Electromagnetic Form Factors of Nucleons with QCD Constraints
Systematic Study of the Space and Time-like Regions

Susumu FURUICHI*
Department of Physics, Rikkyo University, Toshima Tokyo 171-8501, Japan

Hirohisa ISHIKAWA†
Department of EconomyCMikei University, Urayasu Chiba, 279-8550, Japan

Keiji WATANABE‡
Department of Physics, Meisei University, Hino Tokyo 191-8506, Japan

Abstract

Elastic electromagnetic form factors of nucleons are investigated both for the time-like and the space-like momentums under the condition that the QCD constraints are satisfied asymptotically. The unsubtracted dispersion relation with the superconvergence conditions are used as a realization of the QCD conditions. The experimental data are analyzed by using the dispersion formula and it is shown that the calculated form factors reproduce the experimental data reasonably well.

1 Introduction

We have investigated the nucleon electromagnetic form factors by using the dispersion relation, which worked in understanding the low energy data very well. We were able to realize the low energy experimental data for the space-like momentum transfer [1]-[3]. For the low momentum, the vector dominance model is qualitatively valid in explaining the electromagnetic form factors of nucleons except for the \(\rho\) meson mass, which should be taken much smaller than the experimental value. The problem of the \(\rho\) meson mass was solved by taking into account the uncorrelated two pion contribution. The dispersion relation turned out to be very effective for this purpose.

*Present address: Sengencho 3-2-6, Higashikurume Tokyo 203-0012
†e-mail address: ishikawa@meikai.ac.jp
‡e-mail address keijiwatanabe888@yahoo.co.jp; Present address: Akazutumi, 5-36-2, Setagaya Tokyo 156-0044
Phenomenologically, the magnetic and electric form factors of nucleons, \( G_M(t) \) and \( G_E(t) \) respectively, are proportional to the dipole formula \( G_D(t) = 1/(1 + |t|/0.71)^2 \), where \( t \) is the squared space-like momentum transfer expressed in the unit of (GeV/c)^2. The dipole formula represents the experimental data of form factors fairly well for large range of momentum.

To be precise, the experimental data, however, decrease more rapidly than the dipole formula. This is compatible with the prediction of perturbative QCD (PQCD), where the elastic form factors of hardons decrease asymptotically for large squared momentum as compared with the dipole formula [5]: For the boson form factors, \( F(t) \to \text{const}/\ln|t| \) for \( |t| \to \infty \) and for the nucleon form factors, \( F_1^N \) and \( F_2^N \), the charge and magnetic moment form factors, respectively, decrease as \( F_1^N(t) \to \text{const} \frac{1}{t^2} \ln|t|^{-\gamma} \) and \( F_2^N(t) \to \text{const} \frac{1}{t^3} \ln|t|^{-\gamma} \) with \( \gamma \geq 2 \) being a constant. Consequently, \( G_M \) and \( G_E \) decrease as \( t^{-2} \ln|t|^{-\gamma} \).

We showed that the QCD conditions are incorporated by assuming superconvergent dispersion relation (SCDR) for the form factors and investigated the pion and kaon electromagnetic form factors. It was shown that the experimental data are reproduced both for the space-like and the time-like momentum [12, 13, 14].

Different from the boson form factors, for the nucleons the unphysical regions of absorptive parts are not observed for \( s < 4m^2 \), with \( m \) being the nucleon mass. This makes the problem of the nucleon form factors difficult, as was observed by R. Wilson in his review article [4], in which he emphasized the importance of systematic study of the time-like and space-like regions for the nucleon form factors.

It is the purpose of this paper to examine the nucleon electromagnetic form factors to see if it is possible to realize the experimental data with the QCD constraints satisfied, where the space-like and time-like regions are treated on equal footing in the chi square analysis.

Organization of this paper is given as follows: In Sec.2 we summarize on the dispersion relation for the nucleon electromagnetic form factors with the QCD condition imposed. In Sec.3, imaginary parts of the form factors are given for the low, intermediate and asymptotic regions. In Sec.4 remarks concerning numerical analysis are given and the numerical results are summarized in Sec.5. The final section is devoted to general discussions.

### 2 Dispersion relation for the nucleon form factors

As is mentioned in Sec.1, the electromagnetic form factors approach zero asymptotically for \( t \to \infty \). Therefore, we may assume the unsubtracted dispersion relations for the charge and magnetic moment form factors \( F_1^I \) and \( F_2^I \), respectively, with \( I \) denoting the isospin state \( I = 0, 1 \). That is,

\[
F_i^I = \frac{1}{\pi} \int_{t_0}^{\infty} dt' \frac{\text{Im} F_i^I(t')}{t' - t},
\]

\( ^1 \)We adopt the convention \( t < 0 \) for the space-like and \( t > 0 \) for the time-like momentum. We write \( t = -Q^2 \) for the space-like momentum and \( t = s > 0 \) for the time-like momentum.
where the threshold is \( t_0 = 4\mu^2 \). Here \( \mu \) is the pion mass being taken as the average of the neutral and charged pions.

We briefly summarize the asymptotic theorems which are used to incorporate the constraints of PQCD [6], where the proof is given in Ref. [12]. Let \( F(t) \) satisfy the dispersion relation (1), and \( \text{Im} F \) be given as

\[
\text{Im} F(t') = \frac{c}{\ln(t'/Q_0^2)} + O\left(\frac{1}{\ln(t'/Q_0^2)^{\gamma+1}}\right)
\]

(2)

for \( t \to \infty \) with \( \gamma > 1 \). Then \( F(t) \) approaches

\[
F(t) = \frac{1}{\pi} \int_{t_0}^{\infty} dt' \frac{c}{(t'-t)[\ln(t'/Q_0^2)]^\gamma} \rightarrow \frac{c}{\pi(\gamma-1)\ln(|t|/Q_0^2)^{\gamma-1}}
\]

(3)

for \( t \to \pm \infty \).

Generally, if \( \text{Im} F(t) \) tends to

\[
t'^{n+1} \text{Im} F(t') \rightarrow \frac{c}{\ln(t'/Q_0^2)^\gamma} + O\left(\frac{1}{\ln(t'/Q_0^2)^{\gamma+1}}\right)
\]

(4)

for \( t \to \pm \infty \) and the superconvergence conditions

\[
\int_{t_0}^{\infty} dt' t^k \text{Im} F(t') = 0, \quad k = 0, 1, \ldots, n,
\]

(5)

are satisfied, \( F(t) \) given by (1) approaches for \( t \to \pm \infty \)

\[
F(t) = \frac{1}{\pi} \int_{t_0}^{\infty} dt' \text{Im} F(t') \frac{c}{t'^{n+1} \ln(|t|/Q_0^2)^{\gamma-1}},
\]

(6)

which can be proved by using (3), (1) and (5) together with the identity

\[
\frac{1}{t' - t} = -\frac{1}{t} \left(1 + \frac{t'}{t} + \cdots + \left(\frac{t'}{t}\right)^n\right) + \frac{1}{t^{n+1}} \frac{t'^{n+1}}{t' - t}.
\]

The QCD constraints for the nucleon form factors are, therefore, attained by assuming the unsubtracted dispersion relation and the superconvergence conditions for \( \text{Im} F_i^I \)

\[
\frac{1}{\pi} \int_{t_0}^{\infty} dt' \text{Im} F_i^I (t') = \frac{1}{\pi} \int_{t_0}^{\infty} dt' t' \text{Im} F_i^I (t') = 0,
\]

(7)

\[
\frac{1}{\pi} \int_{t_0}^{\infty} dt' \text{Im} F_2^I (t') = \frac{1}{\pi} \int_{t_0}^{\infty} dt' t' \text{Im} F_2^I (t')
\]

= \frac{1}{\pi} \int_{t_0}^{\infty} dt' t'^2 \text{Im} F_2^I (t') = 0,
\]

(8)

where \( \text{Im} F_i^I (t') \) satisfies the asymptotic conditions for \( t' \to \infty \)

\[
t'^i \text{Im} F_i^I (t') \to \text{const}/[\ln(t'/Q_0^2)]^\gamma \quad (i = 1, 2).
\]

(9)
$Q_0$ and $\gamma(\geq 2)$ are constants, the latter of which is written in terms of the anomalous dimension of the renormalization group in QCD.

In addition to the conditions (7) and (8) we impose the normalization conditions at $t = 0$:

$$\frac{1}{2} = \frac{1}{\pi} \int_{t_0}^{\infty} dt' \text{Im} F_1^I(t')/t',$$

$$g^I = \frac{1}{\pi} \int_{t_0}^{\infty} dt' \text{Im} F_2^I(t')/t',$$

where $g^I$ are the anomalous moments of nucleons with the isospin $I$.

### 3 Imaginary part of the form factors

Let us discuss the imaginary parts of nucleon form factors, which are broken up into three parts: The low momentum, the intermediate, and the asymptotic regions.

#### 3.1 Low momentum region

The imaginary parts of the charge and magnetic moment form factors, $\text{Im} F_i^V$, are given in terms two pion contributions

\[
\text{Im}[F_1^V(t)/e] = \frac{m}{2} \left( \frac{t - 4\mu^2}{4m^2 - t} \right)^{1/2} \times \text{Re} \left[ M^*(t) \left\{ f_+^{(-1)}(t) - \frac{t}{4m^2 \sqrt{2}} f_-^{(-1)}(t) \right\} \right],
\]

\[
\text{Im}[2mF_2^V(t)/e] = \frac{m}{2} \left( \frac{t - 4\mu^2}{4m^2 - t} \right)^{1/2} \times \text{Re} \left[ M^* \left\{ \frac{m}{\sqrt{2}} f_-^{(-1)}(t) - f_+^{(-1)}(t) \right\} \right],
\]

where $f_\pm^{(-1)}(t)$ are helicity amplitudes for $\pi\pi \leftrightarrow N\bar{N}$, $M(t)$ is the pion form factor and $\mu$ is the pion mass. For the helicity amplitudes we use the numerical values given by Höhler and Schopper [5] and parameterize $M(t)$ according to them.

\[
M(t) = t_\rho \{ 1 + (\Gamma_\rho/m_\rho d) \} [t_\rho - t - im_\rho^2 \Gamma_\rho (q_\rho/q_\rho)^3 \sqrt{t}]^{-1},
\]

where $m_\rho$ and $\Gamma_\rho$ are the $\rho$ meson mass and width respectively and

\[
t_\rho = m_\rho^2, \quad q_\rho = \sqrt{t_\rho - \mu^2}, \quad d = \frac{3\mu^2}{\pi t_\rho} \ln \frac{m_\rho + 2q_\rho}{2\mu} + \frac{m_\rho}{2\pi q_\rho} \left( 1 - \frac{2\mu^2}{t_\rho} \right).
\]

The imaginary parts thus obtained are denoted as $\text{Im} F_i^H (i = 1, 2)$ hereafter. It must be remarked that the $\rho$ meson contribution is included in the helicity amplitudes of Ref. [5].
3.2 Intermediate region

The intermediate states $4\mu^2 \leq t \leq \Lambda^2$ are approximated by the addition of the Breit-Wigner terms. with the imaginary part parameterized as follow:

$$\text{Im} f_R^{BW}(t) = \frac{g}{(t-M_R^2)^2 + g^2},$$

where

$$g = \frac{\Gamma M_R^2 (M_R^2 + t_{res})^3}{t_{res}^2 (M_R^2 - t_0)^{3/2}} \sqrt{\frac{(t-t_0)^3}{t}} \frac{t^2}{(t+t_{res})^3},$$

where $M_R$ and $\Gamma$ are the mass and width of resonance, respectively. $t_0$ is the threshold $t_0 = 4\mu^2$ and $t_{res}$, being treated as an adjustable parameter, is introduced to cut-off the Breit-Wigner formula.

We write the intermediate part as the summation of resonances

$$\text{Im} F_N^{BW,I} = \sum_n a_n I f_{nR},$$

where $I$ is the isospin and $n$ is the labeling of resonances (see Table I). Here the suffix $i$ denotes $i = 1, 2$, corresponding to the charge and magnetic moment form factors $F_N^1$ and $F_N^2$. The same formulas for $f_{nR}^I$ are used for $i = 1$ and $i = 2$.

3.3 Asymptotic region

To calculate the absorptive part of form factors for the asymptotic region, we need the running coupling constant $\alpha$ for the time-like momentum. We perform the analytic continuation to the time-like momentum by assuming the dispersion relation for $\alpha$; application of the so called analytic regularization [7]-[9].

Let $\alpha_S(Q^2)$ be the running coupling constant calculated by the perturbative QCD as a function of the squared momentum $Q^2$ for the space-like momentum expressed in the Pade form.

$$\alpha_S(Q^2) = \frac{4\pi}{\beta_0} \left[ \ln(Q^2/\Lambda^2) + a_1 \ln\{\ln(Q^2/\Lambda^2)\} 
+ a_2 \frac{\ln\{\ln(Q^2/\Lambda^2)\}}{\ln(Q^2/\Lambda^2)} + \frac{a_3}{\ln(Q^2/\Lambda^2)} + \cdots \right]^{-1}.$$  

Here $\Lambda$ is the QCD scale parameter, and $a_i$ are given in terms of the $\beta$ function of QCD,

$$a_1 = 2\beta_1/\beta_0^2, \quad a_2 = 4\frac{\beta_1^2}{\beta_0^4}, \quad a_3 = \frac{4\beta_2^2}{\beta_0^4} \left( 1 - \frac{\beta_0 \beta_2}{8 \beta_1^2} \right),$$

where

$$\beta_0 = 11 - \frac{2n_f}{3}, \quad \beta_1 = 51 - \frac{19n_f}{3}, \quad \beta_2 = 2357 - \frac{5033}{9} n_f + \frac{325}{27} n_f^2$$  

$\beta_0$, $\beta_1$, and $\beta_2$ are the coefficients of the $\beta$ function. $n_f$ is the number of active quark flavors.
with \( n_f \) being the number of flavor. We perform the analytic continuation of the squared momentum \( Q^2 \) to the time-like region, \( s \), by the replacement in \((19)\)

\[
Q^2 \to e^{-i\pi s}.
\]

Then the effective coupling constant becomes complex; \( \alpha_S(q) = \text{Re}[\alpha_S(s)] + i\text{Im}[\alpha_S(s)] \)

with

\[
\text{Re}[\alpha_S(s)] = \frac{4\pi u}{\beta_0 D(s)},
\]

\[
\text{Im}[\alpha_S(s)] = \frac{4\pi v}{\beta_0 D(s)}.
\]

We write

\[
\alpha_S(s) = \frac{1}{u - iv} = \frac{u + iv}{D},
\]

\[
D = u^2 + v^2,
\]

where

\[
u = \ln(s/\Lambda^2) + \frac{a_1}{2} \ln\{\ln^2(s/\Lambda^2) + \pi^2\}
\]

\[
+ \frac{a_2}{\ln^2(s/\Lambda^2) + \pi^2} \left[ \frac{1}{2} \ln(s/\Lambda^2) \ln\{\ln^2(s/\Lambda^2) + \pi\theta\} \right]
\]

\[
+ \frac{a_3 \ln(s/\Lambda^2)}{\ln^2(s/\Lambda^2) + \pi^2},
\]

\[
v = \pi + a_1 \theta
\]

\[
- \frac{a_2}{\ln^2(s/\Lambda^2) + \pi^2} \left[ \frac{\pi}{2} \ln\{\ln^2(s/\Lambda^2) + \pi^2\} - \theta \ln(s/\Lambda^2) \right]
\]

\[
- \frac{\pi a_3}{\ln^2(s/\Lambda^2) + \pi^2},
\]

and

\[
\theta = \tan^{-1}\{\pi/\ln(s/\Lambda^2)\}.
\]

The running coupling constant is given by the dispersion integral both for the space-like and the time-like momentum

\[
\alpha_R(t) = \int_{Q_0^2}^{\infty} dt' \frac{\sigma(t')}{t' - t}
\]

with

\[
\sigma(t') = 4\pi v/\beta_0 D.
\]

\( \alpha_R(t) \) represented by \((28)\) is called analytically regularized running coupling constant as it has no singular point for \( t < 0 \). The regularization eliminates the ghost pole of \( \alpha_S(Q^2) \) appearing at the point

\[
Q^2 = Q^* = \Lambda e^{u^*},
\]
where $u^* = 0.7659596 \cdots$ for the number of flavor $n_f = 3$. Calculating (25), we find that $\alpha_R(t)$ is approximately given by the simple formula with the ghost pole subtracted

$$\alpha_R(t) \approx \alpha_S(Q^2) - A^*/(Q^2 - Q^*^2),$$  \tag{31}$$

where the residue $A^*$ is

$$A^* = 4\pi^2 e^{u^*} / \left\{ \beta_0 \left( 1 + \frac{a_1}{u} - a_2 \frac{\ln u^*}{u^*} + \frac{a_2 - a_1}{u^*^2} \right) \right\}.$$  \tag{32}$$

We use (31) as the regularized coupling constant; for the time-like momentum we replace $Q^2 = e^{-i\pi s}$ as was mentioned before.

The QCD parts, $F_{QCD,I}^i (i = 1, 2; I = 0, 1)$ are written as follows:

$$F_{QCD,I}^i(t) = \hat{F}_{QCD,I}^i(t) h_i(t),$$  \tag{33}$$

where $\hat{F}_{QCD,I}^i$'s are given as expansion in terms of the running coupling constant

$$\hat{F}_{QCD,I}^i(t) = \sum_{j \geq 2} c_{QCD,I}^{i,j} \{ \alpha_R(t) \}^j,$$  \tag{34}$$

for the space-like momentum ($t < 0$). We multiply the function $h_i(t)$ in (33) to assure the convergence of the superconvergence conditions (7) and (8). The following formula is assumed for $h_i(t)$:

$$h_i(t) = \left( \frac{t - t_Q}{t + t_1} \right)^{3/2} \left( \frac{t_2}{t + t_2} \right)^{i+1} (i = 1, 2),$$  \tag{35}$$

which may be interpreted as the form factor for $\gamma \rightarrow q\bar{q}$ with $t_Q$ being the threshold of the quark antiquark pair. The parameters $t_Q$, $t_1$ and $t_2$ are taken as adjustable parameters and will be determined by the analysis of experimental data.

For the time-like momentum ($t > 0$), we perform the analytic continuation of the regularized effective coupling constant $\alpha_R(t)$ to $\alpha_R(s)$ through the equation

$$\alpha_R(s) = \alpha_R(Q^2 e^{-i\pi s}) = Re[\alpha_R(s)] + i Im[\alpha(s)].$$  \tag{36}$$

We take three loop approximation for the effective coupling constant and express the QCD part as follows:

$$\hat{F}_{QCD,I}^i(s) = \sum_{2 \leq j \leq 4} c_{QCD,I}^{i,j} \{ \alpha_R(t) \}^j,$$  \tag{37}$$

The summation in (37) begins in the second order in the effective coupling constant so as to realize the logarithmic decrease of the nucleon form factors.

Imaginary part of (37) is obtained to be

$$\text{Im} \hat{F}_{QCD,I}^i = 2c_{i,2}^{QCD,I} \text{Re} \alpha_R \text{Im} \alpha_R + c_{i,3}^{QCD,I} [3(\text{Re} \alpha_R)^2 \text{Im} \alpha_R - (\text{Im} \alpha_R)^3]$$

$$+ c_{i,4}^{QCD,I} [4(\text{Re} \alpha_R)^3 \text{Im} \alpha_R - 4\text{Re} \alpha_R (\text{Im} \alpha_R)^3] + \cdots,$$  \tag{38}$$
\[ \text{Im} F_{QCD,I}^I(s) = \text{Im} \hat{F}_{QCD,I}^I(s) h_i(s). \]

We write the low energy part, intermediate resonance part and asymptotic QCD parts of form factors as \( F_{iI}^I, F_{BW,I}^I \) and \( F_{QCD,I}^I \), which are given by the dispersion integral with the imaginary parts (13), (18) and (38), respectively. The form factors \( F_{iI}^I \) are defined by adding them up. We impose the conditions (7) and (8) on \( \text{Im} F_{iI}^I \) so that the QCD conditions are satisfied.

4 Numerical analysis

We analyzed the experimental data of nucleon electro-magnetic form factors for the space-like momentum \( G_N^M/\mu_N, G_N^E/G_D, G_M^N/\mu_M G_D, G_E^N \) and the ratio \( \mu_p G_E^p/G_M^p \) and for the time-like momentum \( |G_p| \) and \( |G_n| \). The parameters in the form factors are determined so that the calculated results realize the experimental data. In addition to the data used in our previous analysis [3], [10] we used the data in Refs.[15] - [38]. In our analysis we treat both of space-like and time-like regions on equal footing in the chi square analysis.

Space-like region:

For \( G_N^N/\mu^N \) and \( G_E^N \) we used the ratio to the dipole formula \( G_D \) as we have done in Refs.[2], [3], [10]. In addition to them we take into account in the chi square analysis the data of the ratio \( 0 < \mu_p G_E^p/G_M^p \) obtained by the polarization experiments.

Time-like region:

Experimentally, the proton and neutron form factors \( |G_p| \) and \( |G_n| \) for the time-like momentum are obtained by using the formula for the cross section \( \sigma_0 \) for the processes \( e + \bar{e} \rightarrow N + \bar{N} \) or \( N + \bar{N} \rightarrow e + \bar{e} \), which is given as

\[ \sigma_0 = \frac{4\pi \alpha^2 \nu}{3s} \left( 1 + \frac{2m_N^2}{s} \right) |G(s)|^2, \tag{39} \]

where \( \alpha \) is the fine structure constant, \( m_N \) and \( \nu \) are the mass and velocity of the nucleon \( N \), respectively. \( |G_N^N| \) are estimated from \( |G| \) under the assumption \( G_M = G_E \) or \( G_E = 0 \) [37]. \( \sigma_0 \) is now expressed in terms of \( G_N^N \) and \( G_E^N \)

\[ \sigma_0 = \frac{4\pi \alpha^2 \nu}{3s} \left( |G_M|^2 |G_N^N|^2 + \frac{2m_N^2}{s} |G_E|^2 \right). \tag{40} \]

Equating (39) and (40), we have

\[ |G|^2 = \frac{|G_N^N|^2 + 2m_N^2 |G_E|^2 / s}{1 + 2m_N^2 / s}. \tag{41} \]

Substituting our calculated result of form factors to the right hand side of (41), we get the theoretical value for \( |G| \), which is compared with the experimental data for the magnetic form factor obtained under the assumption \( G_M = G_E \).
The parameters appearing in our analysis are the following:
Residues at resonances, coefficients appearing in the expansion by the QCD effective coupling constants, cut-offs for the low, intermediate and asymptotic regions $\Lambda_1$, $\Lambda_2$ and $\Lambda_3$, respectively. In addition to them we have parameters in the Breit-Wigner formula and the convergence factor $h$ of QCD contribution, $t_0$, $t_{\text{res}}$, $t_1$, $t_2$, $t_3$.

We have taken the masses and the widths of resonances as adjustable parameters. As the superconvergence constraints impose very stringent conditions on the form factors, it was necessary to take the masses and widths as parameters.

5 Numerical results

We give in Table I and II the results for the parameters; in Table I the masses and widths of resonances and residues at resonance poles obtained by our analysis and in Table II the coefficients $c_{i,j}^{QCD,I}$ ($i = 1, 2; j = 2, 3, 4; I = 0, 1$) in the expansion in terms of the effective coupling constant $\alpha_R$ of QCD defined by (37).

The value of $\chi^2$ is obtained to be $\chi^2_{\text{tot}} = 267.0$, which includes both the data of space-like and time-like regions. The $\chi^2$ value for the space-like data is $\chi^2_{\text{space}} = 233.3$. The total number of data used in the chi square analysis is 209 and the number of parameters is 36. Therefore, $\chi^2_{\text{tot}}/Df = 1.54$. For the time-like momentum, the data of Ablikim et al. [37] (2005) is systematically smaller than that of Antonelli et al. [38] (1998). In the present analysis we restricted ourselves to the (2005) data. We were able to get much better result both for the time-like and for the space-like part than the result obtained by the restriction to (1998) data.

We summarize the parameters obtained by the chi square analysis in Table I and II.

| isospin $I$ | $n$ | mass (GeV) | width (GeV) | $a_1^{I,n}$ (GeV$^2$) | $a_2^{I,n}$ (GeV$^2$) |
|-----------|-----|------------|-------------|----------------------|----------------------|
| $I = 1$   | 1   | 1.367      | 0.324       | -6.7                 | 1.06                 |
|           | 2   | 1.376      | 0.220       | 9.562163             | -17.81168            |
|           | 3   | 1.6096     | 0.26        | -8.323391            | 11.18087             |
|           | 4   | 1.832      | 0.381       | 5.746830             | -5.941032            |
|           | 5   | 2.320      | 0.430       | -0.40                | 0.40                 |
| $I = 0$   | 1   | 1.01945    | 0.426$x10^{-2}$ | -3.302706             | 0.5371316            |
|           | 2   | 1.227      | 0.1609      | 0.6303140$x10^2$     | -1.981666            |
|           | 3   | 1.472      | 0.2123      | -1.734902            | -2.589660            |
|           | 4   | 1.530      | 0.1416      | -2.460598            | 4.147711             |

Table I Parameters obtained by the analysis. Residues at resonances.
Fig.1 Proton form factor for the space-like momentum: Magnetic and electric form factors.

| isospin | $i$ | $c_{i,2}^{QCD, I}$ | $c_{i,3}^{QCD, I}$ | $c_{i,4}^{QCD, I}$ |
|---------|-----|------------------|------------------|------------------|
| $I = 1$ | 1   | $-2.403217$      | $0.220 \times 10^2$ | $-4.40$          |
|         | 2   | $5.072373$       | $-0.510 \times 10^2$ | $0.543 \times 10^2$ |
| $I = 0$ | 1   | $2.964186$       | $-0.195 \times 10^2$ | $-0.687 \times 10^2$ |
|         | 2   | $-6.333821$      | $0.770 \times 10^2$  | $-0.1982 \times 10^3$ |

Table II Coefficients of expansion in terms of the effective of coupling constant QCD defined in (37).

The parameters $t_1$, $t_2$, $t_{res}$, $t_Q$ and $\Lambda_1$ are determined as follows:
$t_1 = 0.2070 \times 10^3$ (GeV/c)$^2$, $t_2 = 0.2240 \times 10^3$ (GeV/c)$^2$, $t_{res} = 0.2082 \times 10^3$ (GeV/c)$^2$
and BW cut = $\Lambda_1 = 0.2600 \times 10^2$ GeV/c. QCD threshold = $\sqrt{t_Q} = 0.2060 \times 10^2$ GeV/c.
We take the number of flavor as $n_f = 3$ and the QCD scale parameter $\Lambda_{QCD} = 0.213$ GeV.

The calculated results are illustrated in Figs.1-4; In Figs.1, 2 we give the results for the space-like momentum for $G_M^p/\mu_p$, $G_E^p$, $G_M^n/\mu_n$, $G_E^n$ and in Fig.3 the ratio of electric and magnetic form factors of proton $\mu_p G_E^p/G_M^p$. In Fig.4 the results for the time-like momentum are illustrated for the proton and neutron form factors $|G_p|$ and $|G_n|$, respectively. Experimental data are taken from [15]-[38].
Fig. 2 Neutron form factors for the space-like momentum: Magnetic and electric form factors.

Fig. 3 Ratio of the electric and magnetic form factors of proton for the space-like momentum.
Fig. 4 Nucleon form factors for the time-like momentum: the proton and neutron form factors.

6 Concluding remarks

We have demonstrated that our superconvergent dispersion relation works in synthesizing the low and the high momentum parts of nucleon electromagnetic form factors for the space-like and time-like momentums as we did for the bosons.

For the space-like momentum we were able to reproduce the experimental data, but for the time-like momentum we did not have very good results. If we restrict ourselves only to the data of space-like momentum, leaving out the time-like data in the chi square analysis, the result for the space-like momentum is improved a little; we have $\chi^2_{\text{space}} = 217$. By using the parameters thus determined, we calculated the time-like part $|G'|$, which turned out to be very large; the value of chi square became as large as $\chi^2_{\text{time}} = 1.4 \times 10^6$. Incorporation of the data for the time-like momentum seems to be necessary in the systematic study of space-like and time-like momentum, although the number of data is limited.

We used the experimental data for the helicity amplitudes obtained by Höher and Schopper in which the contribution from the $\rho$ meson is included. As their data are limited to low $t$ ($\leq 0.779 \text{(GeV/c)}^2$), we do not have sufficient data for the region $s \leq 4m_N^2$. We supplemented the unphysical region for $I = 1$ state by introducing vector bosons with the small mass, $m_V \lesssim 1.4 \text{ GeV/c}^2$. For the isoscalar state we also introduced a vector boson with small mass.

In our calculation we treated all of the vector boson masses and widths as parameters. If they are kept at experimental values, we get poor results. The superconvergence conditions are so strong that the value of $\chi^2$ is very sensitive to the mass and width. The masses are obtained to be smaller than the experimental value and the existence of vector bosons with the masses around $1.2 \sim 1.4 \text{ GeV/c}^2$ are implied.
To conclude the paper we remark on the mass about 1.2 GeV/c^2. Both for \( I = 0 \) and \( I = 1 \) states there are indications of resonances observed by the processes \( e^+ e^- \to \eta \pi^+ \pi^- \), \( \gamma p \to \omega \pi^0 p \) and \( B \to D^* \omega \pi^- \). Incorporation of further resonances may improve results for the time-like momentum.

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