Two-mode squeezed state as Schrödinger-cat-like state

E. Oudot, P. Sekatski, F. Fröwis, N. Gisin, and N. Sangouard

1Department of Physics, University of Basel, CH-4056 Basel, Switzerland
2Group of Applied Physics, University of Geneva, CH-1211 Geneva 4, Switzerland
3Institut für Theoretische Physik, Universität Innsbruck, Technikerstr. 25, A-6020 Innsbruck, Austria
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In recent years, there has been an increased interest in the generation of superposition of coherent states with opposite phases, the so-called photonic Schrödinger-cat states. These experiments are very challenging and so far, cats involving small photon numbers only have been implemented. Here, we propose to consider the two-mode squeezed state as an example of a Schrödinger-cat-like state. In particular, we are interested in various criteria aiming to identify quantum states that are macroscopic superpositions in a more general sense. We show how these criteria can be extended to continuous variable entangled states. We apply them to two-mode squeezed states and argue that they belong to a class of general Schrödinger-cat states. We provide experimental guidelines to verify their macroscopic quantum nature and to measure their size. Our results not only promote two-mode squeezed states for exploring quantum effects at the macroscopic level but also provide direct measures to evaluate their usefulness for quantum metrology.

Introduction — The question of what is a macroscopic quantum state has received quite a lot of attention over the last decade [1]. The motivation is not to address a new question – not at all, as it dates back from the early days of quantum theory [2] – but rather comes from the experimental progress, now allowing one to harness large systems while highlighting their quantum nature. Quantum optics experiments reporting on squeezing operations provide a nice example. They are obtained from a $\chi^2$-nonlinearity and result in largely entangled states. The entanglement can further be detected with homodyne detections, by means of the Duan – Simon criterion [6, 7]. This has been at the heart of many experiments and the more recent progress allows one to demonstrate entanglement in squeezed states with a huge number of photons – so huge that they can be detected with classical power-meters [8–10]. This naturally raises the question of whether squeezed states have macroscopic quantum features – a question of deep relevance because so far the squeezed states have been combined with conditional detections [11–17] for exploring quantum effects in many photon states.

In the literature, there exist different criteria for quantifying the macroscopic quantumness [1, 15, 27]. Typically, this includes a definition that assigns to a quantum state a number, which is here called effective size (or simply size). Surprisingly, none of them unambiguously applies to two-mode squeezed states. Those criteria can be grouped into two categories. The first one addresses the question of whether a two component superposition $|\phi_0\rangle + |\phi_1\rangle$ is macroscopic, i.e., whether $|\phi_0\rangle$ and $|\phi_1\rangle$ are macroscopically distinct. For example, the proposal of Ref. [22] states that two spin states are macroscopically distinct if they can be distinguished from a small number of their spins – as a dead cat and an alive cat can be distinguished from a small number of their cells. We can also refer to the proposals of Ref. [24, 25] defining two states as being macroscopically distinct if they can be distinguished with a coarse-grained measurement – as a dead cat and an alive cat can be distinguished with a detector having a very limited resolution. The second category aims to identify quantum states that are able to show some kind of "macroscopic quantum effect". This term characterizes experimental evidence that can not be explained by an accumulated quantum effect originated at the microscopic level of the system. For pure states, a large variance with respect to given observables and Hamiltonians is a sufficient signature for quantum fluctuations that are persistent on a macroscopic level. For mixed states, one typically uses a convex function that reduces to the variance for pure states. For example, the proposal of Ref. [1] shows how the notion of macroscopicity can be linked to the so-called quantum Fisher information [28]. Focusing on photonic states, both groups have strong limitations. The first ones only apply to states of the form $|\phi_0\rangle + |\phi_1\rangle$ and cannot be used directly to measure the size of continuous variable (cv) states. The second category does not focus on a specific state structure but an unambiguously extension to multimode states is missing [24].

In this letter, we show how well representative measures of each groups can be extended to cv entangled states. These extensions allow one to characterize the macroscopicness of two-mode squeezed states (with $N$ mean photons) and to conclude that they belong to a class of general Schrödinger-cat states. We prove that their effective size is basically the same as superpositions of coherent states with opposite phases $|\alpha\rangle - | - \alpha\rangle$ with $|\alpha|^2 = N$ with the great advantage that they are much easier to create. We show how existing experiments need to be modified to experimentally verify the macroscopic quantum nature of squeezed states. The tools that we propose allow one to evaluate precisely the size of states obtained experimentally as well as their.
usefulness for parameter estimations beyond the classical limit. Aside from their fundamental interest, our results thus have important applications for quantum metrology.

Two-mode squeezed states — The state we are interested in is obtained from a parametric process in which photons from a pump laser decay spontaneously into photon pairs — one in mode 1, its twin in mode 2 — while preserving energy and momentum. The corresponding propagator $\hat{S}(g) = e^{\delta a_i^\dagger a_j - a_j a_i}$ with squeezing parameter $g$, applies straightforwardly on the vacuum if written in the normal order. This results in

$$\psi_{\text{tms}} = (1 - \tanh^2 g)^{1/2} e^{\tanh g a_i^\dagger a_j^\dagger |00\rangle}.$$  \hspace{1cm} (1)

The mean photon number in both mode is $N = 2 \text{tr}(a_i^\dagger a_j |\psi_{\text{tms}}\rangle) = 2 \sinh^2 g$. Furthermore, the variance of the observable $X_1^g - X_2^g$ where $X_i^g = \frac{x_i}{\sqrt{2}} (a_i e^{i\theta} + a_i^\dagger e^{-i\theta})$ is given by

$$V_{\psi_{\text{tms}}} (X_1^g - X_2^g) = \cosh 2g - \sinh 2g \cos(\varphi + \phi).$$  \hspace{1cm} (2)

This indicates that the quadratures $X_1^g - X_2^g$ are correlated whereas $X_1^{\pi/2} - X_2^{\pi/2}$ are anti-correlated. The quantum nature of these correlations can be revealed through the Duan – Simon criterion \cite{23, 26} which states that for any bipartite separable states and any real parameter $a$

$$V_{\text{sep}} \left| a |X_1^\varphi + \frac{1}{a} X_2^\varphi\rangle + V_{\text{sep}} \left| a |X_1^\varphi' - \frac{1}{a} X_2^\varphi'\rangle \right.$$  $$> a^2 (\langle X_1^\varphi, X_2^\varphi \rangle) + \frac{1}{a^2} (\langle X_2^{\varphi}, X_2^{\varphi'} \rangle)$$  $$\geq 2 \text{ for } \phi - \phi' = \Phi - \Phi' = \frac{\pi}{2}$$  \hspace{1cm} (3)

while for a two-mode squeezed state

$$V_{\psi_{\text{tms}}} (X_1^0 + X_2^0) + V_{\psi_{\text{tms}}} (X_1^{\pi/2} - X_2^{\pi/2}) = 2 e^{-2g}.$$

The questions that are at the core of this letter are: Is this state macroscopically quantum? Is the effective size comparable to other photonic states? How can one evaluate it in practice?

Macroscopic distinctness for cv states — While several definitions have been proposed to identify states that are macroscopically distinct \cite{20, 23, 26}, we here focus on the proposal of Ref. \cite{20} based on coarse-grained measurements. This choice is arbitrary to some extent. Note, however, that the extension that we propose below easily applies to the measure of Ref. \cite{23}. The extension of measures of Refs. \cite{21, 23} to two-mode squeezed states is less obvious as they primarily address spin systems but the link between measures for spins and photons presented in \cite{21} might be the way to proceed.

The basic principle of the measure of macroscopicity based on coarse-grained measurement is simple. It can be seen as a game where Alice chooses a state in the set $\{ |\varphi_0\rangle, |\varphi_1\rangle \}$ with equal a priori probabilities and sends it to Bob. Bob has to guess which one has been sent using a coarse-grained measurement only. It can be any measurement provided that its resolution is limited. The quantum superposition state $|\varphi_0\rangle + |\varphi_1\rangle$ is qualified macroscopic if Bob wins the game with a detector having no microscopic resolution. Concretely, if one focuses on a noisy photon counting detector for example, the size of $|\varphi_0\rangle + |\varphi_1\rangle$ is characterized by the noise that one can tolerate to distinguish $|\varphi_0\rangle$ and $|\varphi_1\rangle$.

To extend this measure to cv states, we can mimic its original idea by introducing a 50/50 binning of measurement outcomes. For a two-mode squeezed state in particular, Alice measures her mode with a given quadrature and bins the result with respect to its sign. As Alice’s measurement is assumed to be very accurate, this binning corresponds to equiprobable projections onto two orthogonal subspaces of the measured state. Bob has to guess whether she got a positive or negative outcome by measuring his mode with a noisy measurement. The distinguishability of components that Bob receives is again given by the noise that can be tolerated to win the game. Note that the measurement of correlated quadratures maximizes the probability to correctly guess Alice’s outcome. Concretely, the probability that Alice gets the result $x_1$ and Bob $x_2$ knowing that they measure the quadratures $X_1^g$ and $X_2^g$ is given by $|p(x_1, x_2, \sigma)|^2 = \text{tr}(|\psi_{\text{tms}}\rangle \langle \psi_{\text{tms}}| \delta (X_1^g - x_1) g_\sigma (X_2^g - x_2))$ where $g_\sigma$ stands for the noise of Bob’s measurement device. We assume that $g_\sigma$ is a Gaussian with spread $\sigma$ and zero mean. Hence, the probability that Bob correctly guesses the sign of Alice’s result is given by $P_{\sigma}^{\text{guess}} = \int_{-\infty}^{\infty} |p(x_1, x_2, \sigma)|^2 dx_1 dx_2 + \int_{-\infty}^{\infty} |p(x_1, x_2, \sigma)|^2 dx_1 dx_2$. We find

$$P_{\sigma}^{\text{guess}} = \frac{1}{2} + \frac{1}{\pi} \arctan \left( \frac{\sinh g}{\sqrt{1 + 2\sigma^2 \cosh g}} \right).$$  \hspace{1cm} (4)

We can access the maximum noise $\sigma_{\text{max}}$ that Bob can tolerate to win the game with a fixed probability $P_{\sigma}^{\text{guess}}$ by inverting the previous formula:

$$\sigma_{\text{max}} = \sqrt{-1 + N \left( 1 + N \cotan^2 \left( \frac{1}{2} - P_{\sigma}^{\text{guess}} \right) \right) / 2N},$$  \hspace{1cm} (5)

For comparison, the noise that can be tolerated to win a similar game with the optical Schrödinger-cat state ($|\uparrow\rangle |\alpha\rangle - |\downarrow\rangle |\bar{\alpha}\rangle$) is given by

$$\sigma_{\text{max}} = 1 / (\text{erf}^{-1} \left( P_{\sigma}^{\text{guess}} \right))^2 - \frac{1}{2}.$$  \hspace{1cm} (6)

In both cases, the noise scales like the square root of the photon number. Two-mode squeezed states and
Schrödinger cat states thus belong to the same class of macroscopic states.

Let us now focus on practical considerations. The observation that Alice’s and Bob’s $x$-quadratures of the two-mode squeezed state are “macroscopically” correlated (correlated at a large scale, larger then the detector’s resolution) is at the heart of our generalization of the coarse-grained measure. These correlations can be revealed by measuring the joint probability distribution $|p(x_1, x_2)|^2$ with accurate quadrature measurements. (For simplicity, we introduce $p(x_1, x_2) = p(x_1, x_2, 0)$ which stands for the probability amplitudes without noise.) Although this approach is sufficient to measure the size of a given state in theory, one also has to ensure that those correlations are truly quantum in practice. In mathematical terms, we can always choose the state that is shared by Alice and Bob in the $x$-basis

$$
\rho = \int p(x_1, x_2)\tilde{p}(\tilde{x}_1, \tilde{x}_2)f_1(x_1, \tilde{x}_1)f_2(x_2, \tilde{x}_2),
$$

with $f_1(x, \tilde{x}) = f_2(x, \tilde{x}) = 1 \forall x$. If the shared state is pure, we have $f_1(x_1, \tilde{x}_1) = f_2(x_2, \tilde{x}_2) = 1 \forall x_1, \tilde{x}_1, x_2, \tilde{x}_2$ and the correlations revealed through the probability distribution $|p(x_1, x_2)|^2$ are fully quantum. The violation of the Duan-Simon criterion is then sufficient to attest the quantum nature of the state for the size which is evaluated through $\sigma_{\text{max}}$. But how to certify in practice that the function $f_1(x_1, \tilde{x}_1)$ and $f_2(x_2, \tilde{x}_2)$ are close to one, at least in a certain range?

To do so, we consider the effect of imperfect coherences (decoherence) $f_2(x, \tilde{x}) \neq 1$ on the observed violation of the Duan-Simon witness. Note first that the variance $V(\hat{X}_1^2 + \hat{X}_2^2)$ can be directly obtained from $|p(x_1, x_2)|^2$. For the second term required in Eq. (3), we can show that the variance in presence of decoherence

$$
V(\hat{X}_1^2 + \hat{X}_2^2) = V(\hat{X}_1^2 + \hat{X}_2^2)|_{\text{ideal}} + 2(f_1'f_2'' - f_1''f_2')
$$
equals the ideal-case variance $V(\hat{X}_1^2 + \hat{X}_2^2)|_{\text{ideal}}$ plus a factor containing the first and second derivatives of $f_1$, $f_1' = (\partial_1, f_1(x_1, \tilde{x}_1))|_{x_1 = x_1}$, $f_1'' = \int dx_1 dx_2 p(x_1, x_2)^2(\partial_2, f_1(x_1, \tilde{x}_1))|_{x_1 = x_1}$, etc. Since $V(\hat{X}_1^2 + \hat{X}_2^2)|_{\text{ideal}}$ is positive, we get the following upper-bound on the observed variance

$$
2(f_1'f_2'' - f_1''f_2') \leq V(\hat{X}_1^2 + \hat{X}_2^2).
$$

Note that without further assumptions, we cannot bound the range $\delta$ for which $f(x, x + \delta)$ stays close to one. In words, even if the state of Alice and Bob largely violate the Duan-Simon witness, the state can be arbitrarily close to a separable one and $p(x_1, x_2)$ essentially correspond to classical correlations [31]. However, under the assumption of a Gaussian decay of coherence $f(x, \tilde{x}) = e^{-(x-\tilde{x})^2/(2\gamma)}$, Eq. (7) becomes

$$
\frac{1}{\gamma^2} + \frac{1}{\gamma^2} \leq V(\hat{X}_1^2 + \hat{X}_2^2).
$$

This implies $\min(\gamma_1, \gamma_2) \geq 1/\sqrt{V(\hat{X}_1^2 + \hat{X}_2^2)}$, i.e., if one observes the variance $V_{\text{obs}}(\hat{X}_1^2 + \hat{X}_2^2)$ of the total momentum, we can certify that the correlations $|p(x_1, x_2)|^2$ are quantum at least in the range

$$
x_C = \frac{1}{\sqrt{V_{\text{obs}}(\hat{X}_1^2 + \hat{X}_2^2)}}.
$$

Accordingly if the coherence range $x_C$ is lower than the correlation range as witnessed by $\sigma_{\text{max}}$, one can only claim that the state exhibits quantum correlations within the range $x_C$, which is then the true size of the state. Revealing the size of large quantum states thus requires to reveal narrow variances which is harder and harder as the size increases, cf. below.

**General measures for multimode cv states** — Besides measures for macroscopic distinguishability, there has been recent proposals that aim to go beyond the basic structure $|\psi_0\rangle + |\phi_0\rangle$ [19, 21, 23]. While the measures of Refs. [1, 14, 21] were originally defined for spin systems, the definition of Ref. [24] is directly suitable for cv photonic states. For pure states, these three proposals are comparable since a state $|\psi\rangle$ is called macroscopically quantum if it shows a large variance $V$ with respect to a restricted class of operators. In the spin case, the proposals [1, 19, 21] focus on sums of local operators (henceforth simply called “local operators”), whereas Lee and Jeong [24] define their measure for pure states proportional to $V(X^0) + V(X^{\pi/2})$. In Ref. [1], it was argued that local operators in the spin case play to some extent the same role as quadrature operators in mono-mode photonic systems.

The common feature of the proposals for mixed states is that the measures $1, 13, 21, 24$ are convex in the state, which is an important and natural feature for the present purpose. There are no clear arguments in favor of one of the proposal. Nevertheless, we focus here on the quantum Fisher information (QFI) [29], denoted as $\mathcal{F}_p(X)$ for the state $\rho$ and the operator $X$. Importantly, the QFI is the convex roof of the variance $\mathcal{V}_p$ (up to a factor four), that is, it is the largest convex function that reduces to the variance for pure states. For experiments, it is interesting to note that there exist lower bounds on the QFI based on measurable quantities [31].

The extension to photonic states with $n > 1$ modes is not straightforward. Indeed, a multimode version for the measure of Lee and Jeong was proposed [24]. However, it is additive and hence a bunch of “kitten states” $|\psi^{\alpha}_n\rangle \propto (|\alpha\rangle + |\alpha\rangle)^\otimes n$ (with potentially small $\alpha$ but large $n$) is as macroscopically quantum as a “big” single cat state $|\sqrt{n}\alpha\rangle \propto |\sqrt{n}\alpha\rangle + |\sqrt{n}\alpha\rangle$). Here, we propose instead to use a similar account that has been successfully applied in the spin case [1, 19, 21]. The idea is that
the effective size of a product state is the average value of its components, while entangled states should be able to profit from quantum correlations between the modes. Both requirements are achieved by defining the effective size for $\rho$ as

$$N_{\text{eff}}(\rho) = \frac{1}{4\pi} \max_{\theta} \mathcal{F}_\theta(X_{\theta}),$$

where $X_{\theta} = \sum_{n=1}^{N} X_{\theta}^n$. In words, one maximizes the QFI (or the variance for pure states) with respect to sums of local quadrature operators parametrized by $\theta = (\theta_1, \ldots, \theta_N)$. The examples from above then lead to $N_{\text{eff}}(|\psi_\alpha(\varphi)\rangle) = |\alpha|^2 \tanh |\alpha|^2$ and $N_{\text{eff}}(|\psi_{\pi/2}(\varphi)\rangle) = n|\alpha|^2 \tan n|\alpha|^2$ (cp. to \cite{34}).

We now come to the evaluation of the effective size for the two-mode squeezed state. It is simple to see that the variance is largest for the quadratures that are maximally correlated. For the state (11), these are the operators $\hat{X}_1^0 + \hat{X}_2^0$ and $\hat{X}_1^{\pi/2} + \hat{X}_2^{\pi/2}$. The effective size for each of these choices reads $N_{\text{eff}} = \frac{1}{4\pi} V(\langle \hat{X}_1^0 + \hat{X}_2^0 \rangle^2) = \frac{1}{4\pi} 2g^2 \approx N$, which is approximately the same value as for the cat state $|\alpha \rangle + | -\alpha \rangle$ with $|\alpha|^2 = N$.

In principle, the effective size of a pure state could be determined by witnessing a large variance for sums of quadrature operators. However, for mixed states, a large variance is not sufficient. Indeed, one has to verify a large value of a convex function like the QFI. Since this quantity is typically only accessibly through a full state tomography, one has to find other means to estimate it. Recently, a general lower bound on the QFI has been found \cite{34}. It was shown that for any quantum state $\rho$ and any pair of operators $A, B$, it holds that $V_\rho(A) V_\rho(B) \geq \langle i[A, B] \rangle_\rho^2$, which is a tighter version of the Heisenberg uncertainty relation. Here, we use this inequality to bound the QFI from below. For $B = \hat{X}_1^0 + \hat{X}_2^0$, we set $A = \hat{X}_1^{\pi/2} + \hat{X}_2^{\pi/2}$ and find $i[A, B] = 2$. Hence one has

$$N_{\text{eff}}(\psi_{\text{max}}) \geq 1 \frac{1}{V_{\psi_{\text{max}}}(\langle \hat{X}_1^{\pi/2} + \hat{X}_2^{\pi/2} \rangle^2)}.$$  

For the two-mode squeezed state, the anti-correlations between $\hat{X}_1^{\pi/2}$ and $\hat{X}_2^{\pi/2}$ lead to a reduced variance and therefore to a potentially large value of $N_{\text{eff}}$.

On the difficulty to certify the quantum nature of two-mode squeezed states — The common feature of measures for macroscopicity presented before is the requirement to reveal narrow variances, especially when dealing with large size states. How hard is it in practice? To answer this question, we consider the effect of various experimental imperfections on the observed variance $V_{\psi_{\text{max}}}(\langle \hat{X}_1^{\pi/2} + \hat{X}_2^{\pi/2} \rangle)$.

(i) Consider first a noise along $\hat{X}_0$ which acts on a state $\rho$ as $\rho \rightarrow \int d\lambda h(\lambda)e^{i\lambda \hat{X}_0} \rho e^{-i\lambda \hat{X}_0}$ with characteristic function (noise distribution) $h(\lambda)$ of variance $\Delta^2 h$.

The effect of this noise can be directly absorbed in the statistics of the momentum distribution and leads to the following modification of the variance $V(\langle \hat{X}_1^{\pi/2} + \hat{X}_2^{\pi/2} \rangle) \rightarrow V(\langle \hat{X}_1^{\pi/2} + \hat{X}_2^{\pi/2} \rangle) + \Delta^2 h_1 + \Delta^2 h_2$. Therefore, if the experimental setup suffers from such a noise, we cannot certify state of effective size larger than $N_{\text{eff}} = \frac{1}{1-\eta} \frac{1}{\eta}$.

(ii) Similarly, consider a loss channel with transmission $\eta$. It leads to $V(\langle \hat{X}_1^{\pi/2} + \hat{X}_2^{\pi/2} \rangle) \rightarrow \eta V(\langle \hat{X}_1^{\pi/2} + \hat{X}_2^{\pi/2} \rangle) + (1-\eta)$ and the maximal certifiable size is given by $N_{\text{max}} = \frac{1}{1-\eta} \frac{1}{\eta}$.

(iii) Now consider a phase noise characterized by the variance $\Delta \phi^2 = \int p(\phi) \phi^2 d\phi$. It increases the observed variance according to $V(\langle \hat{X}_1^{\pi/2} + \hat{X}_2^{\pi/2} \rangle) \geq \Delta \phi^2 (\langle \hat{X}_1^0 \rangle + \langle \hat{X}_2^0 \rangle)$. Specifically for the two-mode squeezed state, one has $N_{\text{max}}(\psi_{\text{max}}) = \frac{1}{\Delta \phi^2 (2 \sinh (g) + 1)}$ which decays exponentially with the squeezing parameter (in the limit of large enough $g$).

In each case, we clearly see that it becomes harder and harder to observe narrow variances with two-mode squeezed states as their size increases. This is in agreement with recent results \cite{36, 37} stating that it is difficult to observe the quantum nature of macroscopic states. This naturally raises the question of how the quantumness of two-mode squeezed states with mw powers in each mode has been observed in Refs. \cite{8–10}. We believe that the photons were distributed in many modes, while the quantumness was revealed in a very small number of these modes - likely containing small photon numbers. To confirm our intuition, one would need to measure the autocorrelation function $g^{(2)}(\tau)$ of each mode with a 50/50 beamsplitter and photon counting techniques after appropriate intensity attenuation. If each mode of the two-mode squeezed state is monomode $g^{(2)}(0) = 2$ whereas in the limit of an infinite number of modes $g^{(2)}(0) = 1$ \cite{38}. Redoing similar experiments but guaranteeing monomode emissions would allow one to create Schrödinger-cat-like states, and to prove it from the measures we here propose. Besides the fundamental interest, this would also find applications in quantum metrology. Two-mode squeezed states are indeed known to have the potential to provide an extreme precision for phase estimation \cite{39, 40}. As the formula (11) provides a lower bound on the QFI, it can be used directly to quantitatively estimate the metrologic usefulness of states realized experimentally.

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[1] F. Fröwis and W. Dür, New J. Phys. 14, 093039 (2012).
[2] H. Jeong, M. Kang, and H. Kwon, Optics Communications, in press (2014).
[3] T. Farrow and V. Vedral, Optics Communications, in press (2014).
[4] F. Fröwis, N. Sangouard, and N. Gisin, Optics Communications, in press (2014).
[5] E. Schrödinger, Naturwissenschaften 23, 807 (1935).
[6] L.-M. Duan, G. Giedke, J. I. Cirac, and P. Zoller, Phys. Rev. Lett. 84, 2722 (2000).
[7] R. Simon, Phys. Rev. Lett. 84, 2726 (2000).
[8] Y. Zhang, H. Wang, X. Li, J. Jing, C. Xie, and K. Peng, Phys. Rev. A 62, 023813 (2000).
[9] A. Ourjoumtsev, L. S. Cruz, K. N. Cassemiro, M. Martinelli, and P. Nussenzveig, Phys. Rev. Lett. 95, 243603 (2005).
[10] G. Keller, V. D’Auria, N. Treps, T. Coudreau, J. Laurat, and C. Fabre, Opt. Express 16, 9351 (2008).
[11] A. Ourjoumtsev, R. Tualle-Brouri, J. Laurat, and P. Grangier, Science 312, 83 (2006).
[12] J. S. Neergaard-Nielsen, B. M. Nielsen, C. Hettich, K. Mølmer, and E. S. Polzik, Phys. Rev. Lett. 97, 083604 (2006).
[13] K. Wakui, H. Takahashi, A. Furusawa, and M. Sasaki, Opt. Express 15, 3568 (2007).
[14] N. Bruno, A. Martin, P. Sekatski, N. Sangouard, R. T. Thew, and N. Gisin, Nat Phys 9, 545 (2013).
[15] A. I. Lvovsky, R. Ghobadi, A. Chandra, A. S. Prasad, and C. Simon, Nat Phys 9, 541 (2013).
[16] O. Morin, K. Huang, J. Liu, H. Le Jeannic, C. Fabre, and J. Laurat, Nat Photon 8, 570 (2014).
[17] H. Jeong, A. Zavatta, M. Kang, S.-W. Lee, L. S. Costanzo, S. Grandi, T. C. Ralph, and M. Bellini, Nat Photon 8, 564 (2014).
[18] W. Dür, C. Simon, and J. I. Cirac, Phys. Rev. Lett. 89, 210402 (2002).
[19] A. Shimizu and T. Miyadera, Phys. Rev. Lett. 89, 270403 (2002).
[20] G. Björk and P. G. L. Mana, J. Opt. B: Quantum Semiclass. Opt. 6, 429 (2004).
[21] A. Shimizu and T. Morimae, Phys. Rev. Lett. 95, 090401 (2005).
[22] J. I. Korsbakken, K. B. Whaley, J. Dubois, and J. I. Cirac, Phys. Rev. A 75, 042106 (2007).
[23] F. Marquardt, B. Abel, and J. von Delft, Phys. Rev. A 78, 012109 (2008).
[24] C.-W. Lee and H. Jeong, Phys. Rev. Lett. 106, 220401 (2011).
[25] S. Nimmrichter and K. Hornberger, Phys. Rev. Lett. 110, 160403 (2013).
[26] P. Sekatski, N. Sangouard, and N. Gisin, Phys. Rev. A 89, 012116 (2014).
[27] A. Laghaout, J. S. Neergaard-Nielsen, and U. L. Andersen, Optics Communications, in press (2014).
[28] P. Sekatski, N. Sangouard, M. Stobińska, F. Bussières, M. Afzelius, and N. Gisin, Phys. Rev. A 86, 060301 (2012).
[29] S. L. Braunstein and C. M. Caves, Phys. Rev. Lett. 72, 3439 (1994).
[30] Note that the recent work presented in [27] is an extension of the measures of the first categories to non-qubit states but it is unclear how to extend it properly to the multimode case.
[31] To illustrate that, consider a state \( \rho_\epsilon \) with \( f_1(x, \bar{x}) = f_2(x, \bar{x}) = 1 \) for \( |x - \bar{x}| < \epsilon \) and zero otherwise. The peculiarity of this state is that \( \sigma_1 \) satisfies \( \langle \sigma_1^2 \rangle = \langle \sigma_1^4 \rangle = 0 \) in such a way that the violation of the Duan-Simon criterion by \( \rho_\epsilon \) is independent of \( \epsilon \) unless it is strictly equal to zero. Therefore \( \rho_\epsilon \) with \( \epsilon \) approaching zero is an example of a state that can give an arbitrarily high violation of the Duan criteria, while being arbitrarily close to a separable state.
[32] G. Tóth and D. Petz, Phys. Rev. A 87, 032324 (2013).
[33] S. Yu, arXiv:1302.5311 (2013).
[34] F. Fröwis and N. Gisin, arXiv:1409.4440 (2014).
[35] T. J. Volkoff and K. B. Whaley, Phys. Rev. A 89, 012122 (2014).
[36] F. Fröwis, M. v. d. Nest, and W. Dür, New J. Phys. 15, 113011 (2013).
[37] P. Sekatski, N. Gisin, and N. Sangouard, Phys. Rev. Lett. 113, 090403 (2014).
[38] P. Sekatski, N. Sangouard, F. Bussières, C. Clausen, N. Gisin, and H. Zbinden, J. Phys. B: At. Mol. Opt. Phys. 45, 124016 (2012).
[39] B. C. Sanders and G. J. Milburn, Phys. Rev. Lett. 75, 2944 (1995).
[40] P. M. Anisimov, G. M. Raterman, A. Chiruvelli, W. N. Plick, S. D. Huver, H. Lee, and J. P. Dowling, Phys. Rev. Lett. 104, 103602 (2010).