Forced response of electrostatic harmonic drive to torque fluctuation

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Abstract. In this paper, the radial and tangential electric field forces are given. Based on the electromechanical coupled dynamic equations of the drive system, by generalized force and generalized coordinate, the forced response of the drive system to torque fluctuation are investigated. Changes of the forced response along with the system parameters are given as well. The micro flexible ring radius and clearance between the micro ring and stator, etc. have obvious influences on the forced responses of the drive system. These results are useful in design and manufacture of the micro drive system and can offer some reference for other micro electromechanical systems.

1. Introduction

Micro-electromechanical system (MEMS) can be described as machines constructed of small moving sub-elements that have characteristic dimensions in the range of about 0.5-500 μm. Such devices have potential applications in electronic assembly, medical, microspacecraft and military equipment [1-4]. Author invents an electrostatic harmonic drive as shown in Fig.1. The drive mainly consists of a flexible ring and an outer ring stator. The outer ring stator electrodes are applied to voltage sequentially, and a rotational electric field will result in a periodic elastic deformation of the flexible ring and periodic capacity changes between flexible ring and stator. It produce tangential electric field forces to drive the axis to rotate on which flexible ring is supported. In the drive, integration of the harmonic drive, motor and control can be realized. Compared with piezoelectric and electromagnetic actuation principles [5-6], the electromechanical integrated electrostatic harmonic drive needs neither additional elements like coils or cores, nor special materials like piezoelectric ceramics. It is more favorable for miniaturization of the electromechanical devices. Compared with other electrostatic actuation principles [7-9], the drive does not require fabrication of the teeth on micro elements and its output axis does not wobble. This makes it easier to be fabricated and used. In this paper, from analysis of the system energy, the radial and tangential electric field forces are given. Based on the electromechanical dynamic equations, by generalized force and generalized coordinate, the forced response of the drive system to torque fluctuation are investigated. Changes of the forced response along with the system parameters are given as well. These results are useful in design and manufacture of the micro drive system and can offer some reference for other micro electromechanical systems.

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2. Electromechanical coupled dynamic equations

The dynamic equation of the mechanical system for the drive subjected to electric field force is

\[
\frac{\partial^2 u}{\partial \theta^2} + 2 \frac{\partial^2 u}{\partial \theta \partial t} + \frac{\partial u}{\partial t} = \frac{r^4}{E I_s} \frac{\partial^2 q}{\partial \theta^2} + \frac{r^4}{E I_s} q + \frac{r^4 \rho A}{E I_s} \frac{\partial u}{\partial t}
\]  

(1)

where \( u \) is radial displacement of the micro ring; \( \ddot{u} \) is second derivative of displacement \( u \) with respect to time; \( r \) is the average radius of the ring. \( A \) is its transverse section area; \( \rho \) is material density of the ring; \( E \) is the modulus of elasticity of the micro ring material; \( I_x \) is section modular of the ring, \( I_x = ld^3 / 12 \) ( \( l \) and \( d \) is effective width and the thickness of micro ring, respectively); \( \theta \) is position angle of the micro ring; \( q_r \) is radial load for unit arc length, \( q_t \) is tangential load for unit arc length.

The total radial displacement \( u \) of the micro flexible ring consists of the static displacement \( u_s \) and the dynamic displacement \( \Delta u \)

\[
u = u_s + \Delta u
\]  

(2)

Then, the radial load \( q_r \) per unit arc length on the micro ring consists of the static component \( q_{rs} \) and the dynamic one \( \Delta q_r \)

\[
q_r = q_{rs} + \Delta q_r
\]  

(3)

The tangential load \( q_t \) per unit arc length on the micro ring consists of the static component \( q_{ts} \) and the dynamic one \( \Delta q_t \)

\[
q_t = q_{ts} + \Delta q_t
\]  

(4)

Substituting Eqs. (3) and (4) into Eq (1), yields the following equation

\[
\frac{\partial^2 \Delta u}{\partial \theta^2} + 2 \frac{\partial^2 \Delta u}{\partial \theta \partial t} + \frac{\partial \Delta u}{\partial t} = \frac{r^4}{E I_s} \frac{\partial^2 \Delta q}{\partial \theta^2} + \frac{r^4}{E I_s} \Delta q + \frac{r^4 \rho A}{E I_s} \frac{\partial \Delta u}{\partial t}
\]  

(5)

\[
\frac{\partial^3 u}{\partial \theta^3} + 2 \frac{\partial^3 u}{\partial \theta^2 \partial t} + \frac{\partial^2 u}{\partial \theta \partial t} = \frac{r^4}{E I_s} \frac{\partial^2 q}{\partial \theta^2} + \frac{r^4}{E I_s} q + \frac{r^4 \rho A}{E I_s} \frac{\partial u}{\partial t}
\]  

(6)

Eq.(5) is static differential equation of the micro ring displacements subjected to electric field force. Eq. (6) is dynamic equations of the micro ring subjected to electric field force.
The clearance between the micro flexible ring and the outer stator ring is so small that the capacity between them can be calculated by equation of flat capacitor

\[ C = \beta \cdot \frac{\varepsilon_0 \cdot 2\pi r l}{\pi r (t_o - u_o + d_i / \varepsilon_r)} \]  \hspace{1cm} (7)

where \( \varepsilon_0 \) is permittivity constant of free space, \( \varepsilon_0 = 8.85 \times 10^{-12} \text{C}^2 \cdot \text{N}^{-1} \cdot \text{m}^{-2} \), \( \varepsilon_r \) is the relative dielectric constant of the insulating layer, \( t_o \) is the initial clearance between micro ring and outer ring, \( d_i \) is width of the insulating layer, \( u_o \) is the average radial displacement of the micro ring at central angle \([-\beta, \beta]\).

The electrical field force \( F_e \) between polar plates of capacitor can be given by

\[ F_e = \frac{1}{2} v_o \frac{dC_i}{du} \]  \hspace{1cm} (8)

where \( v_o \) and \( C_i \) are voltage and capacitance of the \( i \)th circuit, respectively.

Assuming that the electrical field force is uniformly distributed on the micro ring, the radial load per unit arc length can be denoted by symbol \( q_{ri} \) which is subjected to micro ring, through the central angle \([-\beta, \beta]\). Combining (7) with (8), the electric load \( q_{ri} \) can be calculated as below

\[ q_{ri} = F_e = \frac{1}{2} v_o \frac{dC_i}{du} \]  \hspace{1cm} (9)

The change of the displacement results in changes of the capacity and electric field force between micro flexible ring and outer stator ring. Then, the total capacity \( C_i \) consists of the static capacity \( C_o \) and the dynamic capacity \( \Delta C_i \). The total electric field force \( F_e \) consists of the static electric field force \( F_o \) and the dynamic electric field force \( \Delta F_e \). The total voltage \( v_o \) consists of the static voltage \( v_o \) and the dynamic voltage \( \Delta v_o \). Substituting these changes into Eqs (8) and (9), neglecting the higher order term, yields

\[ q_{ri} = \frac{v_o^2 \varepsilon_0 l}{2(t_o - u_o + d_i / \varepsilon_r)^2} \quad \text{and} \quad \Delta q_e = \frac{v_o^2 \varepsilon_0 l}{(t_o - u_o + d_i / \varepsilon_r)} \Delta u \]

3. Torque and tangential electric field force fluctuation

Let \( C(\theta) \) denote capacity between the flexible ring and outer ring when the electric field rotates angle \( \theta \). From Fig.2, one knows that when the electric field rotates angle \( \theta \) the capacity between the flexible ring and outer stator is

\[ C(\theta) = 2\left( \int_{\frac{\pi}{\beta}}^{\frac{\pi}{\beta}} \frac{\varepsilon_0 \cdot l \, d\phi}{t_o - u_s + d_i / \varepsilon_r} - \int_{\frac{\pi}{\beta}}^{\frac{\pi}{\beta}} \frac{\varepsilon_0 \cdot l \, d\phi}{t_o - u_s + d_i / \varepsilon_r} + \int_{\frac{\pi}{\beta}}^{\frac{\pi}{\beta}} \frac{\varepsilon_0 \cdot l \, d\phi}{t_o - u_s + d_i / \varepsilon_r} \right) \]  \hspace{1cm} (10)

Thus

\[ \frac{dC(\theta)}{d\theta} = 2\varepsilon_0 l \frac{u(\theta + \beta) - u(\theta - \beta)}{[t_o - u_s (\theta - \beta) + d_i / \varepsilon_r] [t_o - u_s (\theta + \beta) + d_i / \varepsilon_r]} \]  \hspace{1cm} (11)

Above equations are substituted into torque equation \( T = \frac{1}{2} v_o^2 \frac{dC(\theta)}{d\theta} \), then, driving torque \( T \) caused by tangential electric field force can be given as below

\[ T = \varepsilon_0 l v_o^2 \frac{u_s (\theta - \beta) - u_s (\theta + \beta)}{[t_o - u_s (\theta - \beta) + d_i / \varepsilon_r] [t_o - u_s (\theta + \beta) + d_i / \varepsilon_r]} \]  \hspace{1cm} (12)
where static displacement $u_s$ is obtained by Eq.(5).

The torque is caused by tangential electric field force. The tangential electric field force is applied to flexible ring through angle $[-\beta, \beta]$ and $[\pi - \beta, \pi + \beta]$. So the arc length to which electric field force is applied equals $4\beta r$. Then the tangential electric field force per unit arc is

$$q_t = \frac{T}{4\beta r^2} = \frac{e_0\varepsilon_0 \varepsilon_1}{4\beta r} \left[ \frac{u_s(\varphi - \beta) - u_s(\varphi + \beta)}{[t_0 - u_s(\varphi - \beta) + d / \varepsilon_s][t_0 - u_s(\varphi + \beta) + d / \varepsilon_s]} \right]$$

(13)

The tangential electric field force fluctuation is

$$\Delta q_t = \frac{e_0\varepsilon_0 \varepsilon_1 \Delta v}{2\beta r} \left[ \frac{u_s(\varphi - \beta) - u_s(\varphi + \beta)}{[t_0 - u_s(\varphi - \beta) + d / \varepsilon_s][t_0 - u_s(\varphi + \beta) + d / \varepsilon_s]} \right]$$

(14)

where $\Delta v$ is voltage fluctuation.

If the voltage fluctuation changes periodically, driving torque will wave periodically, and the tangential electric field force on the flexible ring will also change periodically. Then, the distribution of the tangential electric field force is

when $-\beta \leq \theta \leq \beta$, $q_t = F \sin \omega_z t$

when $\beta \leq \theta \leq \pi - \beta$, $q_t = 0$

where $F = \frac{e_0\varepsilon_0 \varepsilon_1 \Delta v_m}{2\beta r} \left[ \frac{u(\varphi - \beta) - u(\varphi + \beta)}{[t_0 - u(\varphi - \beta) + d / \varepsilon_s][t_0 - u(\varphi + \beta) + d / \varepsilon_s]} \right]$, here $\Delta v_m$ is magnitude of the voltage fluctuation, $\omega_z$ is frequency of the voltage fluctuation ($\Delta v = \Delta v_m \sin \omega_z t$).

The tangential electric field force can be defined in Fourier series form as

$$\Delta q_t = \frac{2F \sin \omega_z t}{\pi} \left( \frac{\beta + 2}{\beta} \sum_{k=2,4,6,\ldots} \frac{\sin k\beta}{k} \cos k\theta \right)$$

(15)

4. Forced response to torque fluctuation

Let $Q_i(t)$ and $q_i(t)$ denote generalized force and generalized coordinate, then

$$Q_i(t) = \int_{0}^{\tau} \Delta q_i(\theta) d\theta + \int_{\tau}^{\pi} \Delta q_i(\theta) d\theta \quad i = (1,2,\ldots)$$

(16)

$$\dot{q}_i(t) = \frac{1}{\omega_i} \int Q_i(\tau) \sin \omega_i(t - \tau) d\tau + q_i(0) \cos \omega_i t + \frac{q_i(0)}{\omega_i} \sin \omega_i t \quad i = (1,2,\ldots)$$

(17)
where initial values \( q_i(0) \) and \( q_i(0) \) of the generalized coordinate can be determined by initial
c condition, here they are taken as zero, \( \phi_i(\theta) \) is \( i \) th mode function.

Then, above equation is simplified as

\[
q_i(t) = \frac{1}{\omega_i} \int_0^\infty Q_i(\tau) \sin\omega_i(t - \tau) d\tau
\] (18)

The dynamic vibrating displacement of the system is

\[
\Delta u_{d}(\theta,t) = \sum_{i=1}^\infty \phi_i(\theta)q_i(t)
\] (19)

Substituting relative equations into Eq. (16), the generalized force corresponding to mode

\[
Q(t) = \int \Delta q_i[\cos(m_1\theta) + A_{m_1} c_h(m_2\theta)d\theta
\] + \[
\sum_{i=1}^\infty \Delta h[A_{i_1} \cos(n_1\theta) + A_{i_2} \sin(n_1\theta) + A_{i_3} \cos(n_2\theta) + A_{i_4} \sin(n_2\theta)]d\theta = (I_{n_1} + I_{n_2})F\sin\omega_1 t
\] (20)

Substituting generalized forces into Eq. (18), and neglecting free vibration term, generalized
coordinates can be given

\[
q_i(t) = \frac{1}{\omega_i}(L_{n_1} + L_{n_2}) \frac{F}{\omega_i^2 - \omega_i^2} \cos\omega_i t \quad (i = 1, 2, 3, \ldots, \infty)
\] (21)

5. Results and discussions

For the drive system with parameters as shown in Table 1, the forced response of the electrostatic
harmonic drive system to torque fluctuation is analyzed as shown in Fig.3 (here, only the first two
modes are given, and static voltage \( v_0 = 25V \), voltage fluctuation magnitude \( \Delta v_w = 5V \), fluctuating
frequency \( \omega_e = 314rad/s \)). Figure 4 shows changes of the forced responses along with system
parameters (here, only the first mode are given and only vibration magnitude at the point \( \theta = 0^\circ \)
is shown). From figures 3 and 4, following observations are worth noting:

(1) For given periodic torque excitation, the forced responses are also periodic vibration.
(2) As the exciting frequency is near to the natural frequency for the mode 1 of the micro drive
system, the forced response magnitude corresponding to mode 1 is the largest, and the forced
response magnitude corresponding to other modes are all very small.
(3) The vibrating frequencies for different modes are all identical. It is because the vibrating
frequency is only decided by torque exciting frequency.
(4) As the order number of the vibrating modes grows, the vibrating displacement peaks on the
flexible ring increase in the circumferential direction of the ring.
(5) As the radius \( r \) increases, the vibrating magnitude of the micro ring decreases. As the clearance \( t_0 \)
increases, the vibrating magnitude of the micro ring increases. These changes are decided by stiffness
changes of the electrostatic harmonic drive system. As a parameter increases, the stiffness of system
increases, then the vibrating magnitude will decrease with increasing the parameter.

| Table 1 Parameters of the example drive system |
|-----------------------------------------------|
| \( r (mm) \) | \( t_0 (\mu m) \) | \( d (\mu m) \) | \( l (mm) \) | \( d_1 (\mu m) \) | \( \varepsilon_r \) | \( E (Gpa) \) | \( \beta (^\circ) \) |
|-------------------|-----------------|--------------|-----------|-----------------|-------------|-------------|-------------|
| 1                 | 2               | 30           | 1         | 0.5             | 8.4         | 70          | 30          |
6. Conclusions
In this paper, by generalized force and generalized coordinate, the forced response of the drive system to torque excitation are investigated. And changes of the forced response along with the system parameters are given. The studies can be used to design parameters of the drive and remove undesirable dynamic behavior.

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