Holomorphic Tachyons and Fractional D-branes

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Abstract

We study tachyon condensation on brane-antibrane systems in orbifold theories from the viewpoint of boundary string field theory. We show that the condensation of holomorphic tachyon fields generates various fractional D-branes. The boundary N=2 supersymmetry in the world-sheet theory ensures this result exactly. Furthermore, our results are consistent with the twisted RR-charges from detailed calculations of boundary states. We also discuss the generation of RR-charges due to holomorphic tachyon fields on multiple brane-antibrane pairs in flat space.

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1 Introduction

Tachyon fields naturally appear in open string theory if we consider various configurations of D-branes. For example, brane-antibrane systems and non-BPS D-branes in Type II superstring theory indeed have tachyon fields. Since the presence of tachyon means the instability of the system, the condensation of tachyon is very important to know the dynamical aspects of string theory.

Recently, tachyon condensation in open string theory has been intensively studied, pioneered by Sen (for a review see [5]). Sen conjectured that if the tachyon fields on these unstable D-brane systems condense into the bottom of the tachyon potential, then the negative energy density exactly cancels the D-brane tension [3]. After this conjecture was proposed, the tachyon potentials for various unstable brane systems have been studied by applying open string field theories and the off-shell structures for various unstable brane systems have been revealed. Calculations with a good approximation called level truncation scheme have been performed in Witten’s cubic string field theory and Berkovits’s superstring field theory [8]. The obtained tachyon potentials agree with the Sen’s conjecture (for example see [9] and also refer to [10] for a review of string field theory approach to tachyon condensation).

Another open string field theory which has been applied to tachyon condensation is the boundary string field theory (or background independent open string field theory [11, 12]). In this theory one has only to discuss a finite number of string fields because the string fields which have expectation values are considered to correspond to only the relevant and marginal perturbations on the world-sheet. For example, one can calculate the exact tachyon potential [13, 14]. Though the validity of this truncation has not been proved completely, one can compute tachyon condensation for specific tachyon fields without any approximation and the result agrees with the Sen’s conjecture exactly [14]. Motivated by the previous results on the world-sheet σ-model approach [13], this formulation has also been generalized for non-BPS D-branes and brane-antibrane systems in superstring theory.

Most of these recent developments in open string field theories are restricted to unstable D-brane systems in non-compact flat backgrounds. However, if one wants to know the geometrical aspects of tachyon condensation, one should challenge curved backgrounds.

Some results of tachyon condensations in curved space have already been obtained. For example, tachyon condensation as marginal deformations has been studied for Z2-orbifolds [23, 24, 25]. The condensation of holomorphic tachyon fields has been discussed in more general (Ricci flat) Kähler manifolds [26, 27, 28]. For tachyon condensa-
tion in SU(2) WZW model see also [28, 29]. The approach which utilizes noncommutative geometry [30] also has been applied to various compact spaces [28, 31, 32].

Investigations of this problem may also be useful for the understanding of substringy geometry for D-branes, which is called “D-geometry” (for example see reviews [33] and references therein). Since the condensation of topologically non-trivial tachyon field generates lower dimensional D-branes [21, 34], one can regard a D-brane roughly as a tachyon field on higher dimensional space. If one handles the tachyon fields in boundary string field theory (BSFT), this will give another stringy description of D-branes and this will be useful to obtain D-geometry.

If we would like to get a BPS D-brane from a brane-antibrane system, it is natural to require that the tachyon field $T$ should be holomorphic [26, 27, 17]. This fact seems to be correct in BSFT if one takes the large volume limit because the world-sheet theory becomes localized at $T = 0$ [35] after the tachyon condensation. Then this equation gives the “holomorphic cycle” (or divisor) on which the BPS D-brane [30] is wrapped [17]. If one would like to discuss this requirement in the world-sheet theory, holomorphy of tachyon fields is equally stated as the boundary (B-type) $\mathcal{N} = 2$ supersymmetry [37] on the world-sheet [17]. On the other hand, if we consider the backgrounds where stringy corrections do exist, then the above arguments will be modified. Therefore the investigation of tachyon condensation in BSFT with the $\mathcal{N} = 2$ supersymmetry is very interesting in the stringy regions.

As a first step of this, in this paper we discuss tachyon condensations in orbifold theories from the viewpoint of boundary string field theory. We consider tachyon fields which preserve the boundary B-type $\mathcal{N} = 2$ supersymmetry (“holomorphic tachyon”). After the tachyon condensation we obtain various fractional D-branes. We can identify the decay products completely by combining the boundary string field theory with some results from boundary state calculations. From this argument we obtain intriguing identities for the characters of the discrete groups which define orbifolds. The world-sheet extended supersymmetry ensures these results of tachyon condensation exactly. Further if we resolve the orbifold singularities, then the final states are regarded as BPS D-branes which are wrapped on various holomorphic cycles. Thus we see again the correspondence between BPS D-branes and holomorphic tachyon fields. In this paper we mainly discuss only $\mathbb{Z}_N$-orbifolds for simplicity.

The paper is organized as follows. In section 2, we first review some known facts on the relation between tachyon condensation in brane-antibrane systems and $\mathcal{N} = 2$ supersymmetry. Further we discuss tachyon condensation for multiple brane-antibrane pairs and generation of RR-charges on flat space. In section 3, we investigate tachyon condensation on orbifolds from the viewpoint of boundary string field theory. In section 4,
we conclude with some future directions. In appendix A, we show the explicit calculations of boundary state.

2 Tachyon Condensation in BSFT and Holomorphy of Tachyon

In this section we first review tachyon condensation on brane-antibrane systems in flat background within the framework of BSFT [11, 12, 13, 14, 16, 17, 18, 19] and we next investigate the generation of D-brane charges from various brane-antibrane systems. We obtain the topological configurations which generalize the Atiyah-Bott-Shapiro construction [38]. In particular we are interested in the relation between the boundary $\mathcal{N} = 2$ supersymmetry and tachyon condensations, which was first discussed in [17]. Through the paper we use the language of Type IIA theory, but all the arguments can be applied to Type IIB theory straightforwardly.

2.1 Tachyon Condensation on a Brane and an Antibrane

In BSFT for superstring, $\mathcal{N} = 1$ superconformal symmetry is preserved in the bulk of world-sheet, but its conformal symmetry is broken at the boundary due to boundary interactions. In other words, only the boundary can be off-shell and the open string fields are expressed as boundary interactions. Then the boundary interactions which describe the tachyon condensation should naturally preserve $\mathcal{N} = 1$ supersymmetry. To realize this supersymmetry one needs extra fermionic fields [34, 35] on the boundary and this freedom corresponds to Chan-Paton factors of non-BPS D-branes and brane-antibrane systems. They are called boundary fermions and we write them by $\eta, \bar{\eta}$ (complex fermion) for a brane-antibrane and $\eta$ (real fermion) for a non-BPS D-brane. Then the world-sheet action $I$ for a brane-antibrane system in flat space is given by [33, 17, 18, 19]

$$I = I_0 + I_B,$$  \hspace{1cm} (2.1)

$$I_0 = \frac{1}{2\pi\alpha'} \int_{\Sigma} d^2w [\partial_w X^\mu \partial_{\bar{w}} X_\mu + \psi_L^\mu \partial_w \psi_{L\mu} + \psi_R^\mu \partial_w \psi_{R\mu}],$$  \hspace{1cm} (2.2)

$$I_B = \int_{\partial \Sigma} d\tau d\theta \left[ -\bar{\Gamma} \bar{D}_\theta \Gamma + \frac{1}{\sqrt{2\pi}} \Gamma T(X^a) + \frac{1}{\sqrt{2\pi}} T(X^a) \bar{\Gamma} \right],$$  \hspace{1cm} (2.3)

where $w, \bar{w}$ denote the coordinates of (Euclidean) world-sheet $\Sigma$ and $\tau = w + \bar{w}$ denotes the boundary coordinate; we define $X^\mu = X_L^\mu + X_R^\mu, \psi_L^\mu, \psi_R^\mu (\mu = 0 \sim 9)$ as the familiar bosonic and fermionic (left-moving and right-moving) fields on the world-sheet. The
tachyon field \( T(X^a), \overline{T}(X^a) \) depends only on the coordinates \( X^a \) which are along the world-volume of the brane-antibrane.

Here we have used \( \mathcal{N} = 1 \) superspace formulation at the boundary of world-sheet as follows

\[
\begin{align*}
X^\mu &= X^\mu + 2i\theta \psi^\mu \quad (\psi^\mu \equiv \frac{1}{2}(\psi_R^\mu + \psi_L^\mu)|_{\partial \Sigma}), \\
\Gamma &= \eta + \theta F, \\
\overline{\Gamma} &= \overline{\eta} + \theta \overline{F}, \\
D_\theta &= \frac{\partial}{\partial \theta} + \theta \frac{\partial}{\partial \tau}.
\end{align*}
\]

(2.4)

Note that \( \Gamma \) is the \( \mathcal{N} = 1 \) superfield for the boundary fermion \( \eta \).

If we write the boundary interactions \( I_B \) in the component form and integrate out the auxiliary fields \( F, \overline{F} \), then we get:

\[
I_B = \int_{\partial \Sigma} d\tau \left[ \overline{\eta} \dot{\eta} - i \sqrt{\frac{2}{\pi}} \overline{\eta} \psi^\mu \partial_\mu \overline{T} + i \sqrt{\frac{2}{\pi}} \psi^\mu \eta \partial_\mu T + \frac{1}{2\pi} \overline{T} \overline{T} \right].
\]

(2.5)

From this it is easy to see that after the quantization of boundary fermions

\[
\{ \eta, \overline{\eta} \} = 1,
\]

(2.6)

the Chan-Paton factors \( \sigma_+ = \frac{1}{2}(\sigma_1 + i\sigma_2), \sigma_- = \frac{1}{2}(\sigma_1 - i\sigma_2) \) and \( \sigma_3 \) correspond to \( \overline{\eta}, \eta \) and \( [\overline{\eta}, \eta] \), respectively. This explains the correct degree of freedom of Chan-Paton factors (2 \times 2 matrices) for a brane-antibrane. The above action includes only the perturbations which represent the tachyon field \( T, \overline{T} \). Furthermore, one can also incorporate the gauge fields which correspond to the Chan-Paton factors 1 and \( \sigma_3 \), but we will set these fields to zero in this paper.

Also the world-sheet action for non-BPS D-branes can be easily obtained if one applies the descent relation \([23]\). This relation says that if one performs the \( \mathbb{Z}_2 \) projection of the boundary interactions \( \Gamma = \overline{\Gamma} \) for a brane-antibrane, then one gets those for a non-BPS D-brane \([33], [16]\).

Now let us require \( \mathcal{N} = 2 \) world-sheet supersymmetry. For example, this supersymmetry is preserved for Calabi-Yau compactifications as is well-known. In order to investigate D-branes in these examples, it is natural to consider a boundary analog of such an extended supersymmetry, though this is not generic. If we get to an on-shell point after the tachyon condensation, then this supersymmetry will be enhanced into \( \mathcal{N} = 2 \) boundary superconformal symmetries, which are classified into A-type and B-type superconformal symmetry \([37]\). These coincide with the classification of BPS D-branes in Calabi-Yau spaces in the large volume limit \([36]\). Therefore it will be particularly interesting to consider the boundary interactions which preserve this symmetry. Then what
kinds of tachyon fields satisfy this requirement? It was pointed out in the paper [17] that the B-type supersymmetry is not broken if one considers holomorphic tachyon field for brane-antibrane systems.

More concretely, the boundary interaction which preserves B-type \( \mathcal{N} = 2 \) supersymmetry (below we will omit the word ‘B-type’ and simply call this \( \mathcal{N} = 2 \) supersymmetry) can be written [17] as follows (for earlier relevant work see also [39])

\[
I_B = -\int_{\partial \Sigma} d\tau d\theta d\bar{\theta} \Gamma + \int_{\partial \Sigma} d\tau d\theta \frac{1}{\sqrt{2\pi}} \Gamma T(Z^i) + (\text{h.c.}),
\]

(2.7)

where we have employed \( \mathcal{N} = 2 \) boundary superspace \((\tau, \theta, \bar{\theta})\) and the boundary fermionic chiral and antichiral superfield \( \Gamma, \bar{\Gamma} \) are defined in our conventions as

\[
\Gamma = -\frac{i}{\sqrt{2}} \eta + \theta F - \frac{i}{\sqrt{2}} \bar{\theta} \partial_\tau \eta,
\]

\[
\bar{\Gamma} = \frac{i}{\sqrt{2}} \bar{\eta} + \bar{\theta} \bar{F} - \frac{i}{\sqrt{2}} \theta \bar{\partial}_\tau \bar{\eta}.
\]

(2.8)

Note that the tachyon field \( T(Z^i) \) depends only on the holomorphic coordinates \( Z^i = X^{2i-1} + iX^{2i} \) along the world-volume in order to preserve \( \mathcal{N} = 2 \) boundary supersymmetry.

The most interesting issue of \( \mathcal{N} = 2 \) supersymmetry is the fact that the boundary superpotential term \( \sim \int_{\partial \Sigma} d\tau d\theta \Gamma T(Z^i) \) is not renormalized as argued in [17]. On the other hand the kinetic term for the boundary fermion is included in the boundary D-term and will receive quantum corrections. We assume that the contributions from the D-term are not singular and therefore the potential term dominates the D-term after the tachyon condensation \(|T_i| \rightarrow \infty\). For example, let us assume the following holomorphic tachyon field on a \( D2 - \bar{D2} \) which is extended in \( Z^1(\equiv Z) \) direction [17]:

\[
T(Z) = \sum_{k=0}^{p} a_k \cdot Z^k = a_p \prod_{k=1}^{p} (Z - z_k).
\]

(2.9)

Then the values of \( \{z_k\} \) are not renormalized. As Sen and Witten argued in [20, 34], if the tachyon field which has a topological charge does condense, then the corresponding lower dimensional D-branes are generated. In our example the tachyon field (2.9) has the winding number \( p \) and thus \( p \) D0-branes should be produced at each point \( z_k \).

Let us see this in BSFT. In superstring theory the spacetime action \( S \) of BSFT is argued to be identified with the disk partition function \( Z_{\text{disk}} \) [15, 16, 17, 18, 19]

\[
S = Z_{\text{disk}} = \int [DX][D\psi][D\eta][D\bar{\eta}] \exp(-I_0 - I_B).
\]

(2.10)

As the tachyon condenses infinitely \( a_p \rightarrow \infty \), the path integrals around the \( p \) fixed points \( Z = z_k \) give dominant contributions to \( Z_{\text{disk}} \). Then the partition function becomes \( p \) times
that for $p = 1$ case [17]. On the other hand, the boundary perturbation for $p = 1$ can be treated within a free theory. Using the results in [10, 18, 19, 17], one can show

$$
\frac{Z(a_1 = 0) \times (\text{Vol})^{-1}}{Z(a_1 = \infty)} = \frac{1}{2\pi^2\alpha'} = \frac{T_{D2-D2}}{T_{D0}},
$$

(2.11)

where $T_{D0}$ and $T_{D2-D2}$ denote the tension of a D0-brane and of a D2 $-$ D2, respectively; Vol denotes the volume of the D2-brane world-volume. Thus after the condensation of tachyon field (2.9), $p$ D0-branes are produced as expected.

Another way to see this is to compute the RR-couplings of a D2 $-$ D2. As discussed in [10, 18, 19, 41], those couplings for a Dp $-$ Dp system in BSFT are written by using Quillen’s superconnection [42] if we ignore the the contributions from non-abelian transverse scalars [19, 43]. They are given by the following formula

$$
S = T_{Dp} \text{Str} \int C \wedge \exp(2\pi\alpha' F)
$$

$$
2\pi\alpha' F = \left( \begin{array}{c}
2\pi\alpha' F^{(1)} - \bar{T}T \\
(i)\frac{3}{2}\sqrt{2\pi\alpha'} D\bar{T} \\
(i)\frac{3}{2}\sqrt{2\pi\alpha'} DT \\
2\pi\alpha' F^{(2)} - T\bar{T}
\end{array} \right),
$$

(2.12)

where Str is supertrace and $F$ is the field strength of superconnection [4]. Then let us compute the RR-coupling which represents the D0-brane charge in the previous example. As shown in [42], continuous deformations of the tachyon field do not change the result. Therefore we can restrict the form of tachyon field to

$$
T(Z) = a_p \cdot Z^p.
$$

(2.13)

Then the integration in (2.12) along the coordinate $Z^1$ does not depend on $a_p$ and we obtain the following RR-coupling

$$
S_{RR} = (i2\pi\alpha')T_{D2} \int C_{D0} \wedge dT \wedge \bar{d}T \ e^{-T\bar{T}}
$$

$$
= 4\pi\alpha' p^2 T_{D2} \int C_{D0} \int_0^\infty 2\pi r dr \ r^{2p-2} \ e^{-r^2p}
$$

$$
= p \ T_{D0} \int C_{D0},
$$

(2.14)

where we have used the relation $T_{D0} = (2\pi)^p(\alpha')^{\frac{3}{2}}T_{Dp}$; the 1-form field $C_{D0}$ denotes the RR-field which couples to D0-branes. Thus we get $p$ units of D0-brane RR-charge matching with the above result.

Next we would like to comment on the higher dimensional generalization. If the tachyon field $T$ depends only on one coordinate (for example, $Z^1$), then the generalization

3We have replaced $T$ with $\bar{T}$ in the reference [19]. Note also that a factor $-1$ in front of $DT$ is different from eq.(4.8) in [19]. This is because here we assume $T$ anticommutes with any odd-forms.
is trivial. More generally, let us consider the tachyon field \( T(Z^1, Z^2, \cdots, Z^n) \) on a \( D2n - \overline{D2n} \). If the holomorphic function \( T \) is reducible as \( T = T^{(1)} \cdot T^{(2)} \cdots T^{(q)} \), then we obtain the sum of the decay products each corresponding to \( T = T^{(1)}, T = T^{(2)}, \cdots, T = T^{(q)} \). Therefore we can assume the function \( T(Z^1, Z^2, \cdots, Z^n) \) is irreducible. Then we will obtain a \( D(2n - 2) \)-brane wrapping on a codimension two hypersurface \( T = 0 \). However, this may be problematic. In general this configuration of the ‘curved’ D-brane seems to be unstable in spite of its holomorphy since the D-brane is put in flat space and cannot wrap any cycles. It would be interesting to investigate this further, though we mainly discuss the generation of D0-brane charges in this paper.

Before closing this subsection, let us ask what will happen if we do not assume the boundary \( \mathcal{N} = 2 \) supersymmetry. First, one can produce lower dimensional non-BPS D-branes. This requires a kink-like tachyon field and is not holomorphic. Second, one will also be able to produce D0-branes and anti D0-branes at the same time. For example, let us consider the following tachyon field on a \( D2 - \overline{D2} \)

\[
T(Z, \overline{Z}) = a_{q+p,q} Z^{q+p} \overline{Z}^q. \tag{2.15}
\]

In the same way as the above RR-charge computation, one can calculate D0-brane charge of this configuration. The result is \( p \) times that of a D0-brane, which can be also seen intuitively from the fact that the tachyon field \((2.15)\) has the winding number \( p \). One may hastily conclude that the configuration \((2.15)\) generates a system of \((q + p)\) D0-branes and \( q \) anti D0-branes after the tachyon condensation. In fact this boundary interaction \((2.15)\) breaks \( \mathcal{N} = 2 \) supersymmetry and thus should be renormalized. Therefore we can argue that a system of \((q + p)\) D0-branes and \( q \) anti D0-branes for any \( q \) will be produced in a certain limit of the following configurations

\[
T(Z, \overline{Z}) = \sum_{q=0}^{\infty} a_{q+p,q} Z^{q+p} \overline{Z}^q. \tag{2.16}
\]

Generally, these are highly interactive theories and it will be difficult to analyze further.

### 2.2 Tachyon Condensation on Multiple Branes and Antibranes

The above formulation can be generalized for multiple brane-antibrane systems. The path-ordered formulation for these was given in \([19]\). If one wants to construct the corresponding \( \mathcal{N} = 1 \) boundary interaction, one has only to include more than one boundary fermions \([17, 18]\). We write the superfields for them as \( \Gamma_i, \overline{\Gamma}_i \) \((i = 1, 2, \cdots, n)\). The quantization of boundary fermions \( \eta_i \) is written by

\[
\{ \eta_i, \overline{\eta}_j \} = \delta_{ij}. \tag{2.17}
\]
Comparing this with the algebra of $\gamma$ matrices $\gamma_1, \ldots, \gamma_{2n}$:

$$\{\gamma_a, \gamma_b\} = 2\delta_{ab},$$

we get the correspondence

$$\eta_i \leftrightarrow \gamma^+_i \equiv \frac{1}{2}(\gamma_{2i-1} + i\gamma_{2i}), \quad \bar{\eta}_i \leftrightarrow \gamma^-_i \equiv \frac{1}{2}(\gamma_{2i-1} - i\gamma_{2i}).$$

(2.19)

Then we can get $2^n \times 2^n$ Chan-Paton matrices which corresponds to $2^{n-1}$ branes and $2^{n-1}$ antibranes even though for the general number of branes and antibranes, $\mathcal{N} = 1$ boundary superspace formulation is not known. In this formalism, the boundary interactions are expanded as

$$I_B = \int_{\partial \Sigma} d\tau d\theta \left[ -\bar{\Gamma}_i D_{\theta} \Gamma_i^+ + \frac{\Gamma_i T_i(X^a)}{\sqrt{2\pi}} + T_{ijk}(X^a) \bar{\Gamma}_i \Gamma_j \Gamma_k + T_{ijk}(X^a) \bar{\Gamma}_i \Gamma_j \Gamma_k + \cdots + (\text{h.c.}) \right],$$

(2.20)

where we have omitted the summation over the indices $i, j, k, \bar{i}$. Note that in the above equation the fields $T_i, T_{ijk}, T_{ijk}, \ldots$ represent non-abelian tachyon fields on $2^{n-1}$ brane-antibrane pairs. Here the non-abelian gauge fields are neglected again and these correspond to the boundary interactions which include even number of boundary fermionic superfields.

If we are interested in $\mathcal{N} = 2$ boundary supersymmetry, then the above boundary interactions should be constrained. The boundary interactions which represent tachyon fields should be in the boundary superpotential terms $\sim \int_{\partial \Sigma} d\tau d\theta W$. Therefore (i) tachyon fields should be holomorphic (or depend only on $Z_i$) and (ii) the potential terms should involve no anti-chiral superfields $\bar{\Gamma}_i$. For example, the second requirement does not allow the field $T_{ijk}$.

Now let us consider tachyon condensation on $2^{n-1}$ D$(2n)$-branes and $2^{n-1}$ anti D$(2n)$-branes. In such a system there should be decay modes which generate BPS D0-branes following the general arguments in K-theory \[33\]. We assume the following $\mathcal{N} = 2$ boundary interaction \[17\] for simplicity

$$I_B = -\int_{\partial \Sigma} d\tau d\theta d\bar{\theta} \sum_i \Gamma_i^+ \bar{\Gamma}_i + \int_{\partial \Sigma} d\tau d\theta \frac{1}{\sqrt{2\pi}} \sum_i \Gamma_i T_i(Z) + (\text{h.c.}),$$

(2.21)

where the tachyon fields $T_i(Z)$ depend only on the holomorphic coordinate $Z^1, \ldots, Z^n$ of the world-volume. Note that if $n = 1$ or 2, then the general $\mathcal{N} = 2$ boundary interaction can be written as the above form. As argued in \[17\] the condensation of these tachyon fields generally produces a D-brane wrapped on the intersection of hyper-surfaces $T_i(Z) = 0$. 

8
Below we would like to investigate this further. The results will be useful in the next section.

We first turn to the RR-coupling which corresponds to the D0-brane charge. The non-abelian tachyon field $T$ can be written by

$$ \left( \begin{array}{cc} 0 & T \\ T & 0 \end{array} \right) = \sum_i \gamma_i^+ T_i + \sum_i \gamma_i^- \bar{T}_i, \quad \text{(2.22)} $$

where note that $\gamma$ matrices here are not projected into the Weyl representation. Notice that we regard the tachyon fields as holomorphic if $T_i$ are holomorphic functions. Generally this does not mean that the non-abelian tachyon field $T$ in the above matrix is holomorphic in an ordinary sense.

Putting this into eq.(2.12), we get the following RR-coupling in BSFT:

$$ S_{RR} = T_{D(2n)} \text{Str} \int C_{D0} \exp\left( -\sum_{i=1}^{n} |T_i|^2 \right) \wedge \exp\left[ \sqrt{-2 i \pi \alpha'} \sum_{i=1}^{n} (\gamma_i^+ dT_i + \gamma_i^- d\bar{T}_i) \right] $$

$$ = T_{D(2n)} \left( -2 i \pi \alpha' \right)^n \times \frac{1}{(2n)!} \int C_{D0} \exp\left( -\sum_{i=1}^{n} |T_i|^2 \right) \wedge \text{Tr}[\gamma_{2n+1}(\sum_{i=1}^{n} \gamma_i^+ dT_i + \gamma_i^- d\bar{T}_i)^{2n}] $$

$$ = (2 i \pi \alpha')^n T_{D(2n)} \int \exp\left( -\sum_{i=1}^{n} |T_i|^2 \right) C_{D0} \wedge \prod_{i=1}^{n} dT_i \wedge d\bar{T}_i, \quad \text{(2.23)} $$

where the chirality matrix $\gamma_{2n+1} = (i)^{-n} \gamma_1 \gamma_2 \cdots \gamma_{2n}$ was inserted in order to replace the supertrace Str with the ordinary trace Tr. Note that there are no RR-charges other than D0-branes produced from the tachyon fields (2.22) because of the trace over $\gamma$ matrices.

If we assume $T_i$ depends only on $Z_i$, then the above integrations are divided into $n$ independent parts. If one sets the degree of $T_i(Z_i)$ is $p_i$, then the result is given by

$$ S_{RR} = (\prod_{i=1}^{n} p_i) \cdot T_{D0} \int C_{D0}. \quad \text{(2.24)} $$

Thus we can conclude that $(\prod_{i=1}^{n} p_i)$ D0-branes are generated in this case. Furthermore one can show that this configuration has a correct tension in BSFT. To see this one has only to note that the partition function $Z_{\text{disk}}$ is also divided into $n$ independent path integrals for each direction $Z_1, \ldots, Z_n$

$$ Z_{\text{disk}} = \prod_{i=1}^{n} Z_{\text{disk}}^i. \quad \text{(2.25)} $$

Then using the previous result of tachyon condensation on a $D2 - \overline{D2}$, it is easy to see the resulting tension is $(\prod_{i=1}^{n} p_i)$ times that of a D0-brane. In particular the configuration $p^1 = \cdots = p^n = 1$ generates a BPS D0-brane and corresponds to Atiyah-Bott-Shapiro [38] construction of K-theory charges.
Let us turn to other configurations. For simplicity, we set \( n = 2 \) and consider a system which is consist of two D4-branes and two anti D4-branes. We consider the following holomorphic tachyon fields for generic examples

\[
T_1(Z_1, Z_2) = (Z_1)^p(Z_2)^q - a, \quad T_2(Z_1, Z_2) = (Z_1)^r(Z_2)^s - b,
\]

(2.26)

where \( p, q, r, s \) are non-zero integers and we assume \( ps - qr \neq 0 \).

First note that the above configurations include only 0-branes if either \( a \) or \( b \) is not zero. This is because if one calculates the disk partition function in BSFT, one always finds the factor \( \exp(-\sum_{i=1}^2 T_i \bar{T}_i) \) and this means that the degree of freedom will be localized at the points (0-branes) defined by the equations \( T_1 = T_2 = 0 \). One can calculate the number of the points and the result is given by \( |ps - qr| \). This shows the total number of generated D0-branes and anti D0-branes is \( |ps - qr| \) because one fixed point gives the tension of a D0-brane (or anti D0-brane). On the other hand, one can also calculate the D0-brane RR-charge of these configurations with an appropriate change of the variables in the integration (2.23). The result is \( (ps - qr) \) times that of a D0-brane. Thus we can conclude that the above tachyon fields (2.26) generate \( (ps - qr) \) BPS D0-branes unless \( a = b = 0 \). Mathematically one can say that the integration (2.26) counts the number of the (localized) solutions to the algebraic equations \( T_i = 0 \) and this result will hold for general \( n \) and nonsingular polynomials \( T_i \).

Next we consider the singular cases \( a = b = 0 \). The D0-brane RR-charge is again given by \( (ps - qr) \). After the condensation of these tachyon fields, 2-branes will also be generated since the equations \( T_1 = T_2 = 0 \) are satisfied for \( Z_1 = 0 \) or \( Z_2 = 0 \). These 2-branes should be \( \text{D}2 - \overline{\text{D}2} \) systems because this configuration does not have D2-brane RR-charge. The generation of \( \text{D}2 - \overline{\text{D}2} \) is not so surprising. If one assumes \( p = r, q = s = 0 \), then this configuration corresponds to the decay into \( p \) pairs of \( \text{D}2 - \overline{\text{D}2} \) at \( Z_1 = 0 \) as can be seen easily by using \( U(2) \) rotational symmetry of \( (\Gamma_1, \Gamma_2) \). Though we cannot determine how many \( \text{D}2 - \overline{\text{D}2} \) systems will be produced for general \( (p, q, r, s) \), it will be interesting to note that a system which is generically D0-branes can become higher dimensional branes for singular points in the field space of BSFT.

Finally we would like to comment on the relation between various tachyon condensation modes and \( \mathcal{N} = 2 \) boundary (B-type) supersymmetry. In the above arguments on tachyon condensations in brane-antibranne systems, we have not observed the generation of D0-branes and anti D0-branes at the same time\(^4\). As can be seen from this example we

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\(^4\)To see this, let \( l \) be the g.c.d. of \( p \) and \( r \) as \( p = l \cdot \alpha \) and \( r = l \cdot \beta \). Then one obtains \( |q\beta - sa| \) solutions about \( Z_2 \) as \( (Z_2)^{q\beta - sa} = a^\beta \cdot b^{-\alpha} \). After we insert this in \( T_1 = T_2 = 0 \) again, we get \( l \) solutions about \( Z_1 \). Thus we get \( |ps - qr| \) solutions.

\(^5\)Of course, if one adds more boundary fermions with preserving \( \mathcal{N} = 2 \) supersymmetry, then we can
believe that there is an intriguing correlation in general backgrounds between the BPS nature of final objects and the holomorphy of tachyon (or $\mathcal{N} = 2$ supersymmetry). In the next section we will see another evidence of this argument in orbifold theories, which give the simplest examples in curved spaces.

3 Tachyon Condensation on Orbifolds

In this section we discuss tachyon condensation in brane-antibrane systems on orbifolds $[44]$. Mainly we consider the four dimensional orbifolds $\mathbb{C}^2/\mathbb{Z}_N$ ($N \geq 2$), but the similar arguments will be applied to higher dimensional examples or more complicated orbifold projections. The relation between the tachyon condensation in these systems and the equivariant K-theory was discussed in $[34, 45]$. The tachyon condensation from the viewpoint of noncommutative geometry $[30]$ was also discussed for orbifolds $[32]$. Here we investigate this in the framework of BSFT and determine what will be generated after the tachyon condensation precisely. Before we do so, let us first review some useful facts about D-branes on orbifolds $[46, 47]$.

In Type II superstring theory we can consider the orbifold projections $\Gamma$ on $\mathbb{C}^2$ which preserve half of the bulk supersymmetries. This means that the discrete groups $\Gamma$ of the orbifold projections should be subgroups of $SU(2)$ and are known to be classified into A,D,E series. Geometrically, the orbifolds $\mathbb{C}^2/\Gamma$ can be realized in the neighborhoods of the A,D,E singularities in K3 surface. These singularities are due to the vanishing 2-cycles in K3. If they are resolved by blowing up, then one gets ALE spaces (see for example $[48]$). However, in string theory these singularities do not imply physical singularities. Indeed there are B-field fluxes (=twisted NSNS-fields) through the 2-cycles $[49]$ and the worldsheet instantons and various D-branes which wrap these cycles do not become tensionless. Thus the theory is not singular.

Below we concentrate on the A series for simplicity, which are equivalent to the familiar discrete groups $\mathbb{Z}_N$. The action of $\mathbb{Z}_N$ is defined as follows:

\begin{equation}
1, g, g^2, \cdots, g^{N-1} \in \mathbb{Z}_N, \quad (g^N = 1),
\end{equation}

\begin{equation}
g : z_1 \to e^{\frac{2\pi i}{N}} z_1, \quad z_2 \to e^{-\frac{2\pi i}{N}} z_2,
\end{equation}

where $z_1, z_2$ denote the coordinates of $\mathbb{C}^2$. We obtain D0 $\overline{D0}$ systems. What we would like to say here is that we cannot obtain D0 $\overline{D0}$ systems if we start from the minimal number ($= 2^{n-1}$) of D(2n) $-$ D(2n) pairs.

\(^6\)Of course the discrete group $\Gamma$ is completely different from the fermionic boundary superfield $\Gamma$, though we use the same symbol below.
Now let us turn to D-branes on $\mathbb{C}^2/\mathbb{Z}_N$. In this paper we always assume that the D-branes are particle-like in the $\mathbb{R}^{1,5}$ direction. Then BPS $D_p$-branes exist for $p = 0, 2, 4$. In particular $D4$-branes are wrapped on the whole $\mathbb{C}^2/\mathbb{Z}_N$. The $D2$-branes which are parallel to the $z_1$-plane or $z_2$-plane are BPS objects and can be treated with the world-sheet $\mathcal{N} = 2$ supersymmetry.

The open string spectrum of $D_p$-branes on the orbifold can be given by $\Gamma$-projection on the Chan-Paton degree of freedom [46, 47]. In other words, $D_p$-branes on $\mathbb{C}^2/\Gamma$ are classified by the group theoretical representations of $\Gamma$ action on Chan-Paton factors. For $\Gamma = \mathbb{Z}_N$, there are $N$ irreducible representations and we denote these by $\{\rho_\alpha\}$ ($\alpha = 0, 1, 2, \cdots, (N-1)$). The representation $\rho_\alpha$ is defined as the one dimensional representation which gives the phase rotation $\exp\left(\frac{2\pi i \alpha}{N}\right)$. Then we call a $D_p$-brane which corresponds to $\rho_\alpha$ representation a $\alpha$-type $D_p$-brane. These $N$ kinds of D-branes are the most fundamental D-branes. For $p = 0$ they are called fractional D-branes [50], which are identified with the $D2$-branes wrapped on vanishing 2-cycles. It is known that vanishing 2-cycles are also classified by the irreducible representations and a $\alpha$-type $D0$-brane corresponds to a $D2$-brane wrapped on the 2-cycle $[\alpha]$ [50, 51, 52]. Fractional D-branes are fixed at the origin $z_1 = z_2 = 0$ and cannot move from there. The tension of each of them is $\frac{1}{N}$ times that of a bulk D0-brane, which can move freely in the orbifold. On the other hand, for $p = 2, 4$ such a $D_p$-brane has the same tension as the ordinary $D_p$-brane since the $g$-action acts on the world-volume non-locally. These facts can also be verified by using boundary state formalism for orbifold theories (see for example [37, 53, 54, 55, 56, 51, 52, 57, 58]) as we will see in the appendix A.

All the other D-branes in the orbifold theory are regarded as linear combinations of these fundamental $D_p$-branes and correspond to all of the reducible representations as $\rho = \oplus_{\alpha=0}^{N-1} c_\alpha \rho_\alpha$ ($c_\alpha \in \mathbb{Z}$). For example, the regular representation $\rho_{\text{reg}} = \oplus_{\alpha=0}^{N-1} \rho_\alpha$ corresponds to a bulk $D0$-brane. Open strings between a $\alpha$-type $D_p$-brane and a $\beta$-type $D_p$-brane belong to the representation $\rho_\beta \otimes \rho^*_\alpha$, where $^\star$ denotes the complex conjugation. Then the super Yang-Mills theories called quiver gauge theories are realized on the world-volume of BPS D-branes as shown in [14].

Here we are interested in the tachyon field $T$ which comes from the open string between a $\alpha$-type $D_p$-brane and a $\beta$-type anti $D_p$-brane. These open strings belong to $\rho_\beta \otimes \rho^*_\alpha$ with the opposite GSO-projection and the $g(\in \mathbb{Z}_N)$ action is given by

$$g : T(z_1, z_2) \rightarrow e^{\frac{2\pi i}{N}(\alpha - \beta)} \cdot T(e^{\frac{2\pi i}{N}}z_1, e^{-\frac{2\pi i}{N}}z_2). \quad (3.2)$$

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Note that if one changes the orientation of the open strings, then they belong to $\rho_\alpha \otimes \rho^*_\beta$.
3.1 Tachyon Condensation on Orbifolds in BSFT

Now let us investigate the tachyon condensation on orbifolds in BSFT. Again we are interested in holomorphic tachyon fields, which preserve $\mathcal{N} = 2$ supersymmetry. As mentioned in the previous section, the spacetime action of BSFT in flat space is defined as the disk partition function eq.(2.10). If the world-sheet action $I_0 + I_B$ is invariant under a certain transformation of the world-sheet fields $X^a$ and $\Gamma$, we can twist the theory by this symmetry. In particular if we regard $g \in \mathbb{Z}_N$ as the symmetry, then we get the BSFT action for D-branes on orbifolds.

Generation of Codimension Two D-branes

Let us first turn to a D2 $-$ $\overline{\text{D2}}$ pair of which world-volume is defined by $z_2 = 0$. We assume that the D2-brane is $\alpha$-type and the anti D2-brane is $\beta$-type. Note that the branes cannot move from $z_2 = 0$. If we remember the $g \in \mathbb{Z}_N$ action (3.2), the tachyon field should be projected as follows:

$$T(z_1) = e^{\frac{2\pi i}{N}(\alpha - \beta)} \cdot T(e^{\frac{2\pi i}{N} z_1}).$$

(3.3)

In BSFT this can be equally stated that the boundary interaction (2.7) should be invariant under the following transformation

$$g : \Gamma \rightarrow e^{\frac{2\pi i}{N}(\alpha - \beta)} \Gamma, \quad Z_1 \rightarrow e^{\frac{2\pi i}{N} Z_1}.$$

(3.4)

Then the allowed tachyon field can be given by

$$T(z_1) = a_q \cdot (z_1)^{\beta - \alpha + Nq},$$

(3.5)

where $q$ is non-negative integers for $\beta \geq \alpha$ and is positive integers for $\alpha > \beta$. The BSFT action becomes

$$S = \int_{C^2/T} [DZ_1][D\Gamma] \exp(-I_0 - I_B)$$

$$= \frac{1}{N} \int_{C^2} [DZ_1][D\Gamma] \exp(-I_0 - I_B)$$

$$= \left(\frac{\beta - \alpha + Nq}{N}\right) T_{D0} \int dx^0,$$

(3.6)

where we have used the fact that the disk partition function after the condensation of the tachyon field (3.3) is the same as that for $(\beta - \alpha + Nq)$ D0-branes as explained in the previous section. The calculation of bulk RR-charge\footnote{Here “bulk RR-charge” means the RR-charge in the untwisted-sector. Notice that there are also twisted RR-charges which is characteristic of orbifold theories. These charges will be discussed later.} is also in the same way as in
section 2 and the result is \((\beta - \alpha + Nq)/N\) times that of a BPS bulk D0-brane\footnote{Note that one can also consider \(q\) invariant tachyon field \(T = a_q \cdot (\bar{z}_1)^{\alpha - \beta + Nq}\). For these the different \(N = 2\) supersymmetry is preserved and have opposite RR-charge. Thus fractional anti D-branes will be produced from these.}. Thus we can conclude that \(\beta - \alpha + Nq\) fractional D0-branes will be generated at the point \(z_1 = 0\). Then what kinds of fractional branes will be generated? To answer this question completely we need the knowledge of twisted RR-charges and we will return to this in the next subsection. Nevertheless we can obtain some hints from the above arguments. First let us set \(\beta = \alpha\). Then the mode \(q = 0\) corresponds to the decay into the vacuum as the tachyon condenses \(a_0 \to \infty\). This is impossible for other cases \(\beta \neq \alpha\) since the types of the brane and the antibrane are different and they cannot annihilate. Note also that the tachyon field for \(\beta = \alpha\) have \(qN\) zeros and they are invariant by the geometric \(Z_N\) action even if we deform the tachyon field (3.3) by allowed polynomials. Then it is natural to identify these zeros as \(q\) bulk D0-branes. For example, it is obvious that they can move from \(z_1 = 0\). Furthermore, it is not difficult to see that the condensation of the tachyon field (3.3) will generate both \(q\) bulk D0-branes and \((\beta - \alpha)\) fractional D0-branes if we assume \(\beta \geq \alpha\). On the other hand, if we assume \(\beta < \alpha\), then we will obtain both \((q - 1)\) bulk D0-branes and \(N + (\beta - \alpha)\) fractional D0-branes.

Next we turn to tachyon condensation on a \(D4 - \overline{D4}\) pair. In this case we can assume the following tachyon field

\[
T(z_1, z_2) = a_{q,r} \cdot (z_1)^{\beta - \alpha + Nq} \cdot (z_1z_2)^r
\]  

(3.7)

We can apply the RR-coupling formula (2.12) or (2.23) to this. Then it is easy to see that the final state after the tachyon condensation consists of \((\beta - \alpha + Nq + r)\) D2-branes on \(z_1 = 0\) and \(r\) D2-branes on \(z_2 = 0\), each of which corresponds to a irreducible representation.

**Generation of Codimension Four D-branes**

Next we consider two \(D4 - \overline{D4}\) pairs and discuss the generation of D0-branes. We can use the boundary interaction (2.21) with \(i = 1, 2\). To see the matrix representation of tachyon fields \(T_1, T_2\) explicitly, let us use the standard expressions of \(\gamma\) matrices

\[
\gamma_1 = \begin{pmatrix} 0 & \sigma_1 \\ \sigma_1 & 0 \end{pmatrix}, \quad \gamma_2 = \begin{pmatrix} 0 & \sigma_2 \\ \sigma_2 & 0 \end{pmatrix}, \quad \gamma_3 = \begin{pmatrix} 0 & \sigma_3 \\ \sigma_3 & 0 \end{pmatrix}, \quad \gamma_4 = \begin{pmatrix} 0 & -i1 \\ i1 & 0 \end{pmatrix},
\]  

(3.8)

where \(\sigma_1, \sigma_2, \sigma_3\) denote Pauli matrices. Then the non-abelian tachyon field \(T\) is given by

\[
T = \begin{pmatrix} \overline{T_2} & T_1 \\ T_1 & -T_2 \end{pmatrix}.
\]  

(3.9)
Now we assume that the two D4-branes and two anti-D4-branes correspond to the representation \( \rho\alpha \oplus \rho_{\beta+\delta} \) and \( \rho_{\alpha+\delta} \oplus \rho_{\beta} \), respectively. The mod \( N \) integers \( \alpha, \beta, \delta \) are arbitrary. The reason why we restrict to this form is because we want to maintain the \( \mathcal{N} = 2 \) supersymmetry in the presence of the boundary perturbation. Indeed if we assume this form, we can read off from eq.(3.9) the \( g \)-action on boundary fermionic superfields as follows

\[
g : \Gamma_1 \to e^{2\pi i N (\alpha - \beta)} \cdot \Gamma_1, \quad \Gamma_2 \to e^{2\pi i N \delta} \cdot \Gamma_2.
\] (3.10)

Further we can assume that \( \beta \geq \alpha \) and \( \delta \geq 0 \) without losing generality.

The holomorphic tachyon fields are classified into the form eq.(2.26) with \( a = b = 0 \) and in addition they should be \( \mathbb{Z}_N \)-invariant. Here we are interested in the generation of only D0-branes and thus we assume \( q = r = 0 \) below.

Then the tachyon fields are classified into the following form

\[
T_1(z_1, z_2) = a_q \cdot (z_1)^{\beta - \alpha + Nq}, \quad T_2(z_1, z_2) = b_r \cdot (z_2)^{\delta + Nr}.
\] (3.11)

In the same way as the codimension two case, we can conclude that \( (\beta - \alpha + Nq)(\delta + Nr) \) fractional D0-branes will be generated. This can also be regarded as purely fractional D0-branes and bulk D0-branes. The number of the former is given by \( (\beta - \alpha)\delta \mod N \). In particular if one sets \( \delta = 0 \) or \( \beta = \alpha \), then we obtain only bulk D0-branes. This is consistent with the fact that the two branes and the two antibranes have identical type for these cases. The more detailed argument which uses twisted RR-charges will be discussed in the next section.

### 3.2 Tachyon Condensation on Orbifolds and Twisted RR-charges

Here we discuss the previous examples of tachyon condensation on the orbifolds from somewhat different viewpoint: we pay attention to the twisted RR-charges in the orbifold theories.

Generally, an orbifold theory in the closed string sector \([44]\) consists of a untwisted-sector and twisted-sectors. Our orbifold \( \mathbb{C}^2/\mathbb{Z}_N \) possesses \( (N - 1) \) twisted-sectors, which are twisted by \( g, g^2, \cdots, g^{N-1} \). In each of the twisted NSNS-sectors there are four massless scalars and these correspond to the moduli of hyper Kähler geometry. On the other hand, in each of the twisted RR-sectors there is one vector field for Type IIA theory. The RR-charges for these vector fields are called twisted RR-charges.

\[\text{As we saw in section 2, some D2 - D2 systems will be generated for } qr \neq 0. \text{ Note that we cannot deform this as in } (2.26) \text{ because of the orbifold projection.}\]
These charges are carried by D-branes which do not belong to the regular representation. In other words, these represent the geometrical information that the branes are wrapped on some non-trivial 2-cycles in the ALE space. Therefore we argue that the twisted RR-charges should be conserved during the tachyon condensation. Our example is consistent with this claim as we will see below. Note that this claim is in strikingly contrast with the fact that for the untwisted (or equally bulk) RR-charge the generation of lower dimensional D-brane charges does indeed occur. This is due to the non-compactness of the orbifold. If one consider the orbifolded torus, then the untwisted charges should be conserved. Indeed the results from the description of tachyon condensation as the marginal deformation were obtained in $\mathbb{Z}_2$-orbifolded torus $[23, 24, 25]$ and the results are consistent with this.

To see this more generally, let us remember the RR-coupling formula (2.12). For compact space, as shown in $[12]$ the Chern character of the superconnection does not change in cohomology if we shift the value of the tachyon fields continuously. This also supports the above arguments.

Now let us return to our examples in $\mathbb{C}^2/\mathbb{Z}_N$. In principle the calculations of twisted RR-charges are possible in BSFT, but the determination of the normalizations is not so easy. Therefore we calculate the charges in the boundary state formalism. For boundary states in orbifold theories see for example $[37, 53, 54, 55, 56, 51, 52, 57, 58]$. This formalism is useful to know couplings with various fields in closed string sector because the boundary state is the description of a D-brane from the viewpoint of closed string theory. The detailed computations are shown in the appendix A and here we will discuss the results.

The outline of the determination is as follows. First we can find the boundary state which represents a $\alpha$-type $D_p$-brane so as to satisfy the Cardy’s condition $[59]$. Then the total boundary state is given by

$$|D_p(\alpha)\rangle = \sum_{k=0}^{N-1} e^{\frac{2\pi ik\alpha}{N}} |T^{(k)}\rangle.$$  \hspace{1cm} (3.12)

Here we defined the boundary states for untwisted sector $|T^{(0)}\rangle = |U\rangle$ and $k$-th twisted sectors $|T^{(k)}\rangle$ as follows

$$|U\rangle = \frac{T_p}{2} (|U\rangle_{NSNS} + |U\rangle_{RR}),$$

$$|T^{(k)}\rangle = \frac{T'_p}{2} (|T^{(k)}\rangle_{NSNS} + |T^{(k)}\rangle_{RR}),$$  \hspace{1cm} (3.13)

where the two normalization $T_p$ and $T'_p$ can be computed as in eq.$(A.30)$. Next note that in the low energy limit the boundary state for each sector is proportional to a massless

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11 Similar conservation law for D-branes in NS5-brane background was recently discussed in $[29]$. 

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state in the sector. Thus we can read off the coupling to the massless field from the
coefficient of the boundary state for each sector \[60, 51, 52, 57, 58\].

In this way we can compute the twisted RR-charges and the result is as follows for
the \(k\)-th twisted RR-charge \(Q^{(k)}_{\alpha,p}\) of a \(\alpha\)-type D\(_p\)-brane
\[
Q^{(k)}_{\alpha,p} = \frac{1}{N} \cdot e^{\frac{2\pi i}{N} \cdot k} \cdot \left(2 \sin \frac{\pi k}{N}\right)^{1 - \frac{k}{2}} \cdot 2^{\frac{2}{3}} \pi^2 (\alpha')^\frac{1}{3}. \tag{3.14}
\]

Note that the above method cannot determine the phase factors which do not depend on \(\alpha\).

Then let us discuss the twisted RR-charges before and after the tachyon condensation.
First consider the generation of the D\((p-2)\) brane charge from a D\(_p\) \(\overline{D_p}\) by the tachyon
field (3.5). The original D\(_p\) \(\overline{D_p}\) \((p = 2, 4)\) has the
\(k\)-th twisted RR-charge \((Q^{(k)}_{\alpha,p} - Q^{(k)}_{\beta,p})\).
Without losing generality we can assume \(\beta \geq \alpha\). If one notes the following elementary
formula
\[
\left(e^{\frac{2\pi i}{N} \cdot k} - e^{\frac{2\pi i}{N} \cdot \beta}\right) = (-i) \cdot e^{\frac{2\pi i}{N} \cdot k} \cdot 2 \sin \left(\frac{\pi k}{N}\right) \cdot \left(e^{\frac{2\pi i}{N} \cdot \alpha} + \cdots + e^{\frac{2\pi i}{N} (\beta - 1)}\right), \tag{3.15}
\]
then one obtains
\[
Q^{(k)}_{\alpha,p} - Q^{(k)}_{\beta,p} = (-i) \cdot e^{\frac{2\pi i}{N} \cdot k} \cdot \sum_{\mu=\alpha}^{\beta-1} Q^{(k)}_{\mu,p-2}. \tag{3.16}
\]
This shows that the final state after the tachyon condensation on a D2 \(\overline{D2}\) should be
fractional D0-branes of type \(\{\alpha, \alpha + 1, \ldots, \beta - 1\}\) with some bulk D0-branes \[13\]. This is
consistent with the results in the previous subsection that the final state consists of \(q\) bulk
D0-branes and \((\beta - \alpha)\) fractional D0-branes. Combining this with the above arguments
we can determine the final state completely.

For \(p=4\), one can also consider more general tachyon field (3.7). These will produce
the intersecting D2-brane system as mentioned in the previous subsection. Then we can
find that the twisted charges are conserved if the charges of \(r\) D2-branes on \(z_1 = 0\) do
cancel those of \(z_2 = 0\). Note also that this configuration is BPS.

Next we turn to the generation of codimension four D-brane charges from the tachyon
fields (3.11). In the same way as before we obtains the following formula
\[
Q^{(k)}_{\alpha,4} + Q^{(k)}_{\beta+\delta,4} - Q^{(k)}_{\beta,4} - Q^{(k)}_{\alpha+\delta,4} = (i)^2 \cdot \sum_{\mu=\alpha+1}^{\beta} \sum_{\nu=0}^{\delta-1} Q^{(k)}_{\mu+\nu,0}, \tag{3.17}
\]

\[12\] The extra phase \((-i) \cdot e^{\frac{2\pi i}{N} \cdot k}\) can be canceled by the phase factor which cannot be determined from the
calculations in the appendix A because it does not depend on \(\alpha\). The origin of \(e^{\pm \frac{2\pi i}{N} \cdot k}\) is easy to understand.
If one considers the D\(_p\)-D\((p-2)\) open string, then the \(g\)-action on the fermionic zero mode generates the
factor \(e^{\pm \frac{2\pi i}{N} \cdot k}\). Thus one must project the open string as 
\[g = e^{\pm 2\pi i (\frac{1}{2} + \beta - \alpha)}.\]
where we assumed $\beta \geq \alpha$ and $\delta \geq 0$. This decomposition rule is again consistent with the result in the previous subsection. Thus we can conclude that after the tachyon condensation there are $(\beta - \alpha)\delta$ fractional D0-branes and their types are given by the above formula.

In both examples of generating D0-branes if we shift the Kähler moduli and blow up the orbifold singularities, then we get (mutually) BPS D2-branes which are wrapped on the corresponding holomorphic 2-cycles in ALE spaces. Since the Kähler structure is independent from the complex structure, these holomorphic 2-cycles can be “defined” by the the equations $T_i = 0$ in the $A_{(N-1)}$-type hypersurface

$$XY = Z^N, \quad (X = z_1^N, Y = z_2^N, Z = z_1z_2). \quad (3.18)$$

It will be also interesting to discuss the relation between the shift of complex structure and the corresponding tachyon field for these examples and we will leave this for future problem.

### 3.3 Some Comments on Generalizations

Before we close this section, let us comment on some generalizations of our results. First it is easy to see that the generalizations for higher dimensional $\mathbb{Z}_N$-orbifolds $\mathbb{C}^n/\mathbb{Z}_N$ ($n \geq 3$) are straightforward since the above arguments largely depend on the algebraic properties of the discrete groups $\mathbb{Z}_N$.

On the other hand, for other types of orbifolds $\mathbb{C}^n/\Gamma$ the results will be non-trivial. Here we do not investigate these further, but it may be natural to conjecture that the following general relation will hold for each $g \in \Gamma$ with a coefficient $C(g)$

$$\sum_{i=1}^{2n-1} \chi_{\alpha_i}(g) - \sum_{i=1}^{2n-1} \chi_{\beta_i}(g) = C(g) \cdot \sum_{\delta \in \Delta} \chi_\delta(g), \quad (3.19)$$

where $\chi_\alpha(g)$ is the character of $g$ for the irreducible representation $\alpha$ and $\Delta$ denotes a certain subset of irreducible representations which depends on $\alpha_i, \beta_i$. Note that if we return to the $\mathbb{C}^2/\mathbb{Z}_N$ examples, then the character is given by $\chi_\alpha(g^k) = \exp(2\pi i \alpha k/N)$ and the relation eq.(3.19) is equivalent to eq.(3.17). The coefficient $C(g)$ will be due to a phase factor and due to the trace over the zeromodes as in (A.6).

### 4 Conclusions
In this paper we have discussed the boundary string field theory description of tachyon condensation with world-sheet $\mathcal{N} = 2$ supersymmetry. This extended supersymmetry generally requires that the tachyon field should be holomorphic [17]. Therefore it is natural to believe that this constraint is related to the spacetime supersymmetry of final states after the tachyon condensation. We have investigated this issue in two examples.

First we have considered brane-antibrane systems in flat space and discuss the generalization of Atiyah-Bott-Shapiro configuration. In the arguments of these the RR-coupling formula for brane-antibrane systems also played an important role. As a result, we obtained only BPS configurations from the minimal number of brane-antibrane pairs.

Next we have investigated tachyon condensation on $\mathbb{Z}_N$-orbifolds mainly in four dimension. This is one of the simplest examples in curved spaces and most of our arguments can be performed algebraically. In this example we have seen that holomorphic tachyon fields generate various BPS fractional D-branes which are wrapped on various holomorphic cycles. The conservation law of various twisted RR-charges was used to identify the final states.

Finally let us mention some future directions. If one wants to see the generation of lower dimensional D-branes explicitly, it will be useful to construct the (off-shell) boundary states during the tachyon condensation in the same way as in [21, 22, 23, 61]. This will make more clear the generation of fractional D-branes from brane-antibrane systems.

In particular for BPS D-branes on the four dimensional orbifolds (or K3 surface), the world-sheet $\mathcal{N} = 4$ superconformal symmetry is realized [37]. Thus it is intriguing to construct $\mathcal{N} = 4$ boundary interactions and discuss tachyon condensation in BSFT.

As mentioned in section 3.3, it will also interesting to investigate other examples of orbifolds because the consideration of tachyon condensation seems to imply non-trivial relations among the characters of irreducible representations.

We hope to return to these issues in future work.

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A Detailed Boundary State Computations

Here we compute the cylinder amplitudes of open strings between fractional Dp-branes \((p = 0, 2, 4)\) on the orbifold \( \mathbb{C}^2/\mathbb{Z}_N \) in order to get correct normalizations of the boundary states. Similar calculations for \(p = 0\) or for \(\mathbb{Z}_2\)-orbifolds have been performed in various papers, for example \([53, 54, 55, 56, 51, 52, 57, 58]\) (see also \([62]\)). Let us first summarize our conventions.

Conventions for Open String

We define the open string Hamiltonian of world-sheet theory as

\[
H_o = \pi (\alpha' p^\mu p^\mu + N_o + a),
\]

where \(p^\mu\) is the momentum and \(N_o \in \mathbb{Z}\) is the contributions from oscillators; \(a\) denotes the zero energy

\[
a = -\frac{1}{2} \quad \text{(for NS-sector)}, \quad a = 0 \quad \text{(for R-sector)}. \tag{A.2}
\]

The moduli of the cylinder is written by \(t\) and we define \(q = e^{2\pi i \tau}\) as

\[
q = e^{2\pi i \tau} \equiv e^{-2\pi t}. \tag{A.3}
\]

The one-loop amplitude \(Z_{\text{open}}\) of open string between a \(\alpha\)-type D\(p\)-brane and \(\beta\)-type D\(p\)-brane can be written as

\[
Z_{\text{open}} = 1 \frac{N}{N-1} \sum_{k=0}^{N-1} e^{i \frac{2\pi}{N} (\alpha - \beta) k} Z_{\text{open}}^{(k)}, \tag{A.4}
\]

where \(Z_{\text{open}}^{(k)}\) is defined by

\[
Z_{\text{open}}^{(k)} = 2 \int_0^\infty \frac{dt}{2t} \text{Tr}_{NS-R} \left[ g^k \frac{1 + (-1)^F}{2} e^{-2H_o t} \right]. \tag{A.5}
\]

This means the \(\mathbb{Z}_N\)-projection into the states which satisfy \(g = e^{i \frac{2\pi}{N} (\beta - \alpha)}\).

Next let us consider the bosonic zeromodes along \(\mathbb{C}^2/\mathbb{Z}_N\) direction. The traces over these zeromodes become

\[
\text{Tr}(1) = V_p \cdot \left( \int \frac{dk}{2\pi} \right)^p, \quad \text{Tr}(g^k) = \frac{1}{(2 \sin(\frac{\pi k}{N}))^p}, \tag{A.6}
\]

where \(V_p\) denotes the volume of a D\(p\)-brane before the \(\mathbb{Z}_N\)-projection. The second equation follows from the calculation \([62]\)

\[
\int (dz)^2 \langle z | g^k | z \rangle = \frac{1}{2 \sin(\frac{\pi k}{N})} \left( \langle z | z' \rangle \equiv \delta^2(z - z') \right). \tag{A.7}
\]
Then we turn to the fermionic zeromodes in the R-sector along $\mathbb{C}^2/\mathbb{Z}_N$ direction. The action of $g$ on these is defined as follows:

$$g \ket{s_1, s_2} = e^{\frac{2\pi i}{N}(s_1-s_2)} \ket{s_1, s_2}, \quad (A.8)$$

where $s_1, s_2 \in \{\pm \frac{1}{2}\}$ denote the spins of the spacetime fermions. From this one can obtain the zeromode trace in R-sector as

$$\text{Tr}_R(g^k) = e^{\frac{2\pi ik}{N}} + e^{-\frac{2\pi ik}{N}} + 2 = 4 \cos^2\left(\frac{\pi k}{N}\right). \quad (A.9)$$

Below we use the trace $\text{tr}$ over only oscillators (not the bosonic and fermionic zeromodes).

**Formulae of $\theta$-functions**

Here we summarize the formulae of $\theta$-functions. First we define the following $\theta$-functions:

$$\eta(\tau) = q^{\frac{1}{24}} \prod_{n=1}^{\infty} (1-q^n),$$

$$\theta_1(\nu, \tau) = 2q^{\frac{1}{8}} \sin(\pi \nu) \prod_{n=1}^{\infty} (1-q^n)(1-e^{2\pi i \nu} q^n)(1-e^{-2\pi i \nu} q^n),$$

$$\theta_2(\nu, \tau) = 2q^{\frac{1}{8}} \cos(\pi \nu) \prod_{n=1}^{\infty} (1-q^n)(1+e^{2\pi i \nu} q^n)(1+e^{-2\pi i \nu} q^n),$$

$$\theta_3(\nu, \tau) = \prod_{n=1}^{\infty} (1-q^n)(1+e^{2\pi i \nu} q^{n-\frac{1}{2}})(1+e^{-2\pi i \nu} q^{n-\frac{1}{2}}),$$

$$\theta_4(\nu, \tau) = \prod_{n=1}^{\infty} (1-q^n)(1-e^{2\pi i \nu} q^{n-\frac{1}{2}})(1-e^{-2\pi i \nu} q^{n-\frac{1}{2}}), \quad (A.10)$$

where we have defined $q = e^{2i\pi \tau}$.

Then the modular transformations are given as follows

$$\eta(\tau) = (-i\tau)^{-\frac{1}{2}} \eta(-1/\tau), \quad \theta_1(\nu, \tau) = i(-i\tau)^{-\frac{1}{2}} e^{-\pi i \nu^2 / \tau} \theta_1(\nu/\tau, -1/\tau),$$

$$\theta_2(\nu, \tau) = (-i\tau)^{-\frac{1}{2}} e^{-\pi i \nu^2 / \tau} \theta_4(\nu/\tau, -1/\tau), \quad \theta_3(\nu, \tau) = (-i\tau)^{-\frac{1}{2}} e^{-\pi i \nu^2 / \tau} \theta_3(\nu/\tau, -1/\tau),$$

$$\theta_4(\nu, \tau) = (-i\tau)^{-\frac{1}{2}} e^{-\pi i \nu^2 / \tau} \theta_2(\nu/\tau, -1/\tau). \quad (A.11)$$

**Open String Cylinder Amplitudes**

Let us compute the open string cylinder amplitudes $Z_{\text{open}}^{(k)}$. We only consider the two coincident Dp-branes.
For the untwisted part $k = 0$, we obtain

$$Z^{(0)}_{\text{open}} = 2V_{p+1} \int_0^\infty \frac{dt}{2t} (8\pi^2 \alpha')^{-\frac{p+1}{2}} \cdot \frac{\theta_3(0, it)^4 - \theta_4(0, it)^4 - \theta_2(0, it)^4}{2\eta(it)^{12}} \tag{A.12}$$

where $V_{p+1}$ is equal to $V_p$ times the “volume” $V_1$ of time-like direction. Note that in the last expression we have performed the modular transformation. For the $k$-th twisted parts, the result is

$$Z^{(k)}_{\text{open}} = 2 - \frac{3p+5}{2} \pi^{-\frac{p+1}{2}} \frac{\theta_3(0, \nu_k/\pi)^4 - \theta_4(0, \nu_k/\pi)^4 - \theta_2(0, \nu_k/\pi)^4}{2\eta(\nu_k/\pi)^{12}}, \tag{A.13}$$

where $\nu_k = -iks/N\pi$.

Then let us compare these results with those from the boundary state calculations. Before that we summarize the conventions. We use the light-cone gauge in NS-R formulation [63] and closely follow the normalization in [60].

**Conventions for Boundary State**

The closed string Hamiltonian is defined by

$$H_c = \pi \alpha' k^\mu k'_\mu + 2\pi (N_L + N_R) + 4\pi a, \tag{A.14}$$

where $N_L$ and $N_R$ are the contributions from left-moving and right-moving oscillators; $a$ denotes the zero energy

$$a = -\frac{1}{2} + \frac{k}{N} \quad \text{(for NSNS-sector)}, \quad a = 0 \quad \text{(for RR-sector)}. \tag{A.15}$$

Note also that the momentum $k^\mu$ in twisted sectors is always zero along the orbifold direction $C^2/\mathbb{Z}_N$.

Further one can define the propagator $\Delta$ as

$$\Delta = \frac{\alpha'}{2} \int ds e^{-\int s H_c}. \tag{A.16}$$
The boundary state for the untwisted-sector and \( k = 1, 2, \cdots, (N - 1) \)-th twisted-sectors are given by

\[
|U\rangle = \frac{T_p}{2}(|U\rangle_{NSNS} + |U\rangle_{RR}),
\]

\[
|T^{(k)}\rangle = \frac{T_p}{2}(|(T^{(k)})_{NSNS} + |T^{(k)}\rangle_{RR}),
\]

where the constants \( T_p, T_p' \) represent the tension and charges of the D-brane and will be determined later. We have defined \(|U\rangle_{sector}\) and \(|T^{(k)}\rangle_{sector}\) as

\[
|U\rangle_{NSNS} = \frac{1}{2} \int \left(\frac{dk}{2\pi}\right)^{9-p} (|U,+,k^a\rangle_{NSNS} - |U,-,k^a\rangle_{NSNS}),
\]

\[
|U\rangle_{RR} = 2 \int \left(\frac{dk}{2\pi}\right)^{9-p} (|U,+,k^a\rangle_{RR} + |U,-,k^a\rangle_{RR}),
\]

\[
|T^{(k)}\rangle_{NSNS} = \frac{1}{2} \int \left(\frac{dk}{2\pi}\right)^{5} (|(T^{(k)})_{+,+},k^i\rangle_{NSNS} - |(T^{(k)}),-,k^i\rangle_{NSNS}),
\]

\[
|T^{(k)}\rangle_{NSNS} = \int \left(\frac{dk}{2\pi}\right)^{5} (|(T^{(k)})_{+,+},k^i\rangle_{RR} + |(T^{(k)}),-,k^i\rangle_{RR}),
\]

where \( k^a \) and \( k^i \) are momenta of the Dp-brane in the untwisted and twisted sectors, respectively. If we regard \( x^6, \cdots, x^9 \) as the coordinates of \( C^2/\mathbb{Z}_N \), then we can take \( a = 1, \cdots, 9-p \) and \( i = 1, 2, \cdots, 5 \). The explicit forms of \(|U, \pm, k^a\rangle_{sector}\), \(|T^{(k)}, \pm, k^i\rangle_{sector}\) are determined by the requirement that they should satisfy the desirable boundary conditions. These conditions are solved by elementary calculations and the explicit forms are given by “coherent states” of left and right-moving oscillators. Here we show the explicit expression only for \( p = 0 \) in NSNS-sector as follows (we assume \( k < N/2 \) for simplicity of the notation and we define \( T^{(0)} = U \))

\[
|T^{(k)}, \epsilon, \tilde{k}\rangle_{NSNS} = \exp \left[ \sum_{n=1}^{\infty} \left( \frac{1}{n} \sum_{\mu=2}^{5} \alpha_{n-\mu} \tilde{x}_{\mu} + i \epsilon \sum_{r>0} \left( \sum_{\mu=2}^{5} \psi_{\mu-r} \tilde{y}_{\mu-r} \right) \right) \right] \times \exp \left[ \sum_{n=0}^{\infty} \left( \frac{1}{n+k/\mathbb{N}} \alpha_{n-\frac{k}{\mathbb{N}}} \tilde{x}_{-\frac{k}{\mathbb{N}}} + \sum_{n=1}^{\infty} \left( \frac{1}{n-k/\mathbb{N}} \alpha_{n+k/\mathbb{N}} \tilde{x}_{-\frac{k}{\mathbb{N}}} \right) \right) \right]
\]

\[
+ \sum_{n=0}^{\infty} \left( \frac{1}{n-k/\mathbb{N}} \beta_{n+k/\mathbb{N}}, \tilde{y}_{n-k/\mathbb{N}}, \tilde{y}_{n-k/\mathbb{N}} \right) + \sum_{n=1}^{\infty} \left( \frac{1}{n+k/\mathbb{N}} \beta_{n-k/\mathbb{N}} \tilde{x}_{n-k/\mathbb{N}} \right)
\]

\[
+ i \epsilon \left( \sum_{r>0} \eta_{r-k/\mathbb{N}} \tilde{y}_{r-k/\mathbb{N}} + \sum_{r>0} \tilde{y}_{r+k/\mathbb{N}} \tilde{y}_{r+k/\mathbb{N}} \right)
\]

\[
+ i \epsilon \left( \sum_{r>0} \xi_{r+k/\mathbb{N}} \tilde{y}_{r+k/\mathbb{N}} + \sum_{r>0} \tilde{y}_{r-k/\mathbb{N}} \tilde{y}_{r-k/\mathbb{N}} \right) \right]|T^{(k)}, \epsilon, \tilde{k}\rangle_{NSNS}^{(0)},
\]

where we defined the zeromode as \(|T^{(k)}, \epsilon, \tilde{k}\rangle_{NSNS}^{(0)}\). The oscillators \((\alpha_{\mu}, \tilde{\alpha}_{\mu})\) and \((\psi_{\mu}, \tilde{\psi}_{\mu})\) are for bosonic fields \((X_L^{\mu}, X_R^{\mu})\) and for fermionic fields \((\psi_L^{\mu}, \psi_R^{\mu})\) on the world-sheet;
(α, ˜α, β, ˜β) denote the oscillators for (Z_L^1, Z_R^1, Z_L^2, Z_R^2) and (η, ˜η, ξ, ˜ξ) are their superpartners. They follow the canonical (anti)commutation relations

\[ [\alpha_{m+k/N}, \alpha_{n-k/N}] = (m+k/N)\delta_{m,-n}, \quad [\beta_{m+k/N}, \beta_{n-k/N}] = (m-k/N)\delta_{m,-n} \]

\[ \{\eta_{r-k/N}, \eta_{r+k/N}\} = \delta_{r+s}, \quad \{\xi_{r+k/N}, \xi_{r-k/N}\} = \delta_{r+s} \] (A.23)

The expressions for the others are also written almost in the same form as (A.22). For more details we recommend the readers to refer to [55, 51, 52, 57], for example.

We also comment that the above definition (A.20) does not work for \( k = \frac{N}{2} \) because there are extra fermionic zeromodes in twisted NSNS-sector along the orbifold direction. In this case one should change the factor in front of R.H.S. of (A.20) into 1 and the sign in the middle of (A.20) into +.

Next the zeromodes are normalized as follows: for the untwisted and twisted sectors

\[ \langle k^a | k'^a \rangle^{(0)} = V_1 (2\pi)^9 \delta^{9} (k^a - k'^a), \quad \langle k^i | k'^i \rangle^{(0)} = V_1 (2\pi)^5 \delta^{5} (k^i - k'^i). \] (A.24)

Finally we get the total boundary state \(|Dp(\alpha)\rangle\) which describes a \( \alpha \)-type Dp-brane as follows:

\[ |Dp(\alpha)\rangle = \sum_{k=0}^{N-1} e^{2\pi i k \alpha \frac{N}{N}} |T^{(k)}\rangle. \] (A.25)

The phase factors \( e^{2\pi i k \alpha \frac{N}{N}} \) are inserted in order to be consistent with the open string calculations. These are proportional to the charges in twisted-sectors.

**Open-Closed duality**

As argued by Cardy [59], the one-loop amplitude of open string should be equal to the tree level amplitude between two boundary states in closed string. This requirement is called Cardy’s condition and often gives a crucial consistency condition of D-branes. In our case, we can write this requirement as follows:

\[ Z_{\text{open}} = \langle Dp(\beta) | \Delta | Dp(\alpha)\rangle. \] (A.26)

Then comparing this with the eq.(A.4) and using (A.25), we obtain

\[ \frac{1}{N} Z_{\text{open}}^{(k)} = \langle T^{(k)} | \Delta | T^{(k)}\rangle. \] (A.27)

**Boundary State Calculations and Determination of the Normalization**
Now let us compute the cylinder amplitude in the boundary state formalism. The result for untwisted-sector is given by

\[
\langle U | \Delta | U \rangle = V_{p+1} T_p^{2 \alpha'} \sqrt{\frac{2}{\pi}} \int \frac{dk}{2\pi} e^{-\frac{1}{2} \alpha'_k s^2} \cdot \frac{\theta_3(0, is/\pi)^4 - \theta_2(0, is/\pi)^4 - \theta_4(0, is/\pi)^4}{\eta(is/\pi)^{12}}.
\]

(A.28)

For \( k \)-th twisted-sectors we obtain

\[
\langle T^{(k)} | \Delta | T^{(k)} \rangle = \frac{V_1 T_p^{2 \alpha'}}{16} \left( \int \frac{dk}{2\pi} \right)^5 ds e^{-\frac{1}{2} \alpha'_k s^2} \eta(is/\pi)^{-6} \left( (-i) \theta_1(\nu_k, is/\pi) \right)^{-2} \times \left[ \theta_3(0, is/\pi)^2 \theta_3(\nu_k, is/\pi)^2 - \theta_4(0, is/\pi)^2 \theta_4(\nu_k, is/\pi)^2 - \theta_2(0, is/\pi)^2 \theta_2(\nu_k, is/\pi)^2 \right].
\]

(A.29)

Then after we perform the integration in the above equations, we can determine the normalizations \( T_p, T'_p \) from the Cardy’s condition:

\[
T_p = \frac{1}{\sqrt{N}} \cdot 2^{3-p} \pi^{p-1} (\alpha')^{\frac{3-p}{2}},
\]

\[
T'_p = \frac{1}{\sqrt{N}} \cdot 2^{p} \pi^{p-\frac{3}{2}} (\alpha')^{\frac{3}{2}} \cdot \left( 2 \sin \frac{\pi k}{N} \right)^{1-\frac{p}{2}}.
\]

(A.30)

**Tension and Charges**

Finally let us determine the tension \( T_{D_p} \) and \( k \)-th twisted RR-charges \( Q^{(k)}_{\alpha,p} \) of a \( \alpha \)-type \( D_p \)-brane. Generally, one can compute a coupling with a closed string field from the overlap of the boundary state with the corresponding vertex operator as discussed in [60]. Therefore the tension and twisted RR-charges of our example can also be read off from the boundary state \( |Dp(\alpha)\rangle \) (A.25) as follows

\[
T_{D0} = \frac{T_0}{\sqrt{N}}, \quad T_{D2} = \sqrt{N} T_2, \quad T_{D4} = \sqrt{N} T_4,
\]

\[
Q^{(k)}_{\alpha,p} = \frac{1}{\sqrt{N}} \cdot e^{\frac{2\pi i k}{N}} \cdot T'_p,
\]

(A.31)

where the factor \( \frac{1}{\sqrt{N}} \) is needed for the correct normalization of untwisted fields [57]; the different coefficients of the tensions for \( p = 2, 4 \) are due to the facts that the volume factor \( V_{p+1} \) in (A.24) should be divided by \( N \) in physical context. Then it is obvious that the tension of a \( \alpha \)-type \( D0 \)-brane is \( \frac{1}{N} \) times that of an ordinary \( D0 \)-brane in flat space. On the other hand, for a \( \alpha \)-type \( D2 \) or \( D4 \)-brane the tension is the same as that of an ordinary \( D \)-brane. Some aspects of twisted RR-charges were discussed in section 3. Note that the \( D \)-brane also has a untwisted charge and twisted NSNS charges, which are proportional to the tension and the twisted RR-charges, respectively.
References

[1] M. B. Green, “Pointlike states for type 2b superstrings,” Phys. Lett. B 329 (1994) 435 [hep-th/9403040].

[2] T. Banks and L. Susskind, “Brane - Antibrane Forces,” hep-th/9511194.

[3] A. Sen, “Tachyon condensation on the brane antibrane system,” JHEP9808 (1998) 012 [hep-th/9805170].

[4] O. Bergman and M. R. Gaberdiel, “Stable non-BPS D-particles,” Phys. Lett. B 441 (1998) 133 [hep-th/9806153].

[5] A. Sen, “Non-BPS states and branes in string theory,” [hep-th/9904207].

[6] K. Bardakci, “Dual Models and Spontaneous Symmetry Breaking,” Nucl. Phys. B 68 (1974) 331; “Spontaneous Symmetry Breakdown In The Standard Dual String Model,” Nucl. Phys. B 133 (1978) 297.

K. Bardakci and M. B. Halpern, “Explicit Spontaneous Breakdown In A Dual Model,” Phys. Rev. D 10 (1974) 4230; “Explicit Spontaneous Breakdown In A Dual Model. 2. N Point Functions,” Nucl. Phys. B 96 (1975) 285.

[7] E. Witten, “Noncommutative Geometry And String Field Theory,” Nucl. Phys. B 268 (1986) 253.

[8] N. Berkovits, “SuperPoincare invariant superstring field theory,” Nucl. Phys. B 450 (1995) 90 [hep-th/9503009]; “A new approach to superstring field theory,” Fortsch. Phys. 48 (2000) 31 [hep-th/9912121].

[9] V. A. Kostelecky and S. Samuel, “On A Nonperturbative Vacuum For The Open Bosonic String,” Nucl. Phys. B 336 (1990) 263; “The Static Tachyon Potential In The Open Bosonic String Theory,” Phys. Lett. B 207 (1988) 169.

A. Sen and B. Zwiebach, “Tachyon condensation in string field theory,” JHEP0003 (2000) 002 [hep-th/9912249].

N. Berkovits, A. Sen and B. Zwiebach, “Tachyon condensation in superstring field theory,” Nucl. Phys. B 587 (2000) 147 [hep-th/0002211].

[10] K. Ohmori, “A Review on Tachyon Condensation in Open String Field Theories,” [hep-th/0102083].

[11] E. Witten, “On background independent open string field theory,” Phys. Rev. D 46 (1992) 5467 [hep-th/9208027]; “Some computations in background independent off-shell string theory,” Phys. Rev. D 47 (1993) 3405 [hep-th/9210067].
[12] S. L. Shatashvili, “Comment on the background independent open string theory,” Phys. Lett. B 311 (1993) 83 [hep-th/9303143]; “On the problems with background independence in string theory,” hep-th/9311177.

[13] A. A. Gerasimov and S. L. Shatashvili, “On exact tachyon potential in open string field theory,” JHEP0010 (2000) 034 [hep-th/0009103].

[14] D. Kutasov, M. Marino and G. Moore, “Some exact results on tachyon condensation in string field theory,” JHEP0010 (2000) 045 [hep-th/0009148].

[15] E. S. Fradkin and A. A. Tseytlin, “Nonlinear Electrodynamics From Quantized Strings,” Phys. Lett. B 163 (1985) 123;
O. D. Andreev and A. A. Tseytlin, “Partition Function Representation For The Open Superstring Effective Action: Cancellation Of Mobius Infinities And Derivative Corrections To Born-Infeld Lagrangian,” Nucl. Phys. B 311 (1988) 205;
A. A. Tseytlin, “Sigma model approach to string theory effective actions with tachyons,” hep-th/0011033.

[16] D. Kutasov, M. Marino and G. Moore, “Remarks on tachyon condensation in superstring field theory,” hep-th/0010108.

[17] K. Hori, “Linear models of supersymmetric D-branes,” hep-th/0012179.

[18] P. Kraus and F. Larsen, “Boundary string field theory of the D D-bar system,” hep-th/0012198.

[19] T. Takayanagi, S. Terashima and T. Uesugi, “Brane-antibrane action from boundary string field theory,” hep-th/0012210.

[20] A. Sen, “SO(32) spinors of type I and other solitons on brane-antibrane pair,” JHEP9809 (1998) 023 [hep-th/9808141].

[21] M. Frau, L. Gallot, A. Lerda and P. Strigazzi, “Stable non-BPS D-branes in type I string theory,” Nucl. Phys. B 564 (2000) 60 [hep-th/9903123].

[22] Y. Matsuo, “Tachyon condensation and boundary states in bosonic string,” hep-th/0001044.

[23] A. Sen, “BPS D-branes on non-supersymmetric cycles,” JHEP9812 (1998) 021 [hep-th/9812031].

[24] J. Majumder and A. Sen, “Blowing up’ D-branes on non-supersymmetric cycles,” JHEP9909 (1999) 004 [hep-th/9906109]; “Vortex pair creation on brane-antibrane pair via marginal deformation,” JHEP0006 (2000) 010 [hep-th/0003124]; “Non-BPS D-branes on a Calabi-Yau orbifold,” JHEP0009 (2000) 047 [hep-th/0007158].
[25] M. Naka, T. Takayanagi and T. Uesugi, “Boundary state description of tachyon condensation,” JHEP0006 (2000) 007 [hep-th/0005114].

[26] Y. Oz, T. Pantev and D. Waldram, “Brane-antibrane systems on Calabi-Yau spaces,” hep-th/0009112.

[27] R. Tatar, “A note on non-commutative field theory and stability of brane-antibrane systems,” hep-th/0009213.

[28] Y. Hikida, M. Nozaki and T. Takayanagi, “Tachyon condensation on fuzzy sphere and noncommutative solitons,” Nucl. Phys. B 595 (2001) 319 [hep-th/0008023].

[29] Y. Hikida, M. Nozaki and Y. Sugawara, “Formation of spherical D2-brane from multiple D0-branes,” hep-th/0101211.

[30] K. Dasgupta, S. Mukhi and G. Rajesh, “Noncommutative tachyons,” JHEP0006 (2000) 022 [hep-th/0005006];
J. A. Harvey, P. Kraus, F. Larsen and E. J. Martinec, “D-branes and strings as noncommutative solitons,” JHEP0007 (2000) 042 [hep-th/0005031];
J. A. Harvey, P. Kraus and F. Larsen, “Exact noncommutative solitons,” JHEP0012 (2000) 024 [hep-th/0010060].

[31] I. Bars, H. Kajiura, Y. Matsuo and T. Takayanagi, “Tachyon condensation on noncommutative torus,” [hep-th/0010101];
E. M. Sahraoui and E. H. Saidi, “Solitons on compact and noncompact spaces in large noncommutativity,” hep-th/0012259.

[32] E. J. Martinec and G. Moore, “Noncommutative solitons on orbifolds,” hep-th/0101199.

[33] M. R. Douglas, “Two lectures on D-geometry and noncommutative geometry,” hep-th/9901146; “Topics in D-geometry,” Class. Quant. Grav. 17 (2000) 1057 [hep-th/9910170].

[34] E. Witten, “D-branes and K-theory,” JHEP9812 (1998) 019 [hep-th/9810188].

[35] J. A. Harvey, D. Kutasov and E. J. Martinec, “On the relevance of tachyons,” hep-th/0003101.

[36] K. Becker, M. Becker and A. Strominger, “Five-branes, membranes and nonperturbative string theory,” Nucl. Phys. B 456 (1995) 130 [hep-th/9507158].

[37] H. Ooguri, Y. Oz and Z. Yin, “D-branes on Calabi-Yau spaces and their mirrors,” Nucl. Phys. B 477 (1996) 407 [hep-th/9606112].

[38] M.F. Atiyah, R. Bott and A. Shapiro, “Clifford Modules”, Topology 3 (1964) 3.
[39] N. P. Warner, “Supersymmetry in boundary integrable models,” Nucl. Phys. B 450 (1995) 663 [hep-th/9506064].

[40] C. Kennedy and A. Wilkins, “Ramond-Ramond couplings on brane-antibrane systems,” Phys. Lett. B 464 (1999) 206 [hep-th/9905197].

[41] M. Alishahiha, H. Ita and Y. Oz, “On superconnections and the tachyon effective action,” hep-th/0012222.

[42] D. Quillen, “Superconnection and the Chern character,” Topology 24 (1985) 89.

[43] S. Terashima, “A construction of commutative d-branes from lower dimensional non-BPS D-branes,” hep-th/0101087.

[44] L. Dixon, J. A. Harvey, C. Vafa and E. Witten, “Strings On Orbifolds,” Nucl. Phys. B 261 (1985) 678; “Strings On Orbifolds. 2,” Nucl. Phys. B 274 (1986) 285;
L. Dixon, D. Friedan, E. Martinec and S. Shenker, “The Conformal Field Theory Of Orbifolds,” Nucl. Phys. B 282 (1987) 13.

[45] H. Garcia-Compean, “D-branes in orbifold singularities and equivariant K-theory,” Nucl. Phys. B 557 (1999) 480 [hep-th/9812226].

[46] M. R. Douglas and G. Moore, “D-branes, Quivers, and ALE Instantons,” hep-th/9603167.

[47] C. V. Johnson and R. C. Myers, “Aspects of type IIB theory on ALE spaces,” Phys. Rev. D 55 (1997) 6382 [hep-th/9610140].

[48] T. Eguchi, P. B. Gilkey and A. J. Hanson, “Gravitation, Gauge Theories And Differential Geometry,” Phys. Rept. 66 (1980) 213.

[49] P. S. Aspinwall, “Enhanced gauge symmetries and K3 surfaces,” Phys. Lett. B 357 (1995) 329 [hep-th/9507012].

[50] D. Diaconescu, M. R. Douglas and J. Gomis, “Fractional branes and wrapped branes,” JHEP9802 (1998) 013 [hep-th/9712230].

[51] D. Diaconescu and J. Gomis, “Fractional branes and boundary states in orbifold theories,” JHEP0010 (2000) 001 [hep-th/9906242].

[52] T. Takayanagi, “String creation and monodromy from fractional D-branes on ALE spaces,” JHEP0002 (2000) 040 [hep-th/9912157].

[53] F. Hussain, R. Iengo, C. Nunez and C. A. Scrucca, “Interaction of moving D-branes on orbifolds,” Phys. Lett. B 409 (1997) 101 [hep-th/9706186].
[54] A. Sen, “Stable non-BPS bound states of BPS D-branes,” JHEP9808 (1998) 010 [hep-th/9805019];
O. Bergman and M. R. Gaberdiel, “Non-BPS states in heterotic-type IIA duality,” JHEP9903 (1999) 013 [hep-th/9901014];
M. R. Gaberdiel and B. J. Stefanski, “Dirichlet branes on orbifolds,” Nucl. Phys. B 578 (2000) 58 [hep-th/9910109].

[55] M. Billo, B. Craps and F. Roose, “On D-branes in type 0 string theory,” Phys. Lett. B 457 (1999) 61 [hep-th/9902196].

[56] I. Brunner, R. Entin and C. Romelsberger, “D-branes on T(4)/Z(2) and T-duality,” JHEP9906 (1999) 016 [hep-th/9905078].

[57] M. Billo, B. Craps and F. Roose, “Orbifold boundary states from Cardy’s condition,” JHEP0101 (2001) 038 [hep-th/0011060].

[58] M. Bertolini, P. Di Vecchia, M. Frau, A. Lerda, R. Marotta and I. Pesando, “Fractional D-branes and their gauge duals,” JHEP0102 (2001) 014 [hep-th/0011077];
M. Frau, A. Liccardo and R. Musto, “The geometry of fractional branes,” [hep-th/0012033];
P. Merlatti and G. Sabella, “World volume action for fractional branes,” [hep-th/0012193].

[59] J. L. Cardy, “Boundary Conditions, Fusion Rules And The Verlinde Formula,” Nucl. Phys. B 324 (1989) 581.

[60] P. Di Vecchia, M. Frau, I. Pesando, S. Sciuto, A. Lerda and R. Russo, “Classical p-branes from boundary state,” Nucl. Phys. B 507 (1997) 259 [hep-th/9707068].

[61] S. P. de Alwis, “Boundary string field theory the boundary state formalism and D-brane tension,” [hep-th/0101200].

[62] E. G. Gimon and C. V. Johnson, “K3 Orientifolds,” Nucl. Phys. B 477 (1996) 715 [hep-th/9604129].

[63] O. Bergman and M. R. Gaberdiel, “A non-supersymmetric open-string theory and S-duality,” Nucl. Phys. B 499 (1997) 183 [hep-th/9701137].