Unparticle Physics Effects on $D^0 - \bar{D}^0$ Mixing

Xue-Qian Li* and Zheng-Tao Wei†

Department of Physics, Nankai University, Tianjin 300071, China

Abstract

The mixing of $K^0 - \bar{K}^0$, $D^0 - \bar{D}^0$ and $B^0_{(s)} - \bar{B}^0_{(s)}$ provides a sensitive probe to explore new physics beyond the Standard Model. The scale invariant unparticle physics recently proposed by Georgi can induce flavor-changing neutral current and contribute to the mixing at tree level. We investigate the unparticle effects on $B^0 - \bar{B}^0$ and $D^0 - \bar{D}^0$ mixing. Especially, the newly observed $D^0 - \bar{D}^0$ mixing sets the most stringent constraints on the coupling of the unparticle to quarks.

* Email: lixq@nankai.edu.cn
† Email: weizt@nankai.edu.cn
I. INTRODUCTION

It is well-known that scale invariance is broken by renormalization and dimensional parameters in quantum field theories. The concept of scale invariance (and more generally the conformal symmetry) may still play an important role in high energy physics. For an asymptotically free theory, such as QCD, the scale invariance is recovered in the high energy limit. In the concerned practical physics processes of high energies, breaking of scale invariance can be systematically incorporated in the anomalous dimensions of operators using the renormalization group method [1]. It is indicated that the scale invariance in the infrared region may be quite different and less known [2]. But the idea of scale invariance is so simple and attractive that there is no a priori to repel it from our world.

In [3], Georgi proposed that a scale invariant stuff contains no particle, but the so-called unparticle. The unparticle possesses some properties which are different from that of ordinary particles. The first aspect is that it has a non-trivial scale dimension $d_U$. The dimension of unparticle is in general fractional rather than an integral number (the dimension for a fermion is half-integral). The fractional dimension must come from some complicated dynamics whose details are unknown at present. Another aspect is that the free unparticle has no definite mass. That means that the Lorentz-invariant four-momentum square $P^2$ is not fixed for a real unparticle. Georgi observed that unparticle with scale dimension looks like a non-integral number $d_U$ of invisible massless particles [3]. To be consistent with the present experimental observations, the coupling of unparticle to the ordinary Standard Model (SM) matter must be sufficiently weak. However, it may be relevant to the TeV physics and might be explored at the LHC and ILC. The interactions between the unparticle and the SM particles are described in the framework of low energy effective theory and lead to various interesting phenomena. There have been some phenomenological explorations on possible observable effects caused by unparticles [3, 4, 5, 6, 7, 8, 9, 10].

The mixing of $K^0 - \overline{K^0}$, $D^0 - \overline{D^0}$ and $B^0_{(s)} - \overline{B^0_{(s)}}$ is of fundamental importance to test the SM and explore new physics beyond the SM. In the scenarios of new physics, there may exist a flavor-changing neutral current (FCNC) to result in such a mixing which can only be realized via loops in the framework of the SM. Thus this observable could be sensitive to new physics effects. In fact, many authors used to explore evidence of new physics in $B^0 - \overline{B^0}$ (or $B^0_s - \overline{B^0_s}$) mixing because data about the mixing have been available for a long while. In the proposed scenario [3], the unparticle can couple to different flavors of quarks and induce FCNC even at tree level as long as the unparticle is neutral. Thus it will cause new contributions to the particle-antiparticle mixing, $B^0 - \overline{B^0}$, $D^0 - \overline{D^0}$ mixing. Generally, based on physics conjecture, the energy scale concerning unparticle is high that it should cause smaller influence on the $K^0 - \overline{K^0}$ mixing, especially the SM contribution to the mixing obviously dominates. The unparticle effects on $B^0_{(s)} - \overline{B^0_{(s)}}$ mixing had been studied in [6, 7] roughly. Since the $B^0_{(s)} - \overline{B^0_{(s)}}$ mixing parameter $x_{B_{d(s)}}$ is large and generally the
contributions from the SM dominate, and the new physics effect if it exists, is less important, thus the observable is not so sensitive to the new physics. Whereas for the D system, the SM contribution is confirmed to be sufficiently small, and the $D^0 - \bar{D}^0$ mixing parameter (the SM prediction is $x_D < 10^{-3}$ [11]) must not be measured by the present experiments, if there is no new physics. By contraries, if sizable mixing is measured, new physics should exist and make main contributions. It is interesting that recently the $D^0 - \bar{D}^0$ has indeed been measured by the Babar and Belle collaborations [12, 13], which may be a signature of existence of new physics. He and Valencia [14] suggested that the mixing is due to the FCNC in the up-type-quark sector for non-universal $Z'$ model and obtained constraints on the model parameters by fitting the data. Instead, we propose that the unparticle scenario is the new physics which is responsible for the observable $D^0 - \bar{D}^0$ mixing.

In this study, we will investigate the effects of the unparticle physics on the neutral meson mixing including $B_s^0 - \bar{B}_s^0$, $D^0 - \bar{D}^0$ and $K^0 - \bar{K}^0$ mixing and constrain the coupling parameter of the concerned interactions between the unparticles and the SM quarks.

II. $M^0 - \bar{M}^0$ MIXING IN UNPARTICLE PHYSICS

We start with a brief review about the unparticle scenario. It is assumed that the scale invariant unparticle fields emerge below an energy scale $\Lambda_U$ which is at the order of TeV [3]. The interactions of the unparticle with the SM particle are described by a low energy effective theory. For our purpose, the coupling of unparticle to quarks is given by following the standard strategy to construct effective interactions as

$$\frac{c_{S}^{q'q}}{\Lambda_U^{d_U}} q' \gamma_\mu (1 - \gamma_5) q O_\mu + \frac{c_{V}^{q'q}}{\Lambda_U^{d_U - 1}} q' \gamma_\mu (1 - \gamma_5) q O_\mu + h.c. \quad (1)$$

where $O_\mu$ and $O_\mu'$ denote the scalar and vector unparticle fields, respectively. The $c_S^{q'q}$ and $c_V^{q'q}$ are dimensionless coefficients and they depend on different flavors in general. If the $q$ and $q'$ belong to the same up- or down-type quark sectors, the above effective interactions may induce FCNC transitions and provide new physics contribution to the neutral meson mixing. In order to simplify the phenomenological analysis, we use the same coefficient for all flavors, $c_S^{q'q} \rightarrow c_S$ and $c_V^{q'q} \rightarrow c_V$. Relaxing this restriction does not change our conclusions.

In this study, we are only interested in the effects of the unparticle field which serves as an intermediate agent in the FCNC transition, thus it only appears as a propagator with momentum $P$ and scale dimension $d_U$. The propagator for the scalar unparticle field is given by [4, 5]

$$\int d^4 x e^{i P \cdot x} \langle 0 | T O_{\mu}(x) O_{\mu}(0) | 0 \rangle = i \frac{A_{d_U}}{2 \sin(d_U \pi)} \frac{1}{(P^2 + i\epsilon)^{2 - d_U}} e^{-i(d_U - 2)\pi}, \quad (2)$$
where
\[ A_{dU} = \frac{16\pi^{5/2}}{(2\pi)^{2dU}} \frac{\Gamma(dU + 1/2)}{\Gamma(dU - 1)\Gamma(2dU)}. \] (3)

The function \(\sin(dU\pi)\) in the denominator implies that the scale dimension \(dU\) cannot be integral for \(dU > 1\) in order to avoid singularity. The phase factor \(e^{-i(dU-2)\pi}\) provides a CP conserving phase which produces peculiar interference effects in high energy scattering processes \([4]\), Drell-Yan process \([5]\) and CP violation in B decays \([7]\). The propagator for the vector unparticle is similarly given by
\[
\int d^4x e^{iP \cdot x} \langle 0 | TO_{dU}(x)O_{dU}'(0) | 0 \rangle = i \frac{A_{dU}}{2\sin(dU\pi)} \frac{-\eta^{\mu\nu} + P^{\mu}P^{\nu}/P^2}{(P^2 + i\epsilon)^{2-dU}} e^{-i(dU-2)\pi}, \] (4)

where the transverse condition \(\partial_\mu O_{dU}' = 0\) is used.

The neutral meson is denoted by \(M^0(q\bar{q}')\) and its antiparticle \(\overline{M^0}(q'\bar{q})\). The mixing occurs via a transition \(q\bar{q}' \rightarrow q'\bar{q}\) at the quark level. In the SM, these FCNC processes can only be realized at loop orders. The lowest contribution which results in the \(M^0 - \overline{M^0}\) mixing is the box diagrams. With the unparticle scenario, the FCNC transitions can occur at tree level and they are depicted in Fig. 1. The double dashed lines represent the exchanged unparticle fields. There are two diagrams corresponding to t- and s-channel unparticle-exchanges which contribute to the \(M^0 - \overline{M^0}\) mixing.

\[ \Delta m_M \approx 2|M_{12}^M| = \frac{1}{m_M} \langle \overline{M^0}|H_{\text{eff}}(|\Delta F| = 2)|M^0\rangle, \] (5)

where \(|\Delta F| = 2\) represents \(|\Delta B| = 2\) for the \(B^0 - \overline{B^0}\) mixing and \(|\Delta C| = 2\) for the \(D^0 - \overline{D^0}\) mixing. For the D meson system, the above relation is valid under the assumption of CP
conservation. The effective operators which contribute to $\Delta F = 2$ are

$$Q_1 = \bar{q}'\gamma_\mu(1 - \gamma_5)qq'\gamma^\mu(1 - \gamma_5)q,$$

$$Q_2 = \bar{q}'(1 - \gamma_5)qq'(1 - \gamma_5)q.$$  \hfill (6)

We only keep the operators at the tree level and more operators would emerge if QCD corrections are taken into account.

It is noted that the transferred momentum square for t- and s-channels are approximately equal, i.e. $P^2 \approx m_M^2$ for heavy meson system.

Now we are able to give the expressions for the mass difference $\Delta m_M$. The unparticle physics contribution $\Delta m^U_M$ is given as

$$\Delta m^U_M = \frac{5 f^2_M \hat{B}_M}{3 m_M} \frac{A_{d\mu}}{2|\text{sin}d_{U\pi}|} \left( \frac{m_M}{\Lambda_U} \right)^{2d_{d\mu}} |c_S|^2,$$  \hfill (7)

for the scalar unparticle and

$$\Delta m^U_M = \frac{f^2_M \hat{B}_M}{m_M} \frac{A_{d\mu}}{2|\text{sin}d_{U\pi}|} \left( \frac{m_M}{\Lambda_U} \right)^{2d_{d\mu}-2} |c_V|^2.$$  \hfill (8)

for the vector unparticle. Note that in the above expression only the absolute value of the function $\text{sin}d_{U\pi}$ exists. Our results are the same as in [7] and slightly different from [6] by a constant factor. In the above derivations, we have used the relations listed below [15]

$$\langle M^0 | \bar{q}'\gamma_\mu(1 - \gamma_5)qq'\gamma^\mu(1 - \gamma_5)q | M^0 \rangle = \frac{8}{3} f^2_M m^2_M \hat{B}_M,$$

$$\langle M^0 | \bar{q}'(1 - \gamma_5)qq'(1 - \gamma_5)q | M^0 \rangle = \frac{5}{3} f^2_M m^2_M \hat{B}_M.$$  \hfill (9)

where $f_M$ denotes the decay constant and $\hat{B}_M$ is a numerical factor which is related to the non-perturbative QCD and takes different values in various models, but as known, is of order of unity.

Some comments are in order:

(1) The mass difference is proportional to a meson mass dependent factor $m^{2d_{d\mu}}_M$ or $m^{2d_{d\mu}-2}_M$ which comes from the unparticle propagator $\frac{1}{(P^2 - m^2_M)^{2d_{d\mu}-2}}$. This is a peculiar effect caused unparticle physics. The propagator for a heavy particle exchange from other new physics does not depend on the low energy scale $m_M$ in general.

(2) The above analysis is applicable to $B^0 - \bar{B}^0$, $B^0_s - \bar{B}^0_s$ and $D^0 - \bar{D}^0$ mixing. For the K-system, there are large uncertainties due to long-distance effects and the approximations which exist in the theoretical calculations. Thus we will not use the data on $K^0 - \bar{K}^0$ mixing to constrain the unparticle physics parameters.

(3) In this work, following the method commonly adopted in literature to study new physics effects, we assume that the new physics beyond the SM which contributes to the mixing is the unparticle sector. One can write

$$\Delta m^{NP}_M = \Delta m^{exp}_M - \Delta m^{SM}_M,$$  \hfill (10)
where $\Delta m_{NP}^B$ corresponds to the contribution of new physics, i.e. the unparticle in this study. The SM prediction on $\Delta m_B$ has already been precise to two-loop order, and the data are much more accurate than before thanks to the progress in experimental measurements at Babar and Belle. Therefore by the deviation between the SM prediction and measured value, we can set a constraint on the parameters for the unparticle scenario.

Considering an extreme case, let us loosen the above restriction, namely, we postulate that the mixing $B^0 - \bar{B}^0$ is fully due to the unparticle contribution and see what constraints we would obtain on the parameters. Later we will show that such constraints are looser than that from that obtained from $D^0 - \bar{D}^0$ mixing. Therefore, one may not need to take the constraint on the unparticle parameters from the data of $B^0 - \bar{B}^0$ mixing at all.

(4) Because $\frac{\Delta m_{B_s}}{\Delta m_{B_d}} = 34 \gg 1$, $B^0_s - \bar{B}^0_s$ mixing provides a looser constraint compared to the $B^0 - \bar{B}^0$ case.

The unknown parameters about the unparticles are: $\Lambda_U$, $d_U$ and $c_S(c_V)$. In the numerical results, we fix the value of $\Lambda_U$ by $\Lambda_U = 1$ TeV. Other input parameters are: $f_B \sqrt{\hat{B}}_B = 0.2$ GeV [16], $f_D \sqrt{\hat{B}}_D = 0.2$ GeV [15], $\Delta m_{B_d} = 0.507$ ps$^{-1}$ [17]. The recent experiment carried out by the Belle collaborations sets $x_D = \frac{\Delta m_{D}}{\Gamma_D} = (0.80 \pm 0.29(stat.) \pm 0.17(syst.))\%$ for the $D^0 - \bar{D}^0$ [13]. We use $x_D < 10^{-2}$ as the upper bound.

At first, we consider the case with $d_U = 3/2$ and constrain $c_S$ and $c_V$ from $B^0 - \bar{B}^0$ and $D^0 - \bar{D}^0$ mixing. Table I lists the upper bounds for the coupling parameters $c_S$ and $c_V$. The bounds obtained from $D^0 - \bar{D}^0$ are more stringent than that from $B^0 - \bar{B}^0$ especially for the vector coupling $c_V$. This confirms our expectation in the Introduction. The bounds obtained from $D^0 - \bar{D}^0$ mixing are: $|c_S| < 2.1 \times 10^{-2}$ and $|c_V| < 5.0 \times 10^{-4}$.

| TABLE I: The upper bounds of $|c_S|$ and $|c_V|$ with $\Lambda_U = 1$ TeV and $d_U = 3/2$. |
|---|---|---|
| From B-system | From D-system |
| $|c_S|$ | $3.4 \times 10^{-2}$ | $2.1 \times 10^{-2}$ |
| $|c_V|$ | $2.3 \times 10^{-3}$ | $5.0 \times 10^{-4}$ |

Then we consider the case with fixed $c_S$, $c_V$ and study the dependence of the $D^0 - \bar{D}^0$ mixing parameter $x_D$ on the scale dimension $d_U$. Figs. 2 and 3 plot the dependence within the parameter range $1 < d_U < 2$. We find that $x_D$ is very sensitive to $d_U$ and decreases rapidly to zero as $d_U$ increases.

Moreover, we also investigate the case with extending the scale dimension to the region $2 < d_U < 3$ and depict the dependence of $x_D$ on $d_U$ in Figs. 4 and 5. There is no principal difference compared to the $1 < d_U < 2$ case except a considerable change for the coupling parameters $c_S$ and $c_V$ which are required to fit the data.
FIG. 2: The $D^0 - \bar{D^0}$ mixing parameter $x_D$ versus unparticle scale dimension $(1 < d_U < 2)$. The solid line is given for $|c_S| = 1 \times 10^{-2}$ and the dashed line for $|c_S| = 2 \times 10^{-2}$.

FIG. 3: The $D^0 - \bar{D^0}$ mixing parameter $x_D$ versus unparticle scale dimension $(1 < d_U < 2)$. The solid line is given for $|c_V| = 2 \times 10^{-5}$ and the dashed line for $c_V = 5 \times 10^{-5}$.

III. CONCLUSIONS

We have investigated the new physics effects from scale invariant unparticle sectors on the mixing of $B^0 - \bar{B^0}$ and $D^0 - \bar{D^0}$. The exchange of unparticle induces the FCNC transitions at tree level and provides new contribution to the mass difference of the meson mass eigenstates. In principle, FCNC transitions may be caused by other new physics effects which contain heavy massive particles and break the scale invariance. We observe a peculiar effect caused by the exchange of unparticle: the mixing parameter depends non-trivially on the neutral
FIG. 4: The $D^0 - \bar{D}^0$ mixing parameter $x_D$ versus unparticle scaling dimension ($2 < d_U < 3$). The solid line is given for $|c_S| = 10$ and the dashed line for $|c_S| = 20$.

FIG. 5: The $D^0 - \bar{D}^0$ mixing parameter $x_D$ versus unparticle scaling dimension $2 < d_U < 3$. The solid line is given for $|c_V| = 2 \times 10^{-2}$ and the dashed line for $|c_V| = 5 \times 10^{-2}$.

meson mass. This dependence might not occur for the heavy particle exchange from other new physics. We use the data on $B^0 - \bar{B}^0$ and $D^0 - \bar{D}^0$ mixing to constrain the parameters in unparticle scenario. We find that the $D^0 - \bar{D}^0$ mixing provides the most stringent constraint on the coupling of the scalar and vector unparticles to the SM quarks. The upper bounds we obtained from $D^0 - \bar{D}^0$ mixing are: $|c_S| < 2.1 \times 10^{-2}$ and $|c_V| < 5.0 \times 10^{-4}$ if we set the energy scale $\Lambda_U = 1$ TeV and scale dimension $d_U = 3/2$. The dependence of scale dimension $d_U$ shows that the mixing parameter is sensitive to the scale dimension and decreases rapidly by almost two orders of magnitude. The obtained parameters may have important effects
on CP violation in B and D decays.

Acknowledgments

Z. Wei would like to thank Guo-Huai Zhu and Chuan-Hung Chen for valuable discussions. This work was supported in part by NNSFC under contract No. 10475042.

[1] For recent reviews, see: V.M. Braun, G.P. Korchemsky, D. Mueller, Prog. Part. Nucl. Phys. 51, 311-398 (2003) [arXiv: hep-ph/0306057]; S.J. Brodsky, arXiv: hep-ph/0408069.
[2] T. Banks and A. Zaks, Nucl. Phys. B 196, 189 (1982).
[3] H. Georgi, arXiv: hep-ph/0703260.
[4] H. Georgi, arXiv: 0704.2457 [hep-ph].
[5] K. Cheung, W.-Y. Keung, T.-C. Yuan, arXiv: 0704.2588 [hep-ph].
[6] M. Luo, G. Zhu, arXiv: 0704.3532v1 [hep-ph].
[7] C.-H. Chen, C.-Q. Geng, arXiv: 0705.0689 [hep-ph].
[8] Y. Liao, arXiv: 0705.0837 [hep-ph].
[9] G.-J. Ding and M.-L. Yan, arXiv: 0705.0794 [hep-ph].
[10] T.M. Aliev, A.S. Cornell and N. Gaur, arXiv: 0705.1326 [hep-ph].
[11] A.F. Falk, Y. Grossman, Z. Ligeti and A.A. Petrov, Phys. Rev. D 65, 054034 (2002) [arXiv: hep-ph/0110317].
[12] B. Aubert et al. (The Babar Collaboration), arXiv: hep-ex/0703020.
[13] M. Staric, Talk presented at XLII Rencontres de Moriond, La Thuile, Italy, 10-17 March, 2007; K. Abe et al. (The Belle Collaboration), arXiv: hep-ex/0703036.
[14] X. He and G. Valencia, arXiv: hep-ph/0703270.
[15] G. Burdman and I. Shipsey, Ann. Rev. Nucl. Part. Sci. 53, 431-499 (2003) [arXiv: hep-ph/0310076].
[16] A.J. Buras, arXiv: hep-ph/9806471.
[17] W.-M. Yao, et al., Particle Data Group, J. Phys. G33, 1 (2006).