Model-Based Reconstruction for Collimated Beam Ultrasound Systems

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Some of the major challenges:

- Detecting flaws in multilayered objects that can be accessed from only one side.
- Non-linear effects such as reverberations.
- Direct arrival signals.

Collimated-beam systems use carefully crafted acoustic beams with side lobes suppressed and transducer diffraction minimized to provide deep penetration and high spatial resolution.
MAP or Regularized Inversion

- **Forward model:** \( f(x) = -\log p(y|x) \)
- **Prior model:** \( h(x) = -\log p(x) \)
- **MAP or regularized inversion:**
  \[
  \hat{x} \leftarrow \text{arg min}_x \{f(x) + h(x)\}
  \]
Assuming a linear system, we seek to reconstruct an image $x$ using a mathematical model of the form

$$y = Ax + Dg + w$$

- $y$ is the observed data,
- $A$ is the system matrix,
- $D$ a matrix whose columns form a basis for the possible direct arrival signals,
- $g$ is a scaling coefficient vector for $D$,
- $w$ is a Gaussian random vector with distribution $N(0, \sigma^2 I)$. 
Transfer Functions

For the homogeneous medium shown in Fig. 1, the transfer function from point $r_i$ to $r_j$ is

$$G(v, f) = \tau \exp\left\{-(\alpha |v| + j2\pi f) \left(\frac{\|v-r_i\|+\|r_j-v\|}{c}\right)\right\},$$

where

- $\tau$ is the transmittance coefficient of the front surface of the medium,
- $\alpha$ is the attenuation coefficient in s/m, and
- $c$ is the sound speed in m/s in the medium.

So, the multi-layer media shown in Fig. 2, the transfer function from $r_i$ to $r_j$

$$G(v, f) = \prod_{l=1}^{L} \tau_l e^{(\gamma_l(v)|f|+2\pi f T_l(v))}$$

where

- $L$ is the total number of layers,
- $\tau_l$ is the transmittance coefficient of the front surface of the $l^{th}$ layer,
- $\gamma_l(v) = c_l \alpha_l T_l(v)$,
- $c_l$ is the acoustic speed in m/s in the $l^{th}$ layer,
- $\alpha_l$ is the attenuation coefficient in s/m in the $l^{th}$ layer,
- and $T_l(v)$ is the travel time in seconds between the front and back interface of the $l^{th}$ layer.
Based on Snell’s law, the time delay from \( r_i \) to \( v \) to \( r_j \) is given by

\[
T(v) = \sum_{l=1}^{L} \frac{\sqrt{z_{i,l}^2 + \eta_{i,l}^2} + \sqrt{z_{j,l}^2 + \eta_{j,l}^2}}{c_l}
\]

where \( z_{i,l} = \eta_{i,l} \tan(\theta_{i,l}) \) and \( z_{j,l} = \eta_{j,l} \tan(\theta_{j,l}) \), \( l = 1, 2, \ldots, L \).

The height of \( v \) as a function of \( \theta_{i,l} \) is

\[
\sum_{l=1}^{L} z_{i,l} = \eta_{i,1} \tan(\theta_{i,1}) + \ldots + \eta_{i,L} \tan(\theta_{i,L})
\]

From Snell’s law, we know that

\[
\theta_{i,k} = \sin^{-1}\left(\sin(\theta_{i,k-1}) \frac{c_k}{c_{k-1}}\right), \forall k \in \{2, 3, \ldots, L\}.
\]

The effective time delay is then computed using **Binary Search** by finding the angle of refraction and solving for the minimum distance.
**Received signal & system matrix**

- In frequency space, the received signal is proportional to

\[
Y(v, f) = -x(v)S(f) \prod_{l=1}^{L} \tau_l e^{-(\gamma_l(v)|f|+2j\pi f T_l(v))}
\]

where \(x(v)\) in \(m^{-3}\) is the reflection coefficient for the voxel \(v\) and \(S(f)\) the Fourier transform of the transmitted signal.

- Then the time-domain **received signal** for a reflection from location \(v\) is given by

\[
y(v, t) = x(v)h(\gamma(v), t - T(v)),
\]

where

\[
h(\gamma(v), t) = \mathcal{F}^{-1}\{-S(f)e^{-\gamma(v)|f|}\}
\]

and \(\mathcal{F}^{-1}\) is the inverse Fourier transform.

- In order to reduce computation, we make the approximation that

\[
\tilde{h}(\gamma, t) = h(\gamma, t) \text{rect}\left(\frac{t}{t_0} - \frac{1}{2}\right)
\]

where \(t_0\) is a constant based on the assumption that \(h(\gamma, t)\) is equal to zero for \(t > t_0\).

- The signal received at time \(t\) by transducer \(r_j\) in response to the transmission from \(r_i\) is computed by summing over all voxels \(v\) to obtain

\[
\tilde{y}_{i,j}(t) = \sum_v \tilde{h}(\gamma(v), t - T(v))x(v)
\]

- This linear relationship between \(x(v)\) and \(y(t)\) determines a single row of the **system matrix** \(A\) in the time domain.
Define a function $\phi_{s,r}(v)$ that has a value ranging from 0 to 1. Then, we modify $\tilde{y}_{i,j}(t)$ to

$$\tilde{y}_{i,j}(t) = \sum_v \tilde{h}(\gamma(v), t - T(v))\phi(v)^{\beta} x(v)$$

The function $\phi_{s,r}(v)$ depends on the incident and reflected angles and given by

$$\phi(v)^{\beta} = \cos^{\beta} \left( \sum_{p=1}^{L} \theta_{i,p} \right) \cos^{\beta} \left( \sum_{q=1}^{L} \theta_{j,q} \right)$$

(a) A simulated beam profile, $\phi(v)^{\beta}$, with (a) $\beta = 1$ and (b) $\beta = 8$. (c) A real beam profile for a well-collimated source.
Finally, the discretized version of the forward model will be

$$- \log p(y|x, g) = \frac{1}{2\sigma^2} ||y - Ax - Dg||^2 + \text{constant},$$

where

- $y \in \mathbb{R}^{MK \times 1}$ is the measurement,
- $\sigma^2$ is the variance of the measurement,
- $A \in \mathbb{R}^{MK \times N}$ is the system matrix,
- $x \in \mathbb{R}^{N \times 1}$ is the image,
- $D \in \mathbb{R}^{MK \times K}$ is the direct arrival signal matrix,
- $g \in \mathbb{R}^{K \times 1}$ is a vector that scales the columns of $D$ independently,
- $M$ is the number of measurement samples, and
- $N$ is the number of pixels.
We adopt the q-generalized Gaussian Markov Random Field (qGGMRF) for the prior model. With this design, the prior model is

\[
p(x) = \frac{1}{z} \exp \left( - \sum_{\{s,r\} \in C} b_{s,r} \rho(x_s - x_r) \right)
\]

where \( z \) is a normalizing constant, \( C \) is the set of pair-wise cliques, and

\[
p(\Delta) = \frac{|\Delta|^p}{p \sigma_{g,s,r}^p} \left( \frac{\Delta}{T \sigma_{g,s,r}} \right)^{q-p} \right)
\]

where \( \sigma_{g,s,r} = \sigma_0 \sqrt{m_s m_r} \) and \( m_s = 1 + (m - 1) \ast \left( \frac{\text{depth of pixel } s}{\text{maximum depth}} \right)^a \)

Hence,

\[-\log p(x) = \sum_{\{s,r\} \in C} b_{s,r} \rho(x_s - x_r) + \text{constant} .\]
Optimization of MAP cost function

- After combining the forward and prior models, the MAP estimate is given by

\[
(x, g)_{\text{MAP}} = \arg \min_{x \geq 0, g} \left\{ \frac{1}{2\sigma^2} \| y - Ax - Dg \|^2 + \sum_{\{s, r \} \in C} b_{s, r} \rho(x_s - x_r) \right\}
\]

ICD Algorithm Using Majorization Technique

Initialize \( x, e \leftarrow y - Ax \)

For k iterations {

\[
g = (D^t D)^{-1} D^t e
\]

\( e \leftarrow e - Dg \)

For each pixel \( s \in S \) {

\[
\tilde{b}_{s, r} \leftarrow \frac{b_{s, r} \rho'(x_s - x_r)}{2(x_s - x_r)} \\
\theta_1 \leftarrow -e^t A_{s, s} + \sum_{r \in \partial s} \tilde{b}_{s, r} (x_s - x_r) \\
\theta_2 \leftarrow A_{s, s}^t A_{s, s} + \sum_{r \in \partial s} \tilde{b}_{s, r} \\
\alpha^* \leftarrow \text{clip}\left\{ -\frac{\theta_1}{\theta_2}, [-x_s, \infty) \right\} \\
x_s \leftarrow x_s + \alpha^* \\
e \leftarrow e - A_{s, s} \alpha^* 
\]

}
Experimental Results: System geometry
Experimental Results: Synthetic Data Results

• Synthetic data was generated using the K-Wave simulator.

• The red and green dashed lines demonstrate the groove and backwall locations, respectively.
Experimental Results: Real Data Results

SAFT

![SAFT Image]

UMBIR

![UMBIR Image]
Conclusion

- We proposed our multi-layer UMBIR algorithm designed for ultrasonic collimated beam systems.
- We showed the derivation of our modified forward model for multilayered structures and collimated ultrasonic-transducers.
- Our results demonstrated that our UMBIR shows clear improvements over SAFT and is effective for real data applications.
Thank You!
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Recons of the 37 positions. The notch can be seen between position 16 and 25.