Undrained Shear Strength $c_u$ and Undrained Elastic Modulus $E_u$ of Anthropogenic Soils from Laboratory Tests

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Abstract. Undrained shear strength $c_u$ and undrained elasticity modulus $E_u$ are one of the basic mechanical parameters describing soil properties in engineering practice. In a simple way $c_u$ can be established by fall cone test or, similar as the $E_u$ modulus, can be determined from the stress-strain curve obtained from the uniaxial or triaxial compression tests. The paper presents the results of laboratory tests of $c_u$ and $E_u$ parameters carried out on anthropogenic soil in uniaxial compression tests and fall cone tests. The soil used in the study represented different types of materials used in earthworks - containing different share of clay fraction. Tests were provided on the soil in different bulk density and water contents. The paper proposes a method of estimating $E_{50}$ on the basis of cone penetrometer tests. Such test does not require any additional preparation and can be performed directly on the soil compacted in the cylinder of the Proctor's apparatus, which allows for a quick assessment of the soil elasticity parameters.

1. Introduction

Anthropogenic soils are those deposited as a result of human activity. They occur in transport embankments (railway, road), under foundations, backfillings of excavations, in hydrotechnical earth structures: earth dams, floodbanks, embankments of water reservoirs and waste dumps. Different types of earth embankments require in each case the use of soil material of appropriate quality. First of all, it is required that the material is properly compacted in order to obtain a final embankment with sufficiently high values of mechanical parameters determining its stability.

The mechanical behaviour is the most important features of a soil in engineering practice. The mechanical parameters make possible to determine the bearing capacity of the subsoil and the conditions of the settlement of the object. There is a whole range of tests, both laboratory and field, which allow determining mechanical parameters of soil: deformability and strength parameter. In addition to advanced and expensive tests, a number of simple and quick tests, that provide an initial estimation, are also carried out.

The parameters characterizing the strain soil behaviour include oedometric modules determined in uniaxial strain state $E_{oed}$ as well as modules determined in triaxial strain condition, secant modulus $E_{50}$ or undrained secant modulus $E_{u50}$ when it has been determined under undrained test. Definitions
of the secant modulus $E_{50}$ (also $E_{u50}$), uniaxial compressive strength $q_u$, and undrained shear strength $c_u$ are presented on figure 1.

![Figure 1. Definitions of the secant modulus $E_{50}$, uniaxial compressive strength $q_u$, and undrained shear strength $c_u$.](image)

The basic strength parameters of soil, apart from the shear strength parameters of the Coulomb-Mohr model (internal friction angle $\phi$ and cohesion $c$), include: undrained shear strength $c_u$ and uniaxial compressive strength $q_u$. The relationship between these parameters can be determined in simple way, by the formula:

$$c_u = \frac{q_u}{2} \quad (1)$$

Apart of advanced laboratory apparatus to determine soil strength and deformability parameters (e.g. triaxial apparatus) there are some easier test, which can help us evaluate those properties. One of those tests is: fall cone test [1-4]. A fall cone penetration method is a simple and quick test that enables to estimate, apart from liquidity limit ($LL$) [5-7], the undrained shear strength $c_u$ [8-11]. It was originally designed and used in Scandinavia to evaluate $c_u$ (see [1,2,12]). Later, it became much more popular for the determination of the liquidity limit $LL$ [6]. Based on model tests and laboratory experiments, it was estimated that the strength value is directly proportional to the weight of the cone $Q$ [mg/s$^2$] and inversely proportional to the square of the cone indentation $h^2$ [mm$^2$] for small deformations. Therefore, the undrained shear strength [8] can be determined from the formula:

$$c_u = K \frac{Q}{h^2}, \quad (2)$$

where $K$ depends mainly on the cone top angle. Although, the results of the test are influenced by such factors as: soil surface uplift (depends on the consistency of the soil) or roughness of the cone surface [13]. After a number of analyses and tests [1, 5, 8], the $K$ correction factor for the strength determination using this method was theoretically introduced. In tests on undisturbed soil, Hansbo [8] obtained that $K$ value vary between 0.8 and 1.0 for the 30$^\circ$ cone and between 0.2 and 0.25 for the 60$^\circ$ top angle. For remoulded clay, $K$ was given as 0.30 for the 60$^\circ$ cone and no value was obtained for a 30$^\circ$ cone. In the following years, various authors conducted similar studies to determine the $K$-value and
obtained similar results. For instance, Wood [2] obtained $K$ factor as 0.85 for the 30° cone and 0.29 for the 60° cone. The literature also emphasizes the dependence of local experience and soil consistency. Many studies have also been dedicated to determining the $K$-factor value on the basis of theoretical and numerical analysis of fall cone tests [1,12,14,15].

The EN ISO 17892 [11] standard recommends that the $K$-factor should be taken as 0.8. It also gives the relation:

$$c_u = c \frac{gm}{i^2}$$

(3)

where $c$ is a constant depending on the angle of the cone and the consistency of the soil, $g$ is the earth acceleration, $m$ is a cone mass, and $i$ - an indentation. The standard also recommends correcting the value by a $\mu$ coefficient depending on the liquidity limit $LL$ [11]:

$$c_u(\text{corr}) = \left(\frac{0.43}{LL}\right)^{0.45} c_u$$

(4)

The paper presents the results of investigations of $c_u$ and $E_{u50}$ parameters under uniaxial compression and with a fall cone test of anthropogenic soils prepared in the laboratory. The soils used in the study represented different types of materials used in earthworks - compacted with the standard Proctor method. For samples of different moisture and density, the values of undrained shear strength $c_u$ and modulus of elasticity $E_{u50}$ were determined. Based on test results a quick method of estimating $E_{u50}$ from fall cone test is proposed.

2. Material and Method

For the study three samples of anthropogenic, fine-grained (cohesive) soil with different share of fractions were selected, representing different types of materials used in earthworks (see table 1).

**Table 1. A Fractional share of examined soils**

| Fractional share [%] | Sa | Si | Cl |
|----------------------|----|----|----|
| Sample I             | 18 | 58 | 24 |
| Sample II            | 54 | 32 | 14 |
| Sample III           | 83 | 12 | 5  |

For each soil, tests were carried out in Proctor’s apparatus, with standard procedure, to determine the optimum moisture content (6-10 points), and to collect samples for further test. The results of preliminaries tests are shown in figure 2 and in table 2.

**Table 2. The results of tests in Proctor’s apparatus**

| Clay fraction Cl [%] | Maximum dry density $\rho_u$ [g/cm³] | Optimal water content $w_{opt}$ [%] |
|----------------------|--------------------------------------|----------------------------------|
| Sample I             | 24                                   | 1.7                              | 17.4                              |
| Sample II            | 14                                   | 1.95                             | 1.4                               |
| Sample III           | 5                                    | 2.01                             | 9.3                               |
A series of compression tests under uniaxial stress conditions and fall cone tests were carried out on soil samples compacted in the Proctor apparatus at different water contents. The soil parameters: undrained shear strength \( c_u \) (from both method) and \( E_u50 \) modules (from uniaxial compression) were determined, which has allowed the results of both type of test to be compared.

The fall cone test was performed similar to the procedure described in EN ISO [11] for the determination of \( c_u \). A cone with an apex angle of 30 degree was placed on the top of soil sample and allows it to penetrate for 5 seconds. After 5 seconds, the cone deep with an electronic displacement sensor was recorded. The mass of the falling cone was changed in such a way as to ensure the penetration of the cone into the soil in the range of 10-20 mm. The \( c_u \) value was determined from fall cone test according to equation (3) and described further in text as \( c_{u(fc)} \).

A uniaxial compression tests (UX) were done according to the EN ISO [16]. The soil samples were obtained from the cylinder of the Proctor apparatus, after compaction at given water contents. Next, sample was placed between two plates of the press and it was compressed with a constant press feed rate of 18 mm / hour, until the ultimate state of strength was achieved. The \( q_u \) and \( c_u \) (described further in text as \( c_{u(UX)} \)) values were determined according equation (1) see also figure1. The \( E_u50 \) values were determined also from this test using equation [16]:

\[
E_{u50} = \frac{d\sigma}{d\varepsilon_{50}} \tag{5}
\]

where: \( d\sigma \) is the change of vertical stress and \( d\varepsilon_{50} \) is the corresponding strain at stress equal 50% of peak strength value - figure1.

3. Results and discussions

For all 3 samples, the undrained shear strength \( c_u \) results obtained from the fall cone and uniaxial compression tests were compared with the moisture content \( w \) of the specimens. A similar comparison was made for the secant module \( E_{u50} \) determined from the uniaxial compression test. The results are presented in the figures 3, 4, and 5. In the majority of cases a good correlation between the mechanical parameter and the moisture content of the sample was obtained. The worst correspondence was obtained for the strength determined by the uniaxial compression test. The summary of results for optimal water content is presented in table 3.
Figure 3. Undrained shear strength from fall cone test \( c_{u(f_c)} \) versus water content \( w \) for samples with different clay share

Figure 4. Undrained shear strength from undrained compression test \( c_{u(UX)} \) versus water content \( w \) for samples with different clay share

Table 3. The overview of the mechanical property results for optimal water content

|                | Fall cone test |                     | Uniaxial compression test |                     |
|----------------|---------------|---------------------|--------------------------|---------------------|
|                | Undrained shear strength \( c_{u(f_c)} \) [kPa] | Undrained shear strength \( c_{u(UX)} \) [kPa] | Secant modulus \( E_{u50} \) [MPa] |
| Sample I       | 206.2         | 142.05              | 8.38                     |
| Sample II      | 227.8         | 36.38               | 2.55                     |
| Sample III     | 354.3         | 21.59               | 4.95                     |
Next, based on the previous outcomes, the values of strength $c_u$ from both methods were compared (figure 6). However, no good correlation was obtained. Nevertheless, it was observed that the coherence increases with increasing water content. This is most likely due to the fact that for soil with low soil moisture the fall cone test is not relevant for strength assessment. On the other hand, a good correlation between the strain modulus from the uniaxial compression test $E_{u50}$ and the undrained shear strength from the cone penetrometer method $c_u(f_c)$ was observed (figure 7). For further analysis, the results obtained for the water content close to the optimal one and higher for the given soil were used (figure 8). This was done in order to avoid distortion of measurements caused by too low soil moisture in the penetrometer test. As can be seen, after excluding the data determined at low water content, the relationship between these values is even better.
Figure 7. Undrained shear strength from fall cone test $c_{ufc}$ versus secant modulus from undrained compression test $E_{u50}$

Figure 8. Undrained shear strength from fall cone test $c_{ufc}$ versus secant modulus from undrained compression test $E_{u50}$ - after excluding samples with low water content

On the basis of the observed dependence, based on previous studies where attempts were made to assess various geotechnical parameters based on fall cone test [18-21], general conversion formula was proposed to estimate the secant modulus $E_{u50}$ from the cone penetrometer test:

$$E_{u50}(f_c) = \left( Cl + 10 \right) c_{u50}(f_c)$$  (6)

The correction has been made in the formula due to the clay fraction content, as it has a clear influence on the soil behaviour. A comparison of the value determined from the uniaxial compression test and the formula - from the fall cone test is shown in figure 9. In all cases, very good compliance was achieved. The application of such a formula in engineering practice will allow estimate the magnitude of the anthropogenic soil strain modulus without the necessity to perform additional special tests.
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Figure 9. Calculated secant modulus $E_{u50(f)}$ versus secant modulus from undrained compression test $E_{u50}$ - verification of the proposed equation

4. Conclusions
Fall cone test is a quick, simple and cheap method used in laboratory testing of soils. In this work the test was used to determine the mechanical properties of anthropogenic soils. Three samples of mineral soils compacted in a laboratory with different clay fraction content were tested. The fall cone tests were carried out on samples of different moisture content after being compacted in Proctor's apparatus with the standard method.

The presented results showed that it is possible to determine the relationship between the mass and indentation of the cone and the value of the modulus $E_{u50}$. However, it is necessary to introduce a correction for the clay fraction content in the soil. The presented method can be used e.g. to plan the study programme and to estimate the time and costs of laboratory tests.

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