Extremum Seeking-based Iterative Learning Linear MPC

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Abstract—In this work we study the problem of adaptive MPC for linear time-invariant uncertain models. We assume linear models with parametric uncertainties, and propose an iterative multi-variable extremum seeking (MES)-based learning MPC algorithm to learn online the uncertain parameters and update the MPC model. We show the effectiveness of this algorithm on a DC servo motor control example.

I. INTRODUCTION

Model predictive control (MPC) [1] is a model-based framework for optimal control of constrained multi-variable systems. MPC is based on the repeated, receding horizon solution of a finite-time optimal control problem formulated from the system dynamics, constraints on system states, inputs, outputs, and a cost function describing the control objective. MPC has been applied to several applications such as aerospace [2], [3], automotive [4], [5], and mechatronic systems [6], [7]. Since MPC is a model-based controller, its performance inevitably depends on the quality of the prediction model used in the optimal control computation.

In contrast, extremum seeking (ES) control is a well known approach where the extremum of a cost function associated with a given process performance (under some conditions) is found without the need for detailed modeling information, see, e.g., [8]–[10]. Several ES algorithms (and associated stability analyses) have been proposed, [8]–[12], [12]–[14], and many applications of ES have been reported [15]–[19].

The idea that we introduce in this work is that the performance of a model-based MPC controller can be combined with the robustness of an ES-based learning algorithm for simultaneous identification and control of linear time-invariant systems with structural uncertainties. While regulation and identification are seemingly conflicting objectives, by identifying (or re-identifying) the system dynamics online and updating the MPC prediction model, the closed-loop performance may be enhanced relative to a standard standard MPC scheme that uses an inaccurate (or outdated) model. The optimal solution to this trade-off between identification and control is given by a dynamic program [20], which, for many applications, is computationally intractable. As a result, many suboptimal techniques and heuristics have been developed in recent years—often in a receding horizon framework which is suitable for integration with MPC.

In [21], an approximation of the dynamic program is developed for a linear input-output map with no dynamics. Approaches for more complex systems avoid dynamic programming altogether and, instead, sub-optimally trade off between inputs that excite the system and inputs that regulate the state. Excitation signals are usually in the frequency domain, where maximizing the Fisher information matrix can be cast as a semi-definite program [23]. However, design in the frequency domain leads to difficulties with constraints that are more naturally addressed in the time domain, e.g., input (and possibly output) amplitude constraints. While the problem formulation in the time domain is highly non-convex, developing such techniques is desirable, and thus the focus of recent work [24]–[32].

In this preliminary work, we aim at proposing an alternative approach to realize an iterative learning-based adaptive MPC. We introduce an approach for a multi-variable extremum seeking (MES)-based iterative learning MPC that merges a model-based linear MPC algorithm with a model-free MES algorithm to realize an iterative learning MPC that adjusts to structured model uncertainties. This approach is an extension to the recent MPC framework reported in [33], where the author proposed to use MES with model-based nonlinear control to design learning-based adaptive controllers for a class of nonlinear systems.

The paper is organized as follows. We start the paper with some preliminaries in Section II. In Section III we first recall the nominal MPC algorithm and then present the main result of the paper, namely, the iterative learning MPC, with a discussion of the algorithm stability. Section IV is dedicated to a DC servo-motor case study. Finally, we conclude the paper with a brief summary of the results in Section V.

II. NOTATIONS AND PRELIMINARIES

Throughout the paper, $\mathbb{R}$, $\mathbb{Z}_{0+}$, $\mathbb{Z}_{[i,j]}$ denote the set of real numbers, positive integers, and positive integers from $i$ to $j$, respectively. For $x \in \mathbb{R}^N$ we define $||x|| = \sqrt{x^T x}$, we denote by $A_{i,j}$, $i = 1, \ldots, n$, $j = 1, \ldots, m$ the elements of the matrix $A$, and by $[A]_i$ the vector equal to the line $i$ of the matrix $A$. We denote by $||A||_2$ the spectral norm matrix, and by $||x||_A = \sqrt{x^T A x}$. We denote by $x(i[k])$ the value of $x$ at the sample time $i$ and the MPC cycle $k$. In the sequel, when we use the term well-posed optimization problem, we mean that the problem admits a unique solution, which is a continuous function of the initial conditions [34].

III. ITERATIVE LEARNING-BASED ADAPTIVE MPC

A. Control objective

We want to design an adaptive controller that solves regulation and tracking problems for linear time-invariant systems with structural model uncertainties under state, input, and output constraints. In what follows, we first present the nominal MPC problem, i.e., without model uncertainties, and then extend this nominal controller to its adaptive form by merging it with an MES algorithm.

B. Constrained linear nominal MPC

Consider a linear MPC, based on the nominal linear prediction model

\begin{align}
  x(k+1) & = Ax(k) + Bu(k), \\
  y(k) & = Cx(k) + Du(k),
\end{align}

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where \( x \in \mathbb{R}^n \), \( u \in \mathbb{R}^m \), \( y \in \mathbb{R}^p \) are the state, input, and output vectors subject to constraints
\[
\begin{align*}
  x_{\text{min}} &\leq x(k) \leq x_{\text{max}}, \\
  y_{\text{min}} &\leq u(k) \leq y_{\text{max}}, \\
  y_{\text{min}} &\leq y(k) \leq y_{\text{max}},
\end{align*}
\]
with the following assumptions.

C. Learning-based adaptive MPC algorithm

At time \( k \), \( x \in \mathbb{R}^n \), \( u \in \mathbb{R}^m \), \( y \in \mathbb{R}^p \) are the lower and upper bounds on the state, input, and output vectors, respectively. At every control cycle \( k \in \mathbb{Z}_+ \), MPC solves the finite horizon optimal control problem
\[
\min_{U(k)} \sum_{i=0}^{N-1} \| x(i|k) \|^2_{Q_2} + \| u(i|k) \|^2_{R_2}, \\
\text{s.t.} \quad x(i+1|k) = A x(i|k) + B u(i|k), \\
\quad y(i|k) = C x(i|k) + D u(i|k), \\
\quad x_{\text{min}} \leq x(i|k) \leq x_{\text{max}}, i \in \mathbb{Z}_+, \\
\quad u_{\text{min}} \leq u(i|k) \leq u_{\text{max}}, i \in \mathbb{Z}_+, \\
\quad y_{\text{min}} \leq y(i|k) \leq y_{\text{max}}, i \in \mathbb{Z}_+, \\
\quad u(0|k) = k x(0|k) = x(k),
\]
where \( Q_2 \geq 0 \), \( P_2 \geq 0 \), \( R_2 > 0 \) are symmetric weight matrices of appropriate dimensions, \( N \) is the prediction horizon, \( N_o \leq N \) is the control horizon (the number of free control moves), \( N_o \leq N, N_e \leq N - 1 \) are the input and output constraint horizons along which the constraints are enforced. The performance criterion is defined by (3a), and (3d)–(3f) enforce the constraints. Equation (3g) defines the pre-assigned terminal controller where \( K_f \in \mathbb{R}^m \), \( K_f \), so that the optimization effectively is \( U(k) = u(0|k) \).

Although the optimal control problem (3) does not explicitly mention a reference, tracking is achieved by including in the state update equation (1a) the reference prediction dynamics
\[
r_r(k+1) = A_r r_r(k),
\]
and an additional output in (1b) representing the tracking error
\[
y_u(k) = C x(k) - C_r r_r(k),
\]
which is then accounted for in the cost function (3a) as later shown in the example (see also [5] for an example in a real world application). At time \( k \), the MPC problem (3) is initialized with the current state value \( x(k) \) by (3h) and solved to obtain the optimal sequence \( U^*(k) \). Then, the input \( u(k) = u_{\text{MPC}}(k) = u^*(0|k) = [I_m \quad 0 \ldots 0] U(k) \) is applied to the system.

C. Learning-based adaptive MPC algorithm

Consider now, the system (1), with structural uncertainties, such that
\[
x(k+1) = (A + \hat{\Delta} A) x(k) + (B + \hat{\Delta} B) u(k), \\
y(k) = (C + \hat{\Delta} C) x(k) + (D + \hat{\Delta} D) u(k),
\]
with the following assumptions.

Assumption 1: The constant uncertainty matrices \( \Delta A, \Delta B, \Delta C \) and \( \Delta D \), are bounded, s.t. \( \| \Delta A \|_2 \leq l_A \), \( \| \Delta B \|_2 \leq l_B \), \( \| \Delta C \|_2 \leq l_C \), \( \| \Delta D \|_2 \leq l_D \), with \( l_A, l_B, l_C, l_D > 0 \).

Assumption 2: There exists non empty convex sets \( K_A \subset \mathbb{R}^{n \times n} \), \( K_B \subset \mathbb{R}^{n \times m} \), \( K_C \subset \mathbb{R}^{m \times n} \), and \( K_D \subset \mathbb{R}^{p \times m} \), such that \( A + \Delta A \in K_A \) for all \( \Delta A \) such that \( \| \Delta A \|_2 \leq l_A \), \( B + \Delta B \in K_B \) for all \( \Delta B \) such that \( \| \Delta B \|_2 \leq l_B \), \( C + \Delta C \in K_C \) for all \( \Delta C \) such that \( \| \Delta C \|_2 \leq l_C \), \( D + \Delta D \in K_D \) for all \( \Delta D \) such that \( \| \Delta D \|_2 \leq l_D \).

Assumption 3: The iterative learning MPC problem (4) (and the associated reference tracking extension), where we substitute the model with structural uncertainty (6) for the nominal model (4) and (5), is a well-posed optimization problem for any matrices \( A + \Delta A \in K_A, B + \Delta B \in K_B, C + \Delta C \in K_C, D + \Delta D \in K_D \).

Under these assumptions, we postulate the following: If we solve the iterative learning MPC problem (4), where we substitute (6) for (4) and (5), iteratively, such that, at each new iteration we update our knowledge of the uncertain matrices \( \Delta A, \Delta B, \Delta C, \) and \( \Delta D \), using a model-free learning algorithm, in our case the extremum seeking algorithm, we claim that, if we can improve over the iterations the MPC model, i.e., learn over iterations the uncertainties, then we can improve over time the MPC performance, i.e., either in the stabilization or in the tracking. Before formulating this idea in terms of an algorithm, we briefly recall the principle of model-free multi-variable extremum seeking (MES) control.

To use the MES learning algorithm, we define the cost function to be minimized as
\[
Q(\hat{\Delta}) = F(y_u(\hat{\Delta})),
\]
where \( \hat{\Delta} \) is the vector obtained by concatenating all the elements of the estimated uncertainty matrices \( \hat{\Delta} A, \hat{\Delta} B, \hat{\Delta} C, \) and \( \hat{\Delta} D \), \( F : \mathbb{R}^2 \rightarrow \mathbb{R}, F(0) = 0, F(y) > 0 \) for \( y \neq 0 \).

In order to ensure convergence of the MES algorithm, \( Q \) need to satisfy the following assumptions.

Assumption 4: The cost function \( Q \) has a local minimum at \( \Delta^* = \Delta \).

Assumption 5: The original parameter estimate vector \( \hat{\Delta} \) is close enough to the actual parameters vector \( \Delta^* \).

Assumption 6: The cost function is analytic and its variation with respect to the uncertain variables is bounded in the neighborhood of \( \Delta^* \), i.e., there exists \( \xi_2 > 0 \), s.t. \( \frac{\partial Q}{\partial \Delta} (\Delta^*) \leq \xi_2 \) for all \( \Delta \in \mathcal{V}(\Delta^*) \), where \( \mathcal{V}(\Delta^*) \) denotes a compact neighborhood of \( \Delta^* \).

Remark 1: Assumption 4 simply means that we can consider that \( \Delta^* \) has at least a local minimum at the true values of the uncertain parameters.

Remark 2: Assumption 5 indicates that our result will be of local nature, meaning that our analysis holds in a small neighborhood of the actual values of the parameters.

Remark 3: We wrote the cost function (7) as function of the tracking error (5), however, the case of regulation or stabilization can be directly deduced from this formulation by replacing the time-varying reference with a constant reference or an equilibrium point.

Under Assumptions 2, 4, and 6 it has been shown (e.g. [9], [10]), that the MES
\[
\begin{align*}
  \dot{z}_i &= a_i \sin(\omega_i t + \frac{i}{\omega}) Q(\Delta), \\
  \Delta_i &= z_i + a_i \sin(\omega_i t + \frac{i}{\omega}), \quad i \in \{1, \ldots, N_p\}
\end{align*}
\]
with \( N_p \leq m + n + m + p + mn \) is the number of uncertain elements, \( \omega_i \neq \omega_j, \omega_i + \omega_j \neq \omega_k, i, j, k \in \{1, \ldots, N_p\} \), and \( \omega_i > \omega^*, \forall i \in \{1, \ldots, N_p\} \), with \( \omega^* \) large enough, converges to the local minima of \( Q \).

The idea that we want to propose here (refer to [33] where we introduced this concept of learning-based adaptive control for a class of nonlinear systems), is that under Assumptions 1-6, we can merge the MPC algorithm and a discrete-time version of the MES algorithm to obtain an iterative learning MPC algorithm. We formalize this idea in the following iterative algorithm:
ALGORITHM I
- Initialize $z_i(0) = 0$, and the uncertainties’ vector estimate $\hat{\Delta}(0) = 0$.
- Choose a threshold for the cost function minimization $\epsilon_Q > 0$.
- Choose the parameters, MPC sampling time $\Delta T_{mPC} > 0$, and MES sampling time $\Delta T_{mes} = N_E \Delta T_{mPC}$, $N_E > 0$.
- Choose the MES dither signals’ amplitudes and frequencies: $\alpha_i, \omega_i, i = 1, 2, \ldots, N_p$.

WHILE($\elltrue$)
  FOR($\ell = 1, \ell \leq N_E, \ell = \ell + 1$)
    - Solve the MPC problem
      \[
      \begin{align*}
      &\min_{U(k)} \sum_{i=0}^{N-1} \|x(i/k)\|^2_{Q_{zt}} + \|u(i/k)\|^2_{R_{zt}} \quad (9a) \\
      \text{s.t.} \quad &x(i+1/k) = (A + \Delta A)x(i/k) + (B + \Delta B)u(i/k), \quad (9b) \\
      &y(i/k) = (C + \Delta C)x(i/k), \quad (9c) \\
      &x_{\min} \leq x(i/k) \leq x_{\max}, \quad i \in \{1, N_c\}, \quad (9d) \\
      &u_{\min} \leq u(i/k) \leq u_{\max}, \quad i \in \{0, N_u - 1\}, \quad (9e) \\
      &y_{\min} \leq y(i/k) \leq y_{\max}, \quad i \in \{1, N_c\}, \quad (9f) \\
      &u(i/k) = K_f x(i/k), \quad i \in \{0, N_u - 1\}, \quad (9g) \\
      &x(0/k) = x(k), \quad (9h)
      \end{align*}
      \]
    - Update $k = k + 1$.
  END
  IF $Q > \epsilon_Q$
    - Evaluate the MES cost function $Q(\hat{\Delta})$.
    - Evaluate the new value of the uncertainties $\hat{\Delta}$:
      \[
      z_i(h + 1) = z_i(h) + \alpha_i \Delta T_{mes} \sin(\omega_i h \Delta T_{mes} + \hat{\Delta}) Q(\hat{\Delta}) \quad (10a)
      \]
      \[
      \hat{\Delta}(h + 1) = \hat{\Delta}(h + 1) + \alpha_i \sin(\omega_i h \Delta T_{mes} - \hat{\Delta}), \quad i \in \{1, \ldots, N_p\}
      \]
    - Update $h = h + 1$.
  END
  Reset $\ell = 0$
END

D. Stability discussion

As mentioned, in this preliminary work we aim at presenting an algorithm that merges model-based MPC and model-free MES learning, to obtain an iterative learning MPC algorithm, with encouraging numerical results (see, e.g., the case study in Section IV). A rigorous stability analysis of the combined algorithm is out of the scope of this work, but we want to sketch below an approach to analyze the stability of Algorithm I. We propose here to follow the analysis presented in [33], for the case of learning-based adaptive control for some class of nonlinear systems. By Assumptions 1-3, the model structural uncertainties $\Delta A, \Delta B, \Delta C$, and $\Delta D$ are bounded, the uncertain model matrices $A + \Delta A, B + \Delta B, C + \Delta C$, and $D + \Delta D$ are elements of convex sets $K_a, K_b, K_c,$ and $K_d$, and that the MPC problem (8) is well-posed. Based on this, the approach for proving stability is based on establishing a boundedness of the tracking error norm $\|\hat{y}_e\|$ with the upper-bound being function of the uncertainties estimation error norm $\|\hat{\Delta} - \Delta\|$. One effective way to characterize such a bound is to use an integral Input-to-State Stability (iISS) or ISS for time-invariant problems) between the input $\|\hat{\Delta} - \Delta\|$ and the augmented state $\|\hat{y}_e\|$, see, e.g. [33]. If the iISS (or ISS) property is obtained, by reducing the estimation error $\|\hat{\Delta} - \Delta\|$ we also reduce the the tracking error $\|\hat{y}_e\|$, due to the iISS relation between the two signals. Based on Assumptions 5, 6, and 7, we know, e.g. [9], [10], that the MES algorithm (10) converges to a local minimum of the MES cost $Q$, which implies (based on Assumption 4), that the estimation error $\|\hat{\Delta} - \Delta\|$ is reducing over the MES iterations. Thus, finally we conclude that the MPC tracking (or regulation) performance is improved over the iterations of Algorithm I. While obviously this discussion is not a rigorous proof of stability of the iterative learning MPC, it provides an interesting guideline to follow for analyzing the controller stability observed during the test case presented next. A more rigorous proof is currently under development and will be presented in future works.

IV. DC SERVO-MOTOR EXAMPLE

The example studied here is about the angular position control of a load connected by a flexible shaft to a voltage actuated DC servo motor, see [35] and references therein. The states are the load angle and angular rate, and the motor angle and angular rate, the control input is the motor voltage, and the outputs are the load angle and the torque acting on the flexible shaft. The model for the system is

\[
\dot{x}(t) = \begin{bmatrix}
0 & 1 & 0 & 0 \\
-\frac{k_l}{J_m} - \frac{k_g}{J_m} & -\frac{G_m}{J_m} & 0 & 0 \\
0 & 0 & 1 & 0 \\
-\frac{K_m}{R_{A,m}} & 0 & 0 & 0
\end{bmatrix} x_c(t) + \begin{bmatrix}
0 \\
0 \\
0 \\
\frac{K_m}{R_{A,m}}
\end{bmatrix} u_c(t)
\]

\[
y_c(t) = \begin{bmatrix}
1 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 \\
-\frac{k_l}{J_m} & 0 & 0 & 0
\end{bmatrix} x_c(t)
\]

where $x_c \in \mathbb{R}^4$ is the state vector, $u_c \in \mathbb{R}$ is the input vector, and $y_c \in \mathbb{R}^2$ is the output vector. In $R_{A}[9]$ is the armature resistance, $K_m[\text{Nm/A}]$ is the motor constant, $J_m[\text{kgm}^2]$, $\beta_m[\text{Nms/rad}]$, $k_l[\text{Nm/rad}]$, $k_g[\text{Nm/rad}]$, are the inertia, friction and stiffness of load and flexible shaft, $J_m[\text{kgm}^2]$, $\beta_m[\text{Nms/rad}]$, are the inertia and friction of the motor, and $g$ is the gear ratio between motor and load. The nominal numerical values used in the simulations are $R_A = 105\Omega$, $K_m = 100\text{Nm/A}$, $J_m = 25\text{kgm}^2$, $\beta_m = 25\text{Nms/rad}$, $k_l = 1.28 \cdot 10^4\text{Nms/rad}$, $J_m = 0.5\text{kgm}^2$, $\beta_m = 0.1\text{Nms/rad}$. The system is subject to constraints on motor voltage and shaft torque

\[
-78.5 \leq \|y_c(t)\|_2 \leq 78.5, \quad (12a)
\]
\[-220 \leq u_c(t) \leq 220. \quad (12b)
\]

The control objective is to track a time varying load angle position reference signal $r_c(t)$. Following the nominal MPC presented in Section [III-B] the prediction model is obtained by sampling the system with a period $\Delta T_{mPC} = 0.1s$, and the trajectory tracking problem is solved by augmenting the model with the reference prediction model [4], which in this case represents a constant reference

\[
r_c(k + 1) = r_c(k),
\]

where $r_c \in \mathbb{R}$, and with an incremental formulation of the control input, such that

\[
u_c(k + 1) = u_c(k) + \Delta v_c(k)
\]
the shaft torque, is oscillating and is violating its upper and lower
violation of the softened constraints.

Thus, the MPC problem results in a family of quadratic
in (3g). In this case study output constraints (12a) are
violation of the softened constraints.

Next, the MPC cost function is chosen as
\[ Q = \sum_{i=0}^{N-1} \left[ \| y_i(t - i\delta T_{mpc}) \|_2^2 + \| y_{i\delta T_{mpc}} \|_2^2 \right], \]
where \( Q \) is the normalized threshold
\[ Q = 10^3 \] and \( R_u = 0.05 \), prediction, constraints, and
control horizons are \( N = 20 \), \( N_c = N_u = N_w = 4 \), and
\( K_f = 0 \) in (12). In this case study output constraints are
considered as soft constraints, which may be (briefly) violated due
to the prediction model not being equal to the actual model. Thus, in
we add the term \( \rho \sigma^2 \), where \( \rho > 0 \) is a (large) cost weight, and
\( \sigma \) is an additional variable used to model the (maximum) constraint
violation of the softened constraints.

Thus, the MPC problem results in a family of quadratic
programs parameterized by the current state \( \dot{x}(k) \) in (13), with
\( n_x = 16 \) constraints, and \( n_u = 4 \) variables. In the subsequent
simulations, we consider initial state \( x(0) = [0 \ 0 \ 0 \ 0]^T \) and reference
\( r_i(t) = 4.5 \sin(\omega t) \), \( T_{ref} = 20 \text{sec} \).

First, to have a base-line performance, we solve the nominal
MPC problem, i.e. without model uncertainties. We report the
the corresponding results on Figure where it is clear that the desired
load angular position is precisely tracked, without violating the shaft
torque and the control voltage constraints. Next, we introduce the
parametric model uncertainty \( \delta z_\beta = -70 \text{ [Nm/rad]} \). Note that
we purposely introduced a very large model uncertainty, i.e. more
than 100% of the nominal value, to show clearly the bad effect of
this uncertainty on the nominal MPC algorithm and to subsequently
test the iterative learning MPC algorithm on a challenging case.
We first apply the nominal MPC controller to the uncertain model,
we show the obtained performance on Figure where it is clear
that the nominal performance is lost, since the second output, i.e.
the shaft torque, is oscillating and is violating its upper and lower
limits. Furthermore, these oscillations are also present on the control
voltage signal.

Now, we apply the iterative learning MPC Algorithm I, where we set
\( \delta T_{max} = 1.5 T_{ref} \). We choose the MES learning cost function as
\[ Q = \sum_{i=0}^{N-1} \left[ \| y_i(t - i\delta T_{mpc}) \|_2^2 + \| y_{i\delta T_{mpc}} \|_2^2 \right], \]
i.e. the norm of the error in the load angular position and velocity,
plus the norm of the error on the shaft torque. To learn the uncertain
parameter \( \beta_i \), we apply the algorithm, as
\[ z_{\beta_i}(k+1) = z_{\beta_i}(k) + a_{\beta_i} \Delta z_{\beta_i} + \frac{1}{2} \left( Q_{\beta_i}(k+1) - Q_{\beta_i}(k) \right) \]
with \( a_{\beta_i} = 10^{-6} \), \( \omega_{\beta_i} = 0.7 \text{ rad/s} \). We select \( \omega_{\beta_i} \) to be higher
than the desired frequency of the closed-loop (around 0.1 rad/s), to
ensure convergence of the MES algorithm, since the ES algorithms
convergence proofs are based on averaging theory, which assumes
high dither frequencies, e.g. [9], [10]. We chose a small value of the
dither signal amplitude, since we noticed that the MES cost function
has large values due to the large simulated uncertainty, so to
keep the search excursion amplitude small, and converge to a
precise value of the uncertainty \( \delta z_\beta \), i.e. to keep \( a_{\beta_i} Q \) small, we
choose small \( a_{\beta_i} \) (for further explanations on how to tune MES
algorithms please refer to [36]). We also set the MES cost function
threshold \( \epsilon_Q \) to 1.5\( Q_{\text{nominal}} \), where \( Q_{\text{nominal}} \) is the value of
the MES cost function obtained in the nominal-model case with the
nominal MPC, i.e. the base-line ideal case. In other words, we
decide to stop searching for the best estimation of the value of the
uncertainty when the uncertain MES cost function, i.e., the value of
\( Q \) when applying the iterative learning MPC algorithm to the
uncertain model, is less or equal to 1.5 of the MES cost function
in the case without model uncertainties, which represents the best
achievable MES cost function.

The obtained results of the iterative learning MPC algorithm are
reported on Figures and. First, note on Figure that the uncertain
cost function initial value (at the first iteration of the MES
learning) is very high, about 4.5 \( \times 10^6 \), which is about 50 times the
value of the nominal base-line MES cost function value. We see on
Figure that this cost function decreases as expected along the
MES learning iterations to reach a small value after about 25
iterations. This corresponds to the required number of iterations to
learn the actual value of the uncertain parameter as shown on Figure.
Eventually, after the convergence of the learning algorithm, we
see on Figure that the nominal base-line performances of the

MPC are recovered and that the output track the desired reference with smooth signals and without violating the desired constraints.

We also tested the case of multiple uncertainties. We assumed the two uncertainties \( \delta \beta_l = -70, \ [N/m/rad], \delta J_l = -0.2, \ [kgm^2] \). We first show on Figure 6 the performance of the nominal MPC when applied to this uncertain model. It is clear that the nominal MPC cannot cope with the uncertainties effect on the system’s performance, since the shaft torque, is oscillating and is violating its limits. The control voltage signal experiences oscillations, as well. Next, we apply the iterative learning-based MPC, where \( \delta \beta_l \) is learned using (14), and \( \delta J_l \) is learned using the ES equations

\[
\begin{align*}
\delta J_l(k' + 1) &= \delta J_l(k') + a_{\delta J} \delta T_{\text{mes}} \sin(\omega_{\delta J} k' \delta T_{\text{mes}} + \frac{\pi}{2}) Q \\
\delta T_{\text{mes}}(k' + 1) &= \delta T_{\text{mes}}(k') + a_{\delta J} \sin(\omega_{\delta J} k' \delta T_{\text{mes}} - \frac{\pi}{2}),
\end{align*}
\]

with \( a_{\delta J} = 10^{-6} \), and \( \omega_{\delta J} = 0.8 \text{ rad/s} \). Note that we choose a smaller dither amplitude for the \( \delta J_l \) estimation, since the value of the uncertainty on \( J_l \) is smaller, so we need a smaller dither signal amplitude for the search of the uncertain value.

The results of the iterative learning MPC algorithm in this case are reported on Figure 7, 8, 9 and 10. The learning cost function shown on Figure 8 is clearly decreasing and stabilizes after about 30 iterations. The uncertainties are learned and the overall tracking performance is recovered, as shown on Figures 9, 10 and 9 respectively. We notice here that the estimation of the uncertainties has some small residual error, this estimation error can be improved by either fine tuning the MES dither signals’ amplitudes, e.g., by using a time-varying dither amplitude [37], or by choosing other type of MES algorithms with larger domain of attraction, e.g. [12], [38].

V. CONCLUSION

In this paper, we have reported some preliminary results about an MES-based adaptive MPC algorithm. We have argued that it is possible to merge together a model-based linear MPC algorithm with a model-free MES algorithm to iteratively learn structural model uncertainties and thus improve the overall performance of the MPC controller. We have discussed a possible direction to analyze the stability of such algorithms. However, a more rigorous
Fig. 10. Uncertain parameter $\delta J_l$ learning evolution over the learning iterations.

analysis of the stability and convergence of the proposed algorithm is under development, and will be presented in our future reports. We have reported encouraging numerical results obtained on a mechatronics example, namely, a DC servo-motor control example. Future investigations will focus on improving the convergence rate of the iterative learning MPC algorithm, by using different ES algorithms with semi-global convergence properties, e.g., [12], [38], and on extending this work to different types of model-free learning algorithms, e.g. reinforcement learning algorithms, and comparing the learning algorithms in terms of their convergence rate and achievable optimal performances.

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