Transport Properties in Gapped Bilayer Graphene

N. Benlakhouy,1,* A. El Mouhafid,1,† and A. Jellal1,2,‡

1Laboratory of Theoretical Physics, Faculty of Sciences, Chouaïb Doukkali University, PO Box 20, 24000 El Jadida, Morocco
2Canadian Quantum Research Center, 204-3002 32 Ave Vernon, BC V1T 2L7, Canada

(Dated: January 26, 2021)

We investigate transport properties through a rectangular potential barrier in AB-stacked bilayer graphene (AB-BLG) gapped by dielectric layers. Using the Dirac-like Hamiltonian with a transfer matrix approach we obtain transmission and reflection probabilities as well as the associated conductance. For two-band model and at normal incidence, we find extra resonances appearing in transmission compared to biased AB-BLG, which are Fabry-Pérot resonance type. Now by taking into account the inter-layer bias, we show that both of transmission and anti-Klein tunneling are diminished. Regarding four band model, we find that the gap suppresses transmission in an energy range by showing some behaviors look like "Mexican hats". We examine the total conductance and show that it is affected by the gap compared to AA-stacked bilayer graphene. In addition, we find that the suppression in conductance is more important than that for biased AB-BLG.

I. INTRODUCTION

The experimental realization of monolayer graphene (MLG) in 2004 by Novoselov and Geim [1] opened up a new field in physics. Such material has attractive electronic, optical, thermal, and mechanical properties. In particular, the observation of Klein tunneling [2, 3], anomalous quantum Hall effect [1, 4], and optical transparency [5]. This makes graphene a good platform for nanoscale adaptor applications [6]. Bilayer graphene (BLG) is a system formed by two stacked sheets of graphene. Besides that, there are two distinct kinds of stacking: AB-BLG or AB-(Bernal) [7], and AA-BLG. AB-BLG has a parabolic dispersion relation with four bands where two of them touch at zero energy, whereas the other two bands split together by the interlayer hopping parameter $\gamma \approx 0.4 \text{ eV}$ [8]. This structure is much more stable and its high-quality samples are developed and studied theoretically and experimentally [9–13]. AA-BLG has a linear energy gapless spectrum with two Dirac cones switched in energy by the quantity $\gamma_1 \approx 0.2 \text{ eV}$ [14], and because of this AA-BLG attained enormous theoretical interest [15–20]. Such a structure is expected to be metastable, just lately, stable samples were discovered [21–24]. The AB-BLG may have clearly defined benefits than MLG, due to greater possibilities for balancing their physical properties. For reference: quantum Hall effect [9, 25], spin-orbit coupling and transverse electric field [26], transmission probability in presence of electric and magnetic static fields [13, 27], and quantum dots [28].

Experimentally, the evidence of Klein tunneling in MLG was confirmed [3, 29–31], which means that there is no electron confinement, and then a gap must be created to overcome this issue. In fact, many methods of induction a band gap in MLG have been elaborated such as substrates [32–39] and doping with impurities [40, 41]. Regarding AB-BLG, band gap can be realized by applying an external electric field [9, 42] or induced by using dielectric materials like hexagonal boron nitride (h-BN) or SiC [44]. To this end, it is theoretically showed that quantum spin Hall phase can be identified in gapped AB-BLG even when the Rashba interaction approached zero [44].

The introduction of an inter-layer bias to AB-BLG opens a gap in the energy spectrum and has a major effect on electronic properties [29]. Here, we analyze the effects of a biased AB-BLG gapped by dielectric layers to show the impact of band gap on transport properties. In both layers of AB-BLG, band gap is the same allowing to open a gap. Using transfer matrix method together with current density, we calculate transmission and reflection probabilities as well as corresponding conductance. At low-energy, $E < \gamma_1$, and in presence of the band gap $\Delta_0$ we find that Fabry-Pérot resonances [48] strongly appear in the transmission. Now by including also the inter-layer bias $\delta$, we show that the total transmission and anti-Klein tunneling significantly diminished. For energies exceeding the inter-layer coupling $\gamma_1$, $E > \gamma_1$, we obtain a new mode of propagating giving rise to the four transmission channels. In this case, $\Delta_0$ suppresses the transmission in the energy range $V_0 - (\Delta_0 + \delta) < E < V_0 + (\Delta_0 + \delta)$, and shows some behaviors that look like "Mexican hats". Finally we find that the resulting conductance in gapped AB-BLG gets modified compared to gapped AA-BLG. Moreover, we find that the suppression in conductance is more important than that for biased AB-BLG [29] because the energy range for a null conductance increases as long as $\Delta_0$ increase and also the number of peaks get reduced.
The paper is organized as follows. In Sec II we construct our theoretical model describing biased and gapped AB-BLG giving rise to four band energies. In Sec III we explain in detail the formalism used in calculating transmission and reflection probabilities together with conductance. In Sec IV we numerically analyze our results and give different discussions with published works on the topic. Finally, in Sec. V we summarize our main conclusions.

II. THEORETICAL MODEL

In the AB-stacked bilayer graphene the atom $B_1$ of the top layer is placed directly below the atom $A_2$ of the bottom layer with van der Waals inter-layer coupling parameter $\gamma_1$, while $A_1$ and $B_2$ do not lie directly below or above each other. Based on $[29, 44]$ we consider a biased and gapped AB-BLG described by the following Hamiltonian near the point $K$

$$\mathcal{H} = \begin{pmatrix}
V_0 + \vartheta_1 & \nu F \pi \dag & v_3 \pi \\
\nu F \pi & V_0 + \vartheta_2 & \gamma_1 \\
-v_3 \pi & \gamma_1 & V_0 - \vartheta_2 \\
\end{pmatrix}$$

where $\nu_F = \frac{2e^2}{h a} \approx 10^6$ m/s is the Fermi velocity of electrons in each graphene layer, $a = 0.142$ nm is the distance between adjacent carbon atoms, $v_3, 4 = \frac{\nu_F \gamma_3, 4}{\hbar}$ represent the coupling between the layers, $\pi = p_x + ip_y$, $\pi^\dag = p_x - ip_y$ are the in-plan momenta and its conjugate with $p_{x,y} = -i \hbar \partial_{x,y}$, $\gamma_1 \approx 0.4$ eV is the interlayer coupling term. The electrostatic potential $V_0$ of width $d$ (Fig. 1) can be varied on the $i$-th layer using top and back gates on the sample. $\vartheta_1 = \delta + \Delta_0$, $\vartheta_2 = \delta - \Delta_0$ with $\delta$ corresponds to an externally induced inter-layer potential difference, and $\Delta_0$ is the band gap. The skew parameters, $\gamma_3 \approx 0.315$ eV and $\gamma_4 \approx 0.044$ eV have negligible effect on the band structure at high energy $[25, 45]$. Recently, it was shown that even at low energy these parameters have also negligible effect on the transmission $[29]$, hence we neglect them in our calculations.

Under the above approximation and for a barrier potential configuration as depicted in Fig. 1, the Hamiltonian (1) can be written as

$$H = \begin{pmatrix}
V_0 + \vartheta_1 & \nu F \pi \dag & 0 & 0 \\
\nu F \pi & V_0 + \vartheta_2 & \gamma_1 & 0 \\
0 & \gamma_1 & V_0 - \vartheta_2 & \nu F \pi \dag \\
0 & 0 & \nu F \pi & V_0 - \vartheta_1 \\
\end{pmatrix}$$

By considering the length scale $l = \hbar \nu_F / \gamma_1$, which represents the inter-layer coupling length $l = 1.64$ nm, we define the dimensionless quantities: $x \equiv x/l$ and $k_y \equiv k_y$, together with $\delta \equiv \frac{1}{\gamma_1}$, $\Delta_0 \equiv \frac{\Delta_0}{\gamma_1}$, $E \equiv \frac{E}{\gamma_1}$, $V_0 \equiv \frac{V_0}{\gamma_1}$. The eigenstates of Eq. (2) are four-components spinors $\psi(x, y) = [\psi_{A_1}(x), \psi_{B_1}(x), \psi_{A_2}(x), \psi_{B_2}(x)]^\dag$, here $\dag$ denotes the transpose of the row vector. As a consequence of the transnational invariance along the $y$-direction, we have $[H, p_y] = 0$, and then we decompose the spinor as

$$\psi(x, y) = e^{ik_y y} [\psi_{A_1}(x), \psi_{B_1}(x), \psi_{A_2}(x), \psi_{B_2}(x)]^T$$

We solve the time-independent Schrödinger equation $H \psi = E \psi$ to obtain a general solution in the region II and then require $V_0 = \delta = \Delta_0 = 0$ to derive the solutions in the regions I and III. Indeed, by substituting Eq. (2) and Eq. (3) we get four related differential equations

$$-i(\partial_x + k_y)\phi_{B_1} = \varepsilon_1 \phi_{A_1} \hspace{1cm} (4a)$$
$$-i(\partial_x - k_y)\phi_{A_1} = \varepsilon_2 \phi_{B_1} - \phi_{A_2} \hspace{1cm} (4b)$$
$$-i(\partial_x + k_y)\phi_{B_2} = \varepsilon_3 \phi_{A_2} - \phi_{B_1} \hspace{1cm} (4c)$$
$$-i(\partial_x - k_y)\phi_{A_2} = \varepsilon \phi_{B_2} \hspace{1cm} (4d)$$

where we have set $\varepsilon_1 = \varepsilon - \vartheta_1$, $\varepsilon_2 = \varepsilon - \vartheta_2$, $\varepsilon_3 = \varepsilon + \vartheta_2$, $\varepsilon_4 = \varepsilon + \vartheta_1$ and $\varepsilon = E - V_0$. We solve Eq. (4a) for $\phi_{A_1}$, Eq. (4d) for $\phi_{B_2}$ and substitute the results in Eqs. (4b,4c). This process yields

$$\begin{align*}
(\partial_x^2 - k_x^2 + \varepsilon_1 \varepsilon_2) \phi_{B_1} &= \varepsilon_1 \phi_{A_1} \\
(\partial_x^2 - k_y^2 + \varepsilon_3 \varepsilon_4) \phi_{A_2} &= \varepsilon_4 \phi_{B_1}
\end{align*}$$

Then for constant parameters, the energy bands are solution of the following equation

$$[-k^2 + \varepsilon_1 \varepsilon_2] [-k^2 + \varepsilon_3 \varepsilon_4] - \varepsilon_1 \varepsilon_4 = 0 \hspace{1cm} (6)$$

such that $k = \sqrt{k_x^2 + k_y^2}$ and the four possible wave vectors are given by

$$k_x^2 = \sqrt{-k_y^2 + \varepsilon^2 + \Delta_0^2} \pm \sqrt{\varepsilon^2(1 + 4\delta^2) - (\Delta_0^2 + \delta^2)^2} \hspace{1cm} (7)$$

where $s = \pm$ defines the modes of propagation, which will be discussed in numerical section. Therefore, the four energy bands can be derived as
\[
\varepsilon_\pm = s \sqrt{k^2 + \delta^2 + \Delta_0^2 + \frac{1}{2} \sqrt{k^2 (1 + 4\delta^2) + \left(\frac{1}{2} - 2\delta\Delta_0\right)^2}}
\] (8)

result [29]

\[
\varepsilon^s_{\pm | \Delta_0=0} = s \sqrt{k^2 + \delta^2 + \frac{1}{2} \pm \sqrt{k^2 (1 + 4\delta^2) + \frac{1}{4}}} 
\] (10)

Now by comparing (9) and (10), we clearly notice that both quantities \(\delta\) and \(\Delta_0\) are inducing different gaps in the energy spectrum. Certainly this difference will affect the transmission probabilities (Figs. 3, 4) as well as conductance (Fig. 7).

It is known that the perfect AB-BLG has a parabolic dispersion relation with four bands, of which two touch each other at \(k = 0\). In Fig. 2 we show the energy bands as a function of the momentum \(k_y\), for the biased and gapped AB-BLG. We observe that when the AB-BLG is subjected to a gap \(\Delta_0\) and an inter-layer bias \(\delta\) the two bands are switched and placed at \(V_0 \pm \sqrt{\gamma_1^2 + (\delta - \Delta_0)^2}\), and the touching bands are shifted by \(2\delta' = 2(\delta - \Delta_0)\). One should notice that there are two cases related to whether the wave vector \(k_0^+\) or \(k_0^-\) is real or imaginary. Indeed for \(E < \gamma_1\), just \(k_0^+\) is real, and for that reason, the propagation is only possible for \(k_0^+\) mode. However when \(E > \gamma_1\), both \(k_0^+\) and \(k_0^-\) are real which presenting a new propagation mode.

As concerning the eigenspinors in regions II, we show that the solution of Eqs. (5) is a plane generated by

\[
\phi_B^2 = a_1 e^{ik_0^+x} + a_2 e^{-ik_0^+x} + a_3 e^{ik_0^-x} + a_4 e^{-ik_0^-x} \] (11)

where \(a_n\) are coefficients of normalization, with \(n = 1, \cdots, 4\). The remaining components of the eigenspinors can be obtained as

\[
\phi_{A_1}^2 = a_1 \Lambda_{\pm A}^1 e^{ik_0^+x} + a_2 \Lambda_{\pm A}^- e^{-ik_0^+x} + a_3 \Lambda_{\pm A}^- e^{ik_0^-x} + a_4 \Lambda_{\pm A}^- e^{-ik_0^-x} \] (12)

\[
\phi_{A_2}^2 = a_1 \rho^+ e^{ik_0^+x} + a_2 \rho^- e^{-ik_0^+x} + a_3 \rho^- e^{ik_0^-x} + a_4 \rho^- e^{-ik_0^-x} \] (13)

\[
\phi_{B_2}^2 = a_1 \chi^+ \rho^+ e^{ik_0^+x} + a_2 \chi^+ \rho^- e^{-ik_0^+x} + a_3 \chi^- \rho^- e^{ik_0^-x} + a_4 \chi^- \rho^- e^{-ik_0^-x} \] (14)

where we have introduced the quantities \(\Lambda_{\pm A}^\pm = \frac{-ik_x \pm k_y}{\varepsilon - \sigma_1}\), \(\rho^\pm = \frac{(\varepsilon - \sigma_3)(\varepsilon - \sigma_2) - k_x^2 - k_y^2}{\varepsilon - \sigma_1}\), \(\chi_{\pm A}^\pm = \frac{ik_x \pm ik_y}{\varepsilon + \sigma_1}\). In matrix notation, the general solution of our system in region II can be written as

\[
\psi_2(x, y) = G_2 \cdot M_2(x) \cdot C_2 e^{ik_y y} \] (15)

where the four-component vector \(C_2\) represents the coefficients \(a_n\) expressing the relative weights of the different traveling modes, which have to be set according to the propagating region [29]. The matrices \(M_2(x)\) and \(G_2\) are given by

\[
\psi_2(x, y) = G_2 \cdot M_2(x) \cdot C_2 e^{ik_y y} \] (15)
\[
\mathcal{G}_2 = \begin{pmatrix}
\Lambda_+ & 1 & 1 & 1 \\
\rho_+ & \Lambda_+ & \Lambda_- & \Lambda_- \\
\chi_+^\rho_+ & \chi_-^\rho_+ & \chi_+^\rho^- & \chi_-^\rho^-
\end{pmatrix}, \quad \mathcal{M}_2(x) = \begin{pmatrix}
e^{ik^+_x}x & 0 & 0 & 0 \\
0 & e^{-ik^+_x}x & 0 & 0 \\
0 & 0 & e^{ik^-_x}x & 0 \\
0 & 0 & 0 & e^{-ik^-_x}x
\end{pmatrix}, \quad \mathcal{C}_2 = \begin{pmatrix} a_1 \\
a_2 \\
a_3 \\
a_4 \end{pmatrix}
\]
density $J$

$$J = v_F \psi^\dagger \begin{pmatrix} \sigma_x & 0 \\ 0 & \sigma_x \end{pmatrix} \psi$$  \hspace{1cm} (33)

where $\sigma_x$ is the Pauli matrix. Then Eq. (33) gives the incident $J^\text{inc}$, reflected $J^\text{ref}$ and transmitted $J^\text{tr}$ current densities. Finally the transmission $T$ and reflection $R$ probabilities are

$$T^\pm_s = \frac{k_0^\pm}{k_0} |t^s_\pm|^2, \quad R^\pm_s = \frac{k_0^\pm}{k_0} |r^s_\pm|^2$$  \hspace{1cm} (34)

To preserve the probability of current, $T$ and $R$ are normalized as

$$\sum_{i,j} (T^i_j + R^i_j) = 1$$  \hspace{1cm} (35)

where the index $i = \pm$ points to the arriving mode, when the index $j = \pm$ points to the exiting mode. For example in the case of channel $k^\pm$, gives $T^+_s + T^-_s + R^+_s + R^-_s = 1$. As already mentioned, for $E > \gamma$, we have two modes of propagation $(k_0^+, k_0^-)$ leading to four transmissions $T^\pm_s$ and four reflections $R^\pm_s$, through the four conduction bands. For sufficiently enough low energy or in the two-band model, $E < \gamma$, the two modes lead to one transmission $T$ channel and one reflection $R$ channel.

From the transmission probabilities, we can calculate the conductance $G$, at zero temperature, using the Landauer-Büttiker formula

$$G(E) = G_0 \frac{L_y}{2\pi} \int_{-\infty}^{\infty} dk_y \sum_{i,j=\pm} T^i_j(E, k_y)$$  \hspace{1cm} (36)

with $L_y$ the length of the sample in the $y$-direction, and $G_0 = 4e^2/h$. The factor 4 comes from the valley and spin degeneracies in graphene. In order to get the total conductance of the system, we need to sum over all the transmission channels

$$G_T = \sum_{i,j} G^j_i$$  \hspace{1cm} (37)

IV. NUMERICAL RESULTS AND DISCUSSION

In this section, we numerically analyze and discuss our main results. First, we evaluate the transmission probability in the two-band model at normal incidence (i.e. $k_y = 0$). To understand our system more effectively in Fig. 3, we present the effect of the band gap $\Delta_0$ on the transmission as a function of the incident energy $E$ and the width $d$ of the barrier. In the (left panel), we plot the energy dependence of the transmission probability for a barrier of width $d = 10$ nm, $d = 25$ nm, and $d = 100$ nm for biased $\delta = 0$ and unbiased system $\delta \neq 0$ with band gap $\Delta_0$. For $\Delta_0 \neq 0$, we observe appearance of resonances in the transmission probability for the energy range $E < V_0 - \delta'$, $\delta' = \delta + \Delta_0$, which can be attributed to the finite size of the AB-BLG as well as the presence of charge carriers with different chirality. These phenomena are known as Fabry-Pérot resonances [48]. For the energy range $V_0 - \delta' < E < V_0 + \delta'$, there is a bowl (window) of zero transmission for $d = 100$ nm in contrary for $d = 10$ nm and $d = 25$ nm the transmission is not zero. However, for $E > V_0 + \delta'$, the transmission still looks like Ben et al. results [29]. Note that the transmission of width $d = 100$ nm, shows anti-Klein tunneling, which is a direct consequence of the pseudospin conservation in the system. In the (right panel), we plot the width dependence of the transmission probability for the incident energies $E = \frac{1}{5}V_0$, $E = \frac{2}{5}V_0$ and $E = \frac{3}{5}V_0$. It is clearly seen that for $E = \frac{1}{5}V_0$ and $E = \frac{3}{5}V_0$ with $\delta_0 = 0$, $\Delta_0 = 0.01\gamma_1$, resonance peaks show up (see upper panel), which are absent for the case $\Delta_0 = 0$ [29]. In the middle and bottom panel, by taking into account the effect of a finite bias $\delta = 0.01\gamma_1$, we observe a decrease of resonance in the transmission probability, and more precisely when $\Delta_0$ is greater than $\delta$.

To investigate the effect of band gap, for energy greater than the interlayer hopping parameter, $E > \gamma$, in Fig. 4 we show the transmission and reflection channels as a function of the incident energy $E$ and transverse wave vector $k_y$ for potential height $V_0 = \frac{1}{2}\gamma_1$ and width $d = 25$ nm. The superimposed dashed curves indicate different propagating modes inside and outside the barriers. For ungapped and unbiased AB-BLG (pristine AB-BLG), Ben et al. [29] showed that all channels are symmetric with respect to normal incidence, $k_y = 0$, i.e. $T^+_s = T^-_s$ and $R^+_s = R^-_s$. This is due to the valley equivalence, namely the transmission probabilities of electrons moving in the opposite direction (scattering from $k^+$ to $k^-$ in the vicinity of the first valley, and scattering from $k^-$ to $k^+$ in the vicinity of the second valley) are the same. Now as for our case by introducing a gap $\Delta_0 = 0.3\gamma_1$, with a null inter-layer bias, $\delta = 0$, we observe that the transmissions are completely suppressed in the energy range $V_0 - \Delta_0 < E < V_0 + \Delta_0$ due to the absence of traveling modes. In $T^+_s$ channel and for energies smaller than $V_0 - \gamma_1$, we find that the resonances are decreased and Klein tunneling get less incandescent than that seen in [29]. We notice that there is asymmetric in the transmission channels with respect to normal incidence, $T^+_s(k_y) = T^-_s(-k_y)$, but reflection channels still showing symmetric behavior, $R^+_s(k_y) = R^-_s(k_y)$, because the incident electrons back again in an electron state [29]. This is not the case for gapped AA-BLG, whereas $T^+_s$ and $T^-_s$ channels preserve the momentum symmetry [43]. In addition, there is a significant distinction for all reflection channels, $R^+_s$, between gapped AB-BLG and biased AB-BLG. Indeed, in our case we observe that the scales of $R^+_s$ get reduced inside the barrier. It is remarkably seen that our transmission channels, $T^+_s$, showed some bowels in the energy spectrum instead of “Mexican hats” as have been see in [29]. This show that $\Delta_0$ can be used to control the transmission behavior in AB-BLG.
In Fig. 5 we show the density plot of the transmission and reflection channels, for biased and gapped systems, \( \delta = 0.3\gamma_1 \), \( \Delta_0 = 0.3\gamma_1 \). The transmission is completely suppressed in the energy range \( V_0 - \delta' < E < V_0 + \delta' \), \( \delta' = \Delta_0 + \delta \) . We notice that the symmetric inter-layer sublattice equivalence is also broken in this case as seen in Fig. 4. We recall that such symmetry broken can be achieved by taking either \( \delta \neq 0 \) or \( \Delta_0 \neq 0 \), which means that there is violation of invariance under the exchange \( k_y \rightarrow -k_y \) as noted in [29, 49] for AB-BLG, in contrast to the AA-BLG [50]. Therefore, the transmission and reflection probabilities are not symmetric with respect to normal incidence as seen in Fig. 5.

Fig. 6 presents the same plot as in Fig. 5 except that we choose a band gap \( \Delta_0 = 0.5\gamma_1 \) greater than inter-layer bias \( \delta = 0.3\gamma_1 \). In this situation, we notice a significant difference in the transmission and reflection channels. Indeed, we observe that Klein tunneling becomes less than that see for the case \( \Delta_0 = \delta = 0.3\gamma_1 \) in Fig. 5. In addition, it is clearly seen that some resonances disappear for the energy range \( E < V_0 - \delta' \). Moreover, we find that the energy bands are pushed and showed some behaviors look like “Mexican hats”, which are more clear than those see in Fig. 5. These results are similar to those obtained in [51], by analyzing the transmission probabilities for a system composed of two single layer-AB bilayer-two single layer (2SL-AB-2SL) of graphene subjected to strong gate potential. In summary, we observe that all transmissions for \( \delta \neq 0 \) and \( \Delta_0 \neq 0 \) are weak compared to the biased AB-BLG [29], or gapped AB-BLG (Fig. 4) cases.

In Figs. 7 we plot the energy dependence of the corresponding conductance for different values of the band gap and an inter-layer bias \( \delta = 0.3\gamma_1 \). The band gap \( \Delta_0 = 0.3\gamma_1 \) contributed by opening a gap in the energy spectrum of AB-BLG at \( V_0 \pm \Delta_0 \), and this of course reflected on the conductance as shown in Fig. 7(a). The resonances that are clear in the transmission probability show up as peaks, and the total conductance \( G_{\text{Tot}} \) has a convex form. For low energies we have \( G_{\text{Tot}} = G_+^+ \) meaning that the propagation is only via \( k^+ \) mode, while \( k^- \) mode is cloaked in this regime until \( E > V_0 + \Delta_0 \). \( G_- \) starts conducting by making an appearance as a rapid increase in the total conductance. Furthermore, \( G_+^+ = G_+^- = 0 \) since \( T_+^+ = T_+^- = 0 \) at low energy but at \( E = \gamma_1 \) both modes are coupled and \( G_+^+ \) start conducting that is why \( G_{\text{Tot}} \neq G_+^+ \). However the band gap does not break the equivalence in the scattered channels of the conductance such that \( G_+^- = G_+^+ \) still equivalent for all energy ranges (see Fig. 7(a)), in contrast to the
FIG. 5. (Color online) The same as in Fig. 4, but now for the band gap $\Delta_0 = 0.3 \gamma_1$ with $\delta = 0.3 \gamma_1$. The dashed white and black lines represent the band inside and outside the barrier, respectively.

FIG. 6. (Color online) The same as in Fig. 4, but now for the band gap $\Delta_0 = 0.5 \gamma_1$ with $\delta = 0.3 \gamma_1$. The dashed white and black lines represent the band inside and outside the barrier, respectively.

The solid curves correspond to the total conductance and the dashed curves correspond to different contributions of the four transmission channels.

firms that our $\Delta_0$ has a significant impact on the transport properties and differs from that induced by bias in AB-BLG [29]. Instead of contrast, the total conductance of a gapped AA-BLG is approximately unchanged even though the band gap has a significant impact on the intracone transport [43]. Now we involve both of parameters by presenting Figs 7(b) and 7(c) corresponding, respectively, to $\Delta_0 = \delta = 0.3 \gamma_1$, and $\Delta_0 = 0.5 \gamma_1$, $\delta = 0.3 \gamma_1$. As expected we observe large suppression of the conductance in the energy range $V_0 - \delta' < E < V_0 + \delta'$, and hence some peaks are removed with a decrease of the total conductance $G_{Tot}$.

V. SUMMARY AND CONCLUSION

We have theoretically investigated the transport properties through rectangular potential barriers of biased AB-BLG gapped by dielectric layers. By solving Dirac
equation, the four band energies are obtained to be dependent on the band gap $\Delta_0$ together with the inter-layer bias $\delta$. Subsequently, using transfer matrix method we have evaluated the corresponding transmission, reflection probabilities, and conductance. In particular, we have analyzed the transmission probability in the two-band model at normal incidence, (i.e $k_y = 0$), firstly in the presence of $\Delta_0$ and secondly by taking into account $\Delta_0$ and $\delta$. As a result, we have observed that the presence of $\Delta_0$ induces extra resonances appearing in transmission profiles. However by adding $\delta$, we have observed that the transmission decreased more and anti-Klein tunneling in AB-BLG is no longer preserved.

Furthermore, we have obtained a new mode of propagation for energies exceeding the inter-layer coupling $\gamma_1$. In this case, we have showed that the band gap $\Delta_0$ breaks the inter-layer sublattice equivalence with respect to $k_y = 0$. Such asymmetry is apparent in the scattered transmission where it depends on the incident mode. The corresponding conductance does not incorporate this asymmetric, and the locations of their peaks are changed in the presence of $\Delta_0$ compared to $\delta$ [29].

VI. ACKNOWLEDGMENTS

The generous support provided by the Saudi Center for Theoretical Physics (SCTP) is highly appreciated by all authors. A.J. thanks Dr. Michael Vogl for fruitful discussion.

[1] K. S. Novoselov, A. K. Geim, S. V. Morozov, D. Jiang, Y. Zhang, S. V. Dubonos, I. V. Grigorieva and A. A. Firsov, Science 306, 666 (2004).
[2] O. Klein, Z. Phys. 53, 157 (1929).
[3] M. I. Katsnelson, K. S. Novoselov, and A. K. Geim, Nat. Phys. 2, 620 (2006).
[4] Y. B. Zhang, Y. W. Tan, H. L. Stormer, and P. Kim, Nature 438, 201 (2006).
[5] R. Nair, P. Blake, A. Grigorenko, K. Novoselov, T. Booth, T. Stauber, N. Peres, and A. Geim, Science 320, 1308 (2008).
[6] A. H. Castro Neto, F. Guinea, N. M. R. Peres, K. S. Novoselov, and A. K. Geim, Rev. Mod. Phys. 81, 109 (2009).
[7] J. D. Bernal, Proc. R. Soc. A 106, 749 (1924).
[8] Z. Q. Li, E. A. Henriksen, Z. Jiang, Z. Hao, M. C. Martin, J. D. Bernal, Proc. R. Soc. A 106, 749 (2015).
[9] E. McCann and V. I. Fal’ko, Phys. Rev. Lett. 96, 086805 (2006).
[10] A. Rozhkov, A. Sboychakov, A. Rakhmanov, and F. Nori, Phys. Rep. 648, 1 (2016).
[11] T. Ohta, A. Bostwick, T. Seyller, K. Horn, and E. Rotenberg, Science 313, 951 (2006).
[12] M. O. Goerbig, Rev. Mod. Phys. 83, 1193 (2011).
[13] I. Redouani, A. Jellal, and H. Bahlouli, J. Low Temp. Phys. 181, 197 (2015).
[14] I. Lobato and B. Partoens, Phys. Rev. B 83, 165429 (2011).
[15] A. L. Rakhmanov, A. V. Rozhkov, A. O. Sboychakov, and F. Nori, Phys. Rev. Lett. 109, 206801 (2012).
[16] Y. Mohammadi and B. A. Nia, Solid State Commun. 201, 76 (2015).
[17] R.-B. Chen, Y.-H. Chiu, and M.-F. Lin, Carbon 54, 268 (2013).
[18] C.-W. Chiu, S.-C. Chen, Y.-C. Huang, F.-L. Shyu, and M.-F. Lin, Appl. Phys. Lett. 103, 041907 (2013).
[19] I. Redouani and A. Jellal, Mater. Res. Express 3, 065005 (2016).
[20] Y. Zahidi, I. Redouani, and A. Jellal, Physica E 71, 259 (2016).
[21] J.-K. Lee, S.-C. Lee, J.-P. Ahn, S.-C. Kim, J. I. B. Wilson, and P. John, J. Chem. Phys. 129, 234709 (2008).
[22] J. Borysiuk, J. Soltys, and J. Piechota, J. Appl. Phys. 109, 093523 (2011).
[23] P. L. de Andres, R. Ramírez, and J. A. Vergés, Phys. Rev. B 77, 045403 (2008).
[24] Z. Liu, K. Suenaga, P. J. F. Harris, and S. Iijima, Phys. Rev. Lett. 102, 015501 (2009).
[25] E. McCann, Phys. Rev. B 74, 161403(R) (2006).
[26] S. Konschuh, M. Gmitra, D. Koban, and J. Fabian, Phys. Rev. B 85, 115423 (2012).
[27] A. Jellal, I. Redouani and H. Bahlouli, Physica E 72, 149 (2015).
[28] G. Giavaras and F. Nori, Phys. Rev. B 83, 165427 (2011).
[29] B. Van Duppen and F. M. Peeters, Phys. Rev. B 87, 205427 (2013).
[30] A. F. Young and P. Kim, Nat. Phys. 5, 222 (2009).
[31] N. Stander, B. Huard, and D. Goldhaber-Gordon, Phys. Rev. Lett. 102, 026807 (2009).
[32] W.-X. Wang, L.-J. Yin, J.-B. Qiao, T. Cai, S.-Y. Li, R.-F. Dou, J.-C. Nie, X. Wu, and L. He, Phys. Rev. B 92, 165420 (2015).
[33] P. San-Jose, A. Gutiérrez-Rubio, M. Sturla, and F. Guinea, Phys. Rev. B 90, 075428 (2014).
[34] M. Kindermann, B. Uchoa, and D. L. Miller, Phys. Rev. B 86, 115415 (2012).
[35] J. C. W. Song, A. V. Shytov, and L. S. Levitov, Phys. Rev. Lett. 111, 266801 (2013).
[36] J. Jung, A. M. DaSilva, A. H. MacDonald, and S. Adam, Nat. Commun. 6, 6308 (2015).
[37] M. S. Nevius, M. Conrad, F. Wang, A. Celis, M. N. Nair, A. Taleb-Ibrahimi, A. Tejeda, and E. H. Conrad, Phys. Rev. Lett. 115, 136802 (2015).
[38] M. Zarenia, O. Leenaerts, B. Partoens, and F. M. Peeters, Phys. Rev. B 86, 085451 (2012).
[39] S.-Y. Zhou, D.A. Siegel, A.V. Fedorov, A. Lanzara, Phys. Rev. Lett. 101, 086402 (2008).
[40] R. N. Costa Filho, G. A. Farias, and F. M. Peeters, Phys. Rev. B 76, 193409 (2007).
[42] Y. Zhang, T.-T. Tang, C. Girit, Z. Hao, M.C Martin, A. Zettl, M. F Crommie, Y R. Shen, and F. Wang, Nat. 459 820 (2009).
[43] H. M. Abdullah and H. Bahlouli, J. Comput. Sci. 26, 135 (2018).
[44] X. Zhai and G. Jin, Phys. Rev. B 93, 205427 (2016).
[45] E. McCann, D.S.L. Abergel, and V.I. Fal’ko, Solid State Communications 143, 110 (2007).
[46] M. Barbier, P. Vasilopoulos, and F. M. Peeters, Phys. Rev. B 82, 235408 (2010).
[47] Michaël Barbier, P. Vasilopoulos, F. M. Peeters, and J. M. Pereira, Jr, Phys. Rev. B 79, 155402 (2009).
[48] I. Snyman and C. W. J. Beenakker, Phys. Rev. B 75, 045322 (2007).
[49] J. Nilsson, A. H. Castro Neto, F. Guinea, and N. M. R. Peres, Phys. Rev. B 76, 165416 (2007).
[50] H. M. Abdullah, M. A. Ezipi, and H. Bahlouli, J. App. Phy. 124, 204303 (2018).
[51] H. M. Abdullah, B. Van Duppen, M. Zarenia, H. Bahlouli, and F. M. Peeters, J. Phys.: Condens. Matter 29, 425303 (2017).
[52] H. M. Abdullah, A. El Mouhafid, H. Bahlouli, and A. Jellal, Mater. Res. Express 4, 025009 (2017).