Exclusive and diffractive quarkonium – pair production at the LHC and FCC

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The production of a quarkonium – pair in exclusive and diffractive processes in pp collisions at the LHC and FCC energies is investigated. We consider the \( J/\Psi \Psi \) and \( \Upsilon \Upsilon \) production in these processes and present predictions for the transverse momentum and rapidity distributions considering the kinematical ranges expected to be covered by central and forward detectors. Results for the cross sections are also presented. Our results indicate that the double \( J/\Psi \) production is dominated by the exclusive process, while the double \( \Upsilon \) production receive a large contribution of the diffractive process. The impact of the modelling of the gap survival factor on our predictions is discussed.

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In the last years the production of a quarkonium – pair (\( J/\Psi \Psi \) and \( \Upsilon \Upsilon \)) have been the subject of a large number of studies\(^\text{1-22}\). Such analysis were strongly motivated by the experimental data from the D0\(^\text{23}\) Collaboration at Tevatron and the ATLAS, CMS and LHCb Collaborations at the LHC\(^\text{24-26}\), which have indicated that the contribution of the double partonic scattering is not negligible in inclusive proton – proton collisions, where both colliding protons dissociate in the interaction. In addition, the LHCb Collaboration have also released the first data for the exclusive double \( J/\Psi \) production\(^\text{27}\), where both colliding protons remain intact and the final state is characterized by two rapidity gaps, i.e. empty regions in pseudo-rapidity that separate the intact very forward protons from the \( J/\Psi \Psi \) state. The study of the exclusive reactions is expected to improve our understanding about the interplay between the small- and large-distance regimes of Quantum Chromodynamics (QCD)\(^\text{28}\). As pointed out in Refs.\(^\text{29, 30}\), a quarkonium – pair can also be produced in double diffractive reactions, which also are characterized by two rapidity gaps and two intact protons in the final state, but the quarkonium pair is generated by the interaction between gluons of the Pomeron (\( P \)), which is a color singlet object inside the proton. The results presented in Ref.\(^\text{30}\) indicated that the contribution of the double diffractive reaction for the double \( J/\Psi \) production are similar to those derived in Ref.\(^\text{31}\) considering the exclusive production. Such result motivates the analysis to be performed in this paper, where we will present a comparison between the predictions for the double quarkonium production in diffractive and exclusive reactions. In particular, we will update the predictions derived in Ref.\(^\text{30}\) considering a more recent parameterization for the diffractive gluon distribution, which was obtained in Ref.\(^\text{32}\) by fitting the latest HERA data for diffractive ep reactions. Moreover, we will present, for the first time, our predictions for the diffractive double quarkonium production at the energies of the Future Circular Collider (FCC)\(^\text{33}\). In addition, we will extend the formalism present in Ref.\(^\text{31}\) for the double \( \Upsilon \) production and estimate, for the first time, the prediction of this final state in exclusive reactions at the LHC and FCC. Our goal is to determine the kinematical range of dominance of these two reactions channels in order to be able to use future experimental data to constrain the underlying assumptions present in the description of diffractive and exclusive processes.

The typical diagrams that characterize the quarkonium – pair production in double diffractive and exclusive processes are represented in Fig.\(^\text{11}\). Differently from the exclusive case, where only the quarkonium – pair will be present in the final state, we have in the double diffractive process the presence of additional particles in the final state, associated to the remnants of the Pomeron, which dissociates in the interaction. The presence of these particles can be used, in principle, to discriminate the diffractive from the exclusive double quarkonium production. However, due to the large pile-up of events in each bunching crossing expected to be present in the future runs of LHC and FCC, it is not clear if the separation of the diffractive and exclusive events will be possible by measuring the rapidity gaps and counting the number of tracks in the final state. As a consequence, events characterized by two rapidity gaps probably only will be separated by tagging the intact hadrons in the final state using forward detectors, as e.g. the CT-PPS and AFP forward proton spectrometers, associated with the ATLAS and CMS central detectors\(^\text{34}\). As exclusive and double diffractive events are characterized by intact hadrons, it is fundamental to know what is the relative contribution of each of these processes for the quarkonium – pair production at central and forward rapidities as well as for LHC and FCC energies. In what follows we will present a brief review of the formalism proposed to describe these reactions, which is based on the Resolved Pomeron model\(^\text{35}\) in the diffractive case and in the Durham model\(^\text{36}\) in the exclusive case. The cross section for the double diffractive cross section can be estimated using the nonrelativistic QCD (NRQCD) factorization formalism\(^\text{37}\) for the quarkonium production and is given by\(^\text{38}\):

\[
d\sigma(pp \to p + QQ + p) = \sum_n g_p^D(x_1, \mu^2) g_p^D(x_2, \mu^2) \cdot d\hat{\sigma}[gg \to Q\bar{Q}_n + Q\bar{Q}_n] \cdot \langle O_n^Q \rangle \langle O_n^\bar{Q} \rangle, \tag{1}
\]
where \( Q \) represents the quarkonium state, \( g^D \) is the diffractive gluon distribution probed at the scale \( \mu^2 \), which we assume to be equal to \( \mu^2 = M^2 + p_{\perp}^2 \), and \( \Delta \sigma \) is the differential cross section for the \( gg \to QQ_n + QQ_\perp \) subprocess, which is perturbatively calculated considering the production of the heavy quark pair \( QQ \) in an intermediate Fock state \( n \), which does not have to be color neutral. Moreover, the quantities \( \langle O \rangle \) are nonperturbative long-distance matrix elements, which describe the transition of the intermediate \( QQ \) in the physical state \( Q \) via soft gluon radiation and are determined from experimental data. As in Ref. \( \text{[30]} \), we will estimate \( \Delta \sigma \) taking into account of the 31 diagrams that contribute for the color singlet channel, as well as the 72 diagrams for the color-octet channel. Moreover, we will assume the following values for the color – octet large-distance matrix elements: \( \langle O^q_3 \rangle (^1S_1) = 3.9 \times 10^{-3} \text{ GeV}^3 \) \( \text{[38]} \) and \( \langle O_8^q \rangle (^3S_1) = 1.5 \times 10^{-1} \text{ GeV}^3 \) \( \text{[32]} \). In the case of the color – singlet channel, it is possible to express the matrix elements in terms of the square of the radial wave function of the quarkonium \( Q \) at the origin \( (|R_Q(0)|^2) \) which is related to the leptonic decay rate \( \Gamma(Q \to e^+e^-) \) \( \text{[40]} \). Therefore, such quantity can be determined e.g. using the recent PDG data \( \text{[41]} \). Finally, in order to calculate the double diffractive cross section we should to assume a model for the diffractive gluon distribution. As in our previous studies \( \text{[29, 30]} \), we will describe this quantity using the Resolved Pomeron model \( \text{[35]} \), which implies that \( g^D(x, Q^2) \) is defined as a convolution of the Pomeron flux emitted by the proton, \( f^P_E(x_E) \), and the gluon distribution in the Pomeron, \( g^P_F(\beta, Q^2) \), with \( \beta \) being the momentum fraction carried by the partons inside the Pomeron. Such quantities can be constrained using the experimental data from diffractive deep inelastic scattering (DDIS) at HERA. Differently from Ref. \( \text{[30]} \), where we have used the fit B obtained by the H1 Collaboration at DESY-HERA \( \text{[42]} \) several years ago, in what follows we will consider the more recent parameterization obtained in Ref. \( \text{[52]} \) using the high – precision data from H1/ZEUS combined inclusive diffractive cross sections measurements. We will use the fit A proposed in Ref. \( \text{[52]} \), but we have verified that the results obtained using the fit B are very similar.

The central exclusive processes are usually described using the Durham model \( \text{[36]} \), proposed many years ago and extensively discussed in the literature (For a review see, e.g. Ref. \( \text{[43]} \)). In this model, the cross section for the central exclusive production of a quarkonium – pair can be expressed in terms of the skewed unintegrated gluon distributions \( f_g \) and the sub–amplitude for the \( gg \to QQ \) process \( \text{[31]} \). At leading logarithmic approximation, it is possible to express \( f_g(x, x', Q^2, \mu^2) \) in terms of the conventional integral gluon density \( g(x) \) and the Sudakov factor \( T \), which ensures that the active gluons that participate of the hard process do not radiate in the evolution from \( Q_1 \) up to the hard scale \( \mu = m_\perp \equiv \sqrt{M_Q^2 + p_{\perp}^2} \). The amplitude for the \( gg \to QQ \) process can be estimated using the hard exclusive formalism proposed in Refs. \( \text{[44, 45]} \) and considering the non – relativistic approximation. The results presented in Ref. \( \text{[31]} \) demonstrated that the exclusive reaction is only sensitive to the color – singlet component of the meson wave function, do not receiving color – octet contributions. The final expressions for the double \( J/\Psi \) production have been included in the publicly available SuperChic Monte Carlo (MC) \( \text{[48]} \). In order to also estimate the double \( \Upsilon \) production, we have modified the SuperChic and included this final state, which allow us to perform a full MC simulation of quarkonium – pair production in central exclusive processes. As in Ref. \( \text{[31]} \) we have fixed the value of the \( \Upsilon \) wave function at the origin to its leptonic width. Moreover, in our calculations we have considered that the conventional gluon distribution is given by the MMHT2014 parameterization \( \text{[47]} \).

One important open question in the description of central exclusive and diffractive interactions in \( pp \) collisions is the treatment of the soft interactions that are expected to lead to extra production of particles, which will destroy the rapidity gaps in the final state and modify the associated cross sections \( \text{[48]} \). The experimental results from

\[ f_g(x, x', Q^2, \mu^2) = f_g(x_1, Q^1, \mu^1) f_g(x_2, Q^2, \mu^2) f_g(x_3, Q^3, \mu^3) \]

FIG. 1: Typical diagrams for the quarkonium – pair production in double diffractive (left panel) and exclusive (right panel) processes. The blob denoted by \( \langle S^2 \rangle \) represents the gap survival factor associated to absorptive effects (See text for details).
TABLE I: Total cross sections for the quarkonium – pair production in central exclusive processes considering $pp$ collisions at \( \sqrt{s} = 14 \) TeV and different models for the absorptive factor. Values in parentheses are for FCC energies (\( \sqrt{s} = 100 \) TeV).

| Absorptive factor | $\sigma_{J/\Psi}$ [pb] | $\sigma_{\Upsilon}$ [pb] |
|-------------------|-------------------------|-------------------------|
| $\langle S_{\text{eik}} \rangle$ – Model 1 | 13.8 (132.1) | $9.3 \times 10^{-4}$ (8.1 × $10^{-5}$) |
| $\langle S_{\text{eik}} \rangle$ – Model 2 | 48.1 (352.5) | $2.4 \times 10^{-4}$ (2.2 × $10^{-5}$) |
| $\langle S_{\text{eik}} \rangle$ – Model 3 | 37.7 (250.1) | $1.6 \times 10^{-4}$ (1.7 × $10^{-5}$) |
| $\langle S_{\text{eik}} \rangle$ – Model 4 | 20.6 (196.7) | $1.2 \times 10^{-4}$ (1.2 × $10^{-5}$) |
| $\langle S^2 \rangle$ | 0.02 (0.01) | 39.3 (439.2) | $1.6 \times 10^{-4}$ (2.4 × $10^{-4}$) |

Tevatron and LHC for these processes have demonstrated that these additional absorption effects cannot be neglected. Theoretically, the soft rescattering corrections associated to reinteractions (often referred to as multiple scatterings) between spectator partons of the colliding protons imply the violation of the QCD hard scattering factorization theorem for diffraction \cite{49}. Such corrections modify the Resolved Pomeron and Durham predictions for the diffractive and central exclusive processes. The modelling of the soft multiple scattering in diffractive $pp$ collisions have been the subject of several studies during the last years. For example, in Refs. \cite{50,51,52} the authors have proposed to treat these effects using a general purpose Monte Carlo. However, such approaches are still strongly dependent on the treatment of the multiple interactions, the assumptions for the color flow along the rapidity gap as well as the modeling of possible proton excitations. In the case of the exclusive processes, the Durham group have proposed an approach to treat the absorptive corrections associated to the additional soft proton – proton interactions (denoted eikonal factor $S_{\text{eik}}^2$), which are independent of the hard processes, as well the rescatterings of the protons with the intermediate partons that are described by the so-called enhanced factor $S_{\text{enh}}^2$. As discussed in Ref. \cite{31}, the magnitude of the enhanced factor is still uncertain, but it is expected to generate a weaker suppression in comparison to that associated to the eikonal survival factor. In the case of the predictions for the double $J/\Psi$ production in exclusive processes presented in Ref. \cite{31}, the enhanced corrections were not included in the calculations. In what follows we also will assume this approximation. In Table I we present the predictions for the total cross sections for the quarkonium – pair production in $pp$ collisions at the LHC and FCC considering the four different models for the eikonal factor $S_{\text{eik}}^2$ present in the SuperChic MC. One have that the distinct treatments of $S_{\text{eik}}^2$ imply that the predictions can differ by a factor $\approx 3$. Another possible approach, largely used in the literature (See e.g. Refs. \cite{53,54}), is based on the assumption that the hard process occurs on a short enough timescale such that the physics that generate the additional particles can be factorized and accounted by an overall factor, denoted gap survival factor $\langle S^2 \rangle$, multiplying the cross section $\langle S^2 \rangle$. In general the values of $\langle S^2 \rangle$ depend on the energy, being typically of order 0.01 – 0.05 for LHC energies. In particular, for the quarkonium – pair production in double diffractive interactions in $pp$ collisions at $\sqrt{s} = 14$ (100) TeV it is expected to be 0.02 (0.01), i.e. the absorptive corrections are expected to suppress the cross section by a factor 50 (100). For comparison, in Table I we also present the results derived multiplying by $\langle S^2 \rangle$ the SuperChic predictions calculated without the inclusion of the absorptive corrections. For LHC energy and the double $J/\Psi$ production, the resulting predictions are similar to those derived with $S_{\text{eik}}^2$. For FCC energy, the prediction is slightly larger. In contrast, for the double $\Upsilon$ production, we predict larger cross sections assuming the overall factor $\langle S^2 \rangle$ instead of $S_{\text{eik}}^2$. As the modelling, magnitude and universality of absorptive corrections are still a theme of intense debate \cite{55,56,57}, in what follows we will assume that the absorptive corrections for the double diffractive and central exclusive processes can be modelled by the same factor $\langle S^2 \rangle$. Surely such assumption can and must be improved in the future. However, considering the current large theoretical uncertainty in the treatment of the soft interactions, we believe that such simplistic approach allow us, at least, to understand what are the main differences between the diffractive and exclusive quarkonium – pair production associated to the distinct approaches for the hard process. In what follows we will present our predictions for the transverse momentum and rapidity distributions as well for the cross sections considering the diffractive and central exclusive quarkonium – pair production in $pp$ collisions at the LHC and FCC energies. We will present results for the typical rapidity ranges covered by central ($-2.5 \leq y \leq 2.5$) and forward ($2.0 \leq y \leq 4.5$) detectors. For the cross sections we also will present our predictions for $\sqrt{s} = 27$ TeV, which is the center – of – mass energy expected to be achieved in the High – Energy Large Hadron Collider (HE-LHC) \cite{62}. In our analysis we will assume that $|R_{J/\Psi}(0)|^2 = 0.56$ GeV$^2$ and $|R_{\Upsilon}(0)|^2 = 2.21$ GeV$^2$. The predictions for the transverse momentum distribution are presented in Fig. \ref{fig:transverse_momentum} considering $pp$ collisions at $\sqrt{s} = 14$ TeV. We have verified that similar results are obtained for the energies of HE-LHC and FCC, with the main difference being the normalization of the distributions. For the double diffractive production, we have that the distribution decreases with $p_T$ following a power - law behavior $\propto 1/p_T^n$, where the effective power $n$ is dependent of the final state considered. Such behaviour is expected, since the quarkonium – pair in the final state in diffractive interactions is generated in
In Fig. 2 we present our predictions for the rapidity distributions considering pp collisions at LHC (left panels) and FCC (right panels) energies. We have that the diffractive mechanism implies wider distributions. Moreover, our results indicate that the production of a double $J/\Psi$ at midrapidities will be dominated by the central exclusive process, with the dominance increasing with the energy. In contrast, we predict the dominance of the diffractive process in the case of double $\Upsilon$ production at the LHC. For the FCC energy, our results indicate that the contribution of the diffractive and central exclusive mechanisms will be similar.

In Table II we present our predictions for the cross sections considering pp collisions for the center-of-mass energies of the LHC, HE-LHC and FCC, and different rapidity ranges. For the HE-LHC energy we assume that $\langle S^2 \rangle = 0.015$. We predict cross sections of order of pb (fb) in the case of the double $J/\Psi$ ($\Upsilon$) production, which increase with the energy and are smaller in the forward rapidity range. For the central exclusive processes the increasing is steeper, which is expected since the cross section is proportional to the forth power of the conventional gluon distribution while in the DD case the cross section is proportional to the square of the diffractive gluon distribution. In agreement with the results presented in Fig. 3 we have that double $J/\Psi$ production is dominated by the central exclusive production. On the other hand, for the double $\Upsilon$ production at the LHC, the DD process dominates. For

In contrast, in the exclusive production, we have that the typical transverse momentum of the quarkonium – pair is determined by the transferred momentum in the Pomeron - proton vertex. As the exclusive cross section has an $e^{-|t|/\beta}$ behavior, where $\beta$ is the slope parameter associated, the associated $p_\perp$ distribution decreases exponentially at large transverse momentum. Therefore, it is expected that the production of a quarkonium – pair with a large $p_\perp$ should be dominated by the diffractive mechanism. On the other hand, if only events with $p_\perp \leq 1$ GeV are selected, the observed quarkonium - pairs will be mainly produced by the exclusive process. It is important to emphasize that our results also indicate that the contribution of the diffractive process for the double $\Upsilon$ production will not be negligible at small – $p_\perp$.
larger energies, the contribution of the double diffractive and central exclusive processes becomes similar.

Finally, let us summarize our main conclusions. In this paper we have investigated the quarkonium – pair production in double diffractive and central exclusive processes considering pp collisions at the LHC, HE-LHC and FCC energies. For the treatment of the double diffractive production we have used the nonrelativistic QCD (NRQCD) factorization formalism for the quarkonium production and the Resolved Pomeron model to describe the diffractive processes. On the other hand, in the case of central exclusive processes, we have considered the Durham model to describe the

| Energy  | Process                                    | 10 TeV | 20 TeV | 40 TeV |
|---------|--------------------------------------------|--------|--------|--------|
| 14 TeV  | $\sigma_{DD}(pp \to p + J/\Psi J/\Psi+p)$  | 10.2 pb| 3.7 pb | 1.7 pb |
|         | $\sigma_{CEP}(pp \to p + J/\Psi J/\Psi+p)$ | 39.3 pb| 25.9 pb| 5.1 pb |
|         | $\sigma_{DD}(pp \to p + \Upsilon p)$       | 1.2 $\times$10^{-2} pb | 6.5 $\times$10^{-3} pb | 2.7 $\times$10^{-3} pb |
|         | $\sigma_{CEP}(pp \to p + \Upsilon p)$      | 1.6 $\times$10^{-3} pb | 1.3 $\times$10^{-3} pb | 2.7 $\times$10^{-5} pb |
| 27 TeV  | $\sigma_{DD}(pp \to p + J/\Psi J/\Psi+p)$  | 10.3 pb| 3.9 pb | 1.8 pb |
|         | $\sigma_{CEP}(pp \to p + J/\Psi J/\Psi+p)$ | 85.0 pb| 50.9 pb| 18.3 pb|
|         | $\sigma_{DD}(pp \to p + \Upsilon p)$       | 1.5 $\times$10^{-2} pb | 7.3 $\times$10^{-3} pb | 3.0 $\times$10^{-3} pb |
|         | $\sigma_{CEP}(pp \to p + \Upsilon p)$      | 3.9 $\times$10^{-3} pb | 2.8 $\times$10^{-3} pb | 7.5 $\times$10^{-4} pb |
| 100 TeV | $\sigma_{DD}(pp \to p + J/\Psi J/\Psi+p)$  | 11.4 pb| 4.0 pb | 2.0 pb |
|         | $\sigma_{CEP}(pp \to p + J/\Psi J/\Psi+p)$ | 439.2 pb| 222.3 pb| 90.4 pb|
|         | $\sigma_{DD}(pp \to p + \Upsilon p)$       | 4.7 $\times$10^{-2} pb | 2.0 $\times$10^{-2} pb | 8.8 $\times$10^{-3} pb |
|         | $\sigma_{CEP}(pp \to p + \Upsilon p)$      | 2.4 $\times$10^{-2} pb | 1.4 $\times$10^{-2} pb | 5.1 $\times$10^{-3} pb |

TABLE II: Cross sections for the quarkonium – pair production in double diffractive (DD) and central exclusive processes (CEP) considering pp collisions at the LHC, HE-LHC and FCC energies.
interaction. We estimated the rapidity and transverse momentum dependencies of the cross sections for the $J/\Psi J/\Psi$ and $\Upsilon \Upsilon$ production and presented predictions considering the kinematical rapidity ranges probed by central and forward detectors. The absorptive corrections have been included in our calculations assuming a simplistic model to treat the soft rescattering corrections. Our results demonstrated that the contribution of the central exclusive (double diffractive) processes can be separated selecting events where the transverse momentum of the pair is small (large). Our results indicate that the study of the quarkonium – pair production can be useful to test the underlying assumptions present in the description of the double diffractive and central exclusive processes.

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