MASS DISTRIBUTIONS OF STARS AND CORES IN YOUNG GROUPS AND CLUSTERS

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Received 2010 November 16; accepted 2011 April 20; published 2011 June 15

ABSTRACT

We investigate the relation of the stellar initial mass function and the dense core mass function (CMF), using stellar masses and positions in 14 well-studied young groups. Initial column density maps are computed by replacing each star with a model initial core having the same star formation efficiency (SFE). For each group the SFE, core model, and observational resolution are varied to produce a realistic range of initial maps. A clump-finding algorithm parses each initial map into derived cores, derived core masses, and a derived CMF. The main result is that projected blending of initial cores causes derived cores to be too few and too massive. The number of derived cores is fewer than the number of initial cores by a mean factor of 1.4 in sparse groups and 5 in crowded groups. The mass at the peak of the derived CMF exceeds the mass at the peak of the initial CMF by a mean factor of 1.0 in sparse groups and 12.1 in crowded groups. These results imply that in crowded young groups and clusters, the mass distribution of observed cores may not reliably predict the mass distribution of protostars that will form in those cores.

Key words: ISM: clouds – stars: formation – stars: luminosity function, mass function

1. INTRODUCTION

For a star, its mass is the most important parameter regarding its evolution and interaction with the environment. On larger scales, galaxy formation and evolution are also shaped by the mass distribution of stars. Moreover, the initial star mass distribution, the initial mass function (IMF), appears to be strikingly universal with a power-law behavior for masses above 1 $M_\odot$ (Kroupa 2002) and a lognormal shape below (Chabrier et al. 2000). Therefore, the IMF is a key issue and a compulsory theory test in star formation, one of the most basic but still most challenging problems in astrophysics.

It is now well established that star formation occurs in the densest and coldest parts of molecular clouds, known as dense cores (see Ward-Thompson et al. 1994; Kirk et al. 2005; Ward-Thompson et al. 2007). Because cores set the initial conditions for star formation, the core mass function (CMF) is often compared to IMF to provide constraints on mass function (SFE), timescale, and fragmentation. In addition to core density and temperature structures, the CMF may also be a theory test by comparing observed and expected CMFs. Above all, the CMF could answer the still unsolved question of fragmentation, especially regarding massive star formation. The presumed differences between CMFs derived, for example, from the competitive accretion model (Bonnell et al. 1997), monolithic collapse (McKee & Tan 2003), or some in-between models could indeed distinguish these theories. If a strong link between CMF and IMF is found, the IMF origin issue may be answered by directly solving the CMF one.

Recent CMF measurements in low-mass star-forming regions have shown a great likeness in shape between the IMF and CMF (Motte et al. 1998 and Johnstone et al. 2000 in $\rho$ Ophiuchi main cloud, Alves et al. 2007 and Rathborne et al. 2009 in the Pipe Nebula; Testi & Sargent 1998 in the Serpens core; Motte et al. 2001 and Johnstone et al. 2000 in Orion B; and Tothill et al. 2002 in the Lagoon Nebula). These observations confirm the role played by cores as direct precursors to stars and also strengthen the idea of a one-to-one relationship between individual stars and individual prestellar cores. As a consequence, these observations also enhance the idea that the shift between the break or peak masses observed in the IMF and CMF is due to an SFE less than 1. The SFE is the ratio between the final star mass and its direct progenitor core initial mass; the currently favored value is about 1/3 (Alves et al. 2007).

The interpretation of the shift between the IMF and CMF as caused by the efficiency of star formation is being questioned, however. Swift & Williams (2008) have shown through numerical experiments that the comparison between the mass functions of dense cores and stars might distinguish with great difficulty only one among several evolutionary schemes given the current observational accuracy. They add that the best way to distinguish several schemes is to study the mass functions over a wide range of mass scales including the high- or low-mass tails. Moreover, Goodwin et al. (2008) have argued that the CMF derived in Nutter & Ward-Thompson (2007) will best match the IMF derived in Kroupa (2002) if cores are assumed to form binaries or higher-order multiples. Observational results in Perseus (Hatchell & Fuller 2008) have provided some new insight into the possible evolution of a prestellar core. The mass distribution of prestellar and protostellar cores in this region indeed appeared inconsistent with a simple direct one-to-one mapping to the IMF.

In addition to the nature of the shift between IMF and CMF, the similarity in shape between the two distributions could also be interpreted as a consequence of the central limit theorem. It has been argued that the IMF is set through the act of a large number of independent physical processes and variables, which leads to its lognormal shape (Larson 1973; Zinnecker 1984; Adams & Fatuzzo 1996). Therefore, the CMF would also present a lognormal shape since it is produced in a similar way. Moreover, the process of deriving a CMF from observational data adds independent random processes that differ from those used to obtain the IMF (these include random errors on core boundaries, core masses, and core blending which do not occur when objects are point sources as in IMF derivation; see Reid et al. 2010). The lognormal shape of both IMF and CMF would then be independent of a one-to-one relationship between prestellar cores and stars.
These uncertainties about the exact relationship between IMF and CMF appear to remain unresolved in large part because the mass distribution of stars that will actually form from an observed CMF is unavailable. In young stellar groups, however, where stars have not moved much from their birth sites, it is possible to numerically approach what natal cores looked like. The original dense core column density map can be estimated based on the current young stellar population, allowing the derived CMF to be directly compared to the actual local star distribution.

Starting with young stellar object (YSO) distributions divided into groups by Kirk & Myers (2011) in four nearby star-forming regions presenting different degrees of crowding, we recreated a possible original column density map for each group. Each YSO contributes to the global column density map of the group by adding one prestellar core profile centered at the YSO position. This profile encloses a total mass equal to the YSO mass, divided by a fixed SFE, either 0.3 or 1.0. We used two well-known profiles representative of different points of view on star formation: the critically stable Bonnor–Ebert (CSBE) sphere (see Bonnor 1956; Ebert 1955) and the thermal–non-thermal (TNT) sphere (see Myers & Fuller 1992). Mimicking the observational CMF derivation, we used the clumpfind algorithm clfind2d (Williams et al. 1994) to obtain the CMF and directly compare it to the local star mass distribution. Henceforth, we will use “initial” CMF to indicate the core mass distribution obtained from the local star mass distribution by a one-to-one relationship between cores and stars and by a fixed SFE; we will use the term “derived” CMF to refer to the mass distribution of cores identified by clfind2d on the simulated column density maps. The properties obtained from the “derived” CMF will also be called “derived” (e.g., “derived” SFE).

Our main results are that the number of derived cores is underestimated and their masses are overestimated due to blending between cores even in relatively uncrowded regions. This happens with no added background and noise and no variation in the number of stars per initial core; the inclusion of these effects would increase the inconsistency between the original and derived cores. We also find that the blending effects are so strong in crowded regions that they dominate other effects, such as those from different core models used to derive the column density maps.

The data and method used are presented in more detail in Section 2. Results are given in Section 3 and discussed in Section 4.

2. METHOD
2.1. Initial YSO Distributions

This study is based on the 14 groups derived by Kirk & Myers (2011) from catalogs of YSOs in four nearby regions. The regions were divided into groups by using the minimum spanning tree (MST) algorithm (Barrow et al. 1985), following the procedure of Gutermuth et al. (2009). The four analyzed nearby star-forming regions are Taurus, Chamaeleon I, Lupus 3, and IC 348 and all have catalogs extending to late M (or even L0) spectral types. The primary catalogs used for each region are Luhman et al. (2010, for Taurus), Comerón (2008, for Lupus 3), Luhman (2007, for Chamaeleon I), and Lada et al. (2006) plus Muench et al. (2007, for IC 348). For the full catalogs, see Kirk & Myers (2011).

The distances adopted to each region are the same as in Kirk & Myers (2011): 140 pc for Taurus, 200 pc for Lupus 3, 160 pc for Cha I, and 300 pc for IC 348. As stated in Kirk & Myers (2011), these four regions are the only ones within 300 pc containing stars younger than several million years and whose local extinction is low enough to make them observable in the optical/near-IR and have a spectral classification completeness of greater than 90%.

The mass estimation of YSOs was based on the spectral-type procedure outlined in Luhman et al. (2003) using a combination of models from Palla & Stahler (1999), Baraffe et al. (1998), and Chabrier et al. (2000). The completeness limit is about 0.02 M⊙ for all regions.

The MST algorithm used by Kirk & Myers (2011) links all the YSOs of one region in a tree, i.e., a connected graph, with the minimum total branch length. By using a cut-off length determined by the total distribution of branch lengths, the MST algorithm defines these groups in a way that mimics the eye behavior; see Kirk & Myers (2011) for further details.

More details on the YSO spectral types, positions, and mass estimation can be found in Kirk & Myers (2011).

2.2. Going Backward: From YSOs to Cores
2.2.1. Core Models

A column density map was derived from the locations and masses of the YSOs by using different core models and SFE values.

The two models used in this study are the CSBE sphere model (see Bonnor 1956; Ebert 1955) and the TNT model (see Myers & Fuller 1992; Myers 2010). We use these particular models because they are frequently used in the community and each offers a different point of view on star formation, in particular on the link between the enclosed mass and the clump radius. Their exact expressions can be found in the Appendix.

The CSBE model was computed in two ways, which we label the CSBET model and the CSBEP model. For the CSBET model, the temperature was set to 16 K for all groups and the external pressure was allowed to vary. This temperature value is the same as the one measured in the Pipe Nebula (Alves et al. 2007). For the CSBEP model, the external pressure was set to $P/k = 3.0 \times 10^6$ K cm$^{-3}$ for all groups and the temperature was allowed to vary. This pressure value is 30 times the pressure measured in the Pipe Nebula because the Pipe Nebula value was too low to describe most star-forming regions. Typical star-forming regions such as Ophiuchus (Johnstone et al. 2000) and Orion B (Johnstone et al. 2001) have higher values. The adopted pressure causes the core column density profile to peak at $3 \times 10^{11}$ cm$^{-2}$, which is consistent with high-resolution observations of nearby cores (see Alves et al. 2007; Kirk et al. 2006). It is noteworthy that these pressure and temperature choices are consistent with the ones derived by assuming a Bonnor–Ebert profile in Johnstone et al. (2000; $\rho_{\text{Ophiuchi, TNT}} \approx 10–30$ K and $P/k \approx 10^6$–$10^7$ K cm$^{-3}$) and Johnstone et al. (2001; Orion B, $T \approx 20–40$ K and $3 \times 10^5$ K cm$^{-3} \leq P/k \leq 3 \times 10^6$ K cm$^{-3}$). For all models, we assumed a mean particle mass, $m$, of 2.3 times the mass of hydrogen.

Figure 1 displays the column density profiles versus the projected radius for each model for an enclosed mass of 0.5, 1, 3, 5, and 10 M⊙. Whereas the profiles of the CSBET and the TNT models are similar when the enclosed mass is low, they differ at high enclosed masses. Henceforth, we will describe the column density profiles in terms of the “core” component (spiky component) and the “clump” component (the extended halo surrounding the centrally peaked “core” component). At low
masses, the “core” component of the TNT model dominates and is essentially a singular isothermal sphere, similar to the CSBE. At high masses, however, the TNT model still shows a central “core” but also a more extended “clump.” On the other hand, the CSBET model retains the same shape but its peak column density scales inversely with mass while its extent scales in proportion to mass. Thus as mass increases, the CSBET model becomes more extended than the TNT model, while the CSBET peak drops below that of the TNT model at the same radius. The CSBET model, however, does not show any important “core” component. It retains the same shape on the entire mass range like the CSBET does, but the column density peak value is independent of the enclosed mass and is set by the external pressure.

2.2.2. Star Formation Efficiency and Resolution

The SFE parameter \( \epsilon \) is defined as the ratio between the final star mass \( M_* \) and the initial total core mass \( M \) where the star comes from:

\[
M_* = \epsilon M. \tag{1}
\]

We used SFE values of 1.0 and 0.3 to determine the parent core mass for every star. The value of 1.0 was motivated by its use as a reference, whereas 0.3 was chosen to match the SFE value derived in the study of Alves et al. (2007) using dust extinction maps.

Once the parent core mass was obtained, the column density function was separately computed on a spatial grid of step 0.001 pc for each YSO and for each different core model. This step is small enough to allow a good representation of the column density profiles (see Figure 1). The individual column density profiles of the progenitor cores within each stellar group were then summed together to give the global column density map necessary to produce the stars seen today. Finally, the maps were smoothed to reproduce observational resolution due to submillimeter single-dish observations (e.g., the bolometer survey in Perseus by Enoch et al. 2006) or star count extinction (e.g., Lombardi et al. 2006 whose dust extinction maps were used in Alves et al. 2007) with a Gaussian kernel of FWHM 0.5 arcmin and 1.0 arcmin.

The resulting maps can be directly compared with dust extinction maps since the extinction depends primarily on the column density. In contrast, dust emission maps are also sensitive to dust temperature and molecular line maps depend on molecular abundance, velocity, and line excitation.

2.3. Identifying Observable Cores

We derive the CMF that would be observed for each group using a standard clump-finding algorithm. We used the two-dimensional version of the automated algorithm clumpfind (clfind2d; Williams et al. 1994), the same one as used in the Pipe Nebula study of Alves et al. (2007) and Rathborne et al. (2009). The algorithm works by first contouring the data at certain levels set by the user. Then it searches for peaks of column density which denote the cores. Afterward the algorithm follows these peaks down to lower column density values until it reaches the threshold set by the user. Thus it does not assume any core profile as does the Gaussclumps algorithm, for instance, (Stutzki & Guesten 1990) which assumes a Gaussian shape.

To further explain, at each iteration, the algorithm finds all contiguous pixels that have a value between the current level and the next one down. It then assigns them to a pre-existing core or a new one depending on whether they are connected to or isolated from any previously identified core. In the case of blended column density features, a “friends-of-friends” algorithm is used to distribute the pixels between several identified clumps. Eventually, at the final level, a core has to be greater than a certain number of pixels; if not, it is rejected. For further details, see Williams et al. (1994).

Our choice of clfind2d parameters was based on Rathborne et al. (2009), who in turn relied on a clfind2d performance study by Kainulainen et al. (2009). Rathborne et al. (2009) improved on the previous list of cores compiled by Alves et al. (2007), using contours of 2 \( \times \) 1.2 mag with a lowest threshold of 1.2 mag. Alves et al. (2007) used clfind2d on the Pipe Nebula extinction map with a maximum contour level of 6.0 mag, which then led to the truncation of the extinction contouring at 6.0 mag and to the fusion of multiple well-separated extinction peaks into single extinction features. Thus in our study, as in Rathborne et al. (2009), the threshold was set to 1.2 \( \times 10^{21} \) cm\(^{-2}\) and the difference between two levels was 2 \( \times 1.2 \times 10^{21} \) cm\(^{-2}\) (we used the conversion 1 mag = \( 10^{21} \) cm\(^{-2}\)). This parameter choice was motivated by the closer comparison to Rathborne et al. (2009) that it offers.

One of the most important results of the simulations of Kainulainen et al. (2009) is that the degree of crowding within

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**Figure 1.** Column density profiles enclosing 0.5, 1, 3, 5, and 10 \( M_\odot \) vs. the projected radius \( b \) (pc). Each plot displays a different model: left: a CSBE sphere at temperature 16 K (CSBET); middle: a CSBE sphere at external pressure \( P/k = 3.0 \times 10^6 \) K cm\(^{-3}\) (CSBEP); and right: a thermal–non-thermal model (TNT).
a molecular cloud can significantly affect both the measured core parameters and the derived CMF in a more important way than the parameters selected for the core extraction algorithm do. Therefore, we separated two cases in our study—isolated and blended groups—according to the criterion used in the simulations of Kainulainen et al. (2009). According to this criterion, a group is isolated if it displays a value of the ratio $f$ of mean separation to mean diameter above 1.0. By separation we mean the distance measured from the peak position of a core to the peak position of its nearest neighbor. The separation and the diameter values of the cores are defined from clfind2d output (position of the peak and surface for each core). For $f \leq 1$, Kainulainen et al. (2009) showed that the mass determination of individual cores is very uncertain and that the derived CMF may not represent the underlying mass function. For the cores, Kainulainen et al. (2009) used Gaussian profiles, random positions, and masses, whereas we use physical profiles, observed positions, and masses of stellar groups whose most massive stars tend to be in more crowded regions (Kirk & Myers 2011). These differences could strongly influence the measured CMF and increase the value of $f$ under which the mass determination becomes uncertain; it also raises the question of the quality of the determination of the core profile. This ratio $f$ is approximately 2.0 in the Pipe Nebula, because it is a relatively quiescent and sparse region. To avoid confusion, hereafter we refer to $f$ as the “crowding ratio”.

In Rathborne et al. (2009), the larger-scale background was subtracted prior to core identification. Since there is no background added to our maps, and the column density is inferred only from the present-day stars, there is no need to first subtract any background. Without background, it is true that the simulated maps departed from the observed dust extinction ones. Nonetheless, it allows us to skip the background removal step and to study more precisely the effects of the clump-finding algorithm on the actual matter that will eventually accrete on the protostar. Finally, it is worth noting that any additional background will increase the blending of a given region.

3. RESULTS

3.1. Column Density Maps

Figures 2, 3, 4, and 5 display the column density maps of representative groups for each region (Taurus 5 or L1529 for Taurus, IC 348 1 or IC 348-Main for IC 348, Lupus 3 1 or Lupus 3-Main for Lupus 3, and Chal 2 or Chal-South for Chamaeleon I). In Figure 2, the three main steps followed in this study are summarized: starting from a YSO group (left panel), we simulate an initial starless column density map (middle panel), from which the clumpfind algorithm clfind2d isolates cores (right panel). These column density maps show two distinct structural components and are then qualitatively similar to dust extinction.
Figure 3. Illustration of how the parameters (SFE and resolution) affect the Lupus 31 group for the CSBET model. Along each row, each map corresponds from left to right to (SFE = 1.0, resolution = 0.5), (SFE = 1.0, resolution = 1.0), (SFE = 0.3, resolution = 0.5), and (SFE = 0.3, resolution = 1.0). On each map, the initial YSO distribution is shown with a marker of size increasing with YSO mass and colored according to the colorbar shown on the left (in $M_\odot$). Top: the simulated column density map. The blue colorbar indicates the column density in units of $10^{21}$ cm$^{-2}$. Bottom: the resulting cores identified using clfind2d. Each core’s area is shown in a different color.

Figure 4. Illustration of the effect of using different core models in the ChaI group for an SFE of 0.3 and a resolution of 1.0`. Along each row, each map corresponds from left to right to the CSBET, CSBEP, and TNT models. On each map, the initial YSO distribution is shown with a marker of size increasing with YSO mass and colored according to the colorbar shown on the left (in $M_\odot$). Top: the simulated column density map. The blue colorbar indicates column density in units of $10^{21}$ cm$^{-2}$. Bottom: the resulting cores identified using clfind2d. Each core’s area is shown in a different color.
Figure 5. Illustration of the effect of using a different core model in the Cha I 2 group for an SFE of 1.0 and a resolution of 1.0. Along each row, each map corresponds from left to right to the CSBET, CSBEP, and TNT models. On each map, the initial YSO distribution is shown with a marker of size increasing with YSO mass and colored according to the colorbar shown on the left (in $M_\odot$). Top: the simulated column density map. The blue colorbar indicates column density in units of $10^{21}$ cm$^{-2}$. Bottom: the resulting cores identified using clfind2d. Each core’s area is shown in a different color.

maps such as the maps in Kirk et al. (2006), where large-scale structure is seen in extinction maps within which small-scale features seen in thermal emission are embedded. The simulated column density maps similarly show a “clump” component that has a spatial extent typically around 0.3 pc (but in an extreme case like IC 348 1, it can reach 1 pc) and originates from the external part of several summed column density profiles. On top of the clump component appear several “core” components that have a spatial extents less than 0.1 pc, matching the central part of a single core column density profile. This description is particularly accurate when the initial SFE is set to 0.3, which is currently the favored SFE value.

Regarding clumpfind output, one can easily see by looking at the maps in Figures 2–5 that cores are undercounted by clumpfind, although the total mass within one region is recovered. As expected, this effect is much stronger in crowded regions. Figure 2 allows a quick comparison between two extreme cases: a very isolated one (Taurus 5) and a very crowded one (IC 348 1). Whereas Taurus 5 has a nearly one-to-one relationship between progenitor cores and stars, the crowded IC 348 1 shows a much higher number of stars per clumpfind core. Hereafter, we will refer to the number of stars per core as the fragmentation ratio $F$. In IC 348 1, clfind2d is less effective in identifying structures: as can be seen in Figure 2, the cores have highly irregular shapes and appear unphysical.

Figure 3 shows the changes in the column density map of Lupus 3 1 for the CSBET model when the values of resolution and initial SFE change. It shows qualitatively that fewer cores are found with a decrease in initial SFE and/or poorer resolution. A poorer resolution has a stronger effect than a decrease in initial SFE because the number of identified cores always decreases significantly with poorer resolution.

Figures 4 and 5 show the column density maps derived for each of the three models for the Cha I-South or Cha I 2 group side by side, assuming an initial SFE of 0.3 (Figure 4) and 1.0 (Figure 5), and a resolution of 1.0 for both. The maps show a larger difference when the initial SFE is set to 0.3 and when the radii of massive cores are bigger. The difference in blending of sources between the panels can be explained by the relation between the enclosed core mass $M$ and the core radius $R$. The CSBET model yields core radii that increase proportionally to the total enclosed mass much more rapidly than for the CSBEP model, where $R \propto \sqrt{M}$, and the TNT model, where $R \propto M^{0.73}$ (see the Appendix, in exact form $M \propto (AR + BR^{2.7})^2$, but the fit $R \propto M^{0.73}$ is excellent for the present YSO masses). For massive stars, the CSBET model yields the widest column density profiles, which in turn causes a larger degree of core blending.

3.2. Derived Core Properties

Several properties can be derived directly from clumpfind output: the total number of cores $N_{C,\text{tot}}$, the mass of each core $M_C$, the total mass of these cores $M_{C,\text{tot}}$, the radius of each core $R$, and...
Table 1
Star and Core Properties

| Group   | Name     | Model | Param. | \( N_{\text{S, tot}}/N_{\text{C, tot}} \) | \( M_{\text{S, tot}}/M_{\text{C, tot}} \) | \( \beta_{\text{tan}}/\beta_{\text{ave}} \) | \( F^d \) | \( f^e \) |
|---------|----------|-------|--------|-----------------------------------|-----------------------------------|---------------------------------|--------|--------|
| Taurus1 | B209     | CSBET | (1,0)5 | 20/15 | 10.64/10.38 | 1.0/3.1 | 1.33 | 2.87 |
| ...     | ...      | ...   | ...    | ...   | ...       | ...    | ...  | ...   |
| Taurus1 | B209     | CSBEP | (1,0)5 | 20/15 | 10.64/10.27 | 2.0/3.8 | 1.33 | 2.40 |
| ...     | ...      | ...   | ...    | ...   | ...       | ...    | ...  | ...   |
| Taurus2 | L1495E   | CSBET | (1,0)5 | 30/27 | 15.53/15.10 | 1.0/3.1 | 1.11 | 2.36 |
| ...     | ...      | ...   | ...    | ...   | ...       | ...    | ...  | ...   |
| Taurus2 | L1495E   | CSBEP | (1,0)5 | 30/27 | 15.53/14.96 | 2.0/3.1 | 1.11 | 2.06 |
| ...     | ...      | ...   | ...    | ...   | ...       | ...    | ...  | ...   |
| Taurus2 | L1495E   | TNT   | (1,0)5 | 30/28 | 15.53/15.82 | 1.4/3.1 | 1.07 | 2.28 |
| ...     | ...      | ...   | ...    | ...   | ...       | ...    | ...  | ...   |
| Taurus3 | B213     | CSBET | (1,0)5 | 19/16 | 8.08/8.74  | 1.0/3.9 | 1.19 | 3.26 |
| ...     | ...      | ...   | ...    | ...   | ...       | ...    | ...  | ...   |
| Taurus3 | B213     | CSBEP | (1,0)5 | 19/15 | 8.08/7.72  | 2.0/3.8 | 1.27 | 2.92 |
| ...     | ...      | ...   | ...    | ...   | ...       | ...    | ...  | ...   |
| Taurus3 | B213     | TNT   | (1,0)5 | 19/17 | 8.08/8.43  | 1.4/3.2 | 1.13 | 2.32 |
| ...     | ...      | ...   | ...    | ...   | ...       | ...    | ...  | ...   |
| Taurus4 | L1551    | CSBET | (1,0)5 | 24/18 | 22.69/22.29 | 1.0/2.8 | 1.33 | 2.47 |
| ...     | ...      | ...   | ...    | ...   | ...       | ...    | ...  | ...   |
| Taurus4 | L1551    | CSBEP | (1,0)5 | 24/18 | 22.69/22.20 | 2.0/3.0 | 1.33 | 2.23 |
| ...     | ...      | ...   | ...    | ...   | ...       | ...    | ...  | ...   |
| Taurus4 | L1551    | TNT   | (1,0)5 | 24/18 | 22.69/22.30 | 1.4/3.1 | 1.33 | 2.59 |
| ...     | ...      | ...   | ...    | ...   | ...       | ...    | ...  | ...   |
| Taurus5 | L1529    | CSBET | (1,0)5 | 14/9  | 8.25/8.04  | 1.0/3.9 | 1.56 | 4.27 |
| ...     | ...      | ...   | ...    | ...   | ...       | ...    | ...  | ...   |
| Taurus5 | L1529    | CSBEP | (1,0)5 | 14/10 | 8.25/7.99  | 2.0/3.9 | 1.40 | 4.02 |
| ...     | ...      | ...   | ...    | ...   | ...       | ...    | ...  | ...   |
| Taurus5 | L1529    | TNT   | (1,0)5 | 14/10 | 8.25/8.50  | 1.4/3.3 | 1.40 | 4.73 |
| ...     | ...      | ...   | ...    | ...   | ...       | ...    | ...  | ...   |
| Taurus6 | L1536    | CSBET | (1,0)5 | 31/27 | 17.67/17.25 | 1.0/3.6 | 1.15 | 2.86 |
| ...     | ...      | ...   | ...    | ...   | ...       | ...    | ...  | ...   |
| ...     | ...      | ...   | ...    | ...   | ...       | ...    | ...  | ...   |

Notes:
- \( \beta_{\text{tan}}/\beta_{\text{ave}} \) is the ratio of the tangential to the average velocity.
- \( F^d \) is a dimensionless factor.
- \( f^e \) is a dimensionless factor.

Source: Michel, Kirk, & Myers (2011, The Astrophysical Journal, 735:51 (23pp), July 1)
| Group   | Name   | Model | Param.a | \(N_{S,\text{tot}} / N_{C,\text{tot}}^{b} \) | \(M_{S,\text{tot}} / M_{C,\text{tot}}^{b} \) | \(\rho_{\text{int}} / \rho_{\text{int}}^{c} \) | \(F^{d} \) | \(f^{e} \) |
|---------|--------|-------|---------|---------------------------------|---------------------------------|---------------------------------|---------|---------|
| Taurus6 | L1536  | CSBEP | (1,0)^5 | 31/25                           | 17.67/17.10                      | 2.0/3.4                          | 1.24    | 2.64    |
|         |        |       | (1,1)^5 | 31/24                           | 17.67/17.77                      | 1.4/3.2                          | 1.24    | 3.04    |
| Taurus6 | L1536  | TNT   | (1,0)^5 | 31/20                           | 17.67/17.77                      | 1.4/3.7                          | 1.55    | 2.50    |
|         |        |       | (0,3,0)^5 | 30/20                           | 17.67/17.29                      | 1.7/3.5                          | 2.0/6.9 | 1.55    |
| Taurus6 | L1536  | CSBET | (1,0)^5 | 24/21                           | 16.13/15.69                      | 1.0/3.0                          | 2.0/3.0 | 1.39    |
| Taurus7 | L1527  | CSBET | (1,0)^5 | 24/17                           | 16.13/15.55                      | 1.0/3.0                          | 2.0/3.0 | 1.29    |
| Taurus7 | L1527  | CSBET | (1,0)^5 | 24/18                           | 14.98                            | 1.0/3.0                          | 2.0/3.0 | 1.39    |
| Taurus8 | L1517  | CSBET | (1,0)^5 | 16/15                           | 12.27/11.94                      | 1.0/2.4                          | 1.07    | 2.31    |
| Taurus8 | L1517  | CSBEP | (1,0)^5 | 16/15                           | 12.27/11.90                      | 1.0/2.5                          | 1.07    | 2.22    |
| Taurus8 | L1517  | CSBEP | (1,0)^5 | 16/14                           | 11.43                            | 1.0/3.0                          | 1.07    | 2.19    |
| Taurus8 | L1517  | CSBEP | (1,0)^5 | 16/14                           | 11.43                            | 1.0/3.0                          | 1.07    | 2.19    |
| ChaI    | ChaI   | CSBET | (1,0)^5 | 12/10                           | 3.66/3.47                        | 1.0/3.8                          | 1.20    | 2.70    |
| ChaI    | ChaI   | CSBET | (1,0)^5 | 12/10                           | 3.66/3.39                        | 1.0/3.9                          | 1.20    | 2.31    |
| ChaI    | ChaI   | CSBET | (1,0)^5 | 12/10                           | 3.66/3.47                        | 1.0/3.8                          | 1.20    | 2.31    |
| ChaI    | ChaI   | CSBET | (1,0)^5 | 12/10                           | 3.66/3.47                        | 1.0/3.8                          | 1.20    | 2.31    |
| ChaI    | ChaI   | CSBET | (1,0)^5 | 12/10                           | 3.66/3.47                        | 1.0/3.8                          | 1.20    | 2.31    |
| ChaI    | ChaI   | CSBET | (1,0)^5 | 12/10                           | 3.66/3.47                        | 1.0/3.8                          | 1.20    | 2.31    |
| ChaI    | ChaI   | CSBET | (1,0)^5 | 12/10                           | 3.66/3.47                        | 1.0/3.8                          | 1.20    | 2.31    |
| ChaI    | ChaI   | CSBET | (1,0)^5 | 12/10                           | 3.66/3.47                        | 1.0/3.8                          | 1.20    | 2.31    |
| ChaI    | ChaI   | CSBET | (1,0)^5 | 12/10                           | 3.66/3.47                        | 1.0/3.8                          | 1.20    | 2.31    |
| ChaI    | ChaI   | CSBET | (1,0)^5 | 12/10                           | 3.66/3.47                        | 1.0/3.8                          | 1.20    | 2.31    |
| ChaI    | ChaI   | CSBET | (1,0)^5 | 12/10                           | 3.66/3.47                        | 1.0/3.8                          | 1.20    | 2.31    |
| ChaI    | ChaI   | CSBET | (1,0)^5 | 12/10                           | 3.66/3.47                        | 1.0/3.8                          | 1.20    | 2.31    |
| ChaI    | ChaI   | CSBET | (1,0)^5 | 12/10                           | 3.66/3.47                        | 1.0/3.8                          | 1.20    | 2.31    |
| ChaI    | ChaI   | CSBET | (1,0)^5 | 12/10                           | 3.66/3.47                        | 1.0/3.8                          | 1.20    | 2.31    |
| ChaI    | ChaI   | CSBET | (1,0)^5 | 12/10                           | 3.66/3.47                        | 1.0/3.8                          | 1.20    | 2.31    |
| ChaI    | ChaI   | CSBET | (1,0)^5 | 12/10                           | 3.66/3.47                        | 1.0/3.8                          | 1.20    | 2.31    |
| ChaI    | ChaI   | CSBET | (1,0)^5 | 12/10                           | 3.66/3.47                        | 1.0/3.8                          | 1.20    | 2.31    |
| ChaI    | ChaI   | CSBET | (1,0)^5 | 12/10                           | 3.66/3.47                        | 1.0/3.8                          | 1.20    | 2.31    |
| ChaI    | ChaI   | CSBET | (1,0)^5 | 12/10                           | 3.66/3.47                        | 1.0/3.8                          | 1.20    | 2.31    |
| ChaI    | ChaI   | CSBET | (1,0)^5 | 12/10                           | 3.66/3.47                        | 1.0/3.8                          | 1.20    | 2.31    |
| ChaI    | ChaI   | CSBET | (1,0)^5 | 12/10                           | 3.66/3.47                        | 1.0/3.8                          | 1.20    | 2.31    |
| ChaI    | ChaI   | CSBET | (1,0)^5 | 12/10                           | 3.66/3.47                        | 1.0/3.8                          | 1.20    | 2.31    |

Note: \(N_{S,\text{tot}} / N_{C,\text{tot}}^{b} \), \(M_{S,\text{tot}} / M_{C,\text{tot}}^{b} \), \(\rho_{\text{int}} / \rho_{\text{int}}^{c} \), \(F^{d} \), and \(f^{e} \) are parameters and ratios used in the analysis of the data.
and the crowding ratio \( f \) describing whether the cores appear isolated \((f > 1)\) or blended \((f < 1)\). The core radius \( R \) is derived from the core surface returned by clfind2d by assuming a circular projected shape. By counting the number of YSOs that one core spatially encloses, we derive the average number of stars per core, or the “fragmentation ratio” \( F \). We fit a power law to the function \( M_C(R) \) and derive the slope \( \beta \) in the expression \( M_C \propto R^\beta \). These bulk properties describing each group are given in Table 1.

### 3.2.1. Multiple Stars per Core

Before comparing the initial CMFs and the derived CMFs, it is interesting to calculate the average number of stars found in each core. Table 1 shows that even in isolated regions, \( F \) is never strictly equal to one and increases \( f \) decreases, which confirms the legitimacy of the crowding ratio criterion \( f \). Figure 6 shows the average value of \( F \) versus \( f \), where each point plotted is averaged over all three models and both resolutions for the same group. In addition to the spatial crowding, the number of cores identified also depends on the peak column density. A low resolution or a flatter column density profile (as in the CSBEP model) yields fewer cores identified.

For a given set of parameters and group, the CSBEP model almost always leads to the highest fragmentation ratio value of the three models. This may seem surprising given that the CSBEP model has the smallest radii for massive cores and as
Figure 6. Fragmentation ratio $F$ vs. crowding ratio $f$. Each color represents a region (red: Taurus; blue: Chamaeleon; green: Lupus 3; and black: IC 348) and each group is represented by a circle for the SFE value of 1 and by a cross for the SFE value of 0.3. Each point is positioned at the average value over the three models for the two resolutions while the bars at each point represent the minimum and maximum values for the crowding ratio (horizontally) and the fragmentation ratio (vertically) over the three models and two resolutions. Strong crowding ($f \ll 1$) results in a large number of input cores merging in the map, i.e., a high fragmentation ratio $F$.

As a result, the least blended maps for an initial SFE of 0.3. The high fragmentation can be explained by the fact that the peaks of CSBEP profiles are the broadest—the CSBEP model profile does not exhibit a “core” component that is as important as that in the CSBET model for low-mass cores or the TNT model for any core (see Figure 1).

The CSBET model, however, has the highest fragmentation when the initial SFE is 0.3 and thus the core masses are much bigger. The CSBET column density peaks are then smaller and broader (see Figure 1). By contrast, the “core” component of the TNT model remains narrow in both low- and high-mass ranges. The broadness of the column density peaks has a larger effect on the fragmentation than the initial SFE.

Thus, the blending of sources leads to undercounting, which is all the more important when the column density profiles are not spiky enough to lead to a good core identification. Such undercounting appears even in groups where the blending is very low (e.g., Taurus region).

3.2.2. Incomplete Mass Recovery

Now that it appears that the common observational CMF derivation using clfind2d is not reliable for recovering initial cores in blended regions, we want to know how the core mass found by clfind2d relates to the input mass of the YSOs within the identified core. Similarly, we seek the resulting relation between the initial CMF and the derived CMF. The initial CMF corresponds to the local YSO distribution by a one-to-one relationship. We already know that, since clfind2d uses a threshold, there will always be some material that will not be assigned to any core. Hence, the total mass of cores will be less than the total mass of gas in the column density map. A poorer resolution strengthens this effect because the maps are smoother.

In Figure 7 for each derived core, we compare the mass assigned by clumpfind and the mass it should have given the YSOs it encloses. In isolated regions (Taurus and to a lesser extent Chamaeleon I and Lupus 3), for the CSBET and CSBEP models, the most massive derived cores are up to 70% less massive than they should be while the least massive derived cores lack around 30% of their mass. In the same regions, the least massive TNT-derived cores get assigned more material than they should by a factor of 30%. The most massive TNT-derived cores lack up to 50% of their mass. The differences between the models can be explained by the differences in column density profiles: the TNT model always gives a strong core component whereas the CSBEP model shows no strong core component and the CSBET model shows only a strong core component at low masses. In crowded regions (IC 348 or Chamaeleon I and Lupus 3 when the initial SFE is set to 0.3), the mass from the most massive initial cores tends to be spread out sufficiently for the massive cores found by clfind2d to miss 50%–75% of their input mass, with some of that material being added to the surrounding low- and intermediate-mass cores.

The comparison between the initial CMFs and the derived CMFs is thus complicated by these effects. If low-mass-derived cores are less (more) massive than they should be, the derived
CMFs are broadened (narrowed) in this mass range. When the blending is important, the masses assigned to derived cores are not related to the YSOs they enclose, particularly for the CSBET model which yields the most blended maps.

### 3.2.3. Core Radii

In the last two sections, we have shown that the derived CMF depends on the input model for relatively isolated regions because the fragmentation ratio $F$ and the way a core mass is related to the masses of the YSOs it encloses change with the input model (see Figure 7, top left and top middle panels corresponding to a resolution of 0.5'). It is then reasonable to think that one can identify the model used to create a map, at least in isolated regions. The best way to achieve this is to study the relation between the core radius $R$ and the core mass $M_C$. Each input model is defined by a particular power law (CSBET: $M_C \propto R$, CSBEP: $M_C \propto R^2$, and TNT: $M_C \propto R^{1.4}$ for an SFE of 1 given the YSO distributions; $M_C \propto R^{1.6}$ for an SFE of 0.3 given the YSO distributions; see the Appendix).

Figure 8 shows the derived core radius $R$ versus the derived core mass $M_C$, each column corresponding to a region and each line corresponding to a set of initial SFE and resolution.

The power law $M_C \propto R^\beta$ was fit by using a least-squares method. It appears that the derived power law depends on the degree of crowding, the initial SFE, and the resolution rather than the initial model power law. Strong blending yields a higher $\beta$ exponent since low-mass-derived cores get assigned nearby gas which is actually from the external part of more extended and massive nearby derived cores. More strikingly in most cases, the value $\beta$ is around 3, which matches a constant density profile.

It appears then that the blending of cores in crowded regions dominates the relation between core radius and mass; the input model relationship has little effect and one cannot discriminate among initial core models using derived core properties.
We show in this section that the blending, which occurs with no noise and no additional background material, leads to a significant undercounting of cores (from a factor of 1.4 in isolated groups up to a factor of 20 in blended groups; see Figure 6), whose assigned masses do not relate to the YSO they enclose in the case of strong blending (see Figure 7). Moreover, core extraction and mass recovery depend on the shape of the core profile (see Figure 7), but the relationship between the mass $M$ and radius $R$ of a derived core is independent of the input core profile and obeys a power law of $M \propto R^3$ (see Figure 8).

3.3. Derived Core Mass Distributions

We now regard the global scale of a group by comparing the mass function of the derived cores, the derived CMF, and the mass function of the initial cores, i.e., the mass function of the original YSOs of the group prior to SFE, or the initial CMF. Usually observationally derived CMFs are directly compared to the IMF since the local star distribution is not available. Here, however, we do not have to assume anything about the future star distribution.

Figures 9–12 show the binned mass functions of the original YSOs and clfind2d derived cores for each region and each set of parameters. As mentioned earlier, the number of cores identified is always smaller than the number of stars. The discrepancy becomes more severe in crowded regions ($f < 1$), especially for poor resolution and a low initial SFE. In the less blended configurations, the derived core mass distributions and the initial core mass distributions do not show a noticeable shift between their peaks. When the configuration becomes more blended, a shift between the peaks of the distributions appears, with the magnitude varying with the degree of crowding. For instance, in IC 348 for the CSBET model (Figure 12, first row), the shift between the peaks of the two distributions is about a factor of 10, when the initial SFE is set to 1 and the resolution is set to 0.5', and increases to a factor of 30 when the initial SFE is set to 0.3 and the resolution is set to 1'.

The most important parameters for describing the IMF are the peak mass, where the function reaches its maximum (around $0.1 M_\odot$; see Kroupa 2002; Chabrier 2003), the break mass, from which the scale-free power-law behavior seems to hold for the high-mass range (around $1 M_\odot$; see Kroupa 2002; Chabrier 2003), and the slope of the high-mass tail power law (around 2.3; see Salpeter 1955; Kroupa 2002; Chabrier 2003). An observed CMF and an IMF are usually compared using these parameters, the SFE being defined as the ratio (see Equation (1)) between the peak masses or the break masses of the two functions. The SFE derived from the comparison between the derived CMF and the

Figure 8. Each column shows for one region the log of the core radius against the log of the core mass $M_C$. ChaI and Lupus 3 were regrouped as their properties of crowding are much alike. Along each column, each map corresponds respectively to a different choice of parameters (SFE, resolution) and displays the results for each model. The solid lines represent the expected radii given the initial YSO mass distribution of the group, the model, and SFE inputs. The black dotted lines correspond to the fit for each model of the power law $M \propto R^3$. 
initial YSO distribution will be called the “derived” SFE. Here, we reproduce this comparison for the relationship between the derived CMF and the local star mass distribution, matching the initial CMF with the initial SFE shift.

To make a power-law fit of the differential core mass function \( \frac{dN}{dM} \), we used the maximum likelihood estimate (known as MLE; see Clauset et al. 2007 and Pineda et al. 2009 for further details). This method avoids the problems raised both by analyzing the cumulative function and binning the data. As pointed out in Muñoz et al. (2007) and Rosolowsky (2005), the upper mass limit can induce high curvature at the high-mass end in the cumulative function shape. Moreover, cumulative functions can show curvature leading to a multiple power-law fit, even in the case where the underlying CMF can be characterized by a single power law (see Li et al. 2007). On the other hand, binning the data most likely induces information loss (see Rosolowsky 2005). The MLE consists of fitting the following function:

\[
\frac{dN}{dM} = N_{cl} \frac{\alpha - 1}{M_{\text{Break}}} \left( \frac{M}{M_{\text{Break}}} \right)^{-\alpha},
\]

(2)

where \( M_{\text{Break}} \) is the minimum mass value at which power-law behavior holds. \( N_{cl} \) is the number of cores more massive than \( M_{\text{Break}} \), and \( \alpha \) is the power-law exponent of the distribution. The MLE gives an estimate of the exponent and the approximated standard error \( \sigma_\alpha \) on it:

\[
\alpha = 1 + N_{cl} \sum_{i=1}^{N_{cl}} \ln \left( \frac{M_i}{M_{\text{Break}}} \right)^{-1}
\]

(3)

Figure 9. Distributions of masses in the Taurus region. Observed cores are shown with a black line (and shading) while the original stellar distribution is shown in green. Where the SFE is 0.3, the red line shows the stellar distribution shifted by this efficiency. Each row corresponds from top to bottom to the CSBET, CSBEP, and TNT models. Along each row, each figure corresponds from left to right to (SFE = 1.0, resolution = 0.5′), (SFE = 1.0, resolution = 1′), (SFE = 0.3, resolution = 0.5′), and (SFE = 0.3, resolution = 1′).
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Figure 10. Distributions of masses in the Chamaeleon I region. Observed cores are shown with a black line (and shading) while the original stellar distribution is shown in green. Where the SFE is 0.3, the red line shows the stellar distribution shifted by this efficiency. Each row corresponds from top to bottom to the CSBET, CSBEP, and TNT models. Along each row, each figure corresponds from left to right to (SFE = 1.0, resolution = 0.5′), (SFE = 1.0, resolution = 1′), (SFE = 0.3, resolution = 0.5′), and (SFE = 0.3, resolution = 1′).

\[ \sigma_\alpha = \frac{\alpha - 1}{\sqrt{N_\alpha}} \]  \hspace{1cm} (4)

The lower cutoff for the power-law region, \( M_{\text{Break}} \), was determined by a Kolmogorov–Smirnov test. All of these steps were performed using an algorithm based on plfit.py, a python implementation by Adam Ginsburg based on the general algorithm of Clauset et al. (2007).3

Figure 13 shows the fit obtained using the MLE method for Cha I 2 for the TNT model, an initial SFE of 0.3, and a resolution of 1′. It appears that the slope obtained for the CMF is shallower than the one obtained for the star mass function. In the MLE method, a finite size bias can be present when the number of cores of mass above \( M_{\text{Break}} \) is under 50 (Clauset et al. 2007). Because some groups do not have enough cores to allow a good determination of \( M_{\text{Break}} \) and \( \alpha \), we divided the groups into two categories for each initial SFE and core model: blended groups, which have a crowding ratio \( f \leq 1 \) for at least one resolution value, and isolated groups for the remaining ones. The blended groups are listed in Table 2 for each initial SFE and core model. In the isolated groups, there are always more than 50 cores identified. In the blended groups, however, there are fewer than 50 cores above \( M_{\text{Break}} \) when the resolution is 1′ for every model and initial SFE. In these groups the fragmentation ratio \( F \) is high and the total number of cores is very low.

The results of the power-law MLE fit are displayed in Figure 16. For each input SFE (1.0 and 0.3), the derived SFE values are shown in the upper plot and the \( \alpha \) values are shown in the lower plot. The derived SFE is estimated in two ways:

3 See also http://tuvalu.santafe.edu/~aaronc/powerlaws/ and for the python implementation agpy.googlecode.com.
Figure 11. Distributions of masses in the Lupus 3 region. Observed cores are shown with a black line (and shading) while the original stellar distribution is shown in green. Where the SFE is 0.3, the red line shows the stellar distribution shifted by this efficiency. Each row corresponds from top to bottom to the CSBET, CSBEP, and TNT models. Along each row, each figure corresponds from left to right to (SFE = 1.0, resolution = 0.′5), (SFE = 1.0, resolution = 1′), (SFE = 0.3, resolution = 0.′5), and (SFE = 0.3, resolution = 1′).

Table 2

| Model | SFE 1 | SFE 0.3 |
|-------|-------|---------|
| CSBET | IC 348-Main, IC 348-North | L1517, Chal-South, Chal-North, Lupus 3-Main, IC 348-Main, IC 348-North |
| CSBEP | IC 348-Main, IC 348-North | Chal-South, Chal-North, IC 348-Main, IC 348-North |
| TNT   | IC 348-Main, IC 348-North | Chal-South, Chal-North, IC 348-Main, IC 348-North |

SFE\textsubscript{Peak} is the ratio of the peak masses of the stellar and the derived core distributions and SFE\textsubscript{Break} is the ratio of the break masses of the stellar and the derived core distributions. \( M_{\text{Peak}} \) is estimated as the position of the maximum in the mass distributions using a Gaussian kernel; because this does not require discrete data bins, it more faithfully reproduces the detailed structure of the CMF compared to a binned mass function (Silverman 1986). \( M_{\text{Break}} \) and \( \alpha \) are derived from the MLE power-law fit.

The fit results on Figure 16 show how the peak and slope of the derived CMF compare to the peak and slope of the distribution of stars. For a one-to-one relation between derived cores and
stars, with constant SFE, the peak and break masses should follow $M_{\text{peak(stars)}} = (SFE_{\text{peak}}) \times M_{\text{peak(cores)}}, M_{\text{break(stars)}} = (SFE_{\text{break}}) \times M_{\text{break(cores)}}$ and the slopes should be identical. Figures 9–12, however, show that crowding and poor resolution generally result in a one-to-many relation between cores and stars.

In isolated regions, the initial SFE is recovered by both SFE_{Peak} and SFE_{Break} as the differences between the initial SFE and the derived SFE_{Peak} and SFE_{Break} are only a few percent, with the largest difference at a resolution of 1.0. The mass at the peak of the derived CMF exceeds the mass at the peak of the initial CMF by a mean factor of 1.0. In blended regions, however, SFE_{Peak} and SFE_{Break} are not similar to either the input values or one another. For a resolution of 0.5, the SFE_{Peak} value is around 0.1 and the SFE_{Break} value is around 0.15/0.25 for the three models and both initial SFEs. For a resolution of 1.0,
all the models for an initial SFE of 1.0 have an SFE_{Peak} value around 0.03 and an SFE_{Break} around 0.06, while all the models for an initial SFE of 0.3 have both an SFE_{Peak} and an SFE_{Break} of around 0.1. The mass at the peak of the derived CMF exceeds the mass at the peak of the initial CMF by a mean factor of 12.1.

Regarding the slopes, all the derived slopes, both for the isolated and blended groups, are similar to the value of 2.35 from Salpeter (1955) within errors. In comparison to the slope derived in the local star distribution, the slopes are slightly better recovered in the blended regions. In blended regions, the stellar slope value is around 2.4 and the derived core slopes range from 2 to 2.5. In isolated regions, however, the stellar slope value is around 3 and the derived core slopes range from 2.5 to 2.7.

To directly address the links between the different mass ranges and their properties, the resulting cumulative mass functions E(M), the fraction of cores or stars with masses greater than M, are shown in Figure 14 for the isolated groups and in Figure 15 for the blended groups. Each row corresponds to different input and local star distributions, and each column corresponds to a different set of initial SFEs and resolutions. Each panel shows the cumulative mass functions with the power-law MLE fit for the derived cores, the local star distribution shifted by the initial SFE value, and the Chabrier IMF. The shift between the local star distributions and the Chabrier IMF shows that variations from region to region regarding the IMF must be considered when comparing the IMF and CMF, as stated in Swift & Williams (2008).

In isolated regions, the fragmentation ratio is very close to one, which allows a good recovery of the initial SFE by both methods (SFE_{Peak} and SFE_{Break}). Since the fragmentation ratio is not exactly one, some derived cores are actually several initial ones blended together. Since the more massive initial cores have a more extended profile and are almost always positioned in or near the most clustered part of the group (see Kirk & Myers 2011), the blending of initial cores happens more often for massive cores. Therefore, the positions of M_{Peak} and M_{Break} for the derived core distribution are very similar to the ones for the star distribution since only the most massive initial cores are affected by the blending in those isolated regions. The slope, however, is very sensitive to the high-mass tail and the derived CMF slopes are shallower than those of the initial stellar distribution. In Figure 14, the comparison of the star and derived core cumulative mass functions shows that, for a resolution of 0.5 (Columns 1 and 3), the two distributions only depart in the very last part of the high-mass tail because the fragmentation ratio F is very close to one. For a resolution of 1' (Columns 2 and 4), the fragmentation ratio F is less close to one and the shift between the two distributions happens at a lower mass. This does not change the derived SFE, but the slopes of the derived cores and stellar distributions differ more than in the better resolution case.

In blended groups, the blending of initial cores is much more important, which poses a difficulty when trying to derive the SFE and the slope \( \alpha \). The number of derived cores can be very
small (see Figure 15, second panels from the left, where only a few cores are identified). Even when the number of derived cores is sufficient to avoid the MLE fitting finite size bias, the derived SFEs are very different from the initial value (for instance, the CSBET model with an initial SFE of 0.3 has an SFE\textsubscript{peak} of 0.10 and an SFE\textsubscript{Break} of 0.25 for a resolution of 0.5, and 0.09 and 0.12, respectively, for a resolution of 1.0). As can be seen in Figure 15 in all panels, the star and derived core distributions are separated by a shift that is caused by a fragmentation ratio $F > 1$. The shift is wider when the resolution is poor—compare the first and third columns of images with a resolution of 0.5 and the second and fourth columns of images with a resolution of 1.0. This effect is directly translated in the derived SFE values which can drop up to 70% when the resolution changes from 0.5 to 1.0. As in isolated regions, the derived slopes are shallower than the initial ones: as for the isolated groups, several initial cores are blended together producing only one more massive derived core. This happens preferentially for the most massive initial cores because they are larger and tend to be in a more clustered part of the group.

The blending, caused by a spatial crowding and a low resolution, seems to be the crucial parameter when deriving the SFE value. It is well recovered in isolated groups (within a few percent of the input value; see Figures 14 and 16), but not in blended groups, where the SFE is underestimated and independent of the input value (around 0.15 for a resolution of 0.5 and around 0.05 for a resolution of 1); the peak mass of the derived CMF exceeds the peak mass of the initial CMF by a mean factor of 12.3; see Figures 15 and 16). Regarding the recovery of the slope $\alpha$, it is shallower than the initial one even in isolated regions but can always be fitted by a power law whose slopes are similar to the Salpeter value (2.35) within error for both isolated and blended groups (see Figures 14, 15, and 16). The comparison of derived CMFs does not distinguish the input models from one another. In addition to causing the differences between derived core masses and radii compared to the input models, the blending also affects the properties of groups on a global scale, since in no case do we obtain a quantitative recovery of the initial CMF by the derived CMF.

4. DISCUSSION

Starting with YSO masses and positions in four nearby star-forming regions, we carried out simulations to estimate the initial starless core column density maps with a one-to-one relation between YSOs and cores and with different values of SFE. The maps are also smoothed with different resolution values. After running the clump-finding algorithm clfind2d on these maps, we derived the CMFs for both isolated groups and blended groups. If our procedure were perfectly recursive, we would recover every stellar mass that we started with. In no case, however, do we obtain such recovery because of the blending. The blending arises from the spatial crowding of cores but also from the smoothing effect of the resolution. The
The blending issue was already addressed by Hatchell & Fuller (2008). In Perseus, the mass distribution of prestellar cores shows an excess of prestellar cores at low masses. A fragmentation process, and different evolutionary timescales). Blending effects such as the horizontal shift between IMF and CMF, may easily be mistaken for physical effects (low SFE, fragmentation process, and different evolutionary timescales). The difficulty in retrieving the initial stellar distribution properties derived in blended regions when the number of cores is sufficient. The fact that we find a Salpeter power law even in the most blended groups where most of the initial cores are merged together or when the original YSO mass function shows a steeper high-mass tail could be the consequence of the central limit theorem, as argued in Reid et al. (2010).

In isolated groups (crowding ratio $f > 1$), our procedure is qualitatively successful, but undercounts the cores by a typical factor of 0.7. Their masses are globally recovered, as both the derived SFEpeak and SFEBreak are within a few percent of the initial SFE. The fraction of individual core mass recovered, however, can vary significantly. In the case of poor resolution (1′), massive derived cores lack up to 70% of their mass. The recovered mass depends on the core density profile and resolution because clfind2d is sensitive to spiky and not extended structure. The reason for qualitative global recovery is that blending effects tend to be local with only a few cores being merged together at a time.

In blended groups (crowding ratio $f < 1$), this procedure is qualitatively and quantitatively unsuccessful, because progenitor cores blend into much larger clumps, which cannot be parsed into their initial cores. Reasons for this are first that massive cores possess a very extended profile and are almost always positioned in a location of higher than average stellar surface density (see Kirk & Myers 2011), which leads to the blending of core profiles on a more global scale than in isolated groups. Second, clfind2d first identifies the peak of a core and then assigns it an extended structure. This leads to undercounting when individual peaks are merged into the summed column density map. Since poor resolution smooths the peaks, it also worsens the undercounting.

These results suggest that, if stars are represented by progenitor cores with a fixed mass ratio, the distribution of apparent core masses derived from a column density map is an unreliable estimator of the stellar masses, especially in young clusters. Blending effects such as the horizontal shift between IMF and CMF, may easily be mistaken for physical effects (low SFE, fragmentation process, and different evolutionary timescales).

The blending issue was already addressed by Hatchell & Fuller (2008). In Perseus, the mass distribution of prestellar cores shows an excess of prestellar cores at low masses. A possible explanation advanced by Hatchell & Fuller (2008) is the selection effect due to blending. This effect is increased by the fact that all clump-finding algorithms look for peaks and cannot identify a core whose peak is merged with the profile of other cores. The extent of this effect, in comparison to the effect of particular evolutionary schemes, remains unknown in Perseus because of different evolutionary timescales for different cores. The difficulty in retrieving the evolutionary scheme by comparing the IMF and CMF was also addressed by Swift & Williams (2008). This difficulty is increased by the impossibility of directly comparing an observational CMF to the initial stellar distribution (*) (for the initial SFE). The fraction of individual core mass recovered, however, can vary significantly. In the case of poor resolution (1′), massive derived cores lack up to 70% of their mass. The recovered mass depends on the core density profile and resolution because clfind2d is sensitive to spiky and not extended structure. The reason for qualitative global recovery is that blending effects tend to be local with only a few cores being merged together at a time.

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**Figure 16.** Star and core mass distribution properties for input SFEs of 1.0 (upper graph) and of 0.3 (lower graph). Upper panel: the derived values of the SFE by the method of the peak mass (red circle marker) and of the break mass (blue triangle marker); the input value of the SFE is indicated by a green solid line. Lower panel: the derived value of the slope $\alpha$ derived by MLE fitting in the high-mass range (bars), the errors on its derivation (red error bars), the number of cores in the high-mass range (number indicated above each bar); the Salpeter value of 2.35 is indicated by a solid green line. For both panels, the values are ordered according to the degree of crowding (isolated/blended, see Table 2), the input model (CSBET/CSBEP/TNT), and whether the distribution is the initial stellar distribution (*) (for the $\alpha$ values), the distribution for a resolution of 0.5 arcmin (0.5) or the one for a resolution of 1.0 arcmin (1.0).
IMF of stars that will actually form. The present results come from the comparison between the local mass star distribution and the derived CMF, i.e., the “observed” CMF, and it is not possible to distinguish a different evolutionary scheme apart from the SFE in the isolated groups.

The smoothed column density maps are similar to observational dust extinction maps (see Kirk et al. 2006). Moreover, the maps appear to meet the conclusions of Smith et al. (2009), particularly in the blended case. Using smoothed particle hydrodynamic (SPH) simulations of massive star-forming clumps, they found that most of the mass accreted by the massive stars was originally distributed throughout the clump at low densities. The radius values in the maps derived here and the results in Smith et al. (2009) are also similar, because very massive cores reach radii around 0.5 pc and low-mass ones have radii less than half a tenth of a parsec.

Observations, however, can also detect cores which will disperse before they form any stars. Thus, the observed CMF is a mixture of lower mass cores that form no stars and higher mass cores that form multiple stars (see Hatchell & Fuller 2008).
In our column density maps, we did not add any background or additional low-mass cores that will not form stars. Our maps are then less close to observations but show the best possible observable relationship between cores and stars. Any extra background or lower mass cores that form no stars will increase the local blending and strengthen our main results about the crucial role played by blending in the derivation of a CMF.

Another limitation of our results is that the stars could have moved from their birth sites in which case the column density maps are not representative of what the actual initial column density maps would have been. The motions of stars in the first few Myr, however, are subvirial (see Andrés et al. 2007; Offner et al. 2009), i.e., relatively slow, and the configuration of stars should not have changed much since their birth. Even so, net motions of stars are more outward than inward, so our approach probably underestimates the actual blending.

The maps are simulated by adding the column density profile of every progenitor core and thus by assuming an identical formation moment for every star. A way of checking the relevancy of the simulated starless column density maps would be to conduct an SPH simulation from the simulated maps back to a YSO distribution and compare it to present YSO distributions. This SPH simulation should take into account different star formation moments. It would, however, require assumptions about how the original cores are distributed along the line of sight, since we only know the YSO positions in the plane of sky.

By being able to directly compare the derived CMFs to the local stellar distribution, our approach allows good estimates of the undercounting of cores due to blending since we start with a one-to-one relation between stars and simulated progenitor cores. The differences between real star-forming regions and the young clusters used here underestimate the effects of blending in actual clusters.

5. CONCLUSIONS

Starting from YSO masses and positions in four nearby star-forming regions, we carried out simulations to estimate the initial starless core column density maps. After running the clump-finding algorithm clfind2d on these maps, we find that a derived CMF can have the right “shape” to match the IMF but can nonetheless undercount the number of star-forming cores and overestimate their masses due to blending of the initial cores in crowded regions. Such blending can occur even with no additional background material, no noise, and a 100% SFE. Our results are as follows.

1. In no case do we obtain a quantitative recovery of the initial CMF by the derived CMF.
2. The comparison of derived CMFs does not distinguish the input models from one another.
3. The derived high-mass tails can always be fitted by a power law whose slopes are similar to the Salpeter value (2.35) within errors for both isolated and blended groups.
4. Even in isolated groups, derived cores are undercounted by a factor of 1.4. In blended groups, this factor can be as high as 20.
5. The initial SFE is recovered within a few percent in isolated groups, whereas it is not recovered in blended groups where the initial SFE is underestimated and independent of the input SFE (around 0.15 for a resolution of 0.5, around 0.05 for a resolution of 1’).
6. The peak mass of the derived CMF exceeds the peak mass of the initial CMF by a mean factor of 1.0 in isolated regions and by a mean factor of 12.3 in blended regions.
7. Mass recovery depends on the shape of the core profile.
8. The relationship between the mass $M$ and radius $R$ of a derived core is independent of the input core profile and obeys a power law of $M \propto R^3$.

These results suggest that, if stars are represented by progenitor cores with a fixed mass ratio, the distribution of apparent core masses derived from a column density map is an unreliable estimator of the stellar masses, especially in young clusters.

Deriving an accurate CMF in blended regions appears to be a very difficult task. Clump-finding programs, which rely on peak identification, have difficulty identifying the extended part of the core profile in blended maps. Even in isolated groups, the outer part of the cores is not well recovered and we find that the relationship between derived core masses and radii is independent of the input model relationship. Further difficulties are expected to arise in real observations where an extended background is removed since the very extended component of the core could be mistaken for the background or could even be missed if it falls below the detection limit of the observations.

We have examined the mapping of known YSO distributions to their modeled observable progenitor cores. The conclusions of this work should be very similar to what observations would give if the local star distribution was available at the same time as the CMF. We expect the results to underestimate the effects of blending since real observations must deal with background removal and likely a slightly more compact configuration of cores than their present-day locations.

We thank Jaime Pineda for very interesting discussions about the clfind2d clump-finding algorithm, Michael Reid for sending us his paper before publication and for discussions about observational bias in deriving CMF, and Patrick Hennebelle for his comments. We are also thankful to the Smithsonian Astrophysical Observatory for partial support of the visit of M.M.

APPENDIX

COLUMN DENSITY IN TERMS OF MASS AND PROJECTED RADIUS

The initial condensation that produces a protostar is represented here by analytic expressions for the column density $N$ as a function of total mass $M$ and projected radius $r$ for the truncated isothermal sphere of Bonnor (1956) and Ebert (1955), hereafter BE, and for the “TNT” density profile described by Myers (2010).

The density profile of the self-gravitating isothermal sphere is approximated to a high degree of accuracy by the expression of Natarajan & Lynden-Bell (1997),

$$n = n_0 \left( \frac{C}{c^2 + \xi^2} - \frac{D}{d^2 + \xi^2} \right),$$

where $n_0$ is the central maximum density,

$$\xi = \frac{r}{a},$$

is the dimensionless radius, $r$ is the spherical radius, and $a$ is the thermal scale length

$$a \equiv \frac{\sigma}{(4\pi G mn_0)^{1/2}}$$

(A3)
for velocity dispersion $\sigma$. Here, $G$ is the gravitational constant and $m$ is the mean mass per particle, with $C = 50$, $D = 48$, $c^2 = 10$, and $d^2 = 12$.

Integration of the density in Equation (A1) yields expressions for the mass $M$ within radius $r$, and for the column density $N$ at projected radius $b$. The mass is

$$M = \frac{\sigma^3 \mu}{(4\pi m \rho_0)^{1/2}} G^{3/2}, \quad (A4)$$

where the dimensionless mass is

$$\mu \equiv C \left( \xi - \arctan \left( \frac{\xi}{c} \right) \right) - D \left( \xi - \arctan \left( \frac{\xi}{d} \right) \right). \quad (A5)$$

If the sphere is truncated at a fixed radius $R$, the corresponding dimensionless radius is denoted as $\xi_{\text{max}} \equiv R/a$. If the sphere is critically stable, then $\xi_{\text{max}} = 6.46$ and Equation (A5) yields $\mu = 15.85$. This case is known as a CSBE sphere.

The column density through a truncated sphere of total mass $M$ within radius $R$ is then given by

$$N = \frac{\sigma^4 \mu \nu}{2\pi m G^2 M}, \quad (A6)$$

where

$$\nu \equiv \left[ \frac{C}{\gamma} \arctan \left( \frac{\xi_{\text{max}}^2 - \beta^2}{\gamma} \right) - \frac{D}{\delta} \arctan \left( \frac{\xi_{\text{max}}^2 - \beta^2}{\delta} \right) \right], \quad (A7)$$

where

$$\beta \equiv \frac{b}{a} \quad (A8)$$

$$\gamma^2 \equiv c^2 + \beta^2 \quad (A9)$$

and

$$\delta^2 \equiv d^2 + \beta^2. \quad (A10)$$

Note that in Equation (A7), $0 \leq \beta \leq \xi_{\text{max}}$. It is useful to write the scale length in terms of the total mass by eliminating $n_0$ from Equations (A3) and (A4), giving

$$a = \frac{GM}{\mu \sigma^2}, \quad (A11)$$

i.e., $a \propto M$ if $\sigma$ (i.e., $T$) is set to a fixed value (CSBET model), or

$$a = \left( \frac{G}{4\pi \rho} \right)^{0.25} \left( \frac{M}{\mu} \right)^{0.5}, \quad (A12)$$

i.e., $a \propto M^{0.5}$ if $P$ is set to a fixed value (CSBEP model).

Another density profile is obtained from the assumption that infall is equally likely to stop at any moment and from the requirement that the resulting distribution of protostar masses follow the IMF. The profile resembles a superposition of “core” and “clump” density profiles. It is also similar to the TNT model of Myers & Fuller (1992) and to the two-component turbulent core model of McKee & Tan (2003),

$$n = A r^{-2} + B r^{-2/3}, \quad (A13)$$

where $A = 34$ pc$^2$ cm$^{-3}$ and $B = 2700$ pc$^2$/3 cm$^{-3}$ (see Myers 2010).

The mass $M$ within radius $R$ is obtained by integrating Equation (A13),

$$M = 4\pi m \left( 3B R^{7/3} / 7 \right). \quad (A14)$$

Similarly, the column density through a TNT sphere of maximum radius $R$ is

$$N = \frac{2A(1 + \xi^2)^{1/2}}{R} \arctan(\xi) + 2B \left( \frac{R}{(1 + s^2)^{1/2}} \right)^{1/3} \times \int_0^s d\xi (1 + \xi^2)^{-1/3}, \quad (A15)$$

where $s$ is a dimensionless variable, $0 \leq s \leq +\infty$, and where the projected radius is

$$b = R/(1 + s^2)^{1/2}. \quad (A16)$$

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