Computer program for single sampling plans by variables with known lot standard deviation

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Abstract. The paper presents a computer program that can be used at design and application of single sampling plans by variables when the standard deviation of the lot is known. The developed program may be applied for quality characteristics with one single or two specification limits. When the quality characteristics have two specification limits, the program determines if two separate plans with one specification limit may be applied for making the decision about the lot acceptance or rejection, and takes into account all the cases that may appear in this situation. The computer program calculates first the plan parameters and then, using the sample measurements, makes the decision about the lot acceptance or rejection. Besides the program presentation, the paper also presents several examples that cover all cases of its utilization.

1. Introduction

One important aspect of statistical quality control is making the decision about the acceptance or the rejection of the lots of products. In the case when the quality characteristics are continuous, this operation may be accomplished by means of the acceptance sampling plans by variables.

In the last years, some authors studied different types of such sampling plans [1, 2, 3].

The present paper studies the single sampling plans by variables with known lot standard deviation. The plan design and application will be realized according to the $k$ method [4, 5]. The application of this method is presented below.

Depending on the specification limits, there are several cases for the plan application.

If the quality characteristic has one single lower limit of specification $L$, by means of a sample of size $n$ that is selected at random from the lot, it is calculated the statistic $z_L$ using the equation (1):

$$z_L = \frac{\bar{x} - L}{\sigma}$$

In equation (1), by $\bar{x}$ it is denoted the mean of the sample. The decision is made using the acceptance constant $k$: if $z_L \geq k$ the lot will be accepted, and if $z_L < k$ the lot will be rejected.

The second situation, for the quality characteristic that has only an upper limit of specification $U$ is analogous. In this case, the statistic $z_U$ is calculated using the equation (2):

$$z_U = \frac{U - \bar{x}}{\sigma}$$

If $z_U \geq k$ the lot will be accepted, and if $z_U < k$ the lot will be rejected.
In the third situation, for the characteristics with two specification limits \( L \) and \( U \), the application of the single sampling plan by variables becomes more complicated.

When the variability is high compared to the tolerance interval, both the upper and the lower proportion of nonconforming units may have high values at the same time. In order to determine if two plans with a single limit may be applied in this case, a modified procedure from that proposed by Duncan in 1974 may be used. This modified procedure is presented in [5], may be applied as follows:

- First, it is calculated the value of \( z_p \) using the equation (3):
  \[
  z_p = \frac{U - L}{2\sigma}
  \]  

- Then, it is calculated the value of \( p^* \), which is the area under the normal distribution curve situated on the right of \( z_p \). This value represents half of the minimum proportion of defective products.

- The decision is made as follows:
  1) If \( 2p^* \leq p_1/2 \) then two plans with a single limit may be applied;
  2) If \( p_1/2 < 2p^* \leq p_1 \) the specifications limits may be too close and nonconforming items may appear on both sides of the tolerance interval if the distribution is centered. In this case there must be determined the values for the lower and upper proportion of defective items, whose sum gives the value \( p_1 \) when the normal curve is displaced between the limits of specification. The maximum of these two values will will replace the value of \( p_1 \) at the calculation of the plan parameters \( n \) and \( k \);
  3) If \( p_1 < 2p^* < p_2 \) there must be reconsidered the plan specifications;
  4) If \( 2p^* \geq p_2 \) the lot must be directly rejected.

The sample size can be calculated using equation (4) [6]:

\[
 n = \left( \frac{z_\alpha + z_\beta}{z_1 - z_2} \right)^2
\]  

In order to calculate the acceptance constant \( k \) (using equation (7)), there must be calculated first the values of \( k_1 \) and \( k_2 \), using equations (5) and (6) respectively [6]:

\[
 k_1 = z_1 - \frac{z_\alpha}{\sqrt{n}}
\]  

\[
 k_2 = z_2 + \frac{z_\beta}{\sqrt{n}}
\]  

\[
 k = \frac{k_1 + k_2}{2}
\]  

In equations (4) - (7) the values of \( z_\alpha, z_\beta, z_1 \) and \( z_2 \) represent the \((1 - \alpha)\%100\), \((1 - \beta)\%100\), \((1 - p_1)\%100\) and \((1 - p_2)\%100\) percentiles of the standardized normal repartition respectively.

By \( \alpha \) and \( \beta \) there are denoted the producer’s risk value and the customer’s risk value.

The acceptable quality level is denoted by \( p_1 \) and the rejectable quality level is denoted by \( p_2 \).

2. Objectives

Depending on the specification limits, there are three cases for the design and application of the single sampling plans by variables in the case when lot standard deviation \( \sigma \) is known.

These are the following:

- The case of the quality characteristics with one single lower specification limit \( L \);
The case of the quality characteristics with one single upper specification limit \( U \);

The case of the quality characteristics with two specification limits.

Furthermore, depending on the lot variability compared to the tolerance interval, the last case has four different kinds of application.

All these variants of plan application require difficult calculations, especially the second case of the plan with two specification limits, what it requires the development of an algorithm for calculating the value that will replace the acceptable quality level at the calculation of the plan parameters.

The objective of the paper is to develop a computer program that may be used for the design and application of the single sampling plans by variables when the lot standard deviation \( \sigma \) is known.

The program performs the selection of the appropriate case of plan application, calculates the values for the sample size and the constant of acceptance, and using the sample measurements that are provided by the user, will make the decision about the lot acceptance or rejection.

3. Methods

The computer program that may be used for the design and application of the single sampling plans by variables with known lot standard deviation was developed in Visual Basic for Applications for Excel 2019.

The functions and procedures respect the syntaxes presented in [7].

When the program is run, a dialog box will be displayed on the screen (figure 1):

![Create Plan dialog box](Image)

**Figure 1.** The Create Plan dialog box.

The first data that the user of program must record into this dialog box are the inspection conditions: the acceptable quality level \( p_1 \), the rejectable quality level \( p_2 \), the producer’s risk \( \alpha \) and the customer’s risk \( \beta \).

Then, the user must record at least one specification limit: the upper specification limit, or the lower specification limit, or the upper and the lower specification limits (in the presented example the measured characteristic has two specification limits).

At the end, the lot standard deviation \( \sigma \) will be recorded.
In order to calculate and display the plan parameters the command button Calculate Plan must be clicked. This action will run the procedure Calculate_Plan_Click(), that will collect first the data from the dialog box Create Plan.

If the quality characteristic has one single specification limit, the procedure will call further another procedure that calculates the plan parameters for this case.

If the measured characteristic has two limits of specification, the procedure Calculate_Plan_Click() calculates first the value of \( z_p \) using the equation (3). By means of \( z_p \), the value of \( 2p^* \) will be computed. Depending on the value of \( 2p^* \), the procedure can call one in four procedures, each one being developed to correspond to a case defined by the modified procedure that is presented in [5].

The sample size \( n \) is calculated using the equation (4) and the acceptance constant \( k \) by means of equation (7). The results will be recorded into the worksheet Plan. The procedure will also create on the worksheet a table with \( n \) rows. This table will be used by the program user for recording the sample measurements. If the user did not record \( n \) measurements exactly, the program will display an error message.

The data from the dialog box in figure 1 correspond to the most difficult case that can appear at the design of this type of sampling plan. This is a plan with two specification limits that corresponds to the case when \( p_1/2 < 2p^* \leq p_1 \). In order to calculate the value of \( p_{calc} \) that will replace the acceptable quality level \( p_1 \) in the equations used for the plan parameters calculation, a function was developed. In figure 2 it is explained how the algorithm of this function was created.

![Figure 2. The algorithm explained.](image-url)

The position of first distribution in figure 2 corresponds to the minimum proportion of nonconforming items: \( 2p^* \). In order to model the displacement of the distribution, the ordinate axis will be moved to the right, step by step, with the value of a decrement that equals to 0.0001. After each ordinate axis shift, the total proportion of nonconforming products \( p_{tot} \) is calculated. If the closest value of \( p_{tot} \), compared to acceptable quality level \( p_1 \), has been reached then \( p_{calc} \) will take the value of the
last $p_U$ (there is no need to continue the displacement because $p_{tot}$ would continue to increase more above $p_1$), otherwise the shift of the ordinate axis will continue. The algorithm for the function that returns the value of $p_1_{calc}$ is presented in figure 3:

![Diagram](image_url)

**Figure 3.** The algorithm for the function that returns the value of $p_1_{calc}$. 
The listing of the developed function is presented in figure 4:

```vbnet
Function p1_calc(z_p As Double) As Double
Dim Finish As Boolean, p_min As Double
Dim p_L As Double, p_U As Double, p_tot As Double
Dim decrement As Double, dif_min As Double, dif As Double
Dim zL_p1_calc As Double, zU_p1_calc As Double

decrement = 0.0001
p_min = 2 * wf.Norm_S_Dist(-z_p, True)
dif_min = Abs(p_min - p1)
zL_p1_calc = -z_p
zU_p1_calc = z_p
Finish = False

Do
    zL_p1_calc = zL_p1_calc - decrement
    zU_p1_calc = zU_p1_calc - decrement
    p_L = wf.Norm_S_Dist(zL_p1_calc, True)
    p_U = 1 - wf.Norm_S_Dist(zU_p1_calc, True)
    p_tot = p_L + p_U
    dif = Abs(p_tot - p1)
    If dif < dif_min Then
        dif_min = dif
        p1_calc = p_U
    Else
        Finish = True
    End If
Loop While Finish = False
End Function
```

Figure 4. The function that returns the value of \( p_1_{\text{calc}} \).

The result of the run of procedure that uses the function \( p_1_{\text{calc}}() \) is presented in figure 5. All information but the sample measurements was recorded on the worksheet by the procedure.

| A | B | C | D | E | F | G | H | I |
|---|---|---|---|---|---|---|---|---|
| 1 | Single sampling plan by variables with known sigma |  |  |  |  |  |  |  |
| 2 | sigma = 0.0022 |  |  |  |  |  |  |  |
| 3 | U = 340.012 |  |  |  |  |  |  |  |
| 4 |  |  |  |  |  |  |  |  |
| 5 |  |  |  |  |  |  |  |  |
| 6 |  |  |  |  |  |  |  |  |
| 7 | Plan parameters: | Please record the measurements: |  |  |  |  |  |  |
| 8 | n = 24 | # | x |  |  |  |  |  |
| 9 | k1 = 2.082218 | 1 | 340.006 |  |  |  |  |  |
| 10 | k2 = 2.073506 | 2 | 340.003 |  |  |  |  |  |
| 11 | k = 2.077862 | 3 | 340.006 |  |  |  |  |  |
| 12 |  | 4 | 340.002 |  |  |  |  |  |
| 31 |  |  | 23 | 340.002 |  |  |  |  |
| 32 |  |  | 24 | 340.003 |  |  |  |  |

Figure 5. The plan parameters and the sample.
In figure 5 there are presented only the first and the last part of the recorded measurements. After the recording of the sample measurements, the user must click the command button labelled *Decision*. This action will run a procedure that makes the decision about the lot acceptance or rejection. In figure 6 there are presented the results of the run of procedure:

![Figure 6. The decision.](image)

| xbar = | 340.0039 |
| ZL =  | 1.790229 |
| ZU =  | 3.704317 |

The lot is rejected

The procedure that makes the decision, depending on the specification limits of the quality characteristic, will call one of the following three procedures: a procedure for a plan with only one single upper limit of specification, a procedure for a plan with one single lower limit of specification, or a procedure for a plan with two limits of specification.

For the presented case the decision was made by the procedure for the plan with two specification limits. This procedure calculated first the sample mean and then applied two plans with one single specification limit (a plan for each specification limit). There were calculated the statistics $z_L$ and $z_U$, and these values were compared with the acceptance constant $k$. In the presented case the lot was rejected.

4. Results and discussions

The developed program was presented for an operating case that corresponds to a measured characteristic with two limits of specification and the lot variability compared to the tolerance interval corresponds to the case when $p_1/2 < 2p^* \leq p_1$.

The case of the measured characteristic with two limits of specification and $2p^* \leq p_1/2$ is analogous to that presented before, with the difference that in this situation there is no need to calculate a new value for $p_1$. In figure 7 there are presented the results of the program run in a situation of this type. The inspection conditions were: $p_1 = 0.01$, $p_2 = 0.025$, $\alpha = 0.05$, $\beta = 0.1$, $U = 235.015$, $L = 235$ and $\sigma = 0.002$.

In all examples presented in this section, only the first measurements of the samples are presented.

![Figure 7. An example for the case when the measured characteristic has two limits and $2p^* \leq p_1/2$.](image)

| A | B | C | D | E | F | G | H | I | J |
|---|---|---|---|---|---|---|---|---|---|
|   |   |   |   |   |   |   |   |   | Decision |
| 1 | Single sampling plan by variables with known sigma |
| 2 | sigma = | 0.002 |
| 3 | L =  | 235 |
| 4 | U =  | 235.015 |
| 6 | Plan parameters: |
| 7 | n =  | 64 |
| 8 | k1 = | 2.120741 |
| 9 | k2 = | 2.120158 |
| 10 | k = | 2.12045 |
| 11 |  |  |  |  |  |  |  |  | xbar = | 235.0072 |
| 12 |  |  |  |  |  |  |  |  | ZL = | 3.579294 |
|   |  |  |  |  |  |  |  |  | ZU = | 3.920706 |

In the case when the measured characteristic has two limits of specification and $p_1 < 2p^* < p_2$, when the command button *Calculate Plan* is clicked, the program will display a message that proposes the reconsideration of the plan specifications.

In figure 8 it is presented a situation of this type. The plan specifications were: $p_1 = 0.0025$, $p_2 = 0.008$, $\alpha = 0.05$, $\beta = 0.1$, $U = 520$, $L = 519.98$ and $\sigma = 0.0035$. 

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If the measured characteristic has two limits of specification and \(2p^* \geq p_2\), when the command button \textit{Calculate Plan} is clicked, the program will display a message that proposes the direct rejection of the lot. In figure 9 it is presented a situation of this type. The specifications of the plan were: \(p_1 = 0.045, p_2 = 0.065, \alpha = 0.05, \beta = 0.1, U = 75.018, L = 75\) and \(\sigma = 0.007\).

Next, the paper presents a case for a measured characteristic with only one single lower limit of specification (figure 10). The case when the measured characteristic has only an upper limit of specification is analogous. The plan specifications for this case are: \(p_1 = 0.035, p_2 = 0.08, \alpha = 0.05, \beta = 0.1, L = 235\) and \(\sigma = 0.015\).
Figure 10. A measured characteristic with a lower limit of specification.

5. Conclusion
The presented computer program may be used for design and application of single sampling plans by variables when the standard deviation of the lot is known. The quality characteristic may have one or two specification limits.

The function $p_{1\_calc}$ that were developed on the basis of the algorithm presented in the paper may be used to determine the new value of $p_1$ that will be used at the computing of the plan parameters in the case when the measured characteristic has two limits of specification and the lot variability compared to the tolerance interval corresponds to the case $p_1/2 < 2p^* \leq p_1$.

The program selects the appropriate case of application for the plan and, using the inspection conditions, calculates the plan parameters. By means of the calculated parameters and the sample measurements the program makes the decision about the lot acceptance or rejection.

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