT) first and then perform phase unwrapping. The WFT is adopted here to filter the trimmed phase to further reduce noise and smooth the phase. The unwrapped phase obtained directly in the laboratory environment is ideal, but it does not mean that good results are maintained under complex conditions such as outdoors. Therefore, The WFT is optional according to the actual situation.
Fig 2. Schematic representation of calculating required phases. The TCs of superposed OVs here are $\pm 1$. (a1-a4) Intensities of four phase-shifting superposed OVs. (b) Calculated phase. The red dotted lines indicate the trimming area. (c) Trimmed phase. (d) Trimmed phase filtered by WFT (optional). (e1-e2) Unwrapped phase. The color bar 1 indicates the phases range. The color bar 2 indicates the range of intensities.

3. EXPERIMENTAL RESULTS

The experimental setup employed to generate a PPOV is shown in the figure. The laser (NEWPORT N-LHP-151) delivers a collimated Gaussian beam with wavelength of 632.8 nm after a linear polarizer (LP), a half-wave plate (HWP), and a telescope consisting of two lenses (L1, L2) are used for collimation. The combination of the LP and the HWP is served to rotate the laser polarization state along the long display axis of SLM and adjust the power of incident light on SLM. The SLM (UPOLABS HDSLM80R) precisely modulates the incident light via loading a hologram mentioned above, and then the aperture (AP) is used to select the first diffraction order of the beam to avoid other stray light. A CCD camera (NEWPORT LBP2) registers the intensity pattern after L4.
Experimental results of phase unwrapping are depicted in Fig 4. According to the distributions of OAM, in general, all situations can be divided into symmetrical distribution and asymmetrical distribution. These two situations can be subdivided into four categories. In the situation of symmetrical distribution, one is radial node \( p \) existed, and another is high order superposed OVs. In the situation of asymmetrical distribution, one is \( |m_1 - m_2| \leq |m_1| \) or \( |m_2| \), and another is \( |m_1 - m_2| > |m_1| \) and \( |m_2| \). We are going to demonstrate the process of phase unwrapping in different situations below.

In the situation of symmetrical distribution, as an effective complement to the example as shown in Fig 1, we demonstrate the versatility of this method when the radial node \( p \) existed, as shown in Fig 4(a1-e1). At the same time, this method is also suitable for high-order superposed OVs, as depicted in Fig 4(a2-e2) and Fig 4(a3-e3). In the situation of asymmetrical distribution, when \( |m_1 - m_2| \leq |m_1| \) or \( |m_2| \), the phase processed by the phase-shifting
technology is the required phase, and the phase unwrapping is no longer required, as shown in Fig 4(a4-e4). Correspondingly, when \( |m_1 - m_2| \geq |m_1| \) and \( |m_2| \), since the calculated phase is the OV of odd-numbered TCs, the phase cannot be unwrapped, as shown in Fig 4(a6-e6). So, it is necessary to directly subtract the phase from the calculated phase. This operation retains the error of phase-shifting technique for phase calculation. Consequently, it does not affect the result theoretically. Finally, there is a special situation that needs to be introduced: multiple phase unwrapping is required in the process of calculating the phase, as shown in Fig 4(a7-e7, f, g). We have performed phase unwrapping three times in total to get the phase distribution of the fundamental mode. To get the final phase distribution, we also need to multiply the corresponding TCs –5 and 3.

We selected 4 groups of representative phases to calculate the mode purity. The mode purity for an arbitrary field \( \Psi \) can be calculated with[44, 45]:

\[
\Gamma_p = \frac{\sum_{\nu}^\infty C^p_m}{\sum_{\nu}^\infty \sum_{\eta}^\infty C^p_m}, \quad \Gamma_m = \frac{\sum_{\eta}^\infty C^p_m}{\sum_{\nu}^\infty \sum_{\eta}^\infty C^p_m}
\]

(8)

where

\[
C^p_m = \int_0^\infty \left( a^p_m(r, \phi, z) \right) a^p_m(r, \phi, z) r dr
\]

(9)

\[
a^p_m(r, \phi, z) = 1/(2\pi)^{1/2} \int_0^{2\pi} \Psi(r, \phi, z) LG^p_m d\phi
\]

(10)

The calculated mode purity of single component is shown in Fig 5. Whether it is radial decomposition or angular decomposition, the model purity can be calculated by self-interference method. Experimental results show that the purity of the angular mode (OAM spectrums) is one order of magnitude higher than that of the radial mode (radial distributions), which is consistent with the more robustness of the angular distribution of an OV.

![Fig 5 Experimental distributions of single component mode purity. (a) Radial distributions of \( m=2 \) ( \( p=0 \) ) and \( m=-1 \) ( \( p=1 \) ) from superposed OVs \( m=1 \& 2 \) ( \( p=0 \) ) and \( m=\pm 1 \) ( \( p=1 \) ), respectively. OAM spectrums of OVs \( m=-5 \) and \( m=3 \) from superposed OVs \( m=-5 \& 3 \) and \( m=\pm 3 \), respectively.](image)

4. DISCUSSION

In this part, we would like to discuss the effect of experimental error on the results. Firstly, we demonstrate the effect of different exposure times on the measurement. The results show that as the exposure time increases, the measured mode purity decreases slightly, as shown in Fig 6(a). This can be explained that without affecting the sampling, the increase in exposure time causes the noise in the image to increase, so the mode purity is slightly reduced. Secondly, we demonstrate the effect of CCD tilt on the measurement. The results show that as the CCD tilt increases, the measured mode purity decreases slightly, as shown in Fig 6(a). This can be explained that the tilt of CCD affects the symmetry distribution of superposed OVs, which in
turn affects the accuracy of the phase calculation, as shown in Fig 6(b). Finally, we demonstrate the effect of CCD shift on the measurement. The results show that as the CCD shift increases, the measured mode purity decreases slightly, as shown in Fig 6(c). Although the CCD shift does not change the distribution of the superposed OV's, if we still perform mode decomposition at the position shown in 0 pixel, the accuracy of the mode decomposition will be reduced.

![Fig 6 The effect of experimental error on the results. The effect of (a) exposure times, (b) CCD tilt and (c) CCD shift on the measurement of the mode purity.](image)

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Disclosures

The authors declare no conflicts of interest.

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