Error Transfer Analysis of Normalized Multichannel Measurement

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Abstract. In multichannel measurement, multiple signals affect the output result through the normalization process. It is difficult to analyze the error transfer, which represents the relation between the error of input and output, in this kind of measurement, especially in the normalization process. Thus, a new method of error transfer analysis is needed. Taking the primary spectrum pyrometry for example, this paper establishes a model for error transfer analysis in three-channel measurement by introducing the concept of solid angle. In the model, the error transfer is represented by the concept of error magnification, which is a dimensionless ratio of projection area of error sphere on the normalized plane to the cross section area of the error sphere. The error magnification can not be measured actually through experiment, but can be obtained from theoretical calculation. It is useful in the design of measurement system, especially the analysis of influence of different gain in different channel. The influence factors of the error magnification, including the absolute value and relation between them, are analyzed base on the model. The formula for analysis of two-channel and n-channel measurement is also derived from the three-channel model. Finally, some principles of operation and improvements in normalized multichannel measurement are proposed in order to reduce error.

1. INTRODUCTION

The measurement process can be generally expressed as

\[ y = f(x) \]  (1)

Where, \( y \) is output of instrument, \( x \) is the quantity to be measured, \( f \) is the relation between them.

The error transfer means the relation between \( \pm \Delta y \), the error of \( y \) produced in measurement, and \( \pm \Delta x \), the error of \( x \) caused by \( \pm \Delta y \).

\[ x \pm \Delta x = f^{-1}(y \pm \Delta y) \]  (2)

Error transfer can be analyzed as

\[ \Delta x = \frac{df^{-1}}{dy} \Delta y \]  (3)
However, there are many multichannel measurements used in practical application. For example, the multi-wavelength thermometry reviewed by Dai (2004), the multichannel detection of echo from underwater target (Wang et al., 2002), and two-wavelength fluorescent microarray scanning (Fu et al., 2007). Normalization is often necessary in multichannel measurement especially those base on color theory: the primary spectrum pyrometry (Cheng et al., 2004), color recognition (Li et al., 2004), shadow detection (Chen et al., 2006) and plants’ diagnosis (Feng et al., 2007; Dong et al., 2008).

The relation between output signals and the quantity to be measured is

\[ y_i = f_i(x) \quad (i = 1, 2, \ldots, n) \quad (4) \]

The normalization is

\[ Y_i = y_i / \left( \sum_{i=1}^{n} y_i / \sum_{i=1}^{n} f_i(x) \right) \quad (5) \]

Finally, \( x \) can be calculated as

\[ x = g(Y_1, Y_2, \ldots, Y_n) \quad (6) \]

Similarly, the error transfer can be expressed as

\[ Y_i \pm \Delta Y_i = \left( \sum_{i=1}^{n} y_i \pm \Delta y_i \right) / \sum_{i=1}^{n} f_i(x) \quad (i = 1, 2, \ldots, n) \quad (7) \]

\[ x \pm \Delta x = g(Y_1 \pm \Delta Y_1, Y_2 \pm \Delta Y_2, \ldots, Y_n \pm \Delta Y_n) \quad (8) \]

In multichannel measurement, the error propagates to the normalized quantities first and then to the final result. It is difficult to analyze the error transfer in traditional way of derivation because the final error is affected by multi-quantities and nothing significant result can be obtained from the analysis.

Taking the primary spectrum pyrometry for example, a model for error transfer analysis of multichannel measurement will be proposed next, and some principles of design and operation will be obtained. In this model, error transfer is not the exact one mentioned foregoing, it is represented by the parameter error magnification. The error magnification can not be measured actually from experiment, it is used just in theoretical analysis of error transfer, and finding a better design of system.

2. ERROR TRANSFER MODEL OF NORMALIZATION

2.1 Case Analysis of the Primary Spectrum Thermometry

The primary spectrum pyrometry takes three-channel outputs, \( R \), \( G \) and \( B \), of CCD camera as the basis of calculation. The calculation process can be divided into two steps.

(a) The normalization of \((R, G, B)\)

\[ r = \frac{R}{R+G+B}, \quad g = \frac{G}{R+G+B}, \quad b = \frac{B}{R+G+B} \quad (9) \]

(b) Searching for the isotherm where the point \((r, g)\) is located on the normalized plane. The
temperature of this isotherm is the quantity to be measured.
Among them, step (a) is the process of normalization and it is a general operation. Essentially, step (b) is the process of solving equations and its detail is different from other measurement method.
According to the independence and difference of the two steps, we can analyze the error transfer in step (a) first and then consider its influence on step (b).
The error transfer model of normalization in the primary spectrum pyrometry is a three-channel model. But its theory can be extended to two-channel or n-channel measurement.

2.2 The Three-Channel Model
Taking the three-channel outputs $R$, $G$ and $B$ as the coordinate axis, a 3D measurement space $R$-$G$-$B$ can be constructed as shown in Fig.1.

![Fig. 1 3D measurement space R-G-B](image)

In Fig. 1, the cube represents the value range of every channel. The point $P$ represents the true value of measurement. After normalization $r$, $g$ and $b$ are obtained. As the equation $r + g + b = 1$ is satisfied, the point $(r, g, b)$ must locate at the plane $LMN$ (the normalized plane), where $OL = OM = ON = 1$.

If error exists, points of measured value will depart from the point $P$ of true value and a cube centred at $P$ appears. The cube represents the max range of error. But if the probability is considered, a sphere distribution which is the inscribed sphere of the cube is more reasonable.
After normalization the projection of the sphere on the normalized plane is an irregular area which can be regarded as infinitesimal element. In condition of constant error, the larger the projection area is, the bigger the transmission error is.
Now the analysis of error transfer has transformed into a solid geometry problem and it is necessary to introduce the concept of solid angle. Let $\Delta S_0$ be the cross section area of the error sphere and $\Delta S_1$ be the projection area, the solid angle subtended by $\Delta S_0$ and $\Delta S_1$ to $P$ can be expressed as

$$\omega = \frac{\Delta S_0}{OP^2} = \frac{\Delta S_0 \cos \theta}{OT^2} \left(0 \leq \theta \leq \arccos \frac{\sqrt{3}}{3}\right)$$  \hspace{1cm} (10)

Where $\theta$ is the angle between line $OQ$ and $OP$. So the ratio of projection area to cross section area is

$$f = \frac{\Delta S_1}{\Delta S_0} = \frac{h^2 / \cos^3 \theta}{R^2 + G^2 + B^2}$$  \hspace{1cm} (11)

Where, $h$ is the distance between origin $O$ and normalized plane $LMN$ ($h = \sqrt{3}/3$), and cosine is
\[
\cos \theta = h \sqrt{r^2 + g^2 + b^2}. \text{ So}
\]

\[
f = \frac{r^2 + g^2 + b^2}{R^2 + G^2 + B^2} \cos \theta = \frac{\sqrt{3}}{R + G + B} \cos \theta \tag{12}
\]

\(f\) is the ratio of two areas and represents the error transfer in normalization. So it is defined as error magnification. The bigger the \(f\), the larger the transmission error is.

### 2.3 The Two-Channel Model

There is something different for two-channel model as shown in Fig.2.

![Fig. 2 2D measurement space R-G](image)

After normalization, \(r\) and \(g\) are obtained from \(R\) and \(G\). As equation \(r + g = 1\) is satisfied, the point \((r, g)\) must locate at line MN where \(\overline{OM} = \overline{ON} = 1\). If error exists, points of measured value will distribute in a circle centre at point P of true value. The projection of the circle on line MN is a line segment. So the error magnification can be defined as the ratio between length of the line segment and the diameter of the circle. Let \(\Delta d\) be the diameter and \(\Delta l\) be the length of line segment, the angle subtended by \(\Delta d\) and \(\Delta l\) to P can be expressed as

\[
\varphi = \frac{\Delta d}{OP} = \frac{\Delta l \cos \theta}{OT} \left(0 \leq \theta \leq \frac{\pi}{4}\right) \tag{13}
\]

So the error magnification is

\[
f = \frac{\Delta l}{\Delta d} = \frac{h/\cos^2 \theta}{\sqrt{R^2 + G^2}} \tag{14}
\]

Where, \(h\) is the distance between origin O and line MN\((h = \sqrt{2}/2\)), and cosine is \(\cos \theta = h/\sqrt{r^2 + g^2}\). So

\[
f = \frac{\sqrt{r^2 + g^2}}{\sqrt{R^2 + G^2}} \cos \theta = \frac{\sqrt{2}/2}{(R + G)^2} \tag{15}
\]
Obviously, the error magnification in two-channel measurement is similar to the three-channel one.

2.4 The N-Channel Model

From above analysis, it is obvious that the essence of error transfer is projection of dimension reduction. If the number of channel is greater than three, the definition of error magnification can not be derived from model because it is difficult to image the space more than three dimensions. But the definition can be obtained from some common relations.

The basic of the construction of Euclidean space is the distance (Lin, 2004) which can be expressed as

\[ \rho = \sqrt{\sum_{i=1}^{n} (a_i - b_i)^2} \]  

(16)

where \( a_i \) and \( b_j \) are the coordinates of two points in the space.

The definition of error magnification in 2D and 3D space includes three distance values, \( L \) the distance between origin and point of measured value, \( l \) the distance between origin and point of normalized value, \( h \) the minimum distance between origin and all points of normalized value. The relation between error magnification and the three distance values is inducted as

\[ f_n = \left( \frac{l}{L} \right)^{n-1} \times \frac{l}{h} \]  

(17)

where, \( (l/L)^{n-1} \) represents the influence of distance, and \( l/h \) which is the reciprocal of cosine represents the influence of angle.

The minimum distance between origin and all points of normalized value can be derived from Cauchy inequality

\[ \left( \sum_{i=1}^{n} Y_i^2 \right) \left( \sum_{i=1}^{n} Z_i^2 \right) \geq \left( \sum_{i=1}^{n} Y_i Z_i \right)^2 \]  

(18)

Let \( Y_i \) be the normalized value and satisfy \( \sum_{j=1}^{n} y_i = 1 \). When \( Z_i = 1 \), yield

\[ \sqrt{\sum_{i=1}^{n} Y_i^2} \geq \frac{1}{\sqrt{n}} = h \]  

(19)

Finally, the error magnification in n-channel measurement is expressed as

\[ f_n = \sqrt{n} \times \left( \sum_{i=1}^{n} Y_i^2 \right)^{\frac{n-1}{2}} \left( \sum_{j=1}^{n} y_j \right)^{\frac{n-1}{2}} \]  

(20)
3. THE EFFECT OF DISTANCE AND ANGLE

\( f \) is the error magnification and represents the error transfer in normalization. Taking the three-channel for example, it is obvious from Eq. (11) that the distance decreases \( f \) while the angle increases it. When \( \theta = 0 \) is satisfied the relation between \( f \) and distance \( OP \) is shown in Fig.3. When \( R^2 + G^2 + B^2 = 10^4 \) the relation between \( f \) and \( \theta \) is shown in Fig.4.

![Fig. 3 Relation between \( f \) and \( OP \)](image1)

![Fig. 4 Relation between \( f \) and \( \theta \)](image2)

As the error magnification attenuates with increase of square distance, the measured value should be as large as possible but not saturated in measurement. For example, we should prolong the exposure time and increase the aperture while using CCD in multichannel measurement. As every output is increased in same rate, the operation will not change anything in subsequent solving process of equations even the process is unknown.

On the other hand, error magnification increases with increase of angle \( \theta \) in the form of \( \frac{1}{\cos^3 \theta} \). This means that output from every channel had better keep approximate in measurement. As it is necessary to use the instrument function of separate signal gain in order to change the normalized value and the subsequent process will be affected, the complicated analysis will not be given in this paper.

In above analysis it is assumed that the absolute error keeps constant while the system is changed. Research of Xu (2004) and Li (2007) shows that common noise in CCD includes shot noise, dark current noise, transfer noise, reset noise, nonuniform response noise, clutter noise and quantization noise. All these noises are independent of output value except for the shot noise which is proportional to the square root of signal (Xu, 2004). So the assumption is reasonable.

Compared to the error analysis of single signal, the distance corresponds to the absolute value of signal and the error sphere corresponds to the absolute error. It is similar in two methods that if the absolute error keeps constant, the increase of absolute signal reduces the relative error. However, the relation between signals what is represented by angle \( \theta \) should be considered in multichannel measurement. This is the main difference between two methods.

4. CONCLUSIONS

Taking the primary spectrum pyrometry for example, the analysis model of error transfer in normalized multichannel measurement is established and the effect of signals on error magnification is analyzed base on the model in this paper. The analysis points out that outputs should keep large and approximate to each other in order to reduce the error magnification.

Although the accurate relation between error and result can not be obtained, the effect of normalization on error is shown clearly. This method simplifies the analysis of error transfer and plays a guidance role in practical operation. The separate signal gain is useful in reducing error and its effect can also be analyzed base on this model.
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NOMENCLATURE
\( R, G, B \) \hspace{1em} output of red, green and blue channel of CCD
\( r, g, b \) \hspace{1em} normalized signal of red, green and blue channel of CCD
\( \Delta S_0 \) \hspace{1em} the cross section area of the error sphere
\( \Delta S_1 \) \hspace{1em} the projection area
\( \Delta d \) \hspace{1em} the diameter of error circle in two-channel measurement
\( \Delta l \) \hspace{1em} the projection line segment of error circle in two-channel measurement
\( f \) \hspace{1em} error magnification
\( h \) \hspace{1em} the minimum distance between origin and all points of normalized value

Greek Letters
\( \phi \) \hspace{1em} the angle subtended by diameter of error circle in two-channel measurement
\( \omega \) \hspace{1em} solid angle subtended by cross section area of error sphere

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