Chapter 6
Media Information Effect Hampering the Spread of Disease

The media effect is expected to deliver appropriate information to the people, allowing them to prepare some provisions for the spread of a disease in advance. As discussed in Chap. 4, gargling, wearing a mask, or avoiding crowds to maintain a “social distance” may help to considerably reduce the transmission rate. The effect from such social preventive measures can be enhanced by the media through rumor, information obtained through personal networks, SNS, and, of course, mass media.

In this chapter, we quantify the media effect to lighten the disease impact. Shared information is diffused through a human network from one individual to another according to the physics of diffusion. Establishing a theoretical model based on the SIR process and taking account of this media effect, we discuss how its introduction
mitigates the final epidemic size (FES) and improves the vaccination coverage (VC) as well as the average social payoff (ASP).

6.1 Positive Effect of Media Helps to Suppress the Spread of an Epidemic

There are various opinions concerning the pros and cons of the media effect in the wake of COVID-19. Media supports people’s groundless terror by providing rootless information and dubious news, which should be called agitation. This is a negative effect from the media that can worsen a situation. In some cases, a certain authority or nation may intentionally control information by covering-up facts for their own benefit, rather than offering correct information to the worldwide public. However, it is also true that media effectively works to deliver useful information to the people, letting them prepare for oncoming waves of an epidemic in the near future. Such information drives people to be aware of risk and allows them to take some intermediate defense measures (IDMs), such as gargling or wearing a mask, as we discussed in Chap. 4, while people without such useful information are at a relative disadvantage. Information may be brought by the media, but also by neighbors through word-of-mouth. If this is the case, the diffusion of information to implement a mathematical model should be not depicted only by discounting the transmission rate of an epidemic, but should also consider how information propagates on an underlying network connecting individuals.

Spreading “awareness” of a disease can play a significant role in reducing the infection, as has been observed by many previous studies of complex population networks. In order to address the effect of awareness upon an epidemic’s dynamics, this chapter considers the voluntary-vaccination dilemma whose template was introduced in Chap. 3, and has applied in both Chaps. 4 and 5, to characterize the decision-making process in a population.

This chapter integrates a vaccination game with different types of strategy-updating rules for vaccination in a susceptible-infected-recovered (SVIR) process, coupled with an unaware–aware (UA) situation (hereafter: SVIR-UA model) in a heterogeneous network. Unlike usual vaccination games on well-mixed populations based on MFA, we mainly concentrate upon two different types of degree distributions: the Poisson (Erdős–Rényi random (E-R random) graph) and power-law (Barabási–Albert scale-free (BA-SF) network) distributions. This model deals with imperfect vaccination, through which a vaccine only offers perfect immunity with a

---

1There have been many previous studies. Some representative ones are:
Salathe and Bonhoeffer (2008), pp. 1505–1508.
Gómez et al. (2013), p. 028701.
Granell et al. (2014), p. 012808.
Guo et al. (2015), p. 012822.
Wang et al. (2016), p. 29259.
probability of $e$ (effectiveness), and otherwise fails to offer immunity. More importantly, the proposed model treats the situation in which information is defined by the binary states of awareness (A) and unawareness (U), allowing an individual to avoid infection by reducing the disease-transmission rate. To do this, we assume that the portion of aware individuals (information-advantageous) may withdraw or stay safe from crowds or undesirable contamination. Thus, in the present study, we assume that an individual entering an aware state does not require any cost, which is different from what the IDM measures presumed.

### 6.2 Model Structure

#### 6.2.1 Formulation of the SVIR-UA Model$^2$

We use the SVIR model as a base and apply the methodology for considering a spatial structure among individuals, as introduced in Sect. 3.4. Hence, let $US_k(t)$, $AS_k(t)$, $UV_k(t)$, $AV_k(t)$, $UI_k(t)$, $AI_k(t)$, and $R_k(t)$ be the densities of the unaware susceptible, aware susceptible, unaware vaccinated, aware vaccinated, unaware infected, aware infected, and recovered nodes of connectivity (degree) $k$ at time $t$ in a population. These satisfy the following normalization condition:

\[
US_k(t) + AS_k(t) + UV_k(t) + AV_k(t) + UI_k(t) + AI_k(t) + R_k(t) = 1. \quad (6.1)
\]

Figure 6.1 shows all compartments and state transfer in the present model. We presume that there are two spatial structures (say, two-layer networks): a physical network and a virtual network. In a physical network, an epidemic spreads, while in a virtual network, the information (actually “awareness”) is diffused. The disease state is threefold: susceptible, infected, and recovered, with state transfer represented by black arrows. The information state is binary: one is either aware (A) or unaware (U), with state transfer being drawn by a red arrow. The connection between unaware susceptible (US) and aware susceptible (AS), or unaware vaccinated (UV) and aware vaccinated (AV) depicts the virtual interaction, suggesting that information spreads from an unaware state to an aware one. On the other hand, the transitions from US to unaware infected (UI), aware susceptible (AS) to aware infected (AI), UV to UI, and aware vaccinated (AV) to aware infected (AI) show physical interaction, suggesting that the infection spreads. The unaware-susceptible and unaware-vaccinated compartments become UI with the disease-transmission rate $\beta$ [day$^{-1}$ person$^{-1}$], with both unaware-susceptible and unaware-vaccinated individuals becoming aware immediately after infection. Aware-infected individuals decay into the removed class at the recovery rate $\gamma$ [day$^{-1}$]. We presume a rate of information spreading from unaware to aware $\alpha$ [day$^{-1}$ person$^{-1}$]. The aware-

$^2$Kabir et al. (2020), p. 109548.
Fig. 6.1 The structure of the SIR-V-UA network model in which vaccination dynamics are displayed as two layers: a virtual layer and a physical layer.
susceptible and aware-vaccinated individuals become infected at a rate of \((1 - \eta) \beta\), where \(\eta\) is the discount rate of infection with the help of information, implying that the individual takes a costless IDM. The case with \(\alpha = 0\) (no information spreading) recovers the default setting, which is the networked SIR/V with the effectiveness model introduced in Sect. 3.4.3.

The vaccinated population fraction is divided into two groups: perfectly immune and non-immune. The effectiveness of the vaccine is denoted as \(e\) \((0 \leq e \leq 1)\) and the efficiency of IDM is \(\eta\) \((0 \leq \eta \leq 1)\), which accounts for the media effect that ensures that individuals protect themselves from an epidemic in advance. The set of differential equations of the SVIR-UA model under a networked population can be given as

\[
\frac{dU(t)}{dt} = -\beta k U(t) \Theta_1(t) (1 - \alpha k \Theta_2(t)) - \alpha k U(t) \Theta_2(t) (1 - \beta k \Theta_1(t)), \quad (6.2a)
\]

\[
\frac{dA(t)}{dt} = \alpha k U(t) \Theta_2(t) (1 - \beta k \Theta_1(t)) - (1 - \eta) \beta k A(t) \Theta_1(t), \quad (6.2b)
\]

\[
\frac{dV(t)}{dt} = -\beta k (V(t) - eV(0)) \Theta_1(t) (1 - \alpha k \Theta_2(t)) - \alpha k V(t) \Theta_2(t)
\]

\[
\times (1 - \beta k \Theta_1(t)), \quad (6.2c)
\]

\[
\frac{dI(t)}{dt} = -(1 - \eta) \beta k (I(t) - eI(0)) \Theta_1(t) + \alpha k V(t) \Theta_2(t) \Theta_1(t)
\]

\[
\times (1 - \beta k \Theta_1(t)), \quad (6.2d)
\]

\[
\frac{dU(t)}{dt} = \beta k (U(t) - eU(0)) \Theta_1(t) (1 - \alpha k \Theta_2(t)) + \beta k U(t) \Theta_1(t)
\]

\[
\times (1 - \alpha k \Theta_2(t)) - UI(t), \quad (6.2e)
\]

\[
\frac{dA(t)}{dt} = (1 - \eta) \beta k A(t) \Theta_1(t) + (1 - \eta) \beta k (A(t) - eA(0)) \Theta_1(t)
\]

\[
+ UI(t) - \gamma A(t), \quad (6.2f)
\]

\[
\frac{dR(t)}{dt} = \gamma A(t), \quad (6.2g)
\]

where \(\Theta(t)\) refers to the mean probability of infected people, which is illustrated by the spatial structure for both the aware and unaware states. Herein, \(\Theta_1(t)\) and \(\Theta_2(t)\) illustrate the mean probabilities for given link points to an infected node (physical interaction), and the links are associated with unaware-to-aware transitions (virtual interactions), respectively, which takes account of the spatial structure through the degree distribution, of which basic concept was introduced at Eq. (3.36) (introduced for the governing equation; Eq. (3.43)) in Sect. 3.4.3:
We must carefully note the substantial difference between the present model (Eqs. 6.2a, 6.2b, 6.2c, 6.2d, 6.2e, 6.2f, 6.2g) and the networked SIR/V with an effectiveness model presented in Eq. (3.43). Let us confirm these one by one. On the right-hand side of Eq. (6.2a), \((1 - \alpha_k \Theta_2(t))\) is newly multiplied by \(\beta_k U S_k(t)\Theta_1(t)\), which is its counterpart as the right-hand term in the first equation of Eq. (3.43) \((\lambda_k S_k(t)\Theta(t))\). Why does the present model (two-layer spatial structure by the SVIR-UA process) require \((1 - \alpha_k \Theta_2(t))\) to be further multiplied? This is explained below. The term \(\beta_k U S_k(t)\Theta_1(t)\) indicates the extent to which \(US_k\) is infected by contacts with infectious individuals in the physical network (denoted by subscript 1). The focal term should represent newly infected individuals (transferred from S to I) who are not aware. Mathematically, the condition of “and not being aware” requires multiplication by \((1 - \alpha_k \Theta_2(t))\) (where subscript 2 indicates the virtual network). Likewise, the term \(\alpha_k U S_k\Theta_2(t)\) indicates the extent to which \(US_k\) is aware through contacts with aware individuals in the virtual network. Since the focal term must represent the individuals newly becoming aware (from U to A) without being infected, the condition of “and not being infected” mathematically requires multiplication by \((1 - \beta_k \Theta_1(t))\). Let us move to the right-hand terms of Eq. (6.2b). The second term, \((1 - \eta)\beta_k AS_k(t)\Theta_1(t)\), must represent individuals becoming newly infected (from S to I) while being aware. Above, we presumed that “both US and unaware-vaccinated individuals become aware immediately after infected,” meaning that the state-transfer probability is 1, so that the condition of “and being aware” requires nothing to be multiplied. In Eqs. (6.2c) to (6.2g), the same concepts as above are applied to obtain the respective terms. Another quite important point to be carefully confirmed is that the last term on the right-hand side in Eq. (6.2e) does not implement \(-\gamma UI_k(t)\) (and the right-hand side of Eq. (6.2g) does not have \(+\gamma UI_k(t)\) in return), but \(-UI_k(t)\) and in return the third term on the right-hand side of Eq. (6.2f) appears as \(+UI_k(t)\). This is because we can assume that an UI individual must immediately become aware, and never recovers in an unaware state.

As we will precisely define later, committing to a vaccination is costly, and being infected also entails an illness cost that is greater than the vaccination cost. Likewise from Sect. 5.2.1, in the current evolutionary framework, we can classify individuals into four states in terms of their health condition and cost burden: (i) a healthy vaccinator (HV) who pays only the vaccination cost and remains healthy in an epidemic season; (ii) an infected vaccinator (IV) who commits to a vaccination but unfortunately becomes infected and must therefore bear the vaccination cost as well as the infection cost; (iii) a successful free rider (SFR) is a non-vaccinator who does not incur any cost and fortunately can survive without being infected; and (iv) a failed free rider (FFR) is also a non-vaccinator who relies utterly upon the herd

\[
\Theta_1(t) = \frac{\sum_k (k - 1)P(k)(UI_k(t) + AI_k(t))}{\langle k \rangle},
\]

\[
\Theta_2(t) = \frac{\sum_k (k - 1)P(k)(AS_k(t) + AI_k(t) + AV_k(t))}{\langle k \rangle}.
\]
immunity state and tries to free ride without taking any protective provisions but eventually becomes infected, thereby bearing the infection cost themselves.

Presuming a positive vaccination coverage; VC, \( x \), we can observe the level of each compartment when \( t \) (the local timescale, meaning the time in a single season) goes to infinity (\( t \rightarrow \infty \)). Thus, we obtain:

\[
HV_k(x, \infty) = V_k(x, \infty), \quad (6.4a)
\]

\[
SFR_k(x, \infty) = S_k(x, \infty). \quad (6.4b)
\]

To evaluate the other two fractions IV and FFR, we introduce the flux notation \( \varphi_A \rightarrow B \), which indicates the total number of individuals changing the state from A to B (one state to another). At this point, the flux between the aware-vaccinated and aware-infected compartments is denoted by \( \varphi_{AV_k \rightarrow AI_k}(x, \infty) \), that from the unaware-vaccinated to unaware-infected compartments is \( \varphi_{UV_k \rightarrow UI_k}(x, \infty) \), that from the aware-susceptible to aware-infected compartments is \( \varphi_{AS_k \rightarrow AI_k}(x, \infty) \), and that from the unaware-susceptible to unaware-infected compartments is \( \varphi_{US_k \rightarrow UI_k}(x, \infty) \). Thus, we recover Table 6.1.

### Table 6.1 Fractions of four types of individuals

| Strategy/State                  | Healthy | Infected          |
|--------------------------------|---------|-------------------|
| Vaccinated (Cooperator; V)     | \( V_k(x, \infty) \) | \( \varphi_{AV_k \rightarrow AI_k}(x, \infty) + \varphi_{UV_k \rightarrow UI_k}(x, \infty) \) |
| Non-vaccinated; Defector (Free Rider; FR) | \( S_k(x, \infty) \) | \( \varphi_{AS_k \rightarrow AI_k}(x, \infty) + \varphi_{US_k \rightarrow UI_k}(x, \infty) \) |

### Table 6.2 Estimated payoff structure at the end of each epidemic season

| Strategy/State       | Healthy | Infected |
|----------------------|---------|----------|
| Vaccinated (V; C)    | \(-C_r\) | \(-C_r - 1\) |
| Non-vaccinated (NV; D)| 0       | -1       |

6.2.2 Payoff Structure

We should follow Table 3.1, because there is only a single costly strategy, which is vaccination (defined as an active provision). We recall the relative cost of vaccination, \( C_r = C_v / C_i \) (0 \( \leq C_r \leq 1 \)), where the costs for vaccination and infection are denoted by \( C_v \) and \( C_i \), respectively. As we discussed in the previous sub-section, at the end of a single season, using the game-theoretic approach, we can classify all individuals who initially chose either vaccination or free riding (defection) into the four classes depending upon their final health status, whether they are healthy or infected at the equilibrium point. Therefore, we depict the present-payoff structure, Table 6.2, as in Table 3.1.

In order to construct the expected payoff, the ASP \( \langle \pi \rangle \), cooperative payoff \( \langle \pi_C \rangle \) (vaccinated), and defective payoff \( \langle \pi_D \rangle \) (non-vaccinated) are formulated as follows:
\[ \langle \pi \rangle = -Cr \sum_k x_k - (Cr + 1) \sum_k (1 - x_k), \] (6.5a)
\[ \langle \pi_C \rangle = \left\{ -Cr \frac{1}{(k)} \sum_k k p(k) V_k(x, \infty) - (Cr + 1) \frac{1}{(k)} \sum_k k p(k) \left( \phi_{AV_1 \rightarrow AI_k}(x, \infty) \right) + \phi_{UV_1 \rightarrow UI_k}(x, \infty) \right\} / \sum_k x_k, \] (6.5b)
\[ \langle \pi_D \rangle = \left\{ -\frac{1}{(k)} \sum_k k p(k) \left( \phi_{AS_1 \rightarrow AI_k}(x, \infty) + \phi_{US_1 \rightarrow UI_k}(x, \infty) \right) \right\} / \sum_k (1 - x_k). \] (6.5c)

### 6.2.3 Strategy Updating and Global Dynamics

In the present model, we apply individual-based risk assessment (IB-RA) and strategy-based risk assessment (SB-RA), as introduced in Sect. 3.2.2. We use the MFA parameter and set the noise parameter in the Fermi function in each state probability function \( \kappa \) to 0.1.

#### 6.2.3.1 Individual-Based Risk Assessment (IB-RA)

As described in Eq. (3.25), there are eight state-transition-probability functions;
\[
P(HV \leftarrow SFR) = \frac{1}{1 + \exp \left[ \frac{-(0 - (-C_r))}{\kappa} \right]} , \quad P(HV \leftarrow FFR) = \frac{1}{1 + \exp \left[ \frac{-(1 - (-C_r))}{\kappa} \right]}, \]
\[
P(IV \leftarrow SFR) = \frac{1}{1 + \exp \left[ \frac{-(0 - (-C_r - 1))}{\kappa} \right]} , \quad P(IV \leftarrow FFR) = \frac{1}{1 + \exp \left[ \frac{-(1 - (-C_r - 1))}{\kappa} \right]}, \]
\[
P(SFR \leftarrow HV) = \frac{1}{1 + \exp \left[ \frac{-(C_r - 0)}{\kappa} \right]} , \quad P(SFR \leftarrow IV) = \frac{1}{1 + \exp \left[ \frac{-(C_r - 1 - 0)}{\kappa} \right]}, \]
\[
P(FFR \leftarrow HV) = \frac{1}{1 + \exp \left[ \frac{-(C_r - 1)}{\kappa} \right]} , \quad P(FFR \leftarrow IV) = \frac{1}{1 + \exp \left[ \frac{-(C_r - 1 - 1)}{\kappa} \right]). \]

At the end of each epidemic season, everyone can update their strategy depending upon the last season’s payoff. Hence, increases or decreases to VC, \( x \), is inevitable. Here, the independent variable, \( t \), indicates the global timescale, which means the number of elapsed seasons. Since we consider the IB-RA strategy-updating rule for decision-making in each subsequent season, the dynamic equation following this particular rule can therefore be expressed as follows:
\[
\frac{dx_k}{dt} = -V_k(x, \infty) \frac{1}{(k)} \sum_k k p(k) AS_k(x, \infty) P(HV \leftarrow SFR) - V_k(x, \infty) \frac{1}{(k)} \sum_k k p(k) \left( \phi_{AS_1 \rightarrow AI_k}(x, \infty) + \phi_{US_1 \rightarrow UI_k}(x, \infty) \right) P(HV \leftarrow FFR))
\]
\[-(\varphi_{AV} \rightarrow AI_i(x, \infty) + \varphi_{UV} \rightarrow UI_i(x, \infty)) \frac{1}{(k)} \sum_k kp(k) S_k(x, \infty) P(IV \leftarrow SFR)\]
\[-(\varphi_{AV} \rightarrow AI_i(x, \infty) + \varphi_{UV} \rightarrow UI_i(x, \infty)) \frac{1}{(k)} \sum_k kp(k)\]
\[\times (\varphi_{AS} \rightarrow AI_i(x, \infty) + \varphi_{US} \rightarrow UI_i(x, \infty)) P(IV \leftarrow FFR)\]
\[+ S_k(x, \infty) \frac{1}{(k)} \sum_k kp(k) V_k(x, \infty) P(SFR \leftarrow HV)\]
\[+ (\varphi_{AS} \rightarrow AI_i(x, \infty) + \varphi_{US} \rightarrow UI_i(x, \infty)) \frac{1}{(k)} \sum_k kp(k) V_k(x, \infty) P(FFR \leftarrow HV)\]
\[+ S_k(x, \infty) \frac{1}{(k)} \sum_k kp(k) (\varphi_{AV} \rightarrow AI_i(x, \infty) + \varphi_{UV} \rightarrow UI_i(x, \infty)) P(SFR \leftarrow IV)\]
\[+ (\varphi_{AS} \rightarrow AI_i(x, \infty) + \varphi_{US} \rightarrow UI_i(x, \infty)) \frac{1}{(k)} \sum_k kp(k)\]
\[\times (\varphi_{AV} \rightarrow AI_i(x, \infty) + \varphi_{UV} \rightarrow UI_i(x, \infty)) P(FFR \leftarrow IV), \quad (6.6)\]

where $x_k$ represents the fraction of vaccinators (cooperators: C). The product of the two classes, as an example, $V_k(x, \infty)$ and $\frac{1}{(k)} \sum_k kp(k) S_k(x, \infty)$ are probabilities that any two portions of individuals of the two classes participate for imitation (if possible).

6.2.3.2 Strategy-Based Risk Assessment (SB-RA)

As described in Eq. (3.27), there are four state-transition-probability functions:
\[P(HV \leftarrow NV) = \frac{1}{1 + \exp[\frac{1}{\kappa}]} , \quad P(IV \leftarrow NV) = \frac{1}{1 + \exp[\frac{1}{\kappa}]} ,\]
\[P(SFR \leftarrow V) = \frac{1}{1 + \exp[\frac{1}{\kappa}]} , \quad P(FFR \leftarrow V) = \frac{1}{1 + \exp[\frac{1}{\kappa}]} .\]

Following the IB-RA case above, we can establish the dynamic equation following SB-RA as follows:
\[\frac{dx_k}{dt} = -x_k V_k(x, \infty) \frac{1}{(k)} \sum_k kp(k)(1 - x_k) P(HV \leftarrow NV)\]
\[-x_k (\varphi_{AV} \rightarrow AI_i(x, \infty) + \varphi_{UV} \rightarrow UI_i(x, \infty)) \frac{1}{(k)} \sum_k kp(k)(1 - x_k) P(IV \leftarrow NV)\]
\[+(1 - x_k) S_k(x, \infty) \frac{1}{(k)} \sum_k kp(k) x_k P(SFR \leftarrow V)\]
6.2.4 Spatial Structure

We must give an explicit form to the degree distribution for \( P(k) \). In this model, we apply the same topology for the physical network (meaning subscript 1 in Eq. (6.3a)) and the virtual network (meaning the subscript 2 in Eq. (6.3b)). Following Sect. 3.4.1, we presume two typical degree distributions; the Poisson and power-law degree distributions, which are, respectively, associated with the Erdős–Rényi random-graph (E-R random) and the Barabási–Albert scale-free (BA-SF) networks. In the Poisson degree distribution \( P(k) = \exp \left( -\langle k \rangle \right) \frac{\langle k \rangle^k}{k!} \), most of the modes have a connectivity \( k \) close to the mean value \( \langle k \rangle = \sum_k kP(k) \). Although BA-SF’s power law obeys; \( P(k) \sim k^{-3} \), we use the approximation presumed in Sect. 3.4.1: \( P(k) = A/k(k + 1)(k + 2) \).

6.2.5 Initial Condition and Numerical Procedure

To solve the above-stated sets of equations for each season as well as the repeating seasons numerically, an explicit finite-difference method is used. Throughout, the minimum and maximum degrees are assumed to be \( k_{\text{min}} = 3 \) and \( k_{\text{max}} = 100 \), respectively.

Initially, we presumed initial values of \( UV_k(x, 0) = x \), \( US_k(x, 0) = 1 - x \), \( UL_k(x, 0) = 0 \), \( AS_k(x, 0) = 0 \), \( R_k(x, 0) = 0 \), and \( AI_k(x, 0) \approx 0 \) to start a new season \( (x = 0.5, & AI_k(x, 0) = 0.0001) \).

6.3 Results and Discussion

Throughout our simulations, we presume that \( R_0 = R = 2.5 \). By varying the topology of spatial structure (where “well-mixed” means that there is no spatial structure, E-R random and BA-SF, and the average degree is set to \( \langle k \rangle = 4 \), \( \langle k \rangle = 8 \) and \( \langle k \rangle = 12 \), we explore the FES, VC, and ASP when presuming either risk assessment (IB-RA) or strategy-based risk assessment (SB-RA) for the strategy-update rule. All visual results below are presented as a 2D heat map, where the relative vaccination cost \( C_r \) (effectiveness of vaccination, \( e \)) is presented along the X-axis (Y-axis) and ranges...
from 0 to 1. In each figure, panels (*-i), (*-ii) and (*-iii), respectively, show the FES ranging for 0 to 1; VC also ranges from 0 to 1 and the ASP ranges from $-1$ to 0.3

Figure 6.2 shows the default setting where $\alpha = 0$ is presumed, which implies there is no information effect. The top, middle, and bottom panels show different topologies. For BA-SF and E-R random, we presume $\langle k \rangle = 8$. As strategy-update rule, IB-RA is presumed. For the well-mixed case, panels (c-i), (c-ii), and (c-iii) recover what we discussed in Chap. 3; more precisely, panels (1-A), (1-B), and (1-C) in Fig. 3.13 (or panels (3-A), (3-B) and (3-C) in Fig. 3.19).

---

3 Theoretically ASP can be less than $-1$. The possible minimum is $-2$, where an individual pays the vaccination cost, fails to obtain immunity, and is unfortunately infected (paying an additional disease cost).
Figure 6.5 is the counterpart to Fig. 6.2 when presuming SB-RA as an update rule. Thus, for the well-mixed case, panels (c-i), (c-ii), and (c-iii) recover what we discussed in Chap. 3; more precisely, panels (2-A), (2-B), and (2-C) in Fig. 3.13 (or panels (3-A), (3-B), and (3-C) in Fig. 3.21).

Returning to Fig. 6.2, the same general tendency among FES and ASP seems common for different topologies. More precisely, the monotone region of red indicating a pandemic phase in (a-i) (BA-SF) is redder than those in (b-i) (E-R random) and (c-i) (well-mixed), implying that the pandemic in BA-SF is more severe than under the other two topologies. This is likely because hub individuals work as so-called super-spreaders in the case of an SF network, due to a much more homogeneous degree distribution than the other two topologies. One thing that is worthwhile to note is that outside of the blue, epidemic-controlled region in (a-ii), there is a higher VC (more red) than in the other two topologies. This makes sense because, in an SF network, inherent vulnerability against an epidemic drives individuals to commit to vaccination more.
Let us consider Fig. 6.2 (a-i). The boundary between monotone red and the remaining area implies a phase transition between the pandemic (red) and controlled (bluer) regions. Such an abrupt transition of a certain physical system as observed in FES is well-recognized as a first-order phase change. Thus, we can define the critical effectiveness for a certain vaccination cost, \( \eta_{\text{cr}}(C_r) \), bringing this phase transition between the pandemic and the controlled phase (see the closed circle in Fig. 6.2 (a-i)).

Now, let us move on to Fig. 6.3, where BA-SF is considered and its average degree is varied in the top, middle, and bottom rows. More importantly we presume that \( \alpha = 0.4 \), with informational awareness, helps to suppress the spread of disease. In fact, comparing Figs. 6.3 (b-*), and 6.2 (a-*), we can confirm that the information effect is remarkable. FES can be significantly reduced and ASP improved significantly. More importantly, this does not come from a higher VC, but results from the awareness effect through \( \eta \). Notably, panel (b-ii) is almost colored blue except in the region where the vaccination cost is quite low but still has reasonable effectiveness.
We can confirm that the information effect works significantly to minimize the impact of the pandemic. Comparing rows in Fig. 6.3, we can note that this positive effect is greater in a case with a lower average degree, which might be conceivable because a lower average degree allows fewer hub individuals. Observing the closed circles plotted at the exact same position in Fig. 6.2 (a-i), we note that the critical effectiveness is decreased, which may be explicable in terms of what we have discussed above.

Figure 6.4 is the counterpart to Fig. 6.3 when presuming ER-random. We also confirm that informational of awareness helps to significantly reduce the pandemic risk. Figure 6.5 is the counterpart to Fig. 6.2, and Figs. 6.6 and 6.7 are the counterparts to Figs. 6.3 and 6.4, respectively, when SB-RA is presumed for update rule. Observing Figs. 6.5, 6.6, 6.7 and comparing them with Figs. 6.2, 6.3, 6.4, our above discussion assuming IB-RA for the update rule is almost true for the alternate update rule SA-RA.

**Fig. 6.5** Columns 1, 2, present the final epidemic size (*-i), vaccination coverage (*-ii), and average social payoff (*-iii), respectively. The power-law degree distribution (a-*), Poisson degree distribution (b-*), and well-mixed population (c-*) are depicted in the upper, middle, and lower panels for the SB-RA updating rule, respectively. Here, the information-spreading rate is $\alpha = 0.0$ and the other parameters are $\beta = 0.83$, $\gamma = 0.333$, and $\eta = 0.3$. 

We can confirm that the information effect works significantly to minimize the impact of the pandemic. Comparing rows in Fig. 6.3, we can note that this positive effect is greater in a case with a lower average degree, which might be conceivable because a lower average degree allows fewer hub individuals. Observing the closed circles plotted at the exact same position in Fig. 6.2 (a-i), we note that the critical effectiveness is decreased, which may be explicable in terms of what we have discussed above.

Figure 6.4 is the counterpart to Fig. 6.3 when presuming ER-random. We also confirm that informational of awareness helps to significantly reduce the pandemic risk. Figure 6.5 is the counterpart to Fig. 6.2, and Figs. 6.6 and 6.7 are the counterparts to Figs. 6.3 and 6.4, respectively, when SB-RA is presumed for update rule. Observing Figs. 6.5, 6.6, 6.7 and comparing them with Figs. 6.2, 6.3, 6.4, our above discussion assuming IB-RA for the update rule is almost true for the alternate update rule SA-RA.
Let us conclude by summarizing what we learned in this Chapter.

We confirmed that the effect of awareness of a spreading infection can remarkably reduce the number of infected individuals for all updating rules, because such awareness encourages people to protect themselves.

We also presumed that both the disease and information spread on the same network. Although the most effective way to hamper a virus’s spread may be through vaccination, this practice is expensive, costly to oneself, and difficult to apply to inoculate the mass of people. Lethal diseases like Dengue, Chikungunya, AIDS, Plague, and Malaria have no active vaccinations, so only awareness can effect limit their spread. The practice of safe sex, use of mosquito coils or nets, hand washing, mask wearing, and other self-protection measures informed by information play a substantial effect in the epidemic-diffusion model.

**Fig. 6.6** The power-law degree distributions (BA-SF) of SB-RA are presented for the final epidemic size (*-i), vaccination coverage (*-ii), and average social payoff (*-iii). Three different average degrees are depicted in figures (a-*), (b-*), and (c-*), respectively. Here, the information-spreading rate is $\alpha = 0.4$ and the other parameters are set as $\beta = 0.83$, $\gamma = 0.333$, and $\eta = 0.3$.
Some pioneering studies have suggested that presuming that the physical network (on which disease spreads) is different from the virtual network (on which strategy-updating rules and information spread) will yield very different dynamical features. As future work, it would be interesting to establish another vaccination game with an improved SVIR-UA epidemic model based on what we have discussed here, referring to two different social networks for disease and awareness spreading.

Fig. 6.7 The Poisson degree distributions (ER-RG) of SB-RA are presented for varying final epidemic size (*-i), vaccination coverage (*-ii), and average social payoff (*-iii). Three different average degrees are depicted in figures (a-*), (b-*), and (c-*), respectively. Here, the information-spreading rate is $\alpha = 0.4$, and the other parameters are $\beta = 0.83$, $\gamma = 0.333$, and $\eta = 0.3$.

Some pioneering studies have suggested that presuming that the physical network (on which disease spreads) is different from the virtual network (on which strategy-updating rules and information spread) will yield very different dynamical features. As future work, it would be interesting to establish another vaccination game with an improved SVIR-UA epidemic model based on what we have discussed here, referring to two different social networks for disease and awareness spreading.

\[ \text{Fukuda et al. (2015), pp. 47–55.} \]
References

Fukuda E, Tanimoto J, Akimoto M (2015) Influence of breaking the symmetry between disease transmission and information propagation network on stepwise decisions concerning vaccination. Chaos Solitons Fractals 80:47–55

Gómez S, Diaz-Guilera A, Gomez-Gardeñes J, Perez-Vicente CJ, Moreno Y, Arenas A (2013) Diffusion dynamics on multiplex networks. Phys Rev Lett 110(2):028701

Granell C, Gómez S, Arenas A (2014) Competing spreading processes on multiplex networks: Awareness and epidemics. Phys Rev E 90:012808

Guo Q, Jiang X, Lei Y, Li M, Ma Y, Zheng Z (2015) Two-stage effects of awareness cascade on epidemic spreading in multiplex networks. Phys Rev E 91(1-1):012822

Kabir KMA, Kuga K, Tanimoto J (2020) The impact of information spreading on epidemic vaccination game dynamics in a heterogeneous complex network- A theoretical approach. Chaos Solitons Fractals 132:109548

Salathe M, Bonhoeffer S (2008) The effect of opinion clustering on disease outbreaks. J R Soc Interface 5:1505–1508

Wang W, Liu QH, Cai S, Tang M, Lidia A, Braunstein L, Stanley HE (2016) Suppressing disease spreading by using information diffusion on multiplex networks. Sci Rep 6:29259