Type Ia supernovae data with scalar-tensor gravity

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We study the use of type Ia supernovae (SNe Ia) in the context of scalar-tensor theories of gravity, taking as a working example induced gravity, equivalent to Jordan-Brans-Dicke theory. Winking at accurate and precision cosmology, we test the correction introduced by a time variation of the Newton’s constant, predicted by scalar-tensor theories, on the SNe distance modulus relation. We find that for induced gravity the coupling parameter is constrained from \( \xi < 0.0095 \) (95\% CL) using Pantheon SNe data alone down to \( \xi < 0.00063 \) (95\% CL) in combination with \textit{Planck} data release DR3 and a compilation of baryon acoustic oscillations (BAO) measurements from BOSS DR12. In this minimal case the improvements in terms of constraints on the cosmological parameters coming from the addition of SNe data to cosmic microwave background (CMB) and BAO measurements is limited, \( \sim 7\% \) on the 95\% CL upper bound on \( \xi \). Allowing for the value of the gravitational constant today to depart from the Newton constant, we find that the addition of SNe further tightens the constraints obtained by CMB and BAO data on the standard cosmological parameters and by 22\% on the coupling parameter, i.e., \( \xi < 0.00064 \) at 95\% CL. We finally show that in this class of modified gravity models the use a prior on the absolute magnitude \( M_B \) in combination with the Pantheon SNe sample leads to results which are very consistent with those obtained by imposing a prior on \( H_0 \), as happens for other early-type models which accommodate a larger value of \( H_0 \) compared to the ΛCDM results.

\begin{equation}
H(1) - H(2) \Big/ \sqrt{\sigma(H(1)) + \sigma(H(2))}.
\end{equation}

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1 Quantified as \( |H(1) - H(2)| \Big/ \sqrt{\sigma(H(1)) + \sigma(H(2))} \).
we must carefully consider modified gravity’s impact on SNe Ia astrophysics and its implication in terms of cosmological parameter inference.

In this paper, we assess the impact of adding SNe data to cosmological analyses alone and in combination with \textit{Planck} DR3 and BAO from Sloan Digital Sky Survey (SDSS) data to constrain as a working example one of the simplest scalar-tensor gravity model such as induced gravity (IG), equivalent to Jordan-Brans-Dicke [45, 46], described by the action

\[ S = \int d^4x \sqrt{-g} \left[ \frac{\xi}{2} R - \frac{\phi_{\mu\nu}}{2} \partial_{\mu} \phi \partial_{\nu} \phi - V(\phi) + L_m \right] \]

where \( \xi > 0 \) is the coupling parameters, \( \sigma \) is a scalar field, \( R \) is the Ricci scalar, and \( L_m \) is the Lagrangian density for matter fields minimally coupled to the metric. We restrict ourselves to a potential of the type \( V(\phi) \propto \phi^4 \) [39, 47, 48] in which the scalar field is effectively massless and the effective gravitational constant \( G_{\text{eff}} \) between two test masses is [49]

\[ G_{\text{eff}}(z = 0) = \frac{1}{8\pi \xi \sigma_0^2} \frac{1 + 8\xi}{1 + 6\xi(1 + \Delta)^2}. \]

Following Ref. [26] (see also Ref. [50]), we introduce an imbalance \( \Delta \) between the gravitational constant today \( G_{\text{eff}}(z = 0) \) and the Newton constant \( G \). Note that, scalar-tensor theories of gravity that involve a scalar field nonminimally coupled to the Ricci scalar naturally lead to a higher CMB-inferred value of \( H_0 \) [21–23, 26, 50–54] and can also help in interpreting the current tensions in the estimates of cosmological parameters from different observations. With the working example adopted here we can therefore also test the difference between a prior on the absolute magnitude and on \( H_0 \).[2]

Our paper is organized as follows. After this introduction, we describe the distance modulus relation used to derive constraints from SNe Ia data including the correction due to the evolution of the Newton’s constant and we quantify the impact on current cosmological data in Sec. II. In Sec. III, we describe the datasets and prior considered and we present our results in light of SNe Ia data alone and in combination with CMB and BAO data. We study the impact of using a prior on the absolute magnitude, based on the SH0ES calibration, together to the SNe Ia sample in Sec. IV. In Sec. V we draw our conclusions. In the Appendix, we assess the importance of including the correct redshift dependence of the Chandrasekhar mass in the SNe likelihood for the models considered.

\[ \mu_{\text{th}} = m_B - M_B = 5 \log_{10} d_L(z) + 25 \] [mag] (3)

\[ d_L(z) = c(1 + z_{\text{hel}}) \int_0^{z_{\text{cmb}}} \frac{dz'}{H(z')} \] [Mpc] (5)

where \( z_{\text{hel}} \) is the heliocentric redshift and \( z_{\text{cmb}} \) is the CMB redshift corrected by peculiar velocities. Finally, the observed distance modulus \( \mu_{\text{obs}} \) is defined as

\[ \mu_{\text{obs}} = m_B - M_B + \alpha x_1 - \beta c + \Delta M + \Delta B \] [mag] (6)

where \( \alpha \) is the coefficient of the relation between luminosity and stretch, \( x_1 \) is the stretch parameter, \( \beta \) is the coefficient of the relation between luminosity and the color, \( c \) is the color, \( \Delta M \) is a distance correction based on the host-galaxy mass of the SN, and \( \Delta B \) is a bias correction based on simulations. The difficulty in the determination of the cosmological parameters lies in the identification of \( M_B \), as detailed in [59].

The evolution of Newton constant predicted in the context of modified gravity theories induces special effects to the physics of SNe Ia; see Refs. [7, 34–39, 41–44]. The observed magnitude redshift relation of SNe Ia can be translated to luminosity distance-redshift relation which leads to the expansion history \( H(z) \) only under the assumption that SNe Ia behave as standard candles, in particular the constancy in time of the Chandrasekhar mass. The peak luminosity of SNe Ia is proportional to the mass of nickel synthesized which is a fixed fraction of the Chandrasekhar mass. \( M_{\text{Ch}} \sim G^{-3/2} \); see Refs. [35, 41]. Therefore the SN Ia peak luminosity varies like \( L \sim G^{-3/2} \) [60] and the corresponding distance modulus (3) in presence of a varying effective gravitational constant becomes

\[ \mu_{\text{th}}(z) = 5 \log_{10} d_L(z) + 25 + \frac{15}{4} \log_{10} \frac{G_{\text{eff}}(z)}{G} \] [mag]. (7)

II. THE DISTANCE MODULUS RELATION IN SCALAR-TENSOR THEORIES

The peculiarity of SNe Ia is their nearly uniform intrinsic luminosity with an absolute magnitude around \( M \sim -19.5 \) [56, 57], and this allows us to promote SNe Ia to a well-established class of standard candles. To evaluate the underlying best cosmological model, we make use of the distance modulus \( \mu \), derived from the observations of SNe Ia, and we compare it with the theoretical one \( \mu_{\text{th}} \), defined as follows

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\[ \mu_{\text{th}}(z) = 5 \log_{10} d_L(z) + 25 + \frac{15}{4} \log_{10} \frac{G_{\text{eff}}(z)}{G} \] [mag]. (7)

Since the absolute magnitude \( M_B \) is marginalized in Eq. (6) as being a nuisance parameter, its possible redshift dependence may carry useful information about the robustness of the determination of \( H_0 \) using SNe Ia data and about possible modifications of \( G_{\text{eff}}(z) \).

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[2] Note also that a parametric sudden transition of the effective gravitational constant at late times [44, 55] could also reduce the tension in the Hubble constant (see Refs. [16, 28] for a review and comparison of models able to alleviate the \( H_0 \) tension).
III. CONSTRAINTS FROM COSMOLOGICAL OBSERVATIONS IN COMBINATION WITH TYPE IA SUPERNOVAE

In this section, we present the constraints on cosmological parameters including Pantheon SNe to the results presented in Ref. [26] obtained from the combination of Planck 2018 DR3 (hereafter P18) with BAO measurements from BOSS DR12. In addition, we show the constraints on the modify gravity parameters from the compilation of SNe alone.

We consider the full CMB information from Planck DR3 [61, 62] including the low-ℓ likelihood Commander (temperature-only) plus the SimAll (EE-only), the high-multipole likelihood Plik, and the CMB lensing likelihood on the conservative multipole range, i.e., $8 \leq ℓ \leq 400$. The compilation of BAO data includes data from Baryon Spectroscopic Survey (BOSS) DR12 [63] consensus results in three redshift slices with effective redshifts $z_{\text{eff}} = 0.38, 0.51, 0.61$, the measure from 6dF [64] at $z_{\text{eff}} = 0.106$, and the one from SDSS DR7 [65] at $z_{\text{eff}} = 0.15$. We consider the Pantheon sample which is a compilation of 1048 spectroscopically confirmed SNe Ia that gathers different surveys spanning the redshift range $0.01 < z < 2.3$ [59].

We use MontePython4 [66, 67] connected to our modified version of the code CLASS5 [68, 69], i.e., CLASSig [21]. For the Markov chain Monte Carlo (MCMC) analysis including CMB data (P18, P18 + SNe, P18 + BAO, P18 + BAO + SNe), we vary the six cosmological parameters for a flat $\Lambda$CDM concordance model, i.e., $\omega_b, \omega_c, H_0, \tau, \ln (10^{10} A_s), n_s$; for the analysis of SNe compilation alone we vary $\Omega_{\text{cdm}}$ fixing $\Omega_{\text{m}} = 0.0407$. The extra parameters related to the coupling to the Ricci curvature are $\zeta_{\text{IG}} \equiv \ln(1 + 4\xi)$, sampled in the prior range $[0, 0.039]$ and $\Delta \in [-0.3, 0.3]$. We assume two massless neutrino with $N_{\text{eff}} = 2.0328$, and a massive one with fixed minimum mass $m_\nu = 0.06$ eV. We assume adiabatic initial condition for the scalar fluctuations [70]. We set the primordial helium abundance according to the prediction from ParthEnoPE [71, 72] taking into account the effect of a different gravitational constant as a source of extra radiation in $Y_{\text{BBN}}(\omega_b, N_{\text{eff}})$ [26, 51]. We vary also nuisance and foreground parameters for the Planck and Pantheon likelihoods.

In Fig. 1, we compare the marginalized constraints on cosmological parameters for P18 and P18 + BAO including Pantheon SNe. The addition of SNe data slightly improves the constraints on cosmological parameters compared to the P18-only case and P18 + BAO combination, for IG we find

$$
\begin{align*}
\xi < & \begin{cases}
0.0080 & \text{P18 + SNe} \\
0.0068 & \text{P18 + BAO} \\
0.00063 & \text{P18 + BAO + SNe}
\end{cases} \\
\end{align*}
$$

at 95% CL for coupling parameter and for the Hubble constant at 68% CL

$$
\begin{align*}
H_0 = & \begin{cases}
68.8^{+0.8}_{-1.8} & \text{P18} \\
68.7^{+0.6}_{-1.3} & \text{P18 + SNe} \\
68.6^{+0.6}_{-0.9} & \text{P18 + BAO} \\
68.6^{+0.6}_{-0.8} & \text{P18 + BAO + SNe}
\end{cases}
\end{align*}
$$

Relaxing the consistency condition on the current value of the effective gravitational constant, i.e., Eq. (2) with $\Delta$ allowed to vary, we find (see Fig. 2) that the imbalance is constrained at 68% CL to

$$
\Delta = \begin{cases}
-0.032^{+0.029}_{-0.025} & \text{P18} \\
0.002^{+0.037}_{-0.032} & \text{P18 + SNe} \\
-0.022 \pm 0.023 & \text{P18 + BAO} \\
-0.003^{+0.034}_{-0.030} & \text{P18 + BAO + SNe}
\end{cases}
$$

at 95% CL for coupling parameter and for the Hubble constant at 68% CL

$$
\begin{align*}
H_0 = & \begin{cases}
70.2^{+1.2}_{-3.1} & \text{P18} \\
68.7^{+0.8}_{-1.4} & \text{P18 + SNe} \\
68.6^{+0.7}_{-0.9} & \text{P18 + BAO} \\
68.6^{+0.6}_{-0.8} & \text{P18 + BAO + SNe}
\end{cases}
\end{align*}
$$

In this case, the addition of SNe data is appreciable leading to tighter constraints for both the standard parameters and the modified gravity ones; moreover the mean value of the imbalance is more consistent to the Newton constant, i.e., $\Delta = 0$. We find that using the SNe data alone we are not able to put constraints on $\Delta$ on the prior range assumed in the analysis, i.e., $[-0.3, 0.3]$. It is interesting to note that the addition of SNe data to P18 and to P18 + BAO leads to larger uncertainties on $\Delta$. Reducing the degeneracy between the coupling parameter $\xi$ and the total matter density parameter $\Omega_{\text{m}},$

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3 https://github.com/dscolnic/Pantheon
4 https://github.com/brinkmann/montepython_public
5 https://github.com/lesgourg/class_public
the addition of SNe data help in constraining $\xi$. Consequently, the bound on the imbalance $\Delta$, which is not constrained by the SNe data alone, is relaxed due to the partial degeneracy with the coupling $\xi$. We check the value of the $\Delta\chi^2$ with respect to the $\Lambda$CDM case, calculated as $\Delta\chi^2 = \chi^2 - \chi^2_{\Lambda\text{CDM}}$. We find that for IG with $\Delta = 0$ ($\Delta$ allowed to vary) the $\Delta\chi^2$ corresponds to -1.4 (-2.6) for P18 + SNe, -2.2 (-3.6) for P18 + BAO, and -2.4 (-3.7) for P18 + BAO + SNe pointing to a slight improvement of fit for IG. We also calculate the Bayes factor with respect to the $\Lambda$CDM case, calculated as the ratio of the evidences for the extended model with respect to the baseline $\Lambda$CDM model. We compute the evidence directly from our MCMC chains using the method introduced in Ref. [73]. The logarithmic of the Bayes factor $\ln B$ for IG with $\Delta = 0$ ($\Delta$ allowed to vary) corresponds to -0.4 (-1.9) for P18 + SNe, -1.2 (-2.9) for P18 + BAO, and to -1.6 (-2.7) for P18 + BAO + SNe showing no statistical preference for the model analyzed [74]. Note that, the Bayes factor depends on the prior range of the parameters and it is enhanced in presence of parame-

\[ \text{FIG. 1. Marginalized joint 68\% and 95\% CL regions 2D parameter space using the Planck 2018 DR3 alone (blue contours) and in combination with BAO data (orange contours) for IG. The dotted and dashed contours include the Pantheon SNe sample for P18 and P18 + BAO, respectively.} \]
FIG. 2. As in Fig. 1, for \( \Delta \) allowed to vary.

Finally we can project the constraints on \( \Delta \) on the value of the Newton constant which is constrained at 68\% CL to \( G_{\text{eff}}(z = 0)/G = 0.938_{-0.049}^{+0.056} \) for P18, \( G_{\text{eff}}(z = 0)/G = 1.005 \pm 0.071 \) for P18 + SNe, \( G_{\text{eff}}(z = 0)/G = 0.957 \pm 0.045 \) for P18 + BAO, and \( G_{\text{eff}}(z = 0)/G = 1.05_{-0.08}^{+0.05} \) for P18 + BAO + SNe.

IV. THE ABSOLUTE Magnitude AND THE HubBLE Parameter PRIORS

As explained in Refs. [59, 75], there is a degeneracy between \( H_0 \) and \( M \) fitting the distance modulus to a SN sample. For this reason, as pointed out in Refs. [14, 15], it is useful to look at corresponding constraints on the absolute magnitude \( M_B \) of Pantheon SNe Ia sample rather than imposed the \( H_0 \) prior from SH0ES on the Hubble parameter at \( z = 0 \) in order to avoid misleading findings for late-time \( \Lambda \)CDM modifications as shown in Refs. [13, 15, 44, 76]. Indeed, the SH0ES Cepheid photometry [77, 78] and Pantheon SNe peak magnitudes give

\[
M_B = -19.2435 \pm 0.0373 \text{ [mag]}.
\] (13)
that the use of a Gaussian prior on the absolute magnitude while dashed contours include a Gaussian prior on the Hubble parameter. Including the SH0ES information [78] as a Gaussian inverse distance ladder calibration of SNe, as done in previous studies of these scalar-tensor models in Refs. [21–23, 26, 51–53], is fully consistent with the most correct way of using a prior information on the absolute magnitude $M_B$. This result is robust when we allow for $G_{\text{eff}}(z=0) \neq G$. Indeed, we find $M_B = -19.357 \pm 0.021$ ($M_B = -19.347 \pm 0.024$) adding a Gaussian prior on $H_0$ to the combination of P18 + SNe + BAO and $M_B = -19.353 \pm 0.022$ ($M_B = -19.354 \pm 0.023$) when we add a Gaussian prior on $M_B$ for IG with $\Delta = 0$ ($\Delta$ allowed to vary). Without including any extra information, we find $M_B = -19.393 \pm 0.020$ at 68% CL for the same P18 + SNe + BAO combination of datasets for IG in both cases fixing $\Delta = 0$ or varying it.

We find for IG $H_0 = 71.6 \pm 1.1$ km s$^{-1}$ Mpc$^{-1}$ at 68% CL for P18 + SNe + $p(M_B)$ for both $\Delta = 0$ and $\Delta$ allowed to vary; see Fig. 4. For IG with $\Delta = 0$ ($\Delta$ allowed to vary), we find that the tension goes from 2.9$\sigma$ (2.9$\sigma$) to 0.9$\sigma$ (0.9$\sigma$) including the prior on the absolute magnitude. With the addition of BAO, we find for IG with

This corresponds to $H_0 = 73.2 \pm 1.3$ km s$^{-1}$ Mpc$^{-1}$ by fitting the Pantheon sample [59] with the low-redshift expansion to the luminosity distance in a $\Lambda$CDM background [15].

In Fig. 3, we compare the marginalized constraints on cosmological parameters for P18 + BAO + SNe including the SH0ES information [78] as a Gaussian prior on the Hubble parameter $p(H_0)$, corresponding to $H_0 = 73.2 \pm 1.3$ km s$^{-1}$ Mpc$^{-1}$, versus a Gaussian prior on the absolute magnitude $p(M_B)$, corresponding to $M_B = -19.2435 \pm 0.0370$ mag [15]. We conclude that the use of a Gaussian prior on $H_0$ derived from the inverse distance ladder calibration of SNe, as done in previous studies of these scalar-tensor models in Refs. [21–23, 26, 51–53], is fully consistent with the most correct way of using a prior information on the absolute magnitude $M_B$. This result is robust when we allow for $G_{\text{eff}}(z=0) \neq G$. Indeed, we find $M_B = -19.357 \pm 0.021$ ($M_B = -19.347 \pm 0.024$) adding a Gaussian prior on $H_0$ to the combination of P18 + SNe + BAO and $M_B = -19.353 \pm 0.022$ ($M_B = -19.354 \pm 0.023$) when we add a Gaussian prior on $M_B$ for IG with $\Delta = 0$ ($\Delta$ allowed to vary). Without including any extra information, we find $M_B = -19.393 \pm 0.020$ at 68% CL for the same P18 + SNe + BAO combination of datasets for IG in both cases fixing $\Delta = 0$ or varying it.

We find for IG $H_0 = 71.6 \pm 1.1$ km s$^{-1}$ Mpc$^{-1}$ at 68% CL for P18 + SNe + $p(M_B)$ for both $\Delta = 0$ and $\Delta$ allowed to vary; see Fig. 4. For IG with $\Delta = 0$ ($\Delta$ allowed to vary), we find that the tension goes from 2.9$\sigma$ (2.9$\sigma$) to 0.9$\sigma$ (0.9$\sigma$) including the prior on the absolute magnitude. With the addition of BAO, we find for IG with
\[ \Delta = 0, H_0 = 70.2 \pm 0.9 \text{ km s}^{-1} \text{ Mpc}^{-1} \] and for \( \Delta \) allowed to vary \( H_0 = 70.0 \pm 0.8 \text{ km s}^{-1} \text{ Mpc}^{-1} \) both at 68\% CL for P18 + SNe + BAO + \( p(M_B) \). For IG with \( \Delta = 0 \) (\( \Delta \) allowed to vary), we find that the tension goes from 3.2\( \sigma \) (3.2\( \sigma \)) to 1.9\( \sigma \) (2.1\( \sigma \)) including the prior on the absolute magnitude to the combination P18 + BAO + SNe. We have reported this last result for completeness although the tension between \( H_0 \) from P18 + SNe + BAO and from SH0ES is superior to 3\( \sigma \). \footnote{Here the tension in terms of number of \( \sigma \) has been calculated on the Hubble parameter with respect to the reference measure \( H_0 = 73.2 \pm 1.3 \text{ km s}^{-1} \text{ Mpc}^{-1} \).}

The consistency in using a Gaussian prior on \( H_0 \) or on \( M_B \) is connected to smooth modification of the background expansion at late time in these models; see [15].

V. CONCLUSION

The near future of cosmology will be fueled by a huge amount of data that will allow us to extract precise and accurate cosmological information. The capability to test extended cosmological models beyond the minimal \( \Lambda \text{CDM} \) will be possible thanks to the combination of different datasets that will allow one to reduce degeneracies and to shrink parameter uncertainties by testing the evolution of Universe at different redshifts and scales. Thus, a continuous progress in the modeling of cosmological observables for extended models is required, in order to avoid confusing systematics with new physics and avoid parameter bias.

In this paper, we explore the use and importance of SNe data for cosmological parameter inference in modified gravity settings in which the gravitational constant vary with redshift, by taking as a working example induced gravity (IG), equivalent to Jordan-Brans-Dicke by a field redefinition. Particularly, we study general constraints on the coupling parameter from the Pantheon compilation of type SNe Ia alone and in combination with Planck DR3 CMB data and BAO measurements from BOSS. We constrain \( \xi < 0.0095 \) (95\% CL) with Pantheon data, \( \xi < 0.00080 \) (95\% CL) for Planck in combination with Pantheon, and \( \xi < 0.00063 \) (95\% CL) from the combination of Planck, BOSS, and Pantheon. Allowing also the imbalance \( \Delta \) to vary, connected to an effective gravitational constant today different from the value of the bare gravitational constant \( G_{\text{eff}} = G(1 + \Delta)^2 \), the bound on the coupling parameter is slightly relaxed to \( \xi < 0.0096 \) (95\% CL) for Pantheon data, \( \xi < 0.00088 \) (95\% CL) for Planck in combination with Pantheon, and \( \xi < 0.00064 \) (95\% CL) for the combination of Planck, BOSS, and Pantheon. For the imbalance, we find \( \Delta = 0.002^{+0.037}_{-0.032} \) (68\% CL) for Planck in combination with Pantheon and \( \Delta = -0.003^{+0.034}_{-0.030} \) (68\% CL) for the combination of Planck, BOSS, and Pantheon; Pantheon data alone cannot constrain \( \Delta \). In the Appendix A, we show that the correction due to the redshift dependence of the Chandrasekhar mass in these models is small, but appreciable, when SN Ia are added to the combination P18 + BAO.

We also test the use of a prior on the absolute magnitude \( M_B \) instead of a prior on the Hubble constant \( H_0 \) from SH0ES observations for this class of modified gravity models finding that the results do not depend on the choice of prior information. Considering the combination P18 + BAO + SNe + \( p(M_B) \), for IG we find \( H_0 = 70.2 \pm 0.9 \text{ km s}^{-1} \text{ Mpc}^{-1} \) (68\% CL), while \( H_0 = 70.0 \pm 0.8 \text{ km s}^{-1} \text{ Mpc}^{-1} \) (68\% CL) when allowing for the imbalance \( \Delta \) to vary. The robustness to different prior assumptions in \( H_0 \) or \( M_B \) is consistent with the fact that the models studied here alleviate the tension in \( H_0 \) at early times.

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A. Comparison of constraints with uncorrected distance modulus relation

We test here the relevance of the redshift dependence of the Chandrasekhar mass in the SNe distance modulus likelihood. In Figs. 5 and 6, we compare the marginalized constraints using the SNe compilation alone with and without the correction to the distance modulus relation introduced in Eq. (7). When we analyze the SNe data alone, we only allowed \( \Omega_m \) to vary (fixing \( \Omega_b = 0.047 \)) in addition to IG parameters, instead we vary \( \Omega_m \) and \( H_0 \) analyzing the SNe data in combination to the prior on the absolute magnitude \( p(M_B) \). For IG with \( \Delta = 0 \), we find \( \xi < 0.0093 \) (95\% CL) and \( \Omega_m = 0.303 \pm 0.023 \) (68\% CL) without correction while we find \( \xi < 0.0095 \) (95\% CL) and \( \Omega_m = 0.320_{-0.021}^{+0.029} \) (68\% CL) correcting for Eq. (7). For IG with \( \Delta \) allowed to vary, we find \( \xi < 0.0095 \) (95\% CL) and \( \Omega_m = 0.302 \pm 0.022 \) (68\% CL) without correction while we find \( \xi < 0.0096 \) (95\% CL) and \( \Omega_m = 0.317 \pm 0.024 \) (68\% CL) correcting for Eq. (7). When analysing SNe data alone, we find a 0.5\sigma shift in the determination of the mean value of \( \Omega_m \) neglecting the redshift dependence of the Chandrasekhar mass.

In Figs. 7 and 8, we compare the marginalized constraints on cosmological parameters for P18 + SNe + \( p(M_B) \) and P18 + BAO and P18 + BAO + SNe + \( p(M_B) \) with and without the correction to the distance modulus relation introduced in Eq. (7). We do not see any effect
FIG. 5. Marginalized joint 68% and 95% CL regions 2D parameter space using the Pantheon SNe sample for IG (blue contours); orange contours include also a Gaussian prior on the absolute magnitude $p(M_B)$. The dotted and dashed contours include the correction included in Eq. (7) for SNe and SNe + $p(M_B)$, respectively.

FIG. 6. As in Fig. 5, for IG with $\Delta$ allowed to vary.

on means and uncertainties on any cosmological parameter for this combination of datasets. Note that for IG with $G_{\text{eff}}(z = 0) = G$, the difference on the redshift range of the Pantheon SNe sample ($0.01 < z < 2.3$) is smaller than 1%.

We find the same outcome also when we vary the imbalance $\Delta$, see Figs. 9 and 10. In this case the value of the Newton constant is constrained at 68% CL to $G_{\text{eff}}(z = 0)/G = 1.05^{+0.05}_{-0.08}$ for P18 + BAO + SNe and to $G_{\text{eff}}(z = 0)/G = 1.04^{+0.06}_{-0.08}$ for P18 + BAO + SNe + $p(M_B)$ allowing for small correction from the distance modulus relation (7). Without including BAO data, we find at 68% $G_{\text{eff}}(z = 0)/G = 1.005 \pm 0.071$ for P18 + SNe and to $G_{\text{eff}}(z = 0)/G = 1.004 \pm 0.070$ for P18 + SNe + $p(M_B)$. 
FIG. 7. Marginalized joint 68% and 95% CL regions 2D parameter space using the \textit{Planck} 2018 DR3 in combination with Pantheon SNe sample (blue contours) for IG; orange contours include also a Gaussian prior on the absolute magnitude $p(M_B)$.

The dotted and dashed contours include the correction included in Eq. (7) for P18 + SNe and P18 + SNe + $p(M_B)$, respectively.

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FIG. 9. As in Fig.7, for IG with ∆ allowed to vary.

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