Single stage queueing/manufacturing system model that involves emission variable

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Abstract. Queueing is commonly occurred at every industry. The basic model of queueing theory gives a foundation for modeling a manufacturing system. Nowadays, carbon emission is an important and inevitable issue due to its huge impact to our environment. However, existing model of queuing applied for analysis of single stage manufacturing system has not taken Carbon emissions into consideration. If it is applied to manufacturing context, it may lead to improper decisions. By taking into account of emission variables into queuing models, not only the model become more comprehensive but also it creates awareness on the issue to many parties that involves in the system. This paper discusses the single stage M/M/1 queueing model that involves emission variable. Hopefully it could be a starting point for the next more complex models. It has a main objective for determining how carbon emissions could fit into the basic queueing theory. It turned out that the involvement of emission variables into the model has modified the traditional model of a single stage queue to a calculation model of production lot quantity allowed per period.

1. Introduction
A queue system where there is only one service stage is called a single stage queue system. Entity arrival process occurs randomly. An entity must join the queue line and wait for some time if upon its arrival, all the servers are busy. Once there is one server available, one entity in the queue immediately gets the process or service provided by the server. The service takes a certain time, called the service time (or processing time). After completion of service the entity leaves the system (called the departure process). The departure process is generally also a random process.

A single queue modeling is an interesting and important study in technology and management [1]. It is an important foundation for modeling more complex manufacturing systems [1] [3] [4] [5]. It is able to reflect the behavior of a single step or single work station manufacturing system [3]. The assessment of the single queue system will provide a fundamental, important, and useful knowledge for manufacturing system analysis. Discussions about a single queue system include the following cases: random characteristics of natural variables (arrival and departure process), number of servers (single server or multi-server), number of entity types (single class or multi-class), produce-to-order
or produce-to-stock, queue capacity (finite or infinite), queuing priority, processor failures and many others [1] [11] [7] [3].

Knowledge of a single queue system model has been very well established proven by the availability of many fundamental books about it. Some of them are Cohen [1] [8] [9] The discussion of queuing theory also appears in several chapters in books on introductory probability theory and operations research [10] [11] [12] [13] Several more books discuss queuing theories that lead to application in manufacturing systems [6][7][5]

Of the several types of manufactured systems being modeled, Buzacott and Shanthikumar [7] uncovered models for single stage systems that operate to produce to order and produce to stock. The difference between produce to order and produce to stock lies in the availability of the warehouse after the finished workpiece is processed (finished product stock). In the case of produce to order there is no stock of finished products then it is assumed that the finished product is directly sent or received by the customer. While in produce to stock the product is stored in a warehouse before finally taken by customer. This system is the simplest manufacturing system where there is only one machine or process facility (and possibly a worker operating the machine or facility). Interarrival time and service time are important stochastic quantities. Various assumptions of the probability distribution on two quantities can be taken, for example exponential, phase-type, general or otherwise.

However, the authors argue that the modeling of this single queue system should be further developed to cover the global issues facing the world today, that is emissions. Indeed, emissions have become a critical issue. Carbon emissions arise because of the large use of fossil fuels for vehicles, power plants, and industry. Carbon dioxide, CO2, is the main constituent of greenhouse gases. CO2 levels in the atmosphere in 2016 has reached 406 ppm [16] According to Wang and Choi [17] the growth rate of Carbon content in the atmosphere is about 2.2% per year. At that rate, there will be considerable impacts, such as the occurrence of typhoons in various places, etc. [17] Furthermore, if the rate does not decrease, the greenhouse gases will rise up to 530 ppm by 2030.

Inclusion of emission variables into current system analysis models is important when global warming and climate change and its impacts are increasingly felt. In the lotsizing decision of production Wang and Choi [17] and Hua et al. [18] accommodated the amount of these emissions into EOQ-like formulas. In these papers the amount of emissions is expressed in the form of intensive quantities. Similarly, in this paper the inclusion of emission quantities will be made into the model of the single stage queue associated with a single stage manufacturing system.

Given that the issue of emissions has become critical, the single-queue system modeling should pay attention to this. So, some questions to be answered at the end of in this paper is formulated as follows:

a) How do we put emission variables into a single stage queuing/manufacturing system model?

b) How do inclusion of emission variable could give contribution on the analysis of production lotsize?

2. Problem description

This paper will focus the study on inclusion of emission variables into a single queuing systems model that is limited to: single class entity, infinite queue capacity, produce to order, and reliable server. Let the raw material for workparts is assumed to be available in abundance. The queue process occurs if the incoming order (demand) can not be immediately fulfilled because there are previous orders that have not been processed yet.

When an order arrives, a raw material is taken and marked. The marked raw material becomes the system entity. So the time between the occurrence of the order will be the entity’s arrival to the system. For ease of analysis, in this paper the traditional $M/M/1$ queuing model is used where the inter-entity arrivals and service times are distributed exponentially, each having an average rate of $\lambda$ and $\mu$. The capacity of the waiting space is considered infinite (figure 1a) rather than finite (figure 1b).
In the system, one by one entity will be processed by the server. The average arrival rate, $\lambda$, is smaller than the average service rate, $\mu$, so that the system would reach steady state. In such cases the workpiece will generally experience the waiting process before the time comes to be processed.

It is assumed that the queuing system operates using the electrical energy generated from the burning of fossil fuels. Entities are on a queue line facilitated by a powered material handling system, and processed in a CNC-based machining system. So, here we will pay attention to emissions that occur because the work ought to be waiting, and processed on the machine.

3. The model

3.1. The basic model

The formulation of the $M/M/1$ model is widely available in the literature. The model is Markovian, so we can define the states of the system, $n$, as the number of entities in the system. Then a transition diagram can be made (figure 2).

![States transition diagram for $M/M/1$ model](image)

**Figure 2.** States transition diagram for $M/M/1$ model

If the system has reached steady state, then the following equation applies: arrival rate = departure rate, which can be imposed on each successive two state transitions. If $P_n$ is defined as the probability that the system is at state $n$ ($n = 0, 1, 2, ...$), where $\lim_{n \to \infty} P\{N(t) = n\}$ then we can find the equations:

$$\lambda P_0 = \mu P_1$$

and

$$(\lambda + \mu)P_n = \lambda P_{n-1} + \mu P_{n+1} \text{ untuk } n = 1, 2, ...$$

so we get:

$$P_n = (1 - \rho)\rho^n \text{ untuk } n = 0, 1, 2, ...$$

(1)

where

$$\rho = \frac{\lambda}{\mu}$$

(2)

Equation (1) is very useful for deriving important criterion formulas in the $M/M/1$ queuing model such as residence time in the system, number of average entities in the system, etc. On the $M/M/1$ system, the expected value of residence time in the system, $E[J]$ is (see [11]):

$$E[J] = \frac{1}{\mu - \lambda}$$

(3)

the average number of entities in the system, $L$:

$$L = \frac{\lambda}{\mu - \lambda}$$

(4)
and queuing time is calculated with:

\[ E(T_w) = E(T) - \frac{1}{\mu} \]

Hence,

\[ E_w = \frac{\epsilon_w \lambda}{\mu(\mu - \lambda)} \]

and

\[ E_p = \frac{\epsilon_p}{\mu} \]

If the queuing number is called \( L_q \) then the value will be equal to the average entity in the system minus the average entity in the machine, that is:

\[ L_q = \lambda - \rho = \frac{\lambda}{\mu - \lambda} - \frac{\lambda}{\mu} = \frac{\lambda^2}{\mu(\mu - \lambda)} \]  

Viewed as a single stage manufacturing system, the throughput is, \( Q_0 = \lambda \) (at steady state).

3.2. Model with emission variable

The basic model would be extended to accommodate the emission variable. Defined, \( \epsilon_w \) and \( \epsilon_p \), respectively, are the intensive quantities that show how much emission occurs (in kg Carbon/minute/workpiece unit) because the workpiece has to wait and be processed (in this case \( \epsilon_w \ll \epsilon_p \)).

Thus, the amount of emissions that occur when a workpart has to wait, \( E_w \), is:

\[ E_w = \epsilon_w \times \text{average time of waiting} \]

and to be processed, \( E_p \), is:

\[ E_p = \epsilon_p \times \text{average time of processing} \]

Since the average processing rate is \( \mu \), then each entity unit undergoes processing for an average time of \( 1/\mu \). Meanwhile, to get the queue time we need to calculate the residence time of the entities in the system, \( T_r \).

The system will generate total emissions per period of \( (L_q E_w + E_p) T_h \), where \( L_q \) = the average number of queuing entities (entity units), and \( T_h \) = throughput (in entity units per week). \( T_h \) can be viewed as a production quantity or lotsize \( (Q) \) which can be produced for a certain period (e.g., week).

At a time epoch, there would be \( L_q \) units of entities in waiting, and \( \lambda/\mu \) being in process, averagely. Since the amount of emissions in that period is limited to no more than a certain limit (i.e., emission cap, \( C \)), then:

\[ \left[ L_q E_w + (\lambda/\mu)E_p \right] T_h \leq C \]

Substituting \( L_q \), \( E_w \), and \( E_p \) by equations (5) and (6) we get:

\[ \left( \frac{\lambda^2}{\mu(\mu - \lambda)} \right) \left( \frac{\lambda}{\mu} \frac{\epsilon_w}{\mu} \right) + \left( \frac{\lambda}{\mu} \frac{\epsilon_p}{\mu} \right) Q \leq C \]  

The time unit of \( Q \) has to be the same with the time unit of \( C \). For example, if the unit of \( C \) is kg of Carbon per week, then the unit of \( Q \) should be unit of entity per week. Equation (11) implies that once the value of \( C \) is reached the manufacturing system has to immediately re-establish the emission quota ownership for the next period. In order for \( Q \) to be larger (for example, because its own production capacity allows for that), the only way it can be done is that the system immediately provides a larger \( C \) through cap-and-trade mechanism [19] [20]

3.3. Numerical example

A hypothetical case is taken arbitrarily to give a calculation example. Suppose a single stage manufacturing system is modeled as a single \( M/M/1 \) queuing system. The arrival of the order
follows the Poisson process with an average rate of 10 units/hour. The average rate of process in the machine is 15 units/hour. The amount of unit emissions per entity per minute is 0.01 kg Carbon. Whereas, the machine's processing of emissions per unit of entities per minute is 0.075 kg Carbon. It is known that the emission cap is 250 kg Carbon per week.

We will calculate the amount of production quantity per week allowed. It should be noted that the arrival and departure rates have to be expressed in units/mins by dividing them by 60. By entering the known values into equation (11) then calculating the value of $Q$, we get

$$\left[\frac{\left(\frac{10}{60}\right)^2}{\left(\frac{15}{60}\right)(\frac{15}{60} - \frac{10}{60})} + \frac{\left(\frac{10}{60}\right)\left(\frac{0.075}{60}\right)}{\left(\frac{15}{60}\right)}\right] Q \leq 250$$

which gives quantity of production equal to $Q = 815$ units/week. The 250 kg carbon dioxide emissions per week does not allow the manufacturing system to produce over 815 units per week. Although in that week there was still some residual capacity, for example, the new quota could only be used for production for the following week. To increase the amount of production per week, the manufacturing system must raise the quota per week.

4. Experimental design

Based on the above numerical examples, it is interesting to examine the effect of emissions on the value of production lot, as well as the maximum production lot (regardless of emissions). Defined $Q_E$ and $Q_0$ as the production lot if the emissions are noticed and not. Thus, a simple experimental design has been constructed as shown in figure 3.

![Figure 3. Value of $Q_E$ and $Q_0$ at various value of $\epsilon_w$ and $\epsilon_P$](image)

(\(\lambda = 10 \text{ unit/hour} = 1680 \text{ unit/week} ; \mu = 15 \text{ unit/hour} ; C = 250 \text{ kg of Carbon per week})

From figure 3 it is clear that emission regulation forces a manufacturing system to reduce its production lots. The greater the value of the emission intensity the greater the reduction of the lot of production.

5. Conclusion

Emission issues force a manufacturing system to account the amount of emissions it generates so as not to exceed its own quota. Including emission quantities into a simple performance model, in this case, a single stage $M/M/1$ queue stage is a good start to trigger the inclusion of these magnitudes into more complex system models.

In this paper, the author deliberately takes a simplest example of the system, since this topic has never been expressed in the literature. So far, the discussion of the effect of the emission quantity is done by mathematical optimization model, never with the performance model. The amount of emissions can be infiltrated into the traditional $M/M/1$ queuing/manufacturing system model by using an intensive amount of kg of Carbon emissions/kg of activity/minute entities. The two emission driver activities seen are waiting and processing activities. A particular emission cap
forces a manufacturing system to limit the quantity of production that may be produced during a certain period (weeks). This production quantity might be called the emission-controlled production quantity.

6. Development opportunities
The real manufacturing systems are very complex cases. Thus, research with similar or similar ideas can be applied to more complex systems. For example in the case of flow line, transfer line, flexible manufacturing system, and others.

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