CMB power asymmetry and suppression: Two sides of the same coin?

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Abstract

The recent measurements of temperature and polarization of Cosmic Microwave Background (CMB) have improved our understanding of the Universe and have showed a remarkable agreement with the ΛCDM cosmological model. However, scale dependent features like power suppression in the angular power spectrum and hemispherical asymmetry in the temperature field of CMB at large angular scales, hinting at possible departure from the ΛCDM model persist in the CMB data. In this paper we present a physical mechanism linked to possible initial inhomogeneities in the inflationary scalar field that could explain both the observed phenomena.

Initial inhomogeneities lead to non-zero values of anisotropic inflationary parameters, which at leading order cause different amounts of hemispherical asymmetry in the scalar and tensor perturbations. The second order effect of anisotropic inflationary parameters naturally lead to a suppression (enhancement) of power at low multipole $l$ for scalar (tensor) perturbations. This model also predicts several characteristic signatures in the temperature and polarization spectra that are accessible to future CMB missions.

1 Introduction

Exquisite measurements of Cosmic Microwave Background (CMB) have opened an era of precision cosmology. The observed angular power spectra of CMB temperature and polarization (specifically E modes) is best fit within the minimal (6 parameters) ΛCDM model at small angular scales. However at large angular scales ($\theta > 3^\circ$), observations seem to indicate a power suppression in the angular power spectra of temperature [1, 2] and statistical isotropy (SI) violation in the temperature field of CMB in the form of a hemispherical power asymmetry [3, 4]. These features lie beyond the scope of standard cosmology based on an isotropic ΛCDM model. Present in CMB maps from both WMAP and Planck [5], these are unlikely to be linked to observational systematic effect. Recently several models are proposed to produce the hemispherical asymmetry [6, 7].

In this paper we propose a model in which we reconcile both these effects by a single phenomenon that is not difficult to accommodate during the inflationary epoch of the Universe. Our model shows that the initial inhomogeneities give rise to both the diagonal and off-diagonal terms of the covariance matrix that contributes respectively to angular power spectra and Bipolar Spherical Harmonic (BipoSH) coefficients [8] for temperature and polarization. Non-zero BipoSH coefficients lead to different amounts of hemispherical asymmetry in the scalar and tensor perturbations. The unique feature of this model is the modulated tensor field that leaves a potentially detectable signature of SI violation in the B mode polarization of CMB. This can be measured from future missions as shown in

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a recent paper [9]. The other salient feature of this model is a naturally associated low multipole (l) power suppression in the angular power spectra only in the scalar perturbations and mild enhancement of power in the tensor perturbations. This model is readily falsifiable by more refined measurements of temperature and polarization field from future missions.

2 Initial inhomogeneities in the inflationary scalar field

The standard hot big bang cosmological model requires an early accelerating epoch of the Universe called Inflation[10, 11] that is accompanied by generation of the initial scalar and tensor perturbations. Inflationary epoch is well studied in the literature [11] for different types of scalar field potentials and the feasibility of different inflationary models with the recent measurements from Planck are also well explored [12, 13].

Dynamics of the inflationary epoch is governed by the dominant scalar field (inflaton). However, the presence of initial inhomogeneities in the inflaton field does not prevent the onset of inflation if the condition [14]

\[ a \lambda > \sqrt{\frac{8\pi}{3m_{pl}} \frac{\delta \Phi}{\dot{H}} \frac{H^{-1}}{H}} \]  

is satisfied, where, \( a \) is the cosmological scale factor, \( H = \frac{\dot{a}}{a} \) is the Hubble parameter, \( \lambda \) is the comoving wavelength of the initial inhomogeneities and \( \delta \Phi \) is the change in the inflaton field (\( \Phi \)). This equation implies that for inhomogeneities of same order as Planck mass (\( \delta \Phi \approx \sqrt{\frac{2}{8\pi m_{pl}}} \)), the typical wavelength of inhomogeneities in the inflaton field (\( \Phi \)) should be greater than few Hubble radii (\( H^{-1} \)). But for inhomogeneities less than the Planck mass (\( \delta \Phi < \sqrt{\frac{2}{8\pi m_{pl}}} \)), the inhomogeneous patch of the inflaton field can be of the same order as Hubble radii (\( H^{-1} \)). In this case, the inhomogeneous inflaton field at Hubble scales can be described by a direction dependent inflaton field given by

\[ \tilde{\Phi}(\hat{n}) = \Phi + \sum_{LM} \delta \Phi_{LM} Y_{LM}(\hat{n}). \]  

We assume that the dipolar term (\( L = 1 \)) is the most dominant term of the inhomogeneous inflaton field. This is motivated by the presence of initial inhomogeneities at the scales of Hubble radius or greater during inflation, which is readable from Eq. (1). This effect can also be comprehended as a horizon scale modulation in the background inflaton field, over which generated fluctuations are embedded.

The anisotropic inflaton field (\( \tilde{\Phi}(\hat{n}) \)) can lead to mildly different values of the inflaton potential in different directions and hence cause a mild departure from the isotropic Hubble parameter in the early epoch of inflation. Granted negligible contribution from the gradient part by satisfying Eq. (1), the anisotropic Hubble parameter can be calculated by the equation

\[ H^2(\hat{n}) = \frac{8\pi}{3m_{pl}^2} \left[ \frac{\dot{\Phi}^2}{2} + V(\tilde{\Phi}(\hat{n})) \right], \]  

and for the slow roll inflationary models, \( \frac{\dot{\Phi}^2}{2} < V(\tilde{\Phi}(\hat{n})) \), the above simplifies to

\[ H^2(\hat{n}) \approx \frac{8\pi}{3m_{pl}^2} V(\tilde{\Phi}(\hat{n})). \]  

The dependence of the inflationary potential on the inflaton field is model dependent. Hence, the modification in the background inflaton field as shown in Eq. (2) induces a model dependent effect in \( V(\tilde{\Phi}(\hat{n})) \) and hence in \( H(\hat{n}) \). To study the effect of direction dependence, we express Hubble parameter in terms of anisotropic Hubble parameter \( H_a \), which is related to \( \tilde{\Phi} \) differently for different inflationary models. The exact functional dependence of \( H(\hat{n}) \) on \( \tilde{\Phi}(\hat{n}) \) is not required to calculate the effects on the observable quantities. In terms of \( H_a \), we can express the Hubble parameter as

\[ H(\hat{n}, \eta) = H_b(\eta)[1 + \chi \dot{\phi} \hat{n}], \]  

where, \( \chi \equiv \chi(\eta) = H_a/H_b \) and \( \dot{\phi} \) is the direction of the dipole. Here we assume that the anisotropy in the Hubble parameter is a perturbation with the leading order term arising from the dipole. The time derivative of \( H(\hat{n}, \eta) \) can be written as

\[ \dot{H}(\hat{n}, \eta) = \dot{H}_b(\eta)[1 + \xi \dot{\phi} \hat{n}], \]  

where, \( \xi \equiv \xi(\eta) = H_a/\dot{H}_b \). So, the anisotropic
behaviour of the Hubble parameter is quantified by two anisotropic parameters $\chi \ll 1$ and $\xi \ll 1$. These parameters are related through average isotropic inflationary evolution by

$$\dot{\chi} = [\xi - \chi] \frac{\dot{H}_b}{H_b}. \quad (7)$$

These anisotropic Hubble parameters quantify to observable signatures in the Primordial Power Spectrum (PPS) discussed in the following sections.

3 PPS from anisotropic Hubble parameter

Inflation generates the initial seed of scalar and tensor perturbations, which at a later epoch evolves to density perturbations underlying to the observed large scale structure and an yet undetected cosmological Stochastic Gravitational Wave Background (SGWB). Due to the rapid expansion phase of Universe, quantum fluctuations of the inflaton field freezes for the modes that are greater than the Hubble radius during inflation. The leading order fluctuations are Gaussian in nature with zero mean. The matter density fluctuations ($\delta \rho$) and tensor fluctuations ($\delta h$) are related to the power spectrum by

$$\langle \delta \rho^*(\mathbf{k})\delta \rho(\mathbf{k}') \rangle = \frac{2\pi^2}{k^3} P_s(k) \delta(\mathbf{k} - \mathbf{k}'),$$

$$\langle \delta h^*(\mathbf{k})\delta h(\mathbf{k}') \rangle = \frac{2\pi^2}{k^3} P_t(k) \delta(\mathbf{k} - \mathbf{k}'). \quad (8)$$

The power spectra are related to the value of the Hubble parameter at the horizon crossing (denoted by $^{s\ast}$) by [10]

$$P_s(k) = A_s \frac{H^2}{\epsilon} |_{s\ast},$$

$$P_t(k) = A_t H^2 |_{s\ast}, \quad (9)$$

where, $A_s$ and $A_t$ denotes the amplitude for scalar and tensor perturbations respectively and $\epsilon = \frac{\dot{H}_b}{H_b} \ll 1$ is the slow roll parameter. The effect of the anisotropic Hubble parameter (mentioned in Eq. (5) and Eq. (6)) on the PPS can be obtained as

$$\tilde{P}_s(k, \tilde{n}) = - A_s \frac{H_s^4 |1 + \chi \tilde{p}\tilde{n}|^4}{H_b |1 + \xi \tilde{p}\tilde{n}|},$$

$$\tilde{P}_t(k, \tilde{n}) = A_t H_b^2 |1 + \chi \tilde{p}\tilde{n}|^2. \quad (10)$$

The angular dependence of the PPS at the leading order is given as

$$\tilde{P}_{s,t}(k, \tilde{n}) = P_{s,t}(k) \left[ 1 + D^{s,t} \tilde{p}\tilde{n} + Q^{s,t} (\tilde{p}\tilde{n})^2 \right], \quad (11)$$

where, $D^s = 4\chi - \xi$, $Q^s = 6\chi^2 - 4\chi \xi$, $D^t = 2\chi$ and $Q^t = \chi^2$. Similar to $\chi$ and $\xi$, $D^{s,t}$ and $Q^{s,t}$ are also time dependent quantities. As a result, the modification to the PPS due to $D^{s,t}$ and $Q^{s,t}$ are important only at the beginning of the inflation and hence only confined to small wave numbers ($k$). So the PPS given in Eq. (11) affects the CMB field only in the large angular scales (low multipoles). Hence this is a scale dependent effect with different modulation strength for scalar and tensor perturbations. Similar form of power spectrum are also recently studied by several authors [6].

Along with the imprints on PPS, anisotropic parameters also lead to a direction dependent slow roll parameter (or spectral index for scalar and tensor perturbations). The modified slow roll parameter due to anisotropic Hubble parameter is

$$\tilde{\epsilon} = \epsilon [1 + (\xi - 2\chi) \tilde{p}\tilde{n} - 2\chi \xi (\tilde{p}\tilde{n})^2]. \quad (12)$$

At leading order in anisotropic parameters, Eq.(12) becomes

$$\tilde{\epsilon} = \epsilon [1 + (\xi - 2\chi) \tilde{p}\tilde{n}]. \quad (13)$$

This can easily be translated to direction dependent spectral index given by

$$\tilde{n}_s - 1 = (n_s - 1)[1 + (\xi - 2\chi) \tilde{p}\tilde{n}]. \quad (14)$$

4 Signatures of the anisotropic inflation in CMB

The effects of anisotropic inflationary parameters can be measured from both scalar and tensor perturbations. The modified PPS given in Eq. (11) affects both the angular power spectra, $C_l$ and its natural generalisation to BipoSH spectra, $A_{l;MM}$ [8] which can be related to the off-
diagonal terms of the covariance matrix by
\[
\langle X_{lm} X_{l'm'}^{\prime} \rangle = \sum_{JM} A_{ll'}^{JM} \left. \frac{\Pi_0^{\prime}}{\sqrt{4\pi}} \right|_{l'} (-1)^{-m'} C_{i,s't'}^{JM} - s C_{i,m'm'}^{JM},
\]
where \( X = T, E, B \) and \( C_{i,m'm'}^{JM} \) are the Clebsch-Gordan coefficients with \( s = 0 \) for \( TT \) & \( TE \) and \( s = 2 \) for \( EE \) & \( BB \). These are the even parity BipoSH spectra as discussed by Book et al. [15].

4.1 Hemispherical asymmetry in CMB

Using the PPS from Eq. (11), the dipolar \((L = 1)\) BipoSH coefficients for both scalar \((s)\) and tensor \((t)\) perturbations can be obtained as
\[
s,t A_{ll+1}^{JM} = D_{s,t}^{JM} \frac{\Pi_0}{2} [s,t C_{TT} + s,t C_{TT}],
\]
\[
s,t A_{ll+1}^{JM} = D_{s,t}^{JM} [s,t C_{XX} + s,t C_{XX}],
\]
\[
s,t A_{ll+1}^{JM} = D_{s,t}^{JM} [s,t C_{TE} + s,t C_{TE} C_{XX}].
\]

In the above expressions, \( C_{XX} \) denote the angular power spectra of \( X = T, E, B \) for an isotropic inflationary model. The BipoSH spectra for both scalar and tensor perturbations are similar to the other phenomenological models [3, 9]. The amplitude of dipole modulations \( (D_{s,t}^{JM}) \) are different for scalar \((s)\) and tensor \((t)\) as recently considered in the mixed modulation model [9] and is dominant only at large angular scales due to the time dependence. The above expressions of BipoSH spectra are obtained by assuming a constant value of \( D_{s,t}^{JM} \) for low \( k \).

Planck [3] has recently shown that a dipole modulation in the scalar field with an amplitude of 0.07 is sufficient to mimic the observed hemispherical asymmetry. But in the presence of modulated tensor perturbations, the value of modulation in the scalar field can be much lower. Hence, we can assume that \( D^s/2 = (4\chi - \xi)/2 \leq 0.07 \). Using this constraint, we can relate two anisotropic Hubble parameters \((\chi \text{ and } \xi)\) as
\[
D^s = 4\chi - \xi \approx 0.
\]
\[
\dot{H}_a = 4\dot{H}_b, \quad \frac{H_a}{H_b} = \frac{k_f}{k_i}.
\]

This implies that the anisotropic Hubble parameter during inflation decays faster than the isotropic Hubble parameter. Using the condition \( \chi = 4\chi \) in Eq. (7) and expressing \( H^2 \propto k^{n_s-1} \), we parameterize the variation of \( \chi \) in terms of the wave number \( k \) as
\[
\chi_f = \left( \frac{k_f}{k_i} \right)^{3(n_s-1)/2}.
\]

Since \( \chi \) depends upon \( \delta \Phi \), it dominates only at large scales. So, the effect is dominant only upto a small \( k_f \) value and vanishes later.

In Fig. 1, we plot the allowed region for \( \chi \) and \( \xi \) consistent with \( D^s/2 = (4\chi - \xi)/2 \leq 0.07 \). The plot indicates that only a narrow region in these parameters are sufficient to produce the observed hemispherical asymmetry. The accurate measurement of the amplitude of dipole modulation in scalar and tensor perturbations can uniquely specify the value of \( \chi \) and \( \xi \), which makes them important probes for the measurement of the initial fluctuations in the early epoch of inflation.

4.2 Direction dependent spectral index for scalar and tensor perturbations.

As shown in Eq. (14), the anisotropic inflation model leads to a direction dependent spectral index for both scalar and tensor perturbations. This departure from the isotropic behaviour is due to the non-zero value of \( \xi - 2\chi \). This value can be well constrained from the accurate measurement of temperature and polarization power spectra. A direction dependent study of the cosmological parameters were carried out recently by Axelsson et al. [16]. Their analysis show that in the hemisphere centred in the direction of dipole asymmetry \( (\tilde{p}) \), the value of \( \tilde{n}_s \) is \( 0.959 \pm 0.022 \) and in the other hemisphere the value of \( \tilde{n}_s \) is \( 0.989 \pm 0.024 \). From Eq. (14), using an all sky average value of \( \tilde{n}_s = 0.962 \) [17], we can conclude that
\[ \Delta_{n_s} \equiv \frac{\bar{n}_s - 1}{n_s - 1} - 1 = \xi - 2\chi > 0. \]  

(21)

\[
\Delta_{n_s} \approx 2\chi > 0. 
\]  

(22)

As a result, magnitude of the dipole modulation in tensor perturbation and in spectral index are related to a single anisotropic inflationary parameter, \( \chi \). This is a unique signature of this model and can be verified from future measurements.

### 4.3 Power suppression at large angular scales

The effect of anisotropic inflationary parameters on the diagonal terms of the covariance matrix arises due to the modification in the direction independent term of the PPS given by

\[
\hat{P}_{s,t}(k) = P_{s,t}(k) \left[ 1 + \frac{Q_{s,t}}{3} \right], \quad (23)
\]

where, \( Q^s = 6\chi^2 - 4\chi \xi \) and \( Q^t = \chi^2 \). The correction term arises from the all sky average of the \( \langle \hat{p} \hat{n} \rangle^2 \) term of Eq. (11). In Fig. 2(a), we plot the values of \( Q^s/3 \) for the allowed range of the parameters \( \chi \) and \( \xi \), shown in Fig. 1. The plot indicates that for all the allowed values of \( \chi \) and \( \xi \), \( Q^s/3 \) is negative. Also, the time dependence of \( \chi \), \( \xi \) results in different values of \( Q^s/3 \) at different angular scales. As a result, at large angular scales, the value of \( Q^s/3 \) is more negative and at small angular scales it vanishes.

![Figure 1: We plot the allowed range of parameters \( \chi \) and \( \xi \) for different values of \( D^s/2 \) and \( \Delta_{n_s} \). On imposing \( D^s/2 \leq 0.07 \), the parameter space for \( \chi \) and \( \xi \) gets restricted. The condition on \( D^s/2 \) also constrains the possible values of \( \Delta_{n_s} \). In large angular scales, maximum values of \( D^s/2 \) and \( \Delta_{n_s} \) are possible, which then reduces with the decrease in the angular scales.](image1)

![Figure 2: (a) For the allowed region of \( \chi \) and \( \xi \) obtained from Fig. 1, we plot the possible values of \( Q^s/3 = (6\chi^2 - 4\chi \xi)/3 \). Plot indicates that most of the possible region of \( Q^s \) are negative and the value transits from more negative values to zero with decrease in the angular scales. (b) We plot the possible variations of \( D_l^{TT} = l(l+1)C_l^{TT}/2\pi \) due to different values of \( \chi \) and \( k_f \) with \( k_f = 5 \times 10^{-5} \) Mpc\(^{-1}\) using Eq. (20). The suppression in \( D_l \) is scale dependent and is dominant only at low \( l \) depending upon the value of \( k_f \). EE and TE spectra also shows similar affects at low \( l \).](image2)
Hence, while explaining the measurements of hemispherical asymmetry, the anisotropic inflation model naturally leads to a suppression of power at large angular scales in the scalar perturbations. This causes a suppression of power in the large angular scales (low $l$) in CMB angular power spectrum for $TT$, $TE$ and $EE$. In Fig. 2(b), we show the effect of $Q^s/3$ in the temperature power spectrum, $C_l^{TT}$ using CAMB [18]. The scale dependent suppression of power is governed by Eq. (20) and hence is present only at large angular scales. Recent results from WMAP and Planck have already confirmed such effects [1, 2, 17]. Unlike suppression of power in $TT$, $TE$ and $EE$, $B$ mode polarization must show an enhancement of fluctuations at low $l$. This is a unique signature of this model and could be search in the future CMB missions.

4.4 Quadrupolar asymmetry in CMB

Like dipolar ($L = 1$) BipoSH spectra, this model also generates quadrupolar ($L = 2$) BipoSH spectra for both temperature and polarization. The signatures of quadrupolar asymmetry in CMB should also be evident only at the large angular scales (low $l$). The corresponding expressions of BipoSH spectra are

$$s,t A_{lll}[TT] = \frac{Q^s_{2l}}{2} [s,t C_{l}^{TT} + s,t C_{l}^{TT}], \quad (25)$$

$$s,t A_{lll}[XX] = \frac{Q^s_{2l}}{2} [s,t C_{l}^{XX} + s,t C_{l}^{XX}], \quad (26)$$

$$s,t A_{lll}[TE] = \frac{Q^s_{2l}}{2} [s,t C_{l}^{TE} + s,t C_{l}^{TE} C_{l}^{TT} C_{l}^{TT}], \quad (27)$$

where, $Q^s = 6\chi^2 - 4\chi \xi$ and $Q^t = \chi^2$. These expressions are obtained assuming a constant value of $Q^s,t$ at low $k$. The constraint on $\xi$ mentioned in Eq. (19) leads to an amplitude of $Q^s = -10^2 \chi^2$. This implies that the quadrupolar anisotropy from scalar is 10 times stronger from tensor. Hence the amplitude of the quadrupolar BipoSH spectra for $B$ mode polarization is 10 times weaker than the $TT$, $EE$ and $TE$ BipoSH spectra.

5 Predicted observables in future CMB measurements

The effects of anisotropic Hubble parameters on Primordial Power Spectrum (PPS) for scalar and tensor perturbations are calculated in Eq. (11). Our analysis show that at leading order in $\chi$ and $\xi$, the PPS for scalar and tensor perturbations has a dipolar dependence and at second order it has a non-zero quadrupolar dependence, along with a modification in the direction independent term. Another consequence of the anisotropic Hubble parameters is reflected in the direction dependent spectral index as shown in Eq. (14).

The observable signatures due to the modifications in the PPS for scalar and tensor perturbations are calculated in Sec. 4 and also summarised in Table 1. One of the most important signature of this model is the scale dependent dipolar asymmetry in both scalar and tensor perturbations. The amplitude of modulation for scalar and tensor are different and are related to the observed hemispherical asymmetry in the CMB field. This effect can be measured from the temperature and polarization maps using the technique discussed in a recent paper [9]. Another important signature of this model is the direction dependent spectral index. Recently an analysis by Axelsson et al. [16] showed that the data from Planck indicates a direction dependent spectral index, which supports the results from our model. In Fig. 1, we plot the allowed range of values of $\chi$ and $\xi$ which can lead to signatures like hemispherical asymmetry and direction dependent spectral index. The scale dependent nature of these effects leads to different values of these parameters in different angular scales. Both these leading order effects of $\chi$ and $\xi$ on CMB field are measured from the recent measurements by Planck [3]. Signatures of hemispherical asymmetry in the B mode polarization can validate this model.

CMB temperature and polarization field also carries observable signatures from the terms which are second order in $\chi$ and $\xi$ denoted by $Q^s,t$ for scalar ($s$) and tensor ($t$) in the paper. One of the major effect is the suppression of fluctuations at low $l$ for scalar perturbations, which in turn affects CMB power spectra $C_l^{TT}$, $C_l^{EE}$ and $C_l^{TE}$. For all the allowed range of $\chi$ and $\xi$ obtained from Fig. 1, the value of $Q^s/3$ is always


Table 1: Summary of the observed & observable prediction of anisotropic inflation on CMB temperature and polarization field

| Observable effects at low $l$ | $\chi$ and $\xi$ dependence | Relevant observations |
|--------------------------------|-----------------------------|----------------------|
| Dipolar Asymmetry in the scalar part of $TT$, $EE$ and $TE$ | $\frac{D^T}{2} = \frac{4\chi^2 - \xi^2}{2}$ | $\frac{D^T}{2} = 0.07$ as measured from $TT$ spectrum by Planck [3]. |
| Dipolar Asymmetry in the tensor part of $TT$, $EE$, $TE$ and $BB$ | $\frac{D^T}{2} = \frac{\chi}{2}$ | Hemispherical asymmetry in B mode polarization [9]. |
| Direction dependent spectral index | $\Delta_n_s = \xi - 2\chi$ | From the measurements of $TT$ spectra by Planck [16]. |
| Suppression of fluctuations in the $TT$, $EE$ and $TE$ | $Q^2_3 = \frac{(6\chi^2 - 4\chi \xi)}{3}$ | Observed in $TT$ spectrum by both WMAP [1] and Planck [17]. |

negative as depicted in Fig. 2(a). At large angular scales this value is negative and it vanishes at small scales. This causes a scale dependent suppression of power at low $l$ as shown in Fig. 2(b). A similar effect in $C_l^{TT}$ has been observed by both WMAP [1] and Planck [17]. Our model naturally leads to a suppression of fluctuation at low $l$ and also predicts a similar effect in the $EE$ and $TE$ spectrum. However, in the B mode polarization, a mild enhancement of power at low $l$ in $C_l^{BB}$ is expected from this model. This model also leads to a negligible quadrupolar asymmetry in both scalar and tensor perturbations at low $l$.

6 Conclusions

In this paper, we reconcile the two observed large scale anomalies in CMB, namely power suppression and hemispherical power asymmetry by considering the effects of initial inhomogeneities present in the inflaton field. Our analysis show that the initial inhomogeneities can also cause hemispherical asymmetry in the tensor field which can be measured from the future missions [19]. Future CMB missions must probe these effects and can constrain the anisotropic inflationary parameters $\chi$ and $\xi$. Our analysis show that both $C_l$ and BipoSH spectra ($A_{lm}^{JM}$) are affected by the same physical phenomena. Hence estimating $A_{lm}^{JM}$ from the temperature and polarization maps by assuming the $C_l$ associated with isotropic $\Lambda$CDM model is not appropriate and motivates joint estimation of $C_l$ and BipoSH spectra $A_{lm}^{JM}$ for accurate measurement of $\chi$ and $\xi$. Measurements of these parameters will open a new window for probing the initial inhomogeneities present during the early epoch of inflation.

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