This paper reports a study into the dynamics of a vibratory machine composed of a viscoelastically-fixed platform that can move vertically and two identical inertial vibration exciters. The vibration exciters’ bodies rotate at the same angular velocities in opposite directions. The bodies host a single load in the form of a ball, roller, or pendulum. The loads’ centers of mass can move relative to the bodies in a circle with a center on the axis of rotation. The loads’ relative movements are hindered by the forces of viscous resistance.

It was established that a vibratory machine theoretically possesses the following:

- one to three oscillatory modes of movement under which loads get stuck at almost constant angular velocity and generate total unbalanced mass in the vertical direction only;
- a no-oscillation mode under which loads rotate synchronously with the bodies and generate total unbalanced mass in the horizontal direction only.

At the same time, only one oscillatory mode is resonant and exists at the above-the-resonance speeds of body rotation, lower than some characteristic speed.

At the bodies’ rotation speeds:

- pre-resonant; there is a globally asymptotically stable (the only existing) mode of load jams;
- above-the-resonance, lower than the characteristic velocity; there are locally asymptotically stable regimes – both the resonance mode of movement of a vibratory machine and a no-oscillations mode;
- exceeding the characteristic velocity; there is a globally asymptotically stable no-oscillations mode.

Computational experiments have confirmed the results of theoretical research. At the same time, it was additionally established that it would suffice, to enter a resonant mode of movement, to slowly accelerate the bodies of vibration exciters to the above-the-resonance speed, less than the characteristic speed.

The results reported here could be interesting both for the theory and practice of designing new vibratory machines.

Keywords: resonant vibratory machine, Sommerfeld effect, inertial vibration exciter of targeted action, single-mass vibratory machine.
2. Literature review and problem statement

Analytically, the application of the Sommerfeld effect to build resonant vibratory machines was studied in the following works:
- [3] for a pendulum rigidly mounted on the shaft of a low-power DC electric motor installed on one of the platforms in a two-mass system;
- [4] for a wind wheel with an unbalanced mass installed on one of the platforms in a three-mass system;
- [5] for a pendulum rigidly mounted on the shaft of an induction electric motor installed on a platform that fluctuates horizontally.

The Sommerfeld effect was discovered and investigated in rotor machines with passive auto-balancers in the following works:
- [6] for a two-ball auto-balancer with the static balancing of the rotor, which executes spatial movement;
- [7] for two two-pendulum auto-balancers with the dynamic balancing of the rotor, which executes spatial movement;
- [8] for a two-ball auto-balancer within a flat rotor model on isotropic supports.

The studies reported in [3–5] found that unbalanced masses (pendulums, wind wheels, etc.) get stuck at one of the resonance frequencies of platform oscillations using an electric motor under a jammed mode overloads the electrical circuit. The use of an air wheel does not provide for a high efficiency due to the peculiarities of converting air energy into mechanical movement.

The studies reported in [3–5] found that unbalanced masses (pendulums, wind wheels, etc.) get stuck at one of the resonance frequencies of platform oscillations using an electric motor under a jammed mode overloads the electrical circuit. The use of an air wheel does not provide for a high efficiency due to the peculiarities of converting air energy into mechanical movement.

The stability of steady motion modes was investigated by the first Lyapunov method using elements of perturbation theory, the theory of nonlinear oscillations [18] were applied.

The results of the theoretical research were tested using a computational experiment. To this end, the differential equations of vibratory machine movement were integrated over a long period of time, sufficient to set a certain mode of movement.
where $\bar{y}$, $\bar{s}$, $\bar{\omega}$ is the characteristic scale to be selected later. Then the differential equations of motion (1) are reduced to the following form

$$
\ddot{v} + \frac{b}{M_\omega} \dot{v} + \frac{k}{M_\omega} v + \frac{\bar{\omega}}{M_\omega} \ddot{s} = 0,
$$

$$
\ddot{\phi}_j \varepsilon \beta \left( \dot{\phi}_j - n_j \right) + \bar{g} \ddot{\phi}_j \cos \phi_j = 0, 
$$

where a dot above a value denotes a derivative for $t$.

Introduce the characteristic scale and dimensionless parameters:

$$
\bar{\omega} = \sqrt{b/\bar{M}_z}, ~ \bar{\omega} = 2\pi \bar{y} R / \bar{M}_z; \quad
\varepsilon = \frac{2m}{\kappa \bar{M}_z} \beta = \frac{b_0 \bar{M}_z}{2m \bar{\omega}^2}, \quad
h = \frac{\bar{b}}{2M_\omega}, \quad
n_j = \frac{\omega_j}{\omega}. \quad (5)
$$

Then equations (4) take the following form:

$$
\ddot{v} + 2\bar{h} \varepsilon \dot{v} + \bar{s} \dot{v} = 0, 
$$

$$
\ddot{\psi}_j + \varepsilon \beta \left( \dot{\psi}_j - n_j \right) + \bar{g} \ddot{\psi}_j \cos \psi_j = 0, \quad / j = 1, 2. / 
$$

To excite the vibrations of targeted action, the bodies of vibration exciters must rotate at the same speeds $n$ in opposite directions:

$$
n_1 = n, \quad n_2 = -n. \quad (7)
$$

Move to the new coordinates that determine the movement of loads

$$
\psi_1 = \psi_1, \quad \psi_2 = \pi - \psi_x. \quad (8)
$$

Then, taking into consideration (7), (8), the dimensionless differential equations of vibratory machine movement (5) take the following form

$$
\ddot{v} + 2\bar{h} \varepsilon \dot{v} + \bar{s} \dot{v} = 0, 
$$

$$
\ddot{\psi}_j + \varepsilon \beta \left( \dot{\psi}_j - n_j \right) + \bar{g} \ddot{\psi}_j \cos \psi_j = 0, \quad / j = 1, 2. / 
$$

In (9)

$$
\bar{s}_j = \left( \cos \psi_1 - \cos \psi_2 \right) / 2, \quad \bar{s}_j = \left( \sin \psi_1 + \sin \psi_2 \right) / 2. \quad (10)
$$

The derived differential equations (9), with accuracy to designations, coincided with the differential equations of motion of a single-mass vibratory machine [10] for the case of two loads in a (single) vibration exciter.

5. 1. 3. Steady motion modes under which loads rotate in opposite directions, at zero approximation ($\varepsilon=0$)

At $\varepsilon=0$, the system of differential equations (9) takes the following form

$$
\ddot{v} + 2\bar{h} \dot{v} + \bar{s} \dot{v} = 0, \quad \psi_1 = 0, \quad \psi_2 = 0. \quad (11)
$$

Note that the system of differential equations (11) does not include implicit dimensionless time $t$. Therefore, the last two equations allow such a solution in which loads rotate at the same angular speeds $\Omega$ in opposite directions.
\[ \ddot{\psi}_1 = \Omega \dot{\psi}_1 + \psi_0, \quad \ddot{\psi}_2 = \Omega \dot{\psi}_2 - \psi_0. \]  

(12)

In (12), the \( \psi_0 \) parameter determines the angle of rotation at which one load is ahead and the other lags behind the average angle of rotation \( \Omega t \) of two loads.

Then

\[ s_1 = -\sin \psi_0 \sin(\Omega t), \quad s_2 = \cos \psi_0 \sin(\Omega t). \]  

(13)

Taking into consideration (13), the first equation in (11) takes the following form

\[ \ddot{v} + 2h\dot{v} + v = \Omega^2 \cos \psi_0 \sin(\Omega t). \]  

(14)

A partial solution to differential equation (14) takes the following form

\[ \ddot{v} = \frac{\Omega^2 \cos \psi_0}{(1 - \Omega^2)^2 + 4h^2 \Omega^2} \left[(1 - \Omega^2) \sin(\Omega t) - \frac{2h \Omega \cos(\Omega t)}{1 - \Omega^2} \right]. \]  

(15)

Note that one cannot find parameters \( \Omega \) and \( \psi_0 \) at zero approximation.

### 5.1.4. Refining the steady motion modes applying the first approximation

By substituting (12), (15) in the second and third equations in (9), we obtain the following two equations

\[ e\beta(\Omega - n) - \varphi \frac{\Omega^2 \cos \psi_0}{2(1 - \Omega^2)^2 + 4h^2 \Omega^2} \times \left[\pm(1 - \Omega^2) \sin \psi_0 + 2h \Omega \cos \psi_0 \right] = 0, \]  

(16)

where the upper character in \( \pm \) corresponds to the first equation and the lower character \( - \) to the second.

Leave in (16) the non-periodic components (interfering with the frequency of movement of loads), and we obtain

\[ e\beta(\Omega - n) + \frac{\varphi \Omega^2 \cos \psi_0}{2(1 - \Omega^2)^2 + 4h^2 \Omega^2} \times \left[\pm(1 - \Omega^2) \sin \psi_0 + 2h \Omega \cos \psi_0 \right] = 0. \]  

(17)

Subtract the second equation from the first equation in (17), we obtain

\[ \frac{\Omega^2}{2(1 - \Omega^2)^2 + 4h^2 \Omega^2} \sin \psi_0 \cos \psi_0 = 0. \]  

(18)

Condition (18) is met if \( \sin \psi_0 \cos \psi_0 = \sin(2\psi_0)/2 = 0. \)

Hence, we find

\[ 2\psi_0 = 0, \pm \pi, \ldots \]  

(19)

Add the second equation to the first equation in (17), and we obtain

\[ e\beta(\Omega - n) + \frac{\varphi h \Omega^2 \cos^2 \psi_0}{(1 - \Omega^2)^2 + 4h^2 \Omega^2} = 0. \]  

(20)

Consider the following possibilities.

### 5.2. Investigating the stability of steady movement modes

#### 5.2.1. A mode of the synchronous rotation of loads with the bodies of vibration exciters

Introduce the unperturbed motion under a synchronous rotation mode

\[ \psi_1 = n\tau - \pi/2, \quad \psi_2 = n\tau + \pi/2, \quad \ddot{v} = 0. \]  

(24)

Introduce perturbed motion

\[ \psi_1 = n\tau - \pi/2 + x_1, \quad \psi_2 = n\tau + \pi/2 + x_2, \quad \ddot{v} = x_0. \]  

(25)

Linearize differential equations of motion (9) to obtain

\[ \ddot{x}_0 + 2k\dot{x}_0 + x_0 + \frac{d^2}{dt^2} \left[ \frac{x_1 - x_2 \sin(n\tau)}{2} \right] = 0, \]  

\[ \ddot{x}_1 + e\beta x_1 + e\beta x_2 - e\beta \dot{x}_2 \sin(n\tau) = 0. \]  

(26)

Introduce new variables

\[ w = (x_1 + x_2)/2, \quad z = (x_1 - x_2)/2. \]  

(27)

Then the system of differential equations (26) is transformed to the following form

\[ \ddot{x}_0 + 2k\dot{x}_0 + x_0 + \frac{d^2}{dt^2} \left[ z \sin(n\tau) \right] = 0. \]
\[ \ddot{z} + \varepsilon \beta \dot{z} + \varepsilon \dot{x}_0 \sin(\pi t) = 0, \quad \ddot{w} + \varepsilon \beta \dot{w} = 0. \]  

(28)

(28) demonstrates that \( z \) is a slow-changing function. Then the first equation in (28), with an accuracy to the values of the zero order of smallness (for \( \varepsilon \), takes the following form

\[ \ddot{x}_0 + 2\dot{h}_i + x_0 = z n^2 \sin(\pi t). \]  

(29)

A partial solution to equation (29) takes the following form

\[ \ddot{x}_0 = \frac{z n^2}{(1-n^2)^2 + 4h_n^2 n^2} \left[(1-n^2)\sin(\pi t) - 2h_n\cos(\pi t)\right]. \]  

(30)

Then the second equation in (28) takes the following form

\[ \ddot{z} + \varepsilon \beta \dot{z} + \frac{\varepsilon n^2 (n^2 - 1)}{2[(1-n^2)^2 + 4h_n^2 n^2]} z = 0. \]  

(32)

Consequently, the mode of the synchronous rotation of the bodies of vibration exciters (\( \alpha > 1 \)).

5.2.2. Modes of load jamming

Introduce the unperturbed motion under the mode when loads get stick

\[ \ddot{v} = \frac{\Omega^2}{(1-\Omega^2)^2 + 4h_n^2 \Omega^2} \times \left[(1-\Omega^2)\sin(\Omega t) - 2h_n\cos(\Omega t)\right]. \]  

(33)

Introduce the perturbed motion

\[ v = \frac{\Omega^2}{(1-\Omega^2)^2 + 4h_n^2 \Omega^2} \times \left[(1-\Omega^2)\sin(\Omega t) - 2h_n\cos(\Omega t)\right] + x_i, \quad \psi_i = \Omega t + x_i. \]  

(34)

Then

\[ s_y = \sin(\Omega t) + \frac{x_1 + x_2}{2} \cos(\Omega t). \]  

(35)

After linearization, the system of differential equations (9) takes the following form

\[ \ddot{x}_0 + 2\dot{h}_i + x_0 + \frac{d^2}{dt^2} \left(\frac{x_1 + x_2}{2} \cos(\Omega t)\right) = 0, \]  

\[ \ddot{x}_j + \varepsilon \beta \dot{x}_j + \frac{\varepsilon n^2 (n^2 - 1)}{2[(1-n^2)^2 + 4h_n^2 n^2]} x_j + \dot{x}_0 \cos(\Omega t) = 0, \quad /j = 1,2/. \]  

(36)

In the new variables (27), system (36) takes the following form

\[ \ddot{x}_0 + 2\dot{h}_i + x_0 + \frac{d^2}{dt^2} \left(\frac{x_1 + x_2}{2} \cos(\Omega t)\right) = 0, \]  

\[ \ddot{w} + \varepsilon \beta \dot{w} + \varepsilon \left[-\ddot{x}_0 \sin(\Omega t) + \dot{x}_0 \cos(\Omega t)\right] = 0, \]  

\[ \ddot{z} + \varepsilon \beta \dot{z} + \frac{\varepsilon n^2 (n^2 - 1)}{2[(1-n^2)^2 + 4h_n^2 n^2]} z = 0. \]  

(37)

(38) demonstrates that only such an oscillatory mode could be stable under which loads get stuck at the pre-resonant speed (\( \Omega < 1 \)).

The second equation in (37) demonstrates that \( w \) is a slow-changing function. Then the first equation in (37), with an accuracy to the values of the zero order of smallness (for \( \varepsilon \), takes the following form

\[ \ddot{x}_0 + 2\dot{h}_i + x_0 = \omega \Omega^2 \sin(\Omega t). \]  

(39)

A partial solution to equation (39) takes the following form

\[ \ddot{x}_0 = \frac{\omega \Omega^2}{(1-\Omega^2)^2 + 4h_n^2 \Omega^2} \times \left[(1-\Omega^2)\sin(\Omega t) - 2h_n\cos(\Omega t)\right] - \omega \Omega^4 \times \left[(1-\Omega^2)\sin(\Omega t) - 2h_n\cos(\Omega t)\right] \cos(\Omega t) = 0. \]  

(40)

Then the second equation in (37) takes the form

\[ \ddot{w} + \varepsilon \beta \dot{w} + \frac{\varepsilon n^2 (n^2 - 1)}{2[(1-n^2)^2 + 4h_n^2 n^2]} w = 0. \]  

(41)

(39), (41) demonstrate that at the pre-resonant speeds of load jamming (\( \Omega < 1 \)) the perturbation is \( w, x \to 0 \), which corresponds to the asymptotic stability of movement. Thus, among all possible modes under which loads get stick, the steady one is the mode under which loads get stuck at pre-resonance speed (\( 0 < \Omega < 1 \)).

Based on the results reported in [10], the second characteristic speed of rotor rotation is

\[ \bar{n}_2 = 1 + \frac{1}{4h_l} = 1 + \frac{m^2 \omega_0^2}{b_n b}. \]  

(42)

Moreover, there is only one pre-resonance frequency of load jamming (\( \bar{n}_2 \)), and only at speeds lower than \( \bar{n}_2 \) but at any values of other parameters.
5.3. Verifying the results of theoretical research using a computational experiment

Below are the results of integrating the system of differential equations (6) under the initial conditions

\[ y = y = 0; \quad \phi_j = -\pi/2, \]
\[ \dot{\phi}_j = 0, \quad / j = 1,2 /. \]  

(43)

Estimated data:

\[ h = 0.1, \quad \beta = 1, \quad \varepsilon = 0.05. \]  

(44)

The speed of rotation of the shafts varies by the following law

\[ n_t(t) = -n_t(t) = \begin{cases} 2\pi / T, & \text{if } t < T / 2; \\ n, & \text{otherwise}. \end{cases} \]  

(45)

where \( T > 2000 \) and \([0, T]\) is the dimensionless time interval in which the differential equations of motion are integrated.

Taking into consideration (44), formula (42) produces \( \bar{n}_t = 3.5 \). The results of the computational experiment are as follows.

At the pre-resonance speeds of body rotation, the only stable mode of movement is the mode when loads get stuck. This movement is globally asymptotically stable and occurs under any initial conditions. Fig. 2 shows the result of the integration of the system of differential equations (6) at \( n = 0.9 \). On the left are the charts of magnitude changes throughout the integration interval. On the right are the charts of magnitude changes after setting the movement – in the interval \([T-\Delta t, T]\), where \( \Delta t = 0.98T \).

The jamming mode begins to appear even during the acceleration of the vibration exciter bodies.

Fig. 3 shows the results of integrating differential motion equations at the above-the-resonance speeds of rotation of vibration exciter bodies not exceeding \( \bar{n}_t = 3.5 \) (\( n = 3.5 \)).

Under the slow acceleration of the bodies of vibration exciters, the mode under which loads get stuck appears first. Next, the jamming mode maintains stability with an increase in the speed of rotation of bodies to a maximum value not exceeding \( \bar{n}_t = 3.5 \). However, for any \( n \in (1, \bar{n}_t] \), a stable mode can be a mode of the synchronous rotation of loads with bodies. Typically, this mode occurs at the rapid acceleration of the bodies. Thus, at \( n \in (1, \bar{n}_t] \), the locally stable are the two modes of movement of a vibratory machine. Of course, each mode has its own pull zone.

Fig. 4 shows the results of integrating differential motion equations at the above-the-resonance speeds of rotation of the bodies of vibration exciters exceeding \( \bar{n}_t = 3.5 \) (\( n = 3.6 \)).

The only stable steady mode of movement is the mode of the synchronous rotation of loads together with the bodies of vibration exciters. Moreover, with slow acceleration, the mode at which loads get stuck appears first. However, when the speed of rotation \( n \) exceeds the characteristic speed \( \bar{n}_t = 3.5 \), the jam mode loses stability.
Applied mechanics

Applied mechanics establish that when rotating the bodies of vibration exciters greater than \( \tilde{n}_2 \) the proper choice of initial conditions or the acceleration rate of the vibration exciter bodies could ensure the onset of any mode out of two possible stable modes.

To drive a vibratory machine to the resonance mode of movement, it would suffice to slowly accelerate the bodies of vibration exciters to a speed less than \( \tilde{n}_2 \).

It should be noted that the same unbalanced masses can be attached to the bodies of vibration exciters. Then the combined vibration exciter would work as two inertial vibration exciters of targeted action. The first would be formed by loads and would excite slow vibrations at the resonance frequency. The second would be formed by the unbalanced masses on the bodies of vibration exciters and would excite rapid vibrations at the rotation frequency of bodies. Since the differential equations of platform movement are linear in relation to the coordinates of the platform and the total unbalanced mass, we can assume that the conditions of performance of a vibration exciter would not change [10].

This work does not investigate the effect exerted on the performance of the vibration exciter by gravity forces. Also unaddressed are the regions of attraction of the locally stable steady modes of movement of vibratory machines. However, this does not significantly affect the results obtained.

In the future, it is planned to investigate the steady modes of movement of two-mass and three-mass resonant vibratory machines with the translational movement of platforms and a vibration exciter of targeted action.

6. Discussion of results of studying the dynamics of a resonant single-mass vibratory machine with a vibration exciter of targeted action

Our study has shown that a vibratory machine possesses the following:

– one to three oscillatory modes of movement under which loads get stuck at an almost constant angular velocity whose value is determined from equation (23):
– a no-oscillations mode under which loads rotate synchronously with the bodies, and their total unbalanced mass in the vertical direction is zero.

Only under a single oscillation mode do the loads get stuck at an almost constant angular rotational velocity \( \Omega_2 \), less than the resonance oscillation frequency of the platform \( n=1 \). With an increase in the speed of rotation of the bodies of vibration exciters, the load rotation frequency approaches a resonance frequency, which excites intense resonance oscillations. At the same time, the platform oscillation amplitude increases monotonously.

At the pre-resonance speeds of the rotation of vibration exciter bodies, there is a globally asymptotically stable mode of load jamming at the pre-resonance rotation speeds. At the above-the-resonance speeds of rotation of the bodies of vibration exciters, smaller than the second characteristic speed, the locally asymptotically stable are both the mode of load jamming at the pre-resonant speeds of rotation and the mode of the synchronous rotation of loads. At the speeds of rotation of the bodies of vibration exciters greater than the second characteristic speed, the globally asymptotically stable is the mode of the synchronous rotation of loads.

Computational experiments confirm the results of our theoretical studies on the existence and stability of steady motion modes. In addition, the computational experiments establish that when rotating the bodies of vibration exciters at above-the-resonance speeds less than \( \tilde{n}_2 \), the proper choice of initial conditions or the acceleration rate of the vibration exciter bodies could ensure the onset of any mode out of two possible stable modes.

7. Conclusions

1. Vibratory machines theoretically possess the following:

– one to three oscillatory modes of movement under which loads get stuck at almost constant angular velocity and form total unbalanced mass only in the vertical direction;
– a no-oscillations mode under which loads rotate synchronously with the bodies and from total unbalanced oscillations. At the same time, the platform oscillation amplitude increases monotonously.

2. At the pre-resonance speeds of the rotation of vibration exciter bodies, a globally asymptotically stable is the only existing mode of load jamming (at pre-resonance speeds). At the above-the-resonance speeds of the rotation of the bodies of vibration exciters, smaller than the characteristic speed, locally asymptotically stable are both the resonant mode of movement of a vibratory machine and a no-oscillations mode. At the speeds of rotation of the bodies of vibration
exciters greater than the characteristic speed, the globally asymptotically stable is the no-oscillations mode.

3. Computational experiments confirm the results of theoretical research and allow us to establish the following:
   – when rotating the bodies of vibration exciters at above-the-resonance speeds lower than the characteristic speed, the proper choice of initial conditions, or the speed of the acceleration of the bodies of vibration exciters could ensure the onset of both a resonant oscillation mode and a no-oscillations mode;
   – to set a vibratory machine to a resonance mode of movement, it would suffice to slowly accelerate the bodies of vibration exciters to a speed less than the characteristic speed.

Acknowledgments

This paper is funded within the framework of scientific work No. 0119U001173 “Stabilization and stability of motion of an unbalanced rotating carrying body in a free or isolated mechanical system” performed at the Central Ukrainian National Technical University and financed from the state budget.

References

1. Kryukov, B. I. (1967). Dinamika vibrationnyh mashin rezonansnogo tipa. Kyiv: Nauk. dumka, 210.
2. Sommerfeld, A. (1904). Beitrag zum dinamischen Ausbay der Festigkeislehre. Zeitschrift des Vereins Deutscher Ingeniere, 48 (18), 631–636.
3. Lanets, O. V., Shpak, Ya. V., Lozynskyi, V. I., Leonovych, P. Yu. (2013). Realizatsiya efektu Zomerfelda u vibratsijnomu maidanchyku z inertsiynym pryvodom. Avtomatyatsiya vyrobnychykh protessiv u mashynobuduvannyh, 47, 12–28. Available at: http://nbuv.gov.ua/UJRN/Avtomatyzac_2013_47_4
4. Kuzo, I. V., Lanets, O. V., Gurskii, V. M. (2013). Synthesis of low-frequency resonance vibratory machines with an aeroshock drive. Naukovyi visnyk Nationalnoho birnichoho universytetu, 2, 60–67. Available at: http://nbuv.gov.ua/UJRN/Nvngu_2013_2_11
5. Yaroshevich, N., Puts, V., Yaroshevich, T., Herasymchuk, O. (2020). Slow oscillations in systems with inertial vibration exciters. Vibroengineering PROCEDIA, 32, 20–25. doi: https://doi.org/10.21595/vp.2020.21509
6. Ryzhik, B., Sperling, L., Duckstein, H. (2004). Non-synchronous Motions Near Critical Speeds in a Single-plane Auto-balancing Device. Technische Mechanik, 24 (1), 25–36. Available at: https://journals.unb.uni-magdeburg.de/index.php/techmech/article/view/911/888
7. Lu, C.-J., Tien, M.-H. (2012). Pure-rotary periodic motions of a planar two-ball auto-balancer system. Mechanical Systems and Signal Processing, 32, 251–268. doi: https://doi.org/10.1016/j.ymssp.2012.06.001
8. Artyunin, A. I., Eliseev, S. V. (2013). Effect of “Crawling” and Peculiarities of Motion of a Rotor with Pendular Self-Balancers. Applied Mechanics and Materials, 373-375, 38–42. doi: https://doi.org/10.4028/www.scientific.net/amm.373-375.38
9. Filimonikhin, G., Yatsun, V. (2015). Method of excitation of dual frequency vibrations by passive autobalancers. Eastern-European Journal of Enterprise Technologies, 4 (7 (76)), 9–14. doi: https://doi.org/10.15587/1729-4061.2015.47116
10. Yatsun, V., Filimonikhin, G., Dumenko, K., Nevdakha, A. (2017). Search for two-frequency motion modes of single-mass vibratory machine with vibration exciter in the form of passive auto-balancer. Eastern-European Journal of Enterprise Technologies, 6 (7 (90)), 58–66. doi: https://doi.org/10.15587/1729-4061.2017.117683
11. Filimonikhin, G., Yatsun, V., Kryuchenko, A., Hrechka, A., Shcherbyna, K. (2020). Synthesizing a resonance anti-phase two-mass vibratory machine whose operation is based on the Sommerfeld effect. Eastern-European Journal of Enterprise Technologies, 6 (7 (108)), 42–50. doi: https://doi.org/10.15587/1729-4061.2020.217628
12. Yatsun, V., Filimonikhin, G., Pirogov, V., Amosov, V., Luzan, P. (2020). Research of antiresonance three-mass vibratory machine with a vibration exciter in the form of a passive autobalancer. Eastern-European Journal of Enterprise Technologies, 5 (7 (107)), 89–97. doi: https://doi.org/10.15587/1729-4061.2020.213724
13. Jung, D. (2018). Supercritical Coexistence Behavior of Coupled Oscillating Planar Eccentric Rotor/AutoBalancer System. Shock and Vibration, 2018, 1–19. doi: https://doi.org/10.1155/2018/4083897
14. Blekhman, I. I., Rivin, E. I. (1988). Synchronization in Science and Technology. ASME, 255.
15. Pan, F., Yongjun, H., Liming, D., Mingjun, D. (2018). Theoretical Study of Synchronous Behavior in a Dual-Pendulum-Rotor System. Shock and Vibration, 2018, 1–13. doi: https://doi.org/10.1155/2018/9824631
16. Hou, Y., Fang, P. (2015). Synchronization and Stability of Two Unbalanced Rotors with Fast Antirotation considering Energy Balance. Mathematical Problems in Engineering, 2015, 1–15. doi: https://doi.org/10.1155/2015/694145
17. Yaroshevich, N. P., Zabolodets, I. P., Yaroshevich, T. S. (2016). Dynamics of Starting of Vibrating Machines with Unbalanced Vibroexciters on Solid Body with Flat Vibrations. Applied Mechanics and Materials, 849, 36–45. doi: https://doi.org/10.4028/www.scientific.net/amm.849.36
18. Nayfeh, A. H. (1993). Introduction to Perturbation Techniques. John Wiley and Sons Ltd.