About the Law of Sinus in a trigonometry class

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Abstract. The law of sine is an equation of great utility in trigonometry. This type of equation has been applied in various contexts as the analysis of the relationship between angle of formation and sheet thickness of aluminum in processes of embossment, also in an instance in the duality of polar spaces of constant curvature. The applications are made obvious in the sense that it is analyzed and continually ponders the theoretical formality. In this article, we show a our own theorem and corollary which is useful in trigonometry.

1. Introduction
Continuous progress to which we face as a society of knowledge, both in the technological part as academic, demand of us a propositive stance to change. This is possible to achieve provided that reform and transform the mediation of knowledge, as a process of reflection, in which each actor involved is recognized and recognizes the other as a pillar in the construction of appropriate policies to routing processes effectively to society. At this moment we are fortunate because educational institutions tend for the integral development of all the members of the academic community, especially the researchers in training, as the person trained integrally is autonomous in their learning, is able to generate their own criteria based on in the things they observes, looks, listens and reads [1]. On the other hand, although training by competences carries in its DNA the constant search for the development and continuous improvement of people [2], the process of integral formation is continuous, permanent, participative and seeks to harmoniously and coherently develop each and every one of the dimensions of the human being [3] and can not be based solely on the transmission of information or only on the teaching of competencies [4].

One of the spaces in which it is possible to generate improvements in the competences of the students is in the formative research, taking a leading role for this purpose the research seedbeds, which emerge as an alternative in the need of transformation of the pedagogy to the new educational reality taking advantage of the capacity of characteristic astonishment of the young people, impelling them and orienting them so that they develop investigative competences [5], since sometimes it is considered that in the research seedbeds, students must generate and execute directed work proposals to problem-solving [6].

All this must be guaranteed in favor of the development of the ability and capacity of the human being to know how to interpret and appreciate the content of the mind of others in order to understand and learn from others [7], understanding learning as building and restructure models to interpret the information it receives [8] seeking the applicability of it for the development of the community, which is why it is considered of great importance that universities guarantee citizen competence of their students [9].
At the academic level, this type of reflection is inherent both in teachers and students. It is necessary to create strategies that enable innovation in the processes of teaching and learning in the classroom and outside of it, where the topics are not exclusively for the focused subject. Cross-cutting knowledge allows, both teachers and students, to relate the theoretical concepts and the pilot, having as purpose the proposition improvements regarding the context.

Institutional educational projects, middle vocational level, covers subjects of great importance, that initiated the theoretical formalism required to the University level, among them there is Trigonometry, which in its teaching-learning process presents a great variety of conceptions and difficulties associated with the concepts of this area of knowledge, as evidenced by various research carried out in this field in recent decades [10-12]. Reason for which some institutions of higher education, contemplate the inclusion of geometry and trigonometry courses, primarily to strengthen necessary topics in the development of the professionalizing cycle.

Trigonometry is one of the courses of the area of mathematics of wider applicability in physics, engineering, chemistry, aeronautics, among others [13]. In trigonometry, a study is made more formal of the relationship of measures of a triangle, basically between sides and angles. The trigonometric functions because of their journal behavior has been used for modeling seismic waves, sounds, in medicine they have used trigonometry for several studies that involve the implant inclined to patients [14], in the same way, that has been applied to define the relationship between bilateral pedicle screws and vertebral rotation in which vertebral rotation is calculated after a postoperative period [15], trigonometry has also been used to evaluate abduction of lower limbs in newborns [16]. Goniometry is a method commonly used to measure the movement of the joint arch, particularly in relation to range of motion and muscle tone [17-19], similarly, in [20] a method based on trigonometric relations is presented between the distance of the work ports to the optical port and the distance between the optical port and the surgical objective, in order to provide a tool to help the reproducible evaluation of the laparoscopic surgical technique. On the other hand, in the field of engineering, trigonometry has been used to perform various calculations that involve objects not only triangular in shape, in [21-25] the trigonometric leveling method is used to determine the orthometric height of the points located in areas of difficult access, such as mountainous regions. Similarly, trigonometry has been used to determine joint angles in the construction of metamorphic robotic hands with spherical palms [26]. Law of sine and cosine law is used to analyze and find the relationships in triangles that are not rectangles. The law of Sines and law of cosines are of utmost importance in the field of trigonometry as these theorems establish relationships between the three sides and three angles of any triangle [27]. Sine law lays down that the equation is satisfied for a triangle \[28,29\]

\[
\frac{\text{sen}(\alpha)}{a} = \frac{\text{sen}(\beta)}{b} = \frac{\text{sen}(\theta)}{c}
\]

Where \(\alpha, \beta\) and \(\theta\) they are the angles of the triangles and \(a, b\) and \(c\) the corresponding opposite sides. In our university courses it is, a law applied in contexts in which it is possible to observe or construct a triangle associated data, for example the distances, or the calculation of angles of elevation, among others. There are other applications for this law among which is located as a fundamental contribution to the experimental geometry in various processes to increase the thickness of a sheet. For example in the construction of aluminium sheets, where the particular geometry uses the dimensional properties, metrology, the relationship between the thicknesses and angles with which law of the sinus for emboassment can be verified [30]. Figure 1 shows that for a simple conical form, the resulting reduced wall thickness can be determined easily through the lye of the breasts [30].

There are also studies more advanced concerning the use of this law, where arises, advanced as an instance in the duality of polar spaces of constant curvature [31]. This advanced form is represented by:

\[
\frac{\text{sen}(\alpha)}{S_k(a)} = \frac{\text{sen}(\beta)}{S_k(b)} = \frac{\text{sen}(\theta)}{S_k(c)}
\]
Under the same conditions for the angles and sides, just that defines the generalized sine function:

\[
S_k(x) = \begin{cases} 
\frac{\sin(\sqrt{k}x)}{\sqrt{k}} & \text{for } k > 0 \\
x & \text{for } k = 0 \\
\frac{\sinh(\sqrt{|k|}x)}{\sqrt{|k|}} & \text{for } k < 0 
\end{cases}
\]

Figure 1. Relationship between angle of training and using sheet metal thickness law of sine in processes of embossment [30].

A basic principle of teaching mathematical methods is to start using examples so students can easily follow the technique [32]. However, it is important that students understand the formality of this type of theorems, in order to enable them to deal with different ways of thinking. With the generation of these spaces, teaching mediation can become the canal's construction of new theories, the use of known laws helps to discover others using basic methods such as algebraic substitution [27]. There are other relationships in trigonometry that are basically intact, such as the law of the tangents and Mollweide formula. The latter is a set of two relationships between the sides and angles of a triangle, and this formula presents property of symmetry similar to the sine and the cosine law, which implies that they are cyclical in nature and thus for a law to another simply turn the order of sides and angles [33].

2. Mathematical method

Sine law also introduces an ambiguity, since there may be up to two satisfactory answers depending on the data supplied initially. The basic condition is to show three data among which involved an angle, its opposite side, and other any data, that may be another angle or other side. Case to case see the possibilities. In the first case, in which the third Datum is one angle more, immediately have the courage of all the Interior of the triangle angles due to the fact that given two of them, the sum of the three is equal to 180°, i.e. that the third angle will be equal to the difference between 180° and the sum of the two given angles. Under this condition, there are two possibilities; there is no response, case in which it is noted that the bosom of one of the angles has value less than -1 or greater than 1; the other condition is that there is a single response that is satisfied. Now if data given initially offered, an angle, its opposite side, and another side, then there are three conditions, the first is that there is no response or that there is only one like the previous case. However, it is possible that there is another answer. Knowing the angle α, and the sides a and b, we can find the value of the angle β using equation (1), and with these data find the angle missing by the sum of the internal angles 180°. If in addition, the supplementary angle β, call it β', coupled with the given angle α is initially less than 180°, implies that there is another triangle satisfying the initial conditions. Take for example the case in which you want to know the missing data if they β = 50°, b = 5, and c = 6. Of the law of Sines, we have that
\[
\frac{\text{sen}(50^\circ)}{5} = \frac{\text{sen}(\theta)}{6}
\] (4)

It is that \( \theta = 66.82^\circ \). Then it follows easily that \( \alpha = 63.18^\circ \) and \( \alpha = 5.83 \), but as seen, must be \( \theta' = 113.18^\circ \) and adding with the initially given angle is fulfilled that \( \beta + \theta' = 163.18^\circ < 180^\circ \), for which there is a second solution satisfying the initial conditions where \( \alpha = 16.82^\circ \) and \( \alpha = 1.89 \).

A question is to know if given three data that satisfy the condition for the use of the sine law, presents or not two possible solutions. A continuous reflection of this question was the following theorem that does not give a solution to the question above but shows a simple relationship between the Interior and exterior angles of a triangle oblique.

Theorem. The triangle \( ABC \) is oblique, with interior angles \( \alpha, \beta, \) and \( \theta \), and sides \( a, b, \) and \( c \) respectively. They are also \( \alpha', \beta', \) and \( \theta' \) respective exterior angles of the triangle. Given an interior angle, the value of this will always be less than any different to their supplementary exterior angles measure.

Demonstration. Without loss of generality, is the interior angle \( \alpha \). We need to show that \( \alpha < \beta' \) and \( \alpha < \theta' \). Note that \( \alpha + \beta + \theta = 180^\circ \), in the same way as \( \beta' \) is the supplement of \( \beta \), we have to \( \beta' + \beta = 180^\circ \). Then we have that \( \alpha + \beta + \theta = \beta' + \beta \), then \( \alpha + \theta = \beta' \) and given that \( \theta > 0 \), has to be \( \alpha < \beta' \). The case \( \alpha < \theta' \) show is done in a similar way. Of the previous theorem can be inferred in the following immediate corollary.

Corollary. The triangle \( ABC \) is oblique, with interior angles \( \alpha, \beta, \) and \( \theta \), and sides \( a, b, \) and \( c \) respectively. They are also \( \alpha', \beta' \) and \( \theta' \) respective external angles of the triangle. If two interior angles are equal, then under the sine law, there will be a single triangle maximum that meets the basic conditions given.

3. Results
Sine law has been used for various physical applications such as training and thickness of chapas of aluminum, the duality of polar spaces of constant curvature. On the other hand, is inherent need to dig deeper in trigonometry and geometry by their high applicability, because they are disciplines in which the properties of several special types of geometric figures are investigated [34]. For this purpose, it is important to generate spaces for reflection with respect to the theoretical basis, in order to be able to find relationships between the known theorems and other results that may arise from the continuous study of an area, particularly trigonometry.

4. Conclusions
The generation of theorems considered as a law of external nature [35], evidence of the commitment made by the teacher and the student during the academic process, this type of results shows that learning already there under the number of learned topics, but by the strategy of independent learning, which recognizes both the student and the teacher. Such changes in the teaching demands greater commitment by actors of the process, and as way to encourage meaningful learning. The importance of continuing to increase the theoretical tools of a certain area of knowledge, such as trigonometry, allows other researchers to continue their activities based on these results. We have shown that given an oblique triangle the value of any of its interior angles will always be less than any measure of the exterior angles other than its supplementary and in addition to this, another result shown as a corollary of the theorem stated in which we guarantee that if two interior angles are equal, then under the law of the sine, there will be a single triangle at most that satisfies the given basic conditions.

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