WEAK LENSING AS A CALIBRATOR OF THE CLUSTER MASS-TEMPERATURE RELATION

Dragan Huterer$^1$ and Martin White$^2$

$^1$Department of Physics, Case Western Reserve University, Cleveland, OH 44106
$^2$Departments of Astronomy and Physics, University of California, Berkeley, CA 94720

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ABSTRACT

The abundance of clusters at the present epoch and weak gravitational lensing shear both constrain roughly the same combination of the power spectrum normalization $\sigma_8$ and matter energy density $\Omega_M$. The cluster constraint further depends on the normalization of the mass-temperature relation. Therefore, combining the weak lensing and cluster abundance data can be used to accurately calibrate the mass-temperature relation. We discuss this approach and illustrate it using data from recent surveys.

Subject headings: cosmology: theory – large-scale structure of universe

1. INTRODUCTION

The number density of galaxy clusters as a function of their mass, the mass function, and its evolution can provide a powerful probe of models of large-scale structure. Historically the most important constraint coming from the present day abundance of rich clusters has been the normalization of the linear theory power spectrum of mass density perturbations (e.g., Evrard 1989, Frenk et al. 1990, Bond & Myers 1991, Henry & Arnaud 1991, Lilly 1992, Dukhur & Blanchard 1992, Bahcall & Cen 1993, White, Edsall & Frenk 1993, Viana & Liddle 1996, Viana & Liddle 1999, Henry 2000). The normalization is typically quoted in terms of $\sigma_8$, the rms density contrast on scales 8$h^{-1}$ Mpc, with the abundance constraint forcing models to a thin region in the $\Omega_M$-$\sigma_8$ plane.

Since the mass, suitably defined, of a cluster is not directly observable, one typically measures the abundance of clusters as a function of some other parameter which is used as a proxy for mass. Several options exist, but much attention has been focused recently on the X-ray temperature. Cosmological N-body simulations and observations suggest that X-ray temperature and mass are strongly correlated with little scatter (Evrard, Metzler & Navarro 1996, Bryan & Norman 1998, Eke, Navarro & Frenk 1998, Horner, Mushotzky & Schart 1999, Nevalainen, Markovich & Forman 2004). How well simulations agree with observational results is far from clear, and several issues need to be resolved. On the simulation side there are the usual issues of numerical resolution and difficulties with including all of the relevant physics. On the observational side instrumental effects can be important (especially for the older generation of X-ray facilities) in addition to the worrying lack of a method for estimating "the mass". In this respect it is interesting to note that weak gravitational lensing provides a constraint on a very similar combination of $\Omega_M$ and $\sigma_8$. Therefore, the two constraints can be combined to check for consistency of our cosmological model, to provide a normalization for the M–T relation, to probe systematics in either method and/or to measure other parameters not as yet included in the standard treatments.

While the cluster constraint comes primarily from scales of about $R = 10 h^{-1}$ Mpc, current weak lensing surveys constrain somewhat smaller scales. These surveys probe scales between roughly 1 and 10 arcmin, which for source galaxies located at $z \simeq 1$ in a ΛCDM cosmology corresponds to 0.7 $h^{-1}$ Mpc < $R < 7 h^{-1}$ Mpc. Therefore, weak lensing probes slightly smaller scales than clusters. As lensing surveys push to larger scales the overlap will become even better.

In this paper we argue that a natural application of combining the cluster abundance and weak lensing constraints is to calibrate the M–T relation for galaxy clusters (see also Hu & Kravtsov 2002). In Sec. 2 we define the M–T relation and derive how cluster abundance constraints depend on $\Omega_M$ and $\sigma_8$. In Sec. 3 we illustrate how combining the two constraints can fix the normalization of the M–T relation using two recently obtained data sets. Finally, in Sec. 4 we discuss this approach further.

2. THE MASS-TEMPERATURE RELATION

Throughout we shall be interested in the abundance of massive clusters at low redshifts, so we parameterize the M–T relation as

$$M(T,z) = M_{15} \left( \frac{T}{T_\star} \right)^{3/2} \left( \Delta_c E^2 \right)^{-1/2} \left[ 1 - 2\Omega_\Lambda(z) \Delta_c \right]^{-3/2}$$

(1)

where $M_{15} = 10^{15} h^{-1}$ M$_\odot$, $\Delta_c$ is the mean overdensity inside the virial radius in units of the critical density, which we compute using the spherical top-hat collapse model, and $E^2 = \Omega_M (1+z)^3 + \Omega_\Lambda + \Omega_k (1+z)^2$. $T_\star$ is the normalization coefficient that we seek to constrain; it roughly corresponds to the temperature of a $M = 7.5 \times 10^{13} h^{-1}$ M$_\odot$ cluster. If measured in keV, the value of $T_\star$ is precisely equivalent to $\beta$ from Pierpaoli, Scott & White (2002) and is 1.34 $f_T$ of Bryan & Norman (1998).

Let us explore the sensitivity of cluster abundance on...
of collapsed objects $dn$ per mass interval $d\ln M$ [Press & Schechter 1974], we define $N(M, z) = dn/d\ln M$. Further defining $\nu \equiv \delta_c/\sigma(M, z)$, where $\sigma(M, z)$ is the rms density fluctuation on mass-scale $M$ evaluated at redshift $z$ using linear theory and $\delta_c \approx 1.686$ is the linear threshold overdensity for collapse, we have
\[ N(M, z) = \sqrt{\frac{2}{\pi M}} \frac{d\ln \sigma(M, z)}{d\ln \nu} \nu \exp\left(-\nu^2/2\right) \tag{2} \]
where $\rho_M$ is the present-day matter density. Assuming we are dealing with the current cluster abundance, $z \approx 0$. Following Pen (1998), for the mass scales of interest we can approximate $\sigma(M) \propto M^{-\alpha}$ where $\alpha \approx 0.27$ for the currently popular $\Lambda$CDM cosmology.

Let us examine the dependence of $N$ on $\Omega_M$, $\sigma_8$ and $M$. Ignoring the term $d\ln \sigma/d\ln \nu$ (which slowly varies) one obtains
\[ \frac{\delta N}{N} = \frac{\delta \sigma_8}{\sigma_8} (1 - \alpha + \nu^2 \alpha) + \frac{\delta \Omega_M}{\Omega_M} (1 - \alpha + \nu^2 \alpha) \tag{3} \]
Setting the left-hand side to zero and using the fact that $\delta M/M = -3/2 \delta T_c/T_c$ for our fiducial cosmology and massive clusters ($M \sim 10^{15} h^{-1} M_\odot$, or $\nu \approx 2$) we have
\[ T_c \propto (\sigma_8 \Omega_M^{0.5} \nu^{-1})^{-1.1}. \tag{4} \]

Therefore, measurements of the cluster abundance at the present epoch constrain a degenerate combination of $T_c$ and $\sigma_8 \Omega_M^{0.5}$. One of them cannot be determined without knowing the other. Thankfully, weak lensing happens to measure roughly this combination of $\Omega_M$ and $\sigma_8$ accurately, and the orthogonal combination much less accurately (e.g. Bernard et al. 1997). Consequently, weak lensing in conjunction with cluster abundance can be used to constrain $T_c$ quite strongly.

3. WEAK LENSING PLUS CLUSTERS: AN EXAMPLE

As a more concrete example of these ideas, let us examine what value of $T_c$ is required to bring current cluster and weak lensing results into agreement. This analysis will necessarily be illustrative, but is already quite enlightening.

3.1. The cluster data

We compute $\sigma_8$ using a Monte-Carlo method following the steps outlined in Pierpaoli, Scott & White (2002). Since some of the details have changed we sketch the procedure here.

We use the HiFluGCS cluster sample of Reiprich & Böhringer (1999), restricted to clusters with $0.03 < z < 0.10$. For simplicity we do not include ‘additional’ clusters of lower flux/temperature which could scatter into the sample. The cosmic microwave background (CMB) frame redshifts from Struble & Rood (1999) were used when available and so were the two-component temperatures published in Ikebe et al. (2002). For each $\Omega_M$ we sample from a distribution of cosmological parameters including $h$, $n$ and $T_c$ (the normalization of the $M$–$T$ relation).

For each such realization we generate 50 mass functions, where the temperature is chosen from a Gaussian with the mean and variance appropriate to the observational value and errors, and a scatter of 15% in mass at fixed $T$ is assumed for the $M$–$T$ relation. Using the mean values of the $M$–$T$ relation and the $L$–$T$ relation from Ikebe et al. (2002)
\[ L_X = 1.38 \times 10^{35} \left( \frac{kT}{1\text{keV}} \right)^{2.5} h^{-2} \text{W} \tag{5} \]
we compute the volume to which clusters of mass $M$ could be seen above the flux limit $f_{\text{lim}} = 1.99 \times 10^{-14} \text{erg s}^{-1} \text{cm}^{-2}$ of the survey.

For each realization of the mass function we compute the best fitting $\sigma_8$ by maximizing the Poisson likelihood of obtaining that set of masses from the theory with all parameters except $\sigma_8$ fixed. The mass function can be computed using either the Press-Schechter (1974), Sheth-Tormen (1999), or Jenkins et al. (2001) formulae.

We have used the Sheth-Tormen prescription throughout, with the mass variance $\sigma^2(M)$ computed using the transfer function fits of Eisenstein & Hu (1999) and masses converted from $M_{\text{NFW}}$ to $M_{\text{vir}}$ assuming an NFW profile [Navarro, Frenk & White 1997] with $c = 5$. The best fitting $\sigma_8$ is corrected from $z$ to $z = 0$. The mean of the $50$, $z = 0$ normalizations is then taken as the fit for that set of cosmological parameters (since the error from Poisson sampling is completely sub-dominant to the error in the $M$–$T$ normalization we do not keep track of it here).

When quoting a best fit for a given triplet of $(\Omega_M$, $\sigma_8$, $T_c)$, we marginalize (average) over the other cosmological parameters $h$ and $n$.

3.2. The weak lensing data

As an example of weak lensing measurements, we use shear measurements obtained using Keck and William Herschel telescopes [Bacon et al. 2002]. These joint measurements used two independent telescopes covering 0.6 and 1 square degrees respectively, and enabled careful assessment of instrument-specific systematics. The authors compute the shear correlation function, and compare with the theoretical prediction. Assuming the shape parameter $\Gamma = 0.21$, the results are well fit by
\[ \sigma_8 \left( \frac{\Omega_M}{0.3} \right)^{0.68} = 0.97 \pm 0.13 \tag{6} \]
which captures the total 68% CL error: statistical, redshift uncertainty and uncertainty in the ellipticity-shear conversion factor. These results are consistent with other recent measurements of cosmic shear [van Waerbeke et al. 2002, Refregier, Rhodes & Groth 2002, Hoekstra, Yee & Gladders 2002].

3.3. Calibrating the M-T relation

Fig. 4 shows the constraints in the $\Omega_M$–$\sigma_8$ plane. The cluster constraint has been marginalized over $h$ and $n$ as explained above, and plotted for three different values of $T_c$. We have checked that the allowed ranges for $h$ and $n$ are wide enough so that essentially all of the likelihood is contained within those ranges. The weak lensing constraints assume the shape parameter $\Gamma = 0.21$. Note that the constraint regions from the two methods are indeed...
principle enables an accurate determination of the normalization $T_\ast$. The fact that cluster abundance and weak lensing probe different scales opens a possibility that one might be able to secure the agreement between the two methods by varying the shape of the power spectrum or the spectral index $n$ rather than the M–T normalization. Unfortunately the constraints we have combined above have individually been marginalized over $h$ and $n$. Ideally, one would combine the cluster and weak lensing likelihood functions and then marginalize over the relevant parameters to get the probability distribution of $T_\ast$:

$$P(T_\ast) = \int L_{\text{clus}}(T_\ast, \Omega_M, \sigma_8, n, h) \times L_{\text{WL}}(\Omega_M, \sigma_8, n, h) d\Omega_M d\sigma_8 dn dh. \quad (7)$$

Then the results would be manifestly independent of the power spectrum parameters. We do not have the ability to perform such an analysis here.

Note, however, that the scales probed by lensing and clusters are quite close, separated an order of magnitude at most. For example, it would require a spectral tilt of $n \sim 1.2$ to make the recently obtained “low” normalization from cluster abundance ($\sigma_8 \sim 0.6$) agree with the “high” normalization from weak lensing ($\sigma_8 \sim 0.9$), and such a high value of $n$ is already disfavored by recent CMB experiments (Balbi et al. 2000, Netterfield et al. 2002, Pryke et al. 2002, Sievers et al. 2002).

4. CONCLUSIONS

There has been a lot of discussion recently regarding the value of cluster normalization $\sigma_8$. While the “old” results favor $\sigma_8 \sim 1$ (Viana & Liddle 1999, Pierpaoli, Scott & White 2002 and references therein), several new cluster abundance analyses favor a significantly lower normalization (Reiprich & Böhringer 1999, Borgani et al. 2001, Viana et al. 2002, Seljak 2002, Kebe et al. 2002, Bahcall et al. 2002). The lower normalization is also favored by the combined analysis of 2dF Galaxy Redshift Survey and CMB data (Lahav et al. 2001). On the other hand, recent weak lensing results (van Waerbeke et al. 2002, Bacon et al. 2002, Refregier, Rhodes & Groth 2001, Hoekstra, Yee & Gladders 2002) tend to favor a higher value of $\sigma_8$.

The cause of this discrepancy between various measurements has not been identified yet; one candidate is larger than anticipated systematic errors in one or both methods. Another possibility is the bias in the relation between the mass and the observable quantity — temperature or luminosity — used to construct the abundance of clusters.

The cluster abundance constraint on $\sigma_8$ crucially depends on the M–T normalization $T_\ast$. Figure 2 summarizes the current status of our knowledge of $T_\ast$. It shows seven determinations from N-body simulations and three from direct observations, as compiled in Pierpaoli, Scott & White (2002) and Muanwong et al. (2002). The shaded region is roughly our favored range of values of $T_\ast$.

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between the different measurements is apparent, and it also appears that the observed values are systematically higher than the ones obtained from simulations (see Muanwong et al. 2002 for further discussion).

We argue here that the cluster abundance – weak lensing complementarity can be used to cross-check the M–T relation. By combining recent weak lensing constraints from Bacon et al. and the HiFluGCS cluster sample of Reiprich & Böhringer, we have demonstrated the utility of this method. While potential systematic errors in both data sets are still a concern, the example we used prefers relatively low values of the M–T normalization ($T_* \lesssim 1\, \text{keV}$). We conclude that future weak lensing surveys (Vista, LSST, SNAP) combined with new cluster data from Chandra and XMM-Newton observations will provide a strong probe of the M–T relation.

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