One Loop Calculation of Cosmological Constant in a Scale Invariant Theory

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Abstract: We compute the cosmological constant in a scale invariant scalar field theory. The gravitational action is also suitably modified to respect scale invariance. Due to scale invariance the theory does not admit a cosmological constant term. The scale invariance is broken by a recently introduced mechanism called cosmological symmetry breaking. This leads to a nonzero cosmological constant. We compute the one loop corrections to the cosmological constant and show that it is finite.

1. Introduction

In a recent series of papers [1–3], we have investigated a scale invariant extension of the standard model. The basic idea has been introduced earlier by Cheng and collaborators [4–6]. Phenomenological consequences of this model have also been studied in Ref. [7, 8]. The essential idea can be captured by considering a simple model where we include only one real scalar field besides gravity. Hence here we consider only this simple model which displays global scale invariance. The action for this model may be written as,

\[ S = \int d^4 x \sqrt{-g} \left[ \frac{1}{2} g^{\mu \nu} \partial_\mu \Phi \partial_\nu \Phi - \frac{\lambda}{4} \Phi^4 - \frac{\beta}{8} \Phi^2 R \right], \]

where \( R \) is the Ricci scalar. The model has no dimensionful parameter. In order to agree with observations scale invariance has to be broken. In Ref. [1] we argued that it is broken by a new phenomenon, which we called cosmological symmetry breaking. This is inspired by the standard big bang model. Here the universe is described by a time dependent...
solution of the classical equations of motion. At leading order the solution is just the homogeneous and isotropic Friedmann-Robertson-Walker (FRW) model. We argue that in order to describe physical phenomenon we need to make a quantum expansion around this classical background.

In Refs. [1, 2] we found that, assuming the FRW metric with scale factor $a(t)$, this model has a classical solution,

$$a(t) = a_0 \exp(H_0 t)$$  \hspace{1cm} (2)

with

$$\Phi_{cl} = \eta = \sqrt{\frac{3\beta}{\lambda}} H_0,$$  \hspace{1cm} (3)

where $\eta$ is the classical solution of the scalar field $\Phi$ and $H_0$ is the Hubble constant, which is independent of time in the present case. Here we have set the spatial curvature parameter $k = 0$ in the FRW metric.

A basic problem with imposing scale invariance is that it is believed to be anomalous [9, 10]. When we compute the quantum loop corrections, we necessarily need to regulate the action, which introduces a scale. Hence one concludes that scale invariance is broken. We have argued in Refs. [1, 2] that this conclusion need not hold in the present case. The basic point is that here the classical solution $\eta$ itself provides a scale. Here we expand the field $\Phi$ around this classical solution,

$$\Phi(x) = \eta + \phi(x),$$  \hspace{1cm} (4)

where $\phi(x)$ represent the quantum fluctuations. Under scale transformations $\eta$ also scales in exactly the same form as $\Phi$. In dimensional regularization the regulated action in $d = 4 - \epsilon$ dimensions may be written as,

$$S = \int d^d x \sqrt{-g} \left( \frac{1}{2} g^{\mu\nu} \partial_\mu \Phi \partial_\nu \Phi - \frac{\lambda}{4} \Phi^2 \eta^{-2\delta} - \frac{\beta}{8} \Phi^2 R \right).$$  \hspace{1cm} (5)

where $\delta = (d - 4)/(d - 2)$. Here we have essentially used the classical field to introduce the scale required in this action. It is clear that even after regularization the action is invariant under scale transformations. Hence we do not expect the scale invariance to be anomalous. The regularized action, Eq. 5 is slightly different from what we proposed earlier in Ref. [2]. A detailed comparison between the two will be presented in a separate paper. Here we simply point out that the action given in Eq. 5 is exactly scale invariant and well defined as long as $\eta \neq 0$. The details about the transformation rule are given in Ref. [2]. The transformation is similar to what was also proposed in Ref. [11].

An important aspect of a scale invariant theory is that it does not permit a cosmological constant term in the action. Hence this may potentially solve the well known cosmological constant problem [12–18]. Alternate approaches to solving the cosmological constant problem are described in Refs. [12, 19–26]. As shown in Refs. [1, 2] a cosmological constant is generated by the classical solution. Furthermore if the full quantum theory is truly scale invariant, as we claim, we expect that the cosmological constant would be
finite at all orders in perturbation theory. Although this might be anticipated [2] since the regulated theory is scale invariant, an explicit demonstration is necessary since the regularization procedure [2] is somewhat unusual. In the present paper we demonstrate that at one loop the cosmological constant is calculable and finite. As far as we know this is the first demonstration that the loop contributions to the cosmological constant may be finite in a quantum field theory.

In pure gravity theory with Minkowski background it has earlier been found that the one loop divergent contributions to cosmological constant vanish [27]. However this cancellation does not survive at two loops [28]. In the present case, however, we demonstrate this cancellation including a matter field with the background metric being the standard FRW metric. Furthermore the cancellation is a result of a symmetry, i.e. the scale invariance, and hence is expected to extend to all orders.

In the present paper we shall ignore quantum gravity corrections. These can be shown to be higher order in powers of $1/\beta$. Hence they can be ignored consistently.

2. Renormalization

In this section we shall only expand the scalar field. For calculation of the cosmological constant we need to expand the metric also. This will be done later. Here we simply set the metric equal to the FRW metric with a scale factor $a(t)$. We shall assume that the universe is evolving very slowly with time. Hence in the loop integrals, the time dependent factors $a(t)$ will be set to their current values. This is essentially an adiabatic approximation. The Ricci scalar,

$$R = -12H_0^2.$$  

Hence, if we ignore quantum gravity contributions, the term proportional to $R$ simply acts like a mass term. Therefore we can obtain all the counterterms by simply repeating the standard field theoretic analysis for a spontaneously broken theory. We caution the reader that the fundamental mechanism here is very different from spontaneous symmetry breaking.

The Lagrangian in $d = 4 - \epsilon$ dimension (see Eq. 5) in terms of bare field and parameters may be expressed as,

$$\mathcal{L} = \frac{1}{2} g^{\mu\nu} \partial_\mu \phi' \partial_\nu \phi' - \frac{\lambda_0}{4\eta^{2\delta}} \phi'^4 - \frac{\beta_0 R}{8} \phi'^2,$$  

where $\phi' = \eta' + \phi'$. Let $\Phi = \eta + \phi$ be the renormalized field. This is related to $\phi'$ by the field renormalization $Z$,

$$\phi' = \sqrt{Z} \phi.$$  

We also have $\eta' = \sqrt{Z} \eta$ and $\phi' = \sqrt{Z} \phi$. We define the counterterms as,

$$\delta_Z = Z - 1,$$  

$$\delta_\lambda = \lambda_0 Z^{2(1-\delta)} - \lambda,$$  

$$\delta_\beta = \beta_0 Z - \beta.$$
where $\lambda$ and $\beta$ are the physical coupling constants. We can rearrange the Lagrangian as,

$$
\mathcal{L} = \left[ \frac{1}{2} g^{\mu \nu} \partial_{\mu} \Phi \partial_{\nu} \Phi - \frac{\lambda}{4 \eta^{2 \delta}} \Phi^4 - \frac{\beta R}{8} \Phi^2 \right] + \mathcal{L}_{ct},
$$

where the counterterm Lagrangian is given by,

$$
\mathcal{L}_{ct} = \frac{1}{2} \delta Z g^{\mu \nu} \partial_{\mu} \Phi \partial_{\nu} \Phi - \frac{\delta \lambda}{4 \eta^{2 \delta}} \Phi^4 - \frac{\delta \beta R}{8} \Phi^2.
$$

We may use the solution of the classical field equation, $\eta$, to rewrite the Lagrangian as,

$$
\mathcal{L} = \left[ \frac{1}{2} g^{\mu \nu} \partial_{\mu} \phi \partial_{\nu} \phi - \frac{\lambda}{4 \eta^{2 \delta}} \Phi^4 + 6 \eta^2 \phi^2 + 4 \eta \phi^3 + \phi^4 \right] - \frac{\beta R}{8} (\eta^2 + \phi^2)
$$

Here the terms linear in $\phi$ in the leading order Lagrangian, i.e. not including counterterms, vanish if $\beta R/4 = - \lambda \eta^{2-2 \delta}$. Hence $m_{\phi}^2 = 3 \lambda \eta^{2-2 \delta} + \beta R/4 = 2 \lambda \eta^{2-2 \delta}$ where $m_{\phi}$ denotes the mass of $\phi$. We point out that $\eta$ has the same dimensions as the field $\phi$ and hence its dimension is a little different from the dimension of mass. We keep the terms given in the third line of Eq. (14) as these will be relevant when we expand the metric.

It is now easy to obtain the Feynman rules for the various vertices and counterterms. In Table 1 we have listed the Feynman rules.

### 3. One Loop Calculation of Counterterms

In this section we shall evaluate the counterterms defined in the last section (Eqs. (9), (10), (11)) up to one loop level. We shall impose the condition,

$$
< \Phi > = \sqrt{\frac{3 \beta}{\lambda} H_0 \eta^\delta}
$$

at all orders in perturbation theory. Here the symbol $< \Phi >$ means the expectation value of the field $\Phi$ in the lowest energy state when we expand around a nontrivial classical solution. A similar condition may also be imposed in the case of spontaneous symmetry breaking [29]. However in the case of spontaneous breaking the meaning of $< \Phi >$ is the vacuum expectation value of $\Phi$, which is different from our case. The total 1-point amplitude at one loop level is shown in Fig. 1. From Table 1 we get,

$$
i M_{1pt}^{1\text{loop}} = -3i \left( \frac{\lambda \eta}{\eta^{2 \delta}} \right) \frac{\Gamma(\epsilon/2 - 1)}{\sqrt{(4\pi)^{d/2}}} \left( \frac{1}{m_{\phi}^2} \right)^{\epsilon/2-1}. \quad (16)
$$

We set our renormalization condition such that the total 1-point amplitude vanishes, i.e.,

$$
-i(\delta \lambda \eta^{3-2 \delta} + \frac{1}{4} \delta \beta R \eta) - 3i \eta \left( \frac{\lambda}{\eta^{2 \delta}} \right) \frac{\Gamma(\epsilon/2 - 1)}{\sqrt{(4\pi)^{d/2}}} \left( \frac{1}{m_{\phi}^2} \right)^{\epsilon/2-1} = 0,
$$


or,

\[
\left( \delta \lambda \eta^2 + \frac{1}{4} \delta \beta R \eta^{2\delta} \right) = -3 \lambda \frac{\Gamma(\frac{\epsilon}{2} - 1)}{(4\pi)^{d/2}} \left( \frac{1}{m^2_\phi} \right)^{(\epsilon/2)-1} = \frac{6 \lambda^2 \eta^2}{(4\pi)^2} \left[ \frac{2}{\epsilon} + 1 - \gamma - \log \left( \frac{\lambda}{2\pi} \right) \right].
\]

(17)

The last relation fixes \( \delta_\beta \) in terms of \( \delta_\lambda \).

To set \( \delta_\lambda \) we consider the 4-point function. At one loop level, the diverging 4-point contributions arise only from the \( \mathcal{O}(\lambda^2) \) diagrams shown in Fig 2.

Figure 1: 1-point contribution at one loop level.
\[ iM_{1\text{loop}}^{4\text{pt}} = \left( \frac{-6i\lambda}{\eta^3} \right)^2 \cdot i[I(s) + I(t) + I(u)] + \mathcal{O}(\lambda^3), \]  

where

\[ iI(p^2) = \frac{1}{2\eta^{2\delta}} \int \frac{d^d k}{(2\pi)^d} \frac{i}{k^2 - m_\phi^2 (k + p)^2 - m_\phi^2} \]

\[ = \frac{-i \Gamma(\epsilon/2)}{2(4\pi)^2} \int_0^1 dz \left[ \frac{4\pi \eta^2}{m_\phi^2 - z(1 - z)p^2} \right]^{\epsilon/2} \]

\[ = \frac{-i}{2} \frac{1}{4\pi} \left[ \frac{2}{\epsilon} - \gamma - \int_0^1 dz \log \left( \frac{m_\phi^2 - z(1 - z)p^2}{4\pi \eta^2} \right) \right] \]

\[ \equiv \frac{-i}{2} \frac{1}{(4\pi)^2} \left[ \frac{2}{\epsilon} + iI'(p^2). \right] \]

The last line defines \( I'(p^2) \). Here \( \gamma \) is the Euler-Mascheroni constant. Hence,

\[ iM_{1\text{loop}}^{4\text{pt}} = 54i \left( \frac{\lambda}{\eta^3} \right)^2 \left( \frac{1}{(4\pi)^2} \right) \left( \frac{2}{\epsilon} \right) + (-6i\lambda)^2 \cdot i[I'(s) + I'(t) + I'(u)] + \mathcal{O}(\lambda^3). \]  

The order \( \lambda^3 \) and \( \lambda^4 \) contributions come from the diagrams shown in Figs. 3(a) and 3(b) respectively. However it is easy to check that they are finite. These are higher order in \( \lambda \) and we shall ignore them. We can get rid of the infinite term by demanding

\[ \delta \lambda = \frac{9\lambda^2}{(4\pi)^2} \left( \frac{2}{\epsilon} \right) - 6\lambda^2 [I'(s_0) + I'(t_0) + I'(u_0)], \]  

where we have set our renormalization point at \( s = s_0, t = t_0 \) and \( u = u_0 \).

Since the 1-point function vanishes at the one loop level we have only two divergent diagrams for the two point amplitude (Fig. 4). We can write,

\[ iM_{2\text{loop}}^{2\text{pt}}(a) = 18i \left( \frac{\lambda \eta}{\eta^3} \right)^2 \frac{1}{(4\pi)^2} \left( \frac{2}{\epsilon} \right) + (-6i\lambda)^2 \cdot iI'(p^2), \]  

\[ iM_{2\text{loop}}^{2\text{pt}}(b) = -3i \left( \frac{\lambda}{\eta^{2\delta}} \right) \frac{\Gamma(\epsilon/2 - 1)}{(4\pi)^{d/2}} \left( \frac{1}{m_\phi^2} \right)^{\epsilon/2 - 1}. \]

The counterterm for the 2-point amplitude is given by,

\[ i(p^2 \delta Z - 3\delta \lambda \eta^{2-2\delta} - \delta \beta R/4) = ip^2 \delta Z - 2i\delta \lambda \eta^{2-2\delta} - i(\delta \lambda \eta^2 + \delta \beta R \eta^{2\delta}/4) \eta^{-2\delta}. \]

\[ + \text{crosses} \]

Figure 2: 4-point divergent contribution at one loop level.
From Eqs. 22 and 17 we see that the last two terms in the r.h.s. of the above equation cancel all the singularities. As there is no divergent term proportional to $p^2$ we set

$$\delta Z = 0$$  \hspace{1cm} (25)

at one-loop level.

The divergent 3-point contribution comes from the diagrams shown in Fig[3]. It is easy to derive,

$$i\mathcal{M}^{3\text{pt}}_{\text{1\text{loop}}} = 54i\eta \left( \frac{\lambda}{\eta^2} \right)^2 \frac{1}{(4\pi)^2} \left( \frac{2}{\epsilon} \right) + \mathcal{F}.$$  \hspace{1cm} (26)

where $\mathcal{F}$ indicates a finite part. However this infinity does not give rise to any problem as the three point counterterm, shown in Table 1, cancels it precisely.

Finally we compute the constant terms in the Lagrangian, i.e. the terms given in the third line of Eq. 14. These evaluate to,

$$\mathcal{L}_X = -\left( \frac{\delta \lambda}{4\eta^2 \beta} \eta^4 + \frac{\delta \beta R}{8 \eta^2} \eta^2 \right)$$

$$= -\frac{3}{4} \left( \frac{\lambda \eta^2}{4\pi} \right)^2 \left( \frac{2}{\epsilon} \right) + \mathcal{F}_X.$$  \hspace{1cm} (27)

where $\mathcal{F}_X$ denotes the finite part.
4. Cosmological Constant at One Loop

In this section we compute the cosmological constant, $\Lambda$, at one loop. This is defined by the term in the action

$$S_\Lambda = - \int d^d x \sqrt{-g} \Lambda$$  \hspace{1cm} (28)

In our scale invariant theory such a term is absent from the action, as discussed above. However due to cosmological breaking of scale invariance we find a nonzero contribution. We expand the metric such that

$$\bar{g}_{\mu\nu} = g_{\mu\nu} + h_{\mu\nu}$$  \hspace{1cm} (29)

Here we follow the notation of Ref. [30] and denote the full metric as well as other variables with a bar. The symbol $g_{\mu\nu}$ represents the classical metric and $h_{\mu\nu}$ the quantum field. In our case $g_{\mu\nu}$ is simply the FRW metric. Here we shall expand gravity only to first order in $h_{\mu\nu}$. We are only interested in the computation of the cosmological constant which is identified with the one point function in gravity. This would involve the computation of the counter lagrangian proportional to $h = h_{\alpha}^\alpha$ and the one loop graph shown in Fig. 6.

At first order in $h$, we find [30], $\bar{g}^{\mu\nu} = g^{\mu\nu} - h^{\mu\nu}$, $\sqrt{-\bar{g}} = \sqrt{-g}(1 + h/2)$ and

$$\bar{R} = R + h^{\beta;\alpha}_{\beta;\alpha} - h^{\beta;\alpha}_{\beta} - h^{\nu}_{\alpha} R^\alpha_{\nu},$$  \hspace{1cm} (30)

where $R_{\mu\nu}$ is the curvature tensor corresponding to the FRW metric. Here “;” denotes covariant derivatives.

We use the conformal time in the FRW metric. Hence the classical metric becomes,

$$g_{\mu\nu} = a^2 \cdot \text{diagonal}(1, -1, -1, -1).$$  \hspace{1cm} (31)
The Ricci scalar is given in Eq. (6) and the tensor,

$$R_{\mu\nu} = -3H_0^2 g_{\mu\nu},$$

(32)

where $H_0$ is the Hubble constant.

We first consider the counterterm action,

$$S_{ct} = \int d^d x \sqrt{-\bar{g}} \left( -\frac{\delta_\beta R}{8} \Phi^2 - \frac{\delta_\lambda}{4\eta^{2\delta}} \Phi^4 \right)$$

(33)

$$= \int d^d x \sqrt{-\bar{g}} \left( -\frac{\delta_\beta \bar{R}}{8} \eta^2 - \frac{\delta_\lambda}{4\eta^{2\delta}} \eta^4 \right) + \ldots .$$

(34)

We identify the terms proportional to $h$ in the counterterm action. We find, dropping surface terms,

$$S_{ct} = \int d^d x \sqrt{-\bar{g}} h \left[ \mathcal{L}_X - \frac{\delta_\lambda \eta^4}{4\eta^{2\delta}} \right] + \ldots$$

(35)

$$= - \int d^d x \frac{3}{4} \sqrt{-g} \left( \frac{\lambda \eta^2}{4\pi \eta^\delta} \right)^2 \left( \frac{2}{\epsilon} + 1 - \gamma - \log \left( \frac{\lambda}{2\pi} \right) \right) + \ldots ,$$

(36)

where $\mathcal{L}_X$ is defined in Eq. (27). In this equation we have displayed only the divergent terms. We next identify the terms proportional to $h\phi^2$ which would contribute to the one loop diagram with an external $h$ line. We find

$$- \sqrt{-g} \frac{\lambda}{4\eta^{2\delta}} \Phi^4 = -3 \sqrt{-g} \lambda \eta^{2-2\delta} h\phi^2 + \ldots ,$$

(37)

and

$$- \sqrt{-g} \frac{\beta}{8} \Phi^2 \bar{R} = -\sqrt{-g} \frac{\beta}{8} R h\phi^2 - \frac{\beta}{8} \sqrt{-g} (\partial_\beta \phi)^2 \left( \Gamma_{\alpha\gamma}^\beta h_{\gamma}^{\alpha\gamma} + \Gamma_{\alpha\gamma}^\gamma h_{\alpha}^{\gamma} \right) + \ldots .$$

(38)

In the Fourier space the second term on the right hand side would be proportional to $k_\beta$. Here we are working in the adiabatic limit where the background metric is assumed to be very slowly varying. Hence the background metric is taken out of the Feynman integral. A Feynman integral proportional to $k_\beta$ is zero, by symmetric integration. Hence the second term does not contribute.

We now evaluate the one loop contribution to graviton one point function. The terms which do not involve derivatives of $\phi$ give the contribution to the amplitude equal to

$$- \sqrt{-g} \frac{5}{8} \left( \frac{\lambda \eta^2}{4\pi \eta^{2\delta}} \right) \frac{\Gamma(\epsilon/2 - 1)}{(4\pi)^{d/2}} \left( \frac{1}{m_\phi^2} \right)^{\epsilon/2-1}$$

$$= \frac{5}{4} \sqrt{-g} \left( \frac{\lambda \eta^2}{4\pi \eta^\delta} \right)^2 \left[ \frac{2}{\epsilon} + 1 - \gamma - \log \left( \frac{\lambda}{2\pi} \right) \right] .$$

(39)

The KE term for the scalar field gives

$$- \sqrt{-g} \frac{\Gamma(-d/2)}{4(4\pi)^{d/2}} \left( \frac{1}{m_\phi^2} \right)^{-d/2} = - \frac{1}{2} \sqrt{-g} \left( \frac{\lambda \eta^2}{4\pi \eta^\delta} \right)^2 \left[ \frac{2}{\epsilon} + \frac{3}{2} - \gamma - \log \left( \frac{\lambda}{2\pi} \right) \right] .$$

(40)
These two precisely cancel the divergent contributions coming from the counterterm action. Hence we find that the cosmological constant is one loop finite. The one loop correction to the cosmological constant is found to be

\[ \delta \Lambda = \frac{1}{2} \left( \frac{\lambda \eta^2}{4\pi} \right)^2. \]

(41)

5. Conclusions

The cosmological constant remains one of the most serious issues in physics. In quantum field theory it acquires infinite contributions which have to be cancelled at each order in perturbation theory by adding suitable counter terms. In this case quantum field theory is unable to predict its value. We have earlier hypothesized that scale invariance might control this parameter. In this case we are not permitted to include a cosmological constant term in the action. If scale invariance is unbroken then this parameter would be zero at all orders. We argued in earlier papers that scale invariance is broken due to cosmological symmetry breaking [1–3]. This phenomenon generates a nonzero cosmological constant.

We have computed the cosmological constant in a scale invariant scalar field theory. Due to scale invariance we expect it to be finite at all orders. We have explicitly demonstrated this to one loop order in a simple model. We expect this to also hold in a scale invariant standard model [4–6]. A particularly interesting extension of this model is to impose local scale invariance [4–6, 31–38]. This has the interesting prediction that the standard model Higgs particle disappears from the particle spectrum. We expect our mechanism to work in these scale invariant theories also.

References

[1] P. Jain and S. Mitra, Mod. Phys. Lett. A 22, 1651 (2007), hep-ph/0704.2273.
[2] P. Jain, S. Mitra and N. K. Singh, JCAP 0803, 011 (2008), hep-ph/0801.2041.
[3] P. K. Aluri, P. Jain and N. K. Singh, hep-ph/0810.4421.
[4] H. Cheng, Phys. Rev. Lett. 61, 2182 (1988).
[5] H. Cheng and W. F. Kao, MIT preprint Print-88-0907 (1988).
[6] W. F. Kao, Phys. Lett. A 154, 1 (1991).
[7] C.-g. Huang, D.-d. Wu and H.-q. Zheng, Commun. Theor. Phys. 14, 373 (1990).
[8] H. Wei and R.-G. Cai, JCAP 0709, 015 (2007).
[9] S. Coleman and R. Jackiw, Annals of Physics 67, 552 (1971).
[10] J. C. Collins, A. Duncan and S. D. Joglekar, Phys. Rev. D16, 438 (1977).
[11] F. Englert, C. Truffin and R. Gastmans, Nucl. Phys. B117, 407 (1976).
[12] S. Weinberg, Rev. Mod. Phys. 61, 1 (1989).
[13] P. J. E. Peebles and B. Ratra, Rev. Mod. Phys. 75, 559 (2003).
[14] T. Padmanabhan, Phys. Rep. 380, 235 (2003).
[15] E. J. Copeland, M. Sami and S. Tsujikawa, Int. J. Mod. Phys. D 15, 1753 (2006).
[16] S. M. Carroll, W. H. Press and E. L. Turner, Ann. Rev. Astron. Astrophys. 30, 499 (1992).
[17] V. Sahni and A. A. Starobinsky, Int. J. Mod. Phys. D 9, 373 2000.
[18] J. R. Ellis, Phil. Trans. Roy. Soc. Lond. A 361, 2607 (2003).
[19] A. Aurilia, H. Nicolai and P.K. Townsend, Nucl. Phys. B 176, 509 (1980).
[20] J. J. Van Der Bij, H. Van Dam and Y. J. Ng, Physica A 116, 307 (1982).
[21] M. Henneaux and C. Teitelboim, Phys. Lett. B 143, 415 (1984).
[22] J. D. Brown and C. Teitelboim, Nucl. Phys. B 297, 787 (1988).
[23] W. Buchmüller and N. Dragon, Phys. Lett. B 223, 313 (1989).
[24] M. Henneaux and C. Teitelboim, Phys. Lett. B 222, 195 (1989).
[25] A. Daughton, J. Louko and R. D. Sorkin, Talk given at 5th Canadian Conference on General Relativity and Relativistic Astrophysics (5CCGRRRA), Waterloo, Canada, 13-15 May 1993, published in Canadian Gen. Rel. 0181, (1993).
[26] D. E. Kaplan and R. Sundrum, JHEP 0607, 042 (2006).
[27] M. Mueller, Phys. Lett. B133, 385 (1983).
[28] B. K. Sawhill, Phys. Lett. B161, 112 (1985).
[29] An Introduction to Quantum Field Theory, M. E. Peskin and D. V. Schroeder, Westview Press (1995).
[30] G. ’t Hooft and M. J. G. Veltman, Annales Poincare Phys. Theor. A 20, 69 (1974).
[31] T. Padmanabhan, Classical and Quantum Gravity, 2, L105-L108 (1985).
[32] D. Hochberg and G. Plunien, Phys. Rev. D 43, 3358 (1991).
[33] W.R. Wood and G. Papini, Phys. Rev. D 5, 3617 (1992).
[34] J. T. Wheeler, J. Math. Phys. 39, 299 (1998).
[35] A. Feoli, W.R. Wood and G. Papini, J. Math. Phys. 39, 3322 (1998).

[36] M. Pawlowski, Turk. J. Phys. 23, 895 (1999).

[37] H. Nishino and S. Rajpoot, hep-th/0403039

[38] D. A. Demir, Phys. Lett. B 584, 133 (2004).