$R$ Parity violating enhancement of $B_u^+ \to \ell^+\nu$ and $B_c^+ \to \ell^+\nu$

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Abstract

We study the decays $B_u^+ \to \ell^+\nu$ and $B_c^+ \to \ell^+\nu$ in the context of the Minimal Supersymmetric Standard Model (MSSM) with explicit $R$ parity violation. We analyse the correlation between the two decays and show that branching ratios (BRs) for $B_c^+ \to \ell^+\nu$ may be of order 5% (over 40% in one case), without violating current bounds on $B_u^+ \to \ell^+\nu$. Although $B_c$ mesons are inaccessible at the $e^+e^-$ $B$ factories, such large BRs for $B_c^+ \to \ell^+\nu$ would possibly be within experimental observability at LEP and the Tevatron Run II, with much larger yields expected at the hadronic $B$ factories. We also update some earlier bounds on products of $R$ parity violating couplings in the light of new experimental results.

Keywords : Rare decay, R parity violation

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1 Introduction

Theoretical studies of rare decays of $b$ quarks have attracted increasing attention since the start of the physics programme at the $B$ factories at KEK and SLAC. Both $B$ factories are performing admirably and in excess of 60 fb$^{-1}$ of data has been accumulated by each experiment. The much anticipated measurement of $\sin 2\phi_1$ has established CP violation in the $B$ system [1, 2], while more recently evidence for CP violation in $B_d \to \pi^+\pi^-$ has been reported [3].

Many rare decays will be observed for the first time over the next few years, and measurements of their branching ratios (BRs) will be sensitive probes of models beyond the Standard Model (SM). In this paper we are concerned with the so far unobserved decays $B^+_u \to \ell^+\nu$ and $B^+_c \to \ell^+\nu$, where $\ell^+=e^+,\mu^+,\tau^+$ (and charged conjugates). In the SM these purely leptonic decays proceed via annihilation of the $B$ meson to a virtual $W$, and offer the possibility of measuring the decay constant $f_B$. However, in the SM the rates are helicity suppressed, and thus the largest BRs are for $\ell^+=\tau^+$. The first measurement of BR($B^+_u \to \tau^+\nu\tau$) is expected at Belle and BaBar, while the SM BR($B^+_u \to \ell^+\nu$) for $\ell^+=e^+,\mu^+$ is below the experimental sensitivity. $B_c$ mesons cannot be produced at the $e^+e^-$ $B$ factories operating at the $\Upsilon(4S)$, but their search is possible at hadronic machines (Tevatron Run II, LHC-B, BTeV). Although BR($B^+_c \to \ell^+\nu$) is considerably larger than the corresponding BR for $B^+_u$, the suppressed cross-section for $B_c$ production and the larger backgrounds at hadron colliders render isolation of these decays very challenging, assuming the BRs of the SM.

In models beyond the SM there can be sizeable enhancements from new particles, which may bring some of these decays within experimental observability. In particular charged Higgs bosons ($H^\pm$) [4, 5] and SUSY particles with $R$ parity violating interactions [6, 7], can both contribute at the tree–level. We study the prediction for BR($B^+_u \to \ell^+\nu$) and BR($B^+_c \to \ell^+\nu$) in the MSSM with lepton violating trilinear couplings $\lambda_{ijk}$ and $\lambda'_{ijk}$. In particular, we examine the correlation between these decays and show that by allowing a second $\lambda'$ coupling to be non–zero, the correlation observed by previous authors can be avoided. The BR($B^+_c \to \ell^+\nu$) can be enhanced to the level of 5% (over 40% in one case) without violating the bounds on BR($B^+_u \to \ell^+\nu$). BRs of this magnitude may be in reach at the Tevatron Run II and LEP, with plentiful yields at LHC-B and BTeV.

Our work is organised as follows: In section 2 we give a brief introduction to the $B_c$ meson. In section 3 we introduce the annihilation decays $B^+_u, B^+_c \to \ell^+\nu$, while section 4 examines the additional contributions to the decays in the MSSM with $R$ Parity violation. Section 5 presents the numerical results for the BRs and section 6 contains our conclusions.

2 The $B_c$ meson

The $B_c$ meson is the last SM meson to be discovered [8, 9]. It is unique in the sense that it is composed of two heavy quarks of different flavour. It possesses no strong or electromagnetic annihilation decay channels because it carries open flavour. Its mass and lifetime have been estimated by non–relativistic potential models and perturbative QCD [10], which predict $M_{B_c} \approx 6.2$ GeV and $\tau_{B_c} \approx 0.5$ps. Due to its mass it cannot be produced at the $e^+e^-$ $B$ factories running at the $\Upsilon(4S)$. It can, however, be produced at LEP and the Tevatron, which both searched in
the channels $B_c \rightarrow J/\psi \ell \nu$, $J/\psi \pi$. LEP reported a few candidate events but claimed no signal \cite{11-13}. The Tevatron Run I produced the first experimental evidence for the $B_c$ meson, with a sample of 20 $B_c \rightarrow J/\psi \ell \nu$ events corresponding to a background fluctuation of 4.8\,$\sigma$, \cite{14,15}. The measured mass and lifetime were $6.4 \pm 0.4\,$GeV and $0.46^{+0.18}_{-0.16} \pm 0.03\,$ps, respectively, in agreement with the theoretical predictions. This is the only experimental information on the $B_c$ meson, and hence the SM predictions for its BRs have not been verified. Large deviations from the SM rates are still permitted by experiment.

Although measurements with a precision comparable to the $B_u$ and $B_d$ systems are not expected in the near future, if new physics could affect the $B_c$ rates significantly without contradicting existing measurements in other systems, these effects could be tested at the upcoming hadronic $B$ machines where the $B_c$ meson accounts for approximately $10^{-3}$ of the inclusive cross-section for all beauty hadrons.

Possible decay channels for the $B_c$ meson are:

(i) $c$ quark spectator, with $\bar{b}$ quark weak decay: e.g. $B_c \rightarrow J/\psi \ell \nu$, $J/\psi \pi$

(ii) $\bar{b}$ quark spectator, with $c$ quark weak decay

(iii) Annihilation decays: $B_c \rightarrow \ell \nu, q\bar{q}'$

In the SM, (i) and (ii) are expected to amount to around 90\% of the width, with the remaining 10\% coming from the annihilation decays. The latter is a much larger fraction than in the case of $B_u$ decays. From now on we will concentrate on leptonic annihilation decays.

3 The decays $B_u^+ \rightarrow \ell^+ \nu_\ell$ and $B_c^+ \rightarrow \ell^+ \nu_\ell$

In the SM, the purely leptonic decays of $B_u^+$ and $B_c^+$ proceed via annihilation to a $W$ boson in the $s$-channel. The decay rate is given by (where $q = u$ or $c$):

$$\Gamma(B_q^+ \rightarrow \ell^+ \nu_\ell) = \frac{G_F^2 m_{B_q}^2 f_{B_q}^2 |V_{q\ell}|^2}{8\pi} \left(1 - \frac{m_\ell^2}{m_{B_q}^2}\right)^2$$

Due to helicity suppression, the rate is proportional to $m_\ell^2$ and one expects:

$$BR(B_q^+ \rightarrow \tau^+ \nu_\tau) : BR(B_q^+ \rightarrow \mu^+ \nu_\mu) : BR(B_q^+ \rightarrow e^+ \nu_e) = m_\tau^2 : m_\mu^2 : m_e^2$$

These decays are relatively much more important for $B_c$ than $B_u$ due to the enhancement factor $|V_{cb}/V_{ub}|^2(f_{B_c}/f_{B_u})^2$. Searches have already been carried out for some of these decays. The current experimental limits and SM predictions are presented in Table 1. “$x$” signifies that a search cannot be carried out since the $B_c$ meson is kinematically inaccessible at the $B$ factories CLEO, BELLE and BABAR. The symbol “$\otimes$” signifies that no search has yet been performed, although in principle one could have been carried out. This particularly applies to LEP and the Tevatron which can produce both $B_u$ and $B_c$ mesons but have not searched for the majority of the annihilation channels. CLEO and the $B$ factories have already reported strong limits on $B_u^+ \rightarrow \ell^+ \nu$ decays for $\ell^+ = e^+, \mu^+$. The best limit on $B_u^+ \rightarrow \tau^+ \nu$ decays comes from the
Table 1: SM predictions and current experimental limits from various machines.

| Decay               | SM Prediction | CLEO       | Belle       | LEP / Tevatron |
|---------------------|---------------|------------|-------------|----------------|
| $B^{+}_u \to e^+\nu_e$ | $9.2 \times 10^{-12}$ | $\leq 1.5 \times 10^{-5}$ | $\leq 4.7 \times 10^{-6}$ | $\otimes$ |
| $B^{+}_u \to \mu^+\nu_\mu$ | $3.9 \times 10^{-5}$ | $\leq 2.1 \times 10^{-5}$ | $\leq 6.5 \times 10^{-6}$ | $\otimes$ |
| $B^{+}_c \to \tau^+\nu_\tau$ | $8.7 \times 10^{-9}$ | $\leq 8.4 \times 10^{-4}$ | $\otimes$ | $\leq 5.7 \times 10^{-4}$ |
| $B^{+}_c \to e^+\nu_e$ | $2.5 \times 10^{-9}$ | $\times$ | $\times$ | $\otimes$ |
| $B^{+}_c \to \mu^+\nu_\mu$ | $1.1 \times 10^{-9}$ | $\times$ | $\times$ | $\otimes$ |
| $B^{+}_c \to \tau^+\nu_\tau$ | $2.6 \times 10^{-2}$ | $\times$ | $\times$ | (see text) |

L3 collaboration. In the search for $B^{+}_u \to \tau^+\nu$ at LEP $^{[19,21]}$ there is a sizeable contribution from $B^{+}_c \to \tau^+\nu$, which has the same experimental signature $^{[22]}$. Thus the experimental limit constrains the BR of an admixture of $B_u$ and $B_c$, and the effective limit on the process $B^{+}_c \to \tau^+\nu$ depends strongly on the conversion probability $b \to B_c (0.02\%–0.1\%)$ $^{[22]}$. For lower estimates of the conversion probability, the present experimental limit puts a bound of the order of the SM width on $B^{+}_c \to \tau^+\nu$. Therefore future, more precise bounds on $B^{+}_u \to \tau^+\nu$ from high energy machines and/or a better knowledge of the $b \to B_c$ conversion probability will start to put meaningful constraints on BR($B^{+}_c \to \tau^+\nu$).

Neither the LEP collaborations nor the Tevatron Run I searched for the channels $B^{+}_u \to e/\mu^+\nu$ and no limits whatsoever exist for $B^{+}_c \to \ell^+\nu$. While neither LEP nor Tevatron Run I could be competitive with the $B$ factories in setting bounds for $B^{+}_u \to e/\mu^+\nu$, attempts to measure this quantity could actually pick up a signal from $B^{+}_c \to e/\mu^+\nu$. In the next section we shall show that in SUSY models with $R$ parity violation, the decays $B^{+}_c \to \ell^+\nu$ may be enhanced sufficiently to bring them within experimental sensitivity of both LEP and Tevatron, thereby motivating a search in these channels. More specifically, assuming an optimistic $b \to B_c$ conversion probability of $\approx 0.1\%$, a measurement that gives a limit of $10^{-4}$ on the $B^{+}_u \to e/\mu^+\nu$ channels may be able to detect $B^{+}_c \to e/\mu^+\nu$ BRs of $\mathcal{O}(5\%)$.

4 $B^{+}_u, B^{+}_c \to \ell^+\nu_\ell$ in SUSY models with $R$ parity violation

Due to the current meagre experimental information on the decays of $B_c$ mesons, future measurements of their BRs may be a test of models beyond the SM. It is entirely possible that its BRs are very different from the SM predictions. The expected yield of $B_c$ mesons from Run II at the Tevatron is $10^6$, increasing to $10^9$ at LHC-B and BTeV $^{[23,24]}$. These machines will provide the first opportunity to study the $B_c$ meson in detail.

New physics contributions to the decays have been studied by a few authors. $^{[4]}$ considered the effects of $H^\pm$ on the decays $B^{+}_u \to \ell^+\nu_\ell$, while $^{[3]}$ extended this analysis to the case of $B^{+}_c \to \ell^+\nu_\ell$. In both cases the $H^\pm$ contribution modifies the SM prediction by a factor $R$ where:

$$R = 1 - \tan^2 \beta (M_{B_\ell}/M_{H^\pm})^2$$  \hspace{1cm} (3)

Since the SM predictions for $B^{+}_u \to e^+\nu_e/\mu^+\nu_\mu$ are already very small (see Table $^{[4]}$), the $H^\pm$
contribution cannot enhance them to current experimental observability. On the other hand, the $H^\pm$ contribution can saturate the current experimental bounds on $B_u^+ \rightarrow \tau^+ \nu_\tau$. The DELPHI search for $B_u^+ \rightarrow \tau^+ \nu_\tau$ constrains $\tan \beta/M_{H^\pm} \leq 0.46 GeV^{-1}$ (90% c.l.) which strengthens to $\tan \beta/M_{H^\pm} \leq 0.42 GeV^{-1}$ if the $B_c$ contribution is included \[20\]. L3 (who neglects the $B_c$ contribution) \[19\] obtains $\tan \beta/M_{H^\pm} \leq 0.46 GeV^{-1}$, which would become stronger if the $B_c$ contribution were included in their analysis. The $B$ factories with $> 100 fb^{-1}$ will be sensitive to BRs of the order of the SM prediction and thus the first measurement of $B_u^+ \rightarrow \tau^+ \nu_\tau$ is expected, unless its BR is suppressed by some new physics contribution ($H^\pm$ or otherwise).

The MSSM with $R$ Parity violating trilinear couplings predicts extra contributions to $B_q^+ \rightarrow \ell^+ \nu$ from tree level slepton or squark exchange. It has been shown that these contributions can be sizeable \[7\]. The $R$ parity violating superpotential is given by

$$W_R = \frac{1}{2} \lambda_{ijk} L_i L_j E^c_k + \lambda'_{ijk} L_i Q_j D^c_k + \frac{1}{2} \lambda''_{ijk} U^c_i D^c_j D^c_k$$ \hspace{1cm} (4)

Here we do not consider the bilinear term $\mu L_i H_2$ since $\mu_i$ are constrained to be very small by neutrino masses \[25\]. Their contribution to $b$ decays will be heavily suppressed compared to that from the trilinear couplings.

The simplest approach to $R$ parity violating phenomenology is to assume that a single $R$ parity violating coupling in the weak basis is dominant with all others negligibly small. It was shown that such an approach leads to several non-zero $R$ parity violating couplings in the mass basis \[26\]:

$$\bar{\lambda}'_{ijk} = \lambda'_{imn} U_{Ljm} D^c_{Rnk}$$ \hspace{1cm} (5)

Where $\lambda'_{imn}$ represents the weak basis couplings and $\bar{\lambda}'_{ijk}$ the mass basis. In the SM, the left handed up–type and down type quark mixing matrices $U_L$ and $D_L$ form the Kobayashi–Maskawa matrix $V_{KM}$. The right handed down type quark mixing matrix $D_R$ is unobservable in the SM, and therefore the generation mixing in the $R$ parity violating couplings \[5\] is a new observable. Limits on $R$ parity violating couplings depend strongly on assumptions on the absolute quark mixing \[26, 27\].

We follow a frequently used approach and assume all the quark mixing to lie in the up type sector. Thus $U_{Lij} = V_{ij}^{KM}$ and the above equation simplifies to:

$$\bar{\lambda}'_{ijk} = \lambda'_{imn} V_{jm}^{KM} \delta_{kn}.$$ \hspace{1cm} (6)

This results in 3 non-zero $\bar{\lambda}'_{ijk}$.

In order to have $R$ parity violating contributions to the decays $B_q^+ \rightarrow \ell^+ \nu_\ell$ one must assume at least two non-zero trilinear terms in the weak basis. The possible contributions are shown in Figure \[1\].

The $s$–channel contributions ($\sim \lambda \lambda'$) are dominant and will from now on be considered exclusively. Unlike the $H^\pm$ case, there is no $m_t$ Yukawa coupling suppression for these contributions and thus large enhancements for BR($B_u^+/B_c^+ \rightarrow \ell^+ \nu$) for all lepton flavours are possible.

It was shown in \[7\] that the $\lambda \lambda'$ mediated contributions to the decay $B_q \rightarrow \ell \nu_\ell$ have the following couplings:

$$B_{ln}^q = \frac{\sqrt{2}}{4 G_F V_{qb}} \sum_{i,j=1}^3 \frac{2}{m_{\tilde{\ell}_i}^2} V_{qj} \lambda_{imn} \lambda'_{ij3}.$$ \hspace{1cm} (7)
Figure 1: \( R \) parity violating contributions to \( B_q^+ \to \ell^+ \nu_n \). Left: slepton exchange in the \( s \)-channel (\( \sim \lambda_{inl} \tilde{\nu}_{i3}^* \)) Right: squark exchange in the \( t \)-channel (\( \sim \lambda_{n3i}^* \tilde{\nu}_{ij3}^* \)). Note that all four vertices conserve baryon number and violate lepton number, this results in two fermion lines originating from (ending in) the upper (lower) vertex of the right diagram.

where \( m_{\tilde{\ell}_i} \) is the mass of the slepton. For \( q = u, c \) one has the explicit values:

\[
B_{lnu}^u = \sum_{i=1}^{3} \lambda_{inl} (1689\lambda_{i13}^* + 384\lambda_{i23}^* + 1.52\lambda_{i33}^*)
\]

\[
B_{lnu}^c = \sum_{i=1}^{3} \lambda_{inl} (32.7\lambda_{i13}^* + 144\lambda_{i23}^* + 6.1\lambda_{i33}^*)
\]

(With \( m_{\tilde{\ell}_i} = 100 \text{ GeV} \) and PDG–values for the CKM matrix elements.)

Using this approach, [7] showed that the upper limits on \( B_q^+ \to \ell^+ \nu \) constrain several combinations of \( \lambda \lambda' \) each. They assumed a single non–zero \( \lambda \) and \( \lambda' \) each in the weak basis. In this approach, the same couplings mediate \( B_u^+ \to \ell^+ \nu \) and \( B_c^+ \to \ell^+ \nu \), and therefore the experimental limits on \( B_c^+ \to \ell^+ \nu \) induce limits on the corresponding \( B_c \to \ell \nu \) channels which are not yet experimentally constrained. Maximum values of \( \text{BR}(B_c^+ \to \ell^+ \nu) \approx 1\% \) were obtained. While confirming the other results of [7], in their approach we obtain maximum values of \( \text{BR}(B_c^+ \to \ell^+ \nu) \approx 0.1\% \) only.

We will show that in the minimal extension of this approach, i.e. 3 non–zero \( R \) parity couplings in the weak basis, this correlation can be dramatically reduced, resulting in much larger BRs for \( B_c \to \ell \nu \gg 1\% \).

5 Numerical results

In Table 2 we give our results for the possible decay fractions

\[
\mathcal{F} \equiv \Gamma(B_c^+ \to \ell^+_i \nu_n)/\Gamma_{\text{tot}}^{SM}.
\]

for the decays \( B_c^+ \to \ell^+_i \nu_n \). A value of \( \mathcal{F} = 100\% \) means that the BRs are 50\% for the combined SM channels and 50\% for the \( R \) parity violating contribution to the annihilation decay in question; for small \( \mathcal{F} \), \( \mathcal{F} \) corresponds to a branching ratio. We consider only the \( R \)-parity violating contribution. The SM and charged Higgs \( R \)-parity conserving MSSM contributions
Table 2: Couplings and possible $F \equiv \Gamma(B_c^+ \to \ell_i^+ \nu_n)/\Gamma_{\text{tot}}^{SM}$. $F_{\text{opt}}$: respecting single coupling bounds but ignoring the bounds from $B_d \to \ell^+\ell^-$, $F_{\text{con}}$: respecting the bounds from $B_d \to \ell^+\ell^-$. PDG-values and $m_{\ell_i} = 100\,\text{GeV}$ have been used for the numerical parameters.

| Channel | Couplings used | $\lambda_{i1l}$ | $\lambda_{i13}$ | $\lambda_{i23}$ | $F_{\text{opt}}(\%)$ | $F_{\text{con}}(\%)$ |
|---------|----------------|----------------|----------------|----------------|------------------|------------------|
| $B_c^+ \to e_i^+ \nu_n$ | $\lambda_{121} \lambda_{i13}$ | 0.05 | 0.02 | -0.088 | 13.77 | 0.004 |
| | $\lambda_{131} \lambda_{i13}$ | 0.06 | 0.02 | -0.088 | 19.82 | 3.305 |
| | $\lambda_{221} \lambda_{i23}$ | 0.05 | -0.041 | 0.18 | 57.53 | 0.003 |
| | $\lambda_{231} \lambda_{i23}$ | 0.06 | -0.041 | 0.18 | 82.85 | 3.305 |
| | $\lambda_{311} \lambda_{i33}$ | 0.06 | -0.045 | 0.20 | 102.2 | 0.003 |
| | $\lambda_{321} \lambda_{i33}$ | 0.06 | -0.045 | 0.20 | 102.2 | 0.004 |
| $B_c^+ \to \mu_i^+ \nu_n$ | $\lambda_{122} \lambda_{i13}$ | 0.05 | 0.02 | -0.088 | 13.76 | 0.001 |
| | $\lambda_{132} \lambda_{i13}$ | 0.06 | 0.02 | -0.088 | 19.81 | 4.953 |
| | $\lambda_{222} \lambda_{i23}$ | 0.05 | -0.041 | 0.18 | 57.50 | 0.004 |
| | $\lambda_{232} \lambda_{i23}$ | 0.06 | -0.041 | 0.18 | 82.80 | 4.953 |
| | $\lambda_{312} \lambda_{i33}$ | 0.06 | -0.045 | 0.20 | 102.2 | 0.004 |
| | $\lambda_{322} \lambda_{i33}$ | 0.06 | -0.045 | 0.20 | 102.2 | 0.001 |
| $B_c^+ \to \tau_i^+ \nu_n$ | $\lambda_{123} \lambda_{i13}$ | 0.05 | 0.02 | -0.088 | 11.65 | 4.193 |
| | $\lambda_{133} \lambda_{i13}$ | 0.004 | 0.02 | -0.088 | 0.075 | 0.075 |
| | $\lambda_{213} \lambda_{i23}$ | 0.05 | -0.041 | 0.18 | 48.68 | 2.796 |
| | $\lambda_{233} \lambda_{i23}$ | 0.06 | -0.041 | 0.18 | 70.09 | 70.09 |
| | $\lambda_{313} \lambda_{i33}$ | 0.004 | -0.045 | 0.20 | 0.385 | 0.385 |
| | $\lambda_{323} \lambda_{i33}$ | 0.06 | -0.045 | 0.20 | 86.54 | 4.193 |

(which can be up to 20 times as large as the SM contribution) is negligible except for $B_c^+ \to \tau_i^+ \nu_n$, which can reach up to 40% for maximum allowed $R$ (eq. 3).

The second to fourth columns give the values for the $R$–parity violating couplings used in obtaining the optimistic $F_{\text{opt}}$. While respecting single coupling bounds and the limits on $B_u^+ \to \ell_i^+ \nu_n$, some of the combinations $\lambda_{i1l} \lambda'_{i13}$ used would violate the bounds given by the experimental limits for $B_d \to \ell^+\ell^-$. This could easily be avoided by allowing a fourth $R$–parity violating coupling to become non–zero which can induce a cancellation analogous to the $B_u^+ \to \ell_i^+ \nu_n$ case. We show the resulting $F_{\text{opt}}$ in the fifth column of Table 2.

Even when not demanding an additional cancellation and instead directly respecting the bounds from $B_d \to \ell^+\ell^-$, some of the combinations can give quite respectable $F$s (in one case up to 70%) for $B_c^+ \to \ell_i^+ \nu_n$. These conservative $F_{\text{con}}$ are shown in the rightmost column. The larger surviving $F$s of $O(5\%)$ involve products of $R$ parity violating couplings that are constrained by decays involving one $\tau$ and one first or second generation lepton. Since the experimental limits on these channels are typically more than two orders of magnitude weaker than the ones involving only first and second generation leptons, they imply less restrictive bounds.

The largest $F_{\text{con}}$ in the rightmost column of Table 2 ($\lambda_{233} \lambda'_{i23}$ inducing an $F_{\text{con}}$ for $B_c^+ \to \tau_i^+ \nu_n$ of 70%) is not constrained by purely leptonic decays of the $B_d$. The potential bound for
\[ \lambda_{233}' \lambda_{213}' \] would come from the decay \( B_d \to \tau^+\tau^- \). For this decay no experimental limits exist as yet, but it is estimated \([28, 29]\) that a limit of \( 1.5 \times 10^{-2} \) could be extracted from LEP missing energy measurements. This limit would impose a bound of \( \lambda_{233}' \lambda_{213}' < 3.0 \times 10^{-3} \), which is smaller than the product of the single bounds \( (5.4 \times 10^{-3}) \), but still does not further restrict our results involving this combination.

In finding \( F_{\text{con}} \) we have used the results of \([20]\), updating the bounds according to the new experimental limits for \( B_d \to \ell^+\ell^- \) \([17, 4]\). The updated bounds are

- \( 1.46 \times 10^{-5} \) instead of \( 4.6 \times 10^{-5} \) for the combinations inducing \( B_d \to e^+e^- \)
  (i.e. \( \lambda_{121}\lambda_{121}', \lambda_{131}\lambda_{131}', \lambda_{131}\lambda_{131}' \))
- \( 1.79 \times 10^{-5} \) instead of \( 4.6 \times 10^{-5} \) for the combinations inducing \( B_d \to e^+\mu^- \)
  (i.e. \( \lambda_{121}\lambda_{121}', \lambda_{121}\lambda_{121}', \lambda_{121}\lambda_{121}' \))
- \( 9.76 \times 10^{-6} \) instead of \( 2.4 \times 10^{-5} \) for the combinations inducing \( B_d \to \mu^+\mu^- \)
  (i.e. \( \lambda_{122}\lambda_{122}', \lambda_{132}\lambda_{132}', \lambda_{232}\lambda_{232}' \))

The other bounds are unchanged, because there are no new limits for channels involving \( \tau \) leptons or purely leptonic \( B_s \) decays.

A more conservative approach that takes into account experimental uncertainties in the input parameters is adopted in \([32]\). While using the same CLEO limits on \( B_d \to \ell^+\ell^- \), they obtain weaker bounds on the products of the \( R \) parity violating couplings than \([4, 30]\). If we were to follow the approach of \([32]\), the above bounds would weaken and the allowed \( F \)s would become even larger.

For completeness we also update the bounds on combinations of a single non–zero \( \lambda \) and \( \lambda' \) each as derived in \([6]\). The new \textit{Belle} limits \([7]\) on \( B_{\tau} \to e/\mu^- \nu \) change the bounds to

- \( 4.10 \times 10^{-5} \) instead of \( 7.3 \times 10^{-5} \) for the combinations \( \lambda_{131}\lambda_{131}', \lambda_{231}\lambda_{231}' \)
- \( 1.81 \times 10^{-4} \) instead of \( 3.2 \times 10^{-4} \) for the combinations \( \lambda_{131}\lambda_{131}', \lambda_{231}\lambda_{231}', \lambda_{231}\lambda_{231}' \)
- \( 1.14 \times 10^{-2} \) instead of \( 2.0 \times 10^{-2} \) for the combination \( \lambda_{231}\lambda_{231}' \)
- \( 4.82 \times 10^{-5} \) instead of \( 8.7 \times 10^{-5} \) for the combinations \( \lambda_{132}\lambda_{132}', \lambda_{232}\lambda_{232}' \)
- \( 2.12 \times 10^{-4} \) instead of \( 3.8 \times 10^{-4} \) for the combinations \( \lambda_{132}\lambda_{132}', \lambda_{232}\lambda_{232}' \)

Again, the bounds on combinations that induce decays involving \( \tau \) leptons remain unchanged.

We would like to emphasise that the cancellation we employ to respect the experimental bounds on \( B_u^+ \to \ell^+_\ell^- \nu_n \) is not a very finely tuned one, e.g. let us look at the second line of Table \([2]\). For the values quoted, \( \lambda_{131} = 0.06, \lambda_{122}' = 0.02 \) and \( \lambda_{\ell^+\nu} = -0.088 \), the \( \text{BR}(B_u^+ \to e^+_\nu) \) induced by this combination completely vanishes. In general, there are areas in the \( \lambda_{113} - \lambda_{123} \)

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1 BABAR anticipates a sensitivity at the level of \( 10^{-3} \) with a full dataset of \( 300 fb^{-1} \).

2 While finalising this work, a paper appeared \([31]\) that also attempts to update some of these bounds. The authors of that paper, however, do not use the latest \textit{B} factory limits \([17]\) which are considerably stricter than the ones that they employ.
Figure 2: Cancellation in BR($B_u^+ \to \ell_i^+ \nu_\ell$). Left: BR against the $R$ parity violating couplings $\lambda'_{113}$ and $\lambda'_{123}$. Right: BR against $\lambda'_{123}$ ($\lambda'_{113} = 8.17 \times 10^{-3}$), indicating the experimental limit for BR($B_u^+ \to \ell_i^+ \nu_\ell$) and the allowed range for $\lambda'_{123}$. ($\lambda_{131} = 0.06$)

plane where the individual contributions from these couplings interfere destructively. We show this behaviour in the left part of Figure 2.

To respect the most recent Belle bound, we have to reduce BR($B_u^+ \to e_i^+ \nu_e$) to less than $4.7 \times 10^{-6}$. To achieve this, $\lambda'_{123}$ could vary in the interval $[-0.091, -0.085]$. By defining the amount of fine tuning required for a certain cancellation as the width of the allowed interval over the distance of its middle from the origin, this corresponds to a 7\% fine tuning. For the $F$s in the rightmost column (which employ smaller $R$–parity violating couplings to respect the $B_d \to \ell^+\ell^-$ bounds), the fine tuning is even less severe. For our example of the second line, $\lambda'_{123}$ can then vary in the interval $[-0.039, -0.033]$ which corresponds to a 17\% fine tuning. The region in parameter space for which the desired cancellation can occur, is therefore rather large. We illustrate this in the right hand part of Figure 2. In general we find that the channels with $F_{\text{con}} \approx 5\%$ in Table 2 require a fine tuning of $\approx 15\%$; for the $F(B_c^+ \to \tau^+ \nu_\tau) = 70\%$ we require a fine tuning of 4\%.

6 Conclusions

Purely leptonic decays of $B_d$, $B_u$, $B_s$ and $B_c$ mesons are helicity (and in the neutral case also loop) suppressed in the SM with BRs typically smaller than $10^{-10}$ for first generation leptons. Only BR($B_c^+ \to \tau^+ \nu_\tau$) is expected to have a BR of $O(3\%)$. Combinations of two $R$ parity violating couplings can mediate these decays without helicity suppression, and the experimental limits on $B_{d/s} \to \ell^+\ell^-$ and $B_u^+ \to \ell_i^+ \nu_\ell$ induce bounds on products of $R$ parity violating couplings that are much stricter than the products of the individual bounds. Since a single pair of non–zero couplings in the weak basis can induce both $B_u^+ \to \ell_i^+ \nu_\ell$ and $B_c^+ \to \ell_i^+ \nu_\ell$, these decays are correlated, and the rather strict experimental bounds on $B_u^+ \to \ell_i^+ \nu_\ell$ induce maximum BRs for the experimentally unconstrained decay channels $B_c^+ \to \ell_i^+ \nu_\ell$ of $O(0.1\%)$ for the first two generation leptons.

In this paper we have shown that in the minimal extension of this two parameter approach, i.e. allowing three $R$ parity violating couplings to be non–zero, the correlation between the
decays $B^+_u \rightarrow \ell^+_l \nu_\ell$ and $B^+_c \rightarrow \ell^+_l \nu_\ell$ is relaxed and the bounds on the former decay do not restrict the latter anymore. When directly respecting the bounds on products of two couplings from $B_{d/s} \rightarrow \ell^+ \ell^-$ decays ("conservative approach"), we obtain BRs of $\mathcal{O}(5\%)$ for channels involving first and second generation leptons. Assuming an analogous cancellation in the neutral $B$ meson leptonic decays by employing an additional non–zero $R$ parity violating coupling ("optimistic approach"), the partial width of these channels can even be of the order of the total SM–width of the $B_c$ meson. For decays involving $\tau$ leptons, the bounds from $B_{d/s} \rightarrow \ell^+ \ell^-$ are so weak that even when respecting these bounds, $\mathcal{O}(1)$ BRs are possible.

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