Making sense of high dimensional concrete data – a statistical approach

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Abstract. Performance of concrete is dependent on a number of factors. It is difficult to understand the influence and interrelationship among these variables, when there are many. Dimensionality reduction techniques can yield the best possible data interpretation based on the variance in data, without loss of much of original information. This paper presents the application of dimensionality reduction technique for analysis of data and decision making in the field of Concrete Technology.

1. Introduction
Strength, workability and durability of concrete are affected by a large number of variables [1]. It is challenging to develop math models considering all the variables. Hence by using the method of proper orthogonal decomposition (POD), prime variables are selected to reduce time and efforts.

This paper presents the effect of replacement of fine aggregate by mineral admixtures on properties of self-compacting concrete as analyzed and interpreted employing POD.

2. Proper orthogonal decomposition
POD is a dimensionality reduction technique for data organization [2, 3]. It also helps to hierarchize variables based on contribution to variation in the data.

Following sequential steps in POD lead to dimensionality reduction of data. Available data is normalized in order to avoid bias. Correlation matrix is generated for the normalized data. Eigenvalues and eigenvectors are obtained for correlation matrix and data reduction is done based on end objective. Obtained results are interpreted and variables are clustered.

3. Illustrative example
An available data set [4] on use of mineral admixture for replacement of fine aggregate in self-compacting concrete of M60 grade is taken up for demonstration of utility of POD in Concrete Technology. The dataset includes variables namely, water to cement ratio (W/C), Water (W), Fine aggregate (FA), slump spread (S), slump flow time to spread 500 mm diameter (S50), V Funnel test Concrete flow time to flow 100 mm (VT10) and 500 mm (VT50), Blocking ratio from L Box test (LB(H2/H1)), Passing ability from U box test (UB(H2/H1)), cube compressive strength at 28 (CS28) and 90 days (CS90), Split tensile strength value (STS28), Flexural strength (FLS28), Elastic modulus (E), Poisson’s ratio (mu), Water absorption (Wabs), Sorption (Sorp28), Rapid chloride penetration test value (RCPT28), Weight loss due to acid arrack (WL_AA), sulphate solution (WL_SA) and corrosion
(WL_Cor) for different percentage (5 – 25 %) of fine aggregate (FA) replacement with fly ash (F), Ground granulated blast furnace slag (GGBS) and Silica fumes (SF). In all mixes cement content (C), coarse aggregate content (CA) and super plasticizer dosage are kept constant and are equal to 550 kg/m³ and 589 kg/m³ and 11 kg/m³ respectively.

4. Results

POD technique is applied on the chosen data set and results are obtained.

4.1. Correlation matrix

Table 1 to table 3 presents correlation matrix, giving the values of degrees of dependence of output variables on input variables. Correlation value of magnitude greater than 0.6 is taken as significant.

**Table 1.** Correlation Matrix for concrete with fine aggregates replaced by fly ash.

| Variables               | W/C  | W   | FA   | F    |
|-------------------------|------|-----|------|------|
| S                       | 0.99 | 0.99| -0.05| 0.05 |
| S50                     | -0.62| -0.62| -0.33| 0.33 |
| VT10                    | -0.59| -0.59| -0.72| 0.72 |
| VT50                    | -0.70| -0.70| -0.44| 0.44 |
| LB(H2/H1)               | 0.00 | 0.00| -0.56| 0.56 |
| UB(H2/H1)               | -0.85| -0.85| 0.48 | -0.48|
| CS28                    | -0.70| -0.70| 0.62 | -0.62|
| CS90                    | -0.48| -0.48| -0.85| 0.85 |
| STS28                   | -0.77| -0.77| 0.56 | -0.56|
| FIS28                   | -0.83| -0.83| 0.54 | -0.54|
| E                       | -0.82| -0.82| 0.55 | -0.55|
| mu                      | -0.58| -0.58| 0.04 | -0.04|
| Webs                    | 0.15 | 0.15| -0.06| 0.06 |
| Sorp28                  | -0.45| -0.45| 0.18 | -0.18|
| RCPT28                  | -0.48| -0.48| 0.22 | -0.22|
| WL_AA                   | -0.21| -0.21| 0.14 | -0.14|
| WL_SA                   | -0.25| -0.25| 0.18 | -0.18|
| WL_Cor                  | 0.19 | 0.19| 0.20 | -0.20|

**Table 2.** Correlation Matrix for concrete fine aggregates replaced by GGBS.

| Variables               | W/C  | W   | FA   | GGBS |
|-------------------------|------|-----|------|------|
| S                       | 0.75 | 0.75| 0.50 | -0.50|
| S50                     | -0.86| -0.86| 0.37 | -0.37|
| VT10                    | -0.95| -0.95| -0.30| 0.30 |
| VT50                    | -0.76| -0.76| -0.52| 0.52 |
| LB(H2/H1)               | -0.52| -0.52| -0.21| 0.21 |
| UB(H2/H1)               | -0.93| -0.93| 0.23 | -0.23|
| CS28                    | -0.82| -0.82| -0.53| 0.53 |
| CS90                    | -0.39| -0.39| -0.90| 0.90 |
From table 1 it is evident that fly ash content is highly correlated to cube compressive strength both at 28 & 90 days and V funnel concrete flow time (VT10) to flow 100 mm. Table 2 reveals dependence of cube compressive strength at 90 days, split tensile strength, flexural strength and modulus of elasticity are greatly affected by GGBS content.

Use of silica fume as a replacement to fine aggregate influences compressive strength at 28 & 90 days, split tensile strength, flexural strength and elastic modulus. Fine aggregate content replacement by silica fume reduces slump spread as indicated by negative correlation value.

| Variables | W/C   | W   | FA   | GGBS |
|-----------|-------|-----|------|------|
| STS28     | -0.73 | -0.73 | -0.63 | 0.63 |
| FlS28     | -0.66 | -0.66 | -0.70 | 0.70 |
| E         | -0.66 | -0.66 | -0.69 | 0.69 |
| mu        | -0.83 | -0.83 | 0.41  | -0.41|
| Wabs      | 0.84  | 0.84  | -0.28 | 0.28 |
| Sorp28    | 0.75  | 0.75  | -0.39 | 0.39 |
| RCPT28    | 0.85  | 0.85  | -0.35 | 0.35 |
| WL_AA     | 0.88  | 0.88  | -0.36 | 0.36 |
| WL_SA     | 0.81  | 0.81  | -0.44 | 0.44 |
| WL_Cor    | 0.86  | 0.86  | -0.38 | 0.38 |

| Variables | W/C | W   | FA   | SF  |
|-----------|-----|-----|------|-----|
| S         | 0.20| 0.20| 0.97 | -0.97|
| S50       | -0.19| -0.19| -0.90| 0.90|
| VT10      | -0.60| -0.60| -0.74| 0.74|
| VT50      | -0.70| -0.70| 0.36 | -0.36|
| LB(H2/H1) | 0.87| 0.87| 0.00 | 0.00|
| UB(H2/H1) | -0.96| -0.96| 0.24 | -0.24|
| CS28      | 0.19| 0.19| -0.85| 0.85|
| CS90      | -0.50| -0.50| -0.86| 0.86|
| STS28     | -0.67| -0.67| -0.68| 0.68|
| FlS28     | -0.69| -0.69| -0.66| 0.66|
| E         | -0.69| -0.69| -0.67| 0.67|
| mu        | -0.61| -0.61| 0.58 | -0.58|
| Wabs      | 0.18| 0.18| -0.40| 0.40|
| Sorp28    | 0.35| 0.35| -0.52| 0.52|
| RCPT28    | 0.29| 0.29| -0.66| 0.66|
| WL_AA     | 0.46| 0.46| -0.49| 0.49|
| WL_SA     | 0.55| 0.55| -0.37| 0.37|
| WL_Cor    | 0.59| 0.59| -0.42| 0.43|

Table 3. Correlation Matrix for concrete fine aggregates replaced by Silica fumes.
4.2. Eigen values and eigenvectors

Eigenvalue quantifies variation in the data and corresponding eigenvector gives the direction of variation. Number of dimensions to be retained can be decided based on magnitude of eigenvalues, scree plots, elbow plots and parrot plots. In present study, components with eigenvalues greater than 1 are selected for the analysis.

Table 4. Eigenvectors for concrete with fine aggregates replaced by fly ash.

| Variables            | Dim.1  | Dim.2  | Dim.3  |
|----------------------|--------|--------|--------|
| EVs                  | 9.31   | 5.58   | 5.33   |
| W/C                  | -0.91  | 0.04   | 0.38   |
| W                    | -0.91  | 0.04   | 0.38   |
| FA                   | 0.29   | -0.62  | 0.70   |
| F                    | -0.29  | 0.62   | -0.70  |
| S                    | -0.94  | 0.04   | 0.30   |
| S50                  | 0.63   | 0.58   | -0.30  |
| VT10                 | 0.41   | 0.65   | -0.63  |
| VT50                 | 0.58   | 0.38   | -0.62  |
| LB(H2/H1)            | -0.13  | 0.58   | -0.39  |
| UB(H2/H1)            | 0.90   | -0.36  | 0.00   |
| CS28                 | 0.81   | -0.49  | 0.18   |
| CS90                 | 0.20   | 0.51   | -0.74  |
| STS28                | 0.90   | -0.33  | 0.09   |
| FIS28                | 0.92   | -0.38  | 0.05   |
| E                    | 0.92   | -0.38  | 0.06   |
| mu                   | 0.35   | -0.57  | -0.67  |
| Wabs                 | 0.10   | 0.77   | 0.53   |
| Sorp28               | 0.70   | 0.53   | 0.45   |
| RCPT28               | 0.73   | 0.48   | 0.44   |
| WL_AA                | 0.49   | 0.63   | 0.57   |
| WL-SA                | 0.53   | 0.59   | 0.59   |
| WL_Cor               | 0.13   | 0.61   | 0.75   |

Table 5. Eigenvectors for concrete with fine aggregates replaced by GGBS.

| Variables | Dim.1  | Dim.2  | Dim.3  |
|-----------|--------|--------|--------|
| EVs       | 13.40  | 6.59   | 1.03   |
| W/C       | -0.98  | 0.06   | -0.05  |
| W         | -0.98  | 0.06   | -0.05  |
| FA        | -0.08  | -0.97  | 0.20   |
| GGBS      | 0.08   | 0.96   | -0.20  |
| S         | -0.76  | -0.53  | -0.36  |
| Variables | Dim.1 | Dim.2 | Dim.3 |
|-----------|-------|-------|-------|
| S50       | 0.85  | -0.38 | 0.11  |
| VT10      | 0.95  | 0.24  | 0.08  |
| VT50      | 0.82  | 0.43  | -0.31 |
| LB(H2/H1) | 0.55  | 0.30  | 0.73  |
| UB(H2/H1) | 0.85  | -0.29 | 0.01  |
| CS28      | 0.85  | 0.47  | -0.15 |
| CS90      | 0.49  | 0.82  | -0.24 |
| STS28     | 0.79  | 0.59  | 0.06  |
| FIS28     | 0.72  | 0.69  | 0.03  |
| E         | 0.73  | 0.68  | 0.06  |
| mu        | 0.82  | -0.48 | -0.14 |
| W/C       | -0.86 | 0.42  | 0.12  |
| W         | -0.77 | 0.54  | 0.15  |
| FA        | -0.86 | 0.47  | 0.08  |
| SF        | -0.87 | 0.47  | 0.07  |
| S         | -0.81 | 0.56  | 0.06  |
| WL_Cor    | -0.86 | 0.48  | 0.02  |

Table 6. Eigenvectors for concrete with fine aggregates replaced by Silica fumes.

| Variables | Dim.1 | Dim.2 | Dim.3 |
|-----------|-------|-------|-------|
| EVs       | 10.45 | 8.30  | 2.29  |
| W/C       | 0.10  | -0.95 | -0.28 |
| W         | 0.10  | -0.95 | -0.28 |
| FA        | -0.89 | -0.22 | 0.37  |
| SF        | 0.89  | 0.22  | -0.37 |
| S         | -0.88 | -0.39 | 0.24  |
| S50       | 0.95  | 0.30  | 0.03  |
| VT10      | 0.47  | 0.79  | -0.31 |
| VT50      | -0.28 | 0.51  | 0.74  |
| LB(H2/H1) | 0.01  | -0.78 | -0.54 |
| UB(H2/H1) | -0.31 | 0.86  | 0.34  |
| CS28      | 0.91  | -0.10 | -0.02 |
| CS90      | 0.73  | 0.65  | -0.17 |
| STS28     | 0.58  | 0.78  | -0.08 |
| FIS28     | 0.56  | 0.79  | -0.04 |
| E         | 0.56  | 0.80  | -0.05 |
| mu        | -0.81 | 0.56  | -0.09 |
| Wabs      | 0.76  | -0.26 | 0.53  |
| Sorp28    | 0.84  | -0.39 | 0.37  |
### 4.3. Component plots

Eigenvectors are plotted for any two components at a time to obtain component plots. These plots help in visualizing interactions or interplay among variables.

![Component plots](image)

**Figure 1.** Component plots (a) Concrete with fly ash replacement to fine aggregates (b) Concrete with GGBS replacement to fine aggregates (c) Concrete with silica fumes replacement to fine aggregates.

| Variables | Dim.1 | Dim.2 | Dim.3 |
|-----------|-------|-------|-------|
| RCPT28    | 0.92  | -0.29 | 0.22  |
| WL_AA     | 0.79  | -0.50 | 0.28  |
| WL_SA     | 0.71  | -0.61 | 0.33  |
| WL_Cor    | 0.72  | -0.63 | 0.24  |
In figure 1, component plots for the first two components are presented for replacement of fine aggregates with fly ash, GGBS and silica fumes.

From table 4 and figure 1(a), it is seen that fly ash replacement to fine aggregates majorly influence L box test blocking ratio, V funnel test flow time, water absorption and weight loss due to acid exposures & corrosion of reinforcement.

Use of GGBS as replacement to FA largely effects 90 days’ cube compressive strength, flexural strength and modulus of elasticity, as seen in table 5 and figure 1(b).

SF replacement increases early age and long term strengths. From figure 1(c) and table 6 it is evident that, replacement of SF has major effect on durability aspects when compared to Fly ash and GGBS replacement to fine aggregates. Use of SF reduces the slump flow and Poisson’s ratio of SCC.

5. Conclusions
Utility of POD has been highlighted and illustrated in understanding effect of fine aggregate replacement by fly ash, GGBS and silica fumes in self-compacting concrete.

The application of POD greatly helps in identifying variables that influence concrete characteristics and also in quantification of their influence.

References
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