Inverse Kinematics Analysis of Hybrid-Driven Special Five-Bar Mechanism for Shield Machine

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Abstract. To simplify the inverse kinematics model of special five-bar mechanism based on Assur group, a inverse kinematics model is proposed by using vector analysis method and Euler’s formula, which is expressed by all explicit equations and establishes a direct relation between the output motion and the input motion. A numerical example and a computer simulation were conducted to verify the mathematical model. Results show that the motion law of control bar \( l_4 \) obtained directly from the mathematical model is consistent with the result from ADAMS simulation, and the mathematical model of the special five-bar mechanism is correct and can be used to analyze the motion law of the drive bar and the control bar directly according to the motion parameters of the output point.

1. Introduction

A circular cross-section shield machine has been used in tunnel digging over one hundred years since it was invented in 1865\cite{1}. Nowadays, along with the sustainable development and utilization of underground space, it has been found that the circular-section shield machine has many disadvantages such as low efficiency and space waste when being used to dig special cross-section tunnels. If a special cross-section shield machine can be used in digging tunnels of special sections, it can reduce digging area and the earthwork volume, and thus improve working efficiency and space utilization.

For the development of the special cross-section shield machine, the crucial task is the study on its cutting mechanism \cite{2}. A hybrid-driven five-bar mechanism is driven by two types of motors, namely the real-time non-adjustable (RTNA) motor which provides major driving power for the cutting mechanism and the real-time adjustable (RTA) motor which acts as a low power motion modulation device for motion adjustment. These two input motions from the RTNA and RTA motors can produce required output motions through a two-degree-of-freedom (2-DOF) closed-loop mechanism \cite{3}, and its output pathway can be of random shape such as ellipse, arch, rectangle and so on. Therefore, this five-bar mechanism becomes the target mechanism for the special shield machine.

To develop this mechanism, study on the motion law of RTA motor becomes urgent. Kinematics analysis of a mechanism includes mechanism analysis and mechanism synthesis. Mechanism analysis solves unknown kinematic parameters of output members according to the established mechanism model and the known motion parameters of driving components. This is the so-called forward
kinematics analysis i.e. positive solutions. Mechanism synthesis solves mechanism scale parameters or the motions of the driving links based on the motion law of output members, and thus to provide accurate kinematics equations for mechanism real-time control, off-line programming and trajectory planning. This is the so-called inverse kinematics analysis i.e. inverse solution. Therefore, inverse kinematics analysis of a mechanism is very important in mechanism kinematics analysis and control.

In 1958, J Eddie Baker [4] proposed the concept and connection method of a five-bar linkage. In 1992, Tokuz Lale Canan [5] put forward the concept of the hybrid-driven mechanism for the first time. Since then, a lot of progresses in hybrid-driven mechanisms, especially 2-DOF hybrid-driven mechanisms had been made. Iraj Hassanzadeh et al [6] obtained the kinematic equation of a 2-DOF five-bar mechanism based on the governing ideal Euler-Lagrange equations and the Jacobian matrix, by which the motion of the output point with random input motion was studied. Using vector loop-closure equations, M E Kütük et al [7] established the kinematic model of a hybrid-driven seven-bar mechanism, and performed kinematic analysis and MATLAB/Sim Mechanics simulation by using this mathematics model and given servo motor inputs. Although the research was focused on forward mechanism kinematics analysis, it provided a theoretical reference of using vector loop-closure equations for inverse kinematics analysis of the special five-bar mechanism. Yu Hongying et al [8] deduced the forward kinematics and inverse kinematics equations of a planar five-bar mechanism and analyzed the kinematic and dynamic characteristics of the mechanism with the help of Kane dynamic equations. Nonetheless, the subject investigated was the motion of a typical symmetrical five-bar mechanism instead of the motion of the special five-bar mechanism.

All of the above studies were aimed at simple planar five-bar mechanisms, and there were relatively few studies on the motion of the special five-bar mechanisms. By using graphic simulation, Wu Xiaowei [9] studied and analyzed the influence of the initial position of the cutting mechanism on the elliptical trajectory when the special five-bar mechanism was used for elliptical trajectory cutting. Nie Jianjun [10] et al conducted the kinematics analysis of the planar five-bar mechanism and analyzed the motion of a planar linkage by using complex vector method and Assur group, and thus established a kinematic model of the special five-bar mechanism in which the relations between the position, velocity and acceleration about the input and output components were derived. X.W. Wu [11] set up the inverse kinematics model of the special five-bar mechanism based on Assur group. However, the inverse kinematics model of the special five-bar mechanism based on Assur group was very complex, and all the equations derived were implicit equations including unknown parameters, which are difficult to be solved directly. Because the direct relations between the output motions and the input motions were not established, these equations are not suitable for off-line programming of the control system.

In this study, the inverse kinematics model of a special five-bar mechanism is established by using vector analysis and Euler’s formula. All equations are explicit equations, and the motion parameters of the control bar are expressed directly by the motion parameters of the drive bar and output point angular displacement and angular velocity.

2. Methodology

2.1. Physical Model

A hybrid-driven special five-bar mechanism has 6 links, \( l_1, l_2, l_3, l_4, l_5 \) and \( l_6 \) which are hinged through points A, B, C, D and E. Link \( l_1 \) is driven by the RTNA motor which provides major driving power and determines the cutting time. Link \( l_4 \) is driven by the RTA motor which acts as a low power motion modulation device and it adjusts angles \( \theta_4 \) to change the ellipse trajectory. Point P is the output point. Point A is the origin of the Cartesian coordinate system xAy, as shown in Fig. 1.

The known conditions and parameters,

- Drive link \( l_1 \)
- Control link \( l_4 \)
- All link lengths \( l_1, l_2, l_3, l_4, l_5 \) and \( l_6 \)
- Output point P coordinate \((x_p, y_p)\)
Drive link \( l_1 \) angular velocity \( \dot{\theta}_1 \) and angular acceleration \( \ddot{\theta}_1 \)
Unknown parameters to be solved,
Drive link \( l_i \) position
Control link \( l_i \) position, angular velocity and angular acceleration
Other unknown parameters related to other links and output point \( P \).

Figure 1. Physical Model of Hybrid-Driven Special Five-Bar Mechanism.

2.2. Position Equation
From the known conditions, Point \( P \) coordinate \((x_p, y_p)\) may be converted into the polar coordinate \((l_{AP}, \theta_p)\), where \( l_{AP} \) and \( \theta_p \) are respectively the distance of Point \( P \) to the origin and its phase angle in the coordinate system. Therefore,

\[ l_{AP} = \sqrt{x_p^2 + y_p^2} \]
\[ \theta_p = \arctan \left( \frac{y_p}{x_p} \right) \]

The five-bar linkage mechanism needs two vector loop-closure equations to represent the mathematical model of its components. The vector loop-closure equations can be obtained by triangle \( ABP \) and closed polygon \( ABCDEA \),

\[ \overrightarrow{AP} = \overrightarrow{AB} + \overrightarrow{BC} + \overrightarrow{CP} \]
\[ \overrightarrow{AB} + \overrightarrow{BC} = \overrightarrow{AE} + \overrightarrow{ED} + \overrightarrow{DC} \]

The above vector loop-closure equations may be expressed in complex equations:

\[ l_{AP} e^{i\theta_p} = l_1 e^{i\theta_1} + (l_2 + l_6) e^{i\theta_2} \] \hspace{1cm} (1)
\[ l_1 e^{i\theta_1} + l_2 e^{i\theta_2} = l_5 + l_4 e^{i\theta_4} + l_3 e^{i\theta_3} \] \hspace{1cm} (2)

Where \( \theta_1, \theta_2, \theta_3 \) and \( \theta_4 \) are respectively the angular displacement of link \( l_1, l_2, l_3 \) and \( l_4 \) relative to axis \( X \) around its center of rotation; \( \theta_5 \) is the angular displacement of Point \( P \) relative to axis \( X \) and around its center of rotation, i.e, the coordinate origin.

According to Euler's formula, the real parts and imaginary parts at each side of Eq.1 as well as Eq.2 are equal,

\[ l_{AP} \cos \theta_p = l_1 \cos \theta_1 + (l_2 + l_6) \cos \theta_2 \] \hspace{1cm} (3)
\[ l_{AP} \sin \theta_p = l_1 \sin \theta_1 + (l_2 + l_6) \sin \theta_2 \]
\[ l_1 \cos \theta_1 + l_2 \cos \theta_2 = l_5 + l_4 \cos \theta_4 + l_3 \cos \theta_3 \]
\[ l_1 \sin \theta_1 + l_2 \sin \theta_2 = l_5 + l_4 \sin \theta_4 + l_3 \sin \theta_3 \] \hspace{1cm} (4)

Eq.3 may be simplified after the elimination of \( \theta_2 \).
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\[ A \cos \theta_i + B \sin \theta_i + C = 0 \]  

(5)

Where

\[ A = -l_{xp} \cos \theta_p = -x_p'; \]
\[ B = -l_{yp} \sin \theta_p = -y_p'; \]
\[ C = [A^2 + B^2 + l_1^2 - (l_2 + l_0)^2] / (2l_1) = [l_{xp}^2 + l_1^2 - (l_2 + l_0)^2] / (2l_1). \]

Using below half-angle trigonometric formula,

\[ \sin \theta_i = \frac{2 \tan(\theta_i / 2)}{1 + \tan^2(\theta_i / 2)} \]
\[ \cos \theta_i = \frac{1 - \tan^2(\theta_i / 2)}{1 + \tan^2(\theta_i / 2)} \]

By replacing \( \sin \theta_i \) and \( \cos \theta_i \) in Eq.5, a quadratic equation with only unknown parameter \( \theta_1 \) is obtained. Thereby, the solutions for \( \theta_1 \) are obtained as follows:

\[ \theta_1 = 2a \tan \frac{B \pm \sqrt{A^2 + B^2 - C^2}}{A - C} \]

(The optional plus or minus sign in the solution shows that there are two solutions, which are related to the initial pose of the mechanism.)

According to \( \theta_1 \) and Eq.3, \( \theta_2 \) is obtained as follows:

\[ \theta_2 = \tan \frac{B_1 + l_1 \sin \theta_1}{A_1 + l_1 \cos \theta_1} \]

Similarly, \( \theta_3 \) and \( \theta_4 \) is obtained from Eq.4 as follows:

\[ \theta_3 = \tan \frac{B_2 + l_2 \sin \theta_2}{A_2 + l_2 \cos \theta_2} \]
\[ \theta_4 = \tan \frac{B_3 + l_3 \sin \theta_3}{A_3 + l_3 \cos \theta_3} \]

Where

\[ A_0 = l_0 - \cos \theta_i - l_2 \cos \theta_2; \]
\[ B_0 = l_0 - \cos \theta_i - l_2 \cos \theta_2; \]
\[ C_0 = (A_0^2 + B_0^2 + l_1^2 - l_1^2) / (2l_1) . \]

With the solutions to \( \theta_1 \sim \theta_4 \), the Cartesian coordinate of each hinge point of the mechanism is determined.

2.3 Velocity Equation

The first-order derivatives of time are derived from Eq.1 and Eq.2 respectively as follows:

\[ l_{xp} \dot{\theta}_p e^{i\theta_p} = l_1 \dot{\theta}_1 e^{i\theta_1} + (l_2 + l_0) \dot{\theta}_2 e^{i\theta_2}; \]  

(6)

\[ l_1 \dot{\theta}_1 e^{i\theta_1} + l_2 \dot{\theta}_2 e^{i\theta_2} = l_3 \dot{\theta}_3 e^{i\theta_3} + l_4 \dot{\theta}_4 e^{i\theta_4}; \]

(7)

Where \( \dot{\theta}_1, \dot{\theta}_2, \dot{\theta}_3 \) and \( \dot{\theta}_4 \) are the angular velocities of link \( l_1, l_2, l_3, \) and \( l_4 \) relative to its center of rotation respectively; \( \dot{\theta}_p \) is the angular velocity of output point \( P \) relative to its center of rotation, i.e. the coordinate origin.

Multiply both sides of Eq.6 by \( e^{-i\theta_0} \):
Expand the above equation using Euler’s formula to form a new equation. Let the real part at the left side of the new equation equal to that at the right side, and can derive $\dot{\theta}_p$ as follows:

$$\dot{\theta}_p = \dot{\theta}_1 \cdot \frac{l_1 \sin(\theta_1 - \theta_2)}{l_2 \sin(\theta_2 - \theta_3)}$$

Multiply both sides of Eq.6 by $e^{-i\theta_0}$:

$$l_{1p} \dot{\theta}_p e^{i(\theta_0)} = l_{1p} \dot{\theta}_1 e^{i(\theta_0)} + (l_2 + l_3) \dot{\theta}_2 e^{i(\theta_0)}$$

Expand the above equation using Euler’s formula to form a new equation. Let the real part at the left side of the new equation equal to that at the right side, and can derive $\ddot{\theta}_1$ as follows:

$$\ddot{\theta}_2 = \ddot{\theta}_1 \cdot \frac{l_1 \sin(\theta_1 - \theta_2)}{(l_2 + l_3) \sin(\theta_2 - \theta_3)}$$

Similarly, $\ddot{\theta}_3$ and $\ddot{\theta}_4$ are obtained from Eq.7 as follows:

$$\ddot{\theta}_3 = \ddot{\theta}_2 \cdot \frac{l_1 \sin(\theta_2 - \theta_3)}{l_1 \sin(\theta_2 - \theta_3)}$$

$$\ddot{\theta}_4 = \ddot{\theta}_3 \cdot \frac{l_1 \sin(\theta_3 - \theta_4)}{l_1 \sin(\theta_3 - \theta_4)}$$

### 2.4 Acceleration Equation

The first-order derivatives of time are obtained from Eq.6 and Eq.7 respectively as follows:

$$l_{1p} \dot{\theta}_p e^{i\theta_0} - l_{1p} \ddot{\theta}_p e^{i\theta_0} = l_{1p} \dot{\theta}_1 e^{i\theta_0} + (l_2 + l_3) \dot{\theta}_2 e^{i\theta_0}$$

$$l_{1p} \dot{\theta}_p e^{i\theta_0} - l_{1p} \ddot{\theta}_p e^{i\theta_0} = l_{1p} \dot{\theta}_1 e^{i\theta_0} + (l_2 + l_3) \dot{\theta}_2 e^{i\theta_0}$$

Where

$\ddot{\theta}_1$, $\ddot{\theta}_2$, $\ddot{\theta}_3$ and $\ddot{\theta}_4$ are the angular accelerations of link $l_1$, $l_2$, $l_3$, and $l_4$ relative to its center of rotation respectively; $\ddot{\theta}_p$ is the angular acceleration of output point P relative to its center of rotation, i.e. the coordinate origin.

Multiply both sides of Eq.8 by $e^{-i\theta_0}$ and $e^{i\theta_0}$ respectively, and $\ddot{\theta}_p$ and $\ddot{\theta}_2$ are obtained according to Euler’s formula as follows:

$$\ddot{\theta}_p = \frac{l_{1p} \dot{\theta}_1 \sin(\theta_1 - \theta_2) - l_{1p} \dot{\theta}_2 \cos(\theta_2 - \theta_3) + l_{1p} \dot{\theta}_3 \cos(\theta_3 - \theta_4) + (l_2 + l_3) \ddot{\theta}_2}{l_{1p} \sin(\theta_1 - \theta_2)}$$

$$\ddot{\theta}_2 = \frac{l_{1p} \dot{\theta}_1 \sin(\theta_1 - \theta_2) - l_{1p} \dot{\theta}_2 \cos(\theta_2 - \theta_3) + l_{1p} \dot{\theta}_3 \cos(\theta_3 - \theta_4) + (l_2 + l_3) \ddot{\theta}_2}{(l_2 + l_3) \sin(\theta_2 - \theta_3)}$$

Similarly, $\ddot{\theta}_3$ and $\ddot{\theta}_4$ are derived from Eq.9 as follows:

$$\ddot{\theta}_3 = \frac{l_{1p} \dot{\theta}_1 \sin(\theta_1 - \theta_2) + l_{1p} \dot{\theta}_2 \cos(\theta_2 - \theta_3) + l_{1p} \dot{\theta}_3 \cos(\theta_3 - \theta_4) + l_{1p} \dot{\theta}_4 \cos(\theta_4 - \theta_5)}{l_{1p} \sin(\theta_2 - \theta_3)}$$

$$\ddot{\theta}_4 = \frac{l_{1p} \dot{\theta}_1 \sin(\theta_1 - \theta_2) + l_{1p} \dot{\theta}_2 \cos(\theta_2 - \theta_3) + l_{1p} \dot{\theta}_3 \cos(\theta_3 - \theta_4) + l_{1p} \dot{\theta}_4 \cos(\theta_4 - \theta_5)}{l_{1p} \sin(\theta_2 - \theta_3)}$$
3. Simulation analysis and discussion

3.1 Numerical Example

In this study, the same parameters as those used in Ref. 12 are used to verify the mathematical model, i.e. \( l_1=7 \) m, \( l_2=18 \) m, \( l_3=17 \) m, \( l_4=13 \) m, \( l_5=5 \) m, and \( l_6=2 \) m. Drive bar \( l_1 \) rotates at a constant angular velocity \( \dot{\theta}_1=6 \) rad/s. \( t \) is time variable in seconds. The length of major semi-axis \( a \) and semi-minor axis \( b \) of the elliptical motion trajectory of output point P are respectively 20 m and 14 m. The original Cartesian coordinate of point P is (0, 14).

By using MATLAB, all motion parameters of control bar \( l_4 \) are obtained. The curves of \( \theta_4 \), \( \dot{\theta}_4 \) and \( \ddot{\theta}_4 \) are shown in Fig. 2 to Fig. 4 respectively.

![Figure 2. Control Bar \( l_4 \) Angular Displacement \( \theta_4 \) calculated by MATLAB When \( a=20, b=14 \).](image1)

![Figure 3. Control Bar \( l_4 \) Angular Velocity \( \dot{\theta}_4 \) calculated by MATLAB When \( a=20, b=14 \).](image2)

![Figure 4. Control Bar Angular Acceleration \( \ddot{\theta}_4 \) calculated by MATLAB When \( a=20, b=14 \).](image3)

From Fig. 2 to Fig. 4, the angular velocity and acceleration of control bar \( l_4 \) change twice rapidly and simultaneously during a 360-degree rotation period of drive bar \( l_1 \) and both changes occur near the semi-minor axis of the elliptical motion trajectory of output point P. Given that the length of semi-major axis and the semi-minor axis of the elliptical motion trajectory of output point P are respectively 20 m and 14 m, the original Cartesian coordinate of point P is (0, 14), and drive bar \( l_1 \) rotates at a constant angular velocity \( \dot{\theta}_1=6 \) rad/s, control bar \( l_4 \) rotates 360 degrees just at 60s. The angular displacement of control bar \( l_4 \) rapidly increases at time \( t=2-3s \) and \( t=31-32s \), and its angular velocity and angular
acceleration peak at \( \dot{\theta}_{4\max} = 2.78571 \text{ rad/s} \) and \( \ddot{\theta}_{4\max} = 38.7712 \text{ rad/s}^2 \) simultaneously at time \( t = 32.5 \text{ s} \). Immediately after that, the angular velocity of control bar \( l_4 \) drops sharply, and its angular acceleration decrease sharply to \( \ddot{\theta}_{4\min} = -31.3051 \text{ rad/s}^2 \) as minimum.

3.2 Simulation and Verification
In order to verify the mathematical model, an ADAMS model for the special five-bar mechanism is established using the parameters above (See Fig. 5). The curves of \( \theta_4 \), \( \dot{\theta}_4 \) and \( \ddot{\theta}_4 \) calculated by ADAMS are illustrated in Fig. 6 - Fig. 8 respectively.

Figure 5. Physical Model of the Special Five-Bar Mechanism in ADAMS.

Figure 6. Angular Displacement \( \theta_4 \) of Control Bar \( l_4 \) Calculated by ADAMS When \( a=20, b=14 \).

Figure 7. Angular Velocity \( \dot{\theta}_4 \) of Control Bar \( l_4 \) Calculated by ADAMS When \( a=20, b=14 \).

Figure 8. Angular Acceleration \( \ddot{\theta}_4 \) of Control Bar \( l_4 \) Calculated by MATLAB When \( a=20, b=14 \).
In Fig. 6 - Fig. 8, the angular velocity and angular acceleration of control bar \( l_4 \) change twice rapidly and simultaneously during a 360-degree rotation period of drive bar \( l_1 \) and both changes occur near the semi-minor axis of the elliptical motion trajectory of output point P. Given the semi-major axis and the semi-minor axis of the elliptical motion trajectory of output point P are respectively 20 m and 14 m, the original Cartesian coordinate point P is (0,14), and drive bar \( l_1 \) rotates at a constant angular velocity \( \dot{\theta}_1 = 6 \) rad/s, control bar \( l_4 \) rotates 360 degrees just at 60s. The angular displacement of control bar \( l_4 \) rapidly increases at time of \( t=2-3 \)s and \( t=31-32 \)s, and its angular velocity and angular acceleration peak at \( \dot{\theta}_{4\text{max}} = 2.8437 \) rad/s and \( \ddot{\theta}_{4\text{max}} = 40.0459 \) rad/s\(^2\) respectively at time \( t=32.6 \)s. Immediately after that, the angular velocity of control bar \( l_4 \) drops sharply, and its angular acceleration rapidly decreases to \( \dot{\theta}_{4\text{min}} = -31.8072 \) rad/s\(^2\) as minimum.

Comparing the results from the numerical example and the ADAMS simulation, it is determined that all corresponding motion parameters of control bar \( l_4 \) from both methods changes with time \( t \) in the same patterns, and the motion law of control bar \( l_4 \) from the mathematical model that from the ADAMS simulation coincides. Under the given conditions in Section 3 and 4, the maximum parameter deviation from two methods is less than 5%. Therefore, the accuracy of the mathematical model established for the special five-bar mechanism is sufficient.

4. Conclusion
To reduce iterative calculation in the control software of the hybrid-driven special five-bar mechanism, the inverse kinematics model of the mechanism based on vector analysis method and Euler’s formula was established. A numerical example and a computer simulation were conducted to verify the mathematics model. The following conclusions could be drawn:

(1) Comparing the results of the numerical example and ADAMS simulation, it is determined that all corresponding motion parameters of control bar \( l_4 \) from both methods changes with time \( t \) in the same patterns, and the motion law of control bar \( l_4 \) from the mathematical model coincides well with that of the ADAMS simulation. The maximum parameter deviation from two methods is less than 5%. Therefore, the correctness of the mathematical model established for the special five-bar mechanism has been proven.

(2) The inverse kinematics model of the mechanism is expressed in explicit equations and establishes the direct relations between the output motion and the input motions and thus is more suitable for the control system off-line programming of the shield machine. Moreover, the inverse kinematics model derived from vector analysis method and Euler’s formula is simpler. It can directly calculate the motion parameters of the drive bar and the control bar by using the motion parameters of the output point, and it also provides a mathematical basis for the dynamic analysis of the special five-bar mechanism.

(3) The numerical example and ADAMS simulation were only used to verify the correctness of the mathematical model. If the mathematical model is to be used in the control system of actual mechanisms, the calculation accuracy shall be further improved.

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