Vector Manifestation in Hot Matter

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Based on the hidden local symmetry (HLS) Lagrangian as an effective field theory of QCD, we find that the chiral symmetry restoration for hot QCD can be realized through the Vector Manifestation where the $\rho$ meson becomes massless degenerate with $\pi$ as the chiral partner. This is done by including, in addition to the hadronic thermal effects due to the $\pi$- and $\rho$-loops, the intrinsic temperature dependences of the parameters of the HLS Lagrangian through the matching of the HLS with the underlying QCD.

1. Introduction

Vector meson mass in hot and/or dense matter is one of the most interesting physical quantities in studying the hot and/or dense QCD where the chiral symmetry is expected to be restored (for reviews, see, e.g., Refs. \cite{1-4}). The BNL Relativistic Heavy Ion Collider (RHIC) has started to measure several effects in hot and/or dense matter. Especially, the light vector meson mass is important for analysing the dilepton spectra in RHIC. In Refs. \cite{5,2} it was proposed that the $\rho$-meson mass scales like the pion decay constant and vanishes at the chiral phase transition point in hot and/or dense matter.

To study the $\rho$ mass in hot matter it is useful to use models including the $\rho$ meson. Among several such models we use the model based on the hidden local symmetry (HLS) \cite{6} which successfully reproduces the phenomena of $\rho$-$\pi$ system at zero temperature. The HLS model is a natural extension of the nonlinear sigma model, and reduces to it in the low-energy region. It was shown \cite{6} that the HLS model is equivalent to other models for vector mesons at tree level. We should stress here that, as first pointed by Georgi \cite{8} and developed further in Refs. \cite{9-12}, \textit{thanks to the gauge symmetry in the HLS model, we can perform a systematic loop expansion including the vector mesons in addition to the pseudoscalar mesons.}

Several groups \cite{13,14,4} studied the $\rho$ mass in hot matter using the HLS model. Most of them included only the thermal effect of $\pi$ and dropped that of $\rho$ itself. In Ref. \cite{14}, the first application of the systematic chiral perturbation with HLS \cite{6,13} in hot matter was made. There hadronic thermal effects of $\rho$ and $\pi$ were included at one loop and the $\rho$ mass was shown to increase with temperature $T$ at low temperature.

In the analysis done in Ref. \cite{14} the parameters of the Lagrangian at $T = 0$ were used by assuming no temperature dependences of them. When we naively extrapolate the results in Ref. \cite{14} to the critical temperature, the resultant axialvector and vector current correlators do not agree with each other. Disagreement between these correlators is obviously inconsistent with the chiral restoration in QCD. However, the parameters of the HLS Lagrangian should be determined by the underlying QCD. As was shown in Ref. \cite{11}, \textit{the bare parameters of the (bare) HLS Lagrangian defined at the matching scale $\Lambda$ for $N_f = 3$ at $T = 0$ are determined by matching the HLS with the underlying QCD at $\Lambda$ through the Wilsonian matching conditions: This was done by matching the current correlators in the HLS with those derived by the operator product expansion (OPE) in QCD. Since the current correlators by the OPE at non-zero temperature depend on the temperature (see, e.g., Refs. \cite{15,16}), the application of the Wilsonian matching to the hot matter calculation implies that the bare parameters of the HLS do depend on the temperature, which we call the \textit{intrinsic temperature dependences} in contrast to the hadronic thermal effects. We stress here that the above disagreement between the current correlators is cured by including the intrinsic temperature dependences.}

In Ref. \cite{17}, on the other hand, the \textit{vector manifestation (VM)} is proposed as a new pattern of the Wigner realization of chiral symmetry, in which the chiral symmetry is restored at the critical point by the massless degenerate pion (and its flavor partners) and the $\rho$ meson (and its flavor partners) as the chiral partner, in sharp contrast to the traditional manifestation à la linear sigma model where the symmetry is restored by the degenerate pion and the scalar...
meson. It was shown that VM actually takes place in the large $N_f$ QCD through the Wilsonian matching. Since the VM is a general property in the chiral restoration when the HLS can be matched with the underlying QCD at the critical point, it was then suggested [7] that the VM may be applied to the chiral restoration in hot and/or dense matter.

In this paper, we demonstrate that the VM can in fact occur in the chiral symmetry restoration in hot matter, using the HLS as an effective field theory of QCD. Here we determine the intrinsic temperature dependences of the bare parameters of the HLS through the Wilsonian matching in hot matter, and convert them to the intrinsic temperature dependences of the on-shell parameters by including the quantum effects through the Wilsonian RGE’s for the HLS parameters [13,14]. Then, we separately include the hadronic thermal effects to obtain physical quantities by explicitly calculating the $\pi$- and $\rho$- thermal loops.

2. Hidden Local Symmetry

Let us first describe the HLS model based on the $G_{\text{global}} \times H_{\text{local}}$ symmetry, where $G = \text{SU}(N_f)_{L} \times \text{SU}(N_f)_{R}$ is the global chiral symmetry and $H = \text{SU}(N_f)_{\chi}$ is the HLS. The basic quantities are the gauge boson $\rho_\mu$ and two variables

$$\xi_{L,R} = \exp[\imath \sigma / F_\pi] \exp[\imath \pi / F_\pi]$$

where $\pi$ denotes the pseudoscalar Nambu-Goldstone (NG) boson and $\sigma$ [7] the NG boson absorbed into $\rho_\mu$ (longitudinal $\rho$). $F_\pi$ and $F_\sigma$ are relevant decay constants, and the parameter $a$ is defined as $a \equiv F_\sigma^2 / F_\pi^2$. The transformation property of $\xi_{L,R}$ is given by

$$\xi_{L,R}(x) \to \xi'_{L,R}(x) = h(x) \xi_{L,R}(x) g^\dagger_{L,R}$$

where $h(x) \in H_{\text{local}}$ and $g_{L,R} \in G_{\text{global}}$. The covariant derivatives of $\xi_{L,R}$ are defined by

$$D_\mu \xi_L = \partial_\mu \xi_L - ig \rho_\mu \xi_L + i \xi_L \mathcal{L}_\mu$$
$$D_\mu \xi_R = \partial_\mu \xi_R - ig \rho_\mu \xi_R + i \xi_R \mathcal{R}_\mu$$

where $g$ is the HLS gauge coupling, and $\mathcal{L}_\mu$ and $\mathcal{R}_\mu$ denote the external gauge fields gauging the $G_{\text{global}}$ symmetry.

The HLS Lagrangian is given by [3]

$$\mathcal{L} = F^2_\pi \text{tr} [\tilde{\alpha}_{\perp\mu} \tilde{\alpha}^\mu_{\perp}] + F^2_\sigma \text{tr} [\tilde{\alpha}_{\parallel\mu} \tilde{\alpha}^\mu_{\parallel}] + \mathcal{L}_{\text{kin}}(\rho_\mu)$$

where $\mathcal{L}_{\text{kin}}(\rho_\mu)$ denotes the kinetic term of $\rho_\mu$

$$\tilde{\alpha}_{\perp\mu} = (D_\mu \xi_R \cdot \xi^R_L + D_\mu \xi_L \cdot \xi^L_R)/(2i)$$

When the kinetic term $\mathcal{L}_{\text{kin}}(\rho_\mu)$ is ignored in the low-energy region, the second term of Eq. (4) vanishes by integrating out $\rho_\mu$ and only the first term remains. Then, the HLS model is reduced to the nonlinear sigma model based on $G/H$.

At zero temperature $T = 0$, it was shown [8,10] that, thanks to the gauge symmetry in the HLS, we can perform the systematic loop expansion including the vector meson. Here the expansion parameter is a ratio of the $\rho$ meson mass to the chiral symmetry breaking scale $\Lambda_{\chi}$ [8] in addition to the ratio of the momentum $p$ to $\Lambda_{\chi}$ as used in the ordinary chiral perturbation theory. By assigning $O(p)$ to the HLS gauge coupling $g$ [11], the Lagrangian in Eq. (4) is counted as $O(p^2)$, and one-loop quantum corrections obtained from the Lagrangian are counted as $O(p^4)$.

Due to quantum corrections, three parameters $F_\pi$, $F_\sigma$ and $g$ are renormalized at one-loop level, and depend on the renormalization scale $\mu$ [14,15,11]. Furthermore, at non-zero temperature $T > 0$, these parameters have the intrinsic temperature dependences. We write both dependences explicitly as $F_\pi(\mu; T)$, $a(\mu; T)$ and $g(\mu; T)$ [16].

To avoid confusion, we use $f_\pi$ for the physical decay constant of $\pi$, and $F_\pi$ for the parameter of the Lagrangian. Similarly, $M_\rho$ denotes the parameter of the Lagrangian and $m_\rho$ the $\rho$ pole mass. For calculating the hadronic thermal corrections it is convenient to adopt the on-shell renormalization scheme at $T = 0$ as in Ref. [2]. Below, we use the following abbreviated notations:

$$F_\pi = f_\pi(\mu = 0; T)$$
$$g = g(\mu = M_\rho(T); T)$$
$$a = a(\mu = M_\rho(T); T)$$

where $M_\rho$ is determined from the on-shell condition:

$$M_\rho^2 = M_\rho^2(T) = a(\mu = M_\rho(T); T) \times g^2(\mu = M_\rho(T); T) F_\pi^2(\mu = M_\rho(T); T)$$

Then, the parameter $M_\rho$ in this paper is renormalized in such a way that it becomes the pole mass at $T = 0$.

3. Hadronic Thermal Corrections

Here we summarize the hadronic thermal effects to the decay constant of $\pi$ and the $\rho$ mass shown in [17,18,3].

#2 Note that this $\sigma$ is different with the scalar meson in the linear sigma model.

#3 The renormalization scale $\mu$ and the temperature $T$ are independent of each other in the present approach.
where the temperature $T$ is assigned to be of $O(p)$ following Ref. [21].

The decay constant of $\pi$ is defined through the longitudinal component of the axialvector current correlator at the low energy limit [21]. The hadronic thermal corrections from $\pi$ and $\rho$ are summarized as [14]

$$f_\pi^2(T) = F_\pi^2 - \frac{N_f}{2\pi^2} \left[ I_2 - a J_4^2 + \frac{a}{3M_\rho^2} (I_4 - J_4^2) \right],$$

(8)

where $I_n$ and $J_n^m$ ($n, m$: integer) are defined as

$$I_n \equiv \int_0^\infty dk \frac{k^{n-1}}{e^{\omega/T} - 1},$$

$$J_m^n \equiv \int_0^\infty dk \frac{1}{e^{\omega/T} - 1} \omega^m,$$

(9)

with $\omega \equiv \sqrt{k^2 + M_\rho^2}$. When we consider the low temperature region $T \ll M_\rho$ in Eq. (8), only the $I_2$ term remains:

$$f_\pi^2(T) \approx F_\pi^2 - \left( \frac{N_f}{2\pi^2} \right) I_2 = F_\pi^2 - \frac{N_f}{2} T^2/12,$$

(10)

which is consistent with the result in Ref. [20].

We estimate the critical temperature by naively extrapolating the above result to the higher temperature without including the intrinsic temperature dependences. The critical temperature for $N_f = 3$ is approximated as

$$T_c^{(had)}(T = 0) = 2 f_\pi(0) \approx 180 \text{ MeV}.$$

(11)

In Ref. [14] $m_\rho$ is defined by the pole of the longitudinal $\rho$ propagator at rest frame:

$$m_\rho^2(T) = M_\rho^2 - \text{Re} \Pi_{\rho}^{(had)}(p_0 = M_\rho, \vec{p} = 0; T),$$

(12)

where $\text{Re} \Pi_{\rho}^{(had)}$ denotes the real part of the longitudinal component of the $\rho$ two-point function at one-loop level. Inside the one-loop correction $\text{Re} \Pi_{\rho}^{(had)}$ we replaced $m_\rho$ by $M_\rho$, since the difference is of higher order. The resultant thermal corrections are summarized as [14]

$$m_\rho^2(T) = M_\rho^2 - \frac{N_f g_s^2}{2\pi^2} \left[ \frac{a^2}{12} \tilde{G}_2 - \frac{5}{4} J_4^2 - \frac{33}{16} M_\rho^2 F_\pi^2 \right],$$

(13)

where $J_4^2$ is defined in Eq. (4) and $F_\pi^2$ and $\tilde{G}_2$ are defined as

$$F_\rho^2 \equiv \int_0^\infty d\omega \mathcal{P} \frac{1}{e^{\omega/T} - 1} \frac{4k^n}{\omega(4\omega^2 - M_\rho^2)}.$$

$$\tilde{G}_2 =$$

$$\int_0^\infty d\omega \mathcal{P} \frac{1}{e^{\omega/T} - 1} \frac{4k^n}{\omega(4\omega^2 - M_\rho^2)}.$$

(14)

with $\mathcal{P}$ denoting the principal part. From this expression it was shown [14] that there is no $T^2$ term in the low temperature region consistently with the result in Ref. [22].

4. Intrinsic Temperature Dependences

Let us now include the intrinsic temperature dependences of $F_\pi$, $a$ and $g$ (and $M_\rho^2 = ag^2 F_\pi^2$) appearing in Eqs. (8) and (13). To do that, we first determine the bare parameters defined at the matching scale $\Lambda$ by extending the Wilsonian matching [11], which was originally proposed for $T = 0$, to non-zero temperature. We should note that, for the validity of the expansion in the HLS, the matching scale $\Lambda$ must be smaller than the chiral symmetry breaking scale $\Lambda_\chi$. We match the axialvector and vector current correlators in the HLS with those derived in the OPE for QCD at non-zero temperature (see, e.g., Refs. [13, 14]). The correlators in the HLS around the matching scale $\Lambda$ are well described by the same forms as those at $T = 0$ [11] with the bare parameters having the intrinsic temperature dependences:

$$\Pi_A^{(HLS)}(Q^2) = \frac{F_\pi^2(\Lambda; T)}{Q^2} - 2z_2(\Lambda; T),$$

$$\Pi_V^{(HLS)}(Q^2) = \frac{F_\pi^2(\Lambda; T) \left[ 1 - 2g^2(\Lambda; T) z_3(\Lambda; T) \right]}{M_\rho^2(\Lambda; T) + Q^2} - 2z_1(\Lambda; T).$$

(15)

where $M_\rho^2(\Lambda; T) \equiv g^2(\Lambda; T) F_\pi^2(\Lambda; T)$ is the bare $\rho$ mass, and $z_{1,2,3}(\Lambda; T)$ are the bare coefficient parameters of the relevant $O(p^4)$ terms [10, 21]. Matching the above correlators with those by the OPE in the same way as done for $T = 0$ [11], we determine the bare parameters including the intrinsic temperature dependences, which are then converted into those of the on-shell parameters through the Wilsonian RGE’s [13, 14]. As a result, the parameters appearing in the hadronic thermal corrections have the intrinsic temperature dependences. In this way we include both the intrinsic and hadronic thermal effects together into the physical quantities.

5. Vector Manifestation

Now, we study the chiral restoration in hot matter. Here we assume that the chiral broken phase is in the confining phase, i.e., the critical temperature $T_c$ for chiral phase transition is not larger than the critical temperature for confinement-deconfinement phase
transition, and the hadronic picture is valid. When the symmetry is completely restored, the HLS is not applicable. We approach to the critical temperature from the broken phase where the HLS is applicable. At the moment we assume that the expansion parameter \( M_\rho/L_\chi \) is small near \( T_c \). It turns out that it is actually small since \( M_\rho \to 0 \) when \( T \to T_c \), as we will show below. We first consider the Wilsonian matching at the critical temperature \( T_c \) for \( N_f = 3 \) with assuming that \( \langle \bar{q}q \rangle \) approaches to 0 continuously for \( T \to T_c \). In such a case, the axialvector and vector current correlators by the OPE approach to each other, and agree at \( T_c \). Then through the Wilsonian matching we require that the correlators in Eq. (15) agree with each other. As was shown in Ref. [17] for large \( N_f \) chiral restoration, this agreement is satisfied if the following conditions are met:

\[
\begin{align*}
g(\Lambda; T) & \to 0 \quad \text{as} \quad T \to T_c, \\
a(\Lambda; T) & \to 1 \quad \text{as} \quad T \to T_c, \\
z_1(\Lambda; T) & \to z_2(\Lambda; T) \quad \text{as} \quad T \to T_c.
\end{align*}
\]

(16)

As we explained above, the conditions for the bare parameters \( g(\Lambda; T_c) = 0 \) and \( a(\Lambda; T_c) = 1 \) are converted into the conditions for the on-shell parameters through the Wilsonian RGE’s. Since \( g = 0 \) and \( a = 1 \) are separately the fixed points of the RGE’s for \( g \) and \( a \) [3], the on-shell parameters also satisfy \((g, a) = (0, 1)\), and thus \( M_\rho = 0 \).

Let us include the hadronic thermal effects to obtain the \( \rho \) pole mass. Here we extrapolate the result in Eq. (13) to the higher temperature with including the intrinsic temperature dependences of the parameters. Noting that \( \bar{G}_2 \to \pi^2 T^2/6 \), \( J_\mu^2 \to \pi^2 T^2/6 \) and \( M_\rho^2 \bar{F}_3^2 \to 0 \) for \( M_\rho \to 0 \), Eq. (13) for \( M_\rho \ll T \) reduces to

\[
m_\rho^2(T) = M_\rho^2 + g^2 N_f \frac{15 - a^2 \pi^2}{2\pi^2} - \frac{2}{6} T^2.
\]

(17)

Since \( a \simeq 1 \) near the restoration point, the second term is positive. Then the \( \rho \) pole mass \( m_\rho \) is bigger than the parameter \( M_\rho \) due to the hadronic thermal corrections. Nevertheless, the intrinsic temperature dependence determined by the Wilsonian matching requires that the \( \rho \) becomes massless at the critical temperature:

\[
m_\rho^2(T) \to 0, \quad \text{as} \quad T \to T_c.
\]

(18)

since the first term in Eq. (17) vanishes as \( M_\rho \to 0 \), and the second term also vanishes since \( g \to 0 \) for \( T \to T_c \). This implies that, as was suggested in Refs. [1,8], the vector manifestation (VM) actually occurs at the critical temperature. This is the main result of this paper, which is consistent with the picture shown in Refs. [3,8].

We should stress here that the above \( m_\rho(T) \) is the \( \rho \) pole mass, which is important for analyzing the dilepton spectra in RHIC experiment. It is noted [17] that although conditions for \( g(\Lambda; T) \) and \( a(\Lambda; T) \) in Eq. (16) coincide with the Georgi’s vector limit [3], the VM \( (f_\pi \to 0) \) should be distinguished from Georgi’s vector realization [3].

6. Critical Temperature

Let us determine the critical temperature. For \( T > 0 \) the thermal averages of the Lorentz non-scalar operators such as \( \bar{q}_\mu D_{\mu} q \) exist in the OPE [4]. Since these are smaller than the main term \( 1 + \alpha_s/\pi \), we expect that they give only small corrections to the value of \( T_c \), and neglect them here. Then, the Wilsonian matching condition to determine the bare parameter \( F_\pi(\Lambda; T_c) \) is obtained from that in Eq. (4.5) of Ref. [11] by taking \( \langle \bar{q}q \rangle = 0 \) and including a possible temperature dependence of the gluonic condensate:

\[
F_\pi^2(\Lambda; T_c) = \frac{1}{8\pi^2} \left[ 1 + \frac{\alpha_s}{\pi} + \frac{2\pi^2}{3} \langle \bar{q}q \rangle G_{\mu\nu} G^{\mu\nu} \right]_{T_c}.
\]

(19)

The on-shell parameter \( F_\pi(0; T_c) \) is determined through the Wilsonian RGE [19,11] for \( F_\pi \) with taking \((g, a) = (0, 1)\). As for large \( N_f \) [3,8], the result is given by

\[
F_\pi^2(0; T_c) = \frac{F_\pi^2(\Lambda; T_c)}{\Lambda^2} - \frac{N_f}{2(4\pi)^2}.
\]

(20)

We need to include the hadronic thermal effects to obtain the relation between the parameter \( F_\pi(0; T_c) \) and the order parameter \( f_\pi(T_c) \). Here we extrapolate the hadronic thermal effect shown in Eq. (6) to higher temperature with including the intrinsic thermal effect. Then, taking \( M_\rho \to 0 \) and \( a \to 1 \) in Eq. (8), we obtain

\[
0 = f_\pi^2(T_c) = \frac{F_\pi^2(0; T_c)}{N_f T_c^2} - \frac{N_f T_c^2}{24}.
\]

(21)

Here we should note that the coefficient of \( T^2 \) in the second term is a half of that in Eq. (10) which is an

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#4 It is known that there is no Ginzburg-Landau type phase transition for \( N_f = 3 \) (see, e.g., Refs. [11]). There may still be a possibility of non-Ginzburg-Landau type continuous phase transition such as the conformal phase transition [12]. When the Wilsonian matching can be applicable for \( N_f = 2 \), the VM should occur.

#5 We should note that we can take \( T \to T_c \) limit with \( \Lambda \) fixed in Eq. (14) since \( \Lambda \) and \( T \) in the bare parameters of the HLS are independent of each other.
approximate form for $T \ll M_\rho$ taken with assuming that the $\rho$ does not become light. On the other hand, here the factor 1/2 appears from the contribution of $\sigma$ (longitudinal $\rho$) which becomes the real NG boson at $T = T_c$ due to the VM where the chiral restoration in QCD predicts $a \to 1$ and $g \to 0$ for $T \to T_c$. From Eq. (21) together with Eqs. (19) and (20), $T_c$ is expressed as

$$T_c = \sqrt{\frac{24}{N_f} F_\pi(0; T_c)} = \sqrt{\frac{3\Lambda^2}{N_f \pi^2}} \times \left[ 1 + \frac{\alpha_s}{\pi} + \frac{2\pi^2}{3} \frac{\left\langle \frac{2}{\Lambda^4} G_{\mu\nu} G^\mu\nu \right\rangle}{N_f} - \frac{N_f}{4} \right]^{1/2}.$$  

(22)

We estimate the value of $T_c$ for $N_f = 3$. The value of the gluonic condensate near phase transition point becomes about half of that at $T = 0$ [24], so we use $\left\langle \frac{2}{\Lambda^4} G_{\mu\nu} G^\mu\nu \right\rangle_{T_c} = 0.006$ GeV$^4$ obtained by multiplying the value at $T = 0$ shown in Ref. [21] by 1/2. For the value of the QCD scale $\Lambda_{\text{QCD}}$, we use $\Lambda_{\text{QCD}} = 400$ MeV [9] as a typical example. For this value of $\Lambda_{\text{QCD}}$, it was shown [1] that the choice of $\Lambda = 1.1$ GeV provides the predictions in good agreement with experiment at $T = 0$. However, the matching scale may have the temperature dependence. In the present analysis we use $\Lambda = 0.8, 0.9, 1.0$ and 1.1 GeV, and determine $T_c$ from Eq. (22). We show the resultant values in Table 1.

| $\Lambda$ | 0.8 | 0.9 | 1.0 | 1.1 |
|----------|-----|-----|-----|-----|
| $T_c$    | 0.21 | 0.22 | 0.23 | 0.25 |

Table 1 Estimated values of the critical temperature $T_c$ for several choices of the value of the matching scale $\Lambda$. Units of $\Lambda$ and $T_c$ are GeV.

We note that the estimated values of $T_c$ in Table 1 are larger than that in Eq. (11) which is obtained by naively extrapolating the temperature dependence from the hadronic thermal effects without including the intrinsic temperature dependences. This is because the extra factor 1/2 appears in the second term in Eq. (21) compared with that in Eq. (11). As we stressed below Eq. (21), the factor 1/2 comes from the contribution of $\sigma$ (longitudinal $\rho$) which becomes massless at the chiral restoration point.

7. Summary and Discussions

To conclude, by imposing the Wilsonian matching of the HLS with the underlying QCD at the critical temperature, where the chiral symmetry restoration takes place, the vector manifestation (VM) necessarily occurs: The vector meson mass becomes zero. Accordingly, the light vector meson gives a large thermal correction to the pion decay constant, and the value of the critical temperature becomes larger than the value estimated by including only the $\pi$ thermal effect. The result that the vector meson becomes light near the critical temperature is consistent with the picture shown in Refs. [22, 3, 8].

Several comments are in order:

As shown in Ref. [1], in the VM only the longitudinal $\rho$ couples to the vector current near the critical point, and the transverse $\rho$ is decoupled from it. The $A_1$ in the VM is resolved and/or decoupled from the axialvector current near $T_c$, since there is no contribution in the VM of the pion current correlator. We expect that the scalar meson is also resolved and/or decoupled near $T_c$ since it in the VM is in the same representation as the $A_1$ is in. We also expect that excited mesons are also resolved and/or decoupled. The estimated values of $T_c$ shown in Table 1 as well as $T_c^{\text{had}}$ in Eq. (11) may be changed by higher order hadronic thermal effects, as in the chiral perturbation analysis [27]. On the other hand, the VM at $T_c$ is governed by the fixed point and not changed by higher order effects.

The parameter $M_\rho^2$ in Eq. (13) presumably has an intrinsic temperature dependence proportional to $T^2$ through the Wilsonian matching. Since we studied the intrinsic dependences only at $T_c$, we cannot definitely argue how $m_\rho(T)$ falls in $T$. However, we think that $g^2(\Lambda; T)$ vanishes as $(\bar{q}q)^2_T$ near $T_c$ in the VM. If $(\bar{q}q)^2_T$ falls as $(1 - T^2/T_c^2)$ near $T_c$, then the $\rho$ pole mass $m_\rho(T)$ as well as the parameter $M_\rho^2(T)$ vanishes as $(1 - T^2/T_c^2)$ which seems to agree with the behavior of $f_\pi^2(T)$. In such a case the scaling property in the VM may be consistent with the Brown-Rho scaling $m_\rho(T)/m_\rho(0) \sim f_\pi(T)/f_\pi(0)$ [3].

Although we concentrated on the hot matter calculation in this paper, the present approach can be applied to the general hot and/or dense matter calculation.

At present, there are no clear lattice data for the
pole mass in hot matter. Our result here will be checked by lattice analyses in future. [28]

In this paper we performed our analysis at the chiral limit. We need to include the explicit chiral symmetry breaking effect from the current quark masses when we apply the present analysis to the real QCD. In such a case, we need the Wilsonian matching conditions with including non-zero quark mass which have not yet been established. Here we expect that the qualitative structure obtained in the present analysis will not be changed by the inclusion of the current quark masses.

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