Non-Gaussianity and gravitational wave background in curvaton with a double well potential

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Abstract

We study the density perturbation by a curvaton with a double well potential and estimate the nonlinear parameters for non-Gaussianity and the amplitude of gravitational wave background generated during inflation. The predicted nonlinear parameters strongly depend on the size of a curvaton self-coupling constant as well as the reheating temperature after inflation for a given initial amplitude of the curvaton. The difference from usual massive self-interacting curvaton is also emphasized.

PACS numbers: 95.85.Bh, 98.80.Es, 98.80.Cq
I. INTRODUCTION

Cosmic inflation solves various problems in the standard Big Bang cosmology such as flatness, the horizon, and the monopole problems [1]. Simultaneously, the quantum fluctuation of a light scalar field, e.g., inflaton field \( \phi \), generated during inflation is stretched by the rapid cosmic expansion and provides the seed of large scale structure in our Universe [2].

The density perturbation generated in a single field inflation model is scale-invariant and almost Gaussian with the corresponding nonlinearity parameter \( f_{NL} \) much less than unity [3]. This is consistent with the current limit on the local type non-linearity parameter \( f_{NL} \) from the Wilkinson Microwave Anisotropy Probe (WMAP) seven-year data, \(-10 < f_{NL} < 74\) at the 95% confidence level [4]. It is expected that the observational sensitivity is going to improve significantly by the Planck data [5] and using large scale structure data within the near future. The non-Gaussianity could be an important observable to discriminate between various mechanisms of density perturbation generation.

On the other hand, beyond the simple canonical single field slow-roll inflation, the large non-Gaussianity with different shape are generally predicted [6, 7]. There are many models for the generation of the observed density perturbation and a large non-Gaussianity. This can happen during inflation [8–11], at the end of inflation [12–15], preheating [16], or deep in the radiation dominated era [17].

The last case includes the “curvaton” scenario [18–22], where the scalar field is too light to make effects around the inflationary epoch but it might play an important role much later in the early Universe. As the Universe expands, the cosmic expansion rate \( H \) becomes comparable to the mass of the curvaton and the curvaton field starts to oscillate in the radiation dominated era. After that when the expansion becomes less than the decay rate of the curvaton, it decays to light fields and the isocurvature perturbation of the curvaton field becomes adiabatic or mixed with that from the inflaton field. If the curvaton energy density is subdominant at its decay time, the large non-Gaussianity is generated in general [23].

Another important measure is the gravitational wave background produced during inflation [24] parametrized by the tensor-to-scalar ratio \( r_T \), because it could directly indicate the energy scale of inflation. \( r_T \) is related with one of the inflaton’s slow roll parameter \( \epsilon \) as \( r_T = 16\epsilon \), while the density perturbation \( \mathcal{P}_\zeta \propto \epsilon^{-1} \) in a single field inflation model. On the other hand, in the curvaton scenario, the density (scalar) perturbation comes from the
curvaton. Nevertheless the (non-)observation of $r_T$ gives strong constraint on the parameters of the curvaton scenario [25]. The present bound is $r_T < 0.36$ (95% CL) [4], which is expected to be tightened as $r_T \simeq 10^{-1}$ from Planck satellite [5] and $r_T \simeq 10^{-3}$ from DEci-hertz Interferometer Gravitational wave Observatory (DECIGO) [26] and/or the Big Bang Observatory(BBO) [27].

Curvaton scenarios have been often modeled by a scalar field $\sigma$ with a quadratic potential $V = \frac{1}{2} m^2 \sigma^2$. In that case the non-Gaussianity has been studied with the sudden decay approximation [28–30], which shows good agreement with the full numerical approach [31]. Beyond the simplest model of the curvaton, there are various possibilities; the inflaton perturbation may not be negligible [32], the curvaton can have different types of potential [33–36] and there could be multiple curvaton fields [37–39]. Significant effects on non-Gaussianity due to non-quadratic terms can be seen [40–45].

So far, it has been assumed that the mass squared at the origin of field is positive as above. However, there is no reason that the true minimum is located at the origin. Scalar fields have been often introduced for spontaneous symmetry breaking in particle physics models. A moduli field, which is a promising candidate of curvaton [21], or the Peccei-Quinn field, to solve the strong CP problem, usually has the large vacuum expectation value (VEV).

In this paper, we examine a curvaton model with a double well potential where it develops nonvanishing VEV. Since a curvaton needs to develop large expectation value during inflation, throughout this work, we assume that the potential is very flat and the self-coupling constant is small enough. Such a tiny self-coupling scalar field model has been studied in the axion model to solve the isocurvature perturbation and domain wall problems [46, 47]. Due to the flatness the curvaton has a large initial amplitude, $\sigma_* \gtrsim v$, and almost stays there during inflation with negligible movement. After inflation the curvaton field starts to roll down into the minimum, oscillates and decay into radiation. After the curvaton decay, its isocurvature perturbation is transferred to the adiabatic curvature perturbation in the radiation dominated plasma.

We take account of the inflaton perturbation as well as that of the curvaton. Therefore the curvature perturbation shows generalised mixed inflaton-curvaton type [32]. We consider the initial amplitude of the curvaton field is arbitrary but larger than the symmetry breaking scale. In the opposite case where the field starts rolling around hill of the potential, interesting results have been shown [48].
The paper is organized as follows. After describing the model with a double well potential in section II, we consider the cosmological evolution of the scalar field in section III. In section IV we estimate the non-Gaussianity and gravitational wave background of this model. We summarize our results in section V.

II. A CURVATON MODEL WITH A DOUBLE WELL POTENTIAL

We consider a real scalar curvaton model with a double well potential. The Lagrangian density of the field is given by

$$\mathcal{L} = \frac{-1}{2}(\partial \sigma)^2 - V(\sigma),$$  \hspace{1cm} (1)

$$V(\sigma) = \frac{\lambda}{4}(\sigma^2 - v^2)^2,$$  \hspace{1cm} (2)

with $\lambda$ and $v$ being respectively the self coupling constant and the VEV. When the field has a nontrivial expectation value in the potential Eq. (2), the effective mass of it is expressed as

$$V_{\sigma\sigma} = \lambda(3\sigma^2 - v^2)$$  \hspace{1cm} (3)

and the mass at the true vacuum is given by

$$m_{\sigma}^2 = V_{\sigma\sigma}|_{\sigma = \pm v} = 2\lambda v^2.$$  \hspace{1cm} (4)

The decay rate of $\sigma$ depends on its interaction with light particles. If $\sigma$ couples with a light fermion $\psi$ through a Yukawa interaction as $\mathcal{L}_{\text{int}} = y\bar{\psi}\sigma\psi$, the decay rate is roughly given by

$$\Gamma_\sigma \simeq \frac{y^2}{8\pi} m_{\sigma}.$$  \hspace{1cm} (5)

If $\sigma$ does not directly couple with light particles, the decay rate would be expressed as

$$\Gamma_\sigma = C\frac{m_{\sigma}^3}{v^2} = C(2\lambda)^{3/2} v.$$  \hspace{1cm} (6)

with $C$ being a numerical coefficient of including coupling constants and phase volume. This kind of decay rate formula is realized, for instance, for the radial direction of Peccei-Quinn field in the hadronic (KSVZ) axion model \cite{49}.

\footnote{The corresponding curvaton scenario with Peccei-Quinn field in the extension of MSSM has been studied in Ref. \cite{50}.}
III. COSMOLOGICAL EVOLUTION OF $\sigma$

By definition, the curvaton field is subdominant during inflation. Its contribution can be important after inflation in the deep radiation dominated era. For this purpose, we consider the case that $\lambda$ is very tiny. In such a case, the potential is very flat, like chaotic inflation with quartic potential, and hence the fields can develop a large expectation value during inflation.

The equation of motion for the homogeneous part of $\sigma$ is given by

$$\ddot{\sigma} + 3H\dot{\sigma} + \lambda(\sigma^2 - v^2)\sigma = 0.$$  \hspace{1cm} (7)

Before the inflaton decay, the energy density of the Universe is dominated by the inflaton whose equation of motion is given by

$$\ddot{\phi} + 3H\dot{\phi} + \frac{dV}{d\phi} = 0,$$  \hspace{1cm} (8)

and then the Friedmann equation is

$$3M_P^2H^2 = \rho_\phi + \rho_\sigma,$$  \hspace{1cm} (9)

with $\rho_\phi \gg \rho_\sigma$. Here $M_P \simeq 2.4 \times 10^{18}$ GeV denotes the reduced Planck mass. After the inflaton decay, the Universe is dominated by the radiation generated from the inflaton decay.

Then the field equations, instead, are

$$\dot{\rho}_r + 4H\rho_r = 0,$$

$$3M_P^2H^2 = \rho_r + \rho_\sigma.$$  \hspace{1cm} (10)

The curvaton field with a large expectation value during inflation almost stays there until the Hubble parameter $H$ becomes comparable with the effective mass, i.e.

$$H_{os}^2 \simeq V_{\sigma|_{os}} = \lambda(3\sigma_{os}^2 - v^2).$$  \hspace{1cm} (11)

From that time, the curvaton starts to oscillate with the initial amplitude $\sigma_{os}$ and the energy density

$$\rho_\sigma|_{os} = \frac{\lambda}{4}(\sigma_{os}^2 - v^2)^2.$$  \hspace{1cm} (12)

When the initial amplitude of the curvaton is much larger than the location of the minimum as $\sigma_* \gg v$, the evolution of the curvaton is dominated by the quartic potential and the hill
at the origin can be ignored during the oscillation. In this case, the energy density of the curvaton decreased as $a(t)^{-4}$ after the oscillation starts.

After the oscillation amplitude decreases enough so that the field cannot go across the potential hill around the origin, the field can settle down at one of the two degenerate and distinct vacua. Once the curvaton field find one of the minimum, the oscillation amplitude becomes less than of the order of $v$, and the energy density becomes smaller than

$$\rho_\sigma|_v = \frac{\lambda}{4}(\sigma_v^2 - v^2)^2,$$

(13)

with $\sigma_v$ being the amplitude of the order of $v$ that denotes the transition from quartic oscillation to quadratic one occurs. Note that $\sigma_v$ is independent of $\sigma_*$. The ratio of the energy densities given by Eqs. (12) and (13) is scaled by $(a_v/a_{os})^{-4}$, because the quartic term initially dominates. The precise value of $\sigma_v$ does not affect the differentiation by $\sigma_*$ but it may affect the energy density of the curvaton when it decay, since the exact transition epoch between quadratic and quartic affect the evolution of the curvaton energy density.

After the curvaton find a minimum at $\langle \sigma \rangle = v$ or $\langle \sigma \rangle = -v$, its energy density deceases as a pressureless matter $\propto a(t)^{-3}$ since the quadratic potential dominates. Which of the VEV would be realized depends on the initial field expectation value. The dependence is shown as the function defined by $\Theta(\sigma_{os}) \equiv \langle \sigma \rangle / v$ in figure 1.

At the late time $t \gg t_v$ in the deep oscillation period dominated by the quadratic potential, the evolution of $\sigma$ can be well expressed as

$$\sigma(t) \simeq v\Theta(\sigma_{os}) + \frac{\sigma_{2os}}{(m_\sigma t)^{3/4}} \sin m_\sigma t.$$  

(14)

The amplitude of the oscillation $\sigma_{2os}$ can be estimated by using the simple scaling law between $t_{os}$ and $t_v$ for radiation dominated (high $T_R$) and oscillating inflaton dominated (low $T_R$) at $H_{os}$ respectively by

$$\sigma_{2os} \simeq \begin{cases} 
(\sigma_v - v) \left( \frac{\rho_{\sigma}|_{os}}{\rho_{\sigma}|_v} \right)^{3/8} \left( \frac{m_\sigma}{2 \sqrt{3 \lambda \sigma_{2os}^2}} \right)^{3/4}, & \text{for high } T_R, \\
(\sigma_v - v) \left( \frac{\rho_{\sigma}|_{os}}{\rho_{\sigma}|_v} \right)^{3/8} \left( \frac{H_R}{2 \sqrt{3 \lambda \sigma_{2os}^2}} \right)^{1/4} \left( \frac{m_\sigma}{2 \sqrt{3 \lambda \sigma_{2os}^2}} \right)^{3/4}, & \text{for low } T_R,
\end{cases}$$

(15)

with

$$\frac{1}{2t_v} \simeq H_v = m_\sigma \left( \frac{\sigma_{2os}}{\sigma_v - v} \right)^{-4/3},$$

(16)
FIG. 1: The position of VEV with varying initial amplitudes of the curvaton. The vertical axis is the sign of VEV, $\langle \sigma \rangle / v$ and the horizontal axis is $\sigma_*/v$.

Here, the time of reheating can be approximated when the Hubble parameter is similar as the decay rate of inflation $\Gamma_\phi$ and the reheating temperature $T_R$ is estimated by

$$
\Gamma_\phi^2 = H_R^2 = \frac{1}{3M_P^2} \frac{\pi^2}{30} g_* T_R^4.
$$

(17)

We consider $\Gamma_\phi$ or $T_R$ as a free parameter. Equation (15) has been sometime noted as $g_{30}$ or $\sigma_{os}$ in literature. Figure 2 shows the good agreement between analytic approximated solutions Eq. (14) and full numerical solutions. Finally, the $\sigma$ field decays into radiation, when the Hubble parameter $H$ becomes comparable with its decay rate $H \simeq \Gamma_\sigma$.

When $v < \sigma_* < \sigma_v$, the curvaton field starts to oscillate initially in the potential dominated by quadratic term when $H^2 \simeq m_\sigma^2$. Therefore we find that

$$
\sigma_{2os} \simeq (\sigma_* - v\Theta)(m_\sigma/2H_{os})^{3/4}.
$$

(18)

Here we summarize the conditions for the $\sigma$ field to be a viable candidate for curvaton. The curvaton is almost massless and its field value is frozen during inflation. This is expressed by $V_{\sigma\sigma} \ll H_{inf}^2$ and rewritten as

$$
(I) \quad 3\lambda(\sigma_*^2 - v^2) \ll H_{inf}^2.
$$

(19)
FIG. 2: The comparison between analytic solutions (purple line) and numerical solutions (blue line) for various initial values $\sigma_{\text{os}} = 4v$ (left), $10v$ (center), and $50v$ (right). The vertical axis is $\zeta/\sqrt{v}$ and the horizontal axis is $m_\sigma t$. These figures show that analytic solutions well describe asymptotic evolution of the field.

The curvaton energy density is subdominant compared with that of the inflaton during inflation, which is expressed as

$$\frac{\lambda}{4}(\sigma^2 - v^2)^2 \ll 3M_P^2H_{\text{inf}}^2.$$  \hfill (20)

This condition is automatically satisfied from Eq. (19) when $\sigma_* < M_P$. Whether $\sigma$ dominates the energy density of the Universe at the moment of $\sigma$ decay depends on the reheating temperature after inflation $T_R$ determined by the inflaton decay rate $\Gamma_\phi$. We will pursue the details for this in the following subsections. During preheating, the symmetry might be restored and the topological defects could be formed due to the parametric resonance and the large fluctuation of $\langle \delta\sigma^2 \rangle \gg v^2$. This problem can be avoided if the dynamics is pure classical and the initial fluctuations $\delta\sigma/\sigma$ is less than the change of the amplitude of $\sigma$ per one oscillation when the curvaton field settles down to one of two minima of the potential [47]. This condition is easily written down as

$$\frac{\delta\sigma}{\sigma} \sim \frac{H_{\text{inf}}/2\pi}{\sigma_{\text{os}}} < \frac{\Delta A}{A} \sim \frac{H_c}{\omega} \sim \frac{\sqrt{\lambda}v^2/\sigma_{\text{os}}}{\sqrt{\lambda}v},$$  \hfill (21)

where we have used the fact that the curvaton oscillation is dominated by quartic term between the end of inflation and the critical point and thus $\sigma^4_{\text{os}}/\sigma^4_v = H_{\text{os}}^2/H_c^2$. Therefore there is no domain wall problem for a large VEV satisfying

$$\frac{H_{\text{inf}}}{2\pi} < v. \hfill (22)$$

In this paper, we consider this large symmetry breaking scale.
A. A high reheating temperature case

First, we consider the case that the reheating after inflation is completed when $\sigma$ starts to oscillate, which means that the Hubble parameter at reheating $H_R$ is larger than that at the beginning of the oscillation. This condition of high reheating temperature corresponds to $H_R > H_{os}$ or

$$T_R > \left[ \left( \frac{\pi^2}{30} g_* \right)^{-1} 3M_P^2 \lambda (3\sigma_{os}^2 - v^2)^2 \right]^{1/4}.$$  \hfill (23)

The energy density of radiation produced by the inflaton decay at $H_{os}$ is

$$\rho_r \big|_{os} = 3M_P^2 H_{os}^2.$$  \hfill (24)

The energy density of the curvaton and radiation from the inflaton decay at $H \simeq \Gamma_\sigma$ are given by

$$\rho_\sigma \big|_{\Gamma_\sigma} \simeq \frac{\lambda}{4} v^4 \left( \frac{a_{\Gamma_\sigma}}{a_\sigma} \right)^3,$$  \hfill (25)

and

$$\rho_r = 3M_P^2 H_{os}^2 \left( \frac{a_{os}}{a_\sigma} \right)^4.$$  \hfill (26)

The $\sigma$ to radiation ratio is evaluated as

$$r \equiv \frac{\rho_\sigma}{\rho_r} = \frac{\lambda v^4}{12M_P^2 H_{os}^2} \left( \frac{a_\sigma}{a_\sigma} \right)^3 \left( \frac{a_{\Gamma_\sigma}}{a_{os}} \right)^4 \left( \frac{3\lambda v^2}{2} \right)^{1/4}$$

$$= \frac{v^{1/2} \sigma_{os}^{3/2}}{36M_P^2} \left( \frac{\sigma_{os}^2 - v^2}{\Gamma_\sigma^2} \right)^{1/4}$$  \hfill (27)

for the radiation dominated Universe.

For the case of $v < \sigma_{os} < \sigma_v$, we obtain

$$r = \frac{\lambda^{1/4} (\sigma_{os}^2 - v^2)^2}{12M_P^2 (3\sigma_{os}^2 - v^2)^{3/4}} \frac{1}{\Gamma_\sigma^{1/2}}$$  \hfill (28)

for the radiation dominated Universe.

B. A low reheating temperature case

Next, we consider the case that the inflaton still oscillates (we assume the quadratic oscillation) around the minimum and the reheating is not completed yet when $\sigma$ starts to oscillate, $H_R < H_{os}$, which is the opposite condition of Eq. (23).
The energy density of the inflaton $\phi$ at $H_{\text{os}}$ is

$$\rho_\phi|_{\text{os}} = 3M_P^2H_{\text{os}}^2.$$  \hfill (29)

At a late time, the energy density of the curvaton and radiation from the inflaton decay at $H \simeq \Gamma_\sigma$ are given by

$$\rho_\sigma|_{\Gamma_\sigma} \simeq \frac{\lambda}{4}v^4\left(\frac{a_v}{a_{\Gamma_\sigma}}\right)^3,$$  \hfill (30)

and

$$\rho_r = \frac{\pi^2}{30}g_\ast T_R^4\left(\frac{a_R}{a_{\Gamma_\sigma}}\right)^4.$$  \hfill (31)

The energy density ratio of $\sigma$ to radiation at late time $H \simeq \Gamma_\sigma$ is evaluated as

$$r = \frac{v\sigma_{\text{os}}}{36M_P^2}\left(\frac{\pi^2g_\ast T_R^4}{90M_P^2\Gamma_\sigma^2}\right)^{1/4}.$$  \hfill (32)

For the case of $v < \sigma_{\text{os}} < \sigma_v$, we obtain

$$r = \frac{(\sigma_{\text{os}}^2 - v^2)^2}{12M_P^2(3\sigma_{\text{os}}^2 - v^2)}\left(\frac{\pi^2g_\ast T_R^4}{90M_P^2\Gamma_\sigma^2}\right)^{1/4}.$$  \hfill (33)

\textbf{IV. POWER SPECTRUM AND NON-GAUSSIANITY}

The curvaton is light during inflation with Eq. (19) and thus has a Gaussian quantum fluctuation with the amplitude $\delta\sigma_\ast \simeq H_\ast/(2\pi)$. The curvaton field value at the onset of its quadratic oscillation is some function of that at the onset of quartic oscillation $\sigma_{2\text{os}} = \sigma_{2\text{os}}(\sigma_{\text{os}})$. In addition we assume the field value at $t_{\text{os}}$ is same as that at horizon exit $t_\ast$,

$$\sigma_{\text{os}}(\sigma_\ast) = \sigma_\ast.$$  \hfill (34)

Thus we can be expand $\sigma_{2\text{os}}$ around the homogeneous part $\sigma_{2\text{os}}$,

$$\sigma_{2\text{os}}(t,x) = \sigma_{2\text{os}}(t) + \sigma'_{2\text{os}}\delta\sigma_\ast + \frac{1}{2}\sigma''_{2\text{os}}(\delta\sigma_\ast)^2 + \frac{1}{6}\sigma'''_{2\text{os}}(\delta\sigma_\ast)^3 + \ldots,$$  \hfill (35)

where the prime denotes the derivative with respect to $\sigma_\ast$.

The curvature perturbation due to the curvaton density fluctuation is easily calculated using $\delta N$ formalism \[29\,30\]. The nonlinear curvature perturbation of the curvaton field on the uniform curvaton density hypersurface is given by \[30\,52\,53\],

$$\zeta_\sigma = \delta N + \frac{1}{3}\int_{\rho_0(t)}^{\rho(t,x)} \frac{d\rho}{\dot{\rho} + \dot{\rho}},$$  \hfill (36)
where $\delta N$ is the perturbed expansion, $\tilde{\rho}$ and $\tilde{p}$ are the local density and pressure of the curvaton respectively. For the oscillating curvaton field in the expanding Universe, the pressure and the energy density is related by $p = w\rho$ with $w = 0$ when quadratic term dominates and $w = 1/3$ when quartic term dominates. In our case, the curvaton energy density at late times ($t \gg t_v$) is given by

$$\rho_\sigma(t, x) \simeq \frac{m_\sigma^2 \sigma_{2\text{os}}^2(t, x)}{2(mt)^{3/2}},$$

(37)

and it can be expanded around the background value $\sigma_{2\text{os}}(t)$ using Eq. (35).

From this, we can find the curvature perturbation of the curvaton field as

$$\zeta_\sigma = \zeta_{\sigma 1} + \frac{1}{2}\zeta_{\sigma 2} + \frac{1}{6}\zeta_{\sigma 3} + \ldots,$$

(38)

where each terms are expressed as

$$\zeta_{\sigma 1} = \frac{2\sigma'_{2\text{os}}}{3\sigma_{2\text{os}}} \delta \sigma_\star,$$

(39)

$$\zeta_{\sigma 2} = -\frac{3}{2} \left(1 - \frac{\sigma'_{2\text{os}}}{\sigma_{2\text{os}}}\right) \zeta_{\sigma 1}^2 \equiv A_2 \zeta_{\sigma 1}^2,$$

(40)

$$\zeta_{\sigma 3} = \frac{9}{4} \left(2 - 3 \frac{\sigma''_{2\text{os}}}{\sigma_{2\text{os}}} + \frac{\sigma''_{2\text{os}}}{\sigma_{2\text{os}}} \frac{\sigma''_{2\text{os}}}{\sigma_{2\text{os}}} \right) \zeta_{\sigma 1}^3 \equiv A_3 \zeta_{\sigma 1}^3.$$

(41)

On this uniform density surface at the curvaton decay time $t_D$ or $H = \Gamma_\sigma$, we have

$$\rho_r(t_D) e^{4(\zeta - \zeta)} + \rho_\sigma(t_D) e^{3(\zeta - \zeta)} = \rho_{\text{tot}}(t_D),$$

(42)

with the radiation perturbation $\zeta_r$ originated from the inflaton $\phi$. Then, the curvature perturbation after the curvaton decay can be expressed as

$$\zeta = \zeta_1 + \frac{1}{2}\zeta_2 + \frac{1}{6}\zeta_3 + \ldots,$$

(43)

where

$$\zeta_1 = (1 - R)\zeta_{r 1} + R\zeta_{\sigma 1},$$

$$\zeta_2 = (1 - R)\zeta_{r 2} + R\zeta_{\sigma 2} + R(1 - R)(3 + R) (\zeta_{r 1} - \zeta_{\sigma 1})^2,$$

$$\zeta_3 = (1 - R)\zeta_{r 3} + R\zeta_{\sigma 3} + 3R(1 - R)(3 + R) (\zeta_{r 1} - \zeta_{\sigma 1}) (\zeta_{r 2} - \zeta_{\sigma 2}) + R(1 - R)(3 + R)(-3 + 4R + 3R^2) (\zeta_{r 1} - \zeta_{\sigma 1})^2,$$

(44)

and

$$R \equiv \frac{3\rho_\sigma}{4\rho_r + 3\rho_\sigma},$$

(45)
at $t = t_D$. Furthermore it is natural to assume $\zeta \ll \zeta_\sigma$ and $\zeta_\sigma$ is almost Gaussian so that only $\zeta_\sigma_1$ is non-negligible in the expansion of $\zeta_\sigma$. Thus in the above we can approximate $\zeta_\sigma_1 - \zeta_\sigma_2 \simeq -\zeta_\sigma_1$ and $\zeta_\sigma_2 - \zeta_\sigma_1 \simeq -\zeta_\sigma_2$.

The power spectrum is obtained as

$$P_\zeta = (1 - R)^2 P_\zeta + R^2 P_\zeta_\sigma. \quad (46)$$

by using Eqs. (38) and (43). In Eq. (46), the spectrum of radiation and the curvaton is given by

$$P_\zeta_\sigma = \left(\frac{H_*^2}{2\pi|\phi|}\right)^2 \frac{H_*^2}{8\pi^2\epsilon M_P^2},$$

$$P_\zeta_\sigma = \frac{H_*^2}{4\pi^2} \left(\frac{2\sigma_{2os}'}{3\sigma_{2os}}\right)^2$$

with

$$\epsilon \equiv \frac{M_P^2}{2} \left(\frac{V_\phi}{V}\right)^2 \simeq \frac{\dot{H}}{H^2}. \quad (48)$$

We defined $\tilde{r}$ as the ratio of the contribution to the linear perturbation of $\zeta$ from the curvaton to that from the inflaton, i.e.

$$\tilde{r} \equiv \frac{R^2 P_\zeta_\sigma}{(1 - R)^2 P_\zeta_\sigma} = \frac{R^2}{(1 - R)^2}\frac{2\epsilon}{3\sigma_{2os}^2} \left(\frac{2\sigma_{2os}'}{3\sigma_{2os}}\right)^2 M_P^2. \quad (49)$$

In the limit of $\tilde{r} \to \infty$, Eq. (46) becomes that of simple curvaton scenario neglecting the inflaton contribution and in the opposite limit $\tilde{r} \to 0$ the power spectrum has dominant contribution from that of the inflaton. Although for both cases the observable non-Gaussianity is possible [17], the constraint from tensor-to-scalar ratio disfavors the small $\tilde{r}$ region as we will see later.

The scalar spectral index is given by

$$n_s = 1 - 2\epsilon + \frac{2V_{\sigma\sigma}}{3H_*^2}, \quad (50)$$

and we find $2\epsilon \simeq 0.04$ from the WMAP [4] data, $n_s \simeq 0.96$. With this value of $\epsilon$, Eq. (46) and Eq. (47) relate $H_*$ with the primordial power spectrum for the given initial value of $\sigma_{os}$. Therefore we can obtain the each contribution from the curvaton and the inflaton separately and thus $\tilde{r}$ in our scenario.
The tensor perturbation (gravitational wave) is also generated during inflation \[24\]. The tensor to scalar ratio \(r_T\) is given by

\[
r_T = \frac{P_T}{P_\zeta} = \frac{16\epsilon}{(1 - R)^2(1 + r)}
\] (51)

If the amplitude is large enough, the gravitational wave background is detectable through the measurement of the B-mode polarization in the cosmic microwave background (CMB) anisotropy by Planck \[5\] as well as the direct detection by future interferometers such as DECIGO \[26\].

We obtain the nonlinearity parameters

\[
f_{NL} = \frac{5}{6} \bar{r}^2 \left[ 3 + \frac{A_2}{R} - 2 - R \right],
\] (52)

\[
g_{NL} = \frac{25}{54} \bar{r}^3 \left[ \frac{9 + 9A_2 + A_3}{R^2} - \frac{18 + 6A_2}{R} - 4 - 3A_2 + 10R + 3R^2 \right].
\] (53)

Here we have assumed that \(\zeta_r\) is Gaussian so that only \(\zeta_{r1}\) is non-zero and \(\zeta_{r1} \ll \zeta_{\sigma 1}\) which is true in the curvaton scenario when \(\sigma_\star \ll M_P\).

The WMAP data on the power spectrum of scalar and tensor perturbation

\[
\mathcal{P}_\zeta \simeq 2.4 \times 10^{-9}, \quad r_T < 0.36,
\] (54)

as well as the local type non-linearity parameter

\[-10 < f_{NL} < 74,
\] (55)

constrain possible values of \(H_\star, \sigma_\star\) and \(R\).

A. small initial expectation value

For a small initial amplitude of the curvaton field \(v < \sigma_\star < \sigma_v\), the oscillation starts when the quadratic term dominates. In this case, from Eq. (18) with \(\sigma_{os} = \sigma_\star\), we find that

\[
\sigma_{2os}' = \frac{(m_\sigma/2\sqrt{\lambda})^{3/4}}{(3\sigma_{os}^2 - v^2)^{3/8}} \left( 1 - \frac{9\sigma_{os}(\sigma_{os} - v)}{4(3\sigma_{os}^2 - v^2)} \right)
\]

\[
\sigma_{2os}'' = \frac{(m_\sigma/2\sqrt{\lambda})^{3/4}}{(3\sigma_{os}^2 - v^2)^{11/8}} \left( -\frac{9}{2}\sigma_{os} - \frac{9}{4}(\sigma_{os} - v) + \frac{297(\sigma_{os} - v)\sigma_{os}^2}{16(3\sigma_{os}^2 - v^2)} \right),
\] (56)

\[
\sigma_{2os}''' = \frac{(m_\sigma/2\sqrt{\lambda})^{3/4}}{(3\sigma_{os}^2 - v^2)^{11/8}} \left( -\frac{27}{4} + \frac{891\sigma_{os}^2 + (\sigma_{os} - v)\sigma_{os}}{16(3\sigma_{os}^2 - v^2)} - \frac{16929(\sigma_{os} - v)\sigma_{os}^2}{64(3\sigma_{os}^2 - v^2)^2} \right).
\]
The curvature perturbation of the curvaton $\zeta_\sigma$ and the corresponding non-linearity parameters are evaluated from this. Note that there are additional factors coming from the dependence on $H_{2\sigma_{\text{os}}}$ compared to the simple curvaton model with quadratic potential. One thing to note is that $A_2$, defined in Eq. (40), becomes negative when $\sigma_{\text{os}} \gtrsim 1.8v$, which changes the sign of $f_{NL}$ in Eq. (52) for small $R$. The non-trivial behavior from this is shown in the lower part of Figs. 4 and 5.

For the pure quadratic potential limit $\sigma_{\text{os}} \simeq v$, the expression of $\zeta_\sigma$ is reduced to

$$\zeta_\sigma = \frac{2}{3} \frac{\delta \sigma_*}{\sigma_* - v} - \frac{1}{3} \left( \frac{\delta \sigma_*}{\sigma_* - v} \right)^2 + \frac{2}{9} \left( \frac{\delta \sigma_*}{\sigma_* - v} \right)^3. \tag{57}$$

The density perturbation of radiation after the curvaton decay is

$$\zeta = (1 - R)\zeta_r + \frac{2R}{3} \left( \frac{\delta \sigma_*}{\sigma_* - v} \right) + \frac{2}{9} \left( \frac{3}{2R} - 2 - R \right) R^2 \left( \frac{\delta \sigma_*}{\sigma_* - v} \right)^2$$

$$+ \frac{4}{81} \left( - \frac{9}{R} + \frac{1}{2} + 10R + 3R^2 \right) R^3 \left( \frac{\delta \sigma_*}{\sigma_* - v} \right)^3 + \ldots. \tag{58}$$

The nonlinearity parameters are given for $v < \sigma_* < \sigma_v$, from Eqs. (52), (53) and (38),

$$f_{NL} = \left( \frac{\tilde{r}}{1 + \tilde{r}} \right)^2 \frac{5}{6} \left( \frac{3}{2R} - 2 - R \right) > -\frac{3}{2}$$

$$g_{NL} = \left( \frac{\tilde{r}}{1 + \tilde{r}} \right)^3 \frac{25}{54} \left( - \frac{9}{R} + \frac{1}{2} + 10R + 3R^2 \right). \tag{59}$$

The sizable large $f_{NL} \sim 100$ is obtained with a small ratio $R \sim 10^{-2}$.

**B. large initial expectation value**

1. evolution of perturbations

Next, we consider a large initial amplitude of the curvaton field $\sigma_* \gg v$. At the early stage of oscillation, the field evolution is due to the quartic potential and highly nonlinear. For our model, the corresponding quantity $\sigma_{2\sigma_{\text{os}}}$ is analytically related to $\sigma_*$ using Eqs. (15) and (34), ignoring the $\Theta$ part. Then, we obtain

$$\frac{1}{\sigma_{2\sigma_{\text{os}}}} \frac{d\sigma_{2\sigma_{\text{os}}}}{d\sigma_*} \simeq \frac{3}{4\sigma_*},$$

$$\frac{\sigma_{2\sigma_{\text{os}}} \sigma''_{2\sigma_{\text{os}}}}{(\sigma_{2\sigma_{\text{os}}})^2} \simeq -\frac{1}{3},$$

$$\frac{\sigma_{2\sigma_{\text{os}}} \sigma''_{2\sigma_{\text{os}}}}{(\sigma_{2\sigma_{\text{os}}})^3} \simeq \frac{5}{9}. \tag{60}$$
for a high reheating temperature case, and
\[
\begin{align*}
\frac{1}{\sigma_{2os}} \frac{d\sigma_{2os}}{d\sigma_*} & \approx \frac{1}{2\sigma_*}, \\
\sigma_{2os}^\prime & \approx 1, \\
\frac{\sigma_{2os}^\prime}{(\sigma_{2os}^\prime)^2} & \approx -1, \\
\frac{\sigma_{2os}^\prime}{(\sigma_{2os}^\prime)^3} & \approx 3,
\end{align*}
\] (61)

for a low reheating temperature case. The prime denotes the derivative with respect to \(\sigma_*\).

In addition, concerning with the \(\Theta\) part, because of the high nonlinearity, the fluctuation \(\delta\sigma\) also undergoes nontrivial evolution. The equation of motion for \(\delta\sigma\) of superhorizon scale \((k \ll H a)\) is given by
\[
\ddot{\delta}\sigma + 3H \dot{\delta}\sigma + \lambda(3\sigma^2 - v^2)\delta\sigma = 0.
\] (62)

During \(\sigma \approx 0\), \(\delta\sigma\) has effectively the negative mass. This tachyonic instability leads to significant amplification of the fluctuation \(\delta\sigma\), in some cases that the initial value \(\sigma_{os}\) corresponds to the transition of the VEV from \(-v\) to \(v\) in Fig. 1. Figure 3 shows the amplification and evolution of the field fluctuation \(\delta\sigma\) and the density \(\zeta_{\sigma}\) for some \(\sigma_{os}\)s. For cases in which the field \(\sigma\) stays near the origin longer, the amplification is sizable and, with Eq. (16), roughly estimated as
\[
T \equiv \frac{\delta\sigma_{2os}}{\delta\sigma_*} \sim e^{\sqrt{\lambda v}\Delta t} \sim e^{\frac{\sigma_{os}}{v}}
\] (63)

for a high reheating case, as seen in the middle row in Fig. 3 for \(\sigma_{os} = 25v\). Here \(\Delta t\) denotes the period during the tachyonic instability works. The final fluctuation after quadratic oscillation starts is, with the amplification factor \(T\), given by
\[
\delta\sigma|_{2os} = T \frac{d\sigma_{2os}}{d\sigma_*} \delta\sigma_*.
\] (64)

However, this enhancement occurs only for limited conditions of \(\sigma_{os}\) near the VEV transition initial expectation value. Thus, from now on, we consider cases without this enhancement and these enhanced modes will be studied in future works. Then, we obtain the curvature perturbation of the curvaton field
\[
\zeta|_{t \gg t_v} = \begin{cases} 
\frac{1}{3} \frac{\delta\sigma_*}{\sigma_*} - \frac{1}{9} \left(\frac{\delta\sigma_*}{\sigma_*}\right)^2 + \frac{1}{9} \left(\frac{\delta\sigma_*}{\sigma_*}\right)^3, & \text{for high } T_R, \\
\frac{1}{2} \frac{\delta\sigma_*}{\sigma_*} - \frac{1}{4} \left(\frac{\delta\sigma_*}{\sigma_*}\right)^2 + \frac{1}{6} \left(\frac{\delta\sigma_*}{\sigma_*}\right)^3, & \text{for low } T_R.
\end{cases}
\] (65)

This is conserved until the curvaton decay for \(t \gg t_v\), as seen in figure.
FIG. 3: The evolution of $\sigma$ (left), $\delta\sigma$ (center), and $\zeta$ (right). The upper (middle, lower) row corresponds to the results for $\sigma_{_{\text{OS}}} = 5 \left(25, 50\right)$ $v$. Notice that the vertical axis of the right-middle figure is logarithmic scale. Here, we assume $\delta\sigma_*= H_*/(2\pi)$ with $H_* = 10^{-3} v$. The green (purple) line in the right figures expresses the analytic formula of $\zeta$ at $t_{_{\text{OS}}}(t_{_{\text{U}}})$ without including the amplification effect. These show that the error of the analytic formula is just about $O(10\%)$ unless the tachyonic instability is induced as for $\sigma_{_{\text{OS}}} = 5v, 50v$.

2. observables

Since the total curvature perturbation of radiation after the curvaton decay is conserved, it can be calculated at the time of the curvaton decay. For the case of double well potential, i.e. initially quartic term dominates and the mass term becomes dominant before the curvaton decays, the primordial curvature perturbation is obtained, from Eqs. (43) and (65)
with $\sigma_{os} = \sigma_*$, as

$$
\zeta = (1 - R) \zeta_r + \frac{R}{2} \left( \frac{\delta \sigma_*}{\sigma_*} \right) \left( \frac{1}{R} - 2 - R \right) \frac{1}{8} R^2 \left( \frac{\delta \sigma_*}{\sigma_*} \right)^2 
+ \frac{1}{48} \left( - \frac{1}{R^2} - \frac{6}{R} + 2 + 10R + 3R^2 \right) R^3 \left( \frac{\delta \sigma_*}{\sigma_*} \right)^3, \quad \text{for high } T_R,
$$

(66)

and

$$
\zeta = (1 - R) \zeta_r + \frac{R}{3} \left( \frac{\delta \sigma_*}{\sigma_*} \right) \left( -2 - R \right) R^2 \left( \frac{\delta \sigma_*}{\sigma_*} \right)^2 
+ \frac{1}{162} \left( 5 + 10R + 3R^2 \right) R^3 \left( \frac{\delta \sigma_*}{\sigma_*} \right)^3, \quad \text{for low } T_R,
$$

(67)

where we assume that the perturbation of radiation, which has the origin from the inflaton field $\zeta_r$, is Gaussian and these higher order contributions are negligible, compared to those of the curvaton. Here $R$ is evaluated when the curvaton decay as

$$
R \simeq \frac{3\rho_\sigma}{4\rho_r + 3\rho_\sigma} \bigg|_{H = \Gamma_\sigma} = \frac{3r}{4 + 3r} \bigg|_{H = \Gamma_\sigma},
$$

(68)

where $\rho_r$ and $\rho_\sigma$ are the energy densities of radiation and the curvaton respectively and $r$ is the ratio of them, $r \equiv \rho_\sigma / \rho_r$. Note that $\zeta_r$ can be comparable to $R \zeta_\sigma$ with small $R$, which becomes the general mixed inflaton-curvaton scenario [32].

The nonlinearity parameters are given for a large initial amplitude, $\sigma_* \gg \sigma_v$, for high $T_R$ from Eq. (66) as

$$
\begin{align*}
\fnl & = \left( \frac{\tilde{r}}{1 + \tilde{r}} \right)^2 \frac{5}{6} \left( \frac{1}{R} - 2 - R \right) > -2 \\
gnl & = \left( \frac{\tilde{r}}{1 + \tilde{r}} \right)^3 \frac{25}{54} \left( - \frac{1}{R^2} - \frac{6}{R} + 2 + 10R + 3R^2 \right), \quad \text{for high } T_R,
\end{align*}
$$

(69)

and for a low $T_R$ from Eq. (67) as

$$
\begin{align*}
\fnl & = \left( \frac{\tilde{r}}{1 + \tilde{r}} \right)^2 \frac{5}{6} (-2 - R) \\
gnl & = \left( \frac{\tilde{r}}{1 + \tilde{r}} \right)^3 \frac{25}{54} \left( 5 + 10R + 3R^2 \right), \quad \text{for low } T_R,
\end{align*}
$$

(70)

The sizable large $\fnl \sim 100$ is obtained with a small ratio $R \sim 10^{-2}$ for a high $T_R$. However it is impossible to have such a large non-Gaussianity for a low $T_R$ since there are cancellations in the coefficients of inverse of $R$-terms. In this region, the curvaton starts the quartic oscillation, when the Universe is dominated by the oscillating inflaton field.
FIG. 4: Left: The contour plots of $R$ and $r_T$. Here we used $v = 10^{15}$ GeV and $T_R = 10^{12}$ GeV. Right: The contour plot of $f_{NL}$ in the same plane of the left. For the sign of $f_{NL}$ refer to figure 6.

FIG. 5: Left: The contour plots of $R$ and $r_T$. Here we used $v = 10^{15}$ GeV and $T_R = 10^{14}$ GeV. Right: The contour plot of $f_{NL}$ in the same plane of the left. For the sign of $f_{NL}$ refer to figure 6.

In Figs 4 and 5, the contour plots of gravitational wave background and $R$ (left window) $f_{NL}$ (right window) are shown with observational constraints for the decay rate given by Eq. (6) in the plane parameters of $\lambda$ and $\sigma_*$. Here we have fixed VEVs $v = 10^{15}$ GeV, and the reheating temperature $T_R = 10^{12}$ GeV (figure 4) and $T_R = 10^{14}$ GeV (figure 5) separately. The observational constraints include the tensor-to-scalar ratio $r_T = 0.1$ and $10^{-3}$ for the expected sensitivity of B-mode detection by Planck and future instruments such as CMBPol [54], respectively. For given $\sigma_{os}$ and $\lambda$, imposing Eq. (54) on Eq. (46) determines $H_*$ with $\epsilon \simeq 0.02$ from Eq. (50).

As can be seen in the figures, the large $\sigma_*$ or the large $\lambda$ region is excluded by the null detection of gravitational wave. The region with $\tilde{r} < 1$ belongs to this excluded region,
FIG. 6: The plots for $f_{NL}$ (left) with the fixed $\lambda$. Here $v = 10^{15}$ GeV and $T_R = 10^{12}$ GeV (left) or $T_R = 10^{14}$ GeV (right).

which means that in the allowed region the power spectrum is dominated by that from the curvaton in our scenario. Above the diagonal line, written above (curvaton dominated), the curvaton energy dominates the Universe before it decays. In the limit of curvaton domination ($R = 1$) the non-linearity parameter becomes $f_{NL} = -5/3$ or $-5/2$ for high $T_R$ and low $T_R$ respectively. Below the line ‘(curvaton dominated)’, $R$ can be much smaller than 1 and thus there is a chance to obtain large non-Gaussianity for high $T_R$ case. This region appears in figure 5 (Right window), as a wedge shape above $\sigma_{os} > \sigma_v$. In this region, it is possible to generate large non-Gaussianity of the order of 20. On the other hand, for a larger initial amplitude the curvaton oscillation starts before reheating, corresponding to low-$T_R$, and the non-Gaussianity is significantly suppressed because of the cancellation as discussed after Eq. (70). This happens in the ‘(low $T_R$)’ region above the dashed diagonal line in figure 5. For $T_R = 10^{12}$ GeV (figure 4), all the drawn region corresponds to ‘(low $T_R$)’ and the non-Gaussianity is small for $\sigma_{os} > \sigma_v$.

The interesting behavior happens for the initial amplitude $v < \sigma_* < \sigma_v$, where the oscillation of the curvaton starts in the quadratic term dominated potential. There are two regions depending on the sign of $f_{NL}$, positive for $v < \sigma_* \lesssim 1.8v$ and negative for $1.8v < \sigma_* \lesssim \sigma_v$. This difference is due to the evolving effective mass in a double well potential, which is constant in the pure quadratic potential. In the figure 6 we show the plot of $f_{NL}$ depending on $\sigma_*$ for given $\lambda$ for $T_R = 10^{12}$ GeV (left) and $10^{14}$ GeV (right) respectively. The positive $f_{NL}$ with the magnitude of the order of 100 is possible for a small
initial amplitude $v < \sigma_* \lesssim 1.8v$.

V. SUMMARY

We have studied the density perturbation generated by a curvaton whose potential is flat with small self-coupling in a double well type. We have used a large VEV, $v > 10^{15}$ GeV, to avoid the domain wall formation and a larger initial curvaton amplitude $\sigma_*$ which is easily obtained in this flat potential.

We have analyzed the cosmological evolution of the scalar field in this flat double well potential to see the viability of the field as a curvaton to generate the primordial density perturbation to explain the structure formation and the anisotropies in the CMB. With a large initial expectation value $\sigma_{os} \gg v$, the energy density decreases as initially $\propto a^{-4}$ and at late time as $\propto a^{-3}$. This is same as a massive curvaton with self-interaction with vanishing VEV. However, three crucial differences and features appear. One is the tachyonic amplification of the fluctuation by the negative mass squared at the origin, for a particular initial value $\sigma_{os}$ which periodically appear in the parameter space. We have shown that, except for these tuned boundaries, the density perturbation and other nonlinear parameters are well approximated by analytic formula. The second is the suppression of non-Gaussianity even for very subdominant curvaton, if the reheating temperature after inflation is as low as not to satisfy Eq. (23). The other is the non-trivial behavior of $f_{NL}$ even when the curvaton field starts oscillation trapped at one of the minima. We found a successful scenario needs the flat potential with small self-coupling of the field of the order of $\lesssim 10^{-10}$ for a reasonable reheating temperature.

In conclusion, we have found the differences in massive self-interacting curvaton models with and without VEV. In addition, in a double well potential curvaton model, non-linear parameters can not be so large for a large initial field value and a low reheating temperature. Therefore, if both large nonlinearity and B-mode polarization will be detected, the potential of the curvaton or thermal history of the early Universe will be constrained.
Acknowledgments

We would like to thank Tomo Takahashi for valuable communication. This work is in part supported by the Korea Research Foundation Grant funded from the Korean Government (KRF-2008-341-C00008), by the second stage of Brain Korea 21 Project in 2006 (K.Y.C), and by the scientific research grants from Hokkai-Gakuen (O.S). O.S would like to thank Department of Physics at Pusan National University for their warm hospitality where many parts of this work have been done.

[1] A. A. Starobinsky, JETP Lett. 30 682 (1979) [Pisma Zh. Eksp. Teor. Fiz. 30 719 (1979)]; K. Sato, Mon. Not. Roy. Astron. Soc. 195, 467 (1981); A. H. Guth, Phys. Rev. D 23, 347 (1981); A. D. Linde, Phys. Lett. B 108 389 (1982); A. Albrecht and P. J. Steinhardt, Phys. Rev. Lett. 48 1220 (1982).
[2] S. W. Hawking, Phys. Lett. B 115, 295 (1982); A. A. Starobinsky, Phys. Lett. B 117, 175 (1982); A. H. Guth and S. Y. Pi, Phys. Rev. Lett. 49, 1110 (1982).
[3] J. M. Maldacena, JHEP 0305, 013 (2003).
[4] E. Komatsu et al., arXiv:1001.4538 [astro-ph.CO].
[5] http://www.rssd.esa.int/index.php?project=PLANCK
[6] X. Chen, R. Easther and E. A. Lim, JCAP 0706, 023 (2007).
[7] E. Silverstein and D. Tong, Phys. Rev. D 70, 103505 (2004); M. Alishahiha, E. Silverstein and D. Tong, Phys. Rev. D 70, 123505 (2004); X. Chen, M. X. Huang, S. Kachru and G. Shiu, JCAP 0701, 002 (2007).
[8] K. Y. Choi, L. M. H. Hall and C. van de Bruck, JCAP 0702 029 (2007).
[9] M. Zaldarriaga, Phys. Rev. D 69, 043508 (2004).
[10] C. T. Byrnes, K. Y. Choi and L. M. H. Hall, JCAP 0810 008 (2008).
[11] C. T. Byrnes, K. Y. Choi and L. M. H. Hall, JCAP 0902 017 (2009).
[12] D. H. Lyth, JCAP 0511 006 (2005).
[13] L. Alabidi and D. Lyth, JCAP 0608 006 (2006).
[14] M. Sasaki, Prog. Theor. Phys. 120 159 (2008).
[15] A. Naruko and M. Sasaki, Prog. Theor. Phys. 121, 193 (2009).
[16] K. Enqvist, A. Jokinen, A. Mazumdar, T. Multamaki and A. Vaihkonen, Phys. Rev. Lett. 94 161301 (2005); JCAP 0503 010 (2005);
A. Jokinen and A. Mazumdar, JCAP 0604 003 (2006).
[17] for a recent review on local type non-Gaussianity, see e.g., C. T. Byrnes and K. Y. Choi, arXiv:1002.3110 [astro-ph.CO]; D. Wands, Class. Quant. Grav. 27, 124002 (2010).
[18] S. Mollerach, Phys. Rev. D 42 313 (1990).
[19] A. D. Linde and V. F. Mukhanov, Phys. Rev. D 56 535 (1997).
[20] D. H. Lyth and D. Wands, Phys. Lett. B 524, 5 (2002).
[21] T. Moroi and T. Takahashi, Phys. Lett. B 522, 215 (2001) [Erratum-ibid. B 539, 303 (2002)].
[22] K. Enqvist and M. S. Sloth, Nucl. Phys. B 626, 395 (2002).
[23] D. H. Lyth, C. Ungarelli and D. Wands, Phys. Rev. D 67, 023503 (2003).
[24] B. Allen, Phys. Rev. D 37, 2078 (1988);
V. Sahni, Phys. Rev. D 42, 453 (1990).
[25] K. Nakayama and J. Yokoyama, JCAP 1001, 010 (2010).
[26] N. Seto, S. Kawamura and T. Nakamura, Phys. Rev. Lett. 87, 221103 (2001).
[27] J. Crowder and N. J. Cornish, Phys. Rev. D 72, 083005 (2005).
[28] N. Bartolo, S. Matarrese and A. Riotto, Phys. Rev. D 69 043503 (2004).
[29] D. H. Lyth and Y. Rodriguez, Phys. Rev. Lett. 95 121302 (2005).
[30] M. Sasaki, J. Valiviita and D. Wands, Phys. Rev. D 74 103003 (2006).
[31] K. A. Malik and D. H. Lyth, JCAP 0609 008 (2006).
[32] D. Langlois and F. Vernizzi, Phys. Rev. D 70, 063522 (2004);
G. Lazarides, R. R. de Austri and R. Trotta, Phys. Rev. D 70, 123527 (2004);
F. Ferrer, S. Rasanen and J. Valiviita, JCAP 0410, 010 (2004);
T. Moroi, T. Takahashi and Y. Toyoda, Phys. Rev. D 72 023502 (2005);
T. Moroi and T. Takahashi, Phys. Rev. D 72 023505 (2005);
K. Ichikawa, T. Suyama, T. Takahashi and M. Yamaguchi, Phys. Rev. D 78, 023513 (2008).
[33] K. Dimopoulos, G. Lazarides, D. Lyth and R. Ruiz de Austri, Phys. Rev. D 68 123515 (2003).
[34] Q. G. Huang, JCAP 0811, 005 (2008).
[35] P. Chingangbam and Q. G. Huang, JCAP 0904, 031 (2009).
[36] K. Enqvist, S. Nurmi, G. Rigopoulos, O. Taanila and T. Takahashi, JCAP 0911, 003 (2009).
[37] K. Y. Choi and J. O. Gong, JCAP 0706 007 (2007).
[38] H. Assadullahi, J. Valiviita and D. Wands, Phys. Rev. D 76 103003 (2007).
[39] Q. G. Huang, JCAP 0809 017 (2008).
[40] K. Enqvist and S. Nurmi, JCAP 0510 013 (2005).
[41] K. Enqvist and T. Takahashi, JCAP 0809, 012 (2008).
[42] Q. G. Huang and Y. Wang, JCAP 0809 025 (2008).
[43] K. Enqvist and T. Takahashi, JCAP 0912, 001 (2009).
[44] K. Enqvist, S. Nurmi, O. Taanila and T. Takahashi, JCAP 1004, 009 (2010).
[45] C. T. Byrnes, K. Enqvist and T. Takahashi, arXiv:1007.5148 [astro-ph.CO].
[46] A. D. Linde, Phys. Lett. B 259 38 (1991).
[47] S. Kasuya, M. Kawasaki and T. Yanagida, Phys. Lett. B 409, 94 (1997);
   S. Kasuya and M. Kawasaki, Phys. Rev. D 56, 7597 (1997); Phys. Rev. D 58, 083516 (1998).
[48] M. Kawasaki, K. Nakayama and F. Takahashi, JCAP 0901 026 (2009).
[49] J. E. Kim, Phys. Rev. Lett. 43, 103 (1979);
   M. A. Shifman, A. I. Vainshtein and V. I. Zakharov, Nucl. Phys. B 166, 493 (1980).
[50] K. Dimopoulos, G. Lazarides, D. Lyth and R. Ruiz de Austri, JHEP 0305, 057 (2003).
[51] L. Kofman, A. D. Linde and A. A. Starobinsky, Phys. Rev. Lett. 76 1011 (1996).
[52] D. H. Lyth, K. A. Malik and M. Sasaki, JCAP 0505 004 (2005).
[53] D. Langlois, F. Vernizzi and D. Wands, JCAP 0812 004 (2008).
[54] D. Baumann et al. [CMBPol Study Team Collaboration], AIP Conf. Proc. 1141, 10 (2009).