A Predictive Mathematical Model for Water Absorption of Sawdust Ash - Sand Concrete

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ABSTRACT

Saw Dust Ash (SDA) is an industrial waste that has been used by many researchers in concrete to achieve economic and environmental sustainability. In this study, 5% of sand was replaced with SDA to produce concrete with different mix ratios. Scheffe’s simplex theory was used for five mix ratios in a [5,2] experimental design which resulted in additional ten mix ratios. Additional fifteen mix ratios were generated from the initial fifteen, for verification and testing. Concrete cubes of 150mmX150mmX150mm were formed using the thirty concrete mix ratios generated, and soaked in water for 24hours. The water absorptions of cubes from each mix ratio were determined with the standard procedure. The results of the first fifteen water absorption values were used for the calibration of the model constant coefficients, while those from the second fifteen were used for the model verification using Scheffe’s simplex lattice design. A mathematical regression model was formulated from the results, with which the water absorptions were predicted. The model was then subjected to a two-tailed t-test with 5% significance, which ascertained the model to be adequate and fit with an R2 value of 0.8244. The study also revealed that SDA can replace 5% of sand and promote environmental sustainability without significantly changing the water absorption.

Keywords-- Saw Dust Ash, Scheffe’s Simplex Lattice, Sustainability, Water Absorption of Concrete

I. INTRODUCTION

Water absorption of concrete is the moisture content in the Saturated Surface-Dry (SSD) condition [1]. It is usually considered after 24hours for water with normal temperature, or 5hours for hot water. Water absorption procedure has been described in [2]. However, [2] specifies that the age of the concrete specimen would be 28days days old. Where the age is less than 28days old, the water absorption value will be higher, or lower where the age is more than 28days old. On this note several studies [3]–[7] have been carried out on water absorption of concrete of different kinds. [4] carried out a study by incorporating Fine Bottom Ash (FBA) with Normal Fine (NF) aggregates in cement concrete. They found the water absorption to be between 11.2% and 18.4%. In another study, [5] also investigated the water absorption and found the values to be between 4.1% and 4.7% for conventional concrete, but increased to between 4.5% and 12.1 when between 25% and 75% of coarse aggregate was replaced with Palm Kernel Shell (PKS) for 7, 28, and 56days age of concrete. However, [6] postulate that the water absorption of mortar can be greatly affected by the relative humidity, as samples tested at a 50% relative humidity gave water absorption results that are six time greater than those with 80% relative humidity. Another study carried out by [7] shows that the water absorption of concrete starts decreasing with water-cement ratios greater than 0.55, due to the excess water available for hydration, whereby the hydration products of cement had blocked some pores in the concrete. However, water absorption cannot be used to ascertain the quality of concrete with regards to strength and durability [7], [8]. Nevertheless, water absorption can be used to characterize concrete as it gives an indication of pore structure and surface condition of the concrete. The aim of this work is to predict the water absorption of Saw Dust Ash SDA concrete using the Scheffe’s simplex model.

A. Saw Dust Ash in Concrete

Saw Dust Ash is the pulverised form of an industrial waste of saw mills. It has been used in concrete construction for over 30 years [9]. Studies by various authors have shown that the use of SDA as sand replacement could have both economic and environmental benefits. This is so given the environmental issues associated with unsustainable sand mining [10], [11]. Chowdhury et al. [12] credited cement production to be a major source of environmental degradation as every 600kg of cement produced emits about 400kg of CO2. They therefore replaced 10% of cement with SDA without negatively affecting the chloride permeability and thaw resistance of the concrete, but rather increased the water absorption, and decreased the drying shrinkage. Similarly, [13] replaced 10% of fine aggregate with SDA, which resulted in acceptable tensile, flexural, and compressive strengths as well as reduce the amount of wastes in the environment. In [13] it was found that SDA has a specific gravity of 2.5, water absorption of 0.56%, fineness modulus of 1.78, and bulk dry density of 1300kg/m3 as against sand with specific gravity of 2.65, water absorption of 0.45%, fineness modulus of 2.21, and bulk dry density of 1512 kg/m3. After the 10% of sand with SDA, these properties became 2.67, 0.5%, 2.2, and 1436kg/m3 for specific gravity, water absorption, fineness modulus, and bulk dry density respectively. This is a significant indication

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that the mixture of sand and SDA did not significantly change the physical properties of sand, making the mixture adequate for a fine aggregate.

The chemical compositions of SDA as found by [13], [14] all indicate that SDA has a high percentage of SiO$_2$ and small percentages of Al$_2$O$_3$ and Fe$_2$O$_3$, which are similar to those of sand with high percentage of about 95% SiO$_2$. Hence SDA can be used with sand as fine aggregate.

**B. Scheffe’s Simplex Theory**

Several authors [15]–[28] have carried out concrete mixture researches with development of mathematical models, most of which were based on Scheffe’s Simplex theory.

Scheffe’s model is based on the simplex lattice and simplex theory or approach [29]. The simplex approach considers a number of components, $q$, and a degree of polynomial, $m$. The sum of all the $i$th components is not greater than 1. Hence,

$$\sum_{q=1}^{q} x_i = 1$$

(1)

$$x_1 + x_2 + \ldots + x_q = 1$$

(2)

with $0 \leq x_i \leq 1$, the factor space becomes $S_{q-1}$.

According to [29] the $\{q,m\}$ simplex lattice design is a symmetrical arrangement of points within the experimental region in a suitable polynomial equation representing the response surface in the simplex region.

The number of points $C_{m}^{(q+m-1)}$ has $(m+1)$ equally spaced values of $x_i = 0, 1/m, 2/m, \ldots, m/m$. For a 3-component mixture with degree of polynomial 2, the corresponding number of points will be $C_{2}^{(3+2-1)}$ which gives 6 (eq. 3 or eq. 4 below) with number of spaced values, 2+1 = 3, that is $x_i = 0, 1/2, 1$ and design points of (1,0,0), (0,1,0), (0,0,1), (1/2,1/2,0), (1/2,0,1/2), and (0,1/2,1/2). Similarly, for a $\{5,2\}$ simplex, there will be 15 points with $x_i = 0, 1/2, 1$ as spaced values.

The 15 design points are (1,0,0,0,0), (1,0,0,0,0), (1,0,0,0,0), (0,0,0,1,0), (0,0,0,1,0), (1,2,1,2,0,0), (1,2,0,1,2,0), (1,2,0,0,1,2), (0,1,2,1,2,0), (0,1,2,0,1,2), (0,0,1,2,1,2), (0,0,1,2,0,1), (0,1,2,0,1,2), (0,1,2,0,1,2).

$$N = C_{q}^{(q+m-1)}$$

(3)

$$N = C_{n}^{(q+n-1)}$$

(4)

For a polynomial of degree $m$ with $q$ component variables where eq. (2) holds, the general form is:

$$Y = b_0 + \sum b_{ij} x_i x_j + \sum b_{ijk} x_i x_j x_k + \ldots + \sum b_{ijkl} x_i x_j x_k x_l$$

(5)

Where $1 \leq i \leq q$, $1 \leq j \leq q$, $1 \leq i \leq j \leq k \leq q$, and $b_{ij}$ is the constant coefficient.

$x$ is the pseudo component for constituents $i, j,$ and $k$.

When $\{q,m\} = \{5,2\}$, eq. (5) becomes:

$$Y = b_0 + b_{11} x_1 + b_{12} x_1 x_2 + b_{13} x_1 x_3 + b_{14} x_1 x_4 + b_{15} x_1 x_5 + b_{22} x_2 x_2 + b_{23} x_2 x_3 + b_{24} x_2 x_4 + b_{25} x_2 x_5 + b_{33} x_3 x_3 + b_{34} x_3 x_4 + b_{35} x_3 x_5 + b_{44} x_4 x_4 + b_{45} x_4 x_5 + b_{55} x_5 x_5$$

(6)

and eq. (2) becomes:

$$x_1 + x_2 + x_3 + x_4 + x_5 = 1$$

(7)

Multiplying eq. (7) by $b_0$ gives

$$b_0 x_1 + b_0 x_2 + b_0 x_3 + b_0 x_4 + b_0 x_5 = b_0$$

(8)

Multiplying eq. (7) successively by $x_1, x_2, x_3, x_4,$ and $x_5$ and making $x_1, x_2, x_3, x_4,$ and $x_5$ the subjects of the respective formulas:

$$x_2^2 = x_2 - x_1 x_2 - x_1 x_3 - x_1 x_4 - x_1 x_5$$

(9)

$$x_3^2 = x_3 - x_1 x_3 - x_2 x_3 - x_3 x_4 - x_3 x_5$$

$$x_4^2 = x_4 - x_1 x_4 - x_2 x_4 - x_3 x_4 - x_4 x_5$$

$$x_5^2 = x_5 - x_1 x_5 - x_2 x_5 - x_3 x_5 - x_4 x_5$$

(9)

Substituting eq. (8) and eq. (9) into eq. (6) we have:

$$Y = b_0 x_1 + b_0 x_2 + b_0 x_3 + b_0 x_4 + b_0 x_5 + b_1 x_1 + b_2 x_2 + b_3 x_3 + b_4 x_4 + b_5 x_5 + b_1 x_1 x_2 + b_1 x_1 x_3 + b_1 x_1 x_4 + b_1 x_1 x_5 + b_2 x_2 x_2 + b_2 x_2 x_3 + b_2 x_2 x_4 + b_2 x_2 x_5 + b_3 x_3 x_3 + b_3 x_3 x_4 + b_3 x_3 x_5 + b_4 x_4 x_4 + b_4 x_4 x_5 + b_5 x_5 x_5 + b_{11} x_1 - x_1 x_2 - x_1 x_3 - x_1 x_4 - x_1 x_5 + b_{12} x_2 - x_1 x_2 - x_2 x_3 - x_2 x_4 - x_2 x_5 + b_{13} x_3 - x_1 x_3 - x_2 x_3 - x_3 x_4 - x_3 x_5 + b_{14} x_4 - x_1 x_4 - x_2 x_4 - x_3 x_4 - x_4 x_5 + b_{15} x_5 - x_1 x_5 - x_2 x_5 - x_3 x_5 - x_4 x_5 - x_5 x_5$$

(10)

Let

$$b_1 = b_0 + b_1 + b_{11}$$

$$b_2 = b_0 + b_2 + b_{22}$$

$$b_3 = b_0 + b_3 + b_{33}$$

$$b_4 = b_0 + b_4 + b_{44}$$

$$b_5 = b_0 + b_5 + b_{55}$$

$$b_{12} = b_{12} - b_{11} - b_{22}$$

$$b_{13} = b_{13} - b_{11} - b_{33}$$

$$b_{14} = b_{14} - b_{11} - b_{44}$$

$$b_{15} = b_{15} - b_{11} - b_{55}$$

$$b_{23} = b_{23} - b_{22} - b_{33}$$

$$b_{24} = b_{24} - b_{22} - b_{44}$$

$$b_{25} = b_{25} - b_{22} - b_{55}$$

$$b_{34} = b_{34} - b_{33} - b_{44}$$

$$b_{35} = b_{35} - b_{33} - b_{55}$$

$$b_{45} = b_{45} - b_{44} - b_{55}$$

(11)

Substituting eq. (11) into eq. (10) gives

$$Y = \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_3 + \beta_4 x_4 + \beta_5 x_5 + \beta_{11} x_1^2 + \beta_{12} x_1 x_2 + \beta_{13} x_1 x_3 + \beta_{14} x_1 x_4 + \beta_{15} x_1 x_5 + \beta_{22} x_2^2 + \beta_{23} x_2 x_3 + \beta_{24} x_2 x_4 + \beta_{25} x_2 x_5 + \beta_{33} x_3^2 + \beta_{34} x_3 x_4 + \beta_{35} x_3 x_5 + \beta_{44} x_4^2 + \beta_{55} x_5^2$$

(12)

This can be rewritten as:

$$Y = \sum_{i=1}^{5} \beta_i x_i + \sum_{1 \leq i < j \leq 5} \beta_{ij} x_i x_j$$

(13)

Where the response, $Y$ is a dependent variable (Water Absorption of concrete). Eq. (12) is the general
equation for a \( [5,2] \) polynomial, and it has 15 terms, which conforms to Scheffe’s theory in eq. (3).

Let \( Y_i \) denote response to pure components, and \( Y_{ij} \) denote response to mixture components in \( i \) and \( j \). If \( x_i = 1 \) and \( x_j = 0 \), since \( j \neq i \), then

\[
Y_i = \beta_i
\]

Which means

\[
\sum_{i=1}^{5} \beta_i x_i = \sum_{i=1}^{5} Y_i x_i
\]

Hence, from eq. (14)

\[
\begin{align*}
Y_1 & = \beta_1 \\
Y_2 & = \beta_2 \\
Y_3 & = \beta_3 \\
Y_4 & = \beta_4 \\
Y_5 & = \beta_5
\end{align*}
\]

According to [29],

\[
\beta_{ij} = 4Y_{ij} - 2\beta_i - 2\beta_j
\]

Substituting eq. (14)

\[
\beta_{ij} = 4Y_{ij} - 2Y_i - 2Y_j
\]

II. MATERIALS AND METHODS

Water, cement, sand, SDA, and granite were the materials used to produce the concrete. This means there are five components in the concrete mix. SDA was used to partially replace 5% of the fine aggregate (sand).

The first five concrete mix ratios derived from different mix design methods [26], [27] are given as:

- BRE 12 = [0.54 1 1.9475 0.1025 2.95];
- BRE 22 = [0.58 1 2.1185 0.1115 3.21];
- USBR 22 = [0.58 1 2.2515 0.1185 3.29];
- BIS 12 = [0.43 1 1.2065 0.0635 2.88];
- ACI 12 = [0.55 1 1.8335 0.0965 3.09]

These can be put in matrix form as follows:

\[
S = \begin{bmatrix}
0.54 & 0.58 & 0.58 & 0.43 & 0.55 \\
1 & 1 & 1 & 1 & 1 \\
1.9475 & 2.1185 & 2.2515 & 1.2065 & 1.8335 \\
0.1025 & 0.1115 & 0.1185 & 0.0635 & 0.0965 \\
2.95 & 3.21 & 3.29 & 2.88 & 3.09
\end{bmatrix}
\]

Their corresponding pseudo components are given as:

\[
X = \begin{bmatrix}
1 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 1
\end{bmatrix}
\]

with centre points

\[
X_{12} = [0.5 0.5 0 0.0]; \ X_{13} = [0.5 0.5 0 0.0];
\]

\[
X_{23} = [0.5 0.5 0 0.0]; \ X_{24} = [0.5 0.5 0 0.0];
\]

\[
X_{34} = [0.5 0.5 0 0.0]; \ X_{35} = [0.5 0.5 0 0.0];
\]

\[
X_{45} = [0.5 0.5 0 0.0]; \ X_{15} = [0.5 0.5 0 0.0];
\]

According to [29],

\[
S_{ij} = XS_i
\]

Substituting,

\[
\begin{bmatrix}
S_{12} \\
S_{13} \\
S_{14} \\
S_{15} \\
S_{23} \\
S_{24} \\
S_{25} \\
S_{34} \\
S_{35} \\
S_{45}
\end{bmatrix} = \begin{bmatrix}
0.5 & 0.5 & 0 & 0 & 0 \\
0.5 & 0 & 0.5 & 0 & 0 \\
0.5 & 0 & 0 & 0.5 & 0 \\
0.5 & 0 & 0 & 0 & 0.5 \\
0.5 & 0.5 & 0 & 0 & 0 \\
0.5 & 0.5 & 0 & 0 & 0 \\
0.5 & 0 & 0 & 0.5 & 0 \\
0.5 & 0 & 0 & 0 & 0.5 \\
0.5 & 0 & 0 & 0 & 0.5 \\
0.5 & 0 & 0 & 0 & 0.5
\end{bmatrix}
\]

This process is repeated for \( S_{24} \), \( S_{25} \), \( S_{34} \), \( S_{35} \), and \( S_{45} \). Similarly, this process is repeated for an additional 15 (control) points that will be used for the verification of the formulated model. The regular pentagons for the actual components with their corresponding pseudo components are given in Figures 1 and 2 respectively. Tables 1 and 2 mix ratio data generated for the main and verification purposes respectively.

![Simplex Plot for Actual Components](image1.png)

![Simplex Plot for Pseudo Components](image2.png)

Table 1: Model Mix Ratios

| Sample Points | w-c ratio | Actual Components | Pseudo Components |
|---------------|-----------|--------------------|------------------|
| BRE12         | 0.54      | 1.9475 0.1025 2.95 | Y_1 = 0 0 0 0 0 0 |
| BRE22         | 0.58      | 2.1185 0.1115 3.21 | Y_2 = 0 1 0 0 0 0 |
After 24 hours, and when cured in water for 24 hours and weighed again. The concrete cubes were removed from the thirty mix ratios in 150mmX150mmX150mm moulds for each of the two replicate concrete cubes made for each of the samples.

### Table 2: Control Points

| Sample Points | Actual Components | Pseudo Components |
|---------------|-------------------|-------------------|
|               | $w$-c ratio $S_1$ | $S_2$ | $S_3$ | $S_4$ | $S_5$ | $Y_{exp}$ | $w$-c ratio $X_1$ | $X_2$ | $X_3$ | $X_4$ | $X_5$ |
| USBR22        | 0.58             | 1     | 2.2515 | 0.1185 | 3.29 | $Y_3$ | 0       | 0     | 1     | 0     | 0     |
| BIS12         | 0.43             | 1     | 1.2065 | 0.0635 | 2.88 | $Y_4$ | 0       | 0     | 0     | 1     | 0     |
| ACI12         | 0.55             | 1     | 1.8335 | 0.0965 | 3.09 | $Y_5$ | 0       | 0     | 0     | 0     | 1     |
| N1            | 0.56             | 1     | 2.033  | 0.107  | 3.08 | $Y_{12}$ | 0.5     | 0     | 0     | 0     | 0     |
| N2            | 0.56             | 1     | 2.0995 | 0.1105 | 3.12 | $Y_{13}$ | 0.5     | 0     | 0.5   | 0     | 0     |
| N3            | 0.485            | 1     | 1.577  | 0.083  | 2.915 | $Y_{14}$ | 0.5     | 0     | 0     | 0.5   | 0     |
| N4            | 0.545            | 1     | 1.8905 | 0.0995 | 3.02 | $Y_{15}$ | 0.5     | 0     | 0     | 0     | 0.5   |
| N5            | 0.58             | 1     | 2.185  | 0.115  | 3.25 | $Y_{23}$ | 0.5     | 0.5   | 0     | 0     | 0     |
| N6            | 0.505            | 1     | 1.6625 | 0.0875 | 3.045 | $Y_{24}$ | 0       | 0.5   | 0     | 0     | 0.5   |
| N7            | 0.565            | 1     | 1.976  | 0.104  | 3.15 | $Y_{25}$ | 0       | 0     | 0.5   | 0     | 0.5   |
| N8            | 0.505            | 1     | 1.729  | 0.091  | 3.085 | $Y_{24}$ | 0       | 0     | 0.5   | 0.5   | 0     |
| N9            | 0.565            | 1     | 2.0425 | 0.1075 | 3.19 | $Y_{35}$ | 0       | 0     | 0.5   | 0     | 0.5   |
| N10           | 0.49             | 1     | 1.52   | 0.08   | 2.985 | $Y_{45}$ | 0       | 0     | 0     | 0.5   | 0.5   |

### A. Water Absorption of Concrete

The procedure used was that described in [2]. Two replicate concrete cubes were made for each of the thirty mix ratios in 150mmX150mmX150mm moulds and allowed to harden. The concrete cubes were removed from the moulds after 24 hours and weighed. They were later cured in water for 24 hours and weighed again. The water absorption was calculated for each mix ratio with eq. (23) and recorded in Table 3 with the averages determined.

**Water Absorption,$W_a = \frac{W_s - W_d}{W_s}$ (23)**

where, $W_s$ is the weight of cube soaked in water after 24 hours, and $W_d$ is the weight of cube at the SSD state. The water absorption, $W_a$, is in percentage.

### III. RESULTS AND DISCUSSIONS

Sieve analysis was carried out on the fine aggregate mixed with 5% SDA as a preliminary investigation. The particle size distribution of the 5% replacement of sand with SDA is shown in Figure 3, and the fineness modulus calculated below.

**Fineness modulus,**

\[
FM = \frac{0.73 + 4.24 + 14.08 + 43.61 + 80.48 + 97.88}{100} = 2.41
\]
The results of the water absorption are shown in Table 3 below.

Table 3 Water Absorption of Concrete

| Sample Annotation | Water Absorption | AVERAGE W_a (%) |     |     |     |     |
|-------------------|------------------|-----------------|-----|-----|-----|-----|
|                   | Dry mass, W_d (kg) | Soaked mass, W_s (kg) | Absorption, W_a (%) | Dry mass, W_d (kg) | Soaked mass, W_s (kg) | Absorption, W_a (%) |     |     |     |     |
| BRE12             | 7.2              | 7.75             | 7.639 | 7.21 | 7.7      | 6.796 | 7.218 |
| BRE22             | 7.4              | 8                | 8.108 | 7.41 | 8        | 7.962 | 8.035 |
| USBR22            | 7.9              | 8.2              | 3.797 | 7.9  | 8.19     | 3.671 | 3.734 |
| BIS12             | 7.6              | 8                | 5.263 | 7.5  | 8        | 6.667 | 5.965 |
| ACI12             | 7                | 7.45             | 6.429 | 7.05 | 7.5      | 6.383 | 6.406 |
| N1                | 7.1              | 7.71             | 8.592 | 7.14 | 7.15     | 8.053 | 8.322 |
| N2                | 7.2              | 7.8              | 8.333 | 7.1  | 7.6      | 7.042 | 7.688 |
| N3                | 7.3              | 7.8              | 6.849 | 7.3  | 7.95     | 6.781 | 6.815 |
| N4                | 7.25             | 7.8              | 7.586 | 7.25 | 7.79     | 7.448 | 7.517 |
| N5                | 7.3              | 7.79             | 6.712 | 7.31 | 7.78     | 6.430 | 6.571 |
| N6                | 7.4              | 7.82             | 5.676 | 7.41 | 7.81     | 5.398 | 5.537 |
| N7                | 7.45             | 7.79             | 4.564 | 7.4  | 7.8      | 5.405 | 4.985 |
| N8                | 7.12             | 7.8              | 9.551 | 7.2  | 7.82     | 8.611 | 9.081 |
| N9                | 7.22             | 8                | 10.803 | 7.2  | 7.8      | 8.333 | 9.568 |
| N10               | 7.4              | 8                | 8.108 | 7.3  | 7.75     | 6.164 | 7.136 |
| C1                | 7.3              | 7.92             | 8.493 | 7.31 | 7.9      | 8.071 | 8.282 |
| C2                | 7.7              | 8.09             | 5.054 | 8.02 | 8.45     | 5.324 | 5.189 |
| C3                | 7.1              | 7.88             | 10.986 | 7.5  | 7.82     | 4.267 | 7.626 |
| C4                | 7.95             | 8.48             | 6.714 | 7.3  | 7.66     | 4.873 | 5.794 |
| C5                | 7.32             | 7.86             | 7.406 | 7.32 | 7.86     | 7.404 | 7.405 |
| C6                | 7                | 7.41             | 5.857 | 7.05 | 7.55     | 7.092 | 6.475 |
| C7                | 7                | 7.59             | 8.416 | 7    | 7.53     | 7.570 | 7.993 |
| C8                | 7.05             | 7.49             | 6.191 | 7.1  | 7.51     | 5.746 | 5.969 |
| C9                | 7.29             | 7.81             | 7.133 | 7.28 | 7.8      | 7.143 | 7.138 |
| C10               | 7.1              | 7.76             | 9.282 | 7.2  | 7.74     | 7.472 | 8.377 |
| C11               | 7.02             | 7.68             | 9.402 | 7.1  | 7.7      | 8.451 | 8.926 |
| C12               | 7.2              | 7.74             | 7.500 | 7.2  | 7.75     | 7.639 | 7.569 |
| C13               | 7.5              | 8                | 6.667 | 7.52 | 8        | 6.383 | 6.525 |
| C14               | 7.56             | 7.96             | 5.291 | 7.57 | 8.01     | 5.812 | 5.552 |

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A. Model Formulation

The coefficients of polynomial from Table 3, eq. (16), and eq. (18) are:

\[ \beta_1 = 7.218, \beta_2 = 8.035, \beta_3 = 3.734, \beta_4 = 5.965, \beta_5 = 6.406, \]
\[ \beta_{12} = 7.218 - 8.035 = 2 * 7.218 - 7.218 = 7.218 \]

Similarly, eq. (18) shows the comparison between the predicted and experimented values of water absorption using the control (verification) data.

Substituting the above coefficients into eq. (12) gives:

\[ Y = 7.218x_1 + 8.035x_2 + 3.734x_3 + 5.965x_4 + 6.406x_5 + 7.993x_1x_2 + 8.484x_1x_3 + 0.894x_4x_5 + 2.82x_1x_5 + 2.746x_2x_3 - 5.852x_2x_4 - 8.942x_2x_5 + 16.926x_3x_4 + 17.992x_3x_5 + 3.802x_4x_5 \]  (24)

Eq. (24) is the mathematical model to predict the water absorption of concrete using SDA to replace 5% of fine aggregate. Table 4 shows the predictions, while Fig. 4 shows the comparison between the predicted and experimented values of water absorption using the control (verification) data.

| Sample Points | Response Y | w-c ratio | Cement | Sand | SDA | Granite | Water Absorption | Water Absorption |
|---------------|------------|-----------|--------|------|-----|---------|-----------------|-----------------|
| Sample A | X_1 | X_2 | X_3 | X_4 | X_5 | Y_{exp} | Y_{pred} |
| BRE12 | Y_1 | 1 | 0 | 0 | 0 | 0 | 7.218 | 7.218 |
| BRE22 | Y_2 | 0 | 1 | 0 | 0 | 0 | 8.035 | 8.035 |
| USBR22 | Y_3 | 0 | 0 | 1 | 0 | 0 | 3.734 | 3.734 |
| BIS12 | Y_4 | 0 | 0 | 0 | 1 | 0 | 5.965 | 5.965 |
| ACI12 | Y_5 | 0 | 0 | 0 | 0 | 1 | 6.406 | 6.406 |
| N1 | Y_{12} | 0.5 | 0.5 | 0 | 0 | 0 | 8.322 | 8.322 |
| N2 | Y_{13} | 0.5 | 0 | 0.5 | 0 | 0 | 7.688 | 7.688 |
| N3 | Y_{14} | 0.5 | 0 | 0 | 0.5 | 0 | 6.815 | 6.815 |
| N4 | Y_{15} | 0.5 | 0 | 0 | 0 | 0.5 | 7.541 | 7.517 |
| N5 | Y_{23} | 0 | 0.5 | 0.5 | 0 | 0 | 6.571 | 6.571 |
| N6 | Y_{24} | 0 | 0.5 | 0 | 0.5 | 0 | 5.537 | 5.537 |
| N7 | Y_{25} | 0 | 0.5 | 0 | 0 | 0.5 | 4.985 | 4.985 |
| N8 | Y_{34} | 0 | 0 | 0.5 | 0.5 | 0 | 9.081 | 9.081 |
| N9 | Y_{35} | 0 | 0 | 0.5 | 0 | 0.5 | 9.568 | 9.568 |
| N10 | Y_{45} | 0 | 0 | 0 | 0.5 | 0.5 | 7.136 | 7.136 |
| C1 | Y_{C1} | 0.4 | 0.4 | 0.4 | 0 | 0.2 | 8.743 | 8.743 |
| C2 | Y_{C2} | 0 | 0.6 | 0.4 | 0 | 0 | 5.803 | 5.803 |
| C3 | Y_{C3} | 0.8 | 0 | 0.2 | 0 | 0 | 7.937 | 7.937 |
| C4 | Y_{C4} | 0 | 0.4 | 0.6 | 0 | 0 | 5.389 | 5.389 |
| C5 | Y_{C5} | 0.6 | 0 | 0 | 0.4 | 0 | 7.570 | 7.570 |
| C6 | Y_{C6} | 0 | 0 | 0.8 | 0.2 | 0 | 6.888 | 6.888 |
| C7 | Y_{C7} | 0.6 | 0.2 | 0 | 0 | 0.2 | 7.534 | 7.534 |
| C8 | Y_{C8} | 0 | 0.4 | 0 | 0.4 | 0 | 5.969 | 5.969 |
| C9 | Y_{C9} | 0.2 | 0 | 0 | 0.8 | 0 | 7.020 | 7.020 |
| C10 | Y_{C10} | 0 | 0 | 0.2 | 0.8 | 0 | 8.227 | 8.227 |
A. \textit{Test of Adequacy of the Model}

A two-tailed student t-test was carried out at 95\% confidence level, which implies $100 - 95 = 5\%$ significance. Since it is a two-tailed, significance $= 5/2 = 2.5\%$

Hence significance level $= 100 - 2.5 = 97.5\%$

Let D be difference between the experimental and predicted responses

The mean of the difference,

$$D_a = \frac{1}{n} \sum_{i=1}^{n} D_i$$ \hspace{2cm} (25)

The variance of the difference,

$$S^2 = \left(\frac{1}{n-1}\right) \sum_{i=1}^{n} (D - D_a)^2$$ \hspace{2cm} (26)

$$t_{\text{calculated}} = \frac{D_a \sqrt{n}}{S}$$ \hspace{2cm} (27)

Where $n =$ number of observations with degree of freedom $n - 1$. Table 5 shows the details of the t-test results.

### Table 5: Student t-test for water absorption of Concrete

| Sample | $Y_{\text{experimental}}$ | $Y_{\text{predicted}}$ | $D = Y_{\text{exp}} - Y_{\text{pred}}$ | $D_a - D$ | $(D-D_a)^2$ |
|--------|---------------------------|-------------------------|----------------------------------------|-----------|-------------|
| C1     | 8.282                     | 8.743                   | -0.461                                 | 0.344     | 0.118       |
| C2     | 5.189                     | 5.803                   | -0.614                                 | 0.497     | 0.247       |
| C3     | 7.626                     | 7.937                   | -0.311                                 | 0.194     | 0.037       |
| C4     | 5.794                     | 5.389                   | 0.405                                  | -0.522    | 0.273       |
| C5     | 7.405                     | 7.570                   | -0.165                                 | 0.048     | 0.002       |
| C6     | 6.475                     | 6.888                   | -0.413                                 | 0.296     | 0.087       |
| C7     | 7.993                     | 7.534                   | 0.459                                  | -0.576    | 0.332       |
| C8     | 5.969                     | 5.534                   | 0.435                                  | -0.552    | 0.305       |
| C9     | 7.138                     | 7.020                   | 0.118                                  | -0.235    | 0.055       |
| C10    | 8.377                     | 8.227                   | 0.150                                  | -0.267    | 0.072       |
| C11    | 8.926                     | 8.299                   | 0.627                                  | -0.744    | 0.554       |
| C12    | 7.569                     | 8.148                   | -0.579                                 | 0.462     | 0.213       |
| C13    | 6.525                     | 7.614                   | -1.089                                 | 0.972     | 0.944       |
| C14    | 5.552                     | 5.793                   | -0.241                                 | 0.124     | 0.015       |

Figure 4: Comparison between Experimental and Predicted Water Absorption
After successfully replacing fine aggregate with 5% SDA in a concrete blend, the water absorptions determined from the laboratory were between 3.734% and 9.568% which are less than the maximum acceptable values. The concrete was batched from five different mix ratios. A multiple regression model was generated from the resulting water absorption experimental values, using Scheffe’s simplex theory for a (5,2) simplex lattice. A two-tailed student t-test was carried out at 5% significance level, which confirmed the adequacy of the derived model with an excellent fit, given an R² value of 0.8244. The results also confirmed that SDA is a suitable material to replace a small fraction (about 5%) of fine aggregate in a bid to promote environmental sustainability.

### IV. CONCLUSIONS

The R² value of 0.8244 in Figure 5 indicates that the experimental results are highly correlated to the predicted results. This is also an indication that the model is fit and adequate.

![Figure 5: Scatterplot of Predicted vs. Experimental Water Absorption](image)

The experimental water absorption results also confirmed that SDA is a suitable material to replace a small fraction (about 5%) of fine aggregate. A multiple regression model was generated from the experimental water absorption values. The concrete was batched from five different mix ratios. A multiple regression model was generated from the experimental water absorption values, using Scheffe’s simplex theory for a (5,2) simplex lattice. A two-tailed student t-test was carried out at 5% significance level, which confirmed the adequacy of the derived model with an excellent fit, given an R² value of 0.8244. The results also confirmed that SDA is a suitable material to replace a small fraction (about 5%) of fine aggregate in a bid to promote environmental sustainability.

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From the t-table, with ν = 15 – 1 = 14, and β = significance level. t(0.975,14) = 2.145

Since t_{calculated} < t(0.975,14) and lies between 2.145 and 2.145, therefore there is no significant difference between the experimental and predicted responses, H₀ is accepted, and H₁ is rejected. The model is confirmed to be adequate.

\[
S^2 = \frac{\sum(D^2)}{n-2} = \frac{3.257}{5} = 0.6554
\]

\[
S = \sqrt{S^2} = \sqrt{0.6554} = 0.808
\]

\[
t_{calculated} = \frac{\bar{Y} - Y}{S/\sqrt{n}} = \frac{-0.117\sqrt{15}}{0.482} = -0.943
\]

\[
F = \frac{(n-2)S^2}{(n-k-1)S^2_{reg}} = \frac{14 \times 0.6554}{13 \times 0.001} = 12.00
\]

\[
R^2 = 0.8244
\]

\[
y = 0.8845x + 0.9387
\]

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