Vertical convection in turbulent accretion disk and light curves of X-ray Nova A0620-00

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Abstract. We describe phenomenon of X-ray Novae in a model of non-stationary accretion α-disk with account for irradiation and vertical convection in outer disk region. We extended the commonly used disk vertical structure model by adding viscous turbulent energy generation in mixing length theory. This model was used to simulate both optical and X-ray light curves of the 1975 outburst of X-ray Nova A0620-00.

1. Introduction
X-ray novae are close binary systems with relativistic star (a black hole or a neutron star) and a low-mass Roche-lobe-filling star (see reviews [1], [2]). We will consider systems that contains a black hole and a low-mass main-sequence star and show the so-called fast rise and exponential decay (FRED) X-ray light curves. Light curves of such systems can be described by non-stationary evolution of a standard accretion disk ([3], [4]). Analytical solutions of this problem are presented in [5], [6] and [7]. These solutions cannot take into account some of the features of accretion disk structure.

First, outer parts of the disk could be cooler than $10^4$ K and have a zone of partially ionized hydrogen where opacity law differs from that in a hotter region of the disk. Second, vertical convection appears in the zone of partially ionized hydrogen ([8], [9]). Third, the outer part of the disk with temperature $10^4$ K could be irradiated by X-rays from the inner part of the disk that has a temperature of $10^5 \div 10^6$ K.

Here we describe a numerical model that includes these features and apply it to both X-ray and optical light curves of 1975 outburst of X-ray nova A0620–00. In this work we discuss only the behaviour of the descending branches of the light curves.

2. Viscous evolution of accretion disk
Let us write the continuity equation for a geometry thin axially symmetric accretion disk:

$$\frac{\partial \Sigma_0}{\partial t} = -\frac{1}{r} \frac{\partial}{\partial r} (\Sigma_0 v_r r),$$

where $\Sigma_0 = \int_{-\infty}^{\infty} \rho dz$ is the surface density of the disk at radius $r$, $z$ is the axial coordinate, $v_r$ is the radial velocity of the accreting matter.
We assume that the angular momentum is taken away only from outer radius of the disk, in other words, there is no wind, and tidal torque from the secondary star is efficient only for the outer edge of the disk [10]. Then equation of the angular momentum transfer reads

\[ \Sigma_0 \varv_r \frac{\partial (\omega r^2)}{\partial r} = -\frac{1}{2\pi} \frac{1}{r} \frac{\partial F}{\partial r}, \]  

(2)

where \( \omega = \sqrt{GM_x/r^3} \) is the angular velocity of accreting matter, \( G \) is the gravitation constant, \( M_x \) is the mass of the black hole, \( F = 2\pi r^2 \times \int_{-\infty}^{\infty} \alpha P dz \) is the viscous torque in standard accretion disk [4], \( \alpha \) is the turbulent viscous parameter, \( P \) is the pressure.

Combination of equations (1) and (2) yields the diffusion equation:

\[ \frac{\partial \Sigma_0}{\partial t} = \frac{1}{4\pi} \frac{(GM_x)^2}{h^3} \frac{\partial^2 F}{\partial h^2}, \]  

(3)

where \( h = \sqrt{\omega r^2} \) is the specific Keplerian angular momentum.

The last equation is a second order differential equation, so we need two boundary conditions to solve it. The first boundary condition is for the inner edge of the disk where matter falls down on the black hole almost without producing viscous torques:

\[ F_{in} = 0. \]  

(4)

The second boundary condition is due to the assumption that during an outburst the mass transfer rate from the secondary star to the disk is much smaller than accretion rate on the black hole:

\[ \frac{\partial F}{\partial h} \bigg|_{out} = 0. \]  

(5)

Equation (3) with boundary conditions (4, 5) composes a system of equations for two unknown functions: \( F(t, h) \) and \( \Sigma_0(t, h) \). In the next section we will find relations between these functions by solving the vertical structure equations.

3. Vertical structure equations

Equations of the vertical structure of the disk are similar to equations of stellar radial structure. Let us write and discuss them one by one.

3.1. Hydrostatic equilibrium

We simulate the disk on timescales much larger the free fall time and therefore we can assume the hydrostatic equilibrium:

\[ \frac{dP}{dz} = \rho g_z, \]  

(6)

where \( \rho \) is the density, \( g_z \simeq \omega^2 z \) is the free fall acceleration.

3.2. Surface density

The second equation comes from surface density definition:

\[ \frac{d\Sigma}{dz} = \rho. \]  

(7)
3.3. Energy generation
There are two sources of energy in the outer region of the disk: viscous heating $\alpha P$ and thermalization of X-ray photons coming from the inner region of the disk $\epsilon_x$:

$$\frac{dQ}{dz} = \alpha P + \epsilon_x.$$  \hspace{1cm} (8)

We used the same approach as in paper [11] to calculate $\epsilon_x$.

3.4. Energy transfer
When the surface temperature of the disk is higher than $10^4$ K all the energy is transferred by radiation. But if there is a vertical convection in the cold disk region then it could transfer part of energy in the direction from the central plane to the disk surface.

$$\frac{dT}{dz} = \frac{g_z \rho T}{P} \nabla,$$  \hspace{1cm} (9)

where $\nabla \equiv d\log T/d\log P$ is the actual logarithmic gradient. In this work we describe the vertical convection by mixing length model in accretion disks from paper [12].

3.5. Equation of state
We use equation of state of perfect gas to link thermodynamical quantities:

$$P = \frac{\rho RT}{\mu(\rho, T)},$$  \hspace{1cm} (10)

where $R$ is the gas constant, $\mu(\rho, T)$ is the molecular weight of the gas. We assume that accretion matter is a mixture of neutral hydrogen, ionized hydrogen and neutral impurity, so molecular weight $\mu$ depends on thermodynamical state of the gas. In addition we assume that this mixture has solar composition [13].

4. Simulation of the light curves
Solution of the vertical structure equations (6 — 10) provides us with relation between the surface density $\Sigma_0$ and the viscous torque $F$. One can use this relation to solve equation (3) and find temporal evolution of the disk and, as a result, the light curves of the outburst.

We assuming the initial condition in the form:

$$F(h) = \frac{2}{\pi} (h_{\text{out}} - h_{\text{in}}) \dot{M}_0 \times \sin \left( \frac{\pi}{2} \frac{h - h_{\text{in}}}{h_{\text{out}} - h_{\text{in}}} \right),$$  \hspace{1cm} (11)

where $\dot{M}_0$ is the initial accretion rate on the black hole, $h_{\text{in}}$ and $h_{\text{out}}$ are specific angular momenta at the inner and outer edges of the disk, respectively. Here the initial accretion rate $\dot{M}_0$ is the accretion rate at the moment of the peak of X-ray luminosity.

For our simulation of light curves of A0620–00 we used parameters of the binary system from [14] and [15]: mass of the secondary star is 0.4 solar masses, mass of the black hole $M_x$ is 6.6 solar masses, orbital inclination is 51°, orbital period is 0.323 days, the Kerr parameter of the black hole is 0.2.

Results of our simulations are shown in Fig. 1 and Fig. 2. It is seen that the X-ray light curve (Fig. 2) has the secondary peak. We explains it with an additional mass transfer from the secondary star to the disk 43 days after the X-ray luminosity maximum. We need to add 30% of the disk mass to it to explain the observed feature.

The late part of the model optical light curve (Fig. 1) lies lower than observations. It could be explained by adding some light from the secondary star. The star could be irradiated by the X-ray photons from the accretion disk and heated. The dashed line on Fig. 1 shows the light curve for the disk with the star that has a constant luminosity.
Figure 1. Optical light curve of X-ray nova A0620–00. Dots indicates observations [16]. The solid line shows the model with flux from the disk, the dashed line shows the model with optical star with magnitude $m = 13^m$.

Figure 2. Soft X-ray (3 ÷ 6 keV) light curve of X-ray nova A0620–00. Dots indicates observations [17], the solid line indicates model.

5. Results
We showed that the standard model of disk accretion can explain behaviour of light curves of X-ray nova both in the optical and X-rays. In the context of this theory we found that during 1975 outburst of X-ray nova A0620–00 the $\alpha$ parameter of the disk was approximately 0.5. We also found that the distance to the source in our model should be somewhat smaller than in [14] and is approximately 0.85 kpc.

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