A Preliminary Study of Breaking Waves Phenomenon on a Numerical Wave Tank

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Abstract. The detailed prediction of breaking waves transformation may be a main concern within the ocean structures design and development at the top of submerged flat reefs. In this preliminary analysis, phenomenon of solitary wave on the reef can be simply interpreted in two-dimensional numerical wave tank (NWT), based on the computational fluid dynamics (CFD). A more sophisticated method, large eddy simulation (LES) turbulence model together with volume of fluid (VOF) for capturing free-surface, could clarify two-phase incompressible flow Navier-Stoke equations by finite volume approach. The other approaches have been used. Existing model results, wave generation, breaking wave, and velocity distributions, devised the validity of laboratory measurements. This set of tests desired to examine the effect of different reef configurations (reef length, reef height, incident wave height, and manning coefficient) on the solitary wave propagation. Results show that the model can contribute decent predictions of the phenomenon of breaking waves on various submerged flat reef configurations.

1. Introduction

Breaking waves are one of the most complicated hydrodynamic model in a coastal area [1]. In terms of topography such as reef dimensions, reef roughness and environmental loads, the wave-reef interactions are governed by morphological specifications. Additionally, the four important of dimensional reef analysis aspects are reef length (a/h), reef height (b/h), wave non-linearity (H/h), and wave steepness (H/L) (where wave height H, wave length L, and water depth h). Understanding solitary wave generation through the reef is important not only for water depth extremely decreases, but also for the impact of wave-reef interaction. In some theoretical schemes, solitary waves are applied to providing a specific description of shallow-water waves nonlinearity [2]. It is relevant also for representing the propagation of long-period waves in shallow water depth [3].

There are a number of studies for the modelling wave dynamics over reefs, described in nonlinear effects [2,4,5], breaking-wave characteristics together with wave run-up in the reef zone [6-8], wave-structure interaction and reef current velocity distribution [9,10], effects of reef morphology configurations [11,12]. Subsequently, a numerical wave tank (NWT) was capabled of simulating breaking waves, which in many cases can precisely replace physical wave flume. The first study carried out by Lin and Liu [13] was based on experimental results. An additional research is focused on this using this computationally wave breaking, which involves air entrainment [14], wave breaking on sloping beach [15], solitary wave run-up process [16] and hybrid computational scheme of breaking wave dynamics model at specific locations [17].
Furthermore, Smoothed Particle Hydrodynamics (SPH) has been supported the complex surf zone geometries, surf zone energy dissipations and nearshore breaking wave [18]. Despite SPH has become a common technique for ocean engineering problems, it is still being improved the performance related to its boundary conditions and model convergence [19]. Alternatively, newly developed Boussinesq equations are more suitable for simulating breaking waves on complicated reef morphology variations [20]. Even Boussinesq equations are the universal tool due to its computational efficiency, several disadvantages still exists in terms of wave breaking could not be captured well as well as could not resolve breaking wave vertical flow due to polynomial approximation. To improve the issues of simulating the solitary breaking wave over the reefs by using Boussinesq, the study generates NWT based on Large Eddy Simulation (LES) is performed. The LES model could provide a more detailed explanation for significant large-scale unsteadiness flow [21]. Since the solitary wave transformation over a reef has been extensively studied previously, it has been stated that the wave-reef interaction analysis will encourage ocean engineering problems [2].

Knowledge of breaking solitary wave over flat reef experimentally and numerically was derived from Yasuda et al. [5]. Free-surface capturing and velocity distribution was measured by PIV (Particle Image Velocimetry) in various locations. In addition, it numerically modeled wave model with the fully nonlinear BIM (Boundary Integral Method). Although those aforementioned studies described interesting results, this paper received a preliminary study. Compared to a previous study [5], this present study is focussed on parameters for breaking solitary waves on reef morphologic properties at breaking point setup by considering aspects of a two-phase flow model. Then, 2-D NWT computes the Navier-Stokes equations collaboration between the Volume of Fluid (VOF) term for free-surface monitoring [22] together with LES for turbulence approach [21,23], respectively. Compared to a previous study [5], this present study is focussed on parameters for breaking solitary waves on reef morphologic properties at breaking point setup.

The paper will be split in three sections, the first section verify previously the accepted model by the laboratory experiments. The validity of the solitary wave elevations as well as horizontal-vertical velocity distribution around the submerged flat reef is demonstrated. The second section addresses the impact of different submerged reef morphologies on the solitary wave propagation (reef length, reef height, incident wave height, and manning coefficient). It is studied with the development of model applications that exclude laboratory data. Finally, the third section includes the major findings. Overall, the present numerical work devotes a preliminary concept into the assessment of the breaking solitary waves phenomenon on various submerged flat reef configurations.

2. Numerical Background

This numerical model allows high order discretization scheme to be implemented, achieving good numerical accuracy and efficiency. It could be used in several modes, which support incompressible flows to various fluid equations problems. can be operated in several modes corresponding to incompressible flows to different fluid equations cases. A viscous incompressible two-phase flow is known and the Navier-Stokes equation defines it. The main statements are the continuity and momentum equation for two coordinate directions terms (x,y):

$$\frac{\partial U}{\partial t} = 0$$  \hspace{1cm} (1)

$$\frac{\partial u}{\partial t} + \frac{1}{\alpha} \left[ u A_x R \frac{\partial u}{\partial x} + v A_y \frac{\partial u}{\partial y} \right] = - \frac{1}{\rho x} \frac{\partial P}{\partial x} + G_x + f_x - b_x - u \frac{R_{SOR}}{\rho}$$ \hspace{1cm} (2)

$$\frac{\partial v}{\partial t} + \frac{1}{\alpha} \left[ u A_y \frac{\partial v}{\partial x} + v A_y \frac{\partial v}{\partial y} \right] = - \frac{1}{\rho y} \frac{\partial P}{\partial y} + G_y + f_y - b_y - v \frac{R_{SOR}}{\rho} \hspace{1cm} (3)$$
A commonly used VoF method is treated the accurate representation of the free-surface interface between the air and water [24]. The fluid density general description is written as

$$\rho = \alpha \rho_1 + (1 - \alpha) \rho_2$$

(4)

Where $U$ is the average velocity over time $t$, $\rho$ is the fluid density, $\rho_1$ is the density of water (1000 kg m$^{-3}$), $\rho_2$ is the density of air (1 kg m$^{-3}$), $P$ is pressure, $\nu$ is the kinematic viscosity, $g$ is the gravity term, $\left(A_x, A_y\right)$ are fractional flow areas, $\left(G_x, G_y\right)$ are accelerations of object, $\left(f_x, f_y\right)$ are viscous accelerations, $\left(b_x, b_y\right)$ are center of mass injection, $R_{SOL}$ is the center of mass, and $\alpha$ is the volume fraction of water.

Additionally, turbulence in breaking waves across the flat reef is described by the LES model. It is employed the domain to control massive turbulent flow energy. The effects of turbulence to compute are represented by an eddy viscosity associated with a proper longitudinal scale, $\Delta$, as it occurs

$$\Delta = (dx \cdot dy)^{\frac{1}{3}}$$

(5)

Here $dx$ and $dy$ are the $x$-axis and $y$-axis directions grid size, respectively. In order to measure strong velocity fluctuations, the grid is implemented.

Further, the solitary wave generation is based on McCowan’s theory [25] that has more accurate than the Boussinesq concept and is approved by Munk [26] after detailed investigations. A sponge layer method, which is theoretically simple and works effectively, employs an additional damping region to dissipate wave and reflected. The free-surface, velocity and wave absorbing layer for a solitary wave generation are

$$\eta = \frac{N}{M} \frac{\sin \left[ M \left(1 + \frac{\eta}{h_1}\right) \right]}{\cos \left[ 1 + \frac{\eta}{h_1}\right] + \cosh \left[ M \frac{X}{h_1}\right]}$$

(6)

$$M = \frac{N}{\varepsilon} \tan \left[ \frac{1}{2} M \left(1 + \varepsilon\right) \right]$$

(7)

$$N = \frac{2}{3} \sin^2 \left[ M \left(1 + \frac{2}{3} \varepsilon\right) \right]$$

(8)

$$\frac{u(x, y, t)}{c_0} = \frac{1 + \cos \left(\frac{My}{h_1}\right) \cosh \left(\frac{MX}{h_1}\right)}{\cos \left(\frac{My}{h_1}\right) + \cosh \left(\frac{MX}{h_1}\right)^2}$$

(9)

$$\frac{w(x, y, t)}{c_0} = \frac{\sin \left(\frac{My}{h_1}\right) \sinh \left(\frac{MX}{h_1}\right)}{\cos \left(\frac{My}{h_1}\right) + \cosh \left(\frac{MX}{h_1}\right)^2}$$

(10)

$$c = u + c_0$$

(11)
\[
\frac{\partial \eta}{\partial t} + u \cdot \nabla \eta = -\frac{1}{\rho} + \nabla \cdot (\nu \nabla \eta) \eta \left(u - u_{str}ight) \tag{12}
\]
\[
\zeta = \zeta_0 + s \frac{\zeta_1 - \zeta_0}{h_1} \tag{13}
\]

Where wave elevation \( \eta \), incident wave height \( H \), water depth \( h_1 \), \( x \)-axis velocity \( u \), \( y \)-axis velocity \( v \), wave speed \( c \), undisturbed velocity \( \bar{u} \), wave speed \( c_0 = \sqrt{g(h_1+H)} \). \( X = x - ct \), \( \varepsilon = H/h_1 \), \( \zeta_0 \) and \( \zeta_1 \) are damping coefficient at starting (0.0) and end sides \((10/s)\) of sponge layer, respectively, the distance from starting side to end side of sponge layer \( s \) and \( u_{str} \) is background stream velocity.

Beyond that, Wilmott [27] consider and calculates ability of model to determine the efficiency of the computational model in a more realistic way:

\[
skill = 1 - \frac{\sum |Y_a - Y_b|^2}{\sum (|Y_a - \bar{Y}_b| + |Y_a - \bar{Y}_b|)^2} \tag{14}
\]

Here \( Y_a \) is the estimated calculation as well as \( Y_b \) is the measured value. In the overline bar, the average value is shown. The result of higher the skill value, close to one, is the better model performance, and the worse skill at the same time is close to zero.

3. Results and Discussion

3.1. Model verification

This initial concept by Yasuda et al. [5] is the outcome of a series of reef experiments. Such experiments were carried out in a 25 m long, 0.3 m wide and 0.1 m deep laboratory flume. An attached horizontal flat reef 0.24 m high was created 32.91 m beginning at the wave generator. Four wave gauge locations (P1-P4) measured wave surface elevations. In this present computational study, the flat reef profile is only represented by various wave conditions. Helluy et al. [28], 6.0 m long and 0.81 m high, adopted the simple 2D computational region as illustrated in Figure 1.

![Figure 1. A two-dimensional computational set-up.](image)

3.2. Computational set up

The Finite Volume (FV) term is discretized Navier-Stokes equations. In order to overcome Navier-Stokes equations, a VOF technique was checked for breaking waves surface elevation monitoring across the reef as well as the Large Eddy Simulation (LES) turbulence is attached to resolve eddy viscosity on the computational domain.

Furthermore, detailed numerical wave generator and absorber settings of Helluy et al [28] at positions 5 and 1 m from the center of a submerged flat reef is also shown in Figure 2a. The transmitted waves are not overtopped out beyond the domain ascribe to a right solid wall condition. However, both top and bottom domains were used in free wall and solid wall respectively. Further, the LES turbulence characteristic did not arrange the inlet flow boundary due to expecting strong
turbulence and embed a no-slip wall boundary at a submerged flat reef surface. Consequently, the coordinate systems of numerical model are identical to the 2D reef experiment measurements.

The structured grid is applied for discretizing the numerical domain. During minimize total cells, the mesh varies in the $x$ and $y$ direction. The cell sizes $dx$ and $dy$ of the free-surface zone are progressively decreased from left to the right boundary as presented in Figures 2b and 2c, sequentially. Beyond that, grid refinement close to the numerical domain along the submerged flat reef by reducing grid sizes as illustrated in Figure 2.

![Figure 2. Boundary conditions and numerical domain grids.](image)

Since validating the numerical model, the grid independence study is conducted by comparing it with the provided experimental results. The provided experimental results for the grid dependence study are the breaker location ($x_b$) and the breaker height ($H_b$). The grid sensitivity of numerical results is tested where the cases were presumed to be significant. The served uniform grid ($dx$ and $dy$) running between 0.0025, 0.005, 0.010 and 0.020 m are checked.

The breaker location $x_b$ and the breaker height $H_b$ are demonstrated in Figure 3. These computational results are comparable with experimentally breaking location $x/h_l = 35.951$ m at the same time as breaker height $H_b = 0.320$ m for $dx = dy = 0.0025$ m, 0.005 and 0.010 m. With grid size $dx = dy = 0.0025$ m, the selection of current parameters could only be observed as difference of under 1% in the breaking wave parameter. Additionally, the 2D models are computationally quite expensive. The total computational mesh is made up of 3637 numbers of cells.

![Figure 3. The grid study on numerical domain: (a) breaker location $x_b$ and (b) breaker height $H_b$; $x_b$ (Exp) and $H_b$ (Exp) are experimentally values by Yasuda et al. [5]](image)
3.3. Analysis of solitary wave surface

The implemented Grimshaw’s solitary wave theory [29] for the four fixed wave gauges P1-P2-P3-P4, as shown in Figure 1, are under a specific wave condition. For a water depth of $h_1 = 0.205$ m persistently, initial solitary wave while using normalized water depth $H_i/h_1 = 0.33$ is implemented. The gauge P1 is placed in front of the flat reef and the gauge P2 is located in the reef tip intended for measure the solitary wave height. The gauges P3 and P4 are sequentially set from the tip of submerged flat reef at 0.515 and 1.020 m, respectively.

As represented in Figure 4 illustrates the comparison between dimensionless free-surface elevations $(\eta/h_1)$ and dimensionless time series $(t = \sqrt{g/h_1})$ at four limited wave gauge P1-P2-P3-P4 locations represented by Yasuda et al. [5] and Helluy et al [28]. It indicates that all models can approximate well the solitary wave propagation across the submerged flat reef with the skill values greater than 0.80.

Since solitary wave records measured by gauge P1 were used to generates during the simulation, the incident waves with P1 were generated very well in Figure 4a within skill values $= 0.87$. The wave reflection of the wavemaker may create a little discrepancy. When the solitary wave passes from gauge P1 to the tip of the submerged flat reef (Gauge P2), the shoaling effect as shown in Figure 4b is concerned.

Nevertheless, after the incident breaking point (see P3 in Figure 4c) and at the center of submerged flat reef (see gauge P4 in Figure 4d), the skill value declines considerably. It may be fundamentally due to air entrainment of breaking wave processes on the free-surface tracking approach. Wave breaking could be started at a location right after the tip of the submerged flat reef. Then, measured from gauge P3 to P4, the wave propagation time series show wave profiles, as well as solitary wave height, are decreased gradually.

As for breaking wave tracking, Yasuda et al. [5] only captured two locations of breaking surface profile. It is necessary to remember that two points are at the breaking and breaking jet-fall point initiation. These overall arguments between present simulations and experimental breaking wave profiles around the jet is fairly well with the skill values larger than 0.80 as demonstrated in Figure 5. The location of the observed locations on the x-direction is $x/h_1 = 35.59$ where they cannot be separated from each other during the overturning breaking wave process.
Figure 5. Comparison of breaking wave profile at two locations; (a) breaking point and (b) breaking jet-fall. Dashed line: prediction by numerical model and red solid circles: experimental results.

Figure 6 represents the compared horizontal and vertical water particle velocities of dimensionless vertical distribution under the breaking peak point. The skill values at two water particle velocities are larger than 0.75. The horizontal water particle velocity \( u \) may be detected \( x/h = 35.18 \) with the quite well skill values of 0.78, which are transformations from subcritical flow \( (u/\sqrt{gh} < 1) \) to supercritical flow \( (u/\sqrt{gh} > 1) \). This location is important to examine the impact of air bubbles, as demonstrated in Figure 6a.

On the contrary, distribution of vertical water particle velocity \( v \) only took place in the supercritical flow \( (u/\sqrt{gh} > 1) \) zone as denoted in Figure 6b. The numerical and experimental of breaking point dimensionless vertical distribution are similar. This compared result gained the skill values of 0.82. It should be noticed that the vertical distributions under breaking peak points are not zero. Overall, the two present dimensionless distribution models fairly well represent the free-surface elevation time series to obtain the breaking peak point position.

Figure 6. Comparison of dimensionless velocity distribution; (a) horizontal water particle velocity and (b) vertical water particle velocity. Dashed line: prediction by numerical model and red solid circles: experimental results.

3.4. Model implementations

Within this part, the validated breaking wave phenomenon has been investigated the impact on the breaker location \( x_b \) and the breaker height \( H_b \) of some related submerged reef morphologic parameters such as reef length \( L_r \), reef height \( h_r \), incident wave height \( H_i \) and manning coefficient \( n \). Based on the basic submerged flat reef profile (see Figure 1), the six reef length variations are examined within the range of 2, 2.2, 2.4, 2.6, 2.8 and 3.0 m. Then, six submerged flat reef heights \( (0.263, 0.268, 0.273, 0.278, 0.283 \text{ and } 0.288 \text{ m}) \) were examined too. To investigate the roughness effect, six rough submerged flat reef surfaces as denoted in Table 1 are described as manning coefficients \( (0.005, 0.01, 0.015, 0.020, 0.025 \text{ and } 0.030) \).
Table 1. List of 2D computational cases.

| Scenarios | Parameters |
|-----------|------------|
|           | $L_r$ (m)  | $h_r$ (m) | $n$  |
| 1         | 2          | 0.263     | -    |
|           | 2.2        |           |      |
|           | 2.4        |           |      |
|           | 2.6        |           |      |
|           | 2.8        |           |      |
|           | 3          |           |      |
| 2         | 2          | 0.263     | -    |
|           |            | 0.268     |      |
|           |            | 0.273     |      |
|           |            | 0.278     |      |
|           |            | 0.283     |      |
|           |            | 0.288     |      |
| 3         | 2          | 0.263     | 0.0050 |
|           |            |           | 0.0100 |
|           |            |           | 0.0150 |
|           |            |           | 0.0200 |
|           |            |           | 0.0250 |
|           |            |           | 0.0300 |

During the model simulated, all simulations are conducted under the interaction of three incident wave heights ($H_i = 0.131$, 0.262, and 0.328 m), constant water depth ($h_1 = 0.31$ m), and constant water celerity ($c = 2.06$ m/s). Figure 7 designates the first scenario that the breaker location ($x_b$) is more responsive to the change in submerged flat reef length ($L_r$) over the tested range, especially the reef height ($h_r$). In this case, during the overturning breaking process, they cannot be distinguished themselves. The variant of incident wave height appears that $x_b$ decreases consistently with the increase in the submerged flat reef length (Figure 7a) and a slight decline of $x_b$ could only be found under reef height smaller than 0.273 m (Figure 7b). Despite that, the $x_b$ cannot be predicted directly from the reef profile where the incident wave height propagated initially.

![Figure 7](image-url). The predicted breaking wave parameters under $h_1 = 0.310$ m and $h_r = 0.263$ m with varying submerged flat reef length; (a) breaker height and (b) breaker location.
The second scenario, the difference in breaker height ($H_b$) with the submerged flat reef length ($L_r$) and reef height ($h_r$), is illustrated in Figure 8. As represented in Figure 8a, note that the $H_b$ is quite sensitive to the increased $L_r$. It caused by a wider submerged flat reef is far from the wave generation yet and dissipates less wave breaking partially. Further, the $H_b$ is more affected by the change in several $h_r$, as shown in Figure 8b, due to the increasing tip of submerged flat reef reflection of the incident wave. Figure 9 shows that both $x_b$ and $H_b$ reduced gradually the submerged flat reef surface roughness with the increasing manning coefficient ($n$) as well as the solitary wave propagations by all of the four morphologic parameters under the third scenario significantly impacted. Then, the wave energy was dissipated on the rough flat reef as well in $x_b$ and $H_b$. In the case from smooth to rough reef flat, the partial resolution has been appeared for resolving eddy viscosity condition in the flow field.

4. Conclusion

In this paper, a simple 2-D numerical wave tank (NWT), well-validated LES turbulence model for wave-reef interactions, has been investigated numerically to solving Navier-Stokes equations. Then wave generation, free-surface elevation, horizontal and vertical distribution velocity of breaking point profile, experimentally reported by Yasuda et al. [5], are evaluated against the numerical model performance. Further, the varying reef morphology parameters such as the reef length ($L_r$), reef height ($h_r$), incident wave height ($H_i$) and manning coefficient ($n$) are also investigated the impacts of solitary wave transformation which laboratory data are unavailable.

All free-surface elevations captured model propagate solitary wave well (larger than 0.80). Overall validated simulation results show that the breaker location, based on the observations of reef length and reef height within tested ranges, get more impact than breaker height. The breaker location is more sensitive to the variation of reef length and reef height due to overturning breaking process where cannot be predicted the breaking wave propagation directly. While, the breaker height, unless the reef height is more affected, is almost insensitive to variations of the reef length. This is related to the less breaking wave dissipation partially. In addition, the manning coefficient as reef roughness is also another factor for reducing both breaker location and breaker height, respectively.
his paper, the interactions between solitary wave transformation and the submerged flat reef configurations, a 2-D numerical wave tank (NWT), have been investigated numerically to solving Navier-Stokes equations with the LES turbulence model with a well-validated model. The performance of the numerical model is evaluated against the experimental data for the wave generation, free-surface elevation, horizontal and vertical distribution velocity of breaking point profile reported by Yasuda et al. [5]. Further, the varying reef morphology parameters such as the reef length ($L_r$), reef height ($h_r$), incident wave height ($H_i$) and manning coefficient ($n$) are also applied to investigate the impacts of solitary wave propagation which laboratory data are unavailable.

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