Fermi regime.

The approximately Zeno regime does not in general mark the onset of the regime after the initial Zeno regime, the exit from the golden rule (FGR), however, is less readily understood. This quadratic decay can be derived by an expansion of the time-evolution operator as the Zeno decay \[21–23\]. This quadratic decay can be seen when the next (quartic) term in the same expansion.

The onset of the linear decay described by Fermi’s golden rule can take place when the next (quartic) term in the same expansion becomes comparable to the quadratic term \[24\]. The approximate t-quartic decay that follows the initial Zeno dynamics has indeed little in common with Fermi’s t-linear decay. The transition between the Zeno and Fermi regimes is clarified in the present work. We define the onset time \( t_F \) of the golden rule as the time such that when \( t \gg t_F \), the FGR is guaranteed to be valid. The golden rule is usually derived from time-dependent perturbation theory \[22, 26\], which assumes short times, but the derivation uses a mathematical property (a distributional limit) that only comes attened to be valid. The golden rule is usually derived from time-dependent perturbation theory \[22, 26\] (also see Supp. Mat.)

\[
\Gamma(t) = 2\pi \int_0^\infty d\omega \frac{F_i(\omega - \omega_0)}{R(\omega)}.
\]  

Here the reservoir coupling spectrum (RSC) \( R(\omega) \) is given by

\[
R(\omega) = \hbar^{-2} \sum_k |\langle e,0| \hat{H}_f |g,1_k \rangle|^2 \delta(\omega - \omega_k)
\]  

where the state of the reservoir \(|1_k\rangle\) contains one excitation in the mode labelled by \( k \) and \(|0\rangle\) is the vacuum state.

**Introduction.** Dirac’s result \[1\] describing the decay of excited quantum systems as linear in time is so solid and ubiquitous as to having been called the (second) golden rule by Fermi \[2, 3\]. The result is widely used throughout physics, including in nuclear \[4, 5\] and particle \[6, 7\] physics, atomic \[8, 9\], molecular \[10, 11\] and plasma \[12\] physics, biological \[13\] and solid-state \[14–16\] physics, as well as photonics \[17\]. It is valid at short to intermediate times, and naturally merges into the Weisskopf-Wigner exponential decay \[18\] at longer times. Deviations from the golden rule at very short and very long times are rather well understood. Arguments based on the Paley-Wiener theorem exclude the possibility of purely exponential decay at very long times \[19, 20\]. As far as very short times are concerned, the probability that the system survives in its initial excited state is known to decrease quadratically with time, a phenomenon known as the Zeno decay \[21–23\]. This quadratic decay can be derived by an expansion of the time-evolution operator up to the lowest nontrivial order in time. By the same token, the exit from the Zeno regime is understood to take place when the next (quartic) term in the same expansion becomes comparable to the quadratic term \[24\].

The onset of the linear decay described by Fermi’s golden rule (FGR), however, is less readily understood. We first note that, although the Fermi regime is, in ascending order of time scales, the second clearly identified regime after the initial Zeno regime, the exit from the Zeno regime does not in general mark the onset of the Fermi regime. The approximately \( t \)-quartic decay that follows the initial Zeno dynamics has indeed little in common with Fermi’s \( t \)-linear decay. The transition between the Zeno and Fermi regimes is clarified in the present work. We define the onset time \( t_F \) of the golden rule as the time such that when \( t \gg t_F \), the FGR is guaranteed to be valid. The golden rule is usually derived from time-dependent perturbation theory \[22, 26\], which assumes short times, but the derivation uses a mathematical property (a distributional limit) that only comes into play at sufficiently long times. There is no contradiction here as the characteristic times for perturbation theory and the distributional limit are different, but the question of what constitutes ‘sufficiently long times’ for Fermi’s linear regime to take over has seldom been examined in detail. We set out to bridge this gap in the present work. To do so, it will be necessary to distinguish between broadband reservoirs, which typically arise if the environment is open and has a continuous spectrum, and narrowband reservoirs, which typically arise if the environment is confined and has a discrete spectrum.

**General equations.** We consider a two-level system, consisting of a ground state \(|g\rangle\) and an excited state \(|e\rangle\) separated by the energy \( \hbar \omega_0 \), and initially prepared in \(|e\rangle\). Due to its coupling with the reservoir, the system will naturally decay to the ground state \(|g\rangle\), with a survival probability \( P_{\text{surv}}(t) = 1 - \Gamma(t)^t \). The generalised decay rate is given by first-order time-dependent perturbation theory \[22, 26\] (also see Supp. Mat.)

\[
\Gamma(t) = 2\pi \int_0^\infty d\omega \frac{F_i(\omega - \omega_0)}{R(\omega)}.
\]
state of the reservoir. \( \hat{H}_\text{I} \) is the system-reservoir interaction Hamiltonian. The function \( F_t(\omega - \omega_0) \), on the other hand, corresponds to the spectral profile of the two-level system at time \( t \):

\[
F_t(\omega - \omega_0) = \frac{t}{2\pi} \text{sinc}^2 \left( (\omega - \omega_0)t/2 \right).
\]  

(3)

From Eq. (1), the most straightforward derivation of the golden rule is as follows: it is seen that in the large time limit \( t \to +\infty \), \( F_t(\omega - \omega_0) \to \delta(\omega - \omega_0) \) and Eq. (1) gives: \( \Gamma(t) \to 2\pi R(\omega_0) \equiv \Gamma_0 \), which is the natural decay rate given by the golden rule. Note that only the reservoir states of frequency \( \omega_0 \) contribute to the decay in the Fermi regime. This is to be contrasted with the Zeno regime, which appears in the limit of very short times \( t \to 0 \), and for which \( \Gamma(t) \to A \times t \) with \( A = \int_0^{\infty} d\omega R(\omega) \). The Zeno regime can be understood as that in which all the reservoir frequencies featured in the RSC respond in phase to the excitation from the two-level system [27].

The use of the distributional limit in the golden rule begs the question of when the large time limit is reached. When this happens depends on the shape of the RSC function \( R(\omega) \), but this answer is unsatisfactorily vague. In quantum mechanics textbooks [25, 28–31] as well as in Ref. [3], the criterion \( t \gg 2\pi/\Delta\omega \) for the applicability of the golden rule is given, with \( \Delta\omega \) the width of the interval around the transition frequency \( \omega_0 \) in which \( R(\omega) \approx R(\omega_0) \). Specific statements from the literature that illustrate our presentation of the standard criterion are given in the Supp. Mat. As we shall see, this criterion is not valid in the general case.

In order to evaluate the integral (1) in the most general case, it is tempting to try to expand the RSC function around the transition resonance frequency \( \omega_0 \). However, this yields a sum of divergent terms: another approach is needed. We have found it convenient to distinguish between broadband and narrowband reservoirs. Broadband reservoirs feature all frequencies in their RSC (below a high-energy cutoff). They can be treated by adequately modifying the naive series-expansion approach mentioned just above: the RSC function is the product of a function that will be Taylor-expanded around \( \omega_0 \), with a ‘cutoff function’ that excludes very high frequencies from the dynamics. Narrowband reservoirs, on the other hand, only feature a small frequency window. We will treat them here as having a RSC of the Breit-Wigner type. Neglecting threshold effects [32], an approximation which we carefully justify, the dynamics can be solved for such reservoirs through Laplace-Fourier transform techniques.

Broadband reservoirs.— Let us start with the case of broadband reservoirs. They correspond in general to open environments with a continuous spectrum. Examples include the interaction between an atom and the electromagnetic modes in free space [33–35], or that of a two-level system interacting with a sub- or super-Ohmic dissipative bath, a model that describes many different systems [36, 37], and in particular, atoms interacting with a degenerate Fermi gas [38, 39]. In a sufficiently general setting, the RSC can be cast in the form

\[
R(\omega) = \lambda \omega \left( \frac{\omega}{\omega_X} \right)^{\eta-1} F_X(\omega)
\]

(4a)

where the dimensionless parameter \( \lambda \ll 1 \) measures the strength of the system-environment coupling and \( \eta \) is in general a real, nonnegative parameter. Also, \( F_X \) is the cutoff function, that is close to unity up to \( \omega \lesssim \omega_X \) and close to zero from \( \omega \gtrsim \omega_X \) onwards [40], with \( \omega_X \gg \omega_0 \) the cutoff frequency. We emphasise that the cutoff function is not introduced artificially: in the case of electronic transitions in atoms, for instance, it appears from a rigorous calculation of the coupling between electrons and photons beyond the electric dipole approximation [33, 41]. More general broadband reservoirs, for instance in the case of atomic transitions, can be written as a weighted sum of functions of the form given in Eq. (4a), with different powers \( \eta \) [33–35]. From the golden rule, the natural decay rate for a reservoir of the form (4a) reads

\[
\Gamma_0 \approx 2\pi \lambda \omega_0 \left( \frac{\omega_0}{\omega_X} \right)^{\eta-1}.
\]

(4b)

In order to determine the onset of Fermi’s linear decay regime for broadband reservoirs of the form (4a), we derive (see details in Supp. Mat.) analytical expressions of the generalised decay rate (1) by distinguishing three time regimes: (i) the cutoff regime \( \omega_X \ll t \), (ii) the intermediate regime \( \omega_X t \ll 1 \ll \omega_X \), and (iii) the resonant regime \( \omega_X \gg t \) [42]. This will allow for different approximations of the frequency integral in Eq. (1).

The case where \( \eta \) is a strictly positive integer in (4a) has been investigated in detail in Refs. [34, 35] in the resonant regime \( \omega_X t \gg 1 \), and we extend here the results to arbitrary positive values of \( \eta \). In order to do so, we first write \( \omega^n = \omega^{[n]} \omega^{n-[n]} \) in the integral \( \Gamma(t) \) [Eq. (1)], where \([n]\) denotes the integer part of \( n \) and expand \( \omega^{[n]} \) around the transition frequency \( \omega_0 \) at all orders. Moreover, for the calculations, we do not specify the form of \( F_X(\omega) \) in Eq. (4a) (this necessitates an approximate computation of the generalised decay rate (1)), and we will use the notation \( \int_0^{\infty} d\omega F_X(\omega) \equiv C\omega_X \) where \( C \) is typically of the order of unity. The decay rate then features two contributions: a resonant contribution, and a tail contribution.

The first, resonant contribution \( \Gamma^{\text{res}}(t) \) is due to the constant term in the binomial expansion of \( \omega^{[n]} \). For this term, only the part of \( F_t(\omega - \omega_0) \) that probes the reservoir \( R(\omega) \) in Eq. (1) in a frequency range of width \( 2\pi/t \) around \( \omega_0 \) contributes to the decay. We make the approximation [34] that \( F_t(\omega - \omega_0) = t/(2\pi) \) in the interval
\(-\pi/t < \omega - \omega_0 < \pi/t\) and vanishes elsewhere. As mentioned above, we need to distinguish three time regimes (see Supp. Mat.): (i) In the cutoff regime \(\omega_{xt} \ll 1\), (our approximation of) \(F_t\) probes the entire RSC, namely, the frequency range between 0 and \(\omega_{xt}\). This yields

\[
\Gamma_{\text{cut}}^\text{res} (t) \simeq \frac{1}{2\pi} \frac{C}{\eta - [\eta] + 1} \left( \frac{\omega_{xt}}{\omega_0} \right)^{\eta - [\eta] + 1} (\omega_{xt}) \Gamma_0. \tag{5a}
\]

(ii) In the intermediate regime \(\omega_{xt} \ll 1 \ll \omega_{xt}, \Gamma_t\) probes the frequency range between 0 and \(\omega_0 + \pi/t\), to give

\[
\Gamma_{\text{int}}^\text{res} (t) \simeq \frac{1}{2\pi} \frac{C}{\eta - [\eta] + 1} \left( 1 + \frac{\pi}{\omega_{xt}} \right)^{\eta - [\eta] + 1} (\omega_{xt}) \Gamma_0. \tag{5b}
\]

(iii) In the resonant regime \(\omega_{xt} \gg 1\), studied in detail in Refs. [34, 35] in the framework of emission under repeated measurements (the calculations are identical to those of the present free-dynamics case), \(F_t\) probes the frequency range between \(\omega_0 - \pi/t\) and \(\omega_0 + \pi/t\). The result in this regime reads

\[
\Gamma_{\text{res}}^\text{res} (t) \simeq \Gamma_0. \tag{5c}
\]

The second, tail contribution \(\Gamma_{\text{tail}}^\text{cut} (t)\), which only exists for reservoirs for which \(\eta \geq 1\), comes from all the other terms \(k \geq 1\) in the binomial expansion: this term includes a contribution from the off-resonant modes of the reservoir to the decay. This is a crucial point as off-resonant frequencies are often overlooked, and we will see that these modes delay the establishment of the FGR regime compared to the standard criterion \(1 \sim \Delta \omega t_F\). Again, for our calculations, we distinguish the same time regimes (see Supp. Mat.): (i) In the cutoff regime \(\omega_{xt} \ll 1\), the argument of the square cardinal sine in \(F_t\) (3) is much smaller than 1 for the whole frequency range of the reservoir, so that, Taylor-expanding the sine at the lowest order, we obtain

\[
\Gamma_{\text{cut}}^\text{tail} (t) \simeq \frac{1}{2\pi} \frac{C}{\eta + 1} \left( \frac{\omega_{xt}}{\omega_0} \right)^{\eta + 1} (\omega_{xt}) \Gamma_0, \quad \eta \geq 1. \tag{6a}
\]

(ii) In the intermediate regime \(\omega_{xt} \ll 1 \ll \omega_{xt}\), we make the same approximation as just above in the range \(0 \leq \omega \leq 2\omega_0\), and for \(2\omega_0 \leq \omega\), we approximate [28, 34] the square sine by its mean value 1/2. The contribution from the latter interval is by far dominant, and we get

\[
\Gamma_{\text{int}}^\text{tail} (t) \simeq \frac{1}{\pi} \frac{C}{\eta - 1} \left( \frac{\omega_{xt}}{\omega_0} \right)^{\eta - 1} \frac{1}{\omega_{xt}} \Gamma_0, \quad \eta > 1. \tag{6b}
\]

(iii) In the resonant regime \(\omega_{xt} \gg 1\), we approximate the square sine by its mean value 1/2 for all \(\omega\) and we obtain the same result as in the intermediate regime:

\[
\Gamma_{\text{res}}^\text{tail} (t) \simeq \frac{1}{\pi} \frac{C}{\eta - 1} \left( \frac{\omega_{xt}}{\omega_0} \right)^{\eta - 1} \frac{1}{\omega_{xt}} \Gamma_0, \quad \eta > 1. \tag{6c}
\]

It is immediately seen that the results (6b) and (6c) do not apply to the special case \(\eta = 1\). In this case, we have instead

\[
\Gamma_{\text{int}}^\text{tail} (t) \simeq \Gamma_{\text{res}}^\text{tail} (t) \simeq \frac{1}{\pi} C \log \left( \frac{\omega_{xt}}{\omega_0} \right) \frac{1}{\omega_{xt}} \Gamma_0. \tag{7}
\]

We show in the Supp. Mat. that the approximations made to obtain analytical expressions of the decay rate \(\Gamma (t) \simeq \Gamma_{\text{res}}^\text{res} (t) + \Gamma_{\text{tail}}^\text{cut} (t)\) are satisfactory for the three different time regimes, by comparing our results with the numerical computation of Eq. (1).

From these analytical expressions, we now aim at determining the onset time \(t_F\) of the golden rule. We first note from Eqs. (5a) and (6a) that in the cutoff regime \(\omega_{xt} \ll 1\), the decay rate \(\Gamma_{\text{cut}}^\text{res} (t) + \Gamma_{\text{cut}}^\text{tail} (t)\) is proportional to \(t\), yielding the quadratic decay characteristic of the Zeno regime: \(P_{\text{decay}} (t) \equiv 1 - P_{\text{surv}} (t) \propto t^2\). While our treatment does not give detailed information about when the dynamics exits this Zeno regime, it shows that in the intermediate regime \(\omega_{xt} \ll 1 \ll \omega_{xt}\), the Zeno dynamics no longer holds, as seen from Eqs. (5b) and (6b).

The regime in which the golden rule is established depends on the exponent \(\eta\), and we must distinguish two cases. For the reservoirs with \(\eta < 1\), where there are no tail contributions to the decay rate, we can see from Eqs. (5b) and (5c) that the golden rule is already established in the resonant regime \(\omega_{xt} \gg 1\), while it is not yet in the intermediate regime \(\omega_{xt} \ll 1 \ll \omega_{xt}\). Hence we conclude that the golden rule establishes between these two regimes, and we set the onset time \(t_F\) of the golden rule to be

\[
t_F = \frac{1}{\omega_0}, \quad \eta < 1. \tag{8a}
\]

For the reservoirs with \(\eta \geq 1\), where there are tail contributions to the decay rate, the golden rule is established
later, in the resonant regime $\omega_0 t \gg 1$, as seen from Eqs. (5c), (6c) and (7). Explicitly, we see from these equations that the time $t_F$ at which the Fermi regime takes over reads

$$t_F = \frac{1}{\pi} \frac{C}{\eta - 1} \left(\frac{\omega_0}{\omega_0}\right)^{\eta - 1} \frac{1}{\omega_0}, \quad \eta > 1,$$

$$t_F = \frac{1}{\pi} C \log \left(\frac{\omega_0}{\omega_0}\right) \frac{1}{\omega_0}, \quad \eta = 1$$

where $C/ (\eta - 1)$ and $C \log (\omega_0/\omega_0)$ are numerical factors broadly of the order of unity. The case $\eta = 1$ is somewhat specific mathematically, but it is very important physically as it describes atomic electronic transitions of the electric dipole type [33, 34], as well as the case of Ohmic dissipative reservoirs [37], the quantum equivalent of mechanical systems submitted to a friction force proportional to their velocity. Note that Eq. (8a), if extended to $\eta = 1$, agrees with the more specific result Eq. (8c), up to a numerical factor broadly of the order of unity.

There are thus two clearly different cases for the onset of the FGR: as $\eta$ is increased from 0 to 1, the onset of the golden rule always takes place around $t_F = 1/\omega_0$, but as soon as $\eta > 1$, the scaling changes: $t_F$ increases by the factor $(\omega_0/\omega_0)^{\eta - 1}$, which can be very large since in general $\omega > \omega_0$ [33, 41]. This can be visualised in Fig. 1 [43].

Our result is in contradiction with the standard criterion [3, 25, 28–31], which has apparently remained largely unchallenged in the literature. Indeed, as described above, the standard criterion indicates that $t_F \simeq 1/\Delta \omega$ with $\Delta \omega$ the width of the interval around $\omega_0$ in which $R(\omega) \simeq R(\omega_0)$. But since $R(\omega_0 \pm \delta \omega) \simeq R(\omega_0)$ $(1 \pm \eta \times \delta \omega/\omega_0)$ for broadband reservoirs, we can see that $\Delta \omega \simeq \omega_0/\eta$ and hence that if this criterion were true, then we would have $t_F \simeq \eta/\omega_0$, that is, $t_F \sim 1/\omega_0$ for all broadband reservoirs regardless of the exponent $\eta$, to be contrasted with our results [Eq. (8)]. Only for $\eta \leq 1$, does our treatment vindicates the standard criterion.

**Narrowband reservoirs.**— As an archetype of a narrowband reservoir, we consider the case of a Breit-Wigner (also known as Cauchy-Lorentz) reservoir, for which the RSC reads:

$$R(\omega) = \frac{\kappa}{\pi} \frac{g^2}{(\omega - \omega_e)^2 + \kappa^2}. \quad (9)$$

This type of reservoir is encountered when considering the coupling of a quantum emitter with a single resonance, that can be a cavity, a photonic or a plasmonic resonance [44–48]. The resonance is characterized by the frequency $\omega_e$ and width $\kappa$, and the coupling of the atom with this resonance by the coupling strength $g$.

In order to derive an analytical expression of the generalized decay rate $\Gamma(t)$, we make the approximation of extending the integral over the (nonexistent) negative frequencies of the reservoir spectrum in Eq. (1). The validity of this approximation is tested numerically in the Supp. Mat. and shown to work for any realistic plasmonic or photonic reservoir. Considering first the resonant case where the resonance frequencies of the system and the reservoir are matched ($\omega_0 = \omega_e$), we obtain (see Supp. Mat.):

$$\frac{\Gamma(t)}{\Gamma_0} = 1 - \frac{1 - e^{-\kappa t}}{\kappa t}$$

where $\Gamma_0 = 2g^2/\kappa$. From this expression, it is natural to define the time $t_F$ at which the Fermi regime takes over, as the characteristic time of the decaying exponential. We thus take $t_F \equiv 1/\kappa$, which is in agreement with the standard criterion [3, 25, 28–31]. Introducing the quality factor $Q \equiv \omega_e/(2\kappa)$ of the resonance, this criterion can be written as

$$t_F = 2 \frac{Q}{\omega_e},$$

which shows that $t_F$ is independent of the coupling strength $g$, and scales linearly with the quality factor $Q$. Therefore, from Eq. (11), we can conclude that for narrowband reservoirs and in the resonant case, losses (that is, a wide spectrum) will accelerate the onset of the Fermi regime. This is illustrated in Fig. 2, where we show how the decay rate $\Gamma(t)$ calculated from Eq. (10) converges to the golden rule value $\Gamma_0$, for different quality factors $Q$. For a typical plasmonic resonance, $Q \sim 10$ and for the optical frequency $\omega_e = 350$ THz (see e.g. [49]), $t_F = 5.71 \times 10^{-14}$ s, so the Fermi golden rule is established almost immediately. On the other hand, dielectric optical cavities, such as whispering gallery cavities, can have ultrahigh $Q$-factors with values over $10^8$ (see Ref. [50] and Refs. therein). For such a high $Q$-factor, and assuming the same central frequency, the onset time of the golden rule is a few tens of microseconds, which cannot be considered as immediate.

Note that for very short times, we get from Eq. (10): $\Gamma(t) \approx \kappa \Gamma_0/2$, which leads to a quadratic decay for the survival probability $P_{surv}(t)$ characteristic of the Zeno regime.

This criterion remains valid in the presence of detuning: $\Delta \equiv \omega_0 - \omega_e \neq 0$. Indeed, in this case, we obtain (see Supp. Mat.)

$$\frac{\Gamma(t)}{\Gamma_0} = 1 - V \left(\frac{1 - \cos (\Delta t) e^{-\kappa t}}{\kappa t} \right) - (1 - V) \operatorname{sinc} (\Delta t) e^{-\kappa t}$$

where $V \equiv [1 - (\Delta/\kappa)^2]/[1 + (\Delta/\kappa)^2]$ is equal to 1 in the resonant case [and in this case Eq. (12) reduces to Eq. (10)]. In Fig. 3, we show the ratio $\Gamma(t)/\Gamma_0$ calculated from Eq. (12) for different detunings $\Delta$, and we see that the time $t_F$ at which the Fermi regime is established does not change in the presence of detuning.
means that \( \Delta \) (dashed), \( \Delta = 2\kappa \), which gives the linear decay regime predicted by the standard criterion given in the literature. However, we notice (also see Ref. \[36\]) different behaviors around the transition frequency, the RSC can be expanded as

\[ R(\omega_0 \pm \delta\omega) \approx R(\omega_0) \left[ 1 \pm 2(\delta\omega/\kappa)^2 \right], \]

which means that \( \Delta\omega = \kappa/2 \) and hence, that the onset time of the linear decay regime predicted by the standard criterion \([3, 25, 28, 29]\) is confirmed by our result Eq. (11). However, we notice (also see Ref. \[36\]) different behaviors before the onset of the golden rule: as long as \( \Delta \leq \kappa \), the generalized decay rate is always smaller than \( \Gamma_0 \) (Zeno effect), whereas for \( \Delta > \kappa \), the decay rate is greatly enhanced during a short period of time (anti-Zeno effect).

**Discussion and conclusion.**— The time \( t_F \) at which Fermi’s golden rule starts being a good approximation to the decay dynamics of a two-level system was obtained. It was found that, in the case of broadband reservoirs with \( \eta \geq 1 \), \( t_F \propto (\omega_X/\omega_0)^{\eta-1} \times 1/\omega_0 \), in stark disagreement with the standard criterion given in the literature \([3, 25, 28, 29]\), which gives \( t_F \sim 1/\omega_0 \) while for \( \eta \leq 1 \), \( t_F = 1/\omega_0 \), in agreement with the standard criterion (for \( \eta = 1 \) we can use either result, as they agree up to numerical prefactors of the order of unity).

In the case of narrowband reservoirs (here modeled by a Breit-Wigner coupling spectrum), we have obtained the universal result \( t_F = 2Q/\omega_c = 1/\kappa \), where \( Q, \omega_c \) and \( \kappa \) are respectively the quality factor, central frequency and width of the reservoir. As can be seen, a high quality factor causes a late onset of the golden rule regime. Our analysis thus shows that studying the behaviour of the reservoir coupling spectrum around the transition frequency is not sufficient in the general case to determine the onset time of Fermi’s golden rule regime, and emphasises the importance \([27]\) of the oft-overlooked off-resonant frequencies of the reservoir. Only for the slowly-growing broadband reservoirs \( \eta \leq 1 \), and for narrowband reservoirs, for which off-resonant frequencies are a less dominant feature of the reservoir coupling spectrum, did we confirm the standard analysis of the onset of the Fermi regime.

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Note also from Fig. 1 that for $\eta \ll 1$, the generalised decay rate verifies $\Gamma(t) \approx \Gamma_0$ long before $t \sim 1/\omega_0$ but however, $\Gamma(t)$ is not constant even at the lowest order of approximation in this regime, which precludes from including it in the Fermi regime.

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