Gravitational wave sources: reflections and echoes

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Abstract

The recent detection of gravitational waves has generated interest in alternatives to the black hole interpretation of sources. A subset of such alternatives involves a prediction of gravitational wave ‘echoes’. We consider two aspects of possible echoes: first, general features of echoes coming from spacetime reflecting conditions. We find that the detailed nature of such echoes does not bear any clear relationship to quasi-normal frequencies. Second, we point out the pitfalls in the analysis of local reflecting ‘walls’ near the horizon of rapidly rotating black holes.

Keywords: black holes, gravitational waves, echoes, quasi-normal modes

(Some figures may appear in colour only in the online journal)

1. Introduction

The source of the recently detected gravitational waves (GWs) by the LIGO collaboration [1, 2] has been interpreted to be the inspiral and merger of a pair of intermediate mass binary black holes. This interpretation has been viewed as secure since the observed waveform had an excellent fit to the very different physics of early and late inspiral. The early waveform fit the ‘chirp’ pattern [3] of the evolving nearly circular binary orbit driven to smaller radii and higher angular velocity by the loss of energy to outgoing gravitational waves. The late waveform fit the pattern for the quasinormal ringdown (QNR) of the perturbed final black hole, a black hole of the appropriate angular momentum and mass as implied by the merger process [4].

The early pattern is not exclusive to orbiting black holes; it would be no different for a binary of any compact objects of the same masses. What is most important for the black hole
interpretation is the QNR, and the way in which the transition from the early waveform to the QNR agrees with the black hole models of numerical relativity [5].

The importance of the QNR to the black hole interpretation has led to the question of alternative, non-black hole, sources of QNR-like waveforms [6–10]. One recent model of a source is the double wormhole of Cardoso, Franzen and Pani [10] (hereafter CFP). The fact that damped oscillations are not uniquely, or even especially, associated with black holes is not news [11, 12], but a relatively new element of the question is whether the replacement for the black hole may involve reflections and may produce echoes, i.e. delayed repetitions of the QNR-like pattern [10, 13–16]. Indeed, a discovery has been claimed of just such echoes in the gravitational wave detector data [17, 18], though the statistical significance of the claim has been disputed by members of the LIGO collaboration [19].

In this paper, we do not focus exclusively on the LIGO detections, but rather we consider somewhat broadly the physics that lies behind recent claims, the nature of reflections of gravitational waves and echoes that might result from such reflections from surfaces around compact objects. There is, however, a possible relevance to gravitational waveform interpretation: the issue of how closely echoes might be delayed repetitions of an earlier burst. Do echoes, for instance, have the same frequency and damping rate of the late ‘ringing’ in an initial burst? Might differences between the initial burst and its echo contain, at least in principle, interesting information?

Clear answers to these questions may have strong implications for the claims made in [17, 18] since that work relies on the echo signal sharing detailed characteristics with the initial burst from the black hole binary system.

We discuss, in section 2, the general nature of echoes, and connect that issue to the meaning and features of quasinormal (QN) modes. We shall point out the distinction between two very different sources of echoes: on the one hand echoes can result from a feature of the ‘curvature potential’ through which waves propagate [20]; on the other hand echoes can be the result of some sort of ‘wall’ surrounding a compact object.

In section 3, we pay particular attention to the physical meaning of ‘reflection,’ and point out a pitfall in the mathematical analysis of reflection of radiation at a surface around a black hole. We conclude and summarize in section 4.

Throughout, the paper we use the conventions of the textbook by Misner et al [21]. In particular, we use the metric convention $- + + +$, and units in which $G = c = 1$. For simplicity we will, for the most part, use spherical symmetry in examples, so that, for instance, we will give details for Schwarzschild, rather than the astrophysically more relevant rotating Kerr holes. But issues of Kerr holes will be important, and will constitute the motivation, especially in section 3.

2. The nature of echoes from compact objects

2.1. Sources of echoes

At the outset it is important to note that there can be at least two distinct sources of echoes. One source is the spacetime itself, and more specifically the curvature potential through which waves propagate. An example of this is the double light ring model of CFP [10]. In that model, two peaks in the curvature potential act, in effect, as two locations at which wave interactions can be viewed in terms of transmission and reflection. A second source of echoes is some sort of a ‘wall’ that forms an inner boundary of the wave propagation problem, and that replaces

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3 Various authors use different names (‘effective potential,’ ‘scattering potential, ‘curvature potential’) for this potential-like term. See, e.g. Press [20].
the horizon as the boundary [17, 22]. These walls are typically associated with speculations, or specific models, of quantum effects.

It is crucial to emphasize here the difference between formal quasinormal ringing (QNR) and quasinormal-like oscillations (QNR-like). The former refers to an eigenvalue problem for single frequency modes of a system, typically a system characterized by a fairly compact potential for wave propagation. The boundary conditions on the modes involve outgoing radiation, so that the eigenproblem is not self adjoint, and the frequency eigenvalues are complex. In the case of black holes, the boundary conditions are outgoing radiation at infinity and ingoing radiation at the horizon. These complex eigenvalues also show up as poles in the frequency-domain Green function for the system. Typically, there is an infinite spectrum of such modes for any linear system, e.g. for the differential equation for a particular multipole mode of a black hole perturbation field\(^4\).

The QNR-like signals are damped oscillations. In the black hole context these QNR-like waveforms have long been associated with the late-time ‘ringdown’ of perturbed black holes. While this association developed in work on black hole perturbation theory in the 1970s, this ringdown has been seen in all numerical relativity simulations of black hole ringdown, simulations based on the fully nonlinear equations of Einstein’s general relativity. Such QNR-like waveforms typically have very nearly the period of oscillation and the exponential damping rate of the least damped of the quasinormal modes, and there was little attention given to the difference.

In fact, a system with a time dependent source cannot exhibit pure QNR\(^5\). The outgoing signal will always be affected by the time dependence of the source as well as the damped-sinusoid pattern of a QN mode. From the Green’s function point of view, the integral of the source over the Green’s function \(^6\) will include a residue for the QN pole, but will have other contributions. It must be asked, then, why is there such a close apparent correspondence between the late time signal and the least damped QN mode?

Part of the answer is that the correspondence is not always valid. Nollert \([11]\) studied the mathematical problem of evolving initial data in the Schwarzschild spacetime and showed that a class of minor modifications of the problem had no discernible effect on the evolved data, but changed the QN spectrum enormously. More recently, CFP have shown that the QN spectrum of a wormhole consisting of two Schwarzschild ‘funnels’ is enormously different from that of the Schwarzschild black hole, yet the initial QNR-like ringing of the wormhole is almost identical to that of the black hole.

There are, therefore, examples in which there are weak or missing connections between the QN frequencies of a system and the QNR-like ringing exhibited in signals generated by sources. But there are examples in which there is a strong connection and black hole processes fall in that second class. It is important to ask why.

CFP have ascribed the QN frequency and QNR-like ringing to the role of the light-ring. In the case of black holes this is an interesting heuristic insight and one that was first shown to give good estimates of QN frequencies by Goebels \([24]\). It cannot, however, be the complete story. One can, after all, trivially set up a \(1 + 1\) model (one spatial dimension, one time dimension) with outgoing radiation boundary conditions but with no attached concept of a light ring; such a problem will have a QN spectrum and a QNR-like ringing. We have also presented a \(3 + 1\) model with no light ring, yet with QNR-like oscillations \([25]\)\(^6\).

\(^4\) For the Kerr spacetime, which does not have multipole modes in the usual sense, the details of the eigenvalue problem are somewhat different.

\(^5\) QNR can also be evident in the evolution of initial data, but in this case also pure QNR is impossible. The initial data for a pure QN mode is unbounded at infinity and at the horizon.

\(^6\) The problem can be viewed not as applying to a multipole of a perturbation field but as a simple one dimensional problem in and of itself. For more details, see: Khanna and Price \([25]\).
A more general view of the connection between QN and QNR-like mathematics is that
the QNR-like signal is due to ‘scattering’ within a potential. That scattering can account for
the damping of the outgoing radiation. The scattering viewpoint is very insensitive to distant
boundary conditions and should be initiated as the source (initial data or particle motion)
interacts with a peak of a potential.

The scattering viewpoint suggests that a WKB approximation may give good estimates,
but of the frequencies of the QNR-like oscillations, not of the true QN eigenvalues. The WKB
approximation uses an integral over the potential, so it is insensitive to the changes (e.g. those
of Nollert) that greatly change the QN spectrum; the approximation also is insensitive to
distant boundary conditions. To some extent the WKB approximation and the scattering view-
point are conceptually, or heuristically quite close. This can be taken as a partial explanation
of the examples in which the QN frequencies do not agree with the scattering/WKB results.
This disagreement is most pronounced when the curvature potential is not smoothly varying in
space. The condition for success of the WKB approximation is that the spatial rate of change
of the curvature potential is small [26].

The WKB approximation has given fairly good agreement with computed black hole QN
frequencies, but it must be kept in mind that the WKB approximation is a high frequency
approximation, and it is typically applied to wavelengths that are of order of the width of the
potential that affects wave propagation. The situation then is that we can take some comfort
in the WKB approximation giving results in good agreement with computed waveforms, but
must not be surprised in the absence of such agreement.

This scattering viewpoint lets us make some predictions about the nature of the QNR-like
signals in echoes. These will be discussed below.

2.2. A model problem: the Pöschl–Teller potential with a reflecting wall

The work by CFP has provided useful examples of echoes from a potential with two peaks.
This is one of two distinct ways in which echoes can be generated. We will refer to that paper
in arguments below, but here we shall focus on the other general manner in which echoes can
be generated: a reflecting wall. Our specific model will start with the equation

\[ \frac{\partial^2 \Psi}{\partial t^2} - \frac{\partial^2 \Psi}{\partial x^2} + V(x) \Psi = \text{Source.} \]  

(1)

For a Schwarzschild black hole \( \Psi \) is a representation of a multipole of a scalar, electromagnetic
or gravitational perturbation field; the \( x \) coordinate is the Regge–Wheeler [27] tortoise coor-
dinate \( r^* \), and the source term can represent a particle. The potential, in the black hole case
is the curvature potential [20], which falls off as \( 1/r^* \) as \( r^* \to \infty \), and falls off exponentially
as \( r^* \to -\infty \), the location of the horizon.

For our model we will start with the Pöschl–Teller potential

\[ V_{PT}(x) = \frac{1}{\cosh^2 x}. \]  

(2)

We will put this well studied model [28, 29] to a new purpose by imposing nonstandard
boundary conditions. As \( x \to \infty \), we use the standard outgoing condition i.e. \( \Psi \) becomes
proportional to \( \exp[i \omega (x - t)] \). For the other boundary condition, the standard choice is
\( \Psi \propto \exp[i \omega (x - t)] \) as \( x \to -\infty \), which we will call the horizon condition, since it is the
analog of the horizon boundary condition for black holes. But we may also take, as a model

\footnote{See footnote 3.}
for reflection, a ‘wall condition,’ the condition that $\Psi = 0$ at some particular value of $x$, the location of a reflecting wall.

The solution of the system of equations (1) and (2), with the outgoing condition, is proportional to the associated Legendre function

$$\Psi \propto P^{\nu}_{\mu}(\tanh x) \quad \nu = \frac{1}{2}(-1 \pm i \sqrt{3}) \quad \mu = i \omega.$$  \hspace{1cm} (3)

By expressing this in terms of a Gauss hypergeometric function it can be shown that the horizon condition at $x \to -\infty$ is achieved only for

$$\omega = -i \left( n + \frac{1}{2} \right) \pm \frac{\sqrt{3}}{2},$$  \hspace{1cm} (4)

where $n = 0, 1, 2, \ldots$. The frequency of interest, the least damped QN mode, is that for $n = 0$.

In the case of the reflecting wall condition, $\Psi = 0$ at $x_{wall}$, we must search numerically for the complex value of $\omega$ for which

$$P^{(-1 \pm i \sqrt{3})/2}_{\mu}(\tanh x_{wall}) = 0.$$  \hspace{1cm} (5)

This formula is to be applied at the peak of the potential, where the second derivative of the curvature potential is negative, and hence the second term on the right is pure imaginary. In the case of the Pöschl–Teller potential in equation (2) this gives $\omega_{QN} = 1.0987 - i 0.4551$. Note that this result is a reasonable approximation of the pure Pöschl–Teller QN mode.

The WKB prediction $\omega_{QN} = 1.0987 - i 0.4551$ applies to the model with $x_{wall} = -5$ as well as to the pure Pöschl–Teller potential, since both models in figure 1 have the same peak
behavior. Here the WKB approximation is still in the right ballpark for the real part, but orders of magnitude wrong for the imaginary part. This should be expected. The Schutz-Will estimate approximates the effective curvature potential as a parabola near the peak and works best if the turning points, those locations at which $\omega^2 = V(x)$, are close together. For the high QN frequencies in some models, this does not apply; there are not even any turning points. It is not surprising that the real part in the Schutz-Will estimate equation (5), which does not depend delicately on the shape of the potential is widely applicable, though it is surprising how good an estimate it is.

It is worth emphasizing that the WKB method is local; it may be considered to be related to the scattering picture of QNR-like phenomena. It may also be worth emphasizing that in our Pöschl–Teller model there is no meaning to a ‘light ring.’

Results are shown in figure 2 comparing the waveform evolved from an initial Gaussian pulse for both the pure Pöschl–Teller potential, and the Pöschl–Teller potential with a reflecting wall at $x_{\text{wall}} = -20$. In the reflecting wall case there is an initial burst that is essentially indistinguishable in the graph from the burst evolved with the pure Pöschl–Teller potential. This is a particularly clear example of the distinction between a QN oscillation and a QNR-like oscillation. There are multiple QN modes in this model. The one that appears to most relevant for the model with $x_{\text{wall}} = -20$ is $0.936 + i 0.01^8$. The QNR-like first burst, however, accurately traces the pure Pöschl–Teller burst, which has both a QN frequency, and an evolved wave form with $\omega = 0.866 + i 0.5$. Again, we see that the scattering viewpoint is justified, and the real part of the WKB approximation is correct to rough order.

The question remains on the nature of the echoes in the reflecting wall case. It might be expected that later and later echoes would approach more and more closely the true QN

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8 Note the small imaginary part; the other modes have an imaginary part that is even smaller. We enlist a few additional modes here: $0.784 + i 0.0056$, $0.631 + i 0.0024$, $0.476 + i 0.0009$. 
frequency. In figure 2, however, there is no sign that the echoes approach the almost undamped oscillations of the true QN mode.

The relationship of the echoes and the first burst is examined in figure 3. If we consider outgoing radiation to be generated at $x = 0$, and the first echo to be the reflection off the wall at $x_{\text{wall}} = -20$, then the outgoing echo should follow the initial QNR-like burst by a time delay of 40. For that reason, in figure 3 we shift the first echo to an earlier time by 40. For the
same reason we shift the second echo to earlier time by 80. The curves in figure 3 show that the basic idea of a delayed echo is correct, but that the delay time is somewhat larger than 40 for each ‘bounce.’

The exponential damping rate of the first echo is neither the 0.5 of the pure Pöschl–Teller QN, nor the 0.01 of the QN for the reflecting potential, but rather a value around 0.35. The appropriate $\omega$ for the QNR-like echo cannot be extracted with good precision because the late-time portion of the echo is not well approximated by a single damped sinusoid. We have made arguments above that a pure QN oscillation is impossible, since the source has its own time variation. We conjecture that the echoes, QNR-like ringing present in outgoing radiation, are not pure QN oscillations. Independent of that conjecture is the fact that in principle there is a difference between the shape of the initial burst and that of any of the echoes. If we generalize from this one example, we can conclude that the echo waveforms contain important information about the conditions from which the echoes emerge.

To show more evidence in support of our conjecture, we changed the boundary condition from a reflecting wall, i.e. a Dirichlet boundary, to a different condition—one that effectively relates the second time-derivative of the field to the negative of the field itself. The results are shown in figure 4. It is clear that the echoes with this boundary condition are very different from those from the reflecting wall. Therefore, these echoes carry important detailed information about the processes that led to their formation and development. Very recently, i.e. well after this current work was completed, a new preprint from Mark et al [34] appeared that performs an extensive study of echo waveforms in the context of a Schwarzschild hole via a Green’s function approach. While the goal of that effort is to provide a template for the echo signals, they confirm several of the claims and conjectures that we make in this work.

Now, since one is most interested in considering echoes in the context of gravitational waves (as opposed to the scalar case, considered thus far) from a realistic, spinning black hole system, one may attempt to simply go ahead and apply a Dirichlet-type boundary condition at a location close to the horizon on the Weyl scalar $\Psi_4$ and evolve using the Teukolsky...
However, such a naive attempt yields no echoes whatsoever! In the next section, we explain this null result and also sketch out a potential scheme for attempting to implement a proper reflection condition in the context of gravitational waves.

3. Reflections of gravitational waves

In this section we consider the description of reflections at some sort of ‘wall.’ We shall not be, nor need to be specific about the nature of this reflecting wall, except for one requirement. The reflection must be the result of a local condition, and not a condition like the modification of the potential (that is in a loose sense, global). We shall clarify what we mean by this with examples of electromagnetism and gravitational perturbations in the Schwarzschild background.

The fundamental concept we want to present here is that, except for a scalar field, there are many features of a field that are encoded in different mathematical packages. Because a gravitational perturbation, with its 10 degrees of freedom is an unnecessarily complicated way to start, but a scalar perturbation is too simple, the complexity ‘Goldilocks zone’ is occupied by electromagnetic perturbation of a Schwarzschild background.

In this case, at any point in spacetime, there are 6 degrees of freedom that can be considered to be the 3 components of the electric field, and the 3 of the magnetic field; alternatively they can be considered the 6 independent components of the Maxwell 4-tensor.

The partial differential equations for this system, Maxwell’s equations in the Schwarzschild background, are uselessly messy in terms of the individual vector or tensor components. A very effective way of repackaging these quantities is to use the 3 complex fields of the Newman–Penrose (NP) formalism. The asymptotic behaviors of these fields, and hence the argument to be made here, depend crucially on the spin-weight of the fields. For that reason we use here a notation that indexes the fields with their spin-weight [31]. The formal definitions of these complex fields, and their connection to the original NP notation, are given in Appendix A in that reference.

The 3 complex fields can most simply be defined through their relationship to the components on the electric and magnetic fields in the Schwarzschild background. We define \( E^{[r]}, E^{[\theta]}, E^{[\phi]} \) as the orthonormal components of the electric field in the basis given by the standard Schwarzschild \((r, \theta, \phi)\) coordinate system. The components of the magnetic field are similarly defined with a \( B \). The NP projections \( \Phi_{-1}, \Phi_{-1}, \Phi_{+1} \) are related to these \( E, B \) components by

\[
\Phi_{+1} = 2^{-1/2} (1 - 2M/r)^{-1/2} \left[ \left( E^{[\theta]} - B^{[\phi]} \right) + i \left( E^{[\phi]} + B^{[\theta]} \right) \right] \quad (6)
\]

\[
\Phi_0 = -\frac{1}{2} \left( E^{[r]} + i B^{[r]} \right) \quad (7)
\]

\[
\Phi_{-1} = -2^{-3/2} (1 - 2M/r)^{1/2} \left[ \left( E^{[\theta]} + B^{[\phi]} \right) - i \left( E^{[\phi]} - B^{[\theta]} \right) \right]. \quad (8)
\]

These relations point to an important property of the \( \Phi_k \): their relationship to ingoing and outgoing radiation. Consider, for example, the quantities constructed on the right in equation (6). For outgoing electromagnetic radiation, the orthonormal components of the electric and magnetic fields all fall off as \( 1/r \), but \( E^{[\theta]} = B^{[\phi]} \) and \( E^{[\phi]} = -B^{[\theta]} \) to leading order in \( 1/r \), so that to this order \( \Phi_{+1} \) vanishes. It turns out, in fact, that \( \Phi_{+1} \) falls off in the large \( r \) limit as \( 1/r^2 \). More generally, there is a ‘peeling theorem’ for the \( \Phi_k \) that tells us that [32]...
\[ \Phi_k \rightarrow \frac{1}{r^{2+k}}. \]  
(9)

It is then \( \Phi_{-1} \) that describes outgoing radiation. In that sense it plays the role of \( \psi_4 \) in the Teukolsky equation [35], the quantity that describes outgoing radiation.

In the same sense, there is a version of a peeling theorem in the horizon limit. It can be shown [31]\(^9\) that in the horizon limit, i.e. in the limit \( r^* \rightarrow -\infty \),

\[ \Phi_k \rightarrow \exp \left( -kr^*/2M \right). \]  
(10)

The quantity that is dominant in the description of radiation being carried into the horizon is therefore \( \Phi_{+1} \).

Although \( \Phi_{-1} \) is dominant for outgoing radiation, and \( \Phi_{+1} \) for ingoing, each of the \( \Phi_k \) carries all information about the other \( \Phi_k \). To express these relationships it is best to consider individual multipoles and remove the angular dependence. The angular dependence of NP fields is described with spin-weighted spherical harmonics. We denote by a caret (\( \hat{\imath} \)) the function of \( r, t \), multiplying each spin-weighted spherical harmonic. (For the precise procedure for moving angular dependence, see [31].)

These equations, i.e. the Maxwell differential equations, are best expressed in derivatives with respect to retarded and advanced time,

\[ u = t - r^* \quad v = t + r^* \]  
(11)

and in Gaussian-esu units. For an \( \ell \)-pole mode the equations are

\[ 2(1 - 2M/r)^{-1} \partial_u \left( r \hat{\Phi}_{-1} \right) = -\frac{1}{2} \ell(\ell + 1) \hat{\Phi}_0 \]  
(12)

\[ 2(1 - 2M/r)^{-1} \partial_v \left( r^2 \hat{\Phi}_{+1} \right) = r \hat{\Phi}_{+1} \]  
(13)

\[ \partial_u \left( r^2 \hat{\Phi}_0 \right) = r \hat{\Phi}_{-1} \]  
(14)

\[ \partial_u \left[ (1 - 2M/r) r \hat{\Phi}_{+1} \right] = -\frac{1}{2} \ell(\ell + 1)(1 - 2M/r) \hat{\Phi}_0. \]  
(15)

From these equations, second-order wave equations can be formulated for any of the \( \hat{\Phi}_k \), and all information could be extracted from that \( \hat{\Phi}_k \). We could then, in principle, work only with \( \hat{\Phi}_{+1} \) for outgoing radiation. By differentiating with respect to \( u \) we could find \( \hat{\Phi}_0 \), and then with a second differentiation with respect to \( u \) we could find \( \hat{\Phi}_{-1} \).

This nature of the NP formalism, this separation into ingoing and outgoing quantities is crucial to implementing reflection conditions. The example of electromagnetic waves is instructive. The condition on a perfectly conducting surface is the vanishing of the tangential electric component and the normal magnetic component.

For definiteness, let us consider even parity fields; these turn out to involve only the real part of the \( \Phi_k \) quantities. The condition that the locally measured value of \( E^{(\theta)} \) vanish, requires both \( \hat{\Phi}_{+1} \) and \( \hat{\Phi}_{-1} \). For a reflecting surface close to the horizon, i.e. at a large negative value of \( r^* \), This is numerically awkward since in the horizon limit the \( \hat{\Phi}_{+1} \) diverges, and \( \hat{\Phi}_{-1} \) vanishes. If, for example, we use a wave equation for \( \hat{\Phi}_{-1} \), the boundary condition would require, according to the Maxwell equations (12)–(15), both \( \hat{\Phi}_{-1} \) and its second derivative with respect

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\(^9\)See after equation (50) in, Price [31].
to advanced time. Notice too that equation (9) tells us that a similar awkwardness applies to a reflecting surface at large $r$.

Heuristically, this awkwardness can be traced to the fact that the reflection condition involves a balance of ingoing and outgoing radiation, and the NP quantities are specific to one or the other. This suggests that reflection problems are best handled in a computation using the ‘balanced’ NP field $\tilde{\Phi}_0$. From equations (6)–(8) and (12)–(15), it then follows that the no-reflection boundary condition is simply that the derivatives of $r\tilde{\Phi}_0$ with respect to advanced and retarded time are opposites of each other.

We now turn to the problem of reflection of gravitational waves. The analog of the electromagnetic reflection conditions would be some conditions on the transverse traceless components of gravitational strain. We need not know precisely what the reflection condition is, only that it is some local condition on the gravitational strain.

The NP formalism for gravitational perturbations [33] encodes all the information about the Weyl tensor in 5 complex fields, $\Psi_4, \Psi_3, \Psi_2, \Psi_1, \Psi_0$, with properties analogous to the 3 complex electromagnetic fields. In particular, $\Psi_4$ describes outgoing radiation (the other $\Psi_k$ fall off faster than $1/r$ as $r \to \infty$). Similarly, $\Psi_0$ describes ingoing radiation, and the other $\Psi_k$ fall off faster than $\Psi_0$ as $r^* \to -\infty$.

There is an important difference between the NP formalism for gravitational perturbations of the Schwarzschild spacetime and those of the NP formalism for electromagnetic perturbations. For electromagnetism, all the NP projections of the Maxwell tensor are gauge invariant; for gravitational perturbations, only $\Psi_4$ and $\Psi_0$ are gauge invariant. The other $\Psi_k$ change under a perturbative transformation of coordinates or projection tetrads. This is why only $\Psi_4$ and $\Psi_0$ can be uncoupled from the other $\Psi_k$ and made to satisfy single-unknown wave equations.

A wave equation for $\Psi_4$, in the context of Schwarzschild spacetime, uncoupled from the other $\Psi_k$ is known as the Bardeen-Press equation [36]. A physically motivated reflection condition near the horizon will involve both $\Psi_4$ and $\Psi_0$ in a manner analogous to the electromagnetic condition involving $\tilde{\Phi}_-^1$ and $\tilde{\Phi}_+^1$. One possibility for dealing with the local boundary conditions is for example, solve for $\Psi_4$, and from the solution find $\Psi_0$. In electromagnetism, finding $\tilde{\Phi}_+^1$ from $\tilde{\Phi}_-^1$ required two derivatives with respect to advanced time, and was numerically delicate. For gravitational perturbations the situation is worse; finding $\Psi_0$ from $\Psi_4$ requires four differentiations with respect to advanced time.

For gravitational perturbations of the Schwarzschild spacetime with reflection conditions, the difficulty can be avoided by using the Zerilli or Regge–Wheeler equations, which, like $\tilde{\Phi}_0$ in the electromagnetic case, are not skewed to ingoing or outgoing wave propagation. Rapidly rotating black holes, however, do not provide this easy workaround. For gravitational perturbations of the Kerr spacetime, there exist no wave equations analogous to the Regge–Wheeler or Zerilli equations; equations exist only for the gauge invariants $\Psi_4$ and $\Psi_0$. Thus, for studies of reflections from exotic ‘walls’ near the horizon, either a very difficult numerical boundary condition can be implemented, or it can be assumed that the the results for the Schwarzschild background give adequate insight for rapidly rotating holes. The former is much easier to implement in the frequency-domain, as attempted in [37].

Another challenge associated to a study of echoes in rotating spacetimes via the Teukolsky equation arises from the lack of Birkhoff’s theorem there. Does the Teukolsky equation even represent the evolution of perturbations of a compact object other than a Kerr black hole? This important consideration was recently raised by the authors of [34].
4. Conclusions

In this article we sought to clarify two aspects of ‘echoes’ in gravitational wave signals from the late stages of binary inspiral.

The first is the general nature of echoes and their relationship to QN modes. We point out that in a sequence of echoes, later echoes are not copies of the first burst. This potentially has strong implications for the claims made in [17, 18] since that work relies on the echo signal being a repetition of the initial burst. Furthermore, later and later echoes of an infinite string of echoes, do not approach a ringing at a QN frequency. In general, a scattering viewpoint involving the curvature potential, where applicable, gives a better heuristic view of the process of signal generation than a QN analysis or considerations of a light ring.

The second goal of this paper is to warn of a pitfall in using the Teukolsky [35] wave function $\Psi_4$ for analyzing the effect of reflecting ‘walls’ outside the horizon. Simply setting Dirichlet or Neumann conditions on this wave function, for example, is not an expression of a locally reflecting wall. Moreover, as pointed out by [34] the lack of Birkhoff’s theorem in the context of a rotating spacetimes poses serious concerns on whether or not $\Psi_4$ is even the relevant quantity to study.

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