Reissner-Nordstrom Black Hole in Gravity Localized Models

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Abstract

We investigate the possibility of having the electrically charged Reissner-Nordstrom black hole in the gravity localized models in a brane world. It is shown that the Reissner-Nordstrom black hole exists as a solution in the 5D Randall-Sundrum domain wall model if there is the $U(1)$ bulk gauge field. We find that the charged black hole is localized on a 3-brane even if zero mode of the bulk gauge field is not in general localized on the brane in the domain wall model. Moreover, we extend these observations to the higher-dimensional topological defect models with codimension more than one in a general space-time dimension such as the string-like defect model with codimension 2 in six dimensions and the monopole-like defect model with codimension 3 in seven dimensions e.t.c.

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1 Introduction

Black holes have so far played a crucial role in the development of quantum gravity and superstring theory. For instance, the discovery of the Hawking radiation \[1\] that black holes are not, after all, completely black but emit thermal radiation with a definite temperature due to quantum effects has triggered the widespread interest in quantum field theory in curved space-time. And the recent progress of a precise calculation of the black hole entropy and string dualities in superstring theory suggests that black holes do obey the ordinary rules of quantum mechanics so the laws of quantum mechanics do not need to be modified radically down to the Planck scale \[2\]. Thus the study of black holes would give us the most valuable clues of constructing a quantum theory of gravity and a unified theory of fundamental interactions in future.

In recent years, we have witnessed a vast interest in the idea of a brane world where our world is considered to be a 3-brane embedded in a higher dimensional space-time \[3, 4, 5, 6, 7, 8, 9\]. The key point of this scenario is localization of gauge and matter fields on the world-volume of the 3-brane, for which D-branes \[10\] would provide a natural framework. Another key point is that even gravity, which watches the whole space-time structure so lives in the whole space-time, can be also localized on the 3-brane in the sense that a normalizable graviton zero mode trapped by the brane reproduces four dimensional Newton’s law with only a small correction \[7\].

It is then natural to ask whether or not there are black hole solutions on a 3-brane in a well-known gravity localized model in \(AdS_5\), the Randall-Sundrum model \[7\] in a brane world. Actually, in the model, since it turns out that the geometry on the brane is Ricci-flat, one can find the Schwarzschild geometry as a solution to Einstein’s equations \[11\]. On the other hand, the Reissner-Nordstrom black hole satisfies Einstein’s equations with the stress-energy of the electric field as a source, so it appears to be difficult to have the charged black hole in the Randall-Sundrum model. Related to this point, it is recently shown that it is impossible to embed the Reissner-Nordstrom black hole in the gravity localized model via the conventional Kaluza-Klein reduction of the bulk metric \[12\]. This situation is quite unsatisfactory since the charged Reissner-Nordstrom black hole makes its appearance in various modern theories such as supergravity and superstring theories. The aim of this paper is to show that it is indeed possible to have the Reissner-Nordstrom black hole as a solution in gravity localized models, which include the Randall-Sundrum model in case of \(AdS_5\). To do that, we shall introduce the bulk \(U(1)\) gauge field in the models, which is needed to obtain Einstein’s equations with energy-momentum tensor associated with the electric field.

At this stage an interesting question arises with respect to the localization of the bulk gauge field on a 3-brane in the Randall-Sundrum model \[7\]. In the Randall-Sundrum model, we are now familiar with the following fact about the localization of bulk gauge field on the 3-brane by means of a gravitational interaction: zero mode of spin 1 bulk vector field is not localized neither on a brane with positive tension nor on a brane with negative tension \[13, 14\]. Recently, this localization mechanism has been also investigated in higher dimensional topological defect models \[13, 16, 17, 18, 19\] by the present author \[18, 20\]. In the analysis
we have clarified that zero mode of spin 1 vector field is localized on a brane with positive tension only in a string-like defect model with codimension 2, but the other topological defects cannot support zero mode of vector field on a 3-brane like the domain wall in the Randall-Sundrum model.

Accordingly, at first sight we might conclude that although we could succeed in constructing the charged Reissner-Nordstrom black hole on a 3-brane by including the bulk gauge field in the gravity localized models, the black hole, or more precisely, the charge carried by the black hole, cannot stay in our universe and enters in the higher dimensional space-time except the string-like defect model. Surprisingly enough, however, the charged Reissner-Nordstrom black hole can reside in our universe. Why is the charged black hole bound on 3-brane by circumventing the aforementioned no-go theorem? A key word here is 'zero mode'. Although it is certainly true that the constant zero mode of the bulk gauge field is not trapped on a 3-brane, the non-constant zero mode has a possibility of being trapped on the 3-brane. Indeed, this phenomenon precisely occurs in the present situation as will seen later.

This paper is organized as follows. In the next section, we present our setup and derive the equations of motion from an action in a general space-time dimension. In Section 3, we show that there are the Reissner-Nordstrom black holes as solutions to the equations of motion obtained in Section 2. Then, in Section 4, we explain why such charged black holes are localized on a 3-brane. The final section is devoted to discussions.

2 Setup

We would like to consider global topological defects in the extra internal dimensions, which are described by a scalar field with an orthogonal symmetry with the Higgs potential. In the defects, the scalar field takes the vanishing configuration at the core, whereas it has the 'hedgehog' configuration outside the core. In this article, we pay our attention to only the exterior geometry outside the core of the defects.

The action with which we start is that of gravity in general $D$ dimensions, with the conventional Einstein-Hilbert action, a cosmological constant $\Lambda$ and some matter action $S_m$:

$$S = \frac{1}{4\kappa_D^2} \int d^Dx \sqrt{-g} (R - 2\Lambda) + S_m,$$

where $\kappa_D$ denotes the $D$-dimensional gravitational constant with a relation $\kappa_D^2 = 8\pi G_N = \frac{8\pi}{M^2}$ with $G_N$ and $M$ being the $D$-dimensional Newton constant and the $D$-dimensional Planck mass scale, respectively. For simplicity, we set $\kappa_D$ to 1 from now on. In this article, as $S_m$ we shall take the $U(1)$ Maxwell’s action $S_A$ plus the $O(n)$ globally symmetric scalar action $S_\Phi$ with the Higgs potential ($S_m = S_A + S_\Phi$), which are defined as

$$S_A = -\frac{1}{4} \int d^Dx \sqrt{-g} g^{MN} g^{RS} F_{MR} F_{NS},$$

$$S_\Phi = \int d^Dx \sqrt{-g} \left\{-\frac{1}{2} g^{MN} \partial_M \Phi^a \partial_N \Phi^a + \frac{\lambda}{4} (\Phi^a \Phi^a - \eta^2)^2 \right\},$$

(2)
where \( F_{MN} = \partial_M A_N - \partial_N A_M \). And we use the index convention such that \( M, N, ... \) denote \( D \)-dimensional space-time indices, \( \mu, \nu, ... \) \( p \)-dimensional brane ones, and \( a, b, ... \) \( n \)-dimensional extra spatial ones, so the equality \( D = p + n \) holds. Throughout this article we follow the standard conventions and notations of the textbook of Misner, Thorne and Wheeler [21].

Taking a variation of the action (1) with respect to the \( D \)-dimensional metric tensor \( g_{MN} \) leads to Einstein’s equations

\[
R_{MN} - \frac{1}{2} g_{MN} R = -\Lambda g_{MN} + 2T_{MN},
\]

where the energy-momentum tensor associated with Eq. (2) is given by

\[
T_{MN} \equiv T_{MN}(A) + T_{MN}(\Phi) = -\frac{2}{\sqrt{-g}} \delta g_{MN} (S_A + S_\Phi),
\]

\[
T_{MN}(A) = F_{MP} F_{N}^P - \frac{1}{4} g_{MN} F^2,
\]

\[
T_{MN}(\Phi) = \partial_M \Phi^a \partial_N \Phi^a - \frac{1}{2} g_{MN} \partial_P \Phi^a \partial^P \Phi^a + g_{MN} \lambda \left( \Phi^a \Phi^a - \eta^2 \right)^2,
\]

with the contraction over indices being achieved in terms of the \( D \)-dimensional metric tensor \( g^{MN} \). The Bianchi identity gives rise to the conservation law for the energy-momentum tensor

\[
\nabla^M T_{MN} = 0.
\]

Finally, the equations of motion to vector fields \( A_M \) read

\[
\partial_M (\sqrt{-g} g^{MN} g^{RS} F_{NS}) = 0.
\]

Next we shall propose our setup and ansätze in order to solve the equations of motion derived thus far. First of all, let us adopt the following cylindrical metric ansatz:

\[
d s_D^2 = g_{MN} dx^M dx^N = g_{\mu\nu}(x^M) dx^\mu dx^\nu + \tilde{g}_{ab}(x^e) dx^a dx^b = e^{-A(r)} \hat{g}_{\mu\nu}(x^\lambda) dx^\mu dx^\nu + dr^2 + e^{-B(r)} d\Omega_{n-1}^2,
\]

where \( d\Omega_{n-1}^2 \) stands for the metric on a unit \( (n - 1) \)-sphere, which is concretely expressed in terms of the angular variables \( \theta_i \) as

\[
d \Omega_{n-1}^2 = d\theta_2^2 + \sin^2 \theta_2 d\theta_3^2 + \sin^2 \theta_2 \sin^2 \theta_3 d\theta_4^2 + \cdots + \prod_{i=2}^{n-1} \sin^2 \theta_i d\theta_n^2,
\]

where each variable takes the range \( 0 \leq r \leq \infty, 0 \leq \theta_2 \leq 2\pi \) and \( 0 \leq \theta_i (i = 3, 4, \cdots, n) \leq \pi \), and the volume element is given by \( \int d\Omega_{n-1} = \frac{2\pi^{\frac{n}{2}}}{\Gamma\left(\frac{n}{2}\right)} \).
As mentioned above, global defects are regarded as taking 'hedgehog' configuration outside the core in the extra dimensions \( [10] \)

\[
\Phi^a = \eta r^a, \quad (9)
\]

where \( r^a \) is defined as

\[
r^a = (r \cos \theta_2, r \sin \theta_2 \cos \theta_3, r \sin \theta_2 \sin \theta_3 \cos \theta_4, \ldots, r \prod_{i=2}^{n-1} \sin \theta_i).
\]

Then, the energy-momentum tensor \( T^M_N(\Phi) \) takes the form outside the defect core

\[
T^r_r(\Phi) = -\frac{1}{2}(n-1)\eta^2 e^{B(r)}, \\
T^{\theta_i}_{\theta_j}(\Phi) = -\frac{1}{2}(n-3)\eta^2 e^{B(r)} \delta_{ij}, \\
T^{\mu \nu}(\Phi) = -\frac{1}{2}(n-1)\eta^2 e^{B(r)} \delta^{\mu \nu}.
\]

For the field strength \( F_{MN} \) of the \( U(1) \) gauge field \( A_M \), we take an ansatz such that only the nonvanishing components are

\[
F_{t\rho} = -F_{\rho t} = e^{-\frac{A(r)}{2}} f_{t\rho} = -e^{-\frac{A(r)}{2}} f_{\rho t},
\]

where we have denoted the coordinates \( x^0 \) and \( x^1 \) on a \( (p-1) \)-brane as \( t \) and \( \rho \), respectively, and we assume that \( f_{t\rho} \) is a function depending on only the radial coordinate \( \rho \) on a brane. Here we have inserted the nontrivial \( r \)-dependent factor \( e^{-\frac{A(r)}{2}} \) in front of \( f_{t\rho} \) for the purpose of canceling out the \( r \)-dependent factor appearing in \( T^M_N(A) \) as will be seen shortly.

With these ansatzs, after a straightforward but a little tedious calculation, Einstein’s equations \( (3) \) reduce to

\[
e^A \hat{R} - \frac{p(n-1)}{2} A'B' - \frac{p(p-1)}{4} (A')^2 - \frac{(n-1)(n-2)}{4} (B')^2 \\
+ (n-1)(n-2 - 2\eta^2)e^B - 2\Lambda = 0,
\]

\[
e^A \hat{R} + (n-2) B'' - \frac{p(n-2)}{2} A'B' - \frac{(n-1)(n-2)}{4} (B')^2 \\
+ (n-3)(n-2 - 2\eta^2)e^B + pA'' - \frac{p(p+1)}{4} (A')^2 - 2\Lambda = 0,
\]

\[
\hat{R}_{\mu \nu} - \frac{1}{2} \hat{g}_{\mu \nu} \hat{R} = -\Lambda_{\rho} \hat{g}_{\mu \nu} + 2 \hat{t}_{\mu \nu}.
\]

In the above the prime denotes the differentiation with respect to \( r \), and then \( \hat{R}_{\mu \nu} \) and \( \hat{R} \) are respectively the Ricci tensor and the scalar curvature associated with the brane metric.
\( \hat{g}_{\mu\nu} \). Here we have defined the effective cosmological constant on the \((p-1)\)-brane, \( \Lambda_p \), by the equation
\[
- e^A \Lambda_p = \frac{p-1}{2} (A'' - \frac{n-1}{2} A'B') - \frac{p(p-1)}{8} (A')^2 + \frac{n-1}{2} \left\{ B'' - \frac{n}{4} (B')^2 + (n - 2 - 2\eta^2)e^B \right\} - \Lambda.
\]
(16)
In addition, the nonvanishing energy-momentum tensor \( T_{\mu\nu}(A) \) takes the form
\[
T_{\mu\nu}(A) = f_{\mu\lambda} f_{\nu}^{\lambda} - \frac{1}{4} \hat{g}_{\mu\nu} f^2 \equiv \hat{t}_{\mu\nu},
\]
(17)
where the indices are raised by means of \( \hat{g}_{\mu\nu} \). Note that as promised this energy-momentum tensor has no dependency on \( A(r) \) due to Eq. (12).

Furthermore, the conservation law for this energy-momentum tensor (5) reads
\[
\hat{\nabla}_\mu \hat{t}_{\mu\nu} = 0, \quad A' \hat{t}_{\mu\mu} - (n - 1)\eta^2 B'e^B = 0.
\]
(18)
Finally, Maxwell’s equations (6) become
\[
\partial_{\mu}(\sqrt{-\hat{g}} \hat{g}^{\mu\nu} \hat{g}^{\lambda\sigma} f_{\nu\sigma}) = 0.
\]
(19)
Note that the raising and the lowering of the indices are respectively carried out by the brane metric tensor \( \hat{g}_{\mu\nu} \) and \( \hat{g}_{\mu\nu} \) in all the reduced equations of motion, which is crucial for the existence of a black hole solution on a brane.

3 The Reissner-Nordstrom black hole solution

We are now in a position to find the Reissner-Nordstrom black hole on a \((p-1)\)-brane in a warped geometry as a solution to a set of equations obtained so far.

Henceforth, let us confine ourselves to \( p = 4 \), in other words, a 3-brane, from a physical interest. (The generalization to an arbitrary \( p \) is straightforward.) In addition to it, we require an ansatz \( \hat{R} = 0 \). The physical plausibility of this ansatz arises from the reasoning that the scalar curvature of the Reissner-Nordstrom black hole is vanishing although the Ricci tensor is not zero but proportional to stress-energy tensor of the electric field. In what follows, we shall solve a set of equations for the domain wall \( n = 1 \) first, and then for the higher dimensional defects \( n \geq 2 \).

3.1 Domain wall (n=1) in five dimensions (D=5)

In this subsection, we solve a set of equations in the case of \( n = 1 \) and five space-time dimensions.
In this case, under the ansatzs \( p = 4 \) and \( \hat{R} = 0 \), the (rr)-component of Einstein equations, \((13)\), reduces to the form

\[-3(A')^2 - 2\Lambda = 0, \tag{20}\]

whose solution is given by

\[A = cr, \tag{21}\]

with the definition of a constant \( c \equiv \sqrt{-\frac{2\Lambda}{3}} \). Here three remarks are in order. The first is that the positivity of \((A')^2\) requires the negativity of a bulk cosmological constant, i.e., anti-de Sitter space as in the Randall-Sundrum model. Actually, when the brane geometry is completely flat \((\hat{g}_{\mu\nu} = \eta_{\mu\nu})\) and chargeless, the present model becomes equivalent to (a half AdS\(_5\) slice of) the Randall-Sundrum model where a single 3-brane sits at the origin \( r = 0 \) and a fifth dimension is noncompact. The second remark is related to the sign in front of the positive constant \( c \) where we have selected a positive sign to bind the graviton on our 3-brane. The final remark is the integration constant in the right hand side of \((21)\). It is obvious that the constant can be set to zero by the redefinition of the brane coordinates \( x^\mu \).

Then substituting \((21)\) into \((16)\), we find that the effective cosmological constant on the brane vanishes, \( \Lambda_4 = 0 \). Next, under \( \Lambda_4 = 0 \) and the ansatz \( \hat{R} = 0 \), we see from \((15)\) that

\[\hat{R}_{\mu\nu} = 2\hat{t}_{\mu\nu}. \tag{22}\]

Indeed, this equation is consistent with our ansatz \( \hat{R} = 0 \) from Eq. \((18)\).

In consequence, the remaining equations which we should solve are Eqs. \((22)\), \((13)\) and \((19)\). (Note that Eq. \((14)\) stems from the \((\theta, \theta_1)\)-component of Einstein’s equations so this equation does not exist in the case of domain wall.) Provided that we choose \( f_{\rho} = \frac{Q}{\rho} \) with \( Q \) being a constant corresponding to the electric charge, as a special solution these equations give us the electrically charged Reissner-Nordstrom black hole geometry on the brane \[21\]. Accordingly, we arrive at the following line element as desired

\[ds_5^2 = e^{-cr} d\hat{s}^2 + dr^2, \tag{23}\]

where \(d\hat{s}^2\) is the line element of the Reissner-Nordstrom black hole geometry on the brane

\[ds^2 = \hat{g}_{\mu\nu}(x^\lambda) dx^\mu dx^\nu, = -(1 - \frac{2M}{\rho} + \frac{Q^2}{\rho^2}) dt^2 + \frac{1}{1 - \frac{2M}{\rho} + \frac{Q^2}{\rho^2}} d\rho^2 + \rho^2 (d\phi^2 + \sin^2 \phi d\varphi^2). \tag{24}\]

### 3.2 Higher dimensional defects (n ≥ 2) in D dimensions (D = 4 + n ≥ 6)

Now we turn our attention to the more general defect with codimension \( n \) in \( D \) space-time dimensions. A defect with codimension 2 in six space-time dimensions is called a string-like
defect. Similarly, a defect with codimension 3 and 4 in seven and eight space-time dimensions is called a monopole-like and an instanton-like defect, respectively. Of course, all the defects are a 3-brane. (Recall that we have set $p = 4$.) In this case, even if we take the ansatzs $p = 4$ and $\hat{R} = 0$ like the domain wall, Einstein equations (13) and (14) do not reduce to a simple and manageable expression owing to the existence of the nontrivial term $e^B$, which comes from the fact that in the $n$ extra dimensions with $n \geq 2$ the space is not flat but curved. (Note that the extra space with $n = 2$ is marginal since the space is conformally flat, so there is a nontrivial solution in this case, which was discovered by Gregory [15].)

An important observation here is that one must set a function $B(r)$ to a constant because otherwise we encounter a 'naked' singularity at somewhere $r$. This observation can be checked by calculating the bulk scalar curvature and higher orders of the curvature tensor [19]. Thus, we assume the form

$$B = -2 \ln R_0,$$  \hspace{1cm} (25)

where we selected the constant to be a specific form for later convenience.

With the ansatzs $p = 4, \hat{R}$ and (25), Einstein equations (13) and (14) read respectively

$$- 3(A')^2 + (n - 1)(n - 2 - 2\eta^2)R_0^{-2} - 2\Lambda = 0,$$  \hspace{1cm} (26)

$$4A'' - 5(A')^2 + (n - 3)(n - 2 - 2\eta^2)R_0^{-2} - 2\Lambda = 0.$$  \hspace{1cm} (27)

It is straightforward to solve the equations (26) and (27) whose result is given by

$$A = cr,$$  \hspace{1cm} 

$$c = \sqrt{-\frac{2\Lambda}{n + 2}},$$  \hspace{1cm} 

$$R_0^2 = \frac{(n + 2)(n - 2 - 2\eta^2)}{2\Lambda}.$$  \hspace{1cm} (28)

This solution coincides with the one found in Ref. [16].

Note that this solution also has a warp factor and makes sense only in anti-de Sitter space. Interestingly enough, in this case as well, the brane cosmological constant is zero, so the same argument as in the domain wall gives us the Reissner-Nordstrom black hole solution on a 3-brane as a special solution. Consequently, we have the bulk geometry with the Reissner-Nordstrom black hole as the brane geometry

$$ds^2_D = e^{-cr}d\hat{s}^2 + dr^2 + R_0^2d\Omega_{n-1}^2,$$  \hspace{1cm} (29)

where $d\hat{s}^2$, and the constants $c$ and $R_0$ are given by Eqs. (24) and (28), respectively.
4 Localization of Reissner-Nordstrom black hole on a 3-brane

In the previous section, we have shown that the charged Reissner-Nordstrom black hole exists on a 3-brane in the gravity localized models by introducing the $U(1)$ bulk gauge field. But our knowledge of localization of gauge field on the topological defects [13, 14, 18, 20] gives us a fear that the Reissner-Nordstrom black hole which we have found may not be localized on the brane but live in the whole space-time. In what follows, we shall show that the Reissner-Nordstrom black hole is in fact confined to the 3-brane.

As can be seen in Eqs. (23) and (29), the 5D line element (23) has a similar structure to the general one (29). As a result of this fact, it is known that in the both cases, zero mode of the bulk gauge field is not localized on a 3-brane [13, 14, 18, 20]. To save space, we shall explicitly prove that the Reissner-Nordstrom black hole which we have found in the previous section is in fact confined to a 3-brane only for the solution (29). The case of 5D solution (23) is a direct consequence of the proof.

Let us start with the action of $U(1)$ vector field:

$$S_A = -\frac{1}{4} \int d^5x \sqrt{-gg}^{MN} g^{RS} F_{MR} F_{NS}. \quad (30)$$

From (29) with the help of (8), we can calculate

$$\sqrt{-g} = R_0^{n-1} e^{-2A} (\sin \theta_2)^{n-2} (\sin \theta_3)^{n-3} \cdots (\sin \theta_{n-2})^2 (\sin \theta_{n-1}) \sqrt{-\hat{g}}. \quad (31)$$

Then, using Eqs. (12), (29) and (31), we can rewrite the action as follows:

$$S_A = -\frac{1}{4} \frac{2\pi^{\frac{5}{2}}}{4 \Gamma(\frac{5}{2})} R_0^{n-1} \int_0^\infty dr e^{-A} \int d^4x \sqrt{-\hat{g}}_{\mu\nu} \hat{g}^{\lambda\sigma} f_{\mu\lambda f_{\nu\sigma}} + \cdots. \quad (32)$$

The existence of a normalizable mode of gauge field on a 3-brane requires that the coefficient in front of 4D gauge action should be finite [14, 18]. Therefore, provided that we define the nontrivial coefficient in (32) as $I_1$, the requirement that the Reissner-Nordstrom black hole is confined to a 3-brane is that $I_1$ is finite. This is indeed the case as follows:

$$I_1 \equiv \int_0^\infty dr e^{-A} = \int_0^\infty dr e^{-cr} = \frac{1}{c} < \infty. \quad (33)$$

It is quite of importance to consider why $I_1$ becomes finite in the present situation since the corresponding coefficient in a Minkowski flat 3-brane is known to be divergent so that the bulk gauge field cannot be trapped on the brane by means of only a gravitational interaction [14, 18]. Note that an exponential factor $e^{-A}$ in (33) comes from (12). (Recall that as mentioned in Section 1, if we do not have this exponential damping factor as in the flat brane, $I_1$ becomes divergent, leading to the conclusion that zero mode of spin 1 bulk vector field is not localized neither on a brane with positive tension nor on a brane with negative
tension \([13, 14]\). This factor was introduced in order to have the Reissner-Nordstrom black hole as a solution to Einstein’s equations. (See Eqs. \((15), (17)\) and recall an ansatz \(f_{t \rho} = \frac{Q}{\rho^2}\).) Accordingly, the existence of the Reissner-Nordstrom black hole solution naturally causes the localization of the black hole charge on a 3-brane! We believe that this is one of striking phenomena in the gravity localized models. Put differently, a no-go theorem with regard to the non-localization of gauge field \([13, 14]\) holds only when its zero mode is a constant mode with respect to the radial coordinate \(r\), whereas in the case at hand zero mode explicitly depends on the coordinate \(r\) so this no-go theorem is not valid now.

5 Discussions

In this article we have investigated the possibility of having the electrically charged Reissner-Nordstrom black hole in the gravity localized models in a general dimension. To do so, it was necessary to introduce the bulk gauge field in order to make Einstein’s equations with the stress-energy tensor of the electric field. But the story was not so simple since it is well known that the bulk gauge field is not generally localized on a 3-brane. Thus, even if we could construct the charged Reissner-Nordstrom black hole on a 3-brane, the black hole, or more precisely speaking its electric charge, may not be trapped on the brane and propagate freely in the whole space-time. In other words, the charged black hole originally living in our world may lose its charge into the extra space and become the Schwarzschild black hole on a 3-brane as time passes by.

One of striking features is that the charged Reissner-Nordstrom black hole in the gravity localized models is indeed localized on our universe and the localization mechanism is provided by Einstein’s equations in a remarkable way.

In a flat Minkowski \((p−1)\)-brane, spin 1/2 and 3/2 fermionic fields are known to be localized on the brane with the exponentially rising warp factor \([22, 14, 18]\). We wish to know in future whether or not a similar phenomenon to the trapping of the charged black hole could happen in a physical situation relevant to those fermionic fields.

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