$H_\infty$ control of combustion in diesel engines using a discrete dynamics model

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Abstract. This paper proposes a control method for combustion in diesel engines using a discrete dynamics model. The proposed two-degree-of-freedom control scheme achieves not only good feedback properties such as disturbance suppression and robust stability but also a good transient response. The method includes a feedforward controller constructed from the inverse model of the plant, and a feedback controller designed by an $H_\infty$ control method, which reduces the effect of the turbocharger lag. The effectiveness of the proposed method is evaluated via numerical simulations.

1. Introduction
In diesel engines, efforts to reduce both NOx and particulate matter (PM) emissions have increased in recent years due to the need to comply with increasingly stringent emission regulations. To meet these requirements, new technologies such as exhaust gas recirculation (EGR) circuits, variable-geometry turbochargers (VGTs), and exhaust gas aftertreatment systems have been introduced [1, 2, 3]. However, these technologies increase the complexity of the system architecture and the difficulty of the control system design.

Conventional controllers are based on lookup tables compiled from the results of many experiments [4]. The complexity of recent engines has greatly increased the effort of constructing these tables. Premixed charge compression ignition (PCCI) combustion is the inevitable next step, as it achieves high energy efficiency while reducing the engine-out emissions, but it is non-robust and difficult to control [5, 6, 7, 8]. Indeed, PCCI combustion cannot be adequately controlled by conventional control, in either steady-state or transient operation. Moreover, the control of PCCI combustion in transient operation is harder than that in the steady-state condition. Model-based controller design methods offer a promising alternative to traditional control of PCCI [9, 10].

This paper proposes a robust $H_\infty$ combustion control method for diesel engines. The plant model is the discrete dynamics model developed by Yasuda et al., which is implementable on a real engine control unit [11, 12]. We introduce a two-degree-of-freedom control scheme with a feedback controller and a feedforward controller. This scheme achieves both good feedback properties, such as disturbance suppression and robust stability, and a good transient response. The feedforward controller is designed from the inverse model of the plant, and the feedback controller is designed by the $H_\infty$ control method, which reduces the effect of the turbocharger lag. The effectiveness of the proposed method is evaluated by simulations using the discrete dynamics model.
2. Discrete dynamics model

A discrete dynamics model for diesel combustion control has been developed as a future implementable model on a real engine control unit. To reduce the computational cost, this nonlinear discrete-time system is evaluated only at representative points in the engine cycle; namely, the timings of the exhaust valve closing (EVC), the intake valve closing (IVC), the ignition (IGN), the peak pressure (PEAK), and the exhaust valve opening (EVO) (see Fig.1).

Initially, the discrete dynamics model comprised a single injection system; then, the model was extended to include multi-injection systems[11, 12]. In this initial examination, we apply the model only as a single-injection model. The state variable $X_k$, the input $U_k$, and the output $Y_k$ of the single-injection model are presented in Table.1. In terms of these variables, the discrete dynamics model with single injection is expressed as:

$$X_{k+1} = f(X_k, U_k), \quad Y_k = g(X_k, U_k), \quad (1)$$

where

$$X_k = \begin{bmatrix} T_{RG,k} \\ n_{O_2, RG,k} \end{bmatrix}, \quad (2)$$

$$U_k = \begin{bmatrix} Q_{fuel,k} \\ \theta_{INJ,k} \\ P_{boost,k} \end{bmatrix}, \quad Y_k = \begin{bmatrix} W_k \\ P_{PEAK,k} \\ \theta_{PEAK,k} \end{bmatrix}. \quad (3)$$

In the controller design, Eq.(1) is linearized around the equilibrium points $U_0$, $Y_0$, and $X_0$ of the input, output, and state, respectively. The deviations from the equilibrium points are defined as follows:

$$x_k = X_k - X_0, \quad (4)$$

$$u_k = U_k - U_0, \quad (5)$$

$$y_k = Y_k - Y_0. \quad (6)$$

1 The EGR rate is also the input to the discrete dynamics model. However, in this study, we treat the EGR rate as a constant (30%) rather than as an input variable.
Table 1. Definitions of states, inputs, and outputs in the discrete dynamics model

| State | Variable | Description |
|-------|----------|-------------|
| $X_k$ | $T_{RG,k}$ | Temperature of residual gas at EVC [K] |
|       | $n_{O_2,RG,k}$ | Oxygen mole of residual gas at EVC [mol] |

| Input | Variable | Description |
|-------|----------|-------------|
| $U_k$ | $Q_{fuel,k}$ | Fuel injection quantity [mm$^3$] |
|       | $\theta_{INJ,k}$ | Fuel injection timing [degATDC] |
|       | $P_{boost,k}$ | Boost pressure [kPa] |

| Output | Variable | Description |
|--------|----------|-------------|
| $Y_k$  | $W_k$    | Indicated output [kW] |
|        | $P_{PEAK,k}$ | Peak pressure [MPa] |
|        | $\theta_{PEAK,k}$ | Peak pressure timing [degATDC] |

In this article, the equilibrium point $U_0$ of the input is selected as

$$U_0 = \begin{bmatrix} 20 & -4 & 110 \end{bmatrix}^T.$$  \hfill (7)

Thus, $Y_0$ and $X_0$ are obtained as

$$Y_0 = \begin{bmatrix} 4.2582 & 6.5630 & 8.9219 \end{bmatrix}^T,$$  \hfill (8)
$$X_0 = \begin{bmatrix} 5.80 \times 10^2 & 1.49 \times 10^{-4} \end{bmatrix}^T.$$  \hfill (9)

Based on these equilibrium points, we obtained a linearized model with the following state-space representation:

$$x_{k+1} = A x_k + B u_k,$$  \hfill (10)
$$y_k = C x_k + D u_k.$$  \hfill (11)

We also define the transfer function $P$:

$$P[z] = C(zI - A)^{-1}B + D.$$  \hfill (12)

Note that in this case, the direct term $D$ of Eq.(11) is square and nonsingular; therefore, $D^{-1}$ exists.

3. Structure of Control system
In designing the $H\infty$ control system, we made the following assumptions:

(1) The indicated output $W_k$, peak pressure $P_{PEAK,k}$, and its timing $\theta_{PEAK,k}$ can be measured at time $k+1$ by sensors embedded in the cylinder.
(2) The actual boost pressure $P_{\text{boost},k}$ follows the reference $P_{\text{ref},\text{boost},k}$ with a first-order lag, i.e., the following equation holds.

$$P_{\text{boost}}[z] = F_b[z] P_{\text{ref},\text{boost}}[z],$$

where $P_{\text{boost}}[z]$ and $P_{\text{ref},\text{boost}}[z]$ are the $z$-transforms of $P_{\text{boost},k}$ and $P_{\text{ref},\text{boost},k}$, respectively, and $F_b[z]$ is a discrete-time first-order lag filter with a time constant of $T_b$. Here we assume $T_b = 2$.

(3) The boost pressure $P_{\text{boost},k}$ can be measured without a time delay.

By assumption (1), the measurement output as $y_{s,k}$ is given by

$$y_{s}[z] = z^{-1} y[z].$$

Further, by assumption (2), the input $u$ of $P$ is related to the controller output $u_s$ as

$$u[z] = M[z] u_s[z],$$

where

$$M[z] = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & F_b[z] \end{bmatrix}.$$  \hfill (15)

Under the above assumptions, a two-degree-of-freedom control system was proposed. In the block diagram of this control system (Fig.2), $r$ is the reference input to $y$, i.e., the reference to the indicated output, the peak pressure, and the peak pressure timing. To satisfy $y = r$, the feedforward controller $K_{FF}$ generates a feedforward control input $u_{ff}$.

The measurement output $y_s$ is the indicated output, peak pressure and peak pressure timing, and is referenced to $r_s = z^{-1} r$. The delay of $y_s$ relative to $y$ (one sampling step) is corrected by $z^{-1}$ in the reference. The deviation $e_r = r_s - y_s$ is the input to the feedback controller $K_{FB}$, which generates a feedback control input $u_{fb}$.

In general, the feedforward controller is defined by the inverse model of $P$. However, because the dynamics of $P$ exert a small effect, the feedforward controller is defined as the inverse static characteristic of $P$:

$$K_{FF} = D^{-1}.$$  \hfill (16)

Note that the feedforward controller design Eq.(16) excludes the turbocharger lag $F_b$. This omission might degrade the system performance. Therefore, in the next section, we propose an $H_\infty$ controller design method that reduces the effect of $F_b$. 

**Figure 2.** Schematic of the proposed control system
4. Design of $H_\infty$ control system

4.1. $H_\infty$ Control Theory

Given a generalized closed-loop configuration shown in Fig.3, the $H_\infty$ control is formulated as a problem that finds an internally stabilizing controller that minimizes the $H_\infty$ norm from $w$ to $z$ [13]. The transfer matrix $G$ in Fig.3 is referred to as a generalized plant. The $H_\infty$ controller minimizes $H_\infty$ norm, which corresponds to the maximum gain of the transfer function. Therefore, the $H_\infty$ control is essentially a disturbance suppression. Multiple disturbances are imposed to the controlled object in actual systems, and the modeling error can also be regarded as an equivalent disturbance [14]. Therefore, the $H_\infty$ control is a powerful approach because various control problems are reduce to disturbance attenuation problems.

4.2. Design

For disturbance suppression, we consider the generalized plant shown in Fig.4. In this figure, $W_u$, $W_e$, and $W_n$ are weighting functions, and the input side disturbance $w_d$ is suppressed by the controlled output $z_e$. The controlled output $z_u$ evaluates the magnitude of the control input $u$. The signal $w_n$ models the measurement noise, and is also required to satisfy the assumption of the standard $H_\infty$ control problem [13]. Not that as plant $P$ is a discrete-time model, the generalized plant must be express as a discrete-time transfer matrix.

The weighting functions $W_u$, $W_e$, and $W_n$ were determined by trial and error as

\[
W_e = \text{diag} \begin{bmatrix} 20 & 5 & 12 \\ s + 0.5 & s + 0.5 & s + 0.5 \end{bmatrix} \]  
(17)

\[
W_u = \text{diag} \begin{bmatrix} 0.005 & 0.005 & 0.005 \\ \end{bmatrix} . \]  
(18)

\[
W_n = \text{diag} \begin{bmatrix} 0.0001 & 0.0001 & 0.0001 \\ \end{bmatrix} . \]  
(19)

All of the weighting functions were discretized by a pole-zero matching equivalent method.
Next, referring to discrete-time $H_\infty$ control theory, we constructed an $H_\infty$ controller with these weighting functions using the MATLAB robust control toolbox R2012a. The achieved minimum $H_\infty$ norm from $w$ to $z$ was 0.83. Finally, the obtained $H_\infty$ controller was employed as the feedback controller of the two-degree-of-freedom control system in Fig.2, and evaluated in simulations. The simulated plant model was the nonlinear discrete-time model governed by Eq.(1). The reference inputs were increased by 1 unit from the equivalent points as step signals. The time steps of the indicated output, peak pressure, and peak pressure timing were $t = 1$ s, $t = 4$ s, and $t = 7$, respectively.

Fig.5 shows the simulation results. The tracking performance to the reference input in the peak pressure and peak pressure timing was poor, probably because the $H_\infty$ controller design neglected the effect of $F_b$. To investigate this supposition, we redesigned and simulated an $H_\infty$ controller with the same weighting functions and $F_b = 1$. As shown in Fig.6, the outputs of the new controller accurately followed the reference, confirming that the effect of $F_b$ is non-negligible.
Figure 6. Simulation results of Design 1 with $F_b = 1$

4.3. Design 2 (accounting for $F_b$)

In this section, we account for $F_b$ in the $H_\infty$ controller design. Because the designed feedforward controller $K_{FF}$ is the inverse of $P$, we add a feedforward input $u_{ff}$ between $M$ and $P$ in the two-degree-of-freedom control system (Fig.2). However, the actual boost pressure cannot be directly manipulated, so $u_{ff}$ is added to the input side of $M$, as shown in Fig.2. Thus, we estimate the error when $u_{ff}$ is added to the input side of $M$ rather than to the output side of $M$ (as in the original design).

Fig.7(a) presents the extracted block diagram of the input and output portions. Fig.7(a) is transformed to Fig.7(b) by preserving the transfer function from $u_{ff3}$ to $u_3$. From this diagram, we observe that adding $u_{ff}$ to the input side of $F_b$ is equivalent to adding

$$w_{d3} = (F_b - 1)u_{ff3}$$

(20)

to the output side of $F_b$.

Note that the disturbance $w_{d3}$ is not unknown, but can be calculated from $u_{ff}$. Thus, we constructed the generalized plant shown in Fig.8. The main feature of this generalized plant is that the disturbance added to the input side of $F_b$ is the input to the feedback controller as $y_{d3}$.
Figure 7. Equivalent transformation of $u_{ff3}$

Figure 8. Block diagram of Generalized Plant 2

In designing the $H_\infty$ controller, the weighting function $W_e$ was selected as

$$W_e = \text{diag} \left[ \frac{20}{s + 0.5}, \frac{5}{s + 0.5}, \frac{12}{s + 0.5} \right], \quad (21)$$

and the same weighting functions $W_u$ and $W_n$ were those used in Design 1. For these weighting functions, the achieved minimum $H_\infty$ norm was from $w$ to $z$ was 0.98. We then re-simulated the two-degree-of-freedom control system shown in Fig.2 with the feedback input modified to

$$u_{fb} = K_{FB} \begin{bmatrix} e_s \\ w_{d3} \end{bmatrix}$$

$$= K_{FB} \begin{bmatrix} I & 0 \\ 0 & (F_b - 1)R \end{bmatrix} \begin{bmatrix} e_s \\ u_{ff} \end{bmatrix}, \quad (23)$$
where \( R = \begin{bmatrix} 0 & 0 & 1 \end{bmatrix} \).

The simulation results are plotted as solid lines in Fig.9. This figure confirms that the indicated output and peak pressure timing accurately follow the reference. On the other hand, the changes in these two outputs affect the peak pressure response.

To reduce the interference between the peak pressure and the other outputs, we reduced the gain of the (1,1) element of \( W_e \) and increased the gain of the (3,3) element of \( W_e \) as shown below:

\[
W_e = \text{diag} \left[ \frac{12}{s + 0.5}, \frac{15}{s + 0.5}, \frac{12}{s + 0.5} \right]
\]  

(24)

The new simulation results are plotted in Fig.10. Although the tracking performance of the peak pressure improves, the indicated output is disturbed by the step change of the peak pressure. By adjusting the weighing functions, we could not simultaneously minimize the tracking errors in the indicated output and peak pressure. However, these errors might be simultaneously reduced by changing the EGR ratio, which is fixed at 30.

Finally, we evaluated the effectiveness of the \( w_{d3} \) feedback by simulating the case of no \( w_{d3} \) feedback (i.e., by setting \( u_{ff} = 0 \) in the right-hand side of Eq.(23)). As the feedback controller,
Figure 10. Simulation results of Design 2 using the re-tuned weighting functions

we employed the $H_\infty$ controller used in the simulation of Fig.10. The results are shown in Fig.11. The large error in the peak pressure timing confirms the effectiveness of Design 2.

5. Conclusions
In this study, we applied $H_\infty$ control theory to a combustion control system for diesel engines. In the generalized plant, we accounted for the turbocharger lag, which degrades the system performance. The improved tracking performance was demonstrated in simulations. In future studies, we will vary the EGR rate in the $H_\infty$ controller design.

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Figure 11. Simulation results of Design 2 without $w_{d3}$ feedback

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