The analysis of students’ reflective thinking ability viewed by students’ mathematical ability at senior high school

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Abstract. The reflective thinking ability is the students’ ability to actively think through rules cautiously to answer questions. The reflective thinking ability is also one of the higher order thinking skills (HOTS) demanded of learners. Through applying the newest national curriculum, the Indonesian government wants students to master the 21st century skills. The types of skills are communication, collaboration, critical thinking and problem solving, and creativity and innovation. This study aims to reveal the students’ reflective thinking characteristics with the qualitative method and the descriptive design. Data collection used instruments of test and interview. Subjects in this study were three students of class XI at Senior High School. The research subjects, based on the mathematical ability, were divided into three categories, that are high, medium and low. While the reflective thinking ability is divided into four stages: habitual action, understanding, reflection and critical reflection. The results showed that the student with the high level is at the stage of reflection, the student with the medium level is at the stage of understanding, and the student with the low level is at the stage of habitual action. It can be concluded that students’ reflective thinking ability is not yet developed optimally.

1. Introduction
In the face of the 21st century, the Indonesian government issued several policies in the field of education. One of its policies is the application of the 21st century education-based national curriculum, which integrates knowledge capabilities, skills, and attitudes, and Information and Communication Technology (ICT). The main support of this curriculum is mastery of Higher Order Thinking Skills (HOTS). In learning activities, students are expected to be able to predict, design, and estimate the solution of the given problem. In general, the new national curriculum requires every student to have critical thinking and problem solving, communication, creativity and innovation, and collaboration [1].

Reflective thinking is one part of HOTS that is very important for students. Reflective thinking makes the student acts reasonably through active, continuous and mindful thinking to lead to further conclusions. Reflective thinking is also an active activity, and it takes effort to explain things, and tries to connect ideas to gain the deep meaning in applying the right strategy. So, the reflective thinking consists of understanding concept and decision making maturely, when faced with a problem [2,3,4].

The reflective thinking ability can be seen from how to respond to problems with preliminary knowledge, explaining the process done, correcting mistakes found during troubleshooting, and communicating ideas [5]. In the learning process, reflective thinking can make teaching and learning...
activities more meaningful. With reflective thinking, students are not only able to answer questions, but also able to express the processes that occur in his mind when solving the problem [6]. In mathematics learning in particular, reflective thinking is one of the most indispensable ability. Because of the reflective thinking, the mistake factor of students in solving problems can be reduced and can encourage students to obtain the best strategy in achieving the learning objectives [7].

In general, reflective thinking ability is divided into three situations: pre-reflective, reflective, and post-reflective. A pre-reflective situation is a situation where the student experiences confusion or doubt; a reflective situation is a transitional situation from a pre-reflective situation to a post-reflective, or situation of the occurrence of the reflective process itself; whereas the post-reflective situation is a situation in which such confusion or doubt can be answered. Meanwhile, the reflective thinking stage is divided into five stages: suggestions, which are thinking or considering possible solutions; intellectualization, which is analysing the problem deeply to solve it; use one suggestion, that is focus on one opinion or create a strong hypothesis to start troubleshooting; mental elaboration, that is to properly assume an idea and reason one of its parts; and testing the hypothesis, which is to implement a hypothesis that is made in the real situation [2].

The reflective thinking ability on its development, is divided into four stages: habitual action that is activities that are done automatically because it has been studied previously and often used; understanding, that is using the existing knowledge without linking it to other situations; reflection, that is the constant, active consideration, with caution about everything that is believed to be true; and critical reflection, that is the highest level of the reflective thinking ability in which one can change the frame of mind and know the reason for something happening [8,9].

In general learning, mathematical reflective thinking is still rarely introduced by teachers. Teachers still tend to give mathematical formulas and concepts that already exist without inviting students to think to find mathematical formulas and concepts that he learned. So, the study of reflective thinking in learning or problem solving is very important to discuss [10].

In this study, researchers analyse students’ reflective thinking ability in solving transformation questions. The reflective thinking ability is the ability of students to actively consider the various rules that are known with caution to answer questions. The reflective thinking ability indicators used in questions consist of distinguishing between relevant and irrelevant data, evaluating/checking the truth of an argument based on the concepts/properties, and interpreting a case based on the mathematical concepts [11].

From the explanation in the background, researchers are interesting to see the students’ reflective thinking in senior high school. This study aims to reveal the students’ reflective thinking ability characteristics on the topic of the transformation. The description of reflective thinking ability is differentiated based on the level of students’ mathematical ability.

2. Method
This research used the qualitative method with the descriptive design. It was carried out in one of the senior high schools in Bandung in the academic year 2017/2018 on the odd semester after students had studied the topic of Transformation. The subject consisted of three students at senior high school who were divided into three categories of mathematical ability, that are high, medium and low ability. The subject selection was conducted using the purposive sampling technique, where informants were selected based on certain criteria. The criteria are that subject could communicate their ways of thinking clearly in writing and orally based on the recommendations of the partner teacher.

Data collection techniques in this study were an essay test and an interview about the topic of the transformation. The essay test was used to categorize students’ mathematical ability. Then, the interview was conducted deeply to observe the subject’s reflective thinking ability during the completion process. The supporting instruments used in this research are test instruments and interview guides. Data analysis techniques was conducted by: (1) data reduction by choosing the main points that fit the research focus, (2) presenting data in the form of narrative text, and (3) drawing conclusions or verifying data.
3. Results and discussion

The number of questions given in this study was three questions that represent all of the transformation topics in the form of problem solving. The first question represented the reflection and translation with the indicator of answering questions after identifying relevant and irrelevant data, the second question represented the composition rotation on different centres with the indicator of checking the truth of the statement, and the third question represented the dilatation associated with the concept of geometry with the indicator of solving the question after interpreting the case based on the involved mathematical concepts [10].

| Question Number | Level of mathematical ability (subject) |
|-----------------|----------------------------------------|
|                 | High (AF)                              | Medium (KN)      | Low (PA)       |
| 1               | Writing down what is known and asked questions. | Writing down what is known and asked questions. | Writing down what is known and asked questions. |
|                 | Identifying and write out the formulas used. | Identifying and write out the formulas used in the completion. | Identifying and write out the formulas used in the completion. |
|                 | Answering questions with coherence.      | Completing questions coherently. | Completing questions coherently. |
|                 | Completing answers with pictures.        | Completing answers with pictures. | Completing answers with pictures. |
| 2               | Writing down what is known and asked questions. | Writing down what is known and asked questions. | Writing down what is known and asked questions. |
|                 | Identifying and writing out the formulas used in problem solving. | Not resolving the issue. | Writing down the formula used in the completion. |
|                 | Answering questions coherently.          | Completing questions coherently. | Completing questions coherently. |
|                 | Making a procedural mistake in the completion steps. | d. Making concept mistakes on matrix operations. | |
| 3               | Writing down what is known and asked questions. | Writing down what is known and asked questions. | Not writing answers |
|                 | Identifying and writing out the formulas used in the completion. | Identifying and writing out the formulas used in the completion. | Answering questions in a non-coherent manner. |
|                 | Completing questions coherently.         | Answering questions in a non-coherent manner. | |
|                 | Knowing other strategies that can be used to answer questions. | Only knowing one strategy that can be used to answer questions. | |

The data in this study were obtained by observing student answer sheets and clarification process in the interview. The level of students’ mathematical ability was categorized based on the answer sheets considering correct answers. The interview was used to reveal the subject’s reflective thinking ability during solving questions. The three subjects were AF, KN, and PA.
Based on Table 1, it is known that there are some differences in answering questions between high-ability subjects (AF), medium (KN), and low (PA). The differences were, among others, seen from the completion, the ability to answer questions and knowledge related to among their strategies that can be used to answer questions. From the table also obtained that the subject already has the reflective thinking ability. The ability to think is shown by the ability of the subject in: understanding questions by writing down what is known and asked questions; planning a problem solving strategy by writing down the strategy or formula used to answer questions; carrying out the completion according to the planned strategy; and doing reflection by using or knowing other strategy to answer the given questions.

**Table 2. Results of the interview based on the level of student’s mathematical ability.**

|                      | Level of mathematical ability (subject) |
|----------------------|----------------------------------------|
|                      | High (AF)                              |
| a.                   | Understanding the question in such a short time. |
| b.                   | Being able to explain completion steps in detail and realize the mistake. |
| c.                   | Knowing sources of errors. |
| d.                   | Rechecking answers by other means. |
|                      | Medium (KN)                            |
| a.                   | Taking a little longer to understand the question. |
| b.                   | Giving a description of completion steps based on the experience in the classroom. |
| c.                   | Not solving the second question because she forgot the formula. |
| d.                   | Rechecking answers only in the answer to the first question. |
|                      | Low (PA)                               |
| a.                   | Sensing difficulty in understanding the question. |
| b.                   | Feeling difficult in explaining completion steps. |
| c.                   | Not solving the third question because not knowing completion steps. |
| d.                   | Not rechecking answers. |

Based on Table 2 about the outcome of the interview with subjects, it is obtained how students thought in answering questions. The thinking process tends to the reflective thinking that includes understanding questions and planning completion strategies; being able to explain arguments of completion steps, reasons for using rules or formulas; and rechecking completion steps.

In answering questions, there are some differences made by subjects. The subject, that has high level ability (AF), took a short time in reading and understanding questions. In addition, AF was able to explain the reason for selecting strategies and completion steps in detail. She also knew sources of errors in answers. In the answer to the second question, she made procedural errors. She also rechecked answers for each question. On the first question, she matched answers with formulas and with analysis of the geometric figure. For the second question, she did not try other strategies to answer it, because she sensed that the answer is correct. This is one of the causes for her mistake. For the third question, she mentioned alternative strategies that can be used to answer it, although in her answer sheet she used only one strategy.

Unlike AF, KN took a little longer to understand the question. In explaining completion steps, she was not more skilful than AF. KN tended to respond with her appropriate experience got from the learning in the classroom. So, in the second question, she did not answer it because she forgot the suitable formula. Related to her process in rechecking answers, she only did it in the first question because completion steps in the first question are simpler than those in other questions. In the third question, she only knew a strategy for answering it. At the same time, PA who has low level ability sensed difficulty in understanding the question. She said that questions given in the test are different with those given in the classroom. When describing the completion steps, she said that she only answered based on her habit in the classroom. In addition, she was not able to clearly explain rules used in the completion steps. On the third question, she did not answer it because she did not know how to answer it. For each question, she did not recheck answers because she worried about making mistakes.
Based on the results of analysis and discussion, it was found that AF already had the reflective thinking ability at the reflection stage. This is shown by her strategies in answering question. She always started the completion process by understanding and planning the completion strategy. In choosing a strategy or method of completion, she did it with great care and consideration, and was aware of the consequences of the chosen way [8]. But in the considering process, she did not reach the stage of checking the truth of other alternative strategies. The search for alternative strategies that can be used, is still limited to a simple analysis based on geometric shapes. She also always checked back the answer. The reflective thinking ability performed by her, that is one of the characteristics of reflective thinking, is to realize the mistake and fix it [12]. Although in the second question, she made a procedural mistake that she realized during the interview. In an interview, she tried to explain her mistake in answering the question. This showed that she does reflective thinking process by realizing its mistake and trying to fix it [13]. This mistake did not deny her reflective thinking stage. Because students who have reflective thinking with high levels of mathematical ability can make mistakes, so what is seen is the process of thinking in general [14].

Based on the same analysis, the medium skilled subject (KN) already had reflective thinking ability at the understanding stage. This is seen from her process in completion steps. She answered questions based on prior knowledge and understanding without relating it to other concepts. This is seen from her strategies described in the answer sheets. It is also seen from her characters that depend on the formula. As in the second question, she did not answer it because she forgot the formula. Her reflection efforts by rechecking is still lower than AF. KN rechecked the answer only for the short completion steps.

Based on the analysis results, PA’s answers only were based on routine habit that was ever experienced. In the process, she had difficulty in explaining completion steps. According to her, her completion steps were based on her experience when answering the topic of transformation. For understanding the difficult question, she preferred not to answer it. She also did not recheck answers. Therefore, the reflective thinking stage performed by her is at the stage of habitual action.

The reflective thinking ability possessed by the three subjects in this study has not developed optimally. This is due to several factors, including the maturity of individual ages and the less facilitating environment of reflective thinking. Where one’s capacity in reflective thinking is influenced by a stimulus-rich environment [15]. So, the stimulus to develop the reflective thinking ability should start to be continued and expanded. Because in learning activities, reflective thinking is still rare and still difficult to be introduced [16].

Based on the above discussion and analysis, subjects with high mathematical ability have higher reflective thinking ability than medium and low ability subjects. Mastery of mathematical concepts possessed by subjects who has high mathematical, affects in information processing and decision making in answering questions. Although on one of questions, AF made a mistake, but compared to KN and PA, AF had a better consideration ability in processing information. This is different from the KN that only relies on formula, or PA that easily gives up when she faces a question that she does not understand.

4. Conclusions
Based on the results and discussion in this study, the reflective thinking ability of the subject in answering questions has not developed optimally. This is indicated by the following characteristics: (1) the subject that has high mathematical ability, is in the reflection stage by showing the behaviour of understanding question before doing the work, considering completion strategies, being able to explain completion steps, rechecking answers, thinking of alternative strategies, and realizing the mistakes; (2) the subject that has medium mathematical ability, is in the understanding stage by showing behaviour, understanding questions before doing the work, answering questions with the known concepts, relying on memorizing, being able to explain the completion process, and checking answers only for uncomplicated completion steps; and (3) the subject that has low mathematical ability, in the habitual action stage by showing behaviour, understanding questions before doing the
work, answering questions in accordance with the routine experience, not being able to explain completion steps, ignoring difficult questions, and not rechecking answers.

Acknowledgments
We would like to thank to the teacher of State Senior High School 2 Bandung for supporting the research process.

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