Realization of a General Multi-step Quantum Cloning Machine

L. Masullo, M. Ricci and F. De Martini

Dipartimento di Fisica and Istituto Nazionale per la Fisica della
Materia, Università di Roma ”La Sapienza”, p.le A. Moro 5, Roma
I-00185, Italy

Abstract

A general multi-step $N \rightarrow M$ probabilistic optimal universal cloning protocol is presented together with the experimental realization of the $(1 \rightarrow 3)$ and $(2 \rightarrow 3)$ machines. Since the present method exploits the bosonic nature of the photons, it can be applied to any particle obeying to the Bose statistics. On a technological perspective, the present protocol is expected to find applications as a novel, multi-qubit symmetrizer device to be used in modern quantum information networks.
A most relevant limitation in quantum information processing is the impossibility of perfectly cloning (copying) any unknown qubit $|\phi\rangle$ [1]. Even if this process is unrealizable in its exact form, it can be approximated optimally by the so-called universal optimal quantum cloning machine (UOQCM), one which exhibits the minimum possible noise for any possible input state. From a theoretical perspective, two different kinds of universal cloning machines have been developed so far: a deterministic $N \rightarrow M$ UOQCM based on a unitary operator acting on $N$ input qubits and $2(M - N)$ ancilla qubits [2,3] and a probabilistic UOQCM based on a symmetrization procedure involving a projective operator acting on $N$ inputs and $(M - N)$ blank ancilla qubits [4].

In the last years several experimental realizations of the UOQCM for polarization ($\pi-$) encoded photon qubits have been reported. The deterministic UOQCM has been realized by associating the cloning effect with QED stimulated emission [5] while the probabilistic machine has been realized adopting a linear symmetrization protocol [6]. Thus far, only the simplest $1 \rightarrow 2$ cloning processes, i.e. for $N = 1$ and $M = 2$, were realized by both schemes. In particular, the probabilistic process was achieved by exploiting the bosonic character of the photons within a linear Hong-Ou-Mandel interferometer scheme [7].

The present work presents the first generalization of the universal optimal cloning process by the realization of a very general linear procedure to extend the probabilistic protocol to any value of $N$ and $M$ according to a suggestion by Werner. [4]. The validity of this theoretical scheme is supported by the here reported experimental implementations of the $1 \rightarrow 3$ and $2 \rightarrow 3$ probabilistic processes for $\pi-$encoded photon qubits ($\pi-$qubits).

Let us outline first the $N \rightarrow M$ probabilistic cloning theory. Consider $N$ identically prepared unknown qubits in the state $\rho_i = |\phi\rangle \langle \phi|$ as input of the cloning machine while $(M - N)$ blank qubits, i.e. all in the state $\rho_A = \frac{I_2}{2}$, are used as an auxiliary resource. To generate $M$ output clones the machine performs the symmetrization of the output state by applying the projective operator, $\Pi_+^{(M)}$ over the symmetric subspace of $M$ qubits:

$$
|\phi\rangle \langle \phi|^{\otimes N} \xrightarrow{N \rightarrow M} \frac{1}{P_{N \rightarrow M}}[\Pi_+^{(M)}(|\phi\rangle \langle \phi|^{\otimes N} \otimes \frac{\mathbb{I}^{\otimes (M-N)}}{2})\Pi_+^{(M)}] \tag{1}
$$
where $p_{N \rightarrow M} = \frac{1}{2^{M-N}} \frac{1+M}{1+N}$ is the success probability of the procedure. All the identical output clones are described by the same output density matrix $\sigma^{N \rightarrow M} = \mathcal{F}_{N \rightarrow M} |\phi\rangle \langle \phi| + (1 - \mathcal{F}_{N \rightarrow M}) |\phi^\perp\rangle \langle \phi^\perp|$, where $\mathcal{F}_{N \rightarrow M} = \langle \phi| \sigma^{N \rightarrow M} |\phi\rangle = (N + 1 + \beta)/(N + 2)$ with $\beta \equiv N/M \leq 1$ is the ”fidelity” of the optimal cloning process [2,3,8]. In the case of our present experiment, the value of these parameters for the $1 \rightarrow 3$ and $2 \rightarrow 3$ machines are found respectively: $p_{1 \rightarrow 3} = \frac{1}{2}$, $\mathcal{F}_{1 \rightarrow 3} = \frac{7}{9}$, $p_{2 \rightarrow 3} = \frac{2}{3}$, $\mathcal{F}_{2 \rightarrow 3} = \frac{11}{12}$.

In order to implement the process expressed by Eq.1, consider that any generic $1 \rightarrow M$ cloning process can in fact be realized by a chain of $(M - 1)$ intermediate identical machines according to the operatorial identity: $\Pi_+^{(M)} = \Pi_+^{(M)}(\Pi_+^{(M-1)} \otimes I^{(1)}) = \Pi_+^{(M)}(\Pi_+^{(M-1)} \otimes I^{(1)}) (\Pi_+^{(M-2)} \otimes I^{(2)}) \cdots (\Pi_+^{(2)} \otimes I^{(M-2)})$. This is justified by the very definition of the symmetric subspace of $M$ qubits as the smallest subspace in $H_d^{\otimes M}$ spanned by the tensor product vectors $|\phi\rangle^{\otimes M}$ for any $|\phi\rangle \in H_2$, the qubit space with $d=2$. In the above expression $I^{(i)}$ stands for the identity operator in the $i$–qubit space, $H_2^{\otimes i}$. This implies that the symmetrization of $M$ qubits can be carried out step by step e.g. starting from the symmetrization of the two input qubits $\rho$ and $\overline{\rho}$, as shown in Fig. 1a [8]. Precisely, the state $\varrho^{(i)}$ realized at the output of any $i$th machine in the chain, i.e. of the overall $1 \rightarrow i$ device, belongs to the set:

$$\varrho^{(2)} = \Pi_+^{(2)}(\rho \otimes \overline{\rho})\Pi_+^{(2)}, \ldots \varrho^{(i)} = \Pi_+^{(i)}(\varrho^{(i-1)} \otimes \overline{\rho})\Pi_+^{(i)}, \ldots \varrho^{(M)} = \Pi_+^{(M)}(\varrho^{(M-1)} \otimes \overline{\rho})\Pi_+^{(M)} = \Pi_+^{(M)}(\Pi_+^{(M-1)} \otimes I^{(1)}) \cdots (\Pi_+^{(2)} \otimes I^{(M-2)}) [\rho \otimes \overline{\rho}^{(M-1)}] (\Pi_+^{(2)} \otimes I^{(M-2)}) \cdots (\Pi_+^{(M-1)} \otimes I^{(1)}) \Pi_+^{(M)}$$

Note that in the above expressions the input states, i.e. the pure $\rho \equiv |\phi\rangle \langle \phi|$ and the fully mixed $\overline{\rho} \equiv \frac{1}{2}$, can be interchanged leading to the two equivalent configurations shown in Fig.1a: the upper one has been chosen for the present implementations. These schemes are represented by arrays of equal Hong-Ou-Mandel interferometers, each one consisting of a 50/50 beam-splitter (BS) and realizing the qubit symmetrization in Hilbert spaces of increasing dimensions. The theory above can be easily extended to the analysis of any general $N \rightarrow M$ cloning process. Optionally, the procedure could be made to consist of a sequence of linear symmetrization devices acting by inequal cloning steps, e.g. by injection of different mixed states along the chain.
The Fig. 1b shows the experimental apparatus by which the 2-step chain \((1 \rightarrow 3) = (1 \rightarrow 2) + (2 \rightarrow 3)\) UOQCM has been realized.

1 \rightarrow 2 UOQCM. The device realizing the first step state-symmetrization was the beam-splitter \(BS_A\) excited over the two input modes by the 2-qubit state: \(\rho_{in}^{1\rightarrow 2} = |\phi\rangle \langle \phi|_S \otimes \frac{I_A}{2}\). After projection by \(BS_A\) in the symmetric subspace, the output state realized on the output mode \(k_2\) was expressed by:

\[ \rho_{out}^{1\rightarrow 2} \equiv \varrho^{(2)} = \frac{2}{3} |\phi\rangle \langle \phi| + \frac{1}{3} \{\phi\phi\} \langle \{\phi\phi\} | \]

where the notation \(\{\phi\phi\}\) stands for a total symmetric combination of the states \(|\phi\rangle\) and \(|\phi\rangle\). The identical condition realized on mode \(k_1\) was neglected, for simplicity. The two clones \(j = 1, 2\) emitted over \(k_2\) were expressed by the same operators: \(\sigma_j^{1\rightarrow 2} = Tr_{\rho} \rho_{out}^{1\rightarrow 2} = \frac{5}{6} |\phi\rangle \langle \phi| + \frac{1}{6} |\phi\rangle \langle \phi| \)

In the experiment a pair of photons was generated by spontaneous parametric down conversion (SPDC) in a 1 mm thick BBO crystal, cut for Type I phase matching. The two photons, each with wavelength (wl) \(\lambda = 795nm\) and coherence time \(\tau_{coh} \simeq 200fs\), were emitted over two modes \(k_A\) and \(k_S\) respectively in the product state of horizontal (\(H\)) linear polarizations (\(\pi\)): \(|H\rangle_S |H\rangle_A\). Then, on mode \(k_S\) the qubit \(|H\rangle_S\) was \(\pi\)–encoded by an optical waveplate (wp) \(WP_\phi\) into the generic pure-state \(|\phi\rangle_S\), \(\rho_S = |\phi\rangle \langle \phi|_S\), while on mode \(k_A\) the qubit \(|H\rangle_A\) was transformed into a fully mixed-state \(\rho_A = \frac{I_A}{2}\) by a depolarizing channel realized by a stochastically driven Electro-Optics Pockels (EOP) cell, \(P_A\). The two qubits \(\rho_S\) and \(\rho_A\) were then drawn into a linear superposition in \(BS_A\). The exact space-time overlap of the two input modes implying the actual realization of the interference was controlled by the microscopic \(BS_A\) displacement: \(Z_A = 2c\Delta t\). Let’s call ”\(BS_A\)-interference” the condition \(Z_A = 0\) corresponding to maximum interference. By turning on the cloning machine, i.e. setting it in \(BS_A\)-interference, the induced Bose coalescence implied an enhancement by a factor \(R_{1\rightarrow 2} = 2\) of the \(|\phi\phi\rangle\) component in the 2-qubit output state and no enhancement of the \(|\{\phi\phi\}\rangle\) component [6]. The measurement of \(R_{1\rightarrow 2}\) was carried out by a post-selection technique, by the \(\pi\)–analysis setup shown at the r.h.s. of Fig.1b, connected directly to
the output mode $k_2$, by disregarding at this stage the presence of $BS_B$. This $\pi-$analyzer consisted of an output mode selector, realized by a 5 meter long single-mode optic fiber, followed by the wp $WP_\phi^{-1}$ that mapped the output state $|\phi\rangle$ into $|H\rangle$ by counterbalancing the action of the input $WP_\phi$. Finally, by a polarizing-BS, ($PBS$) the $|H\rangle$ and $|V\rangle$ components of the output state were directed respectively over the modes $k_\phi$ and $k_\phi^*$. The mode $k_\phi$ was coupled to the detectors $D_1$, $D_2$, $D_3$ by means of two equal 50/50 beam-splitters $BS_1$, $BS_2$ while the mode $k_\phi^*$ was coupled to $D_1^*,D_2^*$ by $BS_3$. The $|\phi\phi\rangle$ component was identified by detecting coincidence by the $D_{1-2}$ pair sets $[D_1,D_2]$ and $[D_1,D_3]$ while the state $|\{\phi^\bot\phi\}\rangle$ was identified by $[D_1,D_1^*]$. The detectors ($D$) were equal single-photon counters SPCM-AQR14.

Three different input states $|\phi\rangle_S = |H\rangle$, $2^{-\frac{1}{2}}(|H\rangle + |V\rangle)$, $2^{-\frac{1}{2}}(|H\rangle + i|V\rangle)$, identified in the following by $|H\rangle$, $|H + V\rangle$ and $|H + iV\rangle$ respectively, were adopted to test the universality of the device. The cloning process was found to affect only the $|\phi\phi\rangle$ component, as expected, and $R_{1\rightarrow 2}$ was determined as the ratio between the peak value (cloning machine switched on) and the basis value (off). The corresponding experimental values of the cloning fidelity, $F_{1\rightarrow 2} = (2R_{1\rightarrow 2} + 1) / (2R_{1\rightarrow 2} + 2)$ were: $F_{1\rightarrow 2}^{H} = 0.831 \pm 0.001$; $F_{1\rightarrow 2}^{H+V} = 0.833 \pm 0.002$; $F_{1\rightarrow 2}^{H+iV} = 0.830 \pm 0.002$. These values are to be compared with the theoretical value $F_{1\rightarrow 2}^{th} = 5/6 \approx 0.833$ corresponding to the optimal enhancement ratio $R = 2$. Similar results for the $1 \rightarrow 2$ UOQCM have been reported in [6].

$1 \rightarrow 3$ UOQCM. In agreement with the upper configuration shown in Fig.1a, the 50/50 beam-splitter $BS_B$ was the next state-symmetrization device: the whole ($1 \rightarrow 3$) UOQCM is shown in Fig.1b. This $BS$ was excited over the input mode $k_2$ by the output state $\rho_{out}^{1\rightarrow 2}$ of the $1 \rightarrow 2$ UOQCM, Eq. 3, and over the other input $k_B$ by the fully mixed state $\rho_B = \frac{I_B}{2}$. In analogy with the first step experiment, this state was obtained by means of an EOP, $P_B$ acting on a highly attenuated quasi single-photon beam expressed by the $\pi-$qubit $|H\rangle_B$, deflected from the main laser by the mirror $M$, and delayed by $Z_B = 2c\Delta t_B$ via an “optical trombone”. Once again, the condition $Z_B = 0$, dubbed here as ”$BS_B$-interference” condition, was made to correspond to the maximum overlapping of the 2 input modes of $BS_B$. In particular, in no-”$BS_A$-interference” condition, i.e. for $|Z_A| \gg$
2\pi_{coh}, the ”\textit{BS}_B\text{-interference}” corresponded to the maximum overlapping in \textit{BS}_B of the \textit{mixed} states \rho_A and \rho_B. In summary, the overall 1 \rightarrow 3 machine was excited by the input state \rho_{in}^{1\rightarrow 3} = \ket{\phi} \bra{\phi} \otimes \frac{I_B}{2} \otimes \frac{I_B}{2}, i.e. by the pure state \rho_S to be cloned and by two mutually \textit{uncorrelated} mixed states \rho_A and \rho_B. By applying to this state the projector \Pi_{(3)} = |\phi\phi\phi \rangle \langle \phi\phi\phi| + |\{\phi\phi\phi^\perp\} \rangle \langle \{\phi\phi\phi^\perp\}| + |\{\phi^\perp\phi^\perp\} \rangle \langle \{\phi^\perp\phi^\perp\}|, the symmetrized output state is obtained:

\begin{equation}
\rho_{out}^{1\rightarrow 3} \equiv \rho^{(3)} = \frac{3}{6} |\phi\phi\phi \rangle \langle \phi\phi\phi| + \frac{2}{6} |\{\phi\phi\phi^\perp\} \rangle \langle \{\phi\phi\phi^\perp\}| + \frac{1}{6} |\{\phi^\perp\phi^\perp\} \rangle \langle \{\phi^\perp\phi^\perp\}| \quad (4)
\end{equation}

Each one of the identical clones \( j = 1, 2, 3 \) can be thought to be expressed by the reduced density matrix: \( \sigma_j^{1\rightarrow 3} = Tr_{h,k\neq j} \rho_{out}^{1\rightarrow 3} = \frac{7}{9} |\phi \rangle \langle \phi| + \frac{2}{9} |\phi^\perp \rangle \langle \phi^\perp| \). The projection over the symmetric subspace was identified by the measurement of the 3-photon Fock state over the output mode \( k_3 \) by the \( \pi \)-analyzer apparatus already described and shown at the r.h.s. of Fig.1b. The output field emitted over \( k_4 \) was neglected. The \( |\phi\phi\phi \rangle, |\{\phi\phi\phi^\perp\} \rangle \) and \( |\{\phi^\perp\phi^\perp\} \rangle \) components of \( \rho_{out}^{1\rightarrow 3} \) were measured by the 3-fold coincidence events respectively by the detector \( (D) \) sets \{D_1, D_2, D_3\}, \{D_l, D_m, D_n^*\} and \{D_l, D_n^*, D_p^*\} for any \( l, m = 1, 2, 3 \) and \( n, p = 1, 2 \). A little inspection of the circuit leads to the following expectations. In the exact \textit{BS}_B\text{-resonance}, and \textit{na-BS}_A\text{-resonance}, i.e. in the condition ”Bose coalescence” of only the two \textit{mixed} states \rho_A and \rho_B in \textit{BS}_B, an enhancement by a factor \( \Gamma \) of the \( |\phi\phi\phi \rangle \) and \( |\{\phi\phi\phi^\perp\} \rangle \) components should be detected by the \( \pi \)-analyzer. Furthermore, by turning on also the \textit{BS}_A\text{-resonance}, i.e. by setting \( Z_A = Z_B = 0 \), a further enhancement of the \( |\phi\phi\phi \rangle \) component by a factor \( R_{1\rightarrow 3}^1 \) and of the \( |\{\phi\phi\phi^\perp\} \rangle \) component by a factor \( R_{1\rightarrow 3}^2 \) were expected. In summary, the full resonance condition, corresponding to the switching on of the \Pi_{(3)}^1\text{-projector, implied the global enhancements by the factors } \Gamma R_{1\rightarrow 3}^1, R_{1\rightarrow 3}^2 \text{ and } \Gamma \text{ respectively of the components } |\phi\phi\phi \rangle, |\{\phi\phi\phi^\perp\} \rangle \text{ and } |\{\phi^\perp\phi^\perp\} \rangle \text{ of the state } \rho_{out}^{1\rightarrow 3}, \text{ Eq.4. Accordingly, the first step of our strategy consisted of the measurement of } \Gamma. \text{ This was provided by injecting in the apparatus the pure state } |\Psi\rangle_{SAB} \equiv |V\rangle_S |H\rangle_A |V\rangle_B, \text{ by setting } Z_A = Z_B = 0 \text{ and by turning off the mixing EOP devices } P_A \text{ and } P_B. \text{ The value of } \Gamma \text{ was determined by the ratio of the counting rates of the } |\{\phi\phi\phi^\perp\} \rangle \text{ components, the only non vanishing one under}
$|\Psi\rangle_{SAB}$ excitation. These rates were measured at the peak of the detected resonance curves, i.e. with $Z_{SB} = 0$, and far from the peak, with $Z_{SB} \gg 2c\tau_{coh}$, being $Z_{SB} = Z_A - Z_B$ the mutual delay between qubits $S$ and $B$ at $BS_B$. The measured value $\Gamma_{\text{exp}} = 1.66 \pm 0.05$, expressing the degree of indistinguishability attained between photons coming from different sources, SPDC and attenuated laser, was to be compared with the theoretical one $\Gamma_{\text{th}} = 2$.

By restoring the full operation of the overall apparatus under excitation by $\rho_{1\rightarrow3}^{1\rightarrow3}$, $R_{1\rightarrow3}^{1\rightarrow3}$ and $R_{1\rightarrow3}^{2\rightarrow3}$ were determined as the ratios of the $|\phi\phi\rangle$ and $|\{\phi\phi\phi\rangle\}$ component measured, via 3-D coincidences, in resonance, i.e. $Z_B = Z_A = 0$ and out of resonance, i.e. $Z_B = 0$, $Z_A \gg 2c\tau_{coh}$. From these measurements, the fidelity of the overall process could be determined: $F_{1\rightarrow3} = (3\Gamma R_{1\rightarrow3}^{1\rightarrow3} + 4R_{1\rightarrow3}^{2\rightarrow3} + \Gamma)/(3\Gamma R_{1\rightarrow3}^{1\rightarrow3} + 6R_{1\rightarrow3}^{2\rightarrow3} + 3\Gamma)$. The plots of Fig. 2a show the experimental 3-D coincidence results measured by the $\pi-$analyzer for the various state components of the output state: $\rho_{\text{out}}^{1\rightarrow3}$. The experimental values of the fidelity measured by the above procedure under excitation of three different input states $\rho_S = |\phi\rangle \langle \phi|_S$ were: $F_{1\rightarrow3}^{\text{H}} = 0.758 \pm 0.008$; $F_{1\rightarrow3}^{\text{H+V}} = 0.761 \pm 0.003$; $F_{1\rightarrow3}^{\text{H+iV}} = 0.758 \pm 0.008$ to be compared with the optimum value: $F_{1\rightarrow3}^{\text{th}} = 7/9 \approx 0.778$.

The protocol can be easily generalized for any long $UOQCM$ chain by a straightforward repetition of the above procedure, as follows. The output state $\rho_{\text{out}}^{1\rightarrow3}$ of the $1 \rightarrow 3$ $UOQCM$ be injected into one input arm of a further state-symmetrizing 50/50 beam-splitter $BS_C$ while a mixed one-photon state $\rho_C = \frac{|H\rangle_C}{2}$ be injected on the other arm. This state is generated, as previously, by extracting by a further mirror $M$ a highly attenuated beam expressed by the $\pi-$qubit $|H\rangle_C$ and mixing it by an $EOP$, $P_C$. This will result in a $1 \rightarrow 4$ $UOQCM$ apparatus generating the output state $\rho_{\text{out}}^{1\rightarrow4}$. Then again: the output state $\rho_{\text{out}}^{1\rightarrow4}$ of the $1 \rightarrow 4$ $UOQCM$ be injected into one input arm of a state-symmetrizing $BS_D$ while a mixed state $\rho_D = \frac{|H\rangle_D}{2}$...

$2 \rightarrow 3$ $UOQCM$. As a significant variant of the above protocol, the $P_A$ Pockels Cell was removed and the beam-splitter and $BS_A$ was excited over the input modes $k_A$ and $k_S$ by the same pure states: $\rho_A = \rho_S = |\phi\rangle \langle \phi|$. In $BS_A-$resonance condition, i.e. $Z_A = 0$, the $BS_A$
acted as a conventional Hong-Ou-Mandel interferometer emitting over the output mode \( k_2 \) the symmetric Bose state \( |\phi\rangle \langle \phi|^{\otimes 2} \) to be injected, together with \( \rho_B = \frac{1}{2} I_B \) into the \( BS_B \) according to the discussion above: Fig.1b. The input state to this novel \( N \rightarrow M \) UOQCM with \( N = 2 \) and \( M = 3 \) was then expressible as: 
\[
\rho_{in}^{2 \rightarrow 3} = |\phi\rangle \langle \phi|^{\otimes 2} \otimes \frac{1_B}{2},
\]
to be mapped onto the 3 clone output state, according to:
\[
\rho_{in}^{2 \rightarrow 3} \rightarrow \rho_{out}^{2 \rightarrow 3} = \frac{3}{4} |\phi\phi\rangle \langle \phi\phi| + \frac{1}{4} |\{\phi\phi\phi^\perp\rangle \langle \{\phi\phi\phi^\perp\}\|
\]
Each output clone \( j = 1, 2, 3 \) could be expressed by the state \( \sigma_j^{2 \rightarrow 3} = Tr_{h,k\neq j} \rho_{out}^{2 \rightarrow 3} = \frac{11}{12} |\phi\rangle \langle \phi| + \frac{1}{12} |\phi^\perp\rangle \langle \phi^\perp| \).

Once again, the \( |\phi\phi\phi\rangle \) and \( |\{\phi\phi\phi^\perp\}\rangle \) components of \( \rho_{out}^{2 \rightarrow 3} \) were measured by the 3-D coincidence events respectively by the sets \([D_1, D_2, D_3]\) and \([D_l, D_m, D_n]\) for any \( l, m = 1, 2, 3 \) and \( n = 1, 2 \). The plots shown in Fig.2b express the experimental results. In full analogy with the previous discussion, the detected bosonic coalescence was expressed by an enhancement of the detected coincidences by a factor \( R_{2 \rightarrow 3} = 3 \) of the \( |\phi\phi\phi\rangle \) component of \( \rho_{in}^{2 \rightarrow 3} \) whereas no enhancement affected \( |\{\phi\phi\phi^\perp\}\rangle \). Furthermore, in analogy with the \( 1 \rightarrow 2 \) cloning process, the experimental value of \( R_{2 \rightarrow 3} \) was determined as the ratio between the peak resonant value for the \( BS_B \) interferometer \( (Z_B = 0) \) and the out of resonance value: \( Z_B >> 2c\tau \). The universality condition was assessed by injecting the same three different input test states adopted for the previous cases. The experimental values of fidelity were found: 
\[
F_{2 \rightarrow 3}^{U} = 0.895 \pm 0.003; \quad F_{2 \rightarrow 3}^{H+V} = 0.893 \pm 0.003; \quad F_{2 \rightarrow 3}^{H+iV} = 0.894 \pm 0.003
\]
in correspondence with the input states defined above. These values are be compared with the calculated optimal value: 
\[
F_{2 \rightarrow 3}^{th} = \frac{(3R_{2 \rightarrow 3} + 2)}{(3R_{2 \rightarrow 3} + 3)} = (11/12) \approx 0.917.
\]
In all previous experiment, the measured values of \( R_{2 \rightarrow 3} \) were reduced by the unwanted injection of two and three photons in the mode \( k_B \) and by simultaneous emissions of two pairs from SPDC. These spurious events affected the measured value of the \( R_{2 \rightarrow 3} \) factor by a calculated average amount of \( \approx 15\% \).

In summary a very general and efficient linear multi-step optical procedure for the probabilistic \( N \rightarrow M \) optimal universal cloning machine has been proposed together with the successful experimental realization of of the first two steps, i.e. the \( (1 \rightarrow 3) \) UOQCM. Fur-
thermore, the probabilistic \((2 \rightarrow 3) UOQCM\), was also demonstrated by a straightforward variant of the same protocol. This shows that a very similar protocol can be adopted to implement contextually the \(N \rightarrow M\) UOQCM, the \(N \rightarrow (M - N)\) Universal NOT gate and any \textit{programmable anti-unitary map} [9] following the very general symmetrization procedure recently proposed by [10]. Since the present method basically exploits the bosonic nature of the photons, it can be straightforwardly applied to any particle obeying to the Bose statistics. On a more sophisticated technological perspective, the present protocol is expected to find applications as a realization of a general, multi-qubit device based on the state-symmetrization process to be used in modern Quantum Information networks. [11–13].
REFERENCES

[1] W. K. Wootters, and W. H. Zurek, Nature (London) 299, 802 (1982).

[2] V. Bužek, and M. Hillery, Phys. Rev. A 54, 1844 (1996).

[3] N. Gisin, and S. Massar, Phys. Rev. Lett. 79, 2153 (1997).

[4] R. Werner, Phys. Rev. A 58, 1827 (1998).

[5] A. Lamas-Linares, C. Simon, J. C. Howell, and D. Bouwmeester, Science 296, 712 (2002); F. De Martini, V. Bužek, F. Sciarrino, C. Sias, Nature (London) 419, 815 (2002); S. Fasel et al., Phys. Rev. Lett. 89, 107901 (2002); F. De Martini, D. Pelliccia, and F. Sciarrino, Phys. Rev. Lett. 92, 067901 (2004).

[6] M. Ricci, F. Sciarrino, C. Sias, F. De Martini, Phys. Rev. Lett. 92, 047901 (2004); F. Sciarrino, C. Sias, M. Ricci, F. De Martini, Phys. Lett. A 323, 34 (2004); W. T. M. Irvine, A. Lamas-Linares, M. J. A. de Dood, and D. Bouwmeester, Phys. Rev. Lett. 92, 047902 (2004).

[7] C. K. Hong, Z. Y. Ou, nad L. Mandel, Phys. Rev. Lett. 59, 2044 (1987).

[8] D. Bruß, A. Ekert, and C. Macchiavello, Phys. Rev. Lett. 81, 2598 (1998).

[9] M. Hillery, V. Buzek, and M. Ziman, Phys.Rev. A. 65, 022301 (2002).

[10] F. Sciarrino, C. Sias, M. Ricci, F. De Martini, Phys. Rev. A 70, 052305 (2004)

[11] A. Barenco et al., SIAM J. Comput. 26, 1541 (1997).

[12] J.I. Cirac, A.K. Ekert, and C. Macchiavello, Phys. Rev. Lett. 82, 4344 (1999)

[13] M. Ricci et al., Phys. Rev. Lett. 93, 170501 (2004)

Figure Captions:

Fig.1.(a)Linear optical scheme for the realization of the general $1 \to M$ and $M-1 \to M$ Universal Quantum Cloning Machines by a chain of identical symmetrizer beam splitters.
(b) Experimental set-up of a $1 \rightarrow 3$ cloning process.

Fig. 2. Experimental results of the $1 \rightarrow 3$ and $2 \rightarrow 3$ UOQCMs for three input qubits. (a) From the upper to the lower row: data corresponding to $|\phi\phi\phi\rangle$, $|\phi\phi\phi\perp\rangle$ and $|\phi\phi\perp\phi\rangle$ components of $\rho_{out}^{1\rightarrow3}$ measured by 3-fold coincidences. From the left to the right column: data corresponding to the $|H\rangle$, $|H + V\rangle$, $|H + iV\rangle$ input state $\rho_{S}$. (b) From the upper to the lower row: data corresponding to $|\phi\phi\rangle$ and $|\phi\phi\perp\rangle$ components of $\rho_{out}^{2\rightarrow3}$ measured by 3-fold coincidences. From the left to the right column: data corresponding to the $|H\rangle$, $|H + V\rangle$, $|H + iV\rangle$ input state $\rho_{S}$.
\[ |\Phi\rangle = |H\rangle \quad |\Phi\rangle = |H+V\rangle \quad |\Phi\rangle = |H+iV\rangle \]

3-coincidence rate (s' \times 200)

**a)**

1→3 UOQCM

\[ |\Phi\rangle |\Phi\rangle \]

\[ |\Phi\rangle |\Phi\rangle_{\perp} \]

\[ |\Phi\rangle |\Phi\rangle_{\perp} \]

**b)**

2→3 UOQCM

\[ |\Phi\rangle |\Phi\rangle \]

\[ |\Phi\rangle |\Phi\rangle_{\perp} \]

\[ |\Phi\rangle |\Phi\rangle_{\perp} \]