Study on Parameter Identification of Assembly Robot based on Screw Theory

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Abstract. The kinematic model of assembly robot is one of the most important factors affecting repetitive precision. In order to improve the accuracy of model positioning, this paper first establishes the exponential product model of ER16-1600 assembly robot on the basis of screw theory, and then based on iterative least squares method, using ER16-1600 model robot parameter identification. By comparing the experiment before and after the calibration, it is proved that the method has obvious improvement on the positioning accuracy of the assembly robot.

1. Introduction

The accuracy of the assembly robot is one of the important indicators of its performance. The precision of the assembly robot refers to the accuracy of the final movement of the end actuator to the specified desired coordinates, which is an important criterion for measuring the robot [1]. Under normal circumstances the assembly of the robot on the movement accuracy requirements higher than the handling of robots and arc welding robot [2].

At present, the commonly used kinematic model has D-H model [3], which establishes a coordinate system for each link, the base system is connected with 0 link, and the tool coordinate system is built on the end of the assembly robot, and the kinematic model. But there are three problems in the calibration process: the calculation process is more cumbersome; cannot arbitrarily set the base coordinate system and tool coordinate system; the model will cause a certain singularity, does not have continuity and completeness. Compared with the D-H model, the use of spin theory to study the kinematics of the robot has three advantages [4, 5]: the overall description of the robot movement, to avoid the description of the singularity caused by DH; movement described clearly, simplifies the institutional Analysis; problem description and resolution simple and unified, easy and other methods of conversion. Therefore, the study of robot parameter identification is becoming more and more concerned [6-8].

In [9], the kinematic parameter calibration method of the product of exponential (POE) is proposed first, but it is inconvenient to use. In [10] improved the model of [9], given the explicit calibration model. [11] Using the spin theory to differentiate the kinematic parameters, the approximate model was established, but the model was not accurate. In [12], the joint rotation error is regarded as the generalized kinematic motion, and the error model is established, but the consideration of the error source is not comprehensive. In this paper, the calibration parameters of assembly robot parameters are completed based on the explicit calibration model of Screw theory. The experimental results show that the assembly robot positioning accuracy after calibration is improved obviously.

2. Screw theory
Screw theory was first introduced by Brockett [13] into the kinematics modeling of robots. Assuming that a rigid body rotates about the angle $\theta$ about the rotation axis $s$, the corresponding rigid body transformation can be expressed as a matrix exponent:

$$ R = \exp(\hat{s} \theta) $$  \hspace{1cm} (1)

where $\hat{s}$ is an antisymmetric matrix, $R$ is a rotation matrix. Formula (1) can be expanded to:

$$ R = \exp(\hat{s} \theta) = I + \hat{s} \theta + (\hat{s} \theta)^2/(2 !) + (\hat{s} \theta)^3/(3 !) + \cdots = I + \hat{s} \sin \theta + \hat{s}^2/(1 - \cos \theta) $$ \hspace{1cm} (2)

When the rigid body to do the spiral movement, the rigid body on a point $P$ rotation angle $\theta$, and then move in the direction of the axis $d = h\theta$ after the final coordinates can be expressed as:

$$ p(\theta, h) = r + \exp(\hat{s}) (p(0) - r) + hs\theta, s \neq 0 $$ \hspace{1cm} (3)

This equation holds for any $p(0) \in \mathbb{R}^3$, so the form of homogeneous coordinates is:

$$ T = \begin{pmatrix} \exp(\hat{s} \theta) & r(1 - \exp(\hat{s} \theta)) + hs\theta \\ 0 & 1 \end{pmatrix}, s \neq 0 $$ \hspace{1cm} (4)

According to the relative theorem of screw theory, any rigid body transform $T$ can be written as a matrix of exponential form of a motion helix:

$$ T = \exp(\hat{\xi} \theta) = \begin{pmatrix} \exp(\hat{s} \theta) & r(1 - \exp(\hat{s} \theta)) + hs\theta \\ 0 & 1 \end{pmatrix}, s \neq 0 $$ \hspace{1cm} (5)

where $\hat{\xi} = (\hat{s} - s \times r)$

When the rigid body is pure rotation $h = 0$, this time:

$$ T = \exp(\hat{s} \theta) = \begin{pmatrix} r(1 - \exp(\hat{s} \theta)) \\ 0 \end{pmatrix}, s \neq 0 $$ \hspace{1cm} (6)

3. Robot kinematics model

The forward kinematics of the assembly robot can be synthesized by the rigid body motion caused by each joint. According to the screw theory, the motion of each joint is generated by the kinematic rotation of the associated joint axis, which can give its kinematic geometric description. If $\xi$ is used to represent the unit motion axis coordinate of a joint axis, the rigid body motion along the joint axis can be expressed as:

$$ T(\theta) = G_1 G_2 \cdots G_n T_{st}(0) = \exp(\hat{\xi}_1 \theta_1) \exp(\hat{\xi}_2 \theta_2) \cdots \exp(\hat{\xi}_n \theta_n) T_{st}(0) $$ \hspace{1cm} (7)

$$ G_i = \exp(\hat{\xi} \theta) = \begin{pmatrix} \exp(\hat{s} \theta) & r_i(1 - \exp(\hat{s} \theta)) \\ 0 & 1 \end{pmatrix}, i = 1,2,\cdots 6 $$ \hspace{1cm} (8)

$$ \hat{\xi}_i = (\hat{s}_i - s_i \times r_i), i = 1,2,\cdots 6 $$ \hspace{1cm} (9)

where $T(\theta)$ represents the position of the axis after the rotation $\theta$ angle, $T_{st}(0)$ indicates that the robot is in the initial position. $T_{st}(0)$ can be written as a constant rotation of the exponential form, the formula (7) rewritten as:

$$ T(\theta) = \exp(\hat{\xi}_1 \theta_1) \exp(\hat{\xi}_2 \theta_2) \cdots \exp(\hat{\xi}_n \theta_n) \exp(\hat{T}) $$ \hspace{1cm} (10)

The equation (10) is obtained by:

$$ \delta TT^{-1} = \frac{\partial T}{\partial \xi} \delta \xi + \frac{\partial T}{\partial \Gamma} \delta \Gamma T^{-1} $$ \hspace{1cm} (11)

The relationship between the theoretical value $\xi^n$ of the rotation joint coordinate and the actual value $\xi^a$ can be expressed by a fixed rigid body transformation $\exp(\hat{\xi}) = \begin{bmatrix} R & q \\ 0 & 1 \end{bmatrix}$, and the relationship between the kinematic rotations of the rotational force is expressed as:
$$\xi^a = Ad(\exp(\hat{\eta}))\xi^n = \exp(\hat{\eta})\xi^n\exp(-\hat{\eta}) = \begin{bmatrix} R & 0 \\ qR & R \end{bmatrix} \begin{bmatrix} \omega^n \\ v^n \end{bmatrix} = \begin{bmatrix} R\omega^n \\ qR\omega^n + Rv^n \end{bmatrix} \quad (12)$$

where $Ad(\exp(\hat{\eta}))\xi^n$ is the adjoint matrix of spin factor $\xi^n$, the rotation joint satisfies the constraint condition:

$$||\omega|| = ||\omega^a|| = ||R\omega^n|| = ||\omega^a|| = 1, \omega^Tv = 0 \quad (13)$$

$$(\omega^a)^Tv^a = (R(\omega^n)^T)(\hat{\beta}\omega^n + Rv^n) = 0 \quad (14)$$

According to the formula (12), the kinematic equation (10) can be written as the concomitant transformation of the theoretical joint rotation:

$$T^a(\theta) = e^{(\vec{\xi}\eta_1 e^{\eta_2} e^{\eta_3} \cdots e^{\eta_k} e^{\eta_k} \cdots)} e^{(\vec{\xi}\eta_1 e^{\eta_2} e^{\eta_3} \cdots e^{\eta_k} e^{\eta_k} \cdots)}$$

The kinematics equation (15) is substituted into equation (11), and the simplification is obtained:

$$\delta TT^{-1} = (\delta(\vec{\xi}) T^{-1})^T = (\delta(A_1 \cdots A_n) (B_1 \cdots B_n))^T = \left(\delta(A_1 B_1)^T + \cdots + Ad(\prod_{i=1}^{n-1} A_i) \cdot (\delta(A_n B_n))^T\right) \quad (16)$$

where $A_i = e^{\xi_i e^{\eta_i} e^{\eta_i} B_i = e^{\xi_i e^{\eta_i} e^{\eta_i}}$, $i = 1, 2, \ldots, n$.

Since the identity $\delta(e^\eta) e^{-\eta} + e^\eta \delta(e^{-\eta}) = 0$, we obtain:

$$\delta(A_i B_i)^T = (I_n - Ad(A_i)) (\delta e \hat{\eta}_i e^{-\hat{\eta}_i}) \quad (17)$$

According to the explicit expression of the exponential mapping in the literature [14] on the screw differential:

$$\delta(e^\eta) e^{-\eta} = \frac{(\delta(e^\eta) e^{-\eta})^T = \left(1 + 4||\beta|| \sin ||\beta|| - 4\cos ||\beta|| \Psi_i + 4||\beta|| \sin ||\beta|| - 4\cos ||\beta|| \Psi_i^2 + 2\sin ||\beta|| - 2\cos ||\beta|| \Psi_i^4 \right) \delta \eta_i = K_i \delta \eta_i \quad (18)$$

where $||\beta|| = \sqrt{\beta_1^2 + \beta_2^2 + \beta_3^2}$.

The actual position of the robot end effector is $T^a$, and its theoretical position is $T^n$, then $\delta TT^{-1}$ is the deviation from $T^a$ to $T^n$, and the deviation is small enough to be obtained by the first order approximation:

$$\delta TT^{-1} = \log(T^a (T^n)^{-1}) \quad (19)$$

For a set of pose error data, the formula (18) is written as follows:

$$y = Jx \quad (20)$$

where $J_i = \begin{cases} (-\hat{\beta} \ 0 \ 0) & \text{for } i = 1 \\ (-\hat{\beta} \ 0 \ 0) Ad(\prod_{i=1}^{n-1} A_k) \cdot (I_n - Ad(A_k)) T_1 & 1 < i \leq 6 \\ R_g \end{cases}$

Assuming that the robot moves k coordinate points in the workspace, the corresponding k actual pose is obtained by Kinect color camera, and the theoretical pose of each coordinate point is calculated by the robot controller. Then by the formula (19) can be obtained k group of geometric error equation, the simultaneous equations can be obtained for the equations:

$$Y = Lx \quad (21)$$

where $Y = [y_1 \cdots y_k]^T$, $L = [J_1 \cdots J_k]^T$. 

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The corresponding geometric parameter error \( x \) can be obtained by solving the iterative least squares method:

\[
x = (L^TL)^{-1}L^TY
\]  

(22)

4. Parameter identification experiment

Parameter identification experimental equipment is the research group ER16-1600 type assembly robot (figure 1), figure 2 is its structural diagram, the experiment used in the measurement equipment Kinect color camera. Parameter identification results shown in table 1, the experimental compensation before and after the robot positioning accuracy shown in figure 3. The average positioning accuracy of the assembled robot is 3.8558 mm, and the average positioning accuracy of the assembled robot is reduced to 0.70035 mm. After the parameter identification, the positioning accuracy of the assembly robot is obviously improved, and the validity of the compensation method is proved.

![Figure 1. ER16-1600 assembly robot.](image1)

![Figure 2. Motion diagram.](image2)

![Figure 3. Errors’ module.](image3)

| Parameter | Nominal value | Identification results |
|-----------|---------------|------------------------|
| \( \xi_1 \) | \((0,0,1,0,0)^T\) | \((-0.0028,-0.0009,0.0024,0.9920,0.1551,0.0115)^T\) |
| \( \xi_2 \) | \((0,0,-160,0,1,0)^T\) | \((0.0016,0.0101,-159.9909,0.0010,1.0002,0.0083)^T\) |
| \( \xi_3 \) | \((-680,0,-160,0,1,0)^T\) | \((-680.1043,0.0015,-160.0002,0.0073,1.0005,0.0008)^T\) |
| \( \xi_4 \) | \((0,680,0,1,0,0)^T\) | \((0.0005,680.0035,0.0073,1.0004,0.1018,0.0074)^T\) |
| \( \xi_5 \) | \((-680,0,-910,0,1,0)^T\) | \((-679.9508,0.0019,-910.5511,0.2131,1.0108,0.0158)^T\) |
| \( \xi_6 \) | \((0,680,0,1,0,0)^T\) | \((0.2041,680.0426,0.0112,1.0017,1.0254,2.1086)^T\) |
| \( T_{st} \) | \((-1050,0,810,0,0,0)^T\) | \((-1051.0152,0.8304,809.7386,0.0001,0.0009,1.0005)^T\) |
5. Conclusion
The kinematic model of the 6-DOF assembly robot is established by using the spin matrix and the exponential product. The modeling method is simple and has obvious geometric intuition. On this basis, the iterative least squares algorithm is combined with spin theory. Compared with the traditional D-H parameter method, the kinematic derivation process of this method is simpler and the geometric significance is clearer. The algorithm is parallel to the common axis of the 6-DOF series robot, and the algorithm has a certain degree of versatility.

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