Power series distributions in clan structure analysis: new observables in strong interactions

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Abstract

We present a new thermodynamical approach to multiparticle production in high energy hadronic interactions, making use of the formalism of infinitely divisible power series distributions. This approach allows us to define new observables, linked to the system fugacity, which characterise different classes of events.

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1 Introduction

The phenomenological analysis of many-particle final states in hadron-hadron collisions in the GeV region has been successfully carried out[1] using a two-component model: each event is assigned to one of two classes, called ‘soft’ and ‘semi-hard’, which correspond to events without mini-jets and events with mini-jets, respectively. We also assume that the multiplicity distribution (MD) in each class is described by a negative binomial (Pascal) distribution (NBD), of course with different parameters \( \bar{n}, k \) in each class. This model was successful in describing the shoulder in MD’s, the oscillations of high rank moments thereof[1] and also forward backward multiplicity correlations.[2] There are also experimental indications that the two classes behave differently in the TeV region.[3]

In order to extrapolate to the LHC region we have to make some assumptions on the behaviour of the parameters of the two NBD’s which we summarise as follows:

* The overall average multiplicity grows as \( \ln^2 \sqrt{s} \).
* The soft component average multiplicity grows as \( \ln \sqrt{s} \) and the MD obeys KNO scaling (thus \( k_{\text{soft}} \) is approximately constant.)
* The average multiplicity in the semi-hard component is approximately twice as large as in the soft one.
* Three scenarios have been examined for the behaviour of \( k_{\text{semi-hard}} \):
  1. same behaviour as for the soft component, i.e., it is constant (therefore KNO scaling is satisfied);
  2. \( k_{\text{semi-hard}}^{-1} \approx \ln \sqrt{s} \), implying a strong violation of KNO scaling;
  3. it follows a QCD-inspired behaviour; KNO scaling is attained only asymptotically. This last scenario is intermediate between the first two.

Of course, using NBD’s means we can explore the clan structure:[4, 5] the average number of clans \( \bar{N} \) and the average number of particles per clan are defined by

\[
\bar{N} = k \ln \left(1 + \frac{\bar{n}}{k}\right) ; \quad \bar{n}_c = \frac{\bar{n}}{k \ln \left(1 + \frac{\bar{n}}{k}\right)} .
\] (1)

It turns out that the second and third scenarios show a number of clans which is rapidly decreasing with c.m. energy (accompanied by a fast increase of the average number of particles per clan). This is surprising, and in this paper we try to understand the implications of this result at parton level using thermodynamical concepts.

To connect the hadronic and partonic levels we use the generalised local parton-hadron duality (GLPHD),[6] which says that all inclusive distributions are proportional at the two levels of investigation:

\[
Q_{n,\text{hadrons}}(y_1,\ldots,y_n) = \rho^n Q_{n,\text{partons}}(y_1,\ldots,y_n),
\] (2)

which corresponds for NBMD parameters to

\[
k_{\text{hadron}} = k_{\text{parton}}, \quad \bar{n}_{\text{hadron}} = \rho \bar{n}_{\text{parton}}.
\] (3)

GLPHD will be applied separately to soft and semi-hard components.
2 A new thermodynamical approach

The thermodynamical approach to multiparticle production has a long history which cannot be summarised here. We would just like to attract the reader’s attention to the result\[7\] that, to leading order in the allowed rapidity range, the generating function (GF) for the MD has the form of an infinitely divisible distribution (IDD).

Keeping in mind this result, we propose the following approach.

The partition function in the canonical ensemble, $Q_n(V, T)$, for a system with $n$ particles, volume $V$ and temperature $T$, is linked to the partition function in the grand-canonical ensemble with fugacity $z$, $Q(z, V, T)$, by the well known relation

$$Q(z, V, T) = \sum_n z^n Q_n(V, T). \quad (4)$$

Quite in general, in the grand-canonical treatment the probability of finding $n$ particles in a system is given by

$$p(n) = \frac{z^n Q_n(V, T)}{Q(z, V, T)}. \quad (5)$$

That is to say, for a thermodynamical system the MD belongs to the class of power series distributions (PSD’s), and is characterised indeed by the following form:

$$p(n) = \frac{a_n b^n}{\gamma(b)}, \quad (6)$$

with constants $a_n$, $b$.

We therefore propose, given a MD in power series form, the following correspondence with Eq. (5):

$$z \leftrightarrow b,$$

$$Q_n \leftrightarrow a_n,$$

$$Q \leftrightarrow \gamma(b) = p(0)^{-1}. \quad (7)$$

When the PSD is also IDD, then we know it can be cast in the form of a compound Poisson distribution, such that

$$p(0) = e^{-\bar{N}}. \quad (8)$$

In a two-step approach, $\bar{N}$ is average number of objects (clans) generated in the first step. This way of describing the partonic cascade is well known:\[5\] the ancestors (first step) are independent intermediate gluon sources; it is their thermodynamic properties which we want to explore.

In our thermodynamical approach, $\bar{N}$ becomes of fundamental importance since Eq.s (7) and (8) imply

$$\bar{N} = -\ln p(0) = \ln Q; \quad (9)$$

all thermodynamical properties can be obtained by differentiating $\bar{N}$.

From the standard relation $PV = k_B T \ln Q$, we obtain the equation of state

$$PV = \bar{N} k_B T, \quad (10)$$

which says that clans form a classical ideal gas.
The negative binomial (Pascal) distribution belongs to both classes, power series and IDD. The standard form, from which the correspondence with the partition function can be obtained, is the following:

\[
p(n) = \frac{k(k + 1)\ldots(k + n - 1)}{n!} \left(\frac{\bar{n}}{\bar{n} + k}\right)^n \left(\frac{k}{\bar{n} + k}\right)^k.
\]  

The identification we propose in our approach is

\[
a_n = \frac{k(k + 1)\ldots(k + n - 1)}{n!},
\]

\[
b = \frac{\bar{n}}{\bar{n} + k},
\]

\[
\gamma(b) = \left(\frac{k}{\bar{n} + k}\right)^{-k}.
\]  

Notice that \(k(V, T)\) is the canonical partition function for a system with 1 particle; it is in our approach an unknown function of \(V\) and \(T\).

Finally notice that \(b\) is the fugacity \(z\):

\[
z = \frac{\bar{n}}{\bar{n} + k}.
\]  

When the ancestors are created early in the evolution, at larger virtualities and with higher temperature, they tend to follow a quasi-classical behaviour, as the production of a new ancestor is competitive with the increase in gluon population within each clan. This results in a relatively large value of the \(k\) parameter, \(i.e.,\) a small amount of aggregation. When the number of partons per clan is very small (close to 1; \(k\) is very large) then essentially each parton is a clan, and the equation of state reduces basically to that of an ideal gas (quasi-classical behaviour):

\[
PV \approx \bar{n}k_B T.
\]

Via GLPHD, we expect a similar situation to hold at hadron level. This behaviour is qualitatively close to that of soft events as well as of scenario-1 semi-hard events.

When the ancestors are created later in the evolution, at lower virtualities and with lower temperature, they tend to remember their quantum nature, as newly produced gluons prefer to stay together with other clan members rather than initiate a new clan. This results in a relatively small value of the \(k\) parameter, \(i.e.,\) a larger aggregation and larger two-particle correlations. When the number of partons per clan begins to grow, the equation of state for partons becomes more and more different (quasi-quantum behaviour), but that for clans remains that of an ideal gas.

\[
PV = \bar{N}k_B T = k \ln (1 + \bar{n}/k) k_B T.
\]  

Via GLPHD, at hadron level we recognise the behaviour of scenario-2 and scenario-3 semi-hard events.

It is interesting now to calculate some thermodynamical quantities.

The Helmholtz free energy can be rewritten in a form symmetric in \(\bar{n}\) and \(k\):

\[
-\frac{A}{k_B T} = \bar{n}\mu - PV = \bar{n} \ln \left(1 + \frac{k}{\bar{n}}\right) + k \ln \left(1 + \frac{\bar{n}}{k}\right).
\]
The average internal energy is
\[ \frac{U}{k_B T} = k_B T^2 \left( \frac{\partial \bar{N}}{\partial T} \right)_{b,V} = \bar{N} \left( \frac{\partial \ln k}{\partial \ln T} \right)_V. \] (17)

The entropy is
\[ S = \frac{U - A}{T} = k_B \left\{ -\frac{A}{k_B T} + T \left( \frac{\partial k}{\partial T} \right)_V \ln \left( 1 + \frac{\bar{n}}{k} \right) \right\}. \] (18)
which coincides with \(-A/T\) in the limit of \((\partial k/\partial T)_V \to 0\), since also \(U \to 0\).

For further discussion of thermodynamical quantities, see Ref. [8].

3 Clan behaviour as a function of fugacity

Relying on GLPHD, we analyse first experimental data on the fugacity and the related \(a\) parameter: the NBD satisfies the recurrence relation
\[ \frac{(n+1)p(n+1)}{p(n)} = a + bn, \] (19)
where
\[ a = \frac{\bar{n}k}{\bar{n} + k}; \quad b = \frac{\bar{n}}{\bar{n} + k}. \] (20)

From Eq. (12) it is seen that \(b\) is the fugacity. In Figure 1 we show for each component and each scenario the energy variation of the parameters \(a\) and \(b\). The points come from NB fits to experimental MD’s, the lines show the predictions from the extrapolation mentioned in the introduction.

The \(a\) parameter corresponds to the average multiplicity for a classical (Poisson) system. The relative behaviour of \(b\) and \(a = kb\) as the c.m. energy increases can be considered an indication of the relative importance of a behaviour closer to a quantum one, i.e. harder, with respect to a behaviour closer to a quasi-classical, i.e. softer, for a class of events. A very slow increase of \(b\) with c.m. energy and an almost constant behaviour of \(a\) is the main characteristic of the class of soft events and of scenario-1 semi-hard events. A very fast decrease of \(a\) in scenarios 2 and 3 and larger values of the fugacity \(b\) characterise harder events: the assumption of strong KNO scaling violation for the semi-hard component (an extreme point of view with respect to that of scenario 1) implies a completely new panorama.

Then we explore the dependence of clan parameters on the fugacity \(b\), induced by its energy evolution:
\[ \bar{N} = k \ln \left( 1 + \frac{\bar{n}}{k} \right) = -k \ln(1 - b); \] (21)
\[ \bar{n}_c = \frac{\bar{n}}{k \ln (1 + \frac{\bar{n}}{k})} = \frac{b}{(b - 1) \ln(1 - b)}. \] (22)

Notice that the average number of particles per clan only depends on the fugacity \(b\). In Figure 2 we show for each component and each scenario the clan parameters as a function of the fugacity. Again, the points come from fits to experimental data, the solid lines are our extrapolations. The dashed grey lines show the variation of clan parameters with \(b\) at fixed \(k\) (that is, at fixed \(V\) and
Figure 1: Fugacity and $a$ parameter dependence on c.m. energy
\[ \bar{N} = \text{avg number of clans} \]

\[ = k \ln(1 - b) \]

\[ \bar{n}_c = \text{avg num of particles per clan} \]

\[ = \left[ \frac{1}{1/b - 1} \ln(1-b) \right]^{-1} \]

**Figure 2**: Clan parameters dependence on fugacity.
for the following values of $k$: 1 (lowest curve), 3, 7, 30 (highest curve); being $\bar{n}_c$ independent of $k$, only one dashed line is visible in the corresponding graphs.

For the soft and scenario-1 semi-hard components it is shown that $\bar{N}$ is a very slow growing function of the fugacity of the system throughout the ISR region ($b < 0.7$), and then starts to grow quickly; $\bar{n}_c$ as a function of the fugacity has a similar behaviour from $\approx 1.5$ to $\approx 3$.

The decrease of the average number of clans in scenarios 2 and 3 leads again to the conclusion that this behaviour is closer to that of a quantum system than to a classical one, favouring as it does the production of larger clans and therefore of regions of higher particle density.

For a discussion of other parameters, like the void scaling function, see again Ref. [8].

4 Conclusions

By defining a new thermodynamical approach to multiparticle production at parton level we have given the physical meaning of fugacity to a parameter previously used only to describe deviations from Poisson behaviour in multiplicity distributions.

On this basis, we revisited our previous extrapolations to the TeV region of inelastic hadron-hadron collisions and examine the different behaviours of the two classes of events (‘soft’ and ‘semi-hard’).

In the first class, i.e., soft events, the ancestors of the clans are produced earlier, at higher virtuality and when the temperature is higher. Ancestors in these conditions generate little (clans are small). This results in a behaviour closer to that of a classical system (ideal gas).

In the second class, i.e., semi-hard events, the ancestors are produced later in the cascade, at lower virtualities and when the temperature is lower. Ancestors in these conditions are more prolific (clans become larger). This results in a behaviour closer to that of a quantum system (stimulated emission); high density regions exist.

Although we used explicitly in the illustration the NB(Pascal)MD, our result is extensible in principle to any infinitely divisible distribution which also belongs to the class of power series distributions.

The results discussed in this paper bring in the spotlight the concept of clans, which up to now was only applied in a statistical framework. At this point, it becomes important to investigate other physical properties of clans, in order to answer questions like the following ones: can clans be considered observable objects? if so, what are their quantum numbers? do they start to interact among themselves in the TeV region? how will this possibility modify the ideal gas equation of state?

Work in this direction has already begun[9] by studying clan masses, with preliminary indications that the answer to the first question (observable clans) is positive. This can be extremely relevant for the new heavy ion machines where the standard examination of events with tens of thousands of particles may be very problematic.

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