More on R-Union and R-Intersection of Neutrosophic Soft Cubic Set

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Abstract R-unions and R-intersections, R-OR, R-AND of Neutrosophic soft cubic sets are introduced and related properties are investigated. We show that the R-union (R-intersection) of internal neutrosophic soft cubic set is also an internal neutrosophic soft cubic set. We show that the R-union and the R-intersection T-external (I-external, F-external) neutrosophic soft cubic sets are also T-external (I-external, F-external) neutrosophic soft cubic sets. The conditions for the R-intersection of two cubic soft sets to be both an external neutrosophic soft cubic set and an internal neutrosophic soft cubic set. Further we provide a condition for the R-intersection and R union of two T-internal (I-internal, F-internal) neutrosophic soft cubic sets are T-external (I-external, F-external) neutrosophic soft cubic sets.

Keywords: Neutrosophic soft cubic set, T-internal (resp. I-internal, F-internal) neutrosophic soft cubic sets, T-external (resp. I-external, F-external) neutrosophic soft cubic set, R-union, R-intersection of neutrosophic soft cubic set.

1. INTRODUCTION

Every real situation does not have a crisp or an exact solution hence there is some degree of uncertainty. To deal with uncertainty many Mathematician developed many theories. In 1965 Zadeh [19] introduced the concept of Fuzzy set were we consider the degree of belongingness to a set as a membership function. Following him in 1986 Atanassov [3] introduced the degree of non membership and defined intuitionistic fuzzy set. Further researches were done in these fields but these two sets were not enough to meet all the uncertainties in
real physical problems. Hence in 1995 Smarandache [5, 6] coined neutrosophic
logic and neutrosophic sets to deal with truth, indeterminate and falsehood.
On other hand in 1999 Molodtsov [4] introduced soft set which helps the view
an environment in a parameterized manner. Pabita Kumar Majii [5-7] had
combined the Neutrosophic set with soft sets and introduced ‘Neutrosophic
soft set’. Y. B. Jun et al. [16-18] coined cubic set by using a fuzzy set and an
interval-valued fuzzy set, and also extended the concept of cubic sets to the
neutrosophic cubic sets. [1] Introduced neutrosophic soft cubic set and the
notion of truth-internal (indeterminacy-internal, falsity-internal) neutrosophic
soft cubic sets and truth-external (indeterminacy-internal, falsity-internal)
neutrosophic soft cubic sets.

As a continuation of the paper [1] we consider R-unions and R-intersections
of T-external (I-external, F-external) neutrosophic soft cubic sets. We provide
examples to show that the R-intersection and the R-union of T-external (resp.
I-external and F-external) neutrosophic soft cubic sets may not be a T-external
(resp. I-external and F-external) neutrosophic soft cubic set. We also discuss
conditions for the R-union of T-external (resp. I-external and F-external)
neutrosophic soft cubic sets to be a T-external (resp. I-external and F-external)
neutrosophic soft cubic set. Further the condition for the R-intersection of
T-external (resp. I-external and F-external) neutrosophic soft cubic sets to be a
T-external (resp. I-external and F-external) neutrosophic soft cubic set.

2. PRELIMINARIES

2.1 Definition [19] Let E be a universe. Then a fuzzy set μ over E is defined
by $X = \{ \mu_x(x) / x \in E \}$ where $\mu_x$ is called membership function of X and
defined by $\mu_x : E \rightarrow [0,1]$. For each $x \in E$, the value $\mu_x(x)$ represents the degree
of $x$ belonging to the fuzzy set $X$.

2.2 Definition: [16] Let X be a non-empty set. By a cubic set, we mean a
structure $\Xi = \{ \langle x, A(x), \mu(x) \rangle \mid x \in X \}$ in which A is an interval valued fuzzy
set (IVF) and $\mu$ is a fuzzy set. It is denoted by $\langle A, \mu \rangle$.

2.3 Definition: [5] Let U be an initial universe set and E be a set of parameters.
Consider $A \subset E$. Let $P(U)$ denotes the set of all neutrosophic sets of U. The
collection $(F, A)$ is termed to be the soft neutrosophic set over U, where F is a
mapping given by $F : A \rightarrow P(U)$.

2.4 Definition: [9] Let X be an universe. Then a neutrosophic (NS) set $\lambda$ is an
object having the form $\lambda = \{ < x : T(x), I(x), F(x) > : x \in X \}$ where the functions
$T, I, F : X \rightarrow [0,1]$ defines respectively the degree of Truth, the degree of
indeterminacy, and the degree of falsehood of the element $x \in X$ to the set $\lambda$
with the condition.
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2.5 Definition: [15] Let X be a non-empty set. An interval neutrosophic set (INS) A in X is characterized by the truth-membership function $A_T$, the indeterminacy-membership function $A_I$ and the falsity-membership function $A_F$. For each point $x \in X$, $A_T(x), A_I(x), A_F(x) \subseteq [0, 1]$.

For two INS

$A = \{ <x, [A_T^-(x), A_T^+(x)], [A_I^-(x), A_I^+(x)], [A_F^-(x), A_F^+(x)] >: x \in X \}$

And

$B = \{ <x, [B_T^-(x), B_T^+(x)], [B_I^-(x), B_I^+(x)], [B_F^-(x), B_F^+(x)] >: x \in X \}$

Then,

1. $A \subseteq B$ if and only if

   \[ A_T^-(x) \leq B_T^-(x), A_T^+(x) \leq B_T^+(x) \]

   \[ A_I^-(x) \geq B_I^-(x), A_I^+(x) \geq B_I^+(x) \]

   \[ A_F^-(x) \geq B_F^-(x), A_F^+(x) \geq B_F^+(x) \text{ for all } x \in X. \]

2. $A = B$ if and only if

   \[ A_T^-(x) = B_T^-(x), A_T^+(x) = B_T^+(x) \]

   \[ A_I^-(x) = B_I^-(x), A_I^+(x) = B_I^+(x) \]

   \[ A_F^-(x) = B_F^-(x), A_F^+(x) = B_F^+(x) \text{ for all } x \in X. \]

3. $A^c = \{ <x, [A_I^-(x), A_I^+(x)], [A_T^-(x), A_T^+(x)], [A_F^-(x), A_F^+(x)] >: x \in X \}$

4. $A \cap B = \{ <x, [\min \{A_T^-(x), B_T^-(x)\}, \min \{A_T^+(x), B_T^+(x)\}], \]

   \[ \max \{A_I^-(x), B_I^-(x)\}, \max \{A_I^+(x), B_I^+(x)\}], \]

   \[ \max \{A_F^-(x), B_F^-(x)\}, \max \{A_F^+(x), B_F^+(x)\} >: x \in X \}$

5. $A \cup B = \{ <x, [\max \{A_T^-(x), B_T^-(x)\}, \max \{A_T^+(x), B_T^+(x)\}], \]

   \[ \min \{A_I^-(x), B_I^-(x)\}, \min \{A_I^+(x), B_I^+(x)\}], \]

   \[ \min \{A_F^-(x), B_F^-(x)\}, \min \{A_F^+(x), B_F^+(x)\} >: x \in X \}$

2.6. Definition: [1]

Let X be an initial universe set. Let NC(X) denote the set of all neutrosophic cubic sets and $E$ be the set of parameters. Let $A \subseteq E$ then $(P, A) = \{ P(e_i) = \{ x, A_{e_i}(x), \lambda_{e_i}(x) : x \in X \} e_i \in A \}$, where
A_{e_i}(x) = \{ < x, A_{e_i}^T(x), A_{e_i}^I(x), A_{e_i}^F(x) > / x \in X \}, is an interval neutrosophic set, \( \lambda_{e_i}(x) = \{ < x, (\lambda_{e_i}^T(x), \lambda_{e_i}^I(x), \lambda_{e_i}^F(x)) > / x \in X \} \) is a neutrosophic set. The pair (P, A) is termed to be the neutrosophic soft cubic set over X where P is a mapping given by \( p: A \rightarrow NC(X) \).

2.7 Definition: [1]
Let X be an initial universe set. A neutrosophic soft cubic set \((P, M)\) in X is said to be
- truth-internal (briefly, T-internal) if the following inequality is valid
  \[
  (\forall x \in X, e_i \in E) \ (A_{e_i}^T(x) \leq \lambda_{e_i}^T(x) \leq A_{e_i}^{+T}(x)), \tag{2.1}
  \]
- indeterminacy-internal (briefly, I-internal) if the following inequality is valid
  \[
  (\forall x \in X, e_i \in E) \ (A_{e_i}^I(x) \leq \lambda_{e_i}^I(x) \leq A_{e_i}^{+I}(x)), \tag{2.2}
  \]
- falsity-internal (briefly, F-internal) if the following inequality is valid
  \[
  (\forall x \in X, e_i \in E) (A_{e_i}^F(x) \leq \lambda_{e_i}^F(x) \leq A_{e_i}^{+F}(x)). \tag{2.3}
  \]
If a neutrosophic soft cubic set in X satisfies (2.1), (2.2) and (2.3) we say that \((P, M)\) is an internal neutrosophic soft cubic in X.

2.8 Definition: [1]
Let X an initial universe set. A neutrosophic soft cubic set \((P, M)\) in X is said to be
- truth-external (briefly, T-external) if the following inequality is valid
  \[
  (\forall x \in X, e_i \in E) \ (\lambda_{e_i}^T(x) \notin (A_{e_i}^{-T}(x), A_{e_i}^{+T}(x))), \tag{2.4}
  \]
- indeterminacy-external (briefly, I-external) if the following inequality is valid
  \[
  (\forall x \in X, e_i \in E) \ (\lambda_{e_i}^I(x) \notin (A_{e_i}^{-I}(x), A_{e_i}^{+I}(x))), \tag{2.5}
  \]
- falsity-external (briefly, F-external) if the following inequality is valid
  \[
  (\forall x \in X, e_i \in E) \ (\lambda_{e_i}^F(x) \notin (A_{e_i}^{-F}(x), A_{e_i}^{+F}(x))). \tag{2.6}
  \]
If a neutrosophic soft cubic set \((P, M)\) in X satisfies (2.4), (2.5) and (2.6), we say that \((P, M)\) is an external neutrosophic soft cubic in X.
2.9 Definition \[1\]
Let (\(P,M\)) = \(\{P(e_i) = \{< x, A_{e_i}(x), \lambda_{e_i}(x) > : x \in X\} e_i \in M\}\)
and (\(Q,N\)) = \(\{Q(e_i) = B_i = \{< x, B_{e_i}(x), \mu_{e_i}(x) > : x \in X\} e_i \in N\}\) be two neutrosophic soft cubic sets in \(X\). Let \(M\) and \(N\) be any two subsets of \(E\) (set of parameters), then we have the following

1. \((P,M) = (Q,N)\) if and only if the following conditions are satisfied
   a) \(M = N\) and
   b) \(P(e_i) = Q(e_i)\) for all \(e_i \in M\) if and only if \(A_{e_i}(x) = B_{e_i}(x)\) and \(\lambda_{e_i}(x) = \mu_{e_i}(x)\) for all \(x \in X\) corresponding to each \(e_i \in M\).

2. \((P,M)\) and \((Q,N)\) are two neutrosophic soft cubic set then we define and denote \(P\)-order as \((P,M) \subseteq_P (Q,N)\) if and only if the following conditions are satisfied
   c) \(M \subseteq N\) and
   d) \(P(e_i) \leq_P Q(e_i)\) for all \(e_i \in M\) if and only if \(A_{e_i}(x) \subseteq B_{e_i}(x)\) and \(\lambda_{e_i}(x)" \geq \mu_{e_i}(x)\) for all \(x \in X\) corresponding to each \(e_i \in M\).

3. \((P,M)\) and \((Q,N)\) are two neutrosophic soft cubic set then we define and denote \(P\)-order as \((P,M) \subseteq_R (Q,N)\) if and only if the following conditions are satisfied
   e) \(M \subseteq N\) and
   f) \(P(e_i) \leq_R Q(e_i)\) for all \(e_i \in M\) if and only if \(A_{e_i}(x) \subseteq B_{e_i}(x)\) and \(\lambda_{e_i}(x) \geq \mu_{e_i}(x)\) for all \(x \in X\) corresponding to each \(e_i \in M\).

2.10 Definition: \[1\]
Let \((P,M)\) and \((Q,N)\) be two neutrosophic soft cubic sets (NSCS) in \(X\) where \(I\) and \(J\) are any two subsets of the parametric set \(E\). Then we define R-union of neutrosophic soft cubic set as \((P,M) \cup_R (Q,N) = (H,C)\) where \(C = M \cup N\)

\[
H(e_i) = \begin{cases} 
P(e_i) & \text{if } e_i \in M - N \\
Q(e_i) & \text{if } e_i \in N - M \\
P(e_i) \lor_R Q(e_i) & \text{if } e_i \in M \cap N 
\end{cases}
\]

where \(P(e_i) \lor_R Q(e_i)\) is defined as

\[
P(e_i) \lor_R Q(e_i) = \{< x, \max\{A_{e_i}(x), B_{e_i}(x)\}, (\lambda \land \mu_{e_i})(x) > : x \in X\} e_i \in M \cap N
\]

where \(A_{e_i}(x), B_{e_i}(x)\) represent interval neutrosophic sets. Hence
2.11 Definition: [1]
Let \((P,M)\) and \((Q,N)\) be two neutrosophic soft cubic sets (NSCS) in \(X\) where \(M\) and \(N\) are any subsets of parameter's set \(E\). Then we define \(R\)-intersection of neutrosophic soft cubic set as
\[
(P^R, M^R) \cap (Q^R, N^R) = (H, C)\]
where \(C = M \cap N\).

\[
H(e_i) = P(e_i) \wedge_{R} Q(e_i)
\]
and \(e_i \in I \cap J\). Here \(F^R(e_i) = \bigcap^R G(e_i)\) is defined as
\[
P(e_i) \wedge_{R} Q(e_i) = H(e_i) = \{< x, \min\{ A_{e_i} (x), B_{e_i} (x)\}, (\lambda_{e_i} \vee \mu_{e_i})(x) > : x \in X\}
\]
where \(A_{e_i} (x), B_{e_i} (x)\) represent interval neutrosophic sets. Hence
\[
P^R(e_i) \wedge_{R} Q^R(e_i) = \{< x, \min\{ A_{e_i}^T (x), B_{e_i}^T (x)\}, (\lambda_{e_i}^T \vee \mu_{e_i}^T)(x) > : x \in X\}
\]
\[
P^I(e_i) \wedge_{R} Q^I(e_i) = \{< x, \min\{ A_{e_i}^I (x), B_{e_i}^I (x)\}, (\lambda_{e_i}^I \vee \mu_{e_i}^I)(x) > : x \in X\}
\]
\[
P^F(e_i) \wedge_{R} Q^F(e_i) = \{< x, \min\{ A_{e_i}^F (x), B_{e_i}^F (x)\}, (\lambda_{e_i}^F \vee \mu_{e_i}^F)(x) > : x \in X\}
\]

2.12 Definition: [2]
The complement of a neutrosophic soft cubic set
\[(F,I) = \{ F(e_i) = \{< x, A_{e_i} (x), \lambda_{e_i} (x) > : x \in X\} \mid e_i \in I \} \]
is denoted by \((F,I)^C\) and defined as
\[(F,I)^C = (F,I)^C = (F^C, -I)\]
where \(F^C : I \rightarrow NC(X)\) and
\[(F,I)^C = \{(F(e_i))^C = \{< x, A_{e_i}^C (x), \lambda_{e_i}^C (x) > : x \in X\} \mid e_i \in I \} \]
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Definition: 3.1

Let \((P, M) = \{ F(e_i) = \{< x, A_{e_i}(x), \lambda_{e_i}(x) > : x \in X \} \in M \} \) and \((Q, N) = \{ G(e_i) = \{< x, B_{e_i}(x), \mu_{e_i}(x) > : x \in X \} \in N \} \) be NSCS in X. Then

1. R-OR of NSCS is denoted by \((P, M) \lor_R (Q, N)\) and defined as \((P, M) \lor_R (Q, N) = (H, M \times N)\) where \(H(\alpha_i, \beta_i) = P(\alpha_i) \cup_R Q(\beta_i)\) for all \((\alpha_i, \beta_i) \in M \times N\).

2. R-AND of NSCS is denoted by \((P, M) \land_R (Q, N)\) and defined as \((P, M) \land_R (Q, N) = (H, M \times N)\) where \(H(\alpha_i, \beta_i) = P(\alpha_i) \cap_R Q(\beta_i)\) for all \((\alpha_i, \beta_i) \in M \times N\).

Example: 3.2

Let \(X = \{x_1, x_2, x_3\}\) be initial universe and \(E = \{e_1, e_2\}\) parameter’s set. Let \((P, M)\) be a neutrosophic soft cubic set over \(X\) and defined as \((P, M) = \{ P(e_i) = \{< x, A_{e_i}(x), \lambda_{e_i}(x) > : x \in X \} \in M \} \) and

\[
\begin{align*}
\text{X} & \quad P(e_1) & \quad P(e_2) \\
& \quad \langle A_{e_1}(x), \lambda_{e_1}(x) \rangle & \quad \langle A_{e_2}(x), \lambda_{e_2}(x) \rangle \\
x_1 & \quad [0.5,0.6][0.6,0.7][0.5,0.6] & \quad [0.7,0.4,0.6] & \quad [0.3,0.6][0.2,0.7][0.2,0.4] & \quad [0.5,0.2,0.2] \\
x_2 & \quad [0.4,0.5][0.7,0.8][0.2,0.3] & \quad [0.6,0.4,0.2] & \quad [0.3,0.5][0.6,0.8][0.2,0.6] & \quad [0.6,0.5,0.4] \\
x_3 & \quad [0.2,0.3][0.2,0.3][0.3,0.5] & \quad [0.5,0.3,0.5] & \quad [0.4,0.7][0.2,0.5][0.3,0.6] & \quad [0.7,0.3,0.4]
\end{align*}
\]

\((Q, N) = \{ G(e_i) = \{< x, B_{e_i}(x), \mu_{e_i}(x) > : x \in X \} \in N \} \)

\[
\begin{align*}
\text{X} & \quad Q(e_1) & \quad Q(e_2) \\
& \quad \langle B_{e_1}(x), \mu_{e_1}(x) \rangle & \quad \langle A_{e_2}(x), \lambda_{e_2}(x) \rangle \\
x_1 & \quad [0.7,0.9][0.3,0.5][0.3,0.4] & \quad [0.4,0.5,0.6] & \quad [0.4,0.7][0.1,0.3][0.1,0.2] & \quad [0.3,0.4,0.4] \\
x_2 & \quad [0.5,0.6][0.3,0.7][0.1,0.2] & \quad [0.5,0.6,0.6] & \quad [0.4,0.6][0.4,0.7][0.2,0.5] & \quad [0.4,0.7,0.5] \\
x_3 & \quad [0.3,0.4][0.1,0.2][0.2,0.4] & \quad [0.3,0.4,0.6] & \quad [0.5,0.8][0.1,0.4][0.1,0.4] & \quad [0.5,0.6,0.6]
\end{align*}
\]

R-OR is denoted by \((H, M \times N) = (P, M) \lor_R (Q, N)\) where \(M \times N = \{(e_1, e_1), (e_1, e_2), (e_2, e_1), (e_2, e_2)\}\) is defined.
R-AND is denoted by \((H, M \times N) = (P, M) \wedge_R (Q, N)\) where 

\[
M \times N = \{(e_1, e_1), (e_1, e_2), (e_2, e_1), (e_2, e_2)\}
\]

is defined.

| X | \(H(e_1, e_1)\) | \(H(e_1, e_2)\) | \(H(e_2, e_1)\) | \(H(e_2, e_2)\) |
|---|---|---|---|---|
| \(P(e_1) \cap Q(e_1)\) | \(P(e_1) \cap Q(e_2)\) | \(P(e_2) \cap Q(e_1)\) | \(P(e_2) \cap Q(e_2)\) |
| \(x_1\) | [0.7,0.9] | [0.4,0.4,0.6] | [0.5,0.6] | [0.4,0.4,0.6] |
| | [0.6,0.7] | [0.5,0.6] | [0.6,0.7] | [0.5,0.6] |
| | [0.5,0.6] | [0.6,0.7] | [0.5,0.6] | [0.6,0.7] |

**Proposition: 3.3** Let \(X\) be initial universe and \(I, J, L\) and \(S\) subsets of \(E\). Then for any neutrosophic soft cubic sets \(A = (F, I), B = (G, J), C = (E, L), D = (T, S)\) the following properties hold

1. if \(A \subseteq_R B\) and \(B \subseteq_R C\) then \(A \subseteq_R C\).
2. if \(A \subseteq_R B\) then \(B^c \subseteq_R A^c\).
3. if \(A \subseteq_R B\) and \(A \subseteq_R C\) then \(A \subseteq_R B \cap_R C\).
4. if \(A \subseteq_R B\) and \(C \subseteq_R B\) then \(A \cup_R C \subseteq_R B\).
5. if \(A \subseteq_R B\) and \(C \subseteq_R D\) then \(A \cup_R C \subseteq_R B \cup_R D\) and \(A \cap_R C \subseteq_R B \cap_R D\).

Proof: Straight forward.
Theorem 3.4
Let \((P, M)\) and \((Q, N)\) be INCS over \(X\) such that
\[
\max \{A_{i_j}^T(x), B_{i_j}^T(x)\} \leq (\lambda_{i_j}^T \land \mu_{i_j}^T)(x), \quad \max \{A_{i_j}^{-1}(x), B_{i_j}^{-1}(x)\} \leq (\lambda_{i_j}^{-1} \land \mu_{i_j}^{-1})(x),
\]
\[
\max \{A_{i_j}^F(x), B_{i_j}^F(x)\} \leq (\lambda_{i_j}^F \land \mu_{i_j}^F)(x)
\]
for all \(e_i \in M \cap N\) and for all \(x \in X\),
then \((P, M) \cup_R (Q, N)\) is also an INCS.

Proof:
Since \((P, M)\) and \((Q, N)\) is an INCS.

So far \((P, M)\) we have
\[
A_{i_j}^T(x) \leq \lambda_{i_j}^T(x) \leq A_{i_j}^{+T}(x), \quad A_{i_j}^{-T}(x) \leq \lambda_{i_j}^{-T}(x) \leq A_{i_j}^{+T}(x),
\]
\[
\lambda_{i_j}^F(x) \leq A_{i_j}^F(x) \leq \lambda_{i_j}^{F+}(x) \quad \text{for all} \ e_i \in M \quad \text{and for all} \ x \in X.
\]

And for \((Q, N)\) we have
\[
B_{i_j}^T(x) \leq \mu_{i_j}^T(x) \leq B_{i_j}^{+T}(x), \quad B_{i_j}^{-T}(x) \leq \mu_{i_j}^{-T}(x) \leq B_{i_j}^{+T}(x),
\]
\[
\mu_{i_j}^F(x) \leq B_{i_j}^F(x) \leq \mu_{i_j}^{F+}(x) \quad \text{for all} \ e_i \in N \quad \text{and for all} \ x \in X.
\]

\[
(\lambda_{i_j}^T \land \mu_{i_j}^T)(x) \leq \max \{A_{i_j}^T(x), B_{i_j}^T(x)\}, \quad (\lambda_{i_j}^{-T} \land \mu_{i_j}^{-T})(x) \leq \max \{A_{i_j}^{-T}(x), B_{i_j}^{-T}(x)\}, \quad (\lambda_{i_j}^F \land \mu_{i_j}^F)(x) \leq \max \{A_{i_j}^F(x), B_{i_j}^F(x)\}
\]
for all \(e_i \in M\) and for all \(x \in X\). Also given that
\[
\max \{A_{i_j}^{-T}(x), B_{i_j}^{-T}(x)\} \leq (\lambda_{i_j}^{-T} \land \mu_{i_j}^{-T})(x), \quad \max \{A_{i_j}^T(x), B_{i_j}^T(x)\} \leq (\lambda_{i_j}^T \land \mu_{i_j}^T)(x), \quad \max \{A_{i_j}^F(x), B_{i_j}^F(x)\} \leq (\lambda_{i_j}^F \land \mu_{i_j}^F)(x)
\]
for all \(e_i \in M \cap N\) and for all \(x \in X\). Now \((P, M) \cup_R (Q, N) = (H, C)\) where \(M \cup N = C\) and

\[
H(e_i) = \begin{cases} P(e_i) & \text{if} \ e \in M - N \\ Q(e_i) & \text{if} \ e \in N - M \\ P(e_i) \lor_R Q(e_i) & \text{if} \ e \in M \cap N \end{cases}
\]

If \(e \in M \cap N\), then \(P(e_i) \lor_R Q(e_i)\) is defined as
\[
P(e_i) \lor_R Q(e_i) = H(e_i) = \{< x, \max \{A_{e_i}(x), B_{e_i}(x)\}, (\lambda_{e_i} \lor \mu_{e_i})(x), x \in X, e_i \in M \cap N\}
\]

where
\[
P^T(e_i) \lor_R Q^T(e_i) = \{< x, \max \{A_{e_i}^T(x), B_{e_i}^T(x)\}, (\lambda_{e_i}^T \lor \mu_{e_i}^T)(x), x \in X, e_i \in M \cap N\}
\]
\[ P^f(e_i) \vee R \ Q^f(e_i) = \begin{cases} < x, \max\{ A^{f}_{\xi_{i}}(x), B^{f}_{\xi_{i}}(x)\}, (\lambda^{f}_{\xi_{i}} \land \mu^{f}_{\xi_{i}})(x), x \in X, \\ e_i \in M \cap N \end{cases} \]

\[ P^e(e_i) \vee R \ Q^e(e_i) = \begin{cases} < x, \max\{ A^{e}_{\xi_{i}}(x), B^{e}_{\xi_{i}}(x)\}, (\lambda^{e}_{\xi_{i}} \land \mu^{e}_{\xi_{i}})(x), x \in X, \\ e_i \in M \cap N \end{cases} \]

Since \((P, M)\) and \((Q, N)\) are INSCS so from above given condition and definition of an INSCS we can write, \(\max\{ A^{f}_{\xi_{i}}(x), B^{f}_{\xi_{i}}(x)\} \leq (\lambda^{f}_{\xi_{i}} \land \mu^{f}_{\xi_{i}})(x) \leq \max\{ A^{e}_{\xi_{i}}(x), B^{e}_{\xi_{i}}(x)\}\) for all \(e_i \in M \cap N\) and for all \(x \in X\). If \(e_i \in M - N\) or \(e_i \in N - M\) then the result is trivial. Thus \((P, M) \cup_R (Q, N) = (H, C)\) is an INSCS if that \(\max\{ A^{f}_{\xi_{i}}(x), B^{f}_{\xi_{i}}(x)\} \leq (\lambda^{f}_{\xi_{i}} \land \mu^{f}_{\xi_{i}})(x), \max\{ A^{e}_{\xi_{i}}(x), B^{e}_{\xi_{i}}(x)\} \leq (\lambda^{e}_{\xi_{i}} \land \mu^{e}_{\xi_{i}})(x)\), \(\min\{ A^{f}_{\xi_{i}}(x), B^{f}_{\xi_{i}}(x)\} \geq (\lambda^{f}_{\xi_{i}} \lor \mu^{f}_{\xi_{i}})(x), \min\{ A^{e}_{\xi_{i}}(x), B^{e}_{\xi_{i}}(x)\} \geq (\lambda^{e}_{\xi_{i}} \lor \mu^{e}_{\xi_{i}})(x)\) for all \(e_i \in M \cap N\) and for all \(x \in X\). Then \((P, M) \cap_R (Q, N)\) is an INSCS.

**Theorem 3.5**

Let \(\{ P(e_i) = \{< x, A^{f}_{\xi_{i}}(x), \lambda^{f}_{\xi_{i}}(x) > : x \in X\} e_i \in M \}\) and

\(\{ Q(e_i) = \{< x, B^{f}_{\xi_{i}}(x), \mu^{f}_{\xi_{i}}(x) > : x \in X\} e_i \in N \}\) be INSCS in \(X\) satisfying the following inequality \(\min\{ A^{f}_{\xi_{i}}(x), B^{f}_{\xi_{i}}(x)\} \geq (\lambda^{f}_{\xi_{i}} \lor \mu^{f}_{\xi_{i}})(x),\)

\(\min\{ A^{e}_{\xi_{i}}(x), B^{e}_{\xi_{i}}(x)\} \geq (\lambda^{e}_{\xi_{i}} \lor \mu^{e}_{\xi_{i}})(x), \min\{ A^{e}_{\xi_{i}}(x), B^{e}_{\xi_{i}}(x)\} \geq (\lambda^{e}_{\xi_{i}} \lor \mu^{e}_{\xi_{i}})(x)\) for all \(e_i \in M \cap N\) and for all \(x \in X\). Then \((P, M) \cap_R (Q, N)\) is an INSCS.

Proof:

Let \(\{ P(e_i) = \{< x, A^{f}_{\xi_{i}}(x), \lambda^{f}_{\xi_{i}}(x) > : x \in X\} e_i \in M \}\) and

\(\{ Q(e_i) = \{< x, B^{f}_{\xi_{i}}(x), \mu^{f}_{\xi_{i}}(x) > : x \in X\} e_i \in N \}.\) Then by definition of an INSCS we have \(A^{f}_{\xi_{i}}(x) \leq \lambda^{f}_{\xi_{i}}(x) \leq A^{f}_{\xi_{i}}(x),\)

\(A^{e}_{\xi_{i}}(x) \leq \lambda^{e}_{\xi_{i}}(x) \leq A^{e}_{\xi_{i}}(x),\) \(A^{f}_{\xi_{i}}(x) \leq \lambda^{e}_{\xi_{i}}(x) \leq A^{f}_{\xi_{i}}(x)\) for all \(e_i \in M\) and for all \(x \in X\). And \(B^{f}_{\xi_{i}}(x) \leq \mu^{e}_{\xi_{i}}(x) \leq B^{f}_{\xi_{i}}(x),\) \(B^{e}_{\xi_{i}}(x) \leq \mu^{e}_{\xi_{i}}(x) \leq B^{e}_{\xi_{i}}(x)\), \(B^{f}_{\xi_{i}}(x) \leq \mu^{e}_{\xi_{i}}(x) \leq B^{f}_{\xi_{i}}(x)\) for all \(e_i \in N\) and for all \(x \in X\). This implies ,
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\[
\min \{ A_{e_i}^T(x), B_{e_i}^T(x) \} \leq (\lambda^T_{e_i} \lor \mu^T_{e_i})(x), \quad \min \{ A_{e_i}^I(x), B_{e_i}^I(x) \} \leq (\lambda^I_{e_i} \lor \mu^I_{e_i})(x),
\]
\[
\min \{ A_{e_i}^F(x), B_{e_i}^F(x) \} \leq (\lambda^F_{e_i} \lor \mu^F_{e_i})(x), \text{ for all } e_i \in M \cap N \text{ and for all } x \in X.
\]

Also since \((P, M) \cap_R (Q, N) = (H, C)\) where \(M \cap N = C\), \(H(e_i) = P(e_i) \land_R Q(e_i)\) if \(e \in M \cap N\) then \(P(e_i) \land_R Q(e_i)\) is defined as
\[
P(e_i) \land_R Q(e_i) = H(e_i) = \{< x, \min \{ A_{e_i}(x), B_{e_i}(x) \}, (\lambda_{e_i} \lor \mu_{e_i})(x), x \in X, e_i \in M \cap N \}
\]

Given condition \(\max \{ A_{e_i}^T(x), B_{e_i}^T(x) \} \geq (\lambda^T_{e_i} \lor \mu^T_{e_i})(x), \max \{ A_{e_i}^I(x), B_{e_i}^I(x) \} \geq (\lambda^I_{e_i} \lor \mu^I_{e_i})(x), \max \{ A_{e_i}^F(x), B_{e_i}^F(x) \} \geq (\lambda^F_{e_i} \lor \mu^F_{e_i})(x), \text{ for all } e_i \in M \cap N \text{ and for all } x \in X.\)

Thus from given condition and definition of INSCS \(\min \{ A_{e_i}^T(x), B_{e_i}^T(x) \} \leq (\lambda^T_{e_i} \lor \mu^T_{e_i})(x) \leq \min \{ A_{e_i}^T(x) \}
\]
\[
\min \{ A_{e_i}^T(x), B_{e_i}^T(x) \} \leq (\lambda^I_{e_i} \lor \mu^I_{e_i})(x) \leq \min \{ A_{e_i}^I(x) \}
\]
\[
\min \{ A_{e_i}^T(x), B_{e_i}^T(x) \} \leq (\lambda^F_{e_i} \lor \mu^F_{e_i})(x) \leq \min \{ A_{e_i}^F(x) \}
\]

for all \(e_i \in M \cap N \text{ and for all } x \in X.\) Hence \((P, M) \cap_R (Q, N)\) is an INSCS.

**Example: 3.6**

Let \((P, I)\) and \((Q, J)\) be T-external neutrosophic soft cubic sets (T-ENSCS) in \(X\) where
\[
(P, I) = P(e_i) = \{< x, ([0.2, 0.5], [0.5, 0.7], [0.3, 0.5]), (0.7, 0.6, 0.8) > e_i \in I \}
\]
\[
(Q, J) = Q(e_i) = \{< x, ([0.6, 0.8], [0.6, 0.7], [0.7, 0.9]), (0.9, 0.7, 0.3) > e_i \in J \}
\]

for all \(x \in X\).

Then \((P, I)\) and \((Q, J)\) are T-ENSCS in \(X\) and \((P, I) \cup_R (Q, J) = (P, I) \cup (Q, J) = P \cup Q(e_i) = \{< x, ([0.6, 0.8], [0.6, 0.7], [0.7, 0.9]), (0.7, 0.6, 0.3) > e_i \in I \cap J \} \text{ for all } x \in X.\)

\((P, I) \cup_R (Q, J)\) is not an T-ENSCS since
\[
(\lambda^T_{e_i} \land \mu^T_{e_i})(x) = 0.7 \in (0.6, 0.8) = \left( A_{e_i}^T \cup B_{e_i}^T \right)^-(x), \left( A_{e_i}^T \cup B_{e_i}^T \right)^+(x)
\]
From the above example it is clear that R-union of T-ENSCS may not be T-ENSCS. We provide a condition for the R-union of T-external (resp. I-external and F-external) neutrosophic soft cubic sets to be T-external (resp. I-external and F-external) neutrosophic soft cubic set.

**Theorem 3.7**

Let \( (P, M) = \{ P(e_i) = \{< x, A_{e_i}(x), \lambda_{e_i}(x)> : x \in X \} \mid e_i \in M \} \) and \( (Q, N) = \{ Q(e_i) = \{< x, B_{e_i}(x), \mu_{e_i}(x)> : x \in X \} \mid e_i \in N \} \) be T-ENSCSs in X such that

\[
\left( \lambda_{e_i}^T \wedge \mu_{e_i}^T \right)(x) = \begin{cases} 
\max \{ \min \{ A_{e_i}^+(x), B_{e_i}^-(x) \}, \min \{ A_{e_i}^-(x), B_{e_i}^+(x) \} \} 
& \text{if } e_i \in M 
\min \{ \max \{ A_{e_i}^+(x), B_{e_i}^-(x) \}, \max \{ A_{e_i}^-(x), B_{e_i}^+(x) \} \} 
& \text{if } e_i \in N 
\end{cases}
\]

(3.7)

for all \( e_i \in M \) and for all \( e_i \in N \) and for all \( x \in X \). Then \( (P, M) \cup_R (Q, N) \) is also an T-ENSCS.

**Proof**

Consider \( (P, M) \cup_R (Q, N) = (H, C) \) where and \( M \cup N = C \)

\[
H(e_i) = \begin{cases} 
P(e_i) & \text{if } e_i \in M - N 
Q(e_i) & \text{if } e_i \in N - M 
P(e_i) \cup_R Q(e_i) & \text{if } e_i \in M \cap N 
\end{cases}
\]

where \( H(e_i) = P(e_i) \cup_R Q(e_i) \) is defined as

\[
P(e_i) \cup_R Q(e_i) = H(e_i) = \{< x, \max \{ A_{e_i}(x), B_{e_i}(x) \}, (\lambda_{e_i} \wedge \mu_{e_i})(x) > : x \in X, e_i \in M \cap N \}
\]

where \( P^T(e_i) \cup_R Q^T(e_i) = \{< x, \max \{ A_{e_i}^T(x), B_{e_i}^T(x) \}, (\lambda_{e_i}^T \wedge \mu_{e_i}^T)(x) > : x \in X, e_i \in M \cap N \} \)

If \( e_i \in M \cap N \) take \( \alpha_{e_i}^T = \min \{ \max \{ A_{e_i}^+(x), B_{e_i}^-(x) \}, \max \{ A_{e_i}^-(x), B_{e_i}^+(x) \} \} \)

and \( \beta_{e_i}^T = \max \{ \min \{ A_{e_i}^+(x), B_{e_i}^-(x) \}, \min \{ A_{e_i}^-(x), B_{e_i}^+(x) \} \} \)
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\[
\min\{A^{-T}_{e_i}(x), B^{+T}_{e_i}(x)\}. \text{ Then } \alpha_{e_i}^T \text{ is one of } A^{-T}_{e_i}(x), B^{-T}_{e_i}(x), A^+_T A^+_T (x), B^{+T}_{e_i}(x). \text{ Now we consider } \alpha_{e_i}^T = B^{-T}_{e_i}(x) \text{ or } B^{-T}_{e_i}(x) \text{ only as the remaining cases are similar to this one.}

If \( \alpha_{e_i}^T = B^{-T}_{e_i}(x) \) then \( A^{-T}_{e_i}(x) \leq A^+_T (x), \leq B^{-T}_{e_i}(x) \leq B^{+T}_{e_i}(x) \) and so \( \beta_{e_i}^T = A^+_T (x) \). Thus \((A_{e_i}^T \cup B_{e_i}^T)^-(x) = B^{-T}_{e_i}(x) = \alpha_{e_i}^T \geq (\lambda e_i^T \wedge \mu e_i^T)(x). \text{ Hence}

\[
\left(\lambda e_i^T \wedge \mu e_i^T\right)(x) \notin \left((A_{e_i}^T \cup B_{e_i}^T)^-(x), A_{e_i}^T \cup B_{e_i}^T)^+(x)\right).
\]

If \( \alpha_{e_i}^T = B^{+T}_{e_i}(x) \) then \( A^{-T}_{e_i}(x) \leq B^{+T}_{e_i}(x) \leq A^+_T (x) \), and so

\[
\beta_{e_i}^T = \max\{A^{-T}_{e_i}(x), B^{-T}_{e_i}(x)\}. \text{ Assume that } \beta_{e_i}^T = A^{-T}_{e_i}(x) \text{ then we have}
\]

\[
B^{-T}_{e_i}(x) \leq A^{-T}_{e_i}(x) < \left(\lambda e_i^T \wedge \mu e_i^T\right)(x) < B^{+T}_{e_i}(x) \leq A^{+T}_{e_i}(x).
\]

or \( B^{-T}_{e_i}(x) \leq A^{-T}_{e_i}(x) = \left(\lambda e_i^T \wedge \mu e_i^T\right)(x) < B^{+T}_{e_i}(x) \leq A^{+T}_{e_i}(x). \text{ For this case}

\[
B^{-T}_{e_i}(x) \leq A^{-T}_{e_i}(x) < \left(\lambda e_i^T \wedge \mu e_i^T\right)(x) < B^{+T}_{e_i}(x) \leq A^{+T}_{e_i}(x) \text{ it is contradiction to the fact that and are T-ENSCS.}
\]

For the case \( B^{-T}_{e_i}(x) < A^{-T}_{e_i}(x) = \left(\lambda e_i^T \wedge \mu e_i^T\right)(x) \leq B^{+T}_{e_i}(x) \leq A^{+T}_{e_i}(x) \)

we have \( \left(\lambda e_i^T \wedge \mu e_i^T\right)(x) \notin \left((A_{e_i}^T \cup B_{e_i}^T)^-(x), A_{e_i}^T \cup B_{e_i}^T)^+(x)\right) \) because

\[
(A_{e_i}^T \cup B_{e_i}^T)^-(x) = A^{-T}_{e_i}(x) = \left(\lambda e_i^T \wedge \mu e_i^T\right)(x). \text{ Again assume that } \beta_{e_i}^T = B^{-T}_{e_i}(x)
\]

then we have \( A^{-T}_{e_i}(x) \leq B^{-T}_{e_i}(x) \leq \left(\lambda e_i^T \wedge \mu e_i^T\right)(x) \leq B^{+T}_{e_i}(x) \leq A^{+T}_{e_i}(x). \text{ From this we can write}

\[
A^{-T}_{e_i}(x) \leq B^{-T}_{e_i}(x) \leq \left(\lambda e_i^T \wedge \mu e_i^T\right)(x) < A^{+T}_{e_i}(x)
\]

\[
\leq B^{+T}_{e_i}(x) \text{ or } A^{-T}_{e_i}(x) \leq B^{-T}_{e_i}(x) = \left(\lambda e_i^T \wedge \mu e_i^T\right)(x) < B^{+T}_{e_i}(x) \leq A^{+T}_{e_i}(x).
\]
For this case \( A_e^T(x) \leq B_e^T(x) < \left(\lambda_{e_i}^T \land \mu_{e_i}^T\right)(x) < B_{e_i}^+(x) \leq A_{e_i}^+(x) \) it is contradiction to the fact that and are T-ENSCS. And if we take the case 

\[
A_e^T(x) \leq B_e^T(x) = \left(\lambda_{e_i}^T \land \mu_{e_i}^T\right)(x) \leq A_{e_i}^+(x) \leq B_{e_i}^+(x),
\]

we get have 

\[
\left(\lambda_{e_i}^T \land \mu_{e_i}^T\right)(x) \notin \left((A_{e_i}^T \cup B_{e_i}^T)^-(x), A_{e_i}^T \cup B_{e_i}^T\right)^+(x) \text{ because } (A_{e_i}^T \cup B_{e_i}^T)^-(x) =
\]

\[
B_{e_i}^-(x) = \left(\lambda_{e_i}^T \land \mu_{e_i}^T\right)(x).
\]

If \( e_i \in M - N \) or \( e_i \in N - M \), then result is trivial.

Hence \( (P, M) \cup_r (Q, N) \) is T-ENSCS in \( X \).

Similarly we have the following theorems:

**Theorem 3.8**

Let \( (P, M) = \{ P(e_i) = \{ < x, A_{e_i}(x), \lambda_{e_i}(x) > : x \in X \} \mid e_i \in M \} \) and \( (Q, N) = \{ Q(e_i) = \{ < x, B_{e_i}(x), \mu_{e_i}(x) > : x \in X \} \mid e_i \in N \} \) be I-ENSCSs in \( X \) such that

\[
\left(\lambda_{e_i}^I \lor \mu_{e_i}^I\right)(x) \in \left\{ \begin{array}{l}
\max \left\{ \min\{A_{e_i}^I(x), B_{e_i}^I(x)\}, \min\{A_{e_i}^I(x), B_{e_i}^I(x)\}\right\}, \\
\min \left\{ \max\{A_{e_i}^I(x), B_{e_i}^I(x)\}, \max\{A_{e_i}^I(x), B_{e_i}^I(x)\}\right\}
\end{array} \right\}
\]

(3.8)

for all \( e_i \in M \) and for all \( e_i \in N \) and for all \( x \in X \). Then \( (P, M) \cup_r (Q, N) \) is also an I-ENSCS.

**Theorem 3.9**

Let \( (P, M) = \{ P(e_i) = \{ < x, A_{e_i}(x), \lambda_{e_i}(x) > : x \in X \} \mid e_i \in M \} \) and \( (Q, N) = \{ Q(e_i) = \{ < x, B_{e_i}(x), \mu_{e_i}(x) > : x \in X \} \mid e_i \in N \} \) be F-ENSCSs in \( X \) such that
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\[
\left( \lambda_{e_i}^F \vee \mu_{e_i}^F \right)(x) \in \left[ \begin{array}{c}
\max \left\{ \min \{ A_{e_i}^{+F}(x), B_{e_i}^{-F}(x) \}, \min \{ A_{e_i}^{-F}(x), B_{e_i}^{+F}(x) \} \right\}, \\
\min \left\{ \max \{ A_{e_i}^{+F}(x), B_{e_i}^{-F}(x) \}, \max \{ A_{e_i}^{-F}(x), B_{e_i}^{+F}(x) \} \right\}
\end{array} \right]
\]

(3.9)

for all \( e_i \in M \) and for all \( e_i \in N \) and for all \( x \in X \). Then \((P, M) \cup_R (Q, N)\) is also F-ENSCS.

**Corollary: 3.10**

Let \((P, M) = \{ P(e_i) = \{< x, A_{e_i}(x), \lambda_{e_i}(x)> : x \in X \} : e_i \in M \}\) and
\((Q, N) = \{ Q(e_i) = \{< x, B_{e_i}(x), \mu_{e_i}(x)> : x \in X \} : e_i \in N \}\) be NSCSs in \(X\).

Then R-union \((P, M) \cup_R (Q, N)\) is also an ENSCS in \(X\) when the conditions (3.7), (3.8) and (3.9) are valid.

**Example: 3.11**

Let \((P, I)\) and \((Q, J)\) be T-external neutrosophic soft cubic sets (T-ENSCS) in \(X\) where
\((P, I) = P(e_i) = \{< x, ([0.3,0.5],[0.2,0.5],[0.5,0.7]), (0.2,0.3,0.4)> : e_i \in I\},
\((Q, J) = Q(e_i) = \{< x, ([0.7,0.9],[0.6,0.8],[0.4,0.7]), (0.4,0.7,0.3)> : e_i \in J\}
for all \( x \in X\).

Then \((P, I)\) and \((Q, J)\) are T-ENSCS in \(X\) and \((P, I) \cap_R (Q, J) = (P, I) \cap (Q, J) = P \cap Q(e_i) = \{< x, ([0.3,0.5] [0.2,0.5],[0.4,0.7]), (0.4,0.7,0.4)> : e_i \in I \cap J\}\) for all \( x \in X\).

\((P, I) \cap_R (Q, J)\) is not T-ENSCS since

\[
\left( \lambda_{e_i}^T \vee \mu_{e_i}^T \right)(x) = 0.4 \in (0.3,0.5) = \left( A_{e_i}^T \cap B_{e_i}^T \right)^-(x), \left( A_{e_i}^T \cap B_{e_i}^T \right)^+(x)
\]

From the above example it is clear that R-intersection of T-ENSCS may not be an T-ENSCS. We provide a condition for the R-intersection of T-external (resp. I-external and F-external) neutrosophic soft cubic sets to be T-external (resp. I-external and F-external) neutrosophic soft cubic set.

**Theorem 3.12**

Let \((P, M) = \{ P(e_i) = \{< x, A_{e_i}(x), \lambda_{e_i}(x)> : x \in X \} : e_i \in M \}\) and
(Q, N) = \{ Q(e_i) = \{< x, B_{e_i}(x), \mu_{e_i}(x) > : x \in X \} : e_i \in N \} be \ T- ENSCSs in X such that

\[
H(e_i) = \begin{cases}
P(e_i) & \text{if } e \in M - N \\Q(e_i) & \text{if } e \in N - M \\P(e_i) \land_R Q(e_i) & \text{if } e \in M \cap N
\end{cases}
\] (3.12)

for all \( e_i \in M \) and for all \( e_i \in N \) and for all \( x \in X \). Then \( (P, M) \cap_R (Q, N) \) is also an \( \mathcal{T} \)-ENSCS.

Proof:
Consider \( (P, M) \cap_R (Q, N) = (H, C) \) where \( I \cap J = C \) and

\[
H(e_i) = \begin{cases}
P(e_i) & \text{if } e \in M - N \\Q(e_i) & \text{if } e \in N - M \\P(e_i) \land_R Q(e_i) & \text{if } e \in M \cap N
\end{cases}
\]

where \( H(e_i) = P(e_i) \land_R Q(e_i) \) is defined as

\[
P(e_i) \land_R Q(e_i) = H(e_i) = \{< x, \min\{A_{e_i}(x), B_{e_i}(x)\},(\lambda_{e_i} \lor \mu_{e_i})(x)\} : x \in X, e_i \in M \cap N\}
\]

where For each \( e_i \in M \cap N \), Take \( \alpha_{e_i}^T = \min\{\max\{A_{e_i}^{+T}(x), B_{e_i}^{-T}(x)\}, \max\{A_{e_i}^{-T}(x), B_{e_i}^{+T}(x)\}\}\) and \( \beta_{e_i}^T = \max\{\min\{A_{e_i}^{+T}(x), B_{e_i}^{-T}(x)\}, \min\{A_{e_i}^{-T}(x), B_{e_i}^{+T}(x)\}\}\). Then \( \alpha_{e_i}^T \) is one of \( A_{e_i}^{+T}(x), B_{e_i}^{-T}(x), A_{e_i}^{-T}(x) \) and \( B_{e_i}^{+T}(x) \). Now we consider \( \alpha_{e_i}^T = B_{e_i}^{-T}(x) \) or \( B_{e_i}^{+T}(x) \) only as the remaining cases are similar to this one.

If \( \alpha_{e_i}^T = B_{e_i}^{-T}(x) \) then \( A_{e_i}^{-T}(x) \leq A_{e_i}^{+T}(x) \leq B_{e_i}^{+T}(x) \leq B_{e_i}^{-T}(x) \) and so \( \beta_{e_i}^T = A_{e_i}^{+T}(x) \). Then given inequality we have \( (A_{e_i}^{+T}(x) \cap B_{e_i}^{+T}(x)) = A_{e_i}^{+T}(x) = \beta_{e_i}^T \).

\[
< \left( \lambda_{e_i}^{T} \lor \mu_{e_i}^{T} \right)(x).
\]

Thus we have \( \left( \lambda_{e_i}^{T} \lor \mu_{e_i}^{T} \right)(x) \in (A_{e_i}^{+T}(x) \cup B_{e_i}^{+T}(x)) \).

If \( \alpha_{e_i}^T = B_{e_i}^{+T}(x) \) then \( A_{e_i}^{-T}(x) \leq B_{e_i}^{+T}(x) \leq A_{e_i}^{+T}(x) \), and so \( \beta_{e_i}^T = A_{e_i}^{+T}(x) \). Assume that \( \beta_{e_i}^T = A_{e_i}^{-T}(x) \)
then we have \( B^{-T}_{e_i} (x) \leq A^{-T}_{e_i} (x) < \left( \lambda_{e_i}^T \vee \mu_{e_i}^T \right) (x) \leq B^+T_{e_i} (x) \leq A^+T_{e_i} (x) \). So from this we can write \( B^{-T}_{e_i} (x) \leq A^{-T}_{e_i} (x) < \left( \lambda_{e_i}^T \vee \mu_{e_i}^T \right) (x) = B^+T_{e_i} (x) \).

For this case \( B^{-T}_{e_i} (x) < A^{-T}_{e_i} (x) = \left( \lambda_{e_i}^T \vee \mu_{e_i}^T \right) (x) \leq B^+T_{e_i} (x) \leq A^+T_{e_i} (x) \). For contradiction to the fact that and are T-ENSCS.

For the case \( B^{-T}_{e_i} (x) < A^{-T}_{e_i} (x) = \left( \lambda_{e_i}^T \vee \mu_{e_i}^T \right) (x) \leq B^+T_{e_i} (x) \leq A^+T_{e_i} (x) \) we have \( B^{-T}_{e_i} (x) = \left( \lambda_{e_i}^T \vee \mu_{e_i}^T \right) (x) \). Again assume that \( \beta^T_{e_i} = B^{-T}_{e_i} (x) \) then we have \( A^{-T}_{e_i} (x) \leq B^{-T}_{e_i} (x) \leq \left( \lambda_{e_i}^T \vee \mu_{e_i}^T \right) (x) \leq B^+T_{e_i} (x) \leq A^+T_{e_i} (x) \). From this we can write \( A^{-T}_{e_i} (x) \leq B^{-T}_{e_i} (x) < \left( \lambda_{e_i}^T \vee \mu_{e_i}^T \right) (x) < B^+T_{e_i} (x) \leq A^+T_{e_i} (x) \) or \( A^{-T}_{e_i} (x) \leq B^{-T}_{e_i} (x) < \left( \lambda_{e_i}^T \vee \mu_{e_i}^T \right) (x) = B^+T_{e_i} (x) \leq A^+T_{e_i} (x) \). For the case \( A^{-T}_{e_i} (x) \leq B^{-T}_{e_i} (x) < \left( \lambda_{e_i}^T \vee \mu_{e_i}^T \right) (x) < B^+T_{e_i} (x) \leq A^+T_{e_i} (x) \) it is contradiction to the fact that and are T-ENSCS. And if we take the case \( A^{-T}_{e_i} (x) \leq B^{-T}_{e_i} (x) < \left( \lambda_{e_i}^T \vee \mu_{e_i}^T \right) (x) = B^+T_{e_i} (x) \leq A^+T_{e_i} (x) \), we get have \( \left( \lambda_{e_i}^T \vee \mu_{e_i}^T \right) (x) \notin \left( A^T_{e_i} \cup B^T_{e_i} \right) (x) \) because \( \left( A^T_{e_i} \cup B^T_{e_i} \right) (x) = B^+T_{e_i} (x) \) and \( \left( A^T_{e_i} \cup B^T_{e_i} \right) (x) = B^+T_{e_i} (x) \). Hence \( (P, M) \cap_r (Q, N) \) is T-ENSCS in X for \( e_i \in M \cap N \).

Similarly we have the following theorems.

**Theorem 3.12**

Let \( (P, M) = \{ P(e_i) = \{ < x, A_{e_i} (x), \lambda_{e_i} (x) > : x \in X \} \} \) and \( (Q, N) = \{ Q(e_i) = \{ < x, B_{e_i} (x), \mu_{e_i} (x) > : x \in X \} \} \) be I-ENSCSs in X such that
for all \( e_i \in M \) and for all \( e_i \in N \) and for all \( x \in X \). Then \( (P, M) \cap_r (Q, N) \) is also an I- ENSCS.

**Theorem 3.13**

Let \( (P, M) = \{ P(e_i) = \{< x, A_{e_i}(x), \lambda_{e_i}(x)> : x \in X \} \mid e_i \in M \} \) and \( (Q, N) = \{ Q(e_i) = \{< x, B_{e_i}(x), \mu_{e_i}(x)> : x \in X \} \mid e_i \in N \} \) be F- ENSCSs in \( X \) such that

\[
\left(\lambda_{e_i}^I \vee \mu_{e_i}^I\right)(x) \in \left\{ \begin{array}{l}
\max\left\{ \min\{A_{e_i}^+(x), B_{e_i}^-(x)\}, \min\{A_{e_i}^-(x), B_{e_i}^+(x)\} \right\}, \\
\min\left\{ \max\{A_{e_i}^+(x), B_{e_i}^-(x)\}, \max\{A_{e_i}^-(x), B_{e_i}^+(x)\} \right\}
\end{array} \right\}
\]

(3.12)

for all \( e_i \in M \) and for all \( e_i \in N \) and for all \( x \in X \). Then \( (P, M) \cap_r (Q, N) \) is also F- ENSCS.

**Corollary: 3.14**

Let \( (P, M) = \{ P(e_i) = \{< x, A_{e_i}(x), \lambda_{e_i}(x)> : x \in X \} \mid e_i \in M \} \) and \( (Q, N) = \{ Q(e_i) = \{< x, B_{e_i}(x), \mu_{e_i}(x)> : x \in X \} \mid e_i \in N \} \) be NSCSs in \( X \). Then \( (P, M) \cap_r (Q, N) \) is also an ENSCS in \( X \) when the conditions (3.11), (3.12) and (3.13) are valid.

**Theorem 3.15**

Let \( (P, M) = \{ P(e_i) = \{< x, A_{e_i}(x), \lambda_{e_i}(x)> : x \in X \} \mid e_i \in M \} \) and \( (Q, N) = \{ Q(e_i) = \{< x, B_{e_i}(x), \mu_{e_i}(x)> : x \in X \} \mid e_i \in N \} \) be T- external neutrosophic soft cubic sets in \( X \) such that

\[
\min\left\{ \max\{A_{e_i}^+(x), B_{e_i}^-(x)\}, \max\{A_{e_i}^-(x), B_{e_i}^+(x)\} \right\} = \left( \lambda_{e_i}^T \land \mu_{e_i}^T \right)(x)
\]

(3.13)
\[
= \max \left\{ \min \{ A_{e_i}^{+T}(x), B_{e_i}^{-T}(x) \}, \min \{ A_{e_i}^{-T}(x), B_{e_i}^{+T}(x) \} \right\} \tag{3.15}
\]

then the \((P, M) \cap_r (Q, N)\) is both an \(T\)-internal neutrosophic soft cubic set and an \(T\)-external neutrosophic soft cubic set in \(X\).

**Proof:** Consider \((P, M) \cap_r (Q, N) = (H, C)\) where \(M \cap N = C\) where \(H(e_i) = P(e_i) \cap_r Q(e_i)\) is defined as \(P(e_i) \cap_r Q(e_i) = H(e_i)\)

\[= \{ x, \min \{ A_{e_i}^{\lambda_i}(x), B_{e_i}^{\mu_i}(x) \}, (\lambda_i \lor \mu_i)(x) > x \in X \} \quad \text{e}_{i} \in M \cap N \} \]

Where \(P^T(e_i) \cap_r Q^T(e_i) = \{ x, \min \{ A_{e_i}^{T}(x), B_{e_i}^{T}(x) \}, (\lambda_i^{T} \lor \mu_i^{T})(x) > x \in X \} \quad \text{e}_{i} \in M \cap N \} \}

For each \(e_{i} \in M \cap N \) Take \(\alpha_{e_i}^{T} = \min \left\{ \max \{ A_{e_i}^{+T}(x), B_{e_i}^{-T}(x) \}, \max \{ A_{e_i}^{-T}(x), B_{e_i}^{+T}(x) \} \right\} \) and \(\beta_{e_i}^{T} = \max \left\{ \min \{ A_{e_i}^{+T}(x), B_{e_i}^{-T}(x) \}, \min \{ A_{e_i}^{-T}(x), B_{e_i}^{+T}(x) \} \right\} \). Then \(\alpha_{e_i}^{T} \) is one of \(A_{e_i}^{+T}(x), B_{e_i}^{-T}(x), A_{e_i}^{-T}(x), B_{e_i}^{+T}(x)\).

Now we consider \(\alpha_{e_i}^{T} = A_{e_i}^{+T}(x)\), or \(A_{e_i}^{-T}(x)\), only as the remaining cases are similar to this one. If \(\alpha_{e_i}^{T} = A_{e_i}^{+T}(x)\) then \(B_{e_i}^{-T}(x) \leq B_{e_i}^{+T}(x) \leq A_{e_i}^{-T}(x) \leq A_{e_i}^{+T}(x)\), and so \(\beta_{e_i}^{T} = B_{e_i}^{+T}(x)\). This implies that \(A_{e_i}^{+T}(x) = \alpha_{e_i}^{T} = \left( \lambda_{e_i}^{T} \lor \mu_{e_i}^{T} \right)(x)\)

\(= \beta_{e_i}^{T} = B_{e_i}^{+T}(x)\). Thus \(B_{e_i}^{-T}(x) \leq B_{e_i}^{+T}(x) = \left( \lambda_{e_i}^{T} \lor \mu_{e_i}^{T} \right)(x) = A_{e_i}^{-T}(x) \leq A_{e_i}^{+T}(x)\). Which implies that \(\left( \lambda_{e_i}^{T} \lor \mu_{e_i}^{T} \right)(x) = B_{e_i}^{+T}(x) = (A_{e_i}^{T} \cap B_{e_i}^{T})^{+}(x)\)

Hence \(\left( \lambda_{e_i}^{T} \lor \mu_{e_i}^{T} \right)(x) \leq \left( A_{e_i}^{T} \cap B_{e_i}^{T} \right)^{+}(x)\). If \(\alpha_{e_i}^{T} = A_{e_i}^{+T}(x)\) then \(B_{e_i}^{-T}(x) \leq A_{e_i}^{+T}(x) \leq B_{e_i}^{+T}(x)\), and so \(\left( \lambda_{e_i}^{T} \lor \mu_{e_i}^{T} \right)(x) = A_{e_i}^{+T}(x) = (A_{e_i}^{T} \cap B_{e_i}^{T})^{+}(x)\). Hence \(\left( \lambda_{e_i}^{T} \lor \mu_{e_i}^{T} \right)(x) \leq \left( A_{e_i}^{T} \cap B_{e_i}^{T} \right)^{+}(x)\). Consequently we note that \((P, M) \cap_r (Q, N)\) is both an \(T\)-INSICS and an \(T\)-ENSCS in \(X\).
Similarly we have the following theorems

**Theorem 3.16**

If neutrosophic soft cubic set \((P, M) = \{ P(e_i) = \{< x, A_{e_i}(x), \lambda_{e_i}(x) > : x \in X \} \ e_i \in M \} \) and \((Q, N) = \{ Q(e_i) = \{< x, B_{e_i}(x), \mu_{e_i}(x) > : x \in X \} \ e_i \in N \} \)
in X satisfy the following condition

\[
\min \left\{ \max \left\{ A_{e_i}^+(x), B_{e_i}^-(x) \right\}, \max \left\{ A_{e_i}^-(x), B_{e_i}^+(x) \right\} \right\} = \max \left\{ \min \left\{ A_{e_i}^+(x), B_{e_i}^-(x) \right\}, \min \left\{ A_{e_i}^-(x), B_{e_i}^+(x) \right\} \right\}
\]

(3.16)

then the \((P, M) \cap (Q, N)\) is both an I-INSCS and an I-ENSCS in X.

**Theorem 3.17**

If neutrosophic soft cubic set \((P, M) = \{ P(e_i) = \{< x, A_{e_i}(x), \lambda_{e_i}(x) > : x \in X \} \ e_i \in M \} \) and \((Q, N) = \{ Q(e_i) = \{< x, B_{e_i}(x), \mu_{e_i}(x) > : x \in X \} \ e_i \in N \} \)
in X satisfy the following condition

\[
\min \left\{ \max \left\{ A_{e_i}^F(x), B_{e_i}^{-F}(x) \right\}, \max \left\{ A_{e_i}^{-F}(x), B_{e_i}^F(x) \right\} \right\} = \max \left\{ \min \left\{ A_{e_i}^F(x), B_{e_i}^{-F}(x) \right\}, \min \left\{ A_{e_i}^{-F}(x), B_{e_i}^F(x) \right\} \right\}
\]

(3.17)

then the \((P, M) \cap (Q, N)\) is both an F-INSCS and an F-ENSCS in X.

**Corollary: 3.18**

Let \((P, M) = \{ P(e_i) = \{< x, A_{e_i}(x), \lambda_{e_i}(x) > : x \in X \} \ e_i \in M \} \) and \((Q, N) = \{ Q(e_i) = \{< x, B_{e_i}(x), \mu_{e_i}(x) > : x \in X \} \ e_i \in N \} \) be NSCSs in X. Then \((P, M) \cap (Q, N)\) is also an ENSCS and INSCS in X when the conditions (3.15), (3.16) and (3.17) are valid.

**Theorem: 3.19**

Let \((P, M) = \{ P(e_i) = \{< x, A_{e_i}(x), \lambda_{e_i}(x) > : x \in X \} \ e_i \in M \} \) and
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\[(Q, N) = \{ Q(e_i) = \{< x, B_{e_i}(x), \mu_{e_i}(x) > : x \in X \} \mid e_i \in N \}\] be T- INSCSs in \(X\) such that \(\left(\lambda_{e_i}^T \wedge \mu_{e_i}^T\right)(x) \leq \max\{A_{e_i}^{-T}(x), B_{e_i}^{-T}(x)\}\) for all \(e_i \in M\) and for all \(e_i \in N\) and for all \(x \in X\), then \((P, M) \cup_R (Q, N)\) is an T-ENSCS in \(X\).

**Proof:**

Let \((P, M) = \{ P(e_i) = \{< x, A_{e_i}(x), \lambda_{e_i}(x) > : x \in X \} \mid e_i \in M \}\) and \((Q, N) = \{ Q(e_i) = \{< x, B_{e_i}(x), \mu_{e_i}(x) > : x \in X \} \mid e_i \in N \}\) are T- INSCSs in \(X\).

Thus for all \(e_i \in M\), we have \(A_{e_i}^{-T}(x) \leq \lambda_{e_i}^T(x) \leq A_{e_i}^T(x)\) and for all \(e_i \in N\), \(B_{e_i}^{-T}(x) \leq \mu_{e_i}^T(x) \leq B_{e_i}^T(x)\). Since \((P, M) \cup_R (Q, N)\) is defined as \((P, M) \cup_R (Q, N) = (H, C)\) where \(C = M \cup N\),

| \(H(e_i)\) | \(P(e_i)\) | \(Q(e_i)\) | \(P(e_i) \vee_R Q(e_i)\) |
|----------------|---------|---------|------------------|
| \(P(e_i)\)   | If \(e_i \in M - N\) |
| \(Q(e_i)\)   | If \(e_i \in N - M\) |
| \(P(e_i) \vee_R Q(e_i)\) | If \(e_i \in M \cap N\) |

Where \(P(e_i) \vee_R Q(e_i)\) is defined as

\[P(e_i) \vee_R Q(e_i) = \{< x, \max\{A_{e_i}(x), B_{e_i}(x)\}, (\lambda_{e_i} \wedge \mu_{e_i})(x) > : x \in X \} \mid e_i \in M \cap N\}\] where \(P'(e_i) \vee_R Q'(e_i) = \{< x, \min\{A_{e_i}^T(x), B_{e_i}^T(x)\}, (\lambda_{e_i}^T \wedge \mu_{e_i}^T)(x) > : x \in X \} \mid e_i \in M \cap N\\} \). Given condition is \(\left(\lambda_{e_i}^T \wedge \mu_{e_i}^T\right)(x) \leq \max\{A_{e_i}^{-T}(x), B_{e_i}^{-T}(x)\}\) for all \(e_i \in M\) and for all \(e_i \in N\) and for all \(x \in X\). This implies that

\[
\left(\lambda_{e_i}^T \vee_R \mu_{e_i}^T\right)(x) \notin \left(\left(A_{e_i}^T \cap B_{e_i}^T\right)^-(x), (A_{e_i}^T \cap B_{e_i}^T)^+(x)\right) = \left(\max\{A_{e_i}^{-T}(x), B_{e_i}^{-T}(x)\}, \max\{A_{e_i}^T(x), B_{e_i}^T(x)\}\right).
\]

Hence \((P, M) \cup_R (Q, N)\) is T-ENSCS in \(X\).

Similarly we have the following theorems

**Theorem: 3.20**

Let \((P, M) = \{ P(e_i) = \{< x, A_{e_i}(x), \lambda_{e_i}(x) > : x \in X \} \mid e_i \in M \}\) and
Theorem: 3.21

Let \((P, M) = \{ P(e_i) = \{< x, A_{e_i}(x), \lambda_{e_i}(x) > : x \in X \} \mid e_i \in M \}\) and \((Q, N) = \{ Q(e_i) = \{< x, B_{e_i}(x), \mu_{e_i}(x) > : x \in X \} \mid e_i \in N \}\) be T- INSCSs in \(X\) such that \(\lambda_{e_i} \wedge \mu_{e_i} \leq \max\{A^{-I}_{e_i}(x), B^{-I}_{e_i}(x)\}\) for all \(e_i \in M\) and for all \(e_i \in N\) and for all \(x \in X\), then \((P, M) \cup_R (Q, N)\) is an I-ENSCS in \(X\).

Corollary: 3.22

Let \((P, M) = \{ P(e_i) = \{< x, A_{e_i}(x), \lambda_{e_i}(x) > : x \in X \} \mid e_i \in M \}\) and \((Q, N) = \{ Q(e_i) = \{< x, B_{e_i}(x), \mu_{e_i}(x) > : x \in X \} \mid e_i \in N \}\) be INSCSs then \((P, M) \cup_R (Q, N)\) is an ENSCS in \(X\) when the THEOREMS (3.19), (3.20) and (3.21) are valid.

Theorem: 3.23

Let \((P, M) = \{ P(e_i) = \{< x, A_{e_i}(x), \lambda_{e_i}(x) > : x \in X \} \mid e_i \in M \}\) and \((Q, N) = \{ Q(e_i) = \{< x, B_{e_i}(x), \mu_{e_i}(x) > : x \in X \} \mid e_i \in N \}\) be T- INSCSs in \(X\) such that \(\lambda_{e_i} \vee \mu_{e_i} \geq \max\{A^{+T}_{e_i}(x), B^{+T}_{e_i}(x)\}\) for all \(e_i \in M\) and for all \(e_i \in N\) and for all \(x \in X\), then \((P, M) \cap_R (Q, N)\) is T-ENSCS in \(X\).

Proof:

Let \((P, M) = \{ P(e_i) = \{< x, A_{e_i}(x), \lambda_{e_i}(x) > : x \in X \} \mid e_i \in M \}\) and \((Q, N) = \{ Q(e_i) = \{< x, B_{e_i}(x), \mu_{e_i}(x) > : x \in X \} \mid e_i \in N \}\) are T- INSCSs in \(X\).

Thus for all \(e_i \in M\), we have \(A^{+T}_{e_i}(x) \leq \lambda_{e_i}^{+T}(x) \leq A^{-+T}_{e_i}(x)\) and for all \(e_i \in N\), \(NB^{+T}_{e_i}(x) \leq \mu_{e_i}^{+T}(x) \leq B^{-+T}_{e_i}(x)\). Since \((P, M) \cap_R (Q, N)\) is defined as \((P, M) \cap_R (Q, N) = (H, C)\) where \(C = M \cap N\) and

\[
(Q, N) = \{ Q(e_i) = \{< x, B_{e_i}(x), \mu_{e_i}(x) > : x \in X \} \mid e_i \in N \}\)
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\[ H(e_i) = P(e_i) \wedge_R Q(e_i) \quad \text{if } e_i \in M \cap N, \text{ where } P(e_i) \wedge_R Q(e_i) \text{ is defined as } P(e_i) \wedge_R Q(e_i) = \{ x \mid \min\{ A_{e_i}(x), B_{e_i}(x)\}, (\lambda_{e_i} \vee \mu_{e_i})(x) > : x \in X \} \quad e_i \in M \cap N \}\]

where \( P^T(e_i) \wedge_R Q^T(e_i) = \{ x, \min\{ A^T_{e_i}(x), B^T_{e_i}(x)\}, (\lambda^T_{e_i} \vee \mu^T_{e_i})(x) > : x \in X \} \quad e_i \in M \cap N \} \).

Given condition is

\[ \left( \lambda^T_{e_i} \vee \mu^T_{e_i} \right)(x) \geq \min\{ A^{+T}_{e_i}(x), B^{+T}_{e_i}(x) \} \]

for all \( e_i \in M \) and all \( e_i \in N \) and for all \( x \in X \). This implies that

\[ \left( \lambda^T_{e_i} \vee \mu^T_{e_i} \right)(x) = \left( A_{e_i}^T \cap B_{e_i}^T \right)(x), \left( A_{e_i}^T \cap B_{e_i}^T \right)^+(x) \]

Hence \((P, M) \cap_R (Q, N)\) is both an T-ENSCS in \( X \).

**Theorem 3.24**

Let \((P, M) = \{ P(e_i) = \{ x, A_{e_i}(x), \lambda_{e_i}(x) : x \in X \} \quad e_i \in M \}\) and \((Q, N) = \{ Q(e_i) = \{ x, B_{e_i}(x), \mu_{e_i}(x) : x \in X \} \quad e_i \in N \}\) be I- INSCSs in \( X \) such that

\[ (\lambda^I_{e_i} \vee \mu^I_{e_i})(x) \geq \max\{ A^{+I}_{e_i}(x), B^{+I}_{e_i}(x) \} \quad \text{for all } e_i \in M \text{ and for all } e_i \in N \text{ and for all } x \in X \].

Then \((P, M) \cap_R (Q, N)\) is an I-ENSCS in \( X \).

**Theorem 3.25**

Let \((P, M) = \{ P(e_i) = \{ x, A_{e_i}(x), \lambda_{e_i}(x) : x \in X \} \quad e_i \in M \}\) and \((Q, N) = \{ Q(e_i) = \{ x, B_{e_i}(x), \mu_{e_i}(x) : x \in X \} \quad e_i \in N \}\) be F- INSCSs in \( X \) such that

\[ (\lambda^F_{e_i} \vee \mu^F_{e_i})(x) \geq \max\{ A^{+F}_{e_i}(x), B^{+F}_{e_i}(x) \} \quad \text{for all } e_i \in M \text{ and for all } e_i \in N \text{ and for all } x \in X \].

Then \((P, M) \cap_R (Q, N)\) is F-ENSCS in \( X \).

**Corollary: 3.26**

Let \((P, M) = \{ P(e_i) = \{ x, A_{e_i}(x), \lambda_{e_i}(x) : x \in X \} \quad e_i \in M \}\) and
\((Q, N) = \{ Q(e_i) = \{ x, B^e_i (x), \mu^e_i (x) : x \in X \} \mid e_i \in N \}\) be INSCSs then 
\((P, M) \cap_{\mathcal{R}} (Q, N)\) is both an ENSCS in \(X\) when the Theorems (3.23), (3.24) and (3.25) are valid.

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