Superconductors of mixed order parameter symmetry in a Zeeman magnetic field

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We study the effect of Zeeman magnetic field in the superconducting phase with two component order parameter scenario, such as, \(d_{x^2-y^2} + e^{i\beta}\alpha\), where \(\alpha = d_{xy}, s\). This scenario is equivalent to applying magnetic field parallel to \(Cu-O\) planes. In a weak magnetic field, which does not cause much change to the predominant \(d\)-wave, supresses the minor \(\alpha\) component leading to pure \(d\)-wave phase. This observation is in contrary to the effect of magnetic field applied in the \(c\)-direction to the \(Cu-O\) planes which is believed to induce a minor component \(\alpha\) to \(d\)-wave superconductors. We also show that the response of such superconductors to a weak Zeeman magnetic field can be quite different depending on the phase \(\theta\) of the minor component \((\alpha)\).

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\bar{\text{74.25.Nf,74.25.Dw,74.62.-c}}
\]

I. INTRODUCTION

Although the nature of the superconducting pair wave function in high \(T_c\) cuprates is not yet known strong evidences of a major \(d_{x^2-y^2}\) symmetry exists \(\dagger\). Experiments sensitive to the internal phase structure of the pair wave function reported a sign reversal of the order parameter supporting \(d\) wave symmetry \(\ddagger\). Most recently from various experiments and theory it appears that the pairing symmetry of these family could be a mixed one like \(d_{x^2-y^2} + e^{i\beta}\alpha\) where \(\alpha\) could be something in the \(s\) wave family or \(d_{xy}\). There were early questions from tunneling experiments regarding the pure \(d\)-wave symmetry \(\ddagger\) as the data supports an admixture of \(d\) and \(s\)-wave components due to orthorhombicity in YBCO \(\ddagger\). Possibility of a minor but finite \(id_{xy}\) symmetry alongwith the predominant \(d_{x^2-y^2}\) has also been suggested \(\dagger\) in connection with magnetic defects or small fractions of a flux quantum \(\Phi_0 = hc/2e\) in YBCO powders. Similar proposals came from various other authors in the context of magnetic field, magnetic impurity, interface effect etc. \(\dagger\) \(\ddagger\) The experimental result by Krishana et al., was interpreted as a signature of induction of a minor component \(eg, id_{xy}\) or \(is\) in a \(d\)-wave superconductor with the application of magnetic field along the \(c\) axis.

In this work, we study in details the effect of a weak Zeeman magnetic field on superconductors with mixed order parameter symmetry like \(\Delta(k) = \Delta_{d_{x^2-y^2}} + e^{i\beta}\alpha\) with \(\alpha = d_{xy}, s\) for arbitrary \(\theta\). It is well known that such superconductors with \(\theta \neq 0\) and \(\alpha \neq 0\) corresponds to broken time reversal states (BTRS). These BTRS states lift the directional degeneracy of charge currents by admixing a subdominant \(\alpha\)-wave component to the \(d\)-wave pairing state and a spontaneous finite current appears \(\dagger\). An application of Zeeman magnetic field can lift the spin degeneracy leading to suppression of BTRS ; a pure \(d\)-wave occurs with increasing magnetic field. For mixed symmetry as above with \(\theta = 0\) that preserves time reversal symmetry and are nodeful respond differently to the Zeeman field as compared to \(\theta \neq 0\) states. For node-full \(\theta = 0\) state, the local gap \(\Delta(k)\) of small magnitude over the Fermi surface may be destructed with the application of Zeeman field leading to a paramagnet pocket. This although true for \(\theta \neq 0\) states, but such states correspond to fully gapped situation all over the Fermi surface which causes weak response to the magnetic field. A clear picture on the above will be demonstrated in this article. It may be mentioned that the high temperature superconductors are quasi two dimensional in nature and therefore, a magnetic field parallel to the \(Cu-O\) plane does not couple to the orbital motion of the electrons in the plane. Therefore, we shall not consider spin-orbit interaction in this work.

In connection with the discussion of order parameter symmetry in cuprates, we would further like to mention that the proposal of mixed order parameter symmetry got the correct momentum when experimental data on longitudinal thermal conductivity by Krishana et al, \(\dagger\) of \(Bi_2Sr_2CaCu_2O_8\) compounds and that by Movshovich et al, \(\dagger\) showed supportive indication to such proposals. There are experimental results related to interface effects as well as in the bulk that indicates mixed pairing symmetry (with dominant \(d\)-wave) \(\ddagger\), thus providing a strong threat to the pure \(d\) wave models. There were early orginal works as regards to the modification of superconductivity due to application of Zeeman magnetic \(\dagger\) and very recently, the Zeeman suppression was discussed in mesoscopic systems \(\dagger\).

II. MODEL CALCULATION

The free energy of a two dimensional planar superconductor with arbitrary pairing symmetry in presence of a magnetic field may be written as,

\[
F_{k,k'}(h) = -\frac{1}{\beta} \sum_{k,\sigma=\pm} \ln(1 + e^{-\sigma\beta E^+_k}) + \frac{|\Delta_k|^2}{V_{kk'}}
\]
where \( E_k^2 = \sqrt{(\epsilon_k - \mu)^2 + |\Delta_k|^2} + \sigma(g\mu_B/2)B \) are the energy eigen values of a Hamiltonian that describes superconductivity, \((g\mu_B/2)B\) is the magnetic moment of the electrons. This includes the assumption that the Zeeman field raises/lowers the energy of the spin up/down quasi-electrons. This includes the assumption that the Zeeman field raises/lowers the energy of the spin up/down quasi-electrons, \((g\mu_B/2)B\) is the magnetic moment of the electrons. This includes the assumption that the Zeeman field raises/lowers the energy of the spin up/down quasi-electrons. This includes the assumption that the Zeeman field raises/lowers the energy of the spin up/down quasi-electrons.

Using the free energy, Eq. (4), i.e., \( \partial F/\partial |\Delta| = 0 \), to get the gap equation as,

\[
\Delta_k = \sum_{k'} \frac{V_{kk'}}{2} \frac{E_{k'}}{2E_k} \left( \tanh\left(\frac{\beta E_k^+}{2}\right) + \tanh\left(\frac{\beta E_k^-}{2}\right) \right)
\]

where \( \epsilon_k \) is the dispersion relation taken from the ARPES data and \( \mu \) the chemical potential will control band filling through a number conserving equation given below.

Since the applied Zeeman field modifies the SC quasiparticles of spin up and down differently, their occupation probabilities are also modified. The number conserving equation that controls the band filling through chemical potential, \( \mu \) in presence of Zeeman field is given by,

\[
\rho(\mu, T, h) = \sum_k \left[ \frac{1}{2} \frac{1}{E_k} \left( \tanh\left(\frac{\beta E_k^+}{2}\right) + \tanh\left(\frac{\beta E_k^-}{2}\right) \right) \right]
\]

where \( h = (g\mu_B/2)B \). Let us consider that the overlap of orbitals in different unit cells is small compared to the diagonal overlap. Then in the spirit of tight binding lattice description, the matrix element of the pair potential used in the SC gap equation Eq. (3) may be obtained as,

\[
V(q) = \sum_{\vec{R}_3} \sum_{\vec{R}_4} V_{\vec{q}} e^{i\vec{q}\cdot\vec{R}_3} V_0 + V_1 f^d(k) f^d(k') + V_1 g(k) g(k')
\]

\[+ V_2 f^{d_4}(k) f^{d_4}(k') + V_2 f^{s_s}(k) f^{s_s}(k')\]

where in the first result of the equation \( \vec{R}_3 \) labels nearest neighbour and further neighbours, \( \vec{q} \) labels and \( V_n \), \( n = 1, 2 \) represents strength of attraction between the respective neighbour interaction. The first term in the above equation \( V_0 \) refers to the on-site interaction which has an effective attractive value giving rise isotropic \( s \) wave. The second and third terms are responsible for \( d \) and extended \( s \) wave symmetry superconductivity whereas the 4th and the 5th terms are responsible for \( d_{xy} \) and extended \( s_{xy} \) symmetries respectively. We restrict only to singlet pairing states (i.e., \( \Delta(k) = \Delta(-k) \)) as applicable for high temperature superconductors. The momentum form factors are obtained as,

\[
\begin{align*}
  f^d(k) &= \cos(k_x a) - \cos(k_y a) \\
  g(k) &= \cos(k_x a) + \cos(k_y a) \\
  f^{d_4}(k) &= 2 \sin(k_x a) \sin(k_y a) \\
  f^{s_s}(k) &= 2 \cos(k_x a) \cos(k_y a)
\end{align*}
\]

For two component order parameter symmetries as mentioned above, we substitute the required form of the potential and the corresponding gap structure into the either side of Eq. (2) which gives us an identity equation.

Then separating the real and imaginary parts together with comparing the momentum dependences on either side of it we get gap equations for the amplitudes in different channels as,

\[
\Delta_j = \sum_k \frac{V_j}{2} \Delta_k \left( \frac{\beta E_k^+}{2} + \tanh\left(\frac{\beta E_k^-}{2}\right) \right)
\]

where \( j = 1, 2 \) corresponding to two components \( d \) and \( s \) or \( d \) and \( d_{xy} \) symmetries. Considering mixed symmetry of the form \( \Delta(k) = \Delta_{d_{x_2-y_2}}(0) f^d(k) + e^{i\theta} \Delta_s(0) \) one identifies \( \Delta_1 = \Delta_{d_{x_2-y_2}}(0), \Delta_2 = \Delta_s(0), f_1^d = f^d(k), f_2^d = 1 \) and \( V^1 = V_1, V^2 = V_0 \) in Eq. (4). Similarly, for mixed symmetries of the form \( \Delta(k) = \Delta_{d_{x_2-y_2}}(0) f^d(k) + e^{i\theta} \Delta_{d_{xy}}(0) f^{d_{xy}} \Delta_2 = \Delta_{d_{xy}}(0), f_2^d = f^{d_{xy}}(k) \) and \( V^1 = V_1, V^2 = V_2 \) of Eq.(4). The potential required to get such pairing symmetries are discussed in Eq. (5).

We solve self-consistently the above three equations (Eq.4 and Eq.3) in order to study the phase diagram of a mixed order parameter superconducting phase in presence of Zeeman magnetic field. The numerical results obtained for the gap amplitudes through Eqs. (5) will be compared with free energy minimizations via Eq. (3) to get the phase diagrams.

### III. RESULTS AND DISCUSSIONS

We present in this section our numerical results for a set of fixed parameters, e.g., a cut-off energy \( \Omega_c = 500 \) K around the Fermi level above which superconducting condensate does not exist, a fixed transition temperature of the minor component \( T_c = (h = 0) = 24 \) K and the bulk \( T_c = 85 \) K determined by the \( d \)-wave order parameter. In figures 1 and 2 we present results for \( \Delta(k) = \Delta_{d_{x_2-y_2}}(0) f^d(k) + e^{i\theta} \Delta_s(0) \) symmetries for \( \theta = \pi/2 \) and \( \theta = 0 \) respectively. Such symmetries would arise from a combination of two component pair potentials \((2^{nd}, 1^{st}), (2^{nd}, 4^{th})\) terms in Eq. (5) for \( \alpha = s \) & \( d_{xy} \) respectively. The amplitudes of extended \( s \)-wave states like \( s_{x_2+y_2}, s_{xy} \) are found to be finite only towards very low band filling, \( \rho \sim 0 \) for \( \theta = \pi/2 \) hence does not cause any mixing with the predominant \( d \)-wave. Therefore, we shall discuss only the results of \( \theta = 0 \) and \( \theta = \pi/2 \) for \( \alpha = s \) & \( d_{xy} \). These two phases of \( \theta \) can cause important differences. It is known that for any \( \theta \neq 0 \), time reversal symmetry is locally broken at lower temperatures with the onset of the secondary component which correspond to a phase transition to a fully gapped phase for \( h = 0 \), from a partially ungapped phase of \( d_{x_2-y_2} \) symmetry. On the other hand, the \( \theta = 0 \) phase still remains nodeful, although the nodal lines shifts a lot from...
the usual $k_x = k_y$ lines of the $d_{x^2-y^2}$. In Fig. 1(a) and (b) we present the temperature dependencies of the order parameters in the complex mixed symmetry. Curves corresponding to the $d$-wave channels are represented with thinner joining lines of different styles whereas the minor $s$ component is denoted through that of same style but with thicker lines (same strategy will be carried out in other figures as well). For a zero Zeeman field ($h = 0$), the amplitude of the $d$-wave is suppressed with the onset of the minor $s$ component at $T = 24$ K which leads to a kink like structure (cf. the thin solid curve in Fig1 (a)).

![FIG. 1. (a) Amplitudes of the $\Delta_{d_{x^2-y^2}}$ and $\Delta_s$ at a fixed band filling $\rho = 0.85$ as a function of temperature (in K) for $\theta = \pi/2$ (i.e, $d_{x^2-y^2} + i s$) phase in various values of Zeeman field ($h$). With the application of weak magnetic field while the $d_{x^2-y^2}$ remains almost unaffected the transition temperature ($T^*_s$) of the minor $s$ component is suppressed strongly. At a field value of $h = 0.014$ eV the $s$-wave shows a first order transition. (b) For field values $h \geq 0.016$ eV the minor $s$-channel is completely suppressed leading to a pure $d$ wave order parameter. The thermal behaviour of $d$-wave superconductivity in presence of the Zeeman field is presented in Fig1(b). While the amplitudes at $T = 0$ K remains almost unaffected with field, the transition temperatures get affected. A magnetic field induced first order transition is observed at $h = 0.04$ eV.

With application of weak Zeeman field the transition temperature of the minor $s$ state decreases while the zero temperature magnitude remains the same. This causes a shift in the kink like structure in the thermal dependence of the $d$ wave channel towards lower temperature with increasing field and hence a small enhancement in the $d$-wave with field at lower temperature occurs. This point will be clearer from Fig. 5(b) as discussed latter. At a field value $h_c = 0.016$ eV, the $s$ wave component is completely suppressed leading to a pure $d$ wave phase. Thus we have a magnetic field induced transition at lower temperature from fully gapped phase of the $d + is$ state to a partially gapped phase of the $d$-wave. Therefore, in absence or very low magnetic field, there is a transition from a partially gapped phase of the $d$-wave at higher temperature to a fully gapped $d + is$ phase at lower temperatures. With increasing field the fully gapped phase region with respect to temperature decreases and brings back the ungapped phase of the $d$-wave. These phase transitions will have important bearings in the thermodynamic and transport properties.

![FIG. 2. Same as that of figure 1(a) except $\theta = 0$ (i.e, $d_{x^2-y^2} + s$ symmetry). For the $d$-wave channel, its amplitude is found to increase below $T^*_s$ in contrast to $\theta = \pi/2$ case where the $d$-wave amplitude is suppressed. The $s$-wave channel suffers drastic suppression in $T_s^*$ as well as the zero temperature amplitude in contrast to that in Fig. 1(a). Note, the critical field ($h_c$) at which the $s$ component is completely suppressed is $0.013$ eV in contrast to $0.016$ eV in case of $\theta = \pi/2$ (cf. Fig1a).]

In Fig. 1(b) we present the paramagnetic state of the $d$-wave. This phase has also been qualitatively investigated recently by Yang and Sondhi [17] with possibility of pairing with finite momentum. We therefore restrict to present only the details thermal dependence of the $d$-wave superconductivity in presence of Zeeman magnetic field that were not discussed. We show that with increas-
ing field (at lower fields) the zero temperature magnitude of the $d$-wave remains unchanged whereas the $T_c$ is reduced. At higher field, e.g., $h = 0.04$ one sees a first order transition from superconducting state to the normal state with respect to temperature. This behaviour causes a magnetic field induced enhancement of the $2\Delta/k_BT_c$ ratio. This ratio is crucial for many physical properties like specific heat jump etc. and hence expected to have drastic effect with magnetic field.

FIG. 3. Amplitudes of the $\Delta_{d_{x^2-y^2}}$ and $\Delta_{d_{xy}}$ at a fixed band filling $\rho = 0.85$ as a function of temperature (in K) for $\theta = \pi/2$ (i.e., $d_{x^2-y^2} + id_{xy}$ in various values of Zeeman field ($h$). With the application of weak magnetic field while the $d_{x^2-y^2}$ remains practically unaffected the transition temperature ($T^*$) of the minor $\alpha = d_{xy}$ component is suppressed strongly. Similar to that of Fig. 1a, a first order transition is seen in the minor $d_{xy}$ channel for $h = 0.014$ eV.

In Fig.2 we describe the thermal behaviour of superconductors with $d + s$ symmetry in presence of magnetic field. First of all, in absence of magnetic field, the thermal behaviors at lower temperatures is quite different (as we have seen in Fig. 1), the $s$-wave gap opens very fast below $T = 24$ K and also induces a growth to the $d$ at a faster rate than that above $T = 24$ K. While the $s$-wave has about three times the zero temperature value compared to $d + is$ phase, the $d$ wave also have quite larger value. With application of a very small magnetic field the $s$-component is suppressed largely; both its $T_c$ and the zero temperature gap being suppressed. The $d$-wave gap magnitude is also suppressed while its $T_c$ remained practically unchanged with such small values of the magnetic field. More importantly, although both the $d$ and the $s$ channel has larger magnitude in the $d + s$ phase the critical field ($h_c = 0.013$ eV) at which the $s$ component vanishes completely is smaller compared to that for the $\theta = \pi/2$ phase ($h_c = 0.016$ eV) of the mixed symmetry. Distinctly, the response of the Zeeman field to the $d + s$ superconductors is more pronounced than the $d + is$ superconductors. This study therefore also revealed the importance of the phase of the minor component in $d$-wave superconductors with a practical example of effect of Zeeman magnetic field, for the first time. In order to establish the importance of the phase of the minor component we also study the effect of Zeeman magnetic field in $d_{x^2-y^2} + id_{xy}$ and $d_{x^2-y^2} + d_{xy}$ symmetry superconductors respectively. The qualitative as well as quantitative behaviors remain almost same as that shown in figures 1 and 2. Thus establishing that the response of the superconductors with mixed order parameter symmetries with $\theta = 0$ phase of the minor component is stronger.

FIG. 4. Same as that in figure 3 except for $\theta = 0$ i.e $d_{x^2-y^2} + d_{xy}$ phase that preserves the time reversal symmetry. The notable difference is that the minor $d_{xy}$ component although have a magnitude at $T = 0$ K and $h = 0$ three times larger than that for $\theta = \pi/2$ case, the minor component is suppressed at a lower critical field $h_c = 0.013$ eV.

In figures 5 and 6 we concentrate on studying behavior of such mixed order parameter symmetry superconductors in presence of magnetic field as a function of band filling ($\rho$) for $d_{xy}$ and $s$ as minor components respectively. With application of a small field the minor component $\alpha = d_{xy}$ or $s$ is suppressed only at lower fillings and then with increasing field it is suppressed in the optimal doping regime suddenly. Therefore, with increasing field one finds mixed symmetry region as well as pure $d$-wave region with respect to filling – a non-uniform superconductivity. This behavior depends on the nature of the minor component. For $\alpha = s$, the mixed symmetry is possible around half-filling and at around $\rho = 0.7$ in an interme-
With increasing field mixed symmetry regions around both the band fillings shrinks and at a field value of $h = 0.03$ the $s$-wave vanishes leading to a pure $d$-wave phase. It may be noticed the $d$-wave channel is enhanced (cf. Fig 5(b)) in the optimal doping region with intermediate field values as was also mentioned earlier while discussing Fig1 (a). This however does not occur in case of $d_{xy}$. For $d_{xy}$ minor component, the minor component is suppressed strongly only from the lower filling with increasing field and at an intermediate field the mixing is possible only near half filling. The $d$-wave boundary towards lower fillings as well as around half-filling also shrinks with increasing field values in contrast to $\alpha = s$. One always see a sharp transition from a mixed phase to a pure $d$-wave phase or otherway around with respect to band filling irrespective of the minor component in a given magnetic field. At larger fields ($h > 0.03$) when the minor component is suppressed completely i.e., one has a pure $d$-wave phase, the $d$-wave phase also show sharp transition from $d$-wave superconducting state to a normal state (cf. Fig. 6).

In Fig. 6 we study the same as that in Fig. 5 for $d + s$ and $d + d_{xy}$ symmetries. In contrast to the $\theta = \pi/2$ phase of the minor component (as in Fig. 5), the minor components have very large values in the $\theta = 0$ phase although have the same $T^\alpha_c$ as mentioned earlier, in absence of the magnetic field. With very small magnetic field such large amplitudes of the condensation in the minor channel is strongly suppressed (see Fig 6(b) specially). The nature of suppression, as in Fig.5, is different for different minor component. For example, $d_{xy}$ is suppressed only from the lower filling whereas the $s$-wave is suppressed both from the lower filling as well as around the optimal doping.

*FIG. 5. Zero temperature amplitudes of the different pairing channels as a function of band fillings in the $d_{x^2-y^2} + e^{i\phi}\alpha$ picture for several fields with $\theta = \pi/2$. An increase in the amplitude of the $d$-wave channel with some particular field values in the $d_{x^2-y^2}$ is picture near optimal doping is worth noticing. With field increasing the order parameter symmetry changes to pure $d$-wave at optimal and larger doping.*

*FIG. 6. Same as figure 5 except $\theta = 0$. Stronger and rapid suppression of the minor channel with field in contrast to that in Fig. 3. A visible change in the dominant $d$-wave channel is also seen in contrast to that in Fig. 3. Note, in case of $d + s$ picture, (b), the $d$-wave channel shows plateau with respect to band filling.*

Overall, it is very distinct that the magnetic field affects the $\theta = 0$ mixed phase more strongly than the $\theta = \pi/2$ phase. This is due to the fact that the response of applied Zeeman field is paramagnetic with destruction of superconductivity over parts of the Fermi surface where the Zeeman field exceeds the local magnitude of the $k$-dependent gap resulting a spin polarization in the normal electrons. For $\theta = \pi/2$ phase, the nodes are miss-
ing and the Fermi surface is gapped all over, although the gap will have local minima. Thus response of the \( \theta = \pi/2 \) phase is weaker compared to the \( \theta = 0 \) phase. Noticeably, the critical value at which the minor component vanishes completely in all band fillings for the \( \theta = 0 \) phase is \( h_c = 0.025 \) eV whereas that for \( \theta = \pi/2 \) is \( h_c = 0.03 \) eV. (The value of \( h_c \) depends on \( \rho \), as well as \( \alpha \) and \( \theta \)).

Above \( h_c = 0.025 \), the paramagnetic state of the \( d \)-wave superconductors are also presented. The region of \( d \)-wave superconductors shrinks with increasing field around the band filling \( \rho = 0.82 \). The change from \( d \)-wave to spin polarized normal state is very sharp with respect to the band filling. In the \( d + s \) state a plateau has been observed in the \( d \) channel in weak or zero field, similar to the behaviour known for the YBCO systems \[1\]. It may be mentioned that Krishana et al., found the longitudinal thermal conductivity of \( \text{Bi}_2\text{Sr}_2\text{CaCu}_2\text{O}_{8+\delta} \) at lower temperatures (5K to 20 K) decreases with the increase in magnetic field applied along \( c \)-axis. Above a critical value of the magnetic field \( H_k(T) \), the thermal conductivity cease to change with the magnetic field and develops a plateau. It was proposed \[2\] that the \( d_{x^2-y^2} \) pairing state is unstable against the formation of \( d + e^{i\theta} \alpha \) (where \( \alpha = s, d_{xy} \)) in presence of \( H_k(T) \) such that the loss of quasiparticle transport in the thermal conductivity can be explained. In contrast, we started with a \( d + e^{i\theta} \alpha \) picture and application of Zeeman field (which may be mapped as application of magnetic field parallel to the 2D Cu – O plane) without considering the orbital effect did not find any enhancement in the condensation of the minor channel but suppression leading to a pure paramagnetic \( d \) state.

## IV. SUMMARY

In summary we have performed a detailed study on the effect of Zeeman magnetic field on mixed pairing symmetry with predominant \( d \)-wave which seems to be very promising symmetry for the high \( T_c \) systems. Thus we have described the paramagnetic state in the mixed symmetry superconductors and subsequently in the \( d \)-wave superconductors. In particular we established that the phase of the minor component mixed with predominant \( d \)-wave is of immense importance. The \( \theta = 0 \) phase minor component symmetry responds to Zeeman field more profoundly than the \( \theta = \pi/2 \) of the minor component. It will be very interesting to calculate the specific heat, Magnetization, density of states as a function of magnetic field using this model. We argued that the orbital effect is secondary when a magnetic field is applied parallel to the conducting plane. This may indicate that the experimental observation by Krishana et al., involve strong coupling of spins to orbitals due to application of magnetic field perpendicular to the plane at lower temperatures. However, the order parameter alone does not completely determine the thermodynamic property of a system. In Ref. \[4\], some estimations and scaling relations in the change of various physical properties due to Zeeman field are given for a pure \( d \)-wave superconductor. It turns out, a weak Zeeman field does little to the order parameter but may profoundly affect the thermodynamic property of a pure \( d_{x^2-y^2} \) superconductor. Such effects will presumably also remain for mixed superconductors with \( \theta = 0 \) as the gapnodes similar to \( d_{x^2-y^2} \) remains, its effect in the \( \theta = \pi/2 \) phase has to be studied more carefully.

## V. ACKNOWLEDGMENTS

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