A new multiuser MIMO system with sum-rate maximization

Shigenori Kinjo\(^{1\dagger}\) and Shuichi Ohno\(^2\)

\(^1\)Japan Coast Guard Academy,
5–1 Wakaba-cho, Kure, Hiroshima 737–8512, Japan
\(^2\)Graduate School of Engineering, Hiroshima University,
1–4–1 Kagamiyama, Higashi-Hiroshima, Hiroshima 739–8527, Japan
a) kinjo@jcga.ac.jp

Abstract: In this study, a simple and high performance precoding scheme for downlink multi-user MIMO (DL MU-MIMO) systems is investigated. Sorted QR decomposition algorithm is adapted for zero-forcing (ZF) Tomlinson-Harashima precoding (THP) with user-stream permutation in order to select a permutation pattern which maximizes a sum-rate of the DL MU-MIMO system. In addition, allocating unequal number of bits to each user-stream improves the DL performance of the ZF-THP in the MU-MIMO scenario. Numerical results show that the proposed ZF-THP scheme outperforms a conventional MMSE-THP scheme.

Keywords: MU-MIMO, ZF-THP, sum-rate, unequal bit allocation

Classification: Wireless Communication Technologies

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1 Introduction

Downlink multi-user multiple-input multiple-output (DL MU-MIMO) technologies realize space division multiple access (SDMA), which will be combined with orthogonal frequency division multiple access (OFDMA) systems to increase the capacity of next generation broadband mobile communication systems [1]. In this study, we investigate a simple and high performance precoding scheme for the DL MU-MIMO systems.

Each subcarrier of an OFDMA system can be represented as a flat fading channel, referred to as a flat MIMO channel, which has been used in many studies on the MU-MIMO system [2, 3, 4, 5]. A zero-forcing (ZF) Tomlison-Harashima precoding (THP) scheme introduced in [3] is a simple precoding scheme for the flat MIMO channels because QR decomposition (QRD) alone is required as the impactful matrix operation. On the other hand, a minimum mean square error (MMSE) THP scheme proposed in [4] has demonstrated superior bit error rate (BER) performance to other precoding schemes. The computational complexity of the MMSE-THP scheme has been reduced by using Cholesky decomposition; however we have to calculate an inverse MIMO channel covariance matrix as additional matrix operation which is undesirable from practical viewpoint. The ZF-THP is hence attractive scheme to avoid the issue.

Two types of the ZF-THP scheme can be seen in [3], one is for single-user (SU) MIMO systems and another is for the MU-MIMO systems. We have investigated the ZF-THP scheme with sub-stream permutation for the SU-MIMO systems in [6]. We have shown that the proposed scheme outperforms the MMSE approach when we use a permutation pattern which maximizes the total bit-rate and allocate unequal number of bits to each sub-stream. In addition, we have proposed a computationally efficient algorithm, modified sorted QRD (MSQRD), which shows almost the same achievable bit-rate with that of the exhaustive search. As for the MU-MIMO system, the ZF-THP with user-stream permutation has been introduced in [5]. They have used the permutation pattern which minimizes the mean square error (MSE); however the BER performance was inferior to that of the MMSE-THP scheme.

In this study, we focus on the ZF-THP scheme in [5]. Main purpose is to confirm whether the strategy in [6] is also applicable to the ZF-THP in the MU-MIMO scenario to improve the downlink performance. We will show, in the numerical examples, that the MSQRD for the MU-MIMO scenario provides approximately the same achievable sum-rate as the exhaustive search, and average BER is superior to the conventional MMSE-THP scheme in high SNRs when appropriate unequal bit allocation patterns are given.

Notation: \( \mathbb{C} \) denotes a set of complex numbers, and \( \mathbb{C}^{N \times M} \) stands for a set of \( N \times M \) complex matrices. \( [\cdot]^T \), \( [\cdot]^H \), and \( \text{det}(\cdot) \) indicate the transpose of a matrix, the Hermitian transpose of a complex matrix, and the determinant of a matrix. \( \text{diag}\{x\} \) represents a diagonal matrix whose diagonal elements are equal to that of a vector \( x \).
2 ZF-THP for DL MU-MIMO scenario

Fig. 1 illustrates the linear representation of a DL MU-MIMO system using THP [5]. $N$ is the number of transmit antennas and the number of receive antennas. An $N \times 1$ transmitted symbol vector is denoted by $s$, whose $i$th entry is the $i$th user’s transmitted symbol. $E$ is a permutation matrix which changes the order of user-streams [4]. $B \in \mathbb{C}^{N \times N}$ and $F \in \mathbb{C}^{N \times N}$ are a lower triangle precoding matrix and a unitary matrix. $a$ is a perturbation vector, which is produced by modulo operation in the THP [3]. In Fig. 1, $d = s + a$ denotes a modified transmitted signal in the linear representation of the THP scheme.

A received signal vector $y \in \mathbb{C}^{N \times 1}$ can be expressed as

$$ y = Hx + n, \quad (1) $$

where $x \in \mathbb{C}^{N \times 1}$ is a transmitted signal vector, and $n \in \mathbb{C}^{N \times 1}$ is a noise vector whose entries are assumed to be independent additive white Gaussian noise with zero mean and variance $\sigma_n^2$. A MIMO channel matrix is denoted as $H \in \mathbb{C}^{N \times N}$. For the MU-MIMO scenario, the QRD is defined by

$$(EH)^H = QR, \quad (2)$$

whereas $HE = QR$ for the SU-MIMO scenario in [6]. $Q \in \mathbb{C}^{N \times N}$ is a unitary matrix and $R \in \mathbb{C}^{N \times N}$ is an upper triangular matrix whose $ij$th element is $r_{ij}$. And, a LQ decomposition of $EH$ is given by

$$ EH = SF^H, \quad (3) $$

where $F = Q$, and $S = R^H$. When we define a vector

$$ r_d = [r_{11} \quad r_{22} \quad \cdots \quad r_{NN}], \quad (4) $$

and a diagonal matrix, $G = diag(r_d)$, a precoding matrix $B$ is given by

$$ B = G^{-1}S. \quad (5) $$

The diagonal elements of $B$ are unity. From Fig. 1, we can rewrite Eq. (1) as

$$ y = HFB^{-1}Ed + n. \quad (6) $$

The MIMO channel matrix can be decomposed as

$$ H = E^T SF^H = E^T GBF^H. \quad (7) $$

Using Eqs. (6) and (7), we obtain

$$ y = E^TGEd + n = G_p d + n, \quad (8) $$

where

$$ G_p = E^TGE = diag\{G^T r_d\}. \quad (9) $$

Finally, an equalized output is given by

$$ \hat{d} = G_p^{-1} y = d + G_p^{-1}n. \quad (10) $$

Fig. 1. Linear representation of ZF-THP
When we express the diagonal matrix \( G_p \) as
\[
G_p = \text{diag}\{g_{p(1)}, g_{p(2)}, \ldots, g_{p(N)}\},
\]
the \( i \)th entry of \( y \) in Eq. (8) is given by
\[
y_i = g_{p(i)}d_i + n_i;
\]
We can select a permutation matrix \( E \) so as to minimize the MSE [5]; however average BER is inferior to that of the MMSE-THP scheme in [4].

3 Proposed ZF-THP scheme

Instead of minimizing the MSE, we maximize a sum-rate in order to select an appropriate permutation matrix. The sum-rate of the ZF-THP scheme in Fig. 1 is given by
\[
R = \sum_{i=1}^{N} R_i = \log \det(\gamma G_p^2 + I),
\]
where \( R_i = \log(\gamma g_{p(i)}^2 + 1) \) denotes the bit-rate of the \( i \)th user in Eq. (12) when \( \gamma \) is an average SNR per user. We maximize Eq. (13) with respect to the permutation matrix \( E \).

As discussed in [6], we have to resort to the exhaustive search, in which we must repeat the QRD by \( N! \) times because we search the maximum value of \( R \) for all possible permutation matrices. To avoid such a large search, we adapt the MSQRD in [6] for the DL MU-MIMO scenario because executing one QRD is sufficient to obtain an appropriate permutation matrix. The MSQRD for the MU-MIMO scenario is shown in Table I.

Table I. Modified sorted QRD for DL MU-MIMO scenario.

| Step | Description |
|------|-------------|
| 1.   | \( R = 0, Q = HH', E = I \) |
| 2.   | for \( l = 1, 2, \ldots, N \) |
| 3.   | \( m_l = \arg \max_{j\in\{l, \ldots, N\}} \|q_j\|^2 \), exchange columns, \( l \) and \( m_l \), in \( Q, R \) and \( E \) |
| 4.   | \( r_l = \|q_l\| \), \( q_l = \frac{q_l}{r_l} \) |
| 5.   | for \( k = l + 1, \ldots, N \) |
| 6.   | \( r_k = q_l^H q_k \), \( q_k = q_k - r_k \cdot q_l \) |
| 7.   | end |
| 8.   | end |
| 9.   | \( S = R^H, G = \text{diag}\{r_d\}, F = Q, B = G^{-1}S \), Output \( E, B, S \) |

The difference between Table I and the MSQRD in [6] can be seen in the step 1 and 9 as we need to modify the input and output matrices because of the difference in the QRD.

The MSQRD searches for the maximum gain, \( r_l^H \), in the \( l \)th iteration and exchanges the order of the user-streams. According to [6], we can anticipate that the MSQRD in the MU-MIMO scenario also show almost the same performance with the exhaustive search. We omit the discussion on the optimality as it is to be the same one shown in [6]. We will confirm it numerically in the next section.
As shown in [6], the product sum $\prod_{i=1}^{N} r_{ii}^2 = \prod_{i=1}^{N} g_{p(i)}^2$ is constant. It is clear that maximizing $g_{p(i)}^2$ results in maximizing the bit-rate $R_i$ of the $i$th user, and then we can increase the sum-rate. However, the channel gains of other users decrease because of the constraint. This can causes $g_{p(j)}^2 \ll 1$ where $i \neq j$ on the $j$th user. In this case, the $j$th user does not give any impact to the sum-rate as $R_j = \log(g_{p(j)}^2 + 1) \approx \log(1) = 0$. We can therefore remove the $j$th user from the transmission. This suggests that we may assign more bits to users with a large channel gain and assign less or zero bits to users with a small one.

This unequal bit allocation strategy seems to give the unfairness in instantaneous bit-rate among users. However, when we consider that our scheme is to be applied to MU MIMO-OFDMA systems, base stations will manage the subcarriers per user so that each user can keep a required bit-rate.

Here we discuss the computational complexity. Although both the MMSE-THP in [4] and the MSQRD have the same order, $O(N^3)$, of complex multiplications, the proposed algorithm does not need additional matrix calculation such as the inverse of a MIMO channel covariance matrix, which we require in the initial setting of the MMSE-THP. The proposed scheme is therefore more computationally efficient than the MMSE-THP scheme.

4 Numerical results

Numerical simulations have been conducted with a flat MIMO channel model, and their results are shown in Fig. 2. The elements of the MIMO channel matrix $H$ are i.i.d. with the complex Gaussian.

![Fig. 2. Performance comparison.](image)

(a) CDFs. (b) Average BERs.

To confirm the optimality of the MSQRD, we evaluate cumulative distribution functions (CDFs) as a function of sum-rate with $N = 4$ and $\gamma = 10$. Permutation matrices are constructed by using two schemes, the exhaustive search and the MSQRD. The results are shown in Fig. 2(a). The figure shows that the MSQRD exhibits approximately optimal capability in the given conditions.

Fig. 2(b) demonstrates BER performance averaged over the receivers in time varying flat MIMO channels with $N = 4$ and 6. Total number of bits per one downlink transmission are fixed to 16 and 24 in each case. QPSK, 16QAM and...
64QAM are applied as modulation schemes. Forward error collection does not applied. Numerical simulations of the average BER in the flat MIMO channel can be seen in many studies [3, 4] to evaluate basic performance of the precoders. We have referred to them in our evaluation.

In the figure, a bit allocation pattern is denoted as $(m_1, m_2, \ldots, m_N)$ where $m_i$ is the number of bits in the $i$th user-stream after the stream permutation. The average BER of the proposed ZF-THP and the MMSE-THP in [4] are compared. As the receiving SNRs of all users in the MMSE-THP scheme are approximately the same [6], the reasonable bit allocation patterns are (4444) and (444444). On the other hand, the proposed ZF-THP scheme outperforms or is equivalent to the MMSE-THP in the high SNR region when we select the bit allocation patterns of (6640) and (666420), which have provided the best average BER performance in the proposed scheme.

5 Conclusions

In this study, a ZF-THP scheme with user-stream permutation for the DL MU MIMO-OFDMA systems has been investigated. Numerical results have shown that the strategy in [6] for the SU-MIMO scenario is also effective in improving the performance of the ZF-THP scheme in the MU-MIMO scenario.