Nuclear matter hole spectral function in the Bethe-Brueckner-Goldstone approach

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Abstract

The hole spectral function is calculated in nuclear matter to assess the relevance of nucleon-nucleon short range correlations. The calculation is carried out within the Brueckner scheme of many-body theory by using several nucleon-nucleon realistic interactions. Results are compared with other approaches based on variational methods and transport theory. Discrepancies appear in the high energy region, which is sensitive to short range correlations, and are due to the different many-body treatment more than to the specific N-N interaction used. Another conclusion is that the momentum dependence of the G-matrix should be taken into account in any self consistent approach.

Key words:
Nuclear matter, Equation of state, Many-body theory, Hole-line expansion.

PACS: =21.10.Pc, 25.30.Fj, 21.65.+f, 21.60.-n, 21.30.-x

1 Introduction

One of the main features of nuclear systems is the relevance of short range correlations (SRC) due to the hard core part of the nucleon-nucleon (NN) potential. These are relevant for the binding energy of the nuclear system in
general and are probed directly by the high momentum and energy region of
the hole spectral function. In fact, SRC are entirely responsible for this region,
where the mean field and the long range correlations can give only negligable
contributions. Spectral functions are extracted mainly from electron-nucleus
scattering experiments.

Theoretically, many approaches have been proposed for a quantitative de-
scription of SRC and the corresponding spectral functions. The many-body
aspect appears essential for a correct understanding of the correlations, and
Bethe-Brueckner [1] or variational [2,3] methods have been extensively applied.
Although the main features of the spectral functions in the region relevant for
SRC seem to have a similar trend within different approaches, discrepancies
are still present and controversial interpretations of the experimental data ex-
ist in the literature. Recently [4] it was pointed out that the main features
of the nucleon spectral function in nuclear matter can be understood in the
framework of transport theory. In this approach the only ingredient is the
in-medium nucleon-nucleon cross section, which can be, in first approxima-
tion, schematized by a single parameter throughout the entire momentum and
energy range. The self-consistent calculation of the single particle self-energy
and strength function then provides the nucleon spectral function. Therefore,
phase space effects seem to dominate the behaviour of the spectral function.

A close comparison among the results of different approaches appears nec-
essary to extract the characteristics of SRC which determine the strength
function. In particular, the dependence of the results on the particular many-
body theory as well as on the specific nucleon-nucleon interaction used in the
calculations should elucidate the structure of the SRC.

In this letter, we present calculations of the nucleon spectral function in nuclear
matter within the Brueckner scheme using different realistic NN interactions
and compare them with other approaches. We will concentrate the comparison
mainly in the high energy region, where the effect of correlations is dominating.

2 Formalism

The hole spectral function in nuclear matter is defined as:

\[ S(k, \omega) = \sum_f |\langle \Psi_{A-1}^f | a_k | \Psi_A^0 \rangle |^2 \delta \left( \omega - \left( E_{A-1}^f - E_A^0 \right) \right) \]  \hspace{1cm} (1)

where \(|\Psi_A^0\rangle\) is the ground state for \(A\) nucleons with eigenvalue \(E_A^0\), \(|\Psi_{A-1}^f\rangle\) is
a complete set of states for \(A - 1\) nucleons with eigenvalues \(E_{A-1}^f\), and \(a_k\) is
the annihilation operator for the normalized plane wave at momentum \(k\).
In a single-particle description of nuclei only states with \( k < k_F \) are occupied and the final states involved in the sum are at most one hole states. In a correlated nucleus, the wave function of the ground state can have high momentum components, and the structure of the spectral function becomes more complicated.

In nuclear matter, the spectral function corresponding to the nucleon self-energy \( M(k, \omega) = V(k, \omega) + iW(k, \omega) \), is given by the well known result [1]:

\[
S(k, \omega) = -\frac{1}{\pi} \text{Im} G(k, \omega) = -\frac{1}{\pi} \frac{W(k, \omega)}{(-\omega - \frac{k^2}{2m} - V(k, \omega))^2 + W(k, \omega)^2}
\]

(2)

where \( G(k, \omega) \) is the single-particle Green’s function:

\[
G(k, \omega) = \frac{1}{-\omega - \frac{k^2}{2m} - V(k, \omega) - iW(k, \omega)}
\]

(3)

The real and imaginary parts of the self-energy, \( V(k, \omega) \) and \( W(k, \omega) \), are highly off-shell in the considered energy and momentum ranges. In nuclear matter, according to the BBG expansion, the whole set of two-hole line contributions is summed up by the diagrams depicted in Fig. 11 [1], where the wavy lines indicate the Brueckner G-matrix. Since it is known that correlations with higher number of holes give a much smaller contribution to the ground state energy [5–7], it is reasonable to assume that their effect on the spectral function is also of minor relevance, producing mainly an additional background. The first diagram (1.a) corresponds to the standard Brueckner approximation for the nucleon self-energy, and has the following expression

\[
M_1(k, \omega) = \sum_{k' < k_F} \langle kk' | G(e(k') + \omega) | kk' \rangle_a
\]

(4)

where \( e(k) \) is the self-consistent single particle energy and the label \( a \) means antisymmetrization. In the energy range relevant to the hole strength function and for \( k > k_F \), this diagram contributes only to the real part \( V(k, \omega) \) of the self-energy. Indeed, the imaginary part of \( M_1(k, \omega) \) comes only from on-shell transitions from one-particle states \((k > k_F)\) to two-particle one-hole states and this process is not allowed by momentum conservation. The diagram (1.b) takes into account transitions to one-particle two-hole states. It contributes to both real and imaginary parts in the considered energy and momentum range. Explicitly, the imaginary part \( W(k, \omega) \) is given by

\[
W(k, \omega) = \frac{1}{2} \sum_{hh'p} \text{Im} \frac{|\langle kp | G(e(h') + e(h')) | hh'p \rangle_a|^2}{E - e(p) + e(h) + e(h') - i\eta} =
\]
\[
\frac{\pi}{2} \sum_{hh'p} |\langle kp|G(e(h) + e(h'))|hh'\rangle_a|^2 \delta(-\omega + e(p) - e(h) - e(h'))
\]  (5)

where the sum is restricted to states \( h \) and \( h' \) with a momentum smaller than \( k_F \) and \( p \) with momentum larger than \( k_F \).

Once the Brueckner calculation for nuclear matter is performed, the G-matrix and the corresponding self-consistent single particle spectrum are obtained. The G-matrix is then calculated off-shell for the entry energies needed in Eqs. (4,5), which provide the real and imaginary parts of the self-energy. Finally, the spectral function is calculated from Eq. (2).

Equation (5) is similar to the formula used in ref. [4]. The main difference is the inclusion in the latter of the single particle strength functions in the phase space integral, which implies a self-consistent calculation. Furthermore, in that work the square of the in-medium scattering matrix is approximated by a constant average value, which simplifies the calculation considerably. The average scattering matrix is then adjusted in order to reproduce at best the results of ref. [8].

Equation (5) looks also similar to the formula for perturbative corrections introduced in ref. [3] within the variational scheme. In this case, the g-matrix is replaced by the residual interaction between hole states and two-hole one-particle states. The perturbative corrections are essential near the Fermi energy since they allow the hole states to acquire a width. However, in the high energy region, where we are focusing the analysis, their contribution is only marginal. In the variational scheme the main contribution comes from the two-hole one-particle orthogonal correlated intermediate states, which cannot be cast in a formula like Equation (5). Therefore a formal comparison between the BBG and variational approach would be misleading.

As a consequence the numerical comparison among these many-body theories appears necessary.

3 Results

We performed calculations of the nuclear matter spectral function with three different two-body potentials, the Urbana \( v_{14} \) [9], the Argonne \( v_{14} \) [10] and the Argonne \( v_{18} \) [11]. Three-body forces were added, according to the Urbana IX model [12], and adjusted to reproduce the correct saturation point. We checked that the effect of the three-body force on the spectral function is negligible. The Urbana potential was used only to compare with the results of ref. [3] and ref. [4].

In Figs. 2-a and 2-b we show the spectral function calculated at two values of
the momentum, \( k = 2.25 \text{ fm}^{-1} \) and \( k = 3.5 \text{ fm}^{-1} \). The three lines indicate our BBG calculations with different potentials as described in the figure, while the full circles label the results of ref. [3], where the Urbana \( v_{14} \) was used. For both momenta the dependence on the nucleon-nucleon potential appears quite weak, only in some cases a discrepancy is present, which however does not exceed 20%. Larger deviations in the high energy region occur between the BBG results and those of ref. [4]. We can conclude that these discrepancies in the high energy region are due to the many-body treatment and not to the interaction employed.

In Figs. 3-a and 3-b we compare our results (for the Urbana \( v_{14} \) potential) to the fully self consistent calculation of Lehr et al. [4]. In this case the comparison can be more transparent. In fact, the first order calculations of ref. [4], indicated by the dotted lines in the figures, are exactly equivalent to our calculations if the G-matrix in Eq. (5) is replaced by a constant average value independent of momenta, directly related to in-medium nucleon-nucleon cross section [4]. The discrepancy with the BBG calculations is, in this case, only due to this approximation, i.e. to the neglect of the G-matrix momentum dependence. This shows the relevance of the momentum dependent correlations for the determination of the single particle spectral function. The fact that the introduction of the self-consistency moves the high energy tail towards the BBG behaviour appears misleading, since in the BBG calculations no self-consistency has been used. This observation seems equally well applicable to the comparison with the variational calculations [3], where also no self-consistency procedure is included in the many-body scheme.

Of course these conclusions do not imply that the self-consistent procedure is not relevant, but only that it must be carried out with the full momentum dependence of the G-matrix.

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Figure Captions.

Fig. 1. Diagrams contributing to the nucleon self-energy $M(k, \omega) = V(k, \omega) + iW(k, \omega)$. The wavy lines indicate the Brueckner G-matrix. (a) The first diagram is the standard Brueckner approximation $M_1(k, \omega)$ for the nucleon self-energy, eq.(4). This diagram contributes only to the real part $V(k, \omega)$ of the self-energy. (b) This diagram takes into account transitions to one-particle two-hole states. It contributes to both real and imaginary parts of the self-energy in the considered energy and momentum range.

Fig. 2. BBG calculations of the nuclear matter strength function $S(k, \omega)$. The solid line is our result with the Argonne $v_{18}$ potential, the long-dashed line is obtained with Argonne $v_{14}$ and the dot-dashed line with the Urbana $v_{14}$. The black dots are the variational results of Benhar et al. (a) For the momentum $k = 2.25 \text{ fm}^{-1}$. (b) For $k = 3.5 \text{ fm}^{-1}$.

Fig. 3. Comparison between our results and those of Lehr et al., ref. [4]. The dotted curves are obtained in that work after the first iteration step, the solid curves are their fully self consistent calculation; the dashed curves show the variational results of Benhar et al. , ref. [3]. Our BBG results calculated with the Urbana potential are shown here as asterisks. (a) For the momentum $k = 2.26 \text{ fm}^{-1}$. (b) For $k = 3.59 \text{ fm}^{-1}$.
