Influence of Tire Tread Convexity on its Rolling with a Slip

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Abstract. The tire tread (inflated) may have a convex (toroidal) shape. At present, there is practically no data on the influence of this structural parameter of the tire on its power and kinematic characteristics when rolling with a slip. The solution to this problem, given in this paper, is based on the consideration of contact phenomena in the friction pair “elastic wheel – rigid support surface”. As a result, relationships that make it possible to determine the lateral force, the turning moment acting in the contact area, the friction loss in the contact area for wheels with different toroidalities as a function of the slip angle and the longitudinal tangential force realized in the contact area are obtained; conclusions about the influence of the convexity on the lateral force, the stabilizing moment and the power of friction losses in the contact area are drawn.

1. Introduction

Most modern wheeled-vehicle tires have a treadmill (tread) that is nearly cylindrical in shape. The contact patch of such tires with a solid supporting surface is close to rectangular. Along with these, there are tires in which (in the inflated state) the tread has a convex shape (toroidal).

As shown in [1], the toroidality (convexity) of the treadmill of the wheel affects the process of its linear rolling only with small longitudinal tangential forces acting in the contact area.

At the same time, the question of the influence of the toroidalness of the treadmill of an elastic wheel on its rolling with a slip and along a curved path remains unclear.

2. Problem Formulation

In this paper, we solve the problem of determining the power and kinematic characteristics of elastic wheels (which include tires) with a convex treadmill while rolling with a slip. That is, in the case when the velocity vector of the wheel axis makes an angle $\delta$, called the slip angle, with the plane of rotation of the wheel, namely: relationships for finding the magnitude of the lateral force, the stabilizing moment and the friction-loss power in the contact area are derived.

3. Solution

To solve this problem, we will use the methodology and equations obtained earlier [1–7] for a wheel with a cylindrical treadmill (the shape of the contact patch is close to rectangular).
In order to simplify, we assume that the shape of the contact patch is close to elliptical (Figure 1), i.e., the coordinates of the contour line of the contact patch are related by:

\[
\frac{a^2}{A^2} + \frac{y^2}{b^2} = 1,
\]

where \(A\) and \(b\) are the semi-axes of the ellipse, and the normal pressures in both the longitudinal and transverse sections of the contact patch are distributed according to a parabolic law [8]:

\[
q_n = q_{n0} \left(1 - \frac{x^2}{A^2} - \frac{y^2}{b^2}\right), \quad q_{n0} = \frac{2F_n}{\pi Ab}.
\]

**Figure 1.** The shape of the contact patch of an elastic toroidal tire with a rigid supporting surface

As in the above studies, we believe that there are two zones in the contact patch – an adhesion section at the contact entry and a sliding section at the contact exit.

In the adhesion section, the longitudinal and transverse tangential displacements of the point of the wheel treadmill, due to the action of the longitudinal and transverse tangential forces, respectively, are defined as [5]:

\[
u_x = \Delta V_x t = \xi_x (a - x), \quad \nu_y = \Delta V_y t = (a - x) t g \delta \approx \delta (a - x).
\]

The corresponding specific tangential forces will be equal to, respectively:

\[
q_{tx} = \lambda_x u_x = \lambda_x \xi_x (a - x),
q_{ty} = \lambda_y u_y = \lambda_y \delta (a - x).
\]

In these expressions, \(\lambda_x\) and \(\lambda_y\) are the tangential stiffness coefficients of the wheel in the longitudinal and transverse directions:

\[
\xi_x = \frac{\Delta V_x}{V_x} = 1 - \frac{r_{wh}^f}{r_{wh}}
\]
relative loss of wheel speed:

\[
r_{wh} = \frac{V_x}{\omega}
\]
rolling radius:

\[
r_{wh}^f = r_0 + \frac{a^2}{3r_i^2}
\]
free rolling radius (when the longitudinal force in the contact area is zero).

In formula (5), the value of the relative loss of wheel speed is different in different longitudinal sections in the contact area, since in accordance with (7), the relative loss of speed depends on the radius \(r_{wh}^f\), which is different for different longitudinal sections [1]. The tangential stiffness coefficient \(\lambda_x\) is the same for all longitudinal sections.

At some point of contact of the longitudinal section upon reaching the total tangential stress:

\[
q_t = \sqrt{q_{tx}^2 + q_{ty}^2} = (a - x) \sqrt{\lambda_x^2 \xi_x^2 + \lambda_y^2 \delta^2}
\]
grip limit value i.e. \( |q_t| = \mu q_n \), a breakdown occurs and sliding begins. The coordinate of the boundary areas of adhesion and sliding in each of the longitudinal sections, taking into account expressions (2), can be found as:

\[
x_0 = -a \pm \frac{A^2}{\mu q_n} \sqrt{\frac{\lambda_x^2}{\lambda_y^2} + \frac{\lambda_y^2}{\lambda_x^2} + \delta^2}.
\]

The lateral force arising in the contact area can be calculated as:

\[
F_y = \int_{-b}^{+b} t_y \, dy,
\]

where

\[
t_y = \int_{x_0}^{x_0} q_{ty}^{sl} dx + \int_{-a}^{-a} q_{ty} dx
\]

linear lateral force; \( q_{ty}^{sl} \) is the transverse component of the tangential stress in the sliding section. To determine it, it should be taken into account that in the sliding section, the tangential stresses \( q_{ty}^{sl} = \mu q_n \) coincide in the direction with the sliding velocity, and in this case:

\[
q_{ty}^{sl} = \mu q_n \frac{V_y^{sl}}{V_x}, \quad q_{tx}^{sl} = \mu q_n \frac{V_x^{sl}}{V_x}.
\]

To determine \( V^{sl}, V_x^{sl}, V_y^{sl} \), we use the expression obtained in [3], which allows us to determine the average value of the longitudinal sliding velocity:

\[
V_x^{sl} = \bar{\tau}_x V_x, \quad V_y^{sl} = \delta V_x.
\]

As a result:

\[
q_{ty}^{sl} = \mu q_n \frac{\delta}{\sqrt{\xi_x^2 + \delta^2}}, \quad q_{tx}^{sl} = \mu q_n \frac{\xi_x}{\sqrt{\xi_x^2 + \delta^2}}.
\]

Taking into account the relationship obtained for \( q_{ty}^{sl} \), and also taking into account the expression for \( q_{ty} \), \( q_{tx} \) and \( x_0 \) after some transformations, we get the following expression for linear lateral force:

\[
t_y = \delta \left[ \frac{\mu q_n}{\sqrt{\xi_x^2 + \delta^2}} \cdot \frac{2a^3 + 3a^2x_0 - x_0^3}{3A^2} + \frac{\lambda_y}{2} (a - x_0)^2 \right].
\]

Similarly, we find the longitudinal tangential force:

\[
F_x = \int_{-b}^{+b} t_x \, dy,
\]

where linear longitudinal shear force is expressed by the relationship:

\[
t_x = \int_{x_0}^{x_0} q_{tx}^{sl} dx + \int_{-a}^{-a} q_{tx} dx = \bar{\tau}_x \left[ \frac{\mu q_n}{\sqrt{\xi_x^2 + \delta^2}} \cdot \frac{2a^3 + 3a^2x_0 - x_0^3}{3A^2} + \frac{\lambda_x}{2} (a - x_0)^2 \right].
\]

Due to the asymmetry of the distribution of longitudinal and transverse tangential stresses on the contact area, moments arise in the contact patch:

\[
M_{tx} = \int_{-b}^{+b} t_x y dy, \quad M_{ty} = \int_{-b}^{+b} m_y dy,
\]

\[
M_z = M_{tx} + M_{ty}.
\]
where $m_y$ is the elementary moment arising in the longitudinal section due to the asymmetry of the distribution of transverse tangential stresses:

$$m_y = \int_{-a}^{x_0} q_{y}^{sl} x dx + \int_{x_0}^{+a} q_{y}^{sl} x dx. \quad (21)$$

Applying the above formulas, for a wheel with a massive rubber tire (rubber layer 36 mm thick, diameter in the equatorial section $D = 2r = 260$ mm, wheel width $2B = 106$ mm, normal load $F_n = 3500$ N), figures 2–3 show the graphical relationships between $F_y$ and $M_z$, illustrating the effects of wheel toroidality, slip angle and longitudinal traction force (traction coefficient $\psi_x = \frac{F_x}{\mu F_n}$).

![Figure 2](image2.png)

**Figure 2.** The relationship between the lateral force and the slip angle for various values of the curvature of the wheel treadmill and the traction coefficient (longitudinal force)

![Figure 3](image3.png)

**Figure 3.** The relationship between the moment of rotation resistance and the slip angle for various values of the curvature of the wheel treadmill and the traction coefficient (longitudinal force)

To assess the wear intensity across the width of the treadmill of a toroidal elastic wheel rolling with a slip on a rigid base, we use the expression for the linear friction power in the contact area:

$$P_{fr}^{lin.} = \int_{-a}^{x_0} \mu q_n V^{sl} dx \int_{-a}^{x_0} \mu q_n V^{sl} dx = \frac{a - x_0}{a + x_0} \sqrt{\xi^2 + \delta^2} \left( a^2 x_0 - \frac{x_0^3}{3} + \frac{2}{3} a^3 \right). \quad (22)$$
The total friction power, which gives an integrated estimate of the treadmill wear intensity, in turn, can be found as:

\[ P_{fr} = \int_{-b}^{+b} P_{fr}^{\text{in}} \, dy. \]  

(23)

4. Results and Conclusions

The obtained relationships make it possible to calculate the lateral force and stabilizing moment acting in the contact patch of the toroidal wheel rolling with a slip, and the wear intensity of the wheel treadmill at various values of the slip angle \( \delta \), realized in the contact of the longitudinal tangential force \( F_x \) and the radius of curvature of the wheel \( \rho \) in the transverse plane. This, in turn, allows one to determine analytically the coefficient of lateral slip taking into account the acting longitudinal force \( F_x \) [9].

As seen in the above graphs, the influence of the curvature of the treadmill in the transverse direction on the magnitude of the lateral force is not so great. Along with this, the influence of this curvature on the value of the stabilizing moment at small longitudinal tangential forces in the contact area at \( \rho < (2 ... 3) \rho \) is evident.

For a longitudinally elongated contact patch \( \left( \frac{\rho}{r} < 1 \right) \), the lateral force is smaller and the stabilizing moment is greater than for a contact patch elongated in the transverse direction (all else being equal).

The power of friction losses in contact area during wheel rolling with a slip slightly depends on the toroidality of the wheel.

In further works [10–13] the authors also considered other questions of the mechanics of wheel rolling, including along a curved path.

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