An investigation of lucky imaging techniques

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ABSTRACT

We present an empirical analysis of the effectiveness of frame selection (also known as lucky imaging) techniques for high-resolution imaging. A high-speed image recording system has been used to observe a number of bright stars. The observations were made over a wide range of values of \( D/r_0 \) and exposure time. The improvement in Strehl ratio of the stellar images due to aligning frames and selecting the best frames was evaluated as a function of these parameters. We find that improvement in Strehl ratio by factors of 4–6 can be achieved over a range of \( D/r_0 \) from 3 to 12, with a slight peak at \( D/r_0 \sim 7 \). The best Strehl improvement is achieved with exposure times of 10 ms or less, but significant improvement is still obtained at exposure times as long as 640 ms. Our results are consistent with previous investigations but cover a much wider range of parameter space. We show that Strehl ratios of >0.7 can be achieved in appropriate conditions whereas previous studies have generally shown maximum Strehl ratios of \( \sim 0.3 \). The results are in reasonable agreement with the simulations of Baldwin, Warner & Mackay.

Key words: instrumentation: high angular resolution – methods: data analysis – techniques: image processing.

1 INTRODUCTION

The frame selection technique for high-resolution imaging involves the recording of a time series of short exposure images and the selection of the sharpest images out of the series for alignment and combining into a final image. Fried (1978) determined that the probability of obtaining a lucky sharp image (defined as one with wavefront variance less than 1 rad\(^2\)) with a telescope of aperture \( D \) in seeing described by a Fried parameter \( r_0 \) (Fried 1967) is given by

\[
P = 5.6 \exp \left[ -0.1557(D/r_0)^2 \right].
\]

This suggests that there will be more good quality images available at low \( D/r_0 \). The probability of such an image is \( 1 \) in 9 for \( D/r_0 = 5 \) or \( 1 \) in 50 for \( D/r_0 = 6 \). For higher \( D/r_0 \), the probability of a sharp image rapidly decreases, being \( 1 \) in 3800 for \( D/r_0 = 8 \). Since the image quality gain will increase with \( D/r_0 \), this suggests the frame selection technique will work best for \( D/r_0 \sim 6–7 \), this being the largest \( D/r_0 \) at which there is a good chance of finding several high-quality images in a typical image sequence of a few thousand frames.

There have been a number of practical demonstrations of this technique variously described as frame selection (Roggemann & Welsh 1996), lucky imaging (Law, Mackay & Baldwin 2006) or selective image reconstruction (Dantowitz, Teare & Kozubal 2000). Baldwin et al. (2001) demonstrated the ability to obtain diffraction-limited star images at 800-nm wavelength with a 2.5-m telescope.

The technique has been used to image the hemisphere of Mercury that was missed by Mariner 10 (Dantowitz et al. 2000; Cecil & Rashkeev 2007; Ksanfomality & Sprague 2007) and is now widely used by amateur astronomers for planetary imaging.

Interest in the technique is rapidly increasing, in part due to the availability of electron multiplying CCD (EMCCD) technology, which allows rapid readout of CCDs with negligible read noise (Mackay et al. 2001), as well as computers with fast processors and large storage capacity. A number of such systems have recently been demonstrated, for example LuckyCam (Law et al. 2006), AstraLux (Hormuth et al. 2008) and FastCam (Oscoz et al. 2008).

However, previous studies have generally been aimed at obtaining the best possible image resolution and have therefore explored a restricted range of parameters. In this paper, we present observations that explore a wide range of parameter space. We have explored empirically the effects of telescope aperture \( D \), wavelength \( \lambda \), frame exposure time \( t \) and frame selection rate (FSR) on the resulting image quality. Unlike most previous studies which have aimed at exploiting excellent seeing conditions, our observations were obtained in a range of seeing conditions from good to poor. The results

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provide information that can help to optimize the design of future instruments.

2 OBSERVATIONS

The observations were obtained using Macquarie University Selective Imaging Camera (MUSIC) Mk I. The study was carried out as a preliminary stage in the design of a more advanced lucky imaging system that will use an EMCCD camera. The imager used for MUSIC was a Watec 100N monochrome video camera. This camera was chosen because it had adjustable exposure times and adequate sensitivity to observe bright stars. The observations used standard BVRI filters. The camera was either placed directly at the telescope focus or when necessary used with a 2.5 times focal extender. In all cases, the image scale was chosen to ensure that the pixels provided good sampling of diffraction-limited star images.

The camera produces video output with an effective pixel size of 8.6 × 8.3 µm and a format of 768 × 576 pixels. The camera generates video data with interlaced scanning (i.e. each frame consists of consecutive scans of odd rows and even rows stitched together). The video data were recorded using a Data Translation DT3155 frame grabber mounted in a PC system using a 3-GHz Pentium 4 processor. The PC was configured with 1 TB of disk space (two 400-Gb and one 200-GB drives) to record the large data files. The operating system was Fedora Core Linux. A data acquisition software system was developed that enabled data from the video camera to be recorded continuously as three-dimensional FITS files, while being displayed in real time. The software made use of c++ classes developed at the Anglo-Australian Observatory for the Infrared Imager and Spectrograph 2 (IRIS2) project and AAO detector controllers (Shortridge et al. 2004). The image display was based on the European Southern Observatory (ESO) real-time display system (Herlin, Brighton & Bierickehl 1996). The system was capable of recording full-frame video data to disk for extended periods. Typical observations consisted of sequences of 3000–10000 video frames.

MUSIC was used on three telescopes. The observing dates, locations and instruments used are summarized in Table 1. The bulk of the observations were obtained on the 1-m Australian National University (ANU) telescope at Siding Spring Observatory. A small number of observations were obtained on the 3.9-m Anglo-Australian Telescope (AAT, also at Siding Spring) and on the 0.4-m telescope of the Macquarie University Observatory in Sydney.

One aim of the observations was to explore the effects of changes in $D/r_0$, which may be regarded as the seeing-normalized aperture. While natural variation in seeing provides changes in $r_0$, we also varied $D$ by placing masks of different size over the telescope aperture. With the 1-m telescope, we used mask sizes of 75, 30, 20 and 10 cm, with the smaller masks being placed off-centre to avoid the central obstruction of the secondary mirror. With the AAT off-centre masks corresponding to apertures from 40 to 1.0 m were used located at the top of the `chimney' above the primary mirror.

![Table 1. Summary of MUSIC observing runs.](https://academic.oup.com/mnras/article-abstract/398/4/2069/983416)

| Date           | Location           | Instrument     |
|----------------|--------------------|----------------|
| 2005 March 8   | Macquarie University, Sydney NSW | Meade 40 cm |
| 2005 March 11–23 | Siding Spring, NSW | ANU 1 m       |
| 2005 June 25   | Siding Spring, NSW | AAT 3.9 m     |
| 2005 November 9–14 | Siding Spring, NSW | ANU 1 m       |

3 DISCUSSION

3.1 Frame selection rate

The stellar images produced by frame selection shown in Fig. 1 are among our best results. Note that the images in the top row are displayed brightness normalized and logarithmic pixel scale to highlight faint features. In Fig. 1(a), the frames were stacked, simulating a long exposure image. The next image has all frames shift-and-added (FSR = 100 per cent), which is equivalent to tilt correction. The binary is well resolved with a separation of 1.7 arcsec that matches published values. Both stars have a clear central peak.
Frame selection techniques

Figure 1. Images of binary star τ Ophiuchi (1.7-arcsec separation) produced by selective imaging. Images in the top row are brightness normalized and logarithmic pixel scale to highlight the faint halo. The three-dimensional plots in the bottom row are linear scale, and show increasing peaks with decreasing FSR. Images were taken on the AAT, $D = 40$-cm aperture mask, $t = 120$ ms, $\lambda = 0.7$-$\mu$m filter.

### Table 2. Strehl ratios of peaks in images in Fig. 1.

| Peak                  | FSR 100 per cent | FSR 10 per cent | FSR 1 per cent |
|-----------------------|-------------------|-----------------|---------------|
| Primary               | 0.517             | 0.654           | 0.799         |
| Secondary             | 0.362             | 0.547           | 0.674         |
| Secondary/primary     | 69.9 per cent     | 83.7 per cent   | 84.4 per cent |

with a significant halo. The third and fourth images have the best 10 and 1 per cent of frames aligned and combined. This removed those frames with the greatest speckling caused by high-order wavefront distortions. The result is greater flux in the central peak and a fainter halo. In the 1 per cent FSR image there is a dark ring at a radius of 8 pixels from each peak, matching the Rayleigh criterion for a diffraction-limited image. The three-dimensional plots show the diminishing halo and increasing brightness of the peaks with decreasing FSR, which is not shown by the brightness-normalized images. The Strehl ratios for both peaks in each frame-selected image listed in Table 2 verify the quality improvement, with the FSR = 1 per cent image having a Strehl ratio of almost 0.8.

The Strehl ratios listed in Table 2 illustrate an important effect of shift-and-add processing. In the long exposure image, the two peaks are unresolved so no Strehl ratio measurement is possible. In each of the frame-selected images, the Strehl ratio of the secondary peak is higher than that in the long exposure, but in all cases is less than the corresponding primary peak. This is because even frames with high overall sharpness may still contain more than one isoplanatic patch. When frames are co-aligned on the brightest pixel, the Strehl ratio of the primary peak is artificially enhanced. Other peaks may have a different tilt and speckle pattern. Shift-and-adding may still improve the sharpness of these features, but to a lesser extent than the primary peak, as seen in Fig. 1 and Table 2. Our data show a general trend to diminishing Strehl ratio and less Strehl improvement with increasing angular distance from the alignment location. This trend seems to be largely independent of FSR and $D/r_0$. This is in qualitative agreement with the simulations by Baldwin, Warner & Mackay (2008), though a direct quantitative comparison is not possible. However, in all of our observations the secondary peaks showed improvement with frame selection, even over angular separations as large as 100 arcsec.

Figure 2. Strehl ratio versus $D/r_0$ of stellar images binned in $D/r_0$. Open circles are the results of selecting and aligning the best 1 per cent of frames. Filled circles are the results of aligning all images with no frame selection (‘shift-and-add’ processing). Stars are the results of summing frames with no shifts (i.e. giving the equivalent long exposure image).

#### 3.2 Seeing-normalized aperture $D/r_0$

We determined $r_0$ from the FWHM of the long exposure image using the standard relationship $\text{FWHM} = 0.98 \lambda / r_0$. Over all of our observations $r_0$ ranged from 1 to 12 cm, and $D/r_0$ from 2.8 to 30.3. Fig. 2 plots bin-averaged Strehl ratios of stellar image versus $D/r_0$ between 3.0 and 12.0. For $D/r_0 > 12.0$ the trends continue more or less flat. For $D/r_0 < 12.0$ the obvious trend is for higher Strehl ratios in better seeing (large $r_0$) and/or with smaller aperture (small $D$).

With low $D/r_0$, not only are there more good frames to choose from (one could use a quality threshold instead of FSR) but the average frame quality is better, as indicated by higher Strehl ratios of the long exposure images in this region. This makes frame selection most suitable for use with small to medium-sized telescopes at visible wavelengths. This does, however, limit the magnitudes of usable target objects or guide stars.

Fig. 3 shows the quality improvement versus $D/r_0$. The improvement factor was measured by the gain in the Strehl ratio, i.e. the Strehl ratio of a frame-selected image divided by that of the long exposure image derived from the same image cube. Fig. 3 compares the improvement made by pure shift-and-add (100 per cent...
As for Fig. 2, but plotting the improvement in the Strehl ratio
long exposure
and

Long exposure value of about 5 cm, this corresponds

Long exposure values can

Strehl ratios measured in the V and I bands.

| Band | $r_0$ (cm) | D/r_0 | Long exposure Strehl ratios |
|------|------------|-------|-----------------------------|
| V    | Max 7.3    | 12.2  | 25.90                       |
|      | Min 1.2    | 2.1   | 4.64                        |
| I    | V 0.107    | 0.161 |
|      | I 0.008    | 0.008 |

Figure 3. As for Fig. 2, but plotting the improvement in the Strehl ratio compared with the long exposure image for each bin of $D/r_0$. Open circles are the results of selecting and aligning the best 1 per cent of frames. Filled circles are the results of aligning all images with no frame selection ('shift-and-add' processing).

Figure 4. Strehl ratio versus $D/r_0$ for individual observations with exposure time of 4 ms or less. Open circles are the result of selecting and aligning the best 1 per cent of frames. Stars are the result of summing all frames without alignment or selection giving the equivalent long exposure image. The line is the prediction from the simulation of Baldwin et al. (2008). Crosses are previous lucky imaging results as described in the text.

FSR) with that from 1 per cent FSR. The 1 per cent FSR points are consistently higher than those for 100 per cent FSR, confirming the advantage of being more selective. The 1 per cent FSR images show an improvement factor greater than 5 for $D/r_0$ between 4.5 and 7.8, with a small peak at $D/r_0 \approx 7$. This represents the best compromise between the diffraction-limited resolution, which improves with increasing $D$, and seeing-limited resolution that improves with increasing $r_0$. Nevertheless, the peak is not very pronounced, and substantial Strehl ratio improvement is obtained at all values of $D/r_0$.

Table 3 shows the ranges of $r_0$, $D/r_0$ and long exposure Strehl ratios measured in the V and I bands.

Data from our individual observations are displayed in Fig. 4 compared with results from other experiments. In this plot, our data are limited to $t \leq 4$ ms and $D/r_0 \leq 12$, and 1 and 100 per cent FSR. The crosses are previous lucky imaging results (Baldwin et al. 2001; Law et al. 2006; Hormuth et al. 2008) and match our data well, even though our data were generally obtained in poorer seeing but with smaller apertures. However, these previous studies generally targeted the optimum case of $D/r_0 \approx 7$, and achieved maximum Strehl ratios of $\sim 0.3$. Our data show that smaller $D/r_0$ values can be used to achieve higher Strehl ratios of $>0.6$ and in a few cases as high as 0.8. The high Strehl ratios for the images in Fig. 1 were achieved with $D/r_0 = 3.84$.

The line in Fig. 4 is the simulation from Baldwin et al. (2008). It can be seen that the simulation line lies at the upper boundary of the scatter of points. Typically, both our observations and previous results lie below the simulation. This is probably due to aberrations in the optics of the telescopes employed. The simulated case assumes a diffraction-limited telescope. In this case, frame selection will select those images in which the wavefront distortion due to turbulence is minimal. When using a real telescope with aberrations, it is necessary instead to select those frames in which the turbulence-induced wavefront distortions cancel out those due to telescope aberrations. The probability of a ‘lucky image’ in this case is lower than that for a perfect diffraction-limited telescope (Beckers & Rimmele 1996) and hence the image quality gain from frame selection is reduced.

3.3 Colour band $\lambda$.

Because $r_0 \propto \lambda^{5/3}$ (Fried 1966), it was expected that the better Strehl ratios would be achieved at longer wavelengths due to the larger $r_0$ patch. Table 3 shows the ranges of $r_0$, $D/r_0$ and long exposure (stacked) Strehl ratios measured from our observations in the V band (0.55 $\mu$m) and I band (0.80 $\mu$m). Because of the lower range of $D/r_0$ in the I-band images, their average frame quality was better than in other bands, giving higher average Strehl ratios in both the long exposures and the frame-selected images. A smaller number of observations obtained in the B and R bands were consistent with this trend.

3.4 Frame exposure time $t$.

To effectively freeze the turbulence, the frame exposure time must be less than $t_0 \sim r_0/v$, where $v$ is the bulk wind velocity in the region responsible for the turbulence (Kern et al. 2000). However, a small $t$ limits the available targets to the brightest objects, and reduces the signal-to-noise ratio (S/N) of each frame. Thus, it is crucial in frame selection to optimize $t$.

The top panel of Fig. 5 shows Strehl ratios against exposure times for 1 per cent FSR, with the data binned into four ranges of $D/r_0$. The bottom panel plots the improvement in Strehl ratio over the stacked images. Even at the longest times tested (640 ms), the Strehl ratios improved by a factor of 2. Shorter times gave greater improvement, but the curves appear to flatten below 8–10 ms, especially for larger $D/r_0$. Hence, we find that, although selective imaging gives improved image quality for any reasonably short exposure time, if the target is sufficiently bright $t$ should be limited to 10 ms. For our typical $r_0$ of about 5 cm, this corresponds...
However, the Strehl gain is rather insensitive to $r_0$, from 3 to 12. The improvements obtained using the best-quality frames Strehl ratios as high as 0.6–0.8 were obtained. The improvement was greatest when imaging at longer wavelengths due to the larger values of $r_0$. The optimum gain in Strehl ratio over long exposures was found to be about a factor of 6 for achieving larger Strehl ratios than previous results, which have generally been limited to Strehl ratios $<0.3$. The variation of Strehl ratio with $D/r_0$ we observe shows a similar form to, but lies below the simulations of Baldwin et al. (2008). This is probably due to telescope aberrations leading to a lower probability of achieving a lucky sharp image.

4 CONCLUSIONS

Our analysis confirms previous results in showing that substantial improvements in image Strehl ratio can be achieved by selecting and aligning the sharpest frames in a time series of short exposure images. By reducing the telescope aperture to be a few multiples of $r_0$, thereby minimizing $D/r_0$, and by selecting only 1 per cent of the best-quality frames Strehl ratios as high as 0.6–0.8 were obtained. The improvement was greatest when imaging at longer wavelengths due to the larger values of $r_0$. The optimum gain in Strehl ratio over long exposures was found to be about a factor of 6 at $D/r_0 \sim 7$. However, the Strehl gain is rather insensitive to $D/r_0$ with improvements of a factor of 4 or more being obtained over the full range of $D/r_0$ from 3 to 12. The improvements obtained by aligning frames without any selection (shift-and-add processing) are smaller, ranging from 2 to 3.

Frame selection has been found to improve image sharpness over a wide range of frame exposure times. Even with exposure times as long as 640 ms the Strehl ratio was improved by a factor of 2. However, the best gains in Strehl ratio (from 4 to 6) for the Siding Spring site are obtained with $t < 10$ ms. The optimum exposure time may be longer for sites with better seeing or at longer wavelengths.

Our results are consistent with previous lucky imaging studies, but explore a wider range of parameter space and show the potential for achieving larger Strehl ratios than previous results, which have generally been limited to Strehl ratios $<0.3$. The variation of Strehl ratio with $D/r_0$ we observe shows a similar form to, but lies below the simulations of Baldwin et al. (2008). This is probably due to telescope aberrations leading to a lower probability of achieving a lucky sharp image.

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