A Knowledge-Theoretic Analysis of Uniform Distributed Coordination and Failure Detectors

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Abstract

It is shown that, in a precise sense, if there is no bound on the number of faulty processes in a system with unreliable but fair communication, Uniform Distributed Coordination (UDC) can be attained if and only if a system has perfect failure detectors. This result is generalized to the case where there is a bound \( t \) on the number of faulty processes. It is shown that a certain type of generalized failure detector is necessary and sufficient for achieving UDC in a context with at most \( t \) faulty processes. Reasoning about processes’ knowledge as to which other processes are faulty plays a key role in the analysis.

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Periodically coordinating specific actions among a group of processes is fundamental to solving most distributed computing problems, and especially to replication schemes that achieve fault tolerance. Unfortunately, as is well known, it is impossible to achieve coordination in an asynchronous setting even if there can be only one faulty process \cite{FLP85}. This is true even if communication is reliable. As a result, there has been a great deal of interest recently in systems with *failure detectors* \cite{CT96}, oracles that provide suspicions as to which processes in the system are faulty. This interest is heightened by results of Chandra, Hadzilacos, and Toueg \cite{CT96,CHT96} showing that consensus can be achieved with relatively unreliable failure detectors.

Here we consider what kind of failure detectors are necessary to attain *Uniform Distributed Coordination* (UDC) \cite{GT89}. We have UDC of action $\alpha$ if, whenever some process (correct or not) performs $\alpha$, then so do all the correct processes. There are two features that distinguish UDC from consensus. First, if a process that initiates an action is later found to be faulty, in the UDC setting, all the processes must still perform the action. On the other hand, in the case of consensus, the nonfaulty processes can agree not to perform the action. This property of UDC is particularly important in practice. Consider for example, a group of processes implementing fault-tolerant service; actions are executed on behalf of clients and change the state of the service (for example, allocating a scarce resource). In the UDC setting, the service cannot repudiate an action should the member eventually be deemed faulty, as could be the case in consensus. With UDC, the service is required to make that action part of the service’s communal history. From the client’s point of view, the eventual designation of a group member as faulty is irrelevant; indeed, one goal of using replication to implement a service is to mask failures from clients. A second difference between UDC and consensus is that, in consensus, processes must typically choose exactly one out of two actions ("attack" or "retreat"; or, "decide 0" or "decide 1"). On the other hand, in UDC, there is no choice to be made; that is, UDC has no requirement that if action $\alpha$ is ever taken, then of necessity, action $\beta$ is never taken. Thus, UDC suffices whenever actions to be taken by a group can be partitioned into non-conflicting subsets; it requires consenses to decide which of a conflicting set of actions to take.

If we have reliable communication, then it is easy to see that we can attain UDC no matter how many processes may fail. Thus, in this setting, UDC is strictly easier than consensus. Intuitively, consensus requires all the correct processes to agree on a particular action. For example, they must all agree to attack or all agree to retreat (but cannot do both). With UDC, if one process attacks, all the correct processes must attack, and if one retreats, all must retreat. But it is perfectly consistent with UDC for the correct processes both to attack and to retreat.

If communication is unreliable but fair, then we show that we can attain UDC even if there is no bound on the number of process failures (that is, even if there are runs in which all processes may fail) in the presence of *weak* failure detectors, which have the
property that eventually each faulty process is permanently suspected by at least one correct process (weak completeness) and at least one correct process is never suspected (weak accuracy). Chandra and Toueg [CT96] showed that consensus with an arbitrary number of failures is also achievable using weak failure detectors. They considered a setting with reliable communication, but their results apply with essentially no change to a setting where communication is unreliable but fair in an appropriate sense.

Chandra and Toueg observed that by having processes communicate their suspicions, a weak failure detector can be converted to a strong failure detector, which satisfies weak accuracy and strong completeness (all correct processes eventually permanently suspect every faulty process). We further show that, under an assumption about the independence of process failures, in systems with no bound on the number of faulty processes, strong failure detectors are equivalent to perfect failure detectors, which satisfy strong completeness and strong accuracy—no process is suspected until it crashes. (Indeed, under the same conditions, weak failure detectors are equivalent to perfect failure detectors.)

These results tell us that, if there is no bound on failures, then we can attain UDC using what are effectively equivalent to perfect failure detectors. Are perfect failure detectors really necessary? We show that in a precise sense they are. Under quite minimal assumptions, perfect failure detectors can be implemented in a system that attains UDC with no bounds on the number of failures.\(^1\) It is interesting to note that Schiper and Sandoz’ *Uniform Reliable Multicast* [SS93] is a special case of UDC where the only action of interest is reliable message delivery. Schiper and Sandoz implement Uniform Reliable Multicast by using the Isis virtual synchrony model [BJ87], which simulates perfect failure detection. Our results support their need to implement it in this way.

What happens if there is a bound on the number of faulty processes? Gopal and Toueg [GT89] show that UDC is achievable with no failure detectors in systems where fewer than half the processes can fail. Here we generalize these results, providing, for each value of \(t\), a generalized failure detector that we can show is necessary and sufficient to attain UDC if there are at most \(t\) failures. The generalized failure detector we consider reports suspicions of the form “at least \(k\) processes in a set \(S\) of processes are faulty” (although it does not specify which \(k\) are the faulty ones). Such generalized failure detectors may be appropriate when the system can be viewed as consisting of a number of components, and all we can say is that some process in a component is faulty, without being able to say which one it is.

The rest of this paper is organized as follows. In Section 2, we provide the necessary background, reviewing the formal model, failure detectors, the formal language, and the definition of UDC. In Section 3, we present our analysis in the case that there is no bound on the number of faulty processes. Our proof techniques may be of independent interest, since they make nontrivial use of the knowledge-theoretic tools of Fagin et al. [FHMV95].

\(^1\)We remark that our notion of “implement” is stronger than the notion of reduction used by Chandra, Hadzilacos, and Toueg [CT96, CHT96]; see Section 3.
Reasoning about the knowledge of the processes in the system—particularly, their knowledge of which other agents are faulty—plays a key role in the analysis. In Section 4, we extend this analysis to the case where there is a known bound \( t \) on the number of faulty processes; we also introduce our generalized failure detectors. We conclude in Section 5 with a discussion of the results and a comparison of our results to results of Aguilera, Toueg, and Deianov [ATD99] who, in response to the conference version of this paper [HR99], provided an alternative characterization of the type of failure detectors needed to attain UDC. Proofs are relegated to the Appendix.

2 Background

In this section, we briefly discuss the formal model (and, in particular, our assumptions about message delivery), failure detectors, the formal language that we use for expressing coordination, which includes operators for knowledge and time, and the notion of UDC.

2.1 The Model

We adopt the familiar model of an asynchronous distributed system. We assume that there is a fixed finite set \( \text{Proc} = \{p_1, \ldots, p_n\} \) of processes with no shared global clock. These processes communicate with one another by passing messages over a completely connected network of channels. Processes fail by crashing and do not recover, but otherwise follow their assigned protocols. Channels are not reliable. A message that is sent is not necessarily received and, even if it is received, there is no upper bound on message transmission delay. However, channels do not corrupt messages (so that every message received is one that was actually sent) and they are fair, in the sense that if the same message is sent from \( p \) to \( q \) infinitely often and \( q \) does not crash, then the message is eventually received infinitely often by \( q \).

Processes and the environment (or nature) execute actions; corresponding to each action is an event (intuitively, the event of that action occurring). We assume that the events that take place at a particular process are totally ordered, and are recorded in that process’s history. The events recorded in \( p \)’s history include communication events of the form \( \text{send}_p(q, \text{msg}) \) (\( p \) sends message \( \text{msg} \) to \( q \)) and \( \text{recv}_p(q, \text{msg}) \) (\( p \) receives \( \text{msg} \) from \( q \)); internal events, which include events of the form \( \text{do}_p(\alpha) \) (\( p \) executes action \( \alpha \)) and \( \text{init}_p(\alpha) \) (\( p \) initiates \( \alpha \); see Section 2.4); the special event \( \text{crash}_p \), which models the failure of \( p \); and failure-detector events, which are discussed in Section 2.2.

A history for process \( p \), denoted \( h_p \), is a sequence of events corresponding to actions performed by process \( p \). A cut is a tuple of finite process histories, one for each \( p \in \text{Proc} \). A run is a function from time (which we take to range over the natural numbers, for simplicity) to cuts. If \( r \) is a run, we use \( r_p(m) \) to denote \( p \)'s history in the cut \( r(m) \). A pair \((r, m)\) consisting of a run \( r \) and a time \( m \) is called a point. We write \( (r, m) \sim_p (r', m') \) if \( r_p(m) = r'_p(m') \). We say that a run \( r' \) extends a point \((r, m)\) if \( r'(m') = r(m') \) for all
Thus, \( r' \) extends \( (r, m) \) if \( r \) and \( r' \) have the same prefix up to time \( m \). Process \( q \) is faulty in run \( r \) iff \( \text{crash}_q \) is in \( q \)'s history. \( F(r) \) denotes the faulty processes in run \( r \).

We assume that a run \( r \) satisfies the following.

**R1.** \( r(0) = (\langle \rangle, \ldots, \langle \rangle) \) (that is, at time 0, each process's history is empty).

**R2.** \( r_p(m + 1) = r_p(m) \) or \( r_p(m + 1) \) is the result of appending one event to \( r_p(m) \).

**R3.** If \( \text{recv}_q(p, \text{msg}) \) is in \( r_q(m) \), then the corresponding send event \( \text{send}_p(q, \text{msg}) \) is in \( r_p(m) \).

**R4.** If the event \( \text{crash}_p \) is in \( r_p(m) \), then it is the last event in \( r_p(m) \).

**R5.** If the number of occurrences of \( \text{send}_p(q, \text{msg}) \) in \( r_p(m) \) grows unboundedly as \( m \) increases, then either the event \( \text{crash}_q \) appears in \( r_q(m) \) for some \( m \) or the number of occurrences of \( \text{recv}_q(p, \text{msg}) \) in \( r_q(m) \) grows unboundedly as \( m \) increases. (Informally, if in run \( r \) process \( p \) sends \( \text{msg} \) infinitely often to \( q \), then either \( q \) crashes or \( q \) receives \( \text{msg} \) infinitely often.)

When we consider failure detectors, we add further conditions to runs.

A **system** is a set of runs. Systems are typically generated by **protocols** executed in a certain **context**. Formally, a **protocol for process** \( p \) is a function from finite histories to actions. A **joint protocol** is a tuple \( (P_1, \ldots, P_n) \) consisting of a protocol for each process in \( \text{Proc} \). A run \( r \) is consistent with a joint protocol \( P \) if, for all times \( m_1 \), if \( r_p(m_1 + 1) = r_p(m_1) \cdot e \) and \( e \) is an event corresponding to a protocol action, then \( e \) is in fact the event corresponding to the action \( P_i(r_p(m_1)) \). A **context** for us is simply a bound on the number of processes that can fail (if there is such a bound), a specification of properties of failure detectors (see Section 2.2, and a specification of communication properties (whether communication is reliable, fair, etc.). Fagin et al. [FHMV95, FHMV97] give a more general definition of context, but this suffices for our purposes). In a given context, a joint protocol generates the system consisting of all the runs satisfying R1–R5 and the constraints of the context that are consistent with the protocol. We say that a joint protocol has a certain property in a given context if the system it generates in that context has that property. Note that, because all runs in the systems we consider are assumed satisfy R5, we are restricting in this paper to systems where communication is fair, although possibly unreliable.

### 2.2 Failure Detectors

Informally, a **failure detector** [CT96] is a per-process oracle that emits suspicions regarding other processes’ faultiness. The fact that a process \( q \) is suspected by process \( p \)'s failure detector does not mean that \( q \) is in fact faulty. Various failure detectors can be defined by imposing conditions on the accuracy and completeness of suspicions.
Chandra and Toueg [CT96] model failure detectors by assuming a function $H$ such that $H(p, t)$ describes the suspicions of $p$‘s failure detector at time $t$. Chandra and Toueg then assume that processes explicitly query their failure detectors to “learn” those suspicions. Our approach is slightly more general. We model the act of $p$ getting a report $x$ from its failure detector by the event $\text{suspect}_p(x)$. A process $p$ could “get a report from its failure detector” either because it explicitly reads it (as Chandra and Toueg assume) or because the failure detector automatically emits a suspicion. A standard report is one of the form “the processes in $S$ are faulty”, which we model by the report $\text{suspect}_p(S)$. A standard failure detector is one whose reports are standard. In a system with standard failure detectors, at each point $p$, define $\text{Suspects}_p(r, m) = S$ if and only if $\text{suspect}_p(S)$ is the most recent failure-detector event in $r_p(m)$. (If there have not been any reports by time $m$ in $r$, $\text{Suspects}_p(r, m) = \emptyset$.) We will shortly generalize the definition of $\text{Suspects}_p(r, m)$ so that it applies in the presence of (some) nonstandard failure detectors.

The differences between our way of modeling failure detectors and the Chandra-Toueg approach are mainly cosmetic. In the Chandra-Toueg approach, what we are calling a run consists of the actions performed by the processes (including reading the failure detector) and a special tape or “oracle” that describes the responses when the failure detector is read. We have used our approach so as to be able to capture all the behavior of the system in terms of histories, without invoking any extra structure (such as extra tapes). It is easy to translate from runs in the Chandra-Toueg framework to runs in our framework, and vice versa. Given a run in the Chandra-Toueg framework, the corresponding run in our framework uses the event “$\text{suspect}_p(x)$” indicates both that $p$ read its special tape and that the response was $x$. Conversely, given a run in our framework, the corresponding run in the Chandra-Toueg framework has $p$ query its failure detector and receive response $x$ at exactly the points where the event $\text{suspect}_p(x)$ appears in its history.

Although there is a one-to-one mapping between runs in our framework and runs in the Chandra-Toueg framework, the systems (i.e., sets of runs) that we allow are slightly more general than the systems they consider. Chandra and Toueg essentially consider only systems that are a cross-product of the set of possible special tapes and the set of possible actions performed by the processes. That is, no correlation is allowed between the behavior of the processes and the behavior of the failure detector. We do allow correlation, and thus can consider types of failure detectors that Chandra and Toueg cannot (see below). However, it would be easy to extend the Chandra-Toueg framework to allow such correlation.

Consider the following properties of standard failure detectors (the first four are also used by Chandra and Toueg):

**Strong Accuracy**: No process is suspected before it crashes. Formally, for all processes $p$ and $q$ and times $m$, if $q \in \text{Suspects}_p(r, m)$, then $\text{crash}_q$ is in $r_q(m)$.

**Weak Accuracy**: If there is a correct process, then some correct process is never suspected. Formally, if $F(r) \neq \text{Proc}$ then there is some $q \notin F(r)$ such that, for all processes $p$ and times $m$, $q \notin \text{Suspects}_p(r, m)$.
**Strong Completeness:** All faulty processes are eventually permanently suspected by all correct processes. Formally, if \( q \in F(r) \) and \( p \notin F(r) \), then there is a time \( m \) such that for all \( m' \geq m \), \( q \in \text{Suspects}_p(r, m') \).

**Weak Completeness:** Each faulty process is eventually permanently suspected by some correct process. Formally, if \( q \in F(r) \) and \( F(r) \neq \text{Proc} \), then there exists some \( p \notin F(r) \) and a time \( m \) such that, for all \( m' \geq m \), \( q \in \text{Suspects}_p(r, m') \).

**Impermanent Strong Completeness:** All faulty processes are eventually suspected (but not necessarily permanently) by all correct processes. Formally, if \( q \in F(r) \) and \( p \notin F(r) \), then there is some time \( m \) such that \( q \in \text{Suspects}_p(r, m) \).

**Impermanent Weak Completeness:** Each faulty process is eventually suspected (but not necessarily permanently) by some correct process. Formally, if \( q \in F(r) \) and \( F(r) \neq \text{Proc} \), then there is some \( p \notin F(r) \) and time \( m \) such that \( q \in \text{Suspects}_p(r, m) \).

A system \( \mathcal{R} \) is said to satisfy a given property of failure detectors (e.g., weak completeness or impermanent strong completeness) if the failure detectors in every run of \( \mathcal{R} \) satisfy the property.

We remark that impermanent strong and weak completeness cannot meaningfully be captured in the Chandra-Toueg framework because, as we mentioned earlier, Chandra and Toueg do not allow correlation between the behavior of processes and the behavior of the failure detector. The only way that a special tape can guarantee impermanent strong completeness is to ensure that eventually, whenever the tape is constructed, it will report a failure. (Otherwise it might be consulted only at times when it does not report a failure.) Impermanent strong completeness requires a correlation between the special tape and the actions of the processes of a sort not allowed by Chandra and Toueg.

Chandra and Toueg define a **perfect failure detector** as one that satisfies strong completeness and strong accuracy, a **strong failure detector** as one that satisfies strong completeness and weak accuracy, and a **weak failure detector** as one that satisfies weak completeness and weak accuracy. We define an **impermanent-strong failure detector** as one that satisfies impermanent strong completeness and weak accuracy and an **impermanent-weak failure detector** as one that satisfies impermanent weak completeness and weak accuracy.

The definitions above have focused on standard failure detectors, whose reports have the form “the processes in some set \( S \) are faulty”. However, other types of reports can also be used, as long as they can be viewed as saying that the processes in some set \( S \) are faulty. For example, a report of the form “the processes in \( \text{Proc} - S \) are correct” can be clearly viewed as saying the processes in \( S \) are faulty. To make this precise, we say that a failure detector is \( g \)-standard if \( g \) is a function mapping the reports of the failure detector to

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\(^2\)Chandra and Toueg do not require that \( F(r) \neq \text{Proc} \) in their definition of weak accuracy or weak completeness, since they assume that there always is at least one correct process. We have added it here since we allow runs where all processes fail.
subsets of \( \text{Proc} \). Thus, the failure detector that reports that the processes in \( \text{Proc} - S \) are correct (such failure detectors are used in [ATD99], for example) is \( g \)-standard, where \( g(\text{"the processes in } \text{Proc} - S \text{ are correct"}) = S \). If \( p \) has a \( g \)-standard failure detector, define \( \text{Suspects}_p(r, m) = S \) if and only if \( \text{suspect}_p(x) \) is the most recent failure-detector event in \( r_p(m) \) and \( g(x) = S \). Notions of strong accuracy, weak accuracy, and so on now apply to \( g \)-standard failure detectors with no change in the definition. Although we consider only standard failure detectors in this paper, all of our results apply to \( g \)-standard failure detectors as well.

Chandra and Toueg show that a failure detector satisfying weak completeness can be converted to one satisfying strong completeness, while still preserving accuracy properties. Roughly speaking, all processes just communicate and tell each other about the suspicions reported by their original failure detectors; their modified failure detector reports all the suspicions they hear about. The same construction can be used to convert a failure detector satisfying weak impermanent completeness to one satisfying strong impermanent completeness.

We need to be a little careful in making precise in our framework the notion of converting one type of failure detector to another. In the simplest case, given a system \( R \), it is simply a question of considering a system \( R' \) where each run \( r \in R \) is replaced by a run \( r' \in R' \) such that each occurrence of an event of the form \( \text{suspect}_p(x) \) is replaced by a different failure-detector event \( \text{suspect}_p(x') \), reflecting the modified failure detector. However, if the conversion also involves additional communication (as in the conversion from weak completeness to strong completeness), the naive replacement of failure-detector events does not suffice. Nevertheless, the basic notion of conversion remains the same. Namely, we assume that there is some function \( f \) mapping runs to runs such that all the events in \( r \) (except possibly the failure-detector events) appear as events in \( f(r) \), and appear in the same order in \( r \) and \( f(r) \). However, \( f(r) \) may have additional events, including additional communication between processes and new failure-detector events. These new failure-detector events are the ones that we consider in determining whether \( R' \) has failure detectors that satisfy properties such as strong completeness. In Section 3, we use a particular instance of this conversion process to show how systems that allow solutions to UDC can be used to implement failure detectors with certain properties. For now, we leave it to the reader to check that Chandra and Toueg’s conversion from failure detectors satisfying weak completeness to ones satisfying strong completeness can be implemented in this framework. This gives us the following result.

**Proposition 2.1:** A system \( R \) with weak (resp., impermanent-weak) failure detectors can be converted to a system \( R' \) with strong (resp., impermanent-strong) failure detectors, while preserving accuracy properties.

Note that we can trivially convert a failure detector that satisfies impermanent strong completeness to one that satisfies strong completeness by always outputting the list of all previously suspected processes. For convenience, we state this as a separate proposition.
Proposition 2.2: A system $R$ with impermanent-strong failure detectors can be converted to a system $R'$ with strong failure detectors, while preserving accuracy properties.

As we show in Section 3, under a minimal assumption that should surely be satisfied in practice, if there is no bound on the number of faults (i.e., if there are runs where all processes may fail), a failure detector that satisfies weak accuracy must also satisfy strong accuracy. Thus, if there is no bound on the number of faults, then there is essentially no difference between impermanent-weak failure detectors and perfect failure detectors.\(^3\)

2.3 The Formal Language

Our language for reasoning about distributed coordination involves time and knowledge. The underlying notion of time is linear (so our language extends linear time temporal logic). We find it useful to be able to reason about the past as well as the future. Formally, we start with (application-dependent) primitive propositions and close under Boolean combinations, $\Box$, and the epistemic operators $K_p$ for each process $p$.

Following [FHMV95], we define the truth of a formula relative to a tuple $(R, r, m)$ consisting of a system $R$, run $r \in R$, and time $m$. We write $(R, r, m) \models \varphi$ if the formula $\varphi$ is true at the point $(r, m)$ in system $R$. Among the primitive propositions in the language are $send_p(q, msg)$, $recv_q(p, msg)$, $crash(p)$, $do_p(\alpha)$, and $init_p(\alpha)$. The truth of these primitive propositions is determined by the cut in the obvious way; for example, $send_p(q, msg)$ is true at a cut precisely when $send_p(q, msg)$ is an event in $p$’s history component of the cut. $\Box \varphi$ holds at a point if $\varphi$ holds from that point on in the run. Thus, $(R, r, m) \models \Box \varphi$ if and only if $(R, r, m') \models \varphi$ for all $m' \geq m$. As usual, we define $\Diamond \varphi = \neg \Box \neg \varphi$; thus, $\Diamond$ is the dual of $\Box$. It is easy to see that $(R, r, m) \models \Diamond \varphi$ if $(R, r, m') \models \varphi$ for some $m' \geq m$. Finally, $K_p \varphi$ is true if $\varphi$ is true at all the points that $p$ considers possible, given its current history. Formally, $(R, r, m) \models K_p \varphi$ if and only if $(R, r', m') \models \varphi$ for all points $(r', m') \sim_p (r, m)$ such that $r' \in R$. We say a formula $\varphi$ is valid in system $R$, denoted $R \models \varphi$, if $(R, r, m) \models \varphi$ for all points $(r, m)$ in $R$.

In our analysis, we make particular use of local and stable formulas. A formula $\varphi$ is local to process $p$ in system $R$ if at every point in $R$, $p$ knows whether $\varphi$ is true, that is, $\varphi$ is local to $p$ in $R$ if $K_p \varphi \lor K_p \neg \varphi$ is valid in $R$. All formulas describing a process’s local state, for example, $send_p(q, msg)$, $recv_q(p, msg)$, $crash(p)$, and $init_p(\alpha)$, are local to that process. It follows from standard properties of knowledge (see [FHMV95]) that formulas of the form $K_p \varphi$ are also local to $p$, since $K_p(K_p \varphi) \lor K_p(\neg K_p \varphi)$ is valid in every system. A stable formula is one that, once true, remains true; that is, $\varphi$ is stable in system $R$ if $\varphi \Rightarrow \Box \varphi$ is valid in $R$. All of $send_p(q, msg)$, $recv_q(p, msg)$, $crash(p)$, $init_p(\alpha)$, and $\Box \varphi$ are stable.

\(^3\)In the notation of Chandra and Toueg, impermanent-$W \cong$ impermanent-$S \cong S \cong P$ for $t = n - 1$ or $t = n$ failures.
2.4 Distributed Coordination

We are interested in modeling distributed coordination of certain actions among the processes in Proc. The actions may be allocating a resource, delivering multicast messages, or committing a transaction; we are not concerned with the specifics. We are also not concerned here with other requirements such as executing actions in a particular order (e.g., total-order multicast) or not executing conflicting actions (e.g., consensus). We are interested only in the eventual, distributed execution of these actions.

Formally, we assume that each process $p$ has a set $A_p$ of coordination actions that it can initiate. We assume that the sets $A_p$ and $A_q$ are disjoint for $p \neq q$. (Think of the actions in $A_p$ as somehow being tagged by $p$.) The fact that an action $\alpha$ is in $A_p$ does not mean that only $p$ can perform $\alpha$. However, it does mean that only $p$ can initiate $\alpha$; no process can perform $\alpha$ unless $p$ initiates it. We assume that for each action $\alpha \in A_p$, there is a special action init$_p(\alpha)$ of $p$ initiating $\alpha$. The corresponding event init$_p(\alpha)$ can appear only in $p$’s history, and can appear at most once in a run. Formally, for the rest of this paper, we consider only systems $R$ where, for all points $(r, m)$ in $R$ and all actions $\alpha \in A_p$, the event init$_p(\alpha)$ can appear only in $r_p(m)$ and can appear at most once in $r_p(m)$.

Informally, a system satisfies Uniform Distributed Coordination (UDC) of action $\alpha$ if whenever any $p' \in$ Proc executes $\alpha \in A_p$, then so eventually does every correct $q \in$ Proc. In addition, no process performs $\alpha \in A_p$ unless $p$ initiates it. Intuitively, if init$_p(\alpha)$ appears in $p$’s history and $p$ is nonfaulty in run $r$, then all the nonfaulty processes in $r$ should perform $\alpha$. Formally, UDC of $\alpha \in A_p$ holds in a system $R$ if the following three conditions hold:

DC1. $R \models$ init$_p(\alpha) \Rightarrow (\text{do}_p(\alpha) \lor \text{crash}(p))$;

DC2. $R \models \land_{q_1, q_2 \in \text{Proc}} (\text{do}_{q_1}(\alpha) \Rightarrow (\text{do}_{q_2}(\alpha) \lor \text{crash}(q_2)))$;

DC3. $R \models \land_{q_2 \in \text{Proc}} (\text{do}_{q_2}(\alpha) \Rightarrow \text{init}_p(\alpha))$.

Non-Uniform Distributed Coordination (nUDC) requires coordination only if the process that performs $\alpha$ is correct. Thus, nUDC of $\alpha$ holds in a system $R$ if DC1, DC3, and the following hold:

DC2’. $R \models \land_{q_1, q_2 \in \text{Proc}} (\text{do}_{q_1}(\alpha) \Rightarrow (\text{do}_{q_2}(\alpha) \lor \text{crash}(q_2) \lor \text{crash}(q_1)))$.

The next propositions show that, unlike UDC, nUDC is easy to attain, and that reliable communication is significant for UDC. (As we said earlier, all proofs are in the Appendix.)

**Proposition 2.3:** There is a protocol that attains nUDC without the use of failure detectors in every context where communication is fair (although possibly unreliable), even if there is no bound on the number of failures.
Proposition 2.4: There is a protocol that attains UDC without the use of failure detectors in every context where communication is reliable, even if there is no bound on the number of failures.

Propositions 2.3 and 2.4 distinguish UDC and nUDC from consensus. Unlike consensus, both UDC and nUDC are attainable in asynchronous systems with failures (although UDC needs reliable communication); indeed, they are attainable without failure detectors no matter how many processes may fail. However, as we shall see in the next two sections, things change when we consider UDC in a context with unreliable communication.

3 UDC With No Bound on Failures

We start by showing that UDC is achievable in a context with fair but unreliable communication, provided we have impermanent-strong failure detectors.

Proposition 3.1: There is a protocol that attains UDC in every context with strong failure detectors, even if there is no bound on the number of failures.

In light of Proposition 2.1 and 2.2, the following corollary is immediate.

Corollary 3.2: There is a protocol that attains UDC in every context with impermanent-weak failure detectors, even if there is no bound on the number of failures.

Chandra and Toueg [CT96] prove a result analogous to Proposition 3.1 for consensus. They show that consensus is achievable in every context where there are strong failure detectors, at most $n - 1$ failures, and where communication is reliable. Their algorithm works without change even if we have only impermanent-strong failure detectors and allow $n$ failures. Moreover, their algorithm can be modified easily to deal with unreliable, but fair, communication. Thus, unlike UDC, the reliability of communication has no significant impact on the attainability of consensus in these contexts.

We prove in Theorem 3.6 below that under certain assumptions about the context (which include the assumption that there is no bound on the number of failures along with our usual implicit assumption that communication is fair, although possibly unreliable), if processes can perform UDC then they can simulate perfect failure detectors. It follows from Proposition 3.4 below that, under these assumptions, strong failures detectors are equivalent to perfect failure detectors. Thus, we will be proving what is essentially a converse to Proposition 3.1. To prove this result, we need to make precise the notion of “simulating a perfect failure detector.”

“Simulating a perfect failure detector” means that we can convert a system $\mathcal{R}$ to a system $\mathcal{R}'$ with perfect failure detectors, using the same type of conversion as outlined in Section 2.2. We now sketch the conversion. Given a run $r \in \mathcal{R}$, we construct a run $f(r)$ such that
P1. \( f(r)_p(0) = (\langle \rangle, \ldots, \langle \rangle); \)

P2. if \( r_p(m + 1) = r_p(m) \cdot e \) and \( e \) is not a failure-detector event, then \( f(r)_p(2m + 2) = f(r)_p(2m + 1) \cdot e; \) if \( r_p(m + 1) = r_p(m) \cdot e \) and \( e \) is a failure-detector event or if \( r_p(m + 1) = r_p(m) \), then \( f(r)_p(2m + 2) = f(r)_p(2m + 1); \)

P3. \( (f(r))_p(2m+1) = (f(r))_p(2m) \cdot \text{suspect}'_p(S), \) where \( S = \{q : (\mathcal{R}, r, m) \models K_p(\text{crash}(q))\}. \)

Thus, in \( f(r) \), process \( p \)'s history is identical to its history in \( r \) except that the failure-detector events in \( r \) are deleted in \( f(r) \), and, at each odd step in \( f(r) \), \( p \)'s failure detector reports the processes that \( p \) knows have crashed at the corresponding point in \( \mathcal{R} \). Now define system \( \mathcal{R}' = \{f(r) : r \in \mathcal{R}\}. \) We say that \( \mathcal{R} \) can simulate perfect failure detectors if the \( \text{suspect}' \) failure detectors in \( \mathcal{R}' \) are perfect. We shortly give conditions on \( \mathcal{R} \) that guarantee that it can simulate perfect failure detectors.

As observed by Aguilera, Toueg, and Deianov [ATD99], our definition allows the simulating function \( f \) to be noncomputable. Technically, this is not quite right. The input to \( f \) is a run, which is an infinitary object, so it does not even make sense to consider the computability or noncomputability of \( f \). However, it is easy to modify \( f \) so that its input and output are not complete runs, but rather prefixes of runs. Given a prefix of length \( m + 1 \) (i.e., given \( r(0), \ldots, r(m) \) for some run \( m \)), \( f \) returns a prefix of length \( 2m + 2 \). Conditions P1–3 still make sense with that change. With that change, it is clear that \( f \) is computable provided that \( \{q : (R, r, m) \models K_p(\text{crash}(q))\} \) is computable for each \( p, r, \) and \( m \). While it is possible to construct systems where this set is not computable, it will be computable in any “reasonable” system. That is because whether \( K_p(\text{crash}(q)) \) holds at the point \( (r, m) \) is typically determined by some easily characterizable sequence of events in \( p \)'s history. While it is beyond the scope of this paper to characterize when \( f \) is computable (it is not even clear how interesting such a characterization would be), it should be clear that it typically is computable.

The notion of simulation implicitly underlying this definition is more general than, but compatible with, the notion of reduction used by Chandra, Hadzilacos, and Toueg [CHT96]. For example, in this paper, it is shown that if consensus can be solved by means of a failure detector (and there are at most \( t < n/2 \) failures), then that failure detector can be transformed to a particular failure detector called \( \diamond \mathcal{W} \) (for \textit{eventually weak}), which satisfies \textit{eventual weak accuracy} and \textit{weak completeness}; see [CT96] for the precise definition. Since consensus can be solved with \( \diamond \mathcal{W} \) failure detectors, these failure detectors are viewed as the weakest failure detectors for consensus.

The key point is that the results of Chandra, Hadzilacos, and Toueg do not apply if UDC is solved without the use of a failure detector. However, our notion of simulation does not depend on using failure detectors to attain UDC. Thus, it applies in situations where some other type of oracle is used, for example, an oracle that gives limited information about which actions have been initiated, in which case the reductions of Chandra, Hadzilacos, and Toueg may not apply at all.
We next describe some conditions on a system $\mathcal{R}$ that together will suffice to show that $\mathcal{R}$ can simulate perfect failure detectors. Before stating them, we need a definition.

**Definition 3.3:** A formula $\varphi$ local to $q$ is said to be **insensitive to failure by $q$** in $\mathcal{R}$ if for all runs $r, r' \in \mathcal{R}$ and all times $m, m'$, if $r'_q(m') = r_q(m) \cdot \text{crash}_q$, then $(\mathcal{R}, r, m) \models \varphi$ iff $(\mathcal{R}, r', m') \models \varphi$.

Now consider the following five conditions on a system $\mathcal{R}$.

A1. If there exists a run $r_S \in \mathcal{R}$ where all the processes in $S$ crash, and $(r, m)$ is a point in $\mathcal{R}$ such that no process in $\text{Proc} - S$ has crashed, then there is a run $r'$ extending $(r, m)$ such that $F(r') = S$.

A2. For all runs $r_1, r_2 \in \mathcal{R}$ and times $m$, if $F(r_1) = F(r_2) = F$ and $(r_1, m) \sim_q (r_2, m)$ for all $q \notin F$, then there are extensions $r'_1$ and $r'_2$ of $(r_1, m)$ and $(r_2, m)$, respectively, such that all the processes in $F$ crash by time $m + 1$ in $r'_1$ and $r'_2$ and $(r'_1, m') \sim_q (r'_2, m')$ for all $m' \geq m$ and all $q \notin F$.

A3. The formula $K_q \text{init}_p(\alpha)$ is insensitive to failure by $q$.

A4. If $\varphi$ is (a) stable in $\mathcal{R}$, (b) local to some process $p$ in $\text{Proc}$, and (c) insensitive to failure by $p$, then for all points $(r, m)$ in $\mathcal{R}$, if there is some nonempty $S \subseteq \text{Proc}$ such that $(\mathcal{R}, r, m) \models \wedge_{q \in S} \neg K_q \varphi$, then there exists a point $(r', m)$ such that (a) $r'_q(m) = r_q(m)$ for all $q \in S$; (b) for all $q \notin S$, there is a (not necessarily strict) prefix $h$ of $r_q(m)$ such that either $r'_q(m) = h$ or $r'_q(m) = h \cdot \text{crash}_q$ and $q$ crashes by time $m$ in $r$; and (c) $(\mathcal{R}, r', m) \models \neg \varphi$.

A5t. For every $S \subseteq \text{Proc}$ such that $|S| \leq t$, there exists a run $r_S \in \mathcal{R}$ such that $F(r_S) = S$.

We now briefly discuss these conditions and their implications. A1 essentially says that process failures are independent of other events. If it is possible for the processes in $S$ to crash, this may happen at any time in any run. A3 says that a process $q$ cannot learn that $p$ initiated $\alpha$ just by $q$’s crashing. A1 and A3 are properties we would expect to hold of all systems generated by protocols in the contexts of interest to us.

A2 says that it is possible for all the faulty processes in $r$ that have not crashed by time $m$ to crash at the next step. More precisely, if two points $(r, m)$ and $(r', m)$ are indistinguishable to the correct processes in $r$, then there are extensions $r_1$ and $r_2$ of these points that continue to be indistinguishable to all the correct processes in $r$, such that all the faulty processes in $r$ have failed by time $m+1$ in $r_1$ and $r_2$. A2 implicitly assumes that there is no information relevant to the system beyond what is in the correct processes’ states. In particular, this means that there cannot be completely reliable message buffers.

\[\text{Footnote: For those familiar with the notion of distributed knowledge [FHMV95], note that conditions (a) and (c) imply that the processes in } S \text{ do not have distributed knowledge of } \varphi.\]
in the system. For suppose that \( q \) had a message buffer such that once a message was in \( q \)'s buffer, then as long as \( q \) did not crash, \( q \) would eventually receive the message. Consider two runs \( r_1 \) and \( r_2 \) such that \((r_1,m) \sim_q (r_2,m)\), \( F(r_1) = F(r_2) \), and \( F(r_1) \) consists of all processes other than \( q \). Moreover, suppose that there is a message \( \text{msg} \) in \( q \)'s buffer in \((r_1,m)\), but not in \((r_2,m)\). By A2, there are extensions \( r'_1 \) and \( r'_2 \) of \((r_1,m)\) and \((r_2,m)\) such that all processes other than \( q \) crash in round \( m + 1 \) in both \( r'_1 \) and \( r'_2 \) and \((r'_1,m') \sim_q (r'_2,m')\) for all \( m' \geq m \). But this is impossible, since \( q \) receives \( \text{msg} \) in \( r'_1 \) but not in \( r'_2 \). More generally, A2 says that communication is unreliable. It does not hold if the network cannot lose all messages that might be in transit at any given time.

A4 says, among other things, that if each of the processes in \( S \) considers \( \neg \varphi \) possible, where \( \varphi \) is a stable failure-insensitive formula local to some process, then there is a point where \( \neg \varphi \) is true that all the processes in \( S \) simultaneously consider possible. A4 is perhaps the least standard property. It holds if processes are essentially using a full-information protocol (FIP) [Coa86, FHMV95] and if \( R \) places some restrictions on the information they can get from failure detectors. With an FIP, when a process \( p \) sends a message to \( q \), it sends complete information about its state. The following example shows that, without FIPs, A4 can fail to be true. Consider a system \( R \) where processes send messages that are formulas in the language defined in Section 2.3. Moreover, assume that every message sent is true at the time that it is sent. Let \((r,m)\) be a point in \( R \) such that neither \( p \) nor \( q \) has crashed at \((r,m)\), and at some time \( m'' < m \), \( q \) sends a message \( \text{msg} \) to \( p' \), which \( p' \) receives. After receiving \( \text{msg} \), \( p' \) sends a message saying \( \text{crash}(q) \lor \text{send}_q(p', \text{msg}) \), which \( p \) receives by time \( m \). Further suppose that \( p' \) has a perfect failure detector, and there is another run \( r' \) in \( R \) such that \( r_p(m) = r'_p(m') \) and, in \( r' \), process \( p' \) knows that \( q \) has crashed (since its failure detector reported this) and \( q \) does not send \( p' \) the message \( \text{msg} \). It follows that \((R, r, m) \models K_p(\text{crash}(q) \lor \text{send}_q(p', \text{msg})) \land \neg K_p(\text{crash}(q)) \land \neg K_p(\text{send}_q(p', \text{msg})) \). Process \( p \) knows \( \neg \text{crash}(q) \lor \text{send}_q(p', \text{msg}) \) because it received a message from \( p' \) saying this, and messages are known to be truthful in \( R \). Process \( p \) does not know \( \text{crash}(q) \) (since \( q \) actually has not crashed at the point \((r,m)\)) nor does \( p \) know \( \text{send}_q(p', \text{msg}) \) (since \( \text{send}_q(p', \text{msg}) \) is not true at \((r', m')\)). But then A4 does not hold in \( R \) for \( \varphi = \text{send}_q(p', \text{msg}) \) and \( S = \{p\} \). For suppose it did hold. Then there must be a point \((r'', m)\) in \( R \) such that (a) \( r''_p(m) = r_p(m) \), (b) \( r''_q(m) \) is a prefix of \( r_q(m) \) (since \( q \) does not crash in \( r \)), and (c) \((R, r'', m) \models \neg \text{send}_q(p', \text{msg}) \). Since \((R, r, m) \models K_p(\text{crash}(q) \lor \text{send}_q(p', \text{msg})) \), it follows that \((R, r'', m) \models \text{crash}(q) \), violating the assumption that \( r''_q(m) \) is a prefix of \( r_q(m) \).

In this example, \( p' \) did not tell \( p \) all it knew, which is precisely what cannot happen with a full-information protocol. Assuming that \( R \) is generated by an FIP, under reasonable assumptions about the runs in \( R \) (discussed below), \( R \) will satisfy A4. To see why, observe that given \((r,m)\) and \( \varphi \) as in the hypotheses of A4, we can construct the run \( r' \) as follows. First note that \((R, r, 0) \models \neg \varphi \), for otherwise, since \( \varphi \) is stable and local to \( p \), \( \varphi \) would be true at all points in \( R \) and so would \( K_q \varphi \) for all \( q \in \text{Proc} \). Thus, let \( m_p \) be the first time in \( r \) where \( \varphi \) becomes true. If \( m_p > m \), then take \( r' = r \) and \( S = \text{Proc} \); A4 trivially holds in this case. If \( m_p \leq m \), let \( S \subseteq \text{Proc} \) be the processes that do not know
\( \varphi \) at \((r, m)\). If processes are following a full-information protocol, there can be no chain of messages from \( p \) to a process \( q \in S \) between times \( m_p \) and \( m \) in \( r \), for if there were, \( q \) would know \( \varphi \) at \((r, m)\).\(^5\) For each process \( q \in \text{Proc} \), let \( m_q \) be the least time at or before \( m \) at which there is a message chain from \( p \) to \( q \) in \( r \) between \( m_q \) and \( m_q \), if there is such a time; otherwise, we take \( m_q = m + 1 \). Note that for \( q \in S \), we have \( m_q = m + 1 \). We then construct \( r' \) so that \( r'_q(m') = r_q(m') \) for \( m' \leq m_q - 1 \); if \( q \) does not crash in \( r \) between times \( m_q \) and \( m \) inclusive, then \( r'_q(m') = r_q(m_q - 1) \) for \( m' \geq m_q \); otherwise, \( r'_q(m') = r_q(m_q - 1) \cdot \text{crash}_q \) for \( m' \geq m_q \). By construction, we have \( r_q(m) = r'_q(m) \) for \( q \in S \). For \( q' \notin S \), we have that \( r'_q(m) \) is either \( r_q(m_q - 1) \) or \( r_q(m_q - 1) \cdot \text{crash}_q \). The reason we need to add \( \text{crash}_q \) is that the failure detector of some process \( q \in S \) might report that \( q' \) fails in \( r \). Since \( r_q(m) = r'_q(m) \), if \( q' \)'s failure detector is accurate, it must be the case that \( q' \) also fails in \( r' \). As long as \( r' \in \mathcal{R} \), it is easy to see that the point \((r', m)\) satisfies the requirements of (this instance of) A4. For by construction, \( r_q(m) = r'_q(m) \) for \( q \in S \); and for \( q' \notin S \), the construction guarantees that either \( r'_q(m) = r_q(m) \) or \( r'_q(m) = r_q(m_q) \cdot \text{crash}_q \). By choice of \( m_p \), we have that \((\mathcal{R}, r, m_p - 1) \vdash \neg \varphi \). Note that if \((\mathcal{R}, r, m) \vdash \neg \text{crash}(p) \) then \( r'_p(m) = r_p(m_p - 1) \); otherwise, either \( r'_p(m) = r_p(m_p - 1) \) or \( r'_p(m) = r_p(m_p - 1) \cdot \text{crash}_p \). Since \( \varphi \) is insensitive to failure by \( p \), in either case, we have that \((\mathcal{R}, r', m) \vdash \neg \varphi \). Thus, \((r', m)\) satisfies the requirements of A4.

This argument shows that as long as it is the case that, for each formula \( \varphi \) and point \((r, m)\) satisfying the hypotheses of A4, there is a run \( r' \in \mathcal{R} \) as constructed above (actually, it suffices that there is a run in \( \mathcal{R} \) that extends \((r', m)\)), then \( \mathcal{R} \) satisfies A4. Thus, for example, it cannot be the case that the failure detector reports in \( \mathcal{R} \) are correlated with message delivery, so that a report from a failure detector saying that \( p \) is faulty is accurate iff \( p \) did not receive a message from \( q \). We do not attempt to completely characterize the conditions under which \( \mathcal{R} \) satisfies A4 here.

A5t says that any subset of processes of size at most \( t \) may fail in some run. This is a standard assumption in the literature. Note that A5t implies A5p if \( t \geq t' \).

Theorem 3.6 below shows that if \( \mathcal{R} \) attains UDC and satisfies A1–A4 and A5n (or A5n−1) and one other quite innocuous condition, then \( \mathcal{R} \) can simulate perfect failure detectors. The “innocuous condition” is the following: Clearly any solution to UDC should allow a process to initiate any of its actions at any time. To guarantee that \( \mathcal{R} \) can simulate perfect failure detectors, it is necessary that, in each run of \( r \), the correct processes (if there are any) initiate actions infinitely often. That is, for all runs \( r \), if \( F(r) \neq \text{Proc} \), then for all times \( m \), some correct process in \( r \) initiates an action after \((r, m)\). Intuitively, if actions are initiated infinitely often, the correct processes will need to be able to detect failures in order to attain UDC repeatedly. On the other hand, if the correct processes do not initiate actions after some point, there will be no need for

\(^5\)There is a message chain from \( p \) to \( q \) between \( m_p \) and \( m > m_p \) if there is a sequence of messages \( \text{msg}_1, \ldots, \text{msg}_k \) and processes \( p_1, \ldots, p_{k+1} \) such that (a) \( \text{msg}_i \) is sent by \( p_i \) to \( p_{i+1} \) and is received, (b) \( p_{i+1} \) sends \( \text{msg}_{i+1} \) after receiving \( \text{msg}_i \), (c) \( p = p_1 \), (d) \( q = p_{k+1} \), (e) \( p \) sends \( \text{msg}_1 \) at or after \( m_p \), and (f) \( q \) receives \( \text{msg}_{k+1} \) at or before \( m \). If the processes follow a full-information protocol, then when \( p_{i+1} \) receives \( \text{msg}_i \), \( p_{i+1} \) all the stable facts that \( p_i \) knew when \( p_i \) sent \( \text{msg}_i \).
them to detect failures after this point. To see the need for this condition, suppose that no actions are initiated after time 17, even though there are some correct processes in $R$ and UDC is attained for all these actions by time 25. Now consider a process $q$ that fails after time 17. There is no need for processes to know that $q$ has failed, since UDC is not required after time 25.

To summarize, our result can be viewed as saying that under some relatively innocuous assumptions (A1–A3 and the assumption that correct processes initiate actions infinitely often), if any subset of processes may fail ($A5_n$) and the processes are telling each other as much as they can (A4), then being able to attain UDC is tantamount to being able to simulate perfect failure detectors.

Before proving Theorem 3.6, we prove two preliminary results. The first shows that, in the contexts of interest to us, weak accuracy and strong accuracy are equivalent. The second provides a characterization of the facts that must be known by a process before it can perform a coordination action $\alpha$. Specifically, a process must know that if there are any correct processes at all, then one of these knows that $\alpha$ has been initiated.

**Proposition 3.4:** If $R$ satisfies $A1$ and $A5_{n-1}$ then $R$ satisfies weak accuracy iff $R$ satisfies strong accuracy.

It follows from Proposition 3.4 that if $R$ satisfies $A1$ and $A5_{n-1}$, then $R$ has strong failure detectors iff $R$ has perfect failure detectors. (Since $A5_n$ implies $A5_{n-1}$, this is a fortiori the case if $R$ satisfies $A1$ and $A5_n$.)

**Proposition 3.5:** If $R$ satisfies $A1$, $A2$, and $A4$, then

$$R \models \forall_{p, p' \in \text{Proc}} \forall_{\alpha \in A_p} \left[ K_p \left( \text{init}_{p'}(\alpha) \land \bigvee_{q \in \text{Proc}} (K_q \text{init}_{p'}(\alpha) \lor \text{crash}(q)) \right) \rightarrow K_p \left( \bigvee_{q \in \text{Proc}} \square \neg \text{crash}(q) \Rightarrow \bigvee_{q \in \text{Proc}} (K_q \text{init}_{p'}(\alpha) \land \square \neg \text{crash}(q)) \right) \right].$$

We are now ready to state our theorem.

**Theorem 3.6:** Suppose $R$ is the system generated by a protocol that attains UDC, $R$ satisfies $A1$–$A4$ and $A5_{n-1}$, and for each run $r \in R$, if $F(r) \neq \text{Proc}$, then infinitely many actions are initiated in $r$ (i.e., infinitely many events of the form $\text{init}_p(\alpha)$ appear in $r$). Then the system $R^f$ has perfect failure detectors.

There are two issues worth noting regarding Theorem 3.6. First, the alert reader may have noticed an apparent contradiction in our results. Proposition 2.4 states the UDC can be attained in contexts where communication is reliable, without using failure detectors.
detectors. Theorem 3.6 states that, if UDC can be attained, then perfect failure detectors can be simulated. Thus, perfect failure detectors can be simulated in systems where communication is reliable. Since it is well known that Consensus can be attained with perfect failure detectors, this suggests that Consensus can be attained in systems where communication is reliable, regardless of the number of process failures. But this is well known to be false [FLP85].

Our results are correct. We escape from the contradiction because, as we noted earlier, A2 specifically precludes reliable communication. Thus, Theorem 3.6 does not apply to systems of the type considered in Proposition 2.4. This observation does emphasize that our main results apply only to systems where communication is unreliable.

Second, the assumption that infinitely many actions must be initiated in each run of \( R \) in Theorem 3.6 may strike some readers as unduly strong (although it can be argued that a service should expect to operate indefinitely and therefore handle infinitely many requests). In any case, the theorem can be rephrased in a way that might make it more palatable. Consider a context that allows solutions to UDC and satisfies A1–A4 and A5\(_{n-1}\). Then there is a joint protocol \((P_1, \ldots, P_n)\) that, when run in that context, generates a system \( R \) such that \( R^f \) has perfect failure detectors. The proof of this result is essentially identical to that of Theorem 3.6: the joint protocol \((P_1, \ldots, P_n)\) is one where each process that does not crash initiates an infinite number of actions. Indeed, every joint protocol where each process that does not crash initiates an infinite number of actions generates a system \( R \) where \( R^f \) has perfect failure detectors. Thus, the result really shows that in contexts where UDC can be solved, UDC can be used to generate perfect failure detectors.

4 Generalized Failure Detectors

Theorem 3.6 shows that if at as many as \( n - 1 \) processes can fail, then UDC essentially requires perfect failure detectors. On the other hand, as Gopal and Toueg [GT89] show, UDC is attainable without using failure detectors in contexts where there are less than \( n/2 \) failures. We now generalize both of these results, characterizing the type of failure detector needed to attain UDC if there is a bound of \( t \) on the number of possible failures, for all values of \( t \).

A generalized failure detector reports that (it suspects that) at least \( k \) processes in a set \( S \) are faulty.\(^7\) As discussed in the Introduction, such generalized failure detectors are appropriate when processes can observe faulty behavior in some component(s) without being able to tell which processes in the component are actually faulty. We model such generalized suspicions by using events of the form \( \text{suspect}_p(S, k) \), with \( k \leq |S| \).\(^8\) We are

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\(^7\) Despite the name, generalized failure detectors are a special case of failure detector as defined in Section 2.2, as well as being a special case of the failure detectors defined by Aguilera, Toueg, and Deianov [ATD99].

\(^8\) Again, it is not necessary that the report of the failure detector has the form \((S, k)\). We can define
interested in generalized failure detectors that give useful information. Of course, what is “useful” may depend on the application. Given a system $R$ and an upper bound of $t$ on the number of failures that may occur in a run $r$ of $R$, we say that $\text{suspect}_p(S,k)$ is a $t$-useful failure-detector event for $r$ if (a) $F(r) \subseteq S$, (b) $n - |S| > \min(t,n-1) - k$ (or, equivalently, $k > |S| - n + \min(t,n-1)$), and (c) $k \leq |S|$. Intuitively, if a generalized failure detector is “good”, then some of its reports are $t$-useful failure events. Note that if $p$ learns at the point $(r,m)$ that there are $k$ faulty processes in $S$ and $n - |S| > \min(t,n-1) - k$, then $p$ can conclude that, if there are any correct processes at all in $r$, then one of the processes in $\text{Proc} - S$ is correct at $(r,m)$ (although it may not know which one). Just knowing that some processes in a set are correct is not useful in general. For example, if $t < n$, then all processes know that at least $n - t$ processes in $\text{Proc}$ are correct. As we shall see, what makes this fact useful is that $F(r) \subseteq S$.

A generalized failure detector in $R$ is $t$-useful if for all $r \in R$ and processes $p$, we have the following:

**Generalized Strong Accuracy:** if $\text{suspect}_p(S,k)$ is in $r_p(m)$, then there is a subset $S' \subseteq S$ such that $|S'| = k$ and for all $q \in S'$, we have that $\text{crash}_q$ is in $r_q(m)$.

**Generalized Impermanent Strong Completeness:** if $p$ is correct, then there is a $t$-useful failure-detector event for $r$ in $r_p(m)$, for some $m$.

Note that it is trivial to construct a $t$-useful failure detector in a context with at most $t$ failures if $t < n/2$: for each $S \subseteq \text{Proc}$ with $|S| = t$, output $(S,0)$ infinitely often. Suspecting no processes in any subset $S$ trivially satisfies generalized strong accuracy, and in every run $r$ at least one $t$-sized subset of $\text{Proc}$ must contain $F(r)$. Whenever $F(r) \subseteq S$, then $(S,0)$ is a $t$-useful failure-detector event.

Also note that if $\text{suspect}_p(S,k)$ is an $(n-1)$-useful or $n$-useful failure-detector event, then we must have $|S| = k$, since the only way to have $k > |S| - 1$ is to have $k = |S|$. Thus, we can easily convert an $n$-useful or $(n-1)$-useful generalized failure detector to a perfect failure detector, by just reporting $\text{suspect}_p(S')$ at time $m$ in run $r$ if $S'$ is the union of the sets $S$ such that the generalized failure detector has reported $\text{suspect}_p(S,k)$ with $|S| = k$ prior to time $m$. Conversely, we can easily convert a perfect failure detector to an $n$-useful (and hence $(n-1)$-useful) failure detector. Given a history for process $p$, we simply replace each event $\text{suspect}_p(S)$ by the event $\text{suspect}_p(S',k)$, where $S'$ is the union of $S$ together with all the sets that appeared in failure-detector events of the perfect failure detector earlier in the history, and $k = |S'|$. It is easy to see that this gives an $n$-useful failure detector. Thus, the following result generalizes Proposition 3.1 and Gopal and Toueg’s result.

**Proposition 4.1:** There is a protocol that attains UDC in a context with a bound of $t$ on the number of failures and with $t$-useful generalized failure detectors.

$g$-generalized failure detector whose reports can be mapped to pairs $(S,k)$. For ease of exposition, we do not bother doing this.
Table 1: The type of failure detector needed for UDC vs. consensus; † indicates optimality.

|               | $0 < t < n/2$                          | $n/2 \leq t < n - 1$ | $n - 1 \leq t \leq n$ |
|---------------|----------------------------------------|-----------------------|------------------------|
| Reliable      | UDC                                    | no FD                 | no FD                  |
|               | consensus                               | $\diamond W$ †        | Strong †               |
|               |                                        |                       |                        |
| Unreliable    | UDC                                    | no FD                 | $t$-useful †           |
|               | consensus                               | $\diamond W$ †        | Perfect †              |

Since, as observed earlier, it is trivial to construct a $t$-useful failure detector in a context with at most $t$ failures, if $t < n/2$, we get the following result of Gopal and Toueg [GT89] as an immediate corollary to Proposition 4.1.

**Corollary 4.2:** If $t < n/2$, then there is a protocol that attains UDC without failure detectors.

We want a converse to Proposition 4.1 that generalizes Theorem 3.6. We show that if processes can perform UDC in a context with a bound $t$ on the number of failures, then $t$-useful generalized failure detectors can be simulated in that context.

Given system $\mathcal{R}$, construct system $\mathcal{R}^f'$ as follows. Fix an order $S_0, \ldots, S_{2^n-1}$ of the subsets of Proc. Let $\mathcal{R}^f' = \{f'(r) : r \in \mathcal{R} \}$ where, for each run $r \in \mathcal{R}$, $f'(r)$ is constructed exactly as $f(r)$ in Section 3, except that $P3$ is replaced by the following condition.

$P3'$. $(f'(r))_p(2m+1) = (f'(r))_p(2m) \cdot \text{suspect}_p(S_l, k)$, where $l$ is the length of the history $r_p(m+1) \mod 2^n$ and

$$k = \max \{k' : (\mathcal{R}, r, m) \models K_p(k' \text{ processes in } S_l \text{ have crashed})\}.$$

**Theorem 4.3:** Suppose $\mathcal{R}$ is the system generated by a protocol that attains UDC in a context with at most $t$ failures, $\mathcal{R}$ satisfies $A1$–$A4$ and $A5_t$, and for each run $r \in \mathcal{R}$, if $F(r) \neq \text{Proc}$, then infinitely many actions are initiated in $r$. Then $\mathcal{R}^f'$ has $t$-useful generalized failure detectors.

As with Theorem 3.6, we can restate Theorem 4.3 to say that in any context with at most $t$ failures where UDC can be attained, there is a joint protocol $\vec{P}$ that generates a system $\mathcal{R}$ such that $\mathcal{R}^f'$ has $t$-useful failure detectors.

5 Conclusions

We have shown that the problem of Uniform Distributed Coordination in asynchronous systems varies in its complexity both with communication guarantees and with the number of failures that must be tolerated (see Table 1). Unlike consensus (or nUDC, for that
matter), UDC is sensitive to communication guarantees in the contexts that we consider in this paper. This is significant since UDC is likely the only acceptable reliability guarantee for many wide-area and collaborative mobile applications, precisely where reliable communication cannot be assumed.

Note that we have completely characterized the type of failure detector required to attain UDC for all values of $t$. For consensus, it is known that $\Diamond \mathcal{W}$ is necessary and sufficient if $t < n/2$. (Recall that in this case no failure detectors at all are necessary to attain UDC.) While strong (actually, impermanent-strong) failure detectors suffice for consensus for $n/2 \leq t < n$, there is no characterization of exactly the type of failure detector that is required. The notion of $t$-useful failure detectors defined here may prove useful in that regard. We leave exploring this issue for future work.

As we mentioned in the introduction, in a paper written in response to the conference version of this paper, Aguilera, Toueg, and Deianov [ATD99] provided an elegant alternative characterization of the weakest failure detector required for UDC.\(^9\) They show that the weakest failure detector for this problem is one that satisfies strong completeness and a notion of accuracy even weaker than what we have called weak accuracy: if there is a correct process, then at all times, some correct process is not suspected (but a different correct process may be the one that is not suspected at every time). They show that if UDC can be solved with a failure detector $F$, then $F$ can be reduced (i.e., effectively transformed) into this weakest failure detector. Technically, this result is incomparable to our result. On the one hand, it is stronger, in that it gives a failure detector that solves UDC in all contexts (not just ones satisfying A1–A4 and A5\(_t\), which are the only ones considered in this paper), and is in a precise sense the weakest failure detector needed to solve UDC. On the other hand, as we observed in the discussion preceding Theorem 3.6, because our results do not proceed by reduction of one failure detector to another, our results apply even in cases where some technology other than failure detectors is used to solve UDC.

**Appendix – Proofs of Propositions and Theorems**

**Proposition 2.3:** There is a protocol that attains nUDC without the use of failure detectors in every context where communication is fair (although possibly unreliable), even if there is no bound on the failures.

**Proof:** We just sketch the protocol here, since it is so simple. Whenever a process $p$ wants to attain nUDC of action $\alpha$ (i.e., if $\text{init}_p(\alpha)$ is in $p$'s history) $p$ goes into a special nUDC($\alpha$) state. If a process is in an nUDC($\alpha$) state, it performs $\alpha$ and sends an $\alpha$-message repeatedly to all other processes (which, intuitively, tells them to perform $\alpha$). If a process

\(^9\)Actually, their results are given for URB, uniform reliable broadcast, but URB and UDC are isomorphic problems; the $\text{init}$ and $\text{do}$ in UDC correspond to $\text{broadcast}$ and $\text{deliver}$ in URB.
receives an $\alpha$-message, it goes into an $\text{nUDC}(\alpha)$ state, if it has not already done so. It is easy to see that this protocol attains $\text{nUDC}$.\footnote{This protocol, like most of the others we present in this paper, does not have any mechanism for termination. Processes keep sending messages forever. Since message communication is unreliable, it is not hard to show that there is in fact no protocol that attains $\text{nUDC}$ and terminates. We can deal with this problem by adding a heartbeat mechanism [ACT97], but this issue is beyond the scope of this paper.}

**Proposition 2.4:** There is a protocol that attains $\text{UDC}$ without the use of failure detectors in every context where communication is reliable, even if there is no bound on the number of failures.

**Proof:** We proceed just as in the proof of Proposition 2.3, except that before performing the action $\alpha$, a process simply sends a message to all other processes telling them to perform $\alpha$ and inform all other processes if they have not already done so. More precisely, if $\text{init}_p(\alpha)$ is in $p$’s history, $p$ goes into a special $\text{UDC}(\alpha)$ state. If a process is in a $\text{UDC}(\alpha)$ state, it sends an $\alpha$-message to all processes and then performs $\alpha$. If a process receives an $\alpha$ message, it goes into a UDC-state if it has not already done so. Since a process $q$ performs $\alpha$ only after sending out an $\alpha$-message to all processes and, by assumption, communication is reliable, if $q$ performs $\alpha$, then other correct processes will receive the message, and thus also perform $\alpha$, even if $q$ crashes.\footnote{If $p$ has a strongly accurate failure detector rather than just a weakly accurate failure detector, it can actually stop sending messages after performing $\alpha$. This follows from the proof of Proposition 3.1.}

**Proposition 3.1:** There is a protocol that attains $\text{UDC}$ in every context with strong failure detectors, even if there is no bound on the number of failures.

**Proof:** The proof is similar in spirit to that of Proposition 2.3. Whenever a process wants to attain $\text{UDC}$ of action $\alpha$, it goes into a special $\text{UDC}(\alpha)$ state. If a process $p$ is in a $\text{UDC}(\alpha)$ state, it sends an $\alpha$-message repeatedly to all other processes (telling them to perform $\alpha$). Process $p$ performs $\alpha$ if it is in a $\text{UDC}(\alpha)$ state and if, for every process $q$, $p$ receives an acknowledgment from $q$ to its $\alpha$-message or $p$’s failure detector says or has said that $q$ is faulty. However, $p$ continues to send $\alpha$-messages (even after performing $\alpha$) to all processes from which it has not received an acknowledgment until it has received an acknowledgment from all processes (which may never happen).\footnote{If $p$ has a strongly accurate failure detector rather than just a weakly accurate failure detector, it can actually stop sending messages after performing $\alpha$. This follows from the proof of Proposition 3.1.} Every time a process $q$ receives an $\alpha$-message from $p$, $q$ sends an acknowledgment to $p$; it also goes into a $\text{UDC}(\alpha)$ state if it has not already done so.

To show that this protocol attains $\text{UDC}$, it suffices to show that, in every run, (1) if a process $p$ is in a $\text{UDC}(\alpha)$ state, then $p$ will eventually perform $\alpha$ or crash and (2) if $p$ performs $\alpha$ then every correct process performs $\alpha$. To see that (1) holds, suppose that $p$ is in a $\text{UDC}(\alpha)$ state in run $r$ and does not crash. Suppose, by way of contradiction, that $p$ does not perform $\alpha$ in run $r$. That means that there must be some process $q$ such that $p$’s failure detector never reports $q$ as faulty and $p$ does not receive an acknowledgment from $q$. Since $p$ has a strong fault failure detector, if $q$ is faulty, then at some point in $r$, $p$
must receive a report to this effect from its failure detector. Thus, \( q \) must be correct in \( r \). Since \( p \) repeatedly sends an \( \alpha \)-message to \( q \), by R5, \( q \) must receive the message infinitely often. That means it sends an acknowledgment back to \( p \) infinitely often. By R5 again, \( p \) must receive the acknowledgment, contradicting the assumption that it does not.

To see that (2) holds, first note that it holds vacuously if there are no processes correct in \( r \). If there is some process that is correct in \( r \), then since \( p \) has a weakly accurate failure detector, there is some correct process, say \( q^* \), that \( p \) never suspects. Thus, if \( p \) performs \( \alpha \), it must receive an acknowledgment from \( q^* \) to its \( \alpha \)-message. Hence, \( q^* \) goes into a UDC(\( \alpha \)) state and never crashes, so (1) implies that it also performs \( \alpha \). Since \( q^* \) is correct, all correct processes eventually receive an \( \alpha \)-message from \( q^* \) and so perform \( \alpha \).

**Proposition 3.4:** If \( R \) satisfies A1 and A5\(_{n-1}\), then \( R \) satisfies weak accuracy iff \( R \) satisfies strong accuracy.

**Proof:** Let \( R \) satisfy A1, A5\(_{n-1}\), and weak accuracy. If \( R \) does not satisfy strong accuracy, then there is a point \((r, m)\) and processes \( p, q \) such that \( q \in \text{Suspects}_p(r, m) \) and \( q \) has not failed in \( r \). Let \( S' = \text{Proc} - \{q\} \). By A5\(_{n-1}\), there is a run \( r' \) where all the processes in \( S' \) fail. Thus, by A1, there is a run \( r'' \) extending \((r, m)\) such that all the processes in \( S' \) fail in \( r'' \). It follows that \( q \) is the only correct process in \( r'' \). By weak accuracy, we must have that \( q \) is never suspected as faulty in \( r'' \), contradicting the assumption that it is in fact suspected by \( p \).

**Proposition 3.5:** If \( R \) satisfies A1, A2, and A4, then

\[
R \models \forall_{p, p' \in \text{Proc}} \forall_{\alpha \in A_{p'}} \left[ K_p (\text{init}_{p'}(\alpha) \land \forall_{q \in \text{Proc}} (K_q \text{init}_{p'}(\alpha) \lor \text{crash}(q))) \right.
\]

\[
\left. \Rightarrow K_p (\forall_{q \in \text{Proc}} \Box \neg \text{crash}(q) \Rightarrow \forall_{q \in \text{Proc}} (K_q \text{init}_{p'}(\alpha) \land \Box \neg \text{crash}(q))) \right].
\]

**Proof:** Suppose, by way of contradiction, that for some \( p, p' \in \text{Proc} \) and \( \alpha \in A_{p'} \), we have that

\[
(R, r, m) \models K_p (\text{init}_{p'}(\alpha) \land \forall_{q \in \text{Proc}} (K_q \text{init}_{p'}(\alpha) \lor \text{crash}(q))) \land
\neg K_p (\forall_{q \in \text{Proc}} \Box \neg \text{crash}(q) \Rightarrow \forall_{q \in \text{Proc}} (K_q \text{init}_{p'}(\alpha) \land \Box \neg \text{crash}(q))) \tag{1}
\]

Then there must be a point \((r^1, m') \sim_p (r, m)\) such that

\[
(R, r^1, m') \models \text{init}_{p'}(\alpha) \land \bigvee_{q \in \text{Proc}} \Box \neg \text{crash}(q) \land \bigwedge_{q \in \text{Proc}} (\Box \neg \text{crash}(q) \Rightarrow \neg K_q \text{init}_{p'}(\alpha)).
\]

We have \((R, r^1, m') \models K_q \neg \text{init}_{p'}(\alpha) \). Since \( p' \) knows that it initiated \( \alpha \) at \((r^1, m')\), we must have \( p' \in F(r^1) \). Moreover, \( F(r^1) \neq \text{Proc} \), because \((R, r^1, m') \models \forall_{q \in \text{Proc}} \Box \neg \text{crash}(q)\). 21
Let $S - Proc = F(r^1)$. By A4 with $\varphi = \text{init}_p(\alpha)$, there exists a point $(r^2, m')$ such that $(r^2, m') \sim_q (r^1, m')$ for $q \in S$ and $(\mathcal{R}, r^2, m') \models \neg \text{init}_p(\alpha)$. For all $q \in S$, we have that $(r^2, m') \sim_q (r^1, m')$. It follows that no process in $S$ has crashed by $(r^2, m')$. By A1, there exists a run $r^3$ extending $(r^2, m')$ such that $F(r^3) = F(r^1)$. Since $r^3$ extends $(r^2, m')$, we must have $r^3_q(m') = r^1_q(m')$ for all $q \in S$. By A2, there exist runs $r^4$ and $r^5$ extending $r^1$ and $r^3$, respectively, such that $r^4_q(m'') = r^5_q(m'')$ for $m'' \geq m'$. Moreover, all the processes in $F(r^1)$ (and, in particular, $p'$) crash by time $m' + 1$ in $r^4$ and $r^5$. Thus, the event $\text{init}_p(\alpha)$ does not appear in $r^5$, which means that $(\mathcal{R}, r^5, m') \models \wedge_{q \in S} \neg K_q \text{init}_p(\alpha)$. Since $r^4$ and $r^5$ are indistinguishable to such $q$ from $m'$ onward, we have $(\mathcal{R}, r^4, m') \models \wedge_{q \in S} \neg K_q \text{init}_p(\alpha)$. Since $(r, m) \sim_p (r^1, m')$ and $r^4$ extends $(r^1, m')$, we must have $(r, m) \sim_p (r^4, m')$. Hence, we have $(\mathcal{R}, r, m) \models \neg K_p(\Diamond (K_q \text{init}_p(\alpha) \vee \text{crash}(q)))$ for $q \in S$. This gives the desired contradiction to (1).

**Theorem 3.6:** Suppose $\mathcal{R}$ is the system generated by a protocol that attains UDC, $\mathcal{R}$ satisfies A1–A4 and $A_5 \rightarrow \neg \sim$, and for each run $r \in \mathcal{R}$, if $F(r) \neq \text{Proc}$, then infinitely many actions are initiated in $r$ (i.e., infinitely many events of the form $\text{init}_p(\alpha)$ appear in $r$). Then the system $\mathcal{R}^f$ has perfect failure detectors.

**Proof:** It is immediate from the construction that $p$ crashes in $(r, m)$ iff $p$ crashes in $(f(r), 2m)$. It easily follows that $p$’s failure detector satisfies strong accuracy. To show that it satisfies strong completeness, suppose that $p$ is correct and $q$ fails in run $f(r) \in \mathcal{R}^f$ and hence also in run $r \in \mathcal{R}$. Since infinitely many actions are initiated in $r$ (and hence $f(r)$), there must be some action $\alpha$ initiated by some correct process, say $p'$, in $f(r)$ after $q$ has failed. Since $\mathcal{R}$ satisfies UDC, by DC1 and DC2, $p$ must eventually perform $\alpha$ in run $r$, say at time $m$. Moreover, by DC2, $p$ knows that, for each process $q'$ (and, in particular, $q$), $q'$ must eventually either crash or must perform $\alpha$. Using DC3, it easily follows that we must have

\[(\mathcal{R}, r, m) \models K_p(\text{init}_{p'}(\alpha) \land \bigwedge_{q' \in \text{Proc}} \Diamond (K_{q'} \text{init}_{p'}(\alpha) \lor \text{crash}(q')))\]

Since $\mathcal{R}$ satisfies A1, A2, and A4 by assumption, it follows from Proposition 3.5 that

\[(\mathcal{R}, r, m) \models K_p\left(\bigvee_{q' \in \text{Proc}} \Box \neg \text{crash}(q') \Rightarrow \bigvee_{q' \in \text{Proc}} (K_{q'} \text{init}_{p'}(\alpha) \land \Box \neg \text{crash}(q'))\right). \tag{2}\]

By way of contradiction, suppose that $(\mathcal{R}, r, m) \models \Box \neg K_p \text{crash}(q)$. Since $q$ crashes in $r$ before $p'$ initiates $\alpha$, it is easy to show that $(\mathcal{R}, r, m) \models \neg K_q \text{init}_{p'}(\alpha)$. Thus, there must exist a point $(r', m') \sim_p (r, m)$ such that $(\mathcal{R}, r', m') \models \neg \text{crash}(q) \land \neg K_p K_q \text{init}_{p'}(\alpha)$. Since $K_q \text{init}_{p'}(\alpha)$ is stable, local to $q$, and (by A3) insensitive to failures by $q$, by A4, there must exist a point $(r^2, m') \sim_p (r', m')$ such that $r^2_q(m')$ is a prefix of $r^1_q(m')$ and $(\mathcal{R}, r^2, m') \models \neg K_q \text{init}_{p'}(\alpha)$. Since $r^2_q(m')$ is a prefix of $r^1_q(m')$, it is easy to show that $(\mathcal{R}, r^2, m') \models \neg \text{crash}(q)$. Thus, $(r^2, m') \sim_p (r, m)$ and $(\mathcal{R}, r^2, m') \models \neg \text{crash}(q) \land \neg K_q \text{init}_{p'}(\alpha)$.

22
By A5_{n-1} and A1, there is a run r^3 extending (r^2, m') such that all processes except q fail in r^3. Since (r^3, m') \sim (r, m), and
\[(r, r^3, m') \models \Box -\text{crash}(q) \wedge -\text{init}_p(\alpha) \wedge \bigwedge_{q' \in \text{Proc} - \{q\}} \Diamond \text{crash}(q').\]

Since (r, m) \sim (r^2, m') \sim (r^3, m'), this contradicts (2).

**Proposition 4.1:** There is a protocol that attains UDC in a context with a bound of t on the number of failures and with t-useful generalized failure detectors.

**Proof:** To attain UDC of action \( \alpha \), a process goes into a special UDC(\( \alpha \)) state. If a process \( p \) in a UDC(\( \alpha \)) state, it sends an \( \alpha \)-message repeatedly to all other processes from which it has not received an acknowledgment, telling them to perform \( \alpha \). Process \( p \) performs \( \alpha \) at time \( m \) if, by time \( m \), there is a set \( S \subseteq \text{Proc} \) and \( k \leq |S| \) such that (a) it is in a UDC(\( \alpha \)) state, (b) its failure detector has reported suspect_p(S, k), (c) it has received messages from all the processes in \( \text{Proc} - S \) acknowledging \( \alpha \), and (d) \( n - |S| > \min(t, n-1) - k \). Process \( p \) continues to send \( \alpha \)-messages to each \( q \in S \) until it either receives an acknowledgment from \( q \) or knows \( q \) to be faulty. (Note that knowledge is only necessary for the protocol’s termination.) A process that receives an \( \alpha \)-message from \( p \) sends an acknowledgment to \( p \) and goes into a UDC(\( \alpha \)) state if it has not already done so.

To show that this protocol attains UDC, again it suffices to show that, in every run, (1) if a process \( p \) is in a UDC(\( \alpha \)) state, then \( p \) will eventually perform \( \alpha \) or crash and (2) if \( p \) performs \( \alpha \) then every other correct process performs \( \alpha \). For (1), suppose that \( p \) is in a UDC(\( \alpha \)) state in run \( r \) and, by way of contradiction, \( p \) neither performs \( \alpha \) nor crashes. Then \( p \) repeatedly sends an \( \alpha \)-message in \( r \) to every process \( q \). By R5, every correct process \( q \) will get it infinitely often. Since \( q \) acknowledges \( p \)'s \( \alpha \)-message each time it gets it, by R5, \( p \) will eventually get an acknowledgment from every correct process. Since \( p \) has a \( t \)-useful failure detector, if it is correct in \( r \), there will be a \( t \)-useful failure-detector event, say suspect_p(S, k), in \( r_p(m) \) for some \( m \). Since \( p \) eventually gets acknowledgments from all the processes in \( \text{Proc} - S \) (since these, at least, are correct in \( r \)), it will eventually perform \( \alpha \), according to the algorithm. Thus, (1) holds.

To see that (2) holds, the arguments for (1) show that if \( p \) performs \( \alpha \) as a result of the failure-detector event suspect_p(S, k), all the processes in \( \text{Proc} - S \) have received an \( \alpha \) message (and hence are in a UDC(\( \alpha \)) state) and \( \text{Proc} - S \) contains at least one correct process, say \( q \), if there are any correct processes in \( r \). Since \( q \) continues to send \( \alpha \)-messages to all processes from which it has not received an acknowledgment, all the correct processes in \( r \) will eventually be in a UDC(\( \alpha \)) state. It then follows from (1) that all the correct processes will perform \( \alpha \).

**Theorem 4.3:** Suppose \( \mathcal{R} \) is the system generated by a protocol that attains UDC in a context with at most \( t \) failures, \( \mathcal{R} \) satisfies A1–A4 and A5_t, and for each run \( r \in \mathcal{R} \),
if \( F(r) \neq \text{Proc} \), then infinitely many actions are initiated in \( r \). Then \( \mathcal{R}' \) has \( t \)-useful generalized failure detectors.

**Proof:** Again, it is easy to see that each correct process \( p \)'s failure detector satisfies generalized strong accuracy. To show that it satisfies generalized impermanent strong completeness, suppose that \( p \) is correct. Since infinitely many actions are initiated in \( r \) (and hence also in \( f(r) \)), there must be some action \( \alpha \) initiated by a correct process \( p' \) in \( f(r) \) at a time after all the processes in \( F(r) = F(f(r)) \) have failed in \( f(r) \).

Since \( \mathcal{R} \) satisfies UDC, \( p \) must eventually perform \( \alpha \) in run \( r \), say at time \( m \). As in the proof of Theorem 3.6, using Proposition 3.5, we can conclude that

\[
(\mathcal{R}, r, m) \models K_p \left( \bigvee_{q' \in \text{Proc}} \Box \neg \text{crash}(q') \Rightarrow \bigvee_{q' \in \text{Proc}} \left( K_{q'} \text{init}_{q'}(\alpha) \wedge \Box \neg \text{crash}(q') \right) \right). \tag{3}
\]

Suppose, by way of contradiction, that \( p \) does not know that at least \( k = |F(r)| - n + t \) processes have crashed at \( (r, m) \). Then there must be a point \((r^1, m') \sim_p (r, m)\) such that \( k' < k \) processes have crashed by \((r^1, m')\). We must have \((\mathcal{R}, r, m) \models \land_{q \in F(r)} \neg K_q \text{init}_{q'}(\alpha)\), since all the processes in \( F(r) \) crashed in \( r \) before \( p' \) initiated \( \alpha \). Consequently, it follows that \((\mathcal{R}, r^1, m') \models \land_{q \in F(r)} \neg K_q \text{init}_{q'}(\alpha)\). By repeated applications of A4, there is a point \((r^2, m') \sim_p (r^1, m')\) such that \((\mathcal{R}, r^2, m') \models \land_{q \in F(r)} \neg K_q \text{init}_{q'}(\alpha)\). (We are using the fact that \( K_q \text{init}_{q'}(\alpha) \) is stable, local to \( q \), and insensitive to failures by \( q \). Thus, if \( \neg K_q \text{init}_{q'}(\alpha) \) holds for some history of \( q \), it holds for any prefix of that history or a prefix followed by a \( \text{crash}_q \) event.) Thus, the only processes that may know \( \text{init}_{q'}(\alpha) \) in \( r \) are those in \( \text{Proc} - F(r) \). Since \( |F(r)| = n - t + k \), we have that \( |\text{Proc} - F(r)| = t - k \). Thus, at most \( t - k \) processes know \( \text{init}_{q'}(\alpha) \) at the point \((r^2, m')\).

Let \( F_1 \) be the set of processes that have crashed by \((r^1, m')\) and let \( F_2 \) be the set of processes that have crashed by \((r^2, m')\). Since \( r^2_q(m') \) is a prefix of \( r^1_q(m') \) for all \( q \in \text{Proc} \), we must have \( F_2 \subseteq F_1 \). Recall that \( |F_1| = k' < k \). We now proceed much as in the proof of Proposition 3.5 to construct a run extending \((r^2, m')\) in which the processes in \( F(r) - F_1 \) do not crash and and do not learn about \( \text{init}_{q'}(\alpha) \).

By A4, there exists a point \((r^3, m')\) such that \((r^3, m') \sim_q (r^2, m')\) for \( q \in F(r) \) and \((\mathcal{R}, r^3, m') \models \neg \text{init}_{q'}(\alpha)\). As in the previous application of A4, the set of processes that are faulty at the point \((r^3, m')\) is a subset of \( F_1 \) and hence consists of at most \( k' \) processes. By A1 and A5, there exists a run \( r^4 \) extending \((r^3, m')\) such that \( F(r^4) = (\text{Proc} - F(r)) \cup F_1 \). Since \( r^4 \) extends \((r^3, m')\), we must have \( r^4_q(m') = r^2_q(m') \) for all \( q \in F(r) \). By A2, there exist runs \( r^5 \) and \( r^6 \) extending \( r^2 \) and \( r^4 \), respectively, such that \( r^5_q(m'') = r^6_q(m'') \) for \( m'' \geq m' \). Moreover, the processes in \( \text{Proc} - F(r) \cup F_1 \) crash by time \( m' + 1 \). Clearly \( p' \notin F(r) \) (since all the processes in \( F(r) \) crash before \( p' \) initiates \( \alpha \)). Thus, the event \( \text{init}_{q'}(\alpha) \) does not appear in \( r^6 \). It follows that \((\mathcal{R}, r^5, m'' = \land_{q \in F(r)} \neg K_q \text{init}_{q'}(\alpha) \text{ for all } m'' \geq m' \) and \( q \in F(r) \), so \((\mathcal{R}, r^5, m') \models \land_{q \in F(r)} \neg K_q \text{init}_{q'}(\alpha) \land \land_{q \in (\text{Proc} - F(r)) \cup F_1} \neg \text{crash}(q)\). Since \((r, m) \sim_p (r^1, m'), (r^1, m') \sim_p (r^2, m'), \) and \( r^1 \) extends \((r^2, m')\), we must have \((r, m) \sim_p (r^5, m')\). But this gives us the desired contradiction to (3).
Thus, \( p \) must know about at least \( k \) failures at the point \((r, m)\). Let \( S \) be any set containing \( F(r) \). Our transformation from \( \mathcal{R} \) to \( \mathcal{R}^f \) guarantees that eventually there will be a failure-detector event \((S, k)\) in \( p \)'s history, and this is a \( t \)-useful event.

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