Mathematical Modelling for Event Occurrence Rainbow Secondary

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Abstract. Secondary rainbow is a rainbow that formed because light is reflected twice when entering the rain drops and looks right out over the primary rainbow. Rules to formation of the secondary rainbow is same as the primary rainbow when it rains coincide with the sun shining from the other side and the observer is between them. The aims of research are: determine the mathematical model of the secondary rainbow, determine the angle, and the angle secondary rainbow colours using differential calculus. This observation method using literature study which is based on the theoretical framework underlying the ways of solving the problems that include, derivatives, maximum/minimum value, and the properties of light. The results of this research is deviation angle of the mathematical models secondary rainbow:

\[ D_s = 2\theta_1 - 6 \sin^{-1}\left(\frac{n_u \sin \theta_1}{n_a}\right) + 2\pi. \]

This model found the optimum angle of the secondary rainbow is 51 °. Large corner secondary rainbow of colours Red, Orange, Yellow, Green, Blue, Indigo and Violet are respectively 50.57, 51.12, 51.72, 52.42, 52.78, 53.45 and 53.60 (in degrees). It is shown that large rainbow angle for each colours of secondary rainbow are different, it is explains that the secondary rainbow is composed of seven colours.

Keywords: Mathematical model of the secondary rainbow, secondary rainbow angle, differential calculus

1. Introduction
The rainbow is at one and the same time one of the most beautiful visual displays in nature and, in a sense, an intangible phenomenon. It is illusory in that it is not of course a solid arch, but like mirages, it is nonetheless real. It can be seen and photographed, and described as a phenomenon of mathematical physics, but it cannot be located at a specific place, only in a particular direction [1].

The rainbow consists of primary rainbow and secondary rainbow. The single bright arc seen after rain shower or in the spray of a waterfall is the primary rainbow [2]. Whereas secondary rainbows are formed when sunlight is reflected twice by drops of rainwater and radiates out with a sharper angle towards the ground [3]. The order of the color of the secondary rainbow will be the opposite the primary rainbow [4]. This causes the secondary rainbow to look like the reflection of the primary rainbow.

There are many researches about the maodel mathematics like [5;6;7;8;9;10;11;12;13;14;15], but all the model discuss about SIR and SEIR model for disease. In this study, researchers were interested in knowing the mathematical model of the rainbow (in this case the secondary rainbow) through
differential calculus. From the mathematical model can be found the magnitude of the angle of the rainbow and the magnitude of the angle of each colour of the secondary rainbow. Next, the researcher chose and took the title "Mathematical Model for Secondary Rainbow Occurrences".

2. Research Methods
This research was conducted using the literature study method by collecting various references in the form of books, journals and other sources related to differential calculus and optical physics. After that, the reference is used to obtain a mathematical model in the event of a rainbow.

The following are the complete steps that the author uses in this study:
1. Making assumptions, the assumptions in question are:
   a) Ball shaped raindrops.
   b) Sunlight entering the raindrops is free of obstacles.
   c) Illustration is carried out on a two-dimensional plane.
   d) The wavelength and refractive index of each colour are known.
   e) The amount of rainwater in the air is quite large
2. Perform simulations provided that the observer is between the sun and the point of rainwater (sun - observer - rainwater point).
3. Determine the mathematical model that will be used to calculate the angle of the rainbow and the angle of each rainbow colour.
4. Calculate the angle of the rainbow and the angle of each rainbow colour according to the refractive index of each colour.
5. Perform rainbow simulations using the Matlab Application

3. Result and discussion

3.1 Mathematical models for Secondary rainbow occurrences
Secondary rainbow is a rainbow formed because light is reflected twice when it enters the drops of rainwater and is seen just above the primary rainbow. The requirement for the formation of a secondary rainbow is the same as the primary rainbow, which is if there is rain along with the sun shining from the other side and the observer is between them. The position of the observer must also turn his back on the sun or in other words look towards the rain. In addition, the eyes of the day, the observer and the centre of the rainbow arc must also be in a straight line. After all conditions are included, the secondary will be clearly visible with a certain deviation angle. The angle of the secondary rainbow deviation will be determined using a mathematical model.

The mathematical model in the event of a secondary rainbow is an equation or formula in mathematics that is used to find the angle of the secondary rainbow deviation. The following is a description of the determination of a mathematical model which is preceded by a simulation that describes the process of the occurrence of a secondary rainbow.

![Figure 1. Incoming rays](attachment:Figure_1_Incoming_rays.png)
When the sun is shining, the entire surface of the rain drops facing the sun will receive sunlight. However, the rays that can form a secondary rainbow and can be seen by observers on earth are the rays that come into the raindrops at the bottom of the horizontal diameter of rain drops, as shown in Figure 1. Among all the incoming rays, there is one ray which forms the rainbow minimum deviation angle. These rays and angles will be used in this study to formulate and find the angle of the rainbow and the angle of each color of the rainbow.

![Figure 2. Process of forming a rainbow](image)

Figure 2 shows that sunlight coming from the opposite direction enters rainwater drops at point P. Furthermore, rainwater drops behave like small prisms that can disperse the light into seven different colours. After that, the light is refracted towards the point Q, then, the light is then reflected twice, i.e. from point Q to R and from point R to S. the ray then comes out of the drops of rainwater at point S towards the observer.

In general, the process of running the sun when it penetrates out of raindrops and the properties of light that occur in it will be explained by the picture below and then followed by a description of the determination of the mathematical model.

![Figure 3. Process of running the sun](image)

Sunlight that enters into the raindrop with an angle of \( \theta_1 \) as shown in figure 3 is refracted and reflected in drops of rainwater with the same angle, which is \( \theta_2 \). Angles \( \theta_1 \) and \( \theta_2 \) are connected by the law of refraction, as follows:

\[
n_u \sin \theta_1 = n_a \sin \theta_2
\]

With \( n_u \) and \( n_a \) are the air refractive index and the water refractive index respectively.

\[
\sin \theta_2 = \frac{n_u \sin \theta_1}{n_a}
\]

\[
\theta_2 = \sin^{-1}\left(\frac{n_u \sin \theta_1}{n_a}\right)
\]

(1)
Figure 3 also shows, the light entering the water drops is refracted towards point Q and produces a PQ line as the refracted ray. The extension of the reflected beam at the point P towards point O which is the centre of the circle forms the OP line. The OQ line is a line formed between the centres of the circle at point O with a bias point that points to the side of the circle precisely at point Q. Next, the light is reflected to point R so that it forms a QR line. The beam is then reflected again from point R to point S and forms the RS line. The rays then come out of raindrops at point S and intersect with light coming at point T. Because the length of the PQ, QR and RS lines are the same, the refractive angle and reflection angle formed in the rain drops are the same, namely $\theta_2$.

The angle between the incoming ray and the outgoing beam is called the ray deviation angle ($D_\text{s}$).

The relationship between $D_\text{s}$ and angles on the way of light coming up to the outgoing light is given in the following equation.

$$D_\text{s} = \alpha + \beta + \gamma + \delta,$$

because

$$\alpha = \delta = (\theta_1 - \theta_2),$$

$$\beta = \gamma = (\pi - 2\theta_2).$$

then

$$D_\text{s} = (\theta_1 - \theta_2) + (\pi - 2\theta_2) + (\pi - 2\theta_2) + (\theta_1 - \theta_2)$$

$$D_\text{s} = 2\theta_1 - 6\theta_2 + 2\pi$$

Substitution $\theta_2$ form Eq (1), found the Eq (2)

$$D_\text{s} = 2\theta_1 - 6\left(\sin^{-1}\left(\frac{n_w \sin \theta_1}{n_u}\right)\right) + 2\pi$$

Equation (2) is modelling mathematics for secondary rainbow

3.2 Secondary rainbow angle

After determining the mathematical model of the secondary rainbow, then the angle of the secondary rainbow will be searched. However, before determining the angle of the secondary rainbow, the minimum deviation angle will be sought first. The description is as follows

$$D_\text{s} = 2\theta_1 - 6\left(\sin^{-1}\left(\frac{1 \sin \theta_1}{4/3}\right)\right) + 2\pi$$

It is known that the air refractive index and the water refractive index are 1 and 4/3, respectively

$$D_\text{s} = 2\theta_1 - 6\left(\sin^{-1}\left(\frac{3 \sin \theta_1}{4}\right)\right) + 2\pi$$

The minimum deviation angle occurs when $D_\text{s}' = 0$ so that $\frac{d(D_\text{s})}{d\theta_1} = 0$

$$\frac{d(D_\text{s})}{d\theta_1} = \frac{d(2\theta_1)}{d\theta_1} - 6\frac{d}{d\theta_1} \left(\sin^{-1}\left(\frac{3 \sin \theta_1}{4}\right)\right) + \frac{d(2\pi)}{d\theta_1}$$

$$d(2\pi)$$
\[
\frac{d(D_2)}{d\theta_i} = \frac{d(2\theta_i)}{d\theta_i} - 6 \left( \frac{d\left(\sin^{-1}\left(\frac{3\sin \theta_i}{4}\right)\right)}{d\theta_i}, \frac{d\left(\frac{3\sin \theta_i}{4}\right)}{d\theta_i} \right) + \frac{d(2\pi)}{d\theta_i}
\]

\[
0 = 2 - 6 \left( \frac{1}{\sqrt{1 - \left(\frac{3}{4} \sin \theta_i\right)^2}} \cdot \frac{3}{4} \cos \theta_i \right) + 0
\]

\[
2 = 6 \left( \frac{1}{\sqrt{1 - \left(\frac{3}{4} \sin \theta_i\right)^2}} \cdot \frac{3}{4} \cos \theta_i \right)
\]

\[
2 = 6 \left( \frac{1}{\sqrt{1 - \left(\frac{3}{4} \sin \theta_i\right)^2}} \cdot \frac{3}{4} \cos \theta_i \right)
\]

\[
1 = 3 \left( \frac{1}{\sqrt{1 - \left(\frac{3}{4} \sin \theta_i\right)^2}} \cdot \frac{3}{4} \cos \theta_i \right)
\]

\[
\sqrt{1 - \left(\frac{3}{4} \sin \theta_i\right)^2} = \frac{9}{4} \cos \theta_i
\]

\[
\left( \sqrt{1 - \left(\frac{3}{4} \sin \theta_i\right)^2} \right)^2 = \left(\frac{9}{4} \cos \theta_i\right)^2
\]

\[
1 - \left(\frac{3}{4} \sin \theta_i\right)^2 = \frac{81}{16} \cos^2 \theta_i
\]

\[
1 - \frac{9}{16} \sin^2 \theta_i = \frac{81}{16} \cos^2 \theta_i
\]

\[
16 - 9\sin^2 \theta_i = 81(1 - \sin^2 \theta_i)
\]

\[
16 - 9\sin^2 \theta_i = 81 - 81\sin^2 \theta_i
\]

\[
81\sin^2 \theta_i - 9\sin^2 \theta_i = 81 - 16
\]

\[
72\sin^2 \theta_i = 65
\]

\[
\sin^2 \theta_i = \frac{65}{72}
\]

\[
\sin^2 \theta_i \approx 0.902778
\]

\[
\sin \theta_i \approx 0.950146
\]

\[
\theta_i \approx \sin^{-1}(0.950146)
\]

\[
\theta_i \approx 71.831971^\circ
\]

Next, substitute the value \(\theta_i\) to equation (3), we found:
\[ D_s \approx 2(71.831971^\circ) - 6\left(\sin^{-1}\left(\frac{3\sin(71.831971^\circ)}{4}\right)\right) + 2\pi \]

\[ D_s \approx 2(71.831971^\circ) - 6\left(\sin^{-1}(0.712610^\circ)\right) + 2(180^\circ) \]

\[ D_s \approx 143.663942^\circ - 6(45.447641^\circ) + 360^\circ \]

\[ D_s \approx 143.663942^\circ - 272.685849^\circ + 360^\circ \]

\[ D_s \approx 230.978093^\circ \]

So, the minimum deviation angle is \[ D_s \approx 230.978093^\circ \] and occurs when \[ \theta_l \approx 71.831971^\circ \].

The deviation angle obtained means that the light with the angle of arrival that is slightly larger or slightly smaller than will be refracted and reflected at almost the same angle. Therefore, the light coming out of rainwater drops will be concentrated near the minimum deviation angle. This concentration of light coming from near the minimum deviation angle makes the rainbow visible to the observer.

**Figure 4.** Elevation angle between incoming rays and out coming rays

Figure 4 shows that there are two angles formed from the intersection between two lines which each represent the incoming light and the light coming out. The smaller angle in figure 3 \[ 360^\circ - 230.978093^\circ = 129.021907^\circ \] is used to find the elevation angle between the observer and the highest point on the rainbow. That elevation angle is called the secondary rainbow angle.

**Figure 5.** Secondary rainbow angle

In Figure 5 it is clear that the angle of the secondary rainbow or the elevation angle of the observer to the lowest point of the secondary rainbow is around 50.970093°.

### 3.3 Angles on Every Secondary Rainbow Colour

Sunlight that was previously only white will change into many colours when the light enters raindrops. This occurs because sunlight consists of several wavelengths in the form of a colour spectrum, each of which has a different refractive index. The colour spectrum in question is red, orange, yellow, green, blue, indigo and purple. Red has the smallest refractive index while the purple has the largest refractive index. The refraction index of each colour spectrum is complete can be seen in the following table 1 follow: [3]
Table 1. Refractive index of each rainbow colour

| No. | Color | Index Refraction |
|-----|-------|------------------|
| 1   | Red   | 1.3318           |
| 2   | Orange| 1.3339           |
| 3   | Yellow| 1.3362           |
| 4   | Green | 1.3389           |
| 5   | Blue  | 1.3403           |
| 6   | Indigo| 1.3429           |
| 7   | Purple| 1.3435           |

The angle of each rainbow colour can be determined by entering each refractive index value of each colour into the secondary rainbow mathematical model that has been obtained previously. Before determining the angle of the red ray, the minimum deviation angle will be sought in Equation (4):

\[ D_s = 2\theta_1 - 6\left(\sin^{-1}\left(\frac{(1)\sin \theta_1}{1.3318}\right)\right) + 2\pi \]  

(4)

The minimum deviation angle occurs when \( D_s' = 0 \) so that \( \frac{d(D_s)}{d\theta_1} = 0 \)

\[ \frac{d(D_s)}{d\theta_1} = \frac{d(2\theta_1)}{d\theta_1} - 6\left(\frac{\sin^{-1}\left(\frac{(1)\sin \theta_1}{1.3318}\right)}{d\theta_1}\right) + \frac{d(2\pi)}{d\theta_1} \]

\[ \frac{d(D_3)}{d\theta_1} = \frac{d(2\theta_1)}{d\theta_1} - 6\left(\frac{\sin^{-1}\left(\frac{\sin \theta_1}{1.3318}\right)}{d\theta_1}\right) \cdot \frac{d(\sin \theta_1)}{d\theta_1} + \frac{d(2\pi)}{d\theta_1} \]

\[ 0 = 2 - 6\left(\frac{1}{\sqrt{1 - \left(\frac{1}{1.3318}\sin \theta_1\right)^2}} \cdot \frac{1}{\cos \theta_1}\right) + 0 \]

\[ 2 = 6\left(\frac{1}{\sqrt{1 - \left(\frac{1}{1.3318}\sin \theta_1\right)^2}} \cdot \frac{1}{\cos \theta_1}\right) \]
\[
2 = 6 \left( \frac{1}{\sqrt{1 - \left( \frac{1}{1.3318} \sin \theta_1 \right)^2}} \cdot \frac{1}{1.3318} \cos \theta_1 \right)^2 \\
1 = 3 \left( \frac{1}{\sqrt{1 - \left( \frac{1}{1.3318} \sin \theta_1 \right)^2}} \cdot \frac{1}{1.3318} \cos \theta_1 \right)^2 \\
\sqrt{1 - \left( \frac{1}{1.3318} \sin \theta_1 \right)^2} = \frac{3}{1.3318} \cos \theta_1 \\
\left( \sqrt{1 - \left( \frac{1}{1.3318} \sin \theta_1 \right)^2} \right)^2 = \left( \frac{3}{1.3318} \cos \theta_1 \right)^2 \\
1 - \left( \frac{1}{1.3318} \sin \theta_1 \right)^2 = (5.0742) \cos^2 \theta_1 \\
1 - (0.5638) \sin^2 \theta_1 = (5.0742) \cos^2 \theta_1 \\
1 - (0.5638) \sin^2 \theta_1 = (5.0742)(1 - \sin^2 \theta_1) \\
1 - (0.5638) \sin^2 \theta_1 = 5.0742 - (5.0742) \sin^2 \theta_1 \\
(5.0742) \sin^2 \theta_1 - (0.5638) \sin^2 \theta_1 = 5.0742 - 1 \\
(5.0742) \sin^2 \theta_1 - (0.5638) \sin^2 \theta_1 = 5.0742 - 1 \\
4.5104 \sin^2 \theta_1 = 4.0742 \\
\sin^2 \theta_1 = \frac{4.0742}{4.5104} \\
\sin^2 \theta_1 = 0.903290 \\
\sin \theta_1 = 0.950416 \\
\theta_1 \approx \sin^{-1}(0.950416) \\
\theta_1 \approx 71.881561^\circ \\
\]
Substitution the value \( \theta_1 \approx 71.881561^\circ \) into equation (4), then

\[
D_S \approx 2(71.881561^\circ) - 6\left( \sin^{-1}\left( \frac{1 \sin(71.881561^\circ)}{1.3318} \right) \right) + 2\pi \\
D_S \approx 2(71.881561^\circ) - 6(45.531235^\circ) + 2(180^\circ) \\
D_S \approx 143.763122^\circ - 273.187412^\circ + 360^\circ \\
D_S \approx 230.57571^\circ \\
\]
So, the minimum deviation angle is around 230.57571 and occurs when \( \theta_1 \approx 71.881561^\circ \)

Just like when determining the angle of the secondary rainbow, the large elevation angle of the secondary rainbow is

\[
\text{Red} = 180^\circ - (360^\circ - 230.57571^\circ) \\
\text{Red} = 50.57571^\circ \\
\]
This means the observer can see the red color of the rainbow around the angle when turning his back on the sun. In the same way you will get the angle for the other rainbow colors as follows:
Table 2. Secondary rainbow color angle

| No. | Color  | Rainbow Color Angle (°) |
|-----|--------|-------------------------|
| 1   | Red    | 50.575710               |
| 2   | Orange | 51.126388               |
| 3   | Yellow | 51.726011               |
| 4   | Green  | 52.425297               |
| 5   | Blue   | 52.785944               |
| 6   | Indigo | 53.452228               |
| 7   | Purple | 53.605344               |

The results obtained show that the purple color has the largest secondary rainbow angle and the red color has the smallest secondary rainbow angle. The difference in angles in each rainbow color causes the rainbow to appear to have seven different colors.

Figure 6. The difference in angle of secondary rainbow colour

These results also show the differences in the angle of the primary rainbow and secondary rainbow as the results of previous studies by [5] which produce the angle of the rainbow color as follows [5]:

Table 3. Primary rainbow color angle

| No. | Color | Rainbow Color Angle (°) |
|-----|-------|-------------------------|
| 1   | Red   | 42.25                   |
| 2   | Orange| 41.95                   |
| 3   | Yellow| 41.62                   |
| 4   | Green | 41.23                   |
| 5   | Blue  | 41.03                   |
| 6   | Indigo| 40.66                   |
| 7   | Purple| 40.58                   |

3.4. Secondary Rainbow Simulation

Secondary rainbow can be simulated in the Matlab program by entering a mathematical model that has been obtained previously. Here is the process of simulating a secondary rainbow in the Matlab.

Figure 7. The deviation angle of secondary rainbow with using matlab
Figure 7 shows the relationship between the angle of incoming ray and deviation. On the picture, the minimum deviation angle was 4.03 rad = 230.978093°, this is equal to computation from the obtained model.

**Figure 8.** The elevation angle of secondary rainbow with using matlab

In contrast to figure 7, figure 8 shows the incoming ray and elevation angle of the rainbow. Fit to the mathematics model of computation which resulted on minimum-elevation angle of secondary rainbow was about 0.889 rad = 50.970093°.

**Figure 9.** The deviation and elevation angle of secondary rainbow colour with using matlab

Figure 9 illustrate between the angle of incoming ray and elevation angle (left) and deviation angle (right) on each colour of the rainbow. The resulted plot which was not significantly different to the mathematics model, they were 4.0243 rad = 230.57571° for red to 0.88 rad = 50.57571° for purple. And for the rainbow elevation angle (red was 0.88 rad to purple: 0.93 rad) to 0.93 rad = 53.605344°.

### 4. Conclusion

The mathematical model for the occurrence of secondary rainbows is:

\[ D_2 = 2\theta_1 - 6\left(\sin^{-1}\left(\frac{n_n \sin \theta_1}{n_a}\right)\right) + 2\pi \]

Information: \( D_2 \) is Rainbow deviation angle; \( \theta_1 \) is Angle comes; \( n_n \) is Air refractive index and \( n_a \) is Water refractive index.

The secondary rainbow angle is around 50.970093°.

The angles of all secondary rainbow colours are presented in the following table 4.
Table 4. Angular list table of all secondary rainbow colours

| No. | Color  | Refraction Index | Angle of Coming (°) | Minimum Deviation Angle (°) | Angle of Rainbow Color (°) |
|-----|--------|------------------|---------------------|----------------------------|---------------------------|
| 1   | Red    | 1.3318           | 71.881561           | 230.575710                 | 50.575710                 |
| 2   | Orange | 1.3339           | 71.813242           | 231.126388                 | 51.126388                 |
| 3   | Yellow | 1.3362           | 71.739847           | 231.726011                 | 51.726011                 |
| 4   | Green  | 1.3389           | 71.652318           | 232.425297                 | 52.425297                 |
| 5   | Blue   | 1.3403           | 71.608625           | 232.785944                 | 52.785944                 |
| 6   | Indigo | 1.3429           | 71.524410           | 233.452228                 | 53.452228                 |
| 7   | Purple | 1.3435           | 71.505260           | 233.605344                 | 53.605344                 |

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