Normal mode behaviours in solar prominence plasmas

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Abstract. The evolutionary dynamics of normal modes, non-adiabatic magneto-hydrodynamic (MHD) waves, in the prominence plasmas in the presence of kinematic viscosity and inertia-dependent species stratification is herein explored. It judiciously implements a new energy equation devised to describe optically thin radiation losses, thermal conduction, viscosity, heating mechanisms, and so forth. The dispersion analysis demonstrates that the thermal conduction plays as a primary sink for both the wave classes of fast and slow spectral groups. Finally, synoptic indication to further refinements and scopes is concisely stressed.

1. Introduction
The solar prominence plasmas, which are composed of partially ionized gases supported against the solar gravity by the coronal magnetic field, have recently acquired rapidly growing interest among astro-plasma communities. The evolutionary prominence dynamics, via the excitation of collective waves and exotic oscillations, have extensively been investigated [1-3]. The prominence oscillations can be of both small and large amplitudes. These small amplitude oscillations have been detected mainly in the velocity field describable in the framework of magneto-hydro-dynamic (MHD) waves [4, 5]. Besides, the coronal gravity and electromagnetic effects in dictating the viscous prominence kinetics with proper thermodynamic functions are yet to be well explored [6-8]. The real picture of non-adiabatic prominence waves (eigen-modes) can be obtained by considering different unavoidable mechanisms, such as radiative losses, heating, electro-thermal conduction, and so forth.

The inclusion of the dissipative agencies (e.g., viscosity, conduction, radiative cooling, etc.) could explain the spatiotemporal damping [4, 5]. A refined exploration would require a proper energy-transport law integrating all the phenomenological effects. The wave damping mechanisms in viscous stratified prominence plasma have hitherto been remaining an open long-sought goal as far as seen. This paper is motivated by the need for realizing the realistic prominence mode behaviors. We herein propose a theoretical analysis to see mainly the damping factors and behaviors. The results are important in the intriguing coronal seismology and in-situ mensuration of the coronal atmosphere.

2. Physical and mathematical models
We consider an unbounded prominence plasma configuration in the presence of magnetic field acting along the x-direction in homogeneous stratified MHD equilibrium. It includes all the possible realistic effects, such as optically thin radiative losses, viscosity, gravity, etc. A schematic is sketched in figure 1.
The model is setup by a closed set of non-adiabatic MHD equations with all the generic notations [5, 7]. If no source or sink, it includes the following respective conservation laws of mass, force, etc.

\[
\frac{D\rho}{Dt} + \rho \vec{\nabla} \cdot \vec{v} = 0
\]

(1)

\[
\rho \left( \frac{D\vec{v}}{Dt} \right) = -\vec{\nabla} p + (1/\mu) (\vec{\nabla} \times \vec{B}) \times \vec{B} + \rho g \hat{z} + \rho \nu \left\{ \frac{4}{3} \vec{\nabla} \cdot \vec{v} \right\} - \vec{\nabla} \times (\vec{v} \times \vec{v})
\]

(2)

\[
\frac{Dp}{Dt} - (\gamma p/\rho) \left( \frac{D\rho}{Dt} \right) + (\gamma - 1) \left[ \rho L(\rho, T) - \vec{\nabla} \cdot (K \vec{V} T) \right] = 0
\]

(3)

\[
\frac{\partial \vec{B}}{\partial t} = \vec{\nabla} \times (\vec{v} \times \vec{B})
\]

(4)

\[
p = \rho RT/\tilde{\mu}
\]

(5)

\[
\text{here, } D/Dt = \partial / \partial t + v \cdot \vec{\nabla}
\]

represents the material (Lagrangian or convective) derivative associated with the mean MHD flow dynamics and \( \rho \) is the mass density. The symbol \( R \) denotes the ideal gas (molar) constant and \( \tilde{\mu} \) is the plasma degree of ionization, defined as \( \tilde{\mu} = 1/1 + \xi \), with \( \xi = n_i/(n_i + n_e) \), as the relative population density of the ionized species with respect to the total plasma population density. The non-adiabatic effects act via a conduction term, \( \vec{\nabla} \cdot (K \vec{V} T) \), where \( K \) is the thermal conductivity and \( T \) is the temperature (in K). \( \gamma \) is the adiabatic index and \( L \) the heat-loss function[4].

In a homogeneous MHD equilibrium [4, 5], where the temperature and density are uniform, one finds

\[
L(\rho_0, T_0) = 0.
\]

(7)

The function \( L \) measures the difference between an arbitrary heat input and radiative heat loss. Such losses are described by optically thin radiative loss function [4, 5] expressed as

\[
L(\rho, T) = \chi^* \rho T^{\alpha} - h \rho T^{2b}
\]

(8)

Here, \( \chi^* \) and \( \alpha \) are the temperature-dependent piecewise functions accounting for the different radiative loss regimes. Different heating scenarios are specified by the distinct exponents \( a \) and \( b \).

Applying the Fourier-based normal mode analysis over basic structure equations (1)-(6), with all the relevant perturbations evolving as plane waves of the form \( \sim \exp(i\omega t - k \cdot r) \), under noise boundary effects considered, we finally get the linear quintic dispersion relation after decomposition exercises as

\[
a_0 \omega^5 + a_2 \omega^3 + a_3 \omega^2 + a_4 \omega + a_0 = 0
\]

(9)

where the various involved coefficients in the presence of viscous and gravitational effects are as

\[
a_0 = \left[ \frac{i}{\mu \rho_0} \right] \left[ B_{00}^2 p_0 k_0^2 \left( - AT_0 + H \rho_0 \right) \left( k_0^2 + i \left( -i k_0 + \frac{1}{2H_0} \right) \right)^2 \right]
\]

(10)

\[
a_2 = \left[ \frac{1}{\mu \rho_0^2} \right] \left[ B_{00}^2 c_p k_0^2 \mu p \left( -i k_0 + \frac{1}{2H_0} \right) \right] + \frac{4}{3} m \mu p \mu \nu k_0^2 \left( AT_0 - H \rho_0 \right) - i \mu p \nu \left( -i k_0 + \frac{1}{2H_0} \right) \gamma
\]

\[
\left( AT_0 - H \rho_0 \right) + \frac{1}{3} \mu g AT_0 \rho_0 \nu k_0^2 \left( -i k_0 + \frac{1}{2H_0} \right) + \frac{4}{3} AT_0 B_0^2 \mu \nu \left( -i k_0 + \frac{1}{2H_0} \right) \gamma + \frac{4}{3} i \mu g AT_0 \rho_0 \nu k_0^2 \left( -i k_0 + \frac{1}{2H_0} \right) \gamma
\]

\[
+ NT_0 B_0 \nu \left( -i k_0 + \frac{1}{2H_0} \right) + i \mu g AT_0 \rho_0 \nu \left( -i k_0 + \frac{1}{2H_0} \right) \gamma - \frac{4}{3} m \mu p \nu k_0^2 \left( -i k_0 + \frac{1}{2H_0} \right) \gamma \left( AT_0 - H \rho_0 \right) + \mu p \nu k_0^2 \left( AT_0 - H \rho_0 \right),
\]

(11)
The dashed (blue), dotted (red), and dashed-dotted (black) lines link to the three distinct prominence condition results. It is interesting to note that the coefficients \( a_2 - a_5 \) in equation (9) produce all the previous results,[4] but as special cases with the roles of gravity and viscosity entirely turned off. Here, the difference lies in the gravitational effects via \( a_2 \) and \( a_3 \); and dissipative effects via \( a_4 \), \( a_5 \), and \( a_6 \). Clearly, equation (9) is a fifth-degree polynomial in \( \omega \) with a strong dispersive dependency via \( k \). The
principal goal lies in the time damping of MHD waves, and hence, \( k \) is considered as a real, but \( \omega = \omega_R + i\omega_l \) as a complex. Out of all the five roots of equation (9) obtained numerically, only one root is purely imaginary, indicating non-propagatory thermal or condensation mode [9]. The rest of the roots are two conjugate pairs, one for the slow mode and the other the fast mode. Thus, the damping time is \( \tau_D = 1/\omega_l \), the period of the waves is \( T = 2\pi/\omega_R \) (hereafter, \( T \) as period) and the damping per period is \( D_p = \omega_l/\omega_R \).

3. Results
A numerical analysis based on of the dispersion relation (9), yields the results as follows. As in figure 2, with increase in temperature, \( \tau_D \) of the fast wave at first decreases followed by a rapid rise to a peak (at \( k \sim 10^3 \) m\(^{-1}\)), and again decreases. We speculate that \( \tau_D \) of the slow wave decreases in magnitude with increase in temperature. This means, higher the temperature, the more is the time required to damp the waves; and vice versa. Likewise, \( T \) decreases in a similar fashion with respect to \( \tau_D \) of the slow wave, but \( D_p \), in contrast, increases with rise in temperature. Thus, the thermal conduction plays an important role in the damping mechanism. The changes in the damping behaviour is due to the coupled interplay of the intrinsic MHD field (\( B = 10^{-3} \) T) and gravito-electromagnetically induced field (\( B_{GEM} = -(1/c_A)E_{GIP} \sim 10^{-8} \) T) as per the well-known electromagnetic Faraday-Lenz action [7].

4. Conclusions
We have studied the damping behaviour of the normal eigen-modes (MHD waves) in the solar prominence plasmas considering all the realistic non-adiabatic terms in an appropriate energy law. A new generalized dispersion relation (quintic) is derived by employing linear normal mode analysis. It enables us to characterize two distinct classes of modes: the fast wave and slow ramified wave. It is seen that the damping time, \( \tau_D \), of the fast and slow waves decrease with increase in temperature. We further see that the thermal conduction plays an important role in the damping mechanisms. Thus, the joint effects of non-adiabaticity sourced by thermal conduction, radiative losses and heating; alongside viscosity and gravitational stratification, are able to damp both the fast and slow modes. The model formalism presented here may be useful for basic study of the prominence seismology for characterizing the solar structure, morphology and coronal behavior. In a broader sense, it can also be extended to explore the MHD phenomena prevalent in different stellar and astrophysical configurations. For more concretized implications and applications, we finally admit that further refinements are necessary with the proper inclusion of all other prevailing relevant effects [7-9], such as ion-neutral damping, phase mixing, geometric curvature effects, tunnel-induced wave leakage, etc.

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