Estimating water distribution of the rotating sprinkler with pulsating pressure on sloping land

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Abstract: Pulsating pressure plays an important role in improving the poor irrigation quality and the uneven water distribution caused by the terrain slope. Water distribution is one of the key factors in design of the sprinkler irrigation system, however, it is difficult to measure in practice. To provide appropriate technical parameters for the design of sprinkler irrigation system with pulsating pressure on sloping land, a mathematical model was established according to the water conservation principle and finite element idea, and its accuracy was experimentally verified. The model was applied to study the effects of terrain slope, sprinkler arrangement, sprinkler spacing and average pulsating pressure on water distribution on sloping land. The results showed that the water distribution was more favorable under the gentle terrain slope, when slope decreased from 25% to 5%, the uniformity increased from 74.47% to 86.22%. Sprinklers arranged in equilateral triangle and with the spacing close to R₀ had the best water distribution uniformity, the uniformity coefficient (CU) of which was 11.43% and 8.75% higher than that in square and rectangular arrangement, respectively. The CU increased with the increase of the average pulsating pressure. However, the effect of increasing water pressure on promoting the uniformity of water distribution gradually decreases. Therefore, when using the Rainbird R5000 sprinkler on sloping land with pulsating pressure, it is suggested that the sprinkler irrigation systems should be arranged below the terrain slope of 20%, and operated at the average pulsating pressure of 300 kPa. The suitable sprinkler arrangement is the equilateral triangle, and with the spacing of 0.8R₀ to 1.0R₀.

Keywords: sprinkler irrigation, water distribution on sloping land, estimation model, pulsating pressure
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1 Introduction

Sloping land, as one of the most common topographies in the world, covers about one-fourth of the total farmland area in China. Owing to the poor ability of soil water retention and the application difficulty of the traditional surface irrigation technologies on sloping land, these crops are highly susceptible to drought, which led to low and unstable yields.

Sprinkler irrigation has become one of the most widely used irrigation methods on sloping land because of its good applicability to complex terrain and its ability to control the water quantity and irrigation period[1,2]. However, affected by the terrain slope, the water distribution uniformity of sprinkler irrigation on sloping land is worse than that on flat land. The water distribution curve on flat land can be regarded as a series of concentric circles formed by the sprinkler in the center, and water application rates at the same distance from the sprinkler are almost the same. However, on sloping land, the water distribution curve is similar to the shape of an “egg”, and water application rate of uphill is greater than that of downhill at the same distance from the sprinkler, which resulting in poor water application uniformity[3].

In the past decades, the adjustment of sprinkler installation angle and height was usually used to increase water distribution uniformity, but it increases the complexity and difficulty in the construction of sprinkler irrigation system[4,5]. Xiang et al.[6] has provided a device which can adjust throw radius of sprinkler to improve the water distribution uniformity on sloping land. However, this device was only suitable for terrain slope less than 10%. Furthermore, part-circle sprinklers are often used to match the required coverage area on sloping land, but it increases the water application rates in the wetted area and raises the cost of sprinkler system due to the decrease of sprinkler spacing. Water supply technology with pulsating pressure offers a new method for improving the uniformity of water distribution, and it has been applied in irrigation in recent years[7,10]. In sprinkler irrigation, the water distribution uniformity and energy distribution can be improved obviously by using pulsating pressure. The Christiansen uniformity coefficient (CU) value of water distribution for a single sprinkler at pulsating pressure was about 10% higher than that of constant pressure. When sprinklers were placed in rectangular arrangement, CU values of water distribution for pulsating pressure were on average 4.06% higher than that for constant pressure with different arrangements. The water distribution pattern is the basic data that was usually used for the design of sprinkler irrigation system[11,14]. However, limited by experimental conditions, the water distribution pattern on sloping land is too difficult to measure. Calculation of the water distribution pattern becomes an important issue to be resolved in urgent need for the design of sprinkler irrigation systems with pulsating pressure on sloping land.

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The objectives of the paper were to establish a mathematical model for estimating water distribution pattern of a rotating sprinkler with pulsating pressure on sloping land, and the effects of terrain slope, sprinkler arrangement, sprinkler spacing and pulsating water pressure on the water distribution pattern were studied. As a result, the appropriate technical parameter for sprinkler irrigation at pulsating pressure on sloping land was determined.

2 Model construction

2.1 Idea of model construction

Water quantity obtained at different times for any spot in wetted area is different on sloping land, because sprinkler operating pressure changes periodically with time using pulsating pressure to supply water. To calculate the water distribution of a single rotating sprinkler with pulsating pressure on sloping land conveniently, a finite element idea was used to establish the model. Firstly, the wetted area was divided into dozens of micro circular sectors. Secondly, the instantaneous water quantity for the sprinkler nozzle each passing through a micro circular sector on sloping land at pulsating pressure was evaluated by using the radial leg of the measured catch-can data on flat ground at constant pressure, based on the principle of water conservation along the radial direction. Because the duration for nozzle passing through the micro circular sector was very short, resulting in a very small change in sprinkler operating pressure, the pressure for sprinkler running on the micro circular sector could be considered as constant pressure. Thirdly, the water quantity of each micro circular sector was the sum of each rotating period during an irrigation event. Finally, the water distribution of a single rotating sprinkler with pulsating pressure on sloping land was obtained by the combination of water distribution for each micro circular sector.

2.2 Partition of micro circular sectors

The wetted area for a single sprinkler consists of two half ellipses on sloping land at constant pressure as shown in Figure 1. The sprinkler was located at the center of the ellipse. The semi major-axis and semi minor-axis of one-half ellipse is throw radius \( R \) on flat ground and uphill throw \( R_{up} \) on the uphill slope, respectively. The semi major-axis and semi minor-axis of the other half ellipse is downhill throw \( R_{down} \) on the downhill slope and throw radius \( R \) on flat ground, respectively. To calculate the water distribution of a single sprinkler with pulsating pressure on sloping land, the wetted area was divided into \( n \) micro circular sectors with the same angle according to the rotation of the nozzle. The \( n^{th} \) micro circular sectors for the \( n^{th} \) time on sloping land, and the corresponding throw radius on flat ground could be expressed as \( R'_n(m) \). The water quantity conversion coefficient \( \lambda_n(m) \) was introduced in this study and defined as the ratio of instantaneous throw radius on sloping land to that on flat ground, which can be expressed by:

\[
\lambda_n(m) = \frac{R'_n(m)}{R_n(m)} \tag{1}
\]

Because water falling within the \( n^{th} \) micro circular sectors on sloping land is equal to that on flat ground according to the principle of water conservation along the radial leg, water distribution for the \( n^{th} \) micro circular sectors on sloping land can be obtained by \( \lambda_n(m) \) and the measured water distribution for the \( n^{th} \) micro circular sectors on flat ground.

As shown in Figure 2, \( M_n \) is any spot in the \( n^{th} \) micro circular sectors on sloping land, and polar coordinates of \( M_n \) are \((\rho, \theta)\). The instantaneous water application rate at any spot \( M_n \) and instantaneous sprinkler operating pressure for the nozzle passing through the \( n^{th} \) micro circular sectors for the \( m^{th} \) time on sloping land could be expressed as \( P_n(m) \) and \( H_n(m) \), respectively. \( M'_n \) is the corresponding spot of \( M_n \) on flat ground, and its polar coordinates are \( \left(\frac{\rho}{\lambda_n(m)}, \theta\right) \). Then, the instantaneous water application rate at any spot \( M_n \) for the nozzle passing through the \( n^{th} \) micro circular sectors for the \( m^{th} \) time on sloping land can be estimated by Equation (2).

\[
P_n(m) = P'_n(m) \lambda_n(m) = \frac{P'_n(m)R'_n(m)}{R_n(m)} \tag{2}
\]

where, \( P'_n(m) \) is the instantaneous water application rate at the spot \( M'_n \) when the nozzle passes through the \( n^{th} \) micro circular sectors for the \( m^{th} \) time at sprinkler operating pressure of \( H_n(m) \) on flat ground.

2.3 Estimating water distribution for a micro circular sector on sloping land

Each instantaneous throw radius for the nozzle passing through the micro circular sectors on sloping land is different, because the sprinkler operates at pulsating pressure. Instantaneous throw radius could be expressed as \( R'_n(m) \) when the nozzle passes through

\[
\text{image}
\]
water distribution for micro circular sectors in an irrigation event with pulsating pressure on sloping land can be obtained. 

(1) Calculation of $R_o(m)$ and $R_v(m)$

$R_o(m)$ in equation (3) can be calculated as follows:\(^{[15]}\)

a. For uphill slope

$$R_o(m) = R_o(m)\cos\beta[1 - \tan\beta\cot(\theta + \beta)]$$

$$180^\circ < \theta < 360^\circ$$

(4)

b. For downhill slope

$$R_o(m) = R_v(m)\cos\beta[1 + \tan\beta\cot(\theta - \beta)]$$

$$0^\circ < \theta < 180^\circ$$

$$\beta = \frac{180}{\pi}\arctan(\sin\alpha)$$

(5)

(6)

where, $\beta$ is the projected angle (the angle between the projection of water jet trajectory on sloping land and that on level surface), ($\beta$); $\theta$ is the droplet landing angle on flat ground, ($\theta$); $i$ is the terrain slope; $\alpha$ is the sprinkler rotating angle, ($\alpha$), the plus and minus signs are for the term and specify $0^\circ \leq \alpha < 180^\circ$ and $180^\circ < \alpha < 360^\circ$ conditions, respectively.

Figure 3  Sketch of coordinate transformation from flat ground to sloping land

When sprinkler operating pressure is given, and $R_v(m)$ can be measured on flat ground. Additionally, $R_o(m)$ can be estimated by Equation (7)\(^{[16,17]}\)

$$R_v(m) = 4\mu^2H_v(m)\sin^2\gamma\left[\cot\gamma - 0.216\left(\frac{H_v(m)\rho}{d}\right)^{0.4}\right]$$

where, $\mu$ is the flow rate coefficient and $\mu = 0.86$-0.90\(^{[15]}\); $\gamma$ is the nozzle elevation angle, ($\gamma$); $d$ is the nozzle diameter, mm.

It is assumed that the function of pulsating pressure is trigonometric. The instantaneous sprinkler operating pressure $H_v(m)$ for the nozzle passing through the $n^{th}$ micro circular sectors for the $m^{th}$ time can be calculated using equation (8).

$$H_v(m) = H_v + A\sin\left(\frac{2\pi t_v(m)}{T}\right)$$

(8)

where, $H_v$ is the average value of pulsating pressure, kPa; $A$ is the amplitude of pulsating pressure (kPa); $T$ is the period of pulsating pressure, s; $t_v(m)$ is the sprinkler operating time before the nozzle passing through the $n^{th}$ micro circular sectors for the $m^{th}$ time, s.

$t_v(m)$ in Equation (8) can be calculated using Equation (9).

$$t_v(m) = \sum_{i=1}^{n} t_{si}(m)$$

(9)

$t_s(m)$ in Equation (9) can be calculated using Equation (10).

$$t_s(m) = \frac{T_{si}(m)}{N}$$

(10)

where, $T_{si}(m)$ is the sprinkler operating time per rotation at pressure of $H_v(m)$, s. The relationship between $T_{si}(m)$ and $H_v(m)$ was fitted by the measured data.

(2) Calculation of $P_s'(m)$

The relationship between instantaneous water application rate and average water application rate at the spot $M'_s$ for rotating sprinklers could be expressed as Equation (11).

$$P_s'(m) = \frac{P_s'(m)_{\text{average}}T_{si}(m)}{t_s'(m)}$$

(11)

where, $P_s'(m)_{\text{average}}$ is the average water application rate at the spot $M'_s$ under sprinkler operating pressure of $H_v(m)$ on flat ground.

$P_s'(m)_{\text{average}}$ could be calculated by the measured catch-can data on flat ground. Interpolations are performed within the experimentally-obtained catch-can profiles (the radial leg of the catch-cans data measured on flat ground at constant pressure setting for all tests) according to distance from the sprinkler and the sprinkler operating pressure. Thus, there are two interpolations: firstly, along the radial leg of the measured catch-can data; Secondly, between two adjacent pressures from the experimental data.

Linear interpolation is used to obtain the value of $P_s'(m)_{\text{average}}$ at the spot $M'_s$ from the experimental data. Because polar coordinates of $M'_s$ are $\left(\frac{\rho}{\lambda_i(m)\theta}\right)$, the distance from $M'_s$ to the sprinkler is $\frac{\rho}{\lambda_i(m)\theta}$. The average water application rate at the distance of $\frac{\rho}{\lambda_i(m)\theta}$ from the sprinkler and the operating pressure of $H_i$, $P_i$, can be calculated by Equation (12).

$$P_i = \frac{P_1' - P_2'}{d_1 - d_2}\lambda_i(m) + \frac{P_2'd_2 - P_1'd_1}{d_1 - d_2}$$

(12)

where, $P_1'$ is the average water application rate at the distance of $\frac{\rho}{\lambda_i(m)\theta}$ from the sprinkler and the operating pressure of $H_1$; $P_2'$ is the average water application rate at the distance of $\frac{\rho}{\lambda_i(m)\theta}$ from the sprinkler and the operating pressure of $H_2$; $d_1$ and $d_2$ can be known from the experimental data.

Similarly, the average water application rate at the distance of $\frac{\rho}{\lambda_i(m)\theta}$ from the sprinkler and the operating pressure of $H_2$ can be calculated by Equation (13). $H_2$ is also the designated pressure at the sprinkler in experiment which is known.

$$P_2' = \frac{P_1' - P_2'}{d_1 - d_2}\lambda_i(m) + \frac{P_2'd_2 - P_1'd_1}{d_1 - d_2}$$

(13)

where, $P_2'$ is the average water application rate at the distance of $\frac{\rho}{\lambda_i(m)\theta}$ from the sprinkler and the operating pressure of $H_2$; $P_1'$ is the average water application rate at the distance of $\frac{\rho}{\lambda_i(m)\theta}$ from the sprinkler and the operating pressure of $H_2$; $P_2'$ is the average water application rate at the distance of $d_1$ from the sprinkler and the operating pressure of $H_1$; $P_2'd_2 - P_1'd_1$ can be known from the experimental data.

Suppose that $H_1 \leq H_v(m) \leq H_2$, and combined with Equations (12) and (13), then the average water application rate at the
distance of \( \frac{\rho}{\lambda_n(m)} \) from the sprinkler and the operating pressure of \( H_m(m) \) for the spot \( M'_n \) can be calculated by equation (14).

\[
P'_n(m)_{\text{average}} = \frac{P'_{n1} - P'_{n2} H_1}{H_1 - H_2} + \frac{P'_n H_1 - P'_n H_2}{H_1 - H_2} \tag{14}
\]

Substituting Equation (14) into Equation (11), the instantaneous water application rate at the spot \( M'_n \) under sprinkler operating pressure of \( H_m(m) \) on flat ground can be calculated using Equation (15).

\[
P'_n(m) = \left( \frac{\rho}{\lambda_n(m)} - \frac{P'_{n2} - P'_{n1}}{H_1 - H_2} H_m(m) + \frac{P'_n H_1 - P'_n H_2}{H_1 - H_2} \right) \left( T'_n(m) - t'_n(m) \right) \tag{15}
\]

2.4 Estimating water distribution on sloping land

The water distribution for a single rotating sprinkler on sloping land can be obtained by the combination of each micro circular sector in Section 2.3. Additionally, when multiple sprinklers are used on sloping land, the water distribution can be regarded as the superposition of the water distribution of each sprinkler.

3 Model validations

3.1 Experimental design

To verify the model of water distribution for rotating sprinkler with pulsating pressure on a sloping land, an indoor experiment on water distribution of a single rotating sprinkler was conducted at the Irrigation Hydraulics Laboratory of Northwest A&F University, Yangling, China. Figure 4 shows the experimental setup, and Figure 5 shows the relationship among testing apparatus. The testing apparatus consisted of an automatic pressure control system, a stainless steel water tank, a sprinkler, a pressure sensor, height adjustable brackets, steel channels, and catch-cans.

1. Catch-can 2. Stainless steel channel 3. Variable-frequency control cabinet 4. Height adjustable bracket 5. Retaining plastic 6. Pressure sensor 7. Sprinkler

Figure 4 Experimental setup for the sprinkler water distribution on sloping land

Figure 5 Schematic diagram of relationship among testing apparatus

Pulsating pressure was controlled by an automatic pressure control system that consisted of a programmable logic controller (PLC), variable-frequency drive (VFD), and a centrifugal pump with an electric motor. Firstly, the program used for producing pulsating pressure was uploaded to the PLC to control the VFD, which modified the pump motor speed. Parameters of pulsating pressure (such as the function types, maximum pressure, minimum pressure, and function period) can be set in the program. Secondly, the electric motor speed of the centrifugal pump was adjusted by the VFD to produce the expected pulsating pressure. The pressure provided in this study was presented in Figure 6.

Figure 6 Pulsating pressure versus time

The Rainbird R5000 sprinkler was selected for the tests. The diameter of nozzle is 3.0 mm, the jet angle is 10°, and the manufacturer’s recommended operating pressure ranges from 170 to 450 kPa. The pressure transducer was a Xi’an Xinmin model CYB in a range between 0 to 300 kPa at an accuracy of ±0.1%. The pressure transducer wrapped in a plastic bag was installed at the nozzle inlet and connected to a data logger. Pressure was recorded at 5 s intervals by the data logger and the average pressure was calculated for each test.

The experimental terrain slope was artificially constructed by adjustable brackets and steel channels (with a length of 3.0 m and a width of 0.15 m). Steel channels were placed on the brackets, and the height of the brackets was calculated according to the terrain slope. Catch-cans (with a diameter of 10.6 cm and height of 14.0 cm) were placed in the steel channels and arranged at a spacing of 1.0 m×1.0 m. Water distributions of a single Rainbird R5000 sprinkler were measured at different pulsating pressures on the same slope. The slope was 10%, which is expressed as a ratio: increase in elevation over a horizontal distance. To keep the sprinkler operated at normal working pressure, average values of pulsating pressure were set to 250 kPa, 300 kPa, and 350 kPa according to the amplitude of pulsating pressure, which was 50 kPa. The function type of pulsating pressure was set trigonometric with a period of 18 s. There were totally three treatments, each treatment contained two tests, and the duration of each test is 1 h. In the first test, the sprinkler was installed on the top of the slope to record water distribution for downhill slope. In the second test, under exactly the same experimental conditions (the same pressure and height of sprinkler, etc.), the sprinkler was installed at the bottom of the slope to record water distribution for uphill slope. The water distribution for a single sprinkler on sloping land can be obtained by combining the water distribution measured on the uphill and downhill slopes. Each test was repeated three times, and the measured catch-can data were averaged.

Additionally, to provide the measured catch-can data for the model estimating water distribution at pulsating pressure on sloping land, water distributions of a single Rainbird R5000 sprinkler were also measured at constant pressures of 200 kPa, 250 kPa, 300 kPa, 350 kPa and 400 kPa on flat ground. A double radial leg of catch-cans was used such that at each distance from the sprinkler there were two cans, and the measured catch-can data from each pair were averaged. Catch-cans were placed at 0.106-m intervals in the radial direction, and were placed side-by-side. The distance between the center of the first
catch-can and the sprinkler location was 0.21 m. There were 150 catch-cans in the experiment for each radial leg, for a total of 300 catch-cans, and these were enough to extend beyond the wetted radius of tested sprinkler for all pressures. The weight of the water in each catch-can was measured using an electronic balance (Mettler Toledo, model XP10001S; 0.1 g accuracy) to calculate the water application rate.

3.2 Statistical analysis

Christiansen Uniformity Coefficient (CU)\(^{(16)}\) and standard deviation (SD) and were used to evaluate sprinkler water application uniformity with the measured catch-can data. Scheduling coefficient (SC) represents the ratio of area receiving low water application rate to the average water application rate applied through the wetted area. CU, SD and SC are expressed in equation form as:

\[
CU = 100 \left[ 1 - \frac{\sum_{i=1}^{N} P_i - \bar{P}}{\sum_{i=1}^{N} P_i} \right]
\]  

\[
SD = \sqrt{\frac{\sum_{i=1}^{N} (P_i - \bar{P})^2}{N}}
\]  

\[
SC = \frac{\bar{P}}{D_{iq}}
\]

where, \(P_i\) is the water application rate of individual catch-can, mm/h; \(\bar{P}\) is the average measured water application rate of all catch-cans, mm/h; \(D_{iq}\) is the mean value of the lowest one-quarter water application rate of all catch-cans, mm/h.

Catch3D is a mathematical model that can be used to analyze measured performance data of sprinklers in agriculture, including water application uniformity calculation\(^{(19)}\). It was used for some calculations of the results presented herein.

The accuracy of the model can be evaluated by mean bias error, root mean square error and the coefficient of determination, herein referred to as MBE, RMSE and \(R^2\)\(^{(20,21)}\). The closer the \(R^2\) to 1 and the lower the MBE and the RMSE, indicating the higher accuracy of the model is.

\[
MBE = \frac{\sum_{i=1}^{N} P_i - \bar{P}_w}{N}
\]  

\[
RMSE = \sqrt{\frac{1}{N} \sum_{i=1}^{N} (P_i - \bar{P}_w)^2}
\]  

\[
R^2 = 1 - \frac{\sum_{i=1}^{N} (P_i - \bar{P}_w)^2}{\sum_{i=1}^{N} (P_i - \bar{P}_w)^2}
\]

where, \(P_i\) is the simulated water application rate, mm/h; \(\bar{P}_w\) is the measured water application rate, mm/h; \(\bar{P}_w\) is the average of measured water application rate, mm/h.

3.3 Comparison of simulated and measured water distribution

The absolute errors between simulated and measured water distribution for a single sprinkler at different pulsating pressure are shown in Figure 7, the slope was 10%, and the period of pulsating pressure was 18 s. The origin point (0, 0) was the location of the sprinkler. As shown in Figure 7, the simulated water distribution differs little from the measured data in the whole wetted area. At different pulsating pressure, the absolute errors of water distribution in different parts of the wetted area were basically less than 0.45 mm/h, and the maximum error was 1.8 mm/h, which was mainly distributed at the edge of the uphill slope.

![Figure 7](image-url) Absolute errors between simulated and measured water distribution for a single sprinkler

Moreover, Table 1 shows the average value, CU and relative errors of water application rates in wetted area of a single sprinkler with pulsating pressure on sloping land. It can be seen that the value of average water application rate and CU between simulated and measured are very close to each other. The MBE, RMSE and \(R^2\) are -0.046, 0.384, and 0.82, respectively, indicating the difference between simulated and measured water application rates in wetted area varied slightly. It can be concluded that the model could be used to predict water distribution of a single sprinkler with pulsating pressure on sloping land.

| Average pressure /kPa | Amplitude /kPa | Average water application rate /mm·h\(^{-1}\) | CU\(\%\) | Average water application rate /mm·h\(^{-1}\) | CU\(\%\) |
|------------------------|---------------|--------------------------------|-----------|--------------------------------|-----------|
| 250                    | 50            | Simulated 2.40 Measured 2.49 Err\/% 3.61 | Simulated 75.59 Measured 74.55 Err\/% 1.40 |
| 300                    | 50            | Simulated 2.47 Measured 2.61 Err\/% 5.36 | Simulated 77.34 Measured 74.67 Err\/% 3.58 |
| 350                    | 50            | Simulated 2.70 Measured 2.74 Err\/% 1.46 | Simulated 78.00 Measured 78.34 Err\/% 0.43 |
Moreover, in practical, multiple sprinklers are usually used on sloping land. To test the applicability of the model to multiple sprinklers used on sloping land, Table 2 shows the average water application rate, CU and relative errors of water application rate under different arrangements with pulsating pressure on sloping land. The relative errors of average water application rate were all less than 3.1%, the relative errors of CU were all less than 3.9%, and the values of $R^2$ were all greater than 0.82, very close to 1. It proved that this model has a high accuracy in simulating the water distribution of multiple sprinklers with pulsating pressure on sloping land.

### Table 2: Average water application rate, CU and relative error of water application rates in wetted area of overlapped sprinkler

| Arrangement                  | Spacing | Simulated | Measured | Err/% | Simulated | Measured | Err/% | $R^2$ |
|------------------------------|---------|-----------|----------|-------|-----------|----------|-------|-------|
| Square                       | 1.0\(R_0\) | 7.44      | 7.22     | 3.05  | 74.08     | 73.49    | 0.80  | 0.82  |
| Rectangular                  | 1.0\(R_0\)+0.8\(R_0\) | 8.67      | 8.45     | 2.60  | 76.76     | 75.91    | 1.12  | 0.84  |
| Equilateral triangle         | 1.0\(R_0\) | 7.70      | 7.63     | 0.92  | 85.51     | 85.03    | 0.56  | 0.95  |
| Equilateral triangle         | 0.8\(R_0\) | 8.36      | 8.46     | 1.12  | 93.16     | 94.66    | 1.56  | 0.85  |
| Equilateral triangle         | 1.2\(R_0\) | 5.80      | 5.93     | 2.19  | 69.06     | 71.84    | 3.90  | 0.89  |

Note: $R_0$ is the throw radius for Rainbird R5000 sprinkler at the constant pressure of 300 kPa on flat ground.

4 **Model applications**

The above model was applied to study the effects of terrain slope, sprinkler arrangement, sprinkler spacing, and average pulsating pressure on water distribution for Rainbird R5000 sprinkler, which is important for sprinkler irrigation system design[22,23]. The function type, period and amplitude of pulsating pressure were set as trigonometric, 18 s and 50 kPa in the model, respectively. The factors and levels for the simulation are listed in Table 3.

### Table 3: Factors and levels for the simulation

| Slope/% | Sprinkler spacing | Sprinkler arrangement | Average pulsating pressure/kPa |
|---------|-------------------|-----------------------|--------------------------------|
| 5       | 0.6\(R_0\)        | Square                | 250                            |
| 10      | 0.8\(R_0\)        | Equilateral triangle  | 300                            |
| 15      | 1.0\(R_0\)        | Rectangular           | 350                            |
| 20      | 1.2\(R_0\)        | -                     | -                              |
| 25      | 1.4\(R_0\)        | -                     | -                              |

Note: $R_0$ is the throw radius for Rainbird R5000 sprinkler at the constant pressure of 300 kPa on flat ground.

4.1 **Effect of slope on water distribution**

Table 4 lists the throw radius and the average water application rate for a single sprinkler on different slopes, and the average pulsating pressure was 300 kPa. As shown in Table 4, with the increase of terrain slope, the throw radius of uphill decreases, while the throw radius of downhill increase and the ratio of downhill to uphill throw increase. This is mainly because, when the slope increases, the water spray to the uphill will easily be hindered by the terrain slopes, which resulted in a shorter throw radius. While, when the water sprays to the downhill, it is not easily influenced by terrain slopes and has a longer throw radius. The throw radius directly affects the wetted area of the sprinkler. Due to the changes of throw radius, the wetted area decreases on uphill while increases on downhill. In the meantime, quantities of water sprayed from the sprinkler to uphill and downhill were the same. Therefore, the average water application rate increases for uphill, while decreases for downhill. As a result, the ratio of water application rate for downhill to uphill decreases with the increase of the slope, which indicated that the difference of water distribution between uphill and downhill increase and the uniformity of water distribution in the whole wetted area became worse. When the slope changed from 5% to 25%, CU declined from 69.06% to 63.76%.

As for multiple sprinklers, Figure 8 shows the water distribution patterns of three sprinklers with equilateral triangle arrangement on different slopes, and the average pulsating pressure was 300 kPa and sprinkler spacing was 1.0\(R_0\). Locations of the three sprinklers in Figure 8 were coordinates (0, 0), (10, 0) and (5, 8.66). As shown in Figure 8, water distribution patterns are similar on five slopes, that is, most of water is concentrated on the middle and lower parts of the wetted area, and a small quantity of water fell on the top of the wetted area. The area with low water application rate increase with the increase of the slope, and the value of $SC$ are 1.90, 1.96, 2.14, 2.47 and 2.56 for slopes of 5%, 10%, 15%, 20% and 25%, respectively. Because the throw radius of uphill decrease with the increase of the slope, the water from two sprinklers located in coordinates (0, 0) and (10, 0) was difficult to reach the area on the top, and was more concentrated in the middle and edges of the wetted area, which makes the difference of water distribution in the wetted area increased. Therefore, the CU decreases with the increase of the slope. CU are 86.22%, 85.51%, 82.76%, 78.34% and 74.47% for slopes of 5%, 10%, 15%, 20% and 25%, respectively. The CU value for the slope of 20% was lower than 80%, which is the minimum CU recommended by many designers in practice[24]. Consequently, it is recommended that the Rainbird R5000 sprinkler should be operated below the slope of 20% in practice.

### Table 4: Throw radius and average water application rate for a single sprinkler on different slopes

| Slope/% | Uphill throw/m | Downhill throw/m | Ratio of downhill throw to uphill throw | Average water application rate for uphill slope/mm h$^{-1}$ | Average water application rate for downhill slope/mm h$^{-1}$ | Ratio of water application rate for downhill slope to uphill slope | CU/% |
|---------|----------------|------------------|------------------------------------------|------------------------------------------------------------|--------------------------------------------------------------|---------------------------------------------------------------|------|
| 5       | 8.87           | 10.77            | 1.21                                     | 2.33                                                       | 2.19                                                         | 0.94                                                          | 69.06|
| 10      | 8.91           | 11.54            | 1.30                                     | 2.39                                                       | 2.13                                                         | 0.89                                                          | 68.82|
| 15      | 8.90           | 11.89            | 1.34                                     | 2.44                                                       | 2.10                                                         | 0.86                                                          | 68.38|
| 20      | 8.68           | 12.95            | 1.49                                     | 2.47                                                       | 2.02                                                         | 0.82                                                          | 67.98|
| 25      | 8.10           | 14.32            | 1.77                                     | 2.56                                                       | 1.90                                                         | 0.74                                                          | 63.76|
4.2 Effect of sprinkler arrangement on water distribution

Water distribution patterns of sprinklers with three different arrangements by square (1.0R₀ × 1.0R₀), rectangular (0.8R₀ × 1.0R₀), and equilateral triangle (1.0R₀) are shown in Figure 9, and the average pulsating pressure was 300 kPa, the slope was 10%. For square arrangement, the four sprinklers were located at the four vertices. Most water cumulated in the middle and lower part of the wetted area, where the application rate was larger than 8 mm/h. The application rates at the top or edges of wetted area were lower than 6 mm/h and took up 36.4% of the whole wetted area. Compared to the square arrangement, the water application rate of rectangular arrangement is more concentrated. The ratio of areas with low application rate (lower than 6 mm/h) to the whole area decreased to 21.6%, the most obvious reduced area marked A in Figure 8a, while the left and right sides of the wetted area, which marked B and C, still existed in Figure 8b. This is because when sprinklers are arranged in rectangular, it shortens the distance between the sprinklers on the uphill and downhill. Water sprayed from the downhill sprinklers could easily reach the areas near the uphill sprinklers, and the overall water application rate was improved. For equilateral triangle arrangement, the ratio of area with low application rate to the whole area decreased obviously, only took up 8.3%. Areas with low application rate only show on the top of the triangle. The reason is that the equilateral triangle arrangement shortens the distance between sprinkler on uphill to area B and area C. It is concluded that sprinklers with equilateral triangle arrangement have the best water distribution uniformity, with the CU of 85.51%. It is followed by the rectangular arrangement, with the CU of 76.76%. Sprinklers arranged in square has the worst water distribution uniformity and its CU accounts for only 74.08%, which cannot meet the recommended CU values in practice. Therefore, it is recommended to use the equilateral triangle arrangement in the design of the sprinkler irrigation project with the pulsating pressure on sloping land.
4.3 Effect of sprinkler spacing on water distribution

The water application rate and its standard deviation under different sprinkler spacings are shown in Table 5. The slope was 10%, and the average pulsating pressure was 300 kPa. As sprinkler spacing increases, the maximum and minimum values of water application rate in the wetted area decrease gradually, and the minimum value decreases most obviously, which led to the range of the water application rate increases.

Table 5 Water application rate under different sprinkler spacing

| Sprinkler spacing | Min to max/mm·h⁻¹ | Average/mm·h⁻¹ | SD   |
|-------------------|--------------------|----------------|------|
| 0.6R₀             | 7.98-14.13         | 11.29          | 1.752|
| 0.8R₀             | 7.63-10.17         | 8.36           | 0.711|
| 1.0R₀             | 3.29-9.90          | 7.70           | 1.608|
| 1.2R₀             | 2.13-9.70          | 5.80           | 2.190|
| 1.4R₀             | 2.13-8.87          | 4.99           | 2.306|

Analysis of the proportion of water application rate in different intervals with different sprinkler spacings is shown in Figure 10. When the sprinkler spacing is less than or equal to R₀, the water application rate shows a high concentration. Values of water application rate are mainly distributed between average±2 mm/h. When the sprinkler spacings are 0.6R₀, 0.8R₀, and 1.0R₀, the proportions of water application rate between average±2 mm/h are 80.6%, 96.3%, and 86.4%, respectively. When the sprinkler spacing is 1.2R₀, the number of low water application rates increase obviously, and the proportions of water application rate near average value reduced to 57.8%. When sprinkler spacing increases to 1.4R₀, the proportion of water application rates between 2.0-3.0 mm/h, which deviates far away from the average value, reaches more than 40%. It can also be seen in Table 5 that the value of standard deviation of the water application rates in wetted area increase sharply, which indicated that when the sprinkler spacing is greater than R₀, the dispersion degree of water application rates increase, and the uniformity of water distribution become worse. The CU values in five different spacings (0.6R₀, 0.8R₀, 1.0R₀, 1.2R₀, and 1.4R₀) are 83.48%, 93.16%, 85.51%, 69.06% and 61.90%, respectively. This is because when sprinkler spacing was larger than sprinkler throw radius, water sprayed from sprinkler can hardly reach the area beyond the sprinkler throw radius, which lowers the water application rate in these areas, increases the heterogeneity of water distribution and decreases the CU value. When sprinkler spaced greater than 1.2R₀, it cannot meet the requirements of sprinkler irrigation design. In conclusion, for the Rainbird R5000 series sprinkler, the sprinkler spacing should be set between 0.8R₀ and 1.0R₀ when using pulsating pressure on sloping land.

4.4 Effect of pressure on water distribution

Figure 11 shows the water distributions for sprinklers with equilateral triangle arrangement under different average pulsating pressures, the sprinkler spacing was 1.0R₀ and slope was 10%. Most water culminated within the middle part of the wetted area, while the water application rate in the top area is relatively low. As average pulsating pressure increases, the area on the top with low water application rate decreases gradually, the value of SC were 2.34, 1.96 and 1.89 under the three water pressures of 250 kPa, 300 kPa and 350 kPa, respectively. Additionally, except for the area with low water application rate, the water supplication rate in other areas increased, and the proportion of water application rates around the average value in the wetted area increased. Standard deviations were 1.516, 1.435, and 1.393, respectively, and values of CU were 81.75%, 85.51%, and 86.08%, respectively. However, as average pulsating pressure increases, the effect of average pulsating pressure on the uniformity of water distribution is weakened. When water pressure increases from 250 to 300 kPa, CU value increases by 3.76%. When water pressure increases from 300 to 350 kPa, CU value only increases 0.57%, less than a quarter of the former. This is because the relationship between throw radius and average pulsating pressure is fractional power, and the exponent is 0.5, which means the increase rate of sprinkler throw radius is slower than that of water pressure, the area with low water application rate increases slowly with the increase of water pressure, and the effect of increasing water pressure on promoting the uniformity of water distribution gradually decrease. Considering the uniformity of water distribution and operating costs, RainBird R5000 sprinkler should be operated under the average pulsating pressure of 300 kPa.
5 Conclusions

Based on the water conservation principles, and the finite element idea, a mathematical model of water distribution under pulsating water pressure on sloping land was established, and its accuracy was experimentally verified. The model could accurately and reliably simulate the water distribution on sloping land when using pulsating pressure.

The effects of terrain slope, sprinkler arrangement, spacing, and average pulsating pressure on water distribution of sloping land under pulsating pressure conditions were analyzed. The gentler the terrain slope the higher the water distribution uniformity, and when the slope is greater than 20%, the water distribution uniformity cannot meet the sprinkler quality requirements. Sprinklers arranged in equilateral triangle have the best water distribution uniformity, with the uniformity coefficient (CU) of 85.51%, followed by the rectangular arrangement and square arrangement, with CU of 76.76% and 74.08%, respectively. For any increase in sprinkler spacing, the water application rate in the wetted area become scattered, and the uniformity of water distribution got worse with a faster trend. The uniformity of water distribution was increased with the average pulsating pressure, however, the increased extent was limited.

Considering the operational cost of the sprinkler systems and the water distribution uniformity, when the Rainbird R5000 sprinkler is used on sloping land with pulsating pressure, it is suggested that the sprinkler irrigation system should be arranged below the slope of 20%, with an average working pressure of 300 kPa, and the equilateral triangle arrangement is more suitable with sprinkler spacing of 0.8R₀-1.0R₀.

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