A Variant of Earley Parsing

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Abstract. The Earley algorithm is a widely used parsing method in natural language processing applications. We introduce a variant of Earley parsing that is based on a “delayed” recognition of constituents. This allows us to start the recognition of a constituent only in cases in which all of its subconstituents have been found within the input string. This is particularly advantageous in several cases in which partial analysis of a constituent cannot be completed and in general in all cases of productions sharing some suffix of their right-hand sides (even for different left-hand side nonterminals). Although the two algorithms result in the same asymptotic time and space complexity, from a practical perspective our algorithm improves the time and space requirements of the original method, as shown by reported experimental results.

1 Introduction

Earley parsing is one of the most commonly used methods for the (automatic) syntactic analysis of natural language sentences, given a context-free grammar model. This method does not use backtracking, resulting in time and space efficiency, and is quite flexible, in that it does not require the input grammar to be cast in any particular form. Earley parsing was first defined in \cite{6}, in the context of formal language parsing. This method has later been rediscovered in \cite{10,11} from the perspective of application to natural language processing, where it was called active chart parsing. Active chart parsing makes also use of a data structure, called agenda, which allows a more flexible control of competing analyses.

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A considerable number of results and applications regarding Earley parsing have been published in the literature. From a theoretical perspective, improvements of the Earley algorithm have been reported in [9], [15] and [16]. Several reformulations of Earley parsing have also been presented. Most remarkably, in [3] Earley parsing is related to the deterministic simulation of a particular kind of nondeterministic pushdown automaton, and a recursive reformulation of Earley parsing has been proposed in [14].

From the perspective of natural language parsing, the Earley method has been adapted to work with context-free grammars enriched with feature structures in [22], [24] and [7], and to cope with on-line semantic interpretation in [27]. Comparison of Earley parsing with other parsing strategies has been experimentally carried out and reported in [30] and [24].

In this paper we focus on a drawback of the Earley algorithm: the recognition of a production within the input is started by looking for the constituents in its right-hand side, proceeding from left to right. In this process, the algorithm keeps track of the position within the input at which the recognition has started. Since this information is needed only if the whole recognition can be carried to an end, the algorithm behaves in a rather inefficient way in several cases in which production recognition cannot be successfully completed. We propose a variant of the original method, in which the problem is solved by delaying some of the computation until the involved productions have been fully recognized. This is achieved using an idea first presented in [3] in the context of left-corner parsing, as it will be discussed at length in the final section. When applied in the framework of active chart parsing, our technique results in the “inversion” of the fundamental rule [10, 11] that combines a left active edge with a right inactive edge. Although our proposal does not result in an asymptotic improvement of the time and space complexity of the Earley algorithm, reported experimental results provide evidence that in practical cases our method achieves an increase in time and space efficiency.

The remainder of this paper is organized as follows. In Section 2 some preliminaries are discussed. We review the Earley parsing method in Section 3 and then introduce our variant in Section 4. Some empirical results are given in Section 5, and related work is discussed in Section 6.

2 Preliminaries

We introduce the formal notation that will be used throughout the paper.

A string \( w \) is a finite sequence of symbols over some alphabet. We denote as \(|w|\) the length of \( w \), and as \( \varepsilon \) the (unique) string of length zero. The set of all strings over some alphabet \( \Sigma \), \( \varepsilon \) included, is denoted \( \Sigma^* \). A context-free grammar (CFG) is a rewriting system \( G = (V_T, V_N, P, S) \), where \( V_T \) and \( V_N \) are two finite, disjoint sets of terminal and nonterminal symbols, respectively, \( S \in V_N \) is the start symbol, and \( P \) is a finite set of productions. Each production has the form \( A \rightarrow \alpha \) with \( A \in V_N \) and \( \alpha \in (V_N \cup V_T)^* \). The size of \( G \), written \(|G|\), is defined as \( \sum_{(A \rightarrow \alpha) \in P} |A\alpha| \).
We generally use symbols $A, B, C, \ldots$ to range over $V_N$, symbols $a, b, c, \ldots$ to 
range over $V_T$, symbols $X, Y$ to range over $V_N \cup V_T$, symbols $\alpha, \beta, \gamma, \ldots$ to range 
over $(V_N \cup V_T)^*$, and symbols $v, w, x, \ldots$ to range over $V_T^*$. For a fixed grammar, 
the binary relation $\Rightarrow$ is defined over $(V_N \cup V_T)^*$ such that $\gamma A \delta \Rightarrow \gamma \alpha \delta$ whenever $A \rightarrow \alpha$ belongs to $P$. We will mainly use the reflexive and transitive closure of $\Rightarrow$, denoted $\Rightarrow^*$.

3 Earley Parsing

We briefly present here the Earley algorithm, before introducing the variant of 
this method in the next section.

Let $G = (V_T, V_N, P, S)$ be a CFG. We associate with $G$ a set of symbols, 
called dotted items, specified as:

$$I_E = \{ [A \rightarrow \alpha \cdot \beta] \mid (A \rightarrow \alpha \beta) \in P \}. \quad (1)$$

Dotted items are used below to represent intermediate steps in the process of 
recognition of a production of the grammar, where the sequence of symbols in 
between the arrow and the dot indicates the sequence of constituents recognized 
so far at consecutive positions within the input string. More precisely, 
given a production $p : (A \rightarrow X_1X_2 \cdots X_r)$, $r \geq 0$, the process of recognition of 
the right-hand side of $p$ is carried out in several steps. We start from item 
$A \rightarrow \cdot X_1X_2 \cdots X_r$, attesting that the empty sequence of constituents has 
been collected so far. This item represents a prediction for $p$. We then proceed with 
item $A \rightarrow X_1 \cdot X_2 \cdots X_r$ after the recognition of a constituent $X_1$, and so on. 
Production $p$ has been fully recognized only if we reach item $A \rightarrow X_1X_2 \cdots X_r \cdot \cdot \cdot$, 
attesting therefore the complete recognition of a constituent $A$. In active chart 
parsing, items in $I_E$ with the dot not at the rightmost position of the right-hand 
side are used to label the so-called active edges.

Given a string $w = a_1a_2 \cdots a_n$, with $n \geq 0$ and each $a_i$ a terminal symbol, 
we call position within $w$ any integer $i$ such that $0 \leq i \leq n$. In what follows, $E$ 
is a square matrix whose entries are subsets of $I_E$ and are addressed by indices 
that are positions within the input string. Entries are denoted as $E_{i,j}$. The 
insertion by the algorithm of item $[A \rightarrow \alpha \cdot \beta]$ in $E_{i,j}$, $i \leq j$, attests the fact that 
the sequence of constituents in $\alpha$ exactly spans the substring $a_{i+1} \cdots a_j$ of the 
input. (See below for a more precise characterization of the algorithm.) Control 
flow is not specified in the method below, since it is usually regulated by means 
of a data structure called agenda, which directs the incremental construction 
of the table by means of an iteration: starting from an empty table, items are 
added as long as needed, and with the desired priority.

**Algorithm 1 (Earley)** Let $G = (V_T, V_N, P, S)$ be a CFG. Let $w = a_1a_2 \cdots a_n$ 
be an input string, $n \geq 0$, and $a_i \in V_T$ for $1 \leq i \leq n$. Compute the least 
$(n + 1) \times (n + 1)$ table $E$ such that $[S \rightarrow \cdot \alpha] \in E_{0,0}$ for each $(S \rightarrow \alpha) \in P$, and

1. $[A \rightarrow \cdot \gamma] \in E_{i,j}$ if $[B \rightarrow \alpha \cdot A \beta] \in E_{i,j}$, $(A \rightarrow \gamma) \in P$;
2. $[A \rightarrow \alpha a_j \cdot \beta] \in E_{i,j}$ if $[A \rightarrow \alpha \cdot a_j \beta] \in E_{i,j-1}$;
3. $[A \rightarrow \alpha B \cdot \beta] \in E_{i,j}$ if $[A \rightarrow \alpha \cdot B \beta] \in E_{i,k}$, $[B \rightarrow \gamma \cdot \cdot \cdot] \in E_{k,j}$. 

The string \( w \) is accepted if and only if \( [S \rightarrow \alpha \cdot] \in E_{0,n} \) for some \( (S \rightarrow \alpha) \in P \).

The correctness of the algorithm immediately follows from the property below, whose proof can be found in [3] and [4].

**Proposition 1.** In Algorithm 1, an item \([A \rightarrow \alpha \cdot \beta]\) is inserted in \( E_{i,j} \) if and only if the following conditions hold:

A1. \( S \Rightarrow a_1 \cdots a_i A \gamma \), some \( \gamma \); and
A2. \( \alpha \Rightarrow a_{i+1} \cdots a_j \).

For methods cruder than the Earley algorithm, membership of an item in some entry may merely be subject to condition A2, which is sufficient for determining the correctness of the input. However, Earley’s algorithm is more selective, as is apparent from condition A1, which characterizes the so called top-down filtering capability of the method. Condition A1 guarantees that only those constituents are predicted that are compatible with the portion of the input that has been read so far.

Assuming the working grammar as fixed, a simple analysis reveals that Algorithm 1 runs in time \( O(n^3) \). This will be more carefully discussed in the next section.

### 4 A Variant of Earley Parsing

In this section we introduce a variant of Earley parsing that can be obtained by reconsidering the way in which the results of the intermediate steps are stored in the process of production recognition.

Let us focus on the dependence of the running time of Algorithm 1 on the length of the input string. From this perspective, the most expensive step is Step 3. Intuitively, this is the case because there might be \( \mathcal{O}(n^2) \) items that are inserted at this step in some entry of \( E \), and each item can in turn be the result of \( \mathcal{O}(n) \) different combinations of pairs of items already in \( E \). In practice, the total number of different combinations of dotted items attempted by Step 3 when processing an input string dominates the running time of Algorithm 1. The change to the new method consists in a decomposition of Step 3 that results, in some cases, in a reduction of this number. We introduce the basic idea through an example.

Consider a production \( p : (A \rightarrow A_1 A_2 \cdots A_r) \), \( r \geq 3 \). Let \( D \) be a set containing \( d > 2 \) positions within the input string. Assume that the dotted item \([A \rightarrow A_1 \cdot A_2 \cdots A_r]\) has been inserted in the entry \( E_{i,j_1} \), for each \( i \in D \) and for some fixed \( j_1 \). This corresponds to \( d \) constituents \( A_1 \) recognized within the input. Assume also that, for each \( t \) with \( 2 \leq t \leq r - 1 \), a constituent \( A_t \) has been recognized in entry \( E_{j_{t-1},j_t} \). Finally, assume that no constituent \( A_r \) is found.

\[\text{When both the input string and the grammar are taken as input parameters, Algorithm 1 runs in time } \mathcal{O}(|G|^2 n^3). \] An improvement of Algorithm 1 has been presented in [5], running in time \( \mathcal{O}(|G| n^3) \).
We depict the case of $d = 3$, $r = 4$, and assume $D = \{i_1, i_2, i_3\}$. We represent the input string by means of an horizontal line and each dotted item in $E$ by means of an arc; only the relevant positions within the input string are depicted. In the attempt to recognize production $A \rightarrow A_1 \cdots A_4$, the algorithm has created 3 dotted items $[A \rightarrow A_1 \cdot \cdot \cdot A_4]$, one for each position in $D$, depicted by solid arcs above the horizontal line. Since each of these items has a different left position, the Earley algorithm is forced to instantiate 3 independent processes for the recognition of $A \rightarrow A_1 \cdots A_4$. These processes will create the dotted items depicted by the dashed arcs. Note that in collecting the remaining constituents $A_2, A_3, A_4$ the method duplicates the needed effort.

starting at position $j_{r-1}$ (see Figure 1). Under these assumptions, Step 3 will be executed $d(r - 2)$ more times, carrying out $d$ independent recognition processes for $p$, to find out at the end that none of these processes can be successfully completed, because of the lack of constituent $A_r$. The fact that the above recognition processes are independent one of the other is due to the fact that in Step 3 we record the position within the input where each process started (the positions in $D$).

We observe that the left position of $p$ in the input string is needed only if the recognition process of $p$ can be successfully completed, in order to locate the constituent corresponding to the left-hand side of $p$ for use in the remaining analysis of the input. On this basis, we reformulate Step 3 by splitting it up into two substeps. The first substep performs the recognition of $p$ in a forward manner, without maintaining any record of the left position. This is done using an array $U$ in whose entries we store only the suffixes of $p$’s right-hand side that must still be recognized. If the recognition can be successfully completed, we apply the second substep and compute the left positions of $p$ in a backward manner, starting from the rightmost constituent in $p$’s right-hand side and proceeding toward the left, storing the intermediate results in table $T$.

The proposed technique thus delays part of the computation from the former Step 3 until we are granted that $p$ can be successfully recognized. In this way we avoid the computational inefficiency revealed by our example. In fact, whenever
Algorithm 1 has now been split into Steps 2 and 5 of Algorithm 2, which act exactly on Step 1 of Algorithm 1. Step 2 of Algorithm 2 performs more efficiently than the original formulation of Step 3. In fact, since the backward substep proceeds from right to left, constituents \( A_r, A_{r-1}, \ldots \) will be visited only once in the attempt to find all possible left positions for \( p \).

We observe that for the technique described above to work in its full generality, also Step 2 from Algorithm 2 should be split into two substeps. This allows correct treatment of productions containing terminal symbols in their right-hand sides. Finally, it is not difficult to see that the problem described above can be generalized to productions sharing some suffix of their right-hand sides, that is productions of the form \( A \rightarrow \alpha \gamma \) and \( B \rightarrow \beta \gamma \), in cases that \( \gamma \) is, at some position, predicted independently for both productions.

We are now in a position to give a precise specification of the proposed parsing algorithm. Let \( G = (V_T, V_N, P, S) \) be a CFG. We associate with \( G \) a set of symbols, called suffix items, specified as:

\[
I_V = \{[\beta] \mid (A \rightarrow \alpha \beta) \in P\}.
\]

Suffix items serve two different purposes. First, the insertion of suffix item \([\alpha]\) in entry \( U_j \), where \( U \) is a one-dimensional array, means that the process of forward recognition of a production \( A \rightarrow \alpha \beta \), for some \( A \) and \( \alpha \), has been successfully carried out, up to position \( j \) and up to the constituents in the sequence \( \alpha \). In other words, there exists at least one \( i, i \leq j \), such that some dotted item \([A \rightarrow \alpha \beta]\) would have been inserted in \( E_{i,j} \) by Algorithm 1. Second, the insertion of suffix item \([\beta]\) in \( T_{i,j} \) means that at least one production \( A \rightarrow \alpha \beta \), for some \( A \) and \( \alpha \), has been completely recognized and the constituents in the sequence \( \beta \) have been collected backwards so far, spanning the substring \( a_{i+1} \cdots a_j \).

Algorithm 2 (Variant of Earley) Let \( G = (V_T, V_N, P, S) \) be a CFG. Let \( w = a_1a_2\cdots a_n \) be an input string, \( n \geq 0 \), and \( a_i \in V_T \) for \( 1 \leq i \leq n \). Compute the least \( (n+1) \times (n+1) \) table \( T \) and the least \( n+1 \) array \( U \) such that \([\alpha] \in U_0\) for each \((S \rightarrow \alpha) \in P\), and

1. \([\gamma] \in U_j\) if \([A\beta] \in U_j, (A \rightarrow \gamma) \in P\);
2. \([\beta] \in U_j\) if \([a_j\beta] \in U_{j-1}\);
3. \([\beta] \in U_j\) if \([B\beta] \in U_k, (B \rightarrow \gamma) \in P, [\gamma] \in T_{k,j}\);
4. \([\varepsilon] \in T_{m,m}\) if \([\varepsilon] \in U_m\);
5. \([a_j\beta] \in T_{j-1,m}\) if \([a_j\beta] \in U_{j-1}, [\beta] \in T_{j,m}\);
6. \([B\beta] \in T_{k,m}\) if \([B\beta] \in U_k, (B \rightarrow \gamma) \in P, [\gamma] \in T_{k,j}, [\beta] \in T_{j,m}\).

The string \( w \) is accepted if and only if \([\alpha] \in T_{0,n}\) for some \((S \rightarrow \alpha) \in P\).

Step 1 of Algorithm 1 exactly corresponds to Step 1 of Algorithm 2. Step 2 of Algorithm 2 has now been split into Steps 2 and 5 of Algorithm 2, which act...
as forward and backward substeps, respectively. Similarly, Step 3 of Algorithm 1 has been split into Steps 3 and 6 of Algorithm 2. Step 4 of Algorithm 2 is needed to initiate the backward process of recognizing a production, after the forward process has completed recognition of the right-hand side.

The correctness of the method directly follows from the property stated below, which characterizes the presence of suffix items in entries of $U$ and $T$.

**Proposition 2.** In Algorithm 2, an item $[\beta]$ is inserted in $U_j$ if and only if the following conditions hold:

- $A1. \, S \Rightarrow a_1 \cdots a_i A \gamma$, some $i$, $A$ and $\gamma$;
- $A2. \, (A \rightarrow \alpha \beta) \in P$, some $\alpha$; and
- $A3. \, \alpha \Rightarrow a_i+1 \cdots a_j$,

and an item $[\beta]$ is inserted in $T_{j,k}$ if and only if the following conditions hold:

- $B1. \, \text{the conditions A1, A2 and A3 hold};$ and
- $B2. \, \beta \Rightarrow a_{j+1} \cdots a_k$.

The proof of the above statement is similar to that of Proposition 1.

It is not difficult to see that Algorithm 2 has running time $O(n^3)$ (again, we assume the working grammar is fixed). Therefore Algorithms 1 and 2 present the same asymptotic time complexity. For the purpose of more carefully comparing the two algorithms, we give below an alternative to Proposition 2, which characterizes the entries in $U$ and $T$ in terms of the entries in $E$.

**Proposition 3.** In Algorithm 2, an item $[\beta]$ is inserted in $U_j$ if and only if the following condition holds:

- $A1. \, \text{at least one item } [A \rightarrow \alpha \cdot \beta] \text{ is inserted in } E_{i,j} \text{ by Algorithm 1, for some } A, \alpha \text{ and } i,$

and an item $[\beta]$ is inserted in $T_{j,k}$ if and only if the following conditions hold:

- $B1. \, \text{the condition A1 holds};$ and
- $B2. \, \beta \Rightarrow a_{j+1} \cdots a_k$.

This proposition clearly shows that the number of items in $U$ is always smaller than the number of items in $E$: several items $[A \rightarrow \alpha \cdot \beta]$ in $E_{i,j}$ for fixed $j$ but differing $A$, $\alpha$ and $i$ correspond to one single item $[\beta]$ in $U_j$.

On the other hand, the number of items in $T$ may be larger than the number of items in $E$ since for each $[A \rightarrow \alpha \cdot \beta]$ in $E_{i,j}$ we may have $[\beta]$ in several $T_{j,k}$ for distinct values of $k$. Since there may be up to $n$ such $k$ in the worst case, the number of items in $T$ may be up to $n$ times larger than the number of items in $E$.

One example of a CFG were this phenomenon is apparent is the following.

$$
S \rightarrow AB \quad A \rightarrow C \quad B \rightarrow C \quad C \rightarrow aC \quad C \rightarrow \varepsilon
$$
For input $a^n$, some $n$, Algorithm 1 computes $n + 1$ items of the form $[S \rightarrow A \cdot B] \in E_{0,i}$, $0 \leq i \leq n$, and $n + 1$ items of the form $[S \rightarrow AB \cdot] \in E_{0,j}$, $0 \leq j \leq n$. On the other hand, Algorithm 2 computes $n^2 + n$ items of the form $[B] \in T_{i,j}$, $0 \leq i \leq j \leq n$.

We define $|E| = \Sigma_{i,j}|E_{i,j}|$, $|U| = \Sigma_{i}|U_i|$, $|T| = \Sigma_{i,j}|T_{i,j}|$, and summarize the above as follows.

**Proposition 4.** For a fixed CFG and input of length $n$, let $E$ be constructed by Algorithm 1 and $U$ and $T$ by Algorithm 2. Then:

1. $|U| \leq |E|$; and
2. $|T| \leq n \cdot |E|$.

The second part of this proposition seems to suggest that the table size may be much larger for the variant. The empirical data presented by the next section however show that such worst-case behaviour does not seem to occur for the practical grammars at hand.

Based on the number of items that are stored in the respective tables, we can investigate the number of steps that are performed by the two algorithms. We count the number of elementary parsing steps consisting in the derivation of one item in a table from one or more objects, such as productions, input symbols, or other items in a table. For example, in the case of Algorithm 2 every combination of four objects of the form $[B\beta] \in U_k$, $(B \rightarrow \gamma) \in P$, $[\gamma] \in T_{k,j}$, and $[\beta] \in T_{j,m}$ is counted as one elementary parsing step according to Step 6. For a certain CFG and input, let us denote the number of applications of Steps 1, 2 and 3 of the Earley algorithm by $\mathcal{E}_1$, $\mathcal{E}_2$ and $\mathcal{E}_3$. Similarly, we introduce the notation $\mathcal{V}_1, \ldots, \mathcal{V}_6$ for the six steps of the variant. We further define $\mathcal{E} = \mathcal{E}_1 + \mathcal{E}_2 + \mathcal{E}_3 + |\{\alpha \mid (S \rightarrow \alpha) \in P\}|$, and $\mathcal{V} = \mathcal{V}_1 + \mathcal{V}_2 + \cdots + \mathcal{V}_6 + |\{\alpha \mid (S \rightarrow \alpha) \in P\}|$.

Based on condition 1 in Proposition 3 we may conclude that $\mathcal{V}_1 \leq \mathcal{E}_1$, $\mathcal{V}_2 \leq \mathcal{E}_2$ and $\mathcal{V}_3 \leq \mathcal{E}_3$. The number of applications of Step 4 is bounded by the number of items $[\varepsilon] \in U_j$, which is bounded by the number of items $[A \rightarrow \gamma \cdot] \in E_{i,j}$. This in turn is bounded by the number of items $[A \rightarrow \cdot \gamma] \in E_{i,i}$ times the number of $j$ such that $\gamma \Rightarrow a_{i+1} \cdots a_j$. The number of such $j$ is bounded by $n+1$, and the number of $[A \rightarrow \cdot \gamma] \in E_{i,i}$ is bounded by $\mathcal{E}_1$ plus $|\{\alpha \mid (S \rightarrow \alpha) \in P\}|$. Therefore we have $\mathcal{V}_4 \leq (n + 1) \cdot (\mathcal{E}_1 + |\{\alpha \mid (S \rightarrow \alpha) \in P\}|)$

Steps 5 and 6 cannot be applied more than once for each application of Steps 2 and 3 and $[\beta] \in T_{j,m}$, for at most $n + 1$ different values of $m$. Therefore we have $\mathcal{V}_5 \leq (n + 1) \cdot \mathcal{V}_2 \leq (n + 1) \cdot \mathcal{E}_2$ and $\mathcal{V}_6 \leq (n + 1) \cdot \mathcal{V}_3 \leq (n + 1) \cdot \mathcal{E}_3$.

Combining the above, we obtain:

**Proposition 5.** For fixed CFG and input of length $n$, we have $\mathcal{V} \leq (n + 2) \cdot \mathcal{E}$.

In the worst case, the number of steps for the variant may thus be greater than the number of steps for the original Earley algorithm by a factor which is $O(n)$. Again, the data presented by the next section suggest that this consideration has little bearing on practical cases.
5 Empirical Results

We have performed some experiments with Algorithms 1 and 2 for four practical context-free grammars.

The first grammar generates a subset of the programming language ALGOL 68 [28]. The second and third grammars generate fragments of Dutch, and are referred to as the CORRie grammar [29] and the Deltra grammar [23], respectively. These grammars were stripped of their arguments in order to convert them into context-free grammars. The fourth grammar, referred to as the Alvey grammar [4], generates a fragment of English and was automatically generated from a unification-based grammar.

The test sentences have been obtained by automatic generation from the grammars, using a random generator to select productions, as explained in [19]; therefore these sentences do not necessarily represent input typical of the applications for which the grammars were written. Table 1 summarizes the test material.

![Table 1](image)

**Table 1.** The test material: the four grammars and some of their dimensions, the average length of the test sentences (20 sentences of various lengths for each grammar), and the average number of parses per sentence (excluding parses containing cycles, i.e. subderivations of the form $A \rightarrow^+ A$).

| G          | $|V_N|$ | $|P|$ | $|w|$ | Parses |
|------------|--------|------|------|--------|
| ALGOL 68   | 783    | 167  | 330  | 13.7   | $2.6 \times 10^4$ |
| CORRie     | 1141   | 203  | 424  | 12.3   | $2.3 \times 10^4$ |
| Deltra     | 1929   | 281  | 703  | 10.8   | $1.1 \times 10^3$ |
| Alvey      | 5072   | 265  | 1484 | 10.7   | $3.2 \times 10^4$ |

**Table 2.** Dynamic requirements: average time and space per sentence.

| G          | $|V|$ | $|U|$ | $|T|$ | $|U|+|T|$ | $\mathcal{E}$ | $|E|$ |
|------------|------|------|------|----------|-------------|------|
| ALGOL 68   | 2,062| 2,054| 1,362| 2,054    | 2,054       | 1,437|
| CORRie     | 19,164| 15,492| 2,746| 17,450   | 17,450      | 8,361|
| Deltra     | 60,849| 54,238| 4,759| 57,582   | 57,582      | 12,694|
| Alvey      | 47,562| 34,238| 5,398| 47,552   | 47,552      | 6,304|

Table 2. Dynamic requirements: average time and space per sentence.

Our implementation is merely a prototype, which means that absolute duration of the parsing process is little indicative of the actual efficiency of more sophisticated implementations. Therefore, our measurements have been restricted to implementation-independent quantities, viz. the number of elements stored in the parse table and the number of elementary steps performed by the algorithm. In a practical implementation, such quantities will strongly influence the space
and time complexity, although they do not represent the only determining factors. Furthermore, all optimizations of the time and space efficiency have been left out of consideration.

In our experiments we have also considered an alternative way of introducing suffix items $\beta$ (albeit only those with $|\beta| \geq 2$) into the parsing process, namely by first applying a grammar transformation $\tau_2$, and then executing Algorithm 1 as usual. This was motivated by the literature on covers [21, 12], which shows that some complicated parsing algorithms can be simulated by means of grammar transformations and simpler parsing algorithms. We have not found any way to completely simulate Algorithm 2 in this manner, but the following transformation captures some of its behaviour. For an arbitrary grammar $G = (V_T, V_N, P, S)$, we define $\tau_2(G) = (V_T, V_N \cup \{X\}, P', S)$, where $P'$ contains the following productions:

- $A \rightarrow X[\alpha]$ for all $(A \rightarrow X\alpha) \in P$ with $|\alpha| > 1$;
- $A \rightarrow \alpha$ for all $(A \rightarrow \alpha) \in P$ with $|\alpha| \leq 2$;
- $[X\alpha] \rightarrow X[\alpha]$ for all $[X\alpha] \in I_V$ with $|\alpha| > 1$;
- $[XY] \rightarrow XY$ for all $[XY] \in I_V$.

Note that the transformed grammar is in two normal form, which means that the length of right-hand sides of productions is at most 2.

Table 2 presents the costs of parsing the test sentences. These data show that there is a significant gain in space and time efficiency in moving from Algorithm 1 to Algorithm 2. The biggest improvement in the number of parsing steps is observed in the case of the Alvey grammar, where it amounts to a decrease by over 41%. The biggest improvement in the total number of items stored in the tables occurs for de Deltra grammar, where it amounts to a decrease by over 30%. Only for individual sentences for ALGOL 68 was there an increase in time and space, by at most 1.2% and 0.2%, respectively.

In the case of ALGOL 68 and Alvey, it is striking that $T$ is so much smaller than $U$ and $E$. This may be explained by the relatively low level of ambiguity, as compared to the other two grammars (see Figure 1). Both the Earley algorithm and its variant predict many productions in the form of items in $U$ and $E$, but only a limited number of these productions will be recognized in their entirety, resulting in items in $T$. Although less striking in these cases, we see that also for CORRie and Deltra $T$ is smaller than $U$. This suggests that the potential undesirable behaviour of the variant with regard to the original Earley algorithm, as discussed in the previous section, does not occur in practice.

The approach using the grammar transformation is not competitive with the other two approaches. Although the number of steps is sometimes slightly smaller than in the case of Algorithm 1, the space requirements are larger in all cases.

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4 Algorithm 2 avoids any use of items of the form $[A \rightarrow X\cdot Y]$. The same cannot be achieved by means of a grammar transformation and Algorithm 1. An alternative would be to apply some other kind of tabular algorithm to the transformed grammar. See e.g. [20].
6 Concluding Remarks

We have presented a variant of the Earley algorithm and have discussed cases in which it achieves space and time savings with respect to the original algorithm. Our variant is based on the following two main ideas. First, we do not compute left positions of productions until we are granted that production recognition can be completed within the input. Second, we only use suffix items as defined in (2).

The idea of dropping left positions of productions has first been proposed by [13], where a functional realization of left-corner parsing is presented. This idea was rediscovered by [5] and expressed in a more direct way, using a table similar to our table $U$.

The idea of using suffix items has also been proposed in [13]. It has later been rediscovered by [5]. It was also applied to LR parsing in [20]. In the literature on chart parsing, e.g. in [2], one sometimes also finds a weaker form of this idea, where the set of items used in labeling edges is $I_C = \{ [A \rightarrow \beta] \mid (A \rightarrow \alpha \beta) \in P \}$. One observes that, with respect to items $[A \rightarrow \alpha \cdot \beta]$ from $I_E$, the $\alpha$ is omitted as in the case of $I_V$, yet the left-hand side $A$ is retained. If this idea is not combined with the idea of dropping left positions, then the benefit of this is limited to grammars containing many pairs of productions of the form $A \rightarrow \alpha \beta$ and $A \rightarrow \gamma \beta$, with $\alpha \neq \gamma$. The idea of using suffix items is related to the difference between two kinds of Earley parsing for the ID/LP formalism: in [25] the items are of the form $[A \rightarrow \alpha \cdot \beta]$, where $\alpha$ is a string of constituents and $\beta$ is a set of constituent, whereas in [1], both $\alpha$ and $\beta$ are sets. This allows representation of several items according to [25] by a single item according to [1], as has been argued in [17, Section 9.2].

The ideas above rely on productions or items having some suffix in common. Alternatively, one can investigate optimizations that rely on productions that have prefixes in common [18].

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