Non-extensive statistics and a systematic study of meson-spectra at LHC energy $\sqrt{s_{NN}} = 2.76$ TeV

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Received: 27 February 2014 / Revised: 17 March 2014
Published online: 11 April 2014 – © Società Italiana di Fisica / Springer-Verlag 2014
Communicated by Xin-Nian Wang

Abstract. The transverse momentum spectra of secondary pions and kaons, produced in P + P and various central Pb + Pb collisions at $\sqrt{s_{NN}} = 2.76$ TeV at LHC, have been analyzed systematically with an approach based on Tsallis non-extensive statistics. The analytical results have been utilized to determine some of the very important thermodynamical parameters bearing characteristic signatures of the partonic medium produced in such collisions.

1 Introduction

The multiparticle production rate in ultrarelativistic heavy-ion collision experiments and their momentum distributions is very sensitive to and carry useful information on the possible occurrence of a phase transition from a deconfined partonic state, produced due to the depo-
sition of an enormous amount of energy in the vicinity of the centre of mass of two colliding beams, to confined hadronic sates. The analyses of particle spectra with a suitable model/approach, thus, pave the way to get insights on the thermodynamical evolution of such a de-
confined medium and its cooling off through the process of hadronization. Analysis of such a system is statisti-
cal in nature, and Tsallis-generalized non-extensive statistics, instead of usual Boltzmann-Gibbs statistics, has been proven, over the years, to be a good choice to deal with such a system [1–39]. The mechanism behind the emergence of Tsallis-like spectra from such a hot and dense partonic matter, produced immediately after the nuclear interactions at ultrarelativistic energies, is yet to be under-
stood theoretically. However, the possible reasons could be as follows: i) The produced fireball, consisting of partons, may not be fully deconfined or weakly coupled, but strong correlations or $N$-body interactions among its costituents may exist [40,41]. The presence of such long-range correlations may give rise to the non-Markovian nature of the hadronizing system and hence, to Tsallis spectra [20–25]. ii) Besides, the particle spectra from such nuclear inter-
actions are, generally, obtained by averaging, statistically, over a million of events, and the appearance of a power-
law–like tail in hadron spectra, which is reproduced, quite successfully, by Tsallis-generalized statistics, may also be generated due to event-by-event fluctuations [7–18,38] of various observables [42–45] characterising such a hot and dense medium.

Ever since, the data on different hadronic spectra have started pouring in from LHC experiments, different derivatives of Tsallis non-extensive statistics are exten-
sively used to interpret different aspects of the hadroniz-
ing medium by various theoretical groups [38,39,46–57]. Most of these groups, so far, confined their studies mainly to the P + P collisions at all the colliding energies at LHC. In refs. [46–55], analyses of transverse-momentum ($p_T$) spectra of charged hadrons, along with some identi-
fied ones produced in P + P collisions at LHC and in dif-
ferent central nuclear interactions at RHIC energies, were done by developing a “self-consistent” theory on the basis of non-extensive statistics, whereas, in refs. [38,39], particle spectra as a function of $p_T$, energy fraction and lon-
gitudinal momentum fraction for a fixed-event multiplic-
ty were under scanner with the help of another version, called “super-statistics”. However, in refs. [56,57], charged hadron spectra from the Pb + Pb interaction at 2.76 TeV at LHC, along with those produced in P + P, P + P and different A + A interactions over a wide range of colliding energies, were taken into account to find a possible scaling behaviour, called “q-scaling”, of the non-extensivity parameter $q$ extracted from the analyses of all the exper-
imental data at different energies.

The present author along with his collaborators had made efforts [32,33] in studying the impact of Tsallis non-extensive statistics on some of the identified hadronic spectra available from RHIC experiments following the phenomenological prescriptions made in refs. [7,16]. In the present work, a similar task of systematic analysis of

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the meson spectra, available from LHC experiments, has been taken up. Such a systematic study demands to have data for a particular variety, to be analysed, for all the centrality bins of colliding Pb nuclei in addition to the same for P + P collisions at all colliding energies. But, till date, a moderate range of data is available only for √sNN = 2.76 TeV. So, for the present study, we concentrate mainly on a single colliding energy at LHC with two abundant light mesonic varieties — π- and K-mesons. Besides this, the parameters obtained from the analyses have also been used to determine some of the thermodynamical parameters, such as pressure, energy density, trace anomaly, square of velocity of sound, etc., which provide useful information on the equation of state of the fireball produced in ultrarelativistic nuclear collisions, and the results have been further compared with a lattice-QCD-based calculation.

The work is organized as follows: Section 2 presents a brief sketch of the non-extensive statistics and the main working formulae to be used in the present study. The obtained results are reported in next section (sect. 3) with some specific observations. And the last section is preserved for the concluding remarks.

2 Non-extensive statistics and transverse momentum spectra

The generalized statistics of Tsallis is not only applicable to an equilibrium system, but also to non-equilibrium systems with stationary states [8]. As the name “non-extensive” implies, these entropies are not additive for independent systems.

The non-extensive Boltzmann factor is defined as [8]

\[ x_{ij} = (1 + (q - 1)\beta\epsilon_{ij})^{-q/(q-1)}, \]  

where \( \epsilon_{ij} = \sqrt{p_i^2 + m_j^2} \) is the energy associated with the \( j \)-th particle of rest mass \( m_j \) in the momentum state \( i \), \( \beta = 1/T \) is the inverse temperature variable, \( q \) is a measure of degree of fluctuation present in the system and is called non-extensivity parameter; with \( q \rightarrow 1 \), the above equation approaches the ordinary Boltzmann factor \( e^{-\beta\epsilon_{ij}} \).

If \( n_{ij} \) denotes the number of particles of type \( j \) in the momentum state \( i \), the generalized grand canonical partition function is given by

\[ Z = \sum_{(i)} \prod_{j} x_{ij}^{n_{ij}}. \]  

The average occupation number of a particle of species \( j \) in the momentum state \( i \) can be written as [8]

\[ \tilde{\nu}_{ij} = x_{ij} \frac{\partial}{\partial x_{ij}} \log Z = \frac{1}{(1 + (q - 1)\beta\epsilon_{ij})^{q/(q-1)}} \pm 1, \]  

where the – sign is for bosons and the + sign is for fermions.

The probability of observation of a particle of mass \( m_0 \) in a certain momentum state can be obtained by multiplying the average occupation number with the available volume in the momentum space [8]. The infinitesimal volume in the momentum space is given by

\[ d^3 p = E \, dy \, p_T \, dp_T \, d\phi, \]  

where \( E \) is the energy, \( p_T \) is the transverse momentum, \( y \) is the rapidity and \( \phi \) is the azimuthal angle. Hence, the probability density \( w(p_T, y, \phi) \) is given by

\[ w(p_T, y, \phi) \propto \frac{1}{(1 + (q - 1)\beta E)^{q/(q-1)}} E \, dy \, p_T \, dp_T \, d\phi. \]  

Assuming azimuthal symmetry, one would obtain the invariant distribution as

\[ \frac{1}{2\pi} \frac{d^2 N}{p_T \, dp_T \, dy} = C \frac{m_T}{1 + (q - 1)\beta T_{\text{eff}}^{q/(q-1)}} \pm 1, \]  

where \( C \) is a proportionality constant.

Using the relationships \( \beta = \frac{1}{T_{\text{eff}}} \) and \( E = m_T \cos y \), where \( T_{\text{eff}} \) is the effective temperature of the interaction region and \( m_T = \sqrt{m_0^2 + p_T^2} \) is the transverse mass, the invariant yield at mid-rapidity (for \( y \approx 0 \)) will take the form

\[ \frac{1}{2\pi} \frac{d^2 N}{p_T \, dp_T \, dy} = C \frac{m_T}{1 + (q - 1)\frac{m_T}{T_{\text{eff}}}^{q/(q-1)}} \pm 1. \]  

The average multiplicity of the detected secondary per unit rapidity in the given rapidity region can be obtained by the relationship

\[ \frac{dN}{dy} = \int_0^\infty \frac{d^2 N}{dp_T \, dy} \, dp_T \]

\[ = C_1 \int_0^\infty \frac{m_T}{1 + (q - 1)\frac{m_T}{T_{\text{eff}}}^{q/(q-1)}} \, p_T \, dp_T, \]  

where \( C_1 = 2\pi C \).

Hence, the constant \( C_1 \) can be expressed in terms of \( \frac{dN}{dy} \) by the relationship

\[ C_1 = \frac{dN}{dy} \int_0^\infty \frac{1}{1 + (q - 1)\frac{m_T}{T_{\text{eff}}}^{q/(q-1)}} \, p_T \, dp_T. \]  

Combination of eqs. (7) and (9) will provide us with the main working formula for the invariant yield for a
detected secondary and it is given by
\[
\frac{d^2 N}{p_T dp_T dy} = \frac{dN}{dy} \int_{0}^{1} \frac{1}{(1 + (q-1) \frac{m_T}{T_{\text{eff}}})^{q/(q-1)} + 1} \pm 1 \times \frac{m_T}{[1 + (q-1) \frac{m_T}{T_{\text{eff}}}]^{q/(q-1)}}. \tag{10}
\]

Furthermore, it was observed earlier that the parameters \(T_{\text{eff}}\) and \(q\) are strongly correlated, even if they are set free \([16,32]\). So, these two parameters along with the average multiplicity can phenomenologically be correlated by the following relationships \([16]\):
\[
T_{\text{eff}} = T_0 (1 - c (q-1)), \tag{11}
\]
\[
\langle N \rangle \sim n_0 N_{\text{part}} \sim c (q-1), \tag{12}
\]
with \(\langle N \rangle = \frac{4N}{dy}\) and \(c = - \frac{\phi}{D c_p \rho_T}\), where \(D, c_p, \rho, T_0\) are, respectively, the strength of the temperature fluctuations, the specific heat under constant pressure, density, the critical temperature (also called the Hagedorn temperature \([58,59]\)) of the hadronizing system when it is in thermal equilibrium \((q = 1)\) and \(N_{\text{part}}\) is the number of the participant nucleons. Equation (11) describes the fluctuation in the temperature, where it is assumed that the effective temperature \(T_{\text{eff}}\) is the outcome of two simultaneous processes: i) the fluctuation of the temperature around \(T_0\) due to a stochastic process in any selected region of the system and ii) some energy transfer between the selected region and the rest of the system, denoted by \(\phi\) \([16]\). It is absolutely uncertain whether such energy transfers could/should be invariably linked up with flow velocity (normally denoted in the hydrodynamical model texts as “\(u\)”). So, for the sake of calculational simplicity and correctness, we assume the factor \(\phi\), for the present, to be independent of any flow velocity. The fluctuation in the multiplicity is described by eq. (12). The assumption behind this relationship is that if \(N\) particles are distributed in energy according to Tsallis non-extensive distribution \([16]\). In the original work \([16]\), a “\(\sim\)” sign was used in place of “\(\sim\)” on the left-hand side of eq. (12). But, the present modified form of eq. (12) will take into account only the magnitude of difference between the product \(n_0 N_{\text{part}}\) and \(\langle N \rangle\), even if the product term exceeds the average term, and hence, will keep \(c\) positive and, in turn, \(\phi\) negative ensuring that the energy is transferred from the interaction region to the spectators of the non-interacting nucleons \([16]\).

Equation (10), along with the constraints imposed by eqs. (11) and (12) provides the working formula for the present analysis.

Once the parameters \(T_{\text{eff}}\) and \(q\) are in hand, the logarithm of partition function can numerically be calculated by the relationship \([7]\)
\[
\frac{1}{V} \log Z = \frac{1}{(2\pi)^4} \left[ \int d^3 p \int_{m_\pi}^{M} \rho (m) \log \left( \frac{1}{1 - x_{ij}} \right) \right. \\
+ \left. \int d^3 p \int_{m_\rho}^{M} \rho (m) \log (1 + x_{ij}) \right], \tag{13}
\]
where the first term inside the parenthesis belongs to mesons and the second one to baryons, \(V\) is the volume of interaction region and \(\rho (m)\) is the hadronic mass spectrum. The parametrization for \(\rho (m)\) used here is given by \([49]\)
\[
\rho (m) = \gamma m^{-5/2} \left[ 1 + (q_0 - 1) \frac{m}{m_0} \right]^{\frac{1}{q_0 - 1}}, \tag{14}
\]
with the parameter values given by \(\gamma = 5 \times 10^{-3} \text{GeV}^{3/2}\), \(m_0 = 0.607 \text{GeV}\) and \(q_0 = 1.138\). This parametrization is suitable up to \(m = 2.5 \text{GeV}\). So, the upper limit of integration with respect to \(m\) has been kept \(M = 2.5 \text{GeV}\) \([49]\), while the lower limit for the mesonic part is the pion mass \((m_\pi = 0.140 \text{GeV})\) and that for baryons is the mass of proton \((m_\rho = 0.938 \text{GeV})\). A point to be noted is that the above parametrization is used, here, only to avail a continuous description of the hadron mass spectrum over the specified region of hadronic mass.

Now, it is possible to determine, numerically, the values of the following transport coefficients \([49]\), which are essential entities to determine the equation of state of the matter formed after nuclear interactions at very high energies:
\[
\Pi = \frac{T_{\text{eff}}}{V} \log Z, \tag{15}
\]
\[
s = \frac{\partial \Pi}{\partial T_{\text{eff}}}, \tag{16}
\]
\[
\epsilon = \frac{T_{\text{eff}}}{V} \frac{\partial \log Z}{\partial T_{\text{eff}}}, \tag{17}
\]
\[
\alpha = \frac{\epsilon - 3\Pi}{T_{\text{eff}}}, \tag{18}
\]
\[
c_v = \frac{\partial \epsilon}{\partial T_{\text{eff}}}, \tag{19}
\]
\[
c_v^2 = \frac{s}{c_v}, \tag{20}
\]
where \(\Pi, s, \epsilon, \alpha, c_v, c_v^2\) are the pressure of the interaction volume, entropy density, energy density, trace anomaly, specific heat at constant volume and square of the velocity of sound, respectively.

### 3 Results and discussions

The working formula (eq. (10)) was applied, in its present form, to obtain the fits to the data on \(p^{0,\pm}\) and \(K^{\pm}\) production in P + P collisions (fig. 1) at \(\sqrt{s_{NN}} = 2.76 \text{TeV}\),
Fig. 1. Plots of transverse momentum spectra of π and K mesons produced in P + P collisions at $\sqrt{s_{NN}} = 2.76$ TeV. The filled symbols represent the experimental data points [60,61]. The solid curves provide the fits on the basis of the non-extensive approach (eq. (10)).

Fig. 2. Plots of the invariant yield of π mesons produced in different central Pb + Pb collisions at $\sqrt{s_{NN}} = 2.76$ TeV. The symbols represent the experimental data points [62,63], while the solid curves provide the fits on the basis of the non-extensive approach (eqs. (10)–(12)).
Table 1. Values of fitted parameters with respect to the experimental data on meson spectra produced in P + P collision at $\sqrt{s_{NN}} = 2.76$ TeV.

| Meson type | $N_{\text{part}}$ | $n_0$      | $q$      | $T_{\text{eff}}$(GeV) | $\chi^2/\text{ndf}$ |
|------------|-------------------|------------|----------|-----------------------|---------------------|
| $\pi^0$    | 1.914 ± 0.003     | 1.150 ± 0.003 | 0.078 ± 0.004 | 8.335/16              |
| $\pi^+$    | 1.913 ± 0.006     | 1.150 ± 0.004 | 0.078 ± 0.003 | 31.915/36             |
| $\pi^-$    | 2                 | 1.913 ± 0.005 | 1.151 ± 0.003 | 0.078 ± 0.003         | 33.714/36           |
| $K^+$      | 0.244 ± 0.002     | 1.146 ± 0.004 | 0.089 ± 0.005 | 16.643/36             |
| $K^-$      | 0.244 ± 0.004     | 1.146 ± 0.003 | 0.089 ± 0.004 | 14.336/36             |

Fig. 3. Plots of the invariant yield of $K$ mesons produced in different central Pb + Pb collisions at $\sqrt{s_{NN}} = 2.76$ TeV. The symbols represent the experimental data points [63] and the solid curves provide the fits on the basis of the non-extensive approach (eqs. (10)–(12)).

and the corresponding parameter values are given in table 1, where $n_0$ denotes the average multiplicity per unit rapidity of the produced meson variety in the P + P interaction.

The fits for different central Pb + Pb collisions obtained on the basis of eq. (10) along with the constraints given in eqs. (11) and (12) are depicted in figs. 2 and 3. The values of various parameters obtained from the fits are provided in tables 2 and 3. It is observed from the values of $\chi^2/\text{ndf}$, enlisted in tables 1–3, that the performance of the present approach is quite satisfactory in reproducing the experimental data, except for the cases of pion production at 80–90% central collisions.

The values of Hagedorn’s temperature ($T_0$) obtained from the fits for various centralities have been depicted graphically in fig. 4, which exhibits almost constant behaviour for a particular secondary type emitted in the nuclear interactions at LHC energy, 2.76 TeV. The average value of $T_0$ is found to be $0.144 \pm 0.002$ GeV from the analysis of pion spectra, which is in good agreement with the findings of the analysis of pion production in $e^+e^-$ collisions ($T_0 = 0.131$ GeV) [17]. On the other hand, the
Table 2. Values of fitted parameters with respect to the experimental data on pion spectra at different centralities of Pb + Pb collisions at LHC energy $\sqrt{s_{NN}} = 2.76$ TeV.

| Meson type | Centrality | $N_{\text{part}}$ | $\langle N \rangle$ | $c$ | $T_0$ (GeV) | $\chi^2/\text{ndf}$ |
|------------|------------|-------------------|---------------------|-----|-------------|------------------|
| $\pi^0$    | 0–20       | 308               | 499 ± 13            | 1.581 ± 0.002 | 0.145 ± 0.003  | 2.370/5          |
|            | 20–40      | 157               | 248 ± 8             | 1.773 ± 0.002 | 0.144 ± 0.002  | 3.291/5          |
|            | 40–60      | 69                | 105 ± 2             | 2.045 ± 0.005 | 0.144 ± 0.003  | 1.841/5          |
|            | 60–80      | 23                | 32 ± 1              | 2.689 ± 0.005 | 0.141 ± 0.002  | 4.277/5          |
| $\pi^+$    | 0–5        | 382.8             | 755 ± 72            | 0.331 ± 0.006 | 0.145 ± 0.007  | 53.156/38        |
|            | 5–10       | 329.7             | 651 ± 23            | 0.354 ± 0.002 | 0.143 ± 0.002  | 72.973/38        |
|            | 10–20      | 260.5             | 430 ± 12            | 1.385 ± 0.004 | 0.143 ± 0.002  | 25.218/38        |
| $\pi^-$    | 20–30      | 186.4             | 300 ± 6             | 1.587 ± 0.003 | 0.145 ± 0.002  | 28.783/38        |
|            | 30–40      | 128.9             | 210 ± 15            | 1.60 ± 0.03   | 0.145 ± 0.002  | 51.502/38        |
|            | 40–50      | 85.0              | 120 ± 6             | 2.437 ± 0.003 | 0.145 ± 0.007  | 19.514/38        |
|            | 50–60      | 52.8              | 70 ± 2              | 2.781 ± 0.005 | 0.143 ± 0.003  | 19.268/38        |
|            | 60–70      | 30.0              | 38 ± 2              | 3.036 ± 0.003 | 0.143 ± 0.005  | 24.912/38        |
|            | 70–80      | 15.8              | 19.5 ± 0.8          | 3.37 ± 0.02   | 0.15 ± 0.01    | 51.925/38        |
|            | 80–90      | 7.5               | 8.4 ± 0.7           | 4.02 ± 0.02   | 0.18 ± 0.02    | 107.797/38       |

| Meson type | Centrality | $N_{\text{part}}$ | $\langle N \rangle$ | $c$ | $T_0$ (GeV) | $\chi^2/\text{ndf}$ |
|------------|------------|-------------------|---------------------|-----|-------------|------------------|
| $\pi^-$    | 0–5        | 382.8             | 755 ± 63            | 0.331 ± 0.005 | 0.145 ± 0.008  | 48.199/38        |
|            | 5–10       | 329.7             | 651 ± 17            | 0.354 ± 0.002 | 0.143 ± 0.002  | 67.538/38        |
|            | 10–20      | 260.5             | 429 ± 11            | 1.385 ± 0.005 | 0.143 ± 0.003  | 21.182/38        |
| $\pi^-$    | 20–30      | 186.4             | 300 ± 11            | 1.587 ± 0.002 | 0.145 ± 0.002  | 24.163/38        |
|            | 30–40      | 128.9             | 210 ± 12            | 1.60 ± 0.02   | 0.145 ± 0.002  | 47.613/38        |
|            | 40–50      | 85.0              | 120 ± 8             | 2.451 ± 0.003 | 0.145 ± 0.007  | 13.621/38        |
|            | 50–60      | 52.8              | 70 ± 2              | 2.783 ± 0.003 | 0.143 ± 0.004  | 19.420/38        |
|            | 60–70      | 30.0              | 38 ± 2              | 3.036 ± 0.003 | 0.143 ± 0.006  | 27.799/38        |
|            | 70–80      | 15.8              | 19.5 ± 0.7          | 3.32 ± 0.01   | 0.15 ± 0.01    | 52.373/38        |
|            | 80–90      | 7.5               | 8.4 ± 0.7           | 3.96 ± 0.05   | 0.18 ± 0.02    | 112.440/38       |
Table 3. Values of fitted parameters with respect to the experimental data on kaon spectra at different centralities of Pb + Pb collisions at LHC energy $\sqrt{s_{NN}} = 2.76$ TeV.

| Meson type | Centrality | $N_{\text{part}}$ | $\langle N \rangle$ | $c$ | $T_0$ (GeV) | $\chi^2/\text{ndf}$ |
|------------|------------|-------------------|-------------------|-----|-------------|-----------------|
|            | 0–5        | 382.8             | 107 ± 9           | 1.487 ± 0.008 | 0.222 ± 0.008 | 36.611/33 |
|            | 5–10       | 329.7             | 91 ± 4            | 1.335 ± 0.006 | 0.218 ± 0.003 | 22.858/33 |
|            | 10–20      | 260.5             | 69 ± 3            | 1.032 ± 0.008 | 0.220 ± 0.005 | 11.637/33 |
| $K^+$      | 20–30      | 186.4             | 48 ± 3            | 0.734 ± 0.003 | 0.220 ± 0.004 | 9.363/33 |
|            | 30–40      | 128.9             | 33 ± 2            | 0.733 ± 0.005 | 0.217 ± 0.006 | 31.073/33 |
|            | 40–50      | 85.0              | 18 ± 2            | 1.827 ± 0.003 | 0.220 ± 0.007 | 1.148/33  |
|            | 50–60      | 52.8              | 10.5 ± 0.4        | 2.524 ± 0.004 | 0.220 ± 0.008 | 5.213/33  |
|            | 60–70      | 30.0              | 5.4 ± 0.3         | 3.183 ± 0.006 | 0.220 ± 0.007 | 8.672/33  |
|            | 70–80      | 15.8              | 2.5 ± 0.2         | 3.809 ± 0.003 | 0.225 ± 0.002 | 16.688/33 |
|            | 80–90      | 7.5               | 1.01 ± 0.05       | 4.15 ± 0.06  | 0.22 ± 0.01  | 54.042/33 |

| Meson type | Centrality | $N_{\text{part}}$ | $\langle N \rangle$ | $c$ | $T_0$ (GeV) | $\chi^2/\text{ndf}$ |
|------------|------------|-------------------|-------------------|-----|-------------|-----------------|
|            | 0–5        | 382.8             | 107 ± 8           | 1.427 ± 0.005 | 0.222 ± 0.002 | 25.271/33 |
|            | 5–10       | 329.7             | 91 ± 3            | 1.387 ± 0.005 | 0.218 ± 0.003 | 23.098/33 |
|            | 10–20      | 260.5             | 69 ± 5            | 1.032 ± 0.006 | 0.220 ± 0.005 | 8.910/33  |
| $K^-$      | 20–30      | 186.4             | 48 ± 3            | 0.734 ± 0.003 | 0.220 ± 0.005 | 8.016/33  |
|            | 30–40      | 128.9             | 33 ± 2            | 0.733 ± 0.004 | 0.217 ± 0.004 | 30.770/33 |
|            | 40–50      | 85.0              | 18 ± 2            | 1.827 ± 0.003 | 0.220 ± 0.006 | 1.368/33  |
|            | 50–60      | 52.8              | 10.5 ± 0.5        | 2.524 ± 0.006 | 0.220 ± 0.008 | 5.213/33  |
|            | 60–70      | 30.0              | 5.3 ± 0.2         | 3.252 ± 0.005 | 0.220 ± 0.004 | 6.843/33  |
|            | 70–80      | 15.8              | 2.5 ± 0.2         | 3.809 ± 0.004 | 0.225 ± 0.002 | 19.972/33 |
|            | 80–90      | 7.5               | 1.01 ± 0.04       | 4.15 ± 0.06  | 0.22 ± 0.01  | 42.726/33 |

The effective temperature, $T_{\text{eff}}$, and the non-extensive parameter $q$ calculated from the fitted parameters (excluding the errors), are given in tabular form in table 4 and in graphical format in fig. 5 as a function of participant nucleons. Excluding the results from P+P collisions, $T_{\text{eff}}$ decreases while $q$ increases for both the varieties, in general, when going from $N_{\text{part}} = 382.8$ [66–68] for 0–5%
Table 4. Values of $q$ and $T_{\text{eff}}$ obtained from different meson spectra produced at various centralities.

| $N_{\text{part}}$ | $N_{\text{\pi-part}}$ | $T_{\text{eff}}^{\pi^0}$ | $q^{\pi^0}$ | $T_{\text{eff}}^{\pi^+}$ | $q^{\pi^+}$ | $T_{\text{eff}}^{\pi^-}$ | $q^{\pi^-}$ | $T_{\text{eff}}^{K^+}$ | $q^{K^+}$ | $T_{\text{eff}}^{K^-}$ | $q^{K^-}$ |
|------------------|------------------------|--------------------------|-------------|--------------------------|-------------|--------------------------|-------------|--------------------------|-------------|--------------------------|-------------|
| 2                | 3.05                   | 0.078                    | 1.150       | 0.078                    | 1.150       | 0.078                    | 1.151       | 0.089                    | 1.146       | 0.089                    | 1.146       |
| 7.5              | 8                      | 0.053                    | 1.176       | 0.054                    | 1.179       | 0.041                    | 1.196       | 0.041                    | 1.196       | 0.041                    | 1.196       |
| 15.8             | 19                     | 0.067                    | 1.163       | 0.067                    | 1.166       | 0.103                    | 1.142       | 0.103                    | 1.142       | 0.103                    | 1.142       |
| 23               | 32                     | 0.087                    | 1.114       |                          |             |                          |             |                          |             |                          |             |
| 30               | 45                     | 0.07                     | 1.168       | 0.071                    | 1.142       | 0.112                    | 1.036       | 0.112                    | 1.036       | 0.112                    | 1.036       |
| 52.8             | 91                     | 0.080                    | 1.159       | 0.080                    | 1.159       | 0.170                    | 1.090       | 0.170                    | 1.090       | 0.170                    | 1.090       |
| 69               | 127.5                  | 0.108                    | 1.123       |                          |             |                          |             |                          |             |                          |             |
| 85               | 164                    | 0.094                    | 1.146       | 0.094                    | 1.145       | 0.187                    | 1.083       | 0.187                    | 1.083       | 0.187                    | 1.083       |
| 128.9            | 267                    | 0.120                    | 1.109       | 0.120                    | 1.109       | 0.207                    | 1.064       | 0.207                    | 1.064       | 0.207                    | 1.064       |
| 157              | 341                    | 0.114                    | 1.118       |                          |             |                          |             |                          |             |                          |             |
| 186.4            | 415                    | 0.118                    | 1.119       | 0.118                    | 1.119       | 0.208                    | 1.071       | 0.208                    | 1.071       | 0.208                    | 1.071       |
| 260.5            | 625                    | 0.120                    | 1.115       | 0.120                    | 1.116       | 0.203                    | 1.076       | 0.203                    | 1.076       | 0.203                    | 1.076       |
| 308              | 773                    | 0.119                    | 1.115       |                          |             |                          |             |                          |             |                          |             |
| 329.7            | 828                    | 0.139                    | 1.088       | 0.139                    | 1.088       | 0.193                    | 1.087       | 0.193                    | 1.087       | 0.193                    | 1.087       |
| 382.8            | 1014                   | 0.141                    | 1.091       | 0.141                    | 1.091       | 0.194                    | 1.085       | 0.194                    | 1.085       | 0.194                    | 1.085       |

Fig. 5. Plots of the effective temperature $T_{\text{eff}}$ and the non-extensive parameter $q$ as a function of number of participant nucleons in $P + P$ and $Pb + Pb$ collisions at $\sqrt{s_{NN}} = 2.76$ TeV, for the production of secondary $\pi$ and $K$ mesons.
central to \( N_{\text{part}} = 7.5 \) for 80–90% central Pb + Pb collision at 2.76 TeV. A deviation, though very weak, from this trend is observed for \( K \)-meson production in the central collision regions with \( N_{\text{part}} \geq 100 \).

Figure 6(a) depicts graphically the behaviour of average multiplicity (\( \langle N \rangle = dN/dy \)) of the detected secondary with respect to \( N_{\text{part}} \), while figs. 6(b) and (c) represent the same, but this time normalized by pair of participant nucleons (\( N_{\text{part}}/2 \)) and by pair of participant quarks (\( N_{q\text{-part}}/2 \)), respectively. The values of \( N_{\text{part}} \) for different centralities have been obtained from refs. [66–68] and those of \( N_{q\text{-part}} \) (table 4) have been calculated using the PHOBOS Glauber Monte Carlo Simulation [69] with the incorporation of the prescription made in ref. [70] and the method employed in refs. [71,72] into the code. A close inspection of fig. 6(b) and (c) will reveal that the dependence of the normalized yield of both the varieties on \( N_{q\text{-part}} \) is not so prominent compared to that on \( N_{\text{part}} \) for most of the centrality bins, which indicates a nearly linear dependence of \( \langle N \rangle \) on \( N_{q\text{-part}} \). A similar observation was made for meson production at RHIC interactions [72].

In figs. 7 and 8, the nature of various transport coefficients, calculated numerically on the basis of eqs. (15)–(20) and obtained for different centrality classes of Pb + Pb interactions, have been represented graphically. One of the useful parameters, the square of the velocity of sound (\( c_s^2 \)), was calculated from rapidity spectra in four most central Pb + Pb collisions at \( \sqrt{s_{NN}} = 2.76 \) TeV in ref. [73]. These results have also been incorporated in fig. 8(b) to provide a comparison with the results obtained from the present analysis.

Figure 9 is the graphical comparison of our results on pressure, trace anomaly and square of velocity of sound as functions of effective temperature (\( T_{\text{eff}} \)) with those obtained by a calculation on the basis of lattice-QCD thermodynamics [74] with zero chemical potential. It is observed that the values of all the three entities, calculated using the parameters obtained from the fits of both the mesonic spectra, are in moderate agreement with the lattice-QCD–based calculations at high temperature regions. Moreover, the values of the square of the velocity of sound remain nearly constant over the entire range of
effective temperature. Accumulation of multiple data in the close vicinity of a particular temperature is due to the variation in the value of $q$.

4 Conclusions

As $T_{\text{eff}}$ and $q$ are mutually correlated and exhibit a strong contrast behaviour, one expects that an increase in the temperature of the interaction volume will be exhibited by very low value of $q$, i.e., the interaction region will have lesser degree of fluctuations, and will be in the vicinity of thermal equilibrium. The values of effective temperature of the interaction volume, obtained from the analysis of $\pi$ spectra for different centralities, in the present study, are found to be somewhat similar compared to those observed for RHIC interactions [32,33]. However, the same parameter is found to be a bit high for $K$ spectra. But, surprisingly, there is no significant decrement in the associated value of $q$, obtained from kaon spectra at a particular centrality, though the corresponding effective temperature is quite high compared to that obtained for pion spectra. Besides, the values of $q$ remain almost the same with respect to those observed at RHIC energies [32,33] at different centralities. This observation indicates that either the fireball, produced in nuclear collisions at this particular LHC energy, possesses partonic constituents which are correlated mutually through long-range interactions, or the presence of event-by-event fluctuations in the particle production rate. This fact is once again validated from fig. 9, where the obtained values of both the parameters —the trace anomaly and the square of the velocity of sound— are far away from their respective ideal gas limits $\frac{\Pi}{T^4} \rightarrow 0$ and $c_s^2 \rightarrow \frac{1}{3}$.

Hence, from various signatures available from present analyses, like i) the value of the Hagedorn temperature ($T_0 \sim m_\pi$), ii) the linear dependence of $\langle N \rangle$ on $N_{q\text{-part}}$, iii) the presence of long-range correlations and/or multiplicity...
fluctuations, etc.; there are ample reasons to assume that the process of hydrodynamic evolution of the fireball and its subsequent hadronization is quite similar to that of the processes observed at RHIC energies. This inference is in accord with the observations made in ref. [40].

However, one point is to be noted here that we have not incorporated any type of collective transverse flow in our present approach to extract information from the transverse momentum spectra, where, mainly, light mesons have been dealt with. The influence of such collective flow becomes more significant for the production of heavier hadrons as it contributes more to the average transverse momentum, associated with a particular secondary, with an increment in the mass of the produced hadron keeping the average thermal momentum same for all the varieties [75,76]. So, it is quite clear that, in our future endeavour, the present approach may need to be modified, by taking into account the effect of transverse flow.

Fig. 8. Plots of the trace anomaly ($\frac{\epsilon - 3\Pi}{T^4}$, in units of $T^2$) and the square of the velocity of sound ($c_s^2$) calculated for P+P and different central Pb+Pb collisions at LHC energy $\sqrt{s_{NN}} = 2.76$ TeV. The data points with error bars in (b) are the results on the basis of a Landau hydrodynamic model [73].

Fig. 9. Comparison of some thermal parameters obtained in the present work with those from a lattice-QCD calculation [74].
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