Does $J/\psi \to \pi^+\pi^-$ fix the Electromagnetic Form Factor $F_\pi(t)$ at $t = M_{J/\psi}^2$?

J. Milana

Department of Physics, University of Maryland
College Park, Maryland 20742, USA

S. Nussinov

Department of Physics, University of Maryland, College Park, Maryland 20742
and Physics Department, Tel–Aviv University, Ramat–Aviv, Tel–Aviv, ISRAEL

M. G. Olsson

Physics Department, University of Wisconsin
Madison, Wisconsin 53706, USA

We show that the $J/\psi \to \pi^+\pi^-$ decay is a reliable source of information for the electromagnetic form factor of the pion at $t = M_{J/\psi}^2 = 9.6\text{GeV}^2$ by using general arguments to estimate, or rather, put upper bounds on, the background processes that could spoil this extraction. We briefly comment on the significance of the resulting $F_\pi(M_{J/\psi}^2)$.

PACS numbers: 13.40.Fn, 13.20.Gd, 13.20.Cz
It is believed that the pion’s electromagnetic form factor $F_\pi(t)$ can be more reliably calculated for $|t| \gg \Lambda_{QCD}^2$ than the corresponding quantities for the nucleon. However, $F_\pi(t)$ is more difficult to measure. In principle $F_\pi(t)$ can be measured for time like $t$ in $e^+e^-$ colliders via $e^+e^- \rightarrow \pi^+\pi^-$. However, for $t$ values of interest, $|t| \approx 10$GeV$^2$, the above ratio is rather small and may be difficult to extract from the few $e^+e^- \rightarrow \pi^+\pi^-$ events.

At the $J/\psi$ resonance the rate of all interactions is vastly enhanced and branching ratios for rare channels such as the G–parity (or isospin) forbidden $J/\psi \rightarrow \pi^+\pi^-$ can be measured. This rate could fix $F_\pi(t = M_{J/\psi}^2 = 9.6$GeV$^2)$ if the decay proceeds predominantly via the one photon exchange amplitude illustrated in Fig. 1a. The dependence on the charmonium’s wavefunction can be eliminated by comparing the obtained branching ratio $Br(J/\psi \rightarrow \pi^+\pi^-)$ to the leptonic decay rate $Br(J/\psi \rightarrow e^+e^-)$, from which one obtains that

$$\frac{Br(J/\psi \rightarrow \pi^+\pi^-)}{Br(J/\psi \rightarrow e^+e^-)} = \frac{F^2_\pi(M_{J/\psi}^2)}{4}. \quad (0.1)$$

The experimental values

$\begin{align*}
Br(J/\psi \rightarrow e^+e^-) &= (6.27 \pm .20)10^{-2} \\
Br(J/\psi \rightarrow \pi^+\pi^-) &= (1.47 \pm .23)10^{-4}
\end{align*}$

would then imply that

$$F_\pi(M_{J/\psi}^2) = .098 \pm .008. \quad (0.4)$$

This value of $F_\pi(t = 9.6$GeV$^2)$ exceeds most theoretical estimates and also the extrapolations of $F_\pi(t)$ from the normally accepted values at large space–like momentum inferred from $\pi$ electroproduction data (see however [1]).

There are two additional mechanisms contributing to $J/\psi \rightarrow \pi^+\pi^-$:

$$A^{J/\psi \rightarrow \pi^+\pi^-} = A^{\pi}_\gamma + A^{\pi}_{ggg} + A^{\pi}_{\gamma g g g}. \quad (0.5)$$

$A^{\pi}_{ggg}$ is taken to mean the contribution to the amplitude of a purely hadronic process, which perturbatively would be initiated via a three gluon state and hence the nomenclature.
Likewise, $A_{\pi gg}^\pi$ is a mixed hadronic–electromagnetic contribution that would be initiated via a two gluon, one photon intermediate state (Figs. 1b, 1c respectively). In the following we will estimate $A_{\pi gg}^\pi$ and $A_{\pi gg}^\gamma$ and show that both amplitudes fall considerably short of explaining the observed $J/\psi \rightarrow \pi^+ \pi^-$ decay rate, thus justifying Eq. (0.4) above.

(I) $A_{\pi gg}^\pi$. Because the $J/\psi \rightarrow \pi^+ \pi^-$ violates isospin, this purely hadronic process can proceed only via the isospin breaking parameter $m_d^0 - m_u^0$ which appears explicitly in the QCD Lagrangian. Such an amplitude should therefore be suppressed by the small dimensionless factor $\epsilon_I = (m_d^0 - m_u^0)/Q$ with $Q$ some typical momentum in the problem. Rather than rely on any explicit, model dependent calculation, we present the following more general argument by comparing with the SU(3) analog process $J/\psi \rightarrow K\overline{K}$. Since the $J/\psi \rightarrow K\overline{K}$ decay violates SU(3) symmetry, the corresponding purely hadronic decay amplitude $A_{ggg}^K$ will have in this case the explicit small SU(3) breaking suppression factor $\epsilon_{SU(3)} = (m_s^0 - m_d^0)/Q$. Consequently we expect that

$$\frac{A_{\pi gg}^\pi}{A_{ggg}^K} \approx \frac{\epsilon_I}{\epsilon_{SU(3)}} \approx \frac{m_d^0 - m_u^0}{m_s^0 - m_{d,u}^0} \approx .02 - .03 \quad (0.6)$$

where in the spirit of the Vafa–Witten theorem we used the values of Lagrangian or “current” quark masses in estimating the above ratio. There are two $K\overline{K}$ decay modes, $J/\psi \rightarrow K^0\overline{K}^0$ (or $K_s^0\overline{K}_L^0$) and $J/\psi \rightarrow K^+ K^-$. The amplitude $A_{ggg}^K$ is simply given by the former,

$$A_{ggg}^K \approx A_{J/\psi \rightarrow K_s^0\overline{K}_L^0} \quad (0.7)$$

The point is that the one photon and $\gamma gg$ contributions to the $J/\psi \rightarrow K_s^0\overline{K}_L^0$ decay also vanish in the SU(3) limit due to the cancelling contribution of $s, \overline{d}$ quarks of opposite charge. Thus the amplitudes $A_{\gamma gg}^K$ and $A_{\gamma gg}^\pi$ are suppressed both by an explicit $\alpha_E$ and $\epsilon_{SU(3)}$ factors and are hence negligible. Multiplying Eq. (1.3) and Eq. (0.6) with the observed branching rate

$$Br(J/\psi \rightarrow K_s^0\overline{K}_L^0) = (1.1 \pm .14) \times 10^{-4} \quad (0.8)$$
implies that
\[ A_{ggg}^\pi \approx \frac{1}{30} A^{J/\psi \to \pi^+ \pi^-} \] (0.9)
so that it can be safely ignored.

(II) \( A_{ggg}^\pi \): It is very suggestive from a perturbative framework that this process is suppressed by a factor of \( \alpha_s / \pi \) as it involves an extra gluon loop in comparison with the corresponding expression for \( A_\gamma^\pi \). Indeed recent detailed calculations \[11\] using a range of pion wavefunctions \[4\] \[5\] indicate that
\[
\frac{R}{A_{ggg}^\pi / A_\gamma} = \frac{\alpha_s}{\pi} \left\{ \begin{array}{c} 1/20 \\ 1/40 \end{array} \right\} \approx \left\{ \begin{array}{c} 0.45 \\ 0.23 \end{array} \right\}
\] (0.10)
where the smaller \( R \) value corresponds to the use of the more realistic, non–asymptotic pion wave function \[5\] allowing for a larger \( F_\pi(t) \) (which however still falls short by more than a factor of two of explaining \( Br(J/\psi \to \pi^+ \pi^-) \)).

In order however not to rely too heavily on detailed model calculations we would like to obtain a more general, “phenomenological”, estimate for \( A_{ggg}^\pi \). Let us therefore for the moment assume that only \( A_{ggg}^\pi \) contributes to the decay \( J/\psi \to \pi^+ \pi^- \).

Consider first the total inclusive radiative decay of \( J/\psi \) into non–charmed hadrons: \( Br(J/\psi \to \gamma + \text{hadrons}) \). This process can be viewed as \( J/\psi \to \gamma gg \) with the subsequent hadronization of the two gluon system, in the same way that \( J/\psi \to \text{hadrons} \) proceeds via a three gluon initial perturbative state. Thus the ratio
\[
\frac{Br(J/\psi \to \gamma + \text{hadrons})}{Br(J/\psi \to \text{hadrons only})} \approx \frac{Br(J/\psi \to \gamma + gg)}{Br(J/\psi \to ggg)} = \frac{16}{5} \frac{\alpha_E}{\alpha_s} = .07 - .09,
\] (0.11)
is readily \[12\] computed reflecting simply color and symmetrization factors (and where we’ve taken \( 1/3 \geq \alpha_s \geq 1/4 \)). Note that the symmetrization factors enhances the case with the final state photon by a factor of 3. Such an enhancement would in general be absent in the case that the bosons were not final state particles but were instead found in a virtual intermediate state, as we will be using below. Nevertheless, in order to be as conservative as possible, we will use Eq. (0.11) in our estimates without further modification.
For the three gluon system the incorporation of the gluons or the quark pairs (to which they may convert) into hadrons is guaranteed by the basic hypothesis of quark and gluon confinement. However, we are for our purposes interested in the case where the $\gamma gg$ intermediate state converts into hadrons only. For this to happen, the virtual photon must convert into a $q\bar{q}$ pair which will cost an explicit extra factor of $\alpha_E$:

$$Br(J/\psi \to \gamma gg \to \text{hadrons}) = Br(J/\psi \to \gamma gg \to q\bar{q}gg \to \text{hadrons})$$

$$\approx \alpha_E Br(J/\psi \to \gamma + \text{hadrons}) = (5 - 7) \times 10^{-4}. \quad (0.12)$$

We are focussing on a particular exclusive channel, namely a final $\pi^+\pi^-$ state. Thus we need to estimate the probability $f$ that the $q\bar{q}gg$ state in Eq. (0.12) hadronizes specifically into a $\pi^+\pi^-$ state. While it is uncertain how reliably one can directly compute $f$, we will infer an estimate for $f$ from the probability that such a $q\bar{q}gg$ will hadronize into an analogue $\pi\rho$ state, i.e. we will take that

$$f \equiv Br(q\bar{q}gg|_{J/\psi \to \pi^+\pi^-}) \approx \frac{1}{2} Br(q\bar{q}gg|_{J/\psi \to \pi\rho}), \quad (0.13)$$

where the factor of 1/2 reflects the two transverse polarizations of the $\rho$ included in the $\pi\rho$ final state.

Note that the actual branching ratio

$$Br(J/\psi \to \pi^+\rho^-) \approx 0.4\% \quad (0.14)$$

appears to be anomalously large in comparison with the branching ratio to other two body channels. Indeed it has triggered the speculation of the existence of a glueball state in the vicinity of the $J/\psi$. While such speculation is controversial, there is general uniform agreement that the $\pi^+\rho^-$ branching ratio is unusually large. Hence irrespective of the correct explanation for $Br(J/\psi \to \pi^+\rho^-)$, its usage to estimate $A_{\gamma gg}^{\pi}$ must lead to a conservative upper bound. On the other hand, if the glueball resonance scenario is correct, we would be severely overestimating $A_{\gamma gg}^{\pi}$ since such a resonance would clearly not couple to the $\gamma gg$ channel.
Finally, in order to estimate $A_{\pi \gamma \pi \gamma}$, we will (conservatively) ignore the possible unusual behavior of the $\pi \rho$ final state and note that a general two body, light meson exclusive state is expected to be a short distance event. Hence, in order to generate the same $q\bar{q}gg$ state in Eq. (0.13), we need to convert one gluon into a $q\bar{q}$ pair, and thus we will take that

$$\frac{Br(J/\psi \to \gamma gg \to q\bar{q}gg)}{Br(J/\psi \to ggg \to q\bar{q}gg)} = \frac{1}{\alpha_s} Br(J/\psi \to \gamma gg \to hadrons),$$

(0.15)

which will again enhance our estimate for $A_{\pi \gamma \pi \gamma}$ by $1/\alpha_s$. Combining Eqs. (0.13) and (0.15) and inserting (0.12) and (0.14), we obtain that $A_{\pi \gamma \pi \gamma}$ alone would contribute a branching

$$Br_{\gamma gg}(J/\psi \to \pi^+\pi^-) = (3 - 6)10^{-6},$$

(0.16)

which is at least 25 times smaller than the observed value. We hence conclude that even under the most unfavorable scenarios, $A_{\pi \gamma \pi \gamma}$ is less than a 20% correction so that

$$A_{J/\psi \to \pi^+\pi^-} = A_{\pi \gamma} + A_{ggg} + A_{\pi \gamma \pi \gamma} \approx A_{\pi \gamma}.$$

(0.17)

Having established that the $J/\psi \to \pi^+\pi^-$ data implies a fairly large value of $F_\pi(t = M_{J/\psi}^2)$, we briefly turn to some concluding remarks:

(i) Recent results from E760 at Fermilab [15] indicates that the proton’s electromagnetic form factor in the large time–like region is also unusually large (by about a factor of 2 over the space–like data). A substantial imaginary part to hadronic form factors in the time–like region could account for this apparently systematic enhancement.

(ii) We expect that $A_{K \gamma} \approx A_{\pi \gamma}$ as the kaon’s and pion’s electromagnetic form factors should be rather similar at $t = M_{J/\psi}^2$. Since $A_{ggg}^{\pi}$ can be argued to be small along similar lines presented for $A_{ggg}^{\pi}$ and using our previous value for $A_{ggg}^{K}$, Eq. (0.7), we obtain that

$$A_{J/\psi \to K^+K^-} \approx A_{J/\psi \to \pi^+\pi^-} + A_{J/\psi \to K^0\bar{K}^0}.$$

(0.18)

Considering that $A(J/\psi \to ggg \to K\bar{K})$ is expected to have a substantial imaginary part (see [11] for an explicit calculation of an analogous case), there could in general be a large relative phase between the two terms in Eq. (0.18). Thus the latter is quite consistent with the observed branching.
\[ Br(J/\psi \to K^+ K^-) = (2.4 \pm 0.3) \times 10^{-4}. \] (0.19)

Acknowledgements: J.M. thanks S. J. Brodsky and V. Chernyak for useful conversations. S.N. thanks G. Karl for discussions. This work was supported in part by the U.S. Department of Energy under grant No. DE-FG05-93ER-40762.
REFERENCES

[1] C. E. Carlson and J. Milana, Phys. Rev. Lett. 65, 1717 (1990).

[2] L. M. Barkov, et al., Nucl. Phys. B256, 365 (1984); DM2 Collaboration, Phys. Lett. B220B, 321 (1989).

[3] K. Hikasa et al., Phys. Rev. D 45 I.1 (1992) (Particle Data Group).

[4] G. R. Farrar and D. R. Jackson, Phys. Rev. Lett 43, 246 (1979); A. V. Efremov and A. V. Radyushkin, Phys. Lett. B 94, 245 (1980).

[5] V. L. Chernyak and A. R. Zhitnitsky, Phys. Rep. 112, 173 (1984).

[6] C. J. Bebek et al., Phys. Rev. D 17, 1693 (1978).

[7] Indeed our arguments for estimating $A_{\pi}^{ggg}$ will not rely on the specific 3 gluon mechanism, but only on the fact that this is a purely hadronic amplitude.

[8] The Vafa–Witten theorem [C. Vafa, and E. Witten, Nucl. Phys. B234, 173 (1984)] based on QCD inequalities exclude a spontaneous breaking of isospin or other global vectorial symmetries. Any isospin violating purely hadronic amplitude must therefore contain an explicit factor of $m_{u} - m_{d}$.

[9] Recall that in the SU(3) flavor symmetry limit, $A(J/\psi \rightarrow K^{+}K^{-}) = A(J/\psi \rightarrow \pi^{+}\pi^{-})$.

[10] Indeed all electromagnetic properties such as the charge radius of the $K^{0}$ vanish in the SU(3) limit. See e.g., O. W. Greenberg, S. Nussinov, and J. Sucher, Phys. Lett. 70B, 289 (1977).

[11] R. Kahler and J. Milana, Phys. Rev. D 47, R3690 (1993).

[12] P. B. Mackenzie and G. P. Lepage, Phys. Rev. Lett. 47, 1244 (1981).

[13] The modification due to the fact that empirically $Br(J/\psi \rightarrow hadrons\ only) \approx 2/3$ will
cancel in our estimates between Eqs. (1.12) and (1.14) below.

[14] Wei–Shou Hou and A. Soni, Phys. Rev. Lett. 50, 569 (1983); S. J. Brodsky, G. P. Lepage and San Fu Tuan, Phys. Rev. Lett. 59, 621 (1987).

[15] T. A. Armstrong et al., Phys. Rev. Lett. 70, 1212 (1993).
FIGURES

FIG. 1. The three contributions to the decay of charmonium into $\pi^+\pi^-$. Curly lines are gluons, wavy lines photons. (a) is proportional to the pion’s electromagnetic form factor. (b) and (c) are background processes not proportional to $F_\pi(M_{J/\psi}^2)$. 