Lattice QCD and fundamental parameters of the Standard Model

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Abstract. Our present theory for the elemental particles in nature, the Standard Model, consists of 6 leptons and 6 quarks, plus the 4 bosons which mediate the electromagnetic, weak, and strong forces. The theory has several free parameters which must be constrained by experiment before it is entirely predictive. In Nature quarks never appear alone; only bound states of strongly coupled valence quarks (and/or anti-quarks) are detected. Consequently, the parameters governing quark flavor mixing are difficult to constrain by experiment, which measures properties of the bound states. Numerical simulations are needed to connect the theory of how quarks and gluons interact, quantum chromodynamics (formulated on a spacetime lattice), to the physically observed properties. Recent theory innovations and computer developments have allowed us finally to do lattice QCD simulations with realistic parameters. This poster describes the exciting progress using lattice QCD simulations to determine fundamental parameters of the Standard Model.

1. The Standard Model and quark flavor mixing
In the Standard Model of particle physics the 6 quarks (Eqn. 1) change their flavor by emitting or absorbing a $W$ boson, the carrier of the Weak Force. We can test the Standard Model and look for physics beyond the Standard Model by determining the parameters governing flavor-changing interactions as precisely and thoroughly as possible.

\[
\begin{align*}
\text{quarks} : & \begin{pmatrix} u \\ d \\ c \\ s \\ t \\ b \end{pmatrix} \\
\text{leptons} : & \begin{pmatrix} e \\ \nu_e \\ \mu \\ \nu_\mu \\ \tau \\ \nu_\tau \end{pmatrix} \\
\text{bosons} : & (\gamma, g, W, Z)
\end{align*}
\]

Isolated quarks are never seen in experiments; they are confined to exist in bound states generically called hadrons. Although we want to study transitions like the quark decay $b \rightarrow u$, experiments see hadron decays, like a $B$ meson decaying into a $\pi$ meson (Fig. 1).

2. Lattice QCD
Lattice QCD simulations use Monte Carlo methods to numerically create hundreds of representative configurations of the glue field. The computational bottleneck is the inversion of

1 for the HPQCD Collaboration: I. Allison, C. T. H. Davies, A. Gray, E. Gulez, G. P. Lepage, Q. Mason, M. Nobes, J. Shigemitsu, H. Trottier

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Figure 1. To explore the Standard Model, we must understand the weak decays of individual quarks, like an anti-bottom quark decaying into an anti-up quark plus positron and neutrino (top). In experiments, however, we never see a quark alone, but only in a bound state; for example, experiments would measure the decay rate for a $B^0$ meson decaying to a $\pi^-$ meson plus positron and neutrino (bottom).

Figure 2. Cartoon of $B^0 \rightarrow \pi^- e^+ \nu_e$ on the lattice. The anti-$b$ quark (thick cyan) propagates through a sea of glue and virtual quark–anti-quark pairs, respectively depicted by colored dots and circular loops. At some point the anti-$b$ quark turns into an anti-$u$ quark (thin cyan), emitting a $W$ boson which later decays to a positron and neutrino. The $d$ quark (red) pairs with the $\bar{b}$ to constitute a $B^0$ meson, and later with the $\bar{u}$ to be a $\pi^-$. It is a spectator to the weak decay, propagating through and interacting with the colorful sea.

a large, nearly singular matrix, a step which is necessary to include effects of virtual quark–anti-quark pairs. Only recently have methods been developed which allow us to include these effects realistically. The MILC Collaboration has utilized appreciable resources to generate state-of-the-art configurations which have been made public to the scientific community [1]. All of the results presented here use these configurations to compute a wide variety of quantities.

Since the quarks that make up the hadrons are so strongly coupled, hadrons are nonperturbative bound states. It is insufficient to organize a weak coupling perturbation expansion, in which the dominant process is the exchange of 1 gluon between quarks while a 2 gluon-exchange process is subdominant. Since the coupling strength is not small, many gluon-exchange processes are just as important. A more appropriate picture is one of quarks propagating through a sea of glue (Fig. 2). Lattice QCD solves the problem nonperturbatively by numerically simulating important configurations of the glue field.

3. Testing lattice QCD (and getting it right)
Now that effects of virtual quark–anti-quark pairs can be included in lattice QCD simulations, a fair comparison can be made between theory and experiment. Figure 3 shows the before-and-after status of lattice results for a variety of quantities. The left plot shows theory divided by experiment when virtual quark effects are ignored, or “quenched,” and the right panel shows the same quantities when virtual up, down, and strange quarks are included. (Virtual charm, bottom, and top quark effects are negligible.) The right panel achieves for the first time reliable agreement of lattice QCD and experiment within the small errors shown.

Observation of a meson made up of a bottom and a charm quark, the $B_c$ meson, has been made very recently. Lattice QCD predicted the mass of the $B_c$ meson very precisely (within 0.3%)
before the mass was measured experimentally [3]. Now the CDF experiment has confirmed the lattice prediction [4]. The ability of lattice calculations to predict values for observables before experimental measurements adds a great deal of weight to those calculations that are needed because experiment cannot make the measurement.

Another lattice QCD prediction is the shape of the $f_+$ form factor describing the weak decay of a $D$ meson to a $K$ meson plus lepton and neutrino. The Fermilab/MILC collaboration predicted the momentum dependence of $f_+$ using lattice QCD [5], which agrees with the subsequent experimental measurement by the FOCUS collaboration [6].
Figure 6. Constraints on the Standard Model parameters $\bar{\rho}$ and $\bar{\eta}$. The upper left plot shows status circa 2001; the upper right plot demonstrates the dramatic effect that precise Lattice QCD results will have; the lower left plot shows what better experiments can achieve alone; the lower right plot shows the combined result of precision experiment and theory.

4. QCD coupling and quark masses

The determination of the strength of the QCD coupling $\alpha_s$ from lattice QCD $\alpha_s(M_Z, \overline{\text{MS}}) = 0.1170(12)$ is more precise than the accepted world average given by the Particle Data Group $\alpha_s(M_Z, \overline{\text{MS}}) = 0.1187(20)$, and agrees well (Fig. 4). The quark masses have also been computed from first principles by lattice QCD (Fig. 5).

5. Lattice QCD inputs to understanding flavor changing

Figure 6 shows constraints from several experimental measurements on 2 Standard Model parameters, $\bar{\rho}$ and $\bar{\eta}$. If the allowed region disappears due to conflicting constraints, this would signal new physics, beyond the Standard Model. Lattice QCD calculations are necessary to extract the flavor changing properties of quarks from experimental measurements of $B$ meson decays and oscillations. Improved lattice calculations are vital to making full use of the experimental data being taken at the $B$ factories.

The decay $B \to \pi \ell \nu$ (Fig. 1) occurs infrequently compared to other $B$ decays, so it is difficult to measure experimentally. Lattice QCD is needed to determine the shape and absolute normalization of the form factors $f_+ \text{ and } f_0$. The integral of $f_+$ over the momentum transferred to the lepton and neutrino then can be combined with the experimentally measured $B \to \pi \ell \nu$ decay rate and $B^0$ lifetime to determine the Standard Model parameter $|V_{ub}|$. Figure 7 shows preliminary lattice calculations of these form factors by us and our Fermilab/MILC colleagues.

The leptonic decay constants of the $B_d$ and $B_s$ mesons must be determined from lattice QCD. The prospects for determining these experimentally are dim: the $B_d$ meson almost never decays...
Figure 7. (From [8, 9].) $B \rightarrow \pi\ell\nu$ form factors as functions of momentum transferred to the leptons. Determination of the form factors is necessary to extract the Standard Model parameter $|V_{ub}|$ from experimental measurements of the $B \rightarrow \pi\ell\nu$ decay rate.

into a lepton and neutrino without also emitting another hadron, and the leptonic decay of the $B_s$ is forbidden at leading order in the Standard Model. Nevertheless, these quantities enter into the theory describing the quantum mechanical oscillations between $B^0$ and $\bar{B}^0$ mesons. As lattice reduces the uncertainties in $f_{B_d}$ and $f_{B_s}$, the constraint on Standard Model parameters $\rho$ and $\eta$ from $B^0 - \bar{B}^0$ oscillations can be tightened (see green bands in Fig. 6). This is an especially interesting constraint, since oscillations are processes which should be affected indirectly by any new physics particles. Figure 8 shows our progress in determining the light quark mass dependence of the ratio $f_{B_s}/f_{B_d}$.

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