Non-leptonic two-body decays of $\Lambda_b^0$ in light-front quark model

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Abstract

We study the non-leptonic two-body weak decays of $\Lambda_b^0 \to pM$ with $M = (\pi^-, K^-)$ and $(\rho^-, K^{*-})$ in the light-front quark model under the generalized factorization ansatz. By considering the Fermi statistic between quarks and determining spin-flavor structures in baryons, we calculate the branching ratios ($B$s) and CP-violating rate asymmetries ($A_{CP}$s) in the decays. Explicitly, we find that $B(\Lambda_b^0 \to p\pi^-, pK^-) = (4.18 \pm 0.15 \pm 0.30, 5.76 \pm 0.88 \pm 0.23) \times 10^{-6}$ and $A_{CP}(\Lambda_b^0 \to p\pi^-, pK^-) = (-3.60 \pm 0.14 \pm 0.14, 6.36 \pm 0.21 \pm 0.18)\%$ in comparison with the data of $B(\Lambda_b^0 \to p\pi^-, pK^-) = (4.5 \pm 0.8, 5.4 \pm 1.0) \times 10^{-6}$ and $A_{CP}(\Lambda_b^0 \to p\pi^-, pK^-) = (-2.5 \pm 2.9, -2.5 \pm 2.2)\%$ given by the Particle Data Group, respectively. We also predict that $B(\Lambda_b^0 \to pp^-, pK^{*-}) = (12.13 \pm 3.27 \pm 0.91, 2.58 \pm 0.87 \pm 0.13) \times 10^{-6}$ and $A_{CP}(\Lambda_b^0 \to pp^-, pK^{*-}) = (-3.32 \pm 0.00 \pm 0.14, 19.25 \pm 0.00 \pm 0.80)\%$, which could be observed by the experiments at LHCb.
I. INTRODUCTION

It is known that $b$-physics has been providing us with a nature platform to observe CP-violating phenomena, test the heavy quark effective theory, and explore physics beyond the Standard Model (SM). Among the various processes, the weak decays of $\Lambda^0_b$ as a complementary of the $B$-meson ones give us an opportunity to verify the QCD factorization hypothesis. In the recent years, several interesting $\Lambda^0_b$ decay processes have been measured by the LHCb Collaboration, such as the two-body non-leptonic mode of $\Lambda^0_b \to \Lambda \phi$ [1] and the radiative decay of $\Lambda^0_b \to \Lambda \gamma$ [2]. In addition, the direct CP violating rate asymmetries in $\Lambda^0_b \to p \pi^-$ and $\Lambda^0_b \to p K^-$ have also been searched with the most recent data of $(-2.5 \pm 2.9)\%$ and $(-2.5 \pm 2.2)\%$ [3, 4], respectively. One expects that LHCb will accumulate more and more high quality data after the upgrade and lead b-physics into a precision era. There is no doubt that comprehensive studies in the $\Lambda^0_b$ processes are necessary. Particularly, only theoretical estimations could not be able to understand the future LHCb experimental measurements.

To describe the exclusive $\Lambda^0_b$ decays, there are lots of different QCD approaches, such as the generalized factorization approach (GFA) [5, 6], perturbative QCD (pQCD) method [7], light-front quark model (LFQM) [8, 9], MIT bag model (MBM) [10], and light cone sum rule (LCSR) [11].

In this work, we concentrate on the non-leptonic two body decays of $\Lambda^0_b \to p M$ with $M$ representing a pseudoscalar (P) or vector (V) meson in the final states. We use the effective Hamiltonian including the QCD-penguin and electroweak-penguin operators and the effective Wilson coefficients are evaluated at the renormalization scale $\mu = 2.5$ GeV in the NLL precision [12, 13]. To obtain the decay amplitudes, we follow GFA to split the matrix elements into two pieces, resulting in that the only relevant quantities for the decay amplitudes are meson decay constants and baryon transition form factors. The hardest part to get the decay amplitude is to calculate the baryonic transition form factors in $\Lambda^0_b \to p$ because of the complicated baryon structures and the non-perturbative nature of QCD at the low energy scale. To extract the form factors, we use LFQM, which has been widely used in the $B$-meson [14-17] and heavy to heavy baryonic transition systems [18, 19]. The greatest advantage of LFQM is that we can deal the baryon states consistently with different momenta because of the boost invariance property in the light front dynamics. As a trade off, we are only allowed to evaluate the form factors in the space-like region to avoid the
zero-modes or so called Z-graphs, which are hard to be calculated in LFQM. We analyze the branching ratios of the pseudoscalar modes for \( \Lambda^0_b \rightarrow (p\pi^-, pK^-) \) and their corresponding CP-violating asymmetries, and compare them with the current experimental data. We also predict the vector decay modes of \( \Lambda^0_b \rightarrow (p\rho^-, pK^{*-}) \). In particular, we would like to check if the sizable CP-violating rate asymmetry in \( \Lambda^0_b \rightarrow pK^{*-} \), predicted to be as large as 20\% in GFA [5, 6], can be confirmed in LFQM.

This paper is organized as follows. In Sec. II, we present our formalism, which contains the defective Hamiltonians, decay widths and asymmetries, vertex functions of the baryons and baryonic transition form factors in LFQM. We show our numerical results of the form factors, branching ratios and CP asymmetries and compare our results with those in the literature in Sec. III. In Sec. IV, we give our conclusions.

II. FORMALISM

A. Effective Hamiltonians

To study the exclusive two-body non-leptonic processes of \( \Lambda^0_b \rightarrow pM \) with \( M \) being the pesudoscalar \( P = (\pi^-, K^-) \) and vector \( V = (\rho^-, K^{*-}) \) mesons, we start with the effective Hamiltonians of \( b \rightarrow qu\bar{u} \) (\( q = d, s \)) at quark level, given by

\[
H_{\text{eff}} = \frac{G_F}{\sqrt{2}} \left[ V_{ub} V_{uq}^* (C_1 O_1 + C_2 O_2) - V_{tb} V_{tq}^* \sum_{i=3}^{10} C_i O_i \right],
\]

where \( G_F \) is the Fermi constant, \( C_i \) stand for the Wilson coefficients evaluated at the renormalization scale \( \mu \), \( V_{q_1q_2} \) represent the CKM quark mixing matrix elements, and \( O_{1-10} \) are the operators, given as

\[
\begin{align*}
O_1 &= (\bar{q}u)_{V-A}(\bar{u}b)_{V-A}, \\
O_2 &= (\bar{q}_\beta u_\alpha)_{V-A}(\bar{u}_\alpha b_\beta)_{V-A} \\
O_3 &= (\bar{q}b)_{V-A} \sum_Q (\bar{Q}Q)_{V-A}, \\
O_4 &= (\bar{q}_\beta b_\alpha)_{V-A} \sum_Q (\bar{Q}_\alpha Q_\beta)_{V-A} \\
O_5 &= (\bar{q}b)_{V-A} \sum_Q (\bar{Q}Q)_{V+A}, \\
O_6 &= (\bar{q}_\beta b_\alpha)_{V-A} \sum_Q (\bar{Q}_\alpha Q_\beta)_{V+A} \\
O_7 &= \frac{3}{2}(\bar{q}b)_{V-A} \sum_Q e_Q (\bar{Q}Q)_{V+A}, \\
O_8 &= \frac{3}{2}(\bar{q}_\beta b_\alpha)_{V-A} \sum_Q e_Q (\bar{Q}_\alpha Q_\beta)_{V+A} \\
O_9 &= \frac{3}{2}(\bar{q}b)_{V-A} \sum_Q e_Q (\bar{Q}Q)_{V-A}, \\
O_{10} &= \frac{3}{2}(\bar{q}_\beta b_\alpha)_{V-A} \sum_Q e_Q (\bar{Q}_\alpha Q_\beta)_{V-A}.
\end{align*}
\]
Here, $O_{1,2}$, $O_{3-6}$ and $O_{7-10}$ correspond to the tree, QCD and electroweak-penguin loop operators, respectively, while $Q = u, d, s, c, b$ for $\mu = O(m_b)$. To calculate the decays of $\Lambda_b^0 \to pM$, we need to find the matrix elements for the operators, given by

$$\langle pM|H_{\text{eff}}|\Lambda_b^0 \rangle = \sum_{i,j} c_i^{\text{eff}}(\mu) \langle pM|O_{ij}|\Lambda_b^0 \rangle,$$  \hspace{1cm} (3)

where the explicit expressions for the effective Wilson coefficient $c_i^{\text{eff}}(\mu)$ can be found in Refs. [12, 13]. Note that the physical matrix elements on the left-hand side of Eq. (3) should be independent of the renormalization scheme and scale.

Based on GFA, each element can be written as

$$\langle pM|O_{ij}|\Lambda_b^0 \rangle = \langle M|(\bar{u}q)|0\rangle \langle p|(|\bar{b}a)|\Lambda_b^0 \rangle,$$  \hspace{1cm} (4)

where the explicit Dirac and color indices are suppressed. As a result, the decay amplitudes are governed by the mesonic and baryonic transitions separately. For the former, we can use the definitions, given by

$$\langle 0|A^\mu|P\rangle = i f_P p_P^\mu, \quad \langle 0|A^\mu|V\rangle = f_V m_V \epsilon^\mu,$$  \hspace{1cm} (5)

where $f_M (M = P, V)$ are the meson decay constants, $m_M$ correspond to the masses of $M$, and $p^\mu(\epsilon^\mu)$ is the momentum (polarization) vector for $P(V)$. The latter can be related to the baryonic transition form factors, defined by

$$\langle p|V_\mu - A_\mu|\Lambda_b \rangle = \bar{u}_p(p_f) \left( f_1(k^2)\gamma_\mu - i\sigma_{\mu\nu} \frac{k^\nu}{m_{\Lambda_b}} f_2(k^2) + \frac{k_\mu}{m_{\Lambda_b}} f_3(k^2) \right) u_{\Lambda_b}(p_i)$$

$$- \bar{u}_p(p_f) \left( g_1(k^2)\gamma_\mu - i\sigma_{\mu\nu} \frac{k^\nu}{m_{\Lambda_b}} g_2(k^2) + \frac{k_\mu}{m_{\Lambda_b}} g_3(k^2) \right) \gamma_5 u_{\Lambda_b}(p_i),$$  \hspace{1cm} (6)

where $k^\mu = p_i^\mu - p_f^\mu$ and $k^2 = m^2_{P(V)}$. The matrix elements for $O_{5-8}$, which have the $(V - A)(V + A)$ structure, can be calculated by the employment of the Dirac equation after applying the Fierz transformation and factorization, given by

$$\langle pM|(V - A)(V + A)|\Lambda_b^0 \rangle = -2 \langle M|(S + P)|0\rangle \langle p|(S - P)|\Lambda_b^0 \rangle$$

$$= - \left[ R_1\langle p|V_\mu|\Lambda_b^0 \rangle + R_2\langle p|A_\mu|\Lambda_b^0 \rangle \right] \langle M|(V - A)^\mu|0\rangle,$$  \hspace{1cm} (7)

with

$$R_1 = \frac{2m^2_M}{(m_b - m_u)(m_q + m_u)}, \quad R_2 = \frac{2m^2_M}{(m_b + m_u)(m_q + m_u)},$$  \hspace{1cm} (8)
where the quark masses are the current quark ones. Consequently, the amplitude for \( \Lambda_b^0 \to pP \) is written as

\[
A(\Lambda_b^0 \to pP) = i \frac{G_F}{\sqrt{2}} f_P \bar{u}_p(p_f) \left\{ [V_{ub} V_{ts}^* (a_4 + a_{10})][f_1(k^2)(m_{\Lambda_b} - m_p) + g_1(k^2)(m_{\Lambda_b} + m_p)\gamma_5 - V_{tb} V_{tq}^* (a_6 + a_8)]R_1f_1(k^2)(m_{\Lambda_b} - m_p) - R_2g_1(k^2)(m_{\Lambda_b} - m_p)\gamma_5 \right\} u_{\Lambda_b}(p_i), \tag{9}
\]

where \( a_i = c_i^{\text{eff}} + c_i^{\text{eff}} / N_{c_{\text{eff}}} \) for \( i = \text{odd} (\text{even}) \) with the effective color number \( N_{c_{\text{eff}}} \) to parameterize the non-factorizable QCD effects of the octet-octet operators in Eq. (4). Here, \( f_2 \) and \( g_2 \) have no contributions to the amplitude due to the anti-symmetric structure of \( \sigma_{\mu\nu} \), while the terms associated with \( f_3 \) and \( g_3 \) are suppressed by the factor of \( m_P^2/m_{\Lambda_b}^2 \).

Similarly, the amplitude for \( \Lambda_b^0 \to pV \) is given by

\[
A(\Lambda_b^0 \to pV) = i \frac{G_F}{\sqrt{2}} f_V m_V \epsilon^\mu \bar{u}_p(p_f) \left\{ [V_{ub} V_{ts}^* (a_4 + a_{10})] \left[ \left( f_1(k^2) + f_2(k^2) \frac{m_{\Lambda_b} + m_p}{m_{\Lambda_b}} \right) \gamma_\mu - 2f_2(k^2)(p_f)_\mu \right. \\
\left. - \left( g_1(k^2) - g_2(k^2) \frac{m_{\Lambda_b} - m_p}{m_{\Lambda_b}} \right) \gamma_\mu \gamma_5 - 2g_2(k^2)(p_f)_\mu \gamma_5 \right] \right\} u_{\Lambda_b}(p_i). \tag{10}
\]

**B. Decay widths and CP asymmetries**

The decay widths of \( \Lambda_b^0 \to pM (M = P, V) \) can be found from Eqs. (9) and (10), read as

\[
\Gamma(\Lambda_b^0 \to pP) = \frac{p_e}{16\pi} G_F^2 f_P^2 \left\{ \frac{(m_{\Lambda_b} + m_p)^2 - m_P^2}{m_{\Lambda_b}^2} \right\} \left[ V_{ub} V_{ts}^* (a_4 + a_{10}) \right]^2 f_1^2(m_P^2)(m_{\Lambda_b} - m_p)^2 + \frac{(m_{\Lambda_b} - m_p)^2}{m_{\Lambda_b}^2} - \frac{m_P^2}{m_{\Lambda_b}^2} \\
\left[ V_{ub} V_{ts}^* (a_4 + a_{10}) - R_2(a_6 + a_8) \right]^2 g_1^2(m_P^2)(m_{\Lambda_b} + m_p)^2 \right\}, \tag{11}
\]

and

\[
\Gamma(\Lambda_b^0 \to pV) = \frac{p_e}{8\pi} \frac{E_p + m_p}{m_{\Lambda_b}} G_F^2 f_V^2 \left\{ \frac{m_{\Lambda_b} + m_p}{m_{\Lambda_b}} \right\} V_{ub} V_{ts}^* (a_4 + a_{10})^2 \left\{ \frac{2}{m_{\Lambda_b}} - \frac{g_1(m_V^2) - m_{\Lambda_b} - m_p}{m_{\Lambda_b}} + \frac{E_p - m_p}{E_p + m_p} \left( f_1(m_V^2) \right)^2 \\
+ \frac{m_{\Lambda_b} + m_p}{m_{\Lambda_b}} f_2(m_V^2)^2 \right\} \left\{ \frac{2}{m_{\Lambda_b}} - \frac{g_1(m_V^2) - m_{\Lambda_b} - m_p}{m_{\Lambda_b}} + \frac{E_p - m_p}{E_p + m_p} \left( f_1(m_V^2) \right)^2 \right\} \right\}, \tag{12}
\]
where \( p_c \) is the momentum in the center mass frame and \( E_p \) is the energy of the proton.

The direct CP-violating rate asymmetry is defined by

\[
\mathcal{A}_{CP}(\Lambda^0 \to pM) \equiv \frac{\Gamma(\Lambda^0 \to pM) - \Gamma(\bar{\Lambda}^0 \to \bar{p}\bar{M})}{\Gamma(\Lambda^0 \to pM) + \Gamma(\bar{\Lambda}^0 \to \bar{p}\bar{M})}
\]  

(13)

where \( \Gamma(\Lambda^0 \to pM) \) and \( \Gamma(\bar{\Lambda}^0 \to \bar{p}\bar{M}) \) are the decay widths of the particle and antiparticle, respectively.

C. Vertex functions of baryons

In LFQM, a baryon with its momentum \( P \) and spin \( S \) as well as the z-direction projection of spin \( S_z \) are considered as a bound state of three constitute quarks. As a result, the baryon state can be expressed by [20–26]

\[
|B, P, S, S_z\rangle = \int \left\{ d^3 \tilde{p}_1 \right\} \left\{ d^3 \tilde{p}_2 \right\} \left\{ d^3 \tilde{p}_3 \right\} 2(2\pi)^3 \frac{1}{\sqrt{P^+}} \delta^3(\tilde{P} - \tilde{p}_1 - \tilde{p}_2 - \tilde{p}_3)
\]

\[
\times \sum_{\lambda_1, \lambda_2, \lambda_3} \Psi^{SS_\perp}(\tilde{p}_1, \tilde{p}_2, \tilde{p}_3, \lambda_1, \lambda_2, \lambda_3) C^{\alpha\beta\gamma} F_{abc} q^a_\alpha(\tilde{p}_1, \lambda_1) q^b_\beta(\tilde{p}_2, \lambda_2) q^c_\gamma(\tilde{p}_3, \lambda_3),
\]  

(14)

where \( \Psi^{SS_\perp}(\tilde{p}_1, \tilde{p}_2, \tilde{p}_3, \lambda_1, \lambda_2, \lambda_3) \) is the vertex function, which can be formally solved from the Bethe-Salpeter equations by the Faddeev decomposition method, \( C^{\alpha\beta\gamma} (F_{abc}) \) is the color (flavor) factor, \( \lambda_i (\tilde{p}_i) \) with \( i = 1, 2, 3 \) are the LF helicities (3-momenta) of the on-mass-shell constituent quarks, defined as

\[
\tilde{p}_i = (p_i^+, p_i^\perp), \quad p_i^\perp = (p_i^1, p_i^2), \quad p_i^- = \frac{m_i^2 + p_i^2}{p_i^+},
\]

(15)

and

\[
d^3\tilde{p}_i \equiv \frac{dp_i^+ dp_i^\perp}{2(2\pi)^3}, \quad \delta^3(\tilde{p}) = \delta(p^+)\delta^3(p^\perp),
\]

\[
|q^a_\alpha(\tilde{p}, \lambda)| = d^a_\alpha(\tilde{p}, \lambda)|0\rangle, \quad \{ d^{a'}_\alpha(\tilde{p}', \lambda'), d^a_\alpha(\tilde{p}, \lambda) \} = 2(2\pi)^3 \delta^3(\tilde{p}' - \tilde{p})\delta_{\lambda\lambda'}\delta_{a'a}\delta^{a'a},
\]  

(16)

respectively. The internal motions of the constituent quarks are described by the kinematic variables of \( (q_\perp, \xi) \), \( (Q_\perp, \eta) \) and \( P_{tot} \), given by

\[
P_{tot} = \tilde{P}_1 + \tilde{P}_2 + \tilde{P}_3, \quad \xi = \frac{p_1^+}{p_1^+ + p_2^+}, \quad \eta = \frac{p_1^+ + p_2^+}{P_{tot}^+},
\]

\[
q_\perp = (1 - \xi)p_1^\perp - \xi p_2^\perp, \quad Q_\perp = (1 - \eta)(p_1^\perp + p_2^\perp) - \eta p_3^\perp,
\]

(17)

where \( (q_\perp, \xi) \) characterize the relative motion between the first and second quarks, while \( (Q_\perp, \eta) \) the third and other two quarks. The invariant masses of \( (q_\perp, \xi) \) and \( (Q_\perp, \eta) \) systems
are represented by [20]

\[
M_3^2 = \frac{q_2^2}{\xi(1 - \xi)} + \frac{m_1^2}{\xi} + \frac{m_2^2}{1 - \xi},
\]

\[
M^2 = \frac{Q_3^2}{\eta(1 - \eta)} + \frac{M_3^2}{\eta} + \frac{m_3^2}{1 - \eta},
\]

respectively. The vertex function of \(\Psi^{SS_z}(\tilde{p}_1, \tilde{p}_2, \tilde{p}_3, \lambda_1, \lambda_2, \lambda_3)\) in Eq. (14) can be written as [20, 24, 27]

\[
\Psi^{SS_z}(\tilde{p}_1, \tilde{p}_2, \tilde{p}_3, \lambda_1, \lambda_2, \lambda_3) = \Phi(q_\perp, \xi, Q_\perp, \eta)\Xi^{SS_z}(\lambda_1, \lambda_2, \lambda_3),
\]

where \(\Phi(q_\perp, \xi, Q_\perp, \eta)\) is the momentum distribution of the constituent quarks and \(\Xi^{SS_z}(\lambda_1, \lambda_2, \lambda_3)\) represents the momentum-dependent spin wave function, given by

\[
\Xi^{SS_z}(\lambda_1, \lambda_2, \lambda_3) = \sum_{s_1, s_2, s_3} \langle \lambda_1| R_1^\dagger | s_1 \rangle \langle \lambda_2| R_2^\dagger | s_2 \rangle \langle \lambda_3| R_3^\dagger | s_3 \rangle \left\langle \frac{1}{2} s_1, \frac{1}{2} s_2, \frac{1}{2} s_3 | SS_z \right\rangle,
\]

with \(\langle \frac{1}{2} s_1, \frac{1}{2} s_2, \frac{1}{2} s_3 | SS_z \rangle\) the usual \(SU(2)\) Clebsch-Gordan coefficient, and \(R_i\) the Melosh transformation, corresponding to the \(i\)th constituent quark, expressed by [20, 28]

\[
R_1 = R_M(\eta, Q_\perp, M_3, M)R_M(\xi, q_\perp, m_1, M_3),
\]

\[
R_2 = R_M(\eta, Q_\perp, M_3, M)R_M(1 - \xi, -q_\perp, m_2, M_3),
\]

\[
R_3 = R_M(1 - \eta, -Q_\perp, m_3, M),
\]

with

\[
R_M(x, p_\perp, m, M) = \frac{m + xM - i\vec{\sigma} \cdot (\vec{n} \times \vec{p})}{\sqrt{(m + xM)^2 + p_\perp^2}},
\]

where \(\vec{\sigma}\) stands for the Pauli matrix and \(\vec{n} = (0, 0, 1)\).

In principle, one could solve the momentum wave function by introducing the QCD-inspired effective potential like the one-gluon exchange one. However, the equation will become too complicated to be solved in the three-body case. Therefore, we use the phenomenological Gaussian type wave function with some suitable shape parameters to describe the momentum distributions of the constituent quarks. It is also possible to compare the LFQM wave function with the light-cone distribution amplitude to get the QCD renormalization improvement at different energy scales [29]. The baryon spin-flavor-momentum wave function \(F_{abc}\Psi^{SS_z}(\tilde{p}_1, \tilde{p}_2, \tilde{p}_3, \lambda_1, \lambda_2, \lambda_3)\) should be totally symmetric under any permutations
of the quarks to keep the Fermi statistics. The spin-flavor-momentum wave functions of $\Lambda_b^0$ and $p$ are given by

$$|\Lambda_b^0\rangle = \frac{1}{\sqrt{6}} [\phi_3 \chi^{\rho_3}(|dub\rangle - |uwb\rangle) + \phi_2 \chi^{\rho_2}(|dub\rangle - |uwd\rangle) + \phi_1 \chi^{\rho_1}(|dub\rangle - |uwd\rangle)],$$

$$|p\rangle = \frac{1}{3} [\chi^{\rho_3}(|udu\rangle - |dum\rangle) + \chi^{\rho_2}(|udu\rangle - |dum\rangle) + \chi^{\rho_1}(|udu\rangle - |dum\rangle)],$$

(23)

where the explicit forms for $\phi_{1,2}$ can be found in Ref. [21],

$$\phi_3 = \mathcal{N} \sqrt{\frac{\partial q_z \partial Q_z}{\partial \xi \partial \eta}} e^{-\frac{q_z^2}{2\beta_q^2} - \frac{Q_z^2}{2\beta_Q^2}}, \quad \chi^{\rho_3} = \frac{1}{\sqrt{2}} (|\uparrow\downarrow\uparrow\rangle - |\downarrow\uparrow\uparrow\rangle),$$

(24)

and

$$q_z = \frac{\xi M_3}{2} - \frac{m_1^2 + q_{\perp}^2}{2M_3\xi}, \quad Q_z = \frac{\eta M}{2} - \frac{M_3^2 + Q_{\perp}^2}{2M\eta},$$

$$q = (q_{\perp}, q_z), \quad Q = (Q_{\perp}, Q_z),$$

(25)

with $\mathcal{N} = 2(2\pi)^3(\beta_q\beta_Q\pi)^{-3/2}$ and $\beta_q,\beta_Q$ being the normalized constant and shape parameters, respectively.

Here, the baryon state is normalized as

$$\langle B, P', S', S_z'| B, P, S, S_z \rangle = 2(2\pi)^3 P^+ \delta^3(\vec{P}' - \vec{P})\delta_{S_z'S_z},$$

(26)

resulting in the normalization of the momentum wave function, given by

$$\frac{1}{2^2(2\pi)^6} \int d\xi d\eta d^2q_{\perp} d^2Q_{\perp} \phi_3^2 = 1.$$  

(27)

We emphasize that the momentum wave functions of $\phi_i$ are associated with the different shape parameters of $\beta_q$ and $\beta_Q$ in $\Lambda_b^0$. For the proton, the momentum distribution functions are the same, i.e. $\phi = \phi_3 (\beta_q = \beta_Q)$, for any spin-flavor state because of the isospin symmetry.

**D. Baryonic Transition form factors**

The baryonic transition form factors of the $V - A$ weak current are given by Eq. (6) with $P' - P = k$. We choose the frame such that $P^+$ is conserved ($k^+ = 0, k^2 = -k_{\perp}^2$) to calculate the form factors in order to avoid other diagrams involving particle productions, also known as the Z-graphs in the light-front formalism [20, 24]. The matrix elements of the vector and axial-vector currents at quark level correspond to three different effective diagrams as shown in Fig. 1. Since the spin-flavor-momentum wave functions of the baryons are totally
FIG. 1. Feynman diagrams for the baryonic weak transitions, where the bullet of “•” denotes the V-A current vertex, with (a) $p'_1 - p_1 = k$, (b) $p'_2 - p_2 = k$ and (c) $p'_3 - p_3 = k$.

symmetric under the permutation of the quarks indices, we have that $(a) + (b) + (c) = 3(a) = 3(b) = 3(c)$. We only present the calculation for the diagram (c) in Fig. 1, which contains simpler and cleaner forms with our notation $(q_\perp, Q_\perp, \xi, \eta)$. We can extract the form factors from the matrix elements through the relations,

$$
\begin{align*}
&f_1(k^2) = \frac{1}{2P_+}(B_f, P', \uparrow | \bar{q}\gamma^+ c| B_i, P, \uparrow), \\
&f_2(k^2) = \frac{1}{2P_+} \frac{M_B}{k_\perp}(B_f, P', \uparrow | \bar{q}\gamma^+ c| B_i, P, \downarrow), \\
&g_1(k^2) = \frac{1}{2P_+}(B_f, P', \uparrow | \bar{q}\gamma^+ \gamma_5 c| B_i, P, \uparrow), \\
&g_2(k^2) = \frac{1}{2P_+} \frac{M_B}{k_\perp}(B_f, P', \uparrow | \bar{q}\gamma^+ \gamma_5 | B_i, P, \downarrow). \\
\end{align*}
$$

(28)

Note that $f_3$ and $g_3$ are unobtainable when $k^+ = 0$, but they are irrelevant in this work because of the kinematic suppression of $m^2_p/m^2_{\Lambda_u}$ associated with them. The full expressions of the form factors in Eq. (28) are give by

$$
\begin{align*}
&f_1(k^2) = \frac{3}{2^2(2\pi)^6} \int d\xi d\eta d^2q_\perp d^2Q_\perp \Phi(q'_\perp, \xi, Q'_\perp, \eta)\Phi(q_\perp, \xi, Q_\perp, \eta)(F^{def} F_{abc} \delta_{q_f q_\gamma} \delta_{q_\gamma \xi} \delta_{q_\xi g}) \\
&\times \sum_{s_1, s_2, s_3} \sum_{s'_1, s'_2, s'_3} \langle S', \uparrow | s'_1, s'_2, s'_3 \rangle \langle s_1, s_2, s_3 | S, \uparrow \rangle \prod_{i=1,2,3} \langle s'_i | R'_i | s_i \rangle,
\end{align*}
$$

(29)

$$
\begin{align*}
&g_1(k^2) = \frac{3}{2^2(2\pi)^6} \int d\xi d\eta d^2q_\perp d^2Q_\perp \Phi(q'_\perp, \xi, Q'_\perp, \eta)\Phi(q_\perp, \xi, Q_\perp, \eta)(F^{def} F_{abc} \delta_{q_f q_\gamma} \delta_{q_\gamma q_\xi} \delta_{q_\xi \xi} \delta_{q_\xi g}) \\
&\times \sum_{s_1, s_2, s_3} \sum_{s'_1, s'_2, s'_3} \langle S', \uparrow | s'_1, s'_2, s'_3 \rangle \langle s_1, s_2, s_3 | S, \uparrow \rangle \prod_{i=1,2} \langle s'_i | R'_i | s_i \rangle \langle s'_3 | R'_3 \sigma_2 R'_3 \rangle | s_3 \rangle,
\end{align*}
$$

(30)

$$
\begin{align*}
&f_2(k^2) = \frac{3}{2^2(2\pi)^6} \frac{M_B}{k_\perp} \int d\xi d\eta d^2q_\perp d^2Q_\perp \Phi(q'_\perp, \xi, Q'_\perp, \eta)\Phi(q_\perp, \xi, Q_\perp, \eta)(F^{def} F_{abc} \delta_{q_f q_\gamma} \delta_{q_\gamma q_\xi} \delta_{q_\xi \xi} \delta_{q_\xi g}) \\
&\times \sum_{s_1, s_2, s_3} \sum_{s'_1, s'_2, s'_3} \langle S', \uparrow | s'_1, s'_2, s'_3 \rangle \langle s_1, s_2, s_3 | S, \downarrow \rangle \prod_{i=1,2,3} \langle s'_i | R'_i | s_i \rangle,
\end{align*}
$$

(31)
TABLE I. Numerical values of the effective Wilson coefficients with the NDR scheme and \( \overline{\text{MS}} \) at the renormalization scale of \( \mu = 2.5 \text{ GeV} \)

|       | \( b \to d \) | \( \bar{b} \to d \) | \( b \to s \) | \( \bar{b} \to \bar{s} \) |
|-------|----------------|-----------------|------------|----------------|
| \( c_1^{eff} \) | 1.168          | 1.168           | 1.168      | 1.168          |
| \( c_2^{eff} \) | -0.365         | -0.365          | -0.365     | -0.365         |
| \( 10^4c_3^{eff} \) | 238 + 14i      | 254 + 43i       | 243 + 31i  | 241 + 32i      |
| \( 10^4c_4^{eff} \) | -497 - 42i     | -545 - 130i     | -512 - 94i | -506 - 97i     |
| \( 10^4c_5^{eff} \) | 145 + 14i      | 162 + 43i       | 150 + 31i  | 148 + 32i      |
| \( 10^4c_6^{eff} \) | -633 - 42i     | -682 - 130i     | -649 - 94i | -643 - 97i     |
| \( 10^4c_7^{eff} \) | -1.0 - 1.0i    | -1.4 - 1.8i     | -1.1 - 2.2i| -1.1 - 1.3i    |
| \( 10^4c_8^{eff} \) | 5.0            | 5.0             | 5.0        | 5.0            |
| \( 10^4c_9^{eff} \) | -112 - 1.3i    | -113 - 2.7i     | -112 - 2.2i| -112 - 2.2i    |
| \( 10^4c_{10}^{eff} \) | 20             | 20              | 20         | 20             |

\[
g_2(k^2) = \frac{3}{2^2(2\pi)^6} \frac{M_B}{k_\perp} \int d\xi d\eta d^2q_1 d^2Q_\perp \Phi(q'_1, \xi, Q'_1, \eta)\Phi(q_1, \xi, Q_1, \eta)(F_{d_1} \sigma_{d_1} \delta_{d_1} \delta_{d_1}) 
\times \sum_{s_1, s_2, s_3} \sum_{s'_1, s'_2, s'_3} \langle S'_1, \uparrow | s'_1, s'_2, s'_3 \rangle \langle s_1, s_2, s_3 | S, \downarrow \rangle \prod_{i=1,2} \langle s'_i | R'_i \sigma_3 R'_i | s_i \rangle \langle s'_3 | R'_3 \sigma_3 R'_3 | s_3 \rangle . \tag{32} \]

III. NUMERICAL RESULTS

We choose the Wolfenstein parameterization for the CKM matrix with the corresponding parameters, taken to be \( [4] \)

\[
\lambda = 0.22650 \pm 0.00048, \quad A = 0.790^{+0.017}_{-0.012}, \quad \rho = 0.141^{+0.016}_{-0.017}, \quad \eta = 0.357 \pm 0.011 . \tag{33} \]

Under the naive dimensional regularization (NDR) scheme and \( \overline{\text{MS}} \) renormalization, the numerical values of the effective Wilson coefficients \( c_i^{eff}(\mu) \) at \( \mu = 2.5 \text{ GeV} \) for both \( b \to q \) and \( \bar{b} \to \bar{q} \) are given in Table. \( [12, 13] \). For the quark masses in Eq. \( [8] \), we start from the values at \( \mu = 2.0 \text{ GeV} \) given by PDG \( [4] \), and use the renormalization group equation \( [13] \) to determine them at \( \mu = 2.5 \text{ GeV} \), given by

\[
m_b(2.5 \text{ GeV}) = 4.88 \text{ GeV} \quad m_s(2.5 \text{ GeV}) = 88.1 \text{ MeV} \\
m_d(2.5 \text{ GeV}) = 4.42 \text{ MeV} \quad m_u(2.5 \text{ GeV}) = 2.04 \text{ MeV} \tag{34} \]

10
TABLE II. Hadron masses and meson decay constants in units of GeV

| m_Λ_b | m_p | m_π | m_K | m_ρ | m_{K^*} | f_π | f_K | f_ρ | f_{K^*} |
|--------|-----|-----|-----|-----|---------|-----|-----|-----|--------|
| 5.62   | 0.94| 0.14| 0.49| 0.77 | 0.89    | 0.13| 0.16| 0.21| 0.22   |

In Table II, we show our parameter sets of the hadron masses and meson decay constants. In our numerical calculations, we also set the Λ_0^b lifetime to be \( \tau_{Λ_0^b} = 146.4 \text{ fs} \). In this work, the effective color number \( N_{c_{eff}} \) is fixed to be 3.

Since the baryonic transition form factors in LFQM can be only evaluated in the space-like region \( (k^2 = -k_{\perp}^2) \) due to the condition \( k^+ = 0 \), we take some analytic functions to fit \( f_1(2)(k^2) \) and \( g_1(2)(k^2) \) in the space-like region and perform their analytical continuations to the physical time-like region \( (k^2 > 0) \) as in Refs. [21–23, 30–32]. We employ the numerical values of the constituent quark masses and shape parameters in Table III.

By using Eqs. (29)-(32), we compute totally 32 points for all form factors from \( k^2 = 0 \) to \( k^2 = -9.7 \text{ GeV}^2 \). To fit the \( k^2 \) dependences of the form factors in the space-like region, we use the form, given by

\[
F(k^2) = \frac{F(0)}{1 - q_1 k^2 + q_2 k^4},
\]

where \( q_{1,2} \) are the fitting parameters. Our results for the form factors are presented in Table IV and compared with the results from light-cone sum rule (LCSR) [11] where the form factors are parameterized by

\[
f_i(k^2) = \frac{f_i(0)}{1 - q^2/m_{B^*(1^-)}^2} \left\{ 1 + b_i \left( z(k^2, t_0) - z(0, t_0) \right) \right\},
\]

\[
g_i(k^2) = \frac{g_i(0)}{1 - q^2/m_{B^*(1^+)}^2} \left\{ 1 + \tilde{b}_i \left( z(k^2, t_0) - z(0, t_0) \right) \right\}
\]

The definition of these parameters can be found in Ref. [11]. From the table, one has that \( f_1 \approx g_1 \), which agrees with the relation of \( f_1 = g_1 \) in the heavy quark limit. One can also see that \( f_2(k^2 = 0) > f_1(k^2 = 0) \), which is similar to the cases in the \( Λ_c^+ \) decays [21], but...
TABLE IV. Form factors for the transition matrix elements of $\Lambda_b^0 \to p$ in LFQM as well as LCSR with the axial-vector (pseudo-scalar) interpolating current of $\eta_{\Lambda_b}^{A(P)}$.

|       | $f_1$          | $f_2$          | $g_1$          | $g_2$          |
|-------|----------------|----------------|----------------|----------------|
| $F(0)$| 0.132 ± 0.003  | 0.149 ± 0.003  | 0.129 ± 0.003  | −0.020 ± 0.001 |
| $q_1$ | 0.802 ± 0.199  | 0.708 ± 0.189  | 0.7702 ± 0.187 | 0.036 ± 0.137  |
| $q_2$ | 1.164 ± 0.208  | 1.082 ± 0.109  | 1.103 ± 0.190  | 0.732 ± 0.107  |

The authors in Ref. [11] use the opposite sign convention for $f_2$($g_2$).

Different from those in LCSR [11] for the $\Lambda_b^0$ transitions. For LFQM and LCSR, the results of $f_1$($g_1$) are the same, whereas those of $f_2$($g_2$) are different, in the low-momentum transfer region. The predictions of the branching ratios and CP-asymmetries in these two models are similar because the contributions of $f_1$ and $g_1$ are dominated in $\Lambda_b^0 \to pP(V)$. The discrepancies between LFQM and LCSR appear in the $\Lambda_b^0$ semi-leptonic decays. The decay rates in these semi-leptonic decays for LFQM are much smaller than the LCSR ones as well as the data due to the lack of the information from the $B^*(1^{-(+)}\gamma)$ poles in LFQM, which are important in the calculations of $\Lambda_b^0 \to p\ell^+\bar{\nu}_\ell$ via the analytical continuation method. Our predictions of the decay branching ratios and direct CP-violating rate asymmetries are summarized in Table V where the first and second errors contain the uncertainties from the form factors and Wolfenstein parameters in Table IV, respectively. It is interesting to see that our results of $A_{CP}(\Lambda_b^0 \to pV)$ are free of the baryonic uncertainties under the generalized factorization assumption. The reason is that all form-factor dependencies in
$\Gamma(\Lambda_b^0 \rightarrow pV)$ can be totally factorized out within the generalized factorization framework so that they get canceled for the CP-violating rate asymmetries of $A_{CP}(\Lambda_b^0 \rightarrow pV)$ \cite{5, 6}. As a result, the numerical values of $A_{CP}(\Lambda_b^0 \rightarrow pV)$ are QCD model-independent. In the table, we also show the recent experimental data \cite{4} as well as other theoretical calculations in the literature \cite{6–9}, where $pQCD_a$ and $pQCD_b$ represent the conventional and hybrid $pQCD$ approaches \cite{7}, while LFQM$_a$ \cite{8} and LFQM$_b$ \cite{9} refer to those in LFQM with and without the considerations of the QCD and electroweak-penguin loop contributions, respectively. We find that our predictions of $B(\Lambda_b^0 \rightarrow p\pi^-, pK^-)$ are consistent with the experimental data and those in GFA and LCSR. In addition, we have that $B(\Lambda_b^0 \rightarrow pK^-) > B(\Lambda_b^0 \rightarrow p\pi^-)$ as indicated by the data, which is different from those predicted by $pQCD$ \cite{7}, LFQM \cite{8, 9} and MBM \cite{10}. Our results for $B(\Lambda_b^0 \rightarrow (p\rho^-, pK^*)^-)$ also agree with those in GFA \cite{6}. Note that LFQM$_b$ \cite{9} does not consider the loop contributions, so that its results are incompatible with the data as well as all other theoretical ones. For the CP-violating rate asymmetries, we find that $A_{CP}(\Lambda_b^0 \rightarrow p\pi^-) = (-3.60 \pm 0.14 \pm 0.14)\%$, consistent with the data and those from GFA \cite{6}, LFQM$_a$ \cite{8} and MBM \cite{10}, but different from $pQCD$ \cite{7}. On the other hand, we obtain a positive CP-violating rate asymmetry for $\Lambda_b^0 \rightarrow pK^-$, which is the same as GFA, LFQM$_a$ and MBM, whereas the data of $(-2.5 \pm 2.2)\%$ \cite{4} along with those from $pQCD$ is negative. It is interesting to note that, similar to GFA and LFQM$_a$, our result in LFQM also predicts a sizable asymmetry of $\sim 20\%$ in $\Lambda_b^0 \rightarrow pK^{*-}$.

IV. CONCLUSIONS

We have studied the non-leptonic two body decays of $\Lambda_b^0 \rightarrow pM$ with LFQM based on the generalized factorization ansatz. By considering the Fermi statistic between quarks and determining spin-flavor structures in baryons, we have evaluated the baryonic form factors in the LFQM. In particular, we have found that $f_1 \simeq g_1$, which agrees with the requirement in the heavy quark limit, whereas $f_2(k^2 = 0) > f_1(k^2 = 0)$, different from those in the literature for the $\Lambda_b^0$ transitions. It is possible to include the QCD renormalization effect by comparing the LFQM wave function and light-cone distribution amplitude to understand the uncertainty at different energy scales as the study in Ref. \cite{29}, which could be done in our future work. We have compared our form factors with those in LCSR, and shown that they are almost the same in the low-momentum transfer region. However, the LCSR ones
TABLE V. Our results in comparison with the experimental data and those in various theoretical calculations in the literature, where \( B \)s and \( A \)s are in units of \( 10^{-6} \) and \%, respectively. In our results, the first errors come from hadronic uncertainties and the second errors come from the uncertainties of CKM matrix.

|                | \( B(\Lambda^0_b \to p\pi^-) \) | \( B(\Lambda^0_b \to pK^-) \) | \( B(\Lambda^0_b \to pp^-) \) | \( B(\Lambda^0_b \to pK^{*-}) \) |
|----------------|---------------------------------|---------------------------------|-------------------------------|----------------------------------|
| this work      | \( 4.18 \pm 0.15 \pm 0.30 \)   | \( 5.76 \pm 0.88 \pm 0.23 \)   | \( 12.13 \pm 3.27 \pm 0.91 \) | \( 2.58 \pm 0.87 \pm 0.13 \) |
| Data [4]       | \( 4.5 \pm 0.8 \)              | \( 5.4 \pm 1.0 \)              | -                            | -                                |
| GFA [6]        | \( 4.25^{+1.04}_{-0.48} \pm 0.93 \) | \( 4.49^{+0.84}_{-0.35} \pm 0.64 \) | \( 11.03^{+2.72}_{-1.25} \pm 2.45 \) | \( 2.86^{+0.62}_{-0.29} \pm 0.52 \) |
| pQCD \( a \) [7] | \( 4.66 \)                     | \( 1.82 \)                     | -                            | -                                |
| pQCD \( b \) [7] | \( 5.21 \)                     | \( 2.02 \)                     | -                            | -                                |
| LFQM \( a \) [8] | \( 4.30 \)                     | \( 2.17 \)                     | \( 7.47 \)                   | \( 1.01 \)                       |
| LFQM \( b \) [9] | \( 8.90 \)                     | \( 0.718 \)                   | \( 26.1 \)                   | \( 1.34 \)                       |
| MBM [10]       | \( 4.5 \)                      | \( 3.4 \)                      | -                            | -                                |
| LCSR [11]      | \( 3.8^{+1.3}_{-1.0}(2.8^{+1.1}_{-0.9}) \) | -                             | -                            | -                                |

|                | \( A_{CP}(\Lambda^0_b \to p\pi^-) \) | \( A_{CP}(\Lambda^0_b \to pK^-) \) | \( A_{CP}(\Lambda^0_b \to pp^-) \) | \( A_{CP}(\Lambda^0_b \to pK^{*-}) \) |
|----------------|--------------------------------------|--------------------------------------|--------------------------------------|--------------------------------------|
| this work      | \( -3.60 \pm 0.14 \pm 0.14 \)      | \( 6.36 \pm 0.21 \pm 0.18 \)       | \( -3.32 \pm 0.00 \pm 0.14 \)       | \( 19.25 \pm 0.00 \pm 0.80 \)       |
| Data [4]       | \( -2.5 \pm 2.9 \)                 | \( -2.5 \pm 2.2 \)                 | -                                    | -                                    |
| GFA [6]        | \( -3.9^{+0.0}_{-0.0} \pm 0.4 \)   | \( 6.7^{+0.3}_{-0.2} \pm 0.3 \)    | \( -3.8^{+0.0}_{-0.0} \pm 0.4 \)    | \( 19.7^{+0.4}_{-0.3} \pm 1.4 \)    |
| pQCD \( a \) [7] | \( -32 \)                         | \( -3 \)                           | -                                    | -                                    |
| pQCD \( b \) [7] | \( -31 \)                         | \( -5 \)                           | -                                    | -                                    |
| LFQM \( a \) [8] | \( -3.37^{+0.29}_{-0.37} \)       | \( 10.1^{+1.3}_{-2.0} \)           | \( -3.19^{+0.25}_{-0.25} \)         | \( 31.1^{+2.8}_{-1.9} \)            |
| LFQM \( b \) [9] | -                                  | -                                  | -                                    | -                                    |
| MBM [10]       | \( -4.4 \)                        | \( 6.7 \)                          | -                                    | -                                    |
give better results than those in LFQM in the high-momentum transfer region because the analytical continuation method in LFQM misses the information about the $B^*(1^{+(-)})$ poles, which is important in the calculation for $\Lambda_0^b \to p\ell^- \bar{\nu}_\ell$. From these baryonic form factors in LFQM, we have found that $\mathcal{B}(\Lambda_0^b \to p\pi^-, pK^-) = (4.18 \pm 0.15 \pm 0.30, 5.76 \pm 0.88 \pm 0.23) \times 10^{-6}$ and $\mathcal{A}_{CP}(\Lambda_0^b \to p\pi^-, pK^-) = (-3.60 \pm 0.14 \pm 0.14, 6.36 \pm 0.21 \pm 0.18)$% in comparison with the data of $\mathcal{B}(\Lambda_0^b \to p\pi^-, pK^-) = (4.5 \pm 0.8, 5.4 \pm 1.0) \times 10^{-6}$ and $\mathcal{A}_{CP}(\Lambda_0^b \to p\pi^-, pK^-) = (-2.5 \pm 2.9, -2.5 \pm 2.2)$%, respectively, while $\mathcal{B}(\Lambda_0^b \to p\rho^-, pK^{*-}) = (12.13 \pm 3.27 \pm 0.23, 2.58 \pm 0.87 \pm 0.13) \times 10^{-6}$ and $\mathcal{A}_{CP}(\Lambda_0^b \to p\rho^-, pK^{*-}) = (-3.32 \pm 0.00 \pm 0.14, 19.25 \pm 0.00 \pm 0.80)$%, which could be observed by the experiments at LHCb.

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[1] R. Aaij et al. [LHCb], Phys. Lett. B 759, 282-292 (2016)
[2] R. Aaij et al. [LHCb], Phys. Rev. Lett. 123, 031801 (2019).
[3] R. Aaij et al. [LHCb Collaboration], Phys. Lett. B 787, 124 (2018).
[4] P.A. Zyla et al. (Particle Data Group), Prog. Theor. Exp. Phys. 2020, 083C01 (2020).
[5] Y. K. Hsiao and C. Q. Geng, Phys. Rev. D 91, 116007 (2015); C. Q. Geng and Y. K. Hsiao, Mod. Phys. Lett. A 31, 1630021 (2016).
[6] Y. Hsiao, Y. Yao and C. Geng, Phys. Rev. D 95, 093001 (2017).
[7] C. D. Lu, Y. M. Wang, H. Zou, A. Ali and G. Kramer, Phys. Rev. D 80, 034011 (2009).
[8] J. Zhu, Z. T. Wei and H. W. Ke, Phys. Rev. D 99, 054020 (2019).
[9] Z. X. Zhao, Chin. Phys. C 42, 093101 (2018).
[10] C. Q. Geng, C. W. Liu and T. H. Tsai, Phys. Rev. D 102, no.3, 034033 (2020)
[11] A. Khodjamirian, C. Klein, T. Mannel and Y. M. Wang, JHEP 09, 106 (2011)
[12] A. Ali, G. Kramer and C. D. Lu, Phys. Rev. D 58, 094009 (1998)
[13] A. J. Buras, M. Jamin, M. Lautenbacher and P. H. Weisz, Nucl. Phys. B 370, 69 (1992).
[14] C. Q. Geng, C. C. Lih and W. M. Zhang, Phys. Rev. D 57, 5697 (1998); D 62, 074017 (2000).
[15] C. Q. Geng, C. W. Hwang, C. C. Lih and W. M. Zhang, Phys. Rev. D 64, 114024 (2001).
[16] C. Q. Geng and C. C. Liu, J. Phys. G 29, 1103 (2003).
[17] C. Q. Geng, C. C. Lih and C. Xia, Eur. Phys. J. C 76, 313 (2016).
[18] C. K. Chua, Phys. Rev. D 99, 014023 (2019).
[19] C. K. Chua, Phys. Rev. D 100, 034025 (2019).
[20] F. Schlumpf, hep-ph/9211255.
[21] C. Q. Geng, C. C. Lih, C. W. Liu and T. H. Tsai, Phys. Rev. D 101, 094017 (2020).
[22] H. Y. Cheng, C. K. Chua and C. W. Hwang, Phys. Rev. D 70, 034007 (2004).
[23] H. W. Ke, X. Q. Li and Z. T. Wei, Phys. Rev. D 77, 014020 (2008).
[24] W. M. Zhang, Chin. J. Phys. 32, 717 (1994).
[25] H. W. Ke, N. Hao and X. Q. Li, Eur. Phys. J. C 79, 540 (2019).
[26] H. W. Ke, X. H. Yuan, X. Q. Li, Z. T. Wei and Y. X. Zhang, Phys. Rev. D 86, 114005 (2012).
[27] C. Lorcs, B. Pasquini and M. Vanderhaeghen, JHEP 1105, 041 (2011).
[28] W. N. Polyzou, W. Glockle and H. Witala, Few Body Syst. 54, 1667 (2013).
[29] G. Bell, T. Feldmann, Y. M. Wang and M. W. Y. Yip, JHEP 11, 191 (2013).
[30] H. Y. Cheng, C. K. Chua and C. W. Hwang, Phys. Rev. D 69, 074025 (2004).
[31] W. Jaus, Phys. Rev. D 44, 2851 (1991).
[32] W. Jaus, Phys. Rev. D 53, 1349 (1996) Erratum: [Phys. Rev. D 54, 5904 (1996)].