Relativistic light synthesis of femtosecond sawtooth pulses

K. Hu\textsuperscript{1} and H.-C. Wu\textsuperscript{1,2,*}

\textsuperscript{1}Institute for Fusion Theory and Simulation and Department of Physics, Zhejiang University, Hangzhou 310027, China
\textsuperscript{2}IFSA Collaborative Innovation Center, Shanghai Jiao Tong University, Shanghai 200240, China

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Formation of femtosecond sawtooth pulses by high harmonic generation from relativistic oscillating mirrors is studied. For oblique incidence of $p$-polarized laser pulse, one can efficiently control the intensity of the first few harmonics by modifying the plasma density gradient at the plasma-vacuum interface. With appropriate choice of the laser amplitude and incidence angle, the generated harmonics can form a millijoule femtosecond sawtooth pulse. The scheme does not rely on phase manipulation and/or pulse synchronization as in the state-of-art methods. It is also shown that such a sawtooth pulse can boost the generation of ultrashort terahertz pulses in laser-plasma interaction by one order of magnitude, as compared with that from the conventional two-color laser pulse.

Ultrashort terahertz (THz) electromagnetic pulses can be produced from the interaction of fs multicolor pulses with gas targets [1–4]. Martínez et al. pointed out that by using a fs sawtooth pulse one can increase the THz radiation efficiency to 2\%, or 50 times higher than that from the standard two-color configuration [5]. The sawtooth pulse can optimize the driven electron trajectories to maximize their velocities. Realization of fs sawtooth pulses is therefore of high priority.

A sawtooth wave can be formed by linear superposition of harmonic waves, and the wave electric field can be written as

$$E(t) = \sum_{k=1}^{N} f(t) \frac{E_1}{k} \cos(k\omega_1 t - \frac{\pi}{2}),$$

where $f(t)$ is the envelope of the sawtooth pulse, and $E_1$ and $\omega_1$ are amplitude and the frequency of the fundamental harmonic, respectively, and $k$ is the harmonic order. In fact, to a high degree of accuracy the sawtooth waveform can be synthesized with just the fundamental and the next few harmonics. In fact, Martínez et al. [5] found that generation of THz radiation is most efficient when only the first few harmonics are included, since the amplitude saturates with addition of higher harmonics. Thus, the properties of the first few harmonics are of critical importance for THz radiation wave generation.

Sawtooth lasers can be generated by coherent pulse synthesis [6–8], and pulse duration of the order of a nanosecond and energy of about 1 mJ has been achieved [9, 10]. Sub-bands with different frequencies are first obtained from one or several broadband pulses, usually generated by laser interaction with gas or solid targets, and then undergo phase manipulation and amplitude manipulation. A successful synthesis requires manipulation of three parameters: (i) the carrier phase of each sub-band, (ii) the amplitude of each sub-band, and (iii) the relative delay between the sub-bands. Currently, two main schemes are widely adopted [11]. In the first scheme, each sub-band undergoes amplitude and phase manipulation in separate modules before being combined. Since the sub-bands travel over very different optical paths, relative delay and the phase jitter are inevitable, making manipulation of the first two parameters quite challenging. In contrast, in the other approach the sub-bands propagate and are adjusted as a whole, so that it is difficult to accurately adjust the amplitudes and the carrier phases.

In this paper we consider generation of intense fs sawtooth laser pulse by synthesis of relativistically intense laser light through high-harmonic generation (HHG) from relativistic oscillating mirrors (ROM). HHG from intense laser pulses interacting with solid targets has been considered as a promising method for production of bright ultrashort bursts of x-ray and extreme-ultraviolet radiation [12–15]. When an ultraintense laser pulse impinges on a solid target, electrons at the plasma surface are first driven out into vacuum by the laser field, and then pushed back towards the target. As the electrons are pulled out of the plasma, they form an ROM that specularly reflects the driving laser light, producing a pulse consisting of a series of harmonics [16–19]. The first few harmonics can have relativistic intensities.

The major advantage of the proposed scheme is that it avoids problems associated with phase modulation and pulse synchronization. The harmonics generated by ROM are locked in phase: there is no appreciable phase chirping and all harmonics have the same carrier phase [20–22]. Besides, the single laser drive ensures synchronization and collinearity of the harmonics. The peaks of the harmonics coincide and the relative delay is almost zero. That is, synthesis is self accomplished in the process of HHG generation. The only parameter that needs careful modulation is the harmonic amplitude, and this can be achieved by introducing an adjustable exponential density gradient at the target-vacuum interface [22].

We have studied the effects of this gradient length, the target density, laser intensity, and incidence angle. And phase properties of the generated harmonics under our parameters are analysed. Synthesis of several fs relativistic harmonics makes the sawtooth pulse four to five
orders of magnitude shorter and more powerful than that produced by conventional coherent pulse synthesis.

HHG from ROM dominates at relativistic intensities, when the normalized vector potential of the incident laser pulse is close to or larger than unity \cite{17, 22, 23}. According to the selection rule proposed by Lichters et al. \cite{18}, only oblique incidence of \( p \)-polarized laser produces both odd and even harmonics in the same polarization direction, as required for producing a sawtooth pulse. Our main aim is to investigate the parameters that are relevant to the harmonic amplitudes in HHG from ROM. Particle-in-cell (PIC) simulations with the one-dimensional PIC code JPIC \cite{24}, serve as numerical experiments for obtaining the harmonic spectra. In particular, we consider the effects of the \( 1 \) laser amplitude \( a_0 = eE_0/\omega_0 mc \); \( 2 \) electron density \( n_0/n_c \), where \( n_c = e_0 m \omega_0^2 / c^2 \) is the critical density; \( 3 \) angle of incidence \( \theta \); and \( 4 \) density gradient scalelength \( L/\lambda_0 \) at the plasma-vacuum interface, where \( n \propto \exp(x/L) \). Here \( e \) and \( m \) are the charge and the mass of the electron, \( c \) is the speed of light. \( \lambda_0 \) and \( \omega_0 \) are the wavelength and the frequency of the incident laser, respectively.

We start with the case where a \( p \)-polarized \( a_0 = 1 \) laser beam obliquely impinges on a plasma layer at \( \theta = 45^\circ \). The vector potential of the laser is \( \mathbf{a} = a_0 \cos(\omega_0(t - x/c))\exp(-t^2/T^2)\hat{y} \), where \( T = 5\lambda_0/c \) is the duration of the laser. For \( \lambda_0 = 800 \) nm, the FWHM intensity of the laser is 15.75 fs. The plasma layer is of density \( n_0/n_c = 100 \) and initial thickness \( 1.5\lambda_0 \). The density gradient scalelength at the plasma-vacuum surface is \( L/\lambda_0 = 0.6 \). The ions remain fixed, since they move very little on the time scale of interest and thus have negligible effect on the harmonics.

The waveform and the spectrum of the reflected light are shown in Fig. 1. One can see that a nearly Gaussian sawtooth laser pulse is formed. The pulse has 527 GW in peak power and 9.84 mJ in energy. The FWHM of the pulse intensity is 14.9 fs. From Fig. 1(b), one can see that the generated waveform is very similar to that obtained from Eq. (1). This is because the amplitudes of the \( 2 \) to \( 5 \) generated harmonics (normalized to the amplitude of the fundamental harmonic) are \( a_{s2} = 0.51, a_{s3} = 0.26, a_{s4} = 0.16, a_{s5} = 0.064 \), which are not far from the desired values \( a_{sk} = 1/k \), as illustrated in Fig. 1(c). Among them \( a_{s2} \) affects the waveform most, thus requiring particular attention. The harmonics higher than the \( 4 \)th or \( 5 \)th have little effect due to their small amplitudes. In the following, we shall mainly focus on the second harmonic.

Next we consider the effects of the density scalelength for \( L/\lambda_0 = 0.01 \) to \( 0.1 \), for the laser intensities \( a_0 = 0.5, a_0 = 0.75, a_0 = 1, a_0 = 2, \) and \( a_0 = 5 \), with the other parameters the same as in Fig. 1. For a fixed \( a_0 \), increasing the scalelength will first boost up the harmonic amplitude quickly. Then \( a_{s2} \) approaches a saturation value and may
even decrease a little when \( L/\lambda_0 \) gets close to 0.1. The increase can be attributed to smaller restoring force and larger amplitude of the oscillations induced by the longer density gradient. (We have excluded the \( L/\lambda_0 > 0.1 \) cases because the corresponding reflected pulses contain too much noise.) The same trend occurs for the case shown in Fig. 2(b), where the incidence angle is changed to \( \theta = 30^\circ \). Both figures also illustrate that emission of the second harmonic increases greatly for larger laser intensity, especially for longer scalelength. One can see that \( a_{s2} \) falls off significantly for \( a_0 = 0.5 \) and \( a_0 = 0.75 \) when \( L/\lambda_0 > 0.6 \). This can be attributed to the fact that coherent wake emission (CWE) starts to play a role in the process. In order to produce sawtooth pulses stably, we shall set \( a_0 \geq 1 \).

To see how the plasma density is related to the harmonic generation, we have carried out simulations for \( n_0/n_e = 50, 100, \) and 200. In Figs. 2(c) and 2(d), for \( a_0 = 1 \) and 5, respectively, and \( \theta = 45^\circ \), one can clearly see that the plasma density has little effect on the intensity of the second harmonic. This is because the range of the electron bunch’s motion is limited within the density gradient, thus has little to do with the maximum density of the plasma target. Figures 2(e) and 2(f) show the relationship between the incidence angle and the amplitude of the second harmonic. Electrons of the ROM are driven by the ponderomotive force, determined by the vertical component of the laser field, as well as by direct action of the parallel component. PIC simulations show that the total contribution of these two effects boosts the harmonic amplitude to a maximum at \( \theta = 45^\circ \) to \( 60^\circ \).

Another point that should be considered is the relative phases of the harmonics. Previous studies have shown that harmonics have no phase difference in the case \( L/\lambda_0 \ll 0.1 \) [20]. To test if this conclusion still stands when a longer gradient of \( L/\lambda_0 \sim 0.1 \) is introduced, we have analyzed the phases of the first three harmonics in the case shown in Fig. 1(a). The results are visualized in Fig. 3. Each curve represents the waveform of one harmonic, obtained by Fourier transform, containing information of the frequency, amplitude, and phase. One can see that the phase difference of the harmonics is almost zero (marked by the red arrows), in agreement with Eq. (1). That is, in our scheme the phase properties of the harmonics naturally meet the condition for a sawtooth wave.

As discussed above, fs sawtooth pulses can be produced by choosing appropriate parameters such as the density gradient scalelength, laser intensity and incidence angle. As the density gradient scalelength becomes longer, PIC simulations first show a fast rise of harmonic intensities, follow by progressive saturation, when the scalelength is larger then 0.06\( \lambda_0 \). The saturation value increases for larger laser intensity and reaches its maximum for an incidence angle between 45° to 60°. The key point is to adjust the saturation value of the second harmonic to near 0.5 by selecting proper laser amplitude and incidence angle. No phase manipulation and synchronization modules are needed and the generated sawtooth pulse can be directly impinged on the gas jet for THz radiation production. The schematic drawing is given in Fig. 4(a).

Next we check if the obtained sawtooth pulse can boost THz radiation generation from a He target, using 1D PIC simulation. The process of ionization is added to the code and the ionization potentials are 24,587 eV for He\( \rightarrow \)He\(^+\) and 54,416 eV for He\(^+\)\( \rightarrow \)He\(^{2+}\). Two sets of simulations are performed, and each set contains seven simulations with different laser intensities. In the first set, the sawtooth wave shown in Fig. 1(a) is used as the incident pulse, and the field strengths are...
a_{11} = 1.4, a_{12} = 0.6a_{11} = 0.84, a_{13} = 0.4a_{11} = 0.56, 
a_{14} = 0.2a_{11} = 0.28, a_{15} = 0.1a_{11} = 0.14, a_{16} = 0.05a_{11} = 0.07 
and a_{17} = 0.02a_{11} = 0.03. In the experiments, one can adjust the laser strength by changing the distance between the laser’s focusing point and the target. For comparison, in the second set we use Gaussian p-polarized, two-color laser pulses with the temporal profile 
a_{1} \times exp(-t^{2}/T_{1}^{2}) \cos[\omega_{1}(t-x/c) - \pi/2] + 
a_{1}/2 \times exp(-t^{2}/T_{1}^{2}) \cos[2\omega_{1}(t-x/c) - \pi/2]. \) We consider seven values for \( a_{11} \), and set \( \lambda_{1} = 2\pi c/\omega_{1} = 800nm \) and \( T_{1} = 2.74\lambda_{1}/c \) to ensure that these pulses have the same powers and the same maximum field strengths as the corresponding ones in the first set. The simulation box is \( 70\lambda_{1} \) in the x-direction and the cell number per \( \lambda_{1} \) is 100. The He target has a length of \( 10\lambda_{1} \) and its atomic density is \( 0.00125n_{c} \). After full ionization, the plasma density is \( n_{e} = 0.0025n_{c} \), and the corresponding plasma frequency is \( 18.7 \) THz. We concentrate on the backward THz radiation with frequency lower than \( 20 \) THz. The higher frequency radiation is blocked by a low-pass filter.

Figure 4(a) shows the THz radiation for two types of incident lasers. The lasers have a peak amplitude of \( a_{i} = 0.14 \), corresponding to an intensity of \( I_{i} = 4.21 \times 10^{16} \) W/cm\(^2\). For such high laser strength, 99.5% of the electrons contained in neutral He atoms are ionized by the incident laser pulse. It is obvious that the THz radiation in the sawtooth case has a peak electric field of \( 14.24 \) MV/cm, 2 times larger than that in the two-color case. Figure 4(b) summarizes the peak amplitudes of the THz radiation for different laser intensities in both cases. We see that with \( a_{i} > 0.7 \) (\( \sim I_{i} > 10^{16} \) W/cm\(^2\)), the sawtooth pulse obtained from our HHG scheme can effectively increase the THz radiation field strength by 2 to 3 times, compared to that from the two-color pulse. If the spot sizes are the same in both cases, the efficiency boost can be several times to one order of magnitude larger.

The boost cannot be observed when \( a_{i} = 0.03 \), since the laser is too weak and less than 6% of the target electrons are ionized.

We have also carried out a third set of simulations, in which the approximate sawtooth wave generated by the \( a_{0} = 5 \) laser pulse (the green diamonds in Fig. 2(a)) is adopted. Again, the maximum electric fields are the same as in the previous two sets. The results are shown by the blue triangles in Fig. 4(c). Surprisingly, the resulting THz radiation pulses have even larger amplitudes than that in the first set. This can be attributed to the fact that the incident pulses in the third set contain stronger higher-order harmonics as compared to the sawtooth pulses used in the first set.

In summary, in this paper we have presented a robust method for generating a sawtooth pulse via synthesis of relativistic fs harmonics. The latter are generated by laser interaction with plasma targets. By introducing a density gradient scalelength of \( 0.06\lambda_{0} - 0.1\lambda_{0} \) at the plasma-vacuum interface, we can effectively increase the amplitudes of the first few harmonics to meet the requirement for forming a sawtooth wave. By proper choice of laser amplitude and incidence angle, a fs gigawatt sawtooth pulse can be generated. Such a sawtooth pulse can boost the generated THz radiation by several times, compared with that from the two-color pulse with the same power. Finally, we also note that the present scheme completely avoids the complicated process of phase modulation and pulse synchronization that is required in the conventional methods for generating THz radiation.

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