Spin-glass transition in the spin–orbit-entangled $J_{\text{eff}} = 0$ Mott insulating double-perovskite ruthenate

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We have successfully synthesized new Ru4+ double perovskite oxides SrLaInRuO6 and SrLaGaRuO6, which are expected to be a spin–orbit coupled $J_{\text{eff}} = 0$ Mott insulating ground state. Their magnetic susceptibility is much significant than that expected for a single Ru4+ ion for which exchange coupling with other ions is negligible. Their isothermal magnetization process suggests that there are about 20 percent isolated spins. These origins would be the Ru3+/Ru5+ magnetic defects, while the regular Ru4+ sites remain nonmagnetic. Moreover, SrLaGaRuO6 shows a spin-glass-like magnetic transition at low temperatures, probably caused by isolated spins. The observed spin-glass can be interpreted by the analogy of a dilute magnetic alloy, which can be seen as a precursor to the mobile $J_{\text{eff}} = 1$ exciton as a dispersive mode as predicted.

Recent material investigations have revealed novel phenomena driven by spin–orbit coupling (SOC)1–20. The strong SOC splits a $t_{2g}$ band into $J = 3/2$ and $J = 1/2$ bands, which results in the realization of the SOC Mott insulating state. For example, the Ir4+ ions with the $d^5$ electron configuration have an effective orbital moment $L = 1$, resulting in a $J_{\text{eff}} = 1/2$ pseudospin. Such a spin–orbit-entanglement gives rise to unconventional interaction among pseudospins. In the case of $J_{\text{eff}} = 1/2$ pseudospins, the bond-dependent Ising interactions, which have been called the Kitaev interaction in recent years, are realized1. Such a realization of the SOC pseudospin state was first observed in layered perovskite SrIrO3. Consequently, it is theoretically predicted that a honeycomb lattice magnet with $J_{\text{eff}} = 1/2$ pseudospins is a promising host of a quantum spin liquid (QSL). In the realistic compounds, RuCl3 and H3LiIr2O6 exhibit the Kitaev QSL behavior3–6.

In contrast, the $d^4$ electron system (Ru4+, Os4+, and Ir5+) has not been attracted much attention due to an absence of local moments in the ionic ground state. However, $J_{\text{eff}} = 1$ excitations become dispersive modes in a crystal due to moderate superexchange (SE) interactions. These mobile spin–orbit excitons may condense in this situation, which results in a magnetically ordered state7–11. In order to realize such a state, the exchange interaction must overcome a critical value sufficient to exceed the energy gap $\Delta$ between the $J_{\text{eff}} = 0$ and $J_{\text{eff}} = 1$12. Such a condensed state, for which the physicists conceive a terminology—spin–orbit-exciton condensation (SOEC), is analogous to magnon condensation phenomena in a spin dimer system13. One factor differentiating the $d^4$ system from the spin-dimer one is the anisotropy of the strong exchange interaction, which originates from the strong spin–orbit interaction. Therefore, it is expected that a novel condensed phase will be realized.

Although theoretical studies have been enormously advanced to search for anomalous phenomena driven by SOEC, experimental studies have not been carried out due to the lack of model materials. This situation is because the energy scale of $\Delta$ is too large; the 5$d^4$ (Ir5+ and Os4+) compounds should be typically nonmagnetic. Indeed, the weak magnetic anomalies observed in some Ir5+ double perovskites are better explained by the Ir4+ and Ir6+ magnetic defects rather than SOEC14–21. Therefore, a SOEC seems to be much less feasible for 5$d^4$ compounds. On the other hand, SOEC is more likely realizable in 4$d^4$ compounds such as Ru4+, where SOC is smaller than Ir5+ and is comparable to SE. Moreover, the SOC vs. SE competition can be tuned by a lattice control. Therefore, Ru4+ double perovskites would be good model-compounds for a realization of SOEC22.

We report the magnetic properties of novel double perovskites SrLaInRuO6 and SrLaGaRuO6 with Ru4+ ion. These deviate significantly from the single-spin susceptibility expected for Ru4+ ($J_{\text{eff}} = 0$) ions, even though the
distance between the magnetic ions is sufficiently large. Furthermore, the magnetization process up to 60 T demonstrates the presence of about 20 percent isolated spins. These behaviors can be explained as originating from the Ru$^{3+}$/Ru$^{5+}$ magnetic defects. Moreover, only SrLaGaRuO$_6$ shows a spin-glass transition at $T_f \approx 50$ K. We discuss the origin of the observed spin-glass transition from the analogy of the dilute magnetic alloy from the viewpoint of $J_{	ext{eff}} = 0$ physics in SOC Mott insulators.

**Experimental methods**

Polycrystalline samples of SrLaRu$_6$ (M = In, Ga) were synthesized by the conventional solid-state reaction from stoichiometric mixtures of SrCO$_3$, La$_2$O$_3$, M$_2$O$_3$ (M = In, Ga), and RuO$_2$. The obtained samples were characterized by powder X-ray diffraction (XRD) experiments using a diffractometer with Cu Kα radiation. The cell and crystal structure parameters were refined using the Rietveld method using the Rietan-fp version 2.16 software. The temperature dependence of the magnetization was measured using the magnetic property measurement system (MPMS; Quantum Design) equipped in the Institute for Solid State Physics at the University of Tokyo. Magnetization curves up to 60 T were measured using an induction method with a multilayer pulsed magnet at the Institute for Solid State Physics at the University of Tokyo.

**Results**

**Crystal structure.** Figure 1a and b shows powder XRD patterns from thus-obtained samples. All peaks are indexed to monoclinic unit cells based on the space group of P2$_1$/c. The Rietveld analysis converged well with the distorted double perovskite structure shown in Fig. 1c and the structural parameters in Table 1. No deviation from the ratio of Sr:La = 1:1 was detected within the experimental error. We estimate the modified tolerance factor $t_m$ as structural stability in double perovskite using the ionic radii values, yielding $t_m = 0.93834$ and 0.98015 in SrLaInRuO$_6$ and SrLaGaRuO$_6$, respectively. In the condition of $t_m < 1$, the double perovskite-type compounds should crystallize a monoclinic structure. Our samples certainly satisfy the criterion.

![Figure 1](https://example.com/figure1.png)

**Figure 1.** Powder X-ray diffraction patterns of (a) SrLaInRuO$_6$ and (b) SrLaGaRuO$_6$. The observed intensities (red), calculated intensities (black), and their differences (blue) are shown. Vertical bars (green) indicate the positions of the Bragg reflections. (c) Crystal structure of SrLaMRuO$_6$ (M = In, Ga) obtained from crystal structure parameters refined using the Rietveld method. The VESTA program is used for visualization.

| atom   | site | x    | y    | z    | B (Å) |
|--------|------|------|------|------|-------|
| Sr/La  | 4e   | 0.5060 (7) | 0.5346 (3) | 0.252 (1) | 0.23 (4) |
| In     | 2c   | 0    | 1/2  | 0    | 1.3 (3) |
| Ru     | 2b   | 1/2  | 0    | 0    | 1.3 (3) |
| O1     | 4e   | 0.229 (7) | 0.222 (7) | 0.992 (6) | 2.7 (2) |
| O2     | 4e   | 0.324 (7) | 0.704 (6) | 0.963 (4) | 2.7 (2) |
| O3     | 4e   | 0.405 (5) | 0.988 (2) | 0.222 (5) | 2.7 (2) |

**Table 1.** Crystallographic parameters for SrLaInRuO$_6$ and SrLaGaRuO$_6$ (both space group: P2$_1$/c) determined from powder X-ray diffraction experiments. The obtained lattice parameters are $a = 5.6976$ (4), $b = 5.7427$ (4), $c = 8.0669$ (5) Å, and $\beta = 90.05$ (1)$^\circ$ for SrLaInRuO$_6$ and $a = 5.5077$ (4), $b = 5.5538$ (3), $c = 7.8210$ (3) Å, and $\beta = 90.586$ (3)$^\circ$ for SrLaGaRuO$_6$. $B$ is the atomic displacement parameter.
In estimating the valence of the B-site cations at the center of the octahedral MO₆ (M = In, Ga) and RuO₆ in the two double perovskite oxides, we used the bond valence sum BV expressed by the following formula,

\[ BV = \sum_i^N \exp \left( \frac{R_0 - R_i}{0.37} \right) \]

where \( R_0 \) is the empirical bonding parameter, \( R_i \) is the inter-bond cation–anion distance, and \( N \) is the coordination number. The estimated \( BV \) values of In/Ga and Ru ions in SrLaInRuO₆ and SrLaGaRuO₆ are listed in Table 2, which aligns with the expected values.

**Magnetism.** Figure 2a shows the temperature dependence of magnetization \( M/H \) of SrLaMRuO₆ (M = In, Ga) and La₂MgRuO₆ under a magnetic field of 1 T. In this figure, only the results of field cooling (FC) data are shown. The black dotted curve shows the free-spin magnetization of Ru⁴⁺ ions calculated as described in the text. The \( M/H \) curves of SrLaGaRuO₆ under several magnetic fields. In each field, measurements were conducted upon heating after zero-field cooling (ZFC, open circles) and then upon cooling (FC, closed circles). (c) The isothermal magnetization of SrLaInRuO₆ and SrLaGaRuO₆ under a magnetic field up to 60 T at 4.2 K. The black dashed lines represent the best fit by Eq. (4) described in the text.

In estimating the valence of the B-site cations at the center of the octahedral MO₆ (M = In, Ga) and RuO₆ in the two double perovskite oxides, we used the bond valence sum \( B_V \) expressed by the following formula,

\[ B_V = \sum_i^N \exp \left( \frac{R_0 - R_i}{0.37} \right) \]

where \( R_0 \) is the empirical bonding parameter, \( R_i \) is the inter-bond cation–anion distance, and \( N \) is the coordination number. The estimated \( B_V \) values of In/Ga and Ru ions in SrLaInRuO₆ and SrLaGaRuO₆ are listed in Table 2, which aligns with the expected values.

**Magnetism.** Figure 2a shows the temperature dependence of magnetization \( M/H \) of SrLaMRuO₆ (M = In, Ga) under an applied field of 1 T. For comparison, the \( M/H \) data of La₂MgRuO₆ with a similar d⁴ electron configuration is displayed. The magnetic response of SrLaMRuO₆ and La₂MgRuO₆ is quite different despite the similar electronic state of the Ru⁴⁺ single ion. When considered from a crystallographic point of view, these compounds are not expected to have strong magnetic interactions because of the significant separation of Ru⁴⁺ ions. Therefore, it seems strange that the magnetic responses of SrLaMRuO₆ and La₂MgRuO₆ are so different. Kotani theoretically predicted the effective magnetic moment of dⁿ ions (n = 1 ~ 5) as a function of electron filling n, spin–orbit coupling, temperature, and ligand environment. The effective magnetic moment \( \mu_{\text{eff}} \) of low-spin d⁴ in an octahedral environment can be expressed as follows,

\[ \mu_{\text{eff}}^2 = \frac{\lambda^2 (4 + x^2 / 2 - 9 e^{-x^2 / 2} (5x / 2 - 15) e^{-3x^2 / 2})}{x^4 [1 + 3e^{-x^2 / 2} + 5e^{-3x^2 / 2}]} \]

where \( \lambda = \lambda / k_B T \), \( \lambda \) is the spin–orbit coupling interaction, and \( k_B \) is the Boltzmann constant. Thus, the magnetic susceptibility of isolated Ru⁴⁺ ions \( \chi_{\text{calc}} \) can be expressed as follows,

\[ \chi_{\text{calc}} = \frac{N \mu_{\text{eff}}^2}{3 k_B T} \]

where \( \chi_{\text{calc}} \) is the magnetic susceptibility calculated using a value of \( \lambda = 980 \text{ cm}^{-1} \) for Ru⁴⁺ ions. The \( \lambda \) value is expected to be smaller than the completely free-ion value of \( \lambda = 1400 \text{ cm}^{-1} \) determined in the study used Ru⁴⁺ complexes, which origin of \( \lambda \)-shrinking would be covalency. Note that it is necessary to incorporate the effect of the low symmetry field in order to reproduce the susceptibility in distorted Ru⁴⁺ double perovskites since Eq. (2) is calculated in the cubic symmetry field. The \( M/H \) data of La₂MgRuO₆ seemingly
Table 3. Results of fits to the isothermal M using the model described in the text. The parameters $N_{1/2}$ and $N_{3/2}$ are fixed to equal.

| Compound          | $N_{1/2}$ (Ru$^{4+}$) | $N_{3/2}$ (Ru$^{4+}$) | $\chi_{\text{eff}}$ (cm$^3$/mol Oe) |
|-------------------|------------------------|------------------------|-------------------------------------|
| SrLaInRuO$_6$    | 0.1088 (3)             | 0.1088 (3)             | 0.00869 (2)                         |
| SrLaGaRuO$_6$    | 0.0937 (21)            | 0.0937 (21)            | 0.01503 (2)                         |

High-field magnetization. Figure 2c shows the isothermal magnetization $M$ up to 60 T. The $M$-$H$ curves show convex behavior upward, implying an isolated spin different from the van Vleck magnetism of Ru$^{4+}$ pseudospin $J_{\text{eff}}=0$ state. The origin of the isolated spin will be discussed later. The increase in magnetization at high-field regions is due to the van Vleck paramagnetism.

**Discussion**

As described above, the two novel double perovskite ruthenates SrLaInRuO$_6$ and SrLaGaRuO$_6$ are expected to show a van Vleck magnetism of Ru$^{4+}$ pseudospin $J_{\text{eff}}=0$ state. However, the observed $M$-$H$ is considerably larger than a single Ru$^{4+}$ spin, indicating the deviation from the $J_{\text{eff}}=0$ state. In addition, the isothermal magnetization demonstrates the existence of an isolated spin.

A similar enhancement of magnetization has been reported in highly solid-solution double perovskite iridates Sr$_2$Ca$_x$IrO$_6$. An X-ray magnetic circular dichroism (XMCD) measurement demonstrates an emergent partial charge disproportionation (PCD) of Ir$^{5+}$ → 0.5Ir$^{4+}$ + 0.5Ir$^{6+}$ due to a site-randomness. In light of this result, similar Ru$^{4+}$/Ru$^{5+}$ magnetic defects possibly occurs in SrLaInRuO$_6$ and SrLaGaRuO$_6$ due to a similar intrinsic A-site randomness.

The magnetization of isolated spin $M_{\text{iso}}$ follows a Brillouin function, while the van Vleck term $M_{\text{vv}}$ should be proportional to $H$. Both terms would contribute to the observed nonlinear behaviors of the isothermal $M$. Here, in order to separate the contributions of the isolated spins and the van Vleck term, we analyze the $M$ data with a modified Brillouin function,

$$M = M_{\text{iso}} + M_{\text{vv}} = \sum_i N_i g_i \mu_B J \left\{ \frac{2J + 1}{2J} \coth \left( \frac{2J + 1}{2J} \frac{\mu_B H}{k_B T} \right) - \frac{1}{2J} \coth \left( \frac{1}{2J} \frac{\mu_B H}{k_B T} \right) \right\} + \chi_{\text{eff}} H$$

(4)

where $N_i$ represents a scaling factor to account for a finite number of paramagnetic free spins, $g_i$ (≈ 2) is the g-factor, $\mu_B$ is the Bohr magneton, $J$ (≈ 1/2 and 3/2) is the total angular momentum. For the second term, $\chi_{\text{eff}}$ indicates the van Vleck term. The values of $N$ and $\chi_{\text{eff}}$ are summarized in Table 3. Provided that $N_{1/2}$ and $N_{3/2}$ are fixed to equal considering the local charge disproportionation model, the $M$ data up to 60 T fit the Eq. (4). The best fits are shown by the dashed lines in Fig. 2c, with the fitting parameters given in Table 3. Our analysis suggests that ≈ 20% of free spins ($J=1/2$ and 3/2) are present. The orphan spins possibly emerged by the valence being off from tetravalent, which is no evidence from the crystal structural analysis. Although we cannot rule out other origins, these facts support that the PCD model is a good solution. As in the Ir$^{5+}$ system, the PCD-generated isolated spins may be directly detected by XMCD measurements: it is a further issue. In addition, the van Vleck term was found to be more significant for SrLaGaRuO$_6$.

The estimated van Vleck term of SrLaGaRuO$_6$ is larger than SrLaInRuO$_6$. According to Boltzmann statistics, the van Vleck term is proportional to the concentration of $J_{\text{eff}}=1$ exciton. Therefore, the difference in $\chi_{\text{eff}}$ between SrLaInRuO$_6$ and SrLaGaRuO$_6$ is due to the different $\Delta$. In the theoretical prediction, a non-cubic crystal field, generated by a distortion of the RuO$_6$ octahedra, effectively reduces $\Delta$. Here, we introduce the bond angle variance $\sigma$, as a scale parameter of the polyhedral distortion. The $\sigma$-value in the RuO$_6$ octahedra can be parametrized by the following formula,

$$\sigma = \sqrt{\sum_{i=1}^{12} (\phi_i - \phi_0)^2 / m - 1}$$

(5)

where $m$ is the number of O–Ru–O angles, $\phi_i$ is the $i$th bond angle of the distorted coordination-polyhedra, and $\phi_0$ is the bond angle of the coordination polyhedral with $O_6$ symmetry; $\phi_0$ equals 90° for octahedron. Calculations
Ru\textsuperscript{3+} (Ru\textsuperscript{5+})

free spin

Interaction
via \( J_{\text{eff}} = 1 \) exciton

Ru\textsuperscript{4+} (\( J_{\text{eff}} = 0 \))

Figure 3. Schematic of the mechanism of spin-glass induced by isolated spin and \( J_{\text{eff}} = 1 \) excitons, as an analogy of a dilute magnetic alloy. The interaction between free spins mediated by mobile \( J_{\text{eff}} = 1 \) excitons corresponds to the RKKY interaction.

using the atomic position parameters listed in Table 1 yield the \( \sigma \)-values of 7.7976° and 10.2708° for SrLaInRuO\textsubscript{6} and SrLaGaRuO\textsubscript{6}, respectively, indicating a strikingly larger non-cubic crystal field in SrLaGaRuO\textsubscript{6} than SrLaInRuO\textsubscript{6}. Therefore, the concentration of \( J_{\text{eff}} = 1 \) exciton of SrLaGaRuO\textsubscript{6} should be larger than SrLaInRuO\textsubscript{6}, consistent with the large-small relationship of \( \chi_{\text{mv}} \).

Furthermore, it is theoretically predicted that the SE interaction between \( J_{\text{eff}} = 0 \) reduces \( \Delta \). In SrLaInRuO\textsubscript{6} and SrLaGaRuO\textsubscript{6}, the \( J_{\text{eff}} = 0 \) pseudospins interact via the SE interaction through Ru\textsuperscript{4+}–O\textsuperscript{2–}–M\textsuperscript{3+}–O\textsuperscript{2–}Ru\textsuperscript{4+} paths with \( M = \text{In, Ga} \). Thus, it is considered that the difference in the SE interaction between these two systems arises from the filled outermost orbitals, which are 4d and 3d orbitals for SrLaInRuO\textsubscript{6} and SrLaGaRuO\textsubscript{6}, respectively. Therefore, it is reasonable that the SE magnitude is different.

Based on the results so far, it is reasonable to consider that the spin-glass transition in SrLaGaRuO\textsubscript{6} is due to randomly arranged isolated spins. Strangely enough, however, no spin-glass transition has been observed in SrLaInRuO\textsubscript{6} where the isolated spin concentration is comparable. However, it is unlikely that all the 19% localized spins interact strongly in SrLaGaRuO\textsubscript{6} where the Ru–Ru distance is far apart. This fact suggests a difference in the magnitude of the interaction between randomly arranged isolated spins.

The origin of the spin-glass transition in SrLaGaRuO\textsubscript{6} can be inferred by analogy with dilute magnetic alloys. In dilute magnetic alloys, partially arranged magnetic atoms interact with each other via RKKY interactions. As mentioned in the introduction, the \( J_{\text{eff}} = 1 \) excitons become a dispersive mode due to strong SE interactions. In this situation, the mobile \( J_{\text{eff}} = 1 \) exciton may behave like a conduction electron. Therefore, the interaction via a mobile \( J_{\text{eff}} = 1 \) exciton between the free spins in a \( J_{\text{eff}} = 0 \) magnet can be regarded as an RKKY interaction. A schematic diagram of this mechanism is shown in Fig. 3. This interaction should be proportional to the concentration of \( J_{\text{eff}} = 1 \), which is consistent with the presence/absence of spin-glass transition. The feasibility of the spin-glass transition in the category of spin–orbit excitonic magnetism is very interesting and requires further theoretical studies. In the broad context, this finding also suggests that the several magnetic responses in SrLaGaRuO\textsubscript{6} which have been found so far, would be explained by the generated isolated spin model. Thus, we sincerely hope that it should be carefully re-examined.

Summary
We have successfully synthesized new Ru\textsuperscript{4+} double perovskite oxides SrLaInRuO\textsubscript{6} and SrLaGaRuO\textsubscript{6}. The temperature-dependent \( M/H \) and isothermal \( M \) data can be explained by the van Vleck magnetism of \( J_{\text{eff}} = 0 \) states with additional isolated spins possibly generated by the Ru\textsuperscript{4+}/Ru\textsuperscript{5+} magnetic defects. While SrLaInRuO\textsubscript{6} is paramagnetic down to 2 K, SrLaGaRuO\textsubscript{6} shows spin-glass transition at \( T \text{\textsubscript{g}} \sim 50 \) K. We propose that the origin of spin-glass is isolated spins couple via mobile \( J_{\text{eff}} = 1 \) excitons as an analogy of a dilute magnetic alloy. It is expected that the spin-glass transition due to the introduction of isolated spins demonstrates the existence of mobile \( J_{\text{eff}} = 1 \) excitons as dispersive modes as predicted in spin–orbit-entangled d\textsuperscript{4} ions.

Data availability
The datasets generated and analyzed during the current study are available from the corresponding author.

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Author contributions
H.Y. conceived and conducted the experiment(s), performed statistical analysis and figure generation, and wrote the original manuscript. Y.H. designed the experiments, conducted the experiment(s), edited the manuscript, and performed statistical analysis. A.M. and K.K. performed the magnetization measurements using pulsed magnetic fields. H.A.K. administrated project and edited the manuscript. All authors discussed the results and reviewed the manuscript.

Competing interests
The authors declare no competing interests.

Additional information
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