Correspondence between phenomenological and IBM-1 models of even isotopes of Yb

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Abstract: Energy levels and the reduced probability of E2– transitions for ytterbium isotopes with proton number Z = 70 and neutron numbers between 100 and 106 have been calculated through phenomenological (PhM) and interacting boson (IBM-1) models. The predicted low-lying levels (energies, spins and parities) and the reduced probability for E2– transitions results are reasonably consistent with the available experimental data. The predicted low-lying levels (gr–, β1– and γ1– band) produced in the PhM are in good agreement with the experimental data compared with those by IBM-1 for all nuclei of interest. In addition, the phenomenological model was successful in predicting the β2–, β3–, β4–, γ2– and 1+– band while it was a failure with IBM-1. Also, the 3+– band is predicted by the IBM-1 model for 172Yb and 174Yb nuclei. All calculations are compared with the available experimental data.

Key words: ytterbium (Yb), energy levels, model, even-even, isotopes, nuclei

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1 Introduction

The medium-to heavy-mass ytterbium (Yb) isotopes located in the rare-earth mass region are well-deformed nuclei that can be populated to very high spin. Much experimental information on even-odd-mass of Yb isotopes has become more abundant [1–6]. For the heavier A = 174 to 178 nuclei [7], previous work using deep inelastic reactions and Gammasphere have begun to reveal much information about the high-spin behavior of these neutron-rich Yb isotopes. The yrast states in the well-deformed rare-earth region have been described by using the projected shell model [8–14].

Prior to the present work the level structure of the ground band state and low-lying excited states of even-even nuclei has been studied both theoretically and experimentally [15], including the decay, Coulomb excitation and various transfer reactions.

In this study, two calculations for energy levels of 170,172,174,176Yb isotopes have been done by using two different models, the phenomenological model (PhM) and the interacting boson model (IBM-1). Positive parity state energies and the reduced probability of E2– transitions are calculated and compared with the available experimental data. The structure of excited levels is discussed.

2 Theoretical models

The calculations have been performed using the phenomenological and interacting boson models. In the next subsection, we will explain these models.

2.1 Phenomenological model (PhM)

To analyze the properties of low-lying positive parity states in Yb isotopes, the PhM of [16–18] is used. This model takes into account the mixing of states of the gr–, β–, γ– and Kπ = 1+– band. The model Hamiltonian is chosen in the following form

\[ H = H_{\text{rot}}(I^2) + H_{K,K'}^\sigma, \]  (1)

where \( H_{\text{rot}}(I^2) \) is the rotational part of the Hamiltonian, and

\[ H_{K,K'}^\sigma(I) = \omega_K \delta_{K,K'} - \omega_{\text{rot}}(I)(j_x)_{K,K'} \xi(I,K) \delta_{K,K'+1}^\sigma, \]  (2)

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where $\omega_K$ is the band head energy of the rotational band, $\omega_{rot}(I)$ is the rotational frequency of the core, and $(j_x)_{K,K'}$ is the matrix elements which describe the Coriolis mixture between rotational bands:

$$(j_x)_{\gamma,1} = -\sqrt{3} \tau_0, (j_x)_{\gamma,1} = -\sqrt{3} \tau_1, (j_x)_{\gamma,1} = -1 \tau_2$$

and

$$\xi(I,0) = 1 \quad \xi(I,2) = \sqrt{1 - \frac{2}{I(I+1)}}.$$ 

The eigenfunction of Hamiltonian model Eq. (1) has the form

$$|MK\rangle = \left[ \frac{2I+1}{16\pi^2} \right]^{\frac{1}{4}} \left\{ \sqrt{2} \Psi_{I',K} D_{MK}^I(\theta) \right. \right.$$

$$\left. + \sum_{K'} \frac{\Psi_{I',K'} D_{MK,K}^{I',I}(\theta) b_{K'}^+} {\sqrt{1 + \delta_{K',0}}} + \left. (-1)^I b_{K,-K}^+ \right\} \left| 0 \right.$$ 

where $\Psi_{I',K}$ is the amplitude of the mixture of basis states.

Solving the Schrödinger equation one can determine the eigenfunctions and eigenenergies of the positive parity states.

$$(H_{K',K}^{I',I} - \varepsilon_I^{I',I}) \Psi_{I',K} = 0.$$ 

The complete energy of a state is given by

$$E_I^{I'}(I) = E_{rot}(I) + \varepsilon_I^{I',I}.$$ 

The rotating-core energy $E_{rot}(I)$ is calculated by using the Harris parameterizations [19] of the energy and the angular momentum, that is

$$E_{rot}(I) = \frac{1}{2} j_0 \omega_{rot}^2(I) + \frac{3}{4} j_1 \omega_{rot}^4(I),$$

$$\sqrt{I(I+1)} = j_0 \omega_{rot}(I) + j_1 \omega_{rot}^3(I).$$

where $j_0$ and $j_1$ are the inertial parameters of the rotational core.

The rotational frequency of the core $\omega_{rot}(I)$ is found by solving the cubic equation Eq. (7). This equation has two imaginary roots and one real root. The real root is

$$\omega_{rot}(I) = \left\{ \frac{\tilde{I}}{2j_1} + \left[ \left( \frac{j_0}{3j_1} \right)^3 + \left( \frac{\tilde{I}}{2j_1} \right)^2 \right]^{\frac{1}{2}} \right\} \left( \frac{\tilde{I}}{2j_1} + \left[ \left( \frac{j_0}{3j_1} \right)^3 + \left( \frac{\tilde{I}}{2j_1} \right)^2 \right]^{\frac{1}{2}} \right)^{-1},$$

where $\tilde{I} = \sqrt{I(I+1)}$. Eq. (8) gives $\omega_{rot}(I)$ at the given spin $I$ of the core.

2.2 Interacting boson model (IBM-1)

The IBM has become one of the most intensively used nuclear models, due to its ability to describe the low-lying collective properties of nuclei across an entire major shell with a Hamiltonian. In IBM-1 the spectroscopies of low-lying collective properties of even-even nuclei are described in terms of a system of interacting $s$ bosons ($L=0$) and $d$ bosons ($L=2$). Furthermore, the model assumes that the structure of low-lying levels is dominated by excitations among the valence particles outside major closed shells. In the particle space the number of proton bosons $N_p$ and neutron bosons $N_n$ is counted from the nearest closed shell, and the resulting total boson number is a strictly conserved quantity. The underlying structure of the six-dimensional unitary group $SU(6)$ of the model leads to a Hamiltonian, capable of describing the three specific types of collective structures with classical geometrical analogues (vibrational [20], rotational [21], and $\gamma$-unstable [22]) and also the transitional nuclei [23] whose structures are intermediate. The IBM-1 Hamiltonian can be expressed as [22]

$$H = 2\epsilon_s(s^+\tilde{s}) + 2\epsilon_d(d^+\tilde{d})$$

$$+ \sum_{L=0,2,4} \frac{1}{2} (2L+1)^2 C_L [d^+d^+X^+d^+d]^2(0)_0 + \frac{1}{\sqrt{2}}(d^+d^+X^+d^+d)^2(2)_0$$

$$+ \frac{1}{\sqrt{2}}(d^+d^+X^+d^+d)^2(2)_0 + \frac{1}{\sqrt{2}}(d^+d^+X^+d^+d)^2(2)_0$$

$$+ u_2 [(d^+s^+)(d^+d)^2(2)_0 + (d^+s^+)(d^+d)^2(2)_0 + (s^+s^+)(d^+d)^2(2)_0 + (s^+s^+)(d^+d)^2(2)_0),$$

where $(s^+d^+)$ and $(s,d)$ are creation and annihilation operators for $s$ and $d$ bosons, respectively.

The IBM-1 Hamiltonian equation Eq. (9) can be written in the general form as [24]

$$H = \varepsilon_{n_d} + a_0 P^1 + a_1 LL + a_2 QQ + a_3 T3T3 + a_4 T4T4.$$
and hence, no distinction is made between neutron and proton bosons taking results is given in the next section.

3 Result and discussion

In this section, the calculated results are discussed separately for low-lying states of even-even isotopes of Yb, with neutron number from 100 to 106. Our results include energy levels and the reduced probability of E2-transitions.

3.1 Energy levels

To describe the positive parity states in the PhM, we determine the model parameters via the following procedure. In accordance with [25], we suppose that, at low spins, the rotational core energy is the same as the energies of the ground band states.

Description of the parameters:
– the inertial parameters \( j_0 \) and \( j_1 \) of rotational core determined by Eq. (6), using the experimental energy of ground band states up to spin \( I \leq 10 \);
– headband energy of ground \((gr)−\) and \( \beta_1− \) bands taken from experimental data, because they are not perturbed by the Coriolis forces at \( I = 0 \);
– headband energy of \( \gamma− \), \( K^\pi = 1^+ \) bands \((\omega_2 \text{ and } \omega_1+\) and also matrix elements \((K|j_0|K^\pm 1)\) are free parameters of the model. They have been fitted by the least squares method requiring the best agreement between the theoretical energies and experimental data. The fitting parameters of the model are given in Table 1.

Table 1. Parameters used in PhM to calculate energy of low excited states in Yb isotopes.

| \( A \)  | \( (j_0)_{gr} \) | \( (j_0)_{\beta_1} \) | \( (j_0)_{\gamma_1} \) |
|---|---|---|---|
| 170 | 0.186 | 0.394 | 0.728 |
| 172 | 0.275 | 0.978 | 0.325 |
| 174 | 0.185 | 0.400 | 0.085 |
| 176 | 0.200 | 0.540 | 0.400 |

Note: where, \((j_0)_{K^\pi} \) \((K^\pi = 1^+)\) are matrix elements of the Coriolis interactions and \( Q_0 \) is the intrinsic quadrupole moment of the nucleus (in fm\(^2\) units) taken from Ref. [26].

Also, in the present work the rotational limit of IBM-1 has been applied to \( ^{170−178} \text{Yb} \), from the ratio \((E(4^+)/E(2^+))\) it has been found that the \( ^{170−178} \text{Yb} \) isotopes are rotational (deformed nuclei) and these nuclei have been successfully treated as exhibiting the \( SU(3) \) symmetry of IBM-1.

In IBM-1, the \((E(4^+)/E(2^+))\) ratio is 3.33, 2.5 and 2 for the \( SU(3) \), \( O(6) \) and \( U(5) \) limits, respectively. From experimental data, the \((E(4^+)/E(2^+))\) ratios are 3.302, 3.33, 3.329 and 3.319 for \( ^{170−176} \text{Yb} \) isotopes, respectively. Therefore, the \( SU(3) \) limits have been applied to study the properties of the Yb isotopes (see Eq. (11)) and the parameters \( \varepsilon, a_0, a_3 \) and \( a_4 \) vanish \((\varepsilon = a_0 = a_3 = a_4 = 0)\) for the \( SU(3) \) limit.

For the analysis of excitation energies in Yb isotopes we tried to keep to the minimum number of free parameters in the Hamiltonian. The explicit expression of the Hamiltonian adopted in the calculations is [24]:

\[
H = a_1 L + a_2 Q, \quad Q = a_3 L + a_4 Q, \quad Q = a_5 L + a_6 Q, \quad Q = a_7 L + a_8 Q.
\]  

In the framework of IBM-1, for the isotopic chains of Yb with \( Z = 70 \) nuclei, the number of proton boson holes is 6, the number of neutron boson particles which varies from 9 to 11 for \( ^{170−174} \text{Yb} \), and the number of neutron boson holes for \( ^{176} \text{Yb} \) is 10. Table 2 shows the coefficient values which we used in IBM-1. The comparison of calculated energy levels and experimental data of low-lying states of \( ^{170} \text{Yb}, ^{172} \text{Yb}, ^{174} \text{Yb} \) and \( ^{176} \text{Yb} \) isotopes are presented in Figs. 1–4, respectively. The PhM calculations are plotted on the left of the experimental data and the IBM-1 calculations on the right for the \( gr−, \beta_1− \) and \( \gamma_1− \) bands. The experimental data are taken from [26] for all isotopes of Yb and also from [27–30] for \( ^{170−174} \text{Yb} \) and \( ^{176} \text{Yb} \), respectively. From these figures, we can see that our calculated results (energies, spin and parity) in both models are reasonably consistent with experimental data, except that the \( \gamma_1− \) band energies in the IBM-1 calculations for \( ^{172} \text{Yb} \) and \( ^{174} \text{Yb} \) nuclei disagree with the experimental data. Also the phenomenological calculations are in better agreement with the experimental data than from those of IBM-1. In the high spin these figures show the difference between our calculation and the experimental data. Furthermore the phenomenological model predicts the energies, spin and parity of the \( \beta_2−, \beta_3−, \gamma_2− \) and \( 1^+ \) band, as is shown in Tables 1–7, respectively. Finally, we believe that the failure to use a pairing parameter was the cause of the disagreement between the IBM-1 calculations and experimental data; this will be discussed in future studies.

![Fig. 1. (color online) Comparison of calculation energy values by PhM and IBM-1 with experimental data for \( ^{170} \text{Yb} \).

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3.2 Reduced probability of $B(E2)$ transitions

In the PHM, with the wave functions calculated by solving the Shrödinger equation Eq. (4), the reduced probabilities of $E2$ transitions between states $I_i K_i$ and states $I_f K_f$ are calculated [16, 18] as:

$$B(E2;I_i K_i \rightarrow I_f K_f) = \left\{ \begin{array}{c} \sqrt{\frac{5}{16\pi}}Q_0 \psi_{I_i g^r-gr} | \psi_{I_f g^r-gr} \rangle C_{I_i I_f 0,20} \psi_{I_f K_f K_i} } \\
+ \sum_n \psi_{I_i K_i_{i,n} g^r-gr} \psi_{I_f K_f K_i} C_{I_i K_i_{i,n} 20} \psi_{I_f K_f K_i} \sqrt{1 + \delta_{K_i 0,0}} \\
+ \psi_{I_i g^r,K_i} \sum_n m_{K_i} \psi_{I_i K_i_{i,n} g^r-gr} C_{I_i K_i_{i,n} 2K_i} \psi_{I_f K_f K_i} \end{array} \right\}^2, \quad (12)$$

where $m_{K_i} = \langle g^r | i\hbar \langle E2 \rangle | K_i \rangle$ ($K_i = 0^+, 2^+$ and $1^+$) are constants to be determined from the experimental data, $Q_0$ is the internal quadrupole moment of the nucleus, and $C_{I_i K_i_{i,n} 2K_i}$ are the Clebsch-Gordon coefficients.

Table 2. Parameters used in IBM-1 to calculate energy of excited states in Yb isotopes.

| $A$ | $a_1$ | $a_2$ | CHI |
|-----|-------|-------|-----|
| 170 | 0.0094 | -0.0120 | -1.30 |
| 172 | 0.0091 | -0.0112 | -1.20 |
| 174 | 0.0084 | -0.0150 | -1.24 |
| 176 | 0.0089 | -0.0126 | -1.30 |

Table 3. Values of parameters obtained from $\hat{T}(E2)$ operator in program IBMT for calculating the absolute values of $B(E2)$.

| $A$ | $\alpha_2$ | $\beta_2$ |
|-----|------------|------------|
| 170 | 0.1060     | -0.140     |
| 172 | 0.1037     | -0.137     |
| 174 | 0.0960     | -0.126     |
| 176 | 0.0980     | -0.129     |

Another advantage of the interacting d- boson model is the matrix elements of the electric quadrupole operator. The reduced matrix elements of the E2 operator $\hat{T}(E2)$ have the form [20-22]

$$\hat{T}(E2) = \alpha_2 \left[ d' s + s' \bar{d} \right]^2 + \beta_2 \left[ d' \bar{d} \right]^2$$

$$= \alpha_2 \left[ d' \bar{s} + s' \bar{d} \right]^2 + \chi \left[ d' \bar{d} \right]^2 = e_B Q, \quad (13)$$

where $\alpha_2$ and $\beta_2$ are two parameters and $\beta_2 = \alpha_2 e_B$, $\alpha_2 = e_B$ (effective charge). The values of these parameters are presented in Table 3. Then the $B(E2)$ values are given by

$$B(E2;J_i \rightarrow J_f) = \frac{1}{2J_i + 1} |\langle J_f || \hat{T}(E2) || J_i \rangle |^2. \quad (14)$$
For the calculations of the absolute $B(E2)$ values the two parameters $\alpha$ and $\beta$ of Eq. (13) are adjusted according to the experimental $B(E2;2^+_\text{gr} \rightarrow 0^+_\text{gr})$. Table 8 shows the values of the parameters $\alpha$ and $\beta$, obtained in the present calculations. We present our calculated results of the reduced probability of $E2$– transitions of both models, and the comparison of calculated values of $B(E2)$ transitions with experimental data [31] are given in Table 9 for all nuclei of interest. In general, most of the calculated results in both models are reasonably consistent with the available experimental data, except for a few cases that deviate from the experimental data. As mentioned in Table 9 PhM calculations are better than those of IBM-1 when compared with the experimental data, except $B(E2;2^+_\text{gr} \rightarrow 0^+_\text{gr})$ for $^{170}$Yb, $^{174}$Yb and $^{176}$Yb, $B(E2;2^+_\text{gr} \rightarrow 0^+_\text{gr})$, $B(E2;2^+_\text{gr} \rightarrow 2^+_\text{gr})$ for $^{172}$Yb and $B(E2;2^+_\text{gr} \rightarrow 0^+_\text{gr})$, $B(E2;2^+_\text{gr} \rightarrow 4^+_\text{gr})$ for $^{174}$Yb.
Table 8. Energy levels of $^{1+}$ band of Yb isotopes (in MeV).

| I   | $^{170}$Yb (Exp. [26, 27] PhM) | $^{172}$Yb (Exp. [26, 28] PhM) | $^{174}$Yb (Exp. [26, 29] PhM) | $^{176}$Yb (Exp. [26, 30] PhM) |
|-----|-------------------------------|-------------------------------|-------------------------------|-------------------------------|
| 1^+ | 1.606                         | 1.605                         | 2.009                         | 2.006                         |
| 2^+ | 1.832                         | 1.662                         | 2.047                         | 2.059                         |
| 3^+ | --                            | 1.746                         | 2.109                         | 2.138                         |
| 4^+ | --                            | 1.856                         | 2.193                         | 2.242                         |
| 5^+ | --                            | 1.993                         | --                            | 2.371                         |
| 6^+ | --                            | 2.153                         | --                            | 2.523                         |
| 7^+ | --                            | 2.337                         | --                            | 2.697                         |
| 8^+ | --                            | 2.544                         | --                            | 2.893                         |
| 9^+ | --                            | 2.771                         | --                            | 3.109                         |
| 10^+| --                            | 3.018                         | --                            | 3.345                         |
| 11^+| --                            | 3.285                         | --                            | 3.599                         |
| 12^+| --                            | 3.569                         | --                            | 3.871                         |
| 13^+| --                            | 3.871                         | --                            | 4.160                         |
| 14^+| --                            | 4.190                         | --                            | 4.466                         |

Table 9. Values of $B(E2)$ transitions of Yb isotopes (in W.u.).

| $I, K_i \rightarrow I, K_f$ | $^{170}$Yb | $^{172}$Yb | $^{174}$Yb | $^{176}$Yb |
|-----------------------------|------------|------------|------------|------------|
|                             | Exp. [31]  | PhM        | IBM-1      | Exp. [31]  | PhM        | IBM-1      | Exp. [31]  | PhM        | IBM-1      | Exp. [31]  | PhM        | IBM-1      |
| $2^{+}_{gr} \rightarrow 0^{+}_{gr}$ | 201(6)     | 216        | 198.543    | 212(2)     | 212        | 211.689    | 212(2)     | 212        | 211.689    | 212(2)     | 212        | 211.689    |
| $4^{+}_{gr} \rightarrow 2^{+}_{gr}$ | --         | 309        | 280.768    | 301(20)    | 303        | 299.697    | 301(20)    | 303        | 299.697    | 301(20)    | 303        | 299.697    |
| $6^{+}_{gr} \rightarrow 4^{+}_{gr}$ | --         | 340        | 303.549    | 320(30)    | 334        | 324.746    | 320(30)    | 334        | 324.746    | 320(30)    | 334        | 324.746    |
| $8^{+}_{gr} \rightarrow 6^{+}_{gr}$ | 360(30)    | 356        | 309.178    | 400(40)    | 350        | 331.835    | 400(40)    | 350        | 331.835    | 400(40)    | 350        | 331.835    |
| $10^{+}_{gr} \rightarrow 8^{+}_{gr}$ | 356(25)    | 366        | 306.15     | 375(23)    | 359        | 329.971    | 375(23)    | 359        | 329.971    | 375(23)    | 359        | 329.971    |
| $12^{+}_{gr} \rightarrow 10^{+}_{gr}$ | 268(21)    | 372        | 296.956    | 430(60)    | 366        | 322.160    | 430(60)    | 366        | 322.160    | 430(60)    | 366        | 322.160    |
| $14^{+}_{gr} \rightarrow 12^{+}_{gr}$ | --         | 377        | 283.181    | 394^{+60}_{-45} | 370    | 309.724    | 394^{+60}_{-45} | 370    | 309.724    | 394^{+60}_{-45} | 370    | 309.724    |
| $16^{+}_{gr} \rightarrow 14^{+}_{gr}$ | --         | 381        | 265.349    | --         | 374        | 293.311    | --         | 374        | 293.311    | --         | 374        | 293.311    |
| $18^{+}_{gr} \rightarrow 16^{+}_{gr}$ | --         | 383        | 243.819    | --         | 376        | 273.310    | --         | 376        | 273.310    | --         | 376        | 273.310    |
| $20^{+}_{gr} \rightarrow 18^{+}_{gr}$ | --         | 386        | 218.751    | --         | 379        | 249.967    | --         | 379        | 249.967    | --         | 379        | 249.967    |

| $I, K_i \rightarrow I, K_f$ | $^{174}$Yb | $^{176}$Yb |
|-----------------------------|------------|------------|
|                             | Exp. [31]  | PhM        | IBM-1      | Exp. [31]  | PhM        | IBM-1      |
| $2^{+}_{gr} \rightarrow 0^{+}_{gr}$ | 201(7)     | 205        | 199.908    | 183(7)     | 184        | 182.916    |
| $4^{+}_{gr} \rightarrow 2^{+}_{gr}$ | 280(9)     | 294        | 283.321    | 270(25)    | 263        | 258.969    |
| $6^{+}_{gr} \rightarrow 4^{+}_{gr}$ | 370(50)    | 324        | 307.532    | 298(22)    | 290        | 280.618    |
| $8^{+}_{gr} \rightarrow 6^{+}_{gr}$ | 388(21)    | 339        | 315.122    | 300(5)     | 303        | 286.743    |
| $10^{+}_{gr} \rightarrow 8^{+}_{gr}$ | 335(22)    | 348        | 314.533    | --         | 312        | 285.139    |
| $12^{+}_{gr} \rightarrow 10^{+}_{gr}$ | 369(23)    | 354        | 308.624    | --         | 317        | 278.384    |
| $14^{+}_{gr} \rightarrow 12^{+}_{gr}$ | 320(8)     | 359        | 298.941    | --         | 321        | 267.636    |
| $16^{+}_{gr} \rightarrow 14^{+}_{gr}$ | --         | 362        | 285.192    | --         | 324        | 253.459    |
| $18^{+}_{gr} \rightarrow 16^{+}_{gr}$ | --         | 365        | 268.659    | --         | 327        | 236.177    |
| $20^{+}_{gr} \rightarrow 18^{+}_{gr}$ | --         | 367        | 249.249    | --         | 328        | 215.995    |
4 Summary

In this paper, energy levels and the reduced probability of E2– transitions positive parity for $^{170-176}$Yb isotopes with neutron numbers between 100 and 106 have been calculated through PhM and IBM-1 models. The predicted low-lying levels ($\gamma_1$, $\beta_1$, and $\gamma_2$ band) by PhM are in good agreement with the experimental data as compared with those by IBM-1 for all nuclei of interest. In addition, the PhM is successful in predicting the $\beta_2$, $\beta_3$, $\beta_4$, $\gamma_2$ and $1^+$– band while IBM-1 fails. Also, the $3^+$– band is predicted by IBM-1 for $^{172}$Yb and $^{174}$Yb nuclei. All calculations are compared with the available experimental data. Also, the reduced probability of E2– transitions of PhM calculations are better than those of IBM-1 when compared with the experimental data, except $B(E2; 2_+^{gr} \rightarrow 0_+^{gr})$ for $^{170}$Yb, $^{174}$Yb and $^{176}$Yb, $B(E2; 6_+^{gr} \rightarrow 4_+^{gr})$ for $^{172}$Yb, $B(E2; 4_+^{gr} \rightarrow 2_+^{gr})$ and $B(E2; 14_+^{gr} \rightarrow 12_+^{gr})$ for $^{174}$Yb and also $B(E2; 12_+^{gr} \rightarrow 10_+^{gr})$ for $^{170}$Yb.

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