Phase diagram of solid hints new fundamental constant and sees atomic orbital

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Research Article

Keywords: phase diagram, hysteresis, diamond, enormous pressure, solid-state tomography, core of star, fundamental constants, transition temperature, phase boundary, transition-metal oxides, organic superconductor, manganite

Posted Date: December 1st, 2021

DOI: https://doi.org/10.21203/rs.3.rs-1114543/v1

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Cold and pressure transform gas into liquid and then into solid. Van der Waals understood the phase diagram of liquefiable gas with the molecular volume and intermolecular attraction, however, was silent on how solid behaved\(^1\). Unfortunately, solid-state phase diagram have remained uncomprehended mystery; only its straight boundary\(^2,3\) was explained by struggle of order vs. chaos. Here we show that the volume of orbital overlap has its own energy, with the universal density 8.941 eV/Å\(^3\) announced as new fundamental atomic constant that determines the transition temperature \(T_C\). Furthermore, we devised solid-state tomography, valid to 5 TPa, - imaging orbital through the baric dependencies of \(T_C\). Triangle-shaped pattern of the diagram is explained by the only possible way, just as only one plane passes through triangle: -inflation of the intersection volume during the transition determines hysteresis, but its disappearance does triple point; -approaching ions, whose orbitals overlap, curves the line from zero-field-cooling (ZFC) \(T_C\) to triple point; -the straight line between zero-field-heating (ZFH) \(T_C\) and triple point is a consequence of straightening tilting angle. Diamond melting point, calculated from volumes of the tetrahedral covalent bonds, excellently agrees with real; furthermore, the points up to 2 TPa agree with experiment\(^4\). Our findings open up way to interpret antiferromagnetism and steric effect in mono, binary, and ternary transition-metal oxides and sulfides\(^5-11\), and advance in unravelling unconventional superconductivity\(^12,13\), ascertaining the roles of \(\sigma\)- and \(\pi\)-hybridizations. Thereby, the importance of the solid-state tomography for organic conductors\(^12,13\) being high-compressible and interior of stars can scarcely be exaggerated.

In 1873, Van der Waals showed that the volume of molecules of the gas has the matter and they interact between them with the force named later in his honour\(^1\), however, was silent on how solid behaves. Van der Waals’ studies allowed Kamerlingh Onnes\(^14\) to liquefy helium and discover superconductivity in mercury; so unravelling the mystery of phase chart of gas had advanced not only physics of gas and liquid, but also solid. Solid is the last aggregate state of substance, which only changes its structural, electronic, magnetic and other properties till either its temperature has become infinitely close to absolute zero or its destruction under enormous temperature and pressure in the core of star. Unfortunately, up to now the phase chart of solid capable to advance science remains an uncomprehended mystery. Boundaries between the different properties of solid often behave intriguing manner. The phase charts of organic and cuprate superconductors where four different states meet at the multicritical point\(^12\) is especially interesting. The phase diagram of iron no less attractive puts up, however, knotty issue about whether the number of solid phases should be raised to five\(^15\). This work reveals the nature of the phase diagram of solid, that is, answers the question been silent by Van der Waals.

We have asked ourselves why the phase boundaries of Sm\(_{0.55}\)Sr\(_{0.45}\)MnO\(_3\) manganite behave straightforwardly with different slopes along the lines \(BC\) and \(DE\) and curvedly on \(AC\) (Fig. 1). The \(DE\) line energy slope \(\frac{d\mu}{d\mu}T_C/d\mu = 0.99\) [Hund’s rule gives \(\mu = 6\mu_B\) both for Mn\(^{+3}\) and for Mn\(^{+4}\) (Supplementary Table S1)] means that magnetic field only suppresses the thermal agitation and so the Curie point is linear:

\[
T_C = \frac{\mu}{k_B}B\cos\theta.
\]

The \(\mu \cdot B\) angle \(\theta\) is, at the same time, a tilting one between the adjacent Mn\(^{+3}\)O\(_6\)\(^-\) and Mn\(^{+4}\)O\(_6\)\(^-\) octahedrons; along \(DE\) \(\theta_{DE} = \arccos0.99 \approx 8.2^\circ\).
Fig. 1. | The Curie-point-magnetic-field diagram $[T_C(B)]$ of Sm$_{0.55}$Sr$_{0.45}$MnO$_3$.
Ferromagnetism appears at downward and/or rightward crossing of the curve $AC$ but disappears at upward and/or leftward crossing of the line $BC$. So if the triangle-like region $ABC$ has been reached from under $AC$ (above $BC$) it is ferromagnetic (paramagnetic), that is, the nonvolatile memory effect driven both by temperature and by magnetic field takes place. $AC$, $BC$ and $DE$ are seen to excellently describe the Curie points (the red signs at heating and the blue at cooling) both of polycrystalline$^{25,26}$ and of single crystal$^{27}$.

Our attempts to interpret the steep energetic pitch 2.37 of $BC$ as easy as $DE$ failed. To reveal the secret of $BC$ was previously necessary to explain $AC$. We supposed that the transition temperature $T_{tr}$ is proportional to the volume of the double spheric cap $V = 2\pi h^2 (R - h/3)$ formed at the intersection height $h$ of two orbitals with the radius $R$ of their spheric ends (Fig. 2a). So the intersection volume has the energy that equals the phase-transition energy $k_B T_{tr}$. By introducing the energy density $\nu$, $T_{tr} = \frac{2\pi \nu}{k_B} h^2 (R - h/3)$. (2)

The subscript $tr$ points out that Exp. (2) suited for different-type transitions - Curie, Neel, Mott and melting points. The indirect intersection of half-filled orbital with empty through full-filled (that is, diamagnetic) orbital leads to double exchange, but with half-filled to superexchange which, in turn, forbids the Mott’s conductance. The direct intersection of half-filled orbitals creates covalent bond whose energy and number yield the melting point.

Fig. 2. | The biconvex-lens volume of the orbital intersection and the geometric calculation of the intersection height $h_B$. a. The tilting angle $\theta$ between MnO$_6$ octahedrons and magnetic field $B$ determines the Mn$^{3+ (4+)}$ magnetostatic energy $\mu_B B \cos \theta$. b. The blue letterings belonging to the point $A$ yield the distance $r$ from the site Mn$^{3+ (4+)}$ or O$^{2-}$ to the center of the curvature $R$ of the spheric end of its $e_g$ or $2p$ orbital, $r = (a - R + h_A)/\cos \theta_A$. The red letterings belonging to the point $B$ give $h_B = R \cos \theta_B - a_B + R$, which with the $r$ calculated from the blue letterings, transforms into Exp. (4).
Obviously, the intersection height \( h = \varepsilon a \), \( a \) is half a distance between Mn\(^{3+/4+}\) and O\(^2-\) and \( \varepsilon \) a relative striction. A magnetic cation shrinking solid wastes its magnetostatic energy on enhancing the elastic energy of unit-cell volume \( V \): \( \mu \cdot B = VE\varepsilon^2/2 \), \( E \) is Young’s modulus. Then \( \varepsilon = \sqrt{2}\mu \cdot B / VE \) and so \( h = a\sqrt{2}\mu \cdot B / VE \), as ZFC Curie point \( T_C^B \) is nonzero, it needs to add the threshold intersection height \( h_A \) to this. Substitution of \( h = a\sqrt{2}\mu \cdot B / VE + h_A \) into Exp. (2) gives

\[
T_C = \frac{2\pi}{k_B} \left( \frac{2a^2\mu B}{VE} + 2h_A a \frac{\sqrt{2\mu B}}{VE} + h_A^2 \right) \left[ R - \left( a \frac{\sqrt{2\mu B}}{VE} + h_A \right) / 3 \right].
\] (3)

Magnetic field supports magnetism along \( AC \) same as on \( DE \), therefore, the best physical description of \( AC \) is the sum of Exps. (3) and (1) at \( \cos \theta \) linearly turning down from \( 0 \) to \( 0 \leq \cos \theta \leq \theta_{DE} \) (\( \cos \theta \) linearly depends both on temperature \(^{16}\) and on the rare-earth-cation radius \(^{17}\), therefore to preset its magnetic-field linearity is quite reasonable). We found the best fit with the root-mean-square error (RMSE) \( 0.58 \) K at the initial and fitted quantities mentioned in Table 1. So the curvilinearity of \( AC \) is a consequence of the curvilinear field dependence of the intersection volume at the striction, that is, approaching Mn\(^{3+/4+}\) and O\(^2-\) in the paramagnetic state just before appearing the ferromagnetism.

To understand hysteresis \( AB \) is to know why the ZFH Curie point \( T_C^B \) is higher than \( T_C^A \). We think that \( h_A \) triggers double exchange, which removes the Coulomb distortion \(^{18}\) (certainly the chemical distortion, defined by the tolerance factor, remains), the orbital overlap inflating till \( hb \). The geometric constructions (Fig. 2b) for the points \( A \) and \( B \) give

\[
h_b = (a-R+h_A)\cos\theta_a/\cos\theta_b-ab+R,
\] (4)

\( a_b \) is half a distance between Mn\(^{3+/4+}\) and O\(^2-\) at \( B \). Expression (2) with \( h = h_b \) and the entries for \( B \) (Table 1) gives \( T_C^B = 130.07 \) K excellently consistent with the experimental 128.6 K. Thus, at \( T_C^A \), \( h_A \) avalanche-like widens till \( hb \) which generates \( T_C^B \) observable at heating only. Thus, the mystery of hysteresis is due to the changing of intersection volumes during the type-I-like transition, that is, \( T_C^B - T_C^A = \frac{2\pi}{k_B} \left[ h_b^2 (R - h_b/3) - h_A^2 (R - h_A/3) \right] \).

Now we are able to interpret \( BC \) wherein magnetic field, straightening \( \theta \) from \( \theta_b \) to \( \theta_c \) same as \( AC \) (otherwise the triple point is not), orientates the MnO\(_6\) octahedrons, the intersection expanding. As \( \theta \) does not change the spacing Mn\(^{3+/4+}\)-O\(^2-\), Exp. (4) transforms into

\[
h_{BC} = (a_b-R+h_b)\cos\theta(B)/\cos\theta_B-ab+R.
\] (5)

Magnetic field supports magnetism on \( BC \) same as on \( AC \) and \( DE \) because the best graph of \( BC \) is also a sum of Exp. (2) with \( h = h_{BC} \) and Exp. (1). \( BC \) excellently traces the data with RMSE 1.37 K only with the initial entries, without fitting (Table 1). So the linearity of \( BC \) is a consequence of the linear field dependence of the intersection volume at straightening \( \theta(B) \) in ferromagnetic state.

![Fig. 3](image)

**Fig. 3.** \( T_C(P) \) diagram\(^{27}\) fitted by Exp. (6) (the translucent purple curve) with \( E(P) = 325P^{0.3} + E_0. \)
In the case of the pressure diagram, the Hooke law \( P = E(P)\varepsilon \) (the Young’s modulus \( E(P) \) is supposed to depend on pressure as well as temperature\(^{19} \), magnetic order\(^{20} \) and even form\(^{21} \)) gives \( h = aP/E(P) + h_B \) which turns Exp. (2) into

\[
T_{tr} = \frac{2\pi n}{k_B} \left\{ \frac{a^2}{E(P)} + 2h_B \frac{aP}{E(P)} + h_B^2 \right\} \left[ R - \frac{aP}{E(P)} + h_B \right]/3. \tag{6}
\]

Expression (6) with the initial entries (Table 1) and only with fitting the prefactor and power in \( E(P) = 325 P^{19} + E_0 \) excellently fits the \( T_C(P) \) diagram\(^{18} \) in (Sm\( _{11.6}\)Nd\(_{0.4}\))\(_{0.55}\)Sr\(_{0.45}\)MnO\(_3\) (Fig. 3).

Finally, our calculation of the diamond melting-point is the brightest approbation of our discovery. Diamond starts to melt when its atom receives the energy enough to break all its covalent bonds. The diamond \( sp^3 \)-orbitals intersection is the carbon \( p \)-orbital length minus the diamond covalent radius\(^{22} \): \( h = r_p - r_c \), where \( r_p \) is obtained as the carbon-hydrogen distance in methane (“gaseous diamond”) plus the hydrogen covalent radius and minus Bohr radius: \( r_p = r_{CH_4} + r_H^c - r_B \). Expression (2) multiplied by the four covalent bonds with this \( h \) and \( R = r_p/4 \) gives the diamond melting point \( 4157 \text{K} \) in excellent agreement with the real \( 4000 \text{K} \) (Supplementary Table S1). To further affirm our theory, proceeding from the warrantable assumption that the diamond phase persists under enormous pressure\(^{23} \), we calculated the baric dependence of the melting-point in diamond, \( T_C(P) \), through its \( \rho(P) \) taken from Extended Data Table\(^{24} \)1. The baric dependence of the covalent radius

\[
r_c(P) = \sqrt[3]{\frac{8(8+12m_u)}{\rho(P)}}^{1/3}, \tag{7}
\]

\( m_u \) is the atomic mass constant. Then \( h(P) = r_p - \sqrt[3]{\frac{8(8+12m_u)}{\rho(P)}}^{1/3} \) and \( T_{tr}(P) = \frac{2\pi n^2}{5k_B e^2} h^2(P) \left[ 3R - h(P) \right] + \Delta T_{tr}(P) \), calculated for the ellipsoidal intersection (\( r \) and \( R \) are the large and small semiaxes). \( \Delta T_{tr}(P) = P \frac{12m_u}{k_B} \left[ \frac{1}{\rho(0)} - \frac{1}{\rho(P)} \right] \) is the pressure-induced contribution resembling the energy of ideal gas \( U = PdV \), where \( U = k_B T \rho(P) \) and \( dV = 12m_u \left[ 1/\rho(0)-1/\rho(P) \right] \) is the striction of the unit cell per a carbon, and analogous to the magnetostatic addition of Exp. (1); that is, both pressure and magnetic field in themselves increase \( T_C \). Our calculated \( T_C(P) \) are quite satisfactory with the experiment\(^4 \) (Fig. 4a).

![Image](image_url)

**Fig. 4.** | The melting point of diamond up to 2000 GPa and tomography of its \( sp^3 \) orbital up to 5000 GPa. **a.** The background is the cutting from the inset of Fig\(^3 \) 4; red line shows the measurements\(^4 \) and black line the values\(^4 \) calculated assuming a constant specific heat. Blue balls mark our data; the uncertainties are smaller than the symbol size. The values of our graph are quite satisfactory with the lines\(^4 \), but its go does not with. That is, the ellipsoidal shape of the orbital does not work at extremal pressure and realistic fits are necessary. \( T_{tr}(P) = \frac{16\pi n^2}{k_B} \int_0^P R^2(r_c) \, d\rho_P \), derived from Exp (8), gives a possibility to precisely describe \( T_C(P) \) provided \( r_c(P) \) and \( R(r_c) \) are known from other experiments. **b.** Solid-state tomography (pink stars) of \( sp^3 \)-orbital in diamond, obtained from Exp. (8) under pressure up to 5 TPa (pink stars), that is superimposed upon the orbital image. The tomography confirms that even such enormous pressure cannot distort the orbitals and so the strength of their covalent bonds is enough to keep the diamond structure\(^23 \).
At last, the hit of our work is an orbital tomography – the reconstruction of the form of the orbital by means of the layer-by-layer scan of its radii $R$ versus the covalent radii $r_c$, which uses the temperature-pressure diagram as a microscope. The stress-induced decrement of the covalent radius $\partial r_c(P)$ increases the orbital intersection on the disk-shaped layer $2\pi r(\partial r_c(P))$ which induces the increment of the melting energy per half an $sp^3$ covalent bond $k_B\partial T_m(P)/\partial r_c(P) = 2\pi \nu r(\partial r_c(P))$. Then the measurements of the derivative $\partial T_m(P)/\partial r_c(P)$ allow us to profile the orbital via its radii

$$R(r_c) = \frac{1}{4} \frac{k_B\partial T_m(P)}{\pi \nu \partial r_c(P)}$$

(8)

$R(r_c)$ derived from $T_m(P)$ digitized from red line on the inset of Fig. 4 and $r_c(P)$ calculated from Exp. (7) through the same $\rho(P)$ table excellently outline the $sp^3$ orbital (Fig. 4b).

Thus, the announced atomic constant $\nu = 8.941 \text{ eV/Å}^3$ obtained from fitting $AC$ has been also confirmed by drawing $BC$ and $Tc(P)$, the calculation of the hysteresis, the melting point and orbital tomography in diamond at ultrahigh pressures. Our discovery qualitatively explains the diagrams of antiferromagnetics not clear for decades. Pressure, holding the transition-cation planes away from the anion planes in NiS, $V_2O_3$ and RNiO$_3$ ($R = \text{Pr, Nd, Nd}_{0.7}\text{La}_{0.3}$), narrows the $e_g$-$p$ orbital overlappings, so that $T_N$ lowers and the $e_g$ electrons, which cannot migrate because of antiferromagnetism, gradually become current carriers. The BaVS$_3$ diagram become understandable if to take into account that the pressure-induced orbital overlaps in the V-S chains favours to antiferromagnetism which impedes conductance. The linear $T_N(P)$ of the Gd, Pr and Tm orthoferrites means the fulfillment of Hooke law at their small rare-earth-cation radii, but the more radius in the La orthoferrite shortens the Fe-O bond and so inflates the 3$d$ and 2$p$ orbital intersection that causes the lift and nonlinearity of its $T_N(P)$. Expression (6) with the chemical pressure $P_i \sim 1 - y/y_p$ ($y_p$ is a doping index of the paramagnetic composition) instead of $P$ could fit the $T_N(y)$ diagram$^{10}$ of $\text{Ni}_{1-x}\text{Sr}_x\text{Se}_y$. The coincidence of the Sm$_{0.55}\text{Sr}_{0.45}\text{MnO}_3$ Curie point with the MnO Neel point hints at their same intersection volumes, that is, distinct-type transitions can have their near critical points, then certain substances must pose on the near points of the diagram “intersection volume–$Tc$”, revealing that their phase transitions caused by the orbital overlapping. Although the wave function is smear, the outline of orbital is sharp (Fig. 4b), that is, electron is like a son who was forbidden by his father to leave the yard, but he sometimes leaves and returns; so that chemists, specifying the orbital overlapping, can tailor necessary substance properties without complicated quantum simulations.

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**Competing interests** The author declares no competing interests.

**Additional information**

**Supplementary information** The online version contains supplementary material available at https://doi.org/10.1038/ ______.

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**Peer review information** *Nature* thanks __ for their contribution to the peer review of this work.

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Table 1. Initial data and fitted parameters in tracing the phase boundaries on Figs. 1 and 3.

| Curve | Point B | Line BC | Curve Tc(P) |
|-------|---------|---------|-------------|
| $\mu = 6 \mu$ | | | |
| $a = 0.9775 \text{ Å}$ | | | |
| $V = 226.301 \text{ Å}^3$ | | | |
| $R = a/4$ | | | |
| $h_A = 0.027147 \text{ Å}$ | | | |
| $\theta_A = 9.85^\circ$ | | | |
| $E_0 = 25.31 \text{ GPa}$ | | | |
| $v = 8.941 \text{ eV/Å}^3$ | | | |
| $a_B = 0.97628 \text{ Å}$ | | | |
| $\theta_B = 9.5^\circ$ | | | |
| $h_B = 0.0291592 \text{ Å}$ | | | |

$R$, $h_A$, $\theta_A$, $E_0$ and $v$ are fitted parameters of the curve AC. The tilting angle $\theta$ derives from the angle $\alpha$ of the chemical bond Mn$^{3+}$-O$^2$-Mn$^{4+}$: $\theta=(\pi-\alpha)/2$. The normal-condition Young’s modulus $E_0$ is in good agreement with values for manganites\textsuperscript{28}. $R$ is about a fourth of the orbital length that is standard for $d$ and $p$ orbitals\textsuperscript{29}. The values of $a$ and $V$ are calculated from the neutron-diffraction data\textsuperscript{18,30}. $a_B$, $\theta_B$ and $h_B$ are fitted parameters of the point B. The values of $a_B$, $\theta_A$ and $\theta_B$ are in good consistency with the neutron-diffraction data\textsuperscript{18,30}. And $\theta_C$ is in good agreement with $\theta_B$, that is, $\theta_C \geq \theta_B$. All the calculations were carried out in Supplementary Table S1. As seen, the fitted (●) data of AC are initial (■) for BC, B and Tc(P). And the fitted values of B are initial for BC and Tc(P). So, BC is drawn only with the initial data, without fitting. Tc(P) is traced only with fitting the prefactor and power in the pressure-depended Young’s modulus $E(P) = 325P^{1/3} + E_0$. 

$E_0$ is traced only with fitting the prefactor and power in the pressure-depended Young’s modulus $E(P) = 325P^{1/3} + E_0$. 

E(P) = 325P^{1/3} + E_0.
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