Calculation of the staggered spin correlation in the framework of the Dyson-Schwinger approach

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Abstract

Based on the linear response of the fermion propagator with respect to an external field, we first derive a model-independent expression for the staggered spin susceptibility in which the influence of the full pseudoscalar vertex function is included. This expression for the staggered spin susceptibility is quite different from that given in the previous literature. The numerical values of the staggered spin susceptibility are calculated within the framework of the Dyson-Schwinger approach. Our numerical result shows that the nonperturbative dressing effects on the fermion propagator is very important when one studies the staggered spin susceptibility which corresponding to antiferromagnetic correlation in both Nambu phase and Winger phase.

Key-words: QED\textsubscript{3}; Nambu phase; Winger phase; staggered spin susceptibility

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Quantum electrodynamics in (2+1) dimensions (QED$_3$) has attracted much interest over the past few years. It has many features similar to QCD, such as spontaneous chiral symmetry breaking in the chiral limit and confinement [1–13]. Moreover, it is super-renormalizable, so it does not suffer from the ultraviolet divergence which are present in QED$_4$. Due to these reasons it can serve as a toy model of QCD. In parallel with its relevance as a tool through which to develop insight into aspects of QCD, QED$_3$ is also found to be equivalent to the low-energy effective theories of strongly correlated electronic systems. Recently, QED$_3$ has been widely studied in high T$_c$ cuprate superconductors [14–23] and graphene [24–26].

Dynamical chiral symmetry breaking (DCSB) occurs when the massless fermion acquires a nonzero mass through nonperturbative effects at low energy, but the Lagrangian keeps chiral symmetry when the fermion mass is neglected. In a four-fermion interaction model Nambu and Jona-Lasinio first adopted the mechanism of DCSB to generate a nonzero mass for the fermion from nothing solely through interactions [27]. The Dyson-Schwinger equations (DSEs) provide a natural framework within which to explore DCSB and related phenomena. In 1988, T. Appelquist et al. [4] studied DCSB in massless QED$_3$ with $N$ fermion flavors by solving the DSEs for fermion self-energy in leading order of the $1/N$ expansion and found DCSB occurs when $N$ is less than a critical number $N_c$. Later D. Nash showed that the gauge-invariant critical number of fermion flavor still exists by considering higher order corrections to the gap equation [5]. In 1995, P. Maris solved the coupled DSEs with a set of simplified vertex functions and obtained the critical number of fermion flavor $N_c=3.3$ [6, 8]. Recently, in massless unquenched QED$_3$, Fischer et al. [12] self-consistently solved a set of coupled DSEs and obtained $N_{c_{\text{crit}}} \approx 4$ by using more sophisticated vertex ansatz in unquench QED$_3$.

From the differences in the temperature dependence of the Cu and O site relaxation rates in NMR experiment, it is shown that there exist antiferromagnetic correlations in the underdoped cuprates. However, within the slave boson mean field theory of the t-J model, the staggered spin correlation tends to zero in the infrared region. That is to say, the mean field treatments lose a lot of antiferromagnetic correlation. In order to explain the above puzzle, Wen et al. considered the U(1) gauge fluctuations corrections to the staggered spin susceptibility at order $1/N$ in the Algebraic spin liquid (ASL) based on slave boson treatment of t-J model and found that the dressed fermion propagator acquires an anomalous dimension exponent compared with the free spinon propagator in the chiral...
symmetric phase (Wigner phase), which enhances the staggered spin correlation and restores the antiferromagnetic order which is lost at the mean-field level \[19, 22\]. It is clear that the staggered spin susceptibility plays a crucial role in ASL where the anomalous dimension exponent indicates the non-Fermi liquid behavior in the pseudogap phase in effective QED$_3$ theory of cuprate superconductor.

On the other hand, based on the phase fluctuation model \[28\], Franz et al. proposed a new quantum liquid-the Algebraic Fermi liquid (AFL) to describe the pseudogap state in the effective QED$_3$ theory \[29, 30\]. By studying the gauge-invariant response function in AFL, Franz et al. also found that the full staggered spin susceptibility exhibits a nontrivial anomalous dimension exponent to $1/N$ order in Wigner phase. This result is in agreement with the finding of Refs. \[21, 22\]. However, Liu et al. argue that once the fermion acquires a constant mass in DCSB phase (Nambu phase), the spin staggered correlation function defined in Ref. \[22\] is nonzero even at the mean field level and antiferromagnetic order gets restored \[31\].

Up to now, in all the above literatures, the theoretical calculations of the spin susceptibility are usually done in the framework of perturbation theory where only the leading $1/N$ order corrections to the staggered spin correlation function is added to the mean field level. In this paper, we will study the staggered spin susceptibility using a nonperturbative method since the appearance of antiferromagnetic order is a nonperturbative phenomenon.

Our starting point is the Lagrangian density of massless effective QED$_3$ with N-flavored Dirac fermion

$$\mathcal{L} = \sum_{i=1}^{N} \bar{\psi}_i \gamma_\mu (\partial_\mu + ieA_\mu) \psi_i + \frac{1}{4} F_{\mu\nu}^2.$$  \hspace{1cm} (1)

In this context, we are focusing on the case of N=2 since the nature of the effective QED$_3$ theory at N = 2 is most interesting for true electronic systems. In QED$_3$, the dimensional coupling constant $\alpha = e^2$, which provides an intrinsic mass scale similar to $\Lambda_{QCD}$ in QCD. For simplicity, we set $\alpha = 1$ in this paper. The $4 \times 4$ gamma matrices can be defined as $\gamma_\mu = \sigma_3 \otimes (\sigma_2, \sigma_1, -\sigma_3)$, which satisfy the Clifford algebra $\{\gamma_\mu, \gamma_\nu\} = 2\delta_{\mu\nu}(\mu, \nu = 0, 1, 2)$. It is well known that the DSEs provides a successful description of various nonperturbative aspects of strong interaction physics in QCD \[32, 33\]. We naturally expect that it could be a useful nonperturbative approach in the study of spin susceptibilities and in this paper we will give a full formula for the staggered spin susceptibility.
In this work, we shall employ the linear response theory of fermion propagator to study the staggered spin correlation function \cite{29, 30}. Our starting point is the usual QED$_3$ Lagrangian added with an additional coupling term $\Delta \mathcal{L} = \bar{\psi} \gamma_5(\gamma_5) \gamma_\delta \psi_\delta \mathcal{V}(x)$, where $\gamma_5 = \sigma_2 \otimes 1$ and $\mathcal{V}(x)$ is a variable external pseudoscalar field. The fermion propagator $G_{\alpha\beta}[\mathcal{V}](x)$ in the presence of the external field $\mathcal{V}$ can be written as \cite{34, 36}

$$G_{\alpha\beta}[\mathcal{V}](x) = \int \mathcal{D}\bar{\psi} \mathcal{D}\psi \mathcal{D}A \psi_\alpha(x) \bar{\psi}_\beta(0) \exp\{-\int d^3x [\mathcal{L} + \Delta \mathcal{L}]\},$$

where the subscripts denote the spinor indices. If we only consider the linear response term of the fermion propagator $G_{\alpha\beta}[\mathcal{V}](x)$, we obtain

$$G_{\alpha\beta}[\mathcal{V}](x) = \int \mathcal{D}\bar{\psi} \mathcal{D}\psi \mathcal{D}A \psi_\alpha(x) \bar{\psi}_\beta(0) \exp\{-S[\bar{\psi}, \psi, A]\} + \int \mathcal{D}\bar{\psi} \mathcal{D}\psi \mathcal{D}A \int d^3y \psi_\alpha(x) \bar{\psi}_\beta(0) \bar{\psi}_\gamma(y)(\gamma_5) \gamma_\delta \psi_\delta(y) \mathcal{V}(y) \exp\{-S[\bar{\psi}, \psi, A]\} + \cdots, \tag{3}$$

where $G_{\alpha\beta}(x) = \langle 0 | T[\psi_\alpha(x) \bar{\psi}_\beta(0)] | 0 \rangle$ is the fermion propagator in the absence of the external field $\mathcal{V}$, the linear response term of the fermion propagator

$$G^\mathcal{V}_{\alpha\beta}(x) = \int d^3z \langle 0 | T[\psi_\alpha(x) \bar{\psi}_\beta(0) \bar{\psi}_\gamma(z)(\gamma_5) \gamma_\delta \psi_\delta(z)] | 0 \rangle \mathcal{V}(z) \tag{4}$$

and the ellipsis represents terms of higher order in $\mathcal{V}$.

Now we expand the inverse fermion propagator $G^{-1}[\mathcal{V}]$ in powers of $\mathcal{V}$ as follows

$$G^{-1}[\mathcal{V}] = G^{-1}[\mathcal{V}] \bigg|_{\mathcal{V}=0} + \frac{\delta G^{-1}[\mathcal{V}]}{\delta \mathcal{V}} \bigg|_{\mathcal{V}=0} \mathcal{V} + \cdots = G^{-1} + \mathcal{V} \Gamma_P + \cdots, \tag{5}$$

which leads to the following formal expansion

$$G[\mathcal{V}] = G - G \mathcal{V} \Gamma_P G + \cdots. \tag{6}$$

Here, the pseudoscalar vertex $\Gamma_P$ is defined as

$$\Gamma_P(y_1, y_2, z) = \frac{\delta G^{-1}[\mathcal{V}](y_1, y_2)}{\delta \mathcal{V}(z)} \bigg|_{\mathcal{V}=0}. \tag{7}$$

Note that Eq. (7) is a compact notation and its explicit form reads

$$G_{\alpha\beta}[\mathcal{V}](x) = G_{\alpha\beta}(x) - \int d^3y_1 d^3y_2 d^3z G_{\alpha\gamma}(x - y_1) \Gamma_P(y_1, y_2, z) \mathcal{V}(z) \gamma_\delta G_{\delta\beta}(y_2) + \cdots \tag{8}$$

$$= G_{\alpha\beta}(x) - \int d^3z \int \frac{d^3P}{(2\pi)^3} \int \frac{d^3q}{(2\pi)^3} e^{-i(q + 2P)x} e^{iP \cdot z} G_{\alpha\gamma}(q + \frac{P}{2}) \Gamma_P(q, P) \mathcal{V}(z) \gamma_\delta G_{\delta\beta}(q - \frac{P}{2}) + \cdots.$$
Setting $x = 0$ in Eq. (5) and comparing the linear response term in Eq. (9), we obtain

$$
\langle 0 | T[\psi_\alpha(0)\bar{\psi}_\beta(0)\bar{\psi}_\gamma(z)(\gamma_5)\gamma_\delta\psi_\delta(z)] | 0 \rangle
= -\int \frac{d^3P}{(2\pi)^3} \int \frac{d^3q}{(2\pi)^3} e^{iPz}G_{\alpha\gamma}(q + \frac{P}{2})[\Gamma_P(q, P)]_{\gamma\delta}G_{\delta\beta}(q - \frac{P}{2}).
$$

(9)

After multiplying $(\gamma_5)_{\beta\alpha}$ on both sides of Eq. (10) and summing over the spinor indices, we finally obtain

$$
\langle 0 | T[\bar{\psi}_\beta(0)(\gamma_5)_{\beta\alpha}\psi_\alpha(0)\bar{\psi}_\gamma(z)(\gamma_5)\gamma_\delta\psi_\delta(z)] | 0 \rangle
= \int \frac{d^3P}{(2\pi)^3} \int \frac{d^3q}{(2\pi)^3} e^{iPz}Tr[G(q + \frac{P}{2})[\Gamma_P(q, P)]G(q - \frac{P}{2})\gamma_5].
$$

(10)

The staggered spin correlation function in momentum space is

$$
\langle S^z(x)S^z(0) \rangle = \int \frac{d^3P}{(2\pi)^3} e^{ipx}\langle S^z(P)S^z(-P) \rangle,
$$

(11)

where the $z$-component of the electron spin density operator $S^z(x) = \bar{\psi}(x)\gamma_5\psi(x)$ [30]. From Eq. (11) and Eq. (12), one obtain the following model-independent expression for the staggered spin susceptibility

$$
\langle S^z(P)S^z(-P) \rangle = \int \frac{d^3q}{(2\pi)^3} Tr[G(q + \frac{P}{2})\Gamma_P(q, P)G(q - \frac{P}{2})\gamma_5].
$$

(12)

It should be noted that Eq. (13) is quite different from that given in the previous literature [21, 22, 29, 30]. It reduces into the staggered spin correlation defined in Ref. [30] when one uses the bare pseudoscalar vertex $\Gamma_{P0} = \gamma_5$.

From Eq. (13) it can be seen that the staggered spin susceptibility is closely related to the full fermion propagator and the pseudoscalar vertex. Once the full fermion propagator and the pseudoscalar vertex are known, one can calculate exactly the staggered spin susceptibility. However, at present it is still not possible to calculate the full fermion propagator and the pseudoscalar vertex from the first principles of QED$_3$. So, in this work we will adopt the DSE approach to calculate the fermion propagator and the pseudoscalar vertex.

The general form of the dressed fermion propagator is:

$$
G^{-1}(p) = i\gamma \cdot pA(p^2) + B(p^2),
$$

(13)

where $A(p^2)$ is the wave function renormalization and $B(p^2)$ is the self-energy function. As usual, one calls the phase in which $B(p^2) = 0$ the Wigner phase and the phase in which
$B(p^2) \neq 0$ the Nambu phase. Under rainbow approximation, the DSEs for the fermion propagator can be written as:

$$G^{-1}(p^2) = i\gamma \cdot p + \Sigma(p^2) = i\gamma \cdot p + \int \frac{d^3q}{(2\pi)^3} D_{\mu\nu}(p - q)\gamma_\mu G(q)\gamma_\nu, \quad (14)$$

where the gauge boson propagator in Landau gauge reads

$$D_{\mu\nu}(q) = \frac{1}{q^2[1 + \Pi(q^2)]}(\delta_{\mu\nu} - \frac{q_\mu q_\nu}{q^2}), \quad (15)$$

with $\Pi(q^2)$ being the polarization function. The pseudoscalar vertex $\Gamma_P$ satisfies an inhomogeneous Bethe-Salpeter equation, which in the ladder approximation reads

$$\Gamma_P(q, P) = \gamma_5 - \int \frac{d^3k}{(2\pi)^3} \gamma_\mu D_{\mu\nu}(k - q)G(k + \frac{P}{2})\Gamma_P(k, P)G(k - \frac{P}{2})\gamma_\nu. \quad (16)$$

We now focus on the low-energy behavior of the staggered spin susceptibility in Nambu phase and Wigner phase. From Lorentz structure analysis, $\Gamma_P(q)$ can be written as:

$$\Gamma_P(q) = \gamma_5 F(q^2) + i\gamma \cdot q \gamma_5 H(q^2). \quad (17)$$

Substituting Eq. (13) and Eq. (17) into Eq. (12), we finally obtain the staggered spin susceptibility in low energy limit

$$\langle S^z(0)S^z(0) \rangle = \int \frac{d^3q}{(2\pi)^3} \frac{4F(q^2)}{[q^2A^2(q^2) + B^2(p^2)]}, \quad (18)$$

where

$$F(q^2) = 1 + 2 \int \frac{d^3k}{(2\pi)^3} \frac{F(k^2)}{(k - q)^2(1 + \Pi[(k - q)^2])[k^2A^2(k^2) + B^2(k^2)]}. \quad (19)$$

From Eq. (18) we see that once the functions $A(p^2)$, $B(p^2)$ and $F(p^2)$ are known, the staggered spin susceptibility can be calculated. The remaining task is then to calculate these functions by numerically solving the corresponding DSEs.

By numerically solving the coupled DSEs for the fermion propagator, we can obtain the wave-function renormalization $A(p^2)$ and the fermion self-energy function $B(p^2)$. From these we can study the staggered spin susceptibility. In this paper, following Refs. [6, 7], we choose the vertex ansatz $\Gamma_\nu(p, k) = \frac{1}{2}[A(p^2) + A(k^2)]\gamma_\nu$ (the BC$_1$ vertex). Thus in the Landau gauge the coupled DSEs reads

$$A(p^2) = 1 + \frac{1}{p^2} \int \frac{d^3k}{(2\pi)^3} \frac{A(p^2) + A(k^2)}{A^2(k^2)k^2 + B(p^2)} \frac{A(k^2)(p \cdot q)(k \cdot q)/q^2}{[q^2(1 + \Pi(q^2))]}, \quad (20)$$
\[ B(p^2) = \int \frac{d^3k}{(2\pi)^3} \frac{B(k^2)}{[A^2(k^2)k^2 + B^2(k^2)][q^2(1 + \Pi(q^2))]} \cdot \] 

\[ \Pi(q^2) = N \int \frac{d^3k}{(2\pi)^3} \frac{A(k^2)A(p^2)[A(p^2) + A(k^2)]}{q^2[A^2(k^2)k^2 + B^2(k^2)]} \frac{[2k^2 - 4k \cdot q - 6(k \cdot q)^2/q^2]}{[A^2(p^2)p^2 + B^2(p^2)]}, \] 

where \( q = p - k \). By numerically solving the above coupled DSEs, one can obtain the momentum dependence of \( A(p^2) \) (in both Nambu phase and Wigner phase) and \( B(p^2) \) (in Nambu phase), which is plotted in Fig. 1 and Fig. 2, respectively. From Fig. 1 it can be seen that \( A(p^2) \) in Wigner phase has a power law behavior in the infrared region

\[ A(p^2) = cp^{2\kappa}, \] 

where the constant \( c \) and the power \( \kappa \) agree with the result in Ref. \[12\]. Here, we would like to stress that the characteristic anomalous exponents of stressed fermion propagator of the ASL or AFL have been widely studied in pure QED\(_3\) model. It has been conjectured long ago that the vector dressing function \( A(p^2) \) in Landau gauge is given by power laws in the infrared region in the chiral symmetric phase of massless QED\(_3\) \[37\]. But this conjecture has been validated only recently both analytically and numerically in Ref. \[12\] for the \( 1/N_f \) and more complete truncation schemes. The authors of Ref. \[12\] obtained the explicit anomalous dimension of the fermion vector dressing function in the infrared region by numerically

\[ \text{FIG. 1: The momentum dependence of } A(p^2) \text{ with } N = 2 \text{ in both Nambu phase and Wigner phase} \]
solving the full coupled set of DSEs. They found that the corresponding anomalous dimension in Wigner phase is different from the old $1/N_f$ expression given in Ref. [37]. In the DCSB phase the vector dressing function $A(p^2)$ does not have a power law behavior in the infrared region. The reason for this is as follows. In the DCSB phase a second scale, i.e., the dynamically generated fermion mass, emerges. This mass enters the photon polarisation and the equation for the $A(p^2)$ function and prevents a power law in the infrared. The deeper reason is that power laws always correspond to some kind of conformality, which usually does not exist once there exists a scale (the fermion mass) in the system. As discussed in Ref. [30], this change of behavior reflects the difference in physics between the AFL and the AF phase and should lead to a difference between the staggered spin susceptibility in the chiral symmetric phase and that in the DCSB phase, independent of the regularization procedure. In this paper, we try to show that the staggered spin susceptibility in Wigner phase is different from that in Nambu phase, since the anomalous dimension emerges and plays an important role only in Wigner phase.

Before numerically calculating the staggered spin susceptibility, let us analyze the large momentum behavior of the integrand in Eq. (18). From the large momentum behavior of $A(p^2)$, $B(p^2)$ and $F(p^2)$ we see that the staggered spin susceptibility given by Eq. (18) is linearly divergent and this divergence cannot be eliminated through the standard renormal-
ization procedure. This is very similar to the case of chiral susceptibility in QCD, which is quadratically divergent \[38, 39\]. In order to extract something meaningful from the staggered spin correlation, one needs to subtract the linear divergence of free staggered spin susceptibility from expression (18), which is in analogy to the regularization procedure in calculating the chiral susceptibility \[38, 39\]. That is to say, we define the regularized staggered spin susceptibility by

\[
<S_Z^Z(0)S_Z^Z(0)_R = <S_Z^Z(0)S_Z^Z(0)> - <S_Z^Z(0)S_Z^Z(0)>_{\text{free}}, \tag{24}
\]

where the free staggered spin susceptibility \(<S_Z^Z(0)S_Z^Z(0)>_{\text{free}}\) is calculated from Eq. (18) by setting \(F(q^2) = 1, A(p^2) = 1\) and \(B(p^2) = 0\) \[30\].

![FIG. 3: The dependence of \(<S_Z^ZS_Z^Z>_R\) on the number of fermion flavors in both Wigner phase and DCSB phase in the low energy limit](image)

After solving the above coupled DSEs by means of iteration method, we can now calculate the regularized staggered spin susceptibility \(<S_Z^ZS_Z^Z>_R\) given by Eq. (24) in the low energy limit in both Nambu phase and Wigner phase for the case of \(F(q^2) = 1\) and for several different number of fermion flavors. The dependence of \(<S_Z^ZS_Z^Z>_R\) on the number of fermion flavors in both Wigner phase and DCSB phase is shown in Fig. 3. From Fig. 3 it can be seen that when \(N\) approaches the critical number of fermion flavors, which is about 3.3 in the BC1 truncated scheme for DSE, the numerical values of the susceptibility
in Nambu phase and Wigner phase are almost the same. This can be understood as follows. From Fig. 4 it can be seen that when $N$ approaches the critical number of fermion flavors, the fermion propagator in Nambu phase tends to the one in Wigner phase. So the values of the susceptibility in these two phases should tend to be equal, which is what one expects in advance. From Fig. 3 it can also be seen that for small $N$ the susceptibilities in these two phases show apparent difference, and here let us analyze the case of small $N$. A physically interesting case is $N = 2$. For $N = 2$, the susceptibility in Wigner phase is 0.02904, and the susceptibility in Nambu phase is 0.02533. The reason for this can be seen as follows. From Fig. 1 it can be seen that $A(p^2)$ in the two phases coincide in the large momentum region, but show apparent difference in the infrared region. In the infrared region $A(p^2)$ in Nambu phase is constant, whereas $A(p^2)$ in Wigner phase shows a power law behavior. From Fig. 2 it is also seen that $B(p^2)$ in Nambu phase is nearly a constant in the infrared region, while it vanishes when $p^2$ is large enough. Therefore, it is easy to understand that when $N = 2$, due to the difference between $A(p^2)$, $B(p^2)$ in these two phases in the infrared region, the numerical values of the susceptibility in these two phases are different. In addition, our numerical results show that the smaller is $N$, the larger is the difference between the fermion propagators in Nambu phase and Wigner phase (this can be seen by comparing Fig. 2 and Fig. 4). This explains why the smaller is $N$, the larger is the difference between the spin staggered susceptibility in Nambu phase and Wigner phase, as is shown in Fig. 3.

In summary, in this paper, based on the linear response theory of the fermion propagator to an external pseudoscalar field, we first obtain a model-independent integral formula, which expresses the staggered spin susceptibility in terms of objects of the basic quantum field theory: dressed propagator and vertex. When one approximates the pseudoscalar vertex function by the bare one, this expression, which includes the influence of the nonperturbative dressing effects, reduces to the expression for the staggered spin susceptibility obtained using perturbation theory in previous works. After appropriately regularizing the additive linearly divergence, we study the staggered spin susceptibility by numerically solving the coupled DSEs in the low energy limit. Our results indicates that the staggered spin susceptibility enhances and antiferromagnetic correlation gets restored in both Nambu phase and Wigner phase.
FIG. 4: The momentum dependence of $A(p^2)$ with $N = 3$ in both Nambu phase and Wigner phase

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