Life time of superconductive state in long overlap Josephson junction

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The computer simulations of fluctuational dynamics of the long overlap Josephson junction in the frame of the sine-Gordon model with a white noise source have been performed. It has been demonstrated that for the case of constant critical current density the mean life time (MLT) of superconductive state increases with increasing the junction’s length and for homogeneous bias current distribution MLT tends to a constant, while for inhomogeneous current distribution MLT quickly decreases after approaching of a few Josephson lengths. The mean voltage versus junction length behaves inversely in comparison with MLT.

At the present time in the area of quantum calculations serious hopes are connected with the possibility to create qubits on the basis of point 1, and distributed Josephson junctions 2. The main advantage of qubits based on Josephson junctions is the relative simplicity of manufacturing and integrating in one circuit with other possible realizations of qubits 3. It is necessary to point, that at the present time all Josephson junctions are manufactured with the use of electron-beam lithography 4, and can always be considered as distributed. One of the most fundamental problems of all quantum calculations is connected with the destruction of entangled states of qubits through the interaction with the environment. This event is characterized by the time of decoherence, the maximal increase of which is needed for the design of quantum computers. At the same time, due to mathematical difficulties, even in the case of a long Josephson junction in the presence of thermal fluctuations, the life time of superconductive state is not studied well enough (see, e.g., 5-10). Mostly, annular and infinitely long junctions have been considered, but it has been understood, that for short junctions the escape of the whole string over the potential barrier occurs, while for long junctions the creation of kink-antikink pairs is the main mechanism of the escape process. In our recent works, dedicated to investigation of noise-induced errors in Josephson junctions 11,12, only the model of point junction was considered, so no dependences between fluctuational properties and junction’s length were studied. Therefore, at the present time an important problem of investigation of superconductive state life time with the aim to improve noise immunity of long Josephson junctions is not yet solved. In this Letter we present results of numerical investigation of superconductive state life time based on the computer simulation of the sine-Gordon equation with white noise source.

In the frame of the resistive McCumber-Stewart model 13 the phase difference of the order parameter \(\varphi(x, t)\) of long Josephson junction in the overlap geometry (see Fig. 1) is described by the sine-Gordon equation:

\[
\frac{\beta^2 \partial^2 \varphi}{\partial t^2} + \frac{\partial \varphi}{\partial t} - \frac{\partial^2 \varphi}{\partial x^2} = i(x) - \sin(\varphi) + i_f(x, t),
\]

with the following boundary conditions:

\[
\frac{\partial \varphi(0, t)}{\partial x} = \frac{\partial \varphi(L, t)}{\partial x} = \Gamma.
\]

Here the time and the space are normalized to the inverse characteristic frequency \(\omega_c^{-1}\) and the Josephson length \(\lambda_J\), respectively, \(\beta = 1/\alpha^2\) is the McCumber-Stewart parameter, \(\alpha\) is the damping, \(i(x)\) is the bias current density, normalized to the critical current density of the junction, \(i_f(x, t)\) is the fluctuational current density, \(\Gamma\) is the normalized magnetic field, \(L = l/\lambda_J\) is the dimensionless length of the junction. In the case where the fluctuations are treated as white Gaussian noise with zero mean, and the critical current density is fixed, its correlation function is:

\[
\langle i_f(x, t)i_f(x', t')\rangle = 2\gamma\delta(x-x')\delta(t-t'),
\]

where

\[
\gamma = I_T/(J_c\lambda_J)
\]

is the dimensionless noise intensity, \(J_c\) is the critical current density of the junction, \(I_T = 2ekT/h\) is the thermal current, \(e\) is the electron charge, \(h\) is the Planck constant, \(k\) is the Boltzmann constant and \(T\) is the temperature. It should be noted, that in the total critical current \(I_c = \int_0^L J_c(x)dx\) (here \(l = L\lambda_J\) was supposed to be fixed, and the noise intensity in that case was proportional to

\[
\frac{\beta}{\partial \varphi(0, t)} = \frac{\partial \varphi(L, t)}{\partial x} = \Gamma.
\]
the dimensionless length of the junction $L$. In the frame of the present paper we consider more interesting (from the practical point of view) case where the critical current density is held constant and the critical current increases with the junction’s length.

Numerical solution of the sine-Gordon equation \[11\] with boundary conditions \[2\] has been performed on the basis of implicit finite-difference scheme \[14\] with the account of the white noise source. Typical values of discretization steps are $\Delta x = \Delta t = 0.1 - 0.02$, number of realizations are $R = 1000 - 10000$.

In the frame of the present paper we will consider and compare two cases of bias current distribution: homogeneous $i(x) = i_0$ and inhomogeneous one, characteristic for a superconductive thin film \[13\], \[17\]:

$$i(x) = \frac{i_0 L}{\pi \sqrt{x(L-x)}}. \quad (5)$$

For the junctions with small capacitance $\beta \ll 1$ (large damping $\alpha \gg 1$), the mean life time (MLT) of superconductive state is, physically, the mean time till the generation of noise-induced voltage pulse. The MLT is defined as the mean time of phase $\varphi$ existence in the considered interval \[14\], \[17\], where $W(\varphi, x, t)$ is the probability density, and $P(t)$ is the probability that the phase is located in the initial potential well:

$$\tau = \int_0^{+\infty} t w(t) dt = \int_0^{+\infty} P(t) dt, \quad (6)$$

$$w(t) = -\frac{\partial P(t)}{\partial t}, \quad P(t) = \frac{1}{L} \int_0^L \int_{-\pi}^{\pi} W(\varphi, x, t) d\varphi dx.$$ 

First, let us consider the case of homogeneous bias current distribution $i(x) = i_0$. The MLT of superconductive state versus length of a Josephson junction, calculated numerically on the basis of definition \[6\] with the delta-shaped initial distribution at $\varphi_0 = \arcsin(i_0)$, is presented in Fig. 2. As it is seen, the MLT of superconductive state for the case of constant critical current density rises for small lengths $L \leq 1$ and reaches a constant for $L \geq 5$. Such behavior is explained by the existence of two different mechanisms of the string escape over the potential barrier \[3\] (see also \[3\] - \[10\]): it may escape either as a whole, or by forming kink-antikink pairs. For $L \leq 1$ with the increase of the string length, by the interaction of elementary parts of the string with their neighbors, the phase is stronger kept in the initial potential well which leads to the increase of the life time. Even if the kink-antikink pair is fluctuatuionally formed, but the string is too short, the attraction between the kink and antikink outweighs the driving force $i$ and the incipient nucleus collapses returning the string to the initial potential well. If, however, the string is long enough, then the driving force predominates and the nucleus expands, speeding up the escape process. These mechanisms are illustrated in Fig. 3 while for $L < 1$ the first mechanism is predominated, for $L \geq 5$ the second one leads to the saturation of MLT. It is important to mention, that for the considered overlap junction (see boundary conditions \[2\]) not only the creation of kink-antikink pairs (curve 4), but also creation of one (curve 2), two (curve 3), or even more kinks is possible. As it is seen from Fig 2 the constant of saturation depends on thermal noise intensity $\gamma$ (decreases with the increase of $\gamma$), and bias current density $i$ (also decreases with the increase of $i$, because of decrease of the potential barrier height).

![Fig. 2: The MLT of superconductive state of long Josephson junction for the case of homogeneous bias current distribution with $\Gamma = 0$, $\beta = 0.01$: squares - $i_0 = 0.5$, $\gamma = 0.3$; circles - $i_0 = 0.7$, $\gamma = 0.3$; triangles - $i_0 = 0.9$, $\gamma = 0.3$; solid line - $\gamma = 0.2$, $i_0 = 0.7$; dashed line - $\gamma = 0.4$, $i_0 = 0.7$.](image)

![Fig. 3: Phase evolution in the long Josephson junction depending on its length: 1 - $L = 2$, 2 - $L = 5$, 3 - $L = 20$, $t = t_1$, 4 - $L = 20$, $t = t_2 > t_1$.](image)

It is interesting to check the limiting transition of a long junction to a point one for small junction lengths $L \ll 1$. For a point junction in the case of large damping $\beta \ll 1$ the MLT of superconductive state was found analytically \[10\] for arbitrary value of noise intensity. That formula, however, was obtained for the case of constant critical current $I_c$, and the noise intensity in that case...
that in inhomogeneous case there are unstable areas
neous and inhomogeneous bias current distribution is the
6). The main difference between the cases of homoge-
the case of constant bias current distribution (see Fig.
increase for small lengths is absolutely the same as for
string escape. It is obvious, that mechanism of MLT
also be explained by the existence of two mechanisms of
strong maximum for the lengths
bias current density are presented in Fig. 5.
Results of study of the MLT in the case of inhomogeneous
superconductive state dependence on junctions’s length.
profile significantly changes the picture of the life time of
width is much smaller than its length. Such bias current
the formula (5) [13], [15] for the case when the junction
junction is made on the basis of thin superconductive
films, the bias current distribution may be governed by
the coincidence is better for larger noise intensity.
In real situations it is hard to achieve the homogeneous
bias current distribution. Since a long overlap Josephson
junction is made on the basis of thin superconductive
state for the case of a short Josephson junction
limiting case

\[
\tau = \frac{L}{\gamma}\left\{ \int_{\varphi_0}^{\varphi_2} e^{-i(\cos x + i\phi)\frac{L}{\gamma}} \int_{\phi_1}^{x} e^{i(\cos \phi + i\phi)\frac{L}{\gamma}} d\phi dx + \right. \\
+ \int_{\varphi_1}^{\varphi_2} e^{i(\cos \phi + i\phi)\frac{L}{\gamma}} d\phi \cdot \int_{\phi_2}^{\infty} e^{-i(\cos \phi + i\phi)\frac{L}{\gamma}} d\phi \right\}. \tag{7}
\]

where \(\varphi_0\) - coordinate of initial delta-shaped distribution, \(\varphi_{1,2}\) - boundaries of the interval, which define the initial potential well. Indeed, as it is seen from Fig. 4, the results of numerical simulations of a long junction perfectly coincide with the formula (7) not only in the limiting case \(L \ll 1\), but even up to \(L \sim 1\) and for \(L > 1\) the coincidence is better for larger noise intensity.

In real situations it is hard to achieve the homogeneous bias current distribution. Since a long overlap Josephson junction is made on the basis of thin superconductive films, the bias current distribution may be governed by the formula (7) for the case when the junction width is much smaller than its length. Such bias current profile significantly changes the picture of the life time of superconductive state dependence on junctions’s length. Results of study of the MLT in the case of inhomogeneous bias current density are presented in Fig. 5.

The main feature of such dependences is the existence of strong maximum for the lengths \(L \approx 5\). This effect can also be explained by the existence of two mechanisms of the string escape. It is obvious, that mechanism of MLT increase for small lengths is absolutely the same as for the case of constant bias current distribution (see Fig. 6). The main difference between the cases of homogeneous and inhomogeneous bias current distribution is the fact, that in inhomogeneous case there are unstable areas near the edges of a junction \((x \approx 0, L)\) because there the bias current is larger than the critical current according to the formula (7). These areas work as generators of "kink-antikink" pairs. Due to the unstable character of a potential profile near the edges, amount of those pairs is significantly larger than in the junction with homogeneous bias current distribution. So, the influence of the second mechanism on the life time for the inhomogeneous bias current distribution is larger in comparison with the homogeneous one. But the lengths of "switching on" of this mechanism are the same as for the homogeneous case (see Fig. 6).

On one hand, to experimentally observe the predicted above effect of non-monotonous dependence of the MLT of superconductive state versus junction’s length, one needs to perform advanced measurements. On the other hand, it is now believed that the use of more homogeneous bias current distributions may increase the

FIG. 4: Comparison between numerical simulations of MLT (symbols) and formula (7) (lines) for the case of homogeneous bias current distribution with \(\Gamma = 0\), \(\beta = 0.01\), \(i_0 = 0.7\); circles and solid line - \(\gamma = 1\), squares and dashed line - \(\gamma = 0.3\), triangles and dotted line - \(\gamma = 0.2\).

FIG. 5: The MLT of superconductive state of long Josephson junction for the case of inhomogeneous critical current density with \(\Gamma = 0\), \(\beta = 0.01\), \(\gamma = 0.4\) and: circles - \(i_0 = 0.5\), squares - \(i_0 = 0.6\), triangles - \(i_0 = 0.7\).

FIG. 6: Comparison of MLT of superconductive state of long Josephson junction for different cases of bias current distribution with \(\Gamma = 0\), \(\beta = 0.01\), \(\gamma = 0.3\), \(i_0 = 0.7\): circles - inhomogeneous distribution, squares - homogeneous distribution.
power and decrease the linewidth of practical Flux-Flow Oscillators (FFOs), based on long Josephson junctions. There are attempts to increase the homogeneity by the increase either the junction’s width or the width of the idle regions (see [13] for details) and the check of the homogeneity of the current profile is only performed by the measurements of the linewidth. This takes a long time and there are many other different factors which affect the linewidth. So, a reliable tool to detect a homogeneity of bias current distribution in a particular FFO design is needed. To this end, the measurement of a voltage-length characteristic for a fixed value of bias current (slightly smaller than the critical current) can be recommended as such a tool.

The mean noise-induced voltage versus junction’s length demonstrates the “inverse” behavior in comparison with the MLT (see Fig. 7). For the case of constant current density and homogeneous bias current distribution the mean voltage decreases for small lengths \( L \leq 1 \) and reaches a constant for \( L \geq 5 \). For the case of inhomogeneous current distribution the minimum of the mean voltage versus length is observed. As one can see from Fig. 7 the voltages for homogeneous and inhomogeneous cases differ exponentially in the small noise limit for large lengths of Josephson junctions. If, with increase of the junction’s width, the bias current distribution will be more and more close to the homogeneous one, it can be clearly seen in \( V(L) \) dependence.

In this work numerical and qualitative analysis of fluctuational dynamics of long Josephson junctions is presented. It has been demonstrated that for the case of constant critical current density the mean life time (MLT) of superconductive state increases with increasing the junction’s length and for homogeneous bias current distribution MLT tends to a constant for \( L > 5 \). However, for inhomogeneous current distribution MLT quickly decreases after approaching a maximum for lengths around \( L \approx 5 \). Therefore, from the fluctuational stability point of view, there is no reason to increase the length of a long Josephson junction more than \( L \approx 5 \), excepting the cases, where increasing of the junction length can improve other useful properties of Josephson electronic devices. It has also been demonstrated, that the measure of the homogeneity of bias current distribution may be performed on the basis of voltage versus length characteristics for bias current, smaller than the critical one: for junction lengths larger than \( 5 \lambda_j \), the voltages for bias current distribution and the homogeneous one differ exponentially in the limit of small noise intensity.

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FIG. 7: Comparison of mean voltage \( V(L) \) for the different cases of bias current distribution with \( i_0 = 0.9, \Gamma = 0, \beta = 0.01, \gamma = 0.05 \): circles - inhomogeneous case, squares - homogeneous case; \( \gamma = 0.1 \): solid line - inhomogeneous case, dashed line - homogeneous case.

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