Zero point and zero suffix methods with robust ranking for solving fully fuzzy transportation problems

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Abstract. Transportation issue of the distribution problem such as the commodity or goods from the supply to the demand is to minimize the transportation costs. Fuzzy transportation problem is an issue in which the transport costs, supply and demand are in the form of fuzzy quantities. In the case study at CV. Bintang Anugerah Elektrik, a company engages in the manufacture of gensets that has more than one distributors. We use the methods of zero point and zero suffix to investigate the transportation minimum cost. In implementing both methods, we use robust ranking techniques for the defuzzification process. The study result show that the iteration of zero suffix method is less than that of zero point method.

Keywords: Zero suffix method, Zero point method, Fully fuzzy transportation problems

1. Introduction
The current business competition does not only involve between companies but also the networks of the supply chain. Supply chain is a network of companies that works together to create and deliver a product into the end users or consumers\cite{1}. A company that maintains the continuity of production, need to run the production process more efficient and effective. Inventories can be in the form of raw materials that are stored for processing, processed components, processed materials in manufacturing processes and finished items that are stored for sale. Inventories play an important role for the company to run well \cite{2}.

The last part of supply chain management in a company. It is related to the transportation problems. The transportation problem is the study on distribution of a commodity or items from a supply quantity to a demand quantity to minimize the transportation cost. The purpose of the fuzzy transportation problem is to determine the delivery schedule by minimizing the total of fuzzy cost in order to keep the inventory quantity and fuzzy demand.

CV. Bintang Anugrah Elektrik is a company that engages in the manufacture of the gensets, it has more than one distributors. In the delivery activity at CV. Bintang Anugrah Elektrik, the company sends the genset to various cities, therefore CV. Bintang Anugrah Elektrik requires a policy on shipping costs to minimize the cost so that it must be a proper method to solve transportation problem. Transportation problems can be solved by using zero point method and zero suffix method to get the minimum cost.
Ismail Mohideen S. and Senthil Kumar P. used the zero point method of multiplication operation and concluded that it is better than both Vogel’s Approximation method and the modified distribution method [3]. Sharma Gaurav et al. explained that the zero point method as the symmetric procedure for transportation problems, is easy to be applied for any type of transportation problem with an objective function (maximum or minimum). This method provides the decisions for an optimal solution of the logistic problems in the transportation problem [4]. In solving transportation problem, Pandian P. and Natarajan G used zero point method with trapezoidal fuzzy number to find the optimal value of the objective function. This method is the systematic procedure that can be used to make the decisions, when we solve the various types of logistics problems involving fuzzy parameters [5]. Concerning with the fuzzy transportation problem, Samuel E. also studied the method of the increasing of zero point, and concluded that this method was simpler and more efficient compared to VAM, SVAM, GVAM, RVAM, BVAM. He stated that this method was easy to be understood that provided the optimal solution [6].

In the currently, a number of researcher investigate regarding to the transportation solution using zero point method or zero suffix method. Fegade et al. applied the zero suffix method to solve this problem. They inferred that the optimal solution was converted by fuzzy problem to crisp using robust ranking method. Based on fuzzy problem, the total fuzzy cost reaches optimal more effective [7]. Chandrasekaran, S.et al. studied the fuzzy transportation problem using heptagon fuzzy number with the zero suffix method and they found the optimal solution in solving fuzzy problem [8]. Nirmala. G and Anju R used the zero suffix method to solve an optimal solution and they obtained the optimal solution with fewer iterations [9]. Annie Christi M.S and Kumari Shoba. K. applied the robust ranking with the zero suffix method and they got the optimal solution of fuzzy problems accurately and effectively [10]. The aims of the research is to investigate the number of iterations, allocations, optimized solutions of fully fuzzy transportation problems by using Zero Suffix methods and Zero Point methods with robust ranking technique.

2. Preliminaries
In this section, some basic definitions of trapezoidal fuzzy numbers are presented.

2.1 Definition [11]:
Let X denote a universal set. A fuzzy subset $\tilde{A}$ of $X$ is defined as ordered set that its members are as follows:

$$\mu_{\tilde{A}} : X \rightarrow [0,1]$$

which assign to each element $x \in X$ to a real number $\mu_{\tilde{A}}(x)$ in the interval $[0,1]$.

2.2 Definition [5]:
A fuzzy number $\tilde{A}$ is a trapezoidal fuzzy number that is expressed by $(a_1, a_2, a_3, a_4)$ where $a_1, a_2, a_3$ and $a_4$ are real number and the membership function $\mu_{\tilde{A}}(x)$ is given by,

$$\mu_{\tilde{A}}(x; a_1, a_2, a_3, a_4) = \begin{cases} 0 & \text{for } x \leq a_1, \\ \frac{(x-a_1)}{(a_2-a_1)} & \text{for } a_1 \leq x \leq a_2, \\ 1 & \text{for } a_2 \leq x \leq a_3, \\ \frac{(a_4-x)}{(a_4-a_3)} & \text{for } a_3 \leq x \leq a_4, \\ 0 & \text{for } x \geq a_4, \end{cases}$$
2.3 Definition [12]:

Fully fuzzy transport problem (FFTP) is a linear programming problem with the specific structure. If in a transportation issue, all parameters and variables are fuzzy, then it is included as full fuzzy transport issues. The full fuzzy transport issue (FFTP) is formulated as follows:

\[
\text{Min} \tilde{z} \approx \sum_{i=1}^{m} \sum_{j=1}^{n} \tilde{c}_{ij} \tilde{x}_{ij},
\]

subject to

\[
\sum_{i=1}^{n} \tilde{x}_{ij} = \tilde{a}_i \text{ for } i = 1, 2, \ldots, m
\]

\[
\sum_{i=1}^{n} \tilde{x}_{ij} = \tilde{b}_j \text{ for } j = 1, 2, \ldots, n
\]

\[\tilde{x}_{ij} \geq 0 \text{ for all } i \text{ and } j\]

3. Robust Ranking Techniques in Trapezoidal Fuzzy

If \( \tilde{A} \) is the trapezoidal fuzzy number, then the robust ranking can be defined as follows [7]:

\[
R(\tilde{A}) = \int_{0}^{1} (0, 5) \left( a^l_{\tilde{A}}, a^u_{\tilde{A}} \right) d\alpha
\]

where \( \tilde{A} = a_1, a_2, a_3, a_4 \) and

\[a^l_{\tilde{A}}, a^u_{\tilde{A}} = (a_2 - a_1) \alpha + a_1, a_4 - (a_4 - a_3) \alpha\]

4. Zero Point Method and Zero Suffix Method

4.1 The Steps for Solving the Fully Fuzzy Transportation Problem

The steps to solve the fully fuzzy transportation problem by using the zero suffix method and the zero point method with the robust ranking on the trapezoidal fuzzy number can be presented as in the following figure 1:
4.2 Zero Point Method
Steps of Zero Point Method used to find the optimal solution of fuzzy balanced transportation problem [5].

**Step 1**: Construct the fuzzy transportation table of the fuzzy transportation problem, then convert it into the balanced fuzzy transportation table.

**Step 2**: Subtract each row entries of the fuzzy transportation table from the row that has the minimum value.

**Step 3**: Subtract each column entries of the result of fuzzy transportation table after using the Step 2 from the column with minimum value.

**Step 4**: Check if each the fuzzy demand column \( (b_j) \) is less than to the sum of the fuzzy supplies \( (a_i) \) the cost reduction result in the column was zero. Also, check if each the fuzzy supply row \( (a_i) \) was less than to sum of the fuzzy demands column \( (b_j) \) in which the raw value is zero \( (c_{ij} = 0) \). If it is fulfilled, go to step 7, if it is not, go to step 5.

**Step 5**: Close the minimum values with the horizontal line and vertical line that have zero values from the reduction result of the fuzzy transportation table so that some rows or columns are not covered, when they do not fulfill the condition in step 4.

**Step 6**: Construct the revision of the new fuzzy transportation table as follows:

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**Figure 1**: The Fully Fuzzy Transportation Problem for Zero Suffix Method And Zero Point Method
(i) Find the smallest entry of the reduced $w_{ij}$ of fuzzy cost matrix which was not covered by any lines.
(ii) Reduce the smallest reduced costs $w_{ij}$ to all the uncovered costs $c_{ij}$ and add the costs $c_{ij}$ that is covered by the intersection of two lines, then return to step 4.

**Step 7**: Select a reduced cell in the fuzzy transportation table that has the largest of the reduced cost, called $(\alpha, \beta)$. If there are more one value, choose one of this value maximum cost. Say $(\alpha, \beta)$. If there were more than one, then select one.

**Step 8**: Select a cell in the $-\alpha$ row or and $-\beta$ column of the reduced fuzzy transportation table whose the reduced cost zero ($c_{ij} = 0$) and then, select the maximum allocation to the cell. If such cell does not occured for the maximum value, find the next maximum so that such a cell occurs. If such cell does not occur for any value, we select any cell in the reduced fuzzy transportation table in which was reduced cost zero.

**Step 9**: Reform the reduced fuzzy transportation table after deleting on the supply row and demand column.

**Step 10**: Repeat step 7 to step 9 until all the fuzzy supply points are fully used and all the fuzzy demand points are fully received.

**Step 11**: The allocation generates the fuzzy solutions on fuzzy transport issues.

### 4.3 Zero Suffix Method

The zero suffix method is a method for finding the optimal solution of the transportation problems[7]. The steps to solve the transportation problems by using this method can be described as follows.

**Step 1**: Construct the transportation table.

**Step 2**: Subtract each row enteries of the transportation table from the minimum row. Then, subtract each column enteries of the transportation table on a certain minimum column.

**Step 3**: In the matrix cost, there is an entry with zero value in each row and column, then find the suffix value denoted by $S$.

$$ S = \frac{\text{Add the cost of nearest adjacent sides of zero}}{\text{Additional Cost}} $$

**Step 4**: Choose the maximum of $S$ if it has one of them maximum value. If it has two or more of the same value, then choose just one, the cost becomes the allocation of goods by regarding to the demand and supply.

**Step 5**: After step 4, select minimum of $\{a_i, b_j\}$, then allocate it into transportation table. The resulting table should has at least one cost of 0 in each row and column, otherwise repeat step 2.

**Step 6**: Repeat step 3 to step 5 until the optimal solution is obtained. The optimal cost is obtained if the column or row is saturated (suffix values = 0)

### 4.4 Numerical Case

In this section give numerical solution of take a transportation problem for CV. Bintang Anugerah Elektrik using zero point method and zero suffix method. We use robust ranking techniques for the defuzzification process. Next, we give comparison of results of the implication of Zero Point Method and Zero Suffix Method.
Table 1. Fully Fuzzy Transportation Table

| From          | To          | Destination       | Supply          |
|---------------|-------------|-------------------|-----------------|
| West Semarang | Demak       | (35000, 50000,    | (30000, 45000,  |
|               |             | 60000, 75000)    | (2,3,5,7)       |
|               | Purwodadi   | 70000, 90000)    | 55000, 70000)   |
| Source        | Kendal      |                   |                 |
| Temanggung    |             | (30000, 45000,    | (40000, 50000,  |
|               |             | 55000, 70000)    | (1,3,4,5)       |
|               |             | 80000, 100000)   | 75000, 95000)   |
|               | East Semarang| (40000, 45000,    | (40000, 50000,  |
|               |             | 45000, 55000,    | (1,3,4,5)       |
| Demand        |             | 65000, 85000)    | 55000, 70000)   |

Table 1 is a full fuzzy transportation table with form of trapezoid numbers which transportation costs, supply, and demand are quantities fuzzy. Now apply robust ranking technique for the above fuzzy transportation problem

\[
R(\tilde{A}) = \int_0^1 (0,5)(a_1^l, a_1^u) d\alpha
\]

Where \(a_1^l, a_1^u = \{(a_2 - a_1)a + a_1, a_4 - (a_4 - a_3)a\}

\[
R(\tilde{c}_{11}) = R(35000, 50000, 60000, 75000)
\]

\[
= \int_0^1 (0.5)(15000\alpha + 35000 + 75000 - 15000\alpha) d\alpha
\]

\[
= \int_0^1 (0.5)(110000) d\alpha
\]

\[
= 55000
\]

Similarly

\[
R(\tilde{c}_{12}) = 66250, \quad R(\tilde{c}_{13}) = 50000, \quad R(\tilde{c}_{21}) = 75000, \quad R(\tilde{c}_{22}) = 81250, \quad R(\tilde{c}_{23}) = 68750, \quad R(\tilde{c}_{31}) = 55000, \quad R(\tilde{c}_{32}) = 62500, \quad R(\tilde{c}_{33}) = 53750
\]

Rank of all supply

\[
R(\tilde{a}_{11}) = 4.25, \quad R(\tilde{a}_{12}) = 2.5, \quad R(\tilde{a}_{13}) = 3.25
\]

Rank of all demand

\[
R(\tilde{b}_{11}) = 3.25, \quad R(\tilde{b}_{12}) = 4.75, \quad R(\tilde{b}_{13}) = 4
\]

Now, from the robust ranking technique, we have the following crisp value table.
Table 2. Crisp Transportation Table

| Source            | Destination | Demand |
|-------------------|-------------|--------|
| West Semarang     | Demak       | 55000  |
|                   | Purwodadi   | 66250  |
|                   | Kendal      | 50000  |
| Temanggung        |             | 75000  |
| East Semarang     |             | 55000  |
|                   |             | 81250  |
|                   |             | 68750  |
|                   |             | 53750  |
|                   |             | 3,25   |
|                   |             | 4,75   |
| Dummy             |             | 0      |
|                   |             | 0      |
|                   |             | 0      |
|                   |             | 2      |
| Demand            |             | 3,25   |
|                   |             | 4,75   |
|                   |             | 4      |

From table 2, we get the total supply and total demand as follows:

\[
\sum_{i=1}^{n} \tilde{a}_i = 4,25 + 2,5 + 3,25 = 10
\]

\[
\sum_{j=1}^{m} \tilde{b}_j = 3,25 + 4,75 + 4 = 12
\]

It was found that \( \sum_{j=1}^{n} \tilde{a}_i < \sum_{i=1}^{m} \tilde{b}_j \). So the transportation problem was not balanced. Next, to be balanced, \( \sum_{j=1}^{m} \tilde{a}_i = \sum_{i=1}^{m} \tilde{b}_j \). Then we have the following Table 3.

Table 3. Balanced Full Fuzzy Transportation Table

| Source            | Destination | Supply |
|-------------------|-------------|--------|
| West Semarang     | Demak       | 55000  |
|                   | Purwodadi   | 66250  |
|                   | Kendal      | 50000  |
| Temanggung        |             | 75000  |
| East Semarang     |             | 55000  |
|                   |             | 62500  |
|                   |             | 68750  |
|                   |             | 53750  |
|                   |             | 3,25   |
|                   |             | 4,75   |
|                   |             | 4      |
| Dummy             |             | 0      |
|                   |             | 0      |
|                   |             | 0      |
|                   |             | 2      |
| Demand            |             | 3,25   |
|                   |             | 4,75   |
|                   |             | 4      |

Table 3 represents a transportation table has been balanced by adding a dummy row.

4.4.1 Implementation of Zero Point Method

In this section we apply zero point method to get optimum solution of cost transportation problem.
Table 4. Optimal Solution with Zero Point

| From          | To  | Destination | Supply |
|---------------|-----|-------------|--------|
| West Semarang | Demak | 0.25  | 3750   | 4 |
|               | Purwodadi | 2.5  |   | |
|               | Kendal |   |   | |
| Temanggung    | Demak | 1250 | 2.5  | 0 |
|               | Purwodadi |   |   | |
|               | Kendal |   |   | |
| East Semarang | Demak | 3  | 0.25  | 3750 |
|               | Purwodadi |   |   | |
|               | Kendal |   |   | |
| Dummy         | Demak | 7500 | 2  | 1250 |
|               | Purwodadi |   |   | |
|               | Kendal |   |   | |

Table 4 is a transportation table with an optimal value, obtained using the zero point method. By solving zero point method we get the following allocation as follows:

\[ x_{11} = 0.25, x_{13} = 4, x_{22} = 2.5, x_{31} = 3, x_{32} = 0.25, x_{42} = 2 \]

We get the minimum transportation cost as follows:

\[ Z = \sum_{i=1}^{m} \sum_{j=1}^{n} c_{ij}x_{ij} \]

\[ Z = 55000(0.25) + 50000(4) + 81250(2.5) + 55000(3) + 62500(0.25) + 0(2) \]

\[ Z = 597500 \]

4.4.2 Implementation of Zero suffix method

In this section we apply zero suffix method to get optimum solution of cost transportation problem.

Table 5. Optimal Solution with Zero Suffix

| From          | To  | Destination | Supply |
|---------------|-----|-------------|--------|
| West Semarang | Demak | 55000 | 66250 | 4 |
|               | Purwodadi | 50000 | 50000 | |
|               | Kendal |   |   | |
| Temanggung    | Demak | 75000 | 81250 | 68750 |
|               | Purwodadi |   |   | |
|               | Kendal |   |   | |
| East Semarang | Demak | 55000 | 62500 | 53750 |
|               | Purwodadi |   |   | |
|               | Kendal |   |   | |
| Dummy         | Demak | 0 | 2 | 0 |
|               | Purwodadi |   |   | |
|               | Kendal |   |   | |

Table 5 is a transportation table with an optimal value using the zero suffix method. By solving zero suffix method we get the following allocation as follows:

\[ x_{11} = 0.25, x_{13} = 4, x_{22} = 2.5, x_{31} = 3, x_{32} = 0.25, x_{42} = 2 \]

We get the minimum transportation cost as follows:

\[ Z = 55000(0.25) + 50000(4) + 81250(2.5) + 55000(3) + 62500(0.25) + 0(2) \]

\[ Z = 597.500 \]

The results of the implementation of zero point and zero suffix methods are shown in Table 6 as follows.
Table 6. Comparison of the Result of Zero Point Method and Zero Suffix Method

| No | Name of method         | Number of iteration | Allocations | Optimal Solution |
|----|------------------------|---------------------|-------------|------------------|
| 1  | Zero Point Method      | 5                   | $x_{11} = 0.25$  $x_{13} = 4$  $x_{22} = 2.5$  $x_{31} = 3$  $x_{32} = 0.25$  $x_{42} = 2$ | 597500 |
| 2  | Zero Suffix Method     | 4                   | $x_{11} = 0.25$  $x_{13} = 4$  $x_{22} = 2.5$  $x_{31} = 3$  $x_{32} = 0.25$  $x_{42} = 2$ | 597500 |

Table 6 is a comparison table of the zero point method and the zero suffix method by comparing iteration, allocation and optimal solution.

5. Conclusions

The comparison result shows that in Zero Point Method, the total iteration to reach the optimum solution after 5 times with allocation 6 items with the optimal solution is 597,500. While in Zero Suffix Method, the number of iterations to reach the optimum solution are needed 4 times with allocation 6 items, and the optimal solution is 597,500, it is the same value if compared to the first method. It can be concluded that the Zero Suffix Method uses less iteration than Zero Point Method, whereas allocation and minimum optimum solution are the same.

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