Research on Electromagnetic Scattering Characteristics of Combined Conducting and Dielectric Target Above Coastal Environment

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ABSTRACT The electromagnetic (EM) scattering characteristics of target above coastal environment are complicated due to the simultaneous existence of ground region and sea region. Nevertheless, the actual target possesses both conducting part and dielectric part. Such a composite scattering problem involves multiple media parameters and large amount of unknowns. Hence, aiming at accurately and efficiently calculating the EM scattering from combined conducting and dielectric target above coastal environment, the hybrid SIE-KA method is introduced in which single integral equation (SIE) method is utilized to reduce the number of combined target’s unknowns and Kirchhoff approximation (KA) is applied to obtain induced electric and magnetic current on the surface of coastal environment. Moreover, multilevel fast multipole algorithm (MLFMA) is adopted to significantly reduce the computational cost and memory requirement. Several numerical examples demonstrate the accuracy and efficiency of the hybrid method. Finally, the EM scattering characteristics of combined conducting and dielectric target above coastal environment have been studied. The radar cross section (RCS) curves with different category of environment, polarization mode, working frequency, statistic parameters of coastal environment, permittivity of target’s dielectric part, target’s height and target category have been analyzed in detail to provide some useful conclusions. The approach and results can be broadly applied in remote sensing simulation of coastal environment with combined target.

INDEX TERMS Coastal environment, electromagnetic scattering, Kirchhoff approximation, single integral equation, multilevel fast multipole algorithm, combined conducting and dielectric target.

I. INTRODUCTION Recent years have seen the rapid development of metamaterials, plasmonics, metasurface, graphene, AI-based algorithms and high-frequency asymptotic method [10], [11], numerical method [12], [13] and high-frequency asymptotic method [14]. Nevertheless, there are certain disadvantages associated with the application of analytical method: firstly, it is only suitable for handling the symmetrical objects with simple boundary condition, e.g., sphere, cylinder and plane; secondly, it cannot handle several complex media situations, e.g., objects coated with inhomogeneous material, combined conducting and dielectric target or divisional circumstances. On the other hand, numerical methods are widely utilized to analyze the EM scattering characteristics of target and environments. Among them, method of moment (MOM) [15] becomes the representative method due to its great universality and accuracy. Nevertheless, the main drawback of purely MOM is that the high computational complexity and huge memory requirement are unavoidable when handling the scattering...
problems with larger unknowns. Many relevant optimization methods have been proposed which mainly focus on two key points: one is to accelerating process of solving matrix equation by manipulating impedance matrix, e.g., multilevel fast multipole algorithm (MLFMA) [16], banded matrix iterative approach/canonical grid method (BMIA/CAG) [17], multilevel sparse matrix canonical grid method (MSMCG) [18], extended propagation-inside-layer expansion (EPILE) [19], or simplifying matrix structure, e.g., integral equation domain decomposition method (IE-DDM) [20], electric/magnetic current combined-field integral equation and domain decomposition method (JMCFIE-DDM) [21]; the other is to reduce the number of unknowns like by introducing the single integral equation (SIE) [22], which remarkably cut off half of unknowns compared with that using the coupled integral equation. Besides these strategies above, some methods depending on computer technology [23], [24] as parallel computing with CPU or GPU have become practicable way to improve the computation efficiency of numerical method. Previous works focused on the analysis of EM scattering from pure conductor or pure dielectric target. The scattering problems of composite target have attracted many researchers’ interests due to its practical significance. Fuscaldo et al. [25] furnished a convenient analytical framework based on KA for analysis of EM scattering from composite target. Xu et al. [26] utilized a synthetic basis functions method (SBFM) to analyze scattering properties of nonperiodic composite metallic and dielectric structures. Cheng et al. [27] proposed an time-domain fast dipole method to accelerate the time-domain volume-surface integral equation for calculating the transient EM scattering from composite conducting-dielectric target. Li et al. [28] proposed a nonconformal and nonoverlapping DDM based on hybrid volume and surface integral equation (VSIE) to compute the scattering from composite target. However, these works mainly solve the scattering problems of single composite target. When actual environment is taken into consideration with the target, the surface subdivision of rough surface generates a great mass of unknowns, which would lead to unbearable memory requirement. Therefore, aiming at overcoming this problem, the high-frequency asymptotic method is introduced to compute the scattering from the environment part efficiently. The Kirchhoff approximation (KA) [12] is the representative method, which is utilized to obtain the induced electric and magnetic current of rough surface. Some researchers have solved the scattering problem of target with large-scale environment efficiently by hybrid MOM-KA method [29]–[31]. So far, previous scholars have done large quantities of works to explore the EM scattering characteristics of target and environment. Deficiently, actual targets and circumstances are characterized with complex media, which are always simplified or ignored in the research process. Firstly, in terms of target, there are both conducting part and dielectric part existing simultaneously, e.g., stealth aircraft with metallic body and thin dielectric coating, conducting antenna with dielectric radomes, and metal cruise missile equipped with dielectric warhead. Then, in terms of environment, there are diverse regions with different media properties coexisting, e.g., the coastal environment, water zone embedded in land surface, and blocked cultivated land. If such complex conditions are all taken into consideration, the EM scattering problem of target and environment will be more complicated. Previous works mainly focused on the environment and pure sea surface with target. In this paper, the coastal composite rough surface is considered with the combined target, where the composite scattering model is closer to the actual detection situation in coastal environment. The dual-complexity of both combined target and coastal environment makes the EM scattering characteristic be controlled by plenty of factors. Previous works about divisional environment modeling in our research group have been proposed, e.g., the modeling method for the water zone embedded in land surface [32], ground and near sea in adjacent region [33] and island in deep sea [34]. A borderline modeling method of fractal boundary for divisional environment has been proposed in [35]. The core concept of modeling method for divisional rough surface can be divided into three parts: first, adopting Gaussian spectral function to generate the rough surface for ground region; secondly, selecting suitable sea spectrum to simulate the rough surface for sea region; finally, according to the concrete shape of coastline, choosing suitable weighted inverse tangent function to construct the borderline between two regions. A major advantage of this modeling method is that it can guarantee the statistic property of each region remain invariant and connect two areas smoothly. In [34], Pierson-Moskowitz (PM) sea spectrum was chose for build up the surface of deep sea, but in [33], considering the effect of finite water depth in near sea region, JONSWAP spectral function combined with shallow coefficient was utilized to generate the near sea surface. However, there are certain drawbacks associated with the use of this modeling method. Firstly, this is an approximated theoretical model which lacks the validation of practical experiments due to limited experimental condition of our research group. Secondly, it is just a surface model without considering layered structure. Nevertheless, it provides an attempt to simulate the coastal environments for electromagnetic computation. In this present work, we quote the geometrical model [33] to describe the surface properties of coastal environment. Based on the geometrical model of coastal environment, the combined target is introduced to form the composite scattering model. In [36], we have proposed the methodology to analyze the combined target above single environment. However, the hybrid SIE-KA-FMM method proposed in [36] could just be utilized to solve the combined target with PEC surface. In this paper, coastal environment has diverse media properties, which leads to the coexistence of both induced electric current and induced magnetic current on the facet of coastal composite rough surface. Hence, we propose the new version of hybrid SIE-KA method for combined target above coastal environment, where MLFMA is utilized to...
further reduce memory requirement and computation consume. What’s more, the combined target with this kind of divisional environment will lead to more complicated scattering mechanism. It is of practical significance to research such scattering problems. The unique phenomena and conclusions found in this paper will provide a good data support for the field of radar detection and remote sensing.

The rest of paper is organized as following: In Section II, we review the notations and formulations of the SIE-KA and provide the new version adjusting to the coastal environment. We then discuss the acceleration of SIE-KA using MLFMA. Both target and environment are disserted by triangulation. With regard to coastal environment surface generation, the modeling method for the divisional rough surface is introduced referring to Ref. [33]. The combined target includes perfect electric conductor (PEC) part and dielectric part, where \( S_1 \) and \( S_2 \) present their surface respectively. \( S_3 \) presents contacting surface between PEC part and dielectric part. \( S_4 \) presents the coastal environment. \( N_1 \) is the total number of edges in \( S_1 \) which includes the common edges of contacting boundary between \( S_1 \) and \( S_2 \), and \( N_2 \) is the total number of edges in \( S_2 \) which doesn’t include the common edges. \( N_3 \) and \( N_4 \) present the total number of edges in \( S_3 \) and \( S_4 \) respectively.

In this paper, a tapered incident beam is introduced in order to circumvent artificial truncation effects. We utilize the 3-D form of tapered incident beam, and the concrete forms are given in the Equation (8) in Ref. [32]. In Figure 1, \( \theta \) and \( \varphi \) denote pitching angle and azimuth angle, respectively. As described above, when the composite scattering model is illuminated by incident EM wave, there is equivalent electric current or equivalent magnetic current appearing along the surface. On the surface \( S_1 \) of PEC part, only equivalent electric current will be stimulated, which is denoted by \( \mathbf{J}_1 \). Moreover, both equivalent electric current and magnetic current will be stimulated along the dielectric surface, which are denoted by \( \mathbf{J}_2, \mathbf{M}_2 \) on \( S_2 \) and \( \mathbf{J}_{KA}, \mathbf{M}_{KA} \) on \( S_4 \), respectively. As we already known, the equivalent electric current or equivalent magnetic current can be expanded by the RWG basis function [31], thus, the above equivalent currents can be expressed as

\[
P\text{EC region } S_1: \eta_0 \mathbf{J}_1 = \sum_{j=1}^{N_1} a_j \mathbf{f}_{1j} \tag{1}
\]

\[
\text{Dielectric region } S_2: \eta_0 \mathbf{J}_2 = \sum_{j=1}^{N_2} b_j \mathbf{f}_{2j} \tag{2}
\]

\[
\text{Divisional Environment } S_4: \eta_0 \mathbf{J}_{KA} = \sum_{j=1}^{N_4} d_j \mathbf{f}_{4j} \text{ and } M_{KA} = \sum_{j=1}^{N_4} e_j \mathbf{f}_{4j} \tag{3}
\]

where \( \eta_0 \) is wave impedance in free space, which can be written as \( \eta_0 = \sqrt{\mu_0/\varepsilon_0} \). \( a_j, b_j, c_j, d_j \) and \( e_j \) are expansion coefficients. In Ref. [25], the inner surface of dielectric part of combined target is defined as the surface \( S_4 = S_2 + S_3 \), and only effective electric current \( \mathbf{J}_{2e}^{\text{eff}} \) will be stimulated on the inner surface \( S_4 \), which can be expanded by the RWG basis function as follow

\[
-\eta_0 \mathbf{J}_{2e}^{\text{eff}} = \sum_{j=1}^{N_2} g_j \mathbf{f}_{2j} + \sum_{j=1}^{N_3} h_j \mathbf{f}_{3j} \tag{4}
\]

where \( g_j \) and \( h_j \) are expansion coefficients. From the boundary condition of dielectric rough surface, we obtain the following equations that are \( \mathbf{J}_{KA} = \mathbf{n}_4 \times \mathbf{H}_{\text{sur}} \) and \( \mathbf{M}_{KA} = -\mathbf{n}_4 \times \mathbf{E}_{\text{sur}} \), where \( \mathbf{n}_4 \) is the unit normal vector of arbitrary point in coastal environment. \( \mathbf{E}_{\text{sur}} \) and \( \mathbf{H}_{\text{sur}} \) denotes total electric field and total magnetic field of coastal environment, respectively. Total electric field \( \mathbf{E}_{\text{sur}} \) only depends on incident electric field \( \mathbf{E}_i \) and scattering electric field \( \mathbf{E}_{\text{sur}}^{\text{sc}} \) when no target exists above coastal environment. Correspondingly, total magnetic field \( \mathbf{H}_{\text{sur}} \) depends on incident magnetic field \( \mathbf{H}_i \) and scattering magnetic field \( \mathbf{H}_{\text{sur}}^{\text{sc}} \) from pure coastal

![FIGURE 1. Geometry configuration for the EM scattering from a combined conducting and dielectric target above coastal environment.](image-url)
environment. When target is reckoned in, the existence of it will become the secondary incidence resource to illuminate the divisional rough surface. Then, the total electric field $E_{\text{sur}}$ and magnetic field $H_{\text{sur}}$ can be expressed as

$$E_{\text{sur}} = E^{i} + E^{s}_{\text{sur}} + E^{s}_{\text{sur}}$$  \hspace{1cm} (5)

$$H_{\text{sur}} = H^{i} + H^{s}_{\text{sur}} + H^{s}_{\text{sur}}$$  \hspace{1cm} (6)

where $E^{s}_{\text{sur}}$ and $H^{s}_{\text{sur}}$ present incident electric field and incident magnetic field from the combined target respectively, whose expressions can be written as

$$E^{s}_{\text{sur}} = -\eta_{0}L_{01}(J_{1}) - \eta_{0}L_{02}(J_{2}) + K_{02}(M_{2})$$  \hspace{1cm} (7)

$$H^{s}_{\text{sur}} = -K_{01}(J_{1}) - (1/\eta_{0})L_{02}(M_{2}) - K_{02}(J_{2})$$  \hspace{1cm} (8)

Here, we identify the electric field integral operator (EFIO) and magnetic field integral operator (MFIO) as $L_{01}(\cdot)$ and $K_{02}(\cdot)$ respectively, whose expressions are given in the Equation (7) in Ref. [36]. When the relationship between incident wavelength $\lambda$ and curvature radius $\rho$ of arbitrary point in divisional rough surface meets $\rho \gg \lambda$, the Kirchhoff approximation can be applied. The scattering electric field and magnetic field of divisional rough surface can be expressed as

$$E^{s}_{\text{sur}} = \Gamma_{\parallel}E^{i}_{\parallel}e_{\parallel} + \Gamma_{\perp}E^{i}_{\perp}\hat{e}_{\perp} + \Gamma_{\parallel}E^{s}_{\text{sur}}\hat{e}_{\parallel} + \Gamma_{\perp}E^{s}_{\text{sur}}\hat{e}_{\perp}$$  \hspace{1cm} (9)

$$H^{s}_{\text{sur}} = \frac{1}{\eta_{0}}\hat{k}_{t} \times \left( \Gamma_{\parallel}E^{i}_{\parallel}\hat{e}_{\parallel} + \Gamma_{\parallel}E^{s}_{\text{sur}}\hat{e}_{\parallel} + \Gamma_{\perp}E^{i}_{\perp}\hat{e}_{\perp} + \Gamma_{\perp}E^{s}_{\text{sur}}\hat{e}_{\perp} \right) + \Gamma_{\parallel}E^{s}_{\text{sur}}\hat{e}_{\parallel}$$  \hspace{1cm} (10)

where $\hat{k}_{t} = \hat{k}_{t} - 2\hat{n}(\hat{n} \cdot \hat{k}_{t})$ presents the local reflection direction and $\hat{k}_{t}$ presents the local incident direction; Set the local orthogonal system is $(\hat{e}_{\parallel}, \hat{e}_{\perp}, \hat{k}_{t})$; $\hat{e}_{\parallel} = (\hat{n} \times \hat{k}_{t}) / |\hat{n} \times \hat{k}_{t}|$ and $\hat{e}_{\perp} = \hat{e}_{\perp} \times \hat{k}_{t}$ denote the vector of horizontal polarization and vertical polarization respectively; $\Gamma_{\parallel}$ and $\Gamma_{\perp}$ denote horizontal and vertical polarization component of Fresnel reflection coefficient, whose expressions are given in Equation (9) in Ref. [36], where $\epsilon_{r}$ and $\mu_{r}$ denote relative permittivity and relative permeability of local surface; $\theta' = \arccos(-\hat{n} \cdot \hat{k}_{t})$ denotes the local incidence angle. The incident magnetic field can be written as

$$H^{i} = (1/\eta_{0})\hat{k}_{t} \times (E^{i}_{\parallel}\hat{e}_{\parallel} + E_{\text{sur}}^{i}\hat{e}_{\parallel} + E^{s}_{\text{sur}}\hat{e}_{\parallel} + E^{s}_{\text{sur}}\hat{e}_{\perp})$$  \hspace{1cm} (11)

Then, the total electric field $E_{\text{sur}}$ and magnetic field $H_{\text{sur}}$ of divisional rough surface can be obtained with (9), (10) and (11), which can be rewritten as

$$E_{\text{sur}} = E^{i}_{\parallel}\hat{e}_{\parallel} + E^{i}_{\perp}\hat{e}_{\perp} + E^{s}_{\text{sur}}\hat{e}_{\parallel} + E^{s}_{\text{sur}}\hat{e}_{\perp} + \Gamma_{\parallel}E^{i}_{\parallel}\hat{e}_{\parallel} + \Gamma_{\parallel}E^{s}_{\text{sur}}\hat{e}_{\parallel} + \Gamma_{\perp}E^{i}_{\perp}\hat{e}_{\perp} + \Gamma_{\perp}E^{s}_{\text{sur}}\hat{e}_{\perp}$$  \hspace{1cm} (12)

$$H_{\text{sur}} = (1/\eta_{0})\hat{k}_{t} \times (E^{i}_{\parallel}\hat{e}_{\parallel} + E^{i}_{\perp}\hat{e}_{\perp} + E^{s}_{\text{sur}}\hat{e}_{\parallel} + E^{s}_{\text{sur}}\hat{e}_{\perp} + \Gamma_{\parallel}E^{i}_{\parallel}\hat{e}_{\parallel} + \Gamma_{\parallel}E^{s}_{\text{sur}}\hat{e}_{\parallel} + \Gamma_{\perp}E^{i}_{\perp}\hat{e}_{\perp} + \Gamma_{\perp}E^{s}_{\text{sur}}\hat{e}_{\perp}$$  \hspace{1cm} (13)

From the boundary condition of dielectric rough surface, $J_{KA}$ and $M_{KA}$ along the divisional rough surface can be written as

$$J_{KA} = (1/\eta_{0})\hat{n}_{4} \times \left( \frac{k_{t}}{\eta_{0}} \times \left[ (E^{i} + E^{s}_{\text{sur}}) \cdot \hat{e}_{||} + (E^{i} + E^{s}_{\text{sur}}) \cdot \hat{e}_{\perp} \right] \right)$$
$$+ \Gamma_{\parallel}(E^{i} + E^{s}_{\text{sur}}) \cdot \hat{e}_{||} + \Gamma_{\perp}(E^{i} + E^{s}_{\text{sur}}) \cdot \hat{e}_{\perp}$$  \hspace{1cm} (14)

$$M_{KA} = -\hat{n}_{4} \times \left( (1 + \Gamma_{\parallel}) \left( E^{i} + E^{s}_{\text{sur}} \right) \cdot \hat{e}_{||} \right)$$
$$+ \Gamma_{\parallel}(E^{i} + E^{s}_{\text{sur}}) \cdot \hat{e}_{||}$$  \hspace{1cm} (15)

where $\hat{e}_{||} = \hat{e}_{||}$ and $\hat{e}_{\perp} = \hat{e}_{\perp}$ present the dyadic of horizontal and vertical polarization. Substitute (1), (2), (3) and (7), (8) into (14) and (15), then both sides of (14) and (15) are tested by $\hat{n}_{4} \times f_{4i}$, the expressions of (14) and (15) can be written in vector matrix form as follow

$$\{d\} = [D_{41}] \cdot \{a\} + [D_{42}] \cdot \{b\} + [F_{42}] \cdot \{c\} + [W]$$  \hspace{1cm} (16)

$$\{e\} = [E_{41}] \cdot \{a\} + [E_{42}] \cdot \{b\} + [Y_{42}] \cdot \{c\} + [V]$$  \hspace{1cm} (17)

where expressions of matrix elements in (16) can be found in Equation (14) in Ref. [36], and expressions of matrix elements in (17) are taken in these forms

$$E_{4i}(q, j) = \int_{T_{41}} f_{4i}(r) \cdot \left[ (1 + \Gamma_{\parallel})L_{04}(f_{qj}) \cdot \hat{e}_{||} + (1 + \Gamma_{\perp}) \cdot \hat{e}_{\perp} \right] dS$$
$$L_{04}(f_{qj}) \cdot \hat{e}_{\perp} \right] dS, \hspace{1cm} i \in [1, N_{4}], j \in [1, N_{q}], \hspace{1cm} q = 1, 2.$$  \hspace{1cm} (18)

$$Y_{42}(i, j) = -\int_{T_{42}} f_{4i}(r) \cdot \left[ (1 + \Gamma_{\parallel})K_{02}(f_{2j}) \cdot \hat{e}_{||} + (1 + \Gamma_{\perp}) \cdot \hat{e}_{\perp} \right] dS, \hspace{1cm} i \in [1, N_{4}], j \in [1, N_{2}]$$  \hspace{1cm} (19)

$$E_{4i}(r) \cdot \hat{e}_{\perp} \right] dS, \hspace{1cm} i \in [1, N_{4}]$$  \hspace{1cm} (20)

The electric field integral equation (EFIE) in $S_{1}$ of combined target can be written as

$$\hat{n}_{1} \times E^{i}_{1} = -\hat{n}_{1} \times (E^{i} + E^{s}_{\text{sur}-1}), \hspace{1cm} r \in S_{1}$$  \hspace{1cm} (21)

Correspondingly, the EFIE and magnetic field integral equation (MFIE) in $S_{2}$ of combined target can be written as

$$\hat{n}_{2} \times E^{s}_{2} = -\hat{n}_{2} \times (E^{i} + E^{s}_{\text{sur}-1}), \hspace{1cm} r \in S_{2}$$  \hspace{1cm} (22)

$$\hat{n}_{2} \times H^{s}_{2} = -\hat{n}_{2} \times (H^{i} + H^{s}_{\text{sur}-1}), \hspace{1cm} r \in S_{2}$$  \hspace{1cm} (23)

where $E^{i}$ and $H^{i}$ present the scattering electric and magnetic field of combined target, whose expressions are the same as (7) and (8); when incident wave illuminates the surface of coastal environment, the existence of surface will become the secondary incidence resource to illuminate the
combined target. $E^s_{\text{surf}}$ and $H^s_{\text{surf}}$ present incident electric field and incident magnetic field from the divisional rough surface respectively, whose expressions can be written as

$$E^s_{\text{surf}} = -\eta_0 L_{04} (M_{KA}) + K_{04} (M_{KA})$$

$$H^s_{\text{surf}} = - (1/\eta_0) L_{04} (M_{KA}) - K_{04} (J_{KA})$$

By substituting (7) and (24) into (22) and (23), we have

$$\tilde{n}_1 \times E^i = \tilde{n}_1 \times \left[ L_{01} (\eta_0 J_1) + L_{02} (\eta_0 J_2) - \tilde{K}_{02} (M_2) \right]$$

$$+ L_{04} (\eta_0 J_{KA}) - \tilde{K}_{04} (M_{KA})$$

$$\tilde{n}_2 \times E^i = \tilde{n}_2 \times \left[ L_{01} (\eta_0 J_1) + L_{02} (\eta_0 J_2) - (1/2) M_2 - \tilde{K}_{02} (M_2) \right]$$

$$+ L_{04} (\eta_0 J_{KA}) - (1/2) M_{KA} - \tilde{K}_{04} (M_{KA})$$

where $\tilde{K}_{02} (\cdot)$ and $\tilde{K}_{04} (\cdot)$ present the principal value integral of $K_{02} (\cdot)$ and $K_{04} (\cdot)$ respectively. Substitute (1), (2), (3) into (26) and (27), both sides of (26) are tested by $\tilde{n}_1 \times f_1$, both sides of (27) are tested by $\tilde{n}_2 \times f_2$, the expressions of (26) and (27) can be written in vector matrix form as follow

$[Q_{11}] \cdot \{a\} + [Q_{12}] \cdot \{b\} + [P_{12}] \cdot \{c\} + [R_{14}] \cdot \{d\}$

$+ [T_{14}] \cdot \{e\} = \{S_1\}$

$[Q_{21}] \cdot \{a\} + [Q_{22}] \cdot \{b\} + [P_{22}] \cdot \{c\} + [R_{24}] \cdot \{d\}$

$+ [T_{24}] \cdot \{e\} = \{S_2\}$

where expression of matrix element $Q_{11}, Q_{12}, Q_{21}, Q_{22}, P_{12}, P_{22}, R_{14}$ and $R_{24}$ can be found in Equation (20) in Ref. [36]. The matrix element $T_{14}$ and $T_{24}$ are taken in these forms

$T_{pi4}(i,j) = - \int_{T_{pi}} f_{pi}(r) \cdot \tilde{K}_{04}(f_{p4})dS$, $i \in [1, N_p], j \in [1, N_4], p = 1, 2$. (30)

When we substitute (16) and (17) into (28) and (29), only three groups of unknowns remain to be solved: $\{a\}, \{b\}$ and $\{c\}$. For solving $\{a\}, \{b\}$ and $\{c\}$, the fields within the dielectric part of combined target must be taken into consideration. In usual coupled integral equation method the electrical field $E_d$ and magnetic field $H_d$ in the interior region ($S_d = S_2 + S_3$) of combined target’s dielectric part could be calculated by the pair of equivalent currents ($J_2, M_2$), but with the opposite sign by

$$E_d = -\eta_1 L_{1d} (-J_2) + K_{1d} (-M_2)$$

$$H_d = -(1/\eta_1) L_{1d} (-M_2) - K_{1d} (-J_2)$$

Then, utilizing the SIE method, the electric field $E_d$ and magnetic field $H_d$ generated by the effective electric current $J_{2e}^{\text{eff}}$ [22] along the inner surface $S_d$, whose expressions can be written as

$$E_d = -\eta_1 L_{1d} (J_{2e}^{\text{eff}})$$

$$H_d = -K_{1d} (J_{2e}^{\text{eff}})$$

where $\eta_1$ is wave impedance, which can be written as $\eta_1 = \sqrt{\mu_1/\varepsilon_1}$; The operator $L_{1d}(\cdot)$ and $K_{1d}(\cdot)$ are the same as the Equation (7) in Ref. [36] by adjusting the corresponding medium parameters in them, and the operation of integral is carried out over the inner surface $S_d$. The fields in the inner surface $S_d$ satisfy Maxwell’s equations, and the tangential fields are continuous across the surface of dielectric part. From the boundary conditions, we have

$$J_2 = \tilde{n}_2 \times H_d = -(1/2) J_{2e}^{\text{eff}} - \tilde{n}_2 \times \tilde{K}_{1d} (J_{2e}^{\text{eff}})$$

$$M_2 = \tilde{n}_2 \times E_d = -\eta_1 L_{1d} (J_{2e}^{\text{eff}})$$

where $\tilde{K}_{1d}(\cdot)$ presents the principal value integral of $K_{1d}(\cdot)$. The tangential component of electric field on the contacting surface $S_3$ disappears, thus, we have

$$0 = \tilde{n}_3 \times E_d = -\tilde{n}_3 \times \eta_1 L_{1d} (J_{2e}^{\text{eff}})$$

Using RWG basis function, the expansion form of $J_{2e}^{\text{eff}}$ is given in (4), and the expansion coefficient $g_i$ and $h_j$ can be obtained by average value of edge electric current as the same as the Equation (23) in Ref. [36]. Substitute (2) into (35) and (36), then, both sides of (35) and (36) are tested by $\tilde{n}_2 \times f_3$, both sides of (37) are tested by $\tilde{n}_1 \times f_2$, the expressions of (35)-(37) can be written in vector matrix form as follow

$$[b] = [\tilde{M}_{22}] \cdot \{g\} + [\tilde{M}_{23}] \cdot \{h\}$$

$$[c] = [\tilde{N}_{22}] \cdot \{g\} + [\tilde{N}_{23}] \cdot \{h\}$$

$$[0] = [\tilde{B}_{32}] \cdot \{g\} + [\tilde{B}_{33}] \cdot \{h\}$$

where the expressions of matrix elements can be found in Equation (25) in Ref. [36]. By substituting (38), (39) and (40) into (28) and (29), the matrix equation can be rewritten as

$$\begin{bmatrix}
A_{11} & A_{12} & A_{13} \\
A_{21} & A_{22} & A_{23} \\
0 & A_{32} & A_{33}
\end{bmatrix}
\begin{bmatrix}
a \\
g \\
h
\end{bmatrix}
= \begin{bmatrix}
\tilde{S}_1 \\
\tilde{S}_2 \\
0
\end{bmatrix}$$

(41)

where expressions of matrix elements are taken in these forms

$$[A_{11}] = ([Q_{11}] + [R_{14}] \cdot [D_{41}] + [T_{14}] \cdot [E_{41}])$$

$$[A_{12}] = ([Q_{12}] + [R_{14}] \cdot [D_{42}] + [T_{14}] \cdot [E_{42}]) \cdot [\tilde{M}_{22}]$$

$$+ ([P_{12}] + [R_{14}] \cdot [F_{42}] + [T_{14}] \cdot [Y_{42}]) \cdot [\tilde{N}_{22}]$$

$$[A_{13}] = ([Q_{12}] + [R_{14}] \cdot [D_{42}] + [T_{14}] \cdot [E_{42}]) \cdot [\tilde{M}_{23}]$$

$$+ ([P_{12}] + [R_{14}] \cdot [F_{42}] + [T_{14}] \cdot [Y_{42}]) \cdot [\tilde{N}_{23}]$$

$$[A_{21}] = ([Q_{21}] + [R_{24}] \cdot [D_{41}] + [T_{24}] \cdot [E_{41}])$$

$$[A_{22}] = ([Q_{22}] + [R_{24}] \cdot [D_{42}] + [T_{24}] \cdot [E_{42}]) \cdot [\tilde{M}_{22}]$$

$$+ ([P_{22}] + [R_{24}] \cdot [F_{42}] + [T_{24}] \cdot [Y_{42}]) \cdot [\tilde{N}_{22}]$$

$$[A_{23}] = ([Q_{22}] + [R_{24}] \cdot [D_{42}] + [T_{24}] \cdot [E_{42}]) \cdot [\tilde{M}_{23}]$$

$$+ ([P_{22}] + [R_{24}] \cdot [F_{42}] + [T_{24}] \cdot [Y_{42}]) \cdot [\tilde{N}_{23}]$$

$$[A_{32}] = [\tilde{B}_{32}]$$

(42)
\[ A_{33} = [\bar{B}_{33}] \]  
\[ \tilde{S}_1 = (S_1) - [R_{14}] \cdot [W] - [T_{14}] \cdot [V] \]  
\[ \tilde{S}_2 = (S_2) - [R_{24}] \cdot [W] - [T_{24}] \cdot [V] \]

### B. ACCELERATION OF SIE-KA USING THE MULTILEVEL FAST MULTipoLE ALGORITHM

In order to improve the efficiency of the aforementioned hybrid method, MLFMA being an effective accelerating technique can be introduced for accelerating the matrix-vector multiplications (MVMs). Initially, when implementing the MLFMA, an Oct-tree structure needs to be generated. In detail: a box enclosing the whole scattering model is named as the zero level, which is divided into eight smaller boxes named the 1st level; then, each box is recursively divided into finer sub-boxes until the edge length of the finest cube \( D_f \) is about 0.25\( \lambda \), and the final level is the total number of levels \( L_f \). The interactions between these boxes can be divided into the near-region couplings and the far-region couplings, which are calculated by the direct MOM and the fast multipole method (FMM) respectively. The essence of FMM is expression of Green function, which is based on the vector addition theorem.

\[
G(\mathbf{r}_o, \mathbf{r}_{om}) = \frac{e^{-jk|\mathbf{r}_o - \mathbf{r}_{om}|}}{4\pi|\mathbf{r}_o - \mathbf{r}_{om}|} = -\frac{jk}{(4\pi)^2} \int d^2 \hat{k} e^{-j\mathbf{k}\cdot \mathbf{r}_{om}} T(\hat{k}, \hat{\mathbf{r}}_{mn}) e^{j\mathbf{k}\cdot \mathbf{r}_{om}} 
\]

where

\[
T(\hat{k}, \hat{\mathbf{r}}_{mn}) = \sum_{l=0}^{L} (-1)^l (2l + 1) h_l^{(2)}(kr_{mn}) P_l(\hat{k} \cdot \hat{r}_{mn})
\]

Correspondingly, the dyadic expression of Green function can be written as follow

\[
\tilde{G}(\mathbf{r}_o, \mathbf{r}_{om}) = (\mathbf{i} - \nabla \nabla') \frac{e^{-jk|\mathbf{r}_o - \mathbf{r}_{om}|}}{4\pi|\mathbf{r}_o - \mathbf{r}_{om}|} = -\frac{jk}{(4\pi)^2} \int d^2 \hat{k} (\mathbf{i} - \mathbf{k}) e^{-j\mathbf{k}\cdot \mathbf{r}_{om}} T(\hat{k}, \hat{\mathbf{r}}_{mn}) e^{j\mathbf{k}\cdot \mathbf{r}_{om}}
\]

Here, \( \mathbf{r}_o \) resides in cube \( m \) centered at \( \mathbf{r}_m \), \( \mathbf{r}_{om} \) resides in cube \( m' \) centered at \( \mathbf{r}_{m'} \), and their relationships are: \( \mathbf{r}_{om} = \mathbf{r}_o - \mathbf{r}_m \), \( \mathbf{r}_{om} = \mathbf{r}_{om} - \mathbf{r}_{m'}. \) \( h_l^{(2)}(\cdot) \) denotes the spherical Hankel function of the second kind; \( P_l(\cdot) \) denotes the Legendre polynomial of degree \( l \); \( L \) is the number of multipole expansion terms, and it can ensure high precision when set as \( L = kD_f + 5 \ln(kD_f + \pi) \). The expressions of matrix elements in matrix (16) and (17): \( D_{41}, D_{42}, F_{42}, E_{41}, E_{42} \) and \( Y_{42} \) are taken into these forms

\[
D_{4q}(i, j) = \int \int d^2 \hat{k} (U_4^{D_{41}} \cdot T_{mn}(\mathbf{k}_0, \mathbf{r}_{mn})) V_{jm'}^{D_{4q}} + M_{41}^{D_{41}} \cdot T_{mn}(\mathbf{k}_0, \mathbf{r}_{mn}) N_{jm'}^{D_{4q}} \quad q = 1, 2.
\]

The expressions of each element in (55) to (58) are provided in appendix (1-10). Where \( U_{im}^{(s)}, T_{mn}(\cdot) \) and \( V_{jm'}^{(s)} \) are the disaggregation, translation and aggregation matrices on the concrete level respectively. Apply FMM to the integral equation of target, the matrix elements for far-region groups in matrix (28) and (29) \( Q_{11}, Q_{12}, Q_{21}, Q_{22}, P_{12}, P_{22}, R_{14}, R_{24}, T_{14} \) and \( T_{24} \) can be written as

\[
Q_{pq}(i, j) = \int \int d^2 \hat{k} U_{im}^{D_{4q}} \cdot T_{mn}(\mathbf{k}_0, \mathbf{r}_{mn}) V_{jm'}^{D_{4q}} \quad p, q = 1, 2.
\]

\[
P_{12}(i, j) = \int \int d^2 \hat{k} U_{im}^{P_{11}} \cdot T_{mn}(\mathbf{k}_0, \mathbf{r}_{mn}) V_{jm'}^{P_{22}}
\]

\[
P_{22}(i, j) = -(-\delta_{ij}/2) + \int \int d^2 \hat{k} U_{im}^{P_{22}} \cdot T_{mn}(\mathbf{k}_0, \mathbf{r}_{mn}) V_{jm'}^{P_{22}}
\]

\[
R_{pq}(i, j) = \int \int d^2 \hat{k} U_{im}^{R_{pq}} \cdot T_{mn}(\mathbf{k}_0, \mathbf{r}_{mn}) V_{jm'}^{R_{pq}} \quad p = 1, 2.
\]

\[
T_{pq}(i, j) = \int \int d^2 \hat{k} U_{im}^{T_{pq}} \cdot T_{mn}(\mathbf{k}_0, \mathbf{r}_{mn}) V_{jm'}^{T_{pq}} \quad p = 1, 2.
\]

The aggregation processes of in above integral equations are implemented by indirectly calculate these matrices elements with interpolation from the \( (L_f - 1) \)-th level to the second level successively. Translation processes are carried out on each level. Detailed description of the MLFMA can be found in [16], and it will not be repeated here. Based on the above theory and formulations, the hybrid SIE-KA method with MLFMA are formed to calculate EM scattering from three-dimensional arbitrarily shaped combined conducting and dielectric target above coastal environment. We adopt the bi-conjugate gradient method (Bi-CGM) [37] to solve the matrix equation (41) iteratively. When the relative error of unknown electric/magnetic current between the \( n \)-th step and \( (n-1) \)-th step is less than \( 10^{-4} \), the iterative computation stops.
TABLE 1. Memory requirement and computational complexity.

| Method         | Number of unknowns | Memory requirement | Computational complexity |
|----------------|--------------------|--------------------|-------------------------|
| Conventional MOM | $N_1+2N_2+N_3+N_4$ | $o((N_1+2N_2+N_3+N_4)^2)$ | $o((N_1+2N_2+N_3+N_4)^2)$ |
| MLFMA          | $N_1+2N_2+N_3+N_4$ | $o(N_1+N_2+N_3+N_4)$   | $o((N_1+2N_2+N_3+N_4)^2)$ |
| SIE            | $N_1+N_2+N_3+N_4$  | $o((N_1+N_2+N_3+N_4)^2)$ | $o((N_1+N_2+N_3+N_4)^2)$ |
| SIE-MLFMA      | $N_1+N_2+N_3+N_4$  | $o(N_1+N_2+N_3+N_4)$   | $o(N_1+N_2+N_3+N_4)$     |
| SIE-KA         | $N_1+N_2+N_3+N_4$  | $o(N_1+N_2+N_3+N_4)$   | $o(N_1+N_2+N_3+N_4)$     |
| SIE-KA-MLFMA   | $N_1+N_2+N_3+N_4$  | $o(N_1+N_2+N_3+N_4)$   | $o(N_1+N_2+N_3+N_4)$     |

C. MEMORY REQUIREMENT AND COMPUTATIONAL COMPLEXITY

To validate the efficiency of the hybrid SIE-KA-MLFMA method, the memory requirement and computational complexity are necessarily investigated and compared with some other methods, such as conventional MOM with Bi-CGM (based on coupled integral equation method), the SIE method (target by SIE, rough surface by MOM), the hybrid SIE-MLFMA method, the hybrid SIE-KA method and the hybrid SIE-KA-MLFMA method. The comparison of the memory requirement and computational complexity of different methods is shown in Table 1.

As shown in Table 1, conventional MOM has the most number of unknowns, and other four methods have the same number of unknowns. Comparing conventional MOM and SIE, when introducing SIE, the number of unknowns in dielectric part of combined target is cut off by half, meanwhile, the memory requirement and computational complexity reduce correspondingly. When combining SIE with KA, the original dimension of impedance matrix will be reduced and just depend on the number of target’s unknowns ($N_1 + N_2 + N_3$). The memory requirement and computational complexity of SIE-KA is $o(N_1 + N_2 + N_3)^2$. In general, the number of environmental unknowns $N_4$ is much bigger than target’s unknowns ($N_1 + N_2 + N_3$). However, the good performance of SIE-KA only exists on the premise of this condition that the number of target’s unknowns is relatively small. When the number of target’s unknowns increases, high memory requirement and high computational complexity are still the major burden of computational process. Through analysis of this table, the SIE-KA is suitable for solving the composite EM scattering problem with larger environmental unknowns and relatively smaller target’s unknowns. When combining SIE with MLFMA, the hybrid SIE-MLFMA method has better performance comparing with SIE in both memory requirement and computational complexity. But the SIE-MLFMA still has to endure the enormous number of unknowns which contains both combined target and divisional rough surface. In practical radar detection, the personal computer will not suffice the huge memory requirement with the larger number of actual target’s unknowns, and the hybrid SIE-KA-MLFMA method would be more suitable for this application. By applying MLFMA, the SIE-KA-MLFMA has smaller computational complexity and requires less memory. This table shows us three important conclusions: firstly, the SIE is efficient in reducing the number of unknowns; secondly, the introduction of KA allows us to solve a composite EM scattering problem with a large-scale environment; finally, the application of MLFMA makes the composite EM scattering problem with the electrically large targets solvable.

The validation of the hybrid SIE-KA-MLFMA method will be verified by the representative computation examples at the beginning of Section III. Abundant numerical simulations will be further exhibited. Meanwhile, several special phenomena and interesting results will be shown and some important conclusions will be discussed in the section.

III. NUMERICAL SIMULATION AND DISCUSSION

A. MODELING OF COASTAL ENVIRONMENT

The surface of ground region is simulated by using Gaussian spectral function, whose statistic properties depend on the RMS $h_{rms}$ and correlation length $l_x$, $l_y$ (along the x-axis and y-axis direction). When we establish the surface model of sea surface, the concrete location of sea region must be considered. When the waves approach the seashore, the finite depth of this shallow sea region will have effects on the shape and statistic properties of sea surface. Bases on energy flux conservation theorem, shoaling coefficient is introduced by McCormick to connect shoaling effect with sea spectrum [38]. The validation of shoaling coefficient is verified during near sea experiments by Brinkkjær and Jonsson [39]. Therefore, we introduce the finite-depth sea spectrum [40] which combines JONSWAP spectrum and shoaling coefficient to simulate the near sea surface of coastal environment.

The shoaling coefficient $\xi(d)$ is the function of sea depth $d$ and the sea wave number $k$, which can be written in this form

$$\xi(d) = \sqrt{\frac{\cosh^2(kd)}{\sinh(kd) \cosh(kd) + kd}}$$  (64)

The relationship between the sea frequency and $k$ is given as follow

$$f = g_c k (1 + k^2/k_m^2) \tanh(kd)/2\pi$$  (65)

where $k_m$ denotes the wavenumber with minimum phase speed, $k_m = 363.2$ rad/m; $g_c$ is gravity acceleration,
$g_c = 9.81 m/s^2$. The finite-depth sea spectrum is taken in this form

$$S_{\text{Finite}}(f, d) = \xi^2(d) \cdot S_{\text{JONSWAP}}(f)$$  \hspace{1cm} (66)

where $S_{\text{JONSWAP}}(f)$ is the JONSWAP spectrum, and its expression [40] is shown as follow

$$S_{\text{JONSWAP}}(f) = \frac{\alpha g_c^2}{(2\pi)^3 f^5} \exp \left[ -\frac{5}{4} \left( \frac{f m}{f} \right)^4 \right] \exp \left[ -\frac{\gamma - f m}{2\gamma f m} \right]$$  \hspace{1cm} (67)

where $\alpha$ is dimensionless constant, $\alpha = 0.076(g_c X/v_{10})^{-0.22}$; $X$ denotes wind fetch and $v_{10}$ denotes wind velocity on the height of 10.0 m above sea surface. $f_m$ is the frequency corresponding to spectral peaks, $f_m = 3.5 g_c^{0.67}/(X^{0.33} v_{10}^{0.34})$. $\gamma$ is enhancement factor of peak value, whose observation value ranges from 1.5 to 6 and mean value is 3.3. $\sigma_f$ is peak height parameter which is defined as follow

$$\sigma_f = \begin{cases} 
0.07, & f \leq f_m \\
0.09, & f > f_m 
\end{cases}$$  \hspace{1cm} (68)

When we establish the surface of near sea region, the water depth distribution of this region should be considered. Due to the limitation of our group, the measured datum of real near sea’s water depth cannot be obtained easily. Analytically, we can build a sketch map of profile for near sea’s water depth distribution based on the research report of the East China Sea submarine topography [41]. In the East China Sea, the seafloor of continental shelf is divided into inner continental shelf and outer continental shelf. In the region of outer continental shelf, the slope of seabed is stabilized at approximately 1.0°/km, and the water depth is approximately 60 m. But in the region of inner continental shelf (extending from land about 10 km), the slope is ranging from 0 to 0.9 [41]. In this paper, we introduce an approximate way to build the profile model for near sea’s water depth distribution, which is given in Figure 2 in Ref. [33]. According to the approximate model in [34], the slope of inner continental shelf is assumed to be 0.7, where $H_{\text{near}}$ denotes water depth and $l_{\text{shelf}}$ denotes the extended distance of continental shelf. The method is just a simple and approximate way to simulate the water depth distribution in near sea region. If the measured datum of real near sea’s water depth is available, the effect of water depth on sea spectrum will be more practical and reasonable.

Based on the suitable sea spectrum for near sea region, we adopt the theory of composite rough surface modeling [33] to establish the geometrical model for coastal environment. In order to deal with the boundary between ground region and near sea region, the weighted inverse tangent function is utilized, whose expression is shown as follow:

$$f_{\text{com}}(x, y) = f_{\text{sur}1}(x, y) \cdot \frac{\pi/2 + \arctan f_m(x, y)}{\pi} + f_{\text{sur}2}(x, y) \cdot \frac{\pi/2 - \arctan f_m(x, y)}{\pi}$$  \hspace{1cm} (69)

where $f_{\text{sur}1}(x, y)$, $f_{\text{sur}2}(x, y)$ and $f_{\text{com}}(x, y)$ denote height profiles of rough surface $\text{sur}1$, rough surface $\text{sur}2$ and composite rough surface $\text{com}$, respectively; $f_m(x, y)$ denotes the weighted function for borderline modulation, and its concrete form could be confirmed by the structure of actual environment. The sketch map of composite rough surface after treatment of weighted inverse tangent function is shown as follow:

As shown in Figure 2, the weighted inverse tangent function is a key processing to connect two different regions and smooth the borderline. In this paper, we choose the linear boundary (Equation (9) in Ref. [33]) to build the geometrical model of coastal environment.

The working frequency is set as $f = 300 MHz$; Size of coastal environment is $L_x \times L_y \times 30 \lambda \times 30 \lambda$. The root mean square (RMS) is set as $h_{\text{rms}} = 0.5 \lambda$ and the correlation length in $x$-axis and $y$-axis direction are set as $l_x = l_y = 2.0 \lambda$. The wind fetch is set as $X = 50 km$ and wind velocity on the height of 10.0 m above sea surface is set as $v_{10} = 5.0 m/s$. The geometrical model of coastal environment is built as shown in Figure 3, which can marvelously simulate the coastal environment. In this paper, the following numerical examples are calculated based on the assumption that each region of divisional environment is homogeneous medium surface. Thus, the approximated treatment is implemented that the relative permittivity of each region is separately assigned by the borderline.

B. VERIFICATION OF HYBRID SIE-KA-MLFMA METHOD

The example of a combined conducting and dielectric sphere above coastal environment is considered in this subsection.
to validate the accuracy and to illustrate the efficiency of the hybrid SIE-KA-MLFMA method. In the paper, all computation examples are calculated at the working frequency $f = 1.4$GHz (except for example $E$). In this subsection and all following subsections (except for example $E$, $F$ and $G$), all rough surfaces in computation examples are under the same conditions: Size of coastal environment is $L_x \times L_y : 30\lambda \times 30\lambda$; The borderline is $x_0 = 0$; The position of each region is shown in Figure 3. In ground region, the RMS is set as $b_{rms} = 0.5\lambda$, and the correlation length in $x$-axis and $y$-axis direction are set as $l_x = l_y = 2.0\lambda$; In near sea region, the wind fetch is set as $X = 50$km and wind velocity on the height of $10.0$ m above sea surface is set as $v_{10} = 1.0$m/s. The relative permittivity of sea region is set as $\varepsilon_r - sea = (72.55, -j63.02)$ when water’s salinity $s_{water}$ equals to $32.54\%$, temperature is $20^\circ C$, which is computed based on the Debye expression proposed in [42]; The relative permittivity of ground region is set as $\varepsilon_r - gr = (16.33, -j11.68)$ when sand’s content equals to $51.5\%$, clay’s content equals to $13.5\%$, volumetric water content $m_v$ is $30\%$ and temperature is $20^\circ C$, which is computed based on the semi-empirical formulas proposed in [43]. In this paper, incident wave of all computation examples is set as 3-D tapered incident beam, and the tapered parameter $g_x$, $g_y$ and rough surface length $L_x$, $L_y$ along $x$ and $y$ axis direction should satisfy $g_x = L_x/4$ and $g_y = L_y/4$. As analyzed in subsection C of Section III, the hybrid SIE-KA method is not suitable for the scattering problems with large target’s unknowns. Thus, we choose the MLFMA based on conventional coupled integral equation method as the reference algorithm to compare with the hybrid SIE-KA-MLFMA method in aspects of memory requirement and computation time. The vertical-to-vertical (VV) polarization, vertical-to-horizontal (VH) polarization, horizontal-to-vertical (HV) polarization and horizontal-to-horizontal (HH) polarization are considered. The bi-static RCS of the combined target above divisional rough surface is calculated by MLFMA and SIE-KA-MLFMA respectively. The simulation results are determined by one surface realization. In this paper, the computing platform is AMD processor of $2.3$ GHz with $64$ GB RAM. In this computation example, the target is a combined sphere which consists of a PEC hemi-sphere and a dielectric hemi-sphere with permittivity $\varepsilon_r = (4.0, -j2.0)$; Its diameter and height above surface are set as $4.0\lambda$ and $5.0\lambda$, respectively. Incident angles are: $\theta_1 = 45^\circ$, $\phi_1 = 0^\circ$; Observation angle $\theta_2$ is from $-90^\circ$ to $90^\circ$, $\phi_2 = 0^\circ$; The simulation results are shown in Figure 4. In MLFMA, the number of combined target’s unknowns is $43556$, and the number of divisional rough surface’s unknowns is $274745$. By contrast, when introducing SIE, the number of combined target’s unknowns reduces to $31495$. As analyzed in subsection C of Section II, when applying hybrid SIE-KA method, the dimension of impedance matrix mainly depends on the number of combined target’s unknowns instead of the total unknowns, which will bring the great reducing of the memory requirement and improvement of the efficiency. It is observed that the bi-static scattering in hybrid SIE-KA-MLFMA method is in good agreement with MLFMA under both co-polarization and cross-polarization. At some large scattering angles, there are some discrepancies existing. These inconsistencies are unsurprising because of the fact that the KA method [29]–[31] has the deficiency in calculating the scattering from rough surface at large scattering angles.

We choose the simulation result with VV polarization in Figure 4 as the example to compare the memory requirement and simulation time of each algorithm listed in Table 1 in detail. The memory requirements of MOM, SIE and SIE-KA are $57.29$GB, $51.66$GB and $12.18$GB, respectively. The simulation time of MOM, SIE and SIE-KA is $9.12$ h, $8.32$ h and $1.96$ h, respectively. Because the multiple surface realizations have to be carried out to solve the composite scattering problems involving rough surfaces, these above algorithms without acceleration strategy are not competent for solving this kind of scattering problem in terms of memory requirement and computing time. Thus, MLFMA should be applied and the detailed comparison of algorithms with the acceleration by MLFMA is given as follow

The memory requirement and simulation time for combined sphere above divisional rough surface with one surface realization by MLFMA, SIE-MLFMA and SIE-KA-MLFMA are presented in Table 2. The memory requirement of
SIE-KA-MLFMA is 23.44% of SIE-MLFMA, 21.14% of MLFMA. The simulation time of SIE-KA-MLFMA is 18.09% of SIE-MLFMA, 16.71% of MLFMA. The table leads us to conclude that the hybrid SIE-KA-MLFMA method is efficient and accuracy for analyzing the 3-D EM scattering from combined conducting and dielectric target above divisional rough surface.

TABLE 2. Memory requirements and simulation times of The MLFMA, SIE-MLFMA and SIE-KA-MLFMA for combined sphere above divisional rough surface with one surface realization.

| Solver      | Memory Requirement(MB) | Simulation time (s) |
|-------------|------------------------|---------------------|
| MLFMA       | 713.45                 | 4132.89             |
| SIE-MLFMA   | 643.34                 | 5817.21             |
| SIE-KA-MLFMA| 150.82                 | 690.74              |

These numerical results lead us to conclude that the hybrid SIE-KA-MLFMA method is efficient in reducing the number of unknowns and memory requirement. It is also a good way to make the composite EM scattering problems of combined target with large divisional rough surface solvable. Based on this hybrid method, plentiful numerical simulations will be exhibited to show the composite EM scattering characteristics of the combined conducting and dielectric target above divisional rough surface in following subsections, and the effect of multifarious factors on the EM scattering characteristics will be discuss in detail.

C. COMPOSITE SCATTERING CHARACTERISTIC WITH DIFFERENT CATEGORY OF ENVIRONMENT

In this example, the composite scattering characteristics of combined cube above single ground rough surface, single sea surface and divisional rough surface are analyzed. The working frequency is set as $f = 1.4$GHz. In this paper, only VV polarization mode is considered, and the polarization characteristics will be studied carefully in example $D$. The side length of combined cube is 5.0$\lambda$. The position of combined cube is shown in the sketch map of Figure 5. The target’s height above rough surface is 5.0$\lambda$. The dielectric part of combined cube is marked by dark color. The dielectric part’s permittivity of combined cube is set as $\varepsilon_{r1} = (4.0, -j2.0)$. The sizes of three rough surfaces are set as $L_x \times L_y : 30\lambda \times 30\lambda$. Each part’s statistic parameters of divisional rough surface are the same as that in example $B$. The single ground rough surface’s statistic parameters are: $h_{rms} = 0.5\lambda$, $l_x = l_y = 2.0\lambda$. The single sea surface’s statistic parameters are: $X = 50$km, $v_{10} = 1.0$m/s. The relative permittivities of ground region and sea region are set as $\varepsilon_{r-geo} = (16.33, -j11.68)$ and $\varepsilon_{r-sea} = (72.55, -j63.02)$ respectively. Incident angles are: $\theta_i = 45^\circ$, $\phi_i = 0^\circ$; Observation angle $\theta_o$ is from $-90^\circ$ to $90^\circ$, $\phi_o = 0^\circ$. Simulation results are shown in Figure 5.

In Figure 5, when the combined cube is located above single sea surface, there is an obvious peak value existing at the specular scattering direction due to the sea surface’s strong specular scattering characteristic. Meanwhile, there is an obvious backscattering in the scattering angle range from $30^\circ$ to $48^\circ$, because quasi-dihedral structure which consists of the relative flat sea surface and the side face of combined cube will bring strong backscattering. However, when the combined cube is located above single ground rough surface, there is no peak value or strong backscattering existing in the bi-static RCS curve due to the ground rough surface’s strong diffuse scattering characteristic in all angle range. A valuable discovery is that when the combined cube is located above divisional rough surface, the peak value and the strong backscattering also exist near the direction of specular scattering and backscattering, respectively. But this peak value is smaller than that of single sea surface background, since only the half of divisional rough surface is relatively flat. Due to the existence of ground region, the diffuse scattering in other angles of combined cube above divisional rough surface enhances. The scattering mechanism leading to the strong backscattering is similar to sea surface’s condition. The order of backscattering scattering intensity from a combined cube above environment is “single sea surface > divisional rough surface > ground rough surface”. According to the comparison of a combined cube above three kinds of rough surface, we can conclude that the combined cube above divisional rough surface presents a unique scattering characteristic which is a fusion of sea surface’s specular scattering characteristic and ground rough surface’s diffuse scattering characteristic.

D. COMPOSITE SCATTERING CHARACTERISTIC WITH DIFFERENT POLARIZATION MODE

In this example, the composite scattering of a combined cube above divisional rough surface in different polarization mode is analyzed in detail. All concrete parameters of combined cube and divisional rough surface are the same as example $C$. The combined cube is directly above the original point of $xoy$ plane, whose height above rough surface is 5.0$\lambda$. Incident angles are: $\theta_i = 45^\circ$, $\phi_i = 0^\circ$; Observation angle $\theta_o$ is from
The volumetric water content is 30%. The ground region’s relative content equals to 51.5%, clay’s content equals to 13.5%, and divisional rough surface is set as a cube is directly above the original point of the xoy plane, whose height above rough surface is 5.0m. The size of divisional rough surface is set as $L_x \times L_y : 6.0m \times 6.0m$. The statistic parameters of ground region are: $h_{\text{rms}} = 0.1m$, $l_s = l_l = 0.6m$. The statistic parameters of single sea surface’s statistic parameters are: $X = 50km$, $v_{10} = 1.0m/s$. Temperature is set as 20°C. Water’s salinity equals to 32.54%. The sea region’s relative dielectric constants corresponding to the three frequencies are $\varepsilon_r^{\prime-\text{sea}} = (72.88, -j117.92)$, $\varepsilon_r^{\prime\prime-\text{sea}} = (72.55, -j63.02)$ and $\varepsilon_r^{\prime-\text{sea}} = (72.02, -j46.47)$. Sand’s content equals to 51.5%, clay’s content equals to 13.5%, and volumetric water content is 30%. The ground region’s relative dielectric constants corresponding to the three frequencies are $\varepsilon_r^{\prime-\text{gro}} = (16.38, -j7.24)$, $\varepsilon_r^{\prime\prime-\text{gro}} = (16.33, -j11.68)$ and $\varepsilon_r^{\prime-\text{gro}} = (16.26, -j10.28)$. Incident angles are: $\theta_i = 45^\circ$, $\phi_i = 0^\circ$; Observation angle $\theta_s$ is from $-90^\circ$ to $90^\circ$, $\phi_s = 0^\circ$. The bi-static RCS curves with different working frequency are given in Figure 6.

As shown in Figure 6, the curve trend of HV polarization is similar to VH polarization, and there are some discrepancies existing between the HH polarization and VV polarization at some scattering angles. The integral values of bi-static RCS curves in co-polarization mode are much larger than that of cross-polarization mode at all scattering angles. Every bi-static RCS curve has a peak region at the specular direction, but the peak value of co-polarization mode is more obvious. Meanwhile, there is an obvious backscattering in the scattering angle range from 33° to 56° in both VV and HH polarization due to the existence of combined cube and relatively flat sea surface. But when polarization mode is HV polarization or VH polarization, the backscattering is not obvious.

**E. COMPOSITE SCATTERING CHARACTERISTIC WITH DIFFERENT WORKING FREQUENCY**

In this example, the influence of working frequency on the bi-static RCS of a combined cube above divisional rough surface is discussed. The working frequencies are set as: $f_1 = 700MHz$, $f_2 = 1.4GHz$ and $f_3 = 2.1GHz$. The combined cube is directly above the original point of the xoy plane. The side length of combined cube is 0.8 m. The height of combined cube above divisional rough surface is 1.0 m. The size of divisional rough surface is set as $L_x \times L_y : 6.0m \times 6.0m$. The statistic parameters of ground region are: $h_{\text{rms}} = 0.1m$, $l_s = l_l = 0.6m$. The statistic parameters of single sea surface’s statistic parameters are: $X = 50km$, $v_{10} = 1.0m/s$. Temperature is set as 20°C. Water’s salinity equals to 32.54%. The sea region’s relative dielectric constants corresponding to the three frequencies are $\varepsilon_r^{\prime-\text{sea}} = (72.88, -j117.92)$, $\varepsilon_r^{\prime\prime-\text{sea}} = (72.55, -j63.02)$ and $\varepsilon_r^{\prime-\text{sea}} = (72.02, -j46.47)$. Sand’s content equals to 51.5%, clay’s content equals to 13.5%, and volumetric water content is 30%. The ground region’s relative dielectric constants corresponding to the three frequencies are $\varepsilon_r^{\prime-\text{gro}} = (16.38, -j7.24)$, $\varepsilon_r^{\prime\prime-\text{gro}} = (16.33, -j11.68)$ and $\varepsilon_r^{\prime-\text{gro}} = (16.26, -j10.28)$. Incident angles are: $\theta_i = 45^\circ$, $\phi_i = 0^\circ$; Observation angle $\theta_s$ is from $-90^\circ$ to $90^\circ$, $\phi_s = 0^\circ$. The bi-static RCS curves with different working frequency are given in Figure 7.

In Figure 7, with the increase of working frequency, the trend of the bi-static RCS curve keeps stable, however, the scattering energy near the specular scattering direction comes weaker. The backscattering energy also decreases, especially in the scattering angle range from 4° to 37° and from 58° to 90°. Based on the permittivities listed above, we find that working frequency has a little influence on the real part of permittivity but has certain influence on the imaginary part of permittivity. This is one of the reasons causing this phenomenon. The other reason is that when working frequency increases, the divisional rough surface becomes relatively rougher that will enhance the diffuse scattering characteristic of environment. Accordingly, the backscattering will also be effected by the relatively rougher environment.

**F. COMPOSITE SCATTERING CHARACTERISTIC WITH DIFFERENT WORKING FREQUENCY**

In this example, the influence of ground region’s statistic parameters (RMS and correlation length) on the bi-static RCS of a combined cube above divisional rough surface is analyzed in detail. All concrete parameters of combined cube are the same as example C. The combined cube is directly above the original point of the xoy plane, whose height above rough surface is 5.0λ. The wind velocity on the height of 10.0 m above sea surface keeps as $v_{10} = 1.0m/s$. Firstly, we keep the correlation length both in x and y axis direction $l_s$, $l_l$ stable as 2.0λ. The RMSs are set as $h_{\text{rms}}^{\prime\prime} = 0.1\lambda$ and $h_{\text{rms}}^{\prime\prime} = 0.5\lambda$, respectively. Incident angles are: $\theta_i = 45^\circ$, $\phi_i = 0^\circ$; Observation angle $\theta_s$ is from $-90^\circ$ to $90^\circ$, $\phi_s = 0^\circ$. The bi-static RCS curves with different RMS are given in Figure 8.

With the increase of RMS, the surface of ground part which belongs to the divisional rough surface will become rougher. A slight variation in one part may affect the situation as a whole. In Figure 8, with the increase of RMS, the scattering...
In Figure 9, with the increase of correlation length, the peak value of bi-static RCS curve increases slightly, and the variation of backscattering is not notable. In the range of scattering angle from -15° to 30°, the value of bi-static RCS curve shows an obvious decline. The reason causes above phenomenon is that with the increase of correlation length, the correlation of every point on the surface of ground part increases, which will make the ground region relatively flatter than before. Therefore, the diffuse scattering characteristic of whole divisional rough surface will be weakened.

**G. COMPOSITE SCATTERING CHARACTERISTIC WITH DIFFERENT WIND VELOCITY OF NEAR SEA REGION**

In this example, the influence of near sea surface’s statistic parameters (wind velocity on the height of 10.0 m above sea surface) on the bi-static RCS of a combined cube above divisional rough surface is discussed. All concrete parameters of combined cube are the same as example C. The combined cube is directly above the original point of $xoy$ plane, whose height above rough surface is 5.0λ. The statistic parameters of ground region keep as: $h_{rms} = 0.5\lambda$ and $l_x = l_y = 2.0\lambda$. The wind velocities on the height of 10.0 m above sea surface are set as: $v'_{10} = 1.0\text{m/s}$ and $v'_{10} = 5.0\text{m/s}$, respectively. Incident angles are: $\theta_i = 45^\circ$, $\varphi_i = 0^\circ$; Observation angle $\theta_s$ is from $-90^\circ$ to $90^\circ$, $\varphi_s = 0^\circ$. The bi-static RCS curves with different wind velocity are given in Figure 10.

With the increase of wind velocity, the surface’s fluctuation of near sea part which belongs to the divisional rough surface will become more intense. In Figure 10, with the increase of wind velocity, the scattering energy near the specular scattering direction decreases significantly and the backscattering energy also decreases to some extent. The reason causes above phenomenon is that the whole divisional rough surface’s specular scattering characteristic mainly depends on the relatively flat sea surface. Meanwhile, the permittivity of seawater is large. Therefore, the wind velocity also has an obvious effect on the bi-static RCS curve.

**H. COMPOSITE SCATTERING CHARACTERISTIC WITH DIFFERENT RELATIVE PERMITTIVITY OF TARGET’S DIELECTRIC PART**

In this example, the influence of relative permittivity of target’s dielectric part on the bi-static RCS of a combined cube above divisional rough surface is analyzed. All concrete parameters of divisional rough surface are the same as example C. The side length of combined cube is 5.0λ. The combined cube is directly above the original point of $xoy$ plane, whose height above rough surface is 5.0λ. We set two abbreviation Die1 and Die2 to denote the relative permittivity $\varepsilon_{r1} = (4.0, -j2.0)$ and $\varepsilon_{r2} = (8.0, -j4.0)$ respectively.
In Figure 11, we give four combination forms of the combined cube, and the dielectric part is marked by dark color. Incident angles are: $\theta_1 = 45^\circ$, $\varphi_1 = 0^\circ$; Observation angle $\theta_2$ is from $-90^\circ$ to $90^\circ$, $\varphi_2 = 0^\circ$. The bi-static RCS curves with different target’s combination form are given in Figure 11.

In Figure 11, there is an obvious peak value near the specular scattering direction in each curve, but the peak value and its corresponding scattering angle are not varied with the change of the target’s combination form because the specular scattering characteristic mainly depends on the properties of divisional rough surface which has been analyzed intensively in previous computation examples. As shown in Figure 11, we can easily find that the target’s combination form has a great influence on the backscattering from the combined target above divisional rough surface. When the cube is pure PEC, there is the strongest backscattering near the scattering angle $45^\circ$. When the part marked by dark color is replaced by dielectric part with the relative permittivity of Die1, the backscattering decreases remarkably. When the whole cube is replaced by dielectric part with the relative permittivity of Die1, the backscattering is no longer noticeable. These phenomena prove two important conclusions: one is that the dielectric part of an object will reduce its own scattering capability; the other is that the scattering intensity from pure dielectric object is much weaker than that from pure PEC object. What’s more, by comparison of the target’s combination form “Die1+PEC” and “Die2+PEC”, we can find that the backscattering with “Die2+PEC” is stronger than that with “Die1+PEC”. Therefore, third important conclusion can be obtained: the dielectric part in combined cube with bigger relative permittivity will lead to stronger backscattering.

**I. I. COMPOSITE SCATTERING CHARACTERISTIC WITH DIFFERENT TARGET’S HEIGHT**

In this example, the influence of target’s height above the divisional rough surface on the bi-static RCS of a combined cube above divisional rough surface is analyzed. All concrete parameters of divisional rough surface are the same as example C. The side length of combined cube is 5.0$\lambda$. The combined cube is directly above the original point of $xoy$ plane. The target’s heights above the divisional rough surface are set as: $H_{tar}' = 3.0\lambda$, $H_{tar}'' = 5.0\lambda$ and $H_{tar}''' = 7.0\lambda$. Incident angles are: $\theta_1 = 45^\circ$, $\varphi_1 = 0^\circ$; Observation angle $\theta_2$ is from $-90^\circ$ to $90^\circ$, $\varphi_2 = 0^\circ$. The bi-static RCS curves with different target’s height form are given in Figure 12.

In Figure 12, with the increase of target’s height, the value of bi-static RCS curve near the specular scattering direction decreases slightly. The reason causes this phenomenon is that when combined cube is closer to the divisional rough surface, the interaction between target and environment is stronger which will contribute more energy to the specular scattering. In addition, with the increase of target’s height, the curve in the backscattering region has no regular variation in intensity or fluctuation, because such an analogous dihedral construction consisting of combined target and divisional rough surface will produce strong backscattering even if the target’s height changes within a small range.

**J. COMPOSITE SCATTERING CHARACTERISTIC WITH DIFFERENT TARGET’S CATEGORY**

The target category and concrete combination form directly determines the scattering characteristics of a combined object. Furthermore, the scattering characteristics of a combined object will affect the composite scattering characteristics. Firstly, we consider a more complicated cube which is divided into four equal-volume parts along the diagonal plane which are vertical to $xoy$ plane. These four parts are marked from “1” to “4” as shown in Figure 13. The geometrical parameters of combined cube and all concrete parameters of divisional rough surface are the same as example C. We set two abbreviation Die1 and Die2 to denote the relative permittivity $\varepsilon_{1} = (4.0, -j2.0)$ and $\varepsilon_{2} = (8.0, -j4.0)$ respectively. Incident pitching angle is $\theta_1 = 45^\circ$; Observation azimuth angle $\varphi_2$ is from 0$^\circ$ to 360$^\circ$, $\theta_2 = 45^\circ$. The mono-static RCS curves of different combined cube above divisional rough surface in azimuth angle distribution are given in Figure 13.

As shown in Figure 13, the mono-static scattering energy of a PEC cube above divisional rough surface mainly
FIGURE 13. The Mono-static RCS curves of different combined cube above divisional rough surface in azimuth angle distribution.

FIGURE 14. The Mono-static RCS curves of different combined cylinder above divisional rough surface in azimuth angle distribution.

bigger permittivity will lead to stronger backscattering in corresponding azimuth distribution. These three conclusions are accordance with the conclusion of example $H$.

Secondly, we consider a complicated cylinder which is divided into four equal-volume parts along the diagonal plane which are vertical to $xoy$ plane. These four parts are marked from “1” to “4” as shown in Figure 14. The target’s parameters are set as following: diameter of cylinder bottom is $4.0\lambda$, height of cylinder is $4.0\lambda$, and height above surface is $5.0\lambda$ respectively. The concrete parameters of divisional rough surface are the same as example $C$. Incident pitching angle is $\theta_i = 45^\circ$; Observation azimuth angle $\varphi_o$ is from $0^\circ$ to $360^\circ$, $\varphi_o = 45^\circ$. The mono-static RCS curves of different combined cylinder above divisional rough surface in azimuth angle distribution are given in Figure 14.

Comparing with the results in Figure 14, the distribution of mono-static scattering energy from a PEC cylinder above divisional rough surface in azimuth range is relatively uniform. The reason causes this phenomenon is that the profile of the cylinder is curved surface structure. The peaks which towards $\varphi_o = 0^\circ$, $\varphi_o = 90^\circ$, $\varphi_o = 180^\circ$ and $\varphi_o = 270^\circ$ respectively are not obvious as Figure 13. However, the scattering intensity can still be approximately divided into three levels according to the environmental region which the cylinder’s profile faces, the order is “sea surface > divisional rough surface > ground rough surface” which is accordance with the conclusion of example $C$ and example $L$. Then, the effect of combination form of cylinder on the mono-static scattering is analyzed as follow. When the part 4 is replaced by Die1, the scattering intensity has a sharp decline ranging from $\varphi_o = 225^\circ$ to $\varphi_o = 315^\circ$. When the part 1 and part 3 are further replaced by Die1, the sharp decline will occur in the range of $135^\circ$ to $225^\circ$, $0^\circ$ to $45^\circ$ and $315^\circ$ to $360^\circ$ simultaneously. When all parts are replaced by Die1, the scattering intensity in all azimuth angles decreases dramatically. These three noticeable phenomena prove two important conclusions: one is that the dielectric part of an object will cause a sharp decline in the azimuth angle range where this part is located; the other is that the scattering capability of pure dielectric object is much weaker than that of pure PEC object in whole azimuth distribution. Finally, by comparison of the target’s combination form “Die1+PEC+Die1+Die1” and “Die2+PEC+Die2+Die2”, we can find that the scattering intensity of the dielectric part with permittivity Die2 is stronger than that of the dielectric part with permittivity Die1 in corresponding azimuth distribution. Therefore, third important conclusion can be obtained: the dielectric part with concentrates in four directions, and they are respectively $\varphi_o = 0^\circ$, $\varphi_o = 90^\circ$, $\varphi_o = 180^\circ$ and $\varphi_o = 270^\circ$. Among them, the scattering energy at azimuth angle of $0^\circ$ is the largest. The scattering energy corresponding to $90^\circ$ and $270^\circ$ is at the same level which belongs to the second level. The scattering energy corresponding to $180^\circ$ is the weakest. The reason causing these four strong peaks is that each side of cube which is vertical to the environment combines with the rough surface as the quasi-dihedral structure which will bring strong backscattering. It is worth mentioning that when azimuth angle is $0^\circ$, the side of cube faces the sea surface, when azimuth angle is $90^\circ$ or $270^\circ$, the sides of cube are towards the adjacent region between sea and ground, finally, when azimuth angle is $180^\circ$, the side of cube faces ground rough surface. Different environmental surface combined with side of cube will cause different level of backscattering intensity, the order is “sea surface > divisional rough surface > ground rough surface” which is accordance with the conclusion of example $C$. Then, the effect of combination form of cube on the mono-static scattering is analyzed as follow. When the part 4 is replaced by Die1, the scattering intensity has a sharp decline ranging from $\varphi_o = 225^\circ$ to $\varphi_o = 315^\circ$. When the part 1 and part 3 are further replaced by Die1, the sharp decline will occur in the range of $135^\circ$ to $225^\circ$, $0^\circ$ to $45^\circ$ and $315^\circ$ to $360^\circ$ simultaneously. When all parts are replaced by Die1, the scattering intensity in all azimuth angles decreases dramatically. These three noticeable phenomena prove two important conclusions: one is that the dielectric part of an object will cause a sharp decline in the azimuth angle range where this part is located; the other is that the scattering capability of pure dielectric object is much weaker than that of pure PEC object in whole azimuth distribution. Finally, by comparison of the target’s combination form “Die1+PEC+Die1+Die1” and “Die2+PEC+Die2+Die2”, we can find that the scattering intensity of the dielectric part with permittivity Die2 is stronger than that of the dielectric part with permittivity Die1 in corresponding azimuth distribution. Therefore, third important conclusion can be obtained: the dielectric part with
azimuth angles decreases dramatically. These four noticeable phenomena can also prove the three conclusions in the last example. And the results also verify that they are not the unique phenomena existing in combined cube which contains plane structure, the combined cylinder containing curved surface structure also has the same rules.

**K. A COMPOSITE MISSILE MODEL ABOVE DIVISIONAL ROUGH SURFACE**

In this example, a composite missile model is considered in order to analyze the composite scattering from it above divisional rough surface. The geometrical parameters of the composite missile model are shown in Figure 15. The concrete parameters of divisional rough surface are the same as example C. The working frequency is set as \( f = 1.4\text{GHz} \). The composite missile is located above the divisional rough surface with the height of \( 5.0\lambda \), and its head is toward the positive direction of \( x \)-axis. As shown in Figure 15, the nose marked with dark color represents dielectric part with permittivity \( \varepsilon_r = (4.0, -j2.0) \) which is expressed as abbreviation Die1. Incident pitching angle \( \theta_i \) is from \(-90^\circ\) to \(90^\circ\), and incident azimuth angle \( \varphi_i \) is \(0^\circ\). The mono-static RCS curves of the missile model above divisional rough surface are given in Figure 16.

As shown in Figure 16, the legend “PEC+Sur” denotes the mono-static RCS curve of PEC missile above divisional rough surface; the legend “Com+Sur” denotes the mono-static RCS curve of composite missile above divisional rough surface; the legend “Sur” denotes the mono-static RCS curve of divisional rough surface; the legend “PEC” and “Com” denote the mono-static RCS curve of PEC missile and composite missile, respectively.

In Figure 16, in the curve of mono-static RCS of divisional rough surface, we can easily find that there is an obvious peak value at the scattering angle of \(0^\circ\) in spite of the existence of ground region. In general, if the environment is pure ground rough surface, the large roughness will lead to a strong energy distribution region near the scattering angle of \(0^\circ\), rather than a peak value. The unique scattering characteristic of divisional rough surface has been analyzed in detail in example C, and this kind of environment has an important influence on the total field. As we seen, the curves of “PEC+Sur” and “Com+Sur” are similar to the curve of divisional rough surface, and there are just a few discrepancies existing among them at some scattering angles. The effect of the nose’s material changing on the total field is not obvious. Thus, a further comparison has to be made to show the effect of dielectric part on the target’s scattering characteristics.

In Figure 16, the material of missile’s nose will affect the scattering characteristic of the whole target. When the PEC nose is replaced by the Die1, the scattering intensity from the head-on direction (when \( \theta_i = 90^\circ \)) has a sharp decline about \(5.216\text{dB} \). The distribution of scattering intensity varies greatly except for the angle range from \(-30^\circ\) to \(4^\circ\). There are two obvious changes in the angle range: one is from \(-90^\circ\) to \(-60^\circ\), and the other is \(58^\circ\) to \(77^\circ\). These phenomena demonstrate the conclusion that the dielectric material is beneficial to the stealth performance of missile in the head-on direction.

**IV. CONCLUSION**

In this paper, the electromagnetic scattering characteristics of combined target above coastal environment are studied. Due to the multiple media parameters and large amount of unknowns in this composite scattering problem, the hybrid SIE-KA-MLFMA method is proposed. The analysis for the memory requirement and computational complexity of proposed method is discussed in detail, which illustrates the superiority of this hybrid method theoretically. Then, corresponding numerical simulation demonstrates that the proposed algorithm is accurate and efficient. The difference of EM scattering from combined conducting and dielectric target above ground rough surface, sea surface and divisional rough surface are compared. The results of composite scattering show a unique phenomenon which is a fusion of sea surface’s specular scattering characteristic and ground rough surface’s diffuse scattering characteristic. Moreover, the influences of polarization mode, working frequency,
RMS and correlation length of ground region, wind velocity of near sea region, relative permittivity of target’s dielectric part, target’s height and target category on composite EM scattering characteristic are discussed in detail. These phenomena prove several important conclusions: first, specular scattering characteristic mainly depends on the properties of coastal environment; second, the dielectric part of an object will reduce its own scattering capability whether in azimuth or ranging path; third, the scattering intensity from pure dielectric object is much weaker than that from pure PEC object; finally, the dielectric part in combined object with bigger relative permittivity will lead to stronger backscattering.

Hence, the proposed hybrid SIE-KA-MLFA method could be applied as an efficient solver which is suitable for calculating the EM scattering of combined conducting and dielectric object above the coastal environment. In addition, the new laws and characteristics found in these computation examples will have important guidance significance and application value in the field of radar target detection, remote sensing, even stealth technology etc. The main limitations of the proposed method are follows: firstly, it is lack of capability to solve the EM scattering problems of combined target with multi-layered environment; secondly, the uniform dissection rule of the whole scattering model still leads to a large number of environmental unknowns. In order to further reduce the environmental unknowns, the domain decomposition method would be introduced to the improvements of the hybrid method in our future researches. The further works of our research group will focus on the scattering problems about multiple combined target with layered environment or divisional environment with inhomogeneous medium surface, which will simulate more complex situations to explore their inherent laws. The related applications in the field of remote sensing will be studied, and the concrete technologies like radar imaging and target recognition will be further developed in our future works.

APPENDIX

\[ U_{im}^{\text{D44}} = M_{jm}^{\text{D44}} = -\left(\frac{k_0}{16\pi^2}\right)^2 \int_{\Gamma_{\text{di}}} e^{-j\vec{k}\cdot\vec{r}_{\text{m}}} f_{\text{di}}(r_{\text{m}}) \cdot \hat{n}_{\text{di}} \cdot \hat{\gamma} dS \]  

\[ V_{jm}^{\text{D44}} = \int_{\Gamma_{\text{ij}}} e^{j\vec{k}\cdot\vec{r}_{\text{m}}} \left[ f_{\text{ij}}(r_{\text{m}}) \cdot \hat{\gamma}\right] \cdot \hat{n}_{\text{ij}} \cdot \hat{\gamma} dS' \]  

\[ N_{jm}^{\text{D44}} = \int_{\Gamma_{\text{ij}}} e^{j\vec{k}\cdot\vec{r}_{\text{m}}} \left[ f_{\text{ij}}(r_{\text{m}}) \cdot \hat{\gamma}\right] \cdot \hat{n}_{\text{ij}} \cdot \hat{\gamma} dS' \]  

\[ U_{im}^{\text{E44}} = \left(\frac{k_0^2}{16\pi^2}\right)^2 \int_{\Gamma_{\text{di}}} e^{-j\vec{k}\cdot\vec{r}_{\text{m}}} f_{\text{di}}(r_{\text{m}}) \cdot \hat{n}_{\text{di}} \cdot \hat{\gamma} dS \]  

\[ V_{jm}^{\text{E44}} = \int_{\Gamma_{\text{ij}}} e^{j\vec{k}\cdot\vec{r}_{\text{m}}} \left[ f_{\text{ij}}(r_{\text{m}}) \cdot \hat{\gamma}\right] \cdot \hat{n}_{\text{ij}} \cdot \hat{\gamma} dS' \]  

\[ N_{jm}^{\text{E44}} = \int_{\Gamma_{\text{ij}}} e^{j\vec{k}\cdot\vec{r}_{\text{m}}} \left[ f_{\text{ij}}(r_{\text{m}}) \cdot \hat{\gamma}\right] \cdot \hat{n}_{\text{ij}} \cdot \hat{\gamma} dS' \]  

\[ U_{im}^{\text{E22}} = \left(\frac{k_0^2}{16\pi^2}\right)^2 \int_{\Gamma_{\text{di}}} e^{-j\vec{k}\cdot\vec{r}_{\text{m}}} f_{\text{di}}(r_{\text{m}}) \cdot \hat{n}_{\text{di}} \cdot \hat{\gamma} dS \]  

\[ V_{jm}^{\text{E22}} = \int_{\Gamma_{\text{ij}}} e^{j\vec{k}\cdot\vec{r}_{\text{m}}} \left[ f_{\text{ij}}(r_{\text{m}}) \cdot \hat{\gamma}\right] \cdot \hat{n}_{\text{ij}} \cdot \hat{\gamma} dS' \]  

\[ N_{jm}^{\text{E22}} = \int_{\Gamma_{\text{ij}}} e^{j\vec{k}\cdot\vec{r}_{\text{m}}} \left[ f_{\text{ij}}(r_{\text{m}}) \cdot \hat{\gamma}\right] \cdot \hat{n}_{\text{ij}} \cdot \hat{\gamma} dS' \]

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