ON THE ECCENTRICITIES AND MERGER RATES OF DOUBLE NEUTRON STAR BINARIES AND THE CREATION OF “DOUBLE SUPERNOVÆ”

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ABSTRACT

We demonstrate that a natural consequence of an asymmetric kick imparted to neutron stars at birth is that the majority of double neutron star binaries should possess highly eccentric orbits. This leads to greatly accelerated orbital decay, due to the enormous increase in the emission of gravitational radiation at periastron as originally demonstrated by Peters. A uniform distribution of kick velocities constrained to the orbital plane of the presupernova binary would result in ~24% of surviving binaries coalescing at least 10,000 times faster than a circular orbit system. Even if the planar kick constraint is lifted, ~6% of bound systems still coalesce this rapidly. In a nonnegligible fraction of cases it may even be possible that the system could coalesce within 10 years of the final supernova, resulting in what might resemble a “double supernova.” For systems like the progenitor of PSR J0737−3039A, this number is as high as ~9% (in the planar kick model). Whether the kick velocity distribution extends to the range required to achieve this is still unclear. We do know that the observed population of binary pulsars has a deficit of highly eccentric systems at small orbital periods. In contrast, the long-period systems favor large eccentricities, as expected. We argue that this is because the short-period highly eccentric systems have already coalesced and are thus selected against by pulsar surveys. This effect needs to be taken into account when using the scale-factor method to estimate the coalescence rate of double neutron star binaries. We therefore assert that the coalescence rate of such binaries is underestimated by a factor of several.

Subject headings: binaries: close — pulsars: general — waves

1. INTRODUCTION

Beginning with the detection of the Hulse & Taylor (1975) double neutron star, pulsar astronomers have unveiled a small but growing sample of double neutron star binaries over the last 30 years. Owing to the extreme regularity of their radio pulse emissions, these exotic objects have proven to be invaluable scientific laboratories, particularly for tests of general relativity, as they are some of the few known objects for which the effects of gravitational radiation are measurable. In fact, the first confirmation that gravitational radiation existed was through measurements of the orbital decay of the first known double neutron star (Taylor & Weisberg 1989).

Recent surveys have started to unveil a better picture of the double neutron star population. The Parkes multibeam surveys have discovered three probable double neutron star systems (Lyne et al. 2000; Burgay et al. 2003; Faulkner et al. 2005), and Arecibo surveys have detected two more (Wolszczan 1991; Champion et al. 2004). A survey of intermediate latitudes at the Green Bank Telescope detected the “mildly relativistic” binary PSR J1518+4904 (Nice et al. 1996). It is unclear whether the binary PSR B1820−11 consists of two neutron stars or a neutron star and white dwarf (Lyne & McKenna 1989). Although we are still dealing with small number statistics, the population is now large enough to see trends emerging.

A phenomenon of crucial importance to the binary coalescence rate is the kick velocity imparted to a neutron star during its supernova, a dominant factor in determining the eccentricity and coalescence time of the subsequent orbit (Hills 1983; Dewey & Cordes 1987; Bailes 1989). Binary neutron star formation and evolution also depends greatly on this little-understood supernova kick, since a double neutron star system must survive two such kicks in order to remain bound. To some extent, knowledge of the kick velocity distribution can be extracted statistically from measurements of the observed sample of double neutron stars (Willems & Kalogera 2004). The magnitude of the kicks can be inferred by the space velocities of pulsars, which are an order of magnitude higher than those of their progenitors, the OB stars. Investigations of the pulsar population demonstrate that the average kick must be of the order of 400–500 km s⁻¹ (Lyne & Lorimer 1994; Cordes & Chernoff 1998; Arzoumanian et al. 2002; Hobbs et al. 2005).

Further evidence for pulsar kicks comes from precession effects observed in the binary pulsar PSR J0045−7319 (Kaspi et al. 1996) and spin-orbit coupling in the relativistic binaries PSR B1913+16, PSR B1534+12, and PSR J1141−6545 (Weisberg et al. 1989; Stairs et al. 2004; Hotan et al. 2004). Assuming that tidal forces and mass transfer align the spin and angular momentum axes prior to the ultimate supernova explosion, these systems suggest a kick out of the orbital plane has occurred.

Binary pulsars are in some sense fossil records of the progenitor system from which they evolved. We date these systems by the characteristic age of the pulsar and can therefore extrapolate back to the orbital conditions immediately after the final supernova explosion. What prohibits us from going further back in time to derive a unique progenitor is the possibility that the newly born pulsar received an impulsive kick at birth. Since the kick velocity distribution is largely unknown, including its orientation with respect to any reference point, such as the pulsar spin axis, we are left with a rather unsatisfactory family of potential progenitors for any observed neutron star binary (see, e.g., Wex et al. 2000; Willems et al. 2004; Thorsett et al. 2005).

The relativistic nature of these objects has brought attention to the galactic rate of double neutron star coalescence—the merger event at the climax of a long orbital decay and in-spiral due to
gravitational radiation. Such events are some of the most significant anticipated sources of gravitational waves, detectable by interferometers such as the Laser Interferometer Gravitational-Wave Observatory (LIGO; Abramovici et al. 1992). Double neutron star coalescence is also considered a major candidate for explaining the origin of a subset of the mysterious cosmic gamma-ray bursts observed by gamma-ray observatories such as the BATSE instrument aboard the Compton Gamma-Ray Observatory (CGRO; Meegan et al. 1992).

Much research has been conducted into predicting the double neutron star coalescence rate, using Monte Carlo population synthesis models (Belczynski et al. 2002; Tutukov & Yungelson 1993; Lipunov et al. 1997; Portegies Zwart & Spreeuw 1996) or by statistical extrapolation from the observed sample (Curran & Lorimer 1995; Kalogera et al. 2001, 2004). Monte Carlo methods are currently hampered by the vastness of the population parameter space and compounded uncertainties, and statistical methods suffer from the small sample size of known double neutron stars.

It is customary to estimate the event rate of advanced LIGO by estimating the fraction of neutron star binaries we could have detected in pulsar surveys if they were of similar period and luminosity to those known and assigning a “scale factor” to each. For each known neutron star binary the number of similar systems in the Galaxy are derived from the scale factor, thus obtaining a coalescence rate for the Milky Way that can be used to infer an event rate for the local universe by estimating the number of galaxies out to the LIGO detection limit. We refer to this method as the scale factor approach. While limited by small number statistics, the growing population of observed neutron star binaries makes us increasingly confident that coalescence is relatively common in the universe.

The only alternative approach is to attempt to simulate the entire binary pulsar population. Uncertainties in initial conditions and the physics of binary evolution and kicks mean that such estimates vary by more than 2 orders of magnitude (Belczynski et al. 2002). The population synthesis method is therefore of limited use in estimating the merger rate.

For now, the coalescence rate is better estimated by considering the scale factor method; however, when this is done we find the coalescence rate is dominated by the double pulsar PSR J0737–3039A (Burgay et al. 2003). This serves to demonstrate the heavy influence of small number statistics on the uncertainties in the scale factor method.

The scale factor method also suffers from the extent to which the relativistic binary pulsar population remains invisible to pulsar surveys. For instance, if there existed a population of neutron stars that were completely invisible in the radio, we would have no way to estimate their population from radio observations. Similarly, if only 1% of recycled neutron stars emitted radio waves, we would multiply the scale factors currently in use by 100.

Our best estimates of the in-spiral rate require us to think of all the reasons why double neutron stars might be invisible to our surveys and to take them into account. What we explore in this paper is a selection effect that removes binary pulsars from our surveys and has received little attention to date. This is the production of highly eccentric short-period binaries with very short lifetimes produced via favorably oriented kicks in the binary progenitor. These systems are not long-lived, and the coalescence rate of the scale-factor approach needs to be corrected for it. While this effect has been alluded to from time to time (Tutukov & Yungelson 1993), we determine its importance for an arbitrary binary in the presence of a kick velocity that can occupy any of the phase space that will produce a bound orbit, because in this case the problem is scale-invariant.

In this paper we derive the expected distributions of orbital eccentricity for any surviving binary pulsar if the kick velocity is capable of uniformly populating the available phase space after the explosion (§ 2). In §§ 3 and 4 we look at the lifetimes of the surviving systems due to gravitational radiation losses and identify a population of binaries that coalesce very rapidly. These results are used to explain the observed distribution of binary pulsar eccentricities (§ 5), which at long orbital periods favor high eccentricities and at low periods small eccentricities. In § 6 we argue that the coalescence rate should be revised to take this into consideration, particularly for the double pulsar.

2. ECCENTRICITIES

In this section we examine the expected orbital eccentricities of relativistic double neutron stars from a purely analytical point of view, avoiding any assumptions about specifics such as the distributions of pre-supernova masses, kick velocities, and orbital separations. Instead, we present a study of the potential eccentricities of an arbitrary double neutron star system such that the conclusions drawn here can be considered common to the evolution of all double neutron star systems. Indeed, all of our results also hold for any two compact objects (black holes, white dwarfs, or neutron stars) where the final compact object (a black hole or a neutron star) is created in a supernova and receives a kick.

We view the effect of kick velocity through a map of the second neutron star’s post-supernova “velocity space,” our representation of all possible relative velocities that correspond to bound orbits. The boundary of this velocity-space region is circular, with a radius equal to the escape speed of the system (√2 in units of the orbital speed required for the orbit to be circular). Each point in the map corresponds to one possible orbital velocity vector (v, v_tan), where the position (0, 0) would correspond to the circular orbital velocity, with zero radial velocity (v), and a tangential velocity (v_tan) equal to the circular speed.

To calculate the semimajor axis and eccentricity of an arbitrary binary, we derive formulae from two expressions for the total energy E of the system (Hills 1983) and two expressions for the total angular momentum L of the system (Murray & Dermott 1999):

\[ E = \frac{-G M m}{2a}, \]

\[ E = -\frac{G M m}{R} + \frac{1}{2} \mu v^2, \]

\[ L = \mu v R \cos \theta, \]

\[ L = \mu [G M_T a (1 - e^2)]^{1/2}. \]

Here \(\mu = (Mm)/(M + m)\) is the binary system’s reduced mass, derived from the component masses \(M\) and \(m\), \(M_T\) its total mass, \(a\) the semimajor axis, \(e\) its eccentricity, \(v\) the absolute magnitude of the relative orbital velocity immediately after the explosion, oriented at an angle \(\theta\) to the vector that connects the two stars, and \(R\) the initial orbital separation. Note that for our purposes, we only need to describe the condition immediately after the supernova and its associated kick. From these four equations we derive expressions for the semimajor axis and the
eccentricity of an arbitrary binary. We adopt a normalized velocity $v$ in units of the binary system’s circular speed:

$$ a = \frac{R}{2 - v^2}, \quad (5) $$

$$ e = \left[1 - v^2 \cos^2 \theta (2 - v^2)\right]^{1/2}. \quad (6) $$

These formulae are applied to generate a contour map of eccentricities in velocity space. Note that the results are independent of the choice of $R$. A code was written to uniformly populate the velocity-space region with grid points $(v_x, v_y)$, then apply equation (6) to calculate the eccentricity corresponding to each grid point (where each grid point represents one possible velocity). The two-dimensional data were finally plotted as a contour map and are presented here in Figure 1.

In its broader sense, this map applies to any arbitrary binary system. It simply transforms the radial and tangential components of a system’s instantaneous orbital velocity into orbital eccentricity. Over time, an orbit of $e = 0.9$ would trace out the relevant contour in Figure 1. A circular orbit would remain confined to one of two locations (corresponding to prograde and retrograde orbits). The dimensionless “velocities” in this map can always be scaled to absolute velocities for a given system, using its relative circular speed—for example, the relative circular speed of the PSR J0737−3039A progenitor (following its supernova) was of the order of 600 km s$^{-1}$.

Now, for our purposes, Figure 1 is used to describe the potential post-supernova eccentricities of a neutron star binary that receives a supernova kick. Before the supernova, the system’s relative orbital velocity would be located at the coordinate $(v_x, v_y) = (0, 1)$. But this ignores the fact that there is significant mass loss in a supernova explosion. In reality, the presupernova system would be located somewhere above $(0, 1)$, with the ordinate in the tangential direction given by the square root of the mass ratio of the pre-supernova and post-supernova systems. After the supernova and its associated kick, the system’s velocity will be displaced to a new point in Figure 1, given by the vector sum of the initial orbital velocity and the kick velocity. This new velocity will result in a new orbital semimajor axis (calculable from eq. [5]) and a new orbital eccentricity (as illustrated in Fig. 1). Here we have made the reasonable assumption that the outgoing supernova shell’s impact with the companion star has negligible effect on the final orbital configuration.

With this model in mind, one can see the significance of the small size of the low-eccentricity regions in Figure 1. These imply that, for a given progenitor system, there is a rather tight constraint on the kicks that will result in eccentricities below 0.3, compared to the full range of kicks that can produce bound systems. However, this contradicts the current observed sample of relativistic neutron star binaries, in which a majority of systems have eccentricities below 0.3. Furthermore, with observed mean kick magnitudes of 400–500 km s$^{-1}$, we expect systems to be highly prone to being kicked out of the small low-eccentricity “bull’s-eye” region. These facts lend themselves to two possible interpretations: either kick magnitudes are small more frequently than previously thought, or there is a strong selection effect in operation, preventing the observation of high-eccentricity relativistic neutron star binaries.

The two-dimensional eccentricity data generated for Figure 1 are presented as a histogram and cumulative distribution in Figure 2. Here we have also provided eccentricity data for a three-dimensional (“isotropic”) extension of our planar velocity space, generated by uniformly populating a “sphere” of radius $\sqrt{2}$ in velocity space. This allows us to examine the volumetric distribution of eccentricities in velocity space, as we discard the assumption that supernova kicks are restricted to the pre-supernova orbital plane. Of course, equations (5) and (6) remain valid in this isotropic case, as the orbital velocity is always constrained to an orbital plane, even if the orbital plane has been reoriented by the supernova kick.

The eccentricity histogram clearly illustrates a heavy bias toward very high eccentricities in the planar velocity space, with over 50% of all eccentricities greater than 0.8. The distribution...
is less skewed when we allow for kick velocities out of the orbital plane, but the bias is still toward very high eccentricities. We assert that this illustrates a strong inherent tendency toward highly eccentric systems arising as a consequence of supernova kicks.

This finding is not reflected in the number of highly eccentric relativistic binaries in the observed sample of double neutron stars, which tend to have low eccentricities. Analyzing this from the standpoint of kick velocities, the observed systems (most of which have low $e$) seem to have received kicks that are a very minor subset of the total range of nondisrupting supernova kicks. The relative proportion of this subset is seen in the size of the tiny bull’s-eye region of Figure 1. Hence, in the observed sample of neutron star binaries, we would expect low eccentricities to be rare rather than dominant. This contradiction between theory and observation is discussed further in the following sections.

3. LIFETIMES

In this section we examine the probable coalescence times of relativistic double neutron stars, using a velocity-space contour map to illustrate dependence on the supernova kick velocity. As in the previous section, we avoid assumptions about specifics such as the distributions of stellar masses, kick velocities, and orbital separations, and we present our results in units of the circular orbital speed and circular coalescence time (the merger time for an arbitrary system with zero eccentricity, denoted as $\tau_{\text{circ}}$). These generic units have been chosen for their usefulness in comparing binaries in a scale-invariant manner.

In producing the aforementioned contour map of coalescence times, we first generated a family of arbitrary double neutron star systems over all possible post-supernova relative orbital velocities. Computationally, this was achieved by uniformly populating the velocity space with grid points, where each point represents a relative orbital velocity. These velocities were then used to calculate the corresponding semimajor axes and eccentricities, using equations (5) and (6). Both of these factors will greatly influence the rapidity of each system’s orbital decay, with close, highly eccentric binaries being much more volatile than wider, more circular ones. To compute numerical estimates of just how rapidly each system will coalesce, each grid point’s initial orbital parameters were evolved forward in time by numerically integrating the coupled differential equations derived by Peters (1964) for describing orbital decay by relativistic gravitational radiation. A Runge-Kutta fourth-order algorithm was implemented with an adaptive step size, designed to integrate the equations forward to the precise time when the semimajor axis of the system drops to zero, thus determining the coalescence time. After iterating through all grid points, the two-dimensional coalescence time data were used to produce the contour map shown in Figure 3.

The most significant feature of Figure 3 is the large central region within which coalescence times rapidly drop from 0.1 $\tau_{\text{circ}}$ (in units of the coalescence time of a circular system) to $10^{-3}$ $\tau_{\text{circ}}$ and even lower. To put this in perspective, for the orbital parameters of PSR J0737–3039A/B, $10^{-3}$ $\tau_{\text{circ}}$ corresponds to a coalescence time of less than 1000 years. Figure 3 thus demonstrates that a double neutron star system that receives a supernova kick of the order of its circular speed has a very high likelihood of coalescing within an incredibly short time span.

The cumulative distribution of coalescence times is provided in Figure 4, which shows the uncoalesced percentage of the representative population as a function of time. This figure dramatically illustrates the high probability of rapidly coalescing double neutron star systems, with up to $\sim24\%$ of all systems coalescing in less than $10^{-3}$ $\tau_{\text{circ}}$ (equivalent to 8500 years when scaled to the parameters of PSR J0737–3039). After just 0.1 $\tau_{\text{circ}}$, about half of all systems have coalesced. This is an intriguing result, implying that the coalescence rate of double neutron stars is underestimated. These rapidly coalescing systems would be practically undetectable by modern pulsar surveys, since they would not live long enough to give sufficient opportunity for detection. They would, however, be detectable by gravitational wave observatories such as LIGO, which will come online in the near future. We might predict that at short orbital periods there will be a deficit of highly eccentric systems in the radio sample because these are selected against because
of their shorter lifetimes. We now investigate this in the following section.

4. ECCENTRICITY DISTRIBUTION EVOLUTION

Here we analyze the time evolution of the eccentricity distribution of a family of arbitrary double neutron star systems. The computation performed here is similar to that performed in the prior section. To begin, for every possible post-supernova relative orbital velocity (represented computationally by uniformly spread grid points in our velocity space), we calculate an eccentricity and semimajor axis by applying equations (5) and (6). This results in a standard representative family of double neutron star systems, whose orbital parameters we can then numerically integrate forward in time using the coupled differential equations derived by Peters (1964). In this way, the eccentricity distribution of the whole family of possible double neutron star systems can be computed at birth, and at any time afterward. Figure 5 shows our resulting computed eccentricity distributions at three stages in time.

This figure serves to illustrate the rapid coalescence of the vast majority of highly eccentric systems, which were produced in the greatest numbers when all possible bound orbital velocities were represented in our sample. This produces a rapidly evolving eccentricity distribution that changes on very short timescales ($0.01 \tau_{\text{circle}}$) and generally shifts toward lower eccentricities. (Note that the high-eccentricity end of the distribution never quite falls to zero, as there is a small lingering minority of systems in our model that have both high eccentricity and very large semimajor axis, giving them very long coalescence times. These are the systems located very near the boundary of our velocity space in Fig. 1.) While the initial post-supernova eccentricity distribution may be highly skewed, Figure 5 demonstrates that it does not retain its shape for very long, quickly flattening out to a more uniform distribution from which the observed eccentricity distribution of double neutron star systems could be more plausibly derived.

5. OBSERVED POPULATION

Here we examine the observed population of double neutron star binaries, particularly in terms of the eccentricity distribution. Figure 6 shows the semimajor axes and eccentricities of all currently known eccentric ($e > 0.05$) binary pulsar systems residing within the disk of our Galaxy. We have excluded pulsars in globular clusters, where the formation histories are quite different. Relativistic systems (those that will coalesce within a Hubble time) are indicated by dotted circles, and noncoalescing systems are represented by asterisks. Note the comparatively low eccentricities of the relativistic binaries. Pulsar information was obtained from the Australia Telescope National Facility Pulsar Catalog (Manchester et al. 2005).

The majority of relativistic binaries tend to have low eccentricities, which seems fortuitous considering the tiny size of the low-eccentricity bull's-eye region in velocity space as shown in Figure 1. To achieve such low eccentricities, these relativistic binary systems would have had to experience kicks from a very minor subset of the total range of nondisrupting supernova kicks—the remaining majority of nondisrupting kicks would naturally result in higher eccentricities. In other words, while the range of low-eccentricity kick velocities can seem large (especially in the isotropic case), the overall range of nondisrupting kick velocities is always much larger, and hence it seems reasonable to expect that low eccentricities should be rare.

Our analysis of uniformly distributed pulsar kicks finds that surviving binaries should create mostly highly eccentric systems. We contend that supernova kick velocities can easily be large enough in magnitude to kick a binary out of its small low-eccentricity bull's-eye and into the dominant, high-eccentricity region of velocity space (see Fig. 1). These large supernova kicks can be inferred from the abundance of single pulsars roaming the galaxy at high velocities (mean of the order of 400–500 km s$^{-1}$), most likely accelerated by supernova kicks. With regards to our simple kick model, it is almost certain that the real kick velocity distribution would not uniformly populate the available phase space after a supernova explosion, as the kicks in close binaries would have to range from small velocities to higher than 1000 km s$^{-1}$ with equal probability. Nevertheless, trying to guess the exact shape and magnitude of the pulsar kick velocity distribution leads to a plethora of models, and we resist this temptation. Our ignorance of the kick velocity distribution does not invalidate our results, unless pulsars can never receive a sufficiently large kick, of magnitude similar to that of the original pre-supernova orbital velocity. We are inclined to assume that such large kicks can exist, as observed pulsar velocities suggest kicks may be imparted with magnitudes up to and exceeding 1000 km s$^{-1}$ (Cordes et al. 1993).
We propose that the observed population of relativistic double neutron stars exhibits low eccentricities because systems with higher eccentricities have coalesced before they could be detected. An average pulsar is detectable by modern pulsar surveys for approximately 10 Myr; however, we have shown that binary systems have an appreciable likelihood of being kicked into coalescence within 1 Myr or even less. Not only will these systems coalesce rapidly, they will also have orbits so tight that the effects of “Doppler smearing” are too strong for current search algorithms to detect them in some surveys. In summary, the low eccentricities of the relativistic double neutron star systems suggest that a strong selection effect is in operation, and we claim that accelerated decay due to gravitational radiation is the cause.

Faulkner et al. (2005) suggest that there is a correlation reinforced by the discovery of PSR J1756–2251 between orbital eccentricity and the spin period of recycled pulsars. Our results suggest that many highly eccentric systems resulting from close binaries are never seen by pulsar surveys, and that long-period binaries favor high eccentricities. Close binaries transfer matter when the evolutionary timescale of the donor is still long. We therefore expect that short-period progenitors will be spun up to shorter periods than the longer orbital period systems. A natural consequence of this is that the short-period high-e systems are selected against, that very few long-period low-e systems are born, and therefore a “correlation” is observed.

6. DISCUSSION: IMPLICATIONS FOR THE MERGER RATE AND DOUBLE SUPERNOVAE

We have seen that the binary pulsar population is missing highly eccentric binary pulsars in relativistic orbits, which we attribute to their short observable lifetimes. The most reliable estimates of the merger rate of neutron star binaries come from extrapolation of the observed population, which ignores these missing binaries that we expect to be formed in large numbers before dying young. Our calculations suggest that factors of several are required to correct the merger rate for these missing binaries. We have also shown that there is a nonnegligible potential for coalescence times of the order of months if the pulsar kick is of the same order as the pre-supernova orbital velocity. This may mean that even relatively wide pre-supernova orbits can be driven to coalesce in remarkably short times by receiving an appropriately oriented retrograde kick. This phenomenon could reveal itself in the form of a supernova followed closely by a second similarly scaled catastrophic event (the coalescence), where both would be observed at the same position a few months or years apart—what might be described as a double supernova. How soon after the original burst of star formation in the universe might we expect to see such an event? Population III stars—the ancient stars that were the first to form in the history of the universe—may have often occurred in binaries where both stars underwent supernovae, subsequently taking part in an ultrarapid coalescence that could be detected through gravitational radiation emissions (the only known way of observing these objects, otherwise hidden by the epoch of reionization). This idea has been alluded to before (Belczynski et al. 2004); however, our findings emphasize the possibility that such a coalescence could occur very quickly after initial star formation, allowing future gravitational wave detectors to directly study events that occurred within the first few Myr of star formation in the universe.

Throughout this work we have assumed that supernova kicks can be large enough to be comparable to the pre-supernova orbital velocity. If a subset of supernovae produced little or no kick, and the mass loss was low, this could also provide a mechanism to create low-eccentricity binaries (Dewi et al. 2005). The next important phase of this debate will therefore be to determine how many binary pulsars are formed with low eccentricities, and how many high-eccentricity systems have coalesced due to gravitational radiation losses before we had a chance to observe them.

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