Mimicking the cochlea with an active acoustic metamaterial

Matthieu Rupin\textsuperscript{1,3}, Geoffroy Lerosey\textsuperscript{2}, Julien de Rosny\textsuperscript{1} and Fabrice Lemoult\textsuperscript{1,}\textsuperscript{*}

\textsuperscript{1} Institut Langevin, CNRS UMR 7587, ESPCI Paris, PSL Research University, 1 rue Jussieu, F-75005 Paris, France
\textsuperscript{2} Greenerwave, ESPCI Paris Incubator PC’up, 6 rue Jean Calvin, F-75005 Paris, France
\textsuperscript{3} Currently at hap2U, 20 rue du Tour de l’Eau, F-38400 Saint Martin d’Hères, France

E-mail: fabrice.lemoult@espci.psl.eu

Abstract

The human ear is a fascinating sensor, capable of detecting pressures over ten octaves of frequency and twelve orders of magnitudes. Here, following a biophysical model, we demonstrate experimentally that the physics of a living cochlea can be emulated by an active one-dimensional acoustic metamaterial. The latter solely consists on a set of subwavelength active acoustic resonators, coupled to a main propagating waveguide. By introducing a gradient in the resonators’ properties, we establish an experimental set-up which mimics the dynamical responses of both the dead and the living cochlea: the cochlear tonotopy as well as the low-amplitude sound amplifier are reproduced.

For mammalians, the hearing is operated within the inner ear and more precisely inside the cochlea (figure 1(A)) which acts as a fantastic sensor according to its wide frequency range, i.e. ten octaves, and its enormous amplitude dynamics from 0 to 120 dB. This second property is mainly a consequence of the cochlea’s ability to enhance low-amplitude sounds, and it is referred to as the cochlear amplification effect \cite{1,2}. In the cochlea, sounds are converted into flexural waves on the so-called basilar membrane (figure 1(B)). On this membrane lie the hair cells of the organ of Corti (figure 1(C)) that are responsible for the cochlear amplification due to active mechanisms. Several models have been proposed to explain this phenomenon \cite{3–9}, but here we will focus on one in particular \cite{10,11} which requires only a few ingredients which makes it experimentally testable: the hair cells of the cochlea behave as active nonlinear resonators operating at the particular regime of the so called Hopf bifurcation \cite{2,11}. In this work, we propose to first demonstrate experimentally that a gradient index metamaterial permits to mimic the cochlea, and second to implement this active model to study an artificial cochlea based on an active metamaterial \cite{12–18}. We show that it well reproduces the amplification of low amplitude sounds, one of the main characteristics of the mammalian living cochlea.

To begin, we consider the inner-ear wave that travels along the curvilinear path of the cochlea (figure 1(A)). This is a one-dimensional guided wave which vertically displaces the basilar membrane (figure 1(B)), and whose velocity and impedance are governed by the local properties of the cochlear partition. Due to the fact that these properties evolve along a gradient from the base to the apex \cite{19}, the inner-ear wave presents a very interesting spatial profile (figure 1(D)): as the wave penetrates along the cochlea its amplitude increases until it reaches a maximum, followed by a sudden drop corresponding to the end of the propagation. This increase in amplitude is accompanied by a progressive reduction of the local wavelength. Depending on the frequency, the maximum is reached for different positions, thus allowing a spatial coding of the frequency contents, and this is referred to as the tonotopic organization of the cochlea. All of these observations come from post-mortem measurements on the cochlea \cite{19}, but they are also known to apply to the response of the living cochlea to high-amplitude sounds \cite{2,20}. If we move to the low amplitude regime (figure 1(E)), the cochlear wave presents a response which is narrowly peaked in amplitude near the maximum. This effect is known as the cochlear amplification and is a clear evidence of the nonlinear nature of the living cochlea. If everything was linear, decreasing the excitation from 80 to 20 dB, the same multiplication factor should apply to the response of the cochlear wave (dashed red line in figure 1(F)), but thanks to the active processes within the hair cells an enhancement of the low intensity sounds is observed, and it has been measured to roughly 40 dB for the case of gerbils \cite{21} and mice \cite{22} so we expect similar values for the human ear.
Our goal is to demonstrate that a locally resonant metamaterial \cite{12, 18, 23, 24} whose properties evolve with position is a good candidate to reproduce the dynamic response of a dead cochlea. We start by modeling the human cochlea by a 33 mm long one-dimensional propagation medium supporting a single propagation mode with a quadratic dispersion relation (typical of a membrane-like wave), in which we regularly space (every 8 μm) in order to coincide with the number of hair cells’ rows in the human cochlea) subwavelength resonant point scatterers \cite{25} (figure 2(A)). Each point resonator is characterized by its resonance pulsation $\omega_0$ and also by its quality factor which actually has two different contributions: the radiative damping (parameterized by $Q_{\text{rad}}$) governs the coupling of the resonator to the propagating waves inside the waveguide, while the viscous damping ($Q_{\text{loss}}$) impacts on the intrinsic attenuation inside the resonator. Mathematically, the response of a single resonator in the monochromatic case can be expressed through its transmission $T(\omega)$ and reflection $R(\omega)$ coefficients inside the waveguide which take the form of a Lorentzian functions:

$$T(\omega) = \frac{\omega_0^2 - \omega^2 - \frac{\omega_0}{Q_{\text{rad}}}}{\omega_0^2 - \omega^2 - \frac{1}{Q_{\text{rad}}} + \frac{1}{Q_{\text{loss}}}}$$

Figure 1. The human ear. (A) The outer (beige), middle (red) and inner (blue) parts of the human ear. (B) Cross-section of one round of the cochlea showing the scala vestibuli (SV) and the scala tympani (ST), separated by the cochlear partition (CP) which contains the sensory hair cells. (C) These cells are represented in green (inner hair cells) and red (outer hair cells). They are attached to the basilar membrane (BM) and are connected to the tectorial membrane (TM) via their hair bundles. (D) The uncoiled cochlea. The vibration of the stapes of the middle ear (red) creates a guided wave that causes the BM to vibrate. The low (red), medium (green) and high (blue) frequencies travel up to different locations between the apex and the base, forming the tonotopic map. (E) Similar to (D) but for small amplitude excitations: the amplitude peak is narrowed. (F) Basilar membrane response for two different exciting amplitudes: the cochlear amplifier can induce an amplification of 40 dB (for living gerbils and mice).
When packing such identical point scatterers on a subwavelength scale, the waves propagating in the main waveguide interact with them and interesting phenomena occur near the resonance frequency \([18, 26]\). The wave propagation, and notably the dispersion relation, can be calculated through the transfer matrix formalism \([18]\). It exhibits a strongly dispersive behavior with very different responses depending on frequency (figures 2(C) and (D)). For example, for frequencies below the resonance the effective wavelength is shorter compared to freespace, or equivalently the effective wave velocity is smaller \([18, 27, 28]\). In contrast, for frequencies right above the resonance only evanescent waves exist (shaded area), which can be modeled by a so-called single negative metamaterial \([23, 29, 30]\). One can also notice the different impacts of the types of damping on the dispersion relation.

In order to reproduce a cochlear wave, the resonant frequency of each resonator (blue line in figure 2(B)) is now chosen to match the tonotopic law of Greenwood \([31]\). And we choose to equal the two quality factors for each resonator, and we adjust them with a linear fit (red line in figure 2(B)) to the psycho-acoustic measurements of Gold \([32]\). The fact that the resonance frequency and the quality factors evolve with position means that the previously discussed dispersion curves of a medium made of identical resonators now only apply locally. We
thus fall within the frame of a gradient-index metamaterial [33, 34] and more precisely we should observe the so-called rainbow trapping effect, initially proposed in optics [35] and more recently in acoustics [36, 37] as well as the increase of the pressure field from the entrance to a specific position [38, 39]. We analytically compute the solution by solving the multiple scattering problem resulting from an excitation at the basal position for a wide range of frequencies, again from a transfer matrix formalism [18]. The results are presented as a function of the distance in figure E, G and H and as a function of the frequency in figures 2(F), (I) and (J). We note that the solution clearly exhibits the desired behavior, with an increase of the field for a position (frequency) that depends on the frequency (position) and no further propagation after this point (frequency) due to the presence of a bandgap. The accumulated phase from the base to the position where the wave stops remains between 2 and 4 cycles (see figure 2(H)) which is in agreement with the measurements made in different mammals [1] as opposed to some other models that give many cycles [3–7]. Another interesting feature is the fact that we obtain a purely traveling wave with no reflected wave. This comes from the fact that we have adjusted the radiative quality factor and the viscous one in order to reach the critical coupling condition [40]. It should be noted that obtaining this particular regime where the radiative losses exactly compensate for the viscous losses supports a certain deviation in the adjustment of the losses and a wider range of values (for the quality factors) also gives similar results. In a way, this medium can be seen as a broadband perfect absorber similarly to the one obtained in acoustics with Helmholtz resonators [41].

To validate our approach we have experimentally produced such a gradient-index metamaterial. Instead of reproducing the analytical metamaterial we have just computed, we have decided to build a much bigger one in order to observe the phenomena existing at the subwavelength scale of the individual resonators, but the physics remains the same. Our experimental system consists of a main waveguide made from a 10 cm diameter plastic pipe which presents a single propagating mode below the cut-off frequency of 1700 Hz (figure 3(A)). The subwavelength resonant unit cells consist of quarter-wavelength resonators made of 2.8 cm diameter pipes, which are coupled to the side of the main waveguide and regularly spaced with a lattice constant \( a = 4.3 \) cm. We also applied the exponential gradient to the resonance frequencies as in the analytical medium, but we limited ourselves to the shaded area of figure 2(B) and the resonances range from 300 to 800 Hz, all below the cutoff frequency of the second propagating mode within the waveguide. The sub-wavelength nature of the metamaterial is respected for all values of the ratio \( \lambda / a \) (\( \lambda \) is the wavelength) which ranges from 27 for the lowest frequency resonance to 9 for the highest one. As shown in the top view in the inset of figure 3(A), we put a piece of 2 cm thick open-cell sponge in each resonator in order to increase their dissipation.

We first simulate such a medium with COMSOL Multiphysics and obtain the pressure maps of the figure 2(B) for 3 different frequencies. Those maps evidence the fact that the pressure field is maximum at the top end of each resonator, and it also highlights an interesting feature: an input amplitude of one creates an output of roughly 10 thanks to the resonance nature of such a medium. To measure the acoustic pressure in each resonator we therefore place small electret microphones 1 cm below the top of each pipe, and the source is a speaker which emits within the main waveguide (located on the left side of the metamaterial and not visible in the picture). All these elements are linked to an external sound board after being pre-amplified.

In the three panels of figure 3(C) we show the spatial pattern of the measured pressure fields for three different emission frequencies. We plot 6 different phases (in different grays) as well as the envelope (red curves) which corresponds to the maximum of the acoustic pressure at each position. A traveling wave, similar to the cochlear one, exhibiting an enhancement of its amplitude at a frequency-dependent position accompanied by a reduction in wavelength, is obtained, in very good agreement with the simulations. Note that the purely traveling character of the waves is demonstrated by both the absence of perturbations in the envelopes of the waves and by the profiles of the acoustic pressure at different phases which show a forward displacement. This fundamental property of the cochlear wave is in stark contrast with the previous proposals of acoustic rainbow trapping which exhibit standing wave patterns [36, 37]. This condition was fulfilled analytically by equalizing the radiative quality factor and the loss one, but here adding the sponges with less accuracy gives this feature.

Eventually, we show in figures 3(D) and (E) the waves’ envelopes as well as the accumulated phase for all of the measured frequencies, demonstrating the continuous spectral properties while the resonance frequencies are clearly discretized. So far, we have reproduced in a very simple experimental design, based on acoustic resonators, the passive response of the inner ear. In particular, this approach already gives important insights in the key role of dissipation in the existence of a traveling wave in the cochlea.

Next, we aim to reproducing the ear’s properties due to its biological nature and notably the previously mentioned cochlear amplification of low-level sounds. One of the scenarios for the origin of this fascinating ability is the idea that the outer hair cells could play the role of active electromotile resonators; and from now on, we want to test this model in which the active wave is the consequence of nonlinear resonators operating near a Hopf bifurcation [11]. Experimentally, we simply need to turn the passive resonators of the setup of figure 3(A) into active ones. This kind of cochlear design with active resonators were already tested in the context of analog...
integrated models of a cochlea [42–44], but here we voluntarily want to decouple what happens through acoustic waves from what happens in the resonators.

The first step consists in designing such active acoustic resonators. Typically, we place a small speaker at the top end of the acoustic quarter-wavelength resonator (figure 4(A)), whose emission will be linked to the pressure field measured by the microphone placed in the same resonator, one centimeter below. Due to the fact that they are both connected to a numerical audio interface, the feedback loop between them introduces a delay, and therefore we are in the so called field of active delayed resonator [45]. The acoustic pressure \( p(t) \) at the top-end is therefore ruled by the equation:

\[
\ddot{p}(t) + \frac{\omega_0}{Q} \dot{p}(t) + \omega_0^2 p(t) = S(t) + g(t) p(t - \tau),
\]

where \( \omega_0 \) is the natural resonance pulsation of the passive resonator, \( Q \) its quality factor (which stands for both the radiative and the loss terms) and \( S(t) \) corresponds to the source term of the resonator which comes from the pressure field at the other extremity of the resonator. The value of the delay-time \( \tau \) is chosen so that the pole of the monochromatic version of this equation occurs at \( \omega_0 \). Namely, \( \tau \) is the minimum value that we can impose that satisfies the condition: \( \tau = \left( n + \frac{1}{2} \right) \pi / \omega_0 \), \( n \) being an integer. The gain \( g(t) \) is chosen in order to build a Hopf bifurcation as well as a dependence on the exciting pressure field. In our experimental scheme it takes the following form:

\[ \text{Figure 3. Experimental passive cochlear wave in an airborne metamaterial. (A) Photograph of the experimental gradient-index metamaterial for airborne sounds, made from 38 quarter-wavelength acoustic resonators of different heights. (Inset) A 2 cm thick open-cell sponge has been added at the top end of each resonator to tune the dissipation. (B) Simulated pressure field within the medium for 3 different frequencies. (C) Experimental spatial distribution of the acoustic pressure measured in the resonators for 3 monochromatic excitations (red lines correspond to the envelopes while black ones are the real part for 6 different phases). (D) and (E) are the experimental maps of the amplitude’s envelope and the accumulated phase for all of the measured frequencies (the black dotted line represents the position of the maximum of the amplitude for each frequency curve in order to visualize the accumulated phase on panel (E)).} \]
Acoustic pressure, $p_{out}$, measured inside the resonator as a function of normalized frequency, $f/f_0$, $f_0$ is the resonance frequency. It exhibits the nonlinear resonator response to the amplitude of the incoming wave, $p_{in}$. The different curves correspond to different amplitudes of excitation.

$$g(t) = \begin{cases} G_c \left[ 1 - \left( \frac{p(t - \tau)}{P_0} \right)^2 \right] & \text{if } |p(t - \tau)| > P_0 \\ 0 & \text{otherwise} \end{cases}$$

This control parameter $g(t)$ is configured by a characteristic amplitude $P_0$ and the critical gain $G_c$. The latter corresponds to a particular behavior of the feedback loop which creates self-sustained oscillations within the resonator: this is the so-called Hopf bifurcation when $g(t) = G_c$. This happens when the reinjected energy within the resonator from the speaker situated at the top matches the lost one. This is mathematically fulfilled by the condition $G_c = \pm \frac{\omega^2}{Q}$, while this is experimentally adjusted by finding the value that corresponds to the bifurcation. In order to create a cubic nonlinearity for low-amplitude excitations only, this critical gain is multiplied by a coefficient that quadratically depends on the ratio $\frac{p(t - \tau)}{P_0}$. Therefore, the chosen active resonator operates precisely at the Hopf bifurcation for very low amplitudes, while the gain is forced to 0 for high amplitudes which guarantees the stability of the resonator. In between those two regimes, a cubic nonlinearity occurs for $|p(t - \tau)| \approx P_0$.

These three different regimes lead to a highly nonlinear active resonator as shown in the experimental response in figure 4(B). If we consider a small-amplitude wave impinging on a single active resonator, the frequency spectrum of the response corresponds roughly to self-sustained oscillations and it exhibits a very sharp peak (blue line). Then, increasing the exciting amplitude the response broadens and broadens until it corresponds to the passive response of the resonator for the loudest incoming sounds (red line). Thanks to the defined gain in equation (2) which drops to 0 for the highest samples, the resonators are very stable and this experiment is very reproducible. Note that the reference of the units decibel units in those experiments (which we defined as dBsetup) is not the auditory detection threshold of 20 $\mu$Pa but rather the minimum amplitude of detection for the setup defined in arbitrary units.

In order to mimic the cochlear wave, it is necessary to study the collective response of these nonlinear sub-wavelength resonators when they are used as unit cells in the gradient-index metamaterial previously introduced. With this configuration we now enter in the world of active acoustic metamaterials which are known to offer great opportunities to obtain, among other, acoustic diodes [46, 47], switches [48] or more complex sound manipulation [49–51]. This approach is not in complete agreement with the description of the cochlea since the tonotopy is generally attributed to the gradient in the mechanical properties of the basilar membrane, while only the nonlinearity is attributed to the hair cells [2]. In our experiment, the resonators are responsible for both the tonotopy and the active processes. However, our experimental model agrees with a macroscopic model where the inner-ear active traveling wave is the result of coupled active resonators. We therefore arrive at the active setup of figure 5(A), where only 16 resonators were active because of experimental limitations. The pressure fields obtained at three different frequencies for different sound levels are given in figures 5(B)–(D) and deserve several comments. First, for high-amplitude incoming tones (top panels), a
traveling wave very similar to the one obtained in the passive configuration (see figure 3(C)) is retrieved. This corresponds to the passive case where the gain is forced to zero in equation (2). Second, when we decrease the amplitude of the incoming sound, the tonotopic effect is enhanced. Indeed, the amplitude of the wave at the corresponding position (sweet spot) is increased with regard to that of the wave at the entrance of the metamaterial. At the same time, the spatial extent of the amplified region is narrowed. This is consistent with in vivo measurements [1, 21, 22] on mammalian cochleae and also with the model of an active traveling wave proposed by Duke and Jülicher [11]. Again, the nonlinear response of the metamaterial is evidenced by the shape of the spatial pressure distribution which depends on the sound amplitude; this shape reproduces nicely what was predicted by the theoretical models of the inner-ear wave. Nevertheless, we have to cede that we are reaching the limits of our setup since the half effective length roughly equals the spacing between two resonators and the wave in the setup does not look as continuous as in the case of in vivo measurements.

To further investigate the response of this active metamaterial in comparison to what is known about the cochlea, and notably to highlight the hyper-sensitivity at low-amplitude, we have conducted a set of experiments while gradually increasing the excitation amplitude. This permits to draw curves of the sensitivity, defined as the ratio of the acoustic pressure measured inside the resonators $p_{\text{out}}$ to the incoming acoustic pressure $p_{\text{in}}$ (figure 5(A)) for one particular frequency (399 Hz, case of figure 5(B)). Here, the nonlinear behavior of the setup is clearly revealed by the fact that the different curves do not superimpose. At high excitation (red), the amplitude of the sensitivity increases along the curvilinear abscissa of the metamaterial and reaches a maximum of about 10 dB. This increase is solely due to the effect of passive resonances (i.e. the rainbow trapping effect). For the lowest-level incoming sound (blue), however, the sensitivity goes up to 40 dB and the spatial extension of the enhancement sharpens.

In the meantime, those sensitivity curves exhibit behaviors that have barely been observed in vivo; we note, especially at low amplitude of excitation, the presence of nodes and anti-nodes of pressure along the spatial pattern which are the signature of a stationary wave’s presence. This is in contradiction with the common picture of the active traveling wave since a stationary wave is the consequence of waves propagating in opposite directions (from the apex to the base in the inner ear, and vice versa). Those waves are not considered in some models [11], but their presence is consistent with the well-known effect of spontaneous oto-acoustic emission in which the ear emits sounds in absence of excitation [2], a phenomenon which is also observed in our case but not...
presented here. Our measurements clearly show that similar back-propagating waves also exist when the cochlea is excited with low amplitude sounds, while they are absent with high ones. Since we have only one canal, the backward wave is in the same canal as the forward one contrariwise to the reference [52].

The presence of this reversed wave in vivo is not yet fully accepted [53] even if it appears in the data of [54], but we believe that it is a direct consequence of the competition between the active processes and the critical absorption condition: these backward traveling waves are the price to pay for the hyper-sensitivity at a low excitation’s amplitude. Those backward waves are also confirmed by the phase measurements presented in figure 6(B): while for the high amplitudes the phase seem to increase from the base to the apex it is the opposite for the low amplitude sounds meaning that the dominant wave comes from the resonator to the base. In between the transition shows that the accumulated phase is reduced compared to the passive case. Nevertheless, to conclude on the existence of those waves in vivo this would need more systematic investigations and comparisons with existing literature, the scope of future works.

The reliability of the previous results is demonstrated by the plot of figure 6(C), which represents the spatial maximum of $P_{\text{out}}$ as a function of $P_{\text{in}}$ at a given frequency. The three regimes observed in the case of the mammalian ear [1] are retrieved despite some differences in the order of magnitude. At high excitation, the ear has a linear response and the output pressure grows linearly with the incoming pressure. Decreasing the excitation, a change of the slope in the logarithmic scale is observed at 60 dBsetup: an inverse-cubic power law is observed. This effect is typical of a cubic nonlinear resonator such as the one chosen in this study. This power law is compatible with the cubic nonlinearities such as the phantom frequencies of Tartini that have been highlighted in many psychoacoustic studies. At very low amplitudes, the slope recovers a value of 1 (near 20 dBsetup). This is due to the fact that $P_0$ in equation (2) was set to a value significantly higher than that of the threshold, thus ensuring that the self-sustained oscillations are proportional in amplitude to those of the input. This sensitivity response is a characteristic response of mammalian cochlea [1]. Note that due to experimental conditions the gain of 29 dB measured here is lower than the 40 dB observed in nature. To reach this value, one

Figure 6. Nonlinear response analogous to the cochlear amplifier. The frequency of the incoming sound for the 3 panels is 399 Hz (it corresponds to the case of figure 3(B)). (A) Sensitivity defined as the ratio of the acoustic pressure in the resonators $P_{\text{out}}$ to the incoming acoustic pressure $P_{\text{in}}$. This representation shows the hyper-sensitivity to low-amplitude incoming sounds. (B) The accumulated phase corresponding to the same data as the panel (A). (C) Nonlinear dynamic of the acoustic pressure’s evolution in one resonator. This curve is analogous to experimental measurements realized on living cochleae.
would need to generate a larger difference between the active and the passive response of the resonators. This would have the effect of lowering the passive quality factor of the pipes from roughly 10 down to a few units.

Finally, we would like to highlight that we clearly observed the two expected transition points. The low transition is due to the feedback that self-maintains the resonator close to the Hopf bifurcation, as explained in reference [11]. The higher transition that does not appear in this theoretical prediction is a consequence of the resonators’ low energetic response: for the highest excitations their response is negligible, which is consistent with the fact that the hair cells in the cochlea cannot act on big displacements of the basilar membrane.

To conclude, we would like to emphasize that very few ingredients have been used to reproduce the active cochlear wave. First, a dissipative gradient-index locally resonant metamaterial well mimics the passive traveling wave. The active processes, responsible for the hyper-sensitivity to low-amplitude sounds, are simply created by replacing the resonant unit cells of the metamaterial by active resonators, and, in order to reproduce the inner-ear, they are tuned to operate near a Hopf bifurcation. As a result, the response of the medium becomes dependent on the excitation amplitude, and we have reproduced the active amplification of low-level sounds that characterizes the mammalian hearing abilities. Our results confirm that only a few ingredients are required to phenomenologically reproduce the cochlea: by building an active gradient index metamaterial we have reproduced the tonotopy, the cubic nonlinearity as well as the low-amplitude amplification. We have therefore built a very simple experimental platform to investigate phenomena such as phantom Tartini’s frequencies observed in psychoacoustics. Future works will therefore focus on studying polychromatic excitations and see how the different active resonators behave collectively since there are coupled through the main acoustic waveguide.

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ORCID iDs

Geoffroy Lerosey © https://orcid.org/0000-0002-7087-5377
Fabrice Lemoult © https://orcid.org/0000-0001-9757-0760

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