Anomalous GPDs in the photon
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Abstract

We consider deeply virtual Compton scattering (DVCS) on a photon target, in the generalized Bjorken limit, at the Born order and in the leading logarithmic approximation. This leads us to the extraction of the photon anomalous generalized parton distributions (GPDs) [1, 2].

1 Introduction

At high energies, the point-like nature of the photon dominates over its hadronic component. This allows for a purely perturbative evaluation of its structure functions $F_2^\gamma$ and $F_L^\gamma$, which have therefore been the subject of much work during the seventies because, at this time, this unique feature was hoped to allow for an experimental confirmation of the fractional charge of quarks. Still at this time, the photon structure functions were also under intense study due to the completely different behaviour that they exhibit, compared to the nucleons structure functions. Indeed, it had been proven in [3] that $F_2^\gamma$ shows a log $Q^2$ behaviour already at the parton model (PM) level and, what was even more surprising, that this PM result is renormalized when taking into account QCD corrections [4]. A very accurate determination of the photon structure functions is now available [5] (see also [6]) and it is in accord with experiments [7].

On the other hand, for a bit more than ten years, one has been interested in the generalisation of DIS processes to exclusive hard reactions (see [8] for the case with photons). This gave birth to new objects relevant for studies of the nucleons structure: the generalized parton distributions. DVCS on a real photon target looks like a theoretical laboratory for the study of the properties of GPDs. It may also become an interesting process for an electron-photon collider [9, 10].

2 The DVCS process

Deeply virtual Compton scattering on a photon target

$$\gamma^*(q) \gamma(p_1) \rightarrow \gamma(q') \gamma(p_2)$$

involves, at leading order in $\alpha_{em}$, and zeroth order in $\alpha_S$, six one-loop Feynman diagrams: one box, one ‘crossed’ and one ‘cat-ears’ with quarks in the loop (‘quarks’ diagrams), and three similar diagrams with antiquarks (‘antiquarks’ diagrams).

The amplitude of the process $\gamma^* \gamma \rightarrow \gamma \gamma$ with one virtual and three real photons can be written as

$$A \doteq \epsilon_\mu \epsilon_\nu^* \epsilon_1^\alpha \epsilon_2^\beta T^{\mu\nu\alpha\beta},$$

where in our kinematics the four photon polarization vectors $\epsilon(q)$, $\epsilon^*(q')$, $\epsilon_1(p_1)$ and $\epsilon_2^*(p_2)$ are transverse. The tensorial decom-
position of $T^{\mu\nu\alpha\beta}$ reads \[ T^{\mu\nu\alpha\beta} = \frac{1}{4} g_{T}^{\mu\nu} g_{T}^{\alpha\beta} W_{1} + \frac{1}{8} \left( g_{T}^{\mu\alpha} g_{T}^{\nu\beta} + g_{T}^{\mu\beta} g_{T}^{\nu\alpha} - g_{T}^{\mu\nu} g_{T}^{\alpha\beta} \right) W_{2} + \frac{1}{4} \left( g_{T}^{\mu\alpha} g_{T}^{\nu\beta} - g_{T}^{\mu\beta} g_{T}^{\nu\alpha} \right) W_{3}, \]
and it involves three scalar functions $W_{i}$, $i = 1, 2, 3$. The final result of our calculation of the DVCS amplitude can be expressed as an integral over the quark momentum fraction $x$. It reads

$$W_{1} = \frac{e_{q} N_{C}}{2 \pi^{2}} \int_{-1}^{1} dx \frac{2 x}{(x - \xi + i \eta)(x + \xi - i \eta)} \times \left[ \theta(x - \xi) x^{2} + (1 - x)^{2} - \xi^{2} \right]$$
$$\quad + \theta(-x - \xi) x^{2} + (1 + x)^{2} - \xi^{2} \right] \log m^{2},$$
$$W_{2} = 0,$$

and

$$W_{3} = \frac{e_{q} N_{C}}{2 \pi^{2}} \int_{-1}^{1} dx \frac{2 \xi}{(x - \xi + i \eta)(x + \xi - i \eta)} \left[ \theta(x - \xi) x^{2} - (1 - x)^{2} - \xi^{2} \right]$$
$$\quad - \theta(-x - \xi) x^{2} + (1 + x)^{2} - \xi^{2} \right] \log m^{2}.$$

At this point, there are two important things to know about the intermediate steps of the calculation. First of all, each Feynman diagram possesses an UV divergence. These divergences cancel when summing the ‘quarks’ box, crossed and cat-ears diagrams contributions (obviously a similar cancellation also occurs when summing the ‘antiquarks’ diagrams contributions). The second point concerns the cat-ears diagrams. Although it is crucial to include their contributions to cancel UV divergences, they do not lead to any log $m^{2}/Q^{2}$ terms, therefore the handbag dominance interpretation of the leading logarithmic result will be justified.

We now want to interpret this result from the point of view of QCD factorization based on the operator product expansion, yet still in the zeroth order of the QCD coupling constant and in the leading logarithmic approximation. For this, we write for any function $\mathcal{F}(x, \xi)$ the obvious identity:

$$\mathcal{F}(x, \xi) \log m^{2} = \mathcal{F}(x, \xi) \log m^{2}$$
$$\quad + \mathcal{F}(x, \xi) \log M_{F}^{2},$$

where $M_{F}$ is an arbitrary factorization scale. We will show below that the first term with $\log m^{2}/M_{F}^{2}$ may be identified with the quark content of the photon, whereas the second term with $\log M_{F}^{2}/Q^{2}$ corresponds to the so-called photon content of the photon, coming from the matrix element of the two photon correlator $A_{\mu}(\frac{-z}{2}) A_{\nu}(\frac{\xi}{2})$ which contributes at the same order in $\alpha_{em}$ as the quark correlator to the scattering amplitude. Choosing $M_{F}^{2} = Q^{2}$ will allow to express the DVCS amplitude only in terms of the quark content of the photon.

3 QCD factorization of the DVCS amplitude on the photon

To understand the results of Eqs. (1-3) within the QCD factorization, we first consider two quark non local correlators on the light cone and their matrix elements between real photon states:
\[
F^{q} = \int \frac{dz}{2\pi} e^{ixz}(\gamma(p'))|\bar{q}(\frac{z}{2}N)\gamma.Nq(\frac{z}{2}N)|\gamma(p))
\]
and
\[
\tilde{F}^{q} = \int \frac{dz}{2\pi} e^{ixz}(\gamma(p'))|\bar{q}(\frac{z}{2}N)\gamma.N\gamma_{5}q(\frac{z}{2}N)|\gamma(p)) ,
\]
where we note \(N = n/n.p\) and where we neglected, for simplicity of notation, both the electromagnetic and the gluonic Wilson lines.

There also exists the photon correlator \(F^{\nu\mu}(\frac{z}{2}N)F^{\nu N}(\frac{z}{2}N)\) (where \(F^{\nu\mu} = N_{q}F^{\nu\mu}g\)), which mixes with the quark operators [4], but contrarily to the quark correlator matrix element, the photonic one begins at order \(\alpha^{0}_{em}\), as seen for instance in the symmetric case (where \(Z = \frac{3}{2}N\)):

\[
\int \frac{dz}{2\pi} e^{ixz}(\gamma(p_{2}))[F^{\nu\mu}(\frac{z}{2}N)F^{\nu N}(\frac{z}{2}N)]g_{\mu\nu}\gamma(p_{1})
\]

\[
= -g_{T}^{\mu}\epsilon_{\mu}(p_{1})\epsilon_{\nu}(p_{2})(1-\xi^{2})[\delta(1+x)+\delta(1-x)].
\]

The quark correlator matrix elements, calculated in the lowest order of \(\alpha_{em}\) and \(\alpha_{S}\), suffer from ultraviolet divergences, which we regulate through the usual dimensional regularization procedure, with \(d = 4 + 2\epsilon\). We obtain (with \(\frac{1}{\epsilon} = \frac{1}{\epsilon} + \gamma_{E} - \log 4\pi\))

\[
F^{q} = \frac{N_{C}e_{q}^{2}}{4\pi^{2}}g_{T}^{\mu}\epsilon_{\mu}(p_{1})\epsilon_{\nu}(p_{2}) \left[ \frac{1}{\epsilon} + \log m^{2} \right] F(x,\xi),
\]

with

\[
F(x,\xi) = \frac{x^{2} + (1-x)^{2} - \xi^{2}}{1 - \xi^{2}}\theta(1 > x > \xi)
\]

\[
- \frac{x^{2} + (1+x)^{2} - \xi^{2}}{1 - \xi^{2}}\theta(-\xi > x > -1)
\]

\[
+ \frac{x(1-\xi)}{\xi + \xi^{2}}\theta(\xi > x > -\xi)
\]

for the \(\mu \leftrightarrow \nu\) symmetric (polarization averaged) part. A similar result for the antisymmetric (polarized) part can be found in our original publication [2].

The ultraviolet divergent parts are removed through the renormalization procedure involving both quark and photon correlators. The \(\frac{1}{\epsilon}\) terms in Eq. (7) define the nondiagonal element \(Z_{\nu\nu}F_{R}\) of the multiplicative matrix of renormalization constants \(Z\). The \(\frac{1}{\epsilon}\) terms are then subtracted by the renormalization of quark operators. The renormalization procedure introduces a renormalization scale which we here identify with a factorization scale \(M_{F}\) in the factorized form of the amplitude. Imposing the renormalization condition that the renormalized quark correlator matrix element vanishes when the factorization scale \(M_{F} = m\), we get from Eq. (7) for the renormalized matrix element [5]

\[
F_{R}^{q} = \frac{N_{C}e_{q}^{2}}{4\pi^{2}}g_{T}^{\mu}\epsilon_{\mu}(p_{1})\epsilon_{\nu}(p_{2}) \log \frac{m^{2}}{M_{F}^{2}} F(x,\xi),
\]

and a similar result for the antisymmetric case.

These results permit us to define the generalized quark distributions in the photon, \(H_{q}^{q}(x,\xi,0)\), as

\[
F_{R}^{q} = -g_{T}^{\mu}\epsilon_{\mu}(p_{1})\epsilon_{\nu}(p_{2})H_{q}^{q}(x,\xi,0),
\]

\[
\tilde{F}_{R}^{q} = i\epsilon^{\mu\nu\rho\sigma}n_{\rho}(p_{1})\epsilon_{\nu}(p_{2})H_{q}^{q}(x,\xi,0)
\]

and to write the quark contribution to the DVCS amplitude as a convolution of coefficient functions and distributions \(H_{q}^{q}\)

\[
W_{q} = \int_{-1}^{1} dx C_{V}^{q}(x)H_{q}^{q}(x,\xi,0),
\]

\[
W_{\tilde{q}} = \int_{-1}^{1} dx C_{\tilde{A}}^{q}(x)H_{q}^{q}(x,\xi,0),
\]

where the Born order coefficient functions \(C_{V/A}^{q}\) attached to the quark-antiquark symmetric and antisymmetric correlators are the

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usual hard process amplitudes:

\[ C_{V/A}^q = -2e_q^2 \left( \frac{1}{x - \xi + i\eta} \pm \frac{1}{x + \xi - i\eta} \right) . \]

We recover in that way the \( \log \frac{m^2}{M_F^2} \) term in the right hand side of Eq. (4).

The photon operator contribution to the DVCS amplitude at the order \( \alpha^2_{em} \) considered here, involves a new coefficient function calculated at the factorization scale \( M_F \), which plays the role of the infrared cutoff, convoluted with the photon correlator or with its antisymmetric counterpart. The results of these convolutions effectively coincide with the amplitudes calculated in Section 2 with the quark mass replaced by the factorization scale \( m \to M_F \), and leads to the second term in the right hand side of Eq. (4). The triviality of Eq. (4) in fact hides the more general independence of the scattering amplitude on the choice of the scale \( M_F \) which is controlled by the renormalization group equation.

We still have the freedom to fix the factorization scale \( M_F^2 \) in any convenient way. Choosing \( M_F^2 = Q^2 \) kills the logarithmic terms coming from the photon correlator, so that the DVCS amplitude is written (at least, in the leading logarithmic approximation) solely in terms of the quark correlator, recovering a partonic interpretation of the process.

4 Conclusion

We derived the leading amplitude of the DVCS process on a photon target. We have shown that the amplitude coefficients \( W_i \), factorize in the forms shown in Eq. (3), irrespectively of the fact that the handbag diagram interpretation appears only after cancellation of UV divergencies in the scattering amplitude. We have shown that the objects \( H_i^q(x, \xi, t) \) are matrix elements of non-local quark operators on the light cone, and that they have an anomalous component which is proportional to \( \log(Q^2/m^2) \). They thus have all the properties attached to generalized parton distributions.

This work is the first step towards the study of the anomalous generalized parton distributions in the photon. It would be nice to consider the \( \Delta_T \neq 0 \) kinematics. Another logical step beyond this analysis is the inclusion of QCD corrections. Since we have demonstrated the presence of anomalous terms both in the DGLAP and ERBL regions, this can be done by solving the non-homogeneous DGLAP-ERBL evolution equations, which are the generalisation of the non-homogeneous DGLAP equations for the photon structure functions.

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