LETTER

Broadband diffraction of correlated photons from crystal superlattices

Zi S D Toa*, Anna V Paterova and Leonid A Krivitsky

Institute of Materials Research and Engineering (IMRE), Agency for Science, Technology and Research (A’STAR), Singapore 138634, Singapore

* Author to whom any correspondence should be addressed.
E-mail: desmond_toa@imre.a-star.edu.sg

Keywords: photons, superlattices, down-conversion, entangled photons, correlation

Supplementary material for this article is available online

Abstract

Sources of broadband quantum correlated photons present a valuable resource for quantum metrology, sensing, and communication. Here, we report the generation of spectrally broadband correlated photons from frequency nondegenerate spontaneous parametric down-conversion in a custom-designed lithium niobate superlattice. The superlattice induces a nonlinear interference between the pump, signal and idler, resulting in an experimentally observed comb-like emission spanning 0.060 μm and 1.4 μm of spectral bandwidth at 0.647 μm and 3.0 μm wavelengths, respectively. While this broad mid-infrared bandwidth is attractive to quantum metrology and sensing due to the enablement of fast spectral multiplexing for data acquisition, the comb-like structure, achieved without an input frequency comb, offers targeted frequencies for quantum communication applications. In addition to useful technological applications, our concept offers an interesting analogy between optical diffraction in quantum and classical optics.

In classical optics, diffraction refers to phenomena in which a wave encounters an obstacle or aperture, which, in turn, effectively serves as a secondary source of waves [1]. The amplitudes of these secondary waves subsequently interfere. In the case of light waves incident on an array of circular apertures, this results in a characteristic diffraction pattern comprising concentric rings of bright and dark regions that can be observed on the screen.

Here, we consider an analogous phenomenon in quantum optics, in which a coherent laser interacts with an array of nonlinear crystals, referred here as a superlattice. Each superlattice element analogously serves as a secondary source of waves, producing correlated photons via spontaneous parametric down-conversion (SPDC) [2–4]. The wavefunctions of SPDC photons generated in the different elements interfere to produce an interference pattern in wavelength–angular (λ–k) coordinates [5]. Furthermore, similar to conventional diffraction, interference fringe narrowing is observed with an increasing number of nonlinear elements. Thus, one can consider that the pump laser undergoes diffraction (in frequency coordinates) upon encountering the superlattice to give visible signal and mid-infrared (MIR) photons at distinct frequencies.

To demonstrate this effect, we custom-designed and fabricated a bi-periodically poled lithium niobate (LiNbO3) (bPPLN) superlattice. The periodically poled regions, where the SPDC occurs, are separated by similarly poled regions of the LiNbO3 crystal, see figure 1. Thus, two periods, one for domains within stacks and the other for gaps, are operative here. Note that previous realizations of the superlattices had intervening linear media, which resulted in angular modulation of the λ–k spectrum [2]. In contrast, the nonlinear interference in our bPPLN superlattice results in frequency modulated SPDC emission, which is highly relevant to applications in quantum key distribution, quantum metrology, and sensing. For example, utilizing spectral multiplexing, sensors can benefit from an increased measurement speed, improving their competitiveness with existing classical techniques. We also note here that bPPLNs have been used earlier to generate polarization entangled states [6, 7] and for optical wavelength conversion [8]. Note that, in

© 2021 The Author(s). Published by IOP Publishing Ltd
addition to the $\lambda-k$ domain which we use here, one can also demonstrate interference in the time-frequency domain [9,10].

The theoretical study of the SPDC frequency modulation in crystal superlattices was presented in [5]. The closest earlier experimental demonstration used two nonlinear fibers generating frequency degenerate photon pairs [11]. However, the experimental realization of the superlattice with more nonlinear elements was limited by the stability and scalability of the fiber platform. In contrast, our approach possesses a high intrinsic stability, versatility of monolithic crystal enables easy scaling up of the number of nonlinear elements, with up to 86 elements demonstrated in this work. Furthermore, our approach enables us to experimentally demonstrate ultra-broadband frequency modulation with a signal full-width-half-maximum (FWHM) of 0.030 $\mu$m centered at 0.647 $\mu$m from a single crystal. This translates to a corresponding frequency modulation with FWHM of ~0.660 $\mu$m centered at 3.0 $\mu$m in the MIR idler channel, making it relevant to practical sensing and spectroscopic applications demanding ultra-broad bandwidth in the MIR, in particular those exploiting 'undetected photons' [15–20].

The theory for SPDC in layered nonlinear media, such as the biPPLN superlattice here, was previously discussed [3–5, 21–24], but primarily limited to only two nonlinear crystals separated by a linear medium [3,4]. Nonetheless, here we briefly describe the one-dimensional theory.

The two-photon (or biphoton) state generated via SPDC in layers of nonlinear media is given by [24,25]

$$|\psi\rangle = |\text{vac}\rangle + \sum_n \sum_{k_n,k'_n} f_n \hat{a}_{k_n}^\dagger \hat{a}_{k'_n}^\dagger |\text{vac}\rangle,$$

where $f_n (k_n, k'_n)$ is the biphoton field amplitude from the $n$th crystal, $\hat{a}_{k_n}^\dagger$ and $\hat{a}_{k'_n}^\dagger$ are creation operators of the signal and idler photons in the $n$th crystal with wavevectors $k_n$ and $k'_n$ respectively, and $|\text{vac}\rangle$ is the vacuum state [24,26,27].

The biPPLN superlattice is comprised of multiple nonlinear domains with the same material properties. We assume the pump to be a monochromatic plane wave. Thus, the amplitude of a biphoton field is given by $f_n \propto \chi_n |E_p| \exp(\tau_n + t_n) dz D_{n,\lambda} D_{n,i}$, where $\chi_n$ is the signed second-order optical susceptibility of the $n$th nonlinear domain, $\tau_n = \sum_{n=1}^{n-1} t_n$ and $t_n + l_n$ are, respectively, the front and back edge coordinates of the $n$th nonlinear crystal domain of thickness $l_n$. $|E_p|$ is the pump field [23,24], $D_{n,\lambda}$ is the propagation function for the pump, signal, and idler ($\lambda = p,s,i$) wavelengths in the $n$th crystal domain as given by $D_{n,\lambda}(k_n, z) = \exp(-i k_n \lambda z)$, where $k_{n,\lambda}$ is the longitudinal wavevector of the wavelength $\lambda$ inside the $n$th nonlinear layer.

The two-photon intensity (normalized to the square of the crystal length) is thus given by the absolute square of the sum of contributions from the $N$ crystal domains [3,5,24]:

$$I(\omega, \theta) \propto \frac{1}{L} \left| \sum_{n=1}^{N} \chi_n l_n \sin(\Delta_n/2) \exp \left( -\frac{i \Delta_n}{2} + i \sum_{n'=1}^{n} \Delta_{n'} \right) \right|^2,$$

where $\Delta_n = \lambda_n - \lambda_i$ is the frequency difference between the pump and idler, $L$ is the crystal length, and $\chi_n$ is the second-order nonlinear susceptibility of the $n$th layer.

**Figure 1.** Structure of a biPPLN superlattice with eight domains per stack ($n_d = 8$), three gaps ($n_g = 3$) of the same length as a stack ($n_{g,p} = 1$). The white arrows denote the optic axis orientation in each domain. Blue and red colored domains have up and down optic axis orientation, respectively. Green pump laser propagates in the positive $y$ direction. Note that the LiNbO$_3$ material fills the gap. (Inset) Photograph of the LiNbO$_3$ crystal with one poling design illuminated by the pump laser. The bright scattering regions indicate different stacks.
Note that equation (2) also implies that the interference between the correlated signal and idler photons results in the experimentally observed signal frequency modulation.

The length of each domain in the poled region of the LiNbO$_3$ crystal is $l_{\text{domain}} = 5.16 \, \mu m$, chosen to fulfill the SPDC phase-matching conditions. Upon pumping with a 0.532 $\mu m$ continuous wave laser, the structure yields frequency nondegenerate SPDC centered at 0.647 $\mu m$ and 3.0 $\mu m$ for the signal and idler photons, respectively.

The domains are arranged into stacks, such that each stack contains $n_{\text{cl}}$ domains with alternating optic axes between adjacent domains, see figure 1. Thus, the length of each stack is $l_{\text{stack}} = n_{\text{cl}} \times l_{\text{domain}}$. In between two stacks, the intervening gap is specified with a length $m_{\text{gap}}$ times that of the stack and has an optical axis orientation opposite to the adjacent domains of two different stacks. Thus, the length of each gap is $l_{\text{gap}} = m_{\text{gap}} \times l_{\text{stack}}$. Note that stacks with an even number of domains will result in alternating gap optic axis orientations. Each poling design in the crystal is comprised of $n_{\text{gap}}$ gaps and $n_{\text{stack}} = (n_{\text{gap}} + 1)$ stacks. The resultant length of a poling design is thus given by

$$l_{\text{design}} = l_{\text{stack}} \times (n_{\text{gap}} \times m_{\text{gap}} + n_{\text{gap}} + 1),$$

thus, each poling design can be uniquely specified using the parameters $n_{\text{cl}}$, $n_{\text{gap}}$ and $m_{\text{gap}}$, with a fixed $l_{\text{domain}}$.

The specifications for the various poling designs are summarized in table 1.

A 63.5 mm $\times$ 8.2 mm $\times$ 0.5 mm (length $\times$ width $\times$ height) LiNbO$_3$ crystal was designed by us, and custom manufactured by HC Photonics. Three poling designs are presented here, see table 1. The crystal facets are coated with an anti-reflective coating with less than 0.5% reflectance for 0.532 $\mu m$ wavelengths.

In our experiments, the crystal is pumped by a 0.532 $\mu m$ wavelength single longitudinal mode (SLM) continuous-wave Nd:YAG laser (200 mW, linewidth $\lesssim 1$ MHz, Oxxiux SA). A 2 $\times$ beam expander is placed before the crystal to reduce the transverse dimension of the pump beam to account for the finite thickness of the crystal. A fraction of pump photons is down-converted to $\sim 0.647 \, \mu m$ visible signal and $\sim 3.0 \, \mu m$ idler photons via type-0 collinear frequency nondegenerate SPDC. A notch filter is used to block the pump beam after the crystal, while the signal photons are focused onto the slit of an imaging spectrograph (Acton SpectraPro 2300i) using a three-lens system. The spectrum of signal photons is centered at 0.647 $\mu m$ and is recorded using a visible-range electron-multiplying charge-coupled device (EMCCD) camera (Andor iXon 897) at the output of the spectrograph. Camera exposure duration varies from 5 s to 10 s, with a 100 $\times$ electron-multiplying gain set during the data acquisition. The EMCCD sensor has 512 $\times$ 512 pixels with pixel size of 16 $\mu m$, and was kept at a temperature of $-80^\circ C$. All measurements were conducted at the ambient lab temperature of 22 $^\circ C$.

Superlattice spectra simulation code is written in Python 3.7 [28, 29] with refractive indices from Gayer et al [30] set to the ambient lab temperature of 22 $^\circ C$. Furthermore, Gaussian convolution is employed in our spectra, in order to match detection of the 0.532 $\mu m$ SLM pump laser beam over 5 $\times$ 5 pixels of the EMCCD camera.

We observed the ultra-broadband frequency comb-like diffraction, spanning 0.0304 $\mu m$ and 0.0307 $\mu m$ FWHM in signal wavelength, from designs 1 and 3, see figures 2(a) and (c). The overall spectral bandwidth, defined by the ‘envelope’ formed by the maxima of the spectral fringes (green line in figure 2), was similar.

### Table 1. Parameters and experimental results for three poling designs. Agreement between the theoretical model and experimental results is described by a sample Pearson correlation coefficient (SPCC).

| Design parameters | Experimental signal [idler] envelope FWHM ($\mu m$) | Experimental signal [idler] average peak spacing ($\mu m$) | Theoretical signal [idler] average peak FWHM ($\mu m$) | Results |
|-------------------|-----------------------------------------------|-------------------------------------------------|-----------------|---------|
| $l_{\text{design}}$ | $l_{\text{stack}} \times (n_{\text{gap}} \times m_{\text{gap}} + n_{\text{gap}} + 1)$, |
| $l_{\text{domain}}$ | $l_{\text{gap}} = m_{\text{gap}} \times l_{\text{stack}}$. Note that stacks with an even number of domains will result in alternating gap optic axis orientations. Each poling design in the crystal is comprised of $n_{\text{gap}}$ gaps and $n_{\text{stack}} = (n_{\text{gap}} + 1)$ stacks. The resultant length of a poling design is thus given by | $l_{\text{design}} = l_{\text{stack}} \times (n_{\text{gap}} \times m_{\text{gap}} + n_{\text{gap}} + 1)$, |
| $n_{\text{cl}}$ | $n_{\text{gap}}$ and $m_{\text{gap}}$ with a fixed $l_{\text{domain}}$. |
| $l_{\text{design}}$ | $l_{\text{stack}} \times (n_{\text{gap}} \times m_{\text{gap}} + n_{\text{gap}} + 1)$, |
| $l_{\text{domain}}$ | $l_{\text{gap}} = m_{\text{gap}} \times l_{\text{stack}}$. Note that stacks with an even number of domains will result in alternating gap optic axis orientations. Each poling design in the crystal is comprised of $n_{\text{gap}}$ gaps and $n_{\text{stack}} = (n_{\text{gap}} + 1)$ stacks. The resultant length of a poling design is thus given by | $l_{\text{design}} = l_{\text{stack}} \times (n_{\text{gap}} \times m_{\text{gap}} + n_{\text{gap}} + 1)$, |
| $l_{\text{domain}}$ | $l_{\text{gap}} = m_{\text{gap}} \times l_{\text{stack}}$. Note that stacks with an even number of domains will result in alternating gap optic axis orientations. Each poling design in the crystal is comprised of $n_{\text{gap}}$ gaps and $n_{\text{stack}} = (n_{\text{gap}} + 1)$ stacks. The resultant length of a poling design is thus given by | $l_{\text{design}} = l_{\text{stack}} \times (n_{\text{gap}} \times m_{\text{gap}} + n_{\text{gap}} + 1)$, |
| $l_{\text{domain}}$ | $l_{\text{gap}} = m_{\text{gap}} \times l_{\text{stack}}$. Note that stacks with an even number of domains will result in alternating gap optic axis orientations. Each poling design in the crystal is comprised of $n_{\text{gap}}$ gaps and $n_{\text{stack}} = (n_{\text{gap}} + 1)$ stacks. The resultant length of a poling design is thus given by | $l_{\text{design}} = l_{\text{stack}} \times (n_{\text{gap}} \times m_{\text{gap}} + n_{\text{gap}} + 1)$, |
| $l_{\text{domain}}$ | $l_{\text{gap}} = m_{\text{gap}} \times l_{\text{stack}}$. Note that stacks with an even number of domains will result in alternating gap optic axis orientations. Each poling design in the crystal is comprised of $n_{\text{gap}}$ gaps and $n_{\text{stack}} = (n_{\text{gap}} + 1)$ stacks. The resultant length of a poling design is thus given by | $l_{\text{design}} = l_{\text{stack}} \times (n_{\text{gap}} \times m_{\text{gap}} + n_{\text{gap}} + 1)$, |
between these two designs as they share the same number of nonlinear domains in a stack. This envelope FWHM is determined from fitting a Gaussian curve to the experimental fringe peaks, see figures 2(a) and (c). The observed difference in the modulation period is due to different gap lengths between stacks. This has a clear analogy with classical diffraction, in which closer spaced apertures result in the diffraction pattern with less frequent modulation.

The fact that the number of nonlinear domains per stack determines the overall spectral bandwidth was further demonstrated with design 2, see figure 2(b). A larger number of domains per stack led to shrinkage of the overall spectral bandwidth, despite having a similar number of gaps with design 3 and the same gap length multiple as design 1, see figures 2(a) and (c). Thus, broader spectral bandwidth than presented here is achievable with fewer nonlinear domains per stack. Again, these observations are akin to classical diffraction, where gratings with fewer elements produce narrower diffraction patterns. We emphasize that the observed diffraction patterns arise from the quantum interference of the biphoton wavefunction, see equation (2).

The average spectral peak spacing is determined by the number of nonlinear domains per stack, \( n_{nnl} \), and the gap length multiple, \( m_{gap} \). This can be seen by first comparing designs 1 and 3, in which the average peak spacings decreased by a factor of 4 from 0.0041 \( \mu m \) to 0.0011 \( \mu m \) in signal wavelengths when \( m_{gap} \) is increased by the same factor of 4, see table 1. There is also a decrease by a factor of 4 from 0.0041 \( \mu m \) to 0.0010 \( \mu m \) in the average signal peak spacing from design 1 to 2, with which \( n_{nnl} \) is increased by a factor of 4 from 16 to 64. In addition, the average peak spacings of designs 2 and 3 are similar with 0.0010 \( \mu m \) and 0.0011 \( \mu m \) respectively, due to the decrease by a factor of 4 in \( n_{nnl} \) and an accompanying increase, also by a factor of 4, in \( m_{gap} \) when going from designs 2 to 3. Thus, one can consider \( m_{gap} \) to be determining the number of peaks within the spectral envelope set by \( n_{nnl} \).

The theoretical average FWHM of signal peaks was (1) invariant to the superlattice design parameters and (2) about half a pixel width on our camera system (see table 1). This corresponds to about 30 GHz in bandwidth. We have provided the theoretical values, as the spectral bandwidths of peaks for all three designs could not experimentally resolved.

Agreement between observed and simulated spectra was generally good, as evidenced by the match between spectral positions of the comb peaks and the high sample Pearson correlation coefficients (SPCCs) for the three designs, see table 1. In figure S1 (https://stacks.iop.org/QST/7/01LT01/mmedia) of supplemental material, we show the full \( \lambda-k \) spectra produced by the superlattices. The spectra indicate additional angular modulation at larger scattering angles (more than 1° at fixed wavelengths, see also figure S2. These results indicate the richness of the proposed platform for spectral and spatial shaping of the biphoton emission.

In summary, we have thus demonstrated ultra-broadband diffraction in frequency coordinates from the nonlinear interference of SPDC photons. For this, we custom-designed and fabricated biPPLN superlattices, with which SPDC emission spanning 0.060 \( \mu m \) and 1.4 \( \mu m \) of spectral bandwidth around 0.647 \( \mu m \) and 3.0 \( \mu m \) wavelengths, respectively, was demonstrated. This is competitive with recent work by Vanselow et al in which similar bandwidths in the visible and MIR were demonstrated [31]. Conversely, our superlattice design is not limited to specific pump wavelengths arising from requiring matching group indices. In addition, our superlattice design can achieve greater bandwidth by simply decreasing the number of domains per stack. Note that the achievable wavelength range can be further increased via temperature-tuning, see figure S3 in supplemental material. Beyond the demonstration of an interesting analogy between quantum and classical optical diffraction, which is of fundamental interest, these biPPLN superlattices have practical implications. They include but are not limited to emerging applications requiring broad spectral bandwidth [31–36], such as broadband IR sensing [2], metrology [37], and quantum entangled frequency combs [38, 39]. Note that the present design is also applicable to type-II phase-matching, which is more commonly used in quantum optical experiments [40, 41].

For example, our light source can be embedded in a nonlinear interferometer in which mid-IR spectral fingerprints can be readout from the observation of correlated visible range signal photons [37]. The crystal superlattice will provide multiple spectral components in the mid-IR, which could then be read out at once, thus eliminating the need for time consuming spectral scanning. It has not escaped our notice that the designs presented herein might make for easier targeted frequency multiplexing for quantum communication and quantum key distribution [42]. Last but not least, our approach, akin to quantum mode engineering [43, 44], can potentially improve other instrument specifications in quantum hyperspectral IR imaging [15, 16, 18–20].
Figure 2. Normalized experimental (orange line) and simulated (blue line) spectral cross-sections at zero scattering angle for designs (a) 1, (b) 2, and (c) 3, see table 1. The green line is a Gaussian fit to the maxima in the experimental data. Note the ultra-broadband diffraction from designs 1 and 3 and the dependence of the spectral envelope on the parameters of the superlattice. Sub-figure (b) is presented in a different horizontal scale for clarity.
Acknowledgments

The authors acknowledge the support of the Agency for Science, Technology and Research (A*STAR) under its project C210917001. This work was also supported by the A*STAR Computational Resource Centre through the use of its high-performance computing facilities.

Data availability statement

The data that support the findings of this study are available upon reasonable request from the authors.

ORCID iDs

Zi S D Toa https://orcid.org/0000-0003-2890-6579
Anna V Paterova https://orcid.org/0000-0002-2819-7678
Leonid A Krivitsky https://orcid.org/0000-0001-8757-9200

References

[1] Hecht E 2017 Optics (Boston: Pearson)
[2] Paterova A V and Krivitsky L A 2020 Nonlinear interference in crystal superlattices Light Sci. Appl. 9 82
[3] Burlakov A V, Kulik S P, Penin A N and Chekhova M V 1998 Three-photon interference: spectroscopy of linear and nonlinear media J. Exp. Theor. Phys. 86 1090–7
[4] Burlakov A V, Chekhova M V, Klyshko D N, Kulik S P, Penin A N, Shih Y H and Streekalov D V 1997 Interference effects in spontaneous two-photon parametric scattering from two macroscopic regions Phys. Rev. A 56 3214–25
[5] Kitaeva G Kh, Tishkova V V, Naumova I I, Penin A N and Tang S H 2005 Mapping of periodically poled crystals via optical parametric tomography J. Raman Spectrosc. 36 116–22
[6] Kuo P S, Verma V B and Woon N S 2020 Demonstration of a polarization-entangled photon-pair source based on phase-modulated PPLN OSA Continuum 3 295–304
[7] Herrmann H, Yang X, Thomas A, Poppe A, Soler W and Silberhorn C 2013 Post-selection free, integrated optical source of non-degenerate, polarization entangled photon pairs Opt. Express 21 27981–91
[8] Santiago-Cruz T, Sultanov V, Zhang H, Krivitsky L A and Chekhova M V 2021 Entangled photons from subwavelength nonlinear films Opt. Lett. 46 653–6
[9] Atatüre M, Sergienko A V, Saleh B E A and Teich M C 2001 Entanglement in cascaded-crystal parametric down-conversion Phys. Rev. Lett. 86 4013–6
[10] Di Giuseppe G, Atatüre M, Shaw M D, Sergienko A V, Saleh B E A and Teich M C 2002 Entangled-photon generation from parametric down-conversion in media with inhomogeneous nonlinearity Phys. Rev. A 66 013801
[11] Su J, Cui L, Liu Y, Li X and Ou Z Y 2019 Versatile and precise quantum state engineering by using nonlinear interferometers Opt. Express 27 20479–92
[12] Soltész A S, Kumar P, Pertsch T, Sukhorukov A A and Setzpfandt F 2018 LiNbO3 waveguides for integrated SPDC spectroscopy APL Photon. 3 021301
[13] Boes A, Corcoran B, Chang L, Bowers J and Mitchell A 2018 Status and potential of lithium niobate on insulator (LNOI) for photonic integrated circuits Laser Photon. Rev. 12 1700256
[14] O’Brien J, Patton B, Sasaki M and Vučković J 2013 Focus on integrated quantum optics New J. Phys. 15 035016
[15] Paterova A V, Yang H, Toa Z S D and Krivitsky L A 2020 Quantum imaging for the semiconductor industry Appl. Phys. Lett. 117 054004
[16] Paterova A V, Maniam S M, Yang H, Grenci G and Krivitsky L A 2020 Hyperspectral infrared microscopy with visible light Sci. Adv. 6 eabd0460
[17] Vanselow A, Kaufmann P, Zorin I, Heise B, Chrzansowski H M and Ramelow S 2020 Frequency-domain optical coherence tomography with undetected mid-infrared photons Optica 7 1729–36
[18] Kvitkovsky I, Chrzanskowski H M, Avery E G, Bartolomeaus H and Ramelow S 2020 Microscopy with undetected photons in the mid-infrared Sci. Adv. 6 eaba2024
[19] Lahiri M, Łapkiewicz R, Lemos G B and Zeilinger A 2015 Theory of quantum imaging with undetected photons Phys. Rev. A 92 013832
[20] Lemos G B, Borish V, Cole G D, Ramelow S, Łapkiewicz R and Zeilinger A 2014 Quantum imaging with undetected photons Nature 512 409–12
[21] Kitaeva G Kh, Tishkova V V, Naumova I I, Penin A N, Kang C H and Tang S H 2005 Mapping of periodically poled crystals via spontaneous parametric down-conversion Appl. Phys. B 81 645–50
[22] Kitaeva G Kh and Penin A N 2004 Parametric frequency conversion in layered nonlinear media J. Exp. Theor. Phys. 99 272–86
[23] Klyshko D 1994 Parametric generation of two-photon light in anisotropic layered media J. Exp. Theor. Phys. 105 1574–82
[24] Klyshko D 1993 Ramsey interference in two-photon parametric scattering JETP 104 2676–84
[25] Belinsky A and Klyshko D 1994 Two-photon wave-packets Laser Phys. 4 663–89
[26] Zou X Y, Wang L J and Mandel L 1991 Induced coherence and indistinguishability in optical interference Phys. Rev. Lett. 67 318–21
[27] Wang L J, Zou X Y and Mandel L 1991 Induced coherence without induced emission Phys. Rev. A 44 4614–22
[28] Millman K J and Aivazis M 2011 Python for scientists and engineers Comput. Sci. Eng. 13 9–12
[29] Pérez F, Granger B E and Hunter J D 2011 Python: an ecosystem for scientific computing Comput. Sci. Eng. 13 13–21
[30] Gayer O, Sacks Z, Galun E and Arie A 2008 Temperature and wavelength dependent refractive index equations for MgO-doped congruent and stoichiometric LiNbO3, *Appl. Phys. B* 91 343–8

[31] Vanselow A, Kaufmann P, Chrzanowski H M and Ramelow S 2019 Ultra-broadband SPDC for spectrally far separated photon pairs *Opt. Lett.* 44 4658–41

[32] Nasr M B, Giuseppe G D, Saleh B E A, Sergienko A V and Teich M C 2005 Generation of high-flux ultra-broadband light by bandwidth amplification in spontaneous parametric down conversion *Opt. Commun.* 246 521–8

[33] Nasr M B, Carrasco S, Saleh B E A, Sergienko A V, Teich M C, Torres J P, Torner L, Hum D S and Fejer M M 2008 Ultrabroadband biphotons generated via chirped quasi-phase-matched optical parametric down-conversion *Phys. Rev. Lett.* 100 183601

[34] Katamadze K G, Paterova A V, Yakimova E G, Balygin K A and Kulik S P 2011 Control of the frequency spectrum of a biphoton field due to the electro-optical effect *JETP Lett.* 94 262

[35] Kalashnikov D A, Katamadze K G and Kulik S P 2009 Controlling the spectrum of a two-photon field: inhomogeneous broadening due to a temperature gradient *JETP Lett.* 89 224–8

[36] Horoshko D B, Kolebov M I, Gumpert F, Shand I, König F and Chekhova M V 2020 Nonlinear Mach–Zehnder interferometer with ultrabroadband squeezed light *J. Mod. Opt.* 67 41–8

[37] Paterova A, Yang H, An C, Kalashnikov D and Krivitsky I 2018 Measurement of infrared optical constants with visible photons *New J. Phys.* 20 043015

[38] Maltese G, Amanti M I, Appas F, Sinnl G, Lemaitre A, Milman P, Baboux F and Ducci S 2020 Generation and symmetry control of quantum frequency combs *Npj Quantum Inf.* 6 13

[39] Reimer C et al 2016 Generation of multiphoton entangled quantum states by means of integrated frequency combs *Science* 351 1176–80

[40] Thyagarajan K, Lugani J, Ghosh S, Sinha K, Martin A, Ostrowsky D B, Alibart O and Tanzilli S 2009 Generation of polarization-entangled photons using type-II doubly periodically poled lithium niobate waveguides *Phys. Rev. A* 80 052321

[41] Rubin M H, Klyshko D N, Shih Y H and Sergienko A V 1994 Theory of two-photon entanglement in type-II optical parametric down-conversion *Phys. Rev. A* 50 5112–33

[42] Roslund J, de Araújo R M, Jiang S, Fabre C and Treps N 2014 Wavelength-multiplexed quantum networks with ultrafast frequency combs *Nat. Photon.* 8 109–12

[43] Chekhova M V and Ou Z Y 2016 Nonlinear interferometers in quantum optics *Adv. Opt. Photon.* 8 104–55

[44] Ou Z Y and Li X 2020 Quantum SU(1,1) interferometers: basic principles and applications *APL Photon.* 5 080902