Abstract—We consider the distributed computing problem of multiplying a set of vectors with a matrix. For this scenario, Li et al. recently presented a unified coding framework and showed a fundamental tradeoff between computational delay and communication load. This coding framework is based on maximum distance separable (MDS) codes of code length proportional to the number of rows of the matrix, which can be very large. We propose a block-diagonal coding scheme consisting of partitioning the matrix into submatrices and encoding each submatrix using a shorter MDS code. We prove that, up to a level of partitioning, the proposed scheme does not incur any loss in terms of computational delay and communication load compared to the scheme by Li et al. We further show that the assignment of coded matrix rows to servers to minimize the communication load can be formulated as an integer program with a nonlinear cost function, and propose an algorithm to solve it. Numerical results show that, for heavy partitioning, the proposed coding scheme entails only a slight increase in computational delay and/or communication load. Partitioning, on the other hand, allows the use of much shorter MDS codes, thus significantly lowering the decoding complexity.

I. INTRODUCTION

Distributed computing has emerged as one of the most effective ways of tackling increasingly complex computation problems. One of the main application areas is large-scale machine learning and data analytics. Google routinely performs computations using several thousands of servers in their MapReduce clusters. Distributed computing systems bring significant challenges. Among them, the problems of straggling servers and bandwidth scarcity have recently received significant attention. The straggler problem is a synchronization problem characterized by the fact that a distributed computing task must wait for the slowest server to complete its computation. On the other hand, distributed computing tasks typically require that data is moved between servers during the computation, the so-called data shuffling, which is a challenge in bandwidth-constrained networks.

Coding for distributed computing to reduce the computational delay and the communication load between servers has recently been considered in [2], [3]. In [2], a structure of repeated computation tasks across servers was proposed, enabling coded multicast opportunities that significantly reduce the required bandwidth to shuffle the results. In [3], the authors showed that maximum distance separable (MDS) codes can be applied to a linear computation task (e.g., multiplying a vector with a matrix) to alleviate the effects of straggling servers and reduce the computational delay. In [4], a unified coding framework was presented and a fundamental tradeoff between computational delay and communication load was identified. The ideas of [2], [3] can be seen as particular instances of the framework in [4], corresponding to the minimization of the communication load or the computational delay.

The distributed computing problem addressed in [4] is a matrix multiplication problem where a set of vectors \( x_1, \ldots, x_N \) are multiplied by a matrix \( A \). In particular, as in [3], the authors suggest the use of MDS codes, whose dimension is equal to the number of rows of \( A \), to generate some redundant computations. In practice, the size of \( A \) can be very large. For example, Google performs matrix-vector multiplications with matrices of dimension of the order of \( 10^{10} \times 10^{10} \) when ranking the importance of websites [5]. Since the decoding complexity of MDS codes on the packet erasure channel is quadratic (for Reed-Solomon (RS) codes) in the code length [6], for very large matrix sizes the decoding complexity may be prohibitively high.

In this paper, we introduce a block-diagonal encoding scheme for the distributed matrix multiplication problem. The proposed encoding is equivalent to partitioning the matrix and applying smaller MDS codes to each submatrix separately. We prove that up to a certain partitioning level, partitioning does not increase the computational delay or the communication load with respect to the scheme in [4]. The storage design for the proposed block-diagonal encoding can be cast as an integer optimization problem with a nonlinear objective function, whose computation scales exponentially with the problem size. We propose a heuristic solver for efficiently solving the optimization problem, and a branch-and-bound approach for improving on the resulting solution iteratively. We exploit a dynamic programming approach to speed up the branch-and-bound operations. Numerical results are provided that show that a high level of partitioning can be applied (thus using significantly shorter MDS codes and reducing the decoding complexity) at the expense of only a slight increase in the communication load and/or the computational delay.

II. SYSTEM MODEL

We consider the problem of multiplying a set of vectors with a matrix. In particular, given an \( m \times n \) matrix \( A \) and \( N \) vectors...
$x_1,\ldots,x_N$ of length $n$ we want to compute the $N$ vectors $y_1 = Ax_1, \ldots, y_N = Ax_N$. The computation is performed in a distributed fashion using $K$ servers, $S_1,\ldots,S_K$, where each server stores $\mu m$ matrix rows, for some $\frac{1}{K} \leq \mu \leq 1$. Prior to distributing the rows among the servers, $A$ is encoded by an $r \times m$ encoding matrix $\Psi$, resulting in the coded matrix $C = \Psi A$, of size $r \times n$, i.e., the rows of $A$ are encoded using an $(r, m)$ linear code.

Let $q = \frac{Km}{r}$, where we assume that $r$ divides $Km$ and hence $q$ is an integer. The $r$ coded rows of $C$, $c_1,\ldots,c_r$, are divided into $\binom{K}{r}$ disjoint batches, each containing $r/\binom{K}{r}$ coded rows. Each batch is assigned to $\mu q$ servers. Correspondingly, a batch $B$ is labeled by a unique set $\mathcal{T} \subset \{S_1,\ldots,S_K\}$, of size $|\mathcal{T}| = \mu q$, denoting the subset of servers that store that batch, and we write $B_\mathcal{T}$. Server $S_k$, $k = 1,\ldots,K$, stores the coded rows of $B_\mathcal{T}$ if and only if $S_k \in \mathcal{T}$.

### A. Distributed Computing Model

We consider the MapReduce framework described in [4], where we assume that the input vectors $x_1,\ldots,x_N$ are known to all servers at the start of the computation. The overall computation then proceeds in three phases: map, shuffle, and reduce.

1) **Map Phase:** In the map phase, we compute in a distributed fashion coded intermediate values, which will be later used to obtain vectors $y_1,\ldots,y_N$. Server $S$ multiplies the input vectors $x_j$, $j = 1,\ldots,N$, by all the coded rows of matrix $C$ it stores, i.e., it computes

$$Z_j(S) = \{cx_j : \forall c \in \{B_\mathcal{T} : S \in \mathcal{T}\}\}, j = 1,\ldots,N.$$ 

The map phase terminates when a set of servers $G \subseteq \{S_1,\ldots,S_K\}$ that collectively store enough values to decode the output vectors have finished their map computations. We denote the cardinality of $G$ by $g$.

Let $D_k$ denote the delay that server $S_k$ incurs in computing $Z_1(S_k),\ldots,Z_N(S_k)$. We assume that $D_1,\ldots,D_K$ are independent and identically distributed (i.i.d) random variables, and denote by $D(i)$, $i = 1,\ldots,K$, the $i$-th order statistic, i.e., the $i$-th smallest variable of $D_1,\ldots,D_K$. As in [3], running the map phase on a single machine is assumed to take a random amount of time according to the shifted-exponential cumulative probability distribution

$$F(t) = \begin{cases} 1 - e^{-(\frac{t}{N} - 1)}, & \text{for } t \geq \mu N \\ 0, & \text{otherwise} \end{cases}.$$ 

When the algorithm is distributed into $K$ subtasks, the runtime cumulative probability distribution of a subtask is $F(K)$. The expected latency for the fastest $g$ servers to finish their map computations is $E(D(g)) = \mu N \left(1 + \sum_{j=K-g+1}^{K} \frac{1}{j}\right)$ [7].

**Definition 1.** The computational delay $D$ is the average amount of time spent in the map phase per vector $y$.

After the map phase, the computation of $y_1,\ldots,y_N$ proceeds using only the servers in $G$. We define by $Q \subseteq G$ the set of the first $q$ servers to complete the map phase. Each of the $q$ servers in $Q$ is responsible to compute $N/q$ of the vectors $y_1,\ldots,y_N$. Let $V_{S}$ be the set containing the indices of the vectors $y_1,\ldots,y_N$ server $S \in Q$ is responsible for. The remaining servers in $G$ assist the servers in $Q$ in the shuffle phase.

The $(r,m)$ linear code proposed in [4] is an MDS code for which $y_1,\ldots,y_N$ can be obtained from any subset of $q$ servers, i.e., $G = Q$ and $g = q$. The computational delay for the scheme in [4], denoted here by $D_{MDS}$, is thus

$$D_{MDS} = \frac{1}{N} \sum_{j=q}^{\mu q} \left(1 - \frac{1}{j}\right).$$

2) **Shuffle Phase:** In the shuffle phase, intermediate values calculated in the map phase are exchanged between servers in $\mathcal{G}$ until all servers in $Q$ hold enough values to compute the vectors they are responsible for.

As in [4], we allow creating and multicasting coded messages that are simultaneously useful for multiple servers. For a subset of servers $S \subseteq Q$ and $S \subseteq Q \setminus S$, we denote the set of intermediate values needed by server $S$ and known exclusively by the servers in $S$ by $V_{S}$. More formally,

$$V_{S} = \{cx_j : j \in WS S \cap T = \mathcal{Q} = S\}. $$

Let $\alpha_j \triangleq \frac{(n-1)\left(\frac{q}{\mu q}\right)(\frac{q}{\mu q}-1)}{\mu q}$ and $s_q \triangleq \inf(s : \sum_{i=s}^{\mu q} \alpha_i \leq 1 - \mu)$. For each $j \in \{\mu q,\mu q-1,\ldots,s_q\}$, and every subset $S \subseteq Q$ of size $j + 1$, the shuffle phase proceeds as follows.

1) For each $S \subseteq Q$, we evenly and arbitrarily split $V_{S} \setminus S$ into $j$ disjoint segments $V_{S} \setminus S = \{V_{S} \setminus S, S_{1}, \ldots, S_{j}\}$, and associate the segment $V_{S} \setminus S$ with server $S \subseteq S_{j}$.  

2) Server $S \subseteq Q$ multicasts the bit-wise XOR of all the segments associated with it in $S$. More precisely, it multicasts $\bigoplus_{S \subseteq S \setminus \tilde{S}} V_{S} \setminus S$ to the other servers in $S \setminus \tilde{S}$.

For every pair of servers $S, \tilde{S} \subseteq S$, since server $S$ has computed locally the segments $V_{S} \setminus (S \setminus \tilde{S})$, it can cancel them from the message sent by server $\tilde{S}$, and recover the intended segment. We then unicast any remaining needed values until all servers in $Q$ hold enough intermediate values to decode successfully.

**Definition 2.** The communication load, denoted by $L$, is the number of messages per source row and vector $y$ exchanged during the shuffle phase, i.e., the total number of messages sent during the shuffle phase divided by $mN$.

The communication load after completing the multicast phase is $\sum_{j=s_q}^{\mu q} \frac{\alpha_j}{j}$ [4], and for the scheme in [4] the total communication load after unicasting any remaining values becomes

$$L_{MDS} = \sum_{j=s_q}^{\mu q} \frac{\alpha_j}{j} + 1 - \mu - \sum_{j=s_q}^{\mu q} \alpha_j,$$ (1)

As in [4], we consider the cost of a multicast message to be equal to that of a unicast message. In real systems, however, it may vary depending on the network architecture.
3) Reduce Phase: Finally, in the reduce phase, the vectors \( y_1, \ldots, y_N \) are computed. More specifically, server \( S \in Q \) uses the locally computed sets \( Z_c^{(1)}, \ldots, Z_N^{(1)} \) and the received messages to compute the vectors in \( \{y_j : j \in W_S\} \).

### III. BLOCK-DIAGONAL CODING

We introduce a block-diagonal encoding matrix of the form

\[
\Psi = \begin{bmatrix}
\psi_1 & & \\
& \ddots & \\
& & \psi_T
\end{bmatrix},
\]

where \( \psi_1, \ldots, \psi_T \) are \( \frac{r}{T} \times \frac{m}{r} \) encoding matrices of an \((\frac{r}{T}, \frac{m}{r})\) MDS code, for some integer \( T \) that divides \( m \) and \( r \). Note that the encoding given by \( \Psi \) amounts to partitioning the rows of \( A \) into \( T \) disjoint submatrices \( A_1, \ldots, A_T \) and encoding each submatrix separately. We refer to an encoding \( \Psi \) with \( T \) disjoint submatrices as a \( T \)-partitioned scheme, and to the submatrix of \( C = \Psi A \) corresponding to \( \psi_i \) as the \( i \)-th partition. We remark that all submatrices can be encoded using the same encoding matrix, i.e., \( \psi_i = \psi \), \( i = 1, \ldots, T \), reducing the storage requirements, and can be encoded/decoded in parallel if many servers are available. We remark that the case \( \Psi = \psi \) (i.e., the number of partitions is \( T = 1 \)) corresponds to the scheme in \cite{4}, which we will sometimes refer to as the unpartitioned scheme.

#### A. Assignment of Coded Rows to Batches

For a block-diagonal encoding matrix \( \Psi \), we denote by \( c_i^{(1)} \), \( t = 1, \ldots, T \) and \( i = 1, \ldots, r/T \), the \( i \)-th coded row of \( C \) within partition \( t \). For example, \( c_1^{(2)} \) denotes the first coded row of the second partition. As described in Section II, the coded rows are divided into \( \binom{K}{\mu} \times T \) disjoint batches. To formally describe the assignment of coded rows to batches we use a \( \binom{K}{\mu} \times T \) integer matrix \( P = [p_{i,j}] \), where \( p_{i,j} \) describes the number of rows from partition \( j \) that are stored in batch \( i \). Note that, due to the MDS property, any set of \( m/T \) rows of a partition is sufficient to decode the partition. Thus, without loss of generality, we consider a sequential assignment of rows of a partition into batches. For example, for the assignment \( P \) in Example \( 1 \) (see \( 2 \)), rows \( c_1^{(1)} \) and \( c_2^{(1)} \) are assigned to batch 1, \( c_3^{(1)} \) and \( c_4^{(1)} \) are assigned to batch 2, and so on. The rows of \( P \) are labeled by the subset of servers the corresponding batch is stored at, and the columns are labeled by its partition index. The assignment matrix \( P \) must satisfy the following conditions.

1. The entries of each row of \( P \) must sum to the batch size, i.e., \( \sum_{j=1}^{T} p_{i,j} = r/(\mu q) \), \( 1 \leq i \leq \binom{K}{\mu} \).
2. The entries of each column of \( P \) must sum to the number of rows per partition, i.e., \( \sum_{i=1}^{\binom{K}{\mu}} p_{i,j} = \frac{r}{T} \), \( 1 \leq j \leq T \).

We refer to the pair \( (\Psi, P) \) as the storage design.

#### Example 1

\( m = 20, N = 4, K = 6, q = 4, \mu = 1/2, T = 5 \). For these parameters, there are \( r/T = 6 \) coded rows per partition, of which \( m/T = 4 \) are sufficient for decoding, and \( \binom{K}{\mu q} = 15 \) batches, each containing \( r/(\mu q) = 2 \) coded rows. We construct the storage design shown in Fig. \( 1 \) with assignment matrix \( P \):

\[
\begin{bmatrix}
1 & 2 & 3 & 4 & 5 \\
(S_1, S_2) & 2 & 0 & 0 & 0 & 0 \\
(S_1, S_3) & 2 & 0 & 0 & 0 & 0 \\
(S_1, S_4) & 0 & 2 & 0 & 0 & 0 \\
(S_2, S_3) & 0 & 0 & 0 & 0 & 2 \\
(S_2, S_4) & 0 & 0 & 0 & 0 & 2 \\
\end{bmatrix},
\]

where rows are labeled by the subset of servers the batch is stored at, and columns are labeled by the partition index. For this storage design, any \( g = 4 \) servers collectively store at least 4 coded rows from each partition. However, some servers store more rows than needed to decode some partitions, suggesting that this storage design is suboptimal.

Assume \( G = \{S_1, S_2, S_3, S_4\} \) is the set of \( g = 4 \) servers that finish their map computations first. Also, assign vector \( y_i \) to server \( S_i \), \( i = 1, 2, 3, 4 \). We illustrate the coded shuffling scheme for \( S = \{S_1, S_2, S_3\} \) in Fig. \( 2 \).

\( S_1 \) multicasts \( c_1^{(1)} x_3 + c_2^{(1)} x_2 \) to \( S_2 \) and \( S_3 \). Since \( S_2 \) and \( S_3 \) can cancel \( c_1^{(1)} x_3 + c_1^{(2)} x_2 \) respectively, both servers receive one needed intermediate value. Similarly, \( S_2 \) multicasts \( c_2^{(1)} x_3 + c_3^{(1)} x_1 \), while \( S_3 \) multicasts \( c_1^{(2)} x_2 + c_0^{(1)} x_1 \). This process is repeated for \( S = \{S_2, S_3, S_4\}, S = \{S_1, S_3, S_4\}, \) and \( S = \{S_1, S_2, S_4\} \). After the shuffle phase, we have sent 12 multicast messages and 30 unicast messages, resulting in a communication load of \((12 + 30)/20)/4 = 0.525\), a 50% increase from the load of the unpartitioned scheme (0.35, given by \( 1 \)). In this case, \( S_1 \) received additional intermediate values from partition 2, despite already storing enough, further indicating that the assignment in \( 2 \) is suboptimal.

### IV. PERFORMANCE OF THE BLOCK-DIAGONAL CODING

In this section, we analyze the impact of partitioning on the performance. In the following theorem, we show that we can partition up to the batch size without increasing the communication load and the computational delay.

**Theorem 1.** For \( T \leq r/(K/\mu q) \) there exists an assignment matrix \( P \) such that the computational delay and the communication load are equal to those of the unpartitioned scheme.
Proof: The computational delay is unchanged if any \( q \) servers hold enough coded rows to decode all partitions. For \( T = r/(K_{\mu q}) \), the assignment \( P \) consisting of only ones is always valid, and the \( q \) servers in \( Q \) then collectively store \( \mu q m / T \) rows from each partition. Since each coded row is stored by at most \( \mu q \) servers in \( Q \), the servers in \( Q \) always collectively store at least \( \mu q m / T = m / T \) unique coded rows from each partition.

To see that the communication load is unchanged, we note that after having sent all multicast messages, all servers hold the same total number of coded intermediate values regardless of partitioning. Since all servers need \( m / T \) values from each partition, if we can show that each server will hold the same number of coded values from each partition at this point, we have also shown that the communication load is unchanged. First, for the given assignment, all servers store the same number of rows from each partition before the shuffle phase. Second, in the shuffle phase, all servers \( S \in Q \) receive the values in \( \gamma(S) \) for all \( S \subseteq Q \) such that \( |S| = \mu q + 1, \mu q, \ldots, s_q + 1 \). \( \gamma(S) \) is computed from a union of batches and therefore guaranteed to contain an equal number of coded intermediate values from each partition for this assignment. These arguments together show that all servers will hold an equal number of coded values from each partition after all multicast messages have been sent. For \( T \leq r/(K_{\mu q}) \) we can split partitions into smaller partitions until we have exactly \( r/(K_{\mu q}) \) partitions, and repeat the same argument. \( \blacksquare \)

### A. Communication Load and Computational Delay

For the unpartitioned scheme [4], \( G = Q \), and the number of remaining values that need to be unicasted after the multicast phase is constant, regardless of which subset \( Q \) of servers first finish their map computations. However, for the block-diagonal (partitioned) coding scheme, both the cardinality of \( G \) and the number of remaining unicasts may vary.

For a given assignment \( P \) and a specific \( Q \), we denote the number of remaining values needed after the multicast phase by server \( S, S \in Q \), by \( U(S)^{(P)} \), and the total number of remaining values needed by the servers in \( Q \) by \( U_Q(P) \triangleq \sum_{S \subseteq Q} U(S)^{(P)} \). Let \( Q^q \) denote the superset of all sets \( Q \). Then, for a given storage design \((\Psi, P)\), the communication load of the block-diagonal coding scheme is given by

\[
L_{BDC}(\Psi, P) = \sum_{j=1}^{s_q} \alpha_j + \frac{1}{mN} \sum_{Q \in Q^q} U_Q(P).
\]

To evaluate \( U(S)^{(P)} \), we count the total number of intermediate values that need to be unicasted to server \( S \) until it holds \( m / T \) intermediate values from each partition. On the other hand, the computational delay is \( D_{BDC} = \frac{1}{K} E(D_{(g)}) \).

### B. Optimal Assignment

For a given \( \Psi \), the assignment of rows into batches can be formulated as an optimization problem, where one would like to minimize \( L_{BDC} \) over all assignments \( P \). More precisely, the optimization problem is \( \min_{P \in \mathbb{P}} L_{BDC}(\Psi, P) \), where \( \mathbb{P} \) is the set of all assignments \( P \), and where the dependence of \( L_{BDC} \) on \( P \) is nonlinear. This is a computationally complex problem since both the complexity of evaluating the performance of a given assignment and the number of assignments scale exponentially in the problem size.

### C. Decoding Complexity

The decoding complexity of RS codes, a popular class of MDS codes, on the packet erasure channel is quadratic with the code length [4]. The unpartitioned scheme of [4] has decoding complexity \( O(r^2) \). On the other hand, our proposed block-diagonal scheme has decoding complexity \( O(r^2/T) \). By choosing \( T \) large, we can thus significantly reduce the decoding complexity over the scheme in [4].

### V. Assignment Solvers

We propose two solvers for the problem of assigning rows into batches: a heuristic solver that is fast even for large problem instances, and a hybrid solver combining the heuristic solver with a branch-and-bound solver. The branch-and-bound solver produces an optimal assignment but is significantly slower, hence it can be used as stand-alone only for small problem instances. We use a dynamic programming approach to speed up the branch-and-bound solver by caching \( U_Q(S) \) for all \( Q \), \( Q \in Q^q \), indexed by the batches each \( U_Q(S) \) is computed from. This way we only need to update the affected \( U_Q(S) \) when assigning a row to a batch. For all solvers, we first label the batches lexicographically and then optimize \( L_{BDC} \) in [4]. The solvers are available under the Apache 2.0 license [8].

#### A. Heuristic Solver

The heuristic solver creates an assignment matrix \( P \) in two steps. We first set each entry of \( P \) to \( \gamma \triangleq r/(K_{\mu q} \cdot T) \), thus assigning the first \( K_{\mu q} \gamma \) rows of each partition to batches such that each batch is assigned \( \gamma T \) rows. Then, for a given storage design \((\Phi, P)\), the communication load of the block-diagonal coding scheme is given by

\[
L_{BDC}(\Psi, P) = \sum_{j=1}^{s_q} \alpha_j + \frac{1}{mN} \sum_{Q \in Q^q} U_Q(P).
\]

#### B. Branch-and-Bound Solver

The branch-and-bound solver finds an optimal solution by recursively branching at each batch for which there is more than one possible assignment and considering all options. For each branch, we lower bound the value of the objective function of any assignment in that branch and only investigate branches with possibly better assignments.

1) **Branch**: For the first row with remaining assignments of a partial assignment matrix, branch on every available assignment for that row.
We first find a candidate solution using the heuristic solver and then iteratively improve it using the branch-and-bound solver. In particular, we decrement by 1 and then iteratively improve it using the branch-and-bound solver.

C. Hybrid Solver

We introduced a block-diagonal coding scheme for distributed matrix multiplication based on partitioning the matrix into smaller submatrices. Compared to earlier (unpartitioned) schemes, the proposed scheme yields lower decoding complexity with no performance loss up to a level of partitioning. For heavier partitioning, the lower decoding complexity is achieved at the expense of a slight increase in average computational delay and communication load (about 2% and 10%, respectively, with 3000 partitions for a matrix of 6000 rows).

VI. NUMERICAL RESULTS

In Figs. 3 and 4, we plot the performance of the proposed block-diagonal coding scheme with assignment $P$ given by the heuristic and the hybrid solver. We also give the average performance over 100 random assignments. The results shown are averages over 1000 randomly generated realizations of $Q$ as it is computationally infeasible to evaluate the performance exhaustively for larger systems. The error bars show the best and worst performance.

In Fig. 3 we plot the communication load and computational delay normalized by those of the unpartitioned scheme of [4] as a function of the number of partitions $T$. The system parameters are $m = 6000$, $K = 9$, $q = 6$, $N = 6$, and $\mu = 1/3$. For up to $K^{\mu q} = 250$ partitions, the proposed scheme does not incur any loss in performance. For heavier partitioning, a tradeoff between performance and partitioning level is observed. In particular, if the number of partitions is larger than 1000, an increase of the communication load is observed. However, note that with 3000 partitions (the maximum possible), there is only about a 10% increase in communication load. On the other hand, the increase of computational delay with the number of partitions is very small with $P$ from the heuristic solver. A further improvement in communication load can be achieved using the hybrid solver, but at the expense of a possibly larger computational delay. In Fig. 4 we plot the normalized performance for a constant $\mu q = 2$, $\mu m = 2000$ coded rows per server, and $m/T = 10$ rows per partition as a function of the number of servers, $K$. As $K$ grows, the communication load of both solvers converge, but the heuristic solver outperforms the random assignments by approximately 10 percentage points in terms of computational delay.

VII. CONCLUSION

We introduced a block-diagonal coding scheme for distributed matrix multiplication based on partitioning the matrix into smaller submatrices. Compared to earlier (unpartitioned) schemes, the proposed scheme yields lower decoding complexity with no performance loss up to a level of partitioning. For heavier partitioning, the lower decoding complexity is achieved at the expense of a slight increase in average computational delay and communication load (about 2% and 10%, respectively, with 3000 partitions for a matrix of 6000 rows).

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