Quartic Quasi-Topological-Born-Infeld Gravity

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Abstract

In this paper, quartic quasi-topological black holes in the presence of a nonlinear electromagnetic Born-Infeld field is presented. By using the metric parameters, the charged black hole solutions of quasi-topological Born-Infeld gravity is considered. The thermodynamics of these black holes are investigated and I show that the thermodynamics and conserved quantities verify the first law of thermodynamics. I also introduce the thermodynamics of asymptotically AdS rotating black branes with flat horizon of these class of solutions and I calculate the finite action by use of the counterterm method inspired by AdS/CFT correspondence.
I. INTRODUCTION

The paradigm of extra dimensions does much more that excite our imagination. It solves the so called "hierarchy of scales" problem. Several extra-dimensional models have been introduced in the past few years. It is well known that the natural generalization of the Einstein-Hilbert action to higher dimensional spacetime, and higher order gravity with second order equation of motion, is the Lovelock action \cite{1}. Because of the topological origin of the Lovelock terms, the second term of the Lovelock action (the Gauss-Bonnet term) does not have any dynamical effect in four dimensions. Similarly, the cubic term just contributes to the equations of motion in seven dimensions or greater. A modification of higher order Lovelock gravity which contains cubic and quartic terms of Riemann tensor and contributes to the equation of motions in five dimensions is quasi-topological gravity \cite{2–4}. Since the quasi-topological theory contains derivatives of metrics of order not higher than two, the quantization of linearized quasi-topological theory is free of ghosts. Thus, it is natural to study the effects of these higher curvature terms on the properties and thermodynamics of black holes and black branes \cite{3–12}.

It is well known that the rsh of gravitational equations (like Einstein equation) is energy-momentum tensor, which relate to matter and various fields. Among these fields the electromagnetic field is so important. Without exaggeration the linear electromagnetic field is one of the most successful theories of electromagnetic fields. In order to solve the problem of self-energy of electron, the theory of the non-linear electromagnetic field was introduced by Born and Infeld (BI) in 1934 \cite{13}. In the limits of weak fields, the Born-Infeld lagrangian reduces to Maxwell lagrangian plus some small corrections.

If one is to consider the Maxwell fields coupled to a gravitational action, which also includes string generated corrections at higher orders, then, it is natural to consider string generated corrections to the electromagnetic field action as well. It is known that there are Born-Infeld terms which appear as higher order corrections to the Maxwell action. In fact, considering the analogy between the quasi-topological and the Born-Infeld terms, it is worth to include both these corrections simultaneously. In this letter, I will consider the quartic quasi-topological gravity in the presence of nonlinear electromagnetic field and consider the black hole and black brane solutions of this theory.

The outline of this paper is as follows: In Sec. II a brief review of the quartic quasi-
topological gravity in the presence of a nonlinear Born-Infeld electromagnetic field is presented. In section III I consider the charged black holes of quasi-topological gravity in the presence of Born-Infeld electromagnetic field. Section IV is devoted to the investigation of the thermodynamic properties of these solutions and the first law of thermodynamics. In Sec. V I endow the solutions with asymptotically AdS charged rotating black branes and study the thermodynamic properties of them with flat horizon. Then, in Sec. VI the finite action and conserved quantities of the solutions are calculated. Finally, I finish my paper with some concluding remarks in section VII.

II. QUASI-TOPOLOGICAL-BORN-INFELD ACTION

The action of quartic quasi-topological gravity in \((n + 1)\) dimensions in the presence of a nonlinear Born-Infeld electromagnetic field can be written as

\[
I_G = \frac{1}{16\pi} \int d^{n+1}x \sqrt{-g} \left[ -2\Lambda + L_1 + \mu_2 L_2 + \mu_3 X_3 + \mu_4 X_4 + L(F) \right].
\]

where \(\Lambda = -n(n-1)/2l^2\) is the cosmological constant, \(L_1 = R\) is just the Einstein-Hilbert Lagrangian, \(L_2 = R_{abcd} R^{abcd} - 4 R_{ab} R^{ab} + R^2\) is the second order Lovelock (Gauss-Bonnet) Lagrangian, \(X_3\) is the curvature-cubed Lagrangian \[3\]

\[
X_3 = R_{abcd} R^{e f} R^a b + \frac{1}{(2n - 1)(n - 3)} \left( \frac{3(3n - 5)}{8} R_{abcd} R^{abcd} R \right)
- 3(n - 1) R_{abcd} R_{e f}^a R^{de} + 3(n + 1) R_{abcd} R^{ac} R^{bd}
+ 6(n - 1) R_a b R^c R^e a - \frac{3(n - 1)}{2} R_a b R^a R + \frac{3(n + 1)}{8} R^3 \right),
\]

and \(X_4\) is the fourth order term of quasi-topological gravity \[4\]

\[
X_4 = c_1 R_{abcd} R^{cdef} R_{e}^a R_{h}^b + c_2 R_{abcd} R^{abcd} R_{e f} R_{g h} R_{e}^e R_{g h} R_{e}^f + c_3 R R_{ab} R^{ac} R_e^b + c_4 R_{abcd} R^{abcd} (R_{e f}^e R_{g h} R_{e}^f R_{g h} R_{e}^f)^2
+ c_5 R R^{ac} R_{d e} R_{e f} R_{g h} R_{e}^b + c_6 R R_{abcd} R^{ac} R_{d e} R_{e f} R_{g h} R_{e}^b + c_7 R R_{abcd} R^{ac} R_{d e} R_{e f} R_{g h} R_{e}^b + c_8 R R_{abcd} R^{ac} R_{d e} R_{e f} R_{g h} R_{e}^b
+ c_9 R R_{abcd} R^{ac} R_{d e} R_{e f} R_{g h} R_{e}^b + c_{10} R^4 + c_{11} R^2 R_{abcd} R^{abcd} R_{e f} R_{g h} R_{e}^b + c_{12} R^2 R_{abcd} R_{e f} R_{g h} R_{e}^b
+ c_{13} R_{abcd} R^{abef} R_{ef g} R^{g h} R_{e f}^c R_{g h} R_{e f}^{d g} + c_{14} R_{abcd} R^{abef} R_{ef g} R^{g h} R_{e f}^c R_{g h} R_{e f}^{d g}. \]

(3)
with
\[
\begin{align*}
  c_1 &= - (n - 1) \left( n^7 - 3 n^6 - 29 n^5 + 170 n^4 - 349 n^3 + 348 n^2 - 180 n + 36 \right), \\
  c_2 &= - 4 (n - 3) \left( 2 n^6 - 20 n^5 + 65 n^4 - 81 n^3 + 13 n^2 + 45 n - 18 \right), \\
  c_3 &= - 64 (n - 1) \left( 3 n^2 - 8 n + 3 \right) \left( n^2 - 3 n + 3 \right), \\
  c_4 &= - (n^8 - 6 n^7 + 12 n^6 - 22 n^5 + 114 n^4 - 345 n^3 + 468 n^2 - 270 n + 54), \\
  c_5 &= 16 (n - 1) \left( 10 n^4 - 51 n^3 + 93 n^2 - 72 n + 18 \right), \\
  c_6 &= - 32 (n - 1)^2 (n - 3)^2 \left( 3 n^2 - 8 n + 3 \right), \\
  c_7 &= 64 (n - 2) (n - 1)^2 \left( 4 n^3 - 18 n^2 + 27 n - 9 \right), \\
  c_8 &= - 96 (n - 1) (n - 2) \left( 2 n^4 - 7 n^3 + 4 n^2 + 6 n - 3 \right), \\
  c_9 &= 16 (n - 1)^3 \left( 2 n^4 - 26 n^3 + 93 n^2 - 117 n + 36 \right), \\
  c_{10} &= n^5 - 31 n^4 + 168 n^3 - 360 n^2 + 330 n - 90, \\
  c_{11} &= 2 \left( 6 n^6 - 67 n^5 + 311 n^4 - 742 n^3 + 936 n^2 - 576 n + 126 \right), \\
  c_{12} &= 8 \left( 7 n^5 - 47 n^4 + 121 n^3 - 141 n^2 + 63 n - 9 \right), \\
  c_{13} &= 16 n (n - 1) (n - 2) (n - 3) \left( 3 n^2 - 8 n + 3 \right), \\
  c_{14} &= 8 (n - 1) \left( n^7 - 4 n^6 - 15 n^5 + 122 n^4 - 287 n^3 + 297 n^2 - 126 n + 18 \right).
\end{align*}
\]

In the action (1), \(L(F)\) is the Born-Infeld Lagrangian given as [14–19]
\[
L(F) = 4 \beta^2 \left( 1 - \sqrt{1 + \frac{F^2}{2 \beta^2}} \right). \quad (4)
\]
where \(F = F_{\mu \nu} F^{\mu \nu}\), \(F_{\mu \nu} = \partial_\mu A_\nu - \partial_\nu A_\mu\) is the electromagnetic field tensor and \(A_\mu\) is the vector potential. One may note that in the limit \(\beta \to \infty\) reduces to the standard Maxwell form \(L(F) = -F^2\).

**III. CHARGED QUASI-TOPOLOGICAL-BORN-INFELD BLACK HOLE SOLUTIONS**

Now, I introduce the charged black hole solutions of quasi-topological gravity in the presence of nonlinear Born-Infeld electromagnetic field with Lagrangian (4). The metric has the following form:
\[
ds^2 = - f(\rho) dt^2 + \frac{d\rho^2}{f(\rho)} + \rho^2 d\Omega^2. \quad (5)
\]
where
\[
\Omega^2 = \begin{cases} 
  d\theta_1^2 + \sum_{i=2}^{n-1} \prod_{j=1}^i \sin^2 \theta_j d\theta_i^2 & k = 1 \\
  d\theta_1^2 + \sinh^2 \theta_1 d\theta_2^2 + \sinh^2 \theta_1 \sum_{i=3}^{n-1} \prod_{j=2}^i \sin^2 \theta_j d\theta_i^2 & k = -1 \\
  \sum_{i=1}^{n-1} d\phi_i^2 & k = 0
\end{cases}
\]
represents the line element of an \((n - 1)\)-dimensional hypersurface with constant curvature \((n - 1)(n - 2)k\) and volume \(V_{n-1}\). Using the metric (5) and
\[
A_\mu = h(\rho)\delta_\mu^0,
\]
for the vector potential, one can calculate the one dimensional action after integration by parts. One obtains the action per unit volume as
\[
I_G = \frac{(n-1)}{16\pi l^2} \int d\rho d\nu \left[ \rho^n (1 + \psi + \hat{\mu}_2 \psi^2 + \hat{\mu}_3 \psi^3 + \hat{\mu}_4 \psi^4) \right] + \frac{4l^2 \beta^2 \rho^{(n-1)} (1 - \sqrt{1 - \frac{h'^2}{\beta^2}})}{(n-1)}.
\]
(7)
where \(\psi = l^2 \rho^{-2}(k - f)\) and the dimensionless parameters \(\hat{\mu}_2, \hat{\mu}_3\) and \(\hat{\mu}_4\) are defined as:
\[
\hat{\mu}_2 \equiv \frac{(n-2)(n-3)}{l^2} \mu_2, \quad \hat{\mu}_3 \equiv \frac{(n-2)(n-5)(3n^2 - 9n + 4)}{8(2n-1)l^4} \mu_3, \quad \hat{\mu}_4 \equiv \frac{n(n-1)(n-2)^2(n-3)(n-7)(n^5 - 15n^4 + 72n^3 - 156n^2 + 150n - 42)}{l^6} \mu_4.
\]
Variation with respect to \(h(\rho)\) gives
\[
3h'\beta^2 - h'^2 + rh''\beta^2 = 0,
\]
(8)
and therefore one can show that the vector potential can be written as
\[
h(\rho) = -\sqrt{\frac{(n-1)}{2n-4 \rho^{n-2}}} q \Gamma(\eta),
\]
(9)
where \(q\) is an integration constant which is related to the charge parameter and
\[
\eta = \frac{(n-1)(n-2)q^2}{2\beta^2 \rho^{2n-2}}.
\]
In Eq. (9) and throughout the paper, the following abbreviation for the hypergeometric function is used,
\[
\text{\(2F_1\left(\left[\frac{1}{2}, \frac{n-2}{2n-2}\right], \left[\frac{3n-4}{2n-2}\right], -z\right) = \Gamma(z)\).}
\]
(10)
The hypergeometric function $\Gamma(\eta) \rightarrow 1$ as $\eta \rightarrow 0$ ($\beta \rightarrow \infty$) and therefore $h(\rho)$ of Eq. (9) reduces to the gauge potential of Maxwell field.

Varying the action (7) with respect to $\psi(\rho)$ yields

\[
(1 + 2\hat{\mu}_2\psi + 3\hat{\mu}_3\psi^2 + 4\hat{\mu}_4\psi^3) \frac{dN(\rho)}{d\rho} = 0,
\]

which shows that $N(\rho)$ should be a constant. Variation with respect to $N(\rho)$ and substituting $N(\rho) = 1$ gives

\[
\hat{\mu}_4\psi^4 + \hat{\mu}_3\psi^3 + \hat{\mu}_2\psi^2 + \psi + \kappa = 0,
\]

where

\[
\kappa = \hat{\mu}_0 - \frac{m}{\rho^n} + \frac{4l^2\beta^2}{n(n-1)} \left[1 - \sqrt{1 + \eta} - \frac{\eta}{n-2}F(\eta)\right]
\]

and $m$ is an integration constant which is related to the mass of the spacetime. In order to obtain the black hole solutions, I choose two solutions of $f(\rho)$ as

\[
f_1(\rho) = k + \frac{\rho^2}{l^2} \left( \frac{\hat{\mu}_3}{4\hat{\mu}_4} + \frac{1}{2}R - \frac{1}{2}E \right),
\]

\[
f_2(\rho) = k + \frac{\rho^2}{l^2} \left( \frac{\hat{\mu}_3}{4\hat{\mu}_4} - \frac{1}{2}R + \frac{1}{2}K \right).
\]

where

\[
R = \left( \frac{\hat{\mu}_3^2}{4\hat{\mu}_4^2} - \frac{\hat{\mu}_2}{\hat{\mu}_4} + y_1 \right)^{1/2},
\]

\[
E = \left( \frac{3\hat{\mu}_3^2}{4\hat{\mu}_4^2} - \frac{2\hat{\mu}_2}{\hat{\mu}_4} - R^2 - \frac{1}{4R} \left[ \frac{4\hat{\mu}_2\hat{\mu}_3}{\hat{\mu}_4^2} - \frac{8}{\hat{\mu}_4} - \frac{\hat{\mu}_3^3}{\hat{\mu}_4^3} \right] \right)^{1/2},
\]

\[
K = \left( \frac{3\hat{\mu}_3^2}{4\hat{\mu}_4^2} - \frac{2\hat{\mu}_2}{\hat{\mu}_4} - R^2 + \frac{1}{4R} \left[ \frac{4\hat{\mu}_2\hat{\mu}_3}{\hat{\mu}_4^2} - \frac{8}{\hat{\mu}_4} - \frac{\hat{\mu}_3^3}{\hat{\mu}_4^3} \right] \right)^{1/2},
\]

\[
\Delta = \frac{H^3}{27} + \frac{D^2}{4}, \quad H = \frac{3\hat{\mu}_3 - \hat{\mu}_2}{3\hat{\mu}_4} - \frac{4\kappa}{\hat{\mu}_4},
\]

\[
D = \frac{2}{27} \frac{\hat{\mu}_3^2}{\hat{\mu}_4^2} - \frac{1}{3} \left( \frac{\hat{\mu}_3}{\hat{\mu}_4^2} + \frac{8}{\hat{\mu}_4} \right) \frac{\hat{\mu}_2}{\hat{\mu}_4} + \frac{\hat{\mu}_3^2 \kappa}{\hat{\mu}_4^2} + \frac{1}{\hat{\mu}_4^2}
\]

and $y_1$ is the real root of following equation:

\[
y^3 - \frac{\mu_2 y^2}{\mu_4} + \left( \frac{\mu_3}{\mu_4^2} - 4 \frac{\kappa}{\mu_4} \right) y - \frac{\mu_3^2 \kappa}{\mu_4^3} + \frac{4\mu_2 \kappa}{\mu_4^2} - \frac{1}{\mu_4^2} = 0
\]

The metric function $f(\rho)$ for the uncharged solution ($q = 0$) is real in the whole range $0 \leq \rho < \infty$. But for charged solutions, one should restrict the spacetime to the region $\rho \geq r_0$, 


where \( r_0 \) is the largest real root of \( \Delta_0 = \Delta(\kappa = \kappa_0) \), \( R_0 = R(\kappa = \kappa_0) \), \( E_0 = E(\kappa = \kappa_0) \) and \( K_0 = K(\kappa = \kappa_0) \), and \( \kappa_0 \) is

\[
\kappa_0 = \hat{\mu}_0 - \frac{m}{r_0^n} + \frac{4l^2\beta^2}{n(n-1)}[1 - \sqrt{1 + \eta_0} - \frac{\eta_0}{n - 2}F(\eta_0)]
\]  

(22)

where

\[
\eta_0 = \frac{(n - 1)(n - 2)q^2}{2\beta^2 r_0^{2n-2}}.
\]

Performing the transformation

\[
r = \sqrt{\rho^2 - r_0^2} \Rightarrow d\rho^2 = \frac{r^2}{r^2 + r_0^2}dr^2
\]  

(23)

the metric becomes

\[
ds^2 = -f(r)dt^2 + \frac{r^2dr^2}{(r^2 + r_0^2)f(r)} + (r^2 + r_0^2)\sum_{i=1}^{n-1}d\phi_i^2.
\]  

(24)

where now the functions \( \eta, h(r) \) and \( \kappa \) are

\[
\eta = \frac{(n - 1)(n - 2)q^2}{2\beta^2(r^2 + r_0^2)^{2n-2/2}}.
\]

(25)

\[
h(r) = -\sqrt{\frac{(n - 1)}{2n - 4}}\frac{q}{(r^2 + r_0^2)^{(n-2)/2}}\Gamma(\eta).
\]

\[
\kappa = \hat{\mu}_0 - \frac{m}{(r^2 + r_0^2)^{n/2}} + \frac{4l^2\beta^2}{n(n-1)}[1 - \sqrt{1 + \eta} - \frac{\eta}{n - 2}F(\eta)]
\]  

(26)

IV. THERMODYNAMICS OF QUASI-TOPOLOGICAL-BORN-INFELD BLACK HOLES

One can obtain the Hawking temperature of the black hole solutions that are considered in the previous section as:

\[
T_+ = \frac{f'(r_+)}{4\pi}\sqrt{1 + \frac{r_0^2}{r_+^2}}
\]

\[
= \frac{(n - 1)[n\hat{\mu}_0\Upsilon_+^8 + (n - 2)k l^2\Upsilon_+^6 + (n - 4)k^2\hat{\mu}_2 l^4\Upsilon_+^4 + (n - 6)k^3\hat{\mu}_3 l^6\Upsilon_+^2 + (n - 8)k^4\hat{\mu}_4 l^8] + 4\hat{\mu}_5^8\beta^2(1 - \sqrt{1 + \Upsilon_+})}{(\Upsilon_+^6 + 2k\hat{\mu}_2 l^2\Upsilon_+^4 + 3k^2\hat{\mu}_3 l^4\Upsilon_+^2 + 4\hat{\mu}_4 k^3 l^6)4\pi(n - 1)l^2\Upsilon_+}
\]  

(27)
where $\Upsilon_+ = \sqrt{r_+^2 + r_0^2}$ and $r_+$ is the largest real root of $f(r)$.

The entropy density for black hole in quartic quasi-topological gravity becomes,

$$S = \frac{r_+^{n-1}}{4} \left( 1 + \frac{2k(n-1)\hat{\mu}_2}{(n-3)r_+^2} + \frac{3k^2(n-1)\hat{\mu}_3}{(n-5)r_+^4} + \frac{4\hat{\mu}_3k^4}{r_+^6(n-7)} \right) \tag{28}$$

Calculating the flux of the electric field at infinity, one can find the charge of the black hole as

$$Q = \frac{V_{n-1}}{4\pi} \sqrt{\frac{(n-1)(n-2)}{2}} q \tag{29}$$

The electric potential $\Phi$ at infinity with respect to the horizon can be defined by [20, 21],

$$\Phi = A_\mu \chi^\mu \bigg|_{r=\infty} - A_\mu \chi^\mu \bigg|_{r=r_+} \tag{30}$$

where $\chi = \partial/\partial t$ is the null generator of the horizon. The electric potential $\Phi$ can be found as follow:

$$\Phi = \sqrt{\frac{(n-1)}{2(n-2)(r_+^2 + r_0^2)^{(n-2)/2}}} \Gamma(\eta_+). \tag{31}$$

By using the behavior of the metric at large $r$, the ADM (Arnowitt-Deser-Misner) mass of black hole can be arrived. One can easily show that the mass of the black hole is

$$M = \frac{V_{n-1}}{16\pi} (n-1) m. \tag{32}$$

In order to investigate the first law of thermodynamics, I use the expression for the entropy, the charge, and the mass that are given in Eqs. [28], [29] and [32], and keep $f(r_+) = 0$ in the mind, I introduce as

$$M(S, Q) = \frac{(n-1)(r_+^2 + r_0^2)^{n/2}}{16\pi} \left\{ \frac{4\beta^2}{n(n-1)} \left[ 1 - \sqrt{1 + \frac{1}{\beta^2}} + \frac{(n-1)\Xi}{(n-2)} \Gamma(\Xi) \right] + \hat{\mu}_0 + k \frac{l^2}{(r_+^2 + r_0^2)^2} + \hat{\mu}_2k^2 \frac{l^4}{(r_+^2 + r_0^2)^4} + \hat{\mu}_3k^3 \frac{l^6}{(r_+^2 + r_0^2)^6} + \hat{\mu}_4k^4 \frac{l^8}{(r_+^2 + r_0^2)^8} \right\} \tag{33}$$

where

$$\Xi = \frac{16\pi^2 Q^2}{\beta^2(r_+^2 + r_0^2)^{n-1}}.$$

In Eq. [33], $r_+$ is the real root of Eq. [28] which is a function of $S$. I can regard the parameters $S$ and $Q$ as a complete set of extensive parameters for the mass $M(S, Q)$ and define
the intensive parameters $T$ and $\Phi$ conjugate to them. These quantities are the temperature and the electric potential

$$T = \left( \frac{\partial M}{\partial S} \right)_Q, \quad \Phi = \left( \frac{\partial M}{\partial Q} \right)_S.$$  

(34)

It is easy to show that the intensive quantities calculated by Eq. (34) that are obtained by computing $\partial M/\partial r_+$ and $\partial S/\partial r_+$ and using the chain rule, coincide with Eqs. (27) and (31), respectively. So, the thermodynamic quantities calculated in Eqs. (27) and (31) lead to the first law of thermodynamics,

$$dM = TdS + \Phi dQ.$$  

(35)

V. THERMODYNAMICS OF CHARGED ROTATING QUASI-TOPOLOGICAL-BORN-INFELD BLACK BRANES

Now, I apply the spacetime solution (5) for $k = 0$ with a global rotation. One may perform the following rotation boost in the $t - \phi_i$ planes to add angular momentum to the spacetime

$$t \mapsto \Xi t - a_i \phi_i, \quad \phi_i \mapsto \Xi \phi_i - \frac{a_i}{l^2} t$$  

(36)

where $[x]$ is the integer part of $x$ for $i = 1...[n/2]$. The $SO(n)$ rotation group in $n + 1$ dimensions shows the maximum number of rotation parameters. So, the number of independent rotation parameters is $[n/2]$. Therefore, for the flat horizon of the AdS rotating solution with $p \leq [n/2]$, the metric can be written as follows [22]:

$$ds^2 = -f(r) \left( \Xi dt - \sum_{i=1}^{p} a_i d\phi_i \right)^2 + \frac{(r^2 + r_0^2)}{l^4} \sum_{i=1}^{p} \left( a_i dt - \Xi l^2 d\phi_i \right)^2$$

$$+ \frac{r^2 dr^2}{(r^2 + r_0^2) f(r)} - \frac{(r^2 + r_0^2)}{l^2} \sum_{i<j}^{p} (a_i d\phi_j - a_j d\phi_i)^2 + (r^2 + r_0^2) \sum_{i=p+1}^{n-1} d\phi_i.$$  

(37)

where $\Xi = \sqrt{1 + \sum_i^{k} a_i^2 / l^2}$. The vector potential for this solution can be rewritten as

$$A_\mu = -\sqrt{\frac{(n-1)}{2n-4}} \frac{q}{(r^2 + r_0^2)^{(n-2)/2}} \Gamma(\eta) \left( \Xi \delta_\mu^0 - \delta_\mu^i a_i \right) \text{(no sum on } i \text{)}.$$  

(38)

Analytic continuation of the metric, I can obtain the temperature and angular momentum as follows:

$$T = \frac{f'(r_+)}{4\pi \Xi} \sqrt{1 + \frac{r_0^2}{r_+^2}}.$$  

(39)
Calculating the flux of the electric field at infinity, one drives the electric charge per unit volume $V_{n-1}$:
\[ Q = \frac{1}{4\pi} \sqrt{\frac{(n-1)(n-2)}{2}} \Xi q. \]  
\[ (41) \]

According to the fact that $\chi = \partial_i + \sum_i \Omega_i \partial_i$, and using Eq. (30), the electric potential $\Phi$ is obtained as
\[ \Phi = \sqrt{\frac{(n-1)}{2(n-2)}} \frac{q}{\Xi(r^2 + r^2_0)^{(n-2)/2}} \Gamma(\eta_+). \]  
\[ (42) \]

VI. CONSERVED QUANTITIES OF THE SOLUTIONS

In this section, I drive the action and conserved quantities of the solutions. In general, the action and conserved quantities of the spacetime are divergent when evaluated on the solutions. By using the counterterms method and AdS/CFT correspondence, one can consider this divergence for asymptotically AdS solutions of Einstein gravity [23]. The finite action for asymptotically AdS solutions with flat boundary, $\hat{R}_{abcd}(\gamma) = 0$ may be written as follow:
\[ I = I_G + I_b + I_{ct} \]  
\[ (43) \]

In this finite action, the following boundary term makes the Einstein-Hilbert action well-defined [24],
\[ I_b^{(1)} = \frac{1}{8\pi} \int_{\partial M} d^n x \sqrt{-\gamma} K. \]  
\[ (44) \]

and the proper surface term for the Gauss-Bonnet term is [25–28],
\[ I_b^{(2)} = \frac{1}{8\pi} \int_{\partial M} d^n x \sqrt{-\gamma} \left\{ \frac{2\hat{\mu}l^2}{(n-2)(n-3)} J \right\}. \]  
\[ (45) \]

where $J$ is the trace of
\[ J_{ab} = \frac{1}{3} (2KK_{ac}K^c_b + K_{cd}K^{cd}K_{ab} - 2K_{ac}K^{cd}K_{db} - K^2K_{ab}). \]  
\[ (46) \]
and the surface terms for the curvature-cubed term of quasi-topological gravity have been driven in Ref. [6] as

\begin{equation}
I^{(3)}_b = \frac{1}{8\pi} \int_{\partial M} d^n x \sqrt{-\gamma} \left\{ \frac{3\mu_4 l^4}{5n(n-2)(n-1)(n-5)}(nK^5 - 2K^{ab}K_{ab}K^{cd}K_{cd}) + 4(n-1)K_{ab}K^{cd}K_{cd}K_eK_{ce}^d - (5n-6)KK_{ab}[nK^{ab}K^{cd}K_{cd} - (n-1)K^{ac}K^{bd}K_{cd}] \right\}.
\end{equation}

and

\begin{equation}
I^{(4)}_b = \frac{1}{8\pi} \int_{\partial M} d^n x \sqrt{-\gamma} \left\{ \frac{2\mu_4 l^6}{7n(n-1)(n-2)(n-7)(n^2-3n+3)}\left\{ \alpha_1 K^{ac}K_{ac}K_{bd}K^{cd} + \alpha_2 K^{2K^{ab}K_{cd}K^{e}K_{de} + \alpha_3 K^{2K^{ab}K_{ac}K_{bd}K^{ce}K_{de}} + \alpha_4 K^{2K^{ab}K_{cd}K^{e}K_{de}K_{ef} + \alpha_5 K^{2K^{ab}K_{ac}K_{bd}K^{ce}K_{cd}K_{ef} + \alpha_6 K^{2K^{ab}K_{ac}K_{bd}K^{cd}K^{ef} + \alpha_7 K^{2K^{ab}K_{ac}K_{bd}K^{cd}K_{ef}K_{ef} + \alpha_8 K^{2K^{ab}K_{ac}K_{bd}K^{cd}K_{ef}K_{ef}} \right\} \right\}.
\end{equation}

presents a surface term which makes the action of quartic quasi-topological gravity well-defined [8]. And the last term, \( I_{ct} \), which is a functional of the boundary curvature invariants, is counterterm and may be written as follow [29–32]:

\begin{equation}
I_{ct} = -\frac{1}{8\pi} \int_{\partial M} d^n x \sqrt{-\gamma} \frac{(n-1)}{L_{eff}}.
\end{equation}

The conserved quantities associated with the Killing vectors \( \partial/\partial t \) and \( \partial/\partial \phi^i \) can be obtained as

\begin{align}
M &= \frac{1}{16\pi}m \left( n\Xi^2 - 1 \right), \\
J_i &= \frac{1}{16\pi}n\Xi ma_i.
\end{align}

which are the mass and angular momentum of the solution.

By using Gibbs-Duhem relation

\begin{equation}
S = \frac{1}{T}(M - Q\Phi - \sum_{i=1}^{k} \Omega_i J_i) - I,
\end{equation}

one can introduce the entropy per unit volume \( V_{n-1} \) as follow:

\begin{equation}
S = \Xi \frac{r^{n-1}}{4}
\end{equation}

This shows that the entropy obeys the area law for our case where the horizon is flat.
VII. CONCLUDING REMARKS

In this paper, I introduced the quartic quasi-topological gravity in the presence of Born-Infeld field which is a nonlinear electromagnetic field. I computed the charged black hole solutions of this theory. These solutions presented a black holes with one or two horizons or a naked singularity depending on the values of charge and mass parameters. In order to verify the first law of thermodynamics of these black hole solutions, I calculated the thermodynamic quantities $S$, $Q$ and $M$ where the mass was driven as a function of the extensive parameters $S$ and $Q$. Then, I considered the thermodynamics of asymptotically AdS black branes with flat horizon. As in the case of Einstein solutions, the action and the conserved quantities of the quasi-topological solutions are not finite, I used the counterterm method to drive the finite action and the conserved quantities. I presented that the entropy obeys the area law for black branes with flat horizon.

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