Dipole and Quadrupole Electroexcitations of the Isovector $T = 1$ Particle-Hole States in $^{12}\text{C}$

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Electroexcitations of the dominantly $T = 1$ particle-hole states of $^{12}\text{C}$ are studied in the framework of the harmonic oscillator shell model. All possible $T = 1$ single-particle-hole states of all allowed angular momenta are considered in a basis including single-particle states up to the $1f - 2p$ shell. The Hamiltonian is diagonalized in this space in the presence of the modified surface delta interaction. Correlation in the ground state wave functions by mixing more than one configuration is considered and shows a major contribution that leads to enhance the calculations of the form factors. A comparison with the experiment shows that this model is able to fit the location of states and a simple scaling of the results give a good fit to the experimental form factors.

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Nuclear structure models can be successfully tested by comparing the calculated and measured electron scattering form factors. The success of such a model reveals a valuable information about the charge and current distributions of nuclei.

The transverse electric form factor of the $2^+$ level at 16.107 MeV $T = 1$ of $^{12}\text{C}$ were measured by Flanz et al. (1) by means of $180^\circ$ electron scattering over a momentum-transfer range from $q = 0.51$ to $2.05$ fm$^{-1}$. Deutschmann et al. (2) studied the inelastic electron scattering crossing section for the M1 transition to the 15.11 MeV $(1^+, T = 1)$ level and for M2 transition to the 16.58 MeV $(2^-, T = 1)$ level in $^{12}\text{C}$ in the momentum-transfer region $(q = 0.4 - 3.0$ fm$^{-1})$. Electron scattering at 200 MeV on $^{12}\text{C}$ and $^{13}\text{C}$, have been studied by T. Sato et al., (3) to investigate magnetic dipole M1 and electric quadrupole E2 nuclear form factors. The effect of higher configurations wave functions are investigated by Bennhold et al., (4). Donnelly (5) studied the $T = 1$ single-particle-hole states of $^{12}\text{C}$ on the basis of the harmonic oscillator (HO) shell model in the particle-hole formalism developed by Lewis and Walecka, configuration mixing is included via a Serber—Yukawa residual interaction.

In this Letter, we study the isovector $(T = 1)$ transition in $^{12}\text{C}$ which connects the $(J^z = 0^+, T = 0)$ ground state with the $(J^z = 2^-, 3^-$ and $3^+$) isovector states. The ground state is taken to have closed $1s_{1/2}$ and $1p_{1/2}$ shells. The states expected to be most strongly excited from closed-shell nuclei are linearly combination of a configurations in which one nucleon has been raised to a higher shell, forming pure single-particle-hole state.

This approximation is called Tamm-Dancoff approximation (TDA). The dominant dipole and quadrupole states of $^{12}\text{C}$ are considered with the framework of the harmonic oscillator (HO) shell model. The Hamiltonian is diagonalized in the space of the single-particle hole states, in the presence of the modified surface delta interaction (MSDI). The space of the single-particle-hole states include all shells up to $2p_{1/2}$ shell. Admixture of higher configurations is also considered. A comparison of the calculated form factors using this model with the available experimental data for the dominantly $T = 1$ states are discussed.

The ground state of $^{12}\text{C}$ is taken to have closed $1s_{1/2}$ and $1p_{3/2}$ shells, and is represented by $\Psi_0$. A state formed by the promotion of one particle from the shell-model ground state is called a particle-hole state. The particle-hole state of the total Hamiltonian is represented by $\Phi_{J,M}(ab^{-1})$ with labels $(a)$ for particles with quantum numbers $(n_a,e_j)$ and $(b)$ for holes with quantum numbers $(n_b,e_j)$. The state $\Phi_{J,M}(ab^{-1})$ indicating that a particle was vacated from $j_b$ and promoted to $j_a$. The excited state wave function can be constructed as a linear combinations of pure basis $\Phi^z$ as

$$\Psi_{JM}^n = \sum_{ab} \chi_{ab^{-1}}^J \Phi_{JM}(ab^{-1}),$$

where the amplitude $\chi_{ab^{-1}}^J$ can be determined from a diagonalization of the residual interaction. By including the isospin $T$, one now has to solve the secular equation

$$\sum_{ab} [(\hat{a}^\dagger b^{-1} | H | ab^{-1})\,_{JM}T_{T_{T}}, - E_{j} \delta_{ab^{-1},ab^{-1}}] \chi_{ab^{-1}}^{JT_{T}} = 0.$$ (2)

The matrix element of the Hamiltonian is given by

$$\langle \hat{a}^\dagger b^{-1} | H | ab^{-1}\rangle_{JM}T_{T} = (e_a - e_b) \delta_{ab,bb} + \langle \hat{a}^\dagger b^{-1} | V | ab^{-1}\rangle_{JM}T_{T},$$ (3)
where $e_a - e_b$ is the unperturbed energy of the particle-hole pair obtained from energies in nuclei with $A \pm 1$ particles.

The matrix element of the residual interaction $V$ is given by the modified surface delta interaction (MSDI) with the strength parameters $A_0 = 0.8$ MeV, $A_1 = 1.0$ MeV, $B = 0.7$ MeV and $C = -0.3$ MeV $\delta$

$$\langle \hat{a}^\dagger V | \hat{b}^{-1} \rangle_{\text{JMTS}} = -\sum_{JT} (2J + 1)(2T + 1) \times \left\{ \begin{array}{c} j_a \ j_b \ j_f \ \\ j_a \ j_b \ J \end{array} \right\} \langle \hat{a} | V | \hat{b} \rangle \langle \hat{a} \rangle_{JT}. \quad (4)$$

The matrix elements of the multipole operators $T_f$ are given in terms of the single particle matrix elements by $\delta$

$$\langle \Psi_J || T_{J_{ts}} || \Psi_0 \rangle = \sum_{ab} \chi^{J_{ts}}_{ab} \langle a || T_{J_{ts}} || b \rangle, \quad (5)$$

where $t_z = 1/2$ for protons and $-1/2$ for neutrons. The amplitudes $\chi^{J_{ts}}_{ab}$ can be written in terms of the amplitudes $\chi^{JT}_{ab}$ in isospin space as $\delta$

$$\chi^{J_{ts}}_{ab} = (-1)^{T_f - T_i} \left[ \left( \begin{array}{cc} T_f & 0 \\ -T_f & 0 \end{array} \right) \frac{T_i}{T_z} \right] \frac{\sqrt{2} \chi^{JT=0}_{ab}}{2} + 2t_z \left[ \begin{array}{cc} T_f & 0 \\ -T_f & 0 \end{array} \right] \frac{T_i}{T_z} \frac{\sqrt{2} \chi^{JT=1}_{ab}}{2}, \quad (6)$$

where

$$T_z = \frac{Z - N}{2} \quad (7)$$

The single particle matrix elements of the electron scattering operator $T_f$ are those of Ref. $\delta$ with $\eta$ selects the longitudinal ($L$), transverse electric ($E\ell$) and transverse magnetic ($M\ell$) operators, respectively. Electron scattering form factors involving angular momentum transfer $J$ is given by $\delta$

$$|F_J(q)|^2 = \frac{4\pi}{Z^2(2J_i + 1)} \left| \langle \Psi_{J_f} || T_{J_{fs}} || \Psi_{J_s} \rangle \right|^2 \times |F_{J,s}(q)|^2 \quad (8)$$

where $J_i = 0$ and $J_f = J$ for closed shell nuclei and $q$ is the momentum transfer. The last two terms in Eq. (8) are the correction factors for the c.m. and the finite nucleon size (f.s.). $\delta$. The total inelastic electron scattering form factor is defined as $\delta$

$$|F_J(q, \theta)|^2 = |F_J(q)|^2 + \left[ \frac{1}{2} + \tan^2(\theta/2) \right] |F_{J\ell}(q)|^2, \quad (9)$$

where $|F_{J\ell}(q)|^2$ is the transverse electric or transverse magnetic form factor.

The resulting particle-hole states are listed in Table I and Table II together with the values of $J^\pi$, for the positive and negative parity states, respectively. The configuration energies $e_a - e_b$ of these states obtained from the spectra of $^{13}$C and $^{14}$C $\delta$ are also given in these tables.

Higher configurations are included in the calculations when the ground state is considered as a mixture of the $|1s_1/2\rangle^4 (1p_3/2)^8$ and $|2s_1/2\rangle^4 (2p_3/2)^8$ configurations, such that the ground state wave function becomes

$$\langle \Psi_0 \rangle = \gamma \langle \Psi_{00} (1s_1/2)^4 (1p_3/2)^8 \rangle + \delta \langle \Psi_{00} (2s_1/2)^4 (2p_3/2)^8 \rangle \quad (10)$$

with $\gamma^2 + \delta^2 = 1$, $\chi^{JT}_{ab} = \gamma \chi^{JT}_{ab}$ and $\chi^{JT}_{ab} = \delta \chi^{JT}_{ab}$.

The excited states is also assumed as a mixture of the particle-hole configurations, $|a_1 b_1^{-1}\rangle$, $|a_2 b_2^{-1}\rangle$, $|a_2 b_2^{-1}\rangle$ and $|a_1 b_2^{-1}\rangle$, where $|a_1\rangle = |a\rangle = |n_a l_a j_a\rangle$, $|a_2\rangle = |a\rangle = |n_a + 1 l_a j_a\rangle$, $|a_1\rangle = |b\rangle = |n_b l_b j_b\rangle$ and $|a_2\rangle = |b\rangle = |n_b + 1 l_b j_b\rangle$. The configurations which include the higher configurations is called the extended space configurations.

The matrix element given in Eq. (5) is called the model space matrix element, where $a$ and $b$ are defined by the amplitudes given in Table III and Table IV, respectively.

**TABLE I:** The unperturbed energies of the particle-hole positive parity states and the possible $J$ and $T = 1$ values in $^{12}$C.

| Particle-hole configuration $|a^b\rangle$ | Particle-hole configuration energies $(e_a - e_b)$ MeV | $J$ |
|-----------------------------------|------------------------------------------|---|
| $(1p_{1/2}) (1p_{3/2})^{-1}$     | $13.77$                                  | 1, 2 |
| $(1d_{5/2}) (1s_{1/2})^{-1}$     | $33.90$                                  | 2, 3 |
| $(2s_{1/2}) (1s_{1/2})^{-1}$     | $33.14$                                  | 0, 1 |
| $(1d_{3/2}) (1s_{1/2})^{-1}$     | $38.38$                                  | 1, 2 |
| $(1f_{7/2}) (1p_{3/2})^{-1}$     | $25.74$                                  | 2, 3, 4, 5 |
| $(2p_{3/2}) (1p_{3/2})^{-1}$     | $27.37$                                  | 0, 1, 2, 3 |
| $(1f_{5/2}) (1p_{3/2})^{-1}$     | $34.17$                                  | 1, 2, 3, 4 |
| $(2p_{1/2}) (1p_{3/2})^{-1}$     | $33.37$                                  | 1, 2 |

**TABLE II:** The unperturbed energies of the particle-hole negative parity states and the possible $J$ and $T = 1$ values in $^{12}$C.

| Particle-hole configuration $|a^b\rangle$ | Particle-hole configuration energies $(e_a - e_b)$ MeV | $J$ |
|-----------------------------------|------------------------------------------|---|
| $(1p_{1/2}) (1s_{1/2})^{-1}$     | $30.05$                                  | 0, 1 |
| $(1d_{5/2}) (1p_{3/2})^{-1}$     | $17.62$                                  | 1, 2, 3, 4 |
| $(1d_{3/2}) (1p_{3/2})^{-1}$     | $22.11$                                  | 0, 1, 2, 3 |
| $(2s_{1/2}) (1p_{3/2})^{-1}$     | $16.86$                                  | 1, 2 |
| $(1f_{7/2}) (1s_{1/2})^{-1}$     | $42.02$                                  | 3, 4 |
| $(2p_{3/2}) (1s_{1/2})^{-1}$     | $43.65$                                  | 1, 2 |
| $(1f_{5/2}) (1s_{1/2})^{-1}$     | $50.45$                                  | 2, 3 |
| $(2p_{1/2}) (1s_{1/2})^{-1}$     | $49.65$                                  | 0, 1 |
TABLE III: Energies and amplitudes $\chi_{J^T}$ for $J^+ T = 1$ states.

| Particle-hole configuration | E(1$^+$) = 15.54 MeV $\chi_{11}$ | E(2$^+$) = 15.49 MeV $\chi_{21}$ |
|-----------------------------|-----------------------------------|-----------------------------------|
| (1p$_1$/2)(1p$_3$/2)$^{-1}$ | 0.99671                           | 0.99774                           |
| (1d$_5$/2)(1s$_1$/2)$^{-1}$ | 0.00000                           | 0.00321                           |
| (2s$_1$/2)(1s$_1$/2)$^{-1}$ | 0.03900                           | 0.00000                           |
| (1d$_1$/2)(1s$_1$/2)$^{-1}$ | 0.01378                           | 0.03724                           |
| (1f$_7$/2)(1p$_3$/2)$^{-1}$ | 0.00000                           | 0.00892                           |
| (2p$_3$/2)(1p$_3$/2)$^{-1}$ | 0.05340                           | -0.01953                          |
| (1f$_5$/2)(1p$_3$/2)$^{-1}$ | 0.01823                           | 0.03449                           |
| (2p$_1$/2)(1p$_3$/2)$^{-1}$ | 0.04104                           | 0.03828                           |

The extended space matrix element becomes

$$
\langle \Psi_f | T_{J_{T_z}} | \Psi_0 \rangle = \sum_{a_1b_1} \chi_{a_1b_1}^{J_{T_z}} \langle a_1 | T_{J_{T_z}} | b_1 \rangle + \sum_{a_1b_2} \chi_{a_1b_2}^{J_{T_z}} \langle a_1 | T_{J_{T_z}} | b_2 \rangle + \sum_{a_2b_1} \chi_{a_2b_1}^{J_{T_z}} \langle a_2 | T_{J_{T_z}} | b_1 \rangle + \sum_{a_2b_2} \chi_{a_2b_2}^{J_{T_z}} \langle a_2 | T_{J_{T_z}} | b_2 \rangle ,
$$

(11)

where

$$
\chi_{a_1b_1}^{J_{T_z}} = C_1 \chi_{ab-1}^{J_{T_z}}, \\
\chi_{a_1b_2}^{J_{T_z}} = C_2 \chi_{ab-1}^{J_{T_z}}, \\
\chi_{a_2b_1}^{J_{T_z}} = C_3 \chi_{ab-1}^{J_{T_z}}, \\
\chi_{a_2b_2}^{J_{T_z}} = C_4 \chi_{ab-1}^{J_{T_z}}.
$$

(12)

The values of the parameters $C^*$ are given in Table V.

Experimentally the states $1^+$, $2^+$ and $2^-$ are found at 15.11 MeV, 16.11 MeV and 16.58 MeV, respectively. We obtain the values 15.54 MeV, 15.52 MeV and 18.19 MeV for the states $1^+$, $2^+$ and $2^-$, respectively.

The $1^+$ (15.11 MeV), M1 form factor is shown in Fig. 1. The dashed curve represents the calculations with the amplitudes $\chi^{*}$ reduced by a factor 2, to agree with the low $q$ data. The single-particle matrix elements are calculated with the harmonic oscillator wave functions (HO) with oscillator parameter $b = 1.64$ fm to agree with the elastic form factor determination. A best fit to the M1 form factor data is obtained by Hicks et al. with $b = 1.67$ fm.

Karagadlis et al. adopted the value 1.7 fm in describing the form factors for the $0 \rightarrow 2^+$ (4.44 MeV and 16.11 MeV) transitions in $^{12}$C. The low momentum transfer data for the 15.11 MeV $1^+$ state are fitted with $b = 1.9$ fm.

This $b$ value is adopted also by Donnelly, to describe the M1 form factors up to momentum transfer 1.4 fm$^{-1}$. In our work we fix the value of $b = 1.64$ fm and trying to find other contributions that can describe the experimental form factors. When the M1 form factor is calculated with the extended space configurations, the result is shown by the solid curve in Fig. 1. The data are very well described up to momentum-transfer 1.7 fm$^{-1}$, and the form factor is shifted to agree with the location of the experimental diffraction minimum. The calculated form factor decreases too steeply compared to the data at high $q$ values. All previous theoretical treatments shows this behaviour. Our results are consistent with those of Sato et al., where the meson exchange current is
The Coulomb C2 and the transverse E2 form factors are shown in Figs. 2 and 3. The amplitudes $\chi$ have to be reduced also by a factor 2 to fit the low $q$ data. The Coulomb form factor is calculated with core-polarization effects by introducing effective charges $1.25e$ for the protons and $0.5e$ for the neutrons. With these effective charges, the experimental data are very well described for the region of the momentum transfer $q \leq 2.2\, fm^{-1}$. The same agreement is obtained for the E2 form factor.

The transverse M2 form factor for the excitation to the $2^-\, 16.58\, MeV$ state are shown in Fig. 4. The dashed curve represents the model space calculations with $b = 1.64\, fm$, where the data can not be satisfactorily described, especially in the region of the low $q$ values. Due to the absence of an accepted model, Hicks et al. used a simple phenomenological procedure to interpret the data. In our work the data are well described using the extended space configurations and $b = 1.35\, fm$. The amplitudes are quenched by 86%. The results are consistent with those of Hicks et al., a diffraction minimum is obtained at $q \approx 0.6\, fm^{-1}$.

Good agreement is obtained in comparison of the form factors for group of single-particle-hole states with the experimental data. The amplitudes of the transition to the positive-parity states considered in this work have to be reduced by a factor of 2 to describe the low $q$ data. The form factors for the odd-parity particle-hole states are in reasonable agreement with the experiment and need a reduction factor of 1.16 in the amplitudes $\chi$ to yield excellent agreement. Higher configurations are necessary to improve the agreement with the low $q$ values of the form factors.
TABLE V: Values of the parameters \( C_i \) used in the extended space calculations.

| \( J^\pi \) | \( C_1 \) | \( C_2 \) | \( C_3 \) | \( C_4 \) |
|---------|---------|---------|---------|---------|
| 1\(^+\) | 0.92    | -0.27   | -0.27   | 0.078   |
| 2\(^-\) | -0.92   | 0.27    | -0.27   | 0.078   |

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