Dynamic response of isolated Aharonov-Bohm rings coupled to an electromagnetic resonator

Bertrand Reulet , Michel Ramin , Hélène Bouchiat and Dominique Mailly

Laboratoire de Physique des Solides, Associé au CNRS, Bât 510, Université Paris-Sud, 91405, Orsay, France.
C.N.R.S Laboratoire de Microstructures et de Microélectronique, 196, Avenue Ravela, 92220, Bagneux, France

We have measured the flux dependence of both real and imaginary conductance of GaAs/GaAlAs isolated mesoscopic rings at 310 MHz. The rings are coupled to a highly sensitive electromagnetic superconducting micro-resonator and lead to a perturbation of the resonance frequency and quality factor. This experiment provides a new tool for the investigation of the conductance of mesoscopic systems without any connection to invasive probes. It can be compared with recent theoretical predictions emphasizing the differences between isolated and connected geometries and the relation between ac conductance and persistent currents. We observe $\Phi_0/2$ periodic oscillations on both components of the magnetoconductance. The oscillations of the imaginary conductance whose sign corresponds to diamagnetism in zero field, are 3 times larger than the Drude conductance $G_0$. The real part of the periodic magnetoconductance is of the order of 0.2$G_0$ and is apparently negative in low field. It is thus notably different from the weak localisation oscillations observed in connected rings, which are much smaller and opposite in sign.

Mesoscopic metallic rings present a spectacular thermodynamic property: they carry a persistent non dissipative current when they are threaded by a magnetic flux $\Phi$. The existence of such a persistent current is a consequence of the coherence of the electronic wave functions along the ring. However unlike a superconductor, when connected to a voltage source, the same rings present a finite ohmic conductance whose value is close to its classical expectation given by the Drude formula, which depends only on the elastic scattering time (quantum interference effects give rise to contributions which are only a small fraction of this main classical contribution in the metallic diffusive regime). It has already been pointed out a number of times, that the existence of a finite ohmic resistance for a phase coherent sample is not paradoxical when one properly takes into account the influence of the measuring leads. These macroscopic wires connected to the sample play indeed the role of incoherent reservoirs of electrons where thermalisation of the electrons and dissipation take place. Such a strong coupling with a reservoir of electrons can be avoided by studying the current response of a mesoscopic ring to a time dependant flux, which induces an electric field along the ring. Since the early work of Büttiker et al. [1,2] subsequently generalized by a number of authors [3,4,12], it has been shown that the conductance measured by this last technique on an isolated ring is indeed fundamentally different from the conductance of the same sample connected to a voltage source. It essentially depends on the inelastic time $\tau_{\text{el}}$ (which describes the coupling of the electrons to the thermal bath). Furthermore it is strongly related to the presence of persistent currents through the ring.

In its ac version the experiment consists in measuring the complex magnetic susceptibility of the rings $\chi(\omega) = \chi'(\omega) + i\chi''(\omega)$ submitted to a small ac flux superimposed on a dc one $\Phi$. In the linear response limit, this susceptibility is related to the complex ac conductance of the rings $G$ by $\chi(\omega) \propto i\omega G(\omega)$. Let us summarize the main theoretical predictions [13,12]. An important energy scale for the dynamics of the system is its thermalisation time $\gamma^{-1} \approx \tau_{\text{el}}$. In the adiabatic limit $\omega < \gamma$, $3\tau_{\text{el}}(G)$ reduces to the derivative of the persistent current, while a relaxation term occurs at higher frequency. In the continuous spectrum limit $\gamma \gg \Delta$ (where $\Delta$ denotes the mean level spacing) and zero frequency, $\Re(G)$ is given by $\Re(G) = G_0 + \delta G(\Phi)$ where $\delta G \approx G_0 \frac{\Delta}{\gamma} \ll G_0$ is the $\Phi_0/2$ periodic Altschuler Aronov and Spivak (AAS) weak localisation correction, positive in weak field [13,14]. Nevertheless the same quantity in the discrete spectrum limit $\gamma \ll \Delta$ (and in the canonical statistical ensemble corresponding to isolated objects) may present oscillations of opposite sign (for $T < \Delta$, $\omega < \gamma$) and amplitude of the order of $G_0$. These oscillations are predicted to reverse sign and become of the order of $\frac{\Delta}{\gamma}G_0$ when the temperature $T$ increases.

Motivated by these findings, we have designed an experiment to measure the complex ac conductance of an array of GaAs/GaAlAs isolated rings. The discrete spectrum is much easier to reach in these samples, where $\Delta$ is of the order of a few tens of mK, than in metallic ones of comparable sizes, corresponding to $\Delta$ in the microkelvin range. The sample is an array of $10^5$ isolated square rings $2 \mu m$ on a side, made using e-beam lithography. The electronic parameters of the rings are obtained from transport measurements done on connected rings and wires made using the same process as the isolated rings. Moreover, because of depletion effects, the real width of the wires etched in the 2D electron gas is substantially smaller than the nominal one, and must be determined by weak localisation measurements [15]. From these measurements we deduced the following parameters:
\( \Delta = 35 \text{mK} \quad E_c = hD/L^2 = 200 \text{mK} \quad M = 17 \quad l_{tr} = 3 \mu \text{m} \quad M_{eff} = 4 \quad L_\phi(T = 50 \text{mK}) = 7 \mu \text{m} \) where \( M \) denotes the number of channels of the rings, \( M_{eff} \) their effective number, \( l_{tr} \) the transport length and \( L_\phi(T) \) the (temperature dependant) phase coherence length of electrons. The electronic motion is then diffusive along the rings and ballistic in the transverse direction. In terms of frequency, the energies are: \( \Delta = 630 \text{MHz} \quad E_c = 4.2 \text{GHz} \)

This determines the interesting range of frequency: from a few hundreds of megahertz \( (\omega < \Delta) \) to a few gigahertz \( (\omega \approx E_c) \). The inelastic parameter \( \gamma \), of course, cannot be a priori deduced from such transport experiments, since it represents a property of the isolated rings. Nevertheless, assuming that \( \gamma \) is of the order of \( \hbar/\tau_\phi \) (where \( \tau_\phi \) is the phase coherence time of electrons measured by weak localisation in wires having the same width than the rings), we expect that it is smaller than \( \Delta \) below 50 mK (such an assumption is in agreement with the results obtained by Sivan et al. \[16\] on the tunnel spectroscopy measurements of quantum dots).

Our aim was to be able to detect the in-phase and out-of-phase response of this array of rings to a small magnetic excitation. We recall that such an experiment which deals with electrically isolated objects is very different from the ac measurement of the complex conductance of connected rings \[17\]. Since the estimated amplitude of the signal was extremely small, we had to design a special experimental setup. We have used a resonant technique in which the rings are magnetically coupled to an electromagnetic multi-mode resonator, whose performances are affected by the perturbations due to the rings.

The resonance frequencies \( f_n \) and quality factors \( Q_n \) are modified by the presence of the rings according to:

\[
\begin{align*}
\delta f_n &= \frac{2\pi N M^2 L f_n^2}{3m} \Im[G(f_n)] \\
\delta Q_n &= \frac{2\pi N M^2 f_n L}{\Im[G(f_n)]}
\end{align*}
\]

where \( N \) is the number of rings and \( M \) the mutual inductance between one ring and the resonator of self-inductance \( L \). Considerations of coupling optimization between the samples and the detector, have lead us to use the meander strip line resonator depicted on fig. 1 on the top of which the array of rings is deposited. In this geometry each ring is close to the resonating line, which ensures a good mutual coupling between them. The line, open at both ends, has each time its length is a multiple value of \( \lambda_0/2 \), where \( \lambda_0 \) is the electromagnetic wavelength.

Typical superconducting Nb resonators produced on sapphire substrates have a fundamental resonant frequency of 380 MHz and a \( Q = 80,000 \) at temperatures below 1K. The sensitivity of our experiment is determined by the precision with which we can detect a small deviation of \( f_n \) and \( Q_n \), actually: \( \delta f_n/f_n \approx 10^{-9} \) and \( \delta Q_n/Q_n^2 \approx 10^{-10} \), for an injected power of 1 nW, which avoids the heating of electrons and corresponds to an ac magnetic flux less than 0.1 \( \Phi_0 \) through the rings (more technical details will be published somewhere else \[18\]).

Ideally all rings should be exposed to the same ac magnetic field and therefore should have a very well controlled position tightly coupled to the resonator. But for the moment this is very difficult to achieve, since for reasons of lithography the line and the rings are sitting on different substrates. However, as long as we are concerned only with the linear response, it is not required that all the rings experience the same ac field (as soon as it is small enough). The problem of the homogeneity of the dc magnetic field is somewhat more serious. Since the characteristic signature of the effects we are looking for, are periodic oscillations with the dc flux through the rings, it is crucial that they see essentially the same dc field. Due to the Meissner effect, the dc field just above the resonator is strongly inhomogeneous spatially. These field inhomogeneities decrease however exponentially with the distance between the rings and line substrates and are reduced to 10% when a 1.5 \( \mu \) thick, mylar film is inserted between the detector and the rings substrate. We have estimated in this geometry the typical mutual inductance \( M \) between one ring and the resonator to be of the order of 1.5 \( 10^{-13} \) H. We have checked this value by measuring the susceptibility of an array of superconducting aluminum rings. The most serious difficulty we had to overcome in order to realize this experiment is the existence of spurious losses coming from the partially etched GaAlAs top layer of the heterostructure. The first attempt was done with very slightly etched samples, we observed a drop of the quality factor of the resonator from 80,000 to 10. By etching the samples more deeply we could decrease these losses.

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{fig1}
\caption{Optical photography of a piece of the resonator coupled to the GaAs/GaAlAs mesoscopic rings. One notes the two folded Nb lines (1 \( \mu \)m thick, 2 \( \mu \)m wide and 20 cm long) on the sapphire substrate.}
\end{figure}
by a factor 100 and we obtained the results depicted below where \( Q = 1650 \) for the fundamental frequency. It was not possible yet to work on the higher harmonics. We hope to reduce further these residual losses in future experiments.

Let us now describe the magnetic field and temperature dependence of the complex susceptibility of the array of GaAs/GaAlAs rings measured at 310MHz. The measurements were done using two different resonators and were reproducible from one resonator to the next. The dc magnetic field was modulated at 3Hz with an amplitude of 1Gauss. The resulting signals are proportional to the derivatives of \( f_1 \) and \( Q_1 \) with respect to the dc magnetic field. In fig. 2 we show the field dependence of \(-\frac{\partial f_1}{\partial H}\) averaged 40 times. One clearly notes in low field the oscillations associated with the rings superimposed on the linear dependence corresponding to the diamagnetism of the niobium. The 5 Gauss periodicity corresponds to a flux of amplitude \( \Phi_0/2 = h/2e \) in the squares. The oscillations are not visible at fields larger than 10 Gauss which is due to the rather small aspect ratio of the rings, \( 1\Phi_0 \) through the area of the wires corresponds indeed to 15 Gauss. One deduces from fig. 2:

\[
-\frac{\partial f_1}{\partial H} = \alpha H + \beta_1 \sin 4\pi \frac{\Phi}{\Phi_0} + \beta_2 \sin 8\pi \frac{\Phi}{\Phi_0}
\]  

(2)

with \( \alpha = 13\text{Hz/Gauss}^2 \), \( \beta_1 = 27 \pm 2\text{Hz/Gauss} \) and \( \beta_2 = 10 \pm 2\text{Hz/Gauss} \). According to eq. 2 this amplitude of the oscillations corresponds to an imaginary conductance of the order of \( 2.5 \times 10^{-2} \Omega^{-1} \) per ring for which the estimated Drude conductance is \( G_0 = 5 \times 10^{-4} \Omega^{-1} \).

This is clearly shown in fig. 3. The temperature dependence of the \( h/2e \) periodic component of the signal (see fig. 3) is compatible with an exponential decay with a characteristic energy of 200mK over a range of temperature corresponding to \( 1.5\Delta - 2E_c \).

Since the frequency is smaller than the level spacing, we can assume that the variations of \( f_1 \) reflect only the dc orbital magnetism of the rings: \(-\frac{\partial f_1}{\partial H} \propto \frac{\partial^2 f_1}{\partial \Phi^2}\). Thus, the amplitude of the oscillations corresponds to a value of 1.5nA per ring which is of the order of \( 2E_c/\Phi_0 = 1.4\text{nA} \) (the factor 2 standing for spin), to be compared to \( 2\sqrt{E_c/\Phi_0} = 0.5\text{nA} \) or \( 2\Delta/\Phi_0 = 0.2\text{nA} \). However, under the same assumption, our result implies a diamagnetic zero field persistent current. One possible explanation of this sign could be the presence of interactions which modify the orbital magnetism of the rings. The existence of an attractive interaction between electrons in the GaAs/GaAlAs heterojunction, which is necessary to explain our experimental results, is not very likely, but it cannot be completely ruled out. Furthermore we cannot exclude that we are in the regime \( \gamma < \omega \), implying a possible contribution from relaxation processes to \( \Im m(G) \), but according to ref. 11, this last hypothesis does not explain the observed sign either.

The magnetic field oscillations of the dissipative conductance, obtained by integration of \(-\frac{\partial f_1}{\partial H}\), are presented in fig. 3. Their period is also \( h/2e \), and their amplitude, which is an order of magnitude smaller than the oscil-
losses of the background signal which is dominated by the residual contribution to the flux dependence of $f_1$ with Aharonov-Bohm-like oscillations. A quantitative estimation of this effect is clearly needed and is underway.

In conclusion, we have designed an experiment sensitive enough to measure the complex conductance of an array of rings in the relevant frequency range. We have shown evidence of $h/2e$ flux oscillations on both real and imaginary part of the conductance. The sign of the oscillations of the imaginary conductance corresponds to diamagnetism in low field, which is for the moment difficult to interpret. The periodic component of the real part of the conductance has apparently a sign opposite to the weak localisation oscillations measured in connected rings, and is at least 10 times larger.

This work has strongly benefited from the help and suggestions of L. P. Lévy, M. Nardonne, P. Pari and C. Urbina. This work was partly supported by grant from DRET No. 92/181.

1. L. P. Lévy, G. Dolan, J. Dunsmuir and H. Bouchiat, Phys. Rev. Lett. 64, 2074 (1990).
2. V. Chandrasekhar, R. A. Webb, M. J. Brady, M. B. Ketchen, W. J. Gallagher and A. Kleinssasser, Phys. Rev. Lett. 67, 3578 (1991).
3. D. Mailly, C. Chapelier and A. Benoît, Phys. Rev. Lett. 70, 2020 (1993).
4. R. Landauer, Physica Scripta 42, 110 (1992).
5. M. Büttiker, Y. Imry and R. Landauer, Phys Rev. Lett. 96A, 365 (1983).
6. R. Landauer and M. Büttiker, Phys Rev. Lett. 54, 2049 (1985).
7. M. Büttiker, New Techniques and Ideas in Quantum Measurement Theory, edited by D. N. Greenberger, Ann. N. Y. Acad. Sci. 480, 194 (1986).
8. Y. Imry and N. S. Shiren, Phys. Rev. B 33, 7992 (1986).
9. N. Trivedi and D. A. Browne, Phys. Rev. B 38, 9581 (1988).
10. B. Reulet and H. Bouchiat, Phys. Rev. B 50, 2259 (1994).
11. A. Kamenev, B. Reulet, H. Bouchiat and Y. Gefen, Europhys. Lett. 28, 391 (1994).
12. A. Kamenev and Y. Gefen, unpublished.
13. B. L. Altshuler, A.G. Aronov and B. Z. Spivak, Pis’ma Zh. Eksp. Teor. Fiz. 33, 101 (1981) [ JETP Lett. 33, 94 (1981)].
14. D. Y. Sharvin and Y. V. Sharvin, Pis’ma Zh. Eksp. Teor. Fiz. 34, 285 (1981) [ JETP Lett. 34, 272 (1981)].
15. B. Reulet, H. Bouchiat and D. Mailly, unpublished.
16. U. Sivan, F. P. Milliken, K. Milkove, S. Rishton, Y. Lee, J. M. Hong, V. Boegli, D. Kern and M. deFranza, Europhys. Lett. 65, 829 (1994).
17. J. B. Pieper and J. C. Price, Phys. Rev. Lett. 72, 3586 (1994).
18. B. Reulet, H. Bouchiat and D. Mailly, unpublished.
19. V. Ambegaokar and U. Eckern, Phys. Rev. Lett. 65, 381

FIG. 4. Temperature dependence of the $h/2e$ periodic components of $\partial f_1/\partial H$ and $\partial Q_1/\partial H$.
(1990).