Research on Trajectory Prediction Algorithm for Low-altitude Emergency Rescue Aircraft

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Abstract. With the rapid development of general aviation, this aviation solution is widely used in emergencies such as natural disasters, major accidents, and social security problems because of its fast start-up speed, high rescue efficiency, and less restrictions. In this paper, state-dependent mode transition hybrid estimation (SDTHE) is combined with the improved intent inference algorithm (IIA) to obtain a more accurate trajectory prediction algorithm. The algorithm overcomes the shortcoming of the hypothesis that the likelihood function is zero in the interactive multiple model algorithm (IMM), and considers the real-time motion state and mode of the aircraft, so that the final prediction result is more accurate.

1. Research background and research status

General aviation (GA) emergency rescue is fast, with high rescue efficiency and less restrictions. It is a key means of aviation in the emergency rescue system. However, due to its unique design, the GA rescue work is highly risky caused by uncertainties in the operation process. Existing literature on the low-altitude emergency rescue aircraft trajectory prediction algorithm is not much. At present, most scholars and experts predict the trajectory of aircraft by the following two approaches [1]: optimal state estimation and aircraft simulation model establishment. In the first one, the basic and typical algorithm is the KF algorithm. Reference [2] mentioned that the traditional KF algorithm has low accuracy in the trajectory prediction problem. If the trajectory prediction is considered to be a discrete random linear hybrid system estimation, the main analysis method is the IMM algorithm. The most important part for the second approach is to establish the base library. The limitation is that it must have the real-time meteorological environment data. However, it is difficult to achieve for the flight of low-altitude emergency rescue. Due to the special nature of the low-altitude airspace, such as lack of coverage of radar, the safety separation of rescue flights are basically controlled by the pilots. Therefore, the intention of the pilots must be included in the study. Aircraft-related intent inference approaches have been studied for many years and can be broadly divided into two categories: one is subjective intent [3]; the other is objective intent [4]. The existing problem is that the amount of
post-production generated in the inference, which will bring great difficulty to acquire the real-time flight status.

Therefore, the key research content of this paper is to propose a better flight state and flight intention combination algorithm without a basic library. And in the complex and ever-changing emergency rescue environment, how to infer the flight intention of the next moment according to the current flight state, and then use the obtained flight intention to predict the aircraft’s real-time trajectory is the main research content of this paper.

2. Trajectory prediction algorithm based on SDTHE and IIM

2.1. State-dependent mode transition hybrid estimation (SDTHE)

2.1.1 Aircraft motion model. The discrete-time stochastic linear hybrid system shown in the following three equations can be used to represent the motion of the aircraft in a low-altitude rescue environment:

$$ s(t) = A_{s(t)} s(t - 1) + W_{s(t)} (t) $$
$$ z(t) = C_{s(t)} s(t) + \nu_{s(t)} (t) $$
$$ n(t) = \delta (\omega(t - 1), x(t - 1)) $$

Among them, when the system is at time $t$, and $s(t) \in \mathbb{R}^n$, $z(t) \in \mathbb{R}^n$ and $n(t)$ are state variables, observed variables and flight modes.

The probability transfer function $\delta$ of the flight mode is currently not available, so we use the Markov transfer matrix associated with the state: $P \{ \pi_{i,j}(s(t - 1)) = \{ s_{i,j}(t - 1) \} \}$, $i, j = 1, 2, \ldots, r$

$$ \pi_{i,j}(s(t - 1)) \triangleq \rho \left[ m_i(t) \mid m_j(t - 1) \right] $$

2.1.2 SDTHE algorithm. The computational structure of the SDTHE algorithm comes from the IMM algorithm. If the assumptions difficulties of the IMM algorithm are to be overcome, the SDTHE algorithm will increase the time-varying step of the state transition probability, and selectively discard the problem that the mean value of the innovation sequence is zero while calculating the new likelihood probability function. The specific calculation process is: Step 1: input interaction, Step 2: modal transition probability update; Step 3: Kalman filter; Step 4: Modal probability update; Step 5: Mixed output.

2.2. Improved intent inference algorithm (IIIA)

2.2.1 IIIA algorithm under horizontal dimension, and CV mode. The unit vector associated with the intent model $I_j$ is defined as:

$$ e_j(t) = \frac{x_{tp} - x_{at}}{R_{tp}} e_x \frac{y_{tp} - y_{at}}{R_{tp}} e_y $$

(5) $e_x$ and $e_y$ are unit vectors in the x and y directions, point $(x_{tp}, y_{tp})$ and $(x_{at}, y_{at})$ represent the true position of the target $T_p(t)$, $R_{tp}$ is the straight line length from current position of the target to $T_p(t)$. For target for time $t$:

$$ \psi_j(t) = \cos^{-1}(e_j(t), e_j) $$

(6) Within the horizontal coordinate plane, IML can be expressed by the following formula:

$$ \lambda_j(t) = \mathcal{N} \left[ \psi_j(t) - \psi_{\infty}(t) ; 0, \sigma^2 \right], \forall I_j \in M_h $$

(7)

2.2.2 IIIA algorithm under horizontal dimension and CT mode. The IML can be used to infer the actual flight intention of the target before the flight mode transition point, but the rate of change of the IML

$$ \delta_j(t) = \dot{\lambda}_j(t) - \ddot{\lambda}_j(t - 1), \forall I_j \in M_h $$

is required for the flight after this

2.2.3 Intent model function and intentional inference correlation function based on aircraft status. In order to reduce the complexity of the calculation process, this paper uses a function to represent the intent model of the target in two different flight modes, CV or CT, as defined below:

$$ K(I_j) = \left\{ \begin{array}{ll}
\delta \lambda_j(t), & \forall I_j \in M_h, i \neq \hat{\omega}(\xi) \neq \hat{\omega}(\xi - 1), \hat{\omega}(\xi) = CT \\
\Omega_j(t), & \forall I_j \in M_h
\end{array} \right. $$

(9)
The nature of the intent inference problem is actually the issue of the maximum correlation. The result of intent inference is based on the intent of a correlation function $K(I)$ which has the maximum value. The actual flight intent of inference is: 

$$ I = \arg \max_{I \in \theta} K(I) \quad (10). $$

2.3. **Trajectory prediction algorithm based on SDTHE and IIIM**

In order to facilitate the study of the efficiency and accuracy of the new algorithm, this paper makes the following assumptions about the aircraft model:

The calculation procedure for the new algorithm is as follows (predictive time is $\tau$): Obtain the position and speed information of target through ADS-B, and obtain the flight state and mode estimation $\hat{m}(t)$ at the end of time $\tau$ through SDTHE. Combining the heading and TCP information, obtain the intent of the inference $\hat{I}(t)$ and the position of the target $T_p(t)$ through the IIIM algorithm. According to the speed of the aircraft, the time taken for the target to reach the intent node can be calculated.

There are two types of positions contained in the intent area of the IIIM: the point the target will reach when it sets its current heading and the point at which the heading change for $\theta$ passes. Assume $\theta = 45^\circ$, then the part outside the intent area can be defined as the left and right areas of the target. $T_p(t)$ is the targeted intent node. Therefore, according to the flight mode, all position points can be expressed in the following two ways: If the target is in the CV state, then $T_p(t)$ is on the extension of the current heading; If the target is in the CT state, then $T_p(t)$ is on either side of the current heading.

3. **Case Simulation and Analysis**

3.1. **Simulation model**

In two-dimensional network coordinates, the state vector of the target can be expressed as: $\xi = [x \ y \ y \ y]^T$.

The motion model in the horizontal direction can be expressed as:

$$ s(t) = A_{\xi}(\xi_{t-1}) + B_{\xi}(\xi_{t-1}) \begin{bmatrix} \theta \ \nu \end{bmatrix} \quad (11). $$

When $m(t)$ is in CV mode:

$$ A_{\xi} = \begin{bmatrix} 1 & \frac{\sin(\omega T)}{\omega} & 0 & \frac{\cos(\omega T) - 1}{\omega} \\ 0 & \frac{\cos(\omega T)}{\omega} & 0 & -\frac{\sin(\omega T)}{\omega} \\ 0 & \frac{1 - \cos(\omega T)}{\omega} & 1 & \frac{\sin(\omega T)}{\omega} \\ 0 & \frac{\sin(\omega T)}{\omega} & 0 & \cos(\omega T) \end{bmatrix}, \quad B_{\xi} = \begin{bmatrix} T^2/2 & 0 \\ T & 0 \\ 0 & T^2/2 \\ 0 & T \end{bmatrix}. \quad (12), \text{ when } m(t) \text{ is in CT mode:}$$

Then, modal transition matrix of SDTHE is:

$$ \Pi(s) = \begin{bmatrix} \Pi_{11}(s) & \Pi_{12}(s) \\ \Pi_{21}(s) & \Pi_{22}(s) \end{bmatrix} = \begin{bmatrix} 1 - \Pi_{12}(s) & \Pi_{12}(s) \\ \Pi_{21}(s) & 1 - \Pi_{22}(s) \end{bmatrix}. \quad (13).$$

(14) And, $\Pi_{12}(s)$ is the probability from CV to CT, $\Pi_{21}(s)$ is the probability from CT to CV. The horizontal and vertical distances from the target to the TCP point are $h_0$ and $h_{\eta}$, then $\pi_{12}(x)$ is:

$$ \Pi_{12}(s) = d_1 + d_2 \frac{N(\nu_0; 0, \sigma^2_{\nu}) - N(\nu_0; 0, \sigma^2_{\nu})}{N(\nu_0; 0, \sigma^2_{\nu})}. \quad (15)$$

Among them, $d_1 = 0.05$, $d_2 = 0.45$, the value of $\sigma_{\nu}$ and $\sigma_{\eta}$ are related to the flight procedure. $\theta(x)$ is the heading deviation of the target and $\pi_{21}(s) = d_3 + d_4 \frac{N(\theta(s); 0, \sigma^2_{\theta})}{N(\theta(s); 0, \sigma^2_{\theta})}$. Among them, $d_3 = 0.1, d_4 = 0.4, \sigma_\theta = 5$.  

3
The sampling period $T = 2\text{s}$, simulated with the MATLAB programming tool to obtain the results of Figure 1: results (a) aircraft real trajectory, measurement trajectory, estimated trajectory and predicted trajectory, (b) aircraft estimated standard deviation and prediction standard deviation, (c) Mean estimated error mean and predicted error mean.

### 3.2. Numerical examples
Assume that the initial cruising speed of the low-altitude emergency rescue aircraft is 54km/h, the turning rate is $3^{\circ}/\text{s}$, and the flight path consists of five segments: 0-400s: The aircraft maintains heading $180^{\circ}$ and constant-speed straight flight; 400-600s: the aircraft makes a coordinated turn to the left; 600-610s: an un-accelerated straight flight after the aircraft finish the turn; 610-660s: the aircraft makes a coordinated turn to the right; 660-1500: The aircraft's heading remains the same, flying at a constant speed. Among them, the model estimates the noise covariance as:

$$Q_{cr} = \begin{bmatrix} 0.25 & 0 & 0 & 0.25 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 3 & 0 \\ 0 & 0 & 0 & 3 \end{bmatrix}, \quad Q_{ct} = \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix}.$$  

The measurement noise covariance is:

$$R = \begin{bmatrix} 200 & 0 \\ 0 & 200 \end{bmatrix}.$$  

With sampling period $T = 2\text{s}$, and simulated with the MATLAB, results are showed in Figure 1: results (a) real trajectory, measurement trajectory, estimated trajectory and predicted trajectory, (b) estimated standard deviation and prediction standard deviation, (c) estimated error mean and predicted error mean.

The idea of combining SDTHE with IIIA algorithm proposed in this paper is as follows: the real-time position and velocity information of the aircraft at this moment is input into the SDTHE algorithm, and the flight state and mode estimation at the next moment are obtained, and then the IIIA algorithm is substituted according to the heading of the aircraft and the information of the TCPs. Then the next time and target intent node are obtained. The specific results are shown in Table 1.

![Figure 1 Results](image)

**Figure 1** Results(a) aircraft real trajectory, measurement trajectory, estimated trajectory and predicted trajectory, (b) aircraft estimated standard deviation and prediction standard deviation, (c) Mean estimated error mean and predicted error mean.)

| Planned node/(m,m) | Actual time/s passing | The new algorithm/s | Error/s |
|-------------------|----------------------|---------------------|---------|
| N0(2000,10000)    | 0                    | 0                   | 0       |
| N1(2000,5000)     | 333                  | 333                 | 0       |
| N2(2300,3000)     | 427                  | 425                 | -2      |
| N3(3700,3000)     | 520                  | 517                 | -3      |
| N4(4000,2700)     | 613                  | 610                 | -3      |
| N5(4000,-10000)   | 1459                 | 1456                | -1      |

### 4. Conclusion
Based on the IMM and II A algorithms, the accuracy of the aircraft trajectory prediction algorithm is further improved by the SDTHE. The two hypotheses in the IMM filtering algorithm are solved, and the simulation is introduced based on the theoretical research basis. Data was simulated to verify the validity and accuracy of the new algorithm.
5. References

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