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Multi-Objective Ant Colony Optimization Algorithm Based on Discrete Variables

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Abstract. Based on multi-objective optimization of discrete variable, a new multi-objective ant colony optimization algorithm based on discrete variable has been proposed in this paper. The improved multi-objective ant colony algorithm established two databases on the basis of P-ACO algorithm, including sets of feasible solutions and non-feasible solutions, and “the repeated solutions” was replaced with “the special solutions” to acquire the Pareto optimal front-end of the multi-objective problems. This algorithm is not only a better application in solving multi-objective problem in the discrete variables system, but also can be applied to the truss structure optimization successfully. The present paper proposes a multi-objective ant colony optimization algorithm based on discrete variable, which can provide a good performance in program design, arithmetic speed and generality of the proposed method. It is also simple and practical, and suitable for projects in the future.

1. Introduction

Most engineering and scientific problems are multi-objective problems, and there are multiple objectives that conflict with each other. Therefore, how to acquire the optimal solution to these problems has always been the focus of academic and engineering fields. In the 1990s, many researchers successively proposed weighted method, goal programming method, ε-constraint method and other weight-based multi-objective optimization algorithm [1-2]. During this period, multi-objective optimization algorithm and its theoretical research began to draw people’s attention. With the gradual rise of evolutionary algorithms and the increased complexity of multi-objective problems, multi-objective evolutionary algorithms have received more extensive attention, including multi-objective particle swarm optimization algorithm, multi-objective genetic algorithm, multi-objective optimization immune algorithm, multi-objective ant colony optimization algorithm, etc [3-5].

In civil engineering, the design variables of structural multi-objective optimization problems are often discrete, thus multi-objective ant colony optimization algorithm can better meet the requirements in this respect in many multi-objective evolutionary algorithms. Although many researchers have
proposed methods such as: Multiple Objective Ant-Q Algorithm, Ant Algorithm for Bi-criterion Optimization Problem, Multi Colony for Bi-criterion Optimization Problem, Pareto Ant Colony Optimization and Multiple Ant Colony System, etc [6-7]. However, it is undeniable that these algorithms are far from satisfying people’s requirements. Especially for the structural optimization design in civil engineering, the existing multi-objective ant colony algorithm cannot fully meet the engineering requirements in terms of performance and function. Therefore, how to propose a more effective multi-objective ant colony algorithm has been the goal that the researchers have been striving for.

2. Improved multi-objective ant colony algorithm

2.1. P-ACO algorithm

Pareto Ant Colony Optimization [8-9] (hereinafter referred to as P-ACO) was first proposed by Doerner et al. in 2001, mainly to solve the multi-objective portfolio selection problem, and has achieved very good effects. In the P-ACO algorithm, the pheromone concentration matrix is defined by each objective function of the multi-objective problems. The details are as follows:

\[
j = \arg \max_{j \in \Omega(x)} \left\{ \left( \sum_{k=1}^{K} (p_i \cdot \tau_{ij}^k) \right)^{\alpha} \cdot \left[ \eta_i(x) \right]^{\beta} \right\} \quad q \leq q_0 \quad \text{Otherwise}
\]

\[
\rho \cdot \tau_{ij}^k = \left( 1 - \rho \right) \cdot \tau_{ij}^{k-1} + \rho \cdot \tau_0
\]

Where, \( K \) is the number of objective functions; \( \eta_{ij} \) is the pheromone concentration of the original settings on the path \((i, j)\); \( q \) is a random number within the range \([0,1]\); \( q_0 \) is the design parameter specified in the ant colony algorithm \((0 \leq q_0 < 1)\), used as the basis for determining the next node the ant selects at one node; \( \hat{\Omega} \) refers to the probability distribution of the ant colony selecting the next node at one node, which satisfies the following formula:

\[
p_i(x) = \left\{ \begin{array}{ll}
\sum_{k=1}^{K} (p_i \cdot \tau_{ij}^k)^{\alpha} \cdot \left[ \eta_i(x) \right]^{\beta} & i \in \Omega \\
0 & \text{Otherwise}
\end{array} \right.
\]

This probability distribution is based on the parameters \( \alpha \) and \( \beta \), in which \( \alpha \) is the importance degree of the residual information on the path; \( \beta \) is the importance degree of the heuristic information \((\beta \geq 0)\).

For different objective functions in the iterative process, each ant will use the local pheromone to update the pheromone concentration matrix on any boundary of the objective function. The specific formula is as follows:

\[
\tau_{ij}^k = (1 - \rho) \cdot \tau_{ij}^{k-1} + \rho \cdot \tau_0
\]

Where, \( \rho \) is the pheromone residual coefficient; \( \tau_0 \) is the initial pheromone concentration value.

The global pheromone update is made up of ants that find a feasible solution. The pheromone concentration update rules for the unused objective function are as follows:

\[
\tau_{ij}^k = (1 - \rho) \cdot \tau_{ij}^{k-1} + \rho \cdot \Delta \tau_{ij}^k
\]

Where, the value of \( \Delta \tau_{ij}^k \) satisfies:
$$\Delta t^{*}_{ij} = \begin{cases} 
\lambda_i & \text{if } X_j \text{ is a non-feasible solution} \\
\lambda_2 & \text{if } X_j \text{ is a feasible solution and } X_i < X_j \\
\lambda_3 & \text{if } X_j \text{ is a feasible solution and } X_i \text{ and } X_j \text{ are non-dominant} \\
\lambda_4 & \text{if } X_j \text{ is a feasible solution and } X_i > X_j 
\end{cases} \quad (5)$$

Where, \(i, j = 1, 2, 3, \ldots, N\) \((i \neq j)\), \(N\) is the ant colony size; \(\lambda_1, \lambda_2, \lambda_3\) and \(\lambda_4\) \((\lambda_4 > \lambda_3 > \lambda_2 > \lambda_1)\) are the four control parameters of the multi-objective P-ACO algorithm, and \(\lambda_4 = 15, \lambda_3 = 10, \lambda_2 = 5, \lambda_1 = 0\).

### 2.2. Improved multi-objective ant colony algorithm

In order to better adapt to the multi-objective structural design optimization problems, it is necessary not only to reduce the occurrence of repeated solutions in the iterative process, but also to improve the mass of the new solution. To this end, based on the P-ACO algorithm, this paper puts forward the concept of “adding alternative repeated solutions”. The specific methods are as follows:

1. In the P-ACO algorithm, establish a “feasible solution set, FP” and “non-feasible solution set, NFP”, record each new solution one by one, and compare with the new solution generated from each iteration \((NC > 1)\). If the new solution already exists, it will be replaced by a “special solution”. On the contrary, there is no effect.

2. This “special solution” is a solution in the front-end of the current iterative solution of the random selection algorithm, selecting its surrounding solutions in a specific order, and the solution satisfies the condition that not existing in “feasible solution set, FP” and “non-feasible solution set, NFP”, and such a solution is defined as the current “special solution”. This solution will be used to replace the repeated solution that occurs during this iterative process.

3. The weight of the release concentration set by each ant colony for different objective functions in the P-ACO algorithm is cancelled, and the improved P-ACO algorithm adopts the classic AS mode as the selection mode.

### 2.3. Improved multi-objective ant colony algorithm flow chart

1. Initialize the P-ACO algorithm (including: ant colony size, maximum number of iterations, initial pheromone concentration, etc.);

2. Initialize the external “feasible solution set, FP” and “non-feasible solution set, NFP” of the P-ACO algorithm, and both are empty sets;

3. Set the number of iterations: \(NC = 0\);

4. Command: ant \(i = 1\);

5. The current ant \(i\) uses the AS mode to optimize;

6. In the \(i = i + 1\) step, if \(i \leq m\) \((m\) is ant colony size), skip to step (5);

7. Judge whether the new solution is a repeated solution. If the “new solution” \(\in\) FP or “new solution” \(\in\) NFP, randomly generate a new solution to replace the repeated solution around a solution of the current Pareto front-end, otherwise continue;

8. Update the set FP and the set NFP;

9. Update the Pareto front-end PF set;

10. Update the amount of information on different objective function paths in accordance with equations (4) and (5);

11. In step \(NC = NC + 1\), if \(NC < NC_{max}\), skip back to step (4); otherwise, the loop ends and output the result of program calculation.
3. Simulation experiment and data result analysis

3.1. BNH problem illustrative example

The method of this paper is mainly for the multi-objective optimization problem of truss structures based on discrete variables. Therefore, this paper transforms the continuous BNH problem [10] into the discrete problem, that is, divides the value range of continuous variables into $n$ equal parts (i.e.: the value range of $x_1$ and the value range of $x_2$ are divided into 100 equal parts), which converts continuous variables into discrete variables. The parameters of the multi-objective ant colony algorithm are shown in Table 1.

| Parameters | $\alpha$ | $\beta$ | $\rho$ | $Q$ | $m$ | $NC_{\text{max}}$ |
|------------|----------|---------|--------|-----|-----|------------------|
| Values     | 1        | 0.2     | 0.1    | 1   | 30  | 200              |

\[
\begin{align*}
\text{min:} & \quad F(x) = \{f_1(x), f_2(x)\} \\
f_1(x) &= 4x_1^2 + 4x_2^2 \\
f_2(x) &= (x_1 - 5)^2 + (x_2 - 5)^2 \\
\text{s.t.:} & \quad C_1(x) = (x_1 - 5)^2 + x_2^2 \leq 25 \\
& \quad 0 \leq x_1 \leq 5 \\
& \quad 0 \leq x_2 \leq 3 \\
& \quad C_2(x) = (x_1 - 8)^2 + (x_2 + 3)^2 \geq 7 \\
& \quad 0 \leq x_1 \leq 5 \\
& \quad 0 \leq x_2 \leq 3 \\
\end{align*}
\]

The BNH problem is a bivariate constraint problem, and its Pareto front-end is connected for easy calculation and solution. The specific results solved by the method herein are as follows:

![Figure 1: The feasible solutions under discrete domain on BNH problem](image1.png)

![Figure 2: Calculation result on BNH problem](image2.png)

Figure 1 shows all feasible solutions after the BNH problem is transformed into a discrete problem. Figure 2 shows the Pareto front-end solved by the algorithm herein. By comparison with Figure 1, it is indicated that the Pareto front-end acquired by the method herein is completely consistent with the real result, thus the optimization method is trustworthy.

3.2. Multi-objective optimization illustrative example of truss structure

The plane bar truss structure is shown in Figure 3, including: 6 nodes, 10 variables, and the material is aluminum: $E = 68.96 GPa$, $\rho = 27150.8 N/m^3$, the allowable stress of all the bars is $\pm 172\,4MPa$. The downward concentrated force $P = 444.89 kN$ is applied at nodes 2 and 4. The discrete variable set
of sections (unit: $cm^2$) is shown in Table 2. The values of the main control parameters of the improved multi-objective ant colony algorithm are shown in Table 1. The two objective functions are the mass of the structure and the vertical displacement of the model node 2, respectively. In addition, the two types of objective functions in the illustrative examples conflict with each other [11,12].

| Serial No. | Area    | Serial No. | Area    | Serial No. | Area    | Serial No. | Area    |
|-----------|---------|------------|---------|------------|---------|------------|---------|
| 1         | 0.645   | 2          | 3.225   | 3          | 6.450   | 4          | 12.90   |
| 5         | 19.35   | 6          | 25.80   | 7          | 32.25   | 8          | 38.70   |
| 9         | 41.93   | 10         | 45.15   | 11         | 48.38   | 12         | 51.60   |
| 13        | 54.83   | 14         | 58.05   | 15         | 61.28   | 16         | 64.50   |
| 17        | 70.95   | 18         | 77.40   | 19         | 83.85   | 20         | 90.30   |
| 21        | 96.75   | 22         | 103.2   | 23         | 109.7   | 24         | 116.1   |
| 25        | 122.6   | 26         | 129.0   | 27         | 135.5   | 28         | 141.9   |
| 29        | 148.4   | 30         | 154.8   | 31         | 161.3   | 32         | 167.7   |
| 33        | 174.2   | 34         | 180.6   | 35         | 187.1   | 36         | 193.5   |

Figure 3. 10-bar plane truss structure

Figure 4. Pareto optimal solutions and algorithm comparison

This paper not only realizes the application of multi-objective ant colony algorithm in multi-objective optimization design of truss structure, but also simulates the optimization process of the algorithm through MATLAB language. The Pareto front-end of the optimization result is shown in Fig. 4. The multi-objective optimization result acquired by this method is within the range of $[1.373318, 116146.7]$ and $[6.286536, 18231.27]$. The Pareto front-end consists of 344 kinds of solutions, and the "green + line" in Figure 4 is the Pareto front-end searched herein. Since the discrete domains are adopted as the design variable herein, the range and shape of the Pareto front-end are different from the traditional methods based on continuous variables, but the trend is consistent, and the method herein is more suitable for practical engineering.

Through the comparison in Fig. 4, we can see that the Pareto front-end searched by the method herein is not only basically consistent with the results of traditional $\varepsilon$-constraint method (pink ○ circle line), Chebyshev method (bright blue × line), weighted method (bright red + word line) and the multi-objective immune algorithm [12], the distribution is relatively uniform, and the feasible solution of the
Pareto front-end is more complete and diverse. In addition, the method herein has obvious advantages in solution speed, constraints, practicability, etc.

4. Conclusion
According to the aforesaid discussion and the verification of several multi-objective illustrative examples, the proposal of a multi-objective ant colony optimization algorithm based on discrete variables herein is feasible and effective. The analysis and calculation of multiple illustrative examples further prove the feasibility and practicability of the algorithm. The specific analysis conclusions are as follows:

a). The multi-objective ant colony optimization algorithm based on discrete variables is an efficient multi-objective optimization algorithm. It shows good optimization effects in solving multiple objective functions, multi-objective section optimization of truss structure, and multi-objective section optimization of transmission tower structure.

b). For the simple multi-objective function discrete problem, the Pareto optimal front-end solved by the method herein is consistent with the real result, and the whole Pareto front-end solution points are complete and in uniform distribution.

c). Compared with the traditional multi-objective optimization algorithm in solving the multi-objective section optimization problem of truss structure, this algorithm not only possesses the same advantages as solving the multi-objective function, but also takes a relatively short time to acquire the satisfactory Pareto front-end, and the types of the solutions are more diverse.

d). This algorithm shows a good superiority in solving structural multi-objective optimization design problems, thus further proves its application value in practical engineering, and provides theoretical basis and design ideas for multi-objective topology, shape and layout optimization of the truss structure in the future.

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