Migdal’s short range correlations in a covariant model

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Abstract

We construct a covariant model for short range correlations of a pion emerged in nuclear matter. Once the delta-hole contribution is considered an additional and so far neglected channel opens that leads to significant modifications in the vicinity of the kinematical region defined by \( \omega \sim |q| \). We speculate that this novel effect should be important for the quantitative interpretation of charge exchange reactions like \(^{12}\text{C}(^{3}\text{He},t)\).

The pion plays a special role in nuclear physics. A large amount of work has been done to understand the many facets of nuclear pion dynamics \([1,2,3,4,5,6,7,8]\). Nevertheless, there still exists quite a bit of ambiguity as to what is the quantitative form of the pion spectral function in cold nuclear matter. This problem reflects in part the fact that there are no commonly accepted Migdal parameters that describe the short range correlation effects. In particular \( g'_{N\Delta} \) and \( g'_{\Delta\Delta} \) are not too well determined (see e.g. \([9,10]\)). In this short note we focus on the particular aspect how to treat short range correlations in a covariant manner. The latter are required to reduce the strength of the nucleon- and delta-hole contributions to the pion self energy. This avoids for instance a pion condensate at unreasonably small nuclear densities. Most works were performed in a non-relativistic framework with the exception of a series of works by Dmitriev \([5,11,12,13,14,15]\). In the latter works it was demonstrated that a proper covariant treatment of the nucleon-hole term leads to a contribution proportional to \( \omega^2 - q^2 \) rather than to \( q^2 \) as suggested by a non-relativistic treatment. Thus a faithful evaluation of the pion self energy in nuclear matter requires a relativistic treatment at least in the vicinity of \( \omega \sim |q| \). The purpose of this work is to demonstrate that previous works \([5,11,12,13,14,15]\) did not completely succeed in constructing a covariant model for Migdal’s short range...
correlations. The shortcomings of these works will be overcome and a fully covariant generalization of Dmitriev’s model will be presented here. It turns out that the necessary modifications to Dmitriev’s model lead to significant effects in the pion self energy for \( \omega \sim |\vec{q}| \) missed so far.

Following Dmitriev’s original work [5] we consider the interaction of pions with nucleons and isobars in terms of the leading order vertices

\[
\mathcal{L} = \frac{f_N}{m_{\pi}} \bar{\psi} \gamma_5 \gamma^\mu \vec{T} \partial_\mu \phi_{\pi} \psi + \frac{f_\Delta}{m_{\pi}} \left( \bar{\psi} \vec{T} \psi \partial_\mu \phi_{\pi} + \text{h.c.} \right),
\]

(1)
as predicted by the chiral Lagrangian. We use \( T_i^j = \delta_{ij} - \tau_i \tau_j / 3 \) and \( f_N = 0.988 \) and \( f_\Delta = 2 f_N \) in this work. The nucleon and isobar propagators are

\[
S(p,u) = \frac{1}{\vec{p} - m_N + i \epsilon} + \Delta S(p,u),
\]

\[
\Delta S(p,u) = 2 \pi i \Theta(\vec{p} \cdot u) \delta(p^2 - m_N^2) (\vec{p} + m_N) \Theta(k_F^2 + p^2 - (u \cdot p)^2),
\]

\[
S_{\mu\nu}(p) = \frac{-i}{\vec{p} - m_\Delta + i \epsilon} \left( g_{\mu\nu} - \frac{\gamma_\mu \gamma_\nu}{3} - \frac{2 p_\mu p_\nu}{3 m_\Delta^2} - \frac{\gamma_\mu p_\nu - p_\mu \gamma_\nu}{3 m_\Delta} \right),
\]

(2)

where the time like four-vector \( u_\mu \) specifies the nuclear matter frame. For symmetric nuclear matter at rest it follows \( u_\mu = (1, \vec{0}) \) and \( \rho = 2 k_F^3/(3 \pi^2) \).

In (2) we do not consider an additional structure in the isobar propagator which would modify the off-shell properties of the isobar only. This term is not relevant for the development of this work.

It is straight forward to write down the nucleon and delta-hole contributions to the pion self energy,

\[
\Pi_\pi(q,u) = -q^\mu \left( \Pi_{\mu\nu}^{(Nh)}(q,u) + \Pi_{\mu\nu}^{(\Delta h)}(q,u) \right) q^\nu,
\]

(3)

where we follow previous works and disregard vacuum polarization effects. This leads to loop functions of the form

\[
\Pi_{\mu\nu}^{(\Delta h)}(q,u) = \frac{4}{3} \frac{f_\Delta^2}{m_{\pi}^2} \int \frac{d^4p}{(2\pi)^4} i \text{tr} \Delta S(p,u) S_{\mu\nu}(p + q) + (q_\mu \to -q_\mu),
\]

\[
\Pi_{\mu\nu}^{(Nh)}(q,u) = 2 \frac{f_N^2}{m_{\pi}^2} \int \frac{d^4p}{(2\pi)^4} i \text{tr} \left( \Delta S(p,u) \gamma_5 \gamma_\mu \frac{1}{\vec{p} + \vec{q} - m_N} \gamma_5 \gamma_\nu + \frac{1}{2} \Delta S(p,u) \gamma_5 \gamma_\mu \Delta S(p + q) \gamma_5 \gamma_\nu \right) + (q_\mu \to -q_\mu).
\]

(4)

In order to arrive at realistic nucleon-hole and delta-hole contributions to the
pion polarization in nuclear matter it is crucial to introduce short range correlation effects that significantly reduce the contributions of the nucleon-hole and delta-hole diagrams [6]. Most authors would also argue that the loop functions (4) should be multiplied by a suitable form factor that reduces the strength of the loop function at large momenta. Since this would necessarily introduce some ambiguities we refrain from doing this here. The aim of this short note is not to provide a fully realistic pion self energy in nuclear matter rather we suggest to improve a frequently used model as to achieve consistency with covariance. We do, however, incorporate a reasonable spectral distribution of the isobar and fold the delta-hole loop function with a spectral function that describes the $P_{33}$ phase shift of pion-nucleon scattering. The specifics of the spectral function we use here can be found in [16].

A covariant form of the short range correlations was introduced explicitly by Nakano et al. [15]

$$\mathcal{L}_{\text{Migdal}} = g'_{11} \frac{f_N^2}{m_{\pi}^2} \left( \bar{\psi} \gamma_5 \gamma_\mu \gamma_5 \bar{\tau} \psi \right) \left( \bar{\psi} \gamma_5 \gamma_\mu \gamma_5 \bar{\tau} \psi \right)$$

$$+ g'_{22} \frac{f_\Delta^2}{m_{\pi}^2} \left( \bar{\psi}_\mu \bar{T} \psi \right) \left( \bar{\psi} \bar{T} \psi \right) + \left( \left( \bar{\psi}_\mu \bar{T} \psi \right) \left( \bar{\psi} \bar{T} \psi \right) + \text{h.c.} \right) \right)$$

$$+ g'_{12} \frac{f_N f_\Delta}{m_{\pi}^2} \left( \bar{\psi} \gamma_5 \gamma_\mu \bar{\tau} \psi \right) \left( \left( \bar{\psi} \bar{T} \psi \right) + \text{h.c.} \right),$$

(5)

where it is understood that the local vertices are to be used at the Hartree level. The Fock contribution can be cast into the form of a Hartree contribution by a simple Fierz transformation. Therefore it only renormalizes the coupling strength in (5) and can be omitted here. Note that the terms proportional to $\bar{\psi}_\mu \bar{T} \psi$ and $\bar{\psi}_\mu \bar{T} \psi$ of (5) were missed in [15]. They are required to recover the proper non relativistic limit of the short range correlations as introduced by Migdal.

In previous works [5,11,12,13,14,15] the short range correlation effects were introduced in the form,

$$\Pi(q,u) = \frac{\Pi_{Nh}(q^2 \pi^2 + g'_{11} \Pi_{Nh}) + \Pi_{\Delta h}(q^2 \pi^2 + g'_{11} \Pi_{Nh}) - 2 g_{12} \Pi_{Nh} \Pi_{\Delta h}}{(q^2 + g'_{11} \Pi_{Nh}) (q^2 + g'_{22} \Pi_{\Delta h}) - g_{12}^2 \Pi_{Nh} \Pi_{\Delta h}},$$

$$\Pi_{\Delta h}(q,u) = -q^\mu \Pi_{\mu \nu}^{(\Delta h)}(q,u) q^\nu, \quad \Pi_{Nh}(q,u) = -q^\mu \Pi_{\mu \nu}^{(Nh)}(q,u) q^\nu, \quad (6)$$

with $\Pi_{\mu \nu}^{(Nh)}$ and $\Pi_{\mu \nu}^{(\Delta h)}$ taken from (4). As will be demonstrated explicitly below the result (6) is strictly speaking not correct and requires a generalization for

$$q^2 \neq (q \cdot u)^2.$$
The form of (6) was taken over from a corresponding expression obtained by Migdal [6] in the non-relativistic case, but not properly modified for the covariant vertices (1, 5).

It is evident that the expression (6) were correct if the generic loop functions $\Pi^{(Nh)}_{\mu\nu}$ and $\Pi^{(\Delta h)}_{\mu\nu}$ had contributions proportional to $g_{\mu\nu}$ and $q_{\mu} q_{\nu}$ only. However, the most general decomposition of the loop functions involves additional structures proportional to $u_{\mu} q_{\nu}$ and $q_{\mu} u_{\nu}$. To derive the correct generalization of (6) we introduce a transverse projector, $T_{\mu\nu}(q, u)$ and a set of longitudinal projectors $L_{ij\mu\nu}(q, u)$,

\begin{align*}
T_{\mu\nu} &= g_{\mu\nu} - \frac{q_{\mu} q_{\nu}}{q^2} - X_{\mu} X_{\nu}, \quad X_{\mu} = \frac{(q \cdot u) q_{\mu} - q_{\mu} u_{\mu}}{\sqrt{q^2}} - \frac{q^2}{\sqrt{q^2} - (q \cdot u)^2} , \\
L_{11\mu\nu} &= \frac{q_{\mu} q_{\nu}}{q^2}, \quad L_{12\mu\nu} = \frac{q_{\mu} X_{\nu}}{\sqrt{q^2}}, \quad L_{21\mu\nu} = \frac{X_{\mu} q_{\nu}}{\sqrt{q^2}}, \quad L_{22\mu\nu} = X_{\mu} X_{\nu}, \\
L_{ik} \cdot L_{ij} &= \delta_{kl} L_{ij}, \quad L_{ij} \cdot T = 0 = T \cdot L_{ij}, \quad u^2 = 1, \quad X^2 = 1, \quad (7)
\end{align*}

that trivialize the solution of the Dyson equation in the presence of the structures $q_{\mu} u_{\nu}$ and $u_{\mu} q_{\nu}$. The loop functions are decomposed into the complete set of projectors,

\begin{align*}
\Pi^{(Nh)}_{ij} &= \Pi^{(Nh)}_{\mu\nu} L_{ij\mu\nu}, \quad \Pi^{(\Delta h)}_{ij} = \Pi^{(\Delta h)}_{\mu\nu} L_{ij\mu\nu}, \quad (8)
\end{align*}

where only the longitudinal projections are needed in this work. A corresponding decomposition of the Migdal interaction vertices,

\begin{equation}
\gamma_{5} \gamma_{\mu} \otimes \gamma_{5} \gamma_{\nu} = \gamma_{5} \gamma_{\mu} \otimes \gamma_{5} \gamma_{\nu} \left( T_{\mu\nu} + L_{11\mu\nu} + L_{22\mu\nu} \right),
\end{equation}

implies a straight forward generalization of (6). The self energy can be cast into the form of a sum of 11, 33 and 13, 31 components of an appropriate 4×4 matrix that incorporates the additional 2×2 matrix structure introduced in (7,8),

\begin{align*}
\Pi &= - \left[ (1 - J G)^{-1} J \right]_{11} - \left[ (1 - J G)^{-1} J \right]_{33} \\
&\quad - \left[ (1 - J G)^{-1} J \right]_{13} - \left[ (1 - J G)^{-1} J \right]_{31}, \quad (9)
\end{align*}

where the coupling matrix, $G$, and loop matrix, $J$, are
$$G = \begin{pmatrix} g'_{11} & 0 & g'_{12} \\ 0 & g'_{11} & 0 \\ g'_{21} & 0 & g'_{22} \\ 0 & g'_{12} & g'_{22} \end{pmatrix}, \quad J = \begin{pmatrix} \Pi^{(Nh)}_{11} & \Pi^{(Nh)}_{12} & 0 & 0 \\ \Pi^{(Nh)}_{21} & \Pi^{(Nh)}_{22} & 0 & 0 \\ 0 & 0 & \Pi^{(\Delta h)}_{11} & \Pi^{(\Delta h)}_{12} \\ 0 & 0 & \Pi^{(\Delta h)}_{21} & \Pi^{(\Delta h)}_{22} \end{pmatrix}. \quad (10)$$

It is evident that only matrix elements $ij$ with $i, j = 1, 3$ contribute to the self energy since the latter reflect the derivative coupling of the pion baryon vertex structure. In the particular case where $\Pi^{(Nh)}_{12} = \Pi^{(Nh)}_{21} = 0$ and $\Pi^{(\Delta h)}_{12} = \Pi^{(\Delta h)}_{21} = 0$ holds the result (9) reproduces (6).

We proceed and derive explicit representations for the loop matrix $J$ for the standard case of nuclear matter at rest with $u_\mu = (1, \vec{0})$. The nucleon-hole functions read $^2$,

$$\Pi^{(Nh)}_{ij}(\omega, \vec{q}) = \frac{i f_N^2}{m_\pi^2} \Im \int_0^{k_F} \frac{d^3p}{2 p_0 (2\pi)^3} \frac{8 K^{(Nh)}_{ij}(\vec{p} + \vec{q})}{2 p \cdot q + q^2 + i \epsilon} \Theta(p_0 + \omega)$$

$$+ \frac{f_N^2}{m_\pi^2} \Re \int_0^{k_F} \frac{d^3p}{2 p_0 (2\pi)^3} \frac{8 K^{(Nh)}_{ij}(\vec{p} - \vec{q})}{2 p \cdot q + q^2 + i \epsilon} \pm (q_\mu \to -q_\mu), \quad (11)$$

where $q_\mu = (\omega, \vec{q})$, $p_0 = \sqrt{m_N^2 + \vec{p}^2}$ and

$$K^{(Nh)}_{11} = 2 m_N^2, \quad K^{(Nh)}_{12} = K^{(Nh)}_{21} = 0,$$

$$K^{(Nh)}_{22} = \frac{\omega^2 - \vec{q}^2}{q^2} \left(2 \vec{p}^2 + \omega p_0 + \vec{p} \cdot \vec{q} \right). \quad (12)$$

Since the transition loop functions $\Pi^{(Nh)}_{12} = \Pi^{(Nh)}_{21} = 0$ vanish identically it would be justified to ignore the coupled channel structure in the nucleon-hole channel if the delta-hole contributions were neglected altogether. The loop function $\Pi^{(Nh)}_{22} \neq 0$ will be relevant once the delta-hole channel is considered since in the latter channel $\Pi^{(\Delta h)}_{12} \neq 0$ holds. The following result is derived,

$$\Pi^{(\Delta h)}_{ij}(\omega, \vec{q}) = \frac{4 f_\Delta^2}{9 m_\pi^2} \int_0^{k_F} \frac{d^3p}{2 p_0 (2\pi)^3} \frac{8 K^{(\Delta h)}_{ij}(m_N m_\Delta + m_N^2 + (p \cdot q))}{2 p \cdot q + q^2 - m_\Delta^2 + m_N^2 + i \epsilon}$$

$$+ \pm (q_\mu \to -q_\mu), \quad (13)$$

$^2$ The off-diagonal (diagonal) loop function $\Pi_{ij}(\omega, \vec{q}) = \Pi^{(\Delta h)}_{12}(-\omega, \vec{q})$ must be anti-symmetrized (symmetrized) in $\omega$. This follows from the corresponding property of $L^{\mu\nu}_{ij}$. The resulting pion self energy is symmetric in $\omega$. 

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Fig. 1. The pion self energy as a function of $\omega$ and $|\vec{q}|$ evaluated at nuclear saturation density with $k_F = 268$ MeV. The solid line shows the full result including the coupled channel structure of (9), the dashed line follows with $\Pi^{(\Delta h)}_{12} \to 0$ in (9). The dotted line gives the result if one symmetrized the transition isobar-hole loop function $\Pi^{(\Delta h)}_{12}$.

$$
K^{\Delta h}_{11} = 1 - \frac{(q^2 + p \cdot q)^2}{q^2 m^2_\Delta}, \quad K^{\Delta h}_{22} = 1 + \frac{(\omega |\vec{p}| \cos(\vec{q}, \vec{p}) - |\vec{q}|p_0)^2}{m^2_\Delta q^2},
$$

$$
K^{\Delta h}_{12} = K^{\Delta h}_{21} = i \frac{q^2 + p \cdot q}{q^2 m^2_\Delta} (|\vec{q}|p_0 - \omega |\vec{p}| \cos(\vec{q}, \vec{p})).
$$

We refrain from presenting analytic expressions for the final loop functions since the expressions are cumbersome and already published for the 11 components [13,15]. Moreover it is more transparent to work directly with the representations (11, 12, 13).

In Fig. 1) we present the pion self energy (9) and (6) for the choice of parameters $g_{11} = 0.585$ and $g_{12} = 0.191 + 0.051 g_{22}$ with $g_{22} = 0.6$ as suggested in [2] and [15]. We do not study here possible deviations from those values because this is not the point of this work. The figure clearly illustrates significant effects in certain kinematical regions as the result of a proper treatment of the coupled channels. Most striking is the enhancement by about a factor 5 found for the imaginary part of the pion self energy at $\omega \sim 350$ MeV and $|\vec{q}| = 300$ MeV. The inclusion of the transition loop function $\Pi_{12}^{(\Delta h)}$ together with $\Pi_{22}^{(\Delta h)}$
is crucial here. From the form of $\Pi^{(\Delta h)}_{12}$ it follows directly that at $\omega \neq 0$ and $|\vec{q}| = 0$, the kinematical region probed by the quenching of the Gamow-Teller resonance [17], the coupled channel structure discussed here is superficial. In this case the use of the expression (6) is justified since $\Pi^{(\Delta h)}_{12} = 0$ holds. A further interesting limit is $\omega = 0$ with $\vec{q} \neq 0$ as is probed by a possible pion condensate. Here one would expect to recover the algebraic form of the non-relativistic scheme of Migdal for $|\vec{q}| \ll m_N$ and $|\vec{p}| \ll m_N$. This is indeed confirmed if the functions $K^{(\Delta h)}_{ij}$ are expanded in powers of $\frac{|\vec{p}|}{m_N}$, $\frac{|\vec{q}|}{m_N}$.

Then the transition moment, $K^{(\Delta h)}_{12}$, is suppressed by the factor $|\vec{q}|/m_N$ as compared to the leading moment, $K^{(\Delta h)}_{11}$. Therefore we expect only minor corrections when studying pion condensation phenomena within the generalized result (5). Particularly striking are, however, new effects close to the kinematical point $\omega^2 \sim |\vec{q}|^2 \neq 0$ where the transition loop function can no longer be neglected. This kinematical domain is crucial for the peak structure [5,7,14] in the $^{12}\text{C}(^3\text{He},t)$ transfer reaction [18,19].

Acknowledgments

I would like to thank E. Kolomeitsev, C. Korpa and D. Voskresenski for useful discussions.

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