Focusing Waves at Arbitrary Locations in a Ray-Chaotic Enclosure Using Time-Reversed Synthetic Sonas

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Time reversal mirrors have been widely used to achieve wave focusing in acoustics and electromagnetics. A typical time reversal experiment requires that a transmitter be initially present at the target focusing point, which limits the application of this technique. In this letter, we propose a method to focus waves at an arbitrary location inside a complex enclosure using a numerically calculated wave signal. We use a semi-classical ray algorithm to calculate the signal that would be received at a transceiver port resulting from the injection of a short pulse at the desired target location. The quality of the reconstruction is quantified in three different ways and the values of these metrics can be predicted by the statistics of the scattering-parameter \( |S_{21}|^2 \) between the transceiver and target points in the enclosure. We experimentally demonstrate the method using a flat microwave billiard and quantify the reconstruction quality as a function of enclosure loss, port coupling and other considerations.

Introduction. Wave focusing through a strongly scattering medium has been an intriguing research topic in the fields of optics, acoustics and electromagnetics [1]. Its potential applications include medical imaging, ultrasound therapy, communications, and nondestructive testing. In optics, wavefront-shaping has been used to focus light both through and inside strongly scattering media [13]. This method utilizes the spatial degree of freedom in the optical domain by modulating the phase of each pixel in the light beam. One can also achieve focusing in the temporal domain. For example a time-reversal mirror (TRM) [4] has been demonstrated to have good focus for waves passing through strongly scattering media in acoustics [5,7] and in electromagnetics [8,9]. A TRM can work both in open systems with a strongly scattering medium placed between the target and transceiver ports, or in closed billiard systems supporting ballistic propagation of waves in which the wavelength is much smaller than the billiard size. In fact a relatively simple single-channel TRM can be efficiently implemented in ray-chaotic billiard systems [8], and the experiments discussed here are performed in such billiards.

A typical time reversal experiment consists of two steps [6,8,10]. First, in the time-forward step, one injects a short pulse at the target port and collects the resulting long-duration transmitted signal (called the “sona”) at the transceiver port. In the time backward step one time-inverts the previously collected and recorded sona signal and sends it back into the system through the transceiver port, resulting in a time reversed short pulse at the target port. Since an active source must be present at the target location to create the initial signal, and because the sona is unique to that location, this process must be repeated for any location that one desires to focus wave upon. As shown in previous works [11,12], one can relax this constraint to some extent by placing a passive nonlinear object at the desired target location and using its higher harmonic nonlinear response as a “beacon” for later time-reversal. In acoustics, several methods [13,14] have been developed to shift the location of the reconstruction, but these are either limited to small shifts or to the special case geometry of acoustic waveguides, and in both of these cases one must still have a source located at a representative target location to produce a baseline sona signal.

Here we present a method to focus electromagnetic waves at an arbitrary location in a ray-chaotic billiard using an extension of the time reversal technique. This method is successful but has limitations due to wave propagation loss, port coupling mismatch, finite mode density of the billiard, and the existence of chaos in the ray limit. We discuss the effects of these factors by presenting experimental results on high-loss and low-loss billiards, different antennas and frequency ranges (to modify coupling), and modification of the cavity to vary the modal structure. In general, we find that the synthetic sona method can produce good time-reversal focusing at an arbitrary location in experiments with well-coupled antennas.

Synthetic Sona. The construction of the synthetic sona starts with a calculation of ray orbits [15]. Specifically using a ray tracing code, we obtain the trajectories of rays that start from the target point, bounce off of the walls, and arrive at the transceiver port, all with orbit length less than a certain specified upper limit. Each bounce on the billiard wall follows the law of specular reflection. Then for each trajectory \( i \), the orbit length \( L_i \), number of bounces \( n_i \), and ray bundle divergence factor \( D_i \) [16,18], are used to calculate a scaled and time-delayed version of the input signal, \( g(t) \), which is usually a short Gaussian pulse. Summing up contributions from all \( N \) trajectories of length less than the up-
per limit gives the synthetic sono signal \(s_{\text{syn}}(t)\). The calculation is performed in the frequency domain first,
\[ S(\omega) = \sum_{i=1}^{N} G(\omega) e^{-j\omega L_i/c} (-1)^n \sqrt{D_i}, \]
where \(S(\omega)\) and \(G(\omega)\) are the Fourier transforms of \(s_{\text{syn}}(t)\) and \(g(t)\) respectively. Then an inverse Fourier transform of \(S(\omega)\) into the time domain gives \(s_{\text{syn}}(t)\). Fig. 1 shows an example of a calculated synthetic sono from 4 simple orbits linking the target port and transceiver port in a 2D billiard. The above calculation does not include propagation loss. If we assume that the loss is uniform and results in an amplitude decay of \(e^{-1/\tau}\) and also assume that \(\tau\) is approximately frequency-independent, then we can apply an exponential window function to the synthetic sono to simulate the effect of propagation loss \([19]\).

**Experimental Setup.** For our microwave time reversal experiments \([3\ 20]\), billiards are simulated by quasi-2D, ray-chaotic cavities. One of the cavity shapes that we employ \([21\ 23]\) is depicted in the lower right inset of Fig. 1 and is referred to as a ‘bowtie’ shape. We also utilize a superconducting cut-circle shape \([24\ 26]\). We generate a short Gaussian pulse on a 7 GHz carrier frequency, \(g(t)\), inject it into the billiard through the target port, and a signal \(s(t)\), called the sono signal, is measured at the transceiver port (see Fig. 1 inset). This sono signal is then time reversed and sent back into the billiard through the transceiver port. A time-reversed reconstruction \(r(t)\) of the original Gaussian short pulse is then measured at the target port. Due to loss and other factors, \(r(t)\) not only contains a time reversed Gaussian pulse replica, but also has temporal sidelobes (see Fig. 2b).

We carry out the synthetic sono calculation procedure for all orbits with orbit length less than \(10\sqrt{A}\), a total of \(1.2 \times 10^9\) orbits for the bowtie billiard, where \(A\) is the billiard area, and inject the time reversed synthetic sono into the billiard to obtain the result at the target port shown in Fig. 2(d). Only the upper half of the signals are plotted since they are symmetric about the time-axis. Fig. 2(a) and (b) are the measured sono and the time-reversed reconstruction in the traditional time-reversal scheme. The reconstruction signal shows a peak, which is the reconstructed Gaussian pulse, and symmetric sidelobes around the peak. Fig. 2(c) and (d) are the calculated synthetic sono (corresponding to orbits up to four meters long, or 15 ns) and its reconstruction at the target port. There is a significant peak in the reconstruction, but the sidelobes are now unbalanced. This result demonstrates focusing at the target port in the microwave billiard using a purely synthetic sono.

**Reconstruction Quality.** The quality of a time-reversal reconstruction is influenced by several factors, including the propagation loss and port coupling mismatch. In order to examine the effect of these factors and compare reconstructions under different conditions, we must first quantify the reconstruction quality. Here we define the following measures of quality: reconstruction peak-to-peak voltage \(V_{pp}\), focus ratio and transfer ratio.

i) \(V_{pp}\) is the peak to peak voltage of the reconstructed Gaussian pulse. ii) The focus ratio is the average power of the reconstructed short Gaussian pulse divided by the average power of the entire reconstructed signal. The focus ratio measures how the reconstructed pulse stands out from the sidelobes and noise. iii) The transfer ratio is the energy in the entire waveform that is received at the target port divided by the energy in the time-reversed sono signal. This quantifies how efficiently energy is being transferred from the transceiver port to the target port.

To achieve good reconstruction, we need to have high transfer ratio to efficiently deliver energy to the target.
point, and also high focus ratio so that the delivered energy is well focused in a very short time period with minimum sidelobes.

Effects of Loss and Mismatched Port Coupling. Here we discuss two main factors that affect reconstruction quality: propagation loss and port coupling mismatch. Intuitively, a system with higher loss should lose more information during the transmission between the two ports, hence the reconstruction should be of lower quality. However we have also observed that the time-reversal reconstruction in the superconducting cavity can be worse than that in a similar cavity in the normal state, mainly because of antenna coupling issues. Hence propagation loss and port coupling mismatch both affect reconstruction quality and we shall discuss them now.

We find that to good approximation the sona signal amplitude decays exponentially in time as $e^{-t/\tau}$, where $\tau$ is the sona amplitude decay time. In addition, our experimental results on a superconducting 2D raychaotic cavity shows that $s_{\text{normal}}(t) = s_{\text{SC}}(t)e^{-t/\tau_0}$, with $1/\tau_0 = 1/\tau_{SC} - 1/\tau_{\text{normal}}$, where $s_{\text{normal}}(t)$ and $s_{\text{SC}}(t)$ are the sona signals measured in the superconducting and normal state of the cavity, respectively. Based on these results, it can be shown that in the case of the traditional measured sona method, higher loss will result in a scaled down sona signal with faster decay rate, and a scaled down reconstruction signal with smaller sidelobes; the reconstruction will have a smaller $V_{pp}$ and transfer ratio, but a higher focus ratio. See Supplemental Material at [URL will be inserted by publisher] for details of the derivation.

Port coupling can be varied by using a different antenna or using different carrier frequencies. The former modifies the radiation impedance of the port entirely, and the latter uses the fact that radiation impedance is a function of frequency [23, 27]. Both effects lead to a different billiard transfer function $S_{21}(\omega)$, which is the ratio of the transmitted wave amplitude over the incident wave amplitude between the transceiver port (1) and the target port (2). Define the mean transmission $\mu \equiv \langle |S_{21}|^2 \rangle_{\text{avg}}$ averaged over a 2 GHz frequency range surrounding the center frequency of the Gaussian pulse, and $\sigma_n \equiv \frac{\sigma(|S_{21}|^2)}{\mu}$ where $\sigma(|S_{21}|^2)$ is the standard deviation of $|S_{21}|^2$ in the same frequency range as $\mu$. $\mu$ and $\sigma_n$ measure the amplitude and fluctuations of the transmission spectrum $|S_{21}|^2$, respectively.

If we plot $\mu$ and $\sigma_n$ as a function of pulse center frequency together with normalized $V_{pp}$ and focus ratio, as shown in Fig.3 (a) for measured sona and (b) for synthetic sona, we find that $\mu$ and $\sigma_n$ predict the trend of $V_{pp}$ and focus ratio, respectively, in the traditional measured sona method. For the synthetic sonas $\mu$ has a high correlation with $V_{pp}$, although $V_{pp}$ has stronger fluctuations compared to the experimental case. To see the influence of noise on the reconstruction quality, we add Gaussian random noise with 2 mV standard deviation to the sonas and the reconstruction signals. Fig.3 (c) and (d) show that when noise is added, $V_{pp}$ and focus ratio follow $\mu$ in both the traditional measured sona and synthetic sona methods. This is because the average power in the reconstruction signal is mostly determined by the noise power, which is set to a constant, and the focus ratio is now proportional to average power in the reconstructed Gaussian pulse, which is proportional to $V_{pp}$. Hence $\mu$ becomes the only controlling factor in this case. In summary, knowledge of the mean value of transmission between the transceiver and target ports is an excellent predictor of reconstruction quality. The higher the mean $|S_{21}|^2$ the higher the quality of the reconstruction.

Synthetic Sona Duration Constraint. The synthetic sona duration is limited first by the computation cost and accumulation of error in the short orbit calculation. Since the number of orbits increases exponentially with orbit length, while the influence of each orbit decreases exponentially due to loss, it is more efficient to only calculate synthetic sonas with orbits within a length limit, depending on the computational budget. The ray-chaotic property of the billiard ensures that no ray is trapped inside the cavity without eventually reaching a port, but it also makes errors accumulate exponentially, at a rate determined by the largest Lyapunov exponent for the nonlinear map describing the ray trajectories. Hence the later part of the synthetic sona may contain more error than the earlier part.

To see the effects of accumulating errors, we create variations of the bowtie cavity by adding inserts to alter the scattering geometry of some of the walls. The differences between the geometry information of the actual inserts and the one assumed in the synthetic sona calculation are larger than that of the empty bowtie case. To determine the appropriate duration of the synthetic sona,
we apply a windowing function to the full synthetic and measured sona and plot the reconstruction quality (normalized to its saturation value) versus the sona duration in Fig. 4. The windowing function has a 1.5 ns Gaussian-shaped rise and fall, to avoid introducing higher frequency components. For the traditional measured sona method, both $V_{pp}$ and the focus ratio increase monotonically and eventually saturate when a longer sona duration is utilized. The saturation occurs when most of the sona signal with significant amplitude is used for time-reversal. But for the synthetic sona method, the focus ratio is highest when the synthetic sona duration is around $4\sqrt{A}/c (=4.5$ ns) for the bowtie with inserts, where $c$ is speed of light. This is because the shape of the inserts is known with less certainty than that of the empty bowtie, so the accumulation of error is more rapid. The later part of the synthetic sona contributes more to the sidelobes rather than to the reconstruction peak.

The synthetic sona duration is limited, but in order to have a good reconstruction the synthetic sona should be close to the $1/e$ decay time, $\tau$. It can be shown that the duration of the earlier-in-time sidelobe (prior to the reconstruction peak) is determined by the synthetic sona duration, and the later-in-time sidelobe (after the peak) is determined by $\tau$. See Supplemental Material at [URL will be inserted by publisher] for the dependence of the sidelobes on sona duration and $\tau$. If the synthetic sona is significantly shorter than $\tau$, then the reconstruction will have a small focus ratio with large sidelobes on the later side of the reconstruction, but very little sidelobe on the earlier side, causing it to look more like a sona signal rather than a reconstruction. For the bowtie billiard, the synthetic sona length is 15 ns which is close to $\tau = 14$ ns, so it works well. But if we change to a less well-coupled antenna or decrease the propagation loss such that the decay time $\tau$ is much longer, the reconstruction quality drops significantly. This is confirmed with time-reversal experimental results from the superconducting cut-circle cavity which has a very long decay time $\tau \approx 1300$ ns in the superconducting state and $\tau \approx 50$ ns in the normal state. The synthetic sona reconstruction in the superconducting state resembles a typical sona signal with a prominent sidelobe after the peak, while in the normal state it has balanced sidelobes, and thus better focus ratio. The focus ratio is 758 in the normal state with well-coupled antenna, 308 when changed to a less well-coupled antenna, and 123 when it is in the superconducting state with a less well-coupled antenna. For comparison, the focus ratio of a perfect reconstruction without sidelobe and noise in this experimental setup is 3333.

Conclusion. In this paper, we have shown that focusing electromagnetic waves at an arbitrary location inside a ray-chaotic billiard can be achieved by using time-reversed synthetic sonas, calculated from the cavity geometry and location of the wave input and focusing points. The focusing quality is quantified and is influenced by cavity loss and port coupling. To achieve a high quality synthetic sona reconstruction with the optimal focus ratio, the billiard should be fairly lossy, and the synthetic sona duration should be close to the $1/e$ sona amplitude decay time, although it is limited by the computation cost and accumulation of error. In many practical applications systems are lossy (less reverberating), allowing for the synthetic sona to potentially work well. If the reconstruction amplitude or energy transfer is of more concern, then lower loss and better-coupled antennas (large $\mu$) are required.

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[1] A. P. Mosk, A. Lagendijk, G. Lerosey, and M. Fink, NATURE PHOTONICS 6, 283 (2012).
[2] I. M. Vellekoop and A. P. Mosk, OPTICS LETTERS 32, 2309 (2007).
[3] I. M. Vellekoop, E. G. van Putten, A. Lagendijk, and A. P. Mosk, Optics Express 16, 67 (2008).
[4] H. L. Calvo, R. A. Jalabert, and H. M. Pastawski, Phys. Rev. Lett. 101, 240403 (2008).
[5] M. Fink, Phys. Today 50, 34 (1997).
[6] S. M. Anlage, J. Rodgers, S. Hemmady, J. Hart, T. M. Antonsen, and E. Ott, Acta Physica Polonica A 112, 569 (2007).
[7] B. E. Anderson, M. Griffa, C. Larmat, T. J. Ulrich, and P. A. Johnson, Acou. Today 4, 5 (2008).
[8] C. Draeger and M. Fink, Phys. Rev. Lett. 79, 407 (1997).
[9] N. Quiéffin, S. Catheline, R. K. Ing, and M. Fink, J. Acoust. Soc. Am. 115, 1955 (2004).
[10] G. Lerosey, J. de Rosny, A. Tourin, A. Derode, G. Montaldo, and M. Fink, Phys. Rev. Lett. 92, 193904 (2004).
[11] M. Frazier, T. Taddese, T. Antonsen, and S. M. Anlage, Phys. Rev. Lett. 110, 063902 (2013).
[12] M. Frazier, B. Taddese, B. Xiao, T. Antonsen, E. Ott, and S. M. Anlage, Phys. Rev. E 88, 062910 (2013).
[13] H. C. Song, W. A. Kuperman, and W. S. Hodgkiss, The Journal of the Acoustical Society of America 103, 3234 (1998).
[14] S. Conti, P. Roux, and M. Fink, Applied Physics Letters 80, 3647 (2002).
[15] J.-H. Yeh, J. A. Hart, E. Bradshaw, T. M. Antonsen, E. Ott, and S. M. Anlage, Acta Physica Polonica A 112, 569 (2007).
[16] J. Hart, E. Ott, and T. Antonsen, Phys. Rev. E 80, 041109 (2009).
[17] J.-H. Yeh, J. Hart, E. Bradshaw, T. Antonsen, E. Ott, and S. M. Anlage, Phys. Rev. E 81, 025201(R) (2010).
[18] J.-H. Yeh, J. Hart, E. Bradshaw, T. Antonsen, E. Ott, and S. M. Anlage, Phys. Rev. E 82, 041114 (2010).
[19] B. T. Taddese, T. M. Antonsen, E. Ott, and S. M. Anlage, AIP Advances (in press) arXiv:1208.5431.
[20] B. T. Taddese, T. M. Antonsen, E. Ott, and S. M. Anlage, Electronics Letters 47, 1165 (2011).
[21] P. So, S. M. Anlage, E. Ott, and R. N. Oerter, Phys. Rev. Lett. 74, 2662 (1995).
[22] S.-H. Chung, A. Gokirmak, D.-H. Wu, J. S. A. Bridgewater, E. Ott, T. M. Antonsen, and S. M. Anlage, Phys. Rev. Lett. 85, 2482 (2000).
[23] S. Hemmady, X. Zheng, E. Ott, T. M. Antonsen, and S. M. Anlage, Phys. Rev. Lett. 94, 014102 (2005).
[24] S. Ree and L. E. Reichl, Phys. Rev. E 60, 1607 (1999).
[25] B. Dietz, A. Heine, A. Richter, O. Bohigas, and P. Leboeuf, Phys. Rev. E 73, 035201 (2006).
[26] J.-H. Yeh, Z. Drikas, J. G. Gil, S. Hong, B. T. Taddese, E. Ott, T. M. Antonsen, T. Andreadis, and S. M. Anlage, Acta Phys. Polon. A 124, 1045 (2013).
[27] X. Zheng, S. Hemmady, T. M. Antonsen, S. M. Anlage, and E. Ott, Phys. Rev. E 73, 046208 (2006).