Active ReLU Linearized Neural Network based Frequency-Constrained Unit Commitment in Low-Inertia Power Systems

Mingjian Tuo
Student Member, IEEE
Department of Electrical and Computer Engineering
University of Houston
Houston, TX, USA
mtuo@uh.edu

Xingpeng Li
Senior Member, IEEE
Department of Electrical and Computer Engineering
University of Houston
Houston, TX, USA
xli82@uh.edu

Abstract—Conventional synchronous generators are gradually being replaced by inverter-based resources. Such transition introduces more complicated operation conditions, and also imposes challenges for system operators on maintaining system frequency and rate-of-change-of-frequency (RoCoF) security due to reduction in system inertia. To ensure the system wide frequency security, this paper presents an active rectified linear unit (ReLU) linearized neural network (ARLNN) based RoCoF-constrained unit commitment (ARLNN-RCUC) model. A predictor is first trained to predict the highest locational RoCoF based on a high-fidelity simulation dataset. Instead of incorporating the complete trained neural network into unit commitment, a ReLU linearization method is implemented on selected neurons to improve the algorithm efficiency. The effectiveness of proposed ARLNN-RCUC model is demonstrated on the IEEE 24-bus system by conducting time domain simulation on PS/E.

Index Terms—Deep learning, Frequency stability, Low-inertia power systems, ReLU linearization, Rate of change of frequency, Unit commitment.

I. INTRODUCTION

The decarbonization of the electricity generation relies on the integration of converter-based renewable energy sources (RES) over the past decades. In addition, the development of high-voltage direct current (HVDC) transmission systems has resulted in deployment of power-electronic devices [1]. As a result, current power systems may ultimately shift towards power systems where all generation comes from converter-based resources.

The main challenge of power systems stability is insufficient inertia of the system [2]. Due to the retirement and replacement of conventional generation, more generation is coming from converter-based resources. Consequently, the system kinetic energy would decrease significantly, leaving the system more vulnerable to high rate of change of frequency (RoCoF) and large frequency excursion when a loss of generation or a large variation occurs. When RoCoF violates the pre-specified threshold, protection devices would disconnect generators from the grid [3]. The reduced system inertia would require additional fast frequency response services to ensure frequency stability [4]. Studies in [5] show that virtual inertia and altering RoCoF protection are very effective in reducing the frequency stability risk of future converter-based power system in Ireland.

Transmission system operators (TSO) have imposed extra RoCoF related constraints in the conventional unit commitment model to keep the minimum amount of synchronous inertia online [6]. EirGrid has also introduced a synchronous inertial response constraint to ensure that the available inertia is above a minimum limit of 23 GWs in Ireland [7]. The Swedish TSO once ordered one of its nuclear power plants to reduce output by 100 MW to mitigate the risk of loss of that power plant [8].

Following a different strategy, some studies have included frequency related constraints into traditional security-constrained unit commitment (SCUC) formulations. Ref. [9] implemented a system equivalent model-based enhanced frequency stability constrained multiperiod optimal power flow model. In [10], frequency-related constraints were incorporated into SCUC enforcing limits on RoCoF that is derived from a uniform frequency response model. Ref. [11] studied a mixed analytical-numerical approach based on multi-regions and investigated a model combining evolution of the center of inertia and certain inter-area oscillations. However, previous works focus on the collective performance of all generators, they failed to consider the nodal inertial response and the impact of disturbance propagation [12]. To handle higher order characteristics of system dynamics, a deep neural network based RoCoF constrained unit commitment (DNN-RCUC) is proposed in [13]-[14], which incorporates frequency related constraints against the worst-case contingency. However, none of these methods considers locational RoCoF security; and the efficiency of DNN-based algorithms has not been discussed in the literature.

To bridge the aforementioned gaps, an active rectified linear unit (ReLU) linearized neural network based RoCoF-constrained unit commitment (ARLNN-RCUC) model is proposed, in which the derived frequency related constraints are incorporated into SCUC to secure the system locational frequency stability against worst contingencies. Secondly, a model-based data generation approach is used to efficiently generate practical cases for RoCoF predictor training. The generated dataset covers vast ranges of reasonable operating conditions, being able to avoid divergent time domain simulations. Thirdly, we propose a node-of-interest active selecting method to reduce the approximation error of ReLU...
linearization, such method improves the computational efficiency of the DNN-RCUC model while maintaining the model’s high performance.

The remainder of this paper is organized as follows. Section II discusses the power system mathematical based model and RoCoF predictor model thoroughly. Section III presents the methodology of model-based data generation. ReLU linearization approaches and the proposed ARLNN-RCUC model. Section IV shows the simulation results. Section V concludes this paper and presents future work.

II. SYSTEM FREQUENCY DYNAMICS

A. System Equivalent Model

The frequency of the power system is one of the most important metrics that indicate the system stability. Traditionally, the frequency is treated as unique of the whole system, which is derived from the system equivalent model extended from one-machine swing equation. The kinetic energy $E_i$ stored in a rotational mass is proportional to its moment of inertia and the square of its angular velocity:

$$E_i = \frac{1}{2}J_i\omega_i^2$$  \hspace{1cm} (1)

System inertia is defined as the total amount of rotational energy stored in all connected synchronous units; dynamics of generators can be expressed as follows.

$$E_{sys} = \sum_{i=1}^{N} \frac{1}{2}J_i\omega_i^2 = \sum_{i=1}^{N} H_i S_{bi}$$  \hspace{1cm} (2)

where $H_i$ is the inertia constant of the generator in seconds; $J_i$ is the moment of inertia of the shaft in kg·m²/s; $S_{bi}$ is the base power in MVA; and $\omega_i$ is the nominal rotational speed instead of the actual speed of the machine.

For a single generator $i$, the swing equation is expressed as:

$$P_m - P_e = M_i \frac{d\omega_i}{dt} + D_i \Delta \omega_i$$  \hspace{1cm} (3)

where $P_m$ is the mechanical power and $P_e$ is the electrical output power. The swing equation of the one machine model can be then applied to the whole grid [15].

B. Dynamic Model

Only considering the dynamics of the equivalent model in systems would underestimate the actual need for frequency ancillary services, leading to higher locational RoCoF and larger regional frequency deviation than expected.

Using the topological information and the system parameters, the transmission network can be modeled as a graph consisting of nodes (buses) and edges (branches). The oscillatory behavior of each individual bus can be expressed as follows:

$$m_i \ddot{\theta}_i + d_i \dot{\theta}_i = P_{ini} - \sum_{j=1}^{n} b_{ij} \sin (\theta_i - \theta_j),$$  \hspace{1cm} (4)

where $m_i$ and $d_i$ denote the inertia coefficient and damping ratio for node $i$ respectively, while $P_{ini}$ denotes the power input. With inertia on certain nodes $m_i > 0$, it is an approximation model for the swing dynamics of high-voltage transmission network within a short period following the event.

[16]. A network-reduced model with $N$ generator buses can be obtained by eliminating passive load buses via Kron reduction [17]. The phase angle $\theta$ of generator buses can be expressed by the following dynamic equation,

$$M \ddot{\theta} + D \dot{\theta} = P - L \theta$$  \hspace{1cm} (5)

where $M = \text{diag}\{\{m_i\}\}$, $D = \text{diag}\{\{d_i\}\}$; for the Laplacian matrix $L$, its off-diagonal elements are $l_{ij} = -b_{ij}\psi_i(0)\psi_j(0)$, and diagonals are $l_{ii} = \sum_{j=1,j\neq i}^{N} b_{ij}\psi_i(0)\psi_j(0)$. Under the assumption of homogeneous inertia, the frequency deviations at bus $i$ can then be derived [16],

$$\ddot{\theta}_i(t) = \frac{\Delta P}{2\pi m} \sum_{g=1}^{N_g} \beta_{ag} \beta_{bg} \sqrt{\frac{\alpha_g}{m}} \sin \left( \frac{\beta_g \alpha_g}{m} \right) \left[ e^{-\frac{\beta_g t}{\tau}} \sin \left( \frac{\beta_g \alpha_g}{m} \right) \right]$$  \hspace{1cm} (6)

where $\Delta$ denotes average inertia distribution on generator buses; and bus $b$ is where disturbance occurs.

C. RoCoF Prediction

DNN has the ability to amend the limitations of model-based approaches. Given the load forecast $d$ and RES forecast $r$, following a generator contingency $\sigma$, at period $t$, the highest locational RoCoF value of the system $R_i$ is a function with respect to the contingency level, contingency location, system states, unit dispatch, load data and RES profile.

$$R_i = R^r(s_x, u_i, d_x, r_x, m_x)$$  \hspace{1cm} (7)

where $s_x$ denotes the system states, and $u_i$ is the generation dispatch at period $t$. The loss of largest generation not only causes mismatch in system power balance but also degrades the system synchronous inertia, resulting in higher frequency deviation and larger initial RoCoF. The DNN based RoCoF predictor for $R^r$ is then expressed as,

$$\hat{R}_i = \hat{R}^r(x,W,b)$$  \hspace{1cm} (8)

where $x$ is the feature vector; $W$ and $b$ denote the well-trained parameters. Both the magnitude and location of the contingency will have impact on the inertial response. The generator status feature vector for sample $s$ is defined as follows,

$$u_s = [u_{1,s}, u_{2,s}, \ldots, u_{N,s}]$$  \hspace{1cm} (9)

The disturbance feature vector is defined against the loss of then largest generation, the magnitude of the contingency is expressed as,

$$P^u = \max_{g\in G} \{P_{1,s}, \ldots, P_{2,s}, \ldots, P_{N,s}\}$$  \hspace{1cm} (10)

The location of the disturbance is represented by the index of the generator producing maximum power,

$$g^u = \arg\max_{g\in G} \{P_{1,s}, \ldots, P_{2,s}, \ldots, P_{N,s}\}$$  \hspace{1cm} (11)

The information of magnitude and location into the disturbance feature vector as,

$$\sigma^v = [0, \ldots, 0, P^u, \ldots, 0, g^u, \ldots, 0]$$  \hspace{1cm} (12)

Laplacian matrix $L$ of the grid and Fiedler mode value depend on the power-angle characteristics, which are determined by the active power injection [18]. Thus, the active power injection of
all synchronous generator will be encoded into the feature vector.

\[ P_s = [P_{1,s}, \ldots, P_{2,s}, \ldots, P_{N_G,s}] \]  

(13)

The overall feature vector of a sample \( s \) can be then defined as follows,

\[ x_s = [u_s, a^c_s, P_s] \]  

(14)

A fully connected neural network with \( N_h \) hidden layer is considered with the following setting: each layer uses a ReLU activation function as \( \sigma(z) = \max(z, 0) \), which returns zero when the node is with a negative value, implying the node is inactive. The feature vector is fed into the well-trained predictor, and the output layer of the predictor is a linear activation function.

III. METHODOLOGY

A. Model-based Data Generation

A large number of system profiles with very different power injections are utilized in [13] to ensure reliability under vast ranges of operating conditions. However, many randomly sampled power injections are not stable under the worst-case disturbance even with stability predictor applied. Such random injections may lead to divergence during the simulation initialization process. Even if the time-domain simulation is initialized successfully, the transient stability may still suffer from system oscillations as well as large rotor angle differences.

Model-based systematic data generation approach is used to generate reasonable and representative data that will be used to train RoCoF Predictors. Training samples are generated from traditional SCUC (T-SCUC) models and frequency constrained SCUC models [18] over various load and RES scenarios. The T-SCUC is the base model generating dispatching samples given the load forecast and RES forecast. For system equivalent model based RoCoF constrained security constrained unit commitment (ERC-SCUC) model, constraints on system initial RoCoF is introduced to guarantee system aggregated frequency stability against \( G - 1 \) contingency. The location based RoCoF constrained SCUC (LRC-SCUC) introduces locational RoCoF constraints based on the definition of local buses and non-local buses. Nodal RoCoF constraints ensure system stability by imposing limit on locational RoCoF over all buses under all \( G - 1 \) contingency. The detail of all models is presented in [18].

B. DNN-RCUC

To encode the DNN into the MILP SCUC problem, decision variables are introduced to build the disturbance feature vector. Binary variable \( a^c_{g,t} \) is used to indicate the status of largest output power of generator \( g \) in scheduling period \( t \), a big-M method is introduced to express the disturbance vector \( [e_{g,t}, \ldots, e_{N_G,t}] \). Thus, the input feature vector can be expressed as follows,

\[ x_t = [u_{1,t}, \ldots, u_{N_G,t}, e_{g,t}, \ldots, e_{N_G,t}, P_{1,s}, \ldots, P_{N_G,s}] \]  

(15)

RoCoF-limiting constraints can be derived from the pre-trained RoCoF predictor \( \hat{R}(x,W,b) \), the nonlinear constraints can be incorporated into MILP problems by introducing auxiliary binary variables \( a \). The reformulation of DNN-based RoCoF predictor is as follows,

\[ z_q[l,t] \geq \hat{z}_q[l,t], \forall q, l, t, \]  

\[ z_q[l,t] \leq M \alpha_q[l,t], \forall q, l, t, \]  

\[ z_q[l,t] \geq 0, \forall q, l, t, \]  

\[ a_q[l,t] \in [0,1], \forall q, l, t, \]  

\[ \hat{R}_{ct} = z_{l,t}W_{L,t} + b_{L,t}, \forall t, \]  

(16f)

Where \( l \) indicates the index of neurons, and \( q \) represents the network layer; \( z \) and \( \hat{z} \) represent the activated value and pre-activated value of each neuron respectively. The RoCoF related constrained in DNN-RCUC [14] considering threshold \(-\text{RoCoF}_{\text{lim}}\) can be formulated as,

\[ \hat{R}_{ct} \leq -\text{RoCoF}_{\text{lim}}, \forall t, \]  

(16g)

C. Active ReLU Linearization

A neural network is comprised of set of layers of neurons; activation function, such as ReLU, is applied to the result of linear combination of values from neuron nodes [19]. Previous study in [14] has found that incorporation of DNN would introduce multiple binary variables, as explained in section III.B. As a consequence, such reformulation increases the computational burden of the SCUC model. Reference [20] has demonstrated that linearization of ReLU function can greatly reduce the DNN size without too much degradation of classification accuracy.

Approximation of ReLU function is shown in Fig. 1. The weighted sum of input signals to the node is denoted as variable \( x \), and the output of the node is denoted using the variable \( z \). Given the upper and lower bounds \([LB,UB]\) of \( x \). The relationship of \( z \) and \( x \) can then be approximated by a set of constraints \( z \geq 0, z \geq x, \) and \( z \leq \frac{UB(x-LB)}{UB-LB} \). These constraints are all linear equations with constant \( UB \) and \( LB \).

To reduce the approximation error, one mitigation is to add penalty on nonnegative \( z \) in the objective function to push \( z \) to be set at the bottom two sides of the triangle. Intuitively, ReLU linearization is applied to each neuron node, converting a DNN-RCUC model into ReLU linearized neural network based RCUC (RLNN-RCUC). However, this process may introduce approximation error and subsequently result in low prediction accuracy. In this study, we propose a node-of-interest active selecting method to improve the computational efficiency of the reduce the SCUC model while reducing the linearization approximation error. The generated dataset is first fed into the well-trained network, a nodal positivity index \( e^+_m \) is proposed to estimate the percentage of positive preactivated values of neuron node \( m \) in a layer.
\[
\varepsilon_m^\alpha = \sum_{n \in N} \hat{x}_{n,m}^\alpha - \frac{1}{N} \left( \hat{x}_{n,m}^\alpha - \frac{1}{N} \sum_{n \in N} \hat{x}_{n,m}^\alpha \right)
\]

(17)

A threshold of \( \gamma_m^\alpha \geq 0 \) is set to select nodes suitable for ReLU linearization with less approximation error. Thus, \((16a)-(16e)\) for selected neurons can be replaced by \((18b)-(18e)\), which could be applied to convert DNN-RCUC model into an ARLNN-RCUC model.

\[
z_m[t],t \geq \hat{x}_{m,t}^\alpha, \forall m, \forall l, \forall t,
\]

(18b)

\[
z_m[t],t \leq UB \cdot (\hat{x}_{m,t}^\alpha - LB), \forall m, \forall l, \forall t,
\]

(18c)

\[
z_m[t],t \geq 0, \forall m, \forall l, \forall t,
\]

(18d)

\[
a_m[t],t \in [0, 1], \forall m, \forall l, \forall t,
\]

(18e)

IV. CASE STUDIES

A case study on IEEE 24-bus system [21] is provided to demonstrate the effectiveness of the proposed methods. This test system contains 24 buses, 33 generators and 38 lines, which also has wind power as renewable resources. The base case has a total demand from 1,195 MW to a peak of 2,116 MW. Additional deviation ranging from [-20%, 20%] is applied to the base value. The mathematical model-based data generation is operated in Python using Pyomo. Regarding post-contingency frequency limits, RoCoF must be higher than -0.5Hz/s to avoid the tripping of RoCoF-sensitive protection relays, and the optimality gap is set to 0.1%. The PSS/E software is used for time domain simulation and labeling process. Full-scale models with detailed generator dynamics are implemented for more realistic data.

A. Predictor Training

For the DNN layers, the number of neurons is set 10 for each layer. ReLU is used as the activation function. A total of 8,161 samples were collected based on strategies proposed in previous section. The entire dataset is divided into two subsets: 6,529 samples (80%) for training and 1,632 samples (20%) for validation. Fig. 2 presents the evolution of MSE losses on the training and validation sets over the training process of the proposed DNN model. Mean squared error (MSE) decreases as the number of epochs increases.

![Fig. 2. The learning curve of the proposed DNN: MSE losses versus the number of epochs.](image)

TABLE I shows the validation accuracy of the RoCoF predictor under different tolerances. The validation accuracy is 99.41% with 10% tolerance and 93.62% with a smaller tolerance of 5% respectively, implying the high performance and the robustness of the trained predictor.

| Tolerance | 10% | 9% | 8% | 7% | 6% | 5% |
|-----------|-----|----|----|----|----|----|
| Accuracy  | 99.41% | 98.78% | 98.14% | 96.61% | 95.53% | 93.62% |

B. Simulation Results

Fig. 3 and Fig. 4 present the forecast wind power and load for a sample test case in respectively. The total scheduling horizon is 24 hours. Hours 9-12 are selected to be the time instance where frequency related constraints are applied to secure system stability against generator contingency considering high penetration level of intermittent wind generation. The test case has a demand ranging from 1,604 MW to a peak of 1,916 MW. The peak wind generation is 266 MW.

![Fig. 3. Example of wind power profile of the IEEE 24-bus system.](image)

![Fig. 4. Example of load profile of the IEEE 24-bus system.](image)

TABLE II lists the simulation results of the proposed ARLNN-based RCUC model and other benchmark models. It can be observed that DNN-RCUC and the proposed ARLNN-RCUC have the highest operational cost among all models; the extra cost results from the efforts in securing the RoCoF stability at each node. For RLNN-RCUC case, the total cost is same as T-SCUC model where no RoCoF related constraints are considered, implying that the approximation error due to ReLU linearization leads to RoCoF related constraints being non-binding.

| Model           | Total Cost [\$] | Computational Time [s] | Highest RoCoF [Hz/s] |
|-----------------|----------------|------------------------|----------------------|
| T-SCUC          | 1486556.34     | 13.58                  | 0.8053               |
| ERC-SCUC        | 1494340.99     | 20.24                  | 0.6145               |
| LRC-SCUC        | 1615135.45     | 35.25                  | 0.5634               |
| DNN-RCUC        | 1641966.76     | 368.58                 | 0.4985               |
| RLNN-RCUC       | 1486556.34     | 14.24                  | 0.8053               |
| ARLNN-RCUC      | 1641953.75     | 66.78                  | 0.4997               |

After inclusion of the complete ReLU, the resulting computational time of DNN-RCUC is increased to 368.58s.
Incorporation of complete ReLU formulations introduces multiple binary variables into SCUC formulations, subsequently increases the computational time. With active ReLU linearization algorithm, the total computational time is reduced to 66.78 s. Besides the RoCoF related constraints remain binding in the case of ARLNN-RCUC while it fails to bind in the case of RLNN-RCUC case.

Additionally, we run the dynamic simulation of G-1 contingency for all 5 models at hour 9 when the system netload is the lowest. As shown in TABLE III and Fig. 5, the proposed ARLNN-RCUC can maintain RoCoF within safe range following contingency of generator loss. It should be noted that the ERC-SCUC and LRC-SCUC models cannot ensure system RoCoF security under same situation. Similarly, RLNN-RCUC model fails to secure RoCoF stability due to high approximation error.

![Fig. 5. RoCoF evolution of ARLNN-RCUC model.](image)

**TABLE III**

| Hour | 8 | 9 | 10 | 11 | 12 | 13 |
|------|---|---|----|----|----|----|
| T-SCUC | 15 | 15 | 15 | 15 | 15 | 15 |
| ARLNN-RCUC | 15 | 15 | 16 | 13 | 15 | 14 |

**V. CONCLUSIONS**

In this paper, we proposed an ARLNN-RCUC model to secure system RoCoF stability while maintain the computational efficiency of RCUC model. By incorporating DNN into T-SCUC model, system stability could be secured with less conservativeness. However, this process would significantly increases the computational burden of DNN-RCUC model. A node-of-interest active selecting method is proposed to improve the computational efficiency of DNN based RCUC algorithm.

The simulation results on the IEEE 24-bus system indicate that the proposed ARLNN-RCUC can improve power system inertial responses without much increased computational time. Our proposed node-of-interest active selecting method has been proved to significantly reduce the computational time comparing to DNN-RCUC. And such algorithm can also reduce the approximation error of ReLU linearization, maintaining the performance of the proposed method.

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