One-photon exchange contribution to $B^{\pm} \to (\pi^{\pm}, K^{\pm})\ell^{+}\ell^{-}$ decays

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Abstract

A so far neglected long-distance (also called one-photon exchange) contribution to the rare semileptonic $B^{\pm} \to (\pi^{\pm}, K^{\pm})\ell^{+}\ell^{-}$ ($\ell = e, \mu$) transitions is evaluated. Although it does not contribute to solve the possible breaking of lepton-universality observed by LHCb in the $B^{\pm} \to K^{\pm}(\mu^{+}\mu^{-}/e^{+}e^{-})$ ratio, nor provides an important hadronic contamination to their decay rates, its contribution to the branching ratios of the $B^{\pm} \to \pi^{\pm}\ell^{+}\ell^{-}$ transitions turns out to be significant. This hadronic pollution should be taken into account when looking for new physics effects in decays into pions, which suggests to restrict these searches to squared lepton-pair invariant mass in the $(1, 8)$ GeV$^2$ range. The interference of the one-photon exchange contribution with the dominant short-distance one-loop amplitude induces a sizable CP asymmetry in these rare decays, which calls for dedicated measurements.

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I. INTRODUCTION

Rare meson decays are expected to serve as harbinger for New Physics (NP) in experiments at the intensity frontier [1–3]. Better measurements or upper limits on a plethora of rare (semi)leptonic and radiative K and B meson decays in forthcoming experiments, compared to precise Standard Model (SM) predictions, eventually will provide indirect indications of heavier particles with new interactions. Particularly sensitive for NP searches are those decays dominated by short-distance (SD) dynamics where the hadronic uncertainties are well under control. Conversely, precise measurements of these rare decays, combined with non-observation of NP effects, will furnish a better determination of flavor mixing parameters.

Very recently, the LHCb collaboration has reported a deficit in the ratio of muon to electron pairs produced in $B^+ \rightarrow K^+ \ell^+ \ell^-$ decays, in the $(1,6)$ GeV$^2$ region for the squared invariant-mass of the lepton pair [4]. This energy region is cleverly chosen as it excludes long-distance (LD) contributions associated to charmonium and light vector resonance production in $B^-$ decays followed by their conversion to lepton pairs (for definiteness, hereafter we will focus on decays of negatively charged $B$ mesons, unless otherwise indicated). The measured ratio $R_K \equiv B(\bar{B} \rightarrow K^0 \mu^+ \mu^-)/B(\bar{B} \rightarrow K^0 e^+ e^-) = (0.745^{+0.099}_{-0.074} \pm 0.036)$ [4], if confirmed in more refined measurements, would call for lepton universality violating interactions since in the SM $R_K = 1.0003 \pm 0.0001$ [5–8] for the energy region reported by LHCb [4]. Possible explanations involving NP interactions have been suggested as the source of non-universal leptonic interactions [9]. Interestingly, other anomalies have been reported in angular observables of related $B \rightarrow K^* \ell^+ \ell^-$ decays [10] which have been widely discussed in the literature [11].

In the SM, the semileptonic $B^+ \rightarrow P^+ \ell^+ \ell^-$ ($P = K, \pi$) decays are dominated by the SD $b \rightarrow s \ell^+ \ell^-$ transition [3, 8, 12–15]. This elementary process is induced by the electromagnetic and weak penguin (Figure 1a), as well as $W$ boson box (Figure 1b) diagram contributions, which are dominated by loops involving the top quark. In these processes, LD chiral corrections to the hadronic matrix element have been computed for soft momenta of light pseudoscalars (i.e. high-momentum dilepton pairs), where they amount to rather large $O(20, 30\%)$ corrections [16]. Our point here is that, although this kind of corrections will be much smaller on the other energy end (large recoil region for the heavy-to-light B-meson form factors [17]), the high precision of measurements as well as the sharp predictions of the dominating SD contributions suggests LD effects at the GeV scale be taken into account as well for these more energetic $P$ mesons.

As far as we know, the one-photon exchange Feynman diagrams shown in Figures 1c-1d provide a so far neglected LD contribution that must be included in the calculation of these exclusive semileptonic rare decays. As it was shown long ago [18], the $K^\pm \rightarrow \pi^\pm \ell^+ \ell^-$ rare semileptonic decays are dominated by the one-photon exchange contributions analogous to the ones of Figure 1c and 1d: the SD top-quark loop penguin and $W$-box contributions in that case are negligible small [1]. Owing to gauge-invariance, this LD contribution vanishes at lowest order in chiral perturbation theory but this is not the case at higher orders [18, 19]. According to the respective CKM mixing factors in the SD and LD amplitudes, one can
FIG. 1. Short-distance contributions to $B^- \rightarrow P^- \ell^+ \ell^-$ decays are shown in (a) (penguin) and (b) (W box) diagrams. The one-photon, LD, contributions are shown in (c) and (d). The full circle denotes the electromagnetic form factor of the charged pseudoscalar mesons. Other long-distance structure-dependent vector and axial-vector terms do not contribute due to gauge invariance [18].

envision a correction at the percent level for $P = K$ but a slightly higher effect could be expected for $P = \pi$ depending on the limits of the integrated observables. These additional LD contributions could in principle modify the $R_P$ value due to kinematical effects [20].

The purpose of this paper is to quantify these expectations and evaluate the contributions given by diagrams in Figures 1(c,d) for the $B^- \rightarrow P^- \ell^+ \ell^-$ decays for low values of the di-lepton pair invariant mass. Two different sets of inputs are used for the $\pi^-$ and $K^-$ electromagnetic form factors in order to estimate the systematic error of our computation. For these rare $B$ decays, it is found that the one-photon exchange diagrams do not modify lepton universality for the energy range measured by LHCb [4] and that they turn out to be very small in the $P = K$ case, although their effects in the $P = \pi$ rates are more significant.

The new LD contribution that we study has a different dependence on CKM mixing elements than the SD one and also carries a sizable strong phase. Therefore, there is an associated CP violation, which has not been considered before. We find measurable effects both for $P = \pi, K$ on the integrated CP asymmetry and encourage the LHCb and Belle-II Collaborations to measure such asymmetries.

1 We neglect a subdominant $O(G_F^2)$ neutrino-exchange amplitude. This one was checked to give a highly suppressed effect in $K^\pm \rightarrow \pi^\pm \ell^+ \ell^-$ decays [21].
II. DECAY AMPLITUDE

For completeness and later use, we will consider first the SD contribution as given in ref. [5]. Let us choose the following convention for the particle’s momenta:

\[ B^{-}(p_B) \to P^{-}(p_P)\ell^{+}(p_+)\ell^{-}(p_-). \]

The SD contributions (Figures 1a,b) to the decay amplitude can be written, to very good approximation as [5]:

\[ \mathcal{M}_{SD} = \frac{G_{F}q}{\sqrt{2} \pi} V_{ub} V_{ts}^{*} \xi_{F}(q^{2}) p_{B}^{\mu} \left[ F_{V}(q^{2})\bar{\ell}\gamma_{\mu}\ell + F_{A}(q^{2})\bar{\ell}\gamma_{\mu}\gamma_{5}\ell \right], \tag{1} \]

where \( q = p_+ + p_- \) is the total momentum of the lepton pair, the subindex in the CKM matrix element stands for \( D = d, s \) in the case of \( |\Delta S| = 0, 1 \) transitions, and \( F_{V} \approx C_{9} = 4.214 \) and \( F_{A} = C_{10} = -4.312 \) denote the vector and axial Wilson coefficients at NNLO [22] corresponding to the \( O^{Q}_{9} = (\bar{q}L\gamma_{\mu}b)(\bar{\ell}\gamma_{\mu}\ell) \) and \( O^{Q}_{10} = (\bar{q}L\gamma_{\mu}b)(\bar{\ell}\gamma_{\mu}\gamma_{5}\ell) \) operators in the effective weak Hamiltonian for the \( b \to q\ell^{+}\ell^{-} \) transition. For the \( P = \pi \) case, all three products of CKM matrix elements scale as \( \mathcal{O}(\lambda^{3}) \). However, the loop function continues to pick the top as the dominant contribution.\(^{2}\)

Although we are aware that more refined heavy-to-light meson form factors have been developed [12, 23], we will use the classical Ball and Zwicky form factors [24] for our estimates and include the corresponding uncertainty in the errors, as discussed below. The expressions for the \( q^{2}\)-dependent form factors are

\[ \xi_{\pi}(q^{2}) = \frac{0.918}{1 - q^{2}/(5.32 \text{ GeV})^{2}} - \frac{0.675}{1 - q^{2}/(6.18 \text{ GeV})^{2}} + \mathcal{P}_{\pi}(q^{2}), \]
\[ \xi_{K}(q^{2}) = \frac{0.0541}{1 - q^{2}/(5.41 \text{ GeV})^{2}} + \frac{0.2166}{1 - q^{2}/(5.41 \text{ GeV})^{2} + \mathcal{P}_{\pi}(q^{2})}, \]

where the polynomials \( \mathcal{P}_{P}(q^{2}) \) as well as the needed Gegenbauer moments at the required energy scale can be found in Refs. [5, 24].

The LD one-photon exchange amplitude is given by (neglecting the contribution of diagram 1c—which is of order \( (m_{P}/m_{B})^{2} \) with respect to 1d— is, however, a good approximation)

\[ \mathcal{M}_{LD} = \sqrt{2} G_{F}(4\pi\alpha) V_{ub} V_{ts}^{*} f_{B} f_{P} \frac{1}{q^{2}(m_{B}^{2} - m_{P}^{2})} \]
\[ \times \left[ M_{B}^{2} (F_{P}(q^{2}) - 1) - m_{P}^{2} (F_{B}(q^{2}) - 1) \right] p_{B}^{\mu} \bar{\ell}\gamma_{\mu}\ell, \tag{2} \]

where \( f_{X} \) denotes the decay constant of the pseudoscalar meson \( X \) according to the PDG [35] conventions for \( f_{K,\pi,B} \) and \( F_{X}(q^{2}) \) is the electromagnetic form factor of the corresponding meson. Note that the pure scalar QED (point meson approximation), as well as the structure-dependent vector and axial-vector terms, do not contribute to the amplitude as required by gauge-invariance [18]. Thus, the one-photon exchange contribution is sensitive to

\(^{2}\) We are not neglecting the charm and up quark contributions in our numerical analysis, however. We are also including subleading corrections with heavy-meson form factor ratios and other Wilson coefficients to eq. (1), see ref. [5].
the intermediate $q^2$ region of the pseudoscalar form factor, and eventually to their resonance structure.

Due to the vector nature of the one-photon exchange contribution, its amplitude can be absorbed into the contribution of the $O_9$ operator in the SD amplitude under the replacement

$$\xi_P(q^2)F_V \rightarrow \xi_P(q^2)F_V + \kappa_P m_B^2 \left[ \frac{F_P(q^2) - 1}{q^2} \right],$$

where

$$\kappa_P = -8\pi^2 \frac{V_{ub}V_{ub}^*}{V_{tb}V_{tb}^*} \frac{f_B f_P}{m_B^2 - m_P^2}.$$  

Note that $\kappa_P \sim \mathcal{O}(10^{-2}) \times \frac{V_{ub}V_{ub}^*}{V_{tb}V_{tb}^*}$ so that its influence is governed by the CKM factor which is $\sim \mathcal{O}(\lambda^0)$ for $P = \pi$ and $\mathcal{O}(\lambda^2)$ for $P = K$. This suggests a larger effect for $B^+ \rightarrow \pi^- \ell^+ \ell^-$ transitions but a detailed analysis of the electromagnetic meson form factors is needed to confirm these expectations.

III. PSEUDOSCALAR FORM FACTORS

For the electromagnetic form factors of the light pseudoscalar mesons ($P = \pi, K$) we have considered two approaches. On the one hand we have used form factors that are obtained within the frame of Resonance Chiral Theory (RχT) \[26\] and, on the other hand, the phenomenological form factors used by the BaBar Collaboration which employs the Gounaris-Sakurai (GS) parametrization \[27\]. The first approach has the advantage of providing a low-energy behaviour complying with the chiral limit of QCD \[25\], which is a must if we want to get close to thresholds in some of our evaluations \[3\]. Alternatively, the GS parametrizations include more excited resonances and, for this reason, are expected to give a closer description of data at higher energies. In the cases at hand, the bulk of the contribution (even for integrated observables starting at $q^2 = 1$ GeV$^2$) will be given by the $\phi(1020)$ ($\rho(770)$) meson exchange in the K (π) cases. Therefore, we should expect very similar results for the two approaches, RχT being more reliable for observables starting near thresholds and the GS for the much less important higher energy range (up to 6 or 8 GeV$^2$ depending on the channel).

Specifically for $F_\pi(q^2)$, we have used the parametrizations in the last two Refs. in \[30\], which include three isovector resonances ($\rho(770)$, $\rho(1450)$ and $\rho(1700)$) and the resummation of final state interactions encoded in the chiral loop functions. These representations provide good quality fits to Belle data \[28\] for $\tau^- \rightarrow \pi^- \pi^0 \nu_\tau$ decays. Additionally, we have included the characteristic $\rho(770) - \omega(782)$ interference appearing in the neutral channel by multiplying the $\rho(770)$ term by the factor

$$1 - \theta_{\rho\omega} \frac{q^2}{3M_\rho^2 M_\omega^2 - q^2 - iM_\omega \Gamma_\omega},$$

where

3 On the contrary, GS parametrizations introduce spurious phases below thresholds and the decoupling of excited resonances at low energies is spoiled.

4 The numerical values that we have employed in the present analysis are those given in these references.
with $\theta_{\rho\omega} = (-3.3 \pm 0.5) \cdot 10^{-3}$ GeV$^2$ [29]. Other isospin breaking corrections are neglected.

This parametrization is compared to BABAR data [31] in Figure 2, which are available for the energy region $2m_\pi \leq \sqrt{q^2} \leq 3$ GeV. There, the phenomenological GS parametrization includes an additional $\rho$-like excitation. Tiny differences between both parametrizations are seen in the region where the destructive interference between the $\rho(1450)$ and the $\rho(1700)$ resonances is stronger. The comparison also shows the effect of the $R\chi T$ parametrization lacking of the $\rho(2250)$ meson, which is clearly visible in the $(2, 2.3)$ GeV region. These minor differences are taken into account in the final quoted errors.

Analogously, we have considered two parametrization of $F_K(q^2)$ form factor. Since this form factor is completely dominated by the extremely narrow $\phi(1020)$ meson, we have considered the $R\chi T$ prediction with only one multiplet of resonances [32]. On the other hand, the BaBar Collaboration [33] has reported measurements of the kaon form factor from threshold up to 2.5 GeV. Since $|F_K(q^2)|^2$ drops by 6 orders of magnitude in going from the peak to 2.5 GeV (see Figure 3), it should be sufficient to include only the $\phi(1020)$ resonance in its parametrization.

As it can be observed in Figure 3, experimental data for $|F_K(q^2)|^2$ are reasonably well described from threshold up to approximately 1.3 GeV. Deviations at higher energies have a negligible impact in the integrated observables of rare $B$ decays. Above this energy, other resonance structures with very small (and alternating) peaks and dips around the single resonance queue can be observed. These, in turn, are well described using the parameterization.
FIG. 3. $R \chi T$ and GS parametrization (BaBar fit) of the electromagnetic kaon form factor as a function of $m_{ll} = \sqrt{q^2}$ are compared to experimental data from BABAR [33].

quoted in the BaBar paper [33] (which be fitted to the BaBar data), which includes two ϕ, three ρ and three ω excitations in addition to the single lightest vector meson multiplet included in the $R \chi T$ form factor.

Finally, the $B$-meson electromagnetic form factor contribution is suppressed by a factor $m_{B}^{2}/M_{B}^{2}$ in eq. (2) with respect to the ones of lighter mesons. In addition, a dynamical suppression of the form factor itself is expected according to the reduced charged radius of heavy mesons. Just in order to confirm this, we consider the effective Lagrangian coupling $D$-mesons to lighter mesons and external sources in ref. [36] generalized to include $B$ mesons.

It can be seen that, upon requiring a Brodsky-Lepage behaviour [37] of this form factor at infinite momentum transfer, the shape of $F_{B}(q^{2})$ is given just by flavour symmetry, yielding

$$F_{B}(q^{2}) = 1 + \frac{3}{2} q^{2} \left( \frac{1}{M_{\rho}^{2} - q^{2} - i M_{\rho} \Gamma_{\rho}(q^{2})} - \frac{1}{3(M_{\omega} - q^{2} - i M_{\omega} \Gamma_{\omega})} \right), \quad (6)$$

where $\Gamma_{\rho}(q^{2})$ can be found in Refs. [30]. Our numerical evaluations confirm that the effect of this form factor will be completely negligible in rates of rare $B^{-}$ mesons.

The spectra of the lepton pair, normalized to the $B^{-}$ meson decay width are plotted in Figure 4 for low values of the lepton-pair invariant mass. We observe that above the

5 Besides, in the limit of ideal $\omega(782) - \phi(1020)$ mixing the former state does not contribute to $F_{K}(q^{2})$ and only $\rho(770)$ and $\phi(1020)$ remain, with a prominent role of the latter.

6 The procedure employed is consistent with the heavy quark mass limit, as explained in the quoted reference.
muon-pair threshold both spectra are identical. Clearly, the integrated rates in the whole kinematical range would exhibit a trivial breaking of leptonic universality owing to the lower electronic threshold and its enhancement due to the $1/q^2$ dependence in the LD amplitude. However, this will happen both in the well-known SD as well as in the new LD contributions; thus only the numerical evaluation will tell if there is any additional measurable breaking of universality due to LD effects in the considered processes.
TABLE I. Integrated branching ratios of $B^- \to P^- \ell^+\ell^-$ decays for $P = \pi$ (left hand side) and $P = K$ (right hand side). We tabulate separately the short-distance (SD), long-distance (LD) and their interference contributions for the kinematical ranges of interest.

|            | $B^- \to \pi^- \ell^+\ell^-$ | $B^- \to K^- \ell^+\ell^-$ |
|------------|-------------------------------|-----------------------------|
|            | $0.05 \leq q^2 \leq 8 \text{ GeV}^2$ | $1 \leq q^2 \leq 8 \text{ GeV}^2$ | $1 \leq q^2 \leq 6 \text{ GeV}^2$ |
| LD         | $(9.16 \pm 0.15) \cdot 10^{-9}$ | $(5.47 \pm 0.05) \cdot 10^{-10}$ | $(1.70 \pm 0.21) \cdot 10^{-9}$ |
| interf.    | $(-2.62 \pm 0.13) \cdot 10^{-9}$ | $(-2^{+2}_{-2}) \cdot 10^{-10}$ | $(-6 \pm 2) \cdot 10^{-11}$ |
| SD         | $(9.83^{+1.49}_{-1.04}) \cdot 10^{-9}$ | $(8.71^{+1.35}_{-0.90}) \cdot 10^{-9}$ | $(1.90^{+0.69}_{-0.41}) \times 10^{-7}$ |
| Total      | $(1.64^{+0.15}_{-0.11}) \cdot 10^{-8}$ | $(9.06^{+1.36}_{-0.90}) \cdot 10^{-9}$ | $(1.92^{+0.69}_{-0.41}) \times 10^{-7}$ |

IV. BRANCHING FRACTIONS FOR $B^- \to P^- \ell^+\ell^-$

Searches for New Physics at large hadronic recoil in the $B^- \to K^- \ell^+\ell^-$ decays are restricted to the $(1, 6) \text{ GeV}^2$ range of $q^2$ [3]. Therefore we will stick to this region for this channel. The $B^- \to \pi^- \ell^+\ell^-$ decays have just been observed [38] and such studies have not taken place yet. We therefore include our results both for a range starting at $q^2 = 0.05 \text{ GeV}^2$ (basically the muon threshold) and at 1 GeV$^2$. In either case we cut the phase-space integration at 8 GeV$^2$ to avoid the charmonium region. In table I we show the corresponding branching ratios of LD, SD and their interference contributions by considering these different kinematical integration domains.

Several comments concerning these results are in order:

- Our results for the SD contribution to the $B^- \to \pi^- \ell^+\ell^-$ branching ratio are higher than those in ref. [15] because of the different heavy-meson form factors employed. Specifically, in this analysis form factors parameters were fitted to reproduce $B^+ \to \pi^0 \ell^+\nu_\ell$ data, resulting in smaller SD contributions than in other analyses [13, 14] or ours.

- Another source of difference in the SD contributions comes from the updated inputs we are using [34], while older PDG values were employed in earlier analyses. As a result, our numbers for the $\pi$ case are larger by $\sim 5\%$.

- We have not performed a dedicated study of the errors of the SD contributions. Errors quoted in table I are obtained rescaling the errors in Refs. [15] and [6] according to the different central values obtained by them and us. As discussed extensively in these references (see also [12]), the dominating error for the $K$ case comes from the heavy-meson vector form factor, while in the $\pi$ case the choice of the renormalization scale $\mu_b$ basically saturates the overall uncertainty (see, however, [11]).

- Our study of the LD contributions to $B^- \to P^- \ell^+\ell^-$ has been performed with two different sets of form factors in the $(1, 6) (K)$ and $(1, 8) (\pi)$ GeV$^2$ ranges. In the above-GeV intervals, the error has been estimated from the difference between these predictions. When including the region immediately above threshold we have only employed the set of chiral-based form factors estimating the error as the difference
between the results obtained using dispersive form factors and a Guerrero-Pich-like resummation \cite{30}. Analogous procedure has been employed in order to obtain our results for CP violation in the next section.

- The violations of lepton universality induced by kinematical effects on \( R_K \) and \( R_\pi \) are always given by the \( SD \) contribution. The \( LD \) modification is –in all the considered energy ranges– smaller than the error of the \( SD \) contribution. Therefore, \( R_K = 1.0003(1) \) in the (1, 6) GeV\(^2\) range \cite{3} and \( R_\pi = 1.0006(1) \) in the (1, 8) GeV\(^2\) range.

When we add the contributions of \( q^2 \) values above 8 GeV\(^2\), ref. \cite{15}, our branching fraction corresponding to the full kinematical domain becomes \( B(B^- \to \pi^- \ell^+\ell^-) = (2.6^{+0.4}_{-0.3}) \times 10^{-8} \). These results can be compared to available data from Refs. \cite{4,38}:

\[
B(B^- \to \pi^- \mu^+\mu^-) = (2.3 \pm 0.6 \pm 0.1) \times 10^{-8}, \tag{7}
\]

\[
B(B^- \to K^- e^+e^-) = (1.56^{+0.20}_{-0.16}) \times 10^{-7}, \quad \text{for } 1 < q^2 < 6 \text{ GeV}^2. \tag{8}
\]

Our results and experimental data agree within error bars. In the \( B^- \to K^- \ell^+\ell^- \) decays, current errors on the (completely dominating) \( SD \) contribution do not allow to tell whether there is a tension between SM prediction and the LHCb measurement or not. With the smaller error expected on the branching fraction of the \( B^- \to \pi^- \ell^+\ell^- \) decays from the next run of LHC measurements, one might be able to notice a tension between SM predictions and data. It must be noted that a reduction of the current error to less than a half will be able to pinpoint the LD contribution to these decays that we have been discussing.

V. CP VIOLATION

The LD one-photon exchange and the SD contributions to the amplitude of the \( B^\pm \to P^\pm \ell^+\ell^- \) decays have different weak and strong phases, as it can be easily checked from eqs. \((1)\) and \((2)\). Thus, a (partially integrated) CP asymmetry \(^8\)

\[
A_{CP}(P) = \frac{\Gamma(B^+ \to P^+ \ell^+\ell^-) - \Gamma(B^- \to P^- \ell^+\ell^-)}{\Gamma(B^+ \to P^+ \ell^+\ell^-) + \Gamma(B^- \to P^- \ell^+\ell^-)}, \tag{9}
\]

can be generated from the interference of diagrams shown in Figure 1. By inserting the amplitudes of LD and SD contributions in the previous expressions, it can be shown that the width difference has the form (as seen in table \(\|\) interferences are much smaller than the SD and LD contributions):

\[
\Delta_{CP} = \left[ \Gamma(B^+ \to P^+ \ell^+\ell^-) - \Gamma(B^- \to P^- \ell^+\ell^-) \right] = -32\alpha^2 G_F f_P f_B \Im \{ V_{tb} V_{tb}^* V_{ub} V_{ub}^* \}
\]
\[
\times \int dq^2 \int ds_{12} \frac{1}{q^2(M_B^2 - m_P^2)} \left[ 2(P_B \cdot P_\perp)(P_B \cdot P_\perp) - \frac{M_B^2 q^2}{2} \right] \tag{10}
\]
\[
\times \Im \left\{ \xi_F(q^2) F_V(q^2) \left[ M_B^2 - (F_P(q^2) - 1) - m_P^2 (F_B(q^2) - 1) \right] \right\},
\]

\(^7\) A possible large CP violation in the \( B^\pm \to \pi^\pm \ell^+\ell^- \) decays induced by weak annihilation one-photon exchange contribution was proposed for the first time in ref. \cite{39}.

\(^8\) Other CP violating observables can be analyzed analogously.
TABLE II. Our results for $A_{CP}(\pi)$ (in %) are compared to those in ref. [39] for different energy ranges.

| $(q_{\text{min}}^2, q_{\text{max}}^2)$ | Ref. [39] | Our results |
|--------------------------------------|-----------|-------------|
| (1, 8) GeV$^2$                       | 13 ± 2    | 2.5 ± 1.5   |
| (1, 6) GeV$^2$                       | 16 ± 2    | 3.3 ± 2.5   |
| (2, 6) GeV$^2$                       | 13$^+2_{-3}$ | 4.0 ± 2.6   |

where $s_{12} = (p_K + p_\pi)^2$. A straightforward evaluation of the above expression leads to:

$$A_{CP}(P) = \begin{cases} 
(14 \pm 2)\%, & \text{for } P = \pi, \ 0.05 \le q^2 \le 8 \text{ GeV}^2, \\
(2.5 \pm 1.5)\%, & \text{for } P = \pi, \ 1 \le q^2 \le 8 \text{ GeV}^2, \\
(-1.3 \pm 0.4)\%, & \text{for } P = K, \ 1 \le q^2 \le 6 \text{ GeV}^2.
\end{cases} \quad (11)$$

There is no gain in the case $P = K, \ 0.05 \le q^2 \le 6 \text{ GeV}^2$, where the amount of CP violation is reduced to about one half and the corresponding error is only barely smaller. The quoted error in our results stems from the systematic error attributed to parametrizations of the light meson electromagnetic form factors.

In ref. [39], using QCD factorization, the $A_{CP}(\pi)$ asymmetry was predicted to be larger on the basis of a big absorptive contribution generated by a $u$ quark (inside the B meson) which is on-shell after radiating a photon. Our results are compared to theirs in table II. We switch the signs of their results because our conventions for defining $A_{CP}(\pi)$ are opposite. We note that, within our approach, the solely contribution of the B-meson electromagnetic form factor to the CP asymmetry is completely negligible, at the $O(10^{-4})$ level.

This $O(\text{few }\%)$ CP violating figures shall enhance the case for their measurements. Current values at the PDG are well compatible with zero [35] in $B^\pm \to K^\pm \ell^+\ell^-$ (and also in $B^\pm \to K^{*\pm} \ell^+\ell^-$) decays, while this observable is not yet reported in the $B^\pm \to \pi^\pm \ell^+\ell^-$ case.

Therefore, despite the fact that the tree-level one-photon exchange diagrams give a small contribution to the decay rates (specially for the $K$ case), they can generate a non-negligible CP asymmetry within the Standard Model. This CP asymmetry altogether with measurements of the decay rates can be used as a test of New Physics in the rare $B^\pm \to P^\pm \ell^+\ell^-$ decays. This makes us emphasize the need of a dedicated measurement of these observables in the next LHC run at LHCb and in the forthcoming Belle-II experiment.

VI. CONCLUSIONS

In this paper we have considered the one-photon exchange contribution to the rare $B^\pm \to P^\pm \ell^+\ell^-$ decays, with $P = \pi$ or $K$. Its effects in the decay rates of the $P = K$ case turn out to be of order 1% with respect to the (top quark loop dominated) SD contribution for the range $1 \le q^2 \le 6 \text{ GeV}^2$ of the squared lepton pair invariant mass. We do not foresee forthcoming measurements being sensitive to this contribution in the near future. On the contrary, this fact confirms the suitability of this range for new physics searches.

In the case of a $\pi^\pm$ meson in the final state, the corresponding effect turns out to be significant in integrated observables starting close to threshold. This suggests to take –in
analogy to the case of a final state with $K$ – the range $1 \leq q^2 \leq 8 \text{ GeV}^2$ for precision measurements, since the LD contribution is reduced to less than $10\%$ with a negligible uncertainty in that interval. On the other hand, more refined measurements of the fully integrated branching fraction for this decay could be sensitive to our contribution once the error is reduced below a half of the current uncertainty.

Interestingly, the different weak and strong phases of the SD and LD (one-photon exchange) contributions are capable to generate a CP asymmetry. Again, this CP asymmetry is large in the case of a pion in the final state for $0.05 \leq q^2 \leq 8 \text{ GeV}^2$ values of the lepton-pair invariant mass, but also sizable and worth to measure in the $1 \leq q^2 \leq 8 \text{ GeV}^2$ interval. For the kaon case, the range $1 \leq q^2 \leq 6 \text{ GeV}^2$ is optimal for such a search. More refined measurements of this CP asymmetry and of the magnitudes of the decay rates at LHCb and future $B$-superfactories can provide another non-trivial test of the Standard Model or may furnish indications of New Physics.

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