Dynamic excitation and FE analysis to assess the shear modulus of structural timber

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Abstract A total of 99 Silver fir lumbers were tested by means of torsional vibration, to calculate the shear modulus; and free–free flexural vibration, to calculate the Modulus of Elasticity. The first five frequencies of vibration in bending were recorded and used to evaluate the shear modulus by finite elements approach. The present work shows that, thanks to flexural vibration test, it is possible to predict the torsional shear modulus using the Modulus of Elasticity and the first five frequencies of vibration deduced from flexural vibration tests ($r^2 = 0.66$, with high significance level). At present this work is the first one able to predict the shear modulus on structural lumber instead of small specimens, and the work is validated by comparing the results to the shear modulus calculated using another technique (torsional vibration test). This work has an important practical significance since it is carried out on structural lumber. The proposed method offers the possibility to predict the shear modulus with higher accuracy if compared to the European standard EN338, suggesting that flexural excitation test can be used in practice for a more accurate shear modulus prediction.

Keywords Finite elements analysis • Structural timber • Shear modulus • Torsional vibration • Flexural vibration

1 Introduction

The Shear modulus ($G$) of solid timber is usually calculated as a fraction of the bending Modulus of Elasticity ($MOE$): $G = MOE/16$ according to the European Standard EN 338 [1], and $G = MOE/20$ according to the Brazilian Standard [2]. Those standards give only an approximate value of $G$, since $G$ and $MOE$ are poorly related [3]. The $MOE/G$ ratio is affected by great variability: Curry [4] reports an $MOE/G$ ranging between 8 and 40; Doyle [5] found an $MOE/G$ of about 13.5 testing structural lumber of Southern Pine, and Doyle and Markwaldt [6] a value of 11.4. Similar values for this ratio were reported by Samson and Sotomayor-Castellanos [7] ($MOE/G = 15$) and Olsson and Källsner [8] ($MOE/G = 15–17$). Chui [3] reports an $MOE/G = 20$ for small and clear specimens of White Spruce and an $MOE/G$ varying from 25 to 30 for small unclear specimens.
The knowledge of $G$ is basic for the efficiency of the construction procedure [9], it is important when short members are considered within a structure and for members subjected to torsion action; when short timber is loaded in bending because its deflection is also dependent on $G$; it is used to design strengthening intervention and to determine torsional rigidity and lateral torsional stability of structural beams.

$G$ can be also calculated by means of static tests [10]; nevertheless those procedures are difficult to be carried out and time consuming.

Alternative methods for $G$ calculation are based on dynamic excitations.

Torsional vibration test provides a direct $G$ measurement [11], but it is not widely applied because only $G$ can be calculated, and because it requires peculiar skills and expertise to be carried out [11].

Methods based on flexural vibration are considered as a more convenient way for $G$ calculation: $G$ and MOE can be predicted simultaneously and the test is quite simple and fast to carry out. Those methods are based on the approximate Goens solution of the Timoshenko’s vibration equation [12], on which many research works are based [11, 13–18].

It is difficult to compare $G$ values based on different vibration techniques because the anisotropy of wood and the different stress distribution along the lumber in bending or torsion excitation. This fact and the anisotropy of wood makes difficult to compare $G$ calculated using these two different methods. $G$ calculated using torsional excitation is more sensible to the characteristics of the lumber surfaces, while $G$ calculated using bending excitation depends more on the characteristics of the central layers of the lumbers. It is important for an inhomogeneous material like wood, and especially for structural lumbers where localized defects, like knots, may have a different effect on $G$ in function of the defects, dimension, position and the type of excitation. Another way to predict $G$ was proposed by Olsson and Källsner [8] who investigated the capacity of FE analysis to predict the solid lumbers bending modes of vibration and $G$ and proposed this method as a possible alternative for $G$ calculation in the EN 408.

The present work aims to propose an alternative method to calculate $G$ by means of flexural vibration test together with a FE analysis.

2 Materials and methods

2.1 Lumbers

99 lumbers of Silver Fir (Abies alba, Mill.) coming from the Calabria Region (Italy) were used for the tests. The average lumber dimension is $0.05 \times 0.10 \times 2.00$ m$^3$. The moisture content ($MC$) was measured by means of a capacitive hygrometer resulting in the range between 12 and 18% at the time of test. The average density, calculated as weight to volume ratio, was 420 kg/m$^3$. The length over height ratio ($l/h$) was around 20 ($cv = 2\%$). All the lumbers were graded according to the Italian Standard for the visual strength grading of solid timber UNI 11035:2010 [19]. The mains features affecting the strength of material were recorded and here listed:

- Slope of grain: Calculated as the angle between the fiber orientation and the longitudinal axis of timber.
- Knots: Calculated dividing the minimum diameter of a knot by the dimension of the face on which it appears and recording the maximum observed value observed for each lumber.
- Ring shake: Measuring the distance between the pith and the ring shake.
- Checks: Their extension in the longitudinal and transverse directions were measured.
- Ring dimension: Measured on the lumber ends to evaluate the density of wood.

Presence of other features like biological attacks and damages potentially affecting the strength of timber were also observed and recorded.

According to the Italian Standard the lumbers are graded as S1, S2, S3 (corresponding to the strength classes) and R (rejected) in function of the presence and extension of the recorded defects. S1 and S2 correspond to C24 and S3 to C18 strength class according to the European EN 338 [1] (C24: bending strength = 14 MPa, Modulus of Elasticity = 11 GPa; C18: bending strength = 18 MPa, Modulus of Elasticity = 9 GPa). Lumbers were graded as follow: S1 = 9%, S2 = 45%, S3 = 29%, R = 16%. Knots dimension was the most important defects and the primary reason for lumber downgrade. The larger part of rejected lumber presented very large knots. Only one lumber was rejected for the presence of excessive
slope of grain (higher than 18%) and the 95% of lumbers have a slope of grain lower than 5%.

2.2 Torsional vibration test

The tests were carried out according to Divos et al. [11] who tested clear specimens using torsional test and bending excitation, reporting the torsional test as a reliable method for the evaluation of $G$. Similar results were also reported later by Roohnia and Kohantorabi [20] who compared $G$ using flexural and torsional vibration techniques: they reported a coefficient of determination of 0.93 between the $G$ calculated using torsional and flexural vibrations, ascribing the result to the fact they tested small clear and homogeneous specimens.

For the evaluation of $G$ in torsion ($G_t$) each lumber was supported in the middle of the narrow face (randomly selected) and a hammer was used to excite the lumber itself at the edge of the wide face close to one end. The frequency of vibration was recorded by means of a piezoelectric accelerometer (PCB 480E09) fixed to the specimens by bees-wax and connected to a notebook. A schematic representation of the test set up is shown in Fig. 1. The recorded signal was processed by a FFT analyzer freeware software. Since the vibration spectra in torsion is also affected by the bending vibration, the lumber excitation was done in correspondence of the nodal point of the fundamental frequency of vibration ($0.224 \times l$). In such way the influence of the fundamental frequency of vibration in bending was minimized. The fundamental frequency of vibration in torsion was used for the shear modulus calculation using the Eq. 1.

$$G_t = \frac{(2f_t)^2 \rho J_p}{ab^3 c}$$  \hspace{1cm} (1)

where $G_t$ is the shear modulus calculated from the torsional vibration test; $l$ is the lumber length; $f_t$ is the fundamental frequency of vibration; $\rho$ is the density; $J_p$ is the polar moment of inertia; $a$ and $b$ are respectively the thickness and the width of lumber; $c$ is a constant depending on the aspect ratio $ab$. For the samples of this research the constant $c = 0.229$ for $ab = 2$ [11]. The equation was deduced directly from Tymoshenko and Goodier [21] who derived the exact expressions for the stress, strain and dimensional relations of bars in torsion.

2.3 Free-free flexural vibration test

Each lumber was suspended in correspondence of their theoretical nodal points for fundamental frequency of vibration ($0.224 \times l$) using foam support and excited in thickness direction by means of a hammer impact. The mass of the support and the accelerometer and the stiffness of the supports are considered negligible so they cannot affect the mode of vibration in a significant way. Each lumber was tested edgewise with the same orientation than in torsional test. A schematic representation of the test set up is shown in Fig. 2.

The modes of vibration were recorded using the same apparatus described for the torsional tests. Figure 3 shows an example of the FFT spectra obtained from flexural vibration tests. The first 5 frequencies of vibration were recorded ($f_f^1; f_f^2; f_f^3; f_f^4; f_f^5$), the fundamental one was used to calculate the dynamic MOE according to [22] using Eq. 2

$$E_f = \frac{4\pi^2 f_f^1 l^3}{k^4 J m}$$  \hspace{1cm} (2)

where $E_f$ is the Modulus of Elasticity in free–free flexural vibration tests; $f_f^1$ is the fundamental frequency of vibration; $l$ is the lumber length; $J$ is the moment of inertia, $m$ is the mass and $k$ the constant corresponding to the fundamental mode in free–free flexural vibration. The formula was derived from the equation and solution proposed by Bernoulli [23]. The method is valid when the length-to-depth ratio is high ($l/h \geq 20$) and the elastic support influence is ignored.

This is a simple method for the assessment of the elastic properties of a mass [17], widely used in research works and industries as well [24–30]. All the

Fig. 1 Schematic test set up for torsional vibration test. Drawing not in scale

Fig. 2 Schematic test set up for free–free flexural vibration test. Drawing not in scale
five frequencies of vibration were used for $G_t$ prediction.

2.4 FE analysis and $G_t$ prediction

Freeware CALFEM® extension toolbox for MATLAB® was used to perform the FE analysis. The beam model is based on 100 elements and the mesh size was optimized to reach a good calculation convergence. No boundary conditions were applied in order to simulate the free–free condition. The lack of boundary condition provides solutions corresponding to rigid body motions; those solutions were discarded. The FE analysis was used to predict the first five frequencies of vibration ($f_{FE1}$; $f_{FE2}$; $f_{FE3}$; $f_{FE4}$; $f_{FE5}$) for different $MOE$-$G$ combinations (around 10,000 combinations were used; $G$ ranged between 0.2 and 1.5 GPa with 0.026 GPa step; $MOE$ between 5 and 19 GPa with 0.075 GPa step). For those simulations the density used was the value of 420 kg/m$^3$ being the average density of the tested beams.

The predicted frequencies were used to fit five polynomial models, one for each frequency if vibration: $G - MOE - f_{FE1}^3$; $G - MOE - f_{FE2}^3$; $G - MOE - f_{FE3}^{1/2}$; $G - MOE - f_{FE4}^{1/3}$; $G - MOE - f_{FE5}$. An example of the model developed using the fundamental frequency of vibration $f_{FE1}$ is shown in Fig. 4.

The five models were used to calculate $G$ combining the data from the flexural vibration tests, substituting $MOE$ with $E_f$ and $f_{FE}$ with $f_f$, obtaining five 5 values for each lumber ($G_1^f; G_2^f; G_3^f; G_4^f; G_5^f$).

Since the frequencies predicted by the FE analysis are based on the use of the average density, the values of $G_i^f$ were corrected by multiplying them by the factor $\sqrt{420/\rho}$, where $\rho$ is the actual density of each lumber. The whole procedure is illustrated in Fig. 5.
3 Results and discussion

Descriptive statistics for the various $G$ and $E_f$ are reported in Table 1. $G_t$, is poorly related to $E_f$ ($r^2 = 0.05$, Fig. 6). This result is similar to the ones reported by other research works who investigated the relation between $G$ and $MOE$ by means of dynamic tests [3, 31]. The relation between $G_t$ and density gives $r^2 = 0.22$ (Fig. 7), lower than the value of 0.38 reported in [32] obtained by static tests on small and clear specimens; however it is close to the value of 0.22 reported in [8] where structural lumbers of Norway Spruce were tested by means of dynamic excitation.

The mean $G_t$ value is 0.53 GPa (cv = 25%), and the mean $E_f$ is 9.4 GPa. The average $E_f/G_t$ is 19 (minimum = 8 GPa, maximum = 42 GPa (cv = 31%). $E_f/G_t$ is higher than the result obtained applying the EN 338 ($G = MOE/16$), so the method proposed in the EN 338 overestimated $G$.

$G$ predicted using the EN 338 ($G_{EN338}$) is always higher than $G_t$: the difference between the minimum values is around 38% and the difference between the maximum values is around 29%.

Since $G_{EN338}$ is calculated dividing $MOE$ by 16, also the relation between $G_t$ and $E_f$ is poor and not statistically significant.

The capacity of the FE analysis to predict the first five frequencies of vibration is good (see Table 2): maximum difference 4%. The coefficient of variability between the predicted and the observed frequencies of vibration increases from the fundamental to the 5th frequency of vibration. Moreover, the predicted set of frequencies is always different from the observed one.

![Fig. 5 Schematic representation of the procedure for the $G_t$ prediction. Experimental tests on the left and FE analysis procedure on the right](image)

![Fig. 6 Scatter plot and equation of the regression line between $G_t$ and $E_f$. Relation not statistically significant](image)

### Table 1 Descriptive statistics for various physical and mechanical properties

|        | $G_t$ (MPa) | $\hat{G}_t$ (MPa) | $G_{EN338}$ (MPa) | $E_f$ (GPa) | $\rho$ (kg/m$^3$) |
|--------|-------------|-------------------|------------------|-------------|-----------------|
| Min    | 231         | 251               | 372              | 6.0         | 224             |
| Max    | 826         | 706               | 1159             | 18.5        | 587             |
| Average| 527         | 526               | 586              | 9.4         | 420             |
| cv (%) | 25          | 20                | 21               | 21          | 14              |

$G_t$ is the shear modulus calculated from torsional tests. $\hat{G}_t$ is the shear modulus based on FE analysis and the frequencies of vibration in flexural test; $G_{EN338}$ is the shear modulus calculated according to EN 338; $E_f$ is the modulus of elasticity calculated in free–free flexural vibration test; $\rho$ is the density at the time of test.
as a consequence of the inhomogeneity of the wood itself (e.g. presence of knots, fissures, etc.), which is not considered by the FE analysis.

The ability to predict $G_i$, applying the linear models as reported in Fig. 4 (using $E_f$ and a single frequency of vibration as predictors) is poor. The best result was obtained using $\hat{G}_f^1$, calculated applying the polynomial model showed in Fig. 4. Scatter plot and the regression equation are shown in Fig. 8.

If a defect like a large knot has an important effect on $G$, but it is located close to the nodal points for a frequency of vibration in bending, it has negligible effect on the frequency of vibration itself. This explain the poor capacity to predict $G_i$ using a single frequency of vibration. Considering that different frequencies of vibration have different nodal points, a multiple linear model based on several $G$ values, obtained from different modes of vibration in bending, should represent a more precise way to predict the average $G$ of a lumber: $\hat{G}_f^1$, $\hat{G}_f^3$ and $\hat{G}_f^5$ (obtained from the 1st, the 3rd and the 5th vibration modes respectively) were used to predict $G$. The relation between the predicted $G$ ($\hat{G}_i$) and $G_i$ is shown in Fig. 9 ($r^2 = 0.66$, $p$ value <0.01, mean absolute error of about 60 MPa; $\hat{G}_f = -2.6 * \hat{G}_f^1 + 1.9 * \hat{G}_f^5 - 0.3 * \hat{G}_f^5 + 1103.1$).

Divos et al. [11] proposed a model to predict $G_i$ with a very good accuracy ($r^2 = 0.72$) using the method based on flexural vibration proposed by Chui [3].

The higher accuracy of their model is a consequence of the fact that they tested small and clear specimens which are much more homogeneous than structural lumber. The model cannot be used in this work because the method proposed by Chui [3] requires specimens for $l/h \leq 10$.

Roohnia and Kohantorabi [20] predicted $G_i$ with high accuracy ($r^2 = 0.93$) testing small, clear and well oriented specimens, but the method they proposed cannot be used on structural lumber, because it requires material perfectly oriented in two orthotropic directions, radial and tangential.

The average predicted shear modulus ($\hat{G}_i$) is less than 1% lower than $G_i$. Only a slightly improvement is possible using all the $\hat{G}_f$ values ($\hat{G}_f^1$, $\hat{G}_f^3$, $\hat{G}_f^5$) for $G_i$ prediction ($r^2 = 0.67$, $p$ value <0.01). The

![Fig. 7](image) Scatter plot and equation of the regression line between density and $G_i$. Relation statistically significant ($p$ value <0.01)

![Fig. 8](image) Scatter plot and equation of the regression line between knot $G_i$ and $\hat{G}_i$, $\hat{G}_i$ calculated from flexural vibration test ($E_f$ and $f_f$). Relation statistically significant ($p$ value <0.01)

![Fig. 9](image) Predicted shear modulus ($\hat{G}_i$) versus observed $G_i$. Relation statistically significant ($p$ value <0.01)

| $f_f/f_{FE}$ | $f_f/f_{FE}$ | $f_f/f_{FE}$ | $f_f/f_{FE}$ | $f_f/f_{FE}$ |
|-------------|-------------|-------------|-------------|-------------|
| Min         | 1.01        | 0.99        | 0.95        | 0.85        | 0.78        |
| Max         | 1.06        | 1.12        | 1.09        | 1.18        | 1.17        |
| Average     | 1.03        | 1.04        | 1.03        | 1.02        | 1.01        |
| cv (%)      | 1           | 3           | 2           | 6           | 7           |
difference is probably a consequence of the greater influence of the central part of the lumber on $f_1$, $f_2$ and $f_3$ during flexural excitation and on torsional frequency of vibration, while $f_4$, $f_5$ are less affected by the same part of the lumber. The model works pretty well taking into account the characteristics of the tested materials (variable dimensions, different $MC$) and the fact that $G$, calculated using different methods, gives different results. Indeed, the positions of the defects have different influences in bending than in torsion, and the stress distribution in torsion and bending is also different. As expected, the accuracy of the prediction largely depends on the inhomogeneity of the material.

4 Conclusions

The method proposed by the EN 338 for the $G$ prediction confirmed to be inaccurate, since it overestimate the mean $G$ value of about 10%.

A predictive model developed by means of FE analysis, based on flexural vibration test, gives more accurate results: the average predicted $G$ corresponds to the average observed shear modulus (difference less than 1%) and the model predicts $G$ with good accuracy ($r^2 = 0.66$). This is the first work carried out on structural lumber that is validated by comparing the predicted $G$ and the calculated $G$ using a reference method (torsional vibration tests). Since the $ll/h$ ratio of the tested material is within the range of the one used for the bending properties characterization of structural timber (EN 408) and the tests were carried out on structural lumber instead on small specimens the proposed method could be included in the Standard itself, after additional investigation on various species. The proposed method offers an improvement to predict the shear modulus of structural lumber and being based on flexural vibration that is easier to perform, faster and cheaper, it offers a practical alternative for $G$ calculation.

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Compliance with ethical standards

Conflict of interest  The authors declare that they have no conflict of interest.

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