D$^3$ Dark Energy in String Warped Compactification

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We study the evolution of relic D3-branes after the D3/$\overline{D3}$-brane inflation in string warped compactification. The motion of D3-branes can be frozen under certain condition during the radiation/matter domination. These D3-branes can not be released until the D3/$\overline{D3}$-branes potential energy becomes dominated at late time. Subsequently they will move towards to $\overline{D3}$-branes, which play the role of uplifting AdS minimum to dS minimum, near the apex of throats. The annihilation of D3/$\overline{D3}$-branes leads to the disappearance of dS vacua. This process may be regarded as a rapid decay channel of present dS vacua. We discuss the parameter spaces required by this process and calculate the decay time.

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The current set of cosmological data $^1$, $^2$, $^5$ implies that the universe might experience several separate stages of acceleration, in each which the universe is in a near dS state. The earlier accelerated stage driven by inflaton began about 13 billion years ago, followed by a reheating process, in which the universe is reheated to a suitable temperature. While the recent accelerated expansion began approximately 5 billion years ago, which is driven by dark energy constituting about 2/3 of the total energy density of the present universe.

Both stages of near dS expansion of universe have still challenged string theory $^4$, $^5$, $^6$. It might become increasingly clear that dS state could not be stable in any theory of quantum gravity, instead it should be a metastable resonance with its lifetime less than the recurrence time $^7$. Recently, there have been some progresses on constructing the metastable dS vacua in string theory. Based on the warped flux compactification studied by GKP $^8$ in type IIB string theory, KKLT $^9$ stabilized the volume modulus in a supersymmetric AdS minimum by taking into account nonperturbative effects, and then they uplifted this AdS minimum to a metastable dS minimum near the apex of throats. The annihilation of D3/$\overline{D3}$-branes leads to the disappearance of dS vacua. This process may be regarded as a rapid decay channel of present dS vacua. We discuss the parameter spaces required by this process and calculate the decay time.

Background- We firstly review the background on which our work is based. The GKP compactification $^8$ of IIB string theory on a threefold M with 7-branes and O3-planes can be efficiently described as F theory $^{22}$ compactification on a CY fourfold X. 3-form fluxes and D3-branes added are subject to the global tadpole constraint or the global conservation of RR 5-form $F_5$ flux

$$N_{D3} - N_{\overline{D3}} = \frac{1}{2\kappa_{10}^2 T_3} \int_M \mathcal{H}_3 \wedge F_3 - \frac{\chi(X)}{24} = 0 \, , \quad (1)$$

where $\kappa_{10}^2 = (2\pi)^7 (\alpha')^4 g_s^2/2$ is the 10-dimension Planck scale and $T_3 = 1/(2\pi)^3 (\alpha')^2 g_s$ is the 3-brane tension. The Euler number of the CY fourfold $\chi(X)$ gives the effective negative D3-branes charge in IIB string theory of O3-planes and D7-branes wrapped on 4-cycles of M. This must be balanced by the charge from 4-dimension
D3-branes, the wrapped NSNS and RR 3-form fluxes $H_3$ and $F_3$, which also source $F_5$.

Following KKLT [9], the non-perturbative correction $Ae^{-a\rho}$ of $W$ is introduced, which makes it ($W = W_0 + Ae^{-a\rho}$) fix the overall Kähler modulus, in the meantime stabilize the volume modulus $\rho = \sigma + i\alpha$ in a finite and moderately large value with a supersymmetric AdS minimum $\Lambda_{\text{AdS}}$. Then they add some D3-branes. These added D3-branes can be balanced in the tadpole condition (1) by turning on more fluxes. The D3-branes break the supersymmetry and bring an additional term

$$\mathcal{V} = \Sigma_i 2p_i^\beta T_i,$$

(2)

to the effective potential of volume modulus $\sigma$. In Planck unite, $T_3/m_p^4 \sim 1/V_6^2 \sim 1/\sigma^4$ [17], where $V_6$ is the warped volume of compactification manifold, and $\beta$ is the redshift factor, which can be related to the deformation parameter of the conifold tip and given by $\beta^4 \sim \sigma \exp (-\frac{2\alpha}{a\rho})$. Thus Eq. (2) gives a term $\sim 1/\sigma^2$, which leads to an uplift to a metastable dS vacuum.

The background fluxes $M, K$ are given by

$$M = \frac{1}{4\pi^2a'} \int_A F_3 \quad , \quad K = \frac{1}{4\pi^2a'} \int_B H_3,$$

(3)

respectively, where the A cycle is the $S^3$ which is finite at the tip, while the B cycle is the 6-dimensional dual of A. To minimize their energy, the D3-branes have to migrate to the apex of throats, so the energy density per D3-brane added depends on the fluxes. The $i$ in Eq. (2) labels the different throats. There could be large number of 3-cycles in a typical CY manifold, each of which can carry nontrivial fluxes, so the existence of many throats could be expected to be quite generic. In the background with multiple throats the discrete density of vacua can increase dramatically. By adjusting the fluxes in each individual throat, one may tune $\mathcal{V}$ with very high accuracy. Thus for sufficiently fine tuning parameters, the additional term in the potential $\mathcal{V}(\sigma)$ may lift the AdS global minimum to a required local dS minimum. The cosmological applications of the compactification with multiple throats have been considered in Ref. [23, 24, 25] for various aims.

**D3-branes in Dark Energy** - We start with many wandering D3/D3-branes in the bulk. The D3-branes can feel a net radial force proportional to the 5-form flux $F_5$, $F_5(r) = -2T_3F_5(r)$, and thus will be driven to the IR ends of different throats. This force is a sum of gravitation and 5-form contributions. For the D3-branes, both terms cancel. Thus though during random transfer some of D3/D3-branes can annihilate, the case that many D3/D3-branes accumulate rapidly near the apex of different throats may be still expected from a generic initial distribution of D3/D3-branes. These accumulated D3-branes can further attract D3-branes in the bulk, which under certain conditions results in the acceleration of different stages of observable universe.

The inflation of early universe may occur in a throat with a weakly warped factor, inflation throat, in which the D3-branes in the bulk will be driven and move towards the D3-branes near the apex of inflation throat. The D3/D3-branes after the D3/D3-brane inflation annihilate by the tachyon condensation and their energy is released into the Standard Model degrees of freedom of observable universe, see Ref. [23] for a warped reheating. The universe after the reheating may have a KKLT-like vacuum with its value equal to observed cosmological constant, and will experience successive periods of radiation, matter and dark energy domination following the evolution of standard cosmology.

The motion of D3-branes during the radiation/matter domination can be different from that during the D3/D3-brane inflation. From the 4-dimension effective viewpoint, the rolling of a scalar field can be frozen when the Hubble parameter during the radiation/matter domination is larger than the effective mass of this scalar field. Thus similarly in this sense, the motion of D3-branes towards D3-branes can become important only after their potential energy starts to dominate the universe at late time. During the D3/D3-branes potential energy domination, the D3-branes will be driven and annihilate D3-branes near the apex of throat with a strongly warped factor, dark energy throat. This dark energy source is called as D$^3$S dark energy here. The D$^3$S vacuum after the annihilation of D3/D3-branes will have lower value or disappear, see Fig.1 for an illustration.

The potential between a D3-brane and $p \ D$S-branes in

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**FIG. 1:** The illustration of D$^3$S dark energy in CY space. D3-branes are placed in the apex of throats, while some D3-branes wander in the bulk. Initially D3-branes will be driven and move towards to D3-brane in inflation throat, after D3/D3-branes annihilation, the relic D3-branes will be attracted by D3-branes in dark energy throat, and rapidly move out of the inflation throat and into the dark energy throat, subsequently the motion of D3-branes is frozen due to the expansion of radiation/matter-dominated universe. They can not be released until the D$^3$S potential energy starts to dominate the universe.
a warped background takes the form
\[ V(r) = 2p\beta^4 T_3(1 - \frac{\kappa_0^2 \beta^4 T_3}{r^4}), \] (4)
where \( r \) is the distance between D3/\overline{D3}-branes, \( \beta \) is the redshift factor \( \sim \exp(-\frac{2\kappa r}{3pM}) \) and is denoted by \( r_0/R \) in Ref. \[11\], and for conventions, \( (1/2\pi^4)\kappa_0^2 T_3^4 = R^4/N = 4\pi g_s(\alpha')^2 \) has been set here.

To make the kinetic term canonical, we implement the change \( \varphi = \sqrt{T_3} \tau \). The potential (4) can be rewritten as
\[ V(\varphi) = 2p\beta^4 T_3(1 - \frac{1}{2\pi^2} \beta^4 T_3 \varphi^4), \] (5)
where \( \kappa_0^2 T_3^4 = \pi \) has been used. For an uplift to a D-state with its value equal to that of observed cosmological constant,
\[ \Lambda(\varphi) = V(\varphi) - |\Lambda_{\text{AdS}}| \simeq \Lambda_0 \] (6)
is required, where \( \Lambda_0 \) is the present value of cosmological constant of observable universe. To make the relic D3-branes not have an opportunity to annihilate \( \overline{\text{D3}} \)-branes up to today, the condition
\[ \frac{\Lambda''}{\Lambda_0} \sim m_p^2 \frac{\Lambda'(|\varphi|)}{\Lambda_0} \sim \frac{20p^3 T_3^2 \beta^8 \Lambda_0}{\pi^2} \varphi^6 \leq 1 \] (7)
must be satisfied, where \( h_0^2 \sim \Lambda_0/m_p^2 \) is the present Hubble parameter with \( m_p = T_3 V_6^{1/2}/\sqrt{\pi} \) being 4-dimension Planck scale. Eq. (7) gives
\[ \frac{20p^3 T_3^2 \beta^8 \Lambda_0}{\pi^2} \varphi^6 \leq \frac{\varphi^6}{V_6} < 1, \] (8)
where the reason of the latter inequality is that two branes can not be separated by a distance larger than the volume of compactification manifold. For \( |\Lambda_{\text{AdS}}| \gg \Lambda_0 \), \( V(\varphi) \simeq 2p\beta^4 T_3 \simeq |\Lambda_{\text{AdS}}| \) (for \( |\Lambda_{\text{AdS}}| \simeq \Lambda_0 \), \( 2p\beta^4 T_3 \simeq \Lambda_0 \) is also suitable for replacing) can be obtained, thus
\[ \beta^4 \simeq \frac{|\Lambda_{\text{AdS}}|}{2pT_3}. \] (9)
Instituting (9) into (8), we have
\[ p \gtrsim \frac{5}{\pi^3} \frac{|\Lambda_{\text{AdS}}|^2}{T_3 \Lambda_0}. \] (10)
Thus for \( 5|\Lambda_{\text{AdS}}|^2 \leq \pi^3 T_3 \Lambda_0 \), one or several \( \overline{\text{D3}} \)-branes will be enough for the uplifting to a D-state minimum with its value equal to that of observed cosmological constant, while for \( 5|\Lambda_{\text{AdS}}|^2 \gg \pi^3 T_3 \Lambda_0 \), a large number of \( \overline{\text{D3}} \)-branes will be required, see the upper right panel of Fig. 2 for a numerical analysis. Notice that the potential (4) between a D3-brane and \( p \overline{\text{D3}} \)-branes is valid only for small \( p \), while for enough large \( p \), the correction to background will become important.

We then calculate the decay time \( T \) of \( \overline{\text{D3}} \) dark energy in small \( p \) limit. In principle, all wandering D3-branes during the D3/\overline{D3}-brane inflation will be attract into the inflation throat, and after the D3/\overline{D3}-brane inflation the relic D3-branes will move out of the inflation throat and into the dark energy throat. To make this process feasible, the warping of inflation throat should be weaker than that of dark energy throat. Thus the time of exiting from the inflation throat, compared with the time of staying in the dark energy throat, can be neglected. Two points may be further considered for this calculation. One is that during the radiation/matter domination, the motion of D3-branes is nearly frozen due to the expansion of universe, which provides an initial value for time integral. The other is that when the slow rolling condition (7) is broke down, D3-branes will be expected to annihilate \( \overline{\text{D3}} \)-branes rapidly. Thus the majority of the moving time should be from the slow rolling process of D3-brane, in which
\[ 3h_0 \dot{\varphi} + \Lambda'(\varphi) \simeq 0 \] (11)
can be satisfied. Thus
\[ T \simeq \int dt \simeq - \int \frac{3h_0}{\Lambda'(\varphi)} d\varphi. \] (12)
Instituting \( h_0^2 \simeq h_0^2 = \Lambda_0/3m_p^2 \) and Eq. (10) into Eq. (12), we have
\[ T \simeq - \int \frac{\pi^3 \sqrt{\lambda_0}}{4\sqrt{3p}} \frac{\sqrt{\lambda_0}}{m_p \beta^8 T_3^4} \varphi^5 d\varphi \]
\[ = \frac{\pi^3}{24\sqrt{3p}} \frac{\sqrt{\lambda_0} T_3^4}{m_p \beta^8} \varphi_0^6 \]
\[ \lesssim \frac{\pi^3}{24\sqrt{3p}} \frac{\sqrt{\lambda_0} V_6^{1/2}}{\beta^8} \sim O(1) \frac{\sqrt{\lambda_0} V_6^{1/2}}{p/\beta^8}, \] (13)
where $T_3 V_6^2 = \sqrt{3} m_\nu$ and $\pi^6 < V_6$ has been used. Further $\beta$ can be cancelled by using Eq. 9, we obtain

$$T < \mathcal{O}(1) \frac{p T_3^3 V_6^2 \Lambda_0^{1/2}}{|\Lambda_{AdS}|^2}. \quad (14)$$

For a compare, defining $T_0 \simeq 1/h_0 = \sqrt{3} m_\nu/\sqrt{\Lambda_0}$ as the present age of observable universe, we have

$$T \lesssim \frac{\pi^3}{p T_3 / |\Lambda_{AdS}|^2}. \quad (15)$$

Thus for $6|\Lambda_{AdS}|^2 > \pi^3 T_3\Lambda_0$, D$\bar{D}$ dark energy will decay in a cosmological age, while for $6|\Lambda_{AdS}|^2 \leq \pi^3 T_3\Lambda_0$, the decay time of D$\bar{D}$ dark energy will be many times longer than the age of universe. The lower left panel of Fig. 2, after we combine (10) and (15), is plotted, in which only $|\Lambda_{AdS}| > \pi^2/6$, $p = 1$ is taken and for $x \leq \pi^2/6$, $p$ is approximately $(5/\pi^2)x^2$. For larger value of $p$, the solid line of Fig. 2 will rise, and thus the decay time will longer.

Discussion - In KKLT-like compactification, many D$\bar{D}$-branes are placed in different throats and required to implement the uplifts from AdS minimum to dS minimum. For a generic initial distribution, some D$\bar{D}$-branes after the D$\bar{D}$/$\bar{D}$/$\bar{D}$-brane inflation may be left. These relic D$\bar{D}$-branes will be driven and annihilate D$\bar{D}$-branes near the apex of dark energy throats, which may induce a rapid decay of present dS vacua.

We study this process in this note. The main point is that the motion of D3-branes can be frozen during the radiation/matter domination, which may be realized by placing D$\bar{D}$-branes in a strongly warped throat, in which $\Lambda''(\varphi) < h_0^2$ can be ensured. But the parameter spaces for the number $p$ of D$\bar{D}$-branes and the energy scale $|\Lambda_{AdS}|$ of AdS minimum are not arbitrary. We find that only for $6|\Lambda_{AdS}|^2 \leq \pi^3 T_3\Lambda_0$, one or several D$\bar{D}$-branes are suitable for the uplift to an observed value of cosmological constant, while for a larger $|\Lambda_{AdS}|$, more D$\bar{D}$-branes will be required, otherwise D3/$\bar{D}$/$\bar{D}$-branes will annihilate so early that we can not observe the D$\bar{D}$ dark energy. In a model with multiple dark energy throats, whether can one find other uplifting modes of using D$\bar{D}$-branes to relax above conditions? For example, one may firstly uplift AdS minimum to another AdS minimum with lower absolute value by placing some D$\bar{D}$-branes in a throat with a weakly warped factor, then uplift this new AdS minimum to dS minimum by same step but in a throat with a strongly warped factor. However, it seems that this uplifting mode will make the thing worse. Generally, the weakly warped factor will lead to the strong attraction, thus D$\bar{D}$-branes will be firstly driven into these throats with the weakly warped factors and annihilate D$\bar{D}$-branes in these throats.

Generally, D$\bar{D}$ dark energy will inevitably lead to a sharp change of the value of dark energy in the future, either reducing its value or leading to a catastrophic decay to AdS state (The negative cosmological constant will eventually induce a Big Crunch \[26\], see also Ref. \[27\] for a cyclic universe experiencing different AdS minima. The decay time of D$\bar{D}$ dark energy is faster than that of decompactification to 10-dimension Minkowski vacua \[6\] and NS5-brane mediated decay \[17, 18\]. For a larger $|\Lambda_{AdS}|$, its decay time is almost the same order as the present age of universe. Further, it may be interesting to compare D$\bar{D}$ dark energy model with current observational constraints on cosmic doomsday \[28\]. Finally, of course one can also specially design some models which evade the decay channel proposed here, for example, by simply not having relic D3-branes after the D$\bar{D}$/$\bar{D}$/$\bar{D}$-brane inflation, as in KKLT \[5\]. Thus in this case one of the other decay channels to a 10-dimension Minkowski vacuum or a different flux vacuum will have to occur.

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