Measuring transversity densities in singly polarized hadron-hadron and lepton-hadron collisions

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Abstract: We show how the transverse polarization of a quark initiating a jet can be probed by the azimuthal distribution of two hadrons (of large $z$) in the jet. This permits a twist 2 asymmetry in hard processes when only one of the initial particles is polarized transversely. Applications to hadron-hadron and lepton-hadron scattering are discussed.

1 Introduction

A notorious result in perturbative QCD is that many transverse spin asymmetries are zero at the twist-2 level. While this provides an interesting window [1, 2, 3] onto twist-3 effects, it also means that the twist-2 part of the distribution of transversely polarized quarks in a transversely polarized hadron is rather difficult to measure.

In this paper, we suggest a new method for measuring transverse spin effects at the leading (twist-2) level, by probing suitable polarization sensitive properties in jet fragmentation. Our technique jointly probes the distribution of transverse spin (strictly, transversity) of quarks in a transversely polarized hadron and the polarization dependence of the fragmentation of a quark jet. It can therefore be used in an experiment in which only one of the incoming particles is polarized. The polarization dependence is in the azimuthal distribution of low-mass pairs of pions about the jet axis.

One experiment in which these measurements could be made is deeply inelastic scattering of unpolarized leptons on transversely polarized hadrons. The European Muon Collaboration (EMC) has already demonstrated [4] that two particle correlations in the region that we advocate are readily measurable. Therefore, the correlation should be accessible to a polarized version of that experiment. The proposed HERMES experiment at HERA should also be able to do the measurement.

Another obvious experiment is in high $p_t$ particle production in singly polarized hadron-hadron collisions. This would be ideal at the experiment proposed by the RHIC Spin Collaboration [5] for the RHIC collider at Brookhaven, if sufficient resolution could be obtained. A fixed target hadron-hadron experiment might be too low in energy for the reliable application of perturbative QCD.

The normal spin asymmetries measured in singly polarized collisions are higher twist [6], that is, they are suppressed by a power of the large virtuality in the hard scattering. The reason is that transverse spin asymmetries are associated with off-diagonal terms in the spin density matrix of a quark (when a helicity
basis is used). But in QCD, quark helicity is conserved within a typical hard scattering, so that there are no interference terms of the form:

\[
\langle \text{helicity}^+ | \text{final state} \rangle \langle \text{final state} | \text{helicity}^- \rangle.
\]  

This result is true to all orders of perturbation theory provided that

- Quark masses are negligible on the scale of \( p_\perp \).
- All couplings are vector or axial vector. (Thus both QCD and electroweak physics with the photon, \( W \) and \( Z \) are covered.)
- Final state polarizations are not measured.

Examples of this result are the higher twist nature of the transverse spin asymmetries in high \( p_\perp \) inclusive single pion production in singly polarized hadron-hadron collisions and in totally inclusive deeply inelastic scattering (the structure function \( G_2 \) [7]).

To measure the transversity distribution at the twist-2 level in a standard model process, we must probe the transversity of at least two quarks that participate in the hard scattering. In this paper, we propose that one of these quarks can be in the final state: a measurement can be made of the transverse spin state of a final state quark, that is, a quark that initiates a jet. Furthermore, this can be done by examining the correlation of two hadrons of large \( z \) and small relative transverse momentum in the jet. There are several advantages: The experiment can be done in a singly polarized collision. Thus one can go to a kinematic region where dominant process is of valence quarks scattering on relatively small \( x \) gluons, so that the cross section can be large. The subprocess asymmetry can be large, and we expect a substantial asymmetry in the fragmentation.

Another possibility is to measure something like an intrinsic \( k_\perp \) distribution in fragmentation. This has a twist-2 transverse spin asymmetry, and is discussed in a separate paper [8].

Related earlier work can be found in [9, 10]. In addition, similar ideas to ours have been proposed by Efremov, Mankiewicz and Törnqvist [11]. They concentrated on the case of longitudinal polarization, where one has to measure a three particle correlation to get a leading-twist asymmetry.

2 Factorization

2.1 Unpolarized

To define our notation and normalizations, let us recall the factorization theorem for high \( p_\perp \) inclusive single particle production in hadron-hadron collisions \( A + B \rightarrow C + X \). Let the initial-state hadrons have momenta \( p_A \) and \( p_B \), and let the observed hadron have momentum \( p_C \). The factorization theorem reads

\[
E_C \frac{d\sigma}{d^3p_C} = \sum_{abc} \int d\xi_A d\xi_B \frac{dz}{z} f_{a/A} (\xi_A) f_{b/B} (\xi_B) |k_c| \frac{d\hat{\sigma}}{d^3k_c} D_{C/c} (z),
\]

which is illustrated in Fig. 1. The sum is over the various flavors of parton (quarks, antiquarks and gluon) that can participate in the hard scattering process, while \( f_{a/A} \) and \( f_{b/B} \) are the parton densities for the initial hadrons, and \( D_{C/c} (z) \) is the fragmentation function. The hard scattering function \(|k_c| d\hat{\sigma} / d^3k_c\) is for the scattering \( a + b \rightarrow c + X \) at the parton level; it is a purely ultraviolet function, with all mass singularities canceled, so that it can be calculated perturbatively. The variable \( z \) represents the fractional momentum of the measured hadron relative to its parent quark, so that we set \( k_c = z p_C \), when we use the center-of-mass frame of the hard scattering. (Strictly speaking, we should use a light-front definition.) Corrections to Eq. (2) are higher twist; that is, they are suppressed by a power of the transverse momentum of \( C \).

It can be checked that an interpretation of the fragmentation function, with the normalizations indicated, is that \( z D_{C/c} (z) \, dz \) is the number of hadrons of type \( C \) in a parton of type \( c \) that have fractional momentum \( z \) to \( z + dz \). Because of the factor \( z \), it is common to define the fragmentation function to be \( d_{C/c} (z) \equiv \)
\[ z D_{C/c}(z), \text{ rather than } D. \] However, the behavior of \( D \) under Lorentz transformations is simpler, and this will be more convenient for us when we define a function for the fragmentation into two observed hadrons.

It might appear that we have neglected the possibility that the hadron \( C \) has transverse momentum relative to the parton \( c \). However, this is not so. In accordance with the derivation\[12\] of Eq. (2), we have actually integrated over all small values of this transverse momentum, while realizing that the dependence of the hard scattering on small changes in the transverse momentum is higher twist. Large values of this transverse momentum are correctly taken care of by the higher order corrections to the hard scattering function.

### 2.2 Polarized Case

We now make two extensions to Eq. (2). The first is to take account of polarization, and the second is to allow the observed final-state hadron \( C \) to be replaced by a two-particle state.

Polarization is easily taken account of. First, whenever an incoming hadron is polarized, the quarks and gluons entering the hard scattering may themselves be polarized, and must be equipped with a spin density matrix \( \rho \[13, 14, 15\] \). In the case of a massive on-shell spin-\( \frac{1}{2} \) particle, like a proton, \( \rho \) is completely specified by the particle’s Pauli-Lubanski spin vector. But in the case of a massless particle or of a parton, only the density matrix itself may be used, since the Pauli-Lubanski vector is singular in the massless limit.

The density matrix of the final-state parton \( c \) is determined by the initial-state polarization and by the polarization dependence of the hard scattering cross section \( \hat{\sigma} \). If the fragmentation depends on the spin of this parton, then it will have some non-trivial analyzing power. This can be represented by a matrix \( \rho_d \), which we will call the decay matrix. It is the exact analog of a density matrix in the initial state, except that it will be normalized so that a fragmentation with no analyzing power will have \( \rho_d \) equal to the unit matrix.

In this paper, we will consider the case that only one of the initial state hadrons is polarized, and that we make a measurement sensitive to the spin of the final state parton \( c \). Then Eq. (2) must be replaced by

\[
E_C \frac{d\sigma}{d^3p_C} = \sum_{abc} \int d\xi_A d\xi_B \frac{dz}{z} \rho_{\alpha\alpha'} f_{a/A}(\xi_A) f_{b/B}(\xi_B) H_{\alpha\alpha'\beta\beta'}(a + b \rightarrow c + X) \rho_{d,\beta\beta'} D_{C/c}(z),
\]

where \( H \) is the hard scattering cross section with the density and decay matrices factored out. Eq. (3) gives the cross section for the process \( A^\uparrow + B \rightarrow C + X \). Its derivation is a minor generalization of the derivation given in [14] for processes in which both of the initial-state hadrons are polarized.
This equation represents the most general case. Now consider the case that hadron \( A \) is transversely polarized. Then the transversity of a quark in \( A \) will be proportional to the hadron’s transversity. But the gluons will be unpolarized, because of conservation of angular momentum about the axis of collision, as Artru and Mekhfi \[14\] explained. Now, transversity corresponds to the off-diagonal part of the quark’s density matrix. So helicity conservation in the hard scattering implies that we need the final-state parton \( c \) to be a quark, and we need its decay matrix to be off-diagonal, if we are to get a leading-twist asymmetry under reversal of the hadron’s transverse spin. Thus for the spin dependence, the sums over \( a \) and \( c \) in Eq. (3) are only over flavors of quark and antiquark, but the sum over \( b \) is over gluons as well.

The standard measurement is for the observed final-state hadron \( C \) to be a pion. Then there is certainly no sensitivity of the fragmentation function to the polarization of the quark, so that the decay matrix \( \rho_d \) is unity. In this case, the asymmetry in the complete cross section in Eq. (3) is zero. The only asymmetry can come from higher twist corrections, which are in fact \[1, 3\] twist-3. This implies that, at large \( Q \), the asymmetry is roughly \( M/Q \), where \( Q \) is the scale of the hard scattering, say the \( p_{\perp} \) of the produced quark (relative to the beam axis), and \( M \) is some fixed hadronic mass scale.

But Eq. (3) applies to a general hadronic state \( C \). (The proof only requires that the state have an invariant mass much less than \( Q \).) So to get a nonzero asymmetry at the leading twist level, we merely need to find a state that is polarization sensitive. This is what we will do in the next section.

3 Polarization Sensitive Final States

In this section, we discuss a number of candidates for polarization-sensitive final states.

3.1 Proton

In principle we could choose the final-state hadron \( C \) to be a proton and measure its polarization. This should give a substantial asymmetry. Indeed, it is known that in the distribution of partons in a hadron, the flavor and spin states of quarks at large \( x \) reflect the corresponding quantities for their parent hadron. Similarly, in fragmentation, it is known that the ratio of positive to negative pions at large \( z \) in a jet is substantially correlated with the charge of the initiating parton \[15\]. So it would be surprising if the contrary were true for the spin state of hadrons at large \( z \) in a jet.

Unfortunately, the measurement of the spin of a proton of many GeV of energy in a typical high-energy-physics detector is rather hard. (It is not impossible: one could conceive of measuring the rescattering of the proton in the detector and using some polarization sensitive process as a polarimeter.) So we would do better to try a different state for \( C \).

3.2 \( \Lambda \)

The most obvious measurement \[14\] is to pick out \( \Lambda \)s in the final state, since their decay is self analyzing. However, they are produced rarely, and in a simple quark model, the spin of the \( \Lambda \) is carried by its strange quark, which would substantially reduce the asymmetry in the fragmentation. This is by no means an ironclad prediction, and is worth checking.

But the \( \Lambda \) might result from the decay of a \( \Sigma \), in which case the above argument does not hold.

3.3 \( \Sigma^0 \)

A better bet might be to measure the production of \( \Sigma^0 \), with a measurement of the polarization of the \( \Lambda \) in its decay serving to probe the spin of the \( \Sigma \). This seems a more complicated, indirect method.

Note that a final-state gluon may acquire a polarization from the hard scattering, independently of the spin of the initial-state hadrons. But this will produce no asymmetry under reversal of the hadron’s polarization.
3.4 \( \rho \) meson

One could imagine measuring production of \( \rho \) mesons. Now, the density matrix of a massive spin-1 particle can be decomposed into components of helicity 0, 1 and 2, and the off-diagonal part of the density matrix of a quark corresponds to a helicity flip of one unit. Therefore, angular momentum conservation implies that only the helicity 1 part of the density matrix of the \( \rho \) gives the desired a transverse spin asymmetry in the fragmentation. However, there is only one Lorentz invariant in the amplitude of the decay \( \rho \to \pi\pi \), so that measurements can only get at the symmetric part of the density matrix, whereas the helicity 1 part of the density matrix is antisymmetric.

It should be noted that the \( \rho \) meson in fragmentation appears over a rather larger background of continuum \( \pi\pi \) states, and one could certainly have an interference effect. This would appear as a special case of the inclusive two pion measurement, which we will discuss next.

3.5 Inclusive Two Pion

Now let us consider the inclusive production of two pions, but without requiring them to come from the decay of a \( \rho \). In Eq. (3), we can use the following charge states of nonidentical pions:

\[
C = \pi^+\pi^0 \text{ or } \pi^-\pi^0 \text{ or } \pi^+\pi^-.
\] (4)

The derivation of the factorization theorem does not care what kind of a system the measured hadronic final state \( C \) is, provided only that its invariant mass is much less than the natural scale of \( p_\perp \) in the hard scattering. Thus Eq. (3) continues to be valid. We may choose the invariant mass of the two pion system to be in whatever range, say below a GeV or so, that maximizes the asymmetry. Presumably the appropriate range will be where interference effects, e.g., \( \rho - \omega \rho - \text{continuum} \), are important.

One can also measure the symmetric charge combinations, e.g., \( C = \pi^+\pi^+ \), but the same argument as applied to the \( \rho \) will show that the asymmetry in the distribution of the two pions must then be antisymmetric in the two pions, and will therefore have a zero when the two pions have equal values of their longitudinal momentum fractions. This suggests that the overall values of the asymmetry should be less than when the two pions are unequal in charge.

Note also that the EMC [4] measured the distribution of two pion systems in deeply inelastic scattering, as a function of the \( z \) of the pair and of its invariant mass. They observed a substantial \( \rho \) peak, but over a large continuum. Thus the unpolarized part of the fragmentation function to two pions, \( D_{\pi\pi/q}(z) \), is actually known.

To get at the polarization dependence, one must measure the plane of the two pions. For the larger invariant masses, one must remember that the two pions can come out of two different jets, and so there will be a correlation between the plane of the pions and the plane of the scattering, and hence a non-uniform azimuthal distribution. This correlation will have some tail down to low invariant masses. What we are looking for is a spin asymmetry in the azimuthal distribution of the plane. We will examine this in more detail in the next section.

4 Fragmentation Functions

Experimentally, the spin-dependence in Eq. (3) manifests itself in the angular distribution of final-state particles. To exhibit this, we write Eq. (3) in terms of transverse spin vectors instead of density matrices. We define \( s_A \perp \), \( s_a \perp \), and \( s_c \perp \) to be respectively the transverse spins of the initial hadron \( A \), of the quark \( a \) and of the quark \( c \). Our normalization is such that for a fully polarized particle of helicity \( \lambda \) and transverse spin \( s_\perp \), we have \( \lambda^2 - s_\perp^2 = 1 \).

The decay matrix in Eq. (3) can be written in terms of \( \lambda_d \) and \( d_\perp^d \), which represent the analyzing power of the final state \( C \) for the helicity and transverse spin of the quark initiating the jet. The dependence on the spin of the quark \( c \) is then proportional to \( \lambda_c \lambda_d - s_c \perp \cdot d_\perp \), of which we will only need the \( -s_c \perp \cdot d_\perp \) term in this paper.

In Feynman graph calculations of cross sections, the spin-dependence of the hard scattering function can be obtained by taking a trace of Feynman graphs times complex conjugate graphs with projectors of the
the following form \[17\]:

\[
\frac{1}{2} \mathcal{J}(1 + \lambda \gamma_5 + \gamma_5 k_\perp),
\]

which represents the appropriate sum over quark wave functions. The 1/2 represents the spin average used for an initial state, and is omitted for the final state. More modern calculations are done directly in terms of helicity amplitudes \[18\].

Then Eq. (3) can be rewritten as

\[
E_C \frac{d\sigma}{d^3p_C} = \sum_{abc} \int d\xi_A d\xi_B \frac{dz}{z} f_{a/A}(\xi_A) f_{b/B}(\xi_B)
\]

\[
[H(a + b \rightarrow c + X) + A_L \lambda \lambda_H (a + b \rightarrow c + X)
\]

\[
- A_T s_\perp^\mu d_\perp^\nu H_{\mu\nu}(a + b \rightarrow c + X)] \frac{D_{C/c}(z)}{z},
\]

which is linear in the various spin vectors. Here, \(\lambda\) and \(s_\perp\) denote the helicity and transverse spin of hadron \(A\). We have used the result that, in a parity invariant theory like QCD, the helicity and transverse spin of quark \(a\) are proportional to the corresponding quantities for the parent hadron \(A\); the constants of proportionality are denoted by \(A_L\) and \(A_T\). Note that \(A_L\) and \(A_T\) are functions of the parton variable \(\xi_A\), of the flavor \(a\) of the initial quark, and of the Altarelli-Parisi scale \(Q\) at which the parton densities are computed.

We have used parity invariance together with helicity conservation in Feynman graphs with massless quarks to show that the hard-scattering cross section has the form of a spin-independent term \(H\) plus spin-transfer terms proportional to \(\lambda\lambda_H\) and \(s_\perp \cdot d_\perp\).

For a given incoming parton state (particular values of \(\xi_A, \xi_B, a,\) and \(b\)), the outgoing quark \(c\) has helicity

\[
\lambda_c = A_L \lambda H_L H,
\]

and transverse spin

\[
s_\perp^c = A_T s_\perp^a H_{\mu\nu} H. \]

The minus sign with the \(H_{\mu\nu}\) in Eq. (8) arises because the transverse spins are essentially Euclidean vectors.

An alternative form of the spin of the final-state quark is

\[
s_\perp^c = T s_\perp^a = A_T T s_\perp^a. \]

Here, \(s_\perp^a\) and \(s_\perp^c\) are vectors obtained by rotating the initial quark and hadron spin vectors in the plane of the hard scattering, as in Fig. 2. We have used the parity invariance of QCD, as applied to the hard scattering, to show that the final-state quark spin is proportional to the rotated initial-state quark spin, with a coefficient of proportionality \(T\) that we call the spin-transfer coefficient.

### 4.1 Operator Definitions

To be able to derive Feynman rules for calculations with parton distribution fragmentation functions, we need operator definitions of these functions. For the unpolarized case, these were given in \[19\], with a motivation coming from light-front quantization. The definitions involve a light-like vector \(n^\mu\) (a different one for each of the parton densities and fragmentation functions in Eq. (9)). All of these functions are invariant under scaling of their \(n^\mu\) vector, i.e., under \(n^\mu \rightarrow \lambda n^\mu\).

In the case of the distribution functions, this vector together with the parent hadron’s momentum \(p^\mu\) defines the axis of the scattering. If we let the light-like vector for this case be \(n_0^\mu\) then the parton density depends only on the ratio \(\xi = k \cdot n_a/p \cdot n_a\), where \(k^\mu\) is the momentum of the (massless) parton entering the hard scattering. This can be rewritten in terms of the center-of-mass vector as \(\xi = k \cdot N/\sqrt{p \cdot N^2 - p^2 N^2}\).

Similarly, the fragmentation of a quark of momentum \(k^\mu\) to a single hadron of momentum \(p^\mu\) depends on one kinematic variable \(z = p \cdot n/k \cdot n\), where \(n^\mu\) now represents the appropriate light-like vector for the definition of the fragmentation function.
Definitions for the polarized case were given in [13, 15, 20] for the distribution functions. Definitions of the polarized fragmentation functions for the case that the transverse momentum dependence is measured were given in [8]. They are easily modified for our case. The unpolarized fragmentation function is

\[ D_{C/c} \equiv \sum_X \int \frac{dy^-}{12(2\pi)^3} e^{ik^+y^-} \text{Tr} \gamma^+ \langle 0 | \psi_c(0, y^-, 0_{\perp}) | CX \rangle \langle CX | \bar{\psi}_c(0) | 0 \rangle. \]  

The factor of 1/12 is the product of a factor 1/2 which occurs with all such definitions for fermions and a factor 1/6 for an average over the spin and colors states of the quark. The sum over \( X \) means that we sum over all final states \(|CX\rangle\) containing the chosen hadronic system \( C \). Suitable path-ordered exponentials of the gluon field along the line \( y^+ = y_{\perp} = 0 \) must be inserted into this and the next definition to make them gauge-invariant. We are using light-front coordinates such that \( y^\mu y_{\mu} = 2y^+ - y_{\perp}^2 \); thus the vector \( n^\mu \) used above is \( n^\mu = \delta^\mu_+ \).

The definition (10) is easily generalized [8] to give the transverse spin dependence \( s_{c,\perp} \cdot d_{\perp} D \) of the fragmentation of a polarized quark of transverse spin \( s_{c,\perp} \):

\[ d_{\perp} D_{C/c} \equiv \sum_X \int \frac{dy^-}{12(2\pi)^3} e^{ik^+y^-} \text{Tr} \gamma^+ \gamma_5 \gamma_{\perp} \cdot s_{\perp} \langle 0 | \psi_c(0, y^-, 0_{\perp}) | CX \rangle \langle CX | \bar{\psi}_c(0) | 0 \rangle. \]  

These definitions imply that the \( k^- \) and \( k_{\perp} \) of the incoming parton are integrated over. At first sight Eq. (11) depends on two light-like vectors to define the + and − axes. In fact, it is invariant under \( s_{c,\perp}^\mu \rightarrow s_{c,\perp}^\mu + a n^\mu \), so that it actually only depends on one of the light-like vectors: \( s_{c,\perp}^\mu \) has merely to be chosen to satisfy \( s_{c,\perp} \cdot n = 0 \).

### 4.2 Fragmentation to two hadrons

When the observed state \( C \) in the fragmentation function is a single particle with no spin measured, it is easy to see from Eq. (11) that there is no polarization dependence, because from the vectors we have available we cannot construct a non-zero scalar quantity linear in the quark’s transverse spin.\(^2\)

\(^2\)The definition for the helicity dependence of the fragmentation will not concern us in this paper.

\(^3\)This requirement was evaded in [8] by the measurement of the transverse momentum of \( C \) relative to the quark: this effectively introduced another vector that played the role that in this paper is played by the relative transverse momentum within a two particle system for \( C \). Higher twist contributions to the cross section can also avoid this result, since more complicated Dirac matrix structures are then involved in the definition of the relevant fragmentation functions (compare [3]).
But we can construct such a quantity when $C$ is a two-hadron state: it gives dependence on the azimuthal angle between the plane of the two hadrons and the transverse spin vector. We let $p'_C$ and $p''_C$ be the momenta of the two hadrons. The unpolarized fragmentation function, $D_{12/c}$ depends on three kinematic variables: the invariant mass of the pair $m^2_{12} \equiv (p_{C1} + p_{C2})^2$, and the fractional momentum variables $z_1 \equiv p_{C1} \cdot n = \sqrt{p_{C1} \cdot N^2 - m^2_{12} N^2/k_c \cdot N}$ and $z_2 \equiv p_{C2} \cdot n = \sqrt{p_{C2} \cdot N^2 - m^2_{12} N^2/k_c \cdot N}$. Here $k_c$ is the momentum of the parton coming out of the hard scattering, and $N$ is the rest vector of the hard scattering.

Define an axial vector:

$$\Sigma^\mu \equiv \frac{\xi_{\mu03}, n^\alpha p'^{\gamma}_{C1} p''_{C2}}{(p_{C1} + p_{C2}) \cdot n m_{12}}.$$  

(12)

Then we get a scalar by contracting it with the quark spin: $s_{c \perp} \cdot \Sigma$. Hence we can write the spin-dependence of the fragmentation in the form

$$d_{12,c} D(p_1, p_2, k_c, N) = \Sigma^\mu A_{12/c}(z_1, z_2, m_{12}),$$  

(13)

where $A_{12/c}$ is a scalar function. The factorization theorem now becomes

$$E_{C1} E_{C2} \frac{d\sigma}{d^4 p_{C1} d^3 p_{C2}} = \sum_{abc} \int d\xi_A d\xi_B \frac{dz}{z} f_{a/A}(\xi_A) f_{b/B}(\xi_B) [H(a + b \to c + X)D(z_1, z_2, m_{12}) - A_T(\xi_A) A_{12/c}(z_1, z_2, m_{12}) s^\mu_{A \perp} \Sigma^\nu H_{\mu\nu}(a + b \to c + X)].$$  

(14)

No pseudoscalar can be constructed with the helicity $\lambda_c$ and the vectors we have available so far. Thus there is no leading-twist dependence of the fragmentation of a longitudinally polarized quark to two hadrons. So we must set $\lambda_d$ in Eq. (11) to zero.

The scalar product on which the fragmentation depends is

$$-s_{c \perp} \cdot \Sigma = \frac{|k_\perp|}{m_{12}} |s_\perp| \cos \phi,$$  

(15)

where $\phi$ is the angle between the transverse part of the initial hadron’s spin vector and the normal to the plane of the two detected final-state particles. We have defined $k_\perp$ to be the transverse momentum of one of the particles in $C$, relative to the axis defined by $p''_{C1} + p''_{C2}$. To be more precise, we choose a $z$ axis along the spatial part of $p_{C1} + p_{C2}$. We write

$$p_{C1} = (k_\perp, p''_{C1}), \quad p_{C2} = (-k_\perp, p''_{C2}).$$  

(16)

Then

$$\Sigma^\mu = (0, k^-_c, -k^-_c, 0)/m_{12}.$$  

(17)

In the unpolarized case, the plane of the two hadrons 1 and 2 is uniformly distributed in azimuth. But Eq. (13) shows that in the case of transverse polarization there will be a $\cos \phi$ asymmetry.

There are two sources of systematic error in this statement. The first is that the experimental apparatus is likely to be asymmetric in this azimuthal angle about the axis of the jet. To cancel this, one should look for a change in the angular distribution when the spin of the initial-state hadron is reversed.

There is also theoretical error, since, when the relative $k_\perp$ of the final-state hadrons gets large enough, one should consider that the hadrons arise from the fragmentation of two different jets, as in a $2 \to 3$ parton process that is a higher order correction to the lowest order $2 \to 2$ process. This will certainly give a distribution that is not uniform in azimuth. Again, one should look to the asymmetry under reversal of the initial-state spin to avoid these confounding effects. The spin asymmetry will go away at large $k_\perp$, since the 2 measured hadrons will then come from separate jets, and for that situation we have seen that the spin asymmetry is higher twist.

To check that the transverse spin asymmetry of the fragmentation is indeed permitted by the symmetries of QCD, Collins and Ladinsky [21] have calculated the fragmentation function of a quark to two pions in a

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4To get a helicity asymmetry, we need a third measured particle. Such asymmetries were considered by Nachtmann [14], and more recently by Efremov et al. [17]
linear sigma model of quarks and pions. This is of course by no means a perfect model, but it does have the correct chiral symmetries. Collins and Ladinsky do indeed find a large spin asymmetry. To get the asymmetry requires that the quarks have a mass, as is obtained from chiral symmetry, and that there be interference effects with non-trivial phases. These interference effects occur between continuum two-pion production and decay of the sigma resonance. Analogous effects are to be expected in more realistic models.

5 Hard Scattering Calculations

In this section we present the results of lowest order calculations of the spin asymmetries for the hard scattering.

The calculation of the spin transfer coefficient is most easily done by using a helicity decomposition of the scattering amplitudes. We let $A_{\alpha\beta x}^{a\beta'}$ be the helicity amplitude, with $s_a$ and $s_c$ denoting the helicities of quarks $a$ and $c$, and with $x$ denoting the helicity state of all the other partons. Then the hard scattering coefficient in Eq. (3) is

$$H_{\alpha\alpha';\beta\beta'} \propto \sum_x A_{\alpha\beta x}^{a\beta'} A_{\alpha'\beta' x}^{a'\beta'}. \quad (18)$$

Chirality conservation for massless quarks implies that $H_{\alpha\alpha';\beta\beta'}$ is nonzero only when:

- $\alpha = \alpha'$ and $\beta = \beta'$,
- or $a$ and $c$ are both quarks or both antiquarks of the same flavor, $a = c$, and $\alpha = \beta$ and $\alpha' = \beta'$,
- or $a$ and $c$ are the quark and antiquark of each other, $a = -c$, and $\alpha = -\beta'$ and $\alpha' = -\beta$,

Thus the only nonzero terms are $H_{++;++}$, $H_{--;--}$, $H_{+--;+}$, $H_{--;++}$, and $H_{++;--} = H_{--;++}$. The last two of these can only be nonzero if $a = \pm c$. Parity invariance gives $H_{++;++} = H_{--;--}$, $H_{+--;+} = H_{--;++}$, and $H_{++;--} = H_{--;++}$.

Then the spin transfer coefficient is

$$T = \frac{H_{++;--}}{H_{+--;+} + H_{++;--}} = \frac{\sum_x A_{++;+}^{a\beta x} A_{--;-}^{a'\beta' x}}{\sum_x A_{++;+}^{a\beta x} A_{--;-}^{a'\beta' x} + \sum_x A_{+--;+}^{a\beta x} A_{--;-}^{a'\beta' x}} \quad (19)$$

Formulae for the helicity amplitudes can conveniently be found in the book by Gastmans and Wu [18].

5.1 Deeply Inelastic Scattering

Exactly the same principles apply to deeply inelastic lepton scattering as to hadron-hadron scattering, except that one of the incoming particles is a lepton and hence can be treated as a parton. We let $p_A^\mu$ be the momentum of the incoming hadron, and we let $l'^\mu$ and $l'^\mu$ be the momenta of the incoming and outgoing leptons.

The calculation of the spin transfer coefficient to lowest order was already done in [22]. We just use the tree graphs for lepton-quark scattering with photon exchange. The result is

$$T_{eqq} = \frac{4(1 + \cos \theta)}{4 + (1 + \cos \theta)^2} = \frac{1 - y}{1 - y + \frac{1}{2} y^2}. \quad (20)$$

Here $\theta$ is the scattering angle in the lepton-quark center-of-mass, and $y$ is the usual variable $p_A \cdot (l - l')/p_A \cdot l$, which is $(E - E')/E$ in the target’s rest frame, and $(1 - \cos \theta)/2$ in the lepton-quark center-of-mass. The spin transfer coefficient is 100% at small scattering angles, and decreases to zero for exactly backward scattering. But even at 90° ($y = 1/2$), it is 80%.
5.2 Hadron-Hadron Scattering

There are several parton subprocesses \(a + b \rightarrow c + X\), each of which has its own spin transfer coefficient \(T_{abc}\). (Our notation is that of the initial-state partons it is \(a\) that is polarized.) As we have seen, chirality conservation implies that \(T_{abc}\) is zero unless \(a\) is a flavor of (anti)quark and either \(c = a\) or \(c = -a\). Here \(-a\) denotes the antiparticle of \(a\).

Thus at the tree level we need to consider the following processes: \(i + g \rightarrow i + g, i + i \rightarrow i + i, i + \bar{i} \rightarrow i + \bar{i}, i + \bar{i} \rightarrow i + i, i + j \rightarrow i + j, i + \bar{j} \rightarrow i + \bar{j}\). Here \(j\) denotes any flavor of quark that is not the same as \(i\). We took the helicity amplitudes we needed from the book by Gastmans and Wu [18].

For the processes \(qq' \rightarrow qq'\), with different quark flavors, or for \(qg \rightarrow qg\), we have

\[
T_{qq} = T_{qq'} = \frac{4(1 + \cos \theta)}{4 + (1 + \cos \theta)^2} = \frac{1 - y}{1 - y + \frac{1}{2}y^2},
\]

the same as for electron-quark scattering. Here, \(\theta\) is the angle between the incoming polarized quark and the detected outgoing parton. Note that in the factorization formula each of these subprocesses will give two contributions to the cross section, one with \(c = q\), with the spin transfer given by Eq. (21), and one with \(c = q'\) or \(c = g\), with zero spin-transfer. We are assuming that of the incoming quarks, it is the \(q\) that is polarized. The \(q'\) may also be the antiquark of a flavor other \(q\).

For identical quark flavors, we have

\[
T_{qq} = \frac{4(1 + \cos \theta)^2(1 + 2 \cos \theta)}{(11 + \cos^2 \theta)(1 + 3 \cos^2 \theta)} = \frac{(1 - y)^2(3 - 4y)}{(3 - y + y^2)(1 - 3y + 3y^2)},
\]

and

\[
T_{qq} = \frac{4(1 + \cos \theta)(7 - \cos \theta)}{35 + 8 \cos \theta + 10 \cos^2 \theta - 8 \cos^3 \theta + 3 \cos^4 \theta} = \frac{(1 - y)(3 + y)}{3 - 2y + y^2 - 2y^3 + 3y^4},
\]

\[
T_{qq} = -\frac{4(1 + \cos \theta)^2}{35 - 8 \cos \theta + 10 \cos^2 \theta + 8 \cos^3 \theta + 3 \cos^4 \theta} = \frac{(1 - y)^2}{3 - 6y + 13y^2 - 10y^3 + 3y^4}.
\]

For unequal flavor annihilation, and for pure gluon initial states, we get zero:

\[
T_{qqq'} = T_{gg} = 0.
\]

Notice that, with one exception, all of the nonzero coefficients equal unity at zero scattering angle, \(y = 0\). The exception is the coefficient, \(T_{qqq}\).

6 Measurements

We now summarize the measurements that can be done to probe the polarized fragmentation function. The distributions of quark transversity in a hadron and the polarized fragmentation functions are non-perturbative quantities for which we have no detailed predictions. But they should be universal: the same in different processes. Measurements of them will shed light on the chiral properties of hadron wave functions and of the long-distance part of the fragmentation. In the valence region for the distribution and fragmentation functions, it is reasonable to conjecture from our experience with other flavor quantum numbers that there are large transverse spin correlations.

We are effectively trying measuring a spin transfer from the initial state to the final state particle. Suppose temporarily that only one parton subprocess dominates, and that only a small range of the parton variables is important. Then the spin asymmetry in the cross section is a product of the asymmetries in its factors:

\[
A_\sigma = A_f T_H A_D.
\]
where $A_f$ is the transverse spin asymmetry in the parton density, $T_H$ is the spin transfer in the hard scattering subprocess, and $A_D$ is the analyzing power of the fragmentation. The most general case simply involves a weighted average of Eq. (27) over the different subprocess (quark + gluon → quark + gluon, quark + quark → quark + quark, etc) and over the kinematic integrals.

The asymmetry for spin transfer in the hard subprocess can be large, as shown by our calculations. Moreover, in the valence region for the incoming parton, we expect large polarization for the quarks. Similarly, we might expect that at large $z$, say $z > 0.5$, the spin of the final state hadron is highly correlated with the spin of the quark initiating the jet, just as the flavor is correlated. Thus, we might expect a large asymmetry in the overall process.

6.1 Deeply Inelastic Scattering

Here one measures the distribution of two pions in a jet in collisions of unpolarized electrons on transversely polarized protons (or neutrons).

There should be a dependence of the azimuth of the plane of the two pions that reverses sign when the spin of the incoming hadron is reversed. The azimuthal angle is about the jet axis. If the two pions are the leading pions in the jet, it should be sufficient to define the jet axis by the sum of the pions’ momenta. Maximum asymmetry will most likely occur under the following conditions:

- When $x_{Bj}$ is large, so that the quark entering the hard scattering will be highly polarized.
- When the direction of the two pions is roughly that of the direction of the jet predicted by the parton model. This avoids dilution by the zero asymmetry for pions in gluon jets.
- When the fragmentation variable $z$ is large, so that the pions’ quantum numbers follow those of the quark initiating the jet. The two leading pions in the jet are the most obvious candidates.
- When the charges of the pions are unequal. Note that the asymmetry will reverse sign when the flavors of pions of equal longitudinal momentum are exchanged.

The asymmetry can also be measured for other flavor of hadron pairs (e.g., involving kaons), for which there will be corresponding polarized fragmentation functions.

The asymmetry has a kinematic zero when the invariant mass of the pion pair is at threshold. It will be higher twist when the invariant mass of the pair is large, for then the pions will come from different jets.

One test of the polarization dependence of the hard scattering is in the dependence of the asymmetry on the scattering angle $\theta$ in the lepton-quark frame, when $z$ and $x_{Bj}$ are fixed. This results from the $\theta$ dependence of the spin transfer coefficient predicted by QCD in Eq. (20).

It is also possible to perform a measurement of the azimuthal distribution of $\Lambda$ decays; this will give a measurement of the polarization dependence of the fragmentation of quarks to $\Lambda$s.

6.2 Electron-Positron Annihilation

In electron-positron annihilation, one can try measuring the azimuthal correlations between a pair of pions on one jet and a pair in another jet [11]. Again, the pion pairs should be at large fractional momentum: they should be leading pions to get the biggest asymmetry.

The correlation will be proportional to the product of two fragmentation asymmetries, and thus it will give a measurement of the fragmentation asymmetry independently of the parton transversity distribution in initial-state hadrons. But the measurement will require good control of the systematics, since there will be no initial-state spin to reverse.

6.3 Hadron-Hadron Scattering

The measurement of the azimuthal distribution of pion pairs in collisions of unpolarized hadrons on transversely polarized nucleons is very similar to the case of deeply-inelastic scattering. The pion pair will need
to be of low invariant mass (say less than a GeV) but of large transverse momentum relative to the incoming beams. This will ensure that the pair comes from a hard jet. Again, the azimuthal angle is of the pion pair about the jet axis.

A particularly favorable configuration is when the jet is somewhat forward in the overall center-of-mass, so that the hard collision would typically be of a valence quark from the polarized hadron and a small $x$ gluon from the unpolarized hadron. The valence quark would probably be highly polarized, and the large number of small $x$ gluons will enhance the event rate.

As in the case of deeply inelastic scattering, one can examine the dependence of the spin asymmetry on the angle $\theta$ of the hard scattering, in the parton-parton center-of-mass. This would involve measuring opposite jets associated with pion pairs so that the hard scattering can be reconstructed. (To a first approximation we will have two-jet events.)

7 Conclusions

We have shown that the transverse spin of a quark initiating a jet can be probed by the azimuthal dependence of pion pairs, particularly leading pions, in the jet fragments. This allows leading twist spin asymmetries in singly polarized collisions.

Measurements can be made in deeply inelastic lepton-hadron scattering and in hadron-hadron scattering. They will give jointly a transverse polarization asymmetry of quark fragmentation and of the transversity distribution of quarks in a transversely polarized hadron, and will test the spin transfer coefficients predicted by QCD for the hard scattering.

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