Using finite-difference time-domain methods with a Rayleigh approach to model low-frequency sound fields in small spaces subdivided by porous materials

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Abstract: Finite-Difference Time-Domain (FDTD) models are used to predict low-frequency sound fields in small volumes containing a limp panel formed from a porous material which partially or completely subdivides the volume. This porous panel is incorporated into FDTD using a Rayleigh model as proposed by Suzuki et al. However, to accurately reproduce the low-frequency sound field it is found necessary to introduce an additional Moving Frame Model (MFM) to account for motion of the porous panel. For spaces that are completely subdivided by a porous panel, the MFM accounts for a spring-mass-spring resonance that can occur below the lowest acoustic cavity mode. The MFM assumes lumped mass behavior of the porous panel which is coupled to the FDTD update equations that incorporate the Rayleigh model. FDTD is compared against measurements using transient excitation with a pulse input to a loudspeaker in a small reverberant room under three different conditions: (1) empty room, (2) with a mineral fibre panel partially dividing the room, and (3) with a mineral fibre panel completely dividing the room. Close agreement is obtained between experimental results and FDTD incorporating the MFM; this validates the models as well as implementation of the loudspeaker as a hard velocity source.

Keywords: Room, Cabin, Sound field, FDTD, Rayleigh model, Porous material

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1. INTRODUCTION

This paper is concerned with the prediction of low-frequency sound fields in small volume spaces which are partially or completely divided by a porous, limp panel. Such spaces can occur as cabins in automotive, aeronautic and marine vehicles, as well as small rooms inside buildings such as music studios. In such spaces, prediction of transient signals at low frequencies is often important.

For low- and mid-frequency problems in room acoustics, the Finite-Difference Time-Domain method (FDTD) has been shown to have significant potential, although the implementation of frequency-dependent boundary conditions remains an important issue [1,2]. Previous research by Suzuki et al. [3] implemented the Rayleigh model [4] in FDTD for porous materials dividing a space. However, there was no experimental validation.

The purpose of the research in this paper is to experimentally validate the following: (a) FDTD incorporating the Rayleigh model in a room subdivided by a porous panel, (b) the need for an additional moving frame model to simulate the motion of the porous panel in response to the surrounding pressure field and (c) the inclusion of a loudspeaker as a hard velocity source in FDTD with a low-frequency pulse.

For the experimental work, three different room configurations are used: (1) an empty hard-walled room, (2) the room when partially divided by a porous panel and (3) the room when completely divided by a porous panel.

2. FDTD IMPLEMENTATION

2.1. Overview of the FDTD Approach

FDTD is used to solve a system of first-order linear differential equations that govern the propagation of acoustic waves across a space [4] according to:

\[
\frac{\partial p}{\partial t} = -\rho c^2 \nabla \cdot \mathbf{v}
\]  

(1)
where \( \rho \) is the air density, \( c \) is the speed of sound, \( v \) is particle velocity, \( p \) is sound pressure, \( \nabla \) indicates gradient.

Equation (1) is the continuity equation and Eq. (2) is the momentum equation. To solve these equations, the FDTD approach calculates the pressure and particle velocity at a set of points that are offset in space and time using a staggered grid as shown in Fig. 1.

In three dimensions, the discretization of Eqs. (1) and (2) results in the following set of equations [1]:

\[
\begin{align*}
\frac{\partial v_x}{\partial t} &= -\frac{1}{\rho} \nabla p \\
\frac{\partial v_y}{\partial t} &= -\frac{1}{\rho} \nabla p \\
\frac{\partial v_z}{\partial t} &= -\frac{1}{\rho} \nabla p
\end{align*}
\]

2.3. Implementation of the Source

The sound source is a loudspeaker which is implemented in FDTD as a hard velocity source. The loudspeaker cone points upwards into the room (z-direction) and was experimentally characterized to be acting as a piston in the frequency range of interest. For this reason a uniform driving function could be applied over the surface area of the cone on the FDTD grid.

Figure 2 shows the FDTD implementation of the velocity source in the x-direction (horizontal) and the z-direction (vertical). The velocity elements representing the loudspeaker cone are referred to as the driving function. These are all assigned the same velocity value which corresponds to the measured cone velocity with transient excitation from a pulse. The other velocity elements that form the boundary of the loudspeaker cabinet are set to zero to represent rigid boundaries. Implementing the loudspeaker in this way emulates a piston on the surface of a sealed cabinet, since all elements of the diaphragm move with the same particle velocity.

2.4. Boundary Conditions

The boundary conditions used to describe all room surfaces are frequency-independent and given by [6]:

\[
v_x = \frac{p}{Z_x}, \quad v_y = \frac{p}{Z_y}, \quad v_z = \frac{p}{Z_z}
\]

For example, the velocity component in the x-direction can be calculated using the discrete form of Eq. (8):

\[
v_{x,i+1/2}^{n+1} = v_{x,i+1/2}^{n} + \frac{\Delta t}{\rho Z_x} (p_{i+1/2,j,k}^{n} - p_{i,j,k}^{n})
\]

2.5. Rayleigh Model for the Porous Panel

The porous panel is incorporated in FDTD following the implementation proposed by Suzuki et al. [3] using the Rayleigh model. This treats the porous material as a set of parallel narrow tubes embedded in a rigid frame [4]. Assuming these tubes are parallel to the x-axis, the
following equations describe the acoustic field inside each tube:
\[
\frac{\partial p}{\partial t} + \rho c^2 \frac{\partial v}{\partial x} = 0 \tag{10}
\]
\[
\frac{\partial p}{\partial x} + \rho \frac{\partial v}{\partial t} + rv = 0 \tag{11}
\]
where \( v \) is the average particle velocity over the cross section of the air channel and \( r \) is the airflow resistivity of the air channel (Pa·s/m²).

The one-dimensional formulation of the Rayleigh model is generalized to two and three spatial dimensions [3]. In three dimensions, Eq. (10) becomes Eq. (1) and Eq. (11) gives the following set of equations:
\[
\frac{\partial p}{\partial t} + \rho \frac{\partial v_x}{\partial x} + r_s v_x = 0 \tag{12}
\]
\[
\frac{\partial p}{\partial y} + \rho \frac{\partial v_y}{\partial t} + r_s v_y = 0 \tag{13}
\]
\[
\frac{\partial p}{\partial z} + \rho \frac{\partial v_z}{\partial t} + r_s v_z = 0 \tag{14}
\]

The corresponding set of FDTD equations is therefore:
\[
v_{x, i+\frac{1}{2}}^{n+\frac{1}{2}} = \frac{1}{\rho + r_s \Delta t} \left[ -\frac{\Delta t}{\Delta x} (p_{i+1}^n - p_i^n) + \rho v_x^{n-\frac{1}{2}} \right] \tag{15}
\]
\[
v_{y, j+\frac{1}{2}}^{n+\frac{1}{2}} = \frac{1}{\rho + r_s \Delta t} \left[ -\frac{\Delta t}{\Delta y} (p_{j+1}^n - p_j^n) + \rho v_y^{n-\frac{1}{2}} \right] \tag{16}
\]
\[
v_{z, k+\frac{1}{2}}^{n+\frac{1}{2}} = \frac{1}{\rho + r_s \Delta t} \left[ -\frac{\Delta t}{\Delta z} (p_{k+1}^n - p_k^n) + \rho v_z^{n-\frac{1}{2}} \right] \tag{17}
\]

These equations model the sound field inside the porous material. If the airflow resistivity is zero, Eqs. (15), (16) and (17) reduce to Eqs. (4), (5) and (6), respectively.

### 2.6. Moving Frame Model (MFM) for the Porous Panel

The Rayleigh model provides an approximate description of the losses that occur due to the friction between the air particles and the rigid frame of the porous material. However, it is assumed that the rigid frame of the porous panel remains stationary. If the frame of the porous panel is allowed to move and the velocity of the air particles is considered in relation to the frame for the resistive term in Eq. (11), it is possible to account for the motion of the frame in FDTD with a Moving Frame Model (MFM) by rewriting Eq. (11) as
\[
\frac{\partial p}{\partial x} + \rho \frac{\partial v}{\partial t} + rv_{\text{air/frame}} = 0 \tag{18}
\]
where \( v_{\text{air/frame}} \) is the velocity of the air particles relative to the frame velocity which in discretized form is given by:
\[
v_{\text{air/frame}, i+\frac{1}{2}}^{n+\frac{1}{2}} = v_{i+\frac{1}{2}}^{n+\frac{1}{2}} - v_F^{n-\frac{1}{2}} \tag{19}
\]

where \( \Delta p \) represents the pressure difference across the porous panel element, \( m_s \) is the mass per unit area of the panel and \( v_F \) is the frame velocity of the panel element.

Figure 3 shows an example of the lumped mass in the thickness direction of the panel which is represented by two velocity elements. The volume element with mass, \( m \) (shaded grey) is subject to a pressure gradient (which in this example is \( p_{i+1}^n - p_{i-1}^n \)) such that all points within it move with the same velocity, \( v_F \). After some algebraic manipulation, discretisation of Eqs. (20) and (18) results in:
\[
v_F^{n+\frac{1}{2}} = -\frac{\Delta t}{m_s} (p_{i+1}^n - p_{i-1}^n) + v_F^{n-\frac{1}{2}} \tag{21}
\]
\[
v_{i+\frac{1}{2}}^{n+\frac{1}{2}} = -\frac{\Delta t}{\Delta x} (p_{i+1}^n - p_i^n) + \rho v_F^{n-\frac{1}{2}} \tag{22}
\]

If \( m_s \) approaches infinity the frame velocity, \( v_F \), will approach zero and Eq. (22) reduces to Eqs. (15), (16) or (17) depending on the direction being evaluated. The motion of the panel is assumed to be unrestricted in the \( y \)-direction; hence Eqs. (21) and (22) are only used to calculate the panel motion in this direction.

The MFM can be used for porous panels that partially or completely divide a room. For the latter this leads to a spring-mass-spring resonance as shown in Fig. 4. This is an interesting feature as it can cause a general rise in the sound pressure level as well as a distinct peak below the fundamental mode of the empty room.
For the system in Fig. 4, the frequency, $f_0$, at which the spring-mass-spring resonance occurs is given by:

$$f_0 = \frac{1}{2\pi} \sqrt{\frac{k_1 + k_2}{m}}$$

(23)

The moving frame model is implemented in such a way that knowledge of $k_1$ and $k_2$ is not required for calculation of $f_0$. This is because movement of the frame is inherently included in FDTD update Eqs. (21) and (22) regardless of the geometry of the room or whether the porous panel partially or completely divides the room volume.

3. EXPERIMENTAL PROCEDURE

3.1. Reverberant Room

A 13 m$^3$ reverberant room is used for the experimental validation. The walls are brick with a plaster finish and the floor and ceiling are cast in situ concrete slabs. Measurements are carried out in three different room configurations: empty room, the room partially divided by a porous panel and the room completely divided by porous panel. The source and receiver positions remain the same for all the different conditions.

B&K Type 4135 free-field 1/4 inch microphones are used to measure sound pressure levels on a horizontal grid (6 × 8) and a vertical grid (6 × 7), as shown in Figs. 5 and 6. The distance between microphone positions on the grid is 350 mm along the x-axis and 400 mm along the y-axis. The location index in the y- and z-directions is referred to by row number with the location in the x-direction referred to by position number as shown in Fig. 6.

The loudspeaker faces upwards in one corner of the room, with the centre of the cone at a height of 270 mm, at a distance from the side walls of 210 mm in the x-direction and 180 mm in the y-direction. On Figs. 5 and 6, the position of the cone is indicated by a red shaded circle.

FDTD requires impedances for the walls and floors and these are determined experimentally. In the empty room, microphones are positioned at three corners to measure the spectrum with excitation from broadband noise. The 3 dB bandwidths, $\Delta f$, are determined for the first 11 modes of the room. These are used to calculate a damping constant, $\zeta$, for each mode, according to [4]:

$$\zeta = \pi \Delta f$$

(24)

from which the specific acoustic impedance, $Z_{a,s}$, that corresponds to the mth mode is calculated using [7]:

$$Z_{a,s} = \frac{c}{\zeta_m} \left( \frac{\xi_{p,m}}{L_x} + \frac{\xi_{q,m}}{L_y} + \frac{\xi_{r,m}}{L_z} \right)$$

(25)

where $p$, $q$ and $r$ are the mode numbers in the x, y and z directions respectively and take non-negative integer values. The numerators are $\xi_{p,m} = 1$ if $p = 0$ and $\xi_{p,m} = 2$ if $p \neq 0$ and are similarly defined for $\xi_{q,m}$ and $\xi_{r,m}$.
value considered for the frequency-independent specific acoustic impedance was obtained from the average of the specific acoustic impedances obtained from the first 11 room modes. For one of the three corner microphone positions, the specific acoustic impedances obtained for each room mode are indicated in Table 1. This indicates that although there is variation, it is reasonable to use an average value in the FDTD model. The average value of specific acoustic impedance obtained for all the three measurement positions is 224.9. This value was subsequently multiplied by the value of characteristic impedance of air to yield the average characteristic acoustic impedance of the room surfaces needed as input data in Eq. (9).

### 3.2. Porous Panel

The porous panel is formed from 1.2 m × 0.6 m Rockwool slabs (rock fibre) with 100 mm thickness, a bulk density of 100 kg/m³ and porosity of 0.96. The airflow resistivity is taken from previous measurements reported by Hopkins [7] which were carried out according to ISO 9053; these values are 48.820 Pa·s/m² in the thickness dimension and 23.560 Pa·s/m² in the lateral direction. Both values are incorporated in the FDTD model.

The porous panels are held in place with cotton cloth adhesive tape at the joints with the walls and between the slabs of rock fibre. This tape also sealed any air gaps.

### 3.3. Loudspeaker Output

The source used for the measurements is a closed cabinet sub-woofer (Celestion C6S — 10 inch driver) with the electronics removed so that the unit is passive. Preliminary measurements confirmed that the loudspeaker could be modeled as a linear system with pistonic behavior over the frequency range of interest which is below 140 Hz.

The input signal used for the measurements is a Gaussian pulse. As this is modified by the loudspeaker, signal processing and amplification chain, the driving function used in the FDTD models is obtained by measuring the velocity on the centre of the loudspeaker cone in an anechoic chamber. A B&K Type 4393 accelerometer with a weight of 2.4 g is used for the measurement as mass loading is negligible over the frequency range of interest.

The FFT analyser uses the same sampling frequency as the FDTD simulations. The time domain and spectrum level of the velocity driving function is shown in Fig. 7.

### 4. RESULTS

#### 4.1. Empty Room

The first stage in the validation is to compare FDTD with measurements in the empty room. A typical example of the magnitude, phase (wrapped) and impulse response at one of the receiver positions on the horizontal grid is shown in Fig. 8. The peak in the magnitude at 60 Hz
corresponds to the lowest frequency room mode which is \( f_{010} \), where the subscript indicates \( p, q, \) and \( r \) corresponding to the \( x, y \) and \( z \) directions respectively.

Good agreement between FDTD and measurements indicates that the impulse from the loudspeaker has been correctly incorporated as a hard velocity source in FDTD. It also indicates that it is reasonable to use a frequency-independent impedance for all the walls and floors. However, whilst FDTD predicts all the trends of the phase excursions there are occasional discrepancies which become most apparent when the phase wraps at \( \pm 180^\circ \).

4.2. Validation of FDTD with the Moving Frame Model

The next stage is to assess inclusion of the MFM in FDTD with the porous panel through comparison of FDTD results with measurements. Figures 9 and 10 show three
different microphone positions taken from the measurement grids for the room when partially and completely divided by the porous panel respectively. For the room that is partially divided by the porous panel, inclusion of the MFM is necessary to correctly predict the lowest frequency peak in the spectrum, although it has negligible effect at higher frequencies. When the panel completely divides the room volume the results show that the MFM is essential to correctly predict sound pressure levels near the spring-mass-spring resonance otherwise errors up to 20 dB can be incurred. The spring-mass-spring resonance occurs below the first room mode and is calculated to be 23 Hz, although the shallow peak in the measurements is only evident at 27 Hz. There is an indication that the MFM can improve the FDTD prediction outside the damping controlled region (i.e., 3 dB bandwidth) of the room modes, for example, see between 70 Hz and 90 Hz on Fig. 10.

Having demonstrated the validity of FDTD with MFM for the room with the porous panel, the magnitude, phase (wrapped) and impulse response are shown on Figs. 11 and 12 at two different grid positions. As with the empty room, there is close agreement with measurements which

![Fig. 11](image1.png)  
**Fig. 11** FDTD and measured responses on the horizontal grid, Row 1 Position 4.

![Fig. 12](image2.png)  
**Fig. 12** FDTD and measured responses on the horizontal grid Row 6 Position 4.
indicates that there is potential to use the predicted impulse response for the purpose of auralisation.

The final stage of the validation is to compare measurements with FDTD using MFM in terms of the spatial variation of the sound pressure level over the measurement grids. Contour plots of the sound pressure level (dB re \(10^{-5}\) Pa) with 1 Hz FFT lines are shown in Figs. 13 to 16 at the following frequencies: 60 Hz (corresponding to axial mode, \(f_{010}\)), 70 Hz (corresponding to...
to axial mode, $f_{001}$), 112 Hz (corresponding to tangential mode, $f_{110}$) and 131 Hz (corresponding to oblique mode, $f_{111}$). All the measured features relating to the nodal and anti-nodal planes for the axial, tangential and oblique modes are predicted by the FDTD model. In addition there is close agreement between FDTD and measurements over both measurement grids. This demonstrates the ability of FDTD to accurately map the sound field with or without a porous panel that partially or completely divides the room.
5. CONCLUSIONS

This paper investigated the prediction of low-frequency sound fields using FDTD in a small room incorporating a porous panel. Close agreement between FDTD and measurements shows that: (a) the Rayleigh model is valid for a room that is partially or completely divided by a porous panel of fibrous material, (b) an additional moving frame model is required in FDTD to introduce low-frequency panel motion and (c) a loudspeaker driven by a pulse can be accurately modeled in FDTD as a hard velocity source acting as a piston with all other velocity elements forming the cabinet set to zero to represent rigid boundaries.

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