Approach the Fundamental Limit of Orbital Angular Momentum Multiplexing

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Establishing and approaching the fundamental limit of orbital angular momentum (OAM) multiplexing are paramountly important and increasingly urgent for current multiple-input multiple-output research. In this work, we elaborate the fundamental limit in terms of independent scattering channels (or degrees of freedom of scattered fields) through angular-spectral analysis, in conjunction with a transformation of basis. The scattering channel limit is universal for arbitrary spatial mode multiplexing, which is launched by a planar electromagnetic device, such as antenna, metasurface, etc, with a predefined physical size. As a proof of concept, we demonstrate both theoretically and experimentally the limit by a metasurface hologram that transforms orthogonal OAM modes to plane-wave modes scattered at critically separated angular-spectral regions. Particularly, a min-max optimization algorithm is applied to suppress angular spectrum aliasing, achieving good performances in both full-wave simulation and experimental measurement at microwave frequencies. This work offers a theoretical upper bound and corresponding approach route for engineering designs of OAM multiplexing.

I. INTRODUCTION

The growing demand for high data (transmission) rate per unit bandwidth has driven researchers to continuously explore the potential of independent scattering channels. It is well known that upper bound of the data rate is determined by Shannon’s channel capacity [1]; and various spatial mode multiplexing (SMM) strategies have been proposed to approach the scattering channel limit for maximizing the channel capacity [2, 3]. Orbital angular momentum (OAM) mode is an approximate solution to the free-space Helmholtz equation in cylindrical coordinates [4]. In contrast to extensively-used plane-wave mode, OAM mode exhibits unique electromagnetic (EM) properties including nonuniform intensity with a phase singularity, helical wavefront, strong divergence and rich orthogonal modes [5–7], which has been regarded as a promising solution for overcoming the issue of limited frequency channels [8–10]. In the past few years, high capacity communication systems based on OAM multiplexing as well as corresponding OAM generation/detection technologies have drawn great attentions both in optical [11, 18] and EM [19, 29] researches.

However, OAM multiplexing provokes a discussion about its advantages over traditional multi-input multi-output (MIMO) [30, 31] systems and its fundamental scattering channel limit [32–34]. In [32], an intuitive limit is given according to spatial-bandwidth product method in view of the size of focusing lens and width of OAM beams. Similarly, considering the expansion of OAM beam width during wave propagation, the aperture size and propagating distance are deemed to govern the OAM scattering channel limit, where analytical results on the numbers of independent scattering channels have been derived [33]. Based on degrees of freedom of scattered fields [34], the relation between the scattering channel limit and EM source size was clarified by employing multipole expansion of truncated Bessel beams over a sphere surface [34]. Moreover, information theory of metasurfaces [35] is proposed to infer the degrees of freedom of plane-wave multiplexing. Nonetheless, a simple and physically intuitive bound on the number of independent scattering channels and a specific EM device designed to demonstrate the fundamental scattering channel limit would bring new insights into this problem.

In this work, we establish an alternative and accessible way to deduce the scattering channel limit of OAM multiplexing, meanwhile, propose an optimization algorithm to approach the limit in ideal and realistic metasurface designs. The number of independent scattering channels of OAM modes, bounded by the physical size of the generating or detecting aperture, is equivalent to that of plane-wave modes, which could be argued from a mathematical point of view: a transformation of basis,
from OAM modes to plane-wave modes and vice versa, will not change the degrees of freedom (or the number of independent scattering channels) of SMM. Consequently, the fundamental scattering channel limit is generally applicable to any SMM systems. Furthermore, a minimax algorithm could be exploited to mitigate mode crosstalks to a great extent for approaching the fundamental limit of OAM multiplexing.

This paper is organized as follows. In Section II, we elaborate the scattering channel limit of plane-wave modes generated by a 2-D area-constrained source. In Section III, we prove that OAM basis and plane-wave basis share the same scattering channel limit when they are generated by the same size source. After that, minimax optimizations for both ideal and realistic metasurfaces are discussed in Section IV. Then, experiment is conducted to verify the performance of the realistic metasurface at microwave frequencies in Section V. Additionally, the details of minimax optimization are described in Appendix.

II. SCATTERING CHANNEL LIMIT OF PLANE-WAVE MULTIPLEXING

The SMM relies on the independence or orthogonality of EM modes in various bases, such as plane-wave basis and OAM basis. The data rate of the SMM system has been intensively studied in information theory, exhibiting a close connection to wave basis concept in EM theory (the relation between them is discussed in Supplemental Material). Briefly speaking, the data rate critically depends on the number of available orthogonal EM modes scattered by a 2-D planar EM device or generated by a 2-D equivalent source, which can be referred as the fundamental scattering channel limit of the 2-D SMM system. In this section, we first consider the plane-wave basis, in which the derivation of the scattering channel limit (equivalent to the number of independent and distinguishable plane-wave modes generated by an area-constrained source at far-field) is straightforward. Suppose the physical size of the area-constrained 2-D source is \( L_x \times L_y \), which can be regarded as a band-limited source function \( J(r') \). The far-field pattern of this source can be calculated with a dyadic Green’s function as

\[
E_{f_{ar}}(k) = -j\omega\mu_0 e^{-jk_0 r} \int_v \left( a_\theta a_\theta + a_\varphi a_\varphi \right) \cdot J(r') \exp(jk_0 r' \cdot \mathbf{a}_R) d\mathbf{r}',
\]

where \( a_\theta a_\theta \) and \( a_\varphi a_\varphi \) represent the \( \theta \) and \( \varphi \) components of unit dyadic, \( a_R \) denotes the \( R \) component of unit vector, both in the spherical coordinates, and \( k_0 \) is the wave number in free space.

Obviously, the transform from near-field to far-field is a low-pass Fourier transform from the spatial domain to the angular-spectral domain. For plane-wave modes propagating in free space, each mode will occupy a certain \( \Delta k \) area because the source area is finite or constrained. They will approach the distinguishable limit when the corresponding half-power circles of far-field patterns in the \( k \) space are nearly tangent, referred to as Rayleigh’s limit. Hence, on account of the limited angular-spectral region in half space, the number of distinguishable plane-wave modes as independent scattering channels can be easily deduced, inspired by the same concept in [37].

The quantitative analysis is demonstrated as following. The minimum resolutions along the \( x \) and \( y \) directions in the angular-spectral domain are

\[
\Delta k_x = \frac{2\pi}{L_x}, \quad \Delta k_y = \frac{2\pi}{L_y},
\]

where \( L_x \) and \( L_y \) are the side lengths of the source. Thus, the minimum \( k \) elementary area needed for one plane-wave mode propagating in free space is \( \Delta k_x \cdot \Delta k_y \), denoted by the dotted-line pixel in FIG. 1(a). In free space, we have

\[
k_x^2 + k_y^2 + k_z^2 = k_0^2,
\]

where \( k_0 \) is the free space wave number. For a propagating plane-wave mode which obeys \( k_z^2 > 0 \), the total angular-spectral area in the half space is bounded by

\[
k_x^2 + k_y^2 < k_0^2,
\]

marked by the light red circle in FIG. 1(a). Then, the number of independent scattering channels can be obtained as

\[
N \leq \frac{\pi k_0^2}{\Delta k_x \cdot \Delta k_y} = \frac{\pi \Delta k_x \cdot \Delta k_y}{2\pi L_x \cdot 2\pi L_y} = \frac{\pi S}{\lambda_0^2},
\]

FIG. 1. Fundamental scattering channel limit of plane-wave multiplexing. (a) The minimum distance between two dots represents the minimum resolution in \( k \) space, the dotted square formed by adjacent four dots represents the scattering channel for a free-space propagating plane-wave mode and the light red zone inside the circle represents the total available \( k \) space region. (b) Far-field pattern of multiplexed plane-wave modes generated by a square source with side length \( L_x = L_y = 1.12 \lambda_0 \). The radius of depicted \( k \) area is \( k_0 \). The dotted black lines represent the half-power circles and the crosses represent the pre-designed scattering directions of the four generated plane-wave modes.
where $N$ is the fundamental limit of independent scattering channels of plane-wave multiplexing, $S = L_x \cdot L_y$ is the geometric area of the source and $\lambda_0$ is the free space wavelength. This limit considers only one type of polarization, and it should be doubled if dual polarizations are adopted. If more plane-wave modes beyond this limit are added, they will not be distinguishable or angularly resolved. Moreover, evanescent modes are required to support the expanded angular spectrum, which, however, are not suitable for far-field communication.

Here, an example is given for validation. The far-field pattern of four multiplexed plane-wave modes oriented at four pre-designed directions is depicted in FIG. 1 (b). The minimum side length of the source is set as $L_x = L_y = 1.12 \lambda_0$ according to Eq. (5). The far-field is calculated by

$$P_{\text{far}} = \left| \sum_{n=1}^{4} F \left\{ \exp[jk_x(n) \cdot x + jk_y(n) \cdot y] \cdot \Omega(x, y) \right\} \right|^2,$$

where $F$ denotes Fourier transform, $\Omega(x, y)$ is a window function with $\Omega = 1$ ($\Omega = 0$) inside (outside) the source area, $n$ is the index of plane-wave modes, and the corresponding wave numbers of the four pre-designed directions are $k_x(n)/k_0 = [0.6, 0, -0.6, 0]$ and $k_y(n)/k_0 = [0.6, 0, -0.6, 0]$. It can be observed from FIG. 1 (b) that the half-power circle of each mode is nearly tangent to that of nearby modes; and the $k$ space is almost totally occupied with little wide-angle margin, validating Eq. (5).

### III. THE EQUIVALENCE OF SCATTERING CHANNEL LIMIT BETWEEN OAM BASIS AND PLANE-WAVE BASIS

Different from plane-wave basis, spatial waves or structured waves, such as Laguerre-Gaussian beams carrying OAM, form the spatially orthogonal basis which can be multiplexed towards unidirection [6]. Information is encoded (generated) or decoded (detected) in terms of the superposed orthogonal modes via a EM generator or detector involving antenna arrays, metasurfaces, spiral phase plates, etc. Particularly, the EM detector is capable of distinguishing these modes for realizing information retrieval. Intuitively, the structured waves can be infinitely multiplexed in a line of sight (LOS) communication system. However, the maximum number of orthogonal spatial modes is limited by the physical or geometrical size of the EM device.

In this section, we discuss the scattering channel limit of OAM multiplexing and connect it to that of plane-wave multiplexing, in view of the fact that the transformation from a set of OAM modes to a set of plane-wave modes is a basis-to-basis mapping. Both propagating OAM modes and propagating plane-wave modes are complete orthogonal bases in free space, and thus the transformation between the two basis sets is a unitary transformation. Consequently, with the same source size, the two sets of orthogonal basis share the same scattering channel limit given in Eq. (5). To increase the data rate of SMM, if extra OAM modes are added beyond this limit, these added modes are non-orthogonal to (dependent on) the existing modes and cannot be distinguishable for information transmission. This is similar to the situation when increasing the number of antennas beyond a limit, the data rate of an area-constrained SMM system cannot be increased. From the information-theoretical view, the diversity has an upper bound when the source size is fixed [38].

Regarding a specific design of OAM multiplexing, the transformation of basis is simultaneously a OAM mode detecting procedure for information transmission. Here, a metasurface is utilized to realize the transformation of basis benefited from its remarkable advances in OAM multiplexing [39][41]. As shown in FIG. 2, for a set of multiplexed OAM modes with encoded amplitudes propagating towards one direction, each of them can be transformed to one plane-wave mode propagating towards a pre-designed direction with a well-engineered metasurface. As a time-reversal procedure due to reciprocity, multiplexed OAM modes can be generated by impinging the metasurface with plane-wave modes from different directions. Based on the above explanations, the maximum number of independent OAM scattering channels is bounded by the size of the metasurface. Such fundamental scattering channel limit can be approached if corresponding orthogonal plane-wave modes are generated and angularly distinguishable. In this case, the information encoded in the amplitudes of multiplexed OAM modes is transmitted to that in the amplitudes of multiplexed plane-wave modes. Nevertheless, maximizing the number of independent scattering channels is difficult be-
beams are utilized as OAM sources with the radical in-transformed plane-wave modes. Laguerre Gaussian (LG) of high-order OAM modes and suppressing side lobes of initial phases to be optimized for reducing mode crosstalks. However, the methods of calculating the far-field patterns with the input variables are different for the ideal and realistic cases. Technical details and results are discussed as following.

A. Ideal Metasurface

To begin with, we consider the minimum source size for spatial multiplexing of four OAM modes, then extend them to eight modes. In order to match the full-wave EM simulation in the next subsection, the metasurface is designed to be composed of $10 \times 10$ meta-atoms working at 5.088 GHz, in which the meta-atoms refer to the repetitive unit cells of the metasurface. The area of this metasurface is set to be $S = 1.36 \lambda_0^2$ and the side length is $L_x = L_y = 1.17 \lambda_0$. From Eq. (3), the maximum number of independent OAM modes allowed for multiplexing is calculated to be 4.3, leaving negligible margin for the multiplexing of four modes. According to the method proposed in [42], multiplexed OAM modes can be detected by a single metasurface with phase distribution

$$t(r, \phi) = A(r, \phi) \exp \left( j \phi_0(r, \phi) \right) \cdot \left\{ \sum_n \exp \left[ j \left( l_n \phi + k_x(n)x + k_y(n)y \right) \right] \right\}, \quad (7)$$

where $n$ is the index of modes, $l_n$ is the topological charge of OAM modes, $A(r, \phi)$ are the normalized amplitude factors for guaranteeing $|t(r, \phi)| = 1$ and $\phi_0(r, \phi)$ are the initial phases to be optimized for reducing mode crosstalks of high-order OAM modes and suppressing side lobes of transformed plane-wave modes. Laguerre Gaussian (LG) beams are utilized as OAM sources with the radical index $p = 0$ and azimuthal index $l_0$ at the position $z = 0$ cause the transformation of basis by the metasurface is always non-ideal, obstructed by mode crosstalks. Hence, in order to approach the scattering channel limit, minimax algorithm is employed to optimize the metasurface in the next section.

$$LG_{l_0} = \sqrt{\frac{2}{\pi(l_0)!}} \frac{1}{w(0)} \left[ \frac{r \sqrt{2}}{w(0)} \right]^{l_0} \exp \left[ -r^2 \right] \exp[jl_0\phi] \cdot L_0^{l_0} \left( \frac{2r^2}{w^2(0)} \right), \quad (8)$$

where $r$ is the radius, $\phi$ is the azimuthal angle, $L_0$ is the associated Laguerre polynomial and $w(0)$ is the beam waist. It is worth noting that $w(0)$ is set equal to the side length of the metasurface. Next, a phase plane of the same size is digitized into $10 \times 10$ pixels to represent the ideal metasurface and additional phase will be carried when the LG beams pass through the plane. After that, the far-field pattern can be calculated with a Fourier transform [43].

$$E_{\text{far}} = \mathcal{F} \left\{ LG_{l_0}, t(r, \phi) \right\} = \sum_n E_{\text{OAM}(l_n+l_0)}(k_x, k_y), \quad (9)$$

where $\mathcal{F}$ denotes Fourier transform, $t(r, \phi)$ is the phase distribution of the metasurface and $(l_n + l_0)$ represents the orders of generated OAM modes. The normalized far-field intensity pattern can be written as

$$P_{\text{far}}(k_x, k_y) = \frac{E_{\text{far}}^2(k_x, k_y)}{\sum_{k_x,k_y} E_{\text{far}}^2(k_x, k_y)}, \quad (10)$$

where $\sum_{k_x,k_y}$ denotes the sum of power density over all $k$ space. Obviously, far-field intensity $P_{\text{far}}$ is a function of $k_x$ and $k_y$, denoting the power intensities along different directions. By engineering the phase distribution of the metasurface, the OAM mode with $l_0 + l_n = 0$, which has the highest intensity and behaves like a plane wave mode, can be generated towards a pre-designed direction to realize the mode detection and information retrieval. It is straightforward to prove that the transformation (9) from the OAM modes to the plane-wave modes is a unitary transformation.

Unfortunately, the unwanted high-order OAM modes with $l_0 + l_n \neq 0$ are inevitable with this detecting method. As the size of metasurface becomes smaller, the unwanted crosstalks between the high-order modes become worse, especially for the minimum limiting case, which shows a big difference from our previous work [42]. Such side lobes will not only reduce the detecting accuracy but also will also make it difficult to analyze the scattering channel limit. Mathematically, the optimization in this case can be considered as a worst-case tolerance problem, and the key is to find the phase distribution which makes the worst far-field pattern optimal under the incidence of each OAM mode to be multiplexed. Thus we propose a minimax optimization algorithm to suppress the unwanted side lobes as much as possible, details of the optimization algorithm can be found in Appendix, where the input variables are the phases of the discretized pixels (ideal meta-atoms) denoted by $[\phi_1, \phi_2, \cdots, \phi_{100}]$.
multiplexed LG beams are fixed as the input sources and objective functions are constructed with far-field patterns given by Eqs. (7-10).

The comparison between non-optimized and optimized results is depicted in FIG. 3 from which one can see that the side lobes are significantly suppressed, especially for the modes 1 and -1. With the optimization, the normalized far-field intensity patterns under the incidences of multiplexed OAM modes are shown in FIG. 4. For each scattered plane-wave mode, the maximum-intensity point in the $k$ space with a normalized power intensity 1 is set as the center of the half-power circle, and the radius of the half-power circle is set as the median of the distance between the two points with the normalized power intensity 1 and 0.5 in the $k$ space, respectively. Furthermore, the pre-designed scattered directions of the four OAM modes are denoted by the crosses. There will be minor deviations between the designed and realistic scattered directions. For the four OAM modes multiplexing, half-power circle of each scattered plane-wave mode is nearly tangent to that of nearby modes and almost all of the $k$ space is occupied, manifesting that the OAM scattering channel limit is the same as that of the plane-wave case in Section II. Moreover, eight OAM modes multiplexing is also investigated with a larger metasurface whose size is $S = 2.67 \lambda^2_0$, and the theoretical upper limit $N$ in Eq. (5) is calculated to be 8.4. In FIG. 4(b), the red-dot circle denotes an overlapped useless field region resulting from the crosstalks of high-order OAM modes. Fortunately, eight transformed plane-wave modes are angular-spectrally separated in a marginal sense. Therefore, the fundamental scattering channel limit of OAM multiplexing can be efficiently approached with the proposed optimization algorithm.

**B. Realistic Metasurface**

In this subsection, a realistic metasurface is built up, where the input variables, sources, calculation of far-field and error compensations are different from the ideal case. The area of this realistic metasurface is set to be $S = 1.36 \lambda^2_0$ and it is constructed by the same $10 \times 10$ meta-atoms as the ideal case. For flexible phase control and simple printed-circuit-board fabrication, Pancharatnam-Berry meta-atoms are chosen to construct the detecting metasurface for achieving desired geometric phase [46]. Under the incidence of circular-polarized wave, additional phase will be carried by the transmitted cross-polarized wave and the introduced phase by each meta-atom is exactly twice of its rotation angle. It is noticeable that the size of this detecting metasurface is micrometre with the side length $L_x = L_y = 1.17 \lambda_0$, which suggests small size meta-atoms are needed. Based on the design principle in [46], we construct a meta-atom around $0.11 \times 0.11 \lambda^2_0$ by using a substrate with a high relative dielectric constant $\epsilon_r = 12.2$. Using periodic boundary conditions in full-wave EM simulation, this meta-atom achieves nearly 99.8% transmission efficiency at 5.088 GHz as shown in FIG. 5. And the detecting metasurface, composed of $10 \times 10$ meta-atoms, is shown in FIG. 6.

As we adopt the geometric phase based metasurface, all the variables to be optimized are the rotation angles of the meta-atoms represented by $[\theta_1, \theta_2, \cdots, \theta_{100}]$. Also, the far-field calculation is different from that of previous case. In the ideal case, totally confined LG beams and ideal phase plane are utilized, which are not attainable in realistic implementation. For general consideration, circularly polarized plane-wave modes are used to gen-

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**FIG. 3.** Responses of an ideal metasurface for OAM detecting under the incidences of different OAM modes. (a)-(d) Far-field intensity patterns under the incidences of OAM modes $l_0 = 1, 3, -3, -1$ without optimization; (e)-(h) Far-field intensity patterns under the incidences of OAM modes $l_0 = 1, 3, -3, -1$ with optimization.

**FIG. 4.** Responses of the ideal metasurfaces under the incidences of multiplexed OAM modes. (a) Far-field intensity pattern under the incidence of multiplexed OAM modes $l_0 = 1, 3, -3, -1$. The dotted black lines represent the half-power circles of far-field intensity patterns and the crosses represent the pre-designed scattered directions of the four OAM modes (the scattering channel limit of OAM multiplexing with this metasurface is 4.27); (b) Far-field intensity pattern under the incidence of multiplexed OAM modes $l_0 = 1, 3, 5, 7, -7, -5, -3$. The dotted black lines represent the half-power circles of far-field intensity patterns, the dotted red line represents the overlapped mode and the crosses represent the pre-designed scattered directions of the eight OAM modes (the scattering channel limit of OAM multiplexing with this metasurface is 8.4).
The mission matrix becomes atom can be represented by a transmission matrix than using periodic boundary conditions. Each meta-
ations of meta-atoms. Several methods are proposed to breakdown of periodic boundary conditions after rota-
duced by metasurface is always non-ideal because of the design, a challenging problem is that the phase intro-
mission of the four multiplexed OAM modes with the meta-atom. The parameters are \( r = 12.2, p = 7 \) mm, \( h = 3.81 \) mm, \( l = 6.4 \) mm, \( d = 6.7 \) mm and \( w = 1.2 \) mm; (b) Simulated magnitudes of the co-transmission coefficients; (c) Simulated phases of the co-transmission coefficients.

As a result, the transmission matrix of each meta-
atom can be calculated by their rotation angles \( \alpha_1, \alpha_2, \cdots, \alpha_{100} \).

(3) OAM modes with \( l_0 = 1, 2, -2, -1 \) are generated by four metasurfaces with different phase distributions as excitation sources. Even if the generated OAM modes are not pure, the OAM sources \( E_i \) can be sampled from full-wave simulation to avoid the error.

With these methods, we can reduce the errors from the breakdown of periodic boundary conditions and impure OAM sources. Nonetheless, phase error still exists, which could result from the inevitable coupling between meta-
atoms.

The output near-field \( E_o \) can be calculated by multiply-
ifying rotated transmission matrices with the \( E_i \) fields at corresponding meta-atom centers.

\[
\begin{bmatrix}
E_o^{xp}(n) \\
E_o^{yp}(n)
\end{bmatrix} = T(\alpha_n) \cdot \begin{bmatrix}
E_i^{xp}(n) \\
E_i^{yp}(n)
\end{bmatrix},
\]

(13)

where \( n = 1, \cdots, 100 \) is the index of each meta-atom, \( \alpha_n \) is the rotation angle of each meta-atom, and super-
scripts \( xp \) and \( yp \) denote the \( x \)-polarized and \( y \)-polarized components. With the output near-field, far-field can be obtained by Fresnel diffraction [47]

\[
E_{far}(x, y) = e^{i k_o z / i \lambda_o z} \int_{-\infty}^{+\infty} E_o(x', y', 0) \cdot e^{i k_o \frac{1}{2 \pi} [(x-x')^2 + (y-y')^2]} dx' dy',
\]

(14)

where \( z \) is the distance between the source plane and target plane, and \( x' \) and \( y' \) denote the geometric positions at source plane. The left-polarized and right-polarized components are taken as

\[
E_l = \frac{E_{far}^{xp} + i E_{far}^{yp}}{2}, E_r = \frac{E_{far}^{xp} - i E_{far}^{yp}}{2}.
\]

(15)

Assuming the incident wave is left circularly-polarized, the objective polarization will be transformed twice in total (one is for the generation of OAM source and an-
other for detection), which will end up being still left
FIG. 7. Full-wave EM simulations of far-field patterns. (a)-(d) Normalized 3-D patterns under the incidences of OAM modes \( l_0 = 1, 2, -2, -1 \); (e) Normalized 2-D pattern in \( k \) space under the incidence of multiplexed OAM modes. The dotted black lines represent the half-power circles of far-field intensity patterns and the crosses represent the pre-designed scattered directions of the four OAM modes.

circularly-polarized. Hence, the objective far-field intensity is

\[
P_{\text{far}}(k_x, k_y) = \frac{E_l^2(k_x, k_y)}{\sum_{k_x, k_y} E_l^2(k_x, k_y)}. \tag{16}
\]

Then, the optimization algorithm can be constructed similarly to the ideal case, as described in Appendix, where the input variables are the rotation angles of the meta-atoms, superposed E-field of four OAM modes generated by the generating metasurfaces is employed as the input source, and the far-field patterns are calculated with Eqs. (11-16). Optimized far-field patterns are depicted in FIG. 7, where the scattered waves have relatively low side lobes. Far-field patterns without optimization can be found in Supplemental Material, which shows severe side lobes. The response of the detecting metasurface under the incidences of the multiplexed OAM modes is also given as a comparison to the ideal case result in FIG. 4(a). Although there appear some deviations of wave orientations and deformations of wave shapes due to the mode crosstalks, the four modes are generally separated. Hence, the fundamental scattering channel limit of OAM multiplexing can also be approached with the realistic detecting metasurface.

V. MICROWAVE EXPERIMENTS

Experimental facilities and measured results are given in this section. As shown in FIG. 8, experimental facilities mainly consist of circularly-polarized (CP) horn antenna, generating and detecting metasurfaces, supporting frame and near-field scanning system. The circularly-polarized horn antenna has an aperture size of 128×128 mm, which emits left circularly-polarized wave at 5.088 GHz. The OAM modes carrying topological charges \( l_0 = 1, 2, -2, -1 \) are generated with corresponding four generating metasurfaces, then impinge on the optimized detecting metasurface. The distance between the two metasurfaces is fixed as 6 mm by a plastic spacer to avoid severe divergences of OAM modes, while focusing lens should be applied if a long distance communication is considered. A 500×500 mm iron plate with a 70×70 square hole in the center is fabricated as the supporting frame, which is covered by a piece of the same shaped absorption material to reduce the edge scattering of the metal hole. This supporting frame is used for guaranteeing that waves can only pass through the metasurface rather than bypass it. The details of fabricated metasurfaces and supporting frame are given in Supplemental Material. Near-field scanning system is employed for collecting the data of near-field electric components \( E^{xp} \) and \( E^{yp} \), from which left circularly-polarized far-field pattern can be calculated. As testing criteria, the distance between the metasurface and probe is set as 170 mm, and 52×52 data points are sampled over a 800×800 mm plane.

Measured far-field patterns are drawn in FIG. 9. Compared with simulation results, the directions of the four scattered waves show some deviations, and there are also more severe side lobes, especially for modes -1 and -2. From the result of the multiplexed OAM incidences in 2-D \( k \) space, mode crosstalks and wave deformations can be
noticed, while the four modes can be roughly separated.

The measurement error can be largely attributed to the
coupling between the edge of square hole and the upper
surface of metasurface, the reflection between two meta-
surfaces, as well as the fabrication errors of metasurfaces.
To realize the miniaturization of meta-atoms, working
bandwidth has to be sacrificed to a relatively narrow
level, while the high dielectric constant of substrate also
imposes strict constraints for fabrication. Generally, the
deviations in measurement are acceptable as our aim is
to explore the feasibility of approaching the fundamental
limit of OAM multiplexing in the practical design. Al-
though our solution is proved to be reliable in numerical
simulations and experiments, we believe more advanced
meta-atoms should be proposed to realize wider working
bandwidth and simpler fabrication process while main-
taining small sizes, which will take full advantage of the
wave manipulation ability of metasurfaces.

VI. CONCLUSION

In summary, an accessible derivation of the upper scat-
tering channel limit for OAM multiplexing is given by
the transformation of basis and angular-spectral analysis,
which reveals the relation between the number of inde-
pendent scattering channels and the physical size of de-
tecting aperture. Then, the theory is verified with both
ideal optimization case and practical full-wave simul-
atation, where the minimax algorithm is applied successfully
to suppress side lobes. Moreover, microwave experiment
is also carried out to test the performance of optimized
metasurfaces. By using this theory, the ultimate channel
capacity of SMM based communication system can be
precisely estimated. The optimization algorithm is also
useful for practical implementation of SMM, especially
for OAM multiplexing.

APPENDIX
MINIMAX OPTIMIZATION FOR
METASURFACE DESIGNS

The minimax algorithm is used to minimize the pos-
sible loss for a worst (maximum loss) scenario, which is
frequently used in engineering problems [48, 49]. The

\[ \text{minimax}\{f_n(\kappa_1, \kappa_2, \cdots \kappa_{100})\}, \quad n = 1, 2, 3, 4, \quad (17) \]

where \( f_1, f_2, f_3, f_4 \) are the four objective functions under
the incidences of four OAM modes and \( [\kappa_1, \kappa_2, \cdots \kappa_{100}] \)
denotes the input variables, being the phases of pixels
\( [\phi_1, \phi_2, \cdots \phi_{100}] \) for the ideal metasurface case and the ro-
tation angles of meta-atoms \( [\alpha_1, \alpha_2, \cdots \alpha_{100}] \) for the real-
listic metasurface case, respectively. With the input vari-
ables, four objective functions can be constructed with
the normalized far-field patterns \( P_{\text{far}} \) given in Section
IV.

For the construction of objective functions, the objec-
tive power is set as the sum of power included in a \( k \)
circle centered at a pre-designed direction, considering the
fact that power cannot be concentrated on a point but
an area. The corresponding wave numbers of the four
designed directions are \( a_n = k_x(n)/k_0 = [0.6, 0, -0.6, 0] \)
and \( b_n = k_y(n)/k_0 = [0, 0.6, 0, -0.6] \), where \( k_0 \) is the
free-space wave number and \( n = 1, 2, 3, 4 \). The radius of
the \( k \) circle is set as \( 1/3 \) \( k_0 \), which shows the best perfor-
mance after testing. Note that the input powers of the
four incident OAM modes are set to be the same, so that
higher objective power indicates better performance. In
order to search the best performance of the worst case
under the incidences of four OAM modes, reciprocal of
the objective power should be set as objective function.
The final objective functions are

\[
f_n = \left[ \sum_{\frac{4\pi}{\lambda_0} - a_n}^{\frac{4\pi}{\lambda_0} - b_n} P_{far}(k_x, k_y) \right]^{-1}. \tag{18}
\]

We use the fminimax function in MATLAB optimization toolbox to construct the algorithm.

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