Thin-shell wormhole under non-commutative geometry inspired Einstein–Gauss–Bonnet gravity

Nilofar Rahman 1,a, Mehedi Kalam 1,b, Amit Das 2,c, Sayeedul Islam 3,d, Farook Rahaman 4,e, Masum Murshid 1,f

1 Department of Physics, Aliah University, IIA/27, New Town, Kolkata 700160, India
2 Department of Physics, Ashoknagar Vidyasagar Bani Bhaban High School (H.S.), North 24 Parganas, West Bengal 743222, India
3 Department of Mathematics, Sister Nivedita University, Kolkata 700156, India
4 Department of Mathematics, Jadavpur University, Kolkata 700032, India

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Abstract Einstein–Gauss–Bonnet gravity is a generalization of the general relativity to higher dimensions in which the first- and second-order terms correspond to general relativity and Einstein–Gauss–Bonnet gravity, respectively. We construct a new class of five-dimensional (5D) thin-shell wormholes by the ‘Cut–Paste’ technique from black holes in Einstein–Gauss–Bonnet gravity inspired by non-commutative geometry starting with a static spherically symmetric, Gaussian mass distribution as a source and for this structural form of the thin-shell wormhole we have explored several salient features of the solution, viz., pressure–density profile, equation of state, the nature of wormhole, total amount of exotic matter content at the shell. We have also analyzed the linearized stability of the constructed wormhole. From our study we can assert that our model is found to be plausible with reference to the other model of thin-shell wormhole available in literature.

1 Introduction

In 1935, Einstein and Rosen [1] tried to produce a field theory for electrons. Considering a spherically symmetric mass distribution which had already been used for black holes, Einstein and Rosen implemented a coordinate transformation to eliminate the region comprising the curvature singularity. But, the solution demonstrates the existence of bridges through space-time. This bridge can act as a tunnel through which one can travel in space-time to another part of the universe and it is known as Einstein–Rosen bridge. It is also known as Schwarzschild wormhole. Theoretically, this bridge is a shortcut path that minimizes the travel time as well as distance. However, in 1962, Wheeler and Fuller [2] argued that Einstein and Rosen bridge structure is unstable if it connects two parts of the same universe. Therefore, one can argue that though Schwarzschild wormhole exists theoretically, but it is not traversable in both directions. In 1988, Morris and Thorne [3] first ever proposed a wormhole solution of Einstein field equations which was found to be traversable. The solution represents a hypothetical shortcuts between two regions (either of the same Universe or of two separate Universes) connected by a throat. The throat of the wormholes can be defined as a two-dimensional hypersurface of minimal area and for the existence of such an wormhole the null energy condition has to be violated. Hence, all traversable wormholes require exotic matter which can be held responsible for the violation of the null energy condition. The requirements of exotic matter for the existence of a wormhole can be made infinitesimally small by a suitable choice of the geometry [4]. Visser [5] has also proposed another way to minimize the usage of exotic matter to construct a wormhole, and it is known as ‘Cut and Paste’ technique, in which the exotic matter is concentrated at the wormhole throat. In the ‘Cut and Paste’ technique, the wormholes can be theoretically constructed by cutting and pasting two different manifolds to obtain geodesically complete new manifold with a throat placed in the joining shell [6]. Using the Darmois-Israel formalism [7, 8], one can determine the surface stresses of the exotic matter located in thin shell placed at the joining surfaces. This new construction of wormhole is known as thin-shell wormhole and is extremely useful to perform the stability analysis for the dynamic case. Several authors have used this surgical technique (i.e., the Cut and Paste technique) to construct thin wormholes. Poisson and Visser [6] have analyzed the stability of a thin wormhole constructed by joining two Schwarzschild spacetimes. Eiroa and Simeone [9] have constructed the wormholes by applying the cut and paste technique to

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a e-mail: rahmannilofar@gmail.com
b e-mail: mehedikalam@yahoo.co.in
c e-mail: amdphy@gmail.com (corresponding author)
d e-mail: sayeedul.i@snuniv.ac.in
e e-mail: rahaman@associates.iucaa.in
f e-mail: masum.murshid@wbsce.te.ac.in

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two metrics corresponding to a charged black hole solution in low energy bosonic string theory, with vanishing antisymmetric field along with the Maxwell field, whereas they have analyzed cylindrically symmetric thin wormhole geometry associated to gauge cosmic strings [10]. Thibeault et al. [11] have analyzed the stability as well as the energy conditions of five-dimensional spherically symmetric thin-shell wormholes in Einstein–Maxwell theory along with the Gauss Bonnet term.

Now, the Einstein–Gauss–Bonnet (EGB) gravity which is a generalization of Einstein’s general relativity, was originally introduced by Lanczos [12], and it was rediscovered by Lovelock [13]. Amid the larger class of general higher-curvature theories, this theory has some special characteristics having higher derivative terms. Nevertheless, the field equations in this theory are of second-order like that of general theory of relativity. In this theory the Gauss-Bonnet term is present in the low energy effective action of heterotic string theory [14–16], and can be considered as the dimensionally extended version of the four-dimensional Euler density. Also, this term appears in six-dimensional Calabi-Yau compactifications of M-theory [17]. The EGB gravity enables the researchers to study how higher curvature corrections to black hole physics significantly change the qualitative features which are available due to black holes under the background of general relativity.

The Gauss–Bonnet term naturally appears as the second significant term in heterotic string effective action. Also, the noncommutative geometry appears from the study of open string theories. One can show that the gravitational wave signal GW150914 which has been recently detected by LIGO and Virgo collaborations [18], can be used to place a bound on the scale of quantum fuzziness of noncommutative space-time [19]. In a paper [19], author has shown that the leading noncommutative correction to the phase of the gravitational waves produced by a binary system appears at the second order of the post-Newtonian expansion. Also, in another paper [20], the plausibility of using quantum mechanical transitions that is induced by the combined effect of gravitational waves and noncommutative structure to probe the spatial noncommutative nature has been explored. Snyder [21, 22] first introduced the noncommutative spacetime to study the divergences in relativistic quantum field theory. Noncommutative geometry also aims to place General Relativity and the Standard Model on the same footing in order to describe gravity, the electro-weak and strong forces as gravitational forces in a unified spacetime [23]. Noncommutativity substitutes the point-like structures by smeared objects [24–27] and is used to eliminate the divergences which normally appear in general theory of relativity. It is supposed to be intrinsic property of spacetime and hence, does not depend on any particular feature such as curvature.

Although general relativity retains the usual commutative form, the noncommutative geometry leads to a smearing of matter distributions which can be considered as due to the intrinsic uncertainty embodied in the coordinate commutator as

\[ [x_\alpha, x_\beta] = i \theta_{\alpha\beta}. \]

where \( \theta_{\alpha\beta} \) is an anti-symmetric matrix which determines the fundamental cell discretization of spacetime. The standard way of modelling the smearing effect is accomplished by using a Gaussian distribution of minimal length \( \sqrt{\theta} \) due to possible uncertainty. Hence, one can consider a mass \( M \), described by a \( \delta \)-function distribution for a static, spherically symmetric, Gaussian-smeread matter source, in \( D \)-dimensions [28, 29], as

\[ \rho_\theta(r) = \frac{\mu}{(4\pi\theta)^{(D-1)/2}} e^{-r^2/(4\theta)}, \]

where \( M \) is the total mass of the source, may be due to a diffused centralized object such as a wormhole [26] and the particle mass \( \mu \) is supposed to be diffused throughout a region of linear size \( \sqrt{\theta} \). Here, \( \theta \) is the noncommutative parameter which can be taken to be of a Planck length. Thus, the above equation plays the role of a matter source and the mass is smeared around the region \( \sqrt{\theta} \) instead of situated at a particular point.

One can find a number of studies inspired by noncommutative geometry which are available in the literature either under the background of general relativity [30–33] or under modified gravity theories [34–36]. The model of self-sustained traversable wormhole has been studied by Garattini and Lobo [30], whereas Rahaman et al. [31, 32] explored two new wormhole solutions inspired by noncommutative geometry taking two different energy density profile admitting conformal motion. Kuhfittig and Gladney [33] studied charged wormhole inspired by noncommutative geometry with low tidal forces. In another paper authors [34] have studied the wormhole solutions inspired by noncommutative geometry in the framework of \( f(T) \) gravity whereas Hassan et al. have [35] studied the wormhole solutions in teleparallel gravity inspired by noncommutative geometry. In a work Samir et al. [36] have investigated the existence of wormholes by introducing noncommutative geometry in terms of Gaussian and Lorentzian distributions in \( f(R, G) \) gravity. The present paper searches for a solution of five-dimensional (5D) Einstein–Gauss–Bonnet equations in the presence of a static, spherically symmetric Gaussian mass distribution to find a thin-shell wormhole solution inspired by the noncommutative geometry. The starting point is to solve the non-linear Einstein–Gauss–Bonnet equations for a static and spherically symmetric metric for a source as mention by Eq. (2).

The paper is organized as follows: In Sect. 2 we find a general solution which leads to the five-dimensional (5D) spherically symmetric static Einstein–Gauss–Bonnet equations for the source as mentioned by Eq. (2), which further allows us to compute some quantities, viz., the components of stress–energy tensors around the thin shell of a noncommutative inspired thin-shell wormhole in Sect. 3. In Sect. 4 we find the Equation of State (EOS) at the throat of the thin-shell wormhole. We investigate the motion of test particle near the throat to find the nature of wormhole in Sect. 5 whereas Sect. 6 deals with the total exotic matter content at the shell. We analyse the linearized stability of our model in Sect. 7 and finally, in Sect. 8 we pass some concluding remarks.
2 Black holes in non-commutative geometry inspired Einstein–Gauss–Bonnet gravity

The action of Einstein–Gauss–Bonnet gravity in D-dimension [37, 38] can be written as

$$ S = \frac{1}{2\kappa} \int d^D x \sqrt{-g} \left[ \mathcal{L} + \alpha L_{GB} \right] + S_m. $$  \hfill (3)

$S_m$ represents the action associated with matter and $\alpha$ is a coupling constant with dimension of (length)$^2$ which is positive in the heterotic string theory. For the case of general theory of relativity $\mathcal{L} = R$, the Ricci scalar and the second-order Gauss–Bonnet term $L_{GB}$ is given by

$$ L_{GB} = R_{\mu\nu\gamma\delta} R^{\mu\nu\gamma\delta} - 4 R_{\mu\nu} R^{\mu\nu} + R^2. $$  \hfill (4)

Throughout the paper we assume that $8\pi G = c = 1$ and the variation of the above action with respect to the metric $g_{\mu\nu}$ gives the Einstein–Gauss–Bonnet equations [39–41] as

$$ G^E_{\mu\nu} + \alpha G^G_{\mu\nu} = T_{\mu\nu}, $$  \hfill (5)

where $G^E_{\mu\nu}$ is the Einstein tensor, while $G^G_{\mu\nu}$ is explicitly given by [42, 43]

$$ G^G_{\mu\nu} = 2 \left[ -R_{\mu\nu\sigma\tau} R^{\sigma\tau} - 2 R_{\mu\nu\rho\sigma} R^{\rho\sigma} - 2 R_{\mu\rho\nu\sigma} R^{\rho\sigma} + R R_{\mu\nu} \right] - \frac{1}{2} L_{GB} g_{\mu\nu}. $$  \hfill (6)

Here, one can note that the divergence of Einstein–Gauss–Bonnet tensor $G^G_{\mu\nu}$ vanishes and the spherically symmetric vacuum solution [37, 38] for this theory is given by

$$ ds^2 = -f(r) dt^2 + \frac{dr^2}{f(r)} + r^2 d\Omega_D^2, $$  \hfill (7)

where $d\Omega_D^2$ is the line element on the D unit sphere, i.e.,

$$ d\Omega_D^2 = d\theta_1^2 + \sin^2 \theta_1 d\theta_2^2 + \ldots + \prod_{n=1}^{D-1} \sin^2 \theta_n d\theta_n^2 $$  \hfill (8)

and volume of the D unit sphere is given by

$$ \Omega_D = 2 \frac{\pi^{\frac{D-1}{2}}}{\Gamma\left(\frac{D-1}{2}\right)}. $$  \hfill (9)

As we have mentioned earlier that noncommutativity eliminates point like structure in favor of smeared objects in the flat spacetime. The effect of smearing is mathematically implemented with a Gaussian distribution of minimal width $\sqrt{\theta}$ [24]. Here, we find 5-dimensional (5D) black hole solution inspired by the noncommutative geometry in the Einstein–Gauss–Bonnet gravity. For the matter density $\rho$ mentioned before along with the condition $g_{00} = 1/g_{rr}$, two components of the diagonal stress–energy tensor read

$$ T^0_0 = T^r_r = \rho_0(r) = \frac{\mu}{16\pi^2 \theta^2} e^{-r^2/(4\theta)}. $$  \hfill (10)

The Bianchi identity $T^{ab}_b = 0$ [28] reads

$$ 0 = \partial_r T^r_r + \frac{1}{2} g^{00} [T^r_r - T^0_0] \partial_r g_{00} + \frac{1}{2} \sum_i g^{ij} \left[ T^r_i - T^i_r \right] \partial_r g_{ij} $$  \hfill (11)

and due to the condition $g^{ij} \partial_r g_{ij} = 2/r$, one can obtain the energy-momentum tensor in the five-dimensional (5D) spacetime as

$$ T^r_i = T^i_r = \rho_0(r), $$

$$ T^0_0 = T^\phi_\phi = \rho_0(r) + \frac{r}{3} \partial_r \rho_0(r). $$  \hfill (12)

Moreover, Eq. (5) for the matter source as given by Eq. (12), leads to a general solution [29]

$$ f_\pm(r) = 1 + \frac{r^2}{4\theta} \left[ 1 \pm \sqrt{1 + \frac{8\alpha \mu}{r^4 \pi^2} \gamma\left(\frac{2}{4\theta}\right)} \right], $$  \hfill (13)

where $\gamma\left(\frac{2}{4\theta}\right)$ is the lower incomplete Gamma function which is defined as

$$ \gamma\left(\frac{2}{4\theta}\right) \equiv \int_0^{\frac{2}{4\theta}} \mathrm{d} u \, e^{-u}. $$  \hfill (14)
The variation in the metric function $f(r)$ with respect to $r$ (km)

To proceed further, one can define the mass–energy $\mu(r)$ as

$$\mu(r) = \frac{2\mu}{\pi} r^2 \left(\frac{\alpha}{2} + \frac{\alpha}{4} \frac{M}{\sqrt{\theta}} \right),$$

and the total mass-energy ($M$) measured by asymptotic observer [44] is given by

$$M = \lim_{r \to \infty} \mu(r) = \frac{2\mu}{\pi},$$

and the solution as given by Eq. (13) reads

$$f(\pm) = 1 + \frac{r^2}{4\alpha} \left[ 1 \pm \sqrt{1 + \frac{8\alpha M}{r^4} \left(2 + \frac{r^2}{4\theta}\right)} \right],$$

Here, if one consider $M_1 = \frac{M}{\sqrt{\theta}}$, $\alpha_1 = \frac{\alpha}{\sqrt{\theta}}$, and $r_1 = \frac{r}{\sqrt{\theta}}$, then the metric function takes the form

$$f(r) = 1 + \frac{r^2}{4\alpha_1} \left(1 - \sqrt{\frac{8\alpha_1 M_1}{r_1^4} e^{-\frac{r_1^2}{4\alpha_1}} (r_1^2 + 1)} \right),$$

In Fig. 1 the function $f(r)$ has been plotted w.r.t. $r$ for different values of $\frac{M}{\sqrt{\theta}}$ and from the figure it is clear that the difference between the values of $f(r)$ at the inner horizon with that at outer horizon shows an increasing profile with the increase in $\frac{M}{\sqrt{\theta}}$ value. There also exists a critical value for $\frac{M}{\sqrt{\theta}}$ to form a black hole and it is revealed that there is no significant dependence on $\alpha$ at two horizons.

3 Thin-shell wormholes from black holes in non-commutative geometry inspired Einstein–Gauss–Bonnet gravity

The thin-shell wormhole that we are to construct can be done by copying the solutions of two regular black holes and the region can be written as (removing from each of the copies)

$$\Omega^\pm \equiv \{ r \leq a \mid a > r_h \},$$

where $a$ is a constant and $r_h$ is the event horizon of the black hole. As a result of the above operation, the timelike hypersurfaces can be expressed topologically,

$$d\Omega^\pm \equiv \{r = a \mid a > r_h \}.$$

The two timelike hypersurfaces located at $d\Omega^+ = d\Omega^- = d\Omega$ along with two asymptotically flat regions that are connected by a wormhole whose throat is at $d\Omega$. The intrinsic metric upon $d\Omega$ can be written as

$$ds^2 = -dr^2 + a^2(\tau) d\Omega^2,$$

where $\tau$ is the proper time on the junction shell and $a(\tau)$ is the radius of the throat. Using Lanczos equation [7, 8, 45–48] the intrinsic surface stress energy tensor can be obtained as

$$S_j^i = -\frac{1}{8\pi} (k_j^i - \delta_j^i k^i),$$
where \( k_{ij}^\pm = K_{ij}^+ - K_{ij}^- \) is the discontinuity in the second fundamental form. We can write the second fundamental \([49–54]\) form as
\[
K_{ij}^\pm = -n_\nu \left[ \frac{\partial^2 X^\nu}{\partial \xi^i \partial \xi^j} + \Gamma^{\nu}_{\alpha\beta} \frac{\partial X^\alpha}{\partial \xi^i} \frac{\partial X^\beta}{\partial \xi^j} \right],
\]
(23)
where \( n_\nu \) is the unit normal vector to \( d\Omega \) and \( \xi^i \) represents the intrinsic co-ordinates. The parametric equation at the hypersurface \( d\Omega \) is \( f(x^\mu(\xi^i)) = 0 \).

Using the above equation, the normal vector can be expressed as
\[
n^\pm_\mu = \pm \left[ g^{\alpha\beta} \frac{\partial F}{\partial X^\alpha} \frac{\partial F}{\partial X^\beta} \right]^{-\frac{1}{2}} \frac{\partial F}{\partial X^\mu},
\]
(24)
with \( n^\mu n_\mu = +1 \). \( k_{ij} \) can be written as \( k_{ij} = \text{diag}(k_{r1}^+, k_{r2}^+, \ldots, k_{rD}^+) \), as a result of spherical symmetry.

Therefore, the surface-energy tensor has a form as \( S^\mu_j = \text{diag}(-\sigma, p_{\theta_1}, p_{\theta_2}, \ldots, p_{\theta_D}) \), where \( \sigma \) is the surface energy density and \( p \) is the surface pressure. Now, from the Lanczos equation, we can get
\[
\sigma = -\frac{D}{4\pi a} \sqrt{f(a) + \hat{a}^2}
\]
(25)
\[
p = p_{\theta_1} = p_{\theta_2} \ldots = p_{\theta_D} = \frac{1}{8\pi} \frac{2\hat{a} + f'(a)}{\sqrt{f(a) + \hat{a}^2}} - \frac{D - 1}{D} \sigma,
\]
(26)
where the dots and primes denote the differentiation with respect to \( \tau \) and \( a \), respectively.

The conservation equations can be written as
\[
\frac{d}{d\tau} \left( \sigma a^D \right) + p \frac{d}{d\tau} (a^D) = 0
\]
(27)
or,
\[
\dot{\sigma} + D\frac{\dot{\hat{a}}}{\hat{a}} (p + \sigma) = 0,
\]
(28)
which are obeyed by \( \sigma \) and \( p \). For the static configuration having a radius \( a \), we take \( \dot{a} = 0 \) and \( \hat{a} = 0 \). All the calculations are done in five dimensions (5D), i.e., by putting \( D = 3 \). Now, Eq. (25) reduces to
\[
\sigma(a) = \frac{3}{4\pi a} \left[ \frac{4a}{\sqrt{4a^2 - 1}} \right] + 1.
\]
(29)
\[
\sigma_1 = -\frac{3}{8\pi a} \left[ \frac{a^2 - 3(a^2 + \theta)(a^2 + \alpha^2 - 1)}{a^2} \right],
\]
(30)
where \( a_1 = \frac{a}{\sqrt{\theta}} \) and Eq. (26) reduces to
\[
p(a) = \frac{1}{8\pi} \frac{f'(a)}{\sqrt{f(a)}} - \frac{2}{3} \sigma,
\]
(32)
\[
p(a) = \frac{\sqrt{8\alpha M - \frac{2\alpha M}{\alpha^2} \left( a^2 + 4a^2 + 3a^2 \right)} \left( a^2 + 4a^2 + 3a^2 \right)}{a^2} - \frac{a^2}{\alpha^2} \left[ \frac{8\alpha M - \frac{2\alpha M}{\alpha^2} \left( a^2 + 4a^2 + 3a^2 \right)}{a^2} \right].
\]
(33)
Fig. 2 The variation in $\sigma \,(km^{-2})$ with respect to $a \,(km)$

Fig. 3 Pressure $p \,(km^{-2})$ is plotted with respect to $a \,(km)$

Similarly, for $p_1 = \sqrt{\theta} p$ we have

$$p_1 = \frac{4e^{-\frac{a^2}{\theta}} \alpha_1 M_1 (a^2 + 4) - (16\alpha_1 M_1 + 3a_1^4) + (3a_1^4 + 8\alpha_1 a_1^2)}{2\alpha_1 M_1 \left(4e^{-\frac{a^2}{\theta}} (a_1^2+4)\right)} + 1,$$

$$+ \frac{8\pi \alpha_1 a_1^3}{\left(\sqrt{\frac{2\alpha_1 M_1 \left(4e^{-\frac{a^2}{\theta}} (a_1^2+4)\right)}{a_1^2\alpha_1} + 1}\right)} ,$$

(34)

where the function $f(a)$ is given by Eq. (18). The negative sign in the surface density itself implies the existence of exotic matter in the shell which automatically violates the null energy condition as well as the weak energy condition.

In Fig. 2 we have plotted the surface energy density with respect to throat radius $a$ for different values of $\frac{M}{\theta}$ and from the figure it is revealed that as we are increasing the throat radius, the negative values of $\sigma$ increases rapidly and reaches a maximum value and after that it slightly decreases and tends to a saturated value. Figure 3 shows the variation of surface pressure with respect to throat radius $a$ and it is clear from the figure that as we increase the throat radius, the value of $p$ is decreasing rapidly but after a certain point it has no dependence on $\frac{M}{\theta}$ value. Figure 4 represents the violation of the null energy condition which is necessary for the construction of wormhole.

4 Equation of state

The Equation of State (EOS) is given by

$$p(a) = \omega \, \sigma(a) .$$

(35)
Fig. 4 $\sigma p (\text{km}^{-2})$ is plotted with respect to $a$ (km)

\[ \frac{\sigma}{\theta} = 0.1 \]

\[ \frac{\sigma}{\theta} = 0.2 \]

\[ \frac{\sigma}{\theta} = 0.4 \]

Fig. 5 The variation of Equation of State (EOS) parameter with respect to $a$ (km)

Therefore, by virtue of Eqs. (31) and (34) the equation of state parameter $\omega$ can be explicitly written as

\[
\omega = \frac{e^{-\frac{3a^2}{\sqrt{8a^2 + 16aM + 8aM}}} \left( 3a^4 \left( \sqrt{a^3 - 2(\alpha^2 + 4) e^2 \alpha^2 \alpha M + 8aM} - 1 \right) + 8a^2 \sqrt{a^3 - 2(\alpha^2 + 4) e^2 \alpha^2 \alpha M + 8aM} - 16aM \right)}{3a^2 \left( \sqrt{a^3 - 2(\alpha^2 + 4) e^2 \alpha^2 \alpha M + 8aM} - 1 \right) - 4a}.
\]  

(36)

Figure 5 shows the variation of equation of state parameter with respect to throat radius for different values of $\frac{\sigma}{\theta}$ and it reveals that the negative values of $\omega$ decreases as we increase the throat radius. Also, there is no significant dependence on $\frac{\sigma}{\theta}$ values.

5 The gravitational field

To study the nature of the gravitational field of our wormhole we calculate the observer’s four acceleration which is given by

\[ A^\mu = u'^\mu, \]  

(37)

where

\[ u'^\mu = \frac{\partial x^\nu}{\partial \tau} = \left( \frac{1}{\sqrt{f(r)}}, 0, 0, 0 \right). \]  

(38)

Hence, the only non-zero component can be written as

\[ A^r = \frac{f'(r)}{2}. \]  

(39)
Fig. 6 Acceleration $A' (\text{km})^{-1}$ of the test particle with respect to $a \ (\text{km})$

or

$$A' = \frac{e^{-\frac{a^2}{4}} \alpha}{a} \left(4\theta^2 e^{\frac{a^2}{4}} \left(1 - \frac{8\alpha M}{a^2} \left(1 - \frac{1}{\theta^2} \left(\frac{4\alpha a^2}{a^2} + \frac{a^2}{\theta^2} \right) + 1 - 1 \right) \right) - \alpha M \right)}{16\alpha \theta^2 \left(4 - \frac{e^{-\frac{a^2}{4}} \alpha}{a} \left(\frac{2a_1 M_1}{a^2} \left(\frac{4 - \frac{a^2}{\theta^2} (a^2 + 4)}{a^2} + 1 - 1 \right) \right) - \alpha_1 M_1 \right)} + 1. \quad (40)$$

Now $A'_1 = \sqrt{\theta} A'$, therefore,

$$A'_1 = \frac{e^{-\frac{a^2}{4}} \alpha}{a} \left(4\theta^2 e^{\frac{a^2}{4}} \left(1 - \frac{8\alpha M}{a^2} \left(1 - \frac{1}{\theta^2} \left(\frac{4\alpha a^2}{a^2} + \frac{a^2}{\theta^2} \right) + 1 - 1 \right) \right) - \alpha M \right)}{16\alpha \theta^2 \left(4 - \frac{e^{-\frac{a^2}{4}} \alpha}{a} \left(\frac{2a_1 M_1}{a^2} \left(\frac{4 - \frac{a^2}{\theta^2} (a^2 + 4)}{a^2} + 1 - 1 \right) \right) - \alpha_1 M_1 \right)} + 1. \quad (41)$$

For a test particle moving towards the radial direction from rest the geodesic equation can be written as

$$\frac{d^2 r}{d\tau^2} = -\Gamma'' \left(\frac{dr}{d\tau}\right)^2 = -A'. \quad (42)$$

The nature of the wormhole is attractive if $A' > 0$ and repulsive if $A' < 0$ depending on the parameters of equation. We have plotted $A'$ with respect to throat radius in Fig. 6 and it shows that the model of wormhole constructed is attractive in nature under the noncommutative geometry inspired Einstein–Gauss–Bonnet gravity. Also, the attractive nature is decreasing with the increase of throat radius.

6 The total amount of exotic matter

We can determine the total amount of exotic matter in a thin-shell wormhole by using the integral

$$\Omega_\sigma = \int [\sigma + p] \sqrt{-g} d^{D+1} x. \quad (43)$$

A new radial co-ordinate $R = \pm(r - a)$ has been introduced by Eiroa and Simeone [9] as

$$\Omega_\sigma = \int_0^{2\pi} \int_0^\pi \cdots \int_0^\pi \int_{-\infty}^{\infty} [p + \rho] \sqrt{-g} d\rho d\theta_1 d\theta_2 \cdots d\theta_D. \quad (44)$$
Fig. 7 The variation of the total amount of exotic matter $\Omega$ at the shell with respect to $a$ (km)

Fig. 8 Three-dimensional plot of total exotic matter $\Omega$ with respect to total mass $M$ as well as $\alpha$

Since, the radius of the shell is constant it does not exert any radial pressure, so that we have $\rho = \delta(R)\sigma(a)$ and one can write

$$\Omega = \int_0^{2\pi} \int_0^\pi \int_0^{\pi} \rho \sqrt{-g} |_{r=a} d\theta_1 d\theta_2 \ldots d\theta_D = 2a^D \sigma(a) \frac{\pi^{D+1}}{\Gamma\left(\frac{D+1}{2}\right)},$$

and for five dimension (5D) we get,

$$\Omega = \frac{\alpha}{\Omega_1} \sqrt{\frac{8\alpha M\left(1 - e^{-\frac{2}{\Omega_1}} \frac{(a^2 + \alpha_1^4)}{\alpha_1^4}\right)}{4\alpha} + 1} + 1.$$  

(46)

$$\Omega_1 = \frac{3}{2} \pi a^2 \sqrt{\frac{8\alpha M_1 - 2\alpha_1 M_1 e^{-\frac{2}{\alpha_1^2}} (a_1^2 + \alpha_1^4)}{\alpha_1^4} - 1}.$$  

(47)

Figure 7 shows the variation of total exotic matter content with respect to throat radius and from figure we can assert that as we increase the value of throat radius the amount of exotic matter at the shell increases rapidly. So, to minimize the amount of exotic matter content we have to consider the throat of the wormhole near the event horizon.
Fig. 9 Plots of $\beta^2$ as function of $a_0$ for $a = 0.1$

7 Linearized stability

To obtain the stability of the thin-shell wormhole one has to perform a small perturbation around a static solution at $a = a_0$. One can rearrange the Eq. (25) to obtain the equation of motion at the thin-shell region as

$$\dot{a}^2 + V(a) = 0,$$

where the potential can be defined as

$$V(a) = f(a) - \frac{16\pi^2 a^2 \sigma^2(a)}{D^2}.$$  \hspace{1cm} (49)

Now, expanding the potential $V(a)$ into a Taylor series around $a_0$ up to second order we have

$$V = V(a_0) + V'(a_0)(a - a_0) + \frac{1}{2} V''(a_0)(a - a_0)^2 + O[(a - a_0)^3],$$

where the prime (') represents the derivative with respect to $a$.

As we are linearizing around a static solution at $a = a_0$, we have $V(a_0) = 0$ and $V'(a_0) = 0$ and the stable equilibrium configurations correspond to the condition $V''(a_0) > 0$. Here, one can introduce a the parameter $\beta^2(\sigma)$ as the speed of sound which is given by

$$\beta^2(\sigma) = \frac{\partial p}{\partial \sigma} = \frac{p'}{\sigma'},$$

\hspace{1cm} (51)
Plots of $\beta^2$ as function of $a_0$ for $\alpha = 0.2$

The first- and second-order derivatives of $V(a)$ is found to be

$$V'(a) = f'(a) - \frac{32\pi^2 a \sigma}{D^2}[\sigma - D(\sigma + p)]$$

and

$$V''(a) = f''(a) - \frac{32\pi^2}{D^2} \left[\left((D - 1)\sigma + Dp\right)^2 - D\sigma(\sigma + p)(D - 1 + D\beta^2)\right].$$

Here, $V(a_0) = 0$ and $V'(a_0) = 0$ and for a stable wormhole solution one must have $V''(a_0) > 0$, i.e.,

$$\beta^2 \leq \frac{D^2 f''(a_0) - 32\pi^2 (D(p + \sigma)(p + 2\sigma) - 3D\sigma(p + \sigma) + \sigma^2)}{32\pi^2 D^2 \sigma(p + \sigma)}.$$  \tag{54}

Using Eqs. (30) and (33) the above inequality takes the form

$$\beta^2 \leq \frac{a_0^2 (f_0^2) - 2a_0 f(a_0) f_0^2 + (D - 1)f_0^2 + 4(D - 1)f_0^2}{2Df_0(2f_0 - a_0)f_0^2},$$  \tag{55}

where $f_0 = f(a_0)$, $f_0' = f'(a_0)$ and $f_0'' = f''(a_0)$. Figures 9, 10 and 11 show the stability regions of the wormhole solutions. From all these figures one can infer that as one increases the $\frac{M_\theta}{\mathcal{F}}$ value the stability region is shifted from the higher value of throat radius.
to the lower value of throat radius \(a\). Also, there is no significant change in the nature of the stability region as one varies the value of \(\alpha\) which is evident from the figures.

8 Conclusion

In this paper we construct a new thin-shell wormhole from black holes in non-commutative geometry inspired Einstein–Gauss–Bonnet gravity. For that we find out the non-commutative geometry inspired black hole solution in Einstein–Gauss–Bonnet gravity. Then, the ‘Cut and Paste’ technique which was first introduced by Visser [5], is employed to construct the wormhole from the corresponding black hole space-time assumed to be asymptotically flat in nature.

For the above mentioned structural form of a thin-shell wormhole we have noted down several salient aspects of the solution as can be described below:

(1) **Pressure–density profile** The surface energy density with respect to throat radius \(a\) has been plotted in Fig. 2 for different values of \(\frac{M}{\theta}\) and the figure shows that the negative values of \(\sigma\) increases rapidly with the increase of throat radius and reaches a maximum value and after that it slightly decreases and tends to attain a saturated value. Figure 3 shows the variation of surface pressure with respect to throat radius \(a\) and reveals that as we increase the throat radius, the value of \(p\) is decreasing rapidly but...
after a certain point it has no dependence on $\frac{M}{\omega}$ value. Also, Fig. 4 represents the violation of the null energy condition which is a necessary for the construction of wormhole.

(2) **Equation of state** Figure 5 which shows the variation of equation of state parameter with respect to throat radius for different values of $\frac{\theta}{\omega}$ reveals that the negative values of $\omega$ decreases as we increase the throat radius. Also, there is no significant dependence on $\frac{\theta}{\omega}$ values.

(3) **The gravitational field** The acceleration of test particle $A'$ has been plotted with respect to throat radius $a$ in Fig. 6 which shows that the model of wormhole constructed is attractive in nature under the noncommutative geometry inspired Einstein—Gauss–Bonnet gravity as $A'>0$ throughout the entire range of $a$. Moreover, the attractive nature is decreasing with the increase of throat radius.

(4) **The total amount of exotic matter** Figure 8 shows the variation of total exotic matter content with respect to throat radius $a$ and one can assert that as one increases the value of throat radius the amount of exotic matter at the shell increases rapidly. So, to minimize the amount of exotic matter content one has to consider the throat of the wormhole near the event horizon.

(5) **Linearized stability** In Figs. 9, 10 and 11 we have depicted the stability regions of the wormhole solutions for different values of $\alpha$. From all these figures one can infer that as one increases the $\frac{M}{\theta}$ value the stability region is shifted from the higher value of throat radius to the lower value of throat radius $a$ and there is no significant change in the nature of the stability region as one varies the value of $\alpha$.

Finally, we can conclude that our model of thin-shell wormhole which is constructed from the ‘Cut and Paste’ technique under the noncommutative geometry inspired Einstein–Gauss–Bonnet Gravity is found to be plausible with reference to the other model of thin-shell wormhole available in literature.

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