Amplification of mechanical quadratures using weak values

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Abstract
An interferometric arrangement is proposed in which the technique of weak value amplification is implemented in order to enlarge the effect of a single photon on the quadratures of a movable mirror of an optical cavity. The photon interacts weakly with the mirror via radiation pressure and is postselected in the dark port of the interferometer. The real and imaginary parts of weak values of angular momentum type photonic operators produce an amplification of the mirror quadratures, which is large as compared to the scenario in which all photons are taken into consideration, i.e. when no postselection is performed. The effect is studied both for a mirror initialized in a thermal and coherent states. For a thermal state, the weak value amplification effect is boosted with the number of particles of the mirror, which occurs due to the imaginary part of the weak values.

1. Introduction

The weak value of an observable is a complex value that shows up in a weak measurement of the observable followed by postselection [1–5]. Broadly speaking, a weak measurement is a measurement in which the interaction between the measurement device and the system is very weak. Consequently, after the weak measurement a tiny amount of information is obtained, but the quantum state of the system remains unperturbed. Postselection, on the other hand, is a second measurement of a projector (performed sequentially after the first measurement) with two possible outcomes: success or failure.

Studying the results of a (weak) measurement, or simply, analyzing the quantum state of the system, conditioning on a successful postselection [6], is interesting due to the appearance of quantum effects. One example is presented in [7], where it is shown that postselection may increase the quantum Fisher information (about a certain parameter unitarily encoded into the system) in a way ‘not allowed’ by a classical theory (in which operators commute). Another example is precisely the weak value, which, from a purely statistical perspective, may be regarded as the ensemble average of a weak measurement conditional on a successful postselection.

Weak values are objects interesting on their own [8–10], that have been used to study counterfactual paradoxes [11–14], wave-particle duality [15, 16] and are related to contextuality [17]. From the more practical side, weak values have applications in quantum state reconstruction [18–20] and metrology [21–26]. In metrological applications, the benefits rely mostly on a technique that has been called weak value amplification. Turns out that the magnitude of the weak value can lie outside the eigenvalue range, in which case it is said to be anomalous. An anomalous weak value will have therefore a larger effect on the measurement device as compared to a standard measurement, in which case the pointer variable (the ’apparatus needle’) is displaced by a quantity within the eigenvalue range. The increased effect of the system on the measurement device due to anomalous weak values will be referred as the weak value amplification effect. In metrological applications, it is worth mentioning that, although the measurement precision does not improve with the use of weak values, it is possible the reach the same level of precision with fewer data (recall that postselection may fail, i.e. the size of the whole data set will be reduced only to the successfully postselected events) [27–30].
In this article we are interested in applying the weak value amplification effect to enlarge the ‘momentum kick’ given by a single photon to a mechanical oscillator. Amplification of the effect of a single photon on an optomechanical cavity using postselection has been previously studied, e.g. in [31, 32]. In this work, we offer a description of the amplification effect by using weak values, which allows to ‘control’ the amplification by properly choosing the initial and final states of the system. In particular, we will study the interaction between a single photon (a microscopic degree of freedom) and a movable mirror of a Fabry–Pérot cavity (a macro or mesoscopic degree of freedom). Inside the cavity, the photon and the mirror interact via radiation pressure [33–35] which produces optomechanical entanglement [36]. Outside the cavity, the photon may be detected either in the dark or bright port of an interferometric arrangement. Postselection will be considered successful when detection occurs in the dark port. We will study the change of the quadratures of the mirror after the interaction, conditioning on the events for which the postselection was successful. Since the interaction between the mirror and the photon is weak, the change of the mechanical quadratures will depend on weak values of the photonic operators that take part in the interaction.

The structure of this article is as follows. In section 2 we review the weak value amplification effect. Our treatment, however, considers a (slightly) different interaction hamiltonian between the system and the measurement device, as compared to the standard hamiltonian presented in most literature on the subject. In section 3 we show how the previous effect may be applied to the interaction between a single photon and the moving mirror of an optical cavity. In section 4 our results are presented for a mirror initialized in thermal and coherent states. Finally, in section 5 the results are commented and summarized.

2. Weak value amplification

It will be assumed that the measurement device (MD) is a quantum harmonic oscillator, while the target system (or simply, the system) is a $q$-level system (qudit). The initial quantum states of the MD and the system are denoted as $\rho_S$ and $\rho_0$, respectively, while the initial joint state of the system and the MD is the product (uncorrelated) state $\rho_{S,M} = \rho_S \otimes \rho_M$. Expectation values on these initial states will be denoted as $\langle \hat{M} \rangle \equiv \text{Tr}(\hat{M}\rho_{S,M})$ and $\langle \hat{S} \rangle \equiv \text{Tr}(\hat{S}\rho_0)$, where $\hat{M}$ and $\hat{S}$ are any hermitian operators of the MD and the system, respectively. Note that expectation values computed over the initial states have no special subscript.

We will also assume that the interaction between the system and the MD is described by a hamiltonian of the form

$$\hat{H}_{\text{int}} = \tilde{g}(\hat{j}_X \hat{Y} + \hat{j}_Y \hat{X}),$$

where $\hat{j}_X, \hat{j}_Y$ are angular momentum operators of the system, and $\hat{X}, \hat{Y}$ are the quadratures of the MD, which satisfy $[\hat{X}, \hat{Y}] = i$. The real parameter $\tilde{g}$ represents the coupling strength between the system and the MD. As an example, the interaction between a two-level atom (a qubit) and a single cavity mode (a harmonic oscillator), typically described by the Jaynes–Cummings hamiltonian [37], has a similar form to (1) in the rotating wave approximation. It is worth to point out that this hamiltonian differs from the interaction hamiltonian of a measurement, since the latter corresponds to the product of a system variable (the measured observable) and an apparatus variable, and includes a Dirac delta function (or any impulse function) to describe the impulsive character of the measurement. This model is often referred as the von Neumann hamiltonian [2, 38].

When the free evolution of the MD and the system can be neglected, the interaction hamiltonian generates the evolution operator

$$\hat{U} = \exp[-i\int_{t_0}^{t_0+\Delta} \hat{H}_{\text{int}} dt] = \exp[-ig(\hat{j}_X \hat{Y} + \hat{j}_Y \hat{X})],$$

where $(t_0, t_0 + \Delta)$ represents the time interval in which the interaction between the MD and the system occurs. Note that $\int_{t_0}^{t_0+\Delta} \tilde{g} dt = \Delta \tilde{g} = g$.

Let $\hat{R} = \hat{S} \otimes \hat{M}$ be some joint observable of the system and the MD. The expectation value of $\hat{R}$ on the state after the interaction is $\langle \hat{R} \rangle_f \equiv \text{Tr}(\hat{R}\rho_{S,M}\hat{U}^\dagger) = \text{Tr}(\hat{U}^\dagger \hat{R} \hat{U}\rho_{S,M})$. Notice that expectation values computed over the final state have the subscript $f$. The transformation $\hat{U}^\dagger \hat{R} \hat{U}$ can be expanded in powers of $g$ using the well known operator expansion [37],

$$\hat{U}^\dagger \hat{R} \hat{U} = \hat{R} + ig[\hat{j}_X \hat{Y} + \hat{j}_Y \hat{X}, \hat{R}] + \frac{ig^2}{2!}[\hat{j}_X \hat{Y} + \hat{j}_Y \hat{X}, [\hat{j}_X \hat{Y} + \hat{j}_Y \hat{X}, \hat{R}]] + \ldots$$

If the interaction between the MD and the system is sufficiently weak so that the terms up to second order are small as compared with the first two terms of the expansion, then

$$\langle \hat{R} \rangle_f \approx \langle \hat{S} \rangle \langle \hat{M} \rangle + i(g/2)\langle \hat{j}_Y, \hat{S} \rangle \langle \hat{Y}, \hat{M} \rangle + i(g/2)\langle \hat{j}_X, \hat{S} \rangle \langle \hat{Y}, \hat{M} \rangle$$

or
where $[\cdot,\cdot]$ and $\{\cdot,\cdot\}$ are commutators and anticommutators, respectively. We will distinguish now two measurements strategies: (a) without postselection and (b) with postselection. When using the first strategy, we let the MD and the system interact weakly via (2). Then, the measurement device variable, $\hat{M}$, is observed, and $\langle \hat{M} \rangle_f$ is computed by repeating the experiment many times. By obtaining $\langle \hat{M} \rangle_f$, we expect to gain information about the system, as in any standard measurement. This strategy is illustrated in figure 1(a). On the other hand, when employing postselection, the MD and the system interact weakly via (2) and then a measurement of a projector is made. Finally, the measurement device variable $\hat{M}$ is observed. By repeating the experiment many times the average value of $\hat{M}$ is computed, but only considering the trials in which the postselection was successful (a conditional expectation value). The procedure is represented in figure 1(b). We now analyze expression (4) for each strategy.

2.1. Measurement strategy without postselection
In this case $\hat{R} = 1 \otimes \hat{M}$, which means that the variable $\hat{M}$ of the MD is observed in order to extract some information regarding to the system, as was previously explained. Since in this scenario $\hat{S} = 1$, expression (4) reduces to

$$\langle \hat{M} \rangle_f \approx \langle \hat{M} \rangle + ig\langle \hat{j}_y \rangle \langle \hat{Y}, \hat{M} \rangle + ig\langle \hat{j}_y \rangle \langle \hat{X}, \hat{M} \rangle.$$  

(5)

Thus, when $\hat{M} = \hat{X}$, $\langle \hat{X} \rangle_f - \langle \hat{X} \rangle \approx g\langle \hat{j}_y \rangle$, which shows that the change of the $X$ quadrature of the MD will be proportional to the expectation value of $\hat{j}_y$. On the other hand, if $\hat{M} = \hat{Y}$, then it is clear that $\langle \hat{Y} \rangle_f - \langle \hat{Y} \rangle \approx -g\langle \hat{j}_y \rangle$, i.e. the shift of the $Y$ quadrature will be proportional to $\langle \hat{j}_y \rangle$. In general, the change of $\hat{M}$ will be proportional to the expectation values of $\hat{j}_x$ and $\hat{j}_y$. This is what is expected from a weak measurement without postselection: the shift of the pointer will depend linearly on the expectation value of the system variables that appear in the interaction Hamiltonian (1), computed over the initial system state.

2.2. Measurement strategy with postselection
In this scenario we consider first the case $\hat{R} = \hat{P}_\phi \otimes 1$, where $\hat{P}_\phi = |\phi\rangle_S \langle \phi|_S$ is a projector into the pure (normalized) system state $|\phi\rangle_S$. In this situation, $\hat{M} = 1$ and expression (4) becomes

$$\langle \hat{P}_\phi \rangle_f \approx \langle \hat{P}_\phi \rangle + ig\langle [\hat{J}_x, \hat{P}_\phi] \rangle \langle \hat{Y} \rangle + ig\langle [\hat{J}_y, \hat{P}_\phi] \rangle \langle \hat{X} \rangle.$$  

(6)

The expectation value of a projector coincides with the probability to read the eigenvalue 1 of the projector. Therefore, expression (6) corresponds to the probability to read the eigenvalue 1 in a strong measurement of the projector, i.e. if the system and the MD interact weakly via the Hamiltonian (1) and then we measure strongly the projector $\hat{P}_\phi$, the probability to read the eigenvalue 1, and thereby projecting the system into the state $|\phi\rangle_S$, will be given by this expression. This probability is also called the probability to postselect the state $|\phi\rangle_S$ after the weak interaction between the MD and the system. Notice that, when the expectation values of the commutators vanish, then the probability simply corresponds to $\langle \hat{P}_\phi \rangle$, i.e. to the probability to project the initial state into the final state $|\phi\rangle_S$. Note also that the approximation (6) assumes that $\langle \hat{P}_\phi \rangle$ is larger than the higher order terms of the expansion, and thereby can not be arbitrarily small.

Suppose now that we perform a measurement of $\hat{R} = \hat{P}_\phi \otimes \hat{M}$ and ask for the expectation value of $\hat{M}$ conditioned to the cases in which the state $|\phi\rangle_S$ was successfully postselected. The conditional expectation value is given by

$$\text{E}(\hat{M} | f) = \frac{\langle \hat{P}_\phi \hat{M} \rangle_f}{\langle \hat{P}_\phi \rangle_f},$$  

(7)

![Figure 1](image-url)
where the numerator can be evaluated using (4) while the denominator corresponds to (6). The final result is

\[ E(\hat{M}) = \langle \hat{M} \rangle + ig \left( \frac{\langle \hat{J}_x, \hat{P}_y \rangle}{2 \langle \hat{P}_0 \rangle} \cdot \langle [\hat{Y}, \hat{M}] \rangle - g \frac{\langle \hat{J}_x, \hat{P}_y \rangle}{i \langle \hat{P}_0 \rangle} \cdot \text{Cov}(\hat{Y}, \hat{M}) \right) + ig \left( \frac{\langle \hat{J}_y, \hat{P}_x \rangle}{2 \langle \hat{P}_0 \rangle} \cdot \langle [\hat{X}, \hat{M}] \rangle - g \frac{\langle \hat{J}_y, \hat{P}_x \rangle}{i \langle \hat{P}_0 \rangle} \cdot \text{Cov}(\hat{X}, \hat{M}) \right), \]  

(8)

where \( \text{Cov}(\hat{A}, \hat{B}) = \langle [\hat{A}, \hat{B}] \rangle / 2 - \langle \hat{A} \rangle \langle \hat{B} \rangle \) is the covariance between the variables \( \hat{A} \) and \( \hat{B} \). This expression constitutes a direct application of equation (17) of [39] to a hamiltonian of the type (1). It shows that the change of \( \hat{M} \) depends both on the commutators and anticommutators between the system variables \( \hat{J}_x, \hat{J}_y \) and \( \hat{P}_x, \hat{P}_y \), divided by \( \langle \hat{P}_0 \rangle \). Therefore, the change may be larger than the one described by (5). When the initial system state is pure, i.e. \( \rho_S = |\psi\rangle \langle \psi|_S \), these terms are related to the real and imaginary parts of weak values,

\[ \Re(J_{x,w}) = \frac{\langle \hat{J}_x, \hat{P}_y \rangle}{2 \langle \hat{P}_0 \rangle}, \quad \Im(J_{x,w}) = -\frac{\langle [\hat{J}_x, \hat{J}_y] \rangle}{2 \langle \hat{P}_0 \rangle}, \]  

(9)

\[ \Re(J_{y,w}) = \frac{\langle \hat{J}_y, \hat{P}_x \rangle}{2 \langle \hat{P}_0 \rangle}, \quad \Im(J_{y,w}) = -\frac{\langle [\hat{J}_y, \hat{J}_x] \rangle}{2 \langle \hat{P}_0 \rangle}, \]  

(10)

where \( \Re(*) \) and \( \Im(*) \) denote the real and imaginary parts of *, while the complex values \( J_{x,w} \) and \( J_{y,w} \) correspond to the weak values of \( \hat{J}_x \) and \( \hat{J}_y \), respectively, which are defined as

\[ J_{x,w} = \frac{\langle \phi | \hat{J}_x | \psi \rangle}{\langle \phi | \psi \rangle}, \quad J_{y,w} = \frac{\langle \phi | \hat{J}_y | \psi \rangle}{\langle \phi | \psi \rangle}. \]  

(11)

Note that the subscript \( S \) has been omitted to simplify the expressions, but it should be clear that all states and variables refer to the system. The shift of \( \hat{M} \) depends therefore on both weak values, which may be larger than the expectation values appearing in (5). In order to amplify the change of \( \hat{M} \) it is useful to chose quasi orthogonal initial and final states. However, there is a limit to the amplification with weak values: the zero and first order terms of (4) and (6) should dominate over the higher order terms of the expansion.

### 3. Application to optomechanical system

In this section we show how the previous results can be applied to an optomechanical system (OMS). The OMS corresponds to an optical Fabry Pérot cavity with one movable mirror. The center of mass of the mirror is described as a single mode quantum harmonic oscillator of frequency \( \Omega \). The cavity mode is coupled to an infinite collection of external modes, as is depicted in figure 2. The hamiltonian \( \hat{H} \) that describes the dynamics of the whole system (the OMS coupled to the external modes) is given by

\[ \hat{H} = \hat{H}_{\text{OMS}} + \hat{H}_f + \hat{H}_{\text{dis}}, \]

\[ \hat{H}_{\text{OMS}} = \omega_{\text{om}} \hat{a}^\dagger \hat{a} + \Omega \hat{c}^\dagger \hat{c} - g \hat{a}^\dagger \hat{a} (\hat{c}^\dagger + \hat{c}), \]

\[ \hat{H}_f = \int_0^\infty d\omega \hat{b}^{\dagger}_\omega \hat{b}_\omega, \]

\[ \hat{H}_{\text{dis}} = \sqrt{\frac{1}{2\pi}} \left( \hat{a} \int_0^\infty d\omega \hat{b}^{\dagger}_\omega + \hat{a}^\dagger \int_0^\infty d\omega \hat{b}_\omega \right). \]  

(12)

![Figure 2. A single cavity mode interacts with a single mechanical mode (the center of mass of the movable mirror) via radiation pressure. The cavity mode in turn interacts with a set of infinite (sinusoidal) external modes.](image)
The first contribution, $\hat{H}_{\text{OMS}}$, describes the OMS. The cavity and mechanical mode operators are $\hat{a}$ and $\hat{c}$, respectively. The frequency of the cavity mode is denoted as $\omega_{\text{cav}}$. The parameter $g_0$ is the vacuum optomechanical coupling strength, that quantifies the strength of the radiation pressure interaction between a single cavity mode and the moving mirror. The second contribution, $\hat{H}_{\text{f}}$, is the free energy of the external modes. The third term, $\hat{H}_{\text{dep}}$, describes the coupling between the external field and the cavity mode, where $\Gamma$ is the decay rate of the cavity field through the fixed mirror.

We will assume that the initial state of the external field is a (pure) single photon state,

$$|\psi\rangle_{\text{S}} = \int_0^\infty d\omega G(\omega; \omega_0, \varepsilon) \hat{b}^+ |\phi\rangle_{\text{S}}, \quad G(\omega; \omega_0, \varepsilon) = \frac{1}{\pi} \frac{1}{\omega - \omega_0 + i\varepsilon},$$  \hspace{1cm} (13)

where $|G(\omega; \omega_0, \varepsilon)|^2$ is a Lorentzian spectral distribution. The parameter $\omega_0$ is the median of the distribution and $\varepsilon$ is the half-width at half-maximum of the spectrum (a measure of the width of the distribution). The state $|\phi\rangle_{\text{S}}$ is the multimode vacuum state. Experimentally, a signature of the purity of the states produced by a single photon source is given by the Hong-Ou-Mandel (HOM) interference [40, 41].

If the moving mirror starts in a number state $|n\rangle_{\text{M}}$ and the cavity is initially empty $|0\rangle_{\text{cav}}$, the Schrödinger equation can be solved analytically. In an interaction picture with respect to $\Omega \hat{c}^\dagger + \hat{H}_{\text{f}}$, the evolution of the external field and the OMS corresponds to

$$|n\rangle_{\text{M}}|\psi\rangle_{\text{S}} \rightarrow |n\rangle_{\text{M}}|\psi\rangle_{\text{S}} - \sqrt{2} g \hat{J}_+ |\psi\rangle_{\text{S}}|n\rangle_{\text{M}} + \sqrt{2} g \hat{J}_- |\psi\rangle_{\text{S}}^\dagger |n\rangle_{\text{M}}.$$  \hspace{1cm} (14)

The operators $\hat{J}_+, \hat{J}_-$ are ladder (angular momentum type) operators that will create single photons states with a shifted frequency spectrum. We will describe them below, but previously let us comment on different aspects regarding the process described by (14). In the first place, note that the probabilities to excite the oscillator to the state $|n+1\rangle_{\text{M}}$, or to produce a transition to $|n-1\rangle_{\text{M}}$, are proportional to $g^2$, where $g \equiv g_0/\Omega$, is a small scaled optomechanical parameter. Therefore, the initial states will be barely perturbed. Secondly, notice that cavity states do not appear in expression (14). This occurs because the cavity begins and ends in the vacuum state, and therefore it is not necessary to include it explicitly. In the third place, and as it is explained in detail in 6, the scattering process described by (14) is valid under different conditions, which are: (a) the OMS should operate in the resolved side band regime ($\Omega \gg \Gamma$), (b) the radiation pressure interaction should be weak ($g^2 n \ll 1$), (c) the photon should have a small detuning with respect to the cavity frequency, i.e. $\omega_0 = \omega_{\text{cav}} + \delta$, where the detuning is $\delta \equiv -g_0^2/\Omega$, and (d) be nearly monochromatic ($\varepsilon \ll \Gamma$), and (e) the interaction should last for long times ($t \gg \varepsilon^{-1}$). Now, let us turn into the definition of $\hat{J}_+, \hat{J}_-$, considering the following boson creation operators

$$\hat{b}_{-1} \equiv \int_0^\infty d\omega G(\omega; \omega_0 - \Omega, \varepsilon) \hat{b}^+, \quad \hat{b}_0 \equiv \int_0^\infty d\omega G(\omega; \omega_0, \varepsilon) \hat{b}^+, \quad \hat{b}_1 \equiv \int_0^\infty d\omega G(\omega; \omega_0 + \Omega, \varepsilon) \hat{b}^+.$$  \hspace{1cm} (15)

It is easy to note that $|\psi\rangle_{\text{S}} = \hat{b}_0^+ |\phi\rangle_{\text{S}}$, and $|d\rangle_{\text{M}} \equiv \hat{b}_{-1}^+ |\phi\rangle_{\text{S}}$ is a single photon state with the frequency spectrum ‘shifted to the left’ by $\Omega$, while $|u\rangle_{\text{S}} \equiv \hat{b}_1^+ |\phi\rangle_{\text{S}}$ is a single photon state with the frequency spectrum ‘shifted to the right’ by $\Omega$. Since there is no frequency overlap between the states, it is clear that $\langle \psi\rangle_{\text{S}} |u\rangle_{\text{S}} = \langle \psi\rangle_{\text{S}} |d\rangle_{\text{M}} = 0$. Also, all the operators satisfy the usual boson commutation relations, and all the states $|\psi\rangle_{\text{S}}, |u\rangle_{\text{S}}$ and $|d\rangle_{\text{M}}$ are normalized. Using these definitions, we introduce the ladder operators as follows,

$$\hat{J}_+ \equiv \sqrt{2} (\hat{b}_0^+ \hat{b}_{-1} + \hat{b}_{-1}^+ \hat{b}_0), \quad \hat{J}_- \equiv \sqrt{2} (\hat{b}_0^+ \hat{b}_{-1} - \hat{b}_{-1}^+ \hat{b}_0).$$  \hspace{1cm} (16)

Notice that $\hat{J}_+ |\phi\rangle_{\text{S}} = \sqrt{2} |u\rangle_{\text{S}}$ and $\hat{J}_- |\phi\rangle_{\text{S}} = \sqrt{2} |d\rangle_{\text{M}}$, i.e. the ladder operators applied to the initial state simply shift its spectrum to the left or to the right. Making an analogy to a system with angular momentum, the state $|\phi\rangle_{\text{S}}$ would be a state with angular momentum numbers $j = 1, m = 0$, the state $|u\rangle_{\text{S}}$ would be defined by $j = 1, m = 1$ and the state $|d\rangle_{\text{M}}$ by $j = 1, m = -1$.

Thereby, when the initial state of the external field is $|\psi\rangle_{\text{S}}$, the cavity is empty, and conditions a-e are satisfied, then the interaction between the external field and the moving mirror, can be described by an effective evolution operator

$$\hat{U} = \exp[\sqrt{2} g (\hat{J}_+ \hat{c}^\dagger - \hat{J}_- \hat{c})] = \exp[-ig (\hat{J}_+ \hat{Y} + \hat{J}_- \hat{X})],$$  \hspace{1cm} (17)
4. Results

In this section we compare both strategies, assuming that the mirror is prepared in a thermal or coherent state, which belong to the family of Gaussian states.

4.1. Without postselection

The measurement without postselection is described by equation (5). However, in order to account for the free evolution of the mirror, the variable $\hat{M}$ of the right hand side should be replaced by

$$\hat{M}(t) = \exp(-i\hat{M} \omega t) \hat{M} \exp(-i\hat{M} \omega t).$$

Note that given the initial system state (13), the expectation values $\langle \hat{J}_x \rangle = \langle \hat{J}_y \rangle = 0$. Therefore, there is no change of $\hat{M}$ to first order in $g$ besides its free evolution, i.e. $\langle \hat{M} \rangle = \langle \hat{M}(t) \rangle$. However, by using a different initial state $\langle \hat{J}_x \rangle \sim \langle \hat{J}_y \rangle \sim 1$, i.e. the expectation values may be in the order of magnitude of the largest eigenvalue of the variables. Consider the observation of the $X$ or $Y$ quadrature of a mirror initially prepared in a thermal or coherent state. In these cases, expression (5) becomes

$$\langle \hat{X} \rangle_f - \langle \hat{X}(t) \rangle = g [\cos(\Omega t) - \sin(\Omega t)],$$

$$\langle \hat{Y} \rangle_f - \langle \hat{Y}(t) \rangle = g [\sin(\Omega t) + \cos(\Omega t)],$$

which shows that both variables oscillate with an amplitude equal to the coupling constant $g$. Recall that the terms $\langle \hat{X}(t) \rangle$ and $\langle \hat{Y}(t) \rangle$ correspond to the free evolution of the quadratures. For a thermal state both are zero, while for a coherent state $\alpha$ the expectation values are $\langle \hat{X}(t) \rangle = \sqrt{2} [\cos(\Omega t) \Re(\alpha) - \sin(\Omega t) \Im(\alpha)]$ and $\langle \hat{Y}(t) \rangle = \sqrt{2} [\cos(\Omega t) \Im(\alpha) + \sin(\Omega t) \Re(\alpha)]$.

For a thermal or coherent state, the commutators $\{\hat{X}, \hat{M}\}$ and $\{\hat{Y}, \hat{M}\}$, appearing in (5), are in the order of magnitude of the unity, when $\hat{M}$ is any quadrature or rotated quadrature. Therefore, and since $\langle \hat{J}_x \rangle \sim \langle \hat{J}_y \rangle \sim 1$ (i.e. near their maximum values), the amplitude of the oscillations (18) is the coupling constant $g$.

4.2. With postselection

In order to consider the strategy with postselection a method to select a pure state should be implemented. We will consider the interferometric arrangement depicted in figure 3, by which the detection of a photon in the so called dark port will select the state

$$\ket{\phi}_S = \delta e^{i\theta} \ket{\psi}_S - i\sqrt{1 - \delta^2} \ket{d}_S,$$

where $\delta$ is a real parameter between 0 and 1. A justification of the postselected state is presented in Appendix C. According to expression (11), the quantum weak values of $\hat{J}_x$ and $\hat{J}_y$ are
Note that both weak values are complex values with the same amplitude but with a phase difference of $\pi/2$ (using a state $|\phi\rangle$ of a more general form, weak values with different amplitudes and phases can be obtained).

### 4.2.1. Thermal states

Consider again the observation of the $X$ or $Y$ quadrature of a mirror prepared in a thermal state. In this case, expression (8) shows that

$$E(\hat{X}|f) = 2(1 + N) \left( \frac{g}{\delta} \right) \sqrt{\frac{1 - \delta^2}{2}} \sin(\Omega t - \theta),$$

$$E(\hat{Y}|f) = 2(1 + N) \left( \frac{g}{\delta} \right) \sqrt{\frac{1 - \delta^2}{2}} \cos(\Omega t - \theta).$$

(21)  

(22)

By comparing these results to (18), it is easy to note that the amplification depends on (i) the amplitude of the weak value ($\sim\delta^{-1}$) and (ii) the mean number of phonons $N \equiv \langle \hat{n}\rangle$. Therefore, the smaller is the parameter $\delta$ (which will enlarge the weak value) and the larger is the mean number of phonons $N$, the larger will be the amplification.

It is worth noting that the amplification depends mainly on the imaginary part of the weak values, which couple to the covariance terms, as may be seen in (8). For a thermal state, the covariances depend on the mean number of phonons, which produces the term $\sim N/\delta$ in (21) and (22). On the other hand, the real part of the weak value couples to the commutators, whose order of magnitude is equal to unity, which contributes with the term $\sim \delta^{-1}$ in (21) and (22).

The limit on the amplification is given by the validity of the approximations (4) and (6). As shown in Appendix B, the limit is given by $\delta \gg g\sqrt{N}$, while the postselection probability (6) corresponds to $\langle \hat{P}_f \rangle \approx \delta^2$. Thus, a smaller value of $\delta$ will produce a larger amplification at the cost of a smaller postselection probability. On the other hand, a large $N$ will increase the amplification effect, but at the same time will limit the values that $\delta$ can take. Hence, there is a trade-off between $\delta$ and $N$. In any case, as compared to the set of equation (18), the change of the quadratures described by (21) and (22) may be considerable larger, as can be seen in figure 4.
4.2.2. Coherent states

For a coherent state, from equation (8), it is easy to note that

\[ \langle \hat{X}(t) \rangle = 2 \left( \frac{g}{\delta} \right) \sqrt{\frac{1 - \delta^2}{2}} \sin(\Omega t - \theta), \]

\[ \langle \hat{Y}(t) \rangle = 2 \left( \frac{g}{\delta} \right) \sqrt{\frac{1 - \delta^2}{2}} \cos(\Omega t - \theta). \]

Notice that the amplification depends now only on the magnitude of the weak values \( \sim \delta^{-1} \). The mean number of phonons, \( N \), does not show up because in this case the commutators and covariances appearing in expression (8) are both of the order of magnitude of the unity, unlike the previous case of a thermal state where the covariance depended on the number of particles. On the other hand, since the limit on the amplification is defined by the restriction \( g \sqrt{N} \ll \delta \), the amplification may be larger when \( N \) is close to unity (the MD contains a small number of phonons), as shown in figure 5. From an amplification perspective, there is no trade-off between \( N \) and \( \delta \), i.e. the smaller \( \delta \) the larger the amplification. Nevertheless, the postselection probability depends both on \( N \) and \( \delta \), as can be seen in equation (B4). Therefore, the mean number of phonons, although does not affect the amplification, can modify the number of successfully postselected events.

5. Conclusions

In this work we have studied the amplification of the quadratures of a moving mirror that weakly interacts with a single photon, mediated by a Fabry-Pérot cavity, when postselection of photons is performed using an interferometric arrangement. The amplification of the effect of a single photon on the mirror quadratures depends on the weak values of two hermitian operators (\( \hat{J}_x \) and \( \hat{J}_y \)), whose magnitudes can lie outside the eigenvalue range. The magnitude of the weak values is \( \sim \delta^{-1} \), where \( \delta \) is a small parameter that corresponds to the transmittance of a beam splitter located at the exit of the interferometric arrangement, while the postselection probability \( \sim \delta^2 \) is the fraction of light arriving at the dark port of the interferometer. The limit on the amplification by weak values is given by the restriction \( g \sqrt{N} \ll \delta \leq 1 \), where \( g \) is a scaled coupling constant between the photon and the movable mirror of the cavity, and \( N \) is the mean number of particles of the mirror. As compared to a standard strategy that does not postselect photons (but employs all of them) the amplification gain may be large (one or two orders of magnitude larger). On the other hand, the phases of the weak values may be set in order to adjust the time at which the amplification reaches its maximum value. The real parts of the weak values couples to the commutators between the mechanical quadratures, while the imaginary parts to the covariances between the quadratures. For some states, such as thermal states, the latter depends on the mean number of particles of the MD, which increases by \( N \) the weak value amplification effect.
produced by the single photon. Therefore, in these cases, the amplification occurs mainly due to the imaginary part of the weak values. For coherent states, on the contrary, both terms (commutators and covariances) are of the order of the unity and the amplification effect is based only on the anomalous weak values. Since the weak value amplification effect has been useful for parameter estimation in certain experimental situations, the proposed setup might be useful for the estimation of $g$, a parameter that can be small for different optomechanical systems. On the other hand, large weak values may be advantageous for hypothesis testing [42, 43]. Consequently, the weak values described in our setup could possibly be applied to the problem of interaction detection, i.e. to decide whether the interaction between the photon and the mirror actually occurred or was absent. Additionally, weak values have been considered as a quantum effect [44, 45], which makes interesting on its own their application in mesoscopic systems [46].

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**Data availability statement**

The data that support the findings of this study are available upon reasonable request from the authors.

**Appendix A: Interaction between OMS and external modes**

The hamiltonian (12) preserves the total number of photons $\hat{a}^\dagger \hat{a} + \int d\omega (\hat{b}^\dagger \hat{b} + \text{H.c})$. Since the initial state has one photon, the state of the whole system (the external optical modes, the cavity mode and the mechanical mode) at any time, $|\psi(t)\rangle$, will belong to the single photon Hilbert space, and can be expressed as

$$|\psi(t)\rangle = |1\rangle_{\text{av}} \sum_{m=0}^{\infty} A_m(t) |m\rangle_{\text{OMS}} + |0\rangle_{\text{av}} \sum_{m=0}^{\infty} |m\rangle_{\text{OMS}} \int_0^\infty d\omega B_m(\omega, t) \hat{b}^\dagger |\phi_s\rangle \quad (A1)$$

where $|\hat{m}\rangle_{\text{OMS}}$ is a displaced number state [47], defined as $|\hat{m}\rangle_{\text{OMS}} \equiv \exp \left( \frac{\hat{b}^\dagger}{\sqrt{\Omega}} (\hat{c}^\dagger - \hat{c}) \right) |m\rangle_{\text{OMS}}$, while $A_m(t)$ and $B_m(\omega, t)$ are the probability amplitudes that satisfy the normalization condition of the state. The first term describes the situation in which the photon is outside the cavity, the oscillator in some linear combination of displaced states, and the external field in the vacuum state. The second term corresponds to the case in which the photon is outside the cavity and eventually entangled to the oscillator (while the cavity is empty).

Initially, there are no photons inside the cavity, and therefore $A_m(0) = 0$. Also, the photon is initially outside the quantum state (13) and the mirror in some number state $n$ (unentangled with the photon), which means that $B_m(\omega, 0) = \delta_{m,n} G(\omega; \omega_0, \epsilon)$. Given these initial conditions and the hamiltonian (12), the Schrödinger equation, $i\partial \psi(t) / \partial t = \hat{H} |\psi(t)\rangle$, can be solved analytically, i.e. it is possible to obtain exact expressions for the probability amplitudes $A_m(t)$ and $B_m(\omega, t)$. This corresponds to solving the single photon scattering problem by an optomechanical cavity, which was done in [48] by using the Laplace transform method to solve the corresponding differential equations. From that work we take equation (10) to point out that, for long times (t $\gg$ 1 and $\Gamma \gg 1$), the solution (in a rotating frame with respect to the free hamiltonians of the mirror and the external field) is and

$$A_m(t) = 0, \quad B_m(\omega, t) = G(\omega; \omega_0, \epsilon) \delta_{m,n} - i\sqrt{2\pi \Gamma} G(\omega; \omega_0)$$

$$+ [n - m] \Omega \delta_{\langle k \rangle M, \Gamma / 2}, \quad (A2)$$

where $\Delta_{k,m} = \omega_{av} + \Omega (k - m) - \frac{\gamma_0^2}{\Omega}$ and $C_m(k) = \langle m \rangle_{\text{OMS}} |k\rangle_{\text{OMS}} \langle k | n\rangle_{\text{OMS}}$. The first solution, (A2), shows that no photons remain inside the cavity. The second solution, (A3), describes the entangled state of the photon and the mirror. The first contribution in the right hand side of (A3) describes a photon being simply reflected by the cavity, without interacting with the mirror. The second contribution represents the scenario in which the photon enters the cavity and interacts with the mirror takes places.

The Lorentzian functions $G(\omega; \Delta_{k,m}, \Gamma / 2)$ that appear inside the summation in the right hand side of (A3) do not overlap in the resolved sideband regime, $\Gamma \ll \Omega$. If $\omega_0$ is chosen to be $\omega_{av} - \gamma_0^2 / \Omega$ (the photon has a small detuning with respect to the cavity frequency) and the spectral width of the pulse is not larger than the cavity decay rate, $\epsilon \ll \Gamma$, then all terms in the summation, for which $k = n$, will have a negligible overlap with the first factor, $G(\omega_0; \omega_0) + [n - m] \Omega \delta_{\langle k \rangle M, \Gamma / 2}$. Therefore, in this scenario only the term $k = n$ will survive, and we can thereby perform the approximation
Furthermore, if we assume that the pulse is quasi monochromatic, i.e. $\varepsilon \ll \Gamma$, then the product of the two Lorentzian functions in the right hand side can be approximated by $G(\omega; \omega_0) + [n - m]\Omega, \varepsilon)G(\omega = \Delta_{n,m}\Delta_{n,m}\Gamma/2)$, i.e. the second factor is replaced by its maximum value. Consequently,

$$B_m(t) = G(\omega; \omega_0)\varepsilon)\delta_{n,m} - 2C_m(n)G(\omega; \omega_0 + [n - m]\Omega, \varepsilon).$$

When $g\sqrt{N} \ll 1$, then $C_m(n) = 1, C_n-1(m) = -g\sqrt{n}$ and $C_n+1 = g\sqrt{n + 1}$, while the rest of the coefficients are neglectable. Therefore, $B_m(\omega, t) = -g(\omega; \omega_0, \varepsilon), B_{n-1}(\omega, t) = 2g\sqrt{n}G(\omega; \omega_0 + \Omega, \varepsilon)$, and $B_{n+1}(\omega, t) = -2g\sqrt{n + 1}G(\omega; \omega_0 - \Omega, \varepsilon)$, from which the transition (14) can be directly derived by using (A1).

### Appendix B. Weak value amplification regime

The conditional expectation value of the MD variable $\hat{M}$ is given by (7). Both, numerator and denominator, should be expanded up to first order in $g$ to derive expression (8), in which weak values show up. First, we focus on the numerator, $\langle \hat{P}_0\hat{M}\rangle_f$. In sections 3 and 6 it was pointed out that $g\sqrt{N} \ll 1$ allows a first order expansion of the operator (17). Therefore $\hat{U}\hat{P}_0\hat{M}\hat{U}_f$ will have up to second order terms in $g$. Consequently,

$$\langle \hat{P}_0\hat{M}\rangle_f = \delta^2 c_0 - g\delta \sqrt{\frac{1 - \delta^2}{2}} + \delta^2(1 - \frac{\delta^2}{2})c_1,$$

where the coefficients $c_0, c_1$ and $c_2$ depend on the state of the MD. In general,

$$c_0 = \langle \hat{M}(t) \rangle,$n$$

$$c_1 = \langle [\hat{X}, \hat{M}(t)] \rangle \sin(\theta) + i\langle [\hat{X}, \hat{M}(t)] \rangle \cos(\theta),$$

$$c_2 = \langle \hat{X}\hat{M}(t)\hat{X} \rangle + \langle \hat{Y}\hat{M}(t)\hat{Y} \rangle + 2\gamma(\langle \hat{X}\hat{M}(t)\hat{Y} \rangle).$$

In order to reduce expression (B1) to a first order expansion, we have to take into account the state of the MD. For a thermal state (and $\hat{M}$ being any quadrature), it is easy to show that $c_0 = c_2 = 0$. Hence, expression (B1) is automatically of first order. For a coherent state, two conditions will be required: (a) $\delta^2c_0 \gg g^2c_2$ and (b) $g\delta c_1 \gg g^2c_2$. Working out the coefficients it is possible to show that $c_0 \sim \sqrt{N}, c_2 \sim (1 + N^2)$ and $c_2 = \sqrt{N}(N + 1)$. Hence, both conditions will be satisfied when $g\sqrt{N} \ll \delta$ (assuming $N \geq 1$).

Next, let us take into consideration the denominator in (7), i.e. the postselection probability,

$$\langle \hat{P}_0\rangle_f = \delta^2 - 2g\delta \sqrt{\frac{1 - \delta^2}{2}} [\langle \hat{X} \rangle \sin(\theta)]$$

$$+ [\langle \hat{Y} \rangle \cos(\theta)] + 2g^2(1 - \frac{\delta^2}{2})(N + 1).$$

For a thermal state, the first order term vanishes. Therefore, in order to disregard the second order term we need to assume that $\delta^2 \gg g^2N$ (in the scenario $N \geq 1$). Under this assumption, the postselection probability simply corresponds to $\langle \hat{P}_0\rangle_f \approx \delta^2$. For a coherent state, on the contrary, the first order term does not vanish. There will be, as in the previous analysis, two conditions (the zero and first order terms should dominate over the second order term). In the case $N \geq 1$ both conditions are satisfied when $g\sqrt{N} \ll \delta$. In general, for a MD in a coherent state $\sqrt{N}e^{i\beta}$ the postselection probability (B3) will depend linearly on $g$, i.e.

$$\langle \hat{P}_0\rangle_f \approx \delta^2 - 2g\delta \sqrt{1 - \delta^2} \sqrt{N} \sin(\theta - \beta).$$

Consequently, and as a summary, the limit on the amplification is defined by the restriction $g\sqrt{N} \ll \delta$, both for a thermal and coherent states.

### Appendix C: Postselection

The beam splitter (BS) located at the exit of the interferometer, and included in figure 3, is described in figure C1 with more detail. The relationship between the input and output modes is given by
where $t_i$ and $r_i$ are the complex transmittances and reflectances of each side of the beamsplitter, respectively, whose values are given in the caption of figure 3. The transformation of the fields \((C_1)\) allows to transform states between the inner and outer paths of the interferometer. Consider a single photon in the outer arm associated to the dark port, where $r_2$ and $t_1$ denote complex conjugation. Consequently, counting a photon in the dark port is equivalent to select the state \((19)\).

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Figure C 1. unbalanced beam splitter located at the output of the interferometric arrangement presented in figure 3, where the fields $\hat{b}_0$ and $\hat{b}_{-1}$ are mixed coherently.
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