Time Dependent Heterogeneous Vehicle Routing Problem for Catering Service Delivery Problem

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Abstract. The heterogeneous vehicle routing problem (HVRP) is a variant of vehicle routing problem (VRP) which describes various types of vehicles with different capacity to serve a set of customers with known geographical locations. This paper considers the optimal service deliveries of meals of a catering company located in Medan City, Indonesia. Due to the road condition as well as traffic, it is necessary for the company to use different type of vehicle to fulfill customers demand in time. The HVRP incorporates time dependency of travel times on the particular time of the day. The objective is to minimize the sum of the costs of travelling and elapsed time over the planning horizon. The problem can be modeled as a linear mixed integer program and we address a feasible neighbourhood search approach to solve the problem.

1. Introduction

Food catering companies need a logistic system to plan their services to meet the demands of their customers in scheduled time. In operational research the logistic system which involves delivery customers’ demand considering fleet of vehicle and their routing is called Vehicle Routing Problem (VRP). In terms of optimization the objective of this problem is to minimize the total travel cost (proportional to the travel times or distances) and operational cost (proportional to the number of vehicles used) such that to meet customers’ demand in time. Due to its important role in the logistic system, the research domain of VRP has been widely explored (see for example, the survey in [1], [2].

The catering company focused in this paper uses various type of vehicle in its operation to deliver the meals for customers. Each type of vehicle has different capacities. The variant of VRP which considers mixed fleet of vehicles is called Heterogeneous VRP (HVRP), introduced firstly by [3]. This generalization is important in practical terms, for most of customers demand are served by several type of vehicles ([4], [5]). The objective of the HVRP is to find fleet composition and a corresponding routing plan that minimizes the total cost.

So far the most approach proposed for solving the HVRP is heuristics. [6] were the first to tackle the HVRPTW and developed a number of parallel insertions heuristics based on the insertion scheme of [7] and embedding in the calculations of the relevant criteria the acquisition costs of [3]. [8] presented a deterministic annealing metaheuristic for the HVRPTW, outperforming the results of [6], and then [9] developed a linearly scalable hybrid threshold-accepting and guided local search meta heuristic for solving large scale HVRPTW instances. [10] presented an Adaptive Memory Programming solution approach for the HVRPTW that provides very good results in the majority of the benchmark instances examined. [11] proposed a hybrid algorithm for the problem. Their algorithm is composed by an Iterated Local Search (ILS) based heuristic and a Set Partitioning (SP) formulation.

However, in reality, most of its operation, the catering service faces difficulty to be in time to reach its customers, due to traffic congestion or vehicle breakdowns. The common assumption in VRP is that the travel times are constant. Therefore if we use the ordinary VRP model or VRP with time windows (VRPTW) to tackle this difficulty we will get suboptimal or even infeasible solution [12].
VRP which accounts for traffic congestion effect in the model is called time-dependent VRP (TDVRP). This variant of VRP was first formulated by [13] and [14]. They proposed a mixed integer linear programming model and the travel times were computed using step functions. [15] proposed a model based on a node-based time dependent travel speeds for TDVRP. [16] also present a model based on time dependent travel speeds. They show that the time-dependent model provides substantial improvement over a fixed travel times. VRP which consider traffic congestion was also considered by [17]. They discussed the impact of congestion avoidance, such as, selecting alternative routes, in their model. They used modified Djikstra algorithm and a restricted dynamic programming heuristic to solve the problem. [18] use genetic algorithm for solving TDVRP with time windows. They solve the TDVRPTW contains 100 customer nodes, 25 vehicles with capacity of each vehicle is 200. [19] use ant colony algorithm hybridized with insertion heuristic to solve TDVRPTW.

This paper deals with a logistic problem faced by a catering company to deliver daily meals to customers spread across the city of Medan, Indonesia. Customers demand are varied in the amount of meals and time of delivery. Therefore, it is necessarily for the company to plan the time of delivery, and type of vehicle to be used. It should be noted that the scheduled time for the catering problem is so tight. The catering company has a limited number of fleet of vehicles. Therefore it needs to plan a schedule that can organize these vehicles in order to satisfy their customers. Due to the fact that heterogeneous vehicle with different capacities are available, the basic framework of the vehicle routing can be viewed as a Heterogeneous Vehicle Routing Problem with Time Windows (HVRPTW). The coverage area of the operation of this catering company is large, and the traffic condition in Medan city is getting worst, it would be unavoidable to include time dependent in the model. Then we will have a time-dependent HVRPTW (TDHVRPTW) model for the catering problem.

We address a mixed integer programming formulation to model the problem. A feasible neighborhood heuristic search is proposed to get the integer feasible solution after solving the continuous model of the problem.

2. Problem Formulation And Problem Solution
This paper considers a problem faced by a catering service company located in Medan city, North Sumatra Province, Indonesia. Using graph theory, the basic frame work of TDHVRP for the catering problem can be defined as follows. Let $G = (V, A)$ be a directed graph, where $V = \{0, 1, \ldots, n\}$ is the vertex set and $A \subseteq \{(i, j) : i, j \in V, i \neq j\}$ is the set of route. For each route $(i, j) \in A$ a distance (or travel) cost $c_{ij}$ is defined. Vertex 0 (i = 0) and vertex n (i = n) are the depot vertex, center of service, where the vehicle fleet is located. Define $V_c$ is the set of customers’ vertex. Each vertex $i \in V_c$ has a known fixed daily meal demand $q_i \geq 0$, a service time $s_i \geq 0$, and a service time windows $[a_i, b_i]$. In particular, at depot the demand $q = 0$ and service time $t = 0$. As this is a heterogeneous problem, we define a fleet of K vehicles composed by m different type of vehicles, each with capacity $Q_m$. The number of vehicles available for vehicle type m is $n_m$. Define $K_m$ as the set of vehicle type m. Each customer is served by exactly one vehicle with the associated type. At the depot $(i = 0)$, a time window for vehicles to leave and to return to depot is given by $[a_0, b_0]$. The arrival time of a vehicle at customer $i$ is denoted by $a_i$ and its departure time is $b_i$. Vehicle routes are restricted to a maximum duration of $H_k$, $k = 1, \ldots, K$. Each type of vehicle is associated with a fixed cost, $f_m$. Another cost occurs for travelling through route $(i, j) \in A$, defined as $\alpha^m_{ij} = d_{ij}g_{ij}$, where $d_{ij}$ is the distance travelled between route i to route j and $g_{ij}$ a factor cost for travelling, for m type of vehicle. Further more, a fixed acquisition cost $f_k$ is incurred for each of vehicle k in the routes. Each route originates and terminates at the central depot and must satisfy the time window constraints, i.e., a vehicle cannot start servicing customer $i$ before $a_i$ and after $b_i$ however, the vehicle can arrive before $a_i$ and wait for service.
Due to the catering company considered in this research has a lot of customers, the time scheduled for deliveries is so tight. Meanwhile the operation time daily is within busy traffic condition, therefore we should impose travel time as a function of departure time $b_i$ ($t_{ij} \geq 0$) 

The model formulated is necessarily imposed a time-dependent for each delivery. The problem can be called as TDHVRPTW.

Firstly we define the decision variables as follows.

- $x_{0j}^k = \begin{cases} 1 & \text{if vehicle type } k \in K \text{ to deliver from depot to customer } j \in V_c \, ; \\ 0 & \text{otherwise} \end{cases}$
- $x_{ij}^m = \begin{cases} 1 & \text{if vehicle type } m \in K_m \text{ to deliver for } (i, j) \in V', i \neq j \, ; \\ 0 & \text{otherwise} \end{cases}$
- $z_0^m = \begin{cases} 1 & \text{if vehicle type } m \in K \text{ is available and active at depot } \, ; \\ 0 & \text{otherwise} \end{cases}$
- $l_{ij}^m$ Arrival time for vehicle type $m \in K_m$ at customer $i \in V_c$ (non-negative continuous variable)
- $u_{ij}^m$ Duration of service of vehicle type $m \in K_m$ at customer $i \in V_c$ (non-negative continuous variable)

3. The Mathematical Model

The basic model of TDHVRPTW for catering problem can be written mathematically as follows.

In this basic framework the manager of the catering company wants to use the available vehicle for each type efficiently, such that the total cost is minimized. The total cost consists of traveling cost of all vehicle used and the cost for the availability of vehicle in the planning horizon time of a day.

$$\text{Minimize} \quad \sum_{j \in V_c} \sum_{k \in K} c_{0j} x_{0j}^k + \sum_{(i, j) \in V', m \in K_m} c_{ij}^m x_{ij}^m + \sum_{m \in K_m} f_m z_0^m$$

Subject to

$$\sum_{k \in K} x_{0j}^k = 1, \quad \forall j \in V_c$$

$$\sum_{k \in K} \sum_{j \in V'} x_{ij}^k = 1, \quad \forall i \in V_c$$

Constraints (2) and (3) are to ensure that exactly one vehicle regardless their type enters and departs from every customer and from the central depot and comes back to the depot.

$$\sum_{i \neq j} x_{ij}^k - \sum_{i \neq j} x_{ji}^k = 1; \quad \forall j \in V', \forall k \in K$$

A flow conservation equation is necessarily needed to maintain the continuity of each vehicle route on each period of time. This equation is presented in Constraints (4).
Constraint (5) represents that each customer is served only by the available and active vehicle of type m.

\[
x_{ij}^m \leq z_{0}^m, \quad (i, j) \in V_c, \forall m \in K_m
\]  

(5)

Constraints (6) and (7) state the availability of vehicles by bounding the number of route, related to vehicle k for each type, directly leaving from and returning to the central depot, not more than one, respectively.

\[
\sum_{j \in V_c} x_{ij}^k \leq 1; \quad \forall k \in K
\]  

(6)

\[
\sum_{j \in d, d \neq 1} x_{ij}^k \leq 1; \quad \forall k \in K
\]  

(7)

Constraint (8) ensures that each delivery does not exceed the capacity of each type of vehicle.

\[
x_{ij}^m (l_i^m + u_i^m + s_i + t_{ij}) = 0; \quad \forall m \in K_m, (i, j) \in A
\]  

(9)

Constraint (9) establishes the equilibrium among the arrival time, duration of service, service time and travel time at customers in the routes assigned.

\[
l_i^m \leq a_i \sum_{j \in V_c} x_{ij}^m; \quad \forall m \in K_m, i \in V_c
\]  

(10)

\[
a_i \sum_{j \in V_c} x_{ij}^m \leq l_i^m + u_i^m \leq b_i \sum_{j \in V_c} x_{ij}^m; \quad \forall m \in K_m, i \in V_c
\]  

(11)

Constraints (10) and (11) present time window as the committed scheduled time-dependent for each customer.

\[
\sum_{j \in V_c} x_{ij}^m \leq n_m; \quad \forall m \in K_m
\]  

(12)

Constraint (12) guarantees that the number availability of active vehicle does not exceed the number of vehicle available at the central depot of catering company.

The model formulated in equations (1) – (12) is in the form of large scale Mixed Integer Nonlinear Programming (MINLP).

4. The Algorithm
For the integrizing process it is necessarily to partition the non-feasible integer basic variable.

Let

\[
x = [x] + f, \quad 0 \leq f \leq 1
\]

be the (continuous) solution of the relaxed problem, \([x]\) is the integer component of non-integer variable \(x\) and \(f\) is the fractional component.

Stage 1.
Step 1. Get row $i^*$ the smallest integer infeasibility, such that $\delta_* = \min\{f_*, 1 - f_*\}$

(This choice is motivated by the desire for minimal deterioration in the objective function, and clearly corresponds to the integer basic with smallest integer infeasibility).

Step 2. Do a pricing operation

$$v^*_j = v^*_j B^{-1}$$

Step 3. Calculate $\sigma_j = v^*_j \alpha_j$

With $j$ corresponds to

$$\min_j \left( \frac{d_j}{\alpha_j} \right)$$

Calculate the maximum movement of non-basic $j$ at lower bound and upper bound. Otherwise go to next non-integer non-basic or super-basic $j$ (if available). Eventually the column $j^*$ is to be increased from LB or decreased from UB. If none go to next $i^*$.

Step 4.

Solve $B \alpha_j^* = \alpha_j^*$ for $\alpha_j^*$

Step 5. Do ratio test for the basic variables in order to stay feasible due to the releasing of non-basic $j^*$ from its bounds.

Step 6. Exchange basis

Step 7. If row $i^* = \emptyset$ go to Stage 2, otherwise

Repeat from step 1.

Stage 2. Pass1: adjust integer infeasible superbasics by fractional steps to reach complete integer feasibility.

Pass2: adjust integer feasible superbasics. The objective of this phase is to conduct a highly localized neighbourhood search to verify local optimality.

5. Conclusions

The catering company has a lot of customers to be served with a variety of volume of meal container. Therefore the company needs several type of vehicle to carry out the deliveries. This paper is to develop a model of time dependent heterogeneous vehicle routing with time windows problem. This model is used for solving a catering problem of a company located in Medan city, Indonesia. The deliveries depend on the traffic congestion. The result model is in the form of mixed integer non linear programming problem. We will solve the model using the nearest neighbor heuristic algorithm.

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