Is DLA Locally Isotropic or Self-Affine?

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Abstract

We present results of simulations which show unambiguously that DLA clusters are not self-affine, in contrast to frequent claims. The measured observable is the asymmetry of the last step of a walker before he sticks to the growing cluster. Using deposition onto an originally straight line off lattice, we show that this asymmetry tends to zero algebraically with the thickness of the deposit.
Though diffusion limited aggregation (DLA) [1] has been studied very intensively during the last decade [2], its theoretical understanding remains still very unsatisfactory. There have been a number of isolated results such as Halsey’s relations for the generalized dimensions of the growth measure in 2-d [3], or the results of [4] on the shape of large clusters on 2-d regular lattices, but very basic properties of DLA clusters are still unclear.

One such basic question is whether DLA clusters themselves (not the growth measure on them!) are multifractal [5], become space-filling at infinite size [6] or at least show multiscaling [7, 8].

Even more basic is the question whether DLA clusters are isotropic on small scales. On large scales it is of course clear that they are not. Rather, the direction towards the center of the cluster is clearly singled out. But it might well be that this anisotropy is lost as one goes into further and further ramifications of the main branches of the cluster. By looking at their (rather small) clusters, the authors of the first paper on DLA [1] claimed that DLA should indeed be isotropic and self-similar on small scales. But this was questioned by a more systematic study [9], in which a strong anisotropy was found.

Unfortunately, the analysis of [9] is marred by a number of problems. It was done on a square lattice which is known today to lead to serious artifacts [10], a parametrization of the two-point correlation function was used which is neither scale-invariant nor self-affine, and no systematic extrapolation towards the limit
of infinite clusters was attempted. Finally, working in a radial geometry as in [9] induces rather awkward finite-size effects.

The latter can be avoided by using not a single site as a seed of the cluster, but an infinite line (i.e., a finite line with periodic boundary conditions). This is often called diffusion limited deposition (DLD), in order to distinguish it from DLA proper. In DLD, the anisotropy on large scales is even more clearly seen than in DLA, and its origin is well understood. It is mostly on the basis of this large scale anisotropy that it is often claimed [10] that DLA is self-affine.

In the following we shall present numerical data obtained in off-lattice simulations which clearly show that DLD becomes isotropic in the limit of infinite thickness of the deposit, on scales much smaller than this thickness. Since its microscopic structure should be the same as in DLA, this shows that also DLA should become isotropic on small scales.

Before proceeding, we should make a few comments on self-affinity. A set is self-affine (in the statistical sense), if it (resp. its statistical characteristics) is invariant under an affine, i.e. inhomogeneous linear transformation which is not just a isotropic rescaling plus a translation. The prototype of a self-affine critical phenomenon is directed percolation. There, the anisotropy is seen as soon as

\footnote{There exists one study [11] which finds very strong anisotropy effects in DLD also on small scales, but this was done on a square lattice with strong noise reduction. It is almost sure that the results of [11] are lattice artifacts.}
a scale larger than one lattice unit is considered. There is no intermediate range of scales in which lattice effects disappear, but anisotropy is not yet fully pronounced. It is precisely the existence of such a range which is claimed in the present note.

We study DLD off lattice in 2 dimensions, with periodic boundary conditions (period $L$) sidewise. The step length in the diffusion process is variable, depending on the distance from the cluster. Particles are presented by discs of radius $R$. If the distance between the $n$-th diffusing particle and the nearest particle on the cluster (resp. to the flat initial surface) is $r$, the length of the next step is chosen as $l \leq r + \epsilon R$, with $\epsilon = 0.05$. Since $\epsilon > 0$, such a step can lead to a overlap with the cluster. If this happens, the steps length is reduced such that the particle just touches the cluster, the cluster is updated by incorporating the particle, and the angle $\theta_n$ of the last step (relative to the normal to the base line) is recorded (fig.1).

Figure 1: Schematic drawing of a particle approaching and sticking to a deposit during its very early stage.

In fig.2 we show average values of $\cos \theta_n$ against $n$. The lateral size of the system was chosen as $L = 5120R$, and $n$ went up to $n_{\text{max}} = 4 \times 10^5$. If we assume that the density in the deposit decays as $h^{-3}$ with height $h$ [12], this implies that the deposit grows to an average thickness $h_{\text{max}} \approx R(n_{\text{max}}R/L)^{1.43} \approx 10^3R$. This was indeed observed, and it means that all our simulations correspond to the limit
\( h_{\text{max}} \ll L \) in which the system can be regarded as having an essentially infinite lateral extension, and we do not have to worry about finite-\( L \) corrections. The data shown in fig.2 represent the results of 50 such runs.

Figure 2: Log-log plot of \( \langle \cos \theta \rangle \), where \( \theta \) is the angle of a particle’s velocity just before it sticks, against \( nR/L \). In order to reduce statistical fluctuations, data are binned into intervals with \( \Delta n/n = 0.02 \).

Apart from rather strong corrections at small \( n \) (which are easily explained from the fact that the initial flat surface is very different in its microstructure from the later surfaces consisting of small discs), we see a very clear scaling behavior. More precisely, we find

\[
\langle \cos \theta \rangle \sim n^{-\alpha} \tag{1}
\]

with

\[
\alpha = 0.23 \pm 0.03 \tag{2}
\]

This scaling law seems unrelated to any previously observed scaling law in DLA, and by itself should be interesting. But more interesting is that it demonstrates quite convincingly that \( \lim_{n \to \infty} \langle \cos \theta \rangle = 0 \), i.e. that particles approach the cluster isotropically before the stick. On the other hand, the power \( \alpha \) is not very large, i.e. the convergence of \( \langle \cos \theta \rangle \) is very slow. This explains why analyses where no extrapolation towards \( n = \infty \) is made should find strong anisotropies.
Let us now discuss our result in view of other results about DLA. First of all, we stress again that it does not mean that DLA or DLD is isotropic on large scales ($\geq L$). The reason why it cannot be so is well understood.

Secondly, there exists a result which at first seems to contradict our conclusions. That is the finding of [13, 14] that shortest paths in DLA (and presumably also in DLD) have dimension 1. Thus the stems of typical large branches are straight, which seems to be in conflict with our claim that typical branches grow non-straight. The resolution of this conflict is that straight branches will grow longer than curled ones, shield them from further growth, and will finally form the stems of large arms. Thus it is posterior selection which is responsible for the straightness, not the fact that all branches are straight from the outset.

Finally, our claim is fully compatible with the result of [12] that in the limit $h \gg L_{\nu_\perp}$ the deposit has a surface of thickness $\sim L$, i.e. that the structures in this surface are roughly isotropic — which also shows that the deposit is not self-affine.

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