A comparative study on two different approaches of Bulk viscosity in the Polyakov-Nambu-Jona-Lasinio model

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We have gone through a comparative study on two different kind of bulk viscosity expressions by using a common dynamical model. The Polyakov-Nambu-Jona-Lasinio (PNJL) model in the realm of mean-field approximation, including up to eight quark interactions for 2+1 flavor quark matter, is treated for this common dynamics. We have probed the numerical equivalence as well as discrepancy of two different expressions for bulk viscosity at vanishing quark chemical potential. Our estimation of bulk viscosity to entropy density ratio follows a decreasing trend with temperature, which is observed in most of the earlier investigations. We have also extended our estimation for finite values of quark chemical potential.

Keywords: Bulk viscosity; PNJL Model.

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1. INTRODUCTION

Ideal hydrodynamics successfully described the experimental data of transverse momentum ($p_T$) spectra and elliptic flow coefficient $v_2(p_T)$ for different hadrons, produced in the heavy ion collisions. However, one can expect a weakly interacting gas like behavior of the medium, produced in the heavy ion collisions because of the asymptotic freedom of Quantum Chromo Dynamics (QCD). The argument went stronger when one coupled a 2+1-dimensional ideal hydrodynamical model of Quark Gluon Plasma (QGP) phase to a hadron cascade one to properly account for the viscous behavior in late hadronic stage. This eventually led to a concept of perfect fluidity, with an implication to be a strongly coupled plasma.
In Au+Au and Cu+Cu collisions at Relativistic Heavy Ion Collider (RHIC), elliptic flow $v_2$\cite{21,23} is found to be large and not generally consistent with experimental observations upon consideration of different types of initial conditions. The absence of proper knowledge of initial conditions hints about the presence of viscosities in the QGP phase to reproduce $v_2$ data. It is observed\cite{24,25,30} that both shear and bulk viscosities suppress elliptic flow $v_2$. However the smallness of this suppression leads to a prediction of very small specific shear viscosity values\cite{31,32} for the temperature region probed by RHIC or Large Hadron Collider (LHC).

Bulk viscosity $\zeta$ manifests itself by an addition of a diagonal term $\pi \delta^{ij}$ to stress tensor $T^{ij}$ in the local rest frame. This happens due to local isotropic deviations from equilibrium. Bulk viscous pressure $\Pi$ becomes proportional to scalar expansion rate $\theta$ at the location of fluid cell in the Navier-Stoke’s approximation, i.e. $\Pi(x) = -\zeta \theta(x)$. So, expansion is opposed by the bulk term, manifested in the negative value. Thus, for an isotropically expanding fireball, bulk viscosity reduces the radial acceleration, thus decreasing radial flow. The picture is somewhat different in case of shear viscosity. Increase in radial flow by the shear term leads to flatter shape of particle $p_T$ spectra\cite{33} whereas the situation is reverse for bulk viscosity\cite{25,34,35}.

Alongside, in Ref.\cite{42} it has been found that for any reasonably small running coupling constant bulk viscosity is quantitatively less compared to shear viscosity. A list of references on the microscopic calculations of bulk viscosity coefficient is not very small\cite{36,37}. Moore and Saremi\cite{36} and Buchel\cite{37} found that for systems with diverging specific heat near transition temperature $T_c$, the relaxation time can blow up with $1/\zeta$ still remaining finite. Coming to the expected nature of $\zeta$, it should vanish for a system of massless non-interacting quanta due to conformal invariance. Presence of interactions in the system changes the scenario and leads to deviation from this conformal limit. However the deviation remains very small in all regions except near phase transition where strong interactions may lead to large correlation length.\cite{36,39,38} Kinetic theory approaches estimate $\zeta$ to be varying as second power in this deviation. The technique has been applied in relaxation time approximation\cite{40} as well as for systems of photons radiated by massive particles in equilibrium\cite{41}, to which leading order QCD results\cite{42} also agree. For strongly coupled $\mathcal{N}=4$SYM theory this variation however is taken to be linear in the deviation.\cite{43} Computation for bulk viscous effects has been done by few authors considering hadron gas\cite{44,45} as well. Increase of $1/\zeta$ has been observed towards lower temperatures for massive pions\cite{44} in contrast to a decrease for massless pions.\cite{45} However, near phase transition peak like behavior of specific bulk viscosity supports the general arguments\cite{39,47,48} which happens because of long range correlations closely related to the chiral symmetry restoration. In presence of second order phase transition, $1/\zeta$ is supposed to diverge.\cite{40,47} The critical behavior of bulk viscosity in Gross-Neveu and linear sigma models have also been explored in Ref.\cite{19} and Ref.\cite{20} respectively.

In the Nambu-Jona-Lasinio (NJL) model, bulk viscosity coefficients $\zeta$ of quark matter has been calculated by Sasaki et al.\cite{51} Marty et al.\cite{50} Ghosh et al.\cite{55} and
Deb et al.\textsuperscript{56} They have used the expression of bulk viscosity, which is based on the relaxation time approximation (RTA) in kinetic theory approach\textsuperscript{53,54,56} or quasi-particle approach of Kubo formalism.\textsuperscript{55} Xiao et al.\textsuperscript{52} has calculated $\zeta$ in the Polyakov-Nambu-Jona-Lasinio (PNJL) model by using the expression based on the Kubo formalism with sum rule approach. Using the former and latter approaches, Refs.\textsuperscript{42,51,54–57,64,68} and Refs.\textsuperscript{47,48,63,67} have respectively estimated $\zeta$ in different other dynamical models. In this context, our interest lies in the study of both expressions of $\zeta$ under the framework of a common dynamical model and search some equivalence between two approaches. We have chosen Polyakov-Nambu-Jona-Lasinio (PNJL) model as our common dynamics to apply on both approaches of bulk viscosity calculations.

The paper is organized as follows. Next section contains formalism part, where a brief discussion of PNJL model has been mentioned first and then the expression of bulk viscosity in two approaches are addressed. Next, in result section, the equivalence of two expressions of bulk viscosity is mainly investigated in numerical point of view for vanishing quark chemical potential ($\mu$) and then results of finite $\mu$ have also been addressed. Finally we have made summary of our work.

2. Formalism

Kubo Formalism\textsuperscript{72,73} relates viscosity coefficients to the correlation functions of the energy-momentum tensor. For the bulk viscosity coefficient $\zeta$ of QCD matter, the relation is

$$\zeta = \lim_{\omega \to 0} -\frac{\text{Im}G_R(\omega, 0)}{9\omega} = \lim_{\omega \to 0} \frac{\pi \rho(\omega, 0)}{9\omega},$$

where imaginary part of retarded Green function $\text{Im}G_R$ and corresponding spectral density $\rho$, associated with $\zeta$, are defined as

$$-\text{Im}G_R(\omega, 0) = \pi \rho(\omega, 0) = \int_0^\infty dt \int d^3 \vec{r} e^{i\omega t} \langle [\theta_{\mu}(x), \theta_{\mu}(0)] \rangle.\tag{2}$$

The structure of trace of energy-momentum stress tensor $\theta_{\mu}^{\mu}$ for QCD in low-energy theorems at finite temperature and density has been addressed in Ref.\textsuperscript{74} Using standard Kramers-Kronig relation along with some basic thermodynamics and considering the quark and gluon components, we get\textsuperscript{17,18,52}

$$-G_R(0, 0) = 2 \int_0^\infty \frac{\rho(u, 0) du}{u} = -6\left(-f_\pi^2 M_\pi^2 - f_K^2 M_K^2\right) + 16|\epsilon_v| + Ts\left(\frac{1}{c_s^2} - 3\right)$$

$$+ \left(\mu \frac{\partial}{\partial \mu} - 4\right) T^3 \frac{\partial}{\partial T} \left[\frac{T^4}{T^4 + \mu \frac{\partial}{\partial \mu} - 2}\right] \langle m\bar{q}q\rangle_T.\tag{3}$$

To extract bulk viscosity $\zeta$, an ansatz is to be made for the spectral density $\rho$. Since the divergent contribution is deducted in the definition of quantities in right hand side of Eq.\textsuperscript{3} (realized in terms of the first term in right hand side), so the
high frequency perturbative continuum\cite{20} \( \rho(u) \sim \alpha_s u^4 \) is not included in left hand side of the same. In low frequency region, the following ansatz is chosen,
\[
\frac{\rho(\omega, \vec{0})}{\omega} = \frac{9\zeta}{\pi} \frac{\omega_0^2}{\omega_0^2 + \omega^2}
\]

such that it satisfies Eq. (1), where \( \omega_0 \) is the mass scale corresponding to region of validity of perturbation theory. At \( \omega_0 \gg T \), it is expected that the spectral density will become perturbative and temperature-independent. So we obtain,
\[
\zeta = \frac{1}{9\omega_0} \left[ T s \left( \frac{1}{c^2_s} - 3 \right) + \left( \mu \frac{\partial}{\partial \mu} - 4 \right) T^2 \frac{\partial (P_T)}{\partial T} + \left( T \frac{\partial}{\partial T} + \mu \frac{\partial}{\partial \mu} - 2 \right) (m^2 q)_T + 6 \left( f^2_\pi M^2_\pi + f^2_K M^2_K \right) + 16|\epsilon_v| \right]
\]

which is the final expression for bulk viscosity coefficient, \( \zeta \), based on QCD sum rule.

Next, we will come to another possible expression of \( \zeta \), which is based on the relaxation time approximation (RTA) in kinetic theory approach\cite{40,57} Exactly same expression can also be derived from the quasi-particle approach of Kubo formalism, where spectral density of \( \zeta \) can be expressed in terms of one-loop diagram\cite{55,68,69}. The idea of quasi-particle picture is introduced via inclusion of finite thermal width (\( \Gamma \)) in the internal lines of one-loop diagram. Instead of going to their detail derivation, which one can see in Refs.\cite{40,57} for RTA method and in Refs.\cite{55,68} for quasi-particle approach of Kubo formalism, let us come to their final expression for \( \zeta \). For vanishing quark chemical potential (\( \mu = 0 \)), the RTA expression is
\[
\zeta = \frac{12}{T} \int \frac{d^3k}{(2\pi)^3} \frac{f_\pi \left[ 1 - f_\pi \right]}{(E_k)^2 \Gamma} \left[ \left( \frac{1}{3} - c^2 \right) k^2 - c^2 \frac{d}{d\beta^2} \left( \beta^2 m^2 \right) \right]^2,
\]

where the effect of Polyakov loop is absorbed into the PNJL distribution function designated by \( f_\pi \).\cite{25,27} To get the total bulk viscosity, we will have to take the sum over 2+1 flavors. Here, \( E_k = \{k^2 + m^2\}^{1/2} \) is energy of quark and \( \Gamma = 1/\tau \) is its thermal width, which inversely determine its relaxation time \( \tau \) in the medium.

Here in this work, we compute the bulk viscous coefficient using the methodologies of LET and RTA. For our studies, we use the framework of the 2+1 flavor PNJL model taking up to eight quark interaction terms. This model was developed by addition of Polyakov loop to the NJL model\cite{25,27} An insight into its formalism and recent developments can be found in Refs. [25,27,28].

3. Results and Discussions

Let us start our discussion from numerical values of \( \zeta \), obtained from the Eq. (5) at \( \mu = 0 \). These are plotted in the upper panel of Fig. (1) for two different values of \( \omega_0 \), which inversely control the numerical strength of \( \zeta \). Each of them exhibit
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Fig. 1. (color online) $\zeta$ as a function of temperature at vanishing chemical potential. Results of upper and lower panels are obtained from Eqs. (5) and (6), which are based on sum rule and quasi-particle approaches of Kubo relation (RTA in Kinetic theory) respectively.

visible peak structures near the transition temperature $T_c \sim 169$ MeV. The temperature dependence of quantities like $\epsilon - 3P$ and $\langle m\bar{q}q \rangle_T$ are collectively responsible for exploring this peak structure. The former quantity is closely related with speed of sound $c_s$. The deviation of $c_s^2$ from its limiting value, $c_s^2 = 1/3$, measures the violation of conformality, which is basically determined from the term $\left(\frac{1}{3} - c_s^2\right)$, associated with the former quantity. The second quantity $\langle m\bar{q}q \rangle_T$ vanishes in the massless limit and exposes a conformally symmetric medium. Hence, non-zero values of both quantities make a link between the non-zero values of $\zeta$ and violation of conformality. The peak structure of $\zeta$ near $T_c$ indicates maximum breaking of conformal symmetry.

Next, let us come to the results of $\zeta(T, \mu = 0)$ from the Eq. (6), which are plotted
in the lower panel of Fig. 1. Similar to $\omega_0$ of Eq. (5), $\Gamma = 1/\tau$ in Eq. (6) inversely controls the numerical strength of $\zeta$. We have chosen two certain values of $\Gamma$ or $\tau$, which can approximately reproduce the $\zeta$ in same order of magnitude as done by $\omega_0 = 1$ and 1.5 GeV in earlier case. On the basis of this approximate matching, we may get an equivalence between two parameters:

$$\tau = 4 \text{ fm or, } \Gamma = 0.049 \text{ GeV} \equiv \omega_0 = 1.5 \text{ GeV}$$

$$\tau = 7 \text{ fm or, } \Gamma = 0.028 \text{ GeV} \equiv \omega_0 = 1 \text{ GeV} , \hspace{1cm} (7)$$

which are coming from two different approaches. In this context, the reader should focus only on the peak structures near the transition temperature in both approaches because their numerical strength (peak strength) are basically matched. Although, the numerical values of $\zeta$ at temperatures away from $T_c$ are quite different in two approaches. The possible reason is discussed in the next paragraph.

One of the major difference between the two expressions of $\zeta$ is that the conformal breaking terms are folded by (PNJL) distribution function of quark in the RTA expression (8) but it is absent in the LET expression (9). Now, this (PNJL) distribution function of quark always has a suppressing effect at low $T$ after the folding or integration operation. So, this is the mathematical reason for lower values of $\zeta(T)$ at $T < T_c$ in RTA approach than that of sum rule approach. One may expect this kind of folding by the quark distribution function in the LET expression of $\zeta$ if one consider some different kind of ansatz for this spectral density, which may contain this kind of folding. In ref. [48] it is distinctly mentioned that a possible uncertainty in this sum rule approach may be appeared from this ansatz for the spectral density.

In the RTA approach, the terms of conformal symmetry breaking are $(1 - c_s^2)$ and $d_q / d\beta (\beta^2 m_q^2)$. Similar to earlier case, one can check again that both of the terms vanish in the massless limit. The role of $d_q / d\beta (m_qq)_T$ in the sum rule approach is
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Fig. 3. (color online) $\zeta$ as a function of temperature at vanishing chemical potential in the sum rule (upper panel) and quasi-particle (lower panel) approaches. For comparison, Lattice-QCD values of $\zeta$ (squares with error bars) from Ref. [71] are pasted.

equivalently played by the term, $\frac{d}{d\beta} (\beta^2 m^2) = m^2 + T m \frac{dm}{dT}$, of the quasi-particle approach. To understand the temperature dependence of these quantities, we have plotted quark masses (lower panel) and their temperature derivative (upper panel) in the left-hand side of Fig. (2). The peak structures of $\zeta$ are built by the peak structures of $\frac{dm}{dT}$, which basically represent the transition between broken and restored phases of chiral symmetry. In the right hand side of Fig. (2), the entropy density $s$ (upper panel) and the square of speed of sound $c_s^2$ (lower panel) are plotted against temperature axis. Entropy density at high $T$ domain reach to Stefan-Boltzmann limit (SB-limit), $s_{SB} = \frac{3\pi^2}{2} T^3$, under 2+1 flavor consideration. The $s/s_{SB}$ is plotted in the upper-right panel of Fig. (2), where red dotted line is denoting the SB
Fig. 4. (color online) $\zeta$ (upper panel) and $\frac{1}{\zeta}$ (lower panel) as a function of temperature for non-zero quark chemical potential.

limit $s/s_{SB} = 1$. Being inversely related with $\frac{ds}{dT}$, $c_s^2$ exhibits a dip near $T_c$, where rate of increment of $s(T)$ is larger. Hence one naturally gets a peak structure in $(\frac{1}{3} - c_s^2)$ near $T_c$, which contributes in the peak structure of $\zeta$. SB limit of $c_s^2$ is marked by red dotted line in the figure. To display the approach of the thermodynamical quantities towards their SB limit, we have plotted Fig. 2 up to $T = 400$ MeV to cover the high $T$ region.

Next, upper and lower panels of Fig. 3 show the temperature dependence of specific bulk viscosity $\zeta/s$, obtained from two different approaches with the same parameters, taken for $\zeta(T)$ in Fig. 1. In both approaches, we observe decreasing nature of $\zeta/s(T)$ in the region of $T > T_c$. Since $\zeta$ from sum rule approach is quite larger than that from quasi-particle approach at low temperature ($T < T_c$), so $\zeta/s$ increases to large numbers as one decreases the temperature at low temperature
Table 1. List of estimated values of $\zeta/s$ in earlier works at hadronic (2nd column) and quark (3rd column) temperature domain. Dynamics with references and nature of temperature dependence are described in 1st and 4th columns respectively.

| Dynamics | References | $T \leq T_c$ | $T \geq T_c$ | Nature of $T$ dependence |
|----------|------------|--------------|--------------|--------------------------|
| LQCD     | 47, 48, 71 | -            | $\approx 1 - 0$ | Decreasing               |
| HRG      | 67         | $\approx 0.02 - 0.03$ | -            | Decreasing               |
| HRG + HS | 67         | $\approx 0.02 - 0.1$  | -            | Increasing               |
| HRG      | 63         | $\approx 0.01 - 0.035$ | -            | Increasing               |
| HRG + HS | 67         | $\approx 0.025 - 0.12$ | -            | Increasing               |
| HT       | 42         | -            | $\approx 0.002 - 0.001$ | Decreasing               |
| NJL      | 51         | $\approx 0.9 - 0.02$  | $\approx 0.02 - 0.002$ | Decreasing               |
| NJL      | 53         | $\approx 1.7 - 0.13$  | $\approx 0.13 - 0.005$ | Decreasing               |
| NJL      | 53         | -            | $\approx 0.1 - 0.01$  | Decreasing               |
| NJL      | 56         | $\approx 0.61 - 0.11$ | $\approx 0.11 - 0.004$ | Decreasing               |
| LSM      | 57         | $\approx 0.61 - 0.11$ | $\approx 0.11 - 0.004$ | Decreasing               |
| Unitarization | 68   | $\approx 0.04 - 0.027$ | -            | Decreasing               |
| HRG      | 63         | $\approx 0.15 - 0.025$ | -            | Decreasing               |

domain ($T < T_c$). That is why the low $T$ results have not been shown in the upper panel of Fig. [3]. In the lower panel of Fig. [3], results of quasi-particle approach show peak structures at $T_c$ and hence, a discrepancy between the quantitative nature of $\zeta/s(T < T_c)$ in two approaches are observed. This is because the $\zeta(T < T_c)$ from quasi-particle approach is quite smaller than that from sum rule approach. The $\zeta/s$ at $T \geq T_c$ are more or less same in both approaches and quite satisfactorily agree with LQCD results, obtained in Ref. At high temperature domain, $\frac{s}{\zeta}$ in both approaches are tending to be zero. It indicates that both methodologies are obeying the general fact of high temperature QCD, which approaches towards the massless limit to attain its conformal symmetric behavior.

We have extended our investigation for finite quark chemical potential by using Eq. (5) of sum rule approach in Kubo framework. In the upper panel of Figure (4), $\zeta$ is plotted for three different values of quark chemical potential $\mu_q = 50, 100, 150$ MeV keeping charge and strangeness chemical potentials ($\mu_Q$ and $\mu_S$ respectively) fixed at zero and $\omega_0 = 1$GeV. It is seen that with increase of $\mu_q$, peaks of $\zeta$ are shifted towards lower $T$ with higher numerical strength. For different values of chemical potential, the transitions take place at different temperatures in the QCD phase diagram, which causes shifts in the peak positions of bulk viscosity.

The lower panel of Fig. [4] shows the results for $\frac{s}{\zeta}$ as a function of $T$ considering same set of values of chemical potentials, where we again see the increasing nature of $\frac{s}{\zeta}$ with increasing of $\mu$. Similar trend was observed for shear viscosity to entropy density ratio in earlier studies based on the same PNJL dynamics.

To understand our results with respect to earlier works, we have made the table which contain approximate numerical values of $\zeta/s$, extracted from some earlier works. The 1st column contains the information of dynamics for different works and next, the values of $\zeta/s$ at hadronic and quark temperature domain for vanishing quark or baryon chemical potential are put in
and 3rd columns respectively. The nature of the function, $\zeta/s(T)$ for different estimations are mentioned in the last column. We have also divided the table into two part, where references of upper part have used the expression of $\zeta$, based on sum rule approach of Kubo formalism and the references of lower part have used the RTA or quasi-particle expression of $\zeta$. Now most of the works observed decreasing nature of $\zeta/s(T)$ in both $T < T_c$ and $T > T_c$ regime as well as in entire range of $T_{50, 51, 54, 56, 57}$. Our results, based on the Kubo expression of sum rule approach, are supporting this decreasing nature of $\zeta/s(T)$. In numerical point of view, one can relate our results to the divergence nature of $\zeta/s$ near $T_c$, as observed in Refs. $47, 48, 71$. Now, on the other hand, the RTA expression is exhibiting a peak structure in the plot of $\zeta/s$ vs $T$. This kind of peak structure is also observed in the Linear Sigma Model (LSM) calculations $50, 57$ as well as in NJL model calculations $55$. We may assume indication of similar kind of peak structure from the increasing nature of $\zeta/s(T < T_c)$, observed in Refs. $63, 67$ whose calculations are based on the Hadron Resonance Gas (HRG) model without $63$ and with $63, 67$ Hagedorn States (HS). So our investigations on two different expression of $\zeta$ in PNJL model is revealing a qualitative discrepancy between the $\zeta/s(T < T_c)$ of two approaches, because of quantitative differences in their $\zeta(T < T_c)$. However, our $\zeta/s(T > T_c)$ are qualitatively very similar and for present set of values ($\omega_0 = 1, 1.5$ GeV and $\Gamma = 0.049, 0.099$ GeV) in both approaches match quite satisfactorily with results, obtained by Refs. $47, 48, 71$.

4. Summary

In the framework of PNJL model with 2+1 flavor consideration, we have gone through a numerical investigation of two different expressions for bulk viscosity, based on two different approaches. One is Kubo formalism in sum rule approach and another is the quasi-particle approach of Kubo framework or the relaxation time approximation in kinetic theory approach. We have computed the temperature dependence of $\zeta$ and $\zeta/s$ for vanishing quark chemical potential in both approaches, where peak structure in $\zeta$ near $T_c$ and the decreasing nature of $\zeta/s(T)$ (particularly at $T \geq T_c$) are their common numerical outcomes. The peak structure in $\zeta$, which is interpreting the maximum breaking of conformal symmetry, is basically originated from two important quantities, associated with speed of sound and temperature dependent quark masses, which are adopted in both approaches. However, two approaches contain two individual parameters, closely related with spectral width of Kubo formalism, which inversely control the peak strength of $\zeta$. We have tried to observe their role for equivalence in $\zeta$. At low temperature domain ($T < T_c$), we have noticed a numerical discrepancy in $\zeta(T)$ for these two approaches. The main reason is that the RTA expression contains the effect of folding by (PNJL) distribution function of quark, while it is absent in LET expression.

Our finite density results of bulk viscosity are indirectly reflecting the fact of QCD phase diagram - the transition occurs at lower temperature as quark chemical
potential becomes non-zero and increases. This can be realized from the shifting of peak position in $\zeta$ towards the lower temperature, when we increase quark chemical potential.

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