Inverse Compton cooling in Klein-Nishina regime and GRB prompt spectrum

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ABSTRACT

Synchrotron radiation mechanism, when electrons are accelerated in a relativistic shock, is known to have serious problems to explain the observed gamma-ray spectrum below the peak for most Gamma-Ray Bursts (GRBs): the synchrotron spectrum below the peak is much softer than observed spectra. Recently, the possibility that electrons responsible for the radiation cool via Inverse Compton, but in the Klein-Nishina regime, has been proposed as a solution to this problem. We provide an analytical study of this effect and show that it leads to a hardening of the low energy spectrum but not by enough to make it consistent with the observed spectra for most GRBs (this is assuming that electrons are injected continuously over a time scale comparable to the dynamical time scale, as is expected for internal shocks of GRBs). In particular, we find that it is not possible to obtain a spectrum with $\alpha > -0.1$ ($f_\nu \propto \nu^\alpha$) whereas the typical observed value is $\alpha \sim 0$. Moreover, extreme values for a number of parameters are required in order that $\alpha \sim -0.1$: the energy fraction in magnetic field needs to be less than about $10^{-4}$, the thermal Lorentz factor of electrons should be larger than $10^6$, and the radius where gamma-rays are produced should be not too far away from the deceleration radius. These difficulties suggest that the synchrotron radiation mechanism in internal shocks does not provide a self-consistent solution when $\alpha > \sim -0.2$.

Key words: radiation mechanisms: non-thermal - methods: analytical - gamma-rays: bursts, theory

1 INTRODUCTION

The dissipation mechanism responsible for the prompt emission of Gamma-Ray Bursts (GRBs) remains unknown. There have been various ideas put forth to explain it (for a review, see Piran 1999, 2004; Mészáros 2006; Gehrels et al. 2009). One of the main problems is to explain the fact that the majority of GRBs exhibit a spectrum $f_\nu \propto \nu^\alpha$, with $\alpha \sim 0$ below the peak of the spectrum (Preece et al. 2000), whereas the simplest version of the synchrotron model (in the so-called “fast cooling regime”) predicts $\alpha = -1/2$ (see, e.g., Ghisellini et al. 2000). Recently, a modified version of the synchrotron model, in which electrons that radiate below the peak of the spectrum cool via Inverse Compton (IC) in the Klein-Nishina (KN) regime (Derishev et al. 2003) has gained popularity (Bošnjak et al. 2009, Nakar et al. 2009, Wang et al. 2009, Fan 2010, Daigne et al. 2011). The main idea is very simple: Electrons cooling via synchrotron mechanism (or IC in the Thomson regime) exhibit an energy loss rate $\propto \gamma_e^4$, where $\gamma_e$ is the electron Lorentz Factor (LF) and $\delta = 2$. The observed synchrotron spectrum is then $f_\nu \propto \nu^{-(d+1)/2} \equiv \nu^{-1/2}$. However, when the cooling of electrons is dominated by the IC in the KN regime, where the photon-electron interaction cross section scales as $\sim \gamma_e^{-1}$, then $\delta \approx 1$ and $f_\nu \sim \nu^0$. In this paper, we investigate this scenario in detail and explore its consequences.

We analytically study the IC cooling in the KN regime assuming that electrons are injected continuously over a time scale comparable to the dynamical time scale, as is expected for the internal shock model of GRBs (Piran, Shemi & Narayan 1993; Katz 1994; Rees & Mészáros 1994). Recently, Daigne et al. (2011) have provided a detailed numerical calculation of the same context; however, they have assumed that electrons are injected instantaneously in the internal shock. Therefore, our work and the work of Daigne et al. (2011) are complementary.

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Essentially, Daigane et al. (2011) deals with the case when electrons are no longer being injected and they simply cool, which happens when
the internal shock has already passed through the shell. They consider the superposition of emission of many shells, for which the shock has
crossed all of them, and the shells simply adiabatically expand and cool. We, however, consider the shock as it traverses the shell, accelerates
electrons, and these radiate.

The scenario presented in this paper has been considered before (Nakar et al. 2009; Fan 2010) and our results are consistent. However, in
contrast with these works and with the work of Daigane et al. (2011), the work presented here is applied to the prompt phase data of particular
GRBs with $\alpha \approx 0$: we analyze the data of GRB 080916C, a burst detected by the Fermi Satellite, in the context of the scenario described
above, and provide constraints on this scenario based on available $>100$ MeV data.

Recent developments on prompt GRB theory have cast doubt on the internal shock model (see, e.g., Kumar & Narayan 2009, Zou et al.
2009). New alternative models have been proposed to solve the low-energy spectral index problem described above and other prompt
theory issues (see, e.g., Meszaros & Rees 2000, Drenkhahn & Spruit 2002; Lyutikov & Blandford 2003; Giannios 2008; Narayan & Kumar
2009; Kumar & Narayan 2009; Lazari et al. 2009; Beloborodov 2010; Vurm et al. 2011; Meszaros & Rees 2011; Ioka 2010; Ioka et al. 2011;
Zhang & Yan 2011, Bosnjak & Kumar 2012; Pe'er et al. 2012). It is, however, still relevant to critically test the internal shock model, in the
particular case where electrons cool via IC in the KN regime, to assess its feasibility.

It is important to mention that there exists a fraction of GRBs with $0 < \alpha < 1/3$ (only about 25 per cent of GRBs have more than 50
per cent of their spectra with $\alpha$ in this range; see Kaneko et al. 2006), that is, with spectra consistent with synchrotron radiation mechanism;
however the scenario presented here is unable to explain them. In this work we focus on the majority of GRBs, which have $\alpha \approx 0$; in
particular, we study the case of GRB 080916C, which shows $\alpha = -0.02 \pm 0.02$ for most of its duration (Abdo et al. 2009).

We set up our model and present the relevant time scales in Section 2. In Sections 3 and 4, we calculate the effect of IC cooling in the
KN regime on the electron energy distribution and on the observed spectral slope, respectively. In Section 5 we derive the relevant physical
parameters (radius of emission and total luminosity). In Section 6 we apply our results to GRB080916C, and to an average long-duration
GRB. In Sections 7 and 8, we present a Discussion and our Conclusions.

2 ELECTRON COOLING

Let us consider a GRB jet that has bulk LF $\Gamma$ and bolometric $\gamma$-ray luminosity (isotropic equivalent) $L$. The peak of the GRB spectrum ($v f_v$)
in observer frame is $\nu_p$. We define

$$\epsilon_p \equiv \frac{h \nu_p (1 + z)}{m_e c^2},$$

where $z$ is the redshift and $h$, $m_e$ and $c$ are Planck’s constant, the electron mass and the speed of light, respectively.

The case we are trying to explain using the idea of electron cooling via IC cooling in the KN regime is when the observed spectrum
below the peak is $f_v \propto \nu^\alpha$, with $\alpha \sim 0$; $\alpha$ is known as the low energy spectral index. We take $\alpha$ to extend from at least 10 keV to $\nu_p$; 10
keV is roughly the lower energy limit of the GBM detector on board the Fermi satellite.

Let us take the thermal LF of electrons that produce 10 keV photons (via synchrotron process) to be $\gamma_4$, and the LF of electrons producing
photons of frequency $\nu_p$ to be $\gamma_\nu$. We start with the assumption that electrons with LF $\gamma_4 < \gamma_\nu < \gamma_4$ cool primarily via IC in the KN regime.
In order to satisfy this assumption, $\gamma_4$ should be such that

$$\frac{\gamma_4 (1 + z) h \nu_p}{\Gamma} > m_e c^2,$$

which leads to the condition that $\gamma_4 > \epsilon_p^{-1} \Gamma$. We define a variable $\eta_4$ ($\eta_4 \geq 1$) which tells us how deep electrons of $\gamma_4$ are in the KN regime.
With this, $\gamma_4$ is

$$\gamma_4 = \eta_4 \epsilon_p^{-1} \Gamma.$$ (3)

We can find the magnetic field strength in the jet comoving frame, $B$, so that electrons with LF $\gamma_4$ have synchrotron radiation at 10 keV
in the observer frame. The observed synchrotron frequency of electrons of $\gamma_4$ is

$$\nu = \frac{e B \gamma_4^2 \Gamma}{2 \pi m_e c (1 + z)} = (1.15 \times 10^{-8} \text{ eV}) B \gamma_4^2 \Gamma (1 + z),$$

where $c$ is the electron charge. For $\nu = 10$ keV, and using (3), we find the magnetic field

$$B = (8.7 \times 10^2 G)(1 + z) \epsilon_p^{-1} \eta_4^{-2} \Gamma_3^{-3},$$

where here and throughout the paper we use the usual notation $Q_\nu = Q/10^n$, with the exception of $\gamma_e$, $\eta$ and the Compton-Y parameter, $Y$;
in these cases the subscript indicates the log_{10} of the observed synchrotron frequency in eV we are referring to.

The cooling time due to synchrotron radiation for an electron of LF $\gamma_\nu$, in the jet comoving frame, is given by

$$t_{\text{syn}}' = \frac{6 \pi m_e c}{\sigma_T B^2 \gamma_\nu} = (7.7 \times 10^8 s) B^{-2} \gamma_\nu^{-1},$$ (6)
where $\sigma_T$ is the Thomson cross section. For $\gamma_e = \gamma_i$ we find, using (3) and (5), that

$$t'_{\text{syn}} = (1.1 s) \frac{\eta_i^4 \Gamma^3}{(1 + 3) \eta_i^2}. \quad (7)$$

We now calculate the electron cooling time due to IC scattering of $\gamma$-ray photons. The cross section for scattering photons of frequency $\nu_0$ by electrons of $\gamma_i$ is smaller than the Thomson cross section by a factor of $\approx \eta_i$. Thus,

$$t'_{IC} \approx \frac{4 \pi R^2 \Gamma^2 m_e c^2}{\gamma_i (\sigma_T L/\eta_i)} = (15 \text{ s}) \frac{R_{15}^2 \Gamma^3 \eta_i}{\gamma_i L_{53}}, \quad (8)$$

or by substituting (5), we find

$$t'_{IC} \approx (0.15 \text{ s}) R_{15}^2 \Gamma^3 \eta_i \eta_s^{-1}. \quad (9)$$

Note that the IC cooling time is essentially independent of $\gamma_e$ in the KN regime. The reason is that the energy of electrons is $m_e c^2 \gamma_e$ and the IC power in the KN regime is approximately $\propto \gamma_e^3$, therefore, the time scale is almost independent of $\gamma_e$. We will compare these cooling time scales with the dynamical time in the jet comoving frame

$$t'_{dyn} = \frac{R}{c_\Gamma} = (33 \text{ s}) R_{15} \Gamma^{-1}. \quad (10)$$

### 3 EFFECT OF IC COOLING IN KN REGIME ON ELECTRON DISTRIBUTION

The electron energy distribution, $n_e$, in steady state, for electrons of LF $\gamma_e$, is determined from the continuity equation

$$\frac{\partial}{\partial \gamma_e} \left[ \gamma_e n(\gamma_e) \right] = S(\gamma_e) \propto \left( \frac{\gamma_i}{\gamma_e} \right)^{-p}$$

for $\gamma_e \gtrsim \gamma_i$, \quad and $\gamma_e < \gamma_i$, \quad (11)

and the cooling of electrons is determined by

$$-\dot{\gamma}_e = \frac{\sigma_k B^2 \gamma_e^2}{6 \pi m_e c} + \frac{\sigma_{KN} L \gamma_e^2}{4 \pi R^2 \Gamma^2 m_e c^2}$$

where $\sigma_{KN}$ is the KN cross section which we write as $\sigma_{KN} = \sigma_T f(\eta)$, and

$$f(\eta) = \frac{3}{4} \left[ \frac{1 + \eta}{\eta^3} - \frac{2 \eta (1 + \eta)}{1 + 2 \eta} - \frac{\ln(1 + 2 \eta)}{2 \eta^2} - \frac{1 + 3 \eta}{1 + 2 \eta^2} \right]. \quad (13)$$

We also define $\eta$ as $\eta = \epsilon_0 \gamma_e / \Gamma$ analogous to (3), and it indicates how deep electrons of $\gamma_e$ are in the KN regime, defined in (2). In this section we keep the dependence of $\eta$ on $\gamma_e$. As a reminder, $\eta$ was defined for a specific $\gamma_e = \gamma_i$ in the previous section; we will return to that same definition later on. With the use of (13), equation (12) can be rewritten as

$$-\dot{\gamma}_e = \frac{\gamma_i^2}{T_{syn}} + \frac{\gamma_i^2}{T_{IC}} f(\eta), \quad (14)$$

where $T_{syn}$ and $T_{IC}$ are defined as

$$T_{syn} = \frac{6 \pi m_e c}{\sigma_T B^2}, \quad (15)$$

and

$$T_{IC} = \frac{4 \pi R^2 \Gamma^2 m_e c^2}{\sigma_T L}. \quad (16)$$

Note that $t'_{syn} = T_{syn} / \gamma_e$ is the synchrotron cooling time, defined in (3), and $t'_{IC,KN} = T_{IC} \sigma_T / (\sigma_{KN} \gamma_e) = T_{IC} / (f(\eta) \gamma_e)$ is the IC cooling time in the KN regime (see eq. (3) and note that $f(\eta) \sim \eta^{-1}$ for $\eta \gg 1$).

We can define $Y_{KN} \equiv t'_{syn} / t'_{IC,KN} = T_{syn} / (T_{IC} / f(\eta))$, and identify it as the Compton-$Y$ parameter in the KN regime for electrons with LF $\gamma_e$, which are deep in the KN regime as characterized by their parameter $\eta$. The electron cooling rate is now

$$-\dot{\gamma}_e = \frac{\gamma_i^2}{T_{syn}} (1 + Y_{KN}) \quad (17)$$

Since we are interested in the flux below the peak, we consider electrons with LF $\gamma_e < \gamma_i$. Moreover, we need to consider the case $\gamma_i < \gamma_e < \gamma_i$, where $\gamma_e$ is the cooling LF, which is the LF of electrons that cool on a dynamical time, $t_{dyn}$. The case $\gamma_e < \gamma_i < \gamma_i$ gives $f_e \propto \nu^{1/3}$ below the peak, which is impossible to obtain for electrons accelerated in shocks for GRBs (Ghisellini et al. 2000; Kumar & McMahon 2008).

The solution of the continuity equation (11) for the the electron distribution for $\gamma_e < \gamma_i$ is $n_e \propto \gamma_e^{-2}$ and, using (17), the corresponding power-law index of the distribution is

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\[ p_1 \equiv \frac{d \ln n_e}{d \ln \gamma_e} = 2 + \frac{Y_{KN} f(\eta)}{\int f(\eta)(1 + Y_{KN})} \]  
(18)

As mentioned before, \( \eta \) depends on \( \gamma_e \), and using equation (13) we find

\[
\frac{df(\eta)}{d\eta} = \frac{3}{4} \left[ \frac{2\eta(2\eta^4 - 39\eta^2 - 63\eta^2 - 34\eta^3 - (2\eta + 1)^3(\eta^2 - 4\eta - 6) \ln(2\eta + 1)}{2\eta^4(2\eta + 1)^3} \right].
\]  
(19)

For \( \eta \geq 1 \), which we consider here, eq. (19) always yields a negative value, and since \( Y_{KN} \geq 0 \), then 1 \( \leq p_1 \leq 2 \). Since we are interested in \( f_\nu \propto \nu^0 \), we will focus on finding the corresponding \( \eta \) and \( Y_{KN} \) that give \( p_1 \to 1 \), so that \( \alpha \to 0 \). Let us first calculate \( \alpha \) for a given \( p_1 \).

4 SPECTRAL SLOPE WHEN ELECTRON DISTRIBUTION IS CLOSE TO \( \gamma_e^{-1} \)

Let us consider the electron energy distribution to be

\[
n_e \propto \begin{cases} \left( \frac{\gamma_e}{\gamma_i} \right)^{-p_2} & \gamma_e < \gamma_i \\ \left( \frac{\gamma_e}{\gamma_i} \right)^{-p_1} & \gamma_e > \gamma_i. \end{cases}
\]  
(20)

We will take \( p_1 \) to be very close to 1 and \( p_2 \approx 2 \). The exact value of \( p_1 \) can be found with (18).

The synchrotron flux is given by

\[
f_{\nu_o} = A \int_{\nu_o}^{\infty} d\nu_e \nu_e \left( \frac{\nu_o}{\nu(\gamma_e)} \right)^{1/3},
\]  
(21)

where \( A \) is a constant proportional to \( B \), and \( \nu_o \) is the LF of electrons radiating at synchrotron frequency \( \nu_o \), which is

\[
\nu_o = \left( \frac{2\pi m_e c^2}{eB} \right)^{1/2}.
\]  
(22)

The frequency \( \nu(\gamma_e) \) is the synchrotron frequency of electrons with LF \( \gamma_e \)

\[
\nu(\gamma_e) = \frac{eB\gamma_e^2}{2\pi m_e c}.
\]  
(23)

Substituting the last two expressions into (21), we can integrate this expression using (20). By absorbing constants in a new variable \( A' \), we find

\[
f_{\nu_o} = A' \nu_o^{1/3} \frac{\gamma_i^{-1/3}}{p_1 - \frac{1}{3}} \left[ \left( \frac{\nu_o}{\gamma_i} \right)^{-p_1 - 1/3} - \frac{p_2 - p_1}{p_2 - \frac{2}{3}} \right].
\]  
(24)

With this, we can find the spectral index for synchrotron radiation to be

\[
\alpha \equiv \frac{d \ln f_{\nu_o}}{d \ln \nu_o} = \frac{1}{3} - \frac{1}{3} \left( \frac{p_1 - \frac{1}{3}}{p_1 - \frac{2}{3}} \right) \left( \frac{\nu_o}{\gamma_i} \right)^{p_1 - 1/3}.
\]  
(25)

Since we are interested in the low energy spectral index, \( \alpha \), at \( \nu_o = 10 \) keV, \( \frac{\nu_o}{\gamma_i} = \left( \frac{10 \text{ keV}}{\nu_p} \right)^{1/2} \). For example, for \( \nu_p = 1 \) MeV, then \( \frac{\nu_o}{\gamma_i} = 0.1 \), and for \( p_1 = (1.02, 1.1, 1.15, 1.3) \) and \( p_2 = 2 \) we find \( \alpha = (-0.057, -0.089, -0.109, -0.173) \). Note that from (25) \( \alpha < 0 \) for \( p_1 \geq 1 \).

Since we want the spectrum at 10 keV to have \( \alpha \approx 0 \), we will choose the LF of electrons to be \( \gamma_e = \gamma_i \). We will determine how deep these electrons have to be in the KN regime (\( \eta_k \)) and their Compton-\( \gamma \) parameter, \( Y_{KN,A} \), so that we can obtain \( p_1 \to 1 \). We describe the calculation in the next section.

5 PHYSICAL PARAMETERS CONSISTENT WITH \( \alpha \to 0 \)

In this section, we determine the values of \( \eta_k \) and \( Y_{KN,A} \) that are required in order to obtain the low energy spectral index close to zero at 10 keV. The idea is simple: we scan all possible combinations of \( \eta_k \) and \( Y_{KN,A} \) and determine the power-law index of the electron distribution, \( p_1 \), which can be obtained with (13). Next, we use this power-law to find the observed spectrum at 10 keV, \( \alpha \), using (25). We present the results of our parameter search for \( \eta_k \) and \( Y_{KN,A} \) that yield a certain desired value for \( p_1 \) and \( \alpha \) (Fig. 1: Left panel).

For a given \( \eta_k \) and \( Y_{KN,A} \) we can determine the radius, \( R \), at which the emission is produced. Since \( Y_{KN,A} \) is defined as \( Y_{KN,A} = t'^{IC}_{syn} / t'^{IC}_{KN} \), we can use (1) and (5) to determine \( R \) as a function of \( Y_{KN,A} \) and \( \eta_k \). We find
and (8) to determine $\sigma \approx \alpha$. The first constraint that we can place on the radius of emission is given by the photospheric radius, while the largest radius is approximately the deceleration radius (see text). Both figures are for the parameters of GRB080916C (details can be found in the next section).

$$R_{15} \approx 3 \frac{3/2 Y_{\alpha} 1/2 L_{53}}{Y_{\alpha,4}(1 + z)e_\beta^3}.$$  

(26)

We can now calculate $R$ as a function of $\eta_4$ and $Y_{KN,4}$ to find the range of radii where synchrotron radiation should be produced for each value of $\alpha$. In the $Y_{KN,4}$ versus $\eta_4$ plane, lines of constant radius are approximately given by $Y_{KN,4} \sim \eta_4^3$ (Fig. 1: Right panel).

It is important to mention that our analytical results use a simplified approximation for the KN cross section in (5), which is that $\sigma_{KN} \approx \sigma_T/\eta_4$. However, results shown in all figures use the full KN cross section $\sigma_{KN} = \sigma_T f(\eta)$, so that the radius is $R \propto \eta^2 (f(\eta))^{1/2}$, instead of $R \propto \eta^{3/2}$; analytical results are within a factor $\sim 2$ of numerical values.

### 5.1 Constraint on the radius of emission

The first constraint that we can place on the $Y_{KN,4}$ versus $\eta_4$ parameter space is that the cooling time scale for electrons radiating at 10 keV cannot be larger than the dynamical time scale. After all, we need electrons to cool rapidly via IC scatterings in the KN regime to obtain $\alpha \approx 0$. This constraint can be obtained in the following way.

The electron cooling time in the jet comoving frame for electrons of LF $\gamma_4$ is given by $t'_\text{cool} = (1/t'_\text{syn} + 1/t'_\text{IC,KN})^{-1}$. We use (6) and (5) to determine $t'_\text{cool}$. We impose the constraint, as discussed above, that $t'_\text{cool} < t'_\text{dyn}$, and with it, we find an upper limit on the radius of emission, $R_{\text{cool}}$, and we can use it to constraint our $Y_{KN,4}$ versus $\eta_4$ parameter space. An analytical estimate of this maximum radius is provided below.

The cooling of electrons of LF $\gamma_4$ is dominated by the IC cooling in the KN regime. Therefore, we can approximate $t'_\text{cool} \approx t'_\text{IC,KN}$ and then set $t'_\text{cool} < t'_\text{dyn}$, and use (5) and (10) to find

$$R_{\text{cool},15} \approx 220 \frac{L_{53} \Gamma_3}{\Gamma_3^{\gamma_p^3}}.$$  

(27)

Another upper limit on the radius is given by the radius at which the external forward shock sets in, that is, the deceleration radius, $R_{\text{dec}}$. This radius is a function of the total blast wave energy, $E$, the circum-stellar density, $n$, which we assume is a constant, and the bulk LF (see, e.g., Sari, Piran & Narayan 1998)

$$R_{\text{dec},15} = 130E^{1/3} n_0^{-1/3} \Gamma_3^{-2/3}.$$  

(28)

The true upper limit will be given by the minimum of $R_{\text{cool}}$ and $R_{\text{dec}}$.

There is also a lower limit on the radius, given by the photospheric radius,

$$R_{\text{ph}} \approx \frac{L\sigma_T}{8\pi m_n c^3 \Gamma_3} \approx (5.5 \times 10^{10} \text{ cm}) L_{53} \Gamma_3^{-3}.$$  

(29)

Therefore, the radius of emission should lie between $R_{\text{ph}}$ and the minimum of $R_{\text{cool}}$ and $R_{\text{dec}}$.  

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5.2 Constraint on SSC component

The last constraint that we place on the $Y_{KN,i}$ versus $\eta_4$ parameter space is that the total luminosity in the synchrotron-self-Compton (SSC) component at $\sim 1$ GeV, $L_{IC}$, should not exceed the synchrotron luminosity, $L_{syn}$, for consistency with the Fermi data (Abdo et al. 2009), that is $L_{IC}/L_{syn} < 1$. The ratio of luminosities is given by $L_{IC}/L_{syn} \approx Y_{KN,i}$, where $Y_{KN,i}$ is the Compton-Y parameter in the KN regime of electrons radiating at $\gamma_i$ (Nakar et al. 2009). The relationship between $Y_{KN,i}$ and $Y_{KN,4}$, is approximately given by

$$Y_{KN,i} = Y_{KN,4} f_{syn}(\gamma_i) f_{IC}^{\text{max}}(\gamma_i),$$

where $f_{syn}(\gamma_i)$ is the synchrotron flux below the KN frequency, $\nu_{KN}(\gamma_i) = m_e c^2 / (b \gamma_i)$, both quantities in the jet comoving frame. The ratio of these comoving synchrotron fluxes is given by $\gamma_4/\gamma_i = (10 \text{ keV}/\nu_p)^{1/2}$, therefore

$$Y_{KN,i} = Y_{KN,4} \left( \frac{10 \text{ keV}}{\nu_p} \right)^{1/2}. \quad (30)$$

We have found that the analytical calculation of Compton-Y can overestimate its true value by up to a factor of $\sim 10$ (Barniol Duran & Kumar 2011, see, also, Nakar et al. 2009). If we restrict our parameter space to $L_{IC}/L_{syn} < 1$, to avoid conflict with Fermi high energy observations, then these last two considerations translate to a conservative constraint on $Y_{KN,i}$ given by $Y_{KN,i} \lesssim 10$. Solutions that have $Y_{KN,i}$ larger than this limit violate Fermi observations and are ruled out.

To summarize, we calculate the power-law index of the electron energy distribution function, $p_1$, as a function of $Y_{KN,4}$ and $\eta_4$ using (13). With $p_1$ and equation (15), we can determine the lower energy spectral index at 10 keV, $\alpha$. The 2-D space $(Y_{KN,4}, \eta_4)$ can be constrained by calculating the radius of emission that corresponds to each point in the $Y_{KN,4}-\eta_4$ plane. The radius of emission should not be smaller than the photospheric radius nor larger than the minimum of the deceleration radius (eq. (28), and the radius at which the dynamical and cooling time scales are equal, (eq. (27)). We can further constrain the parameter space by ensuring that the amount of energy in the SSC component is not excessive.

6 APPLICATION TO GRB DATA

We present the allowed $Y_{KN,4}-\eta_4$ parameter space for two GRBs. We chose a very energetic Fermi GRB, GRB080916C, and another more “standard” GRB, which we call GRB$_0$. The parameters for GRB080916C are $\epsilon_p = 7.5$, $z = 4.3$, $L_{53} \approx 1$, $\Gamma = 10^3$ and, for most of the duration of the prompt emission of this GRB, $\alpha = -0.02 \pm 0.02$ (Abdo et al. 2009). To find the deceleration radius, $R_{dec}$, we choose $E_{55} \approx 3$ (Kumar & Barniol Duran 2010) and assume $n_0 = 1$, although the dependence on $E$ and $n$ is weak. We choose the parameters for GRB$_0$ to be more typical values of a GRB: $\epsilon_p = 3$, $z = 2$, $L_{53} \approx 10^{-2}$, $\Gamma = 300$, $E_{55} \approx 10^{-2}$, $n_0 = 1$ and, for most of the duration of the prompt emission of this GRB, $\alpha = 0$ (Preece et al. 2000). Following the prescription at the end of last section, we present the results of the allowed parameter space in Fig. 2 (Left panel). Note that we present the parameter space now as a function of $Y_{KN,4}$, instead of $Y_{KN,i}$ (which is a better indicator of $L_{IC}/L_{syn}$), however, they are related by a constant factor, eq. (31).

As can be seen, $\Gamma \rightarrow 0$ as we move into a region where $\eta_4$ and $Y_{KN,4}$ (and consequently, $Y_{KN,4}$) both become larger. What is the maximum possible lower energy spectral index at 10 keV ($\alpha^{max}$) that can obtained? It is given by the value where the maximum of $Y_{KN,4}$ intersects with the maximum allowed radius (see Fig. 3). This intersection gives us the maximum allowed value for $\alpha$ ($\alpha^{max}$) which is found to be $\sim 0.1$ (Table I). Therefore, any of the GRB gamma-ray radiation for which $\alpha > -0.1$ cannot be produced by the synchrotron process in shock heated plasma, where electrons are only accelerated when they cross the shock front and are scattered back to the other side (this however, is not the case if electrons are continuously accelerated while they are traveling downstream or upstream; we do not consider this scenario in this paper).

In Table I we also present the maximum value of $\alpha$ obtained if: (i) We decrease the value of the peak energy as observed during the prompt phase (Abdo et al. 2009), and/or (ii) We decrease the value of the LF of the source, as suggested by several groups (see, e.g., Zou et al. 2011, Hascoët et al. 2011).

The luminosity carried by the magnetic field, $L_B$, as measured by a lab frame observer is

$$L_B = \frac{B^2 L^2 c}{8 \pi} = (1.1 \times 10^{52} \text{erg}) R_{15}^2 (\epsilon_p/\eta_4) \Gamma_3^4 (1+z)^2, \quad (32)$$

where we made use of (5). Therefore, the fraction of energy carried by the magnetic field is

$$\epsilon_B = \frac{L_B}{L} = 0.1 (\epsilon_p/\eta_4)^4 L_{53}^{-1} \left( \frac{R_{15} (1+z)}{\Gamma_3^2} \right)^2. \quad (33)$$

We present the value of $\epsilon_B$ for the $\alpha^{max}$ case (Table I); $\epsilon_B$ is found to be very small, in the range $\sim 10^{-6} - 10^{-4}$. We also provide in Table 1 the value of the LF of electrons that radiate at the peak of the spectrum ($\gamma_i$) in order for the low energy spectral slope to be $\alpha^{max}$; we find $\gamma_i \gtrsim 10^6$.

If we take the observer frame variability time scale of the gamma-ray light curve to be
Figure 2. Left: The solid lines are contours of constant $\alpha$ (from the upper right to the lower left corner $\alpha = -0.1, -0.12, -0.15, -0.2, -0.3$), while the short-dashed lines correspond to constant $R$ value in this 2-D plane; $R = 10^{14}$ cm for the left most line, and the subsequent ones correspond to $10^{12}, 10^{13}, 10^{14}, 10^{15}, 10^{16}$ and $10^{17}$ cm (the largest radius is approximately the deceleration radius). Note that we plot $Y_{KN,i}$ instead of $Y_{KN}$ (which differ only by a constant factor). The maximum allowed value of $Y_{KN,i} \approx 10$, so as to avoid a SSC component excess in the Fermi high energy observations, is plotted as a long-dashed line (the factor of 10 allows for an overestimation in the analytical result). The allowed region is the region below the long-dashed line and to the left of the maximum radius, which has been shaded. The maximum value of $\alpha^{max} = -0.11$ is obtained at the intersection of the maximum allowed radius and the maximum $Y_{KN,i}$. It is clear that $\alpha > -0.1$ is impossible to reach for the data of GRB080916C. Right: The solid lines show $\alpha$ in the $\epsilon_{\epsilon}/\epsilon_{B}$ versus $\eta_4$ plane. $\epsilon_{\epsilon}$ and $\epsilon_{B}$ are the fractions of energy in electrons and magnetic field, respectively, while $\eta_4$ indicates how deep in the KN regime are the electrons radiating at the peak frequency. From top to bottom, $\alpha = -0.20, -0.25, -0.30, -0.35, -0.40, -0.45$. The data in these panels are for GRB080916C, however, the plot for an average long-duration GRB is almost identical.

| GRB   | $\Gamma_3$ | $\epsilon_{\rho}$ | $\eta_4$ | $\gamma_i$ ($\times 10^6$) | $\epsilon_B$ ($\times 10^{-5}$) | $\alpha^{max}$   |
|-------|------------|------------------|---------|-----------------|-------------------|----------------|
| 080916C | 1          | 7.5              | 1300    | 2               | 3                 | -0.11 (-0.14)  |
|       | 0.3        | 7.5              | 16500   | 6               | 0.3               | -0.10 (-0.14)  |
|       | 1          | 2.5              | 400     | 1               | 10                | -0.14 (-0.20)  |
|       | 0.3        | 2.5              | 3400    | 2               | 2                 | -0.13 (-0.20)  |
| 2      | 0.3        | 3                | 2500    | 2               | 2                 | -0.11 (-0.17)  |
|       | 0.1        | 3                | 28000   | 7               | 0.2               | -0.10 (-0.17)  |
|       | 0.3        | 1                | 900     | 1               | 7                 | -0.15 (-0.25)  |
|       | 0.1        | 1                | 5700    | 2               | 1                 | -0.14 (-0.25)  |

Table 1. We present the maximum possible spectral slope, $\alpha^{max}$, for two GRBs: GRB080916C and GRB2; $\alpha^{max}$ is the value of $\alpha$ obtained, where the maximum $Y_{KN,i}$ intersects with the maximum radius (see Fig. 2 Left panel). We present results for different values of the GRB-jet LF and the observed spectral peak ($\epsilon_{\rho}$; see Eq. 1 for definition). In addition, we also present the values for $\eta_4$, $\gamma_i$ and $\epsilon_B$, where the spectral slope is maximum. The value of $\alpha$ does not reach the observed value $\alpha = -0.02 \pm 0.02$ (Abdo et al. 2009) for GRB080916C, nor $\alpha = 0$ for GRB2 for any combination of parameters. At $\alpha^{max}$ we find extreme values for the LF of electrons radiating at the peak and for the energy fraction in the magnetic field. The values of $\alpha^{max}$ where we assume a variability time scale of $\delta t = 0.1$ s are in parenthesis (see next Section).

$$\delta t = \frac{R(1+z)}{2c\Gamma^2}$$

as expected in the internal shock model, then in order for the low energy spectral slope to be $\alpha^{max}$, we find $\delta t \geq 10$ s, since the emission is produced at (or very close to) the deceleration radius. This is very long compared with the observed time scale of 0.1 s or less.

We can calculate the low energy spectral index as a function of $\epsilon_{\epsilon}/\epsilon_B$, where $\epsilon_{\epsilon}$ is the fraction of energy in electrons. This will allow us to compare our results with previous work that uses this ratio. To do this, we use (Ando et al. 2008)

$$Y_{KN,i} = \sqrt{\frac{\epsilon_{\epsilon}/\epsilon_B}{\eta_4}}$$

where $\eta_4$ parameterizes how deep electrons of $\gamma_i$ are in the KN regime, analogous to (3), and it is related to $\eta_4$ as $\eta_4/\eta_4 = \gamma_i/\gamma_4 = (\nu_p/10\text{ keV})^{1/2}$. Using (31) and (35), we find $\epsilon_{\epsilon}/\epsilon_B$ as a function of $\eta_4$ and $Y_{KN,i}$ and calculate $p_1$ using equation (18), and $\alpha$ from equation (25). The result is shown in Fig. 2 (Right panel).

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7 DISCUSSION

Most GRBs have low energy spectral index $\alpha$ between 0 and $-0.1$; $f_\nu \propto \nu^{-\alpha}$. For $\alpha > -0.1$, it is required that the power-law index for electron energy distribution function, $p_i = 3 \ln \Gamma \nu / (3 \ln \nu)$, should be less than 1.15 at $\gamma_e$ corresponding to 10 keV synchrotron photons (in observer frame). For this, two conditions must be satisfied. 1. The Compton-$Y$ parameter for electrons radiating at 10 keV should be $Y_{KNA} \gtrsim 20$ (including KN effect), and 2. The LF of electrons radiating at 10 keV should be $\gtrsim 2000 \Gamma^{-1}$, where $\epsilon_p = \frac{h \nu_{ph}(1+z)}{m_e c^2}$, that is, $\eta_4 \gtrsim 2000$ (see Fig. 1). These conditions apply both for the data of GRB080916C (a highly energetic explosion) and also for an average long duration burst.

Electron index changes sharply, from being close to 1 (when IC in the KN regime dominates) at 10 keV to >2 at the peak of the observed spectrum at $\sim 1$ MeV, that is, the electrons distribution index increases from $\sim 1$ to $>2$, when the electron LF increases by a factor $\sim 10$. In this case, the spectral index for synchrotron radiation at 10 keV is not given by $- (p_i - 1)/2$ (as mentioned in the Introduction); in fact, it is significantly smaller. For instance, when $p_i = 1.02$ (for electrons radiating at 10 keV), $\frac{d \ln f_\nu}{d \ln \nu} = -0.057$ and not $-0.01$ as naively expected; see (25).

Another consequence of the requirement that $\eta_4 \gtrsim 2000$ is that the energy fraction in magnetic field, $\epsilon_B$, is rather small $\epsilon_B \lesssim 10^{-9} R_{15}^2$ (GRB080916C) and $\epsilon_B \lesssim 10^{-7} R_{15}^2$ (for an average burst); see (33). In addition, the LF of electrons radiating at the peak of the spectrum ($\sim 1$ MeV) is $\sim 10$ times larger than that of electrons radiating at 10 keV, therefore, $\gamma_i \gtrsim 2 \times 10^4 \Gamma^{-1}$, which is $\gamma_i \gtrsim 10^6$ almost independent of GRB energy (Table 1). Both values of $\epsilon_B$ and $\gamma_i$ are extreme and their implications will be discussed in the Conclusions.

However, we can also ask: what is $\alpha$ for a reasonable set of parameters? In this case, “reasonable” means two things: (1) The radius of emission of the prompt emission should be between (i) the photospheric radius and (ii) the radius where electrons producing 10 keV synchrotron photons cool on a time scale shorter than the dynamical time via IC scatterings in the KN regime, or the deceleration radius, whichever is smaller, and (2) The energy in the IC component should not be very large, so that the GRB spectrum does not show an IC bump at $\sim 1$ GeV, as the Fermi satellite sees no sign for such an excess. We have calculated the maximum value of $\alpha$ that can be obtained for a highly energetic Fermi burst, GRB080916C, and also for an average GRB and the results are presented in Table 1. We find that the maximum value of $\alpha$ is $\alpha = -0.1$. For this value, the same consequences as discussed above apply, and can be found in Table 1. Namely, that $\eta_4 \sim 10^3$, and this implies a very large value of $\gamma_i$ and an extremely small value of $\epsilon_B$. Moreover, the maximum value of $\alpha$ occurs at a very large radius, close to the deceleration radius, which, will have problems producing variable light curves with $\delta t$ smaller than a few seconds.

Conversely, we can also fix the observed variability time scale of the gamma-ray light curve to be $\delta t = 0.1$ s and determine the radius of emission with (34), as expected in the internal shock model. We can determine the maximum value of $\alpha$ at this radius and its value is presented in parenthesis in Table 1. Notice that this value of $\alpha$ is even further away from the observed value. However, at this radius, the values of $\epsilon_B \sim 10^{-4} - 10^{-5}$ and $\gamma_i \sim 10^3 - 10^4$ are less extreme and in marginal agreement with the internal shock model. Nevertheless, this further accentuates the fact that synchrotron emission, in which electrons cool mainly via IC in the KN regime, in the context of the internal shock model, cannot explain the observed $\sim 0$ spectrum of most GRBs.

Expressing our analytical results as a function of $\epsilon_e / \epsilon_B$ allows us to compare them with previous numerical work. Nakar et al. (2009) (see, also, Fan 2010) have found numerically that with $\epsilon_e / \epsilon_B = 100$ ($10^3$), $\alpha$ cannot exceed $\sim -0.3$ ($-0.2$). In this work, we analytically confirm their results (see right panel of Fig. 2). Daigne et al. (2011) have also found numerically that $\alpha = 0$ is possible in the case when electrons are injected instantaneously. However, in Nakar et al. (2009) and the present work, we have assumed that electrons are injected regularly over a time scale comparable to the dynamical time scale. Nevertheless, Daigne et al. (2011) present numerical results for our scenario in their Fig. 2 (bottom left panel) for $\eta_4 = 100$, which agree with our analytical calculation.

Our work and the work by Daigne et al. (2011), as mentioned before, differ mainly on the chosen time scale at which electrons are injected to the shock, $t_{inj}$. We take $t_{inj} \sim t_{dyn}$, where $t_{dyn}$ is the dynamical time scale, whereas Daigne et al. take $t_{inj} \ll t_{dyn}$. We find a stricter limit on the allowed value of $\alpha$: $\alpha \gtrsim -0.2$, whereas Daigne et al. find that solutions with $\alpha = 0$ can be reached (see their Fig. 2). These results are not in contradiction, since both studies probe two different phases found in the internal shock model. The first phase ($t_{inj} \sim t_{dyn}$) has electrons continuously being injected to the shock as it crosses the shell. The second phase ($t_{inj} \ll t_{dyn}$) corresponds to the case where the shock has already traversed the shell and electrons cool as the shell adiabatically expands. Most of the available energy is dissipated in the first phase; for this reason we have chosen this particular scenario in this paper, which is the common practice in studies of the internal shock model (see, e.g., Piran 1999).

However, when calculating synthetic GRB light curves, Daigne et al. (2011) do follow the dynamics of the shock crossing numerically, and consider a large number of discretized shells on the dynamical time scale. In each collision the electron injection occurs instantaneously, $t_{inj} \ll t_{dyn}$, but the electron injection process over the full simulation is comparable to the dynamical timescale. In this case, when including IC cooling in the KN regime (see their fig. 9), they find $\alpha \gtrsim -0.1$. The difference in our results appears because the IC scatterings in their work do not occur between the same photon and electron distributions we have considered: In Daigne et al. (2011) the scatterings between photons emitted in a shocked region and electrons or photons present in a subsequent shocked region were not considered (see Bošnjak et al. 2009). This affects the cooling of the electrons via IC, allowing them to reach spectra closer to $\alpha = 0$. 

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8 CONCLUSIONS

In this work we have investigated the possibility that the observed low energy GRB prompt spectrum, which is $f_{\nu} \sim \nu^{0}$ (below the peak) for a good fraction of all long duration GRBs (Preece et al. 2000; Kaneko et al. 2006; Pelangeon et al. 2008; Krimm et al. 2009; Ghirlanda et al. 2010), is due to synchrotron radiation from electrons that cool mainly via the IC mechanism in the KN regime (Derishev et al. 2003, Bošnjak et al. 2009, Nakar et al. 2009, Wang et al. 2009, Fan 2010, Daigne et al. 2011).

We present an analytical method to determine the power-law index of the electron energy distribution function, $p_{i}$, that cools via IC cooling in the KN regime as a function of two parameters: $\eta$, which is a measure of how deep electrons of interest are in the KN regime, and $Y_{KN}$, which is the Compton-$Y$ parameter (including KN corrections) for these electrons. We have calculated the observed low energy spectral index for synchrotron radiation, $\alpha$, as a function of these two parameters as well as the power-law index of the electron energy distribution above $\gamma_{i}$ ($\gamma_{i}$ corresponds to the LF of electrons radiating at the peak). We find that $\alpha$ is not simply given by $-(p_{i} - 1)/2$ as naively expected, but it is smaller, which makes it very difficult to explain the observed value of $\alpha \approx 0$ for a good fraction of GRBs.

We find that $\alpha > -0.1$ cannot be obtained for parameters relevant for GRBs, if the radiation mechanism is the synchrotron process and electrons are accelerated in a relativistic shock, where electrons are only accelerated when they cross the shock front and are scattered back to the other side. Therefore, the $\gamma$-ray radiation from a significant fraction of long duration GRBs that have low energy spectral index larger than $-0.1$ cannot be accounted for by this mechanism.

Even $\alpha \approx -0.1$ faces severe difficulties. The large radius for generation of $\gamma$-rays is in conflict with the short variability time ($\lesssim 0.1$ s) of prompt GRB light curve. Moreover, the energy in the magnetic field must be extremely small, $\epsilon_{B} \sim 10^{-6} - 10^{-4}$, and $\gamma_{i} \gtrsim 10^{6}$ for the mechanism to be able to able to harden the spectral slope from $\alpha = -0.5$ to $\sim -0.1$ (Table 1).

It is unlikely that the energy fraction in the magnetic field will be so small ($\epsilon_{B} < 10^{-4}$) in internal shocks. If the central engine of GRBs is powered by accretion onto a black hole, we expect $\epsilon_{B} \sim 1\%$ as magnetic fields of such a strength are likely produced in the accretion disk by the Balbus-Hawley mechanism (Hawley, Gammie & Balbus 1996); for a magnetar based central engine this small $\epsilon_{B}$ is even more surprising.

For the typical thermal LF of electrons to be large, $\gamma_{i} \gtrsim 10^{9}$, in internal shocks where shells collide with a relative LF of a few to 10, it is required that approximately only 1 in $\sim 10^{3}$ electrons are accelerated when they cross the shock-front but they receive $\sim 10\%$ of the total energy. This is in contradiction with the numerical PIC simulations of Sironi & Spitkovsky (2011). Moreover, the $\sim 99.9\%$ of electrons which are not accelerated have a thermal LF of a few thousand due to their interaction with protons (Sironi & Spitkovsky 2011), and these electrons produce a significant IC bump in the spectrum at $\sim 100$ MeV which is not seen for any bursts. The SSC flux of these electrons at 100 MeV will be very large: about a factor of 10 larger than the observed flux.

All these difficulties suggest that the synchrotron radiation mechanism in internal shocks does not provide a self-consistent solution when the low-energy spectral index for GRBs is larger than about $-0.2$.

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