The Unexpected Hanging Problem and a Trivial, but Unexpected, Solution

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ABSTRACT

The Unexpected Hanging Problem is also known as the Surprise Examination Problem. We here solve it by isolating what is logical reasoning from the rest of the human psyche. In a not-so-orthodox analysis, following our tradition (The Liar, Dichotomy, The Sorites and Russell's Paradox), we talk about the problem from a perspective that is more distant than all the known perspectives. From an observational point that is in much farther than all the observational points used until now, the reader can finally see why the problem has been perpetuated as a problem and can also see that the problem was never an actual problem: Once more, we have an allurement. The allurement this time makes us start paying attention to all the complexity of the human psyche when studying problems that involve human feelings. The main finding could be told to be that we have to understand and study more the human psyche, in all its intricacies, also when dealing with problems that seem to belong with exclusivity to Mathematics or Logic.
Keywords: Hanging problem; exam problem; psyche; logic; allurement; surprise drill; surprise examination problem.

1. INTRODUCTION

The Surprise Examination Paradox is also known as the Unexpected Hanging Problem [1].

Let us begin by recalling the paradox. It has many variants, the earliest probably being Lennart Ekbom's surprise drill and the best known to mathematicians (thanks to Quine and Gardner) being an unexpected hanging. We shall give the surprise examination version.

A teacher announces in class that an examination will be held on some day during the following week and moreover that the examination will be a surprise. The students argue that a surprise exam cannot occur. For suppose the exam were on the last day of the week. Then on the previous night, the students would be able to predict that the exam would occur on the following day and the exam would not be a surprise. So it is impossible for a surprise exam to occur on the last day. But then a surprise exam cannot occur on the penultimate day, either, for in that case the students, knowing that the last day is an impossible day for a surprise exam, would be able to predict on the night before the exam that the exam would occur on the following day. Similarly, the students argue that a surprise exam cannot occur on any other day of the week either. Confident in this conclusion, they are of course totally surprised when the exam occurs (on Wednesday, say).

The announcement is vindicated after all. Where did the students' reasoning go wrong?

As Holtzman [3] points out, the unexpected hanging and measurement problems both have unsatisfying or puzzling consequences following apparent sound logic.

In all cases, there is an easy-to-see solution that is attained by extrapolating the boundaries of Logic.

2. DEVELOPMENT

2.1 Modelling the Problem

Regardless if we are considering an exam or an assassination, we have to worry about the days of the week and about how things are being presented to us.

Because of that, the table we present on this page might be of use.

In the hanging case [2], the table must look like Table 1.

In the exam case [1], it does not need to have the divisions (morning, noon and postnoon).

2.2 Some Assumptions and Justifications

We notice that being able to predict the outcome of a not-purely scientific event and therefore being prepared to handle it, is a psychological, and not a logical or mathematical, thing.

Surprise is a non-scientific term, since we cannot really define it to the rigors of Science.

If we want to think of this problem as a scientific problem, we have to get rid of terms that involve feelings and we have to specify things to a scientific level.
Table 1. Hanging problem

| Monday  | Tuesday | Wednesday | Thursday | Friday |
|---------|---------|-----------|----------|--------|
| Morning | Morning | Morning   | Morning  | Morning|
| NOON    | NOON    | NOON      | NOON     | NOON   |
| Postnoon| Postnoon| Postnoon  | Postnoon | Postnoon|

One of the things that we need to specify is whether the week we consider is free of holidays.

We also need to tell how long the operation of hanging takes and the precise time at which the warden/person who hangs leaves work in the case of [2].

There are quite a few factors that would have to be included for us to be able to believe that this problem is a scientific problem.

We would like to highlight, in particular, the work of Kahneman and Tversky [4]. Our natural tendency seems to be inferring things that are not scientifically plausible when the events we study/observe involve repetition. In the same way that happened with the gambler, we can have it happening in this problem because the reasoning has to be repeated for each day of the week.

The wealth that the work of Kahneman and Tversky [4] brings to us is in terms of warning more than anything else: We need to be able to detach ourselves from any possible inference based on our sensations/perceptions.

That is just to stress even more the importance of creating a totally scientific definition for surprise if we are to address this problem in Science.

3. DISCUSSION

If we forget all that however and decide to adopt what we shall call machine-alike approach, we will reach the conclusions of [1] when studying the version of the problem there presented.

The conclusion is then that the exam cannot occur on a Friday if the perspective of the logical observer/solver is considered.

See (1):

*This formalization allows at least the first step of the students’ argument to be carried out: Given this announcement, the students can deduce that the examination will not occur on the last day of the week.*

Notice however that if the student/prisoner can calculate that the exam/hanging cannot occur Friday, then it won’t be a surprise for them anymore when it happens on any day of the week (apart from Friday).

Holtzman [3] does notice that: *R is based on the independence of the reasoning of the analyst and the prisoner (...). A somewhat analogous error in Physics would be to ignore the interaction between measuring system and system being measured.*

The truth is that Psychology goes well beyond our logical reasoning.

Of course the students would be holding some expectation that the exam occurred Friday even if they thought that that were not logical.

As Müller [5] points out, [6] want to solve the paradox on the basis that the teacher’s announcement is not a premise of future action, but rather a promise which may or may not be kept.

3.1 New Premises

Confusion appears right here, when this problem is analysed in the scientific literature, because we seem to grossly disregard human nature.

It is interesting to notice that thinking has actually regressed from Holtzman [3] to Shapiro [7]. Whilst Holtzman [3] notices the existence of at least two points of view and acknowledges some *shades of a certain colour* in the problem, let’s say, Shapiro sees things from only one perspective and in black and white.

It is obvious that a person may say whatever they want and may also do whatever they want, independently of rules, laws, agreements and conventions [8].

There is then always a certain amount of expectation of an exam or a hanging happening on the mentioned Friday.
We know, from our own experience as students, that if a teacher tells us we will have only one official exam and it will happen on the 24th of August, we will expect, like we will hold some amount of expectation, that things do not go precisely as they said, we will hold some expectation that they come, all of a sudden, and tell us that today is exam day because, for instance, they felt the need of giving an extra exam.

This problem is then about Psychology, not Mathematics or Logic.

This problem is about our daily lives.

It is an actual truth that if today is Thursday and the exam may happen today or tomorrow, then we hold some amount of expectation that it happen today and some amount of expectation that it happen tomorrow despite whatever that has been declared.

_Surprise_ is a vague term [9], i.e. a term that does not have a precise definition, crystal clear and unique, for at least our logical thinkers.

Müller [5] says: _I will use here only the probabilistic meaning of "surprise" and "unexpected", namely p<1 and for "expected", p=1. With the probability calculation procedure, clear results are obtained. It is better to avoid psychological expectation, because it can vary for many reasons. Among other things, one may be so blah or well-composed or so certain, that nothing surprises (including certainty based on wrong assumptions) or on the contrary so nervous (including assumptions of uncertainty based on mistaken beliefs) that everything does._

This way, it may be that when we talk about that who promises (teacher/warden), we talk about a person who is extremely rational and believes that the other human beings also are; a person who never changes their actions in the middle of the way or who never does the unexpected, a person who never declares one thing and does another or it may be that we talk about a person who is extremely sensitive, for whom human beings behave in an unexpected way in each and every second, as they themselves do.

The first type of subject, which we shall call machine-alike, would take for granted that the exam can only be on a day that be not Friday if put in the position of the hanged/student. The first type of subject would probably inhabit the universe described in [7].

The second type of subject, which we shall call ultrasensitive, if put in the position of hanged/student, will take for granted that the exam can happen on any day of the week and nothing that is said matters.

Because we wish for measuring surprise, _basically inside of us_, it does not make sense to decide that everyone will think in the same way when facing those situations.

We should probably worry about making sure that the logical thinker is surprised, since nobody else would be making calculations, like nobody else would bother.

Müller [5] seems to forget all subtleties of human nature and consider that we can know whether a person took for granted that something will happen (p=1) or not (p<1).

Lots of religious people would enter depression and start praying, so that nothing would be a surprise for them: They may actually _leave this world_ by the time of the prayers in the case of the hanging.

This process could be called catatonia (light degree of catatonia, temporary catatonia and etc.) in Psychiatry [10].

Brasic [10] defines catatonia as _a state of apparent unresponsiveness to external stimuli in a person who is apparently awake._

A person in catatonic state (not mattering how light) would completely ignore the situation, so that there would be no analysis at all going on in their minds and that means that there would be no possible consideration of any value for the variable p created by Müller [5].

Lots of people give up before trying to reason as well, like they assume that it is all bad or something and never think about anything that relates to the word surprise or the problem that has been proposed.

Some of those would take drugs at this point if drugs were accessible. The intentions would then be alienating themselves mentally, what means that they could not have any decision in what regards the proposal.
This process is called escapism [11].

If a machine-alike subject, say a kamikaze or a terrorist, has a situation like the one described in the Hanging Problem to deal with, they should never experience surprise, for they should always be ready to die ([12], p. 30).

Since that is a permanent state of mind, there is no talk about whether there is a surprise or not.

Machine-alike people would be to one end and ultrasensitive people would be to the other end of the spectrum.

The problem is that, as we previously said, everything is context-related in what comes to human beings: Nobody is to one of those ends at all times, not even a kamikaze or a terrorist.

Only during the time of the mission, if so, will a kamikaze or a terrorist have the mindset we here describe.

Usually people will be in the middle of the spectrum because it has to be a truth that people are usually psychiatrically OK.

Notice that only one of the extremes of the spectrum would be completely logical and could allow for us to use Müller’s [5] variable, p, when describing people’s decision processes.

In this case, we must still remember that it is all context-related, so that only a very small group of people would have choices that we can study by means of Müller’s reasoning [5].

His reasoning is then not good enough to solve this problem.

3.2 On the Way to a Conclusion

Thus, contrary to what we see in [5], we do have to work with what Müller calls psychological expectation if we want to solve this problem.

If we stick to our extremely-rational approach and we then assume that we have nobody of the type terrorist or kamikaze inside of the jail cell and we also assume that the people inside of the cell are conscious at all times, so that they will not pass out or something, since, in this case, the hanging would always be a surprise, not mattering when it happens, we can decide to consider the logical solver’s reasoning.

If we also assume that the warden/teacher is machine-alike, then we agree that they make the impossible not to commit a logical mistake.

In principle, there is no reason for the teacher/warden to drop Thursday before noon as a possibility, since it is assumed that the person could be thinking that either Thursday or Friday would be suitable and therefore they would have no certainty on the date, if nothing else.

Notice that for this solution to work we need to assume that both teams of players are machine-alike: The listeners will reason logically and coldly, will be alert no matter what, therefore will not have fainted, done drugs, engaged in catatonic praying or anything like that and the teacher/warden will always make sure they do not commit any logical mistakes and are completely coherent in terms of what they say and what they do, no matter what.

Bear in mind that we also have to assume that the world does not change for this solution to work: No major catastrophes or incidents may happen during the period (yet a war could start and the complex could then be invaded by enemy forces).

We also have to assume that all prisoners and all students being considered bothered about the proposal and the term surprise.

There are also other things that we need to assume: We talk about a normal week, without holidays or special days of any sort, which could disturb the prison routine; if the warden/teacher gets to be replaced during the week, then the person who replaces them is of the same type.

Notice that if we assume that the teacher/warden will always make sure they are coherent in terms of what they do and say and they do not commit any logical mistakes when speaking or doing things, but we also assume that they never analyze the possible logical analysis of the prisoners/students, then they will hang them/give the exam to them on any day that be not Friday. If we assume that they do analyze the possible logical analysis of the prisoners/students however, then we also assume that they will never hang or apply the exam because the only available option would be Friday (completely surprised), but all would be able to predict that by the end of Thursday, and we then will not have a surprise anymore.

Notice also that, in real life, we could always apply techniques to distract their minds (prisoners/students), so that any day would mean surprise (it suffices that they are not thinking about it).

\[1\] Notice that nobody is really like this all the time. We here talk about someone like a terrorist or a kamikaze on a mission of the type terrorist or kamikaze.

\[2\] Notice that if we assume that the teacher/warden will always make sure they are coherent in terms of what they do and say and they do not commit any logical mistakes when speaking or doing things, but we also assume that they never analyze the possible logical analysis of the prisoners/students, then they will hang them/give the exam to them on any day that be not Friday. If we assume that they do analyze the possible logical analysis of the prisoners/students however, then we also assume that they will never hang or apply the exam because the only available option would be Friday (completely surprised), but all would be able to predict that by the end of Thursday, and we then will not have a surprise anymore.

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and believes plus does the same things; nobody from the classroom or from the cell is replaced during this week, but, if they are, then the person to replace those who leave is of the same type; everyone inside of those premises is in condition of thinking normally (well fed, with basic needs attended to, say toilet, hygiene and etc.), along with others.

At this height of things, perhaps our readers are starting to understand why the problem now goes from what could be called real life to pure theory.

All obviously becomes unbearable: Our intentions with this problem could only be making people acquire an interest in studying Logic or Mathematics by making them use real life as a point of connection, but, after we convert it all properly, there is not much of real life left.

3.3 There is a Very Simple Solution in the Machines World (Only)

Suppose that things are the way machines like them and we therefore have a machine against machine situation.

Now, Friday is definitely ruled out.

A question we said that we must answer is What is a surprise for a machine?

Oh, well, says the logical player, we assume that surprise means incapability of predicting.

Assume it is now the end of the business day, Thursday and the exam/hanging has not yet happened: End of the day, nobody working anymore.

We enter that information into the computer.

The computer then thinks: The exam/hanging can only happen on Friday.

The computer has predicted the result, therefore has been capable of predicting and will therefore not be surprised if it happens on Friday.

Suppose that it is now Wednesday instead.

Nothing has happened and we have the same conditions as before (end of the day and etc.).

We enter this information into the computer system.

The computer then thinks: It may happen on Thursday or Friday.

The computer has been unable to predict the event and therefore we have a surprise if the event happens on Thursday.

The same will happen for each and every day of the week apart from Friday.

In the hanging version of the problem, we just have to reword our lines to in the afternoon on Thursday.

3.4 Forward and Backward Reasoning (Müller's Finding)

We are obviously using the forward or normal thinking, not the backward or unusual thinking.

We could, if wishing, consider Müller's [5] lines to support our choice:

[29] Re. (a)- The type of backward reasoning used here is not valid, since it implies the mistaken premise that the Thursday night or Friday morning p=1 for Friday is a truth-in-itself which can be carried backward. On Wednesday night, p=1/2 (and not p=1) for each of Thursday and Friday.

[30] Thinking that on Wednesday night p=1 for both Thursday and Friday implies two steps: (i) "I have already proven on Thursday night that p=1 for Friday" and (ii) "because this is true on Thursday night, it follows that on Wednesday night p=1 for both Thursday and Friday". It implies backward-in-time causation, which in turn means assuming an (impossible) reversal of the flow of experience or "time reversal" (the flow of experience is the origin of the "time" concept). It is like saying that "I will be taking an airplane to Timbuktu on Friday, therefore I can have dinner in the Timbuktu Grand Hotel on Thursday."

[31] Re. (b)- In forward reasoning: Until Monday morning, the probability of the event occurring is 1/5 for each of the coming days, Monday night it is 1/4 and so forth, Thursday night it is 1/1 for Friday. The probability is a mental (subject-and-object) tool for quantifying forward likelihood of the event. This is so even for researching back in time,
for instance "how likely is it that event X happened 10000 years ago?" (e.g. in studies with radiocarbon dating); the answer to the question is ahead, not backward, in the researcher's flow of experience.

[32] There is a similarity between backward reasoning and playing a videotape backward: It can be done, but in case it is understood as a forward display the content may be clearly false. When a broken wine-glass is shown to assemble itself, collect wine from the carpet and to jump onto the table, the deception is evident to most people. You accept or reject or correct what you see according to what you had previously accepted as real: 'You see what you know', rather than the opposite, 'you know what you see', which implies the traditional ontological idea that what you see is real and given to you in pre-fabricated (MIR) form. {c}

[33] Some other examples are less clear, both on videotapes (the movement of a pendulum, the bouncing of a ball) and in logic (backward reasoning of the type discussed here. Or also reasoning from effects to causes, for instance if something goes wrong that the devil, witchcraft or the butler, must have done it). These fuzzy situations are the ones that may cause problems. Furthermore, in the case of "truth" and "backward reasoning" the same word may be used with more than one meaning, which can contribute to confusion of issues. {b}

[34] The time-reversal concept per se also needs discussion. This notion can help to illustrate reversible quantum processes, a method used by Feynman (likely as a purely technical procedure without ontological implications). But a time-reversal assumption is false if it is taken to mean that experience can flow backward and in that case the concept can be counter-productive. (Kant had in his critical period offered a definition of time and space and of causality, as "apriori intuitions", needed for comprehension. But in his opus postumum, he had changed his opinion to a constructivist one and wrote that the only way to arrive at a (conceptual) "system of the world" is to create it (see [53]. - I am indebted to Ernst von Glasersfeld for this and related references.)

[35] Because experience flows only forward, backward reasoning and playing tapes backward cannot be used for proper reasoning or documentation, but perhaps to discuss problems of logic, or for advertising, entertainment, tricks, or (self-) deception. Time is our conceptual tool for handling, including quantifying, the flow of experience; we can make graphic illustrations in which the time dimension points to the past. But the flow of experience-which is the origin of "time"-cannot be an experience-independent (traditional ontological) entity that can go backward as well as forward. And that time and space are not absolute (or primary) was shown by Einstein.

[36] There is no paradox for a subject who evaluates with which forward probability to expect the event to happen. In 0-D, the implied notion of an impossible truth-in-itself which can be carried backward in an equally impossible time-in-itself is abandoned in favor of a view centered on what people can actually do with the word-concepts they trust.

The thing is that we cannot go backwards and forwards at the same time when solving this problem.

If we do so, however, lots of constraints and concerns must be exhibited for the solution not to contain flaws.

When we say Friday is not possible, in our first move, we are obviously reasoning forwards.

Assuming that Friday is not possible and going back to Wednesday, however, is reasoning backwards.

The problem with that is that we got to Friday by going forwards, not backwards.

We assume we were over Wednesday when we got to the conclusion that we should drop Friday and therefore, as Müller [5] says so well, we will end up having a fallacious argument if insisting with this move.
4. CONCLUSION

The most important lesson we take from the scientific analysis of the Surprise Examination Problem or of the Unexpected Hanging Problem is obviously that Science has strict rules of pertinence.

We must worry about the words we use as much as we worry about the situations we consider when deciding on whether a problem is scientific or not.

The words can confuse us and so can the situations, especially if we can visualize and feel as if we are living them, but Extralogic, purest, will keep us seeing everything as we should.

We are calling Extralogic the Logic that includes drawing a line between what is purely scientific and what contains elements that could be called exclusively human. We are also including the scientific methods to deal with what is not purely scientific in this concept.

Situations that involve emotions and vague terms in general should never be included in Pure Science: We would have to use Psychology, Biology and quite a few other elements that usually contain pieces of knowledge and rules that escape Science, perhaps by much, to analyse those situations instead.

Situations in which a human being cannot be replaced with a computer are usually non-purely scientific.

The statistical world is a world in-between most of the time, like a world between the real and the scientific world. It is then adequate to deal with things that are in the borderline, like part to the set of the purely scientific things and part to the supplementary set to Science.

Statistics is then definitely inside of the concept of Extralogic.

In terms of Science, the more we split and classify things, the better.

The first concern would always be with what is machine-friendly (and therefore can be fully dealt with by logical systems and sometimes at least by Mathematics) and what is in the supplementary set to that of what is machine-friendly (and therefore may be dealt with Logic or should not be considered Science/Pure Science).

The Unexpected Hanging Problem/Surprise Examination Paradox has a very simple solution inside of the machines world: The exam/hanging can happen any day of the week apart from Friday.

The Unexpected Hanging Problem/Surprise Examination Paradox is not a paradox or a problem in real life, in what escapes the machines world, because it is not expected that its lines mean anything there.

Basically, surprise is a vague word, since different people will have different understandings: Some think they need to be happy and therefore that surprise is a positive term; some think they need to simply not be able to predict the event and therefore surprise is a term that directly relates to unexpected.

Its definition in the dictionary is [13]

1. shock, start (informal), revelation, jolt, bombshell, eye-opener (informal), bolt from the blue, turn-up for the books (informal) It is perhaps no surprise to see her attempting a comeback.

2. amazement, astonishment, wonder, disbelief, incredulity, stupefaction To my surprise I am in a room where I see one of my mother's sisters.

There is a huge difference between shock and wonder.

The two mentioned senses have nothing to do with expected and unexpected however.

We ourselves have chosen the sense expected for the sake of having more chances of engaging in a purely scientific discussion.

Müller [5] also chose this sense.

A person saying what the warden/teacher says in real life would be giving no information whatsoever because the students would still believe that the exam may occur on any day of the week, as we here explained and the prisoners would still do what normal human beings do in this situation, which is thinking that it is just a joke, another way in which the warden can humiliate or injure them.

Very rarely would a person in real life be in a playful state of mind and decide to reason logically about what has been said.
Even if they did that, they would never believe their conclusions at the rate of one hundred percent, since everything that has to do with humans seems to be unpredictable.

The conclusion is that this problem is an allurement: It exists to show how human nature is far from being something that we can access or control via mechanical gadgets.

The problem exists perhaps as a counter-example to the claim that natural human actions/events can be fully controlled by Science.

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COMPETING INTERESTS

The author has declared that no competing interests exist.

REFERENCES

1. Chow TY. The Surprise Examination or Unexpected Hanging Paradox; 1998. Retrieved April 7 2014 Available: http://www.math.mit.edu/~tchow/unexpected.pdf
2. Pinheiro MR. The Unexpected Hanging Problem; 2013. Retrieved April 7 2014 Available:http://mathematicalcircle.blogspot.com.au/2013/10/the-unexpected-hanging-problem.html
3. Holtzman JM. A Note on Schrödinger’s Cat and the Unexpected Hanging Paradox; 1988. Retrieved August 28 2014 Available: http://www.jstor.org/stable/687217
4. Tversky A, Kahneman D. Belief in the law of small numbers. Psychological Bulletin. 1971;76(2):105-110. DOI: 10.1037/h0031322
5. Müller HFJ. Effect of Working Ontology on Some Conceptual Puzzles; 2003. Retrieved August 28 2014 Available: http://philsci-archive.pitt.edu/1161/1/57-tamul.htm
6. Margalit A, Bar-Hillel M. Expecting the Unexpected. Philosophy. 1983;13:263-288.
7. Shapiro SC. A Procedural Solution to the Unexpected Hanging and Sorites Paradox; 1998. Retrieved August 28 2014. Available: http://www.jstor.org/stable/2659782
8. Pinheiro MR. Concerning the Solution to The Liar Paradox; 2012. Retrieved August 28 2014. Available:http://nb.vse.cz/kfil/elogo/scienc e/pinheiro12b.pdf
Russell B. Vagueness; 1923. Retrieved April 7 2014 Available:http://www.personal.kent.edu/~rmuhamma/Philosophy/RBwritings/vaguene ss.htm
9. Stout CE. The Psychology of Terrorism: Programs and Practices in Response and Prevention. Greenwood Publishing Group. 2002;4.
10. Surprise. (n.d.) Collins Thesaurus of the English Language–Complete and Unabridged 2nd Edition. 1995;2002. Retrieved August 28 2014. Available: http://www.thefreedictionary.com/surprise

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