Extending the Multiple Traveling Salesman Problem for Scheduling a Fleet of Drones Performing Monitoring Missions

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Abstract—In this paper we schedule the travel path of a set of drones across a graph where the nodes need to be visited multiple times at pre-defined points in time. This is an extension of the well-known multiple traveling salesman problem. The proposed formulation can be applied in several domains such as the monitoring of traffic flows in a transportation network, or the monitoring of remote locations to assist search and rescue missions. Aiming to find the optimal schedule, the problem is formulated as an Integer Linear Program (ILP). Given that the problem is highly combinatorial, the optimal solution scales only for small sized problems. Thus, a greedy algorithm is also proposed that uses a one-step look ahead heuristic search mechanism. In a detailed evaluation, it is observed that the greedy algorithm has near-optimal performance as it is on average at 92.06% of the optimal, while it can potentially scale up to settings with hundreds of drones and locations.

I. INTRODUCTION

Unmanned aerial vehicles (UAVs), or simply drones [1], [2], are used in a plethora of civil applications due to their ease of deployment, low maintenance cost, high-mobility and ability to hover [3]. Such vehicles are utilized for real time monitoring of road traffic, search and rescue operations, civil infrastructure inspection, wireless coverage, remote sensing, delivery of goods, security and surveillance, and precision agriculture [4].

A main advantage of drones is that, in contrast to other vehicles, they are not restricted to traveling over a road network and thus can swiftly move over disperse locations. In order to maximize this ability, their scheduled and coordinated flying is crucial. Thus, in this paper we propose a generalized graph-based solution to schedule drones for monitoring missions. Here, it is important to note that both surveillance and monitoring tasks focus on developing control laws which enable groups of robots to transverse and observe a given domain, but with a slightly different focus. The goal of surveillance is to maximize some measure of coverage or information gathering, while monitoring focuses on ensuring that certain areas of the domain (usually predefined) are visited with a certain frequency.

Recently, drone-related problems have been intensively researched and artificial intelligence (AI) techniques such as heuristic search, optimization, multi-agent systems and machine learning are becoming extremely popular. For example, Kitjacharoenchai et al. [5] study a delivery truck-drone combination problem, where autonomous drones fly from delivery trucks, make deliveries, and subsequently fly to any available delivery truck nearby. Aiming to minimize the arrival time of both trucks and drones at the depot after completing the deliveries, they propose two solutions, one using mixed integer programming and another using insertion heuristics to solve large sized problems. Moreover, Evers et al. [6] study online stochastic UAV mission planning with time windows and time-sensitive targets using heuristic methods. In addition, Delle Fave et al. [7] study the use of drones for aerial imagery collection and they define this problem as one of task assignment where the drones dynamically coordinate over tasks representing the imagery collection requests. They solve the resulting optimization problem using an asynchronous and decentralized implementation of the max-sum algorithm. In a slightly different vein, Ramchurn et al. [8] model the drones as intelligent agents and study the way human-agent collectives can address challenges in disaster response. Specifically, their proposed methodology utilizes crowd-sourcing combined with machine learning to obtain situational awareness from large streams of reports posted by members of the public. This collected information can be utilized to inform human-agent teams in coordinating multi-UAV deployments, as well as task planning for responders on the ground. In a similar vein, Baker et al. [9] study the survivor discovery problem and present a solution based on a continuous factored coordinated Monte Carlo tree search algorithm. Further, Sharafeddine and Islambouli [10] study the use of UAVs as instant recovery devices for cellular networks aiming to decide on the initial location of the UAVs. The authors propose an optimal solution using Mixed Integer Linear Programming techniques as well as an equivalent greedy algorithm. Finally, Menouar et al. [11] outline the possible applications of drones for supporting intelligent transportation systems within a smart city domain.

Against this background, we initially formulate the problem of scheduling drones across a set of locations with monitoring demand in predefined points in time as an Integer Linear Program (ILP) and we solve it offline and optimally. Given the high complexity of the problem and the equivalent limited scalability of the optimal solution, we also develop a greedy algorithm that uses heuristic search. In our experimental evaluation, we
observe that the greedy algorithm has very good scalability and performance close to the optimal. This problem is an extension of the Multiple Traveling Salesman Problem (MTSP) [12], which is based on the well-known Traveling Salesman Problem (TSP) [13]. Compared to the MTSP, in this work (i) all agents do not begin their journey from the same node, and (ii) an agent may pass from one node multiple times in predefined points in time. This problem also shares similarities with the Vehicle Routing Problem [14] but differs in point (ii) mentioned above. The model of this problem is based on [15] where the authors study the scheduling of electric vehicles in a mobility-on-demand scheme. The main differences are that (a) in this work the agents initiate their traveling in an autonomous manner, and (b) the execution of a trip is not directly related to the execution of the previous one.

The rest of the paper is structured as follows: Section II provides a detailed problem formulation, Section III describes the optimal solution of the problem and Section IV the equivalent greedy one. Section V evaluates the proposed solutions in different settings and finally, Section VI concludes this work and provides insights for future work.

II. Problem definition

We define the set of UAVs or drones \( a \in A \subseteq N \) acting as fully cooperative agents. Each agent has its own type defined by a tuple \( p_a = \{n_a^{\text{init}}, n_a^{\text{fin}}, n_a.t, c_a, v_a, v_a^{\text{max}}\} \) where \( n_a^{\text{init}}, n_a^{\text{fin}}, n_a.t \) are the initial, final and current location of the agent, \( c_a, v_a, v_a^{\text{max}} \) are the current and maximum energy level of the agent, \( v_a \) is the current velocity and \( v_a^{\text{max}} \) the maximum velocity of the agent. We consider a set of discrete points in time \( T \subseteq N, t \in T \) where \( t \) is global for the system and the same for all agents.

All agents are supposed to move across an undirected fully connected graph \( G(N,E) \) where \( n \in N \subseteq N \) is a set of nodes and \( \{i,j\} \in E \subseteq N \) is a set of edges. Every edge has a cost \( c_{(i,j)} \in \mathbb{R} \) that denotes the time to travel across an edge, or the equivalent energy required.

The system aims to schedule the agents to pass from specific nodes of the graph at specific points in time. To achieve this, we define the “flying demand” as \( d_{n,t} \in \{0,1\} \) which contains the set of points in time a node \( n \) should be visited by an agent. The trips of all agents over the edges of the graph must be scheduled and coordinated in order for the flying demand of each node to be covered to the maximum extent (i.e., fully covering the demand may be impossible due to insufficient resources). In this setting, we assume that a fully charged battery is enough for a drone to cover all trips concerned and that the agents will not collide while flying over the same edge as they fly in different altitudes. In this work, henceforth, the terms agent, UAV and drone are used interchangeably.

III. Optimal solution

In this section, we model the problem of scheduling drones as an Integer Linear Program (ILP) and we solve it optimally using IBM Ilog CPLEX 12.10. We define 4 decision variables: (1) \( f_{dn,t} \in \{0,1\} \) which denotes whether at least one agent flies over node \( n \) at time \( t \), (2) \( e_{na.a,t} \in \{0,1\} \) which denotes whether agent \( a \) hovers over location \( n \) at time \( t \), (3) \( k_{n,n',a,t} \in \{0,1\} \) which denotes whether agent \( a \) flies across the edge connecting nodes \( n \) and \( n' \) at time \( t \) and (4) \( startT_{n,n',a,t} \in \{0,1\} \) which denotes the time \( t \) an agent \( a \) begins traveling across the edge connecting nodes \( n \) and \( n' \neq n \).

We define an objective function (Eq. 1) which maximizes the demand that is actually covered by the agents. This function consists of the sum of the points in time an agent passed by a node based on the initial demand and the sum of all agents’ location changes multiplied by a very small number \( \mu \). The second sum is always smaller than the first and is subtracted from it, in order to prevent agents from changing locations when they do not need to do so. The reader should note that this function is linearized at run time by CPLEX. This is usually done by adding two extra decision variables and two extra constraints. The same is true for all absolute values used later in the constraints. This objective function is maximized under a number of constraints:

Objective function:

\[
\sum_{n \in N} \sum_{t \in T} (d_{n,t} \times f_{n,t}) - (\sum_{n \in N} \sum_{n' \in N} \sum_{a \in A} (|k_{n,n',a,t+1} - k_{n,n',a,t}|) \times \mu)
\] (1)

Subject to:

Temporal, spatial, and routing constraints:

\[
e_{p_{n,a,t}} = 1, \forall a \in A
\] (2)

\[
e_{p_{na},a,t} = 1, \forall a \in A
\] (3)

\[
\sum_{n \in N} e_{n,a,t} \leq 1, \forall a \in A, t \in T
\] (4)

\[
\sum_{n \in N} \sum_{n' \in N} k_{n,n',a,t} \leq 1, \forall a \in A, t \in T
\] (5)

\[
\sum_{n \in N} \sum_{n' \in N} \sum_{a \in A} (|k_{n,n',a,t+1} - k_{n,n',a,t}|) \times \mu
\] (6)

\[
\sum_{n \in N} \sum_{n' \in N} startT_{n,n',a,t} \leq 1, \forall a \in A, t \in T
\] (7)

\[
e_{p_{na},a,t} - e_{p_{na},a,t+1} \leq \sum_{n' \in N} k_{n,n',a,t},
\forall a \in A, t \geq 0, t \leq T - 1, a \in A
\] (8)
The completion constraints ensure the proper execution of tasks. Thus, if the initial flying demand for a node is to be covered, at least one agent must fly over this location for a certain time period (Eq. 13). Moreover, for each agent and point in time, if this agent initiates a trip at time $t$, then the starting time of this trip is set to $t$ (Eq. 13). For each agent, the total time it flies across edges must be equal to the travel time required for each trip (Eq. 14). Finally, for each agent, location, and point in time, if a trip begins at time $t$ then this agent must travel for a period of time equal to $t + 1 + \text{duration}$ (Eq. 15).

IV. Greedy scheduling

Given that the optimal solution is practical only for small size problems (see Section [7]), here we present a greedy algorithm which applies a one-step look ahead heuristic search mechanism and scales up to problems involving thousands of agents and locations. The algorithm consists of two parts, namely the pre-processing (see Alg. 1) and the main scheduling algorithm (see Alg. 2) parts.

Regarding the pre-processing part, the initial (Alg. 1, lines 1 – 3) and final locations (Alg. 1, lines 4 – 6) of each agent are set. If the start location is different than the final one, the values for the time to travel between these two positions is set to −1 (denoted in the greedy algorithm for simplicity as $e_{pa,t} \in L$) (Alg. 1, lines 7 – 8). Note that these values correspond to the minimum travel time of an agent which is to hover above its initial location until it must start traveling to its final location in order to be there at the last point in time.

Algorithm 1 Pre-processing Phase (Initialization of sets and variables).

Require: $N$ and $A$ and $T$ and $\forall n \in N : n_{init}^a, n_{fin}^a$ and $\forall n \in N, \forall t \in T : d_{n,t}$.

1: for all $(a \in A)$ do
2: for all $(t \in T)$ do {Initialize the location of the agents.}
3: $e_{pa,t=0} = n_{init}^a$
4: for all $(a \in A)$ do
5: if $(n_{init}^a) \neq n_{fin}^a$ then {Set the value for the final location of the agent.}
6: $e_{pa,t} = n_{fin}^a$
7: for all $(t \in T : t \geq |T| - 2 - e_{pa,t} + c_{n,n'} + c_{n',n''}) k t < |T| - 1$ do {If the start location is different that the final one, set the values for the time to travel between these two positions to −1.}
8: $e_{pa,t} = -1$

In terms of the main scheduling algorithm, we initially create a random sequence seq of all agents. Given this, we iterate through this sequence and for each agent and each point in time we take the current location of the agent (Alg. 2, line 4 – 7). If the agent is not currently traveling across two locations, we check whether a flying demand exists for this location and point in time. If
this is the case, the sum of covered demand is increased by one (Alg. 2, lines 9–12). In the next step of this algorithm, we need to decide the next location of the agent. To do so, we calculate the distance and demand for all locations, and we schedule the agent to travel to the closest location, including its current one, where demand exists and it has enough time to fly from there to its final destination (Alg. 2, lines 13–17). If such a new location is found, then the agent is set to travel to this location for a period of time that corresponds to the actual travel time (Alg. 2, lines 18–20), and stay there (Alg. 2, lines 21–22) until it is time to fly to its final destination (Alg. 2, lines 23–25). The intuition of this heuristic is to avoid having agents being idle for a long period of time, but instead immediately route them to locations with demand. Given that each agent has its own initial location, the performance of this algorithm may be affected by the initial random sequence of the agents. Thus, we execute the algorithm multiple times, each time with a different random sequence for the agents. We continue this execution as long as the solution is improving, or until a number of threshold iterations are made without any improvement on the solution (Alg. 2, lines 1–2 and lines 26–29). Note that trying the entire set of possible sequences of agents would demand $|A|!$ executions of the algorithm. From this point onward, we will refer to this algorithm as Greedy.

V. Performance Evaluation

In this section we evaluate our algorithms on a number of settings in order to determine their ability to handle potentially large numbers of locations, points in time, UAVs and demand. In doing so, we consider two settings, one having 7 locations, 49 edges (i.e., a fully connected graph), up to 5 agents, 12 points in time, and 15% of points in time with flying demand for each location, and a larger one with 20 locations, 400 edges, 100 points in time, up to 15 agents, and 15% of points in time with flying demand. In all cases the duration of the trip between any two locations is between 1 and 3 points in time. The first setting is used to evaluate the scalability of the optimal algorithm and the efficiency of the greedy algorithm compared to the optimal. The second setting is used to evaluate the performance of the greedy algorithm and its ability to handle large numbers of locations, agents, and points in time. The evaluation of our algorithms is executed in three main parts:

- **EXP1:** The execution time and the scalability of the optimal and the greedy algorithm.
- **EXP2:** The performance of the optimal and the greedy algorithm in terms of the average percentage of completed flying demand.
- **EXP3:** The sensitivity of the greedy algorithm on the number of locations, UAVs, points in time, and demand.

### Algorithm 2 UAV scheduling algorithm.

**Require:** $N$ and $A$ and $T$ and $\forall a \in A : n_{a}^{\text{init}}, n_{a}^{\text{fin}}$, $\forall n \in N, \forall t \in T : d_{n,t}$ and threshold.

1. $\text{count} = 0, \text{best} = 0$ {Initiate auxiliary variables.}
2. **while** ($\text{count} < \text{threshold}$) **do**
3.   $\text{totalCov} = 0$
4.   **Create a random sequence seq of the systems’ agents.**
5.   **for all** $a \in \text{seq}$ **do**
6.     **Create a random sequence seq of the systems’ agents.**
7.     **for all** $t \in T$ **do**
8.       $\text{curLoc} = e_{p_{a,t}}$
9.       **if** $(d_{\text{curLoc},t} == 1)$ **then** {If there is demand at the current location and point in time}
10.        $\text{totalCov} = \text{totalCov} + 1$ {The demand for the current location and point in time is covered.}
11.        $d_{\text{curLoc},t} = 0$
12.       **for all** $k \in L$ **do**
13.         $\text{if} (d_{k,t+c_{\text{curLoc},k+1}} == 1 \& t + c_{\text{curLoc},k} + c_{k,n_{a}^{\text{fin}}} < t \& c_{\text{curLoc},k} < \text{min})$ **then**
14.            $\text{newLoc} = k, \text{min} = c_{\text{curLoc},k}$
15.         **if** such a location has been found
16.            **for all** $t' \in T : t' \geq t + 1$ and $t' < t + 1 + c_{\text{curLoc},\text{newLoc}}$
17.               $e_{p_{a,t'}} = -1$ {Set the time period that the drone is flying from its current location to the new one.}
18.               **for all** $t' \in T : t' \geq t + 1 + c_{\text{curLoc},\text{newLoc}}$ and $t' < T - 2$
19.                 $e_{p_{a,t'}} = \text{newLoc}$ {Set the time period that the drone is to hover over its new location.}
20.        **if** $(t + 1 + c_{\text{newLoc},n_{a}^{\text{fin}}} == T - 2)$ **then** {If it is time for the drone to fly to its final location.}
21.            **for all** $t' \in T : t' \geq t + 1 + c_{\text{newLoc},n_{a}^{\text{fin}}}$ and $t' < T - 2$
22.               $e_{p_{a,t'}} = n_{a}^{\text{fin}}$
23.        **if** $(\text{best} < \text{result})$ **then** {If the current result is better than the best so far, then update the value for best and set the count equal to zero.}
24.            $\text{best} = \text{totalCov}, \text{count} = 0$
25.       **else**
26.         $\text{count} = \text{count} + 1$

#### A. EXP1: Execution Time and Scalability

Execution time and scalability are typical metrics for scheduling algorithms. As can be seen from Fig. 1, the execution time of the optimal algorithm shows a steep
increase with the number of agents. At the same time, the greedy algorithm has a very low execution time of less than 0.003 seconds for this small setting which shows a low rate of increase as it remains under 0.5 seconds even for the larger setting.

**Fig. 1.** Execution time: Optimal vs greedy algorithm.

**B. EXP2: Performance of the Optimal and the Greedy Algorithms**

In this section, we evaluate our algorithms in terms of average completion of flying demand. In the small setting and as can be seen from Fig. 2, the greedy algorithm performs well with performance reaching 96.81% of the optimal in the best case, 87.04% in the worst case, and 92.06% on average. In the large setting and as can be seen from Fig. 3, the greedy algorithm achieves an approximately 100% completion of flying demand when 15 drones are used. However, even with 8 drones, the coverage is already over 90%, and with 11 drones it is over 99%. Given the limited scalability of the optimal algorithm, we argue that the performance of the greedy algorithm is very satisfactory, making it the best choice for medium- and large-scale settings.

**Fig. 2.** Locations coverage percentage: Optimal vs greedy algorithm.

**Fig. 3.** Locations coverage percentage (greedy algorithm).

**Fig. 4.** Example execution of the greedy algorithm.

**C. EXP3: Sensitivity of the Greedy Algorithm**

The greedy algorithm has already shown to have near optimal performance. However, we also need to evaluate how this performance may be affected by the number of locations, points in time, and density of demand. As can be seen from Fig. 5 when the number of locations increases, the performance of the algorithm gradually drops and close to 100% coverage is achieved only with high number of drones. This occurs because when the locations increase, the total demand also increases. Further, when the locations increase, the drones tend to fly around more as the demand is more spatially scattered. In addition, as can be seen from Fig. 6 when the locations remain fixed, but the number of the points in time increases we observe no major change in the performance of the algorithm. It is important to note that in this case the volume of points in time with demand increases, but the percentage of the total remains fixed at 15%. Thus, from these two experiments we can conclude that the performance of the algorithm is affected by the number of locations, but not from the number of points in time.

Another dimension that has to be examined, is how the greedy algorithm is affected by the density of the demand, in other words the percentage of points in time with fly over demand. As can be seen from Fig. 7 when the density of the demand is high, the performance of the algorithm drops. For instance, if we have 15 drones and 15% demand the coverage is at approximately 100%, but when the demand is at 60% the coverage is at approximately 85%. To explain our decision to experiment with up to 60% demand, we argue that if the demand is too high, this leads to a situation where a drone should constantly fly over each location. In such a setting, a
scheduling algorithm is not really needed.

Fig. 5. Sensitivity of the greedy algorithms when varying the number of agents and locations.

Fig. 6. Sensitivity of the greedy algorithm when varying the number of agents and points in time.

Fig. 7. Sensitivity of the greedy algorithms when varying the number of agents and points in time with demand.

VI. CONCLUSIONS AND FUTURE WORK

In this work we examined the scheduling of drones across a graph. In this vein we extended the well-known multiple-traveling salesman problem by adding the constraint of multiple visits per node at specific points in time. Initially, we formulated the problem as an Integer Linear Program and we solved it offline and optimally. Given that this solution has limited scalability, we also developed a greedy algorithm that uses a one-step look-ahead heuristic function and achieves near optimal performance while also scaling to large settings.

For future work, we aim to handle the limited range of the drones by adding the ability to recharge their batteries between specific routes. We also aim to monitor and manage the flying altitude of the drones to achieve collision avoidance. Finally, we aim to develop an online algorithm for the same problem that will use reinforcement learning techniques.

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