Global Structure of Deffayet (Dvali–Gabadadze–Porrati) Cosmologies

Arthur Lue

Center for Cosmology and Particle Physics
Department of Physics
New York University
New York, NY 10003

Abstract

We detail the global structure of the five-dimensional bulk for the cosmological evolution of Dvali-Gabadadze-Porrati braneworlds. The picture articulated here provides a framework and intuition for understanding how metric perturbations leave (and possibly reenter) the brane universe. A bulk observer sees the braneworld as a relativistically expanding bubble, viewed either from the interior (in the case of the Friedmann–Lemaître–Robertson–Walker phase) or the exterior (the self-accelerating phase). Shortcuts through the bulk in the first phase can lead to an apparent brane causality violation and provide an opportunity for the evasion of the horizon problem found in conventional four-dimensional cosmologies. Features of the global geometry in the latter phase anticipate a depletion of power for linear metric perturbations on large scales.

*E-mail: lue@physics.nyu.edu
I. INTRODUCTION

The gravity theory of Dvali–Gabadadze–Porrati (DGP) is a braneworld theory with a metastable four-dimensional graviton [1]. The graviton is pinned to a four-dimensional braneworld by intrinsic curvature terms induced by quantum matter fluctuations; but as it propagates over large distances, the graviton eventually evaporates off the brane into an infinite volume, five-dimensional Minkowski bulk. As a result, the DGP braneworld theory is a model in a class of theories in which gravity deviates from conventional Einstein gravity not at short distances (as in more familiar braneworld theories), but rather at long distances. Such a model has both intriguing phenomenological [2,3,4,5] as well as cosmological consequences [6,7,8,9,10,11].

A braneworld model of the sort where gravity is modified at extremely large scales is motivated by the desire to ascertain how our understanding of cosmology may be refined by the presence of extra dimensions. Observational pillars of the standard cosmological model, including the cosmic microwave background and large scale structure, provide important tests of theories that seriously modify physics at cosmological scales. Deffayet’s cosmological equations for the DGP model already point to important deviations from the standard model [7,8,9,10,11]. In order to constrain the DGP theory further, understanding the development of large scale structure is necessary. Leakage of gravitational energy into the bulk is a key aspect of the DGP braneworld model, and one expects such leakage to modify the spectrum of density fluctuations at the largest observable cosmological scales. However, a detailed analysis, or even an intuitive understanding, of how spectral power of metric perturbations on the brane fills the bulk and (potentially) reenters the brane depends on the global structure of the brane worldsheet.

In this paper, we articulate the global structure of the five-dimensional bulk for the cosmological evolution of DGP braneworlds as a first step in understanding how large scale structure in the universe is modified. After reviewing the particulars of the model, we take the equations laid out in [6] and show how one may interpret the evolution of the braneworld as a relativistically expanding bubble, viewed either from the interior or the exterior, depending on the specific phase of the theory. We then go on to examine the cosmological time foliations of the bulk and show how peculiarities arise, such as shortcuts through the bulk (leading to effective brane causality violation), and breakdowns of that foliation occur in different regimes of the theory, and discuss how understanding the global geometry of DGP braneworld evolution may offer insight into cosmological perturbations.

\[1\] Such shortcuts seem to be prevalent in braneworld theories. For previous studies of this phenomenon and how such shortcuts relate to four-dimensional Lorentz symmetry violation, see for example [12,13,14,15].
II. PRELIMINARIES

A. The Model and Cosmology

Consider a braneworld theory of gravity with an infinite volume bulk and a metastable brane graviton \[1\]. We take a four-dimensional braneworld embedded in a five-dimensional Minkowski spacetime. The bulk is empty; all energy-momentum is isolated on the brane. The action is

\[ S(5) = -\frac{1}{2} M^3 \int d^5 x \sqrt{-g} \ R + \int d^4 x \sqrt{-g^{(4)}} \ L_m + S_{GH} . \] (2.1)

The quantity \( M \) is the fundamental five-dimensional Planck scale. The first term in Eq. (2.1) corresponds to the Einstein-Hilbert action in five dimensions for a five-dimensional metric \( g_{AB} \) (bulk metric) with Ricci scalar \( R \). The term \( S_{GH} \) is the Gibbons–Hawking action. In addition, we consider an intrinsic curvature term which is generally induced by radiative corrections by the matter density on the brane \[1\]:

\[ -\frac{1}{2} M_P^2 \int d^4 x \sqrt{-g^{(4)}} \ R^{(4)} . \] (2.2)

Here, \( M_P \) is the observed four-dimensional Planck scale (see \[2\] for details). Similarly, Eq. (2.2) is the Einstein-Hilbert action for the induced metric \( g^{(4)}_{\mu\nu} \) on the brane, \( R^{(4)} \) being its scalar curvature. The induced metric is

\[ g^{(4)}_{\mu\nu} = \partial_\mu X^A \partial_\nu X^B g_{AB} , \] (2.3)

where \( X^A(x^\mu) \) represents the coordinates of an event on the brane labeled by \( x^\mu \).

The general time-dependent line element under consideration is of the form

\[ ds^2 = -N^2(t,y)dt^2 + A^2(t,y)\gamma_{ij}dx^i dx^j + B^2(t,y)dy^2 , \] (2.4)

and, the metric components are given by \[3\]

\[ N = 1 + \epsilon|y|\ddot{a} \left( \dot{a}^2 + k \right)^{-1/2} \]
\[ A = a + \epsilon|y| \left( \dot{a}^2 + k \right)^{1/2} \] (2.5)
\[ B = 1 , \]

\[ ^2\text{Throughout this paper, we use } A, B, \ldots = \{0,1,2,3,5\} \text{ as bulk indices, } \mu, \nu, \ldots = \{0,1,2,3\} \text{ as brane spacetime indices, and } i, j, \ldots = \{1,2,3\} \text{ as brane spatial indices.} \]
where \( k = -1, 0 \) or 1 is the intrinsic spatial curvature parameter. We take the total energy-momentum tensor which includes matter and the cosmological constant on the brane to be

\[
T_{AB}^{\text{brane}} = \delta(y) \, \text{diag} \left( -\rho, p, p, p, 0 \right) .
\]  

(2.6)

When the matter content on the brane is specified, the induced scale factor \( a \equiv A(t, y = 0) \) is determined by the Friedmann equations \[6\]:

\[
\sqrt{H^2 + \frac{k}{a^2}} = \frac{1}{2r_0} \left[ \epsilon + \sqrt{\frac{4r_0^2}{3M_P^2} \rho + 1} \right] \]  

(2.7)

and

\[
\dot{\rho} + 3(\rho + p)H = 0 ,
\]  

(2.8)

where we have used the usual Hubble parameter \( H = \frac{\dot{a}}{a} \) and we have defined a crossover scale

\[
r_0 = \frac{M_P^2}{2M_P^3} .
\]  

(2.9)

This scale characterizes that distance over which metric fluctuations propagating on the brane dissipate into the bulk \[1\].

### B. The Coordinate Transformation

Starting with Eqs. (2.4–2.5) when \( k = 0 \), Deruelle and Dolezel \[16\] obtained an explicit change of coordinate \( Y^A = Y^A(X^A) \) to go to the canonical five-dimensional Minkowskian metric

\[
ds^2 = -(dY^0)^2 + (dY^1)^2 + (dY^2)^2 + (dY^3)^2 + (dY^5)^2 .
\]  

(2.10)

See also Ref. \[3\]. The coordinate transformations is

\[
Y^0 = A(y, t) \left( \frac{r^2}{4} + 1 - \frac{1}{4\dot{a}^2} \right) - \frac{1}{2} \int dt \frac{a^2}{\dot{a}^3} \frac{d}{dt} \left( \frac{\dot{a}}{a} \right) ,
\]

\[
Y^5 = A(y, t) \left( \frac{r^2}{4} - 1 - \frac{1}{4\dot{a}^2} \right) - \frac{1}{2} \int dt \frac{a^2}{\dot{a}^3} \frac{d}{dt} \left( \frac{\dot{a}}{a} \right) ,
\]

(2.11)

\[
Y^i = A(y, t) x^i ,
\]

where \( r^2 = x^i x^j \eta_{ij} \), and \( \eta_{ij} \) is here a flat Euclidean three-dimensional metric. The transformation Eqs. (2.11) can be generalized to nonzero spatial curvatures. We describe that generalization in Sec. \[VII\].
III. THE BRANE WORLDSHEET IN A MINKOWSKI BULK

We wish to focus on the early universe of cosmologies of DGP braneworlds, and in particular on the four-dimensional big bang evolution at early times. For clarity, we restrict ourselves to spatially flat braneworlds \((k = 0)\) and radiation domination (i.e., \(p = \frac{1}{3} \rho\)) such that, using Eqs. (2.7, 2.8), \(a(t) = t^{1/2}\) when \(H \gg r_0^{-1}\) using appropriately normalized time units. A more general equation of state does not alter the qualitative picture; changes resulting from altering the spatial curvature are discussed in Sec. VI. Equation (2.7) shows that the early cosmological evolution on the brane is independent of the choice of the \(\epsilon\)–parameter. However, we see that late time evolution depends quite sensitively to the choice of \(\epsilon\). When \(\epsilon = -1\), as \(H(t)\) approaches the value \(r_0^{-1}\) at late times, the evolution of the scale factor transitions between four-dimensional Friedmann–Lemaitre–Robertson–Walker (FLRW) behavior to five-dimensional FLRW behavior. We refer to this phase as the FLRW phase. When \(\epsilon = 1\), as \(H(t)\) approaches the value \(r_0^{-1}\) at late times, the asymptotic state is an empty universe undergoing deSitter expansion with \(H = r_0^{-1}\). We refer to this phase as the self-accelerating phase. See Ref. [6] for details.

The global configuration of the brane worldsheet is determined by setting \(y = 0\) in the coordinate transformation Eq. (2.11). We get

\[
\begin{align*}
Y^0 & = t^{1/2} \left( \frac{r^2}{4} + 1 - t \right) - \frac{4}{3} t^{3/2} \\
Y^5 & = t^{1/2} \left( \frac{r^2}{4} - 1 - t \right) - \frac{4}{3} t^{3/2} \\
Y^i & = t^{1/2} x^i . 
\end{align*}
\]

(3.1)

The locus of points defined by these equations, for all \((t, x^i)\), satisfies the relationship

\[
Y_+ = \frac{1}{4Y_-} \sum_{i=1}^{3} (Y^i)^2 + \frac{1}{3} Y_-^3 , \tag{3.2}
\]

where we have defined \(Y_+ = \frac{1}{2}(Y^0 \pm Y^5)\). Note that if one keeps only the first term, the surface defined by Eq. (3.2) would simply be the light cone emerging from the origin at \(Y^A = 0\). However, the second term ensures that the brane worldsheet is timelike except along the \(Y_+\)–axis. Moreover, from Eqs. (3.1), we see that

\[
Y_- = t^{1/2} , \tag{3.3}
\]

implying that \(Y_-\) acts as an effective cosmological time coordinate on the brane. The \(Y_+\)–axis is a singular locus corresponding to \(t = 0\), or the big bang. \(\footnote{The big bang singularity when \(r < \infty\) is just the origin \(Y_- = Y_+ = Y^i = 0\) and is strictly pointlike. The rest of the big bang singularity (i.e., when \(Y_+ > 0\)) corresponds to the pathological case when \(r = \infty\).} \)
FIG. 1. A schematic representation of the brane worldsheet from an inertial bulk reference frame. The axes $Y_{\pm}$ are defined such that $Y_{\pm} = \frac{1}{2}(Y^0 \pm Y^5)$. The bulk time coordinate, $Y^0$, is the vertical direction, while the axis perpendicular to the $(Y_+, Y_-)$–plane represents all $Y^i$. The big bang is located along the $Y_+$–axis, while the dotted surface is the future lightcone of the event located at $Y^A = 0$. The curves on the brane worldsheet are examples of equal cosmological time, $t$, curves and each is in a plane of constant $Y_-$.

This picture is summarized in Fig. 1. Taking $Y^0$ as its time coordinate, a bulk observer perceives the braneworld as a compact, hyperspherical surface expanding relativistically from an initial big bang singularity ($Y^A = 0$, for all $A$). A single point on that hyperspherical surface moves strictly at the speed of light and maintains an infinite energy density. The worldline of this remnant of the big bang follows the $Y_+$–axis. Note that a bulk observer views the braneworld as spatially compact, even while a cosmological brane observer does not. Simultaneously, a bulk observer sees a spatially varying energy density on the brane, whereas a brane observer sees each time slice as spatially homogeneous.

The results of this section are analogous to those found for the global and causal structure of Randall–Sundrum II (RS2) braneworld cosmologies [17,18]. However, in the RS2 case, one only treats that part of the bulk interior to the brane worldsheet, and the bulk is anti-deSitter. The bulk is strictly Minkowski for DGP braneworlds, with the associated intuitive benefits. In particular, the five-dimensional lightcone is respected, as well as the causal structure of the asymptotic infinities of the spacetime. Inertial bulk observers always have trivial worldlines.

Though the brane cosmological evolution between the FLRW phase and the self-accelerating phase is indistinguishable at early times, the bulk metric Eqs. (2.4–2.5) for each phase is quite distinct. That distinction has a clear geometric interpretation: The FLRW phase ($\epsilon = -1$) corresponds to that part of the bulk interior to the brane worldsheet, whereas the self-accelerating phase ($\epsilon = 1$) corresponds to bulk exterior to the brane worldsheet. The full bulk space is two copies of either the interior to the brane worldsheet (the first phase) or the exterior (the latter phase), as imposed by $Z_2$–symmetry. Those two copies are then spliced across the brane. We examine each case separately.
FIG. 2. A schematic representation of the FLRW brane worldsheet from an inertial bulk reference frame with all $Y^i$–coordinates suppressed. Equal–$|y|$ curves (thin solid lines) are roughly parallel to the brane worldsheet (thick solid line). The dotted lines represent curves of equal cosmological time (equal–$t$). Note that equal–$t$ curves intersect the big bang singularity along the $Y_+–$axis when $|y| = 2t$, and thus extend no further in principle. I.e., the bulk is limited to $|y| \leq 2t$, implying an observer off the braneworld sees a finite-volume bulk.

IV. GLOBAL STRUCTURE OF THE FLRW PHASE

A. Foliation of Equal Cosmological Time Surfaces

If one takes $\epsilon = -1$ in Eq. (2.5), the scale factor in a radiation dominated universe ($p = \frac{1}{3} \rho$) evolves like (using appropriately normalized time units)

\begin{align*}
a(t) &= t^{1/2} \quad \text{when} \quad H \gg r_0^{-1} \quad (4.1) \\
a(t) &= (r_0 t)^{1/4} \quad \text{when} \quad H \ll r_0^{-1}. \quad (4.2)
\end{align*}

By taking $\epsilon = -1$, Eqs. (2.11) implies that one is effectively choosing the interior region of Fig. II as the bulk. Again, an observer goes from one copy of the interior to another as one crosses the brane. The bulk coordinates of an event may be read from Eqs. (2.11). With some algebra, we arrive at

\begin{align*}
Y_- &= t^{1/2} \left( 1 - \frac{|y|}{2t} \right) \\
Y_+ &= t^{1/2} \left[ \left( 1 - \frac{|y|}{2t} \right) \left( \frac{r^2}{4} - t \right) - \frac{4}{3} t \right] \\
Y^i &= t^{1/2} x^i \left( 1 - \frac{|y|}{2t} \right). \quad (4.3)
\end{align*}
Figure 2 depicts equal–$|y|$ curves and equal cosmological time (equal–$t$) curves from the point of view of an inertial bulk observer. One can imagine this image as a slice of Fig. 1 down the middle along the plane determined by $Y_+$ and $Y_-$. The bulk region that corresponds to this phase is the region interior to the brane worldsheet in Fig. 1. We see then that the volume of any spacelike surface in the bulk is finite. Each equal–$t$ curve only extends to a value of $|y| = 2t$. There it intersects the big bang singularity located along the $Y_+$–axis.

B. Apparent Brane Causality Violation

Consider an event, $O$, located on the brane worldsheet. We wish to examine its past lightcone to see all events which are in causal contact with $O$. Take the cosmological coordinates of this event to be $t = t_0, y = 0, x^i = 0$. If one is restricted to motion on the brane, then geodesics are determined by the induced metric Eqs. (2.4–2.5) with $|y| = 0$. The locus of past directed brane null rays are defined by the coordinates $x^i(t)$ such that

$$r(t) = 2(t_0^{1/2} - t^{1/2})$$

for a given cosmological time $t < t_0$. As expected, even as $t \to 0$, the past directed brane lightcone only encompasses a finite region in $r$. However, imagine that certain signals (e.g., graviton degrees of freedom) may travel through the bulk. Since the bulk lightcone is trivial in the coordinates $\{Y^A\}$, one can use Eqs. (3.1) to identify what events on the brane worldsheet intersect the past bulk lightcone of $O$. That locus of brane coordinates is given by the coordinates $x^i(t)$ such that

$$r(t) = \sqrt{\frac{4}{3} \left( t_0^{1/2} - t^{1/2} \right) \left( t_0^{3/2} - t^{3/2} \right) t_0^{1/2} t^{1/2}}$$

where again $t$ is a cosmological time such that $t < t_0$. Note that as $t \to 0$, the past bulk light cone encompasses an arbitrarily large region in comoving coordinate, $r$. Because of the convexity of the brane worldsheet, one can see that only the FLRW phase allows propagation of light signals through the bulk between events on the brane. If signals may travel unimpeded through the bulk, such signals would appear to travel faster than the speed of light on the brane, thus providing an effective brane causality violation. One can see that this conclusion is natural by examining Fig. 1. The past lightcone of any event $O$ on the brane worldsheet clearly encompasses the entire big bang for $r < \infty$, which is located at the origin, $Y^A = 0$. 

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V. GLOBAL STRUCTURE OF THE SELF-ACCELERATING PHASE

A. Foliation of Equal Cosmological Time Surfaces

The self-accelerating phase corresponds to taking $\epsilon = 1$ in Eq. (2.5). With this choice of $\epsilon$, Eq. (2.5) implies the scale factor in a radiation dominated universe evolves as (using appropriately normalized time units)

$$a(t) = t^{1/2} \quad \text{when} \quad H \gg r_0^{-1} \quad (5.1)$$

$$a(t) \sim e^{t/r_0} \quad \text{when} \quad H \ll r_0^{-1} \quad (5.2)$$

The bulk region is the region exterior to the brane worldsheet in Fig. 1. The coordinates of an event in the bulk may again be read from Eqs. (2.11):

$$Y^- = t^{1/2} \left(1 + \frac{|y|}{2t}\right)$$

$$Y^+ = t^{1/2} \left[\left(1 + \frac{|y|}{2t}\right) \left(r^2 + \frac{t}{4} - 4\frac{3}{4}t\right) - \frac{4}{3}t\right]$$

$$Y^i = t^{1/2} x^i \left(1 + \frac{|y|}{2t}\right). \quad (5.3)$$

Unlike the FLRW phase, spatial slices of the Minkowski bulk are infinite in volume. However, there exists a subtlety with equal-time foliation of the big bang for the self-accelerating phase. The cosmological coordinate system Eqs. (2.4) develops singularities when $|y| = 2t$. This condition in the FLRW phase defines a set of points lying on the brane itself and corresponded to the big bang singularity. In the self-accelerating phase, the locus $|y| = 2t$ exists in the Minkowski bulk. Here, the surface $|y| = 2t$ follows the relationship

$$Y^+ = \frac{1}{4Y^-} \sum_{i=1}^{3} (Y^i)^2 - \frac{1}{12}Y_-^3. \quad (5.4)$$

When this relationship is satisfied, $N = 0$. Moreover, one can show that no value of $(y, t)$ covers a region where $Y^+$ is smaller than the value defined by Eq. (5.4).

B. Reversal of Cosmological Time and $N \leq 0$

Indeed, one can show that this coordinate singularity is associated quite generally with the existence of a decelerating scale factor and one may derive intuition based on phenomenologically interesting cosmic evolutions. Consider a general scale factor evolution $a(t)$. Then, one can ascertain the parametric evolution of equal-$|y|$ (and fixed $r^2$) curves with respect to cosmological time, $t$. Using the coordinate transformation Eqs. (2.11), one can track $\dot{Y}^+$ and $\dot{Y}^-$ for fixed $|y|$. We find that
FIG. 3. The evolution of the metric function $N = 1 + |y|\ddot{a}/\dot{a}$ for a given $|y|$. The evolution occurs in three different qualitative phases: (A) inflation, where the scale factor accelerates; (B) reheating, where the scale factor transitions from an accelerating phase and a decelerating phase and therefore $N$ vanishes for sufficiently large $|y|$; and (C) FLRW evolution, where the scale factor decelerates in its growth. In this last phase, $N$ vanishes when $t = \frac{1}{2}|y|$ in a radiation dominated universe.

Thus, $\dot{Y}_+ + \dot{Y}_-$ vanish simultaneously when $N(t, y) = 0$, implying cusps in the equal–$|y|$ curves in the $(Y^5, Y^0)$–plane.

Using a typical cosmological scenario, one can generate a qualitative picture of the evolution of the metric function $N(t, y)$. At early times, we assume an inflationary phase that is approximately deSitter with a Hubble scale, $H_{\text{infl}}$. In this case, the scale factor grows with positive acceleration. Eventually, the inflationary stage ends and reheating occurs. A hot FLRW-like, big bang universe is restored. Figure 3 shows how, for a sufficiently large $|y|$, the metric function $N(t, y)$ develops nodes. One node occurs during the reheating stage, the other during the FLRW stage. When the FLRW stage is radiation dominated, the second node occurs near $t = \frac{1}{2}|y|$. Thus, the minimum $|y|$ for which these nodes develop depends on the earliest time at which FLRW evolution is restored.

So when $N < 0$, both $Y_+$ and $Y_-$ decrease with increasing cosmological time, $t$. This implies a reversal of the direction of future-versus-past for the bulk observer relative to a brane observer. This is not surprising since one can think of $N$ acting as a sort of speed of light. When that value is negative, one is effectively reversing the sign of time.

In order to provide a more intuitive description of the early universe, let us imagine that the big bang is smoothed out by a stage of early inflation that extends arbitrarily far into
FIG. 4. A schematic representation of the self-accelerating brane worldsheet from an inertial bulk reference frame with all $Y^i$-coordinates suppressed. (a) Equal-|$y$| curves passing through the wedge defined by the dotted curve have two cusps. For the part of the curve between the two cusps, $N < 0$, i.e. increasing $t$ implies evolution that is past directed for a bulk inertial observer. (b) Equal cosmological time curves (dotted curves) in the same universe. Each spacetime point within the wedge (thin solid curve) defined by $N = 0$ has three equal time curves passing through it. Outside the wedge, one can assign a unique cosmological time to each spacetime point.

the past. Figure 4 provides a visual description of equal-|$y$| and equal cosmological time curves from the point of view of an inertial bulk observer. Again, imagine taking a slice of Fig. 4 down the middle, but where the brane worldsheet is smoothed out at the origin to include an early inflating stage. Indeed, in order to fill out the region of bulk spacetime not covered by values of $Y_+$ less than that determined by Eq. (5.4), one needs to includes something like this early inflationary stage.

C. Leakage and Depletion of Anisotropic Power

In Sec. VI we saw how null worldlines through the bulk in the FLRW phase could connect different events on the brane. This observation was a consequence of the convexity of the brane worldsheet and the choice of bulk. Conversely, if one chooses the bulk corresponding to the self-accelerating phase, one may conclude that no null lightray through the bulk connects two different events on the brane. Let us elaborate on the consequences of such an observation.

Consider again Fig. 4. Because the brane worldsurface must be timelike and spatially compact, each external past-directed and future-directed lightrays can only intersect the
brane worldsheet once. But perturbations through the bulk must follow these lightrays, since the action Eq. (2.1) dictates that the empty bulk is pure Einstein gravity. From the propagator [1,2,3,19], one can see that gravitational modes on the brane are influenced only by bulk modes at length scales of order $\mathcal{O}(r_0)$ or larger. If one imagines that the initial state of perturbations is localized around the brane, then no bulk perturbations from the brane in the past can reintersect the brane in the sufficiently late future, and that one expects only a depletion of amplitude from the power spectrum of perturbations at large scales.

This argument is subject to the validity of using linearized perturbations on cosmological backgrounds. But recall at the times of interest (when $H \sim r_0^{-1}$), perturbations have condensed into compact objects like galaxies, clusters, etc. Study of compact objects in the DGP braneworld theory [4,5,20] suggest that nonlinear effects may not necessarily decouple, even at large scales. A more detailed study of how perturbation on differing scales interact as well as the importance of nonlinearities in the DGP model is necessary to draw a definitive conclusion on the matter.

VI. NONFLAT SPATIAL GEOMETRIES

For the metric (2.5) with $k \neq 0$, one can find as a change of coordinate $Y^A = Y^A(X^A)$ leading to the canonical Minkowski metric (2.10). It can be defined for $k = 1$ by [8]

$$Y^0 = A(y, t) \tilde{y} + \tilde{z},$$

$$Y^5 = A(y, t) \tilde{y},$$

$$Y^i = A(y, t) \tilde{Y}^i,$$

(6.1)

where $\tilde{Y}^i$ and $\tilde{Y}^5$ are functions of the $x^i$ only and verify

$$(\tilde{Y}^5)^2 + 3 \sum_{i=1}^{3} (\tilde{Y}^i)^2 = 1,$$

(6.2)

which defines a three-dimensional $k = 1$ maximally symmetric space. For $k = -1$, we find [8]

$$Y^0 = A(y, t) \tilde{Y}^0,$$

$$Y^5 = A(y, t) \tilde{y} + \tilde{z},$$

$$Y^i = A(y, t) \tilde{Y}^i,$$

(6.3)

where $\tilde{Y}^i$ and $\tilde{Y}^0$ are function of the $x^i$ only and verify

$$(\tilde{Y}^0)^2 - \sum_{i=1}^{3} (\tilde{Y}^i)^2 = 1,$$

(6.4)
which defines a three-dimensional $k = -1$ maximally symmetric space. The quantities $\tilde{y}$ and $\tilde{z}$ are given by

$$\tilde{y} = \frac{\dot{a}}{\sqrt{\dot{a}^2 + k}}$$  \hspace{1cm} (6.5) $$

$$\tilde{z} = k \int dt \frac{1}{a} \frac{d}{dt} \left( \frac{a}{\sqrt{\dot{a}^2 + k}} \right).$$  \hspace{1cm} (6.6) $$

So, we see that on the brane the coordinate $Y^0$ acts as the cosmological time coordinate when $k = 1$, whereas the coordinate $Y^5$ acts as the cosmological time coordinate when $k = -1$. Compare this to the case when $k = 0$ and the cosmological time is characterized by $Y_\infty = \frac{1}{2}(Y^0 - Y^5)$.

The qualitative features represented in Fig. 1 are the same except for the following. In the $k = 1$ scenario, the big bang is a pointlike singularity at the origin, and the entire braneworld is a hypersphere whose radius is determined by the induced scale factor $a(t)$. The brane worldsheet lies entirely within the light cone of the origin ($Y^A = 0$) such that the surface maintains a spherical symmetry around the $Y^0$–axis. Again, equal cosmological time surfaces are loci of equal $Y^0$ on this brane worldsheet. When $k = -1$, the big bang is composed of two light rays located along the light cone defined by $Y^5 = 0$. Once again, the entire brane worldsheet is located within the light cone of the origin. Equal-cosmological-time surfaces are loci of equal $Y^5$ on the brane worldsheet. These pictures are directly analogous to those found in [18], but where again the bulk internal and external to the brane worldsheet refer to different phases of the theory, and each bulk is strictly Minkowski. Moreover, the types of subtleties shown to exist in the coordinate transformation for the self-accelerating phase discussed in Sec. V carry over. In particular, the mapping of the cosmological coordinate system to the bulk coordinate system is not one-to-one when there exist values of $|y|$ where $N \leq 0$.

\section{VII. CONCLUDING REMARKS}

We examined the global structure of early universe cosmologies for Dvali–Gabadadze–Porrati (DGP) braneworlds. Two distinct phases exist: the Friedmann–Lemaître–Robertson–Walker (FLRW) phase, where late time evolution follows five-dimensional FLRW behavior, and the self-accelerating phase, where the brane asymptotes to late time deSitter expansion with an empty brane. A bulk observer sees the brane as a relativistically expanding, roughly hyperspherical bubble emerging from a pointlike big bang. Depending on the spatial curvature of the \textit{internal} brane cosmology, the brane is strictly hyperspherical (positive spatial curvature, $k = 1$), has one residual big bang point on the hypersphere moving at exactly the speed of light (flat spatial geometry, $k = 0$), or two diametrically
opposed residual big bang points on the hypersphere, also moving at exactly the speed of light (negative spatial curvature, \( k = -1 \)). Note that a bulk observer perceives the brane as compact, even when a cosmological brane observer would not (i.e., the \( k = 0, -1 \) cases). The bulk is two identical copies of the space interior to this compact brane, glued across a \( Z_2 \)-symmetric brane when in the FLRW phase. Correspondingly, the bulk space is two copies of the space exterior to the brane when in the self-accelerating phase.

The FLRW phase exhibits lightlike shortcuts through the bulk that connect different events on the brane. This phenomena can lead to apparent brane causality violation and provides an opportunity for the evasion of the horizon problem. Unlike the big bang of conventional four-dimensional FLRW cosmology, the past (bulk) lightcone of every spacetime event on the brane contains the entire big bang for all comoving coordinate radii \( r < \infty \). Phrasing this statement another way is that the entire brane worldsheet is in the future lightcone of the big bang for \( r < \infty \), where the locus of spacetime events of the big bang with \( r < \infty \) is strictly pointlike. Thus, gravitons emitted from a time arbitrarily close to the big bang may travel through the bulk and may transmit thermal information to all parts of the brane during the evolution of the early universe. The difficulty in this mechanism for evading the horizon problem is that bulk propagation of gravitons is at its least significance precisely at those times (i.e., the early big bang) when we wish to transmit them through the bulk.

The self-accelerating phase does not possess lightlike worldline that connect different events on the brane. This observation implies that density perturbations leaving the brane via the bulk cannot reenter the brane at some later time. At sufficiently large scales, where brane fluctuations are strongly coupled to bulk modes, one anticipates that only depletion of power can occur for metric fluctuations, since there is no mechanism for replenishment of power at these scales. This argument implies a deficit in the power spectrum of linear density perturbations at large scales. The magnitude of this deficit, as well as the validity of this analysis motivated by linearized perturbations, are subject to more quantitative inquiry. Nevertheless, one can see the advantage of developing an understanding of the global structure of DGP braneworlds for gaining insight into their phenomenological consequences.

**ACKNOWLEDGMENTS**

The author would like to thank C. Deffayet, G. Dvali, G. Gabadadze and G. Starkman for helpful discussions. This work is sponsored in part by NSF Award PHY-9996137 and the David and Lucille Packard Foundation Fellowship 99-1462.
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