Dynamics of Metastable Vacua in the Early Universe

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Abstract

We study the question whether a possible metastable vacuum state is actually populated in a phase transition in the early universe, as is usually assumed in the discussion of vacuum stability bounds e.g. for Standard Model parameters. A phenomenological (3+1)-dimensional Langevin equation is solved numerically for a toy model with a potential motivated by the finite temperature 1-loop effective potential of the Standard Model including additional non-renormalizable operators from an effective theory for physics beyond the Standard Model and a time dependent temperature. It turns out that whether the metastable vacuum is populated depends critically on the value of the phenomenological parameter $\eta$ for small scalar couplings. For large enough scalar couplings and with our specific form of the non-renormalizable operators the system (governed by the Langevin equation) always ends up in the metastable minimum.

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1. The symmetry breaking potential of a field theory at vanishing temperature $T$ may have local (energetically disfavoured) minima in addition to the global minimum which defines the true vacuum. This leads to metastability if the system sits at $T = 0$ in a false vacuum and the decay of such a false vacuum may occur in two ways. First it is possible to cross the classical energy barrier which results from the balance between the released volume energy of a single “bubble of the correct vacuum” with the amount of energy stored in the surface of that bubble. Vacuum decay sets in after formation of the first bubble of critical size which requires however a big enough thermal fluctuation or some source of local energy deposition. Alternatively the false vacuum may decay via quantum tunneling and the parameters of the theory determine in general which decay mechanism dominates.

The study of questions concerning the decay of metastable vacua under the influence of fluctuations induced by quantum effects goes back to the work of Krive and Linde [1] who studied vacuum stability under quantum corrections in the $\sigma$-model. The issue of vacuum stability may also be relevant for the Standard Model (see e.g. the review of Sher [2] and references therein) and for model extensions. Requiring that our electro-weak vacuum is the true vacuum leads for a large top mass $m_t \gtrsim 80$ GeV to lower bounds on the Higgs mass [3]. These bounds can be somewhat relaxed by allowing the electro-weak vacuum to be metastable with a lifetime longer than the age of the universe [4], where the dominating decay mode is the classical transition via the formation of a critical bubble triggered by random collisions of ultra high energy cosmic rays. Due to the experimental lower bounds on the Higgs mass metastability is in the Standard Model almost ruled out. It may however still be an interesting issue in different model extensions and is certainly also in general an interesting question in field theory.

Thermal effects were included only in a few cases in studies of metastability (see e.g. [3]-[7]). Most authors concentrated on transitions at vanishing temperature assuming that the universe is already in the false vacuum. This ignores however that the symmetry was restored at high temperature in the early universe and we will discuss in this paper the question how likely or unlikely it is that the system falls into a metastable vacuum with cosmological boundary conditions. Given the physical situation realized in the early universe (or more generally in a system that is cooled through a phase transition), we calculate thus for a theory exhibiting symmetry breaking with metastable vacua at $T = 0$ the probability to end up in the false vacuum in the first place.

To address this topic one needs to study the response of a given field theory to a changing temperature which is obviously a highly non-trivial task[1]. A calculation from first principles would require to self-consistently identify a heat bath, to calculate the coupling between the

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1Since the temperature in the situation considered here explicitly depends on time, one can not take recourse to well established approaches like Langers theory of bubble nucleation. Langers description may however be used in the late stages of the evolution when the time-dependence of the temperature is dynamically no longer important and if the potential barrier separating the vacua is sufficiently high such that a saddle point expansion about the top of the barrier is a good approximation [3].
bath and the system and solve the equations of motion for the macroscopic expectation values that describe the system as the temperature of the heat bath is changed. All this would have to be done in the framework of non-equilibrium field theory. The conceptual foundations for such a study are in principle given (at least for a scalar theory) by Morikawa [9] and Gleiser and Ramos [10] (c.f. also Boyanovsky et. al. [11]). Using the CTP-formulation of field theory by Schwinger and Keldysh [12] they discuss the (perturbative) formulation of an effective equation of motion in a real $\lambda \phi^4$-theory which turns out to resemble the well known Langevin equation in the form
\begin{equation}
\Box \varphi + V'(\varphi) + \eta(\varphi) \dot{\varphi} = \xi(\varphi),
\end{equation}
albeit with rather complicated dissipative and noise contributions.

This makes contact with a number of numerical studies that use equations of the type given in eq. (1) to analyze the decay of false vacua at finite temperature [14]-[21]. In [20] and [21], for example, (3+1)-dimensional theories are studied using an equation of the form
\begin{equation}
\Box \varphi + U'(\varphi, T) + \eta \dot{\varphi} = \xi,
\end{equation}
where the used effective potential $U(\varphi, T)$ is motivated by the high temperature result found in the Standard Model written as
\begin{equation}
U(\varphi, T) = \frac{a}{2} (T^2 - T^2_c) \varphi^2 - \frac{\alpha}{3} T \varphi^3 + \frac{\lambda}{4} \varphi^4,
\end{equation}
where $\eta$ and $\xi$ are related by the fluctuation – dissipation relation
\begin{equation}
\langle \xi(\vec{x}, t) \xi(\vec{x}', t') \rangle = 2\eta T \delta^{(4)}(x - x'),
\end{equation}
The authors studied the system at a fixed (namely the critical) temperature and choose $\eta$ to be 1, arguing that the value of $\eta$ would not affect the final (thermodynamical) state of the system, but only the approach to equilibrium. Here we will use a Langevin equation of the form (2) to model the dynamics of a scalar theory with a symmetry breaking phase transition (at some critical temperature $T_c$) and with metastable vacua for $0 \leq T (\gtrsim T_c)$ in order to calculate the probability of metastable states. As mentioned above the question is whether the metastable states decay during cooling by thermal activation long before the temperature approaches zero or if the system populates the metastable states with reasonable probability which would be the starting of the usual vacuum stability discussion in the Standard Model.

2. We first discuss the choice for the effective potential $U(\varphi, T)$ appearing in equation (3) and for the time dependence of the temperature. Even though we will not aim at a discussion of the full Standard Model, embedded in some specific extension at a given scale $\Lambda$ which renders the physical vacuum metastable, it makes sense to stick as close as possible to the Standard Model potential. For reasons to be discussed below, we will only be interested in theories with a relatively small scalar self-coupling. We chose therefore as
a first contribution to $U(\varphi, T)$ the 1-loop effective potential of the Standard Model in the form (c.f. [13])

$$U_{(1)}(\varphi, T) = -DT_2^2 \varphi^2 + \frac{\lambda}{4} \varphi^4 + B \varphi^4 \ln \frac{\varphi^2}{\sigma^2} +$$

$$+ \sum_{\text{Bosons}} \frac{g_B T^4}{2\pi^2} \int_0^\infty dx x^2 \ln \left(1 - e^{-x^2 + (m_B/T)^2}\right) -$$

$$- \sum_{\text{Fermions}} \frac{g_F T^4}{2\pi^2} \int_0^\infty dx x^2 \ln \left(1 + e^{-x^2 + (m_F/T)^2}\right) \tag{5}$$

where $g_{B,F}$ count the number of degrees of freedom of the corresponding fields and, taking into account only the $SU(2)$ gauge bosons and the top quark,

$$D = \frac{1}{24} \left[6 \left(\frac{m_W}{\sigma}\right)^2 + 3 \left(\frac{m_Z}{\sigma}\right)^2 + 6 \left(\frac{m_t}{\sigma}\right)^2\right]$$

$$B = \frac{1}{64\pi^2\sigma^4} \left(6m_W^4 + 3m_Z^4 - 12m_t^4\right)$$

$$T_2^2 = \frac{m_H^2 - 8B\sigma^2}{4D} \tag{6}$$

with the vacuum expectation value $\sigma = 246$ GeV.

The influence of new physics from an extended model at some scale $\Lambda$ can be parametrized by suitable higher dimensional non-renormalizable operators. For a specific extension these operators can be obtained as usual by connecting tree graphs which contain light scalars by propagators of heavy degrees of freedom. For illustration one can assume heavy scalars with masses $M^2 \sim \Lambda^2 + T^2$ which interact with the light scalars. To obtain the leading contribution to the six point function one connects two quartic vertices which contain one heavy scalar. For small external momenta this results at $T = 0$ in an effective six point function for the light scalar of the form $\varphi^6/\Lambda^2$. Including temperature effects results approximately in $\varphi^6/(\Lambda^2 + T^2)$ and for small temperatures we can finally expand in $T/\Lambda$.

For $\varphi \ll \Lambda$ the discussion can be restricted to the leading higher dimensional operators. While contributions to the two and four point functions can be absorbed into the definition of the usual scalar parameters the leading new terms occur in the 6- and 8-point functions. This procedure yields thus an additional contribution to the potential of the form

$$U_{(2)}(\varphi, T) = \left(1 - \frac{T^2}{\Lambda^2}\right) \left[-\frac{g_6 \varphi^6}{6 \Lambda^2} + \frac{g_8 \varphi^8}{8 \Lambda^4}\right] \tag{7}$$

which is also heuristically suggestive. For the present study $g_6$ and $g_8$ should be viewed as couplings which parametrize new physics. The common $T$-dependent factor has the following properties: it vanishes for $T \to \Lambda$ where the theory is not reliable any more and it becomes unity for $T \to 0$ as in the usual zero temperature field theory.
Finally, the interesting temperatures are of the order of the weak scale, $T \sim \mathcal{O}(100)$ GeV. Taking the time dependence of the temperature to be governed by the (adiabatic) expansion of the universe according to the Friedman equations (curvature effects can be neglected at these temperatures to a very good approximation) we have for a radiation dominated universe with thermodynamical equation of state $\rho = 3p$ that

$$T \propto t^{-1/2}.$$ 

(8)

Using Gaussian white noise obeying the fluctuation – dissipation relation (4) the only free parameters are now the scalar self-coupling $\lambda$ (resp. the Higgs mass $m_H$), the couplings $g_6$, $g_8$ and the scale $\Lambda$, as well as the viscosity coefficient $\eta$. We consider different values of $\lambda$, taking $\lambda$ between $\lambda = 0.03$ (since for the physical Higgs mass one has $m_H^2 = (2\lambda + 12B)\sigma^2$ with $B$ as in eq. (8) which is $B \simeq -0.0045$ if the measured masses are used, $\lambda$ has to be larger than 0.028 for $m_H$ to be real) and – for reasons to be discussed below – $\lambda = 0.044$, corresponding to $m_H$ between 18.4 and 45.1 GeV. For every value of $\lambda$, we choose values of $g_6$ and $g_8$ such that the full effective potential $U(\varphi, T) = U(1)(\varphi, T) + U(2)(\varphi, T)$ for $T = 0$ has in addition to the physical minimum at $\sigma$ a second, deeper one at $\sigma'$ close to the scale of new physics $\Lambda$, which we choose to be at $\Lambda = 4\sigma$. This situation is reminiscent of the scenarios considered in connection with the stability bounds as discussed above. Finally $\eta$ is varied for each $\lambda$ in order to identify possible different dynamical regimes.

3. Before discussing the results, let us give some details of the numerical procedure used. The full details may be found in [22]. We want to perform numerical simulations in our simplified model inspired by the Standard Model in an early universe context. Therefore the system is rewritten in dimensionless quantities and we adopt the choice of [21] by writing explicitly

$$\tilde{x} := a^{1/2}T_2 x \quad , \quad \tilde{t} := a^{1/2}T_2 t$$
$$X \equiv \tilde{\varphi} := a^{-1/4}T_2^{-1/4} \varphi \quad , \quad \theta \equiv \tilde{T} := T/T_2$$
$$\tilde{\alpha} := a^{-3/4} \alpha \quad , \quad \tilde{\lambda}(T) := a^{-1/2} \lambda(T)$$
$$\tilde{g}_6 := a^{-1/2} g_6 \quad , \quad \tilde{g}_8 := a^{-1/2} g_8$$

(9)

where $a = 2D$ and

$$\alpha = \frac{1}{4\pi} \left[ 6 \left( \frac{m_W}{\sigma} \right)^3 + 3 \left( \frac{m_Z}{\sigma} \right)^3 \right].$$

(10)

Omitting the twiddles, in the dimensionless system eq. (2) has the form

$$\frac{\partial^2 X}{\partial t^2} - \nabla^2 X + \eta \frac{\partial X}{\partial t} + \frac{\partial U(X, \theta)}{\partial X} = \xi(x, t)$$

(11)

with the fluctuation – dissipation relation

$$\langle \xi(x, t)\xi(x', t') \rangle = 2\eta \theta \delta(t - t') \delta^{(3)}(x - x').$$

(12)
Instead of evaluating the full expression for the potential \( U \) at any temperature we will for simplicity rely on the high and low temperature expansion (HT and LT) \( \text{[13]} \) in the appropriate regions of \( m/T \). It is well known that both the high and the low temperature expansion work surprisingly well even for \( m/T \) of order 1. Taking into account a field independent contribution to the HT-potential (which is often ignored) HT and LT can therefore be matched in a reasonably smooth manner at \( m = T \). Even though there are different masses entering the potential we may exploit the fact that all mass(es) are proportional to the scalar expectation value and use the top mass to decide upon which of the two regimes is appropriate. The potential of eq. (5) can now be written as

\[
\tilde{U}(1)(X, \theta = 0) = -\frac{1}{2}X^2 + \frac{1}{4}\lambda X^4 + \frac{BX^4}{\sqrt{a}} \ln \left( \left( \tilde{\lambda} + \frac{2B}{\sqrt{a}} \right) X^2 \right)
\]

\[
\tilde{U}_{(1),HT}(X, \theta) = \frac{1}{2}\theta X^2 - \frac{1}{3}\tilde{\lambda} X^3 + \frac{1}{4}(\tilde{\lambda}_T - \tilde{\lambda}) X^4 - \frac{13}{60a^{3/2}} \pi^2 \theta^4
\]

\[
\tilde{U}_{(1),LT}(X, \theta) = -\frac{\theta^4}{\pi^2 a^{3/2}} \left[ 6 \left( \frac{m_t}{\sigma} a^{1/4} \frac{X}{\theta} \right)^2 \cdot K_2 \left( \frac{m_t}{\sigma} a^{1/4} \frac{X}{\theta} \right) \right.
\]

\[
+3 \left( \frac{m_W}{\sigma} a^{1/4} \frac{X}{\theta} \right)^2 \cdot K_2 \left( \frac{m_W}{\sigma} a^{1/4} \frac{X}{\theta} \right)
\]

\[
+ \left[ \frac{3}{2} \left( \frac{m_Z}{\sigma} a^{1/4} \frac{X}{\theta} \right)^2 \cdot K_2 \left( \frac{m_Z}{\sigma} a^{1/4} \frac{X}{\theta} \right) \right]
\]

\[
\tilde{U}(2)(X, \theta) = \left( 1 - \frac{\theta^2}{\Lambda^2 \sqrt{a}} \right) \left[ -\frac{1}{6}\tilde{g}_6 \frac{X^6}{\Lambda^2} + \frac{1}{8}\tilde{g}_8 \frac{X^8}{\Lambda^4} \right]
\]

(13)

where \( K_{\nu}(x) \) are the modified Bessel functions of the second kind and of the order \( \nu \). Since we need later on the first and second derivatives of \( U \), it is more appropriate to use \( K_2 \) explicitly instead of the more familiar exponential representation. The choices for \( g_6 \) and \( g_8 \) depending on \( \lambda \) will be given below. Note the explicit time dependence of the potential appearing in eq. (11) which is a consequence of the \( t \)-dependence of the temperature as given by eq. (8).

This equation is analytically not solvable and we use therefore a discretization scheme for a numerical integration. Consider a hyper-cubic lattice in 3+1 dimensions with \( N_x \) points in each spatial direction separated by the spacing \( \delta x \) and \( N_t \) points in the temporal direction separated by the spacing \( \delta t \). Then \( X_{i,n} \) is the value of \( X \) at the lattice point \((x_i, n)\) where \( i = (i_x, i_y, i_z) \). For sufficiently small spacings the derivatives can be written as differences whose exact form depends on the discretization scheme. Like in \( [24] \), we use the well known staggered leapfog scheme which is of second order accuracy in time. Next the second order differential equation (11) is rewritten as a pair of two first order equations for \( X \) and \( \dot{X} \). Introducing additional lattices for \( \dot{X} \) which are shifted by \( \delta t/2 \) (i.e. \( \dot{X}_{i,n+1/2} \) and \( \dot{X}_{i,n-1/2} \)
and applying the substitution $X_{i,n+1} = X_{i,n} + \delta t \cdot \dot{X}_{i,n+1/2}$ yields

$$
\dot{X}_{i,n+1/2} = \frac{1}{1 + \frac{1}{2} \eta \delta t} \left[ \left( 1 - \frac{1}{2} \eta \delta t \right) \dot{X}_{i,n-1/2} + \delta t \left( \nabla^2 X_{i,n} - \frac{\partial U}{\partial X_{i,n}} + \xi_{i,n} \right) \right].
$$

(14)

The noise term becomes in discretized form

$$
\xi_{i,n} = \sqrt{\frac{2 \eta \theta}{\delta t (\delta x)^3}} G_{i,n}
$$

(15)

and thus the fluctuation – dissipation theorem now reads

$$
\langle \xi_{i_1,n_1} \xi_{i_2,n_2} \rangle = 2 \eta \theta \frac{1}{\delta t} \delta_{n_1,n_2} \frac{1}{(\delta x)^3} \delta_{i_1,i_2},
$$

(16)

where the $G_{i,n}$ are Gaussian distributed random numbers of unit variance. The initial conditions are chosen to be

$$
X_{i,0} = 0 \quad \text{and} \quad \dot{X}_{i,-1/2} = 0,
$$

(17)

which corresponds to a homogeneous system without fluctuations supposedly describing a rapidly and adiabatically cooled (i.e. quenched) state of the early universe before the onset of the electro-weak phase transition. We also impose periodic boundary conditions to the finite lattice, i.e. compactification of the spatial dimensions. The generation of unwanted long range correlations is avoided due to the uncorrelated noise terms.

The simulation requires a few more words of explanation. Since the temperature evolves with time we have to fix the initial and the final temperature, $\theta_{\text{ini}}$ and $\theta_{\text{fin}}$, respectively, which are connected by

$$
\theta_{\text{ini}} \cdot t_{\text{ini}}^{1/2} = \theta_{\text{fin}} \cdot t_{\text{fin}}^{1/2}
$$

(18)

yielding thus the number $N_t$ of time steps $\delta t$,

$$
N_t = \frac{t_{\text{fin}} - t_{\text{ini}}}{\delta t} = \left[ \left( \frac{\theta_{\text{ini}}}{\theta_{\text{fin}}} \right)^{1/\kappa} - 1 \right] \cdot \frac{t_{\text{ini}}}{\delta t}.
$$

(19)

We choose for all calculations $\theta_{\text{ini}} = 1.5$, well above the onset of the phase transition. The final temperature is to a large extent free; one just has to take care that all interesting dynamics has happened already when it is reached. $\theta_{\text{fin}}$ should preferentially not be too small since $N_t$ depends strongly on the ratio $\theta_{\text{ini}}/\theta_{\text{fin}}$. Usually, $\theta_{\text{fin}} = 0.15$ is a good compromise, together with $\delta t = 0.1$. The temperature $\theta(t)$ and thus the potential $U(X, \theta(t))$ with its first and second derivative are calculated at every time step of the simulation. All extrema of $U$ are searched and it is determined whether they are minima or maxima. This fixes the number of phases which may be one, two or three in our model. Note that for $\theta > 0$ the second (or third) minimum may be at field values which are larger than the cutoff $\Lambda$. 


Table 1: Potential parameters (c.f. eq.s (2)-(7) and (9)) and corresponding critical values of $\eta_{\text{crit}}$.

| $\lambda$ | $\tilde{g}_6$ | $\tilde{g}_8$ | $m_H$ (GeV) | $\sigma'$ (GeV) | $\eta_{\text{crit}}$ |
|-----------|--------------|--------------|-------------|----------------|-------------------|
| 0.030     | 0.003941     | 0.063056     | 18.4        | 948.2          | 0.69              |
| 0.031     | 0.003892     | 0.062279     | 21.4        | 941.6          | 0.60              |
| 0.032     | 0.003578     | 0.057254     | 24.1        | 958.2          | 0.54              |
| 0.033     | 0.003558     | 0.056929     | 26.4        | 948.1          | 0.45              |
| 0.034     | 0.003549     | 0.056791     | 28.7        | 938.1          | 0.40              |
| 0.035     | 0.003285     | 0.052557     | 30.7        | 951.2          | 0.34              |
| 0.036     | 0.003290     | 0.052645     | 32.6        | 937.4          | 0.27              |
| 0.037     | 0.003054     | 0.048859     | 34.4        | 949.2          | 0.21              |
| 0.038     | 0.003066     | 0.049053     | 36.1        | 933.0          | 0.15              |
| 0.039     | 0.002806     | 0.044899     | 37.8        | 949.6          | 0.12              |
| 0.040     | 0.002818     | 0.045082     | 39.3        | 930.6          | 0.10              |
| 0.041     | 0.002620     | 0.041916     | 40.9        | 940.6          | 0.08              |
| 0.042     | 0.002436     | 0.038978     | 42.3        | 948.8          | 0.06              |
| 0.043     | 0.002400     | 0.038404     | 43.7        | 933.3          | 0.01              |
| 0.044     | 0.002227     | 0.035640     | 45.1        | 941.3          | –                 |

4. In order to discuss the results of the simulations we need to specify first values for the parameters of the potential. As described above, we assume the non-renormalizable operators introduced in eq. (7) to be generated by new physics with characteristic scale $\Lambda = 4\sigma \simeq 1 \text{ TeV}$. The corresponding couplings will be chosen such that for any given value of $\lambda$ the absolute minimum of the full potential is at $\sigma' \approx \Lambda$ for vanishing temperature. Finally $\lambda$ is varied as mentioned above between 0.03 and 0.044. Table 1 shows the values of the couplings $\lambda$, $\tilde{g}_6$ and $\tilde{g}_8$ together with the corresponding values of $m_H$ and the position of the stable minimum at $T = 0$, $\sigma'$. The corresponding potentials at vanishing temperature for the parameters of table 1 are plotted in figure 1. Note that height of the barrier between the metastable minimum at $\sigma = 246 \text{ GeV}$ and the stable minimum at $\sigma'$ increases with growing $\lambda$. The system is prepared with $X = \dot{X} = 0$ at $\Delta t = 0$, and the temperature is taken to be $1.5 \times T_2$ initially. The temperature is then taken to depend on time according to eq. (8) and the Langevin equation is solved as described above. A typical result of a solution of equation (4) with the above assumptions on the potential and for different values of $\eta$ is displayed in figure 2, where we plot the first and second moment of the dimensionless field, namely the mean value of the field $\langle X \rangle$ and the second moment $\Delta X = \sqrt{\langle (X - \langle X \rangle)^2 \rangle}$ as functions of $t$ for $\lambda = 0.03$ and different values of $\eta$. After an initial phase where the behaviour of the system is rather independent of $\eta$ and where it evolves essentially homogeneously ($\Delta X$ is very small), the behaviour of the first moment shows a strong dependence on the damping parameter $\eta$: For small $\eta$ the system reaches the absolute minimum relatively fast and in a more or less homogeneous fashion, performing damped oscillations around $X = \bar{\sigma}'$. As
Figure 1: The dimensionless potentials $U$ as functions of the dimensionless field $X$ for vanishing temperature with $\tilde{g}_{0,8}$ as given in table 1. With increasing values of $\lambda$ the height of the barrier between the metastable and the true vacuum gets larger.

$\eta$ gets larger, the system remains close to the metastable minimum at $\tilde{\sigma} \approx \tilde{\sigma}'/4$ for longer times and the transition to the absolute minimum, though still occurring for $\eta < 0.7$, takes longer and longer (note how the second moment deviates from 0 for longer times – the system is highly inhomogeneous during the transition). For $\eta = 0.7$, the behaviour changes dramatically: The system is no longer able to cross the potential barrier and to populate the true vacuum, but remains in the metastable minimum with only very small fluctuations. The region of $\eta$ around the critical value $\eta_{\text{crit}} = 0.69$ is resolved in the lower panel of figure 2. We perform the same procedure for the sets of parameters as given in the table, always identifying the critical value of $\eta$ as long as it exists. From table 1, one notices that $\eta_{\text{crit}}$ is a decreasing function of $\lambda$. In figure 3 we display the first moment for a relatively large coupling $\lambda = 0.42$. The critical $\eta$ turns out to be 0.06 in this case. The dynamics of the system differs from the case displayed in figure 2 mostly by more pronounced oscillations around the false vacuum state for small $\Delta t$. These may be understood by noting that first
Figure 2: The first and second moments $\langle X \rangle$ (upper curves) and $\Delta X$ (lower curves) for $\lambda = 0.03$. In the lower panel the dependence on $\eta$ around $\eta_{\text{crit}} = 0.69$ is resolved (note the different timescales).
the values of $\eta$ displayed are small compared to the ones shown in fig. 2, i.e. the system is less damped, and that second the curvature of the potential at the metastable minimum is larger for larger $\lambda$ (c.f. figure 1). By a simple kinematic analogon one expects more pronounced oscillations. Finally, for $\lambda = 0.044$, the critical value of $\eta$ vanishes and the system is no longer able to reach the true vacuum by thermal activation, irrespective of the value of $\eta$.

We have checked finite size effects and found them to be small for the lattice size chosen. In particular the values for $\eta_{\text{crit}}$ found on a lattice with $64^3$ spatial sites are identical up to $\pm 0.01$ with the ones found on a $50^3$ lattice and given in the table. We have also performed calculations with time steps smaller than 0.1 as used for the results given above. Thus e.g. the results for the moments at $\lambda = 0.03$ and $\eta$ around 0.69, where one expects the errors of discretization to be most pronounced, are identical within the numerical precision for $\delta t = 0.1$, 0.05, and 0.03. Too large values for $\delta t$ result of course in large deviations from the behaviour found above. We thus believe that the numerical error on $\eta_{\text{crit}}$ (for the potential used) is $\pm 0.01$ and thus well under control.

5. To conclude, we have discussed the dynamics of the population and decay of metastable vacua in an environment with a time dependent temperature. We have done this using a phenomenological Langevin equation for a scalar field with an effective potential motivated by the Standard Model embedded in some model with characteristic scale $\Lambda \sim 1$ TeV, leading for vanishing temperature to the existence of a stable vacuum state at $\varphi = \sigma' \sim \Lambda$. 

Figure 3: The first moment for $\lambda = 0.042$. 

In particular the values for $\eta_{\text{crit}}$ found on a lattice with $64^3$ spatial sites are identical up to $\pm 0.01$ with the ones found on a $50^3$ lattice and given in the table. We have also performed calculations with time steps smaller than 0.1 as used for the results given above. Thus e.g. the results for the moments at $\lambda = 0.03$ and $\eta$ around 0.69, where one expects the errors of discretization to be most pronounced, are identical within the numerical precision for $\delta t = 0.1$, 0.05, and 0.03. Too large values for $\delta t$ result of course in large deviations from the behaviour found above. We thus believe that the numerical error on $\eta_{\text{crit}}$ (for the potential used) is $\pm 0.01$ and thus well under control.
This is the situation assumed for the derivation of bounds on Standard Model parameters from vacuum stability as considered in the literature [2]. We have discussed the dependence of the dynamics on the (phenomenological) parameter \( \eta \), governing dissipative effects. In particular to answer to the question whether the theory, starting at high temperatures in the symmetric phase, populates the (now metastable) physical vacuum depends critically on this parameter. In the framework studied here, using an equation of motion of the form (2) with Gaussian white noise \( \xi \) related to \( \eta \) by (4) and a potential given by (5)-(7) we find that the metastable vacuum is indeed populated for any value of \( \eta \) if \( \lambda \gtrsim 0.043 \), corresponding to \( m_H \gtrsim 43.7 \text{ GeV} \). We have shown that for smaller values of \( \lambda \) there exists a critical value of the viscosity parameter, \( \eta_{\text{crit}} \), such that for \( \eta < \eta_{\text{crit}} \) the system reaches the stable vacuum by thermal activation before \( T = 0 \), whereas for \( \eta > \eta_{\text{crit}} \) the system will populate the metastable vacuum (being closer to \( \varphi = 0 \) in our case) and is not able to cross the potential barrier by thermal activation.

Clearly the results collected in table 1 depend both on the specific form of the Langevin equation and on the effective potential. However, they highlight the fact that one may under certain conditions hope to learn something about the dynamics of phase separation without detailed knowledge of the parameters of the effective equation of motion. Thus, while for small \( \lambda \) the late time behaviour of the model studied here crucially depends on whether \( \eta \) is larger or smaller than \( \eta_{\text{crit}} \), for large enough quartic coupling the value of \( \eta \) is indeed irrelevant. For a given theory it might thus suffice to calculate the effective potential and its dependence on the temperature if the parameters of the theory are in a range where \( \eta_{\text{crit}} = 0 \). With the specific assumptions made in this work our results thus justify the usual discussion of metastability in the Standard Model. On the other hand we also note that for potentials with multiple minima the final state of the system – neglecting quantum effects, i.e. in a treatment on the basis of Langevin type equations – is not generally independent of \( \eta \) if the system is "quenched", i.e. the temperature is time dependent. This fact should motivate more efforts along the lines of [9], [10] and [11] in order to improve our understanding of the derivation of phenomenological equations of motion and their parameters from field theory.

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