TIME TRAVEL?*

S. Deser

Department of Physics
Brandeis University
Waltham, Massachusetts 02254 U.S.A.

and

R. Jackiw

Center for Theoretical Physics
Laboratory for Nuclear Science
and Department of Physics
Massachusetts Institute of Technology
Cambridge, Massachusetts 02139 U.S.A.

In Memoriam

Gary Feinberg

Extended version of talk presented at
46 LNS 46
Cambridge, MA, May 1992

* This work is supported in part by funds provided by the National Science Foundation under grant #PHY-88-04561 (SD) and the U. S. Department of Energy (D.O.E.) under contract #DE-AC02-76ER03069 (RJ).
To travel into the past, to observe it, perhaps to influence it and correct mistakes of one’s youth, has been an abiding fantasy of mankind for as long as we have been aware of a past. Here are described some recent scientific investigations on this topic.

Before the twentieth century, time travel was discussed only in works of fiction; among innumerable instances, best known are surely the novels of Mark Twain and H. G. Wells. The latter marks a transition: Wells, a graduate of London’s Imperial College of Science and Technology, couched his novel The Time Machine in scientific language, giving us an early work of science fiction.

After 1900, special relativity made scientific discussion of time machines possible. The question may be posed in this way: it makes perfectly good sense to speak about travel that returns to the same point in space. But Einstein and Minkowski tell us that space and time are equivalent, so after a journey can we return to our starting position in time? Here the answer is well-known: the space-time of special relativity is flat and rigid; while time travel into the past is indeed possible, it requires faster-than-light velocities. This is seen as follows.

The Lorentz transformation law of space ($\Delta x$) and time ($\Delta t$) intervals in some initial (inertial) reference frame into corresponding quantities (designated by an overbar) in another frame, moving relative to the original one with frame velocity $v$ along say the $x$-axis, is given by

$$
\Delta \overline{t} = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}
\left(\Delta t - \frac{v}{c^2} \Delta x\right) \quad (1a)
$$

$$
\Delta \overline{x} = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}
\left(\Delta x - v \Delta t\right) \quad (1b)
$$

Since the velocity $u$ of a moving object in the original frame is $\Delta x/\Delta t$, we also have

$$
\Delta \overline{t} = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}
\Delta t \left(1 - \frac{uv}{c^2}\right) \quad (2)
$$
For $\Delta t$ to remain real, the frame velocity $v$ must not exceed the velocity of light $c$, but $\Delta t$ can have a sign opposite to $\Delta t$ if

$$\frac{u}{c} > \frac{c}{v} > 1 \quad (3)$$

One can think of this result as a continuation of the familiar time-dilation story: the faster one travels starting from rest, the slower flows time; time stands still for travel at the velocity of light and runs backward once $c$ is exceeded.

A time machine is now constructed in the following manner. When a faster-than-light object — called a tachyon — is emitted with $u > c$ by a source, moving backwards with speed $v < c$ and satisfying (3), the tachyon will arrive at its goal a time $|\Delta t|$ before it was sent out; then if returned by a similarly backward-moving source, it will arrive at its point of origin at $2|\Delta t|$ before emission. However, there are no known tachyons, and time machines cannot be constructed with the physics and engineering possibilities provided by special relativity. Nevertheless, it should be stressed that there is no logical prohibition against exceeding light velocity. Indeed a hypothetical world in which tachyons exist is physically consistent. Although tachyons have been looked for experimentally none have ever been found; see Ref. [1] for a nice review of the entire subject.

But then we come to Einstein’s general relativity, within which space-time geometry is no longer fixed; indeed it can take all kinds of unexpected configurations, depending on the matter content of the universe. In particular there can be geometries containing paths along which one can travel into the past with velocity less than that of light — such paths are called closed time-like curves. The first solution to Einstein’s theory with closed time-like curves was obtained in 1949 by Gödel and it permits construction of time machines.2
Gödel solved the Einstein gravity equations of general relativity

\[ G_{\mu\nu} + \frac{\Lambda}{2} g_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu}. \]  (4)

The left side is geometrical; it contains the Einstein tensor, \( G_{\mu\nu} \), formed from the metric tensor \( g_{\mu\nu} \) in which is encoded the geometry of space-time, together with a cosmological term. The right side describes matter, \( T_{\mu\nu} \) being its local energy-momentum distribution; matter determines geometry through Eq. (4). (Newton’s \( G \) is the appropriate dimensional proportionality constant.)

Gödel took \( T_{\mu\nu} \) to be a space-time constant, not vanishing only in its time-time component (energy density),

\[ T_{00} = \frac{c^4}{8\pi G} \Lambda > 0. \]  (5)

The metric tensor that then solves Einstein’s equations leads to the space-time interval

\[ ds^2 = g_{\mu\nu} dx^\mu dx^\nu = \left( c dt - \sqrt{\frac{2}{\Lambda}} \left( \cosh \sqrt{\Lambda} r - 1 \right) d\theta \right)^2 - dr^2 - \frac{1}{\Lambda} \sinh^2 \sqrt{\Lambda} r \ d\theta^2 - dz^2, \]  (6)

where \( r, \theta \) are planar circular coordinates, with \( \theta = 0 \) and \( 2\pi \) identified, and there is no interesting structure in the \( z \)-direction. A curve \( x^\mu(\tau) \) is closed and time-like if both \( x^\mu(0) = x^\mu(1) \) (closed) and \( (ds/d\tau)^2 = g_{\mu\nu}(dx^\mu/d\tau)(dx^\nu/d\tau) > 0 \) (time-like). It is therefore clear that a circular path in the Gödel universe for which \( t, r \) and \( z \) remain constant, while \( \theta \) varies from 0 to \( 2\pi \), is closed and time-like provided \( \cosh \sqrt{\Lambda} r > 3 \), i.e., \( r > \frac{2}{\sqrt{\Lambda}} \ln(1 + \sqrt{2}) \).

This result caused great puzzlement, because first of all there is no evidence for time travel — we know of no visitors from the future, and second our classical notions of causality (which to be sure are already challenged by quantum mechanics) prejudice us against considering geometries with closed time-like curves, where effects precede causes. Here is Einstein’s reaction\(^3\)
“Kurt Gödel’s [time machine solution raises] the problem [that] disturbed me already at the time of the building up of the general theory of relativity, without my having succeeded in clarifying it. · · · It will be interesting to weigh whether these [solutions] are not to be excluded on physical grounds.”

A more recent comment is by Hawking.\footnote{4}

“Gödel presented a · · · solution [which] was the first to be discovered that had the curious property that in it it was possible to travel into the past. This leads to paradoxes such as ‘What happens if you go back and kill your father when he was a baby?’ It is generally agreed that this cannot happen in a solution that represents our universe, but Gödel was the first to show that it was not forbidden by the Einstein equations. His solution generated a lot of discussion of the relation between general relativity and the concept of causality.”

Upon further reflection it comes as no surprise that Einstein’s general relativity allows closed time-like curves: in Einstein’s theory geometry is determined from matter by Eq. (4) whose schematic form is

\[
\begin{align*}
\text{space-time} & \quad \Longleftrightarrow \quad \{ \text{distribution of matter} \} \\
\text{geometry} & \quad \text{of} \quad \text{matter} \quad \cdot
\end{align*}
\]

But Einstein’s equations may be read in the opposite direction: pick an interesting geometry — no matter how strange — and in particular one containing closed time-like curves, then determine the (unphysical?) matter distribution that engenders it, thereby “finding” a time machine solution, \textit{i.e.}, redirecting the arrow above:

\[
\begin{align*}
\text{time machine} & \quad \Longrightarrow \quad \{ \text{peculiar matter} \} \\
\text{geometry} & \quad \text{of} \quad \text{matter} \quad \text{“unphysical?”} \quad \cdot
\end{align*}
\]
Our “defense” against these solutions and against the paradoxes they entail is to assert that the exotic matter distributions supporting time machines are unphysical. Thus Gödel’s universe requires a constant and uniform energy density, which clearly is unphysical. The time machines studied by Thorne and his colleagues at CalTech make use of wormholes, another exotic and presumably unphysical form of matter in which a narrow channel — the wormhole — connects distant regions of space-time and is threaded by a closed time-like curve.

The reason for current interest in time travel ideas derives from the recent realization that infinitely long and arbitrarily thin cosmic strings can support closed time-like curves. Cosmic strings are hypothetical, but entirely physical structures that may have survived from a cosmic phase transition, and that may even be responsible for the present-day large scale structure in the universe. Only completely conventional physical ideas are relied upon in cosmic string speculations; cosmologists make use of cosmic strings in model building, and astrophysicists occasionally report sightings, although thus far evidence is inconclusive. Nevertheless, although infinite length and arbitrary thinness are idealizations, one would not view such cosmic strings as unphysical. Therefore, if they indeed give rise to closed time-like curves, there is something new that physicists must confront.

To recognize string-generated closed time-like curves, we need to record the space-time that is produced by a cosmic string. We use coordinates in which an infinitely long and thin cosmic string lies along the $z$-axis through the origin. Its mass per unit length is $m$ and we also endow the string with intrinsic spin $J$ per unit length. The cross-sectional area is assumed to be vanishingly small, so the mass density is proportional to a two-dimensional spatial $\delta$- function, while the spin density is more singular, being produced by a momentum density proportional to derivatives of $\delta$. 
Solving Einstein’s equation (4) (with vanishing cosmological constant $\Lambda$) for a stationary string, carrying the above mass and spin distributions, produces a metric described by the line element

$$ds^2 = g_{\mu\nu}dx^\mu dx^\nu$$

$$= \left( c \, dt + \frac{4}{c^3} GJ d\theta \right)^2 - dr^2 - \left( 1 - \frac{4}{c^2} Gm \right)^2 r^2 d\theta^2 - dz^2 .$$

Indeed this two-parameter $(m, J)$ metric tensor is the general time-independent solution to (4) outside any matter distribution lying in a bounded region on the plane and having cylindrical symmetry. As in the Gödel line element (6), there is no structure in the $z$-direction, while in the perpendicular $(r, \theta)$ plane, with $\theta = 0$ and $2\pi$ identified, the non-trivial geometry supports closed time-like curves when $J$ is non-vanishing: take constant $t$ and $r$ and describe a circle ($\theta$ ranging from 0 to $2\pi$) with sufficiently small radius

$$r < \frac{4GJ}{c^3 - 4Gmc} .$$

By changing coordinates one can hide the presence of the string and make the line element appear locally Minkowskian: with the definitions

$$\tau = t + \frac{4}{c^3} GJ \theta , \quad \varphi = \left( 1 - \frac{4}{c^2} Gm \right) \theta ,$$

(7) becomes

$$ds^2 = (c \, d\tau)^2 - dr^2 - r^2 d\varphi^2 - dz^2 .$$

Now, however, the ranges of these flat-looking coordinates are unconventional: that of $\varphi$ is diminished to $\left( 1 - 4Gm/c^2 \right) 2\pi$, while the new time variable $\tau$, rather than flowing in a smooth and linear fashion, jumps by $8\pi GJ/c^4$ whenever the string is circumnavigated, owing to the identification of $\theta = 0$ and $2\pi$. The defect in the angular range turns the spatial plane
into the surface of a cone with deficit angle determined by $m$, while the time-helical structure, where pitch is proportional to intrinsic spin $J$, is responsible for the closed time-like curves (8).

There is another entirely equivalent framework for understanding the geometry produced by our cosmic string: In the idealization of infinitely long and thin strings, there is no structure along the $z$-axis, hence we can suppress that direction altogether. Then the theory becomes (2 + 1)-dimensional “planar” gravity, governed by the same Einstein equations (4) but in this reduced space. Cosmic string sources now become “point-particles” at the locations where the strings pierce the $z = 0$ plane.

Einstein’s gravity theory in (2 + 1) dimensions is a much-studied model, both for pedagogical reasons — one hopes that the dimensional reduction effects sufficient simplification to permit thorough analysis, while still retaining useful content to inform the physical (3 + 1)-dimensional problem — and also for practical calculation — as explained above, idealized cosmic strings are effectively described by (2 + 1)-dimensional gravity. [Analogously, motion of charged particles in magnetic fields that are constant along the $z$-axis is effectively governed by planar dynamics, as in Landau theory, quantum Hall effect and perhaps high-$T_c$ superconductivity.] We shall use the name cosmon to refer uniformly to infinite cosmic strings in space and to particles on the plane.

A simplifying feature of the lower-dimensional interpretation (always without cosmological constant, although it can be included) is that space-time is locally flat whenever sources $(T_{\mu\nu})$ are absent. The reason for this is that the Riemann curvature tensor, which completely determines geometry, is linearly related to the Einstein tensor $G_{\mu\nu}$, in (and only in) three-dimensions (they are each other’s double duals). Thus at any point where there are no sources, $G_{\mu\nu}$ vanishes by (4), and consequently the three-dimensional Riemann tensor also
vanishes: space-time is Minkowskian. Moreover, when sources are point-cosmons, the space-time is flat everywhere, except at these points, and this is why it is possible to transform the line element (7) to the flat one (10) almost everywhere; only the unconventional range and jump properties of the coordinates remind us that there are sources somewhere. Indeed these coordinate defects contain all the physical information about sources that is accessible in regions outside them.\textsuperscript{10}

Further examination of the geometry generated by cosmons shows that the mass of any individual one (= mass per unit length of a cosmic string) must not exceed $c^2/4G$, so that the deficit angle not exceed $2\pi$ ($m = c^2/4G$ corresponds to the cone closing into a cylinder). The total mass of an assembly of static cosmons, each with acceptable mass, may exceed $c^2/4G$, but then it must precisely equal $c^2/2G$ for space to be non-singular, and space is necessarily closed.\textsuperscript{10} [This is a consequence of the fact that the total mass of static sources in (2 + 1)-dimensional gravity is proportional to the Euler number of the 2-space.] Note in particular: sources, which individually, i.e. locally, are acceptable ($m < c^2/4G$) can give rise to configurations that are globally unacceptable ($c^2/4G < \sum m \neq c^2/2G$); this lesson will be essential in what follows.

In Figs. 1–3 we depict the space and the space-time of a single cosmon using flat coordinates with unconventional ranges. In Fig. 1 the cosmon is spinless; the three-dimensional space is structureless along the $z$-direction (direction of the string), in the perpendicular plane an angular region, of magnitude to $8\pi Gm/c^2$, is excised and the edges identified — the space is a cone. In Fig. 2, the $z$-axis is suppressed, the cosmon is a point-particle and the time axis as well as the world line of the stationary cosmon are indicated; since there is no spin, time flows smoothly from $-\infty$ to $\infty$. Figure 3 depicts the previous situation, but now the cosmon
Fig. 1: Space of a spinless cosmon.

Fig. 2: Space-time of a spinless cosmon.
also carries spin and time acquires a helical structure, as explained above. For more cosmons, the space is an assembly of cones, joined smoothly, as is possible provided each of their deficit angles and the sum is suitably bounded by the limits given above.

Another useful way of describing the space-time created by a cosmon is to notice that after the cosmon is circumnavigated, the flat space-time description requires an additional rotation in space and a jump in time, which effect the coordinate identifications that encode the presence of a massive, spinning cosmon. These identifications may be represented in terms of a Poincaré transformation (spatial rotation or Lorentz boost and space-time translation) on the three-dimensional space-time (since the invariance group of flat space-time is the Poincaré group),

\[ x' = M_{\mu \nu} x^\nu + a^\mu. \]  

Here $\mathbf{x}$ and $x$ are the identified three-vectors, $M_{\mu \nu}$ is a Lorentz transformation, specialized for the single static source to a spatial rotation through the angle $8\pi Gm/c^2$, and $a^\mu$ is the
jump in coordinates, specialized to a jump of $8\pi G J/c^4$ in the time component. A general cosmon configuration, not necessarily a single cosmon at rest but also a moving one or indeed an assembly of cosmons, is also described by such a Poincaré identification, with $x$ and $\bar{x}$ referring to points exterior to the assembly. For example, a spinless cosmon moving with velocity $\mathbf{v}$ through the point $x_0$ leads to the identification

$$\bar{x} = B^{-1}_\mathbf{v} M_m B_\mathbf{v} (x - x_0) + x_0,$$

where $B_\mathbf{v}$ is the Lorentz boost that brings the cosmon to rest and $M_m$ is the mass-determined rotation. Equation (12) is merely the statement that the coordinate $B_\mathbf{v} (x - x_0)$ is identified as in (11), with $a$ vanishing when there is no spin.

As already remarked, owing to the time-helical structure that is required by the Einstein equation, a spinning ($J \neq 0$) cosmon gives rise to closed time-like curves. Now intrinsic spin attached to a cosmic string may still be deemed unphysical (like any classical spin involving derivatives of $\delta$-functions), so we need not worry about closed time-like curves supported by locally spinning cosmons. However, one may ask whether two or more spinless cosmons, moving relative to each other and thus also carrying orbital angular momentum, can still support closed time-like curves. This question was answered affirmatively by Gott at Princeton University.\textsuperscript{11} He found that two spinless cosmons, each moving faster than a critical but subluminal ($v < c$) velocity, do indeed support closed time-like curves.

In fact the question of possible closed time-like curves in a many-spinless cosmon universe had been posed even earlier and answered negatively in the original investigation that first exhibited closed time-like curves in the presence of spinning cosmons.\textsuperscript{10} Briefly, the reasoning there relied on the expectation that the perfectly good Cauchy development of two freely
moving cosmons from $t = -\infty$ should not be spontaneously destroyed at some finite time, when they approach each other, and then be reinstated when they diverge as $t \to +\infty$. Resolution of the apparent contradiction between this expectation and Gott’s result is the most recent development in the time machine story.

Fig. 4: Two-cosmon time machine.

The disposition of cosmons that support closed time-like curves is portrayed in Fig. 4. Two spinless cosmons, each with equal (for simplicity) masses $m$ are located at the center of the figure, and the excised wedges point up and down. Consider now the light path from $A$ to $B$. It is credible that a path surrounding the cosmons and taking advantage of the “missing” space can be shorter than the direct, straight line $AB$. Then cosmons moving in opposite directions with speed $v$, in a manner already described above in connection with the tachyonic time machine, create closed time-like curves, provided $v$ exceeds a critical value $v_{\text{critical}}$, which
nevertheless is less than \( c \), so that neither cosmon is tachyonic. Gott shows that

\[
\frac{v}{c} > \frac{v_{\text{critical}}}{c} = \cos \left( \frac{4\pi G m}{c^2} \right) < 1.
\] (13)

This result, in what purports to be a physically acceptable situation, produced intense interest (not only in the physics community, as is seen from the appended list of semi-popular accounts\(^\text{12}\)) since for the first time time-travel appeared to be consistent with unexceptional physical principles, awaiting only development of engineering possibilities. Fantastic applications obviously come to mind and were discussed, while at the same time some pitfalls inherent in commercial development were noted in a nationally syndicated newspaper feature, published the same day as the analysis in Ref. [13] and reproduced in Fig. 5.

\textbf{Fig. 5:} Pitfall in commercial development of time machines.

But there is a catch: since one needs a \textit{pair} of moving strings, one may ask what is their combined energy and momentum. Now energy and momentum are conserved quantities
that arise from invariance under time- and space-translation. However, when space-time is
as globally complicated — “sewn-together” cones — as it becomes in the presence of strings,
the addition rules for combining energy and momentum become non-trivial and non-linear.
In particular, even though each of the two cosmons is moving sufficiently slowly to be non-
tachyonic, as soon as the velocity of each exceeds the critical value (13) needed to support a
time machine, their center-of-mass becomes tachyonic.

This is seen as follows. We have already remarked that a general cosmon distribution
is characterized by an element of the Poincaré group, as in (11). Concentrating now on the
(2 + 1)-dimensional Lorentz transformation matrix \( M \), we recall that it may be presented as
the exponential of three generator matrices \( \Sigma_\mu \),

\[
M = \exp \left\{ \frac{8\pi}{c^4} GP^\mu \Sigma_\mu \right\} .
\]  

(14)

(This is analogous to presenting a three-dimensional spatial rotation matrix as the exponential
of angular momentum matrices.) The three-vector \( P^\mu \) defines the energy-momentum of a
cosmon assembly. For example, when a single cosmon is at rest, \( M \) is a pure rotation, which

corresponds to

\[
P = 0 , \quad P^0 = mc , \quad \Sigma_0 \equiv \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & -1 & 0 \end{pmatrix} ,
\]

(15)

where the rows and columns are labelled by \((t, x, y)\). For the cosmon assembly to be non-
tachyonic, \( i.e. \) physically constructible, we must require

\[
P^\mu P_\mu \equiv m^2 c^4 > 0 .
\]

(16)

Now Gott’s time machine makes use of two equal-mass \( (m) \) cosmons, moving in op-
posite directions with speed \( v \). Suitably extending (12), we see that the resulting Lorentz
identification that describes it is

\[
M_{\text{total}} = B_v^{-1} M_m B_v B_{-v}^{-1} M_m B_{-v} = B_v^{-1} M_m B_v^2 M_m B_v^{-1}
\]

(17a)
with total energy and momentum defined by

\begin{equation}
M_{\text{total}} = \exp \left\{ \frac{8\pi G}{c^3} \sum_{\mu} p_{\mu} \Sigma_\mu \right\} .
\end{equation}

A straightforward calculation yields the formula for the total mass, \( m_{\text{total}} \),

\begin{equation}
\sin \left( \frac{2\pi Gm_{\text{total}}}{c^2} \right) = \frac{1}{\sqrt{1 - v^2/c^2}} \sin \left( \frac{4\pi Gm}{c^2} \right) ,
\end{equation}

which for small \( Gm/c^2 \) recovers the special relativistic result

\begin{equation}
m_{\text{total}} = \frac{2m}{\sqrt{1 - v^2/c^2}} .
\end{equation}

For real \( m_{\text{total}} \), the left side of (18) is less than unity, so we must have

\begin{equation}
\frac{v}{c} < \cos \left( \frac{4\pi Gm}{c^2} \right) .
\end{equation}

This is precisely opposite to Gott’s criterion (13) for the presence of closed time-like curves! Thus we learn that when (13) is satisfied, each cosmon travels with a velocity below that of light, but the total system possesses a tachyonic energy. It follows that only tachyons can give rise to Gott’s sufficiently rapidly moving pair of cosmons.\(^{13}\) In the absence of such tachyonic sources, energy is unavailable to produce the rapidly moving cosmons. Another perspective is given by formula (18). We see that when \( v = 0 \), \( m_{\text{total}} = 2m \). As \( v \) increases from zero, \( 4Gm_{\text{total}}/c^2 \) increases and approaches unity when \( v \) reaches \( v_{\text{critical}} \), corresponding to a deficit angle \( 2\pi \) in the surrounding space. As the critical velocity is exceeded, the deficit angle surpasses \( 2\pi \), and the system becomes unphysical.

Thus the two opposite claims\(^{10,11}\) about time machines are reconciled:\(^{13}\) just like the special relativistic time machines, which could only be constructed if tachyons exist — but
they do not — so also the cosmic string time machines require tachyonic center-of-mass velocities, and cannot be produced in the absence of tachyons, that is in our world.

Even without mentioning the tachyonic center-of-mass, we can give an equivalent geometrical description of the unphysical nature of Gott’s system: Its exterior geometry necessarily contains closed time-like curves at spatial infinity. Existence of a time machine during that portion of the system’s evolution when the cosmons are sufficiently close requires that closed time-like curves be also present at the boundary of space for all time. Space-times with this property have long been known in classical gravity, where they are produced by unusual identifications of space and time. An example is Misner space, and one can show that the coordinate identifications by means of the Lorentz transformation (17a) when \( v > v_{\text{critical}} \) give rise to such a space. Moreover, the effective energy-momentum tensor responsible for the exterior Gott metric is tachyonic, with energy density replaced by spatial stress.

The above analysis was confirmed in further investigations on the obstacles to constructing Gott’s time machine. Carroll, Farhi and Guth showed that it cannot be manufactured as a subsystem of a normal time-like matter assembly in an open universe. In a universe that is spatially closed, owing to a mass distribution with critical magnitude for closing space (\( \sum m = c^2/2G \)), a pair of cosmons traveling with velocity exceeding \( v_{\text{critical}} \) can be produced; nevertheless a time machine cannot be built here either because the universe dramatically self-destructs just before the cosmons get close enough to engender a closed time-like curve — there is not enough time to travel into the past.

While we are reassured, at least as concerns this most recent string-inspired attempt, that causality will not be violated by time travel, further questions remain and motivate further study. One wonders whether one can a priori and once-and-for-all characterize those mass
distributions that *mathematically* support time travel and therefore should perhaps be deemed unphysical. Indeed, Hawking\(^7\) has encapsulated these ideas in his “Chronology Protection Conjecture” that “the laws of physics do not allow the appearance of closed time-like curves.” But establishing this conjecture requires knowing why the laws of physics prohibit certain forms of matter. That this question is difficult to answer is well-illustrated by the cosmon time machine: each individual cosmon is a perfectly acceptable subluminal and physical form of matter — but fitting exactly two rapidly moving cosmons into a non-singular universe is impossible. Evidently global, rather than local considerations come into the definition of “physical.”

However, it should be remarked that as paradoxical as time-travel may appear (“killing one’s infant father,” *etc.*) we can be assured that mathematical analysis, correctly carried out, will provide a consistent physical picture — which of course need not correspond to reality if conditions for time-travel cannot be physically met. This had been already established for tachyonic effects within special relativity.\(^1\) Recent investigations of physical processes in the presence of closed time-like curves, like those generated by spinning strings\(^{17}\) or by other mechanisms\(^{18}\) confirm that unexpected but not self-contradictory results are found, *e.g.*, perturbative violation of unitarity in interacting quantum field theories.

We study paradoxical aspects of physics — like time machines — to illuminate our understanding of fundamental issues. For example, the very concept of time remains mysterious in physical theory: What gives time its observed direction? Indeed what gives it a definition, in the first place, since general relativity requires space-time to be determined by matter? It should be apparent from the foregoing that here there is still much to learn.
REFERENCES

1. G. Feinberg, *Scientific American* **222**(2), 68 (1970).

2. K. Gödel, *Rev. Mod. Phys.* **21**, 447 (1949).

3. A. Einstein, in *Albert Einstein: Philosopher–Scientist*, P. Schilpp, ed. (Tudor, New York, 1957).

4. S. Hawking, in *K. Gödel, Collected Works*, Vol. II, S. Feferman, ed. (Oxford University Press, New York, 1990).

5. M. Morris and K. Thorne, *Am. J. Phys.* **56**, 395 (1988).

6. Specific, technical objections can also be raised to wormhole-supported time machines. The narrowness of the channel would prevent anything of significant size from squeezing through its opening. Moreover, Hawking\(^7\) has argued that quantum fluctuations around a wormhole generate infinite energy and stress, thereby closing it, but the subject is still being investigated.

7. S. Hawking, in *The Sixth Marcel Grossmann Meeting on General Relativity*, H. Sato, ed. (World Scientific, Singapore, 1992).

8. For a review, see A. Vilenkin, *Physics Reports* **121**, 263 (1985).

9. For reviews, see R. Jackiw, in *Relativity and Gravitation: Classical and Quantum*, J. D’Olivio, E. Nahmad-Achar, M. Rosenbaum, M. Ryan, L. Urrutia and F. Zertuche, eds. (World Scientific, Singapore, 1991), and in *The Sixth Marcel Grossmann Meeting on General Relativity*, H. Sato, ed. (World Scientific, Singapore, 1992).

10. S. Deser, R. Jackiw and G. ’t Hooft, *Ann. Phys.* (NY) **152**, 220 (1984).
11. J. Gott, *Phys. Rev. Lett.* **66**, 1126 (1991).

12. *Time*, 13 May 1991; *Science News*, 28 March 1992; *New Scientist*, 28 March 1992; *Discover*, April 1992; *Science*, 10 April 1992. Only the *New Scientist* and *Science* articles convey accurate information.

13. S. Deser, R. Jackiw and G. ’t Hooft, *Phys. Rev. Lett.* **68**, 267 (1992), and the last paper in Ref. [9].

14. C. W. Misner, in *Relativity Theory and Astrophysics I: Relativity and Cosmology*, J. Ehlers, ed. (American Mathematical Society, Providence, 1967).

15. S. Carroll, E. Farhi and A. Guth, *Phys. Rev. Lett.* **68**, 263, (E) 3368 (1992); see also D. Kabat, MIT preprint CTP#2034 (November 1991). The claim in the first of these papers that time machines can be constructed in closed universes was shown to be false in Ref. [16].

16. G. ’t Hooft, *Class. Quant. Grav.* **9**, 1335 (1992).

17. P. Gerbert and R. Jackiw, *Comm. Math. Phys.* **124**, 495 (1988).

18. J. Friedman, M. Morris, I. Novikov, F. Echeverria, G. Klinkhammer, K. Thorne and U. Yurtsever, *Phys. Rev. D* **42**, 1915 (1990); D. Deutsch, *Phys. Rev. D* **44**, 3197 (1991); J. Friedman, N. Papastamatiou and J. Simon, University of Wisconsin–Milwaukee preprint WISC-MIL-91-TH-17 (1991); J. Hartle, UC Santa Barbara preprint, UCSBTH-92-04 (1992); D. Boulware, University of Washington preprint UW/PT-92-04 (1992).