High accuracy robust attitude tracking control application on quadrotor UAV with the prescribed performance

Jingxin Dou¹, Qiansheng Tang² and Bangchun Wen²

Abstract
In this paper, a high accuracy attitude tracking control is presented for a quadrotor unmanned aerial vehicle (UAV) under the model uncertainties and external unpredictable disturbances by using a robust feedback control method. First, the cascade attitude dynamics of a quadrotor UAV is derived with the lumped disturbances. Then, a backstepping adaptive modified robust integral of the sign of the error (RISE) control is designed to improve the robustness of the control system against the disturbances. For arriving high accuracy tracking under the disturbances, a prescribed performance control algorithm is employed in combination to hold the transient state performance where the convergence rate is faster than the prescribed value, and the maximum overshoot is less than the prescribed value. The stability analysis is carried out to prove that the presented controller could guarantee that the closed-loop system is asymptotically stable, and the tracking feedback errors can converge to an arbitrarily small value near zero. The superior performance of the proposed robust controller is corroborated by comparing with the existed controller.

Keywords
Quadrotor, attitude tracking, robust feedback control, prescribed performance, adaptive modified RISE

Date received: 19 September 2020; accepted: 4 December 2020
Handling Editor: James Baldwin

Introduction
The quadrotor UAVs can achieve autonomous flight, hovering, and backward flying. The quadrotor UAVs have been used in wide applications of civil and military fields due to the excellent flight performances. With these performances, the quadrotor UAVs could be qualified to different sorts of missions such as reconnaissance, rescue, delivery, and so on.¹,² Hence, several groups of researchers have developed a variety of linear and nonlinear control laws to hold the flight performance of the quadrotor UAV in application of different fields.³–⁵ In Zheng et al.,⁶ a second order sliding mode controller was suggested for a quadrotor UAV to perform asymptotic tracking control. In Sarabakha et al.,⁷ a fuzzy state-feedback controller based on the Takagi-Sugeno fuzzy model was designed to guarantee the stability of a quadrotor UAV. In Shirzadeh et al.,⁸ based on the visual servoing system, a vision-based control scheme was investigated for a quadrotor to achieve high moving target tracking performance. In Huo et al.,⁹ an attitude stabilization controller was designed for a quadrotor by using backstepping control law. In Torres et al.,¹⁰ a fuzzy state feedback controller based

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on Takagi-Sugeno technique was presented to stabilize a quadrotor. As is known to all, each control algorithm has its advantages and shortcomings. For instance, the backstepping control approach easily deals with the cascade control model of the quadrotor UAV by recursive technique, but the backstepping control approach is lack to cope with perturbations to hold the robustness. The advantage of the sliding mode control strategy is of rejecting the uncertainties of control system, and has well robustness under disturbance, but the weakness is that the chattering phenomenon could not be eliminated by using the switching function. Hence, varieties of composite control schemes were suggested to obtain high control performance for complex control systems.\textsuperscript{11–15} The flight control system of the quadrotor UAV also is a complex control system which is nonlinearity, coupling, and particularly under-actuated.\textsuperscript{16–18} Hence, the compositions of various control strategies are presented for the quadrotor UAV. A backstepping sliding mode controller was carried out to making well trajectory tracking and ensure the stability for a quadrotor in Kacimi et al.\textsuperscript{19} A fuzzy sliding mode controller was employed to guarantee attitude stabilization of a quadrotor, and a fuzzy approach was use to eliminate chattering in Corchado et al.\textsuperscript{20} A second-order sliding mode control law combined with PID was design to improve attitude tracking performance and convergence rate for a quadrotor in Lee et al.\textsuperscript{21} All of the aforesaid references focus on the control performance of the quadrotor by depending on the dynamic of the control system or ignoring the disturbances.

On the other hand, on account of the widespread disturbances including internal disturbances of control system and external random disturbances exist in the control system, the problem of robust disturbance rejection control has always been a research focus in the theory of control.\textsuperscript{22–25} The control system of quadrotor UAV is more susceptible to disturbances due to under-actuated and special structure. For stability performance of a quadrotor UAV under different types of the disturbances, a backstepping controller based on high-order extend state observer was designed in Shi et al.\textsuperscript{26} In Castillo et al.,\textsuperscript{27} an attitude controller based on disturbance observer was presented for a quadrotor that guarantees tracking performance under aggressive attitude maneuvers and external disturbances. In Huang et al.,\textsuperscript{28} a vessel landing controller based on adaptive sliding mode control law was investigated for a quadrotor with parametric uncertainties. In Eliker and Zhang,\textsuperscript{29} an adaptive backstepping fast terminal sliding mode controller was employed for a quadrotor UAV that is suffering from uncertain parameters and external disturbances. With the anti-disturbance control law, the control schemes of the aforesaid references have guaranteed the stability of control system. However, the transient performances of the quadrotor UAV have received few effort.

For improving the transient performances of control system, several schemes were carried in the theory and applied in a number of applications.\textsuperscript{30–32} In Nguyen and Chen,\textsuperscript{33} a sliding mode controller combined with prescribed performance control law was designed for a positioning system to achieve high precision in the tracking. In Mobayen and Ma,\textsuperscript{34} a finite-time composite nonlinear feedback controller was designed for a chaotic systems with external disturbances and parametric uncertainties which guarantees high performance synchronization. In Zerari and Chemachema,\textsuperscript{35} an adaptive neural network controller combined with the prescribed performance technology was designed to overcome the unknown effects for continuously stirred tank reactor systems. Motivated by the aforesaid research results, an adaptive robust attitude controller integrated with the prescribed performance technology is presented to achieve high accuracy attitude tracking and improve the transient performance for a quadrotor UAV under the disturbances in this paper. Compared with the aforesaid literatures on quadrotor UAV, the principal theory contribution of this paper are summarized as follows:

1) Compared with the previous works, this paper aims to the point on the problem of the high accuracy robust tracking and the transient performance improvement for the quadrotor UAV with the unmodeled dynamics and external disturbances. For dealing with the mentioned question, an adaptive backstepping modified robust integral of the sign of the error control with the prescribed performance is proposed for improving attitude tracking performance of the quadrotor UAV.

2) The attitude stability of the quadrotor UAVs in previous works has been researched, but the transient performances of the quadrotor control systems are often neglected. In this paper, a prescribed performance function is employed for restrain the transient performances. Under the action of the prescribed performance technology, the control system can quickly recover from transient state to stable state during suffering from disturbances, and the control system of the quadrotor UAV also has faster convergence rate and smaller overshoot.

3) Considering the backstepping arithmetic is lack to cope with perturbations, the modified robust integral of the sign of the error control technique are combined for rejecting the disturbances to hold the better robustness in this paper. Compared with the previous RISE law, the modified adaptive RISE is better at reduce
the chattering phenomenon due to a hyperbolic function is employed to replace with the switching term, and an adaptive gain is proposed to compensate the internal and external disturbances by dynamic parameter adjustment.

The paper is organized as follows. The attitude dynamical model of quadrotor system is introduced in Section 2. The robust controller with the prescribed performance and the stability analysis of the proposed controller are given in Section 3. In Section 4, the numerical simulation results are given. Finally, a conclusion is drawn in Section 5.

Mathematical model of quadrotor attitude dynamics

The main characteristics of quadrotor UAV are the vertical takeoff, hovering and super maneuvering flight. The control system of the quadrotor UAV is susceptible to the uncertain disturbances during flight. Hence, during the quadrotor dynamics model established, the disturbances cannot be ignored. Many previous works have introduced the quadrotor modeling clearly. The attitude and fixed coordinate frames in Figure 1 are depicted in Figure 1. The inertial frame and body frame of quadrotor UAV .

\[ \mathbf{Y} = \begin{bmatrix} Y \\ \mathbf{\omega} \end{bmatrix} = \begin{bmatrix} \phi \\ \theta \\ \psi \end{bmatrix}, \quad \mathbf{\omega} = \begin{bmatrix} \omega_x \\ \omega_y \\ \omega_z \end{bmatrix}, \quad \mathbf{Y}_d = \begin{bmatrix} \phi_d \\ \theta_d \\ \psi_d \end{bmatrix} \]

where \( \mathbf{Y} = [\phi \quad \theta \quad \psi]^T \) is the attitude Euler angles, \( \mathbf{\omega} = [\omega_x \quad \omega_y \quad \omega_z]^T \) is the angular velocity with respect to the body-fixed frame, \( \mathbf{Y}_d = [\phi_d \quad \theta_d \quad \psi_d]^T \) is the reference command for quadrotor UAV attitude tracking, and \( \mathbf{e} = [\mathbf{e}_\phi \quad \mathbf{e}_\theta \quad \mathbf{e}_\psi]^T \) is the tracking error for attitude Euler angles.

The quadrotor UAV tends to deviate from the reference command, while it is subject to the uncertain disturbances. The control system is changed from steady state to transient state, and the control performance declines rapidly. Hence, when the control system is suffering from the disturbances, the control scheme must have ability to guarantee that the transient state of the control system must be restrained for returning to the steady state quickly. For ensuring prescribed transient state and restraining bounded tracking error, the prescribed performance control scheme is employed as \[^3\delta\]:

\[ -\eta_t \mathbf{e}_i < \mathbf{e}_i < \mathbf{e}_i, \quad i = \phi, \theta, \psi \]

where, \( 0 \leq \eta_t \leq 1 \), \( \mathbf{e}_i = (\mathbf{e}_{i,0} - \mathbf{e}_{i,\infty}) \) exp \((-k_it) + \mathbf{e}_{i,\infty} \) is performance function, and \( \mathbf{e}_i \) is positive and decreasing,
The time derivative of (6) is calculated as follows:

\[
\dot{e_i} = \frac{1}{2} \left( \frac{1}{\eta_i e_i + e_i} + \frac{1}{\dot{e}_i - e_i} \right) \left( \dot{e}_i e_i - \dot{e}_i e_i \right)
\]

The formula (7) can be rewritten as follows:

\[
\dot{z}_i = \eta_i \dot{e}_i - e_i
\]

where \( \eta_i = \text{diag}(\eta_{i\phi}, \eta_{i\theta}, \eta_{i\psi}) \), \( \dot{e}_i = \text{diag}(\dot{e}_{i\phi}, \dot{e}_{i\theta}, \dot{e}_{i\psi}) \), \( e = \text{diag}(e_{i\phi}, e_{i\theta}, e_{i\psi}) \).

\( e_i, \tilde{e}_i, k_i \) represent positive constants respectively.

For unifying the control objectives, the output error transformation function is defined, which is able to equivalently transformed the constrained tracking error into the unconstrained error.

\[
e_i = \tilde{e}_i S_i(\tilde{z}_i)
\]

where \( \tilde{z}_i \) is the error related to the constrained tracking error into the unconstrained error, \( S_i(\tilde{z}_i) \) is a strictly increasing and smooth function, and satisfies as follows:

\[
\begin{align*}
- \eta_i < S_i(\tilde{z}_i) < 1 \\
S_i(0) = 0, \tilde{z}_i = 0 \\
\lim_{\tilde{z}_i \to -\infty} S_i(\tilde{z}_i) = -\eta_i \\
\lim_{\tilde{z}_i \to +\infty} S_i(\tilde{z}_i) = 1 \\
S_i(\tilde{z}_i) = e^{\tilde{z}_i} - \eta e^{-\tilde{z}_i} \\
\end{align*}
\]

With the properties of the function \( S_i(\tilde{z}_i) \), and based on (5), the inverse transformation of \( S_i(\tilde{z}_i) \) is as follows:

\[
\tilde{z}_i = S_i^{-1} \left( \frac{e_i}{\eta_i} \right)
\]

\[
= \frac{1}{2} \ln \left( 1 + \frac{e_i}{\eta_i e_i} \right) - \frac{1}{2} \ln \left( 1 + \frac{e_i}{\dot{e}_i} \right)
\]

The time derivative of (6) is calculated as follows:

\[
\dot{\tilde{z}}_i = \frac{1}{2} \left( \frac{1}{\eta_i e_i + e_i} + \frac{1}{\dot{e}_i - e_i} \right) \left( \dot{e}_i e_i - \dot{e}_i e_i \right)
\]

The formula (7) can be rewritten as follows:

\[
\dot{z}_i = \eta_i \dot{e}_i - e_i
\]

\( \eta_i = \text{diag}(\eta_{i\phi}, \eta_{i\theta}, \eta_{i\psi}) \), \( e = \text{diag}(e_{i\phi}, e_{i\theta}, e_{i\psi}) \).

Figure 2. Quadrotor UAV attitude control system structure.

**Tracking control design**

For arriving the control objective, the attitude tracking controller is presented for a quadrotor with disturbances, which parallels the backstepping control scheme.

Step 1: Define the dynamic tracking output error as:

\[
z_2 = \omega - \alpha_1
\]

where \( \alpha_1 \) is virtual control input, \( z_2 = [z_{2\phi}, z_{2\theta}, z_{2\psi}]^T \).

Based on (1), (8), (9), and taking the time derivative of (3), the formula (8) is rewritten as:

\[
\dot{z} = -e + \eta(W(Y)z_2 + W(Y)\alpha_1 - \dot{Y}_d)
\]

Based on (10), \( \alpha_1 \) can be designed as:

\[
\alpha_1 = W^{-1} \eta^{-1} (-K_1 z_2 + \eta \dot{Y}_d + e)
\]

where \( K_1 = \text{diag}(k_{11}, k_{12}, k_{13}) \) is positive definite matrix.

Step 2: Derivating with respect to (9),

\[
\dot{z}_2 = \dot{\omega} - \dot{\alpha}_1
\]

Substituting the second equation of (1) into (12), the formula (13) can be obtained as:

\[
J_{z_2} = J_\omega - J_\dot{\alpha}_1
\]

Where \( J_\omega = \dot{W}(\omega)J_\omega + LU + D - J_\dot{\alpha}_1 \).

Defining the auxiliary tracking error \( z_3 \),

\[
z_3 = z_2 + \alpha z_2
\]

where \( \alpha \) is a positive constant.

Substituting (13) into (14),

\[
J_{z_3} = \dot{W}(\omega)J_\omega + LU + D - J_\dot{\alpha}_1 + \alpha J_{z_2}
\]
Based on (15), the proposed attitude tracking controller can be designed as follows:

\[
U = L^{-1}(-W(\omega)J_\omega + J\dot{\alpha}_1 - \alpha Jz_2 - u_R) \tag{16}
\]

For rejecting the lumped disturbance \(D\) which include the unmodeled dynamic and external disturbances, a hyperbolic function is designed to replace with the switching term of the tradition RISE for eliminating the chattering phenomenon. Hence, a modified RISE robust term \(u_R\) is designed as follows:

\[
u_R = (k_s + 1)z_2 - (k_s + 1)z_2(0) \\
+ \int_0^t [(k_s + 1)\alpha z_2(\tau) + \hat{\beta} \tanh(z_2(\tau))] d\tau \tag{17}
\]

where \(k_s = \text{diag}(k_{s1}, k_{s2}, k_{s3})\) is designed parameter matrix, \(\hat{\beta} = \text{diag}(\hat{\beta}_1, \hat{\beta}_2, \hat{\beta}_3)\) is positive switching matrix, and \(\hat{\beta}\) is the estimated value of the desired valued \(\beta\) of the RISE robust term, \(\tanh(z) = (e^{\rho z} - e^{-\rho z})/(e^{\rho z} + e^{-\rho z})\), \(\mu\) is a positive gain.

For compensating the external disturbances, and reducing the constraint of the controller design where the uppers of disturbance and its derivatives are beforehand determined, an adaptive law is employed for the modified RISE robust term \(u_R\) as

\[
\hat{\beta}_i = \zeta_i z_i^T [\alpha e_i \tanh(|z_i|)]^\top \tanh(|z_i|) \cdot \tanh(z_i) \tag{18}
\]

where \(\zeta, \alpha\) are positive gains, \(\epsilon\) is adaptation gain, \(i = \phi, \theta, \psi\).

Substituting the controller (16) into (15),

\[
Jz_3 = D - u_R \tag{19}
\]

Based on (14) and (17), derivative with respect to (19),

\[
Jz_3 = \dot{D} - u_R \\
= \dot{D} - (k_s + 1)z_2 - (k_s + 1)\alpha z_2 - \hat{\beta} \tanh(z_2) \tag{20}
\]

\[
= \dot{D} - (k_s + 1)z_3 - \hat{\beta} \tanh(z_2)
\]

\[\begin{aligned}
L_1 &= \bar{\beta}[z_2(0)]_1 - z_2^T(0)\dot{D}(0) - \int_0^t z_3 [\dot{D} - \bar{\beta} \tanh(z_2)]d\tau \\
L_2 &= z_2^T[\dot{D} - \bar{\beta} \tanh(z_2)] \\
L_3 &= \int_0^t L_2(\tau)d\tau = \int_0^t [z_2^T[\dot{D} - \bar{\beta} \tanh(z_2)]d\tau \\
&= \int_0^t (z_2^T + \alpha z_2^T) [\dot{D} - \bar{\beta} \tanh(z_2)]d\tau \\
&= \int_0^t z_2^T [\dot{D} - \bar{\beta} \tanh(z_2)] - z_2^T(0) [\dot{D} - \bar{\beta} \tanh(z_2)] \\
&+ \int z_2^T D d\tau + \int \alpha z_2^T [\dot{D} - \bar{\beta} \tanh(z_2)]d\tau
\end{aligned} \tag{23}
\]

Based on (23), the following inequality can be obtained as follows:

\[
\int_0^t L_2(\tau)d\tau \leq \|z_2\|_1 \cdot \|\dot{D}\|_2 - \bar{\beta}\|z_2\|_1 \\
+ z_2^T(0)\dot{D}(0) + \bar{\beta}\|z_2(0)\|_1 \\
+ \int (\|z_2\|_1 \cdot \|\dot{D}\|_2 - \alpha \bar{\beta}\|z_2\|_1) d\tau
\]

\[\begin{aligned}
&\leq -z_2^T(0)\dot{D}(0) + \bar{\beta}\|z_2(0)\|_1 \\
&+ \int \alpha \|z_2\|_1 (\delta_1 - \frac{\delta_2}{\alpha} - \bar{\beta})d\tau
\end{aligned} \tag{25}
\]

Delay on the above set condition

\[
\int_0^t L_2(\tau)d\tau \leq \|z_2\|_1 \cdot \delta_1 - \bar{\beta}\|z_2\|_1 - z_2^T(0)\dot{D}(0) \\
+ \bar{\beta}\|z_2(0)\|_1 + \int (\|z_2\|_1 \cdot \delta_2)d\tau \\
+ \int (\|z_2\|_1 \cdot \delta_1 - \alpha \bar{\beta}\|z_2\|_1) d\tau
\]

\[\begin{aligned}
&\leq -z_2^T(0)\dot{D}(0) + \beta\|z_2(0)\|_1 \\
&+ \int \alpha \|z_2\|_1 (\delta_1 + \frac{\delta_2}{\alpha} - \bar{\beta})d\tau
\end{aligned} \tag{25}
\]

\[\begin{aligned}
&\leq -z_2^T(0)\dot{D}(0) + \beta\|z_2(0)\|_1
\end{aligned} \tag{25}
\]
Based on (21) and (25), it is shown that the auxiliary function \( L_{1} \) is nonnegative.

Based on the Lyapunov stability theory, the following Lyapunov function is designed

\[
V = \frac{1}{2} \hat{z}^T \hat{z} + \frac{1}{2} \hat{z}^T \hat{z}_2 + \frac{1}{2} \hat{z}_1^T J \hat{z}_1
\]

where \( \hat{\beta} \) represents the estimate error \( \hat{\beta} = \beta - \beta_{i} \).

The time derivative of (26) is given as

\[
\dot{V} = \hat{\beta}^T \hat{z} + \hat{z}_2^T \hat{z}_2 + \hat{z}_1^T J \hat{z}_1
\]

where \( \hat{\beta} = \beta - \beta_{i} \).

With the squares of the components of \( \hat{z} \) and \( \hat{z}_i \), \( (i = 1, 2) \), the (27) can be obtained as

\[
\dot{V} \leq -K_i \| \hat{z} \|^2 + \frac{1}{2} \| \hat{z}_2 \|^2 + \frac{(\hat{\eta} W)^2}{2} \| \hat{z}_2 \|^2
\]

\[
+\frac{1}{2} \| \hat{z}_1 \|^2 + \frac{1}{2} \| \hat{z}_1 \|^2 - \alpha \| \hat{z}_1 \|^2 - (k_s + 1) \| \hat{z}_1 \|^2
\]

\[
= -\left( K_i - \frac{1}{2} \right) \| \hat{z} \|^2 - \left( \alpha - \frac{1}{2} - \frac{\hat{\eta} W^2}{2} \right) \| \hat{z}_1 \|^2
\]

\[
- \left( k_s + 1 \right) \| \hat{z}_1 \|^2
\]

(28)

Based on the assumed gains, the (28) can be rewritten as

\[
\dot{V} \leq -c \| \hat{z} \|^2
\]

where \( c = \min \left\{ \lambda_{\min}(K_i) - \frac{1}{2}, \alpha - \frac{1}{2} - \frac{\hat{\eta} W^2}{2}, k_s + \frac{1}{2} \right\} \).

Based on the Lyapunov stability theory, the following Lyapunov function is designed

\[
\dot{V} = -\lambda \| \hat{z} \|^2 - D \| \hat{z} \|^2 + L
\]

(26)

where \( \lambda \) and \( D \) are bounded, and then, \( \hat{z} \) is bounded with (8), (10) and (11). Based on the above bounded conditions, it is obtained that \( \alpha_1 \) is bounded. Therefore, all close-loop system signals are bounded. With (26) and (29), it is obtained that \( \lim_{t \to \infty} \| \hat{z} \| = 0 \), and then, \( \lim_{t \to \infty} \| e \| = 0 \). Hence, the proposed control law (16) can guarantee that the asymptotic stability of control system. With the conditions with respect to that \( \hat{z} \) is bounded, the inequality \( -\eta < S(\hat{z}) < 1 \) is held. Hence, \( -\eta e < e < e_1 \) is held that the prescribed performance of the tracking errors are guaranteed.

Thus, Theorem 1 is proven.

**Simulation and analysis**

Aiming at the system of the quadrotor UAV (1) and the controller (16), the proposed control scheme is demonstrated. The value of inertial matrix is \( J = \text{diag}(0.016, 0.016, 0.028) \text{Ns}^2/\text{rad} \) and \( L = \text{diag}(0.2, 0.2, 0.05) \).

In order to validate the designed control scheme, the desired trajectory is selected as \( Y_d = [0, 0, 0]^T \). The attitude dynamics states initially are listed as: \( Y = [\pi/3, -\pi/3, -0.4]^T \), \( \omega_0 = [0, 0, 0]^T \). Considering the vibrations produced by the quadrotor rotors are time-varying, the unmodeled parts excited by the rotors vibrations may be irregular effective. Therefore, the lumped disturbances \( D \) is designed by using the non-linear piecewise function, which includes the unmodeled parts of the attitude system and external disturbances are listed as follows:

\[
D_\phi = 0.6 \sin(0.6t) (1 - \text{sgn}(\sin(0.5t)))
\]

\[
D_\theta = 0.6 \cos(0.6t) (1 - \text{sgn}(\sin(0.4t)))
\]

\[
D_\psi = 0.4 \sin(0.4t) \cos(0.4t) (1 - \text{sgn}(\sin(0.6t)))
\]

The main gains of the proposed control are selected as: \( K_i = [6, 6, 4]^T \), \( K_s = [0.5, 0.5, 0.25]^T \). The desired tracking trajectories of the quadrotor attitude are taken to be zero. The performance function is designed as \( (1.2 - 0.01) e^{-0.1 x} + 0.01 \). The results of the MATLAB simulations are shown and analyzed as follows:

In Figures 3 to 6, the simulation results are shown by using the proposed control scheme. In Figure 3, the attitude angles tracking results are presented with the proposed controller. With the effect of the proposed controller, the three attitude angle values all can arrive the desired trajectory within about 1.6s, and the overshoots are taken to zero nearly. Thus, the setting of the control objective can be achieved. In Figure 4, the attitude angular velocities of the quadrotor UAV are shown. The attitude angular velocities values are held around the zero after 1.6s, it is mean that the control systems reach the stable state under the action of the controller. The attitude tracking errors of the quadrotor UAV are shown in Figure 5. As the show, the three attitude angles tracking errors \( e \) are always limited to the specified range from upper and lower bound of the performance function. The value of tracking errors \( e \) is held at among from -0.004 to 0.004 when the control systems reach the stable state. The converging rate and minimum overshoot are arrived, and the prescribed
performance of the tracking errors is guaranteed. The control inputs of the attitude control system are indicated in Figure 6.

In addition, for validating the robustness and advancement of the proposed control scheme, a backstepping sliding mode controller in Almakhles37 and a traditional RISE-PID controller are carried out respectively for comparison. The quadrotor UAV with the traditional RISE-PID controller or the backstepping sliding mode controller suffering from the

Figure 3. Attitude angle tracking results of the quadrotor UAV with the proposed control scheme: (a) Euler angle $\phi$ tracking result, (b) Euler angle $\theta$ tracking result, and (c) Euler angle $\psi$ tracking result.

Figure 4. Attitude angular velocity of the quadrotor: (a) angular velocity $\omega_\phi$, (b) angular velocity $\omega_\theta$, and (c) angular velocity $\omega_\psi$. 
The three attitude angles tracking results of the quadrotor UAV with the different control schemes are shown in Figures 7 and 8 respectively. As shown from the figures, the severe chattering of the quadrotor attitudes angles is obvious under the influence of the disturbances. The overshoot and the stable time of control system by using the backstepping sliding mode controller are smaller than the traditional RISE-PID controller. However, comparing with the proposed control scheme,
the overshoots and chattering of the control inputs are significantly larger by using both the traditional RISE-PID controller and the backstepping sliding mode controller in the absence of constraints of the prescribed performance function and the modified RISE. When the disturbances change suddenly, the attitude tracking result values have larger fluctuation with the backstepping sliding mode controller than the others, and the time required for the control systems return to the steady state with the traditional RISE-PID controller is longer than the other controllers. In addition, when the control systems reach the stable state, the attitude angles tracking errors of the quadrotor UAV are held at among about from –0.05 to 0.04 with the traditional RISE-PID controller, and from –0.04 to 0.07 with the backstepping sliding mode controller. Hence, the attitude angles tracking performance of the quadrotor UAV with the proposed controller is most robust under the disturbances.

**Conclusion**

In this study, a backstepping adaptive modified RISE feedback controller with the prescribed performance is proposed for high precise attitude tracking of a quadrotor UAV under the model uncertainties and external disturbances. For guaranteeing the transient performance of the control system under the disturbances, a prescribed performance function is employed for imposing the constraints on the attitude Euler angle tracking errors. Under the influence of the prescribed performance functions, the tracking errors can obtain the better transient properties with quicker convergence rate and minimum overshoot, and transient performances is evidently improved to against the disturbances. For improving the robustness, the backstepping control law combined with adaptive modified RISE are presented for attitude tracking. The stability analysis and the numerical simulation results have verified to the effectiveness of the proposed control strategy. It reveals that the proposed controller has better tracking performance than traditional RISE-PID controller or backstepping sliding mode controller. The primary of the future improve work is to devise adaptive finite-time robust tracking controller to guarantee the transient performance for a quadrotor UAV subject to parameter perturbation, time-delay and full states saturation.

**Declaration of conflicting interests**

The author(s) declared no potential conflicts of interest with respect to the research, authorship, and/or publication of this article.

**Funding**

The author(s) disclosed receipt of the following financial support for the research, authorship, and/or publication of this article: This work is supported in part by the National Natural Science Foundation of China (grant number...
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References
1. Ozbek NS, Onkol M and Efe MO. Feedback control strategies for quadrotor-type aerial robots: a survey. Trans Inst Meas Control 2016; 38: 529–554.
2. Lavaei A and Atashgah MAA Optimal 3D trajectory generation in delivering missions under urban constraints for a flying robot. Intell Serv Robot 2017; 10: 241–256.
3. Das H. Dynamic inversion control of quadrotor with a suspended load. IFAC PapersOnLine 2018; 51: 172–177.
4. Muńoz F, González-Hernández I, Salazar S, et al. Second order sliding mode controllers for altitude control of a quadrotor UAS: real-time implementation in outdoor environments. Neurocomputing 2016; 233: 61–71.
5. Bakshi NA and Ramachandran R. Indirect model reference adaptive control of quadrotor UAVs using neural networks. In: Proceedings of the 10th international conference on intelligent systems and control, Coimbatore, India, 7–8 January 2016, pp.7727123. IEEE.
6. Zheng EH, Xiong JJ and Luo JL. Second order sliding mode control for a quadrotor UAV. ISA Trans 2014; 53: 1350–1356.
7. Sarabakha A, Fu C, Kayacan E, et al. Type-2 fuzzy logic controllers made even simpler: from design to deployment for UAVs. IEEE Trans Ind Electron 2018; 65: 5069–5077.
8. Shirzadeh M, Asl HJ, Amirkhani A, et al. Vision-based control of a quadrotor utilizing artificial neural networks for tracking of moving targets. Eng Appl Artif Intell 2017; 58: 34–48.
9. Huo X, Huo MY and Karimi HR. Attitude stabilization control of a quadrotor UAV by using backstepping approach. Math Probl Eng 2014; 2014: 1–9.
10. Torres F, Rabhi A, Lara D, et al. Fuzzy state feedback for attitude stabilization of quadrotor. Int J Adv Robot Syst 2016; 13: 61934.
11. Wang L, Gong HJ and Liu CS. Disturbance observer-based adaptive fault-tolerant dynamic surface control of nonlinear system with asymmetric input saturation. Int J Control Autom Syst 2019; 17: 617–629.
12. Mobayan S, Yazdanpanah MJ and Majd VJ. A finite-time tracker for nonholonomic systems using recursive singularity-free FTSm. In: Proceedings of the 2011 American control conference, San Francisco, CA, USA, 29–1 June–July 2011, pp.1720–1725. IEEE.
13. Mohammad Mehdi F and Mahdi S. Indirect adaptive fuzzy control for flexible-joint robot manipulators using voltage control strategy. J Intell Fuzzy Syst 2015; 28: 1451–1459.

Figure 8. Attitude angle tracking results of the quadrotor UAV with the backstepping-SMC control scheme: (a) Euler angle \(\phi\) tracking result, (b) Euler angle \(\theta\) tracking result, and (c) Euler angle \(\psi\) tracking result.
14. Mobayen S. Design of LMI-based sliding mode controller with an exponential policy for a class of underactuated systems. *Complexity* 2016; 21: 117–124.

15. Muhammad Syahril M and Liu TH. Implementation of predictive controllers for matrix-converter-based interior permanent magnet synchronous motor position control systems. *IEEE J Emerg Sel Top Power Electron* 2019; 7: 261–273.

16. Wang R and Liu JK. Adaptive formation control of quadrotor unmanned aerial vehicles with bounded control thrust. *Chin J Aeronaut* 2017; 30: 807–817.

17. Pazooki M and Mazinan AH. Hybrid fuzzy-based sliding-mode control approach, optimized by genetic algorithm for quadrotor unmanned aerial vehicles. *Complex and Intell Syst* 2018; 4: 79–93.

18. Ali ZA and Li XD. Modeling and controlling of quadrotor aerial vehicle equipped with a gripper. *Meas Control* 2019; 52(5–6): 577–587.

19. Kacimi A, Mokhtar A and Kouadr B. Sliding mode control based on adaptive backstepping approach for a quadrotor unmanned aerial vehicle. *Prz Elektrotech* 2012; 88: 189–193.

20. Zeghlache S, Saigaa D, Kara K, et al. Fuzzy Sliding mode control with chattering elimination for a quadrotor helicopter in vertical flight. *HAIS’12: Proceedings of the 7th international conference on hybrid artificial intelligent systems volume part I*, Sulamanca, Spain, March 2012, pp.125–136. Springer.

21. Lee KU, Choi YH and Park JB. Inverse optimal design for position control of a quadrotor. *Appl Sci-Basel* 2017; 7: 907.

22. Gao S, Ning B and Dong HR. Fuzzy dynamic surface control for uncertain nonlinear systems under input saturation via truncated adaptation approach. *Fuzzy Sets Syst* 2016; 290: 100–117.

23. Mobayen S and Tchier F. Nonsingular fast terminal sliding-mode stabilizer for a class of uncertain nonlinear systems based on disturbance observer. *Sci Iran* 2017; 24: 1410–1418.

24. Shirin Fartash T, Seyed Hossein K and Mehrdad M. Torque ripple minimization and control of a permanent magnet synchronous motor using multi objective extremum seeking. *IEEE-ASME Trans Mechatron* 2019; 24: 2151–2160.

25. Zhu CL, Li CX, Chen XY, et al. Event-triggered adaptive fault tolerant control for a class of uncertain nonlinear systems. *Entropy* 2020; 22: 598.

26. Shi XL, Sun YQ and Shao XL. Robust output feedback trajectory tracking for quadrotors. *Proc IMechE, Part G: J Aerosp Eng* 2019; 233: 1596–1610.

27. Castillo A, Sanz R, Garcia P, et al. Disturbance observer-based quadrotor attitude tracking control for aggressive maneuvers. *Control Eng Pract* 2019; 82: 14–23.

28. Huang YT, Zheng ZW, Sun L, et al. Saturated adaptive sliding mode control for autonomous vessel landing of a quadrotor. *IET Control Theory Appl* 2018; 12: 1830–1842.

29. Eliker K and Zhang WD. Finite-time adaptive integral backstepping fast terminal sliding mode control application on quadrotor UAV. *Int J Control Autom Syst* 2020; 18: 415–430.

30. Panagou D, Stipanovic DM and Voulgaris PG. Distributed coordination control for multi-robot networks using lyapunov-like barrier functions. *IEEE Trans Autom Control* 2016; 61: 617–632.

31. Hashim HA, El-Ferik S and Lewis FL. Neuro-adaptive cooperative tracking control with prescribed performance of unknown higher-order nonlinear multi-agent systems. *Int J Control* 2019; 92: 445–460.

32. Wu H, Liu S, Cheng C, et al. Observer based adaptive double-layer fuzzy control for nonlinear systems with prescribed performance and unknown control direction. *Fuzzy Sets Syst* 2020; 392: 93–114.

33. Nguyen LM and Chen X. Discrete time quasi sliding mode control for piezo-actuated positioning systems: a prescribed performance control approach. *IFAC Papersonline* 2017; 50: 5121–5126.

34. Mobayen S and Ma J. Robust finite-time composite nonlinear feedback control for synchronization of uncertain chaotic systems with nonlinearity and time-delay. *Chaos Solitons Fractals* 2018; 114: 46–54.

35. Zerari N and Chemachema M. Robust adaptive neural network prescribed performance control for uncertain CSTR system with input nonlinearities and external disturbance. *Neural Comput Appl* 2020; 32: 10541–10554.

36. Bechlioulis CP and Rovithakis GA. Robust adaptive control of feedback linearizable MIMO nonlinear systems with prescribed performance. *IEEE Tran Autom Control* 2008; 53: 2090–2099.

37. Almakhles DJ. Robust backstepping sliding mode control for a quadrotor trajectory tracking application. *IEEE Access* 2020; 8: 5515–5525.