Invisibility or unhearability cloaks have been made possible by using metamaterials enabling light or sound to flow around obstacle without the trace of reflections or shadows. Metamaterials are known for being flexible building units that can mimic a host of unusual and extreme material responses, which are essential when engineering artificial material properties to realize a coordinate transforming cloak. Bending and stretching the coordinate grid in space require stringent material parameters; therefore, small inaccuracies and inevitable material losses become sources for unwanted scattering that are decremental to the desired effect. These obstacles further limit the possibility of achieving a robust concealment of sizeable objects from either radar or sonar detection. By using an elaborate arrangement of gain and lossy acoustic media respecting parity-time symmetry, we built a one-way unhearability cloak able to hide objects seven times larger than the acoustic wavelength. Generally speaking, our approach has no limits in terms of working frequency, shape, or size, specifically though we demonstrate how, in principle, an object of the size of a human can be hidden from audible sound.

1. Introduction

For centuries people have dreamt of an invisibility cloak that can make someone indiscernible for the naked eye when hidden underneath it. An ideal cloak would involve the suppression of back reflected light to render an object camouflaged, but the shadow behind it must also diminish for truly being able to make an object disappear. More than a decade ago an approach was brought forward based on transformation optics permitting the path of light to be bent around objects to be hidden [1–7]. This transformational approach to engineer space has ever since been extended to other areas of wave physics for cloaks of unhearability comprising sound and mechanical vibrations [8–13]. Despite those advances, it remains a fundamental challenge to create an unhearability cloak of ultrathin layer width involving minuscule loss-free materials. Here we propose theoretically and demonstrate experimentally that a parity-time symmetric metasurface incorporating acoustic gain and loss can act as such cloak when insonified from one direction. In contrast to transformation acoustics that implies a coordinate-transformation-based deformation of sound through the accurate distribution of the material properties, our approach to perfectly absorb incoming sound and to reemit it behind the hidden object solely implies the engineering of a complex acoustic metasurface-impedance. The use of parity-time symmetry enables unique cloaking properties useful in the audible range but also applicable to hide submarines from sonar detection.

Non-Hermitian systems that respect parity-time (\(PT\)-)symmetry have recently become an active frontier in wave physics due to unprecedented possibilities in guiding both sound and light [14, 15]. Most notably, designing complex eigenstates through appropriate balancing of gain and loss provides an unexpected paradigm for exploring non-Hermitian wave control in flourishing areas such as wave-guiding, sensing, communication, and topological insulators [16–24]. In this work we demonstrate that a sizeable acoustically rigid obstacle (7 times larger than the wavelength) appears hidden to sound waves when coated by a \(PT\) symmetric metasurface due to the cancellation of reflections and reradiation of the impinging field to the far-side. When
adjusting the gain-loss contrast to a point where this scenario is reached, unidirectional invisibility (unhearability) is obtained through a concurrent acoustic stealth and shadow cancellation.

Designing PT symmetry is based on manipulating absorption using judicious structures with gain regions and vice versa. The Hamiltonian commutes with the combined PT operator when loss and gain are equally balanced giving rise to entirely real eigenmode frequencies representing the unbroken or exact phase. When the loss and gain contrast exceed a certain threshold to reach the broken phase, one of the complex eigenmodes exhibits exponential growth while the other one decays exponentially. The transition between these two phases is the non-Hermitian singularity, also known as an exceptional point (EP) where the modes coalesce.

One-way invisibility has been explored already in the context of layered non-Hermitian one-dimensional systems whose physics is well explained by the PT-symmetry breaking transitions of the scattering matrix. The anisotropic transmission resonance (ATR) is associated with the flux-conservation process leading to full transparency, i.e., unity transmission resonance (ATR) is associated with the flux-breaking transitions of the scattering matrix. The anisotropic PT whose physics is well explained by the context of layered non-Hermitian one-dimensional systems coalesce.

Also known as an exceptional point (EP) where the modes approach each other as the eigenvalues tend to a double degeneracy, PT symmetry ceases to exist. The EP is a point in parameter space where the two eigenvalues change sign, and the two corresponding eigenvectors become linearly dependent. The EP is characterized by the vanishing of the transmission coefficient, i.e., $\mathcal{T} = 1$. Under the assumption that the air gap separating the ultrathin PT metasurface as has been previously simulated for microwave radiation [29]. The approach consists in camouflaging the portion of the insonified metasurface through absorption and providing the time-reversed image, i.e., acoustic gain, to the shadow region behind the rigid obstacle (Figure 1). Through PT symmetric field properties associated with sound impinging at and emanating from the metasurface cloak, we provide a theoretical insight into the complex two-dimensional scattering problem.

2. Complete Absorption of Sound

We begin the study by designing the insonified portion of the cloak, whereas the gain portion will be treated afterwards. Hence, in order to engineer complete scattering cancellation to an incoming plane wave we cover the rigid cylinder of radius $a$ by a lossy metasurface as shown in Figure 1. The aim is to impedance match the metasurface to the surrounding air to totally absorb the incident wave without reflection. Under the assumption that the air gap separating the ultrathin cloak from the rigid cylinder is substantially smaller than the wavelength of the sound wave, i.e., $k(b - a) \ll 1$, we can write down the necessary complex surface impedance to fulfill camouflaging of the obstacle (see supplementary information (available here) for derivations):

$$\begin{align*}
\Re(Z_s) &= Z_0 \frac{b}{a \cos \theta}, \\
\Im(Z_s) &= Z_0 \frac{b}{a} k(b - a),
\end{align*}$$

where $\theta$ is the angular position in polar coordinates and $k = 2\pi/\lambda$ where $\lambda$ is the wavelength. Equation (1) states that an impedance match of the metasurface to its surrounding with respect the geometrical parameters $a$ and $b$ is essential in order to achieve complete acoustic energy absorption. In (1), the free space impedance $Z_0 = \rho_0 c_0$ where $\rho_0$ and $c_0$ are the mass density and speed of sound in air, respectively. Interestingly, if one would attempt to engineer scattering cancellation of a sound soft cylinder, e.g., an air cylinder submerged in water, then one would end up with exactly the same surface impedance as found for an equivalent microwave problem [29] (see supplementary material for the derivation). There is a plethora of passive and active metamaterials solutions available capable of completing sound absorption [13]. For the realization of a one-sided compact and lightweight unhearability cloak we use Helmholtz resonators that can be fabricated to absorb sound at broad spectral windows. In the present case, we focus on the audible range and therefore begin the design by engineering sound absorption at a frequency of $f = 3$ KHz although the approach could readily be realized at other desired frequencies. Strong air oscillations in the neck of these resonators in the presence of viscous losses are responsible for efficient energy dissipation. Conclusively, most absorbed acoustic energy is localized at the neck region; therefore, in order to fully camouflage an object, we pattern the rigid obstacle of radius $a = 40$ cm by Helmholtz resonators (Figure 2(a)) and adjust the individual neck parameters $w$ and $t$ accordingly (see supplementary information for their values) to account for the angular variation. By computing the averaged acoustic pressure $\langle p \rangle$ and velocity normal to the resonator surface $\langle \mathbf{v} \rangle$ through full-wave simulations in Comsol, we are able to determine the impedance of the metasurface $Z_s = \langle p \rangle / \langle \mathbf{v} \rangle$. The aforementioned viscothermal losses are introduced in the neck region through the dynamic viscosity $\mu = 1.802 \times 10^{-5}$ kg/ms, the heat capacity $c_p = 1007 J/kgK$, and the thermal conductivity $k = 0.02476 W/mK$. In the absence of acoustic backscattering, we predict total absorption of a plane wave at each individual Helmholtz resonator as displayed in Figure 2(b) via their specific angular position. Correspondingly, we are able to explain full acoustic absorption via surface impedance matching as predicted in (1) where the real part of the relative metasurface impedance scales according to $1/\cos(\theta)$ and its imaginary counterpart approaches zero for a vanishing gap separation $a = b$.

3. Acoustic Gain Adjustment

Perfect absorption removes acoustic backscattering and is the first ingredient of a PT symmetric system. The time-reversed image of this response constitutes acoustic
amplification that we implement with an active electric circuit to control a semicircular array of loudspeakers (Figure 3(a)) [21, 28]. In order to implement sound amplification of equal but opposite strength to the absorbing counterpart, we begin by doing so mathematically. Operating parity $\mathcal{P}$ inversion and time reversal $\mathcal{T}$ on the loss portion of the cloak relates its impedance to the one at the gain portion $Z_s(\theta) = -Z_s^*(\pi + \theta)$ with respect to Figure 3(a). Further, we are able to employ those two operations to the pressure field $P(\pi + \theta)$ at the lossy metasurface, to relate it to its gain counterpart:

$$\mathcal{P}[P(\pi + \theta, t)] = P(\theta, t),$$

$$\mathcal{T}[P(\pi + \theta, t)] = P^*(\pi + \theta, -t).$$

These two actions result in $P(\theta) = \mathcal{PT}[P(\pi + \theta)]$ from which one analytically is able to determine the acoustic intensities residing at either side of the non-Hermitian metasurface cloak (see supplementary information for details)

$$I(\theta) = I(\pi + \theta).$$

Equation (3) signifies that when the $\mathcal{PT}$ symmetric metasurface cloak is irradiated at the loss portion, the acoustic intensity in the nearest vicinity of an individual Helmholtz resonator (located at $(\pi + \theta)$) equals the intensity at the exact opposite active loudspeaker (located at $\theta$). Incidentally, whenever an object appears undetectable to sound or microwaves, the wave intensity is constant in space and equal to the incident wave power flux. This property is extremely useful when adjusting the individual loudspeakers to realize a one-way unhearability cloak. First, as detailed in the supplementary information, we placed the two jointed semishells surrounding the rigid obstacle inside an acoustic $\mathcal{PT}$ symmetric waveguide whose rigid walls are covered by absorbing cotton. The acoustic source is formed by an array of loudspeakers that generate plane waves with frequency $f = 3$ KHz. In order to emit signals from the gain semishell perfectly synchronized with the impinging signal, a microphone measures the incoming sound field in front of the Helmholtz resonators whose phase and amplitude is processed through a phase shifting and amplifier circuit. The adjustment is performed in relation to the discrete and opposite locations of the Helmholtz resonator and loudspeaker couples whose intensity relation is shown in Figure 3(b). Due to size limitations of the source and the geometrical restrictions of the waveguide, our detection range exhibits a unitary intensity relation up to $\pm 48^\circ$ beyond which deviations start to grow. In other words, the subsequent cloaking experiment will be conclusively limited to this range.

4. $\mathcal{PT}$ Symmetry Cloak

Figure 4(a) displays the metasurface cloak in its entirety comprising the jointed non-Hermitian semishells. The obstacle to be cloaked was realized via a cylindrical acrylic shell with an outer diameter of 80 cm. The Helmholtz resonators responsible for the loss components and the active loudspeakers providing acoustic gain were mounted onto the shell. The individual Helmholtz resonators were 3D printed with thermoplastics whose geometrical parameters are tabulated in the supplementary information. At an operation frequency of 3 KHz corresponding to an acoustic wavelength of 11 cm, simulations and experimental measurements display how the pressure waves impinging the undecorated acrylic obstacle back-scatter at the side of irradiation but leave and almost

![Figure 1: (a) A rigid cylinder is covered by an ultrathin $\mathcal{PT}$ symmetric metasurface. The left (right) semicircle metasurface contains acoustic loss (gain) to fully absorb incoming (reemit outgoing) sound waves. (b) Three scenarios are exemplified: no cloak, comprising strong backscattering and a shadow; camouflaging through complete absorption with a lossy semicircle metasurface only; cloaking via $\mathcal{PT}$ symmetry.](image-url)
soundless shadow at the obstacle’s far-side (Figure 4(b)). Contrary to this, when sound irradiates the non-Hermitian semishells that have been tuned to fulfill $\mathcal{PT}$ symmetry adhering to the aforementioned constant intensity setting, the acrylic obstacle, whose diameter is about 7 times larger than the acoustic wavelength, is acoustically camouflaged to match its surrounding via complete absorption, but, more importantly, the acoustic shadow gets eliminated through reconstruction of the impinging wave (Figure 4(c)). The experimental measurements show that, within the test areas, both in front of and behind the obstacle, almost perfect plane waves have been sustained rendering the object to be hidden perfectly unhearable and concealed. Additional simulations confirm that the present design is a stringent one-way implementation for sound at normal incidence (see supplementary information).
Figure 3: (a) The amplifying portion of the metasurface cloak has been decorated by loudspeakers that are controlled by gain circuits. (b) The anisotropic transmission resonance with omnidirectional full transmission and one-sided zero reflection is accompanied by a spatial symmetry of the measured intensity profile $I(\theta) = I(\pi + \theta)$, which was the experimental parameter for the gain adjustment.

Figure 4: (a) Experimental realization of the $\mathcal{PT}$ symmetric metasurface cloak made out of two jointed semishells (radius $a = 40$ cm): (a) lossy Helmholtz resonator array and active loudspeakers controlled by gain circuits. (b) Full-wave simulations of the pressure field of a bare rigid cylinder when insonified from the left by a plane wave at $f = 3$ kHz. The dashed test areas have been experimentally measured. (c) Simulations of the metasurface cloak surrounding the rigid obstacle and the corresponding measurements at the front and the backside of the decorated obstacle.

5. Discussion

Further improvement can be achieved by enlarging the loudspeaker array to launch a near-ideal plane wave. Also, decorating an object of arbitrary shape with gain and loss units in response to a point source or more complicated wave shapes greatly broadens the usage of $\mathcal{PT}$ symmetry based acoustic cloaks. We implemented the proof of concept by means of Helmholtz resonators to suppress back reflected sound via resonant absorption. In analogy to the implemented active gain component, active loss control would enrich the possibility of eliminating backscattering and providing an acoustic camouflage dynamically at a wider spectral range. Yet, increasing the bandwidth of a passive
metasurface could be made possible by combining various resonating unit cells [13]. Extension toward an acoustical concealment of three-dimensional bodies by ultrathin non-Hermitian shells is another avenue worth pursuing.

In conclusion, we have derived a theoretical recipe to realize an acoustic unhearability cloak via $\mathcal{PT}$ symmetry. By combining loss and gain structures, we showed that reflected sound is eliminated from an insonified body to be concealed and how it is reconstructed at the rear side of it. Full-wave simulations and measurement data support the theoretical predictions in creating a cloak based on a single but non-Hermitian shell structure.

Conflicts of Interest

All authors declare that there are no conflicts of interest regarding the publication of this article.

Acknowledgments

This work was supported by the National Key R&D Program of China (Grant No. 2017YFA0303700), the National Natural Science Foundation of China (Grant No. 11634006) and the Innovation Special Zone of National Defense Science and Technology. The authors would like to thank Prof. Jing Li and Prof. Jian-cheng Tao for fruitful discussions. Daniel Torrent acknowledges the support from the MINECO through a Ramón y Cajal grant (Grant no. RYC-2016-21188). Johan Christensen acknowledges the support from the European Research Council (ERC) through the Starting Grant no. 714577 PHONOMET. The authors would like to thank Prof. Jing Li and Andrea Alu for helpful suggestions and stimulating discussions.

Supplementary Materials

(1) Surface impedance derivation. Figure S1: geometry of the metasurface cloak. (2) PT symmetry field. (3) One-way cloak and angular sensitivity. Figure S2: numerical simulation when insonified from the gain side. Figure S3: numerical simulation when insonified at various angles of incidence (4). Experimental implementation. Table I: geometrical parameters of the Helmholtz resonators. Figure S4: experimental implementation. Figure S5: electrical gain circuit. (Supplementary Materials)

References

[1] J. B. Pendry, D. Schurig, and D. R. Smith, “Controlling electromagnetic fields,” Science, vol. 312, no. 5781, pp. 1780–1782, 2006.
[2] D. Schurig, J. J. Mock, B. J. Justice et al., “Metamaterial electromagnetic cloak at microwave frequencies,” Science, vol. 314, no. 5801, pp. 977–980, 2006.
[3] J. Li and J. B. Pendry, ”Hiding under the carpet: A new strategy for cloaking,” Physical Review Letters, vol. 101, Article ID 203901, 2008.
[4] J. Valentine, J. Li, T. Zentgraf, G. Bartal, and X. Zhang, “An optical cloak mode of dielectrics,” Nature Materials, vol. 8, p. 568, 2009.
[5] T. Ergin, N. Stenger, P. Brenner, J. B. Pendry, and M. Wegener, “Three-dimensional invisibility cloak at optical wavelengths,” Science, vol. 328, no. 5976, pp. 337–339, 2010.
[6] X. Ni, Z. J. Wong, M. Mrejen, Y. Wang, and X. Zhang, “An ultrathin invisibility skin cloak for visible light,” Science, vol. 349, no. 6254, pp. 1310–1314, 2015.
[7] A. Alu, “Mantle cloak: invisibility induced by a surface,” Physical Review B: Condensed Matter and Materials Physics, vol. 80, Article ID 245115, 2009.
[8] S. A. Cummer and D. Schurig, ”One path to acoustic cloaking,” New Journal of Physics, vol. 9, article 45, 2007.
[9] D. Torrent and J. Sánchez-Dehesa, “Acoustic cloaking in two dimensions: a feasible approach,” New Journal of Physics, vol. 10, no. 6, Article ID 063015, 2008.
[10] L. Sanchis et al., “Three-dimensional axisymmetric cloak based on the cancellation of acoustic scattering from a sphere,” Physical Review Letters, vol. 110, no. 14, Article ID 141102, 2013.
[11] L. Zigoneanu, B.-I. Popa, and S. A. Cummer, “Three-dimensional broadband omnidirectional acoustic ground cloak,” Nature Materials, vol. 13, no. 4, pp. 352–355, 2014.
[12] N. Stenger, M. Wilhelm, and M. Wegener, “Experiments on elastic cloaking in thin plates,” Physical Review Letters, vol. 108, Article ID 014301, 2012.
[13] S. A. Cummer, J. Christensen, and A. Alu, “Controlling sound with acoustic metamaterials,” Nature Materials, vol. 1, Article ID 16001, 2016.
[14] C. M. Bender and S. Boettcher, “Real spectra in non-hermitian hamiltonians having pt symmetry,” Physical Review Letters, vol. 80, no. 24, pp. 5243–5246, 1998.
[15] C. M. Bender, D. C. Brody, and H. F. Jones, “Complex extension of quantum mechanics,” Physical Review Letters, vol. 89, Article ID 270401, 2002.
[16] K. G. Makris, R. El-Ganainy, D. N. Christodoulides, and Z. H. Musslimani, “Beam dynamics in pt symmetric optical lattices,” Physical Review Letters, vol. 100, Article ID 103904, 2008.
[17] A. Guo, G. J. Salamo, D. Duchesne et al., “Observation of pt-symmetry breaking in complex optical potentials,” Physical Review Letters, vol. 103, Article ID 093902, 2009.
[18] C. E. Ruter, K. G. Makris, R. El-Ganainy, D. N. Christodoulides, M. Segev, and D. Kip, “Observation of parity-time symmetry in optics,” Physical Review Letters, vol. 6, article no 192, 2010.
[19] B. Peng, Ş. K. Özdemir, F. Lei et al., “Parity–time-symmetric whispering-gallery microcavities,” Nature Physics, vol. 10, no. 5, pp. 394–398, 2014.
[20] J. Christensen, M. Willatzen, V. R. Velasco, and M.-H. Lu, “Parity-time synthetic phononic media,” Physical Review Letters, vol. 116, Article ID 207601, 2016.
[21] C. Shi, M. Dubois, Y. Chen et al., “Accessing the exceptional points of parity-time symmetric acoustics,” Nature Communications, vol. 7, Article ID 11110, 2016.
[22] M. Wang, L. Ye, J. Christensen, and Z. Liu, “Valley physics in non-hermitian artificial acoustic boron nitride,” Physical Review Letters, vol. 120, Article ID 246601, 2018.
[23] Z. Zhang, M. Rosendo Lopez, Y. Cheng, X. Liu, and J. Christensen, “Non-hermitian sonic second-order topological insulator,” Non-Hermitian Sonic Second-Order Topological Insulator, vol. 122, Article ID 195501, 1955.
[24] Z. Gong, Y. Ashida, K. Kawabata, K. Takasan, S. Higashikawa, and M. Ueda, “Topological phases of non-hermitian systems,” Physical Review X, vol. 8, Article ID 031079, 2018.

[25] Z. Lin, H. Ramezani, T. Eichelkraut, T. Kottos, H. Cao, and D. N. Christodoulides, “Unidirectional invisibility induced by PT-symmetric periodic structures,” Physical Review Letters, vol. 106, Article ID 213901, 2011.

[26] L. Ge, Y. D. Chong, and A. D. Stone, “Conservation relations and anisotropic transmission resonances in one-dimensional pt-symmetric photonic heterostructures,” Physical Review A, vol. 85, Article ID 023802, 2012.

[27] A. Regensburger, C. Bersch, M.-A. Miri, G. Onishchukov, D. N. Christodoulides, and U. Peschel, “Parity-time synthetic photonic lattices,” Nature, vol. 488, no. 7410, pp. 167–171, 2012.

[28] R. Fleury, D. Sounas, and A. Alu, “An invisible acoustic sensor based on parity-time symmetry,” Nature Communications, vol. 6, Article ID 5905, 2015.

[29] D. L. Sounas, R. Fleury, and A. Alu, “Unidirectional cloaking based on metasurfaces with balanced loss and gain,” Physical Review Applied, vol. 4, Article ID 044005, 2015.