The effect of the size and position of transverse cracks in the natural frequencies of the rods

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Abstract. In this paper discusses the issues related to the development of the acoustic method of free oscillations in relation to the diagnosis of cylindrical products. The results of numerical experiment to determine the oscillation frequencies of rods with defects such as "crack".

1. Introduction
Control methods based on the measurement of natural frequencies are sufficiently widespread to determine the physical and mechanical properties of different kinds of objects, materials and products [1], [2]. It is known that the acoustic method of free oscillations allows you to quickly, cost-effective resources to make decisions about the technical condition of controlled products [3], [4]. Review of publications showed that topical are issues of control of blades blanks of turbine engines, axles of railway cars, rods pumping units of oil wells, the pistons of internal combustion engines, etc. We solve the problem of the method of free oscillations with respect to the control of cylindrical products in the parameters of oscillations.

2. Theoretical analysis
The object of the calculations is cylindrical rods 300 mm long and 16 mm in diameter, mounted on two supporting knife located on the edges of the rod at a distance of 0.22 to its length. Determine the frequency and waveform defect-free rod and bars with cracks of various sizes and location.

Calculations were carried out using finite element method.

The problem of free oscillations provides for a harmonic law changes in the movement of time in the form of:

\[ \{q(t)\} = e^{i\omega t} \{q\} \]  

The system of equations describing the free oscillations is of the form:

\[ \{M\} \{\frac{d^2 q(t)}{dt^2}\} + \{K\} \{q(t)\} = 0, \]  

where [M] - stiffness matrix, [K] - mass matrix.

Substituting (1) into (2) and canceling the common factor, we obtain a system of linear algebraic equations:

\[ [K] \{q\} - \omega^2 [M] \{q\} = 0 \]  

(3)
From a mathematical point of view, equation (3) is a generalized eigenvalue problem whose solution is reduced to finding the spectrum of eigenvalues $\lambda_i = \omega_i^2$ and their corresponding eigenvectors $\{qi\}$. In terms of the mechanics of the eigenvalues $\lambda_i$ is the square of the frequency of free oscillations of $\omega_i$, and the eigenvectors $\{qi\}$ characterize the shape of these oscillations.

Defects imitated incisions of different shapes with free edges, planes coincide with the planes of finite elements. Single defect depth was varied from 1 to 6 mm, the distance from the edge of the rod to the defect varied from 30 mm to 120 mm in increments of 30 mm (Figure 1).

Also studied two rods defects (Figure 2), the parameters of which are given in Table 1.

As an example of calculation results in Table 2 shows the frequencies of the first 28 modes of vibration of the rod without defect and defective four rods with different positions of transverse cracks.

![Figure 1. Model rod with a single defect rods.](image1)

![Figure 2. Model rod with two defects.](image2)
Table 1. Model parameters rods with two defects.

| N  | The depth of the defect | The distance to the defect from the edge of the first rod | Distance from the second defect to bearing supports |
|----|-------------------------|--------------------------------------------------------|--------------------------------------------------|
| 1  | 6 mm                    | 3 cm                                                   | 3 cm                                             |
| 2  | 6 mm                    | 3 cm                                                   | 6 cm                                             |
| 3  | 6 mm                    | 3 cm                                                   | 9 cm                                             |
| 4  | 6 mm                    | 3 cm                                                   | 12 cm                                            |
| 5  | 6 mm                    | 3 cm                                                   | 15 cm                                            |
| 6  | 6 mm                    | 3 cm                                                   | 18 cm                                            |

Table 2. The oscillation frequency of defect-free rod and four rods transverse cracks.

| Number waveforms | The frequencies of the rod without defect | Frequencies of the rods with one defect |
|------------------|------------------------------------------|----------------------------------------|
|                  | Distance from the edge of the rod to the defect | 3 cm | 6 cm | 9 cm | 12 cm |
| 1                | 794.4                                    | 794.87                                 | 792.8                                      | 789.32 | 785.73 |
| 2                | 944.9                                    | 1016.8                                 | 998.43                                    | 986.46 | 955.53 |
| 3                | 1302                                     | 1314.4                                 | 1300.4                                    | 1302.9 | 1309.6 |
| 4                | 1411                                     | 1396                                   | 1320.4                                    | 1340.1 | 1385.7 |
| 5                | 2201.1                                   | 2127.9                                 | 2030.8                                    | 2121.5 | 2127.3 |
| 6                | 2608.9                                   | 3028.6                                 | 2998.4                                    | 3025.7 | 3028.1 |
| 7                | 2997.2                                   | 3152.6                                 | 3152.6                                    | 3126.5 | 3107.4 |
| 8                | 3112.2                                   | 3270.1                                 | 3246.6                                    | 3271   | 3268.3 |
| 9                | 4468.6                                   | 4344.8                                 | 4331.2                                    | 4274.9 | 4185.1 |
| 10               | 6196.4                                   | 6210                                   | 6162.9                                    | 6167.7 | 6156.6 |

3. Conclusions
Results showed that the presence of a defect in the rod change over those frequencies antinodes waveforms which occur in the crack. Frequency, which coincides with the position of the crack node Current waveform, defect-free and have suitable rods substantially coincide.

Having a second defect leads to a shift in the oscillation frequencies corresponding to its size and position. The effect of each defect on the final product controlled oscillation spectrum is determined according to the principle of superposition.

To effectively detect the presence of defects in the product it is advisable to bring it at the same time the maximum number of frequencies and analyze all components of the spectrum.

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