Hadron Spectroscopy with Dynamical Wilson Fermions at $\beta = 5.3^*$

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We present results from simulations of lattice QCD using two flavors of dynamical Wilson fermions at a lattice coupling $\beta = 5.3$ on $16^3 \times 32$ lattices at two hopping parameters, $\kappa = 0.1670$ and 0.1675, leading to $m_{\pi} \approx 0.44$ and 0.33 respectively. We show spectroscopy for S-wave hadrons and compare our results to other recent simulations with dynamical Wilson fermions.

There are two popular ways of discretizing the Dirac operator and action on the lattice. Wilson and staggered fermions each have some advantages and some drawbacks, and neither are entirely satisfactory. Staggered fermions have “good” chiral behavior with one $U(1) \times U(1)$ chiral symmetry, protecting massless quarks. But the spin/flavor assignment is non-trivial and really valid only in the continuum limit, and an exact simulation algorithm requires multiples of four flavors. For Wilson fermions the chiral symmetry is explicitly broken and its recovery requires fine tuning. On the other hand, spin/flavor assignments are as in the continuum theory and the exact algorithm requires only multiples of two flavors. Of course, in the continuum limit, both formulations should lead to identical physics results. It is, therefore, important to check whether this really holds.

To date most simulations with dynamical fermions employed staggered fermions. This is due to the general feeling that Wilson fermions are much more difficult to simulate and that it might be even impossible to achieve light pions on finite lattices. Indeed, in the simulations with Wilson fermions done until now the ratios $m_{\pi}/m_{\rho}$ achieved were $\gtrsim 0.7$, while those for staggered fermions $\gtrsim 0.5$, with the most recent results reported at this conference being $\approx 0.4$.

The HEMCGC ("High Energy Monte Carlo

*Presented by U. M. Heller
Grand Challenge" collaboration attempted to bring dynamical Wilson fermion simulations closer to the status of staggered fermions. Here we report on some preliminary results of this effort.

The simulations were done on the SCRI CM-2, with a lattice size of $16^3 \times 32$, using the fast CMIS inverter, running at a sustained speed of about 3 Gflops on half the machine. We chose a gauge coupling $\beta = 5.3$, somewhat smaller than in typical runs with two flavors of staggered fermions, since the renormalization of the coupling for Wilson fermions is bigger and we did not want too small a lattice spacing and hence too small a physical volume. We used two values of $\kappa$, 0.1670 and 0.1675. For $\kappa = 0.1670$ we used a conjugate gradient residual of $1 \times 10^{-5}$ — we use the normalization conventions of — and, after thermalization, time steps $dt = 0.017$ for 425 trajectories, $dt = 0.02$ for 1000 trajectories and $dt = 0.01$ for the last 100 trajectories. These choices gave acceptance rates of about 60%, 45% and 80% respectively. In total, we measured propagators on 305 configurations, separated by 5 trajectories. For $\kappa = 0.1675$ we used a time step $dt = 0.0069$ throughout. During the warm-up we used a CG residual of $1 \times 10^{-5}$ and observed the acceptance rate dropping from about 80 to $\sim 40\%$. We then lowered the CG residual to $3 \times 10^{-7}$, after some tests, after which the acceptance rate increased to about 90%. Here we measured so far on 159 configurations separated by 3 trajectories.

Masses were extracted from fully correlated $\chi^2$-fits of the propagators to a single exponential. Errors quoted in Table 1 or shown in the figures were obtained from an extrapolation in 1/block-size from blocks of 4 and 8 measurements. An example of the error extrapolation based upon which this choice was made is shown in Fig. 1.

On every configuration analyzed, we measured from two different wall sources, one at $t = 0$ (referred to as “kind” 1) and one at $t = 16$ (“kind” 2). We made fits to the propagators for each “kind” separately and also fits to both “kinds” simultaneously with a common mass but independent amplitudes. As can be seen from Table 2 the two “kinds” are consistent — the simultaneous fits “1 and 2” give a good confidence level — except for the pions. We believe the inconsistency indicates that even the extrapolated errors quoted are still underestimated due to long autocorrelations in simulation time. A sample time history for the pion propagator for $\kappa = 0.1675$ is shown in Fig. 2. A time history of the pion propagator for $\kappa = 0.1670$ already appeared in Fig. 4 of reference. In Fig. 3 we show the effective mass for the two “kinds” of pions at both $\kappa$ values.

With the caveat that our errors might be underestimated in mind, we computed mass ratios, listed in Table 2, and show them — taking the average over “kind” 1 and “kind” 2 — together with older dynamical Wilson data from Ref. in the Edinburgh plot of Fig. 4.

Extrapolating the pion squared masses (using the average over “kind” 1 and “kind” 2) to zero we find a critical kappa value $\kappa_c = 0.1680(6)$. Extrapolating the other particles to $\kappa_c$ we then obtain $m_\rho = 0.435(12)$, $m_p = 0.57(3)$, and $m_\Delta = 0.71(3)$, giving inverse lattice spacings of about 1765, 1645, and 1735 MeV, respectively.

In conclusion, we have brought dynamical Wil-
Table 1
Results from correlated fits of the propagators to a single exponential.

| Particle Kind | κ | range mass | χ²/dof | C.L. |
|---------------|---|------------|--------|-----|
| pion 1        | 0.1670 | 11 – 16   | 0.470( 4) | 20.830/4 | 0.000 |
| pion 2        | 0.1670 | 11 – 16   | 0.450( 4) | 4.483/4  | 0.345 |
| pion 1 and 2  | 0.1670 | 10 – 16   | 0.465( 3) | 52.650/11 | 0.000 |
| pion 1        | 0.1675 | 10 – 16   | 0.322( 9) | 7.396/5  | 0.193 |
| pion 2        | 0.1675 | 9 – 16    | 0.340( 7) | 3.536/6  | 0.739 |
| pion 1 and 2  | 0.1675 | 10 – 16   | 0.312(31) | 14.040/11 | 0.231 |
| rho 1         | 0.1670 | 11 – 16   | 0.656( 7) | 12.752/4 | 0.013 |
| rho 2         | 0.1670 | 9 – 16    | 0.635( 4) | 5.166/6  | 0.523 |
| rho 1 and 2   | 0.1670 | 11 – 16   | 0.650( 6) | 23.600/9 | 0.005 |
| rho 1         | 0.1675 | 5 – 16    | 0.557( 6) | 6.345/10 | 0.786 |
| rho 2         | 0.1675 | 4 – 16    | 0.531( 5) | 12.790/11 | 0.307 |
| rho 1 and 2   | 0.1675 | 6 – 16    | 0.538( 5) | 11.740/19 | 0.896 |
| proton 1      | 0.1670 | 5 – 16    | 0.952( 8) | 8.890/10 | 0.543 |
| proton 2      | 0.1670 | 5 – 16    | 0.994( 7) | 4.822/10 | 0.903 |
| proton 1 and 2| 0.1670 | 9 – 16    | 0.972( 9) | 9.806/13 | 0.710 |
| proton 1      | 0.1675 | 5 – 16    | 0.807(16) | 11.457/10 | 0.323 |
| proton 2      | 0.1675 | 4 – 16    | 0.754( 9) | 14.098/11 | 0.228 |
| proton 1 and 2| 0.1675 | 6 – 16    | 0.804(12) | 16.580/19 | 0.618 |
| delta 1       | 0.1675 | 5 – 16    | 1.028( 8) | 8.905/10 | 0.521 |
| delta 2       | 0.1675 | 6 – 16    | 1.071(10) | 14.259/10 | 0.161 |
| delta 1 and 2 | 0.1675 | 6 – 16    | 1.061( 7) | 22.710/19 | 0.250 |
| delta 1       | 0.1675 | 5 – 16    | 0.935(10) | 14.464/10 | 0.153 |
| delta 2       | 0.1675 | 4 – 16    | 0.838(13) | 4.361/11  | 0.958 |
| delta 1 and 2 | 0.1675 | 7 – 16    | 0.911(24) | 8.827/17  | 0.946 |

son fermion spectroscopy almost up to the level of the staggered one, finding a lightest pion of \( m_\pi \simeq 0.33 \), as compared to about 0.27 for staggered fermions, and \( m_\rho \simeq 0.54 \), compared to about 0.52. While these two flavor simulations are difficult, i.e., CPU time intensive, they come, contrary to the case of staggered fermions, from an exact algorithm. Our results show that light pion masses can be achieved with Wilson fermions on finite lattices. Very preliminary results from a run at \( \kappa = 0.1677 \) indicate that even lighter pions (\( m_\pi \approx 0.25 \) or less) are feasible on present day lattices.

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Figure 2. Time history of the pion propagator at distance 6 for $\kappa = 0.1675$

Figure 3. Effective pion masses

| Particles | Kind | $\kappa$ | Ratio  |
|-----------|------|----------|--------|
| $m_{\pi}/m_{\rho}$ | 1    | 0.1670   | 0.71(1) |
| $m_{\pi}/m_{\rho}$ | 2    | 0.1670   | 0.71(1) |
| $m_{\pi}/m_{\rho}$ | 1 and 2 | 0.1670 | 0.72(1) |
| $m_{\pi}/m_{\rho}$ | 1    | 0.1675   | 0.58(2) |
| $m_{\pi}/m_{\rho}$ | 2    | 0.1675   | 0.64(2) |
| $m_{\pi}/m_{\rho}$ | 1 and 2 | 0.1675 | 0.58(6) |
| $m_{P}/m_{\rho}$ | 1    | 0.1670   | 1.45(2) |
| $m_{P}/m_{\rho}$ | 2    | 0.1670   | 1.56(3) |
| $m_{P}/m_{\rho}$ | 1 and 2 | 0.1670 | 1.49(2) |
| $m_{P}/m_{\rho}$ | 1    | 0.1675   | 1.45(3) |
| $m_{P}/m_{\rho}$ | 2    | 0.1675   | 1.42(2) |
| $m_{P}/m_{\rho}$ | 1 and 2 | 0.1675 | 1.49(3) |

Figure 4. Edinburgh plot for dynamical Wilson fermions. Squares are the new HEMCGC data. All other data come from [7], with crosses for $\beta = 5.4$, diamonds for 5.5 and bursts for 5.6.