The local Hubble flow:

A manifestation of dark energy

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Abstract

Our local environment at $r < 10$ Mpc expands linearly and smoothly, as if ruled by a uniform matter distribution, while observations show the very clumpy local universe. This is a long standing enigma in cosmology. We argue that the recently discovered vacuum or quintessence (dark energy (DE) component with the equation of state $p_Q = who_Q c^2$, $w \in [-1, 0]$) from observations of the high-redshift universe may also manifest itself in the properties of the very local Hubble flow. We introduce the concept of the critical distance $r_Q$ where the repulsive force of dark energy starts to dominate over the gravity of a mass concentration. For the Local Group $r_Q$ is about 1.5 Mpc. Intriguingly, at the same distance 1.5 Mpc the linear and very "cold" Hubble flow emerges, with about the global Hubble constant. We also consider the critical epoch $t_Q$, when the DE antigravity began to dominate over the local matter gravity for a galaxy which at the present epoch is in the
local DE dominated region. Our main result is that the homogeneous dark energy component, revealed by SNIa observations, resolves the old confrontation between the local Hubble flow and local highly non-uniform, fractal matter distribution. It explains why the Hubble law starts on the outskirts of the Local Group, with the same Hubble constant as globally and with a remarkably small velocity dispersion.

1 Introduction

The Hubble diagram for high redshift Type Ia Supernovae (Riess et al. 1998, Perlmutter et al. 1999) and the Boomerang and MAXIMA-1 measurements of the first acoustic peak location in the angular power spectrum of the cosmic microwave background (CMB) (de Bernardis 2000; Jaffe et al. 2000) strongly favor a standard cosmological model having the critical density ($\Omega = \Omega_m + \Omega_\Lambda = 1$) and a dominant $\Lambda$-like, “dark energy’ component at the present epoch ($\Omega_\Lambda^0 \approx 0.7$). In this Letter we argue that the cosmological $\Lambda$ component is also important for understanding the very local Hubble diagram. Namely, there is a recognized riddle in observational cosmology – the linear and quiet Hubble flow in the very clumpy local universe (Sandage et al. 1972; Weinberg 1977; Peebles 1992; Baryshev 1994; Karachentsev & Makarov 1996; Baryshev et al. 1998; Sandage 1999).

We show that the Hubble flow emerges there, at a distance of about 1.5 Mpc, where the antigravity of the dark energy starts to dominate over the gravity of clumpy matter. We suppose that this is not a coincidence but the key to the riddle of the local Hubble law.

2 The enigma of the local Hubble flow

2.1 Formulation of the problem

In the standard cosmology the dynamics described by the Hubble law, $V = H \times r$, is a strict consequence of a uniform distribution of self-gravitating matter (Robertson 1955; Peebles 1993). Hence the predicted linear velocity field is only valid for scales where the universe is uniform. However, a very puzzling fact, long ago noted by Sandage,
Tammann & Hardy (1972; STH), is that Hubble discovered his law in the distance interval $1 - 20$ Mpc, where the galaxies are very clumpy distributed. Indeed, this was deep inside a cell of uniformity, the size of which is still debated, but there are modern estimates from 30 Mpc up to 200 Mpc (Sylos Labini, Montuori & Pietronero 1998; Wu, Lahav & Rees 1999). Inside such cells the galaxies are fractally distributed, being clumpy at all smaller scales. This means that "in fact, we would not expect any neat relation of proportionality between velocity and distance for these [closeby] ... galaxies" (Weinberg 1977).

On the other hand, recent studies of the local volume ($< 10$ Mpc), based on accurate distances, confirm a "cold" linear Hubble flow, with about the global Hubble constant $H_0$ (Sandage 1986; Karachentsev & Makarov 1996; Teerikorpi 1997; Giovanelli et al. 1999; Ekholm et al. 1999). Sandage (1999) recently expressed this surprising situation, as the "extremely local rate [of expansion] is the same as the global rate to better than 10%", and an "explanation of why the local expansion field is so noiseless remains a mystery". Indeed, the velocity dispersion around the local Hubble law is very low ($\sigma_V \leq 50 - 70$ km/s) (Sandage 1986; Karachentsev & Makarov 1996; Ekholm et al. 2000; in the last work Cepheid distances gave $\approx 40$ km/s). That here is a real problem, has been also shown with N-body simulations (Governato et al. 1997): the expected velocity scatter close to galactic clumps like our LG is $150 - 700$ km/s, depending on the CDM model.

One may thus divide the enigma of the local Hubble flow into the following Problems:

1. The linear velocity law appears at distances (1.5 Mpc) which are about 1 percent of the scale where the Universe may finally appear uniform. Why does our highly non-uniform closeby environment expand as if it were uniform? Why is the rate of expansion the same locally and globally, i.e. deep inside and well outside of the cell of uniformity?

2. What makes the local Hubble flow so quiet? Not only does the Hubble law exist, but it has a remarkably small scatter.

### 2.2 Previous attempts to explain the Hubble enigma

In their seminal paper, STH discussed the hierarchical galaxy distribution suggested by de Vaucouleurs (1971). Wertz (1971) and Haggerty & Wertz (1971) had made calcula-
tions on how the Hubble law deviates from linearity inside such a structure. STH did not find the predicted non-linearity and rejected the hierarchical model. However, they emphasized that the co-existence of the linear Hubble law and the large local clouds of inhomogeneous matter still is a dilemma. STH proposed two possible solutions. The linearly expanding universe may be inhomogeneous if its mean density is very low, $<< \rho_{\text{crit}}$. Or it may be filled by a totally uniform dark matter.

Modern redshift surveys, together with accurate measurements of the local velocity field, have sharpened this "Hubble - de Vaucouleurs” paradox (Baryshev et al. 1998). Indeed, the 3-dimensional galaxy maps at scales from 0.1 Mpc up to 20-200 Mpc are well described by a scale invariant fractal distribution (Davis & Peebles 1983; Pietronero 1987; Peebles 1993; Sylos Labini, Montuori & Pietronero 1998; Teerikorpi et al. 1998; Wu, Lahav & Rees 1999). Using new data on large scale structure and on the Hubble law, the solutions mentioned by STH were studied in Baryshev et al. (1998). From the linear theory of the gravitational growth of density fluctuations (without dark energy), they concluded that either $\Omega_m \leq 0.01$ or for the uniform dark matter $\Omega_{\text{dark}} \geq 0.99$.

Nevertheless, these explanations are not sufficient, if considered together with recent cosmological observations which give $\Omega_m \approx 0.3$. Also there is no direct evidence for a high-density uniform dark matter. Hence the riddle still exists.

3 Dark energy and dynamical model

3.1 Vacuum, quintessence, dark energy

Recent detection of the dark energy component provide a natural candidate for a high-density truly uniform background, giving a novel possibility for resolving the Hubble enigma.

In the cosmological theory there are several kinds of dark energy with a positive energy density ($\epsilon_Q = \rho_Q c^2 > 0$) and negative pressure ($p_Q < 0$). The dark energy or quintessence is a common name for: the classical Einstein’s cosmological constant

$^1$Note that Mandelbrot (1982) replaced the old concept of hierarchical galaxy distribution by the more adequate concept of stochastic fractals.
Λ, time-variable cosmological constant \( \Lambda(t) \), cosmological vacuum, effective scalar field and other forms of exotic matter with negative pressure (see reviews by Bahcall et al. 1999; Sahni & Starobinsky 2000). Dark energy is now much discussed and used for interpretation of observational cosmological tests (Wang et al. 2000; Podariu & Ratra 2000). The main reason to consider evolving DE is the “cosmic coincidence”: Why is the rapidly decreasing energy density of matter at the present epoch so close to that of the unchanging vacuum density?

The most striking property of the dark energy having the equation of state \( p_Q = w \epsilon_Q \), with \( w \in [-1, 0) \), is that its gravitating mass

\[
M_Q = \frac{4\pi}{3} (1 + 3w) \rho_Q r^3
\]

is negative for \( w < -1/3 \). This produces cosmological ”antigravity” or a repulsion which accelerates the expansion of the universe. E.g. for Einstein’s \( \Lambda = 8\pi G \rho_{\Lambda}/c^2 \) (cosmological vacuum with \( w = -1 \)), the gravitating mass is \( M_\Lambda = -8\pi \rho_{\Lambda} r^3/3 \).

### 3.2 Dynamical model and the critical distance \( r_Q \)

We consider the dark energy component uniformly spreading everywhere, and existing alongside with luminous and dark matter in the immediate environment of our Galaxy. Hence, the local dynamics is determined by the competition of the gravity of dark (and luminous) matter and the antigravity of the DE. The mass-to-luminosity ratios in systems of different scales suggest that \( M/L \) remains constant for \( r > 0.5 \) Mpc (Bahcall, Lubin, & Dorman 1995), hence dark matter should be distributed like luminous matter on such scales. The correlation analysis of the space distribution of galaxies shows that the mean density of luminous matter (and hence, of dark matter) decreases with increasing scale so that the matter mass increases as \( M_m(r) \propto r^D \), where \( D \), the fractal dimension, is between 1 and 2 at least on scales up to 20 Mpc (Wu, Lahav & Rees 1999). As its density is constant \( M_Q(r) \propto r^3 \), the DE component starts to dominate after some distance \( r_Q \).

For a simple estimate of the scale beyond which vacuum or quintessence dominates, we consider two models that may imitate the environment of the Local Group, where the
local Hubble law is accurately observed. The first model is a generalization of Sandage’s (1986) point-mass model, where a mass $M$ is placed on the dark energy background with density $\rho_Q$. In the second model, a spherical distribution of mass $M_m(r)$ is placed on the DE background.

The dynamics of expansion is described by the equation of motion which is the (1,1)-component of Einstein’s field equations under the assumptions of spherical symmetry, dust-like matter, and cosmological dark energy given by the energy-momentum (EM) tensor $T^i_k = \rho_Q c^2 \text{diag}(1, -w, -w, -w)$:

$$\ddot{r} = -G M_{\text{eff}} / r^2; \quad M_{\text{eff}} = M_m(r) + M_Q(r). \quad (2)$$

Here $M_m(r) = M$, or $M_m(r) = M_\ast (r/r_\ast)^P$, respectively for the first and second model, $M_Q$ is given by Eq. (1).

For the point-mass the critical distance $r_Q$ corresponding to $\ddot{r} = 0$, i.e. the distance where DE antigravity compensates matter gravity, is defined by:

$$r > r_Q = (3M/(4\pi \tilde{w} \rho_Q))^{1/3} \quad (3)$$

where $\tilde{w} \equiv -3w - 1$. For $w \geq -1/3$ there is no repulsive force and $r_Q$ does not exist.

For the second model the dark energy term dominates dynamically at distances

$$r > r_Q = (r_\ast)^{-\frac{2}{3}} M_\ast^{-\frac{1}{3}} (\frac{3}{4\pi \tilde{w} \rho_Q})^{\frac{1}{3}} \quad (4)$$

### 3.3 The temporal behavior of $r_Q$ and critical time $t_Q$

In the case of cosmological constant a galaxy which now is on the border of the vacuum dominated region around a mass clump, previously lied inside the gravity dominated sphere. In terms of comoving coordinates, $r_Q$ shifts outwards for increasing redshift. This prompts us to ask how $r_Q$ behaves if the DE density changes, as in quintessence models, where $r_Q$ may still shift outwards in comoving coordinates, but slower than for the constant vacuum.

We illustrate the shift of $r_Q$ in time for three types of DE models: 1) Einstein’s cosmological constant $\Lambda$, 2) the quintessence with $w = -2/3$, and 3) the case of “coherent” evolution of a quintessence ($\rho_Q = k \rho_{hm}$, where $\rho_{hm}$ is the uniform matter component)
during the late history of the universe, with $w = -2/3$, $k = 1$. The condition that the covariant divergence of the total EM tensor (quintessence + matter) is zero implies that 
\[ \dot{\rho} = -3\left(\rho + \frac{p}{c^2}\right)\dot{a}/a, \]
where $\rho = \rho_Q + \rho_m$ and $p = p_Q$ for dust-like matter ($p_m = 0$). Hence the DE density behaves as
\[ \rho_Q \propto a^{-3\left(\frac{1+k+w}{1+k}\right)} \] (5)
For $1/k = 0$ (no homogeneous matter) Eq.(5) gives the usual (non-coherent) quintessence behavior $\rho_Q \propto a^{-3(1+w)}$.

Let us consider the ratio between two metric distances $r_{gal}/r_Q$, where $r_{gal}$ is the distance to a galaxy which now is in the DE dominated region and takes part in the Hubble flow. The distance $r_{gal}$ is simply proportional to $a$. The behavior of the critical distance $r_Q$ depends on the DE and matter models. E.g. for the point-mass model the ratio is $r_{gal}/r_Q = a^{(-w/(1+1/k))}$, which for $1/k = 0$ gives $a^{-w}$, corresponding either to Einstein’s $\Lambda$ ($w = -1$) or to the quintessence (for $w \leq -1/3$).

We define a characteristic time for the antigravity dominance. It is counted from the critical epoch $t_Q$, corresponding to the critical scale factor $a_Q = a(t = t_Q)$ when DE antigravity starts to dominate over the gravity of matter concentration component for a galaxy which presently ($t = t_0$) is at the distance $r = r_{gal} = 2r_Q^0$, i.e. well within the DE dominated region. E.g. for Sandage’s point-mass model thus defined critical scale factor is
\[ a_Q = 2^{(1+1/k)/w} \] (6)
With $1/k = 0$ this gives $a_Q = 0.5$ for $w = -1$ and $a_Q = 0.354$ for $w = -2/3$, while with $k = 1, w = -2/3$ the critical epoch is at $a_Q = 0.125$. This illustrates the difference in critical times for different quintessence models.

## 4 Specific properties of dark energy dominated dynamics

### 4.1 Structure evolution in DE dominated regions

The theory of structure formation in the general case of an evolving DE component is still under construction, though there are some results for particular cases (Sahni & Starobinsky 2000; Fabris & Goncalves 2000). If during the structure formation period
most of the energy of the universe produces antigravity and hence resists gravitational collapse, it is impossible for the structure to grow at all. On the other hand, if antigravity dominates only a short period, as in the case of the cosmological constant, then even for $\Omega_\Lambda \sim 1$ it has only a slight effect on the infall velocities around growing structures (Peebles 1984; Lahav et al. 1991). This is clear from Eq. (2) which gives for the ratio of DE and point-mass accelerations $g_Q/g_m \propto a^{-3w}$, hence for $w = -1$ the dynamical influence of the constant $\Lambda$-component decreases very fast, while for $w = -2/3$ the antigravity acts longer.

Another important consequence of the DE component is that within $\Lambda$ dominated regions initial peculiar velocities decrease due to adiabatic cooling. The linear analysis of gravitational instability, following Zeldovich (1965), shows that only decreasing modes exist in the vacuum-dominated region (Chernin, Teerikorpi & Baryshev 2000). This property of the accelerated universe is important for understanding the local Hubble flow. Our simple dynamical models led to the notion of the distance $r_Q$ to the border between the matter and DE dominated regions. If a galaxy now beyond $r = r_Q$ also in the past was long enough in the DE dominated region, this would readily explain the smooth Hubble law at such distances. Then initial peculiar velocities induced by the mass of the Local Group (and similar more distant groups) have decreased because of the adiabatic cooling ($\delta V(\text{now}) = \delta V(z)a(z)$).

Therefore a compromise situation is possible when the time-variable DE is dominant during a period, sufficient for the structure growth in early times and for the cooling of velocity dispersion in late times. This means that such DE dominated regions between mass concentrations are "pacific oceans" where the Hubble law appears and the global Hubble constant may be measured even locally.

### 4.2 Restoration of the Hubble flow in volumes with bulk motion

We referred to two properties of the cosmological vacuum, namely uniformity and antigravity. Let us now consider an important third property of the vacuum. It is clear that the medium with the equation of state $p = -\rho c^2$ described by the cosmological constant has EM tensor which is proportional to the metric tensor $T^{k}_{l} = \alpha g^{k}_{l}$. Hence it has the
remarkable mechanical property of vacuum: rest and motion can not be discriminated relative to it. In other words, any matter motion is co-moving to this medium.

When the vacuum dominates over the self-gravity of matter, matter masses like galaxies move as test particles in the antigravity of the vacuum. The vacuum accelerates the motions tending to form a regular kinematic pattern with a linear velocity field. The expansion rate in such a flow depends only on the value of the vacuum density.

These considerations can be applied to a local vacuum-dominated area, no matter how fast is the bulk motion of the area against the CMB. They are valid as well for the large-scale matter distribution; in both cases, the flow will be linear and the rate of expansion will be the same on any scale. And the flow will also be stable against velocity or density perturbations.

5 The critical distance $r_Q$ for the Local Group

Let us calculate the critical distance for the Local Group in the case of the point-mass model and the cosmological vacuum ($w = -1$ and $\rho_Q = \rho_\Lambda$ is constant in time).

The most recent estimate for the mass of the LG gives $M_{LG} = 2 \times 10^{12} M_\odot$ (van den Bergh 1999). The vacuum density from SNIa observations is $\rho_\Lambda \simeq 0.7 \rho_{crit} = 4.7 \times 10^{-30} h_{60}^2 \text{g/cm}^3$. Then the distance $r_\Lambda$, where the vacuum starts to dominate, is $\simeq 1.5$ Mpc in the point-mass model. If one changes $M_{LG}$ or $\rho_\Lambda$ by a factor of two, the critical distance is changed by 26 percent. If one chooses the mass distribution model with fractal dimension $D = 1$ and $M_* = 2 \times 10^{12} M_\odot$ at $r_* = 1$ Mpc, then Eq. (4) gives $r_\Lambda = 1.8$ Mpc.

Hence, $r_\Lambda$ is robustly put into the range from 1 to 2 Mpc. This value is surprisingly close to the distance where the Hubble law emerges, which is 1.5 Mpc (Sandage 1986). Is this just a coincidence? Not at all, we suggest that this is rather a major feature of the local matter flow. Indeed, the dynamical dominance of the dark energy at 1-2 Mpc and beyond means that the homogeneous vacuum provides the dynamical background for the Hubble flow of matter, on these distances.
6 The local and global values of the Hubble constant

Now we can understand Problem I – the near-equality of the local and global Hubble constants – if the undistorted Hubble law is the signature of the dynamical dominance of the uniform DE component. In regions, which have for a sufficient time been DE-dominated, the rate of expansion – in the first (and main!) approximation – is expressed via the DE density: $H_0 \approx (8\pi G \rho_Q/3)^{1/2}$. The lumpy, fractally distributed matter gives only a small correction to this formula for the global Hubble constant, because its average density is much less than the DE density. The very interesting aspect of the dominating uniform DE component is that it allows us to predict the value of the Hubble constant from a local estimate of the dark energy density.

If we identify the distance $r_{\text{start}}$ where the Hubble law starts, with the critical radius $r_Q$ for our Local Group, then the predicted value of $H_0$ is

$$H(\text{predicted}) \approx \left(\frac{2GM_{\text{LG}}}{\tilde{w}}\right)^{1/2} (r_{\text{start}})^{-3/2}$$

where $M_{\text{LG}}$ is the mass of the Local Group. With the ”standard” values $M = 2 \times 10^{12} M_\odot$ and $r_{\text{start}} = 1.5$ Mpc, the Eq.(7) gives for $H_0$ values from 53 km/s/Mpc ($w = -1$) to 74 km/s/Mpc ($w = -2/3$), covering the values obtained for $H_0$ by different research groups. More conservatively, these are upper limits, if in fact $r_Q < r_{\text{start}}$. We regard this agreement with direct measurements of $H_0$ as evidence for the cosmological vacuum and quintessence models. Note that this reasoning for the DE component is based on locally measured quantities ($M_{\text{LG}}$ and $r_{\text{start}}$) and is independent of the supernovae determination of $\Omega_\Lambda$ (which did not depend on the value of $H_0$).

7 The quietness of the local Hubble flow

The possibility to solve Problem II may be illustrated by the following simplified reasoning. At any epoch $a(t)$ and on the scale $r$ one may regard $\delta V \approx H r$ as an upper limit to the scatter around the Hubble law. For models considered in Sec.3.3 the curvature is zero and hence from Eq.(6) it follows that the Hubble constant $H(a)$ depends on the scale factor as $a^{-(3/2)\beta}$, where $\beta = (1 + k + wk)/(1 + k)$. For example in the case of
vacuum, i.e. \( w = -1 \) and \( 1/k = 0 \) we have \( H = \text{const} \), and in the case of dust matter \( k = o \) hence \( H \propto a^{-3/2} \).

So the scatter \( \delta V(a) < H_0 r_0 a^{1-(3/2)\beta} \), where \( r_0 \) is the considered scale at the present epoch. E.g. for \( r_0 = 5 \) Mpc and \( H_0 = 60 \) km/s/Mpc, \( \delta V(a) < 300 \) \( a^{1-(3/2)\beta} \) km/s. This is also valid at the critical epoch \( a_Q \) after which the peculiar velocities within a DE dominated region start to cool down adiabatically, so that \( \delta V(\text{now}) = \delta V(a_Q) \) \( a_Q \). Hence one expects now \( \delta V(\text{now}) < 300 \) \( a_Q^{2-(3/2)\beta} \) km/s.

For simple numerical examples we use the critical epoch \( a_Q \) calculated in Sec.3.3. In the case of the cosmological constant we have \( a_Q = 0.500 \), hence \( \delta V(\text{now}) < 75 \) km/s. For the quintessence with \( w = -2/3 \), \( a_Q = 0.354 \) and \( \delta V(\text{now}) < 63 \) km/s, and for the case of coherent evolution \( w = -2/3, k = 1 \), the critical scale factor \( a_Q = 0.125 \) and now one expects to observe \( \delta V(\text{now}) < 38 \) km/s.

Of course, these numbers just illuminate the trend for different DE models. A complete treatment of the dispersion problem needs high-resolution large-volume simulations of structure formation on the evolving DE background.

8 Conclusions

We conclude that a cosmological dark energy component allows one to understand the emergence of the linear and quiet Hubble flow at the border of the Local Group. The coincidence of the \( r_Q \) with the starting distance of the Hubble law is seen to have the same roots as the “cosmic coincidence” of \( \Lambda \) and matter densities. A summary of our results is:

* We introduce the concept of the critical distance \( r_Q \) where the repulsive force starts to dominate over the gravity of matter. For the Local Group \( r_Q \approx 1.5 \) Mpc in the case of \( \rho_Q = 0.7 \rho_{\text{crit}} \). This is close to the distance where the local Hubble flow emerges. We make this new cosmic coincidence as a starting point for understanding the cold local Hubble flow as reflecting the dark energy dominated region around us.

* In dark energy dominated regions of the universe (“pacific oceans”) the linear Hubble law exists due to the homogeneity of the DE component, with the same Hubble
constant at first approximation determined by the dark energy density. This resolves Problem I that the Hubble law not only starts at the border of the DE dominated region, but it appears with the $H_0$ as expected from the dark energy density.

* The solution of Problem II follows from the fact that a galaxy spending a sufficient time in the DE dominated region looses its initial peculiar velocity which cools down adiabatically. The mysterious quietness of the local Hubble flow does not look at all dramatic in the context of the dark energy driven expansion.

* Our results support a dark energy component which presently dominates the dynamics of the Universe on scales from the Local Group to the Hubble radius. Even the closeby galaxy universe is now seen as a cosmic laboratory where all the physical ingredients of the universe: luminous matter, dark matter, and the dark energy, may be detected. Hence, the very local volume is extremely important for the study of global properties of the universe. Cosmology starts immediately beyond the border of the Local Group!

In other environments the DE dominance may start from smaller or larger distances. So, around the Coma cluster, the vacuum starts dominating from about 20 Mpc. For a cosmologist in Coma, cosmology begins around such a distance. In such DE dominated regions the quiet Hubble flow is produced by the same vacuum density and hence the Hubble constant is the same.

One now realizes that when Hubble found his linear redshift law in the closeby lumpy environment, he actually saw the influence of the mysterious uniform dark energy. Though the value of the vacuum or DE density is still poorly understood in cosmological theory, the observations make one agree with the old words by Lemaître who was the first to associate $\Lambda$ with the vacuum having positive energy density and negative pressure leading to a cosmological repulsion: "Everything happens as though the energy in vacuo were different from zero." (Seitter & Duemmler 1989).

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References

Bahcall, N.A., Lubin, L.M., Dorman, V. 1995, Astrophys.J., 447, L81
Bahcall N., Ostriker J., Perlmutter S., Steinhardt P. 1999, Science 284, 1481
Baryshev Yu. 1994, Astron.Astrophys.Trans., 5, 15
Baryshev Yu., Sylos Labini F., Montuori M., Pietronero L., Teerikorpi P. 1998, Fractals, 6, 231
Chernin A., Teerikorpi P., Baryshev Yu. 2000, Adv. Space Res.(in press)
Davis M., Peebles P.J.E. 1983, ApJ, 267, 465
de Bernardis et al. 2000, Nature, 404, 955
de Vaucouleurs G. 1971, Science, 167, 1203
Ekholm T., Lanoix P., Teerikorpi P., Paturel G., Fouqué P. 1999, A&A, 351, 827
Ekholm T., Baryshev Yu., Teerikorpi P., Hanski M., Paturel G. 2000, A&A (submitted)
Fabris, J.C., Goncalves, S.V.B., 2000, gr-gc/0010046
Giovanelli R., Dale D., Haynes M., Hardy E., Campusano L. 1999, ApJ, 525, 25
Governato F. et al. 1997, New Astr., 2, 91
Haggerty M., Wertz R.J. 1971, MNRAS, 155, 495
Jaffe A.H. et al. 2000, astro-ph/0007333
Karachentsev, I., Makarov, D. 1996, AJ, 111, 794
Lahav O., Lilje P., Primack J., Rees M. 1991, MNRAS, 251, 126
Mandelbrot B. 1982 The Fractal Geometry of Nature (W.H. Freeman )
Peebles P.J.E. 1984, ApJ, 284, 439
Peebles P.J.E., 1992, Lecture in the symposium ”The Cosmic Microwave Background Radiation and the Large-Scale Structure of the Universe after COBE” (Stockholm 1992)
Peebles, P.J.E. 1993 Principles of Cosmology (Princeton: Princeton Univ. Press)
Perlmutter, S. et al. 1999, ApJ, 517, 565
Pietronero L. 1987, Physica, A144, 257
Podariu S., Ratra B. 2000, ApJ, 532, L109
Riess, A.G. et al. 1998, AJ, 116, 1009
Robertson, H.P. 1955, PASP, 67, 82
Sahni V., Starobinsky A., 2000, astro-ph/9904398
Sandage, A. 1986, ApJ, 307, 1
Sandage, A. 1999, ApJ, 527, 479
Sandage A., Tammann G., Hardy E. 1972, ApJ, 172, 253
Seitter, W.C., Duemmler, R. 1989, in Morphological Cosmology (eds. P. Flin, H.W. Dverbeck), Springer, Berlin, pp. 377-387
Sylos Labini, F., Montuori, M., Pietronero, L. 1998, Phys.Rep., 293, 61
Teerikorpi, P. 1997, ARA&A, 35, 101
Teerikorpi, P. et al. 1998, A&A, 334, 395
van den Bergh, S. 1999, A&A Rev., 9, 273
Wang L., Caldwell R.R., Ostriker J.P., Steinhardt P.J. 2000, ApJ, 530, 17
Weinberg, S. 1977 The First Three Minutes (Basic Books, New York), p.26
Wertz J.R. 1971, ApJ, 164, 227
Wu, K.K.S., Lahav, O., Rees, M.J. 1999, Nature 397, 225
Zeldovich, Ya.B. 1965, Advan. Astron. Ap., 3, 241