Influence of regularization in image reconstruction in electrical impedance tomography

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Abstract. The purpose of the application of electrical impedance tomography is to obtain images of areas that are difficult to access inside of the chest and other parts of the human being. Today, this has been applied in other areas of engineering with a view to investigate phenomena for which it is difficult to obtain data for robust research. Electrical Impedance Tomography (EIT) of an inverse problem is nonlinear and ill-conditioned. This requires a careful theoretical approach and practice to get good images. To enhance the images, it is important to be sensitive to various parameters that influence the process of image reconstruction, such as the measured voltage and the current density injected into the electrodes. The impedance contact and current density are both high in a point electrode. To reduce this, a large electrode modeled with a Finite Element Method (FEM) is used. A reduced numbers iterations is found when large electrodes are used. When FEM models are used the performance of the electrical impedance tomography reconstruction algorithm can be improved.

1. Introduction
The electric impedance tomography (EIT) techniques used in images reconstruction are divided into two parts: first the forward problem obtains a measured voltage from injecting a set of standard currents and a known object’s internal conductivity (or resistivity) distribution. Through equations for electric potential and a Finite Element Method (FEM), we obtain the first estimate of the spatial variation of the conductivity within the section for those standard currents injected at the border. Then, we solve the inverse problem. The same patterns of currents are injected again into the border electrodes, using the finite element method (FEM) and the Newton-Raphson method to find the calculated voltages and compare them with the voltages measured with a desired tolerance.

EIT has numerous applications that can be categorized in three main fields: 1. Industrial. This application includes the image of fluid flow in pipelines, measures the distribution of fluid flow in vessels of mixtures and non-destructive testing, such as the arrest of cracks (Cheney et al, 1999; Dickin et al, 1996). 2. Geophysics. These applications are geophysical, measured through the holes and surfaces (Polydorides and Lionheart, 2002). 3. Medicine. EIT is used to monitor heart and lung functions, measures of brain function (Barber 1984, Barber and Brawn 1989), detection of hemorrhage, measurement of the digestive system, detection and classification of tumors in breast tissue and functional imaging of the chest (Eyugoblu et al, 1989 by Harris et al, 1992).

EIT has some limitations that should be circumvented in its adoption in routine medical diagnostics. Its major limitation is its low spatial resolution, noise and susceptibility to errors in the electrodes. In medical applications, there is a large diversity of image characteristics. EIT is unsuitable for the
anatomical images in the same manner as magnetic resonance imaging (MRI) or computed tomography (CT). However, EIT appears promising as a tool for clinical diagnosis. It has an advantage of being relatively inexpensive compared with CT and MRI. EIT is non-invasive, safe, small and easy to carry. It can be used for long periods with low energy cost and maintenance. It may be feasible in the use of continuous monitoring of air entering the lungs also avoiding risks to the patient. It can also be used in monitor gastric lavage or cerebral hemorrhage.

2. Tomography Applied in Industrial Process

The term tomography applied in industrial process (TAPI) refers to a wide range of visualization techniques that are non-invasive relatively new (since the late 80s) and still in development. The goal is to get the TAPI images of cross sections of dynamic industrial processes (Adler, 1996 and Adler, 2004; Asfaw and Adler, 2005). Tomography offers a new way of viewing the internal behavior of industrial processes. The image of the cross section produced by tomography provides an efficient process of information that can be used for viewing, monitoring, mathematical modeling and verification of possible controls. There are many forms of tomography systems such as electricity, ultrasound, radiation, magnetic resonance, microwave and optical.

The tomography methods are still under development. Some challenges remain to be focused on: improving the sensitivity of the spatial resolution of the measures, development of more precise methods of image reconstruction, improving the efficiency of data processing, and suitability designing safe and reliable electronic hardware for use in industrial environments (not just in the lab). EIT is an ill-conditioned problem and to finds stable results if the solution incorporates some form of basic knowledge in problem behavior. In this regard, several requirements must be met to obtain the accuracy of specific images and for direct generation of models that describe more precisely the trajectory made by photon tissue models and precisely recover the inverse images of the optical properties of the tissues (Klose, 2003). Using Newton's optimization system, through the Jacobian (which combines measures of data with optical properties), these algorithms are widely used because they are easy to implement and can easily be generalized for use as the basis for simple and complex image reconstruction. (Dehghani et al, 2003, Yalavarthy, 2007). Another alternative method to reduce the number of parameters includes the use of image reconstruction base (Paulsen 1996) and also the use of mixed algorithms adapted to the environment of specific interest to be refined to higher precision numbers.

Table1. Research applications of fluid.

| Applications         | Phases         | References                                                                 |
|----------------------|----------------|-----------------------------------------------------------------------------|
| Flow in ducts        | Solid–Liquid   | Steverson et al, 2006; Dai, 2004; Norman et al, 2005; Pullum et al, 2006. |
|                      | Gas–Liquid     | Dong et al, 2006; Wang et al, 2006.                                         |
|                      | Liquid–Liquid  | Henningson et al, 2006; Henningson et al, 2007.                              |
| Stirred tanks        | Liquid–Liquid  | Ricard et al, 2005; Kim et al, 2006.                                        |
|                      | Gas–Liquid     | Ricard et al, 2005; Stanley et al, 2005.                                    |
| Packed bed reactor   | Solid–Liquid   | Bolton et al, 2004; Vijayan et al, 2007; Fransolet et al, 2001; Toye et al, 2005; Fransolet et al, 2005. |
2.1. Multiphase Fluid

Industrial applications of industrial tomography with fluids are common for the different phases (liquid, gaseous or liquid particulates), and also for multiphase fluids (when different phases exist in the same fluid of an industrial process). These states are detected by different concentrations of each phase computed based on knowledge of the electrical conductivity of each phase in this case using the electric resistance tomography (ERT) which can be applied to various procedures involving the conductivity of fluids in several processes that involve conductive fluid in the continuous phase. Typical applications are the ERT to view pipes and multiphase fluids transmission in stirred tanks, for which there is commercial applications for a wide range of material types. Recent applications can be seen in Table 1 (Gigu`ere, R., et al., 2008).

3. Models

The model is characterized in two parts. The direct problem and inverse problem are presented below.

3.1. Direct Problem

If $\Omega$ is the area subject to the presence of an electric field (Cheney et al., 1999)

$$\nabla \varphi(x, \omega) \nabla u = 0$$  \hspace{1cm} (1)

Where $x$ is a point inside the region $\Omega$, and $u$ is the electric potential and $\varphi(x, \omega)$ is the admittivity, if electric currents are injected into the boundary surface $\partial\Omega$ then power flow density $J$ is

$$\nabla \varphi \frac{\partial u}{\partial \theta} = J \text{ } \text{em} \partial\Omega_l \quad l=1,2,...L \text{ (electrode number)}$$  \hspace{1cm} (2)

Equations (1) and (2) with the condition $J = 0$ condition (charge conservation) and $u = 0$ (reference voltage), which consists of a current pattern in electrodes is known as solid model. The total flow of current through the electrodes is equal to the integral of the current density, ie:

$$\oint_{\partial\Omega} \nabla \varphi \frac{\partial u}{\partial \theta} \text{d}s = V_i, \quad l = 1, 2... L$$  \hspace{1cm} (3)

The elementary matrix equation is (Vanegas, 2002):

$$[Y_c] = \int_{A} \frac{1}{A} [B]^t[D][B] \text{d}A$$  \hspace{1cm} (4)

Where $y_c$ is the local conductivity matrix, $t$ is the thickness, $B$ is the geometry matrix, $D$ is the properties matrix, $A$ is the area of each element. To calculate the total internal voltages, the global matrix is used.

$$[Y_c]^i[V_i] = [C_i], \quad i = 1, 2... n \text{ (node number)}$$  \hspace{1cm} (5)

Where $[Y]$ is the local conductivity matrix, and $[V_i]$ is the vector of voltage at the points of the finite element mesh. And, $[C_i]$ is the pattern of current applied to the electrodes.

$$[V_i] = [C_i]^t[Y_c^{-1}]$$  \hspace{1cm} (6)

which $[Y_c^{-1}]$ is the inverse global matrix. The conductivity is a current and voltage function. In the direct problem, a set of current patterns are injected into the region and a distribution of conductivity (impedance) and voltages on the electrodes are measured.

The next step is solve the inverse problem.

3.2 The Inverse Problem

The aim of the process of image reconstruction is to get the change in conductivity occurring at a given time interval. In the first direct problem, there is the distribution of conductivity, then the measured voltages and conductivities are used to verify the accuracy of the images generated by the inverse problem.
The answers must be found within the specified tolerance. If not, a new iteration is made until the results are within the desired limit. The inverse problem is not linear and ill-conditioned. To find a satisfactory solution using good algorithm convergence, in this case we have chosen the Newton-Raphson method for its simplicity of implementation and good convergence. In the classical formulation of the Newton-Raphson method, we seek a distribution of conductivity \( \sigma^* \) that minimizes the differences between the measured voltages \( [V_m] \) and calculated voltages \( [V_c] \) to reduce the error

\[
\{e\} = \{V_c\} - \{V_m\}
\]  

(7)

The finite element model for the direct problem

\[
[Y_c'(\sigma)] \times [Vc] = [C]
\]

(8)

Thus the difference between the voltages can be written as:

\[
\{e\} = \{(T)[Y_c'(\sigma)]^{-1}\} - \{V_m\}
\]

(9)

Where \( T \) is the matrix transformation for the equations dimensionally consistent with the voltages calculated in the electrodes,

\[
[V_m]_{hex} = T \{V_m\}_{hex}
\]

(10)

Thus, the equation can be expressed by \( \{f\} = 0 \) for each standard voltage injected. To find the set of equations using the Newton-Raphson method, and using the expanded Taylor series, i.e.

\[
f_i(\sigma + \delta \sigma) = f_i(\sigma) + \Sigma \frac{\partial f_j(\sigma)}{\partial \sigma_j} \delta \sigma_j + O(\delta \sigma^2)
\]

(11)

The terms \( \delta \sigma^2 \) and higher orders do not contribute significantly in making calculations and \( f(\sigma + \delta \sigma) = 0 \) we have a system of linear equations in \( \delta \sigma_j \)

\[
\sum_{j=1}^{N} J_{ij} \delta \sigma_j = -f_i(\sigma)
\]

(12)

Where

\[
J_{ij} = \frac{\partial f_i(\sigma)}{\partial \sigma_j}
\]

(13)

Equation (12) defines an iterative process to calculate the homogeneous conductivity \( \sigma^* \) for each element. At each iteration \( k \), a supposed conductivity \( \sigma_k \) is used in Equation (12) and allows you to update \( \sigma^*(Yorkey et al.1987 a, b) \).

\[
\sigma^{k+1}_i = \sigma^*_i + \delta \sigma_i \quad i = 1, 2 \ldots E
\]

(14)

considering the conductivity has a homogeneous value in each element of the mesh of the finite element method. When the change of the iteration produces \( \sigma^* \) less than the tolerance, the method converges. To find the matrix derived using a resource for finding the derivative of the matrix.

\[
H_{ij} = \frac{\partial (Y^{-1} c_j)}{\partial \sigma}
\]

(15)

Where \( c_j \) = the standard currents, \( Y \) is the matrix of conductivity.

With the relation,

\[
YY^{-1} = I
\]

(16)

we derive and find the relation:

\[
\frac{\partial Y^{-1}}{\partial \sigma} Y + Y^{-1} \frac{\partial Y}{\partial \sigma} = 0
\]

(16)

Multiplying both sides by \( Y^{-1} \) and simplifying:
Finally, substituting equation (17) in Equation (15) defining the sensitivity matrix

\[ H_{ij} = -Y^{-1} \frac{\partial Y}{\partial \sigma} Y C_j \]  

(18)

Where:

\[ \frac{\partial Y}{\partial \sigma} = \text{The derivative of the conductivity with respect to matrix conductivity of the elements}, \]

\[ H_{ij} = \text{the sensitivity matrix function on the elements, (i) and (j) the number of current standard,} \]

\[ Y^{-1} \text{ is the inverse of the global conductivity matrix,} \]

\[ C_j = \text{current standards and j = the number of current standard} \]

The next step is to find the contact model and standard current.

### 3.3 Electrodes Contact Model and current pattern

Besides seeking models to estimate the distribution of electrical impedance and the regularization techniques to improve the performance of the algorithms, in practice it is not enough just to model the interior of the regions which will estimate the parameters. What happens at the border becomes a more important element for the accuracy of the results. Many authors experimentally and theoretically analyze the configuration of the electrodes and applied current patterns. What is the number and size of the electrodes? Should the contact impedance and conductivity of the electrode be modeled?

Consider the area of domain under an electric field discussed in equation 1, with the same definitions of the variables, the admittivity \( \gamma(x, \omega) = \sigma(x, \omega) + i\omega e(x, \omega) \) where \( e \) is the electrical permittivity and \( \omega \) is the angular frequency of the applied current.

The two-dimensional model has some simplifications that do not have significantly influence. The Equation (2) together with the conservation of charge \( (J = 0) \) reference voltage provide a possible electrode model which is called Continuum Model. However, this model is inadequate, since the current density \( J \) is unknown. In practice, only the currents that are injected separately in each pair of electrodes (one takes the current in and the other moves it out) are known.

This density could be considered constant cm each electrode (Gap Model) but still be an inadequate model (Chen et al., 1989).

The important effects to be considered in the model are the discretization of the electrodes and their conductivity. This can be done as follows (Chen, et al. 1989 Samersalo E. et al., 1992): the total current passing through the electrode is equal to the current density through it as described by the Equation (3). where \( C_i \) is the current injected into the i-th electrode and \( \partial \theta \) its corresponding area. Furthermore, this model is null the current density between the electrodes.

The conventional way of modeling the high conductivity of the electrodes is imposing the condition that \( u = V_i \) where \( i = 1, 2 .. L \). This model is known as the Shunt Model.

To improve the modeling, considers the electrochemical effect that occurs in the contact area between the electrode and the contact region, which is a thin layer and has high resistance. The impedance \( z_1 \) of this layer is called the effective contact impedance or impedance surface. To consider this effect, add a potential variation and replace the relation \( u = V_i \) by

\[ u + z_1 Y \frac{\partial u}{\partial \theta} = V_i \quad l = 1, 2 \ldots \]  

(19)

resulting in the so-called Completed Model (Samersalo E. et al., 1992), whose unique solution has been demonstrated and is able to estimate experimental results with up to 1% accuracy.
The Completed Model is the set of equations (1), (2), (3) and (19) together with the conditions of conservation of charge \( \sum_{i=1}^{n} C_i = 0 \) and the definition of potential reference \( \sum_{i=1}^{n} V_i = 0 \). From the introduction of the first notions of visibility and sensitivity (Seager et al., 1984; Seager and Bates, 1985) an index of distinctness was defined (Gisser et al., 1987) from this introduction allowed for the quantitative measuring the ability of a standard current to distinguish between two values of conductivity. Gisser (1987) implements a set of currents that maximizes the distinguishability- Best Pattern (Isaacson, 1986, Valhkonen,1997) demonstrates the influence between the size and number of electrodes to distinguish between two values of conductivity.

3.4 The Model of Electrodes

The modeling of the contact impedance of the electrode-electrolyte interface has been worked out by several authors to obtain a more representative estimate of the distribution of resistivity. Along this lines, we implemented the electrodes model proposed by Hua in the paper: Finite Modeling of Skin-Contact Impedance in Electrical Impedance Tomography (Hua et al., 1993), which also represents the distribution of the electric field at the contact interface, it the dominant effect of the electrode contact impedance, such as discretization of contact impedance and shunt effect and edge. The model consists of a finite element mesh shown below:

![Figure 1- a) Simplified model of electrode skin contact interface, b) model equivalent resistances.](image)

submitted the following boundary conditions:

\[
\int_{S_{4-5-6}} \rho^{-1} \partial \phi / \partial n \, ds = \int_{S_{4-5-6}} J \, ds = C_i
\]

Assuming that the potential across electrode is equal to \( \phi_4 = \phi_5 = \phi_6 \). Using the finite element method (Logan, 1986) with rectangular elements I and II, resulting in:

\[
\begin{bmatrix}
\frac{1}{3a \rho} & \frac{a^2 + t^2}{2} & \frac{a^2 - t^2}{2} & 0 & 0 & 0 \\
\frac{a^2 - t^2}{2} & \frac{a^2 + t^2}{2} & \frac{a^2 - t^2}{2} & 0 & 0 & 0 \\
2(a^2 + t^2) & \frac{a^2 - t^2}{2} & \frac{a^2 + t^2}{2} & \frac{a^2 - t^2}{2} & 0 & 0 \\
\end{bmatrix}
\begin{bmatrix}
\phi_1 \\
\phi_2 \\
\phi_3 \\
\phi_4 \\
\phi_5 \\
\phi_6 \\
\end{bmatrix}
= \begin{bmatrix}
0 \\
0 \\
0 \\
\int_{S_4} J_{4 DL} ds \\
\int_{S_5} J_{5 DL} ds \\
\int_{S_6} J_{6 DL} ds \\
\end{bmatrix}
\]

Where \( a \) is the width of the element, \( t \) is the thickness of the interface in the \( z \) (thickness of the shell element), and \( \rho \) is the resistivity of the interpolation functions or shape of the elements containing geometric information. With the boundary conditions and considering that the thickness \( t \) of the interface is very small compared to half the width of the electrode, as \( t \) and \( \rho \) are unknown variables, a new variable \( \rho' = t \rho \) called impedance contact with dimension \( \Omega \, cm^2 \) is defined. Thus, dividing \( \rho' \) by the area, \( \Omega \) is obtained. While \( t \) is the order of 40\( \mu \)m, the value of the resistivity interface is high, therefore \( \rho' \) is in a moderate range. Hua arrives with some considerations at,
This model is incorporated into the matrix of the global finite element model and will be implemented in solving the inverse problem to estimate the unknown parameters like the impedances at the interface.

3.4 Regularization

The regularization method is to determine the approximate solution in a smooth and consistent way with the experimental observation data why they are mixed to a certain level of noise. The solution is adds smoother additional information, which transforms the ill-conditioned problem in a well-posed one. Among the techniques, there are the methods of Tikhonov regularization, reducing the effects of ill-conditioned systems (Bacrie Cohen et al., 1997), restoring the continuity of data solution (Cheney et al, 1990). The most widely referenced is the method of Tikhonov regularization or Tikhonov-Phillips method. It minimizes the expression of the form:

$$
\min \phi (\sigma) = \frac{1}{2} (V_m - \hat{V}^T (\sigma))' (V_m - \hat{V}^T (\sigma)) + \alpha \left( H^T H \sigma \right) = 0
$$

(23)

where $V_m$ is the vector, $s$, is the measured voltage value, $V_c$, is the initial conductivity and the operator is simulating the voltage distribution to the initial conductivity. If the ideal solution is sought iteratively, this is the distribution of conductivity that minimizes Equation (23).

3.5. Evaluation of Images

A set of image data with the ideal is possible to compare the image reconstruction algorithms. Figure 2 shows the CT reconstructed using generic iterative algorithms (GIA). The finite element method element model was used to reconstruct the images (Gigu`ere, R., et al, 2008).

![Figure 2 – Image reconstruction using generic algorithms (GIA) (Gigu`ere, R., et al, 2008).](image)

The reconstruction in Figure 2 was performed using the Tikhonov and Landweber method with heuristic values for adjustment. The Tikhonov method requires the selection of a regularization which
is heuristically found among those that reach the specified tolerance. The resulting images’ associated heuristic values are selected when the comparison with the reference images are deemed satisfactory. Figure 2 shows this selection based on reference images to compare the quality of images reconstructed with the algorithms listed. Wheeler et al (2002), review several figures for use as a benchmark from those which have been proposed in the literature. Another way to compare is to do experimenting with objects of known capacitance and compare the images obtained with the figures of merit (Adler, 1996; Adler, 2004) to find the most suitable option.

3.6 Analysis of Results

The impedance problem is to find a distribution of conductivity $S_1$ that minimizes the difference found between the voltages calculated using the finite element method and the voltages measured at the border and can be formulated as follows:

$$\{e\} = \{V_e\} - \{V_m\}$$

$$\min \phi_1 = \frac{1}{2} (\overline{V_m} - \overline{V}_e(\sigma))^T (\overline{V} - \overline{V}_e(\sigma))$$

$$\min \phi_2 = \frac{1}{2} \alpha (\sigma^T H^T H) \sigma^T$$

$$\min \phi_2 = \frac{1}{2} \alpha (\sigma^T H^T H) \sigma^T$$

Figure 3-a) Iteration 38 with step: 0.04 and factor adjustment: $\lambda = 1e^{-4}$.

b) From approximately 50th iteration there are no great advances in the iterative process.

$\overline{V}_e$ and $V_e$ are the calculated and measured voltages respectively. The index error $(e)$ is obtained by solving the nonlinear system in the electrical conductivity through the Newton-Raphson method. In Figure 3, an example of the iterative process is shown, we can see that the Figure 38(a) shows an image with conditions to be adopted. In Figure 3(b), from the approximately 50th iteration there are no great advances in the iterative process. In Figure 3(b), a graph with the performance of the iterative process is shown with the adjustment factor $(1e^{-4})$, phi$_1$ is the first term of Equation (23) and is defined in equation (25). In each iteration, comparing the two terms of equation (23) may evaluate the effect of a greater or lesser regularization factor. In the present study, the iterative method of Newton-Raphson was used.

The capacitances obtained were tested to ensure that an adequate reconstruction of the original image maintained. The high values of factor adjustment influencing the image can produce solutions that reduce the data scale to maximum. The reducing adjustment factor scale preserves as much as possible the image to obtain a proper reconstruction. This approach allows for a theoretical interpretation of the image reconstruction algorithm that takes into account the various sources of prior information, such as the magnitude of electronic equipment that can introduce noise and its spatial resolution obtained for the maximum number of electrodes used.
4. Conclusion
The objective of this study was to review the techniques of electrical impedance tomography, highlighting the relevant topic of the impedance of contact electrodes. The contact impedance is high and generates an interference in the readings of voltages and the conductivities of the elements near the electrodes. The control for this effect is accomplished through the use of the Hua model (1993). Their use contributes to the reduction of noise and artifacts in the image. Use the Hua model of electrode (1993) contributes to model the impedance contact to get more accurate conductivity and voltages. The inclusion of this type of electrode improves the results, contributing to find a more appropriate distribution of conductivity images. The Newton-Raphson method used in the solution of this system with nonlinear behavior is quite effective. The method was used in its classical formulation with convergence in a small number of iterations.

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