A Maximum-Likelihood Analysis of Observational Data on Fluxes and Distances of Radio Pulsars: Evidence for Violation of the Inverse-Square Law

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We analyze pulsar fluxes at 1400 MHz ($S_{1400}$) and distances ($d$) extracted from the Parkes Multi-beam Survey. Under the assumption that distribution of pulsar luminosities is distance-independent, we find that either (a) pulsar fluxes diminish with distance according to a non-standard power law, or (b) that there are very significant (i.e. order of magnitude) errors in the dispersion-measure method for estimating pulsar distances. The former conclusion (a) supports a model for pulsar emission that has also successfully explained the frequency spectrum of the Crab and 8 other pulsars over 16 orders of magnitude of frequency, whilst alternative (b) would necessitate a radical re-evaluation of both the dispersion-measure method and current ideas about the distribution of free electrons within our Galaxy.

Recently, superluminal polarization currents, whose distribution patterns move faster than light in vacuo, have been invoked as sources of pulsar emission [1]. This idea is derived from the work of Bolotovskii, Ginzburg and others, who showed that both such superluminal polarization currents do not violate Special Relativity (since the oppositely-charged particles that make them move subluminally) and that they form a bona-fide source term in Maxwell’s equations [2, 3, 4, 5]. The validity of these ideas has been demonstrated in a variety of laboratory experiments [6, 7, 8, 9, 10]. Moreover, by extending the approach to a superluminal polarization current whose distribution pattern follows a circular orbit, it was possible to explain quantitatively several observables from the Crab pulsar, including the spacing and widths of the emission bands at frequencies around 8 GHz, the maximum of the radiation spectrum, and the overall continuum spectrum across 16 orders of magnitude in frequency [1]. Subsequently, successful quantitative fits were carried out for 8 other pulsars [11] and a related superluminal model reproduced the general form of pulsar Stokes parameters [12].

In this Letter, we demonstrate a further prediction for rotating superluminal sources; that there is a component of the emission whose flux $S$ decays with distance $d$ as $S \propto 1/d$ [13, 14], rather than the conventional inverse square law ($S \propto 1/d^2$). Our demonstration employs a Maximum Likelihood Method (MLM) [15] analysis of pulsar observations [16]. The MLM is carried out to circumvent the significant Malmquist bias [17] due to the increasing non-detection of weaker pulsars as $d$ increases.

The pulsar emission component with $S \propto 1/d$ is due to a general property of sources that travel faster than their emitted waves; the relationship between reception time and retarded time is not monotonic and one-to-one [5, 18]. Multiple retarded times [19], or, if the source accelerates, extended periods of retarded time [19], can contribute to the waves received instantaneously, resulting in temporal focusing, i.e., concentration of the energy carried by the waves in the time domain [9, 10, 13]. This effect is well known in acoustics [18, 19, 20]. It is the temporal focusing from the parts of the source that approach the observer at the wave speed and with zero acceleration that leads to the $S \propto 1/d$ flux component [13, 14]. Note that this mechanism does not violate conservation of energy since the enhanced flux detected in some places is compensated exactly by diluted fluxes elsewhere [13, 18]. Moreover, we emphasize that the emission discussed in this paper arises from true superluminal motion; electromagnetic disturbances (polarization currents) that travel faster than the speed of light in vacuo, $c$ [9]. This should not be confused with the apparent superluminal motion of certain radio sources that is thought to arise from relativistic aberration [21].

Our analysis of $S_{1400}$ versus $d$ assumes that the luminosity function of pulsars is uniform throughout the Galaxy, i.e. that the populations of pulsars at various distances from the Earth are similar, each containing a representative spread of pulsar types, sizes and energies. We extract 1400 MHz fluxes ($S_{1400}$) and dispersion measures from the ATNF Pulsar Catalogue [16] (http://www.atnf.csiro.au/research/pulsar/psrcat). To
FIG. 1: (a) The 1109 Galactic pulsars in the Parkes Multibeam Survey plotted as $S_{1400}$ versus distance $d$, where $d$ is determined from the NE2001 interpretation of dispersion measure [23]. Pulsars with periods $P_0 < 0.1$ s are shown as hollow points, and those with $P_0 > 0.1$ s are displayed as filled points. The apparent differences between the distributions of the two sets of pulsars may reflect the fall-off in sensitivity of the Parkes instrument for faster pulsars (see Fig. 2 of Ref. [26]). (b) Cumulative population distribution in $S_{1400}$ for 9 distance bins. The mean distance of each bin is given in the inset key.

eliminate statistical biases from different instruments, we restrict the sample to the 1109 galactic pulsars detected using a single instrument, the Parkes Multibeam Survey [22]. We use the so-called NE2001 [23] model to evaluate $d$ values from the dispersion measures given in the ATNF catalogue; this was shown [23, 24] to give pulsar positions that are more consistent with the known distributions of matter in the Galaxy than previous models.

We first show that the Parkes observations show a strong Malmquist bias due to instrumental issues; consequently, the MLM [15] is essential in making quantitative conclusions about the flux-distance relationship. Fig. 1(a) plots the Galactic pulsars from the Parkes Multibeam Survey as $\log_{10}(S_{1400})$ versus $\log_{10}(d)$. It is already obvious that data are very sparse for $S_{1400} \leq 0.1$ mJy. To assess whether this is an instrumental artefact, or a fundamental property of the pulsar population, we group the pulsar data in bins covering certain distance ranges (e.g. $6.0 \leq d \leq 7.0$ kpc) and plot the cumulative distribution functions $N(S_{1400})$ of each bin in Fig. 1(b). The $d$ bins are chosen so that they cover a reasonably small range of $d$ but contain a large enough population for meaningful statistics ($\sim 100$ pulsars).

Note first that all the cumulative distribution functions in Fig. 1(b) tend to zero at roughly the same $S_{1400}$. This strongly suggests that low-flux part of each cumulative distribution is representative of the roll-off in sensitivity of the instrument, rather than an intrinsic property of each pulsar population. On the other hand, the high-flux sides of the curves in Fig. 1(b) are likely to be more representative of intrinsic properties of the pulsar populations. As such, they should move to lower fluxes as $d$ increases. This does indeed happen, but at a slower rate than the inverse-square law; the 75% points of the functions spread over roughly a factor 2.8 in $S_{1400}$, even though the distance varies by a factor of around 3.3. This is a much smaller spread than that expected for the inverse square law ($\sim 10$).

Simple analysis thus far has suggested that the Parkes Multibeam Survey is subject to a substantial Malmquist bias because it misses a large fraction of pulsars with $S_{1400} \leq 0.4$ mJy and cuts off completely for $S_{1400} < \sim 0.1$ mJy. Both of the latter figures are of the same order of magnitude as the calculated minimum detectable flux of the Parkes instrument ($\sim 0.15$ mJy; see Fig. 2 of Ref. [26]). To make further progress, we require a method that attempts to compensate for missing data, caused by instrumental sensitivity problems, in a systematic way. Originally, Efstathiou et al. [15], and subsequently a number of authors, have demonstrated that the MLM is very suitable for such problems by applying it to red-shifts of very distant objects, a data set which is incomplete due to instrumental problems somewhat analogous to those of the Parkes survey [15]. The Parkes database is especially suitable for treatment, since $S_{1400}$ and $d$ values are essentially independently derived.

Our implementation of the MLM determines the probable luminosity function, $\phi(L)$, based on the (incomplete) observed data, where $L$ is the luminosity. The technique fits a quasi-continuous $\phi(L)$ to the observations under the assumption of an instrumental cut-off [15]. Here, the MLM is used to determine the most likely value of the exponent $n$ in the relationship $S_{1400} \propto d^{-n}$. The intrinsic luminosity function is therefore calculated using the $S_{1400}$ and $d$ values from Parkes Survey for each of the trial exponents ($n_{\text{trial}} \in \{0.5, 1.0, 1.5, 2.0, 2.5, 3.0\}$); this feeds into the self-consistent determination of $\phi(L)$ and the cutoff through the relationship $L_i = S_i d_i^n$, where $S_i = S_{1400}$ for the $i^{\text{th}}$ pulsar at distance $d_i$. The probability that pulsar $i$ is observed in a flux-limited survey is given by

$$p_i \propto \phi(L_i)/\int_{\ln(L_{\text{min}}(d_i))}^{\infty} \phi(L) dL.$$

where $L_{\text{min}}(d_i)$ is the minimum luminosity that a pulsar at distance $d_i$ can have to be detected. We define a likelihood function $L = \prod_i p_i$. Following Ref. [15], we use an approach that does not assume a simple functional for $\phi(L)$. Instead, the luminosity function is parameterized as $N_k$ steps: $\phi(L) = \phi_k$, for $L_k - \Delta L < L < L_k + \Delta L$, with $k = 1, \ldots, N_k$. The maximum likelihood function
assumes the form
\[ \ln L = \sum_{i=1}^{N_p} W(L_i - L_k) \ln \phi_k - \sum_{i=1}^{N_p} \ln \left\{ \sum_{j=1}^{N_k} \phi_j \Delta L H[L_j - L_{min}(d_i)] \right\} + \text{const.} \] (1)

where \( N_p \) is the total number of pulsars in the Parkes survey. Here, \( W(x) = 0 \) for \(-\Delta L/2 \leq x \leq \Delta L/2\), and 1 otherwise, and
\[ H(x) = \begin{cases} 
0, & x \leq -\Delta L/2 \\
(x/\Delta L + 1/2), & -\Delta L/2 < x \leq \Delta L/2 \\
1, & x \geq \Delta L/2. 
\end{cases} \]

The parameters \( \phi_k \) determining the luminosity function are given by the self-consistent set of equations
\[ \phi_k \Delta L = \frac{\sum W(L_i - L_k)}{\sum_{i \geq 2} \phi_i \Delta L H[L_i - L_{min}(d_i)]}, \]
with \( k = 1, \ldots, N_b \). The above equations are solved iteratively to obtain the luminosity function, with the goodness of fit being parameterized by the relative convergence error \( \epsilon = \sum_{b \geq 2} (\phi_b - \phi_{b-1})^2 \). This is basically the relative mismatch between successive iterations; the smaller the value of \( \epsilon \), the better the representation of the data. In all cases, the 10 – 15% or so of pulsars with very high intrinsic luminosity were excluded from the analysis to ensure a quasi-continuous distribution of the luminosity function.

When the complete Parkes data set is used (Fig. 2(a), solid points), we find that the derived luminosity function converges very rapidly for \( n_{\text{trial}} = 1.0 \) and 1.5 with a small \( \epsilon \). The convergence to a putative luminosity function is considerably (\( \sim 10^5 \)) worse when one assumes an unphysical \( n_{\text{trial}} = 0.5, 2.5 \) and 3.0, as well as the commonly accepted inverse-square law \( (n_{\text{trial}} = 2) \). The relative convergence for these values of \( n_{\text{trial}} \) can be somewhat improved by restricting the analysis to a smaller set of pulsars, but is still not comparable to those obtained for \( n_{\text{trial}} = 1.0 \) and 1.5. Overall, the best combination is \( n_{\text{trial}} = 1 \) with 983 pulsars fitted, implying \( S_{1400} \propto 1/d \); the error with \( n_{\text{trial}} = 1.5 \) is somewhat larger, with 980 pulsars fitted. However, the good convergence for both exponents suggests that the observed flux may be a mixture of the \( S_{1400} \propto 1/d \) component and spherically-decaying radiation, both of which are to be expected from a superluminally-rotating source [13]. In slower pulsars, whose light cylinders lie further away from the central neutron star, the superluminally-rotating part of the current distribution may not be dense enough to give rise to a dominant nonspherically-decaying component of the radiation [1].

The main assumption of our analysis thus far is that pulsar populations are similar throughout the Galaxy. This is potentially open to question if there are two distinct populations of pulsars, especially if some property of each population results in different instrumental cut-offs. As shown in Fig. 2 of Ref. [26], the sensitivity of the Parkes Instrument is limited for pulsars with periods \( P_0 < 0.1 \) s; i.e., distant millisecond pulsars are harder to detect than equivalent slower pulsars (see Fig. 1(a)). This might result in an apparent \( d \)-dependent evolution of the characteristics of the detected pulsar population. Second, though recent work [11] suggests that all pulsars possess the same emission mechanism, some opine that millisecond pulsars form a distinct population [27] and might therefore possess a different luminosity function. Both of these concerns can be addressed by excluding pulsars with periods \( P_0 < 0.1 \) s from the MLM fit. The hollow points in Fig. 2(a) show the result; the minimum \( \epsilon \) is obtained with \( n_{\text{trial}} = 1.5 \). For comparison, Fig. 2(b) shows the result of running the MLM on only the 43 pulsars with \( P_0 < 0.1 \) s; once again, the fit is best for \( n_{\text{trial}} = 1.5 \). Therefore, in spite of the separation of pulsars with \( P_0 < 0.1 \) s and \( P_0 > 0.1 \) s, the MLM never obtains a minimum \( \epsilon \) for \( n_{\text{trial}} = 2 \); the errors for \( n_{\text{trial}} = 1, 1.5 \) are smaller, often dramatically so. This suggests that the violation of the inverse-square law by the pulsar population is a robust phenomenon.

Finally, the MLM was tested on two synthetic galaxies of pulsars in which \( S_{1400} \) was constrained to decay as \( d^{-1.5} \) and \( d^{-2} \) (Fig. 2(c)). As all pulsar population models in the literature (e.g., Refs. [24,25]) are both contaminated with assumptions involving the inverse-square law and involve \( \sim 10 \) – 20 adjustable parameters, we derived synthetic pulsar distributions that are consistent with the Parkes database using Bayesian methods [28]. As with the Parkes data, the luminosity function was determined using a range of trial exponents \( (n_{\text{trial}} = 0.5, 1.5, 2 \) and 2.5). In each case, the MLM located the correct value of \( n \), with an accuracy better than \( \pm 0.5 \) (Fig. 2(c)). This gives confidence in our assertion that pulsar flux data in the Parkes survey violate the inverse square law.

In the various implementations of the MLM, the inferred pulsar luminosity function (e.g., Fig. 2(d)) always decreases monotonically [29]. As discussed above, the effects of instrumental insensitivity will lead to an apparent luminosity function that falls off at low \( S_{1400} \). This problem may have led to the commonly-held belief that the pulsar luminosity function is a log-normal distribution [27]. Once one compensates for the loss of data (Fig. 2(d)), the maximum characteristic of a log-normal distribution will be absent.

The MLM [15] therefore finds that the observed 1400 MHz flux of pulsars does not fall off as the conventionally-assumed \( 1/d^2 \), instead returning exponents of either 1 or 1.5. There are two possible conclusions; either the dispersion measure estimates of pulsar
Relative convergence error from the MLM applied to 983 \((n_{\text{trial}} = 0.5, 1)\) or 980 \((n_{\text{trial}} = 1.5, 2, 2.5, 3)\) pulsars from the Parkes Survey versus trial exponent \(n_{\text{trial}}\). Red, hollow diamonds: \(\epsilon\) from the MLM applied to 941 \((n_{\text{trial}} = 0.5, 1)\) or 938 \((n_{\text{trial}} = 1.5, 2, 2.5, 3)\) pulsars with periods \(P_0 > 0.1\) s. (b) MLM fit for the 43 Parkes pulsars with \(P_0 < 0.1\) s. (c) Relative convergence errors for synthetic galaxies in which \(S_{1400} \propto d^{-1.5}\) (red, hollow points) and \(S_{1400} \propto d^{-2}\) (green, filled points) versus \(n_{\text{trial}}\). (d) The inferred luminosity function for 983 pulsars with \(n_{\text{trial}} = 1\); large circles are raw Parkes data; small diamonds are the fitted luminosity function.

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