Discrimination of Dephasing Sources by Continuous Dynamical Decoupling

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(Dated: May 11, 2020)

Decoherence induced by the unwanted noise is one of the most important obstacles to be overcome in the quantum information processing. To do this, the noise power spectral density needs to be accurately characterized and then used to the quantum operation optimization. The known continuous dynamical decoupling technique can be used here as a noise probing method based on a continuous-wave resonant excitation field, followed by a gradient descent protocol. In this paper, we estimate the noise spectroscopy over a frequency range 0-530 kHz for both laser frequency and magnetic field noises by monitoring the transverse relaxation from an initial state $|+\sigma_z\rangle$. In the case of the laser noise, we also research into two dominant components centered at 81.78 and 163.50 kHz. We make an analogy with these noise components and driving lasers whose lineshape is assumed to be Lorentzian. This method is verified by the experimental data and finally helps to characterize the noise.

I. INTRODUCTION

The problem of a system interacting with a noisy environment is of great significance in the field of quantum computing[1]. In general, there are some strategies to fight this induced decoherence and improve the fidelity of quantum algorithms. One is through multibit enoding and error-correction protocols which is of course at the cost of more quibits[2]. And the another method is reducing the amount of environmental noise. This can be realized by applying a reverse compensation signal targeting the interested noise. Or in contrast, reduce the system’s internal sensitivity to noise through the application of coherent control-pulse methods[3–8] or encoding the information to pairs of dressed states which are formed by concatenated continuous driving fields[9–15]. However, the latter two approachs depend on the noise power spectral density(PSD) measurement[16, 17]. To do this, we first review some widely used methods.

Laser frequency noise or phase noise describes how the frequency of a laser output electrical field deviates from an ideal value. This quantity, which is defined to evaluate the short-term stability of a laser, as well as the estimated linewidth of the laser have attracted widespread attention as a fundamental topic about lasers. Generally, the frequency noise features can be revealed through the following two schemes. One is the beat-note method[18]. If the target laser is beat with only one reference laser, we will get an electrical signal that contains noise information of both lasers, but fail to separate their contributions apart. Instead, if two reference lasers are introduced in the experiment, two electrical signals will be monitored. These signals are first mixed down to a lower frequency and then analyzed by a digital cross-correlation method to characterize the frequency noise PSD of the target laser[19]. In other words, to obtain the noise PSD, we have to set up at least other two similiar lasers. This is uneconomic so that has limited applications. In addition, a Rabi spectroscopy scan using a weak and long excitation pluse is also helpful to the noise PSD characterization[20]. But the result can only be a reference as the accuracy is insufficient.

The beat-note and Rabi spectroscopy scan protocols only access to the laser noise. However, for many solid-state qubits, the environmental noise includes magnetic field fluctuations, frequency noise and power fluctuations of laser fields. If the laser power fluctuations are supressed in the experimental setup, the rest two single-axis noise will be dominant, and affect the qubit evolution in a similiar way[21]. For a semiclassical treatment, we obtain the noise induced Hamiltonian $H = \frac{1}{2} f(t)\sigma_z$ by tracing over the environmental degrees, where $f(t)$ represents the overall noise[5]. One simple and effective technique suggested for this noise PSD estimating is pulsed dynamical decoupling (PDD)[22–27]. PDD consists of the applying of $\pi$ pulses, $X_\pi$ or $Y_\pi$, to the system with designed intervals, which revert the coherence decay due to $f(t)$, such as the Carl-Purcell-Meiboom-Gill (CPMG)[28] and Uhrig dynamical decoupling (UDD)[29] sequences. However, the PDD technique may become helpless for noise separation for three reasons. First, unlike magnetic noise, the laser noise acts on the system only when the pulses are applied. This difference makes it difficult to separate them apart using the PDD theoretical model. Moreover, the finite $\pi$-pulse width can not be ignored in the experiments, and this will lead to a limited frequency range. Finally, the accumulated error from $\pi$ pulses would be considerable at high probe frequencies[30]. This makes the protocol inefficient when the fidelity of single $\pi$ pulse operation is not qualified.

An alternative approach is continuous dynamical decoupling (CDD) technique where a continuous driving field is used instead of the $\pi$ pulses in PDD[31, 32]. CDD overcomes the outlined drawbacks of PDD above, and makes it possible to detect both laser and magnetic noise. In fact, CDD is equivalent to the Rabi oscillation method used in ref.[24, 33], which makes use of the generalized Bloch equations[34]. In the rest of this paper, we mainly introduce the PSD discrimination and estimation of the laser and magnetic noises utilizing the CDD approach in a trapped ion system. Note that compared to the

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conventional beat-note protocol, this method is actually handy and economical for characterizing the interested manipulation lasers.

II. THEORETICAL MODEL AND CONTROL SETTING

For the present experiment we use a single $^{40}$Ca$^+$ ion confined in a standard blade trap. A pair of Zeeman sublevels in the $^2S_{1/2}$ and $^2D_{5/2}$ manifolds are normally used as a qubit, as shown in FIG. 1(a). We notice that the magnetic fluctuation induced decoherence is distinguishable for different qubit definition, so we employ $|1\rangle = [^2S_{1/2}, m_j = -1/2 \rangle \leftrightarrow |2\rangle = [^2D_{5/2}, m_j = -1/2 \rangle$ and $|1\rangle \leftrightarrow |3\rangle = [^2D_{5/2}, m_j = -5/2 \rangle$ as magnetic noise and thus laser noise probes, on which the same pulse sequences are performed. In the experiment, we first initialize the qubit to state $|1\rangle$ by the Doppler cooling, sideband cooling as well as optical pumping methods. Then a $\pi/2 \sigma_y$ pulse is applied to rotate the qubit by $90^\circ$ to $+\sigma_y$ axis. During the driving evolution process, the 729 nm laser beam along $\sigma_y$ is turned on for time $t$, followed by another $\pi/2 \sigma_y$ pulse. The final state is discriminated using a florescence detection scheme, as the inset in FIG. 1(b).

FIG. 1. Level structure and control sequence. (a) Scheme of two level systems driven by 729 nm laser. (b) Survival probability $P_s$ as a function of 729 nm-laser strength $\alpha$ for $t = 200 \mu$s and transition $|1\rangle \leftrightarrow |3\rangle$. By checking $\alpha$ with $\Omega$, we find $P_s$ has two local minimum values smaller than 0.5 at $\Omega \approx 82$ kHz and 164 kHz. They can be viewed as two sufficiently strong noise components which do not follow the rules of Eq. (2) and should be modelled in another way. Each point represents 200 experiments, and the error bars denote the error of the mean.

For the driving evolution step, we consider a two level system (TLS) subjected to the noise from manipulation laser and magnetic field fluctuations. For resonant driving along the $\sigma_y$ axis with Rabi frequency $\Omega$, the Hamiltonian describing the qubit system in the rotaing frame can be written as

$$H = f(t)[\cos(\Omega t)\sigma_y + \sin(\Omega t)\sigma_x]/2.$$ (1)

Here, $f(t) = f_h(t) - f_l(t)$ (unit: rad) is the combined noise in time domain, and $f_h(t)$, $f_l(t)$ represents the part from magnetic field fluctuations (laser). Generally, $f(t)$ can be treated as a stationary, Gaussian-distributed function with zero mean, i.e., $\langle f(t) \rangle = 0$. The statistics of this stochastic process $f(t)$ is then fully determined by its autocorrelation function $C(t_1 - t_2) = \langle f(t_1)f(t_2) \rangle$, or equivalently by its power spectral density defined by the Fourier transform of $C(t)$: $S(f) = \int_{-\infty}^{\infty} C(t)e^{-2\pi if}dt$ (Wiener-Khinchin theorem).

In the weak noise limit, the connection between the driving evolution time $t$ and the survival probability $P_s$ of qubit on $|\pm \sigma_y\rangle$ has already been discussed in ref.[35] based on the stochastic Liouville theory and superoperator formalism. When only up to the second order cumulant is considered:

$$P_s(t) = \frac{1}{2} + \frac{1}{2} \exp \left(-\int_0^t df S(f)F(f, \Omega, t)\right),$$ (2)

where $F(f, \Omega, t) = \frac{t^2}{2} \left[ \sin^2((f + \Omega)t) + \sin^2((f - \Omega)t) \right]/2$ is the filter function characterized by $t$, $\Omega$ and peaked at $f = \pm \Omega$ with full width at half maximum (FWHM) $1/t$ Hz. This means the performance of the sequence is sensitive to the characteristics of the noise, and we can get some sketchy but impressive information about the noise strength at Fourier frequency $f = \Omega$ by simply scanning $\Omega$.

III. EXPERIMENTAL SCHEME AND RESULT

With $t = 200 \mu$s, we obtain the survival probability $P_s$ as a function of $\alpha$, which refers to the 729 nm-laser amplitude modulation used in arbitrary wave-form generator (AWG). As shown in FIG. 1(b), there are two dominant components of noise at $\approx 82$ kHz and $\approx 164$ kHz that are too strong to be described by Eq. (2). According to Eq. (2), we will see an exponential decay from 1 to 0.5 with time-dependent rate $\gamma(t) = \frac{1}{2} \int_0^t df S(f)F(f, \Omega, t)$. Generally, this is valid only if the PSD of noise varies gently as a function of $f$. However, FIG. 1(b) suggests that when $f \approx 82$ or $164$ kHz, $P_s$ can be at least down to 0.3, smaller than the limit value 0.5 of Eq. (2). This means that we have to research on these two components of noise by another method.

The research on a strong coupled environment has already been discussed in some articles like ref.[8, 36]. However, in this model, a discrete spectrum assumption is made. They express the PSD as the sum of discrete noises $S(f) = \sum_{k=1}^{N} S_k \delta(f - f_k)$ where $S_k$ is the noise strength at $f_k$. Obviously, it ignores the linewidth of the PSD around $f_k$. This is valid when study some dominant components whose linewidth is relatively small, such as the noise at 50 Hz and the harmonics coming from the power line[8]. In our system, the 82 and 164 kHz components of noise come from the modulation process of the etalon of 729 nm Ti sapphire laser, and have non-negligible linewidth according to FIG. 1(b). Here, we measure the evolution of $P_s$ as a function of time $t$ with $\Omega$ being set to around 82 kHz, as shown in FIG. 2(a). We observe Rabi oscillations between $|\pm \sigma_y\rangle$ which are similar to the case where a TLS is driven by a (off)-resonant laser beam. Therefore, we will make an analogy with these noise components and driving lasers, called laser-like noise (LLN) in the rest of this paper. And there is one important assumption: the laser lineshape of LLN is Lorentzian[32], so that the frequency noise of LLN is white noise with PSD $S_{LLN}(f) = h$ [37]. The relation between $h$ and lineshape of LLN can be expressed as $I(f) = E_h^2/[h^2 + 16\pi^2(f - f_0)^2]$ with the half
width at half maximum (HWHM) $\Gamma = h/4\pi$ Hz[38] and central frequency $f_0 \approx 82$ or 164 kHz whose exact values are needed to be extracted from the experimental data.

The solid curves are fittings using $\Gamma_1$ and $\Gamma_2$. (b) Fitting results of HWHM $\Gamma$ of the $\approx 82$ kHz noise component. Each point represents a fitting result at different $\Omega$. Solid and dashed lines represent the average values of $\Gamma$ for both transitions, then the mean value 0.41 kHz of which is employed to characterize this noise. (c) By preparing the initial state $|1\rangle$, and then applying the resonant 729 nm laser beam driving the transition $|1\rangle \leftrightarrow |3\rangle$ with $\Omega = 79.9$ kHz, we measure the $|1\rangle$ state population $P_1$ to verify the parameters obtained above. The blue points are the experimental data, and the blue curve serves to guide our eyes. The numerical simulation result is represented by the solid red curve, which is consistent with the experimental data. Each point represents 200 experiments, and the standard measurement error bars are not shown in (a) and (c).

We write the Hamiltonian describing a TLS driven by the LLN in the interaction picture referring to the frequency of 729 laser

$$H_{LLN}^{I} = \frac{1}{2} \Omega_0 \sigma_z + \Omega_{LLN} \cos \left( \Omega_0 t + \int_0^t f_{LLN}(\tau) d\tau \right) \sigma_z. \quad (3)$$

Here, $\Omega_{LLN} = E_0/2$, $\Omega_0$ represent the resonant Rabi frequency and central frequency of LLN respectively. $f_{LLN}(t)$ is the frequency noise of LLN in time domain. Transform $H_{LLN}^{I}$ to the interaction picture with $H_0 = \frac{1}{2} \Omega_0 \sigma_z$:

$$H_{LLN}^{I} = -\frac{1}{2} \Delta \sigma_z + \frac{1}{2} \int f_{LLN}(t) \, d\tau \sigma_z + \frac{1}{2} \Omega_{LLN} \sigma_z. \quad (4)$$

where $\Delta = \Omega_0 - \Omega$ is the detuning. Because we assume that the LLN noise is short correlated (white noise), then the Bloch-Redfield theory[34] gives the relationship: $\Gamma_2 = \frac{1}{2} \Gamma_1 + \Gamma_\phi$. Here, $\Gamma_1 = T_1^{-1}$, $\Gamma_2 = T_2^{-1}$ are the longitudinal ($\sigma_z$ axis) and transverse ($\sigma_{x,y}$ plane) relaxation rate respectively. $\Gamma_\phi$ shows the so-called pure dephasing of the qubit and $\Gamma_\phi = \frac{S_{LLN}}{2} = h/2$. When $\Delta \neq 0$, the qubit dynamics is conveniently described in a new eigenbasis $\{|\pm \sigma_z\rangle\}$ rotated from the conventional basis $\{|\pm \sigma_z\rangle\}$ by an angle $\eta = \arctan \frac{\Omega_{LLN}}{\Delta}$.

Here, we define two relaxation rates $\tilde{\Gamma}_1$ and $\tilde{\Gamma}_2$ analogous to $\Gamma_1$ and $\Gamma_2$. They correspond to the decay of longitudinal and transverse parts of the density matrix in the new basis, respectively. Then we obtain[39]

$$\tilde{\Gamma}_1 = \sin^2 \eta \Gamma_\phi + \frac{1 + \cos^2 \eta}{2} \Gamma_1 \quad (5)$$

$$\tilde{\Gamma}_2 = \cos^2 \eta \Gamma_\phi + \frac{\sin \eta}{2} \Gamma_1. \quad (6)$$

As a result we obtain

$$\tilde{\Gamma}_2 = \frac{3 - \cos^2 \eta}{4} \Gamma_1 + \frac{3 + \cos^2 \eta}{2} \sin^2 \eta \Gamma_\phi. \quad (7)$$

The derivation of Eq. (7) utilizes the expression $\tilde{\Gamma}_2 = \tilde{\Gamma}_1 + \tilde{\Gamma}_\phi$. Note the intensity of 729 nm laser is stabilized in our experimental setup, so the longitudinal relaxation effect $\Gamma_1$ is negligible. And only the noise perpendicular to $\sigma_z$ axis ($\Gamma_\phi$) contributes to $\Gamma_1$ and $\Gamma_2$. Thus Eq. (5, 7) can be simplified to the form $\tilde{\Gamma}_1 = \sin^2 \eta \Gamma_\phi$ and $\tilde{\Gamma}_2 = \cos^2 \eta + \frac{1}{2} \sin^2 \theta \eta \Gamma_\phi$, which are used to get the information $\Gamma_\phi$, $\Omega_{LLN}$ and $f_0$ about the line shape $I(f)$ by fitting with the LLN-driving Rabi oscillations, as shown in FIG. 2(a). Data are obtained by measuring $P_1$ as a function of $t$. For each $\Omega$, the LLN results in a Rabi oscillation with angular precession frequency $\Omega_\theta = 2 f_{LLN}^2 + \Delta^2$. To determined these parameters as precise as possible, we average over more than twenty Rabi oscillations for each dominant noise component, and finally obtain $\Gamma = h/4\pi = 0.41$ kHz, $\Omega_{LLN} = 2.56$ kHz, $f_0 = 81.78$ kHz for noise around 82 kHz and $\Gamma = 0.57$ kHz, $\Omega_{LLN} = 3.30$ kHz, $f_0 = 163.50$ kHz for noise around 164 kHz. In FIG. 2(b), an example of the averaged $\Gamma$ is given. The solid red diamonds and solid black dots show the fitted results for transitions $|1\rangle \leftrightarrow |2\rangle$ and $|1\rangle \leftrightarrow |3\rangle$ respectively. The dashed red and solid black lines are fittings of these $\Gamma$. The small difference between transition lines may be resulted from the grey magnetic field or just from the detecting errors. In our opinions, it is resolvable to determine $\Gamma$ just by averaging over the means of two transitions. To verify $I(f)$ obtained above, we measure the Rabi oscillation in the laboratory frame with the 729 nm laser driving the transition $|1\rangle \leftrightarrow |3\rangle$ from initial state $|1\rangle$, and $\Omega = 79.9$ kHz. In FIG. 2(c), the probability of staying on state $|1\rangle$, defined as $P_1$, is plotted versus the driving time $t$. Interestingly, as $t$ changes, $P_1$ experiences several collapses and revivals, which should be a consequence of the phase modulation resulted from the 81.78 kHz noise. The solid red curve is a numerical simulation result using $I(\omega)$ and the obtained parameters. We find it is consistent with the experimental data points except for the first collapse, which may be resulted from the power-noise contribution neglected in this paper.

On the other hand, we study the rest of the noise spectrum by using the standard continuous dynamical decoupling technique. The evolution of $P_1$ is described by Eq. (2). Different from the experiments measuring two dominant noise components above where the 729 nm laser power remains the same in $Y_{1/2}$ and driving evolution processes (see the inset in FIG. 1(b)), here we use laser pulses with different power in these stages, so that we can save measurement time while the PSD $S(f)$ of noise is captured. Let us describe this in more details. We first carefully locate the laser power at a value where
FIG. 3. Noise power spectral density. (a) Survival probability $P_{so}$ of $|+\sigma_z\rangle$ as a function of $t$. The oscillation is resulted from the AC-Stark shift difference explained in the main text. The blue and red curves are numerical simulations using $S_0(f)$ and $S(f)$ given in (b) and (c) respectively. Each point represents 200 experiments, and the error bars are not shown. (b) Experimentally determined noise PSD $\delta_0(f)$ over a range 0–530 kHz. The inset gives the detailed description for the part below 1.5 kHz. (c) An example of $S(f)$ extrapolated from $S_0(f)$ with the gradient descent protocol for transition $|1\rangle \leftrightarrow |3\rangle$. The inset serves to show remarkable differences between $\delta_0(f)$ and $S(f)$ for the frequencies below 1.5 kHz.

We then calculate the gradient of this function $\delta f S(f)$, for any target frequency $f_i$. The gradient is used to update the initial $S(f)$ towards a closer matching of all the experimental and calculated decays. Finally, a comparison of the optimized PSD $S(f)$ (orange diamonds) with $S_0(f)$ (black dots) corresponding to $|1\rangle \leftrightarrow |3\rangle$ is shown in FIG. 3(c). We can easily find that the gradient optimization has the most remarkable effects for the extreme points of $S(f)$, especially the ones between sharp increases and decreases. The reason is simple. In these intervals, the rectangular approximation approach may not be valid anymore because $S(f)$ cannot be replaced by $S(\Omega)$ within the filter bandwidth. After obtaining the optimized $S(f)$, we also verify its correctness. In FIG. 3(a), we give the simulated evolution $P_{so}$ based on $S_0(f)$ (blue curve) and $S(f)$ (red curve) respectively. It is obvious that the red curve matches the data points better, which shows a greater fidelity of $S(f)$.

To separate the noise, we utilize the $S(f)$ for both transitions $|1\rangle \leftrightarrow |2\rangle$ and $|1\rangle \leftrightarrow |3\rangle$ obtained above whose difference can be used to speculate the magnetic field fluctuations. According to the atomic physics, the magnetic field noise-induced energy level shift changes the transition circular frequency by $f_B(t) = \frac{\mu_0 B(t)}{\hbar} [g_j(D_{j\pi/2})m_t - g_j(S_{1/2})m]$. 

The noise PSD is relatively small based on FIG. 1(b), such as $\Omega_\Delta = \Omega = 200$ kHz whose $\pi/2$ pulse lasts 1.25 $\mu$s. Then this kind of $\pi/2$ pulses are employed as $\gamma_{\pi/2}$ in all experiments where different $\Omega$ are used in the driving evolution processes. In this way, we do not need to calculate and check the $\pi/2$-pulse time when we change $\Omega$, so that lots of operation time could be saved. However, it should be mentioned that due to this difference of laser power between $\gamma_{\pi/2}$ and driving evolution stages, thus different AC-Stark shift, a Ramsey-like oscillation will be introduced when we monitor $P_s$ as a function of $t$:

$$P_{so} = \frac{1}{2} + \frac{1}{2} \cos(\delta_\Omega t) \exp \left(-\int_0^\infty df S(f)F(f, \Omega, t) \right),$$

where $\delta_\Omega$ is the AC-Astark shift difference.

FIG. 3(a) shows an example of this oscillation decay. The experimental data are represented by the green dots. We then use the filtering property of the sequence to characterize the noise spectrum. We notice the filter is sufficiently narrow about $\Omega$ so that we can treat the noise as a constant $S(\Omega)$ within its bandwidth $1/\tau$ and approximate Eq. (8) as

$$P_{so} = \frac{1}{2} + \frac{1}{2} \cos(\delta_\Omega t) \exp \left(-\frac{\delta_\Omega^2}{2} t \right).$$

Utilizing this rectangular approximation approach, the data can be fitted to get $S(\Omega)$. As a result, we obtain the noise PSD $S_0(f)$ over a frequency range 0–530 kHz by combining other two dominant noise components. As shown in FIG. 3(b), we observe several peaks at ~4 kHz and 24 kHz belonging to the frequency noise of laser, and a sharp increase below 1 kHz (the inset), which comes from the magnetic field fluctuations. We also observe a stronger noise for transition $|1\rangle \leftrightarrow |3\rangle$ (orange diamonds) than $|1\rangle \leftrightarrow |2\rangle$ (black dots), which is consistent with our assumption.

To extend the accuracy of the measured noise PSD, we propose a gradient descent protocol based on matching to the experimental oscillation decay curves. Define a objective function as the sum of the squared error between the experimentally measured decay $P_s(t_j)$ and the calculated decay rule $P'_s(t_j) = \frac{1}{2} + \frac{1}{2} \cos(\delta_\Omega t) \exp \left(-\int_0^\infty df S'(f)F(f, \Omega, t_j) \right)$ for a given $S'(f)$:

$$J = \sum_{j=1}^M (P_s(t_j) - P'_s(t_j))^2.$$
where $\mu_B$ is the Bohr magneton, $g_j$ the Lande $g$ factors, and $m', m$ are the magnetic quantum numbers of the $D_{5/2}$ and $S_{1/2}$ state, respectively. $\delta B(t)$ represents the magnetic field strength fluctuations. For $^{40}\text{Ca}^+$, $g_j(S_{1/2}) \approx 2$ and $g_j(D_{5/2}) \approx 1.2$, so that the PSD of $\delta B(t)$ can be expressed as $S_{\delta B}(f) = [S_{1+3}(f) - S_{1-2}(f)]/24 \times 0.16\mu_B^2$. And next we obtain the PSD of laser frequency noise by $S_L(f) = S_{1-2}(f) - [S_{1+3}(f) - S_{1-2}(f)]/24$, as shown in FIG. 4. From the discrete data points in FIG. 3(c) to the continuous lines here, we have employed the interpolation method. In FIG. 4, we observe complex PSD for both kinds of noise which cannot be described by a single line-type, such as $1/f^2$ used in many articles. We also see a small interconnected trend between $S_{\delta B}(f)$ and $S_L(f)$ in some peaks, such as $f \approx 4$ and 24 kHz. In our opinions, this is primarily derived from the limited accuracy at these points where the fitting error is considerable.

IV. DISCUSSION

We have employed the continuous dynamical decoupling protocol in this work to carry out the noise spectroscopy. It is based on modulating the system-environment interaction by applying a continuous driving field along $\sigma_z$ axis to the system after initializing the system on $|+\sigma_z\rangle$. This driving field and the corresponding Rabi frequency $\Omega$ create the dressed states $|+\sigma_z\rangle$ and $|\sigma_z\rangle$ between which the transition frequency is $\Omega$ in the interaction picture. In this way, the system will suffer less from the most frequency bands except for those around $\Omega$, which will induce state transitions as long as the noise spectroscopy $S(f)$ is nonnegligible. By using this filtering property of controlling sequences and the gradient descent algorithm, we have directly characterized the environmental noise PSD over a range 0-530 kHz, which in turn enables the design of coherent control method that targets the specific noise. However, for the case of laser noise, there are two dominant components centered at 81.78 and 163.50 kHz that are two strong to use this filter function method. Instead, we propose an approach to regard these noises as driving lasers performed in $\{|+\sigma_z\rangle, |\sigma_z\rangle\}$ space after initialized on $|+\sigma_z\rangle$. To simplify the model, a Lorentzian-lineshape assumption is made. This turns out to be particularly useful to strong noise components whose linewidth can not be neglected. On the other hand, due to the equivalent timescales of both laser noise and magnetic field noise acting on system, we finally obtain PSD of each by encoding the qubit on different Zeeman sublevels of $D_{5/2}$. Especially, this method is of great significance for characterizing the noise in lasers. And we do not need to employ the beatnote protocol anymore for which at least other two similiar lasers are required.

We also observe some shortcomings of this work. Firstly, the insufficient PSD accuracy. We should have sampled Rabi frequencies $\Omega$ more densely and repeated the experimental sequences more times, especially around the extreme points of PSD. But this can only be realized at the cost of more time. In addition, the power fluctuation noise of laser is not considered in this work because we have performed the power stabilization operations in our experimental setup. In general, this is valid, but sometimes it will also bring errors, such as the first collapse in FIG. 2(c). Finally, it should be mentioned that the low frequency (<1kHz) noise can not be characterized accurately using our method. Due to the limited transverse relaxation time $T_2 \approx 2$ ms in our system, it is difficult for us to determine the Rabi frequency below 1 kHz, thus obtaining the corresponding $S(f)$. Considering the fact that the magnetic field noise has a clear peak at low frequencies, additional work targeting this should be of great significance.

ACKNOWLEDGMENTS

This work is supported by the National Basic Research Program of China under Grant No. 2016YFA0301903 and the National Natural Science Foundation of China under Grant No. 61632021.

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