A meshless thermomechanical travelling-slice model of continuous casting of steel

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Abstract. The travelling slice approximation of continuous casting process is widely used in industrial practice to model the heat transfer and solidification. Short computational times that are possible due to the reduction from three to two dimensions compensate for lower accuracy introduced by the approximation. In the same spirit, the travelling slice approximation is also used for the thermomechanical model in the present paper. The model includes contributions from metallostatic pressure, thermal contraction and viscoplastic deformation to describe the stress in the material slice. It incorporates simple models to predict the hot tearing and cracking. The model is solved with a novel strong-form meshless method. It allows flexible adaptive node distribution to accurately describe the behaviour in the solidifying region. In this paper we focus on the two-way coupling of the heat transfer and deformation of the strand to model the air-gap formation in the initial stage of the casting process and to study the emergent behaviour that is the result of this coupling. We find that the inclusion of the air-gap model significantly changes the stress distribution in the corners of the strand because of the reheating caused by the reduced heat-transfer.

1. Introduction
The travelling slice models of continuous casting are widely used in casting practice by both plant operators and caster designers to model the heat transfer during the casting process and to estimate the influence of casting parameters on the position of the solid-liquid interface [1, 2]. The models are also used in optimization of the caster operation, based on productivity, quality, safety, maintenance and resources requirements and can serve also for caster on-line control [3]. The travelling slice model follows a slice of the strand through the caster. Such simplification is possible because of the slow changes in temperature field along the casting direction as compared to the variation in the orthogonal direction. This results in the model where the only mechanism for transfer of heat along the casting direction is the movement of the material slice. Although the travelling slice model introduces significant simplification by reducing the transport mechanisms to only diffusion/forces in perpendicular direction of the casting and to only advection in the direction of the casting. This provides much shorter computational times that offset the reduced accuracy of the physical description. The resulting models have thus achieved widespread use in industry.

The same idea is in the present paper applied also to the mechanical equilibrium in the shell. In the areas where most of the loading is introduced, either through thermal contraction of the material or through the action of the metallostatic pressure on the inside surface of the shell, the problem can be
reduced to finding the stress equilibrium of a two-dimensional slice. Such conditions are encountered in the mould and several very informative papers have been produced on the topic [4–6], with special focus on the air-gap formation in the mould.

In contrast with the established way to solve the stress equilibrium problem with the finite element method, the present paper uses an original strong-form meshless method to discretize the governing equations. This method has been in the past used to solve the problems in thermoelasticity [7, 8] and plasticity [9] and has been applied to develop a model of hot-tearing during DC casting of aluminium [10] and a model of hot-rolling of steel [11]. In the considered problems it matches the performance of the finite element method and outperforms other meshless methods, such as the meshless local Petrov-Galerkin method [8]. The meshless nature of the method simplifies the adaptive node refinement that follows the position of the mushy zone by removing the need to remesh the computational domain. Similar strong-form meshless methods have been successfully applied to several other aspects of casting process and solidification. The thermal and the mechanical models are introduced, and the inter-model coupling is discussed first, followed by a short description of the numerical method. The paper is concluded by the presentation of the results and the discussion of the emergent behaviour of the solidified shell.

2. Model formulation

2.1. Heat transfer

The heat transfer model tackles the solidification problem in mixture enthalpy formulation. The relation between the enthalpy and the temperature, which guides the solidification, is obtained from JMatPro® [12] and is specific to the steel composition under consideration. The heat removal is described via specified heat flux at the surface. The calculation of the heat flux is performed with the resistor model [4], which relates the heat flux to the difference in temperature between the shell wall and the cooling water. Their relation considers the presence of the air-gap, the amount of radiative heat transfer and the conductivity of the mould. The heat transfer model also includes the influence of spray cooling and heat transfer to support rolls in the later stages of simulation. The details of the model can be found in [2].

2.2. Mechanical equilibrium

The mechanical model is stated in small-deformation approximation and considers additive decomposition of the strain into elastic, thermal and viscoplastic parts. The thermal part is determined by

$$
\varepsilon' = I \int_{T_0}^{T} \alpha(T)dT,
$$

where $\alpha(T)$ is the temperature-dependent coefficient of thermal expansion obtained from [12]. The viscoplastic part is given by the Garafalo law

$$
\varepsilon'' = A_\sigma \exp\left(-q / T \right) \left( \frac{\sigma}{\sigma_0} \right)^n
$$

(2)

The parameters $A_\sigma$, $q$, $n$ and $\sigma_0$ are determined from the least squares fit of the high-temperature flow-stress data obtained from JMatPro.

The influence of the metallostatic pressure is introduced through an additional body force term given by $f_b = p_m \nabla s$, where $p_m$ is the metallostatic pressure at the centre of the slice and $s$ is the liquid fraction. The resulting plane strain equilibrium equation for the displacement field $\mathbf{u}$ is

$$
-p_m \nabla s = G \nabla \mathbf{u} + (G + \lambda) \nabla (\nabla \cdot \mathbf{u}) + \nabla \lambda \nabla \cdot \mathbf{u} + \nabla G \left( \nabla \mathbf{u} + (\nabla \mathbf{u})^T \right) - \nabla \cdot \left( G \varepsilon'' \right) - \nabla \cdot \left( (3\lambda + 2G) \varepsilon' \right),
$$

(3)

where $\lambda$ and $G$ are the Lamé parameters.
The influence of the mushy zone is considered in several ways. The influence of metallostatic pressure is introduced by a two-phase analysis of stress in the mushy zone. The changing material properties in the mushy zone are introduced through the temperature dependence, obtained from [12]. All the terms are updated during the simulation with the information from the thermal model.

The boundary conditions for the solid mechanics model are either free-slip, on the parts of the boundary where the shell is determined to be in contact with the mould, or traction-free, on the parts where there exists an air gap between the surface of the shell and the position of the mould.

2.3. Coupling of the models
A two-way coupling is provided between the thermal and the mechanical models. The coupling of the mechanical model is performed explicitly through the temperature field, which determines the thermal deformations, and implicitly, through the fraction of solid, which determines the term describing the influence of the metallostatic pressure. The material parameters related to the elasticity of the mechanical model are temperature dependent.

The coupling of the thermal model with the solid mechanics model is performed through the boundary conditions for the heat transfer to the mould via the calculated air-gap heat flux.

3. The solution procedure
The two components of the model are discretized separately, using the same family of strong-form meshless methods, based on local interpolation with radial basis functions. The method is described in more detail in [2] for the thermal model and in [7] for the solid mechanics model. The basic idea of the method is to first introduce a node arrangement that describes the interior and the boundary of the computational domain. A subdomain consisting of neighbouring nodes of each node in the node arrangement is then selected to construct a local interpolation with radial basis functions, which is then used to calculate the derivatives that are present in the governing equations. Such approach is very flexible with respect to the node positions, dimensionality flexible and highly accurate [13].

In this paper, the interpolation with fifth-order polyharmonic splines, augmented with a second order polynomial, is used over local subdomains, consisting of 13 nodes.

The discretization in time is explicit for the thermal model and implicit for the solid mechanics model. The two models function in lock-step. First, the thermal field is updated and the updated thermal field is used to recalculate the material properties and the thermal loading of the mechanical model. Then the stress equilibrium is calculated with the updated data. After the deformations of the surface are known, they are used to update the heat transfer to the mould. The coupling procedure has no iterations within the time step to converge on either the air-gap size or the temperature field, since the time step \( \Delta t = 2 \times 10^{-3} \) s is limited by the stability of the heat transfer, and is small enough to facilitate a frequent information transfer between the models.

The node arrangement, on which the solid mechanics calculations are performed, is non-uniform and is periodically updated to remain refined near the solidification front. The node arrangement at the beginning of the simulation and at the exit from the mould is shown in figures 1(a) and 1(b), respectively. The node spacing varies from 0.5 mm in the mushy zone to 1 mm in the solid shell and 5 mm in the liquid part of the computational domain. The node arrangement of the thermal model is regular square grid with spacing of 1.84 mm and remains unchanged throughout the simulation.
4. Results

The results presented in this paper are given for continuous casting of a square 184 mm billet of high-strength steel grade 51CrV4. The casting temperature is 1529 °C and the withdrawal rate is 1.44 m/min. The mould geometry is simplified to have a square shape with a taper rate of $3 \times 10^{-4}$, resulting in the dimensions of the mould, changing from 184.3 mm at the top (at vertical position 0 m) to 184 mm at the bottom (at vertical position -1 m). The presentation of the results is split into a set that discusses the influences on the surface of the billet and a set that discusses the cross-section of the billet.

The first set of the results, given in figures 2, 3 and 4, is used to compare the conditions on the surface for the simulations that consider the air-gap and the simulations without the air-gap. The variables of interest are plotted on unwrapped surface of the billet. A cut is performed along a corner and the billet surface is then unwrapped, displaying periodic structure of the plotted values caused by the very high symmetry being present in the solution. In figure 2, the calculated air-gap size is shown. There are two observations that can be made about the results. Firstly, we see that the gap, calculated without the coupling, is larger by approximately a factor of two. Secondly, the evolution of the gap is different. In the uncoupled case, the gap starts to form and reaches certain steady state value that remains essentially the same throughout the mould. The results of the coupled model show a contrasting behaviour. The air gap is formed and increases in the first 100 mm of the simulation and is then slightly reduced in the rest of the simulation.

![Figure 1](image1.png)

**Figure 1.** Illustration of the node arrangement at (a) the start of the simulation, and (b) at the exit from the mould.

![Figure 2](image2.png)

**Figure 2.** The size of the air gap in the mould. (a) The results of the model for the case with air-gap dependent heat transfer model and (b) without the air-gap dependence.
The temperature field on the surface is shown in figure 3. Again, the effect of the inclusion of the coupling is evident. In the case without the coupling, illustrated in the right plot, we can see that the corners of the billet get significantly colder than the centre of the billet and remains so throughout the simulation. The case that includes the coupling, shows a more dynamic behaviour of the corner temperature. At the beginning, before the air gap is formed, the corners are undercooled, but they reheat, after the air-gap is formed. They remain warmer than the centre of the face at the exit of the mould.

![Figure 3](a) Temperature at the surface. (a) The results of the model for the case with air-gap dependent heat transfer model and (b) results without air-gap dependence.

The plots in figure 4 show the heat flux on the surface of the billet. In the plot on the right, the heat transfer only depends on the surface temperature, since the effect of the air-gap is neglected. The plot on the left shows the behaviour of the heat flux that includes the effect of the air-gap. The heat flux is reduced by a factor of 10 in the corners when the air-gap is formed, which results in re-heating of the corners of the billet.

Overall, the results are qualitatively the same as the ones presented in relevant literature [4–6] and they match the observations and experience of the industrial partner. Further quantitative evaluations will follow in the future.

![Figure 4](a) Heat flux at the surface. (a) The results of the model for the case with the air-gap dependent heat transfer model and (b) results without air-gap dependence.
4.1. The interior of the billet
Figures 5 and 6 show the temperature and the equivalent stress over the slice, respectively. These plots are arranged in two rows: the top row displays the state of the coupled model and the bottom row the state of the uncoupled model. The plots in each column are given at specific position of the slice. The first column gives plots at \( z = -0.17 \) m, the second at \( z = -0.41 \) m, and the third at the exit from the mould at \( z = -1 \) m. In the plots, the shape of the slice is deformed by the displacement vector, scaled by a factor of 20, to give an idea about the shape of the gap at the specific positions in the mould.

The plots in figure 5 show the temperature field in the slice. We notice the fact that the re-heating of the corners in the case of coupled model causes slice to have a very gentle temperature profile in the corners, while the sharp temperature profile remains present at the parts of the surface that are in contact with the mould.

This change in the temperature field is reflected on the stress state that is illustrated by the equivalent stress in figure 5. It is noticed that when the air-gap forms in the mould, the increase in the temperature in the corners reduces the thermal strains in the corners, which are the most important contribution in the mechanical loading of the slice. This significantly reduces the equivalent stress that is present in the corners. Large equivalent stress remains present in the parts that remain in the contact with the mould.

![Temperature field](image)

**Figure 5.** Temperature field (in degrees Kelvin) of the cross-section of the billet. The top row shows the results of the coupled and the bottom row the results of the uncoupled model.
5. Conclusions

Implementation of a meshless mechanical travelling slice model for continuous casting of steel is presented. The model uses results of thermal travelling slice to calculate the thermal strains, which are the most important driving term for the mechanical model. The mechanical model is used to calculate the air gap between the mould and the solid shell. This information is used to influence the heat flux between the shell surface and the mould. A comparison of the results that incorporate this two-way coupling and the results that do not use it, shows that the coupling has a stabilising effect. The air gap of the coupled model is smaller than the air gap of the uncoupled model, moreover the equivalent stress in the corner is reduced while the temperature of the corners of the billet is increased. This illustrates that the coupling between the solid mechanics and the thermal conditions in the strand is very important and should be considered in the models. In the future, we will compare the predicted temperature of the shell at the exit of the mould with the measurements to validate the predicted behaviour and to fine-tune the material parameters, present in the resistor model of the heat transfer.

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