Neutron star matter in the quark-meson coupling model in strong magnetic fields

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Abstract

The effects of strong magnetic fields on neutron star matter are investigated in the quark-meson coupling (QMC) model. The QMC model describes a nuclear many-body system as nonoverlapping MIT bags in which quarks interact through self-consistent exchange of scalar and vector mesons in the mean-field approximation. The results of the QMC model are compared with those obtained in a relativistic mean-field (RMF) model. It is found that quantitative differences exist between the QMC and RMF models, while qualitative trends of the magnetic field effects on the equation of state and composition of neutron star matter are very similar.

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I. INTRODUCTION

Recent observations of soft gamma repeaters (SGRs) and anomalous X-ray pulsars (AXPs) have suggested that the surface magnetic field of young neutron stars could be of order $10^{14} - 10^{15}$ G \cite{1}. On the other hand, it is estimated that the interior field in neutron stars may be as large as $10^{18}$ G \cite{2,3}. Motivated by the existence of strong magnetic fields in neutron stars, theoretical studies on the effect of extremely large fields on dense matter and neutron stars have been carried out by many authors \cite{3,4,5,6,7,8,9,10}. The inclusion of hyperons and boson condensation has also been investigated \cite{11,12,13}. It was found that magnetic fields could change the composition of matter dramatically, and the softening of the equation of state (EOS) caused by Landau quantization is overwhelmed by the stiffening due to the incorporation of the nucleon anomalous magnetic moments for field strengths $B > 10^5 B^*_e$ ($B^*_e = 4.414 \times 10^{13}$ G is the electron critical field). So far, most of these studies have employed the relativistic mean-field (RMF) theory, which is a field theoretical approach at hadron level and has been widely used in the description of nuclear matter and finite nuclei \cite{14}. It is well known that various RMF models could provide similar nuclear saturation properties but different behaviors at high density. In Ref. \cite{3}, the model dependence of strong magnetic field effects within the RMF theory has been investigated. It is instructive to study the properties of neutron star matter in the presence of strong magnetic fields by employing various approaches.

In this paper, we adopt the quark-meson coupling (QMC) model for the investigation of strong magnetic field effects on neutron star matter. The QMC model was originally proposed by Guichon \cite{15}, in which the quark degrees of freedom are explicitly taken into account. The QMC model describes a nuclear many-body system as nonoverlapping MIT bags in which quarks interact through self-consistent exchange of scalar and vector mesons in the mean-field approximation. The mesons couple not to point-like nucleons but directly to confined quarks. In contrast to the RMF approach, the quark structure of nucleons plays a crucial role in the QMC model, and the basic coupling constants are defined at quark level. The QMC model has been subsequently extended and applied to various problems of nuclear matter and finite nuclei with reasonable success \cite{16,17,18}. Furthermore, the model has also been used to investigate the properties of neutron stars with the inclusion of hyperons, quarks, and kaon condensation \cite{19,20,21}. It is interesting to extend the QMC
model for the study of a nuclear many-body system in strong magnetic fields, and make a systematic comparison between the QMC and RMF models.

In general, a nuclear many-body problem has to be worked out in the QMC model by two steps. First, one can calculate the nucleon properties in medium by using the MIT bag model with an external scalar field $\sigma$ generated by nuclear medium. Then in the second step, the entire nuclear system is solved at hadron level with the effective nucleon mass as a function of $\sigma$ obtained in the first step. The main difference between the QMC and RMF models is reflected in the dependence of effective nucleon mass on scalar field $\sigma$. In the QMC model, the scalar field plays a vital role in determining the nucleon properties such as effective mass and radius, while the vector fields do not cause any change of nucleon properties but appear merely as energy shifts. When we treat a nuclear many-body system in strong magnetic fields, we assume that the magnetic fields do not change the internal structure of nucleons, then nucleons in meson and magnetic fields could be treated quite similarly as in the RMF models. With the effective nucleon mass obtained at quark level, we can investigate the properties of neutron star matter in strong magnetic fields.

This paper is arranged as follows. In Sec. II, we briefly describe the QMC model for neutron star matter and present relevant equations in strong magnetic fields. In Sec. III, we show and discuss the numerical results in the QMC model and make a systematic comparison with the results of the RMF model. The conclusion is presented in Sec. IV.

II. QUARK-MESON COUPLING MODEL FOR NEUTRON STAR MATTER IN STRONG MAGNETIC FIELDS

The QMC model describes a nuclear many-body system as nonoverlapping spherical bags in which quarks interact through self-consistent exchange of scalar and vector mesons in the mean-field approximation. To perform the calculation for neutron star matter in strong magnetic fields, we first study the nucleon properties with external meson and magnetic fields by using the MIT bag model. The fields in the bag are in principle functions of position, which may cause a deformation of the nucleon bag. For simplicity, we neglect the spatial variation of the fields over the small bag volume, and take the values at the center of the bag as their average quantities $\sigma$. Then the quarks in the bag satisfy the Dirac
The normalized ground state for a quark in the bag is given by

$$\psi_q(r,t) = N_q e^{-i\epsilon_q t/R} \left( \frac{j_0(x_q r/R)}{\beta_q \sigma \cdot \hat{r} j_1(x_q r/R)} \right) \frac{\chi_q}{\sqrt{4\pi}} ,$$

where

$$\beta_q = \sqrt{\frac{\Omega_q - R m^*_q}{\Omega_q + R m^*_q}} ,$$

and

$$N_q^{-2} = 2 R^3 j^2_0(x_q) \left[ \Omega_q(\Omega_q - 1) + R m^*_q/2 \right]/x_q^2 ,$$

with $\Omega_q = \sqrt{x_q^2 + (R m^*_q)^2}$, $m^*_q = m_q + g^3_3 \sigma$. $R$ is the bag radius, and $\chi_q$ is the quark spinor.

The boundary condition, $j_0(x_q) = \beta_q j_1(x_q)$, at the bag surface determines the eigenvalue $x_q$. The energy of a static nucleon bag consisting of three ground state quarks is then given by

$$E_{\text{bag}} = 3 \frac{\Omega_q}{R} - \frac{Z}{R} + \frac{4}{3} \pi R^3 B_{\text{bag}} ,$$

where the parameter $Z$ accounts for zero-point motion, and $B_{\text{bag}}$ is the bag constant. The effective nucleon mass is then taken to be

$$M_N^* = E_{\text{bag}} .$$

The bag radius $R$ is determined by the equilibrium condition $\partial M_N^*/\partial R = 0$. In the present calculation, we take the current quark mass $m_q = 5.5$ MeV. The parameter $B_{\text{bag}}^{1/4} = 210.854$ MeV and $Z = 4.00506$ are determined by reproducing the nucleon mass in free space $M_N = 939$ MeV and the bag radius $R = 0.6$ fm as given in Ref. [20]. We note that $\omega$ and $\rho$ mean fields appear merely as the energy shifts which do not cause any change in nucleon properties, therefore the effective nucleon mass obtained in the QMC model depends on the $\sigma$ mean field only.

For neutron star matter consisting of a neutral mixture of neutrons, protons, electrons, and muons in $\beta$ equilibrium in the presence of strong magnetic fields, the total Lagrangian
density at hadron level can be written as

\[
\mathcal{L}_{\text{QMC}} = \sum_{b=n,p} \bar{\psi}_b \left[ i \gamma_\mu \partial^\mu - q_b \gamma_\mu A^\mu - M_N^* - g_\omega \gamma_\mu \omega^\mu - g_\rho \gamma_\mu \rho_3^\mu - \frac{1}{2} \kappa_b \sigma_{\mu\nu} F^{\mu\nu} \right] \psi_b \\
+ \sum_{l=e,\mu} \bar{\psi}_l \left[ i \gamma_\mu \partial^\mu - q_l \gamma_\mu A^\mu - m_l \right] \psi_l + \frac{1}{2} \partial_\mu \sigma \partial^\mu \sigma - \frac{1}{2} m_\sigma^2 \sigma^2 \\
- \frac{1}{4} W_{\mu\nu} W^{\mu\nu} + \frac{1}{2} m_\omega^2 \omega_\mu \omega^\mu - \frac{1}{4} R_{i\mu\nu} R_i^{\mu\nu} + \frac{1}{2} m_\rho^2 \rho_{i\mu} \rho_i^\mu - \frac{1}{4} F_{\mu\nu} F^{\mu\nu},
\]

(7)

where \( \psi_b \) and \( \psi_l \) are the nucleon and lepton fields, respectively. \( A^\mu = (0, 0, Bx, 0) \) refers to a constant external magnetic field \( B \) along the \( z \)-axis. The field tensors for the \( \omega, \rho, \) and magnetic field are given by \( W_{\mu\nu} = \partial_\mu \omega_\nu - \partial_\nu \omega_\mu, R_{i\mu\nu} = \partial_\mu \rho_{i\nu} - \partial_\nu \rho_{i\mu}, \) and \( F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu. \) It has been argued in Ref. [3] that the contributions from anomalous magnetic moments of nucleons should be considered for large field strengths. Here the anomalous magnetic moments of nucleons are included with \( \kappa_p = \mu_N (g_p/2 - 1) = 1.7928 \mu_N \) and \( \kappa_n = \mu_N g_n/2 = -1.9130 \mu_N, \) where \( \mu_N \) is the nuclear magneton. As for leptons, the effects of anomalous magnetic moments of electrons and muons on the EOS are very small as shown in Ref. [9], and the electron anomalous magnetic moments could be efficiently reduced by high-order contributions from the vacuum polarization in strong magnetic fields [23]. Therefore, we neglect the anomalous magnetic moments of leptons in the present work. The coupling constants at hadron level, \( g_\omega, g_\rho, \) and \( g_\omega, \) are related to the corresponding quark-meson couplings as \( g_\omega = 3g_\omega^2 \) and \( g_\rho = g_\rho^2 \) [22]. The quark-meson coupling constants \( g_\omega^2 = 5.957, g_\rho^2 = 2.994, \) and \( g_\rho^3 = 4.325 \) are determined by fitting the properties of nuclear matter [20], and the meson masses \( m_\sigma = 550 \text{ MeV}, m_\omega = 783 \text{ MeV}, \) and \( m_\rho = 770 \text{ MeV} \) are used in the present calculation.

From the Lagrangian density given in Eq. (7), we obtain the following meson field equations in the mean-field approximation

\[
m_\sigma^2 = -\frac{\partial M_N^*}{\partial \sigma} (\rho_\sigma^p + \rho_\sigma^n),
\]

(8)

\[
m_\omega^2 \omega_0 = g_\omega (\rho_\omega^p + \rho_\omega^n),
\]

(9)

\[
m_\rho^2 \rho_{30} = g_\rho (\rho_\rho^p - \rho_\rho^n),
\]

(10)

while the Dirac equations for nucleons and leptons are given by

\[
(i \gamma_\mu \partial^\mu - q_b \gamma_\mu A^\mu - M_N^* - g_\omega \gamma_\mu \omega_0 - g_\rho \gamma_0 \rho_3^\mu - \frac{1}{2} \kappa_b \sigma_{\mu\nu} F^{\mu\nu} \right) \psi_b = 0,
\]

(11)

\[
(i \gamma_\mu \partial^\mu - q_l \gamma_\mu A^\mu - m_l) \psi_l = 0.
\]

(12)
The energy spectra for protons, neutrons, and leptons (electrons and muons) are given by

\[ E_{\nu,s}^p = \sqrt{k_{f,s}^2 + \left( \sqrt{M_N^* + 2\nu q_p B - s\kappa_p B} \right)^2 + g_\nu \omega_0 + g_\rho \rho_{30}}, \]

(13)

\[ E_s^n = \sqrt{k_{f,s}^2 + \left( \sqrt{M_N^* + \frac{k_{f,s}^2 + k_{f,s}^2}{2} - s\kappa_n B} \right)^2 + g_\nu \omega_0 - g_\rho \rho_{30}}, \]

(14)

\[ E_{\nu,s}^l = \sqrt{k_{f,s}^2 + m_l^2 + 2\nu |q_l| B}, \]

(15)

where \( \nu = n + \frac{1}{2} - \text{sgn}(q) s/2 = 0, 1, 2, \ldots \) enumerates the Landau levels of the fermion with electric charge \( q \). The quantum number \( s = \pm 1 \) are for spin-up and spin-down cases. As discussed in Ref. [3], the Dirac spinors are no longer eigenfunctions of the spin operator along the magnetic field direction \( \sigma_z \) when the anomalous magnetic moments are taken into account. However, \( s \) reduce to the eigenvalue of \( \sigma_z \) as the anomalous magnetic moment tends toward zero. We note that the first Landau level to be occupied is the state with \( \nu = 0, s = +1 \) for the proton and \( \nu = 0, s = -1 \) for the lepton, while the neutron first occupies the state with \( s = -1 \) due to its negative anomalous magnetic moment. This argument is consistent with the results in nonrelativistic approach [3].

The expressions of the scalar and vector densities for protons and neutrons are given by

\[ \rho_p = \frac{q_p B M_N^*}{2\pi^2} \sum_{\nu} \sum_s \frac{\sqrt{M_N^* + 2\nu q_p B - s\kappa_p B}}{\sqrt{M_N^* + 2\nu q_p B}} \ln \frac{k_{f,s}^p + E_f^p}{\sqrt{M_N^* + 2\nu q_p B - s\kappa_p B}} \]

(16)

\[ \rho_v = \frac{q_p B}{2\pi^2} \sum_{\nu} \sum_s k_{f,s}^p, \]

(17)

\[ \rho^n = \frac{M_N^*}{4\pi^2} \sum_s \left[ k_{f,s}^n E_f^n - (M_N^* - s\kappa_n B)^2 \ln \frac{k_{f,s}^n + E_f^n}{M_N^* - s\kappa_n B} \right], \]

(18)

\[ \rho_v^n = \frac{1}{2\pi^2} \sum_s \left\{ \frac{1}{3} k_{f,s}^{n3} - \frac{1}{2} s\kappa_n B \left[ (M_N^* - s\kappa_n B) k_{f,s}^n \right. \right. \]

\[ + E_f^n \left. \left. \left( \arcsin \frac{M_N^* - s\kappa_n B}{E_f^n} - \frac{\pi}{2} \right) \right] \right\}, \]

(19)

where \( k_{f,v,s}^p \) and \( k_{f,s}^n \) are the Fermi momenta of protons and neutrons, which are related to the Fermi energies \( E_f^n \) and \( E_f^n \) as

\[ E_f^n = k_{f,s}^{n2} + \left( \sqrt{M_N^* + 2\nu q_p B - s\kappa_p B} \right)^2, \]

(20)

\[ E_f^n = k_{f,s}^{n2} + (M_N^* - s\kappa_n B)^2. \]

(21)
The chemical potentials of nucleons and leptons are given by

\[ \mu_p = E_p^f + g_\omega \omega_0 + g_\rho \rho_{30}, \]  
\[ \mu_n = E_n^f + g_\omega \omega_0 - g_\rho \rho_{30}, \]  
\[ \mu_l = E_l^f = \sqrt{k_{f,\nu,s}^l m_l^2 + 2\nu |q_l| B}. \]

For neutron star matter with uniform distributions, the composition of matter is determined by the requirements of charge neutrality and \( \beta \)-equilibrium conditions. In the present calculation, the \( \beta \)-equilibrium conditions are expressed by

\[ \mu_p = \mu_n - \mu_e, \]  
\[ \mu_\mu = \mu_e, \]

and the charge neutrality condition is given by

\[ \rho_p^e = \rho_n^e + \rho_\mu^\mu, \]

where the vector density of leptons has a similar expression to that of protons

\[ \rho_v^l = \frac{|q_l| B}{2\pi^2} \sum_{\nu} \sum_s k_{f,\nu,s}^l. \]

We solve the coupled Eqs. (8)-(10), (25), (26), and (27) self-consistently at a given baryon density \( \rho_B = \rho_p^e + \rho_n^e \) in the presence of strong magnetic fields. The energy density of neutron star matter is given by

\[ \varepsilon_m = \varepsilon_p + \varepsilon_n + \varepsilon_e + \varepsilon_\mu + \frac{1}{2} m_\sigma^2 + \frac{1}{2} m_\omega^2 \omega_0^2 + \frac{1}{2} m_\rho^2 \rho_{30}^2, \]

where the energy densities of nucleons and leptons have the following forms:

\[ \varepsilon_p = \frac{q_p B}{4\pi^2} \sum_{\nu} \sum_s \left[ k_{f,\nu,s}^p E_{f}^p + \left( \sqrt{M_N^* + 2\nu q_p B - s k_p B} \right) \ln \frac{k_{f,\nu,s}^p + E_{f}^p}{\sqrt{M_N^* + 2\nu q_p B - s k_p B}} \right], \]
\[ \varepsilon_n = \frac{1}{4\pi^2} \sum_s \left\{ \frac{1}{2} k_{f,s}^n E_{f}^n - \frac{2}{3} s k_n B E_{f}^n \left( \arcsin \frac{M_N^* - s k_n B}{E_{f}^p} - \frac{\pi}{2} \right) - \left( \frac{s k_n B}{3} + \frac{M_N^* - s k_n B}{4} \right) \right\} \times \left[ (M_N^* - s k_n B) k_{f,s}^n E_{f}^n + (M_N^* - s k_n B)^3 \ln \frac{k_{f,s}^n + E_{f}^n}{M_N^* - s k_n B} \right], \]
\[ \varepsilon_l = \frac{|q_l| B}{4\pi^2} \sum_{\nu} \sum_s \left[ k_{f,\nu,s}^l E_{f}^l + (m_l^2 + 2\nu |q_l| B) \ln \frac{k_{f,\nu,s}^l + E_{f}^l}{\sqrt{m_l^2 + 2\nu |q_l| B}} \right]. \]
The pressure of the system can be obtained by

\[ P_m = \sum_i \mu_i \rho_i^i - \varepsilon_m = \mu_n \rho_B - \varepsilon_m, \]  

where the charge neutrality and \( \beta \)-equilibrium conditions are used to get the last equality. Note that the contribution from electromagnetic fields to the energy density and pressure, \( \varepsilon_f = P_f = B^2/8\pi \), should be taken into account in the calculation of the EOS.

III. RESULTS AND DISCUSSION

In this section, we present the properties of neutron star matter in strong magnetic fields by using the QMC model. The effective nucleon masses in the QMC model are obtained self-consistently at quark level, which is the main difference from the RMF model. The comparison of results in several RMF models has been investigated extensively in Ref. [3]. It was found that quantitative differences persist between those models, while qualitative trends of magnetic field effects are very similar. Here we would like to make a systematic comparison between the QMC and RMF models, which are based on different degrees of freedom.

In Fig. 1, we show the matter pressure \( P_m \) as a function of the matter energy density \( \varepsilon_m \) for the field strengths \( B^* = B/B_c = 0, 10^5, \) and \( 10^6 \). The results of the QMC model in the upper panels are compared with those of the RMF model with TM1 parameter set [24] in the lower panels. In Ref. [3], the RMF model with GM3 [25] and several other parameter sets has been used and compared with each other. Here we prefer to make a comparison with the TM1 model, which includes nonlinear terms both for \( \sigma \) and \( \omega \) mesons. The TM1 model was determined in Ref. [24] by reproducing the properties of nuclear matter and finite nuclei including neutron-rich nuclei, and has been widely used in many studies of nuclear physics [13, 26, 27, 28]. We show the results with and without the inclusion of nucleon anomalous magnetic moments in the right and left panels, respectively. In both QMC and TM1 models, the Landau quantization of charged particles causes a softening in the EOS as shown in the left panels, while the inclusion of anomalous magnetic moments leads to a stiffening of the EOS as shown in the right panels. These effects become significant as magnetic fields increase above \( B^* \sim 10^5 \). The softening of the EOS due to Landau quantization can be overwhelmed by the stiffening due to the incorporation of anomalous
magnetic moments with increasing magnetic fields for $B^* > 10^5$. The EOS in the QMC model is slightly stiffer than the one in the TM1 model. We show in Fig. 2 the effective nucleon masses as a function of baryon density $\rho_B$ again for $B^* = 0, 10^5,$ and $10^6$. It is found that the influence of magnetic fields on the effective masses is not observable until $B^* > 10^5$. Without the inclusion of anomalous magnetic moments, the effective masses for $B^* > 10^5$ are smaller than the field-free values, but they become to be larger than the field-free values in strong magnetic fields when the anomalous magnetic moments are included. It is clear that the changes of effective masses in the QMC model are much smaller than those in the TM1 model.

It is instructive to discuss the spin polarization of nucleons and electrons in neutron star matter with the presence of strong magnetic fields. For charged particles such as electrons and protons, the energy spectrum is characterized by the quantum number of Landau level $\nu$ and the spin projection $s$. If we ignore the anomalous magnetic moments of the charged particles, all energy levels except the ground state are doubly degenerate with opposite spin projections. The ground state of electrons (protons) is a single state with $\nu = 0, s = -1$ ($s = +1$). As the field strength increases, the energy gap between Landau levels becomes larger and larger, and at a critical field, all particles only occupy the first Landau level with $\nu = 0, s = -1$ for electrons and $s = +1$ for protons. This is called complete spin polarization which occurs as the energy gap between Landau levels is comparable to the Fermi momentum of the particle. It is the order of $B^* \sim 10^4$ for electrons and protons in neutron star matter at the densities of interest. Note that neutrons can not be altered by external magnetic fields when the anomalous magnetic moments are ignored. It is very interesting to estimate the contributions from anomalous magnetic moments on the spin polarization of nucleons. When the anomalous magnetic moments are included, the double degeneracy of Landau levels for protons disappears, and the energy spectra for both neutrons and protons are spin-dependent. The ground state of protons (neutrons) is spin-up (spin-down). The next level to be occupied for protons is spin-up ($\nu = 1, s = +1$), then the third level may be spin-down ($\nu = 1, s = -1$) or spin-up ($\nu = 2, s = +1$), which should be determined by comparing their energy spectra. Neutrons can be completely polarized with the inclusion of anomalous magnetic moments. The spin-up states begin to be occupied by neutrons for $k_{f,
u}^2 n_2 \gtrsim 4 |\kappa_n| M_N^* B$, so complete spin polarization of neutrons appears at the order of $B^* \sim 10^5$. We define the proton spin polarization as the ratio of spin-up proton...
density to total proton density, \( \rho_p^v(s = +1)/\rho_p^v \). The results with and without the inclusion of anomalous magnetic moments are shown in the middle and left panels of Fig. 3, while the neutron spin polarization defined as the ratio of spin-down neutron density to total neutron density, \( \rho_n^v(s = -1)/\rho_n^v \), is shown in the right panels. It is obvious that both proton and neutron spin polarizations increase with decreasing baryon density and increasing field strength, and a complete spin polarization occurs at a critical \( B^* \) for a given baryon density. By comparing the left and middle panels, one can find that the inclusion of anomalous magnetic moments leads to a significant increase in the proton spin polarization. We note that there is no noticeable difference between the QMC and TM1 models in Fig. 3.

In Fig. 4, the proton fraction \( Y_p = \rho_p^v/\rho_B \) and the muon fraction \( Y_\mu = \rho_\mu^v/\rho_B \) are plotted as functions of the field strength \( B^* \) for the baryon densities \( \rho_B = 0.075, 0.15, 0.30, \) and 0.60 fm\(^{-3} \). One can easily obtain the neutron fraction \( Y_n = \rho_n^v/\rho_B \) and the electron fraction \( Y_e = \rho_e^v/\rho_B \) by the relations: \( Y_n = 1 - Y_p \) and \( Y_e = Y_p - Y_\mu \). The results with and without the inclusion of anomalous magnetic moments are shown in the right and left panels. It is obvious that the composition of neutron star matter depends on both the baryon density \( \rho_B \) and the magnetic field strength \( B^* \). When the external magnetic fields are absent, the composition of matter is dominated by neutrons with a small proton fraction. It is clear that the proton fraction remains unaltered from the field-free case for relatively low fields. As the field strength increases to the value of having complete proton spin polarization, a significant enhancement of the proton fraction is observed due to large reductions in chemical potentials of protons and leptons caused by strong magnetic fields, and then protons become to be dominant for \( B^* > 10^6 \). On the other hand, lepton fractions increase with the increasing of proton fraction by the requirement of charge neutrality. The effect of strong magnetic fields leads to a reduction in the electron chemical potential, so that muons disappear when the electron chemical potential is less than the rest mass of the muon. It is evident that the inclusion of anomalous magnetic moments causes a slight increase in the proton and lepton fractions.

**IV. SUMMARY**

In this paper we have investigated the effects of strong magnetic fields on the properties of neutron star matter within the QMC model. The QMC model describes a nuclear many-
body system as nonoverlapping MIT bags in which quarks interact through self-consistent exchange of scalar and vector mesons in the mean-field approximation. The effective nucleon masses in the QMC model are obtained self-consistently at quark level, which is an important difference from the RMF models. It is clear that the effects of strong magnetic fields become significant only for field strength $B^* > 10^5$. The Landau quantization of charged particles causes a considerable enhancement of the proton fraction and a softening in the EOS, while the inclusion of nucleon anomalous magnetic moments leads to a stiffening of the EOS. With the inclusion of anomalous magnetic moments, a complete spin polarization of fermions occurs at a critical $B^*$ which depends on the baryon density. The complete spin polarization of neutrons appears at higher field than the one of protons.

We have made a systematic comparison between the results of the QMC and RMF models. It is found that quantitative differences exist between these models, while qualitative trends of magnetic field effects on the EOS and composition of neutron star matter are very similar. In the interior of neutron stars hadronic matter may undergo a phase transition to deconfined quark matter. To estimate the phase transition in the presence of strong magnetic fields, the QMC model should be applicable since it describes hadronic matter in terms of quark degrees of freedom.

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FIG. 1: (Color online) The matter pressure $P_m$ versus the matter energy density $\varepsilon_m$ for the field strengths $B^* = 0, 10^5, \text{and} 10^6$. The results of the QMC model with and without the inclusion of anomalous magnetic moments are shown in the upper-right and upper-left panels, respectively. Those in the TM1 model are also shown in the lower panels for comparison.
FIG. 2: (Color online) The effective nucleon mass $M_N^*/M_N$ as a function of the baryon density $\rho_B$ for the field strengths $B^* = 0, 10^5, \text{ and } 10^6$. The results with and without the inclusion of anomalous magnetic moments are shown in the right and left panels, respectively.
FIG. 3: (Color online) The spin polarization of protons (left and middle panels) and neutrons (right panels) as functions of magnetic field strength. The dotted (blue), solid (black), dashed (green), and dashed-dotted (red) lines correspond to $\rho_B = 0.075$, 0.15, 0.30, and 0.60 fm$^{-3}$, respectively. The neutron spin polarization is purely due to the interaction of anomalous magnetic moments with the magnetic field. The inclusion of anomalous magnetic moments leads to an increase in the proton spin polarization.
FIG. 4: (Color online) The proton and muon fractions as functions of magnetic field strength. The dotted (blue), solid (black), dashed (green), and dashed-dotted (red) lines correspond to $\rho_B = 0.075, 0.15, 0.30, \text{and } 0.60 \text{ fm}^{-3}$, respectively. The neutron and electron fractions could be obtained by $Y_n = 1 - Y_p$ and $Y_e = Y_p - Y_\mu$. 