Reexamination of the resonance contributions in

\[ B \rightarrow X_s e^+ e^- \]

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Abstract

With help of the recent developments in the heavy quarkonium physics, we reexamine the long distance(LD) effects in \( B \rightarrow X_s e^+ e^- \) dominantly from the charmonium resonances \( J/\Psi \) and \( \Psi' \) through the decay chains \( B \rightarrow X_s J/\Psi(\Psi') \rightarrow X_s e^+ e^- \). We find that the resonance to nonresonance interference are reduced substantially.

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The rare decay $B \to X_s e^+ e^-$ has been the subject of many theoretical studies in the framework of the standard model and its extensions such as the two Higgs doublet models and models involving supersymmetry \cite{1-8}. It is believed that once $B \to X_s e^+ e^-$ is observed, it will offer a useful probe of the standard model and of its extensions. To this end the relevant branching ratio, the dilepton invariant mass distribution and other distributions of interest should be calculated with sufficient precision before reliable conclusion be drawn.

At present, the effective Hamiltonian for $B \to X_s e^+ e^-$ decays relevant for scale $\mu \approx m_b$ in which the short distance QCD effects taken into account has been calculated to the next-to-leading order \cite{9-11} in the framework of a renormalization group improved perturbation theory, and the Wilson coefficients in the Hamiltonian are presented to have rather small dependence on $\mu$ and $\Lambda_{\overline{MS}}$.

The actual calculation of $B \to X_s e^+ e^-$ involves not only the evaluation of Wilson coefficients of ten local operators but also the calculation of the corresponding matrix elements of these operators relevant for $B \to X_s e^+ e^-$, which has been studied in HQET including the non-perturbative $O(1/m_b^2)$ corrections enhanced the rate for $B \to X_s e^+ e^-$ by roughly 4% \cite{12,13}. A realistic phenomenological analysis should also include the long distance contributions which are mainly due to the $J/\psi$ and $\psi'$ resonances \cite{13-17}. However, there are large uncertainties in modeling estimation of such effects and further study is needed. In the papers \cite{13-17}, the phenomenological parameter $a_2$ has to be input in the sub-amplitude $A(b \to sJ/\Psi)$ to compensate the large discrepancies between theoretical predictions and experimental data for $Br(B \to J/\Psi X_s)$. However, such inputs would lead to overestimation of the LD contributions as it will be presented in what follows.

In this paper, we re-examine the LD contributions $B \to J/\Psi X_s \to X_s e^+ e^-$ stimulated by the recent significant progress in heavy quarkonium physics \cite{18,19}. We will show that the LD contributions can be substantial reduced when the subamplitude $A(b \to sJ/\Psi)$ decomposed into two different parts which do not interfere with each other: color singlet and color-octet parts as in NRQCD \cite{13}.
We will start with the effective Hermitian given as follows [11]

\[ H_{\text{eff}} = -\frac{4G_F}{\sqrt{2}} V_{ts}^* V_{tb} \sum_{i=1}^{10} C_i O_i \]  

(1)

where the operator basis is chosen to be

\begin{align*}
Q_1 &= (\bar{s}_\alpha c_\beta)_{V-A} (\bar{c}_\beta b_\alpha)_{V-A}, \\
Q_2 &= (\bar{s}c)_{V-A} (\bar{e}b)_{V-A}, \\
Q_3 &= (\bar{s}b)_{V-A} \sum_q (\bar{q}q)_{V-A}, \\
Q_4 &= (\bar{s}_\alpha b_\beta)_{V-A} \sum_q (\bar{q}_\beta q_\alpha)_{V-A}, \\
Q_5 &= (\bar{s}b)_{V-A} \sum_q (\bar{q}q)_{V+A}, \\
Q_6 &= (\bar{s}_\alpha b_\beta)_{V-A} \sum_q (\bar{q}_\beta q_\alpha)_{V+A}, \\
Q_7 &= \frac{e^8}{8\pi} m_b \bar{s}_\alpha \sigma^{\mu\nu} (1 + \gamma_5) b_\alpha F_{\mu\nu}, \\
Q_8 &= \frac{e^8}{8\pi} m_b \bar{s}_\alpha \sigma^{\mu\nu} (1 + \gamma_5) T_{\alpha\beta}^a b_\beta G_{\mu\nu}^a, \\
Q_9 &= (\bar{s}b)_{V-A} (\bar{e}e)_{V}, \\
Q_{10} &= (\bar{s}b)_{V-A} (\bar{e}e)_{A},
\end{align*}

(2)

where \( \alpha \) and \( \beta \) denote color indices, \( L \) and \( R \) denote chiral projections \( L(R) = 1/2(1 \mp \gamma_5) \).

The Wilson coefficients can be found in [11, 21–26].

Using the effective Hamiltonian, the SM-based short distance (SD) matrix element for the decay \( b \to se^+e^- \) can be written as

\[ M(b \to se^+e^-) = \frac{G_F \alpha}{\sqrt{2}\pi} V_{ts}^* V_{tb} \left[ \left( C_9^{eff} - C_{10} \right) (\bar{s}\gamma_\mu Lb)(\bar{e}\gamma^\mu Le) + \left( C_9^{eff} + C_{10} \right) (\bar{s}\gamma_\mu Lb)(\bar{e}\gamma^\mu Re) - 2C_7^{eff} (\bar{s}\sigma_{\mu\nu} q^\nu q^\rho m_b Rb)(\bar{e}\gamma^\mu e) \right]. \]  

(3)

Introducing

\[ \hat{s} = \frac{q^2}{m_b^2} = \frac{(p_{e^+} + p_{e^-})^2}{m_b^2}, \quad z = \frac{m_c}{m_b} \]  

(4)

and taking the spectator approximation, one finds

\[ \frac{dBr(B \to X_se^+e^-)}{d\hat{s}} = Br(b \to ce\bar{\nu}) \frac{\alpha^2}{4\pi^2} \frac{V_{ts}^2}{V_{cb}} \left( \frac{1 - \hat{s}}{2z} \right)^2 \left( 1 + 2\hat{s} \right) \left( |\tilde{C}_9^{eff}|^2 + |\tilde{C}_{10}|^2 \right) + \]

2
where
\[
\tilde{C}_9^{\text{eff}} = \tilde{C}_9^{\text{NDR}} \tilde{\eta}(\hat{s}) + h(z, \hat{s}) \left( 3 C_{1(0)} + C_{2(0)} + 3 C_{3(0)} + C_{4(0)} + 3 C_{5(0)} + C_{6(0)} \right) \\
- \frac{1}{2} h(1, \hat{s}) \left( 4 C_{3(0)} + 4 C_{4(0)} + 3 C_{5(0)} + C_{6(0)} \right) \\
- \frac{1}{2} h(0, \hat{s}) \left( C_{1(0)} + 3 C_{4(0)} \right) + \frac{2}{9} \left( 3 C_{3(0)} + C_{4(0)} + 3 C_{5(0)} + C_{6(0)} \right),
\]
and
\[
h(z, \hat{s}) = \frac{8}{9} \ln \frac{m_b}{\mu} - \frac{8}{9} \ln z + \frac{8}{27} + \frac{4}{9} x \\
- \frac{2}{9} (2 + x) |1 - x|^{1/2} \left\{ \begin{array}{ll} \\
\ln \left| \frac{\sqrt{1 - x} + 1}{\sqrt{1 - x} - 1} \right| - i\pi, & \text{for } x \equiv \frac{4 z^2}{s} < 1 \\
2 \arctan \frac{1}{\sqrt{2} - 1}, & \text{for } x \equiv \frac{4 z^2}{s} > 1,
\end{array} \right. \\
h(0, \hat{s}) = \frac{8}{27} - \frac{8}{9} \ln \frac{m_b}{\mu} - \frac{4}{9} \ln \hat{s} + \frac{4}{9} i\pi.
\]
\[
f(z) = 1 - 8 z^2 + 8 z^6 - z^8 - 24 z^4 \ln z, \\
\kappa(z) = 1 - \frac{2 \alpha_s(\mu)}{3\pi} \left[ (\pi^2 - \frac{31}{4})(1 - z)^2 + \frac{3}{2} \right], \\
\tilde{\eta}(\hat{s}) = 1 + \frac{\alpha_s(\mu)}{\pi} \omega(\hat{s}),
\]
with
\[
\omega(\hat{s}) = \frac{2}{9} \pi^2 - \frac{4}{3} \text{Li}_2(\hat{s}) - \frac{2}{3} \ln \hat{s} \ln(1 - \hat{s}) - \frac{5 + 4\hat{s}}{3(1 + 2\hat{s})} \ln(1 - \hat{s}) - \\
\frac{2 \hat{s}(1 + \hat{s})(1 - 2\hat{s})}{3(1 - \hat{s})^2(1 + 2\hat{s})} \ln \hat{s} + \frac{5 + 9\hat{s} - 6\hat{s}^2}{6(1 - \hat{s})(1 + 2\hat{s})}. 
\]

Where \( f(z) \) and \( \kappa(z) \) are the phase-space factor and the single gluon QCD correction to the \( b \rightarrow ce\bar{\nu} \) decay [27, 28] respectively. \( \tilde{\eta} \) represents single gluon corrections to the matrix element of \( Q_9 \) with \( m_s = 0 \) [10, 29].

Now we implement the LD contributions in \( B \rightarrow X_s e^+e^- \). The resonance amplitude, which includes a Breit-Wigner form for the intermediate \( \Psi(nS) \) state, is given by the well known formula
\[
A(B \rightarrow X_s V \rightarrow X_s e^+e^-) = \frac{G_F \alpha}{\sqrt{2\pi}} V_{ts}^* V_{tb} (\bar{s} \gamma_\mu L b)(\bar{e} \gamma^\mu e)
\]
\begin{equation}
\times \left( C_1 + \frac{C_2}{3} \right) \frac{16\pi^2}{3} \frac{f_V^2/m_V^2}{m_V^2 - s - i\Gamma m_V},
\end{equation}

where $f_V$ is the decay constant of vector meson $V$ ($\Psi, \Psi'$) determined by

\begin{equation}
\Gamma(V \to e^+e^-) = \frac{16\pi\alpha^2}{27m_V^3} f_V^2.
\end{equation}

In the numerical calculations in the literature [13–17], the combination of the Wilson coefficients $C_1 + C_2/3$ is treated as a phenomenological parameter $a_2$ with a value 0.24 on the consideration of the experiment data $Br(B \to X_sJ/\Psi) = 0.8 \pm 0.08\%$, which is much larger than the QCD prediction $C_1 + C_2/3 = 0.12$, because the usual calculation using the Wilson coefficient would produce very low predictions of the branching ratio $Br(B \to X_sJ/\Psi)$. However, the theory of heavy quarkonium production and decays has recently undergone a number of significant developments [18, 19], where color-octet mechanism and factorization have been developed. It has been shown that the color-octet contributions compete with the color-singlet contributions in the process $B \to X_sJ/\Psi$ [30] and the predictions agree with the experiment data.

Including the color-octet contributions, one can get

\begin{equation}
A(B \to X_s\Psi \to X_se^+e^-) = \frac{G_F\alpha}{\sqrt{2}\pi} V_{ts}^* V_{tb} \left[ C^{\text{sig}} \langle X_s^1 | \bar{s} \gamma_\mu L | b \rangle \langle \Psi | \bar{c} \gamma^\mu L | X^1 \rangle \\
+ C^{\text{oct}} \langle X_s^8 | \bar{s} \gamma_\mu L T^a | b \rangle \langle \Psi | \bar{c} T^a L | X^8 \rangle \right] \\
\times \bar{e} \gamma^\mu e \frac{16\pi^2}{3} \frac{f_\Psi/m_\Psi^2}{m_\Psi^2 s - i\Gamma m_\Psi} + (\Psi \to \Psi'),
\end{equation}

where $C^{\text{sig}} = (C_1 + C_2/3)$ and $C^{\text{oct}} = 2C_2$. The term proportional to $C^{\text{sig}}$ is the color singlet amplitude and the term proportional to $C^{\text{oct}}$ is the color-octet amplitude. The color-singlet contribution to $B \to X_s l^+l^-$ can be included directly using eq.(13) as a modification to $\tilde{C}_3^{\text{eff}}$ as the usual cases in [13–17]. However, the color-octet contribution can not be treated in such way, because the color-octet amplitude has different color structure from the SD amplitude in eq.(3) and does not interfere with it within NRQCD [18, 19] just as the cases in [30, 32], so, the LD contribution is reduced, especially, in the low $q^2$ region. Based on such an observation, we have

\begin{equation}
\frac{dB_r(b \to se^+e^-)}{ds} = B_r(b \to e\bar{e}\bar{\nu}) \frac{\alpha^2}{4\pi^2} \left| \frac{V_{ts}}{V_{cb}} \right|^2 (1 - \hat{s})^2 \left( 1 + 2\hat{s} \right) \times
\end{equation}
\begin{align*}
\left( |\tilde{C}^{eff}_9 + Y^{res}(\hat{s})|^2 + |\tilde{C}_{10}|^2 + |C_{oct}(\hat{s})|^2 \right) + \\
4 \left( 1 + \frac{2}{\hat{s}} \right) |C^{(0)eff}_7|^2 + 12 C^{(0)eff}_7 \text{ Re}[\tilde{C}^{eff}_9 + Y^{res}(\hat{s})], \tag{16}
\end{align*}

with

\begin{align*}
Y^{res}(\hat{s}) &= C^{sig} \frac{16\pi^2}{3} \frac{f_\Psi^2/m_\Psi^2}{m_\Psi^2 - \bar{s}m_b^2 - i\Gamma_\Psi m_\Psi} + (\Psi \rightarrow \Psi'), \tag{17}
\end{align*}

\begin{align*}
|C_{oct}(\hat{s})|^2 &= C^{2}_{oct} \frac{16\pi^2}{3} \frac{3 \langle O_8^{(3S_1)} \rangle}{2 \langle O_1^{(3S_1)} \rangle} \frac{f_\Psi^4/m_\Psi^4}{(\bar{s}m_b^2 - m_\Psi^2)^2 + \Gamma_\Psi^2 m_\Psi^2} + (\Psi \rightarrow \Psi') \tag{18}
\end{align*}

where \(O_{1,8}^{\Psi,\Psi'}(3S_1)\) are defined in [19]. The other colored \(\bar{c}c\) pairs produced at short distance with different quantum number \(^1S_0\) and \(^3P_J\) contribute to \(J/\Psi, \Psi'\) productions with same Wilson coefficients \(C_{oct}\).

\[\Gamma(b \rightarrow J/\Psi + X) \propto 3C^{2}_{oct}(m_b^2 + 8m_c^2) \langle O_8^{(3S_1)} \rangle \]
\[+ 9C^{2}_{oct} m_b^2 \langle O_8^{(3S_0)} \rangle + 6C^{2}_{oct} (m_b^2 + 8m_c^2) \langle O_8^{(3P_1)} \rangle. \tag{19}\]

The color-octet matrices can be calculated by Lattice calculations or fitted out from experiment data. For simplicity, one can parameterize color octet contributions using only one matrix \(\langle O_8^{(3S_1)} \rangle\) in eq(18). Cho and Leibovich have performed a fit to CDF data including the color-octet contribution and found

\begin{align*}
\langle O_8^{(3S_1)} \rangle &= 1.2 \times 10^{-2} GeV^3, \tag{20}
\langle O_8^{(3S_1)} \rangle &= 7.3 \times 10^{-3} GeV^3. \tag{21}
\end{align*}

Furthermore, using these values, it has been shown that theoretical predictions agree with experiment results of \(Br(B \rightarrow \Psi X_s)\) and \(Br(B \rightarrow \Psi' X_s)\) respectively [30, 34]. The color-singlet matrix elements determined from leptonic decays of \(\Psi\) and \(\Psi'\) are listed as

\begin{align*}
\langle O_1^{\Psi}(3S_1) \rangle &= 1.32 GeV^3, \tag{22}
\langle O_1^{\Psi'}(3S_1) \rangle &= 0.53 GeV^3. \tag{23}
\end{align*}

The numerical results are presented in Fig.1. The thin solid line is the short distance contribution, the dotted line is the result including long distance contribution as the results
in ref \cite{13–17}, the dotted-dash line is our new result. We can see that the LD effects are significantly reduced, especially in the region of theoretical and experimental interesting $\hat{s} < 0.31$ (corresponding the dilepton invariant mass $\sqrt{\hat{s}} < 0.9m_\Psi$), where short distance physic might be extracted safely. Here, we recall that there are large uncertainties in the color matrix elements. However, the uncertainties would not spoil our conclusion if the color-octet mechanism is true and provides solutions to the large heavy quarkonium production data.

Note that in extrapolating the dilepton invariant masses away from the resonance region, no extra $q^2$ dependence is included in the $V_i - \gamma^*(q^2)$ junction. (The $q^2$ dependence written explicitly in eq.(3) is due to the Breit-Wigner shape of the resonance). Introducing the momentum dependence junction strength defined by

$$\langle 0 | \bar{c} \gamma_\mu c | V(q) \rangle = f_V(q^2) \epsilon_\mu$$

and using the formula derived by Terasaki \cite{35}

$$f_V(q^2) = g_V(0)(1 + \frac{q^2}{C_V}[d_V - h(q^2)])$$,

where $C_\Psi = 0.54$, $C_\Psi' = 0.77$, $d_\Psi = d_\Psi' = 0.043$ and

$$h(q^2) = \frac{1}{16\pi^2 r} \left[ -4 - \frac{20r}{3} + 4(1 + 2r)\sqrt{\frac{1}{r} - 1} \arctan \frac{1}{\sqrt{\frac{1}{r} - 1}} \right]$$

with $r = q^2/m_V^2$ for $0 \leq q^2 \leq m_V^2$. For $m_V^2 \leq q^2$, $f_V(q^2) = f_V(m_V^2)$ which is determined by eq(13). Such a consideration leads to another considerable suppression on LD contribution in the low dilepton invariant mass region. The numerical results present in the Fig.1 by a dash line in the region $\hat{s} \leq 0.35$.

In summary, we have investigated the long distance effects in $B \rightarrow X_s e^+ e^-$ including color-octet contribution. We have shown that the LD effects are reduced substantially due to the color-octet amplitude not interfering with the SD amplitude and the color-singlet amplitude. This result is important for extracting short distance physics which is a good probe for the physics beyond the Standard Model. Furthermore, if the momentum dependence of the $V_i - \gamma^*(q^2)$ conversion strength is considered, the long distance effects in the low dilepton
invariant mass region is negligible small. The main uncertainty in this paper stems from the validity of the application of NRQCD factorization formalism to charmonium production from B meson decays [30–32, 34]. NRQCD factorization formalism is well realized in the studies of high \( P_T \) quarkonium production at hadron colliders, however, in the case of B decays, there is very little excess energy left over. In such a situation, one should pay attention to the validity. To our knowledge, the validity of such a particular application of color-octet heavy quarkonium production ideas has never been justified. Given the validity, another problem still remains obscure. The soft gluon emitted or absorbed by the colored \( \bar{c}c \) may obscure dilepton distribution, however, it is unfortunate that little progress has been made in understanding the role played by these soft gluons. Anyway, progress in understanding \( B \to \text{Charmonium} \) decays will improve our predictions on \( B \to X_s e^+e^- \).

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References

[1] W. S. Hou, R. I. Willey and A. Soni, *Phys. Rev. Lett.* **58** (1987) 1608.

[2] B. Grinstein, M. J. Savage and M. B. Wise, *Nucl. Phys.* **B319** (1989) 271.

[3] S. Bertolini, F. Borzumati, A. Masiero and G. Ridolfi, *Nucl. Phys.* **B353** (1991) 591.

[4] A. Ali, T. Mannel, T. Morozumi, *Phys. Lett.* **B273** (1991) 505.

[5] W. Jaus and D. Wyler, *Phys. Rev.* **D41** (1990) 3405.

[6] N. G. Deshpande, K. Panose and J. Trampetić, *Phys. Lett.* **B308** (1993) 322.

[7] A. Ali, G. F. Giudice and T. Mannel, *preprint CERN-TH 7346/94, hep-ph/9408213*. (1994)
[8] C. Greub, A. Ioannissian and D. Wyler, *preprint ZU-TH 25/94*, hep-ph/9408382.

[9] M. Misiak, *Nucl. Phys.* **B393** (1993) 23.

[10] M. Misiak, *Nucl. Phys.* **B461(E)** (1995).

[11] A. J. Buras and M. Münz, *Phys. Rev.* **D52** (1995) 186.

[12] A. Falk, M. Luke and M. J. Savage, *Phys. Rev.* **D49** (1994) 3367.

[13] A. Ali et al., hep-ph/9609443.

[14] C. S. Lim, T. Morozumi and A. I. Sanda, *Phys. Lett.* **B218** (1989) 343.

[15] N. G. Deshpande, J. Trampetić and K. Panose, *Phys. Rev.* **D39** (1989) 1461.

[16] P. J. O’Donnell and H. K. K. Tung, *Phys. Rev.* **D43** (1991) R2067.

[17] M. R. Ahmady, *Phys. Rev.* **D53** (1996) 2483.

[18] E. Braaten and T. C. Yuan, *Phys. Rev. Lett.* **71** (1993) 1673.

[19] G. T. Bodwin, E. Braaten and G. P. Lepage, *Phys. Rev.* **D51** (1995) 1125.

[20] A. J. Buras, M. Jamin and M. E. Lautenbacher, *Nucl. Phys.* **B408** (1993) 209.

[21] A. J. Buras, M. Misiak, M. Münz and S. Pokorski, *Nucl. Phys.* **B424** (1994) 374.

[22] M. Ciuchini, E. Franco, G. Martinelli, L. Reina and L. Silvestrini, *Phys. Lett.* **B316** (1993) 127.

[23] M. Ciuchini, E. Franco, G. Martinelli and L. Reina, *Nucl. Phys.* **B415** (1994) 403.

[24] M. Ciuchini, E. Franco, L. Reina and L. Silvestrini, *Nucl. Phys.* **B421** (1994) 41.

[25] G. Cella, G. Curci, G. Ricciardi and A. Viceré, *Nucl. Phys.* **B431** (1994) 417.

[26] G. Cella, G. Curci, G. Ricciardi and A. Viceré, *Phys. Lett.* **B325** (1994) 227.

[27] N. Cabibbo and L. Maiani, *Phys. Lett.* **B79** (1978) 109.

[28] C. S. Kim and A. D. Martin, *Phys. Lett.* **B225** (1989) 186.
[29] M. Ježabek and J. H. Kühn, Nucl. Phys. B320 (1989) 20.

[30] P. Ko, J. Lee and H.S. Song, Phys. Rev D53 (1996)1409.

[31] G.T. Bodwin, E. Braaten and T.C. Yuan, Phys. Rev D46 (1992)3703.

[32] S. Fleming, et al., Phys. Rev D55 (1997)4098.

[33] P. Cho and Leibovich, Phys. Rev D53 (1996)150.

[34] X. He and A. Soni, Phys. Lett B391 (1997)456.

[35] Terasaki, Nuovo Cimento A66(1981)475.
Figure Captions

Fig.1. The dileptonic invariant mass spectrum for the decay $B \rightarrow X_s e^+ e^-$. The thin solid and the dotted lines correspond to a spectrum without resonances effects, with resonances but phenomenological parameter $a_2$ used as in [13–17]. The dotted dash line is the results including the color-octet effects. The dash line is as same as the dotted-dash line but including the momentum dependent $V - \gamma$ conversion strength effects.
Fig. 1

\[ \frac{d\text{Br} (B \rightarrow X_s e^+e^-)}{d\hat{S}} \times 10^{-5} \]