Electroweak symmetry breaking through bosonic seesaw mechanism in a classically conformal extension of the Standard Model

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Abstract

We suggest the so-called bosonic seesaw mechanism in the context of a classically conformal $U(1)_{B-L}$ extension of the Standard Model with two Higgs doublet fields. The $U(1)_{B-L}$ symmetry is radiatively broken via the Coleman-Weinberg mechanism, which also generates the mass terms for the two Higgs doublets through quartic Higgs couplings. Their masses are all positive but, nevertheless, the electroweak symmetry breaking is realized by the bosonic seesaw mechanism. We analyze the renormalization group evolutions for all model couplings, and find that a large hierarchy among the quartic Higgs couplings, which is crucial for the bosonic seesaw mechanism to work, is dramatically reduced toward high energies. Therefore, the bosonic seesaw is naturally realized with only a mild hierarchy, if some fundamental theory, which provides the origin of the classically conformal invariance, completes our model at some high energy, for example, the Planck scale. The requirements for the perturbativity of the running couplings and the electroweak vacuum stability in the renormalization group analysis as well as for the naturalness of the electroweak scale, we have identified the regions of model parameters. For example, the scale of the $U(1)_{B-L}$ gauge symmetry breaking is constrained to be $\lesssim 100$ TeV, which corresponds to the extra heavy Higgs boson masses to be $\lesssim 2$ TeV. Such heavy Higgs bosons can be tested at the Large Hadron Collider in the near future.
1 Introduction

In the Standard Model (SM), the electroweak symmetry breaking is realized by the negative mass term in the Higgs potential, which seems to be artificial because there is nothing to stabilize the electroweak scale. If new physics takes place at a very high energy, e.g. the Planck scale, the mass term receives large corrections which are quadratically sensitive to the new physics scale, so that the electroweak scale is not stable against the corrections. This is the so-called gauge hierarchy problem. It is well known that supersymmetry (SUSY) can solve this problem. Since the mass corrections are completely canceled by the SUSY partners, no fine-tuning is necessary to reproduce the electroweak scale correctly, unless the SUSY breaking scale is much higher than the electroweak scale. On the other hand, since no indication of SUSY particles has been obtained in the large hadron collider (LHC) experiments, one may consider other solutions to the gauge hierarchy problem without SUSY.

In this direction, recently a lot of works have been done in models based on a classically conformal symmetry, where an additional $U(1)$ gauge symmetry, e.g., $U(1)_{B-L}$, is added [1]-[23]. This direction is based on the argument by Bardeen [24] that the quadratic divergence in the Higgs mass corrections can be subtracted by a boundary condition of some ultraviolet complete theory, which is classically conformal, and only logarithmic divergences should be considered (see Ref. [6] for more detailed discussions). If this is the case, imposing the classically conformal symmetry to the theory is another way to solve the gauge hierarchy problem. Since there is no dimensionful parameter in this class of models, the gauge symmetry must be broken by quantum corrections. This structure fits the model first proposed by Coleman and Weinberg [25], where a model is defined as a massless theory and the gauge symmetry is radiatively broken by the Coleman-Weinberg (CW) mechanism, generating a mass scale through the dimensional transmutation.

In this paper we propose a classically conformal $U(1)_{B-L}$ extended SM with two Higgs doublets. An SM singlet, $B-L$ Higgs field develops its vacuum expectation value (VEV) by the CW mechanism, and the $U(1)_{B-L}$ symmetry is radiatively broken. This gauge symmetry breaking also generates the mass terms for the two Higgs doublets through quartic couplings between the two Higgs doublets and the $B-L$ Higgs field. We assume the quartic couplings to be all positive but, nevertheless, the electroweak symmetry breaking is triggered through the so-called bosonic seesaw mechanism [26] [27] [28], which is analogous to the seesaw mechanism for the neutrino mass generation and leads to a negative mass squared for the SM-like Higgs doublet. A large hierarchy among the quartic Higgs couplings is crucial for the bosonic seesaw mechanism to work at the $U(1)_{B-L}$ symmetry breaking scale. Although it seems unnatural to introduce the large hierarchy by hand, we find that the renormalization group evolutions of the quartic Higgs couplings...
Table 1: Particle contents in our model. $i = 1, 2, 3$ is the generation index.

dramatically reduce the large hierarchy toward high energies. Therefore, once our model is defined at some high energy, say, the Planck scale, the large hierarchy at the $U(1)_{B-L}$ symmetry breaking scale is naturally realized by a mild hierarchy. We also show that the perturbativity of model couplings and the electroweak vacuum stability are maintained up to the Planck scale with a suitable choice of the input parameters. From the naturalness of the electroweak scale, we find the $U(1)_{B-L}$ gauge symmetry breaking scale to be $\lesssim 100$ TeV, which predicts extra heavy Higgs boson masses to be $\lesssim 2$ TeV. Such heavy Higgs boson can be tested at the LHC in the near future.

In the next section, we will define our model, and discuss the $U(1)_{B-L}$ symmetry breaking as well as the electroweak symmetry breaking by the bosonic seesaw mechanism. We also present the mass spectrum of the model. In Sec. 3 we will analyze the renormalization group evolutions for all couplings of the model and present our numerical results. We will see that the hierarchy among the quartic Higgs couplings is dramatically reduced toward high energies. Sec. 4 is devoted to conclusion.

## 2 Model

We consider an extension of the SM with an additional $U(1)_{B-L}$ gauge symmetry. The particle contents of our model are listed in Table 1 where two Higgs doublets ($H_1$ and $H_2$) and one SM singlet, $B - L$ Higgs field ($\Phi$) are introduced. As is well known, the introduction of the three right-handed neutrinos ($N^i$, $i = 1, 2, 3$) is crucial to make the model free from all the gauge and gravitational anomalies. In addition, we impose a classically conformal symmetry to the model, under which the scalar potential is given by

$$V = \lambda_1 |H_1|^4 + \lambda_2 |H_2|^4 + \lambda_3 |H_1|^2 |H_2|^2 + \lambda_4 (H_1^\dagger H_1)(H_2^\dagger H_2) + \lambda_5 |\Phi|^4 + \lambda_{H1\Phi} |H_1|^2 |\Phi|^2 + \lambda_{H2\Phi} |H_2|^2 |\Phi|^2 + \left( \lambda_{\text{mix}} (H_1^\dagger H_2) \Phi^2 + \text{h.c.} \right).$$  

(1)
Here, all of the dimensionful parameters are prohibited by the classically conformal symmetry. In this system, the $U(1)_{B-L}$ symmetry must be radiatively broken by quantum effects, i.e., the CW mechanism. The CW potential for $\Phi$ is described as

$$V_\Phi(\phi) = \frac{1}{4} \lambda_\Phi(v_\Phi) \phi^4 + \frac{1}{8} \beta_{\lambda_\Phi}(v_\Phi) \phi^4 \left( \ln \frac{\phi^2}{v_\Phi^2} - \frac{25}{6} \right),$$

where $\Re[\Phi] = \phi/\sqrt{2}$, and $v_\Phi = \langle \phi \rangle$ is the VEV of $\Phi$. When the beta function $\beta_{\lambda_\Phi}$ is dominated by the $U(1)_{B-L}$ gauge coupling ($g_{B-L}$) and the Majorana Yukawa couplings of right-handed neutrinos ($Y_M$) as shown in Appendix, the minimization condition of $V_\Phi$ approximately leads to

$$\lambda_\Phi \simeq \frac{11}{6\pi^2} \left( 6g_{B-L}^4 - \text{tr} Y_M^4 \right),$$

where all parameters are evaluated at $v_\Phi$. Through the $U(1)_{B-L}$ symmetry breaking, the mass terms of the two Higgs doublets arise from the mixing terms between $H_{1,2}$ and $\Phi$, and the scalar mass squared matrix is read as

$$-\mathcal{L} \approx \frac{1}{2} (H_1, H_2) \begin{pmatrix} \lambda_{H1\Phi} v_\Phi^2 & \lambda_{\text{mix}} v_\Phi^2 \\ \lambda_{\text{mix}} v_\Phi^2 & \lambda_{H2\Phi} v_\Phi^2 \end{pmatrix} \begin{pmatrix} H_1 \\ H_2 \end{pmatrix},$$

where we have assumed a hierarchy among the quartic couplings as $0 \leq \lambda_{H1\Phi} \ll \lambda_{\text{mix}} \ll \lambda_{H2\Phi}$ at the scale $\mu = v_\Phi$. In the next section, we will show that this hierarchy is dramatically reduced toward high energies in their renormalization group evolutions. Because of this hierarchy, mass eigenstates $H'_1$ and $H'_2$ are almost composed of $H_1$ and $H_2$, respectively. Hence, we approximately identify $H'_1$ with the SM-like Higgs doublet. Note that even though all quartic couplings are positive, the SM-like Higgs doublet obtains a negative mass squared for $\lambda_{H1\Phi} \ll \lambda_{\text{mix}}^2/\lambda_{H2\Phi}$, and hence the electroweak symmetry is broken. This is the so-called bosonic seesaw mechanism [26, 27, 28].

In more precise analysis for the electroweak symmetry breaking, we take into account a scalar one-loop diagram through the quartic couplings, $\lambda_3$ and $\lambda_4$, shown in Fig. 1, and the SM-like Higgs doublet mass is given by

$$m_h^2 \simeq -\frac{\lambda_{H1\Phi}}{2} v_\Phi^2 + \frac{\lambda_{\text{mix}}}{2\lambda_{H2\Phi}} v_\Phi^2 + \frac{\lambda_{H2\Phi}}{16\pi^2} (2\lambda_3 + \lambda_4) v_\Phi^2 \simeq \lambda_{H2\Phi} v_\Phi^2 \left[ \frac{1}{2} \left( \frac{\lambda_{\text{mix}}}{\lambda_{H2\Phi}} \right)^2 + \frac{2\lambda_3 + \lambda_4}{16\pi^2} \right],$$

In our analyses, we will take boundary conditions as $\lambda_1(v_\Phi) = \lambda_2(v_\Phi) = \lambda_H(v_\Phi)$, for simplicity, where $\lambda_H$ is a Higgs quartic coupling in the SM.
where we have omitted the $\lambda_{H1\Phi}$ term in the second line, and the observed Higgs boson mass $M_h = 125$ GeV is given by $M_h = m_h/\sqrt{2}$.

In addition to the scalar one-loop diagram, one may consider other Higgs mass corrections coming from a neutrino one-loop diagram and two-loop diagrams involving the $U(1)_{B-L}$ gauge boson ($Z'$) and the top Yukawa coupling, which are, respectively, found to be [4]

$$
\delta m_h^2 \sim \frac{Y^2_{\nu}v_H^2}{16\pi^2}, \quad \delta m_h^2 \sim \frac{y_t^2g_{B-L}v_H^2}{(16\pi^2)^2},
$$

where $Y_\nu$ and $y_t$ are Dirac Yukawa couplings of neutrino and top quark, respectively.

It turns out that these contributions are negligibly small compared to the scalar one-loop correction in Eq. (5). As we will discuss in the next section, the quartic couplings $\lambda_3$ and $\lambda_4$ should be sizable $\lambda_3, \lambda_4 \gtrsim 0.15$ in order to stabilize the electroweak vacuum. The neutrino one-loop correction is roughly proportional to the active neutrino mass by using the seesaw relation, and it is highly suppressed by the lightness of the neutrino mass. The two-loop corrections with the $Z'$ boson is suppressed by a two-loop factor $1/(16\pi^2)^2$. Unless $g_{B-L}$ is large, the two-loop corrections are smaller than the scalar one-loop correction. In Table 2 we summarize typical orders of magnitude for the three corrections for $v_\Phi = 10$ and 100 TeV. For the light neutrino mass, we have adopted the seesaw relation, $m_\nu \sim (Y_\nu v_H)^2/(Y_M v_\Phi) \sim 0.1$ eV, with the SM-like Higgs field VEV, $v_H = 246$ GeV. For both $v_\Phi = 10$ and 100 TeV, we have fixed $\lambda_3 = \lambda_4 = 0.15$, $g_{B-L} = 0.15$ and $Y_\nu = 2.0 \times 10^{-6}$, while we have used $\lambda_{H2\Phi} = 0.01$ ($10^{-4}$) and $Y_M = 0.23$ (0.023) for $v_\Phi = 10$ (100) TeV.

The other scalar masses are approximately given by

$$
M^2_{\phi} = \frac{6}{11}\lambda_4 v_\phi^2, \quad M^2_H = M^2_A = \lambda_{H2\Phi} v_\phi^2 + (\lambda_3 + \lambda_4)v_H^2, \quad M^2_{H^\pm} = \lambda_{H2\Phi} v_\phi^2 + \lambda_3 v_H^2,
$$
Table 2: Typical orders of magnitude of the quantum corrections to the SM-like Higgs doublet mass.

|                  | $v_{\phi}$ | 10 TeV | 100 TeV |
|------------------|------------|--------|---------|
| 1-loop with scalar | $\sim (50 \text{ GeV})^2$ | $\sim (50 \text{ GeV})^2$ |         |
| 2-loop with $Z'$  | $O(1) \text{ GeV}^2$   | $O(10) \text{ GeV}^2$  |         |
| 1-loop with neutrino | $O(10^{-3}) \text{ GeV}^2$ | $O(10^{-3}) \text{ GeV}^2$ |         |

where $M_\phi$ is the mass of the SM singlet scalar, $M_H$ ($M_A$) is the mass of CP-even (CP-odd) neutral Higgs boson, and $M_{H^\pm}$ is the mass of charged Higgs boson. The extra heavy Higgs bosons are almost degenerate in mass. The masses of the $Z'$ boson and the right-handed neutrinos are given by

$$M_{Z'} = 2g_{B-L}v_{\phi},$$

$$M_N = \sqrt{2}y_Mv_{\phi} \simeq \left[ \frac{3}{2N_\nu} \left( 1 - \frac{\pi^2\lambda_\Phi}{11g_{B-L}^4} \right) \right]^{1/4} M_{Z'},$$

where we have used $\text{tr}Y_M = N_\nu y_M$, for simplicity, and $N_\nu$ stands for the number of relevant Majorana couplings. In the following analysis, we will take $N_\nu = 1$ for simplicity, because our final results are almost insensitive to $N_\nu$. In the last equality in Eq. (11), we have used Eq. (3).

3 Numerical results

Before presenting our numerical results, we first discuss constraints on the model parameters from the perturbativity and the stability of the electroweak vacuum in the renormalization group evolutions. In our analysis, all values of couplings are given at $\mu = v_{\phi}$. For $v_{\phi}$ at the TeV scale, we find the constraint $g_{B-L} \lesssim 0.3$ to avoid the Landau pole of the gauge coupling below the Planck scale, while a more severe constraint $g_{B-L} \lesssim 0.2$ is obtained to avoid a blowup of the quartic coupling $\lambda_2$ below the Planck scale. From $g_{B-L} \lesssim 0.2$ and the experimental bound $M_{Z'} > 2.9$ TeV on the $Z'$ boson mass [29, 30], we find $v_{\phi} > 7.25$ TeV. The electroweak vacuum stability, in other words, $\lambda_H(\mu) > 0$ for any scales between the electroweak scale and the Planck scale, can be realized by sufficiently large $\lambda_3$ and/or $\lambda_4$ as $\lambda_3 = \lambda_4 \gtrsim 0.15$. To keep their perturbativity below the Planck scale, $\lambda_3 = \lambda_4 \lesssim 0.48$ must be satisfied, while we will find that the naturalness of the electroweak scale leads to a more severe upper bound.

To realize the hierarchy $\lambda_{H_1\phi} \ll \lambda_{\text{mix}} \ll \lambda_{H_2\phi}$, we take $\lambda_{H_1\phi} = 0$, for simplicity. When we consider $\lambda_{\text{mix}}$ in the range of $0 < \lambda_{\text{mix}} < 0.1 \times \lambda_{H_2\phi}$, the relation between $v_{\phi}$ and $\lambda_{H_2\phi}$ obtained by Eq. (5) is shown in the left panel of Fig. 2. Here, we have fixed $\lambda_3 = \lambda_4 = 0.15$ as an example. The red and blue lines correspond to the lowest value
Figure 2: The relation between $v_\Phi$ and $\lambda_{H2\Phi}$ through Eq. (5) (left panel), and the corresponding extra heavy Higgs boson mass spectrum (right panel). The red and blue lines correspond to $\lambda_{\text{mix}} = 0$ and $\lambda_{\text{mix}} = 0.1 \times \lambda_{H2\Phi}$, respectively. The shaded region shows the perturbativity bound for $g_{B-L} = 0.2$. The vertical lines show the upper bound of $v_\Phi$, at which Higgs mass corrections from the two loop diagrams with the $U(1)_{B-L}$ gauge boson become $(10 \text{ GeV})^2$ for $g_{B-L} = 0.1$ (left) and $g_{B-L} = 0.01$ (right), respectively. Note that $\lambda_{H2\Phi} v_\Phi^2$ is almost constant. Since all heavy Higgs boson masses are approximately determined by $\lambda_{H2\Phi} v_\Phi^2$, they are almost independent of $v_\Phi$, as is shown in the right panel of Fig. 2. The heavy Higgs boson masses lie in the range between 1 TeV and 1.7 TeV, which can be tested at the LHC in the near future.

In Eq. (5), it may be natural for the first term from the tree-level couplings dominates over the second term from the 1-loop correction. This naturalness leads to the constraint of $\lambda_3 = \lambda_4 < 0.26$, which is more severe than the perturbativity bound $\lambda_3 = \lambda_4 \lesssim 0.48$ discussed above. This condition is equivalent to the fact that the origin of the negative mass term mainly comes from the diagonalization of the scalar mass squared matrix in Eq. (4), namely, the bosonic seesaw mechanism.

Now we present the results of our numerical analysis. In Fig. 3 we show the renormalization group evolutions of the quartic couplings. Here, we have taken $\lambda_{H1\Phi} = 0$, and $\lambda_{H2\Phi} = 10^{-2}$ and $10^{-4}$ for $v_\Phi = 10$ TeV (left panel) and 100 TeV (right panel), respectively. The red, green, and blue lines correspond to the running of $\lambda_{H1\Phi}$, $\lambda_{H2\Phi}$ and $\lambda_{\text{mix}}$, respectively. The rightmost vertical line denotes the reduced Planck scale $M_{Pl} = 2.4 \times 10^{18}$ GeV.

\[ \lambda_{\text{mix}} = 0 \text{ and the highest value } \lambda_{\text{mix}} = 0.1 \times \lambda_{H2\Phi}, \text{ respectively}. \]

Although the bosonic seesaw mechanism does not work for $\lambda_{\text{mix}} = 0$, one may consider the electroweak symmetry breaking through the scalar loop correction shown in Fig. 1.
Figure 3: Renormalization group evolutions of the quartic couplings for $v_\phi = 10$ TeV (left) and 100 TeV (right). The red, green, and blue lines correspond to $\lambda_{H1\Phi}$, $\lambda_{H2\Phi}$ and $\lambda_{\text{mix}}$, respectively. The rightmost vertical line shows the reduced Planck scale.

In this plot, the other input parameters have been set as $g_{B-L} = 0.17$ and $\lambda_3 = \lambda_4 = 0.17$ to realize the electroweak vacuum stability without the Landau pole, and $\lambda_\phi = 10^{-3}$. The value of $\lambda_1 = \lambda_2 = \lambda_H$ at $\mu = v_\phi$ has been evaluated by extrapolating the SM Higgs quartic coupling with $M_h = 125$ GeV from the electroweak scale to $v_\phi$. For this parameter choice, the $Z'$ boson and the right-handed neutrinos have the masses of the same order of magnitude as $M_{Z'} = 3.4$ (34) TeV and $M_N = 2.0$ (20) TeV for $v_\phi = 10$ (100) TeV, while the $B - L$ Higgs boson mass is calculated as $M_\phi = 0.23$ (2.3) TeV. As is well-known, $M_\phi \ll M_{Z'}$ is a typical prediction of the CW mechanism. The masses of the heavy Higgs bosons are roughly 1 TeV for both $v_\phi = 10$ TeV and 100 TeV.

In order for the bosonic seesaw mechanism to work, we have assumed the hierarchy among the quartic couplings as $\lambda_{H1\Phi} \ll \lambda_{\text{mix}} \ll \lambda_{H2\Phi}$ at the scale $\mu = v_\phi$. One may think it unnatural to introduce this large hierarchy by hand. However, we find from Fig. 3 that the large hierarchy between $\lambda_{H1\Phi}$ and $\lambda_{H2\Phi}$ tends to disappear toward high energies. This is because the beta functions of the small couplings $\beta_{\lambda_{H1\Phi}}$ and $\beta_{\lambda_{H2\Phi}}$ are not simply proportional to themselves, but include terms given by other sizable couplings (see Appendix for the explicit formulas of their beta functions). This behavior of reducing the large hierarchy in the renormalization group evolutions is independent of the choice of the boundary conditions for $g_{B-L}$, $\lambda_3$, $\lambda_4$ and $\lambda_\phi$. Therefore, Fig. 3 indicates that once our model is defined at some high energy, say, the Planck scale, the large hierarchy among the quartic couplings, which is crucial for the bosonic seesaw mechanism to work, is naturally achieved from a mild hierarchy at the high energy.

We see in Fig. 3 that $\lambda_{\text{mix}}$ is almost unchanged. This is because $\beta_{\lambda_{\text{mix}}}$ is proportional to $\lambda_{\text{mix}}$, which is very small (see Appendix). Hence, the hierarchy between $\lambda_{\text{mix}}$ and the
Table 3: Additional vector-like fermions. $x$ is a real number.

|                   | $SU(3)_c \otimes SU(2)_L \otimes U(1)_Y$ | $U(1)_{B-L}$ |
|-------------------|------------------------------------------|--------------|
| $S_{L,R}$         | (1, 1, 0)                                | $x$          |
| $S'_{L,R}$        | (1, 1, 0)                                | $x-2$        |
| $D_{L,R}$         | (1, 2, 1/2)                              | $x + 2$      |
| $D'_{L,R}$        | (1, 2, 1/2)                              | $x$          |

Figure 4: One-loop diagram due to the additional fermions, which is relevant to $\beta_{\lambda_{\text{mix}}}$. Other couplings get enlarged at high energies. To avoid this situation and make our model more natural, one may introduce additional vector-like fermions listed in Table 3, for example.

Although $x$ is an arbitrary real number, we assume $x \neq 1$ to distinguish the new fermions from the SM leptons. These fermions have Yukawa couplings as

$$- \mathcal{L}_V = Y_{SS} S_L \Phi S'_R + Y_{SD} \overline{S'_R} H_2 D'_L + Y_{DD} \overline{D'_L} \Phi D_R + Y_{DS} \overline{D'_L} H_1 S_L + Y'_{SS} \overline{S'_L} H_2 D'_R + Y'_{SD} \overline{D'_R} \Phi D_L + Y'_{DD} \overline{D'_R} H_1 S_R + \text{h.c.},$$

so that $\beta_{\lambda_{\text{mix}}}$ includes terms of $Y_{SS} Y_{SD} Y_{DD} Y_{DS}$ and $Y'_{SS} Y'_{SD} Y'_{DD} Y'_{DS}$, which are not proportional to $\lambda_{\text{mix}}$. These terms originate the diagram shown in Fig. 4. Accordingly, the minimization condition of $V_\Phi$ is modified to

$$\lambda_\Phi \simeq \frac{11}{6\pi^2} \left[ 6g_{B-L}^2 \frac{1}{2} \left( Y_{SS}^4 + Y_{SS}^4 + 2Y_{DD}^4 + 2Y_{DD}^4 \right) \right].$$

From the conditions $\lambda_\Phi > 0$ and $g_{B-L} < 0.2$, the additional Yukawa contribution should satisfy $Y_{SS}^4 + Y_{SS}^4 + 2Y_{DD}^4 + 2Y_{DD}^4 \lesssim 3 \times (0.4)^4$. Note that vector-like fermions masses are dominantly generated by $v_\Phi$, and they are sufficiently heavy to avoid the current experimental bounds.

\[3\] As another possibility, one may think that some symmetry forbids the $\lambda_{\text{mix}}$ term and it is generated via a small breaking.
Figure 5: Runnings of quartic couplings for $v_\Phi = 100$ TeV with additional vector-like fermions. The input parameters are the same as before.

Fig. 5 shows the runnings of the quartic couplings for $v_\Phi = 100$ TeV with the additional vector-like fermions. The input parameters are the same as before, while we have taken the Yukawa couplings as $Y_{SS} = Y_{SD} = Y_{DD} = Y_{DS} = 0.2$ and $Y'_SS = Y'_SD = Y'_DD = Y'_DS = 0.1$ at $\mu = v_\Phi$, for simplicity. Toward high energies, $|\lambda_{\text{mix}}|$ becomes larger, and the hierarchy with the other couplings becomes mild. We can see that $\lambda_{H1\Phi}$ is negative below $\mu \simeq 10^8$ GeV, because the contributions of additional Yukawa couplings to $\beta_{\lambda_{H1\Phi}}$ are effective below $\mu \simeq 10^8$ GeV. Above the scale, the contribution of $U(1)_{B-L}$ couplings becomes effective, and then $\lambda_{H1\Phi}$ becomes positive. As a result, the large hierarchy at the $U(1)_{B-L}$ symmetry breaking scale can be realized with a mild hierarchy at some high energy. We expect that an ultraviolet complete theory, which provides the origin of the classical conformal invariance, takes place at the high energy.

4 Conclusion

We have investigated a classically conformal $U(1)_{B-L}$ extension of the Standard Model with two electroweak Higgs doublet fields. Through the Coleman-Weinberg mechanism, the $U(1)_{B-L}$ symmetry is radiatively broken, and a mass scale is generated via the dimensional transmutation. This symmetry breaking is the sole origin of all dimensionful parameters in the model, and the mass terms of the two Higgs doublet fields are generated through their quartic couplings with the $B-L$ Higgs field. All generated masses are set to be positive but, nevertheless, the electroweak symmetry breaking is realized by the bosonic seesaw mechanism. In order for the bosonic seesaw mechanism to work,
we need a large hierarchy among the two Higgs doublet masses, which originates from a
total hierarchy among the quartic couplings. Although it seems unnatural to introduce
the large hierarchy by hand at the $U(1)_{B-L}$ symmetry breaking scale, we have found
through analysis of the renormalization group evolutions of the quartic couplings that
this hierarchy is dramatically reduced towards high energies. Therefore, once our model
is defined at some high energy, for example, the Planck scale, in other words, the origin
of the classically conformal invariance is provided by some ultraviolet complete theory at
the Planck scale, the bosonic seesaw mechanism is naturally realized with a mild hierar-
chy among the quartic couplings. The requirements for the perturbativity of the running
couplings and the electroweak vacuum stability in the renormalization group analysis as
well as for the naturalness of the electroweak scale, we have identified the regions of model
parameters such as $g_{B-L}(v_\Phi) \lesssim 0.2$, $0.15 \lesssim \lambda_3(v_\Phi) = \lambda_4(v_\Phi) \lesssim 0.23$, and $v_\Phi \lesssim 100$ TeV.
We have also found that all heavy Higgs boson masses are almost independent of $v_\Phi$, and
lie in the range between 1 TeV and 1.7 TeV, which can be tested at the LHC in the near
future.

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Appendix

The beta functions in the $U(1)_{B-L}$ extended SM

One-loop $\beta$-functions in our model, which are calculated by using SARAH [31], are given as follows:

\begin{equation}
\beta_{g_Y} = \frac{3g_Y^3}{16\pi^2}, \quad \beta_{g_2} = \frac{9g_2^3}{16\pi^2}(-3), \quad \beta_{g_3} = \frac{9g_3^3}{16\pi^2}(-7),
\end{equation}

\begin{equation}
\beta_{g_{B-L}} = \frac{g_{B-L}}{16\pi^2} \left( 7g_{mix}^2 + 8g_{mix}g_{B-L} + \frac{68}{3}g_{B-L}^2 \right),
\end{equation}

\begin{equation}
\beta_{g_{mix}} = \frac{1}{16\pi^2} \left[ g_{mix} \left( 14g_Y^2 + 7g_{mix}^2 + 8g_{mix}g_{B-L} + \frac{68}{3}g_{B-L}^2 \right) + 8g_{B-L}g_Y^2 \right],
\end{equation}

\begin{equation}
\beta_{\sigma_Y} = \frac{y_4}{16\pi^2} \left( -8g_Y^2 + \frac{2}{3}g_{mix}^2 - \frac{5}{3}g_{mix}g_{B-L} - \frac{2}{3}g_{B-L}^2 + \frac{9}{2}y_t^2 \right),
\end{equation}

\begin{equation}
\beta_{\lambda_M} = \frac{Y_M}{16\pi^2} \left( -6g_{B-L}^2 + 4Y_M^2 + 2trY_M^2 \right),
\end{equation}

\begin{equation}
\beta_{\lambda_1} = \frac{1}{16\pi^2} \left[ \frac{3}{8} \left( 2g_Y^4 + g_Y^2 + g_{mix}^2 \right) - 6y_t^4 + \lambda_1 \left( -9g_Y^2 - 3g_Y^2 + 2g_Y^2 + 12y_t^2 \right) + 24\lambda_Y^2 + 2\lambda^3 + 2\lambda_4 + \lambda_4^2 + \lambda_{H1\Phi}^2 \right],
\end{equation}

\begin{equation}
\beta_{\lambda_2} = \frac{1}{16\pi^2} \left[ \frac{3}{8} \left( 2g_Y^4 + g_Y^2 + g_{mix}^2 \right) + 48g_{B-L}^2(g_Y^2 + g_{mix}^2) - 12g_{mix}g_{B-L}(g_Y^2 + g_{mix}^2) + 144g_{mix}^2g_{B-L} - 768g_{mix}g_{B-L} - 1536g_{B-L}^2 - 6y_t^4 + \lambda_2 \left( -9g_Y^2 - 3g_Y^2 + g_{mix}^2 \right) + 48g_{mix}g_{B-L} - 192g_{B-L}^2 + 12y_t^4 + 24\lambda_Y^2 + 2\lambda^3 + 2\lambda_4 + \lambda_4^2 + \lambda_{H2\Phi}^2 \right],
\end{equation}

\begin{equation}
\beta_{\lambda_3} = \frac{1}{16\pi^2} \left[ \frac{3}{4} \left( 2g_Y^4 + (-g_Y^2 + g_Y^2 + g_{mix}^2) \right) + 24g_{mix}^2g_{B-L} - 12g_{mix}g_{B-L}(g_Y^2 + g_{mix}^2) + \lambda_3 \left( -9g_Y^2 - 3g_Y^2 + g_{mix}^2 \right) + 24g_{mix}g_{B-L} - 96g_{B-L}^2 + 6y_t^4 + 12\lambda_1 + 12\lambda_2 + 4\lambda_3 \right] + 4\lambda_1 + 4\lambda_2 + 4\lambda_3 + 2\lambda^2 + 2\lambda_{H1\Phi} + \lambda_{H2\Phi},
\end{equation}

\begin{equation}
\beta_{\lambda_4} = \frac{1}{16\pi^2} \left[ 3g_Y^4 + g_{mix}^2 - 24g_{mix}g_{B-L} + \lambda_4 \left( -9g_Y^2 - 3g_Y^2 + g_{mix}^2 \right) + 24g_{mix}g_{B-L} - 96g_{B-L}^2 + 6y_t^4 + 4\lambda_1 + 4\lambda_2 + 8\lambda_3 + 4\lambda_4 + 4\lambda_{mix} \right],
\end{equation}

\begin{equation}
\beta_{\lambda_{H1\Phi}} = \frac{1}{16\pi^2} \left[ \lambda_{H1\Phi} \left( \frac{9}{2}g_Y^2 - \frac{3}{2}g_{mix}^2 \right) - 24g_{B-L}^2 + 4trY_M^2 + 6y_t^4 + 12\lambda_1 + 8\lambda_4 \right] + 4\lambda_{H1\Phi} + 4\lambda_3 + \lambda_{H2\Phi} + 2\lambda_4 + 8\lambda_{mix}^2 + 12g_{mix}^2g_{B-L},
\end{equation}

\begin{equation}
\beta_{\lambda_{H2\Phi}} = \frac{1}{16\pi^2} \left[ \lambda_{H2\Phi} \left( \frac{9}{2}g_Y^2 - \frac{3}{2}g_{mix}^2 \right) + 24g_{mix}g_{B-L} - 120g_{B-L}^2 + 4trY_M^2 + 12\lambda_1 + 8\lambda_4 \right] + 4\lambda_{H2\Phi} + 4\lambda_3 + \lambda_{H1\Phi} + 8\lambda_{mix}^2 + 12g_{mix}^2g_{B-L} - 192g_{mix}g_{B-L}^2 + 768g_{B-L}^2,
\end{equation}

\begin{equation}
\beta_{\lambda_{mix}} = \frac{\lambda_{mix}}{16\pi^2} \left[ \left( \frac{9}{2}g_Y^2 - \frac{3}{2}g_{mix}^2 \right) + 12g_{mix}g_{B-L} - 72g_{B-L}^2 + 4trY_M^2 + 3y_t^2 + 2\lambda_3 + 4\lambda_4 + 4\lambda_{H1\Phi} + 4\lambda_{H2\Phi} + 4\lambda_{\phi},
\right]
\end{equation}
where $g_{\text{mix}}$ is a kinetic mixing coupling of the $U(1)$ gauges, and we take $g_{\text{mix}}(\nu_\Phi) = 0$ for its boundary condition, so that there is no mixing between $Z$ and $Z'$ bosons. We neglect Dirac Yukawa couplings except top Yukawa coupling $y_t$ in our analysis.

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