The uncertainty: A history in Mathematics

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Abstract. Naturally every phenomenon consists possible has the same concept but has a different character: certainty and uncertainty. Therefore, in mathematics continuously developed several paradigms to understand it. This paper historically reveals the development of an understanding of uncertainty based on certainty, i.e. a framework for seeking the solution.

1. Introduction

Many technology and science problems are subject to uncertainty due to the inherent randomness of natural phenomena and/or to the imperfect knowledge of the variables that determine the functional state of the human-created structures [1]. In universe of discourse (or just universe) \( U \), this is a concept containing all of the objects \( O \) referred to in a discourse or an argument [2, 3]. All terms \( w(s) \) that used to give meaning of the object, or it is knew as the universe of interpretation or domain of discourse.

In line with those meaning of uncertainty, an object in philosophy therefore is a thing, being or concept, where specially entity such as social actor is something that is tangible or intangible, it within the grasp of the senses [4]. In any paradigm [5], the existence of an object based on assumptions, foundations, and implications, as a basic concept in semantic to understand some problems about it whereby an object can be manipulated for example by the commands of a programming language such as a value, variable, function, data structure, or any system [6]. Mathematically, the symbols represent objects, and that is the basic problem semantic also such as an human ontology [7]. In an ontology [8], a root of any formal concept is a set theory and a logic. Fundamentally, the well-defined objects gave the meaning for \( o_1 \) and \( o_2 \) are two different objects that essentially \( o_1 \neq o_2 \ \forall o_1, o_2 \in O \) and an object is basic unit in \( O \) defined to be an item based on an unique item indexed by \( \{1, \ldots, n\} \) where \( o_i = 1 \) if \( i \in I \), and \( o_i = 0 \) otherwise [9]. In other words, logically \( o \) is in \( O \) if it implies \( o \) [10]. A set of integers, \( I \), is a well-defined set to. Indices in here are the information about an object like other kinds of information [11, 12]. Objects are basic units in discrete, but also we understand them in continuous. Therefore, types of objects in the real world are often found to be filled with uncertainty and/or partial truth [13, 14, 15]. This paper proposes a framework to understand the phenomena as a history of mathematic.

2. Review: Toward History

When did a term "set" is defined formally? A question is not yet has an answer. However, at end of the 19th century, Georg Cantor, the founder of set theory [16], gave the following
definition of a set as the beginning of this theory, i.e. “A set is a gathering together into a whole of definite, distinct objects of our perception and of our thought - which are called elements of the set”\(^1\) [17]. The set theory is one of ways for formalizing the concepts, where the notation for specifying set through the concepts of membership and predicator [12]. Conceptually, discussion about the set theory has been proven long enough reviewed where long before, the algebra has been already developed by Muhammad ibn Musa al-Khwarizmi which is generally based on the concept of the set, i.e. kind of abstraction that explained as variables [14, 15]. Therefore, in the world there are some meaning of phenomena in parallel, i.e. *immortality-infinity, change-probability*, and *quantity-number* are in pair such as set theory and logic in parallel in philosophy and mathematics [18].

Axiomatic set theory was developed in 1908 by Ernst Zermelo is to understand simply some problems using the diagrams\(^2\) logically, where basic axiom of set theory stating that for all family \((S_i)_{i \in I}\) of nonempty set there exists a family \((x_i)_{i \in I}\) of elements such that \(\exists x_i \in S_i \forall i \in I\) [19, 20] and then all statements are expressed by symbolism of first order logic (we know it as predicate calculus) by Kenneth Kunen [21] and we call the statements as axioms. However, all of these concepts has been limited to the consideration of atomic statements in rules [22] that cannot be adapted to another one. It also cannot use for dealing with the change on the world where any object exist.

In mathematics and formalization in philosophy, the logical reasoning is the essence of the defining paradigms and for understanding the phenomena in the world [23]. Let \(E\) is a space of event from world, where there are events \(\varepsilon_i, i = 1, 2, \ldots, n\). In classical logic, an inference associated with logical implication to define basically something it is exist[24].

**Definition 1.** An event \(\varepsilon\) is relevant to an object \(o\) if it implies the object, i.e. if the object can be inferred from the event, or

\[
\sigma(\varepsilon \Rightarrow o) = \text{TRUE}
\]

(read : "if \(\varepsilon\) then \(o\)" is true).

For identifying any object, a well-known paradigm of relevance always assigns the vectors for explaining the object in discrete or in continuous. This is to capture the differences of immortality-infinity and quantity-number. In statistics, we always explored the basic measures like average, median, mode, standard deviation, range, etc., to be the vectors for giving the meaning of object in a behaviour [25], for example to find the spreading out randomly through a variance. In optimization, we frequently used a concept of comparing between the vectors for taking a meaning of object in usefulness [26]. However, for understanding about object is found in ever-growing and ever-changing world, the models of the classical statistic and optimization are not convenient for applying it.

### 3. Methodology: A framework based on history

The methodology is a history of innovation [27], and be a history of research on the world [28, 29, 30]. As a history of innovation, the fuzzy set have been widely applied in the fields of knowledge discovery, patterns recognition, artificial intelligence (AI), control systems, etc. [31]. We have already seen some developments of theory in mathematics [32] like fuzzy group [33, 34], fuzzy topology [35], fuzzy calculus [36], or trends in statistics [37] for example fuzzy regression [38] and Bayes methods [39]. Rough set is a relatively new mathematical and AI technique, particularly useful for discovering relationship in data, typically to formalize an information

\(^1\) https://en.wikipedia.org/wiki/Set\(_\text{m}\)\(\text{athematics}\)

\(^2\) Venn diagram
system [40], i.e. a technique that can compared to statistical analysis, particularly discriminant analysis, and machine learning in classical AI.

The new information about uncertainty, the combination of fuzzy sets and rough sets is very significant, and then rough fuzzy sets has been introduced by D. Dubois and H. Prade [41], but it needs more depth research done. However, as a basic of research, in formal we need definition, and then proof of relationship in cause and effect in lemma, proposition, or theorem, and conclusion in a corollary [42, 43]. All of those should be reasonably proven in a logical arrangement of statements or involving algebra in a more straightforward manner. Therefore, a framework for understanding the uncertainty is (i) by constructing the model so that it can be mathematically proven by the derived concept [44]; (ii) by testing the data in accordance with the concept of uncertainty built; (iii) by determining the relationship between the objects associated with the data or between objects and events that reduce subjective impacts of meaning [45].

4. Discussion: Uncertain object in a history space

In mathematics, to be more important that objects be definable in some uniform way. Using a modal of logic, we define the certainty as follows.

**Definition 2.** An object \( o \in O \) is a certainty if there is a vector space \( \Sigma \) be a part of the space of event \( \mathcal{E} \) where \( \sigma(\varepsilon \Rightarrow o) \) is true, i.e.

\[
\Sigma_o = \begin{cases} 
1 & \text{if } o \text{ is true at } \varepsilon \in \mathcal{E} \\
0 & \text{otherwise,}
\end{cases}
\]

such that for all \( \sigma \in \Sigma, \delta(\Sigma) = R \) exist, where \( R \) is a set of real number.

In fact, the objects in the collection could and often do contradict each other in any particular event, and not all the necessary knowledge is available (but be random) [46]. Therefore, the relationships between \( \varepsilon \) and \( o \) in logic is uncertain [47, 48, 49, 50] and degree of uncertainty measured by [51]

\[
P : \sigma(\varepsilon \Rightarrow o) \rightarrow [0, 1]
\]

Thus, the vectors can be generated in probability [52]. This mean that in determining the relevance of an event to an object, the success or failure of an implication relating the true or false values is not enough.

Let us make the scenarios for understanding the uncertainty based on Eq. (2). Considering the binary variable

\[
u_{it} = \begin{cases} 
1 & \text{if generating unit } i \text{ is available in hour } t \\
0 & \text{otherwise.}
\end{cases}
\]

The collection of random variables \( U_i = \{u_{it}|t = 1, 2, \ldots, 24\} \) representing the availability of unit \( i \) throughout a day is a discrete stochastic process. Or, if we denote the velocity of wind blows in time-range \( t \) at a given point \( A \) by \( v^A_t \), then the set of random variables \( V^A = \{v^A_t|t = 1, 2, \ldots, 24\} \) representing the wind blows in \( A \) openly throughout a day is a stochastic legal action of natural without cease [53]. Thus, the different approaches to object-wind and event-hour can generate different vectors, but these showed uncertainty character, as the occurrence theory.

In history, during the 20th century natural science underwent an enormous shift: mathematical interpretation for it was the driving force behind this shift ontologically [31]. There are scientists releasing the physics of the deterministic law language into something less certain as probabilities [54], but the determinism inhibits the true goal of research which is the discovery: If it was no longer possible to study the space of event (natural world) from a
mechanistic viewpoint then it was certainly no longer possible to study other space of event (social facts) in those ways [55]. Therefore, to deal with the problem of uncertainty, the theory of probability has been established and has been successfully applied to many areas of science [56]. However, in spite of its success, theory of probability is not well-full to do of understanding uncertainties in all phenomena [57]. Among the stable conditions of world, the flexible conditions exist in the environment of objects [58, 59, 60]. The probability theory cannot capture the uncertainties resulted from the vagueness of event spaces. New theory is of uncertainty and probability that leading to the tentative nature of discovery were adopted in natural and social phenomena with the concept of falsification introduced by Karl Popper [61].

In 1965, a fuzzy sets theory has been proposed by Zadeh [62] to deal with uncertainty problems, but this theory emphasizes on the differences among the membership degrees of different objects in the same set by taking the membership function as some subjective. The flexible conditions of world evoke the existing notion of indiscernibility [63] and inability to distinguish between objects, because after all the classification or clustering is done formally and mathematically, the application of the fuzzy set remains related to the subjective properties if the data is not tested correctly, and providing an approximation of sets or concepts by means of binary relations, especially that constructed from empirical data [64]. In real life, when dealing with the set, we often have no media of precisely for mark off individual object as different element from each other due to limited capable. So, to handle imperfect data, rough set theory is complements other more traditional theories such as probability theory, stochastic process or occurrence theory, and fuzzy set theory.

On the end of twentieth century, Z. Pawlak was introducing a concept, it called as rough set theory [65]. The rough sets theory is based on partitioning the universe, which deals with uncertain knowledge by the known knowledge, and it is objective. In philosophy the rough set is founded on the assumption that we associate some information with every object of the universe of discourse. However, as with the adequacy of the data as a sample of the population, the relation between disconnected objects caused by fuzzification will be re-explained by the rough set into a new form of classification or clustering. Therefore, need to understand the uncertainty by aligning it to the certainty, and through the modeling of complete uncertainty, taking into account all the parameters by which the appropriateness of approach is testable.

**Conjecture 1.** The uncertainty is the certainty.

Therefore, the rough sets and the fuzzy sets have a common ground.

### 5. Conclusion

Conceptually the problem associated with uncertainty is a problem carried naturally through the certainty. Therefore, formally the model becomes a bridge to overcome uncertainty and every process must be accompanied by testing, so it can be stated that uncertainty is certain.

### References

[1] Nasution M K M 2016 Social network mining (SNM): A definition of relation between the resources and SNA *International Journal on Advanced Science, Engineering and Information Technology* 6(6).

[2] Charles C 2006 Burgess’s ‘Scientific’ arguments for the existence of mathematical objects *Philosophia Mathematica* 14.

[3] Nasution M K M 2014 New method for extracting keyword for the social actor *Lecture Notes in Computer Science (including subseries Lecture Notes in Artificial Intelligence and Lecture Notes in Bioinformatics)* 8397 LNAI(PART 1).

[4] Mahyuddin K M N, Sitompul O S, Nasution S and Ambarita H 2017 New similarity *IOP Conference Series: Materials Science and Engineering* 180(1).

[5] Dawson Jr J W 2006 Why do mathematicians re-prove theorems? *Philosophia Mathematica* 14.
[6] Ramer A and Padet C 1995 Information semantics of possibilistic uncertainty Fuzzy Sets and Systems 69.
[7] Nasution M K M, Sitompul O S, Sinulingga E P and Noah S A 2016 An extracted social network mining Proceedings of 2016 SAI Computing Conference, SAI 2016.
[8] Nasution M K M 2018 Ontologi Ontologi dan Taksonomi 1.
[9] Nasution M K M 2017 Modelling and simulation of search engine Journal of Physics: Conference Series 801(1).
[10] Nasution M K M, Hardi M and Sitepu R 2016 Using social networks to assess forensic of negative issues Proceedings of 2016 4th International Conference on Cyber and IT Service Management, CITSM 2016.
[11] Nasution M K M, Syah R and Elveny M 2017 Studies on behaviour of information to extract the meaning behind the behaviour Journal of Physics: Conference Series 801(1).
[12] Elfida M, Nasution M K M, and Sitompul O S 2018 Enhancing to method for extracting Social network by the relation existence IOP Conference Series: Materials Science and Engineering 300(1).
[13] Nasution M K M 2018 SumutSiana IOP Conference Series: Materials Science and Engineering 309(1).
[14] Nasution M K M and Sawaluddin 2018 Redefining the magic square on numerical characters IOP Conference Series: Materials Science and Engineering 308(1).
[15] Nasution M K M, Sitompul D, and Harahap M 2018 Modeling reliability measurement of interface on information system: Towards the forensic of rules IOP Conference Series: Materials Science and Engineering 308(1).
[16] Malzahn-Chaunier D 2008 Sets, Logic and Maths for Computing Heidelberg, Berlin: Springer.
[17] Cantor G 1895 Beiträge zur Begründung der transfiniten Mengenlehre Annalen Bd. 46.
[18] Nasution M K M 2005 Konsep penelitian dalam teknologi informasi Al-Khawarizmi - Journal of Computer Science 1.
[19] Zermelo E 1908 Untersuchungen über die Grundlagen der Mengenlehre I. Mathematische Annalen 65.
[20] Jean van Heijenoort 1967 Investigations in the foundations of set theory From Frege to Godel: A Source Book in Mathematical Logic.
[21] Kunen K 1980 Set Theory: An Introduction to Independence Proofs Elsevier.
[22] Day M 1977 An axiomatic approach to first law thermodynamics. Journal of Philosophical Logic 6.
[23] Priest G 2005 Towards Non-Being: The Logic and Mephostophiles of Intentionality Exford: Oxford University Press.
[24] Nasution M K M and Noah S A 2012 Information retrieval model: A social network extraction perspective Proceedings - 2012 International Conference on Information Retrieval and Knowledge Management, CAMP'12.
[25] Nasution M K M 2018 Semantic interpretation of search engine resultant IOP Conference Series: Materials Science and Engineering 300(1).
[26] Nasution M K M 2012 The ontology of knowledge based optimization Proceedings of International/National Seminar Matematika dan Terapan (SiManTap).
[27] Nasution M K M 2018 No research without publication: early mining Journal of Physics: Conference Series 978(1).
[28] Nasution M K M, Elveny M, Syah R and Noah S A 2015 Behavior of the resources in the growth of social network Proceedings - 5th International Conference on Electrical Engineering and Informatics: Bridging the knowledge between Academic, Industry, and Community, ICEEI 2015.
[29] Nasution M K M, Sitepu R, Rosmayati, Bakti D and Hardi S M 2018 Research mapping in North Sumatra based on Scopus IOP Conference Series: Materials Science and Engineering 309(1).
[30] Nasution M K M 2018 Indonesia knowledge dissemination: a snapshot IOP Conf. Series: Journal of Physics: Conf. Series 978.
[31] Nasution M K M, Mahfudz S and Elfida M 2010 Diskoveri pengetahuan: Suatu kritik Prosiding Seminar Nasional Ilmu Komputer 2010.
[32] Kandel A and Byatt W J 1978 Fuzzy sets, fuzzy algebra, and fuzzy statistics Proceedings of the IEEE.
[33] Rosenfeld A 1971 Fuzzy groups J Math Anal Appl 35.
[34] Nasution M K M 2004 Grup dan subgrup kabur J Math Anal Appl 35.
[35] Wong C K 1973 Fuzzy Points and Local Properties of Fuzzy Topology Urbana: University of Illinois at Urbana-Champaign.
[36] Goetschel R and Voxman W 1986 Elementary fuzzy calculus Fuzzy Sets and System, 17.
[37] Taheri S M 2003 Trends in fuzzy statistics Austrian Journal of Statistics 32.
[38] Peters G 1994 Fuzzy linear regression with fuzzy intervals Fuzzy Sets and Systems, 63.
[39] Chongfu H 1977 Principle of information diffusion Fuzzy Sets and Systems 91.
[40] Pawlak Z and Skowron A 2007 Rough sets: Some extension Information Sciences 177.
[41] Dubois D and Prade H 1990 Rough fuzzy sets and fuzzy rough sets International Journal of General System 7.
[42] Nasution M K M, Syah R, and Elfida M 2018 Information Retrieval Based on the Extracted Social Network Advances in Intelligent Systems and Computing 662.

[43] Elveny M, Syah R, Elfida M, Nasution M K M 2018 Information Retrieval on social network: An Adaptive Proof IOP Conference Series: Materials Science and Engineering 300(1).

[44] Nasution M K M and Ambarita H 2018 Reliability enumeration model for the gear in a multi-functional machine IOP Conference Series: Materials Science and Engineering 308(1).

[45] Nasution M K M and Noah S A 2011 Extraction of academic social network from online database 2011 International Conference on Semantic Technology and Information Retrieval, STAIR 2011.

[46] Molchanov I 2005 Theory of Random Sets Heidelberg, Berlin: Springer.

[47] Nasution M K M and Noah S A 2010 Superficial method for extracting social network for academics using web snippets Lecture Notes in Computer Science (including subseries Lecture Notes in Artificial Intelligence and Lecture Notes in Bioinformatics) 6401 LNAl.

[48] Nasution M K M and Sitompul O S 2017 Enhancing extraction method for aggregating strength relation between social actors Advances in Intelligent Systems and Computing 573.

[49] Nasution M K M 2018 Social network extraction based on Web: 1. Related superficial methods IOP Conference Series: Materials Science and Engineering 300(1).

[50] Nasution M K M, Sitompul O S and Noah S A 2018 Social network extraction based on Web: 3. The integrated superficial method IOP Conf. Series: Journal of Physics: Conf. Series 978.

[51] Girotto V and Johnson-Laird P N 2004 The probability of conditionals Psychologia, 47.

[52] Skorokhod A V 2005 Basic Principles and Applications of Probability Theory Heidelberg, Berlin: Springer.

[53] Daniel A R 1991 Stochastic simulation and forecasting of hourly average wind speed sequences in Jamaica. Solar Energy 46.

[54] Corbetta P 2003 Social Research: Theory, Method and Techniques London: Sage.

[55] Nasution M K M and Noah S A 2017 Social Network Extraction Based on Web. A Comparison of Superficial Methods Procedia Computer Science 124.

[56] Lubis I and Nasution M K M 2017 Probability Model for Designing Environment Condition Journal of Physics: Conference Series 801(1).

[57] Lubis I, Nasution M K M and Maulina M 2018 Basic framework of urban design based on natural resources IOP Conf. Series: Earth and Environmental Science 126.

[58] Nasution M K M and Maulina M 2018 Calligraphy design for coconut garbage use IOP Conf. Series: Earth and Environmental Science 126.

[59] Nasution M K M, Nuradi T E and Syah R 2017 SumutSiana: A framework for applying ICT to preserve the cultural heritage of Sumatera Utara Indonesia Journal of Telecommunication, Electronic and Computer Engineering 9(2-4).

[60] Syah R, Nuradi T E and Nasution M K M 2017 A framework to apply ICT for bequeathing the cultural heritage to next generation Journal of Physics: Conference Series 801(1).

[61] Popper K 1963 The Open Society and its Enemies London: Routledge.

[62] Zadeh L A 1965 Fuzzy sets Information and Control, 8.

[63] Nasution M K M, Hardi M, Sitepu R and Simulingga E 2017 A method to Extract the Forensic about Negative Issues from Web IOP Conference Series: Materials Science and Engineering 180(1).

[64] Nasution M K M, Hardi M and Syah R 2017 Mining of the social network extraction Journal of Physics: Conference Series 801(1).

[65] Pawlak Z 1982 Rough sets International Journal of Computer and Information Science 11.