Andersen localization and the Planck length as source of disorder

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The role of disorder on wave propagation through the universe is studied. Assuming space fluctuations of the order of the Planck length and the size of the universe as the corresponding localization length for the background radiation, we obtain the exponent $\nu$ (close to unity) in the power law relationship between these quantities. This suggests that the role of Anderson localization is not negligible at cosmological scales.

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I. INTRODUCTION

Anderson localization is a fruitful topic embracing a multitude of wave phenomena going from quantum to classical physics. This is natural since it is related to interference of generic coherent waves. In quantum physics, more explicitly in electronic systems, it has played a major role in our knowledge of mesoscopic aspects and modern aspects of nanosystems. In classical physics, it is related to a broad spectrum of situations including electromagnetic-waves, mechanic-waves, and also to the interface theory called quantum chaos. In this work we consider another application of Anderson localization: its possible role in wave propagation inside the universe.

Today it is usually assumed that the average mass density in space is about $\sim 10^{-28}$ [kg/m$^3$] but, as mentioned in literature, there are obviously matter fluctuations, space-time fluctuations, and then disorder.

We will assume that we have a classical disordered medium and the only relevant length characterizing distances is the position averaged fluctuation $\Delta R$. Moreover, in average the system is assumed to be homogenous and time independent. Consider a wave of frequency $\omega$ propagating in the disordered medium characterized by the velocity $c$ when disorder is not present. That is, we have the wavelength (with a factor $2\pi$) $\lambda = c/\omega$ characterizing the propagating wave.

As said before, in this paper we are concerned with the possible role of Anderson localization for wave propagation in the universe, a subject not yet studied to the best of our knowledge. We will derive a simple relationship between disorder, the wavelength and the so-called localization length (section 2). This relationship will be tested using cosmological parameters (section 3).

II. LOCALIZATION LENGTH FOR CLASSICAL WAVES

It is well known that uncorrelated disorder could produce exponential localization for coherent wave propagation. Namely, the wave-amplitude decays exponentially with a characteristic length $L_e$ called the localization length. This length is generally dependent on the degree of disorder (here $\Delta R$), the dimension $D$ of the system and the frequency $\omega$ (or the wavelength) of the wave. For electronic systems there is a reasonable comprehension of the relation between localization and the respective dimension $D$ through the so-called scaling function. It is generally accepted that for low dimension (smaller than 3) localization holds. Note that eventual dependence on the mean free path is assumed inside the wavelength.

Classically, waves with long wavelength are less localized. This is reasonable since the frequency $\omega$ usually multiplies the disorder (random refraction index). In one dimension there are explicit examples where $L_e \sim \omega^{-2}$.

Considering the above statements (including section I) we conjecture that the localization length for classical systems is given by the generic relationship:

$$\frac{\lambda^*}{L_e} = g_D \left( \frac{\Delta R}{\lambda^*} \right),$$

where $g_D(x)$ is an unknown function depending on the effective dimension $D$. The fact that in equation (1) the dimensionless quantity $\lambda^*/L_e$ (or $\Delta R/\lambda^*$) appears explicitly is easy to understand since we have normalized all distances by the wavelength in the generic wave equation.

When no-disorder exists ($\Delta R = 0$) the localization length must be infinite then necessarily the function $g_D$ satisfies

$$g_D(0) = 0.$$  \hspace{1cm} (2)

It is well known that disorder tends to smooth out some quantities like the density of states and others. In this sense, we will assume that near-zero the function $g$ has a power law behavior, particularly,

$$g_D(x) \sim x^\nu, \text{ for } x \ll 1.$$  \hspace{1cm} (3)

This is the case for some systems where explicitly $\nu = 1$ (see references) and others with a particular random metric.
III. PLANCK LENGTH AS A POSITIONAL DISORDER SOURCE

In this section we consider a simple model where space is treated quantum-mechanically but propagating waves are not. Fluctuations are within the realm of quantum mechanics and they are expected to play a fundamental role at Planck-length scales of the space-time structure. Consider a classical wave of wavelength $\lambda^*$ propagating inside the universe. Assume that quantum fluctuations in the space are of the order of the Planck length $\Delta R \sim 10^{-35}[m]$. On the other hand, the universe then we have evaluated the exponent $\nu$ could be evaluated from (4) as:

$$\nu = 0.955,$$

close to unity. Note that for a particular theoretical model with a random metric the value $\nu = 1$ was found explicitly. The relationship suggested by the expressions (4) (and (5)) correlates parameters like the universe size, the background wavelength and the Planck length.

We can consider that in the exponential distribution of space fluctuations (only relevant space length scale), we conjecture that the localization length satisfies the relation (1). For small frequency, where weak localization is expected, we have the simple power law relationship (4).

IV. CONCLUSIONS

In an averaged-homogenous classical medium supporting waves, where the disorder is characterized by distance fluctuations (only relevant space length scale), we conjecture that the localization length satisfies the relation (4). For small frequency, where weak localization is expected, we have the simple power law relationship (4).

Assuming the origin of space fluctuations as a quantum mechanics phenomenon (order of the Planck length), the background radiation localized at distances of the size of the universe then we have evaluated the exponent $\nu$ (see (4)) giving a number close to unity (5). This suggests that Anderson localization is a phenomenon which must be considered in cosmological problems. Naturally the problem involves general relativity and Anderson localization theory, it is a complex study. From this point of view, this work is essentially destined to put both branches together in a simple way.

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