Distribution of local density of states in superstatistical random matrix theory

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(Date text: October 10, 2018)

Abstract

We expose an interesting connection between the distribution of local spectral density of states arising in the theory of disordered systems and the notion of superstatistics introduced by Beck and Cohen and recently incorporated in random matrix theory. The latter represents the matrix-element joint probability density function as an average of the corresponding quantity in the standard random-matrix theory over a distribution of level densities. We show that this distribution is in reasonable agreement with the numerical calculation for a disordered wire, which suggests to use the results of theory of disordered conductors in estimating the parameter distribution of the superstatistical random-matrix ensemble.
I. INTRODUCTION

The formalism of superstatistics (statistics of a statistics), has recently been proposed by Beck and Cohen [1] as a possible generalization of statistical mechanics. Superstatistics arises as weighted averages of ordinary statistics (the Boltzmann factor) due to fluctuations of one or more intensive parameter (e.g. the inverse temperature). It considers a non-equilibrium system as traveling within its phase space which is partitioned into cells. Within each cell, the system is described by ordinary Maxwell-Boltzmann statistical mechanics, i.e., its statistical distribution is the canonical one $e^{-\beta E}$, but $\beta$ varies from cell to cell, with its own probability density $f(\beta)$. This formalism has been elaborated and applied successfully to a wide variety of physical problems, e.g., in [2, 3, 5, 6, 7, 8, 9, 10, 11].

Superstatistics has been applied to model systems with partially chaotic classical dynamics within the framework of random-matrix theory (RMT) in Ref. [12, 13]. It has provides a possible mechanism for the initial stage of transition of a system out of the state of chaos but fails to reproduce the Poisson statistics that is believed to describe regular systems. In the standard RMT [14, 15, 16, 17], a chaotic system is modeled by an ensemble of random matrices that depends only on the symmetry of the system. For example, a system of spinless particles, which has a time-reversal symmetry is represented by a Gaussian orthogonal ensemble (GOE). The joint distribution of matrix elements of the Hamiltonian $H$ is proportional to $\exp \left[-\eta \text{Tr} \left(H^{\dagger}H\right)\right]$, where $\text{Tr}$ is trace and $H^{\dagger}$ is the Hermitian conjugate of $H$. The parameter $\eta$ is related to the square of the mean level density. This distribution is based on two main assumptions: (i) the matrix elements are independent identically-distributed random variables, and (ii) their distribution is invariant under unitary transformations. For most of the physical systems, however, the phase space is partitioned into regular and chaotic domains. These systems are known as mixed systems. Attempts to generalize RMT to describe such mixed systems are numerous; for a review see [16]. Most of these attempts are based on constructing ensembles of random matrices whose elements are independent but not identically distributed, e.g. in [15, 18, 19, 20, 21]. Thus, the resulting expressions are not invariant under base transformation. The superstatistical generalization follows another route. It keeps base invariance, but violates matrix-elements independence. The intuitive explanation for using superstatistics is based on the ansatz that the spectrum of the system under consideration is partitioned into small cells. Within each cell, the spectrum
is described by an ordinary Gaussian random-matrix ensemble, but ensemble parameter $\eta$ varies from cell to cell. One may define the density of states constituting a single cell as a ‘local density of states’ (LDOS) proportional to $\sqrt{\eta}$. Superstatistics assumes that LDOS and thus the parameter $\eta$ is no more a constant parameter as in the original RMT, but allowed to fluctuate according to a distribution $\tilde{f}(\eta)$. The joint matrix-element distribution is represented as an average over $\exp[-\eta \text{Tr} (H^\dagger H)]$ with respect to the parameter $\eta$. The resulting distribution depends on the matrix elements in the form of $\text{Tr}(H^\dagger H)$, which is base independent as the corresponding distribution in ordinary RMT.

The central question for superstatistics is how to choose the parameter distribution. In superstatistical thermodynamics, one obtains Tsallis’ statistics \cite{24,25} when the inverse temperature $\beta$ has a $\chi^2$ distribution, but this is not the only possible choice. Beck and Cohen give several other possible examples of functions which are possible candidates for $f(\beta)$. Generalized entropies, which are analogous to the Tsallis entropy, can be defined for these general superstatistics \cite{26,27}. Sattin \cite{6} suggested that, lacking any further information, the most probable realization of $f(\beta)$ will be the one that maximizes the Shannon entropy. This latter approach was used in \cite{13} to estimate the distribution $\tilde{f}(\eta)$ of the parameter $\eta$ of RMT.

The object of the present paper is to find out whether the local density of states defined above is related to the local spectral density of states (in the literature, also LDOS) which is relevant to many practical applications in the field of condensed matter physics \cite{28}. There, the notion of LDOS follows from the conjecture that even in a metallic sample there is a finite probability to find “almost localized” eigenstates so that the density of eigenstates is different at different locations in the sample. Here, by localization we shall mean a situation, in which eigenfunctions are localized in the space of ”unperturbed” eigenvalues on a scale which is significantly smaller than the size of the ensemble. Indeed, the size of the region which is populated by an eigenfunction (termed localization length) measures the maximum number of basis state coupled by perturbation (off-diagonal elements). In the state of chaos, the localization length approaches the size of the ensemble, which means that the eigenfunctions become ergodic, i.e., extended over the whole energy shell. Then, the LDOS coincides with the ”global” density of states and both follow Wigner’s semicircular law. As the system departs from chaos, the localization length decreases and the LDOS becomes distinguished from the global one. The main question, which we seek to answer is the following: “Can
we benefit from the achievements of the well developed theory of disordered conductors in estimating the parameter distribution \( \tilde{f}(\eta) \) of the superstatistical ensemble?" Instead of answering this question directly (which is technically quite difficult), we compare the superstatistical distribution \( \tilde{f}(\eta) \) derived in \[13\] from the principle of maximum entropy with the distribution of LDOS deduced by Altshuler and Prigodin \[29\] for disordered conductors. After a brief review of our previous results, given in Section II, we show in Section III that the two distributions have similar shapes especially near the chaotic limit. The distribution \( \tilde{f}(\eta) \) is compared with the distribution of LDOS obtained in the numerical simulation of the closed wire \[30\]. While the agreement of the superstatistical distribution with the numerical experiment is not completely satisfactory, it still demonstrates the analogy between the notions of LDOS in the two disciplines. Section IV shows by comparison with a numerical experiment \[31\] that the level-density distribution obtained for a random wire can be used as a parameter distribution in the superstatistical random matrix theory. The conclusion of this work is formulated in Section V.

II. FORMALISM

For the sake of completeness and clarity, we recall the derivation of the superstatistical model introduced in \[13\]. RMT models the Hamiltonian of a chaotic system in terms of an ensemble of random matrices, whose matrix elements have a Gaussian probability density distribution

\[
P(H) \propto \exp \left[-\eta \text{Tr} (H^\dagger H)\right].
\]  

The mean level density for a GOE with a large dimension \( N \) is given by Wigner’s semi-circle law \[14\]

\[
\rho(\eta, \varepsilon) = \frac{1}{2\pi} \left[N\eta(1-\eta\varepsilon^2/N)\right]^{1/2} \Theta(1-\eta\varepsilon^2/N),
\]  

where \( \varepsilon \) is the eigenvalue of \( H \) and \( \Theta(X) \) is the Heaviside step function. In practical calculations with GOE, one usually avoids the ends of the spectrum and works in a region with a nearly constant level density equal to \( \rho(\eta,0) \sim \sqrt{\eta} \).

The superstatistical generalization models the quantum-number space of a system of mixed regular-chaotic dynamics as made up of many smaller cells that are temporarily in a chaotic phase. Each cell is large enough to obey the statistical requirements of RMT but
has a different distribution parameter $\eta$ associated with it, according to a probability density $\tilde{f}(\eta)$. Consequently, the superstatistical random-matrix ensemble describes the mixed system as a mixture of Gaussian ensembles. Its matrix-element joint probability density distributions obtained by integrating distributions of the form in Eq. (1) over all positive values of $\eta$ with a statistical weight $\tilde{f}(\eta)$,

$$P(H) = \int_0^\infty \tilde{f}(\eta) \frac{\exp \left[ -\eta \text{Tr}(H^\dagger H) \right]}{Z(\eta)} d\eta,$$

(3)

where $Z(\eta) = \int \exp \left[ -\eta \text{Tr}(H^\dagger H) \right] d\eta$. Here we use the "B-type superstatistics" $\tilde{f}(\eta)$. The parameter $\eta$ may be expressed in terms of the local mean level spacing $D$ as

$$D = \frac{c}{\sqrt{\eta}},$$

(4)

where $c$ is a constant depending on the size of the ensemble, which can be evaluated by setting $D = 1/\rho(\eta, 0)$ and using Eq. (2).

In the new framework of RMT provided by superstatistics, the local mean spacing $D$ is no longer a fixed parameter but it is a stochastic variable with probability distribution $f(D)$. Instead, the observed mean level spacing is just its expectation value. The fluctuation of the local mean spacing is due to the correlation of the matrix elements which disappears for chaotic systems. In the absence of these fluctuations, $f(D) = \delta(D - 1)$ and we obtain the standard RMT. Within the superstatistics framework, we can express any statistic $\sigma$ of a (sufficiently chaotic) mixed system that can in principle be obtained from the joint eigenvalue distribution by integration over some of the eigenvalues, in terms of the corresponding statistic $\sigma^{(G)}(D)$ for a Gaussian random ensemble with mean level spacing $D$. The superstatistical generalization is given by

$$\sigma = \int_0^\infty f(D) \sigma^{(G)}(D) dD.$$  

(5)

The remaining task of superstatistics is the computation of the distribution $f(D)$. Following Sattin [6], we use the principle of maximum entropy (MaxEnt) to evaluate the distribution $f(D)$. Lacking a detailed information about the mechanism causing the deviation from the prediction of RMT, the most probable realization of $f(D)$ will be the one that extremizes the Shannon entropy

$$S = -\int_0^\infty f(D) \ln f(D) dD$$

(6)
with the following constraints: (i) The major parameter of RMT is $\eta$ defined in Eq. (1). Superstatistics was introduced in Eq. (3) by allowing $\eta$ to fluctuate around a fixed mean value $\langle \eta \rangle$. This requires the existence of the mean inverse square of $D$, $\langle D^{-2} \rangle = \int_{0}^{\infty} f(D) D^{-2} dD$.

(ii) The fluctuation properties are usually defined for unfolded spectra, which have a unit mean level spacing. We thus require $\int_{0}^{\infty} f(D) dD = 1$. As a result, we obtain

$$f_{\text{MaxEnt}}(D) = C \exp \left[ -\alpha \left( \frac{2D}{D_0} + \frac{D_0^2}{D^2} \right) \right]$$

(7)

where $\alpha$ and $D_0$ are parameters, which can be expressed in terms of the Lagrange multipliers of the constrained extremization, and $C$ is a normalization constant. Substituting this distribution into Eq. (5) one obtains expressions for the (global) level density, the nearest-neighbor-spacing (NNS) distribution and the two-level correlation function [13]. Eq. (5) has recently been applied [34] to obtain a generalization of the well-known Porter-Thomas distribution of transition intensities, relevant for chaotic regimes, for systems with mixed regular-chaotic dynamics. This generalization agrees with the data better than currently available results that can not explain observed shift of the peak position as the system evolves out of the state of chaos.

III. DISTRIBUTION OF LOCAL DENSITY OF STATES

The local density of states plays a central role in the theory of disordered metals [28, 35]. It also known in nuclear physics as strength function [36]. It gives the distribution of basis eigenfunctions in terms of eigenstates of the system. It is obtained by projecting a basis state $k$ onto exact eigenstates $i$ of the Hamiltonian $H$ and then defined in terms of the expansion coefficients $C^i_k$ as

$$\nu_k(E) = |C^i_k|^2 \rho(E),$$

(8)

with $\rho(E)$ as the density of exact eigenstates. Here, the average is taken over a small window of the eigenstates $i$ with energies around $E$. The LDOS has a well-defined classical interpretation as shown by Benet et al. [39]. The unperturbed energy $E_0$ is not constant along a classical trajectory of the full Hamiltonian with a given total energy $H = E$. If we keep the unperturbed energy $E_0$ fixed, the bundle of trajectories of the total Hamiltonian $H$, which reach the surface of the unperturbed Hamiltonian $H_0 = E_0$, has a distribution in the total energy $E$ which is described by a measure $\nu_{E_0}(E)$. In the quantum case, this
measure corresponds to the imaginary part of the retarded Green’s function at energy $E_0$ [40]. Distributions of LDOS are relevant for description of fluctuations of tunneling conductance across a quantum dot [37], of transport phenomena in disordered wires [38], of properties of some atomic spectra [41], and of a shape of NMR line [42]. In random media the LDOS is a random quantity. Altshuler and Prigodin [29] have studied the LDOS distribution in strictly one-dimensional disordered chains. They obtain closed-form expressions for the LDOS distributions in open and closed wires. The normalized distribution for the closed system reads

$$P_{\text{AP}}(\omega, \nu_{\text{AP}}; \nu) = \sqrt{\frac{\omega}{2\pi \nu^3}} \exp \left[ \omega - \frac{1}{2} \omega \left( \frac{\nu}{\nu_{\text{AP}}} + \frac{\nu_{\text{AP}}}{\nu} \right) \right],$$  \hspace{1cm} (9)

where $\omega$ is a positive parameter. The distribution $P_{\text{AP}}(\omega, \nu_{\text{AP}}; \nu)$ has a unit mean value $\nu_{\text{AP}}$ and a variance equal to $1/\omega$. We note that this distribution fulfils the two conditions imposed by Beck and Cohen [1] on the parameter distribution in superstatistics, which are normalization and tending to a delta function as $\omega \to \infty$.

Let us now assume that the LDOS $\nu$ is equal to the inverse of local mean level spacing $D$. In order to justify this assumption, we compare the distribution $P_{\text{AP}}(\nu)$ with the distribution $f_{\text{MaxEnt}}(D)$ of the local mean level densities (7), which has been applied in the superstatistical RMT [13, 34]. For this purpose, we have to translate the distribution $f_{\text{MaxEnt}}(D)$ obtained in the previous section into the context of the problem of LDOS in complex systems. This can be straightforwardly done by substituting $\nu = 1/D$ in Eq. (7). The distribution of LDOS in the superstatistical RMT is then given by

$$P_{\text{MaxEnt}}(\nu) = C \frac{1}{\nu^2} \exp \left[ -\alpha \left( \frac{2\nu_0}{\nu} + \frac{\nu^2}{\nu_0^2} \right) \right],$$  \hspace{1cm} (10)

where $\nu_0 = 1/D_0$. The normalization constant is given by

$$C = \frac{2\sqrt{\pi} \alpha \nu_0}{G^{30}_{03}(\alpha^3|0, \frac{1}{2}, 1)},$$  \hspace{1cm} (11)

where $G^{3,0}_{0,3}(z|b_1, b_2, b_3)$ is Meijer’s G-function [32, 33]. The parameter $\nu_0$ is fixed by requiring that the average value of the LDOS is equal to 1, which yields

$$\nu_0 = \frac{G^{30}_{03}(\alpha^3|0, \frac{1}{2}, 1)}{\alpha G^{30}_{03}(\alpha^3|0, 0, \frac{1}{2})}.$$  \hspace{1cm} (12)

The variance of this distribution is given by

$$\sigma^2_{\text{MaxEnt}} = \frac{G^{30}_{03}(\alpha^3|0, 0, 0) G^{30}_{03}(\alpha^3|0, \frac{1}{2}, 1)}{\left[ G^{30}_{03}(\alpha^3|0, 0, \frac{1}{2}) \right]^2} - 1,$$  \hspace{1cm} (13)
which tends to zero as $\alpha$ tends to $\infty$ (nearly chaotic regime) and behaves as $\alpha^{-3/2}$ at small $\alpha$ (nearly integrable). Figure 1 compares the distributions of LDOS in Eq. (9) and (10) for same variances. The figure clearly suggests the equivalence of the two distributions are nearly equivalent, except at small values of $\alpha$ or $\omega$ that correspond to variances $\sigma^2 > \langle \nu \rangle^2$, which is adapted in the superstatistical approach with nearly integrable systems. This should not make an obstacle for using the Altshuler-Prigodin LDOS distribution in superstatistical RMT. The superstatistical approach is not expected to model a system in the final stage of transition out of chaos, where the system approaches the integrability limit. As previously mentioned, this is a base-invariant approach. Integrable systems by definition have well defined complete set of eigenvalues that yields a diagonal Hamiltonian matrix when used as a basis.

Schomerus et al [30] calculate the probability distribution of the local density of states in a disordered one dimensional conductor or single-mode waveguide. They start from the relation between the LDOS and the retarded Green function, which they have obtained by a numerical simulation for a wire with one or both fixed ends. They have given exact results for the distributions of the local densities of states in one-dimensional localization, contrasting the microscopic length scale (below the wavelength) and mesoscopic length scale (between the wavelength and the mean free path). Their data points are shown in Fig.2 for the case of a closed wire. These data result from a numerical simulation for a wire of length equal to 55 times the mean free path, with the LDOS computed halfway in the wire. Figure 2 also shows the result of calculation using the Altshuler-Prigodin distribution (9) by a dashed line and the distribution in Eq. (9) by a solid line. The mean LDOS is fixed to be equal 1 in the three distributions. While the Altshuler-Prigodin distribution fits the results of numerical simulation better, the superstatistics distribution still provides a reasonable representation of the data.

The absence of quantitative agreement between the predictions of Eqs. (9) and (10) does not imply that the LDOS $\nu$ is not equal to the inverse of local mean level spacing $D$. The Altshuler-Prigodin distribution (9) is derived from an elaborate theory that starts from the relation between the level density and the retarded Green’s function for noninteracting electrons in a wire and arrives at a relation between the microscopic LDOS and the reflection coefficients, and then uses elaborate methods perform the local spatial average that gives the LDOS. It is interesting to note that the distribution in Eq. (10) is derived in [13] from
the principle of maximum entropy with the constraints of fixed $\langle D^{-2} \rangle$ and $\langle D \rangle$ as mentioned above. If we replace the first constraint by another one that requires that $\langle D^{-1} \rangle$, we arrive exactly to the Altshuler-Prigodin distribution. The fact that the Altshuler-Prigodin distribution fits the numerical-experimental data perfectly, as shown in Fig. 1, while the distribution (10) fits the data only qualitatively suggests that $f(D)$ may better be obtained by maximizing entropy under the conditions of fixed $\langle D \rangle$ and $\langle D^{-1} \rangle$.

IV. NEAREST-NEIGHBOR-SPACING DISTRIBUTION

The question that this section tries to answer is whether the distribution of LDOS obtained in the study of disordered metals is suitable for describing the transition out of chaos within the superstatistical approach to RMT. For this purpose we consider a system, which is described by a superposition of random matrix ensembles of different LDOS with probability-density function given by the Altshuler-Prigodin distribution $P_{\text{AP}}(\omega, \nu_{\text{AP}}; \nu)$. Its spectral characteristics are expressed as averages of the corresponding characteristics of a Gaussian random ensemble in analogy with Eq. (5). Then, the superstatistical NNS distribution is given by

$$ p_{\text{AP}}(\omega, s) = \int_0^{\infty} P_{\text{AP}}(\omega, \nu_{\text{AP}}; \nu) \, p_{\text{WD}}(\nu, s) \, d\nu, $$

where $p_{\text{WD}}(\nu, s)$ is a Wigner-Dyson distribution with mean level spacing equal to $1/\nu$,

$$ p_{\text{WD}}(\nu, s) = \frac{\pi}{2} \nu^2 s \exp\left( -\frac{\pi}{4} \nu^2 s^2 \right). $$

We then obtain

$$ p(\omega, s) = \frac{1}{4} \sqrt{2\pi} \nu_{\text{AP}}^2 s \int_0^{\infty} \sqrt{x} \, \exp \left[ \omega - \frac{\omega}{2} \left( x + \frac{1}{x} \right) - \frac{\pi}{4} \nu_{\text{AP}}^2 x^2 s^2 \right] \, dx $$

the parameter $\nu_{\text{AP}}$ is fixed by requiring that the mean-level spacing $\langle 1/\nu \rangle$ is equal to 1 which yields

$$ \nu_{\text{AP}} = 1 + 1/\omega. $$

Unfortunately, we are not able to evaluate the integral in Eq. (16) analytically, but there is no problem in its numerical evaluation.

We shall demonstrate the quality of the NNS distribution in Eq. (16) by applying it to the results of a numerical experiment by Gu et al. [31] on a random binary network. Impurity bonds are employed to replace the bonds in an otherwise homogeneous network.
The authors of Ref. [31] numerically calculated more than 700 resonances for each sample. For each impurity concentration $p$, they considered 1000 samples with totally more than 700,000 levels computed. Figure 3 shows their results for four values of concentration $p$. The figure also shows the best fits obtained for NNS superstatistical distributions obtained using the parameter distributions following the Altshuler-Prigodin formula and the ones obtained in [13] using the principle of MaxEnt. The high statistical significance of the data allows us to assume the advantage of the superstatistical distribution base on the Altshuler-Prigodin LDOS distribution for describing the results of this experiment.

V. SUMMARY AND CONCLUSION

Many characteristics of disordered metals and those of insulators have been understood using RMT. The spectral fluctuations of a disordered metal are well described by random matrix theory, while the fluctuations of a system in the insulating regime follow the Poisson statistics. The nature and the details of the metal-insulator transition, on the other hand, still belong to the most intensively studied problems. Localization of wave functions by disorder can be seen in the fluctuations of the density of states, which can be probed using the tunnel resistance of a point contact. We have previously described the analogous transition between regular and chaotic dynamics within the framework of RMT as a superposition of two statistics, namely one described by the matrix-element distribution $\exp \left[ -\eta \text{Tr} (H^1 H) \right]$ and another one by the probability distribution $\tilde{f}(\eta)$ of the inverse variance of the matrix elements, which is derived from the principle of maximum entropy. In the present investigation, we show an interesting connection between the distribution $\tilde{f}(\eta)$ and the distribution of LDOS which is used in the study of disordered systems. We have also found that the LDOS distribution that follows from $\tilde{f}(\eta)$ is in a reasonable agreement with the numerical simulation of LDOS in a weakly-disordered metallic wire. The agreement is of course not as good as for the distributions obtained by Altshuler and Prigodin for LDOS in one-dimensional disordered systems using a more sophisticated approach. However, we obtain the Altshuler-Prigodin distribution if we modify the (optional) constraints imposed on entropy maximization in our previous work, by requiring a fixed value of $\langle \eta^{1/2} \rangle$ rather than fixing $\langle \eta \rangle$. Therefore, both the Altshuler-Prigodin distribution and the parameter distribution used in our previous work are equally well based on the principle of maximum entropy,
both satisfy the Beck-Cohen criteria, but the comparison with experimental data prefers the former distribution. In conclusion, we suggest to use the Altshuler-Prigodin formula as a parameter distribution that describes the superposition of GOE’s in the superstatistical ensemble. This has been illustrated by an analysis of a high-quality numerical experiment on the NNS distributions of resonance spectra of disordered binary networks.

[1] C. Beck and E. G. D. Cohen, Physica A 322, 267 (2003).
[2] E. G. D. Cohen, Physica D 193, 35 (2004).
[3] C. Beck, Physica D 193, 195 (2004).
[4] C. Beck, Europhys. Lett. 64, 151 (2003).
[5] F. Sattin and L. Salasnich, Phys. Rev. E 65, 035106(R) (2003).
[6] F. Sattin, Phys. Rev. E 68, 032102 (2003).
[7] A. M. Reynolds, Phys. Rev. Lett. 91, 084503 (2003).
[8] M. Ausloos and K. Ivanova, Phys. Rev. E 68, 046122 (2003).
[9] S. Abe and S. Thurner, Phys. Rev. E 72, 036102 (2005).
[10] C. Beck, E. G. D. Cohen, and H. L. Swinney, Phys. Rev. E 72, 056133 (2005).
[11] C. Beck, Physica A 365, 96 (2006).
[12] A. Y. Abul-Magd, Physica A 361, 41 (2006).
[13] A. Y. Abul-Magd, Phys. Rev. E 72, 066114 (2005).
[14] M. L. Mehta, Random Matrices 2nd ed. (Academic, New York, 1991).
[15] F. Haake, Quantum Signatures of Chaos (Springer, Heidelberg, 1991).
[16] T. Guhr, A. Müller-Groeling, and H. A. Weidenmüller, Phys..Rep. 299, 189 (1998).
[17] H-J. Stockmann, Quantum Chaos : an introduction, Cambridge Univ. Press (1999).
[18] N. Rosenzweig and C. E. Porter, Phys. Rev. 120, 1698 (1960).
[19] M. S. Hussein and M. P. Pato, Phys. Rev. Lett. 70, 1089 (1993); Phys. Rev. C 47, 2401 (1993);
    Phys. Rev. Lett. 80, 1003 (1998).
[20] G. Casati, L. Molinari, and F. Izrailev, Phys. Rev. Lett. 64, 1851 (1990).
[21] Y. V. Fyodorov and A. D. Mirlin, Phys. Rev. Lett. 67, 2405 (1991).
[22] G. Le Caër and R. Delannay, Phys. Rev. E 59, 6281 (1999).
[23] K. A. Muttalib and J. R. Klauder, Phys. Rev. E 71, 055101(R) (2005).
[24] C. Tsallis, J. Stat. Phys. 52, 479 (1988).
[25] C. Tsallis, Lect. Notes Phys. 560, 3 (2001).
[26] S. Abe, Phys. Rev. E 66, 046134 (2002).
[27] C. Tsallis and A. M. C. Souza, Phys. Rev. E 67, 026106 (2003); Phys. Lett. A 319, 273 (2003).
[28] A. B. Mirlin, Phys. Rep. 326, 259 (2000).
[29] B. L Altshuler, V. N. Prigodin, Zh. Eksp. Teor. Fiz. 95, 348 (1989) [Sov. Phys. JETP 68, 198 (1989)].
[30] H. Schomerus, M. Titov, P. W. Brouwer, and C. W. J. Beenakker, Phys. Rev. B, 65, 121101(R) (2002).
[31] Y. Gu, K. W. Yu, and Z. R. Yang, Phys. Rev. E 65, 046129 (2002).
[32] A. M. Mathai, A Handbook of Special Functions (Clarendon, Oxford, 1993).
[33] Wolfram Research’s Mathematical Functions 2002, http://functions.wolfram.com
[34] A. Y. Abul-Magd, Phys. Rev. E 73, 056119 (2006).
[35] Y.V.Fyodorov, O. A.Chubykalo, F. M.Izrailev, and G.Casati, Phys.Rev. Lett. 76, 1603 (1996).
[36] A. Bohr and B. Mottelson, Nuclear structure, Vol. 1 (Benjamin, New York, 1969).
[37] Y. Alhassid, Rev. Mod. Phys. 72, 895 (2000).
[38] C. W. J. Beenakker, Rev. Mod. Phys. 69, 731 (1997).
[39] L. Benet, F.M. Izrailev, T.H. Seligman, A. Suárez-Morenod, Phys. Let. A 277, 87 (2000).
[40] G. Casati, B. V. Chirikov, I. Guarneri and F. M. Izrailev, Phys. Lett. A 23, 430 (1996).
[41] P. de Vries and A. Lagendijk, Phys. Rev. Lett. 81, 1381 (1998).
[42] F. C. Fritschij, H. B. Brom, L. J. de Jongh, and G. Schmid, Phys. Rev. Lett. 82, 2167 (1999).
Figure captions

FIG. 1. (Color on line) Distributions of LDOS estimated by using the principle of MaxEnt and previously used in the superstatistical approach to RMT [13] (solid line) compared with the ones obtained by Altshuler and Prigodin for a closed one-dimensional wire [29] (dashed line) that have same variances.

FIG. 2. ((Color on line) Distribution of LDOS in a disordered metal numerically calculated by Schomerus et al. [30] (histogram) compared with the corresponding distribution deduced from the superstatistical approach to RMT [13] (solid line) and that obtained by Altshuler and Prigodin for a closed one-dimensional wire [29] (dashed line).

FIG. 3. (Color on line) Nearest neighbor spacing distributions of geometrical resonances in random network, calculated by Gu et al. [31] compared with the superstatistical distributions in which the parameter distributions are estimated using the MaxEnt principle (solid lines) as well as those based on the Altshuler-Prigodin distribution of LDOS (dashed lines).
Fig 3