Phenomenological Varying Modified Chaplygin Gas
with Variable \( G \) and \( \Lambda \):
Toy Models for Our Universe

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Abstract

This article motivated by the recent articles and results of two authors. Recently, J. Sadeghi and H. Farahani presented a work [1], where they include viscosity and analyze general model, by this way they extended models considered by M. Khurshudyan [2] and [3]. In this article, We tempt to consider varying Modified Chaplygin gas model in case of variable \( G \) and \( \Lambda \). It is well known, that varying \( G \) and \( \Lambda \) gives rise to modified field equations and modified conservation laws. We will consider two different toy models. First model is a Universe with one component phenomenological gas of our consideration, while for the second model we assume existence of a composed fluid of gas and a matter with \( P = \omega(t)\rho_m \). Sign changeable interaction between fluids is accepted. We will analyze important cosmological parameters like EoS parameter of a fluid, deceleration parameter \( q \) of the model.

Introduction

The observations of high redshift type SNIa supernovae [4-6] reveal the speeding up expansion of our universe. The surveys of clusters of galaxies show that the density of matter is very much less than critical density [7], observations of Cosmic Microwave Background (CMB) anisotropy indicate that the universe is flat and the total energy density is very close to the critical \( \Omega_{\text{tot}} \approx 1 \) [8]. In order to explain experimental data concerning to the nature of the accelerated expansion of the Universe a huge number of hypothesis were proposed. For instance, in general relativity framework, the desirable result could be achieved by so-called dark energy: an exotic and mysterious component of the Universe, with negative pressure (we thought that the energy density is always positive) and with negative EoS parameter \( \omega < 0 \). Dark energy occupies about 73% of the energy of our universe, other component, dark matter, about 23%, and usual baryonic matter occupy about 4%. The simplest model for a dark energy is a cosmological constant \( \omega_{\Lambda} = -1 \) introduced by Einstein, but with cosmological constant we faced with two problems i.e. absence of a fundamental mechanism which sets the cosmological constant zero or very small value the problem known as fine-tuning problem, because in the framework of quantum field theory, the expectation value of vacuum energy is 123 order of magnitude larger than the observed value [9]. The second problem known as cosmological coincidence problem, which asks

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why are we living in an epoch in which the densities of dark energy and matter are comparable? Alternative models of dark energy suggest a dynamical form of dark energy, which at least in an effective level, can originate from a variable cosmological constant [10, 11], or from various fields, such as a canonical scalar field [12-17] (quintessence), a phantom field, that is a scalar field with a negative sign of the kinetic term [18-26] or the combination of quintessence and phantom in a unified model named quintom [27-40] and could alleviate these problems. Finally, an interesting attempt to probe the nature of dark energy according to some basic quantum gravitational principles are the holographic dark energy paradigm [41-52] and agegraphic dark energy models [53-55].

Furthermore, since no known symmetry in nature prevents or suppresses a non-minimal coupling between dark energy and dark matter, there may exist interactions between the two components. At the same time, from observation side, no piece of evidence has been so far presented against such interactions. Indeed, possible interactions between the two dark components have been discussed in recent years. It is found that a suitable interaction can help to alleviate the coincidence problem. Different interacting models of dark energy have been investigated. For instance, the interacting Chaplygin gas allows the universe to cross the phantom divide, which is not permissible in pure Chaplygin gas models. This is a recent believe and deep accepted fact among researchers. Several different approaches are taken and intensively developed during these years in order to find an appropriate physics. For some cases it can be thought that not only one certain type of physics is involved and stands behind this phenomena, but rather we are working with a mixture of several physics giving the recent picture of our Universe. There is deep hope concerning to the development of basis of quantum gravity and respectively quantum cosmology. For any case we faced with very hard problem. Despite to the huge number of theoretical models and observational data and possibilities, still we are working in darkness and we can not find that small light: the end of tunnel. Research in theoretical cosmology considered two possible ways to explain later time accelerated expansion of the universe. Remembering the field equations, it becomes clear why. Remember that field equations make connection between geometry and matter content of Universe in a simple way. Therefore there is two possibilities either we should modify matter content which is coded in energy-stress tensor or we should modify geometrical part including different functions of Ricci scalar etc. Different type of couplings between geometry and matter could give desirable effects as well.

Let give some critics concerning to mentioned ways thought to be true. Mater content modification gives possibility to include two types of matter: Dark energy (DE) and dark matter (DM). DE is a fluid with negative pressure and positive energy density (remember this is in classical regime, with no well established quantum gravity coming from investigations of black holes Stephan Howking accepted possibility of existence of dark energy with negative energy density and positive pressure) giving negative EoS parameter defined as \( \omega = p/\rho \). Today, we do not feel lack of models for DE, which could be seen from the number of references given above. However, all of them are phenomenological models and wait to be proved by observational data. The same can be said for DM, which thought to operate on large scales and be responsible for structure formation, evolution etc. There is a thought that these two components are the manifestation of the same "matter". Even if it is true, then there is not any mechanism explaining how it was or it is possible to have such type of separation. Then, if you can accept this possibility, then natural question is what will happen in future i.e. can they recombine together or how they will start to evolve and what are consequences. As a result of imagination, we can propose several mechanism that can make connection between DE and DM and one of them is possibility having speeds faster than speed of light, making connection mysterious for our brain, because simply we can not "see" that. But if they have different origin, then we really face with a wall. However, independent of the mentioned and other possibilities, we have big open problem, because we do not know structure of these matters. Research concerning to the modification matter part of field equations give rise of an understanding that more complex and crazy forms for EoS equation can be considered. Modification of geometrical part gives rise of different modified theories like \( F(R) \), \( F(T) \), \( F(G) \) etc. Models of this origin however contain some future singularities that our Universe should faced, but we do not need to worry, because these are just models and fortunately with appropriate choose of a function we can escape singularities and extend life-time for our Universe. But, there are models also, that can explain accelerated expansion without any DE, for instance, Cardassian Universe [56-61]. In this model, we need modify Friedmann equations and having usual matter is enough.
It is well known that Einstein equations of general relativity do not permit any variation in the gravitational constant $G$ and cosmological constant $\Lambda$ because of the fact that the Einstein tensor has zero divergence and by energy conservation law is also zero. So, some modifications of Einstein equations are necessary. This is because, if we simply allow $G$ and $\Lambda$ to be a variable in Einstein equations, then energy conservation law is violated. Therefore, the study of the effect of varying $G$ and $\Lambda$ can be done only through modified field equations and modified conservation laws.

In this paper we use results of the resent papers [1-3] and extend them to the case of variable $G$ and $\Lambda$. We will consider two different toy models. First model is a Universe with one component phenomenological gas of our consideration, while for the second model we assume existence of a composed fluid of gas and a matter. We use sign changeable interaction between fluids and analyze important cosmological parameters like EoS parameter of a fluid and deceleration parameter $q$ of the model. This article is organized in following way. Section introduction is devoted to introduce basic ideas and gives some general information related to the research field and our motivation. Next section review FRW Universe with variable $G$ and $\Lambda$. In section “Phenomenological Fluid and Model Setup” we recall the basics of origin of the fluid and general settings how problem can be solved. In other sections we investigate parameters of the models and then we give conclusions.

**FRW Universe with variable $G$ and $\Lambda$**

Field equations that govern our model with variable $G(t)$ and $\Lambda(t)$ (see for instance [62]) are,

$$R^{ij} - \frac{1}{2} R g^{ij} = -8\pi G(t) \left[ T^{ij} - \frac{\Lambda(t)}{8\pi G(t)} g^{ij} \right],$$

where $G(t)$ and $\Lambda(t)$ are function of time. By using the following FRW metric for a flat Universe,

$$ds^2 = -dt^2 + a(t)^2 \left( dr^2 + r^2 d\Omega^2 \right),$$

field equations can be reduced to the following Friedmann equations,

$$H^2 = \frac{\dot{a}^2}{a^2} = \frac{8\pi G(t)\rho}{3} + \frac{\Lambda(t)}{3},$$

and,

$$\frac{\ddot{a}}{a} = -\frac{4\pi G(t)}{3} (\rho + 3P) + \frac{\Lambda(t)}{3},$$

where $d\Omega^2 = d\theta^2 + \sin^2 \theta d\phi^2$, and $a(t)$ represents the scale factor. The $\theta$ and $\phi$ parameters are the usual azimuthal and polar angles of spherical coordinates, with $0 \leq \theta \leq \pi$ and $0 \leq \phi < 2\pi$. The coordinates $(t, r, \theta, \phi)$ are called co-moving coordinates.

Energy conservation $T_{ij}^0 = 0$ reads as,

$$\dot{\rho} + 3H (\rho + P) = 0.$$

Combination of (3), (4) and (5) gives the relationship between $\dot{G}(t)$ and $\dot{\Lambda}(t)$

$$\dot{G} = -\frac{\dot{\Lambda}}{8\pi \rho}.$$

Ever since Dirac’s proposition of a possible time variation of $G$, a volume of works has been centered around the act of calculating the amount of variation of the gravitational constant. For instance, observation of spinning-down rate of pulsar $PSRJ2019 + 2425$ provides the result,

$$\left| \frac{\dot{G}}{G} \right| \leq (1.4 - 3.2) \times 10^{-11} \text{yr}^{-1}. $$
Depending on the observations of pulsating white dwarf star G117 – B15A, Benvenuto et al. [63] have set up the astroseismological bound as,

\[-2.50 \times 10^{-10} \leq \frac{\dot{G}}{G} \leq 4 \times 10^{-10} \text{yr}^{-1}.\]  

(8)

For a review to "Large Number Hypothesis" (LNH) we refer our readers to [64] and references therein. Subject of our interest is to consider a modified Chaplygin gas based on a Universe with variable $G$ and $\Lambda$. This model is a phenomenological and we are interested by the evolution of the Universe with this setup. For $\Lambda$ we will consider one of the forms intensively considered in literature and using Eqs (3)-(6) we can recover scale factor, behavior of $G$ and other cosmological parameters including EoS parameter and deceleration parameter $q$, which could describe behavior of the Universe. The starting point for scanning the range of parameters of the model is that acceleration will be caused by a fluid with negative EoS parameter i.e. at the same time we should satisfy $q < 0$ and $\omega < 0$ conditions. Keeping spirit of Dirac's LNH we will consider,

\[\Lambda \propto t^{-2}.\]  

(9)

Other forms for $\Lambda(t)$ were considered over years based on phenomenological approach, some of examples are, for instance, $\Lambda \propto (\dot{a}/a)$, $\Lambda \propto \ddot{a}/a$ or $\Lambda \propto \rho$ to mention a few. As we are interested by toy models we pay our attention to the problem from a numerical investigation point of view and we believe that after some effort we also can provide exact solutions for the problem, which will be done in other forthcoming articles.

**Phenomenological Fluid and Model Setups**

We already mentioned, that matter modification of field equations teach us that we can consider fluids with strange EoS equations. One of the examples is Chaplygin gas given by [65],

\[P = -\frac{B}{\rho},\]  

(10)

where $B$ is a constant. Then, generalized Chaplygin gas (GCG) with the following equation of state [66, 67],

\[P = -\frac{B}{\rho^\alpha},\]  

(11)

with $0 < \alpha \leq 1$. As we can see the GCG is corresponding to almost dust ($P = 0$) at high density which is not agree completely with our Universe. Therefore, modified Chaplygin gas (MCG) with the following equation of state introduced [68-70],

\[P = \mu \rho - \frac{B}{\rho^\alpha},\]  

(12)

where $\mu$ is a positive constant. This model is more appropriate choice to have constant negative pressure at low energy density and high pressure at high energy density. The special case of $\mu = \frac{1}{3}$ is the best fitted value to describe evolution of the universe from radiation regime to the $\Lambda$CDM regime. In the Ref. [2] one of the authors motivated by a series of works [71-73] proposed a model of varying generalized Chaplygin gas and considered its sign-changeable interaction of the form $Q = q(3Hb\rho_m + \gamma\dot{\rho})$ with fluid. As there is an interaction between components there is not energy conservation for the components separately, but for the whole mixture the energy conservation is hold. This approach could work as long as we are working without knowing the actual nature of the dark energy and dark matter as well as about the nature of the interaction. This approach at least from mathematical point of view is correct. The forms of interaction term considered in literature very often are of the following forms: $Q = 3Hb\rho_{dm}$, $Q = 3Hb\rho_{de}$, $Q = 3Hb\rho_{tot}$, where $b$ is a coupling constant and positive $b$ means that dark energy decays into dark matter, while negative $b$ means dark matter decays into dark energy. From thermodynamical view, it is argued that the second law of thermodynamics strongly favors dark energy decays into dark matter. However it was found that
the observations may favor the decaying of dark matter into dark energy. Other forms for interaction term considered in literature are \( Q = \gamma \dot{\rho}_{dm} \), \( Q = \gamma \dot{\rho}_{de} \), \( Q = \gamma \dot{\rho}_{tot} \), \( Q = 3Hb\gamma \rho_i + \gamma \dot{\rho}_i \), where \( i = \{ dm, de, tot \} \). These type of interactions are either positive or negative and can not change sign. However, recently by using a model independent method to deal with the observational data Cai and Su found that the sign of interaction \( Q \) in the dark sector changed in the redshift range of \( 0.45 \leq z \leq 0.9 \). Hereafter, a sign-changeable interaction [74] and [75] were introduced,

\[
Q = q(\gamma \dot{\rho} + 3bH\rho),
\]

where \( \gamma \) and \( b \) are dimensionless constants, the energy density \( \rho \) could be \( \rho_{dm}, \rho_{de}, \rho_{tot} \). \( q \) is the deceleration parameter given by,

\[
q = -\frac{1}{H^2 \ddot{a}} = -1 - \frac{\dot{H}}{H^2}.
\]

This new type of interaction, where deceleration parameter \( q \) is a key ingredient makes this type of interactions different from the ones considered in literature and presented above, because it can change its sign when our universe changes from deceleration \( q > 0 \) to acceleration \( q < 0 \). \( \gamma \dot{\rho} \) is introduced from the dimensional point of view. We would like also to stress a fact, that by this way we import a more information about the geometry of the Universe into the interaction term. This is not only one possibility, in forthcoming articles we hope to provide and consider other forms of sign-changeable interactions. The origin of the fluid considered before, is based on a simple assumption. We proposed the following strategy: somehow, concerning to an unknown physics (even could be very well known) it was possible to separate components of the darkness of the Universe and was found that we have fluid (F) + MCG + Remaining Darkness (RD). Only information which we know about RD is that its EoS parameter \( \omega_x < 0 \). We assume that interaction between MCG and RD gives to born a varying modified Chaplygin gas and that there is not any interaction between F and RD. This assumption allows us still to think that our Universe consists of mixture of a F and a varying MCG gas. Then we will assume that EoS parameter of RD \( \omega_x \) is a function of time: \( \omega(t) = \omega_0 + \omega_1(t \dot{H}) \), which has an explicit time dependence that disappears with the \( t \dot{H} = H \) condition. Which in its turn gives us a modification of \( B \) constant and for the new model it reads as,

\[
B(a) = -\omega(t)B_0a^{\frac{1}{(1+\omega(t))(1+\alpha)}}.
\]

Therefore a fluid of our Universe considered to be a MCG with the following EoS,

\[
P = \mu \rho - \frac{B(a)}{\rho^\alpha}.
\]

In order to have a complete picture of our proposition, we will consider two different models as the following.

1. An Universe with varying \( G \) and \( \Lambda \) with a given \( \Lambda(t) \) and Varying MCG

2. An Universe with varying \( G \) and \( \Lambda \) with a given \( \Lambda(t) \) and a mixture of Varying MCG and a fluid.

For the fluid we assume a EoS \( P_f = \omega(t)\rho_f \), where \( \omega(t) \) is the same as for varying MCG of our consideration. Possible coupling between fluids is modeled by sigh-changeable interaction.

**Models and Cosmological Parameters**

In this section we will start to investigate models and we will start with a single component Universe. Bellow we provide general algorithm which a reader should keep in order to perform numerical research of the problem. For simplicity, we reduce number of parameters of the models assuming some of them known a priori.
One component fluid Universe

Compared with the second model of our interest first model is going to be a simple model. Problem solving strategy is based on a solution of a system of differential equations (4), (5) and (6) with a given $\Lambda(t) \propto t^{-2}$ in our case. We analyze problem numerically and present profiles of cosmological parameters and quantities and give comprehensive analysis of obtained results. Plots are organized in such a way, that they provide good inside into properties of the model.

First panel of 4 plots (Fig. 1) represent behavior of $G(t)$ for different values of model parameters. From the first graph representing behavior of $G(t)$ as a function of $\omega_0$ with $\omega_1 = -1.8$ and $\alpha = 0.3$ as follow: $G(t)$ is an increasing function. For some initial stages of evolution $G(t)$ has almost the same behavior, which starts exhibit differently for latter stages evolution. Behavior of $G(t)$ reveal to be the same for a different values of $\alpha$ for a given values of $\omega_0$ and $\omega_1$. From the third graph, we can understood that with decreasing value of $\omega_1$, after certain time we register decreasing in rate of $G(t)$. Last graph is represent general case for the behavior of $G$.

Second panel of graphs (Fig. 2) represent behavior of deceleration parameter $q$ over time. We have ever accelerated expansion, because $q$ is negative during whole evolution. For different scenarios related to pair of fixed and varying parameters $q$ increases for early stages of evolution, but for later stages of evolution it is decreases. However, with $\omega_1 = -1.8$ and $\alpha = 0.3$ we observed that $q$ continues its decreasing with a law speed in case $\omega_0 = -2.2$. For a fixed values of $\omega_0$ and $\omega_1$ for different values of $\alpha$ we conclude that we have increasing and then decreasing function of $q$.

The same discussion hold for third panel of graphs (Fig. 3) which represent $\omega_{MCG}$ in terms of time.

![Figure 1: One component](image)

Two Component Fluid Universe

If we consider two component fluid Universe, then equations (3) and (4) extended to the following,

$$
\left( \frac{\dot{a}}{a} \right)^2 = \frac{8\pi G(t) \rho_{tot}}{3} + \frac{C}{3t^2},
$$

where $C$ is an arbitrary constant and,

$$
\frac{\ddot{a}}{a} = -\frac{4\pi G(t)}{3} (\rho_{tot} + 3P_{tot}) + \frac{C}{3t^2},
$$

(18)
where we used the equation (9) and define,
\[ \rho_{\text{tot}} = \rho_F + \rho_{\text{MCG}}, \]
\[ P_{\text{tot}} = P_F + P_{\text{MCG}}. \]  
(19)

Therefore the energy-momentum conservation law obtained as the following,
\[ \dot{\rho}_{\text{tot}} + 3(\rho_{\text{tot}} + P_{\text{tot}})H = 0. \]  
(20)

Also we can obtain,
\[ \dot{G} = \frac{2C}{8\pi \rho_{\text{tot}} t^3}, \]  
(21)

**Interacting case**

If we consider interaction between fluid and gas then the conservation energy separate as the following,
\[ \dot{\rho}_{\text{MCG}} + 3H(\rho_{\text{MCG}} + P_{\text{MCG}}) = Q, \]  
(22)

where \( P_{\text{MCG}} \) is given by the relations (12) and,
\[ \dot{\rho}_F + 3H(\rho_F + P_F) = -Q, \]  
(23)

where \( Q \) is interaction term given by the equation (13). Now we should solve a system of differential equations of (18), (21), (22) and (23).

Plots given by the Fig. 4 represent behavior of \( G(t) \) for different values of model parameters. The first graph representing behavior of \( G(t) \) as a function of \( \omega_0 \) which shows increasing function of time but yields to a constant at the late time which is consequence of interaction term. Such behavior is the same for other plots which are graphs representing behavior of \( G(t) \) as a function of \( \alpha, \omega_1 \) and \( \gamma \) respectively. As \( \alpha \) and \( \gamma \) increased then \( G(t) \) increased, but increasing \( |\omega_1| \) decreased \( G(t) \).

Plots of the Fig. 5 represent behavior of deceleration parameter \( q \) over time. As previous case we have ever
accelerated expansion, because $q$ is negative during whole evolution. The last plot shows that varying $\alpha$ is not important and $q$ is totally decreasing function of time. Variation of other parameters show increases of $q$ for early stages of evolution, but for later stages of evolution it is decreases. The first plot shows that increasing $|\omega_0|$ increased the value of $q$ but increasing $|\omega_1|$ decreased one.

Plots of the Fig. 6 represent $\omega_{MCG}$ versus time. We can see that $\omega_{MCG}$ increased in the initial stage to reach maximum at about $0.1 < t < 0.2$ and then decreased to reach a constant value at the late time. This is completely different with the previous case where there is only one component fluid. the second plot shows that increasing $\alpha$ increased $\omega_{MCG}$. The last graph is represent general case for the behavior of $\omega_{MCG}$.

Figure 3: One component

Figure 4: Interacting two components
Discussion

In this paper we consider Modified Chaplygin gas as a cosmological model which unifies dark matter and dark energy. This work which is extended version of previous works [1-3] written based on interesting idea which tells that unknown physics separates dark side of Universe to gas and fluid which may be interact with each other while there is not any interaction between remaining darkness of Universe and fluid. Such interaction yields to varying modified Chaplygin gas. An important point of this paper is that we considered variable $G$ and $\Lambda$.

First of all we studied Universe with variable $G$ and $\Lambda$ and varying modified Chaplygin gas as a simple model. Then we consider two component fluid which involve sign-changeable interaction. We assumed $\Lambda(t)$ proportional to $t^{-2}$ and have numerical analysis of $G(t)$, declaration parameter and $\omega_{MCG}$. In the first case we found that the variation of $\alpha$ is not important for $G(t)$. However we found that $G(t)$ is increasing function of time. The declaration parameter increased at the early stage ant then is decreasing function of time. $\omega_{MCG}$ has similar behavior as $q$. In the second case where we consider sign-changeable interaction $G(t)$ is also increasing function of time but yields to a constant at the late time. The declaration parameter of this case has similar behavior of the previous case. It is seen that the Universe initially undergoes a rapidly falling acceleration followed by a rise in it. At a particular epoch the Universe get into a phase of constant acceleration in which we are presently located. Finally $\omega_{MCG}$ increased at the initial stage and rich to a maximum and finally yields to a constant at the late time.

We can also investigate other cosmological quantities such as scale factor, $\omega(t)$ and density. In the Figs. 7, 8 and 9 we draw these parameters for the case of single component fluid. The first plot of the Fig. 7 shows that increasing $|\omega_0|$ decreased scale factor to a constant. As we can see from the last plot this constant obtained for $\omega_0 = -2.2$, $\omega_1 = -1.8$ and $\alpha = 0.3$. The second plot of the Fig. 7 shows that the scale factor decreased by increasing of $\alpha$. on the other hand plots of the Fig. 9 show that energy density is decreasing function of time.

Scale factor of two components fluid Universe drawn in the plots of the Fig. 10. It shows that increasing $\omega_0$ decreased scale factor. The last graph of the Fig. 10 is represent general case for the behavior of $\alpha$. Plots of the Fig. 11 show that EoS parameter yields to a negative constant at the late time. Finally plots of the Fig.
12 show that energy density is decreasing function of time. In that case the second plot tells that increasing $\alpha$ decreased the energy density.

In this paper we considered modified Chaplygin gas which may be extended to the case of modified cosmic Chaplygin gas with the following equation of state [70],

$$p = \mu \rho - \frac{1}{\rho^\alpha} \left[ \frac{B}{1 + \omega} - 1 + (\rho^{1+\alpha} - \frac{B}{1 + \omega} + 1)^{-\omega} \right],$$  \hspace{1cm} (24)

where $\omega$ is the cosmic parameter. Also one can include shear viscosity and bulk viscosity [76] and investigate effect of them on cosmological parameters.

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Appendix: Single Component fluid Universe

Figure 7: Scale factor of single component fluid Universe

Figure 8: EoS parameter of single component fluid Universe
Figure 9: Energy density of single component fluid Universe

Appendix: Two Components fluid Universe

Figure 10: Scale factor of two component fluid Universe
Figure 11: EoS parameter of two component fluid Universe

Figure 12: Energy density of two component fluid Universe