No radiative corrections to the Carroll-Field-Jackiw term beyond one-loop order

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Abstract
We demonstrate explicitly the absence of the quantum corrections to the Carroll-Field-Jackiw (CFJ) term beyond one loop.

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The Carroll-Field-Jackiw (CFJ) term $\mathcal{L}_{CFJ} = \epsilon^{abcd}k_a A_b \partial_c A_d$, originally introduced in [1], is certainly the most known and studied example of a Lorentz-breaking term. In [2], this term, for the first time, has been shown to arise as a one-loop quantum correction in an appropriate Lorentz-breaking extension of QED, and turned out to be finite. Further, a number of results related to this term, especially to its quantum generation, have been obtained. Besides being finite, the most important feature of this term is the fact that it is ambiguous in the one-loop approximation. From the formal viewpoint, its ambiguity is related with the fact that, actually, the Feynman diagrams corresponding to this term are superficially divergent, so that one faces an undetermined expression like $\infty - \infty$. From the physical viewpoint, the ambiguity of this term in the one-loop approximation is related with the presence of the Adler-Bell-Jackiw (ABJ) anomaly (see [3]). Further, various results for this term have been obtained within different calculation schemes (for a detailed discussion, see f.e. [4] and references therein). However, all these studies have been performed at the one-loop order. Moreover, up to now, there is no examples of higher-loop calculations in Lorentz-breaking theories. In this paper, we take the first step towards this study. Explicitly, our aim in this paper is to argue that the one-loop result for the CFJ term is the unique possible, i.e., that there is no higher-loop CFJ contribution. The proof we develop is based on gauge symmetry grounds.

Our starting point is the simplest Lorentz-breaking extension of QED introduced already in [2]:

$$\mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \bar{\psi} \not{\partial} \psi - \bar{\psi} \gamma^{\mu} A_{\mu} \psi - \bar{\psi} b_{\gamma 5} \psi,$$

in which $b_{\mu}$ is a constant axial four-vector. Actually, it will be easy to see that the proof we present in the paper still holds if we introduce Lorentz-violating terms in the gauge sector of the Lagrangian density (1) as well.

It is well known that, in the theory described by (1), the CFJ term is induced at one loop by radiative corrections arising from Lorentz-CPT violation in the fermionic sector [2]. Here, we will show that the Lorentz-violating contribution to the polarization tensor $\Pi^{\mu\nu}(p)$ in the two-loop approximation is of second order in external momenta and, hence, there is no two-loop correction to the CFJ term. Actually, the result has very solid foundations since the proof is based on the Ward identities for the vertex function $\Gamma^{(n)}_{\mu_1, \cdots, \mu_n}(p_1, \cdots, p_n)$ with $n$ external photon lines, looking like

$$\Pi_{\mu}^{\mu_1} \Gamma^{(n)}_{\mu_1, \cdots, \mu_n}(p_1, \cdots, p_n) = 0,$$

due to transversality of all quantum corrections to any $n$-point function of the gauge field. Here, the indices in the momenta $p_i$ of the external photons take values $i = 1, 2, \cdots, n$. As we will
argue, the method used in the two-loop proof can be generalized to higher orders, leading us to
the conclusion that the CFJ term is not induced beyond one-loop order.

The main premise we assume is that the vertex functions $\Gamma^{(n)}$ are analytic in the limit of zero external momenta. In order to avoid infrared singularities in this limit, we maintain $m \neq 0$ in the fermion sector. At the same time, it is well known that singularities arising from photon propagators inside an arbitrary Feynman diagram do not cause infrared problems. Of course, we also assume that the regularization used to handle divergent integrals is gauge invariant.

Next, we will follow the methodology used in [5] and show that for $n > 2$, the vertex functions $\Gamma^{(n)}_{\mu_1, \cdots, \mu_n}(p_1, \cdots, p_n)$ are $\mathcal{O}(p_1 p_2)$, where $p_1$ and $p_2$ are independent external momenta. Indeed, differentiating (2) with respect to $p_1$ and then setting $p_1$ to zero we find

$$\Gamma^{(n)}_{\mu_1, \cdots, \mu_n}(0, p_2, \cdots, p_n) = 0. \quad (3)$$

Therefore, it follows from the Taylor expansion that

$$\Gamma^{(n)}_{\mu_1, \cdots, \mu_n}(p_1, p_2, \cdots, p_n) = \mathcal{O}(p_1). \quad (4)$$

Since we have chosen $p_1$ and $p_2$ to be independent variables, which is possible for $n > 2$, we can apply these arguments as well to $p_2$ and find

$$\Gamma^{(n)}_{\mu_1, \cdots, \mu_n}(p_1, p_2, \cdots, p_n) = \mathcal{O}(p_2). \quad (5)$$

As the conditions (4) and (5) are independent and must be simultaneously satisfied, we conclude that

$$\Gamma^{(n)}_{\mu_1, \cdots, \mu_n}(p_1, p_2, \cdots, p_n) = \mathcal{O}(p_1 p_2). \quad (6)$$

This is the key relation we need to the proof.

Actually, two-loop Feynman diagrams can be written in terms of the four point vertex function $\Gamma^{(4)}_{\mu_\sigma \rho \nu}(p_1, p_2, p_3, p_4) \equiv \Gamma^{(4)}_{\mu_\sigma \rho \nu}$. We present this relation schematically in Figure 1. A fermion loop within the four-point vertex function is obtained by cutting the internal photon line of the two-loops Feynman diagrams shown in this figure. More precisely, we can write the Lorentz-violating contribution to the polarization tensor at the two-loop order as

$$\Pi^{2\text{loops}}_{\mu \nu}(p) = \int \frac{d^4 q}{(2\pi)^4} \Delta F(q) \Gamma^{(4)}_{\mu_\sigma \rho \nu} \bigg|_{p_3=p_4=0}, \quad (7)$$

where in the expressions above we choose $p_1 = -p_2 = p$ and

$$\Gamma^{(4)}_{\mu_\sigma \rho \nu} \bigg|_{p_3=p_4=0} = \left[ \Gamma^{(4a)}_{\mu_\sigma \rho \nu} + \Gamma^{(4b)}_{\mu_\sigma \rho \nu} + \Gamma^{(4c)}_{\mu_\sigma \rho \nu} + \Gamma^{(4d)}_{\mu_\sigma \rho \nu} + \Gamma^{(4e)}_{\mu_\sigma \rho \nu} + \Gamma^{(4f)}_{\mu_\sigma \rho \nu} + \Gamma^{(4g)}_{\mu_\sigma \rho \nu} \right]_{p_3=p_4=0}. \quad (8)$$
The Feynman diagrams corresponding to each term in this expression are shown in Figure 2. The crosses in the fermion lines in the loop represent an arbitrary number of insertions of the vertex $\bar{\psi} b^\gamma \gamma_5 \psi$. Of course, the proof still hold if we insert an arbitrary number of Lorentz-violating vertices in any of the fermion lines of the diagrams. For simplicity, within our study we have omitted the usual two-loop Feynman diagrams of QED (without Lorentz-violating insertions). Finally, applying the result given in equation (5) for the two-loop contribution given in [S], we immediately conclude that $\Pi^{2\text{loops}}_{\mu\nu}(p)$ is $O(p^2)$. So, the CFJ term is not induced at two-loop order.

In fact, the procedure used at the two-loop order is easily generalized for high orders. For example, three-loops contributions as showed in Figure 3 can be expressed in terms of a Feynman diagram of the six point vertex functions $\Gamma^{(6)}_{\mu\sigma\rho\nu}(p_1, p_2, \cdots, p_6)$. Following this procedure, order-by-order in loops, we shall conclude that there is no corrections to the CFJ term beyond one-loop.

Let us discuss our results. Using the properties of the Feynman diagrams, we proved the absence of a CFJ-like correction at the two-loop order. The essence of our proof can be formulated as follows. Any two-loop contribution to the two-point function of the gauge field can be obtained as a contraction of external legs in some one-loop four-point function of the gauge field. Since, as we already noted, each such four-point function, by gauge invariance reasons, is at least quadratic in external momenta, we note that, after the contraction of some gauge fields into the propagators, the corresponding two-point function of the gauge field will also be at least quadratic in the external momentum and, thus, will not contribute to the CFJ term. Clearly, the same argumentation can be
FIG. 2: Terms with one Lorentz-violation insertion that contribute to the polarization tensor at two-loops written as a sum of Feynman diagram of the four-point vertex function. The diagrams are obtained just by cutting internal lines of two-loops diagrams as those of figure 1. The boxes appearing in fermion lines indicate the insertion of an arbitrary number of the vertex $\bar{\psi}\gamma^{\mu}\gamma^{5}\psi$

FIG. 3: Three loops contribution in the polarization tensor. We obtained a fermion loop of the six-point vertex function by cutting the internal photon lines of the three-loops diagram.

applied for higher-point contributions to the one-loop effective action of the gauge field, which also are at least quadratic in external momenta by gauge symmetry reasons. After performing similar contractions of some external gauge legs to the propagators, we arrive at higher-loop results which are also quadratic in the external momentum and, hence, do not contribute to the CFJ term. Effectively, we proved that the unique possible correction to the CFJ term is of one-loop order. Similar argumentation can be applied to non-Abelian theories as well.

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