FOUR-QUARK STATES AND NUCLEON-ANTINUCLEON ANNIHILATION WITHIN THE QUARK MODEL WITH INSTANTON INDUCED INTERACTION

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Abstract

The spectrum of $q^2\bar{q}^2$, $J^p = 0^+, 2^+$ mesons is discussed. We have shown that due to instanton-induced forces the physical states are strong mixtures of the $SU_f(3)$ group basis states. The cross-sections for annihilation of the $N\bar{N}$ system into $(q^2\bar{q}^2)(q\bar{q})$ mesons are obtained. The $a_0(980)$ meson is considered as $q^2\bar{q}^2$ meson consisting of $9_f$ and $36_f$ plets. The branchings are also predicted for the cross-sections for production of the $a_0(980)$ and tensor $q^2\bar{q}^2$ mesons in $N\bar{N}$ annihilation.

INTRODUCTION

Four-quark mesons were first considered in papers [1], [2] in the MIT version of the bag model. From this consideration it followed in particular that the known $a_0(980)$, $f_0(975)$ mesons may be assigned to the lightest $q^2\bar{q}^2$ states. Now there exist some additional experimental indications of multiquark meson states. Let us mention the most clear of them. In the experiments on vector meson $VV'$ production in $\gamma\gamma$ scattering there are observed resonance-like signals which are probably of the four-quark nature [3]. At the Serpukhov facility, in hadron-hadron scattering the extraordinary C-resonance was discovered in the $\phi\pi$ system with mass $\sim 1.5$ GeV and quantum numbers $(I=1, J^{PC} = 1^{--})$ [4]. Later on, the GAMS group reported on the resonance structure in the $\eta\pi$ channel [5] with exotic quantum numbers $(I=1, J^{PC} = 1^{--}+)$ which is probably a G-partner of the C-resonance.

Rich material for the $q^2\bar{q}^2$ meson spectroscopy can be extracted from $NN \rightarrow (q^2\bar{q}^2)(q\bar{q}) \rightarrow mesons$ reactions [6]. In paper [7] the resonance observed in $p\bar{n} \rightarrow \pi^- X_0(1480) \rightarrow 3\pi^- 2\pi^+$ reaction is analyzed. The decay of $X_0(1480)$ goes mainly through the $\rho^0\rho^0$ system. This resonance possibly also contributes to $\gamma\gamma \rightarrow \rho^0\rho^0$ reaction and was observed by the TACCO, CELLO and JADE groups. Recently the ASTERIX collaboration has presented the data on the annihilation of $p\bar{p}$ into the $\pi^+\pi^-\pi^0$ system where the resonance in the $\pi^+\pi^-$ mass spectrum with a mass $\sim 1565$ MeV and a width 170 MeV is observed. The Crystal Barrel collaboration in LEAR has observed a tensor meson with a width $\sim 120$ MeV in the $p\bar{p} \rightarrow \pi^0 X_2(1515) \rightarrow \pi^0\pi^0\pi^0$ reaction [8]. In many papers the E(1420) meson with a width $\sim 60$ MeV observed for example in $p\bar{p} \rightarrow (K^\pm K_s\pi^\mp)\pi^+\pi^-$ is widely discussed [9], [10]. The decay of the E-meson goes mainly through $a_0(980)\pi$ ($a_0(980) \rightarrow K\bar{K}$). So, if $a_0(980)$ is really a four-quark state, then it is natural...
to suppose that the E-meson is also a four-quark state. The global discussion on all four-quark system has been considered in [12].

All these experimental data need a theoretical analysis to answer the question: are these really the states that may be described as $q^2\bar{q}^2$ states? It is necessary to calculate the $q^2\bar{q}^2$ meson mass spectrum and their wave functions in the quark model where the mixing of hadron states is taken into account and investigate their decay properties.

This paper is concerned mainly with phenomenology: the spectrum of $q^2\bar{q}^2$ mesons (based on [13]), their couplings and the possibility that some $N\bar{N}$ channels are related with these exotic mesons. In the first section of this paper we review the quark model with the interaction induced by the vacuum of Quantum Chromodynamics (QCD). In the second section, in the framework of this model we present the mass spectrum of $q^2\bar{q}^2$ mesons, their couplings and the basis of physical states.

Sections 3, 4 and 5 contain the application of the results obtained. Our primary predictions concern signals indicating the possible four-quark nature of scalar (in particular, the $a_0(980)\pi$ meson), vector and tensor mesons from their production in $N\bar{N}$ reaction. This analysis is quite general. It is based only on the group-theoretical structure of this reaction and does not use a concrete dynamical model. As phenomenological parameters we use the mass of four-quark states predicted in our model and strong decay constants fitted from experiment. We expect the most bright manifestation of the four-quark states in the reaction $N\bar{N} \rightarrow (4q)$ tensor meson + $(q\bar{q})$ vector meson discussed in Sect. 5. Specifically, in the reaction $n\bar{p} \rightarrow \rho^-\rho^-\rho^+$, we predict the appearance of the exotic double charged $E_{\pi\pi}^-$ tensor meson in the invariant mass spectrum of the $\rho^-\rho^-$ system ([13]). This prediction is made for the first time. Another thing is that if the hypothesis of four-quark origin of the $a_0(980)$ is right, then from the observation of the $a_0(980)$ in $p\bar{p} \rightarrow a_0(980)\pi_0$ (Crystal Barel, CELLO) ([14], [15]) and the estimation (5.5) given below it follows that one should expect the intense yield of the exotic $E^-\pi^0\pi^0$ in the reaction $n\bar{p} \rightarrow E^-\rho^{++}$. 
Chapter 1

Quark model with QCD vacuum-induced interaction

In [16] the quark bag model has been formulated with emphasis on the quark-QCD vacuum field interaction. A crucial fact allowing the construction of a realistic model of hadrons is that there is the physical medium surrounding a bag populated by collective intensive vacuum fluctuations. The interaction of fields inside the bag with external vacuum fields completely changes the structure of the standard bag model. Within this approach the bag surface is determined self-consistently by minimizing the total energy of the hadron as a system of Dirac quarks in an external vacuum field.

Let us consider the hierarchy of fields in the QCD vacuum and the bag-vacuum system. As it is known, the QCD vacuum has a complicated structure. Conventionally the nonperturbative fields can be divided in two parts: a short-wave component that provides the helicity sensitive interaction of quarks at small distances and a long-wave component that gives the confinement. In the framework of the instanton liquid model [17, 18] the first part is connected with a fine-granular vacuum structure where the one-instanton interaction with effective size \( \rho_c \sim 1.5 \div 2 \text{ GeV}^{-1} \) dominates. The second component is connected with the long-wave collective excitations of the instanton liquid with wave length \( \lambda \approx R_{\text{hadron}} \approx R_{\text{conf}} \), where \( R \approx 3 \rho_c \) is an average distance between instantons and \( R_{\text{conf}} \approx 5 \div 6 \text{ GeV}^{-1} \) is a confinement radius.

The main assumption of our model [16] is that the QCD vacuum is almost not destroyed by color fields inside the hadron and the interaction of quarks and gluons localized in the bag with vacuum fields defines the hadron structure. We shall regard the bag and fields localized in it as immersed in the physical instanton vacuum, and we shall suggest that quarks give rise to (practically) no distortion of the local properties of this medium. This assumption is analogous to the QCD sum rule (QCD SR) one. In the latter case as a probe, i.e. a nonlocal object selecting the lowest hadron states, the correlator of hadron currents is used like the bag in our quark model. Here, one also suggests that local sources do not perturb the properties of physical vacuum, the sizes of quark and gluon condensates.

There are three different length scales of fluctuations in the bag-vacuum system: small-size instanton fluctuations with characteristic frequency \( \epsilon_i \sim 1/\rho_c \), fluctuations of fields localized inside the bag with frequency \( \epsilon_q \sim \omega_q/R_{\text{bag}} \), and long-wave vacuum fluctuations with \( \epsilon_{\text{vac}} \sim 1/R_{\text{conf}} \). For low-lying excitations of quarks in the bag, the factorization of small, intermediate and large distances occurs: \( 1/\rho_c > > \omega_q/R_{\text{bag}} > > 1/R_{\text{conf}} \), and the use of the effective Lagrangian technique is correct [19]. For the scale \( r \leq \rho_c \) the main effect is the interaction related with tunnelling due to the instanton, and at the scale \( r \sim R_{\text{bag}} \) it is the confinement of quarks in the bag. With this hierarchy of interactions we can regard that the interaction of quarks in the hadron mainly develops on one instanton on the background of external vacuum medium.

Long-wave vacuum fields \( Q(x), A_{\mu}(x) \) satisfy the classical Yang-Mills equations (acting...
on the state of physical vacuum \(|\Omega >\)

\[
(i\hat{D} - m_i)\bar{Q}^i(x) | \Omega > = 0,
\]

\[
\nabla_{\mu} G_{\mu\nu}^b(\bar{Q}^i(x) - \frac{1}{2} \gamma_\nu Q^i(x) | \Omega >, \nonumber
\]

\[
C_{\mu\nu} = \partial_\mu \bar{A}_\nu^a - \partial_\nu \bar{A}_\mu^a + gf^{abc} \bar{A}_\mu^a \bar{A}_\nu^b.
\]

Their solutions are parametrized by singlet renormalization-invariant averages \([20]\):

\[
< 0 | \frac{\alpha_s}{2\pi} : G_{\mu\nu}^a (0) G_{\alpha\beta}^{a\nu} (0) : | 0 > \approx 0.012 \text{ GeV}^4, \qquad (1.2)
\]

\[
< 0 | \frac{\alpha_s^{1/3}}{\pi} : \bar{Q}(0) Q(0) : | 0 > \approx -(250 \text{ MeV})^3, \ldots,
\]

where the sizes of condensates are determined phenomenologically within the current algebra and QCD SR. Dots stands for the operators of higher dimensions and the normal ordering of operators with respect to perturbative vacuum state \(| 0 >\) is implied.

To describe the interaction of valence quarks with long-wave vacuum fields, we make the substitution

\[
q(x) \Theta_V(x) \rightarrow q(x) \Theta_V(x) + Q(x), \quad A_\mu^a(x) \Theta_V(x) \rightarrow A_\mu^a(x) \Theta_V(x) + A_\mu^a(x)\]

in the bag model Lagrangian

\[
\mathcal{L}^{QCD} \Theta_V(x) \rightarrow \mathcal{L}^{QCD} \Theta_V(x) + \Delta \mathcal{L}_{vac}, \quad (1.4)
\]

where

\[
\Delta \mathcal{L}_{vac} = [\bar{q}(x) \Theta_V(x)](\bar{\gamma} \frac{\gamma_\nu}{2} \partial_\nu)Q(x) + \bar{Q}(x)(\bar{\gamma} \frac{\gamma_\nu}{2} \partial_\nu)[q(x) \Theta_V(x)] +
\]

\[
+ g\bar{q}(x) \gamma_\mu \frac{\lambda_\alpha}{2} q(x) A_\mu^a(x) \Theta_V(x), \quad (1.5)
\]

\(Q\) and \(\bar{Q}\) are anticommuting vacuum quark fields, \(A_\mu^a(x)\) is an external gauge field. Localized field components \(q(x)\) and \(A(x)\) are approximated by the solutions of the bag model equations in the spherical cavity approximation \([21, 22]\).

The interaction with the external long-wave vacuum field \((1.5)\) results in an additional energy increasing with bag size \([16]\). As a consequence, there occurs a situation when a further growth becomes impossible (that is, large fluctuations of the bag size are strongly suppressed). Thus, within our model the bag stability is due to the interaction of bag fields with vacuum fields.

The interaction of quarks with the short-wave component of vacuum fields, small-size instantons, is approximated by the effective 't Hooft Lagrangian \([23, 24]\) that in the instanton liquid model is \([17, 23]\):

\[
\Delta \mathcal{L}^{inst} = \sum_{i=u,d,s}^{i=u,d,s} \sum_{i>j} n_c (-k')^2 \{ \bar{q}_i R q_i L, \bar{q}_j R q_j L [1 + \frac{3}{32} \lambda_3^a \cdot \lambda_3^a (1 - \frac{3}{4} \sigma_\mu^a \cdot \sigma_\mu^a)] + (R \leftrightarrow L)\}
\]

where the constant \(k' = \frac{4\pi \rho_C^3}{3} (m_s \rho_C)\) characterizes the interaction strength of a quark with an instanton and is proportional to the instanton volume, \(n_c\) is the instanton density in the vacuum related to the vacuum energy density by \(\rho_C = \frac{\alpha_s}{2\pi} \rho_{vac}\), \(\rho_{vac}\) is an effective size of the instanton in the QCD vacuum, \(\rho_C = (\frac{1}{2} + \rho_{vac})\rho_C\).
in the physical vacuum \cite{24}, \( \langle \emptyset | \hat{Q}_i \hat{Q}_i | \emptyset \rangle \) is a quark condensate. An effective mass takes into account long-range field correlations in the instanton vacuum. The term \((R \leftrightarrow L)\) in \((1.6)\) corresponds to the interaction through an anti-instanton. In addition, averaging over instanton space positions and orientations in color space is implicitly supposed. The averaging over collective variables provides the translation and gauge invariance of instanton interaction. The Lagrangian \((1.6)\) is written for \(SU_f(2)\)-interaction in the \(SU_f(3)\) theory. Selection of \(SU_f(2)\) corresponds to the case when one of the quarks interacts with a vacuum condensate. For the \(qq\) - system one should in \((1.6)\) change

\[ \lambda_q \rightarrow -\lambda_q^T, \quad \sigma_q \rightarrow -\sigma_q^T. \]

Let us note that a specific feature of the effective Lagrangian \((1.6)\) is that only the amplitudes with quarks being in zero fermionic mode states \cite{23} are nonzero:

\[ (\vec{\sigma}_i \oplus \vec{c}_i) | \geq 0, \quad (1.7) \]

where \(\vec{\sigma}_i\) is the spin, \(\vec{c}_i\) is the color (subgroup \(SU_c(2)\)) of an \(i\)-th quark. This yields nontrivial spin-spin forces between zero mode quarks and is especially important for the phenomenology of multiquark states \cite{13,20,28}.

The interaction between quarks with long-wave vacuum fields dominates at distances of an order of the confinement radius \((R_c \sim 1 \text{ fm})\). At intermediate distances \((\rho_c \sim 0.3 \text{ fm})\) the vacuum structure is characterized by high-frequency fluctuations approximated by instantons \((\omega_{\text{inst}} \sim 1/\rho_c)\).

The hadron energy is a sum of the quark kinetic energy and the energies of interactions due to one - gluon and one - instanton exchange and interaction with condensate:

\[ E = E_{\text{kin}} + \Delta E_{\text{OGE}} + \Delta E_{\text{inst}} + E_{\text{vac}}, \quad (1.8) \]

where

\[ E_{\text{kin}} = \frac{1}{R} \sum_{i=1}^{N} [\kappa(m_i R)^2 + m_i^2 R^2]^{1/2}, \quad (1.9) \]

\[ E_{\text{vac}} = -\sum_{i,u,d,s} N_i <0|\hat{q}_i q_i|0 > A_i(m_i R) R^2 + \ldots \quad (1.10) \]

\[ \Delta E_{\text{inst}} = -\frac{\rho_c^2}{R^3} \lambda_0 <H| \sum_{i,j}^{u,d,s} [1 + \frac{3}{32} \lambda^a_{i} \lambda^a_{j} (1 + \vec{\sigma}_i \vec{\sigma}_j)]|H>, \quad (1.11) \]

\[ \Delta E_{\text{OGE}} = -\frac{\alpha_s}{R} \lambda_g <H| \sum_{i>j}^{N} (\vec{\sigma} \lambda^a_i)(\vec{\sigma} \lambda^a_j)|H>, \quad (1.12) \]

Here \(N_i\) is the number of quarks of definite flavor; \(\kappa(m_i R)\) is the eigenvalue of a lowest energy quark in a spherical cavity bag of radius \(R\); \(\vec{\sigma}_i, \lambda^a_i\) are the spin and color matrices of the \(i\)-th quark; \(\lambda_0 \approx 10.085, \lambda_g \approx 0.117, A_i(m_i R)\) are the values of integrals over cavity wave functions. These integrals and the values of the spin - color matrix elements over hadron states \(|H>\) are calculated in \cite{16,19}.

The hadron mass is given by \((1.8)\) with \(R_{\text{bag}} = R_{\text{equi}}\). The equilibrium radius \(R_{\text{equi}}\) for each state is determined by minimizing the energy in zero order in interactions: \(E_0 = E_{\text{kin}} + E_{\text{vac}}\). It is just this part of the energy operator that takes into account the (important) strangeness content of a hadron. It obviously splits the states in strangeness.

For calculating the spectrum of 4\(q\) states we use several approximations. For \(\lambda_0, \lambda_g\) we neglect the dependence of these integrals on strange quark mass and compute them at the zero mass hypothesis. Though \(E_{\text{OGE}}\) and \(E_{\text{inst}}\) have the same physical meaning, each \(\Delta \ll E_{\text{OGE}}\) and \(\Delta \ll E_{\text{inst}}\)
taken into account as perturbation. Dots in (1.10) mean the terms of higher powers of $R$ with the coefficients containing the condensates of higher dimensions \[16, 19\]. The latter are suppressed by the powers of small ratio $(\frac{ω_{vac}}{ω_{q}})^2 \propto (\frac{1}{κ_q})^2 < \frac{1}{4}$ and by a small value of corresponding space coordinate integrals. Moreover, in this work we also neglect the corrections for the spurious center of mass motion. There is nothing to prevent one from extending the approach proposed to terms of order $m_s/E_0$ in $ΔE_{int}$ and $< P^2_{cm} >/E_0^2$ as well as to contributions of vacuum expectation values of the operator of higher dimensions. It can be believed that a contribution of this sort will be helpful in a more detailed study of four - quark states.

In the next part we employ the QCD vacuum induced bag model in the spherical cavity approximation to low - lying $q^2\bar{q}^2$ mesons. It has been successful in describing the ground states of light - quark hadrons \[16, 19\].

The parameters of the model, the strong coupling constant $α_s$, the effective size of instanton $ρ_c$, the value of quark condensate $< q\bar{q} >$, characterizing various contributions, have been chosen according to the QCD sum rules,

$$α_s = 0.7, < q\bar{q} > = (-250 MeV)^3$$

(1.13)

and the instanton liquid model

$$ρ_c = 2 Gev^{-1}.$$  

(1.14)

They fit the ground state spectrum quite well. If the interaction with instantons and condensates is taken into account, we can satisfactorily describe both the mass spectra of hadron ground states and obtain correct values of $π - ρ, N - Δ, Σ - Ξ$ and $η - η'$ splittings \[19\].

An advantage of the bag model is the possibility to apply it to the calculation of $q^2\bar{q}^2$ meson spectrum without any new parameters. Previously \[28, 19\] we have applied this model to the problem of stability of the six- quark singlet $H$ dihyperon. Now we extend our study on $q^2\bar{q}^2$ mesons \[13\] to the identification of exotic states in $N\bar{N}$ reaction.
Chapter 2

Mass spectrum of $q^2\bar{q}^2$ mesons in the quark model with QCD vacuum induced interaction

Pair forces in diquark ($q^2$) and quarkonium ($q\bar{q}$) systems occur due to the interaction via gluon and instanton exchanges. The gluon exchange contribution due to a small hyperfine coupling constant is, as a rule, not important. However, the spectrum of $q^2\bar{q}^2$ mesons depends crucially on nonperturbative small size instanton interaction between quarks. In the framework of the QCD vacuum induced model, just this interaction is responsible for $\Delta - N$, $\pi - \rho$ splittings and permits us to solve $U_A(1)$ problem related with large $\eta'$ mass.

To calculate the multiquark hadron spectrum in the quark model, we should construct the physical state basis where the energy is diagonal. The operators of kinetic energy, $E_{kin}$, and the energy of interaction with a quark condensate, $E_{vac}$, depend only on the quark masses and thus they are diagonal in the basis of states with a definite number of s-quarks, $N_s$, ("magically mixed states" or "magic basis", \[1\]). However, the instanton interaction, $\Delta E_{inst}$, lifts the color - spin - flavor degeneracy and is diagonal in the basis states of the total $SU_{csf}(12)$ - group with definite color-spin-flavor quantum numbers.

A situation with the construction of the physical state basis for four-quark systems is analogous in many respects to the one for a system of $\eta, \eta'$ mesons. First, the physical states of these mesons due to the instanton annihilation process have no definite number of s-quarks, second, they do not enter into any irreducible representation of the flavor group $SU_f(3)$, but they are a mixture of the singlet - $\eta_1$ and octet - $\eta_8$ states determined from the diagonalisation of energy. As a result, $\eta - \eta'$ are splitted in mass. In the MIT model there is no reason for the occurrence of a basis different from the magic one. So it is possible there to classify particles with respect to the $SU_f(3)$ multiplets: $9_f, 36_f, 18_f$. As we shall see below, in the general case the instanton interaction that strongly violates the Okubo - Zweig - Iizuka (OZI) rule mixes irreducible representations of the $SU_f(3)$ group and the separation of particles into flavor multiplets becomes meaningless. It is important that this instanton - induced process goes not only for a color octet $q\bar{q}$ pair (as in the one-gluon annihilation case \[34\], \[35\]), which is $1/N_c$ suppressed but also for a color singlet $q\bar{q}$ pair. As we shall see, the instanton interaction lifts isospin degeneracy inherent in the MIT model.

As to the spin degrees of freedom, the ground state of the $q^2\bar{q}^2$ system can be in $J^p = 0^+, 1^+, 2^+$. Let us first consider scalar four - quark states.

2.1 Scalar $q^2\bar{q}^2$ mesons

We shall construct the basis of $q^2\bar{q}^2$ states. A pair of quarks may be in the following color,
Then from combinations of a diquark and an anti-diquark which are color singlets and obey the Pauli principle we shall construct the basis for $q^2\bar{q}^2$ states [31, 33]:

$$|q^2,\bar{q}^2> = |(q_1^2, q_2^2, q_3^2), (\bar{q}_1^2, \bar{q}_2^2, \bar{q}_3^2)>,$$

$$|1>_{cs} = |(6_c, 3_s, \bar{3}_f), (\bar{6}_c, 3_s, 3_f)>,$$

$$|3>_{cs} = |(6_c, 1_s, 6_f), (\bar{6}_c, 3_s, 6_f)>,$$

$$|2>_{cs} = |(\bar{3}_c, 1_s, \bar{3}_f), (3_c, 1_s, 3_f)>,$$

$$|4>_{cs} = |(\bar{3}_c, 3_s, 6_f), (3_c, 1_s, 6_f)>$$

Taking irreducible flavor representations:

$$3_f \otimes \bar{3}_f = 9_f = 1_f(9) \oplus 8_f(9),$$

$$6_f \otimes 6_f = 36_f = 1_f(36) \oplus 8_f(36) \oplus 27_f(36)$$

we obtain for scalar $q^2\bar{q}^2$ mesons ten possible basis states with a definite flavor:

$$|1>_{cs} 1_f(9), \quad |2>_{cs} 1_f(9), \quad |3>_{cs} 1_f(36), \quad |4>_{cs} 1_f(36),$$

$$|1>_{cs} 8_f(9), \quad |2>_{cs} 8_f(9), \quad |3>_{cs} 8_f(36), \quad |4>_{cs} 8_f(36),$$

$$|3>_{cs} 27_f, \quad |4>_{cs} 27_f$$

The physical states are certain superpositions of these states. Thus, in general, in describing experimental data it is necessary to choose nine mixing angles of these states which are free parameters. So, of main interest is the calculation of these parameters within certain dynamic mechanism leading to a physical basis.

Earlier we have shown that the instanton - induced interaction dominates in the unitary - multiplet mixing [13]. Thus, we shall suggest this interaction as the main dynamic mechanism for the construction of the physical basis. The kinetic energy and vacuum energy are easier calculated in the magic basis. It diagonalizes the strange - quark number operator and is connected with the $SU_f(3)$ basis by the known transformations ([1], Table 10; [4]). The relation between the bases allows us to rewrite either the instanton contribution in the magic basis or the kinetic energy in the $SU_f(3)$ basis. Physical states are found by diagonalisation of the total energy of a hadron.

In Tables. 1, 2, 3, there are the (rounded values of) masses of $q^2\bar{q}^2$ mesons and their decompositions in the magic basis (MIT bag states) obtained in our model. Here and in the following we use the notation accepted in [1]. The exotic states, $E$, mesons not classifiable as flavor octets or singlets, carry a subscript denoting the (pseudoscalar) flavor channel to which they couple and are also labelled by their spin parity and the $SU_f(3)$ multiplet in which they lie. The cryptoexotics, $C$, (four - quark flavor singlets or octets) carry a subscript denoting the pseudoscalar with the same quantum numbers and a superscript denoting the number of $s\bar{s}$ pairs. States at the center of the $SU_f(3)$ weight diagram have no subscript. The asterisk means a higher mass multiplet.

The results presented in Tables show that this model predicts the same range of the masses of $q^2\bar{q}^2$ states as in the MIT model. However, the magic basis states which diagonalize the energy operator in the MIT model are strongly mixed by the instanton interaction which is diagonal that respect to the total $SU_{csf}(12)$ group. This essentially changes the decay properties of four - quark mesons.

To analyze the decays of $q^2\bar{q}^2$ mesons, it is also necessary to consider the basis made up of the mesonic pair $(q\bar{q})(q\bar{q})$ states, for which the following color - spin states are possible :

$$|M > = |(q\bar{q})(q\bar{q}) > = |(q\bar{q})_c, (q\bar{q})_s, (q\bar{q})_c, (q\bar{q})_s >$$

$$|1>_{cs} = |(1_c, 1_s)(1_c, 1_s) >, \quad |2>_{cs} = |(1_c, 2_s)(1_c, 3_s) >.$$
For exotic mesons (\( I > 1 \)) in the basis (2.3) the values of masses and the recoupling coefficients practically coincide with the ones presented in [1]. For the remaining nonexotic \( q^2\bar{q}^2 \) mesons the basis (2.3) has to be supplemented by the flavor recoupling:

\[
\begin{align*}
(K\bar{K})^{I=1}, & & (\pi\eta)_{I=1}, & & (\pi\eta_0)^{I=1}, \\
(K\pi)_{I=1/2}, & & (K\eta)_{I=1/2}, & & (K\eta_0)^{I=1/2}, \\
(\eta_0), & & (\eta_0), & & (X)^{I=0}, \\
(\eta_0\eta), & & (\eta_0\eta), & & (K\bar{K})^{I=0},
\end{align*}
\tag{2.6}
\]

The states (2.3), (2.6) form together the basis of states with definite color-spin-flavor content. In Tables 4, 5, 6 the recoupling coefficients of physical \( q^2\bar{q}^2 \) states in the mesonic pair \((q\bar{q})(q\bar{q})\) states are presented.

The \( q^2\bar{q}^2 \) mesons most probably decay into a pair of \( q\bar{q} \) mesons according to the OZI superallowed diagram (Fig. 1). This means that the process is not accompanied by creation or annihilation of quark-antiquark pairs. The decay width is determined by (see f.i. [33]):

\[
\Gamma_{C_i \rightarrow mm'}(s) = \frac{|< C_i|mm' >|^2}{16\pi M_i} F_{mm'}(s),
\]

\[
< C_i|mm' > = g_0 < C_i|mm' >_c < C_i|mm' >_s < C_i|mm' >_f
\tag{2.7}
\]

where \( < C_i|mm' >_c, < C_i|mm' >_s, < C_i|mm' >_f \) are the color, spin and flavor recoupling coefficients, respectively, which determine the content of a four-quark state, \( C_i \), in terms of a pair of mesons \( MM' \) coupled to the same total quantum numbers. They are collected in Tables 4, 5, 6. As in [1], here we do not consider the suppressed decays (that require the additional creation of a quark pair or the gluon exchange). As a consequence, an \( s \)-wave \( q^2\bar{q}^2 \) meson can only dissociate to two colorless \( s \)-wave \( q\bar{q} \) mesons in a relative \( s \)-wave. The quantity \( F_{mm'}(s) \) is a phase space integral which takes into account the finite widths of reaction products; \( g_0^2/4\pi \) is the OZI super-allowed dimensional strong decay constant of a four-quark state into a meson-meson pair. 

### 2.2 Vector and tensor mesons

As it has been shown in detail in [30], the spectrum of \( J^p = 1^+ \) \( q^2\bar{q}^2 \) mesons has the following features. The main effect of the instanton interaction is that unlike the one-gluon exchange interaction, it mixes multiplets \( 18_f \) with \( 18_f \) and \( 9_f \) with \( 36_f \). Further, the states with different isotopic spins which are irreducible in masses in the MIT model are split by the instanton mechanism (\( \sim 50-200 \) Mev).

At the same time for tensor mesons, the instanton interaction practically does not change the results of the MIT model. In Tables 7, 8 the color-spin and flavor recoupling coefficients for \( q^2\bar{q}^2 \) mesons into a pair of vector mesons \( VV' \) are presented.

So, we obtain (Table 1-8) the basis of physical states of scalar, vector and tensor four-quark mesons and their mass spectrum within the quark model with QCD vacuum induced quark interaction. Next sections are devoted to consideration of the \( 4q \) states as participants in \( NN \) reaction. It should be noted that in [36, 37] the four-quark states were also used in order to describe the resonance-type signals in the \( \gamma\gamma \rightarrow VV' \) reaction.

---

1. The overall coefficients \( g_0 \) and \( h_0 \) below (Eq. (4.3)) parametrize the coordinate space overlap (vertex) integral between a four quark bag and two meson bags. Instead of calculating them directly, which is not a good defined
Chapter 3

Four-quark states in $N\bar{N}$ annihilation

The production of $q^2\bar{q}^2$ mesons can proceed according to the OZI - allowed diagram of Fig. 2a. In this process the annihilating quark - antiquark pair has vacuum quantum numbers: $3P_0, J^{pc} = 0^{++}$. Then it follows that the P-wave of $N\bar{N}$ process will dominate in this diagram. Due to this fact this reaction may be used as a good filter for $q^2\bar{q}^2$ states. As in the most part of the previous section, here we do not use any model space coordinate wave functions. Instead, we parametrize the bag - bag dynamics by some vertex constants ($g_0, h_0$) and pay attention mainly to the internal quantum number structure of processes.

There exists also an alternative way of the $q^2\bar{q}^2$ meson production. In Fig. 2b, the OZI - superallowed diagram that dominates over the diagram of Fig. 2a is shown. Moreover, the processes in Fig. 2b may go in relative S-wave with $s$-wave $q\bar{q}$ mesons being in the $J^{pc} = 0^{-+}$ or $1^{-+}$ states and $q^2\bar{q}^2$ meson being in $0^{++}, 1^{++}$ or $2^{++}$ states. From Fig. 2b it is seen that the diquark from a nucleon with the anti-diquark from an antinucleon produce the $q^2\bar{q}^2$ state which may be easily represented as a superposition of the known basis states. The remaining quark from the nucleon and antiquark from the antinucleon may be also easily combined into the usual meson states. Then, the dynamics of the interaction is reduced to the vertex of Fig. 2c. The formula for the cross-section of the process of $N\bar{N}$ annihilation into mesons is the following:

$$d\sigma(N\bar{N} \to mm') \sim \left| \sum_i \frac{<N\bar{N}|C_i m''>|C_i m''|mm'm''>}{m_i^2 - s + i\sqrt{s}\Gamma_i(s)} \right|^2,$$

(3.1)

where $C_i$ is a four - quark state, $m$ is a meson state, $m_i$ and $\Gamma_i(s)$ are the mass and width of $C_i$, $<C_i m''|mm'm''>|C_i m''>$. To calculate (3.1) we have to know the matrix elements $<N\bar{N}|C_i m''>$. First of all, let us consider the wave function of a nucleon as a $q^2\bar{q}$ system:

$$|N> = |q^2\bar{q} >= |(q_c^2q_s^2c_f^2)(q_cq_sq_f)>,$$

$$|N> = \frac{1}{\sqrt{2}}(|(3c_s3f)(3c_s3f) > + |(3c_s1f)(3c_s1f) >),$$

(3.2)

$$|\bar{N}> = \frac{1}{\sqrt{2}}(|(3\bar{c}_s6f)(3\bar{c}_s6f) > + |(3\bar{c}_s1f)(3\bar{c}_s1f) >).$$

(3.3)

From the group - theoretical point of view the wave function of the $N\bar{N}$ system may be decomposed in a complete set of $|q^2\bar{q}^2, q\bar{q}>$ states. From (3.2) one can easily obtain the expansion:

$$|N\bar{N} > = \sum_i <N\bar{N}|(q^2\bar{q}^2)(q\bar{q}) > |q^2\bar{q}^2, (q\bar{q}) > >_i = \frac{1}{8}(|36t, 9f > |9s, 4s > + |18t, 9f > |3s, 4s > + |9t, 9f > |1s, 4s >),$$
where we take into account that $3_s \otimes 3_s \equiv 9_s = 5_s \oplus 3_s \oplus 1_s$, and so on (see Table 9).

The total spin of the $(q^2 q^2)_{s}(q q)_{s}$ system is defined by the total spin of $N \bar{N}$ that can be $J^p = 0^-, 1^-(1_s, 3_s)$. In Table 9 we present the spin recoupling coefficients $< N \bar{N}|(q^2 q^2)(q q) >_s$.

The flavor recoupling of the $p \bar{p}$ system into the $|(q^2 q^2)_f(q q)_f >$ states has the following form:

$$
|36_f, 9_f >= \begin{cases} 
1/3|E^+_\pi\pi^- > +1/3|C^+_{\pi}\pi^- > \\
-2/3|E^0_{\pi\pi} \pi^0 > -1/3|C^0_{\pi}\pi^0 > + \frac{1}{3\sqrt{6}}|C_\pi^0 > \end{cases}
$$

$$
|9_f, 9_f >= 1/\sqrt{2}(-|C^0\eta > + |C^0\pi^0 >),
$$

$$
|18_f, 9_f >= 1/\sqrt{6}(|\bar{C}^0_{\pi}(18_f)\eta > - |C^0_{\pi}(18_f)\pi^0 >) - \\
-1/\sqrt{3}(|C^+_{\pi}(18_f)\pi^- > + |C^-_{\pi}(18_f)\pi^+ >),
$$

$$
\bar{C}^0_{\pi}(18_f) = 1/\sqrt{2}(C^0_{\pi}(18_f) + C^0_{\pi}(18_f)),
$$

$$
\eta = \frac{u\bar{u} + d\bar{d}}{\sqrt{2}}, \pi^0 = \frac{d\bar{d} - u\bar{u}}{\sqrt{2}}.
$$

Here $E_{\pi\pi}, C_{\pi}, C$ are four-quark states in the magic basis, and the sign of $E^\pm_{\pi\pi}, C^\pm_{\pi}, \pi^\pm$ means the charge of the state.

For the $\bar{p}n$ system we have the expansions:

$$
|36_f, 9_f >= \sqrt{2/27}|E^0_{\pi\pi}\pi^- > +1/\sqrt{27}|C\pi^- > - \\
-\sqrt{2/3}|E^0_{\pi\pi} \pi^0 > + \sqrt{2/3}|C^-_{\pi}\eta > + \\
+2/3|E^-_{\pi\pi} \pi^+ >,
$$

$$
|9_f, 9_f >= |C^0\pi^- >,
$$

$$
|18_f, 9_f >= 1/\sqrt{6}(-|C^0_{\pi}(18_f)\pi^- > + |C^0_{\pi}(18_f)\pi^0 >) + \\
+1/\sqrt{3}|C^-_{\pi}(18_f)\eta > - |C^-_{\pi}(18_f)\eta > - \\
-1/\sqrt{3}(|C^-_{\pi}(18_f)\pi^0 > + |C^-_{\pi}(18_f)\pi^0 >).
$$

The $\bar{p}n$ system has exactly defined isospin quantum numbers $I = 1$, $I_z = -1$ and thus in the expressions (3.9) the $C_{\pi}$ states have definite G-parity.

So we have obtained color, flavor, and spin recoupling of the $N \bar{N}$ wave functions into the $(q^2 q^2)(q q)$ wave functions based only on the group-theoretical considerations. This allows us to formulate a simple model of the $(q^2 q^2)(q q)$ meson production in the $N \bar{N}$ annihilation. Let us proceed to the discussion of particular examples from meson spectroscopy.
Chapter 4

The problem of the $a_0(980)$ meson.

The indication of four-quark nature of the $a_0(980)$ meson as the $C^*_\pi(9f)$ state is has been given in paper [1]. Later on, in ref. [33], the description of the $a_0(980)$ meson as a wide resonance ($\Gamma_{a_0\rightarrow\pi\eta} \sim 300 - 500 MeV$) has been suggested. In this case the narrow peak in the $\pi\eta$ mass spectrum is connected with the threshold influence of the $K\bar{K}$ channel. The influence of other $q\bar{q}^2$ states of the MIT-model basis is supposed to be negligible.

However, in our model, as a consequence of the instanton mechanism, to describe the $a_0(980)$ meson, two states with masses $m_{\text{theor}} = 1100 MeV$ and $m_{\text{theor}} = 1350 MeV$ are essential (Table 1). In Fig. 4a the mass spectrum of the channel $\pi\eta$ in the reaction $K^−p \rightarrow (C(1100) + C(1350))\Sigma^+ \rightarrow (\pi^-\eta)\Sigma^+$ is presented which is defined by the amplitude of $< K^-K^0|C(1100) + C(1350)|\pi^-\eta >$ (Fig. 3):

$$
\frac{dN}{dm_{\pi\eta}} \sim \frac{1}{16\pi} \left| \frac{K^-K^0|C(1100)> <C(1100)|\pi\eta>}{D_c(s)} + \frac{K^-K^0|C(1350)> <C(1350)|\pi\eta>}{D_c(s)} \right|^2,
$$

(4.1)

where

$$
D_c(s) = m_c^2 - s^2 - iE\Gamma_c(\sqrt{s}).
$$

(4.2)

It follows from Table 1 that both $C(1100)$ and $C(1350)$ have a considerable coupling with the states $C^*_\pi(9f)$ and $C^*_\pi(36f)$. The coupling of $36f$ with vector mesons as compared with $9f$ is 20 times stronger. The use of simple considerations of the vector dominance model makes it possible to obtain the experimental value of the width of the decay $a_0(980) \rightarrow (C(1100) + C(1350)) \rightarrow \gamma\gamma$ [39] without additional parameters. If we put $g_0 = 16$, in (2.7), then we obtain

$$
\Gamma_{a_0\rightarrow\gamma\gamma} Br(a_0 \rightarrow \pi^0\eta) \approx 0.19 KeV.
$$

(4.3)

In the experiment [15] the $a_0(980)$ meson in the $p\bar{p} \rightarrow a_0\pi$, $a_0\omega$, $a_0\eta$ reactions has been observed. In this connection it is important to note that in the $N\bar{N}$ recoupling the nonet $9f$ state with isospin $I = 1$ (in the MIT model-$C^*_\pi$) is absent which has been previously interpreted as the $a_0(980)$ meson in ref. [1], [33]. It means that $a_0(980)$ can not be observed in the $N\bar{N}$ annihilation within the suggestions of refs. [1], [33].

In our model the $a_0(980)$ meson is strongly coupled with the $C^*_\pi(36f)$ state which is in the $N\bar{N}$ decomposition (3.4), (3.7). In Figs. 4b, 4c the mass spectrum of the $\pi\eta$ system in the reactions $p\bar{p} \rightarrow a_0\omega$, $p\bar{p} \rightarrow a_0\eta$ is shown. It is defined by:

$$
\frac{dN}{dm_{\pi\eta}} \sim \frac{1}{16\pi} \left| \frac{N\bar{N}|C(1100)\omega > <C(1100)|\omega|\pi\eta\omega >}{D_c(s)} + \frac{N\bar{N}|C(1350)\omega > <C(1350)|\omega|\pi\eta\omega >}{D_c(s)} \right|^2.
$$

(4.4)
\begin{align}
<N\bar{N}|C_\omega> & \sim 1/2(\frac{1}{2\cdot3})_{s,1/3} < C|36_f > h_0, \quad (4.5) \\
<C(1100)|36_f >= & \ 0.577, \quad < C(1350)|36_f >= \ 0.556, \quad (4.6) \\
<C_\omega|\pi\eta\omega > & \approx < C|\pi\eta >, \\
<C(1100)|\pi\eta > = & \ 0.043 g_0, \quad < C(1350)|\pi\eta > = \ 0.237 g_0. \quad (4.7)
\end{align}

where the coefficients in (4.6) are taken from Table 1 and the coefficients in (4.7) are from Table 4. The quantity $g_0^2/4\pi$ is the OZI super-allowed dimensional strong coupling constant of transition of the $NN$ system into four-quark and $q\bar{q}$ mesons. In (4.5) the unpolarized initial nucleons are used:

$$|N\bar{N}>= \frac{1}{2}(\sqrt{3} 3_s \oplus 1_s)$$

(4.8)

Using only the recoupling (3.4), (3.7), we can predict the ratios of the $a_0(980)$ meson production cross-sections for different reaction channels (Table 10).

So, in the framework of our approach, a satisfactory description of the existing experimental data for the $a_0(980)$ production cross sections and the width of the decay $a_0(980) \to \gamma\gamma$ and predictions for the cross-sections in the $NN$ annihilation are obtained. The properties of this meson are unusual owing to its being described as mixing of two states, $C(1100)$, $C(1350)$, and its strong coupling with $C_s(9_f)$ and $C_s(36_f)$.

In Refs. [10], a very interesting solution has been suggested for the $a_0(980)$ – and $f_0(980)$ – meson problem. The authors of this Ref. have found that in the framework of the potential model these mesons probably correspond to a very weakly bound $K\bar{K}$ molecule. Due to this structure, the model predicts strong suppression of the $a_0(980)$ production in the $NN$ reaction. It would be crucial to check experimentally the predictions of the $a_0(980)$ production in the $NN$ reaction to discriminate between the approach suggested in [10] and that discussed in the present paper.
Chapter 5

Tensor $q^2q^2$ mesons in $N\bar{N}$ scattering

The production of $q^2q^2$ tensor mesons is possible only when it is accompanied by the vector meson ($N\bar{N} \rightarrow TV$). The most convincing manifestation of tensor mesons would be the observation of the exotic meson $E\pi\pi(36\pi)$ in the reaction

$$\bar{p}n \rightarrow E_{\pi\pi}^+ \rho^+ \rightarrow (\rho^-\rho^0)\rho^+ \rightarrow \pi^-\pi^0\pi^-\pi^0$$

(5.1)

with the vertex:

$$<\bar{p}n|E^--\rho^+>=\frac{1}{4}\frac{\sqrt{20}}{3\sqrt{3}}h_0$$

(5.2)

and the coupling constant $h_0$.

Let us list other interesting channels of the tensor meson - production:

$$\bar{p}n \rightarrow E_{\pi\pi}^+ \rho^0 \rightarrow (\rho^-\rho^0)\rho^0 \rightarrow (\pi^-\pi^0\pi^-)(\pi^0\pi^-),$$

$$\bar{p}p \rightarrow E_{\pi\pi}^+ \rho^+ \rightarrow (\rho^-\rho^0)\rho^+,\hspace{1cm}$$

(5.3)

$$\bar{p}n \rightarrow E_{\pi\pi}^- \omega \rightarrow (\rho^-\omega)\omega,$$

$$\bar{p}p \rightarrow E_{\pi\pi}^- \rho^+ \rightarrow (\rho^-\omega)\rho^+,$$

$$\bar{p}p \rightarrow E_{\pi\pi}^+ \rho^+ \rightarrow (\rho^+\omega)\rho^-,$$

$$\bar{p}p \rightarrow E_{\pi\pi}^0 \omega \rightarrow (\rho^0\omega)\rho^0.$$  

(5.4)

The ratios of the tensor meson production cross - sections in the $N\bar{N}$ reaction for different channels are presented in Table 11.

From (4.5), (5.2) and (4.8) one can estimate the relative yield of the $E^--\rho^+$ mesons:

$$\frac{\sigma(\bar{p}p \rightarrow a_0\pi)}{\sigma(\bar{p}n \rightarrow E^--\rho^+)} \leq \frac{3}{80}$$

(5.5)

The mass spectrum of the $\rho\rho$ system for a reaction like (5.3) has the form:

$$\frac{dN}{dm_{\rho\rho}} \sim F_{\rho\rho}/\pi \left| \frac{<N\bar{N}|E\rho><E|VV'>}{m_E^2 - s - i(\sqrt{2}\Gamma(s) + a/2)} \right|^2,$$

(5.6)

$$F_{\rho\rho}(s) = 1/\pi^2 \int_{4m_\rho^2}^{(\sqrt{s}-2m_\rho)^2} dm^2 \frac{m\Gamma_\rho(m)}{|D_\rho(m)|^2} \int_{4m_\rho^2}^{(\sqrt{s}-2m_\rho)^2} dm'^2 \frac{m'\Gamma_\rho(m')}{|D_\rho(m')|^2}\rho(s,m,m'),$$

$$m\Gamma_\rho(m) = m_\rho\Gamma_\rho\rho_\rho(m) \left( \frac{q(m)}{q(m_\rho)} \right)^3 \frac{1 + (Rq(m_\rho))^2}{1 + (Rq(m))^2},$$

where $\rho$ is the ratio of the tensor meson production cross-sections.
\[ s = m_{\rho \rho}^2, R = 2 \text{GeV}^{-1}, m_{\rho} = 0.321, \Gamma_{\rho} = 0.154 \text{GeV}, \]
\[ \Gamma_{E \to VV'}(s) = \frac{< E|VV' >^2}{16\pi \sqrt{s}} F_{VV'}(s), \Gamma_E(s) = \sum_{VV'} \Gamma_{EVV'}, \]
(5.7)

\[ < E|\rho \rho > = g_0 \frac{1}{\sqrt{3}}, \]

and the corresponding formula for the \( \rho \omega \) mass spectrum of the system for a reactions \( [5.4] \) is as follows:

\[ \frac{dN}{dm_{\rho \omega}} \sim F_\rho/\pi \left| \frac{< N \bar{N}|C_\rho V'' > < C_\rho |VV' >^2}{m_{C_\rho}^2 - s - i(\sqrt{s}/m_{\rho} + a/2)} \right|^2, \]
\[ F_\rho(s) = 1/\pi^2 \int \frac{dm^2 m\Gamma_\rho(m)}{|D_\rho(m)|^2} \rho(s, m_\omega, m), \]
\[ < C_\rho |\rho \omega > = g_0 \frac{1}{\sqrt{3}}, \]
\[ \Gamma_{C_\rho \to \rho \omega}(s) = \frac{< C_\rho |\rho \omega >^2}{16\pi \sqrt{s}} F_\rho(s). \]
(5.8)

The parameters \( m_E, m_{C_\rho}, g_0, a \) can be extracted from the data of the reaction \( \gamma \gamma \to q^2 \bar{q}^2 \to \rho \rho, \rho \omega \). In paper \[33\], it has been shown that \( m_E \approx 1.44 \text{GeV}, m_{C_\rho} = 1.4 \text{GeV}, g_0/(4\pi) \approx 16.4, a \approx 0.65 \). However, a partial wave analysis of the \( \gamma \gamma \to \rho \rho \) reaction can change these results.

As to scalar charged \( q^2 \bar{q}^2 \) mesons, their production is 20 times suppressed as compared to that of tensor mesons. It is enhanced only if it accompanied by a scalar meson (for example in the reaction \( N \bar{N} \to E \pi \to \rho \rho \pi )\).

The situation for neutral mesons is more difficult. The \( N \bar{N} \) system is strongly coupled with scalar mesons from the nonet \( C^0(9_f) \) and with tensor mesons from \( 36_f (E^0, C) \). We obtain for neutral \( q^2 \bar{q}^2 \) mesons \[3\] :

\[ \bar{p}n \to (E^0_{\pi \pi} + C(36_f) + C(9_f))\rho^- \to \left\{ \begin{array}{c} (\rho^+ \rho^-) \rho^- \\ (\rho^0, \rho^0) \rho^- \\ (\omega \rho) \rho^- \end{array} \right\}, \]
(5.9)

\[ \bar{p}p \to (E^0_{\pi \pi} + C(36_f) + C(9_f))\rho^0 \to \left\{ \begin{array}{c} (\rho^+ \rho^-) \rho^0 \\ (\rho^0, \rho^0) \rho^0 \\ (\omega \rho) \rho^0 \end{array} \right\}, \]
(5.10)

\[ \bar{p}p \to (C(36_f) + C(9_f))\omega \to \left\{ \begin{array}{c} (\rho^+ \rho^-) \omega \\ (\rho^0, \rho^0) \omega \\ (\omega \omega) \omega \end{array} \right\}. \]
(5.11)

The mass spectrum of the \( \rho \rho \) system in reactions \[5.3\], \[5.10\] has the form:

\[ \frac{dN}{dm_{\rho \rho'}}(s) \sim \frac{\rho_{\rho'}(s)}{\pi} < \frac{< N \bar{N}|E_{\pi \pi} V'' > < E|\rho \rho >}{D_E(s)} + \frac{< N \bar{N}|C(36_f) V'' > < C(36_f)|\rho \rho >}{D_{C(36_f)}(s)} + \frac{< N \bar{N}|C(9_f) V'' > < C(9_f)|\rho \rho >}{D_{C(9_f)}(s)} |^2, \]
(5.12)
where

$$< E|\rho \rho > = g_0 \frac{1}{\sqrt{3}} \left\{ \begin{array}{ll} -1/\sqrt{3}, & \text{for } \rho^+ \rho^- \\
1/\sqrt{2}, & \text{for } \rho^0 \rho^0, \end{array} \right. \quad (5.13)$$

$$< C(36f)|\rho \rho > = g_0 \frac{1}{\sqrt{3}} \left\{ \begin{array}{ll} 1/\sqrt{6}, & \text{for } \rho^+ \rho^- \\
1/\sqrt{12}, & \text{for } \rho^0 \rho^0 \\
\sqrt{\frac{3}{4}}, & \text{for } \omega \omega, \end{array} \right. \quad (5.14)$$

$$< C(1350)|\rho \rho > = g_0 \left\{ \begin{array}{ll} 0.472, & \text{for } \rho^+ \rho^- \\
0.334, & \text{for } \rho^0 \rho^0 \\
-0.255, & \text{for } \omega \omega, \end{array} \right. \quad (5.15)$$

$$< p\bar{p}|TV > = h_0 \frac{1}{2} \sqrt{20} \left\{ \begin{array}{ll} -\frac{2}{3\sqrt{3}}, & \text{for } E^0(36f)\rho^0 \\
-\frac{3\sqrt{6}}{7}, & \text{for } C(36f)\rho^0 \\
\frac{3\sqrt{6}}{7}, & \text{for } C(36f)\omega, \end{array} \right. \quad (5.16)$$

$$< p\bar{p}|SV > = h_0 \left\{ \begin{array}{ll} \frac{1}{2}, & \text{for } C(9f)\rho^0 \\
-\frac{1}{2}, & \text{for } C(9f)\omega, \end{array} \right. \quad (5.17)$$

$$< p\bar{p}|TV > = h_0 \frac{1}{2} \sqrt{20} \left\{ \begin{array}{ll} \frac{2}{\sqrt{27}}, & \text{for } E^0(36f)\rho^- \\
\frac{1}{\sqrt{27}}, & \text{for } C(36f)\rho^-, \end{array} \right. \quad (5.18)$$

$$< p\bar{p}|SV > = h_0 \left\{ \begin{array}{ll} \frac{1}{2}, & \text{for } C(9f)\rho^- \\
1, & \text{for } C(9f)\rho^-. \end{array} \right. \quad (5.19)$$

The functions $D(s)$ are defined by (4.2).

In reactions (5.9)-(5.11) the effects of constructive and destructive interference are essential. They occur due to the possibility of several $4q$ states to be in an intermediate state. This situation looks like the effects in the $\gamma \gamma \rightarrow VV$ reaction where a specific behavior of the invariant mass spectrum in the $\gamma \gamma$ system has been observed [3].

In the above expressions all numerical coefficients are taken from Tables 8,9 and Eqs. (3.3)-(3.8). In (5.19), (5.17), (5.19) we have the $C(1350)$ as an example of the scalar meson. These formulae provide a set of predictions on the observation of the tensor mesons in the $N\bar{N}$ annihilation. Thus, we have shown that the $N\bar{N}$ annihilation is a good filter for $0^+, 2^+ q\bar{q}^2$ mesons.

CONCLUSION

The spectrum of $q^2\bar{q}^2$ states has been calculated within the model, that takes into account the interaction between quarks through the nonperturbative vacuum of QCD. The nonperturbative vacuum of QCD is determined by two field components. The first is related to the long - wave vacuum fluctuations of quark and gluon fields (the quark and gluon condensates). The second one is constructed with the effective t’Hooft Lagrangian describing the quark interaction through the vacuum effect of small sizes. The vacuum vector, that incorporates the quark condensates, interacts with the vacuum with the effective coefficient $\lambda = 0.12$. The effective Lagrangian with such parameter is a good approximation for the vacuum energy.
hadron spectrum. The spectrum of multiquark states is of particular interest. Among a great number of the $q^2\bar{q}^2$ states one can find several states with the same quantum numbers which, in general, should be mixed. The presented approach allows us to calculate the mixing angles. These calculations give specific predictions for the production cross sections and the widths of the $q^2\bar{q}^2$ meson decays.

A search for the multiquark states usually requires the features which could distinguish these states from the hybrid, glueball and other ones, to be indicated. The analysis of nucleon-antinucleon annihilation into the mesons permits, from our viewpoint, very easy identification of most of the four-quark states. In particular, we have obtained a satisfactory description of the scalar $a_0(980)$ meson ($N\bar{N} \rightarrow a_0\omega$ data). The nature of the $a_0(980)$ meson can be clarified by measuring the meson-production cross section in different annihilation channels $N\bar{N} \rightarrow a_0\pi$, $a_0\omega$, $a_0\rho$, $a_0\eta$.

From our results we expect also a great number of resonances with the four-quark structure in different annihilation channels into three vector mesons $N\bar{N} \rightarrow (VV')V''$ in the two vector meson mass spectrum. The cross sections of four-quark meson production should satisfy rigorous ratios. This point allows an easy separation of $q^2\bar{q}^2$ mesons from a great number of the observable particles.

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Table 1. Masses of $q^2\bar{q}^2(J^p = 0^+)$ mesons and recoupling coefficients in the magic basis (I=1).

| m, MeV | $C^s_\pi(9)$ | $C^s_\pi(9^*)$ | $C_\pi(36^*)$ | $C_\pi(36)$ | $C^s_\pi(36^*)$ | $C^s_\pi(36)$ |
|---------|---------------|----------------|--------------|--------------|----------------|----------------|
| 1100    | .914          | -.021          | .025         | .577         | -.004          | .050           |
| 1350    | -.438         | -0.034         | -.010        | .556         | .014           | .705           |
| 1700    | .254          | .736           | .084         | -.379        | .017           | .493           |
| 1700    | -.014         | -.139          | .989         | -.030        | .028           | .022           |
| 1800    | -.282         | .661           | .113         | .462         | .028           | -.506          |
| 2050    | .014          | -.027          | -.032        | -.012        | .999           | -.005          |

Table 2. Masses of $q^2\bar{q}^2(J^p = 0^+)$ and recoupling coefficients in the magic basis (I=1/2).

| m, MeV | $C^s_K(9)$ | $C^s_K(9^*)$ | $C_K(36^*)$ | $C_K(36)$ | $C^s_K(36^*)$ | $C^s_K(36)$ |
|---------|------------|-------------|------------|----------|---------------|------------|
| 970     | .958       | .000        | .015       | .271     | .008          | .093       |
| 1400    | -.182      | -.133       | .009       | .821     | -.009         | -.524      |
| 1550    | .000       | .985        | -.034      | .060     | -.015         | -.157      |
| 1990    | .000       | .028        | .997       | -.038    | -.040         | -.010      |
| 2000    | -.222      | .104        | .058       | .496     | -.014         | .831       |
| 2200    | -.012      | .016        | .040       | .012     | .999          | .002       |

Table 3. Masses of $q^2\bar{q}^2(J^p = 0^+)$ and recoupling coefficients in the basis of the MIT model (I=0).

| m      | C (9) | C (9*) | C (36^*) | C (36) | C^s (9) | C^s (9*) | C^s (36^*) | C^s (36) | C^ss (36^*) | C^ss (36) |
|--------|-------|--------|----------|--------|--------|----------|------------|--------|------------|------------|
| 800    | .924  | -.003  | -.030    | -.350  | -.045  | .007     | -.040      | .130   | -.002      | .034       |
| 1140   | -.108 | -.107  | -.039    | -.293  | .872   | -.019    | -.020      | .311   | -.022      | -.176      |
| 1350   | .040  | .978   | .084     | .034   | .150   | .001     | .066       | .059   | .011       | -.062      |
| 1600   | .123  | .071   | -.550    | .584   | .026   | .101     | -.267      | .485   | -.113      | .074       |
| 1700   | .144  | -.077  | .787     | .443   | .063   | .137     | .013       | .367   | -.000      | -.006      |
| 1700   | .045  | -.010  | -.176    | .095   | .100   | .964     | -.016      | -.134  | .020       | .018       |
| 1950   | .130  | -.080  | -.191    | .240   | .046   | -.052    | .886       | .029   | .197       | -.230      |
| 2100   | -.207 | .096   | -.012    | -.312  | -.144  | .115     | .309       | .487   | -.043      | .694       |
| 2350   | -.036 | .021   | -.025    | -.026  | -.029  | -.005    | -.201      | .120   | .969       | .034       |
| 2600   | .181  | -.075  | .021     | .297   | .419   | -.160    | .028       | -.493  | .073       | .650       |
Table 4. Recoupling coefficients for \( q^2 \bar{q}^2 (J^p = 0^+) \) mesons into the pair of \((q\bar{q})(q\bar{q})\) mesons (I=1).

| m, MeV | f     | \(|1>_{cs}\) | \(|2>_{cs}\) | \(|3>_{cs}\) | \(|4>_{cs}\) |
|-------|-------|-------------|-------------|-------------|-------------|
| 1100  | \(K^+K^-\) | -.288 | .026 | .087 | -.274 |
|       | \(K^0\bar{K}^0\) | .288 | -.026 | -.087 | .274 |
|       | \(\pi^0\eta_s\) | .453 | -.029 | -.090 | .345 |
|       | \(\pi^0\eta_0\) | .373 | .121 | .219 | -.364 |
| 1350  | \(K^+K^-\) | .387 | .070 | .113 | -.072 |
|       | \(K^0\bar{K}^0\) | -.387 | -.070 | -.113 | .072 |
|       | \(\pi^0\eta_s\) | .096 | .093 | .234 | -.522 |
|       | \(\pi^0\eta_0\) | .358 | .091 | .233 | -.345 |
| 1700  | \(K^+K^-\) | .130 | -.182 | -.113 | -.387 |
|       | \(K^0\bar{K}^0\) | -.130 | .182 | .113 | .387 |
|       | \(\pi^0\eta_s\) | .267 | .398 | .428 | .109 |
|       | \(\pi^0\eta_0\) | -.241 | -.001 | -.208 | .222 |
| 1700  | \(K^+K^-\) | .001 | .057 | .037 | .024 |
|       | \(K^0\bar{K}^0\) | -.001 | -.057 | -.037 | -.024 |
|       | \(\pi^0\eta_s\) | .021 | -.045 | -.066 | -.059 |
|       | \(\pi^0\eta_0\) | .021 | .730 | -.651 | -.149 |
| 1800  | \(K^+K^-\) | .001 | -.253 | -.342 | .112 |
|       | \(K^0\bar{K}^0\) | -.001 | -.253 | .342 | -.112 |
|       | \(\pi^0\eta_0\) | -.461 | .261 | .167 | .281 |
|       | \(\pi^0\eta_s\) | .302 | .166 | .115 | -.307 |
| 2050  | \(K^+K^-\) | .012 | .380 | -.314 | -.082 |
|       | \(K^0\bar{K}^0\) | -.012 | -.380 | .314 | .082 |
|       | \(\pi^0\eta_s\) | .037 | .512 | -.471 | -.119 |
|       | \(\pi^0\eta_0\) | -.009 | -.026 | .016 | .013 |
Table 5. Recoupling coefficients for $q^2\bar{q}^2(J^p = 0^+)$ mesons into the pair of $(q\bar{q})(q\bar{q})$ mesons (I=1/2).

| m, MeV | f     | $|1 >_{cs}|$ | $|2 >_{cs}|$ | $|3 >_{cs}|$ | $|4 >_{cs}|$ |
|--------|-------|------------|------------|------------|------------|
| 970    | $K^0\pi$ | .575       | -.004      | -.074      | .367       |
|        | $K^0\pi^0$ | -.407      | .003       | .052       | -.260      |
|        | $\pi^0\eta_0$ | -.204      | .071       | .168       | -.458      |
|        | $\pi^0\eta_s$ | .050       | .022       | .033       | -.059      |
| 1400   | $K^+\pi^-$ | .137       | .007       | .097       | -.331      |
|        | $K^0\pi^0$ | -.097      | -.005      | -.069      | .234       |
|        | $\pi^0\eta_0$ | .514       | .170       | .311       | -.359      |
|        | $\pi^0\eta_s$ | -.338      | -.099      | -.207      | .328       |
| 1550   | $K^+\pi^-$ | -.108      | .443       | .453       | .271       |
|        | $K^0\pi^0$ | .076       | -.313      | -.320      | -.191      |
|        | $\pi^0\eta_0$ | .119       | -.330      | -.267      | -.228      |
|        | $\pi^0\eta_s$ | -.102      | -.039      | -.054      | .100       |
| 1900   | $K^+\pi^-$ | .003       | .312       | -.257      | -.051      |
|        | $K^0\pi^0$ | -.002      | -.221      | .182       | .036       |
|        | $\pi^0\eta_0$ | .016       | .626       | -.580      | -.131      |
|        | $\pi^0\eta_s$ | -.034      | -.039      | .006       | .038       |
| 2000   | $K^+\pi^-$ | .002       | .107       | .140       | -.202      |
|        | $K^0\pi^0$ | -.001      | -.076      | -.099      | .143       |
|        | $\pi^0\eta_0$ | .371       | .075       | .092       | -.226      |
|        | $\pi^0\eta_s$ | .535       | .136       | .347       | -.515      |
| 2200   | $K^+\pi^-$ | -.005      | .021       | .000       | -.007      |
|        | $K^0\pi^0$ | .003       | -.015      | .000       | .005       |
|        | $\pi^0\eta_0$ | .014       | .022       | -.025      | -.012      |
|        | $\pi^0\eta_s$ | .042       | .743       | -.645      | -.170      |
Table 6. Recoupling coefficients for $q^2 \bar{q}^2(J^p = 0^+)$ mesons into the pair of $(q\bar{q})(q\bar{q})$ mesons (I=0).

| m, MeV | f          | 1 > cs | 2 > cs | 3 > cs | 4 > cs |
|--------|------------|--------|--------|--------|--------|
| 800    | $\pi^+\pi^-$       | .394   | -.063  | -.162  | .513   |
|        | $\pi^0\pi^0$       | .278   | -.044  | -.115  | .363   |
|        | $K^+K^-$           | .024   | .000   | .045   | -.050  |
|        | $K^0\bar{K}^0$     | .024   | .000   | .045   | -.050  |
|        | $\eta_0\eta_0$     | -.540  | -.053  | -.028  | -.105  |
|        | $\eta_0\eta_s$     | .083   | -.009  | .047   | -.034  |
|        | $\eta_s\eta_s$     | .022   | .004   | .016   | -.021  |
| 1140   | $\pi^+\pi^-$       | -.121  | -.078  | -.072  | -.003  |
|        | $\pi^0\pi^0$       | -.085  | -.055  | -.051  | -.002  |
|        | $K^+K^-$           | .425   | -.004  | -.010  | .183   |
|        | $K^0\bar{K}^0$     | .425   | -.004  | -.010  | .183   |
|        | $\eta_0\eta_0$     | -.134  | -.038  | -.057  | .220   |
|        | $\eta_0\eta_s$     | -.320  | .062   | .211   | -.528  |
|        | $\eta_s\eta_s$     | -.114  | .047   | -.058  | .113   |
| 1350   | $\pi^+\pi^-$       | -.091  | .472   | .409   | .285   |
|        | $\pi^0\pi^0$       | -.064  | .334   | .290   | .202   |
|        | $K^+K^-$           | .038   | .016   | -.047  | .061   |
|        | $K^0\bar{K}^0$     | .038   | .016   | -.047  | .061   |
|        | $\eta_0\eta_0$     | .094   | -.255  | -.336  | -.242  |
|        | $\eta_0\eta_s$     | -.104  | .032   | -.029  | -.050  |
|        | $\eta_s\eta_s$     | -.040  | -.003  | -.032  | .037   |
| 1600   | $\pi^+\pi^-$       | .200   | -.096  | .259   | -.034  |
|        | $\pi^0\pi^0$       | .142   | -.068  | .183   | -.024  |
|        | $K^+K^-$           | .169   | -.090  | .151   | -.141  |
|        | $K^0\bar{K}^0$     | .169   | -.090  | .151   | -.141  |
|        | $\eta_0\eta_0$     | .267   | -.285  | .502   | -.289  |
|        | $\eta_0\eta_s$     | .187   | -.033  | .309   | -.165  |
|        | $\eta_s\eta_s$     | .043   | -.071  | .104   | -.027  |
| 1700   | $\pi^+\pi^-$       | .215   | .231   | -.185  | -.123  |
|        | $\pi^0\pi^0$       | .152   | .164   | -.131  | -.087  |
|        | $K^+K^-$           | .130   | .080   | .108   | -.067  |
|        | $K^0\bar{K}^0$     | .130   | .080   | .108   | -.067  |
|        | $\eta_0\eta_0$     | .215   | .602   | -.248  | -.385  |
|        | $\eta_0\eta_s$     | .152   | -.008  | .047   | -.232  |
|        | $\eta_s\eta_s$     | -.004  | -.001  | -.002  | .004   |
| m, MeV | f   | 1 > cs | 2 > cs | 3 > cs | 4 > cs |
|--------|-----|--------|--------|--------|--------|
| 1700   |     |        |        |        |        |
|        | $\pi^+\pi^-$ | .047  | -.053  | .052   | .006   |
|        | $\pi^0\pi^0$  | .033  | -.037  | .037   | .004   |
|        | $K^+K^-$       | -.092 | .291   | .270   | .272   |
|        | $K^0K^0$       | -.092 | .291   | .270   | .272   |
|        | $\eta_0\eta_0$ | .029  | -.094  | .139   | -.038  |
|        | $\eta_0\eta_5$ | .006  | -.461  | -.444  | -.262  |
|        | $\eta_s\eta_s$ | .013  | .010   | -.006  | -.015  |
| 1950   |     |        |        |        |        |
|        | $\pi^+\pi^-$ | .138  | -.081  | .040   | -.011  |
|        | $\pi^0\pi^0$  | .098  | -.057  | .028   | -.008  |
|        | $K^+K^-$       | .049  | .314   | -.301  | -.079  |
|        | $K^0K^0$       | .049  | .314   | -.301  | -.079  |
|        | $\eta_0\eta_0$ | .072  | -.058  | .227   | -.127  |
|        | $\eta_0\eta_5$ | .008  | .494   | -.368  | -.125  |
|        | $\eta_s\eta_s$ | -.140 | .106   | -.221  | .110   |
| 2100   |     |        |        |        |        |
|        | $\pi^+\pi^-$ | -.203 | .023   | .018   | .013   |
|        | $\pi^0\pi^0$  | -.144 | .017   | .013   | .009   |
|        | $K^+K^-$       | .099  | .198   | .047   | -.201  |
|        | $K^0K^0$       | .099  | .198   | .047   | -.201  |
|        | $\eta_0\eta_0$ | -.089 | -.091  | -.151  | .218   |
|        | $\eta_0\eta_5$ | .321  | .167   | -.069  | -.218  |
|        | $\eta_s\eta_s$ | .445  | .091   | .310   | -.425  |
| 2350   |     |        |        |        |        |
|        | $\pi^+\pi^-$ | -.029 | .001   | .016   | -.002  |
|        | $\pi^0\pi^0$  | -.021 | .001   | .011   | -.001  |
|        | $K^+K^-$       | .024  | -.065  | .090   | -.031  |
|        | $K^0K^0$       | .024  | -.065  | .090   | -.031  |
|        | $\eta_0\eta_0$ | -.000 | -.028  | -.005  | .025   |
|        | $\eta_0\eta_5$ | .054  | -.089  | .125   | -.011  |
|        | $\eta_s\eta_s$ | .061  | .726   | -.613  | -.185  |
| 2600   |     |        |        |        |        |
|        | $\pi^+\pi^-$ | .183  | -.012  | -.011  | -.016  |
|        | $\pi^0\pi^0$  | .130  | -.008  | -.008  | -.011  |
|        | $K^+K^-$       | .012  | -.093  | -.195  | .254   |
|        | $K^0K^0$       | .012  | -.093  | -.195  | .254   |
|        | $\eta_0\eta_0$ | .093  | .087   | .132   | -.207  |
|        | $\eta_0\eta_5$ | -.454 | .038   | -.034  | .068   |
|        | $\eta_s\eta_s$ | .422  | .169   | .217   | -.417  |
Table 7. Recoupling coefficients for $q^2q^2$ mesons into pair of vector color - singlet $VV'$ and color-octet $VV'$ states.

|      | $VV'$ | $VV'$ |
|------|-------|-------|
| $9_f$ | $\sqrt{2/3}$ | $-\sqrt{1/3}$ |
| $36_f$ | $\sqrt{1/3}$ | $\sqrt{2/3}$ |

Table 8. Recoupling coefficients in flavor for tensor mesons into vector mesons.

| flavor st. | $\rho^+\rho^-$ | $\rho^0\rho^0$ | $K^{*+}K^{*-}$ | $K^{*0}K^{*0}$ | $\omega\omega$ | $\omega\phi$ | $\rho^0\phi$ | $\rho^0\omega$ | $\phi\phi$ |
|------------|----------------|----------------|----------------|----------------|----------------|--------------|--------------|--------------|------------|
| $C^0(9_f)$ | $\sqrt{1/2}$ | $1/2$ | $0$ | $0$ | $-1/2$ | $0$ | $0$ | $0$ | $0$ |
| $C^s(9_f)$ | $0$ | $0$ | $1/2$ | $1/2$ | $0$ | $-\sqrt{1/2}$ | $0$ | $0$ | $0$ |
| $C^v(9_f)$ | $0$ | $0$ | $-1/2$ | $1/2$ | $0$ | $0$ | $\sqrt{1/2}$ | $0$ | $0$ |
| $E(36_f)$ | $-\sqrt{1/3}$ | $\sqrt{2/3}$ | $0$ | $0$ | $0$ | $0$ | $0$ | $0$ | $0$ |
| $C^0(36_f)$ | $0$ | $0$ | $0$ | $0$ | $0$ | $0$ | $0$ | $0$ | $1$ |
| $C^s(36_f)$ | $\sqrt{1/6}$ | $\sqrt{1/12}$ | $0$ | $0$ | $\sqrt{3/4}$ | $0$ | $0$ | $0$ | $0$ |
| $C^v(36_f)$ | $0$ | $0$ | $1/2$ | $-1/2$ | $0$ | $0$ | $\sqrt{1/2}$ | $0$ | $0$ |
| $C^s(36_f)$ | $0$ | $0$ | $1/2$ | $1/2$ | $0$ | $\sqrt{1/2}$ | $0$ | $0$ | $0$ |
| $C^{ss}(36_f)$ | $0$ | $0$ | $0$ | $0$ | $0$ | $0$ | $0$ | $0$ | $1$ |

Table 9. The spin recoupling coefficients $< N\bar{N}|(q^2\bar{q}^2)(q\bar{q}) >_s$.

| $J_{total}^{pc}$ | $J^2q^2q\bar{q}$ | TV | SV | VS | SS | SS | VV |
|-----------------|-----------------|----|----|----|----|----|----|
| $1^-(3_s)$ | $(9_s, 4_s)$ | $\sqrt{2\over 3\sqrt{3}}$ | $-{1\over 3\sqrt{3}}$ | $\sqrt{2\over 3}$ | $0$ | $0$ |
| $0^-(1_s)$ | $(9_s, 4_s)$ | $0$ | $0$ | $0$ | $1\over \sqrt{3}$ | $\sqrt{2\over 3}$ |
| $1^-(3_s)$ | $(3_s, 4_s)$ | $0$ | $0$ | $1\over \sqrt{3}$ | $0$ | $\sqrt{2\over 3}$ |
| $0^-(1_s)$ | $(3_s, 4_s)$ | $0$ | $0$ | $0$ | $0$ | $1$ |
| $1^-(3_s)$ | $(1_s, 4_s)$ | $0$ | $1$ | $0$ | $0$ | $0$ |
| $0^-(1_s)$ | $(1_s, 4_s)$ | $0$ | $0$ | $0$ | $1$ | $0$ |

Table 10. The ratios of the $a_0(980)$ meson production cross - sections in $NN$ reaction for different channels, $cos^2\theta \sim 2/3$. $\theta$ is a mixing angle in $\eta\eta'$ system.

| $\sigma(N\bar{N}\rightarrow a_0 \pi)$ | $p\bar{p} \rightarrow a_0 \rho$ | $p\bar{p} \rightarrow a_0 \eta$ | $p\bar{p} \rightarrow a_0 \omega$ | $\bar{p}n \rightarrow a_0 \eta$ | $\bar{p}n \rightarrow a_0 \omega$ |
|---------------------------|-----------------|----------------|----------------|----------------|----------------|
| $\sigma(N\bar{N}\rightarrow a_0 \pi)$ | $1/3$ | $cos^2\theta$ | $1/3$ | $2cos^2\theta$ | $2/3$ |
Table 11. The ratios of the tensor meson production cross-sections in $N\bar{N}$ reaction for different channels.

| Reaction                  | $\sigma(N\bar{N}\rightarrow TV)$ | $\sigma(\bar{p}n\rightarrow E^-\rho^-)$ | $\sigma(\bar{p}n\rightarrow E^-\rho^-)$ |
|---------------------------|-----------------------------------|-----------------------------------------|-----------------------------------------|
| $\bar{p}n \rightarrow E^-\rho^0$ | $1/2$                             | $\bar{p}n \rightarrow C^-\omega$         | $1/2$                                     |
| $\bar{p}p \rightarrow E^-\rho^+$ | $1/4$                             | $\bar{p}p \rightarrow C^-\rho^+$         | $1/4$                                     |
| $\bar{p}p \rightarrow E^+\rho^-$ | $1/4$                             | $\bar{p}p \rightarrow C^+\rho^-$         | $1/4$                                     |
| $\bar{p}p \rightarrow C^0\rho^0$ | $1/4$                             | $\bar{p}p \rightarrow C^0\omega$         | $1/4$                                     |

LIST OF CAPTIONS.

Figure 1. The diagrams of a) $q^2\bar{q}^2$ meson decay and b) $q^2\bar{q}^2$ meson production in $\gamma\gamma$ scattering.

Figure 2. The diagrams of $q^2\bar{q}^2$ meson production in $N\bar{N}$ annihilation a) the OZI - allowed diagram, b) the OZI - superallowed diagram, c) the vertex corresponding to b).

Figure 3. The diagram of $a_0(980)$-meson production in $pK^-$ scattering.

Figure 4. The mass spectrum of the $\pi\eta$ system in reactions a) $K^-p \rightarrow (C(1100) + C(1350))\Sigma^+ \rightarrow (\pi^-\eta)\Sigma^+$ and b) $p\bar{p} \rightarrow a_0\omega$, $p\bar{p} \rightarrow a_0\eta$. 
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