Why there is no spin-orbit inversion in heavy-light mesons?

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Abstract

We show that the absence of spin-orbit inversions in heavy-light mesons can be explained by the chiral radiative corrections in the potential model. A new potential model estimate is given of the masses for P-wave bottom mesons.
I. INTRODUCTION

It was suggested long ago by Schnitzer that the strong spin-orbit interaction of the scalar confining potential would lead to spin-orbit inversions in P-wave heavy-light mesons, with the claim that their observation would confirm the scalar nature of the confining potential [1]. This spin-orbit inversion was later reaffirmed in studies with more sophisticated potential models [2, 3]. However, contrary to these studies, the observed masses of the P-wave charmed mesons do not exhibit spin-orbit inversion.

In this paper we show that if the one loop chiral corrections are taken into account then spin-orbit inversions disappear, and the experimental data can be understood within the potential model. Our result suggests that the absence of spin-orbit inversions in the observed P-wave mesons should not be interpreted as the failure of potential model or the confining potential be of non-scalar type, but, rather, should be regarded as a support for the potential model that is augmented by radiative corrections.

II. THE MODEL

We use the relativistic potential model of heavy-light system [4, 5] based on the chiral quark model, with the axial coupling of the light mesons put in explicitly. The Lagrangian reads

\[ \mathcal{L} = \Psi^\dagger (i\partial_0 - H) \Psi + g_A \bar{\Psi} A \gamma_5 \Psi + \mathcal{L}_\Pi \]

\[ \approx \Psi^\dagger (i\partial_0 - H) \Psi + \frac{gA}{2f_\pi} \bar{\Psi} \gamma^\mu \gamma_5 \Psi \partial_\mu \Pi_{ij} + \mathcal{L}_\Pi, \]  

(1)

where \( \Pi = \sum_{a=1}^{8} \pi^a \lambda^a \) and \( \Psi_i = (u, d, s) \) are the light octet mesons and the light quark fields, respectively, and \( \mathcal{L}_\Pi \) denotes the chiral Lagrangian for the light mesons, and

\[ A_\mu = \frac{i}{2} (\xi^\dagger \partial_\mu \xi - \xi \partial_\mu \xi^\dagger) \]  

(2)

with \( \xi = e^{i\Pi/2f_\pi} \). The Hamiltonian \( H \) is given by

\[ H = H_0 + \frac{1}{M} H_1 + \cdots \]  

(3)

where \( M \) denotes the heavy quark mass. The leading Hamiltonian \( H_0 \) in the heavy quark mass expansion reads

\[ H_0 = \gamma^0 (-i \nabla + m) + V(r) \]  

(4)
where \( m = m_i \delta_{ij} \) and \( V(r) \) denote the constituent quark masses and the potential, respectively.

The spectrum of the resonances in conventional potential model is obtained by solving the Dirac equation from \( H_0 \), followed by time-independent perturbations of the subleading terms. The free parameters of the model are fixed by fitting the predicted masses to those of the observed resonances. The masses obtained this way do not agree well with the P-wave charmed mesons, for instance the mass of \( D_s(2317) \) is much lower than the potential model prediction.

Since the heavy-light mesons are chirally active the masses in the potential model get chiral radiative corrections via the axial coupling. We pointed out in \[6\] that the one loop corrections are sizable, comparable to the \( 1/M \) corrections, and must be incorporated in potential model calculations.

The chiral radiative corrections were successful in understanding why \( D_s(2317) \) and \( D(2308) \) have such close masses. In general, potential models predict, roughly, about 100 MeV larger masses for strange states over their non-strange counterparts, and the gap defined as

\[
\text{gap} \equiv [m(D(0^+)) - m(D(0^-))] - [m(D_s(0^+)) - m(D_s(0^-))] \tag{5}
\]

almost vanishes in potential model, whereas, experimentally, the gap is about 95 MeV. When the chiral radiative corrections are taken into account, however, the potential model predicts a gap that is not only consistent with the experimental value but also insensitive to the ultraviolet (UV) cutoff, with the main contribution to the gap coming from the low energy region of about 250 MeV, far down the cutoff \[6\].

The potential model being nonrenormalizable the loop corrections depend on the regularization scheme chosen. In Ref. \[6\] we introduced a three-momentum UV cutoff regularization, and here we employ the same scheme. Our potential model with radiative corrections is thus the relativistic potential model \([1]\) with the three-momentum cutoff regularization.

### III. SPIN-ORBIT INVERSION

In this paper we take the model in Ref. \[5\] as a reference potential model of conventional type. The model predicts spin-orbit inversions in \( D, D_s \) as well as in \( B, B_s \) mesons. For
example, with the meson states denoted by $H(l, j, J)$, the P-wave states $D_s(1, \frac{1}{2}, 1)$ and $D_s(1, \frac{3}{2}, 1)$, which have the same hyperfine splitting, are predicted to have masses 2605 MeV and 2535 MeV, respectively. However, the experimental values are 2460 MeV and 2535 MeV, respectively, and there is no spin-orbit inversion. Spin-orbit inversions are also predicted in D-wave states as well. For instance, the masses of $D_s(2, \frac{3}{2}, 2)$ and $D_s(2, \frac{5}{2}, 2)$ are, respectively, 2953 MeV and 2900 MeV. The spin-orbit inversions in D-wave states have not been tested yet, but as we shall see, when the radiative corrections are taken into account there should be no spin-orbit inversions in D-wave states.

To understand the absence of spin-orbit inversions within the potential model one must consider a new effect that originates from the terms in the Hamiltonian that do not depend on the heavy quark mass, since the spin-orbit inversions survive in the heavy-quark limit. We shall show that the chiral radiative corrections can be such a new effect.

**IV. RADIATIVE CORRECTIONS FOR P AND D-WAVE STATES**

In Ref. [6] we calculated the loop corrections for S and P-wave states with $j = \frac{1}{2}$ to estimate the mass gap. In this paper we extend the calculation to P-wave states with $j = \frac{3}{2}$ as well as D-wave states with $j = \frac{3}{2}$ and $\frac{5}{2}$. The loop corrections can be obtained similarly as in Ref. [6], to which we refer the readers for details. The loop amplitudes can be most conveniently organized by decomposing the plane wave of the internal light-meson propagator into spherical harmonics. A loop amplitude is then given as a sum over the angular momentum quantum numbers $l_\pi, m_\pi$ and $j_n, m_n$ of the light meson and internal state, respectively, in the Feynman diagram. The summation over $m_\pi, m_n$ can be performed exactly and the final result for the one loop correction to the energy of the P and D-wave states, labeled by the quantum numbers $m = (n, l, j, m_j, q)$, can be written as

$$
\Delta E_{m}^{\text{loop}} = \sum_{n} \sum_{l_\pi, n_\pi} \zeta_{l_\pi} J(m, n, l_\pi) C(l_\pi, l_n, j_n),
$$

where the factor $C(l_\pi, l_n, j_n)$ is given by

$$
C(l_\pi, l_n, j_n) = j_n + \frac{1}{2}
$$
for $l_m = 1, j_m = 1/2$, and

$$C(l_\pi, l_n, j_n) = \left( \frac{3l_\pi(l_\pi - 1)}{2(2l_\pi - 1)} \delta_{l_n,l_\pi-2} + \frac{l_\pi(l_\pi + 1)}{2(2l_\pi + 3)} \delta_{l_n,l_\pi} \right) \delta_{j_n,l_n+\frac{1}{2}} + \left( \frac{l_\pi(l_\pi + 1)}{2(2l_\pi - 1)} \delta_{l_n,l_\pi+1} + \frac{3(l_\pi + 1)(l_\pi + 2)}{2(2l_\pi + 3)} \delta_{l_n,l_\pi+2} \right) \delta_{j_n,l_n-\frac{1}{2}} \quad (8)$$

for $l_m = 1, j_m = 3/2$, and

$$C(l_\pi, l_n, j_n) = \left( \frac{l_\pi(l_\pi + 1)}{2(2l_\pi - 1)} \delta_{l_n,l_\pi+1} + \frac{3l_\pi(l_\pi + 1)(l_\pi + 2)}{2(2l_\pi + 3)} \delta_{l_n,l_\pi+2} \right) \delta_{j_n,l_n+\frac{1}{2}} + \left( \frac{l_\pi(l_\pi + 1)}{2(2l_\pi - 1)} \delta_{l_n,l_\pi+1} + \frac{3l_\pi(l_\pi + 1)(l_\pi + 2)}{2(2l_\pi + 3)} \delta_{l_n,l_\pi+2} \right) \delta_{j_n,l_n-\frac{1}{2}} \quad (9)$$

for $l_m = 2, j_m = 3/2$, and

$$C(l_\pi, l_n, j_n) = \left( \frac{l_\pi(l_\pi - 1)(l_\pi + 1)}{(2l_\pi - 1)(2l_\pi + 3)} \delta_{l_n,l_\pi-1} + \frac{l_\pi(l_\pi + 1)(l_\pi + 2)}{2(2l_\pi + 3)(2l_\pi + 5)} \delta_{l_n,l_\pi+1} \right) \delta_{j_n,l_n+\frac{1}{2}} + \left( \frac{l_\pi(l_\pi - 1)(l_\pi + 1)}{(2l_\pi - 1)(2l_\pi + 3)} \delta_{l_n,l_\pi-1} + \frac{l_\pi(l_\pi + 1)(l_\pi + 2)}{(2l_\pi - 1)(2l_\pi + 3)} \delta_{l_n,l_\pi+1} \right) \delta_{j_n,l_n-\frac{1}{2}} + \frac{5l_\pi(l_\pi - 1)(l_\pi - 2)}{2(2l_\pi - 1)(2l_\pi - 3)} \delta_{l_n,l_\pi-3} \delta_{j_n,l_n+\frac{3}{2}} \quad (10)$$

for $l_m = 2, j_m = 5/2$. Here, following the notation in ref. 6,

$$J(m, n, l_\pi) = -\frac{g_A^2}{8f^2} \int_0^\infty \frac{k^2 dk}{(2\pi)^3 E_\pi} \left[ (E_n^0 - E_m^0) |\rho^{(1)}_{mn}(|\vec{k}|, l_\pi)|^2 + 2\text{Re}[\rho^{(1)}_{mn}(|\vec{k}|, l_\pi)\rho^{(2)*}_{mn}(|\vec{k}|, l_\pi)] + \frac{|\rho^{(2)}_{mn}(|\vec{k}|, l_\pi)|^2}{E_\pi + E_m^0 + E_n^0 - i\epsilon} \right]$$

with

$$\rho^{(1)}_{mn}(|\vec{k}|, l_\pi) = \sqrt{4\pi} \int_0^\infty r^2 dr (f_m(r)g_n(r) - f_n(r)g_m(r)) j_{l_\pi}(kr),$$

$$\rho^{(2)}_{mn}(|\vec{k}|, l_\pi) = \sqrt{4\pi} \int_0^\infty r^2 dr (f_m(r)g_n(r) + f_n(r)g_m(r)) \times (m_m + m_n + 2V_s) j_{l_\pi}(kr),$$

where $E_\pi = \sqrt{k^2 + m^2_\pi}$, with $m_\pi$ denoting the light-meson masses, and $E_{m,n}^0$ and $f_{m,n}(r), g_{m,n}(r)$ are the eigenvalues and radial wave functions of the eigenstates of $H_0$, respectively, $j_{l_\pi}(kr)$ denotes the spherical Bessel functions, and in Eq. 6, $\zeta_\pi$ denotes the SU(3)$_{\text{flavor}}$ factors.
TABLE I: Internal states $jL$ allowed at given $l_{\pi}$ in corrections for a state with $j = 5/2, l = 2$.

The selection rule for the internal states, labeled by $n$, and the quantum number $l_{\pi}$ in the summation in Eq. (6) is given by

$$ l_{\pi} + l_m + l_n = \text{odd integer} \quad (14) $$

and

$$ |j_m - l_{\pi}| \leq j_n \leq j_m + l_{\pi}, \quad (15) $$

which come from parity and angular momentum conservations at the vertex in the loop diagram. As an example, the possible internal states in corrections for a D-wave state with $j = 5/2$ are listed in Table I for low values of $l_{\pi}$.

V. MODIFIED ENERGY LEVELS

We can now see how the spin-orbit inversions are affected by the radiative corrections given in the previous section. For this we focus on the effects of the loop corrections on the existing potential model predictions in Ref. [5].

The new energy levels, denoted by $\bar{E}_m$, that include the radiative corrections can be related to the energy levels of the conventional potential model by

$$ \bar{E}_m = E_m + \delta E_m^0 + \delta E_m^{\text{loop}} \quad (16) $$

where $E_m$ denotes the conventional energy levels which contain the leading order level $E_m^0$ as well as the $1/M$ corrections, and $\delta E_m^0$ denotes the shift in the leading order level $E_m^0$ caused by the shift in the fitted values of the parameters of the model, which was induced by the introduction of radiative corrections $\delta E_m^{\text{loop}}$ in the fitting of the parameters. The relation (16) is valid to the leading order of the loop corrections.
TABLE II: Loop corrections $\delta E_{l,j,q}^{\text{loop}}$ for P and D-wave states in the lowest radial excitations. (Units are in MeV.)

| $\delta E_{1,1,d}^{\text{loop}}$ | $\delta E_{1,1,s}^{\text{loop}}$ | $\delta E_{1,2,d}^{\text{loop}}$ | $\delta E_{1,2,s}^{\text{loop}}$ | $\delta E_{2,3,d}^{\text{loop}}$ | $\delta E_{2,3,s}^{\text{loop}}$ |
|-------------------------------|-------------------------------|-------------------------------|-------------------------------|-------------------------------|-------------------------------|
| -261                         | -183                         | -344                         | -181                         | -257                         | -184                         |
| -184                         | -275                         | -184                         | -257                         | -184                         | -184                         |

Now applying the relation (16) to states with differing $j$ but otherwise same quantum numbers in the heavy-quark limit we have

$$\bar{E}_j - \bar{E}_{j'} = E_j - E_{j'} + \delta E_j^{\text{loop}} - \delta E_{j'}^{\text{loop}} + \delta(E_j^0 - E_{j'}^0),$$

(17)

where the quantum numbers other than $j$ are suppressed. Obviously, $\delta E_j^0$ cannot be obtained without refitting the parameters of the model, which is beyond the scope of the present paper, but, fortunately, the differences $\delta(E_j^0 - E_{j'}^0)$ are expected to be small since $E_j^0 - E_{j'}^0$ are already small, between 20 and 50 MeVs, for those states considered here. We can thus ignore the last term in (17) to obtain

$$\bar{E}_j - \bar{E}_{j'} = E_j - E_{j'} + \delta E_j^{\text{loop}} - \delta E_{j'}^{\text{loop}}.$$

(18)

This equation with the loop corrections is our main tool for the investigation of the spin-orbit inversion.

Let us now focus on the spin-orbit inversions in P-wave states. Using the result in the previous section we obtain numerical values for the loop corrections $\delta E_{l,j,q}^{\text{loop}}$ for strange and non-strange mesons in P and D-wave states (see Table II). The numbers were obtained using the fitted parameter values given in Ref. [5] and the UV cutoff put at 700 MeV. In our model the UV cutoff is a parameter that should be fixed along with other potential model parameters by a fitting similar to that performed in Ref. [5]. In the absence of the fitting we here pick up the preferred value for the cutoff suggested in Ref. [6] where the value 700 MeV was found to give a reasonable size for the loop corrections.

Looking on Table II we notice that the magnitudes of the loop corrections are larger for states with smaller $j$, and this feature will be crucial for understanding the absence of spin-orbit inversions.

We shall first consider the effects of the loop corrections on the P-wave $D_s$ mesons. In the following all the states, labeled as before by $H(l,j,J)$, are in their lowest radial excitations.
We shall assume that the modified energy level for the state $D_s(1, \frac{1}{2}, 0)$ coincides with the experimental mass of $D_s(2317)$, and then estimate the masses of $j = 1/2$ and 3/2 states. Reading the values of the conventional energy levels from Ref. [5] and loop corrections from Table III we find the following new energy levels of the states related by Eq. (18):

\[
\begin{align*}
\bar{E}_{D_s(1, \frac{1}{2}, 1)} & = 2435 \text{ MeV,} \\
\bar{E}_{D_s(1, \frac{3}{2}, 1)} & = 2527 \text{ MeV,} \\
\bar{E}_{D_s(1, \frac{3}{2}, 2)} & = 2573 \text{ MeV.}
\end{align*}
\] (19)

Comparing this result with the experimental values 2460 MeV, 2535 MeV, and 2573 MeV, respectively, we see there is good agreement between the new levels and data, and there are no longer spin-orbit inversions.

Using the same procedure we can obtain the modified energy levels for P-wave $D$ mesons as well, and the result is summarized in Table III. We assumed the mass of $D(1, \frac{1}{2}, 0)$ coincides with the Belle measurement $2308 \pm 36$ for $D(0^+)$ [7]. The Belle mass for $D(1, \frac{3}{2}, 1)$ is $2427 \pm 42$ [7], and considering the large uncertainty in the measured value we find our estimate is consistent with data in $D$ mesons as well. We note, however, that if we use the FOCUS value $2407 \pm 41$ for $D(0^+)$ mass [8] then our estimate is no longer consistent with data.

We can now use Eq. (16), along with Eq. (18), to estimate the masses of the P-wave bottom mesons. We shall first compute the mass for $B_s(1, \frac{1}{2}, 0)$, which is the counterpart of $D_s(2317)$. Since the loop corrections are independent of the heavy quark mass we see that the last two terms in Eq. (16) to be heavy-quark mass independent, so we get

\[
\bar{E}_{B_s(l,j,J)} - E_{B_s(l,j,J)} = \bar{E}_{D_s(l,j,J)} - E_{D_s(l,j,J)}.
\] (20)

Identifying again $\bar{E}_{D_s(1, \frac{1}{2}, 0)}$ with the mass of $D_s(2317)$ we find $\bar{E}_{B_s(1, \frac{1}{2}, 0)} = 5634 \text{ MeV}$. With this energy level we can then compute the levels of other P-wave states following the same procedure used for charmed mesons. The result is summarized in the first row for $\bar{E}$ in Table IV. An interesting feature of our estimation is that for $j = 1/2$ the $B_s$ mesons have almost equal or slightly smaller masses than their non-strange counterparts.

The masses for the P-wave bottom mesons can be obtained in a slightly different way using the measured P-wave charmed meson masses. Identifying the modified energy levels $\bar{E}$ for the charmed states with $m_{\text{expt}}$ in Table III we can use Eq. (20) to obtain the modified
levels for the bottom mesons. The result is given in the second row for $\bar{E}$ in Table IV. Although this method does not employ the chiral loop corrections explicitly the numbers agree well with those from the first approach. This is an encouraging evidence for the consistency of our picture of the heavy-light meson as a potential model bound state with chiral cloud.

Now for D-wave mesons we can use Eq. (18) to compute the mass differences within the families. In Tables V and VI the mass differences $\Delta \equiv \bar{E}_{H(2,j,1)} - \bar{E}_{H(2,\frac{3}{2},1)}$ are summarized. As in P-wave states there are no longer spin-orbit inversions when the loop corrections are incorporated in.

VI. CONCLUSION

We have calculated in relativistic potential model the one loop chiral corrections for the energy levels of the heavy-light mesons in P and D-wave states with $j = 3/2, 5/2$, and shown that the loop corrections can explain the absence of the spin-orbit inversions in charmed mesons. The disappearance of spin-orbit inversions by the loop corrections is not confined to P-wave states only, as we have explicitly shown with D-wave states, and appears to be
TABLE V: Level differences in D-wave charmed mesons. (Units are in MeV.)

|     | Δ      |
|-----|--------|
| D(2, $\frac{3}{2}$, 2) | 38     |
| D(2, $\frac{5}{2}$, 2) | 53     |
| D(2, $\frac{3}{2}$, 3) | 77     |
| $D_s$(2, $\frac{3}{2}$, 2) | 40     |
| $D_s$(2, $\frac{5}{2}$, 2) | 78     |
| $D_s$(2, $\frac{5}{2}$, 3) | 103    |

TABLE VI: Level differences in D-wave bottom mesons. (Units are in MeV.)

|     | Δ      |
|-----|--------|
| $B$(2, $\frac{3}{2}$, 2) | 12     |
| $B$(2, $\frac{5}{2}$, 2) | 33     |
| $B$(2, $\frac{3}{2}$, 3) | 41     |
| $B_s$(2, $\frac{3}{2}$, 2) | 13     |
| $B_s$(2, $\frac{5}{2}$, 2) | 59     |
| $B_s$(2, $\frac{5}{2}$, 3) | 67     |

a generic feature of the potential model. We proposed that the discrepancy between the observed masses and potential model predictions for heavy-light mesons can be remedied once the chiral loop corrections are included, and this allowed us to predict the masses for the P-wave bottom mesons which may be tested in the near future.

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