A novel high-order Voronoi algorithm and its application in affected coverage of health city assessment

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Abstract. The work of China's urban health impact assessment (HIA) is still in the exploratory stage. From the spatial perspective of urban planning, using the natural topological characteristics of Delaunay Triangle, the high Order Voronoi Diagram algorithm of the order Delaunay Triangle data structure is applied in practice. This algorithm can deal with the k nearest neighbors and the mutual distance relationship, which can be effectively applied to the definition of the impact scope of healthy city evaluation, thus providing a new auxiliary method for health city evaluation research.

1. Introduction

The design of urban planning can have a potential positive or negative impact on the health of the population through the shaping of the urban built environment. Public health has long been one of the emerging topics of concern for urban planning scholars [1]. The Health Impact Assessment (HIA) is another very important way of developing a healthy city to quantify the health effects of planning policies or programs on local residents. HIA assesses and predicts health-related issues and severity of urban space and planning projects and has been incorporated into urban planning and design decisions[2] as a tool to improve public health. The integration of health concepts into urban planning is becoming one of the frontier issues of Chinese planning scholars [3]. International exploration has shown that HIA provides an innovative way for planners to solve this problem.

The urban health impact zone is the scope of influence that the evaluation center city can achieve, that is, the maximum regional health range of the central city for the social, economic and environmental factors in the surrounding area. To this end, determining the spatial impact of a city is an important task of urban planning and urban health city evaluation.

Voronoi Diagram is one of the important contents in computational geometry research. At present, there are many mature algorithms, which are widely used in image processing, pattern recognition, urban planning, geographic information, disaster science, ecology and economics. Most applications of Voronoi Diagrams stay in ideal spatial divisions and neighboring queries, which do not satisfy the interaction between actual elements and human cognitive habits. High-Order Voronoi Diagrams are easy to solve \( k > 1 \) nearest neighbor query problem, more practical application. Aurenhammer and Schwarzkopf; Chazelle and Edelsbrunner; Dehne have proposed a more mature high-order Voronoi algorithm[4-6].

This study is still in the exploratory stage of China's urban health impact assessment (HIA) work. From the perspective of urban planning, using the natural topological characteristics of Delaunay Triangle, \( k \) Higher Order Voronoi Diagram Algorithm for Order Delaunay Triangle Data Structure.
This algorithm can deal with the \( k \) nearest neighbors and mutual distance relationship, and can be applied to the definition of the impact range of healthy city evaluation, thus providing a new auxiliary method for health city evaluation research.

### 2. The basic concept of high-order Voronoi

The higher Order Voronoi Diagram is an extension of the Voronoi Diagram, also known as \( k \) Order Voronoi Diagram or \( v(k) \) for short.

**Definition:** set \( n \) discrete point sets \( p = \{p_1, p_2, \ldots, p_n\} \). For a given integer \( k (0 < k < n) \), to form any \( k \) discrete point sets \( p^k = \{p_{i1}, p_{i2}, \ldots, p_{ik}\} \), among them \( p_{i1}, p_{i2}, \ldots, p_{ik} \in p \). The sets can be up to \( l = C_n^k = n!/(k!(n-k)!) \). Write as \( A(k)(P) = \{p^1_k, p^2_k, \ldots, p^n_k\} \). In this way, the sets of points with the smallest distance from the \( k \) points of \( p^k \) constitutes \( k \) Order Voronoi area:

\[
VR_H(p^k_i) = \{q | qp_{i1} \leq |qp_i|, \text{ and } |qp_{i2}| \leq |qp_i|, \ldots, \text{ and } |qp_{ik}| \leq |qp_i|, p_j \in P - p^k_i\}
\]

That is \( VR_H(p^k_i) = \{q | \max_{p_h}(|qp_h|), p_h \in p^k \leq \min_{p_j}(|qp_j|), p_j \in P - p^k_i\} \).

For higher Order Voronoi Diagrams or \( k \) Order Voronoi Diagram, when \( k = 1 \), it is a 1st Order Voronoi Diagram. The vertices and edges of the \( k \) Order Voronoi Diagram are called \( k \) Order Voronoi Vertex and \( k \) Order Voronoi Edges. \( v(k) \) expresses as unordered \( k \) Order Voronoi Diagram, \( v^k \) expresses as \( k \) Order Voronoi Diagram, \( v^{<k>} \) expresses as the nearest point of \( k \) Order Voronoi[7].

### 3. High-order Voronoi algorithm

The Delaunay Triangulation and the Voronoi Diagram are mutually similar, but \( k \) Order Delaunay Triangulation and the \( k \) Order Voronoi Diagrams of the order are not directly dual, but the following relationship exists:

Hypothesis \( p_i, p_j, p_k \) are the any three points of the set \( P \), \( v \) is the intersection in the vertical bisectors of \( \triangle p_i p_j p_k \), that is the center of \( C(p_i, p_j, p_k) \), the circumscribed circle of \( \triangle p_i p_j p_k \). The point set of the circumscribed circle is \( R \subset P \), if \( |R| = k \), then is the Point of set \( P \) corresponding the vertices of \( k + 1 \) Order Voronoi Diagram \( v^{(k+1)} \) versus \( v^{(k+2)} \) Order Voronoi Diagram \( v^{(k+2)} \) [8]. Therefore, any vertices of \( v^{(k)} \) in the triangle corresponds to a triangle, the number of these points in the circumscribed circle is exactly \( k - 1 \) or \( k - 2 \), and these \( k \) Order Triangles and the higher Order Voronoi Diagrams can form such a type of dual relationship. They are keeping \( k \) when the Delaunay Triangle of the order has good triangulation characteristics, a high Order Voronoi Diagram can also be obtained, as follows[9]:

**Input:** Planar discrete point set \( P = \{p_1, p_2, \ldots, p_n\} \) And order \( k (1 < k < n) \)

**Output:** \( k \) Order of Voronoi Diagram \( v^{(k)} \).

1. (1) by point \( x \), discrete points of \( P \) coordinate sorting, construct the initial Delaunay Triangle. Assume that an initial has been generated \( k - 1 \) Order Triangle \( \triangle p_1 p_2 p_3 \).
2. Select either side of \( \triangle p_1 p_2 p_3 \) (arranged counterclockwise), \( \overline{p_1 p_2} \), if ever \( \overline{p_1 p_2} \) scan to the outside circle \( p_4 \), find \( C(p_1, p_2, p_4) \) just included as \( k - 1 \) point, record the \( \triangle p_1 p_2 p_4 \), otherwise continue scanning \( p_1 \) until determined \( k - 1 \) Order Delaunay Triangle \( \triangle p_1 p_2 p_4 \).
3. (3) for the other side \( \overline{p_2 p_3} \), \( \overline{p_3 p_4} \) repeats step 2. Then process the next one \( k - 1 \) Order Triangle until you get all \( k - 1 \) Order Delaunay Triangle.
4. (4) Repeat steps (1), (2), and (3) in the same way, \( k - 2 \) order Delaunay Triangle.
5. (5) by \( k - 1 \) and \( k - 2 \) order Delaunay Triangle gets \( k \) Order Voronoi[8][10].

Figure 1 shows the 1st, 3rd, 5th, and 6th Order Voronoi Diagrams implemented by 20 random experimental points.
4. Actual case application

4.1. Research area

The study area is located in Tiandeng County, southwest of Guangxi, China, and is affiliated to Chongzuo City. Tiandeng County governs 6 towns and 7 townships. The total area of the county is 2,195.23 square kilometers, with a total population of 455,400. There are Zhuang, Han, Yao, Miao, and Yi ethnic groups, of which Zhuang population accounts for 98.9% of the total population. The county towns and 13 townships were selected as research objects. The data is mainly obtained through the 2019 Guangxi Statistical Yearbook, the 2019 China Urban Statistical Yearbook, the practical inquiry website and the bigemap software. The vector data is downloaded in the bigemap software.

4.2. Indicator system

On the basis of the "China Health City Evaluation Index System (2018 Edition)" standard, combined with the practice results of local construction, strictly adhere to the principles of science, comparability, measurability and sustainable development to build a relevant health city indicator system. 21 indicators under the six criteria levels of environment, society, services, population, culture and security are used to evaluate the development of healthy cities, as shown in Table 1.

Table 1 Index system of healthy cities in Guangxi province
| Health evaluation system | (1) The proportion of days with good ambient air quality |
|--------------------------|--------------------------------------------------------|
| Health environment       | (2) Heavy and above pollution days                      |
|                          | (3) Water quality compliance rate of drinking water      |
|                          | (4) Safety guarantee rate of centralized drinking water source |
|                          | (5) Harmless health toilets penetration rate (rural)      |
|                          | (6) National Health County (township) ratio              |
| Health society           | (7) The actual reimbursement ratio of basic medical insurance hospitalization expenses |
|                          | (8) Occupational health check coverage                   |
|                          | (9) Food sampling inspection 3 batches / thousand people |
|                          | (10) Excellent rate of student physical fitness monitoring |
| Health service           | (11) Child health management rate                        |
|                          | (12) Number of public health personnel per 10,000 population |
|                          | (13) Number of beds in medical institutions per 1,000 population |
| Healthy crowd            | (14) Life expectancy per capita                         |
|                          | (15) Child mortality under 5 years old                   |
|                          | (16) Maternal mortality rate                             |
| Health culture           | (16) Smoking rate among people over 15 years old         |
|                          | (17) Residents' health literacy level                    |
|                          | (18) Low coverage                                       |
| Social Security          | (19) Endowment insurance coverage                        |
|                          | (20) Medical insurance coverage                         |
|                          | (21) Unemployment insurance coverage                     |

4.3. Implementation results

Analytic hierarchy process[12-14] the weights of each index are obtained. Each attribute has a different dimension, which will affect the result of data analysis. In order to eliminate the dimension effect between data, data normalization processing is required. For the \( i \)-th attribute value, the following formula can be used. standardization:

\[
F_i = \frac{x_i - \min(x_i)}{\max(x_i) - \min(x_i)}
\]

\( F_i \) is a standard value, and each feature attribute is uniformly mapped to the interval. Using the algorithm of this study, location information \(|P|=14\), taking the order \( k =3 \), Figure 2 to obtain the ideal health city evaluation impact range results.
Figure 2 actual case application results, $k = 3$

5. Conclusion

At present, the health impact assessment of urban planning in China is still in the exploratory stage. The HIA evaluation model takes less into the scope of healthy urban space. The high Order Voronoi Diagram algorithm proposed in this paper and the selection of healthy city evaluation factors are spatially studied for the scope of urban evaluation. Urban planning health impact assessment provides a new means to help decision makers make more favorable public health choices, and thus achieve the goal of improving public health has important practical significance and practical value.

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