Higher order chemical reaction effects on Cu–H₂O nanofluid flow over a vertical plate

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Many fluids used in industries will possess a uniform velocity acting along with it. Although a few researchers have analyzed the fluid flow along with a constant velocity but such modeling in nanofluids is quite new. The novelty of this work is the numerical evaluation of a nanofluid with a constant velocity through a vertical plate in a porous medium under Dufour as well as Soret impacts coupled with a higher order chemical reaction. A rotating MHD nanofluid is investigated for both heat as well as mass transfer. An incompressible, steady-state fluid is subjected to flow through a semi-infinite plate by taking into account viscous dissipation as well as a magnetic field. Flow equations are typically represented by PDEs that are nonlinear and coupled. The PDEs are changed to ODEs by similarity transformation variables. Runge-Kutta method of 4th order accuracy along with shooting technique is employed to solve the converted system of ODEs. Cu−H₂O is used to provide an in-depth analysis of the examined problem. In order to account for practical considerations, the maximum order of the chemical reaction is limited to 3 and a comparative analysis is provided for 1st and 3rd order chemical reactions. For different physical quantities, different numerical values that are obtained using MATLAB are used to analyze various properties regarding the flow. Heat transfer, and mass transfer rates are discussed using graphs and tables. Compared to low order chemical reactions, high order chemical reactions allow higher rates at which the reaction takes place, thus allowing greater rates of heat and mass transfer.

There is a wide range of applications for heat, and mass transfer with higher order chemical reactions through porous media in chemical and water industries. Industrial processes like filtration, distillation, cooling, and drying rely on heat, and mass transfer, in conjunction with chemical reactions. In industries, the rate at which a process occurs is extremely essential; the faster the rate, the lesser time is required in the process, which in turn reduces the amount of time required for storage, resulting in a process that is more efficient and comparatively affordable. The order of a chemical reaction explains the relationship between the concentration of a species and the rate at which the reaction takes place. A reaction is of \( n \)th order if the rate of a reaction is directly proportional to the \( n \)th power of concentration of the species. Chemical reactions occur at a faster rate as the order of the reaction increases. It is necessary to have faster rates of chemical reaction in order to manufacture chemicals such as fertilizers, polymers, and color dyes on a large scale. The effects of higher order chemical reactions in porous media have previously been studied by Rahman et al.1, Mallikarjuna et al.2, Rajani et al.3, Matta et al.4 and Sastry et al.5.

Thermal energy is used extensively in industrial operations including expulsion, papermaking, and cooling computer chips to generate finished products with desired qualities. A significant role for viscous dissipation, and joule heating is to modify temperatures by acting as an energy source. Heat transfer rates change when temperatures do. Several authors Chen6, Alam et al.7, Jaber8, Pandey and Kumar9, Prakash et al.10, Reddy et al.11 have researched this topic extensively.

By passing a fluid through the product, heat can be either transferred away from or transferred towards the product. There must be ways of transferring heat efficiently without wasting significant energy. Some metals, which have high thermal conductivity, can be combined with a liquid, such as water, to transfer heat efficiently. In their research, Choi and Eastman12 used nanometer-sized particles and a base fluid to develop nanofluids, which are colloidal suspensions containing nanometer-sized particles. As a result of its high compatibility, this fluid is capable of inducing or reducing heat transfer. The study of nanofluids to enhance the rates of heat transfer in solar thermal applications has been done by Sheikholeslami13–15.
In their study, Jagadha et al. conducted an investigation on the flow of nanofluid over porous vertical plate taking into account factors like dispersion, radiation, dissipation, chemical reaction, as well as Brownian motion, and some dimensionless number effects. Swain et al. made an analysis of the effect of viscous dissipation, joule heating, magnetic parameter, and suction parameter on the MHD flow of a nanofluid with a higher order chemical reaction. This study was conducted through a stretching surface in a permeable medium.

In an analytical study, Alaidrous and Eid examined how nanofluid moves through a porous stretching sheet by considering the impacts of higher order chemical reaction, radiation, Joule heating, and viscous dissipation. Gopal et al. investigated numerically how an MHD nanofluid flow in a porous stretching sheet is affected by the impacts of higher order chemical reaction as well as viscous dissipation.

Many fluids used in industries will possess a uniform velocity acting along with it. Although a few researchers have analyzed the fluid flow along with a constant velocity but such modeling in nanofluids is quite new. The originality of this work is in the numerical evaluation of a nanofluid travelling at a constant velocity through a vertical plate in porous media while being subjected to Dufour and Soret impacts in conjunction with a higher-order chemical reaction. We analyze the boundary layer equations by incorporating the nanoparticles and the base fluid's thermodynamical properties along with a uniform velocity. Through illustrations of the obtained graphs, we explore how each parameter affects velocity, temperature, and concentration. Using tables, we examine local skin friction coefficient, Nusselt number and Sherwood pertaining to fluid flow parameters.

**Mathematical modeling**

A vertical, semi-infinite plate contained in a porous medium is oriented along the x-axis. Envision a steady state nanofluid flow of a uniform velocity \( U_0 \) in the x-axis and the y-axis is normal to the x-axis as shown in Fig. 1. Nanofluid considered is incompressible, laminar and at steady state rotating along the y-axis with velocity \( \Omega \). A uniform magnetic field \( B_0 \) is taken to be acting along the y-axis which is assumed to be electrically non-conducting. We assume that the magnetic Reynolds number of the flow is taken to be small enough so that the induced magnetic field is negligible in comparison with applied one, so that the magnetic field acts along y-axis. Nanofluid provides an environment for chemical reactions of order \( n \) among species diffusing in it. Species concentrations near the wall and in the free stream significantly affect the flow. Because of this, we take into account the Dufour and Soret effects. Buoyancy is produced by a temperature disparity between the fluid and its surroundings. In this case, we take into account the Dufour and Soret effects. Buoyancy is produced by a temperature disparity between the fluid and its surroundings. In this case, we take into account the Dufour and Soret effects. Buoyancy is produced by a temperature disparity between the fluid and its surroundings. In this case, we take into account the Dufour and Soret effects.

![Flow geometry.](image)

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is base fluid’s thermal expansion coefficient, \( \phi \)

\[
\rho_n (u \frac{\partial w}{\partial x} + v \frac{\partial w}{\partial y}) = \mu_n \frac{\partial^2 w}{\partial y^2} - \frac{\mu_n}{K_s} w + 2\Omega \rho_n (U_0 - u) - \sigma_n B_o^2 w,
\]

(3)

\[
u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \frac{\kappa_n}{(\rho C_p)_n} \frac{\partial^2 T}{\partial y^2} + \frac{D_m K_T}{C_p} \frac{\partial^2 C}{\partial y^2} + \frac{\mu_n}{(\rho C_p)_n} \left[ (\frac{\partial u}{\partial x})^2 + (\frac{\partial w}{\partial y})^2 \right] + \frac{\sigma_n B_o^2}{(\rho C_p)_n} ((U_0 - u)^2 + w^2),
\]

(4)

\[
u \frac{\partial C}{\partial x} + v \frac{\partial C}{\partial y} = \frac{D_m K_T}{T_m} \frac{\partial^2 T}{\partial y^2} - \kappa (C - C_\infty)^n.
\]

(5)

With boundary conditions,

\[
u = 0, v = v_0(x), w = 0, T = T_w, C = C_w \quad \text{at} \quad y = 0
\]

\[
u = U_0, w \rightarrow 0, T \rightarrow T_\infty, C \rightarrow C_\infty \quad \text{as} \quad y \rightarrow \infty.
\]

(6)

Here, \( u, v, w \) are velocities in \( x, y, z \) directions. \( B_0 \) is applied magnetic field, \( C_p \) is specific heat at constant pressure, \( C_i \) is concentration susceptibility, \( C \) is nanofluid’s local concentration, \( \rho_n \) is nanofluid’s concentration on wall, \( C_\infty \) is nanofluid’s concentration in free stream, \( D_m \) is molecular diffusivity, \( g \) denotes acceleration due to gravity, \( K^* \) denotes permeability parameter, \( K_T \) denotes thermal diffusivity ratio, \( \kappa \) denotes chemical reaction parameter, \( n \) is chemical reaction’s order, \( T \) is nanofluid’s local temperature, \( T_w \) is nanofluid’s temperature on the wall, \( T_\infty \) is nanofluid’s temperature in free stream, \( T_m \) is mean fluid temperature.

We now have two velocities \( U_0 \) (a constant velocity acting along with the fluid) and \( u \) (velocity along the x-axis). For the ease of solving the problem, we define a velocity \( u_1, u_1 = U_0 - u \) as per the idea used by Raptis and Pfardikis.

As a consequence of applying the transformation \( u_1 = U_0 - u \), the Eqs. (1 to 5) and the boundary condition (6) are transformed into the following equations.

\[
u \frac{\partial u_1}{\partial x} + \frac{\partial v}{\partial y} = 0
\]

(7)

\[
u_n ((U_0 - u_1) \frac{\partial u_1}{\partial x} + \frac{\partial u_1}{\partial y}) = \frac{\mu_n}{K_s} u_1 - 2\Omega \rho_n w - \sigma_n B_o^2 u_1 - \frac{\rho_n}{(\rho C_p)_n} g(T - T_\infty) - \frac{\rho_n}{(\rho C_p)_n} g(C - C_\infty).
\]

(8)

\[
u_n ((U_0 - u_1) \frac{\partial w}{\partial x} + v \frac{\partial w}{\partial y}) = \frac{\mu_n}{K_s} w + 2\Omega \rho_n u_1 - \sigma_n B_o^2 w,
\]

(9)

\[
u_n ((U_0 - u_1) \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y}) = \frac{\kappa_n}{(\rho C_p)_n} \frac{\partial^2 T}{\partial y^2} + \frac{D_m K_T}{C_p} \frac{\partial^2 C}{\partial y^2} + \frac{\mu_n}{(\rho C_p)_n} \left[ (\frac{\partial u_1}{\partial x})^2 + (\frac{\partial w}{\partial y})^2 \right] + \frac{\sigma_n B_o^2}{(\rho C_p)_n} ((u_1)^2 + w^2),
\]

(10)

\[
u_n ((U_0 - u_1) \frac{\partial C}{\partial x} + v \frac{\partial C}{\partial y}) = \frac{D_m K_T}{T_m} \frac{\partial^2 T}{\partial y^2} - \kappa (C - C_\infty)^n.
\]

(11)

The boundary conditions are

\[
u_1 = U_0, v = v_0(x), w = 0, T = T_w, C = C_w \quad \text{at} \quad y = 0
\]

\[
u_1 \rightarrow 0, w \rightarrow 0, T \rightarrow T_\infty, C \rightarrow C_\infty \quad \text{as} \quad y \rightarrow \infty.
\]

(12)

Thermodynamical properties of nanofluids are

\[
u_n = \frac{\mu_n}{(1 - \phi_n)}, \quad \nu_n = \frac{\mu_n}{(1 - \phi_n)}, \quad \mu_n = (1 - \phi_n) \rho_j + \phi_n \rho_v, \quad (\rho_n) = (1 - \phi_n) \rho_j + \phi_n (\rho_v), \quad (\rho C_p)_n = (1 - \phi_n) (\rho C_p)_j + \phi_n (\rho C_p)_v,
\]

(13)

Here, \( \beta_j \) is base fluid’s thermal expansion coefficient, \( \beta_n \) is nanofluid’s thermal expansion coefficient, \( \rho_j \) is nanoparticle’s thermal expansion coefficient, \( \rho_v \) is base fluid’s density, \( \rho_n \) is nanoparticle’s density, \( \rho_n \) is nanofluid’s density, \( (\rho C_p)_n \) is nanofluid’s heat capacitance, \( \mu_j \) is base fluid’s dynamic viscosity, \( \mu_n \) is nanoparticle’s dynamic viscosity, \( \mu_n \) is nanofluid’s dynamic viscosity, \( \nu_j \) is base fluid’s kinematic viscosity, \( \nu_v \) is kinematic viscosity of nanoparticles, \( \sigma_n \) is nanofluid’s electrical conductivity, \( \nu_n \) is nanofluid’s kinematic viscosity, \( \phi_0 \) is nanofluid’s volume fraction, \( \kappa \) is nanofluid’s conductivity, \( (\rho C_p)_n \) is nanofluid’s specific heat.

The Eqs. (7)–(11) serve as the basic governing equations here. Utilizing transformations (14), the governing Eqs. (7)–(11), as well as the boundary conditions (12) are solved.
\[ \eta = y \sqrt{\frac{U_0}{2\nu x}} g(\eta) = \frac{w}{U_0}, \quad \theta = \frac{T - T_\infty}{T_w - T_\infty}, \quad \phi = \frac{C - C_\infty}{C_w - C_\infty}, \quad \psi = \sqrt{2\nu x U_0 f(\eta)}, \]

\[ u_1 = \frac{\partial \eta}{\partial y} = U_0 f'(\eta) \Rightarrow u = \frac{u_0}{U_0} = 1 - f'(\eta), f_w = v_0(x) \sqrt{\frac{2x}{U_0 v}}. \]

By substituting (13) and (14) in Eqs. (7)–(11),

\[ f'' = \frac{1}{a_1} [-f''(\eta - f) + \frac{a_1 f'}{K} + Rg + \frac{Mf'}{a_2} + \frac{a_3}{a_2} Ri(\theta + N\phi)], \]

\[ g'' = \frac{1}{a_1} [-g'(\eta - f) + \frac{a_1 g'}{K} - Rf' + a_4 Mg], \]

\[ \theta'' = \frac{1}{a_5 - a_6 PrScSoDu} [a_6 (\eta - f) (DuScPrf' - \theta') - MEcPr(f')^2 + g^2] - a_6 a_7 PrEcSo[\theta''(f')^2 + g^2] + a_6 PrDuScy\phi''], \]

\[ \phi'' = \frac{1}{a_5 - a_6 PrScSoDu} [Sc(\eta - f) (a_6 SoPrf' - a_5 \phi') - MEcPrScSo[(f')^2 + g^2] - a_6 a_7 PrEcSo[\phi''(f')^2 + g^2] + a_2 Scy\phi'']. \]

The associated boundary conditions are

\[ f = f_w, f' = 1, g = 0, \theta = 1, \phi = 1 \text{ at } \eta = 0 \]

\[ f' \rightarrow 0, g \rightarrow 0, \theta' \rightarrow 0, \phi \rightarrow 0 \text{ as } \eta \rightarrow \infty, \]

where primes refer to derivatives about \( \eta \); Here

\[ M = \frac{2a_2 B_r^\infty}{U_0 p_f}, K = \frac{2\nu x}{K^\infty U_0}, Ri = \frac{(\nu x)^2 y f y (T_w - T_\infty)}{x(U_0)^2}, N = \frac{T_w - T_\infty}{C_w - C_\infty}, Re_x = \frac{x U_0}{v}, \]

\[ R = \frac{4x \Omega}{U_0}, Pr = \frac{(\mu C_f)}{K_f}, Du = \frac{D_m K_f}{C_f C_p y f}, \]

\[ Sc = \frac{\nu y}{D_m}, So = \frac{D_m K_f}{\nu y T_m C_w - C_\infty}, Ec = \frac{U_0^2}{C_p(T_w - T_\infty)} a_1 = \frac{1}{1 - (1 - \phi_0) 2.5 (1 - \phi_0) + \phi_0 (\frac{2\nu}{\nu})}, \]

\[ a_2 = (1 - \phi_0) + \phi_0 (\frac{P_s}{\rho_f}), a_3 = (1 - \phi_0) + \phi_0 (\frac{(\rho \beta) x}{(\rho \beta) f}), \]

\[ a_4 = \frac{1}{1 - \phi_0} + \phi_0 (\frac{2\nu}{\nu}), a_5 = \frac{1}{(1 - \phi_0) + \phi_0 (\frac{2\nu}{\nu})}, a_6 = \frac{1}{1 - \phi_0 (\rho C_p) y + \phi_0 (\rho C_p) y}. \]

In the above quantities, \( T_w \) denotes temperature on wall and \( C_w \) is concentration on wall; \( \theta \) and \( \phi \) are dimensionless temperature and dimensionless concentration; \( Ec \) is Eckert number; \( M \) symbolizes dimensionless magnetic field parameter; \( K \) is dimensionless permeability parameter; \( y \) symbolizes dimensionless chemical reaction parameter; \( Re_x \) is Reynolds number; \( R \) is rotation parameter; \( Ri \) symbolizes Richardson number; \( Pr \) symbolizes Prandtl number; \( Sc \) symbolizes Schmidt number; \( Du \) and \( So \) symbolize Dufour and Soret effects. For practical applications, local skin friction coefficient, Nusselt number, Sherwood number are relevant physical quantities, that are defined as:

Local Skin friction coefficient, \( C_f = \frac{\tau_w}{\rho w q_w} \Rightarrow \sqrt{2} C_f (1 - \phi_0)^{2.5} = f''(0), \)

Local Nusselt number, \( Nu_x = \frac{x u_x}{K_f (T_w - T_\infty)} \Rightarrow \sqrt{2} \frac{Nu_x K_f}{Re_x} \Rightarrow = - \theta'(0). \)

Local Sherwood number, \( Sh_x = \frac{q_m}{D_m (C_w - C_\infty)} \Rightarrow \sqrt{2} \frac{Sh_x}{Re_x} = - \phi'(0). \)

where \( \tau_w = \mu_n^2 \frac{2u}{\nu} y = 0, \) is the wall shear stress, \( q_w = -K_n^2 \frac{2T}{\nu} y = 0, \) is the wall heat flux, \( q_m = -D_n \frac{2C}{\nu} y = 0, \) is the wall mass flux.
Table 1. Thermodynamical properties of nanofluids (Oztop et al.24).

| Thermodynamical properties | H2O | Cu  |
|---------------------------|-----|-----|
| C_p (J/(kg K))            | 4179 | 385 |
| μ (kg/m²s)                | 997.1 | 8933 |
| α (W/m K)                 | 0.613 | 400 |
| β × 10⁻⁶ (K⁻¹)            | 21   | 1.67 |

Method of solution

In order to solve Eqs. (16)–(20) using the bvp4c package in MATLAB, the construction of functions that give solutions to differential equations with boundary conditions is necessary. Consider, \( f = f(1); f' = f(2); f'' = f(3); g = f(4); g' = f(5); \theta = f(6); \theta' = f(7); \phi = f(8); \phi' = f(9); \)

Eqs. (16)–(19) are transformed into the following first order differential equations.

\[
\begin{align*}
\frac{f'}{f} &= \frac{f(2)}{f(3)} \\
\frac{f''}{f'} &= \frac{f(2)}{f(3)} - \frac{f(f(4))}{f(5)} + \frac{(M)^2(f(6) + Nf(8))}{f(7)} \\
\frac{g'}{g} &= \frac{1}{a_2} \left[ a_6 M f(6) - M f(5) + a_1 M f(4) - \gamma f(9) \right] \\
\frac{\phi'}{\phi} &= \frac{1}{a_2} \left[ a_6 M f(6) - M f(5) + a_1 M f(4) - \gamma f(9) \right] 
\end{align*}
\]

When this problem is viewed as an initial value problem, a numerical solution can be found. MATLAB's bvp4c package has been used to solve these equations.

Results and discussion

Specifically, the intention of this work is to analyze how a rotating nanofluid with a constant velocity interacts with a higher order chemical reaction through a plate in conjunction with a porous medium, viscous dissipation and a magnetic field. Keeping in mind, the real-life applications of the order of chemical reaction, the value of \( n \) is restricted to 3, and a comparison is provided in the following figures for \( n = 1 \) and \( n = 3 \). Additionally, the influence of \( M, Ri, Du, So, Ec, \gamma \) on the velocity of the flow, temperature, and concentration of the nanofluid-Cu—H2O are presented by graphs in the Figs. 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12 and 13. The nanofluid is formed by suspending copper nanoparticles in water with a volume fraction of 0.15. This problem is solved while considering the thermodynamical properties of both Cu and H2O. The thermodynamical properties of Cu and H2O are given in Table 1.

In this paper, for the Figs. 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12 and 13, we have used \( R_i = 4; R = 2; K = 2; Pr = 6.7855; Du = 3; Sc = 0.6; \gamma = 2; f = 0.5; Ec = 1; So = 1; N = 1; \)

Variable parameters are mentioned against the respective graphs.

Figure 2 depicts the velocity for different \( Ri \) and \( n \). For increasing \( Ri \), there is an increase in buoyancy due to which the fluid velocity increases. An increase in \( n \) increases the velocity as well. Figure 3 represents the velocity profile for increasing \( n \) and \( K \). A lower value of \( n \) results in a lower velocity profile. The velocity increases with an increase in \( K \) for a given \( n \) value. Since the ability of the fluid to penetrate into the medium increases with increasing permeability parameter, fluid velocity increases.

The velocity profile in Fig. 4 is illustrated for a variety of \( M \) and \( n \) values. It is evident that velocity decreases as \( n \) increases. Velocity falls when \( M \) increases for a specific value of \( n \). The Lorentz force begins to take control when \( M \) increases, thus lowering the velocity. Figure 5 represents concentration profile for different \( M \) and \( n \). A higher \( n \) value results in higher concentration. The concentration increases with an increase in \( M \) for a given \( n \).

A velocity profile can be observed in Fig. 6 for various values of \( Du \) and \( n \). The increase in velocity is proportionate to \( Du \). The velocity is not affected by an increase in \( Du \) when \( n \) is low. As \( n \) rises, the flow velocity increases at higher \( Du \). At high \( Du \) and high \( n \), the flow velocity is maximum. It is illustrated in Fig. 7 how the temperature changes as \( Du \) and \( n \) increase. Increasing \( n \) seems to increase the temperature of the system when \( Du \) is higher. If \( Du \) is lower, increasing \( n \) does not increase the velocity of the system. An enhancement in \( Du \) increases the molecular collisions from hotter to colder regions, which enhances the temperature.

Figures 8 and 9 reveal velocity and concentration profile for different \( So, n \). For a particular value of \( n \), an increase in \( So \) increases velocity as well as concentration. This is because of the influence of thermal gradients on the diffusing species. Increasing \( n \) rises the velocity profile and the concentration profile.

Kinesis and heat are measured by Eckert number (viscous dissipation parameter). In the presence of viscous fluid stresses, kinetic energy is changed into internal energy. Higher \( Ec \) indicates high kinetic energy resulting in increased fluid vibrations and larger collisions between molecules. The boundary layer region becomes hotter as the amount of molecule collisions increases, dissipating more heat. A more rapid flow and higher temperature are therefore associated with a rise in viscous dissipation parameter. With an increase in \( n \), collisions will increase, resulting in higher velocity and temperature. Figures 10 and 11 illustrate these trends.

Figures 12 and 13 depict velocity and concentration profiles for increasing values of \( \gamma \) and \( n \). An increase in \( \gamma \) translates into a rise in the number of molecules of solute going through chemical reactions, leading to a
Figure 2. Velocity profile for various $K$.

Figure 3. Velocity profile for various $Ri$.

Figure 4. Velocity profile for various $M$. 
Figure 5. Concentration profile for various $M$.

Figure 6. Velocity profile for various $Du$.

Figure 7. Temperature profile for various $Du$. 
Figure 8. Velocity profile for various So.

Figure 9. Concentration profile for various So.

Figure 10. Velocity profile for various Ec.
Figure 11. Temperature profile for various Ec.

Figure 12. Velocity profile for various γ.

Figure 13. Concentration profile for various γ.
reduction in velocity and concentration. Increasing \( n \) rises the velocity profile and the concentration profile despite the opposing effects of the chemical reaction parameter. Table 2 provides a limiting case comparison of our results with the results of Alam et al.\(^7\) with \( \text{Ri} = 4; N = 0.5; R = 0.2; K = 0.5; M = 0.5; \text{Pr} = 0.71; \text{Du} = 0.2; \text{Sc} = 0.6; \gamma = 0; \text{Ec} = 0.01; \text{So} = 0.2; \phi_0 = 0. \)

### Table 2. Comparison of present results with Alam et al.\(^7\) with \( \text{Ri} = 4; N = 0.5; R = 0.2; K = 0.5; M = 0.5; \text{Pr} = 0.71; \text{Du} = 0.2; \text{Sc} = 0.6; \gamma = 0; \text{Ec} = 0.01; \text{So} = 0.2; \phi_0 = 0. \)

| \( \text{Ec} \) | Alam et al.\(^7\) | Present results |
|---|---|---|
| \( f''(0) \) | \( f''(0) \) |
| 0.01 | – 3.691740 | – 3.686071 |
| 0.05 | – 3.721830 | – 3.719738 |
| 0.10 | – 3.761275 | – 3.763524 |
| 0.15 | – 3.802958 | – 3.809364 |
| 0.20 | – 3.847133 | – 3.857470 |
| 0.25 | – 3.894094 | – 3.908065 |

### Table 3. Variation of local skin friction coefficient, Nusselt number, Sherwood number for various values of \( \text{Du} \) for \( n = 1, 2, 3. \)

| \( \text{Du} \) | \( n \) | \( f''(0) \) | \( -\theta'(0) \) | \( -\phi(0) \) |
|---|---|---|---|---|
| \( 1 \) | 1 | – 2.308111 | 0.216378 | 0.841627 |
| 2 | 2 | – 2.355140 | 0.220057 | 0.924967 |
| 3 | 3 | – 2.378237 | 0.225729 | 0.979490 |

### Table 4. Variation of local skin friction coefficient, Nusselt number, Sherwood number for various values of \( \text{So} \) for \( n = 1, 2, 3. \)

| \( \text{So} \) | \( n \) | \( f''(0) \) | \( -\theta'(0) \) | \( -\phi(0) \) |
|---|---|---|---|---|
| \( 1 \) | 1 | – 2.380070 | 0.217950 | 0.827176 |
| 2 | 2 | – 2.356993 | 0.221777 | 0.909210 |
| 3 | 3 | – 2.305027 | 0.229576 | 1.063647 |

As seen, as \( \text{Du} \) increases, it increases \( -\phi'(0) \) and as \( \text{So} \) increases \( -\theta'(0) \) also increase because of Thermo-diffusion and Diffusion-thermo effects created in the fluid.
Conclusion
A comprehensive investigation is performed with a focus on the influences of higher order chemical reaction, magnetic parameter, permeability parameter, viscous dissipation, Dufour and Soret effects on Cu–H₂O nanofluid through a vertical plate. The Cu–H₂O nanofluid is considered to flow through a constant velocity acting along with it. This investigation leads us to the following conclusions.

- Heat and mass transfer are more rapid in higher order chemical reaction than in low order reaction.
- Increasing the values of magnetic parameter $M$, chemical reaction parameter $\gamma$ opposes the nanofluid flow while an increase in Dufour number $Du$, permeability parameter $K$, Eckert number $Ec$ aids to the flow of the nanofluid.
- The temperature of the nanofluid rises as the heat in boundary layer rises due to a rise in Dufour number $Du$ and Eckert number $Ec$.
- The concentration of nanofluid boosts up with an increase in Soret number $So$ while an opposite trend is observed for increasing values of chemical reaction parameter $\gamma$.
- For a particular order of a chemical reaction, the rate of heat transfer increases for increasing Soret number $So$ while an enhancement in Dufour number $Du$ enhances the mass transfer rate.

Moreover, the results of this investigation can be applied to industries manufacturing chemicals such as fertilizers, polymers, and color dyes on a large scale where the faster rates of chemical reactions are significant.

Data availability
The datasets used and/or analysed during the current study available from the corresponding author on reasonable request.

Received: 1 July 2022; Accepted: 9 September 2022
Published online: 11 October 2022

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**Author contributions**
Conception and design of the study done by Dr. R.K.B. and P.K. wrote the main manuscript text and prepared figures and Tables. Both the authors reviewed and finalized the manuscript.

**Competing interests**
The authors declare no competing interests.

**Additional information**
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