Estimates for non-leading distribution functions\textsuperscript{1}

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Abstract

Estimates for leading and non-leading ‘twist’ distribution functions are obtained within the framework of a diquark spectator model using a non-local operator representation.

More details about the method and the results reported here can be found in the long write-up \cite{1}.

1 introduction

In a field-theoretic description of hard scattering processes the information on the hadronic structure is contained in matrix elements of non-local operators (parton correlation functions). For instance, the distribution functions of quarks in hadrons are obtained from these hadronic matrix elements by tracing them with certain Dirac matrices and integration over components of the quark momentum.

Distribution functions are universal in the sense, that they occur in the same form in the description of all hard processes involving the same kind of soft physics. Being, however, of genuine non-perturbative nature, in general, they can not be calculated by perturbative means (with the exception of heavy quark distribution functions). Ultimately, one could wait for lattice gauge theory to provide the answer, but regarding the enormous difficulties those techniques encounter, in the meanwhile, model estimates for the functions may be very useful in predicting cross sections or asymmetries in future experiments.

2 correlations and distribution functions

\begin{equation}
\Phi(p;P,S)
\end{equation}

\textsuperscript{1}Talk presented at the workshop ‘Deep Inelastic Scattering off Polarized Targets: Theory Meets Experiment’, DESY Zeuthen, Germany, Sept. 1-5, 1997
\[
\Phi_{ij}(p, P, S) = \sum_X \int \frac{d^4x}{(2\pi)^4} e^{ip \cdot x} \langle P, S|\bar{\psi}_j(0)|X\rangle \langle X|\psi_i(x)|P, S\rangle.
\] (1)

**inclusive lepton-hadron**

We encounter the quark-quark correlation function in the field theoretical description of, e.g. deep inelastic lepton hadron scattering, whose amplitude squared is depicted by the famous hand-bag diagram (the usual invariants indicated besides it)

\[
Q^2 = -q^2 \quad x_B = \frac{Q^2}{2P \cdot q}.
\]

In fact, in the cross section the correlation function occurs traced with a Dirac matrix, \(\gamma^+\), and integrated over three components of the quark momentum

\[
f_1(x) = \frac{1}{2} \int dp^- d^2p_T \text{Tr}(\Phi \gamma^+) \bigg|_{p^+ = x^+} (2)
\]

which defines a distribution function, \(f_1(x)\), depending only on the longitudinal momentum fraction \(x = p^+/P^+\). The parton model predictions for the structure functions \(F_1\) and \(F_2\) are given (to lowest order) in terms of the distribution functions as

\[
2F_1(x_B) = \frac{F_2(x_B)}{x_B} = \sum_a e_a^2 f_a^1(x_B).
\] (3)

We introduce the notation

\[
\Phi[^\Gamma](x) = \frac{1}{2} \int dp^- d^2p_T \text{Tr}(\Phi |^\Gamma) \bigg|_{p^+ = x^+} (4)
\]

for the quark-quark correlation function traced with a certain Dirac matrix and integrated over \(dp^-d^2p_T\).

**leading twist distribution functions**

Depending on the Dirac matrix involved, different spin properties of the hadronic structure are probed. To leading order (in an \(1/Q\) expansion) the following projections occur in the description of hard processes (\(\lambda\) is the helicity of the hadron, \(S_T\) the transverse part of the spin vector)

\[
\Phi[^{\gamma^+}](x) = f_1(x) \quad \Phi[^{\gamma^+\gamma_5}](x) = \lambda g_1(x) \quad \Phi[^{i\sigma^\alpha+\gamma_5}](x) = S_T^\alpha h_1(x)
\] (5) \(\quad\) (6) \(\quad\) (7)

which have an intuitive probabilistic interpretation\(^2\)

\(^2\)The notation of the functions follows Refs. [5, 6].
interchange

- $f_1(x)$ gives the probability of finding a quark with light-cone momentum fraction $x$ in the “+”-direction (and any transverse momentum).

\[ f_1 = \bullet \]

- $g_1(x)$ is a chirality distribution: in a hadron that is in a positive helicity eigenstate, it measures the probability of finding a right-handed quark with light-cone momentum fraction $x$ minus the the probability of finding a left-handed quark with the same light-cone momentum fraction.

\[ g_{1L} = \bullet \quad - \quad \bullet \quad - \]

- $h_1(x)$ is a transverse spin distribution: in a transversely polarized hadron, it measures the probability of finding quarks with light-cone momentum fraction $x$ polarized along the direction of the polarization of the hadron minus the probability of finding quarks oppositely polarized.

\[ h_1 = \bullet \quad - \quad \bullet \]

twist 3 functions

The subleading (‘higher twist’) functions have no intuitive probabilistic interpretation. Nevertheless, they are well defined via the quark-quark correlation function traced with appropriate Dirac matrices. The pre-factor $M/P^+$ behaving like $1/Q$ in a hard process signals the sub-leading (i.e. ‘twist’ 3) nature of the corresponding distribution functions

\[
\Phi^{[1]}(x) = \frac{M}{P^+} e(x) \quad (8) \\
\Phi^{[\gamma^\alpha \gamma_5]}(x) = \frac{M}{P^+} S_T^\alpha g_T(x) \quad (9) \\
\Phi^{[i\sigma^+-\gamma_5]}(x) = \frac{M}{P^+} \lambda h_L(x) \quad (10)
\]

transverse momentum

In some processes additional information is gained by considering observables which depend on the transverse momentum of an external hadron (like differential cross sections). Typically, these are processes with at least three external momenta, since those cannot be all collinear, in general.

The transverse momentum dependence of the observables is related to the transverse degrees of freedom of partons in the hadrons. Let us examplify the relation for one hadron-inclusive lepton-hadron scattering (see e.g. [6]). The amplitude (squared) is depicted by
\[ [P_h^-, P_h^+, 0_T] \]
\[ [q^-, q^+, q_T \neq 0] \]
\[ [P^-, P^+, 0_T] \]

In a frame where the target momentum, \( P \), and the momentum of the observed hadron in the outgoing channel, \( P_h \), are collinear, the photon momentum will have a transverse component (as indicated besides the diagram). \( \Phi \)

The external transverse momentum, \( q_T \) is related via momentum conservation, i.e., a \( \delta(q_T + p_T - k_T) \) function at the vertex, to the transverse momentum components on the quark lines. Thus, observables differential in the transverse momentum will involve functions like

\[
f_1(x, p_T^+) = \frac{1}{2} \left. \int dp^- \text{Tr}(\Phi \gamma^+) \right|_{p^+=xP^+, p_T} \tag{11}
\]

depending on the longitudinal momentum fraction and the transverse momentum components which are not integrated out. E.g., the differential cross section reveals a behavior

\[
\frac{d\sigma}{dx_B dy dz d^2q_T} \sim \int d^2p_T d^2k_T \delta(p_T + k_T - q_T) f_1(x_B, p_T) \ D_1(z, k_T) \tag{12}
\]

where \( D_1(z, k_T) \) is a transverse momentum dependent fragmentation function which describes the hadronization of the quark. In straightforward analogy we define the functions

\[
\Phi[\Gamma](x, p_T) = \frac{1}{2} \left. \int dp^- \text{Tr}(\Phi \Gamma) \right|_{p^+=xP^+, p_T} \tag{13}
\]

which differs from definition (14) only by the left-out integration over transverse momentum.

\( \square \) leading twist distribution functions \( (p_T \text{ dependent}) \)

Not surprisingly, there are more functions involved due to the additional degrees of freedom. To leading ‘twist’ there are six functions defined by

\[
\Phi[\gamma^+](x, p_T) = f_1(x, p_T^2) \tag{14}
\]
\[
\Phi[\gamma^+ \gamma^5](x, p_T) = \lambda \ g_{1L}(x, p_T^2) + g_{1T}(x, p_T^2) \frac{p_T^\cdot S_T}{M} \tag{15}
\]
\[
\Phi[\sigma^\alpha^+](x, p_T) = S_T^\alpha \ h_{1T}(x, p_T^2) + \frac{p_T^\alpha}{M} \left[ \lambda \ h_{1L}^+(x, p_T^2) + h_{1T}^+(x, p_T^2) \frac{p_T^\cdot S_T}{M} \right] \tag{16}
\]

\( \pentagon \) leading twist distribution functions \( (p_T \text{ dependent}) \)

The reasoning in other frames is similar, one of the external momenta unavoidably will have a transverse component.

\( \triangledown \) leading twist distribution functions \( (p_T \text{ dependent}) \)

Not surprisingly, there are more functions involved due to the additional degrees of freedom. To leading ‘twist’ there are six functions defined by

\[
\Phi[\gamma^+](x, p_T) = f_1(x, p_T^2) \tag{14}
\]
\[
\Phi[\gamma^+ \gamma^5](x, p_T) = \lambda \ g_{1L}(x, p_T^2) + g_{1T}(x, p_T^2) \frac{p_T^\cdot S_T}{M} \tag{15}
\]
\[
\Phi[\sigma^\alpha^+](x, p_T) = S_T^\alpha \ h_{1T}(x, p_T^2) + \frac{p_T^\alpha}{M} \left[ \lambda \ h_{1L}^+(x, p_T^2) + h_{1T}^+(x, p_T^2) \frac{p_T^\cdot S_T}{M} \right] \tag{16}
\]
Of course, all those Dirac projections reduce after integration over transverse momenta to the forms shown before, like for instance, \( \int d^2p_T \Phi^{[\gamma^+]}(x,p_T) = \Phi^{[\gamma^+]}(x) \), etc.

\[\text{\( \bigcirc \) twist 3 functions (\( p_T \) dependent)}\]

Similarly, there is a larger number of ‘twist 3’ functions depending on \( x \) and \( p_T \) which are obtained by the projections:

\[
\begin{align*}
\Phi^{[1]}(x,p_T) &= \frac{M}{P^+} e(x,p_T^2) \\
\Phi^{[\gamma^+]}(x,p_T) &= \frac{p_T^0}{P^+} f^+(x,p_T^2) \\
\Phi^{[\gamma^+\gamma^5]}(x,p_T) &= \frac{MS}{P^+} g^+(x,p_T^2) + \frac{p_T^0}{P^+} \left( \lambda g_L^+(x,p_T^2) + \frac{p_T^\perp \cdot S_T}{M} g_T^+(x,p_T^2) \right) \\
\Phi^{[i\sigma^\perp\gamma^5]}(x,p_T) &= \frac{S^i_T p_T^L - S_T^i p_T^L}{P^+} h^+(x,p_T^2) \\
\Phi^{[i\sigma^+\gamma^5]}(x,p_T) &= \frac{M}{P^+} \left( \lambda h_L(x,p_T^2) + \frac{p_T^\perp \cdot S_T}{M} h_T(x,p_T^2) \right)
\end{align*}
\]

### 3 spectator model

The purpose of our investigation is to obtain estimates for the non-leading (i.e. ‘twist 3’) distribution functions which are experimentally poorly (or not at all) known at present.

To this end we employ a rather simple spectator model with only a few parameters. After fixing the parameters by phenomenological constraints we check that the gross features of the experimentally well-known leading ‘twist’ distribution functions \( f_1(x) \) and \( g_1(x) \) are satisfactorily reproduced.

#### 3.1 ingredients of the model

The ingredients of the model are indicated below:

- The basic idea of the spectator model is to assign a definite mass to the intermediate states occurring in the quark-quark correlation functions

\[
(P - p)^2 = M_R^2
\]

\footnote{for more detailed information about the model see also previous publications [1, 7, 8, 9]}
• The quantum numbers of the intermediate state are determined by the action of the quark field operator on the hadronic state $|P, S\rangle$, i.e. they are the quantum numbers of a diquark system

| spin | isospin |
|------|---------|
| scalar diquark | 0 | 0 |
| axialvector diquark | 1 | 1 |

• The matrix element appearing in the RHS of (1) is given by

$$\langle X_s|\psi_i(0)|P, S\rangle = \left(\frac{i}{\not{p} - m}\right) \Upsilon_{k\ell}^s U_l(P, S)$$

in the case of a scalar diquark, or by

$$\langle X_\lambda^\mu|\psi_i(0)|P, S\rangle = \epsilon^{\ast\lambda}_\mu \left(\frac{i}{\not{p} - m}\right) \Upsilon_{k\ell}^{\mu\nu} U_l(P, S)$$

for a vector diquark. The matrix elements consist of a nucleon-quark-diquark vertex $\Upsilon(N)$, the Dirac spinor for the nucleon $U_l(P, S)$, a quark propagator for the untruncated quark line and a polarization vector $\epsilon^{\ast\lambda}_\mu$ in the case of an axial vector diquark.

• For the nucleon-quark-diquark vertex we assume the following Dirac structures: $^{5}$

$$\Upsilon^s(N) = 1 \ g_s(p^2)$$

$$\Upsilon^{\mu\nu}(N) = \frac{g_a(p^2)}{\sqrt{3}} \gamma_5 \left(\gamma^\mu + \frac{P^\mu}{M}\right)$$

The functions $g_{s/a}(p^2)$ are form factors that take into account the composite structure of the nucleon and the diquark spectator $^{6}$. We use the same form factors for scalar and axial vector diquark:

$$g_{s/a}(p^2) = N \frac{p^2 - m^2}{|p^2 - \Lambda^2|^\alpha}.$$

• The flavor coupling of the proton wave function from a scalar diquark ($S_0$) and an (axial)vector diquark with isospin component $I_3 = 0$ or 1 ($A_0$ or $A_1$, respectively)

$$|p\rangle = \frac{1}{\sqrt{2}} |u \ S_0\rangle + \frac{1}{\sqrt{6}} |u \ A_0\rangle - \frac{1}{\sqrt{3}} |d \ A_1\rangle,$$

leads to the flavor relations

$$f_1^u = \frac{3}{2} f_1^s + \frac{1}{2} f_1^a$$

$$f_1^d = f_1^a$$

$^{5}$a special case of the most general form given in $^{8}$.
and similarly for the other functions. The coupling of the spin has already been included in the vertices.

Putting all ingredients together analytic expression for the quark-quark correlation functions are obtained

\[
\Phi^R(p, P, S) = \frac{N^2}{2(2\pi)^3} \frac{\delta(p^2 - 2P \cdot p + M^2 - M_R^2)}{|p^2 - \Lambda^2|^{2\alpha}} \times (\bar{\psi} + m)(\bar{P} + M) \left(1 + a_R \gamma_5 \right) \left(\bar{\psi} + (3\Omega)\right)
\]

from which the distribution functions can be easily projected.

### 3.2 fixing the parameters

The parameters of the model are fixed as follows: the power in the denominator of the form factor, \(\alpha = 2\), is chosen to reproduce the Drell-Yan-West relation for large \(x\). The mass difference \(M_a - M_s\) is motivated by the \(N - \Delta\) mass difference (with group theoretical factors properly accounted for) and the values for \(\Lambda\) and \(M_s\) reproduce the experimental value for the axial charge, \(g_A\).

| parameter         | fixed by:                                      |
|-------------------|-----------------------------------------------|
| \(\alpha = 2\)    | Drell-Yan-West relation: \(f_1^u(x) \sim (1 - x)^3\) |
| \(M_a - M_s = 200\) MeV | \(N - \Delta\) mass difference               |
| \(M_s = 600\) MeV; \(\Lambda = 0.5\) GeV | \(g_A = 1.26\)                                  |
| \(N\)             | number sum rules \(\int dx f_1^u(x) = 2; \int dx f_1^d(x) = 1\) |

### 4 numerical results

Having fixed the parameters of the model we can present our numerical results.

- The transverse momentum dependent distribution \(f_1^u(x, p_T)\) as obtained from the analytical expression for \(\Phi\) by tracing with \(\gamma^+\) and integration over \(d\vec{p}^-\). The dependence on \(p_T\) is driven by the choice of the form factors Eq.(26) and momentum conservation.
Integrating $f_1(x, p_T)$ over $d^2p_T$ we obtain the usual distribution $f_1(x)$. The values for the first moments $\int_0^1 dx x f_1(x)$ are 0.690 and 0.256 for the $u$ and $d$ quark, respectively. We compare our result with the parametrization from Glück, Reya and Vogt [10] (at the low scale $\mu_{LO}^2 = 0.23 \text{ GeV}^2$).

Note that the lowest moments, $\int f_1(x) dx$ are exactly the same (normalization condition), a fact not immediately apparent from the diagram, since we plot the combination $x * f_1(x)$. The position of the maxima is in fair agreement. Our distribution is narrower due to the non-inclusion of gluons and anti-quarks (which – if included in our model – would have a broadening effect). Thus, we refrain from fine-tuning parameters to obtain a closer agreement with GRV — and are satisfied with agreement in the gross features.

The comparison of our result for $g_1(x)$ with parametrizations taken from the literature [11] reveals a similar agreement in the gross features [1].

Note that the model does not provide a scale dependence; but it is expected to describe physics at a low “hadronic” scale. Thus we compare to distributions found in the literature at scales as low as available.

Having gained some confidence in the model predictions – as well as insight in limitations in accuracy – we can make predictions for the less well known or unknown functions.

\footnote{Note that the model does not provide a scale dependence; but it is expected to describe physics at a low “hadronic” scale. Thus we compare to distributions found in the literature at scales as low as available.}
Yet experimentally completely undetermined is the transverse spin distribution $h_1(x)$. Within our model we obtain an estimate for this function which is numerically close to the result for the helicity distribution $g_1(x)$, but not identical.

- We now turn to the ‘twist 3’ distribution functions. We display $e(x)$ and the combination $g_2(x) = g_T(x) - g_1(x)$. The combination $h_2(x) = 2(h_L(x) - h_1(x))$ is predicted in the model to be just $2 \times g_2(x)$.

- From hermiticity and invariance under parity transformation relations between ‘leading twist’ and ‘subleading twist’ functions can be found like [12, 13]

\[ g_T(x) = g_1(x) + \frac{d}{dx} g_{1T}^{(1)}(x) \]  
\[ h_L(x) = h_1(x) - \frac{d}{dx} h_{1L}^{(1)}(x) \]

involving $p_T$-moments of distribution functions

\[ g_{1T}^{(1)}(x) \equiv \int d^2 p_T \left( \frac{p_T^2}{2M^2} \right) g_{1T}(x, p_T) \]

which, as well, are predicted by the model.

The $p_T$-moment $h_{1L}^{(1)}(x)$ is given by the relation $h_{1L}^{(1)}(x) = -g_{1T}^{(1)}(x)$ within this model.

Note the non-vanishing values of $g_{1T}^{(1)}$ (and correspondingly of $h_{1L}^{(1)}$) at $x = 0$. 
The positivity constraints $|g_1(x)| \leq f_1(x)$ and $|h_1(x)| \leq f_1(x)$ are trivially fulfilled in the model, as well as the Soffer [13] inequality, $2|h_1(x)| \leq (f_1(x) + g_1(x))$.

On the other hand, the non-zero values of $g^{(1)}_{1T}(x=0)$ and $h^{(1)}_{1L}(x=0)$ indicate a small violation of the Burkhardt-Cottingham [14] sum rule, $\int_0^1 dx g_2(x) = 0$, and the Burkardt [15] sum rule, $\int_0^1 dx h_2(x) = 0$. Those violations turn out to be proportional to quark mass effects. Also, the Efremov-Teryaev-Leader [16] sum rule, $\sum_{a \in V} c_a^2 \int_0^1 dx \left( g^a_1(x) + 2g^a_2(x) \right) = 0$, is violated. Probably, those violations are artefacts of the model and might be used to constrain future refinements.

5 conclusions

In the framework of a diquark spectator model we have obtained analytical expressions for quark-quark correlation functions defined as hadronic matrix elements of non-local operators. Distribution functions are given as Dirac projections of the correlators. Numerical results for leading and, in particular, sub-leading ‘twist’ functions have been reported and discussed.

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