Optimum Parameters of a Tuned Liquid Column Damper in a Wind Turbine Subject to Stochastic Load

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Abstract. Parameter optimization for tuned liquid column dampers (TLCD), a class of passive structural control, have been previously proposed in the literature for reducing vibration in wind turbines, and several other applications. However, most of the available work consider the wind excitation as either a deterministic harmonic load or random load with white noise spectra. In this paper, a global direct search optimization algorithm to reduce vibration of a tuned liquid column damper (TLCD), a class of passive structural control device, is presented. The objective is to find optimized parameters for the TLCD under stochastic load from different wind power spectral density. A verification is made considering the analytical solution of undamped primary system under white noise excitation by comparing with result from the literature. Finally, it is shown that different wind profiles can significantly affect the optimum TLCD parameters.

1. Introduction

Remarkable progress in wind turbine technology has been made over the past years with advances in the field of structural and dynamic analysis, which allow for the creation of larger and more efficient wind turbines [1].

The premise of a vibration control device is that if a secondary system composed of a mass, a damping, and a spring is installed on a primary structure with its natural frequency tuned to a very close dominant mode of the primary structure, reductions of the dynamic response can be achieved.

In particular, among energy dissipation systems, tuned liquid column dampers (TLCDs), first proposed by [2] and [3] are emerging in several specialized publications (e.g. [4]–[7]) and have become a good option due to its relatively low cost and good efficiency.

The TLCD operates based on the movement of the liquid column. In this paper, the TLCD has a “U” shape. The TLCD requires no extra mechanism such as springs or joints, furthermore, its geometry may vary according to design needs, making them very versatile devices.

To avoid solving nonlinear simultaneous equations, solutions such as statistical linearization [8] and parameter optimization [9] have been proposed in previous works. Yalla and Kareem [10] derived a closed-form solution for optimized TLCD damping ratio and head loss coefficient. Although the method does not rely on an iterative procedure, in order to solve the minimum variance integrals, a formula is derived indirectly considering some properties of the spectrum of the stationary output of a linear time-invariant system to white noise input. Altay et al. [11] presented an expanded optimization approach, which considers the geometric layout of the damper. A numerical verification is carried out
by stochastic inflow turbulence simulator TurbSim and the aero-elastic dynamic horizontal axis wind turbine simulator FAST.

Optimization procedures have also been proposed for structures with mounted TLCD under harmonic load. By maximizing the reduction of peak structural response under harmonic excitation, Gao [12] designed the optimum tuning parameters for a TLCD for a wide frequency range. Shum [13] proposed a close-form optimal solution by optimizing the response of primary structure at two invariant points using the fixed-point method with a perturbation technique.

This paper aims to propose an optimization approach, based on a global direct search optimization algorithm, in order to find the optimum TLCD parameters for reducing vibration levels in slender structures such as wind turbines, when subject to an arbitrary stationary random wind excitation. Four different wind models are investigated given by a power spectral density (PSD) profile and its effects on the optimum parameters are discussed.

This paper is organized as follow. The mathematical description of TLCD and the wind turbine model are presented in Section 2. Section 3 presents the optimization approach as well as a numerical verification. Section 4 shows the stochastic induced wind load models and a FFT based approach for time history simulation. Finally, Section 5 presents some concluding remarks.

2. TLCD and structure modelling
Considering the TLCD rigidly connected on the primary structure as sketched in Figure 1 and the “U” shaped support with negligible mass when compared to fluid and constant cross section, it is possible to model the structure as a one degree of freedom model with equivalent mass, stiffness and damping.

The equation describing the motion of the fluid and structure is given by

\[ M \ddot{x}(t) + C \dot{x}(t) + Kx(t) = F(t) \]  

where \( M, C, K \) are the mass, damping and stiffness matrices are given by

\[ M = \begin{pmatrix} 1 + \frac{\mu}{\alpha} & \alpha \mu \\ \alpha & 1 \end{pmatrix} \]

\[ C = \begin{pmatrix} 2\omega_e \zeta_e & 0 \\ 0 & 2\omega_a \zeta_a \end{pmatrix} \]

\[ K = \begin{pmatrix} \omega_e^2 & 0 \\ 0 & \omega_a^2 \end{pmatrix} \]  

where the mass matrix is in its dimensionless form, and the system is already linearized. The dimensionless parameters \( \mu = m_a / m_e \) and \( \alpha = b / l \) are the mass ratio and length ration where \( m_a \) and \( m_e \) are the damper and structure mass respectively and \( b \) and \( l \) are the horizontal and total length to the damper’s tube. The \( \omega_e \) and \( \omega_a \) are the structure and damper natural frequencies and we introduce the dimensionless parameter tuning ratio \( \gamma = \omega_e / \omega_a \). Finally, \( \zeta_e \) and \( \zeta_a \) are the structure and damper damping ratio respectively.

3. Parameter optimization criteria
From the linearized system, a frequency response function (FRF) is calculated which will be used in the random vibration analysis. The frequency response functions are obtained by assuming the system in Eq. (1) oscillates under harmonic motion. The system response vector is then given
\[
X(\omega) = [-\omega^2 M + j\omega C + K]^{-1} F(\omega) = H(j\omega) F(\omega)
\]  

(3)

where \( \omega \) stands for the driving frequency, \( H(j\omega) \) is the frequency response function, and \( j \) is the imaginary unit.

Considering the exciting force \( F(t) \) as a stationary random signal with power spectral density (PSD) \( S_y(\omega) \), the structural response is also random and with PSD given by \( S_{yy} = HS_y H^T \) [14] where the superscript \( T \) denotes the matrix transpose.

The desired performance index (cost function) \( J \) will be defined by the mean square response. If the exciting force is zero mean then the response also has zero mean, therefore the mean square response equals the variance, thus

\[
J(\zeta, \gamma) = \int_{-\infty}^{\infty} S_{yy}(\omega) d\omega = \int_{-\infty}^{\infty} H(j\omega) S_y(\omega) H(j\omega)^T d\omega
\]

(4)

where the response PSD \( S_{yy}(\omega) \) is real positive and, therefore, the sufficient and necessary conditions for the optimization are met. For the prescribed frequency range \([\omega_l, \omega_u]\) the optimization problem consists of looking for the parameters that minimize the variance of response PSD, i.e.

\[
\min_{\zeta, \gamma \in \Omega} J(\zeta, \gamma)
\]

s.t. \( \zeta, \gamma \geq 0 \)

\( I = \{i | \omega_i \leq \omega \leq \omega_u\} \)

(5)

where \( \Omega \) is the set of design parameters \( \zeta, \gamma \) satisfying the constraint. To solve the optimization problem we introduce the optimization algorithm.

The Generalized Patter Search (GPS) is a class of direct search methods, originally proposed for unconstrained minimization problems [15], and then extended in its generalized form for problems with bound and linear constraints [16]. GPS is a non-gradient-based algorithm therefore, it is not as strongly affected by random noise in the cost function. However, it requires more function evaluations than gradient-based algorithms to find the true minimum. A more detailed description of the method can be found in [17]. The MATLAB environment is used to perform the GPS algorithm with its included solver \texttt{patternsearch}.

In order to verify if the algorithm and cost function are adequate for solving the optimization problem, a simple case where the closed-form solution is available is investigated.

Considering the case of undamped primary structure \( \zeta_e = 0 \) and white noise excitation, Yalla and Kareem [10] solved the mean variance problem to obtain analytical expressions for optimal parameters. Figure 2 shows the optimized parameters from Yalla and Kareem and the proposed algorithm as a function of the mass ratio for different length ratio. It can be noticed a very good agreement. A slight difference can be seen between the two methods in the optimum damping ratio for large values of mass ratio which can be attributed to tolerance error in the optimization algorithm and the chosen cost function's interval of integration that, in this case, were chosen between 2 and 5 rad/s. However, the optimized damping ratio sensibility does not influence the characteristics of the system. The wind excitation PSDs considered in this paper are detailed in the next section.
Wind profile power spectral density

Wind excitations are highly dynamic, irregular external loads. This section discusses how these could be simulated through the different PSDs such as white noise, Kanai-Tajime, Kaimal and Davenport whose expressions are given:

White Noise:

\[ S_{WN}(\omega) = S_0 \]

Kanai-Tajima:

\[ S_{KT}(\omega) = \left( 1 + 4 \xi_g^2 \left( \omega / \omega_g \right) \right) S_0 \]

\[ / \left[ 1 + \left( \omega / \omega_g \right)^2 + 4 \xi_g^2 \left( \omega / \omega_g \right)^2 \right] \]

Kaimal:

\[ S_{Kai}(\omega) = [4S_0^2 \left( L_g / v_{hub} \right)] \]

\[ / \left[ 1 + (6\omega(L_g / v_{hub}))^{5/3} \right] \]

Davenport:

\[ S_{Dav}(\omega) = 4\kappa L v_{hub}^2 \chi / \left( 1 + \chi^2 \right)^{4/3}, \]

\[ \chi = \omega L / v_{hub} \]

For Kaimal PSD, \( L_g = 340.2 \) m is a scale parameter that involves the wind turbine high and \( v_{hub} = 16 \) m/s is the mean wind velocity. For Davenport PSD, \( \kappa \) is the drag coefficient referred to the mean velocity and \( L \) is the hub height. For Kanai-Tajimi PSD, \( \omega_g = 10.5 \) rad/s and \( \xi_g = 0.317 \) can be interpreted as characteristic frequency and characteristic damping ratio respectively.

Figure 2: Comparing optimized (a) tuning ratio and (b) damping ratio subject to White Noise spectrum as a function of the mass ratio for different length ratio.
Figure 3 shows the obtained optimized TLCD tuning ratio $\gamma_{opt}$ and damping ratio $\zeta_{opt}$ subject to different wind spectra as a function of the mass ratio $\mu$ for a fixed length ratio $\alpha = 0.8$ and 1% primary structural damping. The optimum TLCD damping ratio $\zeta_{opt}$ increases for increasing mass ratio $\mu$, and it is only slightly affected by the choice of wind spectrum. On the other hand, the optimum tuning ratio $\gamma_{opt}$ decreases for increasing mass ratio and it is significantly affected by the choice of wind spectrum. Kaimal and Davenport spectra present almost the same values of tuning ratio, which are overall increasingly smaller for increasing mass ratio, when compared to the white noise spectrum. Furthermore, the choice of wind spectrum can influence the response magnitude and therefore it is relevant for the appropriate choice of optimum TLCD parameters.

5. Concluding remarks
In this paper, it is proposed an optimization approach to find the optimum TLCD parameters for reducing vibration levels in slender structures such as wind turbines, when subject to an arbitrary stationary random wind excitation. The proposed optimization criteria is chosen such that it minimizes the area under the response PSD. Four different wind models, given by a PSD profile, are investigated and it is shown that they can significantly affect the choice of the optimum parameters. A verification of the proposed approach is made considering the case of undamped primary structure under white noise excitation, which has an analytical solution available. The TLCD design parameters can be given by its damping ratio $\zeta$ and tuning ratio $\gamma$. It is shown that the optimum TLCD damping ratio $\zeta_{opt}$ increases for increasing mass ratio and it is only slightly affected by the choice of wind spectrum. Besides, the optimum tuning ratio $\gamma_{opt}$ decreases for increasing mass ratio and it is significantly affected by the choice of wind spectrum.

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