Bounds on QCD axion mass and primordial magnetic field from CMB $\mu$-distortion

Damian Ejlli

INFN, Laboratori Nazionali del Gran Sasso, 67100 Assergi, Italy and Department of Physics, Novosibirsk State University, Novosibirsk 630090 Russia

(Dated: July 22, 2014)

Oscillation of CMB photons into axions can cause CMB spectral distortion in presence of large scale magnetic fields. With COBE limit on $\mu$ parameter and homogeneous magnetic field strength $B \lesssim 3.2$ nG at horizon scale, stronger lower limit on axion mass in comparison with limit of ADMX experiment is found to be $4.8 \times 10^{-5}$ eV $\lesssim m_a$ for the KSVZ axion model. On the other hand using experimental limit on axion mass $3.5 \times 10^{-6}$ eV $\lesssim m_a$ from ADMX experiment together with COBE $\mu$ bound, is found $B \lesssim 53$ nG (KSVZ axion model) and $B \lesssim 141$ nG (DFSZ axion model) for homogeneous magnetic field with coherence length at present $\lambda_B \sim 1.3$ Mpc. Limits on $B$ and $m_a$ for PIXIE/PRISM expected sensitivity on $\mu$ are derived.

The cosmic microwave background (CMB) presents small temperature anisotropy of the order of $\delta T/T \sim 10^{-5}$ on small angular scale and its spectrum is supposed to be slightly distorted due to various mechanisms which might have operated in the early Universe. In general these distortions are described in terms of the so called $\mu$, $i$ and $y$ parameters which their values quantify the type of each distortion. COBE space mission obtained stringent limits on $|\mu| < 9 \times 10^{-5}$ and $|y| < 1.5 \times 10^{-5}$ parameters, thus implying that there might be a very narrow window to look for process leading to spectral distortion. Other planned space missions include PIXIE and PRISM which expect to obtain more stringent limits on $\mu$ and $y$ with respect to COBE of the order of $\mu < 5 \times 10^{-8}$ and $y < 10^{-8}$.

Generally speaking the most popular proposed mechanisms which can create spectral distortion, can be classified as "secondary" mechanisms in the sense that the original CMB spectrum is affected indirectly. Indeed, in these models energy and photon number are injected into the medium from external sources such as decaying dark matter particles, sound waves etc. On the other hand, CMB can also have "primary" spectral distortions which can be disentangled from the secondary ones. An interesting mechanism which can be classified as primary is oscillation of CMB photons into light bosons such as axions, axion-like particles (ALPs) and gravitons. These processes, in cosmological context, are possible in the presence of an external magnetic field where the photon has a vertex coupling with them. In the case of axions the relevant term which describes coupling of photons with axions is given by the interaction Lagrangian density

$$\mathcal{L}_{a\gamma} = -\frac{g_{a\gamma}}{4} F_{\mu\nu} \tilde{F}^{\mu\nu},$$

where $F_{\mu\nu}$ is the electromagnetic field tensor, $\tilde{F}^{\mu\nu}$ is its dual and $a$ is the axion field. In general the coupling constant of axions can be written as

$$g_{a\gamma} = \frac{\alpha_s}{2\pi f_a} \left( \frac{E}{N} - \frac{24 + w}{31 + w} \right),$$

where $\alpha_s$ is the fine structure constant, $f_a$ is the axion decay constant, $E$ is the electromagnetic anomaly associated with axial current and $N$ is the color anomaly. Among of all axion models, two of them namely the KSVZ and DFSZ axion models have been extensively studied in the literature. For KSVZ model we have $E/N = 8/3$ and $E/N = 0$ for DFSZ model. In both models coupling constant of axions to photons $g_{a\gamma}$ is proportional related to axion mass $m_a$. The latter is related with quark masses up ($u$) and down ($d$) and the relation between axion mass $m_a$ and axion decay constant $f_a$ is given by

$$m_a = \frac{m_\pi}{f_a} \frac{w^{1/2}}{1 + w},$$

where $m_\pi = 135$ MeV is the pion mass, $f_\pi \approx 92$ MeV is the pion decay constant and $w = m_u/m_d$ with $m_u, m_d$ being respectively the up and down quark masses. The range of the parameter $w$ is between $0.35 \leq w \leq 0.6$ where in general its standard value is taken $w = 0.56$.

For recent reviews on axions and ALPs see Ref. and for earlier works on axions in cosmology see Ref. The origin of the large scale magnetic field (which makes possible transition of photons into axions), is interesting by itself since its presence, would have enormous impact in several situations in cosmology such as bing bang nucleosynthesis, CMB temperature anisotropy etc.) and in astrophysics (such as cosmic rays deflection etc). Thus, its strength $B_e$ and its direction are of fundamental importance. The most common ways to constrain large scale magnetic field strength have been essentially from CMB temperature anisotropy and Faraday rotation of the CMB. In the former case it is supposed that the external magnetic field would contribute to the total energy density in the Universe and therefore it would be possible that this additional energy density could cause CMB temperature anisotropy. In the latter case the presence of magnetic field would cause polarization of the CMB through the so called Faraday effect, namely the rotation of the polarization plane of the CMB. For a review on large scale magnetic fields see Ref.
In a previous work [16], we obtained tight limits on the ALP parameter space by using coupling of CMB photons with ALPs in primordial magnetic field. In this letter we study oscillation of CMB photons into axions in presence of large magnetic field and derive new limits on axion mass and magnetic field strength. Photon-axion mixing is phenomenologically different from oscillation into ALPs, since in the axion case the two quantities which characterize axions, its mass $m_a$ and coupling constant to photons $g_{a\gamma}$, are directly proportional with each other, while in the ALP case they are in principle unrelated. Consequently, in the case of photon-axion mixing the number of independent parameters is reduced to only $B_c$ and $m_a$ or $g_{a\gamma}$ with respect to photon-ALP mixing. Therefore based on phenomenological or experimental results it would be possible that known one of the parameters $B_c$, or $m_a$, we can constrain the remaining one.

Firstly knowing upper bounds on magnetic field strength at present time we can find limits for the axion mass. In this case case the field strength and coherence length are fixed a priori. Secondly if we know experimental limits on axion mass we can bound the magnetic field strength and discuss about its coherence length a posteriori. In this letter we consider only uniform (homogeneous) magnetic field. The effect on CMB oscillation due to non homogeneous (stochastic) magnetic fields will not be considered. In connection with the first case we use limits on magnetic field from CMB temperature anisotropy and Faraday rotation where field coherence length is greater or comparable with horizon scale. For magnetic field with coherence length comparable with horizon scale, CMB temperature anisotropy gives $B \lesssim 4 \text{ nG}$ [17] and Faraday rotation of Lyman-$\alpha$ forest gives [18], $B \lesssim 1 \text{ nG}$. As far as for the second case we consider existing limits on axion mass to constrain strength of homogeneous magnetic field with coherence length at least comparable with horizon scale during $\mu$ epoch. In the formalism of density matrix which we use below, the magnetic field is assumed homogeneous at given coherence length $\lambda_B$ where field strength changes only due to Universe expansion. Here we adopt rationalized Lorentz-Heaviside natural units, $c = \hbar = k_B = \epsilon_0 = \mu_0 = 1$.

Study of oscillation of CMB photons into axions whith an essential loss of coherence is best formulated in terms of density operator of the system \( \hat{\rho} \) (in our case the system is composed of axions and photons). To the linear order of approximation it satisfies quantum kinetic equation [19]

\[
\frac{d\hat{\rho}}{dt} = -i[H, \hat{\rho}] - \{\hat{\Gamma}, (\hat{\rho} - \hat{\rho}_{eq})\},
\]

where $H$ is the Hamiltonian of photon-axion system including refraction index (first order effects), $\hat{\Gamma}$ is the coherence breaking operator of photons and axions with the background medium, and $\hat{\rho}_{eq}$ is the equilibrium density operator. Since magnetic field mix only the $(\cdot)$ photon state (see below) with the axion, matrix elements of operators $\hat{\rho}$, $\hat{\Gamma}$ and $\hat{H}$ in the basis spanned by two component field $\Psi^T = (A_x, a)$ are respectively given by

\[
\rho = \begin{pmatrix} n_\gamma & \rho_{\gamma a} \\ \rho_{a\gamma} & n_a \end{pmatrix}, \quad \Gamma = \begin{pmatrix} \Gamma_\gamma & 0 \\ 0 & \Gamma_a \end{pmatrix}, \quad H = \begin{pmatrix} M_x & M_{a\gamma} \\ M_{a\gamma} & M_a \end{pmatrix}
\]

where $\rho_{\gamma a} = n_\gamma n_a$ are respectively photon and axion occupation numbers, $\rho_{a\gamma} = \rho_{a\gamma}^* = R + iJ$ with $R$ and $J$ being respectively the real and imaginary part of $\rho_{a\gamma}$. Matrix elements of equilibrium density operator in flavor space are given by equilibrium occupation number $n_{eq} = 1/(e^{\omega} - 1)$ times the identity matrix $I$, $\rho_{eq} = n_{eq} I$ where $x = \omega/T$ with $T$ being the photon temperature. Coherence breaking matrix $\Gamma$ is diagonal in flavor space and its entries are respectively given by the sum of scattering and annihilation/absorption rates of photons ($\Gamma_\gamma$) and axions ($\Gamma_a$). Matrix elements which enter interaction Hamiltonian are respectively [20] $M_x = \omega(n - 1)_x$, $M_{a\gamma} = g_{a\gamma} B_T/2$, $M_a = -m_a^2/2\omega$. Here $B_T$ is the strength of the external magnetic field $B_e$, which is transverse to the direction $\bf{x}$ of the photon/axion propagation. $A_{+,x}$ are the photon polarization states with $+,\times$ being the polarization indexes (helicity) of the photon. The helicity state $+\times$ corresponds to the polarization perpendicular to the external magnetic field and $\times$ describes the polarization parallel to the external field. For the purpose of this work and cosmological epoch which we are interested in, the total refraction index is given by the sum of two main components: the refraction index due to electronic plasma $n_{pla}$ and refraction index due to vacuum polarization $n_{QED}$. The refraction index due to electronic plasma is given by

\[
(n - 1)_{x,+} = \frac{\alpha}{4\pi} \left( \frac{B_T}{B_e} \right)^2 \left[ \frac{14}{45} \right]_x, \left[ \frac{8}{45} \right]_+.\]

where $B_e = m_e^2/e = 4.41 \times 10^{13}$ G is the critical magnetic field.

When total interaction rate which enter the problem is much bigger than expansion rate $\Gamma \gg H$ and photon-axion oscillation frequency $\omega_{osc} \gg H$, equation of motion for density matrix are given by steady state approximation, see Ref. [19] for details. In this case it is possible to express the imaginary part $I$ and real part $R$ through $n_\gamma$ and $n_a$. In this approximation equation of motion for density matrix in FRW metric reduce to the following closed system of first order linear differential equations

\[
(n - 1)_{x,+} = \frac{\alpha}{4\pi} \left( \frac{B_T}{B_e} \right)^2 \left[ \frac{14}{45} \right]_x, \left[ \frac{8}{45} \right]_+.
\]
in \( n_\gamma \) and \( n_a \)

\[
HTn'_\gamma(T) = \frac{4\Gamma M^2_{\gamma}(n_\gamma - n_a)}{4(\Delta M)^2 + \Gamma^2} + \Gamma(n_\gamma - n_{eq}), \tag{7}
\]

\[
HTn'_a(T) = \frac{-4\Gamma M^2_{\gamma}(n_\gamma - n_a)}{4(\Delta M)^2 + \Gamma^2} + \Gamma_a(n_a - n_{eq}). \tag{8}
\]

The system of Eqs. (7) and (8) can be solved numerically or analytically. In the last case it is still difficult to solve it without making any approximation. Let us assume that interaction rate of axions with the medium is negligible \( \Gamma_a \approx 0 \) (for axion mass range considered here this is indeed true) and \( n_\gamma \gg n_a \) with \( n_\gamma \approx n_{eq}^\gamma \). In this case we can write Eq. (8) as

\[
P_a = \frac{n'_a}{n_{eq}^\gamma} = \frac{-4\Gamma M^2_{\gamma}}{HT(4\Delta M)^2 + \Gamma^2}, \tag{9}
\]

All approximations made above are well satisfied since we expect oscillation of CMB photons into axion would produce very small spectral distortion and occupation number of CMB would remain very close to equilibrium. Let us at this point expand the term \( \Delta M(T) \) in power series around the resonance temperature up to first order in \( T \), \( \Delta M(T) \approx k(T)(T - \bar{T}) \) where \( k(T) = d(\Delta M)/dT|_{T=\bar{T}} \).

Assuming that oscillation probability is dominated by the resonance point we can easily integrate Eq. (9) and obtain

\[
P_a(T) = -\frac{2\pi M^2_{\gamma}}{kHT}|_{T=\bar{T}} \tag{10},
\]

where in integrating Eq. (9) each term \( \Delta M, T, k \) and \( H \) has been held constant and equal to their value at \( T = \bar{T} \). Eq. (10) is in general valid in both resonant and non-resonant case but it gives very accurate values of probability in the first case and less accurate results in the latter case. We may also notice that there is no dependence on the interaction rate \( \Gamma = \Gamma_\gamma + \Gamma_a \approx \Gamma_\gamma \) which has been integrated out.

Now in order to calculate \( P_a \) we need to write the temperature dependence of each quantity in Eq. (10). The term \( M_{\gamma}^\gamma \) can be written as

\[
M_{\gamma\gamma}(\bar{T}) = \frac{g_{\gamma\gamma}B_0}{2}\left(\frac{T}{T_0}\right)^2, \tag{11}
\]

where the magnetic field scale with temperature as \( B_T \sim B = B_0(T/T_0)^2 \) (magnetic flux conservation) with \( B_0 \) being the strength of magnetic field at present epoch. The term \( H(\bar{T})\bar{T} \) can be written as \( H(\bar{T})\bar{T} = H(\bar{T}_0)\bar{T}/\Omega_R(T/T_0)^3 \) where \( \Omega_R \) is the present day density parameter of relativistic particles (photons and nearly massless neutrinos). During the \( \mu \) epoch the Universe is radiation dominated. Since we are interested in the period prior to recombination epoch where ionization fraction of free electrons is unity, \( X_e = 1 \) we obtain \( k(\bar{T}) = (3/\bar{T}) \left[ M_{QED}(\bar{T}) - M_{pla}(\bar{T}) \right] \) where \( M_{QED} \) and \( M_{pla} \) are respectively the QED and plasma contributions to the refraction index in \( \Delta M \). Inserting all necessary terms into Eq. (10) we get the following expression for \( P_a \)

\[
P_a(T) = \frac{2\pi}{3H(T)} \frac{M^2_{\gamma}(\bar{T})}{M_{QED}(\bar{T}) + M_{pla}(\bar{T})}, \tag{12}
\]

where in deriving Eq. (12) we have used the fact that for \( T = \bar{T} \) we have \( \Delta M(T) = M_{QED}(\bar{T}) - M_{pla}(\bar{T}) - M_{\mu}(\bar{T}) = 0 \) with \( M_x = M_{QED} - M_{pla} \). We may note that in case \( M_{QED}(\bar{T}) = -M_{\mu}(\bar{T}) \) the denominator of Eq. (12) is zero and the probability goes to infinity. In such case one must consider expansion of \( \Delta M(T) \) up to second order in \( T \) around the resonance temperature \( \bar{T} \). However, for our purpose we do not need it here.

In order to confront Eq. (12) with numerical results and because is more easy to calculate, let us consider the case when \( M_{QED} \ll M_{\mu} \). In the redshift of interest for \( \mu \)-distortion and photon energies considered here, QED term in \( M_{\mu} \) is negligible with respect plasma term and therefore from the resonance condition \( \Delta M = M_x - M_\mu = 0 \) we get

\[
\left(\frac{T}{T_0}\right) = 9 \times 10^6 n_e^{-1/3} \bar{m}_a^{2/3} \text{cm}^{-1}, \tag{13}
\]

where \( \bar{m}_a = m_\mu/eV, n_e \approx 0.88 n_B(T_0) \) is the number density of free electrons at present epoch and \( n_B(T_0) = 2.47 \times 10^{-7} \text{cm}^{-3} \) is the number density of baryons. Eq. (13) is a constraint relation for the axion mass in the resonant case. Inserting all necessary quantities into Eq. (12) we get the following expression for \( P_a \)

\[
P_a(T) = 5.75 \times 10^{-27} x C_{\gamma\gamma} B_{nG}^2 \left(\frac{T}{T_0}\right)^3, \tag{14}
\]

where \( B_{nG} = (B_0/nG) \) and \( C_{\gamma\gamma} \) is defined as

\[
C_{\gamma\gamma} \equiv \left(\frac{E}{N} \times \frac{24 + w}{3 + 1 + w}\right) \frac{1 + w}{w^{1/2}}, \tag{15}
\]

where for \( w = 0.56, \left| C_{\gamma\gamma} \right| \approx 4 \) for \( E/N = 0 \) (KSVZ model) and \( \left| C_{\gamma\gamma} \right| \approx 1.49 \) for \( E/N = 8/3 \) (DFSZ model). It is important to emphasize that Eq. (14) is valid when \( M_{QED} \ll M_\mu \) or

\[
B_{nG}^{1/3} \times x \left(\frac{T}{T_0}\right)^{1/3} \ll 1.23 \times 10^9 \bar{m}_a^{1/3}. \tag{16}
\]

Inserting Eq. (13) into Eq. (16) we get

\[
B_{nG} \ll \frac{4.9 \times 10^{-2}}{\bar{m}_a}, \tag{17}
\]

where we used \( x = 11.3 \) (see below). On the other hand we also need to calculate the axion mass at the resonance
temperature $\bar{T}$ which is given by Eq. (13). Assuming that interested temperature interval is coincident with $\mu$-epoch, $2.88 \times 10^5 \lesssim T/T_0 \lesssim 2 \times 10^6$ the axion mass in this interval is

$$2.66 \times 10^{-6} \text{ eV} \lesssim m_a \lesssim 4.88 \times 10^{-5} \text{ eV}. \quad (18)$$

So, as far as we limit our consideration for magnetic field strength of the order $B_{\text{ng}} \lesssim 10^7$ and axion mass range given by Eq. (18) we can safely use Eq. (14).

In presence of $\mu$-distortions we can expand the photon occupation number for $\mu \ll 1$ as

$$n_\gamma \simeq n_{eq} + \mu \partial_x n_{eq} = n_{eq} - n_{eq}(1 + n_{eq})\mu. \quad (19)$$

Since leakage of photons is due to oscillations into axions, we get the following relation between $P_a$ and $\mu$

$$P_a = \frac{\mu e^{x}}{e^{x} - 1}. \quad (20)$$

Using Eqs. (20), (14) and Eq. (13) we get the following relation between magnetic field and axion mass

$$B_{\text{ng}} = \frac{0.22}{m_a C_{a\gamma}} \left( \frac{\mu e^{x}}{x(e^{x} - 1)} \right)^{1/2}. \quad (21)$$

We can see that Eq. (21) depends on the photon energy $x$ and tighter bound on $B_{\text{ng}}$ or $m_a$ is obtained for higher values of $x$. Indeed, using for example the energy range explored by COBE [3], $1.2 \leq x \leq 11.3$ we get a tighter limit on $B_{\text{ng}}$ at $x = 11.3$

$$B_{\text{ng}} = 6.76 \times 10^{-2} \frac{\sqrt{\mu}}{m_a C_{a\gamma}}. \quad (22)$$

Eq. (22) is our main result which connect three unknown parameters $m_a$, $B_{\text{ng}}$ and $C_{a\gamma}$ with $\mu$ parameter which is determined by experiment. We may notice, that for values of $\mu$ given by COBE [3] and PIXIE/PRISM [4] we have that the bound given by Eq. (17) is indeed well satisfied. Using Eq. (22), in Fig. 1 exclusion plots for $B_{\text{ng}}$ vs. $m_a$ are shown for COBE and expected PIXIE/PRISM limits on $\mu$. We emphasize that our results in resonant case obtained by using Eq. (22) perfectly agree with numerical solution of the system of equation given by Eqs. (7) and (8). The discrepancy between the two is by a factor 1.15 as we have explicitly checked.

In conclusion, in this letter we have studied an indirect mechanism to look for invisible axions, namely through their coupling to two photons where CMB plays the role of photons. Although we do not have direct evidence of CMB spectral distortions, COBE experiments showed that small spectral distortions are indeed allowed. These distortions are created in the early Universe in an epoch when Compton elastic scattering would have been efficient on maintaining kinetic equilibrium. When Compton scattering stop being efficient, the accumulated $\mu$ distortions froze out. Once resonance occur for a given axion mass $m_a$, the amount of spectral distortion after the resonance froze out and is equal to $\mu$. We have explicitly checked that occupation number of axions remains constant after resonance thus implying $\mu$ parameter also froze out.

In Fig. 1 we present our exclusion limits on axion mass and magnetic field strength in resonant case. Is not possible in general to give a definite constrain on $B$ and $m_a$ since none of them is known exactly and moreover only upper limits on $\mu$ parameter exist which relates both. Nevertheless, we can outline important conclusions considering the upper limits of all of them. In general we can base our arguments by simply focusing on Eq. (22). Firstly based on limits on $\mu$ from COBE we can see from Fig. 1 that we can limit the axion mass if we know limits on $B$. For instance in case of KSVZ axion model for $\mu < 9 \times 10^{-5}$ and homogeneous magnetic field with strength $B \lesssim 3.2$ nG we obtain from Eq. (22) that $4.8 \times 10^{-5} \text{ eV} \lesssim m_a$. The limit on magnetic field strength is by a factor 1.2 stronger than that found for homogeneous and anisotropic magnetic field in Ref. [17] and is by a factor 3.2 weaker than that found in Ref. [18] from Faraday rotation of Layman $\alpha$-forest. For the DFSZ axion model the upper limit for homogeneous magnetic field is $B \lesssim 9$ nG which is by a factor 2.5 weaker than KSVZ axion model for the same axion mass. This upper limit on magnetic field for DFSZ model would produce too much CMB temperature anisotropy and makes the DFSZ axion model disfavored with respect to KSVZ axion model. PIXIE/PRISM would put stronger limits with respect to COBE and in particular for homogeneous field with coherence length of Hubble horizon would give $B \lesssim 7.7 \times 10^{-11}$ G for KSVZ axion model and $B \lesssim 2 \times 10^{-10}$ G for DFSZ axion model.

The ADMX collaboration [23] excluded all axion models of being dark matter in the mass region $3.3 \mu \text{eV} - 3.5 \mu \text{eV}$. This mass range lies in the axion mass range considered in this paper, see Eq. (18). Thus, it would be possible to use ADMX limits on axion mass to constrain the magnetic field strength. For example considering the limit $3.5 \mu \text{eV} \lesssim m_a$ we find that magnetic field strength would be (in the case of COBE), $B \lesssim 53$ nG for the KSVZ axion model and $B \lesssim 141$ nG for the DFSZ axion model, see Fig. 1. In the case of PIXIE/PRISM we would have $B \lesssim 1$ nG for KSVZ axion model and $B \lesssim 2.7$ nG for DFSZ axion model. However, knowing upper and/or lower limits for axion mass allows only to constrain the strength of magnetic field and nothing tells about its spatial structure and coherence length. In this case the above limits are valid for homogeneous magnetic fields with at least coherence length of the order of horizon scale during $\mu$ epoch, namely $\lambda_2^H \sim H^{-1}(\zeta_\mu)$ or $\lambda_2^H \sim 3.8 \text{ pc}$ where redshift corresponding to axion mass $m_a \simeq 3.5 \mu \text{ eV}$ is $\zeta_\mu \simeq 3.44 \times 10^5$, see Eq. (13).

The derived limits on homogeneous magnetic field strength are in general stronger than those found from temperature anisotropy [17] and slightly weaker than
FIG. 1. Exclusion plot for the axion parameter space $B - \tilde{m}_a$ in the resonant case due to $\mu$-distortion for $\tilde{m}_a = 2.66 \times 10^{-6} - 4.88 \times 10^{-5}$ eV. In (a) the exclusion plot for COBE [3] upper limit on $\mu$ is shown and in (b) for PIXIE/PRISM [4] expected sensitivity on $\mu$ is shown. In both figures the region above the solid line corresponds to KSVZ axion model ($|C_{\alpha\gamma}| \approx 4$) and the region above dot dashed line correspond to DFSZ axion model ($|C_{\alpha\gamma}| \approx 1.49$).

those found from Faraday rotation [18]. Indeed, at coherence length scale $\lambda_B \sim 1$ Mpc, Faraday rotation of Lyman $\alpha$-forest gives $B \lesssim 10$ nG [18] which is by a factor 5.3 stronger than limit found in KSVZ axion model and by a factor 14.1 stronger than DFSZ axion model (using ADMX limits on axion mass). Limits on axion mass found here in general are of the same order of magnitude with limits found by misalignment mechanism, see Ref. [21] and [10]. Indeed, the lower limit $4.8 \times 10^{-5}$ eV $\lesssim m_a$ for $B \lesssim 3.2$ nG is very close to that found in Ref. [25], namely $m_a \lesssim 76 \mu$ eV - $82 \mu$ eV for CDM axions. According to Ref. [25] an axion within this mass range would explain all dark matter contents in the Universe without requiring other candidates. However, in our case an axion in the mass range $m_a \lesssim 7.6 \mu$ eV - $8.2 \mu$ eV would make non resonant oscillation into CMB photons during the $\mu$-epoch. If misalignment mechanism limits are used for axion mass (non resonant oscillation) such as those in Ref. [25] instead of ADMX limit, the strength of homogeneous magnetic field at $\lambda_B \sim 1$ Mpc would be between $1.4 \times 10^3$ nG - $1.6 \times 10^3$ nG depending on non resonant axion mass. These limits are weaker than those found from Faraday rotation of Lyman $\alpha$-forest and are comparable with limits found from homogeneous Universe, see Ref. [18].

[1] Y. B. Zeldovich and R. A. Sunyaev, Astrophys. Space Sci. 4 (1969) 301. R. A. Sunyaev and Y. B. Zeldovich, Astrophys. Space Sci. 7 (1970) 20.
[2] W. Hu and J. Silk, Phys. Rev. D 48 (1993) 485. J. Chluba and R. A. Sunyaev, Mon. Not. Roy. Astron. Soc. 419 (2012) 1294. R. Khatri and R. A. Sunyaev, JCAP 1209 (2012) 016
[3] D. J. Fissen, E. S. Cheng, J. M. Gales, J. C. Mather, R. A. Shafer and E. L. Wright, Astrophys. J. 473 (1996) 576.
[4] A. Kogut, D. J. Fixsen, D. T. Chuss, J. Dotson, E. Dwek, M. Halpern, G. F. Hinshaw and S. M. Meyer et al., JCAP 1107 (2011) 025.
[5] P. Andre et al. [PRISM Collaboration], arXiv:1306.2259 [astro-ph.CO].
[6] W. Hu and J. Silk, Phys. Rev. Lett. 70 (1993) 2661. J. Chluba, arXiv:1304.6121 [astro-ph.CO].
[7] R. Khatri, R. A. Sunyaev and J. Chluba, Astron. Astrophys. 540 (2012) A124.
[8] J. E. Kim, Phys. Rev. Lett. 43 (1979) 103.
[9] M. A. Shifman, A. I. Vainshtein and V. I. Zakharov, Nucl. Phys. B 166 (1980) 493.
[10] M. Dine, W. Fischler and M. Srednicki, Nucl. Phys. B 189 (1981) 575.
[11] J. Beringer et al. (Particle Data Group), Phys. Rev. D 86, 010001 (2012) and 2013 partial update for the 2014 edition.
[12] A. G. Dias, A. C. B. Machado, C. C. Nishi, A. Ringwald and P. Vaudrevange, JHEP 1406 (2014) 037.
  A. Ringwald, arXiv:1407.0546 [hep-ph].
J. Preskill, M. B. Wise and F. Wilczek, Phys. Lett. B 120 (1983) 127.
L. F. Abbott and P. Sikivie, Phys. Lett. B 120 (1983) 133.
M. Dine and W. Fischler, Phys. Lett. B 120 (1983) 137.
Z. G. Berezhiani, A. S. Sakharov and M. Y. Khlopov, Sov. J. Nucl. Phys. 55 (1992) 1063 [Yad. Fiz. 55 (1992) 1918].
M. Y. Khlopov, A. S. Sakharov and D. D. Sokoloff, Nucl. Phys. Proc. Suppl. 72 (1999) 105.
[13] P. P. Kronberg, Rept. Prog. Phys. 57 (1994) 325.
R. Durrer, P. G. Ferreira and T. Kahniashvili, Phys. Rev. D 61 (2000) 043001
A. Kosowsky and A. Loeb, Astrophys. J. 469 (1996) 1
L. Campanelli, A. D. Dolgov, M. Giannotti and F. L. Villante, Astrophys. J. 616 (2004) 1
D. Paoletti and F. Finelli, Phys. Lett. B 726 (2013) 45
[14] Ya. B. Zel’dovich, JETP, 48, 986 (1965)
K. S. Thorne, ApJ, 148, 51 (1967)
[15] D. Grasso and H. R. Rubinstein, Phys. Rept. 348 (2001) 163
L.M. Widrow, Rev. Mod. Phys. 74, 775 (2002)
M. Giovannini, Int. J. Mod. Phys. D 13, 391 (2004)
R.M. Kulsrud, E.G. Zweibel, Rept. Prog. Phys. 71, 0046091 (2008)
A. Kandus, K.E. Kunze, C.G. Tsagas, Phys. Repts. 505, 1 (2011)
R. Durrer and A. Neronov, arXiv:1303.7121
[16] D. Ejlli and A. D. Dolgov, arXiv:1312.3558 [hep-ph],
[17] J. D. Barrow, P. G. Ferreira and J. Silk, Phys. Rev. Lett. 78 (1997) 3610
[18] P. Blasi, S. Burles and A. V. Olinto, Astrophys. J. 514 (1999) L79
[19] A. D. Dolgov, Phys. Rept. 370 (2002) 333
[20] G. Raffelt and L. Stodolsky, Phys. Rev. D 37 (1988) 1237.
[21] E. Brezin and C. Itzykson, Phys. Rev. D 3 (1971) 618.
W. Y. Tsai and T. Erber, Phys. Rev. D 10 (1974) 492.
[22] P. A. R. Ade et al. [Planck Collaboration], arXiv:1303.5076 [astro-ph.CO].
[23] S. J. Asztalos et al. Phys. Rev. Lett. 104 041301 (2010)
[24] P. Sikivie, Lect. Notes Phys. 741 (2008) 19
[25] E. Di Valentino, E. Giussarma, M. Lattanzi, A. Melchiorri and O. Mena, arXiv:1405.1860 [astro-ph.CO].