Optimum Speed Control of DC Motor applying Lag Lead Compensator

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Abstract
Differential equations describing motor’s dynamic performance are considered in the development of transfer function equations and their Laplace transforms obtained for additional investigation. For the sake of reliability, typical data for two DC Motor models are incorporated within Matlab/Simulink program to demonstrate the elementary time domain performance of the system. A lead lag compensator applied as control method to control the speed of DC Motor. The nonlinear programming optimization algorithm “fmincon; computes a constrained minimum of an objective function of a number of variables starting at a preliminary estimate” used in the optimization routine to reach finest locations for the added pole and zero to attain the optimal parameters for the designed lead lag compensator. A standard test step signal used as a desired input speed to study the effectiveness of the proposed controller based on the performance of the system. In order to validate the controller tracking for speed variation a reference signal with unit step speed changes are included in the simulation studies and the obtained results as the Performance of Optimized Lead Lag compensator for speed changes tracking are presented in figures and tables. Matlab and Simulink are used to carry out the simulation runs and the achieved results were scrutinized and discussed then conclusions deduced that virtuous outcomes reached with the optimal controller’s parameters.

Keywords: DC Motor, Speed Control, Optimum Control, Lead Lag Compensator, fmincon

1- Introduction
The DC motors are widely used in the industrialized control applications. They are flexible for wide range speed control and well designed by adjusting the armature voltage and/or the field current [1]. The drives of DC motor are common in industrialized applications, because of its vast worthy features such as extraordinary starting torque, great response performance, swift braking and stress-free to be linear control [2]. Hussam J. Khasawneh, Osama Abdelaal, Mohammad I. Al Saaideh, and Zaer S. Abo-Hammour [3] explored options of designing compensator. Robust lead lag compensators were investigated by Roberto Zanasi and Stefania Cuoghi [4]. In [5] R. Zanasi, S. Cuoghi and L. Ntogramatzidis, used Nyquist and Nichols planes for designing Lead-Lag Compensators analytically. Discrete Compensators based on inversion formula proposed by R. Zanasi, R. Morselli [6]. Akshay C. Mahakalkar, Gaurav R. Powale, Yogita R. Ashtekar, Dinesh L. Mute adopted root locus approach for compensator design [7]. In [8] Galal A. Hassaan, Mohammed A. AL-Gamil & Maha M. Lashin introduced research study for compensator tuning techniques. The scope of this research work encompass the improvement of linear mathematical model for DC motor, proposal of lead lag compensator, applying fine-tuning techniques for finding optimum parameters for the controllers, execute simulation using MATLAB/SIMULINK for lead lag compensator, controllers’ recitals studied.

2- DC motor model
The torque produced by DC motor cultivates an electromagnetic force exerted into rotor windings, which is proportionate to current flowing through the conductor therefore; torque caused by DC motor is proportionate with current flowing via rotor windings. The amplitude of back electromotive force is proportionate to rotor angular speed as depicted in figure (1) [9].

![Block diagram of DC motor system](image)

Figure 1: Block diagram of DC motor system
From figure (1) a transfer function of DC motor relating rotor angular velocity and armature applied voltage can be derived as:

$$T_m = K_t I_a$$  (1)
\[ E_b = K_b \omega \]  
\( \text{Where } K_t \text{ is torque constant, } K_b \text{ is back electromotive force constant} \)

Differential equation of DC motor circuit is:
\[ L \frac{dI_a}{dt} + RI_a + E_b = E_a \]  
\( \text{Equation of torque is:} \)
\[ J \frac{d\omega}{dt} + B\omega + T_w = T \]  

Laplace Transform of equations (1–4) are:
\[ T_m(s) = K_t I_a(s) \]  
\[ E_b(s) = K_b \omega(s) \]  
\[ (Ls + R) I_a(s) = E_a(s) - E_b(s) \]  
\[ (Js + B) \omega(s) = T(s) - T_w(s) \]

Net voltage across DC motor circuit terminal is \((E_a - E_b)\). The torque revolves the load at \(\omega(s)\) overcoming the disruption torque \(T_w\); the overall transfer function relating rotor angular speed to applied armature voltage given in equation (9).
\[ \frac{\omega(s)}{E_a} = \frac{kt}{(Ls+R)(Js+B)+KtKb} \]  

\( \text{As: } R \text{ is winding resistance (Ohm)} \)
\( L \text{ is winding inductance (Henry)} \)
\( I_a \text{ is armature current (A)} \)
\( I_f \text{ is field current (A)} \)
\( E_a \text{ is applied voltage to armature circuit (V)} \)
\( E_b \text{ is back electromotive force (V)} \)
\( \omega \text{ is motor angular velocity (rad/sec)} \)
\( T_m \text{ is motor torque (Newton-m)} \)
\( J \text{ is moment of inertia (kg m²)} \)
\( B \text{ is friction coefficient (N-m/ (rad/sec))} \)
\( T_w \text{ is disturbance torque (N-m)} \)

3- \textbf{Compensator Control Technique}

A compensator is an added component or circuit that is implanted into a control structure to compensate for incomplete performance. Amongst the many types of compensators, commonly employed compensators are [10]:-

\[ 3.1- \textbf{Lead Compensator} \]

Used for improvement of the system transient response characteristics, increase system stability, and rises the system bandwidth, which involves fast rise time, and slight variation in steady state accurateness.

\[ 3.2- \textbf{Lag Compensation} \]

Used for improvement of the system steady-state error, but at the expenses of reducing the speed of the system response.

\[ 3.3- \textbf{Lead-Lag Compensation} \]

Used for desired improvements in transient response and steady state response together, then in cooperation lead compensator and lag compensator can be employed concurrently. Presenting separate units of lead compensator and lag compensator is uneconomical compared to the case of using a single lag–lead compensator. Lag–lead compensation syndicates the advantages of lag and lead compensations as shown in figure(2)[11].

![Figure (2): lead-lag compensator.](image)
- The transfer function of the compensator is depicted by equation(10):

\[ G_c(s) = \frac{K(s+2)}{(s+3)(s+1)} \]  

- Adding a pole into openloop transfer function, tending to inferior system’s relative stability also increase settling time of the system’s response.
- Adding a zero into openloop transfer function, lean towards enhancing system stability furthermore decrease settling time of the system’s response.
- Physically, adding a zero into feed forward transfer function can be achieved by adding to the system derivative control, and adding a pole into the feed forward transfer function can be attained by adding to the system integral control [12].
The constrained optimization function “fmincon within Matlab” used in the optimization routine to attain best locations for the added pole and zero. The role of this function is to obtain optimal values for the specified parameters based on minimization of a cost function of a number of variables starting at a preliminary estimation [13].

4- Simulation Results and Discussion

The obtained results from the developed optimized Lead Lag compensator control method are documented. Matlab and Simulink [14&15] used to build the simulated system for two models of DC motor speed control by controlling the applied armature voltage. Then simulation runs carried out with two types of reference speed signals the first one is the standard unit step 100 rad/sec and the second one is a variable speed with unit step changes within the motor rated speed. The attained results were discussed, scrutinized, and presented in the following figures:

4.1 DC Motor model (1)

The transfer function of the DC motor model (1) described by equation (11) [16]:

$$\frac{w(s)}{Ea(s)} = \frac{0.2}{0.000005s^2 + 0.002501s + 0.04005}$$ (11)

In figure(3) a Simulink model of Optimized Lead Lag Compensator for speed control of a DC Motor (model 1) used in the simulation studies and the obtained results presented. The Performance of Optimized Lead Lag Compensator for unit step input (model 1) is depicted in figure (4) and the control action is based on adding one pole and one zero to the system transfer function to improve the transient response and steady state response. The nonlinear programming optimization algorithm “fmincon; computes a constrained minimum of an objective function of a number of variables starting at a preliminary estimate” used in the optimization routine to reach finest locations for the added pole and zero [17 – 18]. The system response has the following criteria: overshoot = 1.7%, peak time = 0.035, rise time = 0.0197 second, settling time = 0.04 second, steady state error = 0.1%. In order to validate the controller tracking for speed variation a reference signal with unit step speed changes are included in the simulation studies and the obtained results as the Performance of Optimized Lead Lag compensator for speed changes tracking (model 1) are shown in figure (5).
Fig (3): Block diagram of optimized Lead Lag controller for speed control of a DC Motor (model 1).

Fig (4): Performance of Optimized Lead Lag Compensator for unit step input (model 1).
4.2 DC Motor model (2)

The transfer function of the DC motor model (2) depicted in equation (12) [19]:

\[
\frac{w(s)}{V_a(s)} = \frac{0.5}{0.000004s^2 + 0.0502s + 0.629} \quad (12)
\]

In figure (6) a Simulink model of Optimized Lead Lag Compensator for speed control of a DC Motor (model 2) used in the simulation studies and the obtained results presented. The Performance of Optimized Lead Lag Compensator for unit step input (model 2) is presented in figure (7) and the control action is based on adding one pole and one zero to the system transfer function to improve the transient response and steady state response. The nonlinear programming optimization algorithm “fmincon; computes a constrained minimum of an objective function of a number of variables starting at a preliminary estimate” used in the optimization routine to reach finest locations for the added pole and zero [17-18]. The system response has the following criteria: overshoot = 0.6%, peak time = 0.13, rise time = 0.0403 second, settling time = 0.17 second, steady state error = 0.1%.

In order to validate the controller tracking for speed variation a reference signal with unit step speed changes are included in the simulation studies and the obtained results as the Performance of Optimized Lead Lag compensator for speed changes tracking (model 2) are shown in figure (8).
Fig (6): Block diagram of optimized Lead Lag controller for speed control of a DC Motor (model 2).

Fig (7): Performance of Optimized Lead Lag Compensator for unit step input (model 2).
To summarize the performances of the optimized Lead Lag compensator controllers on model (1) and model (2) the parameters of transient and steady state responses are presented in tables (1) and (2) respectively. Due the variation in the parameters of the above-mentioned two models, this led to variation in electrical time constant, mechanical time constant, natural frequency, and damping ratio between the two systems and the impact of these variations is clearly noticeable on their performances as presented in the following two tables:

| Controller               | Overshoot (%) | Peak Time (sec) | Rise Time (sec) | Settling Time (sec) | Steady State Error (%) |
|--------------------------|---------------|-----------------|-----------------|--------------------|------------------------|
| Optimized Lead Lag       | 1.7           | 0.035           | 0.0197          | 0.04               | 0.1                    |
| compensator              |               |                 |                 |                    |                        |

| Controller               | Overshoot (%) | Peak Time (sec) | Rise Time (sec) | Settling Time (sec) | Steady State Error (%) |
|--------------------------|---------------|-----------------|-----------------|--------------------|------------------------|
| Optimized Lead Lag       | 0.6           | 0.13            | 0.0403          | 0.17               | 0.1                    |
| compensator              |               |                 |                 |                    |                        |

5 Conclusion

The Matlab and Simulink simulation results obtained from the developed optimized Lead-Lag Compensator were investigated. The presented method successfully controlled the speed of DC motor through simulation studies based on models of two different DC motor systems using two types of reference speed signals a unit step reference speed and a step change variable speed reference for speed tracking validation purpose. Comparison of the attained results from this simulation studies applying the above mention control technique concluded that model one has faster response compared to model two due to the mechanical dynamics of each model. Finally the controllers in both cases followed the reference speed and tracked the speed changes successfully.
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