A simulation method of active protection system defeat probability

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Abstract. The past thirty years have seen increasingly rapid advances in the field of Active Protection (AP) technics, and it is widely accepted that the AP system will play a marked role in future combat vehicle design, on account of its contribution to increasing vehicles’ viability without the burden of heavy armor. Depends on their concepts, a conventional AP system may consist of various components range from detectors, sensors, controllers, launchers to countermeasures, and what stands out in the AP system design are the inevitable errors derived from each operating session making the engagement results unpredictable, for example the tracking error of sensors, position error of launcher deployment, etc. The modelling method presented in this paper is a useful account of how to evaluate the AP system errors, and estimate the defeat probability of AP system upon the given boundary conditions.

1. Introduction

1.1. Scope of research
A hard-kill AP system is used to detect and track incoming threats, estimate the engagement point, deploy the launcher to the firing position, and launch the counter-munition at an appropriate time to destroy/neutralize the threats.

Following the first AP system Drozds was invented in the Soviet Union, dozens more of them have come out over the last decades, including the Trophy system developed by Rafael Advanced Defense Systems Ltd., and Arena system developed by at Russia's Engineering Design Bureau and so on, which differ as regards functional concepts, such as method of sensing, build of countermeasure, launching mechanism, and other aspects.

However, all the hard-kill AP systems, in one way or another, are designed to destroy or neutralize the incoming threats by hitting them physically, using fragments (or other lethal elements). Unfortunately, it seems no AP system can make a hundred percent sure that the incoming threats could be defeated in every single scenario. The term ‘defeat probability’ is introduced to qualify how likely the target can be defeated with the specific system boundary conditions.

The method in this paper is developed by introducing error factors during AP system’s operational process, then build the dimensional model connecting the errors as well as ammo model to evaluate the kill probability, finally, give the probability density function of a successful defeat.

1.2. Model introduction
A defeat probability model was developed in MS Excel and @Risk to carry out stochastic target defeat simulations. The model is based on the following principles:

Step 1: Inputs are provided to the model as discussed in section 3.1.

Step 2: The model picks values from the probability density function defined as part of the inputs to the model.

Step 3: The errors in polar coordinates are determined using the launcher as the reference.

Step 4: The errors are translated to cartesian coordinates using the launcher as the reference.

Step 5: The errors are translated to cartesian coordinates using the countermeasure detonation position as reference.

Step 6: The errors are translated to polar coordinates using the countermeasure detonation position as reference.

Step 7: The model determines if the target is within the fragmentation area of the countermeasure.

Step 8: If the target is located within the fragmentation area, the model determines the range between the detonation and the target. Using this range, the defeat probability is determined using the kill probability density function obtained from the warhead concept (refer to table 3).

Step 9: The model repeats steps 1 to 8 for a selected number of times (typically > 1000) to determine the stochastic nature of the defeat probability.

As is illustrated in figure 1, system defeat probability \( P_d \) is a product of the hit probability \( P_h \) and kill probability \( P_k \): by hit probability \( P_h \) the model can determine if the target will be within the fragmentation area, thereby kill probability \( P_k \) will generate the nature of a defeat probability \( P_d \).

![Figure 1. System defeat probability calculation.](image)

The model can be used to carry out how system defeat performance behaves when adjusting the operating scenario inputs on a ‘what-if” basis. This includes the following: finding the optimum launcher/countermeasure detonation distance, finding the optimum countermeasure detonation/target distance, and carrying out sensitivity analysis for the identified input variables.

2. Background

It is postulated that the AP system discussed in this paper has four major subsystems: tracking radar, system controller, directed launcher, and countermeasures. Although there is a variety of ways that an AP system could be made up of, the method given in this paper is always applied.

2.1. AP system working sequence

The sequence of events when an AP system engages with incoming threats is as follow: the radar system detects the targets, then starts tracking and sending cueing data to the system controller. The system controller determines the trajectories of targets on a base of cueing data, and estimates the intercept position of engagement, then deploys directed launcher to the fire position. The directed launcher points to the intercept point on a two-axis basis (both azimuth and elevation), then fire a countermeasure at an appropriate time. The countermeasure denotes when it reaches the intercept point, thereby provides a dense cloud of fragments that targets must pass through, and cause the targets pre-denoted or destroyed.

2.2. Engagement Coordinate
In this scenario, as shown in figure 2, all elements are deemed as mass points, and the coordinate origin \((O)\) is the position of the launcher of countermeasure. Ideally, the countermeasure is fired along \(Z\)-axis direction. After introducing the error factors that affect the interception action, the actual launch direction is vector \(OP\), and \(P\) is where the countermeasure denotes.

![Image](image_url)

**Figure 2.** Engagement coordinate.

The interception vector \(OP\) can be expressed in both cartesian coordinate and polar coordinate. In cartesian coordinate, the intercept position \((x, y, z)\) of \(P\) is used, while in polar coordinate system distance between \(P\) and \(O\) is expressed by symbol \(r\), thereby the angle \(\theta\) between straight-line \(PO\) and \(Z\)-axis is used to describe the error along the countermeasure's launching direction, and the angle \(\Phi\) between vector \(OP\)'s projection on the \((x, y)\) plane and \(X\)-axis shows the error which is perpendicular to the interceptor's dispensing angle.

The following translation of interception vector will be applied, that is, from polar coordinate system to cartesian coordinate system.

\[
\begin{align*}
  x &= r \sin \theta \cos \Phi \\
  y &= r \sin \theta \sin \Phi \\
  z &= r \cos \theta
\end{align*}
\]

And from cartesian coordinate system to polar coordinate system,

\[
\begin{align*}
  r &= \sqrt{x^2 + y^2 + z^2} \\
  \Phi &= \tan^{-1}(\frac{y}{x}) \\
  \theta &= \cos^{-1}\left(\frac{z}{\sqrt{x^2 + y^2 + z^2}}\right)
\end{align*}
\]

### 3. Model description

#### 3.1. Model input

An example of inputs required to the model is shown in the tables below. Table 1 gives operating scenario inputs to the model, including variables related to an AP system design. By adjusting the input, the model can provide different destroy probabilities, thereby help system designers do trade-off of concepts.

| Model Operating Scenario Inputs                  |  
|-----------------------------------------------|---|
| Launcher / countermeasure detonation distance \(\eta_d\) (m) | 20 |
System error input are shown in table 2, it should be noted that the spectrum of error factors is under the consideration of current technics, which means it can be extended in future work, especially when new subsystem or countermeasures are taken into account.

Table 2. System error inputs.

| Countermeasure Variances/ Error Inputs | 1σ | Max | Min | Pdf |
|----------------------------------------|----|-----|-----|-----|
| Countermeasure velocity error Δv_CM (m/s) | | | | |
| Countermeasure dispersion time error Δt_CM (s) | | | | |
| Countermeasure dispersion angular error ΔΦ_CM (rad) | | | | |
| Countermeasure dispersion angular error Δθ_CM (rad) | | | | |
| Launcher angular error ΔΦ_CM (rad) | | | | |
| Launcher angular error Δθ_CM (rad) | | | | |

| Target Tracking Variances / Errors Inputs |
|------------------------------------------|
| Target tracking distance error Δr_T (m) |
| Target tracking velocity error Δv_T (m/s) |
| Target tracking angular error ΔΦ_T (rad) |
| Target tracking angular error Δθ_T (rad) |

| Vehicle / Turret Movement Errors Input |
|----------------------------------------|
| Stability error (tilt, roll, vehicle & turret turning) ΔΦ_STAB (rad) |
| Vehicle velocity error Δv_VEHICLE (m/s) |

| Calculation Delay Errors Input |
|--------------------------------|
| Max avg acceleration error (m/s²) |
| Max deceleration error (m/s²) |
| Range error (m) |

The stochastic nature of the inputs is defined by the model as probability density functions. Two typical examples of inputs defined above are the countermeasure angular dispersion error Δθ_CM, and target tracking angular error Δθ_T, whose probability density functions are presented in figure 3 and figure 4 below.

Figure 3. Countermeasure dispersion angular error Δθ_CM (rad).
This model aims to determine defeat probability for warheads using a high-speed fragment effect and a typical condition is used as input to the simulation for testing purpose, where the warhead is designed with ball-shaped tungsten steel type fragmentation and charge type is PETN, and the input is shown in table 4. The estimate method in kill probability of countermeasure is conceptually derived from the book *Ballistics: Theory and Design of Guns and Ammunition, Second Edition* by D E. Carlucci, et al [1], the paper *Dynamic shape factor for particles of various shapes in the intermediate settling regime* by R. Lau [2], and the paper *Empirical relationships for the terminal settling velocity of spheres in cylindrical columns* by R. Kelilenbeck, et al [3].

Table 3. Input of Warhead defeat characteristics analysis input.

| Warhead defeat characteristics input |  |
|------------------------------------|--|
| Fragment type                      | tungsten steel balls |
| Charge Type                        |  |
| Fragmenting metal weight (kg) [M]  | 0.69 |
| Charge weight (kg) [C]             | 0.69 |
| Number of fragments [N]            | 400 |
| Tungsten weight / volume (kg/m³)   | 17200 |
| Fragment volume (m³)               | 0.00000021 |
| Fragment radius (ball shape) (m)   | 0.00371 |
| Countermeasure cone angle (°) [β]  | 30.00 |
| Shape factor of fragment [K]       | 0.33 |
| Air density (kg/m³) [ρₐ]           | 1.20 |
| Individual fragment mass (kg) [mᵢ] | 0.0017 |
| Drag coefficient [C₃]              | 0.5 |
| Individual fragment cross section (m²) [A] | 0.00004 |

### 3.2. Hit probability model

The reference systems can be divided on the basis of operational elements, into three parts, which are the launcher axis system (x, y, z), the countermeasure axis system (x′, y′, z′), and the target axis system (x″, y″, z″). The transformation between them is required.
As is shown in figure 4, point L is the position of the launcher, and vector $CM_i$ represents the countermeasure trajectory and the vector $\vec{R}$ is the gap between the intercept target position and countermeasure trajectory, that is,

$$
\vec{R} = \vec{T} - CM_i \\
= (T_x, T_y, T_z) - (0, 0, CM_{iz}) \\
= (T_x, T_y, T_z - CM_{iz})
$$

**Figure 5.** Transformation from launcher axis to countermeasure axis.

As is shown in figure 5, the gap vector between the target $T_i$ and the countermeasure trajectory $\vec{R}$ can be represented by,

$$
\vec{R} = \vec{T}_i - CM_i \\
= (0, 0, T_{iz}) - (CM_x, CM_y, CM_z) \\
= (-CM_x, -CM_y, T_{iz} - CM_z)
$$

**Figure 6.** Transformation from target axis to launcher axis.

The relation between target axis and countermeasure axis is illustrated in figure 6, and it is obviously that,

$$
CM + \vec{R} = \vec{T}_i - CM_i
$$

Thus,

$$
\vec{R} = \vec{T}_i - CM_i - CM \\
= (0, 0, T_{iz}) - (0, 0, CM_{iz}) - (CM_x, CM_y, CM_z) \\
= (-CM_x, -CM_y, T_{iz} - CM_{iz} - CM_z)
$$
Figure 7. Transformation from target axis to countermeasure axis.

Using the geometrical relations stated in this section, and combined with the error boundary conditions give in section 3.1, the hit probability $P_h$ can be derived by @Risk as a probability density function of input conditions. This is useful to determine whether the target will be covered within the kill basket of the countermeasure.

3.3. Kill probability model

The kill probability is related to the engagement range between the detonation point of countermeasure and target and this input depends on the design elements of countermeasure, such as the shape of the fragment, type of charge, etc.

The following data in empirical in table 6 is used for single fragment energy to establish the kill probabilities:

| Damage Condition          | $P_k$   | Energy required |
|---------------------------|---------|-----------------|
| Heavy Damage Criteria     | 90%     | 5 kJ            |
| Moderate Damage Criteria  | 50%     | 2.5 kJ          |
| Light Damage Criteria     | 10%     | 1 kJ            |

Warhead defeat characteristics analysis is used to carry out the modelling of kill probability, based on the countermeasure warhead design, as a matrix by $P_k$ and warhead/target distance (refer to table 3).

The initial velocity of the fragment can be calculated by,

$$v_0 = a \times b$$

Where ‘a’ is the Gurney constant for PETN which is 2830 m/s, and ‘b’ can be calculated using the mass ratio of charge weight and fragments $C/M$.

Since the fragment is considered as tungsten steel ball, to simplify the problem, the shape factor ‘K’ of the fragment is set to 0.33. Using the inputs given in table 4, the initial speed of fragments can be derived,

$$v_0 = \sqrt{2E \times \sqrt{\frac{C}{M}}} \left/ \left(1 + K \times \frac{C}{M}\right)\right. = 2450.85 \text{ m/s}$$

The fragment velocity vs. distance matrix is generated from the equation, where ‘d’ is the distance between the countermeasure and the launcher.

$$v_{n+1} = v_n e^{-\rho \alpha C_d Ad/2m_f}$$

By using the following equation, the kinetic energy of each fragment can be derived.
Under the condition given in table 4, the energy decrease of fragments vs. distance matrix is as shown in table 5.

Table 5. Energy vs range matrix.

| Range (m) | Energy (KJ) |
|-----------|-------------|
| 0         | 4.61        |
| 1         | 4.54        |
| 2         | 4.47        |
| 3         | 4.40        |
| 4         | 4.33        |
| 5         | 4.27        |
| 6         | 4.20        |
| 7         | 4.13        |
| 8         | 4.07        |
| 9         | 4.01        |
| 10        | 3.94        |
| 11        | 3.88        |
| 12        | 3.82        |
| 13        | 3.76        |
| 14        | 3.71        |
| 15        | 3.65        |
| 16        | 3.59        |
| 17        | 3.54        |
| 18        | 3.48        |
| 19        | 3.43        |
| 20        | 3.37        |

The probability of a fragment hitting the target as a function of the countermeasure detonation/target range is determined, as refer to table 3.

The target cross-section area can be derived as a function of the interception angle and target geometry characteristics, which can be illustrated in figure 7.
Figure 8. Target cross section area.

When angle $\gamma = 0$, target cross-section area $A_T$ is,

$$A_T = l \times d = A_1$$

When angle $\gamma = 90$, that is,

$$A_T = \pi \left(\frac{d}{2}\right)^2 = A_2$$

For all the other $\gamma$ values, target cross-section area $A_T$ can be derived by,

$$A_T = A_1 \cos \gamma + A_1 \sin \gamma$$

The blast pattern follows the principle as illustrated in figure 8. A cone-pattern cloud (the cone angle of which is represented by $\beta$) of fragments will be provided by the explosion effect of countermeasure.

![Figure 8](image)

Figure 9. Countermeasure frontal area.

The frontal area of fragments can be derived by,

$$A_{CM} = 2\pi r (1 - \cos \beta/2)$$

It is postulated that the number of fragments that loaded into a single countermeasure is represented by ‘N’, and the fragments are evenly distributed in the cloud, thereby the number of fragment-hits on the target’s surface can be estimated by,

$$N_{hit} = N \times \frac{A_T}{A_{CM}}$$

The kill probability of a single fragment $P_{ks}$ can be generated by @Risk as a function of kinetic energy derived in section 3.3 and the empirical data in table 5. Thereby, the kill probability $P_k$ is,

$$P_k = N_{hit} \times P_{ks}$$

The kill probability $P_k$ simulation result as a function of the distance between the countermeasure and the target is as shown in table 6.

Table 6. Kill probability vs. range.

| Range (m) | $A_{CM} (m^2)$ | $N_{hit}$ | $P_k$ (%) |
|-----------|----------------|-----------|-----------|
| 0         | -              | 400.000   | 100.00%   |
| 1         | 2.18           | 6.187     | 100.00%   |
| 2         | 8.73           | 1.547     | 98.88%    |
| 3         | 19.64          | 0.687     | 85.70%    |
| 4         | 34.91          | 0.387     | 48.81%    |
| 5         | 54.55          | 0.247     | 30.96%    |
| 6         | 78.56          | 0.172     | 21.30%    |
| 7         | 106.92         | 0.126     | 15.50%    |
| 8         | 139.66         | 0.097     | 11.75%    |
| 9         | 176.75         | 0.076     | 9.19%     |
| 10        | 218.21         | 0.062     | 7.37%     |
| 11        | 264.04         | 0.051     | 6.02%     |
| 12        | 314.23         | 0.043     | 5.00%     |
| 13        | 368.78         | 0.037     | 4.21%     |
|   | Value   | Error  | Probability |
|---|---------|--------|-------------|
| 14| 427.70  | 0.032  | 3.59%       |
| 15| 490.98  | 0.027  | 3.09%       |
| 16| 558.62  | 0.024  | 2.68%       |
| 17| 630.63  | 0.021  | 2.35%       |
| 18| 707.01  | 0.019  | 2.06%       |
| 19| 787.75  | 0.017  | 1.83%       |
| 20| 872.85  | 0.015  | 1.63%       |

Finally, as is stated above, the system defeat probability $P_d$ can be generated by the product of the hit probability and kill probability,

$$P_d = P_h \times P_k$$

3.4. Simulation results

Under specific boundary conditions, the probability density function of the defeat probability performance value simulation, as is shown in figure 10. From the figure it is evident that the AP system has a 92.8% probability of achieving a defeat probability of 90% or better.

![Defeat probability PDF simulation result.](image)

4. Conclusion

The modelling method presented in this paper was designed to determine the effect of errors introduced during the operating process of AP systems, and provides a comprehensive simulation method towards AP system efficiency evaluation. Given the current stand-off distance for intercepting incoming targets, the system designer may use the model to conduct a system-level trade-off, byways of estimating the effect of errors, and conversely, finding the optimum stand-off distance and countermeasure/target detonation distance, thus the key factors in system design could be identified. New types of technics applied to the AP system design gives rise to the system complexity and as a consequence, more error factors will be brought.

References

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