Modeling and Analysis of Cu-Carbon Nanotube Composites for Sub-Threshold Interconnects

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Abstract The sub-threshold regime is suited for applications requiring ultra-low power consumption with low to medium frequency (tens to hundreds of MHz) of operation. Therefore, this paper presents electrical modeling and comprehensive analysis of copper-carbon nanotube (Cu-CNT) composite interconnects for sub-threshold circuit design. At lower operating frequencies, the effective complex conductivity of Cu-CNT composites in the nanoscale is formulated by developing an analytical model. Based on the proposed equivalent single conductor model, the frequency-dependent resistance and inductance of composite interconnects are computed. Finally, the sub-threshold crosstalk effect, transfer gain, and Nyquist stability of coupled Cu-CNT composite interconnect are analyzed using ABCD matrix approach.

Index Terms Cu–CNT composite, delay, effective conductivity, frequency response, relative stability, sub-threshold.

I. INTRODUCTION

Development of electronic devices is undergoing a paradigm shift with their continued miniaturization [1]. In the current research scenario, due to the exponential scaling of metal-oxide semiconductor (MOS) devices, global interconnect delay is becoming one of the major concerns. More importantly, the reliability issue has become a crucial issue faced by the integrated circuits (ICs) due to rapidly increasing current densities. At nanoscale technology nodes, mechanisms such as surface scattering and grain boundary scattering play a crucial role in the performance estimation of on-chip copper (Cu) interconnects [2], [3], [4]. Such effects are further aggravated below 10 nm since Cu ions start diffusing through Cu-dielectric interface and grain boundary along the interconnect length. This result in discontinuity and void in Cu lines, which leads to the electrical, mechanical, thermal performance degradation and reliability issues during the design of nano-packaging on-chip, interconnects [5]. Diffusion of Cu ions becomes prominent at lower technology nodes because activation energy is smaller than the bulk Cu. Diffusion of Cu ions into the dielectric region creates a leakage path between dielectric and metal layer and it also severely degrades the quality of the dielectric. Therefore, researchers have to look for alternative solutions to this issue.

In the last few decades, carbon nanotubes (CNTs) have attracted a lot of attention as next-generation interconnect technology [6], [7]. CNTs exhibit higher ampacity (maximum current-carrying capacity), larger thermal conductivity, and long mean free path due to their strong C–C bonding in comparison with copper (Cu) counterpart [8], [9], [10]. Thus, for applications mainly in the nanoscale regime, CNTs exhibit superior electrical performance than Cu. Previously, a composite of Cu-CNT was demonstrated in [11]. The fabricated results showed that the thermal conductivity of Cu-CNT composite (395 W m⁻¹ K⁻¹) is comparable with Cu. The Cu-CNT composite possesses excellent compatibility with silicon due to a much-reduced coefficient of thermal expansion mismatch, which is ~10%. More encouraging results
were shown by Subramaniam et al. [12]. They successfully fabricated Cu-CNT composite and demonstrated about 100 × higher ampacity over Cu. Without sacrificing reliability, the composite exhibits superior electrical characteristics with significantly high electromigration resistance and ~13% reduced electrical conductivity. The Cu-CNT composite can be fabricated by electrodeposition of Cu into the pores of premade CNTs. Using a two-stage electroplating and electroless plating process, Cu can then be interlaced into the CNTs [13]. The vertically aligned CNTs were also demonstrated in [14] using chemical vapor deposition and plasma-enhanced chemical vapor deposition techniques. In particular, the Cu-CNT composite is potential in achieving a better trade-off between the performance and reliability and can facilitate a more realistic approach for fabricating on-chip interconnects. However, an in-depth analysis of the sub-threshold performance of Cu-CNT composite based interconnects in ICs which require low to medium performance is still lacking. Our previous research work has successfully established the potential of CNT bundle for sub-threshold interconnects [15]. Thus, it seems reasonable to extend the analysis for Cu-CNT composites to meet the near-future challenges such as higher crosstalk delay and degraded reliability of sub-threshold interconnects, which is the main impetus behind this research study.

In this paper, different composites comprising of Cu-SWCNT bundle (Cu-SWB), Cu-MCWNT bundle (Cu-MWB), and Cu-mixed CNT bundle (Cu-MCB) are considered. The performance of different Cu-CNT composites is computed using Gaussian distribution criterion. It is illustrated that after co-depositing more number of CNTs with Cu, the conductivity of Cu-CNT composites is significantly increased. The electrical performance of Cu-CNT composite interconnects could also be improved by increasing the metallic fraction, as reported in [16], similar to the pure CNTs. The rest of this paper is organized as follows: In Section II, the electrical modeling and characterization of different composite bundles is described with the effective complex conductivity for each bundle computed accurately. Thereafter, a distributed equivalent single conductor (ESC) model of Cu-SWB/ Cu-MWB/ Cu-MCB composite interconnects is presented in Table 1. The probability of distribution of CNTs having a particular diameter in Table 2.

![FIGURE 1. (a) Schematic view and (b) Distributed equivalent single conductor (ESC) model of Cu-SWB/ Cu-MWB/ Cu-MCB composite interconnects.](image)

### II. ELECTRICAL MODEL OF COPPER–CNT COMPOSITES

The schematic of a pair of Cu-CNT composites placed at a distance $d$ over the ground plane is shown in Fig. 1(a). The cross-sectional area of the bundle is considered as 18 nm × 54 nm with an aspect ratio of 3:1. Further, $w$, $t$, and $l$ symbolize the width, thickness, and length of the bundle, respectively. The Cu-CNT composite consists of $G$ types of CNTs having non-uniform diameters with an inter-shell spacing of $\delta = 0.34$ nm. In this analysis, ten different types of CNTs are taken into consideration i.e., $G = 10$ whose diameter $D$ ranges from $D_1 \leq D_i \leq D_{max}$ where, $D_1 = 1$ nm and $D_{max} = 7.12$ nm, are the inner and outer shell diameter of MWCNT, respectively.

The diameter of the $i^{th}$ shell of MWCNT is given as $D_i = D_{1} + 2 \cdot \delta \cdot (i - 1)$, where, $1 \leq i \leq m$

Also, $D_{1} = (D'_{out} - 3D_{out})$ and $G = 3D_{out}/\delta$ (1)

In (1), $m$, $D'_{out}$ and $\sigma_{D_{out}}$ represent the number of shells of MWCNT, the mean diameter and standard deviation, respectively. The CNT filling ratio, $f_{cnt}$ is defined as the fraction of CNTs to total bundle area and is given by [17]

\[
f_{cnt} = \frac{N_c \pi (D_{max} + 0.31 \text{ nm})^2}{4wt}
\]

where, $N_c$ is the total number of CNTs inside the composite bundle. Moreover, a spacing of 0.155 nm between Cu and carbon atoms is taken into account in the model [18]. For realistic analysis, the CNT diameters inside the Cu-CNT composite have been considered to follow Gaussian distribution which is also consistent with [19]. Thus, there is a possibility that CNT diameters lie within the range $(D'_{out} - 3\sigma_{D_{out}}) \text{ to } (D'_{out} + 3\sigma_{D_{out}})$. For the given mean diameter and standard deviation, the number of CNTs having an outer diameter $D_{out}$ in the bundle are obtained as [20]

\[
N(D_{out}) = \frac{N_c \times f_{cnt}}{\sigma_{D_{out}} \sqrt{2\pi} \times \left(D_{out}^2 + \sigma_{D_{out}}^2\right)} \times \left[1 + \left(\frac{D_{out} - D_{out}}{\sigma_{D_{out}}}\right)^2\right]^{-\frac{1}{2}}
\]

For the cross-sectional area of 18 nm × 54 nm, mean diameter and standard deviation considered are 3.71 nm and 1.13 nm, respectively. The distribution of CNTs in Cu-SWB, Cu-MWB, and Cu-MCB composites for different $f_{cnt}$ is presented in Table 1. The probability of distribution of CNTs in Cu-MCB composite having a particular diameter $D_i$ in the range 1 nm to 7.12 nm with $f_{cnt} = 0.2$ and 0.5 is illustrated in Table 2.
TABLE 1. Number of CNTs Corresponding to Filling Ratio in Cu-SWB, Cu-MWB and Cu-MCB Composites

| f_{cnt} | Number of SWCNTs | Number of MWCNTs | Number of SWCNTs | Number of MWCNTs |
|---------|------------------|-----------------|-----------------|-----------------|
| 0.2     | 44               | 18              | 22              | 11              |
| 0.3     | 60               | 25              | 30              | 19              |
| 0.4     | 79               | 34              | 43              | 27              |
| 0.5     | 95               | 47              | 54              | 35              |
| 0.6     | 112              | 60              | 65              | 44              |

TABLE 2. Number of CNTs and Corresponding Diameters in Cu-MCB

| Diameter D_{i}(nm) | f_{cnt} = 0.2 | f_{cnt} = 0.5 |
|---------------------|---------------|---------------|
|                     | SWCNTs        | MWCNTs        | SWCNTs        | MWCNTs        |
| 1.00                | 18            | 0             | 48            | 0             |
| 1.68                | 3             | 0             | 5             | 1             |
| 2.36                | 1             | 1             | 1             | 2             |
| 3.04                | 0             | 1             | 0             | 4             |
| 3.72                | 0             | 1             | 0             | 8             |
| 4.40                | 0             | 1             | 0             | 5             |
| 5.08                | 0             | 1             | 0             | 7             |
| 5.76                | 0             | 1             | 0             | 3             |
| 6.44                | 0             | 3             | 0             | 2             |
| 7.12                | 0             | 2             | 0             | 3             |

A. EFFECTIVE COMPLEX CONDUCTIVITY MODEL OF CU-CNT COMPOSITES AND THEIR COMPARISON

The effective complex conductivity, \( \sigma_{\text{eff}} \) of Cu-CNT composite bundle can be computed as [17]

\[
\sigma_{\text{eff}} = (1 - f_{\text{cnt}}) \sigma_{\text{Cu}} + f_{\text{cnt}}\sigma_{\text{cnt}} \tag{4}
\]

where, \( \sigma_{\text{cnt}} \) is the CNT conductivity and \( \sigma_{\text{Cu}} \) is the conductivity of Cu. The value of \( \sigma_{\text{cnt}} \) can be computed by [17]

\[
\sigma_{\text{cnt}} = \frac{k_{c}}{Z_{\text{cnt}}} \tag{5}
\]

Here, \( Z_{\text{cnt}} \) is the intrinsic impedance of an isolated CNT and \( k_{c} = F_{m} \cdot 1/\pi (D_{\text{out}} + 0.31 \text{ nm})^{2} \)

Substituting \( Z_{\text{cnt}} = R_{\text{cnt}} + j\omega L_{\text{cnt}} \) and (5) into (4), we can obtain

\[
\sigma_{\text{eff}} = (1 - f_{\text{cnt}}) \sigma_{\text{Cu}} + \frac{k_{c}f_{\text{cnt}}R_{\text{cnt}}}{R_{\text{cnt}}^2 + \omega^2 L_{\text{cnt}}^2} - j\frac{k_{c}f_{\text{cnt}}\omega L_{\text{cnt}}}{R_{\text{cnt}}^2 + \omega^2 L_{\text{cnt}}^2} \tag{8}
\]

where, \( R_{\text{cnt}} \) and \( L_{\text{cnt}} \) denote the resistance and inductance of an isolated CNT.

The performance of different composite bundles is investigated in terms of \( \sigma_{\text{eff}} \) and is compared with the pure CNT counterparts in Fig. 2. From Fig. 2(a), it is observed that Re (\( \sigma_{\text{eff}} \)) of Cu-SWB and its pure CNT counterpart remains constant with increase in frequency. This is because for SWCNTs, \( R_{\text{cnt}} \) is much larger than its reactance \( \omega L_{\text{cnt}} \). For Cu-MWB, Cu-MCB, and their pure CNT counterparts, \( R_{\text{cnt}} \) decreases with increase in the number of shells due to the increased \( \lambda_{\text{mfp},i} \). Hence, with increase in frequency, the term \( \omega L_{\text{cnt}} \) dominates over \( R_{\text{cnt}} \) and Re (\( \sigma_{\text{eff}} \)) drops at high frequencies. For the case of Cu-SWB and Cu-MWB composites, Re (\( \sigma_{\text{eff}} \)) at \( f = 100 \text{ kHz} \), is improved by 29% and 37%, respectively as \( f_{\text{cnt}} \) is increased from 0.2 to 0.5. Therefore, the electrical conductivity improves remarkably after co-depositing more number of CNTs with Cu in the nanoscale regime. The break frequency from (8) is calculated as \( R_{\text{cnt}} / 2\pi L_{\text{cnt}} \). For 16 nm
technology node, the values of $R_{\text{cut}}$ and $L_{\text{cut}}$ for MWCNT are computed as 5.91 kΩ and 49.2 nH whereas for SWCNT, these values are 9.85 kΩ and 6.31 pH, respectively. Thus, break frequency for MWCNTs is equal to 19.2 GHz, whereas for SWCNT, $R_{\text{cut}}/2\pi L_{\text{cut}}$ is equal to 248.7 GHz (Fig. 2(b)). In Fig. 2(b), only decreasing behaviour of Im ($\sigma_{\text{eff}}$) is observed where the highest frequency plotted is 500 kHz due to sub-threshold operation.

Im ($\sigma_{\text{eff}}$) of Cu-CNT composite also exhibits a similar trend to that of the corresponding CNT, as it is dependent on $f_{\text{cut}}$ only and increases with $f_{\text{cut}}$ at a particular frequency. Moreover, due to their similar electrical properties, Re ($\sigma_{\text{eff}}$) and Im ($\sigma_{\text{eff}}$) of Cu-MWB and Cu-MCB composites are almost identical and thus, Cu-MCB composites are not included in the following analysis. Moreover, $\sigma_{\text{eff}}$ of Cu-MCB for $f_{\text{cut}} = 0.5$ is validated with the experimental results of [12] and is shown in Fig. 3. It is observed that $\sigma_{\text{eff}}$ of Cu-MCB composites closely resembles the measured data and the error may be attributed to the different alignment of CNTs.

**B. EQUIVALENT SINGLE CONDUCTOR MODEL**

The sub-threshold driver-interconnect-load (Sub-DIL) system incorporating an equivalent single conductor (ESC) model is shown in Fig. 1(b). The CMOS inverter acts as driver and sub-threshold drain current, for rising ramp input voltage is represented by [22], [23]

$$I_d = \mu_n C_{\text{ox}} \frac{W}{L} (\eta - 1) V_{\text{th}}^2 \cdot \exp\left(V_{\text{in}} - V_T / \eta V_{\text{th}}\right) \cdot \left[1 - \exp\left(-V_{ds} / V_{\text{th}}\right)\right]$$

(9)

In (9), $\mu_n$ is the electron mobility, $C_{\text{ox}}$ is the gate-oxide capacitance per unit area, $W$ and $L$ are the effective channel width and channel length respectively, $V_{\text{th}}$ is the thermal voltage, $V_T$ is the threshold voltage, $V_{\text{in}}$ and $V_{ds}$ are the input voltage and drain-to-source voltage respectively and $\eta$ is the sub-threshold slope factor whose value lies between one and two. Under high values of $V_{ds}$ much greater than $V_{\text{th}}$, the factor $[1 - \exp(-V_{ds}/V_{\text{th}})]$ in (9) is close to unity. $I_d$ thus becomes independent of $V_{ds}$ and driver is treated saturated. On the other hand, under low values of $V_{ds}$ much lower than $V_{\text{th}}$, the term $[1 - \exp(-V_{ds}/V_{\text{th}})]$ in (9) can be truncated to

$$I_d = \begin{cases} B_n \cdot \exp \left(\frac{V_{\text{in}} - V_T}{\eta V_{\text{th}}}\right) & \text{for } V_{ds} \geq 4V_{\text{th}} \\ \gamma_n \cdot V_{ds} & \text{for } V_{ds} < 4V_{\text{th}} \end{cases}$$

(10)

In (10), the values of $B_n$ in sub-saturation region and $\gamma_n$ in sub-linear region are given as

$$B_n = \mu_n C_{\text{ox}} \left(\frac{W}{L}\right) (\eta - 1) V_{\text{th}}^2$$

(11)

$$\gamma_n = \mu_n C_{\text{ox}} \frac{W}{L} (\eta - 1) V_{\text{th}}$$

(12)

The process parameters in (10) as obtained from 16 nm BSIM Level-54 CMOS model file are: $\mu_n = 0.03$ m$^2$/V-s, $t_{\text{ox}} = 0.95$ nm, $\eta = 2$. On substituting these values, the driver resistance in sub-threshold region is computed as $R_d = 1/\gamma_n = 21.8$ kΩ.

Next, the parasitics of Cu-CNT composite interconnect as such are dependent on the number of conducting channels $N_{\text{ch}}$ and are function of the CNT diameter of the $i^{th}$ shell, given as

$$N_{\text{ch},i} (D_i) = \begin{cases} x_1 T D_i + x_2 & D_i > D_T / T \\ D_i \leq D_T / T \end{cases}$$

(13)

where, $x_1 = 0.116$ and $x_2 = 0.21$, and $D_T$ is 1300 nm $\cdot$ K at $T = 300$ K. In the ESC model of Cu-CNT composite interconnect, imperfect contact resistance, $R_{\text{ESC}}$ and quantum resistance, $R_q (= h/4e^2)$ behave as its lumped component ($R_1$) [17] as

$$R_1 = \frac{f_{\text{cut}}}{N_c} \times \left( R_{\text{ESC}} + \frac{h}{4e^2 N_{\text{ch},i}} \right)$$

(14)

In addition, the equivalent scattering resistance ($R_{\text{ESC}}$) behaves as distributive component and is dependent on Re ($\sigma_{\text{eff}}$). The per unit length ($p.u.l.$) $R_{\text{ESC}}$ for Cu-CNT composite interconnect is determined by

$$R_{\text{ESC}} = \frac{\rho_{\text{eff}}}{\omega T} = \text{Re} \left( \frac{1}{\sigma_{\text{eff}}} \right) \frac{1}{\omega \cdot t}$$

(15)

where, $\rho_{\text{eff}} = \text{Re} \left( 1/\sigma_{\text{eff}} \right)$ is the effective complex resistivity. The inner and external inducances of Cu-CNT composite interconnect determine its $p.u.l$ inductance ($L_{\text{ESC}}$), given as [10]

$$L_{\text{ESC}} = L_{\text{in}} + L_{\text{ext}} = \text{Im} \left( \frac{1}{\sigma_{\text{eff}}} \right) + \mu_0 \varepsilon_0 \varepsilon_r C_{\varepsilon_{\text{ESC}}}/C_{\varepsilon_{\text{ESC}}/\varepsilon_r}$$

(16)

where, $\mu_0$ and $\varepsilon_0$ are the permeability and permittivity in vacuum, $\varepsilon_r$ is the relative permittivity of the dielectric, and $C_{\varepsilon_{\text{ESC}}/\varepsilon_r}$ is the $p.u.l.$ electrostatic capacitance that appears between the bundle and the ground plane, given as [17]

$$C_{\varepsilon_{\text{ESC}}/\varepsilon_r} = \frac{w}{d + 4 \pi} \ln \left( 1 + \frac{t}{d} \right) + \frac{6}{\pi}$$

$$+ 2 \pi \ln \left[ 1 + \frac{\pi w}{2 (1 + \pi) (t + d)} \right]$$

(17)
The *p.u.l.* quantum capacitance due to quantum electrostatic energies stored inside the channel of CNTs are calculated as

\[ C_{q,ESC} = \left( \frac{4e^2N_c}{h\nu_F} \right) \times \sum_{i=1}^{m} N_{ch,i} \]  

(18)

The total *p.u.l.* equivalent capacitance \( C_{ESC} \) is expressed as

\[ C_{ESC} = \left( \frac{1}{C_{e,ESC}} + \frac{1}{C_{q,ESC}} \right)^{-1} \]  

(19)

Based on (4)–(19), the frequency dependent resistance and inductance of Cu-SWB and Cu-MWB composite interconnects is extracted in Fig. 4 for different \( f_{cnt} \) values. The bundle length is set to be 100 \( \mu \)m. Here, it is seen that co-depositing SWCNTs and/or MWCNTs with Cu significantly suppresses the resistance and inductance of the composite bundled interconnects over isolated CNTs. Moreover, with increase in \( f_{cnt} \), \( R_{ESC} \) and \( L_{ESC} \) of Cu-SWB and Cu-MWB composites decreases, thus resulting in better electrical performance. For a certain value of \( f_{cnt} \), the Cu-MWB composite interconnect exhibits a lower resistance as well as inductance in comparison to the Cu-SWB and isolated CNTs. The reason for this phenomenon is attributed to the presence of significantly larger kinetic inductance in MWCNTs, which is ~two orders larger than the magnetic inductance, or in other words, due to large momentum relaxation time in MWCNTs.

### III. PERFORMANCE ANALYSIS

#### A. CROSSTALK EFFECT

In this section, the prominent crosstalk issue arising in sub-threshold composite Cu-CNT interconnects is examined. The equivalent circuit of three coupled Cu-CNT composite interconnect employing sub-DIL system is shown in Fig. 5. In the Sub–DIL coupled system, lines 1, 3 serve as the aggressor and line 2 as victim. The signal transmission in coupled composite interconnects can be classified into in-phase and out-phase. The two phase modes are defined as \( k_1 = 1, k_2 = -1 \) under out-phase and \( k_1 = k_2 = +1 \) under in-phase.

The \( ABCD \) matrix for the victim sub-threshold composite interconnects is given as [24]

\[
\begin{bmatrix}
A & B \\
C & D
\end{bmatrix} = \left[ \begin{array}{cc}
1 + sC_dR_d & R_d \\
1 & 1 / 2
\end{array} \right] \\
\times \left[ \begin{array}{cc}
cosh (\theta) & Z_0 \sinh (\theta) / Z_0 \\
\sinh (\theta) & \cosh (\theta)
\end{array} \right] \left[ \begin{array}{cc}
1 & R_1 / 2 \\
0 & 1
\end{array} \right]
\]  

(20)

and \( \theta = \sqrt{\left[ R_{ESC} + s \left( L_{ESC} + k_1L_{m12} + k_2L_{m23} \right) \right] sC_{ESC} \Delta x} / (\Delta x) \)  

(21)

\[ Z_0 = \sqrt{\frac{R_{ESC} + s \left( L_{ESC} + k_1L_{m12} + k_2L_{m23} \right)}{s \left( C_{ESC} + (1 - k_1)C_{m12} + (1 - k_2)C_{m13} \right)}} \]  

(22)

where, \( \theta \) and \( Z_0 \) represent the propagation constant and characteristic impedance, respectively, \( L_m \) and \( C_m \) denote the mutual inductance and capacitance, \( \Delta x \) is the infinitesimal segment in the ESC model, and \( C_d \) is the drain diffusion capacitance.

The 50% time delay of victim Cu-CNT composite line is obtained by [25]

\[ \tau_{50\%} = \left( 1.48\xi + e^{-2.9\xi^{1.35}} \right) \sqrt{\frac{L_{equil}(C_{equil} + C_L)}{\xi^2}} \]  

(23)
where, \( C_L \) is the load capacitance, and equivalent resistance \( R_{\text{equiv}} \), inductance \( L_{\text{equiv}} \), capacitance \( C_{\text{equiv}} \) of the decoupled victim line are determined as

\[
R_{\text{equiv}} = R_{\text{ESC}}
\]

\[
L_{\text{equiv}} = L_{\text{ESC}} + k_1 L_{m12} + k_2 L_{m23}
\]

\[
C_{\text{equiv}} = C_{\text{ESC}} + (1 - k_1) C_{m12} + (1 - k_2) C_{m23}
\]  

(24)

Here, \( C_{m12}, C_{m23} \) and \( L_{m12}, L_{m23} \) represent coupling capacitance and mutual inductances between the aggressor and victim lines. The coupling capacitance and mutual inductance can be obtained using expression [26], [27]

\[
C_m = \pi \varepsilon_0 \varepsilon_r / \cosh^{-1} (S_p / D_1)
\]

(25)

\[
L_m = \mu_0 \varepsilon_0 / N_T C_e
\]

(26)

where, \( S_p \) is the spacing between two coupled interconnects, \( N_T \) is the total number of tube counts, \( C_e \) is the electrostatic capacitance of CNTs facing the ground plane. For Cu-CNT composites, the value of \( C_{m12} \) and \( C_{m23} \) is found to be 1.55 fF. This is because \( C_{m12} \) depends upon the spacing between the interconnect lines which is same in Cu-MWB/Cu-MCB/Cu-SWB. The values of \( L_{m12}, L_{m23} \) for Cu-MCB, Cu-MWB and Cu-SWB is determined to be 2.01 pH, 3.16 pH and 3.57 pH respectively, for \( f_{\text{cut}} = 0.5 \). Here, \( \xi \) is the damping factor which can be expressed as

\[
\xi = \frac{R_{\text{equiv}} (C_{\text{equiv}} l + 2 C_L) + 2 (R_1 + R_d) (C_{\text{equiv}} l + C_L)}{4 \sqrt{L_{\text{equiv}} (C_{\text{equiv}} l + C_L)}}
\]  

(27)

Fig. 6 presents the sub-threshold delay performance of Cu-CNT composite and pure CNT interconnects. Here, for SWCNTs, \( f_m = 1/3 \), while \( f_{\text{cut}} = 0.5 \). Moreover, \( R_d = 21.8 \) k\( \Omega \), \( C_d = 1 \) fF, and \( C_L = 1 \) fF, while the relative permittivity, \( \varepsilon_r \) of dielectric is taken as 2. It is evident that Cu-CNT composite interconnects show reduced sub-threshold crosstalk delay than their pure CNT counterparts due to their much improved conductivity. Moreover, the sub-threshold delays of Cu-MWB/Cu-SWB interconnects even with one-half of the mean free path i.e., \( \lambda_{\text{mfp}} = 500 D \) are relatively less influenced as compared with pure SWCNTs and MWCNTs. Hence, with defective CNTs having a dramatically reduced \( \lambda_{\text{mfp}} \), the composite interconnects still demonstrate comparable sub-threshold delay with that for \( \lambda_{\text{mfp}} = 1000 D \). On the basis of bundle arrangements in Fig. 6(b), it is noticed that in comparison to Cu-SWB, since Cu-MWB has more number of conducting channels, therefore, it exhibits a reduced delay due to reduction in the equivalent resistance, inductance and capacitance values. The percentage improvements in the crosstalk delay for Cu-MWB compared to MWCNT for \( \lambda_{\text{mfp}} = 1000 D \) are 67.81%, 45.55%, and 20.12% at 100-, 1000- and 2500 \( \mu \text{m} \) lengths, respectively. The analytical results also match closely with the simulation results.

**B. TRANSFER GAIN**

The input-to-output transfer function \( G(s) \) from (22) is

\[
G(s) = \frac{1}{A + s C_L B}
\]

\[
= \left[ (1 + s R_d (C_d + C_L)) \cosh (\theta l) + (Z_0 R_d + s Z_0 C_l + s^2 Z_0 C_d C_l R_d) \sinh (\theta l) \right]^{-1}
\]

(28)

Using the precise fourth-order Padé’s approximation, the transfer function \( G(s) \) from (28), can be written as

\[
G(s) \approx \left[ 1 + c_1 s + c_2 s^2 + c_3 s^3 + c_4 s^4 \right]^{-1}
\]  

(29)

The coefficients \( c_1, c_2, c_3, \) and \( c_4 \) depend on the parameters of sub-DIL system and can be referred in [15]. The effect of metallic fraction variation on the frequency response of three coupled Cu-SWB composite interconnects under in-phase \((k_1 = k_2 = 1)\) and out-phase \((k_1 = 1, k_2 = -1)\) switching modes is presented in Fig. 7(a). The interconnect spacing is chosen to be 18 nm and the bundle length considered is \( l = 100 \mu \text{m} \). The expression \( R_{\text{equiv}} \) \((2 \pi f_{\text{eq}})\) is utilized to compute the cut-off frequency, \( f_c \). Since the number of conducting channels are lesser in the Cu-SWB composite with \( f_m = 1/3 \), the total inductance is \( \times 3 \) times larger than a densely packed bundle with \( f_m = 1 \), whereas their capacitances remain virtually unchanged. Therefore, the parasitics \( R_{\text{equiv}} \) and \( C_{\text{equiv}} \) associated with Cu-SWB composite interconnect for \( f_m = 1 \) are lower and hence, it exhibits an \( f_c \) of 0.79 MHz, while \( f_c \) in the case
of $f_m = 1/3$ is 0.75 MHz under in-phase switching. Hence, the frequency response can be improved by increasing $f_m$ due to the decreased mutual inductance. Similar variations are also observed under out-phase switching mode. As CNTs with filling ratio of 0.2 were reported in [21], the influence of $f_{cnt}$ on the frequency response of coupled Cu-MWB interconnects is studied in Fig. 7(b). The values of $f_c$ exhibited by Cu-MWB for $f_{cnt} = 0.2$ and $f_{cnt} = 0.5$ are 0.91 MHz and 0.94 MHz under in-phase switching. This is so because increasing $f_{cnt}$ improves the Cu-MWB effective conductivity, and thereby improves the cut-off frequency or bandwidth. Furthermore, the effective capacitance for $(k_1 = k_2 = 1)$ mode is much smaller compared to the $(k_1 = 1, k_2 = -1)$ due to the Miller effect, i.e., $C_{ESC} \ll C_{ESC} + 2C_m/23$. Hence, the values of $f_c$ exhibited by the Cu-MWB and Cu-SWB composite interconnects under out-phase switching are reduced compared to that under in-phase.

Subsequently, comparison of CPU time to compute in-phase and out-phase sub-threshold delays for different Cu-CNT composite configurations using the proposed analytical model and SPICE simulations is shown in Fig. 8.

### C. NYQUIST STABILITY OF COMPOSITE BUNDLES IN SUB–THRESHOLD

A DIL system is said to be stable if its rise time, $T_r$, is higher and peak overshoot voltage, $M_p$, is zero. The $T_r$ and $M_p$ mainly depend on the damping coefficient as presented in (29). In this analysis, SWCNT and MWCNT diameter ranging from 1 nm-2.76 nm and 1 nm-7.12 nm, respectively are taken into account. The quantitative values of $T_r$ and $M_p$ for different filling ratios and metallic fraction of CNTs for Cu-MWB composite are summarized in Table 3. Based on the transfer function $G(s)$ in (30), the Nyquist plots of Cu-MWB composite interconnect wherein length is varied from $10 \mu m \leq l \leq 100 \mu m$, are shown in Fig. 9(a). Based on stability theory, farther the encirclement moves away from the critical point $(-1, 0)$ in complex plane, more the system tends towards stability. It can be observed that by increasing the length of Cu-MWB composite, the critical point $(-1, 0)$ goes towards outside the Nyquist plot and the system becomes more stable. Similarly, in Fig. 9(b), Nyquist diagram shows the impact on relative stability against variations in the filling ratio from $0.3 \leq f_{cnt} \leq 0.5$. With the increase in $f_{cnt}$ value, $N_{ch.i}$ increases, which in turn improves $\sigma_{eff}$ and increases the $C_{ESC}$ of Cu-MWB composite interconnect. Note that $C_{ESC}$ is the governing parameter in determining $T_r$, and with rise in the values of either $C_{ESC}$ and $C_{q,ESC}$ of Cu-MWB, the DIL system tends to damp more promptly, and the system becomes stable.

### IV. CONCLUSION

The novel research work carried out in this paper shows the applicability of three different Cu-CNT composites i.e., Cu-SWB/Cu-MWB/Cu-MCB as interconnects through modeling, simulation and analysis under the sub–threshold region. Using Gaussian distribution, the number of CNTs in Cu-CNT composites is derived. Thereafter, the distributed equivalent single conductor model for composite interconnects is developed. The effective complex conductivity of composites for different filling ratios at much lower operating frequencies is extracted by developing mathematical formulations. It is found that Cu-MWB composite has a larger conductivity than that of the Cu-SWB. For certain $f_{cnt}$, the frequency dependent resistance as well as inductance of Cu-MWB composite interconnects are much lower than Cu-SWB and isolated CNT counterparts. By the virtue of equivalent single conductor model, the sub-threshold performance is computed for three line coupled Cu-SWB/Cu-MWB composite interconnects through analytical $ABCD$ technique and SPICE simulations. It is determined that Cu-MWB leads to improved electrical conductivity, reduced crosstalk, higher bandwidth, and stability and thus, shall be a better choice amongst other composite bundles and pure CNTs. The analytical results also match closely with the
simulation results. Hence, it is concluded that at present time, Cu-MWB can be observed as a feasible solution to meet the near future challenges of on-chip sub-threshold interconnects. The results of our investigations shall be highly lucrative in the realization of Cu-MWB composite interconnects for sub-threshold ICs and can open up a new era for ultra–low power circuits.

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