The harmonic quarks and hadrons up to 1000 MeV

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Abstract

The calculations and analysis of a variance have shown, that masses of mesons up to 1000 MeV strongly correlate with a model spectrum of potential wells at mass of harmonic up-quark 105.6 MeV. It is shown, that probability of casual correspondence of the meson and model spectrums is less than 1 ppm. The harmonic quark model is applied to L=0 hadrons for the analysis of them structures and detection of them filled quark shells. The possible quark structures of all mesons up to 1000 MeV are given. The completely neutral simple boson configurations are found for a threshold a proton-antiproton and the total mass of neutral pion and kaon. Mass relations are fulfilled with precision near 0.01%. It is supposed, that new relations is connected with Higgs mechanism of mass formation. The final precision of calculated masses of harmonic quarks is estimated as 0.005%.

1 Introduction

A many various quark models were applied for investigation of hadrons structures [1]–[6]. The part from them used a relativistic approach, i.e. the masses of quarks are considered small values or even zero [5]. A potentials of various forms frequently use, including the harmonic quark oscillator and degree functions [3, 4, 5]. Others shell quark models were constructed in the no relativistic limit [1, 2, 4, 6]. On this way are received a good results for the mass spectrum of L=1 baryons [6]. There are also other non-standard attempts to attract a shell models for exposition and systematizations of hadrons [7].

In present work for the first time the rigid system of harmonic quarks and model of harmonic potential wells is applied for investigation of hadron structures. In paper [8] the simple equation for masses of quarks is obtained on the basis of two postulates.

The postulate 1.

A quark-antiquark pair of one flavor n can annihilate in part up to a bound state, that defines by the mass equation $m_n \cdot \frac{4}{\pi}$.

We may term this bound state as the full harmonic quark oscillator or quark-oscillator.

The postulate 2.
No stable symmetric state harmoniously annihilating oscillator from a quark-antiquark pair of one flavor can be broken by that or other way and pass at conservation of energy in an unsymmetrical state from a quark-antiquark pair of neighboring flavors.

Simplifying the postulate 2, we actually state the following: the total mass of two quarks with flavors \( n \) and \( n-1 \) is equal to mass of a harmonic oscillator from quark-antiquark pair with flavor \( n \). From here the simple recurrent equation for quark masses follows:

\[
\frac{m_n}{m_{n-1}} = \frac{\pi}{4 - \pi} = 3.659792... \approx 3.66
\]

The equation can be noted and so:

\[
m_n = m_0 \left[ \frac{\pi}{4 - \pi} \right]^n \tag{2}
\]

where \( m_0 \) – mass of a hypothetical quark with zero flavor; \( m_n \) – mass of a quark with flavor \( n \).

These equations relate univalently a masses of all quarks in one spectrum and, thus, all quarks of the harmonic model form the rigid sequence or one set with single ”successor” in each generation. Quarks of the harmonic model have a logarithmic equidistant mass spectrum, and their flavors are serial numbers of levels in this spectrum or a values \( n \) in the Eq. (2). In mathematical aspect there would be enough to postulate an equation for quark masses, i.e. Eq. (1).

Further the masses of harmonic quarks have been calculated with precision \( \approx 0.03\% \). In table 1 these masses of quarks together with energies of their harmonic oscillators are given.

| n (flavor) | 1  | 2  | 3  | 4  | 5  | 6  | 7  |
|-----------|----|----|----|----|----|----|----|
| Quark     | \( d \) | \( u \) | \( s \) | \( c \) | \( b \) | \( t \) | \( b' \) |
| Quark mass, MeV | 28.815 | 105.456 | 385.95 | 1412.5 | 5169 | 18919 | 69239 |
| Oscillator energy, MeV | 36.69 | 134.27 | 491.4 | 1798.4 | 6581 | 24088 | 88158 |

Then in [8, 9] we have shown on examples how the harmonic quarks may be used for description of structures and a masses of some particles. So, the strong results of the harmonic model is the resolution of a riddle of muon mass and interpretation of quark
structure of a neutral pion. The muon is successful attempt of the Nature, suppressing fractional electrical and color charges, explicitly to fix one \( u \)-quark mass state as a lepton. The neutral pion with a large probability is the stationary harmonic oscillator from \( u \)-quark-antiquark pair. Also there was shown as the harmonic quarks may be used for interpretation of quark structure of some light mesons and baryon resonances [9]. Since this article, we transfer to systematic investigations of structure of hadrons with the help of the harmonic quarks and their full oscillators.

2 The harmonic quarks in hadrons

Already in [9] it was scored, that the structure of a fundamental particle \( \eta(548) \) is probably similar to structure of atom of helium. It perhaps consists from two filled shells: the stationary \( s \)-quark oscillator and the external light \( dd \)-quark pair. The analogy was marked also with helium nucleus, which contain two filled shells from two protons and two neutrons. These analogies are deeply symptomatic and expected. Atoms and nucleuses with the filled shells have the greatest binding energy and stability. What should we expect from ensemble of quarks what naturally hadrons are? Quarks of hadron, as ensemble of fermions, will be united in some steady configurations on the basis of the strong coupling. Whereas steady configurations have the minimum energy it is possible to expect, that hadrons first of all will contain also the filled shells. We also shall search for the filled shells in hadronic structures.

2.1 Filled shells

The nuclear ensembles of fermions, with all definiteness have shown us, that the greatest binding energy and the greatest stability is observed at nucleuses with the filled shells, both on protons, and on neutrons. These are so-called twice magic nucleuses – \( ^4\)He, \( ^{16}\)O, \( ^{40}\)Ca. The appropriate configurational numbers for nuclear fermions is 2, 2 + 6... . At transition from a Coulomb interaction in atoms to a strong coupling in nucleuses the considerable role on smaller distances of a spin-orbit interaction was found out also. Whereas a distance between quarks in hadrons is less than a distance between a nucleons in nucleuses the role of a spin-orbit interaction can even increase. For the further analysis we shall be limited only by two filled configurations \( s^2 \) and \( p^6 \) which naturally give numbers 2, 6 and 2 + 6. In mathematical aspect the configuration \( p^6 \) is directly bound to rotational symmetry of three-dimensional space and \( s^2 \) is bound with dot central symmetry. In QCD we deal with color interaction which in mesons exists between a pair of quarks with color and anticolor, i.e. this is a numeral 2. Baryons have 3 valence color quarks and, in view
of anticolors, we have a numeral 6. So we see, completely electro neutral and colorless groups in QCD, for example, from quarks and antiquarks of one flavor can be 2 and 6. That is in the complete correspondence with especially stable nuclear configurations.

So, we shall suppose, that completely neutral configurations with numerals 2 and 6 of quarks of one flavor can form the filled shells with the minimum energy and be part of hadrons. Further we shall suppose also, that the filled \( s^2 \) and \( p^6 \) shells can be composed from a quark-antiquark pairs which are taking place or in a states of a full harmonic oscillators, or in the ”free” states. These shells can be designated as \( 0s^2, 0p^6, 1s^2 \) and \( 1p^6 \). The energy contribution to a hadron of one pair of quarks in states \( 0s^2 \) and \( 1s^2 \) accordingly will be \( \approx 127\% \) and \( \approx 200\% \) from mass of one ”free” quark (Tab.1). Stable two-quark mesons (pions and kaons) place in potential wells of full harmonic \( u- \) and \( s- \) oscillators \[8\]. It allows us to assume that other hadrons are placed also in potential wells of full harmonic oscillators or, that is more truly, in their combinations. The simple potential wells for light and strange mesons are already engaged by pions and kaons. Now, we shall try to construct some system of mesons and then it will be compared with experimental spectrum of meson masses. But before start it is necessary to make some notes.

### 2.2 Notes and Designations

First note is about \( d \)-quark. This quark should have some special properties as against other quarks. \( d \)-quark is the most light and therefore most mobile. It is last quark in a chain of quark decays and their oscillators. It may form at decay of \( u \)-quark or a harmonic \( u \)-oscillator, i.e. it is excitation downwards \[8\], but itself \( d \)-quark is not capable to generate a new quark with smaller mass. \( d \)-quark is as though an appendage or original prolongation \( u \)-quark. In our opinion, in view of activity of parallel chain of quarks \[9\], it also is the reason of isospin properties of \( d \)- and \( u \)-quarks. So, the mass of \( d' \)-quark is approximately defined by the equation \[10\]:

\[
m_{d'} \cong m_d \cdot \cos \theta + m_s \cdot \sin \theta \cong 110 \text{ (MeV)},
\]

where \( \theta \) – Cabibbo angle.

This value, as we see, is practically equal to \( d_\parallel \)-quark mass of parallel chain (105.4 MeV \[19\]) or \( u \)-quark mass of main chain. The subsequent quarks have the complete gang of properties because they have neighbors onto an energy scale both from above and from below. Properties of the heaviest quark are unknown and also should have features. Thus, \( d \)-quark is not completely independent quark. Really, there is no a hadron with mass about 60 MeV (\( 2m_d = 57.6 \text{ MeV} \)) or less; the lightest hadron – a neutral pion – is a harmonic \( u \)-oscillator \[8\].

The second note concerns the sizes of quarks. At the given work we shall not take
into account distinction of quark sizes. Nevertheless, from approximately equal impulses of quarks in a hadron follows, that more heavy quarks should place closer to centre of hadrons.

Let’s enter labels for some quark groups. The quarks of full harmonic oscillators we shall designate by the same symbols, but located in circles, for example, \(u\) or \(s\); while the number, which located in a circle, will inform us about number of harmoniously bound quarks and antiquarks of one flavor. So, \(2_u\) and \(6_s\), will specify the filled \(0s^2\) and \(0p^6\) shells for harmoniously bound groups from two \(u\)- and six \(s\)-quarks accordingly. In essence \(6_s\) group consist of three harmonic oscillators on each of axes of space, but, taking into account, that these three quark-antiquark of a pairs define \(0p^6\) state, it is important to underline it. Let’s also note, that records such as \(2_u\) and \(2_u\) are energy equivalent. Numerals 2 or 6 and symbols without circles we shall designate a groups of quarks in \(s\) and \(p\) states in which quarks have a complete mass.

In view of the notes made above on restricted properties \(d\)-quark and at absence of a mesons with masses below pion we conclude that quark configurations exclusively from \(d\)-quarks are unreal and we shall not view them.

### 3 General simulation

At the first stage the scanning simulation of harmonic quark spectrum has been carried out with using mesons up to 1000 MeV. The purpose of calculation is to define optimal set of harmonic quark masses in relation to a spectrum of actual mesons.

Calculations are carried out under following conditions: the \(u\)-quark mass was varied from 70 up to 270 MeV with a step with 0.2 MeV; the quantity of mesons is equal 14 (antiparticles are excluded). On each step we calculated a standing of potential wells and the sum of deviations of masses of mesons from the nearest wells. Further a root-mean-square variance was calculated and then there were make the normalization on number of potential wells on an interval 0–1000 MeV, i.e. we took into account the current density of potential wells on an energy scale. The normalizing factor is accepted equal to 1 for the set of the harmonic quark masses given in Tab.1. States \(0s^2\) and \(0p^6\) for us are energy degenerate, therefore the configuration \(0s^2 + 0p^6\) consist simply from 4 harmonic oscillators. In result, the model spectrum of potential wells consist exclusively of a sets of the harmonic quark oscillators in quantity from 1 up to 4 for two flavors \(n=2\) and \(n=3\) (\(u\)-quark and \(s\)-quark), namely:

\[
2\@, 4\@, 6\@, 8\@; \\
2\@, 2\@ + 2\@; \ldots; 2\@ + 8\@; \\
4\@, 4\@ + 2\@; \ldots.
\]
The current mass $u$-quark was varied approximately in 3.7 times that follows from a relation [1]. In this case we capture all possible mass variants for the harmonic quarks. The computer calculations, which have been carried out on the complete actual set of mesons up to 1000 MeV, have shown, that the masses of harmonic quarks calculated by us [8] are the optimal, i.e. settle down in a global dispersion minimum. Figure 1 shows the dependence of a variance on the current mass $u$-quark. On the dispersion curve (Fig.1) the global minimum is observed at mass $u$-quark 105.6 MeV. The same value was calculated in [8] by other way. Any other set of harmonic quark masses gives a boosted variance.

Figure 1: The change of a variance depending on mass $u$-quark for mesons up to 1000 MeV.

The dispersion curve has also two local minimums at 92 and 131 MeV. The study of deviations of meson masses from potential wells for minimums has shown, that minimums at 92 and 131 MeV are casual events and have no physical content. Figure 2 illustrates concrete values of deviations of meson masses from potential wells for according masses of $u$-quark.

For mass $u$-quark 105.6 MeV a pions and kaons locate into potential wells while for 92 and 131 MeV it is not observed. So, for 92 MeV pions, kaons and $\eta(548)$ have a deviation more than 20 MeV. For 131 MeV – pions, $\eta(548)$, kaons (J=1), $a(980)$ and $f(980)$. For
\( u \)-quark with mass 105.6 MeV only \( \omega \) and \( \eta(958) \) the deviation has more than 20 MeV.

![Figure 2: The deviation of mesons from harmonic potential wells.](image)

It is important that the global minimum on a dispersion curve keeps position at 105 MeV even if to exclude pions from viewing. Moreover, its keeps position even if to exclude both pions and kaons with \( J=0 \). Thus, we have received one more confirmation of validation for harmonic quark modell and a uniqueness of mass solutions. The dispersion of a global minimum is equal 10.6 MeV. At “lockout” of an isospin on average masses of 9 mesons (\( \pi, K, \eta(548), \rho, \omega, K(J=1), \eta(958), a(980), f(980) \)) the dispersion of a global minimum is equal 13 MeV and \( m_u = 105.6 \) MeV also.

The further simulation was carried out on a generated mass spectrum with random distribution of 9 mesons on an interval 100–1000 MeV. In this case the probability of observation of a variance equal or smaller 13 MeV are lower than 4%. Hence, we may say, that the found correlation between the masses of harmonic quarks and an actual spectrum of meson masses up to 1000 MeV with probability 96%, at least, is not casual. Certainly the actual spectrum of mesons is not casually the distributed and at generation of meson masses we should take into account most simple requirements, for example, obligatory presence about first potential well at least one generated meson or to take into account mass ratios for mesons with simple quark compositions, for example, the charged...
pion and kaon. Therefore, the probability of casual correspondence of the harmonic quarks and actual meson spectrum should be much lower than 4%. So, only at the fixed generation of the second meson (analog $K$) at a potential well $2\bar{s}$ with a deviation no more than 5 MeV the probability of casual observation of the detected legitimacy decreases with 4% down to 0.1%. If analogs $K$ and $\pi$ is generated within $\pm 5$ MeV about wells $2\bar{s}$ and $2\bar{u}$, then the probability decreases with 4% down to 0.001%, i.e. we come near to a level of reliability in 4$\sigma$. Thus, we can state, that a conformity of real and model mass spectrums are observed with probability near to 0.9999. Figure 3 illustrates this physical legitimacy.

Figure 3: The arrangement of the model harmonic wells at mass $u$-quark 105.6 MeV and mesons up to 1000 MeV.

Let’s estimate degree of correspondence of two spectrums by other way. On an interval 1000 MeV the mean distance between wells is $\approx 110$ MeV and, hence, the mean distance from any position on energy scale to a nearest well is lower than $\approx 55$ MeV. For only random distribution on an interval 0–55 MeV a deviations of meson masses from wells would be more or less homogeneously. However all 9 deviations is less than half of it interval and the majority of them does not exceed 5 MeV. Allocation is completely one-sided.

The ratio of an actual deviation of meson mass to all interval 55 MeV is a probability $P_n$ what a casual event will take place in a deviation area. Mathematical expectation for $P_n$ is equal 0.5. On 9 mesons the probability of casual correspondence $P_c$ will equal:

$$P_c = \prod_{n=1}^{9} \frac{P_n}{2} \approx 5 \cdot 10^{-7}$$

Certainly, negligible quantity $P_c$ speaks to us what both meson and model mass spectrums are very strongly connected. Thus we have received the proof of real existence the
Table 2. The energies of simple harmonic quark combinations and masses of some mesons.

| Quark configuration | Configurational energy, MeV | Meson | Mass of a meson, MeV | Difference, MeV |
|---------------------|-----------------------------|-------|---------------------|-----------------|
| 2u                  | 134.27                      | \(\pi^0\) | 134.98              | 0.71            |
|                     |                             | \(\pi^\pm\) | 139.57              | 5.30            |
| 6u                  | 402.81                      | -     | -                   | -               |
| 2s                  | 491.4                       | \(K^\pm\) | 493.65              | 2.25            |
|                     |                             | \(K^0\) | 497.67              | 6.27            |
| 6s                  | 1474.2                      | \(a_0(1450)\) | 1474±19             | -0.2 ± 19       |

harmonic quarks and oscillators. Besides it is obviously that the harmonic quark masses are calculated enough exactly.

4 Possible structures of mesons up to 1000 MeV

Now we can begin detailed study of possible structures of mesons. Apparently (see Fig. 3) mesons place at the following potential wells:

- pions and kaons at single oscillators \(-2_u \ 2_s\);
- \(\eta(548)\) at \(2_u + 6_u\);
- \(\rho(770)\) at \(2_s + 22_u\);
- \(K^*\) at \(2_s + 6_u\);
- \(a(980)\) \(f(980)\) at \(22_s\).

4.1 One filled harmonic shell

So, we shall begin with study of the filled configurations with two and six harmoniously bound \(u\)- and \(s\)-quarks. We could expect presence of potential wells not only for \(2_u\) and \(2_s\), but also for the following simple quark combinations:

- \(6_u\) and \(6_s\).

The depth of potential well of separate harmonic oscillator \(2_u\) and \(2_s\) is accordingly equal 77 and 280 MeV.

In Table 2 these quark configurations are given together with a mesons, which is in these potential wells. Data in Table 2 show, that in potential wells with one quark oscillator are placed a stable mesons of a hadronic spectrum: pions and kaons. That’s right. In potential wells with three oscillators we find only one meson triplet \(a_0(1450)\). The configuration \(6_s\) is filled shell of type \(0p^6\). We could count that a configuration \(6_s\)
is far-fetched. It does not give anything essential, and detection of $a_0(1450)$ in area $\{6\}_s$ is simple concurrence. However we should remember, that in a potential well $\{4\}_s$ with unfilled shell $0p^4$ and with energy 982.8 MeV we also detected a triplet $a_0(983)$ and meson $f(980)$. Nevertheless, in the area of energy $\{6\}_u$ there are no mesons, and we shall try to understand, why it happened.

Configurations $\{6\}_u$ and $\{6\}_s$ differ from each other by energy and number of accessible flavors. At the energy of a configuration $\{6\}_u$ a mesons can consist only from $d$- and $u$-quarks. Except for the filled shell $0p^6$, consisting of three harmonic $u$-oscillators, one more configuration with the same energy and the filled shells is possible: $0s^2(\bar{p}) + 1s^2(u) + 1s^2(d)$. Nothing hinders both configurations immediately to break up to three harmonic oscillators, if energy of $\{6\}_u$ is higher than mass of three $\pi^0$. Any potential barrier at this decay should not be because of a special standing $\pi^0$ in hierarchy of mesons. As we already have assumed earlier in \cite{8} and mentioned above, $\pi^0$ is easiest stable hadron, probably, the unique hadron, which have structure only from one stationary harmonic oscillator. All other applicants with quarks of other flavor, for example, $K^0$ and $D^0$ have neutral antiparticles. So, $K^0$ makes an attempt to turn into the state of harmonic oscillator. However this state is unstable for $K^0$ and with probabilities 50% $K^0$ is returned in a former state or in a state of an antiparticle. In other words, in our opinion, the reason of known oscillations $K^0$ is connected with its unsuccessful attempts to form a stationary harmonic oscillator. Neutral $\pi^0$ is unique in this respect. It simply has no opportunity to turn into any other quark state as against $K^0$ ($s\bar{d}$ or $\bar{s}d$). This unique standing $\pi^0$ probably also is the reason of absence of a neutral meson in a potential well with a configuration $\{6\}_u$ as it is separated without a barrier on 3 stable $u$-oscillator ($3\pi^0$). Besides if a real level $0s^2$ is disposed below the levels $0p^6$, then the filled configuration from two shells will contain 4 $u$-oscillator. It would mean, that real meson may have the mass a little more 537 MeV.

Area of energies $\{6\}_s$ is richer by both an energy opportunities and a quark flavors. Here depth of a potential well is much greater therefore existence of a meson is quite probable. Without taking into account $d$-quarks two configurations with the filled shells and with energy 1474 MeV are here also possible:

$\{6\}_s$ and $\{2\}_s + s\bar{s} + u\bar{u}$.

Last configuration will consist of three filled shells of type $s^2$:

- the first shell from the most bound state, e.g. two harmonic $s$-quarks;
- the second shell from the "free" pair $s$-quarks;
- the third shell from the "free" pair $u$-quarks.

At the account $d$-quarks the number of the filled configurations will considerably increase. Decay of this configuration on 3 strange of an oscillator does not occur because there is no the strange analog $\pi^0$. The strong decay on $3K^0$ is prohibited and as a result in the well we have a resonance $a_0(1450)$. 

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Table 3. Composite combinations from harmonic oscillators and masses of some mesons.

| Quark configuration | Configurational energy, MeV | Meson | Meson mass, MeV | Difference of masses, MeV |
|---------------------|-----------------------------|-------|----------------|-------------------------|
| 2_u + 6_u           | 537.1                       | η(548)| 547.7          | 10.6                    |
| 2_u + 2_s           | 625.7                       | -    | -              | -                       |
| 6_u + 2_s           | 894.2                       | K⁺⁻  | 891.7          | -2.5                    |
|                     |                             | K⁺⁰  | 896.2          | 2.0                     |
| 2_u + 6_s           | 1608                        | π₁   | 1593±8±32      | -15±8±32                |
|                     |                             | X(1600) | 1600±100      | -8±100                  |
| 6_u + 6_s           | 1877.0                      | p̅p  | 1876.54        | -0.46±0.56              |
| 2_s + 6_s           | 1965.6                      | K₂   | 1973±8±25      | 7.4±8±25                |
|                     |                             | K⁰   | 1945±10±20     | -19.4±10±20             |
|                     |                             | X(2000) | -          | -                       |
|                     |                             | f₂(1950) | -         | -                       |

4.2 Two filled harmonic shells

Now we shall transfer to the analysis of quark configurations with two filled shells. Table 3 demonstrates these combinations. They were composed of simple wells which are given in Table 2. Even at absence of actual mesons in the area of the most light mixed configuration 2_u + 2_s the data of Table 3 bring to us a presents. At first, in the potential well from two harmonic filled shells 6_u + 2_s appeared the first strange vector doublets K⁺⁻ and K⁺⁰, and the centre of a well is located precisely between mesons. Second, energy of a configuration 6_u + 6_s is equal precisely to mass of the proton-antiproton pair. This relation is true with an error of definition of quark mass, that was calculated in [8] and equal ≈ 0.03%. This error in recalculation on group 6_u + 6_s is given in Table 3 (±0.56 MeV). It is more than energy difference between p̅p and harmonic group, as this should be at the correct definition of precision with which the masses of harmonic quarks are calculated. The new energy equality gives unique occasion for reflection about the nature of the proton and same time we might hope onto increasing of precision of quark mass calculation. Now we shall sum up the data of Table 3. Except for the configuration 2_u + 2_s all others harmonic configurations with two filled shells successfully work. In double potential wells we are discovered an important mesons and even unexpectedly p̅p a threshold state.

Thus, we can summarize the achieved results which obtained in Tables 2, 3 and also [8].
We managed to locate in harmonic potential wells of full quark oscillations the following hadrons up to 1000 MeV:

- pions, kaons, \(\eta(548)\), \(K^{\pm}\), \(K^*0\), nucleons, \(f(980)\) and \(a(980)\). Besides we shall take into account, that the mass of the first vector meson with latent strangeness \(\omega(782)\) only on 11 MeV is more than mass of the strange pair of quarks \(s\bar{s}\) (772 MeV). The width of a resonance \(\omega(782)\) is 8.4 MeV, i.e. peak begins immediately at reaching of a threshold for \(s\bar{s}\) pair. So now for completeness of a search we should find a place only for two mesons \(\rho(770)\) and \(\eta'\). To find quark configurations of these mesons we shall try to use already found neighboring potential wells with smaller energy. For \(\eta'\) the nearest filled configuration is \(6_u + 2_s\) with energy 894.2 MeV. The difference (64 MeV) is close to mass of \(d\bar{d}\) quark pair (58 MeV). Therefore, the configuration \(\eta'\) could be following: \(2_s + 6_u + d\bar{d}\). Energy of a configuration 951.9 MeV on 6 MeV is less than the mass of \(\eta'\). Then this configuration will consist of three filled shells and it is good that two of them – the heaviest and the most light – same as a quark shells of \(\eta(548)\) meson. Identical external filled shells of both mesons, perhaps, also determine their properties and a title, as \(\eta\) hadrons. The accepting this configuration for the true, we could tell, that the solution of secret of existence and mass \(\eta'\) was reached only with the help of the harmonic quarks. We could go farther and to try to define the mass of \(\eta'\). For this purpose let us to replace the mass of a harmonic oscillator \(2_s\) on mass \(K^0\) and then the mass of \(\eta'\) should be equal 958.1 MeV. For such replacement we have some basis. As we mentioned already \(K^0\) in solo fulfillment make unsuccessful tries to turn into a state of a stationary harmonic oscillator. Presence of other shells can stabilize this process. The part of excess energy of a kaon about 6 MeV (over the strange harmonic oscillator) can be redistributed to external shells. This loss of energy can stabilize the strange oscillator on time life of meson. As shown in [9], the most light \(d\)-quark in \(\pi^\pm\) can have a kinetic energy in some MeV and redistribution of energy from a shell \(2_s\) to a shell \(d\bar{d}\) is quite explained.

We can see, that a shell \(6_u\), not being implemented in solo variant, can be formed at the presence of a potential well of the strange oscillator. Most likely, in this case, the heavy internal shell \(2_s\) with a deep potential well stabilizes the shell \(6_u\) which consist of more light \(u\)-quarks. Then both shells is present in \(K^*\) and \(\eta'\) mesons. These mesons different from kaons and \(\eta(548)\) by shell \(6_u\) only.

Now let’s consider \(\rho(770)\) meson. The filled harmonic well nearest to \(\rho(770)\) is single empty configuration \(2_u + 2_s\) in Table 3. The difference of energies between them \(\approx 140\) MeV is close to masses of pions and \(u\)-oscillator. On Figure 3 we see, that \(\rho(770)\) situated beside with wells \(2_s + 22_u\). Therefore as a first step we can assume this configuration for \(\rho(770)\).

Energy of a configuration is 760 MeV. The actual quark configuration \(\rho(770)\) can be
following for $\rho^0$ and $\rho^\pm$:

$$\begin{align*}
\bar{s}u + u\bar{u} + d\bar{d} \quad \text{and} \quad \bar{s}u + \bar{u}d + u\bar{d} \quad \text{(or} \quad \bar{u}d)\end{align*}$$

Thus, for $\rho^0$ we have a configuration from three filled shells $0s^2(\bar{s}) + 1s^2(u) + 1s^2(d)$.

The $\rho^\pm$ consist from two neutral filled shells $0s^2(\bar{s}) + 0s^2(\bar{u})$ and two valence quarks, as well as in $\pi^\pm$. The interval of possible masses $\rho(770)$ can reach from energy of a well 760 up to 776 MeV. This maximal energy can be received for variant $K^0 + 2\pi^\pm$. In the latter case $K^0$ is a mass analog of a harmonic oscillator $\bar{s}$, in the same sense, as well as in $\eta'$. Experimental medial mass $\rho(770)^\pm$ also is in an interval from 763 up to 776 MeV since it is depends from conditions of observation [11].

Let’s summarize some results of our researches of hadron structures up to 1000 MeV. Using only the filled shells of type $0s^2$ and $0p^6$ from the harmonic quarks and their oscillators, we managed to construct system of potential wells adequate to experimentally observable particles. The obtained data are shown on Figure 3 and are generalized in Table 4.

| Configuration of potential hole | Configurational energy, MeV | Hadron | Conventional or possible configuration of hadron | Hadron mass, MeV |
|---------------------------------|-----------------------------|--------|-----------------------------------------------|------------------|
| $\bar{s}u$                     | 134.27                      | $\pi^0$ | $\bar{s}u$                                    | 134.98           |
|                                |                             | $\pi^\pm$ | $ud, \bar{u}d$                                | 139.57           |
| $\bar{s}s$                     | 491.4                       | $K^\pm$ | $us, \bar{u}s$                                | 493.65           |
|                                |                             | $K^0$   | $s\bar{d} \iff \bar{s}s$                      | 497.67           |
| $\bar{s}u + \bar{s}u$          | 491.4                       | $\eta(548)$ | $? \bar{s}u + ud$ | 547.7           |
|                                |                             |        | $2(ud + \bar{u}d)$                            |                  |
| $\bar{s}s + 2\bar{s}u$        | 760                         | $\rho^\pm$ | $\bar{s}s + \bar{u}\bar{u} + dd$ | $\approx 769$ |
|                                |                             |        | $\bar{s}s + \bar{s}u + (ud$ or $\bar{u}d)$ | $\approx 769$ |
|                                | $2m_s \approx 772$         | $\omega(782)$ | $s\bar{s}$ | $\approx 783$ |
| $\bar{s}s + \bar{s}u$ + $\bar{s}u$ | 894.2                      | $K^{\pm\pm}$ | $(u\bar{s} or \bar{u}s) + \bar{s}u$ | 891.7           |
|                                |                             |        | $K^0$ | $(sd or \bar{s}d) + \bar{s}u$ | 896.2           |
|                                |                             | $\eta'$ | $\bar{s}s + \bar{s}u + dd$ | 958 |
| $s_u + \bar{s}u$               | 938.5                       | $\rho$ or $\bar{p}$ | $? ? ?$ | 938.3           |
| $2\bar{s}s$                   | 982.8                       | $a(980)$ | $\rho(770) + \bar{u}\bar{u}$ | 984.7           |
|                                |                             | $f(980)$ | $s\bar{s} + \bar{u}\bar{u}$ | $\approx 980$ |

All hadrons up to 1000 MeV are submitted in Table 4. Two groups of mesons pay attention pays to itself. These groups are very similar each other under the status of particles:

- $K^{\pm}, K^0$ and $\eta(548)$;
\[ K^{\pm}, K^{*0} \text{ and } \eta'. \]

Both groups differ only by the filled neutral shell 6\( u \) which does not change accessories of mesons to the strange particles or \( \eta \). Besides on an example of \( \eta \)-mesons we may note that the potential well is capable to keep an additional quark-antiquark pair. But we should remember, that there are variants of decisions for \( \eta \) mesons.

### 4.3 Proton-antiproton configuration

Certainly, the most important and unexpected result is obtained for the pair of proton-antiproton. Decay of a configuration \( 6_u + 6_s \) in halves on \( p\bar{p} \) pair can be accomplished by various ways with formation completely vague inside baryon structures.

However, it is very important, that decay happens of two neutral boson configurations \( 0p^6 \), each of which consists of three pairs of full harmonic oscillators.

One possible variant of decay of the symmetric configuration is submitted on Figure 4. The decay generates a proton consisting of three \( u \)-quarks and three \( s \)-quarks so that the total electrical charge is equal +1.

![Figure 4: The decay scheme of two harmonic 0p6-configurations on a proton and an antiproton.](image)

If the main rules of QCD for harmonic bound quarks is same as for free quarks, i.e. their electrical and color charges is equal, then we discover, that the variant of decay on Figure 4 is unique.
If to unite $u$ and $s$ in the pairs $us$, we can receive three partons with a charge $+1/3$. Let’s mark, that in such decay will give in doubling color charges in a proton, as against the concept conventional now. In too time this doubling of color charges gives only the doubling of the total white charge, i.e. this consequence is superable. On the other hand, we can suppose, that one of two groups of a color charges annihilate at decay and at formation of partons, then the color of a proton remains white in the conventional sense.

Because of the complete vagueness of a problem in Table 4 for a proton and an antiproton is conditionally written down the following configuration: $\bar{3}_u + \bar{3}_s$.

4.4 Other boson configurations

Discovery $p\bar{p}$ boson configuration consisting of two filled shells $0p^6(u)$ and $0p^6(s)$, has allowed us to assume, that perhaps there are others important boson configurations, which can generate a hadrons at decay at conservation of total energy. Three simple groups concern to configurations of kind $1p^6$: $6u$, $6s$ and $6u + 6s$ with energies 632.7, 2315 and 2948 MeV.

These electro neutral configurations are also completely colorless groups since each shell $1p^6$ have three colors and three anticolors.

The first configuration, we shall term it $6u$-boson, presents to us a new very important equality of energies.

It is easily to be convinced, that the sum of masses of two most light neutral mesons $\pi^0$ and $K^0$ ($632.649 \pm 0.03$ MeV) is exactly equal the mass of $6u$-boson. $\pi^0$ is truly neutral particle. $K^0$ in long-term (average integrated) sense is truly neutral particle because it is capable to turn into of a state a particle to a state an antiparticle and on the contrary. Thus, we have energy equality of masses of two truly neutral groups. Probability of casual concurrence of energies is close to zero because below 632.7 MeV there are only two simple neutral mesons $\pi^0$ and $K^0$, which represent two simple harmonic oscillators: $\bar{2}_u$ and $\bar{2}_s$. Nevertheless we should estimate a probability of an accidental coincidence of these energies. Estimation of probability of casual event $P$ it is feasible counting upon hit of an integer $u$-quarks on value of the sum of meson masses 632.65 MeV:

$$P \approx 2 \frac{\Delta m_6u}{m_6u} \approx 0.003$$

(4)

where $\Delta m_6u$ – the error with which $6u$-boson mass was defined. Complementary probability (0.997) says to us about some objective order of micro world that gives raises this equality.

All this series of important ”coincidences” testify only that our deductions are true and a manifestation of the harmonic quarks have a true systematical character ($\mu, \pi^0, K^0, \eta(548)$, 548),

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\( K^*, \Delta(1232) \) [2], \( p\bar{p} \), etc.). If we shall estimate the total probability of all casual coincidences on all this array that a result shall completely insignificant and further we will not more return to their discussion. Practically all hadrons up to 1000 MeV concern to \( \text{L=}0 \) background states and the author is sure, that the harmonic quarks and their full oscillators are the tool for the analysis and understanding of these states.

The author believes in possibility, that a found energy equalities are real manifestation of the mechanism of a mass-formation in Higgs sector.

### 4.5 Masses of harmonic quarks

So, we have two important equalities:

\[
m_{p\bar{p}} = 3 \cdot \frac{4}{\pi} (m_u + m_s) \tag{5}
\]

\[
m_{\pi^0} + m_{K^0} = 6m_u \tag{6}
\]

We yet do not understand sense and meaning of equalities (5) and (6). However with believing, that these equalities are true and reflect certain actual physical processes, we can use these equalities for improvement of masses of the harmonic quarks. As the precision of total mass of \( \pi^0 \) and \( K^0 \) exceeds the calculated precision of mass 6\( u \)-boson (632.736±0.18) in 6 times then this give a chance to improve an exactness of quark mass definition up to ± 0.005% (see Table 5).

| Boson       | \( d \) | \( u \) | \( s \) | \( c \) | \( b \) | \( t \) | error,\% |
|-------------|--------|-------|-------|-------|-------|-------|---------|
| \( b\pm \)  | 28.815 | 105.456 | 385.95 | 1412.5 | 5169.4 | 18919 | ±0.030  |
| \( \pi^0 + K^0 \) | 28.811 | 105.441 | 385.89 | 1412.3 | 5168.7 | 18916 | ±0.005  |
| \( p\bar{p} \) | 28.807 | 105.429 | 385.85 | 1412.1 | 5168.1 | 18914 | ???     |

The distinction of quark masses, which was calculated of two new relations (5) and (6), is only 0.01%, but nevertheless it exceeds in two time a relative error of mass definition of group \( \pi^0 + K^0 \). In spite of the fact that the relative precision of proton mass definition is better on the one order, than for a kaon, nevertheless the difference in \( \approx 0.01\% \) is referred to a proton. For this reason, in Table 5 in the cell ”error” for a \( p\bar{p} \) pair are put the symbols ???. This care has some basis because a proton is a stable unique fermion with the considerable magnetic moment as against mesons \( \pi^0 \) and \( K^0 \). Therefore the decay of configuration \( 6_u + 6_s \) probably may be accompanied by some allowances to mass of a proton. At present we do not know neither type of an allowances, nor their
quantities. Existence of the anomalous magnetic moments at an electron or a muon is the quite sufficient reason for caution. Having it in a view, the author recommends using the values of quark masses, obtained of the mass sum $m_{\pi^0} + m_{K^0}$.

From mass relation (6) the important deduction follows about a possible configuration of $\rho(770)$ meson. We built its configuration on the basis of a potential well $2u + 2s$ with addition of a pions. But the potential well $2u + 2s$ in actual meson fulfillment is essence $\pi^0 + K^0$. Taking into account a relation (6) we can note the new variant of configuration for $\rho$-mesons: pion + 6$u$-boson or pion + 1$p^6$(u). Thus, we have received for $\rho$-meson a configuration of a pion with an additional filled shell $1p^6(u)$, which is completely electro-neutral and colorless. At present we have no answer to a question: is which implemented of these two configurations? Nevertheless last configuration seems more preferable with both on simplicity (only two shells) and on likeness to a triplet of pions. Besides this variant has a likeness of shells with the vector mesons $K^*$. 

In the end let’s consider last configuration 6$s$ with energy 2315 MeV. Directly in this area of energy we have three $f$-mesons: $f_2(2300)$, $f_4(2300)$ and $f_2(2340)$. For a configuration 6$s$ the role similar $\pi^0$ in group 6$u$ plays a harmonic oscillator $2s$ or an actual meson $K^0$, which tightly the bound with this oscillator. Then energy of the second participant will be $\approx 1817$ MeV. In this area we shall detect again $\pi^0(1800)$ with $J=0$ also and $f_2(1810)$. It is not so clear, but encouraging results.

We shall not view the configuration $6u + 6s$ with energy 2948 MeV, since it in essence a combination of two above considered configurations. The half of this configuration, i.e. $3u + 3s$, is identical energetically to group $6s$ and is submitted in Table 2.

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