Characterizing the phase profile of a vortex beam with angular-double-slit interference

Ruifeng Liu, Junling Long, Feiran Wang, Yunlong Wang, Pei Zhang, Hong Gao and Fuli Li

Department of Applied Physics, MOE Key Laboratory for Nonequilibrium Synthesis and Modulation of Condensed Matter, Xi’an Jiaotong University, Xi’an 710049, People’s Republic of China

E-mail: zhangpei@mail.ustc.edu.cn and flli@mail.xjtu.edu.cn

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Abstract
The diffracted intensity distribution of a Laguerre–Gaussian beam is studied with an angular-double-slit interferometer. We experimentally demonstrate that the spiral phase structure of a vortex beam can be clearly revealed in this interference geometry, and it gives us an efficient way to distinguish different orders of Laguerre–Gaussian beams. This angular-double-slit interferometer gives us a better understanding of the phase structure of a vortex beam and the interference phenomenon of vortex beams.

Keywords: vortex beam, orbital angular momentum, angular-double-slit interference

(Some figures may appear in colour only in the online journal)

1. Introduction
It is well known that a photon of a vortex beam with an azimuthal phase profile of the form $\exp(\imath l \phi)$ carries a well defined orbital angular momentum (OAM) of $l \hbar$ [1, 2], where $\phi$ is the azimuthal angle and $l$ is an integer which describes the topological charge of vortex light. Because of the singularity on the propagating axis, a dark point exists in the center of the transverse intensity distribution. The OAM’s number of degree of freedom is infinite, so it offers a good source for the quantum information process. It has been used in quantum computation [3–5], quantum cryptography [6] and high capacity free space optical communication [7–9]. It also can be used in optical tweezers [10] to control micro-objects, spiral phase contrast imaging [11], which can enhance the edge contrast, and holographic ghost imaging [12].

Characterizing the topological charge of an optical vortex beam has increasingly attracted attention in recent years, and several methods have been proposed to sort and detect the OAM states. Generally, interference is a convenient way to analyze the phase structure of vortex light [13, 14], such as interfering the measured vortex beam with a uniform plane wave or its mirror image [15, 16], in which the topological charge can be obtained by the number of forks or spiral petals. Some groups reveal the vortex phase profile by diffraction patterns with a special mask. Berkhout et al [17] proved that a multiple-pinhole diffracted system can be used to probe the OAM states of the measured optical vortex. Hickmann et al [18] pioneered a triangular aperture diffraction to measure the vortex phase structure. Beside these, single slits and double slits have also been proposed to analyze the topological change of vortex beams [19–22]. Padgett’s group has proposed two schemes to sort different OAM states under the single photon level. One of the schemes is based on a Mach–Zehnder interferometer with Dove prism [23, 24], and the other scheme is converting OAM states into transverse momentum states [25]. However, most of these proposals are involved with an unexpected complexity of interferometric patterns or a complicated interferometric experimental setup, which makes them difficult to execute in practical application.

Due to the inherent spiral phase distribution $\exp(\imath l \phi)$ of the vortex beam, we propose a scheme which can conveniently characterize the modulus and sign of the vortex beam’s topological charge with an angular double slit. This scheme is analogous to the Young’s double-slit interference in which a constructive or destructive interference pattern can be
obtained according to the phase difference between the double slit. The angular-double-slit interference would give us a better understanding of the phase structure of the vortex beam and the interference phenomenon of vortex beams. Although our experimental setup is similar to the work done by Malik et al [26], what they measured is the OAM spectrum of fields with partial angular coherence and we directly record the interference patterns in the far field.

2. Theory

We start with a simple illumination of the proposed method theoretically. A schematic diagram of the scheme is shown in figure 1 with an angular-double-slit mask. \( \phi_1 \) and \( \phi_2 \) are the two angular positions for two slits on the input plane. \( q_1, q_2 \) and \( q_3 \) are three points on the mask, and \( oq_3 \) is the angular bisector of \( \angle q_1oq_2 \). When a plane wave illuminates the angular double slit, constructive interference patterns occur at the \( o'p_1 \)-axis where \( o'p_1 \) is parallel to \( oq_3 \), because \(|q_1p_1| = |q_2p_1| \) which means the optical path difference from the two slits to point \( p_1 \) is zero. A point \( p_2 \) deviating from the \( o'p_1 \)-axis will experience a constructive or destructive interference pattern according to the optical path difference \(|q_1p_1| - |q_2p_2|\).

If the illuminating light on the mask has a spiral phase front \( \exp(i\ell \varphi) \), the phase difference between \( q_1p_1 \) and \( q_2p_2 \) is

\[
\Delta \Phi = l\Delta \varphi + 2\pi \frac{|q_1p_1| - |q_2p_2|}{\lambda},
\]

where \( \Delta \varphi = \varphi_2 - \varphi_1 \). If \( \Delta \Phi = N\pi \) (where \( \lambda \) is the wavelength of the illuminating light) and \( N \) is an even number, constructive interference patterns occur at the axis of \( o'p_1 \). As shown in figure 1, the angular double slit is symmetrical with respect to the \( o'p_1 \)-axis, so the interference patterns on the \( o'p_1 \)-axis are only determined by the phase difference \( l(\varphi_2 - \varphi_1) \). For a given vortex beam with OAM number of \( l \), we can get constructive or destructive interference patterns on the \( o'p_1 \)-axis depending on the angular difference of the angular double slit. For simplicity of operation, we can fix one slit at \( \varphi_1 = 0 \), and rotate the other slit \( \varphi_2 \) continuously. Thus, we will obtain a periodic constructive or destructive interference pattern at the \( o'p_1 \)-axis, where the \( o'p_1 \)-axis is in the \( \varphi_2/2 \) direction. If the period is \( 2\varphi_0 \), the topological charge of the vortex beam will be determined by \( l = \pi/\varphi_0 \), except that constructive interference patterns always occur when \( l = 0 \).

In the cylindrical coordinate system, Laguerre–Gaussian (LG) modes and Bessel beams are kinds of vortex beam which possess OAM. LG modes are characterized by two mode indices. The radial structure is mainly determined by the azimuthal mode index \( l \) whereas the phase structure is described by the azimuthal mode index \( \ell \). To simplify the numerical calculation, we set \( p = 0 \) and the complex amplitude of LG modes is proportional to

\[
E_l(r, \varphi) \propto \exp \left( -\frac{r^2}{\omega_0^2} \right) \exp(-il\varphi),
\]

where \( \omega_0 \) is the waist size of the beam.

We first assume that the aperture angle and radius of angular-slit \( \varphi_1 \) in figure 1 are \( \Delta\theta \) and \( r_0 \), respectively. In order to get a concise analytical result, we assume that the single angular slit can be approximately treated as a triangular aperture and angular \( \Delta\theta \ll 2\pi/l \). So the spiral phase \( \exp(-il\varphi) \) in the slit is nearly a constant, which means that we can get the Fraunhofer diffracted field of this triangular aperture with a plane wave. After we get the single-slit diffraction, we can calculate the two slits’ interference by just adding their complex amplitudes together.

The far field complex amplitude for a triangular aperture can be expressed as

\[
u(r', \varphi') \propto \frac{r_0}{y} e^{-i\pi(x'+x')\Delta\theta/2} \sin\left( \pi \frac{x'+y}{2} \right) \left( \pi \frac{x'-y}{2} \right) e^{-i\pi l \varphi'}
\]

\[
= \frac{r_0}{y} e^{-i\pi(x'-x')\Delta\theta/2} \sin\left( \pi \frac{x'-y}{2} \right) \left( \pi \frac{x'+y}{2} \right) e^{-i\pi l \varphi'}.
\]

The diffracted pattern of the other angular slit is \( u(r', \varphi' + \Delta\varphi) \), which has a similar form to equation (3). In this way, we can get the angular-double-slit interference complex amplitude when a vortex beam with spiral phase \( \exp(-il\varphi) \) illuminates the slits:

\[
u_l(r', \varphi') = u(r', \varphi') + u(r', \varphi' + \Delta\varphi)e^{-i\Delta\varphi}.
\]

This is the superposition of the two single triangular slits with angular phase difference \( \Delta\varphi \). In the following numerical simulation and experiments, we focus on the diffraction patterns which are composed of the first-order diffraction of the single triangular slit.

Figure 2 shows the numerical simulation results from equations (3) and (4) with different topological charges \( l = 0–5 \). In the simulation process, we rotate the second angular single slit and fix the first angle on the \( x \)-axis. The angular-single-slit width is \( 10^\circ \). When \( l = 0 \), there will always be a constructive interference pattern along the angular bisector of the angular double slit whatever the value of \( \Delta\varphi \). This is analogous to angular-double-slit interference with a plane wave. If the illuminating light has an OAM \( l = 1 \), there is a \( \pi \) phase difference when \( \Delta\varphi = \pi \) and a destructive interference can be observed at the angular bisector direction.
Figure 2. Numerical simulation results of the angular-double-slit interference patterns with different OAM states. $\Delta \varphi$ is the angular difference of the angular double slit, $\varphi_2 - \varphi_1$.

3. Experiment

The experimental setup of angular-double-slit interference is shown in figure 3. The light emitted from the He–Ne laser is expanded with two lenses, and then illuminates a computer-generated hologram with controllable pixels written in an LCOS-SLM X10468 spatial light modulator (SLM). The inset of figure 3 shows the mask written on the SLM, and the mask is a combination of an angular double slit and a hologram grating which generates higher-order LG modes. The first-order diffracted beam is chosen with an aperture, then a charge coupled device (CCD) is used to record the diffracted patterns. In the experimental process, one angular slit is fixed on the x-direction and the other angular slit rotates around the center of the SLM screen.

Figure 4 shows some experimental results of the angular-double-slit interference. As in the analysis above, a constructive interference pattern always can be observed at the angular bisector direction of the angular double slit when the illuminating light is OAM $l = 0$. The dashed lines in figure 4 denote the destructive and constructive interference directions of the angular double slit. When $l = 2$, the destructive and constructive interference patterns can be observed at the angular double-slit bisector when $\Delta \varphi = \pi/2$ and $\Delta \varphi = \pi$, respectively. So it is obvious that the OAM states can be determined by the destructive and constructive interference patterns of the angular double slit.

4. Conclusion

In conclusion, we demonstrate theoretically and experimentally that the interference patterns of the vortex beam after passing through an angular double slit can be used to characterize the modulus and sign of the vortex beam’s topological charge. With a unique intersection angle $\Delta \varphi = \varphi_2 - \varphi_1$ of the angular double slit, destructive or constructive interference patterns occur in the $\Delta \varphi/2$ direction, which can be used to determine the OAM states. The sign of the OAM states can be obtained from the movement direction of the interference patterns. Our proposal does not need a complicated interferometric setup or complicated interference patterns. We note that, although previous works observed the interference fringes with multiple-pinhole [17], triangular aperture [18], double-slit and single-slit experiments (in orthogonal coordinates, but not in the polar coordinates) [19, 20], our proposal offers us a direct understanding to the spiral phase profile of the vortex beam. For a more precise characterization of the topological charge of the vortex beam,
Figure 4. Experimental results of the angular-double-slit diffracted patterns with different OAM states. The dashed lines in the insets tilt $\Delta \varphi/2$ to the horizontal direction.

Figure 5. Angular-double-slit diffracted patterns with different OAM states $l = \pm 4$. The first and third columns are numerical simulation, and the others are experimental results.

we can rotate the second slit full circle to obtain the period of constructive or destructive interference patterns, and then determine the phase profile of vortex beam. We believe this proposal could be useful in the applications of quantum communication processing and astronomy [17] with vortex beams.

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