Local and Global Gravity*

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Abstract

Our long experience with Newtonian potentials has inured us to the view that
gavity only produces local effects. In this paper we challenge this quite deeply in-
grained notion and explicitly identify some intrinsically global gravitational effects. In
particular we show that the global cosmological Hubble flow can actually modify the
motions of stars and gas within individual galaxies, and even do so in a way which can
apparently eliminate the need for galactic dark matter. Also we show that a classical
light wave acquires an observable, global, path dependent phase in traversing a gravi-
tational field. Both of these effects serve to underscore the intrinsic difference between
non-relativistic and relativistic gravity.

1 Introduction

Since its very inception gravitational theory has always been regarded as being intrinsically
local. Specifically, with the Newtonian gravitational potential falling as $1/r$, the motion of
any given system is then primarily determined by the most nearby gravitational sources,
with static, spherically symmetrically distributed distant matter actually making no net
contribution at all. Thus in a static, spherically symmetric Newtonian universe only nearby
sources are relevant. Moreover, the advent of Einstein’s general theory of relativity actually
only served to reinforce this viewpoint, since the metric inside a static (or even a radially
vibrating) spherically symmetric shell is found to be flat in Einstein gravity, to thus again
permit us to locally ignore the rest of a universe of the same symmetry. Thus firstly with
potentials which fall to zero at large distances and then subsequently with the analogous
relativistic notion of asymptotic flatness, a view of gravity has emerged that only local sources
are relevant. In this paper we examine and challenge this longstanding and by now quite
ingrained notion by identifying some explicitly global aspects of gravity, both in standard

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Einstein gravity itself, as well as in the recently advanced fourth order conformal invariant alternate theory of gravity (see e.g. [1]), a theory in which individual gravitational sources are found to actually make asymptotically non-vanishing contributions to the geometry, and in which the global cosmological Hubble flow actually has local consequences, to thus give gravity a quite Machian flavor.

Since a great deal of our gravitational intuition comes from Newtonian celestial mechanics, we begin in Sec. (2) with an analysis of the observational basis for inferring that non-relativistic potentials actually fall at large distances; and show that rising ones are not merely not excluded by available data, but that a phenomenological analysis of the systematics of galactic rotation curves indicates that (linearly) rising potentials may in fact even be favored. Moreover, our specific study reveals that not one but in fact two linear potentials appear to be operative in galaxies, one which scales linearly with the amount of matter in a given galaxy, and a second one which intriguingly has a universal, galaxy independent normalization of order the Hubble scale. In Sec. (3) we show that relativistic covariance does not in fact lead us unambiguously to standard Einstein gravity, and that it is possible to consider other, equally covariant, pure metric theories of gravity, theories which could then have totally different asymptotic properties. In particular we show that the two phenomenologically found linearly rising galactic potentials of Sec. (2) even have a natural home in the conformal gravity alternative where they can both be produced, one locally by the luminous matter within a given galaxy and the other globally by the universal Hubble flow due to the rest of the galaxies in the universe. In Sec. (4) we show that the COW (Colella, Overhauser and Werner) quantum-mechanical neutron beam interferometry experiment in a background gravitational field has a purely classical counterpart, with classical light waves also experiencing global, path dependent interference phase shifts in background gravity, to thus yield another global, expressly relativistic effect; and with the interference being found to be accreditable to the gravitational bending of light, this effect is then one which will occur in any covariant, pure metric based theory of gravity. From the various specific studies of this paper there thus emerges a global character to gravity of a surprisingly far reaching nature.

2 Local and global non-relativistic gravity

In Newtonian gravity the $-MG/r$ potential has four very remarkable properties. Firstly, it permits the replacement of a static, spherically symmetric source of non-zero radius by a point source located at its center. Secondly, it provides for the mutual cancellation of the potentials of distant, static, spherically symmetrically distributed matter sources, so that such matter has no net local effect at all. Thirdly, since $-1/4\pi r$ is the Green's function of the $\nabla^2$ operator, the $-MG/r$ potential also emerges as the exterior solution to the second order Poisson equation, viz.

$$\nabla^2 \phi(r) = 4\pi G \rho(r) \ , \ \phi(r > R_0) = -\frac{4\pi G}{r} \int_0^{R_0} dr' \rho(r') r'^2$$

for a static, spherically symmetric source of radius $R_0$ and mass density $\rho(r)$, to thus connect the Newtonian potential to a dynamical equation of motion; and uniquely so in fact within
the framework of second order equations, since no other power behaved function $r^{-n}$ is simultaneously both the input Green’s function and the output potential to a second order differential equation of motion except $n = 1$. And fourthly, of course, the $-MG/r$ potential provides an excellent first approximation to the celestial mechanics of the solar system as well as to all non-relativistic gravitational phenomena on all smaller distance scales (such as terrestrial), with its only real shortcoming in this regard being the phenomenological existence of the (small) planetary orbit precessions which a $1/r$ potential strictly forbids. 

Now even though all four of these features have been instrumental to our thinking and to our accepting of Newtonian gravity, in a sense the first three features are somewhat moot with only the fourth one actually bearing directly on observation. Thus, for instance, while it is very nice to be able to replace the sun by a point source, nonetheless this is only a calculational convenience. Moreover, even while a static, spherically symmetric universe may indeed decouple locally, nonetheless we do not actually live in such a universe, with there currently being no such analogous decoupling theorems for other possible cosmologies such as comoving, expanding, globally topologically non-trivial ones. And finally, even if we accept the whole four features as representing basic properties of gravitational theory, such Newtonian considerations then lead us to the notion that gravitational sources are to have negligible effect at infinity, a conclusion which even though it is widely accepted has nonetheless never in fact actually been tested, with it not being at all clear as to how it might actually even be tested since the universe in which we live is not in fact asymptotically flat. The locality of gravity is for the moment then essentially a preference of the community rather than an established fact.

Nonetheless, all of this Newtonian wisdom notwithstanding, as of today the issue of whether or not this same $1/r$ potential law actually continues to fit data on distance scales much larger than that of the solar system is actually still a live and yet to be resolved one. Specifically, it is found that on galactic distance scales the velocities of galactic stars and hydrogen gas (and particularly the gas since it is generally distributed in galaxies out to much farther distances from galactic centers than the stars) do not show any sign at all of the Keplerian fall off associated with a $1/r$ potential. Indeed, as can be seen from Fig. (1) which shows a typical set of galactic rotation curves, at the largest detected distances (distances which are well beyond the detected galactic luminous matter regions) the observed orbital rotational velocities are way above the falling luminous Newtonian expectation, and are themselves not only far from falling, but in fact are generally seen to be close to flat or in some cases to even be rising. Now while this shortfall in the velocities has prompted the notion of galactic dark matter (viz. something purely local as per the standard Newtonian reasoning) which is to then provide some additional $1/r$ potential contributions which are to bring the rotation curve predictions back into line with data, nonetheless this appeal to dark matter is only makable provided it is a priori known that Newton’s law of gravity actually continues to hold on these very big distance scales. However, as of today, there has not yet been any independent confirmation of Newton’s law of gravity on these distance scales whatsoever, i.e. one which only involves expressly falsifiable predictions based on unambiguously established matter sources with observationally established density distributions, since galactic dark matter always has to be invoked, to thus show the complete circularity of the reasoning.

Moreover, even if one accepts dark matter as a general concept, in reality one also has to make a guess
In fact, if galactic rotation curves were the only data available to us (i.e. if we had no solar system information at all) we would not be able to extract out a \(1/r\) potential law at all. The continued use of Newton’s law of gravity on galactic distance scales for the moment thus rests on a not yet completely secure foundation. However, if one is actually prepared to contemplate trying to go beyond Newtonian gravity (and therefore by implication to go beyond its covariant Einstein generalization, a point we address in detail below), then not only does one have to deal with the difficult issue of actually challenging the standard theory at all, the only way to modify Newton galactically would be to have to have some new effect which would then dominate over Newton at these large distances, thus making the Newtonian term no longer dominant at infinity and thereby opening up the possibility that gravity might then have global effects. Despite the fact that this possibility goes completely against our local Newtonian intuition, we shall now show that it actually has some observational support.

While a first glance at Fig. (1) reveals some strikingly flat rotation curves,\(^2\) it is more instructive for fundamental theory to look not at the total velocities, but rather at the velocity discrepancies, viz. the excess of the total velocity over the luminous Newtonian contribution. As we see, these discrepancies themselves are far from flat, and in fact are actually growing with distance \(R\) from the center of each galaxy. Now while this must be the case for galaxies whose total velocities are flat (since the discrepancy is the excess over the falling luminous Newtonian contribution), what makes this remark so significant is that it also applies to the low luminosity galaxies as well (the galaxies are presented in order of increasing luminosity in Fig. (1)), galaxies whose total velocities are currently found to predominantly be rising, and to not in fact be flat.\(^3\) The fact that the discrepancies are rising out to the last detected points in each and every galaxy in the set (a set whose luminosities vary by a factor of more than 1000 - see Table (1) which lists some specific input data) thus emerges as a universal feature of the data even as the total velocities as a set do not show any common universal underlying trend.\(^4\)

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\(^2\)This set of galaxies has been identified by Begeman, Broeils and Sanders\(^2\) as possessing particularly reliable data which exhibit the basic pattern of departure from the luminous Newtonian expectation that has so far been observationally obtained, with their paper giving complete data references.

\(^3\)While none of the low luminosity galaxies currently show any flat rotation curve region at all, there is a noticeable turnover in one of these galaxies, viz. DDO 154, the galaxy which is altogether the least luminous of the whole sample. However, since this is the most gas dominated galaxy in the sample, random gas pressures could be making a substantial contribution to motions in the turnover region. We shall thus ignore any possible ramifications of these last few points here, though clearly if this turnover proves to be a real trend which is then reproduced in other low luminosity galaxies, it would eventually have to be accounted for.

\(^4\)The flatness of the total velocities of the higher luminosity galaxies is so striking that the rise in the lower luminosity galaxies has generally been discounted by the community as being in any way representative of a possible trend. And even though the ratios of the velocity discrepancies to the luminous Newtonian contributions are actually greater in the low luminosity galaxies than in the higher luminosity ones (thus incidentally forcing dark matter model fits to require that the dark to luminous mass ratio be bigger for the lower luminosity galaxies - an assumption of dark matter theory which is still not all that well understood), nonetheless it is still taken as a given that the total velocities in the low luminosity galaxies will eventually flatten off. However, the fact that the discrepancies are currently rising in all the galaxies suggests a possibly different asymptotic outcome, with there currently being no observational support at all for the widely held
In order to universally quantify this qualitative velocity discrepancy trend, it is very instructive \cite{3, 4} to evaluate the total centripetal acceleration \((v^2/c^2 R)_{last}\) at the last data point in each galaxy (except for DDO 154 for which we use the last point before the turnover). As we can see from the fourth column in Table (1) \((v^2/c^2 R)_{last}\) (a completely model independent quantity) turns out to be remarkably universal, varying only by a factor of 5 or so over the sample, a variation which is altogether less than the factor of 1000 or so by which the luminosity varies in the same sample, so that \((v^2/c^2 R)_{last}\) emerges as a quantity which contains universal information. Moreover, we see a small but clear trend with increasing mass in the centripetal accelerations. And in fact we find \cite{4} that we can parameterize all the 11 total accelerations listed in Table (1) according to the one universal three component relation

\[
(v^2/c^2 R)_{last} = \gamma_0/2 + \gamma^* N^*/2 + \beta^* N^*/R^2
\]

where the two new universal constants \(\gamma_0\) and \(\gamma^*\) take numerical values 3.06 \times 10^{-30} \text{ cm}^{-1} (a value strikingly close to the inverse of the Hubble radius) and 5.42 \times 10^{-41} \text{ cm}^{-1} respectively, where \(\beta^* = 1.48 \times 10^5 \text{ cm}\), and where \(N^*\) is the total amount of visible stellar (and gaseous) material in solar mass units in each galaxy. Before venturing to discuss the possible significance of this new, purely phenomenological, model independent regularity, it is important to realize that there is nothing in any way special about the actual magnitudes of the radial coordinates, \(R\), of the last detected points in the 11 galaxies, since their locations are fixed purely by the instrumental limits of the various detectors used in measuring the various gas surface brightnesses and not fixed by any dynamics associated with the galaxies themselves. Thus the magnitude of each last measured radial \(R\) (a quantity which varies from 8 kpc to 40 kpc or so over the sample) is essentially arbitrary for the galaxies, and yet \(v^2/R\) can nonetheless still be universally parameterized. As far as we can see, with the luminous Newtonian contribution being decidedly non-leading at the farthest data points, the only obvious way that this could in fact occur would be if \(v^2\) were in fact growing universally with \(R\) so that the magnitude of \(v^2/R\) would not in fact depend on where the last detected points just happened to be located within galaxies. This pattern is clearly not one that one would expect with flat rotation curves, or even in fact think to look for in such a paradigm, and would instead seem to point to potentials which if anything are actually growing (linearly) with distance rather than falling in the familiar Newtonian manner. Moreover, if we provisionally identify \(\gamma^* c^2 r/2\) as the linear potential put out by a typical star such as the sun (our conformal gravity study below will actually justify this identification), then the numerical value for \(\gamma^*\) as inferred from Eq. (2) is so small that the contribution made by such a linear potential to solar system dynamics is then totally negligible, to thus yield us phenomenological linear potentials for stars which only start to become competitive with their Newtonian ones on galactic distance scales, these intriguingly being the distance scales where the standard theory first has to appeal to dark matter. Thus we see from purely phenomenological considerations alone that it is somewhat unwarranted to yet claim the validity of Newton’s law of gravity on distance scales much larger than the solar system ones on which it was first established, and that despite a preconceived prejudice to the contrary, currently available data do not in fact exclude potentials which grow rather than fall with distance. In fact, since the data even appear to lend some support to this possibility, it is to the theoretical belief that the discrepancies (in both low and high luminosity galaxies) will in fact all be flat asymptotically.
underpinnings needed for such an option that we now turn, underpinnings which in fact will enable linear potentials to account for the systematics of the velocity discrepancies of all the data points in Fig. (1) and not just merely of those of the last ones.

3 Local and global relativistic gravity

Even though we have just made a phenomenological case for rising potentials, nonetheless, since it was based entirely on non-relativistic reasoning, in order for it to have any possible fundamental significance it is necessary to show that it is compatible with relativity. Now as regards relativity itself, it is important to realize that the validity of the equivalence principle (the cornerstone of relativistic gravity) is secured once the metric tensor $g^{\mu\nu}(x)$ is identified as the gravitational field, with no commitment being needed as to the specific equations of motions which $g^{\mu\nu}(x)$ is to then obey, save only that they be covariant. The principle of general relativity thus requires only that the gravitational action be a general coordinate scalar function of the metric, with there being no less than an infinite number of such possible candidate scalar actions involving derivatives of the metric of arbitrarily high order. Motivated by the successes of Newtonian gravity Einstein himself opted for a theory based on second order derivatives of the metric, viz.

\[ R^{\mu\nu} - g^{\mu\nu} R^\alpha_\alpha / 2 = -8\pi G T^{\mu\nu} \]  

an equation of motion whose weak gravity limit is the second order Poisson equation of Eq. (1), to thus both yield Newton’s law of gravity and to provide some relativistic corrections to it, corrections which then completely account for the problem of the precessions of the planetary orbits to which we referred earlier.

On an even deeper level the metrication of gravity also provided for a coupling of light to gravity, and since light travels in waves which fill all space, it thus gave gravity some global aspects absent in its coupling to material point particles. Since the global aspects of the coupling of light to gravity will prove central to our discussion below of the COW interferometry experiment, it is of some value to discuss the nature of this coupling from the point of view of the equivalence principle. Even though the coupling of light to gravity is motivated by Einstein’s elevator, this is not actually the origin of the coupling. Specifically, while one can conclude from an analysis of a freely falling observer in an elevator that such an observer could not distinguish between a uniformly accelerating elevator and a uniform gravitational acceleration, that observer is only unable to make such distinction because of the equality of inertial and gravitational masses. From this fact it does not necessarily follow that the same observer could not make any distinction for light since light is massless, and thus may not couple to gravity at all (i.e. the equation of motion could degenerate into zero equals zero), with the equivalence principle then simply not applying to it, and with light then bending in an accelerated coordinate system but not in a gravitational field, an inelegant but nonetheless logical option. As an alternative to starting by simply postulating that there is an equivalence principle for both material particles and for light, one can instead

\[ A specific model where this would in fact be the case would be a scalar theory of gravity in which the source of the scalar field is the trace of the energy momentum tensor, a quantity which simply vanishes for light. The author is indebted to Dr. A. Chodos for alerting him to such a model. \]
proceed just as one does in special relativity. Thus in the same way that special relativity is based on the requirement that all uniformly moving observers have to agree on the same physics, then the analogous starting requirement in general relativity would be to require that non uniformly moving observers also all have to agree on the same physics, to thus lead us to general coordinate invariance (but not yet necessarily to curved space since the assumption of coordinate invariance alone does not suffice to force the Riemann tensor to actually be non-zero). Moreover, in and of itself the principle of coordinate invariance does not tell us how the gravitational field itself is to be described, since at this point it could be just some arbitrary field defined over spacetime just like the electromagnetic field (with gravitational sources being treated analogously to electromagnetic ones and with the universe then being covariant but Riemann flat). It is only the further, independent assumption that the gravitational field is not any old field but is instead the very special one associated with the spacetime metric that then leads us to a covariantly coupled metric theory of gravity (and thus a covariant coupling to electromagnetism as well) in which geodesic motion and the equivalence principle are then output to the theory (rather than input) for both massive and massless particles, so that light then does indeed obey the equivalence principle after all. It is in this way that gravity thus acquires global features.

As regards the standard classic tests of Einstein gravity, they are all nicely contained within the familiar Schwarzschild metric $R_{\mu\nu} = 0$ vacuum metric solution

$$dr^2 = (1 - 2MG/c^2r)c^2dt^2 - dr^2/(1 - 2MG/c^2r) - r^2d\Omega$$

exterior to a static, spherically symmetric gravitational source of mass $M$. This solution then establishes asymptotic flatness as a characteristic feature of standard gravity, to thus dovetail with our Newtonian intuition. However, the successes of the Schwarzschild metric notwithstanding, it was pointed out by Eddington in the very early days of general relativity that these same tests could also be met should some higher derivative of $R_{\mu\nu}$ vanish rather than $R_{\mu\nu}$ itself. Eddington thus raised the issue of how unique the Einstein equations actually are, and despite the longevity and seriousness of the question, the relativity community seems to have somewhat ignored the issue, as well as the directly related one we noted above, namely that covariance alone is not sufficient to select between metric theories of differing order.

Recognizing the need to find a principle which would enable us to make such an unambiguous selection, Mannheim and Kazanas suggested a candidate principle, namely conformal invariance (viz. invariance of the geometry under any and all local stretchings of the form $g_{\mu\nu}(x) \rightarrow \Omega^2(x)g_{\mu\nu}(x)$), the local scale invariance now thought to be enjoyed by the other fundamental interactions, the strong, the electromagnetic, and the weak, theories all of whose coupling constants are dimensionless and all of whose mass scales are dynamical.

If no physical field were ever associated with the metric, there would then be no equation of motion with which to fix the curvature of spacetime at all, be it flat or not. Thus once given the imposition of a principle of general covariance it would appear that the metric would then have to be dynamical.

The solution to the planetary precession problem could in principle have been found within the framework of Newtonian gravity by a small departure from strictly $1/r$ potentials. It was only the experimental detection of the gravitational bending of light which assured that conceptual departures from Newtonian gravity were unavoidable.

Independent of one’s views regarding conformal invariance itself, readers who object to it are still not free to use the Einstein equations until they come up with some other candidate principle which would
conformal gravity (whose action is the conformal invariant \( I_W = -\alpha \int d^4x (-g)^{1/2} C_{\lambda\mu\nu\kappa} C^{\lambda\mu\nu\kappa} \))
where \( C_{\lambda\mu\nu\kappa} \) is the conformal Weyl tensor and \( \alpha \) is a purely dimensionless coupling constant) the equations of motion which are to replace the Einstein equations take the form

\[
g^{\mu\nu}(R_\alpha^\alpha)^{\beta;\beta}/2 + R^{\mu\nu;\beta;\beta} - R^{\mu\beta;\nu;\beta} - 2R^{\mu\beta}R^\nu_\beta + g^{\mu\nu}R_\alpha^\alpha R^{\alpha\beta}/2
-2g^{\mu\nu}(R_\alpha^\alpha)^{\beta;\beta}/3 + 2(R_\alpha^\alpha)^{\mu;\nu}/3 + 2R_\alpha^\alpha R^{\mu\nu}/3 - g^{\mu\nu}(R_\alpha^\alpha)^2/2 = T^{\mu\nu}/4\alpha
\]

and admit of the Schwarzschild vacuum solution just as Eddington had warned us. However, because they are fourth order rather than second order equations these equations admit of other static, spherically symmetric vacuum solutions as well, with the most general such one being found to take the form

\[
-g_{00} = 1/g_{rr} = 1 - \beta(2 - 3\beta\gamma)/r - 3\beta\gamma + \gamma r - kr^2
\]

where \( \beta, \gamma, \) and \( k \) are integration constants. The metric of Eq. (6) thus both contains and generalizes the Schwarzschild solution of standard gravity, recovering the standard Newton-Schwarzschild phenomenology when the \( \gamma \) and \( k \) terms are sufficiently small, while departing from it at sufficiently large enough distances through the presence of noneother than a linearly rising potential, a potential which is of just the type suggested by the phenomenological analysis we presented above in Sec. (2). However, unlike the Schwarzschild solution, the metric of Eq. (6) is not asymptotically flat, and in order to explore the implications of this fact for matter sources, it is necessary to relate the coefficients of this metric to interior properties of the source.

To this end we note that in a static, spherically symmetric geometry the equations of motion of Eq. (5) remarkably reduce without any approximation whatsoever to

\[
\nabla^4 B(r) = (rB)^{\prime\prime\prime}/r = f(r)
\]

where \( B(r) = -g_{00}(r) \) and where \( f(r) = 3(T^0_0 - T^r_r)/4\alpha B(r) \) to thus yield a fourth order Poisson equation rather than the familiar second order one of the standard theory. In its turn Eq. (7) is readily integrated and yields (see also)

\[
B(r > R_0) = -\frac{1}{6r} \int_0^{R_0} dr' f(r')r'^4 - \frac{r}{2} \int_0^{R_0} dr' f(r')r'^2
\]

as its exact exterior solution (up to an uninteresting \( w - kr^2 \) particular integral term). With the recovering of the Newtonian potential in Eq. (8) we thus, in principle at least, divorce the Newtonian potential from the second order Poisson equation, to show that the second then uniquely select out Eq. (3), this being the challenge to the standard theory raised in and still not yet responded to. Moreover, given the fact that the great appeal of Einstein gravity is that it provides an explanation for the equality of the inertial and gravitational masses by means of a fundamental principle, it is thus quite extraordinary that the choice of the particular equations of motion of Eq. (3) would then be made without the analogous guidance of some other comparable underlying fundamental principle.

This same solution was also found by Riegert, and is contained in his PhD thesis (University of California at San Diego, 1986) in which conformal gravity is studied quite extensively, particularly with regard to its quantum aspects. This thesis also provides some quite extensive references to earlier conformal gravity work.
order Poisson equation is thus only sufficient to give the Newtonian potential but not in fact necessary (a state of affairs which will persist until some fundamental principle is identified which uniquely selects out the standard Newton-Einstein theory). Additionally from Eq. (8) we see that our desired linearly rising potential term does indeed couple to matter, to show that in principle a local matter distribution of finite radius $R_0$ can in fact have a global effect at infinity, our Newtonian intuition notwithstanding. In this way then higher order gravity theories thus become global. Having now motivated such rising potentials, we turn now to explore their possible observational implications.

In order to analyze dynamics based on rising potentials, beyond the issue of the fact that the form of the potential is different from that of the Newtonian one, the more fundamental difference is that in the same way that a given source now has dynamical implications at large distances, all the farthest sources from it then also have an effect on it, so it is now no longer possible to ignore the rest of the universe. Thus, for galactic rotation curves for instance, we need to examine the gravitational effect due to the linear potentials of the stars and gas within each galaxy, and also we need to examine the gravitational effect of the Hubble flow of the rest of the universe on motions within individual galaxies.¹⁰

As regards first the linear potential contribution within a given galaxy, for weak gravity it is readily calculated in exactly the same manner as the net Newtonian potential contribution of a galaxy. Thus integrating the individual stellar potentials

$$V^*(r) = -\beta^* c^2/r + \gamma^* c^2 r/2$$

of Eqs. (3) and (8) (we redefine the coefficient of the stellar Newtonian term as $\beta^*$) over an infinitesimally thin galactic optical disk with luminous surface matter distribution $\Sigma(R) = \Sigma_0 \exp(-R/R_0)$ and total number of stars $N^* = 2\pi \Sigma_0 R_0^2$ in the standard non-relativistic weak gravity way, then yields [11] the centripetal acceleration

$$v^2/R = g_{\text{lum}} = g_{\text{lum}}^\beta + g_{\text{lum}}^\gamma$$

where

$$g_{\text{lum}}^\beta = (N^* \beta^* c^2 r/2R_0^3)[I_0(r/2R_0)K_0(r/2R_0) - I_1(r/2R_0)K_1(r/2R_0)]$$

and

$$g_{\text{lum}}^\gamma = (N^* \gamma^* c^2 r/2R_0)I_1(r/2R_0)K_1(r/2R_0)$$

to give a net acceleration which behaves asymptotically as $(v^2/c^2 R)_{\text{last}} = \gamma^* N^*/2 + \beta^* N^*/R^2$. We thus nicely generate a $\gamma^* N^*/2$ term (an explicitly extensive function of the total amount of visible matter $N^*$ within each galaxy) just as needed for our phenomenological Eq. (2). However, we still lack the $\gamma_0/2$ term which is also phenomenologically required. Since this latter acceleration term is independent of the mass of a given galaxy, and since its numerically fitted value given in Sec. (2) is of order the inverse Hubble radius, we can thus anticipate that it must not arise from within a galaxy at all but must instead come from the global Hubble flow of the universe itself.

¹⁰In passing we note that since $-r/8\pi$ is both the Green’s function and output potential to the fourth order Poisson equation, it plays the same role for $\nabla^4$ which $-1/4\pi r$ plays for the $\nabla^2$ operator.

¹¹Since the rest of universe is only known to decouple in Einstein gravity when it is static and spherically symmetric, even for Einstein gravity an expanding, comoving universe could in principle affect motions within individual galaxies, our Newtonian intuition notwithstanding.
In order to try to evaluate any possible contribution of the Hubble flow to individual motions within galaxies, we are first faced with the difficulty that in general (and in standard Einstein gravity in particular) trying to rewrite the comoving Robertson-Walker metric (a cosmological solution to both Einstein gravity and to conformal gravity [12], [13]) in Schwarzschild coordinates is extremely complicated and anyway not particularly illuminating. However, this task is greatly simplified in conformal gravity because of its additional underlying conformal invariance. Specifically, it is found [6] that the general coordinate transformation

$$r = \rho/(1 - \gamma_0 \rho/4)^2, \quad t = \int d\tau/R(\tau)$$

(13)
effects the metric transformation

$$(1 + \gamma_0 r)c^2 dt^2 - \frac{dr^2}{(1 + \gamma_0 r)} - r^2 d\Omega \to \frac{(1 + \rho \gamma_0/4)^2}{R^2(\tau)(1 - \rho \gamma_0/4)^2} \left( c^2 d\tau^2 - \frac{R^2(\tau)(d\rho^2 + \rho^2 d\Omega)}{(1 - \rho^2 \gamma_0^2/16)^2} \right)$$

(14)
to yield a metric which we recognize as being a conformally transformed Robertson-Walker metric. Now while only the Robertson-Walker metric itself is a cosmological solution to Einstein gravity, in conformal gravity not only is Robertson-Walker an allowed cosmological solution, but so is any metric which is conformal to it. Thus we see that a static, Schwarzschild coordinate linear potential metric is coordinate equivalent to the conformally equivalent class of allowed conformal gravity cosmologies with scale factor $R(\tau)$ and (explicitly negative) 3-space scalar curvature $k = -\gamma_0^2/4$. Now, as regards this equivalence, we note that in a geometry which is both homogeneous and isotropic about all points, any observer can serve as the origin for the coordinate $\rho$; thus in his own local rest frame each observer is able to make the above general coordinate transformation with the use of his own particular $\rho$. Moreover, since the observer is also free in conformal gravity to make arbitrary conformal transformations as well, that observer will then be able to see the entire Hubble flow appear in his own local static coordinate system as a universal linear potential with a universal acceleration $\gamma_0 c^2/2$ coming directly from the spatial curvature of the universe (a quantity which incidentally is nicely time independent unlike the time dependent Hubble parameter itself). Since the internal orbital motions of the stars and gas can be discussed in each galaxy’s own rest frame, we thus find that in each such rest frame, each orbiting particle in that specific galaxy will then precisely see the overall Hubble flow acting as a local static universal linear $\gamma_0 r$ potential just as desired. (Of course, in the rest frame coordinate system associated with a given galaxy, any observer on any other galaxy will see an extremely complicated non-static geometry. However, in his own rest frame that observer is still free to use Eq. (14) with the selfsame universal acceleration to describe the internal motions in his own galaxy, which is just what is needed for that galaxy’s rotation curve.) We thus establish a cosmological origin for the universal $\gamma_0 c^2/2$ acceleration needed for Eq. (2), while also identifying a crucial difference between relativistic and non-relativistic reasoning.

\[\text{12In passing we note that in the cosmology discussed in [12], [13] an open universe with explicitly negative k was in fact realized, with such a universe being found to not suffer from the flatness problem found in the standard cosmology.}\]
Specifically, in strictly Newtonian physics the only effect of any background would be to put tidal forces on individual galaxies, forces that would not account for the rotational motions of stars and gas but only to a departure therefrom. Relativistically however, since the background produces an effect at the center of each galaxy, the background therefore contributes to the explicit rotational motions of the stars themselves, to thus yield a previously unappreciated but nonetheless apparently quite general consequence of curvature which enables the stars and gas in galaxies to serve as test particles for probing the Hubble flow.

In order to now combine the above local and global linear potentials we need to embed each local galaxy into the global Hubble flow and solve the gravitational equations of motion of Eq. (5) in the presence of $T_{\mu \nu}^{\text{local}} + T_{\mu \nu}^{\text{global}}$. Given the fact that gravity is weak within galaxies, we shall as a first approximation simply add the local and global metrics given above in Eqs. (11), (12) and (14) (it is the very presence of $T_{\mu \nu}^{\text{local}}$ and its associated local geometry (viz. standard static Schwarzschild coordinates) which dictates the appropriate general coordinate transformation needed for Eqs. (13) and (14)), to yield the total weak gravity acceleration

$$v^2/R = g_{\text{tot}} = g_{\beta}^{\text{lum}} + g_{\gamma}^{\text{lum}} + \gamma_0 c^2/2$$

which can now be directly applied to data. With $\gamma_0$ and $\gamma^*$ taking the fixed numerical values given earlier in Sec. (2), the fits reduce to just one free parameter per galaxy, viz. the standard optical disk mass to light ratio (or equivalently the total amount of stars and gas per galaxy, $N^*$, in solar mass units). Since, unlike dark matter theory, our theory is based on parameters with an absolute scale, it is thus very sensitive to distance determinations to galaxies. Consequently, we first calculate the total velocity predictions (the dotted curves) in Fig. (1) using the distances quoted in [2]. Then, again following [2] we allow for typical uncertainties in the adopted distances to give modest distance shifts of up to ±15% or so. (While larger shifts can actually improve the fits a little in some cases, we have not allowed for shifts of more than this except for NGC 1560 for which a distance estimate of 3.7 Mpc (+23%) has actually been reported in the literature.) With the indicated percentage shifts in adopted distance, with the fitted $M/L$ ratios listed in Table (1), and with $g_{\gamma}^{\text{gal}}$ of Eq. (10) being calculated solely from the known luminous galactic matter (viz. stars and gas), we then obtain (the details of the fitting are given in [4]) the full curve fits of Fig. (1), with the dashed and dash-dotted curves showing the velocities that the Newtonian $g_{\beta}^{\text{lum}}$ and linear $g_{\gamma}^{\text{lum}} + \gamma_0 c^2/2$ terms would separately produce. No dark matter is assumed, and as we can see from the fits, none would appear to be needed. Despite the fact that our model is a highly constrained one with very few free parameters, it nonetheless appears to have captured the essence of the data (our fits have smoothed out some of the structure in the data since we treat the radial dependence of the optical disks as single exponentials for simplicity), and phenomenologically our fitting would thus appear to be competitive with that of both the standard dark matter model and the MOND [14] alternative. Moreover, if our theory is in fact correct, then it provides us with an actual determination of the scalar curvature of the universe, something which years of intensive work has yet to accomplish in the standard theory.

As we can see from Fig. (1), at the shifted adopted distances $(v^2/c^2 R)_{\text{last}}$ is indeed remarkably well fitted by $\gamma_0/2 + \gamma^* N^*/2 + \beta^* N^*/R^2$ (this being the asymptotic limit of $g_{\text{tot}}/c^2$), to thus recover Eq. (3) for each and every galaxy in our sample; and we see that
even while the quantity $\gamma^* N^*/2$ does vary enormously with luminosity over our sample, nonetheless the $\gamma_0/2$ term overwhelms it in all but the largest galaxies, so that $(v^2/c^2 R)_{last}$ only shows a mild (but nonetheless significant) dependence on galactic mass. Since we find that the same $N^*$ as extracted out from $(v^2/c^2 R)_{last}$ also serves to accurately normalize the luminous Newtonian contribution in the inner rotation region where it is dominant, we thus see that the parameters extracted out from $(v^2/c^2 R)_{last}$ alone then provide for a complete accounting of the entire rotation curves with no further adjustment of parameters being needed at all, a fitting which stands in sharp contrast to dark matter halo fitting where no less than 22 halo parameters are used to fit the data set of Fig. (1).

While we have thus made a first case for a possible role for cosmology in elucidating the systematics of galactic rotation curves, (the data of Fig. (1) are certainly not rejecting the idea), nonetheless this idea has to be explored further especially on distance scales larger than galactic where background cosmological effects should be even more prominent. In conclusion then, we would like to state that the conformal gravity theory would appear capable of explaining the general systematics of galactic rotation curves in a completely natural manner, and that our study suggests that rising rather than flat rotation curves is actually the paradigm, with the luminous Newtonian contribution having inadvertently masked that fact in the higher luminosity galaxies. Moreover, through the cosmological connection we have presented, we believe we have made a case for the existence of a universal linear potential associated with the cosmological Hubble flow (and even more generally a case against the widespread belief that whatever is responsible for the rotation curve discrepancies is itself a purely local galactic phenomenon), an intriguing possibility which appears to enable us to circumvent the need for galactic dark matter.

4 Global aspects of wave propagation in a gravitational field

Having now explored some global gravitational aspects on large astrophysical distance scales, we turn next to an initially unlikely place to identify another one, namely a neutron interferometry experiment performed in a terrestrial laboratory. In a landmark series of experiments [17], [18] Colella, Overhauser and Werner (COW) and their subsequent collaborators (see e.g. Refs. [19], [20] for overviews) detected the modification of the phase of a neutron beam as it traverses the earth’s gravitational field, to thus realize the first experiment which in-

\[ \frac{\gamma_0}{2} + \frac{\gamma^* N^*}{2} / \left( \beta^* N^*/R^2 \right) \] \[ \text{decreasing as } N^* \text{ increases, we thus see that departures from the luminous Newtonian expectation are the biggest in the low luminosity galaxies. This then naturally parallels the analogous trend found phenomenologically for the luminosity dependence of the dark to luminous mass ratio in dark matter models to which we referred earlier. In passing we also recall that even though it is not all that clear as to just how dark matter is in fact clever enough to always manage to appropriately match itself to the luminous matter content of individual galaxies, the fact that there is such a trend with luminosity has nonetheless spawned a folklore theorem that it is impossible to replace dark matter by any new theory with only one new scale. As Eq. (2) now shows, it is, however, possible to replace dark matter by a theory with two.}

\[ \text{A first application [13], [14] of conformal gravity to cosmological nucleosynthesis has actually encountered a possible problem with conformal cosmology so far having difficulty generating sufficient primordial deuterium.} \]
volved both quantum mechanics and gravity. A typical generic experimental set up is shown in the schematic Fig. (2) in which a neutron beam from a reactor is split by Bragg or Laue scattering at point $A$ into a horizontal beam $AB$ and a vertical beam $AC$ (we take the Bragg angle to be $45^\circ$ for simplicity and illustrative convenience in the following), with the subsequent scatterings at $B$ and $C$ then producing beams which Bragg scatter again at $D$, after which they are then detected. If the neutrons arrive at $A$ with velocity $v_0$ (typically of order $10^5$ cm sec$^{-1}$) and $ABCD$ is a square of side $H$ (typically of order a few centimeters), then the phase difference between the two split beams is given by

$$\phi_{COW} = \phi_{ACD} - \phi_{ABD} = -\frac{mgH^2}{\hbar v_0}$$

(16)

to lowest order in the acceleration $g$ due to gravity $[17]$, and is actually observable despite the weakness of gravity ($mgH/(mv_0^2/2) \approx 10^{-7}$ in typical experimental conditions), since even though $\int \vec{p} \cdot d\vec{r}$ only differs by the minute amount $m(v_{CD} - v_{AB})H = -mgH^2/v_0 \approx 10^{-26}$ erg sec between the $CD$ and $AB$ paths, nonetheless this quantity is not small compared to Planck’s constant, to thus give an detectable fringe shift (compared to traversing the same interferometer with the plane of $ABCD$ horizontal) even for $H$ as small as a few centimeters. Quantum mechanical interference based experiments thus provide a level of sensitivity for probing background classical gravity way in excess of that achievable in classical Newtonian gravity based laboratory experiments of the same characteristic dimensions.

The detected COW phase is extremely intriguing for two reasons. First, it shows that it is possible to distinguish between different paths which have common end points, with the explicit global ordering in which the horizontal and vertical sections are traversed leading to observable consequences. And second, it yields an answer which explicitly depends on the mass of the neutron even while the classical neutron trajectories (viz. the ones explicitly followed by the centers of the wave packets of the quantum mechanical neutron beam) of course do not. The COW result thus invites consideration of whether the detected ordering is possibly another example of the kind of global topological effects which are characteristic of quantum mechanics, and of whether quantum mechanics actually respects the equivalence principle. As we shall see, the ordering effect is in fact already present in the propagation of classical light waves around the interferometer with classical light undergoing interference in a classical gravitational background, to show that classical gravity is intrinsically global. Once classical waves undergo interference in some given background, it then follows from wave particle duality that quantum mechanical matter waves must inherit the same interference pattern in the same background, so that the neutron COW phase derives from global classical gravity rather than from topological quantum mechanics. Also we will see that this same wave particle duality will enable us to establish that the mass dependence of the neutron beam COW phase is purely kinematic with the equivalence principle then continuing to hold in the presence of quantum mechanics.

To address these issues specifically we have found it convenient to carefully track a light wave as it traverses the interferometer (the discussion parallels that of $[21]$ which monitors the motion of the neutron around the loop). Since the polarization of the light wave is not relevant to our considerations, it suffices to look at solutions to the scalar Klein-Gordon equation $\partial^\mu \partial_\mu \phi - m^2 \phi = 0$ ($\partial^\mu \phi$ denotes $\partial\phi/\partial x^\mu$) in generic background fields of the form $d\tau^2 = B(r)c^2dt^2 - d\vec{r}^2/B(r) - r^2d\Omega$ where $B(r) = 1 - 2MG/c^2r$ near the surface of the earth (in both Einstein gravity and conformal gravity since the linear potential contribution
is completely negligible here). On defining $\phi(x) = \exp(iT(x))$ we find that the phase of the wave obeys $T^\mu T_\mu = i T^\mu_\mu$. In the eikonal or ray approximation the $i T^\mu_\mu$ term can be dropped, so that the phase $T(x)$ is then seen to obey the classical massless particle Hamilton-Jacobi equation $T^\mu T_\mu = 0$, an equation whose solution is the stationary classical action between relevant end points. In the eikonal approximation we can also identify $T^\mu$ as the wavefront normal $dx^\mu/dq$ where $q$ is a convenient affine parameter which can be used to measure distances along ray trajectories. On introducing the wave number $k^\mu = dx^\mu/dq$, we can then set $T(x) = \int k_\mu dx^\mu$, to thus recover the standard expression for the phase of a classical wave. Additionally, since the covariant differentiation of the Hamilton-Jacobi equation yields $(T^\mu T_\mu)_\nu = 2 T^\mu T_\mu, \nu = 2T^\mu (T_\nu, \mu + \partial_\mu T_\nu - \partial_\nu T_\mu) = 2T^\mu T_\nu, \mu = 2k^\mu k_\nu T_\mu = 0$, the identification $k^\mu = dx^\mu/dq$ then yields the massless particle geodesic equation \[22\]. In the eikonal approximation then we see that light rays travel on the curved space geodesics. However, while the identification $T^\mu = k^\mu$ nicely puts $k^\mu$ on the light cone, it also causes $\int k_\mu dx^\mu = \int (dx_\mu/dq) dx^\mu$ to vanish identically and thus not change along trajectories, to initially suggest that there is no detectable interference. However, since the same argument would lead us to the conclusion that a Young double slit experiment with classical light would also not yield any interference pattern, it is thus instructive to explicitly identify why it is that the double slit experiment does in fact display interference.

In a double slit experiment with classical light, light from a source $S$ goes through slits $Q$ and $R$ (see Fig. (3)) to form an interference pattern at points such as $P$, with the distance $\Delta x = QT$ representing the difference in path length between the two beams. Given this path difference, the phase difference between the two beams is usually identified as $k\Delta x$, from which an interference pattern is then readily calculated. However, because of this path difference, the $SQP$ ray takes the extra time $\Delta t = \Delta x/c$ to get to $P$, to thus give a net change in the phase of the $SQP$ beam equal to $k\Delta x - \omega\Delta t$, a quantity which actually vanishes for rays on the light cone, just as had been noted above. The relative phase of the two light rays in the double slit experiment thus does not change at all as the two beams traverse the interferometer, and cannot thus be the cause of the interference pattern. However, because of this time delay, the $SRP$ beam actually arrives at $P$ at the same time as an $SQP$ beam which had left the source at a time $\Delta t$ earlier. Thus if the source is coherent over these time scales, then relative to the $SRP$ beam the $SQP$ beam then carries an additional $+\omega\Delta t$ phase from the very outset, and it is this specific additional phase which is then responsible for the detected interference.\[15\] We thus see that the double slit device itself actually produces no phase change for light. Rather, the choice of point $P$ on the screen is a choice which selects which time delays at the source are relevant at each $P$, with any general interference pattern for light thus not only always involving the time delay at the source, but also in fact always requiring one. Consequently, we now need to determine whether light waves also experience any analogous time delay when they traverse an interferometer placed in a gravitational field.

In order to determine whether there is any such time delay, we need to determine the light ray geodesics in the gravitational field of the earth. To this end it is convenient to rewrite

\[15\]Since this phase is equal and opposite to the $-\omega\Delta t$ phase which the same beam acquires during its propagation to $P$, the phase $k\Delta x$ can still serve as the final observable phase difference for purposes of actually calculating the explicit interference pattern, though this specific phase is non-zero only if there is in fact a time delay.
the Schwarzschild metric in terms of a Cartesian coordinate system $x = r\sin\theta\cos\phi$, $y = r\sin\theta\sin\phi$, $z = r\cos\theta - R$ erected at a point on the surface of the earth. With $z$ being normal to the earth’s surface, to lowest order in $x/R$, $y/R$, $z/R$, $MG/c^2R$ (where $M$ is the mass of the earth and $R$ its radius) the Schwarzschild metric is then found [23] to take the form

$$\text{d}r^2 = f(z)c^2\text{d}t^2 - dx^2 - dy^2 - dz^2/f(z)$$

(17)

where $f(z) = 1 - 2MG/c^2R + 2gz/c^2$ and where $g$ denotes $MG/R^2$. Since the non-relativistic geodesics associated with the metric of Eq. (17) are given by $\ddot{x} = 0$, $\ddot{y} = 0$, $\ddot{z} = -g$, we see that this metric nicely describes a constant gravitational acceleration. Moreover, since the coordinate transformation [19]

$$ct' = c^2\sinh(gt/c)f^{1/2}(z)/g \quad \text{and} \quad z' = c^2(\cosh(gt/c)f^{1/2}(z) - f^{1/2}(0))/g$$

(18)

(a transformation which reduces to)

$$ct' = ct(1 - gR/c^2) + tgz/c \quad \text{and} \quad z' = z(1 + gR/c^2) + gt^2/2 - gz^2/2c^2$$

(19)

to lowest non-trivial order in $g$) transforms the metric of Eq. (17) into the flat Cartesian metric

$$\text{d}r^2 = c^2\text{d}t'^2 - dx^2 - dy^2 - dz^2$$

we see that the metric of Eq. (17) thus nicely incorporates the equivalence principle, not only for material particles, but necessarily also for light (explicitly because of covariance in fact). For the metric of Eq. (17) the Hamilton-Jacobi equation takes the form of the light cone condition

$$f(z)(k^0)^2 - (k^1)^2 - (k^2)^2 - (k^3)^2/f(z) = 0$$

(20)

while the massless particle geodesic equations take the form

$$k^0 = c\text{d}t/\text{d}q = \alpha_0/f(z) \quad \text{and} \quad k^1 = dx/\text{d}q = \alpha_1$$

$$k^2 = dy/\text{d}q = \alpha_2 \quad \text{and} \quad k^3 = dz/\text{d}q = (\alpha_0^2 - (\alpha_1^2 + \alpha_2^2)f(z))^{1/2}$$

(21)

where the $\alpha_i$ are integration constants. From these equations we recognize the existence of the gravitational frequency shift (since $k^0$ depends on $z$), the gravitational time delay ($dz/dt$ depends on $z$), and the gravitational bending of light ($dz/dx$ depends on $z$ if $\alpha_1 \neq 0$), with all of these effects thus participating in the motion of a classical light wave around the $ABCD$ interferometer loop.

While the geodesic equations of Eq. (21) enable us determine the trajectories of the light rays in between the various scattering surfaces, in order to be able to do the complete calculation we also need to know the rules for the scattering of light at the individual crystal surfaces, i.e. we need to determine the Bragg scattering rules in curved space, something which we can do via a sequence of coordinate transformations. Specifically, consider a ray which takes a time $T$ to travel vertically from $A$ to $C$ in the rest frame of the interferometer in the presence of the curved space metric of Eq. (17). According to Eq. (21) it arrives at $C$ with four momentum $k^\mu_{in} = (\alpha_0/f(H), 0, 0, \alpha_0)$. From the point of view of the accelerating flat space observer associated with Eq. (18), to lowest order in $g$ the point $C$ is moving upward with velocity $v = gT' = gT = gH/c$ at that instant, and the wave has momentum $k^\mu_{in} = \alpha_0(1 + gR/c^2)(1, 0, 0, 1)$ to this same order. To an observer who is moving vertically
downward with this same velocity \(v\) at that same instant, the scattering crystal appears to be at rest and the wave appears to have momentum \(k_{in}^\mu = \alpha_0(1 + gR/c^2)(1 - v/c)(1, 0, 0, 1)\) to lowest order. In this frame the wave undergoes standard rest frame flat space Bragg scattering and is thus reflected into the horizontal direction with momentum \(k_{out}^\mu = \alpha_0(1 + gR/c^2)(1 - v/c)(1, 1, 0, 0)\). Applying next a Lorentz transformation with velocity \(v\) vertically upwards transforms back to the coordinate system of Eq. (13) where the outgoing wave then has momentum \(k_{out}^\mu = \alpha_0(1 + gR/c^2)(1 - v/c, 1 - v/c, 0, v/c)\) to lowest order in \(g\). Finally, returning to the original coordinate system of Eq. (17) yields a wave with momentum \(k_{out}^\mu = (\alpha_0/\mu(H), \alpha_0/f^{1/2}(H), 0, 0)\) which is nicely seen to obey the light cone constraint of Eq. (20) to this order. In the original coordinate system we thus see that the general Bragg scattering rule in curved space is that the wave undergoes no change in the magnitude of \(k^0\) in a Bragg scattering, that it emerges with an angle of reflection equal to the angle of incidence, and that the magnitudes of the spatial components of the outgoing momentum take whatever values are needed to keep the outgoing wave on the light cone. Armed with this result we can now finally track light rays around the interferometer loop.

Explicit calculation (to lowest non-trivial order in \(g\) throughout) indicates that the light ray which goes up vertically from \(A\) to \(C\) sets out with momentum \((\alpha_0/f(0), 0, 0, \alpha_0)\) and arrives at \(C\) with a momentum \((\alpha_0/f(H), 0, 0, \alpha_0)\) after a travel time \(t(AC)\) given by

\[
t(AC) = H(1 - gH/c^2 + 2gR/c^2)/c
\]

On scattering at \(C\) the wave is then reflected so that it starts off toward \(D\) with momentum \((\alpha_0/f(H), \alpha_0/f^{1/2}(H), 0, 0)\). On its flight gravitational bending causes it to dip slightly so that it arrives not at \(D\) but rather at the point \(D_1\) with coordinates \((H - gH^2/2c^2, H - gH^2/2c^2)\), with the \(CD_1\) segment taking a time \(t(CD_1)\) given by

\[
t(CD_1) = H(1 - 3gH/2c^2 + gR/c^2)/c
\]

The wave which starts horizontally from \(A\) with momentum \((\alpha_0/f(0), \alpha_0/f^{1/2}(0), 0, 0)\) arrives not at \(B\) but at the point \(B_1\) with coordinates \((H - gH^2/2c^2, -gH^2/2c^2)\) and with a momentum \((\alpha_0/f(0), \alpha_0/f^{1/2}(0), 0, -\alpha_0gH/c^2f^{1/2}(0))\). The \(AB_1\) segment takes a time \(t(AB_1)\) given by

\[
t(AB_1) = H(1 - gH/2c^2 + gR/c^2)/c
\]

At \(B_1\) the scattered wave sets off toward \(D\) with momentum \((\alpha_0/f(0), -\alpha_0gH/c^2f(0), 0, \alpha_0)\) and arrives not at \(D\) or \(D_1\) but rather at the point \(D_2\) with coordinates \((H - 3gH^2/2c^2, H - 3gH^2/2c^2)\), with the \(B_1D_2\) segment taking a time \(t(B_1D_2)\) given by

\[
t(B_1D_2) = H(1 - 2gH/c^2 + 2gR/c^2)/c
\]

As regards the light wave path around the loop, we thus see that the small vertical \(gH^2/2c^2\) dip during each of the two horizontal legs causes each ray to travel a distance \(gH^2/2c^2\) less in the horizontal than it would have done in the absence of gravity. As regards the two vertical legs, we note that even though the \(AC\) leg is completely in the vertical, since the \(B_1D_2\) ray starts its leg with a small horizontal momentum (which it acquired because of the small dip in the prior \(AB_1\) leg), during this leg the ray changes its horizontal coordinate by an amount \(gH^2/c^2\), thereby causing it to reach \(D_2\) after having also traveled a distance \(gH^2/c^2\)
less in the vertical than it would travel in the \( AC \) leg. Consequently, there is a spatial offset \((gH^2/c^2, gH^2/c^2)\) between \( D_1 \) and \( D_2 \) which means that these particular beams are not able to interfere. Moreover, we also see that there is not anyway any time delay between the arrival of the two beams since \( t(AC) + t(CD_1) - t(AB_1) - t(B_1D_2) = 0 \).

Before discussing the issue of this spatial offset, it is instructive to ask where the beams would have met had there been no third crystal at \( D \) to get in the way. Explicit calculation shows that they would in fact have met at the asymmetric point \( D \) would have met had there been no third crystal at \( D \).

The rays would thus have met at \( D_3 \) with time delay \( t(AC) + t(CD_3) - t(AB_1) - t(B_1D_3) = -2gH^2/c^3 \), with both the time delay and the fact that the rays would meet at an asymmetric point rather than on the \( AD \) axis thus directly revealing the explicit nature of the global ordering effect which a background gravitational field has on a closed loop path.

As regards the spatial offset between the two beams, because of it the \( AB_1D_2 \) path interferes not with the \( ACD_1 \) path, but rather with the indicated nearby offset \( A_1C_1D_2 \) path, a very close by path which in fact is found to lie a distance \( gH^2/c^2 \) vertically below \( AB \), an offset distance which is well within the resolution of any incident beam. However, because of the spatial offset between \( D_1 \) and \( D_2 \), the \( AB_1D_2 \) path beam has to travel an extra horizontal distance \( A_2A = gH^2/c^2 \) to first get to the interferometer (to therefore provide an analog to the distance \( \Delta x = QT \) in the double slit experiment, with \( A_1 \) and \( A_2 \) acting just like the pair of slits \( Q \) and \( R \)). This spatial offset itself leads to a time delay \( A_2A/c \), so that finally observable interference is produced, with the two beams acquiring a net phase shift \( -\alpha_0gH^2/c^2 \) due entirely to the \( A_2A \) segment alone. On recognizing that \( \alpha_0 \) is the value of \( k^0 \) at \( z = 0 \) we may set it equal to \( 2\pi/\lambda \) where \( \lambda \) is the wavelength of the incident beam, to finally obtain for the phase shift \( \Delta \phi_{\text{CL}} = -2\pi gH^2/\lambda c^2 \) where \( \text{CL} \) denotes classical light. Now while \( H \) would have to be of the order of \( 10^5 \) cm for \( \Delta \phi_{\text{CL}} \) to actually be detectable in a Bragg scattering interferometer of the same sensitivity as the neutron COW experiment, nonetheless we can still identify this phase shift as an in principle, completely classical effect which reveals the intrinsically global nature of classical gravity.

Now that we have obtained \( \Delta \phi_{\text{CL}} = -2\pi gH^2/\lambda c^2 \) it is instructive to compare it with \( \Delta \phi_{\text{COW}} \). If we introduce the neutron de Broglie wavelength \( \lambda_n = h/mv_0 \), we may rewrite the neutron \( \Delta \phi_{\text{COW}} \) phase of Eq. (14) in the form \( -2\pi gH^2/\lambda_n v_0^2 \). Comparison with \( \Delta \phi_{\text{CL}} \) thus reveals a beautiful example of wave particle duality, with the quantum mechanical neutron matter wave inheriting its interference aspects from the behavior of the underlying classical wave. Thus even while \( \Delta \phi_{\text{COW}} \) does depend on the mass of the neutron, its dependence is

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16 Since the sign of the time is unchangeable by coordinate transformations, the fact of a time delay between the two beams at the source is thus a covariant indicator for interference.

17 While this is still sizable for an interferometer, its interest lies in the fact that it allows us to detect, in principle at least, the gravitational bending of light using laboratory sized distance scales rather than solar system sized ones. Thus it would also be of interest to see what dimension interferometer might serve as a gravitational wave detector.

18 The reason why \( \Delta \phi_{\text{COW}} \) actually depends on \( m \) at all was given in [21] where it was shown that an analogous spatial offset \( gH^2/v_0^3 \) and time delay \( gH^2/v_0^3 \) occur for neutron beams, with the neutron COW
strictly a passive, kinematic one with gravity only coupling to neutron matter waves via their de Broglie wavelengths. Thus the neutron mass has no dynamical consequences in Eq. (10) with the explicit spatial offset and time delay needed to produce interference actually being completely independent of the mass of the neutron. Consequently, the COW experiment would appear to be completely compatible with the equivalence principle.

Now while we have shown that the COW phase does not derive from topological quantum mechanical considerations, it is nonetheless somewhat puzzling that the resulting COW phase is proportional to the area enclosed by the \( ABCD \) loop, since proportionality to the area is a characteristic topological signature. However, in such topological cases, the area in question is typically related to the flux threaded by some electromagnetic field, whereas in the gravitational case the gravitational field is parallel to the loop not perpendicular to it, with there instead being a gravitational field gradient in the plane of the loop. In order to underscore this remark, it is instructive to consider the familiar Michelson Morley experiment, only with one of the interferometer arms vertical rather than horizontal. In such a situation the crystal at \( B \) in Fig. (2) would be replaced by a vertical mirror and that at \( C \) by a horizontal one, with the beams both splitting and recombining at \( A \) to then be detected at a location below \( A \). Such a device would then have no enclosed area. However, due to gravitational bending in the \( AB \) and \( BA \) legs, there would still be a spatial offset at \( A \) between the returning \( BA \) and \( CA \) beams of order \( gL^2/c^2 \) where \( L \) is the \( AB \) length. Consequently, compared to a horizontal Michelson Morley interferometer, one with one arm vertical would undergo a detectable phase shift\(^19\) of order \( 2\pi gL^2/\lambda c^2 \), even though \( L^2 \) is not a measure of any relevant interferometer area. Rather, the bending distance in any arm is given by the length of that arm multiplied by the only available dimensionless perturbation parameter appropriate to the problem, viz \( gL/c^2 \), with the ensuing spatial offset \( gL^2/c^2 \) itself thus being quadratic in \( L \) even while it has the dimension of length rather than of area. (Noting that since the \( AB \) path sweeps out a small (triangular) area in going to \( B \) and back, and that similarly the enclosed COW experiment loop differs slightly from the ideal \( ABCD \) loop, to the extent that any specific area is relevant, it would be the one which represents the difference between the interferometer paths associated with the vertical and horizontal configurations.) Like the Michelson Morley case, the COW phase shift is thus explicitly seen to not be of topological origin, but rather, just like the impact of the Hubble flow on galactic rotation curves, is instead seen to be a further manifestation of the global nature of gravity, a thus characteristic feature of relativistic gravity which seems not to have been given all that phase then being given as the associated net change in the neutron action due to traversing the loop of Fig. (2). Specifically, it was noted that even while the stationary trajectories of the classical \(-mc \int d\tau \) action (viz. the trajectories explicitly followed by the centers of the quantum mechanical neutron beam wave packets) are themselves totally independent of \( m \) (the equivalence principle) to thus give a spatial offset whose magnitude is independent of \( m \), nonetheless the actual value of the classical action itself in any such trajectory does nonetheless depend on \( m \), though only as a purely kinematic overall multiplying factor whose presence does not affect the position of the minimum of the action. And, even though the actual value of the stationary classical neutron point particle action is not observable classically, nonetheless it does become observable quantum mechanically as the phase of the neutron wave function, a phase whose overall normalization explicitly (kinematically) depends on \( m \).

\(^{19}\)To be precise, the gravitational bendings during the \( AC \) and \( CA \) legs which would occur when \( AC \) is in the horizontal no longer occur when \( AC \) is in the vertical, so they can then no longer offset the bendings which occur in the horizontal \( AB \) and \( BA \) legs.
much emphasis in the literature. This work has been supported in part by the Department of Energy under grant No. DE-FG02-92ER40716.00.
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Table (1)

| Galaxy | Distance (Mpc) | Luminosity ($10^9 L_{B\odot}$) | ($v^2/c^2 R$)$_{last}$ ($10^{-30} cm^{-1}$) | Shift (%) | ($M/L$) ($M_{\odot}L_{B\odot}^{-1}$) |
|--------|----------------|-------------------------------|---------------------------------|-----------|-----------------------------------|
| DDO 154 | 3.80 | 0.05 | 1.51 | -11 | 0.71 |
| DDO 170 | 12.01 | 0.16 | 1.63 | -07 | 5.36 |
| NGC 1560 | 3.00 | 0.35 | 2.70 | +23 | 2.01 |
| NGC 3109 | 1.70 | 0.81 | 1.98 | 0.01 | |
| UGC 2259 | 9.80 | 1.02 | 3.85 | +15 | 3.62 |
| NGC 6503 | 5.94 | 4.80 | 2.14 | 3.00 | |
| NGC 2403 | 3.25 | 7.90 | 3.31 | +15 | 1.76 |
| NGC 3198 | 9.36 | 9.00 | 2.67 | -15 | 4.78 |
| NGC 2903 | 6.40 | 15.30 | 4.86 | +14 | 3.15 |
| NGC 7331 | 14.90 | 54.00 | 5.51 | -16 | 3.03 |
| NGC 2841 | 9.50 | 20.50 | 7.25 | 8.26 | |

Figure Captions

Figure (1). The predicted rotational velocity curves associated with conformal gravity for each of the 11 galaxies in the sample. In each graph the bars show the data points with their quoted errors, the full curve shows the overall (adopted distance adjusted) theoretical velocity prediction (in km sec$^{-1}$) as a function of distance from the center of each galaxy (in units of $R/R_0$ where each time $R_0$ is each galaxy’s own stellar optical disk scale length), while the dashed and dash-dotted curves show the velocities that the Newtonian and the linear potentials would separately produce. The dotted curves show the total velocities that would be produced without any adopted distance modification. No dark matter is assumed.

Figure (2). The paths followed by waves in a COW type interferometer.

Figure (3). The paths followed by waves in a double slit experiment.
Figure 1
