Analysis and design of generalized class-E rectifier

Asiya 1, 2a), Tatsuki Osato 1, Xiuqin Wei 3, Kien Nguyen 1, and Hiroo Sekiya 1b)  

1 Graduate School of Advanced Integration Science, Chiba University  
1-33 Yayoi-cho, Inage-ku Chiba 263-8522, Japan  
2 School of Physics and Electronic Engineering, Xinjiang Normal University  
Urumqi, Xinjiang, China  
3 Department of electrical Engineering, Chiba Institute of Technology University  
2-17-1 Tsudanuma, Narashino Chiba 275-0016, Japan  

a) asiya@chiba-u.jp  b) sekiya@faculty.chiba-u.jp

Received July 20, 2019; Revised November 18, 2019; Published April 1, 2020

Abstract: This paper presents a semi-analytical expression of the generalized class-E rectifier. Effect of low output-filter inductance is included in the theoretical formula presented in this paper. The harmonic components are considered in the current flowing through the output-filter inductance. For obtaining the theoretical waveforms, the coefficients of each frequency component are obtained by numerical calculations. By using the semi-analytical expression, characteristics of the class-E rectifier can be comprehended in a theoretical manner. By varying the output-filter inductance, it is possible to improve the rectifier characteristics, such as larger power output capability and zero input reactance, compared with the traditional class-E rectifiers. This paper also presents two examples of designs and implementations of resonant converters with the class-E rectifier. The validity of the semi-analytical expression and design strategies were confirmed from the quantitative agreements with experimental and PSpice simulation results.

Key Words: generalized class-E rectifier, power output capability, input reactance, resonant converters

1. Introduction

Resonant rectifiers have important roles in many applications such as wireless power transfer (WPT) systems [1–7], rectifier part of the resonant DC-DC converters [8–16], and energy harvesting circuits [17]. The resonant rectifiers are suitable for high-frequency and high-efficiency operations due to the soft switching [1–15, 17]. For designs of the resonant rectifier, it is important and helpful to comprehend typical characteristics, for example, AC-to-DC transfer function, peak voltage across the rectifier diode, and power output capability. These characteristics give criteria for component selections, cost reduction, and proper rectifier topology selections. The input resistance and reactance of the rectifier are also important and useful for designs of resonant DC-DC converters and WPT.
systems, in particular. This is because the input reactance of the rectifier affects the resonant inverter operations significantly in these systems [18–20].

The class-E rectifier [1, 21] is one of the typical resonant rectifiers, which achieves high power-conversion efficiency at high frequencies. Because the class-E rectifier has a single rectifier diode in the circuit topology, conduction loss on the diode can be reduced compared with bridge-type rectifiers such as the class-D rectifier [22, 23]. The \( L - C - R \) low-pass filter is included in the class-E rectifier, which works as the output filter to extract the direct component of the diode voltage. In the traditional class-E rectifier, the infinite output-filter inductance is assumed for ignoring the ripple of the output-filter inductance current [1–15, 17, 18]. In this case, the design degree of freedom is small to achieve rectifier-performance improvement.

For example, the traditional class-E rectifier always has capacitive input reactance and it is impossible for the input reactance to be zero [1–15, 17, 18]. Recently, several papers reported that the design of the class-E rectifier with low output-filter inductance. Even though the harmonic components are included in the output-filter inductance current, the performance improvement, such as pure resistive input impedance, can be obtained by considering low output-filter inductance [10, 24, 25]. Additionally, by enabling to use the output-filter inductance as a design parameter, a designer has one more degree of freedom for the class-E rectifier designs. The similar discussion with low input-filter inductance is also a hot topic in the class-E inverter [24, 26]. There is, however, a lack of detailed investigation of the class-E rectifier with low output-filter inductance.

This paper presents a semi-analytical expression of the generalized class-E rectifier, taking into account low output-filter inductance. The harmonic components are included in the current flowing through the output-filter inductance. For obtaining the theoretical waveforms, the coefficients of each frequency component are obtained by numerical calculations. By varying the output-filter inductance, it is possible to obtain improved characteristics, which are larger power output capability and zero input reactance, compared with the traditional class-E rectifier. This paper also presents two examples of designs and implementations of resonant converters with the class-E rectifier. The validity of the semi-analytical expression was confirmed from the quantitative agreements with experimental and PSpice simulation results.

2. Class-E rectifier

2.1 Circuit topology and the operation principle

Figure 1 shows a circuit-topology of the class-E current-driven rectifier, which consists of AC current source \( i_I = I_m \sin(\omega t + \phi) \), rectifier diode \( D \), shunt capacitance \( C_D \), and low-pass filter \( L_f - C_f - R \). Figure 2 shows example waveforms of the class-E resonant rectifier. When the difference of the input current \( i_I \) and the filter-inductance current \( i_L \) is positive, the diode is in OFF-state. During the rectifier-diode OFF-state, no current flows through the diode. The current, however, flows through the shunt capacitance, which generates the diode voltage. Because the input current is AC current, the voltage across the diode is a pulse-type shape waveform as shown in Fig. 2. When the diode voltage reaches zero, the diode turns ON. During the diode ON-state, the difference of the filter inductance current and the input current flows through the diode and the diode voltage and the current flowing...
through the shunt capacitance are almost zero. When the difference of the output-filter inductance current and the input current is zero, the diode turns OFF again. At the diode turn-OFF instant, the voltage and the current at the shunt capacitance are zero. Therefore, zero voltage and low \( \frac{dv_D}{d(\omega t)} \) turn-OFF switching is achieved, which is called the class-E zero-voltage and zero-derivative voltage switching (ZVS/ZDS). Because of the class-E ZVS/ZDS, the class-E rectifier operates high power-conversion efficiency at high frequencies. Additionally, low switching noise is achieved in the class-E rectifier. The class-E rectifier always satisfies the class-E ZVS/ZDS conditions. Therefore, it is unnecessary to consider the soft switching at the rectifier design. It is, on the other hand, necessary to satisfy the given rectifier characteristics, for example, duty ratio and output voltage.

2.2 Output low-pass filter characteristics

In the class-E rectifier, the direct component is extracted from the diode voltage by applying the \( L_f - C_f - R \) low-pass filter, which is the same as the output voltage. In most previous designs of the class-E rectifier, the output-filter inductance is assumed to be sufficiently large so that the ripple of filter-inductance current \( i_L \) can be ignored as shown in Fig 2(a) [1–15, 17, 18]. The characteristic of the class-E rectifier is, however, limited with high output-filter inductance. For example, the class-E rectifier with high \( L_f \) always has the input capacitance [1–15, 17, 18]. At the design of the resonant converter and the WPT system, no input reactance of the rectifier makes the design of the system simple and easy.

The turn-ON and turn-OFF timings of the rectifier diode are determined by the voltage and the current at the diode, which cannot be forced from the outside like the MOSFET driving. Additionally, the output voltage depends on the diode voltage. When the output-filter inductance is assumed to be sufficiently high, we have just one parameter, which is shunt capacitance, for adjusting the duty ratio and the output voltage. The design degree of freedom is insufficient.

Recently, some papers pointed out that the class-E rectifier obtains the additional performances, such as resistive input impedance, by considering low output-filter inductance [10, 24, 25]. When the output-filter inductance becomes low, the harmonic components are included in the output-filter current as shown in Fig 2(b). By using the output-filter inductance as a design parameter, the rectifier design is more flexible. This is because a designer has one more degree of freedom for rectifier designs.
The similar discussion is also a hot topic in the class-E inverter \([24, 26]\). There is, however, no detailed investigation of the class-E rectifier with low output-filter inductance.

3. Circuit analysis

3.1 Assumption

For the rectifier operation analysis, the following assumptions are given.

1. All the passive components work as a linear element.
2. All the components have no parasitic elements.
3. The diode works ideally. Therefore, zero-switching time, zero on-resistance, and infinite off-resistance are assumed.
4. The diode is in OFF-state for \(0 < \omega t < 2\pi (1 - D)\), and ON-state for \(2\pi (1 - D) < \omega t < 2\pi\), where \(D\) is ON-duty ratio. The duty ratio is determined by the diode voltage and current.
5. The AC current source generates a pure sinusoidal input current, namely

\[
i_I = I_m \sin(\omega t + \phi) = a_1 \cos(\omega t) + b_1 \sin(\omega t),
\]

where

\[
I_m = \sqrt{a_1^2 + b_1^2},
\]

and

\[
\phi = \tan^{-1} \frac{a_1}{b_1},
\]

are the amplitude and the phase-shift of the input current, respectively.

3.2 Diode voltage and current

The current flowing through the output-filter inductance is expressed by

\[
i_L = a_{20} + \sum_{k=1}^{N} \left[ a_{2k} \cos(k\omega t) + b_{2k} \sin(k\omega t) \right],
\]

where \(N\) is the number of harmonic components to be considered. During the diode-OFF state, the currents flowing through the diode and the shunt capacitance are

\[
i_D = 0, \quad \text{for} \quad 0 < \omega t < 2\pi (1 - D)
\]

and

\[
i_{CD} = i_L - i_I
\]

\[
= a_{20} - a_1 \cos(\omega t) - b_1 \sin(\omega t) + \sum_{k=1}^{N} \left[ a_{2k} \cos(k\omega t) + b_{2k} \sin(k\omega t) \right], \quad \text{for} \quad 0 < \omega t < 2\pi (1 - D)
\]

respectively. The diode voltage \(v_D\) is produced by the current \(i_{CD}\), namely

\[
v_D = \frac{1}{\omega C_D} \int_0^\omega i_{CD} d(\omega t)
\]

\[
= \frac{1}{\omega C_D} \left\{ a_{20}\omega t - a_1 \sin(\omega t) + b_1 \cos(\omega t) - b_1 + \sum_{k=1}^{N} \frac{1}{k} \left[ a_{2k} \sin(k\omega t) - b_{2k} \cos(k\omega t) + b_{2k} \right] \right\}.
\]

\[
\text{for} \quad 0 < \omega t < 2\pi (1 - D)
\]

When the diode is in ON-state, the currents flowing through the shunt capacitance and the diode are expressed as
\[i_{C_D} = 0, \quad \text{for } 2\pi(1 - D) < \omega t \leq 2\pi \]  
(8)

and

\[i_D = i_L - i_o = a_{20} - a_1 \cos(\omega t) - b_1 \sin(\omega t) + \sum_{k=1}^{N} [a_{2k} \cos(k\omega t) + b_{2k} \sin(k\omega t)], \quad \text{for } 2\pi(1 - D) < \omega t \leq 2\pi \]  
(9)

respectively. From the assumption 3, the diode voltage during diode ON-state is

\[v_D = 0. \quad \text{for } 2\pi(1 - D) < \omega t \leq 2\pi \]  
(10)

From the above, the diode voltage and current waveform can be summarized as

\[v_D = \begin{cases} 
\frac{1}{\omega C_D} \left\{ a_{20} \omega t - a_1 \sin(\omega t) + b_1 \cos(\omega t) - b_1 + \sum_{k=1}^{N} \frac{1}{k} [a_{2k} \sin(k\omega t) - b_{2k} \cos(k\omega t) + b_{2k}] \right\}, & \text{for } 0 < \omega t \leq 2\pi(1 - D) \\
0, & \text{for } 2\pi(1 - D) < \omega t \leq 2\pi 
\end{cases} \]  
(11)

\[i_D = \begin{cases} 
0, & \text{for } 0 < \omega t \leq 2\pi(1 - D) \\
a_{20} - a_1 \cos(\omega t) - b_1 \sin(\omega t) + \sum_{k=1}^{N} [a_{2k} \cos(k\omega t) + b_{2k} \sin(k\omega t)], & \text{for } 2\pi(1 - D) < \omega t \leq 2\pi 
\end{cases} \]  
(12)

By applying the Fourier expansion to Eq. (11), we have a formula of the diode voltage, which is defined in the range of 0 < \omega t \leq 2\pi as

\[v_D = c_0 + \sum_{k=1}^{N} [c_k \cos(k\omega t) + d_k \sin(k\omega t)]. \]  
(13)

The resulting equations of \(c_k\) and \(d_k\) are shown in the Appendix.

### 3.3 Output current and AC-to-DC transfer function

The current flowing through the output filter components have a relationship of

\[i_L = i_{C_f} + i_o, \]  
(14)

where

\[i_{C_f} = \omega C_f R \frac{di_o}{d(\omega t)}, \]  
(15)

is the current flowing through the output-filter capacitance and \(i_o\) is the output current. From Eqs. (4), (14) and (15), the output current of the rectifier can be expressed as

\[i_o = a_{20} + \sum_{k=1}^{N} \left[ a_{2k} - k\omega C_f R b_{2k} \frac{1}{1 + (k\omega C_f R)^2} \cos(k\omega t) + \frac{k\omega C_f R a_{2k} + b_{2k}}{1 + (k\omega C_f R)^2} \sin(k\omega t) \right]. \]  
(16)

Because the DC output voltage is the average value of the output voltage, we have

\[V_o = RI_o = \frac{R}{2\pi} \int_{0}^{2\pi} i_o d(\omega t) = Ra_{20}. \]  
(17)

From Eq. (17) and \(P_I = P_o = I_o V_o\), the AC-DC current transfer function is expressed as

\[M_{1R} = \frac{I_o}{I_{rms}} = \sqrt{\frac{2a_{20}}{a_1^2 + b_1^2}}, \]  
(18)

where \(I_{rms}\) is the root mean square value of the input current, which is \(I_{rms} = I_m/\sqrt{2}\) due to assumption 5.
3.4 Current-coefficient derivations

By considering to the closed circuit loop of \( D - L_f - R \) and Eq. (16), another expression of \( v_D \) can be obtained as

\[
v_D = -v_L - v_o = -v_L - Ri_o
\]

\[
= -Ra_{20} - \sum_{k=1}^{N} \left\{ \frac{Ra_{2k} - k\omega C_f R^2 b_{2k}}{1 + (k\omega C_f R)^2} + k\omega L_f b_{2k} \right\} \cos(k\omega t)
\]

\[
+ \left[ \frac{k\omega C_f R^2 a_{2k} + Rb_{2k}}{1 + (k\omega C_f R)^2} - k\omega L_f a_{2k} \right] \sin(k\omega t) \right\}. \tag{19}
\]

where \( v_L \) is the voltage across the output-filter inductance. From the coefficient comparisons between Eqs. (13) and (19), \( 2N + 1 \) algebraic equations are obtained, namely

\[
\begin{align*}
&c_0 + Ra_{20} = 0, \\
&c_k + \frac{Ra_{2k} - k\omega C_f R^2 b_{2k}}{1 + (k\omega C_f R)^2} + k\omega L_f b_{2k} = 0, \quad \text{for } k = 1, 2, \cdots N \\
&d_k + \frac{k\omega C_f R^2 a_{2k} + Rb_{2k}}{1 + (k\omega C_f R)^2} - k\omega L_f a_{2k} = 0. \quad \text{for } k = 1, 2, \cdots N 
\end{align*} \tag{20}
\]

When the diode voltage reaches zero, the diode turns ON. From Eq. (11), we have

\[
v_D(2\pi(1 - D)) = \frac{1}{\omega C_D} \left( a_{20} 2\pi(1 - D) + a_1 \sin(2\pi D) + b_1 \cos(2\pi D) - b_1 \right)
\]

\[
+ \sum_{k=1}^{N} \frac{1}{k} \left[ -a_{2k} \sin(2\pi kD) - b_{2k} \cos(2\pi kD) + b_{2k} \right] = 0. \tag{21}
\]

Also, the diode current reaches zero when the diode turns OFF. Therefore, from Eq. (12), we have

\[
i_D(2\pi) = a_{20} - a_1 + \sum_{k=1}^{N} a_{2k} = 0. \tag{22}
\]

From the above, \((2N + 3)\) equations are obtained in (20), (21), and (22). For deriving the analytical waveforms, it is necessary to determine \((2N + 4)\) unknown parameters, which are current coefficients \(a_{2k}, b_{2k}, a_{20}, a_1, b_1\), and duty ratio \(D\). Here, the amplitude of the input current \(I_m = \sqrt{a_1^2 + b_1^2}\) is added to the \((2N + 3)\) algebraic equations, and the amplitude value is given as a design specification. In this case, we have \((2N + 4)\) equations. The duty ratio \(D\) is in the sine and cosine functions in Eq. (21). In addition, \(c_k\) and \(d_k\) in Eq. (20) also include the unknown parameter \(D\) in the sine and cosine functions as given in Eqs. (A-1)–(A-3) of the Appendix. \(I_m = \sqrt{a_1^2 + b_1^2}\) is also a nonlinear equation for \(a_1\) and \(b_1\). Therefore, it is necessary to solve \((2N + 4)\) nonlinear simultaneous algebraic equations for obtaining the coefficients of the harmonic components and the duty ratio. By solving the equations, all the coefficients and the duty ratio can be fixed uniquely under the condition that the operating frequency and all the components value are given. In this paper, Newton’s method is adopted for solving the algebraic equations.

3.5 Power output capability

The reverse peak value of the diode voltage \(V_{DM}\) appears during the period \(0 < \omega t \leq 2\pi(1 - D)\) when the diode voltage satisfies

\[
\frac{dv_D}{d(\omega t)} = \frac{1}{\omega C_D} \left( a_{20} - a_1 \cos(\omega t) - b_1 \sin(\omega t) + \sum_{k=1}^{N} \left[ a_{2k} \cos(k\omega t) + b_{2k} \sin(k\omega t) \right] \right) = 0. \tag{23}
\]

Similarly, the peak value of the diode current \(I_{DM}\) appears during the period \(2\pi(1 - D) < \omega t \leq 2\pi\) when the diode current satisfies
$$i_t = I_m \sin (\omega t + \phi)$$

Fig. 3. Equivalent model with the input impedance of the rectifier.

$$\frac{d i_D}{d(\omega t)} = a_1 \sin(\omega t) - b_1 \cos(\omega t) + \sum_{k=1}^{N} k b_{2k} \cos(k \omega t) - a_{2k} \sin(k \omega t) = 0. \quad (24)$$

By solving Eqs. (23) and (24) for $\omega t$, the angular time when the reverse-peak-value of the diode voltage and the peak-value of the diode current can be obtained numerically. In Eq. (24), however, there is a possibility that there is no solution of $\omega t$ in the range of $2 \pi (1 - D) < \omega t \leq 2 \pi$. In this case, the peak value of the current appears at $\omega t = 2 \pi (1 - D)$. By substituting the obtained value of $\omega t$ to Eq. (12), the peak value of the diode current $I_{DM}$ can be obtained. The power output capability of the class-E rectifier is derived from

$$e_p = \frac{V_o I_o}{V_{DM} I_{DM}}. \quad (25)$$

3.6 Input impedance

Figure 3 shows an equivalent circuit of the class-E rectifier with the input impedance of the rectifier $Z_i = R_i + j X_i$. The input resistance and reactance can be obtained based on the fundamental component of the input voltage and input current. The input voltage of the rectifier is equal to the reverse value of the diode voltage. The fundamental component of $v_D$, which is expressed as $v_{D1}$ is regarded as the fundamental reverse input voltage for the impedance derivation. Therefore, we have

$$v_I = -v_{D1} = -c_1 \cos(\omega t) - d_1 \sin(\omega t) = v_{R_i} + v_{X_i}, \quad (26)$$

where

$$v_{R_i} = R_i I_m \sin(\omega t + \phi), \quad (27)$$

and

$$v_{X_i} = -X_i I_m \cos(\omega t + \phi), \quad (28)$$

are the voltage across the input resistance and reactance, respectively. From the Eqs. (1), (2) and (26), the input resistance and reactance are expressed as

$$R_i = \frac{a_1 c_1 + b_1 d_1}{a_1^2 + b_1^2}. \quad (29)$$
Fig. 5. Duty ratio of the diode as functions of (a) $\omega C_D R$ for fixed $\omega C_f R$ and (b) $\omega C_f R$ for fixed $\omega C_D R$.

![Graph showing duty ratio vs. $\omega C_D R$ and $\omega C_f R$](image)

Fig. 6. Ratio of the reverse peak value of the diode voltage to the output voltage $V_{DM}/V_o$ as functions of (a) $\omega C_D R$ for fixed $\omega C_f R$, (b) $D$ for fixed $\omega C_f R$, and (c) $\omega C_f R$ for fixed $\omega C_D R$.

![Graph showing ratio of reverse peak voltage vs. $\omega C_D R$, $D$, and $\omega C_f R$](image)

and

$$X_i = \frac{b_1 c_1 - a_1 d_1}{a_1^2 + b_1^2}, \quad (30)$$

respectively.

4. Characteristic investigation

This section investigates characteristics of the generalized class-E rectifier. In this section, the characteristics of power output capability, AC-to-DC current transfer function, and input impedance are shown for various output-filter inductance. Regardless of the output-filter inductance, the cut-off frequency of the low-pass filter is fixed for 0.1$f$, in which $f$ means the operating frequency of the circuit. Namely, large $C_f$ means low $L_f$.

When the number of the harmonic components $N$, which is proportional to the accuracy of the waveforms, increases, the computation cost also increases. Hence, there is a trade-off relationship between the waveform accuracy and the computation cost in selecting the value of $N$. Figure 4 shows the example waveforms of the current $i_L$ in Eq. (4) and the diode voltage $v_D$ in Eq. (11) for $\omega C_D R = 0.2, \omega C_f R = 100$, and $D = 0.6$ for fixed $N$. From Fig. 4 and the consideration with the computation cost, $N = 5$ is adopted for following characteristic investigations in this paper.

4.1 Relationship between duty ratio and shunt capacitance

Figure 5 shows the ON-duty ratio of the rectifier diode as functions of $\omega C_D R$ for fixed $\omega C_f R$, and $\omega C_f R$ for fixed $\omega C_D R$. As seen in Fig. 5(a), $D$ decreases as the increase in $\omega C_D R$. It is, however, seen from Fig. 5(b) that the duty ratio is insensitive to $\omega C_f R$ as well as the output-filter inductance. The output-filter inductance is sufficiently high to obtain low-ripple current when $\omega C_f R = 0.01$. We
confirmed that the characteristic curve for $\omega C_f R = 0.01$ is almost the same as those from the previous analysis for infinite filter inductance in [21].

4.2 Maximum diode voltage and current

Figure 6 shows the ratio of the reverse peak diode voltage to the output voltage $V_{DM}/V_o$ as functions of $\omega C_D R$ for fixed $\omega C_f R$, duty ratio for fixed $\omega C_f R$, and $\omega C_f R$ for fixed $\omega C_D R$. It is seen from Fig. 6(a) that $V_{DM}/V_o$ increases monotonically as the decrease in $\omega C_D R$. Namely, $V_{DM}/V_o$ increases as the increase in the duty ratio. High-reverse-peak value of the voltage across the diode limits the diode selectivity. In this sense, the range of $D < 0.6$ is useful, in which the reverse peak value of the diode voltage is less than four times of the output voltage. Additionally, the $V_{DM}/V_o$ is insensitive to $\omega C_f R$ for fixed duty ratio as shown in Fig. 6(b). The insensitivity of $V_{DM}/V_o$ to $\omega C_f R$ can be also confirmed from Fig. 6(c).

Figure 7 shows the ratio of the peak value of the diode current to the output current $I_{DM}/I_o$ as functions of $\omega C_D R$ for fixed $\omega C_f R$, (b) $D$ for fixed $\omega C_f R$, and (c) $\omega C_f R$ for fixed $\omega C_D R$. It is seen from Fig. 7(b) that $I_{DM}/I_o$ is insensitive to $\omega C_f R$ in the range of $D < 0.6$. In this range, $I_{DM}/I_o$ increases monotonically as the decrease in the duty ratio. The diode selection is also limited by the maximum current flowing through itself. Therefore, it can be stated from Fig. 7(b) that the range of $D > 0.2$ is useful, in which the peak value of the diode current is less than 10 times of the output current. The insensitivity of $I_{DM}/I_o$ to $\omega C_f R$ is comprehended from Fig. 7(c). From the discussions of Figs. 6 and 7, it can be stated that the diode voltage and current waveforms depend on the shunt capacitance strongly and insensitive to the output-filter inductance. Additionally, the characteristics of the class-E rectifier are discussed in the range of $0.2 < D < 0.6$ hereafter. It is seen from Fig. 5 that the range of $0.1 < \omega C_D R < 10$ corresponds to the range of $0.2 < D < 0.6$.

4.3 Power output capability

The power output capability expresses the cost performance of the diode. The increase in the power output capability leads the cost reduction. Figure 8 shows the power output capability as functions of $\omega C_D R$ for fixed $\omega C_f R$, duty ratio for fixed $\omega C_f R$, and $\omega C_f R$ for fixed $\omega C_D R$. It is seen from Fig. 8 that the maximum power output capability of the class-E rectifier with low output-filter inductance becomes large compared with sufficiently high output-filter inductance. This result is one of the major advantages of considering low output-filter inductance. It is seen from Fig. 8(a) and (b) that the maximum power output capability is obtained around $\omega C_D R = 0.4$, namely around $D = 0.55$ regardless of $\omega C_f R$. Figure 9(a) and (b) show $\omega C_f R$ for obtaining the maximum power output capability at a given $\omega C_D R$ and the duty ratio, respectively. Additionally, Fig. 9(c) shows $\omega C_D R$ for obtaining the maximum power output capability at a given $\omega C_f R$. From these graphs, it can be clarified that the largest power output capability $c_{p_{max}} = 0.109$ can be obtained when $\omega C_D R = 0.396$ and $\omega C_f R = 126.7$ with $D = 0.553$, while that with sufficiently high output-filter inductance is
Fig. 8. Power output capability as functions of (a) $\omega C_D R$ for fixed $\omega C_f R$, (b) $D$ for fixed $\omega C_f R$, and (c) $\omega C_f R$ for fixed $\omega C_D R$.

Fig. 9. The relationship of normalized shunt capacitance, duty ratio, and normalized output-filter capacitance for obtaining maximum output power capability $c_{p_{\text{max}}}$. (a) $\omega C_f R$ and $c_{p_{\text{max}}}$ at a given $\omega C_D R$. (b) $\omega C_f R$ and $c_{p_{\text{max}}}$ at a given $D$. (c) $\omega C_D R$ and $c_{p_{\text{max}}}$ at a given $\omega C_f R$.

Fig. 10. Plots of AC-to-DC current transfer functions as functions of (a) $\omega C_D R$ for fixed $\omega C_f R$, (b) $D$ for fixed $\omega C_f R$, and (c) $\omega C_f R$ for fixed $\omega C_D R$.

$c_p = 0.098$ in [21]. Namely, 11% power output capability increment can be obtained by considering the low output filter inductance.

4.4 AC-to-DC current transfer function

Figure 10 shows plots of the AC-to-DC current transfer function as functions of $\omega C_D R$ for fixed $\omega C_f R$, duty ratio for fixed $\omega C_f R$, and $\omega C_f R$ for fixed $\omega C_D R$. It is seen from Fig. 10(a) that $M_{IR}$ becomes small for large $\omega C_D R$. This is because the large reactive current flows through the class-E rectifier and the amplitude of the input current increases. This is a reason why the maximum diode current is large in Fig. 7. Therefore, $M_{IR}$ decreases in the range of $D < 0.4$ as shown in Fig. 10(b).
Fig. 11. Normalized input resistance as functions of (a) $\omega C_D R$ for fixed $\omega C_f R$, (b) $D$ for fixed $\omega C_f R$, and (c) $\omega C_f R$ for fixed $\omega C_D R$.

Fig. 12. Normalized input capacitance and inductance as functions of (a) $\omega C_D R$ for fixed $\omega C_f R$, (b) $D$ for fixed $\omega C_f R$, and (c) $\omega C_f R$ for fixed $\omega C_D R$.

is seen from Figs. 10(b) and (c) that the low output-filter inductance contributes to adjust the value of AC-to-DC current transfer function in the useful range.

4.5 Input resistance and input reactance

Figure 11 shows $\omega C_D R_i$ as functions of $\omega C_D R$ for fixed $\omega C_f R$, duty ratio for fixed $\omega C_f R$, and $\omega C_f R$ for fixed $\omega C_D R$. It is seen from Figs. 11(a) and (b) that the input resistance is always smaller than the load resistance in the class-E rectifier. The peak value of $\omega C_D R_i$ is high as the filter inductance decreases. The maximum value of $\omega C_D R_i$ for $\omega C_f R = 100$ is about four times as high as that for $\omega C_f R = 0.01$. Because the $\omega C_D R_i$ is sensitive to the duty ratio as shown in Fig. 11(b), it is possible to realize various input resistance by adjusting the shunt capacitance and output filter inductance.

Figure 12 shows normalized input reactance as functions of $\omega C_D R$ for fixed $\omega C_f R$, duty ratio for fixed $\omega C_f R$, and $\omega C_f R$ for fixed $\omega C_D R$. In Fig. 12, the input reactance is expressed as the input capacitance or input inductance as

$$C_i = \frac{a_1^2 + b_1^2}{\omega(b_1 c_1 - a_1 d_1)}, \quad X_i > 0 \quad (31)$$
It is seen from Fig. 12 that the class-E rectifier with sufficiently high output-filter inductance, e.g. $\omega C_f R = 0.01$, has always capacitive reactance. When the output-filter inductance becomes low, however, there is a point that the input capacitance is infinity and inductive reactance appears, which is a special characteristic due to low output-filter inductance. Considering the useful range of the duty ratio $0.43 < D < 0.6$, it is seen from Fig. 12(b) that both the inductive and capacitive reactance can be designed in the class-E rectifier by adjusting the output-filter inductance. It is also possible to design the resistive class-E rectifier, whose input reactance is zero. Figure 13 shows the plots of the normalized output-filter capacitance as functions of the normalized shunt capacitance and duty ratio for achieving the resistive class-E rectifier. It is seen from this figure that it is possible to design the resistive class-E rectifier in the wide range of duty ratio.

5. Experimental verifications

This section evaluates the validity of the semi-analytical expressions as well as the performance evaluations in the previous section. For the evaluations, two DC-DC resonant converters, including the class-E rectifier, were designed and implemented. One converter has the class-E rectifier with the largest power output capability and another with no input reactance.

5.1 Class-E\textsuperscript{2} DC-DC converter with the largest power output capability class-E rectifier

Figure 14 shows a circuit topology of the class-E\textsuperscript{2} DC-DC converter [9, 21]. For the design the converter, operating frequency is $f = 6.78$ MHz, input voltage $V_I = 20$ V, the duty ratio of the inverter switch $D_S = 0.5$, loaded quality factor of the inverter $Q = 10$, and load resistance $R = 20$ Ω were specified. From Fig. 9, the largest power output capability $c_p = 0.109$ can be obtained for $\omega C_f R = 0.396$ and $\omega C_f R = 126.7$. In the parameters, the input resistance and the input inductance are $R_i = 18.6$ Ω and $L_i = 334$ nH, respectively. By using the $R_i$ and $L_i$, it is possible to design the class-E inverter part following the design procedure shown in [14, 21]. Table I gives the theoretical prediction and experimental measurements values of the implemented converter. In this table, the component values were measured by the Keysight E4991A impedance analyzer. In the
Table I. Theoretical and measurement values of the class-E² DC-DC converter.

| Parameters | Analytical | Measurement | Difference |
|------------|------------|-------------|------------|
| $f$        | 6.78 MHz   | 6.78 MHz    | 0.00 %     |
| $V_I$      | 20.0 V     | 20.0 V      | 0.00 %     |
| $L_{C}$    | 23 μH      | 23 μH       | 0.00 %     |
| $D_S$      | 0.5        | 0.51*       | 0.00 %     |
| $C_S$      | 230 pF     | 228 pF      | 0.86 %     |
| $L_0$      | 4.04 μH    | 4.03 μH     | 0.25 %     |
| $C_0$      | 142.45 pF  | 142 pF      | 0.32 %     |
| $C_{D}$    | 464 pF     | 460 pF      | 0.86 %     |
| $L_f$      | 370 nH     | 368 nH      | 0.54 %     |
| $C_f$      | 148.7 nF   | 148 nF      | 0.47 %     |
| $R$        | 20.0 Ω     | 20.0 Ω      | 0.00 %     |
| $V_{DM}$   | 60V        | 58V         | 3.33 %     |
| $I_{DM}$   | 1.67A      | 1.60A       | 4.70 %     |
| $c_p$      | 0.109      | 0.104       | 4.58 %     |
| $\eta$     | 100 %      | 85.4 %      | 14.6 %     |

* The measured duty ratio is the duty ratio of the function generator of Tektronix AFG3022, whose signal was the input signal of the MOSFET driver.

![Fig. 15](image)

Fig. 15. Experimental (black line), PSpice simulation (purple line) and theoretical (green line) waveforms of class-E² DC-DC converter. Vertical: $D_r$: 10 V/div, $v_s$ and $v_D$: 50 V/div, $i_t$: 1 A/div, $i_L$: 2 A/div, $v_o$: 20 V/div, and Horizontal: 40 ns/div.

experiment, an IRF510 MOSFET and a STPS5H100 diode were used as the switching devices. The input signal was generated by AFG3022C function generator. Figure 15 shows experimental, PSpice simulation, and theoretical waveforms of the class-E² DC-DC converter. The experimental waveforms were measured using a Tetronix MDO4034C oscilloscope with the current probe of Tetronix TCP2020. It is seen from the obtained waveforms that the experimental and the simulation waveforms showed the quantitative agreements with the theoretical waveforms even though the rectifier filter inductor current includes harmonic components. The harmonic components in the filter inductor current mean that the filter inductance is not sufficiently large like the previous researches [1–15, 17, 18]. As shown in the phase shift between $i_t$ and $v_D$, the rectifier worked inductively because of the effect of low filter inductance. Additionally, the class-E ZVS/ZDS conditions at turn-on instant were achieved at the inverter switch. It can be stated from this result that the input resistance $R_i$ and the input inductance $L_i$ are comprehended accurately from the semi-analytical expression. The obtained results showed the validities of the semi-analytical waveform expression and characteristic investigations in this paper.
Table II. Theoretical and measurement values of class $\Phi_2$-E converter.

| Parameters | Analytical | Measurement | Difference |
|------------|------------|-------------|------------|
| $f$        | 1 MHz      | 1 MHz       | 0.00 %     |
| $V_I$      | 20.0 V     | 20.0 V      | 0.00 %     |
| $L_C$      | 2.58 $\mu$H | 2.58 $\mu$H | 0.00 %     |
| $D_S$      | 0.38       | 0.38*       | 0.00 %     |
| $C_S$      | 4.25 nF    | 4.12 nF     | 3.10 %     |
| $L$        | 2.04 $\mu$H | 2.07 $\mu$H | 1.45 %     |
| $C$        | 3.11 nF    | 3.11 nF     | 0.00 %     |
| $L_0$      | 21.7 $\mu$H | 21.9 $\mu$H | 0.92 %     |
| $C_0$      | 1.17 nF    | 1.17 nF     | 0.00 %     |
| $C_D$      | 3.98 nF    | 3.98 nF     | 0.00 %     |
| $D$        | 0.5        | 0.51        | 2.00 %     |
| $L_f$      | 3.22 $\mu$H | 3.25 $\mu$H | 0.93 %     |
| $C_f$      | 786 nF     | 803 nF      | 2.12 %     |
| $R$        | 20.0 $\Omega$ | 20.0 $\Omega$ | 0.00 %     |
| $\eta$     | 100 %      | 87.7 %      | 12.3 %     |

* The measured duty ratio is the duty ratio of the function generator of Tektronix AFG3022C, whose signal was the input signal of the MOSFET driver.

In the experiments, 13.89 V output voltage could be obtained with 85.4% power conversion efficiency at 6.78 MHz operation.

5.2 Resonant converter with class $\Phi_2$ inverter and class-E resistive rectifier

The resistive class-E rectifier is applied to the class $\Phi_2$-E DC-DC converter in this paper. Figure 16 shows the class $\Phi_2$-E DC-DC converter. Operating frequency $f = 1$ MHz, input voltage $V_I = 20$ V, inverter switch duty ratio $D_S = 0.38$, loaded quality factor of the inverter $Q = 10$, and load resistance $R = 20 \Omega$ were specified for the converter design.

For the rectifier design, zero input reactance and $D = 0.5$ were given as strict conditions. From Fig. 13, we have $\omega C_D R = 0.50$ and $\omega C_f R = 98.9$ for obtaining $X_i = 0$. For these parameters, the input resistance is estimated as $R_i = 27.2 \Omega$. With the estimated $R_i$, the class-$\Phi_2$ inverter was designed following the design procedure in [19, 25, 26].

Table II gives the theoretical prediction and experimental measurements of the implemented converter. It is seen from this table that both the input and output inductances $L_I$ and $L_f$ are lower than the resonant inductance $L_0$. This is because both the class-$\Phi_2$ inverter and resistive class-E rectifier can be designed only when the inductance is low. In the experiment, an IRF530 MOSFET and a STPS5H100 diode were used as the switching devices.

Figure 17 shows experimental, PSpice simulation, and theoretical waveforms of the class-$\Phi_2$-E DC-DC converter. It is seen from the waveform of the rectifier input current $i_f$ and the diode voltage $v_D$ in Fig. 17 that there was no phase shift between them, which means that the design of the resistive class-E rectifier was succeeded. It is also seen that the inverter MOSFET and the rectifier diode turned ON simultaneously. This is because the class-$\Phi_2$ inverter has also no reactance component in the output filter. Similar to Fig. 15, the waveforms in Fig. 17 also showed the quantitative agreements of waveforms among experiment, PSpice simulation, and theoretical prediction, which showed the
validity of the semi-analytical expression and obtained performance curves in this paper. In the experiment, the resonant converter has 13.25 V output with 87.7% power conversion efficiency at 1 MHz operation.

From two experimental results and the above discussions, it can be stated that the semi-analytical expression presented in this paper has high accuracy and it is helpful for the performance comprehension and design of the generalized class-E rectifier.

6. Conclusion
This paper has presented a semi-analytical expression of the generalized class-E rectifier. Concretely, the effects of low output-filter inductance are considered with the theoretical formula. For obtaining the theoretical waveforms, it is necessary to solve the algebraic equations numerically. However, fruitful information of the class-E rectifier is obtained from the theoretical formula presented in this paper. The theoretical predictions suggest that the low output-filter inductance gives rectifier characteristics improvements, such as larger power output capability and no input reactance, compared with the traditional class-E rectifiers. Two resonant converters with the class-E rectifier were designed and implemented. One converter is the class-E^2 DC-DC converter with the class-E rectifier having the largest power output capability. Another converter is the class-Φ^2-E DC-DC converter with the resistive class-E rectifier. The experimental and PSpice simulation results of both converters showed the quantitative agreements with the theoretical predictions, which denoted the validity of the semi-analytical expression and characteristic investigations presented in this paper.

Appendix
The resulting equation of \( c_k \) and \( d_k \) are expressed as the following equations.

\[
c_0 = \frac{1}{2\pi} \int_0^{2\pi} v_D d(\omega t)
= \frac{1}{2\pi \omega C_D} \left\{ 2a_{20} \pi^2 (1 - D)^2 + a_1 \cos(2\pi D) - a_1 - b_1 \sin(2\pi D) 
- 2b_1 \pi (1 - D) + \sum_{k=1}^{N} \left[ \frac{2a_{2k}}{k^2} \sin^2(k\pi D) + \frac{b_{2k}}{k^2} \sin(2k\pi D) + \frac{b_{2k}}{k^2} 2\pi (1 - D) \right] \right\}, \quad (A-1)
\]

\[
c_k = \frac{1}{\pi} \int_0^{2\pi} v_D \cos(k\omega t) d(\omega t)
\]
\begin{equation}
\begin{align*}
= & \frac{1}{\pi \omega C_D} \left( \frac{a_{2k}}{2k^2} \sin^2(2k\pi D) - \frac{b_{2k}}{4k^2} \left[ 4k\pi(1 - D) + \sin(4k\pi D) \right] + \frac{b_1}{k} \sin(2k\pi D) \right) \\
- & \frac{2}{k^2} \left[ k\pi(1 - D) \sin(2k\pi D) + \sin^2(k\pi D) \right] + \sum_{n=1, n \neq k}^{N} \left\{ \frac{a_{2n}}{n} \left[ \sin^2((k + n)\pi D) - \frac{\sin^2((k - n)\pi D)}{k - n} \right] \\
+ \frac{b_{2n}}{2n} \left[ \frac{\sin((k + n)2\pi D)}{k + n} + \frac{\sin((k - n)2\pi D)}{k - n} - 2\sin(2k\pi D) \right] \right\} + q, \tag{A-2}
\end{align*}
\end{equation}

and
\begin{equation}
\begin{align*}
& d_k = \frac{1}{\pi} \int_{0}^{2\pi} v_D \sin(k\omega t) d(\omega t) \\
= & \frac{1}{\pi \omega C_D} \left( \frac{a_{2k}}{4k^2} \left[ 4k\pi(1 - D) + \sin(4k\pi D) \right] - \frac{b_{2k}}{2k^2} \sin^2(2k\pi D) + \frac{2b_1}{k} \sin^2(k\pi D) \right) \\
& - \frac{1}{k^2} \left[ 2k\pi(1 - D) \cos(2k\pi D) + \sin(2k\pi D) \right] + \sum_{n=1, n \neq k}^{N} \left\{ \frac{a_{2n}}{2n} \left[ \frac{\sin((k + n)2\pi D)}{k + n} - \frac{\sin((k - n)2\pi D)}{k - n} \right] \\
+ \frac{b_{2n}}{n} \left[ \frac{\sin^2((k + n)\pi D)}{k + n} + \frac{\sin^2((k - n)\pi D)}{k - n} + 2\sin(2k\pi D) \right] \right\} + p, \tag{A-3}
\end{align*}
\end{equation}

where
\begin{equation}
q = \begin{cases} 
- \frac{a_1}{4} \sin^2(2\pi D) + \frac{b_1}{4} \left[ 4\pi(1 - D) - \sin(4\pi D) \right], & \text{for } k = 1 \\
\frac{a_1}{4} \left[ \frac{\sin^2((k - 1)\pi D)}{k - 1} - \frac{\sin^2((k + 1)\pi D)}{k + 1} \right] & \text{for } k > 1
\end{cases}
\tag{A-4}
\end{equation}

and
\begin{equation}
p = \begin{cases} 
- \frac{a_1}{4} \left[ 4\pi(1 - D) + \sin(4\pi D) \right] + \frac{b_1}{2} \sin^2(2\pi D), & \text{for } k = 1 \\
\frac{a_1}{2} \left[ \frac{\sin((k - 1)2\pi D)}{k - 1} - \frac{\sin((k + 1)2\pi D)}{k + 1} \right] + b_1 \left[ \frac{\sin^2((k + 1)\pi D)}{k + 1} + \frac{\sin^2((k - 1)\pi D)}{k - 1} \right] & \text{for } k > 1
\end{cases} \tag{A-5}
\end{equation}

References

[1] H. Chapter6-Sekiya, “Inverter/rectifier technologies on WPT systems,” Book Wireless power transfer: theory, technology, and applications, ed. N. Shinohara, pp. 83–111, The Institution of Engineering and Technology, London, 2018.

[2] M. Liu, Y. Qiao, S. Liu, and C. Ma, “Analysis and design of a robust class \(E^2\) DC-DC converter for Megahertz wireless power transfer,” IEEE Transactions on Power Electronics, vol. 32, no. 4, pp. 2835–2845, April 2017.

[3] S. Aldhaler, D.C. Yates, and P.D. Mitcheson, “Design and development of a class EF2 inverter and rectifier for Multimegahertz wireless power transfer systems,” IEEE Transactions on Power Electronics, vol. 31, no. 12, pp. 8138–8150, December 2016.

[4] T. Nagashima, X. Wei, E. Bou, E. Alarcon, M.K. Kazimierzuk, and H. Sekiya, “Analysis and design of loosely inductive coupled wireless power transfer system based on class-E2 DC/DC converter for efficiency enhancement,” IEEE Circuit and System, vol. 62, no. 11, pp. 2781–2789, November 2015.

[5] M. Liu, C. Zhao, J. Song, and C. Ma, “Battery charging profile-based parameter design of 6.78 MHz class-\(E^2\) wireless charging system,” IEEE Transactions on Industrial Electronics, vol. 64, no. 8, pp. 6169–6178, August 2017.

[6] G. Kkelis, D.C. Yates, and P.D. Mitcheson, “Class-E half wave zero dv/dt rectifiers for inductive power transfer,” IEEE Transactions on Power Electronics, vol. 32, no. 11, pp. 8322–8337, November 2017.
[7] M. Liu, M. Fu, and C. Ma, “Parameter design for a 6.78-MHz wireless power transfer system based on analytical derivation of class E current driven rectifier,” *IEEE Transactions on Power Electronics*, vol. 31, no. 6, pp. 4280–4291, June 2016.

[8] Z. Zhang, J. Lin, Y. Zhou, and X. Ren, “Analysis and decoupling design of a 30 MHz resonant SEPIC converter,” *IEEE Transactions on Power Electronics*, vol. 31, no. 6, pp. 4536–4548, June 2016.

[9] T. Nagashima, X. Wei, M.K. Kazimierczuk, and H. Sekiya, “Steady-state analysis of isolated class-E$^2$ converter outside nominal operation,” *IEEE Transactions on Industrial Electronics Society*, vol. 64, no. 4, pp. 3227–3238, November 2016.

[10] Y. Guan, Y. Wang, W. Wang, and D. Xu, “Analysis and design of high-frequency DC/DC converter based on a resonant rectifier,” *IEEE Transactions on Industrial Electronics*, vol. 64, no. 1, pp. 8492–8503, November 2017.

[11] K. Lee and J. Ha, “Analysis and design of resonant rectifier for high-frequency DC-DC converters,” *Proc. APEC 2017*, pp. 2475–2480, May 2017.

[12] Y. Huang, S. Tan, and S.Y. Hui, “Multiphase-interleaved high step-up DC/DC resonant converter for wide load range,” *IEEE Transactions on Power Electronics*, vol. 34, no. 8, pp. 7703–7718, August 2019.

[13] S. Park, and J.R. Davila, “Duty cycle and frequency modulations in class-E DC-DC converters for a wide range of input and output voltages,” *IEEE Transactions on Power Electronics*, vol. 33, no.12, December 2018.

[14] H. Sekiya, J. Lu, and T. Yahagi, “Design of generalized class E$^2$ dc/dc converter,” *IEEE International Journal of Circuit Theory and Applications*, vol. 31, no. 3, pp. 229–248, August 2003.

[15] K. Jin, L. Gu, and J. Wang, “A 10-MHz resonant converter with synchronous rectifier for low-voltage applications,” *IEEE Transactions on Power Electronics*, vol. 34, no. 4, pp. 3339–3347, April 2019.

[16] H. Hase, H. Sekiya, J. Lu, and T. Yahagi, “Resonant DC-DC Converter with Class-E Oscillator,” *IEEE Transactions on Circuits and Systems I: Regular Papers* vol. 53, no. 9, pp. 2025–2035, 2006

[17] Q. Li, J. Wang, and L. Ding, “A wide input amplitude range, highly efficient rectifier for low power energy harvesting systems,” *NOLTA IEICE*, vol. 5, no. 4, pp. 499–511, October 2014.

[18] K.H. Lee, E. Chung, Y. Han, and J.IK. Ha, “A Family of high-frequency single-switch DC-DC converters with low switch voltage stress based on impedance networks,” *IEEE Transaction on Power Electronics*, vol. 32, no. 4, pp. 2913–2924, April 2017.

[19] Y. Yanagisawa, Y. Miura, H. Handa, T. Ueda, and T. Ise, “Characteristics of isolated dc-dc converter with class phi-2 inverter under various load conditions,” *IEEE Power Electronics Regular Paper/Letter/Correspondence*, vol. 30, no. 6, pp. 3200–3214, June 2015.

[20] L. Roslaniec, “Design of single-switch inverters for variable resistance/load modulation operation ,” *IEEE Applied Power Electronics*, vol. 30, no. 6, pp. 3200–3214, June 2015.

[21] M. K. Book-Kazimierczuk, *Resonant Power converters*, A Willey Intercience publication, New York, 1995.

[22] G. Kkelis, D.C. Yates, and P.D. Mitcheson, “Comparison of current driven class-D and class-E half-wave rectifiers for 6.78 MHz high power applications,” *Proc. WPTC’2015*, pp. 13–15, May 2015.

[23] X. Wei, H. Sekiya, T. Nagashima, and M.K. Kazimierczuk, “Steady-state analysis and design of class-D ZVS inverter at any duty ratio,” *IEEE Transactions on Power Electronics*, vol. 31, no. 1, pp. 394–406, January 2016.

[24] J.A. Santiago-Gozalez, K.M. Elbaggari, and K. Khurram, “Design of class E resonant rectifiers and diode evaluation for VHF power conversion,” *IEEE Transactions on Power Electronics*, vol. 30, no. 9, pp. 4960–4972, September 2015.

[25] X. Ren, Y. Zhou, D. Wang, X. Zou, and Z. Zhang, “A 10-MHz isolated synchronous class-$\Phi 2$ resonant converter,” *IEEE Transactions on Power Electronics*, vol. 31, no. 12, pp. 8317–8328,
January 2016.

[26] K. Kitazawa, X. Wei, A. Katsuki, and M. Hirokawa, “Analysis and design of the class-Φ₂ inverter,” *Proc. IEEE Industrial Electronics Society*, pp. 21–23, December 2018.

[27] M.K. Kazimierczuk and K. Puczko, “Exact analysis of class E tuned power amplifier at any Q and switch duty cycle,” *IEEE Transactions on Circuits and Systems*, vol. 34, no. 2, pp. 149–159, February 1987.