Dispersive Manipulation of Paired Superconducting Qubits

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We combine the ideas of qubit encoding and dispersive dynamics to enable robust and easy quantum information processing (QIP) on paired superconducting charge boxes sharing a common bias lead. We establish a decoherence free subspace on these and introduce universal gates by dispersive interaction with a LC resonator and inductive couplings between the encoded qubits. These gates preserve the code space and only require the established local symmetry and the control of the voltage bias.

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Superconducting nano-circuits consisting of charge boxes (CB) are among the most promising candidates for a quantum computer. Coherent control of a single charge qubit and, long decoherence time and, more recently, coupled two qubit systems have been demonstrated. But despite this encouraging experimental progress, there are serious difficulties with superconducting QIP which may appear insurmountable. The first is the severe decoherence experienced by these macroscopic qubits, which are coupled to a large number of degrees of freedom in their environment including control circuitry. A few methods have been employed to reduce the decoherence, but they usually require sophisticated manipulation or significant overhead in the control circuitry. The second major difficulty comes from the imperfect control realizable in solid state systems. Specifically, one finds it difficult to achieve controllable couplings between superconducting qubits, since the commonly used hard-wired inductive or capacitive couplings are untunable. Great effort has been exercised to realize controllable couplings, but they usually require sophisticated manipulation or significant overhead in the control circuitry.

The requirement to reduce decoherence and the desire for the easiest manipulation apply to all QIP implementations. Unfortunately, it is not always easy to accomplish both - actually the goals are often contradictory - since reducing decoherence may require extra complication in the manipulation.

In this work we show how to achieve both goals. Using a closely placed pair of charge boxes (PCB) sharing a common bias lead as the logic qubit, we can encode information in a fashion immune to collective noise, which is the dominating decoherence source in our setting. We introduce LC resonators inductively coupled to the PCBs whose virtual excitations allow us to manipulate the PCB dispersively; all interactions will be off resonance, without energy transfer, and thus a logical qubit stays within its encoding space even during manipulation. By inductively coupling the CBs and taking advantage of dispersive dynamics again, controlled phases can be induced between logical qubits.

Combining dispersive dynamics and encoding offers a new method for QIP. It overcomes the major difficulties of superconducting CBs in a realistic and very simple fashion. The only control required after initialization is to change the voltage bias of the PCB. Another advantage of our method is that it relies only on noise symmetry over short distance and so this is a realistic technique for scalable QIP over large systems. Though we focus on superconducting QIP in this paper, the general principle of dispersive manipulation of encoded qubits will be valuable for many other QIP systems.

Charge qubits and the dominating noise. – In its simplest form, the charge qubit is just a superconducting island voltage biased through a Josephson junction. The Hamiltonian for the CB system is \(H_{CB} = E_c(n - n_g)^2 - E_J \cos \phi\), where the charging energy \(E_c = (2e)^2/2C\), \(C\) being the total capacitance of the island, is much greater than the Josephson energy \(E_J = I_c\Phi_0/2\pi\), and \(\phi\) is the phase drop across the junction. When biased close to \(n_g = C_v/V_g/2e = 1/2\) (\(C_g\) is the gate capacitance), it provides an effective two-state system which can be used as a qubit. In the spin 1/2 notation,

\[
H_{CB} = \frac{1}{2} B^x \sigma^x - \frac{1}{2} B^y \sigma^y ,
\]  

where the spin up or down states correspond to \(n=0\) or \(n=1\) excess Cooper-pairs on the CB. The effective field \(B^x = E_c(2n_g - 1)\) can be tuned by changing the gate voltage \(V_g\).

In order to control \(B^z = E_J\), we use a flux biased small dc-SQUID to replace the junction, whose critical current is maximal \((I_c = I_c^0)\) when the flux bias is off and vanishes \((I_c = 0)\) when it is \(I_c/2 = \Phi_0/4\).

The dominating decoherence sources in this system are circuit noise in the voltage bias and charge fluctuations in the background (known as “charge noise”), as indicated in Fig. 1(a). The circuit noise is described by the well known “Spin-Boson” model. The charge noise is less well understood, but it is now generally believed to be caused by fluctuations of the impurity charges in the substrate. Taking into account the noise sources, we have the total Hamiltonian for the system as \(H = H_{CB} + H_Z + H_B + H_{int}\), where \(H_Z\) and \(H_B\) are the Hamiltonian of the circuit fluctuations,
modelled as a collection of harmonic oscillators [19], and the Hamiltonian of the background charge. Since these noise perturb the voltage bias of the CB, the interaction Hamiltonian \( H_{\text{int}} \) has the form \( H_{\text{int}} = \sigma_i^z \tilde{E}^Z + \sigma_i^z \tilde{E}^B \), where \( \tilde{E}^Z \) and \( \tilde{E}^B \) are environment operators (with the coupling strengths included). Here we focus on the fluctuations in the voltage bias, the dominating source of decoherence and neglect noise in the \( B^z \) field (the critical current), as practised customarily [3, 18, 19]. This is because charge qubits are insensitive to flux noise and the effect of the fluctuation in the \( I_c \) suppression field is secondary to the bias voltage variations discussed above. Detailed treatment of the two voltage noise sources and their influence on the charge qubit system can be found in the literature [18, 19]. For our purpose, the nature of the environment and specific form of \( \tilde{E}^Z \) and \( \tilde{E}^B \) are not essential, therefore we do not go into detail here.

Paired charge boxes and DFS encoding. – As shown in Fig. 1(a), we use two capacitively coupled identical CBs \((a, b)\) with a common bias-lead as an encoded qubit. The small capacitive coupling, \( C_c \ll C_L \), is not essential for the encoding, but necessary for the encoded two qubit gates. The Hamiltonian of the PCB system is

\[
H_{\text{PCB}} = \sum_{i=a,b} \frac{1}{2} (B^2 \sigma_i^z - B_i^z \sigma_i^x) - \gamma \sigma_i^z \sigma_i^x, \quad \text{where} \quad \gamma = \frac{C_c}{C_L} E_c \text{is the coupling energy.}
\]

Since the two CBs share the same lead, obviously they are biased at the same voltage and they will experience the same circuit noise. In addition the nano-scale charge islands are put very close to each other. Therefore, they will experience the same charge fluctuations too (more discussion on this point will be given later). Hence the CBs experience “collective decoherence,” meaning the noise sources couple symmetrically to them, which naturally gives rise to “decoherence-free encoding.” For a review of decoherence-free subspace, see [20] and references therein. Here we have the simplest case of the DFS, with \( |0\rangle = |\downarrow_a \downarrow_b\rangle \) and \( |1\rangle = |\uparrow_a \downarrow_b\rangle \) as the decoherence-free logical states. The way this works can easily be seen: as a consequence of the collective decoherence the coupling between the PCB and the environment is

\[
(\sigma_i^+ \sigma_i^-) \ast (\tilde{E}^B + \tilde{E}^Z), \text{which annihilates the two logical states given above. Therefore the PCB system will not get entangled with the environment if it is initialized and kept in the DFS. Physically, the encoded qubit’s immunity to noise stems from the fact that the CBs acquire random but opposite phases.}
\]

To prepare the system in the DFS, we bias the PCB far off the degeneracy point \( n_g = 1/2 \). In the spin-1/2 picture, this corresponds to applying a strong field in the \( z \) direction. At low temperatures, the spins will line up with the field, and the PCB relaxes to the state \( |\downarrow_a \downarrow_b\rangle \). Keeping both \( B^z \) off, we change \( B^z \) (same for \( a \) and \( b \)) to \(-2\gamma \). This cancels the bias of \( a \) on \( b \) making its total \( B^{z \text{tot}} = B^z + 2\gamma = 0 \). After that we turn on some \( B^x \). After a time \( \pi / B^x \), the state of \( b \) will become \( |\uparrow_b\rangle \), and the system is prepared in the DFS state \( |\downarrow_a \uparrow_b\rangle \). Now we turn off \( B^x \) as well, and from now on the \( B^z \) fields for both CBs will remain off, by biasing the dc-SQUIDS of the PCB at \( \Phi_0/2 \). Since \( B^z \) fields will remain off and need not be tuned after the initialization, the leads tuning \( I_c \) of the PCB could be heavily filtered to keep out the noise once the system is initialized. Alternatively we could make use of the noise free constant flux-bias techniques such as that demonstrated in [21]. In practice it can be difficult to suppress \( I_c \) of the PCB precisely to 0 due to the finite self inductance of the dc-SQUID. However as shown in [22] if low self inductance dc-SQUIDS are used (\( L_{\text{LC}} / \Phi_0 \ll 1 \) the \( B^z \) field at \( \Phi_0/2 \) bias point is negligibly small compared to \( B^x \) fields used for computation and can thus be safely dropped. Many schemes [1, 5, 13] rely on this fact too. One notices that the logical states \( |0\rangle \) and \( |1\rangle \) are always degenerate regardless of the voltage bias \( n_g \), therefore there is no evolution in the idle mode, regardless of the voltage bias or noise in it. To readout the state of the PCB, a measurement of its CB \( a \) or \( b \) will suffice, which is readily accomplished with developed techniques [13].

As seen above, in realizing DFS encoding with the PCB, we lose considerable freedom in manipulating the system. First, in order to guarantee symmetrical coupling to the circuit noise, the two CBs share the same lead and hence they are always biased at the same voltage. More importantly, we must ensure that operations on the PCB do not drive the system out of the DFS, otherwise the immunity to noise is lost [20]. This is why we must keep the \( B^z \) fields for the CBs off: they flip the states of a single CB and hence do not preserve the DFS. Therefore the only control left is the voltage bias of the PCB, which clearly is insufficient for universal QIP on the PCB. To deal with this difficulty, we introduce a LC resonator inductively coupled to the PCB system:

The LC resonator inductively coupled to the PCB. – As shown in Fig. 1(b), we inductively couple the dc-SQUIDS of the two CBs in the PCB system symmetrically to an initially unexcited LC circuit. Even in the ground state its vacuum fluctuations bias the dc-SQUIDS of the PCB off \( \Phi_0/2 \) making charge tunnelling possible. The Hamiltonian for the PCB-LC system is

\[
\mathcal{H} = \mathcal{H}_{\text{PCB}} + \mathcal{H}_{\text{LC}} + \mathcal{H}_{\text{coup}}, \quad \text{where} \quad \mathcal{H}_{\text{LC}} = \omega a^\dagger a, \quad \text{and} \quad \mathcal{H}_{\text{coup}} = -ig(a - a^\dagger)\sigma_x \text{ with } g = \frac{1}{2} M_I c_0 \sqrt{\frac{\hbar}{M_c 2I_c}}. \text{ Here } \omega
\]
and $L$ refer to frequency and inductance of the LC-resonator; $M$ is the mutual inductance between SQUID and resonator. When the PCB and LC are far off resonance, the effect of the LC can be neglected. On the other hand, when we tune the bias of the PCB such that it is close to being in resonance with the LC resonator, within the framework of Rotating Wave approximation the above Hamiltonian becomes

$$H = H_{PCB} + H_{LC} - ig\sum_{i=a,b}(\sigma_i^+ a - \sigma_i^- a^\dagger),$$

where $\sigma_i^\pm = (\sigma_i^x \pm i\sigma_i^y)/2$ are the ladder operators. Notice that $H_{PCB}$ contains a coupling term, in contrast to the standard Jaynes Cummings Model which modifies the dynamics significantly, as will be seen below.

Let $\delta = B^2 - \omega$ be the detuning. If we let the PCB and the (initially un-excited) LC resonator interact right in resonance, i.e., $|\delta - 2\gamma| \ll g \ll \omega$ or $|\delta + 2\gamma| \ll g \ll \omega$, state transfer occurs between the PCB and the LC resonator [13, 14]. This is not allowed in our scheme, since it will drive the PCB out of the DFS. Besides, once the LC resonator is excited, we will have additional decoherence due to the finite quality of the LC resonator [13, 14]. Therefore, we only work in the dispersive region, $g \ll |\delta \pm 2\gamma| \ll \omega$. In this case, the PCB and the LC resonator cannot exchange energy because of the large detuning. However the virtual excitation of the LC resonator gives rise to an effective interaction between the two CBs [24].

$$H_{eff} = \frac{g^2}{\delta + 2\gamma} + \frac{g^2}{\delta - 2\gamma}(\sigma_a^+ \sigma_b^- + \sigma_a^- \sigma_b^+), \quad (2)$$

where the first term describes the Stark shift, which can be neglected since it is the same for both logical states, and the second term is the effective exchange interaction caused by the exchange of a virtual photon. It preserves the DFS (it changes $|\uparrow_a\downarrow_b\rangle$ to $|\downarrow_a\uparrow_b\rangle$ and vice versa) and acts as a logical X gate on the PCB. Starting from $|0\rangle = |\downarrow_a\uparrow_b\rangle$, letting the PCB evolve under the effective Hamiltonian for a time $t = \frac{\pi(\delta - 2\gamma)}{2\gamma}$ or $\frac{\pi}{\lambda}$, we can swap the states of $a$ and $b$ or generate a maximally entangled state between them. The LC resonator is initially in the vacuum state and will not be excited due to the dispersive interaction with the PCB; therefore unlike in previous schemes [13, 14] we are not subject to decoherence caused by its finite quality. Also, notice that our dispersive scheme is fundamentally different from the previous method of using the virtual excitation of a large LC circuit capacitively coupled to all the qubits [8], in which the LC-frequency is much larger than the CB-energies and the coupling strength is tuned by changing the $E_i$’s of the qubits. In our scheme the two energies are close (though the detuning is large) and the $E_i$’s are always on; the coupling strength is controlled by simply changing the detuning. Our scheme takes advantage of the quantum exchange effect (Eq. (2)) assisted by a virtually excited quantum state of the LC-qubit system [24, 25]. It offers noise protection with simplest operation using very realistic parameters (see below), which was not available in previous schemes.

To realize $SU(2)$ on the PCB, we still need a phase gate. This can be accomplished by using another LC resonator, not depicted in Fig. (a) and at a frequency $\omega'$ very different from $\omega$, inductively coupled to the dc-SQUID of only $a$ (or $b$) in the PCB. Then when we tune the voltage bias of the PCB such that it interacts with this LC resonator dispersively ($g \ll |\delta' \pm 2\gamma| \ll \omega'$), a phase gate will be obtained due to the Stark shift $\frac{\gamma^2}{\delta' + 2\gamma}|1\rangle\langle 1|$. The effect of the previous LC resonator can be neglected because it is far off resonance with the PCB. However for the purpose of universal QIP this second LC resonator is not absolutely necessary [26], as will be shown below.

**Inductively coupled PCB arrays.** – To realize universal QIP on the PCBs, we need a scheme to couple them. We notice that different PCBs will experience different noise as they are biased by different leads. Those far apart are susceptible to different charge noise too. So we only have what we call “local” DFS; this only relies on noise symmetry over a few, here two, physical qubits, as is inevitably the only realistic case for scalable QIP, and previously discussed methods [24] do not apply. Stringent restrictions are put on the two qubit coupling in order to preserve the DFS. A capacitive coupling between $1b$ and 2a, for example, cannot be used, as this would cause the noise in PCB1’s lead to leak asymmetrically into PCB2. Furthermore neither PCB may leave its DFS during its evolution.

Here we discuss a new approach that allows scalable QIP based on local DFS. We couple the PCBs inductively, using a small mutual inductance $M^\prime$ between their dc-SQUIDS, as shown in Fig. (b). As the dc-SQUIDS are biased at $\Phi_0/2$ the coupling Hamiltonian is $\lambda \sigma_a^\dagger \sigma_b^\dagger$, where the coupling strength $\lambda = \frac{1}{2}M^\prime R_{1b} R_{2a}$ (chosen much smaller than $E_0^\prime$, the unsuppressed Josephson Energy of the SQUIDS).

Obviously the PCBs in the array in Fig. (b) can be initialized in their DFSs just as described before. Now, if all the PCBs are biased at the same voltage, they will get out of their DFSs due to resonant interactions by the above coupling term. Therefore, we bias the odd numbered PCBs at the degenerate point $n_a = 1/2$, giving a $B^2 = 0$ and the even numbered PCBs at some other point (but far off resonance with their LC resonators) giving a large $B^2$. Because of the large detuning, $B^2 \gg \lambda$, the coupling Hamiltonian has no effect in the idle mode [12]. One might worry that this different biasing will introduce a difference between the phase frequencies of the PCBs, but it is not the case since the DFS states always have the energy $\gamma$ regardless of the voltage bias of the PCBs. When we want to do a controlled phase gate between PCB 1 and 2, we tune their biases near some common target value very different from their previous values (such that they do not interact with other neighboring PCBs) and the LC frequencies. Assuming the fields are $B^2_1$ and $B^2_2$, we work in the dispersive region such that the states of the PCBs do not change except that dispersive phases are obtained due to the virtual transitions. For instance, the energy of the state $|0_1 0_2\rangle$ will be shifted by $\Delta E_{0102}(\lambda \sigma_a^\dagger \sigma_b^\dagger |m\rangle = \frac{\lambda^2}{\gamma^2 + \Delta^2} |m\rangle$, where $|m\rangle = |\downarrow_1\downarrow_2\uparrow_2\uparrow_2\rangle$ is the virtually excited intermediate state and $\Delta = B^2_1 - B^2_2$ is the detuning. Other phases can be calculated too, giving in the basis $|0_1 0_2\rangle, |0_1 1_2\rangle, |1_1 0_2\rangle, |1_1 1_2\rangle$ an effective Hamiltonian $\text{diag}\left(\frac{\lambda^2}{\gamma^2 + \Delta^2}, \frac{-\lambda^2}{\gamma^2 + \Delta^2}, \frac{\lambda^2}{\gamma^2 + \Delta^2}, \frac{\lambda^2}{\gamma^2 + \Delta^2}\right)$, which reduces to $\text{diag}\{0, 0, 0, \frac{\lambda^2}{\gamma^2 + \Delta^2}\}$ in the dispersive region.
λ ≪ |4γ − Δ| ≪ |4γ + Δ| ≪ |4γ ± (B_1^2 + B_2^2)|. This gives a CPHASE(α) gate \( \text{diag}\{1,1,e^{-i\alpha}\} \), where \( \alpha = \frac{x^2}{4\gamma - \Delta} \) with \( t \) being the evolution time.

Notice that in the above implementation of the CPHASE gate, the capacitive coupling \( \gamma \) plays an essential role. As can be easily checked the above procedure does not give an entangling gate if \( a \) and \( b \) do not bias each other (\( \gamma = 0 \)). This is because \( |0_1 0_2\rangle \) and \( |1_1 1_2\rangle \) would acquire opposite energy shifts \( \pm \lambda^2/\Delta \), due to the fact that the energy differences between the initial and intermediate states are opposite for these two cases. Thus the role of the capacitive coupling \( \gamma \) can be understood by an interesting "parity argument:" under the exchange of the states of \( a \) and \( b \) for both PCBs, the energy mismatch (the denominator in the perturbative calculation) due to the detuning \( \Delta \) is odd. This symmetry is broken by the coupling between \( a \) and \( b \), which is even under this operation (as is obvious from the form \( -\gamma\sigma_z^a\sigma_z^b \)).

Another point of interest is that, a phase gate on a PCB can be implemented by using the CPHASE(α) and the X gate, as is easily recognized by the identity \( e^{i\alpha X} = e^{i\alpha}(X_2 \cdot \text{CPHASE}\{2\alpha\})^2 \). Therefore, for the purpose of universal QIP on the PCB array the LC resonator giving the phase gate is not absolutely necessary, as long as the system has more than 1 qubit \[26\]. This is potentially beneficial in reducing the hardware.

**Discussion.** – In the above we described our scheme of QIP with PCBs based on encoded qubits and dispersive dynamics, which requires only tuning the voltage bias of the PCBs. Our scheme can prevent decoherence from collective noise. The circuit noise obviously couples symmetrically to the CBs of the PCB. The charge noise requires some caution, since its exact nature is still a topic of debate \[17, 22\]. The early experiments in \[17\] clearly show that the charge noise on close by islands are correlated. The conclusion drawn from this observation, that the charge noise stems mostly from sources in the substrate was further substantiated by \[28\]. This has important consequences, because it suggests that it is possible to engineer the environment for desired noise configurations. Indeed, analysis in \[17\] shows that high noise correlations can be achieved for properly designed geometry and layouts of the charge islands. Simple environment engineering was already successful \[28, 29\] in various contexts.

For our scheme to work, the CBs must be located within a distance smaller than the wavelength of the background charge fluctuations, so that they experience the same noise. This seems to be realistic, since the advance in device fabrication allows to make smaller structures, and more importantly the fact that the charge noise originates from the substrate makes it possible to engineer the environment for the desired noise symmetry \[17, 23\]. For instance, if we put the PCB on an electrode instead of the substrate \[28\], the charge impurities will be located far away from the PCB, and thus couple symmetrically to the CBs.

Though our scheme eliminates the effect of the collective charge noise on the PCB, the decoherence time will be finite as there are other non-collective noise in the system not dealt with by our prescription, for instance dissipation due to the finite impedance of the junction and the noise in its critical current. The effect of the virtually excited states and the fluctuation of the dispersive energies must be evaluated carefully too, though some results were obtained previously \[20, 31\]. Qualitatively, as shown by simple analysis based on Master equations the number of operations allowed in our dispersive scheme increases by \( \Delta/g \gg 1 \) (here, \( \Delta \) the effective detuning and \( g \) the coupling strength) as compared to the usual scheme based on resonant Rabi manipulations, if the same coupling strength \( g \) is assumed \[33\]. The PCBs do not experience decoherence in the idle mode, which is favorable for a large system in which only a fraction of the qubits undergo active manipulation at the same time. Therefore, our scheme can reduce the error rate of the PCBs below the threshold for error correction schemes \[33\] and thus make superconducting QIP feasible. Detailed calculation of the decoherence time for a realistic PCB system will be reported elsewhere \[32\].

Another practical concern is that the CBs in a PCB will not be completely identical due to the imperfect fabrication. Because only local symmetry is required, this problem is less significant since fabrication variations tend to happen at large scales and experiments show that closely spaced charge-boxes can be equal beyond experimental resolution \[17\]. In addition, the error induced by non-identical qubits was shown to be higher order in the symmetry breaking \[32\]. Therefore, we conclude that the technological problem of imperfect fabrication is already solved to the extent needed for our scheme.

**Parameters.** – Finally, we give some parameters for the experimental consideration. We use small CBs closely spaced with a total capacitance \( C_i \approx 0.16 fF \) and charging energy \( E_c \approx 500 GHZ \). A mutual capacitance \( C_c = 5 aF \) gives \( \gamma \approx 7.5 GHZ \). A mutual inductance \( M = 7pH \) between the PCB and LC with \( L = 50nH \) and \( \omega/2\pi = 200 GHZ \) gives \( g \approx 0.25 GHZ \) for \( I_c = 40 nA \). Tuning the bias of the PCB close to the LC frequency with a detuning \( \delta \approx -12.5 GHZ \) results in an exchange interaction with the strength \( \frac{g^2}{2c^2} \approx \frac{g^2}{25 MHz} \), corresponding to a period of \( 40 ns \). During the idle mode, we bias the odd numbered PCBs at \( B^z = 0 \) and the even numbered ones at \( B^z \approx 100 GHZ \). Since low self inductance de-SQUIDS should be used, we choose \( M'/I_c \approx 10^{-3} \), \( M'' \approx 50pH \) and \( M'' \approx 120GHz \) gives the coupling strength \( \lambda \approx 0.22 GHZ \). Tuning the biases of the neighboring PCBs both to about \( 400 GHz \) with a detuning \( \Delta \approx 28 GHz \) gives a CPHASE gate at the rate \( \frac{\lambda^2}{\gamma^2 - \Delta^2} \approx 25 MHz \). The above parameters are well within the reach of the current technology \[35\].

In conclusion, we have discussed a technique for robust and easy superconducting QIP. By combining the ideas of encoding and dispersive manipulations, we protect the charge qubits from the dominating decoherence and realize universal QIP on the encoded qubits with minimal control. Besides the great potential of solving the fundamental difficulties in superconducting QIP, we expect the general idea of dispersive manipulation of encoded qubits to be of interest to other physical systems, such as atomic and other solid state systems \[36\].

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