Cosmic horizon for GeV sources and photon-photon scattering

G. V. Vereshchagin\textsuperscript{1,2,3}

\textsuperscript{1} ICRANet, p.le della Repubblica, 10, 65122 Pescara, Italy and
\textsuperscript{2} ICRA and Deptartment of Physics, University of Rome “Sapienza”,
p.le A. Moro 5, 00185 Rome, Italy and
\textsuperscript{3} ICRANet-Minsk, National Academy of Sciences of Belarus,
Nezavisimosti av. 68, 220072 Minsk, Belarus

ABSTRACT

Photon-photon scattering of gamma-rays on the cosmic microwave background has been studied using the low energy approximation of the total cross section by Zdziarski & Svensson (1989); Svensson & Zdziarski (1990). Here, the cosmic horizon due to photon-photon scattering is accurately determined using the exact cross section and we find that photon-photon scattering dominates over the pair production at energies smaller than 1.68 GeV and at redshifts larger than 180.

Subject headings: Cosmic rays; Background radiation.

1. Introduction

Photon-photon scattering $\gamma_1 \gamma_2 \rightarrow \gamma'_1 \gamma'_2$ is a nonlinear electrodynamical process, allowed by quantum electrodynamics, but much less well known compared to pair production from two photons, $\gamma_1 \gamma_2 \rightarrow e^+ e^-$. The latter has not yet been directly observed, but has well known astrophysical implications [Stecker et al. 1992; Franceschini et al. 2008; Ruffini et al. 2010]. The total cross section of photon-photon scattering in the low energy approximation can be found in most textbooks on the topic, see e.g. Berestetskii et al. (1982). The exact cross section for arbitrary energies has been determined numerically, see e.g. Karplus & Neuman (1951); De Tollis (1965).

Photon-photon scattering involving cosmic microwave background (CMB) photons has been considered in the cosmological context in Zdziarski & Svensson (1989); Svensson & Zdziarski (1990). Using the low energy approximation these authors obtained analytical expressions for the cosmic horizon, i.e., the redshift as a function of particle energy found by equating the optical depth to unity. In the limit of large redshifts in the Einstein-de-Sitter universe they found

$$z = 5.002 \times 10^3 T_{2.7}^{4/5} h_{50}^{2/15} \varepsilon_{\text{obs}}^{-2/5},$$  \phantom{1} (1)$$

where the dimensionless observed energy $\varepsilon_{\text{obs}} = E_{\text{obs}}/(m_e c^2)$ of the gamma-ray photon is expressed in terms of the electron rest mass energy $m_e c^2$, the temperature $T$ of the cosmic microwave background is normalized to 2.7 K, and the Hubble parameter is $H_0 = 50h_{50}$ km/s/Mpc.
It was recognized that the slope of the relation \(1\) differs slightly from the slope of the Fazio-Stecker relation \(2\) corrected by Zdziarski & Svensson \(3\) to read

\[
z = 8.84 \times 10^3 \epsilon_{\text{obs}}^{-0.478}.
\]

The horizon relations for pair production from two photons \(\gamma_1 \gamma_2 \rightarrow e^+ e^-\) and for the photon-photon scattering \(\gamma_1 \gamma_2 \rightarrow \gamma'_1 \gamma'_2\) were determined, and found to have a crossing point at the approximate redshift \(z_{cr} \simeq 3 \times 10^2\). The authors concluded that photon-photon scattering dominates over pair production at larger redshifts.

In this paper we revisit the derivation of the cosmic horizon relation for photon-photon scattering on the CMB photon background by considering the exact cross section found by Dicus et al. \(4\), instead of the approximate one valid only in the low energy limit. One might argue that the difference between the exact cross section and its low energy approximation would be small even near the pair production threshold, but in fact the ratio between the exact and approximate cross sections at the threshold is 7.26. We emphasize that due to the very similar slopes of the two functions \(1\) and \(2\), even a small change in the cross section results in a significant shift of the crossing point \(z_{cr}\). In this paper it is shown that the above mentioned crossing point is located at a lower redshift than previously determined, namely \(z_{cr} \simeq 180\), and the corresponding photon energy is 1.68 GeV.

These new results are essential for photon propagation from sources located at very high redshifts, above 100. Specifically, such photons are present in models involving exotic particles, which decay into photons in a high redshift universe, see e.g. Mapelli et al. \(5\); Poulin et al. \(6\) and references therein.

2. Exact cross section for photon-photon scattering

The approximate cross section for photon-photon scattering in the low energy approximation is given by

\[
\sigma = \frac{7 \times 139}{345^3 \pi} \alpha^4 r_0^2 \epsilon_{CM}^6,
\]

where \(\alpha\) is the fine structure constant, \(r_0\) is classical electron radius, \(\epsilon_{CM} = \sqrt{\epsilon_1 \epsilon_2 (1 - \cos \theta) / 2} = \sqrt{x (1 - \cos \theta) / 2}\) is the center-of-momentum energy, \(\epsilon_1 = h \nu_1 / m_e c^2\) and \(\epsilon_2 = h \nu_2 / m_e c^2\) are, respectively, the dimensionless energies of the high energy photon and the CMB photon, \(h\) is Planck’s constant, \(\nu\) is the photon frequency, \(m_e\) the electron mass and \(c\) the speed of light. The cosmic horizon is obtained taking into account both the cosmic evolution of the CMB and the cosmological redshift of the high energy photon as follows. First, the cross section is averaged over all angles and integrated over the photon energy using the isotropic distribution function for the CMB photons. Then the result is integrated over distance (redshift) to obtain the optical depth as a function of the redshift of the source and the energy of the observed photon. Equating this optical depth to
unity results in the relation between the redshift of the source and the observed energy. Equation (1) was obtained precisely in this way.

Instead of using the approximate cross section (3), we take the exact cross section represented by the dotted curve in Fig. 1. It is important to emphasize that the exact form of the cross section near the threshold for pair production at \( x \equiv \varepsilon_1 \varepsilon_2 = 1 \) is crucial. The solid curve in Fig. 1 represents the angle averaged cross section. This function is integrated further with the photon distribution function and it is the value near its peak which determines the dependence of the optical depth on the particle energy and distance. It is clear that averaging over angles makes the cross section smoother and shifts the peak to higher values of the variable \( x \).

Fig. 1.— Total cross section of the photon-photon scattering as a function of the variable \( x \equiv \varepsilon_1 \varepsilon_2 \) for head on collisions with \( \vartheta = \pi \) (dotted curve). The solid curve shows the angle averaged cross section, see Eq. (4). The dashed line shows the low energy approximation \( \sim x^3 \).
3. The optical depth and the cosmic horizon for photon-photon scattering

The computation of the optical depth is straightforward, for details see e.g. Ruffini et al. (2016). The optical depth is given by

\[
\tau = 4\pi \frac{c}{H_0} \left( \frac{h}{m_e c} \right)^3 \left( \frac{kT_0}{m_e c^2} \right)^3 \left( \frac{1}{y_0} \right)^3 \int_0^z \frac{dz'}{(1 + z')^4 H(z')} \times \\
\int_0^\infty \frac{x^2 dx}{\exp(x/y) - 1} \int_0^\pi \sigma(x, y, z', \vartheta) (1 - \cos \vartheta) \sin \vartheta d\vartheta,
\]

where the variables \( x = \varepsilon_1 \varepsilon_2 \) and \( y = \varepsilon_2 kT/(m_e c^2) \) depend on the redshift, the index “0" refers to the observed photon at redshift \( z = 0 \),

\[
H(z) = [\Omega_r(1 + z)^4 + \Omega_M(1 + z)^3 + \Omega_\Lambda]^{1/2},
\]

and \( \Omega_r = 8.4 \times 10^{-5} \), \( \Omega_M = 0.3089 \) and \( \Omega_\Lambda = 0.6911 \) are the present normalized densities of radiation, matter and dark energy, respectively, while \( H_0 = 67.7 \) km/s/Mpc. We compute the integral (4) numerically, using the latest cosmological parameters given by the Planck Collaboration et al. (2016).

The result for optical depth \( \tau = 1 \) is shown in Fig. 2 by dashed curve as a function of the energy \( E = h\nu_1 \) of the high energy photon observed today on Earth. Also shown is the cosmic horizon for the pair production from two photons, computed in Ruffini et al. (2016). The high redshift (low energy) asymptotes for cosmic horizons shown in Fig. 2 are power laws given approximately by

\[
z = 0.786 \left( \frac{E}{E_{BW}} \right)^{-0.405},
\]

for photon-photon scattering and by

\[
z = 0.257 \left( \frac{E}{E_{BW}} \right)^{-0.488},
\]

for pair production from two photons, respectively, where \( E_{BW} = (m_e c^2)^2/kT_0 \simeq 1.11 \times 10^{15} \) eV.

The photon-photon scattering starts to dominate over pair production at energies smaller than \( E_{cr} = 1.68 \) GeV and redshifts larger than \( z_{cr} \simeq 180 \). It is important to underline that, unlike pair production by two photons, photon-photon scattering of a high energy photon on low energy background is a process that “splits” the high energy photon into two photons, each of which carries away on average half of the initial energy. This makes the mean free path shown in Fig. 2 also equivalent to the energy loss distance. For the sake of comparison also the result corresponding to \( \tau = 5 \) is shown by dash-dotted curve.

In Fig. 2 the cosmic horizon due to extragalactic background light (EBL) is represented as the dotted curve. It is clear that the dominance of the photon-photon scattering occurs at energies lower than those relevant for the absorption by the EBL, and at much larger redshifts.
Fig. 2.— Cosmic horizon (defined from the condition $\tau = 1$) for photon-photon scattering (dashed curve) and for pair production from two photons (solid curve) as function of energy $E = h\nu$ of the high energy photon measured today on Earth. The dotted curve is the cosmic horizon due to the extragalactic background light, taken from Inoue et al. (2013). For comparison also the condition $\tau = 5$ for photon-photon scattering is shown by dash-dotted curve.

4. Conclusions

The photon-photon scattering at cosmological distances is revisited using the recently obtained exact cross section rather than the low energy approximation adopted in previous work. Since the exact cross section near the pair production threshold is larger than the approximate one obtained in the low energy limit, the dominance of the photon-photon scattering over pair production by two photons occurs at smaller redshifts than previously thought, namely redshifts larger than $z_{cr} \simeq 180$. This corresponds to energies smaller than $E_{cr} = 1.68 \text{ GeV}$. These results are relevant for high energy photons produced during the Dark Ages which follows the decoupling of matter and radiation, e.g. by photons resulting from the decay of unstable particles.

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