The Forward-Backward Asymmetry induced $CP$ Asymmetry of $B^\pm \to \pi^\pm \pi^+ \pi^-$

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Abstract

$CP$ violation of the decay $B^\pm \to \pi^\pm \pi^+ \pi^-$ in the $f_0(500) - \rho(770)^0$ interfering region is analysed. The Forward-Backward Asymmetries (FBAs) and the corresponding $CP$ asymmetries FB-CPAs are particularly investigated. To isolate the CPV caused by the interference of different partial wave more cleanly, we also introduce the direct-CPV-subtracted FB-CPA. Based on the LHCb data, we extract the FBAs, FB-CPAs, direct-CPV-subtracted FB-CPA, as well as the regional CPAs with invariant mass of the $\pi^+ \pi^-$ pair in the range $0.2 \text{ GeV}/c^2 < \sqrt{s_{\text{low}}} < 1.8 \text{ GeV}/c^2$. It is found that the (direct-CPV-subtracted) FB-CPAs are quite large in the $f_0(500) - \rho(770)^0$ interfering region, which confirms that the interference of the intermediate resonances $f_0(500)$ and $\rho(770)^0$ plays an important role for the $CP$ violation of the three-body decay channel $B^\pm \to \pi^\pm \pi^+ \pi^-$. 

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I. INTRODUCTION

Charge-Parity ($CP$) violation was discovered by J.W. Cronion and V.L. Fitch in the neutral kaon system in 1964 [1]. It is closely related to the matter-antimatter symmetry in our universe [2], and is one of the most basic and important properties of the weak interaction. In the Standard Model (SM), $CP$ violation ($CPV$) results from the weak complex phase in the Cabibbo-Kobayashi-Maskawa (CKM) matrix that reflects the transitions of different generations of quarks [3, 4]. To date, $CPV$ has been observed in the $K$, $B$ and $D$ meson systems [1, 5–11], all of which are consistent with the KM mechanism of SM [3].

The study on $CPV$ in multi-body decays of the bottom and the charmed hadrons plays an increasingly important role in both testing the KM mechanism of SM and looking for new sources of $CPV$. Interestingly, large $CP$As regionalized in part of the phase space in some three-body decay channels of $B$ mesons have been reported by LHCb [12–15]. Meanwhile, the integrated $CP$As are relatively small due to the cancellation among different parts of the phase space. Among these three-body decay channels, $B^\pm \rightarrow \pi^\pm \pi^+ \pi^-$ is one of the most extensively studied. Amplitude analysis of this decay channel show that $\rho(770)^0$ is the dominant resonance [16–19]. The large regional $CPA$ observed by LHCb located right in part of the $\rho(770)^0$ region where the angle (in the rest frame of $\rho(770)^0$) of the two pions with the same charge with $B$—which will be denoted as $\theta$ in this paper—is smaller that $90^0$ [13, 14]. Theoretical analysis indicates that the aforementioned large regional $CPA$ can be explained by the interference of $\rho(770)^0$ with the nearby resonance $f_0(500)$, where the corresponding amplitudes are respectively $P$- and $S$-waves [20–25].

Although it can be well explained by the interference of $f_0(500)$ and $\rho(770)^0$, the large regional $CPA$ in $B^\pm \rightarrow \pi^\pm \pi^+ \pi^-$ entangles all kinds of contributions other than the aforementioned $f_0(500) - \rho(770)^0$ interference, such as contributions from interference between the tree and penguin part corresponding to $\rho(770)^0$. Recently, one of the authors (Z.H.Z.) proposed to study $CPV$ induced by the interference of the nearby resonance through the Forward-Backward Asymmetry (FBA) of the final particle and the FBA induced $CP$ Asymmetry (FB-$CPA$) [26]. One advantage of this method is that it can isolated $CPV$ caused by the interfering effect of the nearby resonances (respectively corresponding to even- and odd-waves) [27].

The large data sample allows LHCb to study the $CPV$ caused by the interference of the $S$- and $P$-waves in more detail. To do this, the event yields are allocated into bins according to $\cos \theta > 0$ or $\cos \theta < 0$. In the way, the corresponding regional $CP$As can be systematics studied [13].
However, the analysis of the FBAs and the FB-CPAs are absent in the LHCb’s previous works. This motivate us to performed the analysis of the FBAs and the FB-CPAs of $B^\pm \to \pi^\pm \pi^+ \pi^-$ in this paper, based on the LHCb’s data in Refs. [13] and [14].

This paper is organized as follows. In Sec. II, we first illustrate the definitions of FBA and FB-CPA in detail. In Sec. III, based on the data sample corresponding to an integrated luminosity of 3.0 fb$^{-1}$ collected by the LHCb detector [13], we extract the regional CPAs, the FBAs, the FB-CPAs, as well as the newly introduced CP observables direct-CPV-subtracted FB-CPAs. A fit of cos $\theta$-dependence of CPA with only the inclusion of the amplitudes corresponding to $f_0(500)$ and $\rho(770)^0$ is presented at the end of this section. In Sec. IV, we briefly give the conclusion.

II. THE FORWARD-BACKWARD ASYMMETRY INDUCED CP ASYMMETRY

We first illustrate the definition of several CPA observables for the decay process $B^\pm \to \pi^\pm \pi^+ \pi^-$. In the c. m. frame of one of the $\pi^+\pi^-$ pair with low invariant mass, the aforementioned angle $\theta$ between the two $\pi^-$’s are illustrated in FIG.1. In practice, the invariant mass of the $\pi^+\pi^-$ pair is separated into small intervals. For each interval, it can be further separated into two small regions according to cos $\theta > 0$ or cos $\theta < 0$, which will be denoted as $\Omega_i^+$ or $\Omega_i^-$, respectively, where the subscript $i$ is the label of the small interval. The regional CPAs, which have been repeatedly dealt with in the literature both theoretically and experimentally, are defined as

$$A_{\Omega_i^{\pm}}^{CP} = \frac{N_{\Omega_i^{\pm}} - \bar{N}_{\Omega_i^{\pm}}}{N_{\Omega_i^{\pm}} + \bar{N}_{\Omega_i^{\pm}}},$$

for the region $\Omega_i^{\pm}$, where $N_{\Omega_i^{\pm}}$ and $\bar{N}_{\Omega_i^{\pm}}$ are the event yields of $B^- \to \pi^- \pi^- \pi^+$ and $B^+ \to \pi^+ \pi^+ \pi^-$ in the region $\Omega_i^{\pm}$, respectively. Of course, the regional CPA for $\Omega_i \equiv \Omega_i^+ + \Omega_i^-$ is defined as

$$A_{\Omega_i}^{CP} = \frac{N_{\Omega_i} - \bar{N}_{\Omega_i}}{N_{\Omega_i} + \bar{N}_{\Omega_i}},$$

where $N_{\Omega_i}$ and $\bar{N}_{\Omega_i}$ are respectively the event yields of $B^- \to \pi^- \pi^- \pi^+$ and $B^+ \to \pi^+ \pi^+ \pi^-$ in $\Omega_i$. 

3
The FBA of the interval \( i \) of \( B^- \rightarrow \pi^-\pi^+\pi^- \) is defined as the relative event yields difference between \( \Omega_i^+ \) and \( \Omega_i^- \):

\[
A_i^{FB} = \frac{N_{\Omega_i^+} - N_{\Omega_i^-}}{N_{\Omega_i^+} + N_{\Omega_i^-}},
\]

(3)

The FB-CPA of the interval \( i \) is defined as

\[
A_{CP,i}^{FB} = \frac{1}{2} \left( A_i^{FB} - \overline{A_i^{FB}} \right),
\]

(4)

where \( A^{FB} \) represents the FBA of the interval \( i \) for \( B^+ \rightarrow \pi^+\pi^+\pi^- \). Comparing with the regional CPAs, the FB-CPA is free from the assumption of equal production of \( B^- \) and \( B^+ \) [28], which reduces the corresponding systematic uncertainties.

Alternatively, one can define the CPA corresponding to FBA as

\[
A_{CP,i}^{FB,alt} = \frac{A_i^{FB} - \overline{A_i^{FB}}}{A_i^{FB} + \overline{A_i^{FB}}},
\]

which is similar with the CPAs defined by the decay width. However, there are good reasons for us not to use this definition here. Mathematically, since neither \( A_i^{FB} \) nor \( \overline{A_i^{FB}} \) are positive-definite, the CPA defined in the above equation is not bounded in \((-1, 1)\). In other words, \( A_{CP,i}^{FB} \) is normalized, while \( A_{CP,i}^{FB,alt} \) is not. To be more specific, \( A_{CP,i}^{FB,alt} \) is in fact the relative FB-CPA with respect to the CP-averaged FBA, \( A_{i,CP}^{FB,CP-av} \equiv \frac{1}{2}(A_i^{FB} + \overline{A_i^{FB}}) \), which can be clearly seen from a transformed expression: \( A_{CP,i}^{FB,alt} = A_{CP,i}^{FB} / A_{i,CP}^{FB,CP-av} \). Hence the aforementioned alternative definition is questionable when one what to make a comparison with other CPAs, such as the regional ones. From this perspective, the definition of Eq. (4) is more reasonable for the usage in this paper.

\(^1\) Similar story happens in other cases, such as the hyperon decays, where the CPA corresponding to the decay
The nonzero of FBA is caused by the interference of the odd- and even-waves \[27\]. To see this, one express the decay amplitude as
\[
A = \sum_l a_l P_l(\cos \theta),
\]
the FBA is then
\[
A^{FB} = \frac{1}{\sum_j \left[\langle |a_j|^2 \rangle / (2j + 1) \right]} \sum_{\text{even } l \atop \text{odd } k} f_{lk}(\langle a_l a_k^* \rangle),
\]
where \( f_{lk} = \frac{(-)^{(l+k+1)/2} \pi!}{2\pi^{l+k+1}((l-k)/2)!((l+k)/2)!} \) \[32\]. Consequently, the FB-CPA provide an effective procedure to isolate CPV corresponding to the interference of odd- and even-waves. Strictly speaking, however, the FB-CPA contains also CPVs corresponding to the difference between the decay width of CP-conjugate processes. This can been seen from the denominator of the above equation, which is proportional to the decay width. Hence, the difference between the decay width of the CP-conjugate processes, which is in fact the origin of the direct CPV, can also contribute to FB-CPA. To eliminate this, one can introduce an observable, which will be called as the direct-CPV-subtracted FB-CPA, taking the form
\[
\tilde{A}^{FB}_{CP}(\tilde{\alpha}) = \frac{\sum_{\text{even } l \atop \text{odd } k} f_{lk}(\langle a_l a_k^* \rangle - \langle \bar{a} \bar{a}_k^* \rangle)}{\sum_j \left[\langle |a_j|^2 \rangle / (2j + 1) \right] + \sum_j \left[\langle |\bar{a}_j|^2 \rangle / (2j + 1) \right]},
\]
For the current situation, the direct-CPV-subtracted FB-CPA of the interval \( i \) can be expressed as
\[
\tilde{A}^{FB}_{CP,i} = \frac{N_{\Omega^-} - N_{\Omega^+}}{N_{\Omega^-} + N_{\Omega^+}} - \frac{\tilde{N}_{\Omega^-} - \tilde{N}_{\Omega^+}}{\tilde{N}_{\Omega^-} + \tilde{N}_{\Omega^+}},
\]
based on the assumption that the \( B^- \) and \( B^+ \) are equally produced.

III. FBA AND FB-CPA ANALYSIS BASED ON LHCb DATA IN \( B^\pm \to \pi^\pm \pi^+ \pi^- \)

A detailed analysis of the event distributions of the decay \( B^\pm \to \pi^\pm \pi^+ \pi^- \) has been performed by LHCb based on a data sample corresponding to an integrated luminosity of 3.0 fb\(^{-1}\) \[13\]. Based parameter \( \alpha \) are defined in the literature as \( A^{\alpha}_{CP} = \frac{\alpha + \bar{a}}{\alpha - \bar{a}} \), while an alternative definition which is similar with Eq. \(4\) was presented in Ref. \[29\]. According to the logic here, the former should be view as the relative \( \alpha \)-induced CPA with respect to the CP-averaged decay parameter \( \alpha_{CP} \equiv \frac{1}{2}(\alpha - \bar{\alpha}) \), while the latter is the \( \alpha \)-induced CPA: \( A^{\alpha-\text{ind.}}_{CP} \equiv \frac{1}{2}(\alpha + \bar{\alpha}) \). Of course, we are not saying that the latter definition of CPA is better. On the contrary, despite non-normalised, the former does have some distinct advantages. For example, the former defined relative CPA can be measured through \( \Lambda_c^+ \to \Lambda(\to \rho \pi^+)h^+ \) \[30, 31\] based only on the CP symmetry assumption for the decay \( \Lambda_c^+ \to \Lambda h^+ (h^+ = K^* \text{ or } \pi^+) \), while the latter can not.
on the data of LHCb, we can extract the regional CPAs, the FBAs, the FB-CPAs, as well as the direct-CPV-subtracted FB-CPAs in a wide region of the $\pi^+\pi^-$ pair with lower invariant mass: $0.2 \text{ GeV}/c^2 < \sqrt{s_{\text{low}}} < 1.8 \text{ GeV}/c^2$, which are presented in FIGs. 2 and 3.

FIG. 2: The regional CPAs, $A_{CP}^{\Omega^+}$, and $A_{CP}^{\Omega^-}$, and $A_{CP}^\Omega$ of the decay channel $B^\pm \to \pi^\pm \pi^+\pi^-$ extracted from the LHCb data in Ref. [13] for $\sqrt{s_{\text{low}}}$ from 0.2 GeV/$c^2$ to 1.8 GeV/$c^2$. The dotted, dashed, and solid lines are $A_{CP}^{\Omega^+}$, $A_{CP}^{\Omega^-}$, and $A_{CP}^\Omega$, respectively.

FIG. 2 shows the corresponding regional CPAs of each bins with width of 0.05 GeV/$c^2$ lying in the range $0.2 \text{ GeV}/c^2 < \sqrt{s_{\text{low}}} < 1.8 \text{ GeV}/c^2$, while FIG. 3 shows the FBAs, the FB-CPAs, and the direct-CPV-subtracted FB-CPAs. The errors are estimated with only the inclusion of the statistical uncertainties of the event yields estimated according to $\sqrt{N}$. Both of the two figures show interesting behaviour around the $f_0(500) - \rho(770)^0$ interference region, i.e., $0.45 \text{ GeV}/c^2 < \sqrt{s_{\text{low}}} < 0.75 \text{ GeV}/c^2$. 
FIG. 3: The FBAs ($A_i^{FB}$), the FB-CPAs ($A_i^{FB, CP}$), and the direct-CPV-subtracted FB-CPAs ($\tilde{A}_i^{FB, CP}$) of the decay channel $B^\pm \to \pi^\pm \pi^+ \pi^-$ for $\sqrt{s_{\text{low}}}$ from 0.2 GeV/$c^2$ to 1.8 GeV/$c^2$. The dotted and dashed lines are $A_i^{FB}$ and $\overline{A_i^{FB}}$, respectively. The solid and dash-dotted lines are $A_i^{FB, CP}$ and $\tilde{A}_i^{FB, CP}$, respectively.

FIG. 4: The CPAs, $A_i^{FB, CP}$, $\overline{A}_i^{FB, CP}$, $A_i^\Omega$, $A_i^{\Omega^+}$, and $A_i^{\Omega^-}$ of the decay channel $B^\pm \to \pi^\pm \pi^+ \pi^-$ for $\sqrt{s_{\text{low}}}$ from 0.45 GeV/$c^2$ to 0.75 GeV/$c^2$. The upper and lower solid lines are $A_i^{FB, CP}$ and $\overline{A}_i^{FB, CP}$, respectively. The dotted, dashed, and dash-dotted lines are $A_i^\Omega$, $A_i^{\Omega^+}$, and $A_i^{\Omega^-}$, respectively.
One can see from FIG. 2 that the regional CPAs $A_{CP}^{\Omega^+}$ are quite large in the $f_0(500) - \rho(770)^0$ interference region. For the FBAs, the FB-CPAs, and the direct-CPV-subtracted FB-CPAs in FIG. 3, one can see that there are big differences between $A_i^{FB}$ and $\overline{A}_i^{FB}$, resulting in a large $A_{CP,i}^{FB}$. For a more detailed and transparent comparison, we present all the five CPAs, $A_{CP}^{\Omega_i}$, $A_{CP}^{\Omega^+}$, $A_{CP}^{\Omega^-}$, $A_{CP,i}^{FB}$, and $\overline{A}_{CP,i}^{FB}$ in FIG. 4.

The first thing one can see from FIG. 4 is that $A_{CP}^{\Omega_i}$, $A_{CP,i}^{FB}$, and $\overline{A}_{CP,i}^{FB}$ are quite large throughout the whole $f_0(500) - \rho(770)^0$ interference region. Besides, the differences between $A_{CP}^{\Omega_i}$ and $A_{CP}^{\Omega^-}$ are very large too. However, when summed up the event yields of the region $\Omega_i$, the resulting CPAs $A_{CP}^{\Omega_i}$ are much smaller. This means that the CP violation in this region is dominated by the interference of $S$ - and $P$-waves amplitudes, i.e., the interference between the amplitudes corresponding to $f_0(500)$ and $\rho(770)^0$ respectively. This conclusion can also be verified from the small difference between $A_{CP,i}^{FB}$ and $\overline{A}_{CP,i}^{FB}$, since their difference reflects the CPV within the $S$- or $P$-waves.

| $A_{CP}^{FB}$ | $\overline{A}_{CP}^{FB}$ | $A_{CP}^{\Omega_i}$ | $A_{CP}^{\Omega^+}$ | $A_{CP}^{\Omega^-}$ |
|---------------|-------------------------|--------------------|--------------------|-------------------|
| 0.224 ± 0.012 | 0.194 ± 0.013           | 0.099 ± 0.013      | 0.405 ± 0.020      | -0.074 ± 0.017    |

Amplitude analysis of this decay channel $B^\pm \rightarrow \pi^-\pi^+\pi^\pm$ showed that $\rho(770)^0$ plays a dominant role and that the interference of of $\rho(770)^0$ and $f_0(500)$ also contributes significantly [14]. The CPVs, correspondingly, get contribution from the interference of the tree and penguin parts of the $S$- or $P$-wave amplitudes, which are in fact the origin of direct CPAs of the decay channel $B^\pm \rightarrow \pi^\pm f_0(500)$ or $B^\pm \rightarrow \pi^\pm \rho(770)^0$, and the interference between $S$- and $P$-waves amplitudes. All of the aforementioned contributions of CPV present in the regional CPAs, $A_{CP}^{\Omega^+}$ and $A_{CP}^{\Omega^-}$. While for the region $\Omega_i$, the contribution from the $S$- and $P$-waves interfering term cancel out totally. Meanwhile, theoretical analysis in Sec. III shows that the (direct-CPV-subtracted) FB-CPAs contain only the contributions from the interference between the $S$- and $P$-waves amplitudes. The large FB-CPAs and the direct-CPV-subtracted FB-CPAs in FIG. 4 indicates with no doubt that the CPVs corresponding to the interference between $S$- and $P$-waves amplitudes dominant in the $f_0(500) - \rho(770)^0$ interference region. This can also be seen from the numerical values of $A_{CP}^{\Omega_i}$, $A_{CP}^{\Omega^+}$, $A_{CP}^{\Omega^-}$, and $A_{CP}^{\Omega^-}$, of the whole region presented in TABLE. I, from which one can see that
the $A_{CP}^{FB}$ and $\tilde{A}_{CP}^{FB}$ are much larger than $A_{CP}^{O}$.

The interference of the $S$- and $P$-waves can also explain the fine structures of the cos $\theta$-dependencies of the regional CPAs. FIG. 5 presents the cos $\theta$-dependencies of the regional CPAs measured by LHCb for $0.62 \text{ GeV}/c^2 < \sqrt{s_{\text{low}}} < 0.78 \text{ GeV}/c^2$, where cos $\theta$ is equally divided into 30 bins [14]. From FIG. 5 one can clearly see that the regional CPAs depend on cos $\theta$ strongly. It has been pointed out that the tendency of the regional CPAs of taking opposite signs for cos $\theta > 0$ and cos $\theta < 0$ is due to the interference of $f_0(500)$ and $\rho(770)^0$ [13]. Indeed, one can fit the data in FIG. 5 quite well by the inclusion of only $f_0(500)$ and $\rho(770)^0$. The fitted curve is also shown in FIG. 5.

To fit the data, the decay amplitude is parameterized as

$$\mathcal{A}_{B^- \to \pi^- \pi^+ \pi^-} = \cos \theta R_1 \frac{c_\rho e^{i\delta_\rho}}{s_{\text{low}} - m_\rho^2 + i m_\rho \Gamma_\rho} + R_2 \frac{c_f e^{i\delta_f}}{s_{\text{low}} - m_f^2 + i m_f \Gamma_f},$$

(9)

$$\overline{\mathcal{A}}_{B^+ \to \pi^- \pi^+ \pi^+} = \cos \theta R_1 \frac{\bar{c}_\rho e^{i\delta_\rho}}{s_{\text{low}} - m_\rho^2 + i m_\rho \Gamma_\rho} + R_2 \frac{\bar{c}_f e^{i\delta_f}}{s_{\text{low}} - m_f^2 + i m_f \Gamma_f},$$

(10)

where $c_i$ and $\delta_i$ ($i = \rho, f$) represent the corresponding amplitudes and the relative phases contribution of component $\rho(770)^0$ and $f_0(500)$, respectively, $R_1 = \sqrt{s_{\text{low}} - m_\rho^2 \sqrt{\frac{(m_\rho^2 - s_{\text{low}} + m_\pi^2)^2}{s_{\text{low}}}} - m_\rho^2}$, $R_2 = \sqrt{s_{\text{low}} - m_f^2 \sqrt{\frac{(m_f^2 - s_{\text{low}} + m_\pi^2)^2}{s_{\text{low}}}} - m_f^2}$, where $s^* = \frac{m_\rho^2 + m_\pi^2}{2}$ and $m_B$, $m_\pi$, $m_\rho$, and $m_f$ are the masses of $B$, $\pi$, $\rho(770)^0$, and $f_0(500)$, respectively. The parameters used in Eqs. (9) and (10) are listed in TABLE II, and are borrowed from Ref. [33]. The fitted parameters are presented in TABLE III, where $c_\rho$, $\delta_\rho$ and $\bar{c}_\rho$ are fixed at $c_\rho = 1$, $\delta_\rho = \bar{\delta}_\rho = 0$. The goodness of the fit is 1.10.

### TABLE II: Input parameters used in Eqs. (9) and (10).

| Parameters   | Value (in GeV) |
|--------------|----------------|
| $m_B$        | 5.279          |
| $m_\pi$      | 0.139          |
| $m_\rho(770)^0$ | 0.775         |
| $\Gamma_\rho(770)^0$ | 0.149       |
| $m_{f_0}(500)$ | 0.563         |
| $\Gamma_{f_0}(500)$ | 0.350        |
FIG. 5: Dependence of regional CPAs on $\cos \theta$ for $0.62 \text{ GeV}/c^2 < \sqrt{s_{\text{low}}} < 0.78 \text{ GeV}/c^2$, where $\cos \theta$ is divided into 30 bins. The fitted curve is also shown in this figure.

| Parameter | Value   | Parameter | Value   |
|-----------|---------|-----------|---------|
| $c_\rho$  | 1       | $\delta_\rho$ | 0.05 $\pm$ 0.02 |
| $\delta_\rho$ | 0       | $\delta_f$ | $-0.64 \pm 0.06$ |
| $c_f$     | 0.49 $\pm$ 0.07 | $\delta_f$ | $-0.64 \pm 0.06$ |
| $\delta_f$ | $-0.64 \pm 0.06$ | $\delta_f$ | $1.36 \pm 0.31$ |

IV. SUMMARY AND CONCLUSION

In this paper, the FBA, the FB-CPA, the direct-CPV-subtracted FB-CPAs and the regional CPAs for the three-body decay of $B$ meson $B^\pm \rightarrow \pi^\pm \pi^+ \pi^-$ are analysed based on the data of LHCb [13, 14]. We focus on the CPAs in the $f_0(500) - \rho(770)^0$ interfering region. It is found that the FB-CPAs are quite large in this region. According to the theoretical analysis, the large FB-CPA and the direct-CPV-subtracted FB-CPAs can be explained by the interference of the amplitudes.
corresponding to \( f_0(500) \) and \( \rho(770)^0 \). Moreover, the analysis indicates that the interference between the amplitudes of \( f_0(500) \) and \( \rho(770)^0 \) is the main contribution to \( CPVs \) in this region. The aforementioned interference can even explain more detailed structures of \( CPVs \) in this region, according to the fitting result of FIG. 5.

In conclusion, the interference of intermediate resonances can play an important role in \( CPV \) of three-body decay of bottom meson. The FB-\( CP \)A and the direct-\( CPV \)-subtracted FB-\( CP \)A, according the theoretical analysis and the data-based analysis of this paper, are ideal observables to study \( CPVs \) coursed by the interference of intermediate resonances.

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