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GYRO-MAGNETIC RELATIONS AND MASSES OF STARS
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Abstract

The calculations in Thomas-Fermi approximation show that in a gravitational field each cell of ultra dense matter inside celestial bodies obtains a very small positive electric charge. A celestial body is electrically neutral as a whole, because the negative electric charge exists on its surface. On the order of magnitude the positive volume charge is very small ($10^{-18}e$ only). But it is sufficient to explain the occurrence of magnetic fields of celestial bodies and the existence of a discrete spectrum of steady-state values of masses of stars and pulsars.

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We cannot measure magnetic fields of the majority of stars, which are distant far from us. Therefore, the existence of magnetic fields for the majority of stars can be considered only hypothetically. However, the magnetic field of the Sun is known over than a hundred years, and in the last decades the astronomers managed to measure magnetic fields for a number of stars (so-called $A_p$-stars) \cite{1} and some pulsars \cite{2}. It is interesting to construct a model describing the generation of magnetic fields by stars and to compare it with the data of the astronomers. The mechanism examined below is based on the gravity-induced electric polarization of matter. It is capable to explain also the generation of magnetic fields by planets \cite{3}, however, in the case of stars, this mechanism works in the purest manner.

The action of gravity on metals has often been a topic of discussion before \cite{4}--\cite{9}. The basic result of these researches is reduced to the statement that gravity induces inside a metal an electric field with an intensity

$$\vec{E} \simeq \frac{m_i \vec{g}}{e},$$  \hspace{1cm} (1)

where $m_i$ is the mass of an ion,
$\vec{g}$ is gravity acceleration,
e is the electron charge.

This field is so small that it is not possible to measure it experimentally. It is a direct consequence of the presence of an ion lattice in a metal. This lattice is deformed by gravity and then the electron gas adapts its density to this deformation. The resulting field becomes very small.

Under superhigh pressure, all substances transform into ultradense matter usually named nuclear-electron plasma \cite{10}. It occurs when external pressure enhances the density of matter several times \cite{10, 11}. Such values of pressure exist inside celestial bodies.

In nuclear-electron plasma the electrons form the degenerated Fermi gas. At the same time, the positively charged ions form inside plasma a dense packing lattice \cite{12, 13}. As usually accepted, this lattice may be replaced
by a lattice of spherical cells of the same volume. The radius \( r_s \) of such a spherical cell in plasma of the mass density \( \gamma \) is given by

\[
\frac{4\pi}{3} r_s^3 = \left( \frac{\gamma}{m_i} \right)^{-1} = \frac{Z}{n},
\]

where \( Z \) is the charge of the nucleus, \( m_i = Am_p \) is the mass of the nucleus, \( A \) is the atomic number of the nucleus, \( m_p \) is the mass of a proton, and \( n \) is the electron number density

\[
n = \frac{3Z}{4\pi r_s^3}.
\]

The equilibrium condition in matter is described by the constancy of its electrochemical potential \([10]\). In plasma, the direct interaction between nuclei is absent, therefore the equilibrium in a nuclear subsystem of plasma (at \( T = 0 \)) looks like

\[
\mu_i = m_i \psi + Ze\varphi = \text{const}.
\]

Here \( \varphi \) is the potential of an electric field and \( \psi \) is the potential of a gravitational field.

The direct action of gravitation on electrons can be neglected. Therefore, the equilibrium condition in the electron gas is

\[
\mu_e = \frac{p_F^2}{2m_e} - (e - \delta q)\varphi = \text{const},
\]

where \( m_e \) is the mass of an electron and \( p_F \) is the Fermi momentum.

By introducing the charge \( \delta q \), we take into account that the charge of the electron cloud inside a cell can differ from \( e \). A small number of electrons can stay on the surface of a plasma body where the electric potential is absent. It results that the charge in a cell, subjected to the action of the electric potential, is effectively decreased on a small value \( \delta q \).

The electric polarization in plasma is a result of changing in density of both nuclear and electron gas subsystems. The electrostatic potential of the arising field is determined by the Gauss’ law

\[
\nabla^2 \varphi = \frac{1}{r^2 \frac{d}{dr}} \left[ r^2 \frac{d}{dr} \varphi \right] = -4\pi \left[ Ze\delta(r) - en \right].
\]
where the position of nuclei is described by the function $\delta(r)$. According to the Thomas-Fermi method, $n$ is approximated by

$$n = \frac{8\pi}{3\hbar^3} p_F^3. \quad (7)$$

With this substitution, Eq.(6) is converted into a nonlinear differential equation for $\varphi$, which for $r > 0$ is given by

$$\frac{1}{r^2} \frac{d}{dr} \left( r^2 \frac{d}{dr} \varphi(r) \right) = 4\pi \left[ \frac{8\pi}{3\hbar^3} \right] [2m_e(\mu_e + (e - \delta q)\varphi)]^{3/2} \cdot (8)$$

It can be simplified by introducing the following variables [10]:

$$\mu_e + (e - \delta q)\varphi = Z e^2 u \quad (9)$$

and $r = ax$,

where

$$a = \left( \frac{9\pi^2}{128Z} \right)^{1/3} a_0$$

with $a_0 = \frac{\hbar^2}{m_e e^2}$ = Bohr radius.

With the account of Eq.(4)

$$Ze^2 u = \text{const} - \frac{m_i \psi}{Z} - \delta q \varphi. \quad (10)$$

Then Eq.(8) gives

$$\frac{d^2 u}{dx^2} = \frac{u^{3/2}}{x^{1/2}}. \quad (11)$$

In terms of $u$ and $x$, the electron density within a cell is given by [10]

$$n_{TF} = \frac{8\pi}{3\hbar^3} p_F^3 = \frac{32Z^2}{9\pi^3 a_0^3} \left( \frac{u}{x} \right)^{3/2} \quad (12)$$

Under the influence of gravity the charge of the electron gas in a cell becomes equal to

$$Q_e = 4\pi e \int_0^{r_s} n(r)r^2 dr = \frac{8\pi e}{3\hbar^3} \left[ 2m_e Z e^2 a \right]^{3/2} \frac{4\pi a^3}{3} \int_0^{x_s} x^2 dx \left[ \frac{u}{x} \right]^{3/2}. \quad (13)$$
\[ Q_e = Ze \int_0^{x_s} x dx \frac{d^2u}{dx^2} = Ze \int_0^{x_s} dx \frac{d}{dx}\left[ \frac{du}{dx} - u \right] = Ze \left[ x_s \frac{du}{dx} \right]_{x_s} - u(x_s) + u(0). \]  

(14)

At \( r \to 0 \) the electric potential is due to the nucleus alone \( \varphi(r) \to \frac{Ze}{r} \). It means that \( u(0) \to 1 \) and each cell of plasma obtains a small charge

\[ \delta q = Ze \left[ x_s \frac{du}{dx}\right]_{x_s} - u(x_s) = Ze x_s^2 \left[ \frac{d}{dx} \left( \frac{u}{x} \right) \right]_{x_s}. \]  

(15)

For a cell placed in a point \( R \) inside a star

\[ \delta q = Z e r_s^2 \left[ \frac{d}{dR} \left( \frac{u}{r} \right) \right] \left[ \frac{dR}{dr_s} \right]. \]  

(16)

Considering that gravity acceleration \( \vec{g} = -\frac{d\varphi}{dR} \) and the electric field intensity \( \vec{E} = -\frac{d\varphi}{dR} \)

\[ \frac{dr_s}{dR} = \frac{r_s^2}{e} \left( \frac{m_i \vec{g}}{Z} + \delta q E \right). \]  

(17)

This equation has the following solution

\[ \frac{dr_s}{dR} = 0 \]  

(18)

and

\[ \frac{m_i}{Z} \vec{g} + \delta q E = 0. \]  

(19)

In plasma, the equilibrium value of the electric field on nuclei according to Eq.(11) is determined by Eq.(11) as well as in a metal. But there is one more additional effect in plasma. Simultaneously with the supporting of nuclei in equilibrium, each cell obtains an extremely small positive electric charge.

As \( div \vec{g} = -4\pi G m_i \) and \( div \vec{E} = 4\pi n \delta q \), the gravity-induced electric charge in a cell

\[ \delta q = \sqrt{G} \frac{m_i}{Z} \simeq 10^{-18} e, \]  

(20)

where \( G \) is the gravity constant.
However, because the sizes of bodies may be very large, the electric field intensity may be very large as well

$$\overrightarrow{E} = \frac{\overrightarrow{g}}{\sqrt{G}}. \quad (21)$$

In accordance with Eqs. (18, 19), the action of gravity on matter is compensated by the electric force and the gradient of pressure is absent.

Thus, a celestial body is electrically neutral as a whole, because the positive volume charge is concentrated inside the charged core and the negative electric charge exists on its surface and so one can infer gravity-induced electric polarization of a body.

3

At the surface of the core, the electric field intensity reduces to zero. The jump in electric field intensity is accompanied at the surface of the core by the pressure jump $\Delta p(R_N)$. It leads to the redistribution of the matter density inside a star. In a celestial body consisting of matter with an atomic structure, density and pressure grow monotonously with depth. In a celestial body consisting of electron-nuclear plasma, the pressure gradient inside the polarized core is absent and the matter density is constant. Pressure affecting the matter inside this body is equal to the pressure jump on the surface of the core

$$p = \Delta p(R_N) = \frac{E(R_N)^2}{8\pi} = \frac{2\pi}{9} G \gamma^2 R_N^2, \quad (22)$$

where $R_N$ is the radius of the core.

One can say that this pressure jump is due to the existence of the polarization jump or, which is the same, the existence of the bound surface charge formed by an electron pushed out from the core and making the total charge of the celestial body equal to zero.

Because the electron subsystem of plasma inside a star is the relativistic Fermi gas, we can write its equation of state

$$p = \frac{(3\pi^2)^{1/3}}{4} \frac{\hbar c \gamma^{4/3}}{m_p^{4/3} \beta^{1/3}}. \quad (23)$$
where $\beta \cdot m_p$ is the mass of the matter related to one electron of the Fermi gas system, and

$m_p$ is the proton mass.

Because of the electroneutrality, one proton should be related to electron of the Fermi gas of plasma. The existence of one neutron per proton is characteristic for a substance consisting of light nuclei. The quantity of neutrons grows approximately to 1.8 per proton for the heavy nuclei substance. Therefore, it is necessary to expect that inside stars

$$2 < \beta < 2.8.$$  \hspace{1cm} (24)

As pressure inside a star is known (Eq.(22)), from Eq.(23) it is possible to determine a steady-state value of mass of a star

$$M_\star = \zeta A_\star^{3/2} \frac{m_p}{\beta^2}.$$  \hspace{1cm} (25)

This mass is expressed by dimensionless constants only

$$A_\star = \left( \frac{\hbar c}{G m_p^2} \right) = 1.54 \cdot 10^{38}$$  \hspace{1cm} (26)

$$\zeta = (1.55 \pi)^{1/2} \simeq 5,$$

and the slowly varying parameter $\beta$ (Eq.(24)).

The masses of stars can be measured with a considerable accuracy, if these stars compose a binary system. There are almost 200 double stars whose masses are known with the required accuracy [15]. Among these stars there are giants, white dwarfs, and stars of the main sequence. Their averaged mass is described by the equality

$$\langle M_\star \rangle = (1.36 \pm 0.05) M_\odot,$$  \hspace{1cm} (27)

where $M_\odot$ is the mass of the Sun.

The center of this distribution (Fig.1) corresponds to Eq.(25) at $\beta \simeq 2.6$.

It is interesting to note that the ”biography” of such a star appears much poorer than in the Chandrasekar model.

Temperature does not influence the parameters of relativistic plasma. Therefore, a star with a mass close to the steady-state value (Eq.(25)) is in a stable equilibrium not depending on temperature. It should not collapse with a temperature decreasing. The instability of a star can arise with burning
Figure 1: Mass distributions of stars and pulsars from the binary systems \[15, 17\]. The curve shows Eq. (25).
out of light nuclei - deuterium and helium - and with a related increasing of $\beta$. This growth leads to the reduction of a steady-state value of mass (Eq. (25)) and, probably, to the distraction of stars with greater masses.

4

As the density of matter inside a relativistic star is constant, it is possible to assume that it equals the mean density of the Sun and to estimate a star radius

$$ R \simeq \left( \frac{M_\star}{\frac{4\pi}{3} \gamma_\odot} \right)^{1/3}, $$

(28)

where $\gamma_\odot$ is the mean density of the Sun.

It allows one to calculate the momentum of a star as the momentum of a sphere with a constant density

$$ I = \frac{2}{5} M_\star R^2, $$

(29)

and at a known frequency of rotation $\Omega$ to calculate its angular momentum

$$ L = \frac{2}{5} M_\star \Omega R^2. $$

(30)

In this model the magnetic moment of a star is created by the rotation of a star as a whole. Thus, it is composed of two parts. One is the magnetic moment of the layer of electrons placed on the external surface of a star

$$ \mu_- = -\frac{1}{3} \left( \frac{4\pi}{3} \rho R^3 \right) \Omega R^2. $$

(31)

The second component of the magnetic moment is created by the positively charged core

$$ \mu_+ = \frac{1}{5} \left( \frac{4\pi}{3} \rho R^3 \right) \Omega R^2. $$

(32)

The summary moment is
Figure 2: The observed values of the magnetic moments of celestial bodies vs. their angular momenta. On the ordinate, the logarithm of the magnetic moment over $Gs\cdot cm^3$ is plotted; on the abscissa the logarithm of the angular momentum over $erg\cdot s$ is shown. The solid line illustrates Eq.(34). The dash-dotted line is the fitting of the observed values.
It is remarkable that the gyromagnetic relation of a star, i.e., the relation of its magnetic moment to the angular momentum, is expressed through world constants only

$$\vartheta = \frac{\mu \Sigma}{L} = \frac{\sqrt{G}}{3c}. \quad (34)$$

The measurements permit us to define the frequency of rotation and magnetic fields for a number of stars [1]. It appears enough to check up the considered theory, since masses of stars and their momenta are determined inside the theory (Eq.(27) and (Eq.(30))). The magnetic moments as functions of their angular momenta for all celestial objects (for which they are known today) are shown in Fig.2. The data for planets are taken from [10], the data for stars are taken from [1], and for pulsars - from [2]. As it can be seen from this figure with the logarithmic accuracy, all celestial bodies - stars, planets, and pulsars - really have the gyromagnetic ratio close to the universal value (Eq.(34)). Only the data for the Moon fall out, because its size is too small to create an electrically polarized core.

5

Apparently, the considered theory is quite true for pulsars which consist, as it is supposed, from the neutron substance with an addition of electrons and protons [10]. As this substance is a relativistic one, there is a fair definition of a steady-state value of mass Eq.(25). The astronomers measured masses of 16 radio-pulsars and 7 x-ray pulsars included in a double system [17]. According to this data, the distribution of masses of pulsars is

$$\langle M_{\text{pulsar}} \rangle = (1.38 \pm 0.03) M_\odot. \quad (35)$$

The center of this distribution corresponds to Eq.(25) at $\beta \simeq 2.6$.

The gyromagnetic relations are measured for three pulsars only [2]. These values are in a quite satisfactory agreement with Eq.(34) (Fig.2). For the majority of pulsars [18], there are estimations of magnetic fields obtained using a number of model assumptions [2]. It is impossible to consider these
Figure 3: The estimated values of the magnetic moments of pulsars [18] vs. their angular momenta. Solid line is Eq.(34). Axes are like in Fig.2.
data as the data of measurements, but nevertheless they also are in some agreement with Eq.(34), (Fig.3).

For planets the situation is more difficult. First, inside planets the substance forms not relativistic electron-nuclear plasma, but nonrelativistic electron-ion plasma. It has different equation of state\cite{10} leading to a more complex expression for the stable mass of a planet core than the expression of Eq.(23) for stars. Second, a noncharged layer at the surface of the core can take a significant part of a planet’s volume and it is impossible to neglect a role of this stratum. However, it can be seen from Fig.2 that the gyromagnetic relations of planets are also in the quite satisfactory agreement with Eq.(34).

The detailed calculation for the Earth\cite{3} gives for the magnetic moment $4 \cdot 10^{25} Oe \cdot cm^3$, which is almost exactly twice smaller than the measured value $8.05 \cdot 10^{25} Oe \cdot cm^3$. Thus, it is possible to assume that the basic component of the magnetic moment of planets is induced by the same mechanism which is working in stars.
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