New Constraint on the Parameters in Cabibbo-Kobayashi-Maskawa Matrix of Wolfenstein’s Parametrization

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Abstract

Based on the relation between CP-violation phase and the other three mixing angles in Cabibbo-Kobayashi-Maskawa matrix postulated by us before, a new constraint on the parameters of Wolfenstein’s parametrization is given. The result is consistent with the relative experimental results and can be further put to the more precise tests in future.

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From the discovery of CP-violation by an experiment on $K^0$ decay in 1964 [1], more than thirty years have passed. Due to the physical importance of the violation of this discrete symmetry, such as the fact that it is one of the necessary ingredients to explain the dominance of matter over anti-matter in our universe [2] and it could well be a sensitive probe for new physics beyond the standard model, it has aroused both experimental and theoretical physicists a great interests, many experiments have been done or are being done, number thousands of papers about it have been published.

To understand how CP symmetry is violated, a lot of theories have been established. The theories can be classified into three types [3,4]: superweak [5], milliweak [6,7] which including the standard Kobayashi-Maskawa (KM) model and millistrong [1,8]. In the three-generation standard model, CP violation originates from the single phase naturally occurring in the Cabibbo-Kobayashi-Maskawa (CKM) quark-mixing matrix [9,10]. Actually, this phase is introduced somewhat artificially, and people have been thinking it is independent of the other three angles for many years and never doubted whether they have a intrinsic relation [11].

Furthermore, many researchers have set up a wide variety of quark mass matrices [12-15], the purpose is to construct the CKM matrix and to find the relationships between quark masses and mixing angles together with the CP violation phase. In fact, all the works can be taken as an effort to reduce the number of free parameters presented in the standard model, because, it is evident that the less free parameters, the less uncertainty.

In Ref.[16], we have found that the CP-violation angle and the other three mixing angles satisfy the following relation

$$\sin \frac{\delta}{2} = \sqrt{\frac{\sin^2 \theta_1 + \sin^2 \theta_2 + \sin^2 \theta_3 - 2(1 - \cos \theta_1 \cos \theta_2 \cos \theta_3)}{2(1 + \cos \theta_1)(1 + \cos \theta_2)(1 + \cos \theta_3)}} \tag{1}$$

where $\theta_i$ ($i = 1, 2, 3$) are the corresponding angles in the standard KM parametrization matrix

$$V_{KM} = \begin{pmatrix} c_1 & -s_1 c_3 & -s_1 s_3 \\ s_1 c_2 & c_1 c_2 c_3 - s_2 s_3 e^{i\delta} & c_1 c_2 s_3 + s_2 c_3 e^{i\delta} \\ s_1 s_2 & c_1 s_2 c_3 + c_2 s_3 e^{i\delta} & c_1 s_2 s_3 - c_2 c_3 e^{i\delta} \end{pmatrix} \tag{2}$$

with the standard notations $s_i = \sin \theta_i$ and $c_i = \cos \theta_i$ are used.

The geometry meaning of Eq.(1) is very clear, $\delta$ is the solid angle enclosed by three angles $\theta_1$, $\theta_2$ and $\theta_3$ which relate to the mixing angles among three generations, or the
area to which the solid angle corresponds on a unit sphere. It should be noted that, to make \( \theta_1, \theta_2 \) and \( \theta_3 \) enclosed a solid angle, the condition
\[
\theta_i + \theta_j > \theta_k \quad (i \neq j \neq k \neq i. \ i, j, k = 1, 2, 3)
\]
must be satisfied.

Now, we find that the CP-violation seems to originate in a geometry reason. The deeper dynamic mechanism resulting in this kind of geometry is not clear yet, however, we guess that it maybe closely relate to the noncommutative rotation property of the \( SU(2) \) gauge group, which describes the part of weak interaction in the standard model. It should be referred here, people have recognized that the CP violation parameter \( \epsilon \) is related to a certain area \([15,17]\) more than ten years ago, but the relation between this area and the geometry constructed by three mixing angles have not been recognized yet.

There are other parametrization forms of KM matrix, among them the form introduced by Wolfenstein \([18]\) is the most frequently used one \([19-21]\) in the usual references. It reads
\[
V_W = \begin{pmatrix}
1 - \frac{1}{2} \lambda^2 & \frac{1}{2} \lambda & A \lambda^3 (\rho - i \eta + i \eta \lambda^2) \\
-\lambda & 1 - \frac{1}{2} \lambda^2 - i \eta A^2 \lambda^4 & A \lambda^2 (1 + i \eta \lambda^2) \\
A \lambda^3 (1 - \rho - i \eta) & -A \lambda^2 & 1
\end{pmatrix}.
\]

(3)

So, it is necessarily to convert the new constraint Eq.(1) which represented by four angles \( \delta, \theta_1, \theta_2, \theta_3 \) into the one represented by Wolfenstein’s parameters \( A, \lambda, \rho, \eta \). It is easy to do so if we take use of the following translation prescription between KM’s and Wolfenstein’s parameters \([11]\)

\[
s_1 \approx \lambda, \quad c_1 \approx 1 - \frac{\lambda^2}{2}
\]

(4)

\[
s_2 \approx \lambda^2 A((\rho - 1)^2 + \eta^2]
\]

(5)

\[
s_3 \approx (\rho^2 + \eta^2)^{1/2} A \lambda^2
\]

(6)

\[
sin \delta \approx \frac{\eta}{(\rho^2 + \eta^2)^{1/2} ((\rho - 1)^2 + \eta^2)^{1/2}}.
\]

(7)

From Eq.(1), we obtain
\[
sin \delta = \frac{(1 + \cos \theta_1 + \cos \theta_2 + \cos \theta_3)\sqrt{\sin^2 \theta_1 + \sin^2 \theta_2 + \sin^2 \theta_3 - 2(1 - \cos \theta_1 \cos \theta_2 \cos \theta_3)}}{(1 + \cos \theta_1)(1 + \cos \theta_2)(1 + \cos \theta_3)}.
\]

(8)

Substituting Eqs.(4-6) to Eq.(8) and expanding the right hand side of Eq.(8) in powers of \( \lambda \), with a little more complicated calculation, when approximate to the order of \( \lambda^5 \), we get
\[
sin \delta = \frac{A \sqrt{(1 - \rho)^2 + \eta^2 + (\rho^2 + \eta^2) \lambda^3 + A [(1 - \rho)^2 + \eta^2 + (\rho^2 + \eta^2) - 2 A^2 (1 - 2 \rho)^2] \lambda^5}}{2 \sqrt{2}}
\]

(9)
Identify the right hand sides of Eq.(9) and Eq.(7), we have
\[ \eta \frac{(\rho^2 + \eta^2)^{1/2}}{(1 - \rho)^2 + \eta^2)^{1/2}} \approx \frac{A\sqrt{(1 - \rho)^2 + \eta^2 + (\rho^2 + \eta^2)}}{2\sqrt{2}} \lambda^3. \] (10)

Here, in comparison with \( \lambda^3 \), we have neglected the term of order \( \lambda^5 \).

Eq.(10) is the new constraint on CP-violation and quark-mixing represented by Wolfenstein’s parameters approximate to the order of \( \lambda^3 \). It is the central result of this paper.

In following, we want to give a simple numerical analysis. Let
\[ x = (\rho^2 + \eta^2)^{1/2} \] (11)
\[ y = [(1 - \rho)^2 + \eta^2]^{1/2} \] (12)
then
\[ \eta = \frac{1}{2} \sqrt{2(x^2 + y^2) - (x^2 - y^2)^2 - 1} \] (13)

Substituting Eqs.(11-13) to Eq.(10), we arrive
\[ \frac{\sqrt{2(x^2 + y^2) - (x^2 - y^2)^2 - 1}}{xy} = \frac{A\lambda^3}{\sqrt{2}} \sqrt{x^2 + y^2} \] (14)

Fixing \( \lambda = 0.22 \) and \( A = \frac{4}{5} \), if we take \( y = 0.54 \sim 1.40 \) as input, then \( 0.22 \sim 0.46 \) for \( x \) is permitted. Hence, we find that the results are well in agreement with the experimental analysis [22]
\[ x = \sqrt{\rho^2 + \eta^2} = 0.34 \pm 0.12 \] (15)
and
\[ y = \sqrt{(1 - \rho)^2 + \eta^2} = 0.97 \pm 0.43. \] (16)

In summary, we have worked out a new constraint on the parameters of Wolfenstein’s parametrization of KM matrix. Its results are consistent with the relative experimental results and can be further put to the more precise tests in the future.

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