Two Populations and Models of Gamma Ray Bursts

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Abstract

Gamma-ray burst statistics are best explained by a source population at cosmological distances, while spectroscopy and intensity histories of some individual bursts imply an origin on Galactic neutron stars. To resolve this inconsistency I suggest the presence of two populations, one at cosmological distances and the other Galactic. I build on ideas of Shemi and Piran (1990) and of Mészáros and Rees (1993) involving the interaction of fireball debris with surrounding clouds to explain the observed intensity histories in bursts at cosmological distances. The distances to the Galactic population are undetermined because they are too few to affect the statistics of intensity and direction; I explain them as resulting from magnetic reconnection in neutron star magnetospheres.
1. Introduction

Attempts to explain all the observed gamma-ray bursts (GRB) with a single population of sources have become progressively more difficult. On one hand, their distribution on the sky has been observed, with steadily improving precision (Atteia, et al. 1987; Meehan, et al. 1992), to be isotropic, an observation which is naturally explained (Usov and Chibisov 1975; Goodman 1986, Paczyński 1986, Mao and Paczyński 1992, Fenimore et al. 1992, Piran 1992) if they are at cosmological distances. On the other hand, a number of GRB have been reported to show (Higdon and Lingenfelter 1990) spectral features at a few tens of KeV and at about 400 KeV, which are readily interpretable as cyclotron lines and the two-photon positron annihilation line from the surface of magnetized neutron stars at Galactic distances, but which are inexplicable at cosmological distances. If their validity is accepted, the data appear irreconcilable.

The problem is further complicated by the fact that straightforward models of radiation transport in GRB at cosmological distances (Goodman 1986; Paczyński 1986) predict very brief bursts of radiation with thermalized spectra, in contradiction to observation, while attempts (Brainerd 1992; Katz 1992) to explain the spatial anisotropy and \( \log N \) vs. \( \log S \) or \( V/V_{\text{max}} \) distributions of GRB in Galactic models require the assumption of a spherically symmetric halo of \( \sim 100 \) Kpc radius. Finally, the soft gamma repeaters (SGR) introduce additional confusion. The fact of their repetition and the identification of one of them (March 5, 1979; Cline 1980) with a supernova remnant in the LMC point strongly to a Galactic population, while the presence of spectral features and an 8 second periodicity (March 5, 1979) indicate origin on a magnetic neutron star. However, it is unclear that SGR should be considered GRB at all because their properties, including their spectra, are very different, and arguments made for SGR may be irrelevant to the problems of GRB.

As a first step towards resolving the apparent inconsistencies, I consider the obvious possibility that there are two distinct populations of GRB. A cosmological population \( C \) accounts for most GRB, and explains the statistics of isotropy and the \( \log N \) vs. \( \log S \) or
$V/V_{\text{max}}$ distributions. A population G includes those GRB with spectral lines (a minority), and originate on neutron stars at Galactic distances. An individual GRB cannot be assigned to a population unless it shows spectral lines, but the majority (probably overwhelming) of those without spectral lines must be members of population C. The SGR may be members of population G. I do not consider more exotic possibilities, such as GRB arising within the Oort cloud (Ruderman 1975), because they all seem far-fetched, even though I myself have discussed one of them (Katz 1993).

In the absence of information about the intensity statistics and angular distribution of population G alone, it is not possible to discriminate between a disc and a halo origin. It will be important to obtain this information, which may be reducible from archival data.

In §2 I discuss a possible mechanism for GRB at cosmological distances, building on recent suggestions by others. In §3 I more briefly discuss magnetic reconnection models of GRB at Galactic distances. The March 5, 1979 event in the LMC poses an acute problem of gamma-gamma pair production (Carrigan and Katz 1992) which must be faced, whether or not GRB of Population G have comparable distances and luminosities, and even if there is no population G. §4 contains a summary discussion. Unfortunately, unambiguous observational tests of the ideas discussed here will not be easy.

2. Population C: GRB at Cosmological Distances

The well-known failure of straightforward fireball models to explain the spectral and temporal properties of GRB led Shemi and Piran (1990) to consider neutrino-produced fireballs loaded with small (but not zero) amounts of ordinary matter; they found that the fireball could couple nearly all of its energy to the matter and (with the right values of the parameters) could accelerate it to relativistic velocity. Mészáros and Rees (1993) then pointed out that the interaction of this relativistic debris with surrounding matter might be characterized by times consistent with the range of GRB rise times and durations ($10^{-3}–10^3$ sec).
I build on these ideas. The environments of GRB’s at cosmological distances are open to much speculation (for example, are they low density galactic halos or dense nuclei of galaxies?), but the strong clumpiness of interstellar matter is a consequence of immutable atomic physics (cooling rates), and isolated discrete clouds are likely under a very wide range of conditions. The rarity of GRB makes it possible to assume favorable conditions, if these lie in the range of plausibility; there is no great difficulty if a considerably larger number of fireballs occurring in less favorable circumstances do not produce observable GRB.

GRB at cosmological distances require the radiation of $\sim 10^{51}$ erg in observable gamma-rays. The complex chain of processes which lead to gamma-ray emission must be moderately efficient when the parameters do have favorable values, because the energy radiated as neutrinos by neutron star collapse, formation, or coalescence is unlikely to exceed $0.3M_{\odot}c^2 \approx 5 \times 10^{53}$ erg, and may be considerably less. Thus, while we are entitled to assume favorable circumstances to explain the rare observable GRB, when these circumstances occur the resulting processes must be reasonably efficient. The fraction of neutrino energy converted to an electromagnetic fireball is small. Efficient conversion requires neutrino-neutrino collisions at angles in excess of $90^\circ$, but the neutrinos are generally expanding outward in a neutrino fireball, with velocity vectors which are tending toward radial outflow. Optimal head-on collisions are particularly rare. The conversion of electromagnetic energy to particle kinetic energy also has an efficiency $< 1$. The final conversion to observable gamma-rays is the hardest part of the problem; it is easy to see how this could fail entirely.

Relativistic invariants alone limit the amount of kinetic energy available for radiation by fireball debris. If a relativistic debris cloud with speed $\beta_F c$, Lorentz factor $\gamma_F$ and proper mass per unit area $\sigma$ sweeps up a proper mass per unit area $\alpha \sigma$, then the efficiency
of radiation of the debris kinetic energy can be as large as

\[ \epsilon = \frac{\alpha}{\alpha + \gamma_F (1 - \beta_F)}. \]  

(1)

Values of \( \epsilon > 1/2 \) are obtained for \( \alpha > 1/\left[\gamma_F (1 + \beta_F)\right] \approx 1/(2\gamma_F) \). Collisions with a very broad range of clouds of circum-fireball matter are consistent with efficient conversion of kinetic energy to radiation. This is fortunate, because efficient production of GRB requires that common circum-fireball environments produce observable GRB in most directions; it is not possible to insist on fortuitous geometries or on special values of the parameters.

Most of the kinetic energy will become available when the debris has swept up only a very little matter. If the energy of the explosion is \( Y \) then the proper mass of debris is \( Y/\gamma_F c^2 \), and half the kinetic energy will become available when the swept-up proper mass is \( Y/2\gamma_F^2 c^2 \). In a uniform medium of density \( \rho \) this will occur at an interaction radius

\[ r_I = \left( \frac{3Y}{8\pi \gamma_F^2 c^2 \rho} \right)^{1/3} \sim 2 \times 10^{15} \text{ cm}, \]  

(2)

where the numerical estimate assumed \( Y = 10^{52} \text{ erg}, \rho = 10^{-24} \text{ g/cm}^3 \) and \( \gamma_F = 10^4 \).

The hardest part of the problem is turning the kinetic energy of the relativistic debris into the observed gamma-rays. The collision length of relativistic protons in ordinary matter is about 50 g/cm\(^2\), or about 100 Mpc at typical interstellar densities. Clearly, some collective process is necessary, and it must couple the proton and ion energy into that of electrons, which radiate more readily. Even relativistic electrons do not radiate rapidly under interstellar conditions; the radiation length of a 10\(^{13}\) eV electron (corresponding to equipartition with a \( \gamma_F \sim 10^4 \) proton) for Compton scattering on a 3\(^\circ\)K black body radiation field is \( \sim 10^{23} \) cm, excessive by many orders of magnitude.

In order to obtain short pulses of radiation at distances of order those given by (2) it is necessary that a coherent relativistically expanding front of radiating particles be directed nearly toward the observer. It is not sufficient that individual particles be observed only when directed towards the observer, a condition met by most relativistic radiation
processes. Therefore, ambient magnetic fields must not deflect particles significantly from their initial spherical expansion. This condition will be satisfied if the magnetic energy \( E_{mag} \) in the interaction sphere of radius \( r_I \) is very much less than the debris energy, so the debris can sweep away the ambient magnetic field without significant deflection. If equipartition is assumed between the ambient magnetic field \( B_{ISM} \) and an ambient turbulent velocity field \( v_{ISM} \), then \( E_{mag}/Y \sim v_{ISM}^2/\gamma_F c^2 \ll 1/\gamma_F^2 \), so the ambient field may safely be ignored.

a) Shock Structure

When the debris shell collides with a cloud of ambient matter the resulting flow may be complex. If the shell and the cloud each initially had uniform density and velocity and negligible (on a relativistic scale) temperature, the geometry is slab-symmetric, and all bulk velocities are normal to the planes of symmetry, then the resulting shock structure is shown in Figure 1. There are two shocks S1 and S2 and, in general, a contact discontinuity CD separating shocked fireball debris from shocked cloud. The equations relating the conditions in the four regions are cumbersome, except in the special symmetric case in which debris and cloud initially had the same composition and proper density. In this case, which I assume, there is no contact discontinuity and conditions in regions 2 and 3 are identical, as are those in regions 1 and 4.

The relativistic shock conditions (Landau and Lifshitz 1959) may be used to determine physical conditions. The thermodynamic variables are the proper internal energy density \( e \), the proper pressure \( p \), and the proper enthalpy density \( w = e + p \). In the unshocked cloud \( e_1 = n_1 m_a c^2 \), where \( n_1 \) is the proper atomic number density and \( m_a \) is the proper mass per atom, and \( p_1 = 0 \). In the shocked cloud \( p_2 = e_2/3 \gg e_1 \) in the extreme-relativistic (ER) limit. I shall refer to the frame of the unshocked interstellar material as the local observer’s frame; transformation to our frame requires application of the cosmological redshift. Then, to lowest nontrivial order in \( e_1/e_2 \ll 1 \), the velocities of the fluids with respect to the shock
front S1 are
\[
\frac{v_1}{c} = \sqrt{\frac{(p_2 - p_1)(e_2 + p_1)}{(e_2 - e_1)(e_1 + p_2)}} \approx 1 - \frac{e_1}{e_2}
\]  
(3)
and*
\[
\frac{v_2}{c} = \sqrt{\frac{(p_2 - p_1)(e_1 + p_2)}{(e_2 - e_1)(e_2 + p_1)}} \approx \frac{1}{3} \left(1 + \frac{2e_1}{e_2}\right).
\]  
(4)

The velocity discontinuity \(v_{12}\) between fluids 1 and 2, measured in the frame of either, is obtained from the relativistic expression for the subtraction of velocities
\[
\frac{v_{12}}{c} = \frac{v_1/c - v_2/c}{1 - v_1v_2/c^2} \approx 1 - 2\frac{e_1}{e_2};
\]  
(5)
this velocity is also the velocity \(v_{2L}\) of shocked fluid 2 in the local observer’s frame. The velocity \(v_1\) is also the speed of the shock S1 in that frame.

Fluids 2 and 3 have the same velocity and, given our assumptions that \(n_1 = n_4\) and \(e_1 = e_4\), the same values of the thermodynamic variables. Then the velocity of fluid 3 in the frame of shock S2 is \(-v_2\), and the speed of fluid 4 in that same frame is \(-v_1\). The expressions for combinations of relativistic velocities may then be used to obtain the following results in the local observer’s frame:
\[
\frac{v_3}{c} = \frac{v_{12}}{c} \approx 1 - 2\frac{e_1}{e_2},
\]  
(6)
\[
\frac{v_{S2}}{c} \approx 1 - 4\frac{e_1}{e_2},
\]  
(7)
\[
\frac{v_4}{c} \approx 1 - 2\left(\frac{e_1}{e_2}\right)^2.
\]  
(8)

It is now possible to calculate \(e_2\) from the debris Lorentz factor \(\gamma_F\), defined in the local observer’s frame, using Equation 8:
\[
\gamma_F \equiv \frac{1}{\sqrt{1 - (v_4/c)^2}} \approx \frac{e_2}{2e_1},
\]  
(9)
* Note the assertion in the first edition of Landau and Lifshitz that in the ER limit \(v_2 \to c/3^{1/2}\) is a typographical error; the correct limit \(v_2 \to c/3\) is given in later editions.
\[ e_2 \approx 2n_1m_a c^2 \gamma_F. \]  

(10)

The Lorentz factor \( \gamma_{2L} \) of the shocked material in the local observer’s frame is obtained from \( v_{2L} = v_{12} \), Equations 5 and 10:

\[ \gamma_{2L} \approx \gamma_1^{-1/2} F. \]  

(11)

The detailed mechanics of the shock are obscure, but must be collisionless in order to form a shock at all. The shocked matter need not be in thermodynamic equilibrium. Heating of the shocked material by plasma instabilities is the source of dissipation; the distribution functions of particle energies will not be (relativistic) Maxwellians, but are more likely to be power laws. The distribution of energy between electrons and ions is uncertain. In a highly relativistic shock, as we expect here, electrons and ions are kinematically very similar (identical in the ER limit), so I will assume that the distribution of particle energies is independent of species; roughly half the post-shock energy resides in electrons. Any neutral matter does not interact with the shock, so that the density \( n_1 \) refers only to the ionized component. At distances \( \sim r_I \) (Equation 2) the interstellar material will largely have been ionized by the flash of radiation associated with the fireball or by collision with debris.

Using the shock jump conditions for the proper enthalpy \( w \), \( w_1 = n_1m_a c^2 \), and the ER limit \( w_2 \approx 4e_2/3 \) yields the proper atomic density

\[ n_2 \approx 2 \left( \frac{n_1 c_2}{m_a c^2} \right)^{1/2}. \]  

(12)

Define \( \gamma_2 \) by the relation

\[ \gamma_2 \equiv \frac{e_2}{n_2m_a c^2}; \]  

(13)

then the mean energy (in the frame of fluid 2) per particle is \( \gamma_2 \mu m_p c^2 \), where \( \mu m_p \) is the mean proper mass per particle. For pure ionized hydrogen \( \mu = 0.5 \), while for the usual cosmic abundances (fully ionized) \( \mu = 0.62 \). The mean Lorentz factor of an electron (in
the frame of fluid 2) is
\[ \gamma_{2e} = \gamma_2 \frac{\mu m_p}{m_e}. \]  
(14)

The proper density \( n_2 \) (Equation 12) may be rewritten, using Equation 13, as
\[ n_2 \approx 4n_1 \gamma_2, \]  
(15)
reproducing the result \( n_2 = 4n_1 \) for a strong but nonrelativistic shock \( (\gamma_2 \to 1) \) in a gas with an adiabatic exponent of 5/3. The Lorentz factor \( \gamma_2 \) is found from its definition (Equation 13), and Equations 9 and 15:
\[ \gamma_2 \approx \frac{1}{2} \left( \frac{e_2}{e_1} \right)^{1/2} \approx \left( \frac{\gamma_F}{2} \right)^{1/2}. \]  
(16)
In the local observer’s frame most of the particles are narrowly collimated in the direction of the motion of fluid 2, and the typical Lorentz factor is larger than those given in Equations 14 and 16 by a factor \( \sim \gamma_{2L} \) (Equation 11). The angular width of collimation depends on the angular distribution of the particle momenta in the proper frame of fluid 2, which is unknown. If this is isotropic, then the locally observed angular width (for electrons as well as ions)
\[ \theta_0 \sim \gamma_{2L}^{-1} \approx \gamma_F^{-1/2}. \]  
(17)
Note that this is a much broader angular distribution than the locally observed radiation pattern from a single particle, whose Lorentz factor is \( \sim \gamma_F \) (ions) or \( \sim \gamma_F m_p/m_e \) (electrons).

b) Time Dependence

The geometry of radiation from an advancing spherical shock is shown in Figure 2. The distant (but cosmologically local) observer may first see a flash of radiation from the fireball itself, whose arrival time is taken as \( t = 0 \). If the fireball is a consequence of the merger of binary neutron stars, as often assumed, the initial pulse includes bursts of neutrino and gravitational radiation, as well as electromagnetic radiation. Their emission is
essentially simultaneous, although their arrival may be affected by dispersion arising from plasma refraction, neutrino rest mass (if they have any), etc. The initial electromagnetic flash is expected to have a thermal spectrum and to be extremely brief ($\ll 10^{-4}$ sec) because of the small size of the fireball (Goodman 1986; Carrigan and Katz 1992); if, as assumed here, the fireball energy is largely converted (Shemi and Piran 1990) to kinetic energy of debris this initial flash may be unobservably faint. However, if it is observed the time interval between it and the rest of the GRB is an important constraint on the emission geometry.

Radiation emitted from a point $(r, \theta)$ on the expanding spherical shell arrives at the observer at a time

$$t \approx \frac{r(1 - \cos \theta)}{c} + r \left( \frac{1}{v} - \frac{1}{c} \right) \approx \frac{r(1 - \cos \theta)}{c} + \frac{r}{u},$$

where $v$ is the shell’s expansion velocity and the parameter (dimensionally but not physically a velocity) $u \equiv vc/(c - v)$. If the angular distribution of radiated intensity, measured in the local observer’s frame, is $f(\theta')$, where $\theta'$ is the angle from the normal to the radiating surface, then the energy $dE$ radiated by a patch of area $dA$ is

$$dE = f(\theta') dA. \tag{19}$$

Radiation directed toward the observer has $\theta' = \theta$. Using $dA = 2\pi r^2 \sin \theta d\theta$ and $dt = r \sin \theta d\theta/c$ yields the observed power

$$P(t, r) = \frac{dE}{dt} = 2\pi cr f(\theta). \tag{20}$$

The function $f(\theta)$ is proportional to a convolution of the angular distribution of the radiating particles and their radiation pattern; as previously discussed, the latter is expected to be narrower than the former. A plausible guess is then

$$f(\theta) \propto \frac{1}{\theta^2 + \theta_0^2}, \tag{21}$$
where the angular width $\theta_0 \sim \gamma_F^{-1/2}$ is essentially the same as that of the momentum distribution (Equation 17). The pulse shape is then obtained from Equations 20 and 21, using Equation 18 to eliminate $\theta$:

$$P(t, r) \propto \begin{cases} \left( \frac{t - \frac{r}{u}}{2c} + \theta_0^2 \right), & t \geq \frac{r}{u} \\ 0, & t < \frac{r}{u} \end{cases}$$ (22)

where the approximation $\cos \theta \approx 1 - \theta^2/2$ has been used. $P(t, r)$ has been plotted in Figure 3, where the dimensionless parameter $\tau \equiv 2ct/r\theta_0^2$ has been defined.

The pulse form of Equation 22 should be regarded only as an envelope, for the actual pulse shape will be modulated by the spatial distribution of matter which the debris shell sweeps up. One striking feature of Equation 22 is its abrupt rise, consistent with the observed rapid rises of some GRB. The characteristic width of this function is

$$\Delta t \sim \frac{r\theta_0^2}{2c} \sim \frac{r}{\gamma_F c}.$$ (23)

Use of $r \sim r_I$ (Equation 2) and $\gamma_F \sim 10^4$ yields $\Delta t \sim 10$ sec, the right order of magnitude for the duration of GRB. Much longer or shorter $\Delta t$ may be possible for plausibly different values of the parameters, particularly the cloud density, which is uncertain even to order of magnitude.

The debris shell and shock propagate into a very heterogeneous medium. The effects of structure in $\theta$ are shown by the dashed lines in Figure 3, which assume a cloud uniform in the range $\theta_1 \leq \theta \leq \theta_2$, with abrupt boundaries. A more realistic gradual density profile or shape would produce a gradual rise and decay; the abrupt rise remains if (and only if) the cloud includes the line $\theta = 0$. Thus this abrupt rise is expected for some, but perhaps not all, GRB, in accord with observations.

A complete intensity profile of a GRB requires the integration of Equation (22) over $r$:

$$P(t) = \int_0^{ut} P(t, r)g(r) \, dr,$$ (24)
where the weighting function $g(r)$ includes both the fact that the energy available for radiation falls off for $r > r_I$ and the effects of clumpiness of the ambient matter as a function of $r$. The observable region is a narrow half-cone of apical angle $\sim \theta_0$: unless the scale of spatial structure is $< r_I \theta_0 \sim 2 \times 10^{13}$ cm, clumpiness is more likely to be apparent as a function of $r$ than of $\theta$, justifying the use here of Equation 22 which ignored any dependence of density on $\theta$.

In the absence of spatial heterogeneity $g(r)$ may be taken to impose a cutoff at $r \approx r_I$, so that

$$P(t) \approx \int_0^{\min(r_I, ut)} P(t, r) \, dr \propto \int_0^{\min(r_I, ut)} \frac{r^2 \, dr}{2ct + r \left( -\frac{2e}{u} + \theta_0^2 \right)}. \tag{25}$$

The integral is elementary, but cumbersome, and of limited quantitative interest because of the artificiality of the assumed uniform density; the rise time is $r_I/u$. Two possibilities should be distinguished:

1. For a shock S1 propagating through a homogeneous medium $v/c \approx 1 - e_1/e_2 \approx 1 - 1/(2\gamma_F)$ and $u \approx 2\gamma_F c$. Then, for $r_I$ given by Equation 2 and $\gamma_F = 10^4$, the rise time is several seconds, given by Equation 23 and inconsistent with very rapid rise times. This corresponds to a long GRB pulse envelope.

2. On the other hand, the fireball debris propagates through a vacuum or very low density intercloud medium with a speed $v/c = (1 - 1/\gamma_F^2)^{1/2}$, so that $u \approx 2\gamma_F^2 c$; impacts upon small discrete clouds scattered within a region of size $\sim r_I$ will only introduce a time-width $\sim 10^{-3}$ sec, or less. This might not measurably broaden the abrupt rise given by Equation 22 and in Figure 3.

Integration over $r$ introduces two broadenings, one $O(r_I/2\gamma_F^2 c)$ associated with the entire emission region of size $\sim r_I$, where the low intercloud density is appropriate, and another $O(r_c/2\gamma_F c)$ associated with individual clouds of size $r_c \ll r_I$. These broadenings may each be much less than the envelope width (Equation 23), permitting an observed signal resembling that of Figure 3. Several sub-pulses may be observed if the debris shell collides with several isolated clouds in a medium dense enough to slow the intercloud shock,
so that interpulse times are $O(r_I/2\gamma_F c)$.

The actual situation is much more complicated than can be discussed here. For example, debris shell and clouds need not have the same proper densities, and each is likely to be spatially heterogeneous. Shocks propagating through heterogeneous media vary their strength, and produce associated continuous rarefactions and compressions. It is plausible that the complex structure of observed GRB could be explained by the interaction of relativistic debris shells with clumpy media, but more quantitative results would require numerical relativistic hydrodynamic calculations.

c) Radiation

The hardest part of the problem is turning electron kinetic energy into the observed radiation. Even though the typical Lorentz factor of an electron in the local observer’s frame is $\sim \gamma_2 L \gamma_2 e \sim \mu \gamma_F m_p/m_e$ (from Equations 11, 14, and 16), their rate of synchrotron radiation and Compton scattering in plausible interstellar magnetic and radiation fields is low. I therefore make the radical suggestion that the collisionless shock produces approximate equipartition between the magnetic energy density and the particle energy density. The proper magnetic field $B_2$ and energy density in fluid 2 are then, using Equation 10,

$$\frac{B_2^2}{8\pi} = \zeta e_2 \approx 2\zeta n_1 \gamma_F m_a c^2,$$

(26)

where $\zeta \leq 1/2$ is a phenomenological parameter describing the approach to equipartition. The synchrotron energy loss time for an electron with Lorentz factor given by Equation 14, assuming no correlation between the direction of the electron momentum and that of the magnetic field, is then, taking pure hydrogen composition

$$t_{r2} \approx \zeta^{-1} \left( \frac{n_1}{1\text{ cm}^{-3}} \right)^{-1} \gamma_F^{-3/2} 1.1 \times 10^7 \text{ sec.}$$

(27)

The radiating volume is moving toward the observer with a bulk Lorentz factor $\gamma_2 L$ (Equation 11), so that application of a Lorentz transform yields the local observer’s measured
radiation time
\[ t_{\text{obs}} = \gamma_{2L}(t_{r2} - v_{2L}^2 t_{r2}/c^2) \]
\[ = t_{r2}/\gamma_{2L} \]
\[ \approx \zeta^{-1} \left( \frac{n_1}{1 \text{ cm}^{-3}} \right)^{-1} \gamma_F^{-2} 1.1 \times 10^7 \text{ sec.} \] (28)

For a plausible interstellar cloud density \( n_1 > 1 \text{ cm}^{-3} \) and \( \gamma_F \approx 10^5 \), \( t_{\text{obs}} \) may be a millisecond or less. This justifies the assumption, made implicitly in the discussion of GRB rise times and pulse lengths, that shock-accelerated electrons radiate instantaneously; properly, the pulse profiles predicted by Equations 24 and 25 should be convolved with a broadening function which includes the radiation time, and which has a width \( t_{\text{obs}} \) in the local observer’s frame.

The characteristic frequency of synchrotron radiation, measured in the frame of fluid 2, is obtained from standard expressions using Equations 14 and 26. For pure hydrogen the result is
\[ \nu_2 \sim \zeta^{1/2} \left( \frac{n_1}{1 \text{ cm}^{-3}} \right)^{1/2} \gamma_F^{3/2} 3 \times 10^{11} \text{ sec}^{-1}, \] (29)

while Lorentz transformation to the local observer’s frame, using Equation 11, yields
\[ \nu_{\text{obs}} \sim \zeta^{1/2} \left( \frac{n_1}{1 \text{ cm}^{-3}} \right)^{1/2} \gamma_F^2 3 \times 10^{11} \text{ sec}^{-1}. \] (30)

MeV photons may be observed for \( \zeta = 1/2 \) if \( n_1 \sim 1 \text{ cm}^{-3} \) and \( \gamma_F \sim 4 \times 10^4 \), for example.

It has also been observed (Fishman 1993) that many GRB, or subpulses within them, show a progressive spectral softening with time. This is qualitatively explained using Figure 2 and Equation 18. If the radiation field is isotropic in the frame of fluid 2 (as will be the case if the particle distribution and magnetic field directions are isotropic) and has a characteristic photon energy, then in the local observer’s frame the spectral hardness above this characteristic spectral peak will be a decreasing function of \( \theta \). Higher frequency photons are preferentially observed from smaller values of \( \theta \), which arrive earlier in the burst or sub-pulse, while lower frequency photons are observed over a wider range of \( \theta \) and hence over a longer time. A quantitative prediction for the spectral evolution with time
could be made by numerical integration of the synchrotron emission function, but would depend on (uncertain) assumptions made regarding the energy and angular distributions of the radiating electrons.

3. Population G: Galactic GRB

GRB which show spectral features, typically around a few tens of KeV and at 400 KeV, have long been identified with Galactic magnetic neutron stars and are inexplicable at cosmological distances. Their distances cannot be determined from available data, and could be < 100 pc, ~ 100 Kpc, or anything in between. The familiar arguments concerning the mechanisms of GRB at Galactic distances center on two issues: the source of energy, and the physical conditions in the emitting region. The problems are harder, the greater the assumed distances. The observation of the March 5, 1979 event at a likely distance of 55 Kpc (Cline 1980) forces the consideration of distances of that order, and of correspondingly high luminosities, even though it is unclear whether it (a SGR) was a member of the Population G of GRB, or represented a distinct third class of objects.

The central problem of distant and luminous gamma-ray sources is gamma-gamma pair production (Cavallo and Rees 1978; Schmidt 1978; Katz 1982; Epstein 1985; Carrigan and Katz 1992). This process does not permit the escape of a large luminosity of MeV gamma-rays from a small region unless they are collimated, and thus excludes many models of GRB or SGR at Galactic halo or cosmological distances. It is well known that this problem is avoided in a collimated relativistic outflow of radiating matter, a consideration which led to the popularity of fireball models, in which an opaque cloud of radiation and pair gas adiabatically expands and cools until its particles’ velocity vectors are collimated outward. However, fireballs are conspicuously incapable of producing low redshift (400 KeV) annihilation lines, line features at tens of KeV, or the observed long and complex time structure. The interaction of fireballs with their environment may solve the temporal problem, as discussed in §2, but offers no hope of solving the spectral problem. The case
for magnetic neutron stars for GRB or SGR with spectral lines remains as strong as the data.

It is usually assumed that the radiating region of a GRB or SGR in a neutron star model is dominated by pair plasma, with \( n_+ \gg n_i \), where \( n_+ \) and \( n_i \) are the positron and ion densities, respectively. This assumption is made, in analogy to fireballs, even for non-fireball models, perhaps because the threatened gamma-gamma pair production catastrophe seems a likely source of dense pair plasma, and because the observed annihilation line requires a source of positrons. However, the assumption of pair dominance may not be justified in non-fireball models, such as are required to explain Population G GRB. If sufficient gamma-ray collimation is present to avoid a gamma-gamma pair production catastrophe, then the production of pairs may be negligibly small. When the observation of annihilation radiation provides empirical evidence for the production of some positrons, it should be remembered that an observably narrow annihilation line requires temperatures \(< 50\) KeV, and may be produced by a comparatively small number of positrons precipitated onto the cool neutron star surface; a hot pair plasma does not produce a recognizable annihilation line.

If a pair plasma is not an expanding fireball, it must be trapped on magnetic field lines (Carrigan and Katz 1992). Gravitation is unimportant for pairs, so they fill a magnetosphere (presumably of a magnetic neutron star). However, they quickly (in a free-flight time) precipitate onto the stellar surface, where they annihilate, because they more rapidly radiate their transverse momentum by the cyclotron process; even if the radiation density is sufficient to maintain most leptons in excited Landau (magnetic) states (a condition satisfied under only the most extreme conditions), their interaction with the radiation field destroys their transverse adiabatic invariant. In this magnetosphere-filling geometry the emergent radiation, by whatever process, is not collimated, and gamma-gamma pair production imposes its usual limits on the emergent flux of MeV gamma-rays.

It may be more satisfactory to consider an electron-ion plasma with only a small
admixture of positrons (sufficient to produce the observed annihilation line), so that \( n_+ \ll n_i \). An electric field may accelerate the electrons, which radiate by bremsstrahlung or by the cyclotron process after elastic scattering on the ions raises them to excited Landau states. Because most of the leptons are negative, they form a broadly collimated beam; if they are relativistic the resulting radiation is similarly collimated and there is no gamma-gamma pair production catastrophe or limit (other than a Planck function at an effective temperature characterizing the electron distribution function) on the emergent intensity.

In contrast, an electric field acting on a pair gas heats it but imparts no net momentum to the leptons; two counterstreaming beams of gamma-rays readily produce pairs, rapidly achieving equilibrium with them and limiting the emergent intensity. A minority admixture of positrons in an electron-ion plasma produces only a proportionately small countercurrent of gamma-rays to those produced by the electrons. This countercurrent removes an equal current of electron-produced gamma-rays by pair production, but the remaining electron-produced gamma-rays form a collimated beam and escape comparatively freely, suffering little or no (depending on the degree of collimation) gamma-gamma pair production. This is a consequence of the net momentum imparted to the lepton-photon system by the electric field, in analogy to the momentum imparted to a sector of a fireball by adiabatic expansion.

It is possible to make simple rough estimates of the parameters of an electrically heated ion-electron sheet plasma, which might be the source region of a GRB of Population G, following Katz (1993). Consider a sheet of thickness \( L \), be composed of positive ions of charge \( Z \) and density \( n_i \) (\( n_e = Zn_i \)), have transverse optical depth \( \tau \) and temperature \( T \), and radiate power per unit area \( P \). Define the dimensionless power per unit area \( p \equiv P\hbar^3/(m_e^4c^6) \), dimensionless thickness \( \ell \equiv Lm_e^2/c^2 \) and temperature \( t \equiv k_BT/(m_ec^2) \), where these quantities have been scaled to values characteristic of a relativistic electron (or pair) gas. The characteristic radiant intensity \( m_e^4c^6/\hbar^3 = 4.3 \times 10^{35} \text{ erg}/(\text{cm}^2 \text{ sec}) \) and length \( c^2/(m_ec^2) = 2.8 \times 10^{-13} \text{ cm} \) (the classical electron radius). The optical depth is

\[
\tau \sim n_e\sigma_0 L, \tag{31}
\]
where the characteristic cross-section $\sigma_0 \equiv e^4/(m_e^2 c^4)$. At nonrelativistic energies the appropriate cross-section is the Thomson cross-section $8\pi\sigma_0/3$, but at semi-relativistic energies of interest $\sigma_0$ may be a fair approximation. The observation of a nonthermal spectrum implies that $\tau$ cannot much exceed unity, but $\tau \sim 1$ and $\tau \ll 1$ are each possible.

The large field of a magnetized neutron star has a number of effects. It enters the argument of the effective Coulomb logarithm in collisional processes, typically reducing it to $\ln \Lambda \approx \ln(k_B T m_e c/(\hbar eB))$ (Katz 1982). Both bulk motion and current flow are restricted to be parallel to the field lines, justifying the assumption of thin sheet geometry and making the field distribution nearly force free ($\vec{J} \times \vec{B} = 0$). Perhaps most important, it means that any electron energy resulting from motion perpendicular to the field is immediately radiated. Even in conditions characteristic of the March 5, 1979 event at 55 Kpc the radiation density is far below Planckian (for $t \sim 1$), so that electrons may be assumed to be in their ground magnetic state until collisionally excited, and then to radiate as if in vacuum. The radiation rate per unit area may therefore be estimated using standard expressions for elastic scattering:

$$P \sim n_e n_i k_B T \left( \frac{k_B T}{m_e} \right)^{1/2} \sigma_0 Z^2 \left( \frac{m_e c^2}{k_B T} \right)^2 L \sim \frac{n_e^2 e^4}{m_e c} L Z \ln \Lambda t^{1/2}.$$  \hspace{1cm} (32)

This expression may be inverted, using the definitions of $p$, $\ell$, $t$, and $\alpha \equiv e^2/(\hbar c)$, to give

$$n_e \sim \left( \frac{pt^{1/2}}{t\alpha^3 Z \ln \Lambda} \right)^{1/2} \frac{m_e^3 c^3}{\hbar^3}.$$  \hspace{1cm} (33)

the characteristic density $m_e^3 c^3/\hbar^3 = 1.8 \times 10^{31}$ cm$^{-3}$. An alternative expression for $n_e$ is obtained from Equation 31:

$$n_e \sim \frac{\tau m_e^3 c^3}{\ell \alpha^3 \hbar^3}.$$  \hspace{1cm} (34)

Equating the expressions (33) and (34) yields a result for the thickness:

$$\ell \sim \frac{\tau^2 Z \ln \Lambda}{pt^{1/2} \alpha^3}.$$  \hspace{1cm} (35)
Very roughly, $p \sim 10^{-4}$ for the March 5, 1979 event, if in the LMC, and $p \sim 10^{-7}$ for a typical observed GRB if at 100 Kpc distance, corresponding to $L \sim 1.4\tau^2$ cm and $L \sim 1.4 \times 10^3\tau^2$ cm, respectively, where $Z = 26$, $t = 0.1$, and $\Lambda = 10$ were taken. The densities are correspondingly high.

If the radiating sheet is driven by magnetic reconnection, as is plausible, then the radiated power may be related to the electrical work done:

$$P \sim \sigma_{el}E^2L,$$

where $E$ is the electric field (properly, its component parallel to $\vec{B}$) and a nonrelativistic expression (Spitzer 1962) is used for the (cgs) electrical conductivity:

$$\sigma_{el} \approx 2 \left( \frac{2}{\pi} \right)^{3/2} \frac{(k_B T)^{3/2}}{m_e^{1/2}e^2\xi Z \ln \Lambda},$$

where the parameter $\xi \geq 1$ is a correction factor which allows for the possibility of anomalous (plasma instability) resistivity when current densities and electron drift velocities are large and for the decrease in conductivity when electron velocities approach $c$. The characteristic conductivity $m_e c^3/e^2 = 1.1 \times 10^{23}$ sec$^{-1}$. Equations (36) and (37) may be combined with the definitions of the various dimensionless parameters to give the electric field, current density, and electron drift velocity:

$$E \sim \left( \frac{po^3Z \ln \Lambda}{\tau t^{3/2}n_e m_e e^2} \right)^{1/2};$$

$$j = \sigma_{el}E \sim \frac{pl\alpha^3}{\tau \xi^{1/2} \ln \Lambda} \frac{m_i^3 c^7}{e^5};$$

$$v_{dr} = \frac{j}{n_e e} \sim \frac{ct^{1/2}}{\xi^{1/2}} \sim \frac{1}{\xi^{1/2}}.$$  

The characteristic current density $m_e^3c^7/e^9 = 6.5 \times 10^{38}$ esu/(cm$^2$ sec). The drift velocity is thus comparable to the electron thermal velocity unless $\xi \gg 1$; ion-acoustic instability is likely unless $\xi > m_i/(Zm_e)$.  

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GRB mechanisms based on magnetic reconnection, as in Solar flares, have been qual-
itative discussed for many years (Ruderman 1975). They may explain GRB energetics
and phenomenology (Katz 1982). Magnetic reconnection by sheet currents provides a nat-
ural explanation of the electrically heated sheets discussed in this section. If this model
is assumed, then Maxwell’s equations provide an additional relation among \( j \), \( L \), and the
magnetic field \( B \), permitting further constraints to be placed on the parameters. Because
\( \vec{j} \) is parallel to \( \vec{B} \), the direction of \( \vec{B} \) rotates across a sheet current without changing its
magnitude \( B \). If the total angle of rotation across a uniform current sheet is \( \pi \), then

\[
L = \frac{Bc}{4j}.
\]

Defining the usual characteristic magnetic field \( B_c \equiv m_e^2 c^3/(\varepsilon h) = 4.4 \times 10^{13} \) gauss and
the dimensionless parameter \( b \equiv B/(4B_c) \), Equation 41 may be rewritten

\[
\ell \sim \frac{\tau b \zeta^{1/2} Z \ln \Lambda}{pt\alpha^2}.
\]

Equating this expression to equation (35) yields

\[
\tau \sim \frac{b\alpha \zeta^{1/2}}{t^{1/2}}.
\]

For plausible \( t \sim 1 \) and \( b \sim 0.02 \), \( \tau \sim 10^{-4} \zeta^{1/2} \); either the emission region is very optically
thin or the resistivity is dominated by plasma wave scattering, and is far in excess of its
independent particle value. Either or both of these possibilities is acceptable.

The electric field (Equation 38) may be evaluated, using Equation 34 for \( n_e \) and
Equation 35 for \( \ell \), and defining the characteristic electric field \( E_c \equiv m_e^2 c^3/(\varepsilon h) \equiv B_c = 4.4 \times 10^{13} \) cgs:

\[
E \sim \left( \frac{p\zeta Z \ln \Lambda m_e^4 c^5}{\ell t^{3/2} h^3} \right)^{1/2}
\sim \frac{p\alpha^2 \zeta^{1/2}}{\tau t^{1/2}} E_c.
\]

If the energy release is driven by magnetic reconnection it is proper to use Equation 43 for
\( \tau \), yielding

\[
E \sim \frac{p\alpha}{b} E_c.
\]
At Galactic halo distances the brightest GRB may have $E$ sufficient to produce vacuum breakdown into a pair gas (Smith and Epstein 1993), but in less intense or closer GRB this will not occur, and the resistively heated ion-electron plasma discussed here may be sufficient. On a microscopic level, of course, the power is determined by the magnetic field strength and configuration, and by the mechanisms of plasma resistivity which drive reconnection.

The observed non-thermal gamma-ray spectrum of GRB requires the presence of a nonthermal distribution of “runaway” electrons, consistent with the large $v_{dr}$ and collective processes discussed above. It may be relevant that values of $t \sim 1$, consistent with the radiation of the bulk of the power of GRB at $\sim$ MeV energies, correspond to a maximum in the conductivity and therefore, under conditions of a fixed potential drop, to a maximum in the power dissipation.

If $Z \gg 1$, $\sigma_{el}$ increases approximately linearly with $n_+$ in the range $Zn_i < n_+ < Z^2n_i$ because of the increasing density of charge carriers without a corresponding decrease in their scattering length. This is in contrast to the usual near-independence of density of $\sigma_{el}$. Pair production thus may provide a natural thermostat at $t \sim 1$, reducing the power dissipated in regimes at the pair production threshold by increasing the conductivity under conditions of constant current. It is evident, of course, that observable cyclotron and annihilation lines require much lower values of $t$, and are plausibly produced by energy and positrons precipitated on the dense surface layers of the neutron star, cooled by black-body radiation.

4. Discussion

The fundamental problem of GRB phenomenology is the apparent inconsistency between their spatial distribution, which points strongly toward a cosmological origin, and the spectral features observed in some GRB, apparently inconsistent with such an origin. In this paper I have tried to reconcile these apparently contradictory data by assuming
two disjoint populations of GRB. Because the argument for a cosmological Population C is statistical, while the argument for a Galactic Population G is based on the observation of spectral lines from only a minority of GRB (perhaps from only a minority of that Population), it is difficult to assign an individual GRB to either population, unless it shows spectral lines or is identified with another astronomical object (the SGR of March 5, 1979 is the only good extant example of such an identification).

Fortunately, the models discussed in this paper predict another potential distinguishing characteristic. GRB in Population C produce their radiation by the synchrotron process of relativistic electrons. In the magnetic field is ordered the radiation will be linearly polarized. GRB in Population G produce their radiation by the cyclotron process of semi-relativistic electrons, Coulomb scattered into excited Landau states. This radiation is elliptically polarized, with a substantial circular component. In contrast, radiation produced by annihilation in a pair gas, such as the initial burst from a fireball or the trapped pair gas discussed by Carrigan and Katz (1992), is unpolarized.

The predicted characteristic frequency of gamma-ray emission in fireball debris impact models (Equation 30) is fairly sensitive to $\gamma_F$. The value of $\gamma_F$ depends on physical conditions within the fireball (Shemi and Piran 1990), and a wide range of $\gamma_F$ would be expected, with a corresponding range in spectra. In particular, smaller values of $\gamma_F$ would lead to X-ray, ultraviolet, or visible bursts, which should be searched for. Bursts of lower frequency radiation should have longer durations and smoother time histories (Equation 28), and perhaps also lower efficiencies, as accelerated electrons undergo adiabatic expansion before they radiate their energy.

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Figure Captions

Figure 1: Flow geometry in frame of contact discontinuity CD. S1 and S2 are shocks and numbers denote regions of fluid.

Figure 2: Emission geometry.

Figure 3: Radiation pulse emitted at radius $r$, plotted for $\theta_0^2 = 20c/u$. Dashed lines denote rise and fall for a cloud sharply circumscribed between $\theta_1 = (\tau_1\theta_0^2 - 2c/u)^{1/2}$ and $\theta_2 = (\tau_2\theta_0^2 - 2c/u)^{1/2}$. Note that abrupt rise at $\tau = 2c/(\theta_0^2u)$ is obtained for a uniformly filled medium, independent of the angular distribution of radiation, and does not require a cloud bounded in angle. Units of the ordinate are arbitrary.