Quantum hyper-computing by means of evanescent photons

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Abstract. Nowadays quantum computation is probably considered as the most promising way to achieve superfast calculation performances to be implemented in the next generation of hyper-computers. Nevertheless, the realization of an hypercomputer, namely a system able to perform countable infinite computational steps within a finite time, is usually considered not allowed due to the Heisenberg uncertainty principle that limits the energy required by such steps. This limitation affects quantum algorithms as well as conventional computation. In this paper we show it is possible to bypass such limitation by considering a quantum system making use of evanescent photons produced by some physical processes such, for example, that occurring inside optical components made of metamaterials. This proposal opens very interesting prospects towards a deeper understanding of quantum information and computation as well as to the realization of an actual hypercomputer system and its applications to frontier fields such as Artificial Intelligence.

1. Introduction

Quantum computation makes direct use of quantum-mechanical features, such as states superposition and entanglement, to perform operations on data. Whereas conventional computation, implemented by digital computers, require data to be encoded into binary digits (a bit can have only one of two values, 0 or 1), quantum computation uses quantum bits or qubits (a quantum bit or qubit can exist in a superposition of two bits) to represent data and perform operations on these data. The idea of quantum computing was first introduced by Richard Feynman in 1982 \cite{1}.

A single qubit can then represent a one, a zero, or any quantum superposition of these two qubit states; moreover, a pair of qubits can be in any quantum superposition of 4 states, and three qubits in any superposition of 8. In general, a quantum computer with \( n \) qubits can be in an arbitrary superposition of up to \( 2^n \) different states simultaneously, while a normal computer can be in only one of these \( 2^n \) states at any one time \cite{2}.

In particular, a generic quantum state can be considered as the quantum superposition of two basic states \( |0\rangle \) and \( |1\rangle \) (eigenvectors of some suitable quantum observable), namely

\[
|\psi\rangle = \cos \frac{\phi}{2} |0\rangle + e^{i\theta} \sin \frac{\phi}{2} |1\rangle
\]

(1)

where \( \phi \) and \( \theta \) are the angles that identify the “direction” of the state vector \( |\psi\rangle \) in polar coordinates.

A quantum computer operates by setting up the qubits in a controlled initial state that represents the problem at hand and by manipulating them with a fixed sequence of quantum logic gates. The
sequence of gates to be applied is called a quantum algorithm. The calculation ends with measurement of all the states, collapsing each qubit into one of the two pure states, so the outcome can be at most represented by \( n \) classical bits of information. An example of an implementation of qubits for a quantum computer could start with the use of particles with two spin states: “up” and “down”. In general every quantum system characterized by at least one physical observable \( O \) which is time-conserved and have two sufficiently spaced eigenvalues can be can be mapped onto an effective spin-1/2 system.

A quantum computer with a given number of qubits is fundamentally different from a classical computer composed of the same number of classical bits. For example, to represent the state of an \( n \) - qubit system on a classical computer would require the storage of \( 2^n \) complex coefficients. Although this fact may seem to indicate that qubits can hold exponentially more information than their classical counterparts, care must be taken not to overlook the fact that the qubits are only in a probabilistic superposition of all of their states. This means that when the final state of the qubits is measured, they will only be found in one of the possible configurations they were in before measurement. Moreover, it is incorrect to think of the qubits as only being in one particular state before measurement, since the fact that they were in a superposition of states before the measurement was made directly affects the possible outcomes of the computation [3].

If we consider, for example, a three-bit register device, its state at any time is described by a probability distribution \( P(S) \) (where \( S \) indicates a given bit-register state) over the \( 2^3 = 8 \) different three-bit strings 000, 001, 010, 011, 100, 101, 110, 111. For a deterministic classical digital computer, this distribution is such as, at every instant, we can have \( P(S_i) = 1 \) for a given value of \( i \in \{2^3\} \), and \( P(S_j) = 0 \) for \( j \neq i \) so that at every instant it is in a certain definite state. On the contrary if we consider it as to be a probabilistic quantum computer, then there is a possibility of it being in any one of a number of different states. We can describe this probabilistic state \( |\Psi\rangle \) by eight nonnegative numbers \( P(S_i) \) namely

\[
|\Psi\rangle = P(S_i)|0\rangle|0\rangle|0\rangle \pm B|0\rangle|0\rangle|1\rangle + C|0\rangle|1\rangle|0\rangle \pm D|0\rangle|1\rangle|1\rangle \pm E|1\rangle|0\rangle|0\rangle \pm F|1\rangle|0\rangle|1\rangle + G|1\rangle|1\rangle|0\rangle \pm H|1\rangle|1\rangle|1\rangle
\]

with the constraint \( \sum_i P(S_i) = 1 \).

Sometimes ago, Deutsch [4] proposed the idea of a quantum Turing machine in which quantum computation parallelism would be realized through superposition principle of quantum mechanics basing on which such theoretical device could be able to process many inputs simultaneously so performing a speeded-up calculation. So, he suggested such quantum Turing machines could be able to perform certain type of calculation very faster than classical computer.

Shor [5] firstly and Grover [6] later shown there are some specific types of algorithms that quantum computers are certainly able to solve better than classical ones such, for example, prime number factorization and database search, problems of great importance in the fields of computer science and cryptography respectively. Nowadays, one of the most promising and interesting research field is represented by the possible application of the quantum computing schemes to the develop of Artificial Intelligence (AI).

AI has two main goals: the first one, more general, is to understand the features and dynamics of intelligence (also by analyzing the human and animal behavior), the second one, more specific, is to develop and build intelligent machines. In both the cases it is fundamental to design and perform very fast and powerful calculation techniques, so that today computation plays a crucial role in the field of AI. Due to the recent developments of quantum computational techniques, it is generally believed that such field, as it is, could be able to give to AI the computational support needed to allow it to develop fast.
Nevertheless, at the moment, it is very difficult to realize and implement quantum parallel algorithms for a large part of scopes as, for example, the application to Artificial Intelligence, that are more useful than the corresponding classical counterparts.

This is also due to the known difficulties related to quantum decoherence that affects quantum systems in a superposition state. A possible solution to this problem could be, in principle, to increase computational speed without necessarily considering quantum superposition of states.

It is commonly believed that theoretical limitations arising from fundamental physics will broke Moore’s law about microprocessor performances in the near future. In fact, according to Feynman’s analysis [7], the computational speed is limited by the minimum amount of energy required to transport a bit of information, in an irreversible way, between two devices. This limitation also affects quantum computation in which computational speed as a function of required energy is limited by energy dissipation during the process. A possible way to overcome this limit could be to realize a so-called “accelerated Turing machines”, also known as “Zeno machine” (ZM), that is a hypothetical computational device able to perform a countable infinite number of computational steps within a finite time interval.

A ZM can be substantially thought like a Turing machine (TM) whose calculation steps are in a certain sense identical except for the time taken for their execution. It has been shown this kind of speeded-up calculation can solve the famous “halting problem” (HP) for the TM in a finite interval of time.

Conceptual schemes for hypercomputation are somehow based on performing infinitely many computational tasks in a finite time. Without a loss of generality, we can consider a ZM to be identical to a TM with its input, output and storage tapes, executing the first step in $\frac{1}{2}$ hour, the second in a $\frac{1}{4}$ hour, the third in a 1/8 hour and so on. An important question related to hypercomputation is the existence of a physical process able to perform such type of calculation and solve the HP such, for example, a quantum process or a relativistic machine. In fact, whatever be the notion of “computability” we assume, computation is, in principle, a physically achievable process.

The concept of a ZM is allegedly just in contrast with the Heisenberg uncertainty principle since the energy required to perform a calculation exponentially increase as the computational step is accelerated. For this reason, the ZM has been generally considered, so far, just like a mere mathematical concept unable to be practically realized.

The realization of hypercomputing quantum systems, not suffering from the limitations above described, could solve some of the main current theoretical and practical issues related to the application of quantum computing schemes to AI, like for example, the realization of quantum neural networks.

In this paper, on the contrary, we analyze and propose the theoretical feasibility of a device like a ZM, by considering the evanescent photons of electromagnetic field produced inside physical systems made of metamaterials (MTMs) as the result of coherent dynamics of Quantum Vacuum.

In the physical realization of quantum computing, namely quantum computers, qubits are made up of controlled particles and the related means of control (e.g. devices that trap particles and switch them from one state to another), we here propose the realization of quantum computational systems in which traditional circuits are then replaced with optical ones.

We show such new kind of computers, based on the use of evanescent photons and MTMs technology, could be able to perform accelerated computation whose features could be interesting in the light of a deeper understanding of quantum computation itself as well as its application to frontier fields like AI and many other related fields.

2. The “evanescent” electromagnetic field and its interpretation as tachyon field

Total reflection of electromagnetic waves is well known. It occurs when an electromagnetic wave propagates within a medium of refraction index $n_1$ and reflects on a second medium characterized by a refraction index $n_2 < n_1$ at an angle $\theta_i > \theta_c$ ( $\theta_c$ being the so-called “critical angle”). In this case we
should have, according to Snell law, that all the incident energy is reflected within the first medium. Nevertheless, as experimentally shown, an electromagnetic perturbation, whose propagation is restricted in the neighborhood of the separating surface between two media, is still present in the second medium and is known as “evanescent” wave (EW).

If we consider a reference system where the surface of separation of the two media is in the $xy$ -plane and the incidence plane is in the $xz$ -plane, by indicating with $\vec{k}^T$ the wave vector associated to the transmitted wave, we can write $\vec{k}^T = k_x \vec{u}_x + k_z \vec{u}_z$, where the vector components are given by (we assume $c = 1$):

\begin{align}
  k_x &= \omega n_x \sin \theta_x = \omega n_x \sin \theta_1 \\
  k^y &= 0 \\
  k_z &= \sqrt{n_2^2 - n_x^2 \sin^2 \theta_1}
\end{align}

We see the structure of “transmitted” wave will depend on the sign of the argument in the root of equation (5). In particular, if $0 \leq \sin \theta_x \leq \left( \frac{n_x}{n_1} \right)$, $k^z \in \mathbb{R}$, otherwise, when $n_x \sin \theta_x > n_2$, it is a purely imaginary number and we have an EW transmitted in the second medium. In the particular case $n_2 = 1$, we have $k^z > \omega$ and

\begin{equation}
  k^z = i \omega \sqrt{n_x^2 \sin^2 \theta_1 - 1} \equiv i \vec{k}
\end{equation}

It is interesting to observe that, in this case, the scalar product $\vec{k}^T \cdot \vec{k}^T = \omega^2$, as it always occurs in the case of propagating electromagnetic waves, while the norm $\| \vec{k}^T \| > c$.

This is due to the “anomalous” value assumed by some components of $\vec{k}^T$ (in this case $k^z$) whose value is larger than $\omega$, whereas this could not be the case for a “standard” homogeneous propagating e.m. wave. Furthermore, the presence of an imaginary component causes the EW to have a progressive behavior along the z-axis, characterized by exponentially decreasing amplitude, namely, for the transmitted electric field

\begin{equation}
  \vec{E}(x,z,t) = \vec{E}^T \exp[-i \omega t] \exp[i (k^x x + \omega t)]
\end{equation}

where $\vec{E}^T$ is the amplitude of the transmitted field. This exponentially decreasing behavior makes the wave practically detectable within a limited distance from the propagation point so we can define a “propagation depth” $\delta$ of EW as the distance where its amplitude is reduced to the $\vec{E}^T / e$ of its initial value, namely

\begin{equation}
  \delta = \frac{1}{k^z} = \frac{\lambda}{2\pi \sqrt{n_x^2 \sin^2 \theta_1 - 1}}
\end{equation}

$\lambda$ being the wavelength of the incident wave.

If a third medium (denser than the second) is placed after the second one at a distance $d$ from the first interface, the wave vector becomes real again in this medium and the total reflection amplitude is reduced, giving rise to a transmitted component that “escapes” into the last medium and its amplitude decreases exponentially with $d$ (“Frustrated Total Internal Reflection”).

More generally, inside a material medium the propagation of e.m. waves is described by the Maxwell equation

\begin{equation}
  n^2 \frac{\partial^2 \phi}{\partial t^2} - \nabla^2 \phi(\vec{x},t) = 0
\end{equation}
where \( \phi_i(\vec{x},t) \) is a scalar component of electric or magnetic field and \( n = \sqrt{\varepsilon \mu} \) is the refraction index of the medium (equal to 1 in vacuum).

For a periodic motion we have

\[
\phi_i(\vec{x},t) = \phi_i(\vec{x},0)e^{i\omega t}
\]

where \( \phi_i \) is the amplitude of the i-th scalar component of the field. Using equation (10) in equation (9), the latter becomes the Helmholtz equation

\[
\nabla^2 \phi_i(\vec{x},t) + n^2 \omega^2 \phi_i(\vec{x},t) = 0
\]

that can be solved assuming the plane waves solution

\[
\phi_i(\vec{x}) = \bar{\phi}_i e^{-i\vec{k}\cdot\vec{x}}
\]

that, substituted in equation (11), gives a relation between \( k \) and \( n \)

\[
k^2 = n^2 \omega^2
\]

We note the (one-dimension) stationary Schrödinger equation (we assume, in the following, \( \hbar = 1 \))

\[
-\frac{1}{2m} \frac{\partial^2}{\partial x^2} + V(x) \psi(x) = E \psi(x)
\]

is just in the form of the Helmholtz equation namely

\[
\frac{\partial^2 \psi}{\partial x^2} + 2m \left( E - V \right) \psi = 0
\]

and using again the plane wave assumption (12) for the wave function \( \psi \), we have

\[
k^2 = 2m \left( E - V \right)
\]

From equation (13) we see that if \( n \) assumes an imaginary value, the wave number \( k \) becomes imaginary too and analogously, from equation (16), if \( E < V \) the wave numbers in quantum packet are imaginary as well. We remember that the condition \( E < V \) is just what required for the tunneling effect to occur to a quantum particle “confined” by a potential barrier of “depth” \( V \).

We then notice the refraction index in optics plays the same role of the barrier potential in the quantum-tunneling phenomenon so suggesting the idea that evanescent waves could be actually tunneling modes of e.m. waves. The spreading of e.m. waves across “impenetrable” opaque optical barriers is then analogous to the phenomenon of quantum tunneling experienced by particles through a potential barrier.

The tunneling of evanescent modes of e.m. waves are currently interpreted, from a quantum viewpoint, as the tunneling of virtual photons \([8]\) through the corresponding potential barrier. As already pointed out \([3]\), such tunneling photons associated to evanescent field can be characterized by a superluminal group velocity or, according to the commonly accepted picture, by a negative square mass of the photons belonging to it.

This important result about the group velocity of evanescent e.m. waves can be easily verified within the Fresnel’s theory that predicts the \( k^y \) component of the propagation vector \( \vec{k} \) normal to the reflection plane is imaginary while the component \( k^x \) parallel to the incident plane is larger than \( \omega \). In fact, if we consider the energy – momentum relation

\[
\left( k^y \right)^2 + \left( k^x \right)^2 = \omega^2
\]

and its global variation

\[
k^x \delta k^x + k^y \delta k^y = \omega \delta \omega
\]

so we have, according to the Rayleigh’s formula for the group velocity of EW wave

\[
v^i = \frac{k^i}{\omega}, \quad i = x, y
\]
When $k^r$ is real, as for ordinary progressive waves, the (19) reduce to the de Broglie relations, while, when $k^r > 0$ and $k^\lambda$ is imaginary, as in the case of EW, we have $\psi^\lambda$ imaginary and
\[ \psi^\lambda > 1 \] (20)
since $k^r > \omega$ for EW, namely evanescent photons are “tachyon – like” quantum particles, characterized by a faster than light group velocity.

The assumption about imaginary mass (or, equivalently, imaginary velocity within the Fresnel theory) of superluminal particles is no justified by any physical hypothesis, and has important adverse consequences since the imaginary mass is a not measurable quantity. This critical issue is obviously also present in the quantum picture of evanescent field within which evanescence modes are considered as virtual particles.

Nevertheless, we have already shown [9] how the analysis of the mathematical group $SO(4,C)$ of rotation in the complex plane allows us to deduce from it, using symmetry properties only, two complete orthochrones isomorphic Lorentz groups named $\tilde{L}_s^r$ and $\tilde{L}_s^\lambda$, which respectively correspond to subluminal and superluminal set of coordinates transformations (respectively indicated by $x^\mu$ and $X^\nu$), characterized by real metrics of signature $(-+++)$ and $(+---)$.

The properties of $\tilde{L}_s^r$ allow us to define the tachyonic referential frames (TRF) characterized by the real metric
\[ ds^2 = G_{\mu\nu}dx^\mu dx^\nu \quad \mu, \nu = 1, 2, 3, 4 \] (21)
where
\[ \left\{ G_{\mu\nu} \right\} = \begin{pmatrix}
-1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1
\end{pmatrix} \] (22)
On this basis we can also define a “tachyonic matter” whose properties originate from the metric (21) so that for such a tachyon we have [9]
\[ \tilde{p}^2 - \tilde{E}^2 = \mu^2 \] (23)
where the superscript indicates the quantities referred to the tachyonic reference system (TRF) and the tachyon rest mass $\mu$ has a real physically measurable value, even within an ordinary reference frame (ORF) in which the “usual” Lorentz metric holds, through a momentum measurement [9].

It is very important to stress equation (23) is not just the result of inserting an imaginary mass inside the usual Einstein energy-momentum relation but the consequence of adopting the space-like metric given by equation (21) [9].

Following this model, a space-like relativistic kinematics and dynamics have been developed and the energy-momentum relation as well as the expressions of the four-velocity and four-force have been calculated for tachyons of real rest mass [9].

Furthermore, the energy-momentum equation (23) has been used [9] to write a Klein-Gordon like field equation for a spinless tachyonic quantum wave-function $\tilde{\Psi}(X_1, X_2, X_3, X_4 = i\tilde{T})$, namely
\[ \left[ -\frac{\partial^2}{\partial \tilde{t}^2} + \frac{\partial^2}{\partial X_1^2} + \frac{\partial^2}{\partial X_2^2} + \frac{\partial^2}{\partial X_3^2} \right] \tilde{\Psi}(X_\mu) = -\xi^2 \tilde{\Psi}(X_\mu) \] (24)
where $\xi^2 = \mu^2$. The existence of such tachyonic field is also compatible with the principle of relativity as well as with that of macroscopic causality [9].

As we’ll discuss in the following, such tachyonic properties of evanescent photons or, more correctly, their being tachyons, allow us to propose their use for the realization of a new kind of
hypercomputation process suitable to be realized through a new kind of quantum computer system. In the following we’ll refer to evanescent photons also as tachyons of rest mass $\mu$ [9].

3. Evanescent photons in condensed matter

3.1. Preliminary considerations

In several previous works we have shown [10,11] that, under suitable boundary conditions, the quantum vacuum fluctuations are able to couple so strongly with a matter system, through its proper resonances, to induce the system to “runaway”, through a “Superradiant Phase Transition” (SPT), from the perturbative ground state, characterized by the quantum zero point oscillations of e.m. field and matter, towards a more stable (true) ground state, named the Coherent Ground State (CGS), in which both the e.m. and matter field oscillate in phase with each other at a common frequency $\omega_{coh}$. The resulting coherent state is characterized by a collective common behavior of the quantized e.m and matter fields appearing as a macroscopic quantum object in which atoms and molecules lose its individuality to become part of a whole electromagnetic field + matter entangled system, similar, in many regards, to that characterizing a Bose-Einstein Condensate (BEC).

In particular, one of the most remarkable consequences of the coherent phase transition is the formation, inside the macroscopic quantum coherent e.m. + matter system, of the so-called “Coherence Domains” (CDs), namely the smallest spatial regions in which the coherent evolution of the e.m. + matter field takes place, where the coherent dynamics determines an extended oscillating polarization field able to correlate a high number of elementary matter electric dipoles so generating stable and ordered structures in macroscopic spatial regions as the CDs.

The e.m. field coherently interacting with the matter field inside CDs shows a very special feature [10,11] namely the frequency $\omega_{coh}$ of photons (superradiant photons) belonging to it is lower than the frequency $\omega_0$ of free electromagnetic field originally interacting with matter system.

For these photons the Einstein – De Broglie relation is just the equation (23), so they can be considered as tachyons described by the space-like metric (21) in a TRF and having a real rest mass (see section 2).

The tachyon rest mass $\mu$ depends on the features of the matter system coherently interacting with the e.m. field “condensed” from quantum vacuum, namely [9]

$$\mu = \left(\frac{\sin 2\xi}{a}\right)^2 \left(\frac{2\pi}{3} \frac{N}{V} e^{\omega_{coh}}\right)^{\frac{1}{2}}, \quad (25)$$

where $N/V$ is the density, $e$ the electron charge, $a$ and $\xi$ two real parameters describing the CGS of the given system.

The condition $\omega_{coh} < \omega_0$ [3,10,11] also means the coherent e.m. field is “trapped” inside CDs, except for an evanescent tail of e.m. field escaping from the CD’s boundaries.

A very important feature of the coherent e.m. field so generated inside condensed matter concerns its high stability with respect to quantum and thermal fluctuations that characterize the PGS of the non-interacting matter and e.m field and that make the system to decohere [3,10-13]. Furthermore, in many systems [3,10-13] this stability is expected to persist also at room temperature as long as the coherent fraction of system components is still sufficiently high. If such conditions are satisfied, some coherent field quanta (tachyons) can be “extracted” from CDs without the system coming out of the coherent behavior.

The possibility to use such type of evanescent photons to realize a quantum hypercomputing system will be discussed in the following.
4. Quantum computation by means of evanescent photons

4.1. Preliminary considerations

Large-scale quantum computers will be able to solve certain problems much more quickly than any classical computer using the best currently known algorithms, like integer factorization using Shor’s algorithm or the simulation of quantum many-body systems. There exist quantum algorithms, such as Simon’s algorithm, which run faster than any possible probabilistic classical algorithm [1]. Given sufficient computational resources, a classical computer could be built to simulate any quantum algorithm although quantum computation does not violate the Church–Turing thesis.

Nevertheless, the quantization of evanescent field and its features allow us to consider for its quanta, namely for evanescent photons, the possibility they undergo, like “ordinary” photons, to tunnel effect. The idea of using tunneling photons to realize signal-processing devices has been already tested [14] by considering the transmission of light through nanometer-scale pinholes in a gold film covered by a nonlinear dielectric, which revealed that transmittance of a nanopore or nanopore array at one wavelength could be controlled by illumination with a second, different, wavelength.

This opens the door to optical signal processing devices, such as all-optical switches realized on a microscopic scale which can manipulate single electrons, atoms or photons. If the atoms can be localized at distances smaller than the radiation wavelength, they can be coherently coupled by photons and an entangling quantum logic gate can be realized.

By applying this technology, the computer gate consisting of a large number of small, interconnected electromagnetic ion traps created for both memory and logical processing by manipulating atoms with the tunneling photon, may be realized. The possibility of nano-switching by using evanescent photons in optical near-field was also proposed by T. Kawazoe and co-workers [15].

By utilizing this technology, it is considered that the computation using quantum tunneling photons has the possibility to achieve much faster computational speed, and the influence of energy cost due to the uncertainty principle which prevents speeding up the quantum computation can be overcome as suggested in following.

In particular, as we have seen, EW tunneling photons have superluminal group velocity and in this paper we discuss the idea of a computer system consisting of quantum gates that use quantum tunneling photons to perform logical operations.

Benioff showed that the computation speed was close to the limit imposed by the time-energy uncertainty principle [16]. Margolus and Levitin extended this result to a system with an averaged energy $\langle E \rangle$, which takes a time interval at least equal to $\langle E \rangle = \pi/(2\Delta t)$ to perform logical operations [17].

Summing over all logical gates of operations, the total number of logic operations per second is no more than

$$\Delta t \approx \frac{\pi N}{2 \langle E \rangle}$$

where $\Delta t$ is an operational time of an elementary logical operation and $N$ is the number of consisting gates of the computer. From equation (26) the energy spread for the quantum tunneling photon (abbreviated QTP, hereafter) gate becomes $\frac{1}{1/\beta(\beta-1)}$ times the energy spread for the logical gate using particles moving at sub-luminal speed including photons.

Then the total number of logic operations per second for QTP gates can be given by

$$\Delta t \approx \frac{N}{\beta(\beta-1)} \frac{\pi}{2 \langle E_\alpha \rangle}$$

where $\langle E_\alpha \rangle$ is an averaged energy for QTP gates. The uncertainty in the momentum of tunneling photons moving at the superluminal speed can be given by
where $\mu$ is an absolute value of the mass for the tunneling photon moving at superluminal speed and $\omega$ is an angular frequency of the photon. The velocity of the tunneling photon can be then estimated as

$$v \approx \left(1 + \frac{1}{\sqrt{\omega \Delta t}}\right)$$

(29)

from the energy and the momentum for a tachyonic particle of rest mass $\mu$, respectively given by

$$E = \frac{\mu}{\sqrt{v^2 - 1}}$$

(30)

$$p = \frac{\mu \omega}{\sqrt{v^2 - 1}}$$

(31)

where $E = \omega$ [9].

If we denote the tunneling distance (size of the barrier) by $d$, the time for a photon tunneling across the barrier can be roughly estimated to be $\Delta t = d/v$, then the velocity of the tunneling photon can be given by

$$v \approx \left(1 + \frac{1}{2 \omega d} + \frac{1}{\sqrt{\omega d} + 1} \frac{1}{4 \omega^2 d^2} \right)$$

(32)

The ratio of the minimum energy required for computation by QTP gates and the conventional computation can be then given, by equating equation (26) and equation (27), as

$$R = \frac{<E_\text{c}>}{<E_\text{c}>} \approx \frac{1}{\beta (\beta - 1)}$$

(33)

where

$$\beta \approx 1 + \frac{1}{2 \omega d} + \frac{1}{\sqrt{\omega d} + 1} \frac{1}{4 \omega^2 d^2}$$

(34)

The tunneling distance depends on the physical process involved in the generation of evanescent photons. For example, it has been estimated [3] that if the wavelength of the tunneling photon is in the far infrared region ($\lambda \sim 10^2 \text{nm}$), the energy cost of computation for the computer, which consists of QTP gates, reaches to $10^{-6}$ times smaller than that of conventional computer systems.

In recent years, many studies on the quantum computation were conducted and it was recognized that the computational speed of quantum computing was much higher than that of conventional silicon processors. But the energy cost due to the uncertainty principle which prevents speeding-up of the quantum computation was not considered.

From the theoretical analysis of the energy limit of the quantum computer system which utilizes tunneling photons, it can be shown the energy loss of computation by utilizing superluminal tunneling photons is much lower than that of conventional silicon processors. Moreover, this superluminal effect can eventually speed up computers significantly because it can compensate interconnect delays inside logic gates, which can never be fully eliminated from any real electronic components, and bringing overall transmission rate closer to the ultimate speed limit [18], which actually boost the speed of a signal traveling on an electromagnetic wave for achieving high performance computers.

The problem for conducting quantum computation is related to the decoherence of quantum states. The qubit calculations of the quantum computer are performed while the quantum wave function is in a state of superposition between states, which is what allows it to perform the calculations using both 0 & 1 states simultaneously.

But this coherent state is generally very fragile and difficult to maintain. The slightest interaction with the external world would cause the system to decohere. This is the problem of decoherence,
which is a stumbling block for quantum computation. Therefore the computer has to maintain the coherent state for making calculations. If we let $T$ to be the relaxation time of a single qubit and $\Delta t$ is the operation time of a single logical gate, the figure of merit $\gamma$ for computation can be defined as $\gamma = t/\Delta t$ [19], which is on the order of the number of qubits times the number of gate operations. As a superposition state of the $L$-qubits system would cause decoherence approximately $2^L$ times faster than a superposition state of one qubit [20], then the relaxation time of the $L$-qubits system can be roughly estimated to be $2^{-L}$ times the relaxation time of a single qubit computation.

The minimum energy required to perform quantum computation for the $L$-qubits system can be then given by

$$E_0 \approx \frac{\nu_{\text{g}L}}{T} 2^L$$  \hspace{1cm} (35)$$

where $\nu_{\text{g}}$ is the number of gate operations.

Similar to this equation, the minimum energy required to perform quantum computation utilizing superluminal particle can be estimated as [21]

$$E'_0 \approx \frac{\nu_{\text{g}L}}{\beta(\beta - 1)T} 2^L$$  \hspace{1cm} (36)$$

Supposing that $E_0 = E'_0$, an increase of qubit size to perform computation by superluminal evanescent photon compared with the conventional computation can be given by

$$\Delta L \approx \frac{\log_2[\beta(\beta - 1)]}{1 + 1 / L \log 2}$$  \hspace{1cm} (37)$$

when satisfying $\Delta L < L$.

From which, we can estimate a meaningful increase of qubit size of the quantum computation utilizing superluminal evanescent photons compared with the conventional computer system.

Nevertheless, as we have seen, one of the most important problem remains the decoherence of the quantum system used to implement calculations. Considering evanescent photons can circumvent this important limitation.

### 4.2. Decoherence time in quantum computation with evanescent photons

If we consider a quantum computer system utilizing evanescent photons (tachyons), each computational gate could be entangled with each other, according to the property of non-locality, to exert influences outside the extent of the processor. This is due to the quantum nature of the tachyon field generated inside CD and to its evanescent tail “escaping” from it and capable to interact with similar fields associated to the neighbourhood CDs.

According to Zurek, the density matrix $\rho(x,x')$ of the particle in the position representation evolves by the master equation [22]

$$\dot{\rho} = -i[H,\rho] - \gamma(x - x') \left[\frac{\partial}{\partial x} - \frac{\partial}{\partial x'}\right] \rho - 2m\gamma k_B T(x - x')^2 \rho$$  \hspace{1cm} (38)$$

where $H$ is the particle Hamiltonian, $\gamma$ is the relaxation rate, $k_B$ is the Boltzmann constant, $T$ the temperature and $m$ the mass of the field. He has also shown the off-diagonal peaks of the density matrix, $\rho(x,x') = \phi(x)\phi^*(x')$ will decay at the rate ($\Delta x \simeq x' - x$)

$$\frac{d}{dt}(\rho^{-}) \approx \frac{2m\gamma k_B T}{(\Delta x)^2} \rho^{-} = \frac{\rho^{-}}{\tau_D}$$  \hspace{1cm} (39)$$

from which, quantum coherence will disappear on a time scale given by
Thus the decoherence time in a quantum processor utilizing evanescent photons is obtained as
\[ \tau_D \approx \tau_R \left( \frac{1}{\Delta \sqrt{2mk_BT}} \right)^2 \]  
\[ \tau_D \approx \tau_R \left( \frac{1}{\Delta \sqrt{2mk_BT}} \right)^2 \left[ \beta(\beta - 1) \right]^2 \]  
where \( \beta \) is given by equation (34).

If the error rate is small enough, it is thought to be possible to use quantum error correction, which eliminates the errors due to decoherence, thereby allowing the total calculation time to be longer than the decoherence time.

5. Hypercomputation by evanescent photons

Hypercomputation, which can complete infinitely many steps of computation, refers to models of computation that go beyond, or are incomparable to, Turing computability. The term "hypercomputation" was introduced in 1999 by Jack Copeland and Diane Proudfoot [23]. This includes various hypothetical methods for the computation of non-Turing-computable functions.

The Church–Turing thesis states that any function that is algorithmically computable can be computed by a Turing machine. Hypercomputers compute functions that a Turing machine cannot and which are, hence, not computable in the Church-Turing sense.

Feynman defined the required energy per step of computation as [1]
\[ \text{energy per step} = k_B T \frac{f - b}{(f + b)/2} \]  
where \( k_B \) is Boltzmann’s constant, \( T \) is a temperature, \( f \) is a forward rate of computation and \( b \) is backward rate. Supposing there in no energy supply and the parameters \( f \) and \( b \) are fixed during the computation, we can consider the infinite computational steps given by
\[ E_1 = kE_0, E_2 = kE_1, \ldots, E_n = kE_{n-1}, \ldots \]  
where we let the initial energy of computation to be \( E_0 = k_0 T \), \( k = 2(f - b)/(f + b) \), and \( E_n \) is the energy for the n-th computational step. From the above relations we have \( E_n = k^n E_0 \), and then the energy loss for each computational step becomes
\[ \Delta E_1 = E_0 - E_1 = (1 - k)E_0 \]  
\[ \Delta E_2 = E_1 - E_2 = (1 - k)kE_0 \]  
\[ \vdots \]  
\[ \Delta E_n = E_{n-1} - E_n = (1 - k)k^{n-1}E_0 \]  
According to Lloyd [24], a quantum system with average energy \( \Delta E \) requires at least a time \( \Delta t \) to evolve to an orthogonal state given by
\[ \Delta t = \frac{\pi}{2\Delta E} \]  
from which, if setting \( E = \Delta E_1 \) into equation (45), the total energy required to perform the infinite steps is equal to \( E_0 \). The total time needed for the computation with infinite steps then becomes [3]
\[ T = \sum_{n=1}^{\infty} \Delta t_n = \frac{\pi}{2E_0} \sum_{n=1}^{\infty} \frac{1}{(1 - k)k^{n-1}} \]  
As the infinite sum (46) diverges to infinity when \( 0 < k < 1 \), the Feynman model of computation requires infinite time to complete the calculation.
Hence an accelerated Turing machine cannot be realized for computers utilizing ordinary particles due to the constraint imposed by the time–energy uncertainty principle.

If we consider the relativistic expressions of energy and momentum of a tachyon of rest mass $\mu$ and velocity $v$

$$E = \frac{m_0}{\sqrt{1 - v^2}} \quad (47)$$

and

$$p = \frac{m_0 v}{\sqrt{1 - v^2}} \quad (48)$$

the relation between energy and momentum can be written as

$$E = \frac{p}{v} \quad (49)$$

from which, we have, after differentiating

$$\frac{v \Delta p - p \Delta v}{v^2} = \Delta E \quad (50)$$

Supposing we can assume $\Delta v / v^2 \approx 0$, equation (50) can be simplified as

$$\Delta p \approx v \Delta E \quad (51)$$

According to Park [25], the uncertainty relation for a superluminal particle can be written as

$$\Delta p \cdot \Delta t \approx \frac{1}{v - v'} \quad (52)$$

where $v$ and $v'$ respectively indicate the velocities of the particle after and before their measurements. By substituting equation (51) into equation (52), we obtain the uncertainty relation for superluminal particles, namely

$$\Delta E \cdot \Delta t \approx \frac{1}{\beta(\beta - 1)} \quad (53)$$

when we let $v' = 1$ and $\beta = v$. If we then consider superluminal particles, the time required for the quantum system to perform the computation becomes, from the uncertainty principle for superluminal particles given by equation (51) [16]

$$T = \sum_{n=1}^{\infty} \Delta t_i = \frac{\pi}{2E_0} \sum_{n=1}^{\infty} \frac{1}{\beta_n(\beta_n - 1)(1 - k)k^{n-1}} \quad (54)$$

where $\beta$ is given by

$$\beta_n = \sqrt{1 + \frac{\mu^2}{E_n^2}} = \sqrt{1 + \frac{\mu^2}{k^{2n}E_0^2}} \quad (55)$$

From equation (54) and equation (55), we calculate the computation time can be accelerated (figure 1).
Figure 1. Time required to conduct computation at each step by using "tachyon-like" particles ($k = 1/2$).

Numerical calculations have shown that the infinite sum given by equation (54) converges to a certain value when $0 < k < 1$, as shown in figure 2, in which the horizontal line represents the parameter $\gamma = \mu / E_0$, and the vertical line the time to complete infinite step of calculations.

Figure 2. Computational time required when using "tachyon-like" particles.

From these calculation results, it is clear that an accelerated TM can be in principle realized by using, within the Feynman computational model, superluminal particles instead of subluminal ones. Thus, contrary to the conclusion currently drawn from Feynman model of computation related to ordinary particles, we have shown the use of superluminal particles can overcome the theoretical questions posed by such model, so allowing the realization of an accelerated TM.

6. A novel computational architecture for quantum hypercomputation

Basing on the assumption that the evanescent photon is a measurable and experimentally accessible superluminal particle of real rest mass [9], it is possible to show [26] a tubule structure constructed by MTMs can achieve quantum bit (qubit) computations on large data sets, which would account for the high performance of the computations and very low energy consumption if compared with the conventional silicon processors.

It therefore seems highly plausible that macroscopic quantum ordered dynamical systems of evanescent photons in the processor could play an essential role in realizing long-range coherence in a computing system.

As already shown by Ziolkowski superluminal pulse propagation, which permits consequent superluminal exchange without a violation of macroscopic causality, is possible in electromagnetic MTMs [27].

These MTMs achieve desired effects by incorporating structural elements of sub-wavelength sizes, i.e. features that are actually smaller than the wavelength of the waves, giving rise to a negative refractive index. Such materials then allow the creation of "super lenses" that can have a spatial resolution below that of the incident wavelength.

If a material medium possesses a negative refractive index, the generation of evanescent photons is enhanced, and these propagate without loss according to the properties of a MTM [28].

Since the fundamental building block of modern electronic computers is the transistor, the possibility to implement the above process in order to realize a quantum super-computer would require to replace electronic components with optical ones, namely the realization of an equivalent optical transistor.
This could be achieved by using materials characterized by a non-linear refractive index. In particular, we know about some materials for which the intensity of incoming light affects the intensity of the transmitted one through the material in a similar manner to the voltage response of an electronic transistor. Such an "optical transistor" [29] can be used to create optical logic gates, which in turn are assembled into the higher-level components of the computer's CPU. These will be nonlinear crystals used to manipulate light beams in order to control other ones.

These devices make possible to implement the so-called “photonic logic”, namely the use of photons (light) in logic gates such as NOT, AND, OR, NAND, and etc. in which the switching is obtained by using nonlinear optical effects when two or more signals are combined.

The resonators, like those composing MTMs, can be particularly useful in photonic logic, since they allow a build-up of energy from constructive interference, thus enhancing optical nonlinear effects by using semiconductors inside the sub-structure of an MTM.

The conceptual scheme of superluminal computation so far emerged is then shown in Fig. 3 and it is composed by three elements: 1) an evanescent photon generator, 2) a quantum processor and 3) an holographic memory.

![Fig. 3. Schematic diagram of superluminal computing.](image)

The quantum processor and the holographic memory can be implemented by the substructures of MTMs when assembled according to a tube-like structure. An example of an evanescent photon field “generator” is shown in Fig. 4.

![Fig. 4. Structure of evanescent photon generator.](image)
It can be realized through tube-like structures, by coupling two or more electromagnetic elements, such as optical waveguides, close together so that the evanescent field generated by one element does not decay too much before it reaches the next one.

By means of these waveguides, the evanescent field gives rise to propagating-wave modes, thereby connecting the wave from one waveguide to the next. Mathematically, the process is the same as that of quantum tunneling.

7. Outlook and further considerations

It is known that an accelerate Turing machines allow us to “compute” some functions which are not Turing-computable like, for example, the HP [30], described as: “given a description of an arbitrary computer program, decide whether the program finishes running or continues to run forever”.

A Turing machine then cannot decide if an arbitrary program halts or runs forever. Some proposed hypercomputers can simulate the program for an infinite number of steps and tell the user whether the program halted. The HP for Turing machines can be easily solved by an accelerated Turing machine using a pseudocode algorithm like the following:

begin program;
write 0 on the first position of the output tape;
begin loop;
simulate 1 successive step of the given Turing machine on the given input;
if the Turing machine has halted, then write 1 on the first position of the output tape and break out of loop;
end loop;
end program.

From our results, we can theoretically build, by using superluminal particles, an “oracle machine” namely an abstract machine used to study decision problems within a single operation. Such machine can perform all of the usual operations of a TM, and also asking the oracle itself to obtain a solution to any instance of the computational problem for that oracle. For example, if the problem is a decision problem for a set A of natural numbers, the oracle machine supplies the oracle with a natural number, and the oracle responds with “yes" or "no" stating whether that number is an element of A.

Given a device that tell you in advance whether a given computer program would halt, or go on running forever, you would be able to prove or disprove any theorem whatsoever about integers like, for example, the Goldbach Conjecture (GC) or the Riemann Hypothesis (RH).

You would simply show this “Oracle” a program that would loop through all the integers, testing every possible set of values and only halting if it came to a set that violated the conjecture. Such TMs then could perform countable infinite number of computational steps within a finite time.

In principle, the use superluminal particles for computer technology could allow us, in the near future, to build up a hypercomputer device whose impact on the understanding of quantum computing and its physical basis as well as to its application to the development of strategic fields as, first of all, that of AI systems, could be unimaginable.

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