BLDC motor identification based on optimal control model in the state space

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Abstract. A control model for a BLDC motor in a rotating coordinate system in the state space under vector control has been developed. The problem of identifiability of this model is solved for different values of the load moment. It is proposed to use identification in terms of the correspondence of the mathematical model to the results of the operation of the object. At each step, the determinant of the extended matrix is calculated, which is compared with a constant that numerically divides the space of the state matrices. Thus, the operation of the drives itself makes it possible to determine its identifiability. As a criterion for the optimality of the identification algorithm, a decision-making optimality criterion is chosen in combination with an identifiability criterion for an optimal control algorithm by the criterion of minimum quadratic form. The vector-matrix model of drives in the state space is presented taking into account the relative accuracy of measuring the state of the information-measuring subsystem of drives. It is proposed for practical problems to determine the identifiability criterion by modeling the state matrix for cases when the state matrix parameters exit the space of realizable parameters of serviceable drives. The research results obtained can be used to build diagnostic systems for robot drives.

1. Introduction
To ensure high reliability of robot drives, an effective diagnostic system is required. For the diagnosis of robot drives, an algorithm is proposed for deciding on their identifiability based on a discrete nonlinear control model in the state space.

The method of identification in the state space has been actively developed over the past two decades and has been successfully implemented in many industries. One of the first P. Eickhoff performed the theoretical justification of identification, developed algorithms and methods of identification [1,2]. The identification of dynamical systems is devoted to the work of the following authors: D. Gropp [3], L. Ljung [4], E.P. Sage and J.L. Melsa [5,6], and among Russian authors - Ya.Z. Tsypkina [7], N.S. Reibman [8], Sh.E. Steinberg [9] and others.

R. Beard developed an observer-based defect detection scheme [10]. Jones continued these studies and developed the Beard-Jones Fault Detection Filter [11]. In the 1980s and early 1990s, the main approaches to quantitative diagnostics were developed: an observer-based approach, a parameter estimation method, etc. Some important works in this direction are Frank [12], Isermann [13], and Basville and Nikiforov [14]. The developed methods are well theoretically justified and are classic diagnostic methods. These techniques are based on analytical redundancy, which is a model that describes the diagnosed technical system. On diagnostics of robot drives, articles and monographs

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have been published [15–27], where approaches are considered both on the basis of the parametric approach and on the basis of continuous and discrete models of drives in the state space.

2. Development of a mathematical model of the BLDC motor

The aim of the work is to study the identifiability of the BLDC motor model, which can be used for the diagnosis.

The novelty of this study lies in an approach that takes into account criteria such as controllability, observability, real-time identifiability for various desired angular displacements, velocities and accelerations in the state space during vectorial control of the BLDC motor.

The mathematical basis of vector control by a drive is differential equations that describe the drive equally correctly in both dynamics and statics. Due to the adequacy of control in dynamics, vector control, unlike scalar control, makes it possible to build highly dynamic and precision AC electric drives that provide the highest accuracy and speed of regulation. In addition, when vector control is used, the representation of three-phase quantities in the form of generalized vectors is used and control systems are constructed in rotating coordinates.

To solve the identification problem, a dynamic BLDC motor model was developed in a rotating coordinate system (d, q) oriented along the rotor magnetic axis. On the basis of Kirchhoff’s 2nd law, an equation is compiled that describes the electrical part of the BLDC motor in the coordinate system d, q rotating with the rotor speed \( \omega \):

\[
\bar{u}_s = \bar{i}_s R_s + L_s \frac{d\bar{i}_s}{dt} + j \omega L_s \bar{i}_s + j \omega \Phi_0
\]  

(1)

where \( \bar{u}_s = u_{sd} - ju_{sq} \) is the resulting voltage vector on the stator winding; \( R_s, L_s \) are active resistance and total inductance of the stator phase; \( \Phi_0 = \Phi_{0d} + j \Phi_{0q} \) is a vector of the BLDC motor flow; \( j \omega L_s \bar{i}_s \) is a voltage drop due to the stator scattering flux; \( j \omega \Phi_0 \) is a voltage drop due to the main magnetic flux of the BLDC motor, which is created by the excitation of the rotor.

The discrete model of the BLDC motor is presented in the form of a vector-matrix equation in the state space.

\[
\begin{bmatrix}
I_d(k+1) \\
I_q(k+1) \\
\omega(k+1) \\
\theta(k+1)
\end{bmatrix} =
\begin{bmatrix}
1 - T \frac{R_s}{L_d} & T \omega(k) & 0 & 0 \\
-T \omega(k) & 1 - T \frac{R_s}{L_q} & T - \frac{\psi}{L_q} & 0 \\
0 & \frac{\psi}{J} & 1 - T \frac{F}{J} - T \frac{M(k)}{j \omega(k)} & 0 \\
0 & 0 & T & 1
\end{bmatrix}
\begin{bmatrix}
I_d(k) \\
I_q(k) \\
\omega(k) \\
\theta(k)
\end{bmatrix}
+ \begin{bmatrix}
\frac{T}{L_d} & 0 & 0 & 0 \\
0 & \frac{T}{L_q} & 0 & 0 \\
0 & 0 & \frac{T}{J} & 0 \\
0 & 0 & 1 & 0
\end{bmatrix}
\begin{bmatrix}
I_d(k) \\
I_q(k) \\
\omega(k) \\
\theta(k)
\end{bmatrix}
\]  

(2)

where \( I_d(k+1) \) is the projection of the stator current onto the \( d \) axis at the time instant \( k + 1 \); \( I_q(k+1) \) is the projection of the stator current onto the \( q \) axis at time moment \( k + 1 \); \( \omega(k+1) \) is the angular velocity of the BLDC motor at time \( k + 1 \); \( \theta(k+1) \) is the angular displacement of the BLDC motor at time \( k + 1 \); \( T \) is the sampling interval, the time between \( k + 1 \) and \( k \) samples; \( R_s \) is the active resistance of the stator winding BLDC motor; \( L_q, L_d \) are the stator inductances of the BLDC motor along the \( q \) and \( d \) axes; \( p \) is the number of pairs of the BLDC motor poles; \( \psi \) is the magnetic flux induced by permanent magnets in the stator winding; \( F \) is the coefficient of viscous friction in the supports of the BLDC motor; \( J \) is the moment of inertia; \( M \) is the electromagnetic moment of the BLDC motor; \( I_d(k) \) is the projection of the stator current onto the \( d \) axis at time \( k \); \( I_q(k) \) is the projection of the stator current onto the \( q \) axis at time \( k \); \( \omega(k) \) is the angular velocity of the BLDC motor at time \( k \); \( \theta(k) \) is the angular displacement of the BLDC motor at time \( k \);
U_q, U_d are the projections of the stator voltage on the q and d axes.

The equation at the output of the BLDC motor is written as:

\[
\begin{bmatrix}
I_d \\
I_q \\
\dot{\omega} \\
\dot{\theta}
\end{bmatrix} =
\begin{bmatrix}
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1
\end{bmatrix}
\begin{bmatrix}
I_d \\
I_q \\
\omega \\
\theta
\end{bmatrix}
+ 
\begin{bmatrix}
\xi_{I_d} \\
\xi_{I_q} \\
\xi_{\omega} \\
\xi_{\theta}
\end{bmatrix}
\]

where \([\xi_{I_d}, \xi_{I_q}, \xi_{\omega}, \xi_{\theta}]^T\) is the vector of measurement errors; 
\([I_d, I_q, \dot{\omega}, \dot{\theta}]^T\) is the measured state vector.

The required torque is calculated by the formula (4):

\[
M(k) = \varepsilon(k)J = \omega(k+1) - \omega(k)T
\]

The angular velocity of the BLDC motor is calculated by the formula (5):

\[
\dot{\omega}(k) = \frac{\theta(k) - \theta(k-1)}{T}
\]

The quadratic functional of quality, which determines the energy of control and displacement, is expressed as follows

\[
I = \frac{1}{2} \int_{t_0}^{t_f} (x^T Q x + u^T G u) dt, Q \geq 0, G > 0,
\]

where Q and G are positive arbitrarily defined matrices. The matrices Q and G are chosen arbitrarily; It is not always possible to obtain a solution of the equation for finding u. It is proposed to select these matrices by selection or simulation modeling [1], [2].

The solution for the quality criterion (6), minimizing the control and displacement energy, is determined by the following expression [1], [2].

\[
u = -G^{-1}B^T K x
\]

where K is the Cauchy matrix, K = K^T, which can be found by solving the Riccati equation [1]

\[
-K = Q + A^T K + K T A - K B G^{-1} B^T K, \quad K(t_0) = 0
\]

In abstract-theoretical analysis, identifiability is a particular case of observability. Identifiability is understood as the receipt or refinement of the model of a real object from experimental data. For a linear stationary system at each step of linearization, the criterion of identifiability are determinants of the expanded matrix:

\[
\det \left[ C_k^T : A_k^T C_k^T : (A_k^T)^2 C_k^T : (A_k^T)^3 C_k^T \right] > \gamma,
\]

where \(\gamma\) is the threshold value of the determinants determined by the identification object and close to zero.

3. The results of the simulation of BDT

To study identifiability, a model was built in the SimInTech program and the simulation results were analyzed.

The BLDC motor parameters: stator winding resistance \(R_s = 1.9\) Ohm, stator winding inductance along the d axis \(L_d = 0.0018\) H, stator winding inductance along the q axis \(L_q = 0.0018\) H, inertia moment \(J = 0.0000024\), \(F = 0.001\), \(k = 0.001\), number of pole pairs \(p = 8\), \(fi = 0.001\), angular velocity \(\omega_0 = 220\) rad/s, sampling interval \(T = 0.005\) s.

The discrete model of BLDC motor for calculating determinants of state matrices was built in the software package “Environment for Dynamic Modeling of Technical Systems SimInTech” (No. 2379 in the Unified Register of Russian Programs) developed by 3V Service [28].

A discrete BLDC engine model is constructed for calculating determinants of state matrices. The results of modeling changes in the determinants of the extended state matrix are shown in Figure 1.
Figure 1. Determinants of matrix of state.

The simulation results show that when the torque increment is 0.12 Nm, the determinant of the state matrix tends to zero. This means that the BDTT model becomes unidentifiable. From this we can conclude that too much torque is applied or the parameters of the state matrix leave the space of realizable parameters of serviceable drives.

4. Conclusion
This article has developed a nonlinear discrete model of BLDC in matrix-vector form based on optimal control. The state matrix takes into account the torque that must be provided by the BLDC. Identification criterion of BLDC is calculated as the minimum determinant of the extended state matrix. When the value of the identifiability criterion is less than the threshold value, the BLDC model becomes unidentifiable. In this case, we can conclude that too much torque is applied or that the parameters of the BLDC state matrix leave the space of realizable parameters of serviceable drives.

Acknowledgement
The reported study was funded by RFBR, project number 18-08-00772 A.

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