**β-delayed proton emission from $^{11}$Be in effective field theory**

Wael Elkamhawy,1, * Zichao Yang,2, † Hans-Werner Hammer,1,3, ‡ and Lucas Platter2, 4, §

1 Institut für Kernphysik, Technische Universität Darmstadt, 64289 Darmstadt, Germany
2 Department of Physics and Astronomy, University of Tennessee, Knoxville, TN 37996, USA
3 ExtreMe Matter Institute EMMI, GSI Helmholtzzentrum für Schwerionenforschung GmbH, 64291 Darmstadt, Germany
4 Physics Division, Oak Ridge National Laboratory, Oak Ridge, TN 37831, USA

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We calculate the rate of the rare decay $^{11}$Be into $^{10}$Be + $p + e^- + \bar{\nu}_e$ using Halo effective field theory, thereby describing the process of beta-delayed proton emission. We assume a shallow $1/2^+$ resonance in the $^{10}$Be-$p$ system with an energy consistent with a recent experiment by Ayyad et al. and obtain $b_p = 4.9^{+2.9}_{-2.0} (\text{exp.}) 	imes 10^{-6}$ for the branching ratio of this decay, predicting a resonance width of $\Gamma_R = (9.0^{+4.8}_{-2.3} (\text{exp.}) + 5.3 (\text{theo.}))$ keV. Our calculation shows that the experimental branching ratio and resonance parameters of Ayyad et al. are consistent with each other. Moreover, we analyze the general impact of a resonance on the branching ratio and demonstrate that a wide range of combinations of resonance energies and widths can reproduce branching ratios of the correct order. Thus, no exotic mechanism (such as beyond the standard model physics) is needed to explain the experimental decay rate.

**Introduction.** Halo nuclei display a large separation of scales between a few loosely bound halo nucleons and a tightly bound core [1–4]. The emergence of the halo degrees of freedom is a fascinating aspect of nuclei away from the valley of stability. The halo nucleons in the core potential spend most of their time in the classically forbidden region outside of the range of the core potential. This is analog to the tunnel effect. But since the halo nucleons are bound to the core, they always have to come back into the core potential. This separation of scales can be used to treat these systems using an effective field theory (EFT) approach called Halo EFT [5–7]. Common to all EFTs is that observables are described in a systematic low-energy expansion and that the accuracy of a calculation can be systematically improved. Halo EFT has been applied to a number of observables, including electromagnetic capture reactions and photodisassociation processes [8–14].

Here we will consider, for the first time, the weak decay of the valence neutron of the halo nucleus $^{11}$Be into the continuum, $^{11}$Be $\to$ $^{10}$Be + $p + e^- + \bar{\nu}_e$, within Halo EFT.

First experimental results for this rare decay mode were presented in Refs. [15, 16]. Riisager et al. [17] measured a surprisingly large branching ratio for this decay process, $b_p = 8.3(9) 	imes 10^{-6}$, which could only be understood in their Woods-Saxon model analysis if the decay proceeds through a new single-particle resonance in $^{11}$B. Their measured branching ratio is also more than two orders of magnitude larger than the cluster model prediction by Baye and Tursunov [18]. This led Pfützner and Riisager [19] to suggest that β-delayed proton emission in $^{11}$Be is also a possible pathway to detect a dark matter decay mode as proposed by Fornal and Grinstein [20]. More recently, this branching ratio was remeasured by Ayyad et al. [21] as $b_p = 1.3(3) 	imes 10^{-5}$, similar in size to the previous measurement. They also presented new evidence for a low-lying resonance in $^{11}$B with resonance energy $E_R = 0.196(20)$ MeV and width $\Gamma_R = 12(5)$ keV. Using these parameters, the authors calculated the decay rate in a Woods-Saxon model assuming a pure Gamow-Teller transition. They obtained $b_p = 8 	imes 10^{-6}$, which has the correct order of magnitude but is only consistent within a factor of two with their experimental result. The work by Ayyad et al. was criticized in a recent comment by Fynbo et al. [22]. A new experiment by Riisager et al. [23] gives an upper limit of $b_p \leq 2.2 \times 10^{-6}$ for the branching ratio but some questions remain due to inconsistencies between different measurements. In conclusion, the branching ratio for β-delayed proton emission in $^{11}$Be remains an important unsolved problem.

The ground state of $^{11}$Be is a well-understood S-wave halo nucleus. From the ratio of the one-neutron separation energy of $^{11}$Be and the excitation energy of the $^{10}$Be core, one can extract the expansion parameter for a description with the core and valence neutron as effective degrees of freedom, $R_{\text{core}}/R_{\text{halo}} \approx 0.4$ [8]. Here $R_{\text{core}}$ and $R_{\text{halo}}$ are the length scales of the core and halo, respectively. In principle, both the $^{10}$Be core and the halo neutron can β-decay. Since the half-life of the neutron ($T_{1/2} = 10$ min) is much shorter than the half-life of the core ($T_{1/2} = 10^6$ a), it is safe to assume that for β-delayed proton emission it is always the halo neutron that decays in the halo picture. Therefore, one would naively expect the nucleus to emit this proton due to the repulsive Coulomb interaction: $^{11}$Be $\to$ $^{10}$Be + $p + e^- + \bar{\nu}_e$. This process, called β-delayed proton emission, has well-defined experimental signatures. However, it is also known that short-distance mechanisms such as the decay into excited states of $^{11}$B (that are beyond the halo interpretation) dominate the total β-decay rate of $^{11}$Be [24, 25].

Halo EFT offers a new perspective on β-delayed proton emission from $^{11}$Be by providing a value for the decay rate with a robust uncertainty estimate. It uses the
appropriate degrees of freedom and parametrizes the decay observables in terms of a few measurable parameters. Thus, it is perfectly suited for the theoretical description of low-energy processes such as $\beta$-delayed proton emission from halo nuclei. Kong and Ravnadal [26] used these ideas to successfully describe the inverse process of pp-fusion into a deuteron and leptons. In contrast to the previous calculation in Ref. [18], we will use new experimental input parameters and put additional emphasis on the uncertainties associated with using effective degrees of freedom. The halo neutron can $\beta$-decay through both the Gamow-Teller and Fermi operators. The Fermi operator can only connect states in the same isospin multiplet. If all neutrons in $^{11}$Be contribute to the $\beta$-decay, this implies that the final state must have $T = 3/2$ for a Fermi transition. No such states are currently known in $^{11}$B within the $\beta$-decay window. However, due to the halo character of $^{11}$Be we expect that only the halo neutron decays, such that the final state has no definite isospin. Thus, we will keep our analysis general and consider both the scenarios of Gamow-Teller and Fermi decay as well a pure Gamow-Teller decay in the following. Specifically, we will show that based on the measured branching ratio, a low-lying resonance is the likely reason for the large partial decay rate, confirming the suggestion of Ref. [17]. Furthermore, in $^{11}$B, we explore the impact of the resonance energy and width on the decay rate and show that the recent results for the resonance energy and width of a low-lying resonance are consistent with the experimentally measured branching ratio.

In order to keep our presentation self-contained, we start by summarizing the concepts of Halo EFT for $S$-wave halo nuclei. We discuss the calculation of decay rates with and without resonant final state interactions and then display our results. Note that these are EFTs for two different scenarios. Formally, we perform calculations up to corrections of order $R_{\rm core}/R_{\rm halo}$ in both scenarios but because of the different physics assumptions these cannot be directly compared. We conclude with a summary.

**Theoretical foundations.** The Halo EFT Lagrangian $\mathcal{L}$ for $^{11}$Be as well as the low-lying resonance in $^{11}$B up to next-to-leading order can be written as

$$\mathcal{L} = \mathcal{L}_0 + \mathcal{L}_d,$$

where $\mathcal{L}_0$ is the free Lagrangian of the $^{10}$Be core, neutron and proton

$$\mathcal{L}_0 = c^\dagger \left( i\partial_t + \frac{\nabla^2}{2m_c} \right) c + n^\dagger \left( i\partial_t + \frac{\nabla^2}{2m_n} \right) n + p^\dagger \left( i\partial_t + \frac{\nabla^2}{2m_p} \right) p,$$

with $c$, $n$, and $p$ the core, neutron and proton fields, respectively. The masses of core, neutron and proton are denoted by $m_c = 9327.548$ MeV, $m_n = 939.565$ MeV and $m_p = 938.272$ MeV. The $S$-wave core-neutron as well as core-proton interaction are described by $\mathcal{L}_d$, which reads

$$\mathcal{L}_d = d_{\rm Be}^\dagger \left[ \eta \left( i\partial_t + \frac{\nabla^2}{2M_{nc}} + \Delta \right) + \Delta \right] d_{\rm Be}$$

$$+ d_B^\dagger \left[ \tilde{\eta} \left( i\partial_t + \frac{\nabla^2}{2M_{pc}} + \Delta \right) + \Delta \right] d_B$$

$$- g \left[ c^\dagger n^\dagger d_{\rm Be} + \text{H.c.} \right] - \tilde{g} \left[ c^\dagger p^\dagger d_B + \text{H.c.} \right],$$

where $d_{\rm Be}$ and $d_B$ are dimer fields, with spin indices suppressed, that represent the $J^P = 1/2^+$ ground state of $^{11}$Be and the $J^P = 1/2^+$ low-lying resonance in $^{11}$B, respectively, while $M_{nc} = m_n + m_c$ and $M_{pc} = m_p + m_c$.

The renormalization of the low-energy constants for $^{11}$Be has been discussed in Ref. [8]. Here, we will briefly summarize the relevant results to define our notation. Due to the non-perturbative nature of the interaction, we need to resum the self-energy diagrams to all orders. After matching the low-energy constants for $^{11}$Be appearing in Eq. (2) to the effective range expansion, we obtain the full two-body $T$-matrix

$$T_0(E) = \frac{2\pi}{m_R} \left[ \frac{1}{a_0} - r_0 m_R E - \sqrt{-2m_R E - i\epsilon} \right]^{-1}.$$

where $m_R$ is the reduced mass, and $a_0$, $r_0$ are the $S$-wave $^{10}$Be$-n$ scattering length and effective range, respectively. The residue at the bound state pole of Eq. (3) is required to calculate physical observables, $Z = \frac{2\pi r_0}{m_F} \left( 1 - r_0 \gamma_0 \right)$, with $\gamma_0 = \frac{1}{\sqrt{1 - 2r_0/a_0}} / r_0 \equiv \sqrt{2m_R S_n}$ the binding momentum of the $S$-wave halo state, and $S_n$ the one-neutron separation energy of the halo nucleus.

In order to investigate $\beta$-delayed proton emission from $^{11}$Be, we include the weak interaction current allowing transitions of a neutron into a proton, electron and antineutrino which corresponds to the hadronic one-body current. Moreover, we have to consider hadronic two-body currents that appear in the dimer formalism once the effective range is included. The corresponding Lagrangian is given by

$$\mathcal{L}_{\text{weak}} = - \frac{G_F}{\sqrt{2}} \frac{\mu}{\nu} \left( (J^+_{\mu})^{1b} + (J^+_{\nu})^{2b} \right),$$

where $\mu = \bar{u}_c \gamma^\mu (1 - \gamma^5) v_p$ and $(J^+_{\mu})^{1b} = (V_\mu^1 - A_\mu^1) + i(V_\mu^5 - A_\mu^5)$ denote the leptonic and hadronic one-body currents, respectively. Here the hadronic one-body current is decomposed into vector and axial-vector contributions. At leading order, the contributions to this current are $V_\mu = N^1 \frac{x_\mu^2}{2} N$, $A_\mu = g_A N^1 \frac{x_\mu^2}{2} \sigma_k$, where $|g_A| \simeq 1.27$ is the ratio of the axial-vector to vector coupling constants [27]. Terms with more derivatives and/or more fields (many-body currents) will appear at higher orders. The first and second term give the conventional Fermi and Gamow-Teller operators, respectively. Including resonant core-proton final state interactions, we have to take into account a two-body current with known coupling.
constants which arises from gauging the time derivative of the dimer fields appearing in Eq. (2). It is also decomposed into vector and axial-vector contributions and reads

\[
(J_{\mu}^V)^{2b} = \begin{cases} -d_1^b \mu = 0, \\ g_A d_1^b \sigma_k \mu = k = 1,2,3. \end{cases}
\]  

(5)

In addition, there is also an unknown contribution usually denoted as \(L_{1A}\) that normally appears at the same order. However, in the case with Coulomb interaction, this piece is suppressed by \((R_{\text{core}}/R_{\text{halo}})^{1/2}\) compared to the two-body current in Eq. (5).\(^1\) Therefore, it contributes only at NNLO allowing us to make predictions up to NLO. Note that our power counting including resonant final state interactions implies a suppression of \((R_{\text{core}}/R_{\text{halo}})^{1/2}\) going from order to order instead of \(R_{\text{core}}/R_{\text{halo}}\) as in the case without resonant final state interactions.

**Weak matrix element and decay rate.** We ignore recoil effects in the \(\beta\)-decay and take both the Gamow-Teller and Fermi transitions whose operator coefficients have been factored out and \(\Theta\) is the Heaviside step function. Moreover, \(Z\) and \(\eta\) are the fine structure constant and heavy mass, respectively. For the leading contributions resulting from diagrams (a) and (b) of Fig. 1, we assume this to be a good approximation since the \(^{10}\)Be core is far away from the decaying valence neutron due to the small one-neutron separation energy. For consistency, we use the same Sommerfeld factor in diagram (c) of Fig. 1 although it contains interactions of all particles at a single space-time point. However, since diagram (c) is subleading, we also expect a deviation of subleading order. In order to confirm this expectation, we have performed an explicit calculation using \(Z = Z_p + Z_c\) for all diagrams, which leads to a change of order 30% in the decay rate. Thus, this effect is beyond the 40% accuracy of our calculation (see below) and would enter at higher orders. If a pure Gamow-Teller transition is considered, the factor 1 + 3\(g_A^4\) is replaced by 3\(g_A^2\). This results in a reduction of the decay rate by 17%.

**Beta-strength sum rule.** The so-called Fermi and Gamow-Teller sum rules (also collectively known as beta-strength sum rule) count the number of weak charges that can decay in the initial state. We will require that this beta-strength sum rule is fulfilled exactly at each order within our EFT power counting. The beta-strengths are related to the comparative half-life of a decay, the so-called \(ft\) value given by

\[
ft = \frac{B}{B_F + g_A^2 B_{\text{GT}}},
\]  

(8)

where \(B = 2\pi^3 \ln 2/(m_p^3 G_F^2)\) is the \(\beta\)-decay constant. In this paper, we will use the value \(B = 6144.2\ s\ [28, 29]\). With \(B_{\text{GT}} = 3B_F\), we find

\[
B_F = \frac{B}{(1 + 3g_A^4)} \frac{1}{ft}.
\]  

(9)

The inverse \(ft\) value is directly related to the transition matrix element \(\mathcal{M}\) of \(^{11}\)Be into \(^{10}\)Be + \(p\),

\[
\frac{1}{ft} = \frac{1}{B} \frac{\mathcal{M}^2}{2\pi^2} \int dE \frac{m_R \sqrt{2m_R E}}{m_p} \mathcal{A}(p)^2.
\]  

(10)

For a transition into the continuum, the sum rule is exactly fulfilled when integrating the differential beta-
strengths
\[ \frac{dB_F}{dE} = \frac{1}{2\pi^2} m_R \sqrt{2m_R E} |A(p)|^2, \]  
\[ \frac{dB_{GT}}{dE} = 3 \frac{dB_F}{dE}, \]
over the whole continuum leading to the sum rules \( B_F = 1 \) and \( B_{GT} = 3 \). In the halo picture, we therefore expect beta-strengths \( B_F \) and \( B_{GT} \) to be at most 1 and 3, respectively, when integrating over the available \( Q \)-window. At LO where the full non-perturbative solution for a zero-range interaction is used in the incoming as well as outgoing channel, the sum rule is always satisfied. At NLO where range corrections are included, the sum rule puts strong constraints on the ranges in the incoming and outgoing channels such that only certain combinations are allowed.

**Hadronic current without resonant final state interactions.** The amplitude for the charge changing weak transition of a two-body system is illustrated as diagram (a) of Fig. 1. It was first calculated in pionless EFT by Kong and Ravndal [26]. The corresponding hadronic current can be written as [30]
\[ A_{CS}^{(a)}(p) = -ig\sqrt{Z}C(\eta_p)e^{i\sigma_0} \frac{2m_R}{p^2 + \gamma_0^2} e^{2\eta_p \arctan(p/\gamma_0)}, \]  
where \( \sigma_0 \) is the Coulomb phase and \( C^2(\eta_p) \) is the Sommerfeld factor from Eq. (7). In the \(^{10}\text{Be} - p\) system, the Sommerfeld parameter is \( \eta_p = \alpha Z_p Z_c m_R/|p| = k_C/|p| \), with \( Z_p = 1 \) and \( Z_c = 4 \).

**Hadronic current with resonant final state interactions.** The current (13) includes only the final state interaction from the exchange of Coulomb photons. We now consider resonant final state interactions whose signature is a low-lying resonance in the \(^{10}\text{Be} - p\) channel up to NLO. These contributions are shown as diagrams (b) and (c) of Fig. 1. Diagram (c) contributes only at NLO to the amplitude. It arises from a two-body current (with known coupling strength) that appears as a result of the energy-dependent interactions used in the initial state (see Eq. (2)) and the final state (see Ref. [31]). The thin double line together with the shaded ellipses that represent Coulomb Green’s functions as depicted in diagram (b) essentially combine to the strong scattering amplitude \( T_{CS} \) given either in Eq. (14) or (20) [26, 31].

The degrees of freedom in Halo EFT are the emitted outgoing proton and \(^{10}\text{Be}\). Our treatment of the resonance follows Ref. [31]. The corresponding strong scattering amplitude modified by Coulomb corrections is [31]
\[ T_{CS} = \frac{-4\pi/m_R}{(r_0^C - \frac{1}{3k_C})(p^2 - k_R^2) + \frac{p^2}{3k_c} - 4k_C H(\eta_p)}, \]  
with \( H(\eta_p) = \text{Re}[\psi(1 + i\eta_p)] - \ln \eta_p + \frac{i}{2\eta_p} C^2(\eta_p) \), with the digamma function \( \psi(z) \). The parameters in Eq. (14) are directly related to the complex pole momentum \( k^* = k_R - ik_I \):
\[ -\frac{1}{\alpha_0^C} = - \left( r_0^C - \frac{1}{3k_C} \right) \frac{k_R^2}{2}, \]  
\[ r_0^C = \frac{2\pi k_C}{k_R k_I} \frac{1}{\pi^2 k_C/k_R - 1} + \frac{1}{3k_C}, \]
where \( \alpha_0^C \) and \( r_0^C \) are the Coulomb-modified scattering length and effective range, respectively. Within our power counting, the parameters \( k_C, k_R, k_I \) as well as \( \gamma_0 \) scale as \( 1/R_{\text{halo}} \) implying that both Coulomb-modified scattering parameters \( \alpha_0^C \) and \( r_0^C \) scale as \( R_{\text{halo}} \). Note that we include \( r_0^C \) despite this scaling only at NLO, since the range \( r_0 \) in the incoming channel scales as \( R_{\text{core}} \). The inclusion of both ranges at the same order guarantees that the beta-strength sum rule is satisfied at both LO and NLO.

The diagrams (b) and (c) of Fig. 1 lead to
\[ A_{CS}^{(b)} = -ig\sqrt{Z}4m_R^2 C(\eta_p)e^{i\sigma_0} IT_{CS}, \]  
\[ A_{CS}^{(c)} = -ig\sqrt{Z}4m_R^2 C(\eta_p)e^{i\sigma_0} \left( \frac{\sqrt{r_0^C r_0^C}}{8\pi} \right) T_{CS}, \]
with the complex-valued integral
\[ I = \int \frac{d^3 q}{(2\pi)^3} C^2(\eta_q)e^{2\eta_q \arctan(q/\gamma_0)} \frac{1}{q^2 + \gamma_0^2} \frac{1}{p^2 - q^2 + i\epsilon}. \]  
(19)

The total amplitude \( A \) is the sum of the amplitudes with and without resonance \( A = A_{CS}^{(a)} + A_{CS}^{(b)} + A_{CS}^{(c)}. \) At LO, the Coulomb-modified effective range in the \(^{10}\text{Be} - p \) system is zero and the amplitude reduces to
\[ T_{CS} = -\frac{2\pi}{m_R} \left( \frac{1}{-1/\alpha_0^C - 2k_C H(\eta_p)} \right). \]  
(20)
Results without resonant final state interactions. We consider two scenarios: beta-delayed proton emission with and without resonant final state interactions from a low-lying resonance in $^{11}\text{Be}$. We start with the first scenario and use the one-neutron separation energy of $^{11}\text{Be}$ $S_n = 0.5016$ MeV [25]. In Fig. 2, we plot the differential decay rate $d\Gamma/dE$ as a function of the kinetic energy $E$ of the outgoing hadrons. The solid line gives the result obtained by Baye and Tursunov [18]. The dash-dotted line shows the EFT result with an uncertainty band obtained by adding an uncertainty of order $R_{\text{core}}/R_{\text{halo}} \approx 40\%$ from higher order corrections where we use the smallest value of $R_{\text{halo}}$ given by $1/r_0$ while we estimate $R_{\text{core}}$ by the effective range $r_0$ as a conservative estimate. The remaining curve includes resonant final state interactions and will be discussed below.

For the branching ratio, we obtain $b_p = \Gamma/\Gamma_{\text{total}} = (1.31 \pm 0.51) \times 10^{-8}$ where the EFT uncertainty is again estimated to be of the order of $40\%$. Correspondingly, we obtain for the decay rate $\Gamma = (6.6 \pm 2.6) \times 10^{-10}$ s$^{-1}$. Baye and Tursunov [18] obtain $\Gamma = 1.5 \times 10^{-9}$ s$^{-1}$ which differs by a factor of 2.3 from our result. We note, however, that they used a Woods-Saxon potential with Coulomb interactions tuned to reproduce $^{11}\text{B}$ properties in the final state. Both theoretical results are significantly smaller than the experimental results reported in Refs. [15–17, 21].

Results with resonant final state interactions. We now discuss the second scenario including final state interactions. In Fig. 3, we show the possible resonance parameter combinations that fulfill the beta-strength sum rule. The dash-dotted line is the result at LO where the effective range in the incoming channel as well as the Coulomb-modified effective range in the outgoing channel are zero. At NLO, we use $r_0 = 2.7$ fm determined in Ref. [8] from the measured $B(\text{E1})$ strength for Coulomb dissociation of $^{11}\text{Be}$. The one-neutron separation energy as well as the effective range of $^{11}\text{Be}$ determine the Coulomb-modified effective range in the outgoing channel to be $r_0^C = 1.5$ fm. The sum rule is then satisfied to very good approximation for a wide range of Coulomb-modified scattering lengths in the outgoing channel. The square shows the experimentally measured resonance parameter combinations given in Ref. [21]. We note that the value of $r_0^C$ is determined independently from the experimental resonance parameters. Our NLO curve depicted as the dashed line corresponding to $r_0^C = 1.5$ fm exhibits combinations of $E_R$ and $\Gamma_R$ that are in agreement with this measurement as indicated by the overlap of the square and the curve.

In Fig. 4, we show the results for the decay rate as a function of the resonance energy at NLO while using the corresponding resonance width that satisfies the sum rule as shown in Fig. 3. The black line represents the decay rate obtained moving along the NLO curve in Fig. 3 while the red shaded envelope gives the theoretical uncertainty estimated from the counterterm contribution in the axial current scaling with $R_{\text{core}}/R_{\text{halo}} \approx 40\%$. The green bands show the experimentally measured branching ratio and resonance energy of Ref. [21]. The horizontal blue dashed line denotes the result of the model calculation carried out in Ref. [21] whereas the horizontal blue dash-dotted line gives the upper bound of Ref. [23]. Comparing our results with Ref. [23], we find that resonance energies $E_R \gtrsim 0.214$ MeV give results compatible with this upper bound. The corresponding resonance widths can be read off in Fig. 3. When comparing our results with Ref. [21], we find that the low-lying resonance measured in Ref. [21] with $E_R = 0.196(20)$ MeV and width $\Gamma_R = 12(5)$ keV is consistent with their experimentally measured branching ratio as indicated by the overlap of the square and the red shaded band. According to Fig. 3, we determine the width corresponding to the resonance energy $E_R = 0.196(20)$ MeV as $\Gamma_R = (9.0^{+4.8}_{-3.3}(\exp.)^{+3.5}_{-2.2}(\text{theo.}))$ keV, which agrees well with the experimental value. At LO, the resonance width scales as $k_C^2/m_R$ whereas at NLO this value is enhanced by a factor of $1/(1 - 3k_Cr_0^C)$. This enhancement for Coulomb halos is well known [32–34]. Using $E_R = 0.196(20)$ MeV, we calculate the logarithm of the comparative half-life $\log(ft) = 3.0$ with $B_{\text{GT}} = 2.88$ and $B_F = 0.96$ for a decay including both Gamow-Teller and Fermi transitions and $\log(ft) = 3.1$ with $B_{\text{GT}} = 2.88$ for a pure Gamow-Teller transition. The latter result can be compared to $\log(ft) = 4.8(4)$ calculated by Ayyad et al. [21] which was obtained using a pure Gamow-Teller transition as well, but is significantly larger than our result. This large $\log(ft)$ value was also criticized in the comment by Fynbo et al. [22]. Ayyad et al. corrected the value to $\log(ft) = 2.8(4)$ in their recent erratum [21]. This new value is now in good agreement with our result.
Using the half-life for $^{11}\text{Be}$ given in Ref. [25] we convert the Halo EFT result for $E_R=0.196(20)$ MeV and $\Gamma_R=(9.0^{+4.8}(\text{exp.})_{-3.2}(\text{theo.}))$ keV into the final result for the branching ratio $b_p=4.9_{-2.9}^{+5.6}(\text{exp.})_{-0.9}^{+4.0}(\text{theo.})\times 10^{-6}$. The corresponding differential decay rate is shown by the dashed line in Fig. 2.

**Conclusion.** In this paper, we considered $\beta$-delayed proton emission from $^{11}\text{Be}$. We compared the scenario with no strong final state interactions with the scenario of a resonant enhancement in the final $^{10}\text{Be}--\text{n}$ channel up to NLO. In the case of no strong final state interactions, we obtained results that are in qualitative agreement with Baye and Tursunov with remaining small differences that can be explained by the different treatment of the final state channel. Including a low-lying resonance with the energy measured in Ref. [21] results in a resonance width and partial decay rate in agreement with this experiment. Thus, our model-independent calculation supports the experimental finding of a low-lying resonance. Furthermore, we have explored the sensitivity of the partial decay rate to the resonance energy and decay width and found that this problem is fine tuned, i.e., only certain combinations of width and resonance energy can reproduce the partial decay rate. In contrast to the model calculation in Ref. [21], we included both, Fermi and Gamow-Teller transitions. However, if a pure Gamow-Teller decay is considered, their partial decay rate can also be reproduced with slightly smaller resonance parameters. Thus, our result implies that $^{11}\text{Be}$ is not a good laboratory to detect dark neutron decays since no exotic mechanism is needed to explain the partial decay rate.

The uncertainties are largely determined by higher order contributions of the EFT expansion. The next contribution within our power counting that we did not include is a counterterm contribution in the axial current scaling with $R_{\text{core}}/R_{\text{halo}}$. Uncertainties of the $S$-wave input parameter (the one-neutron separation energy) do not impact the total uncertainty significantly. Therefore, we estimate the uncertainty in the final decay rate to be approximately $R_{\text{core}}/R_{\text{halo}}\approx 40\%$. Experimental data with higher precision could be used to constrain the $^{10}\text{Be}--\text{n}$ and $^{10}\text{Be}--\text{p}$ interactions. It will be interesting to test whether the inclusion of this resonance changes the Halo EFT predictions for deuteron induced neutron transfer reactions off $^{11}\text{Be}$ which were investigated in Ref. [36].

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2 See Ref. [35] for another recent theoretical calculation in support of this resonance.
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