Interferometric angular characterization of ion-exchanged glass binary phase plates

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Abstract. An interferometric characterization of the phase angular shifting produced by rotation of an ion-exchanged glass binary phase plates is presented. The inverse WKB method is used as a starting point to fabricate the phase plates, because such a method can only characterize the phase shift for normal incidence of the light on the plate. A complete phase angular shifting characterization is made by a Mach-Zehnder interferometer where a four-step phase shifting method is used for acquisition of data and a modified Carré algorithm is applied to data processing. The theoretical phase shifting is calculated by using the phase accumulated by a plane wavefront propagating in an axial graded-index media modelling an ion-exchanged glass phase plate. Experimental results present a good agreement with theoretical predictions.

1. Introduction
Binary phase plates are key optical elements in many branches of optics and photonics such as optical imaging, optical metrology \cite{1}, optical communications \cite{2}, and so on. In order to check the importance of such plates, we briefly centre our attention, for instance, on optical communications where significant efforts are being made to increase optical network capacity as data traffic continues to grow in an amazing way. One promising candidate to further increase fiber capacity is mode-division multiplexing in few-mode fibers, which uses transverse space modes as a new information-bearing dimension \cite{2,3}. A fundamental optical element for spatial multiplexing is a modal transformer, which converts the fundamental mode LP01 of a single mode fiber into the excited modes (LP11, LP21) of a few modes fiber. Modal transformation can be implemented in free-space by using a multi-region phase plate which is one of the main building blocks of these modal transformers. Such plates present two values (binary) of the phase in two or more regions. To prevent modal cross-talk at the modal multiplexing stage, high quality modal transformation is desired. In consequence, fabrication of the mentioned phase plates must provide an accurate, stable and steep phase-shift between regions. Other examples in optical imaging, such as diffractive elements (Fresnel lenses and so on), optical metrology and so on can be found, and again phase plates must also provide such an accurate, stable and steep phase-shift between regions.
In a recent work we presented the fabrication and characterization of phase plates fabricated by highly-uniform Ag$^+/Na^+$ ion-exchange (IE) glass but without paying attention to their angular properties, that is, the phase shift undergone by an ion-exchanged glass binary phase plate under rotation. These angular properties can be interesting to get a more accurate phase or even to change the phase with a specific purpose. In this work we present a preliminary study of these angular properties of IE plates. On the other hand, ion-exchange in glass is a well-established technique for materials processing [4], integrated optics [5], and so on, and it presents a high versatility and precision. The first advantage of this technique is that IE phase plates are monolithic. This ensures that the change of phase is achieved inside the material, which provides an excellent uniformity and robustness. Likewise ion-exchange technique allows to easily control the fabrication parameters such as time ($t$), temperature ($T$) and ionic concentration ($c$), which results in a very accurate phase step. In fact, a precision in the phase step as high as $\lambda/60$ can be achieved [8]. Moreover, under different models for the (gradual) refractive index change, compatible with the ion-exchange physical processes, we can determine and optimize the fabrication parameters ($t, T, c$) to obtain IE binary phase plates for several wavelengths, in particular, in soda-lime glasses by Ag$^+/Na^+$ ion-exchange. Although phase plates can be characterized by the Inverse WKB (IWKB) method [5], the angular properties only can be obtained by using an interferometric technique to which a phase recovery algorithm is applied, in particular a modified Carrè algorithm. We have designed the fabrication of ion-exchanged glass phase plates by using the IWKB method [8] in order to obtain, for instance, a modal transformer to LP11 mode from a LP01 mode. The results show that a rotation can compensate small errors of the phase step due to different reasons, therefore to characterize and analyze the angular behaviour of a ion-exchanged glass binary phase plate is an important goal. The plan of the paper is as follows: in section 2 the theoretical angular phase shift of a ion-exchanged (graded-index) glass plate is obtained. In section 3 the fabrication of ion-exchanged glass plates is described and fabrication parameters are obtained in order to produce particular phase plates, that is, binary plates with two regions with phases shifted a certain amount. In section 4 the interferometric device, the acquisition data and the results and their analysis are presented. In section 5 conclusions are presented.

2. Phase Angular Shift of an Ion-Exchanged Glass Plate

The well-developed thermal ion-exchange technique in a glass substrate of refractive index $n_s$ and thickness $h$ produces an axial graded-index profile $n(z)$, with $z$ the normal direction to the substrate. It is common to choose a normalized ionic concentration $c(z, t)$, that is, $c(0, t) = 1$ and the obvious boundary condition $c(\infty, t) = 0$. The ionic concentration $c(z, t)$ is obtained by solving the unidimensional nonlinear diffusion equation [5], and solutions can be written in the form $c(z/d)$, where $d = 2\sqrt{Dt}$ is the so-called effective depth and $D$ the diffusion coefficient, with $h \gg d$, in general. Accordingly, the refractive index profiles achieved by a thermal ion-exchanged process, after a particular time $t$, can be written, in a good approximation, as

$$n(z) = \begin{cases} n_s + \Delta n \frac{c(z/d)}{} & z \in [0, h] \\ 1 & z \notin [0, h] \end{cases}$$

(1)

where $\Delta n$ is the change of index at surface and the following condition $\Delta n \ll n_s$ is assumed. Eq.(1) represents a $z$-axial index profile of a graded-index medium, in particular, of an IE plate.

We are interested in obtaining the phase $\Delta \Phi$ introduced to a plane wavefront under propagation in such a graded-index IE plate. For that purpose, let us consider, a light beam with a plane wavefront incident with an angle $\phi_0$ (with wavevector included in the XZ plane) on a plane-parallel IE plate of axial refractive index profile $n(z)$ and thickness $h$ as shown in Fig.1. The plane wavefront at the exit of the plate is defined by the pointed line joining points $C$ and $D$, then if we consider light of wavelength $\lambda_0 = 2\pi/k_o$ the phase accumulated for such a
plane wavefront from the point $A$ up to the point $C$ is given, according to the Fermat principle, by the expression

$$
\Phi_{AC} = \Phi_{AB} + \Phi_{BC} = k_o \int_0^h n(z) \sqrt{1 + \dot{x}^2} \, dz + k_o BC,
$$

(2)

where $\dot{x} \equiv dx/dz$. From the first term of the right hand member of Eq.(2) the differential equation for the ray trajectory from $A$ to $B$ can be obtained (Euler-Lagrange equation), that is,

$$
\dot{x}(z) = \frac{\sin \phi_o}{(n_s^2 - \sin^2 \phi_o)^{1/2}}.
$$

(3)

An approximation to order $O(\Delta n^2)$ can be made in the ray trajectory by taking into account the assumption $\Delta n \ll n_s$, that is, by integrating Eq.(3) we obtain

$$
x(z) \approx \frac{\sin \phi_o}{(n_s^2 - \sin^2 \phi_o)^{1/2}} z - \frac{\sin \phi_o n_s \Delta n}{(n_s^2 - \sin^2 \phi_o)^{3/2}} \int_0^z c(z'/d) \, dz' + O(\Delta n^2).
$$

(4)

Note that $A'B \equiv x(h)$, where $A' = (0, 0, h)$. Next, by inserting Eq. (3) into $\Phi_{AB}$, that is, in the first term of the right hand member of Eq.(2), and after a long but straightforward calculation, the following result is obtained:

$$
\Phi_{AB} = \frac{k_o n_s}{(n_s^2 - \sin^2 \phi_o)} \{ n_s h + \Delta n \int_0^h c(z/d) \, dz \} - \Phi_o,
$$

(5)

where $\Phi_o = k_o n_s \Delta n \sin^2 \phi_o (n_s^2 - \sin^2 \phi_o)^{-3/2} \int_0^h c(z/d) \, dz$. Likewise, the phase between point $B$ and $C$ is given by $\Phi_{BC} = k_o (A'D - A'B) \sin \phi_o = k_o (h \tan \phi_o - x(h)) \sin \phi_o$, therefore by using Eq.(4) at $z = h$, the following result is obtained:

$$
\Phi_{BC} = k_o h \tan \phi_o \sin \phi_o - \frac{k_o \sin^2 \phi_o h}{(n_s^2 - \sin^2 \phi_o)^{1/2}} + \Phi_o.
$$

(6)

Finally, by adding Eqs.(5) and (6) we obtain the phase $\Phi_{AC}$ accumulated by the incident plane wave after propagation through the graded index plate. In the same way, the phase accumulated for $\Delta n = 0$ can be obtained. Therefore the phase angular shift $\Delta \Phi$ produced by the graded-index plate is given by the following expression

$$
\Delta \Phi(\phi_o) = \Phi_{AC} - \Phi_{AC}(\Delta n = 0) = \frac{k_o n_s \Delta n}{(n_s^2 - \sin^2 \phi_o)^{1/2}} \int_0^h c(z/d) \, dz.
$$

(7)
By taking into account the phase shift for normal incidence, that is, \( \Delta \Phi (\phi_o = 0) \), we obtain the following phase shifting angular law

\[
\Delta \Phi (\phi_o) = \frac{n_s \Delta \Phi (0)}{(n_s^2 - \sin^2 \phi_o)^{1/2}} = \frac{\Delta \Phi (0)}{\cos \phi_s}.
\]

(8)

where \( \phi_s \) is the angle formed by the wavevector (or ray) in the substrate, that is, far from the surface \( z = 0 \) where \( n(z) \approx n_s \). Expression (8) shows that the phase step \( \Delta \Phi (\phi_o) \) can be changed by a simple rotation of the plate. Note that for normal incidence, that is, \( \phi_o = 0 \), a minimum value of the optical phase is obtained. Therefore, it can not be reduced, which indicates that the fabrication process must be designed to make sure that a phase value equal or less to the needed one is achieved. Indeed, by increasing the incidence angle \( \phi_o \) the phase step can be enlarged to the required value. In particular for a glass with \( n_s \approx 1.5 \) the maximum theoretical increasing \( (\phi_o = \pi/2) \) is about 34\%, and, for instance, if \( \phi_o = \pi/3 \) the increasing is about 22\%. Obviously if the phase take values \( \Delta \Phi = \delta + 2m\pi \equiv \delta \pmod{2\pi} \), with \( m \) a positive integer and \( \delta \in [0, 2\pi] \), then the increased phase step due to rotation will be much larger according to the value of \( m \).

3. Fabrication of Ion-Exchanged Glass Plates

The ion-exchange process in the most common glasses is nonlinear, that is, the diffusion coefficient of the ions depends on the concentration. In these cases solutions of the nonlinear diffusion equation are needed. We have proven [8] that the following function provides an approximate solution fitting much better to the experimental results, that is,

\[
c(z, t) = \frac{1}{\alpha} \log(1 + (e^\alpha - 1) \text{erfc}(z/d)), \quad d = 2\sqrt{Dt}
\]

(9)

where \( \alpha \) accounts for the degree of nonlinearity. Let us return to the problem of obtaining the phase introduced by a graded-index plate. From equation (7) the general expression of the phase shift is given by

\[
\Delta \Phi (\phi_o, t) = \frac{2\pi n_s \Delta n d}{\lambda_0 (n_s^2 - \sin^2 \phi_o)^{1/2}} I(\alpha, h),
\]

(10)

where

\[
I(\alpha, h) = \int_0^{h/d} \frac{\log(1 + (e^\alpha - 1) \text{erfc}(z'))}{\alpha} dz'.
\]

(11)

with \( z' = z/d \). Note that the phase grows linearly with the effective depth \( d \), that is, with the ion-exchange time. Moreover, in most practical cases, we can consider \( h \gg d = 2\sqrt{Dt} \), that is, we can formally take \( h/d \approx \infty \), but still this integral \( I(\alpha) \) does not have a closed-form solution. However, it can be seen by numerical integration that \( I(\alpha, \infty) \approx 0.1306 \alpha + 0.5725 \) with an accuracy better than 0.6\%. Taking into account this result equation (10) can be rewritten as follows

\[
\Delta \Phi (\phi_o, t) \approx \frac{2\pi n_s \Delta n d}{\lambda_0 (n_s^2 - \sin^2 \phi_o)^{1/2}} (0.1306 \alpha + 0.5725).
\]

(12)

This equation gives a good approximation (better than 1\%) of the phase step for typical nonlinear ion-exchange processes. Therefore we will design the fabrication of the guide by using the refractive index profile given by equation (1) when the concentration function given by equation (9) is taken into account. For that, we have to use the so called IWKB method [5, 6], that is, the phase plate can be also considered as an integrated multimode waveguide, that is, whith several guided modes. Thus, by a standard technique based on coupling guide-prism we measure the effective indices and hence, by using the IWKB method, the index profile is obtained [5, 7].
must stress that the phase plates can only be characterized by the IWKB method for normal incidence. Moreover, it is also important to check the IWKB method by an alternative method as the interferometric one, which will be described in the next section. In short, the IWKB method allows us to have a starting point for the angular characterization of the plates.

The plate that we have characterized angularly was fabricated by following the procedure described in a previous work [8]. The substrates were plane-round soda-lime glasses of 25.3 mm of diameter and 1 mm thickness (refractive index \( n_s = 1.5106 \) for \( \lambda = 632.8 \) nm). After lithography they were immersed in a 5% AgNO\(_3\)/NaNO\(_3\) salt melt at 340 °C for ion-exchange took place during \( t = 1624.5' \). By one hand, we chose this time because it leads to a large phase shift for \( \lambda = 632.8 \) nm and therefore a more convenient angular phase shift characterization can be made. In effect, by using the IWKB method we obtain an index profile which has \( \Delta n = 0.087303 \), \( \alpha = 3.527 \) and \( d = 40.925 \) \( \mu m \). Inserting this values in Eq. (12) it gives \( \Delta \Phi = 11.67 \pi \simeq 11 \pi + (2/3) \pi \), for normal incidence. Such a large phase shift will reduce the signal-to-noise ratio in the subsequent angular interferometric measures. On the other hand, we have also chosen the mentioned time of fabrication because the phase shift reaches a value close to \( \Delta \Phi \sim 5 \pi \) for \( \lambda = 1550 \) nm, which can be used, for instance, to implement a mode converter to LP11 from a LP01 mode for optical communications.

4. Angular characterization by optical interferometry

In the above section, by using the IWKB method [8] we have designed the fabrication of an ion-exchanged glass phase plates in order to obtain a binary phase plate with a large phase shift, in particular, \( \Delta \Phi = 11.67 \pi \simeq 12 \pi - (1/3) \pi \) for \( \lambda = 632.8 \) nm. As commented, the phase shift of the phase plates can be characterized by the inverse IWKB method only for normal incidence, therefore their angular phase shifting properties will be obtained by using an interferometric method to which a phase recovery algorithm is applied. To that end, we have used a Mach-Zehnder interferometer to analyze such angular phase shifting properties of IE plates by measuring fringe shifting.

![Mach-Zehnder interferometer for angular characterization of a IE phase plate](image-url)
4.1. Interferometric device

The experimental set-up is a Mach-Zehnder interferometer in a rectangular configuration (32 cm long arm, 23 cm short arm) as shown in Fig. 2. The illumination with gaussian beam from a He-Ne laser ($\lambda=632.8$ nm) collimated with transversal section of 2”. The alignment of this interferometer was performed using an Equilateral Hyperbolic Zone Plate designed specifically for this task [8]. In the first step two focal crosses should overlap and next, in a second step, fine alignment is achieved by observing in a plane out of focus and minimizing the number of the fringe in the lobes of the interference pattern, provided the rest of the field is free of fringes. Next, when the IE plate is inserted linear fringes are obtained. In Figure 3 we show a particular interferogram where we can see the phase step produced by the IE plate.

![Interferogram](image)

**Figure 3.** Example of an interferogram produced by a IE phase plate.

The determination of angular properties of the phase plates, requires the proper alignment of it with respect to the incident beam. In order to achieve this, the back reflected beam, produced for a small surface of the plate coated by aluminium (and with triangular shape for indicating the border of the ion-exchanged zone) is used. This reflected beam is carefully aligned back through all the previous optical elements till the laser window. In this point the use of a piezo electric stage (Thorlabs NF5DP20S), in closed-loop with piezo controller and strain gauge reader, situated in a corner of the interferometer, is necessary to introduce the proper nano displacement to produce a controlled phase change in the optical path between the two arms of the interferometer. After three steps of this stage, and four interferograms, captured by a CMOS camera (Thorlabs DCC1545M 1280x1024 pixels) and a macro lens (Canon EF 100mm 1:28 USM ), it is possible recover the phase map of the image using a proper algorithm.

The angular displacement was carried out by means a rotational stage (Newport 495 Series with the PMC200 digital controller) in steps of fraction of degree till 15 degrees. After every rotation the process to recover the phase map is repeated again, by using the piezo electric stage, between every interferogram of the four ones which are necessary for every angular position.

4.2. Data adquisition

As commented, the phase step $\Delta \Phi$ was measured for each orientation of the plate by an interferometric analysis based on the Carré algorithm [8]. The measurements have been made directly on the interferferograms. The algorithm was applied to four interferograms obtained after respective phase steps $\Phi_R$ of the reference beam. Although $\Phi_R$ was adjusted with the piezo-electric stage, it was also recovered in a more accurate way from the interferograms themselves [9]. Then $\Phi_R$ was used to calculate the phase (mod $2\pi$) between the object and reference beams at each point of the plate, which is shown as a false color image in Fig.4. In order to
Figure 4. Phase (false color) difference (mod $2\pi$) for an incidence angle of $4^\circ$.

make easier both the unwrapping of this phase and the interpretation of the interferograms, a slight horizontal tilt was introduced between the interfering beams, which results in the vertical fringes of the interferograms. That is, inside the exchanged region, the unwrapped phase is a ramp across each row of the image and a constant along each column. The same happens outside this region, but a discontinuity occurs at the transition. By a suitable average of the phase in two rectangular regions at both sides of the straight frontier, the height of the discontinuity $\Delta \Phi (\text{mod} 2\pi)$ was obtained. The above procedure was repeated for a range of incidence angles from zero to 15 degrees. In order to obtain the real value of the phase step at normal incidence $\Delta \Phi(0)$, a multiple of $2\pi$ was added to the interferometric result to make it coincide with the WKB estimate. The same value was added to the phase steps for the remaining incidence angles, since $\Delta \Phi$ does not cross any multiple of $2\pi$.

4.3. Results and Analysis

As, for sake of simplicity, we have made the angular characterization for 633.8 nm (He-Ne Laser), then a IE plate with phase $\Delta \Phi (\text{mod} 2\pi)$ has to be considered. In particular, for normal incidence $\Delta \Phi(0) = (1.67 \pm 0.02)\pi (\text{mod} 2\pi) \simeq -(1/3)\pi (\text{mod} 2\pi)$ is the measured interferometric phase. Likewise, by using the IWKB method we had estimated, for normal incidence, a total phase equal to $\Delta \Phi(0) = 12\pi - (1/3)\pi$.

Figure 5. Phase versus incidence angle for a IE phase plate.

In Figure 5 we present the experimental results: phase shift versus incidence angle ($\Delta \Phi - \phi_o$),
and the theoretical curve corresponding to equation (7). The agreement between theoretical and experimental values is quite satisfactory. Note, for instance, that a rotation up to, approximately, $\phi_o \simeq 14^\circ$ provides an additional phase shift $\pi/6$, therefore the new phase shift has been increased up to $\Delta \Phi(14^\circ) = 12\pi - (1/6)\pi$. Likewise, note that very precise phase change can be made by rotating, and such precision is increased with the wavelength because the phase under normal incidence is reduced.

5. Conclusions
As a conclusion, we have shown by a interferometric technique that the phase shift of a IE phase plate is increased under rotation. Moreover, it is a simple way to introduce a precise phase change as the requires angle can be adjusted with a great accuracy. Accordingly, when small errors of the phase step has to be corrected then a rotation of the plate can be used whenever the experimental phase is lower than the required one. On the other hand, if the phase takes values $\Delta \Phi(\text{mod } 2\pi)$ then the increased phase step due to rotation is much larger which can be of interest in optical metrology applications. It has been proven that the agreement between the theoretical results provided by the phase shifting angular law and the experimental ones show a good agreement, therefore the IWKB method can be used to determine the normal incidence phase $\Delta \Phi(0)$ and next the phase shift angular law can be applied to predict the phase shift.

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