Three Dimensional Imaging of Proton in Basis Light-Front Quantization

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We employ an effective Hamiltonian that includes the transverse and longitudinal confinement and the one-gluon exchange interaction with fixed coupling constant. By solving the eigenvalue equation in basis light-front quantization (BLFQ), we generate the light-front wavefunctions (LFWFs) for the nucleon in the valence quark Fock space. Fitting the model parameters, we obtain high quality descriptions of electromagnetic form factors and radius for proton while the results deviate somewhat from experimental data for neutron.

Keywords: BLFQ, LFWF, FFs, PDFs, GPDs

1. Introduction

Basis light-front quantization (BLFQ) is a nonperturbative approach which is developed for solving bound state problems in quantum field theories\textsuperscript{3,13}. This approach has been successfully applied to QED\textsuperscript{3,14} and QCD\textsuperscript{5,13} systems. In our work, we apply the BLFQ approach to the nucleon and study the electromagnetic form factors. As a Hamiltonian formalism, we adopt a light-front effective Hamiltonian, which includes the holographic QCD confinement potential supplemented by longitudinal confinement\textsuperscript{5,13} along with the one-gluon exchange interaction with a fixed coupling constant. The light-front wave functions (LFWFs) are obtained by diagonalizing the effective Hamiltonian and used to calculate the electromagnetic form factors.
Electromagnetic form factors are crucial for probing the structure of the nucleon. In the light-front formalism, the Dirac and Pauli form factors, \( F_1(Q^2) \) and \( F_2(Q^2) \), are defined with the longitudinal vector current \( J^+_1 \),

\[
\langle P + q, \uparrow | J^+_1(0) | P, \uparrow \rangle = \frac{F_1(Q^2)}{2P^+},
\]

where \( Q^2 = -q^2 = q^2 \) is the square of the momentum transfer, and \( M \) is the nucleon mass. The ket \( | P, S_z \rangle \) represents the physical state that can be expanded in terms of the wave functions \( | x_i P_i, k_i, \lambda_i \rangle \).

The flavor form factors can be written as the overlap of light-front wave functions,

\[
G_E^i(Q^2) = F_1^i(Q^2) - \frac{Q^2}{4 \ast M^2} F_2^i(Q^2), \quad G_M^i(Q^2) = F_1^i(Q^2) + F_2^i(Q^2).
\]

The \( i = P \) or \( N \) represents the proton or neutron, and \( F_{1/2}^{i} = \sum_{f} e_f F_{1/2}^{f/i} \) is the Dirac (Pauli) form factors of the nucleon. And the electromagnetic radii of the nucleon can be obtained from

\[
(r_E^2)^i = -\frac{dG_E^i(Q^2)}{dQ^2} \bigg|_{Q^2=0}, \quad (r_M^2)^i = -\frac{6}{G_M^i(0)} \frac{dG_M^i(Q^2)}{dQ^2} \bigg|_{Q^2=0}.
\]

2. Hamiltonian Formalism

BLFQ solves the eigenvalue equation of the light-front Hamiltonian \( P^- | \beta \rangle = P^-_\beta | \beta \rangle \), which leads to the eigenvalue \( P^-_\beta \) and the associated eigenvectors of the bound state. In our work, we only consider the lowest Fock-sector for the expansion of the nucleon, and employ an effective
Hamiltonian $P_{\text{eff}}$ which is given by

$$P_{\text{eff}} = \sum_i \frac{m_i^2 + p_i^2}{x_i} + \frac{1}{2} \sum_{i,j} \left( \kappa_T^4 x_i x_j r_{ij}^2 + \frac{\kappa_L^4}{(m_i + m_j)^2} \partial_x (x_i x_j \partial_x) \right)$$

$$+ \frac{1}{2} \sum_{i,j} \frac{C_F 4 \pi \alpha_s}{Q^2} \bar{u}_{s'}(k_i') \gamma^\mu u_s(k_i) \bar{u}_{s'}(k_j') \gamma^\mu u_s(k_j),$$

where $m_{ij}$ is the constituent mass of quarks and the $i,j = 1, 2, 3$ label the Fock particles. For each single-particle basis state, we employ the discrete plane-wave basis ($k$) in the longitudinal direction and 2D harmonic oscillator (2DHO) basis ($n$ and $m$) in the transverse direction. Besides, a single quantum number ($\lambda$) presents the helicity degree of freedom.

For the nucleon, proton (or neutron) is the lowest eigenstate, denoted by $|P^\Lambda\rangle$, where the $\Lambda$ indicates helicity of the nucleon. In momentum space, the LFWFs are written as

$$\Psi^\Lambda (x_i, k_i, \lambda_i) = \sum_{n_1, m_1, n_2, m_2, n_3, m_3} \left( \psi^\Lambda (k_i, n_i, m_i, \lambda_i) \right)$$

$$\times \prod_i \frac{\sqrt{2}}{b(2\pi)^{2}} \left( \frac{n_i!}{(n_i + |m_i|)!} \right) e^{-p_{i\perp}/(2b^2)} \left( \frac{|p_{i\perp}|}{b} \right)^{|m_i|} \sqrt{\frac{p_i^2}{b^2}} e^{im\theta}. \ (7)$$

Here, $b$ is an HO basis parameter with the dimension of mass, and $L_{n_m}^{m_l}(p_i^2/b^2)$ is the generalized Laguerre polynomial.

### 3. Numerical Results

In this paper, we set the model parameters $m_q/OGE = 0.2$ GeV, $m_q/k = 0.3$ GeV, $\kappa_T = 0.284$ GeV, $\kappa_L = 0.373$ GeV and $\alpha_s = 1.0 \sim 1.2$. In Fig 1(a), the Sachs form factors of the proton show an agreement with the experimental data, except for $G^P_M$ in the low $Q^2$ region. At $Q^2 = 0$, $G^P_M(0)$ gives the anomalous magnetic moments. Our calculations show that the anomalous magnetic moments of the proton ($G^P_M(0) = 2.443 \pm 0.027$) is somewhat different with the experimental measurements ($G^P_M(0) = 2.79$).

In Fig 1(b), we show the Sachs form factors of the neutron and compare them with the experimental data revealing a significant difference. Especially, at $Q^2 = 0$, the $G^N_M(0) = -1.405 \pm 0.026$ is disagree with the experimental data ($G^N_M(0) = -1.91$).

We also calculate the electromagnetic radii of the nucleons, which we show in Table 1. The BLFQ results are in a good agreement with the experimental data.
Fig. 1. The Sachs form factors for the proton (a) and neutron (b). Nucleon Sach’s FFs $G^{P/N}_E(Q^2)$ (upper panel) and $G^{P/N}_M(Q^2)$ (lower panel) are functions of $Q^2$. The bands are BLFQ results reflecting our $\alpha_s$ uncertainty of 10%. The experimental data are taken from Ref.18.

Table 1. Electromagnetic radii of the nucleon. Our results are compared with the experimental data.17

|                  | $\langle r^P_E \rangle /$(fm) | $\langle r^P_M \rangle /$(fm) | $\langle r^N_E \rangle^2 /$(fm$^2$) | $\langle r^N_M \rangle /$(fm) |
|------------------|--------------------------------|--------------------------------|---------------------------------|-----------------------------|
| BLFQ             | 0.85 ± 0.05                    | 0.88 ± 0.03                    | −0.09 ± 0.17                    | 0.90 ± 0.03                  |
| Exp. Data        | 0.833 ± 0.010                  | 0.777 ± 0.016                  | −0.116 ± 0.0022                 | 0.862±0.009                  |

4. Conclusion

In our work, we produce the light-front wave functions by solving the eigenvalue equation of light-front Hamiltonian, and evaluate the electromagnetic form factors of the nucleon. We observe the proton form factors are in a reasonable agreement with the experimental data. The neutron form factors show a significant issue in the low $Q$ region. We also compare the electromagnetic radii of the nucleon with the experimental data.
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