Market Dynamics and Indirect Network Effects in Electric Vehicle Diffusion

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Abstract—The diffusion of electric vehicles (EVs) is studied in a two-sided market framework consisting of EVs on the one side and EV charging stations (EVCSs) on the other. A sequential game is introduced as a model for the interactions between an EVCS investor and EV consumers. A consumer chooses to purchase an EV or a conventional gasoline alternative based on the upfront costs of purchase, the future operating costs, and the availability of charging stations. The investor, on the other hand, maximizes his profit by deciding whether to build charging facilities at a set of potential EVCS sites or to defer his investments.

The solution of the sequential game characterizes the EV-EVCS market equilibrium. The market solution is compared with that of a social planner who invests in EVCSs with the goal of maximizing the social welfare. It is shown that the market solution underinvests EVCSs, leading to slower EV diffusion. The effects of subsidies for EV purchase and EVCSs are also considered.

Index Terms—Two-sided market; indirect network effects, product diffusion, electric vehicles, EV charging services.

I. INTRODUCTION

The diffusion of electric vehicles in the United States has had mixed results so far. Annual new EV sales increased nearly 7-fold from about 18,000 in 2011 to 119,000 in 2014. However, the market share of EVs was only about 0.73% in the new vehicle market [1]. The reason behind the growth of the EV market, or the lack of it, is multifaceted. The growth is driven partially by the increasing awareness of the environmental impacts of gasoline vehicles (GVs), the superior design and performance of some EVs, and, by no small measure, the subsidies in the form of tax credits provided by the federal and state governments. On the other hand, the EV industry faces stiff skepticism due to the high purchase cost of EVs, the limited driving range, and the lack of adequate public charging facilities.

A similar trend exists in the deployment of public charging services. By the end of 2014, the U.S. has built about 9000 charging stations with about 22,000 charging outlets [2], due in part to the direct and indirect investments of federal and local governments. For example, the Department of Energy (DoE) of the United States has provided 230 million dollars in 2013 to establish 13,000 charging stations [3]. It is hoped that such investments will stimulate the EV market, driving its market share toward long-term growth and stability.

Fig. 1: EVs and public charging stations in Metropolitan Statistical Areas in 2013 [1].

A. Summary of results

The growth trends of EVs and EVCSs have strong temporal and geographic couplings as shown in Fig. 1. This could be in part due to the indirect network effects: consumers’ EV adoption on one side of the market is affected by the availability of charging stations, on the other, whereas the level of EVCS investment is affected by the size of EV stock.
the EVCS investment? How do the indirect network effects affect the market dynamics? What are the implications of indirect network effects on the public policy?

We introduce a perfect and complete sequential game model for the two-sided EV-EVCS market with an investor as the leader and each consumer the follower. Through the profit maximization, the investor decides whether to build EVCSs at sites chosen (optimally) from a list of candidate sites or to defer his investment with an earned interest. The candidate sites are heterogeneous; each site may have a different favorability rating and different capital costs. The optimal investment decision also includes the optimal pricing of charging services.

Observing investor’s decision which defines the locations of EVCSs and the charging prices, the consumer decides whether to purchase an EV or a gasoline alternative. If the choice is an EV, the consumer also decides the preferred charging service.

We provide a solution to the sequential game that includes the optimal decisions for the consumers and the investor. A random utility maximization (RUM) model [4] with two different distributions of the consumer vehicle preference is considered.

Under the assumption of the type I extreme value preference distribution, we show that the optimal purchasing decision is a threshold policy on the consumer vehicle preference difference. A closed-form expression for the EV market share \( \eta_c \) is obtained, where \( \eta_c \) is a function of the EV price, the investor’s decisions on the number/locations of charging stations, and the charging prices at those locations. The obtained closed-form solution allows us to examine how the investor’s decisions and the cost of EVs affect the overall EV market share.

To obtain the optimal investment decision, we first study the optimal operation decision of the investor by fixing the set of EVCS sites. We show that the optimal pricing for EV charging at these sites is such that profits generated from these sites are equal. We show further that the optimal pricing increases logarithmically with the density of EV charging sites.

The optimal decision on choosing which EVCS sites to build (or deferring investment) is more complicated and is combinatorial in nature. We provide a greedy heuristic and show that the heuristic is asymptotically optimal as the density of EVCS sites increases.

Under the assumption of the uniform vehicle preference, similar results are obtained in a conference version of the present paper [5]. The optimal purchasing decision of consumers is a threshold policy on the vehicle preference and a dead zone effect on EVCS density is observed. Specifically, when the density of EVCSs is lower than some threshold, the EV market share is zero.

B. Related works and organization

There is an extensive literature on two-sided markets and indirect network effects for various products; see e.g., the compact disc (CD) player and CD title markets [6], the video game console and video game markets [7, 8, 9], the hardware and software markets [10], the credit card market [11], and the yellow page and advertisement markets [12]. Rochet and Tirole in [13] firstly proposed a restrictive definition to distinguish between one-sided and two-sided markets in the context of charge per usage. Caillaud and Jullien pointed out in [14] that, one side of the market always waits for the action from the other side. It is thus critical for players to take the right move during the initial stages of the product diffusion.

There is a growing literature on the EVCS investment from the operation research and engineering perspectives. For example, the charging station deployment has been formulated as an optimization problem from the social planner’s point of view in [15, 16, 17]. A location competition problem of charging stations is considered in [18], where a discrete decision model similar to this paper is used. Efficient design of large scale charging is presented in [19] and the competition of charging operations is considered in [20].

The work of Li et al. [21] and the current paper represent the first analysis of the two-sided EV and EVCS market and the related indirect network effects. The work in [21] focuses on the empirical study of indirect network effects whereas the current paper focuses on the theoretical analysis. This paper builds upon and extends the work of [5]. In particular, this paper extends the model in [5] by allowing (unobserved) vehicle preferences to have a type I extreme value distribution consistent with the discrete choice model [22]. Also new in this paper are the comparison between the market solution and the decisions of the social planner and the effects of subsidies for EV purchase and EVCS investments.

This paper is organized as follows. The structure of the two-sided market and a sequential game model are described in Sec. [II] The solution to the game is obtained by backward induction. In Sec. [III] the consumers’ model and the optimal decisions are stated. The investor’s model and optimal strategy are presented in Sec. [IV] as well as the social welfare optimization. Discussions about different effects of subsidies and the difference between the private market solution and the socially optimal solution are presented in Sec. [V] Sec. [VI] concludes the paper.

II. A Sequential Game Model

In this section, we formulate the two-sided market as a two-player sequential game with perfect and complete information. We first introduce the basic structure of the EV-EVCS market, define the players of the game, and specify the decision process.

A. Two-sided market structure

A two-sided market typically has a structure as illustrated in Fig. 2, where we use a generic hardware-software market as an example to describe its basic components.

A two-sided market includes a set of platforms, say, MacBook™ and the OS X operating system as a hardware-software platform by Apple Inc. vs. Dell’s Inspiron™ and the Windows 8 as an alternative.

On the one side of the platforms is the consumer who makes her purchase decisions based on her platform preferences, the costs of the platforms, and the available softwares for different
platforms. On the other side of the platforms are the software developers who invest time and money in developing softwares for one particular platform or multiple platforms. The software developer makes his decision based on, among other factors, the costs of developing softwares and the popularity of platforms.

For the two sided EV-EVCS market studied in this paper, we consider two platforms: one is the EV as the hardware and the EVCS as the software. The other is the traditional gasoline vehicle as the hardware and the gas station as the software. On the one side of the platforms are the consumers who decide which type of vehicles to purchase based on the cost of EV, the available charging stations, and the cost of charging. On the other side of the platforms is an investor who decides to build and operate charging stations or to defer his investment and earn interest at a fixed rate.

**B. The investor’s decision model**

We assume that the investor is both the builder and the operator of the EVCSs. The investor’s decision has two components: the first is an investment decision on whether to build EVCSs from a list of candidate sites or to defer his investment. The second is an operation decision on pricing the charging services at those built locations.

Let $\mathcal{C} = \{s_1, \ldots, s_N\}$ be the set of candidate sites for charging stations known to the investor. Site $s_i = (f_i, c_i)$ has two attributes: the favorability rating $f_i$ and the marginal operating cost $c_i$. For example, a site at a shopping center may be more attractive than a location that is less frequently visited by consumers. The costs of building and operating such sites are also different.

Given $\mathcal{C}$ and the utility functions of the consumer, the investor’s decision is denoted by $(\mathcal{C}, \tilde{\rho}) \in 2^\mathcal{C} \times \mathbb{R}^{|\mathcal{C}|}$ where $\mathcal{C} \subseteq \mathcal{C}$ is the set of locations selected to build charging stations and $\tilde{\rho} = (\rho_1, \ldots, \rho_{|\mathcal{C}|}) \in \mathbb{R}^{|\mathcal{C}|}$ the vector of charging prices at the built stations.

Assuming the consumer maximizes her utility, the investor chooses the investment sites and charging prices to maximize the investment profit within his budget $B$. The investment optimization is stated as

$$\max_{\mathcal{C}, \tilde{\rho}} \quad \Pi(\mathcal{C}, \tilde{\rho}) - \sum_{i=1}^{|\mathcal{C}|} F(s_i)$$

subject to

$$\sum_{i=1}^{|\mathcal{C}|} F(s_i) \leq B$$

where $\Pi$ is the operational profit and $F(s_i)$ the building cost of station $i$.

**C. The consumer’s decision model**

A consumer observes the investor’s decision on the location set of charging stations $\mathcal{C} = \{s_1, \ldots, s_{N_E}\}$ and the charging price vector $\tilde{\rho} = (\rho_1, \ldots, \rho_{N_E})$ where $N_E$ is the number of charging stations. The consumer first chooses the type of vehicle to purchase. If the choice is an EV, the consumer also decides on the location of charging. The action of the consumer is given by $\{\nu, k\}$ where $\nu \in \{E, G\}$ is the vehicle choice (either EV or GV) and $k \in \{0, 1, \ldots, N_E\}$ the preferred charging station. We include $k = 0$ for the home charging option. The consumer chooses $\{\nu, k\}$ by maximizing the overall vehicle utility that includes the charging utility for the EV purchase.

For the vehicle choice, we assume a widely adopted discrete choice model with random utility functions [18]. The consumer utility model of purchasing a vehicle is assumed as follows.

$$V_E = \beta_1 \mathbb{E}(U_E) - \beta_2 p_E + \Phi + \epsilon_E$$

$$V_G = \beta_1 \mathbb{E}(U_G) - \beta_2 p_G + \Phi + \epsilon_G$$

where $U_E$ is the (random) charging utility of consumer’s best choice defined in (5), $\mathbb{E}(U_E)$ the expected maximum charging utility, $p_E$ the price of an EV, $\Phi$ the utility of owning a vehicle, and $\epsilon_E$ a random vehicle preference of EV. Variables $\mathbb{E}(U_G)$, $p_G$, and $\epsilon_G$ are similarly defined for the gasoline vehicle. The consumer’s decision is then defined by

$$\max \{V_E, V_G\}.$$
D. The sequential game model

The sequential game structure of the two-sided EV-EVCS market is summarized as follows.

- The investor’s decision is defined by the optimization in (1). Specifically, given the set of locations, \( \mathcal{C} \), the investor decides to invest (build and operate) charging stations at a subset \( \mathcal{C} \subseteq \mathcal{C} \) and determines the charging price \( \hat{p} \).
- When \( \mathcal{C} = \emptyset \), the investor defers his investment and earns interest at a fixed rate.
- The consumer’s decision is defined by (3-5). Specifically, having observed the investor’s decision, \( \{ C, \hat{p} \} \), the consumer chooses \( \nu \in \{ E, G \} \). If \( \nu = E \), the consumer also chooses charging stations to charge by maximizing her charging utility.

The dynamic game is solved by backward induction. In particular, we first consider the consumer’s decision by fixing the investor’s choice of charging locations and charging prices. The optimal consumer’s decision is given in Sec. III. In Sec. IV the optimal investor’s decision is presented.

III. CONSUMER DECISIONS

A. Consumer Decision Model and Assumptions

We first summarize the assumptions on the consumer model given in Sec II-C.

A1. Consumers are identical and their decisions are statistically independent. Without loss of generality, we focus on the decision of a single consumer.

A2. The average charging demand is normalized to 1.

A3. The random preference of charging station \( i \), \( \epsilon_i \), is independent and identically distributed (IID) and follows the type I extreme value distribution with the probability density function (PDF)

\[
f(\epsilon) = e^{-\epsilon}e^{-e^{-\epsilon}}.
\]

A4. The random preference of vehicles \( \epsilon_E \) and \( \epsilon_G \) are statistically independent.

The type I extreme value distribution is widely used in the discrete choice model. McFadden firstly introduced it in the consumer choice theory and showed it leads to the multinomial logit distribution across choices.

B. Consumer Decisions and EV Market Share

The main result in this section is the structure of the optimal vehicle decision and the characterization of the EV market share as shown in the following theorem.

**Theorem 1 (Consumer choice):**

1. If the vehicle preferences, \( \epsilon_E \) and \( \epsilon_G \), follow the type I extreme value distribution, the optimal consumer decision is a threshold policy on the difference of the vehicle preferences \( \epsilon_E - \epsilon_G \):

\[
\begin{align*}
\text{if } \epsilon_E - \epsilon_G \geq \tau_e & \quad \text{purchase electric vehicles} \\
\text{if } \epsilon_E - \epsilon_G < \tau_e & \quad \text{purchase gasoline vehicles}
\end{align*}
\]

where

\[
\tau_e = \beta_1 \mathcal{E}(U_G) - \beta_2 p_{G} - \beta_1 \ln \left( \sum_{i=0}^{N_E} \exp(\alpha_i f_i - \alpha_2 p_i) \right) + \beta_2 p_E.
\]

The EV market share is given by

\[
\eta_e = \frac{q^\beta_1}{q^\beta_1 + C},
\]

where \( q = \sum_{i=0}^{N_E} \exp(\alpha_i f_i - \alpha_2 p_i) \), \( C = \exp(\beta_1 \mathcal{E}(U_G) - \beta_2 p_{G} + \beta_2 p_E) \).

2. If the vehicle preference of EV is uniformly distributed with \( \epsilon_E \sim U(0, 1) \) and \( \epsilon_G = 1 - \epsilon_E \), the optimal consumer decision is a threshold policy on the realization of the consumer preference \( \epsilon_E \):

\[
\begin{align*}
\text{if } \epsilon_E \geq \tau_u & \quad \text{purchase electric vehicles} \\
\text{if } \epsilon_E < \tau_u & \quad \text{purchase gasoline vehicles}
\end{align*}
\]

where

\[
\tau_u = \left\{ \left[ \beta_1 \mathcal{E}(U_G) - \beta_2 p_{G} - \beta_1 \ln \left( \sum_{i=0}^{N_E} \exp(\alpha_i f_i - \alpha_2 p_i) \right) + \beta_2 p_E + 1 \right] / 2 \right\}^0.
\]

The EV market share is given by

\[
\eta_u = 1 - \tau_u.
\]

3. Under both assumptions, the charging service market share captured by charging station \( i \) is given by

\[
P_t = \frac{\exp(\alpha_i f_i - \alpha_2 p_i)}{\sum_{k=0}^{N_E} \exp(\alpha_k f_k - \alpha_2 p_k)} \geq q_i.
\]

**Proof:** To derive the optimal consumer vehicle decision from (13), we first compute the expected maximum charging utility from (5) using the type I extreme value distribution of \( \epsilon_i \). Specifically,

\[
\mathbb{E}(U_E) = \ln \left( \sum_{k=0}^{N_E} \exp(\alpha_k f_k - \alpha_2 p_k) \right) \geq \ln \left( \sum_{k=0}^{N_E} q_k \right) = \ln(q),
\]

where \( q_k = \exp(\alpha_k f_k - \alpha_2 p_k) \).

Next, by substituting \( \mathbb{E}(U_E) \) into (2), the consumer’s optimal vehicle choice is given by a threshold policy on \( \epsilon_E - \epsilon_G \).

In particular, the consumer purchases an EV if

\[
\epsilon_E - \epsilon_G \geq \beta_1 \mathcal{E}(U_G) - \beta_2 p_{G} - \beta_1 \ln \left( \sum_{i=0}^{N_E} \exp(\alpha_i f_i - \alpha_2 p_i) \right) + \beta_2 p_E.
\]

Under the assumption of uniform distribution, by substituting \( \epsilon_G = 1 - \epsilon_E \) into (12), we have (8).

From (22) (chapter 4), the EV market share and the EVCS market share are given by (7) and (10).

With Theorem 1 we can examine the trend of EV market share as a function of the charging station density, the charging price, and the price of EV.

First, under both assumptions, the expressions of \( \eta_e \) and \( \eta_u \) indicate that the EV market share is an increasing and concave function of charging station number \( N_E \), which is also validated in Fig. 3 and 4. The concavity of market share
implies that the marginal effect of building charging facilities decreases. In addition, the market share accelerates faster to 1 with a lower EV price.

Under the uniform distribution assumption, the market share, $\eta_u$, has a dead zone effect in the density of charging stations. Specifically, there is a critical $N_E$ below which the market share is zero. With a lower EV price, the market share escapes from the dead zone and achieves 1 faster. As shown in Fig. 4, the critical density of charging stations grows as a “convex” function of EV price.

Under the extreme value distribution assumption, there is no dead zone effect. The EV market share $\eta_e$ is always positive. However, if we treat $\eta_e \leq 5\%$ as a launch failure of EV, there is a critical density below which EV is considered failed. In Fig. 5 the critical density of EVCSs is shown to has a “convex” shape in terms of EV prices.

IV. INVESTOR DECISIONS

After the discussion about the consumer model and her decision, we now focus on the investor decision model that includes the selection of the charging stations locations and the optimal pricing of charging.

A. Investor Decision Model and Assumptions

We make the following assumptions about the investor model:

B1. We consider a single investor who also operates all charging stations. This implies the monopolistic competition in the charging service market.

B2. We assume that the deferred investment earns interest at a rate of $\gamma$.

B3. The investor knows the utility functions of the consumers.

To solve the optimization in (13), we proceed with backward induction: in Sec IV-B we find the optimal pricing with fixed EVCS locations, and in Sec IV-C we optimally choose the charging station locations.

B. Optimal Charging Price

Given the set of charging station locations $C$, the investor determines the optimal charging price $\hat{\rho}$ to maximize the total operation profit. Specifically, the investor has the following optimization.

$$\max_{\hat{\rho}} \Pi = \max_{\hat{\rho}} \sum_{i=1}^{N_E} P_i(\hat{\rho})(\rho_i - c_i)$$

where $\eta(\hat{\rho})$ is the expected EV market share given in Theorem 1 (here we make the dependency on charging price explicit), $P_i(\hat{\rho})$ the market share of station $i$, i.e., the fraction of EV owners who charge at station $i$, and $c_i$ the marginal
convenient which not only attracts consumers to purchase EVs but also encourages them to charge outside home. The EV market share $\eta_j \to 1$ and the fraction of charging at home $P_0(\hat{\rho}^*_j) \to 0$. Based on this trend, we have the convergence of the marginal charging profit shown in the following theorem.

**Theorem 3 (Charging price convergence):** For fixed set of charging stations $C = \{(f_i, c_i), i = 1, \cdots, N_E\}$, the across charging stations uniform profit, $r_j \triangleq \rho^*_{t,j} - c_i$, $j = e, u$, has the following convergence.

1) Under the type I extreme value vehicle preference distribution assumption,

$$
\lim_{v \to \infty} v \partial r_e / \partial v = 1/\alpha_2,
\lim_{v \to \infty} r_e / \ln v = 1/\alpha_2; \tag{16}
$$

2) Under the uniform vehicle preference distribution assumption,

$$
\lim_{v \to \infty} r_u = 2/\alpha_2 \beta_1, \tag{17}
$$

where $v = \sum_{i=1}^{N_E} v_i = \sum_{i=1}^{N_E} \exp(\alpha_1 f_i - \alpha_2 c_i)$ is the sum of the exponential systematic utility.

**Proof:** See Sec. VII-A.

Under the type I extreme value vehicle preference distribution assumption, the charging profit is strictly increasing when the number of charging facilities goes to infinity and the increasing rate is decreasing to zero. Under the uniform vehicle preference distribution assumption, the profit converges to a constant. In both cases, when $\alpha_2$ increases, consumers are more sensitive to the charging price, the profit decreases to $0$.

### C. Optimal Charging Station Locations

After the discussion about the optimal charging price, we consider the choice of charging station locations. Given the set of location candidates $C = \{s_i = (f_i, c_i), i = 1, \cdots, N_L\}$, the investor has the following optimization.

$$\max_{C \subseteq \mathcal{C}} \left\{ \Pi(C, \rho^*_j(C)) - \sum_{i=1}^{\vert C \vert} F(s_i) \right\} \tag{18}$$

subject to $\sum_{i=1}^{\vert C \vert} F(s_i) \leq B$.

where $F(s_i)$ is the building cost of charging station $s_i$ and $\Pi(C, \rho^*_j(C))$ the operational profit.

In general, the optimal investment decision from (18) requires combinatorial search for $C$, which is not tractable. However, the convergence of the optimal charging prices across charging stations in Theorem 3 makes it possible to separate the price decision and the location choice, which leads to a linear complexity heuristic algorithm.

The Greedy Investment Algorithm (GIA) given in Algorithm 1 first ranks the charging stations by the exponential systematic part of the charging utility, $v_i = \exp(\alpha_1 f_i - \alpha_2 c_i)$. It then adds charging stations to the investment list one at a time in the decreasing order of exponential systematic utility $v_i$ until either the budget is exhausted or the cumulated profit starts to decrease.

By ignoring the dependency of charging locations in the marginal charging profit $\rho^*_{t,j} - c_i$ in (18), the GIA is not optimal in general. As $N_E$ increases, however, the marginal
charging profit increases and converges, which makes the algorithm asymptotically optimal.

Theorem 4 (Asymptotic optimality): If the building costs of charging stations are constant, i.e., $F(s_i) = (1 + \gamma)F_0$ (the interest rate), the greedy algorithm is asymptotically optimal (as $N \to \infty$) under both the type I extreme value distribution and the uniform distribution assumption.

**Proof:** See Sec. VII-B.

After obtained the optimal set of charging stations $C^*$ and the optimal charging price vector $\tilde{\rho}_j^*$, the investor will make the investment if the investment profit $(\Pi(C^*, \tilde{\rho}_j^*) - \sum_{i=1}^{\lvert C^* \rvert} F(s_i))$ is positive. Otherwise, the investor will defer his investment and earn interest at rate $\gamma$.

To make $(\Pi(C^*, \tilde{\rho}_j^*) - \sum_{i=1}^{\lvert C^* \rvert} F(s_i))$ positive, the EV price and the building costs of charging stations need to be low enough, which implies that the subsidies for EV purchase and charging stations are necessary to the successful launch of EV.

**D. Social welfare optimization**

We now consider the difference between the solution of the private market defined in Sec. IV.C and that of a social planner who makes investment decisions based on social welfare maximization.

Recall the investor utility $S_I(C, \tilde{\rho}_j)$ and the consumer utility $S_C(C, \tilde{\rho}_j)$ given in (1) and (2):

\[
S_C(C, \tilde{\rho}_j) = E(\max\{V_E(C, \tilde{\rho}_j, \epsilon_E), V_G(\epsilon_G)\}),
\]

\[
S_I(C, \tilde{\rho}_j) = \Pi(C, \tilde{\rho}_j) - \sum_{i=1}^{\lvert C \rvert} F(s_i).
\]

Under the type I extreme value vehicle preference distribution assumption, the consumer utility is stated as

\[
S_C(C, \tilde{\rho}_e) = \ln\left[\sum_{i=0}^{\lvert C \rvert} \exp(\alpha_1 f_i - \alpha_2 \rho_e)\right] \beta_1 C_1 + C_2],
\]

where $C_1 = \exp(-\beta_2 p_E + \Phi)$ and $C_2 = \exp(\beta_1 E(U_G) - \beta_2 p_G + \Phi)$.

Under the uniform vehicle preference distribution assumption, the consumer utility is stated as:

\[
S_C(C, \tilde{\rho}_u) = [\eta_u(C, \tilde{\rho}_u)]^2 + \beta_1 E(U_G) - \beta_2 p_G + \Phi - \frac{1}{2}.
\]

Assume that the social planner can not determine the charging price or the vehicle price, he can only determine the set of charging stations to build. The social planner’s decision is stated as:

\[
\max_{C \subseteq \mathcal{C}} \{S_C(C, \tilde{\rho}_j(C)) + S_I(C, \tilde{\rho}_j(C))\}
\]

subject to $\sum_{i=1}^{\lvert C \rvert} F(s_i) \leq B$,

where $\tilde{\rho}_j(C)$ is the optimal charging price vector determined by the charging station operators given the charging station locations.

The GIA can also be applied to solve for the social welfare optimized investment in charging stations. The following theorem characterizes the difference between the social welfare optimal solution and the market solution.

**Theorem 5 (Social welfare):** Let $C^*$ be the optimal set of charging stations determined by the investor, and assume $\lvert C^* \rvert \gg 1$. Let $C^{**}$ be the optimal charging locations determined by the social planner. Under both the type I extreme value distribution and the uniform distribution assumptions, $\lvert C^{**} \rvert > \lvert C^* \rvert$.

**Proof:** See Sec. VII-C.

Theorem 5 implies that the monopolistic market solution tends to under-build charging stations. The under-provision of EVCSs and lower adoption of EVs relative to the socially optimal outcomes are due to two types of market failures: market power and indirect network effects (or externalities). We assume a monopoly in the EVCS provision. This will lead to under-provision of EVCSs and a higher charging price than a competitive solution. This will in turn lead to a lower EV adoption. Therefore introducing competition in EVCS provision will help EV diffusion. While this form of market failure is a result of our model setup and can be relaxed, the second form of market failure is inherent in the EV market as empirically confirmed in [20]. Indirect network effects are externalities which are not accounted for in individual investment and purchase decisions. They will lead to a wedge in socially optimal outcomes and market outcomes, which justifies government interventions. For example, government can provide subsidies to charging station investors as the U.S. DOE does through various funding programs or mandate the provision of charging stations in real estate development as recently implemented in China [23].

**V. DISCUSSION**

**A. Effects of subsidy**

We consider here the effects of subsidy, either to EV consumers or to the investor of EV charging stations. The results obtained in Sec. III and Sec. IV provide the basis for the numerical results presented here. In the simulation, coefficients from [21] are used and the type I extreme value vehicle preference distribution case is considered.

Fixing the total policy budget as 230 million dollars, we vary the weight of subsidy for EV purchase among the total policy budget. A bisection algorithm is applied to search for
the subsidy amounts for each EV and EVCS so that the constraints of total budget and budget weight are satisfied.

In Fig. 7 the EV market share against subsidy weight with different values of $\beta_2$ is plotted. In the utility model (2), $\beta_2$ represents the consumer sensitivity to the EV price. When $\beta_2$ is large, consumers care about the characteristics of charging facilities less than the EV price. Increasing the EV subsidy dramatically boosts up the EV market share. On the other hand, when $\beta_2$ is close to 0, consumers mainly concern about the charging services and the subsidy for EV purchase plays a tiny role in the EV market share evolution.

Similar impact exists in the EVCS market. As shown in Fig. 8 when $\beta_2$ is big, large EV subsidy weight also drives the investor to invest more charging facilities because of the EV popularity. When $\beta_2$ is close to 0, putting more weight to EV subsidy discourages investment in EVCSs. Fewer charging facilities are invested thus more subsidies for EV purchase, on the contrary, draws down the EV market share.

B. Socially optimal solution vs. private market solution

In Sec. IV-D, the analytical result shows the socially optimal solution requires to build more charging facilities than the private market solution. In this section, a numerical result is presented to illustrate the difference.

As shown in Fig. 9 when the EV price is low, both solutions give a high market share of EV. When the EV price increases, the EV market share as a socially optimal solution drops slower than as a private market solution, the reason for which can be found in Fig. 10. In Fig. 10, when the EV price is low, both solutions give a large density of EVCSs. When the EV price increases, the charging demand shrinks because the EV market share drops. In the private market solution, less charging facilities will be built due to the decreasing operational profit. However, the social planner will build more charging facilities to maintain relatively high consumer utility.
More EVCSs attract more consumers to purchase EVs thus slow down the drop of the EV market share.

This is an intuitive result that at the EV launch stage, the social planner should focus on investing in EVCSs to satisfy consumers and maximize the social welfare because it is non-profitable for the market participants to build any.

VI. CONCLUSION

In this paper, the two-sided market problem of EV-EVCS is considered. A sequential game is formulated to analyze the indirect network effects between the charging station investor and consumers. The optimal operation decision of charging stations is shown as locational equal profit pricing. An asymptotic optimal algorithm of investment decision is proposed which reduces the computation complexity significantly. The social welfare optimization is discussed and it is shown that the socially optimal solution requires more charging stations than the market outcomes. The numerical results are presented to illustrate the impact of government subsidy policy and the difference between the private market solution and the socially optimal solution.

As an analytical approach to understanding the market dynamics of EV diffusion, this paper assumes a stylized model for both the consumers and the investor. Here we aim to capture major factors in the interactions between the consumers and the investor, including the EV price, the coverage of the charging stations, and the price of charging. Ignored in the model includes several nontrivial and practically significant factors. For instance, the price of EV is assumed exogenous, and the EV consumers and charging stations are mostly homogeneous (except that the favorability rating, the operating cost, and pricing of charging are different across locations). A multi-stage counter part of this work is to be reported in the future.

VII. APPENDIX

Here the proofs under the assumption of the type I extreme value vehicle preference distribution are presented. The uniform preference distribution proof can be found in [5]. The subscript $j$ is dropped in this section for convenience.

A. Proof of Theorem 3

In Theorem 3 the optimal charging price is shown to generate uniform profits across charging stations. Denote the uniform profit by $r \triangleq \rho_j - \epsilon_j$, the sum of the exponential systematic utility by $v = \sum_{i=1}^{N_E} v_i = \sum_{i=1}^{N_E} \exp(\alpha_j f_i - \alpha_j \epsilon_j)$ and the ratio of the utility and the exponential profit by $\kappa = v / \exp(\alpha_j r)$. Equation (12) can be rewritten as

$$g(v, r) \triangleq \frac{\alpha_j \beta_j r (1 - \eta)(1 - P_0) + \alpha_j \rho_0 - 1}{C v \exp(\alpha_j f_0 - \alpha_j \epsilon_0)} = \beta_j \ln(v/\kappa) \frac{C}{(\alpha_j f_0 - \alpha_j \epsilon_0) + \ln(v/\kappa) + \alpha_j \rho_0 - 1} - 1 = 0,$$

where $C = \exp(\beta_j E(U_C) - \beta_2 P_C + \beta_3 \rho_E)$ and $\epsilon_0 = \exp(\alpha_j f_0 - \alpha_j \epsilon_0)$. Note the first term in the left hand side of (19), $\alpha_j \beta_j r (1 - \eta)(1 - P_0)$, is positive. We can conclude that as $v \rightarrow +\infty$, $\kappa \rightarrow +\infty$. Otherwise, the second term $\ln(v/\kappa) \frac{\rho_0}{\alpha_j f_0 - \alpha_j \epsilon_0}$ will dominate which violates (19).

As the sum of exponential systematic utility $v$ increases, the charging service at the EVCSs is more convenient, thus more consumers tend to purchase EVs and prefer to charge at the public charging stations. $\kappa \rightarrow +\infty$ gives the convergence of the market share $\eta$ and $P_0$, which is stated as follows.

**Lemma 1:**

$$\lim_{v \rightarrow +\infty} \eta = \lim_{v \rightarrow +\infty} \left(\frac{(\alpha_j + \kappa)^2}{(\alpha_j + \kappa)^2 + \alpha_j \rho_0}\right) = 1,$$

$$\lim_{v \rightarrow +\infty} P_0 = \lim_{v \rightarrow +\infty} \frac{\rho_0}{\alpha_j f_0 - \alpha_j \epsilon_0} = 0. \quad (20)$$

**Proof:** Since $q_0$ and $C$ are constant, $\beta_1 > 0$ and $\kappa \rightarrow +\infty$, $\eta \rightarrow 1$ and $P_0 \rightarrow 0$.

A direct result of Lemma 1 is that the uniform profit $r \rightarrow +\infty$ as $v \rightarrow +\infty$. Otherwise (19) does not hold any more.

By applying the implicit function theorem (IFT) to function $g(v, r)$, we get the derivative of profit $r$ with respect to the exponential systematic utility $v$,

$$\frac{\partial v}{\partial r} = \frac{-\partial g(v, r)/\partial v}{\partial g(v, r)/\partial r} = \frac{1}{\alpha_j v + 1}, \quad (21)$$

where

$$h = \frac{\beta_j (1 - \eta)(1 - P_0) + P_0}{\alpha_j \beta_j \rho_0 (1 - \eta)(1 - P_0) - \alpha_j \beta_j r (1 - \eta)(1 - P_0) + \alpha_j \rho_0 (1 - P_0)}.$$

The numerator of $h$, $\beta_j (1 - \eta)(1 - P_0) + P_0$, converges to 0 as $v \rightarrow +\infty$. If $\beta_1 > 1$, the denominator of $h$ can be rewritten as

$$\alpha_j \beta_j \rho_0 (1 - \eta)(1 - P_0) - \alpha_j \beta_j r (1 - \eta)(1 - P_0) + \alpha_j \rho_0 (1 - P_0),$$

where the last equality is because of (19). In (22), the first term converges to 1 and the second term is positive as $v \rightarrow +\infty$.

If $\beta_1 \leq 1$, the denominator of $h$ can be rewritten as

$$\alpha_j \beta_j \rho_0 (1 - \eta)(1 - P_0) - \alpha_j \beta_j r (1 - \eta)(1 - P_0) + \alpha_j \rho_0 (1 - P_0),$$

where the last equality is because of (19). In (23), the first term converges to $\beta_1$ as $v \rightarrow +\infty$ and the second term is positive.

In both cases, the denominator is bounded below from zero as $v \rightarrow +\infty$. Thus $h \rightarrow 0$ and $v \frac{\partial r}{\partial v} \rightarrow 0 \frac{1}{\alpha_j}$ in (21).

By the l'Hôpital’s rule, since $r \rightarrow +\infty$ as $v \rightarrow +\infty$,

$$\lim_{v \rightarrow +\infty} \frac{r}{\ln v} = \lim_{v \rightarrow +\infty} \frac{\partial r/\partial v}{1/v} = \lim_{v \rightarrow +\infty} \frac{\partial^2 r/\partial v^2}{-v^{-2}} = \frac{1}{\alpha_j},$$

which completes the proof.

Note when $v > \tilde{v}_1$ for some $\tilde{v}_1 > 0$, $h > 0$. $r(v)$ is strictly increasing in $v$ and $v \frac{\partial r}{\partial v} < \frac{1}{\alpha_j}$ when $v > \tilde{v}_1$.

B. Proof of Theorem 2

First, fixing the number of charging stations to build as $N_E$, we examine where to build these stations. Denote the sum of exponential systematic utility by $v = \sum_{i=1}^{N_E} v_i$ and the uniform
charging profit by \( r = \rho^*_i - c_i \). The operational profit of the investor can be stated as
\[
\Pi(v) = r(v)\eta(v) \sum_{i=1}^{N_E} P_i(v) = r(v) \left[ \frac{q_0 + \kappa(v)}{q_0 + \kappa(v)} \right]^{b_1} (q_0 + \kappa(v))^{b_2} + C q_0 + \kappa(v),
\]
(25)

where \( C = \exp(\beta, U_G) - \beta_2 P_G + \beta_3 P_E \), \( q_0 = \exp(\alpha_1 f_1 - \alpha_2 r) \), and \( \kappa(v) = v \exp(-\alpha_2 r) \). The derivative of \( \kappa(v) \) with respect to \( v \) is stated as
\[
\frac{\partial \kappa(v)}{\partial v} = \exp(-\alpha_2 r)(1 - \alpha_2 v) \frac{\partial r}{\partial v}
\]
(26)

which is strictly positive when \( v > \bar{v}_1 \) according to the discussion in Sec. VII-A. So the operational profit \( \Pi(v) \) is strictly increasing in \( v \) when \( v > \bar{v}_1 \).

The second order derivative of \( \Pi(v) \) with respect to \( v \) is stated as
\[
\frac{\partial^2 \Pi}{\partial v^2} = \frac{1}{\eta} \left( v^2 \frac{\partial^2 \eta}{\partial v^2} \right) (1 - P_0) \left( -2 \frac{\partial P_0}{\partial v} \right)^2 \\
\quad + \frac{1}{\eta} \left( \frac{\partial P_0}{\partial v} \right) \left( 1 - \eta \right) \frac{\partial^2 \eta}{\partial v^2} \left( 1 - P_0 \right) + P_0 \\
\quad + \frac{\partial r}{\partial v} \left( 1 - \eta \right) \frac{\partial \kappa}{\partial v} \left( 1 - 2 \eta - P_0 \right) + (\eta + 3) P_0 - 1.
\]
(27)

As \( v \to +\infty \), \( v^2 \frac{\partial^2 \eta}{\partial v^2} \to -\frac{1}{\eta} \), \( \eta \to +\infty \) and \( P_0 \to 0 \). So when \( v > \bar{v}_2 \) for some \( \bar{v}_2 > 0 \), the first term in (27) is negative and bounded above from zero. The second and last term are negative. The third term is positive but converging to zero. The second order derivative of \( \Pi(v) \) is negative and we have the concavity of \( \Pi(v) \) stated in the following lemma.

**Lemma 2:** Asymptotically, \( \Pi(v) \) is a decreasing and concave function of \( v \).

The monotonicity of \( \Pi(v) \) implies that, if given two station candidates \( j \) and \( j' \), fixing the other \( (N_E - 1) \) stations, the one with larger \( v_j = \exp(\alpha_1 f_1 - \alpha_2 c_i) \), \( i \in \{j,j'\} \) should be built because the building costs are the same. So we have the optimal strategy about where to build stations as follows.

**Lemma 3:** Fixing the number of stations to build as \( N_E \), the asymptotically optimal strategy of building is to pick \( N_E \) candidates with largest \( v_i = \exp(\alpha_1 f_1 - \alpha_2 c_i) \).

Next, after we sort the \( N_L \) candidate locations by \( v_i \), we can present the cost \( \sum_{i=1}^{N_E} F(s_i) = (1 + \gamma)F_0N_E \) as a function of \( v = \sum_{i=1}^{N_E} v_i \). We can treat \( v \) as a continuous variable and write \( \tilde{F}(v) = \tilde{F}(v) = (1 + \gamma)F_0N_E + (v - \sum_{i=1}^{N_E} v_i)(1 + \gamma)F_0/v_{N+1} \), if \( v_{N+1} < v \leq v_{N+1} \). Since \( v_i \geq v_{i+1} \), the cost \( \tilde{F}(v) \) is a piecewise linear convex function of \( v \). The partial derivative \( \partial \tilde{F}(v)/\partial v \) is piecewise constant and increasing in \( v \).

The trends of \( \Pi(v) \), \( \tilde{F}(v) \) and the derivatives are plotted in Fig. 11 and 12. In Fig. 11, \( \nu^* \) is the optimal point to maximize the profit \( (\Pi(v) - \tilde{F}(v)) \). In Fig. 12 shows the derivative of \( \tilde{F}(v) \) is increasing and the marginal profit \( \frac{\partial \Pi(v)}{\partial v} \) is decreasing when \( v \) is large enough. The last cross point of the derivatives of \( \Pi(v) \) and \( \tilde{F}(v) \) is the optimal point. Combining Lemma 2 and 3 we have the asymptotic optimality.

**C. Proof of Theorem 5**

Denote the sum of consumers utility and investor’s operational profit by \( S_W(v) = S_C(v) + \Pi(v) \), the social planner is maximizing \( (S_W(v) - \tilde{F}(v)) \).

Fig. 11: Profit and cost of charging stations.

The consumers’ utility, \( S_C(v) \), can be rewritten as a function of the total exponential systematic utility \( v \) as follows.
\[
S_C(v) = \ln([q_0 + v \exp(-\alpha_2 r)]^{b_1} C_1 + C_2),
\]
which is increasing in \( v \). So
\[
\frac{\partial S_W(v)}{\partial v} = \frac{\partial S_C(v)}{\partial v} + \frac{\partial \Pi(v)}{\partial v} > \frac{\partial \Pi(v)}{\partial v}.
\]

We plot the derivative of the social welfare as well as that of the investor utility in Fig 13. The optimal social welfare point \( v^{**} \) is also the cross point of \( \frac{\partial \tilde{F}(v)}{\partial v} \) and \( \frac{\partial S_W(v)}{\partial v} \). Since \( \frac{\partial S_W(v)}{\partial v} > \frac{\partial \Pi(v)}{\partial v} \), it is always true that \( v^{**} \geq v^* \), which implies the socially optimal solution requires more charging stations than the private market outcomes.

**REFERENCES**

[1] Hybrides.com, “2011-2013 Plug-in Vehicles monthly sales dashboard,” Available at: http://www.hybrides.com/december-2014-dashboard/.

[2] U.S. Department of Energy, “Electric Vehicular Charging Station Locations,” Available at: http://www.afdc.energy.gov/fuels/electricity_locations.html.

[3] Electric Transportation Engineering Corporation and the U.S. Department of Energy, “Electric vehicle public charging-time vs. energy,” Available at: http://www.thevproject.com/cms-assets/documents/106078-254667.tvse.pdf.

[4] J. Marschak, “Binary-choice constraints and random utility indicators,” in Proceedings of a Symposium on Mathematical Methods in the Social Sciences, vol. 7, pp. 19-38, 1960.
Fig. 13: Derivatives of social welfare and investor utility.

[5] Z. Yu, S. Li, and L. Tong, “On Market Dynamics of Electric Vehicle Diffusion,” in Proc. of the 52nd Annual Allerton Conference on Communication, Control, and Computing, Oct. 2014.

[6] N. Gandal, M. Kende, and R. Rob, “The dynamics of technological adoption in hardware/software systems: The case of compact disc players,” RAND Journal of Economics, vol. 31, pp. 43–61, 2000.

[7] M. T. Clements and H. Ohashi, “Indirect network effects and the product cycle: Video games in the u.s., 1994–2002,” The Journal of Industrial Economics, vol. 53, no. 4, pp. 515–542, 2005.

[8] K. S. Corts and M. Lederman, “Software exclusivity and the scope of indirect network effects in the u.s. home video game market,” International Journal of Industrial Organization, vol. 27, no. 2, pp. 121–136, 2009.

[9] Y. Zhou, “Failure to Launch in Two-Sided Markets: A Study of the US Video Game Market,” 2014. Working paper. Available at: http://www.docstoc.com/docs/161439721/Failure-to-Launch-in-Two-Sided-Markets-A-Study-of-the-US-Video.

[10] J. P. H. Dubé, G. J. Hitsch, and P. K. Chintagunta, “Tipping and concentration in markets with indirect network effects,” Marketing Science, vol. 29, no. 2, pp. 216–249, 2010.

[11] M. Armstrong and J. Wright, “Two-sided markets, competitive bottlenecks and exclusive contracts,” Economic Theory, vol. 32, no. 2, pp. 353–380, 2007.

[12] M. Rysman, “Competition between networks: A study of the market for yellow pages,” The Review of Economic Studies, vol. 71, no. 2, pp. 483–512, 2004.

[13] J. Rochet and J. Tirole, “Two-sided markets: an overview,” 2004. IDEI working paper.

[14] B. Cailiaud and B. Jullien, “Chicken & egg: competition among intermediation service providers,” Rand Journal of Economics, vol. 34, no. 2, pp. 309–328, 2003.

[15] S. Ge, L. Feng, H. Liu, and L. Wang, “The planning of electric vehicle charging stations in the urban area,” in 2nd International Conference on Electronic & Mechanical Engineering and Information Technology, (Yichang, China), pp. 2726–2730, September 2011.

[16] I. Frade, A. Riberiro, G. Goncalves, and A. P. Antunes, “Optimal Location of Charging Stations for Electric Vehicles in a Neighborhood in Lisbon, Portugal,” Transportation Research Record: Journal of the Transportation Research Board, vol. 2252, pp. 91–98, 2011.

[17] F. He, D. Wu, Y. Yin, and Y. Guan, “Optimal deployment of public charging stations for plug-in hybrid electric vehicles,” Transportation Research Part B: Methodological, vol. 47, pp. 87–101, Jan. 2013.

[18] V. Bernardo, J.-R. Borrell, and J. Perdiguerò, “Fast charging stations: Network planning versus free entry,” 2013.

[19] S. Chen and L. Tong, “Iems for large scale charging of electric vehicles: Architecture and optimal online scheduling,” in IEEE Third International Conference on Smart Grid Communications, SmartGridComm 2012, Taibei, Taiwan, November 5-8, 2012, pp. 629–634, 2012.