Turbulent transport in tokamak plasmas with rotational shear

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Nonlinear gyrokinetic simulations have been conducted to investigate turbulent transport in tokamak plasmas with rotational shear. At sufficiently large flow shears, linear instabilities are suppressed, but transiently growing modes drive subcritical turbulence whose amplitude increases with flow shear. This leads to a local minimum in the heat flux, indicating an optimal \( \mathbf{E} \times \mathbf{B} \) shear value for plasma confinement. Local maxima in the momentum fluxes are also observed, allowing for the possibility of bifurcations in the \( \mathbf{E} \times \mathbf{B} \) shear. The sensitive dependence of heat flux on temperature gradient is relaxed for large flow shear values, with the critical temperature gradient increasing at lower flow shear values. The turbulent Prandtl number is found to be largely independent of temperature and flow gradients, with a value close to unity.

Introduction. Experimental measurements in magnetic confinement fusion devices indicate that sheared mean \( \mathbf{E} \times \mathbf{B} \) flows can significantly reduce and sometimes fully suppress turbulent particle, momentum, and heat fluxes \cite{1,2}. Since these turbulent fluxes determine mean plasma density and temperature profiles, their reduction leads to a local increase in the profile gradients. This increase can be dramatic: transport barriers in both the plasma core and edge have been measured with radial extents on the order of only tens of ion Larmor radii \cite{3}. The associated increase in core density and temperature results in increased fusion power. Thus, understanding how shear flow layers develop and what effect they have on turbulent fluxes is both physically interesting and practically useful.

This Letter reports a numerical study of the influence of sheared toroidal rotation on turbulent heat and momentum transport in tokamak plasmas. Two main effects of sheared toroidal rotation were identified in previous numerical work \cite{4,5}: suppression of turbulent transport by shear in the perpendicular (to the mean magnetic field) velocity and linear destabilization due to the parallel velocity gradient (PVG). While the former observation indicates that a finite flow shear improves plasma confinement, the latter raises the question of whether more shear is always beneficial. Below we report that the PVG-driven linear instability \cite{9} is stabilized at sufficiently large flow shear values, consistent with fluid theory in slab geometry \cite{10}. Correspondingly, fluxes decrease with increasing flow shear as the linear stabilization point is approached. However, beyond this point, transiently growing modes driven by the PVG give rise to subcritical turbulence. The fluxes associated with this turbulence increase with flow shear. This implies an optimal flow shear for each temperature gradient; the fact that the minimum heat flux value is finite indicates that there is a maximum attainable temperature gradient that can be maintained for a given heat flux. Additionally, the observed presence of maxima in the momentum fluxes admits the possibility of bifurcations in the flow shear (and thus the temperature gradient).

In the absence of flow shear, a small increase in temperature gradient leads to a large increase in heat flux (“stiff transport”). Recent experimental evidence \cite{11} suggests that flow shear may reduce this sensitivity in configurations with low magnetic shear. Our results indicate that at low flow shear values, both the critical temperature gradient for the onset of turbulence and the stiffness increase. At high flow shear values, the opposite behavior is observed (the stiffness and critical temperature gradient both decrease).

Model. A closed set of evolution equations \cite{12,13} for mean plasma density, pressure, and toroidal angular momentum is obtained by taking moments of the kinetic equation and applying the \( \delta f \) gyrokinetic ordering \cite{14,15}. These moment equations relate the evolution of the mean quantities to particle, momentum, and heat fluxes, which are typically dominated by turbulent contributions. We restrict our attention to electrostatic fluctuations and assume a modified Boltzmann response \cite{10} for the electron distribution. The resultant particle flux is identically zero, and the radial components of the turbulent heat flux of species \( s \), \( Q_s \), and toroidal angular momentum flux, \( \Pi \), are given by

\[
Q_s = \int d^3v \left( \frac{m_s v^2}{2} \right) \left( \mathbf{v}_E \cdot \nabla \psi \right) \delta f_s \quad (1)
\]

\[
\Pi = \sum_s m_s R^2 \int d^3v \left( \mathbf{v} \cdot \nabla \phi \right) \left( \mathbf{v}_E \cdot \frac{\nabla \psi}{|\nabla \psi|} \right) \delta f_s, \quad (2)
\]

where \( \psi \) is a flux-surface label, \( \phi \) the toroidal angle, \( m_s \) the particle mass, \( v \) its velocity in the frame of mean flow, \( R \) its major radius, \( \mathbf{v}_E \) the fluctuating \( \mathbf{E} \times \mathbf{B} \) drift velocity, \( \delta f_s \) the deviation of the distribution function from a local Maxwellian, and the overline denotes a spatial average over a thin annular region encompassing a given flux surface.

The distribution function \( \delta f_s \) appearing in Eqs. (1) and (2) is calculated by solving the standard \( \delta f \) gyrokinetic equation in the limit where the plasma flow speed, \( u \), is ordered comparable to the ion thermal speed,
in this “high-flow” regime, the flow velocity is constrained to be 
\[ \mathbf{u} = R^2 \omega(\psi) \nabla \phi, \] 
where \( R \) is the major radius and \( \omega \) is the rotational frequency \[13, 17\]. We consider a subsidiary expansion in low Mach number \( \rho_i/L \ll M \ll 1 \), where \( L \) is a typical macroscopic length scale (e.g., the minor radius of the torus) and \( \rho_i \) is the ion Larmor radius. However, we allow for flow gradients of order \( 1/ML \), and so neglect terms proportional to \( \omega \) (such as Coriolis and centrifugal drifts) but retain those proportional to \( d\omega/d\psi \) (flow shear). The resulting equation is

\[
\frac{dh}{dt} + (v_{\parallel} \hat{b} + v_B + \langle v_E \rangle_R) \cdot \nabla h - \langle C[h]\rangle_R =
- \langle v_E \rangle_R \cdot \nabla \psi \left( \frac{dF_0}{d\psi} + \frac{mv_B}{T} \frac{RB_0}{B} \frac{d\omega}{d\psi} F_0 \right) + \frac{eF_0}{T} \frac{d\varphi}{dt}.
\]

where \( d/dt = \partial/\partial t + \mathbf{u} \cdot \nabla \), \( h = \delta f + (e\varphi/T)F_0 \) is the deviation of \( \delta f \) from a Boltzmann response, \( e \) is particle charge, \( \varphi \) is the electrostatic potential, \( \mathbf{v}_E = (c/B^2)B \times \nabla \varphi \), \( T \) is temperature, \( F_0 \) is a Maxwellian distribution of peculiar velocities, \( C \) is the collision operator, \( B \) is the magnetic field strength (with \( B_0 \) the toroidal component), \( \mathbf{v}_B = \hat{b}/\Omega \times \left( v_{\parallel} \hat{b} + v_{\perp} \nabla B / 2B \right) \) contains magnetic drifts, \( \Omega \) is the Larmor frequency, \( \hat{b} = B / B \), and \( \langle \cdot \rangle_R \) denotes the average over gyro-angle at fixed guiding center position \( R \).

Eq. 3 is solved in the rotating reference frame using the local, nonlinear gyrokinetic code \( GS2 \) \[8, 19\]. The toroidal velocity is expanded in \( \psi \) about its value at the center of the simulation domain, \( \psi_0 \), giving \( \mathbf{u} \approx R^2 (\psi - \psi_0) (d\omega/d\psi) \nabla \phi \). The local approximation assumes that \( d\omega/d\psi \) is constant across the simulation domain. Consequently, the effect of shear flow is accounted for by a single parameter, \( \gamma_E = (\psi/q) (d\omega/d\psi) \frac{R_0}{\nu_{th}} = (M/q) (d \ln \omega / d \ln r) \), where \( q \) is the safety factor, \( R_0 \) is the major radius at the center of the flux surface, and \( r \) is the half-diameter of the flux surface (both measured at the height of the magnetic axis).

We study a system whose magnetic geometry corresponds to the widely-used Cyclone base case \[20\] (unshifted, circular flux surface with \( q = 1.4 \), magnetic shear \( \dot{s} = d\ln q / d\ln r = 0.8 \), \( r/R_0 = 0.18 \), and \( R_0/L_n = 2.2 \), with \( L_n^{-1} = -d \ln n / dr \)). The \( \mathbf{E} \times \mathbf{B} \) shearing rate, \( \gamma_E \), and the normalized inverse temperature gradient scale length, \( \kappa \equiv R_0/L_T \), were varied over a wide range of values in a series of linear and nonlinear simulations.

**Linear stability.** The average linear growth rates, \( \gamma \), obtained from these simulations are given in Fig. 1. We see that \( \gamma \) decreases rapidly with \( \gamma_E \) for small values of \( \gamma_E \) before increasing to a local maximum and subsequently decreasing to zero. For \( \gamma_E \gtrsim 0.25 \), the system is linearly unstable only in the presence of both temperature and parallel flow gradients (cf. \[5, 8\]). The system becomes linearly stable for large values of \( \gamma_E \), with the critical \( \gamma_E \) for stability, \( \gamma_E(c) \), increasing approximately linearly with \( \kappa \). Beyond \( \gamma_E(c) \), there are no linear unstable modes. This result is qualitatively similar to the prediction from fluid theory in a slab \[10\], where no linear instability is possible when \( \gamma_E/\kappa \) exceeds a certain critical value.

**Heat flux.** The turbulent heat flux calculated from nonlinear simulations is given in Fig. 3. For all \( \gamma \) values, the heat flux qualitatively follows the same trend as growth rates when \( \gamma_E < \gamma_E(c) \). For \( \gamma_E > \gamma_E(c) \), nonlinear simulations initialized with low-amplitude noise develop no turbulent transport. However, for finite initial fluctuation amplitudes, one finds that the turbulence does not necessarily decay away: for sufficiently large values of \( \kappa \) and initial amplitude, the flux reaches steady state values in excess of those found for \( \gamma_E \) just below \( \gamma_E(c) \). The flux then increases monotonically with \( \gamma_E \). For the range of \( \gamma_E \) considered here, the heat flux associated with a given temperature gradient is thus minimized at a finite shearing rate.

**Subcritical turbulence.** Because the turbulence is present in the absence of linear instability, we refer to it as subcritical. When \( \gamma_E > \gamma_E(c) \), linear simulations exhibit transient growth, with order unity increases in the initial fluctuation amplitudes over times of several \( R_0/\nu_{th} \) before subsequent decay (Fig. 3). The duration of growth decreases with increasing flow shear, but the transient growth rate increases so that the amplification factor of the initial perturbation amplitude grows with flow shear (cf. \[19\]). This transient amplification provides an energy source for the turbulence, which can be maintained by the nonlinearity through redistribution of energy amongst other modes. We have identified the PVG term in Eq. 3 (the \( d\omega/d\psi \) term on the RHS) as the driver of the subcritical turbulence, as the latter is no longer present when the PVG is artificially set to zero.

**Momentum flux.** The toroidal angular momentum flux also mimics the linear growth rate, except near \( \gamma_E =\)
Stiff transport. A serious impediment to confinement is the sensitive dependence of the heat flux on small changes in \( \kappa \). This stiffness of the transport makes it difficult to increase \( \kappa \), and therefore the core temperature, beyond the critical value, \( \kappa_c \), at which turbulence is excited. Recent experimental results indicate that stiffness, \( dQ_i/d\kappa \), may be reduced at low magnetic shear, \( \hat{s} \), and large values of \( \gamma_E \) [23]. For the Cyclone base case considered here (\( \hat{s} = 0.8 \)), we find a complicated dependence of stiffness on flow shear, as illustrated in Fig. 5. At low values of \( \gamma_E \) (\( \lesssim 0.3 \)), the critical \( \kappa \) shifts to higher values, but the stiffness increases. This makes sense intuitively: one expects that for \( \kappa/\gamma_E \to \infty \), the heat flux will tend to the curve corresponding to \( \gamma_E = 0 \). For this
to happen, the value of \( dQ_i/d\kappa \) at \( Q_i = 0 \) must increase to compensate for the increase in \( \kappa_c \).

For \( 0.3 \lesssim \gamma_E < \gamma_E^{(c)} \), both \( \kappa_c \) and the profile stiffness decrease. This is a result of a change in the nature of the linear instability, which is now driven by the PVG. The relaxation of stiffness is modest, and it only occurs for \( \kappa \) near \( \kappa_c \). When \( \gamma_E > \gamma_E^{(c)} \), \( \kappa_c \) initially shifts upwards with increasing \( \gamma_E \), and stiffness increases for \( \kappa \) near \( \kappa_c \). However, for even larger \( \gamma_E \), both the stiffness and \( \kappa_c \) decrease. This is to be expected, as the subcritical turbulence is driven by the velocity, not temperature, gradient and thus has a weaker dependence on \( \kappa \).

**Conclusions.** We have shown that PVG-driven turbulence exists at large flow shears, but only if the initial fluctuation amplitudes are sufficiently large. This has a number of potentially important implications. First, it indicates that linear stability analysis is insufficient to determine critical gradients in rotating plasmas. Furthermore, it shows that the system can undergo hysteresis: The fluctuations associated with a given flow shear and temperature gradient pair depend on the path taken to obtain that pair. As an example, consider the pair \( \gamma_E = 1, R_0/L_T = 10 \). From Fig. 2, we see that if \( R_0/L_T = 10 \) is obtained at \( \gamma_E < \gamma_E^{(c)} \) before increasing \( \gamma_E \) to unity, there will be large heat and momentum fluxes. In this case, there is an optimal flow shear for confinement \( (\gamma_E \approx 1) \) and a maximum attainable temperature gradient for a given power input. However, if \( \gamma_E = 1 \) is obtained at \( R_0/L_T \gtrsim 7.5 \) before increasing \( R_0/L_T \) to ten, then the fluxes will be zero because the initial amplitude is not large enough to excite the subcritical turbulence. Thus it may be beneficial to increase flow shear on a time scale short compared to the energy confinement time.

The existence of local maxima of the toroidal angular momentum flux provides the potential for bifurcations in \( \gamma_E \) and corresponding bifurcations in temperature gradient. However, a detailed transport analysis is necessary to definitively observe such a bifurcation for experimentally relevant conditions. Finally, we note that the \( \mathbf{E} \times \mathbf{B} \) suppression and PVG drive terms differ by a factor proportional to \( (qR_0/r)(B_0/B) \). Thus our results, which should be qualitatively robust, may undergo considerable quantitative variation with changes in magnetic configuration.

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