Teaching quantum mechanics in high-school — Discipline-Culture approach

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Abstract. This study presents a new approach to teaching quantum mechanics in high school physics courses facing the extremely deficient and fragmented treatment of quantum mechanics at present. The suggested curriculum adopts the new paradigm of discipline-culture in representing physics knowledge. In accordance, the curriculum is structured in nucleus-body-periphery emphasizing the principles (nucleus), their illustration (body) and their contrast with the alternative pictures in classical mechanics (periphery). The new curriculum for 30 teaching hours is presented. The results of an experimental application to 46 12th grade students in four groups shows a positive impact on students’ conceptual knowledge and students’ nature-of-science understanding. The benefits of the applied approach and limitations of the study are presented.

1. Introduction
Quantum Theory (QT) is one of the most important developments in modern physics. However, in many countries including ours, the physics curriculum of high schools includes only a few fragments from the historical dawn of the theory such as the photo-electric phenomenon, Bohr’s model of the discrete spectrum of the Hydrogen atom, and radioactivity. Assessment is commonly reduced to using a couple of formulas to solve simple quantitative problems. The teaching normally misses the conceptual contrast between classical and quantum theories representing essentially a different worldview. Students do not receive a holistic picture, a structural hierarchical account of this new fundamental theory of physics. School educators often excuse this situation by the special mathematical and conceptual difficulty of QT making it impossible to present in a very short slot left in the curriculum (about 50 hours in our country) and to the students lacking the required mathematical tools. That is in contrast to university instruction during three semesters and using advanced math [1]. The context of high school makes a similar university-style teaching impossible and requires an essentially different approach in order to reach a sufficiently representative conceptual understanding of QT which is currently missing. At the same time, we need to avoid oversimplification which may distort the actual meaning of QT and mislead the students [2]. Teaching QT in high school should emphasize the new principles which are different from all physics previously learned by the students, instead of teaching QT starting from classical concepts and then explaining the differences [3]. In this study we suggest to reach this goal by applying discipline-
culture organization of physics theory [4]. We have developed an alternative curriculum of QT in high school and experimentally applied it. We will present some results of the experiment.

2. Curricular background
Our extensive review of QT curricula in use at school level revealed a spectrum of approaches to teaching the subject at this level of instruction. It detected the problems of knowledge representation with respect to QT which match to the reported pitfalls of students’ understanding [5]. Introductory university courses often go through showing the historical steps of consolidation of QT and try to present mathematical foundations in a yearlong course [1]. School educators usually do not intend to present a full picture and limit the teaching to certain topics that look feasible to deal with and are considered important. Thus, Petri and Niedderer [6] sought semi qualitative understanding of electron behavior in the atom using a classical analogy of string behavior. In contrast, others omit classical analogies [7] and promote the perspective of ensembles for the topic of elementary particles ([8], [9]). Still others go straight to the advanced account of the path integral approach to represent quantum behavior qualitatively [10]. In any case, however, it seems clear that even a reasonable historical approach [11] is impractical for high schools due to the lack of time and the required background in electro- and thermodynamics. Some other approaches applied remain fragmentary (lacking holistic organization of knowledge elements) even if they draw on a more extended teaching as available in Germany and Italy (13th grade).

The concepts of wave and particle are used to account for classical objects, whereas quantum objects are essentially different even if using the same notions. QT treats objects of different nature and so requires a new notion – ‘quantum’ [12]. We introduce the features of quanta referring to their properties of particle and wave nature and relate to those in classical physics. The quantum result is naturally strange and restricted to a specific reality of the micro-world. It was documented that students often mix wave and particle aspects in various ways combining these two models though they are contradictory in the macro-world [13]. The school teaching approaches to QT we know of lack an inclusive hierarchically organized structure drawing on certain fundamental principles. These teaching approaches do not consider a “big picture” which would define the wave function, quantum states and other fundamental concepts [14]. In this sense, our study is innovative and fills the lacuna in research on teaching quantum physics. Our curriculum also pioneers using certain level of mathematics. Though it avoids using complex numbers or partial differential equations it addresses Schrödinger’s equation, in symbolic presentation and enable students to do some quantitative accounts of quantum situations. We expect an impact of this curricular change to the corresponding well known confusion of students at the undergraduate as well as secondary school level.

3. Discipline-Culture approach
Lacking the overall theoretical structure in school physics is, however, not specific for QT teaching. Though taught in volume, mechanics and electromagnetism curricula, commonly lack that account too. Responding to this situation, Tseitin and Galili [4] suggested addressing a hierarchical structure of fundamental physical theories to be introduced in physics teaching. The suggested perspective emphasizes the standard disciplinary structure of fundamentals-applications in presenting a physical theory and extends it to a cultural triadic structure of nucleus-body-periphery, termed discipline-culture (DC) (figure 1). We adopted this approach in construction of a new curriculum for teaching QT. In accordance, there appeared the basic principles of the QT – the nucleus, the corresponding illustrative phenomena and experiments – the body, and the periphery that included the rival corresponding ideas either from classical physics or early interpretations of quantum phenomena. All these contents were provided in qualitative form in order to fit the constraints of school reality. Yet, we did not completely neglect the quantitative-computational approach, and adjusted the mathematics required for introductory learning.

Figure 1. The triadic structure of DC approach to representing physical theory.
Teaching started with considering the double slit experiments for light and matter. Computer simulation supported demonstrating the wave-particle duality of matter and light clearly emerged in this pair of experiments. The conceptual focus was reached in preferring this case of two states [paths] instead of dealing with a free particle with a continuous spectrum of states. The new meaning of particle “wavity” as related to a probability distribution obtained in applying interference contrasted to classical picture of two matter waves interference. The particle nature of quanton appeared in its appearance in a point of the screen. We introduced the idea of the wave function as a set of coefficients in the linear combination of states of the quanton (two numbers). The principle of superposition was stated as the central and unique feature of the micro-world. Its contrast with the macro-world was further illustrated with the Schrödinger cat thought experiment.

Table 1 represents our curriculum in accordance with the DC structure and hierarchy. The stream of teaching runs through the topics presented the middle column. Yet, the teaching continuously refers to the correspondent items on the left, the principles of the QT stated explicitly, and contrasts the quantum account with the corresponding item from the right column, the rival account from the periphery of QT. This way we stimulate construction of cultural content knowledge [15] by the students regarding the subject matter taught. This means that we expect adequate students' knowledge of QT as a fundamental physical theory with its principles and applications. The contrast with alternative accounts addresses topics of classical mechanics, optics, electromagnetism and early interpretations of quantum world later rejected (such as Schrödinger matter waves).

**Table 1.** The curriculum according to the DC approach.

| Nucleus                          | Body                                         | Periphery                                      |
|----------------------------------|----------------------------------------------|------------------------------------------------|
| Particle-wave duality            | Einstein account of photoelectric phenomenon| Classical waves of light and matter             |
| Light duality                    | Thomson Jr. Electron diffraction             | Light interference in double slit experiment (Young) |
| Matter duality (de Broglie)      | Double slit experiment for light Compton scattering |                                               |
| Quantum particle – Quanton       |                                              |                                               |
| Physical state – pure and mixed  | Schrödinger’s cat thought experiment          | Classical state (x, p), motion, trajectory     |
| Principle of superposition of    | Stern-Gerlach two-state system account       | Classical determinism and uncertainty          |
| states                            | Mach-Zehnder interferometer                  | Schrödinger matter waves                       |
| Wave function, probability and   | Heisenberg position-momentum uncertainty     | Momentum and Energy conservation               |
| measurement.                     |                                              | Bohr model                                     |
| Uncertainty (indeterminacy)      |                                              | Physical quantities and Equation of motion     |
| principle                        |                                              |                                               |
| Operators of physical observables| Tunneling (Radioactivity)                    | Classical potential well, stability           |
| and equation of state            |                                              |                                               |
| Schrödinger equation (symbolic   |                                              |                                               |
| form)                            |                                              |                                               |


The DC-based curriculum implies learning of the triple faceted subject matter continuously combining basic principles, concept definitions, illustrative applications, compared with parallel alternative understanding from the past and present.

An example of this triadic approach is the concept of wave-particle duality – one of the most important features of QT. The nucleus includes probabilistic meaning of wavity, the body knowledge includes phenomena illustrating interference and diffraction, and the periphery includes misconceptions and alternative understandings, like the concept of the electron as "smeared cloud" or the electron as a tiny marble ball. In our course, students were guided to extend their knowledge of the wavity of matter and discrete nature of light toward a concept of the unified wave-particle nature of the world. This concept of wavity of a quantum object holds that the meaning of its location is subjected to interference of a certain wave entity, which is called “the wave function”. The probability to find the particle in any location is considered as taking a state. The probability of such event is determined by the wave function.

4. Research Questions
RQ1(developmental):
   a) What principles should be chosen to represent the uniqueness of the QT (its nucleus) in the form appropriate for learning by high school students?
   b) What applications of the chosen principles, experiment and phenomena (the body of the QT) could illustrate the chosen principles in the way feasible for learning in high school?
   c) What are the correspondent alternatives to the chosen for presenting principles from classical physics theories (the periphery of the QT) to be addressed in the cultural teaching of the QT?
RQ2 (assessment): What are the features of new knowledge (conceptions) that high school students developed following the experimental teaching?
RQ3: (assessment) What is the impact of teaching QT within the applied approach on students’ views on the nature of science (NOS)?
RQ4: (assessment) How did the inclusion of computational aspects of QT affect students' conceptual understanding? What are the teachers' attitudes about it?

5. The experimental teaching
The first goal of the research (RQ1) was to construct the curriculum. It included literature review, interviews with disciplinary experts (4 university professors of physics and 2 university professors of science education) and pilot teaching of six hours in a high school class. The literature review included 5 books of "modern physics", papers on teaching QT, some of which were presented above, and a review of curricula in eight countries. Following integrative analysis of the review, interviews and pilot teaching we arrived to the required nucleus of QT, chose the correspondent illustrative examples (the body knowledge) and the specific elements of periphery (Table 1). The performed triangulation reinforced the content validity of the developed curriculum and the assessment tests we applied.

To answer RQ2-4, we modified the constructed curriculum to fit 30 hours, which was the allotted time as provided by the school. The experimental teaching addressed the sample of four groups of 12th grade students who chose AP course in physics. There was another two pilot groups of 15 and 6 students, and two experimental groups: one of 11 students (E1), and the other of 14 students (E2). All the students were from the same school and took all the population available at that school. The teachers were the organic teachers of those classes.

| Fermions and bosons Pauli principle | Atomic structure and the Periodic Table of elements Photons and Laser | Matter particles, single type of mass |
|------------------------------------|-------------------------------------------------|-------------------------------------|
| Non locality Quantum entanglement  | Bell theorem and Aspect experiment EPR paradox  | Locality principle Hidden variables theory |

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6. Method

To assess students' understandings, pre- and post- questionnaires were developed. The questionnaires comprised nine open questions. They were administered to two of the experimental groups (E1 and E2) and two control groups. One control group (C1) was of 11 high school 12th grade students who studied the “standard curriculum” (waves, geometrical and physical optics, issues in “modern” physics) mentioned earlier. The other control group (C2) was of 21 university students at the end of the second year of their graduate studies, having completed their first course in quantum mechanics.

Our recruiting of control groups faced certain difficulties. Firstly, due to the recent changes in the policy of the Ministry of Education, the “standard curriculum” reduced the status of modern physics as a subject for matriculate examination causing significant decrease of teaching attention. This change made difficult to find appropriate control group for a fare comparison. Secondly, the university students could not help as they faced essentially different curriculum. However, the limited validity of control groups does not devaluate our pre-post comparison. The significance of the assessment of the particular approach to teaching QT to be considered for adoption is shown as effective. That is, in light of the pessimism shown with respect to teaching QT by science educators.

We created several activities for the students (group E2) to be active through learning, and we collected their responses. At the end of the program we gave them a written exam with both conceptual and computational questions. In this way, we evaluated the understanding of quantum computational ability, conceptual understanding, and the ability of near transfer of the developed skill to the novel context. Some students (n=8) from group E2 were randomly selected and were interviewed.

In addition, the lessons were audio (E1) and video (E2) recorded.

In addition, we asked 14 high school teachers about their view of teaching quantum physics in high school in a computational approach.

The data from the questionnaires and interviews were analyzed using qualitative research methods, based on the "grounded theory" approach ([16],[17]).

7. Findings

In this section we present some preliminary results possessing indicative validity. As mentioned above, we examined the effect of teaching quantum physics through the specific curricular structure in two major aspects: students’ understanding of quantum theory (RQ2) and the features of physics knowledge (NOS) (RQ3).

We asked students to explain the wave-particle duality, one of the central principles of the QT. We classified the responses to four categories.

1. The account which addresses the knowledge of particle-wave duality in the probabilistic sense. The following illustrates this category in the post-questionnaire (pseudonyms are used).

   David (E1): The particle has particle properties – location, momentum (determined) and energy, as well as wave properties. The important thing about these wavy properties is that the particle is in superposition of different states, which, in classical sense, all exist [but] with different probabilities.

   The next two categories of responses were of the kind of phenomenological description without quantum features. Students who responded this way, described the phenomena of interference and diffraction.

   2. Missing the probabilistic nature of particle wavity. One of the graduate students answered:

   Dan (C2): Particles have wave properties such as diffraction and interference.

   3. Alternative (or wrong) understanding of the wave-matter duality, such as “particles are energy waves” or the confusion of probability distribution with spatial distribution of matter. Another graduate student answered:

   Ofry (C2): The material can behave in a wavy manner, and be smeared slightly in space.

   Both answers showed confusion of the probabilistic meaning of particle wavity and did not specify the role of wave function. The confusion regarding the distribution of probability reminds the confusion of particles moving in a wavy trajectory [12], [18].
4. Lack of explanatory answer beyond declaration of "duality".

In group E1, seven out of eleven students (63.6%) defined the waviness of matter as the principles of QT (its nucleus), i.e. as a superposition of states with probabilistic meaning (category 1). Two students provided answers which can be ascribed to the periphery of QT (category 3), and two students did not respond. As to the group C2 (graduate students), eight (out of 21; 38%) responded in accord with category 1, five addressed correlation of particle characteristics, mainly interference and diffraction (category 2), six brought irrelevant associations, and two did not respond. We see here, despite the small sample, the advantage of the students exposed to the experimental teaching with respect to the important aspects in QT.

In order to test the near-transfer understanding, after the instruction, which addressed the double-slit experiment, the students of group E2 were asked about the Mach-Zehnder interferometer. In the class, the students learned about the two-slit experiment with and without a detector in one slit (that is, with different wave functions collapse). In parallel, within the test, students were asked about the output in the spectrometer if a single photon is sent through it with or without measuring in one of the two tubes. The students were also asked to predict the output under the classical account. Thirteen out of fourteen students (92.8%) knew that there would be constructive or destructive interference due to the superposition of states if there is no detector on the way, and there would be no interference if such a detector was placed. This was a clear success in the near transfer of understanding the quantum behavior. Also, 9 of the 14 students (64.3%) correctly used the notion of "superposition" in their response, and 13 out of 14 students (92.8%) correctly used the term "wave function collapse" in their answer. These results were encouraging and indicative.

Interestingly, in their responses, 12 students (of 14) compared between the situations, even though they were not asked to do that. The output of the Mach-Zehnder interferometer in its different modifications is presented in Table 2.

**Table 2. The output of the Mach-Zehnder interferometer according to classic and quantum accounts**

| With detector | Classical understanding | Quantum understanding |
|---------------|------------------------|----------------------|
|               | a. particle – no interference | b. particle – no interference |
| Without detector | c. particle – no interference | d. wave – interference |

Seven of the students compared between the two classical situations (a and c), which implies that wave function collapse is irrelevant in the classic account. Three students compared the situations with the detector (a and b), and correctly stated that these situations produce the same outcome (the understanding that measurement causes the wave function collapse and so the classical output). Two students compared between the situations (a, b, and c) and justified the similarity with the collapse of the wave function. The ability to compare between the two accounts and the transfer from the two-slit experiment to the novel situation indicates mature conceptual understanding on behalf of the students.

Another transfer of understanding was found when the students learned about quantum cryptography (the BB84 protocol) but were asked about quantum coin toss. Nine of the fourteen (64.3%) students (E2) were able to describe the process and its results employing different states and measurement bases.

In RQ4, we wondered whether instruction in the computational approach affected conceptual understanding. The instruction included Dirac notation and its using in problem solving. To test students' conceptual understanding, we asked them whether the state \( |\psi\rangle = a|1\rangle + b|2\rangle \) is a wave. Seven students of group E2 were asked this question (seven out of the eight students who agreed to be interviewed). Six students replied positively indicating understanding the quantum waviness as a superposition of states:

*Muli: Yes. It expresses different probabilities to get a certain trait. Until I measure, I will not know what the state is. Not "I do not know", but know that this is the state.*

We inferred here that the student grasped the main meaning of quantum waviness as a superposition of several states in the sense of their coexistence, as characterizes a quanton. Yoni clarified:
Yoni: It is not exactly a wave because it has only two states, it is not spread in space. It is a kind of wave because it is a superposition.

Here, Yoni realized that the meaning of waviness is superposition of states, and he emphasized its new sense in contrast with other [classical] waves, which do “spread in space”.

Within the investigation of RQ4, we tested the correlation between conceptual understanding and the ability of quantitative account of a physical problem solving. Students were scored between 0 and 10 in the computational part of the final test. In parallel, students were scored on the conceptual part of the test. The grades were classified as low in the interval of (0-5), medium – (6-7) and high – (8-10).

Out of the fourteen students (E2), five received a high score both in the quantitative and conceptual assessments. Two students received a medium grade on the quantitative part. Of them, one received a medium grade on the conceptual understanding test, and the other – the high score. Seven students received a low grade on the quantitative part. Of them, on the conceptual part two received high scores, two – medium, and two – low scores. These results indicate positive correlation between the conceptual understanding and the ability of quantitative problem solving. This effect definitely deserves a more comprehensive investigation.

To answer QR3, we analysed the responses to NOS questions. With regard to the status of being proved in physics, six students (out of ten) stated in the pre-questionnaire that proof is obtained by an experiment, and none noted that a claim could be demonstrated mathematically. In the post-questionnaire, six remained requiring empirical evidence, but four mentioned the possibility of a rigorous mathematical proof. None of the students mentioned both.

Another question addressed explanations, whether it is possible that different valid explanations in physics could be equally correct. The answers (categorized by two researchers independently with full agreement) pointed to "minimalistic" and "expanded" perceptions of students. Within the minimalistic approach, the students argued that two valid explanations of the same phenomenon might differ in examples, but not in the basic principles. The students of the expanded perception, however, argued that the explanations may disagree in basic principles they draw on. Both are correct though different. The students split between these two understandings. In the pre-assessment, three students took the expanded approach, and seven – the minimalistic one. In the post-assessment, however, five adopted the expanded view and six stated the minimalistic one. Three students changed their response from minimalistic to expanded, and one – from the expanded to minimalistic. No significant change, therefore, was registered. This aspect, thus, requires further refinement.

The workshop for teachers provided another resource in our study. In it, the teachers were exposed to the triadic structure of a theory and possible activities for its demonstration. We asked the teachers about their views on involving quantitative aspects in teaching quantum physics. Teachers’ views showed a range. Some of them stated that mathematics is an inseparable in physics understanding (5 of 14 teachers):

Yair: Without quantitative instruction, the understanding remains vague with many conceptual errors.

Of them, 2 of 14 teachers emphasized that involving mathematics in teaching quantum, even complicated, is important for understanding:

Noam: In classical topics, it is important and possible [to do quantitative problem solving]. Mathematics is the language of physics and therefore it is a necessary control of understanding. Quantum mathematics is so complex that a student may lose oneself within it, but if the student studied quantitatively, s/he would at least understand its essence and principles.

Still others thought that in teaching quantum physics is not mathematics important (6 of 14 teachers):

Dotan: In classical physics, it is very important to reach the ability to solve quantitative questions. In quantum physics, it is best to reach a level of [conceptual] understanding.
Similarly, a teacher argued for less math in addressing quantum in favor of instigating curiosity and providing an idea:

Shai: It is also important to teach modern physics in terms of students' interest. Leave the calculations to classical physics and give a taste of more and curiosity to modern physics.

8. Discussion and conclusions
Though limited in time to 30 hours instead of the planned 50, we consider our experiment succeeded to provide clear indication that the new approach to school teaching of QT may succeed in providing students' literacy in the desired domain of knowledge. The positive impact of the DC approach to the meaningful learning of quantum physics was recorded. The rich curricular content (Table 1) by itself present a breakthrough deserving recognition of the adequacy of the considered representation of QT content. Students' understanding of wave-particle duality, superposition principle, the meaning of quantum probability, and their abilities to provide quantitative accounts of quantum situations improved.

We got preliminary evidence that despite of severely limited teaching time and the formal tools (mathematical knowledge), there is a way of constructing valid knowledge of quantum physics at school. Our way does not follow the historical development of QT. We argue for the deductive holistic exposure of quantum physics, emphasizing the new ontological and epistemological principles, which created a new physical picture of the world. The DC approach suggests a new curricular design by addressing the contents in conceptual triads, principles – new phenomena accounts – the older principle replaced.

Comparing and contrasting with alternatives (periphery elements) produces good results both in understanding basic concepts, ability of theory based knowledge organization, distinguishing between principles and applications, recognition of validity area.

The comparison between the research group and the undergraduate students revealed that even the university students who learned QT, missed some of its important aspects. Those were known to the school students who learned in the DC approach. It is mainly with respect to the principles seemingly less addressed in undergraduate teaching.

Our study showed that the tested teaching affected students' NOS understandings indicating potential in that respect in physics teaching. We observed that triadic approach enabled students to understand basic principles, which helped them in coping with novel problems they never faced.

We have clarified the directions for further research. It will examine the stability of our results in bigger samples and thus, the validity of our claims. We need more evidence for the correlation between computational abilities and conceptual understanding following the new instruction. This will reinforce our support for the teaching of QT in high school. This aspect presents a general problem of physics education and is especially striking in learning quantum physics. There is evidence that the DC approach is efficient and justifies the effort to introduce the new approach to physics curriculum.

9. References

[1] Serway R A et al 2005 Modern Physics. Thomson Brooks/Cole, USA
[2] Levrini O and Fantini P 2013 Sci. & Edu. 22 1895–1910
[3] Pospiech G 2000 Phys. Edu. 35 393
[4] Tseitlin M and Galili I 2005 Sci. & Edu. 14 235-261
[5] Ireson G 1999 European J. of Phys. 20 193-199
[6] Petri J and Niedderer H 1998 Int. J. of Sci. Edu. 20 1075–1088
[7] Greca I M and Freire O 2003 Sci. & Edu. 12 541-557
[8] Fischler H and Lichtfeldt M 1992 Int. J. of Sci. Edu. 14 181–190
[9] Muller R and Wiesner H 2002 American J. of Phys. 70 200-209
[10] Malgieri M et al 2014 European J. of Phys. 35 1-21
[11] Born M 1962 Atomic Physics (New York: Hafner)
[12] Levy-Leblond J M 2003 Sci. & Edu. 12 495-502
[13] Bungum B et al. 2016 In: J Lavonen et al. eds. Proceedings of the ESERA 2015 Conference. Science education research: Engaging learners for a sustainable future, Part 1, pp. 11-19. Helsinki, Finland.
[14] Krijtenburg-Lewerissa et al 2017 Phy. Rev. Phys. Edu. Res. 13 010109
[15] Galili I 2012 Sci. & Edu. 21 1283-1316
[16] Strauss A and Corbin J M 1997 Grounded theory in practice. Sage
[17] Shkedi A 2003 Words of meaning: Qualitative research-theory and practice. Tel-Aviv: Tel-Aviv university Ramot. (Hebrew)
[18] Olsen R V 2002 Int. J. of Sci. Edu. 24 565-574