In the framework of $U(3)_L \times U(3)_R$ chiral theory of mesons, $\pi K$ elastic scattering is studied. The s-wave and p-wave scattering lengths ($a_0^{3/2}$, $a_0^{1/2}$, $a_1^{3/2}$, and $a_1^{1/2}$) and phase shifts ($\delta_0^{3/2}$, $\delta_0^{1/2}$, $\delta_1^{3/2}$, and $\delta_1^{1/2}$), the total elastic scattering cross section $\sigma_{\text{tot}}(\pi^+K^+)$ and $\sigma_{\text{tot}}(\pi^-K^+)$, and the p-wave cross section $\sigma_p(\pi^-K^+)$ are calculated. Theoretical results are in agreement with the experimental data. There is no new parameter in this study.
The chiral SU(3)$_L \times$ SU(3)$_R$ symmetry of quantum chromodynamics (QCD) Lagrangian in the limit of $m_u = m_d = m_s = 0$ is spontaneously broken to SU(3)$_V$ with the appearances of eight Goldstone pseudoscalar-particles ($\pi, K, \eta$). At low energies, these Goldstone Bosons play very important role in chiral dynamics of the strong interaction. It is well known that the reactions $\pi\pi \rightarrow \pi\pi$ and $\pi K \rightarrow \pi K$ scattering are strongly related to chiral dynamics. The investigation of $\pi\pi$ scattering has a long history [1, 2, 3, 4, 5, 6, 7] and it tests all kinds of chiral theory of meson physics. $\pi K$ scattering is another test of chiral dynamics. Chiral symmetry has been exploited in the study of $\pi K$ scattering[6, 8, 9, 10, 11]. In Ref. [8, 11] chiral perturbation theory (ChPT) has been applied to $\pi K$ scattering. In Ref.[6] resonances are incorporated.

In Refs.[7, 12] a chiral theory of mesons including pseudoscalar, vector, and axial-vector mesons (in brief U(3)$_L \times$ U(3)$_R$ theory below) has been proposed. This theory is phenomenologically successful [13, 14, 15, 16, 17]. It has been applied to study $\pi\pi$ scattering[7]. Theoretical results agree well with data. In terms of $\pi K$ scattering three coefficients of the chiral perturbation theory have been determined[17]. In this paper we investigate $\pi K$ scattering in the framework of this effective chiral theory of mesons.

The three invariant quantities of $\pi(p_1) + K(p_2) \rightarrow \pi(p_3) + K(p_4)$ are defined as

\begin{align*}
s &= (p_1 + p_2)^2 = (p_3 + p_4)^2, \\
t &= (p_1 - p_3)^2 = (p_2 - p_4)^2, \\
u &= (p_1 - p_4)^2 = (p_2 - p_3)^2,
\end{align*}

with $s + t + u = 2(m^2_{\pi} + m^2_K)$. $T^{3/2}(s, t, u)$ and $T^{1/2}(s, t, u)$ are the two independent isospin amplitudes of $\pi K$ scattering

\begin{align*}
T^{3/2}(s, t, u) &= T_{\pi K}(s, t, u), \\
T^{1/2}(s, t, u) &= \frac{3}{2} T_{\pi K^{-}}(s, t, u) - \frac{1}{2} T^{3/2}(s, t, u).
\end{align*}

(1)

The partial wave amplitudes are defined as

\begin{align*}
T^I_l(s) &= \frac{1}{32\pi} \int_{-1}^{1} d\cos \theta P_l(\cos \theta) T^I(s, t, u).
\end{align*}

(3)
In terms of real phase shifts $\delta_l^I(s)$, $T_l^I(s)$ is expressed as

$$T_l^I(s) = \frac{\sqrt{s}}{2q} \frac{1}{2i} (e^{2i\delta_l^I(s)} - 1),$$

$$q = \frac{1}{2}\sqrt{s}[s - (m_K + m_\pi)^2][s - (m_K - m_\pi)^2].$$

(4)

Here $q$ is the pion’s or kaon’s momentum and $\theta$ is the scattering angle in the frame of center of mass.

At low energies (small $q$), the $\pi K$ scattering length $a_l^I$ is defined

$$\text{Re } T_l^I(s) = \frac{\sqrt{s}}{2} q^2 (a_l^I + O(q^2)).$$

(5)

All the meson vertices involved in $\pi K$ scattering are derived from the real part of the effective Lagrangian (eq.(3) in Ref.[12]). The vertices $\mathcal{L}^{\pi\pi KK}$, $\mathcal{L}^{\pi\pi\rho}$, $\mathcal{L}^{\rho KK}$, and $\mathcal{L}^{\rho KK^*}$ are involved in $\pi K$ scattering

$$\mathcal{L}^{\pi\pi KK} = \frac{1}{f_\pi^2} \left( 1 - \frac{6c}{g} \right) \pi_i \partial_\mu \pi_i K_a \partial^\mu K_a - \frac{1}{6f_\pi^2} \left( 1 - \frac{6c}{g} \right) (m_\pi^2 + m_K^2) \pi_i \pi_i K_a K_a$$

$$+ \frac{1}{f_\pi^2} \left\{ \frac{1}{\pi^2} \left( 1 - \frac{2c}{g^2} \right)^2 \frac{4c^2}{g^2} - \frac{8c^4}{g^2} \right\} \partial_\mu \pi_i \partial_\nu \pi_i \partial_\mu K_a \partial^\nu K_a$$

$$+ \frac{1}{f_\pi^2} \left\{ \frac{1}{\pi^2} \left( 1 - \frac{2c}{g^2} \right)^2 \left( 1 - \frac{4c}{g^2} \right) + \frac{8c^4}{g^2} \right\} \partial_\mu \pi_i \partial_\nu \pi_i \partial_\mu K_a \partial^\nu K_a$$

$$- \frac{2}{f_\pi^2} (1 - \frac{6c}{g}) \epsilon_{ijk} f_{abk} \partial_\mu \pi_i \partial^\mu K_a K_b$$

$$+ \frac{1}{f_\pi^2} \left\{ \frac{1}{\pi^2} \left( 1 - \frac{2c}{g^2} \right)^2 \left( \frac{16c^2}{g^2} + \frac{8c^4}{g^2} - 2 \right) \right\} \epsilon_{ijk} f_{abk} \partial_\mu \pi_i \partial_\nu \pi_j \partial_\mu K_a \partial^\nu K_b,$$

(6)

$$\mathcal{L}^{\pi\pi\rho} = f(q^2) \epsilon_{ijk} \rho^i_\mu \pi_j \partial_\mu \pi_k,$$

(7)

$$\mathcal{L}^{\rho KK} = f(q^2) f_{ab} \rho^i_\mu K_a \partial_\mu K_b,$$

(8)

$$\mathcal{L}^{\rho KK^*} = f(q^2) f_{ab} K^a_\mu (\partial_\mu \pi_i K_b - \pi_i \partial_\mu K_b),$$

(9)

where

$$f(q^2) = \frac{2}{g} (1 + \beta q^2), \quad \beta = \frac{1}{2\pi^2 f_\pi^2} \left\{ (1 - \frac{2c}{g^2})^2 - 4\pi^2 c^2 \right\}.$$

(10)

The amplitudes of this process from the contact terms are derived from $\mathcal{L}^{\pi\pi KK}(\text{with the index } D)$

$$T^2_{\pi K}(u, t, s)_D = T_{\pi K^+}(s, t, u)_D,$$
Where \( f_\pi \) is the decay constant of the pion, \( g \) is the universal coupling constant of the \( U(3)_L \times U(3)_R \) theory, and it has been fixed \( g=0.35 \) in Refs. [7, 12]. Following equations have been used in deriving eqs. (11,12),

\[
\frac{F^2}{f_\pi^2}(1 - \frac{2c}{g}) = 1,
\]

\[
c = \frac{f_\pi^2}{2gm_\rho^2}.
\]  

(13)

The resonance parts of the two amplitudes are obtained from the vertices(7,8,9)

\[
T^2_\pi(s,t,u)_D = \frac{2}{f_\pi^2}(1 - \frac{6c}{g})(m_\pi^2 + m_K^2 - s) - \frac{2}{3f_\pi^2}(1 - \frac{6c}{g})(m_\pi^2 + m_K^2)
\]

\[+ \frac{1}{\pi^2 f_\pi^4}(1 - \frac{2c}{g})^2 \frac{4c^2}{g^2} - \frac{8c^4}{f_\pi^4 g^2}](t - 2m_\pi^2)(t - 2m_K^2)
\]

\[+ \frac{1}{\pi^2 f_\pi^4}(1 - \frac{2c}{g})^2(1 - \frac{4c^2}{g^2}) + \frac{16c^4}{f_\pi^4 g^2}(s - m_\pi^2 - m_K^2)^2
\]

\[+ \frac{1}{\pi^2 f_\pi^4}(1 - \frac{2c}{g})^2 \frac{4c^2}{g^2} - \frac{8c^4}{f_\pi^4 g^2}](u - m_\pi^2 - m_K^2)^2,
\]

(11)

\[
T^4_\pi(s,t,u)_D = \frac{3}{2}T_{\pi-K^+}(s,t,u)_D - \frac{1}{2}T^{3/2}(s,t,u)_D,
\]

\[
T^4_\pi(s,t,u)_D = \frac{1}{f_\pi^2}(1 - \frac{6c}{g})(2m_\pi^2 + 2m_K^2 + s - 3u) - \frac{2}{3f_\pi^2}(1 - \frac{6c}{g})(m_\pi^2 + m_K^2)
\]

\[+ \frac{1}{f_\pi^2} \{ \frac{1}{\pi^2}(1 - \frac{2c}{g})^2 \frac{4c^2}{g^2} - \frac{8c^4}{f_\pi^4 g^2} \}(t - 2m_\pi^2)(t - 2m_K^2)
\]

\[+ \frac{1}{f_\pi^2} \{ \frac{1}{\pi^2}(1 - \frac{2c}{g})^2 \frac{3}{2} - \frac{6c}{g} - \frac{8c^2}{g^2} \} + \frac{28c^4}{g^2} \}(u - m_\pi^2 - m_K^2)^2
\]

\[+ \frac{1}{f_\pi^2} \{ \frac{1}{\pi^2}(1 - \frac{2c}{g})^2 \frac{8c^2}{g^2} - \frac{2c}{g} - \frac{1}{2} \} - \frac{20c^4}{g^2} \}(s - m_\pi^2 - m_K^2)^2.
\]

(12)
\[-\frac{1}{4} f^2(u) \frac{1}{u - m_{K^*}^2} [s - t - \frac{(m_{\pi}^2 - m_{K^*}^2)^2}{m_{K^*}^2}], \tag{15}\]

where $\Gamma(s)$ is the decay width of $K^*$

$$\Gamma(s) = f^2(q^2) \frac{q^3}{8\pi s}, \tag{16}$$

$q$ is defined in eq.(4). It is necessary to point out that in the amplitudes(11,12,14,15) all the parameters have been fixed in Refs.[7,12] and there is no new parameter.

The scattering lengths can be derived from the amplitudes(11,12,14,15). However, the leading terms of the s-wave scattering lengths are proportional to either $m_\pi$ or $m_K$. The quark mass term $\bar{\psi}M\psi$($M$ is the quark matrix) contributes to the s-wave scattering lengths too. The effective Lagrangian with quark mass term has been derived in Ref.[17]. Using the formalism developed in Ref.[17], to the leading order in quark masses the contribution of the quark mass term to $\pi K$ scattering can be found from

$$L = \frac{1}{2} \int \frac{d^Dp}{(2\pi)^D} \frac{m}{p^2 + m^2} Tr(\hat{u}M + Mu), \tag{17}$$

where $u = exp\{i\gamma_5 \frac{2}{f_\pi}(\tau_i \pi_i + \lambda_a K_a)\}$ and $\hat{u} = exp\{-i\gamma_5 \frac{2}{f_\pi}(\tau_i \pi_i + \lambda_a K_a)\}$, $m$ is a parameter defined in [7]. Eq.(7) leads to

$$L = \frac{1}{3f_\pi^2} <\bar{\psi}\psi> \{\frac{3}{2}(m_u + m_d) + m_s\}(2\pi^+\pi^- + \pi^0\pi^0)(K^+K^- + K^0\bar{K}^0), \tag{18}$$

$<\bar{\psi}\psi>$ is the quark condensate and defined in Ref.[17]

$$<\bar{\psi}\psi> = -\frac{mD N_c}{(2\pi)^D} \int d^Dp \frac{1}{p^2 + m^2}, \tag{19}$$

where $N_c$ is the number of color. To the leading order in quark masses pion and kaon masses are expressed as[17]

$$m_{\pi}^2 = -\frac{4}{f_\pi^2} <\bar{\psi}\psi> (m_u + m_d),$$

$$m_{K^+}^2 = -\frac{4}{f_\pi^2} <\bar{\psi}\psi> (m_u + m_s),$$

$$m_{K^0}^2 = -\frac{4}{f_\pi^2} <\bar{\psi}\psi> (m_d + m_s). \tag{20}$$
From Eqs. (18,20) we obtain

$$\mathcal{L} = \frac{1}{3f_\pi^2}\left( m_{\pi}^2 + \frac{1}{2}(m_{K^+}^2 + m_{K^0}^2) \right) (2\pi^+\pi^- + \pi^0\pi^0)(K^+K^- + K^0K^0).$$  \hspace{1cm} (21)

Using eqs. (3) and (5), the s-wave and p-wave scattering lengths from the contact parts of the amplitudes (11,12) and Eq. (21) are found

$$a_{0/2}^{3/2} = \frac{1}{2\pi} \frac{m_{\pi}m_K}{f_\pi^2(m_{\pi} + m_K)} \left[ \frac{1 - 6c}{g} + \frac{c}{f_\pi^2 gm_{\pi} + m_K} \right] + \frac{1}{\pi^2 f_\pi^2} \frac{(1 - 2c)^2 m_{\pi}^2 m_K^2}{g^2},$$  \hspace{1cm} (22)

$$a_{1/2}^{3/2} = -\frac{1}{6\pi} \frac{1}{f_\pi^2} \left[ \frac{1 - 2c}{g} \right] \frac{4c^2 - 8c^4}{f_\pi^2 g^2} \frac{(m_K - m_{\pi})^2}{m_K + m_{\pi}},$$  \hspace{1cm} (23)

$$a_{0/2}^{1/2} = \frac{1}{\pi f_\pi^2} \left[ (1 - 6c) \frac{m_{\pi}m_K}{m_{\pi} + m_K} + \frac{c m_{\pi}^2 + m_K^2}{2g m_{\pi} + m_K} \right] + \frac{1}{2\pi^3 f_\pi^4} \frac{(1 - 2c)^2 (1 - 4c)}{g^2} \frac{m_{\pi}^2 m_K^2}{m_{\pi} + m_K},$$  \hspace{1cm} (24)

$$a_{1/2}^{1/2} = \frac{1}{4\pi f_\pi^2} \left[ (1 - 6c) - \frac{1}{6\pi} \frac{1}{f_\pi^2} \left[ (1 - 2c)^2 \frac{4c^2}{g^2} - \frac{8c^4}{f_\pi^4 g^2} \right] \frac{m_{\pi}^2 + m_K^2}{m_{\pi} + m_K} \right] + \frac{1}{3\pi} \frac{1}{f_\pi^4} (1 - 2c)^2 \left[ \frac{3}{2} - \frac{6c}{g^2} - \frac{8c^2}{g^2} + \frac{28c^4}{f_\pi^4 g^2} \right] \frac{m_{\pi}m_K}{m_{\pi} + m_K}.$$  \hspace{1cm} (25)

From the resonance parts of the amplitudes (14,15), we obtain

$$a_{0/2}^{3/2} = -\frac{m_{\pi}m_K}{\pi g^2 m_{\rho}^2 (m_{\pi} + m_K)} - \frac{m_{\pi} + m_K}{4\pi g^2 m_{K^*}^2} \left[ 1 + \beta(m_{\pi} - m_K)^2 \right]^2,$$  \hspace{1cm} (26)

$$a_{1/2}^{3/2} = -\frac{m_{\rho}^2 + 4m_{\pi}m_K + 8\beta m_{\pi}m_K m_{\rho}^2}{6\pi^2 m_{\rho}^2 (m_{\pi} + m_K)} + \frac{1 + \beta^2(m_{\pi} - m_K)^2}{6\pi^2 g^2 m_{K^*}^2 (m_{\pi} + m_K)} - \frac{\beta^2(m_{\pi}^2 + m_K^2)}{3\pi g^2 (m_{\pi} + m_K)} + \frac{(1 + \beta m_{K^*}^2)^2 (m_{\pi}^2 + m_K^2)}{3\pi g^2 m_{K^*}^2 [m_{K^*}^2 - (m_K - m_{\pi})^2] (m_{\pi} + m_K)},$$  \hspace{1cm} (27)

$$a_{0/2}^{1/2} = -\frac{2m_{\pi}m_K}{\pi g^2 m_{\rho}^2 (m_{\pi} + m_K)} + \frac{m_{\pi} + m_K}{8\pi g^2 m_{K^*}^2} \left[ 1 + \beta(m_{\pi} - m_K)^2 \right]^2 - \frac{3(m_K - m_{\pi})^2 [1 + \beta(m_{\pi} + m_K)^2]^2}{8\pi g^2 m_{K^*}^2 (m_{\pi} + m_K)},$$  \hspace{1cm} (28)

$$a_{1/2}^{1/2} = -\frac{m_{\rho}^2 + 4m_{\pi}m_K + 8\beta m_{\pi}m_K m_{\rho}^2}{3\pi g^2 m_{\rho}^2 (m_{\pi} + m_K)} + \frac{1 + \beta^2(m_{\pi} - m_K)^2}{2\pi g^2 [m_{K^*}^2 - (m_{\pi} + m_K)^2] (m_{\pi} + m_K)} - \frac{1 + \beta^2(m_{\pi} - m_K)^2}{12\pi g^2 m_{K^*}^2 (m_{\pi} + m_K)} + \frac{\beta^2(m_{\pi}^2 + m_K^2)}{6\pi g^2 (m_{\pi} + m_K)} - \frac{(1 + \beta m_{K^*}^2)^2 (m_{\pi}^2 + m_K^2)}{6\pi g^2 m_{K^*}^2 [m_{K^*}^2 - (m_K - m_{\pi})^2] (m_{\pi} + m_K)}.$$  \hspace{1cm} (29)
Numerical calculations show that the vector resonance exchanges are dominant in πK scattering. The numerical results of the scattering lengths are listed in Table 1. The data can be traced back from Ref. [6]. $a^I_0$ (s-wave) and $a^I_1$ (p-wave) are given in units of $m_\pi^{-1}$ and $m_\pi^{-3}$ respectively. The theoretical predictions of $a^{1/2}_1$, $a^{1/2}_0$, and $a^{3/2}_0$ are in reasonable agreement with the experimental data. It is found that in $a^{3/2}_1$ the $\rho$ and $K*$ resonances have opposite signs, which leads to strong cancellation, so the value of $a^{3/2}_1$ is very small.

From eq.(4), the phase shifts of the partial waves of πK scattering are defined as

$$\delta^I_0(s) = \arctan \left( \frac{\text{Im} T^I_0(s)}{\text{Re} T^I_0(s)} \right),$$

(30)

Using the unitarity constraint on the partial-wave amplitude of elastic scattering[6], at the leading order it is found

$$\delta^I_1(s) = \arctan \left( \frac{2q}{\sqrt{s}} \text{Re} T^I_1(s) \right).$$

(31)

The isospin 1/2 p-wave phase shifts $\delta^{1/2}_1$ are shown in Fig. 1 and Fig. 2. $\delta^{1/2}_1$ is dominated by the $K^*(892)$ resonance. The theoretical results are in good agreement with the data. The isospin 3/2 p-wave phase shifts are very small(Fig. 3). This is consistent with the result presented in Ref. [6].

The s-wave phase shifts are shown in Fig. 4 and Fig. 5 respectively. Theoretical predictions of $\delta^{1/2}_0$ and $\delta^{3/2}_0$ are in agreement with the data within the error bars.

The differential cross section of elastic πK scattering is expressed as

$$\frac{d\sigma}{dt} = \frac{1}{64\pi s q^2} |T|^2,$$

(32)

| theoretical results | experimental data |
|---------------------|-------------------|
| $a^{1/2}_0$         | 0.13              |
| $a^{3/2}_0$         | -0.05             |
| $a^1_0$             | 0.017             |
| $a^{3/2}_1$         | $-1.68 \times 10^{-4}$ |

Table 1: The s-wave and p-wave πK scattering lengths.
The total cross section of πK scattering is written as

\[ \sigma_{\text{tot}} = \frac{1}{32\pi s} \int_{-1}^{1} d\cos\theta |T|^2. \]  

(33)

Using Eqs.(11,12,14,15), the total cross sections are calculated. Theoretical results of \( \sigma_{\text{tot}}(\pi^+ K^+) \) are in the range of 0.63 mb \( \sim \) 2.0 mb, which agrees with the experimental value \( \sigma_{\text{tot}}^{3/2} \sim 1.8 \) mb given in Ref. [21].

The amplitude of \( \pi^- K^+ \) scattering has \( I=1/2 \) and \( I=3/2 \) two components. \( \delta_{\text{I}}^{3/2} \) is very small (Fig. 3), so it can be ignored in the calculation of the p-wave cross section in terms of phase shifts. Thus, we have

\[ \sigma_p(\pi^- K^+) = \frac{16\pi}{3q^2} \sin^2 \delta_{\text{I}}^{1/2}. \]  

(34)

In Fig. 7, the p-wave cross section of \( \pi^- K^+ \) elastic scattering is shown. The predictions of \( \sigma_p(\pi^- K^+) \) are in good agreement with the data.

Our calculation shows that the contributions from s-wave and p-wave are dominant in the \( \pi K \) elastic scattering at low energies, which is consistent with one claimed in Ref. [21].

To conclude, in terms of the U(3)_L \times U(3)_R theory of mesons a parameter free study of πK scattering is presented. Theoretical results of the s-wave and p-wave scattering lengths, the phase shifts, and the cross sections are in good agreement with the data. The vector resonances are dominant in πK scattering.

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Caption

**FIG. 1** The I=1/2 p-wave $\delta_1^{1/2}(s)$ for $\sqrt{s} \leq 0.86$ GeV. The solid line denotes the theoretical results. The data are from Ref. \textsuperscript{22} (solid squares) and Ref. \textsuperscript{23} (solid inverted triangles).

**FIG. 2** The I=1/2 p-wave $\delta_1^{1/2}(s)$ for $0.70$ GeV $\leq \sqrt{s} \leq 1.1$ GeV. For the notations, see Fig. 1.

**FIG. 3** The I=3/2 p-wave $\delta_1^{3/2}(s)$ for $\sqrt{s} \leq 1.0$ GeV. The solid line denotes the theoretical results.

**FIG. 4** The I=1/2 s-wave $\delta_0^{1/2}(s)$ for $\sqrt{s} \leq 0.86$ GeV. The solid line denotes the theoretical results. The key to the data is as follows: Ref. \textsuperscript{20} (solid circles), Ref. \textsuperscript{21} (solid diamonds), Ref. \textsuperscript{22} (solid squares), Ref. \textsuperscript{24} (open circles).

**FIG. 5** The I=3/2 s-wave $\delta_0^{3/2}(s)$ for $\sqrt{s} \leq 0.95$ GeV. The solid line denotes the theoretical results. The data are from Ref. \textsuperscript{22} (solid squares) and Ref. \textsuperscript{25} (solid triangles).

**FIG. 6** The total elastic cross section $\sigma_{\text{tot}}(\pi^- K^+)$ for $\sqrt{s} \leq 1.05$ GeV. For the notations, see Fig. 4. The solid line is obtained by using eq. (30), and the dot-dashed line is from the partial wave method.

**FIG. 7** The p-wave elastic cross section $\sigma_p(\pi^- K^+)$ for $\sqrt{s} \leq 1.0$ GeV. The solid line denotes the theoretical results. The data are from Ref. \textsuperscript{20} (solid circles).
FIG. 4
FIG. 5

Phase shift $\delta_0^{3/2}$ (degrees)

Energy (GeV)

0 0.6 0.65 0.7 0.75 0.8 0.85 0.9 0.95

-15 -10 -5 0
