Analytical approximations for the extrapolation of lattice data

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Guideline

- Motivation
- Dissection of the $O(p^6)$ amplitude
- A Viennese recipe
- An example: $F_K / F_\pi$
- Conclusions
- Outlook
### Motivation

• At low-energies, ChPT is the EFT of SM → LECs

\[
\begin{align*}
O(p^2) &\rightarrow 2 \\
O(p^4) &\rightarrow 10 \\
O(p^6) &\rightarrow 94
\end{align*}
\]

[Gasser, Leutwyler’84’85]

• Lattice calculations involve different quark masses → good for LECs
  • LO+NLO → O.K.
  • NNLO → only recently

[Bernard, Passemar’10]
[MAILC Col. ’09]

• For comparison with lattice, dependence on masses should be known

• We propose analytical approximations of chiral SU(3) amplitudes for the extrapolation of lattice data to the physical masses.
• We determine NNLO LECs.
ChPT at $O(p^6)$: Dissection of $Z_6$

ChPT in meson sector $\rightarrow \quad Z = Z_2 + Z_4 + Z_6 + \ldots, \quad O(p^2, p^4, p^6, \ldots)$

NNLO functional $Z_6$ of $O(p^6)$

- Tree diagrams of $O(p^6) \rightarrow g$
- Irreducible $\rightarrow a, b, d$
- Reducible $\rightarrow c, e, f$

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Dissection of $\mathbb{Z}_6$: irreducible

Diagrams:
- $a,b,d \rightarrow$ divergent (dim. reg.)
- $a+b+d \rightarrow$ local diverg (renorm th)

[Bijnens et al ‘00]
Dissection of $Z_6$: irreducible

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- $a,b,d \rightarrow$ divergent (dim. reg.)
- $a+b+d \rightarrow$ local diverg (renorm th)
  - [Bijnens et al '00]
- Chiral sym $\rightarrow$ div absorbed by “g“ $\rightarrow$ scale $\mu$

$$L = \frac{1}{(4\pi)^2} \ln \frac{M^2}{\mu^2}$$

$$Z_{6}^{a+b+d+g} = \int d^4x \left\{ C_a^r(\mu) + \frac{1}{4F_0^2} \left[ 4\Gamma_a^{(1)} L - \Gamma_a^{(2)} L^2 + 2\Gamma_a^{(L)}(\mu) L \right] \right\} O_a(x)$$

$$+ \frac{1}{(4\pi)^2} \left[ L_i^r(\mu) - \frac{\Gamma_i}{2} L \right] H_i(x; M) + \frac{1}{(4\pi)^2} K(x; M)$$
Dissection of $Z_6$: irreducible

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• $a,b,d \rightarrow$ divergent (dim. reg.)
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\[
L = \frac{1}{(4\pi)^2} \ln M^2/\mu^2
\]

\[
Z_{a+b+d+g}^{a+b+d+g} = \int d^4x \left[ C_a^r (\mu) + \frac{1}{4 F_0^2} \left( 4 \Gamma_a^{(1)} L - \Gamma_a^{(2)} L^2 + 2 \Gamma_a^{(L)} (\mu) L \right) \right] O_a (x)
\]

\[
+ \frac{1}{(4\pi)^2} \left[ L_i^r (\mu) - \frac{\Gamma_i}{2} L \right] H_i (x; M) + \frac{1}{(4\pi)^2} K(x; M)
\]
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- $a,b,d \rightarrow$ divergent (dim. reg.)
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\[
L = \frac{1}{(4\pi)^2} \ln \frac{M^2}{\mu^2}
\]

\[
Z_{6}^{a+b+d+g} = \int d^4x \left\{ C_a^r(\mu) + \frac{1}{4F_0^2} \left( 4\Gamma_{a}^{(1)}L - \Gamma_{a}^{(2)}L^2 + 2\Gamma_{a}^{(4)}(\mu)L \right) \right\} O_a(x)
\]
\[
+ \frac{1}{(4\pi)^2} \left\{ L_i^r(\mu) - \frac{\Gamma_i}{2} L \right\} H_i(x;M) + \frac{1}{(4\pi)^2} K(x;M)
\]
Dissection of $\mathbb{Z}_6$: reducible

Diagrams:
- $c+e+f \rightarrow$ finite and scale independent
  
  $L = \frac{1}{(4\pi)^2} \ln \frac{M^2}{\mu^2}$

\[
Z_{6}^{c+e+f} = \int d^4x d^4y \left[ \left( L_i^r (\mu) - \frac{\Gamma_i}{2} L \right) P_{i,a} (x) + F_{a} (x; M) \right] G_{a,b} (x, y)
\]

\[
\left[ \left( L_j^r (\mu) - \frac{\Gamma_j}{2} L \right) P_{j,a} (x) + F_{\beta} (x; M) \right]
\]
Dissection of $\mathbb{Z}_6$: reducible

Diagrams:
- $c+e+f \rightarrow$ finite and scale independent
  [Bijnens et al '00]

\[
L = \frac{1}{(4\pi)^2} \ln M^2/\mu^2
\]

\[
Z_6^{c+e+f} = \int d^4x d^4y \left[ \left( L^r_i(\mu) - \frac{\Gamma_i}{2} L \right) P_{i,a}(x) + F_{a}(x; M) \right] \left[ \left( L^r_j(\mu) - \frac{\Gamma_j}{2} L \right) P_{j,a}(x) + F_{\beta}(x; M) \right] G_{a,\beta}(x, y)
\]
Approximation

\[ Z_{\text{app}} \]

is M-dependent through chiral logs, \( M = M_k \).

\[ Z_6 = Z_6^{\text{app}} = Z_6^{a+b+d+g} + Z_6^{c+e+f} \]

\[ \frac{K(x; M)}{F_a(x; M) \times F_b(x; M)} \rightarrow 0 \]

- Scale independent (reliable determination of renormalized LECs)
- Large-\( N_c \) behaviour included:
  - Leading \( (C_i, L_j^k) \)
  - NLO (\( L_i \times 1 \)-loop)
  - All chiral logs at NNLO
Recipe

The amplitude for a given observable can be determined:

- Calculate tree+one-loop diagrams (d,e,f,g)
- In tree-level $O(p^6)$ (g):
  
  \[ C_a^r(\mu) \rightarrow C_a^r(\mu) + \frac{1}{4F_0^2} \left( 4\Gamma_a^{(1)}L - \Gamma_a^{(2)}L^2 + 2\Gamma_a^{(L)}(\mu)L \right) \]

- Collect $L_i$ and $L_iL_j$ and extract chiral logs
- Replace (in f):
  
  \[ L_i^r(\mu)L_j^r(\mu) \rightarrow \left( L_i^r(\mu) - \frac{\Gamma_i}{2}L \right) \left( L_j^r(\mu) - \frac{\Gamma_j}{2}L \right) \]

And (in d,e):

\[ L_i^r(\mu) \rightarrow L_i^r(\mu) - \frac{\Gamma_i}{2}L \]
Application to lattice data: \( \frac{F_K}{F_\pi} \)

Why \( \frac{F_K}{F_\pi} \)?

- At \( \mu=0.77\text{GeV} \), genuine two-loop \( \sim 0.5\% \) [Amoros et al.'00][Bernard, Passemar '10]
- BMW collaboration \( \rightarrow 13 \) points \( (m_\pi<450\text{MeV}) \) [Dürr et al '10]

\[
\left| \frac{F_K}{F_\pi} \right|_{\text{BMW}} = 1.192(7)_{\text{stat}}(6)_{\text{syst}}
\]

\[
\left| \frac{F_K}{F_\pi} \right|_{\text{fit}} = 1.198 \pm 0.005
\]

\[
L^r_5 = (0.76 \pm 0.09) \cdot 10^{-3}
\]

\[
C_{14}^r + C_{15}^r = (0.37 \pm 0.08) \cdot 10^{-3} \text{GeV}^{-2}
\]

\[
C_{15}^r + 2C_{17}^r = (1.29 \pm 0.16) \cdot 10^{-3} \text{GeV}^{-2}
\]

Why fit with \( L_5 \)?

- At \( O(p^4) \) only \( L_5 \)
- At \( O(p^6) \) \( L_5^2 \) is leading in \( 1/N_c \)
- Ranging \( M=M_K(1\pm0.2) \), \( L_5 \) and \( F_K/F_\pi \) don't change, \( C_i \)'s in 2-\( \sigma \)

\[
M_\eta \leq M_K(1.2)
\]

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Conclusions

- From $Z_6$ we propose analytic approx to chiral SU(3) amplitudes
  - User-friendly extrapolation formulas for lattice data
  - Determination of higher-order LECs (specially if lattice simulations use $m_\bar{s}$ lighter than the physical case)
- Approx $\mu$ independent
- Include all chiral logs+LO+NLO terms in $1/N_C$
- Approach useful when genuine two-loop small ($large-N_C$ counting): $F_K/F_{\pi\pi}$
Outlook (work in progress)

- Further example: $K\pi$ vector form factor $f_+(t)$
- Extended approximation (still scale indep) including: