A simple memristive jerk system

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Abstract
A simple memristive chaotic jerk system with one variable to represent the internal state is found. The proposed equilibria-free memristive system yields hidden chaotic oscillation in a narrow parameter space. A circuit is constructed that models the jerk system, and it shows agreement with the predicted oscillation. The new memristive jerk system appears to be one of the algebraically simplest memristive chaotic systems.

1 | INTRODUCTION

Memristors have attracted great interest in engineering for their potential applications [1–5]. They have been used as a dynamic element to generate chaotic signals. The presence of a memristor usually leads to a 4-D system [6–10], although only a 3-D system is required for chaos. Several 3-D memristive chaotic systems have been reported [11–15], but in those cases, the memristor was implemented with a complicated operational-amplifier-based equivalent device.

Jerk systems are a simple type of dynamical system that can generate chaos [16–20]. Such a compact structure is composed of a couple integral operation units in series. Some jerk systems are chaotic when they contain a nonlinearity from quadratic terms [16–18], an exponential function [19] or a cubic term [20]. All other jerk systems that produce chaos based on the memristor are 4-D [21, 22]. The novelty of this work is that compared with prior studies, we aim to construct a completely 3-D jerk memristive circuit. The most important challenge in this work is introducing a suitable memristor to break the existing oscillation in a 2-D structure and finally bring chaos.

In addition, many memristive systems exhibit chaotic oscillation combined with other states because of the integral effect from the memristor [6–10, 21–25]. Unlike these memristive jerk systems, however, a second-order jerk structure with a memristor is explored that is both simple and robust for giving chaos. In Section 2, the model is given and analyzed with basic dynamical analysis. In Section 3, the circuit is built for proving the theoretical analysis. Finally, a short conclusion is made to summarize the work.

2 | SYSTEM MODEL

A simple chaotic jerk oscillator containing a memristor as one of the state variables was found by an exhaustive computer search based on the Euler method. Suppose there is a 2-D jerk structure \( \dot{y} = z, \dot{z} = f(y, z) \), and introduce a flux-controlled memductance \( W(x) \) in it. The following system is found for producing chaos:

\[
\begin{align*}
\dot{x} &= y, \\
\dot{y} &= z, \\
\dot{z} &= -z - az^2 - W(x)y + b.
\end{align*}
\]

where the flux-controlled memductance \( W(x) = 1.3x^2 - 1 \) is introduced in the \( z \)-dot equation. Here the variables \( y \) and \( z \) are the external system variables, and \( x \) is the internal variable in...
the memristor and indicates the magnetic flux. When \( a = 0.239 \) and \( b = 1 \), System (1) produces chaos with Lyapunov exponents \((0.0529, 0, -1.0529)\) after a time of \( t = 2e7 \) and a corresponding Kaplan–Yorke dimension of \( D_{KY} = 2.0502 \) for initial condition \((0, -2, -2)\), as shown in Figure 1, whose basins of attraction (in the \( z = 0 \) plane) are shown in Figure 2.

This system is fairly delicate with chaos in only a narrow range of the parameter space, as indicated in Figure 3. The internal variable \( x \) comes from the integration of the system state variable \( y \). System (1) is asymmetric with a speed of volume contraction determined by a derivative proposed by Lie:

\[
\nabla V = \frac{\partial x}{\partial x} \dot{x} + \frac{\partial y}{\partial y} \dot{y} + \frac{\partial z}{\partial z} \dot{z} = -1 - 2tx.
\]

In addition, System (1) has no equilibria [26–30], and therefore the attractor is hidden [31–35].

More interesting is that unlike other memristive systems [8–10], System (1) outputs relatively stable oscillations except when switching between chaos and sliding initial values that agree with the plots of the dynamical region and basins of attraction. To further verify this, the solution based on offset boosting under a fixed initial condition is used for diagnosing multi-stability [36, 37]. Taking offset-boosting \( d \) in the dimension \( y \to y + d \), it is shown that when offset \( d \) varies in \([-5, 5]\) except for the long transient process, System (1) remains chaotic unless it is dragged sliding with the initial condition, as shown in Figure 4.

The embedded memristor is defined as

\[
\begin{cases}
\dot{x} = y, \\
W(x) = 1.3x^2 - 1, \\
i_M = W(x)y.
\end{cases}
\]

Flux-dependent memductance is related to the internal variable \( x \), which is of quadratic degree [13, 38, 39] and is determined by the system variable \( y \):

\[
W(x) = 1.3x^2 - 1 = 1.3\left( \int_{-\infty}^{x} yds \right)^2 - 1 = 1.3\left( \int_{0}^{x} yds \right)^2 - 1 + W_0
\]
where \( W_0 = 1.3((\int_{-\infty}^t y ds)^2 - (\int_0^t y ds)^2) \). Element memductance and the corresponding theoretical loop of pinched hysteresis are plotted in Figure 5.

3 | JERK CIRCUIT IMPLEMENTATION

Obtaining an analog circuit to realize System (1) is a relatively easy way to introduce a memristor into the operational amplifier-based integration circuit. First, we construct a 2-D jerk main structure. Second, an equivalent circuit is designed for the applied memristor without resorting to another amplifier-based integration element. The basic principle is based on the characteristics of virtual break and virtual short of an operational amplifier. An analog circuit based on Equation (1) is designed as shown in Figure 6, and according to the Kirchhoff law, the circuit equations can be written as follows:

\[
\begin{align*}
\dot{y} &= \frac{1}{R_3C_2}z, \\
\dot{z} &= -\frac{z}{R_1C_1} - \frac{z^2}{R_2C_1} - \frac{W(x)y}{C_1} + \frac{V_b}{R_0C_1}
\end{align*}
\]

Figure 4 Dynamical behaviors in System (1) with \( a = 0.239, b = 1 \), and initial condition \((x_0, y_0, z_0) = (0, -2, -2)\) when the offset parameter \(d\) varies within \([-5, 5]\) (a) Average values of variables, (b) Lyapunov exponents

Figure 5 The memductance and pinched hysteresis loop

Figure 6 Circuit schematic of the memristive jerk oscillator

Figure 7 Equivalent element of the flux-dependent memristor
The experimental constraints of memductance, the inherent pinched hysteresis effect, and the experimental phase portraits agree well with the numerical simulation, proving the system dynamics and the effectiveness of the hardware circuit. As a main element, the equivalent memristor can potentially have a great effect on the performance of the jerk system, which is dominantly determined by two analog multipliers and one operational amplifier. These three components define the memristor applied in this work, and therefore some other physical memristor models (e.g., the HP memristor) cannot guarantee chaos in the 3-D jerk structure.

4 | CONCLUSIONS

By introducing a memristor into a second-order jerk structure, chaotic oscillation is found in a 3-D jerk system. The proposed simple memristive jerk system has only six terms while without any equilibria, one of which is quadratic. Circuit experiments show the same oscillation, and thus they agree with the numerical simulation. When flux-controlled memductance is revised as $W(x) = 1.3 | x | - 1$, a minor parameter adjustment ($a = 0.432, b = 1$) can still recover chaos with Lyapunov exponents $(0.0328, 0, -1.0332)$ and a corresponding attractor dimension of $D_{KY} = 2.0321$, which simplifies the circuit realization. Compared with other memristive systems [10, 40], this system is also unique for its robust chaotic oscillation, although it is hidden [41, 42]. This feature is attractive for its application in chaos-based communication or image encryption. Future work on this circuit can investigate the introduction of other memristors such as the HP memristor into this proposed jerk structure for chaos.

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REFERENCES

1. Chua, L.O.: Memristor—the missing circuit element. IEEE Trans. Circuit Theory. 18(5), 507–519 (1971)
2. Tour, J.M., He, T.: The fourth element. Nature. 453, 42–43 (2008)
3. Wu, K., Wang, X.: Enhanced memristor-based MNNs performance on noisy dataset resulting from memristive stochasticity. IET Circ. Device. Syst. 23(5), 704–709 (2019)
4. Fernando, C., Alon, A., Marco, G.: Nonlinear dynamics of memristor oscillators. IEEE Trans. Circuits Syst. 58(6), 1323–1336 (2011)
5. Zhao, Q., Wang, C., Zhang, X.: A universal emulator for memristor, memcapacitor, and meminductor and its chaotic circuit. Chaos. 29(1), 013141 (2019)
6. Bao, B., Xu, J., Liu, Z.: Initial state dependent dynamical behaviors in memristor based chaotic circuit. Chin. Phys. Lett. 27(7), 070504 (2010)
7. Bao, B., et al.: Chaotic memristive circuit: equivalent circuit realization and dynamical analysis. Chin. Phys. B. 20(12), 120502 (2011)
8. Zhou, L., Wang, C., Zhou, L.: A novel no-equilibrium hyperchaotic multi-wing system via introducing memristor. Int. J. Circ. Theor. App. 46(1), 84–98 (2018)
9. Wang, C., et al.: Memristor-based neural networks with weight simultaneous perturbation training. Nonlinear Dyn. 95(4), 2893–2906 (2018)
10. Li, C., et al.: Complicated dynamics in a memristor-based RLC circuit. Eur. Phys. J. S. T. 228(10), 1925–1941 (2019)
11. Iroh, M., Chu, L.O.: Memristor oscillators. Int. J. Bifurc. Chaos. 18(11), 3183–3206 (2008)
12. Muthuswamy, B., Chu, L.O.: Simplest chaotic circuit. Int. J. Bifurc. Chaos. 20(5), 1567–1580 (2010)
13. Muthuswamy, B.: Implementing memristor based chaotic circuit. Int. J. Bifurc. Chaos. 20(5), 1335–1350 (2010)
14. Sun, J., et al.: Generalised mathematical model of memristor. IET Circ. Device. Syst. 10(3), 244–249 (2016)
15. Kim, H., et al.: Memristor emulator for memristor circuit application. IEEE Trans. Circuits Syst. 59(10), 2422–2431 (2012)
16. Gottlieb, H.P.W.: Question 38. What is the simplest jerk function that gives chaos. Am. J. Phys. 64(5), 525 (1996)
17. Sprott, J.C.: Some simple chaotic jerk functions. Am. J. Phys. 65(6), 537–543 (1997)
18. Sprott, J.C.: Simple chaotic systems and circuits. Am. J. Phys. 68(8), 758–763 (2000)
19. Sprott, J.C.: A new class of chaotic circuit. Phys. Lett. 266(1), 19–23 (2000)
20. Malasoma, J-M.: What is the simplest dissipative chaotic jerk equation which is parity invariant. Phys. Lett. 264(5), 383–389 (2000)
21. Njitacke, Z.T., et al.: Coexistence of multiple attractors and crisis route to chaos in a novel memristive diode bridge-based Jerk circuit. Chaos Solitons Fract. 91, 180–197 (2016)
22. Kengne, J., Negou, A.N., Tchiotsof, D.: Anti-monotonicity, chaos and multiple attractors in a novel autonomous memristor-based jerk circuit. Nonlinear Dyn. 88(4), 1–20 (2017)
23. Li, C., et al.: A memristive chaotic oscillator with increasing amplitude and frequency. IEEE Access. 6, 12945–12950 (2018)
24. Zhang, X., Jiang, W.: Construction of flux-controlled memristor and circuit simulation based on smooth cellular neural networks module. IET Circ. Device. Syst. 12(3), 263–270 (2018)
25. Yuan, F., Wang, G., Wang, W.: Dynamical characteristics of an HP memristor based on an equivalent circuit model in a chaotic oscillator. Chin. Phys. B. 24(6), 207–215 (2015)
26. Nazarimehr, F., et al.: A new four-dimensional system containing chaotic or hyper-chaotic attractors with no equilibrium, a line of equilibria and unstable equilibria. Chaos Solitons Fract. 111, 108–118 (2018)
27. Li, C., Sprott, J.C.: Coexisting hidden attractors in a 4-D simplified Lorenz system. Int. J. Bifurcat. Chaos. 24(3), 1450034 (2014)
28. Pham, V.-T., et al.: Constructing a novel no-equilibrium chaotic system. Int. J. Bifurcat. Chaos. 24(5), 1450073 (2014)
29. Jafari, S., Sprott, J., Golpayegani, S.M.R.H.: Elementary quadratic chaotic flows with no equilibrium. Phys. Lett. A. 377(9), 699–702 (2013)
30. Maaita, J., et al.: The dynamics of a cubic nonlinear system with no equilibrium point. Nonlinear Dyn. 257923 (2015)
31. Leonov, G.A., Vagaitsev, V.I., Kuznetsov, N.V.: Localization of hidden Chua’s attractors. Phys. Lett. A. 375(23), 2230–2233 (2011)
32. Leonov, G.A., Vagaitsev, V.I., Kuznetsov, N.V.: Hidden attractor in smooth Chua systems. Physica D. 241(18), 1482–1486 (2011)
33. Leonov, G.A., Kuznetsov, N.V.: ’Hidden attractors in dynamical systems. From hidden oscillations in Hilbert–Kolmogorov, Aizerman, and Kalman problems to hidden chaotic attractor in Chua circuits’. Int. J. Bifurcat. Chaos. 23(1), 1330002 (2013)
34. Leonov, G.A., Kuznetsov, N.V.: Algorithms for searching for hidden oscillations in the Aizerman and Kalman problems. Dokl. Math. 84(1), 475–481 (2011)
35. Zhang, X., Wang, C.: Multiscroll hyperchaotic system with hidden attractors and its circuit implementation. Int. J. Bifurc. Chaos. 20(9), 1950117 (2019)
36. Li, C., Wang, X., Chen, G.: Diagnosing multistability by offset boosting. Nonlinear Dyn. 90(2), 1335–1341 (2017)
37. Li, C., Sprott, J.C.: Variable-boostable chaotic flows. Optik. 127(22), 10389–10398 (2016)
38. Bao, B., Liu, Z., Xu, J.: Transient chaos in smooth memristor oscillator. Chin. Phys. B. 19(3), 030510 (2010)
39. Bao, B., Liu, Z., Xu, J.: Steady periodic memristor oscillator with transient chaotic behaviors. Electron. Lett. 46(3), 228–230 (2010)
40. Li, Z., Zeng, Y.: A memristor oscillator based on a twin-T network. Chin. Phys. B. 22(4), 040502 (2013)
41. Jiang, H., et al.: Hidden chaotic attractors in a class of two-dimensional maps. Nonlinear Dyn. 85(4), 2719–2727 (2016)
42. Pham, V.T., et al.: Constructing a novel no-equilibrium chaotic system. Int. J. Bifurcat. Chaos. 24(5), 1450073 (2014)

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