The importance of radiative scattering in heated heavy ion plasmas

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The transport of energy in heated plasmas requires the knowledge of the radiation coefficients. These coefficients consist of contribution of bremsstrahlung, photoionisation, bound-bound transitions and scattering. Scattering of photons on electrons is taken into account by the model of Thomson, Klein-Nishina and the first order angular momentum of Klein-Nishina. It is shown that radiative scattering becomes an important part of energy transport in high temperature plasmas. Moreover, the contribution of transport correction to scattering is taken into account. The physics is discussed on the example of a heated plutonium plasma at different particle densities and temperatures in radiative equilibrium.
I. INTRODUCTION

Calculating energy losses by radiative effects in heated plasmas requires the knowledge of the radiation transport coefficients. In local thermal equilibrium these coefficients strongly depend on the electron density, the averaged ionisation stage and the ion density at different stages of ionisation. These contributions are estimated by the eq. of Saha [1]. The required ionisation potentials are taken from literature [2]. Additionally, depression effects which lead to a lowering of the ionisation potentials are considered. The radiation transport coefficients are specific for each constituent of a plasma and include absorption, emission and scattering contributions. It is necessary to distinguish between the process of absorption and scattering. A process is called scattering when a photon interacts with a scattering centre (e.g. atom, ion, electron) and emerges from the interaction into a new direction without altered energy (Thomson scattering) or a slightly altered energy (Compton scattering). Scattering processes mainly depend upon the radiation field and are only weakly coupled to the thermodynamic properties of the plasma. A process is called absorption, when a photon is destroyed by conversion of its energy (wholly or partly) into thermal energy of the plasma [5]. Sometimes those absorption processes are called true absorption. Absorption processes convert the photon energy directly into thermal energy of the gas [5]. The physics of radiative scattering and absorption is discussed in the example of a heated plutonium plasma in a range of temperature between 10 eV and 100 keV. The problem of radiative absorption is discussed widely in literature, e.g [11], whereas scattering is observed rarely, e.g. [12]. Within this report, the results for the overall frequency integrated averaged mean free paths in radiative equilibrium in the limit of dilute and dense plasmas taking scattering on electrons at rest in transport correction into account are presented. The knowledge of the frequency spectra integrated mean free paths is of importance in the diffusion limit of the radiation transport theory, e.g [9].

II. THEORY

The required electron and ion densities in local thermal equilibrium can be determined from the Saha eq. [1]. Assuming non-relativistic and non-degenerated electrons, it is shown [13] that this eq. can be written in form of the non-linear eq.

\[ \bar{r} + \sum_{j=1}^{Z} \frac{1}{\bar{r}^j} (\bar{r} - j) \prod_{r=1}^{j} K_r^D = 0, \]  

(1)

where

\[ K_r^D = \frac{2 (2\pi m_e k_B T)^{3/2}}{h^3} \exp \left( \frac{-I_r + D}{k_B T} \right), \]  

(2)

\[ \bar{r} = n_e / n_p \]  

is the averaged stage of ionisation, \( n_p \) is the initial particle density, \( n_e \) the electron density \( k_B \) the Boltzmann constant, \( T \) the temperature, \( I_r \) the ionisation potential, \( D \) the depression energy, \( m_e \) the electron mass, \( h \) the Planck constant and \( Z \) the atomic number. The participation probabilities \( n_r^* = n_r / n_p \) of the \( r \)-times ionised atoms are obtained using the recursive relation

\[ n_r^* = \frac{K_r^D}{n_p \bar{r}^{n_r^* - 1}}, \quad r = 0, \ldots, Z, \]  

(3)

where \( n_r \) and \( n_{r-1} \) are the ion densities at the ionisation levels \( r \) and \( r - 1 \), respectively. \( n_0^* \) has to be determined iteratively. The ionisation potentials are depressed by free electrons in the vicinity of an ion. It is shown that the depression term \( D \) can be written as [6]

\[ D = 1.161 \times 10^{-10} n_p^{1/3} \text{ (eV)}. \]  

(4)

\( n_p \) is given in \( \text{cm}^{-3} \). The radiative absorption contribution consists of bound-bound, photo effect and (inverse) bremsstrahlung contribution. The bound-bound input has been neglected in the current case. The bremsstrahlung absorption contribution, also called free-free absorption, reads [1]
\[
\Sigma_{ff}(u) = 0.767 \times 10^{-47} \frac{r^3 n_p^2}{u^3 k_B T}, \quad \text{(cm}^{-1})
\]

where \( u = h\nu/k_B T \) and \( \nu \) is the photon frequency. For the case of hydrogen-like ions the combined bremsstrahlung and photo effect absorption cross section, also called bound-free absorption, can be written as [1]

\[
\Sigma_a(u) = 0.96 \times 10^{-7} \frac{n_p}{T^2} \sum_{r=0}^{Z-1} n_r^* (r+1)^2 F_r(u), \quad \text{(cm}^{-1})
\]

where \( n_r^* \) is given by [3]. The function \( F_r(u) \) is given by

\[
F_r(u) = \begin{cases} 
\frac{1}{u^3} \exp \left( u - \frac{I_{r+1}}{k_B T} \right) & : u \leq \frac{I_{r+1}}{k_B T} \\
\frac{2}{u^3} \frac{I_{r+1}}{k_B T} & : u > \frac{I_{r+1}}{k_B T}.
\end{cases}
\]

(7)

The temperature \( T \) is measured in Kelvin. Due to the quantum mechanical predictions of spontaneously and induced emitted photons, the \textit{true} absorption contribution has to be corrected in the following way, e.g. [3]

\[
\Sigma'_a = \Sigma_a \left( 1 - \exp(-u) \right).
\]

(8)

More precise data for the above given absorption coefficient in range of photon energy \( h\nu = 100 - 2000 \) eV are obtained by Henke et al. [7]. Besides absorption, photons can interact with matter by scattering. The probability that a photon having frequency \( \nu \) and flight direction \( \Omega \) is scattered to frequency \( \nu' \) and direction \( \Omega' \) within the intervals \( d\nu \) and \( d\Omega \) in travelling a distance \( ds \) is described by \( \sigma_s(\nu \rightarrow \nu', \Omega \cdot \Omega') \ d\nu \ d\Omega \ ds \), e.g. [4]. The quantity \( \sigma_s(\nu \rightarrow \nu', \Omega \cdot \Omega') \) is called the differential scattering cross section. Within the Klein-Nishina model this quantity reads [4]

\[
\sigma_s(\nu \rightarrow \nu', \mu) = \frac{3}{16\pi} \frac{1}{(1 + \gamma(1-\mu))^3} \left[ 1 + \gamma + \gamma^2 - (\gamma + 2\gamma^2)\mu + (1 + \gamma + \gamma^2)\mu^2 - \gamma\mu^3 \right]
\times \delta \left( \nu' - \frac{\nu}{1 + \gamma(1-\mu)} \right) \left( \frac{8\pi e^4}{3m_e^2 c^2} \right),
\]

(9)

where \( \gamma = h\nu/m_ec^2 \) is a dimensionless energy, \( h \) is the Planck action, \( \nu \) the frequency of the incoming photon, \( \nu' \) the frequency of the photon after scattering, \( c \) the speed of light, \( e \) is the electron charge and \( \mu = \Omega \cdot \Omega' \). Since one asks for the scattering probability of photons with all possible frequencies \( \nu' \) for all possible angles \( \mu \) the differential scattering cross section [9] is evaluated over all frequencies and scattering directions. In that way one obtains the macroscopic Klein-Nishina cross section

\[
\Sigma_s(\gamma) = 2\pi n_e \int_{-1}^{1} d\mu \int_{0}^{\infty} d\nu' \sigma_s(\nu \rightarrow \nu', \mu) = \frac{3}{4} n_e \left[ \frac{1 + \gamma}{\gamma^3} \left( \frac{2\gamma[1+\gamma]}{1+2\gamma} - \log(1+2\gamma) \right) + \frac{1}{2\gamma} \log(1+2\gamma) - \frac{1 + 3\gamma}{(1 + 2\gamma)^2} \right] \left( \frac{8\pi e^4}{3m_e^2 c^2} \right).
\]

(10)

The Klein-Nishina scattering contribution reaches a maximum at around 5 keV for a plutonium plasma at normal particle density. Refer to fig. [11]. By increasing temperature the scattering absorption decreases. This is due to the upcoming pair-production at very high temperatures. For later purpose the first angular momentum of [9] is introduced. The multiplication of [9] by \( \mu \) followed by an integration over all photon frequencies and scattering directions leads to
\[ A_1(\gamma) = 2\pi n_e \int_{-1}^{1} d\mu \int_{0}^{\infty} du' \mu \sigma_s(\nu \rightarrow \nu', \mu) \]
\[ = \frac{3}{4} \frac{n_e}{\gamma^4(1 + 2\gamma)^2} \left[ (4\gamma^5 - 27\gamma^3 - 37\gamma^2 - 18\gamma - 3) \log(1 + 2\gamma) - 6\gamma^5 + 16\gamma^4 \right. \]
\[ + 46\gamma^3 + 30\gamma^2 + 6\gamma \right] \left( \frac{8\pi e^4}{3m^2e^2c^2} \right). \]  

Neglecting the dependency of scattering on \( \gamma \) in (10), the scattering of photons on free electrons at rest is described by the macroscopic Thomson coefficient [4]

\[ \Sigma_{Th} = \sigma_{Th} n_e = \frac{8\pi e^4}{3m^2e^2c^2n_e}, \]  

where \( \sigma_{Th} \) is the frequency independent microscopic Thomson coefficient. \( \Sigma_{Th} \) is valid in very low photon energy limits only. Scattering does not appear spontaneously. Hence, the scattering contribution must not be corrected similar to \( \Sigma_{sh} \). Quantum-mechanical corrections applied to the absorption coefficient using the Gaunt factor [4] are of very small effect only and have not been considered here. Often, it is of practical interest having radiation constants depending on temperature and material density only. For such cases photon frequency averaged methods have been developed. Of importance are the one-group photon energy coefficients. These coefficients are integrated over all photon energy frequencies. For the cases of optical thin and thick plasmas in radiative equilibrium the weight functions of Planck and Rosseland are widely used, e.g. [8]. In an optical thick regime the photon is destroyed in the vicinity of its emergence. The averaged mean free path used in this environment is called the Rosseland mean free path \( \lambda_R \) and is defined as [1]

\[ \lambda_R = \Sigma_{R}^{-1} = \frac{15}{4\pi^4} \int_{0}^{\infty} du \frac{u^4 \exp(-u)}{\Sigma_{tr} (1 - \exp(-u))^{-2}}, \]  

where \( \Sigma_{tr} \) is the transport cross section. \( \Sigma_R \) is the Rosseland mean absorption coefficient. A regime is called optical thin when the averaged mean free path is in range or above of the dimensions of the underlying physical system. In contrast to (13) the transport cross section has to be averaged by the weighting function of Planck. In that case the averaged mean free path \( \lambda_P \) is given by [1]

\[ \lambda_P = \Sigma_{P}^{-1} = \frac{\pi^4}{15} \left( \int_{0}^{\infty} du \Sigma_{tr} \frac{u^3 \exp(-u)}{1 - \exp(-u)} \right)^{-1}. \]  

\( \Sigma_P \) is the Planck mean absorption coefficient. The extension of the formulae (13) and (14) to a multi-group approach is straight-forward. The transport cross section is defined by [4]

\[ \Sigma_{tr} = \Sigma'_{a} + \Sigma_{s} - A_1. \]  

\( A_1 \) vanishes when scattering is taken into account in the Thomson limit. In that case the transport cross section is equal to the total cross section

\[ \Sigma_{tr} = \Sigma_{tot} = \Sigma'_{a} + \Sigma_{s}. \]  

Bernstein [10] proved mathematically that \( \Sigma_P \) forms an upper limit for \( \Sigma_R \) to within a factor of close to unity. For scattering this relation is shown in fig. [1].

III. RESULTS

The results presented in the last section are applied to a heated plutonium plasma at different densities. Under normal circumstances, the particle density of plutonium (\( \alpha \)-phase) is \( n_p = 4.87 \times 10^{22} \text{ cm}^{-3} \). Figure [1] shows the influence
of the scattering contribution of the models of Thomson and Klein-Nishina depending on temperature for a plutonium plasma at normal particle density. At high temperature the scattering absorption by the model of Thomson saturates to a constant due to the fully ionisation of the plasma. The wave-like structure is a result of the ionisation behaviour of plutonium.\footnote{13} Figure (2) shows the Planck and Rosseland averaged mean free path of the pure bremsstrahlung $\Sigma_{ff}$, the combined photo-effect and bremsstrahlung contribution $\Sigma_a$, the total cross section $\Sigma_{tot}$ and the transport cross section $\Sigma_{tr}$. The calculations have been performed with a density of one percent of the normal particle density, fig. (2a) and (2b), at solid state density, fig. (2c) and (2d) and 100-times compression compared to the solid state density, fig. (2e) and (2f). In a dilute plasma the free electrons are weakly influenced by the electric field of the surrounding ions. In contrast to an electron being in the field of an ion, the photon energy absorbed by an electron which does not belong to the vicinity of an ion is not converted into thermal energy. The lower the ion density of the plasma the more important scattering becomes. This effect is shown in fig. (2). The Thomson limit is valid in a small range of temperature only. At a temperature of approximately 1 keV the Klein-Nishina scattering becomes important. Refer to fig. (1). Especially in thin plasmas scattering significantly changes the absorption behaviour of radiation and cannot be neglected. Furthermore, fig. (2) shows the relevance of absorption by the photo-effect at mid-ranged temperatures. This contribution eases at very high temperatures when the plasma becomes fully ionised.

ACKNOWLEDGMENTS

I am grateful to R. KÜLHEIM and A. GROSSMANN giving corrections and comments.

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FIG. 2. Averaged mean free paths for the bremsstrahlung, $\Sigma_{ff}$, the combined bremsstrahlung and photo-effect, $\Sigma_a$, the total cross section, $\Sigma_{tot}$, and the transport cross section, $\Sigma_{tr}$, in a plutonium plasma at different particle densities depending on temperature.