Abstract

We discuss some known approaches and results as well as few new ideas concerning origins and nature of neutrino mass. The key issues include (i) connections of neutrino and charged fermions masses, relation between masses and mixing, energy scale of new physics behind neutrino mass where possibilities spread from the Planck and GUT masses down to a sub-eV scale. The data hint two different new physics involved in generation of neutrino mass. Determination of the CP phase as well as mass hierarchy can play important role in identification of new physics. It may happen that sterile neutrinos provide the key to resolve the riddle.

Keywords:
Neutrino masses and mixing, CP-violation, Quark-lepton unification

1. The riddle

There is something hidden and beyond the standards which

- strongly suppresses,
- badly confuses, and
- violates the law, or maybe, doesn’t (which is difficult to prove).

And probably the first and the second are because of the third. What is this?

Adapted to physicists this would sound as follows: What is behind of

1. Smallness of neutrino mass in comparison to masses of the charged leptons and quarks; weak (or no) mass hierarchy of neutrinos.

2. “Unusual” lepton mixing pattern with two large mixing angles (one being close to maximal) and one small which differs from the quark mixing;

3. Plausible violation of the lepton number.

Connected questions (mostly addressed to experiment): What is the type of mass spectrum (quasi-degenerate, hierarchical) and what is the mass ordering? What is nature of neutrino masses: Dirac versus Majorana, “hard” or “soft” (i.e., medium dependent)? Recall that in oscillation experiments we probe the dispersion relations and not masses immediately. Effective neutrino masses in oscillation experiments, in beta decay, in cosmology and $\beta\beta$-decay can be different.

Does the nature of neutrino mass differ from the nature of the quarks and charged lepton masses? Indeed, usual neutrino masses can be strongly suppressed, e.g. by the seesaw, so that “unusual contributions” dominate. Are sterile neutrinos, if exist, relevant for the solution of the riddle?

I will discuss some existing approaches and results (see also reviews), and present some new points. But before that let me challenge the riddle. Do we understand and interpret the data correctly? Are we asking right questions formulating the riddle? Is whole the story with neutrino mass misleading? For instance, concerning smallness of neutrino mass: Is it normal or special?
Special: comparing masses within the third fermion generation we have

\[ \frac{m_3}{m_\tau} \approx 3 \cdot 10^{-11}, \] (1)

and similar ratios are for other generations if neutrino spectrum is hierarchical.

Normal: neutrinos have no clear generation structure as well as the correspondence “light flavor - light neutrino mass”, especially if the mass hierarchy is inverted or spectrum is quasi-degenerate. Therefore comparison (1) can be misleading. Furthermore,

\[ \frac{m_3}{m_e} \approx \frac{m_e}{m_t} \approx 3 \cdot 10^{-6}, \] (2)

i.e., the same ratio. So, neutrino masses can be treated as a continuation of the mass spectrum of charged fermions with certain gap, probably due to neutrality of neutrinos. This appears even more plausible if originally the two heavier neutrinos had masses in the kev and MeV range, but due to some (new?) mechanism were suppressed by 3 - 6 orders of magnitude.

It is not excluded that the correct solution of the riddle (or the key to the solution) already exists among hundreds of approaches, models, mechanisms, schemes, etc. The problem is then to identify the correct solution. At the same time something fundamental can be missed.

In what follows I will make an assessments of several existing approaches and results in Sec. 2. Scales and scenarios of new physics will be discussed in Sec. 3. In Sec. 4 we consider mixing and CP-violation. Sterile neutrinos as the key to solve the riddle will be mentioned in Sec. 5. We conclude with some guesses.

2. Facts and Arguments

The most important aspects of the riddle include the following.

1. Leptons and Quarks: The riddle is formulated as comparison of neutrino mass and mixing with masses and mixing of quarks. There is no solution of the riddle of quark masses. Can we then solve the neutrino mass riddle? Do the efforts make sense? Yes, if

(i) neutrino mass generation and generation of the charged lepton and quark masses are independent. Examples of the corresponding mechanisms include: Higgs triplet [3], Radiative mechanisms [4,5], Seesaw type III [6], etc.

(ii) we try to explain only the difference of masses and mixing of neutrinos and quarks, and not the whole masses and mixing pattern.

(iii) we still hope (as it was before) that neutrinos will uncover something simple and insightful which will allow us to solve the fermion mass riddle in general.

2. Masses and Mixing: Should the mixing be included in the riddle? In the quark sector the answer is affirmative: relation between masses and mixing [7]

\[ \sin \theta_c \approx \frac{m_d}{m_s} + \ldots \] (3)

exists and Fritzsch (modified) ansatz gives its generalization to 3 generations. In the lepton sector there is no clear answer in view of observed approximate Tri-Bi-Maximal (TBM) mixing [8]. In the residual symmetry approach which explains the mixing [9], there is no connection between masses and mixing (at least in the lowest order). Mixing follows from the form invariance of the mass matrices independently of mass eigenvalues.

On the other hand, maximal mixing can be associated with quasi-degenerate mass states.

3. Mixing of quarks and leptons:

(i) completely related, with the only difference that originates from the Majorana nature of neutrinos;

(ii) Partially related, as it often appears in models with seesaw type I [10], quark-lepton unification, GUT, also with the seesaw type II [3].

(iii) Largely unrelated, if neutrino masses are generated by Higgs triplet, radiative mechanisms, seesaw type II and III.

4. The riddle and the Dark Universe. It can be deep connection of the neutrino mass riddle with other problems: Dark Energy and Dark radiation, baryon asymmetry in the Universe and inflation. E.g., the same symmetry can be responsible for smallness of the neutrino masses and stability of the dark matter particles. So, the solution may come from “heaven” or from completely unexpected side.

3. The riddle and new physics

Now, especially after first run of LHC we have the riddle of new physics: where it is?

3.1. Two types of new physics

It seems leptons “know” about quark mixing”. At the same time there is something qualitatively new in the lepton sector. Probably there are two types of new physics behind neutrino mass and mixing:

1. “The CKM type new physics” which is common for quarks and leptons. It is responsible for small quark mixing and hierarchical structure of the Dirac masses.
2. “Neutrino new physics” – an additional structure in the lepton sector, e.g. the Majorana mass matrix of the right handed neutrinos which realizes the see-saw mechanism. It is responsible for smallness of neutrino mass and large lepton mixing.

These two types are different but should somehow know about each other. A counter example: seesaw with degenerate RH neutrinos.

3.2. Scales and scenarios of new physics

The energy scales of proposed new physics behind neutrino mass $\Lambda_{NP}$ spread over 28 orders of magnitude: from the sub-eV up to the Planck scale. Three possibilities are motivated somehow:

1. GUT-Planck mass scale appears as

$$\Lambda_{NP} = \frac{V_{EW}^2}{m_\nu}.$$  \hspace{1cm} (4)

It is along with the unification line: high scale seesaw, $m_\nu = -m_D^2 M_R^2 m_D$, quark-lepton symmetry (analogy), GUT.

Here there are several possibilities:

a) The heaviest RH neutrino has $M_3 \sim M_{GUT} \approx 10^{16}$ GeV. This can be realized in the 3ν context in the presence of mixing.

b) $M_R = (10^5 - 10^{14})$ GeV, which can be obtained in the double seesaw mechanism as $M_{GUT}^2/M_{Pl}$ \[11\].

Gauge coupling unification, Leptogenesis and probably BICEP-II are in favor of this possibility.

c) $M_R = (10^{16} - 10^{18})$ GeV, which can be realized if many ($\sim 10^2$) heavy singlets (RH neutrinos) exist as is expected from string theory [12].

The GUT-Planck scale scenario has, however, the problem. The simplest seesaw implies new physical scale

$$M_R = m_D^2/m_\nu \approx 10^{14} \text{ GeV} \ll M_{Pl}.$$  \hspace{1cm} (5)

Correction to the Higgs boson mass due to coupling with RH neutrinos equals [13][14]

$$\delta m_H^2 = \frac{y^2}{(2\pi)^2} M_R^2 \log \left( \frac{q}{M_R} \right) \approx \frac{M_D^2 m_\nu}{(2\pi V_{EW})^2} \log \left( \frac{q}{M_R} \right),$$

where $V_{EW}$ is the electroweak VEV and $y$ is the Yukawa coupling. For usual seesaw with $M_R \sim 10^{14}$ GeV, one would get $\delta m_H^2 = (10^{13} \text{ GeV})^2$. The straightforward way to solve the problem is to reduce the scale of seesaw mechanism so that $M_R \sim 10^7$ GeV. This implies small yukawa couplings according to equation above:

$$y = \frac{2\pi \delta m_H}{M_R \sqrt{\log(m_H/M_R)}} \lesssim 2 \cdot 10^{-5},$$  \hspace{1cm} (5)

which in turn requires explanation. Another problem is that the mass $M_R$ is below the lower limit from successful leptogenesis: $M_R > 10^8$ GeV [15].

Possible solution could be some new physics which leads to cancellation of the $v_R$ contribution to $m_{H^\pm}$. E.g. due to loop with new scalars which have the couplings with usual Higgs as sneutrinos in SUSY. This is a kind of “ad hoc supersymmetry”. Cancellation will be absent at the two loop level. But this is enough to suppress the contribution $\delta m^2_{H}$, so that $M_R$ can satisfy the leptogenesis bound. Stronger cancellation at high loop level will require essentially reconstruction of complete SUSY. (See also [16].)

2. The electroweak - LHC scale:

$$\Lambda_{NP} = V_{EW} \pm E_{LHC}.$$  \hspace{1cm} (6)

The lower edge is motivated by already existing scale, whereas the upper one - mostly by logic of “looking under the lamp”.

Here there is no hierarchy problem (even without SUSY). New particles at (0.1 - few) TeV scale are expected which can be tested at LHC. LFV decays can be at the level of sensitivity of the present experiments. The low scale mechanisms include

1. Low scale seesaw, vSM [17], low scale LR symmetry model [18], R-parity violating SUSY with neutrino as RH neutrino [19], inverse seesaw with very small lepton violation term.

2. Radiative mechanisms with one, two, three loops; high dimensional operators; radiative see-saw.

3. Small VEV: Higgs triplet, new Higgs doublets. Some connection to Dark Matter can be realized. vSM deserves special attention in view of possible (although controversial) observation of the astrophysical 3.5 kev X-line and non-observation of new physics at LHC and other experiments. In vSM everything is below EW scale, and correspondingly, nothing is up to the Planck scale. This implies very small neutrino Yukawa couplings. The RH sector consists of two heavy RH neutrinos of few 100 MeV - GeV mass with extremely small (below eV) splitting. These neutrinos generate masses of active neutrinos via seesaw, and the lepton asymmetry in the Universe via oscillations. They can be produced in B-decays (BR $\sim 10^{-10}$) [20].

The third RH neutrino, $\nu_s$, has mass (3 - 10) keV and very small mixing with active neutrinos. It composes the “cooled” warm dark matter in the Universe and its radiative decays explains the 3.5 kev photon line. Higgs inflation can be realized here [21].

Several features pose doubts in this minimal scenario: in particular, extremely small splitting of $M_2$ and
M_3, and “decoupling” of ν_s from generation of the active neutrino masses. Indeed, contribution of ν_s to the masses of active neutrinos equals
\[ \delta m_a = \frac{1}{4} \sin^2 2\theta a m_s \approx (3 - 4) \cdot 10^{-7} \text{eV}, \] (7)
which is much smaller than the smallest relevant term of the mass matrix: \( \sim \sin \theta_1 \sqrt{\Delta m_{21}^2} \sim 4 \cdot 10^{-3} \text{eV} \). So, essentially this neutrino decouples from “seesaw” which indicates that ν_s is not normal RH neutrino. It can be that the standard (high scale) seesaw is realized with three RH neutrinos and ν_s is an additional state which mixes very weakly with neutrino system.

3. The eV - sub eV scale:
\[ \Lambda_{NP} \sim m_{\nu_s} \] (8)
that is, the neutrino mass itself can be the fundamental scale of new physics, and not just spurious quantity made of some other scales as in see-saw. This can be related to the dark sector of the Universe, dark energy, MAVAN as possible realization [22], existence of new relativistic (dark radiation). It is less explored possibility.

Very light dark sector may include (i) new scalar bosons (majoron, axions), (ii) new fermions (sterile neutrinos, baryonic neutrinos [23]), (iii) new gauge bosons (e.g. dark photons) [24]. This sector may be related to the eV-scale seesaw with RH neutrinos for LSND/MiniBooNE/reactor/Ga anomalies [25].

Tests of such a possibility include 5th force searches experiments; searches for modification of dynamics of neutrino oscillations, that is, checks of standard oscillation formulas, etc..

4. Mixing and CP-violation
4.1. PMNS and CKM

In a spirit of two types of new physics and partial relations of quark and lepton mixing we can assume that
\[ U_{PMNS} = U^{T}_{CKM} U_X, \] (9)
where \( U_{CKM} \) follows from the charged leptons or Dirac matrices of neutrinos in the flavor basis. This is the “CKM type new physics” which generates hierarchical structure (similar to \( V_{CKM} \)) and determined (as in Wolfenstein parametrization) by powers of \( \lambda \sim \sin \theta_C \). \( U_X \) comes from new “neutrino structure” and it is related to mechanism of neutrino mass generation which explains smallness of neutrino mass. It should be fixed to reproduce correct lepton mixing angles. Since \( V_{CKM} \approx I, U_X \approx U_{TBM} \).

The prediction from (9) is
\[ \theta_{13} \approx \frac{1}{\sqrt{2}} \theta_C. \] (10)
It has been obtained at purely phenomenological level in [26] and in the context of QLC [27]. The prediction is obtained if
\[ U_X = U^{23}(\pi/2) U_{12} \] (11)
with \( U_{12} \) being arbitrary. Maximal (or nearly maximal) 2-3 rotation is needed to explain nearly maximal \( \nu_\mu - \nu_\tau \) mixing. Special cases are \( U_X = U_{BM} \), which is realized in the QLC [27], and \( U_X \approx U_{TBM} \) in the so called TBM-Cabibbo scheme [28].

From (9) and (11) we obtain
\[ U_{PMNS} \approx U_{12}(\theta_C) U^{23}(\pi/2) U_{12}. \] (12)
To reduce this matrix to the standard form one needs to permute \( U_{12}(\theta_C) \) and \( U^{23}(\pi/2) \) which leads to appearance of \( U_{13}(\theta_C / \sqrt{2}) \). It gives also small deviation of the 2-3 mixing from maximal one.

It should be stressed however that the same value of 1-3 mixing can be obtained in other ways with completely different implications. Some possibilities are
\[ \sin^2 \theta_{13} = A \frac{\Delta m_{21}^2}{\Delta m_{31}^2}, \quad A = O(1), \] (13)
which follows from “naturalness” - an absence of fine-tuning in the mass matrix [29].
\[ \sin^2 \theta_{13} \approx \frac{1}{2} \cos^2 2\theta_{13} \] or \( \theta_{13} \approx \sqrt{2}(\pi/4 - \theta_{23}) \)
from relation between deviation of the 2-3 mixing from maximal. It was predicted in model with \( T' \) symmetry [30] but may also follow from the universal \( \nu_\mu - \nu_\tau \) symmetry violation [31]. Another interesting relation is
\[ \sin^2 \theta_{13} \approx \frac{1}{4} \sin^2 \theta_{12} \sin^2 \theta_{23}, \] (14)
which is analogous of the quark relation \( V_{ub} = 0.5V_{us}V_{cb} \). This may follow from a kind of Fritzsch ansatz for mass matrices (with texture zeros, \( U(1) \) symmetry, etc.). This implies similar structure of mass matrices of neutrinos and charged fermions but with different expansion parameter \( A_\ell \), furthermore the latter satisfies
\[ A_\ell \approx 1 - A_q. \] (15)
Expectations from this scenario are the normal mass hierarchy, certain relations between masses and mixing; flavor alignment in the mass matrix.
4.2. CP-phase prediction

Let us use the relation (9) which gives correct prediction for 1-3 mixing to get some generic results on the CP-phase. First, we assume that $U_{CKM}$ is the only source of CP-violation (similarly to what happens in the quark sector). So, there is no CP violation in $U_X$ (similar possibility has been considered previously in [33].) Then one gets [34]

$$\sin \theta_{13} \sin \delta_{CP} = (- \cos \theta_{23}) \sin \theta_{13}^L \sin \delta_q.$$  

(16)

Since $\sin \theta_{13} \sim \lambda$, $\sin \theta_{13}^L \sim \lambda^3$ and $\delta_q = 1.2 \pm 0.08$ rad, we obtain from (16)

$$\sin \delta_{CP} \sim \lambda^2 \sim 0.046,$$  

(17)

or $\delta_{CP} = \delta$ or $\pi + \delta$, where

$$\delta \approx \frac{\sin \theta_{13}^L}{\sin \theta_{13}} \cos \theta_{23} \sin \delta_q.$$  

(18)

Thus, if leptons have the same origin of CP-violation as quarks, the leptonic CP violation phase is small (unobservable) or very close to $\pi$ which can be observed in atmospheric neutrinos.

There are two implications of this result:

1). If future measurements show that the phase $\delta_{CP}$ deviates substantially from 0 or $\pi$, new sources of CPV beyond CKM should exist (e.g. from the RH neutrino sector), or another framework is realized.

2). New sources may have specific symmetries or structures which lead to particular values of $\delta_{CP}$, e.g. $-\pi/2$. Then CP from the LH rotation which diagonalizes the Dirac mass matrix gives just small corrections.

In general, the phase $\delta_{CP}$ can be large. Neglecting terms of the order $\sim \lambda^3$ we obtain [34]

$$\sin \delta_{CP} = \frac{\sin(a_d + \delta_c)V_{ud}[X_{e3}]}{s_{13}} - \sin a_c|V_{cd}|[X_{\mu3}],$$

where $a_d$, $\delta_c$, and $a_c$ are parameters of the RH neutrinos. Some special values of $\delta_{CP}$ can be obtained under certain assumptions. If, e.g., $X_{e3} = 0$, we have $\sin \delta_{CP} \approx - \sin a_c$. Furthermore, if $a_c = \pi/2$, then $\delta_{CP} \approx 3\pi/2$. One can easily find structure of the RH neutrino mass matrix which leads to these equalities.

In the seesaw type-I $U_X$ is the matrix which diagonalizes [34]

$$M_X = -m_D^{diag} U_R^* (M_R)^{-1} U_R^* m_D^{diag}.$$  

(19)

Here $m_D = U_3 m_D^{diag} U_R^*$, and we assume that $m_D^{\nu/\mu} \sim m_D^{\mu/\nu}$.

In contrast to quarks for the Majorana neutrinos the RH rotation that diagonalizes $m_D$ becomes relevant and contributes to $\delta_{CP}$. Important special case is model with L-R symmetry. In the L-R symmetric basis $U_X = U_R U_S$ with $U_S$ being the matrix which comes from seesaw. Due to the L-R symmetry: $U_R = U_L \sim V_{CKM}^\dagger$, and we assume that there is no CP violation in $M_R$. CP violation in $U_R$ is small. However, it turns out that seesaw itself can enhance this small CPV effect, so that resulting phase in the PMNS matrix is large [34]. The seesaw enhancement of the CP violation is related to strong hierarchy of the mass eigenvalues of $m_D$.

5. Steriles and neutrino portal

Effect of different sterile neutrinos on the 3ν structure can be parametrized as

$$m_\nu = m_\alpha + \delta m_\alpha,$$  

(20)

where the first term is the original active neutrino mass matrix, e.g., from see-saw, whereas the second one is the induced mass matrix due to mixing with sterile. Notice that $m_\alpha = 0.025$ eV in the case of hierarchical mass spectrum.

Three cases are phenomenologically motivated which correspond to different mass scales:

1). The keV mass scale sterile (νMSM): $\delta m_\alpha \ll m_\alpha$. Sterile neutrino decouples from generation of the light neutrino masses, as we said above.

2). The eV mass sterile with mixing required by LSND/MiniBooNE results in $\delta m_\alpha \sim m_\alpha$. This is not a small perturbation, $\delta m_\alpha$ can change structure (symmetries) of the original mass matrix completely. In general, it can be origin of difference of $U_{PMNS}$ and $U_{CKM}$.

3). meV steriles: $\delta m \ll m_\alpha$ [35]: again it can be considered as very small perturbation of the 3ν system.

Is $\nu_\tau$ the key to the solution of the riddle? Various issues we have discussed here (neutrino new physics, scales, symmetries) are related in one way or another to sterile neutrinos.

In general, we can wright effective neutrino interactions as

$$1 \Lambda^{8(F) - 3/2} LHF,$$  

(21)

where $H$ is the Higgs doublet, $F$ is the fermionic operator, which is singlet of the SM symmetry group, and $m(F)$ is the dimension of $F$. Through this “portal” neutrinos get mass and new physics may show up.

6. Instead of conclusion

Warning: Formulation of the riddle here may be misleading and we may misinterpret the data.
Some guesses: Two different types of new physics are involved in explanation of data: the CKM type common to quarks and leptons and physics responsible for smallness of neutrino mass and large lepton mixing. It makes sense to identify the second one, which explains the difference between quarks and leptons masses and mixing. Still generation of quark and neutrino masses can be essentially independent.

New physics at:
- high (GUT) scale: still updating;
- EW scale: wait and see LHC14 results;
- sub eV - eV scale: interesting and worth to explore further.

New neutrino physics may have certain symmetries which leads to specific values of mixing angles and CP phase. The CP-phase from the CKM part is strongly suppressed.

Sterile neutrinos (in one way or another) may turn out to be the key to the solution of the riddle.

References

[1] G. Altarelli and F. Feruglio, New J. Phys. 6 (2004) 106; R. N. Mohapatra and A. Y. Smirnov, Ann. Rev. Nucl. Part. Sci. 56 (2006) 569; G. Altarelli and F. Feruglio, Rev. Mod. Phys. 82 (2010) 2701; G. C. Branco, R. G. Felipe and F. R. Joaquim, Rev. Mod. Phys. 84 (2012) 515; S. F. King and C. Luhn, Rept. Prog. Phys. 76 (2013) 056201, S. F. King, A. Merle, S. Morisi, Y. Shimizu and M. Tanimoto, New J. Phys. 16 (2014) 045018.

[2] F. Capozzi, G. L. Fogli, E. Lisi, A. Marrone, D. Montanino and A. Palazzo, [arXiv:1312.2878] [hep-ph].

[3] W. Konechny and W. Kummer, Phys. Lett. B 70 (1977) 433; M. Magg and C. Wetterich, Phys. Lett. B 94 (1980) 61; J. Schechter and J. W. F. Valle, Phys. Rev. D 22 (1980) 2227; R. N. Mohapatra and G. Senjanovic, Phys. Rev. D 23 (1981) 165; G. B. Gelmini and M. Roncadelli, Phys. Lett. B 99 (1981) 411.

[4] K. S. Babu, Phys. Lett. B 203 (1988) 132, A. Zee, Phys. Lett. B 93 (1980) 389 [Erratum-ibid. B 95 (1980) 461], K. S. Babu, A. Patra and S. K. Rai, Phys. Rev. D 88, 055006 (2013).

[5] E. Ma, Phys. Rev. D 73 (2006) 077301 [hep-ph/0601275].

[6] R. Foot, H. Lew, X. G. He and G. C. Joshi, Z. Phys. C 44, 441 (1989).

[7] R. Gatto, G. Sartori and M. Tonin, Phys. Lett. B 28 (1968) 128.

[8] P. F. Harrison, D. H. Perkins and W. G. Scott, Phys. Lett. B 530 (2002) 167, L. Wolfenstein, Phys. Rev. D 18 (1978) 958.

[9] C. S. Lam, Phys. Rev. D 78 (2008) 073015, Phys. Rev. Lett. 101 (2008) 121602, C. S. Lam, Phys. Rev. D 87 (2013) 013001, W. Grimus, L. Lavoura and P. O. Ludl, J. Phys. G 36 (2009) 115007, W. Grimus and L. Lavoura, JHEP 0904 (2009) 013.

[10] P. Minkowski, Phys. Lett. B 67 (1977) 421, T. Yanagida, in Proc. of Workshop on Unified Theory and Baryon number in the Universe, eds. O. Sawai and A. Sugamoto, KEK, Tsukuba, (1979); M. Gell-Mann, P. Ramond and R. Slansky, in Supergravity, eds. P. van Nieuwenhuizen and D. Z. Freedman (North Holland, Amsterdam 1980), S. L. Glashow, in Quarks and Leptons, Cargese lectures, eds M. Levy, (Plenum, 1980, New York) p. , 707; R. N. Mohapatra and G. Senjanovic, Phys. Rev. Lett. 44, (1980) 912.