Nuclear symmetry energy with mesonic cross-couplings in the effective chiral model

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The effective chiral model is extended by introducing the contributions from the cross-couplings between isovector and isoscalar mesons. These cross-couplings are found to be instrumental in improving the density content of the nuclear symmetry energy. The nuclear symmetry energy as well as its slope and curvature parameters at the saturation density are in harmony with those deduced from a diverse set of experimental data. The equation of state for pure neutron matter at sub-saturation densities is also in accordance with the ones obtained from different microscopic models. The maximum mass of neutron star is consistent with the measurement and the radius at the canonical mass of the neutron star is within the empirical bounds.

I. INTRODUCTION

Over the last decade or so there has been an extensive work and debate dedicated to understanding the behavior of nuclear symmetry energy theoretically as well as experimentally, both at low and high densities. This knowledge is helpful in understanding both finite nuclei and nuclear matter aspects such as Neutron Stars (NS) and supernovae dynamics, related to neutron-rich domain. It also helps in understanding the strong forces at the fundamental level at higher densities. Currently available data on nuclear masses and giant dipole polarizability have constrained the values of symmetry energy and its slope parameter to \( J \sim 32 \text{ MeV} \) and \( L \sim 50 - 80 \text{ MeV} \) at nuclear saturation density \( (\rho \sim 0.16 \text{ fm}^{-3}) \). However, little is known about their behavior at other densities. Motivated by this, theoretically one tries to modify the basic interactions so as to match with the experimental data wherever available. The different variants of the Relativistic Mean Field Models (RMF) could reach out to these values only when the contributions from the cross-coupling of \( \rho \) meson to the \( \sigma \) or \( \omega \) mesons were included.

The models based on chiral symmetry was introduced by Gell-Mann & Levy \([1]\). The importance of chiral symmetry in the study of nuclear matter was emphasized by Lee & Wick \([2]\). However, the linear chiral sigma models fail to describe properties of finite nuclei. In such models, the normal vacuum jumps to a chirally restored normal vacuum (Lee-Wick vacuum) \([12, 13]\). This phenomenon is referred to as chiral collapse problem \([14]\) and it can be overcome mainly in two ways. One of the approaches is to incorporate logarithmic terms of the scalar field in chiral potentials \([15, 16]\) which prevents the normal vacuum from collapsing. This class of chiral models are phenomenologically successful in describing finite nuclei \([21, 22]\). However, these models explicitly break the chiral symmetry and are divergent when chiral symmetry is restored \([15]\).

Alternatively, the chiral collapse problem is prevented by generating the isoscalar-vector meson mass dynamically via Spontaneous Symmetry Breaking (SSB) by coupling the isoscalar-vector mesons with the scalar mesons \([23, 24]\). However, the main drawback of all these models was the unrealistic high nuclear incompressibility \((K)\). Later on, in several attempts, the higher order terms of scalar meson field \([26, 27]\) were introduced to ensure a reasonable \( K \) at saturation density. The non-linear terms in the chiral Lagrangian can provide the three-body forces \([28]\) which might have important roles to play at high densities. The effective chiral model has been used to study nuclear matter aspects such as matter at low density and finite temperature \([29]\), NS structure and composition \([30]\) and nuclear matter saturation properties. As emphasized in Ref. \([27]\), the model parameters are constrained and related to the vacuum expectation value of the scalar field. Since the mass of the isoscalar-vector meson is dynamically generated, practically there are very few free parameters to adjust the saturation properties. However, this type of models had a couple of drawbacks. They yield the symmetry energy slope parameter, \( L \sim 90 \text{ MeV} \), which is a little too large. Also, the symmetry energy at 0.1 \( \text{fm}^{-3} \) baryon density is \( \sim 22 \text{ MeV} \), which is lower than the presently estimated value \([1, 31]\).

In the present work, we employ the effective chiral model in which chiral symmetry breaks spontaneously. We extend this model by including the cross-couplings of \( \sigma \) and \( \omega \) mesons with the \( \rho \) meson. We would like to see whether these terms in the interaction help in fixing the values of symmetry energy and its slope parameter at the saturation density. We study the effects of the cross-couplings on the Equation of State (EoS) for Asymmetric Nuclear Matter (ANM). The effects of the crustal EoS on the mass and the radius of NS are evaluated using the method suggested recently by Zdunik et al. \([32]\).

The paper is organized as follows. We briefly describe...
In Section II we construct three different models with no cross-coupling, the \( \sigma - \rho \) cross-coupling and the \( \omega - \rho \) cross-coupling and corresponding results are discussed. Conclusions are drawn in Section IV.

II. THE MODEL

The complete Lagrangian density for the effective chiral model which includes the various cross-coupling terms is given by,

\[ \mathcal{L} = \mathcal{L}_t + \mathcal{L}_x, \tag{1} \]

where,

\[
\mathcal{L}_t = \bar{\psi}_B \left[ \left( i \gamma_\mu \partial^\mu - g_\omega \gamma_\mu \omega^\mu - \frac{1}{2} g_\rho \rho_\mu \vec{r} \gamma^\mu \right) \psi_B + \frac{1}{2} \left( \partial_\mu \vec{r} \right) \partial^\mu \vec{r} + \partial_\mu \sigma^\mu \right]
- \frac{\lambda}{4} \left( x^2 - x_0^2 \right)^2 - \frac{\lambda_b}{6 \pi^2} \left( x^2 - x_0^2 \right)^{3/2} - \frac{\lambda_c}{8 \pi^2} \left( x^2 - x_0^2 \right)^2 - \frac{1}{4} F_{\mu \nu} R^\mu \psi + \frac{1}{2} g^2 x^2 (\omega_\mu \omega^\mu) + \frac{1}{2} m_\rho^2 \rho_\mu \rho^\mu,
\tag{2} \]

and

\[
\mathcal{L}_x = \eta_1 \left( \frac{1}{2} g^2 \rho_\mu \rho^\mu \right) + \eta_2 \left( \frac{1}{2} \rho_\mu \rho_\nu \rho^\mu \rho^\nu \omega_\mu \omega^\mu \right). \tag{3} \]

Here, \( \psi_B \) is the nucleon isospin doublet interacting with different mesons \( \sigma, \omega \) and \( \rho \). The \( b \) and \( c \) are the strength for self couplings of scalar fields. The \( \gamma^\mu \) are the Dirac matrices and \( \tau \) are the Pauli matrices. \( \mathcal{L}_t \) (Eq. 2) is the original Lagrangian given in Ref. 30. Note that potential for the scalar fields \( (\pi, \sigma) \) are written in terms of a chiral invariant field \( x \) given by \( x^2 = x_0^2 + \sigma^2 \).

\( \mathcal{L}_x \) (Eq. 3) is the new additional piece we add to the original Lagrangian given in 30. It contains cross-coupling terms between \( \rho \) and \( \omega \) and also between \( \rho \) and \( \sigma \). The coupling strength for \( \sigma - \rho \) and \( \omega - \rho \) are given by \( \eta_1 \rho_\sigma^2 \) and \( \eta_2 \rho_\rho^2 \) respectively. The interaction of the scalar \( (\sigma) \) and the pseudo-scalar \( (\pi) \) mesons with the isoscalar-vector meson \( (\omega) \) generates a dynamical mass for the \( \omega \) meson through SSB of the chiral symmetry with scalar field obtaining the vacuum expectation value \( x_0 \). Then the mass of the nucleon \( (m) \), the scalar \( (m_\sigma) \) and the vector meson mass \( (m_\omega) \), are related to \( x_0 \) (vacuum expectation of \( x \)) through

\[
m = g_\sigma x_0, \quad m_\sigma = \sqrt{2 \lambda x_0}, \quad m_\omega = g_\omega x_0, \tag{4} \]

where, \( \lambda = \frac{m_\pi^2 - m_\rho^2}{2 f_\pi} \) and \( f_\pi = x_0 \) is the pion decay constant, which reflects the strength of SSB. In Eq. 3 when \( \eta_1 \neq 0 \) there is a cross-interaction between \( \rho \) and \( \sigma \).

Hence a fraction of \( \rho \) meson mass will come from SSB. The mass of \( \rho \) meson \((m_\rho)\) in this model then will be related to vacuum expectation of \( x \) through

\[
m_\rho^2 = m_\rho^2 + \eta_1 \rho_\rho^2 x_0. \tag{5} \]

In the mean field treatment the explicit role of pion mass is ignored and hence \( m_\pi = 0 \) and mesonic field is assumed to be uniform, i.e., without any quantum fluctuation. Then, the isoscalar-vector field \( \omega \) is of the form \( \omega_\mu = \omega_0 \delta^\mu_\mu \), where \( \delta^\mu_\mu \) is Kronecker delta. Note that \( \omega_0 \) does not depend on space-time but it depends on baryon density \( (\rho) \). The vector field \( (\omega) \), scalar field \( (\sigma) \) and isovector field \( (\rho_\sigma^2) \) equations (in terms of \( Y = x/x_0 = m^*/m \)) are, respectively, given by:

\[
\left[ m_\omega^2 (1 - Y^2) + \eta_2 C_\omega m_\rho^2 (\rho_\sigma^2) \right] \omega_\mu = g_\omega \rho_\rho \rho_\mu,
\tag{6}
\]

\[
(1 - Y^2) - \frac{b}{m^2 C_\omega} (1 - Y^2)^2 + \frac{c}{m^4 C_\omega^2} (1 - Y^2)^3 + \frac{2 C_\sigma m_\omega^2 (\rho_\sigma^2)}{m^2} + \frac{2 \eta_1 C_\sigma C_\rho m_\rho^4 (\rho_\sigma^2)^2}{C_\omega m^2} + \frac{2 C_\rho \rho_\rho (\rho_\sigma^2)^2}{m^2} = 0, \tag{7}
\]

\[
m_\rho^2 \left[ 1 - \eta_1 (1 - Y^2) C_\rho / C_\omega + \eta_2 C_\omega \omega_\omega \rho_\rho^2 \right] \rho_\rho = \frac{1}{2} g_\rho (p_\rho - \rho_\omega). \tag{8} \]

The quantity \( \rho_\rho \) and \( \rho_\sigma \) are the baryon and the scalar density defined as,

\[
\rho = \frac{\gamma}{(2\pi)^3} \int_0^{k_F} d^3 k, \tag{9}
\]

\[
\rho_\sigma = \gamma \left( \frac{2\pi^3}{m^*} \right) \int_0^{k_F} \frac{m^*}{\sqrt{m^* + k^2}} d^3 k, \tag{10} \]

where, \( k_F \) is the baryon fermi momentum and \( \gamma \) ( for example, \( \gamma = 4 \) for Symmetric Nuclear Matter (SNM)) is the spin degeneracy factor. \( C_\sigma \equiv g_\sigma^2 / m_\sigma^2 \), \( C_\omega \equiv g_\omega^2 / m_\omega^2 \) and \( C_\rho \equiv g_\rho^2 / m_\rho^2 \) are the scalar, vector and isovector coupling parameters. The energy density \( (\epsilon) \) and pressure \( (p) \) for a given baryon density \( (\text{in terms of } Y = m^*/m) \) in this model is obtained from the stress-energy tensor, which is given as

\[
\epsilon = \frac{1}{2} \sum_{k_\rho, k_\sigma} \int_0^{k_F} k^2 \sqrt{k^2 + m^2} dk + \frac{m^2}{8 \pi^2} (1 - Y^2)^2 \frac{b}{12 C_\sigma C_\omega} (1 - Y^2)^3 + \frac{c}{16 \pi^2 C_\sigma^2} (1 - Y^2)^4 + \frac{1}{2} m_\rho^2 \omega_\omega Y^2 \frac{1}{2} m_\rho^2 \left[ 1 - \eta_1 (1 - Y^2) C_\rho / C_\omega + 3 \eta_2 C_\omega \omega_\omega \rho_\rho^2 \right] (\rho_\rho^2)^2, \tag{11}
\]

\[
p = \frac{1}{3 \pi^2} \sum_{k_\rho, k_\sigma} \int_0^{k_F} \frac{k^4}{\sqrt{k^2 + m^2}^2} dk - \frac{m^2}{8 \pi^2} (1 - Y^2)^2 \frac{b}{12 C_\sigma C_\omega} (1 - Y^2)^3 - \frac{c}{16 \pi^2 C_\sigma^2} (1 - Y^2)^4 - \frac{1}{2} m_\rho^2 \omega_\omega Y^2 \frac{1}{2} m_\rho^2 \left[ 1 - \eta_1 (1 - Y^2) C_\rho / C_\omega + 3 \eta_2 C_\omega \omega_\omega \rho_\rho^2 \right] (\rho_\rho^2)^2. \tag{12} \]

For SNM we have to set \( k_n = k_p = 0 \). As our present knowledge of nuclear matter is mainly confined
to normal nuclear matter density ($\rho_0$), coupling constants $C_\sigma \equiv g_\sigma^2/m_\sigma^2$ and $C_\omega \equiv g_\omega^2/m_\omega^2$ are not free parameters in the Eqs. [11][12]. To obtain $C_\sigma$ and $C_\omega$, we solve the field equations (Eqs. [9][3]) self consistently while satisfying the nuclear saturation properties. Note that for different values of $Y = x_0/x = m^*/m$, we get different values of $C_\sigma$ and $C_\omega$.

After inclusion of cross interactions $L_\times$ (Eq. [35]) the modified symmetry energy $S(\rho)$ in this model is

$$S(\rho) = \frac{k_F^2}{6\sqrt{k_F^2 + m^2}} + \frac{C_\sigma k_F^2}{12\pi^2(m^*/m_\rho)^2} + \frac{C_\omega k_F^3}{3\pi^2(m^*/m_\rho)^2} \left( \frac{2\omega_C^2 C_\omega k_F^2}{27m^2 Y^2(m^*/m_\rho)^2} \right)$$

where, $m^2 = m_\rho^2 (1 - \eta_1 (1 - Y^2) \sigma_\rho / C_\rho + \eta_2 \rho \omega^2)$ and $k_F = (3\pi^2 n / 2)^{1/3}$. The coupling parameters $C_\sigma, \eta_1$ and $\eta_2$ can be evaluated numerically by fixing symmetry energy $S(\rho)$ and its slope parameter $L$ at saturation density ($\rho_0$). Without cross-couplings ($\eta_1 = \eta_2 = 0$) we revert back to the Lagrangian given in [30].

The symmetry energy can be expanded in Taylor series around saturation density ($\rho_0$) as

$$S(\rho) = J_0 + L\epsilon_1 + \frac{K_{\text{sym}}}{2} \epsilon_1^2 + \frac{1}{6} Q_{\text{sym}} \epsilon_1^3 + O(\epsilon_1^4),$$

where, $\epsilon_1 = \frac{\rho - \rho_0}{\rho_0}$. The symmetry energy coefficient at $\rho_0$ is $J_0$ and the other coefficients are defined at $\rho_0$ as [34].

$$L = 3\rho \frac{\partial S(\rho)}{\partial \rho} \bigg|_{\rho = \rho_0},$$

$$K_{\text{sym}} = 9\rho \frac{\partial^2 S(\rho)}{\partial \rho^2} \bigg|_{\rho = \rho_0},$$

$$Q_{\text{sym}} = 27\rho^3 \frac{\partial^3 S(\rho)}{\partial \rho^3} \bigg|_{\rho = \rho_0}. $$

Similarly, the nuclear incompressibility ($K$) of ANM can also be expanded in terms of $\delta$ at $\rho_0$ as $K(\delta) = K + K_\sigma \delta^2 + O(\delta^4)$, where $\delta = \frac{(\rho - \rho_0)}{\rho}$ is the isospin asymmetry and $K_\sigma$ is given by

$$K_\sigma = K_{\text{sym}} - 6L - \frac{Q_0L}{K}. $$

where $Q_0 = 27\rho^3 \frac{\partial^3 S(\rho)}{\partial \rho^3} \bigg|_{\rho = \rho_0}$ in SNM.

### III. RESULTS AND DISCUSSION

As can be seen from the preceding section that the EoS of the SNM are determined by the coupling parameters $C_\sigma, C_\omega, b$ and $c$ (Eqs. [11][12]). The values of these coupling parameters and resulting SNM properties at the saturation density are listed in Table III. The values of the model parameters lie in the stable region [35].

The density dependence of symmetry energy $S(\rho)$ is obtained by using three different variants of the present model. We consider the case of no cross-coupling (NCC), the $\sigma - \rho$ cross-coupling (SR) and the $\omega - \rho$ cross-coupling (WR). Since the NCC model has only one free parameter (i.e., $C_\rho$) there is not enough freedom to vary $J_0$ and $L$ independently. However, the SR and WR models can provide some flexibility to adjust them. Note that, in comparison to the earlier models (i.e., NCC type), the inclusion of cross-couplings have important implications on $S(\rho)$. The effects of the cross-couplings grow stronger at high densities which are relevant for the study of NS properties.

In Table I we list the values of symmetry constants ($C_\sigma, \eta_1$ and $\eta_2$) and the resulting nuclear matter properties: $J_0, L, K_{\text{sym}}, Q_{\text{sym}}$ and $K_\sigma$ at the saturation density $\rho_0$ and $J_1$ - the symmetry energy at $\rho_1 = 0.1$ fm$^{-3}$. For the NCC, $C_\rho$ is adjusted to yield $J_0 = 32.5$ MeV. For SR(WR) model, the value of $C_\rho$ and $\eta_1$($\eta_2$) are adjusted to yield $J_0 = 32.5$ MeV and $L = 65$ MeV. These values are compatible with $J_0 = 31.6 \pm 2.66$ MeV and $L = 58.9 \pm 16$ MeV obtained by analyzing various terrestrial experimental informations and astrophysical observations [35]. It may be noted that the value of $J_1$ obtained for the NCC model shows a significant deviation from 24.1 ± 0.8 MeV [1] and 23.6 ± 0.3 MeV [31] obtained by analyzing the experimental data on isovector giant resonances, whereas, $J_1$ is in good agreement in case of SR and WR models.

### Table I: List of the model parameters determined from the properties of SNM such as, energy per nucleon $E_0$, nuclear incompressibility $K = 247$ MeV and the nucleon effective mass $Y = m^* / m = 0.864$ at the saturation density $\rho_0 = 0.153$ fm$^{-3}$. The scalar and vector meson coupling parameters are $C_\sigma = g_\sigma^2/m_\sigma^2$ and $C_\omega = g_\omega^2/m_\omega^2$ respectively. $B = b/m^*$ and $C = c/m^*$ are the parameters for the higher order self-couplings of the scalar field with $m$ being the nucleon mass. The nucleon, $\omega$ meson and $\sigma$ meson masses are 939 MeV, 783 MeV and 469 MeV respectively.

| $C_\sigma$ (fm$^2$) | $C_\omega$ (fm$^2$) | $B$ (fm$^2$) | $C$ (fm$^4$) |
|-------------------|-------------------|----------|----------|
| 7.057             | 1.757             | -5.796   | 0.001    |

### Table II: The values of the coupling constants $C_\rho, \eta_1$ and $\eta_2$ are determined from various symmetry energy elements. The mass of the $\rho$ meson is 770 MeV. The values of $C_\rho$ are in units of fm$^2$, $\eta_1$ and $\eta_2$ are dimensionless. All the symmetry energy elements are in units of MeV.

| Parameters | NCC | SR | WR |
|------------|-----|----|----|
| $\rho_0$ | 5.14 | 12.28 | 6.08 |
| $\eta_1$ | 0 | -0.79 | 0 |
| $\eta_2$ | 0 | 0 | 6.49 |

| Nuclear Matter | | | |
|----------------|-----------------|-----------------|-----------------|
| $J_0$ | 32.5 | 32.5 | 32.5 |
| $J_1$ | 22.30 | 24.49 | 23.68 |
| $L$ | 87 | 65 | 65 |
| $K_{\text{sym}}$ | -20.09 | -59.16 | -204.78 |
| $Q_{\text{sym}}$ | 58.73 | 356.11 | -88.04 |
| $K_\sigma$ | -434 | -368 | -513 |
The value of $L$ obtained with NCC model is also a little too large. By inclusion of cross-couplings (SR and WR models) the value of $L$ is reduced by $\sim 25\%$ keeping $J_0$ fixed. In what follows, we shall present our results for the density dependence of symmetry energy, EoSs for the SNM and PNM and the NS properties obtained using the NCC, SR and WR models. We shall also compare our EoSs and the density dependence of symmetry energy with those calculated for a few selected RMF models, namely, NL3 [38], IUFSU [39], BSP [10] and BKA22 [9]. The NL3 model does not include any cross-coupling, the IUFSU and BSP models include the cross-coupling between $\omega$ and $\rho$ mesons, while, BKA22 model is obtained by including the coupling of $\rho$ mesons with the $\sigma$ mesons.

A lot of progress, both theoretically and experimentally, has been made to constrain symmetry energy at sub saturation densities. We consider the data from three important sources: simulations of low energy Heavy Ion Collisions (HIC) in $^{112}\text{Sn}$ and $^{124}\text{Sn}$ [10]; nuclear structure studies by excitation energies to Isobaric Analog States (IAS) [41] and ASY-EOS experiment at GSI [42]. The density dependences of the symmetry energy for NCC, SR, WR and selected RMF models are displayed in Fig. 1. For comparison we have depicted the IAS [41], HIC $\text{Sn+Sn}$ [10] and ASY-EOS [42] data in the figure. It is evident that in the absence of any cross-couplings (NCC), the behavior of symmetry energy as a function of density is not very much compatible with those obtained by analyzing diverse experimental data. Remarkably the SR model satisfies all the above mentioned constraints. None of the considered RMF models satisfy all the symmetry energy constraints. The effects of various cross-couplings on the symmetry energy grow stronger at $\rho > \rho_0$. The symmetry energy is effectively low in WR model compared to NCC and SR models. Thus one may expect significant differences in the properties of NS obtained for the SR and WR models. This will be explored later in the paper.

The symmetry energy elements $L$ and $K_{\text{sym}}$ predominantly determine the value of $K_\tau$ (Eq. 18) which is required to evaluate the incompressibility of ANM. In Fig. 2 we compare our values of $K_\tau$ with various Skyrme and RMF model predictions in $K$ vs $K_\tau$ plot [13]. The dashed lines represent the constraints on $K_\tau$ from $-840$ MeV to $-350$ MeV [14] and $K$ from $220$ MeV to $260$ MeV [17] which have been determined using various experimental data on isoscalar giant monopole resonances. All the three models NCC, SR and WR satisfy these bounds of $K$ and $K_\tau$. It is to be noted that the models with a larger nuclear incompressibility ($K$) tend to have lower $K_\tau$ value. As can be seen from Fig. 2 several Skyrme models but only three RMF models (NLC, DDM1 and DDM2) satisfy the bounds for $K$ and $K_\tau$ simultaneously. The values of $L$ for the nonlinear model NLC with constant coupling is $107.97$ MeV [10] and that for the DDME models with density dependent coupling constants are $51 - 55$ MeV [10]. The value of $L$ for NLC model is very large compared to presently accepted range. We have also looked into the values of $K_\tau$ and $K$ for the several nonlinear RMF models [10]. Among them a few models (BSR type) have $L$ between $60 - 70$ MeV and satisfy the constraints on $K$ and $K_\tau$. These models includes $\sigma - \rho$ and $\omega - \rho$ both cross-couplings.

In Fig. 3 we plot low density EoS for PNM for all of our three models (NCC, SR and WR). The low density behavior of energy per neutron for SR model is in good agreement with those calculated for a few selected RMF models, namely, NL3, IUFSU, BSP and BKA22.

Figure 1: (Color Online) Symmetry energy as a function of scaled density ($\rho/\rho_0$) is plotted for three different variants of the effective chiral model as labeled by NCC, SR and WR obtained in the present work and are compared with those for a few selected RMF models NL3, IUFSU, BSP and BKA22. The constraints on the symmetry energy from IAS [11], HIC $\text{Sn+Sn}$ [10] and ASY-EOS experimental data [42] are also displayed. The inset shows the blown up behavior of symmetry energy at low densities.

Figure 2: (Color Online) The values of $K$ and $K_\tau$ from different models as labeled in [43] are compared with our models (NCC, SR and WR). The vertical and horizontal dashed lines represent the empirical ranges for $K$ and $K_\tau$ respectively.
agreement with the results obtained by microscopic calculations [51, 52] as shown by the shaded region. The PNM EoS for NCC and WR models do not have much overlap with the shaded region. The results for few selected RMF models are also displayed in the figure. Only the BSP model shows marginal overlap with the shaded region. In Ref. [53] two different families of systematically varied models with σ − ρ and ω − ρ cross-couplings have been employed to study the low density behavior of asymmetric nuclear matter. It was found that none of the models with σ − ρ cross-coupling satisfy the low density behavior of the PNM as predicted by Hebeler et al. [54]. However this constraint on the PNM EoS at low densities is satisfied by a couple of RMF models with ω − ρ cross-coupling having \( L \sim 45−65 \) MeV. The EoS with the current parameterization is compared in Fig. 4 with the experimental flow data obtained from the HIC [54] for SNM and PNM EoSs. The later one is constructed theoretically with two extreme parameterizations, the weakest (Asy soft) and strongest (Asy stiff) of symmetry energy as proposed in [55] and as reported in [56]. The SNM EoS is identical for all of our three models, since, the SNM properties are same. It is passing well through the experimental HIC data. In case of the PNM, the resulting EoSs for NCC and SR models pass through the upper end of HIC-Asy soft and lower end of HIC-Asy stiff, whereas, the PNM EoS for the WR model passes through the HIC-Asy soft only. As can be seen from Fig. 4 that the influence of cross-couplings in the effective chiral model at high density is quite strong in comparison to RMF models with similar type of cross-couplings. The PNM EoS for the WR model is quite softer than BSP and IUFSU at high densities. Similar differences can also be seen in the case of SR and BKA22 models.

We extend our analysis to study the mass-radius relationship for static NS composed of beta equilibrated charge neutral matter. The EoS for the core is obtained from the effective chiral model, and the effects of crustal EoS at low densities on the mass and the radius of NS are considered in two different ways. We model the crust EoS by using BPS EoS in the density range \( \rho \sim 4.8 \times 10^{-9} \) fm\(^{-3}\) to \( 2.6 \times 10^{-4} \) fm\(^{-3}\). The crust and the core are joined using the polytropic form \( p(\epsilon) = a_1 + a_2 \epsilon^\gamma \), where the parameters \( a_1 \) and \( a_2 \) are determined in such a way that the EoS for the inner crust for a given \( \gamma \) matches with that for the inner edge of the outer crust at one end and with the edge of the core at the other end. The polytropic index \( \gamma \) is taken to be equal to 4/3. For \( \gamma = 4/3 \), the values of radius \( R_{1.4} \) corresponding to the canonical mass of NS for the NL3 [57, 58] and IUFSU [59] RMF models are with in \( \sim 2\% \) in comparison to those obtained by treating the inner crust in the Thomas Fermi approach [50]. Alternatively, we estimate the contributions of the crust EoS to the NS radius and mass using the core crust approximation approach given in [52] referred hereafter ZFH method. This method enables one to estimate total mass and radius of a NS including the crust contributions very accurately for NS mass larger than 1 M\(_\odot\). In the ZFH method the radius and the mass of NS are given by

\[
R = \frac{R_{\text{core}}}{1 - (\alpha - 1)(R_{\text{core}}^{\gamma} / 2GM - 1)} \tag{19}
\]

\[
M = M_{\text{crust}} + M_{\text{core}} \tag{20}
\]
Table III: The maximum mass and radius of NS composed of $\beta-$ equilibrated matter are listed. The total mass and radii following the ZFH method are obtained by using Eqs. [19]-[21]. These are compared with the ones calculated from the BPS and polytropic EoSs for the outer and inner crusts, respectively. $\rho_{cc}/\rho_0$ is the scaled transition density. $M_{\text{max}}$, $R_{\text{max}}$, $R_{1.4}$ are the NS maximum mass, radius at maximum mass and the radius at $1.4 \, M_\odot$, respectively.

| $\rho_{cc}/\rho_0$ | Model | BPS+polytropic EoS | ZFH method |
|-------------------|-------|--------------------|-------------|
|                   |       | $M_{\text{max}}$  | $R_{\text{max}}$ | $R_{1.4}$ |
| 0.3               | NCC   | 1.97               | 11.55        | 13.31     | 1.97       | 11.48     | 13.12     |
|                   | SR    | 1.97               | 11.24        | 12.75     | 1.97       | 11.20     | 12.71     |
|                   | WR    | 1.84               | 10.74        | 12.22     | 1.84       | 10.67     | 12.03     |
| 0.4               | NCC   | 1.97               | 11.64        | 13.57     | 1.97       | 11.48     | 13.12     |
|                   | SR    | 1.97               | 11.28        | 12.87     | 1.97       | 11.21     | 12.72     |
|                   | WR    | 1.84               | 10.83        | 12.41     | 1.84       | 10.67     | 12.03     |
| 0.5               | NCC   | 1.97               | 11.77        | 13.90     | 1.97       | 11.50     | 13.13     |
|                   | SR    | 1.97               | 11.35        | 13.04     | 1.97       | 11.24     | 12.72     |
|                   | WR    | 1.84               | 10.92        | 12.62     | 1.84       | 10.67     | 12.03     |

with,

$$M_{\text{crust}} = \frac{4\pi P_c R_{\text{core}}^4}{GM_{\text{core}}} (1 - \frac{2GM_{\text{core}}}{R_{\text{core}}^2}).$$

In the above equations $\alpha = (\mu_{cc}/\mu_0)^2$, $\mu_{cc}$ and $\mu_0$ are the chemical potential at transition density ($\rho_{cc}$) and at neutron star surface respectively. $R_{\text{core}}$ and $M_{\text{core}}$ are the radius and mass of NS core. $P_{cc}$ is pressure at transition density. The transition density ($\rho_{cc}$) is mostly in the range 0.4 to 0.6 $\rho_0$ for $L$ typically ranging from 30 to 120 MeV [60]. In the present work we have taken $\rho_{cc}/\rho_0 = 0.3, 0.4$ and 0.5.

Comparison of the results of the two approaches is given in Table III. The maximum mass of the NS is sensitive neither to the methods used to estimate the crust effects nor to the choice of transition density. The WR model, which includes $\sigma - \rho$ cross-coupling, does not satisfy the maximum mass constraint as imposed by PSR J0348+0432 ($M = 2.01 \pm 0.04 \, M_\odot$) [61]. This disfavors the WR model. The values of $R_{1.4}$ obtained using BPS EoS for the outer crust and polytropic EoS for the inner crust are little too large compared to those for the ZFH method. We find that by including $\sigma - \rho$ coupling (SR) $R_{1.4}$ are smaller compared to the NCC model which does not include any cross-coupling term. The radius of NS is sensitive to transition density. Using the strong correlation between transition density ($\rho_{cc}$) and $L$, we found the values of $\rho_{cc}$ to be $0.061 \, \text{fm}^{-3}$ ($\sim 0.4 \, \rho_0$) for NCC and $0.077 \, \text{fm}^{-3}$ ($\sim 0.5 \, \rho_0$) for SR and WR models respectively [59]. The mass radius relationship for the NS for all of our three models obtained using respective values of the transition densities are plotted in Fig. 5. The dashed lines are obtained using the ZFH method in which the effects of the crust EoS were approximated and the solid lines are obtained using BPS and the polytropic EoSs for the outer and the inner crust respectively. It is found that the value of $R_{1.4}$ is decreased by $\sim 0.5$ km in SR model compared to NCC model. The $R_{1.4}$ of SR is consistent with $11.9 \pm 1.22$ km (90% confidence) obtained by constraining symmetry energy at saturation density from various experimental information and theory [34]. The NS maximum mass $M_{\text{max}} = 2.79, 1.94, 2.02, 2.04 \, M_\odot$ and the radius $R_{1.4} = 14.66, 12.49, 12.64, 13.28$ km for the selected RMF models NL3, IUFSU, BSP and BKA22 respectively. The RMF models such as IUFSU and BSP with $\omega - \rho$ cross-coupling readily yield $M_{\text{max}} \sim 2 \, M_\odot$, since, the softening of the EoS due to the inclusion of this cross-coupling is not as strong as in the case of effective chiral model.

Results obtained for the SR model can be summarized in the following way. It yields symmetry energy $J_0 = 32.5$ MeV, symmetry energy slope parameter $L = 65$ MeV, nuclear incompressibility $K = 247$ MeV and the asymmetry term of nuclear incompressibility $K_{\omega} = -368$ MeV at saturation density $\rho_0 \sim 0.153 \, \text{fm}^{-3}$. It also yields symmetry energy $J_1 = 24.49$ MeV at density 0.1 fm$^{-3}$, NS maximum mass $1.97 \, M_\odot$ and radius $R_{1.4} = 12.72$ km. All these values are within presently accepted range. The SR model also satisfies all the discussed constraints from microscopic calculations for low density PNM EoS, density dependence of symmetry energy, HIC data for SNM EoS and HIC-Asy stiff data for PNM EoS.

The contributions of the exotic degrees of freedom, such as hyperons, kaons etc. to the properties of NS are not considered in the present work. In general, the presence of strange particles softens the EoS and reduce the NS maximum mass. In particular, the inclusion of hyperons in the effective chiral model (i.e. NCC type) tend to reduce the NS maximum mass by $\sim 0.3 \, M_\odot$ [59]. The influence of hyperons on the NS properties, however, are very sensitive to the choice of the meson-
IV. CONCLUSION

We have extended the effective chiral model by including the contributions from $\sigma - \rho$ and $\omega - \rho$ cross-couplings. The inclusion of cross-couplings involving $\rho$ meson has helped to improve overall behavior of the density dependence of the symmetry energy.

We have discussed three different variants of effective chiral model in this paper. The model with no cross-coupling (NCC), $\sigma - \rho$ cross-coupling (SR) and $\omega - \rho$ cross-coupling (WR). NCC model yields the value of symmetry energy slope parameter ($L = 87$ MeV) which is a little too large and symmetry energy at crossing density $0.1\text{ fm}^{-3}$ ($J_1 = 22.3$ MeV) which is low compared to presently estimated values. The low-density behavior of PNM EoS for both NCC and WR models does not match well with the range of values proposed by microscopic calculations \[51, 52\]. The WR model gives NS maximum mass to be $1.86\,M_\odot$ which is very less than the values suggested by the microscopic models \[51, 52\]. The NS maximum mass is $1.97\,M_\odot$ which is consistent with the observational constraint. The value of $R_{1.4}$ is within the empirical bounds. The SR model satisfies all the discussed constraints which suggest that the inclusion of $\sigma - \rho$ cross-coupling in the effective chiral model is dispensable. We have also compared our results with a few selected RMF models. In general, it is found that the effects of various cross-couplings within the RMF models are weaker compared to those in the effective chiral model. This effects are more prominent for the models with $\omega - \rho$ cross-coupling.

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\[1\] L. Trippe, G. Colo, and E. Vigezzi, Phys. Rev. C77, 061304 (2008).
\[2\] P. Möller, W. D. Myers, H. Sagawa, and S. Yoshida, Phys. Rev. Lett. 108, 052501 (2012).
\[3\] M. B. Tsang et al., Phys. Rev. C86, 015803 (2012).
\[4\] X. Roca-Maza, M. Centelles, X. Viñas, B. K. Agrawal, G. Colo, B. K. Agrawal, N. Paar, J. Piekarewicz, and D. Vretenar, Phys. Rev. C88, 024316 (2013).
\[5\] X. Viñas, M. Centelles, X. Roca-Maza, and M. Warda, Eur. Phys. J. A50, 27 (2014).
\[6\] X. Roca-Maza, X. Viñas, M. Centelles, B. K. Agrawal, G. Colo', N. Paar, J. Piekarewicz, and D. Vretenar, Phys. Rev. C92, 064304 (2015).
\[7\] C. Mondal, B. K. Agrawal, J. N. De, and S. K. Samaddar, Phys. Rev. C93, 044328 (2016).
\[8\] B. G. Todd-Rutel and J. Piekarewicz, Phys. Rev. Lett. 95, 122501 (2005).
\[9\] B. K. Agrawal, Phys. Rev. C81, 034323 (2010).
\[10\] B. K. Agrawal, A. Suhaksono, and P. G. Reinhard, Nucl. Phys. A882, 1 (2012).
\[11\] M. Gell-Mann and M. Levy, Nuovo Cim. 16, 705 (1960).
\[12\] T. D. Lee and G. C. Wick, Phys. Rev. D9, 2291 (1974).
\[13\] T. D. Lee and M. Margulies, Phys. Rev. D11, 1591 (1975), [401(1974)].
\[14\] A. W. Thomas, P. A. M. Guichon, D. B. Leinweber, and R. D. Young, Prog. Theor. Phys. Suppl. 156, 124 (2004).
\[15\] R. J. Furnstahl and B. D. Serot, Phys. Lett. B316, 12 (1993).
tra, Phys. Rev. C74, 055803 (2006), [Erratum: Phys. Rev.C75,029903(2007)].
[31] X. Roca-Maza, M. Brenna, B. K. Agrawal, P. F. Bortignon, G. Colò, L.-G. Cao, N. Paar, and D. Vretenar, Phys. Rev. C87, 034301 (2013).
[32] J. L. Zdunik, M. Fortin, and P. Haensel, Astron. Astrophys. 599, A119 (2017).
[33] J. Dong, W. Zuo, and J. Gu, Phys. Rev. C91, 034315 (2015).
[34] J. M. Lattimer and Y. Lim, Astrophys. J. 771, 51 (2013).
[35] L.-W. Chen, B.-J. Cai, C. M. Ko, B.-A. Li, C. Shen, and J. Xu, Phys. Rev. C80, 014322 (2009).
[36] P. K. Sahu, K. Tsubakihara, and A. Ohnishi, Phys. Rev. C81, 014002 (2010).
[37] B.-A. Li and X. Han, Phys. Lett. B727, 276 (2013).
[38] G. A. Lalazissis, J. Konig, and P. Ring, Phys. Rev. C55, 540 (1997).
[39] F. J. Fattoyev, C. J. Horowitz, J. Piekarewicz, and G. Shen, Phys. Rev. C82, 055803 (2010).
[40] M. B. Tsang, Y. Zhang, P. Danielewicz, M. Famiano, Z. Li, W. G. Lynch, and A. W. Steiner, Phys. Rev. Lett. 102, 122701 (2009), [Int. J. Mod. Phys.E19,1631(2010)].
[41] P. Danielewicz and J. Lee, Nucl. Phys. A922, 1 (2014).
[42] P. Russotto et al., Phys. Rev. C94, 034608 (2016).
[43] H. Sagawa, S. Yoshida, G.-M. Zeng, J.-Z. Gu, and X.-Z. Zhang, Phys. Rev. C76, 034327 (2007), [Erratum: Phys. Rev.C77,049902(2008)].
[44] J. R. Stone, N. J. Stone, and S. A. Moszkowski, Phys. Rev. C89, 044316 (2014).
[45] J. M. Pearson, N. Chamel, and S. Goriely, Phys. Rev. C82, 037301 (2010).
[46] T. Li et al., Phys. Rev. C81, 034309 (2010).
[47] S. Shlomo, V. M. Kolomietz, and G. Colò, Eur. Phys. J. A 30, 23 (2006).
[48] M. Dutra, O. Lourenço, S. S. Avancini, B. V. Carlson, A. Delfino, D. P. Menezes, C. Providência, S. Typel, and J. R. Stone, Phys. Rev. C90, 055803 (2014).
[49] N. Alam, B. K. Agrawal, M. Fortin, H. Pais, C. Providência, A. R. Raduta, and A. Sulaksono, Phys. Rev. C94, 052801 (2016).
[50] G. Colò, U. Garg, and H. Sagawa, Eur. Phys. J. A50, 26 (2014).
[51] A. Gezerlis and J. Carlson, Phys. Rev. C81, 025803 (2010).
[52] K. Hebeler, J. M. Lattimer, C. J. Pethick, and A. Schwenk, Astrophys. J. 773, 11 (2013).
[53] N. Alam, H. Pais, C. Providência, and B. K. Agrawal, Phys. Rev. C95, 055808 (2017).
[54] P. Danielewicz, R. Lacey, and W. G. Lynch, Science 298, 1592 (2002).
[55] M. Prakash, T. L. Ainsworth, and J. M. Lattimer, Phys. Rev. Lett. 61, 2518 (1988).
[56] G. Baym, C. Pethick, and P. Sutherland, Astrophys. J. 170, 299 (1971).
[57] J. Carriere, C. J. Horowitz, and J. Piekarewicz, Astrophys. J. 593, 463 (2003).
[58] J. Piekarewicz, F. J. Fattoyev, and C. J. Horowitz, Phys. Rev. C90, 015803 (2014).
[59] F. Grill, H. Pais, C. Providência, I. Vidaña, and S. S. Avancini, Phys. Rev. C90, 045803 (2014).
[60] C. Ducoin, J. Margueron, C. Providencia, and I. Vidana, Phys. Rev. C83, 045810 (2011).
[61] J. Antoniadis et al., Science 340, 6131 (2013).
[62] S. Weissenborn, D. Chatterjee, and J. Schaffner-Bielich, Nucl. Phys. A881, 62 (2012).
[63] A. Sulaksono and B. K. Agrawal, Nucl. Phys. A895, 44 (2012).
[64] D. Bizarro, A. Rabhi, and C. Providência, arXiv, 1502.04952.