Measurement of the recoil polarization in the $p(\vec{e}, e'\vec{p})\pi^0$ reaction at the $\Delta(1232)$ resonance

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The recoil proton polarization has been measured in the $p(\vec{e}, e'\vec{p})\pi^0$ reaction in parallel kinematics around $W=1232\text{MeV}$, $Q^2 = 0.121\text{(GeV/c)}^2$ and $\epsilon = 0.718$ using the polarized c.w. electron beam of the Mainz Microtron. Due to the spin precession in a magnetic spectrometer, all three proton polarization components $P_x/P_e = (-11.4 \pm 1.3 \pm 1.4)\%$, $P_y = (-43.1 \pm 1.3 \pm 2.2)\%$, and $P_z/P_e = (56.2 \pm 1.5 \pm 2.6)\%$ could be measured simultaneously. The Coulomb quadrupole to magnetic dipole ratio CMR = $(-6.4 \pm 0.7_{\text{stat}} \pm 0.8_{\text{syst}})\%$ was determined from $P_x$ in the framework of the Mainz Unitary Isobar Model. The consistency among the reduced polarizations and the extraction of the ratio of longitudinal to transverse response is discussed.

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The interaction between the three constituent quarks in a nucleon is mediated by gluons and, at long range, pions. Both exchange bosons produce a tensor force and, consequently, the nucleon wave function might have a D-state admixture \( \langle \rangle \), as is the case for the deuteron. Such an admixture results in an intrinsic deformation of the nucleon \( \langle \rangle \). However, the spectroscopic quadrupole moment of the nucleon must vanish due to its spin \( 1/2 \). In the case of the first excited state \( \Delta(1232) \) with spin \( 3/2 \) the short life-time prohibits a direct measurement of the quadrupole moment. On the other hand, in the electromagnetic \( N \rightarrow \Delta(1232) \) excitation a D-admixture will be visible as small quadrupole admixtures to the dominating \( M_1 \) transition. In the decay of the \( \Delta(1232) \) resonance into the \( N \pi \) channel, this quadrupole mixing is associated with non-zero electric quadrupole to magnetic dipole (EMR) and Coulomb quadrupole to magnetic dipole ratios (CMR). These can be defined as \( \text{EMR} = \Im m\{E_{1\pm}^\pi/M_{1\pm}^\pi\} \) and \( \text{CMR} = \Im m\{S_{1\pm}^\pi/M_{1\pm}^\pi\} \), where the pion multipoles, \( A_{1\pm}^f \), are characterized through their magnetic, electric or longitudinal (scalar) nature, \( A = M, E, S, \) the isospin, \( I \), and the pion-nucleon relative angular momentum, \( l_\pi \), whose coupling with the nucleon spin is indicated by \( \pm \).

Recently, the EMR was determined at squared 4-momentum transfer \( Q^2 \equiv 0 \) with linearly polarized real photons in the reaction \( p(\gamma, p)^{n\pi} \). The result is \( \text{EMR} = (2.5 \pm 0.2 \pm 0.2)\% \), in agreement with the combined partial wave analysis from \( p(\gamma, p)^{\pi^0, \eta} \) and \( p(\gamma, \pi^+)\eta \) which allows an isospin decomposition of the multipoles \( \langle \rangle \).

The determination of the longitudinal quadrupole mixing requires pion electroproduction experiments. In the one-photon exchange approximation the \( N(e, e'\pi)N \) cross section can be split into photon flux and the virtual photon cross section. Without target or recoil polarization, the latter is given by

\[
\frac{d\sigma}{dt} = \lambda \cdot [R_T + \epsilon_L R_L + \sqrt{2\epsilon_L(1+\epsilon)} R_{LT} \cos \Phi + \epsilon R_{TT} \cos 2\Phi + P_x \sqrt{2\epsilon_L(1+\epsilon)} R_{LT} \sin \Phi].
\]

The structure functions, \( R_\lambda \), parameterize the response of the hadronic system to the various polarization states of the photon field, which are described by the transverse and longitudinal polarization, \( \epsilon \) and \( \epsilon_L \), respectively, and by the longitudinal electron polarization, \( P_x \). The ratio \( \lambda = \Phi_\gamma/\Phi_\pi \) is determined by the pion cm momentum \( \Phi_\pi \) and \( k_\gamma = \frac{1}{2}(W - m_\pi^2/\sqrt{W}) \), which is the photon equivalent energy for the excitation of the target with mass \( m_\pi \) to the cm energy \( W \). \( \Phi \) denotes the tilt angle between electron scattering and reaction plane.

Due to the smallness of the \( E_{1\pm}^\pi \) and \( S_{1\pm}^\pi \) multipoles compared to the dominating \( M_{1\pm}^\pi \) amplitude, all experimental information so far is based on the extraction of \( \Re E_{1\pm}^\pi M_{1\pm}^\pi \) and \( \Re S_{1\pm}^\pi M_{1\pm}^\pi \) from the \( R_{TT} \) and \( R_{LT} \) structure functions \( \langle \rangle \). The interference terms are closely related to the EMR and CMR, respectively. Due to the unavoidable non delta-resonant contributions in the structure functions, it is important to measure the resonant amplitudes in several different combinations with the background amplitudes.

This possibility is offered by double polarization observables \( \langle \rangle \), which furthermore benefit from their insensitivity to experimental calibration uncertainties. The general cross section for the \( p(\gamma, e\pi^+)\eta^0 \) reaction with longitudinally polarized electrons and measurement of the recoil proton polarization is composed of 18 structure functions \( \langle \rangle \). In parallel kinematics, where the proton is detected along the direction of the momentum transfer, the three components of the proton polarization take the simple form \( \langle \rangle \) (here the notation of ref. \( \langle \rangle \) is used):

\[
\begin{align*}
\sigma_0 P_x &= \lambda \cdot P_x \cdot \sqrt{2\epsilon_L(1-\epsilon)} R_{LT}^\rho \\
\sigma_0 P_y &= \lambda \cdot \sqrt{2\epsilon_L(1+\epsilon)} R_{LT}^\rho \\
\sigma_0 P_z &= \lambda \cdot P_z \cdot \sqrt{1-\epsilon^2} R_{TT}^\rho.
\end{align*}
\]

The proton polarization independent cross section \( \sigma_0 \) is dominated by \( |M_{1\pm}^\pi|^2 \). The axes are defined by \( \hat{y} = \hat{k}_i \times \hat{k}_f/|\hat{k}_i \times \hat{k}_f| \), \( \hat{z} = \hat{q}/|\hat{q}| \) and \( \hat{x} = \hat{y} \times \hat{z} \), where \( \hat{k}_i \) and \( k_f \) are the momenta of incoming and scattered electron, respectively.

The structure functions of Eq. \( \langle \rangle \)-\( \langle \rangle \) can be decomposed into multipoles of the \( n^0\rho \) final state, which are related to the isospin multipoles by \( A_{1\pm} = A_{1\pm}^f + \frac{2}{3} A_{3\pm}^f \). Displaying only the leading resonance terms with the dominant \( M_{1\pm}^\pi \) amplitude, the polarization components simply read

\[
\begin{align*}
\sigma_0 P_x &= \lambda \cdot P_x \cdot \eta \cdot \Re \{4S_{1\pm}^\pi M_{1\pm}^\pi + n.l.o.\} \\
\sigma_0 P_y &= -\lambda \cdot c_+ \cdot \eta \cdot \Im m\{4S_{1\pm}^\pi M_{1\pm}^\pi + n.l.o.\} \\
\sigma_0 P_z &= \lambda \cdot P_z \cdot \sqrt{1-\epsilon^2}|M_{1\pm}^\pi|^2 + n.l.o.,
\end{align*}
\]

where \( \eta = \omega_{cm}/|q_{cm}| \) denotes the ratio of cm energy and momentum transfer and \( c_\pm = \sqrt{2\epsilon_L(1 \pm \epsilon)} \). As a consequence of the \( M_{1\pm}^\pi \) dominance, \( \Re M_{1\pm}^\pi \) vanishes very close to the resonance position \( W = 1232 \) MeV. In this case the non-leading order (n.l.o.) contributions to \( P_x \) are mainly determined by interferences of imaginary parts of background amplitudes with \( \Im m M_{1\pm} \). Since the background is predominantly composed of almost purely real amplitudes, only small n.l.o. contributions to \( P_x \) are expected. The situation is different for \( P_y \), because here all Born multipoles contribute, including higher partial waves. The large \( P_y \approx 40\% \) that has recently been measured at MIT-Bates \( \langle \rangle \) should, therefore, not a priori be interpreted as evidence for non-resonant (imaginary!) background amplitudes in \( P_x \).

Eqs. \( \langle \rangle \) and \( \langle \rangle \) show that, in the \( p\pi^0 \) channel, CMR = \( \Re \{S_{1\pm}^\pi M_{1\pm}^\pi \}/|M_{1\pm}^\pi|^2 \) can be almost directly determined.
through either $P_x$ or the polarization ratio $P_x/P_e$. Furthermore, the ratio $R_L/R_F$ of longitudinal to transverse response is model-independently accessible without Rosenbluth-separation.

At the Mainz Microtron MAMI a $p(\vec{e}, e'\vec{p})\pi^0$ experiment with longitudinally polarized electron beam and measurement of the recoil proton polarization has been performed. The polarized electrons were produced by photoemission from strained GaAsP crystals using circularly polarized laser light. During the experiment the beam helicity was randomly flipped with a frequency of 1 Hz in order to eliminate instrumental asymmetries. Longitudinal polarization after acceleration was achieved by fine-tuning of the energy of the microtron to 854.4 MeV. A 5 cm thick liquid hydrogen target was used. Beam currents up to 15 $\mu$A with an average polarization of $P_e = 75\%$ were available.

The scattered electrons were detected at $\theta_e = 32.4^\circ$ in Spectrometer B of the three spectrometer setup of the A1-collaboration. At $Q^2 = 0.121$ (GeV/c)$^2$ a range of invariant energies of $W = 1200$ to 1260 MeV was covered. The recoil protons were detected in Spectrometer A at an angle of $\theta_p = 27^\circ$ with a coincidence time resolution of 1 ns. The unobserved $\pi^0$ was identified with a resolution of 3.8 MeV (FWHM) via its missing mass. After all cuts, the experimental background was reduced to 0.4 $\%$ of the accepted $\pi^0$ events and thus neglected.

The proton polarization was measured with a focal plane polarimeter using inclusive $p^{-12}C$ scattering. The detector package of Spectrometer A, consisting of two double planes of vertical drift chambers for particle tracking and two planes of scintillators for triggering purposes, was supplemented by a 7 cm thick carbon scatterer followed by two double planes of horizontal drift chambers. This setup allowed the eventwise reconstruction of the proton carbon scattering angles $\Theta_C$ and $\Phi_C$ with a resolution of 2 mrad. The proton polarization could be extracted from the azimuthal modulation of the cross section

$$
\sigma_C = \sigma_{C,0} \left[ 1 + A_C (P_{fp}^y \cos \Phi_C - P_{fp}^x \sin \Phi_C) \right].
$$

$\sigma_{C,0}$ denotes the polarization-independent part of the inclusive cross section and $A_C$ the analyzing power, which was parameterized according to $\Theta_C < 18.5^\circ$ and according to $\Theta_C \geq 18.5^\circ$.

The two polarization components $P_{fp}^y$ and $P_{fp}^x$ are measured behind the spectrometer’s focal plane. It is possible to determine all three components at the electron scattering vertex due to the spin precession in the spectrometer and the additional information provided by the helicity flip of the electron beam. The spectrometer’s ‘polarization-optics’ is described by a 5-dimensional spin precession matrix. It was checked through a series of measurements of the focal plane polarization of protons elastic $p(\vec{e}, e'\vec{p})$ scattering where the polarization is given by electron kinematics and the proton elastic form factors. From these measurements also the absolute value of $P_e$ was determined, because in this case the error of $A_C$ cancels out in the determination of the quantities $P_x/P_e$ and $P_z/P_e$.

The spin-precession matrix was used for the extraction of the recoil proton polarization in the $p(\vec{e}, e'\vec{p})\pi^0$ experiment. In order to account for the finite acceptance around nominal parallel kinematics ($W = 1232$ MeV, $Q^2 = 0.121$ (GeV/c)$^2$, $\epsilon = 0.718$), averaged polarizations $P_{\pi^0}^{\text{MAID}}$ were generated with the Mainz Unitary Isobar Model (MAID2000) for the event population of the experiment. With the ratios of nominal to averaged polarization, $P_{x,y,z} = P_{\pi^0}^{\text{MAID}}/P_{x,y,z}^{\text{MAID}} = 1.247, 1.238$ and 0.946, respectively, the experimental polarizations were extrapolated to nominal parallel kinematics: $P_{x,y,z} = P_{\pi^0}^{\text{MAID}} P_{x,y,z}$. An additional systematic error is assigned to the polarizations due to the model uncertainty in $P_{x,y,z}$. It is estimated by a ±5 $\%$ variation of the $M_{2\pi}$ multipole and a ±50 $\%$ variation of the other multipoles in MAID.

The results are summarized in Table 1 along with the statistical and systematical errors. $P_x$ and $P_z$ are given normalized to the beam polarization. Under the assumptions that the n.l.o. corrections in Eqs. (3) and (6) as well as the isospin-1/2 contributions in CMR can be neglected, the quantities $S = 1 - P_x/P_e$ and $R = 4P_y/P_xP_e$ of the last two columns can be identified with the CMR.

In contrast to the beam helicity independent $P_y$, false systematic asymmetries and systematic errors of the analyzing power have no impact on $P_x/P_e$ and $P_z/P_e$. On the other hand, the error caused by drifts of the beam polarization only affects $P_x/P_e$ and $P_z/P_e$. The uncertainty of the spin tracing affects all three polarization components and remains the only experimental systematic error in the ratio $R$, where both the electron beam polarization and the analyzing power drop out.

The results for the three polarization components are shown in Figure 1 along with MAID2000 calculations. The curves represent the calculations for four values of the CMR: 0, −3.2 $\%$, −6.4 $\%$ and −9.6 $\%$. $P_x$ is most sensitive to the CMR and from this component $\text{CMR} = (−6.4 \pm 0.7_{\text{stat}} \pm 0.8_{\text{syst}})$ $\%$ is extracted. The result for the MAID2000 analysis of the ratio $P_x/P_e$ is $\text{CMR} = (−6.8 \pm 0.7_{\text{stat}} \pm 0.8_{\text{syst}})$ $\%$. Both values agree very well and are also close to $S$ and particularly to $R$. Apparently, the various interference terms in the n.l.o. corrections of Eqs. (3) and (6) are small or cancel to a large extent. From the variation of the multipoles in MAID2000 a model dependence of the order of 1 $\%$ absolute of the extracted CMR is estimated. The 20 $\%$ discrepancy between MAID and the measured $P_y$ is not expected to further affect the extracted CMR, because $P_y$ is more sensitive to interferences other than the CMR.
In order to compare our result to those from previous $\pi^0$ electroproduction experiments with unpolarized electrons, Figure 2 shows CMR instead of CMR. The MAID2000 analysis of $P_x$ yields CMR = ($-6.6\pm 0.7_{\text{stat}}\pm 0.8_{\text{syst}}$)%. The fact that CMR and CMR agree so well demonstrates how accurately, at the resonance position, the $p\pi^0$ channel yields the CMR which is defined in the isospin $\frac{1}{2}$ channel. Our result agrees with older data \cite{10} and preliminary new Bonn results \cite{14}, but a recent ELSA result \cite{11} seems to be incompatible with all other data. Whether this has experimental or statistical origin, or points to an unexpected $\Im m S_\mu$, background contribution — which was neglected in \cite{11} — is still undecided \cite{20}.

The ratio $R_L/R_T$ of longitudinal to transverse response can be obtained in various ways from the so-called reduced polarizations (RPs) $\chi_{x,y,z}$ \cite{27}. However, the results vary significantly. The smallest value, $R_L/R_T = (4.7\pm 0.4_{\text{stat}}\pm 0.6_{\text{syst}})$%, is extracted from the quadratic sum $\chi_x^2 + \chi_y^2$ of the transverse RPs and the largest one, $R_L/R_T = (12.2^{+1.7}_{-1.6_{\text{stat}}}^{+2.9}_{-2.7_{\text{syst}}})$%, from $\chi_z$ alone. Despite the non-linear error propagation in $R_L/R_T(\chi_z)$, the probability is only a few percent that this discrepancy has purely statistical origin. As a consequence, the consistency relation between the transverse and the longitudinal RPs derived in \cite{13} seems to be violated. This presently prohibits a reliable extraction of $R_L/R_T$ but stresses the importance of a simultaneous measurement of all polarization components with further improved accuracy.

In summary, we have measured the recoil proton polarization in the reaction $p(\vec{e}, e' \vec{p})\pi^0$ at the energy of the $\Delta(1232)$ resonance. Due to the spin precession in the magnetic spectrometer all three polarization components $P_x$, $P_y$ and $P_z$ were simultaneously accessible. From $P_x$ the Coulomb quadrupole to magnetic dipole ratio was determined in the framework of the Mainz Unitary Isobar Model as CMR($Q^2 = 0.121 \text{GeV}^2$) = ($-6.4\pm 0.7_{\text{stat}}\pm 0.8_{\text{syst}}$)%, which is in good agreement with the result obtained from the polarization ratio $P_x/P_z$. There is only a moderate model dependence expected to remain. However, the consistency relation among the polarization components seems to be violated.

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TABLE I. Results for the recoil proton polarization in nominal parallel kinematics. The quadratic sum of the systematic error contributions yields the total systematic error. The extrapolation to nominal parallel kinematics as well as the ratios $S$ and $R$, which approximate the CMR, are explained in the text.

| Observable  | $P_x/P_e$ (%) | $P_y/P_e$ (%) | $P_z/P_e$ (%) | $S$ (%) | $R$ (%) |
|------------|---------------|---------------|---------------|---------|---------|
| Measurement| $-11.4$       | $43.1$        | $56.2$        | $-5.2$  | $-6.4$  |
| stat. error| $±1.3$        | $±1.3$        | $±1.5$        | $±0.6$  | $±0.7$  |
| false asymm.| $-5.2$        | $-6.4$        |               |         |         |
| $\delta P_e$| $±0.45$       | $±2.25$       | $±0.21$       |         |         |
| $\delta A_C$| $0.87$        | $0.81$        | $0.48$        | $0.62$  |         |
| spin precession | $1.05$        | $0.45$        | $0.81$        | $0.48$  | $0.62$  |
| $\delta \rho$| $±0.9$        | $±1.2$        | $±1.0$        | $±0.41$ | $±0.51$ |
| total systematic | $1.4$          | $2.2$         | $2.6$         | $0.7$   | $0.8$   |

FIG. 1. Measured polarization components in comparison with MAID2000 calculations. The dashed, dot-dashed, full and dotted curves correspond to CMR = 0, $-3.2$, $-6.4$, $-9.6\%$, respectively. The MAMI data (full circles) are shown with statistical and systematical error. For the Bates $P_y$ (cross) only the statistical error is indicated, the value is rescaled in $\epsilon$ and, though measured at the same $Q^2$, slightly shifted for clarity.

FIG. 2. Result for $\Re\{S_{1+}^* M_{1+}\}/|M_{1+}|^2$ as extracted from $P_x/P_e$ of this experiment (full circle) with statistical and systematical error, compared to unpolarized measurements from DESY, NINA, the Bonn synchrotron \[8\] (open circles) and ELSA \[11\] (open square), where only the statistical errors are indicated.