Comment on ‘Experimental Entanglement Swapping: Entangling Photons That Never Interacted’

Entanglement swapping has been experimentally demonstrated by Pan et al. using two parametric downconverters and linear optical elements [5]. This experiment was first proposed (in a slightly modified form) by Zukowski, Zeilinger, Horne and Ekert [2], who stressed the possibility of so-called event-ready detections using entanglement swapping. Indeed, Pan et al. note that their experiment for the first time gives the possibility of event-ready detections. However, in this comment we will show that with the current state of technology event-ready detections can not be performed this way.

For event-ready detections we need a way of deciding that a maximally polarisation entangled state left the apparatus. This is equivalent to having an outgoing state

\[ \rho_{\text{out}} = \langle \text{Bell} \rangle \langle \text{Bell} \rangle + O(\xi) \]  

conditioned on detector coincidences, with |Bell⟩ any of the four polarisation Bell states (|Ψ^±⟩, |Φ^±⟩) and ξ \ll 1.

In Refs. [2,5], entanglement swapping is described in terms of two anti-symmetric polarisation Bell states (|Ψ^−⟩_ab \otimes |Ψ^−⟩_cd which, after a Bell detection of modes b and c is turned into a Bell state in modes a and d. In the case of the experiment of Pan et al. (see Fig. 1) modes b and c are sent into a beam-splitter. A coincidence in the detectors D_a and D_b identifies a |Ψ^−⟩ Bell state. Modes a and d should now be in the |Ψ^−⟩ Bell state as well. However, as has been pointed out previously [2,5], parametric down-converters do not produce |Ψ^−⟩ Bell states. Instead, there is a strong pollution of vacuum and small contributions from higher down-conversions which invalidate the above description of entanglement swapping.

To lowest non-trivial order the (unnormalised) states (with linear polarisations along x and y axes) leaving the apparatus after a two-fold coincidence conditioned on the four polarisation settings in D_a and D_b are:

\[ \langle \phi(x,y) \rangle_{\text{ad}} = |0, y^2 \rangle - |y^2, 0 \rangle \]  
\[ \langle \phi(x,y) \rangle_{\text{ad}} = |0, xy \rangle - |xy, x \rangle + |x, y \rangle - |xy, 0 \rangle \]  
\[ \langle \phi(x,y) \rangle_{\text{ad}} = |0, xy \rangle + |y, x \rangle - |x, y \rangle - |xy, 0 \rangle \]  
\[ \langle \phi(x,y) \rangle_{\text{ad}} = |0, x^2 \rangle - |x^2, 0 \rangle \]

where \( |\phi_{(i,j)}\rangle_{\text{ad}} \) is the outgoing state conditioned on an i-polarised photon in D_a and a j-polarised photon in D_b (i, j ∈ {x, y}). Here, for instance, |y^2⟩ is a y-polarised mode in a 2 photon Fock state. No polarisers were used in the Bell state detection of Pan et al., and the state leaving the apparatus is a random mixture ρ of these four states. This mixed state is different from the outgoing state Pan et al. describe in Ref. [5].

Using the Peres-Horodecki partial transpose criterion it can be shown that ρ is indeed entangled (ρ has negative eigenvalues). However, it can not be used for event-ready detections of polarisation entanglement since the states in Eq. (2) are not of the form of Eq. (1).

Can we turn any of the states in Eq. (2) into the form of Eq. (1)? Additional photon sources are not allowed since that would take us beyond the entanglement swapping protocol. It can easily be verified that there is no linear optical transformation which takes any of the states in Eq. (2) to any of the four Bell states. Still we need to bring a two-photon state to a two-photon Bell state. Ordinary detectors destroy photons, so we need at least sufficiently good polarisation independent quantum non-demolition (QND) measurements or a quantum computer of some kind. Furthermore, the correlations (constituting the entanglement) must be preserved. Such QND detectors correspond to technology not yet available. This means that event-ready detections are not yet possible using entanglement swapping with parametric down-conversion.

To conclude, we have calculated the state leaving the apparatus of the entanglement swapping experiment by Pan et al. Contrary to the description of the outgoing state in Ref. [5], this is a random mixture of four (entangled) states. None of them is suitable for event-ready detections with present technology.

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