Inhomogeneities in dusty universe - a possible alternative to dark energy?

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Abstract

There have been of late renewed debates on the role of inhomogeneities to explain the observed late acceleration of the universe. We have looked into the problem analytically with the help of the well known spherically symmetric but inhomogeneous Lemaitre-Tolman-Bondi (LTB) model generalised to higher dimensions. It is observed that in contrast to the claim made by Kolb et al the presence of inhomogeneities as well as extra dimensions can not reverse the signature of the deceleration parameter if the matter field obeys the energy conditions. The well known Raychaudhuri equation also points to the same result. Without solving the field equations explicitly it can, however, be shown that although the total deceleration is positive everywhere nevertheless it does not exclude the possibility of having radial acceleration, even in the pure dust universe, if the angular scale factor is decelerating fast enough and vice versa. Moreover it is found that introduction of extra dimensions can not reverse the scenario. To the contrary it actually helps the decelerating process.

Keywords : accelerating universe ; inhomogeneity; higher dimensions
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1 INTRODUCTION

Following three discoveries in the last century our concept of cosmological evolution has undergone a sea change - in contrast to Einstein’s idea of a static universe Hubble and Slipher (1927) established that it is in fact expanding. Further the CMBR spectra as well as primordial nucleosynthesis studies in the sixties show that the observable universe was in an extremely hot dense state in the past and has since been expanding for the last 13.5 Gyr. as dictated by Einstein’s theory. Finally from the high redshift supernovae data in the last decade [1] we know that when interpreted within the framework of the standard FRW type of universe (homogeneous and isotropic) we are left with the only alternative that the universe is now going through an accelerated expansion with baryonic matter contributing only five percent of the total budget. Later data from CMBR studies [2] further corroborate this conclusion which has led a vast chunk of cosmology community ([3] and references therein) to embark on a quest to explain the cause of the acceleration. The teething problem now confronting researchers is the identification of the mechanism that triggered the late inflation. Workers in this field are broadly divided into two groups - either modification of the original general theory of relativity or introduction of any mysterious fluid in the form of an evolving cosmological constant or a quintessential

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type of scalar field. So far the main emphasis for explaining the recent acceleration rests on the assumption of a homogeneous FRW type of model. But measurements of average matter density from the different cosmic probes on supernovae ($\Omega_M \sim 0$), galaxy distributions ($\Omega_M \sim 0.3$) and CMBR ($\Omega_M \sim 1$) point to a highly confusing picture of the universe such that at least one of the two assumptions must be wrong. As is well known theorists attempt to evade the problem by introducing a cosmological constant (vacuum energy) such that values now become for best fits as supernovae ($\Omega_M \sim 0.3$ and $\Omega_\Lambda \sim 0.7$), galaxy distributions ($\Omega_M \sim 0.3$) and CMBR ($\Omega_M + \Omega_\Lambda \sim 1$). But the popular explanation with the help of a cosmological constant is beset with serious theoretical problems because absence of acceleration at redshifts $z \geq 1$ implies that the required value of the cosmological constant is approximately 120 orders of magnitude smaller than its natural value in terms of Planck scale [4]. As for the alternative quintessential field we do not in fact have a theory that would explain, not to mention predict, the existence of a scalar field fitting the bill without violating the realistic energy conditions. Moreover we can not generate this type of a scalar field from any basic principles of physics. So there has been a resurgence of interests among relativists, field theorists, astrophysicists and people doing astroparticle physics both at theoretical and experimental levels to address the problems emanating from the recent extra galactic observations without involving a mysterious form of scalar field by hand but looking for alternative approaches based on sound physical principles. Alternatives include, among others, higher curvature theory, axionic field and also Brans-Dicke field. Some people attempted to look into the problem from a purely geometric point of view - an approach more in line with Einstein’s spirits. For example, Wanas [5] introduced torsion while Neupane [6] modifies the spacetime with a warped factor in 5D spacetime in a brane like cosmology and finally addition of extra spatial dimensions in physics [7]. But the problem with Wanas’ model is that the geometry is no longer Riemannian. On the otherhand the fact that the warped spacetime always generates acceleration is fairly wellknown and it follows from the wellknown Raychaudhuri equation also. Moreover all the conceptual problems relating to brane models are present in Neupane’s model.

In this context one important thing should not escape our attention. One intriguing fact in the framework of the standard FRW model is that the accelerating phase coincides with the period in which inhomogeneities in the matter distribution at length scales $< 10$ Mpc become significant so that the Universe can no longer be approximated as homogeneous at these scales. One should note that homogeneity and isotropy of the geometry are not essential ingredients to establish a number of relevant results in relativistic cosmology. One need not be sacrosanct about these concepts so far as the relativistic cosmology is concerned. For instance, from the early sixties to the early seventies ([8] and references therein), a research program on the singularity properties of general cosmological solutions has been conducted without relying on the isotropy and on the homogeneity of the geometry. The theme of the present article is somehow opposite to the one analysed in [8] where the emphasis was on the role of the inhomogeneities (and anisotropies) in the proximity of a cosmological singularity. A link between inhomogeneities and cosmological acceleration has been pursued in various studies in recent past, although the journey is not free from serious controversies creeping up time to time. There have been
arguments, based on perturbative estimates, that the backreaction of super horizon inhomogeneities on the cosmological expansion is significant and could cause acceleration \cite{9} when observed from the centre of perturbation. This idea is later supplemented by Wiltshire \cite{10} and also Carter et al \cite{11} where the observed universe is assumed to be an underdense bubble in an Einstein-de Sitter universe and it was shown that from observational point of view their results become very similar to the predictions of $\Lambda$CDM model. However, the validity of the perturbative ansatz is questionable in that the claimed acceleration is later shown to be due to the result of extrapolation of a specific solution to a regime where both the perturbative expansion breaks down and and the constraints are violated \cite{12}. Again we know \cite{13} that in a matter dominated nonrotating model where particles interacting with one another move along geodesic lines it is always possible to define a coordinate system which is at once synchronous ($g_{00} = 1$) and comoving. With this input Hirata and Seljak \cite{14} conclusively showed from Ray Chaudhuri equation that in a perfect fluid cosmological model that is geodesic, rotation-free and obeys the strong energy condition ($\rho + 3p \geq 0$), a certain generalisation of the deceleration parameter $q$ must be always non-negative. But even with the perturbation considered by Kolb et al the vorticity vanishes and consequently Kolb’s claim is flawed. On the other hand, Iguchi et al \cite{15} did obtain simulated acceleration in Lemaitre Tolman (LT) models with $\Lambda = 0$ that obey the conditions set by Hirata et al, which subsequently led Vanderveld et al\cite{16} to draw attention to this apparent contradiction between these two conclusions. However in a recent communication Krasinski et al \cite{17} showed that LT models that simulate accelerated expansion also contain a weak singularity, and in this case the derivation of HS breaks down. In addition to this, there are other singularities that tend to arise in LT models, and Vanderveld et al have failed to find any singularity-free models that agree with observations. So the apparent contradiction is resolved. On the other hand Hansson et al \cite{18} argued that when taking the real, inhomogeneous and anisotropic matter distribution in the semi-local universe into account, there may be no need to postulate an accelerating expansion of the universe at all despite recent type Ia supernova data. In fact inhomogeneous structure formation may alleviate need for accelerating universe.

Here we would like to understand if an inhomogeneous spacetime, filled with incoherent matter, can be turned into an accelerating universe at later times in the framework of a higher dimensional spacetime. The inhomogeneities considered in the present investigation may arise during an early inflationary stage when quantum mechanical fluctuations of the geometry and of the inflaton field are inside the Hubble radius. Depending upon the parameters of the inflationary phase, the initial quantum fluctuations will be amplified leading to a quasi-flat spectrum of curvature perturbations that accounts, through the Sachs-Wolfe effect, for the tiny temperature ripples detected in the microwave sky by several experiments.

As pointed out earlier from Raychoudhury equation it can be shown that in a dust dominated universe there must be a non-vanishing vorticity in order to obtain a negative deceleration parameter. This conclusion negates Kolb’s idea of presenting inhomogeneity as the possible candidate to explain late time acceleration. Also the possibility that the full non-perturbative solutions of the Einstein’s equation for inhomogeneous model can exhibit accelerated expansion was recently proved wrong
by Alnes et al [19] who tried to examine whether spherically symmetric inhomogeneous universe with dust accepts negative deceleration parameter and showed that no physically realistic solution will allow that.

Following Alnes et al we are motivated to see whether introduction of extra spatial dimensions in the inhomogeneous dust distribution can give any additional input in the direction of explaining acceleration in the recent past. Multidimensional space-time is believed to be particularly relevant in the context of cosmology. Moreover in a recent communication [20] it is argued that quantum fluctuations in 4D spacetime do not give rise to dark energy but rather a possible source of the dark energy is the fluctuations in the quantum fields including quantum gravity inhabiting extra compactified dimensions. Here we have the extra advantage that the exact solution for a spherically symmetric inhomogeneous distribution of a matter dominated universe for an \((n+2)\)-dimensional space-time with \(n \geq 2\) is earlier given by us [21].

The solution here also carries two free, spatially dependent functions \(f\) and \(M\).

The motivation of the present work is twofold. Firstly we do away with the assumption of any extraneous scalar field with faulty energy conditions but rather confine ourselves to a clearly physical parameter- inhomogeneous distribution. Secondly inspired by many recent successes of higher dimensional theories we here take a \((n + 2)\) dimensional spacetime as our geometry. However, one should point out at the outset that although the spacetime we have taken here for simplicity has been utilised in the literature by a number of authors in the past (for example, see [22] and references therein) it is open to criticism and needs further refinement in future work. We have organised our paper as follows: We develop the mathematical tools in section 2 and following analogous 4D results define the relevant astrophysical parameters suitable for the inhomogeneous, anisotropic model. This particularly applies to Hubble parameter, which unlike the homogeneous, isotropic case lacks a precise definition. Without exactly solving the field equations here we have been able to show in a general way that with realistic matter field the average volume expansion should always be decelerating. But if the radial expansion decelerates fast enough the angular expansion accelerates even in pure dust case and vice versa. But the presence of extra dimensions does not help matters, rather it acts as an impediment. With the help of our exact solution we find exact expression of the deceleration parameter in section 3 to see that no acceleration is possible in this case. The above result is confirmed in section 4 with the help of well-known Raychaudhuri equation. The paper ends with a discussion in section 5.

2 Mathematical formalism

The \((n + 2)\) dimensional metric for a spherically symmetric inhomogeneous space-time was first given by given by Banerjee et al [21]

\[
\text{ds}^2 = \text{dt}^2 - \frac{R^2(r, t)}{1 + f(r)} \text{dr}^2 - R^2(r, t) \text{d}X_n^2
\]

where \(dX_n^2\) represents an n-sphere with

\[
dX_n^2 = d\theta_1^2 + \sin^2 \theta_1 d\theta_2^2 + \ldots + \sin^2 \theta_1 \sin^2 \theta_2 \ldots \sin^2 \theta_{n-1} d\theta_n^2
\]
and the scale factor, \( R(r,t) \) depends both on space and radial coordinates \((r,t)\) respectively. A prime overhead denotes \( \partial/\partial r \) and a dot denotes \( \partial/\partial t \).

Here \( f(r) \) is an arbitrary function of \( r \) associated with the curvature of \( t-\text{const.} \) hypersurface subject to the restriction

\[
1 + f(r) > 0
\]

(3)

For mathematical simplicity we here take the higher dimensional metric with the topology \( R^{n+2} \), which is however not very realistic. As mentioned in the Introduction this type of metric, although frequently used in literature [22], suffers from the disqualification that we do not get reduction of physical quantities (say, deceleration parameter) for an effective 4D universe as dimensional reduction is not possible. In what follows we shall consider a dust distribution and all the dimensions including the extra ones are treated on same footing. The original 4D metric was first studied by Lemaitre, Tolman and Bondi and later has been used in various astronomical and cosmological contexts. The space time (1) is a generalisation of the well known LTB metric for the \((n+2)\) dimensions. Relevant to point out that it reduces to the \((n+2)\) dim. generalised FRW metric given earlier by Chatterjee et al [23] in the limit \( R(r, t) = a(t) r \) and \( f(r) = k r^2 \) where \( a \) is the FRW scale factor and \( k \) is the curvature constant.

A comoving coordinate system is taken such that \( u^0 = 1, u^i = 0 \quad (i = 1, 2, \ldots, n + 1) \) and \( g^{\mu\nu} u_\mu u_\nu = 1 \) where \( u_i \) is the \((n+2)\)-dimensional velocity. The energy momentum tensor for a dust distribution in the above defined coordinates is given by

\[
T^\mu_\nu = \rho_M(r, t) \delta^\mu_0 \delta^\nu_0 - \rho_\Lambda \delta^\mu_\nu
\]

(4)

where \( \rho_M(r, t) \) is the matter density and we have also kept the vacuum energy \( \rho_\Lambda \) for generality. The fluid consists of successive shells marked by \( r \), whose local density is time-dependent. The function \( R(t, r) \) describes the location of the shell marked by \( r \) at the time \( t \). Through an appropriate rescaling it can be chosen to satisfy the gauge

\[
R(0, r) = r
\]

(5)

The metric (1) with Einstein’s field equations and energy momentum tensor given by equation (4) gives the following independent differential equations as

\[
\frac{n(n-1)}{2} \frac{\dot{R}^2 - f(r)}{R^2} + \frac{n}{2} \frac{2\dot{R} \dot{R} - f'(r)}{\dot{R}R'} = 8\pi G (\rho_M + \rho_\Lambda)
\]

(6)

\[
\frac{n(n-1)}{2} \frac{\ddot{R}^2 - f(r)}{R^2} + n \frac{\ddot{R}}{R} = 8\pi G \rho_\Lambda
\]

(7)

One can integrate the last equation by defining \( \dot{R} = p(R) \) such that the scale factor \( R \) itself becomes the independent variable. The equation (7) now reduces to a Bernoulli type first order differential equation as

\[
p' = g(R)p + h(R)p^{-1}
\]

(8)
where \( g(R) = \frac{1-n}{2} R \) and \( h(R) = \frac{8\pi G \rho}{R} + \frac{(n-1)f}{R} \). Following the standard method of solving this type of equation we finally get

\[
\frac{\dot{R}^2}{R^2} = \frac{M(r)}{R^{n+1}} + \frac{f(r)}{R^2} + \frac{16\pi G \rho \Lambda}{n(n+1)}
\]

where \( M(r) \) is an arbitrary function of integration and depends on \( r \). From the above equations it further follows that for pure matter field (\( \rho_\Lambda = 0 \)) the equation (9) reduces to

\[
\dot{R}^2 = f(r) + \frac{M(r)}{R^{n-1}}
\]

which, again, gives

\[
\frac{n M'}{2 R^2 R^n} = 8\pi G \rho_M
\]

such that \( M(R) \) is non negative, being a measure of the mass content for the n-sphere upto the comoving radius \( r \). The generalized mass function \( M(r) \) of the fluid can be chosen arbitrarily. It incorporates the contributions of all shells up to \( r \) and determines the energy density through equation (11). Because of energy conservation \( M(r) \) is independent of \( t \). Moreover the actual dependence of the arbitrary functions \( M(r) \) and \( f(r) \) are determined by the specific nature of the inhomogeneities of our model considered.

Inhomogeneous distributions being always a bit obscure one can attempt greater transparency via usage of familiar physical quantities like Hubble constant, \( H \) and also the density parameter, \( \Omega_M \) from equation (9) through analogy with the well-known homogeneous FRW equations generalised to \( (n+2) \) dimensions

\[
H^2(t) = \frac{\dot{a}^2}{a^2} = H_0^2 \left[ \Omega_M \left( \frac{a_0}{a} \right)^{n+1} + \Omega_\Lambda + \Omega_c \left( \frac{a_0}{a} \right)^2 \right]
\]

where \( a_0 = a(t_0) \) is the current value of the scale factor. Comparing equations(9) and(12) one can now identify the local Hubble constant as

\[
H(r, t) = \frac{\dot{R}(r, t)}{R(r, t)}
\]

and local matter density via

\[
M(r) = H_0^2 \Omega_M(r) R_0^2(r)
\]

where

\[
f(r) = H_0^2(r) \left[ \Omega_M(r) + \Omega_\Lambda(r) - 1 \right] R_0^2(r)
\]

where \( R_0 = R(r, t_0) \), \( H_0 = H(r, t_0) \) and \( \Omega_\Lambda(r) = \frac{16\pi G \rho \Lambda}{n(n+1)H_0^2(r)} \). The equation (9) can now be recast as

\[
H^2(r, t) = H_0^2 \left[ \Omega_M(r) \left( \frac{R_0}{R} \right)^{n+1} + \Omega_\Lambda(r) + \Omega_c(r) \left( \frac{R_0}{R} \right)^2 \right]
\]

where \( \Omega_c(r) = 1 - \Omega_\Lambda(r) - \Omega_M(r) \). One should note at this stage that although seemingly same the essential difference of this generalised LTB expressions from the
standard FRW case is that all the quantities here depend on spatial coordinate also and when \( n = 2 \) they reduce to the 4D LTB case. This is true even for the gauge freedom exercised in (5). While for the FRW case the present value of the scale factor \( a(t_0) = a_0 \) can be chosen to be any positive number the analogous LTB scale factor \( A(r t_0) \) can be chosen to be any smooth and invertible positive function. From the relations (6) and (7) a little mathematical exercise yields

\[
\frac{n}{n+1} \frac{\ddot{R}}{R} + \frac{1}{n+1} \frac{\dot{R}'}{R'} = -\frac{8\pi G}{n(n+1)}[(n-1)\rho_M - 2\rho_\Lambda]
\]

The equation (17) tells us that the average acceleration in our case is generally negative unless \( \rho_\Lambda > \frac{n-1}{2} \rho_M \). Given the fact that the current observational value of \( \Lambda \) is too small the last inequality is a remote possibility. It has not also escaped our notice that even for a pure matter dominated model the radial acceleration \( \frac{\dot{R}'}{R} \) is possible if our angular scale factor is decelerating fast enough and vice versa. Another important fallout is the role of the extra dimensions in the dynamical process. The equation (16) shows that as the no. of dimensions increases the possibility of achieving acceleration in the model further recedes. We shall subsequently see that this result also follows from the well known RayChowdhuri equation as well.

The volume expansion rate for our \((n+2)\) dim. metric is defined through the \((n+2)\)-velocity of the fluid, \( u^a \) as

\[
(n+1)H = u^a_{,a} = u_{a;b} g^{ab} = u_{a;b} h^{ab}
\]

where

\[
h^{ab} = g^{ab} + u^a u^b
\]

While the definition works perfectly well for a FRW like homogeneous distribution of matter it is always a bit ambiguous to define the deceleration parameter of an inhomogeneous anisotropic model because the relation (18) does not take into account the directional preference of the metric. For example, Tolman-Bondi has a preferred direction, being the radial one. We can still give an operational definition to the average 'volume acceleration' of our model. For inhomogeneous model the directional preference need to be emphasized in the expression for expansion. We define a projection tensor \( t^{ab} \) that projects every quantity perpendicularly to the preferred spacelike direction \( s^a \) (and of course the timelike vector field, \( u^a \)) such that

\[
t^{ab} = g^{ab} + u^a u^b - s^a s^b = h^{ab} - s^a s^b
\]

For our metric (1),

\[
p^a = \frac{\sqrt{1 + f(r)}}{R'} \nabla
\]

and the tensor projects every physical quantity in a direction \( \perp \) to \( s^a \). One can now define the invariant expansion rates as

\[
H_r = u_{a;b} s^a s^b = \frac{\dot{R}'}{R'}
\]

\[
H_\perp = \frac{1}{n} u_{a;b} t^{ab} = \frac{\dot{R}}{R}
\]
so that

$$H = \frac{n}{n + 1} H_{\perp} + \frac{1}{n + 1} H_r$$

(24)

Evidently the above definition gives a sort of averaging over the various directions for our anisotropic model.

If one relaxes the condition of any particular preferred direction (like the radial one as in LTB model) one can explore the definition of the Hubble parameter in a more transparent way considering its directional dependence [24] as follows:

$$H = \frac{1}{n + 1} u^a + \sigma_{ab} J^a J^b$$

(25)

where $\sigma_{ab}$ is the shear tensor and $J^a$ a unit vector pointing in the direction of observation. For an observer located away from the centre of the configuration it gives for our LTB case

$$H = \frac{\ddot{R}}{\dot{R}} + \left( \frac{\dddot{R}}{\dot{R}} - \frac{\ddot{R}}{\dot{R}} \right) \cos^2 \theta$$

(26)

where $\theta$ is the angle between the radial direction through the observer and the direction of observation. Naturally when the two directions coincide, $\theta = 0$ we get $H = H_r$ and for $\theta = \pi/2$ it is $H = H_\theta$. A definition of deceleration parameter in a preferred direction can also be given in terms of the expansion of the Luminosity distance $D_L$ in powers of redshift of the incoming photons. For small $z$ one gets

$$q = -\frac{\dot{H} d^2 D_L}{dz^2} + 1$$

(27)

For $\theta = 0$ and $\theta = \pi/2$ the acceleration is respectively

$$q_r = -\left( \frac{\dot{R}}{\dot{R}} \right)^2 \left[ \frac{\ddot{R}}{\dot{R}} - \frac{\sqrt{1 + f}}{\ddot{R}} \left( \frac{\dddot{R}}{\dot{R}} - \frac{1}{2} \left( \frac{\ddot{R}}{\dot{R}} \right)^2 \right) \right]$$

(28)

$$q_\perp = -\left( \frac{\dot{R}}{\dot{R}} \right)^2 \frac{\ddot{R}}{\dot{R}}$$

(29)

We shall subsequently see in section 4 that deceleration parameter defined this way has an important difference from what we later get in equation (48). Here the parameters do not depend solely on local quantities as opposed to the acceleration parameter of (48). For example we get via equation(10)

$$q_\perp = (n - 1) \frac{M(r)}{R^{n+1}} \frac{1}{H_{\perp}^2}$$

(30)

Thus the equation (30) tells us that here the deceleration parameter $q_\perp$ depends on the total mass function and not on the local energy density of (48).

All the expressions reduce to the familiar Tolman-Bondi case when $n = 2$. It is very difficult to find a general solution of the equation(10). But for the 4D case the scale factor may be expressed in a parametric form for $f(r)$ (-1, 0, +1). But for $n > 2$ the equation can not be expressed in a parametric form as the system of equations become elliptic.
3 Higher dimensional LTB metric

Case I \((f = 0)\)

Since the WMAP data\(^{25}\) shows that the universe is spatially flat to within a few percent we can take \(f = 0\) to get the globally flat solution in \((n + 2)\) dimensions as

\[
R = \left(\frac{n + 1}{2}\right)^{\frac{2}{(n+1)}} M^{\frac{1}{(n+1)}} (t - t_0)^\frac{2}{(n+1)}
\]  

(31)

where \(t_0(r)\) is some arbitrary integration function of \(r\).

Hence

\[
\dot{R} = \left(\frac{n + 1}{2}\right)^{\frac{(1-n)}{(n+1)}} M^{\frac{1}{(n+1)}} (t - t_0)^\frac{1-n}{(n+1)}
\]  

(32)

\[
\ddot{R} = \left(\frac{n + 1}{2}\right)^{\frac{(1-n)}{(n+1)}} M^{\frac{1}{(n+1)}} \left(\frac{1-n}{1+n}\right) (t - t_0)^\frac{-2n}{(n+1)}
\]  

(33)

So,

\[
q = -\frac{\ddot{R}}{R \left(\frac{\dot{R}}{R}\right)^2} = (n - 1) \left(\frac{n + 1}{2}\right)^\frac{2}{(n+1)}
\]  

(34)

Thus \(n < 1\) is needed for acceleration. Further the fig 1 shows that with dimensions deceleration also increases. So addition of extra dimensions is, at least for this model, counterproductive.

Case II \((f \neq 0)\)

As is well known that in the 4D spacetime we do not get the solution of the equation(10) in a closed form for \(f(R) \neq 0\). At best one gets a parametric form of solutions. But a positive thing in higher dimensional cosmology lies in the fact that at least in 5D case\((n = 3)\)we get an analytic solution as

\[
R = \left[f(t-t_0)^2 - \frac{M}{f}\right]^\frac{1}{2}
\]  

(35)

The above equation has been utilised to extensively study the shell crossing and shell focussing singularity generally associated with any inhomogeneous collapse(see

Figure 1: \(q\) vs \(n\) and \(\dot{R}\) vs \(t\)
ref [21] for thorough discussion) such that the 5D case \((n = 3)\) gives

\[
\dot{R} = -M \left[ f(t - t_0)^2 - \frac{M}{f} \right]^{-\frac{3}{2}} \tag{36}
\]

\[
q = -\frac{\dot{R}}{R} \frac{1}{(\frac{R}{R})^2} = \frac{M}{f^2(t - t_0)^2} \tag{37}
\]

Thus for deceleration parameter to be negative, \(M\), which is identified as the mass density parameter must be negative. But equation (11) demands that \(M'(r)\) must be positive which in turn demands \(M(r) \geq 0\). Hence dust dominated spherically symmetric model even in higher dimension does not allow acceleration with a physically realistic matter field.

## 4 Generalised Raychaudhuri Equation

It may not be out of place to address the situation discussed in the last section with the help of the well known Ray Chaudhuri equation\([26]\), which in general holds for any cosmological solution based on Einstein’s gravitational field equations. In an earlier work \([27]\) one of us extended the Ray Chaudhuri equation, the null congruence condition and also the focussing theorem to \((n + 2)\) dimensions to study how inclusion of extra spatial dimensions alters the possibility of occurrence of singularities in physically realistic situations. We later applied the generalised equation to specific higher dimensional cosmological problems. Before writing the generalised equation proper the following definitions in \((n+2)\) dimensions may be in order

\[
\sigma_{\mu\nu} = u_{(\mu;}^\nu) - \frac{1}{n + 1} \Theta (g_{\mu\nu} + u_{(\mu} u_{\nu)}) + u_{(\mu} u_{\nu)} j^j u^j \tag{38}
\]

where for our model given by equation(1)

\[
\Theta = u^\nu_{;\mu} = \frac{\dot{R}'}{R'} + n \frac{\dot{R}}{R} \tag{39}
\]

\[
\omega_{\mu\nu} = u_{[\mu;\nu]} - a_{[\mu} u_{\nu]} \tag{40}
\]

Here \(\Theta\) is the rate of volume expansion and \(\sigma_{\mu\nu}\) is the rate of shearing.

For our LTB metric generalised to \((n + 2)\) dimensions this gives

\[
\sigma_1^1 = \frac{n}{n + 1} \left( \frac{\dot{R}'}{R'} - \frac{\dot{R}}{R} \right) \tag{41}
\]

and

\[
\sigma_2^2 = \sigma_3^3 = \ldots = \sigma_{n+1}^{n+1} = -\frac{1}{2} \sigma_1^1 \tag{42}
\]

This finally gives for our \((n + 2)\) dim. model the shear scalar as
This reduces to the 4D value \([28]\) of shear scalar for \(n = 2\). It may be noted that here \((\mu, \nu)\) run from zero to \((n+1)\).

With the help of the above definitions we get after a long but straight calculation the well known Ray Chaudhuri equation generalised to \((n+2)\) dimensions as

\[
\Theta_{\mu \nu} = \dot{v}^\mu - 2(\sigma^2 - \omega^2) - \frac{1}{n+1} \Theta^2 + R_{\nu \alpha} v^\nu v^\alpha
\]  

(44)

In view of Einstein’s equations the last term in the equation(44) may be replaced by \(-8\pi G[T_{\nu \alpha} - \frac{2}{n} T]\), where \(G\) is now the \((n+2)\) dim. gravitation constant. With matter field expressed in terms of mass density and 3D and higher dimensional pressures the \((n + 2)\) dimensional Ray Chaudhury equation is finally given by,

\[
\dot{\Theta} = -2(\sigma^2 - \omega^2) - \frac{1}{(1+n)} \Theta^2 - \frac{8\pi G}{n} [(n - 1)\rho + 3p + (n - 2)p_e]
\]  

(45)

in a co moving reference frame. Here \(p\) and \(p_e\) are the 3D and extra dimensional isotropic pressure respectively.

With the help of equation(24) we get an expression for an effective deceleration parameter as

\[
q = -\frac{\dot{H} + H^2}{H^2} = -1 - \frac{\dot{\Theta}}{\Theta^2}
\]  

(46)

which allows us to write,

\[
\Theta^2 q = 2(n + 1)\sigma^2 + \frac{8\pi G}{n} (n + 1) [(n - 1)\rho + 3p + (n - 2)p_d]
\]  

(47)

One should look at this relation with caution. While working in higher dimensional cosmology it is envisaged that following dimensional reduction as the 3D expands the extra dimensions shrink to a microscopic size as to be invisible with the existing low energy limit so that all physical quantities (e.g. deceleration parameter) become effectively four dimensional. As we have to sacrifice dimensional reduction for mathematical simplicity nothing of that sort takes place in our case, which is definitely a major defect of the present analysis and need to be rectified in a future work. Although in reference [9] Kolb et al claimed that the vorticity of the LTB metric is nonvanishing our careful calculations show that \(\omega = 0\) even in the generalised LTB model. As we are dealing with a LTB cosmology the fluid is vorticity free and pressure less in standard \(4D\) as well as higher dimension. So we finally get for the particular case of generalised LTB model\((p = p_e = 0)\)

\[
\Theta^2 q = 2(n + 1)\sigma^2 + \frac{8\pi G}{n} (n + 1)(n - 1)\rho
\]  

(48)

As we can see \(\rho, \Theta^2\) and \(\sigma^2\) all are positive we can conclude \(q\) is positive. So the addition of extra dimensions has no qualitative impact in determining the signature
of the deceleration parameter. Let us analyse the last equation a little more thoroughly. For the flat $(n+1)D$ space with $f = 0$ a long but straightforward calculation shows that

$$\theta = \frac{1}{(t - t_0)} \left[ \frac{2(M'_M)(t - t_0) - t'_0}{(M'_M)(t - t_0) - t'_0} \right]$$

(49)

and

$$\sigma^2 = \frac{n^2(n + 4)}{4(n + 1)^2(t - t_0)^2} \left[ \frac{t'_0}{(t - t_0)M'_M - t'_0} \right]^2$$

(50)

Thus with zero curvature, the expansion scalar does not depend on the dimension but falls off as $1/t$ with time. This, in our opinion, is not a generic result but holds for this specific case only. But the shear scalar does depend on the dimension, falling off faster as dimension increases. From equation (50) it appears that shear vanishes when $t'_0 = 0$. In fact in the standard treatment of the LTB metric the radial coordinate is taken such that the initial energy density is perceived as homogeneous. Moreover $f(r)$ is taken as zero to provide consistency with the matter dominated flat model. The inhomogeneity is introduced through the function $t_0(r)$ that appears in the integration of the field equations and determine the local bigbang time. When we plug in all these expressions in the equation (48) we find that the deceleration parameter still increases with the increase in dimension as evident from the fig.2 and we can not get any flip in its signature. This finding is also in line with our observation in the last section.

Our investigations in both the sections leave the impression that physically acceptable inhomogeneous models with a realistic matter field are unable to account for observations. But we argue that inhomogeneous models like LTB include FRLW models as a subclass. Thus, if the FLRW models are considered good enough for cosmology, then the LT models can only be better: they constitute an exact perturbation of the FLRW background, and can reproduce the latter as a limit with an arbitrary precision. The most serious misconception emanates from the realm of accelerating universe itself. One can have a very good fit with observations even with $q < 0[29, 30]$. An important point to remember is that the expansion rate of the universe is not a quantity that is directly observable. It is inferred indirectly through the observations only after one assumes a model for the expansion of the universe. Thus, instead of trying to explain why the expansion is 'accelerating', one should try to explain the data themselves directly in terms of observable quantities.
One such observable is the Luminosity distance-redshift relation, which is probed by supernova observations. While within the framework of homogeneous and isotropic model this relation can only be explained if the expansion rate is accelerating, this is not the case with inhomogeneous models like the LTB one. As discussed in section x the added freedom of having a position dependent expansion in LTB models allows one to explain the data without the need for the expansion to accelerate locally. The explanation will then be that the expansion rate is highest at \( r = 0 \) and decreases with distance from the centre, since the oldest supernova are also furthest away \[31\]. In fact Iguchi et al \[15\] show that models with \( M(r) > 0 \) can reproduce the relationship of a \( \Lambda CDM \) model up to \( z \sim 1 \), but not for higher redshifts. In this work, however, we have not so far tried to confront our higher dimensional model with actual SNIa observations and leave that for a future work to ascertain if the addition of extra dimensions makes a better fit with experimental data.

5 Discussion

The main shortcoming of the present analysis is the choice of the topology of the spacetime itself. Unlike the brane inspired models where observable matter is trapped on a brane but the extra space has macroscopic size or the \textquoteleft space-time-matter' theory of Wesson \[32\] where extra dimensions is a product space with non-compact macroscopic size but matter field is a manifestation of the higher dimensional effect we here, for simplicity, \textit{naively} take a spacetime with topology \( R^{n+2} \) such that all the spatial dimensions are taken on the same footing. This may be the case in the very early universe before the cosmology underwent the compactification transition. But here we are discussing a late scenario where the universe is manifestly 4D with extra dimensions presumably compactified below planckian length. As our spacetime is not amenable to the desired feature of dimensional reduction, in a certain sense the relevance of this model for the current scenario is somewhat obscure. As a future exercise one should address the problem such that the extra space forms a compact manifold with symmetry group \( G \) such that \( (n+1) \) spatial symmetry group is a direct product of \( O(3) \times G \) and not the simple \( O(n + 1) \).

Aside from the above defect and presumably a host others we have made a preliminary attempt to see if the presence of inhomogeneities or extra spatial dimensions separately or jointly can achieve late acceleration without the aid of any extraneous unphysical quintessential scalar field or an evolving cosmological constant. In fact the notion of accelerating expansion has subtleties in inhomogeneous cosmology. The conventional definition of the deceleration parameter through equation (17) is bizarre, if not confusing. It actually accounts for local volume increase during the expansion. One should note that for inhomogeneous or anisotropic model it pertains to a sort of averaging over various directions. For LTB model we find that this average expansion rate is always decelerating for positive energy density. This is also corroborated by the wellknown Raychaudhuri equation. It has not escaped our notice that presence of extra dimensions actually favours the decelerating process. However, without explicitly solving the field equations we find that the radial or the angular acceleration is possible even in pure dust distribution if any one of them
decelerates fast enough even in LTB model. In recent past several authors [33] for example, Darabi as also Szydlowski et al have claimed to get accelerating model with the help of extra dimensions. While the first case deals with primordial inflation, the second author has assumed a negative pressure in the fifth dimension $p_5 < 0$ in his 5D metric to achieve the late acceleration. This is already discussed by Panigrahi and Chatterjee in an earlier work [34]. But introduction of an unphysical negative pressure even in extra dimension does not lead us any nearer to real physics. It only shifts the unphysical input from the 3D space to the extra internal fifth dimension. To end the section a final remark may be reemphasised regarding the apparent accelerated expansion of the universe. To explain the SNIa observations the concept of accelerated expansion of the universe need to be invoked only for a FRW type of model. But one should point out that the Luminosity distance- Redshift relation, not the accelerated expansion is the quantity that can be directly measured. And within inhomogeneous models, as discussed at the end of the last section one gets better fit without the need to introduce the local accelerated expansion and consequent hypothesis of any extraneous, unphysical matter field with large negative pressure.

In the coming years we all hope to learn much more about inflation and observed cosmic acceleration of the universe (attributed to dark energy) from the highly refined computations and sophisticated observations. More data from different cosmic probes as also LHC experiments at CERN might help to address questions about the early universe and the high energy frontiers. The years ahead will certainly bring even more twists, breakthroughs and surprises in gravity and cosmology research and give more insight into the vexed problem of origin of dark energy.

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