Scattering experiments have revolutionized our understanding of nature. Examples include the discovery of the nucleus [R. G. Newton, *Scattering Theory of Waves and Particles* (1982)], crystallography [U. Pietsch, V. Holý, T. Baumbach, *High-Resolution X-Ray Scattering* (2004)], and the discovery of the double-helix structure of DNA [J. D. Watson, F. H. C. Crick, *Nature* 171, 737–738]. Scattering techniques differ by the type of particles used, the interaction these particles have with target materials, and the range of wavelengths used. Here, we demonstrate a two-dimensional table-top scattering platform for exploring magnetic properties of materials on mesoscopic length scales. Long-lived, coherent magnonic excitations are generated in a thin film of yttrium iron garnet and scattered off a magnetic target deposited on its surface. The scattered waves are then recorded using a scanning nitrogen vacancy center magnetometer that allows subwavelength imaging and operation under conditions ranging from cryogenic to ambient environment.

While most scattering platforms measure only the intensity of the scattered waves, our imaging method allows for spatial determination of both amplitude and phase of the scattered waves, thereby allowing for a systematic reconstruction of the target scattering potential. Our experimental results are consistent with theoretical predictions for such a geometry and reveal several unusual features of the magnetic response of the target, including suppression near the target edges and a gradient in the direction perpendicular to the direction of surface wave propagation. Our results establish magnon scattering experiments as a platform for studying correlated many-body systems.

**Significance**

This work describes a general scattering platform that uses magnons to explore the underlying properties of target materials. In this work we show how both phase and amplitude of magnons can be imaged using a nitrogen vacancy center magnetometer and how the scattered pattern of waves can be used to infer geometric and magnetic properties of a target material. To demonstrate this new experimental methodology we use a permalloy disk as our target and show that even with such a simple target unexpected behavior is observed. In addition, we provide a theoretical framework to reconstruct the properties of the target.

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serve such a role (SI Appendix). Coherent generation of magnons in YIG is well established scientifically (10) with a high degree of control and tunability in phase, amplitude, and wavelength spanning several nanometers to hundreds of micrometers. We generate magnons in YIG using a micro stripline deposited on the surface of a 100-nm-thick YIG grown on Gd$_3$Ga$_5$O$_{12}$ substrate (SI Appendix) (Fig. L1). By driving a microwave current through the stripline, coherent magnonic excitations are launched at the frequency of the microwave and at a wavelength set by the underlying magnonic dispersion, $\omega(k)$, in YIG (18). In the presence of an external magnetic field pointing along the stripline (Fig. 1 B, Left) this geometry launches magnons with k-vector perpendicular to the microwave current direction.

Detection of Scattered Waves

A key component of a scattering platform is the ability to image the scattered waves. Currently, there are several established techniques for imaging magnons in YIG, including Brillouin light scattering (12, 13), optical Kerr microscopy (19), and resonant X-ray microscopy (20). Here, we demonstrate the use of a single nitrogen vacancy (NV) center in diamond as a local sensor for magnonic excitations sensitive to both amplitude and phase with nanometer resolution.

The low-energy manifold of an NV center consists of an $S = 1$ spin triplet. Its ground state corresponds to $m_s = 0$ and its excited states consists of the $m_s = \pm 1$ states. At zero magnetic field, the $m_s = 0$ state is split from the excited state by 2.87 GHz. Application of a finite magnetic field along the NV axis splits the $m_s = \pm 1$ states by $2\gamma_B B_{ext}$, allowing for static magnetic field detection (Fig. 1B, Right). As a scanning probe, NV center microscopy (21, 22) has recently been used to image spin textures of skyrmions (23), noncollinear antiferromagnets (24), magnetic domains in 2D materials (25), and viscous current flow (26, 27).

Propagating magnons, including the scattered magnons due to the target, generate local time-varying magnetic fields above the YIG. These can be detected by an NV center if the frequency of the magnons matches that of the electron spin resonance (ESR) of the NV center. We use an external magnetic field to tune the ESR frequency of the NV center to match that of the excited magnons (Fig. 1B). Under these conditions, the NV center undergoes transitions from its ground state to one of its excited states (typically the $m_s = -1$ state) and its corresponding fluorescence will reflect the occupation of the NV center. In the $m_s = 0$ state, the fluorescence is strong, and it is weaker in the excited states. Under weak continuous drive the fluorescence is proportional to the intensity of the driving field, which in turn is directly proportional to the amplitude of the magnonic excitation at that location. Fig. 1E shows the fluorescence of an NV center as a function of the magnon frequency and $B_{ext}$. A clear decrease in NV fluorescence can be seen when the magnon frequency matches the NV ESR frequency. When the driving alternating magnetic field generated by the magnons is strong and coherent, Rabi oscillation of the NV center can be detected (Fig. 1E, Inset). When the excitation frequency matches the ferromagnetic resonance (FMR) of YIG, additional suppression of fluorescence can be observed in Fig. 1E. This effect results from FMR generated magnons and associated magnetic field noise at the NV ESR frequencies (28).
We determine the phase of the propagating coherent magnons using an interference scheme (SI Appendix). In the rotating frame of the NV center, the phase of the oscillating field determines the axis along which the spin rotates (Fig. 1C). To determine this axis, and hence the phase, we apply another radio frequency (RF) reference field that is uniform in space, both in amplitude and phase, and has the same ESR frequency. This is achieved using a wire antenna situated several tens of micrometers away from the sample (loop in Fig. 1D). The total AC field driving the NV center is a vector sum of the field generated locally by the magnon and the reference field:

\[
B_{\text{total}} = B_{\text{magnon}} + B_{\text{ref}} = \text{Re}\left\{ e^{i(kx-\omega t)} + e^{-i(kx+\phi)} \right\} = \text{Re}\left\{ e^{i(kx+\phi)} \left[ e^{-i(kz+\omega t)} \right] \right\},
\]

[1]

Here, \( k \) is the wavenumber, \( \omega \) is the drive frequency, and \( \phi \) is the phase difference between reference field and magnon field. The amplitude of both signals is normalized to 1 for intuitive illustration (Fig. 1C and D). By scanning the NV probe across the sample we observe an increase of fluorescence at locations where the magnon field is exactly out of phase with the reference field (Fig. S4). These peaks in fluorescence recur each time we move a distance corresponding to one wavelength of the magnons. As we vary the phase difference of the two microwave sources we are able to capture the real space propagating component of the magnons (Fig. 2B). A full movie can be seen in Movie S1. Extracting the wavelength of magnons as a function of frequency allows us to directly extract the dispersion relation of the magnons (Fig. 2C and F). We determine the dispersion down to a wavelength of 640 nm (Fig. 2D and E) limited only by the inefficient generation of magnons by the stripline at shorter wavelengths described in detail below. Even with this simple RF waveguide design (SI Appendix) we nevertheless are on par with the optical detection techniques using visible light (29–32).

We first employ our NV-based magnon detection scheme to characterize the generation of magnons from the stripline (33, 34). At a given drive frequency, \( \omega \), only magnons of wavevector \( k \) that satisfy the dispersion relation of the magnetic medium are launched. However, the excitation efficiency associated with a particular \( k \) is also set by the spatial geometry of the stripline. While magnons with wavelengths larger than the width of the stripline can be easily excited, this is not true for magnons with wavelengths considerably shorter than this width. The excitation efficiency of the stripline as a function of \( k \) is governed by the normalized Fourier transform of the spatial microwave field generated by the rectangular stripline, \( H(k) \) (Fig. 3A). Besides the fact that NV center measures the magnetic field generated by magnons of this wavevector, our measured value is further scaled by \( D(k,z) \propto k e^{-kz} \), also known as the filter function (35). Both \( H(k) \) and \( D(k,z) \) are shown in Fig. 3A. The total measured magnetic field is therefore expected to be given by

\[
|B_{\text{magnon}}| = A \times D(k,z) \times H(k),
\]

[2]

where \( A \) is a prefactor accounting for the NV axis oriented at an angle relative to the YIG and \( z \) is the distance between NV and YIG surface (Fig. 1B, Left Inset). Fig. 3D shows both ESR and Rabi measurements due to magnons excited at different frequencies. The oscillatory function due to \( H(k) \) is clearly visible. Fig. 3F shows the ESR response along the NV ESR line taken at different heights above the YIG. A shift in weight of the ESR spectrum to lower \( k \) is visible for higher \( z \) in accordance with the expected NV filter function. Ultimately, what we are doing in the section is to experimentally determine the point spread function of the “detector” to be used in the following scattering experiment.

Fig. 2. Phase imaging of coherent magnons and their dispersion. (A) Spatial image of NV fluorescence under continuous drive of both the stripline and remote antenna. The bright fluorescence signal corresponds to destructive interference of the reference RF signal from the antenna and magnon signals (SI Appendix). (B) Evolution of the magnon wavefront observed by shifting the relative phase (0 to 2\( \pi \)) of the reference source relative to the signal supplied to the stripline. (C) Linecut of an interference pattern generated with magnon frequency at 2.3 GHz corresponding to a wavelength of 2.35\( \mu \)m. (D) Imaging magnons with short wavelength. Magnons with wavelength down to 660 nm can easily be resolved. (E) Line average of image from D. (F) Magnon dispersion extracted from the fluorescence phase maps.
Interaction of Magnons with a Target Material

We now turn to describe the interaction of magnonic waves with a target material. For our target we use a 100-nm-thick permalloy (Py) disk with 5-μm diameter deposited directly onto the YIG. Coherent magnonic plane waves are launched using our microwave stripline. Upon impinging on the Py disk, as described below, magnons are scattered, and the coherent sum of the scattered and unscattered magnons is measured (Fig. 4A). The modulation in intensity observed suggests that the scattering of magnons is a coherent process. Additionally, we find that the scattered magnons are confined within an angular opening 2θ, which is a direct consequence of the anisotropic nature of the magnon dispersion (36–38). A second scattering map containing information on the local scattering phase is generated by applying an additional RF signal from the distant antenna (Fig. 4D). The RF field due to the antenna interferes with the magnetic field generated by the scattered and unscattered waves, thereby producing an image that encodes information about the local phase of the scattered waves.

Reliable Extraction of Target Material Properties

The most prominent features in the wave pattern of Fig. 4A and D are 1) negligible backscattering and 2) confinement of the scattered wave to a cone ahead of the target. These qualitative features of the scattered wave are determined entirely by the dispersion relation of the Damon–Eshbach surface waves (DESW), as has been extensively studied (10, 39–42). The negligible backscattering is due to the “field displacement non-reciprocity” of the free DESW, which implies that waves will be localized on either the top or bottom surfaces of the magnetic film, depending on the direction of propagation. The scattering cone is a direct consequence of the specific dispersion relation of the DESW. The isofrequency curves of the DESW dispersion asymptote to a cone in momentum space whose opening angle is given by

\[ \theta_c = \sin^{-1} \left( \frac{\omega - \omega_m}{\omega_m} \right) \]

\[ = \sin^{-1} \left( \frac{\omega - \omega_m}{\omega_M} \right) \]

Here, \( \omega \) is the mode frequency, \( \omega_m = \gamma_r B_{sat} \) and \( \omega_M = \gamma_r M_S \) (SI Appendix). The group velocity \( v_g \) is normal to isofrequency curves, and energy flow will therefore be limited to a cone in real space, with opening angle \( \theta_c \) with respect to the x axis. For the frequency shown in Fig. 4A (\( f = 2.18 \) GHz), the opening angle obtained using Eq. 3 is \( \theta_c \approx 34^\circ \). This is in reasonable agreement with the measured opening angle, \( \theta_c \approx 28^\circ \pm 2^\circ \). Furthermore, it can be shown (39) that the mode density of DESW diverges upon approaching \( \theta_c \), which explains the precipitous rise in the amplitude of the scattered wave observed near the critical angle in Fig. 4A and D. Beyond these gross features, an intricate structure in the phase profile and contrast of the scattered wave can be easily discerned. These details provide unique and detailed information about the target which we exploit below.

To provide a quantitative model of the magnon scattering by the Py disk we add a spatially localized AC source term to the wave equation for the DESW. We then compute the Green’s function for this wave equation, from which we can determine the scattered wave for an arbitrary source. Conversely, equipped with the Green’s function and experimental wave patterns we can invert the problem to determine the scattering potential. In the spirit of conventional scattering experiments, we adopt the latter approach to interrogate magnetic properties of the target.

In general, the scattered wave is given by the convolution

\[ B_{sw}(r) = \int d^3 r' G(r - r') \delta_d [V \cdot \mathbf{m}(r')] \]

where \( G(r) \) is the Green’s function, \( V \cdot \mathbf{m} \) is the source, and subscript \( d \) denotes a direction orthogonal to the NV axis. In principle, given \( B_{sw} \) and \( G \), the source term is completely determined by inverting the convolution.
However, for the current experimental setup we find that the presence of significant background as well as noise prohibit carrying out this procedure explicitly (SI Appendix). The inversion is therefore done by fitting the “source”, i.e., $V \cdot \mathbf{m}$, to best match the intensity pattern seen in the experiment. As basis functions for the fit we take the Gaussian function $e^{-\left(x^2+y^2\right)/\sigma^2}$ and its derivatives up to eighth order in both directions, where $\sigma$ is fixed to be the radius of the Py disk. We separate the source into real and imaginary components (Fig. 4 B and C, Left and Right Insets), to reflect a possible phase shift of the scattered wave with respect to the incident wave. Results of the analysis for the phase-resolved images are presented in Fig. 4. Note that we shifted the overall phase of the source in a way that makes it easier to separate components with different symmetries. The simplest model of magnetization dynamics of the target excited by the DESW corresponds to magnetic moment of the disk precessing around the direction of the static field, i.e., in the $xz$ plane. Since the disk is very thin in the $z$-direction, the dominant contribution to $V \cdot \mathbf{m}$ should come from the $x$-derivative of the $x$-component of magnetization. This should produce a dipolar pattern for the source aligned along the $x$ axis. Fig. 4B, Right Inset shows that the imaginary part of the source is indeed of the dipolar type, and it is almost an order of magnitude larger than the real part (Fig. 4B, Left Inset). The real part of the source has a quadrupolar character, which implies that the source has an additional gradient along the $y$ axis. We remind the readers that the incident wave is propagating along the $x$ axis, and hence this $y$-gradient cannot be related to the spatial profile of the incident wave. It appears to be an intrinsic feature of the target revealed by our scattering experiment. We comment on several other interesting features of our analysis. While we achieve an excellent fit of our model to the phase images, using the same parameters for the corresponding amplitude images does not automatically produce a good fit. We expect that understanding this discrepancy requires introducing a more sophisticated version of the Py disk/YIG interaction, including renormalization of static susceptibility parameters of YIG below the target. Second, we find the best agreement between theoretically simulated phase images and the experimental data when the target is represented using Gaussian basis functions, rather than functions with sharper edges. This suggests that response of the Py target near the disk edges is strongly suppressed. Finally, experiments have been done in the regime where the diameter of the target is of the order of the magnon wavelength. One can then expect the disk to develop spatial gradients in the direction of propagation of DESW. Surprisingly, we find that the main response of the target corresponds to magnetization of the entire disk oscillating as a whole.

Our magnonic scattering platform provides a way for exploring mesoscopic materials. While clearly suited for magnetic target materials, the time-varying magnetic fields generated by the magnons can also lead to strong interaction with other phases of matter such as superconductors, topological insulators with conducting surface states, spin liquids, and more, thereby providing new insights into such phases that are unattainable by other methods.

Data Availability. All study data are included in the article and/or supporting information.
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