Strong field physics in condensed matter

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What is nice about condensed matter

1. Eco(nomy) friendly
   - QED vacuum
   - Insulator

2. Full of dreams
   - I have a patent for a RRAM device. $$$$?
   - You may earn $ with your nice theoretical ideas!

Schwinger mechanism (Heisenberg-Euler)

\[ E_{th} = \frac{m^2}{e} = 1.3 \times 10^{16} \text{V/cm} \]

Dielectric breakdown

\[ E_{th} = \frac{\Delta^2}{t_{hop}a} \sim 1 \text{eV/Å} \sim 10^8 \text{V/cm} \]
List of correspondences

| High energy | Condensed matter |
|-------------|------------------|
| Schwinger mechanism | (Landau-)Zener breakdown |
| Heisenberg-Euler’s Effective action = | non-adiabatic geometric phase |
| vacuum polarization | extended Berry’s phase theory of polarization |
| pair creation in interacting systems (e.g. QCD) | many-body Schwinger-Landau-Zener mechanism in strongly correlated insulators |
| cf) next talk by S. Nakamura | F_{th} calculated exactly |
| Dirac particles in circularly polarized light | Photovoltaic Hall effect |
| Part 1 | Part 2 |
| Part 3 | Part 3 |

\( \approx \) control of the parity anomaly in 2+1 Dirac systems via light
Part 1 Effective action

Path integral\[-\text{Dirac fermion}\]

\[ L = \bar{\psi} (i \partial + e A - m) \psi \]

\[ \mathcal{L}(A_{\text{ext}}) = -i \ln \int D[\psi, \bar{\psi}] e^{i \int d^D x L(A_{\text{ext}})} \]

\[ = -i \ln \text{Det} [i \partial + e A - m] \]

\[ \Gamma = \Gamma_A + \Gamma_B + \Gamma_C + \ldots + \Gamma \]

\[ \Gamma / V \sim \frac{e^2 E^2}{4\pi^3} \exp \left( -\frac{\pi m^2}{e E} \right) \]

Vacuum instability

\[ H_0 \quad \begin{array}{c} \hline \end{array} \quad H(t) \quad \text{time} \]

\[ |0\rangle \quad \begin{array}{c} \hline \end{array} \quad |\Psi(t)\rangle \]

Groundstate-to-groundstate amplitude

\[ \Xi(t) = \langle 0; \phi(t) | \hat{T} e^{-i \int_0^t H(\phi(s)) ds} | 0; \phi(0) \rangle e^{i \int_0^t E_0(\phi(s)) ds} \]

\[ \mathcal{L} = \lim_{t \to \infty} \frac{-i}{t L^d} \ln \Xi(t) \quad \text{g.s. of } H(\phi) \]
In electric fields momentum flows as \[ \vec{k} + \vec{A}(t) \]

\[ e^{i\beta} \sqrt{p} \]

\[ e^{i\Delta E + i\gamma} \sqrt{1 - p} \]

**Landau-Zener tunneling**

\[ p = e^{-\pi \frac{\Delta^2}{F}} \]

\[ \gamma \quad \text{Aharonov-Anandan (Stokes) phase} \]
Effective Lagrangian for band insulators

\[ \text{Re } \mathcal{L}(F) = -F \int_{\text{BZ}} \frac{dk}{(2\pi)^d} \frac{\gamma(k)}{2\pi}, \]

\[ \text{Im } \mathcal{L}(F) = -F \int_{\text{BZ}} \frac{dk}{(2\pi)^d} \frac{1}{4\pi} \ln[1 - p(k)], \]

TO, H. Aoki, Phys. Rev. Lett. 95, 137601 (2005)

e.g.) Dirac band

non-adiabatic Berry’s phase (Aharonov-Anandan, Stokes phase)

\[ \gamma(k) = \frac{1}{2} \text{Im} \int_{0}^{\infty} ds \frac{e^{-i(\Delta_{\text{band}}(k)/2)^2 s}}{s} \left[ \cot(vFs) - \frac{1}{vFs} \right] \]

cf.) Y. Kayanuma, Phys. Rev. B. 47, 9940 (1993)

Landau-Zener formula for the tunneling probability

\[ p(k) = \exp\left[ -\pi \frac{(\Delta_{\text{band}}(k)/2)^2}{vF} \right] \]

Heisenberg-Euler-Schwinger result is recovered.
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| pair creation in interacting systems (e.g. QCD) | theory of polarization |
| Dirac particles in circularly polarized light | many-body Schwinger-Landau-Zener mechanism in strongly correlated insulators |
| | $F_{th}$ calculated exactly |

**Part 1**

**Part 2**

| Part 3 |
|--------|
| Photovoltaic Hall effect |
| = control of the parity anomaly in 2+1 Dirac systems via light |
Let’s calculate the tunneling probability $p$ in a strongly correlated insulator, exactly.

This is difficult since, ...

Properties of the system depend on the scale.

QCD asymptotic freedom

Mott insulator scaling theory
no confinement
Hubbard model in an electric field

\[ H(t) = -\sum_i \left[ e^{i\phi(t)} c_{i+1}^\dagger c_i^\sigma + e^{-i\phi(t)} c_{i\sigma}^\dagger c_{i+1}^\sigma \right] + U \sum_i n_i^\uparrow n_i^\downarrow \]

\[ \phi(t) = Ft \quad \text{time dependent phase} \]

Four fermion interaction (repulsion)

Exactly solvable via Bethe ansatz

Doublon-hole pair (quark-anti-quark)

Ring geometry

electric field induced by a time-dependent phase
Scale dependent quantum tunneling

Small systems

Infinite system

Drude weight

\[ D(L) = e^{-L/\xi} > 0 \]

Small systems are perfect metal (similar to asymptotic freedom)

\[ p = e^{-\pi \frac{\Delta(L)^2}{D(L)F}} \]

\[ \rightarrow p = e^{-\pi \frac{F_{th} DDP}{F}} \]

\[ L \rightarrow \infty \]
The Dykhne-Davis-Pechkas theory of quantum tunneling

Dykhne JETP (1962), Daviis, Pechukas, J.Chem.Phys. (1976)

Tunneling probability

\[ p = \exp\left(-2\text{Im}S_{1,2}/\hbar\right) \]

\[ S_{1,2} = \int_{t_0}^{t*} dt'[E_2(\Phi(t')) - E_1(\Phi(t'))] \]

singularity = Energy crossing in complex time

\[ E_2(t^*) = E_1(t^*) \]

Evolution to the other Riemann surface

Similar ideas appear in instanton calculus (WKB) in quantum field theory
non-Hermitian Hubbard model

\[ H(t) = - \sum_i \left[ e^{i\phi(t)} c_{i+1}^\dagger c_i + e^{-i\phi(t)} c_i^\dagger c_{i+1} \right] + U \sum_i n_{i\uparrow} n_{i\downarrow} \]

\[ \phi(t) = F t \]

Imaginary time

Non-Hermitian Hubbard model

\[ H(t) = - \sum_i \left[ e^{\Psi} c_{i+1}^\dagger c_i + e^{-\Psi} c_i^\dagger c_{i+1} \right] + U \sum_i n_{i\uparrow} n_{i\downarrow} \]

left hopping ≠ right hopping

ground state Bethe ansatz solution

Fukui, Kawakami, Phys. Rev. B 58, (1998)
Nakamura, Hatano, J. Phys. Soc. Jpn. 75 (2006)

We extended the 1-string solution to non-Hermitian.

T. Oka, H. Aoki, Phys. Rev. B 81, 033103 (2010)
Tunneling probability for the 1d Hubbard model

\[ p_{th}^{DDP} = \exp \left( -\pi F_{th}^{DDP} / F \right) \]

\[ F_{th}^{DDP} = \frac{2}{\pi} \int_0^{b_{cr}} (E_1 - E_0) \frac{d\Psi}{db} db \]

\[ = \frac{2}{\pi} \int_0^{\sinh^{-1} u} 4 \left[ u - \cosh b + \int_{-\infty}^{\infty} d\omega \frac{e^{\omega} \sinh b J_1(\omega)}{\omega (1 + e^{2u|\omega|})} \right] \]

\[ \times \left[ 1 - \cosh b \int_0^{\infty} d\omega \frac{J_0(\omega) \cosh(\omega \sinh b)}{1 + e^{2u\omega}} \right] db. \]

\[ u = U/4 \]

cf) In Dirac it was \( E_{th} = m^2 / e \)

T. Oka, H. Aoki, Phys. Rev. B 81, 033103 (2010)
Comparison with numerical, experimental results (time-dependent density matrix renormalization group)

Good agreement!

TO, H. Aoki, Phys. Rev. B 81, 033103 (2010)
TO, H. Aoki, Phys. Rev. Lett. 95, 137601 (2005)
For small $U$, the threshold is similar to that of a free fermion with a gap. (the large $U$ asymptote is different. $F_{th}^{DDP} \propto \Delta$)

Pair creation = sees only small scale physics?

Is this also true in QCD?
List of correspondences

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- Schwinger mechanism = (Landau-)Zener breakdown
- Heisenberg-Euler’s Effective action = non-adiabatic geometric phase
- vacuum polarization = extended Berry’s phase
- pair creation in interacting systems (e.g. QCD) = theory of polarization
- Dirac particles in circularly polarized light = Photovoltaic Hall effect

**Condensed matter**

- Part 1
  - many-body Schwinger-Landau-Zener mechanism in strongly correlated insulators
  - $F_{\text{th}}$ calculated exactly

- Part 2
  - cf) next talk by S. Nakamura

- Part 3
  - = control of the parity anomaly in 2+1 Dirac systems via light
Question:
What happens to Dirac particles in circularly polarized light?

High energy answer:
the Volkov state
D. M. Volkov, Z. Phys. 94, 250 1935

Condensed matter answer:
A dynamical gap opens at the Dirac point of graphene making it a photo-induced topological insulator.

TO, H. Aoki, PRB 79, 081406 (R) (2009)

The two are different:
in graphene, speed of massless fermion $< <$ speed of light
momentum of light $\mathbf{l}^\mu = (1, 0, 0, 1)$
$2+1$ Dirac + circular polarization

\[ i\partial_t \psi_k = \begin{pmatrix} 0 & k + A e^{i\Omega t} \\ -\bar{k} + A e^{-i\Omega t} & 0 \end{pmatrix} \psi_k \]

\[ k = k_1 - i k_2 \quad A = F/\Omega \]

Floquet’s quasi-energy

a mass scale is introduced dynamically

“Dynamical topological gap”

\[ \kappa = \frac{\sqrt{4A^2 + \Omega^2} - \Omega}{2} \sim A^2/\Omega \]
Kubo-formula for photo-induced transport

\[ \sigma_{ab}(A_{ac}) = i \int \frac{d\mathbf{k}}{(2\pi)^d} \sum_{\alpha, \beta \neq \alpha} \frac{[\epsilon_{\beta}(\mathbf{k}) - f_{\alpha}(\mathbf{k})]}{\epsilon_{\beta}(\mathbf{k}) - \epsilon_{\alpha}(\mathbf{k})} \langle \langle \Phi_{\alpha}(\mathbf{k}) | J_b | \Phi_{\beta}(\mathbf{k}) \rangle \rangle \langle \langle \Phi_{\beta}(\mathbf{k}) | J_a | \Phi_{\alpha}(\mathbf{k}) \rangle \rangle \]

\[ J_{dc}^i = \sigma_{ij}(A_{ac}) E_{dc}^j \]

Large \( A_{ac} \) small \( E_{dc} \)

\[ \epsilon_{\alpha} \text{ Floquet's quasi-energy} \]
\[ f_{\alpha} \text{ occupation fraction} \]

inner product = time average

\[ \langle \langle \Phi_{\alpha} | \Phi_{\beta} \rangle \rangle = \frac{1}{T} \int_{0}^{T} \langle \Phi_{\alpha}(t) | \Phi_{\beta}(t) \rangle \]

2+1 Dirac fermion has a parity anomaly

"Furry picture"

TO and H. Aoki, Phys. Rev. B 79, 081406 (R) (2009)
Extended Thouless-Kohmoto-Nittingale-Nijis formula for photo-induced Hall conductivity (photo-induced Chern form)

\[
\sigma_{xy}(A_{ac}) = e^2 \int \frac{d\mathbf{k}}{(2\pi)^d} \sum_{\alpha} f_{\alpha}(\mathbf{k}) \left[ \nabla_{\mathbf{k}} \times A_{\alpha}(\mathbf{k}) \right]_z
\]

photo-induced gauge field \( A_{\alpha}(\mathbf{k}) \equiv -i \langle \langle \Phi_{\alpha}(\mathbf{k}) | \nabla_{\mathbf{k}} | \Phi_{\alpha}(\mathbf{k}) \rangle \rangle 

Floquet states (time-dependent solution)

photo-induced Berry’s curvature for graphene

TO and H. Aoki, Phys. Rev. B 79, 081406 (R) (2009)
Static current in circularly polarized light

DC-component of the current

\[ \mu_L - \mu_R = V > 0 \]

Parallel current \( J_x \)

Hall current \( J_y \)

Photo-induced Hall conductivity

\[ J_y = G_{xy} V \]

\[ G_{xy} \propto F^2 \]

Experimentally observable!

\[ G \]

\[ G_{xx} \]

\[ G_{xy} \]

IV-characteristics

\[ F = 0.02 \]

\[ F = 0.01 \]

\[ F = 0.00 \]
List of correspondences (summary)

High energy

Schwinger mechanism = (Landau-)Zener breakdown

Heisenberg-Euler’s Effective action = non-adiabatic geometric phase

Part 1

vacuum polarization = extended Berry’s phase

theory of polarization

Part 2

pair creation in interacting systems (e.g. QCD)

\( \sim \) many-body Schwinger-Landau-Zener mechanism in strongly correlated insulators

\( F_{th} \) calculated exactly

cf) next talk by S. Nakamura

Part 3

Dirac particles in circularly polarized light

\( \neq \) Photovoltaic Hall effect

= control of the parity anomaly in 2+1 Dirac systems via light
DDP path for the Hubbard model

T. Oka, H. Aoki, Phys. Rev. B 81, 033103 (2010)
all charge rapidities become complex

\[
L k_j = 2\pi I_j + L \phi + \sum_{\alpha=1}^{N_L} \theta(\sin k_j - \lambda_{\alpha}),
\]

\[
\sum_{j=1}^{L} \theta(\sin k_j - \lambda_{\alpha}) = 2\pi J_{\alpha} - \sum_{\beta=1}^{N_L} \theta \left( \frac{\lambda_{\alpha} - \lambda_{\beta}}{2} \right),
\]

|1; \phi\rangle = |0; \phi\rangle = \frac{C}{\pi + i b_{\sigma}}

groundstate studied in
Fukui, Kawakami, Phys. Rev. B 58, 160501 (1998)
Floquet’s method (a way to obtain the Volkov’s solution)
circular polarization

\[ i\partial_t \psi_k = \begin{pmatrix} 0 & k + Ae^{i\Omega t} \\ \bar{k} + Ae^{-i\Omega t} & 0 \end{pmatrix} \psi_k \]

\[ k = k_1 - ik_2 \quad A = F/\Omega \]

Floquet Hamiltonian (Fourier transform)

\[ H^{mn} = \frac{1}{T} \int_0^T dt H(t) e^{i(m-n)\Omega t} + m\delta_{mn}\Omega I \]

Floquet Hamiltonian

\[ H^{Floquet}_{CP} = \begin{pmatrix} \Omega & k & 0 & A & 0 & 0 \\ \bar{k} & \Omega & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & k & 0 & A \\ A & 0 & \bar{k} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -\Omega & k \\ 0 & 0 & A & 0 & \bar{k} & -\Omega \end{pmatrix} \]

infinite dim.

TO, H. Aoki, PRB 79, 081406 (R) (2009)
circular polarization

\[
H_{CP}^+ = \begin{pmatrix}
\Omega & k & 0 & A & 0 & 0 \\
-k & \Omega & 0 & 0 & 0 & 0 \\
A & 0 & \bar{k} & 0 & 0 & 0 \\
0 & 0 & 0 & -\Omega & k & 0 \\
0 & 0 & \bar{A} & 0 & -\bar{k} & -\Omega
\end{pmatrix}
\]

level repulsion with \(-\Omega\)

level repulsion with \(\Omega\)

\[
\sim \begin{pmatrix}
\kappa/2 & k \\
-k & -\kappa/2
\end{pmatrix}
\]

"Dynamical topological gap"

\[
\kappa = \frac{\sqrt{4A^2 + \Omega^2} - \Omega}{2} \sim A^2/\Omega
\]