RNS model from a new angle for Maximal Gauge Symmetry $SU(3)_C \otimes SU(2)_L \otimes U(1)_{Y_W}$

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Abstract

We consider the RNS model without the GSO conditions. Unlike the gauginos, the ground state fermion in this case is a complex, spinor representation of $SU(3)_C \otimes SU(2)_L \otimes U(1)_{Y_W}$. We identify the open string tachyon with the Higgs boson of the Standard Model.

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In the supersymmetric formulation of the $D = 10$, $N = 1$ string theory in the light-cone gauge, the GSO conditions are in built and the odd $G$ parity states like tachyons are mapped out. Thus, the ground states in the bosonic sector and the fermionic sector are identified as the gauge bosons and the gauginos of a particular chirality. They are connected by a $N = 1$ supersymmetric transformation and both of them transform in the adjoint representation of the gauge group. Since the coincident D-brane - anti-D-brane pair has a tachyonic mode [1], we consider the open string sector of the RNS model without the GSO conditions. However, the light-cone gauge condition: $\psi^+ = 0$ can be imposed on physical states as in Gupta-Bleuler formalism [2, 3], to make the norms of the states positive definite. We demonstrate how the longitudinal and the time components of the world-sheet fermions add the weak hypercharge to the ground state fermions. Since the representations become complex after the addition of the $U(1)$ charge, the gauge group should be unitary. Therefore, the only possibility is that the six dimensional internal manifold is the product space $\mathbb{C}P^2 \otimes S^2$ isomorphic to the $SU(3)_C \otimes SU(2)_L$ without any $U(1)$ factor. The color is, however, confined, as the $CP^2$ is not spin. We demonstrate that there should be three families of fermions by calculating the index of the Dirac operator at $\mathbb{C}P^2 \otimes S^2$. We identify the open string tachyon with the Higgs boson of the Standard Model. Thus, the tachyon condensation will lead to the spontaneous breaking of the $SU(2)_L$ gauge symmetry. States in the open string sector with spin $> 1$ cannot be defined when the tachyon condensation is incomplete and there is some residual vacuum energy. Because it breaks the conformal invariance of the world-sheet action and only the subgroup $OSp(1|2)$ of the super-Virasoro algebra can be defined consistently. The Regge slope also vanishes. Thus, the fundamental physical entities should be point objects. The remnant vacuum energy causes changes in the masses of different particles to different extents.

Following the notations of [4], we can define a vertex operator through the relation

$$V'_s = [F_m, W]_\pm$$

where $F_m$ are the Fourier modes of the supercurrent $\psi \partial x$ and the vertex $W$ has the conformal dimension $\frac{1}{2}$.

The $W$ vertex for the emission of the ground state fermion can be written as

$$W_s = \Theta e^{i p.X}$$

where $\Theta$ is the product of the four spin operators for the bosons we get after the bosonizations of four pairs of transverse components of the world-sheet fermions. Hence, from (1) the corresponding vertex operator will be

$$V'_s = [F_m, W_s]$$

$$= \psi.p \Theta e^{i p.X}$$
We choose a particular local Lorentz frame to set
\[ p^\mu = (\pm \frac{1}{\sqrt{2}}, 0, 0, \frac{1}{\sqrt{2}}, 0, 0, 0, 0, 0, 0) \] (4)
The first four components are the momenta conjugate to the coordinates of the Minkowski spacetime and the rest momenta conjugate to the coordinates of the six dimensional internal space. It is obvious that only \( \psi^3 \) and \( \psi^0 \) contribute to the expression for \( \psi.p \). We replace \( \psi^0 \) by \( i\psi^0 \) in the action to make the kinetic energies of both \( \psi^3 \) and \( \psi^0 \) positive and define the boson \( Y \) through the relation:
\[ \frac{(\psi^3 + i\psi^0)}{\sqrt{2}} = e^{\pm iY}. \] (5)
Thus, we can write
\[ V'_s = \frac{(\psi^3 + i\psi^0)}{\sqrt{2}} e^{ip.X} e^{\pm iY} \Theta. \] (6)
The world-sheet action can be written as
\[ S = \int d^2 \sigma (\partial^\alpha X. \partial_\alpha X + \partial^\alpha Y \partial_\alpha Y + \cdots). \] (7)
Assuming a mode expansion of \( Y \) similar to \( X \), we write
\[ Y = y + Y_W \tau + \text{oscillator terms}, \] (8)
to interpret \( y \) as the curled up coordinate to define the \( U(1)_{YW} \) gauge symmetry and the momentum \( Y_W = \pm 1 \) conjugate to it as the weak hypercharge of the state.

The ground state: \( e^{\pm iY} \Theta \) is again \( 2.2^4 = 32 \) fold degenerate. The sign of the hypercharge of the particle is obviously opposite to that of the anti-particle.

From (6) it is evident that the spinor representation is complex. Hence, the gauge group should be unitary. Thus, the six dimensional internal space can only be \( CP^2 \otimes S^2 \cong SU(3)_C \otimes SU(2)_L \).

Though \( CP^2 \) does not admit spinors, \( CP^2 \otimes S^2 \) does. To see it, we note that the index of the Dirac operator on a six dimensional space of Euclidean signature is \( \chi \), where \( \chi \) is the Euler number of the manifold. Since the Euler number of a product manifold is the product of its Euler numbers, the same for the product space \( CP^2 \otimes S^2 \) is
\[ \nu_+ - \nu_- = \frac{\chi_{CP^2} \cdot \chi_{S^2}}{2} \] (9)
\[ = \frac{3 \cdot 2}{2} = 3 \]
\[ \nu_+ = \nu_- \]

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The Betti-Hodge numbers for \( CP^n \) are given by
\[ b_{pq} = \delta_{p,q} \quad p \leq n \]
\[ = 0 \quad p > n \] (10)

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\[ b_{pq} = \delta_{p,q} \quad p \leq n \]
\[ = 0 \quad p > n \]
Therefore, there will be three families of four component left-handed Weyl spinors on $CP^2 \otimes S^2$ that can be identified with the conjugate isospinors:

\[
\begin{bmatrix}
\nu_L \\
e_L
\end{bmatrix}
\begin{bmatrix}
X_L \\
Y_L
\end{bmatrix}
\]

Though we can easily identify the first isospinor as leptons, the second one cannot be identified with the quarks, because both the isospinors carry integral hypercharges. The second isospinor is, therefore, more like the proton and the neutron.

From (7) it is obvious that the weak hypercharge of (6) has no invariant meaning. Since $V'_s$ has unit conformal dimension, the invariance of the action under local rotations of coordinate axes in the $(X,Y)$ plane will allow us to set $(Y_W, p)$ to any desired values subject to the condition:

\[Y_W^2 + p^2 = 1\]  \hspace{1cm} (14)

Hence, for $S^2 \cong CP^1$ they are given by the matrix

\[
b_{pq}(S^2) = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}
\]  

and those for $CP^2$ are given by the matrix

\[
b_{pq}(CP^2) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}
\].

From the Kunneth formula

\[
b_{pq}(M \otimes N) = \sum_{r,m,s,n} b_{rs}(M) b_{mn}(N),
\]  \hspace{1cm} (11)

subject to the constraints $r + m = p$ and $s + n = q$, we can write the Betti-Hodge numbers for $CP^2 \otimes S^2$ as

\[
b_{pq}(CP^2 \otimes S^2) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}
\]

After proper addition of cells, the matrix can be identified with the Hodge diamond of a six dimensional Calabi-Yau Manifold:

\[
\begin{bmatrix}
1 & 0 & 0 & 1 \\
0 & 3 & 0 & 0 \\
0 & 0 & 3 & 0 \\
1 & 0 & 0 & 1
\end{bmatrix}
\]

Thus, we can write

\[
\nu_+ - \nu_- = b_{11} - b_{21} = 3
\]  \hspace{1cm} (12)
Thus, we can write \((Y_W, p) = (\pm g, p_{CP2})\), where \(p_{CP2}\) is the momentum conjugate to the coordinates of \(CP^2\) to give color charges to the massless quarks. Hence,

\[
Y_W^2 + p^2 = g^2(1 + 8) = 1
\]

Or,

\[
g = Y_W = \frac{1}{3}
\]

It is obvious that massless right-handed spinors with hypercharges differing from those of (6) and (16) do not fit into our model.

While arriving at the last equation, we focused on local aspects and not on the topology of \(CP^2\), which is not spin. Therefore, color helicities should always be confined. This is perhaps reflected in the equation (13).

We should identify the open string tachyon with the Higgs boson to complete the Standard model. According to Sen’s conjecture, when the tachyon settles in the stable vacuum after the condensation, the negative vacuum energy should cancel the brane tensions exactly, restoring the supersymmetry. But it stays in a false vacuum before rolling down to the stable one. During this period, the vacuum should possess a positive energy, however small it may be.

To study the effects of this vacuum energy, we consider the nonlinear sigma model \([6]\) that we regularize by dimensional regularization to yield

\[
S = -\lim_{\epsilon \to 0} \frac{1}{4\pi\alpha'} \int d^{2(1+\epsilon)} \sigma \sqrt{\eta^{\alpha\beta}} \partial_{\alpha} X \partial_{\beta} X
\]

\[
= -\lim_{\epsilon \to 0} \frac{1}{4\pi\alpha'} \int d^{2(1+\epsilon)} \sigma e^{\phi} g_{\mu\nu} \partial X^\mu \partial X^\nu
\]

\[
= -\lim_{\epsilon \to 0} \frac{1}{4\pi\alpha'} \int d^{2(1+\epsilon)} \sigma e^{\phi} (\eta_{\mu\nu} - R_{\mu\nu\sigma} x^\sigma)
\]

\[
\partial X^\mu \partial X^\nu
\]

\[
= -\lim_{\epsilon \to 0} \frac{1}{4\pi\alpha'} \int d^{2(1+\epsilon)} \sigma [\partial X \partial X - \frac{1}{2\epsilon} \alpha' R_{\mu\nu}]
\]

\[
\partial X^\mu \partial X^\nu (1 + \epsilon \phi)
\]

\[
= -\lim_{\epsilon \to 0} \frac{1}{4\pi\alpha'} \int d^{2(1+\epsilon)} \sigma [\partial X \partial X - \frac{1}{2\epsilon} \alpha' \lambda \eta_{\mu\nu}]
\]

\[
\partial X^\mu \partial X^\nu (1 + \epsilon \phi)
\]

\[
= -\lim_{\epsilon \to 0} \frac{1 - \frac{\alpha' \lambda}{2\pi}}{4\pi\alpha'} \int d^{2(1+\epsilon)} \sigma [\partial X \partial X]
\]

\[
- \frac{\alpha' \lambda}{2(1 - \frac{\alpha' \lambda}{2\pi})} \phi \partial X \partial X]
\]

Reparametrization invariance of the world-sheet action allows us to set \(h_{\alpha\beta} = \eta_{\alpha\beta} e^{\phi}\) in [13]. We used Riemann normal coordinates to write \(g_{\mu\nu} = g_{\mu\nu}(X) = \)
\[ \eta_{\mu\nu} - R_{\mu\rho\nu\sigma}(X_0) x^\rho x^\sigma \] in (19) \((X = X_0 + x)\) are locally inertial coordinates at \(X_0\) and put the logarithmically divergent contraction \(\lim_{\sigma \to \sigma'} < x^\rho(\sigma)x^\sigma(\sigma') > = \frac{\alpha'}{2 \epsilon} \). We used the lambda-vacuum solution to the Einstein field equation \(\text{[7]}\) to write \(R_{\mu\nu} = \lambda \eta_{\mu\nu}\) in (21), where \(\lambda\) is proportional to the energy of the false vacuum per unit volume.

The \(\phi\) dependent term in (22) breaks the conformal invariance of the world-sheet action. Hence, one can define only the subalgebra \(OSp(1|2)\) of the super-Virasoro algebra consistently. Therefore, there will be spin \(\leq 1\) states only. This together with the fact that the Regge slope \(-\frac{\alpha'}{1 - 2 \epsilon}\) vanishes in the limit \(\epsilon \to 0\) for finite \(\lambda\) suggests that fundamental physical entities will behave like point-objects and not strings. The \(\phi\) dependent term in (22) also shifts the ground state energy by an amount \(\propto \lambda \phi_0\). It changes the masses of different particles to different extents. Since constraints like (14) are no longer applicable, the existence of right-handed spinors cannot be ruled out.

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