Thermal model for Pb+Pb collisions at $\sqrt{s_{NN}} = 2.76$ TeV with explicit treatment of hadronic ground states

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Abstract. Various explanations of the anomalous proton to pion ratio at the LHC are discussed. The special emphasis is set on the Cracow thermal model with single freeze-out. This model allows to get a good agreement for both the mean hadron multiplicities and the spectra. Moreover, the values of the fit parameters indicate the possibility of pion Bose condensation in the most central collisions at the LHC. Therefore, a modification of the thermal framework is proposed that explicitly allows for the condensation in the ground state. The generalised model makes a link between equilibrium and non-equilibrium thermal models. It also suggests that the pion condensation may be formed in the central collisions.

1 Introduction

Statistical models are used as the standard tools for the analysis of heavy-ion and elementary ($e^+e^−$, $p\bar{p}$, etc.) collisions. These models give a very good description of mean multiplicities of many hadron species using only few parameters, for example, see [1–6]. Therefore, it is quite surprising that the new data from the LHC do not agree with the thermal model prediction for proton abundances [9]. Among possible explanations of this problem there are: hadronic re-scattering effects in the final stage [10], incomplete list of hadrons [11,12], flavor hierarchy at freeze-out [13], and the non-equilibrium hadronization [14,15], see also [16]. Herein, we will focus on the latter explanation, because, as we have shown before in [7,8], it offers a plausible description of the transverse momentum spectra of the produced hadrons.

Surprisingly, hydrodynamic models have problems to reproduce the pion spectra at the LHC as well. The low-$p_T$ pion spectra show enhancement by about 25%–50% with respect to the predictions of different hydrodynamic models, see the compilation shown by ALICE in Refs. [20,21]. One can notice that the pions and protons are anti-correlated. If a model explains protons, it typically underestimates pions. On the other hand, if a model explains pions, then it overestimates protons. More recent papers also illustrate this issue [22,23]. Only in Ref. [24] the pions are described in the satisfactory way, however, no results for the protons are given in this work.

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2 Cracow single-freeze out model

The Cracow single-freeze out model \cite{17,19} allows to solve the problem with the proton/pion ratio and the problem with the pion spectrum \cite{17,18}. The model includes all well established resonances from the PDG. The masses of resonances and their decays are implemented in the THERMINATOR Monte-Carlo code \cite{25,26}. The primordial distribution in the local rest frame has the form:

\[
\frac{dN}{d^3 p} = g_i \int \frac{d^3 p}{(2\pi)^3} \gamma_i^{-1} \exp \left( \frac{1}{\sqrt{m_i^2 + p^2/T}} \right) \pm 1, \tag{1}
\]

where \(g_i = 2s_i + 1\) is the degeneracy connected with the spin \(s_i\) of the \(i\)th particle, \(p\) is the particle momentum, \(m_i\) – mass, and \(T\) is the system temperature. The factor \(\gamma_i\) is expressed by the numbers of light quarks, \(N_q^i\), antiquarks, \(N_{\bar{q}}^i\), strange quarks, \(N_s^i\), strange antiquarks \(N_{\bar{s}}^i\); baryon and strange charges of the particle – \(B_i, S_i\), and the corresponding chemical potentials, \(\mu_B\) and \(\mu_S\):

\[
\gamma_i = \left( N_q^i + N_{\bar{q}}^i \right) \left( N_s^i + N_{\bar{s}}^i \right) \exp \left( \frac{\mu_B B_i + \mu_S S_i}{T} \right). \tag{2}
\]

At the LHC the chemical potentials \(\mu_B\) and \(\mu_S\) are so small that one can set them zero. However, the introduction of the parameters \(\gamma_q\) and \(\gamma_s\) is equivalent to the appearance of the non-equilibrium chemical potentials \(\mu/T = \ln \gamma\):

\[
\gamma_i \approx \left( N_q^i + N_{\bar{q}}^i \right) \left( N_s^i + N_{\bar{s}}^i \right) \exp \left( \frac{\mu_q N_q^i + \mu_{\bar{q}} N_{\bar{q}}^i}{T} + \mu_s N_s^i + \mu_{\bar{s}} N_{\bar{s}}^i \right). \tag{3}
\]

They are connected with the conservation of the sum of the number of quarks and antiquarks during the hadronization process. Similarly, the usual baryon and strange chemical potentials \(\mu_B\) and \(\mu_S\) are connected with the conservation of the difference of the quark and antiquark numbers. Such an effective quark number conservation may appear due to rapid cooling and hadronization of the fireball. Then the system has no time to equilibrate and the numbers of quarks and antiquarks are larger than the equilibrium values.

We note that, the non-equilibrium model may account for hypothetical heavy particles that decay into multi-pion states \cite{11,12}. Equation (3) may describe the equilibrium \(p + \bar{p}\) annihilation into 3 pions. One can also notice that the \(\gamma_i\) factor is different for each particle. Some particles are enhanced, while the other are suppressed, compared to the equilibrium case. Therefore, Eq. (3) resembles the modification factors that are obtained in the hadron gas with rescattering effects \cite{10}. Equation (3) obviously separates the strange and non-strange particles. Therefore, it is similar to the model with two separate freeze-outs for strange and non-strange particles proposed in Ref. \cite{13}. A QCD mechanism of gluon condensation may also lead to a similar effect: the creation of low momentum gluons which transform into pions in the condensate \cite{27,28,29}.

We consider two physics scenarios: the equilibrium case (EQ), where \(\gamma_q = \gamma_s = 1\), and the full non-equilibrium case (NEQ) with \(\gamma_q\) and \(\gamma_s\) treated as free parameters. The spectra are calculated from the Cooper-Frye formula with the special freeze-out hypersurface:

\[
\frac{dN}{dy d^2 p_T} = \int d\Sigma \rho d f(p \cdot u), \quad \sqrt{r^2} = x^2 + y^2 + z^2, \quad x^2 + y^2 \leq r_{\max}^2, \tag{4}
\]

assuming the Hubble-like flow \(u^\mu = x^\mu/\tau_f\).

The system volume, temperature, \(\gamma_q\), and \(\gamma_s\), are taken from the papers \cite{14,15} and approximated by the polynomials, for details see \cite{18}. The combination of the freeze-out time, \(\tau_f\), and the maximum
radius squared, $r_{\text{max}}^2$, gives the system volume per unit rapidity, $V = \pi \tau_f r_{\text{max}}^2$. Therefore, the ratio $r_{\text{max}}^2/\tau_f$ is the only one additional parameter in the model that determines the shape of the spectra.

The Cracow model allows to fit the spectra of pions and kaons with very good accuracy only in NEQ model [7]. Surprisingly, the proton spectrum comes out right without extra fitting as a bonus. The same fit gives the very good agreement for the spectra of $K^0_S$, $K^*(892)^0$, $\phi(1020)$ mesons and a satisfactory agreement for the heavy strange particles from the most central to very peripheral collisions [8]. As we already mentioned in Introduction, the simultaneous fit of the pion and proton spectra is very difficult, and the difference between EQ and NEQ models drastically increases at low $p_T$, see Fig. 1. However, even more surprising fact is that the long living $\phi(1020)$ and the very short living $K^*(892)^0$ come out right from the fit done for pions and kaons only [7, 8]. It is a very strong argument either for the absence of the long rescattering phase after the freeze-out or for the effective parametrization of the re-scattering phase by Eq. (3).

### 3 Pion condensation

There is an upper bound on $\gamma_q$ and $\gamma_s$ because of Bose-Einstein condensation, when the singularities appear in the Bose-Einstein distributions of primordial pions and kaons [1]. For pions, the value of $\gamma_s$ is irrelevant, and we find

$$\gamma_q^{\text{critic}} = \exp\left(\frac{m_{\pi^0}^2}{2T}\right).$$

The fits to the ratios of hadron abundances yield $\gamma_q$ which is very close to the critical. It is equivalent to the pion chemical potential

$$\mu_\pi = 2T \ln \gamma_q \approx 134 \text{ MeV},$$

which is very close to the $\pi^0$ mass, $m_{\pi^0} \approx 134.98$ MeV. It may lead for the condensation of the substantial part of $\pi^0$ mesons.
Figure 2. The non-equilibrium parameters from the paper [8], NEQ, are compared to the new fit in SHARE [31] using the equilibrium model, EQ, and the non-equilibrium model with the possibility of Bose condensation, BEC. The left panel shows the system volume, while the right panel shows the system temperature.

If chemical potential approaches the mass of a particle, $\mu \to m$, the zero momentum level, $p_0 = 0$, and other low lying quantum states become important. Therefore, one should consider the summation over the low momentum states explicitly. One can show that in the thermodynamic limit, $V \to \infty$, one may keep only the $p_0 = 0$ term and start the integration from zero [30]:

$$N = \frac{g}{\exp \left( \frac{m-\mu}{T} \right) - 1} + V \int_0^\infty \frac{d^3p}{(2\pi)^3} \frac{g}{\exp \left( \frac{\sqrt{p^2 + m^2} - \mu}{T} \right) - 1} = N_{\text{cond}} + N_{\text{norm}}$$

where $N_{\text{cond}}$ is the number of particles in the Bose condensate and $N_{\text{norm}}$ is the number of particles in normal states. We have added the condensation term from [5] to the latest version of SHARE [31], because it is the model that was used to obtain our input parameters, $V$, $T$, $\gamma_q$, $\gamma_s$. The obtained non-equilibrium model with the possibility of Bose condensation we call BEC.

The $\pi^0$ mesons will condense first, because they are the lightest particles. The $\pi^0$ multiplicity is not measured in Pb+Pb collisions at the LHC yet. Therefore we add the estimate for the number of $\pi^0$ mesons as $\pi^0 = (\pi^+ + \pi^-)/2$ and fit it together with all other available particle multiplicities. The results are shown in Figs. 2 and 3. We checked that the measured $\pi^0$ spectrum [32] agrees with our estimate. The data exist only for the range $p_T \gtrsim 700$ MeV. It gives just about $1/3$ of the total expected $\pi^0$ multiplicity. Therefore the measurement of the low $p_T$ spectrum of neutral pions is crucially important to judge about the Bose condensation.

One can see that the BEC and NEQ volumes coincide within the errors, while the EQ volume is substantially larger. This is in agreement with the calculations of other authors [14,16] in the EQ and NEQ models. The temperature in EQ is almost constant and is between $150\rightarrow 160$ MeV, as is expected for the equilibrium. However the BEC model demonstrates an interesting centrality dependance. In most central collisions it is close to the temperature in NEQ, while at very peripheral collisions it approaches the EQ temperature. Such a behavior is possible if pion condensation is stronger at small centrality. The behavior of $\gamma_q$ and $\gamma_s$ parameters indeed supports this assumption, see Fig. 3.
At small centralities the $\gamma_q$ and $\gamma_s$ values in BEC are close to those in NEQ, while at high centrality both $\gamma_q$ and $\gamma_s$ approach unity. The $\gamma$’s in BEC are also always smaller than in NEQ. It could help to describe the heavy strange particles, as noted in [8]. However, the error bars are too big to start the cumbersome analysis of the spectra with the new BEC parameters. At first, one has to find a way to make more precise calculations [33].

4 Conclusions

The non-equilibrium thermal model combined with the single freeze-out scenario explains very well the spectra of light particles. It eliminates the proton anomaly and explains the low-$p_T$ enhancement of pions. This enhancement may be interpreted as a signature of the onset of pion condensation in heavy-ion collisions at the LHC. Since the difference between equilibrium and non-equilibrium models strongly increase at low $p_T$, it would be interesting to see the measurements of the charged pion spectrum at smaller values of $p_T$ than those available at the moment. The same is even more important for the $\pi^0$ meson spectrum, because neutral pions condense first.

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