New Quadratic Baryon Mass Relations

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Abstract

By assuming the existence of (quasi)-linear baryon Regge trajectories, we derive new quadratic Gell-Mann–Okubo type baryon mass relations. These relations are used to predict the masses of the charmed baryons absent from the Baryon Summary Table so far, in good agreement with the predictions of many other approaches.

Key words: flavor symmetry, quark model, charmed baryons, Gell-Mann–Okubo, Regge phenomenology

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The investigation of the properties of hadrons containing heavy quarks is of great interest for understanding the dynamics of the quark-gluon interaction. Recently predictions about the heavy baryon mass spectrum have become a subject of increasing interest \([1 - 11]\), due to current experimental activity of several groups at CERN \([12]\), Fermilab \([13]\) and CESR \([14, 15]\) aimed at the discovery of the baryons so far absent from the Baryon Summary Table \([16]\). Recently, for the LHC, B-factories and the Tevatron with high luminosity, several experiments have been proposed in which a detailed study of heavy baryons can be performed. In this connection, an accurate theoretical prediction for the baryon mass spectrum becomes a guide for experimentalists. To calculate the heavy baryon mass spectrum, potential models \([1, 17 - 22]\), nonrelativistic quark models \([23 - 25]\), relativistic quark models \([6]\), bag models \([26 - 29]\), lattice QCD \([30 - 32]\), QCD spectral sum rules \([33]\), heavy quark effective theory \([11, 34 - 36]\), chiral perturbation theory \([2]\), chiral quark model \([1]\), SU(4) skyrmion model \([37]\), group theoretical \([10, 38, 39]\) and other approaches \([3 - 5, 7, 8, 40 - 44]\) are widely used.

The charm baryon masses measured to date are\(^1\) \([16]\)

\[
\begin{align*}
\Lambda_c &= 2285 \text{ MeV}, \\
\Sigma_c &= 2453 \pm 1 \text{ MeV}, \\
\Xi_c &= 2468 \pm 2 \text{ MeV}, \\
\Omega_c &= 2704 \pm 4 \text{ MeV}, \\
\Sigma_c^* &= 2521 \pm 4 \text{ MeV}, \\
\Xi_c^* &= 2644 \pm 2 \text{ MeV}.
\end{align*}
\]

An observation of the \(\Xi_c' = 2563 \pm 15 \text{ MeV}\) was reported by the WA89 Collaboration \([12]\). The \(\Omega_c^*\), as well as double- and triple-charmed baryons, have not yet been observed.

Almost all very recent calculations very consistently predict the mass of the \(\Xi_c^*\) to be around 2580 MeV \([2, 1, 4, 8, 11]\) (see also \([20, 11, 43]\)). Similarly, the mass of the \(\Omega_c^*\) is very consistently predicted to be around 2770 MeV \([2, 1, 4, 7, 9, 11]\) (see also \([1, 21, 26, 30, 41]\)). Predictions for the double- and triple-charmed baryon masses are less definite.

Here we wish to extend the approach based on the assumption of (quasi)-linearity of the Regge trajectories of heavy hadrons in the low-energy region, initiated in our previous papers for heavy mesons \([40, 47]\), to baryons. We shall show that new quadratic Gell-Mann–Okubo type baryon mass relations can be obtained, and used to predict the missing charmed baryon masses. As we shall see, the predicted masses are in good agreement with the results of many other approaches, which should add confidence to an experimental focus on the predicted ranges.

\(^1\)For \(\Sigma_c^*\) we take the uncertainty weighted average of the results of ref. \([13]\), \(2530 \pm 5 \pm 5 \text{ MeV}\), and the most recent results by CLEO \([14]\), \(2518.6 \pm 2.2 \text{ MeV}\).
Let us assume, as in [46, 47], the (quasi)-linear form of Regge trajectories for baryons with identical \( J^P \) quantum numbers (i.e., belonging to a common multiplet). Then for the states with orbital momentum \( \ell \) one has (\( i, j, k \) stand for the corresponding flavor content)

\[
\ell = \alpha'_{kii} m^2_{kii} + a_{kii}(0),
\]

\[
\ell = \alpha'_{kji} m^2_{kji} + a_{kji}(0),
\]

\[
\ell = \alpha'_{kjj} m^2_{kjj} + a_{kjj}(0).
\]

Using now the relation among the intercepts [48, 49, 50],

\[
a_{kii}(0) + a_{kjj}(0) = 2a_{kji}(0),
\]

one obtains from the above relations

\[
\alpha'_{kii} m^2_{kii} + \alpha'_{kjj} m^2_{kjj} = 2\alpha'_{kji} m^2_{kji}.
\]

In order to eliminate the Regge slopes from this formula, we need a relation among the slopes. Two such relations exist,

\[
\alpha'_{kii} \cdot \alpha'_{kjj} = \left(\alpha'_{kji}\right)^2,
\]

which follows from the factorization of residues of the \( t \)-channel poles [51, 52, 53], and

\[
\frac{1}{\alpha'_{kii}} + \frac{1}{\alpha'_{kjj}} = \frac{2}{\alpha'_{kji}},
\]

which may be derived by generalizing the corresponding relation for quarkonia based on topological expansion and the \( q\bar{q} \)-string picture [50] to the case of a baryon viewed as a quark-diquark-string object [57].

For light baryons (and small differences in the \( \alpha' \) values), there is no essential difference between these two relations; viz., for \( \alpha'_{kji} = \alpha'_{kii}/(1 + x), \ x \ll 1 \), Eq. (4) gives \( \alpha'_{kjj} = \alpha'_{kii}/(1 + 2x) \), whereas Eq. (3) gives \( \alpha'_{kjj} = \alpha'_{kii}/(1 + x)^2 \approx \alpha'/(1 + 2x) \), i.e., essentially the same result to order \( x^2 \). However, for heavy baryons (and expected large differences from the \( \alpha' \) values for the light baryons) these relations are incompatible; e.g., for \( \alpha'_{kjj} = \alpha'_{kii}/2 \), Eq. (3) will give \( \alpha'_{kjj} = \alpha'_{kii}/4 \), whereas from Eq. (4), \( \alpha'_{kjj} = \alpha'_{kii}/3 \). One therefore has to choose between these relations in order to proceed further. Here, as in [46, 47], we use Eq. (4), since it is much more consistent with (2) than is Eq. (3), which we tested by using measured light-quark baryon masses in Eq. (2). Kosenko and Tutik [40] used the relation (3) and obtained much higher values for the charmed baryon masses than the measured ones (e.g., \( \Omega_c = 2788 \text{ MeV} \)) and those predicted by most other approaches (see Table I). The reason for this is that lower values for the Regge slopes, as illustrated by the example above, lead to higher values

\[\text{This structure is known to be responsible for the slopes of baryon trajectories being equal to those of meson trajectories [54, 55, 53].}\]
for the masses. We shall justify our choice of Eq. (4) in more detail in a separate publication [57].

It is easy to see that the following relation solves Eq. (4):

\[ a_{i_n, j_s, k_c}^*(0) = a^*(0) - \lambda^*_s j_s - \lambda^*_c k_c, \quad a^*(0) \equiv a_{3,0,0}^*(0), \quad (5) \]

\[ \frac{1}{\alpha_{i_n, j_s, k_c}^*} = \frac{1}{\alpha^*_s} + \gamma_s^* j_s + \gamma_c^* k_c, \quad \alpha_s^* \equiv \alpha_{s,3,0,0}^*, \quad i_n + j_s + k_c = 3, \quad (6) \]

where \( i_n, j_s, k_c = 1, 2, 3 \) are the numbers of \( n-, s-, \) and \( c- \) quarks, respectively, which constitute the baryon, and the sub- and superscript \( \star \) allows for possible differences between multiplets (such as \( \frac{1}{2}^+ \) octet and \( \frac{3}{2}^+ \) decuplet).

It then follows from (6) that

\[ \alpha_s' = \alpha_s, \quad (7) \]

\[ \alpha_s = \frac{\alpha_N'}{1 + \gamma_N^s \alpha_N'}, \quad (8) \]

\[ \alpha_c' = \alpha_c, \quad (9) \]

\[ \alpha_c = \frac{\alpha_N'}{1 + \gamma_c^N \alpha_N'}, \quad (10) \]

\[ \alpha_s' = \alpha_s', \quad (11) \]

\[ \alpha_s' = \frac{\alpha_N'}{1 + (\gamma_N^s + \gamma_c^N) \alpha_N'}, \quad (12) \]

\[ \alpha_c = \frac{\alpha_N'}{1 + (2 \gamma_c^N) \alpha_N'}, \quad (13) \]

\[ \alpha_s' = \alpha_s', \quad (14) \]

\[ \alpha_s' = \frac{\alpha_N'}{1 + 2 \gamma_c^N \alpha_N'}, \quad (15) \]

\[ \alpha_s' = \frac{\alpha_N'}{1 + 3 \gamma_c^N \alpha_N'}, \quad (16) \]

\[ \alpha_s' = \frac{\alpha_N'}{1 + \gamma_c^N \alpha_N'}, \quad (17) \]

\[ \text{where we use } \star = N \text{ to represent the } \frac{1}{2}^+ \text{ multiplet, and with } \star = \Delta \text{ to represent the } \frac{3}{2}^+ \text{ multiplet}, \]

\[ \alpha_s' = \alpha_s', \quad (18) \]

\[ \alpha_s' = \frac{\alpha_N'}{1 + \gamma_s^N \alpha_N'}, \quad (19) \]

\[ \alpha_s' = \frac{\alpha_N'}{1 + \gamma_c^N \alpha_N'}, \quad (20) \]

\[ \text{The notation has changed here, as compared to Eqs. (1)-(4); e.g., } a_{\text{odd}}(0) \equiv a_{3,0,0}(0), \quad a_{\text{even}}(0) \equiv a_{2,1,0}(0), \quad \text{etc.} \]
Consider first the $J^P = \frac{3}{2}^+$ baryons. Introduce, for simplicity,

\[ x \equiv \gamma_s \alpha'_\Delta, \quad y \equiv \gamma_c \alpha'_\Delta. \]  

It then follows from (5)-(13) that

\[ \Delta^2 = \Sigma^2_{s} - \lambda_s^\Delta = \Xi^2_{s} - 2\lambda_s^\Delta = \Omega^2_{s} - 3\lambda_s^\Delta \]

\[ = \Xi^2_{c} - \lambda_c^\Delta = \Omega^2_{c} - 2\lambda_c^\Delta - \lambda_c^\Delta \]

\[ = \Xi^2_{cc} - 2\lambda_c^\Delta = \Omega^2_{cc} - \lambda_c^\Delta - 2\lambda_c^\Delta \]

\[ = \Omega^2_{ccc} - \lambda_c^\Delta. \]  

Note that there are four unknown parameters for each multiplet. By eliminating them, i.e., $x, y, \lambda_s^\Delta, \lambda_c^\Delta$, from the above nine equalities, we can obtain five relations for baryon masses; e.g.,

\[ \Omega^2 - \Delta^2 = 3 (\Xi^2 - \Sigma^2), \]  

\[ \Omega^2_{ccc} - \Delta^2 = 3 (\Xi^2_{cc} - \Sigma^2), \]  

\[ \Omega^2_{ccc} - \Omega^2 = 3 (\Omega^2_{cc} - \Omega^2). \]  

\[ (\Xi^2_{c} - \Delta^2) + (\Omega^2_{cc} - \Xi^2) = 2 (\Xi^2_{c} - \Sigma^2), \]  

\[ (\Omega^2_{cc} - \Xi^2_{cc}) + (\Xi^2 - \Delta^2) = 2 (\Xi^2 - \Sigma^2). \]  

However, just four of them are linearly independent, because of an invariance of the nine equalities under simultaneous permutation ($x \leftrightarrow y, \lambda_s \leftrightarrow \lambda_c$).

Here only Eq. (25) can be tested, since Eqs. (26)-(29) contain the baryon masses not measured so far. For Eq. (25), one obtains (on GeV$^2$) $1.280 \pm 0.005$ vs. $1.300 \pm$
0.030, taking the electromagnetic mass splittings as a measure of the uncertainty (since electromagnetic corrections are not included in our analysis).

The analysis may be easily repeated for the \( J^P = \frac{1}{2}^+ \) baryons, leading to the following two independent mass relations,

\[
\left( \Sigma_c^2 - N^2 \right) + \left( \Omega_c^2 - \Xi^2 \right) = 2 \left( \tilde{\Xi}_c^2 - \Sigma_c^2 \right), \tag{30}
\]

\[
\left( \Omega_{cc}^2 - \Xi_{cc}^2 \right) + \left( \Sigma_c^2 - N^2 \right) = 2 \left( \tilde{\Xi}_c^2 - \Sigma_c^2 \right), \tag{31}
\]

where

\[
\Sigma' \equiv a \Lambda^2 + (1 - a) \Sigma^2, \tag{32}
\]

\[
\Sigma_c' \equiv b \Lambda_c^2 + (1 - b) \Sigma_c^2, \tag{33}
\]

\[
\tilde{\Xi}_c \equiv c \Xi_c^2 + (1 - c) \Xi_c'^2 \tag{34}
\]

are introduced to distinguish between the states having the same flavor content and \( J^P \) quantum numbers, and \( a, b, c \) are not known \textit{a priori}. In order to establish the values of \( a, b \) and \( c \), we use the following relation for the intercepts of the \( \frac{1}{2}^+ \) baryon trajectories in the non-charmed sector \[58\],

\[
2 \left[ a_N(0) + a_\Xi(0) \right] = 3a_\Lambda(0) + a_\Sigma(0), \tag{35}
\]

which has been subsequently generalized to the charmed sector by replacing the \( s \)-quark by the \( c \)-quark, as follows \[40\]:

\[
2 \left[ a_N(0) + a_{\Xi_{cc}}(0) \right] = 3a_{\Lambda_c}(0) + a_{\Sigma_c}(0). \tag{36}
\]

It then follows from the corresponding relations based on (1),(2) that, respectively,

\[
\alpha'_N N^2 + \alpha'_\Xi \Xi^2 = 2\alpha_{\Sigma'} \left( \frac{3}{4} \Lambda^2 + \frac{1}{4} \Sigma^2 \right), \quad \alpha_{\Sigma'} \equiv \alpha'_\Lambda = \alpha'_\Sigma, \tag{37}
\]

\[
\alpha'_N N^2 + \alpha'_{\Xi_{cc}} \Xi_{cc}^2 = 2\alpha_{\Sigma'_c} \left( \frac{3}{4} \Lambda_c^2 + \frac{1}{4} \Sigma_c^2 \right), \quad \alpha_{\Sigma'_c} \equiv \alpha'_{\Lambda_c} = \alpha'_{\Sigma_c}, \tag{38}
\]

and therefore

\[
\Sigma' = \frac{3}{4} \Lambda^2 + \frac{1}{4} \Sigma^2, \tag{39}
\]

\[
\Sigma'_c = \frac{3}{4} \Lambda_c^2 + \frac{1}{4} \Sigma_c^2, \tag{40}
\]

i.e., in the relations (32),(33) \( a = b = \frac{3}{4} \). It is also seen that the only parameter which is responsible for different weighting of the states having the same flavor content and \( J^P \) quantum numbers is the isospin of the state. Thus, since both \( \Xi_c \) and \( \Xi'_c \) have equal isospin \( (I = \frac{1}{2}) \), they should enter a mass relation with equal weights, i.e., in Eq. (34) \( c = 1/2 \), and

\[
\tilde{\Xi}_c \equiv \frac{\Xi_c^2 + \Xi'_c^2}{2}. \tag{41}
\]
Equations (25)-(31), with (39)-(41), are new quadratic baryon mass relations. In the following, we shall make predictions for the baryon masses not measured so far using these relations.

For the $\frac{1}{2}^+$ baryons, in the approximation of equality of the slopes in the light quark sector, $\alpha'_N \cong \alpha'_{\Sigma'} \cong \alpha'_\Xi$ (i.e., $\gamma_{N}^{N} \alpha'_{N} \ll 1$ in Eqs. (7),(8)), it follows from (37) that
\[ 2 \left( N^2 + \Xi^2 \right) \cong 3 \Lambda^2 + \Sigma^2, \]  
(42)
which is a relation obtained by Oneda and Terasaki in the algebraic approach to hadronic physics [59] which holds with an accuracy of $\sim 1.5\%$: (in GeV) 5.235±0.015 vs. 5.160 ± 0.010. Similar approximation for the $\frac{3}{2}^+$ baryons leads, through (2), to relations
\[ \Omega^2 - \Xi^*2 \cong \Xi^*2 - \Sigma^*2 \cong \Sigma^*2 - \Delta^2, \]  
(43)
which have long been discussed in the literature [49, 59, 60, 61] and hold with a high accuracy, as well as (42).

The mass of the $\Xi_c^*$ can now be obtained from Eqs. (30),(39)-(41). Using the measured masses of the states entering these relations, one finds
\[ \Xi_c^* = 2569 \pm 6 \text{ MeV}. \]  
(44)
The mass of the $\Omega_c^*$ is obtained from (28):  
\[ \Omega_c^* = 2767 \pm 7 \text{ MeV}. \]  
(45)
One sees that the value for the $\Xi_c^*$ mass (44) lies within the interval provided by experiment [12]. Both (44) and (45) are consistent with the values 2580 and 2770 MeV, respectively, predicted by almost all very recent calculations [2, 4, 5, 7, 11].

Now, we have two (independent) relations for the $\frac{3}{2}^+$ baryons, Eqs. (26) or (27), and (29), to make predictions for the three unknown masses of the $\Xi_{cc}^*$, $\Omega_{cc}^*$, and $\Omega_{ccc}$. Similarly, we have one relation for the $\frac{1}{2}^+$ baryons, Eq. (31), to make predictions for the two unknown masses of the $\Xi_{cc}$ and $\Omega_{cc}$. In order to obtain two additional relations (for each of the two multiplets), we shall use the approximation of equality of the slopes in the light quark sector referred to above. Indeed, we have fitted the three, vector meson, octet baryon, and decuplet baryon mass spectra simultaneously, by using a common value of $x$ in Eq. (24) and similar relations for vector mesons and octet baryons for all three multiplets. Our results are shown in Table I (the calculation is completed when $\lambda^c_{\Delta}$ becomes zero first of the three $\lambda$'s). It is seen that the best simultaneous fit corresponds to $x = 0.05 \pm 0.01 \ll 1$, and therefore the approximation of equality of the slopes in the light quark sector is completely justified.

For the $\frac{1}{2}^+$ baryons, it then follows from (7)-(10) (with $\gamma_{N}^{N} \alpha'_{N} \ll 1$) that
\[ \alpha'_\Xi \cong \alpha'_{N}, \quad \alpha'_{\Xi_{cc}} \cong \alpha'_{\Xi_{cc}}. \]  
(46)
We now apply the procedure developed for mesons in [46] to baryons, using the following relations based on (2) and (46),

\[ \alpha_N^2 N^2 + \alpha_{\Xi_{cc}}^2 \Xi_{cc}^2 = 2\alpha_{\Sigma_{cc}}^2 \Sigma_{cc}^2, \]
\[ \alpha_N^2 \Xi^2 + \alpha_{\Xi_{cc}}^2 \Xi_{cc}^2 = 2\alpha_{\Sigma_{cc}}^2 \Sigma_{cc}^2, \]
\[ \frac{1}{\alpha_N} + \frac{1}{\alpha_{\Xi_{cc}}} = \frac{2}{\alpha_{\Sigma_{cc}}}, \]

and obtain a sixth power relation for the \( \frac{1}{2}^+ \) baryon masses:

\[ \left( \Xi^2 \Sigma_{cc}^2 - N^2 \Xi_{cc}^2 \right) \left( \Xi^2 - N^2 \right) + \Xi_{cc}^2 \left( \Xi_{cc}^2 - \Sigma_{cc}^2 \right) \left( \Xi^2 - N^2 \right) = 4 \left( \Xi^2 \Sigma_{cc}^2 - N^2 \Xi_{cc}^2 \right) \left( \Xi_{cc}^2 - \Sigma_{cc}^2 \right). \quad (47) \]

The same procedure applied for the \( \frac{3}{2}^+ \) baryons leads to a similar sixth power relation for the \( \frac{3}{2}^+ \) baryon masses:

\[ \left( \Xi^* \Sigma^*_{cc} \right) \left( \Xi^* - \Delta^2 \Xi_{cc}^* \right) + \Xi^*_{cc} \left( \Xi_{cc}^* - \Sigma^*_{cc} \right) \left( \Xi^* - \Delta^2 \right) = 4 \left( \Xi^* \Sigma^*_{cc} \right) \left( \Xi^*_{cc} - \Sigma^*_{cc} \right). \quad (48) \]

Equations (47) and (48) yield the following values for the masses of the \( \Xi_{cc} \) and \( \Xi_{cc}^* \):

\[ \Xi_{cc} = 3610 \pm 3 \text{ MeV}, \quad (49) \]
\[ \Xi_{cc}^* = 3735 \pm 17 \text{ MeV}. \quad (50) \]

The values for the masses of the \( \Omega_{cc} \) and \( \Omega_{cc}^* \) can now be obtained from Eqs. (29) and (31), respectively:

\[ \Omega_{cc} = 3804 \pm 8 \text{ MeV}, \quad (51) \]
\[ \Omega_{cc}^* = 3850 \pm 25 \text{ MeV}. \quad (52) \]

The remaining value for the \( \Omega_{ccc} \) mass is obtained either from (26) or (27):

\[ \Omega_{ccc} = \begin{cases} 
4930 \pm 45 \text{ MeV} \quad \text{from (26)}, \\
4928 \pm 70 \text{ MeV} \quad \text{from (27)}. 
\end{cases} \quad (53) \]

Both results are consistent, as they should be.

The effect on the \( \frac{1}{2}^+ \) and \( \frac{3}{2}^+ \) baryon spectra of setting \( x = 0 \) in Eqs. (24) and corresponding relations for \( \frac{1}{2}^+ \) baryons is negligible (\( \leq \) few MeV), except for the splitting between nonstrange and singly strange baryons (see (42),(43)). Even in this case the absolute size of this splitting is small, and so the included error is not more than 2%. More significantly, this does not affect the multiply strange and charm states by more than 1%.

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Our results are shown in Table II, together with the predictions of many other approaches. One sees that our predictions for the charmed baryon masses done in the Regge framework are in good agreement with those of different approaches. In particular, the predicted value for the $\Xi'_c$ lies in the range provided by experiment [12], and is in close proximity to 2580 MeV, consistent with the very recent predictions [2, 4, 7, 8, 11]. The predicted value for the $\Omega'_c$ mass is in close proximity to 2770 MeV, consistent with almost all very recent calculations [1 – 5, 7, 9, 11].

As remarked by Kaidalov [50], the relations (2),(4), on which our mass predictions are based, have such a structure that a variation of $\alpha'_{kji}$ by 10-15 % leads only to about 1% change in the values of masses $m_{kji}$. Thus, although our calculation of the baryon masses in the double- and triple-charm sectors is based on the assumption of equality of the slopes in the light quark sector, we expect our results to be insensitive to any further adjustment of the values of these slopes.

Extension of the present framework to the beauty sector, and predictions for the masses of the beauty baryons will be the subject of a separate publication.

We note (from Table II) with interest that our results are closest to those derived using a quark-diquark model [43]. Agreement between such a model and linear Regge trajectories is expected from both the QCD area law of the Wilson loop [57] and string approach [54]. We plan to investigate this further in the future.
| x  | $\lambda^p_s$ | $\lambda^N_s$ | $\lambda^\Delta_s$ | $K^*$ | $\phi$ | $\Sigma^2$ | $\Xi$ | $\Sigma^*$ | $\Xi^*$ | $\Omega$ |
|----|----------------|----------------|---------------------|-------|--------|-----------|-------|-----------|--------|--------|
| 0  | 0.219          | 0.422          | 0.420               | 900   | 1015   | 1.304     | 1314  | 1392      | 1536   | 1667   |
| 0.010 | 0.209       | 0.407          | 0.395               | 899   | 1015   | 1.302     | 1315  | 1390      | 1534   | 1669   |
| 0.020 | 0.201       | 0.392          | 0.371               | 899   | 1016   | 1.299     | 1316  | 1388      | 1533   | 1670   |
| 0.030 | 0.192       | 0.377          | 0.348               | 898   | 1017   | 1.296     | 1317  | 1386      | 1532   | 1671   |
| 0.040 | 0.182       | 0.363          | 0.326               | 897   | 1017   | 1.295     | 1318  | 1385      | 1531   | 1672   |
| 0.050 | 0.175       | 0.350          | 0.305               | 897   | 1018   | 1.293     | 1319  | 1383      | 1530   | 1673   |
| 0.060 | 0.167       | 0.336          | 0.285               | 897   | 1018   | 1.291     | 1319  | 1382      | 1529   | 1673   |
| 0.070 | 0.159       | 0.323          | 0.266               | 896   | 1018   | 1.289     | 1320  | 1382      | 1529   | 1674   |
| 0.080 | 0.152       | 0.310          | 0.249               | 896   | 1019   | 1.287     | 1320  | 1381      | 1529   | 1675   |
| 0.100 | 0.137       | 0.286          | 0.215               | 895   | 1019   | 1.284     | 1321  | 1381      | 1529   | 1677   |
| 0.150 | 0.104       | 0.232          | 0.140               | 894   | 1019   | 1.281     | 1323  | 1381      | 1529   | 1676   |
| 0.200 | 0.075       | 0.185          | 0.077               | 894   | 1019   | 1.280     | 1324  | 1383      | 1530   | 1673   |
| 0.277 | 0.038       | 0.122          | 0               | 896   | 1018   | 1.282     | 1323  | 1392      | 1535   | 1667   |

Table I. Simultaneous fit to the vector meson, octet baryon, and decuplet baryon spectra, through the relations

$$
\rho^2 = \frac{K^{*2}}{1 + x} - \lambda^p_s = \frac{\phi^2}{1 + 2x} - 2\lambda^p_s,
$$

$$
N^2 = \frac{\Sigma^2}{1 + x} - \lambda^N_s = \frac{\Xi^2}{1 + 2x} - 2\lambda^N_s,
$$

$$
\Delta^2 = \frac{\Sigma^*^2}{1 + x} - \lambda^\Delta_s = \frac{\Xi^*^2}{1 + 2x} - 2\lambda^\Delta_s = \frac{\Omega^2}{1 + 3x} - 3\lambda^\Delta_s,
$$

as compared to the measured values:

$$
K^{*0} = 896 \text{ MeV}, \quad \phi = 1019 \text{ MeV},
$$

$$
\Sigma^2 = 1.290 \pm 0.003 \text{ GeV}^2, \quad \Xi = 1318 \pm 3 \text{ MeV},
$$

$$
\Sigma^* = 1385 \pm 2 \text{ MeV}, \quad \Xi^* = 1533.5 \pm 1.5 \text{ MeV}, \quad \Omega = 1672.5 \text{ MeV}.
$$

The input parameters are:

$$
\rho = 769 \text{ MeV}, \quad N = 939 \text{ MeV}, \quad \Delta = 1232 \text{ MeV}.
$$

$\lambda$'s are measured in GeV$^2$. 

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| Reference | $\Xi_c$  | $\Xi_{cc}$ | $\Omega_{cc}$ | $\Omega^*_c$ | $\Xi^*_c$ | $\Omega^*_{cc}$ | $\Omega_{cc}$ |
|-----------|--------|---------|-------------|------------|------------|----------------|------------|
| Present work | $2569 \pm 6$ | $3610 \pm 3$ | $3804 \pm 8$ | $2767 \pm 7$ | $3735 \pm 17$ | $3850 \pm 25$ | $4930 \pm 45$ |
| [1] | | | | | | | |
| [2] | 2579 | | | | | | |
| [3] | | | | | | | |
| [4] | $2580 \pm 20$ | $3660 \pm 70$ | $3740 \pm 80$ | $2770 \pm 30$ | $3740 \pm 70$ | $3820 \pm 80$ | $2770 \pm 10$ |
| [5] | 2582 | 3676 | 3787 | 2775 | 3746 | 3851 | |
| [6] | | 3660 | 3760 | 3810 | 3890 | | |
| [7] | $2580 \pm 10$ | | | | | | |
| [8] | 2583 | | | | | | |
| [9] | 2593 | | | | | | |
| [11] | $2581 \pm 2$ | | | | | | $2761 \pm 5$ |
| [17] | 2510 | 3550 | 3730 | 2720 | 3610 | 3770 | 4810 |
| [18] | 2532 | | | 2780 | | | 5026 |
| [19] | 2566 | 3605 | 3730 | 2830 | 3680 | 3800 | 4793 |
| [20] | 2579 | 3645 | 3824 | 3733 | 4837 | | |
| [21] | 2558 | 3613 | 3703 | 2775 | 3741 | 3835 | 4797 |
| [22] | 3710 | | | | | | 4923 |
| [23] | 2590 | | | 2805 | | | |
| [25] | 2608 | | | 2822 | | | |
| [26] | 2530 | 3511 | 3664 | 2764 | 3630 | 3764 | 4747 |
| [27] | | | | | | | 5040 |
| [28] | 2500 | | | 2710 | | | |
| [29] | 2467 | | | 2659 | | | |
| [30] | | | | $2767 \pm 35$ | | | |
| [32] | $2570^{+6}_{-3-6}$ | | $2660^{+5}_{-3-7}$ | | | | |
| [33] | $3630 \pm 50$ | $3720 \pm 50$ | | $3735 \pm 50$ | | $3840 \pm 50$ | |
| [35] | | $3742$ | | $3811$ | | | |
| [36] | $2570$ | $3610$ | $3710$ | $2740$ | $3680$ | $3760$ | $4730$ |
| [37] | $2596$ | $3752$ | $3934$ | $2811$ | $3793$ | $3964$ | $5127$ |
| [38] | $2600$ | $3725$ | $3915$ | $2811$ | $3783$ | $3953$ | $5106$ |
| [39] | $2690$ | $3700$ | $3960$ | $2810$ | $3768$ | $3931$ | $5019$ |
| [40] | $2616$ | $3837$ | $4036$ | | | | |
| [41] | $2583$ | | | | | | $2772$ |
| [42] | $2542$ | $3710$ | $3852$ | $2798$ | $3781$ | $3923$ | $5048$ |
| [43] | $2578$ | $3661$ | $3785$ | $2782$ | $3732$ | $3856$ | $4895$ |
| [44] | $2584$ | $3758$ | $3861$ | | | | |

Table II. Comparison of predictions for the charmed baryon masses not measured so far (in MeV).

Potential models: [1,17-22]
Chiral perturbation theory: [2]
Relativistic quark model: [6]
Chiral quark model: [9]
Heavy quark effective theory: [11,35,36]
Nonrelativistic quark models: [23,25]
Bag models: [26-29]
Lattice QCD: [30,32]
QCD spectral sum rules: [33]
SU(4) skyrmion model: [37]
Group theoretical models: [38,39]
Other models: [3-5,7,8,40-44]

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