Multiplicity of photohadronization and photon–hadron scaling violation

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The method of scaling transformations permitting to carry out the reconstruction of cross sections of $\gamma N$ and $\gamma\gamma$ interactions on the basis of cross sections of nucleon–(anti)nucleon interactions is suggested. The photon–hadron scaling violation is a consequence of dependence of scaling transformation parameter $\bar{n}(s)$ on the energy. The universal function $\bar{n}(s)$ is interpreted as the multiplicity of photohadronization. This function is established by processing the data on cross sections of nucleon–(anti)nucleon interactions in the low energy region $\sqrt{s} < 20$ GeV and is extrapolated to the high energy region up to $\sqrt{s} \sim 200$ GeV. The results of the reconstruction of $\gamma N$ cross sections at high energies and of $\gamma\gamma$ ones at all energies are in a remarkable agreement with available experimental data.

PACS numbers: 13.60.Hb, 13.85.Lg.

I. INTRODUCTION

In the present work we investigate the problem of the photon–hadron scaling violation. Analysis of available experimental data on cross sections of $\gamma N$ interactions shows that the scaling violation can be completely described within the framework of the multiple photohadronization model. The model development has led to the discovery of a new photon–hadron symmetry which can be called the local photon–hadron scaling. The question is the relation between cross sections of $\gamma N$ and $\gamma\gamma$ interactions and the ones of hadron–hadron interaction:

$$\sigma_{\gamma N}(s) = \bar{n}(s) \cdot \sigma_{\gamma N}^{(0)}(s/\bar{n}(s)),$$
$$\sigma_{\gamma\gamma}(s) = \bar{n}^2(s) \cdot \sigma_{\gamma\gamma}^{(0)}(s/\bar{n}^2(s)),$$

where $s$ is the energy squared in the center-of-mass system, $\sigma_{\gamma N}(s)$ and $\sigma_{\gamma\gamma}(s)$ are cross sections of $\gamma N$ and $\gamma\gamma$ interactions, $\sigma_{\gamma N}^{(0)}$ and $\sigma_{\gamma\gamma}^{(0)}$ are so-called calibration functions whose form is determined on the basis of processing the data on cross sections of nucleon–(anti)nucleon interactions. The parameter $\bar{n}(s)$ is the universal function describing the photohadronization multiplicity. This function and available hadron data allow to reconstruct the total cross sections of $\gamma N$ and $\gamma\gamma$ interactions up to energies $\sqrt{s_{\gamma N}}, \sqrt{s_{\gamma\gamma}} \sim 1000$ GeV via the scaling transformation (1).

In Section 2 we bring forward a more accurate formulation of scaling relations and construct the calibration function $\sigma_{\gamma N}^{(0)}$. In Section 3 we propose the model of multiple photohadronization. The analytical form of the photohadronization multiplicity, $\bar{n}(s)$, is imported from QCD calculations. Parameters of this function together with a criterion of its universality are established in Section 4. In Section 5 the same function is used for the reconstruction of cross section of $\gamma\gamma$ interaction.

II. EXPERIMENTAL DATA

Total cross sections of $\gamma p$ interaction are measured in a wide energy region from $\sqrt{s} \sim 1$ GeV to $\sqrt{s} = 210$ GeV. There are direct accelerator data of high precision and completeness at low energies. In what follows, we shall use the data obtained in [1] at $\sqrt{s} = 5.93 \pm 18.54$ GeV. In this region the typical value of the cross section is $\sigma_{\gamma p} \approx 115 \mu b$. At $\sqrt{s} \sim 200$ GeV there are results of four measurements obtained in DESY:

| $\sqrt{s}$, GeV | 180 | 210 | 200 | 210 |
|-----------------|-----|-----|-----|-----|
| $\sigma_{\gamma p}$, $\mu b$ | $143 \pm 4$ | $154 \pm 16$ | $165 \pm 2.3$ | $174 \pm 1$ |

The measurement at $\sqrt{s} = 180$ GeV and two ones at $\sqrt{s} = 210$ GeV have been carried out by ZEUS collaboration [2,3]. The cross section at $\sqrt{s} = 200$ GeV was measured by H1 collaboration [4]. Database [3] includes H1 result and one of ZEUS ones: $\sigma_{\gamma p} = 174 \pm 1 \mu b$ at $\sqrt{s} = 210$ GeV (two other results are likely to be understated). Further, database [3] includes cross sections measured by Baksan collaboration [6] in intermediate energy region:

| $\sqrt{s}$, GeV | 45 | 54 | 75 | 134 |
|-----------------|----|----|----|-----|
| $\sigma_{\gamma N}$, $\mu b$ | $128 \pm 14$ | $146 \pm 15$ | $148 \pm 23$ | $152 \pm 80$ |

Experimental data on cross sections of $\gamma p$ interaction are presented in Fig.1. Fig.2 displays data on cross sections of $\gamma\gamma$ interaction that we shall discuss in Section 5. As it is evident from the data shown in Fig.1, there is a rapid rise of $\gamma p$ cross section with the rise of energy. This fact is known as the effect of the photon–hadron scaling

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violation. The problems of photon–hadron scaling and its violation were discussed, for example, in [3, 4, 5]. It is customary to refer to the relation below, which follows from additive quark model, as “scaling”:

\[ \sigma_{\gamma p}^{(0)}(s) = P_{\gamma - v} \times \frac{2}{3} \bar{\sigma}_{NP}(3s/2), \]

where \( P_{\gamma - v} = 1/250 \) is the probability of photon hadronization to vector mesons \( \rho(770), \omega(782), \phi(1019) \), and

\[ \bar{\sigma}_{NP} = \frac{1}{4} (\sigma_{pp} + \sigma_{pp} + \sigma_{np} + \sigma_{np}). \]

Factors of \( 3/2 \) appear in (2) due to recalculation of the interaction cross section of a 3–quark system with a 3–quark one to the interaction cross section of a 2–quark system with a 3–quark one [5]. The quark diagram representing \( \gamma N \) interaction in the photon–hadron scaling mode is shown in Fig.3a.

We shall call the function \( \sigma_{\gamma p}^{(0)}(s) \) (2) the calibration curve for the cross section of \( \gamma p \) interaction. Clearly, the deviation of measured cross sections \( \sigma_{\gamma p}(s) \) from the calibration curve is a quantitative characterization of the scaling violation.

For constructing the calibration curve in the energy region \( \sqrt{s} = 5.93 - 22.97 \text{ GeV} \), we used data from [10, 11]. In the region \( \sqrt{s} = 30.4 - 62.7 \text{ GeV} \), where \( \sigma_{pp} = \sigma_{np} \), \( \sigma_{pp} = \sigma_{pp} \), with good accuracy, we used only the data on \( \sigma_{pp}(s) \) and \( \sigma_{pp}(s) \) from [12, 13]. For collider energies \( \sqrt{s} = 200, 546, 900, 1800 \text{ GeV} \), the \( pp \) data from [14, 15, 16, 17] were used. Experimental data, used to construct the calibration curve (after scaling transformation (2)), are presented in Fig.1. In the same figure, the fit of the data on averaged cross sections (3), subjected to the scale transformation \( 3s_{NP}/2 = s_{NP} \equiv s \), is shown. The fitting function and values of fitting parameters are

\[ \sigma_{\gamma p}^{(0)}(s) = C_{\gamma p} \ln \frac{s}{s_0} + A_{\gamma p} \left( 1 + \frac{s^{(1)}_{\gamma p}}{s + s^{(2)}_{\gamma p}} \right), \]

\[ C_{\gamma p} = 0.5777 \mu b, \quad \sqrt{s_0} = 2.198 \text{ GeV}, \]

\[ A_{\gamma p} = 95.65 \mu b, \]

\[ \sqrt{s^{(1)}_{\gamma p}} = 2.774 \text{ GeV}, \quad \sqrt{s^{(2)}_{\gamma p}} = 3.589 \text{ GeV}. \]

It should be stressed that \( \sigma_{\gamma p}^{(0)}(s) \) has the status of an experimentally established function. Its further usage in the analytical form (4) is motivated, first of all, by high quality of the fit. Additional physical grounds related to, for example, Froissart asymptotics have a certain sense, but they do not have crucial meaning for the problem under discussion.

**III. SCALING VIOLATION MODEL**

Photon–hadron scaling violation has a simple interpretation and, as we will show below, it is described by the

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**FIG. 1: Total cross section of \( \gamma N \) interaction. \( \sigma_{\gamma p}^{(0)}(s) \) is the calibration curve (4). The upper line is the cross section \( \sigma_{\gamma N}(s) \) according to (11).**

**FIG. 2: Total cross section of \( \gamma \gamma \) interaction. \( \sigma_{\gamma p}^{(0)}(s) \) is the calibration curve (4). The upper line is the cross section \( \sigma_{\gamma N}(s) \) according to (11).**

**FIG. 3: Quark diagrams of \( \gamma N \) interaction: a) photon–hadron scaling mode, b) multiple photonn–hadronization mode.**
universal function whose form is predicted by Quantum Chromodynamics (QCD). The main idea of the suggested model is illustrated in Fig.3b. We assume that the photon hadronization in the nonperturbative vacuum

(i) firstly, takes place before the interaction of hadronization products with a nucleon,

(ii) secondly, as a result of hadronization, a hadron cluster (mainly mesons) with total quantum numbers $J^{PC} = 1^{-}$ appears.

As one can see from Fig.3b, the multiplicity of $\gamma N$ interaction is produced with both the multiplicity of photohadronization process and the multiplicity of particles produced in the interaction of one of the mesons of the cluster with the nucleon. The latter means that the multiplicity of the photohadronization process is described by a function which has the same mathematical structure as the multiplicity function of processes of hadron–hadron interactions and $e^+e^-$ annihilation to hadrons. There are several different representations of the multiplicity function which are consistent with the experimental data. QCD calculations result in the functions of the following form:

$$\tilde{n}(s) = a + b \exp\left[ c\sqrt{\ln(s/\Lambda^2)} \right],$$

$$\tilde{n}(s) = b \ln^n(s/\Lambda^2) \exp\left[ c\sqrt{\ln(s/\Lambda^2)} \right],$$

where $\Lambda$ is a cutoff parameter whose value is determined by the hadronization threshold; $a$, $b$ and $c$ are fitted parameters. The first of the functions (5) was suggested in [18], the second one — in [19]. Note the functions (5) coincide with each other at $a = 0$. Using these functions to describe the multiplicity of photon hadronization we discovered that in the experimental data fitting the value of the parameter $a$ automatically turns out to be small and the value of cutoff parameter $\Lambda$ is automatically close to $\Lambda_{QCD} \approx 2 m_\pi$. The results of the four–parameter fit turn out to be statistically equivalent to the results of the two–parameter fit over parameters $b$ and $c$ with fixed values of $a = 0$, $\Lambda = 2m_\pi$. Because of this, we suggest the following formula for the cross section of $\gamma N$ interaction for the model presented in Fig.3b:

$$\sigma_{\gamma N}(s) = \tilde{n}(s) \cdot \sigma_{\gamma p}^{(0)}(s/\tilde{n}(s)), \quad \tilde{n}(s) = b \exp\left[ c\sqrt{\ln(s/4m_\pi^2)} \right].$$

The function $\sigma_{\gamma p}^{(0)}(s_n)$ in [8] depends on its argument $s_n \equiv s/\tilde{n}(s)$ in the same way as the function [14] depends on the variable $s$. The scaling transformation $s \rightarrow s_n$ takes into account a redistribution of the energy of the primary photon between products of its hadronization. The values of parameters $b$ and $c$ are established by comparing the model with the experimental data.

Note that [10] belongs to the additive quark model. The starting point is the expression for the cross section of an interaction of a nucleon, considered as a 3–quark system, with a 2–quark system. This expression should be subjected to the scaling transformation to the interaction cross section of a nucleon with a 2$\bar{n}$-quark system. The violation of photon–hadron scaling appears because the average number of quarks, which are produced in a photon hadronization and interact with a nucleon, increases with the energy.

IV. DATA FIT. A CRITERION OF UNIVERSALITY OF THE MECHANISM OF PHOTON-HADRON SCALING VIOLATION

We carried out two fitting procedures. In the first one we took into account all data on cross sections of $\gamma p$ interaction over the whole energy range plotted in Fig.1. The following values for the fitted parameters have been obtained:

$$b = 0.713 \pm 0.011, \quad c = 0.148 \pm 0.006,$$

$$\chi^2/\text{dof} = 0.949. \tag{7}$$

In second procedure we used only the low-energy data obtained in [1] in the region $\sqrt{s} = 5.93 - 18.54$ GeV. The results are

$$b = 0.714 \pm 0.025, \quad c = 0.147 \pm 0.013,$$

$$\chi^2/\text{dof} = 0.819. \tag{8}$$

As is seen from (7), (8), the values of fitted parameters obtained in different procedures are practically the same. When we extrapolate the fitting curve built on "low–energy" parameters (8) into the region of intermediate and high energies, it almost coincides with the curve whose parameters (7) were obtained by fitting all the data.

The possibility of the prediction of quantitative characteristics of photon–hadron scaling violation is a distinguishing and quite natural feature of the suggested model. Indeed, at the parton level, patterns of statistical process of photohadronization have to be completely formed at comparatively low energies — only slightly greater than the production threshold of strange hadrons. Parton cascades evolution has to be the universal function of the interaction energy whose analytical form is the same in the whole energy range.

The agreement of results of fits (7) and (8) can be interpreted as an argument supporting the hypothesis of multiple photohadronization and in the same time as a confirmation of the possibility to describe this phenomenon via the universal function of an energy of $\gamma N$ interaction.

V. RECONSTRUCTION OF A CROSS SECTION OF $\gamma\gamma$ INTERACTION

Dependence of a cross section of $\gamma\gamma$ interaction in the photon–hadron scaling approximation is described by the following formula

$$\sigma_{\gamma\gamma}^{(0)}(s) = P_{\gamma \gamma}^{2} \times \frac{4}{9} \sigma_{pp}(9s/4). \tag{9}$$
We use in (9) only cross sections of $\bar{p}p$ interactions, taking into account the similarity of quark–antiquark structures of $\bar{p}p$ and $\gamma\gamma$ pairs. Factors 9/4 appear in (9) as the result of recalculation of the interaction cross section of 3–quark systems $\bar{p}p$ to the one of 2–quark systems $VV$.

For constructing the calibration curve $\sigma_{\gamma\gamma}^{(0)}(s)$, we used all experimental data on $\sigma_{\gamma\gamma}(s)$ at $\sqrt{s} \geq 2.79$ GeV. After applying scaling transformation (9) to the data, we fitted them using the formula with Froissart asymptotics. The results are:

$$ \sigma_{\gamma\gamma}^{(0)}(s) = C_{\gamma\gamma} \ln \frac{s}{s_0} + A_{\gamma\gamma} \left(1 + \frac{s_{\gamma\gamma}^{(1)}}{s + s_{\gamma\gamma}^{(2)}}\right) $$

$$ C_{\gamma\gamma} = 1.541 \text{ nb}, \quad \sqrt{s_0} = 2.198 \text{ GeV}, \quad A_{\gamma\gamma} = 264.525 \text{ nb}, $$

$$ \sqrt{s_{\gamma\gamma}^{(1)}} = 2.855 \text{ GeV}, \quad \sqrt{s_{\gamma\gamma}^{(2)}} = 3.385 \text{ GeV}. $$

Asymptotically at $s \to \infty$, cross sections in all channels of nucleon–(anti)nucleon interactions are equal. Therefore the following relation holds:

$$ C_{\gamma\gamma} = \frac{2}{3} P_{\gamma \to \nu} C_{\gamma\mu} $$

Thereby the parameter $C_{\gamma\gamma}$ in (11) is fixed by the results of the fit (9). The parameter $s_0$ is taken to be common in (9) and (11) because this assumption does not contradict the experimental data. In fact, only three parameters $A_{\gamma\gamma}$, $s_{\gamma\gamma}^{(1)}$, $s_{\gamma\gamma}^{(2)}$ were fitted.

The experimental data on cross sections of $\gamma\gamma$ interaction (10) are presented in Fig.2. The scaling transformed experimental data, that were used for constructing the calibration curve (9), are shown in the same figure. The fit results of the data from the formula (11) show that this formula has the status of an experimentally established function.

Within the framework of the hypothesis of multiple photohadronization the formula for a cross section of $\gamma\gamma$ interaction is built by an obvious scaling transformation

$$ \sigma_{\gamma\gamma}(s) = \bar{n}(s) \cdot \sigma_{\gamma\gamma}^{(0)}(s/\bar{n}(s)), $$

$$ \bar{n}(s) = b \exp \left[ c \sqrt{\ln(s/4m_0^2)} \right]. $$

The function $\sigma_{\gamma\gamma}^{(0)}(s_0)$ in (11) depends on its argument $s_0 \equiv s/\bar{n}^2(s)$ in the same way as the function (11) depends on the variable $s$. In (11) and (11) $\bar{n}(s)$ is the same function with the same values of parameters $b$ and $c$. In such a way, one can reconstruct cross sections of $\gamma\gamma$ interaction using the processed experimental data on cross sections of $\gamma N$ interaction (see the corresponding curve in Fig. 2). The reconstruction results agree with the experimental data at the level of $\chi^2 \sim 0.5$. Only cross section values for $\sqrt{s} = 120.4$ and $158.7$ GeV noticeably deviate from the prediction of formula (12). However, these cross sections are measured with large systematic errors.

VI. DISCUSSION AND CONCLUSIONS

Cross sections of $\gamma N$ and $\gamma\gamma$ interactions can be obtained from hadron cross sections via scaling transformations (2), (6) and (9), (11). The photon–hadron scaling violation is completely described by dependence of the scaling transformation parameter $\bar{n}(s)$ appearing in (9), (11) on the energy. The universal function $\bar{n}(s)$ is interpreted as the multiplicity of a photohadronization. Parameters of this function are determined by processing the experimental data on cross sections of $\gamma p$ interaction in the region of low energies. Once the function $\bar{n}(s)$ is known, cross sections of $\gamma N$ interaction at high energies and the ones of $\gamma\gamma$ interaction at all energies can be reconstructed using the suggested model in complete agreement with available experimental data.

The results obtained in the present work show that the photon–hadron scaling problem can be formulated in new terms. Formulas (2) and (9) express the idea of the global scaling and numerical parameters in these formulas (2/3 and 4/9 respectively) make sense of parameters of global scaling transformations. Formulas (6) and (11), which describe experimental data, allow us to introduce a peculiar photon–hadron symmetry, namely, the local photon–hadron scaling with scaling transformation parameters $2\bar{n}(s)/3$ and $4\bar{n}^2(s)/9$ in $\gamma N$ and $\gamma\gamma$ channels respectively. It is important that the dependence of local scaling transformation parameters on the interaction energy is determined by the universal function $\bar{n}(s)$. The idea of local photon–hadron scaling is expressed explicitly by formulas connecting photon and hadron cross sections (we use (2), (6) and (9), (11))

$$ \sigma_{\gamma N}(s) = P_{\gamma \to \nu} \times \frac{2\bar{n}(s)}{3} \sigma_{Np} \left( \frac{3s}{2\bar{n}(s)} \right), $$

$$ \sigma_{\gamma\gamma}(s) = P_{\gamma \to \nu} \times \frac{4\bar{n}^2(s)}{9} \sigma_{pp} \left( \frac{9s}{4\bar{n}^2(s)} \right). $$

As one can see from Figures 1 and 2, the experimental data support the model of local photon–hadron scaling up to energies $\sqrt{s_{\gamma N}} \approx 200$ GeV and $\sqrt{s_{\gamma\gamma}} \approx 100$ GeV. Formulas (12) and available hadron data allow one to predict the photon cross sections up to energies $\sqrt{s_{\gamma N}}, \sqrt{s_{\gamma\gamma}} \sim 1000$ GeV. The validity of local photon–hadron scaling in the region of LHC and NLC energies is a prerogative of future experiments.

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