Perturbative Fragmentation of Leptoquark into Heavy Lepto-Quarkonium

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Abstract

The fragmentation function of a scalar leptoquark into possible S-wave bound states with a heavy anti-quark is calculated to the leading order in perturbative QCD for the high energy processes at large transverse momenta. The one-loop equations for the $q^2$-evolution of moments of the fragmentation function due to the hard gluon emission by the leptoquark are derived. The integral probabilities of fragmentation are evaluated.

1 Introduction

An abundant discussion about a possible appearance of new physics at HERA [1,2,3] has provided an important and useful experience for a handling of almost-real properties attributed to interactions beyond the Standard Model. So, if a leptoquark [4] is the correct interpretation for the excess in the HERA data at high $Q^2$, $x$, then the former has the total width $\Gamma$, which is much less than the QCD-confinement scale, $\Gamma_{LQ} \ll \Lambda_{QCD}$ [5]. The latter fact means that before a decay, the color-triplet leptoquark $LQ$ can be bound with quarks in the lepto-hadrons: $(\bar{qLQ})$-baryons or $(q_1q_2LQ)$-mesons, which are quite exotic states as well as the double LQonia: $(\overline{LQ_1}LQ_2)$.

It is an attractive and interesting problem to study the spectroscopy, production and decays of such hadrons as a possible window of new physics independently of a success or fall off the treatment of the HERA events.

The description of leptoquarks bound with light quarks is a subject of Leptoquark Effective Theory, which can be developed as a straightforward continuation of analogous Heavy Quark Effective Theory [6] with taking a care on the spin structure of leptoquark. The heavy leptoquarkonia: $(\bar{bLQ})$ and $(\bar{cLQ})$, can be considered in the framework of Non-Relativistic QCD [7] keeping in mind again the spin features.

In this work we discuss the high energy production of heavy lepto-quarkonium containing a scalar leptoquark.

The model-independent pair production of free leptoquarks in hadronic collisions was considered in [8] with account for the next-to-leading order QCD corrections. Attaching the result to the Tevatron search for the scalar leptoquarks [9], the authors have found the constraint $m_{LQ} > 190$ GeV.

At high transverse momenta, the dominant production mechanism for the heavy leptoquarkonium bound states is the leptoquark fragmentation, which can be calculated in perturbative QCD [10] after the isolation of soft-binding factor extracted from the non-relativistic potential models [11, 12]. The corresponding fragmentation function is universal for any high energy process for the direct production of lepto-quarkonia.
In the leading $\alpha_s$-order, the fragmentation function has a scaling form, which is the initial one for the perturbative QCD evolution caused by the emission of hard gluons by the lepto-quark before the hadronization. The corresponding splitting function differs from that for the heavy quark because of the spin structure of gluon coupling to the lepto-quark.

In this work, the LO-scaling fragmentation function is calculated in Section 2. The limit of infinitely heavy lepto-quark, $m_{LQ} \to \infty$, is obtained from the full QCD consideration for the fragmentation. The splitting kernel of the DGLAP-evolution is derived in Section 3, where the one-loop equations of renormalization group for the moments of fragmentation function are obtained and solved. The mentioned equations are universal, since they do not depend on whether the leptoquark will bound or free at low virtualities, where the perturbative evolution stops. The integrated probabilities of leptoquark fragmentation into the heavy lepto-quarkonia are evaluated in Section 4 with making the use of non-relativistic wave-functions for the bound states. The results are summarized in Conclusion.

2 Fragmentation function in leading order

The contribution of fragmentation into the direct production of heavy lepto-quarkonium has the form

$$d\sigma[l_H(p)] = \int_0^1 dz \ d\hat{\sigma}[LQ(p/z), \mu] \ D_{LQ\to l_H}(z, \mu),$$

where $d\sigma$ is the differential cross-section of lepto-quarkonium with the 4-momentum $p$, $d\hat{\sigma}$ is that of the hard production of leptoquark with the scaled momentum $p/z$, and $D$ is interpreted as the fragmentation function depending on the fraction of momentum carried out by the bound state. The value of $\mu$ determines the factorization scale. In accordance with the general DGLAP-evolution, the $\mu$-dependent fragmentation function satisfies the equation

$$\frac{\partial D_{LQ\to l_H}(z, \mu)}{\partial \ln \mu} = \int_z^1 \frac{dy}{y} \ P_{LQ\to LQ}(z/y, \mu) \ D_{LQ\to l_H}(y, \mu),$$

(1)

where $P$ is the kernel caused by the emission of hard gluons off the leptoquark leg before the production of heavy quark pair. Therefore, the initial form of fragmentation function is determined by the diagram shown in Fig.1, and, hence, the corresponding initial factorization scale is equal to $\mu = 2m_Q$. Furthermore, this function can be calculated as an expansion in $\alpha_s(2m_Q)$. The leading order contribution is evaluated in this Section.

Consider the fragmentation diagram in the system, where the momentum of initial lepto-quark has the form $q = (q_0, 0, 0, q_3)$ and the lepto-quarkonium one is $p$, so that

$$q^2 = s, \ p^2 = M^2.$$
In the static approximation for the bound state of leptoquark and heavy quark, the quark mass is expressed as \( m_Q = rM \), and the leptoquark mass equals \( m = (1 - r)M \).

The matrix element has the form

\[
\mathcal{M} = -\frac{2\sqrt{2}\pi\alpha_s}{3\sqrt{3}M^2} \frac{R(0)}{r(1 - r)(s - m^2)} (q'^\mu + (1 - r)p'^\mu) \rho_{\mu\nu} \bar{q} \gamma^\nu (\hat{p} - M) l_H \mathcal{M}_0, \tag{2}
\]

where the sum over the gluon polarizations is written down in the axial gauge with \( n = (1, 0, 0, -1) \)

\[
\rho_{\mu\nu}(k) = -g_{\mu\nu} + \frac{k_\mu n_\nu + k_\nu n_\mu}{k \cdot n},
\]

with \( k = q - (1 - r)p \). The spinors of \( l_H \) and \( \bar{q} \) correspond to the lepto-quarkonium and heavy quark associated to the fragmentation. \( \mathcal{M}_0 \) denotes the matrix element for the hard production of leptoquark at high energy, \( R(0) \) is the radial wave-function at the origin.

Define

\[
z = \frac{p \cdot n}{q \cdot n}
\]

The fragmentation function is determined by the expression

\[
D(z) = \frac{1}{16\pi^2} \int ds \theta \left( s - \frac{M^2}{z} - \frac{m_Q^2}{1 - z} \right) \frac{|\mathcal{M}|^2}{|\mathcal{M}_0|^2},
\]

in the limit of high energies \( q \cdot n \to \infty \). Then one can straightforwardly find

\[
D(z) = \frac{8\alpha_s^2}{27\pi} \frac{|R(0)|^2}{M^3r^2(1 - r)^2} \frac{z^2(1 - z)^2}{(1 - (1 - r)z)^6} \left[ (1 + r^2)(1 + (1 - r)^2z^2) - 2(1 - r)^2(1 + rz) \right], \tag{3}
\]

which tends to

\[
\tilde{D}(y) = \frac{8\alpha_s^2}{27\pi} \frac{|R(0)|^2}{m_Q^4} \frac{(y - 1)^2}{y} \left( \frac{4}{y^6} + \frac{1}{y^4} \right), \tag{4}
\]

at \( r \to 0 \) and \( y = (1 - (1 - r)z)/(rz) \). The coefficient at the \((y - 1)^2\) term is the same as in the fragmentation of heavy quark with the mass \( m \) into the S-wave states of the heavy quarkonium at \( y \to 1 \), if one excepts the factor related with the wave-function of final state. The limit of \( \tilde{D}(y) \) is in agreement with the general consideration of \( 1/m \)-expansion for the fragmentation function \( \tilde{D}(y) \), where

\[
\tilde{D}(y) = \frac{1}{r} a(y) + b(y).
\]

Eq.(4) determines the \( a(y) \)-function explicitly.

### 3 Hard gluon emission

The one-loop contribution of hard gluon emission can be calculated in the way described in previous Section. Then the splitting kernel of the leptoquark is equal to

\[
P_{LQ \to LQ}(x, \mu) = \frac{4\alpha_s(\mu)}{3\pi} \left[ \frac{2x}{1 - x} \right]_+, \tag{5}
\]
where the “plus” denotes the ordinary action: \( \int_0^1 dx f_+(x) \cdot g(x) = \int_0^1 dx f(x) \cdot [g(x) - g(1)] \).

The scalar leptoquark splitting function can be compared with that of the heavy quark

\[
P_{Q \rightarrow Q}(x, \mu) = \frac{4\alpha_s(\mu)}{3\pi} \left[ \frac{1 + x^2}{1 - x} \right]_+,
\]

which has the same normalization factor at \( x \rightarrow 1 \).

Further, multiplying the evolution equation by \( z^n \) and integrating over \( z \), one can get from eq.(1) the \( \mu \)-dependence of moments \( a_n \) for the fragmentation function to the one-loop accuracy of renormalization group,

\[
\frac{\partial a_n}{\partial \ln \mu} = -8\alpha_s(\mu) \left[ \frac{1}{2} + \ldots + \frac{1}{n + 1} \right] a_n, \quad n \geq 1. \tag{6}
\]

At \( n = 0 \) the right hand side of (6) equals zero, which means that the integral probability of leptoquark fragmentation into the heavy lepto-quarkonium is not changed during the evolution, and it is determined by the initial fragmentation function calculated perturbatively in previous Section.

Figure 2: The QCD-evolution for the averaged fraction of scalar leptoquark momentum, as it is developed in the fragmentation with account for the gluon emission to the scale \( \mu \), characterizing the hard production of leptoquark, from the initial value chosen \( \mu_0 = 2m_c \).

The solution of eq.(6) has the form

\[
a_n(\mu) = a_n(\mu_0) \left[ \frac{\alpha_s(\mu)}{\alpha_s(\mu_0)} \right]^{\frac{16n}{\beta_0} \left[ \frac{1}{2} + \ldots + \frac{1}{n + 1} \right]}, \tag{7}
\]

where one has used the one-loop expression for the QCD coupling constant

\[
\alpha_s(\mu) = \frac{2\pi}{\beta_0 \ln(\mu/\Lambda_{QCD})},
\]
Figure 3: The w-function for the leptoquark fragmentation into the heavy lepto-quarkonium versus the fraction of \( r = m_Q/m \).

where \( \beta_0 = 11 - 2n_f/3 \) with \( n_f \) being the number of quark flavors with \( m_q < \mu < m_{LQ} \).

Relation (7) is universal one, since it is independent of whether the leptoquark is free or bound at the virtualities less than \( \mu_0 \). In this work we include the evolution for the fragmentation into the heavy lepto-quarkonium.

As one can see in Fig.2, the leptoquark can lose about 20% of its momentum before the hadronization.

4 Integral probabilities of fragmentation

As has been mentioned above the evolution conserves the integral probability of fragmentation, which can be calculated explicitly from eq.(3)

\[
\int dz \, D(z) = \frac{8\alpha_s^2}{27\pi} \frac{|R(0)|^2}{m_Q^3} \, w(r),
\]

\[
w(r) = \frac{[7 + 30r + 20r^2 + 20r^3 - 75r^4 - 2r^5 + 30r(1 + r + 3r^2 + r^3) \ln r]}{15(1 - r)^7}.
\]

The function of \( w(r) \) is shown in Fig.3 at low \( r \).

To estimate numerically the yield of \( (\bar{b}LQ) \) and \( (\bar{c}LQ) \), one has to evaluate the radial wave function at the origin from the non-relativistic potential models. As was found in [15, 16], the Martin and Buchmüller–Tye potentials give close results for the heavy quarkonia with the reduced mass about \( m_c \sim 1.5 \text{ GeV} \). The characteristics of charmed and beauty lepto-quarkonia are presented in Tab.1, where we have used the Martin potential in the limit of infinitely heavy leptoquark, \( m \gg m_Q \), so that the reduced mass equals the heavy quark mass, and the level
energy is given by the sum of quark mass and the binding energy evaluated numerically from
the Schrödinger equation.

Table 1: The radial wave-functions at the origin, level energies and average sizes of heavy
lepto-quarkonia, as evaluated in Martin potential.

| level         | \( R(0) \), GeV\(^{3/2} \) | E, GeV | \( \langle r \rangle \), fm |
|---------------|-------------------------------|--------|--------------------------|
| 1S(\( \bar{c}LQ \)) | 1.61                          | 1.023  | 0.27                     |
| 2S(\( \bar{c}LQ \)) | 1.20                          | 1.606  | 0.58                     |
| 1S(\( \bar{b}LQ \)) | 3.43                          | 4.039  | 0.16                     |
| 2S(\( \bar{b}LQ \)) | 2.56                          | 4.594  | 0.35                     |

Then one finds for the probabilities of leptoquark fragmentation into the 1S-states with
\( b \) and \( c \)-quarks: \( P_w(b) = 1.22 \cdot 10^{-4} \) and \( P_w(c) = 2.16 \cdot 10^{-3} \), respectively, at \( m_b = 4.9 \) GeV,
\( m_c = 1.5 \) GeV, \( \alpha_s(2m_b) = 0.18 \), \( \alpha_s(2m_c) = 0.26 \) and \( m = 245 \) GeV.

The perturbative fragmentation function in the leading \( \alpha_s \)-order is shown in Fig.4 at \( r = 0.02 \). It is quite a hard distribution, which becomes softer with the evolution (see Fig.4).

Figure 4: The fragmentation function of leptoquark into the heavy lepto-quarkonium, the \( N \)-
factor is determined by \( N = \frac{8\alpha_s^2}{27\pi} \frac{|R(0)|^2}{M^2r^2(1-r)^2} \), the initial function – solid line, for the \( (\bar{b}LQ) \)-state
with \( r = 0.02 \), the fragmentation function including the evolution – dashed line, at the scale
\( \mu: \frac{8\alpha_s}{3\pi} \ln \frac{\mu}{\mu_0} = 0.25 \).
5 Conclusion

In this work the dominant mechanism for the production of possible bound states of a scalar leptoquark with a heavy anti-quark is considered for high energy processes at large transverse momenta, where the fragmentation contributes as the leading term. The corresponding fragmentation function of scalar leptoquark into the heavy lepto-quarkonium can be calculated in perturbative QCD, so that for the S-wave states one finds

\[ D(z) = \frac{8\alpha_s^2}{27\pi} \frac{|R(0)|^2}{M^3 r^2(1-r)^2} \frac{z^2(1-z)^2}{(1-(1-r)z)^6} \left[ (1+r^2)(1+(1-r)^2 z^2) - 2(1-r)^2(1+r)z \right], \]

where \( r \) is the ratio of heavy quark mass to the mass of the bound state. In the infinitely heavy leptoquark limit, \( D(z) \) has the form, which agrees with what expected from the general consideration of \( 1/m \)-expansion for the fragmentation functions.

The hard gluon corrections caused by the splitting of scalar leptoquark are taken into account so that the evolution kernel has the form

\[ P_{LQ \rightarrow LQ}(x, \mu) = \frac{4\alpha_s(\mu)}{3\pi} \frac{2x}{1-x}, \]

which results in the corresponding one-loop equations for the moments of fragmentation function (see eqs. (6), (7)).

The integral probabilities of scalar leptoquark fragmentation into the charmed and beauty lepto-quarkonia are of the order of \( 10^{-3} \) and \( 10^{-4} \), correspondingly.

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