Modeling the image obtained by a system of vortex harmonic lenses

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Abstract. The possibility of obtaining images using a system based on vortex diffraction lenses is considered. A system of two vortex harmonic lenses was simulated. In the system the total intensity distribution was calculated using the Fresnel transform. The width of the point spread function, which is 66 μm, is obtained. The influence of spatial filters on the result of this system is considered.

1. Introduction

Vortex lenses are optical devices that generate vortex light beams. Such lenses transform a plane wave front into a helical continuous wave front. The distance between two adjacent turns of such a front will be equal to the wavelength, and the lens itself is a spiral structure with a decreasing period, which in the focal plane at incident plane wave forms an intensity distribution in the form of a ring. In [1], we consider vortex diffraction lenses that were used to form vortex beams in the focal plane. Also, as described in [2] and [3], in which the formation of Bessel beams by vortex axicons was considered. In addition, the formation and application of vortex beams is discussed in articles [4-11]. In this paper, we will consider a system of two such optical elements with the same size, but different sign of topological charges located at a double focal distance from each other. In fact, it is similar to the classical afocal system with the replacement of conventional lenses with vortex ones. The output of such a system should produce a fairly high-quality image, which is verified in our study.

2. Modeling of vortex lenses

The phase of a vortex harmonic lens corresponds to the formula:

\[ t(r, \lambda_0, \phi) = \frac{1}{\Delta N} MOD_m \lambda_0 \left( \frac{r^2}{2f} - n\phi \right), \quad m = 1,2,3, \ldots \]

where \( n \) is the topological charge, and \( r, \phi \) are the polar coordinates, and \( f \) is the focal length of the lens. The obtained phase functions of such vortex lenses are shown in figure 1. Figure 1 a show the phase function of a conventional diffraction lens for comparison.

The lenses in Fig.1 were calculated for a wavelength of 488 nm. Fig. 1bcd shows the phase functions of lenses with topological charges 1,2,3, respectively. When using one such lens, the image generated will inevitably be worse than the image formed by the lens with the phase function shown in
Fig. 1a. But if you combine two such lenses in the system, the parameters of which were specified above, it is possible to obtain a high-quality image.

![Fig. 1a: Vortex lenses](image)

Fig. 1. The phase plane of vortex lenses with parameters $\lambda_0 = 488$ nm, $\Delta N = 11$ microns and $n = 0$ (a), $n = 1$ (b), $n = 2$ (c), $n = 4$ (d).

3. Formatting the text

The system was modeled using a chain of Fresnel transformations (Fig. 2). After each Fresnel transformation, the resulting distribution was multiplied by the phase function of the corresponding element.

![Fig. 2: Modeling diagram](image)

As the source image, sections of the standard lighting test pattern were used to study the lenses. The results of modeling the formation of such sections are shown in figure 3.

Based on the brightness of adjacent lines in the resulting images, you can determine the dependence of the frequency-contrast characteristic of the lens on the frequency of lines using the formula:

$$T = \frac{(E_{max} - E_{min})/(E_{max} + E_{min})}{(L_{max} - L_{min})/(L_{max} + L_{min})}$$
where $E_{\text{max}}$, $E_{\text{min}}$ is the maximum and minimum illumination of the resulting image, and $L_{\text{max}}$, $L_{\text{min}}$ the maximum and minimum brightness of the original. Based on this expression, a graph of the modulation transfer function was obtained, which is shown in figure 4.

![Figure 3](image3.png)

**Figure 3.** Image modeling of a striped image to determine the MTF with a dimension of 9 microns per pixel and $n = 1$.

![Figure 4](image4.png)

**Figure 4.** Modulation transfer function.

In addition, a direct simulation of the image formation of a point source located in the front focal plane of the first element was performed. Figures 5-6 show the results of numerical simulation of an 11-micrometer source located on the optical axis (Fig. 5) and on the edge of the field of view (Fig. 6) for systems containing lenses with different topological charges.

![Figure 5](image5.png)

**Figure 5.** PSF on the optical axis of the system for $n = 0$ (a), $n = 1$ (b), $n = 4$ (c).

![Figure 6](image6.png)

**Figure 6.** PSF at a distance equal to 1 mm from the optical axis of the system for $n = 0$ (a), $n = 1$ (b), $n = 4$ (c).

As can be seen from the results obtained, on the optical axis of the system, the point scattering function of the vortex lens is visually similar to the point scattering function of a normal lens. However, aberrations similar to the classic coma-type aberration are observed outside the optical axis.

### 4. Filtration

Optical image processing, as opposed to image construction, involves interfering with the process. This intervention can be performed in different ways. Its practical application is based on the ability of
optical systems to perform General linear transformations of incoming data. In the case of the system in figure 2, by placing a filter in the focal area of the system's lenses, we can apply it to the Fourier image of the picture. When the amplitude is zero in the Central Fourier region of the image \((r < 0.1 \text{ nm})\), a low-pass filter can be obtained (Fig. 7), and when zeroing in the periphery \((r > 0.1 \text{ nm})\), we get a high-frequency filter (Fig. 8). The Fourier phase of the image can be influenced using a phase bandpass filter (Fig. 9).

![Figure 7](image1.png)  
**Figure 7.** The result of a low-pass filter when using lenses with topological charges for \(n= 0 \) (a), \(n=1 \) (b), \(n= 2 \) (c), \(n= 4 \) (d).

![Figure 8](image2.png)  
**Figure 8.** The result of a high-pass filter when using lenses with topological charges for \(n= 0 \) (a), \(n=1 \) (b), \(n= 2 \) (c), \(n= 4 \) (d).

![Figure 9](image3.png)  
**Figure 9.** The effect of a band-pass phase filter when using lenses with topological charges \(n= 0 \) (a), \(n=1 \) (b), \(n= 2 \) (c), \(n= 4 \) (d).

5. **Conclusion**  
The paper considers the possibility of obtaining images using a system of vortex diffraction lenses. It is shown that the image quality obtained in such a system is comparable to the simplest refractive lenses. The results of modeling a system consisted on two vortex harmonic lenses, in which the total intensity distribution was calculated using the Fresnel transform, are obtained. The modulation transfer function and the point scattering function of such a system are obtained. Shown are differences between point scattering functions using lenses with different numbers of vortices and for different distances from the point to the optical axis. The influence of various spatial filters on the system under consideration is considered.

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