Hadron resonance gas with repulsive mean field interaction: Thermodynamics and transport properties

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We discuss the interacting hadron resonance gas model to describe the thermodynamics of hadronic matter. While the attractive interaction between hadrons is taken care of by including all the resonances with zero width, the repulsive interactions are included by considering density-dependent mean field potentials. The bulk thermodynamic quantities are confronted with the lattice quantum chromodynamics simulation results at zero as well as at finite baryon chemical potential. We further estimate the shear and bulk viscosity coefficients of hot and dense hadronic matter within ambit of this interacting hadron resonance gas model.

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I. INTRODUCTION

Understanding the phase diagram of strongly interacting matter is one of the important and challenging topics of research currently in strong interaction physics—both theoretically as well as experimentally. The theoretical framework describing a nuclear matter at a fundamental level is quantum chromodynamics (QCD). At low temperature ($T$) and low baryon chemical potential ($\mu$) the fundamental degrees of freedom of QCD are colorless hadrons while at high temperature and high baryon density the fundamental degrees of freedom are colored quarks and gluons. Lattice quantum chromodynamics (LQCD) simulations at zero chemical potential suggest a crossover transition for the QCD matter from hadronic phase to a quark-gluon-plasma (QGP) phase\cite{1-6}. At zero chemical potential, the chiral crossover temperature is estimated to be $T_c \sim 156$ MeV\cite{7}. While LQCD simulations at vanishing chemical potential has been quite successful, LQCD simulations at finite $\mu$ has been rather challenging particularly at high $\mu$ leading to large uncertainties in estimating the transition line in the $T - \mu$ plane of QCD phase diagram \cite{8}. At small $\mu$, however, precise computation of the transition line has been carried out recently\cite{9-11}.

The low energy effective models of QCD, viz., Nambu-Jona-Lasinio model\cite{12,13}, Quark-Meson-Coupling model\cite{14} etc., provide a reasonable theoretical framework to explore the strongly interacting matter below QCD transition temperature, $T_c$. These models are based on certain QCD symmetries and they are tremendously successful in describing many features of QCD phase diagram at zero as well as at finite baryon density. Apart from these symmetry based models another model which has also been tremendously successful in de-

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scribing low temperature hadronic phase of QCD is the hadron resonance gas model (HRG). The hadron resonance gas model is the statistical model of QCD describing the low temperature hadronic phase of quantum chromodynamics. This model is based on so-called Dashen-Ma-Bernstein theorem which allows us to compute the partition function of the interacting system of hadrons in terms of scattering matrix\[15\]. Using this \(S\)-matrix formulation of statistical mechanics it can be shown that if the dynamics of thermodynamic system of hadrons is dominated by narrow-resonance formation then the resulting system essentially behaves like a noninteracting system of hadrons and resonances\[16–18\]. This ideal HRG model, despite its success in describing hadron multiplicities in heavy-ion collision\[19–27\], fails to account for the short range repulsive interactions between hadrons. It has been shown that the repulsive interactions modeled via excluded volume can have significant effect on thermodynamic observables, especially higher order fluctuations\[28–30\] as well as in the context of statistical hadronization\[31\]. One possible way to include these repulsive interactions is through van der Waals excluded volume procedure\[32, 33\]. Another approach is to treat the repulsive interactions in mean field way\[34, 35\]. Recently, the relativistic mean field approach has been used to calculate the fluctuations of conserved charges\[36\]. These authors discussed the repulsive mean field interactions which are present only at finite baryon density. They showed the deviations of higher order fluctuations estimated using the ideal HRG can be accounted by means of repulsive interactions treated in mean field way. The failure of ideal HRG model to explain the thermodynamical observable can be attributed to the fact that at high temperature and density the relativistic virial expansion up to second order virial coefficient cannot be reasonable approximation and the validity of HRG model needs to be checked against its agreement with LQCD results.

In the past few decades relativistic and ultra-relativistic heavy-ion collision experiment has provided a unique opportunity to study the phase diagram of QCD. The relativistic hydrodynamics has been tremendously successful in simulating the evolution of matter created in HIC experiments \[37–45\]. In the relativistic hydrodynamic simulations the coefficient of shear and bulk viscosities influence various observables, \(\text{viz.}\), the flow coefficients, the transverse momentum distribution of produced particles. In fact, it has been found that a finite but very small shear viscosity to entropy ratio should be included in the hydrodynamic description to explain elliptic flow data\[46, 47\]. Further, \(\eta/s\) obtained using AdS/CFT correspondence \[48\] has put the lower bound on its value equal to \(\frac{1}{4\pi}\) called the Kovtun-Son-Starinets (KSS) bound. This interesting finding has motivated many theoretical investigations of this ratio to understand and derive rigorously from a microscopic theory \[49–63\]. The bulk viscosity coefficient \(\zeta\) has also been realized to be important to be included the dissipative hydrodynamics. During the expansion of the fireball, when the temperature approaches the critical temperature, \(\zeta\) can be large and give rise to different interesting phenomena like cavitation when the pressure vanishes and hydrodynamic description breaks down \[64, 65\]. The effect of bulk viscosity on the particle spectra and flow coefficients have been investigated \[66–68\] while the interplay of shear and bulk viscosity coefficients have been studied in Refs. \[69–71\]. The coefficient of bulk viscosity has been estimated for both the hadronic and the partonic systems \[72–103\].

Hydrodynamic simulation of the matter created in HIC collision requires information regarding equation of state (EoS) as well as the transport coefficients. In this work we analyze the QCD equation of state of hadronic matter at finite baryon chemical potential. We employ the hadron resonance gas model to estimate all the thermodynamic quantities. While the attractive interactions between hadrons are accounted by including all the resonance
states up to 2.25 GeV, the short range repulsive interaction among hadrons are treated in mean field approach. We call this model relativistic mean field hadron resonance gas model (RMFHRG). The RMFHRG differs from the Walecka type mean field models in the sense that in former the repulsive mean field interactions are present even at zero baryon density unlike the later case. We will also estimate the shear and bulk viscosity coefficient of hadronic matter within ambit of RMFHRG.

We organize the paper as follows. In Sec. II we compute the pressure and other bulk thermodynamical quantities for the interacting hadron resonance gas with a repulsive mean field interaction. In sec III, we shall discuss the results for the thermodynamics and confront the same with the lattice simulation results both at zero and finite chemical potential. We shall then estimate the viscosity coefficients for hot and dense hadronic matter within ambit of HRG model with mean field interactions. Finally, in sec IV, we summarize the findings of the present investigation and give a possible outlook.

II. HADRON RESONANCE GAS MODEL WITH A REPULSIVE MEAN FIELD POTENTIAL

Thermodynamical properties of hadron resonance gas model can be deduced from the grand canonical partition function defined as

\[ Z(V,T,\mu) = \int dm [\rho_b(m) \ln Z_b(m,V,T,\mu) + \rho_f(m) \ln Z_f(m,V,T,\mu)] \]  

(1)

where \( \rho_b \) and \( \rho_f \) are the mass spectrum of the bosons and fermions respectively. We assume that the hadron mass spectrum is given by

\[ \rho(m) = \sum_a^\Lambda g_a \delta(m - m_a) \theta(\Lambda - m) \]  

(2)

where \( g_a \) is the degeneracy and \( m_a \) is the mass of \( a \)th hadronic species. This discrete mass spectrum consists of all the experimentally known hadrons with cut-off \( \Lambda \). One can set different cut-off values for baryons and mesons.

The short range repulsive interaction among hadrons can be treated in mean field approach where the single particle energy is written as\[34, 35]\n
\[ \varepsilon_a = \sqrt{p^2 + m_a^2} + U(n) \]  

(3)

where \( n \) is the total hadron number density and the potential energy \( U \) represents repulsive interaction. For any arbitrary inter hadronic potential \( V(r) \), the potential energy, after integrating space co-ordinates, is given by

\[ U(n) = Kn \]  

(4)

where \( K \) is a constant which depends on the form of inter hadron potential.

The total hadron number density is

\[ n(T,\mu) = \sum_a n_a \]  

(5)
where \( n_a \) is the number density of \( a \)th hadronic species given by

\[
    n_a = \int d\Gamma_a \frac{1}{e^{\left(\frac{\varepsilon_a - \mu}{T}\right)} \pm 1}
\]

where \( d\Gamma_a \equiv \frac{g_a d^3p}{(2\pi)^3} \) and \( \mu \) is the baryon chemical potential and \( +(-) \) sign corresponds to baryons (mesons). In the Boltzmann approximation momentum integration can be readily done and one can obtain much simpler expression for the number density as

\[
    n_a = \frac{g_a}{2\pi^2} m_a^2 T K_2 \left( \frac{m_a}{T} \right) e^{-\frac{K_n}{T}}
\]

(7)

The total hadron number density is then

\[
    n(T, \mu) = \sum_a \frac{g_a}{2\pi^2} m_a^2 T K_2 \left( \frac{m_a}{T} \right) e^{-\frac{K_n}{T}}
\]

(8)

Eq. (8) is the self consistent equation for number density which can be solved numerically. The total hadron energy density is

\[
    \epsilon(T, \mu) = \sum_a \int d\Gamma_a \frac{\varepsilon_a}{e^{\left(\frac{\varepsilon_a - \mu}{T}\right)} \pm 1} + \phi(n)
\]

(9)

where \( \phi(n) \) represents the correction to the energy density in order to avoid double counting the potential. It can be determined using the condition that

\[
    \varepsilon_a = \frac{\partial \epsilon}{\partial n_a}
\]

(10)

Taking the derivative of Eq. (9) we get

\[
    \frac{\partial \phi}{\partial n} = -\sum_a \int d\Gamma_a \frac{\partial \varepsilon_a}{\partial n_a} e^{\left(\frac{\varepsilon_a - \mu}{T}\right)} \pm 1
\]

(11)

Using Eq. (4) we get

\[
    \frac{\partial \phi}{\partial n} = -K n
\]

(12)

and hence

\[
    \phi(n) = -\frac{1}{2} K n^2
\]

(13)

Pressure of the hadron gas can now be readily written as

\[
    P(T, \mu) = \mp T \sum_a \int d\Gamma_a \ln(1 \mp e^{\left(\frac{\varepsilon_a - \mu}{T}\right)}) - \phi(n)
\]

(14)

It is worth noting that the effective interaction model we are considering is different from the relativistic Lagrangian model. In later case the repulsive mean fields are present only at non zero baryon density, while in former case the repulsive interactions are present even at zero baryon density.
FIG. 1: Scaled pressure (left panel) and the interaction measure (right panel) in RMFHRG and ideal HRG. The lattice data is taken from Ref. [8].

III. RESULTS AND DISCUSSION

In the hadron resonance gas model it is customary to include all the hadrons and resonances up to certain cut-off $\Lambda$. We include all the mesons and baryons up to $\Lambda = 2.25$ GeV listed in the Ref. [104]. We choose the repulsive mean field parameter for baryons $K_B = 450$ MeV·fm$^3$ [36]. For mesons we choose smaller value $K_M = 50$ MeV·fm$^3$.

Fig. 1 shows the scaled pressure and the interaction measure estimated within ambit of RMFHRG (blue solid curve). The dashed curve corresponds to ideal HRG results while the marker circles with error bars correspond to lattice QCD simulations results [8]. We note that the effect of including the repulsive mean field interaction is to suppress the thermodynamical quantities compare to their ideal HRG estimation counterpart (dashed magenta curve). While the HRG pressure (Fig. 2(a)) starts to deviate from the lattice results at $T \sim 160$ MeV, the RMFHRG estimations agrees with the lattice results all the way up to 190 MeV. It won’t be reasonable to push the HRG model results above the QCD transition temperature ($T_c$) which LQCD estimates to lie in the range 155 – 160MeV [7]. The reason for this is twofold. First, the HRG approximation of the hadronic matter might break down at high density near and above $T_c$. Second, the hadrons won’t exits above $T_c$. But recent study [29] shows that the hadrons do not melt quickly as one would expect on the basis of ideal HRG model. In this study the authors has analyzed the possible improvement of ideal hadron resonance gas model by including the repulsive interactions between baryons. If one includes the attractive and repulsive interactions between the baryons through van der Waals parameters, while keeping the meson gas ideal, the pressure of the hadron gas agrees with the LQCD data all the way above $T_c$. We may similarly conclude that the inclusion of repulsive mean fields might push the validity of HRG model well above $T_c$. Nonetheless, we do not have any other strong reason to believe so except for the apparent agreement with the LQCD results.
Unlike pressure the interaction measure is somewhat below LQCD results above $T = 150$ MeV. It is quite established fact that the so called heavy Hagedorn states contribute significantly to the energy density which are missing in our model. The rapid rise of energy density can be explained by extending ideal HRG model by including continuum Hagedorn states along with the discrete states above cut-off $\Lambda$ in the density of states\[^77\].

Fig. 2 shows the scaled pressure and interaction measure at finite baryon chemical potentials estimated within ambit of RMFHRG. We note that the RMFHRG is in reasonable agreement with LQCD results even at finite baryon density. Further, the interaction measure is in better agreement with the LQCD results at finite density than at $\mu = 0$ case. However, while making this observation, we have to keep in mind that the lattice data of Ref.[8] is estimated at order $\mu^2$. Fig.3 shows the adiabatic speed of sound at finite baryon density. The RMFHRG estimations of $C_s^2$ is within error bars of LQCD results. Further, the $C_s^2$ has minimum at $T = 155$ MeV for $\mu = 0$ and at $T = 140$ MeV for $\mu = 300$ MeV which is in very close agreement with the LQCD results.

The coefficients of shear and bulk viscosities can be extracted from relativistic Boltzmann equation. In the relaxation time approximation of the Boltzmann equation the shear ($\eta$) and bulk viscosity ($\zeta$) coefficients for the hadronic matter are given by\[^{49, 50}\]

$$\eta = \frac{1}{15T} \sum_a \int \frac{d^3p}{(2\pi)^3} \frac{p^4}{E_a^2} \{\tau_a f_a^0 + \bar{\tau}_a \bar{f}_a^0\}$$ (15)

$$\zeta = \frac{1}{T} \sum_a \int \frac{d^3p}{(2\pi)^3} \left\{\tau_a f_a^0 \left[ E_a C_{n_B}^2 + \left( \frac{\partial P}{\partial n_B} \right) \varepsilon - \frac{p^2}{3E_a} \right]^2 \right\}$$

$$+ \bar{\tau}_a \bar{f}_a^0 \left[ E_a C_{n_B}^2 - \left( \frac{\partial P}{\partial n_B} \right) \varepsilon - \frac{p^2}{3E_a} \right]^2$$ (16)
where \( f^0 \) is the equilibrium distribution function and \( C_{nB}^2 \) is the speed of sound at constant number density. Further, in Eqs. (16) and (15), \( \tau_a \) is the relaxation time for \( a^{th} \) hadronic particle species while the barred quantities corresponds to that of anti-particles. In this work we shall use thermally averaged relaxation time which for a given species \( 'a' \) is given by

\[
\tau_a^{-1} = \sum_b n_b \langle \sigma_{ab} v_{ab} \rangle. \tag{17}
\]

In the above, the sum is over all particles (b) other than the particle \( 'a' \) with which the scattering takes place; \( \sigma_{ab} \) is the total scattering cross section for the process \( a(p_a) + b(p_b) \rightarrow c(p_c) + d(p_d) \) and \( v_{ab} \) is the relativistic relative velocity given by

\[
v_{ab} = \frac{\sqrt{(p_a \cdot p_b)^2 - m_a^2 m_b^2}}{E_a E_b} \tag{18}
\]

Further, \( n_b \) is the number density for particle species \( 'b' \) given, with \( 'g_b' \) as the corresponding degeneracy factor, as

\[
n_b = \frac{g_b}{(2\pi)^3} \int d^3 p f_b(p) \approx \frac{g_b T^3}{(2\pi)^2} (\beta m)^2 K_2 (\beta m) \sinh(\beta \mu) \tag{19}
\]

where, the last step is written down in the Boltzmann approximation with \( K_2(x) \) being the modified Bessel function of second order. Finally, the thermal average cross section \( \langle \sigma_{ab} v_{ab} \rangle \)
is given as
\[ \langle \sigma_{ab}v_{ab} \rangle = \frac{\int d^3p_a d^3p_b \sigma_{ab}v_{ab} f_a(p_a)f_b(p_b)}{\int d^3p_a d^3p_b f_a(p_a)f_b(p_b)}. \] (20)

In case of relativistic mean field model, the parameter \( K \) as in Eq. (4), can be related to the scattering cross section as follows. The potential energy in Eq. (3) due to inter-particle interaction (potential) \( V(r) \) is given by
\[ U(n) = n \int d^3r V(r) = Kn \] (21)

In Born approximation, the scattering amplitude \( f(\theta, \phi) \) for a particle with mass \( m \) that encounters a scattering potential \( V(r) \) is given by \[ f(\theta, \phi) = -\frac{m}{2\pi} \int d^3r V(r) = -\frac{mK}{(2\pi)} \] (22)

and thus the cross section is given by
\[ \sigma = 4\pi \left( \frac{mK}{2\pi} \right)^2 \] (23)

Then the thermal averaged cross section can be written as \[ \langle \sigma_{ab}v_{ab} \rangle = \frac{\sigma}{8m_am_b^2K_2(\beta m_a)K_2(\beta m_b)} \int_{(m_a+m_b)^2}^{\infty} dS \frac{[S-(m_a-m_b)^2]}{\sqrt{S}} \frac{[S-(m_a+m_b)^2]}{\sqrt{S}} K_1(\beta \sqrt{S}) \] (24)

where \( \sqrt{S} \) is the centre-of-mass energy. It may be relevant here to mention that while the cross section is independent of temperature and chemical potential, the thermal averaged cross section \( \langle \sigma v \rangle \) in general depends upon temperature and chemical potential. However, in the Boltzmann approximation \( \langle \sigma v \rangle \) is independent of \( \mu \). After evaluating the thermal averaged relaxation time for each species, we estimate the viscosity coefficients using Eqs. (15) and (16).

Fig. 4 shows the ratio of shear viscosity to entropy density as a function of temperature. We have compared the ratio \( \eta/s \) estimated within ambit of RMFHRG with various other model calculations [48, 57, 75, 94, 95]. Red dashed curve corresponds to Chapman-Enscog method with constant cross sections [57]. Dashed green curve corresponds to relativistic Boltzmann equation in relaxation time approximation. The thermodynamic quantities in this model has been estimated using scaled hadron masses and coupling (SHMC) model [95]. Brown dashed curve corresponds to estimations made using relativistic Boltzmann equation in RTA. The thermodynamic quantities are estimated within excluded volume hadron resonance gas model (EHRG) [94]. Dot-dashed orchid curve corresponds to the \( \eta/s \) of meson gas estimated using chiral perturbation theory [75]. While the ratio \( \eta/s \) in our model is relatively large at low temperature as compared to other models it rapidly falls and approaches closer to the Kovtun-Son-Starinets (KSS) bound at \( T \sim 170 \text{ MeV} \).

Fig. 5 shows the ratio of bulk viscosity to entropy density as a function of temperature. In Fig. 5(a) blue solid curve corresponds to RMFHRG compared with that of EHRG model (dashed magenta curve) [94] and SHMC model [95]. Note that the ratio \( \zeta/s \) is smaller when the repulsive interactions are treated in a mean field way. In Fig. 5(a) shows the ratio \( \zeta/s \) at
finite baryon chemical potential. At low temperature the ratio is larger at finite $\mu$ it drops below that of $\mu = 0$ case at high temperature. This observation may be attributed to the fact that the entropy density rises much faster than that of $\zeta$ itself at finite baryon density as compared to that of zero baryon density case.

In the context of heavy nucleon-nucleon (NN) collision experiments viscosity coefficients can be estimated along freeze-out curve by finding the beam energy ($\sqrt{S_{NN}}$) dependence of the temperature and chemical potential. This is extracted from a statistical thermal model description of the particle yield at various $\sqrt{S_{NN}}$ [108–110]. We use parametrization of freeze out curve $T(\mu_B)$ given in Ref.[109] as

$$T(\sqrt{S_{NN}}) = c_+(T_{10} + T_{20}\sqrt{S_{NN}}) + c_-(T_{0}^{\text{lim}} + \frac{T_{30}}{\sqrt{S_{NN}}})$$  \hspace{1cm} (25)

$$\mu(\sqrt{S_{NN}}) = \frac{a_0}{1 + b_0\sqrt{S_{NN}}}$$  \hspace{1cm} (26)

where $T_{10} = -34.4$ MeV, $T_{20} = 30.9$ MeV/GeV, $T_{30} = -176.8$ GeV MeV, $T_{0}^{\text{lim}} = 161.5$ MeV, $a_0 = 1481.6$ MeV and $b_0 = 0.365$ GeV$^{-1}$. The functions $c_+$ and $c_-$ smoothly connects the different behaviors of centre-of-mass energies.

Fig.(6) shows viscosity coefficients, $\eta/s$ and $\zeta/s$ along the freeze-out curve. It can be noted that the fluidity measures rapidly falls at low $\sqrt{S}$ and then it remains almost constant at higher $\sqrt{S}$ values. This indicates that the matter produced in heavy-ion collision experiments with wide range of collision energies can exhibit substantial elliptic flow.
FIG. 5: Left panel shows bulk viscosity to entropy density ratio estimated within RMFHRG and compared with other model estimations. Right panel shows $\zeta/s$ at two different baryon chemical potentials.

FIG. 6: Viscosity coefficients along the freeze-out curve. The freeze-out parametrization is taken from Ref.[109].

IV. SUMMARY

In this paper we confronted the interacting hadron resonance gas model (RMFHRG) with LQCD at zero as well as finite density. The repulsive interaction between hadrons are treated in the mean field approach. The thermodynamic quantities estimated within RMFHRG are
found to be in reasonable agreement with LQCD at zero as well as finite chemical potential. The agreement of interaction measure $\epsilon - 3P/T^4$ estimated within RMFHRG is rather poor above $T = 145$ MeV. In fact the interaction measure rise very rapidly near $T_c \sim 156$ MeV. This rapid rise of energy density can be explained by extending ideal HRG model by including continuum Hagedorn states along with the discrete states. We used this RMFHRG equation of state to estimate the shear and bulk viscosity coefficients of hadronic matter. We found the reasonable agreement of both the viscosity coefficients with previous results. The shear viscosity to entropy density ratio $\eta/s$ estimated within RMFHRG is large at low temperature as compared to other calculations. This behavior is due to smaller cross section of mesons in our model. But $\eta/s$ estimated in our calculation rapidly drops at high temperature and approach KSS bound at $T \sim 170$ MeV. We further found that $\eta/s$ at finite chemical is smaller in magnitude as compared to that of zero chemical potential but the overall behavior as a function of temperature do not change. We also found the reasonable agreement of the ratio $\zeta/s$ with previous results. Finally, we have estimated viscosity coefficients along the free-out line. We found that both the ratios, $\eta/s$ and $\zeta/s$, attains constant value at high $\sqrt{S}$ values. This indicates that the matter produced in heavy-ion collision experiments with wide range of collision energies can exhibit substantial elliptic flow.

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