CHARGINO PAIR PRODUCTION AT CERN LEP II

S.Y. Choi

Department of Physics, Yonsei University, Seoul 120-749, Korea

Abstract

Charginos are expected to be the lightest observable supersymmetric particles in many of the supersymmetric models. In the scenario that lighter charginos are pair-produced at CERN LEP II, we present a straightforward procedure for determining the SUSY parameters, \( \tan \beta, M_2, \mu \), and the electron sneutrino mass \( m_{\tilde{\nu}_e} \) up to a four-fold discrete ambiguity, although a large number of unknown SUSY parameters are involved in chargino decays.

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One of the main goals of the CERN $e^+e^-$ collider at LEP II is to search for signs of weak-scale supersymmetry (SUSY). Among many SUSY particles that might be found, the chargino, a mixture of the $W$-ino and charged Higgsinos, is of particular interest. From the theoretical point of view, charginos are expected to be lighter than gluinos and most sfermions, and for this reason, chargino searches have been well studied. The current lower bound on the lighter chargino mass is about 90 GeV. Chargino discovery studies have shown that the discovery reach will extend nearly to the LEP II kinematical limit. Moreover, if kinematically accessible, charginos have a large cross section throughout SUSY parameter space and produce a clean signal in certain decay modes. As the chargino pair production cross section rises rapidly above threshold, each step in collider energy holds the promise not only of chargino discovery, but also of detailed SUSY studies from chargino events.

The purpose of the present work is to estimate the capability of LEP II to determine the parameters of SUSY in lighter chargino pair production in the Minimal Supersymmetric Standard Model (MSSM). This issue has been studied by Leike, Diaz and King and recently by Feng and Strassler. Our approach here is to employ the most dominant chargino decay modes to extract the full possible information on chargino polarizations, while avoiding theoretical assumptions at high energy scales, and exploiting the fact that the chargino-pair production process depends on only a small subset of the SUSY parameters. In this case, although chargino decays are very complicated with many diagrams involved, certain final-state angular correlations, which are experimentally identifiable, allow us to construct three additional observables besides the production cross section so that all the SUSY parameters relevant to chargino pair production are determined up to a discrete ambiguity.

The MSSM includes the usual matter superfields and two Higgs doublet superfields $\hat{H}_1$ and $\hat{H}_2$, which give masses to the isospin $-\frac{1}{2}$ and $+\frac{1}{2}$ fields, respectively. These two superfields are coupled in the superpotential through the term $-\mu e_{ij} \hat{H}_1^i \hat{H}_2^j$, where $\mu$ is the supersymmetric Higgs boson mass parameter. The ratio of the two Higgs scalar vacuum expectation values is defined to be $\tan \beta \equiv \langle H_2^0 \rangle / \langle H_1^0 \rangle$. There are two chargino mass eigenstates that result from the mixing of the electroweak gauginos $\tilde{W}^\pm$ with the Higgsinos due to the spontaneous electroweak symmetry breaking. The chargino mass term is

$$- \mathcal{L}_m = (\tilde{W}_R^- \tilde{H}_{2R}^-) \begin{pmatrix} M_2 & \sqrt{2m_W \cos \beta} \\ \sqrt{2m_W \sin \beta} & \mu \end{pmatrix} \begin{pmatrix} \tilde{W}_L^- \\ \tilde{H}_{1L}^- \end{pmatrix} + \text{h.c.} \quad (1)$$

We assume that the gaugino masses $M_i$ ($i = 1, 2$) and the parameters $\mu$ and $\tan \beta$ are real so that $CP$ violation plays no role in chargino events. Then the $2 \times 2$ complex chargino mass matrix can be diagonalized by two orthogonal matrices $O_L$ and $O_R$ defined by two rotation angles $\phi_L$ and $\phi_R$, respectively, which appear at the couplings between the gauge boson $Z$ and the charginos.

In general, studying MSSM is complicated because of many free parameters so
that we make the following assumptions; (i) $R$ parity is conserved, (ii) the lightest supersymmetric particle (LSP) is the lightest neutralino $\tilde{\chi}^0_1$, (iii) sfermions have masses beyond the LEP II kinematical limit, and (iv) the intergenerational mixing in the sfermion and quark sectors is small and may be neglected. Under our assumptions, for almost all values of parameters, charginos decay to three-body final states consisting of an LSP and either two quarks or two leptons at the LEP II c.m. energy.

One typical observable which is crucial to our analysis is the chargino mass $m_{\tilde{\chi}^\pm_1}$. The mass $m_{\tilde{\chi}^\pm_1}$ along with the LSP mass $m_{\tilde{\chi}^0_1}$ can be measured by the dijet energy distribution in hadronic chargino decays\textsuperscript{[1]}. The end points of the dijet energy $E_{jj}$ and mass spectra are completely determined by $m_{\tilde{\chi}^\pm_1}$ and $m_{\tilde{\chi}^0_1}$ with the maximal and minimal dijet energies, $E_{\text{max}}$ and $E_{\text{min}}$, as

$$m_{\tilde{\chi}^\pm_1} = \sqrt{\frac{E_{\text{max}} E_{\text{min}}}{E_{\text{max}} + E_{\text{min}}} \sqrt{s}}, \quad m_{\tilde{\chi}^0_1} = \sqrt{\frac{1 - 2(E_{\text{max}} + E_{\text{min}})}{\sqrt{s}}}.$$ \hspace{1cm} (2)

The distribution of the invariant mass $m_{jj}$ is between zero and $m_{\tilde{\chi}^\pm_1} - m_{\tilde{\chi}^0_1}$. If at least two of the three end points are sufficiently sharp to be well measured, they can be used to precisely determine $m_{\tilde{\chi}^\pm_1}$ and $m_{\tilde{\chi}^0_1}$.

The amplitude for producing charginos which decay to a certain final state does not factorize into production and decay amplitudes due to the fact that charginos are spin-$\frac{1}{2}$ objects so that the angular correlations of chargino decay products are affected by the underlying structures of the production processes. In the present work, we will consider the production process $e^+e^- \rightarrow \tilde{\chi}^-_1 \tilde{\chi}^+_1$ followed by the decays $\tilde{\chi}^-_1 \rightarrow \tilde{\chi}^0_1 f_1 \bar{f}_2$ and $\tilde{\chi}^+_1 \rightarrow \tilde{\chi}^0_1 f_3 \bar{f}_4$ where $f, f'(g, g')$ are for quarks and/or leptons, and then we can write the amplitude for the sequential process in the narrow width approximation as a multiplication of the production helicity amplitudes $T_{\sigma,\lambda,\bar{\lambda}}$ and two decay helicity amplitudes $D_\lambda$ and $\bar{D}_{\bar{\lambda}}$ for the negative and positive charginos, where $\sigma = \pm$ is the electron helicity, $\lambda, \bar{\lambda}$ are the negative and positive chargino helicities. With the fermion masses neglected, $\tilde{\chi}^-_1 \rightarrow \tilde{\chi}^0_1 f_1 \bar{f}_2$ includes two sfermion-exchange diagrams and one $W$-boson exchange, and the decay amplitudes depend on only the invariant mass of two final fermions because the effects of the sfermions can be well approximated by point propagators\textsuperscript{[3]} under the assumption that sfermions have masses beyond the LEP II kinematical limit.

In order to evaluate the chargino decay helicity amplitudes, we take the chargino rest frames and introduce the angular variables ($\theta^*, \phi^*$) and ($\theta^*, \bar{\phi}^*$) for the two-fermion systems in the negative and positive chargino decays, respectively. Maintaining only the angular dependence and integrating the decay distributions over the invariant masses $q^2_W$ and $\bar{q}^2_W$, we find that when the $f_3 \bar{f}_4$ system is charge-conjugate

\hspace{1cm} a\ An alternate determination of $m_{\tilde{\chi}^\pm_1}$ can be provided by an energy scan at the chargino production threshold\textsuperscript{[4]}. \hspace{1cm}
to the $f_1 \tilde{f}_2$ system, the decays for polarized negative and positive charginos are described by the decay density matrices

$$
\rho_{\lambda\lambda'} = \frac{D_{\lambda} D_{\lambda'}^*}{\sum_{\lambda} D_{\lambda} D_{\lambda'}^*} = \frac{1}{2} \left( \begin{array}{cc} 1 + \kappa \cos \theta^* & \kappa \sin \theta^* e^{i\phi^*} \\ \kappa \sin \theta^* e^{-i\phi^*} & 1 - \kappa \cos \theta^* \end{array} \right),
$$

$$
\bar{\rho}_{\bar{\lambda}\bar{\lambda}'} = \frac{\bar{D}_{\bar{\lambda}} \bar{D}_{\bar{\lambda}'}^*}{\sum_{\bar{\lambda}} \bar{D}_{\bar{\lambda}} \bar{D}_{\bar{\lambda}'}^*} = \frac{1}{2} \left( \begin{array}{cc} 1 - \kappa \cos \bar{\theta}^* & \kappa \sin \bar{\theta}^* e^{i\bar{\phi}^*} \\ \kappa \sin \bar{\theta}^* e^{-i\bar{\phi}^*} & 1 - \kappa \cos \bar{\theta}^* \end{array} \right),
$$

(3)

respectively. The parameter $\kappa$, which determines the angular dependence of the decay distributions, is a function of chargino and neutralino mixing parameters, chargino and neutralino masses, and sfermion masses so that there is a very wide variety in estimating the parameter value. Therefore, we consider $\kappa$ as a phenomenological parameter in our analysis, which is not fixed.

Combining the production and decay amplitudes yields a five-dimensional differential cross section consisting of a kinematical factor and an angular-dependent part $\Sigma(\Theta; \theta^*, \phi^*; \bar{\theta}^*, \bar{\phi}^*)$ where $\Theta$ is the $\tilde{\chi}^-_1$ production angle. The angular dependence $\Sigma$ for the case that two two-fermion systems are charge-conjugate to each other is decomposed into eight independent parts:

$$
\Sigma = \Sigma_{\text{unpol}} + (\cos \theta^* + \cos \bar{\theta}^*)\kappa P + \cos \theta^* \cos \bar{\theta}^* \kappa^2 Q \\
+ (\sin \theta^* \cos \bar{\phi}^* - \sin \theta^* \cos \phi^*)\kappa W \\
+ (\cos \theta^* \sin \bar{\theta}^* \cos \bar{\phi}^* - \cos \theta^* \sin \theta^* \cos \phi^*)\kappa^2 X \\
+ \sin \theta^* \sin \bar{\theta}^* \cos(\phi^* + \bar{\phi}^*)\kappa^2 Y + \sin \theta^* \sin \bar{\theta}^* \cos(\phi^* - \bar{\phi}^*)\kappa^2 Z,
$$

(4)

where the eight distribution functions describing chargino production are defined in terms of the production helicity amplitudes $T_{\sigma,\lambda\bar{\lambda}}$ (see Ref. 7 for the definition of these distribution functions.).

Certainly not all of the distribution functions are experimentally measurable. As for the reconstruction problems in chargino production and decays we have to take into account the following aspects. First of all, because the LSP is not detected, there exists at least a two-fold discrete ambiguity in determining the scattering angle $\Theta$ for the hadronic decay modes of the negative and positive charginos. So, we consider the distributions integrated over the scattering angle $\Theta$. Nevertheless, $\cos \theta^*$, $\cos \bar{\theta}^*$, and $\sin \theta^* \sin \bar{\theta}^* \cos(\phi^* - \bar{\phi}^*)$ are measurable experimentally through the relations

$$
\cos \theta^* = \frac{1}{\beta q^*} \left( \frac{E}{\gamma} - E^* \right), \quad \cos \bar{\theta}^* = \frac{1}{\beta \bar{q}^*} \left( \frac{\bar{E}}{\gamma} - \bar{E}^* \right), \\
\sin \theta^* \sin \bar{\theta}^* \cos(\phi^* - \bar{\phi}^*) = \cos \vartheta - \frac{(E - E^*) (\bar{E} - \bar{E}^*)}{\beta^2 q \bar{q} \gamma},
$$

(5)

where $(E, q)$ and $(\bar{E}, \bar{q})$ are the energy and absolute momentum of two two-fermion systems in the laboratory frame, $\gamma = \sqrt{s}/2m_{\tilde{\chi}^\pm_1}$, the angle $\vartheta$ is the angle between
the two hadronic systems, and the superscript \( \ast \) denotes energy and momenta in the chargino rest frames. Secondly, \( P \) is parity-odd and \( C \)-odd so that its determination requires charge identification of charginos. This can be accomplished by the mixed mode of the leptonic and hadronic decays of the negative and positive charginos where only the lepton charge is needed to be identified. On the other hand, the \( C \)-even \( Q \) and \( Z \) do not require charge identification so that the dominant hadronic decay mode for both negative and positive charginos can be used. Therefore, if the total cross section \( \sigma_{\text{tot}} \) and the total leptonic and hadronic branching ratios of the chargino decays are experimentally determined, we can have three additional independent observables integrated over the scattering angle \( \Theta \); \( \kappa \langle P \rangle \), \( \kappa^2 \langle Q \rangle \), and \( \kappa^2 \langle Z \rangle \). However, the parameter \( \kappa \) is not known so that it should be factored out by taking the ratios of those observables. Consequently, there are four reconstructible observables:

\[
\begin{align*}
m_{\tilde{\chi}_1^{\pm}}, & \quad \sigma_{\text{tot}}(e^+e^- \rightarrow \tilde{\chi}_1^+\tilde{\chi}_1^-), \quad \frac{\langle P \rangle^2}{\langle Q \rangle}, \quad \frac{\langle Z \rangle}{\langle Q \rangle}.
\end{align*}
\]

(6)

Since the chargino-pair production is described by the four parameters, \( \cos 2\phi_L \), \( \cos 2\phi_R \), \( \tan \beta \), and \( m_{\tilde{\nu}} \), these four independent observables are expected to allow us to determine the SUSY parameters, \( \tan \beta \), \( M_2 \) and \( \mu \) (almost) completely. In order to explicitly demonstrate the possibility of measuring the SUSY parameters, we make a case study with the assumption that the experimentally measured values of the four observables at \( \sqrt{s} = 200 \text{ GeV} \) are

\[
m_{\tilde{\chi}_1^{\pm}} = 90 \text{ GeV}, \quad \sigma_{\text{tot}} = 2.5 \text{ pb}, \quad \frac{\langle P \rangle^2}{\langle Q \rangle} = -0.05 \text{ pb}, \quad \frac{\langle Z \rangle}{\langle Q \rangle} = 0.1.
\]

(7)

For the ideal case with no experimental errors the observed values yield \( m_{\tilde{\nu}} = 197 \text{ GeV}, \cos 2\phi_L = 0.14 \) and \( \cos 2\phi_R = 0.03 \).

However, both the systematic and statistical errors should be considered in real experimental situations. Also, the resolution power of the observables constructed from angular correlations depends on the unknown parameter \( \kappa \) rather strongly, and so its genuine estimates require prior understandings about the quantities such as the neutralino mass matrix and sfermion mass spectra. Fig. illustrates how those errors may affect the determination of the cosines of two mixing angles under the assumption that \( \Delta \sigma_{\text{tot}} = \pm 0.2 \text{ pb}, \Delta[\langle P \rangle^2/\langle Q \rangle] = \pm 0.02 \text{ pb}, \) and \( \Delta[\langle Z \rangle/\langle Q \rangle] = \pm 0.02 \) for \( \sqrt{s} = 200 \text{ GeV} \) with a precisely measured chargino mass \( m_{\tilde{\chi}_1^{\pm}} = 90 \text{ GeV} \).

From the measured values of \( \cos 2\phi_L \), \( \cos 2\phi_R \) and \( m_{\tilde{\chi}_1^{\pm}} \), we can extract the allowed values for \( \tan \beta \), \( M_2 \) and \( \mu \). Defining \( x \) and \( y \) satisfying \( M_2 = m_W(x + y) \) and \( \mu = m_W(x - y) \), we find that there are four different solutions of \( x \) and \( y \) for give \( \cos 2\phi_L \) and \( \cos 2\phi_R \): 

\[
(a) \quad \begin{cases} 
  x = \pm \cot(\phi_R - \phi_L) \sin \left( \beta - \frac{\pi}{4} \right), \\
  y = \pm \cot(\phi_R + \phi_L) \cos \left( \beta - \frac{\pi}{4} \right),
\end{cases}
\]
Figure 1: A contour plot for $\sigma_{tot} = 2.5 \pm 0.2$ pb, $\langle P \rangle^2/\langle Q \rangle = -0.05 \pm 0.02$ pb, and $\langle Z \rangle/\langle Q \rangle = 0.1 \pm 0.02$ for $m_{\tilde{\chi}^\pm_1} = 90$ GeV and $m_\nu = 197$ GeV at $\sqrt{s} = 200$ GeV.

\[
(b) \begin{cases} 
  x = \pm \cot(\phi_R + \phi_L) \sin \left( \beta - \frac{\pi}{4} \right), \\
  y = \pm \cot(\phi_R - \phi_L) \cos \left( \beta - \frac{\pi}{4} \right), 
\end{cases}
\]  

(8)

Depending on whether it is larger or smaller than the unity, the value of $\tan \beta$ is determined from $m_{\tilde{\chi}^\pm_1}$, $\cos 2\phi_L$ and $\cos 2\phi_R$, through the relations

\[
\tan \beta = \begin{cases} 
  \mp \frac{p^2 - q^2 \pm 2pq \sqrt{1+\eta \sqrt{p^2+q^2-(1+\eta)p^2q^2}}}{2(1+\eta)p^2q^2-(p-q)^2} & \text{for } \tan \beta \geq 1 \\
  \mp \frac{p^2 - q^2 \pm 2pq \sqrt{1+\eta \sqrt{p^2+q^2-(1+\eta)p^2q^2}}}{2(1+\eta)p^2q^2-(p+q)^2} & \text{for } \tan \beta \leq 1, 
\end{cases}
\]

(9)

where the overall $\mp$ is for (a) and (b) in Eq. (8), $\eta = m_{\tilde{\chi}^\pm_1}/m_W$, $p^2 - q^2 = \sqrt{(1 - \cos^2 2\phi_L)(1 - \cos^2 2\phi_R)}$, $pq = |\cos 2\phi_L - \cos 2\phi_R|/2$, and $p^2 + q^2 = 1 - \cos 2\phi_L \cos 2\phi_R$. Certainly only the $\tan \beta$ value satisfying each condition should be taken in Eq. (9). To conclude, there exists at most a four-fold discrete ambiguity in determining $\tan \beta$, $M_2$, and $\mu$ in the pair production of lighter charginos at CERN LEP II.

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