Self-consistent description of Λ hypernuclei in the quark-meson coupling model

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March 31, 2022

Abstract

The quark-meson coupling (QMC) model, which has been successfully used to describe the properties of both finite nuclei and infinite nuclear matter, is applied to a study of Λ hypernuclei. With the assumption that the (self-consistent) exchanged scalar, and vector, mesons couple only to the u and d quarks, a very weak spin-orbit force in the Λ-nucleus interaction is achieved automatically. This is a direct consequence of the SU(6) quark model wave function of the Λ used in the QMC model. Possible implications and extensions of the present investigation are also discussed.

\textit{PACS:} 24.85.+p, 21.80.+a, 21.65.+f, 24.10.Jv, 21.60.-n, 21.10.Pc

\textit{Keywords:} Quark-meson coupling, Λ hypernuclei, Spin-orbit force, Λ-nucleus potential, Relativistic mean fields, Effective mass

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In earlier work we addressed the question of whether quarks play an important role in finite nuclei \([1, 2]\). This involved quantitative investigations of the properties of closed-shell nuclei from \(^{16}\text{O}\) to \(^{208}\text{Pb}\). These calculations were performed within the quark-meson coupling (QMC) model, first suggested by Guichon [3], where the interaction between nucleons involved the exchange of scalar (\(\sigma\)) and vector (\(\omega\) and \(\rho\)) mesons self-consistently coupled to the quarks within those nucleons. Within the model it has proven possible to successfully describe not only the properties of infinite nuclear matter [4], but also the properties of finite nuclei [1, 2]. Blunden and Miller [5], and Jin and Jennings [6] have made similar studies of the QMC model and some phenomenological extensions. One of the most attractive features of the QMC model is that it does not involve much in the way of additional complications to Quantum Hadrodynamics (QHD) [7]. Furthermore, it produces a reasonable value for the nuclear incompressibility [4].

Recently, with an extended version of the QMC model which took account of the quark structure of the \(\omega\) and \(\rho\) mesons as well, we studied the properties of finite nuclei in a unified manner, as well as the hadron mass changes in finite nuclei [8].

Here we develop the model further to study the properties of \(\Lambda\) hypernuclei. One of the main purposes of this article is to report the first results for \(\Lambda\) hypernuclei calculated with a version of the QMC model extended to flavor SU(3), which takes account of the quark structure of the bound \(\Lambda\) as well as the bound nucleons. Furthermore, we treat both the \(\Lambda\) hyperon sitting outside of the closed-shell nuclear core, and the nucleons in the core, fully self-consistently, in the relativistic mean field approach. Extensive results for \(\Lambda\), \(\Sigma\) and \(\Xi\) hypernuclei together will be reported elsewhere [9].

Within the Born-Oppenheimer approximation, one can derive equations of motion for a \(\Lambda\) hypernucleus in the QMC in the same way as has been done for normal nuclei[1-2]. Such an approach may provide us with important information about the hyperon-nucleon interaction, the deep nuclear interior and a possible manifestation of the quark degrees of freedom via the Pauli principle at the quark level. As an example, the very weak spin-orbit interaction for \(\Lambda\) hypernuclei, which had been phenomenologically suggested by Boussey and Hüfner [11], was first explained by Brockman and Weise [11] in a relativistic Hartree model, and directly confirmed later by experiment [12]. However, a very strong SU(3) breaking effect was required to achieve this small spin-orbit force. An explanation in terms of quark and gluon dynamics was made by Pirner [13]. Alternatively, Noble [14] showed that this smallness of the spin-orbit force could be also realized without any large breaking of SU(3) symmetry, if an \(\omega\Lambda\Lambda\) tensor coupling was introduced, analogous to the anomalous magnetic moment of the \(\Lambda\). However, Dover and Gal [15] questioned whether the tensor coupling of the \(\omega\) meson to the \(\Lambda\) could be related to the anomalous magnetic moment, because the spin of \(\Lambda\) is entirely carried by the s quark in a naive SU(6) valence quark model, and this s quark couples exclusively to the \(\phi\) meson according to the OZI rule. Later, Jennings [16] pointed out that within Dirac phenomenology the tensor coupling of the \(\omega\) meson to the \(\Lambda\) could be introduced in such a way as to guarantee that the direct \(\omega\) coupling to the spin of the \(\Lambda\) was zero – as one would expect from a simple quark model. The resulting spin-orbit force agrees with the result obtained in the present work on the basis of an explicit treatment of the quark structure of the \(\Lambda\) moving in vector and scalar fields that vary in space. Many other studies of the properties of hypernuclei have been made using relativistic, mean field theory [11]-[27]. There has also been a great deal of experimental work [12, 28, 29, 31, 32].

As a second example, it has also been discovered that there is an overbinding problem in the light \(\Lambda\) hypernuclei, and the existence of a repulsive core or the necessity of a repulsive three-body force have been suggested to overcome the problem [33]. The origin of this overbinding, which could not be explained easily in terms of traditional nuclear physics, was ascribed to
the Pauli principle at the quark level by Hungerford and Biedenharn [34]. Investigations of this repulsive core in the L-nucleus system have been made by Takeuchi and Shimizu, and others [35, 36], based on a nonrelativistic quark model.

In the light of these earlier investigations, it seems appropriate to investigate the (heavier) hypernuclear systems quantitatively using a microscopic model based on quark degrees of freedom. For this purpose, the QMC model (which is built explicitly on quark degrees of freedom) seems ideally suited, because it has already been shown to describe the properties of finite nuclei quantitatively.

Using the Born-Oppenheimer approximation one can derive mean-field equations of motion for a hypernuclear in which the quasi-particles moving in single-particle orbits are three-quark clusters with the quantum numbers of a hyperon or a nucleon. One can then construct a relativistic Lagrangian density at the hadronic level [1, 2], similar to that obtained in QHD, which produces the same equations of motion when expanded to the same order in \(v/c\):

\[
\mathcal{L}^{HY}_{QMC} = \overline{\psi}_N(\vec{r}) \left[ i \gamma \cdot \partial - M_N^*(\sigma) \right] \psi_N(\vec{r}) + \overline{\psi}_Y(\vec{r}) \left[ i \gamma \cdot \partial - M_Y^*(\sigma) \right] \psi_Y(\vec{r}) + \frac{1}{2} \left[ (\nabla \sigma(\vec{r}))^2 + m^2_{\sigma}(\vec{r})^2 \right] + \frac{1}{2} \left[ (\nabla \omega(\vec{r}))^2 + m^2_{\omega}(\vec{r})^2 \right] + \frac{1}{2} (\nabla A(\vec{r}))^2, \tag{1}
\]

where \(\psi_N(\vec{r}) (\psi_Y(\vec{r}))\) and \(b(\vec{r})\) are respectively the nucleon (hyperon) and the \(\rho\) meson (the time component in the third direction of isospin) fields, while \(m_\sigma, m_\omega, m_\rho\) are the masses of the \(\sigma, \omega, \rho\) mesons. \(g_\omega\) and \(g_\rho\) are the \(\omega-N\) and \(\rho-N\) coupling constants which are related to the corresponding (u,d)-quark-\(\omega\), \(g^\omega_{ud}\), and (u,d)-quark-\(\rho\), \(g^\rho_{ud}\), coupling constants as \(g_\omega = 3g^\omega_{ud}\) and \(g_\rho = g^\rho_{ud} [1, 4]\).

In an approximation where the \(\sigma, \omega, \rho\), and quark mesons couple only to the u and d quarks (ideal mixing of the \(\omega, \phi\), and \(\rho\) mesons, and the OZI rule are assumed), the coupling constants in the hyperon sector are obtained as \(g^\omega_\omega = (n_0/3)g_\omega\) and \(g^\rho_\rho = g^\rho_{ud}\), with \(n_0\) being the number of the valence u and d quarks in the hyperon \(Y\). \(I^Y_3\) and \(Q_Y\) are the third component of the hyperon isospin and its charge, respectively. The field dependent \(\sigma-N\) and \(\sigma-Y\) coupling strengths predicted by the QMC model, \(g_\sigma(\sigma)\) and \(g^\sigma_Y(\sigma)\), are defined by,

\[
M_N^*(\sigma) \equiv M_N - g_\sigma(\sigma) \sigma(\vec{r}); \tag{2}
\]

\[
M^*_Y(\sigma) \equiv M_Y - g^\sigma_Y(\sigma) \sigma(\vec{r}); \tag{3}
\]

where \(M_N (M_Y)\) is the free nucleon (hyperon) mass. Note that the dependence of these coupling constants on the applied scalar field must be calculated self-consistently within the quark model. Hence, unlike QHD, even though \(g^\sigma_Y(\sigma)/g_\sigma\) may be \(2/3\) in free space, this will not necessarily be the case in nuclear matter. More explicit expressions for \(g_\sigma(\sigma)\) and \(g_\sigma(\sigma)\) will be given later. From the Lagrangian density in Eq. (1), one gets a set of equations of motion for the hypernuclear system,

\[
[i \gamma \cdot \partial - M_N^*(\sigma) - (g_\omega \omega(\vec{r}) + g_\rho \tau^N_3 b(\vec{r}) + \frac{e}{2} (1 + \tau^N_3) A(\vec{r}) ) \gamma_0] \psi_N(\vec{r}) = 0, \tag{4}
\]

\[
[i \gamma \cdot \partial - M_Y^*(\sigma) - (g^\omega \omega(\vec{r}) + g^\rho I^Y_3 b(\vec{r}) + eQ_Y A(\vec{r}) ) \gamma_0] \psi_Y(\vec{r}) = 0, \tag{5}
\]

\[
(-\nabla^2 + m^2_{\sigma}) \sigma(\vec{r}) = -[\frac{\partial M_N^*(\sigma)}{\partial \sigma}] \rho_\sigma(\vec{r}) - [\frac{\partial M_Y^*(\sigma)}{\partial \sigma}] \rho^Y_\sigma(\vec{r}),
\]
\[ \equiv g_\sigma C_N(\sigma) \rho_s(\vec{r}) + g_\sigma^Y C_Y(\sigma) \rho_s^Y(\vec{r}), \]
\[ (-\nabla_s^2 + m_s^2) \omega(\vec{r}) = g_\omega \rho_B(\vec{r}) + g_\sigma^Y \rho_B^Y(\vec{r}), \]
\[ (-\nabla_s^2 + m_s^2) b(\vec{r}) = \frac{g_\rho}{2} \rho_3(\vec{r}) + g_\sigma^Y I^Y_{s} \rho_B^Y(\vec{r}), \]
\[ (-\nabla_s^2) A(\vec{r}) = e\rho_p(\vec{r}) + eQ_Y \rho_B^Y(\vec{r}), \]

where, \( \rho_s(\vec{r}) \), \( \rho_B(\vec{r}) \), \( \rho_s^Y(\vec{r}) \), \( \rho_3(\vec{r}) \) and \( \rho_p(\vec{r}) \) are the scalar, baryon, third component of isovector, and proton densities at the position \( \vec{r} \) in the hypernucleus \[.\] On the right hand side of Eq. (1), a new, and characteristic feature of QMC beyond QHD \[7, 19, 22\] appears, namely, \( -\frac{\partial M_s^*(\sigma)}{\partial \sigma} = g_\sigma C_N(\sigma) \) and \( -\frac{\partial M_s^*(\sigma)}{\partial \sigma} = g_\sigma^Y C_Y(\sigma) \), where \( g_\sigma \equiv g_\sigma(\sigma = 0) \) and \( g_\sigma^Y \equiv g_\sigma^Y(\sigma = 0) \). The effective mass for the hyperon \( Y \) is defined by,
\[ \frac{\partial M_s^*(\sigma)}{\partial \sigma} = -n_0 g_\sigma^0 \int_{bag} d\vec{y} \overline{\psi}_{u,d}(\vec{y}) \psi_{u,d}(\vec{y}) \equiv -n_0 g_\sigma^0 S_Y(\sigma) = -\frac{\partial}{\partial \sigma} \left[ g_\sigma^Y(\sigma) \sigma \right], \]
with the MIT bag model quantities \[1, 4\],
\[ M_s^*(\sigma) = \frac{n_0 \Omega^*(\sigma) + (3 - n_0) \Omega_s^* - z_s}{R_s^*}, \]
\[ S_Y(\sigma) = \frac{\Omega_s^* / 2 + m_{u,d}^*(\sigma) R_s^*(\sigma - 1)}{\Omega_s^* / (\Omega_s^* - 1) + m_{u,d}^*(\sigma) R_s^*/2}, \]
\[ \Omega^*(\sigma) = \sqrt{x_s^2 + (R_s^* \sigma_{u,d}^*)^2}, \quad \Omega_s^* = \sqrt{x_s^2 + (R_s^* \sigma_{u,d}^*)^2}, \quad m_{u,d}^*(\sigma) = m_{u,d} - g_\sigma^0 \sigma(\vec{r}), \]
\[ C_Y(\sigma) = S_Y(\sigma) / S_Y(0), \quad g_\sigma^0 \equiv n_0 g_\sigma^0 S_Y(0) / S_N(0) \equiv \frac{n_0}{3} g_\sigma \Gamma_{Y/N}. \]

Here, \( z_s, B, x, x_s \) and \( m_{u,d,s} \) are the parameters for the sum of the c.m. and gluon fluctuation effects, bag pressure, lowest eigenvalues for the \((u, d)\) and \(s\) quarks, respectively, and the corresponding current quark masses with \( m_u = m_d \). \( z_N \) and \( B \) (\( z_Y \)) are fixed by fitting the nucleon (hyperon) mass in free space.

The bag radii in-medium, \( R_{s,Y} \), are obtained by the equilibrium condition \( dM_{s,Y}^*(\sigma) / dR_{s,Y} \big|_{R_{s,Y}=R_{s,Y}} = 0 \). The bag parameters calculated and chosen for the present study are, \( (z_N, z_A) = (3.295, 3.131) \), \( (R_N, R_A) = (0.800, 0.806) \) fm (in free space), \( B^{1/4} = 170 \) MeV, \( (m_u, m_d, m_s) = (5, 5, 250) \) MeV. The parameters associated with the \( u \) and \( d \) quarks are those found in our previous investigations \[2\]. The value for the mass of the \( s \) quark in the MIT bag was chosen to be 279 MeV, in order to reproduce the mass of the \( \Lambda \) in that model \[20\]. However, the final results turn out to be insensitive to the values of many of these parameters \[1, 2\]. The value for \( \Gamma_{Y/N} \) turned out to be almost unity for all hyperons, so we can use \( \Gamma_{Y/N} = 1 \) in practice \[8\]. At the hadron level, the entire information on the quark dynamics is condensed in \( C_{s,Y}^N(\sigma) \) of Eq. (8). Furthermore, when this \( C_{s,Y}^N(\sigma) = 1 \), which corresponds to a structureless nucleon or hyperon, the equations of motion given by Eqs. (4)-(8) can be identified with those derived from QHD \[19, 21, 22\], except for the terms arising from the tensor coupling and the non-linear scalar field interaction introduced beyond naive QHD. The parameters at the hadron level, which are already fixed by the study of infinite nuclear matter and finite nuclei \[2\], are as follows: \( m_u = 783 \) MeV, \( m_d = 770 \) MeV, \( m_s = 418 \) MeV, \( e^2/4\pi = 1/137.036 \), \( g_\rho^2/4\pi = 3.12 \), \( g_\rho^2/4\pi = 5.31 \) and \( g_\rho^2/4\pi = 6.93 \).

For practical calculations, it has been found that the scalar densities \( C_{s,Y}^N(\sigma) \) can be parametrized as a linear form in the \( \sigma \) field \[1, 2, 3\],
\[ C_{s,Y}^N(\sigma) = 1 - a_{s,Y} \times (g_\sigma \sigma(\vec{r})), \]
where \( g_\sigma \sigma = (3g_\sigma^2 S_N(0)) \sigma \) in MeV, and the value for the \( \Lambda \) hyperon is, \( a_\Lambda = 9.25 \times 10^{-4} \) (MeV\(^{-1}\)) for \( m_u = 5 \) MeV, \( m_s = 250 \) MeV, \( R_N = 0.8 \) fm \( \simeq R_\Lambda \). This parametrization works very well up to about three times normal nuclear density, \( \rho_B \simeq 3\rho_0 \), with \( \rho_0 \simeq 0.15 \) fm\(^{-3}\). Using this parametrization, one can write down the explicit expression for the effective mass of the hyperon \( Y \):

\[
M_Y^*(\sigma) = M_Y - g_Y^\sigma(\sigma(\vec{r})) \simeq M_Y - \frac{n_0}{3} g_\sigma \left[ 1 - \frac{a_Y}{2}(g_\sigma(\sigma(\vec{r}))) \right] \sigma(\vec{r}).
\]  

(13)

The origin of the spin orbit force for a composite nucleon moving through scalar and vector fields which vary with position was explained in detail in Ref.[1] – c.f. sect. 3.2. The situation for the \( \Lambda \) is different in that, in an SU(6) quark model, the \( u \) and \( d \) quarks are coupled to spin zero, so that the spin of the \( \Lambda \) is carried by the \( s \) quark. As the \( \sigma \)-meson is viewed here as a convenient parametrization of two-pion-exchange and the \( \omega \) and \( \rho \) are non-strange, it seems reasonable to assume that the \( \sigma \), \( \omega \) and \( \rho \) mesons couple only to the \( u \) and \( d \) quarks. The direct contributions to the spin-orbit interaction from these mesons (derived in sect. 3 of Ref.[1]) then vanish due to the flavor-spin structure. Thus, the spin-orbit interaction, \( V_{S,O}^\Lambda(\vec{r})\vec{l} \cdot \vec{s} \), arises entirely from the Thomas precession:

\[
V_{S,O}^\Lambda(\vec{r})\vec{l} \cdot \vec{s} = \frac{1}{2} \vec{v}_\Lambda \times \frac{d\vec{v}_\Lambda}{dt} \cdot \vec{s} = -\frac{1}{2M_\Lambda^2(\vec{r})} \left( \frac{d}{dr} [M_\Lambda^*(\vec{r}) + g_\omega^\Lambda \omega(\vec{r})] \right) \vec{l} \cdot \vec{s},
\]

(14)

where, \( \vec{v}_\Lambda = \vec{p}_\Lambda/M_\Lambda^* \), is the velocity of the \( \Lambda \) in the rest frame of the \( \Lambda \) hypernucleus, and the acceleration, \( d\vec{v}_\Lambda/dt \), is obtained from the Hamilton equations of motion applied to the leading order Hamiltonian of the QMC model [1]. Because the contributions from the effective mass of the \( \Lambda \), \( M_\Lambda^*(\vec{r}) \), and the vector potential, \( g_\omega^\Lambda \omega(\vec{r}) \), are approximately equal and opposite we quite naturally expect a very small spin-orbit interaction for the \( \Lambda \) in the hypernucleus. Although the spin-orbit splittings for the nucleon calculated in QMC are already somewhat smaller than those calculated in QHD [2], we can expect even much smaller spin-orbit splittings for the \( \Lambda \) in QMC, by comparing the expressions obtained for the spin-orbit potentials between the nucleon and \( \Lambda \) [1]. (See also Fig. 2.) In order to include the spin-orbit potential of Eq. (14) correctly, we added perturbatively the correction due to the vector potential, \( -\frac{2}{2M_\Lambda^2(\vec{r})r} \left( \frac{d}{dr} g_\omega^\Lambda \omega(\vec{r}) \right) \vec{l} \cdot \vec{s} \), to the single-particle energies obtained with the Dirac equation, Eq. (2). This is necessary because the Dirac equation corresponding to Eq. (1) leads to a spin-orbit force which does not correspond to the underlying quark model, namely:

\[
V_{S,O}^\Lambda(\vec{r})\vec{l} \cdot \vec{s} = -\frac{1}{2M_\Lambda^2(\vec{r})} \left( \frac{d}{dr} [M_\Lambda^*(\vec{r}) - g_\omega^\Lambda \omega(\vec{r})] \right) \vec{l} \cdot \vec{s}.
\]

(15)

This correction to the spin-orbit force, which appears naturally in the QMC model, may also be modelled at the hadronic level of the Dirac equation by adding a tensor interaction.

We are now in a position to discuss the results. As already mentioned, the calculations are fully self-consistent for all fields appearing in Eqs. (1)–(4). In principle, the existence of the \( \Lambda \) outside of the nuclear core breaks spherical symmetry, and one should include this in a truly rigorous treatment. We neglect this effect, since it is expected to be of little importance for spectroscopic calculations [37, 38]. However, we do include the response of the nuclear core arising from the self-consistent calculation, which is significant for a description of the baryon currents and magnetic moments, and a pure relativistic effect [38, 39]. We will discuss later the self-consistency effects between the nuclear core and the \( \Lambda \).
In Fig. 1, we show the effective masses of the nucleon and Λ as well as the baryon densities calculated for (a): $^{17}_{\Lambda}O$, (b): $^{41}_{\Lambda}Ca$ and (c): $^{209}_{\Lambda}Pb$. The results are for the $1s_{1/2}$ Λ state, where effects of the Λ on the whole system are expected to be the largest. The effective masses in the hypernuclei, $^{17}_{\Lambda}O$, $^{41}_{\Lambda}Ca$ and $^{209}_{\Lambda}Pb$ behave in a similar manner as the distance $r$ from the center of each nucleus increases (the baryon density decreases). The decrease of the baryon densities around the center in $^{17}_{\Lambda}O$ in Fig. 1(a), which one might think a shortcoming of the present treatment, is also expected in nonrelativistic potential model calculation [10].

The calculated Λ single-particle energies for the closed-shell nuclear core plus one Λ configuration are listed in Table 1, with the experimental data [29, 32]. At a glance, one can easily see that the spin-orbit splittings in the present calculation are very small for all hypernuclei. These small spin-orbit splittings, tend to be smaller as the baryon density increases, or the atomic number increases. This smallness of the spin-orbit splitting is a very promising achievement in the present approach. Concerning the single-particle energy levels, although a direct comparison with the data is not precise due to the different configurations, the calculated results seem to lead to overestimates. In order to make an estimate of the difference due to this different configuration, we calculated the following quantity. By removing one $1p_{3/2}$ neutron in $^{16}O$, and putting a Λ as experimentally observed, we calculated in the same way as for $^{17}_{\Lambda}O$-which means the nuclear core was still treated as spherical. In this case the calculated energy for the $1s_{1/2}$ Λ is -19.9 MeV, to be compared with the value -20.5 MeV of the present calculation. For larger atomic numbers, the difference is smaller. (See also Table 4.) We also attempted the calculation using the scaled coupling constant, $0.93 \times g^{\Lambda}_{\sigma}(\sigma = 0)$, which reproduces the empirical single-particle energy for the $1s_{1/2}$ in $^{41}_{\Lambda}Ca$, -20.0 MeV [25]. The results obtained using this scaled coupling constant are also listed in Table 1, denoted by $^{41}_{\Lambda}Ca^*$ and $^{209}_{\Lambda}Pb^*$. Then, one can easily understand that the QMC model does not require a large SU(6) (SU(3)) breaking effect in this respect (7 %) to reproduce the empirical single-particle energies. Furthermore, the model still achieves the very small spin-orbit interaction for Λ in hypernuclei, without introducing the tensor coupling of the ω meson, which may be contrasted with the QHD type calculations [22]. We note that the spin-orbit splittings for the Λ (or hyperon) single-particle energies in hypernuclei are not well determined by the experiments [28]-[32].

In Table 2, we list the calculated binding energy per baryon, $-E/A$, rms charge radius, $r_{ch}$, and rms radii of the Λ and the neutron and proton distributions ($r_{\Lambda}$, $r_n$ and $r_p$, respectively), for the $1s_{1/2}$ and $1p_{3/2}$ Λ configurations. The rms charge radius is calculated by convolution with a proton form factor [2]. For comparison, we also give these quantities without a Λ-i.e., for normal finite nuclei. The differences in values for finite nuclei and hypernuclei listed in Table 2 reflect the effects of the Λ, through the self-consistency procedure. One can easily see that the effects of the Λ become weaker as the atomic number becomes larger, and the Λ binding energy becomes smaller.

Regarding the effects of the Λ on the core nucleons, we show also in Fig. 2 the core-nucleon single-particle energies for $^{40}Ca$, and $^{41}_{\Lambda}Ca$ for a $1s_{1/2}$ Λ state. It is interesting to see that the effects of the Λ work differently on the protons and neutrons in $^{41}_{\Lambda}Ca$. The existence of the Λ makes the scalar and baryon densities larger, and the scalar and vector potentials become stronger. As a consequence, the relative strength between the scalar, vector and Coulomb potentials change in $^{41}_{\Lambda}Ca$, compared to those in $^{40}Ca$. From the calculated results for the proton rms radius, $r_p$, and neutron energy shift shown in Fig. 2, we can conclude that the Coulomb potential increases more than the gain in the difference between the scalar and vector potentials. We can also say that the scalar potential increases more than the vector potential, by observing the neutron single-particle energies and the rms radius of the neutron, $r_n$. (See also Table 4.)
Table 1: A single-particle energies (in MeV) for closed-shell nuclear core plus one Λ configuration. The values for $^{41}\Lambda$Ca* and $^{209}\Lambda$Pb* are calculated using the scaled coupling constant $0.93 \times g_\Lambda^\sigma(\sigma = 0)$, which reproduces the empirical single particle-energy for the $1s_1/2$ in $^{41}\Lambda$Ca, -20.0 MeV [29]. For reference, we list the experimental data for $^{16}\Lambda$O and $^{40}\Lambda$Ca of Ref. [29] denoted by $a$, and for $^{89}\Lambda$Y and $^{208}\Lambda$Pb of Ref. [32] denoted by $b$, respectively. Spin-orbit splittings are not well determined by the experiments. (* fit)

|          | $^{17}\Lambda$O | $^{16}\Lambda$O | $^{41}\Lambda$Ca | $^{41}\Lambda$Ca* | $^{40}\Lambda$Ca | $^{40}\Lambda$Ca | $^{91}\Lambda$Zr | $^{89}\Lambda$Y | $^{209}\Lambda$Pb | $^{209}\Lambda$Pb* | $^{208}\Lambda$Pb |
|----------|-----------------|-----------------|------------------|------------------|-----------------|-----------------|-----------------|----------------|-----------------|-----------------|-----------------|
| $1s_{1/2}$ | -20.5           | -12.5           | -27.7            | -20.0            | -20.0           | -29.3           | -32.8           | -22.5           | -35.9           | -27.4           | -27.0           |
| $1p_{3/2}$ | -9.2            | -18.9           | -12.6            | -20.8            | -26.4           | -31.9           | -23.8           | -22.0           | -31.9           | -23.8           | -22.0           |
| $1p_{1/2}$ | -9.1            | -2.5            | -18.8            | -12.5            | -12.0           | -20.8           | -26.4           | -16.0           | -31.9           | -23.8           | -22.0           |
| $1d_{5/2}$ | (1p)            | -9.5            | -4.7             | (1p)             | -11.8           | -19.3           | -27.1           | -19.5           | (1p)            | (1p)            | -27.1           |
| $2s_{1/2}$ | -8.0            | -3.6            | -10.3            | -17.4            | -25.4           | -17.9           | -21.8           | -14.7           | -21.8           | -14.7           | -21.8           |
| $1d_{3/2}$ | -9.4            | -4.7            | -11.8            | -19.2            | -9.0           | -27.1           | -19.5           | -17.0           | -27.1           | -19.5           | -17.0           |
| $1f_{7/2}$ | -2.8            | -11.6           | (1d)             | -21.8           | -14.7           | (1d)            | -21.8           | -14.7           | (1d)            | (1d)            | (1d)            |
| $2p_{3/2}$ | -9.4            | -19.4           | -12.6            | -21.7           | -14.6           | -12.0           | -21.7           | -14.6           | -12.0           | -14.6           | -12.0           |
| $1f_{5/2}$ | -11.5           | -2.0            | -21.7           | -14.6           | -12.0           | (1f)            | -19.4           | -12.6           | (1f)            | (1f)            | (1f)            |
| $2p_{1/2}$ | -9.3            | (1f)            | -19.4           | -12.6           | -12.6           | (1f)            | -19.4           | -12.6           | (1f)            | (1f)            | (1f)            |
| $1g_{9/2}$ | -3.7            | -16.0           | -9.5            | -15.9           | -9.4            | -7.0           | -15.9           | -9.4            | -7.0           | -15.9           | -9.4            |
| $1g_{7/2}$ |              | -9.8            | -4.0            | -9.8            | -4.0            | -9.8            | -9.8            | -4.0            | -9.8            | -4.0            | -9.8            |
| $1h_{11/2}$ |              | -13.2           | -7.1            | -13.2           | -7.1            | -13.2           | -7.1            | -13.2           | -7.1            | -13.2           | -7.1            |
| $2d_{5/2}$ |              | -13.2           | -7.1            | -13.2           | -7.1            | -13.2           | -7.1            | -13.2           | -7.1            | -13.2           | -7.1            |
| $2d_{3/2}$ |              | -9.7            | -3.9            | -12.1           | -6.2            | -6.9            | -1.8            | -6.9            | -1.8            | -6.9            | -1.8            |
| $3s_{1/2}$ |              | -5.6            | -1.1            | -5.6            | -1.1            | -5.6            | -1.1            | -5.6            | -1.1            | -5.6            | -1.1            |
| $2f_{7/2}$ |              | -6.8            | -1.7            | -6.8            | -1.7            | -6.8            | -1.7            | -6.8            | -1.7            | -6.8            | -1.7            |
| $3p_{1/2}$ |              | -5.6            | -1.1            | -5.6            | -1.1            | -5.6            | -1.1            | -5.6            | -1.1            | -5.6            | -1.1            |
| $1i_{13/2}$ |             | -3.4            | -    | -3.4            | -    | -3.4            | -    | -3.4            | -    | -3.4            | -    |
Table 2: Binding energy per baryon, $-E/A$ (in MeV), rms charge radius, $r_{ch}$, and rms radii of the $\Lambda$, $r_\Lambda$, neutron, $r_n$, and proton, $r_p$ (in fm). (* fit)

| $^A_\Lambda$ state | $-E/A$ | $r_{ch}$ | $r_\Lambda$ | $r_n$ | $r_p$ |
|---------------------|--------|----------|-------------|-------|-------|
| $^{17}_\Lambda$O    | 6.75   | 2.85     | 2.22        | 2.58  | 2.73  |
| $^{17}_\Lambda$O    | 6.09   | 2.82     | 2.95        | 2.61  | 2.70  |
| $^{16}_\Lambda$O    | 5.84   | 2.79     | 2.64        | 2.67  |       |
| $^{41}_\Lambda$Ca   | 1s1/2  | 7.83     | 3.52        | 2.55  | 3.30  | 3.42  |
| $^{41}_\Lambda$Ca   | 1p3/2  | 7.57     | 3.51        | 3.17  | 3.32  | 3.41  |
| $^{40}_\Lambda$Ca   |        | 7.36     | 3.48*       |       | 3.33  | 3.38  |
| $^{49}_\Lambda$Ca   | 1s1/2  | 7.75     | 3.54        | 2.61  | 3.63  | 3.45  |
| $^{49}_\Lambda$Ca   | 1p3/2  | 7.53     | 3.54        | 3.24  | 3.64  | 3.44  |
| $^{48}_\Lambda$Ca   |        | 7.27     | 3.52        |       | 3.66  | 3.42  |
| $^{91}_\Lambda$Zr   | 1s1/2  | 8.04     | 4.29        | 3.02  | 4.29  | 4.21  |
| $^{91}_\Lambda$Zr   | 1p3/2  | 7.98     | 4.28        | 3.66  | 4.30  | 4.21  |
| $^{90}_\Lambda$Zr   |        | 7.79     | 4.27        |       | 4.31  | 4.19  |
| $^{209}_\Lambda$Pb  | 1s1/2  | 7.39     | 5.50        | 3.79  | 5.67  | 5.43  |
| $^{209}_\Lambda$Pb  | 1p3/2  | 7.37     | 5.49        | 4.50  | 5.67  | 5.43  |
| $^{208}_\Lambda$Pb  |        | 7.25     | 5.49        |       | 5.68  | 5.43  |

Finally, we show the scalar and vector potential strength for $^{17}_\Lambda$O, $^{41}_\Lambda$Ca and $^{209}_\Lambda$Pb in Fig. 3. The difference between the scalar and vector potentials near the center of the hypernuclei is typically $\sim 35 - 40$ MeV. This value is slightly ($\sim 5 - 10$ MeV) larger than that calculated by Ma et al. [26], which seems to be the specific origin of the overbinding in the present calculation. Although our calculations are based on quark degrees of freedom, in the effective hadronic model the whole dynamics of the quarks and gluons are absorbed into the parameters and coupling constants appearing at the hadron level. In this sense, the effect of the Pauli principle at the quark level [34, 35], is not included between the quarks in the $\Lambda$ and the nucleons. It seems necessary to incorporate the Pauli principle at the quark level, or some equivalent effect which would produce a repulsive core in hypernuclear systems, if one is to reproduce the experimental binding energies in the QMC model.

In summary, we have reported the first results for $\Lambda$ hypernuclei calculated with the QMC model extended to the flavor SU(3). The very small spin-orbit force for $\Lambda$ in hypernuclei was achieved. This is a very promising feature and a direct consequence of the SU(6) quark model wave function and the quark structure of the $\Lambda$ in the QMC model. However, the calculated single-particle energies for the $\Lambda$ tend to be overestimated in comparison with the experimental data. In order to overcome this overbinding problem, it may be necessary to introduce a repulsive core due to the Pauli principle at the quark level, or some equivalent effect. This is a challenging problem for future work.

We would like to thank R. Brockmann for helpful discussions concerning the calculation, and P.A.M. Guichon for useful comments. This work was supported by the Australian Research Council, and A.W.T and K.S. acknowledge support from the Japan Society for the promotion of Science.
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Figure 1: (a)

Figure 1: (b)
Calculated baryon densities, $\rho_B$, and effective masses of the nucleon denoted by, $N$, and the $\Lambda$ hyperon denoted by, $\Lambda$, in hypernuclei for (a): $^{17}_\Lambda$O, (b): $^{41}_\Lambda$Ca, and (c): $^{209}_\Lambda$Pb. All cases are for the $1s_{1/2} \Lambda$ state.
Figure 2: Nucleon single-particle energies for $^{40}\text{Ca}$, and $^{41}_{\Lambda}\text{Ca}$ for the $1s_{1/2}$ $\Lambda$ state.
Figure 3: Calculated scalar and vector potential strength for \(^{17}\Lambda\)O, \(^{41}\Lambda\)Ca and \(^{209}\Lambda\)Pb.