Construction Method of Backbone Grid Based on Survivability in Power Grid

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Abstract. Identifying backbone grid has great significance to the differential planning of power grid, which can improve the ability to withstand natural disasters effectively. In this paper, the problem of exploring backbone grid is formulated as a mixed integer nonlinear programming problem and the optimization target should balance the survivability and cost. Then, the problem of searching for backbone grid is solved with improved biogeography-based optimization algorithm, and several improvement strategies are employed to enhance the convergence rate and precision of the proposed method. The simulation verifies that the mathematical model can identify backbone grid appropriately and the improved algorithm can figure out backbone grid more effectively.

1. Introduction
The frequent occurrence of extreme natural disasters has brought a great threat to the safe operation of power grids, causing serious social and economic impacts around the world. The report of NERC shows that the reliable operation of some critical lines is significant to preventing large-scale blackouts and guaranteeing the supply of important loads [1]. The backbone grid of Quebec, Canada, guaranteed 80% of the power supply in the disaster from snow and ice in 1998 [2], which provided a new way for the power system to cope with extreme natural disasters.

Different from the vulnerability assessment based on complex network theory, which identifies vulnerable nodes and lines from topology structure aspect, the backbone grid pays more attention to the overall power supply capability of the system. The methods based on graph theory set importance indexes of lines and nodes, and build backbone grid by those lines and nodes that have higher indexes [3]. Nevertheless, this method lacks the assessment of system’s capacity to resist external disasters on the whole. The method based on complex network theory search for vulnerable lines and nodes from the physical structure [4], ignoring the running state of the system. The survivability theory [5-6], emerging in recent years, researches the running state of the system after its safety has been destroyed, i.e. it studies whether the system can provide services continually in case of sudden accidents or attacks. And this theory is applicative for evaluating the capability of backbone grids.

In this paper, a method of searching backbone grids based on survivability is proposed. The paper is organized as follows. In section II, an index system model of survivability is established, which can evaluate the comprehensive disaster resistance ability of backbone grid in three aspects: resistibility, fluctuation of operation state and recovery after disasters. In section III, the mathematical model of backbone grid is described. In section IV, the method for exploring backbone grid based on improved
biogeography-based optimization algorithm (IBBO) is proposed. Then, simulation results and analysis are discussed in section V. Finally, a conclusion is given in section VI.

2. Index System of survivability

System’s survivability is a key means to measure the backbone-grid’s power supply capability during serious natural disasters, and also an important index to evaluate the backbone-grid’s ability to help the system to resume normal running state or finish the reconfiguration. So, some indexes are used to constitute the index system of survivability together.

The index system is divided into three levels: comprehensive-class index, i.e. index of survivability; first-class index, i.e. the indexes of resistibility, recovery and fluctuation of operation state; second-class index, i.e. some sub-indexes of first-class indexes.

2.1. Index of Resistibility

During serious natural disasters, some critical lines, nodes and stable topology structure play a key role in power supply, and the capacity of load which get continuous power supply is an intuitive expression of the power supply capability of the system. So, line survival rate, node survival rate and load survival rate are used to constitute the index of resistibility.

Let \( L_0 \) and \( N_0 \) to be the number of lines and nodes respectively in the original system, and \( Q_0 \) represent the whole load. Because of natural disasters, the number of failure lines is \( L_1 \), the number of failure nodes is \( N_1 \), and the loss of load is \( Q_1 \).

2.1.1. Node survival rate. The network cohesion degree [7] is used as node importance, and based on this, node survival rate can be described as follows:

\[
\alpha_n = \frac{\sum_{i=1}^{N_0} s_i}{\sum_{i=1}^{N_0} s_i}
\]

(1)

Where, \( S_I \) is the node importance.

2.1.2. Line survival rate. The average value of network cohesion degree of nodes at both ends of the line is regarded as the line importance. And the line survival rate is as follows:

\[
\alpha_c = \frac{\sum_{j=1}^{L_0} c_j}{\sum_{j=1}^{L_0} c_j}
\]

(2)

Where, \( C_I \) is the line importance.

2.1.3. Load survival rate

\[
\alpha_l = \frac{(Q_0 - Q_1)}{Q_0}
\]

(3)

2.2. Index of Recovery

Building backbone grid is an emergency measure when the original system is out of order, so the grid should serve the reconfiguration of the original system. Therefore, the index of recovery is set to reflect whether the retained backbone grid can restore power supply and the degree of restoration. The index of recovery consisted of generator power reserve and active power margin of line.

Let \( G_0 \) to be the number of generators of the original system, and \( G_1 \) represents the number of generators that broken down.

2.2.1. Generator power reserve rate. In recent years, the proportion of wind turbines connected to power grid is increasing, and the uncertainty of wind power have adverse effects on the stable operation of power grid. Therefore, taking the uncertainty of wind power into account, the generator power reserve rate can be described as follows:
\[ \beta_y = 1 - \frac{G_y - G_{y\text{max}}}{G_{y\text{max}}} - \int_{-\infty}^{x_1} \varphi(x) \, dx \]

Where, \( G_{y\text{max}} \) is the maximum installed capacity; \( x_1 \) is current generator output; \( \varphi(x) \) is probability density function of generator’s active power, and for traditional generator, \( \int_{-\infty}^{x_i} x_i \varphi(x) \, dx_i = G_i \).

2.2.2 Active power margin of line. Considering the line importance, active power margin of line is as follows:

\[ \beta_x = \left( \sum_{j=1}^{L_{\text{max}}} \frac{P_j - P_{j\text{max}}}{(L_j - L_{j\text{max}})c_j} \right) / \sum_c c_j \]

Where, \( P_{j\text{max}} \) is the maximum transmission capacity of line; \( P_j, \, \varphi(P_j) \) are active power and probability density function respectively; \( c_j \) is line importance.

2.3. Index of Fluctuation of Operation State

Node survival rate and line survival rate mainly reflect the survivability of the backbone grid in terms of topology structure. However, grid-connected wind turbines will seriously affect the stability of system. Therefore, the average fluctuation rate of bus voltage and active power of branch are improved to measure the stability of system.

2.3.1. The average fluctuation rate of bus voltage

\[ \lambda_{\text{U}} = \int_{-\infty}^{u_{\text{U}}} \frac{u_{\text{U}} - u_i}{[u_{\text{U}} - u_{\text{U}\text{min}}]} \varphi(u_i) \, du_i + \int_{u_{\text{U}}}^{\infty} \frac{u_i - u_{\text{U}}}{[u_{\text{U}} - u_{\text{U}\text{max}}]} \varphi(u_i) \, du_i \]

Where, \( u_i, \, u_{\text{U}}, \, u_{\text{U}\text{max}} \) and \( u_{\text{U}\text{min}} \) are current voltage amplitude, rated voltage amplitude, upper and lower limits of bus voltage respectively; \( \varphi(u_i) \) represents the probability density function of bus voltage.

After calculating the \( \lambda_{\text{U}} \) of all buses, average fluctuation rate of bus voltage can be obtained by taking node importance as weight.

\[ \lambda_{\text{U}} = \sum_{i=1}^{N_N} s_i \lambda_{\text{U}_i} / \sum_{i=1}^{N_N} s_i \]

2.3.2 The average fluctuation rate of active power of branch

\[ \eta_{\text{P}} = \int_{-\infty}^{P_{\text{P}}} \frac{P_{\text{P}} - P_j}{[P_{\text{P}} - P_{\text{P}\text{min}}]} \varphi(P_j) \, dP_j + \int_{P_{\text{P}}}^{\infty} \frac{P_j - P_{\text{P}}}{[P_{\text{P}} - P_{\text{P}\text{max}}]} \varphi(P_j) \, dP_j \]

Where, \( P_j, \, P_{\text{P}}, \, P_{\text{P}\text{max}} \) and \( P_{\text{P}\text{min}} \) are current active power, rated power, upper and lower limits of active power of branches respectively; \( \varphi(P_j) \) represents the probability density function of branch power.

Similarly, average fluctuation rate of branch power can be obtained by taking line importance as weight.

\[ \eta_{\text{P}} = \sum_{j=1}^{L_L} c_j \eta_{\text{P}_j} / \sum_{j=1}^{L_L} c_j \]

2.4. Index of Survivability

Let \( R_s \) to be a first-class index, and it’s value can be calculated as follows:

\[ R_s = m \times \frac{1}{\eta} \| R \| + n \times \| R \| \]

(10)
Where, \( R = (r_1, r_2, ..., r_\eta) \) denotes the set of second-class index that belongs to \( R_s \); \( m \) and \( n \) are weight coefficients which obey the constraint \( m + n = 1 \).

Index of survivability can be calculated as follows:

\[ S_{surv} = S_{resis} + S_{reco} - S_o \]  

Where, \( S_{resis} \), \( S_{reco} \), \( S_{st} \) are the indexes of resistibility, recovery and fluctuation of operation state.

3. Mathematical Model of Backbone Grid

Backbone grid is a set of lines and nodes that aims to ensure the continuous power supply of important loads under specific operation modes, and is also the basis for rapid recovery or reconfiguration. So, some conditions need to be satisfied for the backbone grid: 1) Some crucial generators, lines and loads should be contained; 2) Satisfying the security operation constrains of power grid and the connectivity of network topology; 3) Considering the economy, the weighted sums of lines and nodes should be the minimum; 4) The index of survivability should be the maximum.

From the perspective above, the problem of constructing backbone grid can be formulated as a mixed integer nonlinear programming problem (MINLPL), whose mathematical model can be described as follows:

\[
\begin{align*}
\min \ f &= a \cdot \frac{X + Y}{X_o + Y_o} - b \cdot S_{surv} \\
\text{s.t.} \ X &= \sum_{i=1}^{l} x_i \\
Y &= \sum_{j=1}^{n} y_j \\
\phi(X,Y) &= 1 \\
g(P,Q,U,\theta) &= 0 \\
h(P,Q,U,\theta) &\leq 0
\end{align*}
\]

Where, \( x_i \) and \( y_j \) mean the operation state of lines and nodes (1 means in service and 0 means out of service); \( l, n \) are the number of lines and nodes employed in backbone grid respectively; \( X_o, Y_o \) are the number of lines and nodes employed in original system; \( \phi(X,Y) = 1 \) means that the system satisfying the constrains of connectivity; \( g(P,Q,U,\theta) = 0 \) and \( h(P,Q,U,\theta) \leq 0 \) are the security operation constrains of power grid; \( a, b \) are weight coefficients which obey the constraint \( a + b = 1 \).

4. Exploring Backbone Grid Based on BBO

4.1. Improved Biogeography-based Optimization Algorithm

Biogeography-based optimization (BBO) is a heuristic search algorithm based on species migration [8]. Habitat suitability index (HSI) is used to describe the degree of habitat suitability for population survival, i.e. the value of objective function of the problem to be optimized. The variables that assess habitability are called suitability index variables (SIVs), i.e. elements in candidate solutions. The migration of species between habitats represents the information sharing among solutions, while, the mutation of individual species brings new elements to existing solutions and realizes information updating to obtain the optimal solution. This paper improves the traditional BBO to speed up the convergence rate and improve the efficiency.

4.1.1 Modified migration model. The traditional BBO algorithm replaces the SIV of the inferior solution with the SIV of the superior solution, which easily leads to the homogenization of the solution and makes the algorithm fall into the local optimum. So the mutation strategy of difference algorithm [9] is drawn into and ameliorated as follows:

\[
X_i(SIV) = X_{wor}(SIV) + F \cdot (X_{i1}(SIV) - X_{i2}(SIV)) + F \cdot (X_{i3}(SIV) - X_{i4}(SIV))
\]
Where, $X_{\text{best}}(\text{SIV})$ is optimal solution; $X_{r1}(\text{SIV}), X_{r2}(\text{SIV}), X_{r3}(\text{SIV})$ and $X_{r4}(\text{SIV})$ are four different stochastic solutions. Developing and exploring ability can be changed by adjusting the value of $F$.

4.1.2. Random perturbation operator. In the evolution process of BBO, the probability of obtaining high-quality solutions is not enough. Therefore, random perturbation operator is introduced to perturb the high-quality solutions and improve the diversity of habitats.

$$X_i(\text{SIV}) = \begin{cases} X_i(\text{SIV}) + \alpha \cdot (X_i(\text{SIV}) - X_k(\text{SIV})) & \text{rand} < \rho \\ X_i(\text{SIV}) & \text{else} \end{cases}$$

(14)

Where, $X_k(\text{SIV})$ is a stochastic solutions; $\alpha, \rho$ are perturbation amplitude and perturbation probability.

4.1.3. Repairing strategy based on minimum spanning tree. The backbone grid is a connected power system, but the traditional BBO can’t guarantee the connectivity of the system represented by the solution vector after iteration. So, in order to reduce the number of invalid solutions and speed up the algorithm, repairing strategy based on minimum spanning tree is used. The specific steps are as follows:

Let graph (a) to represent the initial system.
1) Search all connected sub-blocks of graph A and judging the connectivity by Dijkstra Algorithms. If graph A is a connected network, go to step 5), otherwise go to step 2);
2) Each sub-block is regarded as aggregated point, and new graph named B is formed together with other non-selected nodes and adjacency matrix is obtained.
3) The minimum spanning tree among sub-blocks and other important nodes should be found in graph B.
4) Let M and N to be a pair of adjacent sub-blocks, while, P and Q are their node set respectively. According to the adjacency matrix of graph A, node a, b in P and Q respectively and their connecting line should be found to realize the shortest path connection between M and N. Update graph A and go to step 1);
5) Set the decision variable of newly added lines to 1.

4.2. Procedure of Algorithm

Taking line operation status as input variable, the implementation process for exploring backbone grid can be illustrated as follow:

1) Input primal data both for power system and IBB operation.
2) Randomly generate initial population that satisfy constraints; evaluate the HSI of each habitat and rank them, then retain K best solutions.
3) Obtain the immigration rate and emigration rate; migration and mutation should be completed;
4) Randomly perturb the first half population;
5) Check whether the solution satisfies the constrains of topological connectivity; repair connectivity of the disconnected solution;
6) Recalculate and rank HIS; update the solution set;
7) If the termination criterion of algorithm is satisfied, go to step 8), otherwise go to step 3);

Output results and transform the optimal solution to the optimal backbone grid.

5. Case studies

5.1. Case 1

To illustrate the effectiveness of the proposed method, IBBO is used to explore backbone grid of the IEEE 39-node system which has 10 generators and 46 lines (including 12 transformer branches). Preset important nodes and lines including: lines: 3-4 and 4-5; generators: 31, 38 and 39; load buses: 3, 4, 7, 16, 21, 25, 26, 27 and 29.
The parameters of IBBO: habitat size: 50; maximum iterations: 200; maximum immigration rate and emigration rate; maximum mutation rate; global migration rate.

The simulation result is shown in Fig. 1 and Fig. 2, where, “■”, “●” and “○” are the symbols of generators, loads and contact node respectively. And the red thickened lines and endpoints are the retained branches and nodes of the backbone grid.

Fig. 1 is the optimal backbone grid scheme based on the model shown in formula (12), while Fig. 2 is the solution using the minimum of the number of lines and nodes as objective. Both of the two solutions employ 20 lines and 21 nodes, which means that only by strengthening these selected lines, the capability of power supply for key loads can be greatly improved during natural disasters. In Fig. 1, further simulation shows that the loss of any non-important preset nodes will destroy the topological connectivity or operational security of the backbone grid, which conforms to the economy principle.

Comparing the two results, the survivability of Fig. 1 is 0.8765 and the scheme guarantees 68.6% of the load’s power supply with 43.5% of the lines. Whereas the survivability of Fig. 2 is 0.7221, and 62.7% of the load is retained with 43.5% of the lines. It shows that the proposed algorithm can search the backbone grid considering both the survivability and economy.
Figure clearly illustrates that IBBO obtains the smallest optimal value of objective function, and its convergence rate is faster than the others, which prove that the improvement strategy is effective.

Fig.4 is a comparison of the number of effective SIV in the iterative procedure before and after introducing the connectivity repair strategy. Obviously, without repairing strategy, only about 20% of the SIV are connected, and the repairing strategy makes all the SIV satisfy the constraints of connectivity. So, the efficiency of the algorithm is greatly improved.

5.2. Case 2
To further verify the effectiveness of the algorithm and analyze the impact of wind power on the backbone grid, the IEEE 118 bus system is used to simulate, which consists of 118 nodes and 179 branches. Critical branches and nodes are set as shown in Tab.1.

Table 1 Set of critical branches and nodes

| Class            | Set                                      |
|------------------|------------------------------------------|
| Branches         | L28-29, L34-37, L69-75, L78-79, L82-83, |
|                  | L88-89                                   |
| Generators       | B12, B25, B49, B69, B89                 |
|                  | B2, B3, B7, B11, B13, B14, B16, B20, B21, B28, B29, |
|                  | B33, B35, B39, B41, B43, B44, B45, B47, B48, B50, |
|                  | B51, B52, B53, B60, B67, B75, B78, B79, B82, B83, |
|                  | B86, B88, B94, B96, B106, B117, B118    |
| Load nodes       |                                          |

Two contrast scenes are set up: 1) scene of traditional power grid, i.e. there is no wind turbine generator set; 2) scene of randomness which take wind power into account. In scene 2), 50% of the capacity of traditional units is replaced by wind turbines at generator nodes 10, 12, 25, 49, 77, 80 and 89 while total installed capacity is unchanged. The wind speed satisfies Weibull distribution \( W(c, k) = W(11,2) \), and Monte Carlo Method is used to simulate the random output scene of wind turbines. 

The results of two different scenes are shown in Fig.5 and Fig.6.

Figure 5. Optimal backbone grid scheme for IEEE 118-bus power system.

Figure 6. Optimal backbone grid scheme for wind power integrated IEEE 118-bus power system.

In Fig.5, the optimal backbone covers all the key lines, important generators and load. And the 65 lines selected in the scheme are 36.31% of the original system. That means, strengthening small amount of lines can ensure the supply of most loads. Fig. 6 is the optimal backbone grid obtained according to scene 2, which includes 68 lines and all requisite nodes. Compared Fig.5 with and Fig.6, it is found that in Fig.6, a number of generator nodes such as node 1, 6, 46 and node 92 and their main output branches are added, and the node 10 which has wind turbines are discarded. So, the proportion of wind power in the backbone grid is reduced and the impact of wind power fluctuation on power flow and node voltage is cut down. Meanwhile, the number of output branches of generators nodes
(such as 12 and 49) being selected into the backbone grid is significantly reduced, while their surrounding load is supplied by the nearby traditional generators, which also weakens the influence of fans on the system’s survivability. These results prove that the proposed method can explore backbone grid effectively.

6. Conclusion
In this paper, the mathematical model of backbone grid is analyzed and a search method based on BBO is proposed. The problem of exploring backbone grid is formulated as a MINLPL, while the optimization target is to minimize the weighted sums of lines and nodes and maximize survivability. Several strategies are employed to improve the convergence rate and precision of BBO. The effectiveness of the proposed method is verified through the simulation on IEEE 39-bus power system and IEEE 118-bus power system.

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References
[1] NERC. August 14, 2003 Blackout: NERC actions to prevent and mitigate the impacts of future cascading blackouts[R]. Princeton, New Jersey: NERC, 2004.
[2] Gilles T, Gingras J, Pierre J. Designing a reliable power system: hydro-quebec’s integrated approach[J]. Proceedings of the IEEE, 2005, 93(5): 907-917.
[3] HONG Yunfu. Transmission network differential planning based on complex network theory. [D]. Beijin: North China Electric Power University, 2012(in Chinese).
[4] YANG Wenhui, BI Tianshu, HUANG Shaofeng, et al. An approach for critical lines identification based on the survivability of power grid[J]. Proceedings of the CSEE, 2011, 31(7): 29-35(in Chinese).
[5] SANGJOON P, JIYOUNG S, BYUNGGI K. A survivability strategy in mobile networks[J]. IEEE Trans. on Vehicular Technology, 2006, 55(1):328-340.
[6] Zhao Yijie, Liu Dichen, Wu Jun, et al. Survivability evaluation of backbone network based on linear discriminant analysis and principal component analysis[J]. Power System Technology, 2014, 38(2): 388-394(in Chinese).
[7] Liu Yan, Gu Xueping. Node importance assessment based skeleton-network reconfiguration[J]. Proceedings of the CSEE, 2007, 27(10): 20-27(in Chinese).
[8] Dan Simon "Biogeography-Based Optimization" IEEE Trans. Comp. Vol.12.No.6 December 2008.
[9] Storn R, Price K V, Lampinen J. Differential evolution about practical approach to global optimization. Berlin, Germany: Springer-Verlag, 2005.
[10] Nara K, Shiose A, Kitagawa M, et al. Implementation of genetic algorithm for distribution systems loss minimum re-configuration[J]. IEEE Transactions on Power Systems, 1992, 7(3): 1044-1051.
[11] J. Kennedy R. Eberhart "Particle swarm optimization" Proc. 1995 IEEE Conference on Neural Networks Conf. pp. 1942-1948.