Classical Electrodynamics without Fields and the Aharonov-Bohm effect

Eugene V. Stefanovich
2255 Showers Drive, Apt. 153, Mountain View, CA 94040, USA
eugene.stefanovich@usa.net
March 19, 2008

Abstract

The Darwin-Breit Hamiltonian is applied to the Aharonov-Bohm experiment. In agreement with the standard Maxwell-Lorentz theory, the force acting on electrons from infinite solenoids or ferromagnetic rods vanishes. However, the interaction energies and phase factors of the electron wave packets are non-zero. This allows us to explain the Aharonov-Bohm effect without involvement of electromagnetic potentials, fields, and topological properties of space.

1 Introduction

In recent proposals to reformulate classical Maxwell-Lorentz electrodynamics, electric and magnetic interactions propagate instantaneously \[1,2\] and the free electromagnetic field is an approximation for a large ensemble of discrete quantum particles - photons \[3,4,5\]. These ideas of "field-less" electrodynamics are attractive for several reasons. First, the traditional continuous field description of radiation is in conflict with corpuscular properties of light that are evident in all kinds of single-photon experiments, in the photo-electric effect, etc. Second, the traditional notions of the momentum and energy contained in electromagnetic fields lead to divergences, “4/3 problem” and other paradoxes \[6,7,8,9\], which can be avoided in the field-less description. Third, the usual assumption of the retarded character of the Coulomb and magnetic interactions has not been confirmed by experiment\[1\] and this assumption results in the paradox of

\[1\] Of course, there exist indirect interactions between particles transmitted by real photons emitted, absorbed, and scattered by accelerated charges \[1,2,10\]. It is well-established that these indirect interactions propagate with the speed of light \(c\), and they are responsible for radar, radio, TV, etc. signals. We will not discuss the electromagnetic radiation effects in this paper.
energy non-conservation [11]. On the other hand, the indirect support for field-less electrodynamics is provided by numerous experiments [12, 13, 14, 15, 16, 17, 18, 19], which can be interpreted as an evidence of instantaneous action-at-a-distance. It was also demonstrated that such a superluminality does not violate the principle of causality [20, 24], and that action-at-a-distance potentials can be a reasonable alternative to the general relativistic description of gravity [21].

First attempts at Hamiltonian formulations of electrodynamics without fields were undertaken by Darwin [22] and Breit [23]. They found that electromagnetic effects in the $(1/c)^2$ approximation can be represented by instantaneous interparticle forces. The relativistic invariance of this approach was established in [24]. The Darwin-Breit Hamiltonian was successfully applied to various electromagnetic problems, such as the fine structure in atomic spectra [25, 26], superconductivity and properties of plasma [27, 28, 29].

The Aharonov-Bohm effect [30, 31, 32, 33] is usually believed to be an indication of the fundamental importance of electromagnetic potentials and fields in nature. Although, several non-conventional explanations of this effect were suggested in the literature [34, 35, 36], as far as I know, there were no attempts to interpret this effect in terms of direct interactions between particles. In this paper we would like to fill this gap and to suggest a simple description of the Aharonov-Bohm effect within the Darwin-Breit action-at-a-distance theory. This explanation does not involve the notions of electromagnetic potentials, fields, and non-trivial space topologies.

In section 2 we briefly discuss the relativistic Hamiltonian quantum mechanics and formulation of the Darwin-Breit theory as a classical limit of the dressed particle version of quantum electrodynamics (QED) [37, 10]. In section 3 we apply the Darwin-Breit Hamiltonian to interactions of a moving point charge with solenoids and ferromagnets. The new approach to the Aharonov-Bohm effect is discussed in section 4.

2 Relativistic Hamiltonian dynamics

One class of problems characteristic to Maxwell-Lorentz electrodynamics is related to the apparent non-conservation of total observables (energy, momentum, angular momentum, etc.) in systems of interacting charges. Indeed, in the theory based on Maxwell’s equations there is no guarantee that total observables are conserved, that Newton’s third law of action and reaction is valid, and that total energy and momentum form a 4-vector quantity. Suggested solutions of these paradoxes [6, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47] involved such ad hoc constructions as “hidden momentum”, the energy and momentum of electromagnetic fields, ”Poincaré stresses”, etc.

2 The explanation of this paradox suggested in [11] is not satisfactory, because it assumes interaction of overlapping light waves, which is known to be negligibly small from QED.
In theoretical physics it is well established that conservation laws are consequences of the invariance of observations with respect to inertial transformations of reference frames. These transformations are elements of the Poincaré group. So, the conservation of total observables and their correct transformation properties can be guaranteed in an approach based on a (Hamiltonian) theory of representations of the Poincaré group. In quantum mechanics, such an approach is realized within Wigner-Dirac theory [48, 49, 50] in which dynamics of any isolated physical system is described as a representation of the Poincaré Lie algebra by Hermitian operators in the Hilbert space of states. Representatives of ten basis elements (generators) of the Poincaré Lie algebra are identified with observables of the total linear momentum $P$, total angular momentum $J$, total energy (the Hamiltonian) $H$, and ”boost” operator $K$. In the instant form of Dirac’s dynamics [49], these generators have the form

$$
\begin{align*}
P &= P_0 \\
J &= J_0 \\
K &= K_0 + Z \\
H &= H_0 + V
\end{align*}
$$

where the non-interacting parts $P_0, J_0, K_0, H_0$ are simply sums of one-particle generators, and interactions are contained in the potential energy $V$ and potential boost $Z$ operators.

The commutator of any observable $F$ with the Hamiltonian $H$ determines the time evolution of this observable in the Heisenberg picture of quantum mechanics

$$
\frac{dF(t)}{dt} = \frac{i}{\hbar} [H, F]
$$

Then the conservation of observables $H, P$ and $J$ follows automatically from their vanishing commutators with $H$. These conservation laws hold true independent on interactions that may be present in the multiparticle system. This simple fact is not at all obvious in the Maxwell-Lorentz theory, which, for example, has serious difficulties in explaining the conservation of the total angular momentum of a moving capacitor in the Trouton-Noble experiment [7, 41, 39, 51, 52, 53].

There are three essential steps [10] that need to be made in order to arrive at the Darwin-Breit Hamiltonian from QED, which is rightly considered the most accurate physical theory in existence. First, the ”dressed particle” approach [55, 56, 37] should be applied, which allows one to formulate the quantum field theory in terms of

---

3This operator does not have interpretation as a common mechanical observable, however it is closely related to observables of the center-of-mass position and spin [10].
physical (rather than "bare") particles and avoid ultraviolet divergences. This leads to the perturbation expansion of the potential energy operator \( V \) whose terms are direct particle interactions. In the second perturbation order the interaction is a sum of two-particle terms, so it is possible to consider only the two-particle sector of the Fock space and express the interaction energy as a function of particle charges \( q_i \), positions \( \mathbf{r}_i \), momenta \( \mathbf{p}_i \), spins \( \mathbf{s}_i \), and energies \( h_i = \sqrt{m_i^2 c^4 + p_i^2 c^2} \). Second, in the classical limit \( (\hbar \rightarrow 0) \) commutators of operators are replaced by Poisson brackets \( \{ \ldots , \ldots \} \). Finally, for low-velocity processes of classical electrodynamics one can represent all quantities as series in powers of \( 1/c \) and leave only terms of order not higher than \((1/c)^2\). In these approximations the set of generators (1) - (4) takes the form

\[
\begin{align*}
\mathbf{P}_0 &= \mathbf{p}_1 + \mathbf{p}_2 \\
\mathbf{J}_0 &= [\mathbf{r}_1 \times \mathbf{p}_1] + \mathbf{s}_1 + [\mathbf{r}_2 \times \mathbf{p}_2] + \mathbf{s}_2 \\
H_0 &= h_1 + h_2 \\
&\approx m_1 c^2 + m_2 c^2 + \frac{p_1^2}{2m_1} + \frac{p_2^2}{2m_2} - \frac{p_1^4}{8m_1^3 c^2} - \frac{p_2^4}{8m_2^3 c^2} \\
V &\approx V_{\text{Coulomb}} + V_{\text{Darwin}} + V_{\text{spin-orb}} + V_{\text{spin-spin}} \\
V_{\text{Coulomb}} &= \frac{q_1 q_2}{4\pi r} \\
V_{\text{Darwin}} &= -\frac{q_1 q_2}{8\pi m_1 m_2 c^2 r} \left( (\mathbf{p}_1 \cdot \mathbf{p}_2) + \frac{(\mathbf{p}_1 \cdot \mathbf{r})(\mathbf{p}_2 \cdot \mathbf{r})}{r^2} \right) \\
V_{\text{spin-orbit}} &= \frac{q_1 ([\bar{\mu}_2 \times \mathbf{r}] \cdot (\mathbf{p}_2 - 2\mathbf{p}_1)) - q_2 ([\bar{\mu}_1 \times \mathbf{r}] \cdot (\mathbf{p}_1 - 2\mathbf{p}_2))}{8\pi m_1 c r^3} \\
V_{\text{spin-spin}} &= \frac{(\bar{\mu}_2 \cdot \mathbf{r}_1)}{4\pi r^3} - \frac{3(\bar{\mu}_2 \cdot \mathbf{r})(\bar{\mu}_1 \cdot \mathbf{r})}{4\pi r^5} \\
K_0 &= -\frac{h_1 \mathbf{r}_1}{c^2} - \frac{[\mathbf{p}_1 \times \mathbf{s}_1]}{m_1 c^2 + h_1} - \frac{h_2 \mathbf{r}_2}{c^2} - \frac{[\mathbf{p}_2 \times \mathbf{s}_2]}{m_2 c^2 + h_2} \\
&\approx -m_1 \mathbf{r}_1 - m_2 \mathbf{r}_2 - \frac{p_1^2 \mathbf{r}_1}{2m_1 c^2} - \frac{p_2^2 \mathbf{r}_2}{2m_2 c^2} + \frac{1}{2c^2} \left( \frac{[\mathbf{s}_1 \times \mathbf{p}_1]}{m_1} + \frac{[\mathbf{s}_2 \times \mathbf{p}_2]}{m_2} \right) \\
Z &\approx -\frac{q_1 q_2 (\mathbf{r}_1 + \mathbf{r}_2)}{8\pi c^2 r}
\end{align*}
\]

The Hamiltonian \( H = H_0 + V \) is the Darwin-Breit Hamiltonian for two charged spinning particles. It contains the familiar Coulomb energy (6) and the Darwin potential (7). We use the Heaviside-Lorentz system of units and denote \( \mathbf{r} \equiv \mathbf{r}_1 - \mathbf{r}_2 \) throughout this paper. This form of the Darwin-Breit Hamiltonian can be found in eqs. (9.50) - (9.55) in [10], where contact terms proportional to \( \delta(r) \) are not relevant for classical mechanics and can be omitted. Also here we express the potential energy of interaction in terms of particles’ magnetic moments \( \bar{\mu}_i = e\mathbf{s}_i/(m_ic) \) which correspond to the gyromagnetic ratio \( g \approx 2 \) characteristic for electrons.

\[\text{footnote text}\]
energy \([\text{(7)}]\) \([22]\), which is responsible for magnetic interactions between charged particles. Further relativistic corrections are given by the spin-orbit \([\text{(8)}]\) and spin-spin \([\text{(9)}]\) interactions. A straightforward computation shows that Poisson brackets of the above generators satisfy the Poincaré Lie algebra relationship within \((1/c)^2\) approximation \([24, 57, 58]\).

3 Interaction between charges and magnetic dipoles

In the Aharonov-Bohm effect \([30]\) a charged particle (e.g., an electron labeled by the index 1) interacts with an infinite solenoid or ferromagnetic rod (which can be represented as a collection of point magnetic moments 2). This experiment is not sensitive to the electron’s spin orientation, and the charge of the solenoid/rod is zero. Then we can assume \(\vec{\mu}_1 = 0\) and \(q_2 = 0\), omit interaction terms \((6), (7), (9)\), and write the simplified Hamiltonian for the system "point magnetic dipole + charge" \([5]\):

\[
H = \frac{p_1^2}{2m_1} + \frac{p_2^2}{2m_2} - \frac{p_1^4}{8m_1^3c^2} - \frac{p_2^4}{8m_2^3c^2} + \frac{q_1[\vec{\mu}_2 \times \vec{r}] \cdot \vec{p}_2}{8\pi m_2 cr^3} - \frac{q_1[\vec{\mu}_2 \times \vec{r}] \cdot \vec{p}_1}{4\pi m_1 cr^3} \tag{10}
\]

The time derivative of the first particle’s momentum can be obtained from the Hamilton’s equation of motion

\[
\frac{d\vec{p}_1}{dt} = \{H, \vec{p}_1\} = -\frac{\partial H}{\partial \vec{r}_1}
= \frac{q_1[\vec{p}_1 \times \vec{\mu}_2]}{4\pi m_1 cr^3} - \frac{3q_1([\vec{p}_1 \times \vec{\mu}_2] \cdot \vec{r})\vec{r}}{4\pi m_1 cr^5} - \frac{q_1[\vec{p}_2 \times \vec{\mu}_2] \cdot \vec{r}}{8\pi m_2 cr^3} + \frac{3q_1([\vec{p}_2 \times \vec{\mu}_2] \cdot \vec{r})\vec{r}}{8\pi m_2 cr^5}
\]

The time derivative of the second particle’s momentum follows from the law of conservation of the total momentum \(\{[\vec{P}, H] = 0\)\)

\[
\frac{d\vec{p}_2}{dt} = \{\vec{p}_2, H\} = \{\vec{P} - \vec{p}_1, H\} = -\{\vec{p}_1, H\} = -\frac{d\vec{p}_1}{dt} \tag{11}
\]

This is the third Newton’s law of action and reaction, which holds exactly in the instant form of dynamics, and there is no need to invoke such dubious notions as “hidden momentum” and/or momentum of electromagnetic fields in order to enforce this law.

\(^5\)Here we drop the first two terms in \((5)\), which are the rest energies of the two particles and do not have any effect on dynamics.
It is difficult to measure momenta of particles and their time derivatives in experiment. It is much easier to measure velocities and accelerations, e.g., by the time-of-flight technique \[59\]. The velocity of the charged particle \(1\) is obtained from the Hamilton’s equation

\[
v_1 \equiv \frac{dr_1}{dt} = \{r_1, H\} = \frac{\partial H}{\partial \mathbf{p}_1} = \frac{p_1^0}{m_1} - \frac{p_1^0 \mathbf{p}_1}{2m_1^3c} - \frac{q_1 [\mathbf{\mu}_2 \times \mathbf{r}]}{4\pi m_1^3c r^3}
\]

This relationship is interaction-dependent because the interaction energy in (10) is momentum-dependent. From (11), (12), and vector identity \(a \times [b \times c] = b (a \cdot c) - c (a \cdot b)\) we obtain the acceleration of the charged particle interacting with the magnetic moment at rest \((p_2 = 0)\)

\[
a_1 \equiv \frac{d^2 r_1}{dt^2} = \{v_1, H\}
\]

\[
\approx \frac{p_1}{m_1} - \frac{q_1 [\mathbf{\mu}_2 \times \mathbf{r}]}{4\pi m_1^3 c r^3} + \frac{3q_1 [\mathbf{\mu}_2 \times \mathbf{r}] (\mathbf{r} \cdot \dot{\mathbf{r}})}{4\pi m_1^2 c r^5}
\]

\[
= \frac{q_1 [\mathbf{p}_1 \times \mathbf{\mu}_2]}{2\pi m_1^3 c r^3} - \frac{3q_1 [\mathbf{p}_1 \times \mathbf{\mu}_2] \cdot \mathbf{r}}{4\pi m_1^2 c r^5} + \frac{3q_1 [\mathbf{\mu}_2 \times \mathbf{r}] (\mathbf{r} \cdot \mathbf{p}_1)}{4\pi m_1^2 c r^5}
\]

\[
= -\frac{q_1 [\mathbf{p}_1 \times \mathbf{\mu}_2]}{2\pi m_1^3 c r^3} + \frac{3q_1 [\mathbf{p}_1 \times \mathbf{r}] (\mathbf{\mu}_2 \cdot \mathbf{r})}{4\pi m_1^2 c r^5}
\]

\[
\approx \frac{q_1}{m_1 c} [v_1 \times \mathbf{B}]
\]

This agrees with the standard Lorentz force formula if another standard expression (see eq. (5.56) in \[60\])

\[
\mathbf{B} = -\frac{\mathbf{\mu}_2}{4\pi r^3} + \frac{3(\mathbf{\mu}_2 \cdot \mathbf{r}) \mathbf{r}}{4\pi r^5}
\]

is used for the "magnetic field" of the magnetic moment 6

In Appendix A we show that the same "magnetic field" is created by a small circular loop with current. The "magnetic field" of infinitely long thin solenoid or infinitely long ferromagnetic rod can be obtained by integrating (14) along the length of the

6We write "magnetic field" in quotes, because in the Darwin-Breit approach there are no fields (electric or magnetic) having independent existence at each space point. There are only direct inter-particle forces, and in eq. (14) \(r_1\) and \(r_2\) are coordinates of two particles, rather than general points in space.
It is easy to show that this integral vanishes. This agrees with the prediction of Maxwell’s theory that a charge is moving without acceleration in the vicinity of an infinite magnetized solenoid/rod. In particular, this result is consistent with the lack of "time lag" in experiments \[59\].

## 4 The Aharonov-Bohm effect

Let us consider the following idealized version of the Aharonov-Bohm experiment (see fig. 1): An infinite solenoid or ferromagnetic rod with negligible cross-section and linear magnetization $\mu$ is erected vertically in the origin (grey arrows). The electron wave packet is split into two parts (e.g., by using a double-slit) at point $A$. These subpackets travel on both sides of the solenoid/rod with constant velocity $v_1$, and the distance of the closest approach is $R$. The subpackets rejoin at point $B$, where the interference is measured. (The two trajectories $AA_1B_1B$ and $AA_2B_2B$ are denoted by broken lines.) The distance $AB$ is sufficiently large, so that electron’s path can be assumed parallel to the $y$-axis everywhere

$$\mathbf{r}_1(t) = (\pm R, v_1 t, 0)$$

(15)

Experimentally it was found that the interference of the two wave packets at point $B$ depends on the magnetization of the solenoid/rod [31]. In the preceding section we demonstrated that electron’s acceleration is zero. Therefore the Aharonov-Bohm
effect cannot be explained as a result of classical forces \[34, 61, 62, 63\]. To resolve this paradox it is sufficient to mention that the representation of the wave packet as a point moving through space along the trajectory \[15\] is an oversimplification. A more complete description of the electron’s wave function should also include the overall phase factor

\[
\psi_1(\mathbf{r}, t) \approx e^{i \frac{\hbar}{\sqrt{2}} S(t)} \delta(\mathbf{r} - \mathbf{r}_1(t))
\]

The action integral \(S(t)\) for the one-particle wave packet that traveled between time points \(t_0\) and \(t\) is

\[
S(t) \equiv \int_{t_0}^{t} \left( \frac{m_1 v_1^2(t')}{2} - V_1(t') \right) dt'
\]

where \(V_1(t)\) is the contribution to the particle’s energy due to the external potential. Then the interference of the ”left” and ”right” wave packets at point \(B\) will depend on the relative value of phase factors accumulated by them along the path \(AB\)

\[
\phi = \frac{1}{\hbar} (S_{\text{right}} - S_{\text{left}})
\]

Let us now calculate the relative phase shift in the geometry of fig. \[1\]. The kinetic energy term in \((16)\) does not contribute, because velocity remains constant for both paths. However, the potential energy of the charge

\[
V = \int_{-\infty}^{\infty} \frac{dz}{4\pi c(x^2 + y^2 + z^2)^{3/2}} = \frac{q_1([\mu \times v_1] \cdot \mathbf{r}_1)}{2\pi c(x^2 + y^2)}
\]

is different for the two paths. For all points on the ”right” path the numerator of this expression is \(-q_1 \mu v_1 R\), and for the ”left” path the numerator is \(q_1 \mu v_1 R\). Then the total phase shift is

\[
\phi = \frac{1}{\hbar} \int_{-\infty}^{\infty} \frac{q_1 \mu R v_1}{\pi c(R^2 + v_1^2 t^2)} dt = \frac{e \mu}{\hbar c}
\]

\[7\]Here we integrate the last term in eq. \((10)\) along the length of the solenoid and notice that the mixed product \(([\mu \times v_1] \cdot \mathbf{r}_1)\) is independent on \(z\).
This phase difference does not depend on the electron’s velocity and on the value of $R$. However, it is proportional to the rod’s magnetization $\mu$. So, all essential properties of the Aharonov-Bohm effect are fully described within the Darwin-Breit direct interaction theory. In our description the Aharonov-Bohm effect is a quantum phenomenon, however, in contrast to traditional views, this effect does not prove the existence of scalar and/or vector electromagnetic potentials, and it is not essential whether the solenoid/rod is infinite (so that it induces a multiple-connected topology of space) or not. The latter point is supported by experiments with finite-length magnetized nanowires, which exhibit the phase shift similar to that characteristic for infinite solenoids/rods.

I am thankful to Dr. Peter Enders for helpful comments and discussions.

A Appendix. Current loop and charge

Let us calculate the interaction energy between a neutral circular current loop of small radius $a$ and a point charge in the geometry shown in fig. 2. There are three types

---

8 These results were derived for thin ferromagnetic rods and solenoids, however the same arguments apply to cylindrical rods and solenoids of any cross-section. A cylindrical rod can be represented as a bunch of thin rods. A cylindrical solenoid also can be represented as a bunch of thin solenoids. In this representation, the currents cancel out in the interior, where neighboring components touch each other, and only currents on the outside surface have effect on the charge $q_1$. The same phase shift formula can be obtained for toroidal solenoids, which were used in Tonomura’s experiments.  

9
of charges in this problem: First, there is the charge $q_1$ located at a general point in space $r_1 = (r_{1x}, r_{1y}, r_{1z})$ and having arbitrary momentum $p_1 = (p_{1x}, p_{1y}, p_{1z})$. Also there are positive charges of immobile ions in the metal uniformly distributed along the loop with linear density $\rho_3$ and negative charges of conduction electrons having linear density $\rho_2 = -\rho_3$.

Let us introduce a few simplification in this problem. First, we are not interested in the interaction between charge densities $\rho_2$ and $\rho_3$. Second, the Coulomb interactions of charges $1 \leftrightarrow 2$ and $1 \leftrightarrow 3$ cancel each other. Third, it can be shown that if the current loop moves with total velocity $v$, then $v$-dependent interactions (7) of the charge 1 with electrons and ions in the loop also cancel each other. Therefore, we can assume that the current loop is stationary in the origin, and that only electrons in the loop are moving with velocity $v_2 \approx p_2/m_2$ whose tangential component is $u$, as shown in fig. 2. Finally, the spins of ions and electrons in the wire are oriented randomly, therefore Hamiltonian terms (8) - (9), being linear with respect to particle spins, vanish after averaging. Then the potential energy of interaction between the charge 1 and the loop element $dl$ is given by the Darwin’s formula

$$V_{dl-q1} \approx -\frac{q_1 \rho_2 dl}{8\pi m_1 c^2} \left( \frac{(p_1 \cdot v_2)}{r} + \frac{(p_1 \cdot r)(v_2 \cdot r)}{r^3} \right)$$

In the coordinate system shown in fig. 2 the line element in the loop is $dl = ad\theta$ and $v_2 = (-u \sin \theta, u \cos \theta, 0)$. In the limit $a \to 0$ we can approximate

$$r^{-1} \approx \frac{1}{r_1} + \frac{a(r_{1x} \cos \theta + r_{1y} \sin \theta)}{r_1^3}$$
$$r^{-3} \approx \frac{1}{r_1^3} + \frac{3a(r_{1x} \cos \theta + r_{1y} \sin \theta)}{r_1^5}$$

The full interaction between the charge and the loop is obtained by integrating $V_{dl-q1}$ on $\theta$ from 0 to $2\pi$ and neglecting small terms proportional to $a^3$

$$V_{\text{loop-q1}} \approx -\frac{aq_1 \rho_2}{8\pi m_1 c^2} \int_0^{2\pi} d\theta \left[ (-up_{1x} \sin \theta + up_{1y} \cos \theta) \left( \frac{1}{r_1} + \frac{a(r_{1x} \cos \theta + r_{1y} \sin \theta)}{r_1^3} \right) \right. \left. + \right.$$

$$(-ur_{1x} \sin \theta + ur_{1y} \cos \theta)((p_1 \cdot r_1) - p_{1x}a \cos \theta - p_{1y}a \sin \theta) \times$$

$$\left( \frac{1}{r_1^3} + \frac{3a(r_{1x} \cos \theta + r_{1y} \sin \theta)}{r_1^5} \right) \left. \right]$$

$$\approx -\frac{a^2 uq_1 \rho_2 [r_1 \times p_1]_z}{4m_1 c^2 r_1^3}$$

10
Taking into account the usual definition of the loop’s magnetic moment \( \mu_2 = \pi a^2 \rho_2 u/c \) (see eq. (5.42) in [60]) whose direction is orthogonal to the plane of the loop, we find that for arbitrary position and orientation of the loop

\[
V_{\text{loop}-q_1} \approx -\frac{q_1[\vec{\mu}_2 \times \vec{r}] \cdot \vec{p}_1}{4\pi m_1 c r^3}
\]

which agrees with the spin-charge interaction in (8) when \( p_2 = 0 \). Therefore, the acceleration of the charge \( q_1 \) moving in the field of the current loop is also given by eq. (13).

References

[1] A. E. Chubykalo, R. Smirnov-Rueda. Action at a distance as a full-value solution of Maxwell equations: basis and application of separated potential’s method. Phys. Rev. E, 53:5373, 1996.

[2] J. H. Field. Classical electromagnetism as a consequence of Coulomb’s law, special relativity and Hamilton’s principle and its relationship to quantum electrodynamics. Phys. Scr., 74:702, 2006. http://www.arxiv.org/abs/physics/0501130v5.

[3] J. H. Field. On the relationship of quantum mechanics to classical electromagnetism and classical relativistic mechanics. http://www.arxiv.org/abs/physics/0403076.

[4] A. C. de la Torre. Understanding light quanta: Construction of the free electromagnetic field. http://www.arxiv.org/abs/quant-ph/0503023v2.

[5] R. Carroll. Remarks on photons and the aether. http://www.arxiv.org/abs/physics/0507027v1.

[6] F. Rohrlich. Self-energy and stability of the classical electron. Am. J. Phys., 28:639, 1960.

[7] J. W. Butler. A proposed electromagnetic momentum-energy 4-vector for charged bodies. Am. J. Phys., 37:1258, 1969.

[8] E. Comay. Decomposition of electromagnetic fields into radiation and bound components. Am. J. Phys., 65:862, 1997.

[9] J. Franklin. The nature of electromagnetic energy, 2007. http://www.arxiv.org/abs/0707.3421v2.
[10] E. V. Stefanovich. Relativistic quantum dynamics, 2005. http://www.arxiv.org/abs/physics/0504062v7.

[11] A. Kislev, L. Vaidman. Relativistic causality and conservation of energy in classical electromagnetic theory. http://www.arxiv.org/abs/physics/0201042v1.

[12] G. C. Giakos, T. K. Ishii. Rapid pulsed microwave propagation. *IEEE Microwave and Guided Wave Letters*, 1:374, 1991.

[13] A. Enders, G. Nimtz. Evanescent-mode propagation and quantum tunneling. *Phys. Rev. E*, 48:632, 1993.

[14] A. M. Steinberg, P. G. Kwiat, R. Y. Chiao. Measurement of the single-photon tunneling time. *Phys. Rev. Lett.*, 71:708, 1993.

[15] A. Ranfagni, D. Mugnai. Anomalous pulse delay in microwave propagation: A case of superluminal behavior. *Phys. Rev. E*, 54:5692, 1996.

[16] D. Mugnai, A. Ranfagni, R. Ruggeri. Observation of superluminal behaviors in wave propagation. *Phys. Rev. Lett.*, 21:4830, 2000.

[17] K. Wynne, D. A. Jaroszynski. Superluminal terahertz pulses. *Optics Letters*, 24:25, 1999.

[18] W. D. Walker. Experimental evidence of near-field superluminally propagating electromagnetic fields. http://www.arxiv.org/abs/physics/0009023v1.

[19] A. L. Kholmetskii, O. V. Mishevitch, R. Smirnov-Rueda, R. I. Tzonchev, A. E. Chubykalo, I. Moreno. Experimental evidence on non-applicability of the standard retardation condition to bound magnetic fields and on new generalized Biot-Savart law. *J. Appl. Phys.*, 101:023532, 2007. http://www.arxiv.org/abs/physics/0601084v1.

[20] E. V. Stefanovich. Is Minkowski space-time compatible with quantum mechanics? *Found. Phys.*, 32:673, 2002.

[21] E. V. Stefanovich. A Hamiltonian approach to quantum gravity, 2006. http://www.arxiv.org/abs/physics/0612019v9.

[22] C. G. Darwin. The dynamical motions of charged particles. *Phil. Mag.*, 39:537, 1920.

[23] G. Breit. The effect of retardation on the interaction of two electrons. *Phys. Rev.*, 34:553, 1929.
[24] S. Coleman, J. H. Van Vleck. Origin of "hidden momentum forces" on magnets. *Phys. Rev.*, **171**:1370, 1968.

[25] V. B. Berestetskii, E. M. Livshitz, L. P. Pitaevskii. *Quantum electrodynamics*. Fizmatlit, Moscow, 2001. (in Russian).

[26] E. Breitenberger. Magnetic interactions between charged particles. *Am. J. Phys.*, **36**:505, 1968.

[27] H. Essén. A study of lattice and magnetic interactions of conduction electrons. *Phys. Scr.*, **52**:388, 1995.

[28] H. Essén. Darwin magnetic interaction energy and its macroscopic consequences. *Phys. Rev. E*, **53**:5228, 1996.

[29] H. Essén. Magnetism of matter and phase space energy of charged particle systems. *J. Phys. A: Math. Gen.*, **32**:2297, 1999.

[30] Y. Aharonov, D. Bohm. Significance of electromagnetic potentials in quantum mechanics. *Phys. Rev.*, **115**:485, 1959.

[31] R. G. Chambers. Shift of an electron interference pattern by enclosed magnetic flux. *Phys. Rev. Lett.*, **5**:3, 1960.

[32] A. Tonomura, N. Osakabe, T. Matsuda, T. Kawasaki, J. Endo, S. Yano, H. Yamada. Evidence for Aharonov-Bohm effect with magnetic field completely shielded from electron wave. *Phys. Rev. Lett.*, **56**:792, 1986.

[33] N. Osakabe, T. Matsuda, T. Kawasaki, J. Endo, A. Tonomura, S. Yano, H. Yamada. Experimental confirmation of Aharonov-Bohm effect using a toroidal magnetic field confined by a superconductor. *Phys. Rev. A: Math. Gen.*, **34**:815, 1986.

[34] T. H. Boyer. Darwin-Lagrangian analysis for the interaction of a point charge and a magnet: Considerations related to the controversy regarding the Aharonov-Bohm and Aharonov-Casher phase shifts. *J. Phys. A: Math. Gen.*, **39**:3455, 2006. http://www.arxiv.org/abs/physics/0506181v1.

[35] G. Spavieri, G. Cavalleri. Interpretation of the Aharonov-Bohm and the Aharonov-Casher effects in terms of classical electromagnetic fields. *Europhys. Lett.*, **18**:301, 1992.

[36] G. C. Hegerfeldt, J. T. Neumann. The Aharonov-Bohm effect: the role of tunneling and associated forces, 2008. http://www.arxiv.org/abs/arXiv:0801.0799v2.
[37] E. V. Stefanovich. Quantum field theory without infinites. *Ann. Phys. (NY)*, **292**:139, 2001.

[38] J. M. Keller. Newton’s third law and electrodynamics. *Am. J. Phys.*, **10**:302, 1942.

[39] L. Page, N. I. Adams Jr. Action and reaction between moving charges. *Am. J. Phys.*, **13**:141, 1945.

[40] W. Shockley, R. P. James. "Try simplest cases" discovery of "hidden momentum" forces on "magnetic currents". *Phys. Rev. Lett.*, **18**:876, 1967.

[41] W. H. Furry. Examples of momentum distributions in the electromagnetic field and in matter. *Am. J. Phys.*, **37**:621, 1969.

[42] Y. Aharonov, P. Pearle, L. Vaidman. Comment on "Proposed Aharonov-Casher effect: Another example of an Aharonov-Bohm effect arising from a classical lag". *Phys. Rev. A*, **37**:4052, 1988.

[43] E. Comay. Exposing "hidden momentum". *Am. J. Phys.*, **64**:1028, 1996.

[44] E. Comay. Lorentz transformation of a system carrying "hidden momentum". *Am. J. Phys.*, **68**:1007, 2000.

[45] O. D. Jefimenko. A relativistic paradox seemingly violating conservation of momentum law in electromagnetic systems. *Eur. J. Phys.*, **20**:39, 1999.

[46] A. L. Kholmetskii. On momentum and energy of a non-radiating electromagnetic field, 2005. http://www.arxiv.org/abs/physics/0501148v2.

[47] V. Hnizdo. On linear momentum in quasistatic electromagnetic systems, 2004. http://www.arxiv.org/abs/physics/0407027v1.

[48] E. P. Wigner. On unitary representations of the inhomogeneous Lorentz group. *Ann. Math.*, **40**:149, 1939.

[49] P. A. M. Dirac. Forms of relativistic dynamics. *Rev. Mod. Phys.*, **21**:392, 1949.

[50] S. Weinberg. *The Quantum Theory of Fields, Vol. 1*. University Press, Cambridge, 1995.

[51] G. Spavieri, G. T. Gillies. Fundamental tests of electrodynamic theories: Conceptual investigations of the Trouton-Noble and hidden momentum effects. *Nuovo Cim.*, **118B**:205, 2003.
[52] S. A. Teukolsky. The explanation of the Trouton-Noble experiment revisited. Am. J. Phys., 64:1104, 1996.

[53] O. D. Jefimenko. The Trouton-Noble paradox. J. Phys. A: Math. Gen., 32:3755, 1999.

[54] J. D. Jackson. Torque or no torque? Simple charged particle motion observed in different inertial frames. Am. J. Phys., 72:1484, 2004.

[55] O. W. Greenberg, S. S. Schweber. Clothed particle operators in simple models of quantum field theory. Nuovo Cim., 8:378, 1958.

[56] A. V. Shebeko, M. I. Shirokov. Unitary transformations in quantum field theory and bound states. Phys. Part. Nucl., 32:15, 2001. http://www.arxiv.org/abs/nucl-th/0102037v1.

[57] F. E. Close, H. Osborn. Relativistic center-of-mass motion and the electromagnetic interaction of systems of charged particles. Phys. Rev. D, 2:2127, 1970.

[58] R. A. Krajcik, L. L. Foldy. Relativistic center-of-mass variables for composite systems with arbitrary internal interactions. Phys. Rev. D, 10:1777, 1974.

[59] A. Caprez, B. Barwick, H. Batelaan. A macroscopic test of the Aharonov-Bohm effect, 2007. http://www.arxiv.org/abs/0708.2428v1.

[60] J. D. Jackson. Classical electrodynamics. J. Wiley and Sons, 3rd edition, 1999.

[61] T. H. Boyer. The paradoxical forces for the classical electromagnetic lag associated with the Aharonov-Bohm phase shift, 2005. http://www.arxiv.org/abs/physics/0506180v1.

[62] T. H. Boyer. Comment on experiments related to the Aharonov-Bohm phase shift, 2007. http://www.arxiv.org/abs/0708.3194v1.

[63] T. H. Boyer. Unresolved classical electromagnetic aspects of the Aharonov-Bohm phase shift, 2007. http://www.arxiv.org/abs/0709.0661v1.

[64] G. Matteucci, D. Iencinella, C. Beeli. The Aharonov-Bohm phase shift and Boyer’s critical considerations: New experimental result but still an open subject? Found. Phys., 33:577, 2003.