UNIFORM ASYMPTOTICS FOR THE TAIL OF THE DISCOUNTED AGGREGATE CLAIMS WITH UTAI CLAIM SIZES

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Abstract. This paper considers a risk model, where the price process of the investment portfolio is described by a geometric Lévy process. When the claim sizes are UTAI, the paper obtains the uniform asymptotics of the tail probability of the discounted aggregate claims and the finite-time ruin probability for the claim sizes with dominated varying distributions. The obtained results extend some existed results.

1. Introduction

In this paper, we consider a risk model, where the claim sizes \( \{X_n, n \geq 1\} \) are a sequence of nonnegative and identically distributed, but not independent random variables (r.v.s) with common distribution \( F \). The inter-arrival times \( \{\theta_n, n \geq 1\} \) constitute another sequence of independent and identically distributed (i.i.d) nonnegative r.v.s. The claim arrival times \( \tau_n = \sum_{k=1}^{n} \theta_k, n \geq 1 \) and \( \tau_0 = 0 \) constitute a renewal counting process

\[
N(t) = \sup\{n \geq 0 : \tau_n \leq t\}, \quad t \geq 0,
\]

which represents the number of claims up to time \( t \) and it has a finite mean function \( \lambda(t) = E[N(t)] \to \infty \) as \( t \to \infty \). We assume that the price process of the investment portfolio is a geometric Lévy process \( \{e^{R_t}, t \geq 0\} \) with Lévy process \( \{R_t, t \geq 0\} \), which begins with zero and owns independent and stationary increments. This assumption about the price process has been extensively applied in mathematical finance. One can see [8]–[19].

Suppose that \( \{X_n, n \geq 1\}, \{\theta_n, n \geq 1\} \) and \( \{R_t, t \geq 0\} \) are mutually independent. We use

\[
D(t) = \sum_{k=1}^{\infty} X_k e^{-R_{\tau_k}} 1_{\{\tau_k \leq t\}} \tag{1.1}
\]
to present the discounted aggregate claims up to time $t \geq 0$, in which the indicator function of event $E$ is denoted by $1_E$. The discounted value of the surplus process with stochastic return on investments of an insurance company is described as

$$U(t) = x + \int_{0}^{t} c(s) e^{-Rs} ds - D(t), \text{ for any } t \geq 0,$$

where $x \geq 0$ denotes the initial risk reserve of the insurance company and $c(t)$ is the density function of premium income at time $t$. Assume that the premium density function $c(t)$ is bounded, i.e., there exists some positive constant $H$ such that $0 \leq c(t) \leq H$ for all $t \geq 0$. For this renewal risk model, the finite-time ruin probability up to time $t \geq 0$ can be defined as

$$\psi(x, t) = P\left( \inf_{s \in [0, t]} U(s) < 0 \mid U(0) = x \right).$$

In this paper, we consider the asymptotics for the tail probability of the discounted aggregate claims, which hold uniformly for each $t$, such that $\lambda(t)$ is positive. For this, as in [14], define $\Lambda = \{ t : 0 < \lambda(t) \leq \infty \} = \{ t : P(\theta_1 \leq t) > 0 \}$. If let $\underline{t} = \inf \{ t : \lambda(t) > 0 \} = \inf \{ t : P(\theta_1 \leq t) > 0 \}$ then

$$\Lambda = \begin{cases} (\underline{t}, \infty) & \text{if } P(\theta_1 = \underline{t}) = 0, \\ (\underline{t}, \infty) & \text{if } P(\theta_1 = \underline{t}) > 0. \end{cases}$$

In order to simplify the investigation, we assume that $\underline{t} = 0$. For any $T \in \Lambda$, set $\Lambda^T = [0, T]$.

In this paper, all limit relationships hold as $x$ tends to $\infty$, unless noted otherwise. For two positive functions $m(x)$ and $n(x)$, we denote $m(x) \lesssim n(x)$ or $n(x) \lesssim m(x)$ if $\limsup x m(x)/n(x) \leq 1$; if $\lim m(x)/n(x) = 1$, then write $m(x) \sim n(x)$; if $\lim m(x)/n(x) = 0$ then write $m(x) = o(n(x))$. For a distribution $V$ on $(-\infty, \infty)$, let $\overline{V}(x) = 1 - V(x)$ be its tail.

This paper mainly discusses the upper tail asymptotic independent claim sizes. A sequence of $\{ \xi_n, n \geq 1 \}$ is called to be upper tail asymptotic independent (UTAI) if for any $x \in (-\infty, \infty)$ and $n \geq 1$, $P(\xi_n > x) > 0$, and it holds for any $i \neq j \geq 1$ that

$$\lim_{\min \{ x, y \} \to \infty} P(\xi_i > x \mid \xi_j > y) = 0$$

(see [10]).

In the following, we introduce some subclasses of heavy-tailed distributions. A distribution $V$ on $(-\infty, \infty)$ is called to be heavy-tailed distribution, if for any $\lambda > 0$, $\int_{-\infty}^{\infty} e^{\lambda y} V(dy) = \infty$. A distribution $V$ is said to belong to the dominated varying distribution class, which is denoted by $V \in \mathcal{D}$, if for any $0 < y < 1$,

$$\limsup \overline{V}(xy)/\overline{V}(x) < \infty.$$ 

A distribution $V$ on $(-\infty, \infty)$ is said to belong to the long-tailed distribution class, which is denoted by $V \in \mathcal{L}$, if for any $y \in (-\infty, \infty)$,

$$\lim \overline{V}(x+y)/\overline{V}(x) = 1.$$
For a distribution $V$ on $(-\infty, \infty)$, we denote its upper Matuszewska index by
\[ J_V^+ = -\lim_{y \to \infty} \frac{\log V_*(y)}{\log y} \quad \text{with} \quad V_*(y) := \liminf_{x \to \infty} \frac{V(xy)}{V(x)} \quad \text{for} \quad y > 1, \]
and $L_V = \lim_{y \downarrow 1} V_*(y)$. By these definitions, we know that $V \in \mathcal{D} \iff V_*(y) > 0$ for some $y > 1 \iff J_V^+ < \infty$ (see [1]).

Reviewing the history of research in the discounted aggregate claims, when $R_t = rt$ for some $r \geq 0$ and all $t \geq 0$, there are many researchers investigating ruin probabilities, such as [2], [6], [11]–[14], [17], [18], [20], [21] and so on.

When $\{R_t, t \geq 0\}$ is a Lévy process, [15] studied the risk model where the claim sizes and the inter-arrival times are two sequences of i.i.d r.v.s and they are mutually independent. [9] considered a dependent risk model, where the claim sizes and the inter-arrival times are also two sequences of i.i.d r.v.s, but there exists a dependence structure between the claim sizes and the inter-arrival times. [19] still considered the case that the claim sizes and inter-arrival times are independent. When the claim sizes are UTAI r.v.s with common distribution belonging to the class $\mathcal{L} \cap \mathcal{D}$, [19] gave the uniform asymptotics of the tail probability of the discounted aggregate claims.

In this paper, we will still investigate the risk model, where the claim sizes and the inter-arrival times are independent. We mainly consider the UTAI claim sizes and extend the result of [19] from the distribution of the claim sizes $F \in \mathcal{L} \cap \mathcal{D}$ to $F \in \mathcal{D}$. This will extend the scope of the applications of the main result.

This paper will suppose that the Lévy process $\{R_t, t \geq 0\}$ is right continuous with left limits. Let $E[R_1] > 0$, then $R_t$ drifts to $\infty$ almost surely as $t \to \infty$. We define the Laplace exponent for the Lévy process $\{R_t, t \geq 0\}$ as
\[ \phi(z) = \log E[e^{-zR_1}], \quad z \in (-\infty, \infty). \]
If $\phi(z)$ is finite then for any $t \geq 0$,
\[ E[e^{-zR_t}] = e^{t\phi(z)} < \infty \]
(see, e.g. Proposition 3.14 of [3]).

Now we present the main result of this paper.

**THEOREM 1.1.** For the discounted aggregate claims (1.1), suppose that the claim sizes $\{X_n, n \geq 1\}$ are UTAI r.v.s with common distribution $F \in \mathcal{D}$. If $R_t \geq 0$ almost surely for any $t \geq 0$ then, for each fixed $T > 0$
\[ \int_{0^-}^t P(X_1e^{-R_s} > x) \lambda(ds) \lesssim P(D(t) > x) \lesssim L_F^{-1} \int_{0^-}^t P(X_1e^{-R_s} > x) \lambda(ds) \]
holds uniformly for all $t \in \Lambda^T$. 
COROLLARY 1.1. Under the conditions of Theorem 1.1, if $F \in \mathcal{D}$ then, for each fixed $T > 0$,
\[ L_F \int_{0^-}^{t} P(X_1 e^{-R_s} > x) \lambda (ds) \lesssim \psi(x,t) \lesssim L_F^{-1} \int_{0^-}^{t} P(X_1 e^{-R_s} > x) \lambda (ds) \]
holds uniformly for all $t \in \Lambda^T$.

REMARK 1.1. [19] also investigated the discounted aggregate claims (1.1) for heavy-tailed claim sizes. When the distribution of the claim sizes $F \in \mathcal{L} \cap \mathcal{D}$, Theorem 2.1 of [19] obtained the uniform asymptotics of the tail of the discounted aggregate claims. It is well known that $\mathcal{L} \cap \mathcal{D} \subsetneq \mathcal{D}$, for example the Peter and Paul distribution $F(x) = \sum_{k:2^k \leq x} 2^{-k}$, $x \geq 0$. Then $F \in \mathcal{D}$ but $F/ \notin \mathcal{L} \cap \mathcal{D}$. For the detailed analysis one can see Goldie [5] and Example 1.4.2 of [4]. Thus Theorem 1.1 extends the scopes of the distributions of claim sizes from the class $\mathcal{L} \cap \mathcal{D}$ to the class $\mathcal{D}$.

2. Proofs of main results

Before giving the proof of Theorem 1.1 and Corollary 1.1, we firstly present some lemmas. The first lemma can be obtained from Proposition 2.2.1 of [1] and Lemma 3.5 of [16].

LEMMA 2.1. For a distribution $V$ on $(-\infty, \infty)$, if $V \in \mathcal{D}$ then for each $p > J_V^+$, (1) there exist positive constants $C_1$ and $D_1$ such that the inequality $V(y) \lesssim V(x)^{-p}$ holds for all $x \geq y \geq D_1$; (2) $x^{-p} = o\left(\frac{\lambda(x)}{V(x)}\right)$.

The following lemma is attributed to [14].

LEMMA 2.2. For the renewal counting process $\{N(t), t \geq 0\}$, any $v > 0$, and each fixed $T > 0$, it holds that
\[ \limsup_{x \to \infty} \sup_{t \in \Lambda^T} \lambda^{-1}(t) E\left[N^v(t) I_{\{N(t) > x\}}\right] = 0. \]

The following lemma can be obtained from Theorem 1 of [22].

LEMMA 2.3. Suppose that $\{\xi_k, k \geq 1\}$ are UTAI and nonnegative r.v.s with distributions $V_k \in \mathcal{D}$, $k \geq 1$, respectively. The random weights $\{\Theta_k, k \geq 1\}$ are a sequence of nonnegative r.v.s and are independent of $\{\xi_k, k \geq 1\}$. For some fixed integer $n \geq 1$, let $E\Theta_k^p < \infty$, $1 \leq k \leq n$ for some $p > \max\{J_{V_k}^+, 1 \leq k \leq n\}$. Then it holds that
\[ \sum_{k=1}^{n} P(\Theta_k \xi_k > x) \lesssim P\left(\sum_{k=1}^{n} \Theta_k \xi_k > x\right) \lesssim L_n^{-1} \sum_{k=1}^{n} P(\Theta_k \xi_k > x), \]
where $L_n = \min\{L_{V_k}, 1 \leq k \leq n\}$. 

Proof of Theorem 1.1. By (3.1) of the proof of Theorem 2.1 of [19], we get that there exists some positive constant $C_2$, such that for sufficiently large $x$ and all $t \in \Lambda^T$,

$$
\int_{0}^{t} P(X_1 e^{-R_s} > x) \lambda(ds) \geq C_2 \overline{F}(x) \lambda(t).
$$

(2.2)

For each integer $m \geq 1$, all $t \in \Lambda^T$ and $x > 0$,

$$
P(D(t) > x) = P \left( \sum_{k=1}^{\infty} X_k e^{-R_{\tau_k}} 1_{\{\tau_k \leq t\}} > x \right)
$$

$$
= \sum_{n=1}^{\infty} P \left( \sum_{k=1}^{n} X_k e^{-R_{\tau_k}} 1_{\{\tau_k \leq t\}} > x, N(t) = n \right)
$$

$$
= \left( \sum_{n=1}^{m} + \sum_{n=m+1}^{\infty} \right) P \left( \sum_{k=1}^{n} X_k e^{-R_{\tau_k}} > x, N(t) = n \right)
$$

$$
=: I_1 + I_2.
$$

For $I_2$, by Lemma 2.1 and (3.3) of the proof of Theorem 2.1 of [19], for any $p > J_F^*$, it holds uniformly for all $t \in \Lambda^T$ that

$$
I_2 \lesssim C_1 \overline{F}(x) E \left[ (N(t))^{p+1} 1_{\{N(t) > m\}} \right],
$$

which combining with (2.2) and Lemma 2.2 yields that

$$
\lim \limsup_{m \to \infty} \sup_{x \to \infty} \frac{I_2}{\int_{0}^{t} P(X_1 e^{-R_s} > x) \lambda(ds)} \leq \lim \limsup_{m \to \infty} \sup_{x \to \infty} \frac{I_2}{C_2 \overline{F}(x) \lambda(t)}
$$

$$
\leq \frac{C_1}{C_2} \lim_{m \to \infty} \sup_{t \in \Lambda^T} \lambda^{-1}(t) E \left[ (N(t))^{p+1} 1_{\{N(t) > m\}} \right]
$$

$$
= 0.
$$

(2.3)

Next we estimate $I_1$. Let $H(y_1, \ldots, y_{n+1})$ be the joint distribution of random vector $(\tau_1, \ldots, \tau_{n+1})$, $n \geq 1$. Obviously, for all $1 \leq n \leq m$, $t \in \Lambda^T$ and $x > 0$,

$$
P \left( \sum_{k=1}^{n} X_k e^{-R_{\tau_k}} > x, N(t) = n \right)
$$

$$
= \int_{\{0 \leq s_1 \leq \ldots \leq s_n \leq t, s_{n+1} > t\}} P \left( \sum_{k=1}^{n} X_k e^{-R_{\tau_k}} > x \right) H(ds_1, \ldots, ds_{n+1}).
$$

(2.4)

By Lemma 2.3 and (2.3), we get that

$$
\sum_{k=1}^{n} P \left( X_k e^{-R_{\tau_k}} > x, N(t) = n \right) \lesssim P \left( \sum_{k=1}^{n} X_k e^{-R_{\tau_k}} > x, N(t) = n \right)
$$

$$
\lesssim L_F^{-1} \sum_{k=1}^{n} P \left( X_k e^{-R_{\tau_k}} > x, N(t) = n \right)
$$
holds uniformly for all \( 1 \leq n \leq m, t \in \Lambda_T \) and sufficiently large \( x \).

For all \( t \in \Lambda_T \) and \( x > 0 \),

\[
I_3 := \sum_{n=1}^{m} \sum_{k=1}^{n} P(X_k e^{-R \tau_k} > x, N(t) = n)
\]

\[
= \left( \sum_{n=1}^{\infty} - \sum_{n=m+1}^{\infty} \right) \sum_{k=1}^{n} P(X_k e^{-R \tau_k} > x, N(t) = n)
\]

\[
=: I_4 - I_5.
\]

Therefore, it holds uniformly for all \( t \in \Lambda_T \) and sufficiently large \( x \) that

\[
I_3 \preceq I_1 \preceq \bar{L}^{-1} F I_3.
\] (2.5)

For \( I_4 \), it holds for all \( t \in \Lambda_T \) and \( x > 0 \) that

\[
I_4 = \sum_{k=1}^{\infty} \sum_{n=k}^{\infty} P(X_k e^{-R \tau_k} > x, N(t) = n)
\]

\[
= \sum_{k=1}^{\infty} P(X_k e^{-R \tau_k} > x, N(t) \geq k)
\]

\[
= \int_{0^-}^{t} P(X_1 e^{-R_s} > x) \lambda(ds)
\] (2.6)

and

\[
I_5 \leq \sum_{n=m+1}^{\infty} \sum_{k=1}^{n} P(X_k > x) P(N(t) = n)
\]

\[
= \bar{F}(x) \sum_{n=m+1}^{\infty} n P(N(t) = n)
\]

\[
= \bar{F}(x) E[N(t) 1_{\{N(t) > m\}}].
\] (2.7)

By (2.2), (2.7) and Lemma 2.2, we have that

\[
\lim_{m \to \infty} \limsup_{x \to \infty} \sup_{t \in \Lambda_T} \frac{I_5}{\int_{0^-}^{t} P(X_1 e^{-R_s} > x) \lambda(ds)}
\]

\[
\leq \lim_{m \to \infty} \limsup_{x \to \infty} \sup_{t \in \Lambda_T} \frac{\bar{F}(x) E[N(t) 1_{\{N(t) > m\}}]}{\int_{0^-}^{t} P(X_1 e^{-R_s} > x) \lambda(ds)}
\]

\[
\leq \lim_{m \to \infty} \limsup_{x \to \infty} \sup_{t \in \Lambda_T} \frac{\bar{F}(x) E[N(t) 1_{\{N(t) > m\}}]}{C_2 \bar{F}(x) \lambda(t)}
\]

\[
= 0.
\] (2.8)

By (2.6) and (2.8), it holds uniformly for all \( t \in \Lambda_T \) that

\[
I_3 \sim \int_{0^-}^{t} P(X_1 e^{-R_s} > x) \lambda(ds).
\] (2.9)
Thus, by (2.5) and (2.9) it holds uniformly for all $t \in \Lambda^T$ that
\[ \int_{0^-}^{t} P\left(X_1 e^{-R_s} > x\right) \lambda(ds) \preceq I_1 \preceq L_F^{-1} \int_{0^-}^{t} P\left(X_1 e^{-R_s} > x\right) \lambda(ds), \]
which combining with (2.3) gives that
\[ \int_{0^-}^{t} P\left(X_1 e^{-R_s} > x\right) \lambda(ds) \preceq P(D(t) > x) \preceq L_F^{-1} \int_{0^-}^{t} P\left(X_1 e^{-R_s} > x\right) \lambda(ds) \]
holds uniformly for all $t \in \Lambda^T$. This completes the proof of Theorem 1.1. 

Proof of Corollary 1.1. Next, we follow the line of the proof of Corollary 2.1 of [19] to prove Corollary 1.1. For the upper bound of $\psi(x, t)$, by Theorem 1.1 we know that
\[ \psi(x, t) \leq P(D(t) > x) \preceq L_F^{-1} \int_{0^-}^{t} P\left(X_1 e^{-R_s} > x\right) \lambda(ds) \tag{2.10} \]
holds uniformly for all $t \in \Lambda^T$. Then we will deal with the lower bound of $\psi(x, t)$. For any $0 < \varepsilon < 1$ and sufficiently $x$,
\[ \psi(x, t) = P\left(\inf_{s \in [0, t]} \left\{ D(s) - \int_{0^-}^{s} c(h)e^{-R_h}dh \right\} > x \right) \]
\[ \geq P(D(t) > x + HT) \]
\[ \geq P(D(t) > (1 + \varepsilon)x) \]
\[ \preceq \int_{0^-}^{t} \int_{0}^{1} P(X_1 u > (1 + \varepsilon)x) P(e^{-R_s} \in du) \lambda(ds) \]
\[ = \int_{0^-}^{t} \int_{0}^{1} \frac{\mathcal{F}((1 + \varepsilon)x/u)}{\mathcal{F}(x/u)} \mathcal{F}(x/u) P(e^{-R_s} \in du) \lambda(ds) \]
\[ \geq \inf_{u \in [0, 1]} \frac{\mathcal{F}((1 + \varepsilon)x/u)}{\mathcal{F}(x/u)} \int_{0^-}^{t} \int_{0}^{1} \mathcal{F}(x/u) P(e^{-R_s} \in du) \lambda(ds) \]
\[ \preceq \mathcal{F}_x(1 + \varepsilon) \int_{0^-}^{t} P\left(X_1 e^{-R_s} > x\right) \lambda(ds) \]
holds uniformly for all $t \in \Lambda^T$. Note that the facts that the positive Lévy process $\{R_t, t \geq 0\}$ has nondecreasing paths and $0 \leq c(t) \leq H$ are used in the second step, and Theorem 1.1 is used in the fifth step. Let $\varepsilon \to 0$, we have
\[ \psi(x, t) \preceq L_F \int_{0^-}^{t} P\left(X_1 e^{-R_s} > x\right) \lambda(ds) \tag{2.11} \]
holds uniformly for all $t \in \Lambda^T$. Combining (2.10) and (2.11), we finish the proof of Corollary 1.1. 

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