Research Article

Some Examples of Materialist Dialectics in the Concept of Higher Mathematics

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Some examples of dialectics philosophy in higher mathematics are illustrated in this paper. Firstly, the principle of interconversion between quality and quantity in dialectics philosophy is quantified by the mathematical definition of the limit theory. Secondly, some natural and social phenomena imply the definition of continuous function in incremental form and it is a new explanation for the Zeno paradox. Finally, the dialectics relationship between the local change and the whole change of some variables is discussed in the differential median theorems.

1. Introduction

Higher mathematics is a basic mathematical course for undergraduates, including limit theory for a function, derivative and differential, indefinite integral, multiple integral, and so on. At present, many textbooks are based on the limit theory to discuss differential and integral. Differential is a method to discuss the rate and variation of variable change, including the calculation of derivative and differential. Integral, including the concepts of indefinite integral and definite integral, is the inverse operation of derivative and differential. The calculus is widely used in many fields, such as seeking instantaneous velocity in physics, doing work with variable force, etc.; in economics, marginal analysis, elasticity analysis, etc.; in geometry, the slope of a curve, the area of a trapezoid with curved edges, etc. It not only provides scientific principles and calculation methods for natural phenomena, but also expounds dialectical ideology and philosophy of life from a new perspective. As the mathematician Demollins pointed out [1], “without mathematics, we cannot see through the depth of philosophy; without philosophy, we cannot see through the depth of mathematics; and without both, people cannot see through anything.” In ancient Greece, Heraclitus, Socrates, Zeno, Plato, Aristotle, and so on discovered and used “Dialectics” firstly. After thousands of years of repeated honing, the mathematical thinking method and “Dialectics” are skillfully combined to form a unique philosophical thinking method, that is mathematical dialectics [2, 3]. However, to learn dialectics in calculus well is to understand materialist dialectics more accurately from a new viewpoint. Therefore, the integration of the elements of philosophical education into the teaching of professional knowledge of calculus not only reflects the course characteristics and philosophical background of calculus but also is the teaching goal of the fundamental task of establishing morality and cultivating talents of the general secretary of the internship. However, “it is not easy to teach the philosophical course well, because it is very demanding.” “The key to running the philosophical theory course well lies in the teachers, and the key lies in giving full play to the teachers’ enthusiasm, initiative, and creativity.” [4] Therefore, it is a high demand and high-tech teaching work to integrate the elements of philosophical education into calculus teaching. According to the research work of above literature and the teaching experience of calculus in recent years, several examples of calculus in ideological and philosophical education are discussed in this paper, mainly including the principle of interconversion...
between quality and quantity by the mathematical definition of limit theory; some explanations for natural, social phenomena, and the Zeno paradox by the definition of continuous function in incremental form; and the dialectics relationship between the local change and the whole change by the differential median theorems.

2. Quantifying the Principle of Interconversion between Quality and Quantity in Dialectics from Limit Theory

The idea of the limit theory has been born in the sprouting period of calculus. For example, the problem of stick cutting in “Tianxia chapter” of Mei and Zhang-Hua [5–7] (Zhuangzi (About BC369–BC286): His name is Zhuang Zhou. During the Warring States period, he was born in Song Guo. He is the representative of the Taoist school, the successor and developer of Lao Tzu’s philosophical thought, and the founder of the pre-Qin Zhuangzi school. Their philosophy was “Lao Zhuang philosophy.” His main works include “Free and Unfettered Travel,” “Qi Wu Lun,” “Tianxia Pian,” and so on, which are included in Zhuangzi: “the hammer of one foot cuts half every day, and it will last forever.” That is to say, the wooden stick of one foot cuts half every day, and its length decreases continuously, and tends to zero. Another example is Liu Hui’s cyclotomic method [8–11] (Liu Hui (About 225–295): He is one of the great mathematicians during the Wei and Jin Dynasties of China. His masterpieces are “nine chapters arithmetic note” and “island arithmetic Sutra.” He presents the methods of mouhe square cover, weight difference, cyclotomic method, and so on) in the period of the Three Kingdoms, which used the inscribed regular polygon to calculate the area of a circle. In Figure 1, it can be seen that with the increasing number of sides of inscribed regular polygon, the area of regular polygon trends to the area of circle. According to the above examples, if something changes, such as the length of the stick, the area of the positive polygon, etc., it can be recognized as a variable. The above two examples show the trend of their change; that is, with the increase of independent variables \( n \) such as number of edges of a positive polygon and number of days, dependent variable an (such as area and stick length) trends to a constant. This is called the limit of series. It can be understood that when the independent variable \( n \) is large enough, the distance between the dependent variable (denoted as \( a_n \)) and the constant (denoted as \( a \)) is much smaller. So, the mathematical definition of a number sequence limit is given in the following Definition 1.

**Definition 1.** (see [12]) If a number sequence \( a_n \) is limited by a constant \( a \), it is equivalent to that for an arbitrary given positive and sufficient small number \( \varepsilon > 0 \), there exists a sufficient large natural number \( N \), when \( n > N \), \( |a_n - a| < \varepsilon \) holds.

The dialectics philosophy in Definition 1 can be interpreted distinctly by Figure 2. Firstly, a point \( a \) is given on the number axis and the distance from point \( a - \varepsilon \) and \( a + \varepsilon \) to \( a \) is \( \varepsilon \). If something’s change can be recognized as a number sequence \( a_n \) and it is limited by a constant \( a \), it is equivalent to that the variable \( a_n \) changes with the increase of \( n \) and \( a_n \) trends to a constant \( a \). According to the description of Definition 1, for an arbitrary given positive number \( \varepsilon > 0 \), there exists a natural number \( N \), when \( n > N \), \( |a_n - a| < \varepsilon \). That means, if \( n > N \), \( a - \varepsilon < a_n < a + \varepsilon \). Its implications can be intuitively understood from Figure 2. If \( n > N \), \( a_n \) is located between \( a - \varepsilon \) and \( a + \varepsilon \). That is the quality which \( a_n \) satisfies \( (|a_n - a| < \varepsilon) \). For an given positive number \( \varepsilon > 0 \), there exists a natural number \( N \), when \( n > N \), \( a_n \) satisfies the quality \( (|a_n - a| < \varepsilon) \). On the other words, \( a_n \) changes with the increase of \( n \). \( a_1 \) does not necessarily satisfies the quality \( (|a_1 - a| < \varepsilon) \). \( a_2, a_3, \ldots, a_N \) do not necessarily satisfies the quality \( (|a_n - a| < \varepsilon) \). That is, they are not located between \( a - \varepsilon \) and \( a + \varepsilon \), as shown in Figure 2. With the increase of \( n \), if \( n > N \), all \( a_n \) satisfy the quality \( (|a_n - a| < \varepsilon) \). That is, if \( n > N \), all \( a_n \) are located between \( a - \varepsilon \) and \( a + \varepsilon \), also as shown in Figure 2.

According to the above analysis, the variable \( a_n \) changes with the increase of natural number \( n \). Before the \( N \) term, \( a_n \) changes in quantity. After the \( N \) term, \( a_n \) satisfies the quality \( (|a_n - a| < \varepsilon) \). So, it can be seen that the mathematical symbol language of a number sequence limit implicaes the principle of qualitative change caused by quantitative change in dialectics. Moreover, when quantitative change of \( a_n \) leads to qualitative change, it also point out the degree of quantitative change. That means there are qualitative changes in \( a_n \) after reaching the \( N \) term. After qualitative change in \( a_n \), it shows a stable state, that is, \( n > N \), all \( a_n \) are located between \( a - \varepsilon \) and \( a + \varepsilon \), as shown in Figure 2. This shows that the variables have better controllability. Mathematically, it is called convergence. The given positive number \( \varepsilon \) is relatively fixed. But it is arbitrarily small, and it reveals the measure of approximation between \( a_n \) and \( a \). If the given \( \varepsilon \) changes, the quality which \( a_n \) satisfies \( (|a_n - a| < \varepsilon) \) also changes. Therefore, \( \varepsilon \) can be understood as a measure of the quality which \( a_n \) satisfies \( (|a_n - a| < \varepsilon) \). That is, there exists quantitative change in the quality. Based on the above analysis, Definition 1 implicates the principle of interconversion between quality and quantity in dialectics. It provides the measure of the principle, including the degree of quantitative change (number \( N \)) and measure of the quality which \( a_n \) satisfies \( (|a_n - a| < \varepsilon) \). It also implicates another philosophical element, that is, dialectical relationship between the absoluteness and relativity, including relative fixation and arbitrariness of \( \varepsilon \). Some work implicates this principle, such as oil exploration,
Definition 2. Assume that a function \( y = f(x) \) is defined in a neighborhood at \( x_0 \). Define \( \Delta x \) as \( \Delta x = x - x_0 \), where \( x \) is near the point \( x_0 \) and \( \Delta y = f(x) - f(x_0) \). If \( \Delta x \to 0 \), \( \Delta y \to 0 \), that is, \( \lim_{\Delta x \to 0} \Delta y = 0 \), then the function \( y = f(x) \) is continuous at the point \( x_0 \).

It is easy to see that Definition 2 implicates the mathematical model of the continuous phenomena. If Definition 2 is discussed by dialectical philosophy, there are some interesting findings. There are two sides for everything in dialectical philosophy. And so are the continuous phenomena. Its influence cannot be ignored. For example, in a short period of time, both positive progress caused by hard work and negative progress caused by slack and carelessness are all very small. These are not easy to attract our attention.

For example, the learning process of college students is continuous. If the learning tasks are completed on time every day, although every progress is small, not only the examination can be passed easily after a semester, but also it will affect the learning of follow-up courses. The people’s reaction to the continuous phenomena is obtuse due to continuous principle. But small changes accumulated day by day will eventually become a significant qualitative change, such as "the bank of thousands of miles was destroyed by the ant nest." As a mathematical model of continuous principle, continuous function reveals that calculus is derived from nature, from our daily life and it also implicates materialist dialectics, that is quantitative change leads to qualitative change.

Moreover, Definition 2 of continuous functions also has an important role in solving the Zeno paradox "Flying Arrow is Not Moving" [13, 14]. Zeno pointed out that since the arrow has a temporary position at any moment in its flight, it is no different from immobility in this position [15, 16]. Time is a point on the timeline. The moment represents a very short period of time. No matter how short is a period of time, the time is continuous. The flying arrow is a fusion of time and space. It occupies a space or a position in the space at every time of the flight. Mathematically, each time can be denoted by a symbol \( t \), and a position in the corresponding space can be represented by \( y(t) \). In materialist dialectics, movement refers to the changes and processes of all things and phenomena. Quiescence is the relative state of the motion. In physics, motion and quiescence are relative. An object is called moved if its position changes relative to an object of reference. Also, an object is called static if its position is unchanged. For the flying arrow from time \( t_0 \) to time \( t \), its corresponding position is from position \( y(t_0) \) to \( y(t) \). Assume that \( \Delta t \) is introduced for the variation at the time \( t_0 \), that is \( \Delta t = t - t_0 \), and \( \Delta y \) denotes the variation of the location variable \( y \), that is \( \Delta y = y(t) - y(t_0) \). So, the variation \( \Delta t \) is a period of time. The moment means that the variation \( \Delta t \) is small enough. Moreover, the variation \( \Delta y \) can implicate the motion and quiescence of the objects. If the variation \( \Delta y \) is equal to 0 for any moment, no matter how short is a period of time, this means the position of an object is unchanged at time \( t_0 \). On the other words, the object is static. If the variation \( \Delta y \) is always not equal to 0 for any moment, no matter how short is a period of time, this means the position of an object is changed at time \( t_0 \). On the other words, the object is motional. According to the continuity of flying arrow, if \( \Delta t \to 0 \) and \( \Delta y \to 0 \). However, \( \Delta y \) is always not equal to 0 for any moment, no matter how small is the \( \Delta t \). In mathematics, Zeno's paradox is obviously wrong. So, the mathematical model of continuous phenomena also has an important role in explaining Zeno's paradox.

4. Establishing the Relationship between the Local and the Whole from the Mean Value Theorem

Another content with philosophical elements in calculus is the mean value theorem. The differential mean value theorems are important theorems which implicate the
relationship between functions and derivatives, and it is also the theoretical basis of calculus. It includes Lagrangian mean value theorem, Cauchy mean value theorem, and so on.

Lagrangian mean value theorem [12]: if a function \( f(x) \) is continuous on a closed interval \([a, b]\) and it is differentiable in the open interval \((a, b)\), then there exists more than one point \( \xi \) in the open interval \((a, b)\) \( (a < \xi < b) \) so that

\[
\frac{f(b) - f(a)}{b - a} = f' (\xi).
\]  

(1)

Cauchy mean value theorem [12]: if functions \( f(x) \) and \( g(x) \) are continuous on a closed interval \([a, b]\) and they are derivative in the open interval \((a, b)\), and \( g(x) \neq 0 \) for every point \( x \in (a, b) \); then, there exists more than one point \( \xi \) in the open interval \((a, b)\) \( (a < \xi < b) \) so that

\[
\frac{f(b) - f(a)}{g(b) - g(a)} = \frac{f' (\xi)}{g' (\xi)}.
\]  

(2)

Roll mean value theorem [12]: if a function \( f(x) \) is continuous on a closed interval \([a, b]\) and it is differentiable in the open interval \((a, b)\), \( f(a) \neq f(b) \) then, there exists more than one point \( \xi \) in the open interval \((a, b)\) \( (a < \xi < b) \) so that

\[
f' (\xi) = 0.
\]  

(3)

These implicate the relationship between the function value \( f(x) \) and its derivative value \( f' (\xi) \). The derivative value \( f' (\xi) \) is the instantaneous change rate of the function \( f(x) \) at a point \( x \), which denotes the local feature of the function \( f(x) \). These differential mean value theorems play an important role in dialectic relationship between the local and the whole. This association is quantitative. The mean variation ratio \( \frac{g(4) - g(1)}{(f(4) - f(1))} = \frac{0.1 L/km}{0.1 \text{ ml/m}} \) is 1. However, during the drive, it will be found that the instantaneous fuel consumption is 0.1 ml/m at some time \( \xi \) between 1:00 and 4:00.

Based on the above example analysis, the differential mean value theorem reveals the quantitative relationship between the whole variation of the variables and the local derivatives. This also illustrates the dialectical relationship between whole and local in dialectics. Observing equations (1) and (2) from left to right, the whole is composed of local, properties of the whole can be obtained at the local points. Observing equations (1) and (2) from right to left, the local points are in the whole, and the whole variation will also determine the local points.

The relationship among these differential mean value theorems is derived from special to general. Roll mean value theorem is a special case of the Lagrangian mean value theorem, and the Lagrangian mean value theorem is a special case of Cauchy mean value theorem. This also implicates the dialectical relationship between local and whole. On the one hand, Roll theorem is a part case of the Lagrange and Cauchy theorem, which can be derived from the Lagrange and Cauchy theorem. On the other hand, the conclusions of Lagrange theorem and Cauchy theorem can also be proved by Roll theorem. This implies the properties of the whole can also be derived by the part. So, these differential mean value theorems are the dialectical relationship of unified coexistence. According to the above discussion, a profound dialectical relationship between local and the whole can be shown from these differential mean value theorems.

5. Discussion and Conclusion

In this paper, some examples of dialectics philosophy in higher mathematics are discussed, based on the definition of series limit, differential mean value theorems, and continuum function. Firstly, the Definition 1 of series limit implicates the principle of interconversion between quality and quantity in dialectics. It provides the measure of the principle, including the degree of quantitative change (number \( N \)) and measure of the quality which \( a_n \) satisfies \((|a_n - a| < \epsilon)\). It also implicates another philosophical element, that is dialectical relationship between the absoluteness and relativity, including relative fixation and arbitrariness of \( \epsilon \). Secondly, the differential mean value theorem reveals the quantitative relationship between the whole variation of the variables and the local derivatives. This also illustrates the dialectical relationship between whole and local in dialectics. Finally, as a mathematical model of continuous phenomena, the continuous function reveals that high mathematics is derived from nature and our daily life, and it also implicates materialist dialectics, that is, quantitative change leads to qualitative change. Explained by the definition of continuous function, Zeno paradox is obviously wrong. However, there are many concepts and principle in high mathematics which are rich in ideological and philosophical elements and profound in thought. The work should be continued and more new discoveries will be explored in our future research.
Data Availability

No data were used to support this study.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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