Effective mass suppression in dilute, spin-polarized two-dimensional electron systems

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(Dated: July 8, 2008)

We report effective mass ($m^*$) measurements, via analyzing the temperature dependence of the Shubnikov-de Haas oscillations, for dilute, interacting, two-dimensional electron systems (2DESs) occupying a single conduction-band valley in AlAs quantum wells. When the 2DES is partially spin-polarized, $m^*$ is larger than its band value, consistent with previous results on various 2DESs. However, as we fully spin polarize the 2DES by subjecting it to strong parallel magnetic fields, $m^*$ is unexpectedly suppressed and falls even below the band mass.

PACS numbers: 71.18.+y, 73.43.Qt, 72.25.Dc

In a crystalline solid, electrons moving in the periodic potential of ions are described as quasi-particles with a band effective mass, $m_b$, which is inversely proportional to the curvature of the energy-vs-wavevector (band) dispersion. In the presence of electron-electron interaction, in the Fermi liquid description, the electrons can still be treated as quasi-particles but with a further renormalized effective mass, $m^*$. Numerous studies, both experimental and theoretical, have indeed reported that for low-disorder, dilute, two-dimensional electron systems (2DESs), $m^*$ is typically larger than $m_b$ at very low densities ($n$), and increases as $n$ is reduced and the system is made more interacting [1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12]. Note that the parameter $r_s$, defined as the ratio of the Coulomb to kinetic (Fermi) energy, increases as the 2DES is made more dilute [13]. Here we report $m^*$ measurements in dilute 2DESs confined to AlAs quantum wells. Our main finding, shown in Fig. 1, is that $m^*$ depends not only on $n$ but also on the spin-polarization of the 2DES. When the 2DES is partially spin-polarized, the measured $m^*$ is larger than $m_b$, consistent with most previous reports on other 2DESs. When we fully spin polarize the 2DES by applying a parallel magnetic field, however, $m^*$ is strongly suppressed to values below $m_b$.

Our AlAs quantum well samples were grown, using molecular beam epitaxy, on semi-insulating (001) GaAs substrates. The AlAs wells in these samples have widths of 11 nm (sample A), 12 nm (sample B), or 15 nm (samples C and D). They are bounded by AlGaAs barriers and are modulation-doped with Si [14]. In our samples, thanks to a combination of residual and applied uniaxial in-plane strain [14], the electrons occupy one conduction band minimum (valley) with an anisotropic (elliptical) Fermi contour, characterized by transverse and longitudinal band effective masses, $m_0 = 0.205m_e$ and $m_0 = 1.05m_e$, where $m_e$ is the free electron mass. This means that the relevant (density-of-states) band effective mass in our 2DES is $m^*_0 = \sqrt{m_0 m_b} = 0.46m_e$; this is the value to which we normalize and report all our measured masses.

The samples were Hall bar or van der Pauw mesas, fitted with back and front gates to tune $n$. For the density range $0.55 - 4.8 \times 10^{11} \text{ cm}^{-2}$, the low-temperature mobilities for the samples are between 0.9 and 6 m$^2$/Vs when current is passed along the low-mobility (longitudinal) direction of the occupied valley; for current along the high-mobility (transverse) direction, the mobility is typically 3 to 5 times larger than the above values. Transport measurements were performed using standard low-frequency lock-in techniques, and the samples were cooled in either a dilution refrigerator or a $^3$He cryostat with base temperatures ($T$) of 20 mK and 0.3 K, respectively. In both cryostats, the samples were mounted on a tilting stage so that the angle $\theta$ between the sample normal and the direction of the magnetic field could be varied; we define the total magnetic field as $B_{tot}$, and the two components by $B_\perp$ and $B_\parallel$ (see Fig. 2 inset).

Figure 2 shows the evolution of magnetoresistance traces for sample B at $n = 1.15 \times 10^{11} \text{ cm}^{-2}$ as it is tilted in the magnetic field. The simple Landau level (LL) diagram shown in the inset depicts the quantized energy
levels split by the cyclotron ($E_C$) and Zeeman ($E_Z$) energies and explains this evolution. For this density, at $\theta = 0^\circ$, $E_Z \equiv 3E_C$. This leads to a LL "coincidence" near $\nu = 4$ and the resistance exhibits a maximum rather than a minimum. As the sample is tilted away from $\theta = 0^\circ$, $B_{||}$ increases and enhances the ratio of $E_Z/E_C$, making LLs of opposite spin go through coincidences. This causes, e.g., the $\nu = 5$ resistance minimum at $\theta = 0^\circ$ to become a maximum (at $\theta = 46.8^\circ$) and then turn into a strong minimum at larger $\theta$ when the 2DES becomes fully spin-polarized. Note that for each $\nu$, there is a $\theta$ beyond which the system becomes completely spin-polarized.

Data of Fig. 2 reveal a stark contrast between the traces taken at small and large $\theta$: the resistance oscillation centered around a given $\nu$ becomes much stronger once $\theta$ is increased beyond the last coincidence angle (for that $\nu$) and the 2DES is fully spin-polarized. This observation provides a qualitative hint that the effective mass is suppressed when the spins are all polarized.

To verify this conjecture we performed $m^*$ measurements on this sample at the same density at a large tilt angle ($\theta = 84.6^\circ$) as shown in Fig. 3(a). We followed the usual procedure for mass determination from the $T$-dependence of the amplitude ($\Delta R$) of the Shubnikov-de Haas (SdH) resistance oscillations. In the simplest picture $\Delta R$ is given by the Dingle expression:

$$\Delta R/R_o = \exp(-\pi/\omega_c\tau_q)\xi/sinh(\xi),$$

where $R_o$ is the non-oscillatory component of the resistance and $\tau_q$ is the single-particle (quantum) lifetime. In the simplest picture, both $R_o$ and $\tau_q$ are assumed to be $T$-independent. Here we make this assumption and will return to its consequences later in the paper. The factor $\xi/sinh(\xi)$ represents the $T$ induced damping where $\xi = 2m^*k_BT/\hbar\omega_c$ and $\omega_c = eB_{||}/m^*$ is the cyclotron frequency. We define $\Delta R = (R_{max} - R_{min})$ as indicated in Fig. 3 (for the lowest $T$ trace) and plot $\Delta R/T$ vs. $T$ on a semi-log plot and fit the data to the above Dingle expression to determine $m^*$. Figure 3(b) shows such a plot for $\nu = 5$. The values of $m^*$ so obtained are shown in the inset to Fig. 3(a). Evidently, $m^*$ for the fully spin-polarized 2DES is well below the band mass! In Fig. 3(c), in a "Dingle plot" of $\Delta R/(\xi/sinh(\xi))$ vs. $1/B$, we observe an approximately linear behavior, implying a field-independent $\tau_q = 2.9$ ps.
Next, we present data showing the variation of \( m^* \) with spin-polarization at a fixed \( n \). In Fig. 4(a) we show traces for sample A taken at \( n = 2.56 \times 10^{11} \text{cm}^{-2} \) and \( \theta = 73.2^\circ \). At this \( \theta \), the 2DES is fully spin-polarized for \( \nu < 7 \). We show the measured \( m^* \) for \( \nu = 3, 4, \) and 5 in Fig. 4(b); the inset to Fig. 4(a) shows the \( T \)-dependence of \( \Delta R/T \) for \( \nu = 3 \). In Fig. 4(b) we also include \( m^* \) deduced for the partially spin-polarized case from similar magnetoresistance data (not shown) at \( \theta = 27.1^\circ \); the \( \Delta R/T \) vs. \( T \) plot for \( \nu = 8 \) for this \( \theta \) is also shown in Fig. 4(a) inset. Clearly, \( m^* \) in the partially polarized case is much larger than \( m^* \) for the fully polarized case. The energy ladder diagrams in Fig. 4(b) schematically show the positions of the various LLs and the Fermi energy \( (E_F) \) for two representative cases whose data are shown in Fig. 4(a) inset. Note that when the 2DES is partially spin-polarized, its LLs could be staggered so that \( E_F \) can be in an energy gap whose magnitude is smaller than \( \hbar \omega_c \).

In our experiments, for each partially spin-polarized case, we carefully chose \( \theta \) (via coincidence measurements) to ensure that the spin-up and spin-down levels overlap, as shown in the left ladder in Fig. 4(b). Note also that for the case shown (\( \nu = 8 \)), the 2DES spin-polarization \( P \), defined as the difference between the number of occupied up and down spin levels divided by the total number of occupied levels, is \( P = 0.25 \).

Figure 1 summarizes our results for four samples of different well-widths, densities, and mobilities, measured in seven separate cooldowns, and over a wide range of \( \nu \). The data also include different measurement geometries, e.g., orienting the current and/or \( B || \) along the major or minor axis of the occupied conduction band Fermi ellipse. For example, for sample A, \( B || \) was applied along the minor axis of the occupied conduction-band ellipse while for B, C and D, \( B || \) was along the major axis. As is clear from Fig. 1, the \( m^* \) suppression is observed in all cases and is independent of such parameters. We have preliminary data indicating that the \( m^* \) suppression is also observed in fully spin-polarized narrow AlAs quantum wells (well width < 5nm) where the 2D electrons occupy an out-of-plane conduction band valley with an isotropic in-plane Fermi contour and \( m_b = 0.205m_\text{e} \). The fact that we observe the suppression in both of these two 2DESs suggests that the suppression is general.

However, a few words of caution regarding \( m^* \) determination from the amplitude of the SdH oscillations are in order. As is generally done, in our analysis we have assumed that \( R_0 \) and \( \tau_0 \) or, equivalently, the Dingle temperature \( T_D = \hbar/2\pi k_BT_\omega \), are \( T \)-independent. Such an assumption indeed appears reasonable for data of Fig. 4(a) where the background resistance in the field range where we analyze the \( T \)-dependence of the SdH oscillations is essentially independent of \( T \). But note in Fig. 3 that the background resistance depends on \( T \). More generally, we typically observe an enhancement of \( m^* \) (over \( m_b \)) when the \( T \)-dependence of the background resistance is “metallic”, while the \( m^* \) suppression ensues when the 2DES has turned “insulating” following the application of a sufficiently large \( B || \) to fully spin-polarize it (e.g., in Fig. 3, the 2DES is fully spin-polarized for \( B_{\text{tot}} > 2.3 \text{T} \)). To check the possible consequences of the \( T \)-dependence of resistance background on the deduced \( m^* \), we analyzed our data in two additional ways. Suppose we assume that \( T_D \) is independent of \( T \) but, to account for the \( T \)-dependence of the background resistance, we define \( R_\text{ave} \) as the average resistance near the oscillation, i.e., \( R_\text{ave} = (R_{\text{max}} + R_{\text{min}})/2 \) (see Fig. 3(a)). Such a procedure leads to \( m^* \) values which are about 6% smaller than those indicated in Fig. 3. In a third scenario, we might assume that \( R_\text{ave} \) and \( T_D \) have the same \( T \)-dependence; this may be a reasonable assumption, since \( R_\text{ave} \) and \( T_D \) are both expected to be proportional to the scattering rate. To implement such an assumption, we use an iterative procedure to find a value for \( m^* \) that leads to a \( T \)-dependence for \( T_D \) such that the ratio \( R_\text{ave}/T_D \) is \( T \)-independent.

![FIG. 4: (color online). Dependence of \( m^* \) on spin-polarization for sample A at a fixed density of \( n = 2.56 \times 10^{11} \text{cm}^{-2} \). (a) \( T \)-dependence of SdH oscillations at \( \theta = 73.2^\circ \). Traces were taken at \( T \approx 0.30, 0.87, 1.23, \) and 1.66 K. Inset: Plots of \( \Delta R/T \) vs. \( T \) and fits to the Dingle expression at \( \nu = 3 \) (fully spin-polarized) and \( \nu = 8 \) (partially spin-polarized); note that the \( \nu = 8 \) data were taken at \( \theta = 27.1^\circ \). In (b), we show the deduced \( m^* \) as a function of \( B_{\text{tot}} \). The closed and open circles correspond to the full and partial spin-polarizations, respectively. The two ladder diagrams depict the positions of the Landau levels and the Fermi energy \( (E_F) \) for the cases of \( \nu = 3 \) and 8 whose data are shown in the inset to (a).](image_url)
We find that $m^\ast$ from this method are about 11% larger than $m^\ast$ indicated in Fig. 3. We emphasize that all the $m^\ast$ values reported in our manuscript come from data sets whose analysis via the three procedures described above yield consistent $m^\ast$ values. In particular, our main conclusion that $m^\ast$ for the fully spin-polarized case is suppressed is borne out in all three procedures.

Interaction-induced re-normalization of $m^\ast$ in dilute interacting 2DESs has been studied widely \cite{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13}. Experimental studies on partially spin-polarized 2DESs confined to Si-MOSFETs \cite{1, 2, 3, 4, 5, 6} or narrow AlAs quantum wells \cite{7} have reported that $m^\ast$ is significantly enhanced with respect to $m_b$ and increases as $n$ is lowered. For GaAs 2DESs, too, an enhancement of $m^\ast$ has been reported at very low densities ($r_s \gtrsim 3$) \cite{8}, but, at high densities $m^\ast$ is slightly suppressed compared to $m_b$ \cite{8, 11}; there is also some qualitative theoretical explanation for this non-monotonic behavior (see Ref. \cite{8}).

It is intuitively clear that, besides $r_s$, the spin-polarization of the 2DES should also affect the $m^\ast$ re-normalization since it modifies the exchange interaction. However, most previous studies have ignored this role. One notable exception is the experimental work by Shashkin et al. \cite{6} where measurements of $m^\ast$ revealed no dependence on spin-polarization for $0 < P \leq 1$. In our study, the "partially-spin-polarized" regime corresponds to $0.14 < P < 0.33$ and indeed, in this range, we find no dependence of $m^\ast$ on $P$. We observe the strong mass suppression only when the 2DES is pushed well beyond the full spin-polarization field limit, i.e. when the applied magnetic field is well above what is needed for complete spin-polarization \cite{11}. For example, for the densities in Figs. 3 and 4, the fields needed for complete spin-polarization of the 2DES are around $2.3 \, \text{T}$ (Fig. 3) and $5.7 \, \text{T}$ (Fig. 4), whereas our first suppressed mass is reported at around $4.9 \, \text{T}$ and $7.3 \, \text{T}$, respectively.

Motivated by the results of Ref. \cite{6}, recent theoretical work \cite{11, 12} has addressed the role of spin-polarization on $m^\ast$ re-normalization. In Ref. \cite{11} it is concluded that, although $m^\ast$ should in principle depend on spin-polarization, in a valley-degenerate system such as the one studied in Ref. \cite{6}, the dependence may be too weak to be experimentally measurable \cite{20}. In Ref. \cite{12}, which deals with a single-valley 2DES, however, a rather strong dependence of $m^\ast$ on spin-polarization is reported. There is even a hint that for a fully spin-polarized 2DES $m^\ast$ may fall below $m_b$. But there are some qualitative discrepancies between the predictions of Ref. \cite{12} and our experimental results. For example, $m^\ast$ for a fully spin-polarized 2DES is predicted to decrease with increasing $n$ and become smaller than $m_b$ only at large values of $n$ ($r_s \lesssim 2$) \cite{12}. In contrast, our data suggest that the $m^\ast$ suppression is observed in a large density range and may even be more pronounced at smaller densities. An understanding of the $m^\ast$ suppression we observe in a dilute, single-valley, fully spin-polarized 2DES therefore awaits future theoretical and experimental developments.

We thank the NSF for financial support. Part of this work was done at the NHMFL, Tallahassee, which is also supported by the NSF. We thank E. Palm, T. Murphy, J. Jaroszynski, S. Hannahs and G. Jones for assistance. We also express gratitude to A.H. MacDonald, E. Tutuc, and R. Winkler for illuminating discussions.

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[20] In agreement with this conclusion, in an AlAs sample where two conduction band valleys are occupied, we do not observe a suppression of $m^\ast$ upon full spin-polarization [T. Gokmen et al., unpublished].