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Analysis of Progressive Failure Mechanism of Rock Slope with Locked Section Based on Energy Theory

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Received: 2 February 2020; Accepted: 3 March 2020; Published: 3 March 2020

Abstract: Progressive failure in rock bridges along pre-existing discontinuities is one of the predominant destruction modes of rock slopes. The monitoring and prediction of the impending progressive failure is of great significance to ensure the stability of the rock structures and the safety of the workers. The deformation and fracture of rocks are complex processes with energy evolution between rocks and the external environment. Regarding the whole slope as a system, an energy evolution equation of rock slope systems during progressive failure was established by an energy method of systemic stability. Then, considering the weakening effect of joints and the locking effect of rock bridges, a method for calculating the safety factor of rock slopes with a locked section was proposed. Finally, the energy evolution equation and the calculation method of safety factor are verified by a case study. The results show that when the energy dissipated in the progressive failure process of rock bridges is less than the energy accumulated by itself, the deformation energy stored in the slope system can make the locked section deform continuously until the damage occurs. The system energy equal to zero can be used as the critical criterion for the dynamic instability of the rock slope with locked section. The accumulated deformation energy in the slope system can promote the development of the cracks in the locked section, and the residual energy in the critical sliding state is finally released in the form of kinetic energy, which is the main reason for the progressive dynamic instability of rock slopes.

Keywords: slope engineering; energy; instability criterion; safety factor; structural plane; stability analysis

1. Introduction

The stability of open pit slopes is one of the basic problems in the cross-field of geotechnical engineering and mining engineering. Accurate analysis of the stability status of open pit slopes can not only eliminate the potential hazards of landslide in time, but also provide a reference for the prediction of landslide. The failure process of a slope rock mass with cracks is mainly controlled by the rock bridge and structural plane [1,2]. The rock bridge refers to the discontinuous part, which plays a key bearing role on the potential sliding surface of the slope, which can also be called the locked section [3,4]. Rock slope stability is highly affected by the geometrical configuration and mechanical behavior of the geological discontinuities located in the rock slope. Previous studies [5,6] demonstrated that rock slope failure is frequently controlled by a complex combination of discontinuities that facilitate kinematic release. That is, massive rock slope instability inherently requires the evolution of natural discontinuities through a gradual transition from a discontinuous–continuous to a fully discontinuous medium.
Currently stability calculation for slopes with cracks are performed with different methods, including limit equilibrium method [7–9], numerical simulation way [10–12], and limit analysis approach [13–16]. Among these methods, the limit equilibrium method (LEM) of slices has attracted considerable attention, because of its simplicity and accuracy [7,17–22]. In this method, the ratio of resisting to driving forces on a potential sliding surface is defined as the factor of safety. The limit equilibrium techniques are the most commonly used analytical methods to investigate the stability of landslides. A slope is considered safe only if the calculated safety factor clearly exceeds unity. However, except in the obvious case of a high degree of joint continuity, these method often results in a conservative evaluation of slope stability [23]. Researchers [24–27] have suggested that most discontinuities are not fully persistent and that a complex interaction occurs between pre-existing discontinuities and brittle fracture propagation through intact rock bridges during rock slope failure. The shear strength along potential failure surfaces is hence determined partly by the failure through intact rock and partly by shears along discontinuities.

In fact, the slope failure is driven by energy. The energy released instantaneously during the failure process of the locked section is the main reason for the sudden failure of the rock slope with a locked section [4,28]. The energy method takes the whole slope system as the research object, and solves the stability coefficient in the form of virtual energy change, thus avoiding the complex mechanical analysis. The slope is constantly exchanging energy with the external environment and obeys the conservation of energy principle in the process of slope deformation, which can be expressed as an energy balance equation [29].

Many efforts have been made in the application of the energy principle to limit analysis of slope stability [30–36]. Mauldon and Ureta [30] gave up the traditional idea of limit equilibrium analysis, and put forward a new method to solve the slope stability problem based on the principle of minimum potential energy of stability equilibrium system. Donald and Chen [31] proposed a slope stability analysis method that is based on the energy principle. They started the calculation by establishing a compatible velocity field and obtained the factor of safety by the energy-work balance equation. Wang et al. [32] and Chen et al. [33,34] developed the applicability of this method in slope stability analysis. Gao et al. [35] proposed a numerical analysis method of rock slope stability based on the principle of minimum energy dissipation in view of the main shortcomings of the numerical analysis method of rock slope stability. Tu et al. [29] derived an energy balance equation that can express the energy change from the conservation of energy principle when the strength reduction method is used to analyze slope stability, and then analyzed the relationship between the energy change and slope failure. Gao et al. [36] proposed a new fracture criterion for rock with cracks based on energy principles, and studied the effect of crack propagation on the failure process of a rock slope.

However, most of these studies are only based on the energy principle to analyze the stability of intact rock slope or the influence of crack propagation on the main joint of slope. The effect of the rock bridge in the slope, which is one of the primary determinant of rock slope stability, has not been studied in these studies. In this paper, according to the progressive characteristics of the failure process of the locked section, regarded the whole slope as a system, an energy evolution equation of slope systems was established by an energy method of systemic stability. Considering the weakening effect of joints and the locking effect of rock bridges, we proposed a method for calculating the safety factor of rock slopes with locked sections, and analyzed some influential factors such as slope sizes and rock mass mechanics parameters.

2. Establishment of Mechanical Model

In many cases, the critical sliding instability of a large rock slope with a locked section is controlled by the shear failure of the locked section. The simplified model of a rock slope with a locked section is shown in Figure 1.
The established locked section sliding mechanical (LSSM) model is shown in Figure 2. The angle of the slope is $\theta$, degree; the length of the locked section is $L$, m; the average height of the slope is $H$, m; the thickness of the sliding mass is $h$, m; and the mass of the sliding body is $M$, kg. Before sliding away from the bedrock, the slope is an elastic medium with an elastic modulus of $G$. The elastic deformation of the slope under the action of gravity is $u$ and the gravity of the sliding body is $Mg$. When brittle fracture occurs in the locked section, the peak strength will change into residual strength. It is assumed that the peak-residual strength drop will act on the middle and lower sliding body in the form of equivalent elastic force $P$ when the upper locked section of the slope breaks.

The fracture propagation process of the rock mass in the locked section is highly progressive. The criterion between the shear force of the sliding body under the action of the gravity component and the displacement of the slope body along the sliding surface is as follows [37,38]:

$$T(u) = \frac{G_0 L}{h} u \exp\left[\left(\frac{u}{u_0}\right)^m\right] = \lambda u \exp\left[\left(\frac{u}{u_0}\right)^m\right]$$

(1)

where $T(u)$ is the shear force of the locked section, kN; $G_0$ is the initial shear modulus of elasticity, MPa; $h$ is the thickness of the sliding mass, m; $m$ is the brittleness index; $u$ is the elastic deformation of the slope under the action of gravity, m; $u_0$ is the displacement at the starting point $O$ of the instability of the locked section, m; and $\lambda$ is the shear stiffness, $\lambda = \frac{G_0 L}{h}$, MPa.

The relation curve between shear force and tangential displacement of the locked section is shown in Figure 3.
The average shear displacement of each section slope under the action of shear force is as follows:

$$u_1 = \frac{1}{H} \int_0^H u_1(y) dy = \frac{T(u)}{2HLG} \int_0^H (2yH - y^2) dy = \frac{HT(u)}{3LG}$$

By deriving the shear force in Equation (2), the incremental relationship between the slope sliding force and the average displacement can be obtained as follows:

$$\delta T(u) = \frac{3GL}{H} \delta u_1$$

3. Energy Balance Relationship of Slope System

The breakthrough of the fracture in the rock bridge core area means the loss of the strength of the locked rock mass [39,40]. The energy released in the process of residual stress difference is the main reason for the sudden failure of the locked rock mass. Therefore, the calculation and analysis of the energy evolution process of the locked slope system based on the principle of energy conservation is conducive to a correct understanding of the failure evolution mechanism of the locked slope system.

The expression of shear stress on slope is as follows:

$$\tau_{xy} = \tau_{yx} = \tau \left(1 - \frac{y}{H}\right) = \tau \left(1 - \frac{y}{H}\right)$$

The shear deformation energy produced in the slope body is as follows:

$$U_1 = \frac{L}{2G} \int_0^H \frac{T(u)^2}{L^2} \left(1 - \frac{y}{H}\right)^2 dy = \frac{HT(u)^2}{6GL}$$

The dissipated shear energy when the locked section deforms is as follows:

$$U_p = \int_0^\delta T(u) \delta u$$

As shown in Figure 3, under the action of sliding force, when the deformation reaches the peak value, the slope changes from quasi equilibrium state to dynamic instability state. In this process, the deformation of the locked section is $\delta u$ and the dissipated shear energy is $T(u)\delta u$ ($\delta u > 0$). When the slope system is controlled by multiple locked sections, the energy lost in the middle and upper locked sections will act on the last locked section in the form of equivalent elastic performance. According to the law of conservation of energy, when the last locked section breaks, the dissipated shear deformation energy of the slope system will decrease. In other words, with the increase of the number of locked
sections, the equivalent elastic energy obtained by the slope system will increase, and the slope instability process will be more sudden.

Assuming that there are \( k \) locked sections on the sliding surface of the slope, the deformation of each locked section is the same when it breaks. With the progressive failure of the locked section on the sliding surface, the brittle evolution index rate of the rock mass is 1.48. When the last locked section breaks, the shear energy to be dissipated is as follows:

\[
U_2 = \frac{1}{1.48^{k-1}} U_p \quad (7)
\]

In the progressive failure process of the locked section, the slope needs to release energy. When the slope is in the critical dynamic instability state, if the dissipated energy is greater than the shear deformation energy accumulated by the slope, it needs additional energy from external force to make the slope continue to slide. Therefore, the energy equation of progressive failure of slope system is as follows:

\[
\delta U = \delta U_1 + \delta U_2 + \delta W \quad (8)
\]

where \( U_1 \) is the shear deformation energy accumulated by the slope; \( U_2 \) is the dissipated energy; \( W \) is the additional energy from external force.

The shear strain energy variation of slope system is as follows:

\[
\delta U_1 = \frac{H T(u)}{3GL} \delta T(u) = \frac{T(u)}{k_1} \delta T(u) \quad (9)
\]

The energy dissipated per unit deformation of the locked section is as follows:

\[
\delta U_2 = T(u) \delta u \quad (10)
\]

Combining Equations (9) and (10), and substituting them into Equation (8), the following equation can be obtained:

\[
\frac{\delta U}{\delta u} = \frac{T(u) \cdot T'(u)}{k_1} + T(u) + J \quad (11)
\]

where \( k_1 \) is the shear rigidity of slope, \( k_1 = \frac{3GL}{H} \); \( J \) can be expressed as the input rate of external energy, \( J = \frac{W}{\delta u} \).

When \( \delta U < 0 \), the energy accumulated in the locked section is not enough to cause damage; when \( \delta U = 0 \), the slope system with locked section is in quasi equilibrium state; when \( \delta U > 0 \), the shear energy stored in the locked section can make the locked section deform continuously until the slope is destroyed. Therefore, \( \delta U = 0 \) can be used as the critical criterion of slope dynamic instability under the control of locked section.

4. Calculation and Analysis of Starting Velocity of Slope Instability

4.1. Calculation Formula of Starting Velocity

When the slope is unstable under the action of gravity, the displacement in \( y \) direction is not considered, only the displacement in \( x \) direction is considered. The overall deformation of the slope is as follows:

\[
u_1 = \frac{H T(u)}{3LG} \quad (12)
\]

The displacement component along the direction of gravity is expressed as:

\[
u_g = \frac{H T(u)}{3LG} \sin \theta \quad (13)
\]
The work done by the whole deformation of the slope under the action of gravity is as follows:

\[ \delta W = M g \frac{H T(u)}{3 L G} \sin \theta \delta u \]  

(14)

\[ I = \frac{\delta W}{\delta u} = T(u) \frac{M g H}{3 L G} \sin \theta \]  

(15)

As shown in Figure 3, point \( j \) is the starting point of slope instability with the locked section, and point \( s \) is the ending point of slope instability with the locked section. In the process of sliding from point \( j \) to point \( s \), energy consumption is required for fracture of the locked section, and the shear deformation energy in slope is released. When the displacement reaches point \( s \), the energy consumption of the slope system is as follows:

\[ dU_2 = \frac{1}{1.48^{k-1}} \lambda \mu \exp \left( -\frac{\mu}{\mu_0} \right) d\mu \]  

(16)

In the whole process of landslide, the energy of shear deformation not consumed will be released in the form of kinetic energy, and the energy change of the slope system is as follows:

\[ \Delta U = \int_{u_j}^{u_s} (dU_1 + dU_2 + dW) \]  

(17)

When the brittleness index of rock mass \( m = 1 \), the expressions of \( dU_1 \) and \( dW \) are as follows:

\[ dU_1 = \frac{H \lambda^2}{3 L G} \left( u - \frac{u^2}{u_0} \right) \exp \left( -\frac{2u}{u_0} \right) d\mu \]  

(18)

\[ dW = \frac{M g \sin \theta}{3 L G} \lambda \mu \left[ \exp \left( -\frac{u}{u_0} \right) - \exp \left( -\frac{\mu}{u_0} \right) - \exp \left( -\frac{\mu}{u_0} \cdot \frac{u}{u_0} \right) \right] d\mu \]  

(19)

Combining Equations (16) to (19), we can get the energy evolution equation of the slope system during progressive failure of the locked section as follows:

\[ \Delta U = \int_{u_j}^{u_s} \left[ \frac{H \lambda^2}{3 L G} \left( u - \frac{u^2}{u_0} \right) \exp \left( -\frac{2u}{u_0} \right) + \frac{M g \sin \theta}{3 L G} \lambda \mu \left[ \exp \left( -\frac{u}{u_0} \right) - \exp \left( -\frac{\mu}{u_0} \right) - \exp \left( -\frac{\mu}{u_0} \cdot \frac{u}{u_0} \right) \right] \right] d\mu \]  

(20)

The flow chart for evaluation of slope stability by energy method is shown in Figure 4. According to the kinetic energy theorem, the energy variation of the slope system meets the following equation:

\[ \Delta U = \frac{1}{2} M v^2 \]  

(21)

where \( M \) is the mass of the sliding body, kg; \( v \) is the starting velocity of the locked section when the sliding body breaks, m/s.

According to Equation (21), the expression of the starting velocity in the process of dynamic instability of the slope with locked section can be obtained as follows:

\[ v = \sqrt{\frac{2 \Delta U}{M}} \]  

(22)
4.2 Analysis of Calculation Results

As shown in Figure 5, Washan pit of Nanshan mine is an open-pit mine that mainly mines iron ore located in Maanshan city, Anhui Province, China. The rock mass in the upper part of the slope is seriously broken, and the hard rock in the lower part intersects with the weak structural plane, forming a series of tension cracks, so it is easy to slide along the joint plane. In order to ensure the safety of the residual ore recovery process, a movement and surveying radar (MSR-300) was used to monitor the surface displacement of the slope for three years. The MSR-300 radar system and the monitoring area are shown in Figures 6 and 7, and the main technical parameters of radar system are shown in Table 1.

Figure 5. Washan pit of Nanshan mine.
In 2014, a large-scale high-speed landslide occurred in the Washan pit, and the whole process presented a strong dynamic destructiveness. According to the investigation, the total volume of the sliding mass in the southeast side of the slope is $3.3 \times 10^5$ m$^3$, the average unit weight of the rock mass is $2.5 \times 10^4$ N/m$^3$, the length of the effective locked section in the sliding surface is 3.7 m, the average effective height of the sliding mass is 5 m, the effective thickness of the potential sliding zone is 0.8 m, the elastic modulus of the rock mass is 6.01 GPa, and the Poisson’s ratio is 0.25. According to Equation (20), the energy evolution of the south slope system in the process of instability is shown in Figure 8.

It can be seen from Figure 8 that in the process of failure evolution of the slope with locked section, the energy of the slope system experienced the process of “loss-accumulation-release”. Point $j$ marks the fracture of the locked section, and the instantaneous release of residual deformation energy in the slope system with the locked section fracture becomes the main reason for dynamic failure of the rock slope. From Figure 3, it can be calculated that the displacement of point $j$ is $1.56 \times 10^{-3}$ m, and that of point $s$ is $8.91 \times 10^{-3}$ m. According to Equation (20), the residual energy of slope system in the shadow part is calculated by Matlab integration as follows:

$$\Delta U = 2.71 \times 10^9$$ J
According to Equation (22), the starting velocity during slope instability can be calculated as follows:

\[ v = \sqrt{\frac{2\Delta U}{M}} = 2.57 \text{ m/s} \]

The instantaneous displacement and velocity of the sliding body monitored by the radar are shown in Figures 9 and 10. It can be seen that the instantaneous maximum displacement of the slope reaches 135 mm, the instantaneous starting speed of the slope sliding reaches 2.86 m/s, and the sliding process of the slope with the locked section is highly sudden. This is in good agreement with the starting speed calculated by theory. It shows that the formula of slope energy evolution based on the process of energy accumulation and the release of the slope system can accurately describe the progressive dynamic instability process of the locked slope.

**Figure 8.** Energy evolution process of slope system in Washan pit.

**Figure 9.** Displacement curve of slope instability process.

**Figure 10.** Speed curve of slope instability process.
5. Calculation and Analysis of Safety Factor

5.1. Calculation Formula of Safety Factor

The rock mass with locked section is cut by an uncertain number of discontinuous joints, and the thickness of the discontinuous joint is negligible relative to its own length or the length of the specific engineering rock mass. Therefore, the discontinuous joint can be used as Griffith fracture for mechanical analysis. Assuming that the locked section and the discontinuous joint are arranged regularly, the established mechanical model is shown in Figure 11. Under the compression shear stress, the mechanical model can be described as follows: In the vertical boundary, the load is uniformly distributed, and in the upper boundary of the rock mass, the normal stress is $\sigma_n$, and the shear stress is $\tau$. The length and average width of the discontinuous joint are $2m$ and $2n$ respectively, and the length of the rock bridge is $l$. The length of the discontinuous joint is much larger than its width, and the minimum of joint thickness can be 0 m.

![Figure 11. Stress analysis of rock mass with locked section.](image)

Assuming that the fracture is on the infinite disk of isotropic elastic continuous medium, the stress is uniformly distributed along the upper boundary of the rock mass, so it can be regarded as plane strain problem. According to Equations (23) and (24), the average normal stress and average shear stress at point $x$ of the rock mass are calculated as follows:

$$\sigma(x) = \sigma_n \cdot \frac{x}{\sqrt{x^2 - m^2}} (x \geq m) \tag{23}$$

$$\tau(x) = \tau \cdot \frac{x}{\sqrt{x^2 - m^2}} (x \geq m) \tag{24}$$

Taking the center of the crack initiation as the origin of $x$, the stress distribution at the locked section can be regarded as the superposition of the stress of adjacent joints.

$$\sigma_{n,R}(x) = \sigma_n \cdot \left( \frac{x}{\sqrt{x^2 - m^2}} + \frac{x'}{\sqrt{x'^2 - m^2}} \right) (m \geq x \geq m) \tag{25}$$

$$\tau_{n,R}(x) = \tau \cdot \left( \frac{x}{\sqrt{x^2 - m^2}} + \frac{x'}{\sqrt{x'^2 - m^2}} \right) (m \geq x \geq m) \tag{26}$$

where, $x' = 2a + l - x$.

Assuming that the shear strength parameters of homogeneous intact rock mass are known, the shear strength that can be borne at a certain point in the locked section can be obtained by local stress $\sigma_{n,R}(x)$ and $\tau_{n,R}(x)$:

$$\tau_{n,R}(x) = \sigma_{n,R}(x) \tan \varphi_R + c_R \tag{27}$$
The maximum tangential force that the rock mass with locked section can bear can be obtained by the following integral.

\[ T = \int_{0}^{m+1} \tau_{n,R}(x)dx = \frac{m+1}{m} \left( \frac{x}{\sqrt{\varepsilon^2 - m^2}} + \frac{x'}{\sqrt{\varepsilon'^2 - m^2}} \right) dx \]

\[ = \frac{1}{m} \tan \varphi_R \int_{0}^{m+1} \left( \frac{x}{\sqrt{\varepsilon^2 - m^2}} + \frac{x'}{\sqrt{\varepsilon'^2 - m^2}} \right) dx + c_R \]

where \( l \) is the length of rock bridge.

The average normal stress \( \overline{\sigma}_{n,R} \) and average shear stress \( \overline{\tau}_{r,R} \) of the rock bridge are as follows:

\[ \overline{\sigma}_{n,R} = \frac{1}{l} \int_{0}^{m+1} \sigma_{n,R}(x)dx = \frac{\overline{\sigma}_n}{m+1} \int_{0}^{m+1} \left( \frac{x}{\sqrt{\varepsilon^2 - m^2}} + \frac{x'}{\sqrt{\varepsilon'^2 - m^2}} \right) dx \]

\[ \overline{\tau}_{r,R} = \frac{1}{l} \int_{0}^{m+1} \tau_{r,R}(x)dx = \frac{\overline{\tau}_r}{m+1} \int_{0}^{m+1} \left( \frac{x}{\sqrt{\varepsilon^2 - m^2}} + \frac{x'}{\sqrt{\varepsilon'^2 - m^2}} \right) dx \]

Substituting Equations (29) and (30) into Equation (28), the following equation can be obtained:

\[ \overline{\tau}_{r,R} = \overline{\tau}_{n,R} \tan \varphi_R + c_R \]

The relationship between the uniformly distributed load on the boundary of rock mass and the average stress of local load on the locked section is as follows:

\[ \overline{\sigma}_n = \frac{l}{2m+1} \overline{\sigma}_{n,R} + \frac{2m}{2m+1} \overline{\sigma}_{n,D} = (1 - k_l) \overline{\sigma}_{n,R} + k_l \overline{\sigma}_{n,D} \]

\[ \overline{\tau}_r = \frac{l}{2m+1} \overline{\tau}_{r,R} + \frac{2m}{2m+1} \overline{\tau}_{r,D} = (1 - k_l) \overline{\tau}_{r,R} + k_l \overline{\tau}_{r,D} \]

where \( \overline{\sigma}_{n,D} \) and \( \overline{\tau}_{r,D} \) are the average values of the local normal stress \( \sigma_l(x) \) and shear stress \( \tau_r(x) \) of the continuous section respectively.

In the rock mass with locked section, the average normal stress at the locked section, the average normal stress at the continuous section and the uniformly distributed load on the rock mass boundary can be approximately considered as equal, namely:

\[ \overline{\sigma}_{n,R} \approx \overline{\sigma}_{n,D} \approx \overline{\sigma}_n \]

Assuming that the intact rock is isotropic, the shear strength of the locked section can be obtained by Equation (31). According to the Mohr-Coulomb criterion, the continuous section of the rock mass with locked section can be considered as having no cohesive force, so the shear stress of the continuous section is as follows:

\[ \overline{\tau}_{r,D} = \overline{\sigma}_{n,D} \tan \varphi_D \]

where \( \varphi_D \) is the internal friction angle of the rock mass.

Substituting Equations (31) and (35) into Equation (33), the following equation can be obtained:

\[ \overline{\tau}_r = (1 - k_l) \overline{\sigma}_{n,R} \tan \varphi_R + k_l \overline{\sigma}_{n,D} \tan \varphi_D + (1 - k_l) c_R \]

From Equation (34), the following equation can be obtained:

\[ \overline{\tau}_r = (1 - k_l) \overline{\sigma}_n \tan \varphi_R + k_l \overline{\sigma}_n \tan \varphi_D + (1 - k_l) c_R \]
It is defined that the ratio of the area of the continuous area of the rock mass with locked section to the total area of the rock mass is the through ratio \( k_p \), and \( k_p \) is used to replace \( k_t \), and it can be obtained that:

\[
\tau_r = (1 - k_p)\sigma_n \tan \phi_n + k_p\sigma_n \tan \phi_D + (1 - k_p)c_R
\]

(38)

For the rock mass with locked section, all the rock bridges that are not connected in the shear plane have locking effect on the sliding of the sliding body, while the shear strength of the rock mass at the connecting plane has weakening effect. The shear strength of the locked section in the sliding plane of the slope with locked section is much higher than that of the fully connected structural plane. Therefore, for the stability analysis of the slope with locked section, both the weakening effect of the through plane and the lock effect of the locked section should be considered. Ignoring the influence of the surface shape of the structural plane on the shear strength of the structural plane, considering the gravity of the slope, the stability of the slope rock mass under the locking action is analyzed. The local stress of the locked section at the sliding surface of the slope with the locked section is shown in Figure 12.

![Figure 12. Local stress diagram of locked section.](image)

Assuming that the structural plane in the rock mass of the slope is a Beacher disc joint model, the extension length of the joint in the rock mass during each excavation step is obtained through the stochastic joint network simulation technology. The radius after the extension of the joint disc can be calculated as follows:

\[
\rho' = \frac{2I_c}{\pi}
\]

(39)

where \( I_c' \) is the half trace length of the joint.

Through the radius of the joint disc, the continuous rate of the slope rock mass in each excavation process can be obtained as follows:

\[
k_p = \frac{\sum I_d}{\sum I_d + \sum I_R}
\]

(40)

where \( I_d \) is the area of continuous structural surface, \( I_R \) is the area of locked section.

Under the action of self-gravity, according to Equation (38), the maximum shear force that can be borne by the locked section of the slope is:

\[
T = (1 - k_p)G_N \tan \varphi_{dR} + k_pG_N \tan \varphi_{dD} + (1 - k_p)c_R I_D
\]

(41)

where \( \varphi_{dR} \) and \( \varphi_{dD} \) are the internal friction angles of the rock bridge and the structural plane on the equivalent structural plane, and \( c \) is the cohesion of the equivalent structural plane.

When the angle of slope is greater than the angle between the equivalent structural plane and the horizontal plane, the gravity of slope sliding body is obtained according to the geometric relationship as follows:

\[
G = \frac{(h \cdot \cot \theta - h \cdot \cot \beta) \cdot \gamma h}{2} = \frac{rh^2}{2} \cdot \frac{\sin(\beta - \theta)}{\sin \beta \sin \theta}
\]

(42)
\( G_N = G \times \cos \theta, \ G_T = G \times \sin \theta \)  

(43)

where \( G \) is the gravity of slope sliding body, \( \gamma \) is the unit weight of slope rock mass, \( h \) is the height of slope, \( \beta \) is the angle of slope, \( \theta \) is the angle between the equivalent structural plane and the horizontal plane.

Substituting Equations (42) and (43) into Equation (41), the following equation can be obtained:

\[
T = \frac{rh^2}{2} \cdot \frac{\sin(\beta - \theta)}{\sin \beta \sin \theta} \cdot \cos \theta \cdot \left[ (1 - k_p) \cdot \tan \varphi_{dR} + k_p \tan \varphi_{dD} \right] + (1 - k_p) c_{dR} \frac{h}{\sin \theta} 
\]

(44)

Under the action of self-gravity, in order to judge whether the slope is stable under the lock action of rock bridge, the expression of safety factor is defined as:

\[
F = \frac{T}{G_T}
\]

(45)

where \( G_T \) is the sliding force along the direction of structural plane, \( T \) is the maximum shear resistance of rock bridge.

Substituting Equation (44) into (45), the expression of safety factor is obtained as follows:

\[
F = \cot \theta \left[ (1 - k_p) \cdot \tan \varphi_{dR} + k_p \tan \varphi_{dD} \right] + \frac{2 \sin \beta (1 - k_p) c_{dR}}{\gamma h \sin(\beta - \theta) \sin \theta}
\]

(46)

It can be seen from Equation (46) that the stability of a rock slope with a locked section is related to the angle of slope, the continuity of structural plane, the mechanical parameters of equivalent structural plane, and the angle of structural plane and the height of slope. When \( F > 1 \), the sliding force along the structural plane is less than the maximum shear strength that the rock bridge can bear, and the slope is safe and stable; when \( F < 1 \), the sliding force along the structural plane has exceeded the maximum shear strength that the rock bridge can bear, and the slope is in a state of instability; when \( F = 1 \), the sliding force is equal to the maximum shear strength that the rock bridge can bear, and the slope is in the limit equilibrium state.

5.2. Analysis of Calculation Results

Table 2 shows the calculation parameters of rock slope with locked section under various working conditions. By controlling a single variable, each parameter is substituted into Equation (46) to calculate the change of slope stability safety factor with each basic parameter. The relationship curves between the safety factor and different parameters are shown in Figure 13.

| Figure Number | \( h/(m) \) | \( \gamma/(kN/m^3) \) | \( \beta(°) \) | \( k_p \) | \( \varphi_{dR}(°) \) | \( \varphi_{dD}(°) \) | \( c_{dR}/MPa \) | \( \theta(°) \) | Variables |
|---------------|-------------|----------------|-------------|--------|----------------|----------------|----------------|------------|-------------|
| 13a           | 100–360     | 25             | 0           | 0.8    | 40            | 30            | 0.2            | 25         | \( h \)     |
| 13b           | 200         | 20–42          | 0           | 0.8    | 40            | 30            | 0.2            | 25         | \( \gamma \) |
| 13c           | 200         | 25             | 40–70       | 0.8    | 40            | 30            | 0.2            | 25         | \( k_p \)   |
| 13d           | 200         | 25             | 40          | 0.1–0.8| 40            | 30            | 0.2            | 25         | \( \varphi_{dR} \) |
| 13e           | 200         | 25             | 40          | 0.8    | 6–42          | 30            | 0.2            | 25         | \( \varphi_{dD} \) |
| 13f           | 200         | 25             | 40          | 0.8    | 40            | 20–50         | 0.2            | 25         | \( c_{dR} \) |
| 13g           | 200         | 25             | 40          | 0.8    | 40            | 30            | 0.05–0.25      | 25         | \( \theta \) |
| 13h           | 200         | 25             | 40          | 0.8    | 40            | 30            | 0.2            | 15–30      |              |
In this section, we will calculate the safety factor of slope by using various limit equilibrium methods. As shown in Table 3, safety factor derived from Equation (46) is very close to the value calculated by the other limit equilibrium methods. Compared with the commonly used limit analysis method (LEM), and compare the results with the safety factor calculated by the method proposed in this paper. The calculation results of slope/W software are shown in Figure 14. Table 3 shows a comparison of the safety factor calculated by Equation (46) and that obtained by using four different methods: Fellenius method, simplified Bishop method, simplified Janbu method and Morgenstern–Price method. As shown in Table 3, safety factor derived from Equation (46) is very close to the value especially suitable for the calculation of the safety factor of a rock slope with locked section.

Figure 13. Relation curve between safety factor and different parameters. (a) slope height, (b) bulk density of rock mass, (c) slope angle, (d) continuity rate, (e) internal friction angle of rock bridge, (f) internal friction angle of structural plane, (g) cohesion, (h) structural plane angle.
As shown in Figure 13, the stability of the slope with locked section decreases with the increase of slope height, rock volume weight, slope angle, slope discontinuity and structural plane inclination. For a specific slope, the influence of the change of slope height, rock mass bulk density and slope angle on the safety factor of slope stability is less than the influence of the continuity rate of slope structural plane and the inclination angle of structural plane on the stability of slope; with the increase of friction angle in the rock bridge, friction angle in structural plane and cohesion of structural plane, the safety factor of slope increases gradually. The influence of friction angle and cohesion of structural plane on the stability of slope is more significant. Therefore, when analyzing the stability of such a slope, it is necessary to establish an accurate three-dimensional network model, taking into account not only the locking effect of rock bridge on rock mass, but also the weakening effect of joints on the mechanical properties of rock mass.

6. Discussion

In this section, we will calculate the safety factor of slope by using various limit equilibrium methods (LEM), and compare the results with the safety factor calculated by the method proposed in this paper. The calculation results of slope/W software are shown in Figure 14. Table 3 shows a comparison of the safety factor calculated by Equation (46) and that obtained by using four different methods: Fellenius method, simplified Bishop method, simplified Janbu method and Morgenstern–Price method. As shown in Table 3, safety factor derived from Equation (46) is very close to the value calculated by the other limit equilibrium methods. Compared with the commonly used limit analysis methods, the safety factor calculation method proposed in this paper can not only consider the weakening effect of discontinuous joints, but also reflect the locking effect of the locked section. The new method has the advantages of clear physical meaning and simple calculation process, and is especially suitable for the calculation of the safety factor of a rock slope with locked section.

![Figure 14. Calculation results of Slope/W software.](image)

| $h_1$ (m) | $\beta_1$ (°) | $\theta_1$ (°) | $\gamma_1$ (kN/m$^3$) | $k_p$ | $q_{4B}$ (°) | $c_{4B}$ (MPa) | Fellenius | Bishop | Janbu | M-P | Equation (46) |
|-----------|---------------|---------------|-----------------------|------|--------------|---------------|-----------|--------|-------|-----|-------------|
| 267       | 34            | 43.35°        | 27                    | 0.48 | 28.36        | 0.87          | 1.394     | 1.432  | 1.390 | 1.429 | 1.400       |

It should be noted that the energy evolution equation and safety factor formula proposed in this paper also have some limitations. The energy evolution equation of slope system based on the law of energy conservation is only applicable to the rock slope with locked section. In the stress analysis of slope, only the effect of gravity is considered, and the influence of other external loads on the slope failure evolution process is not considered. Moreover, the formula of safety factor is based on the assumption that the intact rock is isotropic. In the establishment of the mechanical model of the rock
slope with locked section, we consider the discontinuous joints as cohesionless materials, and do not consider the influence of the surface morphology of the structural plane on the shear strength.

7. Conclusions

Based on the principle of energy and considering the progressive characteristics of the failure process of the slope with locked section, this paper calculates and analyzes the instability mechanism of the slope with locked section, and establishes the method of solving the safety factor of the slope with locked section on the basis of considering the joint weakening and rock bridge locking effect, and analyzes the influencing factors, and draws the conclusion as follows:

(1) In the process of progressive failure of slope with locked section, when the loss energy is greater than the accumulated energy of the slope, the slope will not be damaged without external force, and the slope is in a stable state; when the system energy of the slope is equal to zero, the slope is in a quasi-equilibrium state; when the loss energy is less than the accumulated energy of the slope, the stored strain energy can make the locked section deform continuously, which eventually leads to slope failure. Therefore, the system energy of zero can be regarded as the critical criterion of dynamic instability of slope with lock.

(2) The accumulated deformation in the slope system with locked section can cause the failure of the locked section, and the residual energy in the critical sliding state is finally released in the form of kinetic energy, which is the main reason for the gradual and sudden instability of the slope with locked section.

(3) The stability of the slope with locked section is related to the height of the slope, the unit weight of the rock mass, the angle of the slope, the continuity and angle of the structural plane of the slope, the mechanical parameters of the locked section and the through section, etc.; the influence of the height of the slope, the unit weight of the rock mass and the angle of the slope on the stability of the slope is less than the influence of the continuity and angle of the structural plane of the slope. In contrast, the influence of internal friction angle and cohesion on slope stability is more significant.

Author Contributions: Conceptualization, Q.G.; methodology, Q.G.; validation, Q.G. and Y.Z.; formal analysis, J.P. and Y.Z.; investigation, J.P. and Y.Z.; resources, Q.G. and M.C.; writing—original draft preparation, J.P. and Y.Z.; writing—review and editing, J.P.; supervision, M.C.; project administration, Q.G. and M.C.; funding acquisition, Q.G. and M.C. All authors have read and agreed to the published version of the manuscript.

Funding: This research was funded by the National Key Research and Development Program of China (Grant No. 2018YFE0101100) and the Fundamental Research Funds for the Central Universities (Grant No. FRF-TP-18-015A3).

Conflicts of Interest: The authors declare no conflict of interest.

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