Creation and dissipation of magnetic fields in non-ideal GRMHD simulations

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Abstract. Compact objects, such as neutron stars and black holes, are characterized by the presence of strong magnetic fields that are crucial to explain the high-energy emission from these sources. Such magnetic fields may be subject to complex evolution inside the hosting relativistic plasmas, like dynamo-like processes amplifying initial seed fields in accretion disks or in the early stages of neutron star formation, or dissipative reconnection events in thin current sheets, as believed to occur in the magnetospheres of magnetars. Here we present a unified treatment of these non-ideal effects within the framework of general relativistic magnetohydrodynamics (GRMHD). Results of recent numerical simulations obtained with the \textsc{ECHO} code will be briefly discussed for selected test cases.

1. Introduction

The theoretical modeling of the coalescence of neutron stars giving rise to multimessenger signals, such as gravitational waves, the associated gamma-ray burst and the emission from a kilonova [1], as well as of the accretion and emission properties of the plasma surrounding supermassive black holes, like the recently observed one in M87 [2], are both based on numerical simulations in the General Relativistic MagnetohydroDynamics (GRMHD) approximation. However, mostly due to the complexity of the simulations, the plasma is usually regarded as ideal, for which the Ohm’s law is simply a condition of vanishing electric field in the comoving frame of the fluid, and, precisely as for classical MHD, only the magnetic field needs to be evolved together with the fluid quantities. Astrophysical situations should instead allow also for the dissipation of magnetic energy, often occurring in nature in the form of rapid conversion into heat and particle acceleration combined to a change in the magnetic topology (reconnection) [3], or for the dynamo-like amplification of magnetic fields from low initial values up to equipartition with the fluid (say in terms of energy density or pressure) [4].

As far as reconnection is concerned, the quest is how to convert the magnetic energy on the rapid ideal timescales, while classical MHD reconnection theories are known to provide rates that depend on the much slower diffusive timescales, which are too slow to explain observations such as flaring activities. However, a recent analysis has shown that there exists a critical threshold on current sheet’s thickness, beyond which the \textit{tearing} modes evolve on the fast macroscopic Alfvénic timescales [5], and this has been confirmed by means of numerical simulations in classical MHD [6, 7, 8]. In the relativistic case, the study of the tearing instability has been also performed...
[9], showing that reconnection may indeed occur on the relativistic ideal timescales $\sim L/c$ for strongly magnetized plasmas where the Alfvén velocity approaches the speed of light $c$, as required for instance in Pulsar Wind Nebulae [10, 11].

The (exponential) amplification of initial seed magnetic fields in relativistic plasmas is another very important topic in astrophysics, from the conditions in the early Universe [12] to the interior of (proto) neutron stars [13, 14]. Another situation in which amplification of fields is necessary is inside accretion disks. An ideal effect is the magneto-rotational instability (MRI) [15], while dynamo action in a turbulent plasma is also often invoked, for which an electromotive force with a term proportional to the magnetic field arises in Ohm’s law, in analogy to what is supposed to be at work in stellar interiors, needed to explain the solar cycle [16]. For relativistic plasmas, in the last years a novel mechanism of quantum origin has gained increasingly more attention, namely the Chiral Magnetic Effect (CME), due to an imbalance between left- and right-handed fermions. This has been recognized in semi-metals and it is most likely at work in the quark-gluon plasma formed in heavy-ion collision experiments, where the highest magnetic fields in nature, up to $B \sim 10^{18}$ G, are produced [17]. This effect is expected to survive even at large hydrodynamical/MHD scales, where the chiral imbalance leads to an electric current parallel to an external magnetic field, which is precisely the same mechanism of an $\alpha$-dynamo action in classical MHD, so that in the relativistic regime we expect a very similar treatment [18].

Here we describe a novel, unified treatment for the inclusion of all these non-ideal magnetic effects within the ECHO code for GRMHD, and we briefly discuss recent numerical results obtained for the relativistic tearing instability in thin current sheets, where fast reconnection is achieved as required to explain flaring high-energy events in magnetar coronae or pulsar winds, and for the amplification of magnetic fields in thick accretion disks, as those needed to model the dynamics and the emission around the supermassive black hole of M87, and, presumably, of SGR A*, the black hole in our Galaxy.

2. A unified treatment of magnetic non-ideal effects in the ECHO code for GRMHD

The Eulerian Conservative High-Order code, ECHO, is a finite-difference shock-capturing scheme for the GRMHD system of conservation laws, based on the 3+1 formalism of numerical relativity and working in any spacetime metric [19]. The code employs high-order spatial reconstruction and derivation algorithms and a simple two-wave Riemann solver at cell interfaces. In the following we describe the implementation of the magnetic non-ideal effects, namely the inclusion of the resistive and dynamo-type (mean field or chiral) terms within the GRMHD system of equation. We follow the unified treatment of [20, 18], starting from the covariant generalized Ohm’s law

$$e^\mu = \eta j^\mu + \xi b^\mu,$$

where the 4-vectors $e^\mu$, $b^\mu$, and $j^\mu$ are, respectively, the electric field, magnetic field, and current density in the frame comoving with the fluid. In the above relation $\eta$ is the resistivity of the plasma and $\xi$ is the dynamo coefficient. When both coefficients are zero we recover the ideal MHD condition of a vanishing comoving electric field.

In the $3 + 1$ formalism, needed for numerical integration, any four-dimensional spacetime metric is split according to

$$ds^2 = -\alpha^2 dt^2 + \gamma_{ij}(dx^i + \beta^i dt)(dx^j + \beta^j dt),$$

where $\alpha$ is the lapse function, $\beta^i$ the shift vector and $\gamma_{ij}$ is the 3-metric, used to raise/lower the indexes of any spatial three-dimensional vector or tensor. Within this formalism, the system of
dynamo-resistive GRMHD equations in conservative form is (here $\epsilon_0 = \mu_0 = c = 1$)
\begin{align*}
\partial_t (\sqrt{\gamma} D) + \partial_i [\sqrt{\gamma} (\alpha u^k - \beta^k) D] &= 0, \\
\partial_t (\sqrt{\gamma} S_i) + \partial_i [\sqrt{\gamma} (\alpha S^k_i - \beta^k S_i)] &= \sqrt{\gamma}[\frac{1}{2} \alpha S lm \partial_t \gamma_{lm} + S_k \partial_i \beta^k - (D + \mathcal{E}) \partial_i \alpha], \\
\partial_t (\sqrt{\gamma} \mathcal{E}) + \partial_i [\sqrt{\gamma} (\alpha S^k - \alpha v^k D - \beta^k \mathcal{E})] &= \sqrt{\gamma}[\frac{1}{2} \alpha S lm (\beta^k \partial_t \gamma_{lm} - \partial_t \gamma_{lm}) + S_i \partial_i \beta^m - S^k \partial_k \alpha], \\
\partial_t (\sqrt{\gamma} E^k) - \sqrt{\gamma} \epsilon^{ijk} \partial_j (\alpha B_k - \epsilon_{klm} \beta^m E^i) + \sqrt{\gamma} q (\alpha v^i - \beta^i) &= -\sqrt{\gamma} \alpha \Gamma \eta^{-1} \left[ E^i + \epsilon^{ijk} v_j B_k - E^j v_j v^i - \xi (B^i - \epsilon^{ijk} v_j E_k - B^j v_j v^i) \right], \\
\partial_t (\sqrt{\gamma} B^i) + \sqrt{\gamma} \epsilon^{ijk} \partial_j (\alpha E_k + \epsilon_{klm} \beta^m B^i) &= 0,
\end{align*}
with the additional non-evolutionary constraints
\[ \partial_t (\sqrt{\gamma} E^k) = \sqrt{\gamma} q, \quad \partial_t (\sqrt{\gamma} B^k) = 0. \]

In the above expressions, $D = \rho \Gamma$ is the mass density in the laboratory frame, $S_i = \omega \Gamma^2 v_i + \epsilon_{ijk} E^j B^k$ the momentum density, $S_{ij} = \omega \Gamma^2 v_i v_j + p \gamma_{ij} - E_i E_j - B_i B_j + u_{\text{em}} \gamma_{ij}$ the stress tensor, $\mathcal{E} + D = \omega \Gamma^2 - p + u_{\text{em}}$ the total energy density, $\rho$ the mass density in the comoving frame, $p$ the enthalpy per unit volume, $v$ the fluid 3-velocity and $\Gamma$ its corresponding Lorentz factor, $u_{\text{em}} = \frac{1}{2} (E^2 + B^2)$ the electromagnetic energy density, $E^i$ and $B^i$ the electric and magnetic fields, $q$ the charge density, whereas $\gamma$ and $\epsilon_{ijk}$ are respectively the determinant and the Levi-Civita pseudo-tensor of the 3-metric. In order to close the system, an equation of state must be also specified, for instance that of a perfect fluid, $p = (\gamma - 1) \epsilon \rho$, where $\epsilon$ is the thermal energy per unit mass, or equivalently
\[ w = \rho + \frac{\dot{\gamma}}{\gamma - 1} \rho = \rho + \dot{\gamma}_1 \rho, \]
and $\dot{\gamma}$ is the adiabatic index ($\dot{\gamma} = 4/3$ and $\dot{\gamma}_1 = 4$ for a relativistic fluid).

The solenoidal constraint for the magnetic field is enforced through the UCT (Upwind Constrained Transport) method based on a staggered representation of magnetic field components [21, 22], allowing the preservation of the condition to machine accuracy for a second order scheme or up to its nominal spatial accuracy when higher order methods are employed. On the other hand, charge conservation is not enforced similarly using a staggered representation of the electric field components, see the discussion in [23], but we simply replace $\sqrt{\gamma} q$ in the equation for the electric field using Gauss’ law. For an ideal plasma with $\eta \to 0$, both equations become redundant and the electric field becomes a derived quantity, so that in fluxes we can simply use $E^i = -\epsilon^{ijk} v_j B_k$.

The most delicate numerical step in GRMHD conservative schemes is the recovery of the set of fluid primitive variables
\[ \mathcal{P} = [\rho, v^i, p] \]
from the set of conservative variables, those evolved in time by the system, which are
\[ \mathcal{U} = \sqrt{\gamma} [D, S_i, \mathcal{E}, E^i, B^i]. \]

This necessarily involves a nonlinear procedure in the relativistic case, mainly due to the ubiquitous presence of the Lorentz factor $\Gamma$ in the definition of the above quantities. Moreover, in the non-ideal resistive case $\eta \neq 0$ we must face the problem that the source term in the evolution equation for $E^i$ is typically stiff, being proportional to $\eta^{-1}$ (regardless of the presence of a dynamo term proportional to $\xi$), so that an implicit step in the time integration procedure is required. Let us now split the GRMHD system as
\[ \partial_t \mathcal{X} = \mathcal{Q} \mathcal{X} \mathcal{U} + \mathcal{R} \mathcal{X} \mathcal{U}, \quad \partial_t \mathcal{Y} = \mathcal{Q} \mathcal{Y} \mathcal{U}, \]
the maximum growth rate for the instability is
\[ \mathcal{L}_p \]
Precisely as for classical MHD, it is proved that, for given sheet’s length
were performed in the magnetically-dominated case [28] and in the two-fluid approximation [29].
three-dimensional simulations in the resistive relativistic MHD regime, by [9]. Previous investigations
free or pressure-balanced initial equilibrium) was investigated, for the first time using two-
the linear and nonlinear phases of the tearing instability of thin current sheets (either in force-
robust one and has been recently adopted in other resistive relativistic MHD codes [23, 27].
It is convenient to embed the solution of the implicit step within the inversion from
calculated by inverting the energy equation as
\[ \tilde{w}(\tilde{u}^j) = \frac{\Gamma[E + D - \frac{1}{2}(E_i E^i + B_i B^i)] - D/\gamma_1}{\Gamma^2 - 1/\gamma_1}, \]
so that we can evaluate the function \( f_i(\tilde{u}^j) \) and its Jacobian as
\[ f_i(\tilde{u}^j) = \tilde{w}(\tilde{u}^j)\tilde{u}_i + \epsilon_{ilm}E^l(\tilde{u}^j)B^m - S_i, \quad J_{ij} = \frac{\partial f_i}{\partial \tilde{u}^j} = \tilde{w}\gamma_{ij} + \tilde{u}_i\frac{\partial \tilde{w}}{\partial \tilde{u}^j} + \epsilon_{ilm}\frac{\partial E^l}{\partial \tilde{u}^j}B^m, \]
to be plugged into the iterative scheme. The updated velocity at the iteration \( k+1 \) is then
\[ \tilde{u}^j_{(k+1)} = \tilde{u}^j_{(k)} - [J_{ij}^{(k)}]^{-1}f_i^{(k)}, \]
and iterations are repeated until the desired accuracy is reached. This three-dimensional
Newton-Raphson scheme based on the vanishing of momentum equations and using the \( \tilde{u}^j \)
variables, first introduced by [20] and refined in [26] (where the expression for the analytical
Jacobian components are provided, including the dynamo terms), has proved to be the most
robust one and has been recently adopted in other resistive relativistic MHD codes [23, 27].

3. Fast reconnection: the ideal relativistic tearing instability
The linear and nonlinear phases of the tearing instability of thin current sheets (either in force-
free or pressure-balanced initial equilibrium) was investigated, for the first time using two-
dimensional simulations in the resistive relativistic MHD regime, by [9]. Previous investigations
were performed in the magnetically-dominated case [28] and in the two-fluid approximation [29].
Precisely as for classical MHD, it is proved that, for given sheet’s length \( L \) and width \( a \ll L \),
the maximum growth rate for the instability is
\[ \gamma_{\text{max}} \tau_c \simeq 0.6 c_A S^{-1/2}(a/L)^{-3/2}, \]
where \( \tau_c \) is the light crossing time over the length \( L \), the Lundquist number and the relativistic
Alfvén speed, defined in terms of the equilibrium quantities, are, respectively
\[ S = \frac{L c_A}{\eta}, \quad c_A = \frac{B_0}{\sqrt{\rho_0 + 4p_0 + B_0^2}}, \]
Figure 1. Adapted from [9]. Magnetic fieldlines and velocity magnitude (in units of c, according to the colorbar) at time $t = 20\tau_c$ (left panel) and maximum Alfvénic Mach number in the domain as a function of time $t/\tau_c$ (right panel), for a simulation with $a/L = S^{-1/3}$, $S = 10^6$, $\sigma_0 = B_0^2/\rho_0 = 1$, $\beta_0 = 2P_0/B_0^2 = 1$, $c_A = 0.5$. The solid line refers to an initial force-free equilibrium, while the dashed line to a pressure-balance equilibrium.

where we have assumed an ideal gas law with $\gamma = 4/3$. Hence, the growth rate is generally slow, and, for a given $a/L$, the dependency on $S^{-1/2}$ has been confirmed numerically. However, in any dynamical process current sheets have a fast decreasing $a/L$ ratio and $\gamma_{\text{max}}$ is expected to become increasingly larger. We have proved that, when when the critical threshold of $a/L = S^{-1/3}$ is reached (for instance $a = 0.01L$ for $S = 10^6$), the instability becomes indeed ideal, with

$$\gamma_{\text{max}}\tau_c \simeq 0.6 c_A,$$

independently on the value of $S$ (provided this is large enough). Therefore, the classical Sweet-Parker scenario, for which the diffusive region scales as $a/L \sim S^{-1/2}$, much thinner than the critical width, is likely to be never realized in nature, as the current sheet itself disrupts in the elongation process due to the tearing instabilities occurring on the same ideal timescales on which the magnetic configuration evolves.

In the nonlinear phase the formation of secondary reconnection plasmoids is observed, that soon start to move and merge, producing jet-like features and shocks along the sheet. Due to the periodicity assumed along the sheet’s length, in the final configuration we have a single large plasmoid feeded by trans-Alfvénic jets originating from an X-point (see figure 1). Several runs have been performed, up to the the extreme case of $\sigma_0 = B_0^2/\rho_0 = 50$, $\beta_0 = 2P_0/B_0^2 = 0.01$, $c_A = 0.98$, and invariably the dynamics follow a kind of universal evolution in terms of the Alfvén time is $\tau_A = L/c_A$. In particular, the transition from the smooth linear phase to the very rapid nonlinear one occurs at $t \simeq 8\tau_A$, where the growth rate of the instability increases abruptly from $\gamma_{\text{max}}$ (which already may occur on the ideal timescales) to even higher values, in a sort of explosive behaviour.

Such fast reconnection mechanism may be at the origin of the sudden release of magnetic energy observed in magnetars, due to the rearrangement of the complex topology of the currents and magnetic fields [30, 31, 32, 33], as well as of the gamma-ray flares from the Crab Nebula, most probably originated near the termination shock of the wind where the plasma magnetization is stronger [34, 35, 11, 36].
Figure 2. Adapted from [26]. The growth of the magnetic field (averaged over the accretion disk volume) as a function of time, normalized against $P_c$, the rotation period of the center of the disk (left panel), and the distribution of the poloidal field inside the disk for Run1q at $t = 6P_c$ (right panel). In all simulations $\eta_{\text{max}} = 10^{-3}$, while $\xi_{\text{max}} = 3 \times 10^{-2}$ for Run1, $\xi_{\text{max}} = 4 \times 10^{-2}$ for Run3, $\xi_{\text{max}} = 2 \times 10^{-2}$ for Run5. The label ‘q’ indicates a run with a quenching term.

4. A mean-field dynamo model for magnetized accretion disks

The recent imaging of the M87 black hole at millimeter wavelengths by the Event Horizon Telescope (EHT) collaboration [2] has triggered a renewed interest in numerical codes and models for the accretion of magnetized plasma in the ideal GRMHD regime [37], though future simulations will probably move towards the more general resistive case [27], allowing for magnetic dissipation. In this context, an important aspect to consider is the magnetic field amplification inside the accretion disk feeding the black hole, that must reach a level near to equipartition in order to explain the observation of the synchrotron emission in radio [38]. This amplification is usually attributed to MRI, which triggers the development of an MHD turbulent cascade, which in turn is responsible for enhanced viscosity, transport and dissipation of angular momentum and dynamo mechanisms [15]. However, global 3D simulations of the accretion scenario resolving MRI are quite expensive, especially in the relativistic regime.

A way to overcome this difficulty is to resort to the more economic mean-field dynamo approach [16], relying on the non-ideal Ohm’s law described in section 2, where the effect of the MRI-induced turbulence is simply incorporated within the coefficients $\eta$ (enhanced dissipation) and $\xi$ (the effective dynamo term). In the following we report the recent findings of [26], where we have shown for the first time in GRMHD how fully nonlinear axisymmetric simulations are able to describe the magnetic field amplification in thick accretion tori by the mean-field dynamo action, going beyond the simple kinematic approach employed in [39].

In particular, we have shown how the dynamo process is able to produce an exponential growth of an initial seed magnetic field up to equipartition values, when the instability tends to saturate even in the absence of artificial quenching effects (required for a kinematic dynamo). Before reaching the final saturation stage we observe a secondary regime of exponential growing, where the magnetic field increases more slowly due to accretion, which is modifying the underlying fluid equilibrium (see figure 2). Both the resistivity and dynamo terms are actually functions of the local disk density and position. By varying the maximum value of the dynamo function we obtain different growth rates, though the field seems to saturate at approximately the same level, at least for the limited range of parameters explored. In particular, the growth rate of the linear phase is proportional to the ratio $\xi_{\text{max}}/\eta_{\text{max}}$, see [26] for additional details.
The presence of a quenching term, needed to halt the field growth in kinematic simulations and to avoid local high magnetization regions in the present fully dynamical simulations, does not affect much the asymptotic values of the average field. For reasonable values of the central mass density and the commonly employed recipes for synchrotron emission by relativistically hot electrons, our model is able to reproduce naturally the observed flux of Sgr A*, the next target for EHT.

5. Conclusions
In the present paper we have shown how non-ideal magnetic effects such as reconnection and mean-field dynamo action may be modelled numerically by adopting a unified method for 3 + 1 GRMHD codes. The system of equations is stiff due to source terms proportional to the large conductivity coefficient $\eta^{-1}$, so that an implicit step is needed in the time integration Runge-Kutta routine. This is conveniently incorporated within a 3D Newton-Raphson inversion algorithm to recover primitive variables from conservative ones, as described in [20] and in better details in [26], where the analytical expressions for the Jacobian are given for both resistive and dynamo contributions. Recent results obtained with the ECHO code are also briefly described.

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