Global Quantum discord of multi-qubit states

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(Dated: May 22, 2014)

PACS numbers: 03.67.Mn, 03.65.Yz, 05.70.Fh

I. INTRODUCTION

Quantifying the multipartite quantum correlations is a very challenging and still largely open question [1–5]. For bipartite case, entanglement and quantum discord have been widely accepted as two fundamental tools to quantify quantum correlations [6, 7], and quantum discord captures more quantum correlations than entanglement in the sense that a separable state may have nonzero quantum discord. Generalizations of bipartite quantum discord to multipartite states have been considered in different ways [8]. In [5], Rulli and Sarandy proposed a measure for multipartite quantum correlations, called global quantum discord (GQD), which can be seen as a generalization of bipartite quantum discord [9, 10] to multipartite states. GQD is always nonnegative and its use is illustrated by the Werner-GHZ state and the Ashkin-Teller model [5].

In this paper, we provide an equivalent expression for GQD, and give an interpretation of GQD (Sec.III). We derive the analytical expressions of GQD for two classes of multi-qubit states. The phenomena of sudden transition and freeze of GQD are also discussed.

II. GLOBAL QUANTUM DISCORD (GQD)

We briefly review the definition of GQD proposed in [5].

Consider two systems $A_1$ and $A_2$ (each of them is of finite dimension), the symmetric quantum discord of a state $\rho_{A_1A_2}$ of the composite systems $A_1A_2$ is

$$D(\rho_{A_1A_2}) = \min_{\Phi} [I(\rho_{A_1A_2}) - I(\Phi_{A_1A_2}(\rho_{A_1A_2}))].$$

In Eq.(1),

$$I(\rho_{A_1A_2}) = S(\rho_{A_1}) + S(\rho_{A_2}) - S(\rho_{A_1A_2}),$$

is the mutual information of $\rho_{A_1A_2}$, $\min$ is taken over all locally projective measurements performing on $A_1$, $\Phi_{(\cdot)}$ denotes a locally projective measurement performing on the system $(\cdot)$, $S(\cdot)$ is the Von Neumann entropy, and $\rho_{A_1}, \rho_{A_2}$ are reduced states of $\rho_{A_1A_2}$.

$D(\rho_{A_1A_2})$ is a natural extension of the original definition of quantum discord which defined over all projective measurements performing only on $A_1$ or $A_2$ [9, 10].

Since the mutual information $I(\rho_{A_1A_2})$ can be expressed by the relative entropy

$$I(\rho_{A_1A_2}) = S(\rho_{A_1A_2}) - S(\rho_{A_1}) - S(\rho_{A_2}),$$

hence, Eq.(1) can also be recasted as

$$D(\rho_{A_1A_2}) = \min_{\Phi} [S(\rho_{A_1A_2}) - S(\rho_{A_1}) - S(\rho_{A_2}) - S(\Phi_{A_1A_2}(\rho_{A_1A_2}))].$$

Note that the relative entropy of state $\rho$ with respect to state $\sigma$ ($\rho$ and $\sigma$ lie on the same Hilbert space) is defined as

$$S(\rho||\sigma) = tr(\rho \log_2 \rho) - tr(\rho \log_2 \sigma).$$

Further, Eq.(1) can also be rewritten as

$$D(\rho_{A_1A_2}) = \min_{\Phi} [S(\rho_{A_1A_2}) - S(\rho_{A_1}) - S(\rho_{A_2}) - S(\Phi_{A_1A_2}(\rho_{A_1A_2}))].$$

The definition of GQD is a generalization of bipartite symmetric quantum discord. Consider $N$ ($2 \leq N < \infty$) systems $A_1, A_2, \ldots, A_N$ (each of them is of finite dimension), the GQD of state $\rho_{A_1A_2...A_N}$ on the composite system $A_1A_2...A_N$ is defined as

$$D(\rho_{A_1A_2...A_N}) = \min_{\Phi} [S(\rho_{A_1A_2...A_N}) - S(\rho_{A_1}) - S(\rho_{A_2}) - \ldots - S(\rho_{A_N}) - S(\Phi_{A_1A_2...A_N}(\rho_{A_1A_2...A_N}))].$$

It has been proved that $D(\rho_{A_1A_2...A_N}) \geq 0$ for any state $\rho_{A_1A_2...A_N}$ [5]. Also, it is easy to see that $D(\rho_{A_1A_2...A_N})$ keeps invariant under any locally unitary transformation.

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III. AN EQUIVALENT EXPRESSION FOR
GLOBAL QUANTUM DISCORD

In this section, we provide an equivalent expression for GQD. We first state two mathematical facts as the lemmas below.

Lemma 1. For any square matrix (with finite dimension) $A$, let $\overline{A}$ be the matrix whose diagonal elements are the same with $A$, and other elements are zero. $B$ and $\overline{B}$ are defined similarly. Then

\[
tr(AB) = tr(\overline{A} \overline{B}),
\]

(8)

\[
tr(Af(B)) = tr(\overline{A}f(\overline{B})),
\]

(9)

where $f(\cdot)$ is any function.

Lemma 2. Let $\rho_{A_1A_2...A_N}$ be a state on Hilbert space $H_{12...N}$, $\rho_{A_1}$, $\rho_{A_2}$, ..., $\rho_{A_N}$ be the reduced states of $\rho_{A_1A_2...A_N}$ on Hilbert spaces $H_1$, $H_2$, ..., $H_N$, respectively. Suppose $\sigma_{A_1}$, $\sigma_{A_2}$, ..., $\sigma_{A_N}$ are states on $H_1$, $H_2$, ..., $H_N$, respectively. Then it holds that

\[
tr[\rho_{A_1A_2...A_N} \log_2(\sigma_{A_1} \otimes \sigma_{A_2} \otimes \ldots \otimes \sigma_{A_N})] = \sum_{i=1}^{N} tr[\rho_i \log_2 \sigma_i].
\]

(10)

Proof. We only prove the case of $N = 2$, the proof of $N > 2$ is similar. When $N = 2$, we need to prove

\[
tr[\rho_{A_1A_2} \log_2(\sigma_{A_1} \otimes \sigma_{A_2})] = tr[\rho_{A_1} \log_2 \sigma_{A_1}] + tr[\rho_{A_2} \log_2 \sigma_{A_2}].
\]

(11)

It is known that $\rho_{A_1A_2}$ can be written as [12]

\[
\rho_{A_1A_2} = \sum_j c_j \rho_{1j} \otimes \rho_{2j},
\]

(12)

where $\{c_j\}$ are real numbers, $\rho_{1j}$, $\rho_{2j}$ are all Hermite matrices. For $\sigma_{A_1}$, $\sigma_{A_2}$, there exist unitary matrices $U_1$ and $U_2$ such that $D_1 = U_1 \sigma_{A_1} U_1^+$, $D_2 = U_2 \sigma_{A_2} U_2^+$ are all diagonal, where $\pm$ denotes adjoint. Note that

\[
\log_2(D_1 \otimes D_2) = (\log_2 D_1) \otimes I_2 + I_1 \otimes (\log_2 D_2),
\]

(13)

where $I_1$, $I_2$ are the identity operators on $H_1$, $H_2$, respectively. Then Eq.(11) can be directly verified. \hfill \Box

With the help of Lemma 1 and Lemma 2, we can get an equivalent expression for GQD defined by Eq.(7).

Theorem 1. The GQD of a state $\rho_{A_1A_2...A_N}$ defined by Eq.(7) can also be expressed as

\[
D(\rho_{A_1A_2...A_N}) = \min_\Phi [I(\rho_{A_1A_2...A_N}) - I(\Phi_{A_1A_2...A_N}(\rho_{A_1A_2...A_N}))],
\]

(14)

where, the mutual information

\[
I(\rho_{A_1A_2...A_N}) = \sum_{i=1}^{N} S(\rho_i) - S(\rho_{A_1A_2...A_N}).
\]

(15)

Proof. From lemma 1, we have

\[
S(\rho_A | \Phi_A(\rho_A)) = -S(\rho_A) + S(\Phi_A(\rho_A)).
\]

(16)

Note that

\[
I(\rho_{A_1A_2...A_N}) = \sum_{i=1}^{N} S(\rho_i) - S(\rho_{A_1A_2...A_N})
\]

\[
= S(\rho_{A_1A_2...A_N}) S(\rho_{A_1A_2...A_N}).
\]

(17)

Together with Lemma 2, we can easily prove Theorem 1. \hfill \Box

From Theorem 1, we see that, GQD of a state is just the minimal loss of mutual information over all locally projective measurements. This interpretation of GQD is consistency with the symmetric quantum discord in Eq.(1), as well as the original definition of quantum discord for bipartite states.

For a special case, we consider a state $\rho_{A_1A_2...A_N}$ whose reduced states $\rho_{A_1}$, $\rho_{A_2}$, ..., $\rho_{A_N}$ are all proportional to the identity operator. In such case, the GQD of $\rho_{A_1A_2...A_N}$ can be remarkably simplified. We state it as Theorem 2, its proof is easy.

Theorem 2. An $N$-partite state $\rho_{A_1A_2...A_N}$, if its reduced states $\rho_{A_1}$, $\rho_{A_2}$, ..., $\rho_{A_N}$ are all proportional to the identity operator, then the GQD of $\rho_{A_1A_2...A_N}$ can be expressed as

\[
D(\rho_{A_1A_2...A_N}) = -S(\rho_{A_1A_2...A_N}) + \min_\Phi S(\Phi_{A_1A_2...A_N}(\rho_{A_1A_2...A_N})).
\]

(18)

IV. TWO CLASSES OF $N$-QUBIT STATES WHICH ALLOW ANALYTICAL EXPRESSIONS OF GQD

We consider two classes of $N$-qubit states which allow analytical expressions of GQD. We first recall two mathematical facts.

Lemma 3. [13] Group homomorphism of $U(2)$ to $SO(3)$.

For any two-dimensional unitary matrix $U$, there exists a unique real three-dimensional orthogonal matrix $R$ with determinant 1, such that for any real three-dimensional vector $\vec{r} = (r_x, r_y, r_z)$, it holds that

\[
U \vec{r} \cdot \vec{\sigma} = R(\vec{r}) \cdot \vec{\sigma}.
\]

(19)

Conversely, For any real three-dimensional orthogonal matrix $R$ with determinant 1, there exists (not unique) a two-dimensional unitary matrix $U$, fulfills Eq.(19).

In Eq.(19), $\vec{r} \cdot \vec{\sigma} = r_x \sigma_x + r_y \sigma_y + r_z \sigma_z, \vec{\sigma} = \{\sigma_x, \sigma_y, \sigma_z\}$ are Pauli matrices; $R(\vec{r}) = (R\vec{r})^T$ is a real three-dimensional vector, here $t$ denotes matrix transpose.

Lemma 4. [14] Monotonicity of entropy function under majorization relation.

For given $\{p_1, p_2, ..., p_n\}$, $\{q_1, q_2, ..., q_n\}$, satisfy

\[
1 \geq p_1 \geq p_2 \geq ... \geq p_n \geq 0, \sum_{i=1}^{n} p_i = 1,
\]

\[
S(\rho_1) \leq S(\rho_2)
\]

(20)

\[
S(\rho_1 | \rho_2) = S(\rho_1) - S(\rho_2) \geq 0.
\]

(21)

where $S(\rho)$ is the von Neumann entropy of the state $\rho$.
Then we get
\[ \Phi_{A_{1}A_{2}...A_{N}}(|\psi\rangle\langle\psi|) = \frac{1}{2} \left[ \bigotimes_{i=1}^{N} I + \gamma_{i} \Pi_{i} \cdot \sigma_{z} \right] + \bigotimes_{i=1}^{N} \alpha_{i} + i \beta_{i} \Pi_{i} \cdot \sigma_{z} + \bigotimes_{i=1}^{N} \alpha_{i} - i \beta_{i} \sigma_{z}. \] (33)
Eq.(34) is a diagonal matrix with \(2^N\) diagonal elements
\[ \frac{1}{2} \left[ \bigotimes_{i=1}^{N} I + \gamma_{i} \Pi_{i} \cdot \sigma_{z} + \bigotimes_{i=1}^{N} \alpha_{i} + i \beta_{i} \Pi_{i} \cdot \sigma_{z} + \bigotimes_{i=1}^{N} \alpha_{i} - i \beta_{i} \sigma_{z} \right]. \] (35)

where, \(n_{1}, n_{2}, ..., n_{N} \in \{0, 1\}\). Consequently, \(\Phi_{A_{1}A_{2}...A_{N}}(\rho)\) have \(2^{2N}\) eigenvalues
\[ \frac{1}{2^{2N}} - \mu \left[ \bigotimes_{i=1}^{N} I + \gamma_{i} \Pi_{i} \cdot \sigma_{z} + \bigotimes_{i=1}^{N} \alpha_{i} + i \beta_{i} \Pi_{i} \cdot \sigma_{z} + \bigotimes_{i=1}^{N} \alpha_{i} - i \beta_{i} \sigma_{z} \right]. \] (36)

where, \(n_{1}, n_{2}, ..., n_{N} \in \{0, 1\}\).

Note that in Eq.(36), the eigenvalues corresponding to \(\{n_{1}, n_{2}, ..., n_{N}\}\) and \(\{1 - n_{1}, 1 - n_{2}, ..., 1 - n_{N}\}\) are equal. Together with Lemma 4, we then assert that when \(\gamma_{i} = 1\) for all \(i\), \(S(\Phi_{A_{1}A_{2}...A_{N}}(\rho))\) achieves its minimum, since when \(\gamma_{i} = 1\) for all \(i\), \(\Phi_{A_{1}A_{2}...A_{N}}(\rho)\) have \(2^{2N}\) eigenvalues
\[ \left\{ \frac{1 - \mu}{2^{2N}}, \frac{1 - \mu - \mu}{2^{2N}}, \frac{1 - \mu - \mu - \mu}{2^{2N}}, ..., \frac{1 - \mu - \mu - \mu - \mu}{2^{2N}} \right\}. \] (37)

So, by Eq.(37) and Eq.(29), Theorem 3 can be proved.
\[ \square \]
We consider the behavior of Eq.(23) with large number \( N \). Let \( N \to \infty \), we get Corollary 1 below.

**Corollary 1.** When \( N \to \infty \), Eq.(23) approximates

\[
D_{\infty}(\rho) = \mu. \tag{38}
\]

We remark that, Eq.(38) is somehow a good approximation of Eq.(23). In fact, when \( N = 10 \), the deviation is less than \( 10^{-2} \); \( N = 14 \), the deviation is less than \( 10^{-3} \); \( N = 17 \), the deviation is less than \( 10^{-4} \). Fig.1 shows Eq.(23) for \( N = 2, 3, 5, \infty \), respectively.

We also remark that, when \( N = 3 \), Eq.(23) is the same to the result obtained in [5]. When \( N = 2 \), Eq.(23) returns the result of Werner state considered in [9].

![Graph](image_url)

**FIG. 1:** GQD of Werner-GHZ state in Eq.(22) when \( N = 2, 3, 5, \infty \), respectively.

We next consider another class of \( N \)-qubit states.

**Theorem 4.** For \( N \)-qubit state

\[
\rho = \frac{1}{2^N}(I \otimes N + c_1 \sigma_x^N + c_2 \sigma_y^N + c_3 \sigma_z^N), \tag{39}
\]

the GQD of \( \rho \) is

\[
D(\rho) = f(\rho) - g(\rho). \tag{40}
\]

In Eq.(40),

\[
f(\rho) = -\frac{1+c}{2} \log_2 \frac{1+c}{2} - \frac{1-c}{2} \log_2 \frac{1-c}{2},
\]

\[
c = \max(|c_1|, |c_2|, |c_3|); \tag{42}
\]

when \( N \) is odd,

\[
g(\rho) = -\frac{1+d}{2} \log_2 \frac{1+d}{2} - \frac{1-d}{2} \log_2 \frac{1-d}{2},
\]

\[
d = \sqrt{c_1^2 + c_2^2 + c_3^2}; \tag{44}
\]

when \( N \) is even,

\[
g(\rho) = -1 - \sum_{j=1}^{4} \lambda_j \log_2 \lambda_j, \tag{43}
\]

\[
\lambda_1 = [1 + c_3 + c_1 + (-1)^{N/2}c_2]/4, \tag{45}
\]

\[
\lambda_2 = [1 + c_3 - c_1 - (-1)^{N/2}c_2]/4, \tag{46}
\]

\[
\lambda_3 = [1 - c_3 + c_1 - (-1)^{N/2}c_2]/4, \tag{47}
\]

\[
\lambda_4 = [1 - c_3 - c_1 - (-1)^{N/2}c_2]/4. \tag{48}
\]

In Eq.(39), \( I \) is the \( 2 \times 2 \) identity operator, \( \{c_1, c_2, c_3\} \) are real numbers constrained by \( d \in [0,1] \) (when \( N \) is odd) or \( \lambda_1, \lambda_2, \lambda_3, \lambda_4 \in [0, 1] \) (when \( N \) is even).

**Proof.** Obviously, for state \( \rho \) in Eq.(39), its reduced states \( \rho_{A_1}, \rho_{A_2}, \ldots, \rho_{A_N} \) are all proportional to identity operator. Thus, we can calculate \( D(\rho) \) according to Theorem 2. We then need to calculate \( S(\rho) \) and \( S(\Phi_{A_1A_2\ldots A_N}(\rho_{A_1A_2\ldots A_N})) \). It is easy to see that state \( \rho \) has nonzero elements only on the principle diagonal and the antidiagonal, so is the matrix \( (\rho - xI^{\otimes N}) \). By the Laplace theorem in linear algebra, \( M = \det(\rho - xI^{\otimes N}) \) can be expanded as the multiplications of \( 2N-1 \) determinants of \( 2 \times 2 \) matrices,

\[
M = \det(\rho - xI^{\otimes N}) = \prod_{j=1}^{2N-1} \det \left( \begin{array}{cc} M_{jj} & M_{j,2N-j} \\ M_{2N-j,j} & M_{2N-j,2N-j} \end{array} \right). \tag{50}
\]

where

\[
M_{jj} = \frac{1}{2N} + (-1)^{n_1+n_2+\ldots+n_N} \frac{c_3}{2} - x, \tag{52}
\]

\[
M_{2N-j,j,2N-j} = \frac{1}{2N} + (-1)^{N+n_1+n_2+\ldots+n_N} \frac{c_3}{2} - x, \tag{53}
\]

\[
M_{2N-j,j} = \frac{c_1}{2N} + x(-1)^{n_1+n_2+\ldots+n_N} \frac{c_2}{2N}, \tag{54}
\]

\[
M_{2N-j,2N-j} = \frac{c_1}{2N} + x(-1)^{N+n_1+n_2+\ldots+n_N} \frac{c_2}{2N}, \tag{55}
\]

\( \{n_1, n_2, \ldots, n_N\} \in \{0, 1\} \).

Then by direct calculation of \( \det(\rho - xI^{\otimes N}) = 0 \), we get the eigenvalues of \( \rho \). That is, when \( N \) is even, the eigenvalues of \( \rho \) are

\[
\left\{ \frac{1+c_3}{2N} \pm \frac{1}{2N} \right\} + \frac{(-1)^{N/2}c_2}{2N} - x \tag{56}
\]

each of them possesses multiplicity \( 2N-2 \). When \( N \) is odd, the eigenvalues of \( \rho \) are

\[
\left\{ \frac{1}{2N} \right\} + \sqrt{\frac{c_1^2 + c_2^2 + c_3^2}{2}} \tag{57}
\]

each of them possesses multiplicity \( 2N-1 \).

\( S(\rho) \) can then be directly calculated by Eqs.(56-57).
We now calculate \( \Phi_{A_1A_2...A_N}(\rho) \). After the measurement \( \{\Pi_i = (\alpha_i, \beta_i, \gamma_i)\}_{i=1}^N \) as labelled in the proof of Theorem 3, from Eqs.(26-28), we have

\[
\Phi_{A_1A_2...A_N}(\rho) = \frac{1}{2^N} [I^\otimes N + (c_1 \prod_{i=1}^N \alpha_i + c_2 \prod_{i=1}^N \beta_i + c_3 \prod_{i=1}^N \gamma_i) \otimes I].
\]  

(58)

From Lemma 3, for any \( \{\Pi_i\}_{i=1}^N \) we can always perform a locally unitary transformation on \( \Phi_{A_1A_2...A_N}(\rho) \) (\( S(\Phi_{A_1A_2...A_N}(\rho)) \) keeps invariant) such that \( \Pi_i \cdot \sigma \) is transformed to \( \sigma_z \) for all \( i \), then Eq.(58) becomes

\[
\frac{1}{2^N} [I^\otimes N + (c_1 \prod_{i=1}^N \alpha_i + c_2 \prod_{i=1}^N \beta_i + c_3 \prod_{i=1}^N \gamma_i) \otimes I].
\]  

(59)

Eq.(59) is a diagonal matrix, so its eigenvalues are

\[
\{ \frac{1}{2N} [1 \pm (c_1 \prod_{i=1}^N \alpha_i + c_2 \prod_{i=1}^N \beta_i + c_3 \prod_{i=1}^N \gamma_i)] \},
\]  

(60)

each of them has multiplicity \( 2^{N-1} \).

From Lemma 4, we find that minimizing \( S(\Phi_{A_1A_2...A_N}(\rho)) \) is equivalent to maximizing

\[
|c_1 \prod_{i=1}^N \alpha_i + c_2 \prod_{i=1}^N \beta_i + c_3 \prod_{i=1}^N \gamma_i|,
\]  

(61)

over all possible \( \{\Pi_i\}_{i=1}^N \).

Suppose \( \{\Pi_i\}_{i=1}^{N-1} \) are given, then Eq.(61) can be written as

\[
|\alpha_N, \beta_N, \gamma_N) \cdot (c_1 \prod_{i=1}^{N-1} \alpha_i, c_2 \prod_{i=1}^{N-1} \beta_i, c_3 \prod_{i=1}^{N-1} \gamma_i)|.
\]  

(62)

So over all possible \( \Pi_N = (\alpha_N, \beta_N, \gamma_N) \) with \( \alpha_N^2 + \beta_N^2 + \gamma_N^2 = 1 \), Eq.(62) achieves the maximum

\[
(c_1^2 \prod_{i=1}^{N-1} \alpha_i^2 + c_2^2 \prod_{i=1}^{N-1} \beta_i^2 + c_3^2 \prod_{i=1}^{N-1} \gamma_i^2)^{1/2}.
\]  

(63)

Suppose \( \{\Pi_i\}_{i=1}^{N-2} \) are given, because \( \alpha_{N-1}^2 + \beta_{N-1}^2 + \gamma_{N-1}^2 = 1 \), so the maximum of Eq.(63) over all possible \( \Pi_{N-1} = (\alpha_{N-1}, \beta_{N-1}, \gamma_N) \) is

\[
(max\{c_1^2 \prod_{i=1}^{N-2} \alpha_i^2, c_2^2 \prod_{i=1}^{N-2} \beta_i^2, c_3^2 \prod_{i=1}^{N-2} \gamma_i^2\})^{1/2}.
\]  

(64)

The maximum of Eq.(64) over all possible \( \{\Pi_i\}_{i=1}^{N-2} \) apparently is

\[
c = max\{|c_1|, |c_2|, |c_3|\}.
\]

(65)

By Eqs.(56,57,60,65), we then complete this proof. □

We remark that, when \( N = 2 \), Theorem 4 recovers the result of 2-qubit Bell-diagonal state which was first obtained in [11].

We know the original quantum discord may manifest the phenomena of sudden transition and freeze. With the analytical result of Theorem 4, we assert that GQD can also manifest such interesting phenomena. We make this assertion more clear by giving an example. Let \( N \) qubits be in the state Eq.(39), and let any qubit undergo a local phase damping

\[
E_0 = \sqrt{1-p/2} I, \quad E_1 = \sqrt{p/2} \sigma_z.
\]  

(66)

After this channel, the state \( \rho \) in Eq.(39) becomes

\[
\rho(p) = \frac{1}{2N} (I^\otimes N + c_1(p)\sigma_x^\otimes N + c_2(p)\sigma_y^\otimes N + c_3(p)\sigma_z^\otimes N),
\]  

(67)

where

\[
c_1(p) = c_1(1-p), \quad c_2(p) = c_2(1-p), \quad c_3(p) = c_3.
\]  

(68)

So, \( D(\rho(p)) \) can be calculated by Theorem 4. Therefore, similar to the bipartite case discussed in [15, 16], it can be found that, the sudden transition occurs if and only if

\[
0 < |c_1| < max\{|c_1|, |c_2|\},
\]

(69)

and freezing GQD may occur when \( N \) is even.

V. CONCLUSION

In summary, we provided an equivalent expression of global quantum discord (GQD). From this equivalent expression, we gave an interpretation of GQD as the minimal loss of mutual information over all locally projective measurements. This interpretation is consistency with the original quantum discord. We obtained the analytical expressions of GQD for two classes of multi-qubit states, each of them possesses high symmetry. By the analytical expressions of these states, we discussed some behaviors of GQD, including the asymptotic behavior of GQD when \( N \) tends infinity, and the phenomena of sudden transition and freeze of GQD.

This work was supported by the Fundamental Research Funds for the Central Universities of China (Grant No.2010scu23002). The author thanks Qing Hou and Bo You for helpful discussions.

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