Thermodynamics of Hořava-Lifshitz black holes

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Abstract

We study black holes in the Hořava-Lifshitz gravity with a parameter $\lambda$. For $1/3 \leq \lambda < 3$, the black holes behave like the Lifshitz black holes with dynamical exponent $0 < z \leq 4$, while for $\lambda > 3$, the black holes behave like the Reissner-Nordström type black hole in asymptotically flat spacetimes. Hence, these all are quite different from the Schwarzschild-AdS black hole of Einstein gravity. The temperature, mass, entropy, and heat capacity are derived for investigating thermodynamic properties of these black holes.
1 Introduction

Hořava has proposed a renormalizable theory of gravity at a Lifshitz point [1], which may be regarded as a UV complete candidate for general relativity. Recently, the Hořava-Lifshitz gravity theory has been intensively investigated in [2, 3, 4, 5, 6, 7, 8, 9, 10] and its cosmological applications have been studied in [11, 12, 13, 14, 15, 16].

Introducing the ADM formalism where the metric is parameterized [17]

\[
 ds_{ADM}^2 = -N^2 dt^2 + g_{ij}(dx^i - N^i dt)(dx^j - N^j dt), \tag{1}
\]

the Einstein-Hilbert action can be expressed as

\[
 S_{EH} = \frac{1}{16\pi G} \int d^4x \sqrt{gN} \left( K_{ij}K^{ij} - K^2 + R - 2\Lambda \right), \tag{2}
\]

where \( G \) is Newton’s constant and extrinsic curvature \( K_{ij} \) takes the form

\[
 K_{ij} = \frac{1}{2N} \left( g_{ij} - \nabla_i N_j - \nabla_j N_i \right). \tag{3}
\]

Here, a dot denotes a derivative with respect to \( t \).

The action of the \( z = 3 \) Hořava-Lifshitz with a parameter \( \lambda \) is given by

\[
 S_{HL} = \int dt d^3x \left( \mathcal{L}_0 + \mathcal{L}_1 \right),
\]

\[
 \mathcal{L}_0 = \sqrt{gN} \left\{ \frac{2}{\kappa^2} (K_{ij}K^{ij} - \lambda K^2) + \frac{\kappa^2 \mu^2 (\Lambda_W R - 3\Lambda^2_W)}{8(1 - 3\lambda)} \right\}, \tag{4}
\]

\[
 \mathcal{L}_1 = \sqrt{gN} \left\{ \frac{\kappa^2 \mu^2 (1 - 4\lambda)}{32(1 - 3\lambda)} R^2 - \frac{\kappa^2}{2w^4} \left( C_{ij} - \frac{\mu w^2}{2} R_{ij} \right) \left( C^{ij} - \frac{\mu w^2}{2} R^{ij} \right) \right\}. \tag{5}
\]

where \( C_{ij} \) is the Cotton tensor defined by

\[
 C^{ij} = \epsilon^{ijk} \nabla_k \left( R^j_l - \frac{1}{4} R \delta^j_l \right). \tag{6}
\]

Comparing \( \mathcal{L}_0 \) with Eq. (2) of general relativity, the speed of light, Newton’s constant, the cosmological constant, parameter \( \lambda \) are determined by

\[
 c = \frac{\kappa^2 \mu}{4} \sqrt{\frac{\Lambda_W}{1 - 3\lambda}}, \quad G = \frac{\kappa^2}{32\pi c}, \quad \Lambda = \frac{3}{2} \Lambda_W, \quad \lambda = 1. \tag{7}
\]

The equations of motion were derived in [18] and [19], but we do not write them due to the length.

In this work, we investigate the Hořava-Lifshitz black hole solutions and their thermodynamic properties.
2 Lifshitz black holes

Considering $N^2 = \tilde{N}^2 f(r)$ and $N^i = 0$, a spherically symmetric solution could be obtained with a metric ansatz proposed by Lü-Mei-Pope (LMP) \cite{LMP1, LMP2, LMP3, LMP4}:

$$ds^2_{\text{LMP}} = -\tilde{N}^2(r) f(r) dt^2 + \frac{dr^2}{f(r)} + r^2(d\theta^2 + \sin^2\theta d\phi^2),$$  

which implies that

$$K_{ij} = 0, \quad C_{ij} = 0.$$  

The first condition of $K_{ij} = 0$ means that the embedding is trivial for a spherically symmetric, static solution and the non-zero Cotton tensor $C_{ij}$ is not necessary to obtain a spherically symmetric solution. Taking the Lagrangian $L_0$ only, we obtain the Schwarzschild-AdS (SAdS) black hole whose metric function is given by

$$f = 1 - \frac{\Lambda_W}{2} r^2 - \frac{m}{r},$$  

with $\tilde{N}^2 = 1$. The simplest way to obtain a spherically symmetric solution for the Hořava-Lifshitz gravity is to substitute the metric ansatz (8) into the action. Then, let us vary the reduced action with respect to functions $\tilde{N}$ and $f$. This is possible because the metric ansatz shows all the allowed singlets which are compatible with the $SO(3)$ action on the $S^2$. The reduced Lagrangian is given by

$$L = \frac{\kappa^2 \mu^2 \Lambda_W \tilde{N}}{8(1 - 3\lambda)} \left[ 2(1 - f - rf') - 3\Lambda_W r^2 + \frac{\lambda - 1}{2\Lambda_W} f'^2 - \frac{2\lambda(f - 1)}{\Lambda_W r} f' + \frac{(2\lambda - 1)(f - 1)^2}{\Lambda_W r^2} \right].$$  

The first two terms come from $L_0$, while the remaining terms come from higher order Lagrangian $L_1$ with $C_{ij} = 0$. Thus, in the limit of $-\Lambda_W \to \infty$, we expect to find the SAdS black hole \cite{SAdS}. However, this limit is the other limiting case, compared to the asymptotically flat spacetime limit of $-\Lambda_W \to 0$ in anti-de Sitter spacetimes. Hence, we should be careful to treat the limiting case of $-\Lambda_W \to \infty$ in Hořava-Lifshitz gravity.

Introducing a newly radial coordinate $x = \sqrt{-\Lambda_W r}$, we have LMP black hole solutions where $f$ and $\tilde{N}$ are determined to be

$$f = 1 + x^2 - \left(\sqrt{-\Lambda_W m}x\right)^{p_\pm(\lambda)}, \quad p_\pm(\lambda) = \frac{2\lambda \pm \sqrt{6\lambda - 2}}{\lambda - 1}, \quad (12)$$

$$\tilde{N} = x^{q_\pm(\lambda)}, \quad q_\pm(\lambda) = -\frac{1 + 3\lambda \pm 2\sqrt{6\lambda - 2}}{\lambda - 1}, \quad (13)$$

where $m$ is a mass parameter related to the ADM mass\footnote{Our definition of mass parameter is slightly different from Cai-Cao-Ohta \cite{CCO} where a mass parameter $m$ was defined by $\sqrt{mx^{p(\lambda)}}$. In this case, the mass dimension seems to be incorrect. Also, the previous definition of $mx^{p(\lambda)}$ has a problem to derive the entropy.}. For the solution to be real, it requires $\lambda \geq 1/3$. We mention that $p_+$ and $q_+$ exponents are discarded because they
Figure 1: Exponent graphs of $p(\lambda)$ and $q(\lambda)$. Here we observe $p(1/3) = -1$, $p(1/2) = 0$, $p(1) = 1/2$, $p(3) = 1(\bullet)$, and $p(\infty) = 2$, while we observe $q(1/3) = 3$, $q(1/2) = 1$, $q(1) = 0$, $q(3) = -1(\bullet)$, and $q(\infty) = -3$. For $1/3 \leq \lambda < 3$ (L), the Lifshitz black holes ($0 < z \leq 4$) appear, while the Reissner-Nordström type black holes (RN) are shown for $\lambda > 3$.

are singular at $\lambda = 1$. In this work, thus we make replacements: $p_-(\lambda) \rightarrow p(\lambda)$ and $q_-(\lambda) \rightarrow q(\lambda)$ with $-\Lambda_W = 1/l^2 = 1$ for simplicity. Then, $x = r/l$ becomes a convenient variable for describing the LMP black holes. Hereafter we call the Hořava-Lifshitz black holes as the LMP black holes.

Before proceeding, since the solution of (12)-(13) is similar to the charged dilaton solution in three dimensional AdS spacetimes \([23, 24]\), we may use this idea to study and Hořava-Lifshitz black holes and their thermodynamics. As is shown in Fig. 1, for $1/3 \leq \lambda < \infty$, we have two bounds of $-1 \leq p(\lambda) < 2$ and $-3 < q(\lambda) \leq 3$. The first bound implies that the $m$-term plays a role of the mass term because its exponent is always less than “2”, the second term of AdS spacetimes. Importantly, the latter bound appears because higher order curvature terms like $R^2$ and $R_{ij}R^{ij}$ are present, reflecting the Hořava-Lifshitz gravity.

We would like to mention the three cases of $\lambda = 1/3$, $\lambda = 1/2$, and $\lambda = 1$. In the case of $\lambda = 1/3$, we have $f = 1 + x^2 - \frac{r^2}{mr}$ and $\tilde{N} = x^3$ with $x = \frac{r}{l}$, whereas for SAdS case, $f = 1 + x^2 - \frac{mr}{l}$ and $\tilde{N} = 1$. Hence, we expect that its thermodynamic property is different from that of SAdS even though the metric functions $f$ are the same. For $\lambda = 1/2$, we have $f = x^2$ and $\tilde{N} = x$ which may imply that its thermodynamics is marginally defined. Also, the case of $\lambda = 1$ seems to be familiar as $f = 1 + x^2 - \frac{\sqrt{mr}}{l}$ and $\tilde{N} = 1$. However, the LMP black holes in (12) are not yet completely classified and understood.

In order to understand the LMP black holes, it is necessary to introduce Lifshitz black holes whose asymptotic form takes the form with dynamical exponent $z$ \([25, 26, 27, 28, 29]\).
Table 1: Summary for the Lifshitz black holes in the Hořava-Lifshitz black holes.

| $\lambda$ | 1/3 | 1/2 | 1 | 3 |
|-----------|-----|-----|---|---|
| $q$       | 3   | 1   | 0 | −1 |
| $z$       | 4   | 2   | 1 | 0 |
| $p$       | −1  | 0   | $\frac{1}{2}$ | 1 |
| $f(r)$    | $1 + x^2 - \frac{l^2}{mr}$ | $1 - m + x^2$ | $1 + x^2 - \sqrt{\frac{mr}{l}}$ | $1 + x^2 - \frac{mr}{l}$ |
| $N^2(r)$  | $\tilde{N}^2 f x^6 (1 + x^2 - \frac{l^2}{mr})$ | $x^2 (1 - m + x^2)$ | $1 + x^2 - \sqrt{\frac{mr}{l}}$ | $1 - \frac{m}{r} + \frac{1}{\Lambda W r^2}$ |

\[ ds^2_{\text{Lifshitz}} = -x^{2z} F(r) dt^2 + \frac{1}{H(r)x^2} dr^2 + r^2 d\Omega_{d-2}^2, \]  
(14)

where $F(r)$ and $H(r)$ are functions of radial coordinate $r$ with

\[
\lim_{r \to \infty} F(r) = \lim_{r \to \infty} H(r) = 1.
\]  
(15)

Comparing the LMP black holes (12) with the Lifshitz black holes (14) shows the correspondence

\[
\tilde{N}^2 f = x^{2(q+1)} \frac{f}{x^2} \to x^{2z} F(r), \quad f \to H(r)x^2
\]  
(16)

which leads to two relations

\[
z = q + 1, \quad F(r) = H(r) = \frac{f}{x^2}.
\]  
(17)

As is shown in Table 1, we have four interesting cases with Lifshitz asymptotics. For $1/3 \leq \lambda \leq 1/2$, the Hořava-Lifshitz (LMP) back holes are similar to the non-rotating BTZ black holes, while for $1/2 \leq \lambda < 3$, they behave the rotating BTZ black hole. They all belong to the Lifshitz black holes with $0 < z \leq 4$. However, we observe that for $\lambda = 3$, the lapse function $N^2$ takes the metric function for the Reissner-Nordström black hole when interpreting $\frac{1}{-\Lambda W}$ to be the charge of $Q^2$. Hence, for $\lambda > 3$, they seem to be completely different from the Lifshitz black holes, Reissner-Nordström type black holes. In order to confirm the mentioned conjecture, we need to study thermodynamic properties of the LMP black holes.

3 Thermodynamics of Hořava-Lifshitz black holes

In order to explore the properties of Hořava-Lifshitz back holes, let us first study the Hawking temperature of these black holes because it is independent of the mass definition. The
Figure 2: Temperature graphs of $T(x_\pm, \lambda)$ with $x_\pm = r_\pm$ and $l = 1$. Left graphs for Lifshitz black holes with $1/3 \leq \lambda < 3$: $T(x_\pm, \lambda)$ for $\lambda = 1/3, 1/2, 0.7, 1, 2$ from top to bottom. We observe that for extremal points, $T(0, 1/3) = T(0, 1/2) = T(0.40, 0.7) = T(0.58, 1) = T(0.85, 2) = 0$. Right graph for Reissner-Nordström type black hole with $\lambda \geq 3$: $T(x_\pm, \lambda)$ for $\lambda = 3, 4$. $T(1, 3) = T(1.11, 4) = 0$ at extremal points and $T_m(3.63, 4) = 0.05$ at the maximum point.

The temperature for $\lambda \neq 1$ is defined by

$$T = \frac{(\bar{N}^2 f')}{4\pi} \sqrt{-g^{tt}g^{rr}} \bigg|_{x=x_+} = \frac{x_+^{(\lambda)} \left[ (2 - p(\lambda))x_+^2 - p(\lambda) \right]}{4\pi r_+},$$

while for $\lambda = 1$ and SAdS black hole, they lead to

$$T_{\lambda=1} = \frac{3x_+^2 - 1}{8\pi r_+}, \quad T_{SAdS} = \frac{3x_+^2 + 2}{8\pi r_+}.$$  \hspace{1cm} (19)

Firstly, we find the extremal black holes from the condition of $T = 0$ as

$$x_e = 0, \text{ for } \frac{1}{3} \leq \lambda \leq \frac{1}{2}; \quad x_e = \sqrt{\frac{2\lambda - \sqrt{6\lambda - 2}}{-2 + \sqrt{6\lambda - 2}}}, \text{ for } \lambda > \frac{1}{2}.$$  \hspace{1cm} (20)

As is shown in Fig. 2, $T$ is completely different from $T_{SAdS}$ because the former is qualitatively similar to the BTZ black hole with $T = 0$ extremal temperature \cite{32, 33}, while the latter has a minimum temperature $T_{\text{min}}(\sqrt{\frac{2}{3}}) = \frac{\sqrt{3}^2}{2\sqrt{2}\pi}$ \cite{34, 35} with its shape of ⌂. For $\lambda = 1$, there exists an extremal point at $x_e = 1/\sqrt{3}$ which the temperature vanishes. Hence, for $1/2 < \lambda < 3$, the solution interpolates between AdS$_2 \times S^2$ in the near-horizon geometry of extremal black hole and Lifshitz at asymptotic infinity. On the other hand, for $1/3 \leq \lambda \leq 1/2$, one could not have this connection because the corresponding black hole is a massless black hole located at the origin of coordinate $x_e = 0$ which is similar to the non-rotating BTZ black hole \cite{36}, even though they are asymptotically Lifshitz.

Secondly, from the condition of $dT/dr_+ = 0$, we can easily estimate asymptotic behavior of the temperature because it provides a checking point to test the presence of maximum
As shown in Fig. 2, we find the constant asymptotic behavior of temperature at
\[ \lambda = 3 \]  
and thus, the maximum temperature always exists for
\[ \lambda > 3. \]  

For \( \lambda > 3 \), we find completely different temperature because their asymptotic behavior is similar to that of Reissner-Norström black hole. We remind that \( \lambda = 3(z = 0) \) is the edge of Lifshitz black holes and thus, it is not strange for \( \lambda > 3 \) to find the Reissner-Norström-type black hole in asymptotically flat spacetimes.

It is not clear that the ADM mass for Hořava-Lifshitz black holes could be derived from mass parameter \( m \) using the Hamiltonian formulation without ambiguity. The reason is because we do not know clearly what is asymptotically Lifshitz with dynamical exponent \( z \geq 0 \). Up to now, we may calculate the ADM mass only for asymptotically flat and anti-de Sitter spacetimes. However, Cai-Cao-Ohta have tried to derive masses of LMP black holes and its topological black holes \[^{[38]}\]. We also use this method to derive the mass as
\[ \begin{align*}
M(r_+, \lambda) &= \frac{\pi \kappa^2 \mu^2}{2 \sqrt{6\lambda - 2}} \frac{(-\Lambda)^{2p(\lambda)} + \frac{2(\lambda)}{\lambda} m^{2p(\lambda)}}{2^{2p(\lambda)}} \\
&= \frac{\pi \kappa^2 \mu^2}{2 \sqrt{6\lambda - 2}} \frac{-\Lambda (1 + x_+^2)^2}{x_+^{2p(\lambda)}}. 
\end{align*} \]

As is depicted in Fig. 3, we have mass for Lifshitz black holes with \( 1/3 \leq \lambda < 3 \). For \( 1/3 \leq \lambda \leq 1/2 \), there is no minimum point which satisfies \( dM/dx_+ = 0 \) except at \( x_+ = 0 \), implying the non-rotating BTZ black hole. For \( \lambda = 3, 4 \), we find the mass behavior which is similar to that of Reissner-Nordström black holes.

The heat capacity is an important thermodynamic quantity which tell us the stability of the LMP black holes. A heat capacity defined by \( C = \left( \frac{dM}{dT} \right)_\lambda \) takes the form
\[ C(x_+, \lambda) = \frac{4 \pi^2 \kappa^2 \mu^2}{\sqrt{6\lambda - 2}} \left( \frac{(1 + x_+^2)[(2 - p(\lambda))x_+^2 - p(\lambda)]}{(2 - p(\lambda))(1 + q(\lambda))x_+^2 + p(\lambda)(1 - q(\lambda))} \right), \]
while for \( \lambda = 1 \) and SAdS cases, they have
\[ C_{\lambda=1}(x_+) = 2 \pi^2 \kappa^2 \mu^2 (1 + x_+^2) \left( \frac{3x_+^2 - 1}{3x_+^2 + 1} \right), \quad C_{SAdS} = 8 \pi x_+^2 \left( \frac{3x_+^2 + 2}{3x_+^2 - 2} \right). \]

[^{[38]}]: However, recently, a boundary stress-energy tensor approach was used to derive the ADM mass of Lifshitz black holes, especially for 3D Lifshitz black hole from the new massive gravity \[^{[37]}\].
Figure 3: Mass graphs of $M(x_\pm, \lambda)$. Left graphs for Lifshitz black holes: $M(x_\pm, \lambda)$ for $\lambda = 2, 1, 0.7, 1/2, 1/3$ from top to bottom (along $M$-axis). For $1/3 \leq \lambda \leq 1/2$, there is no minimum point which satisfies $dM/dx_+ = 0$ except at $x_+ = 0$. Right graph for Reissner-Nordström type black holes: $M(x_\pm, \lambda)$ for $\lambda = 3, 4$.

Figure 4: Heat Capacity graphs of $C(x_\pm, \lambda)$. Left graphs for Lifshitz black holes: $C(x_\pm, \lambda)$ for $\lambda = 1/3, 1/2, 0.7, 1, 2$ from top to bottom. We check that for extremal black holes, $C(0.4, 0.7) = C(0.58, 1) = C(0.85, 2) = 0$. Right graph for Reissner-Norström type black holes: $C(x_+, 4)$ shows a blow-up point at $x_m = 3.63$, which shows a feature of transition from stable region to unstable region. $C(x_+, 3)$ is an increasing function of $x_+$. 

As is shown in Fig. 4, for Lifshitz black holes, all heat capacities go to $\infty$ as $x_+ \to \infty$. We observe that the Hořava-Lifshitz (LMP) black hole with $1/3 \leq \lambda < 3$ are always thermodynamically stable because their heat capacities are always positive for $x_+ \geq x_e$. On the other hand, for $\lambda > 3$, the heat capacity blows up at the minimum point of $x_m = 3.63$. Hence, small black holes of $x_+ < x_m$ are stable because their heat capacities are positive, while large black hole of $x_+ > x_m$ are unstable because their heat capacities are negative. This indicates a feature of Reissner-Norström type black holes.

Finally, the entropy of Hořava-Lifshitz black holes could be found by assuming that the
first law of thermodynamics holds
\[ dM = T dS. \] (26)

Using this first law, we derive an entropy
\[ S = \frac{\sqrt{2}(-\Lambda)}{\sqrt{3\lambda - 1}} \left( \frac{A}{4} - \frac{\pi}{\Lambda} \ln \frac{A}{4} - S_0 \right), \] (27)
with \( A/4 = \pi r_+^2 \) and \( S_0 = \frac{\pi \ln \pi}{\Lambda} \). The main feature of the LMP black holes has the entropy with a logarithmic term [20, 38]. As is shown in Fig. 5, there is no drastic change in the entropy when changing from Lifshitz to Reissner-Norström type black holes.

However, for \( \lambda > 3 \), we have a different form of the entropy
\[ S = \frac{\sqrt{2}(-\Lambda)}{\sqrt{3\lambda - 1}} \left( \frac{A}{4} + \pi Q^2 \ln \frac{A}{4} - S_0 \right) \] (28)
because we may interpret \( \frac{1}{-\Lambda w} \) to be the charge of \( Q^2 \) in the Reissner-Norström type black holes.

### 4 Discussions

We study thermodynamics of the LMP black holes in the \( z = 3 \) Hořava-Lifshitz gravity according to the coupling constant \( \lambda \). For \( 1/3 \leq \lambda \leq 1/2 \), the black holes behave the non-rotating BTZ back hole because of their extremal point at \( x_e = 0 \), while for \( 1/2 < \lambda < 3 \) including \( \lambda = 1 \), the black holes behave the rotating BTZ back hole because of their extremal point at \( x_e \neq 0 \). However, all these black holes belong to asymptotically Lifshitz. For
\( \lambda > 3 \), we have found Reissner-Norström type black holes. We confirm this classification by studying thermodynamic properties of LMP black holes.

Even though we have started with the \( z = 3 \) Hořava-Lifshitz gravity, we have Lifshitz black hole with dynamical exponent \( z \) and Reissner-Norström type black holes. Frankly speaking, as far as is concerned on obtaining the LMP black holes, we were working with \( z = 2 \) Hořava-Lifshitz gravity without the Cotton tensor \( (C_{ij} = 0) \). We suggest that various black hole solutions are found from this gravity because it is a non-relativistic gravity theory.

On the other hand, there are Lifshitz black holes obtained from relativistic higher derivative gravities, where the higher derivative terms are not considered as perturbative corrections to Einstein-Hilbert action. These are included \( z = 3 \) Lifshitz black hole from the 3D new massive gravity \([31]\), \( z = 3/2 \) Lifshitz black hole in 4D spacetimes \([39]\), and higher dimensional Lifshitz black holes \([40]\).

Consequently, our main results are the interpretation of the Hořava-Lifshitz (LMP) black holes with \( \lambda \geq 1/3 \) as the Lifshitz black holes with dynamical exponent \( z \) for \( 1/3 \leq \lambda < 3(0 < z \leq 4) \) and the Reissner-Nordström black hole with charge \( -\frac{1}{\Lambda_{W}} \) for \( \lambda > 3 \).

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**References**

[1] P. Horava, Phys. Rev. D **79** (2009) 084008 [arXiv:0901.3775 [hep-th]].

[2] P. Horava, JHEP **0903** (2009) 020 [arXiv:0812.4287 [hep-th]].

[3] P. Hořava, Phys. Rev. Lett. **102** (2009) 161301 [arXiv:0902.3657 [hep-th]].

[4] A. Volovich and C. Wen, JHEP **0905** (2009) 087 [arXiv: 0903.2455 [hep-th]].

[5] J. Kluson, JHEP **0907** (2009) 079 [arXiv:0904.1343 [hep-th]].

[6] H. Nikolic, [arXiv:0904.3412 [hep-th]].
[7] H. Nastase, arXiv:0904.3604 [hep-th].

[8] K. I. Izawa, arXiv:0904.3593 [hep-th].

[9] G. E. Volovik, JETP Lett. 89 (2009) 525 arXiv:0904.4113 [gr-qc].

[10] B. Chen and Q. G. Huang, Phys. Lett. B 683 (2010) 108 arXiv:0904.4565 [hep-th].

[11] G. Calcagni, JHEP 0909 (2009) 112 arXiv:0904.0829 [hep-th].

[12] T. Takahashi and J. Soda, Phys. Rev. Lett. 102 (2009) 231301 arXiv:0904.0554 [hep-th].

[13] S. Mukohyama, JCAP 0906 (2009) 001 arXiv:0904.2190 [hep-th].

[14] R. Brandenberger, Phys. Rev. D 80 (2009) 043516 arXiv:0904.2835 [hep-th].

[15] Y. S. Piao, Phys. Lett. B 681 (2009) 1 arXiv:0904.4117 [hep-th].

[16] X. Gao, arXiv:0904.4187 [hep-th].

[17] R.L. Arnowitt, S. Deser and C.W. Misner, The dynamics of general relativity, “Gravitation: an introduction to current research”, Louis Witten ed. (Wilew 1962), chapter 7, pp 227-265, arXiv:gr-qc/0405109.

[18] H. Lu, J. Mei and C. N. Pope, Phys. Rev. Lett. 103 (2009) 091301 arXiv:0904.1595 [hep-th].

[19] E. Kiritsis and G. Kofinas, Nucl. Phys. B 821 (2009) 467 arXiv:0904.1334 [hep-th].

[20] R. G. Cai, L. M. Cao and N. Ohta, Phys. Rev. D 80 (2009) 024003 arXiv:0904.3670 [hep-th].

[21] R. G. Cai, Y. Liu and Y. W. Sun, JHEP 0906 (2009) 010 arXiv:0904.4104 [hep-th].

[22] E. O. Colgain and H. Yavartanoo, JHEP 0908 (2009) 021 arXiv:0904.4357 [hep-th].

[23] K. C. K. Chan and R. B. Mann, Phys. Rev. D 50 (1994) 6385 [Erratum-ibid. D 52 (1995) 2600] arXiv:gr-qc/9404040.

[24] Y. S. Myung, Y. W. Kim and Y. J. Park, Eur. Phys. J. C 58 (2008) 617 arXiv:0809.1933 [gr-qc].
[25] S. Kachru, X. Liu and M. Mulligan, Phys. Rev. D 78 (2008) 106005 [arXiv:0808.1725 [hep-th]].

[26] U. H. Danielsson and L. Thorlacius, JHEP 0903 (2009) 070 [arXiv:0812.5088 [hep-th]].

[27] R. B. Mann, JHEP 0906 (2009) 075 [arXiv:0905.1136 [hep-th]].

[28] G. Bertoldi, B. A. Burrington and A. Peet, Phys. Rev. D 80 (2009) 126003 [arXiv:0905.3183 [hep-th]].

[29] G. Bertoldi, B. A. Burrington and A. W. Peet, Phys. Rev. D 80 (2009) 126004 [arXiv:0907.4755 [hep-th]].

[30] K. Balasubramanian and J. McGreevy, Phys. Rev. D 80 (2009) 104039 [arXiv:0909.0263 [hep-th]].

[31] E. Ayon-Beato, A. Garbarz, G. Giribet and M. Hassaine, Phys. Rev. D 80 (2009) 104029 [arXiv:0909.1347 [hep-th]].

[32] M. Banados, C. Teitelboim and J. Zanelli, Phys. Rev. Lett. 69 (1992) 1849 [arXiv:hep-th/9204099].

[33] M. Banados, M. Henneaux, C. Teitelboim and J. Zanelli, Phys. Rev. D 48 (1993) 1506 [arXiv:gr-qc/9302012].

[34] J. Crisostomo, R. Troncoso and J. Zanelli, Phys. Rev. D 62 (2000) 084013 [arXiv:hep-th/0003271].

[35] Y. S. Myung, Phys. Lett. B 624 (2005) 297 [arXiv:hep-th/0506096].

[36] Y. S. Myung, Phys. Lett. B 638 (2006) 515 [arXiv:gr-qc/0603051].

[37] O. Hohm and E. Tonni, arXiv:1001.3598 [hep-th].

[38] R. G. Cai, L. M. Cao and N. Ohta, Phys. Lett. B 679 (2009) 504 [arXiv:0905.0751 [hep-th]].

[39] R. G. Cai, Y. Liu and Y. W. Sun, JHEP 0910 (2009) 080 [arXiv:0909.2807 [hep-th]].

[40] E. Ayon-Beato, A. Garbarz, G. Giribet and M. Hassaine, arXiv:1001.2361 [hep-th].