Diffractive scattering at high energies

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Abstract

In this work we analyse applicability of regge-based models to hadron interactions. Commonly-used extensions of pure regge model are found to be failed. Low constituent model is introduced and found to be consistent with hadron-hadron and hadron-photon data.

1 Introduction

In this work we review contemporary methods for description of diffraction processes at high energy. In the soft region of kinematical variables QCD has not been solved, so, phenomenological approaches are traditionally used.

The key object of investigation in such approaches is Pomeron. Pomeron is defined as reggeon with largest intercept at canonical regge model or as synonym of diffraction ("diffracton") in more contemporary context. First context of pomeron can be used as basis for the second context, and we get "full" eikonalized pomeron made as the sum of n-pomeron cuts. Another way is to write up propagator of pomeron and vertex pomeron-hadron and pomeron-pomeron functions, fitted to experimental values.

In this work we consider existing models of pomeron and compare them with our low constituent model.

In section 3 we investigate applicability of the quasi-elastic model for the description of hadron-hadron and hadron-photon diffractive interactions.

Main goal of subsection 3.1 section is to estimate eikonal parameter $c$ from the elastic and total cross-section data.

In subsection 3.2 we consider processes of the single diffraction in the framework of the quasi-eikonal model. As compared with Ref. [11] and Ref. [12] we consider all non-enhanced diagrams.

In this work we also consider low constituent model (section 4), where there is only basic quark-gluon states and interactions. This model leads us to the non-local pomeron, but it has clear interpretation of the pomeron flux renormalization.

Applicability of considered models to photon-hadron interactions is discussed in section 5.

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Main problems to be solved

Regge non-enhanced phenomenology well describes total and elastic cross-sections in the Donnachie-Landshoff parameterization, see Fig. 2, taken from [18] and Fig. 2, taken from [14]. This model has problems with unitarity. At first, total cross-sections rise too fast, breaking Fruassar limit [6]:

\[
\sigma_{tot} = \frac{1}{s} \text{Im} A(s, 0) < \pi \frac{m_{\pi}^2}{2} \ln^2(s)
\]

At contemporary collider energies this fact is not very annoying, because observable cross-sections is much lower Fruassar limit.

But unitarity is break up at s-channel, where we have limit on profile function \(J(s, b):\)

\[
J(s, b) = 2 \text{Im} \, \chi_{pp}(s, b) < 1
\]

Reconstruction of profile function shown at Fig. 3.

It is clear, that pure Regge model break up unitarity already at LHC energies. Unitarity at s-channel is closely connected with sum rule for elastic cross-sections derived in [7]:

\[
I_0(s) = \frac{1}{2} \int_{-\infty}^{0} \sqrt{\frac{ds}{s}} dt \rightarrow 1
\]

One can see from Fig. 4, \(I_0(s)\) close to unity at Tevatron energies.

Low-energy single diffraction data is also well described by regge phenomenology with supercritical pomeron, but at the region of Tevatron energies it fails to describe data on single diffraction dissociation. The main problem is that total
Figure 2: The Donnachie-Landshoff fit for the differential elastic $pp$ cross section

single diffraction cross-section rise considerably weaker than it is predicted by $Y$-like Regge diagrams involving only three pomerons. This fact is clearly seen from Fig.5 extracted from Ref.[15], there "Standard flux" corresponds to $Y$-like Regge diagram.

Many ways were suggested to solve this problem. First way is two-variant (Ref.[10] and Ref.[16]) pomeron flux renormalization model, where we consider the equation for cross section of single diffraction

$$\frac{d^3\sigma}{dM^2dt} = f_{P/p}(x,t)\sigma_{Pp}(s)$$

and pick out the factor, named as 'pomeron flux'

$$f_{P/p}(x,t) = K\xi^{1-2\alpha_P(t)}$$

Renormalization of the pomeron flux is made by inserting dependence of $K$ either on $s$ (as in Ref.[10]) either on $x,t$, as in Ref.[16]. This phenomenological approach well describes CDF data on single diffraction, but we need more theoretical bases for extrapolation to higher energies.

The second way is straight-forward account of screening corrections (Ref.[11] and Ref.[12]). This way seems to be more natural, but we need to introduce additional parameters and make some assumptions about Regge diagram technics. In Ref.[11] and Ref.[12] only some parts of sufficient diagrams were accounted by going to the impact parameter space $b$ and replacement initial "Borhn" factor $\chi(s,b)$ to eikonalized amplitude $(1 - e^{-\mu x(s,b)})$. In addition, the central $Y$-like diagram was modified to account low-energy processes and in Ref.[12] the dependence of pomeron intercept on energy was introduced.
Figure 3: Profile function $J(b, s)$ at $\sqrt{s} = 546 GeV$ (solid line), $\sqrt{s} = 1800 GeV$ (dashed line), $\sqrt{s} = 14 TeV GeV$ (dotted line).

Figure 4: Dependence of $I_0(s)$ (eq. (3)) on $\sqrt{s}$, last point - extrapolation for LHC. Figure is gotten from [7].
Figure 5: The total single diffraction cross section for $p(\bar{p}) + p \rightarrow p(\bar{p}) + X$ vs $\sqrt{s}$ compared with the predictions of the renormalized pomeron flux model of Goulianos [10] (solid line) and the model of Gotsman, Levin and Maor [11] (dashed line, labeled GLM); the latter, which includes "screening corrections", is normalized to the average value of the two CDF measurements at $\sqrt{s} = 546$ and 1800 GeV.
3 Applicability of the quasi-eikonal model for the description of hadron-hadron diffractive interactions

3.1 Estimation of eikonal parameter $c$ from the elastic and total cross-section data.

Let’s consider processes of elastic scattering in the framework of the eikonalization procedure.

There is two view on the eikonalization procedure. At first, eikonalization was considered as a natural way to come from Regge behavior $\sigma_{tot}(s) \sim s^\Delta$ to Froussoir-like behavior $\sigma_{tot}(s) \sim \ln(s)^2$. Both pure Regge model and black-disk model well describe total cross-section data. From the other hand, data on elastic cross-sections $\frac{d\sigma}{dt}(s,t)$ can be described assuming only one Regge pole (pomeron) at low and moderate $|t| < 1 GeV^2$. Here we will investigate possibility of eikonalization model to describe elastic scattering data at non-zero transferred momenta.

It is convenient to write up eikonalized amplitude as

$$A(s, \vec{k}) = \frac{4}{t} \frac{1}{c} \int \left\{ e^{ic\chi(b)} - 1 \right\} J_0(bk) bdb$$

where $\chi(b)$ is eikonal profile function and $a(s, \vec{k})$ is generic amplitude.

In the Regge model eikonal profile $\chi(b)$ can be calculated as

$$\chi(b) = -\frac{c^2}{8\pi} \frac{g_ag_b}{R_a^2 + R_b^2 + \alpha'Y + i\pi\alpha'/2} e^{-\frac{1}{4} R_a^2 + R_b^2 + \alpha'Y + i\pi\alpha'/2} e^{\Delta(Y + \frac{b^2}{2})}$$

(7)

To estimate eikonalization parameters, we will try to describe experimental data on total and elastic cross sections

$$\sigma_{tot} = \frac{1}{s} ImA(s, 0)$$

$$\frac{d\sigma}{dt} = \frac{1}{16\pi s^2} |A(s, \vec{k})|^2$$

(8)

at the domain of $0.2 GeV^2 < t < 1 GeV^2$ and $\sqrt{s} \geq 53 GeV^2$, there we have no non-vacuum reggeons and no perturbative $\frac{1}{s}$ behavior.

At first stage we will start from conventional Regge picture, and will vary all parameters in (7) at given $c$. We take only $\frac{d\sigma}{dt}$ data here to concentrate on the $t$ dependence of $A(s, t)$ and to avoid problems with phase of $A(s, t)$. As the result, we get function of likely-hood $\frac{\chi^2}{n.d.f.}$, depending on $c$, see Fig.6.

Dependence of $\frac{d\sigma}{dt}$ at some critical values of $c$ is given at Fig.7 and Fig.8.
Figure 6: Dependence of likelyhood function on c
Figure 7: $\frac{d\sigma}{dt}$ at $c = 0$ (pure pomeron) and at optimal value $c = 0.125$. Difference between graphics is not viewable, so we give one figure.
Figure 8: $\frac{d\sigma}{dt}$ at $c = 0.4$ (region of hardly divergence of experimental data and eikonal model)
It is clear, that this eikonal model with exponential pomeron vertex can describe experimental data only at low region \( c \leq 0.2 \). Including of total cross-section does not change this situation, function \( \frac{\chi^2}{n_d.f.} \) rises even more quickly and has no minimum at non-zero \( c \). Theory predict, that \( c \) is high, \( c \sim 1.5 \) and there is strong limit \( c > 1 \), so we have serious problem. We can try to reject exponential form of pomeron vertex, but we have no other assumptions about functional form of the vertexes.

It is natural to extract eikonal profile function \( \chi(b) \) from experimental data and test it regge-like behavior To make it, we approximate observed amplitude \( A(s,t) \) by regge-like form:

\[
\begin{align*}
A(s, t) &= A(s, 0)e^{B(s)t} \\
A(s, 0) &= s\sigma_{tot}(s)(\rho + i) \\
B(s) &= \frac{B_{real}(s)}{2} + \frac{i\pi \alpha'}{2}
\end{align*}
\]  

there \( B_{real}(s) \) is logarithmic slope of \( \frac{d\sigma}{dt} \) at given \( s \). Reliable of this parametrisation is clear from Fig. We have only to test stability of our results on the changes of unobservable imaginary part of \( \beta(s) \).

Calculations is trivial enough, and we have for the generic amplitude

\[
\begin{align*}
a(s, t) &= \frac{4\pi s}{i\alpha'} \int \ln \left( 1 + \frac{i\alpha'^2 A(s,0)}{8\pi s B(s)} e^{-\frac{2}{4\pi}} \right) J_0(bk) bdb
\end{align*}
\]

Imaginary part of logarithm we define to be small at high \( b \) and to be continuous at all \( b \).

Dependence of the reconstructed generic amplitude on \( t \) is shown at Figs. We don’t include reconstructed amplitude at trivial \( c = 0 \) here.

From the canonical Regge theory, we expect for \( a(s, t) \) to be

\[
a(s, t) = e^{\frac{\pi \alpha(t)}{2}} \beta(t)s^{\alpha(t)}
\]  

there residue \( \beta(t) \) is real. We see, that our reconstructed amplitudes does not satisfy to equation at high \( c \), because we have \( s \)-depended phases and (it is more serious), zeroes moving with \( s \). Moreover, it is clear from analyzeof \( s \)-dependence of \( a(s, t) \) (see Fig.), that \( a(s, t) \) does not scale as \( s^{\alpha(t)} \) at high \( c \).
Figure 9: Generic amplitude $a(s, t)$, divided by $s$, reconstructed for $c=0.75$ at energies $\sqrt{s} = 1800\, \text{GeV}$ (dotted line), $\sqrt{s} = 546\, \text{GeV}$ (dashed line), $\sqrt{s} = 65\, \text{GeV}$ (solid line).
Figure 10: Generic amplitude $a(s, t)$, divideded on $s$, reconstructed for $c=1.2$ at energies $\sqrt{s} = 1800\,GeV$ (dotted line), $\sqrt{s} = 546\,GeV$ (dashed line), $\sqrt{s} = 65\,GeV$ (solid line)
Figure 11: Dependence of $a(s, t)$ on $\ln(s)$ at $c = 0.75$ (solid line) and $c = 1$ (dashed line)
This result is stable under our assumptions about the phase of $A(s, t)$, at $c > 0.75$ we can choose any small imaginary part of $B(s)$ in equations (9) without any influence on the reconstructed amplitude $a(s, t)$. It is possible, that $s$-dependence phase of $a(s, t)$ is arisen from wrong assumptions about phase of the $A(s, t)$, but moving zeroes is stable at high $c$ ($c \geq 1$).

We can consider most general properties of the pomeron cuts. Many theoretical models (see [17] for review) leads us to the next form for the two-pomeron cut

$$A^P_2(s, t) = \frac{i}{16\pi^2} \int_0^\infty dt_1 dt_2 \frac{\alpha_2(\lambda(t_1, t_2))}{\lambda(t_1, t_2)} (N(t, t_1, t_2))^2 A^P(s, t_1) A^P(s, t_2)$$

(12)

Amplitude $A^P(s, t)$ is mostly imaginary, so cut $A^P_2$ have opposite sign to pole $A^P(s, t)$. As it was stated above (Figs. [8] [9]), $A^P(s, t)$ is large enough, close to its asymptotic value. So, at high enough $N(t, t_1, t_2)$ cuts will lead to changing of the exponential generic form of the pole (log-plot became non-linear with up-to convexity) and to existance of zeroes, which is moving with $s$. No of this phenomena is observed, so $N(t, t_1, t_2)$ must be small.

Straightforward estimation of eikonal parameter was done by A.Donnachie and P.V.Landshoff at [8] and [9], by requiring that the dip discovered in CERN ISR $pp$ elastic scattering occurs at the correct value of $t$. Donnachie-Landshoff value 0.4 is, actually, the upper bound on eikonal parameter, because we consider soft non-perturbative pomeron eikonalization, but dip at $t \sim 1\mathrm{GeV}^2$ generated by perturbative triple gluon exchange [5], which is out of our work.

So, we can state, that eikonalization parameter $c$ is limited by $c < 0.5$, if we assume exponential form of the generic pomeron and by $c < 1$ if we make no assumptions about form of the pomeron residue. Anyway, this results is in contrast with the theory expectations and strongly limit eikonalization models, such as restoring of unitarity and renormalization of the pomeron flux in the single diffraction.

### 3.2 Description of the single diffraction data in the framework of the quasi-eikonal model.

Quasi-eikonal model, considered in this work, is standart enough. We use reggeon diagram technic with reggeon propogator $s^{\alpha(t)}$, model gauss vertexes of the interaction of $n$ pomerons with hadron

$$N_h(k_1, \ldots, k_n) = g_h (g_h c_h)^{n-1} \exp \left(-R_h^2 \sum_{i=1}^n k_i^2 \right)$$

and the vertex corresponding to the transition of $l$ pomerons into $m$ pomerons under the $\pi$-meson exchange dominance assumption

$$\Lambda(k_1, \ldots, k_m) = r (g_p c_\pi)^{m-3} \exp \left(-R_e^2 \sum_{i=1}^m k_i^2 \right).$$

(14)

Here $g_h$ is the pomeron-hadron coupling, $c_h$ is the corresponding shower enhancement coefficient, $R_h$ and $R_e$ are the radii of the pomeron-hadron and pomeron-pomeron-pomeron interactions, respectively, $k_i$ are the pomeron transverse momenta. Integration on the pomeron momenta is made trivial in the impact parameter space representation and we only have to sum on then numbmers of pomerons, attached to the same vertexes.

As compared with Ref. [11] and Ref. [12], where only part of sufficient diagrams was accounted (see Fig. [12]), in this paper we account all non-enhanced absorptive corrections to the Y-diagram contribution, shown in Fig. [12].
Because low-energy corrections rapidly decrease with energy, we account only pomeron contributions, but in all sufficient diagrams, as it was done in Ref.[19] and Ref.[20]. It gives us possibility to normalize cross-section of the single diffraction to CDF data and make theoreticaly based predictions for cross-section of the single diffraction at LHC energies.

The contribution $f_{n_1 n_2 n_3 n_4 n_5}$ of each diagram in Fig.12b can be written in a rather simple form

$$f_{n_1 n_2 n_3 n_4 n_5} = \frac{(-1)^{n_1 n_2 n_3 n_4 n_5 + 1}}{c_1 c_2 c_3 c_4 c_5} \frac{8 \pi^3}{c_1 c_2 c_3 c_4 c_5} \left[ \frac{c_1 c_2 c_3 c_4 c_5 \Delta Y}{8 \pi (R_a^2 + R_b^2 + \alpha Y)} \right]^{n_1 + n_2}$$

$$\left[ \frac{c_1 c_2 c_3 c_4 c_5 \Delta Y}{8 \pi (R_a^2 + R_b^2 + \alpha Y)} \right]^{n_3} \frac{1}{\text{det} F} e^{-\frac{1}{\text{det} F}}$$

Here $Y = \ln(s), \ y = \ln(M^2)$. Then inclusive cross section is

$$\frac{d^3\sigma}{dp^3} = \frac{\pi s}{M^2} \sum_{n_1, n_2, n_3=1}^{\infty} \sum_{n_4, n_5=0}^{\infty} f_{n_1 n_2 n_3 n_4 n_5}$$

Our method differs from early work Ref. [20], where all parameters but vertex $r$ were fixed. In this work here we vary all parameters. Parameters were varied with natural limitations, i.e. all parameters were varied above its conventional values. We don’t consider very high or very low values of pomeron intercept and slope, which can be compensated by other parameters.

Another difference as compared with Ref.[20] is the fact, that we use data on total and elastic (differential) cross-sections and data on total single-diffraction cross-
sections. So, we can fix parameters of the model with higher precision and with account of its one-to-one correlations.

The model under consideration doesn’t include possible contributions of low-lying reggeons, so we limit considered energies by \( \sqrt{s} > 52 \text{GeV} \) for elastic and total cross-sections. As was shown in \[21\], modern data don’t give us possibility to distinct simple pole model with total cross-sections \( \sigma_{\text{tot}} = A s^\Delta \) and eikonalized models with \( \sigma_{\text{tot}} = C + D \ln(s) \) or \( \sigma_{\text{tot}} = E + F \ln(s)^2 \). But we can reliably determine parameters of the model \( R_h, g_h, \Delta, \alpha' \) from elastic and total cross-section data at fixing \( c_h \).

We use CDF data on single diffraction for analysis.

CDF data \[22\] was presented as a result of the monte-carlo simulations based on the general formula:

\[
\frac{d^2\sigma}{d\xi dt} = \frac{1}{2} \left[ \frac{D}{\xi^2} e^{(b_0 - 2\alpha'_SD \ln \xi)t} + I\xi^\gamma e^{bt} \right]
\]  

(17)

Taken CDF data is shown in Table 1.

Table 1: CDF fit-parameters from reference [1].

| \( \sqrt{s} \) | \( \sqrt{s} = 546 \text{GeV} \) | \( \sqrt{s} = 1800 \text{GeV} \) |
|----------------|-----------------|-----------------|
| \( D \)        | 3.53 \pm 0.35   | 2.54 \pm 0.43   |
| \( b_0 \)      | 7.7 \pm 0.6     | 4.2 \pm 0.5     |
| \( \alpha'_SD \) | 0.25 \pm 0.02   | 0.25 \pm 0.02   |
| \( \epsilon \) | 0.121 \pm 0.011 | 0.103 \pm 0.017 |
| \( I \)        | 537^{+498}_{-280} | 162^{+160}_{-85} |
| \( \gamma \)   | 0.71 \pm 0.22   | 0.1 \pm 0.16    |
| \( b' \)       | 10.2 \pm 1.5    | 7.3 \pm 1.0     |

This parameters are experimental points tested in our model. Let’s mark, that low-lying reggeons contribution, corresponding to second addend um in (17), isn’t accounted in our calculations and we have to model only parameters \( D, b_0, \alpha'_SD, \epsilon \). We calculate this parameters in the region \( 0.05 < t < 0.1; 0.99 < x < 0.995 \), where we have the most reliable CDF data and contribution of the low-lying reggeons is minimal.

We have to mark, that this data is not precise because of the following reasons:

1. CDF single diffraction data have low statistics and narrow kinematical region, where the data was taken;
2. At each energy \( \sqrt{s} = 546\text{GeV} \quad \sqrt{s} = 1800\text{GeV} \) 6 highly correlated parameters are introduced, and it makes calculations unstable;
3. Fixing of effective pomeron slope on the common value \( \alpha'_SD = 0.25 \) is obliged.

Unreability of the data in Table 1 is clearly seen from analysis of dependence of \( D \) on energy from \( \sqrt{s} = 546\text{GeV} \) to \( \sqrt{s} = 1800\text{GeV} \). As defined \[22\],

\[
D = G(0)s^\Delta
\]  

(18)
here $G(0)$ doesn’t depend on $s$, and $\Delta > 0$. In accordance with this definition, parameter $D$ must increase when energy increases, but in CDF data it decreases.

Total single diffraction cross-sections are well experimentally defined and don’t depend on the model, used in analysis of basic data (detectors counts)

\begin{align*}
\sigma_{SD}(\sqrt{s} = 546 GeV) &= 7.89 \pm 0.33 mb \\
\sigma_{SD}(\sqrt{s} = 1800 GeV) &= 9.46 \pm 0.44 mb
\end{align*}

(19)

We include these two points in $\chi^2$ test with larger weights, than points shown in Table 1.

Because total and elastic cross sections, on one side, and single diffraction cross sections, on other side, have different types, we vary parameters $r, R_\pi$ and $c_\pi$ to achieve the best agreement with data in Table 1 and data (19), fixing at each step parameters $\Delta, \alpha', g_h$ and $R_h$ from total and elastic cross-sections.

Results.

In the end of optimization process we’ve got next parameter set: $g_p^2 = 75.0538, \Delta = 0.0868089, R_p^2 = 1.94755, \alpha = 0.148963, c_p^2 = 2.03954, r = 0.111525, R_\pi^2 = 0.173682, c_\pi^2 = 6.00989$

Following total single diffraction cross sections were calculated at these parameters:

\begin{align*}
\sigma_{SD}(\sqrt{s} = 546 GeV) &= 7.5 mb \\
\sigma_{SD}(\sqrt{s} = 1800 GeV) &= 10 mb
\end{align*}

(20)

Corresponding differential characteristics of differential single diffraction cross sections are enumerated in Table 2:

Table 2: Differential characteristics of differential single diffraction cross sections in our model.

|                  | $\sqrt{s} = 546 GeV$ | $\sqrt{s} = 1800 GeV$ |
|------------------|-----------------------|------------------------|
| $D$              | 2.9628                | 3.08731                |
| $b_0$            | 5.32553               | 5.27187                |
| $\alpha'_{SD}$   | 0.294999              | 0.270871               |
| $\epsilon$      | 0.0580572             | 0.0549202              |

Calculated differential characteristics are very close to ones from the triple-pomeron model, so the following relation is satisfied

\[ \frac{d^3\sigma}{dM^2dt} = f_{P/p}(x, t)\sigma_{P/p}(s) \]

(21)

where

\[ f_{P/p}(x, t) = K(s)\xi^{1-2\alpha_p(t)} \]

(22)

is renormalized pomeron flux. As compared with standard triple-pomeron model the dependence of factor $K$ on energy $s$ is introduced. This dependence provides slowing on the rise of the single diffraction cross section with energy.

Dependence of renormalizing factor $K(s)$ on energy is shown on Fig.13.

As we have noted in section 3.1, there are inconsistencies in estimating $c_p$. From one side, there are theoretical indications, that $c_p > 1$. Such high values of $c_p$
lead to significant divergence of dependence $\frac{d\sigma}{dt}$ on $t$ from exponential behavior $e^{-bt}$ already at $t \sim 0.2 GeV^2$. It is known from experiment, that elastic cross-section falls exponentially on $t$ up to $t \sim 1 GeV^2$. This inconsistence is clearly seen from Fig.14.

From this fact of independence of logarithmic slope on $t$ we conclude, that $c_p << 1$. To explain slow rise of $\sigma_{SD}$ with energy we have to assume very high $c_\pi$, $c_\pi c_p \gg 1$ at $g_\pi \sim g_p$. It gives desired value of the fraction $\frac{\sigma(\sqrt{s}=1800 GeV)}{\sigma(\sqrt{s}=546 GeV)} \sim 1.2$, but leads to very high values of logarithmic slope $b \sim 50 GeV^{-2}$ (situation will be even more worse, than in the case of elastic cross-section, shown on Fig.14). Solving of this problem by precise adaptation of $R_\pi$ is unusable, because it leads to highly differing from experiment and depended on $M^2$ and $t$ values of $\alpha'$ and $\epsilon$.

At $c_p > 1$ we don’t need $c_\pi$ in so high values, and logarithmic slope $b$ is back to values about ones, not tens. So, we must return to the theoretically based area $c_p > 1$ and limit considered area of elastic scattering by $t < 0.2 GeV^2$.

We see, that goog agreement of quasi-eikonal model with experiment is achieved on the border of the allowed region of parameters $[c_p, c_\pi]$ (see. Fig.15).
Figure 14: Elastic cross sections $\frac{d\sigma}{dt}$ for reaction $p + p \rightarrow p + p$. Theoretical curve is at energy $\sqrt{s} = 1800\text{GeV}$. Experimental points are taken at energies from ISR to Tevatron.

From one side, it gives stability of the calculated parameters. From the other side this model has no reserve of stability. If fraction of the cross sections $\frac{\sigma(\sqrt{s} = 1800\text{GeV})}{\sigma(\sqrt{s} = 546\text{GeV})}$ will be defined more precisely and will be found in the region 1.1 ÷ 1.15 (it is minimal value, which is consistent with existed data), then for description of this data we will be obliged to decline either describing elastic and total cross sections or describing logarithmic slope of the single diffraction on $t$.

The main difficulty in quasi-eikonal description of single diffraction is the same as in elastic scattering, we can not construct good full Pomeron propagator as sum of powers of simple-pole Pomerons. Resulting sum is far from Regge-like behavior, while experiment is very Regge-like with exeption of overall normalisation in single diffraction.

In the hadron-hadron interaction domain we suggest another description of pomeron flux renormalization.

We start from Regge-based estimation of probability (at given impact parameter $b$) for pomeron to be emitted with momenta $xP_L, \bar{k}$ and to interact with production of hadron system with mass $M^2$:

$$P(\text{Regge}) = f_{P/P}(x, t)P_{P^p}(M^2)$$

(23)

For this value we have good phenomenological triple-pomeron approach with paramters extracted from low-energy fit or from elastic rescattering analisys.

Resulting event can include soft rescattering processes, see Fig. 10.

Surely, we observe only events without soft-rescattering. To estimate rescattering effects, write up (23) as sum:

$$P(\text{Regge}) = P(\text{Regge}\mid \text{without soft rescatterings}) + P(\text{Regge}\mid \text{with soft rescatterings})$$

(24)

Observable is $P(\text{Regge}\mid \text{without soft rescatterings})$. We assume, that probability
Figure 15: Allowed region of parameters $[c_p, c_x]$. 

Elastic cross section slope is good enough

Direction to slower rise of SD cross section with $S$

Allowed region of $[C_p, C_{pi}]$

SD cross section slope is good enough
of rescattering is independent of Regge part, and so,

\[ P(\text{Regge}|\text{without soft rescatterings}) = P(\text{Regge}) \times P(\text{without soft rescatterings}) \]  

(25)

Last factor is usually named as survival gap probability (SGP).

Eq. (25) leads to estimation

\[ \sigma_{SD}(s) = \sigma^{\text{Regge}}(s) \times \text{SGP}(s) \]  

(26)

Term \( \sigma^{\text{Regge}}(s) \) is triple-pomeron estimation, based on (23) or (4).

Surely, survival gap probability SGP is much larger for single diffraction than for di-jet of higgs production because single diffraction is perireal. Energy dependence of SGP on \( s \) seems to be independent on process, and can be extracted from higgs production estimations. We have used Ref. [5] calculations for dependens SGP on \( s \) (see Fig. 17) and finded

\[ \text{SGP}(s) = C s^\epsilon \]  

(27)

\[ \epsilon \sim 0.11 \]  

(28)

This value of power is very close to soft Pomeron intercept \( \Delta \sim 0.1 \). So, it follows from (20) that observable \( \sigma_{SD}(s) \) rise like \( s^\Delta \), not Regge-like \( s^{2\Delta} \). So, this picture gives good numerical results and can be easily connected to more strict methods, but it fails in description of photon-hadron interactions, where there is no soft rescatterings at all.
4 Applicability of the low constituent model for the description of hadron-hadron diffractive interactions

We consider the three-stage model of hadron interaction at the high energies.

On the first stage before the collision there is a small number of partons in hadrons. Their number, basically, coincides with number of valent quarks and slow increases with the rise of energy owing to the appearance of the bremsstrahlung gluons.

On the second stage the hadron interaction is carried out by gluon exchange between the valent quarks and initial (bremsstrahlung) gluons and the hadrons gain the colour charge.

On the third step after the interaction the colour hadrons fly away and when the distance between them becomes more than the confinement radius \( r_c \), the lines of the colour electric field gather into the tube of the radius \( r_c \). This tube breaks out into the secondary hadrons.

Because the process of the secondary hadrons production from colour tube goes with the probability 1, module square of the inelastic amplitudes corresponds to the elastic amplitude with the different number partons.

This processes are schematically drawn on the left side of Fig.18.

Processes of the single diffraction are naturally discribed in this model. To get colourless final state of the incoming hadron, part of the incoming bremsstrahlung must be colourless in the sum with exchanged coulomb gluon. So, the part of the longitudinal momenta of the incoming hadron \( x = \frac{M^2}{s} \), which is equal to the sum of \( x_i \) exchanged bremsstrahlung gluons, will be transferred. At the final stage after hadronization we get the system composed of the leading hadron, lost part of the moments \( x \), and rapidity separated hadrons with mass \( M^2 = s(1 - x) \). This processes are schematically drawn on the right side of Fig.18.

To estimate the rise of cross section of single diffraction with energy, we will compare leading terms of \( \ln(s) \) for single diffraction and total cross sectons. Leading diagrams for processes of single diffraction are the same as in the case of the total cross section. To describe processes of single diffraction we must fix colours of exchanged gluons to get colourless final states. This condition cancels constant term in sum of \( \ln(s) \) powers for single diffraction cross section. In the case of the
Figure 18: Leading by powers of \( \ln(s) \) diagrams in low constituent model for total cross section (left part) and single diffraction cross section (right part).

single diffraction coefficient at \( \ln(s) \) will be suppressed by factor \( \frac{1}{8} \) in comparison to expression for total cross section, because of \( t \)-channel exchange colourless condition. Coefficient at \( \ln^2(s) \) will be suppressed by factor \( \frac{1}{256} \). Coefficients at different powers of \( \ln(s) \) will be also suppressed by kinematical factor \( K \). It arises because the area of integration on longitudinal momentums of exchanged gluons is limited by the condition, that full exchanged momentum must be in the range \( \frac{1.4 GeV^2}{s} ... 0.15 \) by the common definition of \( \sigma_{SD} \). We assume, that these kinematical factors are the same for coefficients at \( \ln(s) \) and \( \ln^2(s) \). Therefore, if we take decomposition of cross sections on powers of \( \ln(s) \) like

\[
\sigma_{tot} = A_1 + B_1 \ln(s) + C_1 \ln^2(s) \\
\sigma_{SD} = B_2 \ln(s) + C_2 \ln^2(s)
\]

then the next conditions take place

\[
B_2 = \frac{K}{8} B_1 \\
C_2 = \frac{K}{256} C_1
\]

\[
F_{\text{color suppression}} = \frac{B_2}{B_1} \frac{C_2}{C_1} = 32
\]

Last fraction we can call the colour suppression factor. From the CDF\textsuperscript{22} experimental data we have \( B_1 = 1.147, C_1 = 0.1466 \), and from total cross section data analysis we have \( B_2 = 0.598, C_2 = 0.00214 \). For the relation interested to us (33) we have

\[
F_{\text{color suppression}} \text{ experimental} = \left( \frac{B_2}{B_1} \frac{C_2}{C_1} \right)_{\text{experimental}} \sim 36
\]

Precision of the last value is low, but we can state, that \( F_{\text{min}} \approx 10 \). Therefore term proportional \( \ln^2(s) \) is highly suppressed in comparison with the total
cross section, and this effect can be simply explained in our model by existence of color factor in (31)-(32).

This model gives the same values of pomeron slope for elastic and single diffraction cross sections. This slope value is $\alpha' \sim 0.16..0.26$ and consistent with CDF data. This fact is another confirmation of the low constituent model.

5 From hadron-hadron to photon-hadron diffractive interactions

Differences at diffractive properties of hadron-hadron and hadron-photon are traditionally explained in the terms of rescattering effects. Breakdown of factorization from HERA to TeVatron diffractive data is explained by renormalisation of pomeron flux (Goulionas like) either or by low survival gap probability in hadron-hadron interactions at high energies. Anyway, this explanations state that at hadron-hadron interactions rescatterings is high (eikonal parameter $c > 1$), and in photon-hadron case rescatterings is suppressed by factor $1/\alpha_{EM}$.

This model gives good description of di-jet diffractive production at TeVatron compared to HERA one, but failed to explain constancy of the ratio of diffractive to total cross-sections for photon-hadron interactions at fixed $Q^2$ and $M^2$ (and thus $\beta$ fixed), see Fig.5 taken from [1].

Standard triple-Regge theory predicts an increase as $(W^2)^\epsilon$, in clear disagreement with data.

As discussed in [1], no adequate explanation of this fact was found.

Standart rescattering based solution is failed, because rescattering effects is suppressed. Saturation-based model ([3], [4] and references therein) is half-phenomenological and saturation does not seem to be adequate in this wide region of masses $W^2$ and $M^2_X$.

In our low constituent model there is no difference in describing hadron-hadron and photon-hadron single diffraction, because color suppression factor in equation (33) does not depend on the nature of diffracting systems.
Figure 19: Experimental data for the ratio $\sigma^{DD}/\sigma_{tot}$ taken from Ref. [2].
6 Acknowledgements

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