Reheating in non-minimal derivative coupling model

H. Mohseni Sadjadi * and Parviz Goodarzi
Department of Physics, University of Tehran,
P. O. B. 14395-547, Tehran 14399-55961, Iran

May 10, 2014

Abstract

We consider a model with non-minimal derivative coupling of inflaton to gravity. The reheating process during rapid oscillation of the inflaton is studied and the reheating temperature is obtained. Behaviors of the inflaton and produced radiation in this era are discussed.

1 Introduction

To solve some problems of standard model of cosmology and particle physics, such as the horizon problem, flatness, isotropy and homogeneity of the universe, the absence of magnetic monopoles and so on, and to make the big bang cosmology more consistent with astrophysical data, the inflation theory, which considers an epoch of accelerated expansion for the early universe, was introduced [1]. One straightforward way to describe this era, is to consider a slowly rolling scalar field, dubbed inflaton, whose energy density was dominated by its potential during inflation [2,3]. At the end of inflation the universe was cold, so there must be a procedure through which the scalar field decayed to particles which became thermalized and reheated the universe. This could be realized by decaying of the scalar field, e.g. during coherent oscillation in the bottom of the potential [4].

At first sight, it seems that a candidate for this scalar field may be the Higgs boson. But parameters of the standard model do not agree with those required for inflaton [3]. So to reconcile the slow roll inflation with standard model parameters, a framework in which the Higgs boson is non minimally coupled to Ricci scalar has been introduced in [5].

Recently a model comprising a non minimal coupling between the derivatives of the Higgs boson and Einstein tensor has been proposed, which besides its capacity to explain the inflationary phase, is also safe of quantum corrections and unitary violation problem [6]. In this context, the nonminimal derivative coupling may allow the model to describe the acceleration

* mohsenisad@ut.ac.ir

1
as well as the super-acceleration of the universe [8]. Coupling the Einstein tensor to kinetic term of inflaton, as we will see later, enhances the gravitational friction during slow-roll and this allows us to consider more general steep potentials, such as Higgs potential, without contradiction with the CMB observations or collider experimental bound [6].

In [7], this model was employed to study the natural inflation, where the inflaton is assumed to be a pseudo-Nambu-Goldstone boson. In this framework, the global shift symmetry is broken at a scale $f$, giving rise to the inflaton mass. For small field values, the potential is stable against radiative corrections, but slow roll requires that $f$ becomes much larger than the Planck scale, giving rise to eta problem. In [7], it was shown that non-minimal derivative coupling allows to take $f \ll M_P$, without introducing new degrees of freedom, and protects the tree-level shift invariance of the scalar field as well as the perturbative aspects of the theory.

Similar models including nonminimal derivative coupling between a scalar field and gravity have also been used to study the late time evolution of the universe by considering the scalar field as the dark energy [9].

In this manuscript, following [6] we assume that the inflation is implemented by a scalar field with non-minimal derivative coupling to gravity, with a power law potential, and study the reheating process in this model, which to our knowledge, was not studied before. First, we review briefly the inflationary epoch and then study quasi-periodic motion of the inflaton at the end of the slow roll. We consider the decay of the scalar field to ultra-relativistic particles (radiation) via a phenomenological source during coherent rapid oscillation and find the reheating temperature.

2 Non-minimal derivative coupling model and inflation

An action describing a scalar field coupled non-minimally to gravity via its kinetic term is given by [6]:

$$ S = \int \left( \frac{M_P^2 R}{2} - \frac{1}{2} g^{\mu \nu} \partial_\mu \varphi \partial_\nu \varphi + \frac{w}{2} G^{\mu \nu} \partial_\mu \varphi \partial_\nu \varphi - V(\varphi) \right) \sqrt{-g} d^4x, \tag{1} $$

where $G^{\mu \nu}$ is the Einstein tensor, $w$ is a constant with the dimension of inverse mass squared, and $M_P = 2.4 \times 10^{18} GeV$ is the reduced Planck mass. In the absence of terms containing more than two time derivatives, additional degrees of freedom are not produced in this theory.

In the spatially flat Friedmann-Lemaître-Robertson-Walker (FLRW) spacetime, the Friedmann equation is

$$ H^2 = \frac{1}{6M_P^2} \left( 1 + 9wH^2 \right) \dot{\varphi}^2 + 2V(\varphi), \tag{2} $$

\footnote{We use units $\hbar = c = 1$ through the paper.}
and the scalar field equation of motion is given by

\[(1 + 3wH^2)\ddot{\phi} + 3H(1 + 3wH^2 + 2wH)\dot{\phi} + V'(\phi) = 0,\]  

(3)

where "dot" denotes derivative with respect to time and "prime" denotes derivative with respect to \( \phi \).

Using the energy momentum tensor derived from (1), the energy density and the pressure of the scalar field are obtained as

\[\rho_\phi = (1 + 9wH^2)\dot{\phi}^2 + V(\phi),\]

\[P_\phi = \frac{\dot{\phi}^2}{2} - V(\phi) - \frac{w}{2}(3H^2 + 2HH)\dot{\phi}^2 - 2wH\ddot{\phi}\ddot{\phi},\]  

(4)

respectively. \( \rho_\phi \) and \( P_\phi \) satisfy the continuity equation

\[\dot{\rho}_\phi + 3H(\rho_\phi + P_\phi) = 0.\]  

(5)

In the presence of another component, with the energy density \( \rho_R \) and the pressure \( P_R \), interacting with the scalar field via the source term \( Q \), the continuity equation becomes

\[\dot{\rho}_\phi + 3H(\rho_\phi + P_\phi) = -Q,\]

\[\dot{\rho}_R + 3H(\rho_R + P_R) = Q.\]  

(6)

In the inflationary era, we take the field \( \phi \) as the inflaton, and assume that the universe is dominated by only this scalar field. But in the subsequent epochs, one must also take into account the presence of other components such as radiation.

In the following we adopt the high friction condition \[10\]

\[wH^2 \gg 1.\]  

(7)

The slow roll regime is characterized by

\[\ddot{\phi} \ll 3H\dot{\phi}, \quad \frac{|\dot{H}|}{H^2} \ll 1,\]

\[9wH^2\dot{\phi}^2 \ll 2V(\phi),\]  

(8)

yielding

\[H^2 \simeq \frac{V(\phi)}{3M_p^2},\]

\[\dot{\phi} \simeq -\frac{V'(\phi)}{9wH^3}\]  

(9)
The slow-roll conditions are satisfied provided that
\[
\frac{V''(\varphi)}{V'(\varphi)} \ll \frac{3w}{M_P^4},
\]
\[
\frac{V'^2(\varphi)}{V^2(\varphi)} \ll \frac{2w}{M_P^4}.
\]
(10)

In comparison with minimal models, conditions (10) may be satisfied by more steep potentials in the high friction limit (7) \cite{10}.

For the potential
\[
V(\varphi) = \lambda \varphi^q,
\]
(11)
these relations require \(\varphi^{q+2} \gg \frac{M_P^4}{w} \). In the slow-roll era, where \(\dot{H} \ll H^2\) holds, the requiring that the model be outside of the quantum gravity regime implies that \(R \simeq 12H^2 \ll \frac{M_P^4}{w}\). This condition, in terms of the potential, is rewritten as
\[
V(\varphi) \ll \frac{M_P^4}{8}.
\]
(12)
The number of e-folds is given by
\[
N = \int_{\varphi_e}^{\varphi_0} \frac{H \, dt}{V'(\varphi)} = \frac{w}{M_P^4} \int_{\varphi_e}^{\varphi_0} \frac{V^2(\varphi)}{V'(\varphi)} \, d\varphi,
\]
(13)
where \(\varphi_e (\varphi_0)\) is the value of the scalar field at the end (the beginning) of the slow-roll inflation.

By assuming \(\varphi_e \ll \varphi_0\), one can estimate the e-folds number for a chaotic inflation with potential (11) as
\[
N \simeq \frac{w\lambda}{q(q + 2)M_P^4} \varphi_0^{q+2}.
\]
(14)

When the conditions (10) cease to be valid
\[
\frac{V''(\varphi)}{V'(\varphi)} \simeq \frac{3w}{M_P^4},
\]
\[
\frac{V'^2(\varphi)}{V^2(\varphi)} \simeq \frac{2w}{M_P^4},
\]
(15)
the slow-roll inflation ends. For power law potential (11) and \(q \sim \mathcal{O}(1)\), (15) gives
\[
\varphi_e^{q+2} \simeq \frac{q^2 M_P^4}{3w\lambda}.
\]
(16)
3 Quasi-periodic evolution

In this part we try to study the dynamics of the inflaton after the end of slow-roll. In this era, conditions (8,10) cease to be valid and quasi-periodic evolution of the scalar field about the bottom of the potential begins. In fig. (11), using numerical methods, oscillation of the field $\varphi$ is depicted for quadratic potential

$$V(\varphi) = \frac{1}{2}m^2 \varphi^2.$$  \hspace{1cm} (17)

This figure shows that, at the first stages after the slow-roll, the amplitude of the scalar field drops down rapidly. Later, during a phase of rapid oscillation of the field, the rate of decrease of the amplitude becomes much less than the oscillation frequency. To get an insight about the solutions in this epoch, we proceed in the same way as [11] and consider the power law potential (11), with an even integer $q$.

During the quasi-periodic evolution where the amplitude and the frequency of oscillation are time dependent, the scalar field is presented as [11]

$$\varphi(t) = \Phi(t) \cos \left( \int A(t) dt \right),$$  \hspace{1cm} (18)

where, the amplitude $\Phi(t)$ is given by

$$V(\Phi(t)) = \lambda \Phi^q(t) = \rho_\varphi(t).$$  \hspace{1cm} (19)

This equation means that the potential, when is evaluated at the amplitude, gives all the energy. $A(t)$ is some function of time which may be determined as follows: By taking a time derivative of (18), we easily obtain

$$\sin \left( \int A(t) dt \right) = \frac{\dot{\varphi}}{A\Phi} + \frac{\dot{\Phi}}{A\Phi^2}.$$  \hspace{1cm} (20)
\[(\ref{eq:20}), \text{ and } (\ref{eq:18}) \text{ result in} \]
\[A^2 = \frac{\dot{\varphi}^2 \left(1 - \frac{\dot{\varphi}}{\ddot{\varphi}}\right)^2}{\Phi^2 - \varphi^2}. \quad (\ref{eq:21})\]

The continuity equation and eqs. \[(\ref{eq:4})\] lead to:
\[\dot{\rho}_\varphi = -3H[\dot{\varphi}^2(1 + 3wH^2 - w\dot{H}) - 2wH\dot{\varphi}\ddot{\varphi}], \quad (\ref{eq:22})\]

By substituting \(\ddot{\varphi}\) from the equation of motion \[(\ref{eq:3})\] into the above equation we arrive at
\[\dot{\rho}_\varphi = -3H[(9wH^2 + 3w\dot{H})\dot{\varphi}^2] + 2\dot{\varphi}V_\varphi. \quad (\ref{eq:23})\]

By using \(\frac{\dot{\varphi}}{\varphi} = \frac{1}{q\rho_\varphi}\) derived from \[(\ref{eq:19})\], we deduce
\[\left|\frac{\varphi}{\dot{\varphi}}\right| = \left|\frac{6\varphi}{q\rho_\varphi} \sqrt{\frac{w(\rho_\varphi - V(\varphi))}{2}} (3H^2 + 2\dot{H}) + \frac{2V(\varphi)}{\rho_\varphi}\right| , \quad (\ref{eq:24})\]

where \(+(-)\) corresponds to \(\dot{\varphi} > 0(<0)\). This expression is much less than unity, \(\left|\frac{\varphi}{\dot{\varphi}}\right| \ll 1\), whenever the slow-roll is ceased and

\[\Phi \ll \left(\frac{qM_p^2}{6\sqrt{w}\lambda}\right)^{\frac{1}{w+2}}, \quad (\ref{eq:25})\]

becomes valid. This is opposite to slow-roll conditions (see eqs. \[(\ref{eq:8},\ref{eq:10})\] and their subsequent discussion). In this regime

\[A^2 \approx \frac{\dot{\varphi}^2}{\Phi^2 - \varphi^2} = \frac{2(\rho_\varphi - V(\varphi))}{9wH^2(\Phi^2 - \varphi^2)} = \frac{2M_p^2 (\rho_\varphi - V(\varphi))}{3w\rho_\varphi(\Phi^2 - \varphi^2)}. \quad (\ref{eq:26})\]

\[\left|\frac{\dot{\varphi}}{\varphi}\right| \ll \left|\frac{\dot{\varphi}}{\rho_\varphi}\right|\] is the stage of rapid oscillation or high frequency regime, i.e. when \(\left|\frac{\dot{\varphi}}{\varphi}\right| \ll A\). In this epoch we have

\[\left|\frac{\dot{\varphi}}{\Phi}\right| = \left|\frac{2\dot{H}}{qH}\right| = \left|\frac{1}{q\rho_\varphi}\right| \ll A, \quad (\ref{eq:27})\]

showing that the amplitude, the Hubble parameter and the energy density decrease slowly during one period of oscillation.

The time average of energy density over an oscillation cycle (from \(t\) to \(t + T\)) is

\[\langle \rho_\varphi(t) \rangle := \frac{\int_t^{t+T} \rho_\varphi(t')dt'}{T}, \quad (\ref{eq:28})\]
where $T$ is the period. As the amplitude decreases very slowly, we deduce $\langle \rho(\phi(t)) \rangle = V(\Phi(t))$. This result is in agreement with (19), because from (27), it is clear that $\rho(\phi)$ changes insignificantly during a period.

During the rapid oscillation of the scalar field, the parameter $\gamma$, defined by

$$
\gamma = 1 + \frac{\langle \rho(\phi) \rangle}{\langle \rho(\phi) \rangle}.
$$

(27)

is determined as follows

$$
\gamma = \frac{\langle 3wH^2\dot{\phi}^2 - \frac{d(wH^2)}{dt} \rangle}{\langle \rho(\phi) \rangle}.
$$

(29)

To obtain the above equation we have used (4). Using (4), (29) can be rewritten as

$$
\gamma = \frac{2}{3} \frac{\langle \rho(\phi) - V(\phi) \rangle}{\langle \rho(\phi) \rangle}.
$$

(30)

In the above computations, we have converted time integration to $\phi$ integration, and the variable change $x = \frac{\phi}{\Phi}$ was applied. The same method for $w = 0$, results in $\gamma = \frac{2q}{q+6}$ [11]. This constant is the effective value of $\gamma$ in rapid oscillation era, i.e., in this era the scalar field behaves as a barotropic fluid. To elucidate this point we proceed as [11]. Taking time average of (5), and by using (30), we obtain

$$
\dot{\langle \rho(\phi) \rangle} + \frac{2q}{q+2} H \langle \rho(\phi) \rangle = 0.
$$

(31)

Using the definition of time average, we obtain $\langle \dot{\rho}(\phi) \rangle = \frac{\delta \rho(\phi)}{T}$, where $\delta \rho(\phi)$ is the change of $\rho(\phi)$ over the period $T$. Hence

$$
\frac{\delta \rho(\phi)}{T} + \frac{2q}{q+2} H \langle \rho(\phi) \rangle = 0.
$$

(32)

In the high frequency regime $\frac{\delta \rho(\phi)}{T} \approx \dot{\rho}(\phi)$, and we arrive at

$$
\dot{\rho}(\phi) + \frac{2q}{q+2} H \rho(\phi) = 0.
$$

(33)

In the following, as in (33), we will not use the symbol $<>$ for the oscillating scalar field, e.g., by $\rho(\phi)$ we mean the time averaged of the scalar field energy density in the sense explained above.
From eq. (19) we get an approximate equation for $\Phi$ evolution

$$\dot{\Phi} \approx -\frac{2}{q+2}H\Phi.$$ 

(34)

From (34) and (27) it is obvious that $H \ll A$. This means that the expansion rate is much less than the oscillation frequency. Effectively the scale factor is $a(t) \propto t^{\frac{q+2}{q}}$ and the Hubble parameter is given by $H = \frac{q+2}{q} t^{-1}$. $\rho$ satisfies

$$\frac{d(\rho a^{3\gamma})}{dt} = 0,$$

(35)

corresponding to $\rho_{\varphi} \propto t^{-2}$ which is $q$ independent.

4 Particle production

In this part we try to study inflaton decay to ultra-relativistic particles (radiation) in the rapid oscillatory phase. This decay is due to the interaction between the inflaton and produced particles. To study this decay, a phenomenological interaction (see(6)),

$$Q = \Gamma \dot{\varphi}^2,$$

(36)

where $\Gamma$ is a positive constant, was proposed by [12]. Afterwards, in [13], a Lagrangian, including bosonic and fermionic fields and their interactions with the inflaton, was introduced, and it was shown that the effect of particle production can be explained by adding a polarization operator to the inflaton mass term. There was shown that, for quadratic potential, the role of polarization operator may be mimicked by the phenomenological friction term (36), in high frequency regime. However, by taking under consideration the back reaction of the quantum effects on the evolution of the inflaton field [14], and also considering possible decays of the inflaton to other particles [15], (36) can no more be deduced from a Lagrangian.

Other phenomenological models with temperature and field dependent friction coefficient term, $Q = \Gamma(T, \varphi)\dot{\varphi}^2$, have been also considered in the literature [16].

However, as the nature of the inflaton and also primordial produced particles are unknown, we have not yet an exact expression for the form of interaction. In our study, we adopt the widely used phenomenological interaction (36), which reduces significantly computational complexity.

In the presence of the source term (36), the inflaton evolution is given by

$$3wH^2\ddot{\varphi} + 3H(3wH^2 + 2w\dot{H})\dot{\varphi} + V'(\varphi) = -\Gamma\dot{\varphi}.$$ 

(37)
In the following we consider the power law potential (11). Using (4) and with the same method used in (30), one can obtain

\[ \langle 9wH^2 \dot{\varphi}^2 \rangle = \frac{2q}{q + 2} \rho_\varphi. \]  

(38)

Inserting the above relation in the first equation of (6), and by replacing \( P_\varphi + \rho_\varphi \) with its time average over an oscillation cycle, we arrive at

\[ \dot{\rho}_\varphi + 3H\gamma \rho_\varphi + \frac{\gamma \Gamma}{3wH^2} \rho_\varphi = 0. \]  

(39)

We note again that in the above equation, all the values must be regarded as the time averaged values over one oscillation. This relation is only valid on large time with respect to the period of fast oscillation. The second term (friction term) describes dilution resulted from the universe expansion, while the third term corresponds to particle production during the coherent oscillation of the inflaton.

Comparing (39) with the corresponding minimal model relation

\[ \dot{\rho}_\varphi + 3H\gamma_m \rho_\varphi + \gamma_m \Gamma \rho_\varphi = 0, \]  

(40)

where \( \gamma_m = \frac{2q}{q + 2} \), shows that, in the limit \( wH^2 \gg 1 \), the decrease of \( \rho_\varphi \) (due to particle production) and as we will see the reheating temperature are very less than the corresponding values in the minimal model. Note (39) is true only for \( wH^2 \gg 1 \), so (40) cannot be derived from (39) by simply setting \( w = 0 \).

The solution of (39) is

\[ \rho_\varphi \propto a^{-3\gamma} \exp \left[ -\frac{\Gamma \gamma}{3w} \int_{t_{osc.}}^{t} H^{-2}(t')dt' \right], \]  

(41)

where \( t_{osc.} \) is the time when the oscillation commences. In the beginning of particle production, the universe is dominated by the scalar field, and we assume \( \Gamma \ll wH^3 \), but \( H \) is decreasing and this approximation ceases to be valid later, when the second and the third term in (39) acquire the same order of magnitude (we will denote this time by \( t_{rh} \)). So, with our assumptions, and in the scalar field dominated era it is safe to use the approximation \( H = \frac{2}{3\gamma t} \) until \( t = t_{rh} \). The behavior of the Hubble parameter, during rapid oscillation, can also be investigated via numerical method. E.g. in fig.(2), \( H \) is plotted for the quadratic potential (17) and by using Friedmann and continuity equations. So let us write (41) as

\[ \rho_\varphi = \Xi t^{-2} \exp(-\frac{\Gamma \gamma^3}{4w} t^3), \]  

(42)

where \( \Xi = \rho_{\text{osc.}} p_{\text{osc.}} e^{-\frac{3}{4}} \rho_{\text{osc.}}^2 t_{\text{osc.}}^2 \) and \( \rho_{\text{osc.}} = \rho(t_{\text{osc.}}) \). The decrease of \( \rho_\varphi \) is due to particle production which is encoded in the exponential term and also due
Figure 2: $h := \frac{H}{m}$ in terms of dimensionless time $\tau = mt$, for $\{wm^2 = 10^8, \frac{\Gamma}{m} = 10^{-2}\}$, with initial conditions $\{\phi(1) = 0.056, \dot{\phi}(1) = 0\}$, for the quadratic potential.

to the term $t^{-2}$ corresponding to dilution via the universe expansion. The dilution term is independent of $q$ as was explained after eq. (35). For larger values of $\frac{1}{wM_p}$ the decay rate is faster.

Comparing this result with what was obtained before for $\{w = 0, \gamma = 0.5\}$ model [17],

$$\rho_\phi = \rho_{osc}(\frac{t_{osc}}{t})^2 \exp[-\Gamma(t - t_{osc})],$$  \hspace{1cm} (43)

shows that the decrease rate of $\rho_\phi$, due to the expansion of the universe, is the same. But in the presence of $w$, the rate of particle production is decreased. We note again that our result (42) is true only for $wH^2 \gg 1$, so we cannot obtain (43) from (42) by setting $w = 0$.

The radiation satisfies

$$\dot{\rho}_R + 4H\rho_R = \Gamma\gamma \frac{\rho_\phi}{3wH^2},$$ \hspace{1cm} (44)

To study the evolution of $\rho_R$ during scalar field dominated epoch, i.e., when $H^2 \approx \frac{1}{3M_P^2}\rho_\phi$ we write (44) in the form

$$\dot{\rho}_R + 4H\rho_R = \frac{\Gamma\gamma M_P^2}{w},$$ \hspace{1cm} (45)

whose approximate solution is given by

$$\rho_R = \frac{3\Gamma\gamma^2 M_P^2}{(8 + 3\gamma)w}[1 - (\frac{t_{osc}}{t})^{\frac{8 + 3\gamma}{3\gamma}}].$$ \hspace{1cm} (46)

To derive the above equation we have assumed that after the slow-roll, the universe was cold: $\rho_R(t = t_{osc}) = 0$. In fig. (3), $\rho_R$ is plotted for the quadratic potential [17], showing that the radiation density increases monotonically in the rapid oscillation phase. However this behavior is valid
Figure 3: dimensionless energy density \( \rho_{\phi} m^2 M_p^2 \) (points) and \( \rho_R m^2 M_p^2 \) (line) in terms of dimensionless time \( \tau = m t \), for \( \{ w m^2 = 10^8, \Gamma_m = 10^{-2} \} \) with \( \rho_{osc.} = 8.3 \times 10^{-8} m^2 M_p^2 \), for the quadratic potential

only for \( \rho_{\phi} \gtrsim \rho_R \), and in the radiation dominated era we expect that \( \rho_R \) decreases.

Based on Friedmann and continuity equations, the behavior of \( \rho_R \) is also depicted in fig. (4), via numerical method for the quadratic potential (17). This figure shows that the increase of \( \rho_R \) continues until \( \rho_R \simeq \rho_{\phi} \). After \( t = t_{rh} \), i.e. from the beginning of radiation dominated era, \( \rho_R \) begins to decrease.

Figure 4: dimensionless energy density \( \rho_{\phi} m^2 M_p^2 \) (initially upper graph) and \( \rho_R m^2 M_p^2 \) in terms of dimensionless time \( \tau = m t \), for \( \{ w m^2 = 10^8, \Gamma_m = 10^{-2} \} \), with initial conditions \( \{ \phi(\tau = 1) = 0.056, \phi'(\tau = 1) = 0 \} \), for the quadratic potential.

Relativistic particles interact quickly (with respect to the expansion rate of the universe) with each other to become in a thermal equilibrium characterized by temperature \( T_r \), given by

\[
\rho_R = \frac{\pi^2}{30} g_* T_r^4,
\]

(47)
where $g_*$ is the total number of effectively massless degrees of freedom. If $T_r$ is greater than the electroweak scale $T_r > 300 \text{GeV}$ then $g > 106.75 \ [18]$.

The reheating time, $t_{rh}$, is given by

$$\Gamma \simeq 9wH^3(t_{rh}),$$

i.e., when the second and third terms of (49) acquire a same order of magnitude, as it was mentioned before. At this order of time, $\rho_R(t_{rh})$ has also the same order of magnitude as $\rho_\varphi(t_{rh})$. At the reheating time we have $H \simeq \left(\frac{\Gamma}{wH} \right)^{\frac{1}{4}}$, so the use high friction condition (4) during the reheating era is safe only when

$$w\Gamma^2 \gg 1.$$  \hspace{1cm} (49)

For $t \gtrsim t_{rh}$, almost all the energy of the inflaton is transferred to newly produced particles and the universe becomes radiation dominated.

Now, we can estimate the reheating temperature defined by $T_{rh} = T(t_{rh})$. From (46), and $\frac{t_{osc.}}{t_{rh}} \ll 1$, we obtain

$$T_{rh} \simeq 1.89 \left(\frac{\gamma}{8 + 3\gamma}\right)^{\frac{1}{4}} g_*^{-\frac{1}{4}} M^\frac{1}{2}_P \left(\frac{\Gamma}{w}\right)^{\frac{1}{2}}.$$

(50)

Note that the above equation could also be obtained by using $H^2 \simeq \frac{1}{3M^2_P}\rho_R$, and (47). This temperature is specified only by parameters of the system and is independent of initial conditions. In the absence of non-minimal derivative coupling, i.e. for $w = 0$, and for $\gamma = 0.5$, the radiation energy density is obtained as [17]

$$\rho_R = \frac{M^2_P\Gamma}{10\pi^4}\left[1 - \left(\frac{t_{osc.}}{t}\right)^{\frac{1}{2}}\right].$$

(51)

$\rho_R$ increases rapidly from $\rho_R = 0$ to its maximum value and then decreases again, so the maximum temperature is occurred in the beginning of $\varphi$ oscillation before reheating (see fig.4, depicted for the potential (17)). This is in contrast to our model, where as it can be seen from (46), $\rho_R$ increases continuously in $\varphi$ oscillation epoch until $\rho_R \simeq \rho_\varphi$. In $w = 0$ model, reheating temperature is determined by [17]

$$T_{rh}(w = 0) \simeq 1.2g_*^{-\frac{1}{2}} M^\frac{1}{2}_P \Gamma^\frac{1}{4}.$$  \hspace{1cm} (52)

Therefore $T_{rh}(wH^2 \gg 1) \ll T_{rh}(w = 0)$, provided that a same $\Gamma$ is taken into account for both theories. Note that as the reheating process is realized after inflation, $T_{rh}$ must be below the GUT scale: $T_{rh} < 10^{16} \text{GeV}$.

Based on astrophysical data, we are able to estimate the relation of reheating temperature and the number of e-folds. To do so, we proceed as follows: Consider a length scale $l$ which at time $t = t_*$, in the inflation era, left the Hubble radius:

$$l = \frac{a_0}{a(t_*)} \frac{1}{H(t_*)}$$
Figure 5: $\frac{2\pi}{m_s M_P}$, in terms of dimensionless time $\tau = mt$, for $w = 0$, $\Gamma_m = 10^{-2}$ with $\rho_{osc} = 8.3 \times 10^{-8} m^2 M_P^2$, for the quadratic potential.

The number of e-folds from $t_e$ to the end of the slow-roll inflation, denoted by subscript $\epsilon$, may be expressed as

$$N_\epsilon = \ln \left( \frac{a_e}{a_\epsilon} \right) = \ln \left( \frac{a_e a_{rh} a_{eq} a_0 H_0 H_\epsilon}{a_{rh} a_{eq} a_0 a_s H_s H_0} \right),$$

(54)

where 0, $rh$, and $eq$ subscripts denote present, reheating, and matter-radiation equality densities epochs respectively. As we have seen, during rapid oscillation (from $t_e$ until $t_{rh}$), the universe is dominated by a scalar field whose the effective equation of state parameter is given by $\gamma - 1$, whence

$$N_\epsilon = 62 + \ln \left( \frac{a_0 H_0}{a_e H_e} \right) + \ln \left( \frac{V_4}{10^{16} \text{GeV}} \right) + \frac{1}{4} \ln \left( \frac{V_\epsilon}{V_e} \right) - \left( \frac{1}{3\gamma} - \frac{1}{4} \right) \ln \left( \frac{V_e}{\rho_{rh}} \right).$$

(55)

The right hand side expressions are determined as follows: By setting $a_0 = 1$, and adopting the WMAP (pivot) scale $l = \frac{k}{a} = 500 \text{Mpc}$ [20], we arrive at

$$\ln \left( \frac{a_0 H_0}{a_s H_s} \right) = \ln \left( \frac{H_0}{k} \right) = \ln \left( \frac{0.7}{6} \right).$$

(56)

The present Hubble parameter is $H_0 = \frac{7}{30000} \text{Mpc}^{-1}$ [20]. To obtain $V_\epsilon$, we consider the scalar perturbations. The spectral index $n_s$ is [10]

$$n_s - 1 = \frac{M_P^2}{w H^2} \left[ \frac{2 V''(\varphi)}{3 V(\varphi)} - \frac{4 V'(\varphi)}{3 V(\varphi)^2} \right].$$

(57)

The quantities in the right hand side must computed at the time of horizon crossing $c_s k = aH$ during slow-roll inflation, where $k$ is the comoving wavenumber and $c_s$ is the sound speed of scalar perturbation. In the limit
$wH^2 \gg 1$ we have $c_s \simeq 1$ [21], and for the potential (11), we deduce

$$V_e = \lambda \frac{2}{\sqrt{\pi} \sqrt{2}} \left( \frac{2M_p^4 q(q+2)}{w(1-n_s)} \right) \frac{q}{\sqrt{2}}.$$  \hspace{1cm} (58)

Based on WMAP data [20],

$$n_s = 0.968 \pm 0.012.$$  \hspace{1cm} (59)

From $t_s$ until $t_e$, the slow-roll approximation is valid, hence $V_e$ may be estimated with the help of (15), as:

$$V_e = \lambda \frac{2}{\sqrt{\pi} \sqrt{2}} \left( \frac{q^2 M_p^4}{w} \right) \frac{q}{\sqrt{2}}.$$  \hspace{1cm} (60)

$\rho_{rh}$ is specified by (48) or equivalently by (47) and (50):

$$\rho_{rh} = \frac{4.16 M_p^2 \gamma}{8 + 3\gamma} \left( \frac{\Gamma}{w} \right)^3.$$  \hspace{1cm} (61)

In the slow-roll regime, as expected, we have $V_1^4 \simeq V_e^4$, $V_e^4 \lesssim 10^{16} GeV$ also holds [19] ($10^{16} GeV$ is the GUT scale). The reheating is provided by the inflaton energy, hence $\rho_{rh} \lesssim V_e$. So finally we expect to have $N^*_e < 60$. Only for a prompt reheating, $N \approx 60$ may be possible.

As an example, for the quadratic potential $V(\varphi) = \frac{1}{2} m^2 \varphi^2$ where

$$V_e = \frac{\sqrt{2m M_p^2}}{\sqrt{w}},$$

$$V_* = \frac{\sqrt{6m M_p^2}}{\sqrt{(1-n_s)w}},$$

$$\rho_{rh} = 0.15 M_p^2 \left( \frac{\Gamma}{w} \right)^3.$$  \hspace{1cm} (62)

$N^*_e$ depends on the parameters of the model, i.e. $\Gamma$, $w$ and $m$, e.g. choosing $wm^2 \simeq 10^8$, $\frac{\Gamma}{m} \simeq 10^{-2}$ (in agreement with [19]), and setting $m = 10^{-6} M_P$ [2], we find $N^*_e = 43.41$.

5 Conclusion

As a summary, we briefly discussed inflation in the framework of a non-minimal derivative coupling model proposed in [6], and then studied a gap in the literature: the reheating process in this framework. We investigated inflaton evolution in quasi-periodic oscillation at the end of slow-roll. We allowed the scalar field to decay to ultra-relativistic particles (radiation) via
a phenomenological source term. We obtained the reheating temperature which was independent of initial conditions.

We showed that the energy density of radiation, during oscillatory era and when it is smaller than the energy density of inflaton, increases monotonically. This behavior is in contrast to ordinary inflation theory where the maximum temperature occurs before reheating. We confirmed our results via numerical methods.

References

[1] A. H. Guth, Phys. Rev. D 23, 347(1981).
[2] A. Linde, Particle Physics and Inflationary Cosmology (Harwood, Chur, Switzerland, 1990)
[3] A. Linde, Phys. Lett. B 129, 177 (1983).
[4] A. Albrecht, P. J. Steinhardt, M. S. Turner, and F. Wilczek, Phys. Rev. Lett. 48, 1437 (1982); L. Abbott, E. Farhi, and M. Wise, Phys. Lett. B 117, 29 (1982); A. Doglov and A. Linde, Phys. Lett. B 116, 329 (1982).
[5] F. L. Bezrukov and M. E. Shaposhnikov, Phys. Lett. B 659, 703 (2008); F. Bezrukov, A. Magnin, M. Shaposhnikov, S. Sibiryakov, JHEP01(2011)016.
[6] C. Germani and A. Kehagias, Phys. Rev. Lett. 105, 011302 (2010); C. Germani and Y. Watanabe, JCAP 07(2011)031; A. Banijamali and B. Fazlpour, JCAP01(2012)039.
[7] C. Germani and A. Kehagias, Phys. Rev. Lett. 106, 161302 (2011).
[8] H. M. Sadjadi, Phys. Rev. D 83,107301 (2011).
[9] G. Gubitosi and E. V. Linder, Phys. Lett. B 703, 113 (2011); A. Banijamali and B. Fazlpour, Phys. Lett. B 703, 366 (2011); S. V. Sushkov, Phys. Rev. D 80, 103505 (2009); E. N. Saridakis and S. V. Sushkov, Phys. Rev. D 81, 083510 (2010).
[10] C. Germani and Y. Watanabe, JCAP 07(2011)031; C. Germani, L. Martucci, and P. Moyassari, Phys. Rev. D, 85, 103501 (2012); C. Germani, arXiv:1112.1083v1 [astro-ph.CO].
[11] Y. Shtanov, J. Traschen, and R. Brandenberger, Phys. Rev. D 51, 5438 (1995).
[12] A. Albrecht, P. J. Steinhardt, M. S. Turner, and F. Wilczek, Phys. Rev. Lett. 48, 1437 (1982).
[13] L. Kofman, A. Linde, and A. A. Starobinsky, Phys. Rev. D 56, 3258 (1997).

[14] L. Kofman, arXiv:astro-ph/9605155.

[15] D. S. M. Alves and G. M. Kremer, JCAP 0410 (2004) 009.

[16] J. Yokoyama and A. Linde Phys. Rev. D 60, 083509 (1999); E. W. Kolb, A. Notari, and A. Riotto, Phys. Rev. D, 68, 123505 (2003).

[17] E. Kolb and M. Turner, The Early Universe (Addison-Wesley Publishing Company, Redwood City, California, 1990).

[18] J. Mielczarek, Phys. Rev. D 83, 023502 (2011).

[19] A. R. Liddle and D. H. Lyth, Cosmological Inflation and Large-Scale Structure (Cambridge University Press 2000); A. R. Liddle and D. H. Lyth, Phys. Rept. 231, 1 (1993).

[20] E. Komatsu et al., Astrophys. J. Suppl. 192, 18 (2011), arXiv:1001.4538 [astro-ph.CO].

[21] C. Germani and A. Kehagias, JCAP 05(2010)019; S. Tsujikawa, arXiv:1201.5926v1 [astro-ph.CO].