Binary collisions in Fermi systems obey two fundamental symmetries corresponding to the space and time inversion and to the interchange of particles and holes. We show that beyond the local and instant approximation of scattering-in and -out integrals of a kinetic equation, only one of symmetries can be explicit while the other has to be covered by a constraint. This constraint, derived from the optical theorem, allows one to convert at need an explicit particle-hole symmetric form to the space-time symmetric form and vice versa. We implement this constraint to heavy ion reactions, where simulation algorithms require the space-time symmetry while former theories offer kinetic equations with the explicit particle-hole symmetry.

Realistic simulations of heavy ion reactions in the non-relativistic energy range have achieved a level on which basic problems with a reliable treatment of the Boltzmann equation (BE) or of the quantum molecular dynamics (QMD) are already settled. The field is now open for a refinement of underlying equations of motion. Three kinds of improvements are at hand: momentum-dependent mean field, medium effects on binary collisions, and non-local binary collisions. If implemented separately, each kind is covered by a corresponding theory. The mean field follows from the Landau theory of Fermi liquids. The medium effects have been derived within the many-body statistics. The non-local collisions are supported by the theory of gases. In realistic studies all the three kinds should appear together, but none of the mentioned theories covers this general case.

The kinetic equation with the Landau-type mean-field and in-medium non-local collisions has been derived from non-equilibrium Green’s functions. Although this kinetic equation includes all the three kinds of corrections, it is not suited for the numerical simulations because of the symmetry reason. In the BE or QMD simulations each single collision event plays a dual role of the scattering-in and -out, so that the -in process naturally emerges as the space-time mirror of the -out process. Conversely, only theories offering non-local collisions with the space-time symmetry can be implemented. In the kinetic equation of the non-local corrections to scattering-out are not the space-time mirror of the scattering-in as a consequence of the particle-hole symmetry of non-equilibrium Green’s functions. Indeed, the particle-hole symmetry demands that hole-hole collisions (scattering-out) have the same scattering rate, including non-local corrections, as the particle-particle collisions (scattering-in). Accordingly, an explicit particle-hole symmetry excludes an explicit space-time symmetry.

The explicit space-time symmetry is absent not only in but in any theoretical treatment of non-local scattering integrals. The problem has not been recognized, however, due to non-transparent forms of non-local corrections (in fact not aimed for simulations). We anticipate that in all mentioned approaches the collisions obey both symmetries, the particle-hole symmetry is explicit while the space-time symmetry is hidden in the optical theorem. In this letter we derive a constraint with which one can interchange roles of symmetries making the space-time one explicit and the particle-hole one hidden. Although we focus on simulations of heavy ion reactions, the discussed problem exceeds merits of the nuclear physics as the symmetry of non-local collisions in Fermi systems is a general question. The key step of our approach can be easily translated to any formalism favoured in other fields of physics because it is a straightforward substitution based on the optical theorem.

To demonstrate the conflict between the particle-hole and the space-time symmetry, we discuss the system of hard spheres used in model studies of heavy ion reactions without and with the Pauli blocking. Without the Pauli blocking the kinetic equation proposed by Enskog has non-local scattering integrals in which two colliding particles are displaced by their diameter $D$.

$$\frac{\partial f_1}{\partial t} + \frac{k}{m} \frac{\partial f_1}{\partial r} = \int dP \left[ f_3 - f_1 f^* \right].$$  (1)

Here $f_1 = f(k, r, t)$, $f_2 = f(p, r + \Delta_2, t)$, $f_3 = f(k - q, r, t)$, $f_4 = f(p + q, r - \Delta_2, t)$, $dP$ is the differential cross section, and $\Delta_2 = \frac{m}{|q|} D$ is the displacement. The essential part of the Enskog approach is a symmetry between the scattering-in and -out. If the scattering-out describes a process $k, p \rightarrow k - q, p + q$, its conjugate scattering-in process, $k - q, p + q \rightarrow k, p$ is its space and time mirror. Both symmetry operations are necessary. The time inversion reverses the process but it also flips directions of momenta which are then flipped back by space inversion. Due to the space inversion, the displacements in...
the scattering-in and -out integrals have opposite signs.

Let us try to introduce the Pauli blocking of final states by hole distributions. From phase-space trajectories one expects that the final states have the same displacements as the initial ones, therefore the right hand side of (1) should be extended as

\[ \int dP \left[ (1 - f_1)(1 - f_2^-)f_3f_4^- - f_1f_2^- (1 - f_3)(1 - f_4^-) \right]. \quad (2) \]

This scattering integral violates the particle-hole symmetry. Indeed, with the interchange of particles and holes the scattering-in and -out interchange their roles, therefore the directions of displacements appear reversed. This demonstrates the conflict between space-time and particle-hole symmetries.

The conflict follows from incompatible concepts of the scattering-out in the classical and the quantum statistics. One has to recognize that a realistic collision has a finite duration and to compare these two concepts in the time picture. Within the classical picture of the scattering-out, the instant of the transition is attributed to the beginning of the collision when the particle enters particle-particle correlations. Related to the instant of transition, the collision process happens in the past, i.e., before the instant of the transition. The classical particle faces its partner of the scattering-out in front of it, while the quantum hole leaves its hole-partner behind.

To break the ties of causality, we need the anti-causal expansion. In the optical theorem, the anti-causal side, the time cuts of the retarded and advanced functions restrict internal time integrals to the past while on the right hand side to the future. As we will see, the change in the order of operators results in the space-time inversion of non-local corrections.

Further progress depends on the approximation of the T-matrix. Let us first approximate the T-matrix by Bruckner’s reaction matrix for which the two-particle spectral function, \( A = (G^*G^>) \), allows only for unoccupied states. The optical theorem then yields

\[
\{G^>, \Sigma^<\} - \{G^<, \Sigma^>\} = \{G^>, G^> \circ T_R(G^<G^<)T_A\} - \{G^<, G^< \circ T_A(G^>G^>)T_R\}. \quad (5)
\]

The first and second terms correspond to the scattering-in and -out integrals, respectively. Expression (5) has the desired explicit space-time symmetry contrary to explicit particle-hole symmetry of (1).

To see it in detail, we substitute (1) or (2) into the Kadanoff and Baym equation and apply the quasiclassical and quasiparticle approximations. Keeping internal gradients of the scattering integrals to the linear order, one arrives at the kinetic equation

\[
\frac{\partial f_1}{\partial t} + \frac{\partial \varepsilon_1}{\partial k} \frac{\partial f_1}{\partial r} + \frac{\partial \varepsilon_1}{\partial k} \frac{\partial f_1}{\partial k} = \int dP^- f_3^- f_4^- (1 - f_1)(1 - f_2^-)
- \int dP^\pm (1 - f_1^\pm)(1 - f_2^-) f_3 f_4^\pm.
\]

* The theorem represents two alternative expressions of the anti-hermitian part of the T-matrix, \( M = i(T_R - T_A) \). From the ladder equation in the differential form, \( T_{R,A}^{-1} = V - G_{R,A} \), with \( G_{R,A} \) given by the time cut of the spectral function, \( A = i(G_R - G_A) \), follows \( i(T_R^{-1} - T_A^{-1}) = -\mathbb{A} \). Multiplying \( M \) by \( T_R^{-1} \) one finds \( T_R^{-1}M = i - iT_R^{-1}T_A \). Substituting \( T_R^{-1} = T_A^{-1} + iT \) one finds the causal side of the optical theorem \( M = T_R^{-1} \). The anti-causal side, \( M = T_A^{-1}T_R \), follows from \( M T_R^{-1} = i - iT_A T_R^{-1} \) and the same substitution.
where superscripts + and − correspond to (9) and (12), respectively, and denote signs of non-local corrections: $f_1 \equiv f(k, r, t)$, $f_2^\pm \equiv f(p, r \pm \Delta_2, t)$, $f_3^\pm \equiv f(k \pm q \pm \Delta_K, r \pm \Delta_3, t \pm \Delta_3)$, and $f_4^\pm \equiv f(p + q \pm \Delta_K, r \pm \Delta_4, t \pm \Delta_4)$. The $dP^+$ is obtained from $dP^-$ by the flip of signs of all $\Delta$’s involved. More details about the limiting procedure and the differential cross section $dP^-$ the reader can find in [12]. One can see that with superscript − the scattering-out is the particle-hole mirror of the scattering-in, while with + it is the space-time mirror. All non-local corrections are given by derivatives of the scattering phase shift $\phi = \text{Im} \ln T_R(\Omega, k, p, q, t, r)$ [12, 14, 15].

$$\Delta_1 = \frac{\partial \phi}{\partial \Omega}, \quad \Delta_K = \frac{1}{2} \frac{\partial \phi}{\partial r}, \quad \Delta_3 = -\frac{\partial \phi}{\partial k},$$

$$\Delta_2 = \frac{\partial \phi}{\partial p} - \frac{\partial \phi}{\partial q}, \quad \Delta_4 = -\frac{\partial \phi}{\partial k} - \frac{\partial \phi}{\partial q} \quad (7)$$

The collision is of finite duration $\Delta_t$ during which particles can gain momentum $\Delta_K$ due to the medium effect on the collision. Three displacements $\Delta_{2,3,4}$ correspond to initial and final positions of two colliding particles, one position being fixed at $r$. The kinetic equation is justified to the linear terms in the $\Delta$’s which directly corresponds to the linear approximation in gradients.

Without many algebraic details it can be indicated why the change of the causal picture into the anti-causal one results in the flipped signs of all non-local corrections. Writing the T-matrices as products of the amplitude and the phase, $T_R = |T|e^{i\phi}$ and $T_A = |T|e^{-i\phi}$, one can see that the interchange of retarded and advanced T-matrices merely flips the sign of the phase shift $\phi$. As all $\Delta$’s depend linearly on $\phi$, the non-local corrections to the anti-causal scattering-out have signs reversed compared to the causal one.

Equivalency of both forms of kinetic equation (9) can be expressed by a simple constraint. Expanding the scattering-out to the linear terms,

$$\int dP^\pm (1 - f_3^\pm)(1 - f_2^\pm) f_1 f_2^\pm = I_{\text{out}} \pm \sum \alpha C_{\alpha}^\alpha \Delta_\alpha, \quad (8)$$

where $I_{\text{out}}$ is the local and instant scattering-out of the BE and $C_{\alpha}^\alpha$ are coefficients of non-local corrections, one finds that amplitudes of anti-causal and causal coefficients equal while signs are opposite. Since both forms are equivalent, one is satisfied with the linearization of the scattering-out.

$$\sum \alpha C_{\alpha}^\alpha \Delta_\alpha = 0. \quad (9)$$

Perhaps, it is not necessary to remind that constraint (9) is a direct consequence of the optical theorem (9). The constraint gives us a freedom to express the non-local scattering-out as the particle-hole or the space-time mirror of the scattering-in, or if of interest, to suppress the non-local corrections to the scattering-out at all.

Now the theory is ready for implementation into the BE or QMD simulations. In Fig. 1 we present the results of the BE simulation obtained with a modified BUU code of (9). The non-local corrections are evaluated from formulas (9) and the Paris potential with no adjustable parameter. In accordance with (9), for the scattering-out we have used the standard local approximation of the cross section and the local Pauli blocking. In Fig. 1 one can see that the production of high energetic protons is enhanced due to non-local corrections. The effect of non-local collisions is more pronounced for larger impact parameter and for forward/backward angles where it achieves experimentally detectable values. A comparison with experiment and more technical details will be published elsewhere [17] together with the non-local QMD simulations.

![Fig. 1. The angular resolved and absolute (lower right) BUU proton spectra for central collision of $^{209}$Bi+$^{136}$Xe at 55 MeV/A with and without non-local corrections. Three different impact parameter are chosen correspondingly. The local BUU code has been kindly provided us by W. Bauer.](image)
tion because the conservation laws require equal scattering rates of in-and-out processes but for $|T| \neq |T'|$ the rates result different.

The full particle-hole symmetry is obtained within the Galitskii-Feynman approximation for which the two-particle spectral function $\mathcal{A} = (G^+ G^+ - G^- G^-)$ corresponds to the Pauli blocking of internal states $(1-f)(1-f) - ff = 1 - f - f$. From the optical theorem, one then finds a modification of

$$
\int dP^\pm (1 - f_{3}^\pm - f_{4}^\pm) f_{1} f_{2}^\pm = I_{1-f-f} \pm \sum_{\alpha} C_{1-f-f}^\alpha \Delta_{\alpha},
$$

(10)

with a constraint $\sum_{\alpha} C_{1-f-f}^\alpha \Delta_{\alpha} = 0$. This allows us to write two equivalent kinetic equations with either space-time or particle-hole symmetric scattering integrals

$$
\frac{\partial f_{1}}{\partial t} + \frac{\partial \bar{\varepsilon}_{1}}{\partial k} \frac{\partial f_{1}}{\partial r} - \frac{\partial \bar{\varepsilon}_{1}}{\partial r} \frac{\partial f_{1}}{\partial k} = \int dP^- f_{3} f_{4}^\pm (1 - f_{1} - f_{2}^\pm)
$$

$$
- \int dP^\pm (1 - f_{3}^\pm - f_{4}^\pm) f_{1} f_{2}^\pm.
$$

(11)

Again, only one of the symmetries can be explicit, nevertheless, the constraint guarantees that both symmetries are fulfilled at the same time.

The kinetic equation is not suitable for numerical treatment by recent codes, because of negative scattering rates for $1 - f - f < 0$. Recent codes deal exclusively with the Pauli blocking of form $(1 - f)(1 - f)$ which keep the scattering rates positive. Numerical studies of non-local corrections are thus limited to the Bruckner approximation.

We note that on the level of analytic theories, two equivalent kinetic equations settle the conflict between the space-time and particle-hole symmetries. The space-time symmetric form is more convenient for studies of the conservation laws because it allows one to employ symmetry operations developed within the classical theory of gases. Since the non-local corrections to the scattering integrals make the conservation laws appreciably more complex, see the explicit space-time symmetry offers a vital simplification.

In summary, for non-local collisions the scattering integrals cannot have explicit space-time and particle-hole symmetries at the same time. If one of the symmetries is made explicit, the other is covered by a constraint following from the optical theorem. Under the condition that the Pauli blocking of internal and final states are consistent, this constraint simply states that the non-local corrections to the scattering-out vanish. From the practical point of view, this constraint gives us a freedom to select the explicit symmetry of non-local corrections in the scattering-out at convenience, or to suppress them.

With recent simulation algorithms, tractable non-local corrections are restricted to Bruckner’s approximation of the T-matrix. We do not find this limitation discouraging since Bruckner’s theory has been quite successful in studies of the equilibrium nuclear matter.

This work was supported from the GAASCR, Nr. A1010806, the GACR, Nos. 202960098 and 202960021, the BMBF, Nr. 06R0884, and the Max-Planck-Society.

[1] G. F. Bertsch and S. D. Gupta, Phys. Rep. 160, 189 (1988).
[2] J. Aichelin, Phys. Rep. 202, 235 (1991).
[3] C. Gale, G. Bertsch and S. Das Gupta, Phys. Rev. C 35, 1666 (1987).
[4] T. Alm, G. Röpke, A. Schnell, N. H. Kwong and H. S. Köhler, Phys. Rev. C 53, 2181 (1996).
[5] E. C. Halbert, Phys. Rev. C 23, 295 (1981).
[6] R. Malfliet, Nucl. Phys. A 420, 621 (1984).
[7] G. Kortemeyer, F. Daiflin and W. Bauer, Phys. Lett. B 374, 25 (1996).
[8] P. Danielewicz, Ann. Phys. (NY) 152, 239 (1984).
[9] W. Botermans and R. Malfliet, Phys. Rep. 198, 115 (1990).
[10] T. Alm, G. Röpke and M. Schmidt, Phys. Rev. C 50, 31 (1994).
[11] S. Chapman and T. G. Cowling, The Mathematical Theory of Non-uniform Gases, (Cambridge University Press, Third edition 1990).
[12] V. Špička, P. Lipavský and K. Morawetz, Phys. Lett. A 240, 160 (1998).
[13] K. Baerwinkel, Z. Naturforsch. 24a 22 and 38 (1969).
[14] F. Laloe, J. Phys. France 50, 1851 (1989).
[15] P. J. Nacher, G. Tastevin and F. Laloe, Ann. Phys. (Leipzig) 48, 149 (1991); J. Phys. I 1, 181 (1991).
[16] Th. Bornath, D. Kremp, W. D. Kraeft and M. Schlange, Phys. Rev. E 54, 3274 (1996).
[17] K. Morawetz, V. Špička, P. Lipavský, G. Kortemeyer, Ch. Kuhrts and R. Nebauer, subm. nucl-th/9810043.