Effect of Spatial and Temporal Traffic Statistics on the Performance of Wireless Networks

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Abstract

The traffic in wireless networks has become diverse and fluctuating both spatially and temporally due to the emergence of new wireless applications and the complexity of scenarios. The purpose of this paper is to explore the impact of the wireless traffic, which fluctuates violently both spatially and temporally, on the performance of the wireless networks. Specially, we propose to combine the tools from stochastic geometry and queueing theory to model the spatial and temporal fluctuation of traffic, which to our best knowledge has seldom been evaluated analytically. We derive the spatial and temporal statistics, the total arrival rate, the stability of queues and the delay of users by considering two different spatial properties of traffic, i.e., the uniformly and non-uniformly distributed cases. The numerical results indicate that although the fluctuations of traffic when the users are clustered, reflected by the variance of total arrival rate, is much fiercer than that when the users are uniformly distributed, the stability performance is much better. Our work provides a useful reference for the design of wireless networks when the complex spatio-temporal fluctuation of the traffic is considered.

Index Terms

Traffic, delay, queueing theory, stability, stochastic geometry

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This research was supported by the National Natural Science Foundation of China (NSFC) grant No. 61701183.

Part of this work was submitted to the IEEE GLOBECOM 2018 [1].
I. INTRODUCTION

A. Motivations

The emergence of various smart devices and wireless applications, such as real-time wireless gaming, smart grid, free-viewpoint video, advanced manufacturing and Tactile Internet [2], [3], has led to diversified traffic and quality-of-service (QoS) requirements. For example, the voice traffic in the wireless networks is typically delay-sensitive and symmetric in uplink and downlink, while the data and video traffic are generally loss-sensitive and asymmetric in uplink and downlink, which are IP-based and can tolerate certain delay [4]. A meaningful and practically relevant problem is to meet the QoS requirements of diversified applications, which is also one of the most significant goals for the 5G wireless networks [5].

With the continuous evolution of wireless networks, the tremendous traffic and its dynamic variations have become more and more significant in affecting the performance of wireless networks. The pattern of the traffic in a wireless network determines whether a resource block of a base station (BS) is occupied or not, which then shapes the interference pattern in the wireless network. The interference pattern, in turn, affects the performance of the transceiver links in the wireless network, which directly determines the service process of the traffic. Therefore, the traffic and the service provided by the wireless networks are highly coupled with each other. Modeling and analysis of the traffic are essential to design and configure the wireless networks to match the network service with the traffic.

The spatial distribution and the temporal variation of wireless traffic are much more complicated than before. For example, in a cellular network, the wireless traffic during a holiday or a weekend is generally lighter than that during a weekday, and the traffic during the midnight is generally lighter than that during the day time. Meanwhile, the spatial distributions of the wireless traffic are also very different between different regions. For example, the traffic in the business regions could be much heavier during the day time than that during the midnight, and it is reversed in the residential region. The integrated analysis of the traffic with spatial and temporal fluctuation requires to appropriately model both the spatial distribution and the temporal variation of the traffic in the wireless networks. Although the effect of the wireless traffic has been studied extensively in wireless networks, most of them only consider one aspect of the traffic [6]–[8]. The works only modeling the spatial distribution usually use the tools from the stochastic geometry and model the spatial distribution of users by either the uniform
(such as the Poisson point process (PPP)) or the nonuniform point processes. Other works only modeling the temporal variations of traffic usually use the queueing theory to model the arrival process of the packets as stochastic processes. To model both aspects of the traffic, which is necessary for analyzing the wireless networks, requires the combination of stochastic geometry and queueing theory, which brings in more complexities and difficulties in the modeling and evaluation of the wireless networks. To our knowledge, very few works have focused on the effect of spatio-temporal fluctuation of traffic in wireless networks.

In this paper, considering different spatial distributions and temporal properties of wireless traffic, we propose to model the traffic with spatial and temporal fluctuation by combining the tools from stochastic geometry and queueing theory. We explore the statistics of traffic, the total arrival rate, the stability of queues and the delay for different configurations of the wireless networks. Our work can provide a meaningful reference for the theoretical analysis of wireless networks when the complex spatio-temporal fluctuation of the traffic is considered.

B. Related Works

Related works are summarized as follows. The works in [6], [7] discussed the patterns and the spatio-temporal characteristics of wireless traffic in the practical cellular networks. In [6], the authors presented the analysis of traffic measurements collected from commercial cellular networks in China and proposed a spatial traffic model which generated large-scale spatial traffic variations by a sum of sinusoids. In [7], the authors quantitatively characterized the spatio-temporal distribution of mobile traffic and presented a detailed visualized analysis and the work [9] revealed that the traffic in a cellular network is typically unbalanced, changing not only in the time domain but also in the spatial domain. In [10], the authors extracted and modeled the traffic patterns of large scale towers deployed in a metropolitan city. In [11], the authors showed some cell phone activity patterns based on the cell phone data which consists of telecommunications activity records in the city of Milan from Telecom Italia Mobile and the patterns demonstrated that the mobile traffic of urban ecology were clustered in both time field and spatial field. Based on the above works, we can infer that both the spatial distribution and the temporal variation of traffic are irregular, and there is a clear need for new analytical methods to explore the properties of irregular distributions and variations of the up-to-date traffic.

Most existing works related to the theoretical analysis of traffic only study one aspect of the traffic. In [12], the probability density function (pdf) of the number of users in each Voronoi
cell was derived by modeling the locations of users as a homogeneous PPP [13]. The works in [14], [15] extended such pdfs to the case of multi-tier heterogeneous networks. In [16], [17], the authors summarized the applications of the point processes in wireless networks, where the average behavior over many spatial realizations of a network can be evaluated appropriately. In [18], the authors compared the traditional square grid model with the PPP model in terms of coverage, and they discussed the average achievable rate in the general case.

In [19], [20], the authors simultaneously modeled the spatio-temporal arrival of traffic and considered the traffic generated at random spatial regions, other than modeling the flow at each independent user. As for the temporal arrival of packets, the analysis was generally based on the queueing theory where the traffic is modeled as random arrival of packets. In [21], [22], the authors discussed three kinds of scheduling policies, i.e., the random scheduling, the first-input-first-output (FIFO) scheduling and the round-robin scheduling, and compared the delay performance under different scheduling policies. In [23], the authors combined the stochastic geometry and the queueing theory to describe the network with spatial irregularity and temporal evolution. In [24]–[28], a discrete-time slotted ALOHA system with multiple terminals was described, where each terminal had an infinite buffer to store the incoming packets. Due to the random arrival of traffic, the status (idle or busy) of terminals changes over time. The serving rates also change with the statuses of terminals since the idle terminal will not cause any interference to other terminals occupying the same frequency and temporal resources.

C. Contributions

In order to effectively analyze the relationship between traffic with spatio-temporal fluctuation and network performance in wireless networks, we establish a tractable model which characterizes both the spatial distribution and the temporal variation of traffic. The spatial distribution of traffic could be described by the locations of users. Ignoring the mobility of users, the temporal variation of traffic is described by a random arrival process of packets for each user and the dynamic serving process of arrived packets. We first consider the uniformly distributed traffic and assume that both the users and the BSs are distributed as a PPP. Then, we switch to the non-uniformly distributed case where the locations of users are modeled as a Poisson cluster process (PCP). Considering the variation of traffic over time, we compare the difference between the two cases and explore the relationship between the traffic and the delay to gain insight about
the factors that affect the performance of the wireless networks. The main contributions of this paper can be summarized as follows:

- A novel analytical framework based on the combination of stochastic geometry and queueing theory is proposed to capture the spatial and temporal fluctuations of the traffic in wireless networks.
- The spatial and temporal statistics of traffic, the stability of queues and the delay of users are derived by considering different properties of traffic (for example, the uniformly and non-uniformly distributed cases).
- The numerical analyses based on the theoretical results are investigated to gain insights. The effect of various parameters of the traffic is discussed in terms of meeting the delay and stability requirements. We found that although the fluctuations of traffic when the users are clustered, reflected by the variance of total arrival rate, is much fiercer than that when the users are uniformly distributed, the stability performance is much better.

The remaining part of the paper is organized as follows. In Section II we briefly simulate two practical traffic patterns and present the system models. Then, we study the statistics of traffic in Section III and derive the success probability, achievable rate and delay in Section IV. The unstable probability of queue is derived and analyzed in Section V. Numerical results are presented in Section VI to gain insights. Finally, Section VII concludes the paper.

II. SYSTEM MODEL

A. Network Structure

We first plot two patterns that reflect the spatial and temporal characteristics of the traffic in the real scenarios. In Figure 1 we plot the traffic density of instant messaging (IM) and web browsing at 9AM and 3PM in randomly selected dense urban areas. As shown in Figure 1 there are some “hot spots” varying in both temporal and spatial domain and the spatially clustering property is consistent with the findings in [8]. Figure 2 shows the traffic volume about three different service types during one day in the randomly selected cells, which is the same as the findings in [8]. Specially, our data sets are based on a large number of practical traffic records from China Mobile in Hangzhou, an eastern provincial capital in China via the Gb interface of 2G/3G cellular networks or S1 interface of 4G cellular networks [29]. Based on the characteristics of the real traffic in these figures, we propose an analytical model and study the effect of traffic on the performance of wireless networks.
Fig. 1. The cellular network traffic density of instant messaging (IM) and web browsing at 9AM and 3PM in the randomly selected dense urban areas.

Fig. 2. The traffic volume about three different service types during one day in the randomly selected cells.

We consider a wireless network that consists of one tier of BSs and one tier of users, as illustrated in Figure 3. The locations of BSs are modeled as a homogeneous PPP $\Phi_b = \{y_i\}$ with intensity $\lambda_b$, denoted by $\Phi_b \sim \text{PPP}(\lambda_b)$. The users are modeled by another point process $\Phi_u = \{x_i\}$. The transmit power of the BSs is $P_b$. The time is slotted into discrete time slots.
The spatial distribution of users is divided into two cases as follows.

- Case 1: the locations of users form a homogeneous PPP of intensity $\lambda_u$ as shown in the first graph in Figure 3.
- Case 2: the locations of users are distributed as a PCP as shown in the second graph in Figure 3. The centers of the clusters which are the parent points are assumed to be distributed according to a stationary PPP $\Phi_p$ of intensity $\lambda_p$. The users are uniformly scattered according to an independent PPP $\Phi_x$ of intensity $\lambda_c$ in the circular covered area of radius $r_c$ centered at each parent point $x \in \Phi_p$, which are called the daughter points. Thus, the distribution of all users is

$$\Phi_u = \bigcup_{x \in \Phi_p} \Phi_x. \quad (1)$$

In this case, the number of users in a typical cluster is a Poisson random with parameter $\pi r_c^2 \lambda_c$, and the intensity of all users is $\lambda_u = \pi r_c^2 \lambda_c \lambda_p$.

![Fig. 3. Illustration of the network model with BSs and users. Uniformly distributed traffic is modeled by the PPP in the top graph and non-uniformly distributed traffic is modeled by the PCP in the bottom graph.](image)

We assume that the packets arrival process for each user $x_i \in \Phi_u$ is an independent Bernoulli process. The arrival rate of packets is denoted by $\xi_i$ and it is the probability of a packet at user $x_i$ in any given time slot. The size of each packet is assumed to be fixed, and a BS requires exactly one time slot to deliver one packet. Each BS maintains an independent buffer of infinite size for each user within its coverage to save the incoming packets. The number of queues at each BS equals to the number of users within the coverage of this BS. The delay requirement

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is the delay that a user can tolerate. Considering the various latency requirements of users, we assume that the mean delay requirement is $\beta$. In this paper, the probability distribution of $\xi_i$ is denoted by $g(\xi_i)$. The delay requirement $\beta$ can be divided into three ranks, i.e., small value $\beta_s$, medium value $\beta_m$ and large value $\beta_l$, which correspond to the strict latency requirement, medium latency requirement and loose latency requirement respectively.

B. Channel and SIR

Considering the propagation loss, we assume that the links between the serving BS and the users experience Rayleigh fading with unit mean. Then, the received signal power of a user at a distance $l$ from its serving BS is $P_bhl^{-\alpha}$ where $h \sim \text{Exp}(1)$. In other words, the power fading coefficients in different time slots are independent identically distributed and are constant in one time slot. The fading coefficients of interference links are denoted by $\{g_i\}$. Since most of the wireless networks are interference limited, we ignore the noise power in the analysis. If the signal-to-interference ratio (SIR) at a user is larger than a constant threshold $\theta$, the receiver can successfully decode the packets. Otherwise, the packet will be failed for decoding, and the failed packets will be added into the head of the queues and wait to be scheduled again.

Without loss of generality, we introduce the random scheduling in which each user is served randomly by the associated BS with equal probability. We assume that the users always access their nearest BSs. A typical user is assumed to be located at the origin, and its serving BS is located at $y_0$. Let $l_0$ be the distance between the typical user and its associated BS $y_0$, and $\{L_i\}$ be the distances between the typical user and the interfering BSs. The SIR of the typical user at a distance $l_0$ from its serving BS is

$$\text{SIR} = \frac{P_bh_0l_0^{-\alpha}}{\sum_{y_i \in \Phi_b \setminus y_0} P_bg_iL_i^{-\alpha}}.$$  

(2)

III. TRAFFIC STATISTIC

In order to evaluate the spatial and temporal fluctuation of the traffic, we will elaborate from two aspects, i.e., the probability distribution of the number of users served by a BS and the variance of total arrival rate. Firstly, we introduce the following lemma.

**Lemma 1.** The pdf of the coverage area $S$ of a BS is approximated as [12], [30]

$$f_S(x) \simeq \frac{343}{15} \sqrt{\frac{3.5}{\pi}} (x\lambda_b)^{2.5} \exp(-3.5x\lambda_b) \lambda_b.$$  

(3)
In the following, we compare the traffic statistics in the uniformly and non-uniformly distributed users. Firstly, we derive the pdf of the number of users under the case where the users are modeled by the PPP.

**Lemma 2.** Let $N$ be the number of users in a given cell with area $S$ when the users form PPP. The pdf of $N$ is

$$
P(N = k) = e^{-(\lambda_u S)}(\lambda_u S)^k / k!.\quad (4)
$$

**Proof:** The pdf of $N$ can be obtained by the definition of two-dimensional Poisson point process.

Then we derive the pdf of the number of users in the case where the users are distributed as a PCP. A general cluster process is generated by taking a parent point process and daughter point processes, and translating the daughter processes to the position of their parent. The number of positions of parent points forms a uniform PPP of intensity $\lambda_p$ and the daughter points are uniformly scattered on the circle of radius $r_c$ centered at each parent point. The intensity of the PCP is $\lambda_u = \lambda_p \lambda_c \pi r_c^2$.

**Lemma 3.** Let $N$ be the number of users in a given cell with area $S$ when the users obey a PCP. The pdf of $N$ is

$$
P(N = k) = \sum_{a=0}^{\infty} \mathbb{P}(N_p = a, N_d = k) = \sum_{a=0}^{\infty} \frac{e^{-\lambda_p S}}{a!} \left( \frac{\lambda_p S e^{-\lambda_c \pi r_c^2}}{a!} \right)^a \frac{(\lambda_c a \pi r_c^2)^k}{k!},
$$

where $N_p$ is the number of parent points, and $N_d$ is the number of daughter points.

**Proof:** In this lemma, we assume that all the clusters are in area $S$, and that all daughter
point processes are disjoint each other. Thus, the pdf of $N$ is

$$
\mathbb{P}(N = k) = \sum_{a=0}^{\infty} \mathbb{P}(N_p = a, N_d = k)
$$

$$
\overset{(a)}{=} \sum_{a=0}^{\infty} P(N_p = a) P(N_d = k | N_p = a)
$$

$$
\overset{(b)}{=} \sum_{a=0}^{\infty} e^{-\lambda_p S} \left(\lambda_p S\right)^a \frac{e^{-\lambda_c a \pi r_c^2}}{k!} \frac{\left(\lambda_c a \pi r_c^2\right)^k}{k!}
$$

$$
= \sum_{a=0}^{\infty} e^{-\lambda_p S} \left(\lambda_p S e^{-\lambda_c \pi r_c^2}\right)^a \left(\lambda_c a \pi r_c^2\right)^k \frac{1}{k!}
$$

(6)

where (a) follows from the total probability formula and (b) follows from the independence of the numbers of users in disjoint clusters.

![Fig. 4. The PMF of the number of users in the two cases where the users form a PPP or a PCP with the same $\lambda_u$ in a given area $S$.](image)

In Figure 4, we plot the PMF of the number of users in the cases where users form a PPP or a PCP with the same $\lambda_u$ in a cell with area $S$. We observe that, when the cell area $S$ is given, the probability that the number of users is either very small or very large in the PCP case is larger than that in the PPP case. This can be partly attributed to the fact that the users in the PCP case appear to be more grouped and thus, the number of users tends to the extreme values, either very
small or very large. Meanwhile, the probability in PPP is more homogeneous and centralized so the probability is larger than that in the PCP case when the number of users is middle. In both cases, as the value of $\lambda_u S$ increases gradually, the peak value of the PMF decreases since the PMF is more dispersed when the mean number of users increases. Meanwhile, as shown in circle 1, the probability when the number of users is large increases due to the increase of the mean number of users $\lambda_u S$.

Let $\xi_{j,\text{total}}$ be the total arrival rate of packets in the coverage area $S_j$ of a BS $y_j$, we get

$$\xi_{j,\text{total}} = \mathbb{E}(g_j(\xi))N_j,$$

(7)

where $\mathbb{E}(g_j(\xi))$ and $N_j$ are the average arrival rate and the total number of users in area $S_j$, respectively. The mean total arrival rate in area $S_j$, $\mathbb{E}(\xi_{j,\text{total}}) = \mathbb{E}(g_j(\xi))\mathbb{E}(N_j)$, is

$$\mathbb{E}(\xi_{j,\text{total}}) = \mathbb{E}(g_j(\xi))\mathbb{E} \left[ (N_j | S) \right] = \mathbb{E}(g_j(\xi)) \frac{\lambda_u}{\lambda_b},$$

(8)

The variance of total arrival rate in area $S_j$ can be described as $D(\xi_{j,\text{total}}) = \mathbb{E}(\xi_{j,\text{total}}^2) - \left[ \mathbb{E}(\xi_{j,\text{total}}) \right]^2$.

The variance of the total arrival rate in the uniformly distributed case is denoted by $D_P(\xi_{j,\text{total}})$. In the non-uniformly distributed case, the variance is denoted by $D_C(\xi_{j,\text{total}})$. With the above mean and variance of the total arrival rate, we obtain the following lemma.

**Lemma 4.** The variance in the uniformly distributed case is

$$D_P(\xi_{j,\text{total}}) = \mathbb{E}(g_j(\xi))^2 \left( 0.2857 \frac{\lambda_u^2}{\lambda_b^2} + \frac{\lambda_u}{\lambda_b} \right).$$

(9)

The variance in the non-uniformly distributed case is

$$D_C(\xi_{j,\text{total}}) = \mathbb{E}(g_j(\xi))^2 \times \left( 0.2857 \frac{\lambda_u^2}{\lambda_b^2} + \frac{\lambda_u}{\lambda_b} \left( \pi r_c^2 \lambda_c + 1 \right) \right).$$

(10)

**Proof:** When the locations of users form a PPP with density $\lambda_u$, the variance of the total arrival rate is

$$D_P(\xi_{j,\text{total}}) = \mathbb{E}(\xi_{j,\text{total}}^2) - \left[ \mathbb{E}(\xi_{j,\text{total}}) \right]^2$$

$$= \mathbb{E}_{S_j} \left[ \mathbb{E}_P(\xi_{j,\text{total}}^2) \right] - \left[ \mathbb{E}(g_j(\xi)) \frac{\lambda_u}{\lambda_b} \right]^2.$$

(11)
In (11), the mean $E_P (\xi_j^{2, \text{total}})$ is

$$E_P (\xi_j^{2, \text{total}}) = E_P \left[ \left[ E (g_j (\xi)) \right]^2 N_j^2 \right]$$

$$= \left[ E (g_j (\xi)) \right]^2 \sum_{k=0}^{\infty} k^2 P \left( N_j = k \right)$$

$$= \left[ E (g_j (\xi)) \right]^2 \left( \lambda_u^2 S^2 + \lambda_u S \right). \quad (12)$$

The sum $\sum_{k=0}^{\infty} k^2 P \left( N_j = k \right)$ in (12) can be computed as

$$\sum_{k=0}^{\infty} k^2 P \left( N_j = k \right) = \sum_{k=0}^{\infty} k^2 e^{-\lambda_u S} \frac{(\lambda_u S)^k}{k!}$$

$$= e^{-\lambda_u S} \lambda_u S \sum_{k=0}^{\infty} k \frac{(\lambda_u S)^{k-1}}{(k-1)!}$$

$$= e^{-\lambda_u S} \lambda_u S \left( \lambda_u Se^{\lambda_u S} + e^{\lambda_u S} \right)$$

$$= \lambda_u^2 S^2 + \lambda_u S. \quad (13)$$

By utilizing (12) in equation (11), the variance of total arrival rate can be derived as follows.

$$D_P (\xi_j^{\text{total}}) = \left[ E (g_j (\xi)) \right]^2 \left[ E_{S_j} (\lambda_u^2 S^2 + \lambda_u S) - \frac{\lambda_u^2}{\lambda_b^2} \right]$$

$$\overset{(a)}{=} \left[ E (g_j (\xi)) \right]^2 \left[ \int_0^{\infty} \left( \lambda_u^2 x^2 + \lambda_u x \right) f_{S_j} (x) \, dx - \frac{\lambda_u^2}{\lambda_b^2} \right]$$

$$= \left[ E (g_j (\xi)) \right]^2 \left( 0.2857 \frac{\lambda_u^2}{\lambda_b^2} + \frac{\lambda_u}{\lambda_b} \right), \quad (14)$$

where (a) follows from the pdf of $S_j$, and the expression of $f_{S_j}$ is given by (3).

The integral in (14) is

$$\int_0^{\infty} \left( \lambda_u^2 x^2 + \lambda_u x \right) f_{S_j} (x) \, dx = \int_0^{\infty} \left( \lambda_u^2 x^2 + \lambda_u x \right) \frac{343}{15} \sqrt{\frac{3.5}{\pi}} (x \lambda_b)^{2.5} e^{-3.5 \lambda_b x} \lambda_b \, dx$$

$$\overset{(a)}{=} \frac{343}{15} \sqrt{\frac{3.5}{\pi}} \lambda_b^{2.5} \left( \lambda_u^2 \frac{\Gamma(5.5)}{(3.5 \lambda_b)^{5.5}} + \lambda_u \frac{\Gamma(4.5)}{(3.5 \lambda_b)^{4.5}} \right)$$

$$\approx 1.2857 \frac{\lambda_u^2}{\lambda_b^2} + \frac{\lambda_u}{\lambda_b} \quad (15)$$

where (a) follows from the integral $\int_0^{\infty} x^m e^{-\beta x} \, dx = \frac{\Gamma(r)}{\beta^r}$, $r = \frac{m+1}{\beta}$. And, $\Gamma (x) = \int_0^{\infty} t^{x-1} e^{-t} \, dt$ denotes the standard gamma function.

When the users are distributed as a PCP with density $\lambda_u = \pi r^2 \lambda c \lambda_p$, the variance of total arrival rate is

$$D_C (\xi_j^{\text{total}}) = E_{S_j} \left[ E_C (\xi_j^{2, \text{total}}) \right] - \left[ E (g_j (\xi)) \frac{\lambda_u}{\lambda_b} \right]^2. \quad (16)$$
In the above equation (16), the value of $\mathbb{E}_C (\xi_j^2_{total})$ is

$$
\mathbb{E}_C (\xi_j^2_{total}) = \mathbb{E} (g_j (\xi))^2 \mathbb{E}_C (N_j^2)
= \mathbb{E} (g_j (\xi))^2 \left[ \mathbb{D}_C (N_j) + (\mathbb{E}_C (N_j))^2 \right],
$$

(17)

where $N_j = \sum_{i=1}^{N_p} N_{Ci}$ is a Compound Poisson random variable and $N_{Ci} = N_{di}$ is the number of users in the $i$th cluster. According to the properties of Compound Poisson random variable, we have $\mathbb{E}_C (N_j) = \mathbb{E} (N_p) \mathbb{E} (N_C)$ and $\mathbb{D}_C (N_j) = \mathbb{E} (N_p) \left[ \mathbb{D} (N_C) + (\mathbb{E} (N_C))^2 \right]$. $N_p$ and $N_C$ are Poisson random variable with mean $S\lambda_p$ and $\pi r_c^2 \lambda_c$, respectively. Therefore, the mean and variance of $N_C$ is $\mathbb{D} (N_C) = \mathbb{E} (N_C) = \pi r_c^2 \lambda_c$, and the mean of $N_p$ is $\mathbb{E} (N_p) = S\lambda_p$.

Then, the equation (17) can be derived as

$$
\mathbb{E}_C (\xi_j^2_{total}) = \mathbb{E} (g_j (\xi))^2 \left[ \mathbb{E} (N_p) \mathbb{D} (N_C) + \mathbb{E} (N_p) (\mathbb{E} (N_C))^2 + (\mathbb{E} (N_p) \mathbb{E} (N_C))^2 \right]
= \mathbb{E} (g_j (\xi))^2 \left[ S\lambda_u + S\lambda_u \pi r_c^2 \lambda_c + S^2 \lambda_u^2 \right].
$$

(18)

Similar to that in (14), we have

$$
\mathbb{D}_C (\xi_j^2_{total}) = \mathbb{E} (g_j (\xi))^2 \left[ \mathbb{E}_{S_j} \left( S\lambda_u + S\lambda_u \pi r_c^2 \lambda_c + S^2 \lambda_u^2 \right) - \frac{\lambda_u^2}{\lambda_b^2} \right]
= \mathbb{E} (g_j (\xi))^2 \left( 0.2875 \frac{\lambda_u^2}{\lambda_b^2} + \frac{\lambda_u}{\lambda_b} \left( \pi r_c^2 \lambda_c + 1 \right) \right).
$$

(19)

In Figure 5, we plot the variances of total arrival rate in a given area $S$ in the uniformly and non-uniformly cases. We observe that the variance of total arrival rate increases when $\lambda_u$ increases. This is because when the number of users increases, the difference of the arrival rate among the users will be greater. In particular, we observe that the variance of total arrival rate is larger in the non-uniformly distributed case than that in the uniformly distributed case. We also compare the variance in the non-uniformly distributed case for different $r_c$ and $\lambda_c$. When the radius of cluster $r_c$ is the same, the larger the density of cluster $\lambda_c$, the larger the variance of total arrival rate will be. And when the density of cluster $\lambda_c$ is the same, the larger the radius of cluster $r_c$, the larger the variance of total arrival rate will be. The reason is that when increasing the number of users in a cluster, the difference in the arrival rate for each user also increases.

IV. SUCCESS PROBABILITY, ACHIEVABLE RATE AND DELAY

In this section, we analyze the effect of traffic on the success probability, achievable rate and delay in the network. The success probability is the probability that a scheduled packet is
successfully delivered. The achievable rate is the average amount of packets that a typical user successfully receives in each time slot. The delay consists of queueing delay and transmission delay in number of time slots. Since the network is static, i.e., the deployment of BSs is generated first and keeps unchanged hereafter, the success probabilities of various links are different. The various success probabilities result in different achievable rate and delay for different users.

We consider a typical user at the origin and a typical BS located at \( y_0 \) associated with the typical user. Let \( l \) be the distance between the typical user at the origin and the nearest BS \( y_0 \). The pdf of \( l \) can be derived according to a simple fact that the null probability of a 2-D Poisson process in an area \( A \) is \( e^{-\lambda_b A} \). Thus, the pdf of \( l \) is \[ f_l (l) = 2\pi \lambda_b l e^{-\lambda_b \pi l^2} . \] (20)

The status of BSs in the network can be either busy or idle. We assume that the probability of each interfering BS being busy is \( q \) in each time slot \( t \) and the SIR at the typical user in the time slot \( t \) is denoted by \( \text{SIR}_t \). Then, the \( \text{SIR}_t \) can be described as

\[
\text{SIR}_t = \frac{P_b h_0 l_0^{-\alpha}}{\sum_{y_i \in \Phi_b \setminus y_0} \frac{P_b g_i L_i^{-\alpha}}{\prod_{y_i \in \Phi_b \setminus y_0}}},
\] (21)
where $\Pi_{y_i}$ is the indicator function defined such that $\Pi_{y_i} = 1$, if the interfering BS $y_i$ is active, and $\Pi_{y_i} = 0$, otherwise. Then, the probability of $\Pi_{y_i} = 1$ is $q$ and the probability of $\Pi_{y_i} = 0$ is $1 - q$. The above result (21) can be obtained by (2).

A. Success Probability

The SIRs at the typical user in different time slots are independent random variables. However, in order to distinguish it from (2), we still keep the subscribe $t$ in the following discussions. The success probability conditioned on $\Phi_b$ is

$$P_{s|\Phi_b} \triangleq P(\text{SIR}_t > \theta | \Phi_b),$$

(22)

where $\theta$ is the threshold.

In the random scheduling strategy, if there are $N$ users within the coverage of the considered cell, the typical BS will randomly choose one user from the $N$ users with probability $1/N$ in each time slot. For the typical user $x_0$, the arrival rate of packets is $\xi_0$, and the service rate (number of packets transmitted successfully per time slot) is $P_{s|\Phi_b}/N$. According to the property of G/G/1 queue, the probability that the queue at the user $x_0$ is not empty is given by

$$P[\text{Queue is not empty}] = \min \left\{ \frac{N\xi_0}{P_{s|\Phi_b}}, 1 \right\},$$

(23)

where $\xi_0$ is the arrival rate of packets for the typical user $x_0$. Since the probability of the typical BS being active is equivalent to the probability $P[\text{Queue is not empty}]$. Thus, the probability that the typical BS is active equals $\min \left\{ \frac{N\xi_0}{P_{s|\Phi_b}}, 1 \right\}$. Then, we get a fixed-point equation

$$\min \left\{ \frac{N\xi_0}{P_{s|\Phi_b}}, 1 \right\} = q',$$

(24)

where $q'$ is the probability of the typical BS being busy conditioned on $\Phi_b$.

We assume the distance between the typical user and the associated BS to be $l_0$ whose pdf is given by (20), and the distribution of interfering BSs forms a PPP on the whole space $\mathbb{R}^2$. In order to obtain the closed-form expression of $q$, we get an approximate equation of (24)

$$\min \left\{ \frac{N\xi_0}{P_s}, 1 \right\} = q,$$

(25)

where $P_s$ is given in Lemma 5.
Lemma 5. The solution of equation (25) can be expressed as
\[
q^* = \begin{cases} 
\frac{N\xi_0\sin\left(\frac{\pi}{N}\right)}{\sin\left(\frac{\pi}{N}\right) - N\xi_0\theta\pi}, & \text{if } 0 < \xi_0 < \frac{\sin\left(\frac{\pi}{N}\right)}{N\left(\sin\left(\frac{\pi}{N}\right) + \theta\pi\right)} \\
1, & \text{if } \xi_0 \geq \frac{\sin\left(\frac{\pi}{N}\right)}{N\left(\sin\left(\frac{\pi}{N}\right) + \theta\pi\right)}
\end{cases}
\]  
(26)

where \( \sin(\cdot) \) is the sinc function.

Proof: Combined [17] and [18], the success probability can be expressed as
\[
P_{s|\theta_0} = \exp\left(\frac{\pi\lambda_0 q\theta^2\pi^2}{2\theta}\right).
\]  
(27)

Thus, the success probability can be evaluated as
\[
P_s = \mathbb{E}_{\theta_0} \left[ P_{s|\theta_0} \right] = \int_{0}^{\infty} e^{-x\lambda_0\theta^2\pi^2} \frac{2}{\sin\left(\frac{\pi}{N}\right)} f_{\theta_0}(x) \, dx
\]  
(28)

Plugging \( P_s \) into (25), we get an equation
\[
\min \left\{ \frac{N\xi_0 \left(\sin\left(\frac{\pi}{N}\right) + q\theta\pi\right)}{\sin\left(\frac{\pi}{N}\right)}, 1 \right\} = q.
\]  
(29)

Considering solving the above equation (29), we discuss two cases, \( \frac{N\xi_0 \left(\sin\left(\frac{\pi}{N}\right) + q\theta\pi\right)}{\sin\left(\frac{\pi}{N}\right)} < 1 \) and \( \frac{N\xi_0 \left(\sin\left(\frac{\pi}{N}\right) + q\theta\pi\right)}{\sin\left(\frac{\pi}{N}\right)} \geq 1 \). In order to facilitate the understanding of the equation solving process, we give the geometric solution of the equation. In Figure 6, the red lines show the range of solution for equation (29) and the red dot indicates the intersection of the curve \( q = \frac{N\xi_0 \left(\sin\left(\frac{\pi}{N}\right) + q\theta\pi\right)}{\sin\left(\frac{\pi}{N}\right) - N\xi_0\theta\pi} \) and straight line \( q = 1 \). The abscissa of the red dot is indicated by \( B_0 \).

First, we discuss the case \( \frac{N\xi_0 \left(\sin\left(\frac{\pi}{N}\right) + q\theta\pi\right)}{\sin\left(\frac{\pi}{N}\right)} < 1 \). According to (25), we have
\[
\frac{N\xi_0 \left(\sin\left(\frac{\pi}{N}\right) + q\theta\pi\right)}{\sin\left(\frac{\pi}{N}\right)} = q.
\]  
(30)

Solving the equation, we get \( q^* = \frac{N\xi_0 \sin\left(\frac{\pi}{N}\right)}{\sin\left(\frac{\pi}{N}\right) - N\xi_0\theta\pi} \) and \( 0 < q^* < 1 \). We get a conclusion, if \( 0 < \xi_0 < \frac{\sin\left(\frac{\pi}{N}\right)}{N\left(\sin\left(\frac{\pi}{N}\right) + \theta\pi\right)} \), the solution of the equation is \( q^* = \frac{N\xi_0 \sin\left(\frac{\pi}{N}\right)}{\sin\left(\frac{\pi}{N}\right) - N\xi_0\theta\pi} \).

Then, we discuss the second case \( \frac{N\xi_0 \left(\sin\left(\frac{\pi}{N}\right) + q\theta\pi\right)}{\sin\left(\frac{\pi}{N}\right)} \geq 1 \). The equation (29) can be equal to \( \frac{N\xi_0 \left(\sin\left(\frac{\pi}{N}\right) + q\theta\pi\right)}{\sin\left(\frac{\pi}{N}\right)} \geq 1 \) and \( q = 1 \). We get the second conclusion that the solution of the equation is \( q^* = 1 \), if \( \xi_0 \geq \frac{\sin\left(\frac{\pi}{N}\right)}{N\left(\sin\left(\frac{\pi}{N}\right) + \theta\pi\right)} \).
According to (26) and (28), the approximated success probability of the typical user is

$$
\tilde{P}_s = \begin{cases} 
1 - \frac{N\xi_0\theta^2}{\text{sinc}\left(\frac{\theta}{\alpha}\right)}, & \text{if } 0 < \xi_0 < \frac{\text{sinc}\left(\frac{\theta}{\alpha}\right)}{N\left(\text{sinc}\left(\frac{\theta}{\alpha}\right) + \theta^2\right)} \\
\frac{\text{sinc}\left(\frac{\theta}{\alpha}\right)}{\text{sinc}\left(\frac{\theta}{\alpha}\right) + \theta^2}, & \text{if } \xi_0 \geq \frac{\text{sinc}\left(\frac{\theta}{\alpha}\right)}{N\left(\text{sinc}\left(\frac{\theta}{\alpha}\right) + \theta^2\right)}
\end{cases}.
$$

(31)
From the above expression, we observe that the approximated success probability is not only related to the path loss exponent but also related to the distribution of users and the threshold of SIR. By analyzing the expression (31), several observations can be obtained as follows.

1) When the arrival rate of the packets satisfies the condition $0 < \xi_0 < B_0$, the success probability is related to the path loss exponent, the distribution of users and the threshold of SIR. When $\xi_0$ or $N$ increases, the success probability will decrease. This is because the success probability will decrease due to the competition increase of the users requesting service. When the path loss exponent increases, the success probability will increase, which is attributed to the less competition due to the decrease of the messages of service requests. As the threshold increases, the success probability decreases gradually. This is because the communication requirements are higher but the communication conditions are unchanged.

2) When the arrival rate of the packets satisfies the condition $\xi_0 \geq B_0$, the success probability is only related to the path loss exponent and the threshold of SIR. As the path loss exponent increases, the success probability will increase gradually and reach a maximum value. It implies that the interferences from other BSs are dominant when the arrival rate is large enough, and the effects of path loss on the interferences are larger than on the signal. As the threshold increases, the success probability will decrease due to the same reason as 1).

**B. Achievable Rate**

In the following, we evaluate the achievable rate in units of bits/Hz for the typical user $x_0$ where adaptive modulation/coding is used so each user can set their rate. The achievable rate can be measured by the number of packets successfully received at the typical users in each time slot. We assume that the typical user can successfully receive packets only when the SIR is larger than $\theta$.

Because the users can set their rate adaptively, the number of packets transmitted in the communication links is assumed to equal $\log_2(1 + \theta)$ in each time slot. Therefore, by utilizing (31), we get the achievable rate for the typical user $x_0$ which is

$$
\tau = \begin{cases} 
\frac{(\text{sinc}(\frac{2}{\alpha}) - N\xi_0\theta^2)\log_2(1+\theta)}{\text{sinc}(\frac{2}{\alpha})}, & \text{if } 0 < \xi_0 < \frac{\text{sinc}(\frac{2}{\alpha})}{N(\text{sinc}(\frac{2}{\alpha}) + \theta^2)} \\
\frac{\text{sinc}(\frac{2}{\alpha})\log_2(1+\theta)}{\text{sinc}(\frac{2}{\alpha}) + \theta^2}, & \text{if } \xi_0 \geq \frac{\text{sinc}(\frac{2}{\alpha})}{N(\text{sinc}(\frac{2}{\alpha}) + \theta^2)}
\end{cases}
$$

(32)

Observing the equation (32), we can find that the achievable rate is related to the path loss exponent, the distribution of users and the threshold of SIR. The achievable rate decreases linearly
with $N$ or $\xi_0$, for $0 < \xi_0 < B_0$. The linear decrease comes from the increase of users service requests. When the arrival rate of the packets satisfies the condition $\xi_0 \geq B_0$, the achievable rate will only related to the path loss exponent and the threshold of SIR, which means the BS reaches its maximum capacity and the BS will be active all the time.

In Figure 8, we plot the achievable rate $\tau$ with respect to the SIR threshold $\theta$ (dB) for different path loss exponents and arrival rates of packets. As it can be observed from the curves, it seems counterintuitive that the achievable rate increases when the path loss exponent increases. The increase comes from the increase of success probability, and it means that the large path loss exponent is not always bad for the achievable rate. When the threshold of SIR $\theta < 0$, the achievable rate is very small and has similar trends in all curves. This is because the low threshold results in the huge competition due to almost all users in coverage area request service at the same time slot. When the arrival rate of packets is 0.001, the achievable rate increases along with the increase of threshold and only has a little decrease for $\alpha = 2.5$ since the achievable rate is decided by the first half of (32). When the arrival rate of packets is 0.01, the achievable rate initially increases along with the increase of threshold and reaches a peak value, but it starts to decreases as the threshold continue to increase and will tend to 0 as $\theta \to \infty$. In this case, the achievable rate is decided by both the first and last half of (32). The trend means that the threshold of SIR is not always opposite with the achievable rate and the appropriate increase of the threshold of SIR can help improve the achievable rate.

### C. Statistic of Delay

In the following, we discuss the mean delay for random scheduling policy and analyze the relationship between the delay performance and the traffic.

In the random scheduling, the average service rate of the typical user, denoted by $\mu = \hat{P}_s/N$, is

$$\mu = \begin{cases} \frac{\text{sinc}(\frac{\alpha}{2}) - N\xi_0 \theta^2}{N \text{sinc}(\frac{\alpha}{2})}, & \text{if } 0 < \xi_0 < \frac{\text{sinc}(\frac{\alpha}{2})}{N(\text{sinc}(\frac{\alpha}{2}) + \theta^2)}, \\
\frac{\text{sinc}(\frac{\alpha}{2})}{N(\text{sinc}(\frac{\alpha}{2}) + \theta^2)}, & \text{if } \xi_0 \geq \frac{\text{sinc}(\frac{\alpha}{2})}{N(\text{sinc}(\frac{\alpha}{2}) + \theta^2)}. \end{cases}$$

Due to the independence of the transmissions in different time slots, the probability for successfully transmitting a packet in any time slot can be considered as $\mu$. Thus, the queueing system at the typical transmitter is a discrete-time Geo/G/1 queue in which the retrial time has a general
distribution, and the server begins to search the next customer to serve [31] after each service completion.

In the equivalent Geo/G/1 queue, the arrival process of packets is a geometric arrival process since it is a Bernoulli process. The service rate is $\mu$ and the service times of the packets are i.i.d. with geometric distribution.

**Theorem 1.** In random scheduling, the mean delay $D_{\xi_0}$ can be expressed as

$$D_{\xi_0} = \begin{cases} 
\frac{(1-\xi_0)N\text{sinc}(\frac{2}{\alpha})}{\text{sinc}(\frac{2}{\alpha}) - N\xi_0\theta^\alpha - N\xi_0\text{sinc}(\frac{2}{\alpha})}, & \text{if } N < \frac{\text{sinc}(\frac{2}{\alpha})}{\xi_0\text{sinc}(\frac{2}{\alpha}) + \xi_0\theta^\alpha} \\
\infty, & \text{if } N \geq \frac{\text{sinc}(\frac{2}{\alpha})}{\xi_0\text{sinc}(\frac{2}{\alpha}) + \xi_0\theta^\alpha} \end{cases}. \quad (34)$$

**Proof:** From [31 Corollary 2 eq.7], we get the mean time that a customer spends in the system as

$$W = \beta_1 + \frac{2\bar{p}(\beta_1 - 1) [1 - A(\bar{p})] + p\beta_2}{2 [p + \bar{p}A(\bar{p}) - p]}.$$  \quad (35)

Combined with our own queue system, we get $\beta_1 = 1/\mu$, $\beta_2 = 2/\mu^2 - 2/\mu$, $p = \xi_0$, $A(\bar{p}) = 1$.
and $\rho = p\beta_1$. By utilizing the above formulas, we get the mean delay as

$$D_{\xi_0} = \begin{cases} \frac{1-\xi_0}{\mu - \xi_0}, & \text{if } \mu > \xi_0 \\ \infty, & \text{if } \mu \leq \xi_0 \end{cases}. \quad (36)$$

Plugging $(33)$ into $(36)$, we get the result in $(34)$.

When the mean delay is finite and $D_{\xi_0} < \beta$, we get the following inequality

$$\frac{(1 - \xi_0) N \text{sinc} \left( \frac{2}{\alpha} \right)}{\text{sinc} \left( \frac{2}{\alpha} \right) - N \xi_0 \theta \frac{2}{\alpha} - N \xi_0 \text{sinc} \left( \frac{2}{\alpha} \right)} < \beta. \quad (37)$$

After simplifying the above inequality, we obtain

$$N < \frac{\text{sinc} \left( \frac{2}{\alpha} \right)}{\frac{1}{\beta} (1 - \xi_0) \text{sinc} \left( \frac{2}{\alpha} \right) + \left( \xi_0 \theta \frac{2}{\alpha} + \xi_0 \text{sinc} \left( \frac{2}{\alpha} \right) \right)}. \quad (38)$$

In order to facilitate the above expression, we define

$$A_1 = \frac{\text{sinc} \left( \frac{2}{\alpha} \right)}{\xi_0 \text{sinc} \left( \frac{2}{\alpha} \right) + \xi_0 \theta \frac{2}{\alpha}}, \quad (39)$$

$$A_2 = \frac{\text{sinc} \left( \frac{2}{\alpha} \right)}{\frac{1}{\beta} (1 - \xi_0) \text{sinc} \left( \frac{2}{\alpha} \right) + \left( \xi_0 \theta \frac{2}{\alpha} + \xi_0 \text{sinc} \left( \frac{2}{\alpha} \right) \right)}, \quad (40)$$

where $A_1 > A_2$ and $\beta$ is the delay requirement of the typical user.

By analyzing the above inequalities, we obtain several conclusions as follows.

1) When the number of users in a typical cell $N$ satisfies the condition $N < A_2$, the queue is stable and the delay requirement can be satisfied. By observing the expression of $A_2$, we obtain that the larger the value of $\beta$ is, the greater the value that $N$ will be. In other words, when the delay requirements of users are low, the typical cell can accommodate more users and satisfies their delay requirements.

2) When $N$ satisfies the condition $A_2 < N < A_1$, the queue is stable but the delay requirements of users can not be satisfied, i.e., the arriving users can be successfully served by the associated BS, but the delay requirements of users can not be met.

3) When $N$ satisfies the condition $N > A_1$, the queue is not stable, i.e., the users will be blocked in the system and can not be successfully served by the associated BS.
V. Unstable Probability

In this section, we derive the expression of the unstable probability of the typical queue in the wireless network and discuss the unstable probability under two different kinds of distributions of \( \xi_0 \). Specially, we derive and give the general expression of unstable probability under the two distributions. Combined with the distributions of the number of users, we get the unstable probability in PPP and PCP, respectively. The related simulations are given in section VI.

In the Geo/G/1 queue, when the arrival rate of packets \( \xi_0 \) is over the service rate of the system \( \mu \), the queue will be unstable. Therefore, the unstable probability can be evaluated as follows.

**Theorem 2.** The unstable probability of the typical queue in the network is

\[
P_{us} = 1 - \sum_{k=1}^{\infty} P(\xi_0 \leq f(k))P(N = k) - P(N = 0),
\]

where \( f(k) = \frac{\sin(\frac{\alpha}{2})}{k(\sin(\frac{\alpha}{2}) + \theta \frac{\alpha}{2})} \) and \( P(N = k) \) is the pdf of \( N \) given by (4) and (5).

**Proof:** According to (44), the unstable probability can be derived explicitly as

\[
P_{us} = P\left(N \geq \frac{\sin(\frac{\alpha}{2})}{\xi_0(\sin(\frac{\alpha}{2}) + \theta \frac{\alpha}{2})}\right)
\]

\[= 1 - \sum_{k=1}^{\infty} P(\xi_0 \leq f(k))P(N = k) - P(N = 0),
\]

where \( P(\xi_0 \leq f(k)) \) can be evaluated by the cumulative distribution function of \( \xi_0 \) and \( P(N = k) \) can be obtained by (4) or (5).

Since the arrival rate of the packets \( \xi_0 \) may obey different distribution, we consider the unstable probability under exponential distribution and uniform distribution, respectively.

First, we consider the arrival rate \( \xi_0 \sim E(\lambda) \), the unstable probability can be derived as

\[
P_{us} = 1 - \sum_{k=1}^{\infty} \left(1 - e^{-\frac{\lambda \sin(\frac{\alpha}{2})}{k(\sin(\frac{\alpha}{2}) + \theta \frac{\alpha}{2})}}\right)P(N = k) - P(N = 0).
\]

When the arrival rate satisfies \( \xi_0 \sim U(0, b) \), the unstable probability can be written as

\[
P_{us} = 1 - \sum_{k=a}^{\infty} \left(\frac{\sin(\frac{\alpha}{2})}{kb(\sin(\frac{\alpha}{2}) + \theta \frac{\alpha}{2})}\right)P(N = k) - \sum_{k=0}^{a-1} P(N = k),
\]

where the condition is \( \frac{\sin(\frac{\alpha}{2})}{a(\sin(\frac{\alpha}{2}) + \theta \frac{\alpha}{2})} \leq b < \frac{\sin(\frac{\alpha}{2})}{(a-1)(\sin(\frac{\alpha}{2}) + \theta \frac{\alpha}{2})} \).
According to (43) and (4), when the users form a PPP, the unstable probability is

\[
P_{us} = 1 - \sum_{k=1}^{\infty} \left(1 - e^{-k \frac{\lambda_0 S}{\sin(c(\frac{\alpha}{\lambda_0 S}) + \theta_2)}}\right) \left(\sum_{a=0}^{\infty} \frac{e^{-\lambda_p S} \left(\frac{\lambda_0 S}{\lambda_p S} \left(\lambda_0 S\right)^a}{a!} \left(\frac{\lambda c a \pi r_c^2}{k!}\right)^k\right) e^{-\lambda_0 S} - \lambda_0 S.
\]

(45)

When the users are distributed as a PCP, combined (43) with (5), the unstable probability is

\[
P_{us} = 1 - \sum_{k=1}^{\infty} \left(1 - e^{-k \frac{\lambda_0 S}{\sin(c(\frac{\alpha}{\lambda_0 S}) + \theta_2)}}\right) \left(\sum_{a=0}^{\infty} \frac{e^{-\lambda_p S} \left(\frac{\lambda_0 S}{\lambda_p S} \left(\lambda_0 S\right)^a}{a!} \left(\frac{\lambda c a \pi r_c^2}{k!}\right)^k\right) e^{-\lambda_0 S} - \lambda_0 S.
\]

(46)

Conditioned on \(\xi_0 \sim U(0, b)\), when the users form a PPP, the unstable probability is

\[
P_{us} = 1 - \sum_{k=m}^{\infty} \left(1 - e^{-k \frac{\lambda_0 S}{\sin(c(\frac{\alpha}{\lambda_0 S}) + \theta_2)}}\right) \left(\sum_{a=0}^{\infty} \frac{e^{-\lambda_p S} \left(\frac{\lambda_0 S}{\lambda_p S} \left(\lambda_0 S\right)^a}{a!} \left(\frac{\lambda c a \pi r_c^2}{k!}\right)^k\right) e^{-\lambda_0 S} - \lambda_0 S.
\]

(47)

Similarly, when the users form a PCP, the unstable probability can be written as

\[
P_{us} = 1 - \sum_{k=m}^{\infty} \left(1 - e^{-k \frac{\lambda_0 S}{\sin(c(\frac{\alpha}{\lambda_0 S}) + \theta_2)}}\right) \left(\sum_{a=0}^{\infty} \frac{e^{-\lambda_p S} \left(\frac{\lambda_0 S}{\lambda_p S} \left(\lambda_0 S\right)^a}{a!} \left(\frac{\lambda c a \pi r_c^2}{k!}\right)^k\right) e^{-\lambda_0 S} - \lambda_0 S.
\]

(48)

The simulations of (48) will be showed in Figure 12 and Figure 13.

VI. NUMERICAL EVALUATION

In this section, we evaluate the delay and the stability numerically. The numerical results demonstrate the impact of different system parameters on the delay and stability performance. Our numerical evaluations validate the claims in the above discussions and help to gain insights to improve the performance of wireless networks.

A. Evaluations of the delay performance

In Figure 9, we plot the conditional mean delay \(D_{\xi_0}\) given by (34) with respect to the arrival rate of packets \(\xi_0\) for different path loss exponents. We observe that, when the value of \(\xi_0\) is small, as \(\xi_0\) increases, the conditional mean delay \(D_{\xi_0}\) increases gradually since the waiting time is longer due to the increase of the number of arrival packets in a slot time. When the value of \(\xi_0\) is larger than a certain value, the conditional mean delay \(D_{\xi_0}\) will be infinite which means that the queue is unstable. This can be interpreted as that the service ability of BSs are limited.
and the system will be blocked when the traffic is overloaded in each time slot. As the path loss exponent $\alpha$ increases, the conditional mean delay $D_{\xi_0}$ decreases since the waiting time is shorter due to the decrease of the number of arrival packets.

In Figure 9, we plot the conditional mean delay $D_{\xi_0}$ given by (34) as functions of the number of users $N$ for different path loss exponents and observe a similar trend for conditional mean delay shown in Figure 9. We observe that, as the number of users increases, the conditional mean delay also increases gradually, while as the path loss exponent $\alpha$ increases, the conditional mean delay $D_{\xi_0}$ decreases. In Figure 9 when the arrival rate of packets is 0, the conditional mean delay will not be 0. This is because when the arrival rate of packets is 0, the number of users in the cell may not be 0. However, in Figure 10 when the number of users is 0, the conditional mean delay will also be 0. This is because when the number of users in a cell is 0, the traffic in the cell must be 0 and there is no users service in the cell. When the number of users is larger than a certain value, the conditional mean delay will be infinite which means the number of users in the cell exceeds the number of users that the BS can accommodate.
The number of users $N$

The dealy $D$

$\alpha = 2.5$  
$\alpha = 3$  
$\alpha = 4$

Fig. 10. Effect of the number of users $N$ on the conditional mean delay $D_{\xi_0}$ given by (34) for different values of $\alpha$ ($\xi_0 = 0.001, \theta = 10$).

B. Evaluations of the unstable probability

In Figure 11, we plot the unstable probability $P_{us}$ given by (43) as functions of the density of users $\lambda_u$ for different path loss exponents $\alpha$. The arrival rate of packets $\xi_0$ follows an exponential distribution with mean 0.01. As the density of users $\lambda_u$ increases, the unstable probability $P_{us}$ increases due to the increase of the number of users in a given area $S$. When the density of users $\lambda_u$ increases to a large value, the unstable probability $P_{us}$ will approach to 1. It means that the BS reaches its maximum accommodation capacity and the users can not be served successfully by the BS in this case. However, as the path loss exponent $\alpha$ increases, the unstable probability $P_{us}$ decreases gradually since the loss of packets at the communication links will reduce the number of packets arriving the queue and further reduce the unstable probability of the queue. Noted that, when the density of users $\lambda_u$ is very small, the unstable probability in the uniformly distributed case is less than that in the non-uniformly distributed case under the same value of $\lambda_u$. When the density of users continues to increase, the unstable probability in the uniformly distributed case will exceed that in the non-uniformly distributed case. Thus, when the arrival rate follows an exponential distribution, the stability of queue in the uniformly distributed case
The density of users $\lambda_u$ on the unstable probability $P_{us}$ for different path loss exponents $\alpha$ and various distributions of users when the arrival rate follows an exponential distribution with mean 0.01. The parameters are set as $\theta = 10$, $S = 10m^2$, $r_c = 1m$, $\lambda_p = \frac{1}{\pi} m^{-2}$ and $\lambda_c = 1.1\lambda_u m^{-2}$.

Fig. 11. Effect of the density of users $\lambda_u$ on the unstable probability $P_{us}$ for different path loss exponents $\alpha$ and various distributions of users when the arrival rate follows an exponential distribution with mean 0.01. The parameters are set as $\theta = 10$, $S = 10m^2$, $r_c = 1m$, $\lambda_p = \frac{1}{\pi} m^{-2}$ and $\lambda_c = 1.1\lambda_u m^{-2}$.

is better than that in the non-uniformly distributed case for small $\lambda_u$, and it is reversed for large $\lambda_u$.

In Figure 12 we plot the unstable probability $P_{us}$ given by (44) as functions of the density of users $\lambda_u$ for different path loss exponents $\alpha$. The arrival rate of packets $\xi_0$ follows a continuous uniform distribution denoted by $U(0,0.02)$. Similar to the results in Figure 11 as the density of users $\lambda_u$ increases, the unstable probability $P_{us}$ increases and as the path loss exponent $\alpha$ increases, the unstable probability $P_{us}$ decreases. We can observe a similar trend for the unstable probability as that in Figure 11. It further demonstrates the conclusion that the stability of queue in the uniformly distributed case is better than that in the non-uniformly distributed case for small $\lambda_u$, and it is reversed for large $\lambda_u$. Contrast with Figure 11 we can observe that the unstable probabilities in Figure 12 are higher than those in Figure 11 when the parameters are the same except the distribution of $\xi_0$. This indicates that the stability of queue is better when the $\xi_0$ follows an exponential distribution.

In Figure 13 we plot the unstable probability $P_{us}$ given by (43) and (44) as functions of the density of users $\lambda_u$ for different PCP parameters and various distributions of arrival rate.
The density of users $\lambda_u$ on the unstable probability $P_{us}$ for different path loss exponents $\alpha$ and various distributions of users when the arrival rate follows a continuous uniform distribution denoted by $U(0, 0.02)$. The parameters are set as $\theta = 10, S = 10m^2, r_c = 1m, \lambda_p = \frac{1}{11\pi} m^{-2}$ and $\lambda_c = 1.1\lambda_u m^{-2}$.

The arrival rate of packets $\xi_0$ follows a continuous uniform distribution denoted by $U(0, 1)$ or an exponential distribution with mean 0.5. The unstable probability in PCP 1 is larger than that in PCP 2 when $\lambda_u < 1$, which means the users in PCP 2 are more likely to be served successfully than those in PCP 1. This is because when the distribution is PCP 1, the number of users experiencing strong interference will increase drastically which leads to the increase of the unstable probability. When the user distribution is PCP 2, the probability that the users experiencing strong interference is smaller and the unstable probability will also be smaller. The stability of queue is better when the $\xi_0$ follows an exponential distribution compared to it in $\xi_0 \sim U(0, 1)$. In particular, when the path loss exponent $\alpha = 2.5$, the interval shown in circle 1 is small, but when the path loss exponent is $\alpha = 4$, the interval shown in circle 2 is bigger.

Therefore, when the density of users $\lambda_u$ is medium, the stability of queue is not dominated by the distributions of $\xi_0$ but dominated by the path loss exponent.
The density of users $\lambda_u$

The unstable probability of the queue $P_{us}$

PCP 1 $\alpha = 2.5$, $\xi_0 \sim U(0, 1)$

PCP 1 $\alpha = 4$, $\xi_0 \sim U(0, 1)$

PCP 2 $\alpha = 2.5$, $\xi_0 \sim U(0, 1)$

PCP 2 $\alpha = 4$, $\xi_0 \sim U(0, 1)$

PCP 1 $\alpha = 2.5$, $\xi_0 \sim E(2)$

PCP 1 $\alpha = 4$, $\xi_0 \sim E(2)$

PCP 2 $\alpha = 2.5$, $\xi_0 \sim E(2)$

PCP 2 $\alpha = 4$, $\xi_0 \sim E(2)$

Fig. 13. Effect of the density of users $\lambda_u$ on the unstable probability $P_{us}$ for different path loss exponents $\alpha$ when the arrival rate follows either a continuous uniform distribution denoted by $U(0, 1)$ or an exponential distribution with mean 0.5. The parameters are set as $\theta = 10, S = 10m^2, r_c = 1m, \lambda_p = \frac{1}{1m^2}, \lambda_c = 1.1\lambda_u m^{-2}$ for PCP 1 and $\theta = 10, S = 10m^2, r_c = 1m, \lambda_p = \frac{1}{1m^2} m^{-2}, \lambda_c = 1.1m^{-2}$ for PCP 2.

VII. CONCLUSION

In this paper, we consider a tractable model to analyze the effect of spatio-temporal traffic on the wireless network. By considering a network consisting of one tier of BSs and one tier of users, we compared the distributions of users in the uniformly distributed case and the non-uniformly distributed case and derived the pdf of the number of users, the variance of total arrival rate, the success probability, the achievable rate, the conditional mean delay and the unstable probability of queue. Exact expressions were obtained for the novel model and based on the expressions, we discussed the effect of spatio-temporal traffic on network delay and the stability of queue.

From the numerical study, we observe that the fluctuations of total arrival rate are greater in the non-uniformly distributed case than that in the uniformly distributed case. The stability of queue in the non-uniformly distributed case is better than that in the uniformly distributed case, and the stability of queue is better when the arrival rate follows an exponential distribution. Our analyses reveal the differences between the uniformly and non-uniformly distributed cases and
provide insights on the design of wireless networks.

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