Proliferation in Cycle

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In the contracting phase with \( w \simeq 0 \), the scale invariant spectrum of curvature perturbation is given by the increasing mode of metric perturbation. In this paper, it is found that if the contracting phase with \( w \simeq 0 \) is included in each cycle of a cycle universe, since the metric perturbation is amplified on super horizon scale cycle by cycle, after each cycle the universe will be inevitably separated into many parts independent of one another, each of which corresponds to a new universe and evolves up to next cycle, and then is separated again. In this sense, a cyclic multiverse scenario is actually presented, in which the universe proliferates cycle by cycle. We estimate the number of new universes proliferated in each cycle, and discuss the implications of this result.

Recently, it has been found that the contraction of universe with the state parameter \( w \simeq 0 \) can lead to a nearly scale invariant spectrum of primordial perturbation [1,2], also see [3,4,5] and earlier [6] for tensor perturbation, which can be responsible for the seed of observable universe after bounce. This spectrum of curvature perturbation is actually given by the increasing mode of metric perturbation, in which the increasing mode of metric perturbation is inherited by the constant mode of curvature perturbation in its \( k^2 \) order. Thus this equals that it is amplified on super horizon scale, up to bounce. The amplitude that it is amplified to is determined by the bounce scale, and thus in some sense the e-folding number before the end of contracting phase.

Recently, the cyclic scenario, in which the universe experiences the periodic sequence of contractions and expansions, has been reawakened, since it brings the different insights for the origin of observable universe. There has been lots of studies for oscillating or cyclic universe [1,10,12,13,11,14,15,16,17,18], also [19] for a review. In principle, it is interesting to include our observable universe in such a cyclic universe. The nearly scale invariant spectrum of primordial perturbation is required for the structures in our universe. Thus in this sense, we should consider the effect of perturbation on a cyclic universe. In another viewpoint, it is generally expected that the background of cycle universe should be homogenous all along during all cycles. However, since the perturbation is increased during the contraction of each cycle, it is required to check whether such homogeneity can be still preserved as expected.

The contracting phase with \( w \simeq 0 \) is suitable for the observable universe after bounce. In principle, it can be considered to involve in a cyclic scenario. We, in this paper, will explore that if the contracting phase with \( w \simeq 0 \) is included in each cycle of a cycle universe, what occurs. We find that the amplitude of perturbation modes that are generated in previous cycle and still stay on super horizon scale will be inevitably amplified to about order one at about beginning time of following cycle, which will render the different parts of global universe in this cycle evolve not anymore synchronously and decouple each other. This implies that after each cycle the universe will be separated into many parts independent of one another, each of which corresponds to a new universe and evolves up to next cycle, and then is separated again. This result shows that the universe proliferates cycle by cycle, which looks like a cyclic multiverse. We will calculate the number of new universes proliferated in each cycle, and investigate how it is affected by other factors. How this cyclic multiverse scenario incorporates the second law of thermodynamics is also discussed.

We will regard the beginning of the contracting phase as the beginning of a cycle, in each cycle the universe will experience the contraction, bounce, and expansion, successively, and then arrive at the turnaround, which signals the end of a cycle. For generality of the result, we will not involve the building of some special cyclic models, however, which is actually not difficult. We will begin with the review of the perturbation spectrum generated during the contraction with \( w \simeq 0 \) [1,2]. In principle, the contraction with \( w \simeq 0 \) can be easily implemented by introducing a scalar field \( \varphi \) with the suitable exponential potential, which leads a scale solution for \( \varphi \), see e.g. [2]. In this case, the motion equation of curvature perturbation is

\[
\ddot{u}_k + \left( k^2 - \frac{z''}{z} \right) u_k = 0,
\]

where \( u_k \) \(\equiv \zeta_k \) and the prime denotes the derivative with respect to the conformal time \( \eta \), and \( z = \varphi'/h \), where \( h \) is the Hubble parameter. For \( w \simeq 0 \), we have \( a \sim \eta^2 \) and \( h \sim 1/\eta^3 \). The scale solution of evolution of \( \varphi \) gives \( \varphi/h \) constant, which leads \( z \sim \eta^2 \). Thus we have \( \frac{z''}{z} \sim \frac{2}{\eta^2} \).

\[1\] The similar phenomena has been discussed in Ref. [20] for cyclic universe [5], in which the increasing mode of metric perturbation is inherited by the constant mode of curvature perturbation in leading order, which is controversial [2,21,22,23,21,22,24,23], and is different from that discussed here, in which the increasing mode of metric perturbation is inherited by the constant mode of curvature perturbation in its \( k^2 \) order, which certainly occurs for the bounce connecting the contracting and expanding phases, e.g. [25].
which is actually the same as that for inflation, in which $a \sim 1/\eta$ leads $z \sim 1/\eta$ and thus $\frac{v_k}{z} \sim \frac{v_k}{\eta}$. This gives the spectrum $n_s \simeq 1$ is scale invariant, and the amplitude of perturbation is

$$P_{\zeta}^{1/2} \simeq k^{3/2} \left| \frac{h_k}{z} \right| \sim \frac{h_h}{m_p}, \quad (2)$$

where the factor of order one has been neglected and $h_h$ is determined by the energy scale $\rho_h$ of field around the bounce point, $h_h \simeq \sqrt{\frac{2}{m_p}}$. The amplitude of this perturbation spectrum is actually led by the increasing mode of metric perturbation $\Phi$ [2], see also Ref. [3] for more details. This means the amplitude of perturbation begins to be amplified gradually on super horizon scale, up to the bounce, since the perturbation mode leaves the horizon. This will be formulized in term of the e folding number in the following.

The e folding number of mode with some wavelength $\sim 1/k_s$, which leaves the horizon before the end of contracting phase, is defined by

$$N \equiv \ln \left( \frac{a_s h_h}{a_s h_s} \right), \quad (3)$$

which is actually not the e folding number of scale factor, but is that of primordial perturbation, and $k_s$ is the last mode to be generated. We generally have $N \sim 50$, which is required by observable cosmology. In term of $a \sim \eta^2$ and $h \sim 1/\eta^2$, we have $a \sim 1/h^{2/3}$. Thus the e folding number during contraction can be deduced, which is $N_{con} = \ln \left( \frac{h_h}{m_p} \right)^{1/3}$. We substitute it into Eq.(2) to cancel $h_h$, and obtain

$$P_{\zeta}^{1/2} \simeq \frac{h_s}{m_p} e^{3N_{con}}. \quad (4)$$

This result implies that after the perturbation denoted by $k_s$ leaves the horizon, the multiple that its amplitude is amplified is given by the e folding number, while initially, i.e. it just leaves the horizon, its amplitude is approximately given by $\frac{h_s}{m_p}$. This shows that if we regard the initial amplitude of perturbation outside horizon as $\frac{h_s}{m_p}$, then hereafter during contraction the multiple that it is amplified on super horizon scale will be determined by the e folding number before the end of contracting phase, which is the result required for following arguments.

The entropy is inevitably increased during each cycle, which is the requirement of the second law of thermodynamics, thus it is expected that the maximal values which the scale factor can arrive at in ordinal cycles should larger and larger. In this case, there are certainly some modes that leave the horizon during the contraction of each cycle but can not reenter into the horizon during the expansion of corresponding cycle. These modes will desine to stay on super horizon scale all along up to next cycle, see the upper panel of Fig.1. We will regard the previous and current cycles as $i$ and $j$ cycles for convenience, respectively, in which $j = i + 1$. The perturbation during the contraction after turnaround satisfies Eq.(1). Thus if the perturbation mode is initially deep in the horizon of current cycle, its amplitude after bounce in current cycle will be given by Eq.(2), i.e.

$$P_{\zeta(i)}^{1/2} \simeq \frac{h_h}{m_p} \quad (5)$$

where the subscript $i$ denotes the amplitude of perturbation after bounce in $i$ cycle, which is that leaves the horizon during the contraction of $i$ cycle and reenters into the horizon during the expansion of corresponding cycle, thus same meaning for $P_{\zeta(j)}^{1/2}$. Here, the amplitude calculated is that of perturbation induced by the quantum fluctuation of background field in current cycle. However, in general it will be interfered by those perturbations that enter into the horizon during the expansion of previous cycle. Thus the actual amplitude can be larger, dependent of the expansion behavior and matter contents of previous cycle, and the spectrum is also not scale invariant any more. In this case, if we hope that some of universes after proliferation could be same as ours, it seems a period of dark energy in previous cycle or inflation after bounce in current cycle is required, since it helps to push those baneful modes to outside of our observable universe. This will be discussed in details in coming works.

The amplitudes of those modes initially on super horizon scale at current cycle should be calculated by using Eq.(1) again, however, in which instead the initial condition is given by that at the time of turnaround of the previous cycle. This value is $\frac{h_h}{m_p}$, since the amplitude of perturbation on super horizon scale after the bounce of previous cycle is dominated by the constant mode, and thus is unchanged, which will be expected to keep all along up to the turnaround of this cycle. However, after the turnaround, the modes on super horizon scale will be not unchanged any more. In term of the analogy with Eq.(4), in $j$ cycle the amplitude of perturbation mode all along on super horizon scale is given by

$$P_{\zeta(j)}^{1/2} \left| \frac{h_h}{m_p} e^{3N_{con}} \right. \simeq P_{\zeta(i)}^{1/2} e^{3N_{con}}, \quad (6)$$

where Eq.(5) has been applied, and $P_{\zeta(j)}^{1/2} |_i$ denotes the amplitude of perturbation, which leaves the horizon during the contraction of $i$ cycle and reenters into the horizon in the expanding phase of following $j$ cycle. Thus we see that in $j$ cycle it will be further amplified with the proceeding of contracting phase.

The amplitude of perturbation responsible for seeding large scale structure of observable universe is $P_{\zeta(i)}^{1/2} \sim 10^{-5}$. Thus if $P_{\zeta(i)}^{1/2} \sim 10^{-5}$ is required in previous cycle, we can see when $N \simeq \frac{5}{4} \ln 10 \sim 3$, $P_{\zeta(j)}^{1/2} \sim 1$ in $j$ cycle, where it should be noticed that when $P_{\zeta(j)}^{1/2} |_i$ approaches 1, the enhancement of nonlinear effect will make the required $N$ less. Thus in fact this means that nearly at the
beginning time of $j$ cycle, the modes on super horizon scale will have the amplitude be about order one. This will lead to $\frac{a}{h_j} \sim 1$ on corresponding super horizon scale at this time. In this case, it is obviously impossible that the different regions of global universe will evolve synchronously, even if it is synchronous in previous cycle. This indicates that the global universe at the beginning time of this cycle will be separated into many different parts, each of which will evolve independently of one another, up to bounce. While inside any given part, all perturbation modes origin from the interior of horizon, which is causally correlative. Thus in this sense, each of such parts actually corresponds to a new universe.

In principle, each of these new universes will experience the contraction, bounce and expansion, hereafter all or some of them will enter into next cycle and proliferate again, and then the above course is repeated again. This means the proliferation will inevitably occur cycle by cycle. Thus we can have a cyclic multiverse scenario. In this cyclic multiverse, the experience of each universe after proliferation is generally not expected to be synchronous. Thus when some universe are in a period of matter domination, it is possible that there are many other universes which are in the period of contraction or bounce or others. There is also the proliferation of global universe in chaotic eternal inflation \[32, 33\], in which it is induced by the large quantum fluctuation of inflaton field in its horizon scale, which occurs efold by efold. Here, however, the proliferation is induced by the cyclical amplification of perturbation on super horizon scale, which is in classical sense, thus it occurs cycle by cycle.

The number of new universes proliferated at the beginning time of each cycle is \[2\]

\[
N \simeq \left( \frac{k_j}{k_i} \right)^3 = \left( \frac{a_j h_j}{a_i h_i} \right)^3,
\]

where $k_i$ and $k_j$ denote the modes with maximal wavelengths leaving the horizon at $i$ cycle and at following $j$ cycle, respectively, and thus $1/k_i$ and $1/k_j$ are the corresponding wavelengths, respectively. In this sense, $a_{i,j}$ correspond to the magnitudes of scale factors at the beginning time in $i$ and $j$ cycle, respectively, which actually equals to the maximal magnitudes which the scale factors expand to in previous cycle. $h_{i,j}$ are the turnaround scales at corresponding cycles. The turnaround scale is generally same for each cycle. Thus we have $N \simeq \left( \frac{a_j}{a_i} \right)^3$.

We can see that only when $a_j > a_i$ is there the appearing of many new universes, while when $a_j = a_i$, $N = 1$. This means for cyclic universe with equal cycles, the global universe in previous cycle will not be separated into many new universes in current cycle. The reason is simple, because there are not the perturbation modes staying on super horizon scale all along up to next cycle, see the lower left panel of Fig.2. It can be noticed that after the proliferation in $j$ cycle the scale factor of each new universe is approximately $a_j$. Thus for each of them, it can be expected that what happened in $i$ cycle will be repeated, which looks like that plotted in the lower panel of Fig.1. However, it should be reminded that this is only an ideal case, since, as has been mentioned, it is possible that those modes that enter into the horizon during the expansion of $i$ cycle and then leave it during the contraction of $j$ cycle may be also amplified to about order one. In this case, it seems the universe will be split into smaller and smaller parts, which will ultimately end the cycle. This in some sense indicates that in cyclic universe a gradually growing cycle is significant for the continuance of cycle.

We can assume that after the bounce of each cycle the universe enters into the expansion dominated by radiation and then matter. Thus we have the radiation en-

\[\text{FIG. 1: The sketch of } \ln \left( \frac{a_i}{a_j} \right) \text{ with respect to the time. The blue lines denote the evolutions of perturbation modes and the shade regions denote the contracting phases. The upper panel shows the evolutions of perturbation modes in } i \text{ and following } j \text{ cycles for the case that there is not the proliferation in each cycle. There are certainly some modes that leaves the horizon in } i \text{ cycle but can not enter into the horizon during the expansion of corresponding cycle. These modes will destined to stay on super horizon scale all along up to } j \text{ cycle. The amplitude of these perturbations will be amplified to about order one at about beginning time of } j \text{ cycle. This leads the universe separated into many independent new universes, each of which has the initial scale factor equal to that in previous cycle. Thus for each universe in cycle, it seems that the evolution of } \ln \left( \frac{a_i}{a_j} \right) \text{ with the time actually looks like the lower panel. In principle, at the bounce and turnaround points, } h = 0, \text{ thus there is a divergence for } \frac{a_i}{a_j}, \text{ which is not plotted here and Fig.2. However, the discussions are not affected by this neglect.}\]
entropy $S \approx a^3 T^3$, which can be enhanced by the decay of some relic massive particles, where $T$ is the corresponding temperature. For the period of matter domination, $S$ corresponds to the CMB entropy. Though there are the increase of super horizon perturbations, it can be showed that the entropy in scalar and tensor perturbations is smaller than that of CMB \[34, 35\]. Thus around the turnaround we have $S_{i,j} \approx a^{3j} T_i^3$, where $S_i$ and $S_j$ denote the entropy at the beginning time in $i$ and $j$ cycles, respectively. In this sense, $S_i$ is actually the maximal value that the radiation entropy in $i$ cycle can increase to. $T_{i,j}$ can be related to the turnaround scale $h_{i,j}$ by $h_{i,j} \approx \frac{a_{i,j}}{m_p}$. Thus combining these results with Eq.\[7\], then taking $h_j \approx h_i$,

$$N \approx \frac{S_j}{S_i} \quad \quad (8)$$

can be obtained, which shows that the number of new universes proliferated in $j$ cycle is determined by the net increasing amount of radiation entropy in $i$ cycle.

This result indicates that after the proliferation in each cycle, each new universe in this cycle actually has the entropy equal to $S_i$. This can be thought as if there is a net increase of the entropy during some cycle, then in following cycle these net entropy will be assigned to each new universes proliferated such that the entropy of each of them is equal to that at the beginning time of previous cycle. In this sense, it seems the problem of entropy increase suffering the cyclic universe may be alleviated, since if one begins with any given point in cyclic multiverse and look along cycles, he will find in himself observable universe that though the total entropy increases in each cycle, there is not net increase of the entropy from one cycle to next. Here the total entropy is the sum of entropy of all universes at some spacelike hypersurface, which is obviously increased all along, due to the second law of thermodynamics. In some sense, it seems that here the second law of thermodynamics is incorporated in such a fashion in which the increase of total entropy is explained as the increase of the number of universes.

The inflation can be imagined to occur in some cycles. However, regardless whether it occurs before turnaround, see the upper right panel of Fig.2, or after bounce, see the upper left panel of Fig.2, there will be more proliferated parts in next cycle, i.e. new universes. The reason is a nearly exponential expansion makes the scale factor in corresponding cycle larger, thus in term of Eq.\[7\], for the fixed turnaround scale, $N$ will be larger. From Eq.\[7\], we can obtain

$$N \approx \left( \frac{a_j}{a_i} \right)^3 \approx \left( \frac{a_j}{a_{i,j}} \right)^3 e^{3N_{inf}}, \quad \quad (9)$$

where $a_{i,j}$ denotes the maximal value that the scale factor can reach if there is not such an inflation, and $N_{inf}$ is the e fold number of inflation, which may be given by taking the Hubble parameter constant in Eq.\[3\]. Thus with the increase of the e folding number of inflation, the number of new universes proliferated in following cycle will increase exponentially. This result is natural, since it is generally expected that during the reheating after inflation there will be lots of entropy relaxed, and the ratio of the entropy after reheating to that before inflation is approximately $e^{3N}$, thus in this sense it is actually the enhancement of entropy before and after inflation that leads the increase of the number of new universes proliferated in following cycle, which is consistent with Eq.\[9\]. The inflation after bounce was firstly studied in Refs. \[36, 37\], in which the imprint of bounce on CMB has been showed. It can be noticed that if the inflation occurs before the turnaround in some cycle, it actually corresponds to the period of dark energy domination in previous cycle, see the upper right panel of Fig.2.

It has been showed that each of new universes proliferated in each cycle will evolve with the initial entropy $S_i$ and the initial scale factor $a_i$ up to next cycle, and then proliferate in this cycle, and again each of new universes still has the initial entropy $S_i$ and the initial scale factor $a_i$ for following evolution. Thus in this cyclic multiverse scenario the cycle is actually eternal. However, this eternity seems not for past, an initial condition might still be required. The reason is that the number of universes increases with each cycle by a finite factor, which inevitably leads that in the past the number of universes reduces to one, thus there is again a big bang singularity. The principal motivation for cyclicity is to avoid the big bang singularity. This seems be lost here. However, whether the cyclic universe avoids the big bang singularity is still a disputed issue. In principle, if we consider the second law of thermodynamics, since the entropy is increased cycle by cycle, the length of cycle must continuously increase by a finite factor cycle by cycle. Thus if we back along cycles, we will certainly find a less and less length of cycle, up to ‘0’ at a finite time. In this sense, the cyclic universe actually dose not avoid the big bang singularity. Here, the proliferation of universe cycle by cycle is based on the increase of the length of cycles, and thus the increase of entropy. In some sense, it equals to that here the increase of the length of cycles in usual cyclic universe is transferred to the increase of the number of universes. Thus if we think that in a cyclic universe with the increase of entropy, the problem of singularity remains, then it is same here. In principle, the origin of this initial single universe needs to be explained. However, of course, also it may be the one eternally existing in the past, for example, origining from a steady state background, like discussions in Ref. \[15\].

The initial magnitude of scale factor of this initial universe determines that of scale factor of each universe in multiverse, for example, if we find the initial magnitude of scale factor is $a_i$ in $i$ cycle, we will have $a_i$ for this initial single universe. Thus in order to have an observable universe like ours in corresponding cycle, it must be large. This seems to add a requirement for initial condition. However, this can be relaxed as follows. In general, the turnaround scale might be not same for each cycle,
for example, if $h_j < h_i$, it will be possible that the initial magnitude $a_j$ of scale factor of the new universes proliferated is larger than that of previous cycle, see the lower right panel of Fig.2, since $a$ will continue to expand from the scale $h_j$ to $h_j$, where $h_j$ is the scale equal to the turnaround scale in previous cycle. In this case, in term of Eq.(7), $N_{(h_j<h_i)}$ proliferated will be less. This can be estimated by assuming that the evolution between $h_j$ and $h_j$ is dominated by matter,

$$N_{(h_j<h_i)} \simeq \left( \frac{h_i}{h_j} \right)^{\frac{1}{2}} N \simeq \left( \frac{h_i}{h_j} \right)^{\frac{1}{2}} N,$$

(10)

where $a \sim 1/h^{2/3}$ for the period of matter domination has been applied.

When the turnaround scale in some cycle is enough low, for example $h_j \rightarrow 0$, it seems that in following cycle the initial magnitude of scale factor of each universe can be infinite large. However, the case is not so. When $h_j \rightarrow 0$, $1/(a_j h_j) \sim 1/h_j^{1/3}$ is large so that $1/(a_j h_j) \gtrsim 1/(a_i h_i)$. In this case the modes that are generated in previous cycle but can not enter into the horizon during the expansion of corresponding cycle will be inevitable to enter into the horizon of present universe, while these modes actually have amplitude $\frac{\Delta a}{a} \sim 1$. When these metric perturbations enter into the horizon, they will certainly induce the fluctuations of energy density on the corresponding horizon scale, which will render the corresponding regions gravitational collapse. This corresponds to set an upper limit for the initial magnitude of scale factor of each universe proliferated in following cycle. This result shows that the information before two cycles is inaccessible to the observers in observable universe of any given cycle. Thus if we are in some cycle of this cyclic multiverse, we can at most see the modes produced in previous cycle and thus information.

However, it can be noticed that this outcome is actually dependent of the bounce scale. Here the bounce scale is high, thus the amplitude after bounce in each cycle is large, which will be inevitably amplified to order one in following cycle. However, if the bounce scale is enough low, it is also possible that the amplitude of perturbation leaving the horizon in some cycle can not reach order one till several cycles. This will lead some possible and interesting observations, which will be explored in coming works.

In conclusion, it is found that if the contracting phase with $w \simeq 0$ is included in each cycle of a cycle universe, after each cycle the universe will be inevitably separated into many parts independent of one another, each of which corresponds to a new universe and evolve up to next cycle, and then is separated again. Thus a cyclic multiverse scenario is actually presented, in which the universe proliferates cycle by cycle. This scenario are leaded by the amplification of metric perturbation on super horizon scale cycle by cycle, which can be general, since for the contracting phase with $w \simeq 0$ the increasing mode of metric perturbation is inherited by the constant mode of curvature perturbation in its $k^2$ order is universal. We estimate the number of new universes proliferated in each cycle, which, for same turnaround scale, is determined approximately by the net increasing amount of radiation entropy in previous cycle. This in some sense incorporates the second law of thermodynamics in such a fashion in which the increase of total entropy is explained as the increase of the number of new universes.

We have showed that the global configuration of cyclic universe is more complex than expected ever, which actually shows itself a cyclic multiverse. Though the arguments given here seems slightly ideal, it might have captured some essentials of full answer. It can be noticed that in general the background evolution of contracting phase with $w \simeq 0$ is not an attractor. The relevant discussions, also involving the landscape, is being ordered.

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