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Anomalous $U(1)$ Cut
Abstract

We have proposed a very attractive scenario of Grand Unified Theories (GUTs). It employs the supersymmetry (SUSY) and an anomalous $U(1)$ symmetry whose anomaly is canceled via the Green-Schwarz mechanism. In this scenario, the doublet-triplet splitting problem is solved and the success of the gauge coupling unification in the minimal SU(5) GUT is naturally explained with sufficiently stable nucleon. Realistic fermion Yukawa matrices can also be realized simultaneously. In addition, a horizontal symmetry helps to solve the SUSY-flavor problem.
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Chapter 1

Introduction

The standard model (SM) of particle physics is a very successful model which explains hundreds of precise measurements. Theoretically, however, it has many issues to explain. For instance, we have not understood the reason why the absolute value of the electric charge of electron coincides with that of proton very accurately. The radiative correction induces very large mass to the Higgs scalar, and thus it looks unnatural that the electroweak (EW) symmetry breaking scale is so small ($\sim O(100\text{GeV})$). In addition, the SM contains many parameters ($\sim O(10)$), some of which have hierarchical structure, e.g. top Yukawa is much larger than up Yukawa as $\frac{Y_t}{Y_u} \sim 10^{-5}$. Also, it does not treat gravity at all. By such reasons, many authors do not consider it as the most fundamental theory, and have proposed various scenarios for the physics beyond the SM. Among them, the supersymmetric grand unified theory (SUSY-GUT) is one of the most famous scenarios.

SUSY-GUTs realize very beautiful unifications of matter fields and forces, and can give natural solutions for many problems of the SM. In addition, the simplest scenario realizes the unification of the gauge coupling constants though, unfortunately, it is (almost) excluded by non-observation of nucleon decay. However they still have some problems to solve. One of the biggest problems is the so-called doublet-triplet splitting (DTS) problem. And realizing realistic Yukawa matrices of quarks and leptons is also a big issue.

I and my collaborators showed almost all problems of SUSY-GUTs are solved with the aid of an anomalous U(1) gauge symmetry ($U(1)_A$)\cite{1, 2}, whose anomaly is assumed to be canceled via the Green-Schwarz (GS) mechanism\cite{3}. In this scenario, we introduce all the possible interactions that respect the symmetry of the theory, and their coupling constants are assumed to be of order one in unit of the cutoff scale of the theory. This means that the definition of a model is given by the definition of a symmetry, and there is no need to fix each coupling constant if precise analysis is not needed. From such a natural assumption, the DTS problem is solved\cite{4-8} and the success of the gauge coupling unification (GCU) in the minimal SU(5) GUT is naturally explained while nucleon is sufficiently stable\cite{9, 10, 11}. Realistic fermion Yukawa matrices, including the neutrino bi-large mixing angles, can also be realized simultaneously\cite{4, 5, 6, 12}. In addition, a horizontal symmetry helps to solve the SUSY-flavor problem in E$_6$ models\cite{13, 14}. This
analysis may help to construct a realistic $E_8$ unification model.

In this thesis, we summarize these studies of SUSY-GUTs with an anomalous U(1) symmetry (anomalous U(1) GUTs). In §2 some fundamental ideas of the SM, GUT, SUSY and anomalous U(1) symmetry are briefly reviewed. In §3 the starting point and some significant features of the anomalous U(1) GUT scenario are explained. Some concrete models based on SO(10) or $E_6$ gauge symmetry are examined in §4. The role of horizontal symmetries in the anomalous U(1) GUT scenario is discussed in §5. §6 is for summary.
Chapter 2

SM, GUT, SUSY and Anomalous U(1) Symmetry

2.1 Standard Model

The SM is a renormalizable gauge theory based on $G_{SM} = SU(3)_C \times SU(2)_L \times U(1)_Y$ gauge symmetry. The corresponding gauge bosons $G$, $W$ and $B$ mediate the strong, weak and electro-magnetic (EM) interactions. It contains a matter sector of quarks and leptons and a Higgs sector that breaks $SU(2)_L \times U(1)_Y$ down to the electro-magnetic $U(1)_{EM}$ symmetry.

The matter sector consists of three sets of the left-handed quark doublet $Q = (u_L, d_L)$, the right-handed up-type quark $U^c = u^c_R$, the right-handed down-type quark $D^c = d^c_R$, the left-handed lepton doublet $L = (\nu_L, e_L)$ and the right-handed charged lepton $E^c = e^c_R$. It is convenient to add the right-handed neutrino $N^c$ in order to explain non-vanishing neutrino masses reported in Refs. [15, 16]. Their quantum numbers are shown in the Table 2.1 in the left-handed basis. Hereafter, we often use the characters in the first column of the table for representing the quantum numbers in the corresponding third column.

Interestingly, this set of fermions is anomaly free. Namely, all the triangle anomalies including the mixed anomalies are canceled. Anomaly cancellations of $SU(3)^3$ and $SU(2)^3$ are trivial and $U(1)^3$, $[SU(3)]^2 \times U(1)$, $[SU(2)]^2 \times U(1)$ and $[\text{gravity}]^2 \times U(1)$ anomalies are evaluated by

\begin{align}
\text{tr}(Q^3_Y) &= 6 \times \frac{1}{6} + 3 \times \left( -\frac{2}{3} \right)^3 + 3 \times \frac{1}{3} + 2 \times \left( -\frac{1}{2} \right)^3 + 1 = 0, \\
\text{tr}\{T^a_C, T^b_C\} Q_Y &= \frac{1}{2} \left( 2 \times \frac{1}{6} + \left( -\frac{2}{3} \right) + \frac{1}{3} \right) = 0, \\
\text{tr}\{T^a_L, T^b_L\} Q_Y &= \frac{1}{2} \left( 3 \times \frac{1}{6} + \left( -\frac{1}{2} \right) \right) = 0, \\
\text{tr}(Q_Y) &= 6 \times \frac{1}{6} + 3 \times \left( -\frac{2}{3} \right) + 3 \times \frac{1}{3} + 2 \times \left( -\frac{1}{2} \right) + 1 = 0.
\end{align}

In this way, anomalies of the quark sector and those of the lepton sector cancel out. This
Table 2.1: The quantum numbers for the participants of the SM.

| spin | (SU(3)_C, SU(2)_L, U(1)_Y) |
|------|-----------------------------|
| G    | \[(8,1)_0\]                 |
| W    | \[(1,3)_0\]                 |
| B    | \[(1,1)_0\]                 |
| Q    | \[(3,2)_{1/6}\]            |
| U^c  | \[(3,1)_{-3/2}\]           |
| D^c  | \[(3,1)_{1/3}\]            |
| L    | \[(1,2)_{-1/2}\]           |
| E^c  | \[(1,1)_1\]                |
| (N^c)| \[(1,1)_0\]                |
| H    | \[(1,2)_{1/2}\]            |

The fact seems to indicate that quarks and leptons have something to do with each other. This gives one of the motivations to consider GUTs, which realize unification between quarks and leptons.

The Higgs sector consists of only one doublet scalar, \( H = (H^+, H^0) \). The renormalizable Higgs potential is

\[
V = \mu^2 |H|^2 + \frac{\lambda}{2} (|H|^2)^2. \tag{2.5}
\]

If the parameter \( \mu^2 \) is negative, i.e. \( \mu^2 = -m^2 \), the equation of motion (EOM) requires \( |H|^2 = m^2/\lambda \equiv v^2/2 \). (\( \lambda \) should be positive for the stability of the theory.) The gauge transformation of SU(2)_L \times U(1)_Y can make the vacuum expectation values (VEVs) of the other components than the real part of \( H^0 = \frac{1}{\sqrt{2}} (h + i\chi) \) vanishing. This means that these modes are the Numb-Goldstone (NG) modes which are eaten through the Higgs mechanism. In fact by this VEV, three gauge bosons acquire masses proportional to the VEV \( v \). This is understood by examining the gauge interaction of the Higgs, which is given as

\[
|D_\mu H|^2 = \left| \left( \partial_\mu + ig^a \frac{\tau^a}{2} W^a_\mu + ig' Q_Y B_\mu \right) H \right|^2, \tag{2.6}
\]

where \( \tau^a \)'s are the Pauli matrices and \( Q_Y \) is the generator of the hypercharge. We can see that the \( W \) boson \( W^\pm_\mu = \frac{1}{\sqrt{2}} (W^1_\mu \pm iW^2_\mu) \) and the \( Z \) boson which is a linear combination of \( W^3 \) and \( B \), \( Z_\mu \propto gW^3_\mu - g'B_\mu \) acquire \( m_W = \frac{\sqrt{2}}{2} v \) and \( m_Z = \frac{\sqrt{g^2 + g'^2}}{2} v \), respectively. The remaining gauge boson \( A_\mu \propto g'W^3_\mu + gB_\mu \), which is the gauge boson of the EM interaction, remains massless.

Note that quarks and leptons are vector-like under the remaining EM symmetry. Thus, they are expected to acquire masses proportional to the positive power of \( v \). In fact, they acquire masses through the Yukawa interactions:

\[
\mathcal{L}_{\text{Yukawa}} = (Y_U)_{ij} Q_i U^c_j H + (Y_D)_{ij} Q_i D^c_j H^\dagger + (Y_E)_{ij} L_i E^c_j H^\dagger, \tag{2.7}
\]
where $Y_f$'s, $f = U, D, E$, are $3 \times 3$ Yukawa matrices related with the mass matrices by $M_f = Y_f v$. Their elements are complex and therefore they have eighteen real parameters per a matrix. On the other hand, we can rename the fields that have a common quantum number, e.g. $u_{Ri} \to V_{uRij} u_{Rj}$ by a unitary matrix $V_R$. By using these degrees of freedom, we can diagonalize the Yukawa matrices as $Y_f \to V_f Y_f V_f^\dagger = Y_f^{\text{diag}}$. In this mass basis, mixings between generations appear only in the charged current interactions generated by the $W$ boson, as $\bar u_L \Gamma^\mu W^+_{\mu} d_L \to \bar u_L V_{uL}^\dagger \Gamma^\mu W^+_{\mu} V_{dL} d_L$. Thus, the quark mixing is parameterized by only one unitary matrix
\begin{equation}
V_{\text{CKM}} = V_{uL}^\dagger V_{dL}.
\end{equation}

Note that we still have degrees of freedom to rotate the phases of quarks although the common phase rotations of quarks because of the accidental baryon number symmetry. This means that five phases of $V_{\text{CKM}}$ are not physical parameters but three angles and one phase are the physical parameters. Note that the neutral current interaction does not change the flavors at the tree level. Furthermore, even if we consider quantum corrections, flavor changing neutral current (FCNC) processes are suppressed very much by the GIM mechanism, which is consistent with the experimental results.

As for the lepton sector, if the right-handed neutrinos are not introduced, neutrino cannot have mass. Then, all the neutrinos are degenerate and we can rename them freely and thus the lepton mixing vanishes, which is inconsistent with the experiments\cite{15, 16}. Thus we introduce three right-handed neutrinos, $\nu_R$. Because these $\nu_R$'s are neutral under $G_{\text{SM}}$, they can acquire Majorana masses much larger than the weak scale. Then in the effective theory at the weak scale, $\nu_R$'s are almost absent, leading to very tiny left-handed neutrino masses (Seesaw mechanism). This is consistent with the neutrino experiments if the Majorana scale is around $O(10^{13-15})$ GeV.

**Problems**

The SM explained above is consistent with almost all the experimental results if we assign the appropriate values to the parameters by hand. The number of parameters is much less than that of the experimental results and thus the SM is very successful. Nevertheless, we are not satisfied with it. It looks unnatural that the Higgs mass parameter $\mu$ in (2.5) is the order of the weak scale when there are other scale much larger than the weak scale, such as the Majorana scale and Planck scale. This is because scalar masses are not protected by any symmetry against the quantum correction and they suffer very large correction. This means that we need fine-tuning between a tree level mass and quantum corrections. This problem is called as the hierarchy problem. Furthermore, the Yukawa matrices also have hierarchical structures, although the degree of the hierarchy is much milder than that in the Higgs mass. The hypercharges are assigned to be integers when they are multiplied by six, in spite of the fact that charges of a U(1) symmetry can take any values. In addition, it cannot explain why the strong interaction conserves the $CP$ symmetry. Cosmologically, it cannot explain the baryon asymmetry in universe, it has
no candidates for the dark matter, and we do not understand why the dark energy is so small. Of course, the quantum gravity is not treated at all.

### 2.2 GUT

As in the SM, the gauge symmetry that is observed by experiments may be different from the symmetry of the theory. And chiral fields of the original symmetry may acquire masses if they become vector-like under the reduced symmetry. Such masses should be proportional to the symmetry breaking scale, and the vector-like pairs decouple from the low energy effective theory (if the masses are not so small). This idea is employed in GUTs. Namely, we can extend the gauge symmetry $G_{\text{SM}}$ to a GUT symmetry $G$ that contain $G_{\text{SM}}$ as a subgroup. If $G$ is a semi-simple group, the charge quantization can be explained. The simplest example is SU(5). In this case, three forces of the SM are unified into a single force. The additional gauge bosons $X(3,2)_{5/6}$ and $\bar{X}$ of the adjoint representation of SU(5)

$$24 \rightarrow G + W + B + X + \bar{X}$$

(2.9)

acquire masses of the order of the SU(5) breaking scale.

This unification of forces requires unification of the gauge coupling constants. Unfortunately, the gauge coupling unification (GCU) is not so good in non-SUSY GUTs, although the gauge couplings tend to approach each other, as shown in the Figure 2.1.

The unification of forces also requires unification of matter or introduction of additional matter. Surprisingly, the matter sector of the SM can be unified without introducing any additional matter fields in SU(5) GUTs:

$$10 \rightarrow Q + U^c + E^c,$$  

(2.10)

$$5 \rightarrow L + D^c,$$  

(2.11)

$$1 \rightarrow N^c.$$  

(2.12)

In SO(10) GUTs, they can be unified further as

$$16 \rightarrow 10 + 5 + 1,$$  

(2.13)

that is, each generation, including the right-handed neutrino, can be unified into a single multiplet. In $E_6$ GUT, we need additional matter fields 10 and 1 of SO(10) to embed 16 into the fundamental multiplet of $E_6$ 27 which is decomposed in terms of SO(10) as

$$27 \rightarrow 16 + 10 + 1.$$  

(2.14)

Note that, even in SU(5) GUTs, quarks and leptons are contained a common multiplet. This means the baryon number symmetry is not valid, and nucleon is no longer stable. This is the most impressive prediction of GUTs. In fact, the $X$ boson induces the proton decay $p^+ \rightarrow e^+ + \pi^0$. Because nucleon decay has not been observed\[17\], the mass of the $X$ boson must be larger than $\mathcal{O}(10^{15})$GeV. Thus, the SU(5) breaking scale has to also be larger than that scale, as indicated by the Figure 2.1.
Figure 2.1: The gauge coupling flows in the SM: Here we adopt $\alpha_1^{-1}(M_Z) = 59.47$, $\alpha_2^{-1}(M_Z) = 29.81$, $\alpha_3^{-1}(M_Z) = 8.40$.

In contrast to the matter sector, Higgs cannot be unified because there are only one doublet Higgs in the SM, and we have to introduce additional Higgs fields. The simplest possibility is to embed the doublet Higgs into $5$ Higgs. In this case, from the decomposition we find that the partner is color triplet Higgs $H_C$. This colored Higgs also induces nucleon decay, although their coupling to the matter field is Yukawa interaction and thus the contribution is very small. In SO(10) GUTs, the smallest multiplet is a real $10$ representation which is decomposed in terms of SU(5) as

$$10 \rightarrow 5 + \bar{5},$$

and can be embedded into $27$ in $E_6$ models.

The Yukawa interactions are given as

$$10, 10, 5 \rightarrow [Q_i U_j^c H + U_i^c E_j^c H_C + \{i \leftrightarrow j\}] + Q_i Q_j H_C,$$

$$10, \bar{5}, 5^\dagger \rightarrow Q_i D_j^c H^\dagger + E_i^c L_j H^\dagger + Q_i L_j H_C^\dagger + U_i^c D_j^c H_C^\dagger,$$

in the SU(5) model, which are further unified as

$$16, 16, 10 \rightarrow 10, 10, 5 + [10, \bar{5}, 5^\dagger + 5, 1, 5 + \{i \leftrightarrow j\}]$$

in the SO(10) model, which is further unified as

$$27, 27, 27 \rightarrow 16, 16, 10 + 10, 10, 1 + [16, 10, 16 + 10, 1, 10 + \{i \leftrightarrow j\}]$$
### Table 2.2: Superpartner and supermultiplet.

| scalar $\leftrightarrow$ spinor $\leftrightarrow$ vector | supermultiplet |
|--------------------------------------------------------|----------------|
| squark $\leftrightarrow$ quark                          | chiral         |
| slepton $\leftrightarrow$ lepton                         | chiral         |
| Higgs $\leftrightarrow$ higgsino                         | chiral         |
| gaugino $\leftrightarrow$ gauge boson                    | vector         |

in the $E_6$ model. Thus, the Yukawa matrices of the down-type quarks and charged leptons are related with each other at the SU(5) breaking scale as $Y_L = Y_D^c$ in the SU(5) model. In the SO(10) and $E_6$ models, the Yukawa matrix of the up-type quarks is also related as $Y_D = Y_U$ at the SO(10) breaking scale. These relations are bad because of the experimental relation

\[
m_c \sim \frac{1}{3} m_b \gg m_t, \\
m_c \sim \frac{1}{3} m_b \gg m_t,
\]

(2.20)

and they are called as wrong GUT relation. Note that from Eqs. (2.16) and (2.17), we can find that we get baryon number violating interactions $QQQ_L$ and $U^c E^c U^c D^c$ after the colored Higgs $H_C$ is integrated out.

In this way, GUTs can realize very beautiful unifications of forces and of matter fields and can give a natural reason why the hypercharges are quantized. In addition, they give testable predictions, such as nucleon decay and GUT relations of gauge coupling unifications and of Yukawa couplings, although these are not good unfortunately. On the other hand, some problems of the SM are still not solved. Among them, the hierarchy problem becomes a more concrete problem, because there appears the very large GUT scale as indicated by the Figure 2.1. In the next section, we introduce SUSY in order to solve the hierarchy problem.

## 2.3 SUSY

SUSY is the symmetry that exchanges a boson and a fermion (See, for example, Refs. [13, 14]). This means that each field in the SM has a superpartner which has a different spin from that of the SM particle by $\frac{1}{2}$, as shown in the Table 2.2. There appears two kinds of supermultiplets: the vector supermultiplet, $V$, which contains a gauge boson and the corresponding Majorana gaugino (and an auxiliary field $D$), and the chiral supermultiplet, $\Phi$, which contains a Weyl fermion and a complex scalar field (and an auxiliary field $F$). Note that, the sign of the quantum correction to the Higgs mass by a boson is opposite to that by a fermion. Thus, when the boson and fermion have the same quantum numbers,
e.g. mass and coupling, the quantum correction vanishes so that the hierarchy problem is solved.

At this stage, the Higgs field is treated equally as quarks and leptons. We have to introduce the fermionic partner of Higgs which contribute the anomalies so that the anomalies become non-zero. The simplest way to cancel the anomalies is to introduce one more doublet Higgs that has opposite hypercharge $-\frac{1}{2}$. These two Higgs fields are also required for giving masses to both the up-type and down-type quarks because of the holomorphy of the superpotential. Thus, we introduce two Higgs doublets $H_u(1, 2)^\frac{1}{2}$ and $H_d(1, 2)^{-\frac{1}{2}}$.¹ This supersymmetric extension of the SM is called as the minimal supersymmetric standard model (MSSM). In the MSSM, the Yukawa interaction is given by the same expression as Eq.(2.7) if we exchange $L$ by superpotential $W$, $H$ by $H_u$ and $H^\dagger$ by $H_d$ and we interpret each field as a superfield. Because there are two Higgs doublets, there appears an additional parameter that is the ratio of the VEVs of these Higgs fields, $\tan \beta \equiv \frac{\langle H_u \rangle}{\langle H_d \rangle}$. When $\tan \beta$ is large, bottom Yukawa and tau Yukawa is also large.

Because we have not observed any superpartners yet, SUSY must be broken. In order to keep the quantum correction smaller than TeV scale, SUSY should be broken softly, that is, only the interactions whose coefficients have positive mass dimensions are allowed, and the mass scale is around the weak scale ($\lesssim \mathcal{O}(1)\text{TeV}$). This assumption for solving the hierarchy problem leads to an amazing result. The gauge couplings meet with each other very accurately at a very high scale, the usual GUT scale $\Lambda_G \sim 2 \times 10^{16}\text{GeV}$, if we take the normalization of $U(1)_Y$ same as that of the SU(5) model, as shown in the Figure 2.2. This fact seems to imply the existence of SUSY-GUTs as a more fundamental theory.

### 2.3.1 SUSY-Flavor Problem

If we believe the success of GCU seriously, the SUSY breaking scale should not be far away from the weak scale. This means the superpartners will be discovered by TeV scale experiments. In addition, they can give considerable contributions to the low energy precise measurements through loop effects, especially to the processes that the SM has small contributions. Note that the FCNC processes are very suppressed in the SM as mentioned in §2.1. In fact, the FCNC processes have already given severe constraints to some soft parameters. For example, the off-diagonal elements of sfermion mass-squared matrices can make large contributions to FCNC processes through the diagrams shown in the Figure 2.3. The most severe constrains come from the $K^0-\bar{K}^0$ mixing and the $\mu \to e\gamma$ process. Also, the $D^0-\bar{D}^0$ mixing, $B^0-\bar{B}^0$ mixing, $b \to s\gamma$ and $\tau \to \mu\gamma$ processes give constraints. However they are much weaker than the former constraints, and thus we consider only the former constrains in this thesis for simplicity. Thus, the constrained

¹Because these two Higgs are vector-like, they could have a very large mass. Then, they would decouple from the low energy effective theory. Thus, the mass parameter $\mu$ has to be around the weak scale. But it looks unnatural that the SUSY parameter $\mu$ has the same scale as the SUSY breaking scale. This is called as the $\mu$ problem. A solution for this problem is discussed in Ref.20 in the context of anomalous $U(1)$ GUT.
Figure 2.2: The gauge coupling flows in the MSSM: Here we adopt, $\alpha_1^{-1}(M_Z) = 59.47$, $\alpha_2^{-1}(M_Z) = 29.81$, $\alpha_3^{-1}(M_Z) = 8.40$.

Figure 2.3: Diagrams that may induce FCNC processes: This figure is shown in the Figure 12 of Ref. [19].
parameters are \((1, 2)\) elements of sfermion mass-squared matrices in the mass basis where the fermion Yukawa matrices are diagonalized. They are defined by using the fermion mixing matrices \(V_{f\chi}\) as
\[
m^2_{f\chi}^{\text{diag.}} = V^\dagger_{f\chi} m^2_{f\chi} V_{f\chi}\tag{2.22}
\]
for each flavor \(f = U, D, E\) and chirality \(\chi = L, R\).

Roughly speaking, there are three ways to suppress the off-diagonal elements of \(m^2_{f\chi}^{\text{diag.}}\):

- **The degenerate solution:**
  If \(m^2_{f\chi} \propto 1_3\), where \(1_3\) is the \(3 \times 3\) identity matrix, then the off-diagonal elements are not induced even after \(V_{f\chi}\) are operated.

- **The alignment solution:**
  If \(m^2_{fL} \propto Y_f Y_f^\dagger\) and \(m^2_{fR} \propto Y_f^\dagger Y_f\), then \(m^2_{f\chi}\)'s are diagonalized by \(V_{f\chi}\)'s.

- **The decoupling solution:**
  The first and second generations which couple to Higgs very weakly may be heavy as \(O(10)\) TeV. Then, the contributions are suppressed by the heavy masses.

The second solution is not easy to realize, and the third one is not sufficient by itself. Thus, we consider the first solution.

The degeneracy can be realized if a flavor-blind mediation mechanism of the SUSY breaking, such as the gauge mediation, gaugino mediation and anomaly mediation, is realized. In this case, the SUSY-flavor problem is a problem of the SUSY breaking and/or mediation mechanism, which we do not treat so much in this thesis. There are another way to realize the degeneracy. It is to introduce non-abelian flavor symmetry (horizontal symmetry) and to embed the first and second generations into a single multiplet of the horizontal symmetry. Then, the horizontal symmetry ensures that they have a common mass in the symmetric limit because they belong to a common multiplet. In fact, we have not observed such a symmetry, and therefore the horizontal symmetry is broken, lifting the degeneracy. It is discussed whether sufficient degeneracy can be obtained or not in anomalous U(1) GUT scenario in \(\S 5.1\).

### 2.3.2 Nucleon Decay

In SUSY theories, there appear two kinds of nucleon decays: the nucleon decay via dimension 4 operators and that via dimension 5 operators. Of course, nucleon decay would occur via effective 4-Fermi operators whose mass dimension is 6. In non-SUSY theories, there are only fermionic matter fields and thus the effective operators are suppressed by the second power of a very large scale, such as the GUT scale and Planck scale. In contrast, there are also scalar partners and thus the large scale may be replaced by the SUSY breaking scale.
via Dim. 4 operators

Because both Higgs and matter fields are chiral superfields and $H_u$ has the same quantum number as $L$, $H_d$ and $L$ cannot be distinguished. This means the following baryon number violating Yukawa interaction is allowed by the symmetry:

$$W_{\Delta L=1} = \frac{1}{2} \lambda^i L_i L_i E^c_k + \lambda^{ij} L_i Q_j D^c_k + \mu L_i H_u.$$  \hfill (2.23)

$$W_{\Delta B=1} = \frac{1}{2} \lambda^{ijk} U^c_i D^c_j D^c_k.$$  \hfill (2.24)

Integrating out $D^c$ whose mass is around the SUSY breaking scale, we get effective 4-Fermi interactions which are suppressed by only the second power of the SUSY breaking scale, leading to destructively rapid nucleon decay. These dangerous interactions can be forbidden by introducing a $Z_2$ symmetry, called as $R$-parity.\(^2\) The parity assignment is that all the matter superfields have odd parities and the other superfields have even parities. Then, all the terms in the superpotential (2.23) and (2.24) are odd under $R$-parity and therefore are forbidden.

Note that $R$-parity can remain exact. Then the lightest superpartner (LSP) which is the lightest particle possessing odd $R$-parity defined in above footnote cannot decay and thus is stable. The LSP is a candidate for the cold dark matter, favored by the observations.

via Dim. 5 operators

The effective operators $QQQL$ and $U^c E^c U^c D^c$, induced by the colored Higgs exchange in §2.2 become dimension 5 operators when they appear in superpotential, namely two of the fields are scalars and the other two are fermions. Then the operators are suppressed by only first power of a large scale $M$, leading to rapid nucleon decay. Because the scalar matter fields have to be transformed into the fermionic matter fields through a superpartner exchange, the rate of nucleon decay depends strongly on the SUSY breaking parameters\(^2\). However, if we do not allow fine-tuning, the large scale ($\times$ coefficient $y$) should satisfy the relation $y/M < 1/\mathcal{O}(10^{26})\text{GeV}$\(^2\). In the case of the colored Higgs exchange, the coefficient $y$ is small due to the small Yukawa coupling, but we need typically $M > M_P \sim \mathcal{O}(10^{19})\text{GeV}$. Note that, even when the colored Higgs is absent, the physics of the Planck scale may induce the effective operators. And if the coefficient $y$ is $\mathcal{O}(1)$, nucleon decay would occur too rapidly. Thus, it looks natural that each field has a suppression factor as in the case where the Froggatt-Nielsen (FN) mechanism acts (See §2.5).

\(^2\)This $Z_2$ symmetry is not an $R$-symmetry in this sense. But we can assign the parities so that each element of a superfield does not have a common parity, by redefining $R$-parity as $Z_2(-1)^{2s}$ where $s$ is the spin. This additional factor has no physical meanings as far as we consider only Lorentz invariant interactions.
2.4 SUSY-GUT

Employing SUSY, the hierarchy problem can be solved also in GUTs, keeping the beautiful structures of GUTs, e.g. the unifications of forces and of matter fields. In addition, GCU is realized accurately as shown in Figure 2.2, supposing the colored Higgs has a mass around the GUT scale $\Lambda_G \sim 2 \times 10^{16}$GeV.

Unfortunately, such a colored Higgs mass is too light to suppress nucleon decay sufficiently. On the other hand, if the colored Higgs mass is sufficiently large $\sim M_P$ so that the nucleon decay via the colored Higgs exchange is suppressed, GCU is spoiled. Of course it is possible to restore it by introducing other parameters and adjusting them. For example, generally GUTs based on a symmetry that has a rank larger than 4, e.g. SO(10) or E6, have several symmetry breaking scales which can be used for restoring GCU. Alternatively, we can introduce additional multiplets whose mass spectrum does not respect the SU(5) symmetry. In such cases, however, GCU is not a prediction but a constraint of models, and one of the motivations to consider SUSY-GUTs is lost. This is one of the problems of SUSY-GUTs. This issue is discussed in §3.2.

2.4.1 Doublet-Triplet Splitting Problem

Another problem of SUSY-GUTs is the so-called DTS problem. As mentioned above, the doublet Higgs fields have to be light ($\sim O(100)$GeV) while the colored partners must be superheavy ($> \Lambda_G$). It is difficult to realize such a mass splitting within multiplets. Let us illustrate this by using the minimal SU(5) SUSY-GUT as an example. Here, we introduce a pair of $\mathbf{5}$ and $\bar{\mathbf{5}}$ Higgs which contain the MSSM doublet Higgs, $H_u$ and $H_d$, respectively:

\[
\begin{align*}
\bar{H}(\mathbf{5}) &= (\bar{H}_C, H_d), \\
H(\mathbf{5}) &= (H_C, H_u).
\end{align*}
\]

SU(5) is broken down to $G_{SM}$ by the following VEV of an adjoint Higgs $A(24)$:

\[
\langle A \rangle = \begin{pmatrix} 2v \mathbf{1}_3 & 0 \\ 0 & -3v \mathbf{1}_2 \end{pmatrix}, \quad v \sim O(10^{16})\text{GeV}.
\]

Then, the mass term of $H$ and $\bar{H}$ is given as

\[
W_{DT} = \bar{H}[m + A]H,
\]

where $m$ is a mass parameter. From this mass term, we find that the colored Higgs mass $m_C$ and the doublet Higgs mass $\mu$ are given as

\[
\begin{align*}
m_C &= m + 2v > 10^{16}\text{GeV}, \\
\mu &= m - 3v \sim 10^2\text{GeV}.
\end{align*}
\]

This required fine-tuning of at least $O(10^{-14})$ between the parameter $m$ and the VEV $v$. 

15
In the following, for simplicity, we aim to realize $\mu = 0$ as the first approximation, and assume that $\mu$ becomes $O(\Lambda_{SB})$ when we take the SUSY breaking into account as shown in Ref. [20]. This is one of the biggest problems of SUSY-GUTs. And many authors have proposed solutions for this problem. Here, we show some of them, although there are other possible solutions [23, 24].

**Dimopoulos-Wilczek mechanism**

The solution that we mainly employ in anomalous U(1) GUTs is the Dimopoulos-Wilczek (DW) mechanism [25]. This mechanism may be realized GUTs based on a symmetry that contains the U(1)$_{B-L}$ symmetry, such as SO(10) or E$_6$. Because the doublet Higgs fields have vanishing U(1)$_{B-L}$ charges while the colored Higgs fields have non-vanishing charges, the generator of U(1)$_{B-L}$ operates only on the colored Higgs. This means that if an adjoint Higgs $A$ acquires a non-vanishing VEV only in the direction, that is, the VEV is proportional to the generator, then the VEV contributes only to the colored Higgs mass. This is easily understood if we write an explicit form of the DW-VEV:

$$\langle A \rangle_{B-L} = \tau_2 \times \text{diag}(v, v, v, 0, 0), \quad (2.31)$$

in SO(10) models.

Note that the vector multiplets of SO(10) couple to the adjoint multiplet anti-symmetrically:

$$10 \times 10 = 1_s + 45_a + 54_s, \quad (2.32)$$

where the index “s” denotes that the coupling is symmetric and “a” denotes that the coupling is anti-symmetric. This means we need one more vector Higgs $H'(10)$ in addition to $H(10)$ that contains $H(5)$ and $\bar{H}(\bar{5})$ of the minimal SU(5) SUSY-GUT. The mass term $H'H'$ is required to give mass to the additional doublet Higgs of $H'$, while the mass terms $HH$ and $HH'$ must be forbidden because they contribute to the mass of the MSSM doublet Higgs:

$$W_{DT} = \bar{H}A H' + m H'H' (+ H'A H') \quad (2.33)$$

$$= (H(5), H'(5)) \left( \begin{array}{cc} 0 & \langle A \rangle \\ \langle A \rangle & m(\langle A \rangle) \end{array} \right) \left( \begin{array}{c} \bar{H}(\bar{5}) \\ \bar{H}'(\bar{5}) \end{array} \right). \quad (2.34)$$

Because $\langle A \rangle$ does not contribute to the doublet masses, we find that one pair of the doublets are indeed massless.

In this way, the DTS problem can be solved by the DW mechanism. However, it is difficult to realize the DW-VEV (2.31), and usually it needs fine-tuning.

**Sliding Singlet mechanism**

The sliding singlet mechanism [26] is the smartest solution that dynamically achieves DTS.

This mechanism was originally proposed in the context of SU(5) [26], in which an singlet field $Z(1)$ is introduced and the following terms are allowed in the superpotential:
\[ W_{SS} = \bar{H}(A + Z)H. \]  

Here, the adjoint Higgs \( A(24) \) is assumed to have the VEV (2.27). Since the doublet Higgs fields have non-vanishing VEVs \( \langle H_u \rangle \) and \( \langle H_d \rangle \) to break \( SU(2)_L \times U(1)_Y \) into \( U(1)_{EM} \), the minimization of the potential,

\[ V_{SUSY} = |F_H|^2 + |F_{\bar{H}}|^2 = \left( |\langle \bar{H} \rangle|^2 + |\langle H \rangle|^2 \right) \times |-3v + \langle Z \rangle|^2, \]

leads to the vanishing doublet Higgs mass \( \mu = (\langle A \rangle + \langle Z \rangle)^2 = -3v + \langle Z \rangle = 0 \) by sliding the VEV of \( Z \).

Unfortunately, this DTS is known to fail if the SUSY breaking is taken into account. For example, the soft SUSY breaking mass term \( \tilde{m}^2 |Z|^2 (\tilde{m} \sim \Lambda_{SB}) \) shifts the VEV \( \langle Z \rangle \) by an amount of \( \delta \langle Z \rangle \sim \tilde{m}^2 v \langle H \rangle^2 + \tilde{m}^2 \sim \Lambda_{G} \) to minimize the potential. Thus the DTS is spoiled by the SUSY breaking effect in this mechanism.

This is caused by the fact that the terms \( |F_H|^2 + |F_{\bar{H}}|^2 \) give only a mass of order \( \langle H \rangle \) to \( Z \), which is the same order as (or smaller than) the SUSY breaking contribution. Because this mass parameterizes the stability of \( \langle Z \rangle \) against other contributions to the potential, \( e.g. \) SUSY breaking effects \( \tilde{m}^2 |Z|^2 \), soft terms of order \( \Lambda_{SB} \) easily shift the VEV from that in the SUSY limit by a large amount. This can be avoided if large VEVs \( \langle H \rangle \) and \( \langle \bar{H} \rangle \) (larger than \( \sqrt{\Lambda_{SB} \Lambda_{G}} \)) which give a larger mass to \( Z \) are used, and the VEV of \( Z \) is stabilized against SUSY breaking effects. Of course, such large VEVs of the MSSM doublet Higgs are not consistent with the experiments. But large VEVs are acceptable for other Higgs fields that break a larger gauge group into \( G_{SM} \), and SU(6) models are examined in Ref. [27, 28].

We have extracted the essence of this mechanism to make this mechanism applicable in more generic cases by a group theoretical analysis [8]. The essential idea of this mechanism is as follows:

- A mass term of a certain component that has a non-vanishing VEV may vanish because of the EOM as in the previous SU(5) example.

- If a mass parameter of a certain component is guaranteed to be the same as that of the component with non-vanishing VEV, then the mass parameter also vanishes.

- If the non-vanishing VEVs are sufficiently large, the mass hierarchy is stable against possible SUSY breaking effects.

---

3 Here, the contributions to the potential from \( F_A \) and \( F_Z \) are neglected because they are of order \( (\langle HH \rangle)^2 \). In this sense, the doublet Higgs mass \( \mu \) does not vanish exactly but may become of order SUSY breaking scale \( \Lambda_{SB} \).

4 The soft term \( \tilde{m}ZF_Z \) also destabilizes the sliding singlet mechanism because this term alters the contribution of \( F_Z \) to the scalar potential as \( |\bar{H}H + \tilde{m}Z|^2 \), which is the order of \( \Lambda_{SB}^2 \Lambda_{G}^2 (\gg \Lambda_{SB}^2) \) if \( \langle H \rangle \) is order of the weak scale. Such a term is induced by loop effects through the coupling between \( Z \) and the color triplet Higgs. Therefore, even if the terms \( \tilde{m}ZF_Z \) and \( \tilde{m}^2 |Z|^2 \) are absent at the tree level, this problem cannot be avoided.
For instance in $E_6$ models, the $U(1)_{B-L}$ charges of the doublets contained in the SO(10) 10 component of 27 are the same as that of the SO(10) 1 component, namely they are zero. Thus, if the mass term of $\Phi(27)$ and $\Phi'(\overline{27})$ is a function of an adjoint Higgs $A(78)$ and a singlet Higgs $Z(1)$ as

$$W_{SS} = \bar{\Phi}'f(A,Z)\Phi,$$

(2.37)

$A$ acquires non-vanishing VEV in the direction of $U(1)_{B-L}$ generator (DW-VEV) and $1_\Phi$ acquires non-vanishing VEV, then the sliding singlet mechanism can work. Namely, EOM of $1_\Phi'$ makes $\langle Z \rangle$ slided so that $f(0,Z) = 0$, leading to the vanishing mass term of the doublets. In Ref. [8], a more detailed analysis has been made.

### Missing Partner mechanism

The missing partner mechanism [29] is also proposed in the context of SU(5). The idea is that, if we introduce additional Higgs possessing representations that contain a triplet but not doublets, it is possible to give mass only to the triplet Higgs. For instance, 50 contains a triplet but does not contain doublets, and 75 Higgs can connect the 50 Higgs to the 5 Higgs through the interaction $50 \cdot 75 \cdot 5$. We can show the situation schematically as

$$\begin{pmatrix} 3 \\ 2 \end{pmatrix}_5 \leftrightarrow \begin{pmatrix} 3 \\ \text{others} \end{pmatrix}_{50} \leftrightarrow \begin{pmatrix} 3 \\ \text{others} \end{pmatrix}_{\overline{50}} \leftrightarrow \begin{pmatrix} 3 \\ 2 \end{pmatrix}_5,$$

(2.38)

where the arrows represent that the pointed components have non-vanishing mass terms. Of course, we have to forbid the direct mass term $5 \cdot \overline{5}$ which gives a large mass to the doublet Higgs. In this way, relatively large representations, such as 50 and 75, are required to realize this mechanism in SU(5) models. These fields may make the unified gauge coupling divergent below the Planck scale.

On the other hand, this mechanism can be realized in a simpler way in flipped SU(5) models [30], where the gauge symmetry is not a simple group: SU(5)$_F \times U(1)_F$. In this model, the SM fields are embedded in a way the right-handed fields, i.e. SU(2)$_R$ doublets, are “flipped” as

$$10_1 = (Q, D^c, N^c),$$

(2.39)

$$\overline{5}_{-3} = (U^c, L),$$

(2.40)

$$1_5 = \overline{E^c}.$$

(2.41)

SU(5)$_F \times U(1)_F$ is broken down into $G_{SM}$ by the VEV of $C(10_1)$ (and $C(\overline{10}^{-1})$ for the $D$-flatness). It is interesting that in the flipped SU(5) models, adjoint Higgs fields are not required. As for the MSSM Higgs, $H_u$ and $H_d$ are “flipped”, while the colored partners, $H_C$ and $\overline{H}_C$ are not “flipped” as

$$\bar{H}(\overline{5}_{-2}) = (\bar{H}_C, H_u),$$

(2.42)

$$H(5_{-2}) = (H_C, H_d),$$

(2.43)

where $(\bar{H}_C, H_u, H_C, H_d)$ have the same quantum number of the SM gauge group as $(D^c, \overline{L}, \overline{D^c}, L)$, respectively. Note that 10$_1$ contains $D^c$ but does not contain $L$. In fact,
the superpotential
\[ W_{\text{MP}} = CCH + \bar{C} \bar{C} H, \]
gives masses proportional to \( \langle C \rangle \) to the triplet Higgs while the doublet Higgs fields remain massless. We have generalized the flipped model to an SO(10)\( _F \times U(1)_{F'} \) model\(^7\).

### 2.5 Anomalous U(1) Symmetry

Finally, we make comments on U(1)\(_A\). This is often introduced as a simple way to realize the FN mechanism\(^{31}\), which can explain the hierarchy in the Yukawa matrices.

It is known U(1)\(_A\) sometimes appears in the low energy effective theory of a string theory\(^2\). The anomaly of U(1)\(_A\) is cancelled via the GS mechanism\(^3\), for example a non-linear transformation of the dilaton multiplet
\[ S \rightarrow S + i \frac{\delta_{\text{GS}}}{2} \Lambda, \]
where \( \Lambda \) is a gauge transformation parameter, cancels the anomaly. In this sense, the anomalous U(1) symmetry is not anomalous in total.

To be more precise, the gauge kinetic terms in SUSY theories are written as
\[ \mathcal{L}_{\text{gauge}} = \frac{1}{4} \int d^2 \theta \left[ k_A S W^\alpha W_{\alpha A} + k_a S W^a W_{a A} \right] + \text{h.c.}, \]
by using the SUSY field strengths of U(1)\(_A\) and the gauge symmetry \( G_a, W_A, W_a \). Here, \( k_A \) and \( k_a \)'s are the Kac-Moody levels of U(1)\(_A\) and \( G_a \)'s, respectively. The gauge coupling constants \( g_a \) are given by the VEV of the Dilaton field \( S \) as \( k_a \langle S \rangle = 1/g_a^2 \).

The “anomalous” U(1) transformation induces a shift \( \Delta \mathcal{L} \propto \int d^2 \theta [i C_a W^\alpha W_{\alpha a}] \) for each \( G_a \), where \( C_a = \text{Tr}_{G_a} T(R)^2 Q_A \) is the mixed-anomaly. Because the shift induced by (2.45) is common for each \( G_a \) except for \( k_a \), the ratio \( C_a/k_a \) must be common for each \( G_a \) in order that the anomalies are canceled. Taking the [U(1)\(_A\)]\(^3\) and gravitational anomalies, we get the following GS relations\(^4\):
\[ 2\pi^2 \delta_{\text{GS}} = \frac{C_a}{k_a} = \frac{1}{3k_A} \text{tr} Q_A^3 = \frac{1}{24} \text{tr} Q_A, \]
where \( Q_A \) is U(1)\(_A\) charge.

It is known that the Fayet-Iliopoulos (FI) D term proportional to the anomaly \( \delta_{\text{GS}} \) is induced radiatively. Because the Kähler potential \( K \) for \( S \) must be a function of \( S + S^\dagger - \delta_{\text{GS}} V_A \) where \( V_A \) is the vector superfield of U(1)\(_A\), the FI D-term is given as
\[ \int d^4 \theta K(S + S^\dagger - \delta_{\text{GS}} V_A) = \left( -\frac{\delta_{\text{GS}} K'}{2} \right) D_A + \cdots \equiv \xi^2 D_A + \cdots, \]
where we fix the sign of \( Q_A \) so that \( \xi^2 > 0 \). If \( K(x) = -\ln(x) \) as calculated in Ref.\(^{31}\), \( \xi^2 \) is approximated as
\[ \xi^2 = \frac{g_s^2 \text{tr} Q_A}{192\pi^2}, \]
where $g_s^2 = 1/\langle S \rangle$.

The relation (2.47) is an important relation in models that employs $U(1)_A$. But they can be adjusted by introducing some singlet fields with appropriate charges. In particular in GUT models, $a$ runs only one index, and it looks easier to satisfy this relation. Thus, in the following, we do not take care of this relation, for simplicity. As for FI $D$-term, the charge assignments of the models discussed in this thesis give a large $\xi^2$ if we calculate it by the relation (2.49). Thus, we have to assume $K'$ is smaller than the simple case.$^5$ In any case, we simply assume $\xi^2$ is a desirable value in the following.

**Froggatt-Nielsen mechanism**

$U(1)_A$ is often used for realizing the FN mechanism which can give hierarchical factor in effective coefficients even if the original theory does not have such hierarchy.

Let us illustrate this mechanism by using the up-type Yukawa interactions $Q_i U^c_j H$ as an example. If an additional $U(1)$ symmetry is imposed and charges $(q_i, u^c_j, h)$ are assigned for $(Q_i, U^c_j, H)$, the Yukawa interactions are generally forbidden by the $U(1)$ invariance. Their charges may be compensated by a singlet field $\Theta$, called as the FN field, whose $U(1)$ charge is $-1$ as

$$L \ni y_{ij} \left( \frac{X}{\Lambda} \right)^{q_i + u^c_j + h} Q_i U^c_j H, \quad (2.50)$$

where $X = \begin{cases} \Theta & q_i + u^c_j + h > 0 \\ \Theta^\dagger & q_i + u^c_j + h < 0 \end{cases}$.

In this way, we get suppression factors $\lambda^{q_i + u^c_j + h}$ from the theory that has no hierarchy originally. If we assign, for instance, $h = -2n, (q_1, q_2, q_3) = (u^c_1, u^c_2, u^c_3) = (n + 3, n + 2, n)$, the up-type Yukawa matrix is given as

$$Y_U = \begin{pmatrix} \lambda^6 & \lambda^5 & \lambda^3 \\ \lambda^5 & \lambda^4 & \lambda^2 \\ \lambda^3 & \lambda^2 & 1 \end{pmatrix}, \quad (2.54)$$

where we omit the $O(1)$ coefficients $y_{ij}$. This gives tolerable mass spectrum and mixings when $\lambda \sim \sin \theta_C \sim 0.2$, although $m_u/m_c$ is rather large.$^5$

$^5$It may be possible to assume a tree level FI $D$-term that cancel with the loop contribution.
Note that the suppression factors are determined by U(1) charges, and thus we can consider that each field carries its own suppression factor. This leads to the factorizable form of $Y_U$ if we forget the $\mathcal{O}(1)$ coefficients $y_{ij}$:

$$
Y_U \sim (\lambda^3, \lambda^2, 1) \times \begin{pmatrix} 
\lambda^3 \\
\lambda^2 \\
1
\end{pmatrix}.
$$

(2.55)

And this helps to suppress the dangerous nucleon decay via dimension 5 operators induced through the physics around $M_P$ in SUSY theories, as discussed in §2.3.2.

**SUSY-zero mechanism**

In SUSY theories, $\Theta^\dagger$ cannot appear in the superpotential $W$, due to the holomorphy of $W$. This means that negative charges in $W$ cannot compensated by $\Theta^\dagger$. Thus, negatively charged operators are forbidden by the U(1) invariance while positively charged operators can appear in the effective theory below $\langle \Theta \rangle$. This is the SUSY-zero mechanism. This mechanism constrain the form of the superpotential strongly. As shown below, anomalous U(1) GUT scenario makes full use of this mechanism.

**Problems**

As shown above, U(1)$_A$ can be used as a powerful tool for analyzing models, especially in SUSY theories. But it also has some issues. In order that interactions are allowed by U(1)$_A$, the U(1)$_A$ charges must be quantized, but we have no reason the charges are quantized.$^6$

Another problem is related with the SUSY-flavor problem. Usually we assign different charges to different generations to reproduce generation dependent hierarchies. Then, sfermions have generation dependent masses if U(1)$_A$ $D$ term is not zero, and thus the $D$ term must be very small compared to a universal contribution. Thus we have to consider a SUSY breaking mechanism and a mediation mechanism that does not induce a large U(1)$_A$ $D$ term.

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$^6$This may give a hint for a more fundamental theory as the quantization of the hypercharges does.
Chapter 3

Anomalous U(1) GUT

In this chapter, we show some amazing features of the anomalous U(1) GUT scenario.

3.1 Starting Point

One of the most basic assumptions of this scenario is that we introduce “generic interaction”. Here “generic interaction” implies that we introduce all possible interaction terms that respect the symmetry of the model, and that their coupling constants are $O(1)$ in the unit of the cutoff scale $\Lambda$ of the model.\(^1\) This means that infinite number of interaction terms are introduced. Thus, it is indeed difficult to give predictions on many of precision measurements. However, we can still predict the order of magnitude of the parameters related to the low energy physics at the level of order magnitude. Another consequence of the assumption is that the definition of a model is given, except for a few parameters, by the definition of a symmetry: a symmetry group, matter content and representations of the matter fields under the symmetry. This means that, as far as the order-of-magnitude arguments are concerned, the parameters of the models are essentially the anomalous U(1) charges, whose number is the same as that of the superfields.\(^2\)

Another important assumption is made for the vacuum structure of the model:

$$\langle O_i \rangle \sim \begin{cases} 
\lambda^{-o_i} & \text{for } o_i \leq 0 \\
0 & \text{for } o_i > 0 
\end{cases}.$$ \hspace{3cm} (3.1)

Here, we denote GUT singlet operators (“G-singlets”) as $O_i$’s and their anomalous U(1) charge as $o_i$’s,\(^3\) and $\lambda(\ll 1)$ is the VEV of the FN field. The validity of this assumption is discussed in \[3.1.1\]. In such vacua, the SUSY-zero mechanism acts, and the number of

\(^1\)Hereafter, we often use this unit without notice.

\(^2\)If we wish to make a precise analysis, we should fix a large number of parameters (more than those in the SM). Thus, we concentrate ourselves on discussions of order of magnitude in the following. Note that, however, it is still non-trivial whether it is possible or not to reproduce the correct values of the parameters in the SM, because we assume they are all $O(1)$.

\(^3\)Throughout this thesis, we denote all the superfields and chiral operators by uppercase letters and their anomalous U(1) charges by the corresponding lowercase letters.
the relevant interaction terms is reduced, so that we can make analysis of models in spite of the infinite number of interactions. In addition, if the symmetry contains the SU(5) symmetry of Georgi-Glashow and the MSSM is realized below a certain energy, the gauge coupling unification (GCU) of the minimal SU(5) SUSY-GUT is naturally explained as shown in §3.2. In this way, this assumption plays a crucial role in this scenario.

3.1.1 Vacuum Structure

Singlet fields

At first, let us consider the simplest case, where the gauge symmetry is only the anomalous U(1) symmetry and there are no fractional charges. We denote the fields with positive charges as \( Z^+_i \)'s \((i = 1, \cdots, n^+) \), and the fields with negative ones as \( Z^-_i \)'s \((i = 1, \cdots, n^-) \).

Supposing \( \langle Z^+_i \rangle = 0 \) for all \( i \) as in Eq. (3.1), each \( F \)-flatness condition with respect to \( Z^-_i \) is automatically satisfied, because the terms in the \( F \)-flatness condition contain at least one of \( Z^+_j \)'s to compensate the negative charge, \( z^-_i \). In addition, terms that contain more than two of \( Z^+_i \)'s never contribute to the \( F \)-flatness conditions, so that we can analyze the vacuum structure only by considering terms that contain one of \( Z^+_i \)'s. It is worthwhile to note that because the number of such terms is finite, we can make an analysis in spite of the infinite number of terms. Now, we can write the superpotential that would give non-trivial \( F \)-flatness conditions as

\[
W_1 = \sum_i W_{Z^+_i},
\]

(3.2)

where \( W_{Z^+_i} \) consists of all the terms that contain one \( Z^+_i \) and no other positively charged fields. The non-trivial \( F \)-flatness conditions and \( D \)-flatness condition are given by

\[
F_{Z^+_i} = \frac{\partial W_{Z^+_i}}{\partial Z^+_i} = 0, \quad D_A = g_A \left( \sum_i z^-_i |Z^-_i|^2 + \xi^2 \right) = 0,
\]

(3.3)

where \( g_A \) is the gauge coupling constant of the anomalous U(1) symmetry. Among the \( F \)-flatness conditions, one of them is written by others, because the anomalous U(1) invariance makes \( \sum_i z^+_i Z^+_i F_{Z^+_i} = 0 \) hold. Although the \( D \)-flatness condition is a real condition while the \( F \)-flatness conditions are complex ones, the Higgs mechanism eats one real degree of freedom,\(^4\) and we can see the \( D \)-flatness condition also gives one complex condition. Thus, the number of the independent conditions is \( n^+ \). On the other hand, the number of degrees of freedom is \( n^- \). Hence, we can expect that when \( n^- \geq n^+ \), all the conditions can be satisfied. This means the vacua with \( \langle Z^+_i \rangle = 0 \) can be one of the SUSY preserving vacua. When \( n^- > n^+ \), there would appear some flat directions. When \( n^- = n^+ \), there would be no flat directions and all the fields would have superheavy masses.

\(^4\)Here, we ignore the degree of freedom of the dilaton multiplet. If we take account of the dilaton multiplet, there would be a superlight axion.
Of course, the other vacua with \( \langle Z_i^+ \rangle \neq 0 \) also exist. In such vacua, however, the SUSY-zero mechanism and the FN mechanism do not act, and we cannot know there is an anomalous U(1) symmetry at high energy. Thus, we assume here that the vacua with \( \langle Z_i^+ \rangle = 0 \) are selected so that we can examine the implication of the anomalous U(1) symmetry to low energy physics.

As for the magnitudes of the VEV of \( Z_i^- \)'s, they have to be smaller than the coefficient of the Fayet-Iliopoulos term due to the \( D \)-flatness condition. If the coefficient is small, as in string theories where the term is induced radiatively, \( \langle Z_i^- \rangle \)'s are also small. In this case, if we assume the relation in Eq.(3.1), the VEVs become larger as the (negative) charges become larger. Thus, \( \xi^2 \) in Eq.(3.3) is mainly compensated by the VEV of the field with the largest negative charge. Let us call the field as the FN field, \( \Theta \), and fix the normalization of the anomalous U(1) charge so that \( \theta = -1 \). Then, the VEV of the FN field is given as

\[
\langle \Theta \rangle = \lambda \sim \xi \ll 1. \quad (3.4)
\]

Below this scale, the \( F \)-flatness conditions are written as

\[
0 = F_{Z_i^+} = \lambda Z_i^+ \left( \sum_j \lambda z_i^- Z_j^- + \sum_j \sum_k \lambda z_i^+ z_k^- Z_j^- Z_k^- + \cdots \right), \quad (3.5)
\]

where we omit all the coefficients which are \( \mathcal{O}(1) \) due to the assumption of the generic interaction.\footnote{Hereafter, we often omit \( \mathcal{O}(1) \) coefficients without notice.} Defining \( \tilde{Z}_i^- \)'s by \( \tilde{Z}_i^- \equiv \lambda z_i^- Z_i^- \), the above conditions becomes

\[
0 = F_{\tilde{Z}_i^+} = \lambda \tilde{Z}_i^+ \left( \sum_j \tilde{Z}_j^- + \sum_j \sum_k \tilde{Z}_j^- \tilde{Z}_k^- + \cdots \right), \quad (3.6)
\]

which generally leads to solutions with \( \langle \tilde{Z}_i^- \rangle \sim 1 \) if these \( F \)-flatness conditions determine the VEVs. Thus, the \( F \)-flatness conditions require

\[
\langle Z_i^- \rangle \sim \lambda^{-z_i^-}. \quad (3.7)
\]

This relation is exactly the same one in Eq.(3.1). Thus, the assumption of the relation (3.1) is self-consistent, and therefore such vacua may be SUSY vacua. And we assume one of the vacua is selected as the vacuum of the model.

The above argument is for the simplest case. The argument should be changed slightly, when the Higgs sector has a structure by which the difference between the number of non-trivial \( F \)-flatness conditions and that of the degrees of freedom of non-vanishing VEVs is changed. Such a structure can be realized by imposing a certain symmetry, such as \( Z_2 \) parity, or by introducing rational number charges. For example, when the number of \( Z_i^+ \)'s with odd \( Z_2 \)-parity is different from that of \( Z_i^- \)'s with odd \( Z_2 \)-parity, the difference is changed by taking vanishing VEVs of the \( Z_2 \)-odd fields. In such cases, the number of the relevant fields should be considered. Then an essentially same argument can be applied.
Non-singlet fields

Next, let us consider more general cases where the symmetry contains a GUT symmetry in addition to the anomalous U(1) symmetry, and Higgs fields possessing non-trivial representations are introduced. Even in such cases, the same arguments can be applied if we use a set of independent $G$-singlets instead of the singlet fields $Z_i$’s. We can determine the VEVs of $G$-singlets $O_i$’s from the same superpotential as in Eq.(3.2), replacing $Z_i^+$ by a set of independent $O_i$’s with positive charges, although this is not easy. The calculation is simplified if all the fields $\Phi_i^+$’s (including non-singlets) with positive charges have vanishing VEVs. In such cases, the VEVs are determined only by the following part of the superpotential:

$$W = \sum_{i}^{n_+} W_{\Phi_i^+},$$  \hspace{1cm} (3.8)

where $W_{\Phi_i^+}$ consists of the terms that are linear in $\Phi_i^+$ and does not contain the other fields with positive charges. Note that, however, some of non-singlet fields with positive charge can have non-vanishing VEVs, while all the $G$-singlets with positive charge should have vanishing VEVs. For example, let us introduce a pair of fields possessing (anti-)complex representation $\mathbf{R}$, $\Phi(\mathbf{R})$ and $\bar{\Phi}(\bar{\mathbf{R}})$.$^6$ If we set $\phi = -3$ and $\bar{\phi} = 2$, then the $G$-singlet $\Phi\bar{\Phi}$ can have non-vanishing VEV, which means that $\Phi$ with positive charge $\phi = 2$ has a non-vanishing VEV. In such cases, it is not guaranteed that the $F$-flatness conditions of fields with negative charges are automatically satisfied. We have to take account of the part of the superpotential that includes positively charged fields with non-vanishing VEVs, e.g. $\bar{\Phi}$, in addition to those linear in fields with vanishing VEVs, in order to determine the VEVs. In both cases, the $G$-singlets $O_i$ with negative charges have non-vanishing VEVs, $\langle O_i \rangle \sim \lambda^{-\alpha_i}$, if the $F$-flatness conditions determine the VEVs. For example, the VEV of the $G$-singlet $\bar{\Phi}\Phi$ is given as $\langle \Phi\bar{\Phi} \rangle \sim \lambda^{-(\phi+\bar{\phi})}$.

An essential difference appears in the $D$-flatness condition of the GUT symmetry, which requires

$$|\langle \Phi \rangle| = |\langle \bar{\Phi} \rangle| \sim \lambda^{-(\phi+\bar{\phi})/2}.$$ \hspace{1cm} (3.9)

Note that these VEVs are also determined by the anomalous U(1) charges, but they are different from the naive expectation $\langle \Phi \rangle \sim \lambda^{-\phi}$. We can interpret the difference to be generated by the FN mechanism induced by a U(1) symmetry which is a subgroup of the symmetry group. More detailed analysis is made in §3.2.1.

Another important difference may appear in the $D$-flatness condition of the anomalous U(1) symmetry,

$$D_A = g_A \left( \xi^2 + \sum_i \phi_i |\Phi_i|^2 \right) = 0.$$ \hspace{1cm} (3.10)

When the $G$-singlet that has the largest negative charge is a composite operator, such as $\Phi\bar{\Phi}$ where $\phi + \bar{\phi} = -1$, the $D$-flatness condition is approximated as $\xi^2 + \phi |\Phi|^2 + \bar{\phi} |\bar{\Phi}|^2 \sim 0$.  

$^6$Hereafter, we denote a field $\Phi$ possessing a representation $\mathbf{R}$ under the symmetry of the model as $\Phi(\mathbf{R})$.
The $D$-flatness condition of the GUT symmetry leads to $|\Phi| = |\bar{\Phi}|$. This means $|\Phi| = |\bar{\Phi}| = \xi$. In this case, because $\hat{\Phi}\Phi$ plays the role of the FN field, the unit of the hierarchy becomes $\langle\hat{\Phi}\Phi\rangle = \lambda \sim \xi^2$. This relation is different from the previous one in Eq. (3.4) and implies that even if $\xi$ is not so small, the unit of the hierarchy is strong.

In summary, we have the following:

- We assume $G$-singlets with positive total charge have vanishing VEVs so that the FN mechanism and the SUSY-zero mechanism act effectively.
- The $F$-flatness conditions of $G$-singlets with positive charges determine the VEVs of $G$-singlets $O_i$’s with negative charges $o_i$’s as $\langle O \rangle \sim \lambda^{-\alpha}$, while the $F$-flatness conditions of $O_i$’s with negative charges are automatically satisfied.
- The part of the superpotential that determines the VEVs is expressed as $W = \sum_i W_{O_i^+}$, where $W_{O_i^+}$ is linear in $O_i^+$ that has positive charges, and does not contain any other fields with positive charges. When all the fields $\Phi_i^+$’s (including non-singlets) with positive charges have vanishing VEVs, the part can be written as in Eq. (3.8). If some of $\Phi_i^+$’s have non-vanishing VEVs, however, the part of the superpotential $W_{NV}$ that includes only the fields with non-vanishing VEVs must be taken into account.
- When the operator is a composite operator, e.g. $\hat{\Phi}\Phi$, a $D$-flatness condition of the GUT symmetry requires $|\langle \Phi \rangle| = |\langle \bar{\Phi} \rangle| \sim \lambda^{-(\phi + \bar{\phi})/2}$.
- The $G$-singlet with the largest negative charge plays the role of the FN field, $\Theta$. When the $G$-singlet is just a singlet field, the VEV is given as $\langle \Theta \rangle \sim \xi$, which is determined from $D_A = 0$. When the $G$-singlet is a composite operator, e.g. $\Theta \sim \hat{\Phi}\Phi$, the VEV is given by $\langle \Theta \rangle \sim \xi^2$.
- If the number of the independent $G$-singlets with negative charges equals that of the independent $G$-singlets positive charges, generically no massless fields appear.

### 3.2 Gauge Coupling Unification

We have shown that the success of the gauge coupling unification (GCU) in the minimal SU(5) SUSY-GUT is naturally explained in the anomalous U(1) GUT scenario in Ref.[11]. In this section, we show the argument.

As mentioned in §2.3, the hierarchical three gauge couplings meet with each others at the usual GUT scale $\Lambda_G$ in the MSSM, if the suitable SUSY breaking scale $\Lambda_{SB}$ is assumed. This is a very significant result and it is sometimes regarded as an evidence supporting the validity of the existence of SUSY-GUT. Unfortunately, however, the nucleon decay via the colored Higgs exchange is predicted to occur rapidly enough to be observed in present experiments, while it has not been observed yet[17]. In many GUTs, suppression of this nucleon decay is incompatible with the success of GCU[32, 33, 34]. It may be
possible to realize both the suppression and GCU by adjusting parameters by hand, but in such models, GCU is not a prediction but a constraint on the models. It is, however, desirable to construct a model where such adjustments emerge in a natural manner. A few models that realize such adjustments have been proposed\cite{24, 35}. In these models, the MSSM is realized as the effective theory below the unification scale \( \Lambda_U \) that is defined as the scale where the three gauge coupling constants meet with each other. In contrast to these models, in anomalous U(1) GUTs, the MSSM is realized not around \( \Lambda_U \) but below a scale much smaller than \( \Lambda_U \), and the mass spectrum of superheavy fields does not respect the SU(5) symmetry. In addition, the unified gauge group \( G \) may have a higher rank than SU(5) and there are several gauge symmetry breaking scales. Nevertheless, there appear no adjustable parameters that affect the condition for GCU, except for one parameter. This parameter corresponds to the mass of the color Higgs in the minimal SU(5) SUSY-GUT. We can show GCU occurs if this parameter takes an appropriate value which corresponds to the usual GUT scale mass of the colored Higgs. In this sense, we can say that the success of GCU in the minimal SU(5) SUSY-GUT is completely reproduced in the anomalous U(1) GUT scenario.

We introduce a useful concept of “effective charge” in §3.2.1. This concept makes very clear the discussion of GCU in the anomalous U(1) GUT scenario and the following analyses. Then we examine GCU at 1-loop level. Numerical analyses at 2-loop level are shown in Ref.\cite{10}.

### 3.2.1 Effective Charge

In many models where the FN mechanism acts, magnitudes of coefficients of interactions are determined by the simple sum of the charges of the relevant fields. For example, the coefficient of the Yukawa interaction \( HQU \) is given as \( \lambda^{|h|+|q|+|u|} \).

This feature is common for anomalous U(1) GUTs as far as VEVs of \( G \)-singlets are given by Eq.\eqref{3.1}. Let us consider an effective interaction \( X_1 X_2 \cdots X_N \). This interaction term may have contributions from several interaction terms such as \( X_1 X_2 \cdots X_N Z_1 Z_2 \cdots Z_N \) where \( G \)-singlets \( Z_i \) acquire non-vanishing VEVs. An important point is that orders of magnitude of such contributions are common, \( \lambda^{x_1+x_2+\cdots+x_N} \). This is because \( X_1 X_2 \cdots X_N Z_1 Z_2 \cdots Z_N \) has a coefficient of order \( \lambda^{x_1+x_2+\cdots+x_N+z_1+z_2+\cdots+z_N} \), and the VEVs of \( Z_i \)'s cancel the extra factor \( \lambda^{z_1+z_2+\cdots+z_N} \) thanks to the VEV relation in Eq.\eqref{3.1}. In this way, all the effective interactions have coefficients determined by the simple sum of the charges of the relevant fields, as far as VEVs of \( G \)-singlets are related.

Note that such a feature is generally broken when VEVs of non-singlet operators are related. As shown in §3.1.1, non-singlet fields may acquire VEVs different from the naively expected values in the same way as \( G \)-singlets. For instance, let us consider an SO(10) model. A pair of (anti-)spinor Higgs, \( C(\mathbf{16}) \) and \( \bar{C}(\mathbf{\overline{16}}) \) acquire non-vanishing VEVs when \( c + \bar{c} \leq 0: |\langle C \rangle| = |\langle \bar{C} \rangle| \sim \lambda^{-\frac{1}{2}(c+\bar{c})} \). To be more precise, \( \langle C \rangle \) is directed in the \( 1_5 \) component of the decomposition

\[
\mathbf{16} \rightarrow \mathbf{10}_1 + \mathbf{5}_3 + \mathbf{1}_5 \quad (3.11)
\]
in terms of SU(5)_{GG} \times U(1)_V (\subset SO(10)), and \langle C \rangle is directed in the conjugate component 1_{-5}. They are generally different from the expected values \lambda^{-c} and \lambda^{-\bar{c}} for G-singlets. Note that they are also written by anomalous U(1) charges, but they are not written only by their own charge. Let us examine the effect of VEVs of such non-singlet fields on effective interactions. The effective mass term between the SU(5) 5 component of a field T(10) and the SU(5) \bar{5} component of a field \Psi(16) is given by the interaction T\Psi C. Substituting the VEV of C, we find that the effective mass is written as

\lambda^{\bar{c}} \langle C \rangle \sim \lambda^{\psi + t + \frac{3}{2}(c-\bar{c})},

(3.12)

which is not written by the simple sum of the charges of the relevant fields. The discrepancy \Delta c \equiv \frac{1}{2}(c-\bar{c}) appears through the VEV of non-singlet field C, especially through that in a direction with non-vanishing charge of the additional U(1)_V. And the magnitude of the discrepancy is proportional to the U(1)_V charge. This is led from the assumption that all the G-singlets, which have vanishing U(1)_V charge, acquire VEVs given by Eq.(3.1). In fact, the magnitudes of the discrepancies in the VEVs of C and \bar{C} have opposite signs: \langle C \rangle \sim \lambda^{-c+\Delta c} and \langle \bar{C} \rangle \sim \lambda^{-\bar{c}-\Delta c}. This observation shows that the discrepancies are generated through the FN mechanism by U(1)_V, and thus discrepancies in coefficients of effective interactions can be written by the simple sum of the U(1)_V charges of the relevant fields. This means we can consider each field has characteristic discrepancy proportional to its U(1)_V charge, and we can define “effective charge” as \tilde{\phi} \equiv \phi + Q_\phi \Delta_V for a field \Phi, where Q_\phi is the U(1)_V charge of \Phi. Namely, a new hierarchical factor \lambda^{\Delta_V} is generated by the other FN mechanism for each U(1)_V charge. Then, we can determine coefficients of effective interactions by the simple sum of these “effective charges” of the relevant fields, even if VEVs of non-singlets are related. The coefficient \Delta_V is determined as \Delta_V = -\Delta c so that the similar relation to that in Eq.(3.11) holds for non-vanishing VEVs: \langle C \rangle \sim \lambda^{-c} and \langle \bar{C} \rangle \sim \lambda^{-\bar{c}}. Then, the effective charges of 5 of T and of \bar{5} of \Psi are given as \tilde{t} = t + \frac{2}{3} \Delta c and \tilde{\psi} = \psi + \frac{2}{3} \Delta c. Thus, we can see that the effective mass of this 5-\bar{5} pair, (3.12), is indeed given by the simple sum of \tilde{t} and \tilde{\psi} as \lambda^{\tilde{t} + \tilde{\psi}}.

The extension of the concept of effective charges to more general situation is straightforward. If there are several Higgs fields that break U(1)_V, the new hierarchical factor \lambda^{\Delta_V} is determined by the Higgs fields with the largest VEVs which dominate the D-flatness condition of SO(10). In the case where the GUT symmetry G has larger rank than 4, such discrepancies appear through VEVs of non-singlet fields, especially through those in directions with non-vanishing charges of additional U(1)_k’s of G ≃ SU(5) × \prod_k U(1)_k. And the magnitude of the discrepancy in an effective operator is given by a linear combination of the set of the charges of the effective operator. Thus, we can define “effective charge” of \Phi with U(1)_k charge Q^k_\phi as

\tilde{\phi} \equiv \phi + \sum_k Q^k_\phi \Delta_k.

(3.13)

Namely, a new hierarchical factor, \lambda^{\Delta_k}, is generated by each U(1)_k, and its magnitude is determined by the Higgs fields with the largest VEVs that break U(1)_k so that the similar

\textsuperscript{7}Hereafter, we denote the effective charge of a field by using the tilded lowercase letter.
relation to that in Eq. (3.14),
\[
\langle \Phi \rangle \sim \lambda^{-\tilde{\phi}}
\] (3.14)
holds for non-vanishing VEVs. Then, we can determine coefficients of effective interactions by the simple sum of these “effective charges” of the relevant fields, even if VEVs of non-singlets are related. Note that the effective charges respect SU(5) symmetry, because all the U(1)_Y's respect this symmetry.\(^8\) These features of the effective charge play an essential role in the following discussion of GCU.

For example, the masses of superheavy fields \(X_i\) are easily evaluated as
\[
m_{\text{eff}} \lambda \tilde{x}_i + \tilde{x}_j,
\] (3.15)
unless the mass terms are forbidden by some mechanism, such as the SUSY-zero mechanism. Therefore, the determinants of the mass matrices \(M_I\) of superheavy fields, which appear in the expressions of the gauge coupling flows, are written as
\[
det M_I = \lambda^{\Sigma_i \tilde{x}^I_i},
\] (3.16)
where \(I\) is the index denoting the SM irreducible representations. Note that \(\det M\) can be calculated using the simple sum of the effective charges of the massive fields. The ratio of the determinants for each pair of the SM multiplets \(I\) and \(I'\) contained in a single multiplet of SU(5), \(\frac{\det M_I}{\det M_{I'}}\), appears in the relations for GCU. Because the effective charges respect SU(5) symmetry, the contributions of the SU(5) multiplet whose \(I\) and \(I'\) components are both massive cancel. Hence, only the effective charges of massive modes whose SU(5) partners are massless contribute. This can be reinterpreted as meaning that only the effective charges of the massless modes appear in the ratios, that is,
\[
\frac{\det M_I}{\det M_{I'}} = \frac{1/\lambda^{\Sigma_i \tilde{x}^I_i}}{1/\lambda^{\Sigma_i \tilde{x}^{I'}_i}},
\] (3.17)
where \(i\) runs over the massless modes.

### 3.2.2 Gauge Coupling Unification

Now, we carry out an analysis based on the renormalization group equations (RGEs) up to 1-loop level. Here, we consider the most general situation, in which the GUT symmetry \(G\) is successively broken into \(G_{\text{SM}}\) as
\[
G(\equiv H_0) \xrightarrow[\Lambda_1]{\lambda_1} H_1 \xrightarrow[\Lambda_2]{\lambda_2} \cdots \xrightarrow[\Lambda_N]{\lambda_N} G_{\text{SM}}(\equiv H_N).
\] (3.18)

\(^8\)We have flexibility to define the effective charge other than \(\langle \Phi \rangle \sim \lambda^{-\tilde{\phi}}\). Even if we introduce a new hierarchy for each unbroken U(1), e.g. the hypercharge U(1)_Y, such a hierarchy does not appear in the SM invariant interactions. By using this flexibility, we can define the effective charge in a more convenient manner.
Table 3.1: The correction to the renormalization coefficients $\Delta b_{ai}$ by each vector-like pair of chiral multiplet.

| $I$ | $Q + Q$ | $U^c + U^c$ | $E^c + E^c$ | $D^c + D^c$ | $L + L$ | $G$ | $W$ | $X + X$ |
|-----|---------|-------------|-------------|-------------|---------|-----|-----|--------|
| $\Delta b_{1I}$ | $\frac{1}{3}$ | $\frac{5}{3}$ | $\frac{5}{5}$ | $\frac{5}{5}$ | $\frac{5}{5}$ | $0$ | $0$ | $5$ |
| $\Delta b_{2I}$ | $3$ | $0$ | $0$ | $1$ | $0$ | $2$ | $3$ |
| $\Delta b_{3I}$ | $2$ | $1$ | $0$ | $1$ | $0$ | $3$ | $0$ | $2$ |

First, the conditions for GCU are given by

$$\alpha_3(\Lambda) = \alpha_2(\Lambda) = \frac{5}{3}\alpha_Y(\Lambda) \equiv \alpha_1(\Lambda),$$

and the gauge couplings at the cutoff scale $\Lambda$ are given by

$$\alpha^{-1}_a(\Lambda) = \alpha^{-1}_a(\Lambda_{SB}) + \frac{1}{2\pi} \left( b_a \ln \left( \frac{\Lambda_{SB}}{\Lambda} \right) + \sum_i \Delta b_{ai} \ln \left( \frac{m_i}{\Lambda} \right) + \sum_n \Delta a_n \ln \left( \frac{\Lambda_n}{\Lambda} \right) \right),$$

where $a = 1, 2, 3$, $\Lambda_{SB}$ is the SUSY breaking scale, $(b_1, b_2, b_3) = (33/5, 1, -3)$ are the renormalization group coefficients of the MSSM, $\Delta b_{ai}$'s are the corrections to the coefficients caused by the massive fields with masses $m_i$'s (see Table 3.2.2 for concrete values), and the last term is the correction due to the restoration of the gauge symmetry above each symmetry breaking scale $\Lambda_n$:

$$\Delta a_n = -3T_a \left[ H_{n-1}/H_n \right] + T_a \left[ NG_a \right] = -2T_a \left[ NG_n \right].$$

Here, NG$_n$ denotes the NG modes that are absorbed through the Higgs mechanism at the scale $\Lambda_n$, and $T_a$'s are the Dynkin indices of a representation $R$, defined as

$$\text{Tr}(T_AT_B) = T[R]\delta_{AB},$$

where $T_A$'s are the generators in $R$. The $n$-th NG modes, NG$_n$, reside in $H_{n-1}/H_n$, and thus the second equality in Eq. (3.21) is derived.

By using the fact that in the MSSM the three gauge couplings meet at the scale $\Lambda_G \sim 2 \times 10^{16}$ GeV, the relations expressing unification, $\alpha_a(\Lambda) = \alpha_b(\Lambda)$, become

$$(b_a - b_b) \ln(\Lambda_G) + \sum_I (\Delta b_{ai} - \Delta b_{bi}) \ln(\det M_I) + \sum_n (\Delta a_n - \Delta b_n) \ln(\Lambda_n) = 0,$$

where $I$ runs over the SM irreducible representations. Because the sum of $\Delta b_{ai}$'s over an SU(5) multiplet is independent of $a$, the second term in Eq. (3.23) can be written in
terms of the ratios of the determinants of the mass matrices in (3.17), and therefore in
terms of the contributions from the massless modes, as mentioned above. For example
for \[ \overline{5} \] representation, the second term is
\[
(\Delta b_{aL} - \Delta b_{bL}) \ln (\det M_L) + (\Delta b_{aD^c} - \Delta b_{bD^c}) \ln (\det M_{D^c}),
\]
(3.24)
and we know
\[
\Delta b_{aL} + \Delta b_{aD^c} = \Delta b_{bL} + \Delta b_{bD^c},
\]
(3.25)
thus (3.24) becomes
\[
(\Delta b_{aL} - \Delta b_{bL}) \ln \left( \frac{\det M_L}{\det M_{D^c}} \right),
\]
(3.26)
which is written by the ratio of the determinants. In terms of the “effective mass” of
massless modes, which is defined as \( m_{\text{eff}} \equiv \lambda \tilde{x} + \tilde{y} \) even when \( \tilde{x} + \tilde{y} < 0 \), the second term in
(3.28) can be written as
\[
\sum_{i=\text{massless}} (T_a [i] - T_b [i]) \ln \left( \frac{1}{m_{\text{eff}}^i} \right).
\]
These massless modes consist of two types, physical massless modes, such as the MSSM
doublet Higgs (\( H_u \) and \( H_d \)), and unphysical NG modes. From (3.21), we can see that
the contribution of the latter type is cancelled by that of the last term in Eq. (3.23) if the
conditions
\[
m_{\text{eff}}^{\text{NG}_n} \sim \Lambda_n^{-2}
\]
(3.27)
hold. These conditions are satisfied when the vacuum structure satisfies (3.1), because
\( m_{\text{eff}}^{\text{NG}_n} \) is the coefficient of the bilinear term of the \( n \)-th NG modes, \( \Phi \) and \( \tilde{\Phi} \) \( (\tilde{\phi} = \tilde{\phi}) \), and
therefore \( m_{\text{eff}}^{\text{NG}_n} \sim \lambda^{2\tilde{\phi}} \), and from (3.14), \( \Lambda_n \sim \lambda^{-\tilde{\phi}} \).

When (3.27) holds, only the physical massless modes contribute to the conditions for
GCU, and they are independent of the details of the Higgs sector, such as the field content
and the symmetry breaking pattern. In particular, if all the fields other than those in the
MSSM become superheavy, only the MSSM doublet Higgs fields \( H \) contribute, and we have
\[
(b_a - b_b) \ln(\Lambda_G) + (\Delta b_{aH} - \Delta b_{bH}) \ln \left( \frac{1}{m_{\text{eff}}^H} \right) = 0,
\]
(3.28)
for all combinations \((a, b)\). These relations lead to \( \ln(\Lambda_G) = \ln(m_{\text{eff}}^H) = 0 \), and thus
\[
\Lambda \sim \Lambda_G, \tilde{h}_u + \tilde{h}_d \sim 0.
\]
(3.29)
The first relation here simply defines the scale of the theory. The cutoff scale \( \Lambda \) is taken as
the usual GUT scale, \( \Lambda_G \). This is also the case in the minimal SU(5) SUSY-GUT, where
the scale at which SU(5) is broken is also taken as \( \Lambda_G \). The second relation in (3.29)
corresponds to that for the colored Higgs mass in the minimal SU(5) GUT, because the
effective colored Higgs mass is obtained as \( m_{\text{eff}}^{H^c} \sim \lambda \tilde{h}_u + \tilde{h}_d \). Therefore, we have no tuning
parameters for GCU other than those in the minimal SU(5) SUSY-GUT. Note that (when
\( \tilde{h}_u + \tilde{h}_d = 0 \)) if we calculate gauge couplings at a low energy scale in the anomalous U(1)
GUT scenario with any cutoff scale (for example, the Planck scale) and use them as the initial values, the three running gauge couplings calculated in the MSSM meet with each others at the cutoff scale. In this way, we can naturally explain GCU in the minimal SU(5) SUSY-GUT.

3.2.3 Nucleon Decay

As shown in the previous subsection, the success of GCU in the minimal SU(5) SUSY-GUT is reproduced in the anomalous U(1) GUT scenario. Then, it may seem that the same problem as in the minimal SU(5) SUSY-GUT arises in anomalous U(1) GUTs: the nucleon decay via dimension 5 operators tends to be too rapid. In fact the same problem arises if we take $\tilde{h}_u + \tilde{h}_d = 0$. This condition corresponds to the condition that the colored Higgs should have a mass around $\Lambda_G$, and looks required for GCU. Note that, however, the relation $\tilde{h}_u + \tilde{h}_d \sim 0$ does not imply $\tilde{h}_u + \tilde{h}_d = 0$, because there is an ambiguity involving $\mathcal{O}(1)$ coefficients. As mentioned in §3.2.1, contributions to masses from higher-dimensional interactions are not suppressed in contrast to the usual situation. For instance, the VEV of an adjoint Higgs $A$, $\langle A \rangle$, which breaks SU(5) symmetry, contributes to the mass of $X$ and $\bar{X}$ through higher-dimensional interactions, $\lambda x X + \bar{x} X + n a \bar{X} A X$. The orders of such contributions are the same as that from the mass term $\lambda x X + \bar{x} X$, because $\langle A \rangle \sim \lambda^{-a}$. Therefore the $\mathcal{O}(1)$ coefficients do not respect SU(5) symmetry at all. This allows a non-zero value of $\tilde{h}_u + \tilde{h}_d$. If $\tilde{h}_u + \tilde{h}_d$ is negative, the nucleon decay via dimension 5 operators is suppressed. The suppression requires the effective mass of the colored Higgs $m_{H^c}^{\text{eff}} \sim \lambda^{x+a} \bar{X} X$, and therefore $\tilde{h}_u + \tilde{h}_d \leq -3$ is needed. Note that the physical masses of the colored Higgs are smaller than $\Lambda$, although the effective mass is larger, $m_{H^c}^{\text{eff}} > \Lambda$, as shown in concrete models in the next chapter.

In this way, the nucleon decay via dimension 5 operators can be suppressed if we take $\tilde{h}_u + \tilde{h}_d \leq -3$. On the other hand, the nucleon decay mediated by gauge bosons is enhanced compared to the usual GUTs. This is because the cutoff scale $\Lambda$ is required to be around the usual GUT scale $\Lambda_G \sim 2 \times 10^{16}\text{GeV}$ by the condition for GCU (3.29), and the unification scale $\Lambda_U$ is smaller than $\Lambda$ and thus than $\Lambda_G$. In fact in anomalous U(1) GUTs, $\Lambda_U$ is given by the VEV of the Higgs $A$ that breaks SU(5) symmetry as $\langle A \rangle \sim \lambda^{-a} \Lambda \ll \Lambda \sim \Lambda_U$. If we choose $a = -1$ and $\lambda \sim 0.22$ as typical values, the proton lifetime can be roughly estimated, using a formula in Ref.[32] and a recent result provided by a lattice calculation for the hadron matrix element parameter $\alpha$, as

$$\tau_p (p \to e\pi^0) \sim 1 \times 10^{34} \left( \frac{\Lambda_A}{5 \times 10^{15} \text{GeV}} \right)^4 \left( \frac{0.01(\text{GeV})^3}{\alpha} \right)^2 \text{yrs}.$$  

This value is near the present experimental limit[17]. Thus, anomalous U(1) GUTs predict

\[9\text{Note that the slowest proton decay via dimension 6 operators is obtained when } a = 0, \text{ and the value must be the same as that in the usual GUT scenario. However, when } a = 0, \text{ generally terms of the form } \int d^2 \theta A^a W_\alpha W^\alpha \text{ are allowed, where } W_\alpha \text{ is a SUSY field strength. This makes it impossible to realize natural GCU. The most natural way to forbid these terms is to choose } a \text{ to be negative, which leads to a shorter proton lifetime.}\]
that the proton decay $p \rightarrow e + \pi$ will be observed in future experiments.

3.2.4 Summary

In this section, we have shown the success of GCU in the minimal SU(5) SUSY-GUT is completely reproduced in the anomalous U(1) GUT scenario. Usually, if we adopt a simple group whose rank is higher than that of the standard gauge group (for example, SO(10), $E_6$, SU(6), etc.), GCU can always be realized by tuning the additional degrees of freedom related with the several scales of Higgs VEVs. However, in the anomalous U(1) GUT scenario, all the charges of the Higgs fields, except that of the MSSM doublet Higgs, are cancelled in the relations for GCU (3.29), and therefore we have no tuning parameters for GCU other than those in the minimal SU(5) SUSY-GUT. This result is independent of detail of models. In fact the assumptions of the argument are following:

1. The unification group $G$ is simple.
2. The VEV relation (3.1) holds.
3. Below a certain scale, the MSSM is realized.

If these assumptions are realized, the argument can be applied even if the scenario does not employ anomalous U(1) symmetries$^{10}$ and/or there are some flat directions. The second assumption is naturally realized in the GUT scenario with anomalous $^{11}$ U(1) gauge symmetry as shown in §3.1.1. Moreover, some of the above conditions can be weakened. For example, even when the gauge group is non-simple, GCU is realized if the charge assignment respects SU(5) symmetry.

Finally, we would like to make comments on the magnitudes of the gauge coupling constants, which we have not taken care of in this section. If there appear many Higgs below $\Lambda_G$, the gauge couplings tend to become large, and the analysis based on perturbation made in this section may not be applicable. Thus, we have to be careful that the gauge coupling at cutoff scale is in the perturbative region when we construct models.

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$^{10}$In this case, $\alpha_i$’s are not charges, but merely certain numbers that we assign for fields.

$^{11}$We can use non-anomalous U(1) symmetry instead of anomalous U(1) symmetry if 3 conditions in the Introduction are satisfied. However, since we don’t know such models, we adopt anomalous U(1) symmetry in this letter.
Chapter 4

Models

4.1 SO(10) Models

In this section, we introduce the SO(10) models discussed in Ref.[4].

4.1.1 Realization of DW VEV

Before examining concrete models, let us show how the Dimopoulos-Wilczek (DW) type of VEV of an adjoint Higgs $A$, $\langle A \rangle \propto \tau_2 \times \text{diag.}(1,1,1,0,0)$, which is proportional to the generator $B-L$, can be realized in anomalous U(1) GUTs. As mentioned in §2.4.1, usually the realization needs fine-tuning, but it is possible to realize the DW VEV as the result of an equation of motion. For example, this turns out to be the case when we introduce an additional adjoint Higgs $A'$ and assign its anomalous U(1) charge so that $(0 <) -3a \leq a' < -4a$. As mentioned in §3.1.1, the superpotential that is relevant for determining this VEV is the part that is linear in the positively charged Higgs field. Thus, the relevant part in this case is in general written as

$$W_{A'} = \lambda^{a'+a} A'A + \lambda^{a'+3a} ((A'A)_1 (A^2)_1 + (A'A)_{54} (A^2)_{54}),$$

where the suffixes 1 and 54 indicate the representation of the composite operators under the SO(10) gauge symmetry, and we omit $O(1)$ coefficients. We can choose a gauge where the VEV of $A$ is written as $\langle A \rangle = \tau_2 \times \text{diag.}(x_1, x_2, x_3, x_4, x_5)$. In this gauge, the $F$-flatness condition of the $A'$ field requires $x_i (\alpha \lambda^{-2a} + \beta (\sum_j x_j^2) + \gamma x_i^2) = 0$, where $\alpha$, $\beta$ and $\gamma$ are $O(1)$ parameters. Here, the last term comes from the interaction $(A'A)_{54} (A^2)_{54}$. This EOM gives only two solutions $x_i^2 = 0$ and $x_i^2 = -\alpha \gamma^{-N} \lambda^{-2a}$, where $N$ is the number of $x_i \neq 0$ solutions and $N = 0,1,\ldots,5$. The DW VEV is obtained when $N = 3$. Note that the higher-dimensional terms $A'A^{2L+1}$ ($L > 1$) are forbidden by the SUSY-zero mechanism, and it is difficult to forbid them by other symmetries, such as discrete symmetries, because $A^2$ should be a singlet under such symmetries in order to allow both $A'A$ and $A'A^3$. If such terms are allowed, the number of possible VEVs other than the DW VEV becomes larger, and thus it becomes less natural to obtain the DW VEV. This is a crucial point.
in the anomalous U(1) GUT scenario, and the anomalous U(1) gauge symmetry plays an essential role in forbidding the undesirable terms.

In this manner, we can realize the DW VEV without fine-tuning. In order to make sure that the DTS problem is indeed solved, we have to examine the whole Higgs sector. In particular, the rank reducing VEV, e.g. VEVs of spinor Higgs $C$ and $\bar{C}$, should not couple to $W_A'$ while these VEVs and the VEV of $A$ should couple with each others in order to avoid the pseudo NG (PNG) modes. As shown in the next subsection, the decoupling can be realized by the SUSY-zero mechanism, and the avoidance of PNG can be achieved through the Barr-Raby mechanism[36].

### Table 4.1: Typical values of anomalous U(1) charges.

|   | non-vanishing VEV | vanishing VEV |
|---|------------------|---------------|
| 45 | $A(a = -1, -)$   | $A'(a' = 3, -)$ |
| 16 | $C(c = -3, +)$   | $C'(c' = 2, -)$ |
| 16 | $\bar{C}(\bar{c} = 0, +)$ | $\bar{C}'(\bar{c}' = 5, -)$ |
| 10 | $H(h = -3, +)$   | $H'(h' = 4, -)$ |
| 1  | $\Theta(\theta = -1, +), Z(z = -2, -), \bar{Z}(\bar{z} = -2, -)$ | $Z'(s = 3, +)$ |

#### 4.1.2 Higgs Sector of SO(10) Models

In order to break SO(10) down to $G_{SM}$, we need at least an adjoint Higgs $A(45)$ and one pair of spinor Higgs $C(16)$ and $\bar{C}(\overline{16})$. The gauge singlet operators $A^2$ and $\bar{C}C$ must have negative total anomalous U(1) charges to obtain non-vanishing VEVs, as discussed in §3.1.1. Then, they cannot have interaction terms, especially mass terms, by themselves. Thus, we have to introduce corresponding conjugate fields possessing positive charges $A'(45)$, $\bar{C}'(\overline{16})$ and $C'(16)$ in order to give masses to all the Higgs fields.\(^1\) When we employ the DW mechanism to solve the DTS problem, we need two vector Higgs $H(10)$ and $H'(10)$, one of which has a vanishing self mass term, i.e. $H^2$ is forbidden. This mass term is forbidden by the SUSY-zero mechanism if $h < 0$. Then, $H'$ must have positive charge to allow the interaction $HAH'$.\(^2\) This is, in a sense, a minimal set of (non-singlet) Higgs content, and this minimal content is enough to construct realistic models.

An example of charge assignments of the content of the Higgs sector including singlet Higgs is shown in Table 4.1.

---

\(^1\)Strictly speaking, since some of the Higgs fields are eaten by the Higgs mechanism, in principle, a smaller number of positive fields can give superheavy masses to all the Higgs fields. Here we do not examine this possibility.

\(^2\)In this case, the mass term $HH'$ cannot be forbidden by the SUSY-zero mechanism and we have to introduce an additional symmetry, for example $Z_2$ symmetry.
VEV determination

As mentioned in §3.1.1, the superpotential required by determination of the VEVs can be written as

\[ W = W_{A'} + W_{Z'} + W_{C'} + W_{\bar{C}'} + W_{H'} + W_{NV}. \] (4.2)

Here, \( W_X \) denotes the terms linear in the positive charged field \( X \), which has vanishing VEV. And \( W_{NV} \) contains terms consisting of only unprimed fields, i.e. fields possessing non-vanishing VEVs. They are given as

\[
\begin{align*}
W_{A'} &= \lambda^{a'+a}A'A + \lambda^{a'+3a}((A'A)_{1}(A^2)_{1} + (A'A)_{54}(A^2)_{54}) \\
W_{Z'} &= \lambda^{c'+c}Z'((\bar{C}C) + \lambda^{-(c+\ell)} + \lambda^{-(c+\ell)+2a}A^2) \\
W_{C'} &= \bar{C}((\lambda^{c'+a}A + \lambda^{c'+z}Z)C') \\
W_{\bar{C}'} &= \bar{C}'((\lambda^{c'+a}A + \lambda^{c'+z}Z)C) \\
W_{H'} &= \lambda^{b+a+k'H'AH},
\end{align*}
\] (4.3) (4.4) (4.5) (4.6) (4.7)

for the charge assignment of Table 4.1. Note that terms including two fields with vanishing VEVs, because they do not include the products of components that are exclusively singlets under \( G_{SM} \) and thus we can safely ignore them.

\( W_{A'} \) is the same as that in Eq. (4.4), and the argument in §3.1.4 can be applied. Note that the rank reducing Higgs \( C \) and \( \bar{C} \) do not appear in \( W_{A'} \) thanks to the SUSY-zero mechanism. Thus, we can realize the DW VEV without fine-tuning. The VEV \( \langle A(45) \rangle_{B-L} = \tau_3 \times \text{diag}(\nu, \nu, v, 0, 0) \), breaks \( SU(10) \) into \( SU(3)_{C} \times SU(2)_{L} \times SU(2)_{R} \times U(1)_{B-L} \).

The \( F \)-flatness condition of \( Z' \) requires \( \langle \bar{C}C \rangle \sim \lambda^{-(c+\ell)} \). The magnitude of \( \langle C \rangle \) and \( \langle \bar{C} \rangle \) are determined by the \( D \)-flatness condition of \( SO(10) \) as \( |\langle C \rangle| = |\langle \bar{C} \rangle| \sim \lambda^{-(c+\ell)/2} \), as in §3.1.1.

Next, we discuss the \( F \)-flatness conditions of \( C' \) and \( \bar{C}' \), which realize the alignment of the VEVs \( \langle C \rangle \) and \( \langle \bar{C} \rangle \) and results in masses for the PNG fields. This simple mechanism was proposed by Barr and Raby [36]. The \( F \)-flatness conditions \( F_{C'} = F_{\bar{C}'} = 0 \) give \( (\lambda^{a-z}A + Z)C = \bar{C}(\lambda^{a-z}A + \bar{Z}) = 0 \). Recall that the VEV of \( A \) is proportional to the \( B-L \) generator \( Q_{B-L} \) (precisely, \( \langle A \rangle = \frac{3}{2}vQ_{B-L} \)), and that the spinor representation \( 16 \) is decomposed into \( (3, 2, 1)_{1/3}, (\bar{3}, 1, 2)_{-1/3}, (1, 2, 1)_{-1} \) and \( (1, 1, 2)_{1} \) under \( SU(3)_{C} \times SU(2)_{L} \times SU(2)_{R} \times U(1)_{B-L} \). Since \( \langle \bar{C}C \rangle \neq 0 \), \( Z \) is fixed such that \( Z \sim -\frac{3}{2}\lambda vQ_{B-L}^{0} \), where \( Q_{B-L}^{0} \) is the \( B-L \) charge of the component of \( C \) that has non-vanishing VEV. Once the VEV of \( Z \) is determined, no other component fields can have non-vanishing VEVs, because they have different charges \( Q_{B-L} \). If the component that obtains a non-zero VEV is \( (1, 1, 2)_{1} \) (and therefore \( Z \sim -\frac{3}{2}\lambda v \)), the gauge group \( SU(3)_{C} \times SU(2)_{L} \times SU(2)_{R} \times U(1)_{B-L} \) is broken down to the SM gauge group. Once the
direction of the VEV $\langle C \rangle$ is determined, the VEV $\langle \bar{C} \rangle$ must be directed in the same direction, because of the $D$-flatness condition. Therefore, $\langle \tilde{Z} \rangle \sim -\frac{3}{2} \lambda v$.

Finally the $F$-flatness condition of $H'$ leads to vanishing VEVs of the color-triplet Higgs, $\langle H_T \rangle = 0$.

Now, all VEVs have been fixed as $\langle X^+ \rangle = 0$, (4.8) $\langle A \rangle = \tau_2 \times \text{diag}(v, v, v, 0, 0)$, $v = \lambda^{-a}$, (4.9) $\langle C \rangle = \langle N_c^c C \rangle = \lambda^{-c + \bar{c}}$, (4.10) $\langle \bar{C} \rangle = \langle N_c^c \bar{C} \rangle = \lambda^{-c + \bar{c}}$, (4.11) $\langle D, H \rangle = 0$, (4.12)

where $X^+$ denotes all the positively charged Higgs. The symmetry breaking pattern is given as

$$\text{SO}(10) \rightarrow \text{SU}(3)_C \times \text{SU}(2)_L \times \text{SU}(2)_R \times \text{U}(1)_{B-L} \quad \text{at} \quad \lambda^{-a} \Lambda$$

$$\rightarrow \text{SU}(3)_C \times \text{SU}(2)_L \times \text{U}(1)_Y \quad \text{at} \quad \lambda^{-\frac{1}{2}(c + \bar{c})} \Lambda.$$  

The parameter that parametrizes the effective charges is written as $\Delta c = \frac{1}{2}(c - \bar{c})$.

There are several terms that must be forbidden for the stability of the DW mechanism. For example, $H^2$, $HZH'$ and $HZH'$ induce a large mass of the doublet Higgs, and the term $\bar{C}A'AC$ would destabilize the DW VEV of $\langle A \rangle$. We can easily forbid these terms using the SUSY-zero mechanism. For example, if we choose $h < 0$, then $H^2$ is forbidden, and if we choose $\bar{c} + c + a + a' < 0$, then $\bar{C}A'AC$ is forbidden. Once these dangerous terms are forbidden by the SUSY-zero mechanism, higher-dimensional terms that could also become dangerous (for example, $\bar{C}A'A^3C$ and $\bar{C}A'C\bar{C}AC$) are automatically forbidden. This is another attractive property of the anomalous $U(1)$ GUT scenario. The dangerous terms which should be forbidden are

$$H^2, HH', HZH', \bar{C}A'C, \bar{C}A'AC, \bar{C}A'ZC, A'A^4, A'A^5,$$  (4.13)

and the terms required to realize DTS are

$$A'A, A'A^3, HAH', \bar{C}'(A + Z)C, \bar{C}(A + Z)C', S\bar{C}C.$$  (4.14)

Here we denote both $Z$ and $\tilde{Z}$ as “$Z$”. In order to forbid (4.13) but not (4.14), we introduce $Z_2$ parity and assign charges like as in Table 4.1.

Of course, the above conditions are necessary but not sufficient. To determine whether a given assignment actually works well, we have to examine the mass matrices of the Higgs sector.

**Mass spectrum of the Higgs sector**

Here, we examine the mass matrix of the Higgs sector in Table 4.1 for each representation of the SM to show that all the extra fields indeed acquire superheavy masses and the
MSSM is realized at low energy scale. For this purpose, we have to take into account not only terms detailed in the previous section but also terms that contain two fields with vanishing VEVs.

Under the decomposition \( \text{SO}(10) \supset \text{SU}(5) \supset \text{SU}(3)_C \times \text{SU}(2)_L \times \text{U}(1)_Y \), the spinor 16, vector 10 and adjoint 45 are decomposed in terms of the representations of the SM group as

\[
16 \rightarrow \left[ Q + U^c + E^c \right]_{10} + [D^c + L] + \left[ N^c \right]_{1},
\]

\[
10 \rightarrow \left[ D^c + L \right] + \left[ \bar{D}^c + \bar{L} \right],
\]

\[
45 \rightarrow \left[ G + W + X + \bar{X} + N^c \right] + \left[ Q + U^c + E^c \right] + \left[ \bar{Q} + \bar{U}^c + \bar{E}^c \right] + \left[ N^c \right].
\]

First, we examine the mass spectrum of 5 and 5 of SU(5). Considering the additional terms \( \lambda^{2h} H'H', \lambda^{c+\bar{c}} C'C, \lambda^{c+\bar{c}} \bar{C}' \bar{C}H', \lambda^{d+d+\bar{d}+\bar{d}} ZC' \bar{C}H, \lambda^{d+d+\bar{d}+\bar{d}} \bar{C}' \bar{C}H' \) and \( \lambda^{2d+h} \bar{C}^2 H' \), the mass matrices \( M_I \) \((I = D^c, L)\) are given by

\[
M_I = \begin{pmatrix}
\bar{I} & I & H & C & H' & C' \\
H & 0 & 0 & \lambda^{h+h+a} \langle A \rangle & 0 \\
C & 0 & 0 & \lambda^{h+2\bar{c}} \langle C \rangle & \lambda^{\bar{c}+\bar{d}} \\
H' & \lambda^{h+h+a} \langle A \rangle & 0 & \lambda^{2\bar{c}} \langle C \rangle & \lambda^{h'+e+\bar{c}} \langle C \rangle \\
C' & \lambda^{h+\bar{c}+\bar{c}} \langle C \rangle & \lambda^{c+\bar{e}} & \lambda^{h'+\bar{c}+\bar{e}} \langle C \rangle & \lambda^{c+\bar{e}} \\
\end{pmatrix},
\]

where the vanishing elements result from the SUSY-zero mechanism. Substituting the scales of non-vanishing VEVs, we can find that the non-vanishing elements are written as a simple sum of the effective charges of the relevant fields. It is worthwhile examining the general structure of the mass matrices. The first two columns and rows correspond to fields with non-vanishing VEVs that have smaller charges, and the last two columns and rows correspond to fields with vanishing VEVs that have larger charges. Therefore, it is useful to divide the matrices into four \( 2 \times 2 \) matrices as

\[
M_I = \begin{pmatrix}
0 & A_I \\
B_I & C_I \\
\end{pmatrix}.
\]

We can see that the ranks of \( A_I \) and \( B_I \) are reduced to 1 when the VEV \( \langle A \rangle \) vanishes. This implies that the rank of \( M_L \) is reduced, and actually it becomes 3. However, the ranks of \( A_{D^c} \) and \( B_{D^c} \) remain 2, because the field \( A \) becomes non-zero on \( D^c \). Therefore, DTS is realized. The mass spectrum of \( L \) is obtained as \((0, \lambda^{2h}, \lambda^{c+\bar{c}}, \lambda^{d+c})\). The massless modes of the doublet Higgs are estimated to be

\[
\bar{L}_H, \ L_H + \lambda^{h-c+\Delta_c} L_C.
\]

The elements of the matrices \( A_I \) and \( B_I \) become generally larger than the elements of the matrices \( C_I \) because the total effective charges of the corresponding pair of fields in \( A_I \)
and $B_I$ are smaller than those in $C_I$. Therefore, the mass spectrum of $D^c$ is essentially estimated by the matrices $A_{D^c}$ and $B_{D^c}$ as $(\lambda^{h+h'}, \lambda^{h+h'}, \lambda^{e+e'}, \lambda^{\alpha+c})$. Note that in order to realize proton decay, we have to pick up at least one element of $C_I$. Because such an element is generally smaller than the mass scale of $D^c$, proton decay is suppressed. In fact, the colored Higgs effective mass that appears in the expression of proton lifetime is estimated as $(\lambda^{h+h'})^2/\lambda^{2h'} = \lambda^{2h}$, which is larger than the cutoff scale, because $h < 0$.

Next, we examine the mass matrices for the representations $I = Q, U^c$ and $E^c$, which are contained in the 10 of SU(5), where the additional terms $\lambda^{2a'} A'A'$, $\lambda^{e+e'} \bar{C}'C'$, $\lambda^{\alpha+a'+e} \bar{C}'A'C'$ and $\lambda^{e+e'+e} \bar{C}'A'C'$ must be taken into account. The mass matrices are written as

$$M_I = \begin{pmatrix} \bar{I} & A & C & A' & C' \\ A & 0 & 0 & \lambda^{a'+a}\alpha_I & \lambda^{e+e'+a}\langle C \rangle \\ C & 0 & 0 & 0 & \lambda^{e+e'+a}\langle C \rangle \\ A' & \lambda^{a+a'}\alpha_I & 0 & \lambda^{2a'} & \lambda^{e+e+a}\langle C \rangle \\ C' & \lambda^{e+e'+a}\langle C \rangle & \lambda^{e+e'+a}\beta_I & \lambda^{e+e'+a}\langle C \rangle & \lambda^{e+e'} \end{pmatrix},$$

(4.19)

where $\alpha_Q = \alpha_{U^c} = 0$ and $\beta_{E^c} = 0$, because there are NG modes in symmetry breaking processes $SO(10) \rightarrow SU(3)_C \times SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ and $SU(2)_R \times U(1)_{B-L} \rightarrow U(1)_Y$. Defining $2 \times 2$ matrices as in the $I = L, D^c$ case, we can easily find that the ranks of $A_I$ and $B_I$ are reduced. Thus for each $I$, the $4 \times 4$ matrices $M_I$ have one vanishing eigenvalue, which corresponds to the NG mode that is eaten through the Higgs mechanism. The mass spectrum of the remaining three modes is $(\lambda^{e+e'}, \lambda^{e+e'}, \lambda^{2a'})$ for the color-triplet modes $Q$ and $U^c$, and $(\lambda^{a+a'}, \lambda^{a+a'}, \lambda^{e+e'})$ for the color-singlet modes $E^c$.

Finally, we examine the mass matrices for the representations $I = G, W$ and $X$, which are contained in the 24 of SU(5). Considering the additional term $\lambda^{a+a'} A'A'$, the mass matrices $M_I (I = G, W, X)$ are given as

$$M_I = \begin{pmatrix} \bar{I} & A & A' \\ A & 0 & \alpha_I \lambda^{a+a'} \end{pmatrix} \begin{pmatrix} \lambda^{a+a'} \end{pmatrix}. \quad (4.20)$$

Two $G$ and two $W$ acquire masses $\lambda^{a+a}$. Because $\alpha_X = 0$, one pair of $X$ is massless and this massless mode is eaten through the Higgs mechanism. The other pair has a rather light mass of $\lambda^{2a'}$.

In this way, all the extra fields indeed acquire superheavy masses and the physical massless modes are only two doublet Higgs. The gauge coupling unification is realized, as mentioned in [3.2]. Therefore, the Higgs sector goes well. The next issue is about the matter sector, where fields are odd under the R-parity while those of the Higgs sector are assigned even R-parity. Such an assignment of the R-parity guarantees that the argument regarding VEVs in [3.1.1] does not change if these matter fields have vanishing VEVs.
4.1.3 Matter Sector of SO(10) Models

In this section, we show how realistic mass matrices of quarks and leptons are realized in the anomalous U(1) GUT scenario.

In that scenario, higher-dimensional terms give contributions of the same order as renormalizable terms. Thus the order of coefficient of each term in the low energy effective theory respects the GUT symmetry, but the precise value of the coefficient does not respect the symmetry at all if GUT breaking VEVs can couple to the term. This means that the wrong GUT relation between the down-type quarks and charged leptons can be easily avoided. Unfortunately, it is difficult to avoid the wrong GUT relation between the down-type quarks and up-type quarks if we employ the minimal content of the matter sector, three 16 representations \( \Psi_i (\psi_1 \geq \psi_2 \geq \psi_3) \). To avoid the relation, we introduce an additional matter field \( T \) in the 10 representation, which is vector-like so that no exotic particles are expected at low energy while it can modify the origin of each generation in the \( \bar{5} \) sector. The 5 component of \( T \) acquire a mass with a linear combination of \( \bar{5} \) components of \( \Psi_i \) and \( T \) as

\[
\begin{pmatrix}
\bar{5}_\psi_1 \\
\bar{5}_\psi_2 \\
\bar{5}_\psi_3 \\
\bar{5}_T
\end{pmatrix} = \begin{pmatrix}
\lambda t + \psi_1 + \Delta c \\
\lambda t + \psi_2 + \Delta c \\
\lambda t + \psi_3 + \Delta c \\
\lambda 2t
\end{pmatrix}
\begin{pmatrix}
\bar{5}_\psi_1 \\
\bar{5}_\psi_2 \\
\bar{5}_\psi_3 \\
\bar{5}_T
\end{pmatrix},
\]

through the interaction terms \( T\Psi C \) and \( T^2 \). Thanks to the factorization property of the FN mechanism, the ratio of elements of the mass matrix is determined by the effective charges of \( \bar{5} \) fields. Thus, fields possessing the smallest effective charges become the main modes of the massive \( \bar{5} \). If \( t > \bar{\psi}_3 \), the three light modes \( (\bar{5}_1, \bar{5}_2, \bar{5}_3) \) are written as \( (\bar{5}_\psi_1, \bar{5}_T + \lambda^{t-\bar{\psi}_3} \bar{5}_\psi_3, \bar{5}_\psi_2) \). In this case, the origins of each generation of up-type quarks and down-type quarks are different, and thus they have different hierarchical structures. Note that in the framework of the FN mechanism, the mixing angles of quarks and leptons are determined by the difference in their effective charges. In our case, the difference in the \( \bar{5} \) sector which gives lepton mixing is smaller than that in the 10 sector which gives quark mixing. This means that the \( \bar{5} \) sector has milder hierarchy than the 10 sector has. In addition, in that framework, this also means that the lepton mixing angles are larger than the quark mixing angles. These properties are consistent with experiments.

Quark mass matrices

The Dirac mass matrices for quarks and leptons are obtained from the interaction

\[
\lambda^{\psi_1 + \psi_2 + \Delta \psi} \psi_i \psi_j H.
\]
The mass matrices for the up-type quarks are independent of \( T \) and written as

\[
M_U = \begin{pmatrix}
\lambda^2 (\psi - \psi_3) & \lambda (\psi_1 + \psi_2 - 2\psi_3) & \lambda (\psi_1 - \psi_3) \\
\lambda (\psi_1 + \psi_2 - 2\psi_3) & \lambda (\psi_2 - \psi_3) & \lambda (\psi_2 - \psi_3) \\
\lambda (\psi_1 - \psi_3) & \lambda (\psi_2 - \psi_3) & 1
\end{pmatrix} \lambda^{2\psi_3 + h} \langle H_u \rangle.
\] (4.24)

If we take \( 2\psi_3 + h = 0 \) to reproduce the large top Yukawa coupling and \( \psi_1 - \psi_3 = 3, \psi_2 - \psi_3 = 2, \lambda \sim \sin \theta_C \sim 0.2 \) to get correct orders of the CKM matrix elements,

\[
U_{\text{CKM}} = \begin{pmatrix}
1 & \lambda & \lambda^3 \\
\lambda & 1 & \lambda^2 \\
\lambda^3 & \lambda^2 & 1
\end{pmatrix},
\] (4.25)

namely \((\psi_1, \psi_2, \psi_3, h) = (n + 3, n + 2, n, -2n)\), the Yukawa matrix of up-type quarks is given as

\[
Y_U = \begin{pmatrix}
\lambda^6 & \lambda^5 & \lambda^3 \\
\lambda^5 & \lambda^4 & \lambda^2 \\
\lambda^3 & \lambda^2 & 1
\end{pmatrix}.
\] (4.26)

This gives a bit too large up Yukawa coupling, and we need a fine-tuning of \( \mathcal{O}(10\%) \) to get a correct value of the coupling.

Next, let us examine down-type quarks. \( \bar{5}_T + \lambda \bar{t} \bar{\psi}_3 \bar{5}_{\psi_3} \) receives contributions from \( \Psi \bar{\Psi} H \) through \( \bar{5}_{\psi_3} \) and from \( \Psi TC \) through \( \bar{5}_T \) if there is the following Higgs mixing

\[
H_d = \cos \gamma L_H + \sin \gamma L_C,
\] (4.27)

as in the example of Table 4.1 (See (4.18)). When the Higgs mixing is given as \( \overline{\text{H}} \), the magnitudes of these contributions are the same, and the Yukawa matrix of down-type quarks is given as

\[
Y_D = \lambda^2 \begin{pmatrix}
\lambda^4 & \lambda^{\bar{t}-\bar{\psi}_3+1} & \lambda^3 \\
\lambda^3 & \lambda^{\bar{t}-\bar{\psi}_3} & \lambda^2 \\
\lambda & \lambda^{\bar{t}-\bar{\psi}_3-2} & 1
\end{pmatrix}
\] or \( \lambda \bar{t} \bar{\psi}_3 \begin{pmatrix}
\lambda^6-(\bar{t}-\bar{\psi}_3) & \lambda^5-(\bar{t}-\bar{\psi}_3) & \lambda^3 \\
\lambda^5-(\bar{t}-\bar{\psi}_3) & \lambda^4-(\bar{t}-\bar{\psi}_3) & \lambda^2 \\
\lambda^3-(\bar{t}-\bar{\psi}_3) & \lambda^2-(\bar{t}-\bar{\psi}_3) & 1
\end{pmatrix}
\] (4.28)

This gives a realistic ratio \( m_s/m_b \) if \( 1 \lesssim \bar{t} - \bar{\psi}_3 \lesssim 3 \).

Note that if the SU(2)_R symmetry was exact, the CKM matrix would be a unit matrix. This is due to the cancellation between the contributions from up-type quarks and down-type quarks. Thus, we need SU(2)_R breaking effects in quark mass matrices in order to get a non-trivial CKM matrix. The additional matter field \( T \) introduces such an effect, but it is not sufficient. The simplest interaction for that purpose is \( \Psi \bar{\Psi}_j H \bar{C} \bar{C} \). If such an interaction is allowed for the (1, 2) component, which requires \( c + \bar{c} \geq -5 \), the cancellation can be avoided.

Because the ratio of the top Yukawa coupling and the bottom Yukawa coupling is \( \lambda^2 \), \( \tan \beta \equiv \langle H_u \rangle / \langle H_d \rangle \) is predicted to be moderately large: \( \lambda^2 m_t(L_G)/m_b(L_G) \sim 5 \).
Lepton mass matrices

The Yukawa matrices in the lepton sector are the transposes of $Y_D$, except for an overall factor $\eta$ induced by the renormalization group effect:

$$Y_{E,N} = \lambda^2 \left( \begin{array}{ccc} \lambda^4 & \lambda^3 & \lambda \\ \lambda^{\Delta+1} & \lambda^\Delta & \lambda^{\Delta-2} \\ \lambda^3 & \lambda^2 & 1 \end{array} \right) \eta.$$  \hspace{1cm} (4.29)

The right-handed neutrino masses come from the interaction $\lambda^\psi_i + \psi_j + 2\bar{c} \Psi_i \Psi_j \bar{C} \bar{C}$ as

$$M_R = \lambda^\psi_i + \psi_j + 2\bar{c} \langle \bar{C} \rangle^2 = \lambda^{2n-2\Delta c} \left( \begin{array}{ccc} \lambda^6 & \lambda^5 & \lambda^3 \\ \lambda^5 & \lambda^4 & \lambda^2 \\ \lambda^3 & \lambda^2 & 1 \end{array} \right).$$  \hspace{1cm} (4.30)

Therefore the neutrino mass matrix is estimated as

$$M_\nu = Y_D M_R^{-1} Y_D^T \langle H_u \rangle^2 = \lambda^{4-2n+2\Delta c} \left( \begin{array}{ccc} \lambda^2 & \lambda^{\tilde{\psi}_3-1} & \lambda^{\tilde{\psi}_3-2} \\ \lambda^{\tilde{\psi}_3-1} & \lambda^{2(\tilde{\psi}_3-4)} & \lambda^{\tilde{\psi}_3-2} \\ \lambda^{\tilde{\psi}_3-2} & \lambda & 1 \end{array} \right) \langle H_u \rangle^2 \eta^2. \hspace{1cm} (4.31)$$

This is the so-called Seesaw mechanism. This neutrino mass matrix was easily calculated by using effective charges without considering the right-handed neutrinos. For example, the $(3,3)$ element is given by $\bar{5} \Psi_2 \bar{5} \Psi_2 \bar{5} H \bar{5} H$, whose coefficient is given by $\lambda^2(\tilde{\psi}_2 + H) = \lambda^{2n+2+\Delta c - 2n + \frac{5}{2} \Delta c}$. From these mass matrices in the lepton sector, the MNS matrix is obtained as

$$U_{\text{MNS}} = \left( \begin{array}{ccc} 1 & \lambda^{1/2} & \lambda \\ \lambda^{1/2} & 1 & \lambda^{1/2} \\ \lambda & \lambda^{1/2} & 1 \end{array} \right) \hspace{1cm} (4.32)$$

when $\tilde{t} - \tilde{\psi}_3 = 5/2$, i.e.

$$t = n + \frac{1}{2}(c - \bar{c} + 5). \hspace{1cm} (4.33)$$

This gives bi-large mixing angles for the neutrino sector, because $\lambda^{1/2} \sim 0.5$. We then obtain the prediction $m_{\nu_2}/m_{\nu_3} \sim \lambda$, which is consistent with the experimental data \cite{15, 16}:

$$1.9 \times 10^{-3}\text{eV}^2 \leq \Delta m_{\text{atm}}^2 \leq 3.6 \times 10^{-3}\text{eV}^2,$$

$$7.4 \times 10^{-5}\text{eV}^2 \leq \Delta m_{\text{solar}}^2 \leq 8.5 \times 10^{-5}\text{eV}^2. \hspace{1cm} (4.34)$$

The relation $U_{e3} \sim \lambda$ is also an interesting prediction of this matrix. Comparing it with the global fit to neutrino oscillations which gives an upper limit $U_{e3} < 0.15$ at 90\% confidence level \cite{37}, we can expect that $U_{e3}$ will be measured in near future. Also, the normal hierarchy, $m_{\nu_1}/m_{\nu_3} \sim \lambda^2$ is another prediction clashing any hope to observe the neutrinoless double $\beta$ decay in near future.

---

\[\text{Note that this operator has negative charge and is thus forbidden by the SUSY-zero mechanism. In consequence, we need the right-handed neutrinos in order to get this effective mass term.}\]
If we define a parameter $l$ as $4 - 2n + c - \bar{c} = -(5 + l)$, it is given by using the heaviest light neutrino mass $m_{\nu_3}$ as
\[
\lambda^l \sim \lambda^{-5} \frac{\eta^2 \langle H_u \rangle^2}{m_{\nu_3} \Lambda}.
\] (4.35)

The parameter $\eta$ is roughly estimated as
\[
\eta \langle H_u \rangle \sim \eta \langle H_d \rangle \tan \beta \sim m_l \frac{\Lambda G}{m_{\nu_3} \Lambda} \sim m_{\nu_3} \Lambda \sim 200 \text{GeV}.
\] (4.36)

For the following set of parameters, $l = -3$, $\eta \langle H_u \rangle = 200 \text{GeV}$, $\Lambda = 2 \times 10^{16} \text{GeV}$, $\lambda = 0.2$, we get the masses
\[
m_{\nu_3} \sim 5 \times 10^{-2} \text{eV},
\] (4.37)
\[
m_{\nu_2} \sim 1 \times 10^{-2} \text{eV},
\] (4.38)
\[
m_{\nu_1} \sim 2 \times 10^{-3} \text{eV},
\] (4.39)

which are consistent with the experimental results (4.34).

## 4.2 E$_6$ Models

### 4.2.1 E$_6$ Unification of the Higgs Sector

We have shown that the DTS mechanism discussed in the previous section can be extended to E$_6$ unification in Refs. [5, 6]. Here, we examine a simple extension of the Higgs sector of the SO(10) models to E$_6$ models[5].

In order to break the E$_6$ gauge group into the standard gauge group, we introduce the following Higgs content:

1. Higgs fields that break E$_6$ into SO(10): $\Phi(27)$ and $\bar{\Phi}(27)$ ($\langle \Phi(1, 1) \rangle = \langle \bar{\Phi}(1, 1) \rangle$).

2. An adjoint Higgs field that breaks SO(10) into SU(3)$_C \times$SU(2)$_L \times$SU(2)$_R \times$U(1)$_{B-L}$: $A(78)$ ($\langle 45_A \rangle = \tau_2 \times \text{diag}(v, v, v, 0, 0)$).

3. Higgs fields that break SU(2)$_R \times$U(1)$_{B-L}$ into U(1)$_Y$: $C(27)$ and $\bar{C}(27)$ ($\langle C(16, 1) \rangle = \langle \bar{C}(16, 1) \rangle$).

Here, $X(R)$, $R_{1X}$ and $X(R_1, R_2)$ denote a field $X$ possessing the E$_6$ representation $R$, the component of $X$ possessing $R_1$ of SO(10) and the component of $X$ possessing $R_2$ of SU(5) contained in $R_1$ of SO(10), respectively. Of course, the anomalous U(1) charges of the gauge singlet operators, $\bar{\Phi}$, $\bar{C}$ and $A^2$, must be negative.

Naively thinking, it seems that we would have to introduce at least the same number of superfields with positive charges like the Higgs introduced above in order to make the superfields with positive charges massive. We find, however, that this is not the case, because some of the Higgs fields with non-vanishing VEVs are absorbed through the
Higgs mechanism. Actually, when the $E_6$ gauge group is broken to SO(10) by the non-vanishing VEV $|\langle \Phi \rangle| = |\langle \bar{\Phi} \rangle|$, the fields $16_\Phi$ and $\overline{16}_\bar{\Phi}$ are absorbed through the Higgs mechanism. Therefore, if two additional $10$'s of SO(10) in the Higgs sector with non-vanishing VEVs can be massive, we can then save one pair of $27$ and $\overline{27}$ with positive charges. At first glance, such a mass term may seem to be forbidden by the SUSY-zero mechanism. Actually, if all fields with non-vanishing VEVs had negative anomalous U(1) charges, their mass term would be forbidden. As discussed in §3.1.1, however, some of the fields with positive charges can have non-vanishing VEVs if the total charges of $G$-singlets with non-vanishing VEVs are negative. For example, we can set $\phi = -3$ and $\bar{\phi} = 2$. Since $\bar{\Phi}$ has positive charge, the term $\bar{\Phi}^3$ is allowed, and it induces a mass of $10_\delta$ through the non-vanishing VEV $\langle \bar{\Phi} \rangle$. If the term $\bar{\Phi}^2\bar{C}$ is allowed, masses for the two $10$'s, $10_\delta$ and $10_{\bar{C}}$, are induced, so that we can save one pair of $27$ and $\overline{27}$ Higgs.

An example for the field content in the Higgs sector is given in Table 4.2. The symbols $\pm$ denote the quantum numbers for a $Z_2$ parity symmetry which is introduced for the same reason as in the SO(10) models. Here, the Higgs field $H$ of the SO(10) model is contained in $\Phi$, and the $G$-singlet $\bar{\Phi}\Phi$ can play the same role as the FN filed $\Theta$. This $E_6$ Higgs sector has the same number of superfields with non-trivial representations as in the SO(10) Higgs sector, in spite of the fact that the larger group $E_6$ requires additional Higgs fields to break $E_6$ to SO(10).

### Table 4.2: The typical values of anomalous U(1) charges.

| Non-vanishing VEV | Vanishing VEV |
|-------------------|---------------|
| $A(a = -1, -)$    | $A'(c' = 4, -)$ |
| $\Phi(\phi = -3, +)\, C(c = -6, +)$ | $\bar{C}'(c' = 7, -)$ |
| $\bar{\Phi}(\bar{\phi} = 2, +)\, \bar{C}(\bar{c} = -2, +)$ | $\bar{C}'(c' = 8, -)$ |
| $Z_2(z_2 = -2, -), Z_5(z_5 = -5, -), Z_5(\bar{z}_5 = -5, -)$ | |

**DTS and alignment**

Generally, in $E_6$ GUT, the interactions in the superpotential that are made of only $27$ and $\overline{27}$ are written in terms of the units $27^3, \overline{27}27$ and $\overline{27}^3$. Note that terms like $27^3$ or $\overline{27}^3$ do not contain the product of singlet components of $G_{SM}$. Therefore, we can ignore these terms when considering SM-like vacua, while these terms can constrain the existence of vacua other than SM-like vacua. This point is discussed below.

The important terms in the superpotential to determine the VEVs are

$$ W = W_{A'} + W_{C'} + W_{\bar{C}'} + W(\bar{\Phi}). \quad (4.40) $$

Since we have a positively charged field $\bar{\Phi}$ that has a non-vanishing VEV, we have to take into account only such $W(\bar{\Phi})$’s that include only fields possessing non-vanishing VEVs.

---

4Strictly speaking, a linear combination of $\Phi$, $C$ and $A$ and of $\bar{\Phi}$, $\bar{C}$ and $\bar{A}$ becomes massive through the super-Higgs mechanism. The main modes are $16_\delta$ and $\overline{16}_\bar{\delta}$, respectively.
Since $\Phi\Phi$ and $\Phi C$ have negative total charges, the superpotential has essentially terms like $27^3$. Therefore, the superpotential $W(\Phi)$ constrains vacua other than SM-like vacua.

Let us discuss first the VEVs of $\Phi$ and $\bar{\Phi}$. When $\phi + \bar{\phi} \leq 0$, they have non-vanishing VEVs, and the $D$-flatness condition of $E_6$ requires $\langle \Phi \rangle = \langle \bar{\Phi} \rangle$, up to phases. The VEV of $\bar{\Phi}$ can be rotated by the $E_6$ gauge transformation into the following form:

$$\langle \bar{\Phi} \rangle = \begin{pmatrix} \bar{u} \\ 0 \\ \bar{u}_1 \\ \bar{u}_2 \\ 0 \end{pmatrix} \begin{cases} \text{SO(10) singlet (real)} \\ \text{SO(10) 16} \\ \text{the first component of SO(10) 10 (complex)} \\ \text{the second component of SO(10) 10 (real)} \\ \text{the third to tenth components of SO(10) 10}. \end{cases}$$ (4.41)

For simplicity, we adopt a superpotential of the form

$$W(\bar{\Phi}) = \bar{\Phi}^2 + \bar{\Phi}^3 \bar{C}.$$ (4.42)

Then, the $F$-flatness conditions of $10_C$ and $1_C$ lead to $1_\Phi 10_\Phi = 0$ and $10^2_\Phi = 0$, respectively. Thus, two type of vacua are allowed: $\bar{u} \neq 0, \bar{u}_1 = \bar{u}_2 = 0$ and $\bar{u} = 0, \bar{u}_1 = i\bar{u}_2 \neq 0$. This implies that a non-vanishing VEV of $1_\Phi$ requires the vanishing of the VEV $\langle 10_\Phi \rangle$. In this vacuum, $E_6$ is broken down to SO(10). Moreover, in this vacuum, $10_C$ has vanishing VEV, because of the $F$-flatness conditions for $10_\Phi$. Interestingly enough, a vacuum alignment occurs naturally. In the following, for simplicity, we often write $\lambda^n$ in place of the operators $(\bar{\Phi}\Phi)^n$, though these operators are not always singlets.

The superpotential $W_{A'}$ is in general written as

$$W_{A'} = \lambda^{a' + a} A'A + \lambda^{a' + 3a} A'A^3 + \lambda^{a' + a + \bar{\phi} + \phi} \bar{\Phi} A'A \Phi$$

$$+ \lambda^{a' + 3a + \bar{\phi} + \phi} \bar{\Phi} A'A^3 \Phi,$$ (4.43)

under the condition, $-3a + \bar{\phi} + \phi \leq a' < -5a$. Here we assume $c + \bar{c}, c + \bar{\phi}, \bar{c} + \phi < -(a' + a)$ to forbid the terms $CA'AC$ (which destabilizes the DW form of the VEV of $A$), $CA'A\Phi$ and $\bar{\Phi}A'AC$ (which may lead to undesirable vacua in which $\langle \bar{C} \rangle = \langle C \rangle = 0$ by the $F$-flatness conditions of $16_{A'}$ and $16_{A'}$). If $A$ and $(\Phi, \bar{\Phi})$ were separated in the superpotential, PNG fields would appear. Because the terms $\bar{\Phi}A'\Phi$ and $\bar{\Phi}A'A^3\Phi$ connect $A'$ and $A$ with $\Phi$ and $\bar{\Phi}$, the PNG fields acquire non-zero masses. Moreover, these terms realize the alignment between the VEVs $|\langle \Phi \rangle| = |\langle \bar{\Phi} \rangle|$ and $\langle A \rangle$. Note that these terms are also important to induce the term $(45_A 45_A)_{54}(45^2_A)_{54}$, which is not included in the term $A'A^3$, because of a cancellation (see Appendix A). In terms of SO(10), which is not broken by the VEV $|\langle \Phi \rangle| = |\langle \bar{\Phi} \rangle|$, the effective superpotential is given as

$$W_{A'}^{\text{eff}} = 45_A'((1 + 1^2_A + 45^2_A + 16_A 16_A) 45_A$$

$$+ 16_A')(1 + 1^2_A + 45^2_A + 16_A 16_A) 16_A$$

$$+ 16_A'((1 + 1^2_A + 45^2_A + 16_A 16_A) 16_A$$

$$+ 1_A 1_A'((1 + 1^2_A + 45^2_A + 16_A 16_A),$$ (4.44)
and the $F$-flatness conditions are written

\[
0 = \frac{\partial W}{\partial 45_A} = (1 + 1^2_A + 45^2_A + \overline{16}_A 16_A) 45_A, \quad (4.45)
\]

\[
0 = \frac{\partial W}{\partial 16_A} = (1 + 1^2_A + 45^2_A + \overline{16}_A 16_A) 16_A, \quad (4.46)
\]

\[
0 = \frac{\partial W}{\partial 16_A} = \overline{16}_A (1 + 1^2_A + 45^2_A + \overline{16}_A 16_A), \quad (4.47)
\]

\[
0 = \frac{\partial W}{\partial 1_A} = 1_A (1 + 1^2_A + 45^2_A + \overline{16}_A 16_A). \quad (4.48)
\]

The terms in each parenthesis of Eqs. (4.45)-(4.48) look common because we omit the coefficients. Indeed they are common due to an $E_6$ relation that holds when $A$ and $(\Phi, \bar{\Phi})$ are not coupled with each others. On the other hand, when they are coupled, such an $E_6$ relation is absent, and generally, the values in the parentheses of Eqs. (4.46), (4.47) and (4.48) are not zero, leading to $\langle 1 \rangle = 0$. We have two possibilities for the VEV of $1_A$: one vacuum with $\langle 1 \rangle = 0$ and the other vacuum with $\langle 1 \rangle \neq 0$. In the latter vacuum, the DW mechanism in $E_6$ GUT does not work, because the non-vanishing VEV $\langle 1 \rangle$ directly gives its bare mass to the doublet Higgs. Therefore, the former vacuum in which $\langle 1 \rangle = 0$ is favorable to realize DTS. Note that if the term $\Phi A' \Phi$ is allowed, the vacuum $\langle 1 \rangle = 0$ disappears. This destroys the realization of DTS. Here, this term is forbidden by $Z_2$ parity. As in the SO(10) case, we have several possibilities for the VEV of $45_A$, one of which is the DW VEV $\langle 45_A \rangle_{B-L} = \tau_2 \times \text{diag}(v, v, v, 0, 0)$, where $v \sim \lambda^{-a}$. These VEVs break the SO(10) into $SU(3)_C \times SU(2)_L \times SU(2)_R \times U(1)_{B-L}$.

Next, we discuss the $F$-flatness conditions of $C'$ and $C''$, which not only determine the scale of the VEV $\langle \bar{C}C \rangle \sim \lambda^{-(c+\tilde{c})}$ but also realize the alignment of the VEVs $\langle C \rangle$ and $\langle \bar{C} \rangle$. For simplicity, we assume that $\langle 1 \rangle = \langle 1 \rangle = 0$, though there may be vacua in which these components have non-vanishing VEVs. Then, since $\langle 10 \rangle = \langle 10 \rangle = 0$ by the above argument, only the components $16_{C'}$ and $\overline{16}_{C'}$ can have non-vanishing VEVs. The superpotential to determine these VEVs can be written as

\[
W_{C'} = \lambda^{c+\tilde{c}} \bar{\Phi} \left[ \lambda^{c+\tilde{c}} \bar{C} AC + \lambda^{2c+2\tilde{c}} \bar{\Phi} A C C \\
+ \lambda^{c+\tilde{c}} f_1 \left( \Phi \bar{\Phi}, A, Z_i \right) C + f_2 \left( \Phi \bar{\Phi}, A, Z_i \right) C' \\
+ \lambda^{c+\tilde{c}} \bar{C} f_3 \left( \Phi \bar{\Phi}, A, Z_i \right) C' \right] C',
\]

\[
W_{C''} = \lambda^{c+\tilde{c}} \bar{C} f_4 \left( \Phi \bar{\Phi}, A, Z_i \right) \Phi + \lambda^{c+\tilde{c}} f_5 \left( \Phi \bar{\Phi}, A, Z_i \right) C'.
\]

Here, $f_i$’s are certain functions whose forms are easily found and $Z_i$ represents $Z_2$, $Z_3$ and $\bar{Z}_5$. Note that these give common values for each multiplet of $SU(3)_C \times SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ and generally give different values for different multiplet, because these are functions of $A$, $\Phi$, $\bar{\Phi}$ and singlet fields. The vacua are $\langle \bar{C}C \rangle = 0$ and $\langle \bar{C}C \rangle \neq 0$. In the desired vacuum $\langle \bar{C}C \rangle \neq 0$, the $F$-flatness conditions of $16_{C'}$ and $\overline{16}_{C'}$ give non-trivial conditions, which cause the alignment of the VEVs $\langle A \rangle$ and $\langle C \rangle (\langle \bar{C} \rangle)$, as in the SO(10) case. Then,
the above four \( F \)-flatness conditions with respect to \( 1_{C'}, 1_{\bar{C}'}, 16_{C'} \) and \( \overline{16}_{\bar{C}'} \) determine the scale of the four VEVs \( \langle \overline{C} \rangle \sim \lambda^{-(c+\bar{c})} \), \( \langle Z_i \rangle \sim \lambda^{-\bar{z}_i} (i=2,5) \) and \( \langle \overline{Z}_5 \rangle \sim \lambda^{-\bar{z}_5} \). The VEVs \( |\langle C \rangle| \sim \lambda^{-c+\bar{c}} \) break \( SU(2)_{R} \times U(1)_{B-L} \) into \( U(1)_{Y} \).

Now, all the VEVs are determined as

\[
\langle \Phi \rangle = \langle \Phi(1,1) \rangle = \lambda^{-\frac{c+\bar{c}}{2}},
\]

\[
\langle \bar{\Phi} \rangle = \langle \bar{\Phi}(1,1) \rangle = \lambda^{-\frac{c+\bar{c}}{2}},
\]

\[
\langle 45 \rangle_A = \tau_2 \times \text{diag}(v,v,v,0,0), v = \lambda^{-a},
\]

\[
\langle C \rangle = \langle C(16,1) \rangle = \lambda^{-\frac{c+\bar{c}}{2}},
\]

\[
\langle \bar{C} \rangle = \langle C(\overline{16},1) \rangle = \lambda^{-\frac{c+\bar{c}}{2}},
\]

and all the other VEVs are zero. The symmetry breaking pattern is given by

\[
E_6 \rightarrow SO(10)
\]

\[
\rightarrow SU(3)_C \times SU(2)_L \times SU(2)_R \times U(1)_{B-L}
\]

\[
\rightarrow SU(3)_C \times SU(2)_L \times U(1)_Y
\]

at \( \lambda^{-\frac{c+\bar{c}}{2}} \Lambda \)

at \( \lambda^{-a} \Lambda \)

at \( \lambda^{-\frac{1}{2}(c+\bar{c})} \Lambda \).

The parameters that parametrize the effect of the additional FN mechanism are written as \( \Delta \phi = \frac{1}{2}(\phi - \bar{\phi}) \) for \( U(1)_{V'} \) and \( \Delta c = \frac{1}{2}(c - \bar{c}) \) for \( U(1)_Y \).

**Mass Spectrum of the Higgs Sector**

Since all the VEVs are fixed, we can derive the mass spectrum of the Higgs sector.

\( E_6 \) representations are decomposed in terms of \( SO(10) \times U(1)_{V'} \) as

\[
27 = 16_1 + 10_{-2} + 1_4,
\]

\[
78 = 45_0 + 16_{-3} + \overline{16}_3 + 1_0,
\]

which are further decomposed into \( SU(5) \) representations as Eqs. (4.15).

In the following, we study how the mass matrices of the above fields are determined by anomalous \( U(1) \) charges. Note that for the mass terms, we must take into account not only the terms given in the previous argument but also the terms that contain two fields with vanishing VEVs (see Appendix B).

Before going into details, it is worthwhile examining the NG modes that are eaten through the Higgs mechanism, because in some cases, it is not obvious that there are vanishing eigenvalues in the mass matrices. There appear the following NG modes:

1. \( 16 + \overline{16} + 1 \) of \( SO(10) \) (namely, \( Q + U^c + E^c + h.c. + N^c \)) in the breaking \( E_6 \rightarrow SO(10) \).

2. \( Q + U^c + X + h.c. \) in the breaking \( SO(10) \rightarrow SU(3)_C \times SU(2)_L \times SU(2)_R \times U(1)_{B-L} \).

3. \( E^c + h.c. + N^c \) in the breaking \( SU(2)_R \times U(1)_{B-L} \rightarrow U(1)_Y \).
Namely, there are NG modes possessing $2 \times (10, \overline{10}), (5, \overline{5})$ and $4 \times 1$ of SU(5) and $(X, X)$.

First, we examine the mass matrices of $24$ in SU(5). Considering the additional term $A^2$, we get the following mass matrices $M_I$, $I = G, W, X$:

$$
M_I = \begin{pmatrix}
I/\bar{I} & 45_A & 45_{A'} \\
45_A & 0 & \alpha_I \lambda^d+a \\
45_{A'} & \alpha_I \lambda^d+a & \lambda^{2a'}
\end{pmatrix},
$$

(4.58)

where $\alpha_X = 0$ and $\alpha_I \neq 0$ for $I = G, W$. One pair of $X$ is massless and is eaten through the Higgs mechanism. The mass spectra are $(0, \lambda^{2a'})$ for $I = X$ and $(\lambda^{a'+a}, \lambda^{a'+a})$ for $I = G, W$.

Next, we examine the mass matrices for the representations $I = Q, U^c$ and $E^c$, which are contained in $10$ of SU(5). The mass matrices $M_I$ are written as

$$
M_I = \begin{pmatrix}
16_\phi & 16_\bar{C} & 16_A & 45_A & 16_{C'} & 16_{A'} & 45_{A'} \\
16_\phi & 0 & 0 & 0 & 0 & \lambda^{c+\phi} & \lambda^ {\phi+a'-\Delta \phi} & 0 \\
16_{C'} & 0 & 0 & 0 & 0 & \beta_I \lambda^{c_e+\bar{c}} & \lambda^{a'+\Delta c} & 0 \\
16_A & 0 & 0 & 0 & 0 & \lambda^{c_e+a+\Delta \phi} & \lambda^{a'+c-\Delta c} & 0 \\
45_A & \lambda^{c_e+a} & \beta_I \lambda^{c_e+\bar{c}} & \lambda^{a'+c-\Delta c} & \lambda^{c_e+a+\Delta \phi} & \lambda^{\phi+\Delta c} & \lambda^{2a'} & \lambda^{2a'+\Delta \phi} \\
45_{A'} & \lambda^{c_e+\bar{c}} & 0 & \lambda^{a'+c} & \lambda^{a'+\Delta \phi} & \lambda^{a'+c-\Delta c} & \alpha_I \lambda^{a'+a} & \lambda^{2a'} & \lambda^{2a'+\Delta \phi-\Delta c}
\end{pmatrix},
$$

(4.59)

where we have used the relations $\lambda^\phi \langle \Phi \rangle \sim (\lambda^\phi \langle \Phi \rangle)^{-1} \sim \lambda^\Delta c$ and $\lambda^c \langle C \rangle \sim (\lambda^c \langle C \rangle)^{-1} \sim \lambda^{c_e} (\Delta \phi = \frac{1}{2}(\phi - \bar{\phi}), \Delta c = \frac{1}{2}(c - \bar{c}))$. Because one pair of $10$ and $\overline{10}$ (whose main modes are $16_\phi$ and $\overline{16}_\phi$) is eaten through the Higgs mechanism in the process of breaking $E_6$ to SO(10), we can simply omit $16_\phi$ and $\overline{16}_\phi$ during the derivation of the mass spectrum. Then, the mass matrices can be written in the form of four $3 \times 3$ matrices as

$$
M_I = \begin{pmatrix}
0 & A_I \\
B_I & C_I
\end{pmatrix}
$$

(4.60)

as in the SO(10) case. We can find that the ranks of $A_I$ and $B_I$ reduce to two because $(\alpha_I = 0, \beta_I \neq 0)$ for $I = Q, U^c$ and $(\alpha_I \neq 0, \beta_I = 0)$ for $I = E^c$, where the vanishing values are due to the NG theorem. The mass spectra become $(0, 0, \lambda^{a'+a}, \lambda^{a'+a}, \lambda^{c_e+\bar{c}}, \lambda^{c_e+\bar{c}}, \lambda^{2a'})$ for $I = Q, U^c$ and $(0, 0, \lambda^{a'+a}, \lambda^{a'+a}, \lambda^{a'+a}, \lambda^{a'+a}, \lambda^{c_e+\bar{c}})$ for $I = E^c$.

Finally, we examine the mass matrices of $5$ and $\bar{5}$ in SU(5) and show how DTS is realized. Considering the additional terms, we write the mass matrices $M_I$ for the representations $I = D^c, L$ and their conjugates as

$$
M_I = \begin{pmatrix}
0 & 0 & A_I \\
B_I & C_I & D_I \\
E_I & F_I & G_I
\end{pmatrix},
$$

(4.61)
\[
A_I = \begin{pmatrix}
I/\bar{I} & 10_C & 10_C' & \overline{16}_C & \overline{16}_C'
\end{pmatrix}, \quad (4.62)
\]
\[
B_I = \begin{pmatrix}
I/\bar{I} & 10_\Phi & 10_C & \overline{16}_C & \overline{16}_A
\end{pmatrix}, \quad (4.63)
\]
\[
C_I = \begin{pmatrix}
I/\bar{I} & 10_\Phi & 10_C & \overline{16}_C & \overline{16}_\Phi
\end{pmatrix}, \quad (4.64)
\]
\[
D_I = \begin{pmatrix}
I/\bar{I} & 10_C' & 10_C & \overline{16}_C & \overline{16}_A
\end{pmatrix}, \quad (4.65)
\]
\[
E_I = \begin{pmatrix}
10_C' & 10_C & \overline{16}_C & \overline{16}_A
\end{pmatrix}, \quad (4.66)
\]
\[
F_I = \begin{pmatrix}
10_\Phi & 10_C & \overline{16}_C & \overline{16}_\Phi
\end{pmatrix}, \quad (4.67)
\]
\[
G_I = \begin{pmatrix}
10_C' & 10_C' & \overline{16}_C & \overline{16}_A
\end{pmatrix}, \quad (4.68)
\]
where \( S_{D^c} \neq 0 \) and \( S_L = 0 \). We can see that the rank of \( A_L \) is three, which is smaller than the rank of \( A_{D^c} \). This implies that the rank of \( M_L \) is smaller than the rank of \( M_{D^c} \), and the rank of the matrix \( M_I \) is actually 10 for \( I = D^c \) and 9 for \( I = L \). One pair of massless fields, 5 and 5 (whose main modes are \( 16_\Phi \) and \( \overline{16}_\Phi \)), gives the NG mode, which is eaten through the Higgs mechanism during the breaking from \( E_6 \) to SO(10). The other
massless mode for \(I = L\) is identified as the so-called MSSM doublet Higgs. The massless mode is given as

\[
H_u \sim \bar{L}(10_\phi) + \lambda^{\phi-c} \bar{L}(10_C),
\]

\[
H_d \sim L(10_\phi) + \lambda^{\phi-c} L(10_C).
\]

As noted above, \(16_\phi\) and \(\bar{16}_\phi\) are eaten through the Higgs mechanism, and \(10_\phi\) and \(10_C\) can become massive through the matrix \(C_I\), whose elements are generally larger than the elements of \(B_I\), \(D_I\) and \(F_I\). Thus their masses can be estimated as \((\lambda^{\phi+\bar{c}-\Delta\phi}, \lambda^{\phi+\bar{c}-\Delta\phi})\).

With this observation, we ignore the matrices \(B_I\), \(C_I\), \(D_I\) and \(F_I\) and consider only \(A_I\), \(E_I\) and \(G_I\) in the following argument. Because the elements of \(A_I\) and \(E_I\) are generally larger than those of \(G_I\), we can estimate the mass spectrum of the other modes of \(D^c\) from \(A_{D^c}\) and \(B_{D^c}\) as \((\lambda^{c^c+\bar{c}+\Delta\phi}, \lambda^{c^c+\bar{c}+\Delta\phi}, \lambda^{c^c+\bar{c}+\Delta\phi}, \lambda^{c^c+\bar{c}+\Delta\phi}, \lambda^{c^c+\bar{c}+\Delta\phi}, \lambda^{c^c+\bar{c}+\Delta\phi}, \lambda^{c^c+\bar{c}+\Delta\phi}, \lambda^{c^c+\bar{c}+\Delta\phi})\), and that of \(L\) as \((0, \lambda^{c^c+\bar{c}+\Delta\phi}, \lambda^{c^c+\bar{c}+\Delta\phi}, \lambda^{c^c+\bar{c}+\Delta\phi}, \lambda^{c^c+\bar{c}+\Delta\phi}, \lambda^{c^c+\bar{c}+\Delta\phi}, \lambda^{c^c+\bar{c}+\Delta\phi}, \lambda^{c^c+\bar{c}+\Delta\phi})\). As in the SO(10) models, in order to realize proton decay, we have to pick up at least one element of \(C_I\), which is generally smaller than the mass scales of \(D^c\)'s, leading to suppressed proton decay via dimension 5 operators. The effective mass of the colored Higgs is estimated as \((\lambda^{c^c+\phi+\Delta\phi})^2 / \lambda^{2c^c+\Delta\phi} = \lambda^{2\phi+\Delta\phi}\), which is usually larger than the cutoff scale. For example, for the typical charge assignment in Table 4.2, \(2\phi + \Delta\phi = -17/2\).

It is worthwhile to summarize the required terms and the undesirable terms. There are several terms which must be forbidden in order to realize DTS:

1. \(\Phi^3, \Phi^2C, \Phi^2C', \Phi^2C'Z\) induce a large mass of the doublet Higgs.

2. \(\bar{C}A'C, \bar{C}A'AC, \bar{C}A'\Phi\) would destabilize the DW form of \(\langle A\rangle\).

3. \(\bar{C}A'AC, \bar{C}A'AC, \bar{C}A'AC, \bar{C}A'AC, \bar{C}A'AC\) lead to the undesirable VEV \(\langle 16_C \rangle = 0\), unless another singlet field is introduced.

4. \(A'A^n (n \geq 4)\) make it less natural to obtain a DW VEV.

In contrast, the following terms are necessary:

1. \(A'A, \bar{A}A'A^3\Phi\) to obtain a DW VEV \(\langle A\rangle\).

2. \(\Phi^2AC''\) for DTS.

3. \(C''(A + Z)C, C(A + Z)C''\) to achieve alignment between the VEVs \(\langle A\rangle\) and \(\langle C\rangle\) and to give superheavy masses to the PNGs.

4. \(\Phi A'\Phi\) to realize alignment between the VEVs \(\langle A\rangle\) and \(\langle \Phi\rangle\) and to give superheavy masses to the PNGs.

5. \(\Phi^3, \Phi^2C\) to give superheavy masses to two \(10\) of SO(10).
In this way, all the extra fields other than one pair of doublet Higgs indeed acquire superheavy masses. GCU is realized, as mentioned in §3.2. For this Higgs sector, however, the unified gauge coupling at the cutoff scale tends to become large, even when the matter sector is the minimal one, \( i.e. \Psi_i(27) (i = 1, 2, 3) \). An example of the gauge coupling flows is shown in Fig.4.1. The value of the unified gauge coupling strongly depends on the actual charge assignment, and if all anomalous U(1) charges become smaller, the unified gauge coupling at the unified scale \( \lambda^{-a} \) becomes smaller.\(^5\) For example, we can adopt half-integer charges as \( a = -1/2, a' = 5/2, \phi = -3, \bar{\phi} = 2, c = -5, \bar{c} = -1, c' = 13/2, \bar{c'} = 13/2, z_i = -i/2(i = 3, 7, 11) \), where the half integer charges play the same role as the \( Z_2 \) parity, and \( \psi_1 = 9/2, \psi_2 = 7/2, \psi_3 = 3/2 \) with odd \( R \)-parity in the matter sector. For this charge assignment, the unified gauge coupling at the cutoff scale is smaller than in the previous model. However, because the unification scale \( \lambda^{-a} \) is larger than that of the previous model, the model predicts a longer lifetime of the nucleon, which is roughly estimated as

\[
\tau_p(p \to e^+\pi^0) \sim 1 \times 10^{35} \left( \frac{\Lambda_U}{10^{16} \text{ GeV}} \right)^4 \left( \frac{0.01 \text{ GeV}^3}{\alpha} \right)^2 \text{ yrs.}\tag{4.71}
\]

\(^5\)This means a large \( a' + a \) is disfavored by this fact. Note that \( \Phi A' A \Phi \) is required to give large masses to would-be PNG modes, leading to \( a + a' \geq - (\phi + \bar{\phi}) \). Thus, we cannot take \( \phi + \bar{\phi} \) so small, and therefore \( \langle \Phi \rangle \) and \( \langle \bar{\Phi} \rangle \) cannot be so small.
This predicted value is significantly longer than the present experimental lower bound.

Another way to maintain the unified gauge coupling in the perturbative region is to reduce the number of Higgs. This is the topic of the next subsection.

Before examining this possibility, let us mention how these models can be compatible with the matter sector, which is discussed in §4.2.3. The relevant parameters are essentially in a number of two. The first one is $l$, which parametrizes the neutrino mass scale as

$$m_{\nu_3} \sim \lambda^{-l+5} (H_u)^2 \frac{\eta^2}{\Lambda},$$

and is also introduced in SO(10) models in §4.1.3. The other is the one that parametrizes the lepton mixing, $r$, which is defined as

$$r \equiv \frac{\lambda c \langle C \rangle}{\lambda \phi \langle \Phi \rangle}. \ (4.73)$$

This $r$ corresponds to $3 - (\tilde{t} - \tilde{\psi}_3)$ of SO(10) models in §4.1.3. For $\lambda \sim 0.2$, allowed values of these parameters are

$$-1 < l < -4, \quad 0 < r < 3/2. \ (4.74) \quad (4.75)$$

In this case, they are given as

$$l = -2 \left( \tilde{\psi}_2 + \Delta c + \phi \right) - 5, \ (4.76)$$

$$r = \Delta c - \Delta \phi, \ (4.77)$$

where $\Delta \phi = \frac{1}{2} (\phi - \bar{\phi})$ and $\Delta c = \frac{1}{2} (c - \bar{c})$. The charge assignment shown in Table 4.2 and $(\psi_1, \psi_2, \psi_3) = (9/2, 7/2, 3/2)$ yield $r = \frac{1}{2}$ and $l = -2$. Thus, it can be consistent with the matter sector.

### 4.2.2 Simpler E\(_6\) Higgs Sector

In the previous E\(_6\) model, $C(16)$ and $\tilde{C}(16)$ of the SO(10) model are embedded into the 27 field and the $\overline{27}$ field, respectively. However, they may also be embedded into the 78 field, resulting in simpler E\(_6\) models\[6]. Here, we examine this alternative embedding.

Since we introduce two adjoint Higgs $A'$ and $A$, we have two kinds of possibilities for reducing the Higgs sector.

1. The VEV $\langle 16_{A'} \rangle$ or $\langle \overline{16}_{A'} \rangle$ is non-vanishing.
2. The VEV $\langle 16_A \rangle$ or $\langle \overline{16}_A \rangle$ is non-vanishing.

Note that it must be forbidden that 16 and $\overline{16}$ have non-vanishing VEVs simultaneously, which destabilizes the DW form of VEVs. For example, if the VEVs $\langle 16_{A'} \rangle$ and $\langle \overline{16}_{A'} \rangle$ are non-zero, the interactions $A^m$ destabilize the DW form of VEVs because $F_{45_{A'}}$ includes the VEVs $\langle 16_{A'} \rangle$ and $\langle \overline{16}_{A'} \rangle$. At first glance, such an asymmetric VEV structure would be forbidden by the $D$-flatness conditions. But it is shown below that such an interesting VEV can satisfy the $D$-flatness conditions.
Table 4.3: Typical values of anomalous U(1) charges.

|     |        |
|-----|--------|
| 78  | $A(a = -1) A'(a' = 5)$ |
| 27  | $\Phi(\phi = -5) C''(c' = 7)$ |
| 27  | $\bar{C}(\bar{c} = -6) \bar{\Phi}(\bar{\phi}' = 6)$ |
| 1   | $\Theta(\theta = -1) Z_i(z_i = -1)$ for $i = 1$ to 5 |
|     | $Z'(z' = 6)$ |

**Possibility 1:** $\langle 16_A' \rangle \neq 0$

The typical Higgs content is represented in Table 4.3.

Suppose that among the above Higgs fields, only $45_A$, $1_\Phi$, $\overline{16}_C$ and $16_{A'}$ have non-vanishing VEVs such as

$$
\langle 45_A \rangle = \tau_2 \times \text{diag.}(v, v, v, 0, 0) \quad (v \sim \lambda^{-a}) \quad (4.78)
$$

$$
||1_\Phi|| = \left| \langle \overline{16}_C \rangle \right| = \left| \langle 16_{A'} \rangle \right| \sim \lambda^{-\frac{1}{3}(\bar{c} + a' + \phi)} \quad (4.79)
$$

As mentioned above, if $\phi + a' + \bar{c} < 0$, the $G$-singlet $\bar{C}A'\Phi$ can have a non-vanishing VEV, which means that $A'$ has a non-vanishing VEV. Actually, this vacuum satisfies the relations $\langle \text{tr } A'^m \rangle = 0$ and $\langle \bar{C}A'\Phi \rangle \sim \lambda^{-(\bar{c} + a' + \phi)}$, which are consistent with the VEV relation (3.1). And this vacuum satisfies not only the $D$-flatness conditions for SO(10) but also that of $U(1)_V$:

$$
D_{V'} : 4|1_\Phi|^2 - 3|16_{A'}|^2 - |\overline{16}_C|^2 = 0. \quad (4.80)
$$

Therefore, this vacuum satisfies all the $E_6$ $D$-flatness conditions.

Next we discuss the $F$-flatness conditions to know how such a vacuum can be obtained. For simplicity, we assume that any component fields other than $45_A$, $1_\Phi$, $\overline{16}_C$ and $16_{A'}$ have vanishing VEVs. To determine the VEV of $45_A$, it is sufficient to consider the superpotential

$$
W_{A'} = A'A + A'A^3 + A'A^4 + A'A^5. \quad (4.81)
$$

Here, for simplicity, singlet fields $Z_i$'s and coefficients are not written explicitly. The $F$-flatness condition of $45_{A'}$ leads to the DW VEV, $\langle 45_A \rangle \sim \tau_2 \times \text{diag.}(v, v, v, 0, 0)$. (Here $A'A^5$ is needed to avoid the “factorization problem”, as shown in Appendix A) Because the positively charged field $A'$ has a non-vanishing VEV $\langle 16_{A'} \rangle \neq 0$, the $F$-flatness conditions of the negatively charged fields may become non-trivial conditions. Fortunately, in this model, there is no such non-trivial condition. For example, $F_{\overline{16}_A} = 0$ is trivial because $\overline{16}_A$ is a NG mode in the superpotential $W_{A'}$.

The $F$-flatness condition of $Z'$, which is obtained from the superpotential

$$
W_{Z'} = Z'(1 + \bar{C}A'\Phi + f_Z(A, Z_i)), \quad (4.82)
$$

where $f_Z$ is a certain function of $A$ and $Z_i$'s, leads to

$$
\langle \bar{C}A'\Phi \rangle \sim \lambda^{-(\bar{c} + a' + \phi)}. \quad (4.83)
$$

53
The $D$-flatness conditions of $\text{SO}(10)$ and $\text{U}(1)_V$ lead to

\[ |\langle 1_\Phi \rangle| = |\langle 16_C \rangle| = |\langle 16_{A'} \rangle| \sim \lambda \frac{1}{3}(\bar{c} + a' + \phi), \tag{4.84} \]

which correspond to the desired vacuum shown in Eq. (4.79).

The $F$-flatness conditions of $C'$, which are obtained from the superpotential

\[ W_{C'} = \bar{C}(1 + Z_i + A + A'(f_C(A, Z_i) + \bar{C}A'\Phi))C', \tag{4.85} \]

where $f_C$ is another function of $A$ and $Z_i$'s, are written as

\[ F_{16_{C'}} = (1 + Z_i + A)\bar{16}_C = 0, \tag{4.86} \]
\[ F_{1_{C'}} = (f_C(A, Z_i) + \bar{C}A'\Phi)\bar{16}_C16_{A'} = 0. \tag{4.87} \]

These conditions realize an alignment between the VEVs $\langle 45_A \rangle$, $\langle 16_C \rangle$ and $\langle 16_{A'} \rangle$ by shifting the VEVs of the singlet fields $Z_i$, and as a result, the PNG fields become massive.

The $F$-flatness condition of $16_{A'}$, which is obtained from the superpotential

\[ W_{A'} = A'(f_A(A, Z_i) + \bar{C}A'\Phi)A', \tag{4.88} \]

where $f_A$ is another function of $A$ and $Z_i$'s, also realizes an alignment between $\langle 45_A \rangle$ and $\langle 16_{A'} \rangle$.

It is interesting to note that in this model, the “generalized sliding singlet mechanism” in §2.4.1 is naturally realized. The $F$-flatness conditions of $\Phi'$, which are obtained from the superpotential

\[ W_{\Phi'} = \Phi'(1 + Z_i + A + A'(f_\Phi(A, Z_i) + \bar{C}A'\Phi))\Phi, \tag{4.89} \]

where $f_\Phi$ is another function of $A$ and $Z_i$'s, are written as

\[ F_{1_{\Phi'}} = (1 + Z_i)1_\Phi = 0, \tag{4.90} \]
\[ F_{16_{\Phi'}} = (f_\Phi(A, Z_i) + \bar{C}A'\Phi)1_\Phi16_{A'} = 0. \tag{4.91} \]

At first glance, the component field $10_\Phi$, which includes a pair of doublet Higgs, seems to have a mass term from the superpotential $\Phi'(1 + Z_i + A)\Phi$. However, this mass term is a function of $A$ and singlets, and the doublet Higgs has the same quantum number under the generator $\langle A \rangle$ as the component field $1_\Phi$ which has non-vanishing VEV. This is the condition that the generalized sliding singlet mechanism can take place, and the $F$-flatness condition Eq. (4.90) ensures that the masses of the doublets also vanish. Note that the other components of $\Phi$ have different charges, and have superheavy masses. As a result, DTS is realized. In this mechanism, we need not introduce the $Z_2$ symmetry that is required in the DW mechanism.

In the above model, for intelligibility, we introduced a positively charged singlet $Z'$ in order to fix the VEV $\langle \bar{C}A'\Phi \rangle \sim \lambda^{-1}(\bar{c} + a' + \phi)$. However, one of the non-trivial $F$-flatness conditions of $1_{C'}, \bar{16}_{A'}$ and $\bar{16}_{\Phi'}$ can play the same role as $Z'$. If we do not introduce the field $Z'$, the number of the negatively charged singlet fields $Z_i$'s becomes four.
It is worthwhile to note how to determine the anomalous U(1) charges. In order to realize DTS, the terms
\[ A' \bar{A}^5, \Phi' A \Phi, \bar{C}(A + Z)C' \]  
must be allowed, and the term
\[ \bar{C} A'^2 \Phi \]  
must be forbidden. These requirements can be rewritten as inequalities. We determined the charges in order to satisfy those inequalities.

Unfortunately, we have not found any realistic matter sector with such a Higgs sector. Actually, the mixing parameter \( r \) of (4.73), which is obtained as
\[ \lambda_r \equiv \frac{\lambda_{a'} \langle 16 \rangle_{A'}}{\lambda \langle \Phi \rangle} = \lambda_{a'} \langle 16 \rangle_{A'} = \frac{1}{2}(2a' - \bar{c} - \phi) \]  
in this case, must be around 1/2 in order to obtain bi-large neutrino mixings, but it looks difficult to realize, because \( 2a' - \bar{c} - \phi \gg 1 \).

**Possibility 2: \( \langle 16_A \rangle \neq 0 \)**

Here, we consider another possibility in which the \( C(16) \) of the SO(10) model is embedded into the negatively charged adjoint Higgs \( A(78) \). This possibility is more promising, because the condition for a realistic matter sector, \( 2a - \bar{c} - \phi \sim 1 \), can be realized. The content of the Higgs sector is the same as in the previous possibility, except for the charges and the number of singlets.

To begin with, we examine the \( D \)-flatness conditions. Because \( \langle 45_A \rangle \neq 0 \) and \( \langle 16_A \rangle \neq 0 \), the \( D \)-flatness condition in the \( 16 \) direction gives a non-trivial condition. In order to compensate the contribution from \( A \) in the condition, \( \Phi \) and/or \( \bar{C} \) must have non-zero VEV in both \( 1 \) and \( 16 \) (\( \overline{16} \)) components. Therefore, non-trivial \( D \)-flatness conditions are

\[
D_{V+V'} : \quad |\Phi|^2 = |16_A|^2 + |\bar{C}|^2, \\
D_{V} : \quad |\overline{16}\bar{C}|^2 = |16_A|^2 + |16\Phi|^2, \\
D_{\overline{16}} : \quad 45_A^* 16_A = 1\Phi^* 16\Phi - \overline{16}\bar{C}^* \bar{1}\bar{C}. 
\]  

In addition, we suppose that the VEVs
\[
\langle \overline{16}\bar{C} \rangle \langle 16_A \rangle \langle \Phi \rangle \sim \langle \overline{16}\bar{C} \rangle \langle 45_A \rangle \langle \Phi \rangle \\
\sim \langle 1\bar{C} \rangle \langle 45_A \rangle \langle \Phi \rangle \\
\sim \lambda^{-(\bar{c}+a+\phi)} \equiv \lambda^{-3k}, \\
\langle 45_A \rangle \sim \lambda^{-a} 
\]  
are obtained from \( F \)-flatness conditions as is generally expected.\(^6\) From these conditions except for one \( D \)-flatness condition Eq.(4.97), the orders of VEVs are determined as

\(^6\)Strictly speaking, if three conditions in Eq.(4.98) were determined by \( F \)-flatness conditions, the \( F \)-flatness and \( D \)-flatness conditions would become over-determined. Therefore, only two of the three
\[
\begin{array}{|c|c|}
\hline
78 & A(a = -1, +) A'(a' = 5, +) \\
27 & \Phi(\phi = -3, +) C'(c' = 6, -) \\
27 & \Phi'(\phi' = 5, +) \bar{C}(\bar{c} = 0, -) \\
1 & \Theta(\theta = -1, +) Z_i(z_i = -1, +) (i = 1, 2) \\
\hline
\end{array}
\]

Table 4.4: Typical values of anomalous U(1) charges.

follows:

\[
\langle \mathbf{16}_C \rangle \sim \langle \mathbf{16}_A \rangle \sim \langle \mathbf{1}_\phi \rangle \sim \lambda^{-k} \equiv \lambda^{-a} \lambda^r, \\
\langle \mathbf{1}_C \rangle \sim \langle \mathbf{16}_\phi \rangle \sim \lambda^{a-2k} \sim \lambda^{-a} \lambda^{2r}, \\
\langle \mathbf{45}_A \rangle \sim \lambda^{-a},
\]

for \( \lambda^{-a} \gg \lambda^{-k} \). Here, \( r = a - k \) is the mixing parameter, introduced in §4.2.3. For these VEVs, the effective charges can be defined and therefore the natural gauge coupling unification is realized. Taking into account Eq. (4.97), it may appear that this condition requires \( r = 0 \). However, since \( r \) should be small (\( \sim 1/2 \)) to explain the bi-large neutrino mixings and there is an ambiguity due to order one coefficients, Eq. (4.97) can be satisfied even if \( r > 0 \). To be more precise, Eq. (4.97) has the form

\[
\lambda^{-2a} + r = \lambda^{-2a} + \frac{1}{2},
\]

and the r.h.s can become \( 2\lambda^{-2a} + \frac{1}{2} \), allowing \( r = 1/4 \). And the ambiguities in \( \mathcal{O}(1) \) coefficients leaves room for a larger \( r \).

Next, we examine \( F \)-flatness conditions. The typical charge assignment of the Higgs sector is represented in Table 4.4. Here the VEVs are again determined by

\[
W = W_{A'} + W_{\Phi'} + W_{C'},
\]

where

\[
W_{A'} = A' + A^3 + A^4 + A^5, \\
W_{\Phi'} = \Phi'(1 + A + Z_i + A^2 + AZ_i + Z_i^2)\Phi, \\
W_{C'} = \bar{C}(1 + A + Z_i + \cdots + (\bar{C}\Phi)^2)C'.
\]

As in the previous model, the \( F \)-flatness condition of \( \mathbf{45}_A \) leads to the DW VEV, \( \langle \mathbf{45}_A \rangle \sim \tau_2 \times \text{diag}(v, v, v, 0, 0) \). The \( F \)-flatness condition of \( \mathbf{1}_\Phi \) makes the \( E_6 \) singlet part in the parenthesis of Eq. (4.105) vanish, leading to vanishing doublet mass terms by the generalized sliding singlet mechanism. The \( F \)-flatness condition of \( \mathbf{16}_\Phi \) gives a factored equation

\[
(1 + A + Z_i) [\mathbf{45}_A \mathbf{16}_\Phi + \mathbf{16}_A \mathbf{1}_\phi] = 0,
\]

conditions are determined by \( F \)-flatness conditions. Then, another solution,

\[
\langle \mathbf{1}_A \rangle \sim \langle \mathbf{16}_A \rangle \sim \lambda^{-a} \ll \langle \mathbf{1}_\phi \rangle \sim \langle \mathbf{16}_\phi \rangle \sim \langle \mathbf{1}_C \rangle \sim \langle \mathbf{16}_C \rangle,
\]

may be allowed, through which the natural gauge coupling unification will not be realized. Though the \( \mathcal{O}(1) \) coefficients determine which vacuum is realized, the desired vacuum is obtained in some (finite) region of the parameter space for the \( \mathcal{O}(1) \) coefficients.
which can be checked by an explicit calculation based on $E_6$ group theory. The above two $F$-flatness conditions are satisfied by shifting the VEVs of two singlets $Z_i$. The two $F$-flatness conditions of $1_C$ and $16_C$ and the three $D$-flatness conditions in Eqs. (4.93)-(4.97) determine the five VEVs $16_A$, $1_\Phi$, $16_{\bar{\phi}}$, $1_{\bar{C}}$ and $16_{\bar{C}}$. It is straightforward to analyse the mass matrices of Higgs to check that all modes are superheavy except for one pair of doublet Higgs contained in $10_\phi$.\(^7\)

Now, we examine if the conditions are compatible with the matter sector, in which we introduced the same three superfields as in §4.2.3. Applying the same discussion to this case, the parameters $r$ and $l$ will be given as

$$
\lambda^r \sim \frac{\lambda^e(16C)}{\lambda^o(1_\Phi)} \sim \frac{\lambda^{a+\phi}(16_A)(1_\Phi)}{\lambda^o(1_\Phi)} = \lambda^{a-k},
$$

(4.108)

$$
\lambda^{-(5+l)} \sim \lambda^{4+\phi-2\bar{c}}(16_{\bar{C}})^{-2} \sim \lambda^{4+\phi-2\bar{c}+2k}.
$$

(4.109)

For example, a set of charges $(a, \phi, \bar{c}) = (-1, -3, 0)$ in Table 4.4 gives $(r, l) = (1/3, -10/3)$, which is allowed, as shown in §4.2.3.

In this model, we need $A' A^5$, $\bar{\Phi}' A Z \Phi$, $\bar{C} C \Phi C'$ (4.110) and have to forbid $\bar{C} A' \Phi$, $\bar{\Phi}' C \Phi$. (4.111)

However, it is difficult to forbid $\bar{C} A' \Phi$ while allowing $A' A^5$ by the SUSY-zero mechanism for small $r(= \frac{1}{3}(\bar{c} + \phi - 2a))$. Therefore we need another mechanism to forbid $\bar{C} A' \Phi$, e.g. we have to introduce an additional $Z_N$ symmetry. In $W_{\Phi'}$, the simplest superpotential that one could imagine, $W_{\Phi'} = \bar{\Phi}' A \Phi$, is not consistent with the $D$-flatness conditions for these VEVs (4.100)-(4.102). This is again a characteristic of the $E_6$ group, and therefore, $E_6$ breaking effects such as $\bar{\Phi}'(AZ + A^2)\Phi$ are needed.

As in the models discussed in §4.2.1, the half-integer charges of matter supermultiplets play the same role as R-parity in this model. Another charge assignment $(a, \phi, \bar{c}, z_i, a', \bar{d}', \bar{c}') = (-1, -3, 1/3, -1, 5, 5, 23/3)$ gives another example of a consistent model, which requires no additional $Z_N$ symmetries.

**4.2.3 Matter Sector**

In this subsection, we review the matter sector of $E_6$ models discussed in Ref.[12].

**E-Twisting Mechanism**

Let us first recall the so-called E-twisting mechanism, which is naturally realized in $E_6$ unification models[38]. In the case of $E_6$, the $16$ and $10$ representations of $SO(10)$ are

\(^7\)We would emphasize in this model that all the singlet fields also become superheavy, while in the models treated in §4.2.1 one massless singlet field appears.
automatically included in a fundamental multiplet 27 of E$_6$, which is decomposed under E$_6$$\supset$SO(10)$\supset$SU(5) as
\[
27 \rightarrow \underbrace{[(16, 10) + (16, 5) + (16, 1)]}_{16} + \underbrace{[(10, 5) + (10, 5)]}_{10} + [(1, 1)].
\] (4.112)
where the representations of SO(10) and SU(5) are explicated above. Thus, even with the minimal matter content, i.e. $\Psi_i(27)$ ($i = 1, 2, 3$), there appear extra fields: three 10’s and three 1’s in addition to the three 16’s. Because these extra fields are vector-like, they do not change the number of light fields, but they can change the family structure, as in the SO(10) models in 4.1.3. Note that in SO(10) models, we have to introduce an additional 10 field to realize a twisting family structure.

In terms of SU(5), there are three 5 and six 5. Among them, three pairs of (5, 5) become heavy.\(^8\) Indeed, the Higgs fields $\Phi$ and $C$ can yield such masses. The superpotential that gives large masses for (5, 5) pairs are written as
\[
W = \lambda^\psi_i + \psi_j + \epsilon \Psi_i \Psi_j C + \lambda^\psi_i + \psi_j + \phi \Psi_i \Psi_j \Phi.
\] (4.113)
The VEV $\langle \Phi(1, 1) \rangle$ gives the $3 \times 3$ mass matrix of $\Psi_i(10, 5)\Psi_j(10, 5)$ pairs as
\[
M_\Phi = \begin{pmatrix}
\Psi_1(10, 5) & \Psi_2(10, 5) & \Psi_3(10, 5)
\end{pmatrix}
\begin{pmatrix}
\lambda^{\psi_1} & \lambda^{\psi_1 + \psi_2} & \lambda^{\psi_1 + \psi_3} \\
\lambda^{\psi_1 + \psi_2} & \lambda^{\psi_2} & \lambda^{\psi_2 + \psi_3} \\
\lambda^{\psi_1 + \psi_3} & \lambda^{\psi_2 + \psi_3} & \lambda^{\psi_3}
\end{pmatrix}
\lambda^\phi \langle \Phi(1, 1) \rangle,
\] (4.114)
while the VEV $\langle C(16, 1) \rangle$ gives the mass terms of $\Psi_i(16, 5)$ and $\Psi_j(10, 5)$ as
\[
M_C = \begin{pmatrix}
\Psi_1(10, 5) & \Psi_2(10, 5) & \Psi_3(10, 5)
\end{pmatrix}
\begin{pmatrix}
\lambda^{\psi_1} & \lambda^{\psi_1 + \psi_2} & \lambda^{\psi_1 + \psi_3} \\
\lambda^{\psi_1 + \psi_2} & \lambda^{\psi_2} & \lambda^{\psi_2 + \psi_3} \\
\lambda^{\psi_1 + \psi_3} & \lambda^{\psi_2 + \psi_3} & \lambda^{\psi_3}
\end{pmatrix}
\lambda^c \langle C(16, 1) \rangle.
\] (4.115)
Then, the full mass matrix is proportional to
\[
\begin{pmatrix}
\Psi_1(10, 5) & \Psi_2(10, 5) & \Psi_3(10, 5) \\
\Psi_1(16, 5) & \Psi_2(16, 5) & \Psi_3(16, 5) \\
\Psi_3(10, 5) & \Psi_3(10, 5) & \Psi_3(10, 5)
\end{pmatrix}
\begin{pmatrix}
\lambda^{2\psi_1 + r} & \lambda^{\psi_1 + \psi_2 + r} & \lambda^{\psi_1 + \psi_3 + r} \\
\lambda^{\psi_1 + \psi_2 + r} & \lambda^{\psi_2 + \psi_3 + r} & \lambda^{\psi_2 + \psi_3 + r} \\
\lambda^{\psi_1 + \psi_3 + r} & \lambda^{\psi_2 + \psi_3 + r} & \lambda^{2\psi_3 + r}
\end{pmatrix}
\lambda^r \langle C(16, 1) \rangle.
\] (4.116)
where we have defined the parameter $r$ as
\[
\lambda^r \equiv \frac{\lambda^c \langle C(16, 1) \rangle}{\lambda^\phi \langle \Phi(1, 1) \rangle}.
\] (4.117)

\(^8\)The possible right-handed neutrino modes $\Psi_i(16, 1)$ and $\Psi_i(1, 1)$ also acquire large masses, but here we concentrate on the family structure of 5.
which we use frequently in E\(_6\) models. Note that some of the matrix elements may vanish by the SUSY-zero mechanism, but for the moment, we assume that no such zeros appear in the superpotential. In general, there are three massless modes among the six \(\bar{5}\) fields by solving the above \(3 \times 6\) matrix. However, since the matrix has hierarchical structure, we can easily find which \(\bar{5}\)'s remain massless. It is determined by their effective charges, therefore by the parameter \(r\), so that fields possessing smaller effective charges become massive. We can classify all the cases as follows:

1. \(\psi_1 - \psi_3 < r : (16_{\psi_1}, 16_{\psi_2}, 16_{\psi_3})\) type.

2. \(0 < r < \psi_1 - \psi_3 : (16_{\psi_1}, 16_{\psi_2}, 10_{\psi_1})\) type.

3. \(\psi_3 - \psi_1 < r < 0 : (16_{\psi_1}, 10_{\psi_1}, 10_{\psi_2})\) type.

4. \(r < \psi_3 - \psi_1 : (10_{\psi_1}, 10_{\psi_2}, 10_{\psi_3})\) type.

When \(0 < r < \psi_1 - \psi_3\), we can realize different family structures for up-type quarks and down-type quarks so that we can reproduce realistic Yukawa matrices as in SO(10) models where \(\bar{t} > \bar{\psi}_3\). Now, we consider this case, where we can write the three massless modes \((\bar{5}_1, \bar{5}_2, \bar{5}_3)\) as

\[
\begin{align*}
\bar{5}_1 &= 16_{\psi_1} + \lambda^{\psi_1-\psi_3} 16_{\psi_3} + \lambda^{\psi_1-\psi_2+r} 10_{\psi_2} + \lambda^{\psi_1-\psi_3+r} 10_{\psi_3}, \\
\bar{5}_2 &= 10_{\psi_1} + \lambda^{\psi_1-\psi_3-r} 16_{\psi_3} + \lambda^{\psi_1-\psi_2} 10_{\psi_2} + \lambda^{\psi_1-\psi_3} 10_{\psi_3}, \\
\bar{5}_3 &= 16_{\psi_2} + \lambda^{\psi_2-\psi_3} 16_{\psi_3} + \lambda^{\psi_1} 10_{\psi_2} + \lambda^{\psi_2-\psi_3+r} 10_{\psi_3},
\end{align*}
\]

where the first terms on the right-hand sides are the main components of these massless modes, and the other terms are mixing terms with heavy states, \(16_{\psi_3}, 10_{\psi_2}\) and \(10_{\psi_3}\).

If we use SUSY-zero coefficients, various types of massless modes can be realized. For example, if \(\psi_1 + \psi_3 + \phi < 0\), SUSY zeros appear, and the Yukawa terms \(\Psi_3 \psi_i \Phi\) \((i = 1, 2, 3)\) are forbidden. Hence, when \(2\psi_2 + \phi > 0\) the mass matrix \(M_\Phi\) becomes

\[
M_\Phi = \begin{pmatrix} \Psi_1(10, 5) & \Psi_2(10, 5) & \Psi_3(10, 5) \\ \Psi_2(10, 5) & \lambda^{2\psi_1} \psi_1 + \psi_2 & 0 \\ \Psi_3(10, 5) & 0 & 0 \end{pmatrix} \lambda^{\phi v},
\]

and the massless mode \(10_{\psi_4}\) does not mix through non-diagonal mass matrix elements with any other \(\bar{5}\) field. We call such a massless field an “isolated” field. There are various different patterns of massless modes containing the “isolated” fields. For example, if the conditions \(2\psi_2 + \phi \geq 0, 2\psi_3 + c \geq 0\) and \(\lambda^{2\psi_1+\phi} \langle 1_\Phi \rangle > \lambda^{\psi_1+\psi_2+c} \langle 16_C \rangle\) are satisfied in addition to the above condition \(\psi_1 + \psi_3 + \phi < 0\), we have the pattern \((16_{\psi_1}, 16_{\psi_2}, 10_{\psi_4})\), i.e.,

\[
\begin{pmatrix} \bar{5}_1 \\ \bar{5}_2 \\ \bar{5}_3 \end{pmatrix} = \begin{pmatrix} \Psi_1(16, \bar{5}) + \cdots \\ \Psi_2(16, \bar{5}) + \cdots \\ \Psi_3(10, 5) \end{pmatrix},
\]

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which has been adopted in Ref. [38]. Note that $\bar{5}_3$ has no components from $(16, \bar{5})$ and therefore we need the Higgs mixing (4.128) to make $\bar{5}_3$ massive. Here, we do not consider such isolated fields and thus assume
\begin{align*}
0 \leq \psi_1 + \psi_3 + c, \\
0 \leq \psi_1 + \psi_3 + \phi,
\end{align*}
for simplicity.

**Quark mass matrices**

Because the E-twisting mechanism does not change the structure of the 10 sector of SU(5), the mass matrix for up-type quarks are given in essentially the same way as in SO(10) models. Their Yukawa interactions are obtained from the interaction
\[ \lambda^{\psi_i + \psi_j + \phi} \Psi_i \Psi_j \Phi \]
and $\lambda^{\psi_i + \psi_j + c} \Psi_i \Psi_j C$ if there are Higgs mixing
\[ H_u \sim 10_\Phi + \lambda^{c - \phi} 10_C, \]
as [4.126]. They give contributions of the form $16_\Psi 16_\Psi 10_\Phi$. These contributions are of the same order and can be written like [4.124]. Thus, we take $(\psi_1, \psi_2, \psi_3, \phi) = (n + 3, n + 2, n, -2n)$ for the same reason as explained in §4.1.3. Then, we get the Yukawa matrix of up-type quarks as
\[ Y_U = \left( \begin{array}{cccc}
\lambda^6 & \lambda^5 & \lambda^3 \\
\lambda^5 & \lambda^4 & \lambda^2 \\
\lambda^3 & \lambda^2 & 1
\end{array} \right). \]

Down-type quarks have contributions from the same interactions as up-type quarks, and when another Higgs mixing
\[ H_d = \cos \gamma L_{10} + \sin \gamma L_{16_C} \]
exists, the interaction $\Psi C$ gives the other condition that gives a contribution of the form $16_\Psi 10_\Psi 16_C$. When $\sin \gamma \sim \lambda^{\psi - c - r}$, these contributions are of the same order. Because we assume $0 < r < \psi_1 - \psi_3 = 3$, the massless modes of the $\bar{5}$ sector have essentially the same structure as in SO(10) models from §4.1.3. And, from the mixing (4.119), we can find $\tilde{t} - \bar{\psi}_3 \leftrightarrow \psi_1 - \psi_3 - r$. Thus, the Yukawa matrix of down-type quarks is given as
\[ Y_D = \lambda^2 \left( \begin{array}{ccc}
\lambda^4 & \lambda^{4-r} & \lambda^3 \\
\lambda^5 & \lambda^{5-r} & \lambda^2 \\
\lambda & \lambda^{1-r} & 1
\end{array} \right). \]

As mentioned in §4.1.3, we need SU(2)$_R$ breaking effect in order to get non-trivial CKM matrix. In the SO(10) models, we have to allow a higher-dimensional interaction
such as $\Psi_i \Psi_j HCC$. On the other hand, in E\(_6\) models, the Higgs mixing (4.126) (if the mixing breaks SU(2)\(_R\)) or (4.128) is enough to make CKM matrix non-trivial, and we get the correct orders for the CKM matrix elements:

$$U_{\text{CKM}} = \begin{pmatrix} 1 & \lambda & \lambda^3 \\ \lambda & 1 & \lambda^2 \\ \lambda^3 & \lambda^2 & 1 \end{pmatrix}.$$  \hfill (4.130)

**Lepton mass matrices**

The Yukawa matrix of charged leptons is again the transpose of \(Y_D\), except for an overall factor \(\eta\) induced by the renormalization group effect:

$$Y_E = \lambda^2 \begin{pmatrix} \lambda^4 & \lambda^3 & \lambda \\ \lambda^{A+1} & \lambda^A & \lambda^{A-2} \\ \lambda^3 & \lambda^2 & 1 \end{pmatrix} \eta.$$  \hfill (4.131)

As for the neutrino sector, we have a 3\(\times\)6 Dirac mass matrix and a 6\(\times\)6 Majorana mass matrix, because there are six right-handed neutrinos. Thus we have to make calculations using large matrices in order to get a contribution to the Seesaw mechanism. However, the discussion in §3.2.1 ensures that such a contribution is of the same order as that of the higher-dimensional interaction $\bar{5}_2 \tilde{5}_2 5_5 \Phi$ and can be estimated by the simple sum of the effective charges as

$$M_\nu = \lambda^{4-2n+2\Delta c} \begin{pmatrix} \lambda^2 & \lambda^{2-r} & \lambda \\ \lambda^{2-r} & \lambda^{2-2r} & \lambda^{1-r} \\ \lambda & \lambda^{1-r} & 1 \end{pmatrix} \langle H_u \rangle^2 \eta^2,$$  \hfill (4.132)

which leads to the following MNS matrix:

$$U_{\text{MNS}} = \begin{pmatrix} 1 & \lambda^r & \lambda \\ \lambda^r & 1 & \lambda^{1-r} \\ \lambda & \lambda^{1-r} & 1 \end{pmatrix}.$$  \hfill (4.133)

In this way, we get the same matrices as in §4.1.3 by the exchange $\tilde{t} - \tilde{\psi}_3 \rightarrow 3 - r$, thanks to the effective charge. This means that the same discussion for neutrino given in §4.1.3 can be applied here. Namely, we can reproduce bi-large mixing angle when $r = 3 - (\tilde{t} - \tilde{\psi}_3)$, that is

$$c - \bar{c} = \phi - \bar{\phi} + 1,$$  \hfill (4.134)

and if we define the parameter \(l\) as

$$\phi - \bar{\phi} = 2n - 9 - 2r - l,$$  \hfill (4.135)

\(l\) is expressed by using the heaviest light neutrino mass \(m_{\nu_3}\) as

$$\lambda' \sim \lambda^{-5} \eta^2 \langle H_u \rangle^2 \frac{m_{\nu_3}}{\Lambda}.$$  \hfill (4.136)
These parameters should have values within the following range:

\[ -1 < l < -4, \quad (4.137) \]
\[ 0 < r < 3/2. \quad (4.138) \]

**Suppression of FCNC processes**

The crucial difference between SO(10) models and E\(_6\) models are that in E\(_6\) model the non-diagonal elements of the sfermion mass-squared matrix can be suppressed to some extent. This is because the first and second generation of the \(\bar{5}\) sector is contained in a single multiplet \(\Psi_1\), and thus they are degenerate in the symmetric limit. To be more precise, they behave as doublets under an SU(2) subgroup of E\(_6\). We call the subgroup as SU(2)\(_E\). This subgroup is broken by VEVs, \(\langle A \rangle, \langle \Phi \rangle, \langle \bar{\Phi} \rangle\) and so on. This means that the rates of FCNC processes are proportional to these VEVs. Unfortunately, these VEVs are usually too large to suppress the FCNC processes so that they cannot be observed in the present experiments. Thus, the SUSY-flavor problem can be softened but cannot be solved in the E\(_6\) models. A sufficient suppression can be obtained in models which employ a horizontal symmetry, as discussed in the next chapter.
Chapter 5

Models with Horizontal Symmetry

We have several reasons for introducing a horizontal symmetry $G_H$. One of them is to understand the origin of the flavor violation in Yukawa couplings of quarks and leptons. Actually many studies have been made along this direction\cite{40-42}.

The second reason is to unify quarks and leptons in fewer multiplets, though it is strongly related with the first motivation. Usual GUTs with SU(5), SO(10) or E\textsubscript{6} gauge group can make unification of quarks and leptons within one generation. In order to unify all the quarks and leptons into a single multiplet, a larger gauge group such as SO(12 + 2\textit{n}), E\textsubscript{7}, or E\textsubscript{8} is required, though these unified groups cannot realize the chiral matter in 4D theories. However, actually it is possible in higher dimensional field theories, and in that cases, a horizontal symmetry may appear in the effective 4D theories.

The third reason is to solve the SUSY-flavor problem\cite{43}. A non-abelian flavor (horizontal) symmetry can potentially solve this problem. If the first two generation fields become a doublet under the flavor symmetry, $\Psi_a(a = 1, 2)$, the sfermion masses of the first two generation fields become universal, unless the flavor symmetry is broken. This is important in solving the SUSY-flavor problem.

5.1 Horizontal Symmetry for SUSY-Flavor Problem

In this section, we examine the idea that the SUSY-flavor problem can be solved when a horizontal symmetry is introduced. We follow the argument given in Ref.\cite{13}. Of course, in order to obtain realistic hierarchical structures of Yukawa couplings, the flavor symmetry must be broken, for example by a VEV $\langle F_a \rangle$. Then, generally the universal sfermion masses are lifted by the breaking effect. Various models in which the breaking effects can be controlled have been considered in the literature\cite{2,44,45,46}. However, in GUT models with bi-large neutrino mixings, the universality of sfermion masses of the first two generations is not sufficient to solve the SUSY-flavor problem. This is because the large mixings and the $O(1)$ discrepancy between the sfermion masses of the third generation fields and those of the first two generation fields lead to too rapid FCNC processes. The E\textsubscript{6} unification can naturally solve this problem\cite{13}. The essential point is that in the E\textsubscript{6} unification all the three light modes of $\bar{\mathbf{5}}$ fields come from the first two
generation fields $\Psi_a(27)$ as shown in §4.2.3. Therefore, all the three light modes of $\bar{5}$ have universal sfermion masses, which are important in solving the SUSY-flavor problem.

5.1.1 U(2) Models

Let us show how the large mixings and the $\mathcal{O}(1)$ discrepancy lead to too rapid FCNC processes. For simplicity, we consider a simple model with a global horizontal symmetry $U(2)$, under which the three generations of quarks and leptons, $\Psi_i = (\Psi_a, \Psi_3)$ ($a = 1, 2$), transform as $2 + 1$, while the Higgs field $H$ is a singlet. Then only the Yukawa interaction $\Psi_3 \Psi_3 H$ is allowed by the horizontal symmetry, which accounts for the large top Yukawa coupling. Suppose that the $U(2)$ horizontal symmetry is broken by the VEVs of a doublet $\langle \bar{F}_a \rangle = \delta^2 V$ and of an anti-symmetric tensor $\langle A^{ab} \rangle = \epsilon^{ab} v$ ($\epsilon^{12} = -\epsilon^{21} = 1$) as

$$U(2)_H \rightarrow V U(1)_H \rightarrow v$$

The ratios of the VEVs to the cutoff scale $\Lambda$, $\epsilon$ $\equiv$ $V/\Lambda$ $\gg$ $\epsilon' \equiv v/\Lambda$, give the following hierarchical structure of the Yukawa couplings as

$$Y_{U,D,E} \sim \begin{pmatrix} 0 & \epsilon' & 0 \\ \epsilon' & \epsilon^2 & \epsilon \\ 0 & \epsilon & 1 \end{pmatrix}.$$ (5.2)

Moreover, the $U(2)_H$ symmetric interaction $\int d^4\theta \Psi^a \Psi_b S^\dagger S$, where $S$ has a non-vanishing VEV $\langle S \rangle \sim \theta^2 \tilde{m}^2$ and should not be confused with the dilaton field discussed in §2.5, leads to approximate universality of the first and second generation sfermion masses:

$$\tilde{m}^2_{U,D,E} \sim \tilde{m}^2 \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 + \epsilon^2 & \epsilon \\ 0 & \epsilon & 1 + R_{U,D,E} \end{pmatrix}.$$ (5.3)

Here $R_{U,D,E}$ is $\mathcal{O}(1)$, because $\Psi_3$ has nothing to do with $\Psi_a$, $\epsilon^2$ comes from higher dimensional interactions, such as

$$\int d^4\theta (\Psi_a \bar{F}^a)^\dagger \Psi_b \bar{F}^b S^\dagger S,$$ (5.4)

through a non-vanishing VEV $\langle F \rangle$. We have neglected the contributions from $\epsilon'$. The important parameters which are constrained by the FCNC processes are defined as

$$\delta_{f_\chi} \equiv V_{f_\chi}^\dagger \frac{\tilde{m}^2_{f_\chi} - \tilde{m}^2}{\tilde{m}^2} V_{f_\chi},$$ (5.5)

where $V_{f_\chi}$ ($f = U, D, E$, $\chi = L, R$) is a diagonalizing matrix for fermions. The constraints are given as, for example,

$$\sqrt{|\text{Im}(\delta_{D_L})_{12}(\delta_{D_R})_{12}|} \leq 2 \times 10^{-4} \left( \frac{\tilde{m}_Q}{500 \text{ GeV}} \right),$$

$$|\text{Im}(\delta_{D_R})_{12}| \leq 1.5 \times 10^{-3} \left( \frac{\tilde{m}_Q}{500 \text{ GeV}} \right),$$ (5.6)
at the weak scale from $\epsilon_K$ in the $K$ meson mixing, and

$$|(\delta_{E_L})_{12}| \leq 4 \times 10^{-3} \left(\frac{\tilde{m}_L}{100 \text{ GeV}}\right)^2$$

(5.7)

from the $\mu \to e\gamma$ process.

As shown above, the $U(2)_H$ symmetry indeed realizes not only hierarchical Yukawa couplings but also approximately universal sfermion masses of the first two generation fields. These mass matrices lead to the relations

$$\frac{\tilde{m}_2^2 - \tilde{m}_1^2}{\tilde{m}^2} \sim \frac{m_{F_2}}{m_{F_3}},$$

(5.8)

where $m_{F_i}$ and $\tilde{m}_i$ are the masses of the $i$-th generation fermions and of the $i$-th generation sfermions, respectively. Unfortunately, these predictions of this simple model are too large for the $\bar{5}$ sector which has milder hierarchy in fermion masses, leading to too rapid FCNC processes. Furthermore, even if we can manage to make the $\epsilon$ contributions in (5.3) harmless, e.g. by forbidding the higher dimensional interactions (5.4) by hand, the bi-large mixings in the lepton sector lead to too rapid FCNC processes through the $O(1)$ contribution $R_{\bar{5}}$. To be more precise, even if we can realize $\Delta_{f_{\chi}} \equiv \frac{\tilde{m}_{\bar{5}}^2 - \tilde{m}^2}{m^2} = \text{diag}(0, 0, R_{f_{\chi}})$, the large mixings in the $\bar{5}$ sector, such as

$$V_{\bar{5}} = \begin{pmatrix} 1 & \lambda^r & \lambda \\ \lambda^r & 1 & \lambda^{1-r} \\ \lambda & \lambda^{1-r} & 1 \end{pmatrix},$$

(5.9)

where $r \sim \frac{1}{2}$, induce a large $(\delta_{f_{\chi}})_{12}$ as

$$\delta_{\bar{5}} = R_{\bar{5}} \begin{pmatrix} \lambda^2 & \lambda^{2-r} & \lambda \\ \lambda^{2-r} & \lambda^{2-2r} & \lambda^{1-r} \\ \lambda & \lambda^{1-r} & 1 \end{pmatrix}.$$  

(5.10)

Comparing with the experimental constraints (5.6) and (5.7), which suggest $(\delta_{f_{\chi}})_{12} \lesssim \lambda^4$ for $\tilde{m}_Q \sim 500\text{GeV}$, $\tilde{m}_L \sim 100\text{GeV}$, we find that the $(1, 2)$ component of (5.10) is too large.

In addition, the hierarchical Yukawa couplings predicted by this simple model are similar for the up-quark sector, the down-quark sector, and the charged lepton sector, which is inconsistent with the experimental results. Moreover, in the neutrino sector, it seems to be difficult to obtain the large neutrino mixing angles.

### 5.1.2 $E_6 \times SU(2)_H \times U(1)_A$ Models

Note that all the problems mentioned in §5.1.1 arise from the $\bar{5}$ sector, and also that the $\bar{5}$ sector has twisted family structure in the models discussed in §4.1.3 and §4.2.3. By this twist, we can get milder hierarchies and thus larger mixings in the $\bar{5}$ sector, even though the original hierarchy is similar to that in the $10$ sector. Another consequence of
the twist is that a smaller SU(2)_H breaking can reproduce milder hierarchies of Yukawa couplings in the \( \bar{5} \) sector than in 10 sector. Because the lift from the degeneracy is given by the SU(2)_H breaking, we can avoid the disfavored relation (5.8). Furthermore, in \( E_6 \) models, all the generation of \( \bar{5} \) come from \( \Psi_1(27) \) and \( \Psi_2(27) \) which behave as doublets under SU(2)_H in \( E_6 \times SU(2)_H \) models. This means that, in these models, all the \( \bar{5} \)'s are degenerate in the SU(2)_H symmetric limit, and it is expected \( R_{\bar{5}} \ll 1 \).

Let us illustrate this by using an example of \( E_6 \) models employing anomalous U(1) symmetry and SU(2) horizontal (gauge) symmetry shown in Table 5.1. Here we omit singlet fields and additional \( Z_N \) symmetries for simplicity. Note that all the \( E_6 \) charged Higgs are singlets under SU(2)_H. This means that the discussion in §4.2.1 can be applied, and the effect of SU(2)_H appears only on \( \tilde{\psi}_a \) as

\[
\tilde{\psi}_1 = \psi_a + \Delta f, \quad \tilde{\psi}_2 = \psi_a - \Delta f,
\]

where \( \Delta f \equiv \frac{1}{2}(f - \bar{f}) = \frac{1}{2} \). Thus, from this charge assignment together with Eqs. (4.76) and (4.77), we can find that

\[
Y_U = \begin{pmatrix}
\lambda^7 & \lambda^6 & \lambda^{3.5} \\
\lambda^6 & \lambda^5 & \lambda^{2.5} \\
\lambda^{3.5} & \lambda^{2.5} & 1
\end{pmatrix}, \quad Y_D = \lambda^2 \begin{pmatrix}
\lambda^5 & \lambda^4 & \lambda^{3.5} \\
\lambda^4 & \lambda^3 & \lambda^{2.5} \\
\lambda^{1.5} & \lambda^{0.5} & 1
\end{pmatrix},
\]

which lead to

\[
U_{\text{CKM}} = V_{10} = \begin{pmatrix}
1 & \lambda & \lambda^{3.5} \\
\lambda & 1 & \lambda^{2.5} \\
\lambda^{3.5} & \lambda^{2.5} & 1
\end{pmatrix}, \quad U_{\text{MNS}} = V_{\bar{5}} = \begin{pmatrix}
1 & \lambda & \lambda^{1.5} \\
\lambda & 1 & \lambda^{0.5} \\
\lambda^{1.5} & \lambda^{0.5} & 1
\end{pmatrix}.
\]

Note that the main modes of the three generations in the \( \bar{5} \) sector are given by \( (\bar{5}_1, \bar{5}_2, \bar{5}_3) \sim (16_{\Psi_1}, 16_{\Psi_2}, 10_{\Psi_1}) \).

The sfermion mass-squared matrices are written as

\[
\tilde{m}_f^2 = \begin{pmatrix}
\tilde{m}_{f_R}^2 & A_f^\dagger \\
A_f & \tilde{m}_{f_L}^2
\end{pmatrix},
\]

where \( A_f \) is the \( A \)-term matrix. In the following discussion, we restrict our consideration on the mass mixings through \( \tilde{m}_{f_R}^2 \), because a reasonable assumption, e.g. SUSY breaking

| \( E_6 \) | \( SU(2)_H \) | \( U(1)_A \) |
|-----|-----|-----|
| \( 27 \) | \( 27 \) | \( 27 \) |
| \( 78 \) | \( 78 \) | \( -1 \) |
| \( 27 \) | \( 27 \) | \( 5 \) |
| \( 27 \) | \( 27 \) | \( -4 \) |
| \( 27 \) | \( 27 \) | \( 9 \) |
| \( 27 \) | \( 27 \) | \( -2 \) |
| \( 27 \) | \( 1 \) | \( -3 \) |

Table 5.1: Typical values of anomalous U(1) charges of non-singlet fields.
in the hidden sector, leads to an $A_f$ that is proportional to the Yukawa matrix $Y_f$. The corrections $\Delta f_\chi$ in this model are approximately given as

$$\Delta_{10} = \begin{pmatrix} \lambda^5 & \lambda^6 & \lambda^3 \lambda^{3.5} \\ \lambda^6 & \lambda^5 & \lambda^{2.5} \\ \lambda^{3.5} & \lambda^{2.5} & R_{10} \end{pmatrix}, \quad \Delta_{5} = \begin{pmatrix} \lambda^5 & \lambda^6 & \lambda^{5.5} \\ \lambda^6 & \lambda^5 & \lambda^{4.5} \\ \lambda^{5.5} & \lambda^{4.5} & R_{8} \end{pmatrix}, \quad (5.15)$$

where, for example, $(\Delta_{5})_{12}$ is derived by using the interaction $\int d^4 \theta \lambda^f \bar{f} (\Psi \Phi) S \bar{S}$. Note that $m_{\delta_{10}} = m_{\delta_{5}} = m_{\delta_{8}}$. Namely, the Yukawa hierarchy in the superpotential becomes milder, improving the undesirable relations (5.8). The constrained parameters $\delta_{f_\chi}$ are approximated as

$$\delta_{10} = R_{10} \begin{pmatrix} \lambda^5 & \lambda^6 & \lambda^{3.5} \\ \lambda^6 & \lambda^5 & \lambda^{2.5} \\ \lambda^{3.5} & \lambda^{2.5} & 1 \end{pmatrix}, \quad \delta_{5} = R_{5} \begin{pmatrix} \lambda^3 & \lambda^2 & \lambda^{1.5} \\ \lambda^2 & \lambda & \lambda^{0.5} \\ \lambda^{1.5} & \lambda^{0.5} & 1 \end{pmatrix}, \quad (5.16)$$

at the GUT scale. As discussed above, all the sfermion masses for $\tilde{5}$ become equal at the leading order in this model. $R_{5}$ is given by SU(2)$_H$ breaking effects, such as $\langle A \rangle$, $\langle \Phi \rangle$ and $\langle \bar{\Phi} \rangle$, through interactions such as $\int d^4 \theta \Psi \bar{\Phi} \Psi \Phi S \bar{S}$. $R_{5}$ is $O(\lambda^2)$ while $R_{10}$ is $O(1)$ in this model. This reduces the lower limits for the scalar quark masses to satisfy the FCNC constraints to an acceptable level, 250GeV.

**Extension to SU(3)$_H$**

It is interesting that the SU(2)$_H$ horizontal symmetry in the previous model can be extended to SU(3)$_H$. In such models, the three generations of quarks and leptons can be unified into a single multiplet, $\Psi(27,3)$. Assuming that the horizontal gauge symmetry SU(3)$_H$ is broken by the VEVs of two pairs of Higgs fields $F_i(1, 3)$ and $\bar{F}_i(1, 3)$ ($i = 2, 3$) as

$$|\langle F_i \rangle| = |\langle \bar{F}_i \rangle| \sim \delta_i^0 \lambda^{-\frac{1}{2}}(f_i + \bar{f}_i), \quad (5.17)$$

the effect of SU(3)$_H$ can be parameterized by using the following two parameters,

$$\Delta f_3 = \frac{1}{2}(f_3 - \bar{f}_3), \quad (5.18)$$

$$\Delta f_2 = \frac{1}{2}(f_2 - \bar{f}_2) \quad (5.19)$$

as

$$\bar{\psi}_i \equiv \psi - \Delta f_i, \quad \bar{\psi}_1 \equiv \psi + \Delta f_2 + \Delta f_3, \quad (5.20)$$

and thus $\bar{\psi} = \psi + \frac{1}{2}\Delta f_3$. This parametrization corresponds to that in the previous model as

$$\bar{\Psi}_3 \leftrightarrow \bar{\Psi} \bar{F}_3, \quad (5.21)$$

$$\bar{\Psi} \bar{F} \leftrightarrow \bar{\Psi} \bar{F}_2, \quad (5.22)$$

$$\bar{\Psi} \bar{F} \leftrightarrow \bar{\Psi} F_2 F_3, \quad (5.23)$$

$$\Delta f \leftrightarrow \frac{1}{2}(2\Delta f_2 + \Delta f_3). \quad (5.24)$$
Note that in order to realize an $O(1)$ top Yukawa coupling, which is given by $\langle \bar{F}_3^{\dagger} F_3 \rangle^2$, $SU(3)_H$ must be broken at the cutoff scale, namely, $\langle F_3 \rangle \sim 1$ which is realized when $f_3 + \bar{f}_3 = 0$. To obtain the same mass matrices of quarks and leptons as in the previous $E_6 \times SU(2)_H$ model, the effective charges must be taken as $(\hat{\psi}_1, \hat{\psi}_2, \hat{\psi}_3) = (11/2, 9/2, 2)$. For example, a set of charges $(f_3, \bar{f}_3, f_2, \bar{f}_2) = (2, -2, -3, -2)$ and $\psi = 4$ satisfies the above conditions.

### 5.1.3 Discussions

For both $E_6 \times SU(2)_H$ and $E_6 \times SU(3)_H$ models, we have introduced a Higgs sector that breaks $E_6$ to $G_{SM}$ where all the fields have trivial quantum numbers for the horizontal symmetry. This means, as mentioned above, that the discussion in §4.2.1 can be applied to this case. Thus, it is possible to obtain complete $E_6 \times SU(2)_H$ and $E_6 \times SU(3)_H$ anomalous U(1) GUTs, where the degeneracy of the sfermion masses of $5$ fields is naturally obtained. Note that, additional fields that are not singlets under the horizontal gauge symmetry are required for anomaly cancellation. It is interesting to introduce non-singlet Higgs fields under the horizontal gauge symmetry. This is the subject of §5.2.

Before examining that possibility, let us make comments on phenomenology of these models. Because the $SU(3)_H$ symmetry must be broken at the cutoff scale to realize an $O(1)$ top Yukawa coupling, the degeneracy of the sfermion masses between the third generation fields $\Psi_3$ and the first and second generation fields $\Psi_a (a = 1, 2)$ is not guaranteed. Therefore, the $E_6 \times SU(3)_H$ models gives the same predictions for the structure of sfermion masses as $E_6 \times SU(2)_H$ models. Roughly speaking, all the sfermion fields have nearly equal masses, except the third generation fields included in $10$ of $SU(5)$. It must be an interesting subject to study the predictions on FCNC processes (for example, $B$-physics) from such a special structure of the sfermion masses. More precisely, this degeneracy is lifted by $D$-term contributions of $SU(3)_H$ and $E_6$. Though the contributions strongly depend on the concrete models for SUSY breaking and on GUT models and some of them must be small in order to suppress the FCNC processes, it is important to test these GUT models with precisely measured masses of sfermions, as discussed in Ref.[50].

### 5.2 Horizontal Symmetry in $E_6$ Higgs Sectors

In this section, we investigate $E_6$ models with an anomalous U(1) gauge symmetry whose Higgs sectors have non-trivial quantum numbers for the horizontal symmetry, $SU(2)_H$ or $SU(3)_H$[14]. Because $E_6$ contains $SU(2)_E$, these models may realize well-suppressed FCNC processes as suggested in §5.1. In this sense, $E_6$ models seem more promising than $SO(10)$ models which are examined in detail in Ref.[14]. Unfortunately, however, if both $E_6$ and the horizontal symmetry are simultaneously broken, it is difficult to obtain realistic models in which the FCNC processes are sufficiently suppressed. The point is as follows. In order to sufficiently suppress the FCNC processes with the horizontal symmetry, the scale at which the horizontal symmetry is broken should be smaller than $\lambda^2$. (In this section, we
take $\lambda \sim \sin \theta_C \sim 0.22$, and we do not fix the anomalous $U(1)$ charge of the FN field to $-1$ but $\langle \Theta \rangle \sim \lambda^\theta$.) Generally, in anomalous $U(1)$ GUTs, it is difficult to obtain a smaller $E_6$ breaking scale than $\lambda^2$, as mentioned in §4.2.1. Therefore, if both $E_6$ and the horizontal symmetry are broken by a VEV of a single field, that is, both the symmetries are broken at the same scale, then the suppression of the FCNC processes does not become sufficient, although $SU(2)_H$ help to ameliorate the SUSY-flavor problem.

Nevertheless, such a possibility still deserves to be examined, by the second reason discussed at the beginning of this chapter. In particular, if we assume that $E_8$ is the unified group of a more fundamental theory, it seems natural that some of $27(\overline{27})$ Higgs fields also have non-trivial quantum numbers for the horizontal symmetry.

### 5.2.1 $E_6 \times SU(2)_H \times U(1)_A$ Models

Motivated by the decomposition $E_8 \supset E_6 \times SU(3)_H \supset E_6 \times SU(2)_H$, under which $248$ of $E_8$ is decomposed as

$$248 \rightarrow (78,1) + (1,8) + (27,3) + (\overline{27},3)$$

$$+ (78,1) + (1,3 + 2 + 2 + 1) + (27,2 + 1) + (\overline{27},2 + 1),$$

we assign non-trivial representations of the horizontal symmetry only to $27, \overline{27}$ and/or $1$ Higgs fields. Note that, in the matter sector, $\Psi_1(27)$ and $\Psi_2(27)$ are treated as a doublet, and the difference of their effective charges should correspond to the Cabibbo angle, $\tilde{\psi}_1 - \tilde{\psi}_2 \sim 1$. This means that the difference of the effective charges of the two components of doublets should be also around 1 as far as the effective charge is well-defined. In Table 4.4, we have introduced two $27(\Phi, C')$ and two $\overline{27}(\overline{C}, \overline{C}')$, where the difference of anomalous $U(1)$ charges of two fields each is much larger than 1. Thus, it is difficult to unify the Higgs sector of the model, and we concentrate on the Higgs sector of Table 4.2 which contain

$$78 : A, A'$$

$$27 : \Phi, C, C'$$

$$\overline{27} : \overline{\Phi}, \overline{C}, \overline{C}'$$

and some singlets. From the same reason as for the previous models (Table 4.4), it is difficult to embed the primed fields into a doublet if we aim to suppress the FCNC processes not assuming the universal soft mass. If we take $(\Phi, C)$ as a doublet under $SU(2)_H$, the Yukawa interaction for the top quark, $\Psi_3\Psi_3\Phi$, is forbidden by the horizontal symmetry so that it is difficult to realize an $O(1)$ top Yukawa. Thus the remaining possibility is to embed $\overline{\Phi}$ and $\overline{C}$ into a doublet as $\overline{C}_a = (\overline{\Phi}, \overline{C})$.

The Higgs content we consider below is summarized in Table 5.2. All the non-vanishing VEVs are shown, and their magnitudes are expressed with parameters $\Delta \phi$ etc.. For simplicity, we assume that any component fields other than those shown in Table 5.2 have vanishing VEVs. We also assume that the following three $G$-singlets have non-vanishing
Table 5.2: The Higgs content of $E_6 \times SU(2)_H \times U(1)_A$ models except for singlets: Here $SU(2)_H$ doublets are denoted by the index $a$. One or more discrete symmetries are introduced when needed.

VEVs as in the VEV relations (3.1) from three $F$-flatness conditions:

\[
\Phi \bar{C} F \sim \lambda^{-(\phi+\bar{c}+f)} \equiv \lambda^{-3k}, \quad (5.28)
\]

\[
C \bar{C} \bar{F} \sim \lambda^{-(c+\bar{c}+f)}, \quad (5.29)
\]

\[
\bar{F} F \sim \lambda^{-(f+\bar{f})}, \quad (5.30)
\]

where the parameter $k$ should not be confused with $k$ given in §4.2.2. In addition to the three relations, three $D$-flatness conditions

\[
|1_\phi|^2 = |1_{\bar{C}_1}|^2, \quad (5.31)
\]

\[
|16_C|^2 = |\overline{16}_C|^2, \quad (5.32)
\]

\[
|1_{\bar{C}_1}|^2 + |F_1|^2 = |\overline{16}_C|^2 + |F_2|^2 \quad (5.33)
\]

determine three parameters, $\Delta \phi$, $\Delta c$ and $\Delta f$, in terms of the anomalous U(1) charges. Roughly speaking, there are four possible cases as follows:

1. $1_{\bar{C}_1} \sim \overline{16}_C \geq \bar{F}_1, F_2$. \quad (5.34)
2. $\bar{F}_1 \sim \overline{16}_C \geq 1_{\bar{C}_1}, F_2$. \quad (5.35)
3. $\bar{F}_1 \sim F_2 \geq 1_{\bar{C}_1}, \overline{16}_C$. \quad (5.36)
4. $1_{\bar{C}_1} \sim F_2 \geq \bar{F}_1, \overline{16}_C$. \quad (5.37)

As for the second and third cases, the horizontal symmetry breaking scale is larger than the GUT breaking scale $\langle 1 \rangle$. As discussed in §4.2.1, $\langle 1 \rangle$ does not seem so small as $\lambda^2$. Therefore, an $SU(2)_H$ breaking scale larger than $\langle 1 \rangle$ is not sufficient for the suppression of the FCNC processes. For simplicity, we concentrate on the fourth case in the following discussion, but a similar discussion can be applied to the other cases. In the fourth case, we get

\[
1_\phi = 1_{\bar{C}_1} \sim F_2 \sim \lambda^{-k}, \quad (5.38)
\]

\[
\bar{F}_1 \sim \lambda^{-(f+\bar{f})+k}, \quad (5.39)
\]

\[
16_C = \overline{16}_C \sim \lambda^{\frac{1}{2}[-(c+\bar{c})+f-k]}, \quad (5.40)
\]
in other words,
\[ \Delta f = \frac{2f - \phi - \bar{c}}{3} = f - k, \]  
\[ \Delta \phi = \frac{2\phi - f - \bar{c}}{3} = \phi - k, \]  
\[ \Delta c = \frac{c - \bar{c} + \Delta f}{2}. \]  
(5.41)

The conditions for the fourth case \((F_2 \geq \bar{F}_1, 16\bar{c}_2)\) to be realized are given by
\[ 0 < -k \leq -\frac{1}{2}(f + \bar{f}), -c - \bar{c} + f, \]  
which are also written as
\[ f < \Delta f \leq \frac{1}{2}(f - \bar{f}), -c - \bar{c} + 2f. \]  
(5.44)

In addition, as shown in \(\S 4.2.1\) the following conditions are required phenomenologically:

- The parameter \(r\) for the neutrino mixings should be around \(1/2–3/2\).
- The parameter \(l\) for the neutrino mass scale should be around \(-2–3\).
- In order to realize the DTS, \(C'\Phi\Phi\) must be allowed, while \(C'AC\Phi\) must be forbidden.
- \(\bar{C}F\bar{C}F\bar{C}\bar{F}\), which corresponds to \(\bar{\Phi}\bar{\Phi}\bar{C}\) in \(\S 4.2.1\) must be allowed in order to avoid undesirable massless modes.
- In order to give mass to would-be PNG modes, \(A'\Phi\bar{C}\bar{F}\) must be allowed.
- For the gauge coupling unification, a smaller effective mass of the colored Higgs, \(m_C^{\text{eff}} \sim \lambda^{2\phi + \Delta \phi}\), is preferred. In the model displayed in Table \(\text{4.2}\) \(m_C^{\text{eff}} \sim \lambda^{-8.5}\).
- In order to reproduce the realistic quark mass matrices, the SU(2)_R symmetry must be broken in the Yukawa couplings. SU(2)_R breaking VEVs \(\langle C \rangle = \langle \bar{C} \rangle\) can be picked up through the SM Higgs mixing \(\bar{C}'\bar{C}\bar{F}\Phi\bar{F}\) is required), or through higher dimensional interactions (for example, \(\Psi \bar{C}C\Psi \bar{F}\Phi\)).

These conditions are rewritten in terms of the anomalous \(U(1)\) charges as
\[ \frac{1}{2} \leq r = \frac{1}{2}(c - \phi) + \Delta f \leq \frac{3}{2} \]  
\[ -2 \geq l = -5 - 2(\psi - \Delta f - \psi_3) + \phi + 2\Delta c \geq -3 \]  
\[ c < \phi \]  
\[ \bar{f} \geq -3\bar{c} - 2f \]  
\[ 0 \leq a' + a + \phi + \bar{c} + f \geq 0 \]  
\[ 2\phi + \Delta \phi \geq -8.5 \]  
\[ 2\psi + \phi + c + \bar{c} + \bar{f} \geq 0 \text{ or } \bar{c}' \geq -2\phi - \bar{c} - \bar{f} - a, \]  
(5.46) (5.47) (5.48) (5.49) (5.50) (5.51) (5.52)
Note that the first condition is not consistent with the third conditions if \( \tilde{\psi}_1 - \tilde{\psi}_2 = 1 \), that is, \( \Delta f = \frac{1}{2} \) to reproduce the suitable value of the Cabibbo angle. There are three ways to avoid this inconsistency:

1. To relax the first requirement.
   For example, \( r = \frac{1}{4} \) is not an unacceptable choice, although rather large ambiguity of \( \mathcal{O}(1) \) coefficients are needed to reproduce the large atmospheric neutrino oscillation.

2. To set \( c \geq \phi \) and introduce an additional discrete symmetry to forbid \( C'AC\Phi \).
   If \( c = \phi \) is taken, the relation \( r = \frac{1}{2} \) is obtained.

3. To give up the effective charge.
   This strategy is examined in detail in Ref.\[14\].

Here, we construct realistic models, along the first and second strategies. In the following, we consider only the case with \( \Delta f = \frac{1}{2} \), for simplicity.

**Strategy 1:** \( c < \phi \) \((r < \frac{1}{2})\)

The relation \( r = \frac{1}{2}(c - \phi + 1) \) indicates that smaller \( \phi - c(>0) \) leads to larger \( r \) bounded from above by \( 1/2 \). Therefore, if \( \phi - c \) is taken as the minimum unit of \( U(1)_A \) charge, then \( r \) acquires the closest value to \( \frac{1}{2} \). Therefore, the smaller unit leads to the closer value of \( r \) to \( \frac{1}{2} \). Here, we introduce half integer \( U(1)_A \) charges and take \( \theta = -\frac{1}{2} \), which gives \( r = \frac{1}{4} \).

In the fourth vacuum (5.37), the SU(2)\(_{H}\) breaking scale is the same as the E\(_6\) breaking scale, because the VEVs \( \langle 1_\Phi \rangle = \langle 1_{\bar{C}_1} \rangle \sim \langle F_2 \rangle \sim \lambda^{-k} \) break simultaneously SU(2)\(_{H}\) and E\(_6\). In order to suppress the FCNC processes, a smaller SU(2)\(_{H}\) breaking scale is preferable, while a smaller E\(_6\) breaking scale leads to a larger effective mass of the colored Higgs, which may spoil the success of the gauge coupling unification and/or result in gauge couplings in the non-perturbative region, as noted in \( \S 4.2.1 \). Taking account of the above conflict, we take \( k = -1 \) here. Thus, the relation \( k = f - \Delta f \) leads to \( f = -\frac{1}{2} \).

Then, the condition for the vacuum structure (5.37), Eq.(5.44), and the condition (5.49) give a relation

\[
2k - f \geq -3\bar{c} - 2f, \tag{5.53}
\]

that is, \( \bar{c} \geq \frac{5}{6} \). Because \( 3k = \bar{c} + f + \phi \), larger \( \bar{c} \) with \( k \) and \( f \) fixed leads to smaller \( \phi \) and thus a larger mass of the colored Higgs, which leads to less natural explanation for the success of the gauge coupling unification. Therefore, we adopt \( \bar{c} = 1 \), which leads to \( \phi = -\frac{7}{2} \) and \( c = -4 \).

Now, Eq.(5.44) and \( \bar{f} \geq -3\bar{c} - 2f \) lead to \( -2 \leq \bar{f} \leq -\frac{2}{3} \). And we take \( \bar{f} = -2 \).

As for \( a \), both \( a = -1/2 \) and \( a = -1 \) are possible. The former yields relatively large FCNC processes because \( \langle A \rangle \) breaks the SU(2)\(_{E}\) symmetry which guarantees the universality of masses of three \( \bar{5} \) sfermion fields. Therefore, we take \( a = -1 \), though the gauge couplings may become in the non-perturbative region.

Table 5.3 shows an example (and those of the strategy 2). The sign \( \pm \) denotes the parity under the additional \( Z_2 \) symmetry that plays the same role as does the \( Z_2 \) symmetry introduced in Table 4.2.
Next, let us examine the second strategy. With the aid of an additional discrete symmetry, we can forbid the interaction $C'AC\Phi$ while we allow $C'\Phi C\Phi$ even when $\phi \leq c$ which always leads to $r \geq \frac{1}{2}$. For example, consider another $Z_2$ symmetry that only $C$ and $Z_C$ have odd parity. Here $z_C < \phi - c$ is required to forbid $C'AC\Phi Z_C$ and to allow $C'\Phi C\Phi$.

In this analysis, we also introduce half-integer charges. Then, as in the previous strategy, we set $(k, f, \bar{c}, \phi, \bar{f}, a) = (-1, -\frac{1}{2}, 1, -\frac{1}{2}, -2, -1)$. For these charges, Eq. 5.44.

Table 5.3: Examples of the charge assignments for the first and second strategies: This charge assignment yields $r = \frac{1}{2} + \frac{c - \phi}{2}$ and $l = -5 - c$. When $c \geq \phi$, we impose an additional $Z_2$ symmetry and introduce a singlet field $Z_C$.

| C | non-vanishing VEV | vanishing VEV |
|---|---|---|
| 78 | $A(a = -1; -)$ | $A'(a' = 5; -)$ |
| 27 | $\Phi(\phi = -7/2; +)$, $C(c = -4, -7/2, -3, -5/2; +)$ | $C'(\epsilon' = 8; -)$ |
| 27 | $\bar{C}_a(\bar{c} = 1; +)$ | $\Psi_3(\psi = 7/4; +)$, $\Psi_a(\psi = 17/4; +)$ |
| 1 | $\bar{F}_a(\bar{f} = -2; +)$, $F_a(f = -1/2; +)$ | $C'(\epsilon' = 11/2; -)$ |
| 27 | $\Theta(\theta = -1/2; +)$, $Z_i(z_i = -3/2; -)$ | |
| 1 | $Z_C(z_C = -1, -1/2, -1, -3/2; +)$ | |

$\int d^4\theta S^\dagger S[|\Psi F|^2 + |\Psi F^\dagger|^2 + \Psi^\dagger A^2 \Psi].$ (5.55)
requires \( c \leq k - \bar{c} + f = -\frac{5}{2} \), which leads to \( c = -\frac{3}{2}, -3, -\frac{5}{2} \). \((a', c', z)\) are also determined as in the previous strategy. We set \( z_C \) as the largest negative value satisfying \( \phi > c + z_C \), and \( \bar{c}' \) is determined to allow \( C'ZC \).

Table 5.3 summarizes the charge assignments.

Here, the matter fields \((\Psi_3, \Psi_a)\) are also shown. From their charges, we can find that \( l = -3/2, -2, -5/2 \) and that \( \Psi \Phi C Z C \bar{C} \bar{F} \) is allowed, which is important to introduce SU(2) \( \text{breaking in Yukawa couplings. Again, the } R\text{-parity is automatically conserved. The effective mass of the colored Higgs is given as } \lambda^{-1/2} \). And the parameter \( \delta_{10} \) and \( \delta_5 \) are given by the same expression as in Eqs. (5.54).

### 5.2.2 \( E_6 \times SU(3)_H \times U(1)_{A} \) Models

In this subsection, we consider \( E_6 \times SU(3)_H \) model, where three \( \Psi \)’s and three \( \overline{27} \)’s \((\bar{C}, \bar{F}, \bar{C}')\) from a triplet and an anti-triplet of SU(3)_H, respectively. In this case, the anomaly of SU(3)_H of the matter sector is cancelled by that of the three \( \overline{27} \)’s, in contrast to the case of \( \overline{5}, \overline{2} \) where some additional fields must be introduced for the anomaly cancellation.

In order to yield the large top Yukawa coupling, SU(3)_H should be broken near the cutoff scale. Suppose that SU(3)_H is broken to SU(2)_H at the cutoff scale by developing the VEVs \( \langle E \rangle = \langle \bar{E} \rangle \sim \lambda^{-\frac{1}{2}(e+\bar{e})} = 1, \) i.e. \( e + \bar{e} = 0 \). Then it can be shown that the effective theory with SU(2)_H can be identified with a certain SU(2)_H model that have the same U(1)_A charges as the effective charges in the effective SU(2)_H model. The essential point is that all the interactions in the SU(2)_H model can be induced from the interactions in the SU(3)_H model. This is a characteristic feature of models where a symmetry is broken at the cutoff scale. For example, \( \lambda^{2\psi_3+\phi}\Psi_3\Phi \) in SU(2)_H model can be obtained from the interaction \( \lambda^{2\bar{e}+\phi}\Psi \bar{E} \Psi \bar{E} \Phi \) by developing the VEV \( \langle \bar{E} \rangle \sim 1 \). Note that the coefficient of the effective interaction is determined by the effective charges, that is, \( \lambda^{2\psi_3+\phi} \langle \bar{E} \rangle^2 \sim \lambda^{2\bar{e}+\phi} \), where \( \psi_3 \) is the effective charge of \( \Psi_3 \) of the effective SU(2)_H model. Therefore, the total charge of an interaction in the SU(3)_H models is nothing but the total effective charge of the corresponding interaction in the effective SU(2)_H model because SU(3)_H is broken at the cutoff scale. Thus, if a term is forbidden by the SUSY-zero mechanism in the SU(3)_H model, the corresponding term in the SU(2)_H model is also forbidden by the SUSY-zero mechanism. Hence, the effective SU(2)_H model can be described by the SU(2)_H model. Conversely, if an SU(2)_H model is found in which the U(1)_A charges are the same as the effective charges of an SU(3)_H model, then an SU(3)_H model can be found straightforwardly. Note that for SU(2)_H models, the arguments in the previous section can be applied, which makes the discussion much simpler.

### SU(2)_H models for SU(3)_H models

In order to extend the horizontal symmetry to SU(3)_H, the difference \( m = \psi - \psi_3 \) is required to be the same as \( m \equiv \bar{c}' - \bar{c} \). The charge assignments shown in Table 5.3 have discrepancy between \( m = \psi - \psi_3 = \frac{3}{2} \) and \( m = \bar{c}' - \bar{c} = \frac{5}{2} \). Note that phenomenologically another choice is to assign odd parity to \( \bar{C} \) and determine \( \bar{c}' \) that \( \bar{C}'ZC \) is allowed. This choice is convenient for embedding the model into \( E_6 \times SU(3)_H \) model, and we consider this possibility later.
viable value of $m$ is around $\frac{5}{2} - 3$. Thus, models with smaller $\bar{m}$ is needed. Since $\left( k + \Delta f, a - \frac{1}{2}, \phi - c - \frac{1}{2}, -c - z_C - z_i \right)$ in $\S 5.2.1$ $\bar{m}$ is written as

$$\bar{m} = \left( \frac{1}{2} - \phi - \left( a - \frac{1}{2} \right) \right) - \bar{c} = 2 \times \frac{1}{2} - 3k + f - a = 2 \times \frac{1}{2} + \Delta f - 2k - a.$$  

(5.56)

This means that in order to obtain a smaller $\bar{m}$, larger $a$ and $k$ are required. We can construct such models (see Table 5.4), although the suppression of the FCNC processes becomes milder:

$$\delta_{10} \sim \left( \lambda^2 \begin{array}{c} \lambda^2 \\ \lambda^3 \\ \lambda \end{array} \right), \quad \delta_5 \sim \left( \lambda^{1+r} \begin{array}{c} \lambda^2 \\ \lambda^2 - r \lambda \\ \lambda \end{array} \right).$$  

(5.57)

| non-vanishing VEV | vanishing VEV |
|-------------------|--------------|
| $A(a = -1/2; -)$  | $A'(a' = 3; -)$ |
| $\Phi(\phi = -7/2; +), C(c = -4, -7/2; +)$ | $C'(c' = 15/2; -)$ |
| $\bar{C}_a(\bar{c} = 2; +)$ | $\Psi_3(\psi_3 = 7/4; +), \Psi_a(\psi = 19/4; +)$ |
| $\bar{F}_a(\bar{f} = -2; +), F_a(f = 0; +)$ | $\bar{C}'(c' = 5; -)$ |
| $\Theta(\theta = -1/2; +), Z_i(z_i = -1; -)$ | $Z_C(z_C = -1/2; +)$ |

Table 5.4: Examples of the charge assignments of SU(2)$_H$ models that can be embedded into SU(3)$_H$ models: When $c \geq \phi$, we impose an additional $Z_2$ symmetry and introduce a singlet field $Z_C$.

In order to improve the suppression of the FCNC processes, we have to change some assumptions. If we employ the other choice of $Z_2$ parity introduced in $\S 5.2.1$ for $\bar{c}'$ as in the footnote there, we can set $\bar{c}' = -c - z_i$ instead of $\bar{c}' = -c - z_C - z_i$, so that $\bar{C}'(A + Z)C$ is allowed. This can reduce $\bar{m}$. We thus can construct a model that can be embedded into an SU(3)$_H$ model with suppression of the FCNC process to the same level as in the models introduced in $\S 5.2.1$ (See Table 5.5). Actually, the parameters $\delta_{10}$ and $\delta_5$ have the same expression as in the Eqs.(5.54).

SU(3)$_H$ models

Now, we treat SU(3)$_H$ models. The Higgs content is summarized in Table 5.6. Each component of the triplet $\Psi_\alpha$ and the anti-triplet $\bar{C}_\alpha$ can be chosen as

$$(\Psi_1, \Psi_2, \Psi_3) \sim (\Psi E F, \Psi \bar{F}, \Psi E)$$  

(5.58)

$$(\bar{C}_1, \bar{C}_2, \bar{C}_3) \sim (\bar{C} \bar{E} \bar{F}, \bar{C} \bar{F}, \bar{C} E),$$  

(5.59)
Table 5.5: Another example of the charge assignments of SU(2)$_H$ models that can be embedded into SU(3)$_H$ models: This charge assignment yields $r = 1$ and $l = -5/2$.

Table 5.6: The Higgs content of $E_6 \times SU(3)_H \times U(1)_A$ models expect for singlets: Here SU(3)$_H$ triplets and anti-triplets are denoted by the lower and upper index $\alpha$, respectively. One or more discrete symmetries are introduced when needed.

and the effective charge of each element is given as

\[
\begin{align*}
\psi &= (\psi + \Delta f + \Delta e/2, \psi - \Delta f + \Delta e/2, \psi - \Delta e) \\
\bar{c} &= (\bar{c} - \Delta f - \Delta e/2, \bar{c} + \Delta f - \Delta e/2, \bar{c} + \Delta e).
\end{align*}
\]

(5.60) (5.61)

This means that, providing $e = -\bar{c} = \Delta e$ and integrating out $E$ and $\bar{E}$, we get an SU(2)$_H$ model where $(\psi, \psi_3, \bar{c}, \bar{c}', \bar{f}, f)$ are given as $(\psi + e/2, \psi_3 - e, \bar{c} - e/2, \bar{c}' + e, \bar{f} - e/2, f + e/2)$ in terms of the charges in the SU(3)$_H$ model. Conversely, we can construct an SU(3)$_H$ model with $e = -\bar{c} = 2$ as shown in Table 5.7 from an SU(2)$_H$ model in Table 5.5. Here, parity assignment of the additional $Z_2$ symmetry for (anti)triplet fields $(\Psi, \bar{C}, F, \bar{F}, E, \bar{E})$ is $(-, +, +, -, -, -)$, so that $\bar{C}_a (a = 1, 2)$ and $\Psi_a (\alpha = 1, 2, 3)$ have even parity while $\bar{C}_3$ has odd parity, and the others have the same parity as in the SU(2)$_H$ model. This parity plays essentially the same role as that in the SU(2)$_H$ model in Table 5.5. The FCNC processes are suppressed to the same level as in models in Table 5.3. This charge assignment yields $r = 1$ and $l = -5/2$. Odd quarter integer charge of the matter field $\Psi_a$ guarantees that the $R$-parity is automatically conserved.

---

\[\text{As for the } Z_2\text{-parities discussed below, we can find those of each component from Eqs. (5.58) and (5.59). In addition, for example, } \bar{C} \bar{E} \bar{C} \Phi \text{ and } \bar{C} \bar{E} \bar{F} Z_C \text{ (whose charges are usually smaller than that of } \bar{C} \bar{E} \bar{F}) \text{ may pick up } C_1 \text{ component with odd } R\text{-parity.} \]
| Non-vanishing VEV | Vanishing VEV |
|------------------|---------------|
| 78 $A(a = -1; -)$ | $A'(a' = 5; -)$ |
| 27 $\Phi(\phi = -7/2; +)$, $C(c = -5/2; +)$ | $C'(c' = 8; -)$ |
| 27 $\bar{C}^\alpha(\bar{c} = 2; +)$ | $\Psi_\alpha(\psi = 15/4; -)$ |
| 1 $F_\alpha(f = -3/2; +)$, $F^\alpha(\bar{f} = -1; -)$ | |
| 1 $E_\alpha(e = 2; -)$, $\bar{E}^\alpha(\bar{e} = -2; -)$ |
| 1 $\Theta(\theta = -1/2; +)$, $Z_i(z_i = -3/2; -)$ |
| 1 $Z_C(z_C = -3/2; +)$ |

Table 5.7: An example of the charge assignments of SU(3)$_H$ models.

### 5.2.3 Summary and Discussion

Here, we have investigated E$_6$ SUSY-GUTs with an anomalous U(1) symmetry and an SU(2)$_H$ or SU(3)$_H$ horizontal symmetry, where some of GUT-breaking Higgs fields belong to non-trivial representations of the horizontal symmetry. We have found it possible to unify the Higgs sectors for the GUT symmetry and the horizontal symmetry. It is interesting that for SU(3)$_H$ models, SU(3)$_H$ gauge anomaly is cancelled between the triplet matter $\Psi_\alpha$ and the anti-triplet Higgs $\bar{C}^\alpha$.

Unfortunately, the unification of the Higgs sectors of the GUT symmetry and the horizontal symmetry yields in too rapid FCNC processes. This is because in the anomalous U(1) GUT scenario, E$_6$ breaking scale is difficult to be smaller than $\lambda^2$, which is the sufficient value for suppressing the FCNC processes. This fact may mean that another mechanism is required to realize the universality of sfermion masses, or that the fields in the Higgs sector of the GUT symmetry do not have non-trivial quantum numbers under the horizontal symmetry. However, we hope that the arguments in this section give a hint to find out a realistic E$_8$ unification.
Chapter 6

Summary

In this thesis, we have introduced a very interesting scenario. This is a kind of SUSY-GUT scenario that employs an anomalous $U(1)$ gauge symmetry, whose anomaly is assumed to be canceled via the Green-Schwarz mechanism. With the aid of this $U(1)$ symmetry, almost all the problems of usual SUSY-GUTs can be solved simultaneously:

- The Doublet-Triplet Splitting problem can be solved via the Dimopoulos-Wilczek type of VEV, with no fine-tuning.
- The success of the gauge coupling unification in the minimal $SU(5)$ SUSY-GUT is naturally explained.
- The nucleon decay via dimension 5 operators can be suppressed, while that via dimension 6 operators is predicted to be enhanced compared to usual SUSY-GUTs.
- Realistic fermion Yukawa matrices can be reproduced. In particular, the neutrino bi-large mixings can be realized in the (almost) minimal matter content.

Surprisingly, these consequences are led from natural assumptions:

- We introduce the “generic interaction”. Namely, we introduce all the possible interaction terms that respect the symmetry of the model, including non-renormalizable terms. In addition, their coupling constants are $O(1)$ in the unit of the cutoff scale $\Lambda$, and we do not introduce hierarchical structures.
- We assume one of the vacua shown in (3.1) is selected as the vacuum of the model.

This means that the definition of a model is given, except for a few parameters, by the definition of a symmetry: a symmetry group, matter content and representations of the matter fields under the symmetry. And thus, the parameters of the models are essentially the anomalous $U(1)$ charges, whose number is the same as that of the superfields ($\sim O(10)$). We have illustrated these characters by using concrete models based on $SO(10)$ or $E_6$. 

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Also we have examined the role of the horizontal symmetry in anomalous U(1) GUT scenario. One of the motivations to introduce a horizontal symmetry is to solve the SUSY-flavor problem. If we construct models so that all the Higgs that is charged in E_6 are singlets under the horizontal symmetry, the SUSY-flavor problem may be solved in E_6 models. For this purpose, however, we have to assume the D terms of the horizontal symmetry are very suppressed compared to F terms. If we aim to construct models so that some 27 (27) Higgs also have nontrivial representations under the horizontal symmetry, the SUSY-flavor problem is not solved sufficiently but ameliorated. Nevertheless, it is still worthwhile to introduce a horizontal symmetry, because it can realize a further unification of matter fields. We hope that the study introduced in §5.2 gives a hint in finding a realistic E_8 unification model.

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Appendix A

Factorization

As mentioned in § 4.2.1, the naive extension of DTS in the SO(10) models into the E\textsubscript{6} models does not work. In the SO(10) DTS, the interaction \((A'A)_{54}(A^2)_{54}\) plays an essential role. In the E\textsubscript{6} models, however, the term \(A'A^3\) does not include the interaction \((45_A'45_A)_{54}(45_A^2)_{54}\). Therefore the superpotential

\[
W_{A'} = \lambda a' A' + \lambda a' a A' (A'A)_{1}(A^2)_{1} + (A'A)_{650}(A^2)_{650} \tag{A.1}
\]

does not realize the DW VEV naturally. Here, we show that the term \(A'A^3\) of E\textsubscript{6} actually does not include the interaction \((45_A'45_A)_{54}(45_A^2)_{54}\) of SO(10).

The VEV of SO(10) adjoint Higgs can be represented as \(\langle A \rangle = \tau_2 \times \text{diag.}(x_1, x_2, x_3, x_4, x_5)\), thanks to the SO(10) rotation and D-flatness condition. In this gauge,

\[
A' A = 2 \sum_i x'_i x_i, \tag{A.2}
\]

\[
(A'A)_{54}(A^2)_{54} = 2 \sum_i x'_i x^3_i - \frac{2}{5} \left( \sum_i x'_i x_i \right) \left( \sum_j x^2_j \right) \tag{A.3}
\]

In the same manner, the VEV of E\textsubscript{6} adjoint Higgs can be represented in the form \(\langle 1_A \rangle = y, \langle 16_A \rangle = \langle \overline{16}_A \rangle = 0, \langle 45_A \rangle = \tau_2 \times \text{diag.}(x_1, x_2, x_3, x_4, x_5)\). In this gauge, the VEV \(\langle A \rangle\) can be represented as 27 \times 27 matrix as

\[
\langle A \rangle = \begin{pmatrix}
\frac{2\sqrt{3}}{\sqrt{3}} y \\
0 \\
\theta^{MNT}_{16} {T}^{M N}_{16} + \frac{1}{2\sqrt{3}} y {1}_{16} \\
0 \\
\theta^{MNT}_{10} {T}^{M N}_{10} - \frac{1}{\sqrt{3}} y {1}_{10}
\end{pmatrix} \tag{A.4}
\]

Here, \(T^i_{M N}\) is the \(i \times i\) matrix representation of SO(10) generators and the summation of the indices \(M\) and \(N\) is understood from 1 to 10 with \(M > N\). Also, \(1_i\) is the \(i \times i\) unit
matrix. Explicitly, we have
\begin{equation}
(T_{10}^{MN})_{KL} = -i (\delta^{M}_{K} \delta^{N}_{L} - \delta^{M}_{L} \delta^{N}_{K}), \quad (A.5)
\end{equation}
\begin{equation}
(T_{16}^{MN})_{\alpha\beta} = \frac{1}{2} (\sigma^{MN})_{\alpha\beta} \\
= \frac{1}{4i} ([\gamma^{M}, \gamma^{N}] P_{R})_{\alpha\beta}, \quad (A.6)
\end{equation}
\begin{equation}
\theta^{MN} = \begin{cases} 
    x_n & M + 1 = N = 2n, \ (n = 1, \ldots, 5) \\
    0 & \text{otherwise},
\end{cases} \quad (A.7)
\end{equation}

where the $\gamma^{M}$ are SO(10) $\gamma$-matrices and $P_{R}$ is the right-handed projector, which can be written as
\begin{align}
\gamma^{1} &= \tau_{1} \otimes 1 \otimes 1 \otimes 1 \otimes 1, \quad (A.8) \\
\gamma^{2} &= \tau_{3} \otimes 1 \otimes 1 \otimes 1 \otimes 1, \quad (A.9) \\
\gamma^{3} &= \tau_{2} \otimes \tau_{1} \otimes 1 \otimes 1 \otimes 1, \quad (A.10) \\
\gamma^{4} &= \tau_{2} \otimes \tau_{3} \otimes 1 \otimes 1 \otimes 1, \quad (A.11) \\
\gamma^{5} &= \tau_{2} \otimes \tau_{2} \otimes \tau_{1} \otimes 1 \otimes 1, \quad (A.12) \\
\gamma^{6} &= \tau_{2} \otimes \tau_{2} \otimes \tau_{3} \otimes 1 \otimes 1, \quad (A.13) \\
\gamma^{7} &= \tau_{2} \otimes \tau_{2} \otimes \tau_{2} \otimes \tau_{1} \otimes 1, \quad (A.14) \\
\gamma^{8} &= \tau_{2} \otimes \tau_{2} \otimes \tau_{2} \otimes \tau_{3} \otimes 1, \quad (A.15) \\
\gamma^{9} &= \tau_{2} \otimes \tau_{2} \otimes \tau_{2} \otimes \tau_{2} \otimes \tau_{1}, \quad (A.16) \\
\gamma^{10} &= \tau_{2} \otimes \tau_{2} \otimes \tau_{2} \otimes \tau_{2} \otimes \tau_{3}, \quad (A.17) \\
\gamma^{11} &= i \gamma^{1} \gamma^{2} \gamma^{3} \gamma^{4} \gamma^{5} \gamma^{6} \gamma^{7} \gamma^{8} \gamma^{9} \tau^{10} \\
&= \tau_{2} \otimes \tau_{2} \otimes \tau_{2} \otimes \tau_{2} \otimes \tau_{2}, \quad (A.18) \\
P_{R} &= \frac{1 + \gamma^{11}}{2}. \quad (A.19)
\end{align}

In this basis, we have
\begin{align}
\theta^{MN} T_{16}^{MN} &= \frac{1}{2} \left( 1 \tau_{2} \otimes 1 \otimes 1 \otimes 1 \otimes 1 \\
&+ x_{2} 1 \otimes \tau_{2} \otimes 1 \otimes 1 \otimes 1 \\
&+ x_{3} 1 \otimes 1 \otimes \tau_{2} \otimes 1 \otimes 1 \\
&+ x_{4} 1 \otimes 1 \otimes 1 \otimes \tau_{2} \otimes 1 \\
&+ x_{5} 1 \otimes 1 \otimes 1 \otimes 1 \otimes \tau_{2} \right) P_{R}, \quad (A.20) \\
&\equiv B, \quad (A.21) \\
\theta^{MN} T_{10}^{MN} &= \tau_{2} \otimes \text{diag} (x_{1}, x_{2}, x_{3}, x_{4}, x_{5}) \quad (A.22) \\
&\equiv C. \quad (A.23)
\end{align}

Before beginning the calculation, we should determine what coupling can occur in the term $A'A^{3}$ of E$_{6}$. Because $78 \times 78 = 1_{s} + 78_{a} + 650_{s} + 2430_{s} + 2925_{a}$, $A'A^{3} \ni$
\((A'A)_{1}(A^2)_{1}, (A'A)_{650}(A^2)_{650}, (A'A)_{2430}(A^2)_{2430}\). On the other hand, because of the completeness,
\[
(A_1 A_2)_{2430} (A_3 A_4)_{2430} = \sum_{l=1.78,650,2430,2925} \lambda_l (A_1 A_4)_l (A_3 A_2)_l. \tag{A.24}
\]

Therefore,
\[
(A'A)_{2430}(A^2)_{2430} = \sum_{l=1.650,2430} \lambda_l (A'A)_l (A^2)_l, \tag{A.25}
\]

which implies that the above three couplings are not independent, and it is sufficient to examine the first two. They are essentially described as \(\text{tr}A'A\text{tr}A^2\) and \(\text{tr}A'A^3\) in matrix language. If the desirable coupling existed, it would apparently be included only in \((A'A)_{650}(A^2)_{650}\) and \(\text{tr}A'A^3\). Thus we can conclude that it does not exist if \(\text{tr}A'A^3\) does not include \(\sum_i x'_ix^3\).

From (A.4), we find
\[
\text{tr}A'A = \frac{4}{3} y'y + \text{tr}_{16} \left[ B'B + \frac{1}{2\sqrt{3}} B'y + \frac{1}{2\sqrt{3}} y'B + \frac{1}{12} y'y \right] \\
+ \text{tr}_{10} \left[ C'C - \frac{1}{\sqrt{3}} C'y - \frac{1}{\sqrt{3}} y'C + \frac{1}{3} y'y \right] \\
= \left( \frac{4}{3} + \frac{16}{12} + \frac{10}{3} \right) y'y + \left( 16 \frac{1}{2^2} + 2 \right) \sum_i x'_ix_i \\
= 6 \left( y'y + \sum_i x'_ix_i \right). \tag{A.26}
\]

Similarly,
\[
\text{tr}A'A^3 = \frac{16}{9} y'y^3 + \text{tr}_{16} \left[ B'B^3 + 3 \frac{1}{12} (B'By^2 + y'By^2) + \frac{1}{144} y'y^3 \right] \\
+ \text{tr}_{10} \left[ C'C^3 + 3 \frac{1}{3} (C'y^2 + y'yC^2) + \frac{1}{9} y'y^3 \right] \\
= \frac{16}{9} y'y^3 + \frac{16}{2^4} \left( 3 \sum_i x'_ix_i \sum_i x_i^2 - 2 \sum_i x'_ix_i^3 \right) \\
+ \frac{3}{12} \frac{1}{2^2} \left( \sum_i x'_ix_i y^2 + y'y \sum_i x'_ix_i \right) + \frac{1}{144} y'y^3 \\
+ \left[ 2 \sum_i x'_ix_i^3 + \frac{3}{3} \left( 2 \sum_i x'_ix_i^2 + y'y^2 \sum_i x'_ix_i \right) + \frac{10}{9} y'y^3 \right] \\
= 3 \left( y'y + \sum_i x'_ix_i \right) \left( y^2 + \sum_i x_i^2 \right) \\
= \frac{1}{12} \text{tr}A'A\text{tr}A^2. \tag{A.27}
\]
Thus, it is explicitly shown that the desirable coupling does not exist because of the group theoretical cancellation between the contributions from the tr\textsubscript{16} part and the tr\textsubscript{10} part.

There are several solutions, and the simplest one is to use the term $\Phi A' A^3 \Phi$. At first glance, it seems to have no effect, because $\Phi \Phi$ is written as

$$
\Phi \Phi = \begin{pmatrix}
\langle \Phi \Phi \rangle & 0 & 0 \\
0 & 0_{16} & 0 \\
0 & 0 & 0_{10}
\end{pmatrix}.
$$

(A.28)

However this form is a special combination of $(\Phi \Phi)_1$, $(\Phi \Phi)_{78}$ and $(\Phi \Phi)_{650}$. In fact, we have

$$
\begin{pmatrix}
\langle \Phi \Phi \rangle & 0 & 0 \\
0 & 0_{16} & 0 \\
0 & 0 & 0_{10}
\end{pmatrix} = \frac{\langle \Phi \Phi \rangle}{54} \left[ 2 \begin{pmatrix}
1 & 0 & 0 \\
0 & 0_{16} & 0 \\
0 & 0 & 0_{10}
\end{pmatrix} + 3 \begin{pmatrix}
4 & 0 & 0 \\
0 & 0_{16} & 0 \\
0 & 0 & -2 \times 0_{10}
\end{pmatrix}
\right. \\
\left. + \begin{pmatrix}
40 & 0 & 0 \\
0 & -5 \times 0_{16} & 0 \\
0 & 0 & 4 \times 0_{10}
\end{pmatrix}
\right],
$$

(A.29)

where the three matrices on the r.h.s. are proportional to the SO(10) singlets of $1$, $78$ and $650$, respectively. Since the interactions for each representation have independent couplings, generically the cancellation is not realized with no fine-tuning.

There are several other solutions for this problem. The essential ingredient is the interaction between $A' A^3$ and some other operator whose VEV breaks E\textsubscript{6}, because the cancellation is due to a feature of the E\textsubscript{6} group. We now introduce some of these solutions.

- Allowing the higher-dimensional term $A' A^5$. Since $\langle A^2 \rangle$ breaks E\textsubscript{6}, the cancellation can be avoided, which can be shown by a straightforward calculation. Since the number of solutions of the F-flatness conditions increases, it becomes less natural to obtain a DW VEV. But the number of vacua is still finite.

- Introducing additional adjoint Higgs fields $B'$ and $B$, and giving $B$ the VEV pointing to an SO(10) singlet direction. Then $B$ plays the same role as the above $(\Phi \Phi)_{78}$. Examining the superpotential

$$
W = B'B + \Phi B' \Phi,
$$

(A.30)

the desired VEV $\langle 1_B \rangle \neq 0$ and $\langle 45_B \rangle = 0$ is easily obtained.
In this appendix, we give the operators that induce the mass matrices of superheavy particles in E\(_6\) models.

First, we examine the operator matrix \( O_{24} \) of 24 in SU(5), which induces the mass matrices \( M_I \) (\( I = X, G, W \)),

\[
O_{24} = \begin{pmatrix}
I \backslash \bar{I} & 45_A(-1) & 45_A'(4) \\
45_A(-1) & 0 & A'A \\
45_A'(4) & A'A & A'^2
\end{pmatrix}, \tag{B.1}
\]

where the numbers in the parentheses denote typical charges.

Next, we examine the operator matrix \( O_{10} \) of 10 in SU(5), which induces the mass matrices \( M_I \) (\( I = Q, U^c, E^c \)),

\[
\begin{pmatrix}
\Phi(16)(2) & \bar{16}_C(2) & \bar{16}_A(2) & 45_A(1) & \bar{16}_C(8) & \bar{16}_A(4) & 45_A'(4) \\
0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\bar{C}'A\Phi & \bar{C}'A\Phi & \bar{C}'A\Phi & \bar{C}'A\Phi & \bar{C}'A\Phi & \bar{C}'A\Phi & \bar{C}'A\Phi \\
\Phi A'A\Phi & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0
\end{pmatrix}, \tag{B.2}
\]

where we have given only one example, even if there are several corresponding operators.

Finally, we examine the operator matrix \( O_{5} \) of 5 and \( \bar{5} \) in SU(5), which induces the mass matrices \( M_I \) (\( I = L, D^c \)),

\[
O_5 = \begin{pmatrix}
0 & 0 & A_5 \\
B_5 & C_5 & D_5 \\
E_5 & F_5 & G_5
\end{pmatrix}, \tag{B.3}
\]
\[ A_5 = \begin{pmatrix}
1 & 10_{\bar{C}}(7) & 10_{\bar{C}}(8) & 16_{\bar{C}}(8) & 16_{A'}(4) \\
10_{\phi}(-3) & C'A\Phi^2 & \bar{C}'(A + Z)\Phi & 0 & 0 \\
10_{\phi}(-3) & 0 & C'(A + Z)\bar{C} & 0 & 0 \\
16_{C}(-6) & 0 & 0 & \bar{C}'(A + Z)\bar{C} & 0 \\
16_{A}(-1) & 0 & \bar{C}'A\bar{C} & \bar{C}'A\Phi & A'A' \\
\end{pmatrix}, \quad (B.4) \]

\[ B_5 = \begin{pmatrix}
1 & 10_{\phi}(-3) & 10_{C}(-6) & 16_{\bar{C}}(8) & 16_{A}(-1) \\
10_{\phi}(-3) & 0 & 0 & 0 & 0 \\
10_{\phi}(-3) & 0 & 0 & 0 & 0 \\
10_{\phi}(2) & 0 & 0 & 0 & \bar{C}'A^2\bar{C} \\
\end{pmatrix}, \quad (B.5) \]

\[ C_5 = \begin{pmatrix}
1 & 10_{\phi}(2) & 10_{C}(-2) & 16_{\bar{C}}(8) \\
10_{C}(-2) & \bar{C}'C' & 0 & 0 \\
16_{\phi}(-3) & 0 & 0 & 0 \\
10_{\phi}(2) & \bar{C}'A\Phi C' & \bar{C}'A\Phi^2 & \bar{C}'A\Phi C & \bar{C}'A\Phi C \\
\end{pmatrix}, \quad (B.6) \]

\[ D_5 = \begin{pmatrix}
1 & 10_{\phi}(-3) & 10_{C}(-6) & 16_{\bar{C}}(8) & 16_{A}(-1) \\
10_{\phi}(-3) & C'(A + Z)\bar{C}' & C'A\Phi C & C'A\Phi^2 & C^2A'A\bar{C} \\
10_{\phi}(2) & \bar{C}'(A + Z)\bar{C}' & \bar{C}'A\Phi C' & \bar{C}'A\Phi C & \bar{C}'A\Phi C \\
16_{\phi}(7) & 0 & 0 & \bar{C}'(A + Z)\bar{C}' & \bar{C}'A\Phi C' \\
16_{A'}(4) & 0 & 0 & 0 & A'A' \\
\end{pmatrix}, \quad (B.7) \]

\[ E_5 = \begin{pmatrix}
1 & 10_{\phi}(2) & 10_{C}(-2) & 16_{\bar{C}}(8) \\
10_{\phi}(2) & \bar{C}'(A + Z)\bar{C}' & \bar{C}'(A + Z)\bar{C}' & \bar{C}'(A + Z)\bar{C}' & \bar{C}'(A + Z)\bar{C}' \\
10_{C}(-6) & 0 & 0 & 0 & 0 \\
16_{\phi}(7) & C'A\Phi^2 & C'A\Phi C & C'A\Phi C & C'A\Phi C \\
16_{A'}(4) & 0 & 0 & 0 & A'A' \\
\end{pmatrix}, \quad (B.8) \]

\[ F_5 = \begin{pmatrix}
1 & 10_{\phi}(2) & 10_{C}(-2) & 16_{\bar{C}}(8) \\
10_{\phi}(2) & \bar{C}'(A + Z)\bar{C}' & \bar{C}'(A + Z)\bar{C}' & \bar{C}'(A + Z)\bar{C}' & \bar{C}'(A + Z)\bar{C}' \\
10_{C}(-2) & C'A\Phi C' & C'A\Phi C' & C'A\Phi C' & C'A\Phi C' \\
16_{\phi}(7) & 0 & 0 & 0 & A'A' \\
16_{A'}(4) & 0 & 0 & 0 & \Phi A'A\Phi \\
\end{pmatrix}, \quad (B.9) \]

\[ G_5 = \begin{pmatrix}
1 & 10_{\phi}(2) & 10_{C}(-2) & 16_{\bar{C}}(8) & 16_{A'}(4) \\
10_{\phi}(2) & \bar{C}'(A + Z)\bar{C}' & \bar{C}'(A + Z)\bar{C}' & \bar{C}'(A + Z)\bar{C}' & \bar{C}'(A + Z)\bar{C}' \\
10_{C}(-2) & C'A\Phi C & C'A\Phi C & C'A\Phi C & C'A\Phi C \\
16_{\phi}(7) & 0 & 0 & 0 & A'A' \\
16_{A'}(4) & 0 & 0 & 0 & \Phi A'A\Phi \\
\end{pmatrix}, \quad (B.10) \]
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