Enhanced nonperturbative photon-pair conversion in small-angle laser collisions

Pisin Chen$^{1,2,3}$ and Lance Labun$^1$

$^1$ Leung Center for Cosmology and Particle Astrophysics
National Taiwan University, Taipei, 10617 Taiwan
$^2$ Department of Physics and Graduate Institute of Astrophysics
National Taiwan University, Taipei, 10617 Taiwan
$^3$ Kavli Institute for Particle Astrophysics and Cosmology
SLAC National Accelerator Laboratory, Menlo Park, CA 94025, USA

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We show a new scheme of nonperturbative pair production by high energy photons ($\omega \gtrsim m$) in a strong external field is achievable at the next high intensity laser experiments. The pair momentum is boosted and for $\omega \gtrsim 1.2m$ the pair yield is increased when the external field is formed by two laser pulses converging at a small angle. These characteristics are nonperturbative in origin and related to the presence of magnetic field in addition to electric field. By enhancing the signal over perturbative backgrounds, these features allow the employment of above-threshold photons $\omega > 2m$, which further increases the pair yield. We note the close relation of this photon-pair conversion mechanism to spontaneous pair creation, recommending it as an accessible stepping stone experiment using state-of-the-art or soon-to-be laser technology.

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Introduction. In QED, spontaneous pair production is the hallmark of nonperturbative QED since Sauter [1], Heisenberg, Euler [2] and Schwinger [3] achieved some of the first calculations [4–7]. Observation of this QED process would shed light on related nonperturbative quantum field theory (QFT) processes such as the flux-tube model of particle creation in hadron collisions [8] and Seiberg-Witten brane-anti-brane pair creation in string theory [9, 10]. The nonperturbative production probability is exponentially suppressed by the field magnitude $|\vec{E}| = m^2 c^2/e\hbar \simeq 1.32 \times 10^{15} V/m$. For a single-mode oscillating field, the exponential is continuously related to the nonlinear, $N$-photon process [11], which goes as a power law [11, 12]. The SLAC E-144 experiment achieved the multiphoton process with $N \simeq 5$ [13], and observation of the nonperturbative $N \gg 1$ process is now a realistic goal thanks to progress in high laser intensity technology.

In coming years, the ELI [14], ICAN [15] and Texas Petawatt [16] facilities are expected to achieve laser intensities corresponding to $|\vec{E}| \sim (10^{-4} - 10^{-3})|\vec{E}_{c1}|$, nearing but still a few orders of magnitude from the “critical” field at which spontaneous production is rapid [5, 17]. For this reason, several ways to increase the production rate have been studied [18, 19], a promising avenue being the introduction of high frequency photons, which can convert into pairs when propagating in a high intensity external field [20]. Known as “pair conversion” when the external field is the Coulomb field of a nucleus, this process plays an important role in high energy astrophysics [21]. Since the frequency of the external field is much smaller than the electron mass, pair conversion of a single photon requires absorption of $N \gg 1$ quanta from the external field, meaning it is nonperturbative even when the photon $\omega > 2m$. As discussed below, nonperturbative pair conversion shares analytic structure with spontaneous pair production, but differs enough to be achievable at near-future laser facilities, offering an experimental stepping stone of independent theoretical interest.

However, there are limitations to the simple setup of photons in a pure electric field created by counter-propagating laser pulses [20]. The yield of pairs is proportional to the number of high frequency photons, and a large number of photons $(10^{10})$ must be injected to compensate the exponential suppression when the laser field strength is significantly below $|\vec{E}_{c1}|$. This is above the photon density at which perturbative pair production becomes likely $(N_{\gamma}/(\mu m))^3 \sim 10^9$ results in 1 perturbative pair/meter of flight) meaning the invariant mass of two photons in a bunch must be $s = k_1 \cdot k_2 < 4m^2$. These limitations are removed if we can enhance the production rate and/or distinguish nonperturbatively produced pairs from perturbative ones.

In this work, we show how to simultaneously enhance nonperturbative pair conversion and give the produced pairs a characteristic large momentum (rapidity) that is determined by the geometry of the high intensity lasers. Large pair rapidity is achieved by boosting the center of momentum frame of the process: nonperturbatively produced pairs inherit momentum from the external field, as seen by considering it diagrammatically as absorption of $N \gg 1$ soft photons [22]. For spontaneous production, the center of momentum frame of produced pairs coincides with the rest frame of the external field [24]. For pair conversion, the high energy photon also contributes momentum. To take advantage of this fact, we set two high intensity laser pulses to converge at a small angle $\phi$. The superposed laser fields create a total field that
is both off the photon shell (necessary for nonperturbative pair creation) and at a high momentum relative to the lab, controlled by \( \phi \). A novel aspect when considering nonperturbative pair conversion is that for photon frequency near threshold \( \omega \gtrsim m_c \), small \( \phi \) increases the number as well as the momentum of the produced pairs.

We anticipate several advantages to this scheme. First, the relationship between pair momentum and the control parameter \( \phi \) offers a signature to establish the nonperturbative origin of the pairs. Second, creating pairs at high momentum may help control the kinematics of secondary production, that is cascades \cite{25, 26}. If so, we improve the chances to identify momentum signatures characteristic of nonperturbatively produced pairs \cite{27, 29}.

**Nonperturbative pair conversion.** To help explain the mechanism enhancing pair conversion, we first recall features of the spontaneous production mechanism, which is applicable to fields with frequency \( \omega \ll m \). The expected number of pairs per unit volume per unit time is given by the first term of the Schwinger series \cite{30, 31}.

\[
\frac{N_{ee}}{VT} = \frac{a^2 \beta n}{4\pi^4} \frac{a}{a} \coth \left( \frac{b\pi}{a} \right) e^{-\pi m_e^2/a}.
\]

The (invariant) rate depends on the invariants,

\[
a^2 = e^2(\sqrt{S^2 + P^2} - S),
b^2 = -e^2(\sqrt{S^2 + P^2} + S),
\]

which are the squared eigenvalues of the field tensor \( eF_{\mu\nu} \) written in terms of the scalar and pseudoscalar invariants \( 2S = (\vec{B}^2 - \vec{E}^2) \) and \( \mathcal{P} = -\vec{B} \cdot \vec{E} \). Here \( a, b \) are the electric and magnetic field strengths in the field rest frame, which explains why \( a \) appears in the exponent. The presence of magnetic field aligned with the electric field enhances the rate, seeing as \( x \coth x \geq 1 \) with equality only in the \( x \to 0 \) limit. However \( a, b \) are constrained if both electric and magnetic fields are supplied by laser pulses.

In a general reference frame \( a, b \) are simultaneously nonvanishing if and only if \( \mathcal{P} \neq 0 \). Since \( \mathcal{S} = \mathcal{P} = 0 \) for a single plane wave, we must superpose two or more laser fields to have large \( a \) and possibly also parallel \( b \) to achieve significant pair production. For \( x \gg 1 \), \( x \coth x \sim x \), hence increasing \( \mathcal{P} > \mathcal{S} \) produces a linear enhancement of the pair yield. Since the lab frame field energy is finite, experiments with laser pulses face a trade-off between magnetic field energy and electric field energy. For fixed total energy density, the yield Eq. \([1]\) is maximized by optimizing the field for larger \( a \), corresponding to making \( \mathcal{S} \) as large and negative as possible. Similarly, thinking of boosting the spontaneously produced pairs to high energy, the finite lab-frame energy density is shared between field rest-energy density and momentum. Consequently there is a trade-off between the energy and the yield of the produced pairs \cite{24}.

In contrast, magnetic field assisted pair conversion can enhance the overall discovery potential without sacrificing a produced pair yield, because it involves a new invariant

\[
\chi^2 = |eF_\mu^\nu k^\mu|^2 \rightarrow |e\vec{E}|^2(k_0^2 - k_z^2) + |e\vec{B}|^2(k_x^2 \cos^2 \theta + k_y^2 + k_z^2 \sin^2 \theta).
\]

In the second line we evaluate the invariant in a coordinate system with \( \vec{E} \) aligned in the \( z \)-direction and \( \vec{B} \) at an arbitrary angle \( \theta \) in the \( x-z \)-plane. \( \chi \) is maximized when the photon travels in the \( x \) or \( y \) directions, and if in the \( x \) direction, we should choose \( \theta = 0 \), i.e., \( |\vec{B}| |\vec{E}| \). With this choice, \( \chi \) reduces to the product of the field energy density and the photon frequency.

Pair conversion is described by the imaginary part of the photon polarization tensor \( \mathcal{P}_{\mu\nu} \) evaluated in an external field. Seeking the total pair yield, we average over photon polarizations, which just requires the trace of the polarization tensor. This saves diagonalizing the polarization tensor in a general external field with both \( \vec{E}, \vec{B} \neq 0 \), but introduces an \( \mathcal{O}(0.1) \) error in the yield considering that the source of photons may be partially polarized. The general form of \( \mathcal{P}_{\mu\nu} \) is given in \cite{32}. It turns out that we need only the two transverse components in the tensor decomposition, because the longitudinal components do not contribute to trace; the 0 component is assumed to have no nontrivial solutions to the lightcone condition \( k^2 + \Pi_0 = 0 \), hence no propagating modes, and the projection tensor associated to the third space-like component is zero under the trace.

We define the polarization-averaged inverse absorption length

\[
\tilde{\kappa} = \frac{1}{\omega^2} \sum_{\sigma} \mathcal{S}_{\Pi_\sigma},
\]

where \( \sigma = ||, \perp \) runs over transverse polarizations. The absorption probability is then the exponent of \( \kappa \) times the distance \( L \) the photons propagate in the external field, and the number of pairs produced equals the number of photons absorbed,

\[
N_{ee} = (1 - e^{-\tilde{\kappa}L}) N_\gamma.
\]

For the case \( \vec{B} = 0 \), \( \mathcal{S} \Pi \) is evaluated to high accuracy using contour integration and the saddle-point approximation to resum the poles \cite{21}. Pending quantitative study of the general \( \vec{E}, \vec{B} \neq 0 \) case, we adapt the same procedure, and since the method has been presented before, we will give the details elsewhere \cite{33}. We have checked that taking the limit \( b \to 0 \) reproduces at each step the results of \cite{21}. The final result is

\[
\sum_{\sigma} \mathcal{S}_{\Pi_\sigma} = \frac{\alpha a}{2} \left( \left| s_1 \left( \frac{bs_1}{a} \sin(b a/s_1) - \frac{s_2}{\sinh s} \right) \right|^{-1/2} \left( \left( \frac{b}{a} \right) \sin(b s/a) \cos^2(b s/2) + \frac{\sinh s}{\cosh^2(s/2)} \right) \right)^{-1/2} \times \frac{e^{-i b s/a} (b s/a)}{\sinh s_1 \sinh(b s/a)} (N_1(b/a, 1) - N_2(1, b/a))
\]
with

\[ \Phi = m^2 - \frac{v_1^2}{2} \cos vbs - \cos bs + \frac{v_2^2}{2} \cosh v as - \cos as, \]

\[ N_s(x, y) = 2 \cos(xs_a) \left( \frac{\cosh(ys_a) - 1}{\sinh^2(ys_a)} \right). \]  

(7)

All expressions are evaluated at the saddlepoint \( s_a \), which we solve for numerically as the solution to the transcendental equation

\[ \frac{1}{1 + \cosh s_a} + \frac{2m^2}{\tilde{v}^2} = \frac{1}{1 + \cos bs/a} \]  

(8)

The scalars \( v_1^2, v_2^2 \) are derived from the invariant decomposition of the photon momentum vector, and with the approximation that \( k^2 = 0 \) in the external field,

\[ v_1^2 = v_2^2 = (k^\mu F_{\mu}^\nu)^2 / (a^2 + b^2) = \chi^2 / (a^2 + b^2) = \tilde{v}^2. \]  

(9)

In the exponential factor \( \exp(\mp 2\pi s/a) \), the leading contribution in the low frequency limit \( \omega \to 0 \) is the first \( m^2 \) term, which produces \( e^{-\pi m^2/\omega} \) dependence like Eq. (11). For larger values of the invariant \( \chi \), however, the second and third factors \( \propto \chi^2/a^2 \) can compensate small \( a/m^2 \).

Using Eq. (9), we have evaluated the pair yield with different external field geometries. Similar to the constant field case Eq. (11), nonzero \( b \) enhances the total yield. Although \( b \) is present in the exponent \( \Phi \) and the enhancement grows faster than linear for large \( b/a \), the pair yield is again maximized by optimizing the field for the \( a \) invariant. Moreover, the limit \( b \to 0 \) is smooth, meaning small \( b \neq 0 \) is a small positive correction to pair yields.

**Boosting the pairs.** To determine the momentum of the produced pairs, we first go to the rest frame of the high intensity field, which has been treated as classical in the preceding calculation of pair production. The field rest frame is the frame in which its 3-momentum vanishes, with the electromagnetic 4-momentum defined covariantly from the energy-momentum tensor

\[ P^\mu = T^{\mu
u} u_\nu \rightarrow \begin{cases} T^{00} = \frac{1}{2}(\vec{E}^2 + \vec{B}^2) \\ T^{0i} = \vec{E} \times \vec{B} \end{cases} \]  

(10)

where \( u_\nu \) is a 4-vector defining the observer. Taking \( u_\nu = (1, 0) \) means the observer is at rest in the Lorentz frame being considered, and then this definition produces the usual Poynting vector \( \vec{S} \). In strong fields \( |\vec{E}| \sim m^2/e \), QED significantly modifies the Maxwell energy-momentum tensor \( \vec{T} \); however, aiming at the next generation of experiments attaining fields \( |\vec{E}| < 0.1 |\vec{E}_c| \), we can omit these corrections.

The magnitude \( P^\mu P_\mu \) is invariant

\[ \sqrt{P^\mu P_\mu} = \mu = \frac{1}{2}(a^2 + b^2) \]  

(11)

showing the energy density in the rest frame (the mass density) depends only on invariants Eq. (11). The transformation to the field rest frame is obtained from the condition that \( \vec{E}' \times \vec{B}' = 0 \), prime denoting quantities in the rest frame. Plugging in the Lorentz transformation for \( \vec{E}', \vec{B}' \) in terms of \( \vec{E}, \vec{B} \) and making the Ansatz, \( \beta = C(\vec{E} \times \vec{B}) \), we find a quadratic equation for \( C \) with two solutions \( C^{-1} = T^{00} + \mu \) where \( \mu \) is the mass density defined in Eq. (11). The requirement \( \beta^2 < 1 \) means only the \((+}\) solution is physical, and the boost velocity to the field rest frame is

\[ \beta = C(\vec{E} \times \vec{B}), \quad C = (T^{00} + \mu)^{-1} \]  

(12)

Since for light-like fields, the energy density equals the momentum density \( T^{00} = |\vec{E} \times \vec{B}| \), we see that smaller \( \mu \) means a larger boost, \( \beta^2 \to 1 \).

For a simple case to study the boost, we consider two converging laser pulses with equal intensity and frequency, and momentum vectors satisfying \( \vec{P}_1 \cdot \vec{P}_2 \propto \cos \phi > 0 \). Due to the exponential suppression, pair production is significant only in regions where the (total) field invariants are maximized. We calculate the invariants of the total field as from converging plane waves: by comparison to a realistic pulse model \[35,36\], corrections are subleading in the small focusing parameter \( \Delta = \lambda/2\pi R \), where \( \lambda \) is the laser wavelength and \( R \) the radius of the pulse waist. In the overlap region of two converging plane waves, either the magnitude of the net electric field or net magnetic field is larger, depending on how closely aligned the plane formed by \( \vec{P}_1, \vec{P}_2 \) is with the plane formed by the two polarization vectors \( \vec{E}_1/|\vec{E}_1| \). We assume the polarizations of the laser pulses are chosen to maximize \( S \), so as to maximize pair yield, according to the discussion above. In this case, \( S = 2|\vec{E}|^2 \sin^2 \phi, \quad \mathcal{P} = 0 \). These values are exact for two converging plane waves, and valid to leading order in \( \Delta \) for (quasi-)circularly polarized laser pulses \[35\]. Since \( a^2 = |S| \), the invariant \( a \) in the exponent in Eq. (11) is small in the interest limit of small \( \phi \), which suppresses spontaneous production.

The invariants of the combined laser fields give \( \mu = 2|\vec{E}|^2 \sin^2 \phi \), so that, as expected, a smaller convergence angle (more light-like total field), means a larger boost to the rest frame and a larger rapidity for produced pairs. Lastly, we need the momentum of the produced pairs as they appear in the rest frame of the field. To achieve the highest boost, we inject the high energy photons co-propagating with the net momentum of the high intensity field. Considering the production in the field rest frame, we observe that the photon has frequency and momentum \( \omega' = |\vec{k}'| \ll m_e \), and with \( \vec{k} \) arranged to be perpendicular to \( \vec{E} \), the momentum of the tunneling state is transverse to the field providing the tunneling potential. Therefore, in the relevant adiabatic limit, the pair materializes with zero longitudinal momentum, \( p_l = 0 \), the \( \parallel \) direction defined by the \( \vec{E} \) field vector. The mean value of the transverse momentum is determined by momentum conservation as the momentum of the photon \( \langle p_\perp \rangle = \vec{k}' \) (evaluated in the field rest frame).
Using additivity of rapidities, we find the mean rapidity (Lorentz factor \(\gamma_{ee}\)) of the produced pairs

\[
y_{ee} = y_F + \sinh^{-1} \left( \sqrt{ \frac{1 - |\beta|}{1 + |\beta|} \omega } \right), \quad \gamma_{ee} = \cosh y_{ee} \tag{13}
\]

where the field rapidity is \(y_F = \cosh^{-1} (1/\sqrt{1 - \beta^2})\), \(\beta\) given in Eq. (12). Here \(\omega\) is the photon frequency in the lab frame, and the cofactor in the argument of arcsinh is the Doppler factor for the shift to the field rest frame. For large boosts, i.e. small \(\phi\), the second term is subleading, and \(\gamma_{ee} \simeq \gamma F \sim 1/4 |\sin^2 \phi|\).

Figure [1] shows the pair Lorentz factor as a function of convergence angle of the two laser pulses. For comparison, we plot the relative pair yield, normalized to the yield for head-on pulses \(\phi = \pi\). The angle dependence is sensitive to the seed photon energy. At \(\omega \lesssim m\), head-on pulses produce the highest yield, because smaller \(\phi\) reduces \(a\). For \(\omega \gtrsim 1.2m\), the yield increases for small \(\phi\), because decreasing \(a\) increases \(\vec{v}^2/m^2\), which becomes dominant in determining the pair yield.

The impact of high pair momentum on cascade development is seen considering the radiation length \(\xi_{ee} = \alpha^{-1} p_0 \lambda^{-2/3} \sim 2 \times 10^{-4} \gamma_{ee}^{2/3} \mu m \tag{14}\) with \(p_0\) the electron or positron energy and \(\lambda\) from Eq. (3). For a relativistic electron traveling (initially) orthogonal to the electric field, \(\lambda \sim |\vec{E}| p_0\) and we obtain the scaling relation on the right. To suppress cascades, we must have \(\xi_{ee} > \lambda \sim 1 \mu m\) the laser pulse length scale. This requires \(\gamma_{ee} \gtrsim 10^7\), corresponding to \(\phi \sim 10^{-3}\). Even achieving \(\xi_{ee} \sim 0.1\lambda\) should significantly reduce cascade development. Achieving the high gamma factor facilitates search for the momentum signatures associated with spontaneously produced pairs \([24, 27–29]\), as well as being interesting in its own right for the production high energy electron bunches.

For yield estimates, we consider example parameters based on ELI expectations: For the propagation length \(L\) in Eq. (4), we note that a 10 PW–50 J pulse is 1.5 \(\mu m\) long, and take \(\sim 50\%\) efficiency for the fields to be near peak magnitude, which leads to \(L = 0.75 \mu m\), dropping the order 1 \(\phi\)-dependent geometric factor. In figure 2, pair yields normalized to the number of high frequency photons \(N_e\) are shown as a function of laser field strength, for different values of seed photon frequency and laser convergence angle. This shows in absolute scale the advantage in having near threshold seed photons: yield decreases with \(\phi\) for \(\omega \lesssim m\), but increases with \(\phi\) for \(\omega \gtrsim 1.2m\). We can consider above threshold photons, such as \(\omega = 8m\), in conjunction with a small convergence angle, since we may be able to identify nonperturbatively produced pairs by their large initial energy.

**Conclusions.** To summarize, we have calculated non-perturbative photon-pair conversion in converging laser pulses. This configuration takes advantage of the exponential enhancement due to the high frequency seed photon \(\omega \gtrsim m\) at the same time as boosting the momentum of the produced pairs. Since the pair energy is directly related to the experimental control parameter, the laser convergence angle \(\phi\), the correlation \(\gamma_{ee} \sim \phi^{-2}\) provides an identifying feature of nonperturbatively produced pairs. We have found that for photons with \(\omega \gtrsim 1.2m\), decreasing the convergence angle significantly enhances the pair yield over the counter propagating
cases. For \( \omega = 8m \), the enhancement is 6 orders of magnitude around the expected ELI laser field strength \( |\vec{E}| \approx 0.006|\vec{E}_0| \). The \( \phi \)-dependent momentum boost provides a signature to identify the nonperturbatively produced pairs from possible backgrounds.

On the other hand, the relative scalings of the pair-conversion and spontaneous processes must be studied quantitatively to determine an optimum experimental strategy [39]. For instance, the pair conversion process depends on the length \( L \) of the high intensity field, whereas the spontaneous process scales with volume \( L^3 \). Moreover, some laser energy must be diverted to create high frequency photon bunch. By enhancing the pair-conversion yield, our scheme strengthens the case for this avenue toward discovering nonperturbative pair production, especially since pushing a little farther to threshold high frequency photon bunch. By enhancing the pair-conversion process essential in high energy astrophysics.

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[40] \( D^\mu \) is the momentum density, and the field momentum and rest frame are quasi-local quantities, which should be considered as integrated over a mesoscopic volume chosen smaller than the length scale over which the field varies, but larger than the length scale associated with pair formation. Since the laser and pair production scales are widely separated, with the laser wavelength \( \lambda \sim (1 \text{eV})^{-1} \) much greater than the pair formation length \( \lambda_{ce} \) defined by \( \Delta V = |e\vec{E}|\lambda_{ce} = m_e \), we can clearly choose a mesoscopic length scale \( \ell \) satisfying \( \lambda_{laser} > \ell \gg \lambda_{laser}(m_e^2/|e\vec{E}|) \).
which defines the quasi-local volume.