Coordinated Multicell Beamforming and Power Allocation for Massive MIMO with low-resolution ADC/DAC

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Abstract—In this work, we present a solution for coordinated beamforming and power allocation when base stations employ a massive number of antennas equipped with low-resolution analog-to-digital and digital-to-analog converters. We address total power minimization problems of the coarsely quantized uplink (UL) and downlink (DL) communication systems with target signal-to-interference-and-noise ratio (SINR) constraints. By combining the UL problem with the minimum mean square error combiners and by deriving the Lagrangian dual of the DL problem, we prove the UL-DL duality and the existence of no duality gap even with the coarse quantizers. Inspired by the strong duality, we devise a fixed-point algorithm to determine the optimal UL transmit powers, then linearly amplify the UL combiners with proper weights to acquire the optimal DL precoder. Simulation results evaluate the proposed method in terms of total power consumption and achieved SINR.

Index Terms—Coordinated multipoint, joint beamforming and power allocation, low-resolution ADC/DAC, total transmit power minimization, UL-DL strong duality

I. INTRODUCTION

Massive multiple-input-multiple-output (MIMO) has been considered as a key technique for next-generation communication systems because of its advantage in spectral efficiency [1]. Because each antenna is followed by power-hungry analog-to-digital converters (ADCs) and digital-to-analog converters (DACs), however, considerable power consumption has arisen as a bottleneck of realistic implementation. Consequently, transceivers with low-resolution quantizers have been gathering increased momentum [2]–[11]. Moreover, in multicell systems, the in-cell and out-of-cell interferers as well as the non-negligible quantization error should be included in designing lots of communication building blocks.

To examine the low-resolution systems, state-of-the-art data detector and channel estimation have been developed for low-resolution ADCs [2]–[5]. The authors in [4] proposed a learning-based detector with an artificial noise to overcome an issue of 1-bit ADCs, stochastic resonance. In [5], a coding-theoretic approach was given to perform soft detection and its refinement under 1-bit ADCs. Resolution-adaptive ADCs design and the corresponding ADC bit-allocation algorithm were derived in [6]. For tractability, an additive quantization noise model (AQNM) was used in [7]–[9] with informative analyses. Low-resolution DAC systems have also been investigated [10], [11]. In [10], it was shown that achievable rates of 3-4 bits DACs are comparable to infinite-resolution DACs. The AQNM was also utilized in [11] to approximate the uplink (UL) and downlink (DL) achievable rates in full-duplex systems.

As one of the key ingredients of modern cellular systems, a coordinated multipoint (CoMP) design across base stations (BSs) has shown a significant gain in communication performance [12]–[15]. In [12], beamforming (BF) and power allocation (PA) in UL CoMP were developed by using a fixed-point iteration method. Considering DL as a virtual UL, UL-DL CoMP BF and PA were further proposed in [13] using only local measurement in a distributed fashion. The authors in [14] further attained Lagrangian-based duality for multiuser MIMO systems and proposed a distributed method with less complexity load on users and BSs. A vision on the possible combinations of massive MIMO and CoMP architectures was described in [15] achieving higher throughput.

In this paper, we integrate the low-resolution converters into the CoMP BF and PA designs. Under the non-negligible quantizer error, we first write the UL and DL problems, whose purposes are to minimize total transmit power with individual signal-to-interference-and-noise ratio (SINR) constraints. We then prove that the UL-DL duality holds under the coarse quantizer by showing that the UL problem combined with the minimum mean square error (MMSE) equalizer is equal to the Lagrangian dual of the DL problem. We further acquire no duality gap by converting the DL problem to a second-order cone program, which is strictly feasible. With the strong duality, we devise an iterative algorithm to solve the UL problem in a distributed manner with convergence to an optimum. We also state that an optimal DL BF can be a mixture of the UL combiner and weights computed from the UL result. Numerical results show that the proposed design outperforms a conventional method in terms of total power and achievable SINR.
Notation: \( \mathbf{A} \) is a matrix and \( \mathbf{a} \) is a column vector. \( \mathbf{A}^H \) and \( \mathbf{A}^T \) denote conjugate transpose and transpose. \( [\mathbf{A}]_{i,j} \) and \( \mathbf{a}_i \) are the \( i \)-th row and column vectors of \( \mathbf{A} \). We denote \( a_{i,j} \) as the \( (i,j) \)-th element of \( \mathbf{A} \) and \( a_i \) as the \( i \)-th element of \( \mathbf{a} \). \( \mathcal{C}\mathcal{N}(\mu, \sigma^2) \) is a complex Gaussian distribution with mean \( \mu \) and variance \( \sigma^2 \). The diagonal matrix \( \text{diag}(\mathbf{A}) \) has \( \{a_{i,i}\} \) at its diagonal entries, and \( \text{diag}(\mathbf{a}) \) has \( \{a_i\} \) at its diagonal entries. A block diagonal matrix is presented as \( \text{blkdiag}(\mathbf{A}_1, \ldots, \mathbf{A}_N) \). \( \text{eig}_\mathbb{R}(\mathbf{A}) \) and \( \text{eig}_\mathbb{C}(\mathbf{A}) \) denote the maximum and minimum eigenvalues of \( \mathbf{A} \). \( \| \mathbf{A} \| \) is L2 norm. \( \mathbf{I}_N \) is a \( N \times N \) identity matrix and \( \mathbf{0}_N \) is a \( N \times 1 \) zero vector.

II. System Model

We consider a multicell multi-user MIMO network with \( N_c \) cells, \( N_u \) single-antenna users per cell as shown in Fig. 1 with time division multiplexing (TDD) assumption. Users in cell \( i \) mainly communicate with a designated BS in cell \( i \) (BSi) equipped with \( N_h \) antennas. We assume that the BSs have low-resolution ADCs and DACs with the same b-bits precision.

A. Uplink System

Each user \( u \) in cell \( i \) transmits signal \( x_{u}^{ni} = \sqrt{\lambda_i,u} s_{u}^{ni} \) where \( \lambda_i,u \) and \( s_{u}^{ni} \) are transmit power and a symbol, respectively. The channel between user \( u \) in cell \( i \) and the BS is \( \mathbf{h}_{i,j,u} \in \mathbb{C}^{N_h} \). The received signal at BS \( i \) is expressed as

\[
\mathbf{r}_{i} = \mathbf{H}_{i} \mathbf{x}_{i}^{ni} + \sum_{j \neq i} \mathbf{H}_{i,j} \mathbf{x}_{j}^{ni} + \mathbf{n}_{i}^{ni}
\]

where \( \mathbf{H}_{i,j} \in \mathbb{C}^{N_h \times N_h} \) is the channel between BS \( i \) and users in cell \( j \), whose \( u \)-th column is \( \mathbf{h}_{i,j,u} \). \( \mathbf{x}_{u}^{ni} \in \mathbb{C}^{N_h} \) and \( \mathbf{s}_{u}^{ni} \in \mathbb{C}^{N_h} \) are the transmit signal and vector symbols of the \( N_u \) users in cell \( i \), whose \( u \)-th entries are \( x_{u}^{ni} \) and \( s_{u}^{ni} \), respectively. \( \mathbf{A}_{u} = \text{diag}(\lambda_1,u, \ldots, \lambda_{N_u},u) \) collects the transmit power of the users in cell \( i \), and \( \mathbf{n}_{i}^{ni} \sim \mathcal{C}\mathcal{N}(0,\mathbf{I}) \) is the additive white Gaussian noise at BS \( i \). We assume that \( \mathbf{s}_{u}^{ni} \sim \mathcal{C}\mathcal{N}(0,\mathbf{I}) \), \( \forall i \). (1) is merged in a compact form as

\[
\mathbf{r}_{i} = \mathbf{H}_{i} \mathbf{A}_{i}^{1/2} \mathbf{s}_{i}^{ni} + \mathbf{n}_{i}^{ni}
\]

where \( \mathbf{h}_{i} = [\mathbf{H}_{i},1, \ldots, \mathbf{H}_{i,N_c}] \in \mathbb{C}^{N_h \times N_c N_u} \). \( \mathbf{A}_{i} = \text{blkdiag}(\mathbf{A}_{1,i}, \ldots, \mathbf{A}_{N_c,i}) \in \mathbb{C}^{N_h \times N_c N_u} \), and \( \mathbf{s}_{i}^{ni} = \begin{bmatrix} \mathbf{s}_{i,1}^{ni} \\ \vdots \\ \mathbf{s}_{i,N_u}^{ni} \end{bmatrix} \in \mathbb{C}^{N_h \times N_u} \).

For analytical tractability, we adopt the AQNM [7], [16] to have a linear approximation of a non-linear quantizer derived from a scalar MMSE quantizer. Then the quantized signal vector can be given by

\[
Q(\mathbf{r}_{i}) \approx \mathbf{q}_{i,u} = \alpha \mathbf{H}_{i} \mathbf{A}_{i}^{1/2} \mathbf{s}_{i,u}^{ni} + \beta \sum_{j \neq i} \mathbf{H}_{i,j} \mathbf{A}_{i}^{1/2} \mathbf{s}_{j,u}^{ni} + \alpha \mathbf{n}_{i,u}^{ni} + \mathbf{q}_{i,u}^{ni}
\]

where \( Q(\cdot) \) is an entry-wise quantizer of the imaginary and real parts. The quantizer gain \( \alpha \) is a function of \( b \), and defined as \( \alpha = 1 - \beta \), where \( \beta = \frac{\mathbb{E}[r_{i,u}^{ni}r_{i,u}^{ni}]}{\mathbb{E}[r_{i,u}^{ni}]} [16], [17] \). \( \beta \)'s are quantified in Table 1 in [17] for \( b \leq 5 \) assuming \( \mathbf{s}_{i,u}^{ni} \sim \mathcal{C}\mathcal{N}(0,\mathbf{I}) \), \( \forall i \). The quantization noise \( \mathbf{q}_{i,u}^{ni} \) is uncorrelated with \( \mathbf{r}_{i,u} \) and follows \( \mathcal{C}\mathcal{N}(0,\mathbf{C}_{q_{i,u}^{ni}q_{i,u}^{ni}}) \) with covariance of [7], [16].

\[
\mathbf{C}_{q_{i,u}^{ni}q_{i,u}^{ni}} = \alpha \beta \text{diag}(\mathbf{H}_{i} \mathbf{A}_{i}^{-H} + \mathbf{I})
\]
Proof. \(\mu\) is equal to the Lagrangian dual of quantizer error and by proving that the problem in (12)-(13) is a Lagrangian multiplier. We rewrite (10) to the UL SINR in (6), the constraints in \(P1\) are simplified as \(\alpha \lambda_{i,u,h_{i,i,u},h_{i,i,u}^H} + \alpha \lambda_{N_b} + \alpha \beta \text{diag}(H_i A H_i^H)\). Then, the linear MMSE equalizer \(f_{i,u}\) can be expressed as

\[
f_{i,u} = C_{z_{i,u},z_{i,u}}^{-1} h_{i,i,u}.
\]

Applying (10) to the UL SINR in (6), the constraints in \(P1\) become equivalent by replacing \(\mu_{i,u}\) with covariance of \(C_{z_{i,u},z_{i,u}} = \alpha^2 \sum_{(j,v) \neq (i,u)} \lambda_{j,v,h_{j,j,v},h_{j,j,v}^H} + \alpha \lambda_{N_b} + \alpha \beta \text{diag}(H_i A H_i^H)\). Here, (11) implies that \(\alpha^2 \lambda_{i,u,h_{i,i,u},h_{i,i,u}^H} C_{z_{i,u},z_{i,u}} - \lambda_{i,u} \lambda_{i,u} I_{N_b} \geq 0\). Rearranging the indices from (i, u) to (j, v, i) of (i, u, j) \(P2\) becomes equivalent to

\[
\max_{\mu_{i,u}} \sum_{i,u} \mu_{i,u},
\]

s.t. \(K_i(A) \leq \alpha \left(1 + \frac{1}{\gamma_{i,u}}\right) \mu_{i,u} h_{i,i,u} h_{i,i,u}^H,\)

for all \(i, u\) where

\[
K_i(A) = \lambda_{i,u} + \alpha \sum_{j,v} \lambda_{j,v,h_{j,j,v},h_{j,j,v}^H} + \beta \text{diag}(H_i A H_i^H).
\]

A. Uplink and Downlink Duality

By integrating the quantization error terms, we broaden the duality of the UL and DL power minimization problems for infinite-resolution quantizers \([14]\) to low-resolution regime.

**Theorem 1** (Duality). The uplink transmit power minimization problem \(P1\) in (4)-(5) equals to the Lagrangian dual of the downlink transmit problem \(P2\) in (7)-(8).

Proof. We use the MMSE combiners that maximize the SINR to simplify the constraints. Let \(z_{i,u}\) be the interference-plus-noise term of the quantized signal in (2) with covariance of \(C_{z_{i,u},z_{i,u}} = \alpha^2 \sum_{(j,v) \neq (i,u)} \lambda_{j,v,h_{j,j,v},h_{j,j,v}^H} + \alpha \lambda_{N_b} + \alpha \beta \text{diag}(H_i A H_i^H)\). Then, the linear MMSE equalizer \(f_{i,u}\) can be expressed as

\[
f_{i,u} = C_{z_{i,u},z_{i,u}}^{-1} h_{i,i,u}.
\]

Applying (10) to the UL SINR in (6), the constraints in \(P1\) are simplified as \(\alpha^2 \lambda_{i,u,h_{i,i,u},h_{i,i,u}^H} C_{z_{i,u},z_{i,u}} - \lambda_{i,u} I_{N_b} \geq 0\). We then multiply both sides with \(h_{i,i,u} h_{i,i,u}^H\) and rearrange as

\[
h_{i,i,u}^H (\alpha^2 \lambda_{i,u,h_{i,i,u},h_{i,i,u}^H} C_{z_{i,u},z_{i,u}} - \lambda_{i,u} I_{N_b}) h_{i,i,u} \geq 0.
\]

Therefore, we can design the precoder to be

\[
\sum_{i,u} \mu_{i,u}
\]

s.t. \(K_i(M) \geq \alpha \left(1 + \frac{1}{\gamma_{i,u}}\right) \mu_{i,u} h_{i,i,u} h_{i,i,u}^H,\)

for all \(i, u\) where

\[
K_i(M) = I_{N_b} + \alpha \sum_{j,v} \mu_{j,v,h_{j,j,v},h_{j,j,v}^H} + \beta \text{diag}(H_i M H_i^H).
\]

Although we have the Lagrangian dual of \(P2\) in (17)-(18) and the UL problem in (12)-(13), they have the flipped objectives and constraints. However, optimal solutions of both cases can be found at the active constraints. Since (17)-(18) and (12)-(13) have the same objective value at active constraints, they become equivalent by replacing \(\mu_{i,u}\)'s in (17)-(18) with \(\lambda_{i,u}\)'s.

Noting that \(\alpha \to 1\) as \(b \to \infty\), the results pave the way for a generalized understanding of the UL-DL duality derived in \([14]\) by extending it to any quantization resolution. To propose an algorithm that solves \(P1\) and \(P2\), and to prove its optimality, we show strong duality between \(P1\) and \(P2\).

**Corollary 1.** No duality gap exists between \(P2\) and its dual

Proof. We first show that \(P2\) is an instance of a second-order cone programming. Let \(W\) be defined as \(W = [W_1, \cdots, W_{N_c}]\), then the DL problem (7)-(8) is rewritten as

\[
\min_{W,P_o} P_o
\]

s.t. \(\Gamma_{i,u}^{dl} \geq \gamma_{i,u}, \forall i, u\)

\[
\text{Tr}(W^H W) \leq P_o
\]

where \(P_o\) is a positive slack variable. As noted in \([18],[19]\), we take a diagonal phase shifting on the right of the precoder of each cell as \(W_i \text{diag}(\phi_{i,1}, \cdots, \phi_{i,N_c})\) for \(i = 1, \cdots, N_c\). Therefore, we can design the precoder to be \(W_i h_{i,j,v} h_{i,i,u} \geq 0, \forall i, u\) without changing the objective nor the constraints. Using (15), we rewrite the quantization term in (9) as

\[
\sum_{j,v} h_{j,j,v} C_{q_j^H q_j} h_{i,i,u} = \alpha \beta \sum_{j,v} W_{j,v}^H \text{diag}(h_{j,j,v}, h_{j,j,v}^H) W_{j,v}.
\]
Let $D_{i,u} = \text{diag}(h_{j,i,u}h_{j,i,u}^H)$, $W_{BD} = \text{blkdiag}(W_1, \ldots, W_N)$, and $\tilde{W}_{BD} = \text{blkdiag}(\{(I_{N_i} \otimes W_1), \ldots, (I_{N_i} \otimes W_N)\})$. Using (22), the SINR constraints in (20) becomes
\[
\alpha^2 \left(1 + \frac{1}{\gamma_{i,u}}\right) |w_{i,u}^H h_{i,u}|^2 \geq \left\| \alpha W_{BD}^H \text{vec}(h_{1,i,u}, \ldots, h_{N_i,i,u}) \right\|^2 \left(1 - \alpha \right) \left| W_{BD}^H \text{vec}(D_{1/2}^{i,u}, \ldots, D_{1/2}^{N_i,i,u}) \right| \forall i,u.
\]
Because we force $w_{i,u}^H h_{i,u}$ to be non-negative, we can take square root on both sides of (23). In addition, (21) is reformulated as $\|\text{vec}(W)\| \leq \sqrt{C}$. Thus, the problem in (19)-(21) can be modified to the standard second order conic program (18).

Next, (7) is strictly feasible because given a solution $W$, it can be scaled by a factor of $c > 1$ satisfying the constraints. Thus, the strong duality holds between (1) and (7).

**B. Distributed Iterative Algorithm**

We specify solutions using the strong duality, and develop an iterative algorithm that finds the solutions for $P1$ and $P2$ concurrently. We further prove optimality and convergence.

**Corollary 2.** The optimal transmit power for the uplink total power minimization problem (4) is derived as
\[
\lambda_{i,u} = \frac{1}{\alpha \left(1 + \frac{1}{\gamma_{i,u}}\right) h_{i,i,u}^H K_i^{-1}(A) h_{i,i,u}}
\]
where $K_i(A) = I_{N_i} + \alpha \sum_{j,i,u} \lambda_{j,i,u} h_{j,i,u} h_{j,i,u}^H + (1 - \alpha) \text{diag}(H_i A H_i^H)$ with the linear MMSE equalizer given as
\[
f_{i,u} = \left[ \alpha^2 \sum_{(j,v) \neq (i,u)} \lambda_{j,v} h_{j,v} h_{j,v}^H + \alpha I_{N_i} + \alpha \beta \text{diag}(H_i A H_i^H) \right]^{-1} h_{i,u}.
\]

**Proof.** We use $\lambda_{i,u}$ instead of $\mu_{i,u}$ since we showed that they are equivalent. The derivative of the Lagrangian (16) with respect to $w_{i,u}$ is given as
\[
\frac{\partial \mathcal{L}(w_{i,u}, \lambda_{i,u})}{\partial w_{i,u}} = 2 \alpha \left( I_{N_i} - \alpha \left(1 + \frac{1}{\gamma_{i,u}}\right) \lambda_{i,u} h_{i,i,u} h_{i,i,u}^H \right) w_{i,u}
- \alpha \sum_{j,v} \lambda_{j,v} h_{j,v} h_{j,v}^H + \beta \text{diag}(H_i A H_i^H) \right) w_{i,u}.
\]

Setting (26) to zero, we have (24), i.e., the Lagrangian multiplier that meets the stationary condition. Also, all the constraints in $P2$ are active at (24), hereby satisfying the complementary slackness. Thus, (24) is the optimum of $P1$.

Given that we finally have the optimal transmit power, we can compute the optimal UL MMSE combiner in (10). As a function of these factors, we design the optimal DL precoder.

**Corollary 3.** With proper weights, the optimal DL precoders are linearly proportional to the UL MMSE receiver, i.e.,
\[
w_{i,u} = \sqrt{\tau_{i,u}} f_{i,u} \forall i,u, \text{ and } \tau_{i,u} \text{ is derived as } \tau = \Sigma^{-1} l_{N_i,N_i},
\]
where $\tau = [\tau_1^T, \tau_2^T, \ldots, \tau_{N}^T]^T$ with $\tau_i^T = [\tau_{i,1}, \tau_{i,2}, \ldots, \tau_{i,N_i}]^T$, and
\[
\Sigma = \begin{pmatrix}
\Sigma_{1,1} & \Sigma_{1,2} & \cdots & \Sigma_{1,N_i} \\
\Sigma_{2,1} & \Sigma_{2,2} & \cdots & \Sigma_{2,N_i} \\
\vdots & \vdots & \ddots & \vdots \\
\Sigma_{N_i,1} & \Sigma_{N_i,2} & \cdots & \Sigma_{N_i,N_i}
\end{pmatrix},
\]
where each element of matrix $\Sigma_{i,j} \in \mathbb{R}^{N_i \times N_j}$ is given as
\[
\Sigma_{i,j}^{(u,v)} = \left\{ \begin{array}{c}
\frac{\alpha^2 |f_{i,u}^H h_{i,u}|^2}{\gamma_{i,u}} \\
-\alpha \beta |f_{i,u}^H \text{diag}(h_{i,u}, h_{i,u}) f_{i,u} | \text{ if } i = j \text{ and } u, v, \\
-\alpha \beta |f_{j,u}^H h_{j,u}^H |^2 \\
-\alpha \beta |f_{j,u}^H \text{diag}(h_{j,u}, h_{j,u}) f_{j,u} | \text{ otherwise.}
\end{array} \right.
\]

**Proof.** Based on the Lagrangian dual, the global optimum happens when the constraints satisfy active constraints. By replacing $w_{i,u}$ in (9) with $\sqrt{\tau_{i,u}} f_{i,u}$, the constraints of the DL problem in (8) satisfy the following equality conditions:
\[
\frac{\alpha^2}{\gamma_{i,u}} |w_{i,u}^H h_{i,u}|^2 \geq \frac{\alpha^2}{\gamma_{i,u}} |w_{i,u}^H h_{i,u}|^2 - \alpha^2 \sum_{j \neq i} |w_{j,u}^H h_{j,u}|^2 \geq \sum_{j \neq i} |w_{j,u}^H h_{j,u}|^2 - \sum_{j \neq i} \mu_{j,u} C_{q_j q_i} h_{j,u}^H \\
= \alpha^2 |f_{i,u}^H h_{i,u}|^2 - \alpha^2 \sum_{j \neq (i,u)} |f_{j,u}^H h_{j,u}|^2 \tau_{j,v} \\
= \alpha^2 |f_{i,u}^H h_{i,u}|^2 - \alpha^2 \sum_{j \neq (i,u)} |f_{j,u}^H h_{j,u}|^2 \tau_{j,v} \\
= 1, \quad \forall i, u,
\]
where $(\alpha)$ is from (22) and $w_{i,u} = \sqrt{\tau_{i,u}} f_{i,u}$. We cascade the conditions $\forall i, u$ in a matrix form: $\Sigma \tau = 1$ and $\tau = \Sigma^{-1}$.  

We devise the unified algorithm to solve both UL and DL problems. We first solve (24) in UL problem, however, the main drawback is that all the transmit powers engage in the computation of an individual transmit power. Therefore, we employ an iterative standard algorithm based on (24) [14], [18], [20] to find the optimal UL solution. Let $\lambda_{i,u}^{(n)}$ be the result of $n$th iteration, and $A^{(n)}$ be a collection of $\lambda_{i,u}^{(n)}$'s. The algorithm is described as follows:
Step 1. Initialize $\lambda_{i,u}^{(0)}$, $\forall i, u$.

Step 2. Iteratively update $\lambda_{i,u}^{(n+1)}$ until converges using (24) as

$$\lambda_{i,u}^{(n+1)} = \frac{1}{\alpha (1 + \frac{1}{\tau_{i,u}}) h_{i,u}^H K_i^{-1} (A(n)) h_{i,u}}, \forall i, u.$$  

Step 3. Find the UL MMSE combiner $f_{i,u}$ in (25) with $\lambda_{i,u}$.

Step 4. Compute the DL precoder $w_{i,u}$ based on Corollary 1. $K_i$ is a covariance matrix of received signals which may be estimated using local measurements at BSs, [14], hence Step 2 does not entail the explicit inter-cell channel knowledge. The individual scaling weight $\tau_{i,u}$ that achieves the target SINR can be obtained using a per-user update algorithm [21], whose convergence is guaranteed [20]. Each step of the algorithm evolves $\tau_{i,u}$ while assuming other $\tau_{i',u'}$’s are fixed. Thus, the proposed algorithm can be deployed in a distributed manner.

Corollary 4 (Convergence). For any initialization $\lambda_{i,u}^{(0)}$, $\forall i, u$, the proposed fixed-point iterative algorithm converges to an unique fixed point at which total transmit power is minimized.

Proof. We exploit the standard function [20]. We need to show that $\mathcal{F}_{i,u}(\Lambda)$ is a standard function which meets the followings:

- (positivity) If $\lambda_{i,u} \geq 0$, $\forall i, u$, then $\mathcal{F}_{i,u}(\Lambda) > 0$.
- (monotony) If $\lambda_{i,u} \geq \lambda_{i,u}'$, $\forall i, u$, then $\mathcal{F}_{i,u}(\Lambda) \geq \mathcal{F}_{i,u}(\Lambda')$.
- (scalability) For $\rho > 1$, $\rho \mathcal{F}_{i,u}(\Lambda) > \mathcal{F}_{i,u}(\rho \Lambda)$.

It can be shown that $\mathcal{F}_{i,u}(\Lambda(n))$ satisfies the properties by carefully following the proof in Appendix II in [18].

IV. SIMULATION RESULTS

We evaluate the derived theoretical results and the proposed quantization-aware iterative CoMP algorithm (Q-iCoMP). As a benchmark, we test the quantization-aware per-cell iterative algorithm (Q-Percell) by adapting the per-cell algorithm in [13] to low-resolution converters. For the Q-Percell, each BS first finds a solution based on the iterative process in [13] by considering the inter-cell interference as fixed noise. Once the BSs have solutions for the considered noise power, BSs update the noise power and iterate until the solution converges.

We consider two networks with $N_c \in \{2, 7\}$, which we call as a light network and a dense network, respectively. For $N_c = 2$, two cells are next to each other. For $N_c = 7$, the center cell is surrounded by the other six cells. Each BS is in the center of each hexagonal cell with $N_u$ users. The distance between adjacent BSs is 2 km and a user is at least 100 m away from the BSs. For small scale fading, we assume Rayleigh fading with a zero mean and unit variance. For large scale fading, we use the log-distance pathloss in [22]. We consider 2.4 GHz carrier frequency, 10 MHz bandwidth, 8.7 dB lognormal shadowing variance, and 5 dB noise figure. We assume the same target SINR $\gamma$ for all users over all cells.

Fig. 2 shows the cumulative density function (CDF) of the achieved SINR of users over all cells for $\gamma = 0$ dB, $b = 3$, and $N_b = 64$ with (a): $(N_c = 2, N_u = 2)$ and (b): $(N_c = 7, N_u = 4)$. The total transmit powers of the Q-iCoMP and the Q-Percell are annotated in the legend. The Q-iCoMP shows a clear spike at 0 dB with the least total transmit power for both (a) and (b). Therefore, we can find out that the Q-iCoMP properly controls the transmit power so that the achieved SINRs are no more and no less than what the system requires. The Q-Percell achieves similar results with the increment in total transmit power for (a). In the case of (b), although the Q-Percell delivers the implausible power, only 85% of users recorded the exact target SINR whereas around 10% of users cannot achieve the target SINR. This result shows that the Q-Percell is only feasible with the limited number of BSs and users. Further, about 5% of have higher SINR than the target, which may be caused by excessive power due to the lack of BS coordination. We emphasize that the proposed method outperforms the conventional approach in a complicated configuration with more interferers.

Fig. 3 shows total transmit power for the target SINR. We consider the variations in the number of quantization bits and BS antennas, i.e., $b \in \{2, 3, \infty\}$ and $N_b \in \{16, 128\}$, for the dense network with $N_c = 7$ and $N_u = 4$. For $N_b = 16,$
the Q-Percell suffers from the implausible power consumption even with the infinite-resolution at the low SINR requirement. Even though the Q-iCoMP also ends up showing divergence in total transmit power at the medium to high target SINRs with the small number of quantization bits, the Q-iCoMP shows a much slower rate of divergence compared to the Q-Percell. For $N_b = 128$, the Q-Percell has the similar trend as the case with $N_b = 16$, showing the divergence at the medium SINR. In contrast, the Q-iCoMP achieves the target SINRs for all users without divergence for $b \geq 3$ in the considered SINR targets. On both $N_b \in \{16, 128\}$, the Q-iCoMP achieves significant power gain over the Q-Percell. Further, increasing the number of BS antennas from 16 to 128 provides more than 10 dB gain for each user SINR. Therefore, the proper coordination in designing BF and PA is essential when deploying a massive antenna arrays with low-resolution quantizers.

V. CONCLUSION

This paper investigated the CoMP BF and PA for a multicell network with low-resolution ADCs and DACs. Incorporating the effect of non-trivial quantization error, we showed the strong duality between the UL and DL total transmit power minimization problems under the individual SINR constraints in the low-resolution regime. Using the duality, the fixed-point algorithm was developed to solve the UL and DL problems with the coarse quantization. The proposed algorithm determines optimal solutions for the UL and DL problems in a distributed fashion without explicit out-of-cell channel information. Via simulations, we demonstrated that the proposed iterative design is more effective than the conventional approach in terms of the total power consumption and achieved SINR. The performance gain becomes significant especially when we augment the number of antennas and cells. Therefore, proper coordination is required when we consider the massive number of antennas with low-resolution ADCs and DACs.

REFERENCES

[1] T. L. Marzetta, “Noncooperative Cellular Wireless with Unlimited Numbers of Base Station Antennas,” IEEE Trans. on Wireless Commun., vol. 9, no. 11, pp. 3590–3600, 2010.
[2] J. Choi, J. Mo, and R. W. Heath, “Near maximum-likelihood detector and channel estimator for uplink multiuser massive MIMO systems with one-bit ADCs,” IEEE Trans. Comm., vol. 64, no. 5, pp. 2005–2018, Mar. 2016.
[3] C. Studer and G. Durisi, “Quantized massive mu-mimo-ofdm uplink,” IEEE Trans. on Commun., vol. 64, no. 6, pp. 2387–2399, 2016.
[4] J. Choi, Y. Cho, B. L. Evans, and A. Gatherer, “Robust learning-based ML detection for massive MIMO systems with one-bit quantized signals,” in IEEE Global Commun. Conf., 2019, pp. 1–6.
[5] Y. Cho and S.-N. Hong, “One-bit successive-cancellation soft-output (oss) detector for uplink mu-mimo systems with one-bit adcs,” IEEE Access, vol. 7, pp. 27172–27182, 2019.
[6] J. Choi, B. L. Evans, and A. Gatherer, “Resolution-Adaptive Hybrid MIMO Architectures for Millimeter Wave Communications,” IEEE Trans. on Signal Process., vol. PP, no. 99, pp. 1–1, 2017.
[7] O. Orhan, E. Erkip, and S. Rangan, “Low power analog-to-digital conversion in millimeter wave systems: Impact of resolution and bandwidth on performance,” in IEEE Info. Theory and App. Work., Feb. 2015, pp. 191–198.
[8] L. Xu, X. Lu, S. Jin, F. Gao, and Y. Zhu, “On the Uplink Achievable Rate of Massive MIMO System with Low-Resolution ADC and RF Impairments,” IEEE Commun. Lett., vol. 23, no. 3, pp. 502–505, March 2019.
[9] J. Choi, G. Lee, and B. L. Evans, “Two-Stage Analog Combining in Hybrid Beamforming Systems With Low-Resolution ADCs,” IEEE Trans. on Signal Process., vol. 67, no. 9, pp. 2410–2425, 2019.
[10] S. Jacobsson, G. Durisi, M. Coldrey, T. Goldstein, and C. Studer, “Quantized Precoding for Massive MU-MIMO,” IEEE Trans. on Commun., vol. 65, no. 11, pp. 4670–4684, 2017.
[11] J. Dai, J. Liu, J. Wang, J. Zhao, C. Cheng, and J.-Y. Wang, “Achievable rates for full-duplex massive MIMO systems with low-resolution ADCs/DACs,” IEEE Access, vol. 7, pp. 24344–24353, 2019.
[12] F. Rashid-Farrokhli, L. Tassiulas, and K. R. Liu, “Joint optimal power control and beamforming in wireless networks using antenna arrays,” IEEE Trans. on Commun., vol. 65, no. 10, pp. 3131–3134, 2018.
[13] F. Rashid-Farrokhli, K. R. Liu, and L. Tassiulas, “Transmit beamforming and power control for cellular wireless systems,” IEEE J. on Sel. Areas in Commun., vol. 16, no. 8, pp. 1437–1450, 1998.
[14] H. Dahrouj and W. Yu, “Coordinated beamforming for the multicell multi-antenna wireless system,” IEEE Trans. on Wireless Commun., vol. 9, no. 5, pp. 1748–1759, 2010.
[15] V. Jungnickel, K. Manolakis, W. Zirwas, B. Panzner, V. Braun, M. Lossow, M. Sernad, R. Apelrã¶nd, and T. Svensson, “The role of small cells, coordinated multipoint, and massive MIMO in 5G,” IEEE Commun. Mag., vol. 52, no. 5, pp. 44–51, 2014.
[16] A. K. Fletcher, S. Rangan, V. K. Goyal, and K. Ramchandran, “Robust predictive quantization: Analysis and design via convex optimization,” IEEE Journal Sel. Topics in Signal Process., vol. 1, no. 3, pp. 618–632, 2007.
[17] L. Fan, S. Jin, C.-K. Wen, and H. Zhang, “Uplink achievable rate for massive MIMO systems with low-resolution ADC,” IEEE Comm. Letters, vol. 19, no. 12, pp. 2186–2189, Oct. 2015.
[18] A. Wiesel, Y. C. Eldar, and S. Shamai, “Linear precoding via conic optimization for fixed MIMO receivers,” IEEE Trans. on Signal Process., vol. 54, no. 1, pp. 161–176, 2005.
[19] W. Yu and T. Lan, “Transmitter optimization for the multi-antenna downlink with per-antenna power constraints,” IEEE Trans. on Signal process., vol. 55, no. 6, pp. 2646–2660, 2007.
[20] R. D. Yates, “A framework for uplink power control in cellular radio systems,” IEEE J. on Sel. Areas in Commun., vol. 13, no. 7, pp. 1341–1347, 1995.
[21] G. J. Foschini and Z. Miljanic, “A simple distributed autonomous power control algorithm and its convergence,” IEEE Trans. on Veh. Technol., vol. 42, no. 4, pp. 641–646, 1993.
[22] V. Erceg, L. J. Greenstein, S. Y. Tjandra, S. R. Parkoff, A. Gupta, B. Kulic, A. A. Julius, and R. Bianchi, “An empirically based path loss model for wireless channels in suburban environments,” IEEE J. on Sel. Areas in Commun., vol. 17, no. 7, pp. 1205–1211, 1999.