Measurements of the CKM angle $\gamma/\phi_3$ at $B$-factories

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Abstract

The CKM angle $\gamma/\phi_3$ have been measured by two B-factories, the PEPII collider for the BaBar experiment and the KEKB collider for the Belle experiments. The present paper reports recent progress in $\gamma/\phi_3$.

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1 Introduction

The $CP$ violation in the Standard Model is explained by the quark mixing matrix of the charged current weak interaction. The matrix is as known as Cabibbo-Kobayashi-Maskawa (CKM) matrix [1]. The element has the irreducible complex phase which causes $CP$ violation in $K$ and $B$ meson systems [2, 3, 4]. The unitarity of CKM matrix has bring some constrains of the elements $V_{i,j}$ to draw triangles in the complex plane, where $i$ and $j$ stand for the up-type quark ($u, c, t$) and down type quark ($d, s, b$), respectively. One of the triangles which has the sides in same order of magnitude is suitable place to measure the elements precisely. The angles, $\alpha/\phi_2$, $\beta/\phi_1$ and $\gamma/\phi_3$ has been measured by two $B$-factories, the PEPII collider for the BaBar experiment [5] and the KEKB collider for the Belle experiments [6], where $\alpha/\phi_2 = \text{Arg}[-(V_{td}V_{tb}^*)/(V_{ud}V_{ub}^*)]$, $\beta/\phi_1 = \text{Arg}[-(V_{cd}V_{cb}^*)/(V_{td}V_{tb}^*)]$ and $\gamma/\phi_3 = \text{Arg}[-(V_{ud}V_{ub}^*)/(V_{cd}V_{cb}^*)]$.

Current most worst precision of the angle is angle $\gamma/\phi_3$. Improvement of the precision is eagerly awaited. The $\gamma/\phi_3$ can be obtained mainly using the process $B \to D^{(*)}K^{(*)}$ involved with interference with Cabibbo-suppressed $b \to u$ and Cabibbo-favored $b \to c$ quark transition [7]. Some methods to extract $\gamma/\phi_3$ had been suggested so far: GLW [8], ADS [9], Dalitz [10, 11] analyses. The GLW analysis uses $D_0$ meson decays to the $CP$ eigenstates. The ADS analysis uses Cabibbo-favored $D^0$ and doubly Cabibbo-suppressed $D^0$ meson decays. The Dalitz method uses Cabibbo-allowed three-body $D$ meson decays.

2 GLW analysis and the recent measurements

Gronau, London and Wyler (GLW) [8] proposed analysis method to extract the angle $\gamma/\phi_3$ with $D^0$ and $\bar{D}^0$ decay into $CP$ eigenstate such as $K^+K^-$ or $K_S\pi^0$, etc. The observables, double ratio and asymmetry, are defined as below.

\[
R_{CP\pm} \equiv \frac{2B(B^- \to D_{CP\pm}K^-) + B(B^+ \to D_{CP\pm}K^+)}{B(B^- \to D^0K^-) + B(B^+ \to \bar{D}^0K^+)} = 1 + r_B^2 \pm 2r_B \cos \delta_B \cos \phi_3
\]  

(1)

\[
A_{CP\pm} \equiv \frac{B(B^- \to D_{CP\pm}K^-) - B(B^+ \to D_{CP\pm}K^+)}{B(B^- \to D_{CP\pm}K^-) + B(B^+ \to D_{CP\pm}K^+)} = \pm 2r_B \sin \delta_B \sin \phi_3/R_{CP\pm}
\]  

(2)

where the $D_{CP\pm}$, $r_B$ and $\delta_B$ represents the $D$ meson decay into the $CP$ eigenstate of even(+) and odd(-), ratio of amplitudes between $B^- \to \bar{D}^0K^-$ and $B^- \to D^0K^-$ defined $r_B \equiv |A(B^- \to \bar{D}^0K^-)/A(B^- \to D^0K^-)|$, the difference of strong phase, respectively.
The $B^\pm \to D_{CP}^{(*)} K^{(*)\pm}$ had been measured BaBar [12, 13, 14] and Belle [15] collaboration. The BaBar recently has reported the measurement of $B^\pm \to D_{CP}^{(*)} K^{\pm}$ following $D_{CP}^+ \to K^+ K^-$ and $\pi^+ \pi^-$ for CP-even, $D_{CP}^- \to K_{S\pi}^0, K_{S\pi}^0, K_{S\omega}$ and $K_{S\phi}$ for CP-odd eigenstate with the full dataset of $467 \times 10^6 \ U(4S) \to B\bar{B}$ decays [16]. The results of combined $D$ sub-decays as for the each $CP$ eigenstate are as below.

$$A_{CP^+} = 0.25 \pm 0.06 \pm 0.02$$

$$A_{CP^-} = -0.09 \pm 0.07 \pm 0.02$$

$$R_{CP^+} = 1.18 \pm 0.09 \pm 0.05$$

$$R_{CP^-} = 1.07 \pm 0.08 \pm 0.04$$

(3)

where the first error is statistical, the second error is systematic. The parameter $A_{CP^+}$ is different from zero with a significance of $3.6 \sigma$ standard deviations. The result indicates the direct $CP$ violation followed Dalitz analysis results in the latter Section. Also, they remove the $D_{CP}^- \to K_{S\phi}$ subsample with those from the $B^\pm \to D K^\pm$, $D \to K_{S\pi}^0$ to compare the Dalitz analysis [23]. The measured $A_{CP^\pm}$ and $R_{CP^\pm}$ parameters can determine the $x_\pm$ and $y_\pm$ defined in Eq. 10 with the relation as below.

$$x_\pm = \frac{[R_{CP^+}(1 + A_{CP^+}) - R_{CP^-}(1 + A_{CP^-})]}{4}$$

(4)

The obtained $x_\pm$ are consistent with the Dalitz analysis result as below.

$$x_+ = -0.057 \pm 0.039 \pm 0.015,$$

$$x_- = 0.132 \pm 0.042 \pm 0.018.$$  

(5)

3 ADS analysis and the recent measurements

Atwood, Dunietz and Soni (ADS) [9] proposed analysis method to extract the angle $\gamma/\phi_3$ with Cabibbo-favored $D^0$ decay (CFD) and doubly Cabibbo-suppressed $D_0$ decay (DCSD) to adjust the interfering amplitudes have comparable magnitudes through the $B^- \to D^{(*)} K^{(*)-}$ decays, where $D^{(*)}$ means $D^{(*)0}$ and $D^{(*)0}$. The final state $f$ of CFD (DCFD) from $D$ meson can be used for ADS such as $D^0 \to K^- \pi^+$, $K^- \pi^+ \pi^0$ ($D_0 \to K^+ \pi^-$, $K^+ \pi^- \pi^0$), etc. The observables, double ratio and asymmetry, are defined as below.

$$R_{ADS} = \frac{\mathcal{B}(B^- \to [f]DK^-)}{\mathcal{B}(B^- \to [\bar{f}]DK^-)} + \frac{\mathcal{B}(B^+ \to [f]DK^+)}{\mathcal{B}(B^+ \to [\bar{f}]DK^+)}$$

$$= r_B^2 + r_D^2 + 2r_B r_D \cos(\delta_B + \delta_D) \cos \phi_3$$

(6)

$$A_{ADS} = \frac{\mathcal{B}(B^- \to [f]DK^-) - \mathcal{B}(B^+ \to [\bar{f}]DK^+)}{\mathcal{B}(B^- \to [\bar{f}]DK^-) + \mathcal{B}(B^+ \to [f]DK^+)}$$

$$= 2r_B r_D \sin(\delta_B + \delta_D) \sin \phi_3 / R_{ADS}$$

(7)
where \( r_D = |A(D^0 \to f)/A(\bar{D}^0 \to f)| \) and \( \delta_D \) is strong phase difference between \( \bar{D}^0 \to f \) and \( D^0 \to f \).

The \( B^\pm \to D^{(*)}K^{(*)\pm} \) had been measured by BaBar \cite{17,18} and Belle collaboration \cite{19}. The BaBar recently has reported the measurement of \( B^\pm \to D^{(*)}K^\pm \) following \( D^* \to D\gamma, D\pi^0 \) and \( D \to K\pi \) with the full dataset of \( 467 \times 10^6 \, \Upsilon(4S) \to B\bar{B} \) decays \cite{20}. The results are as below.

\[
\begin{align*}
A_{DK} &= -0.86 \pm 0.47 \pm 0.12 \\
R_{DK} &= 0.011 \pm 0.006 \pm 0.002 \\
A_{(D\gamma)K}^* &= -0.36 \pm 0.94 \pm 0.25 \\
R_{(D\gamma)K}^* &= 0.013 \pm 0.014 \pm 0.008 \\
A_{(D\pi^0)K}^* &= 0.77 \pm 0.35 \pm 0.12 \\
R_{(D\pi^0)K}^* &= 0.018 \pm 0.009 \pm 0.004
\end{align*}
\] (8)

The BaBar also has updated the \( B^\pm \to DK^\pm \) following \( D \to K\pi\pi^0 \) with dataset of \( 474 \times 10^6 \, \Upsilon(4S) \to B\bar{B} \) decays \cite{21}. The result is

\[
R_{ADS} = (9.1 \pm 8.2 \pm 14.4 \pm 3.7) \times 10^{-3}
\] (9)

The Belle also has reported the measurement of \( B^\pm \to DK^\pm \) following \( D \to K\pi \) with the full dataset of \( 772 \times 10^6 \, \Upsilon(4S) \to B\bar{B} \) decays \cite{22}. The results are as below.

\[
\begin{align*}
A_{DK} &= -0.39 \pm 0.26 \pm 0.04 \\
R_{DK} &= 0.0163 \pm 0.0044 \pm 0.0007
\end{align*}
\] (10)

The measured \( R_{DK} \) indicates the first evidence of the signal with a significance of \( 4.1\sigma \) standard deviations.

## 4 Dalitz analysis and the recent measurements

Dalitz analysis with \( D \) meson decay into \( CP \) eigenstate of three-body decay \( K_S h^+ h^- \) proposed by Giri, Grossman, Soffer, Zupan \cite{10} and Bondar \cite{11} as a effective method to extract the angle \( \gamma/\phi_3 \), where \( h^\pm \) represents charged light hadrons such as pion and kaon. The advantage of the method is only use of Cabibbo-allowed \( D \) decays.

### 4.1 Model-dependent Dalitz analysis and measurements

The BaBar and Belle had reported \( \gamma/\phi_3 \) measurement with the model-dependent Dalitz analysis \cite{23,24,25}. The model-dependent Datliz analysis uses the isobar
model \[26\] which assume three-body decayed $D$ meson proceed through the intermediate two-body resonances. The total amplitude of Dalitz plane can be represent with two amplitudes of $D^0$ and $\overline{D}^0$ decays into same final state of $K_S h^+ h^-$ as below.

$$f_{B^+} = f_D(m_+^2, m_-^2) + r_B e^{i\phi_3 + i\delta_B} f_D(m_-^2, m_+^2) \quad (11)$$

where $m_+^2 = m_{K_S h^+}^2$, $m_-^2 = m_{K_S h^-}^2$. The $f_D(m_+^2, m_-^2)$ consists of the summed amplitudes of intermediates which comes in the Dalitz plane and single non-resonant amplitude as follows.

$$f_D(m_+^2, m_-^2) = \sum_{j=1}^N a_j e^{i\xi_j} A_j(m_+^2, m_-^2) + a_{NR} e^{i\xi_{NR}} \quad (12)$$

Where $a_j$ and $\xi_j$ are the amplitude and phase of the matrix element, $A_j$ is the matrix element of the $j$-th resonance, and $a_{NR}$ and $\xi_{NR}$ are the amplitude and phase of the non-resonant component.

The Belle had reported the result of $B^\pm \to D^{(*)} K^\pm, K_S \pi^+ \pi^-$ using 18 two-body amplitudes of isobar model with the dataset of $657 \times 10^6 \ Upsilon(4S) \to \bar{B}\bar{B}$ decays \[27\].

$$\phi_3 = (78.4 \pm 10.8 \pm 3.6 \pm 8.9)^\circ \quad (13)$$

where the first error is statistical, the second is systematic, the third error is model uncertainty.

The BaBar has reported the result of $B^\pm \to D^{(*)} K^\pm, D^* \to D \pi^0$ and $D \gamma, D \to K_S \pi^+ \pi^-$ and $K_S K^+ K^-$ using isobar model with improved 8 two-body amplitudes in each $D$ meson sub-decays with the dataset of $468 \times 10^6 \ Upsilon(4S) \to \bar{B}\bar{B}$ decays \[28\].

$$\gamma = (68 \pm 14 \pm 4 \pm 3)^\circ \quad (14)$$

where the first error is statistical, the second is systematic, the third error is the model uncertainty.

### 4.2 Model-independent Dalitz analysis and measurements

The model-independent Dalitz analysis had proposed by Giri, Grossman, Soffer and Zupan \[10\], and further developed by Bonder and Poluektov \[29, 30\].

Assume that the Dalitz plot is divided into $2N$ bins symmetrically to the exchange $m_-^2 \leftrightarrow m_+^2$. The bins are denoted by the index $i$ ranging from $-N$ to $N$ (excluding 0); the exchange $m_+^2 \leftrightarrow m_-^2$ corresponds to the exchange $i \leftrightarrow -i$. Then the expected number of events in the bins of the Dalitz plot of $D$ from $B^+ \to D K^+$ is

$$N^\pm_i = h_B(K_i + r_B^2 K_{-i} + 2\sqrt{K_i K_{-i}}(x_i c_i + y_i s_i)) \quad (15)$$

4
where $K_i$ is the number of events in the bins in the Dalitz plot of the $D^0$ in a flavor eigenstate, $h_B$ is the normalization constant, $x_\pm$ and $y_\pm$ are

$$x_\pm = r_B \cos(\delta_B \pm \phi_3),$$
$$y_\pm = r_B \sin(\delta_B \pm \phi_3).$$

(Coeficients $c_i$ and $s_i$, which include the information about the cosine and sine of the phase difference given by

$$c_i = \frac{\int_{D_i} \sqrt{p_D} \cos(\Delta \delta_D (m_D^2, m^2) dD}}{\sqrt{\int_{D_i} p_D dD \int_{D_i} \sqrt{p_D} dD}},$$
$$s_i = \frac{\int_{D_i} \sqrt{p_D} \sin(\Delta \delta_D (m_D^2, m^2) dD}}{\sqrt{\int_{D_i} p_D dD \int_{D_i} \sqrt{p_D} dD}}.$$  (17)

These averaged strong phases, $c_i$ and $s_i$, in each bin can be extracted from the quantum-correlated $D^0$ decays from $\psi'(3770) \to D\overline{D}$ process. The measurement had been performed by CLEO collaboration [31].

The first model-independent Dalitz analysis of $B^\pm \to DK^\pm$, $D \to K_S \pi^+ \pi^-$ has been performed by Belle [32]. The results are as below

$$x_- = +0.095 \pm 0.045 \pm 0.014 \pm 0.017,$$
$$y_- = +0.137 \pm 0.053 \pm 0.019 \pm 0.029,$$
$$x_+ = -0.110 \pm 0.043 \pm 0.014 \pm 0.016,$$
$$y_+ = -0.050 \pm 0.052 \pm 0.011 \pm 0.021,$$  (18)

where the first error is statistical, the second is systematic, the third error is the uncertainty from $c_i$ and $s_i$. The $(\phi_3, r_B, \delta_B)$ was extracted from $(x_-, y-, x_+, y_+)$ using the frequentist treatment with the Feldman-Cousins. The results are

$$\phi_3 = (77.3 \pm 15.1 \pm 4.2 \pm 4.3)^\circ$$
$$r_B = 0.145 \pm 0.030 \pm 0.011 \pm 0.011,$$
$$\delta_B = (129.9 \pm 15.0 \pm 3.9 \pm 4.7)^\circ,$$  (19)

where the third error is uncertainty from $c_i$ and $s_i$.

5 Conclusion

The measurement precision of $\gamma/\phi 3$ have been progressed according to the data accumulation at B-factories, and have accelerated by the newly developed efficient physics methods and analysis techniques. Though current statistics of ee colliders is over the 1.2 billion $BB$ pairs, but it is still too small to exploit the $\gamma/\phi 3$ and the new physics over there. Current most precise determination is brought by the Dalitz analyses.
Both the model-independent and improved model-dependent analysis pushed down the systematic limitation and open up the possibilities of much higher precision determination at super B factories in near future. Furthermore, the recent updated ADS result indicates observation of the Cabibbo-suppressed $D$ decays which may brought us the competitive determination with the Dalitz analyses. Needless to say, it is important the various approaches including GLW method should be performed since single analysis can’t constrain the $\gamma/\phi_3$ together with the other variables, sufficiently.

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