We study the temperature profile, pion spectra and HBT radii in central symmetric and boost-invariant nuclear collisions using a super hybrid model for heavy-ion collisions (SONIC) combining pre-equilibrium flow with viscous hydrodynamics and late-stage hadronic rescatterings. In particular, we simulate Pb+Pb collisions at $\sqrt{s} = 2.76$ TeV, Au+Au, Cu+Cu, Al+Al, and C+C collisions at $\sqrt{s} = 200$ GeV and Au+Au, Cu+Cu collisions at $\sqrt{s} = 62.4$ GeV. We find that SONIC provides a good match to the pion spectra and HBT radii for all collision systems and energies, confirming earlier work that a combination of pre-equilibrium flow, viscosity and QCD equation of state can resolve the so-called HBT puzzle. For reference, we also show p+p collisions at $\sqrt{s} = 7$ TeV. We make tabulated data for the 2+1 dimensional temperature evolution of all systems publicly available for the use in future jet energy loss or similar studies.
I. INTRODUCTION

With the advent of gauge/gravity duality, it has become possible to effectively simulate far-from-equilibrium thermalization in central (and smooth) nuclear collisions [1]. Combining this pre-equilibrium dynamics with hydrodynamics [2] and a late-stage hadronic cascade [3], one obtains a 'Super hybrid mOdel simulationN for relativistic heavy-Ion Collisions' (SONIC for short) that effectively has only a limited number of parameters, namely those specifying the properties of the incoming nuclei, the speed of sound, and shear and bulk viscosities in the quark–gluon plasma. In this work, we use this model to study symmetric nuclear collisions of different nuclei (Pb, Au, Cu, Al, C) at collision energies ranging from $\sqrt{s} = 2.76$ TeV to $\sqrt{s} = 62.4$ GeV. We study the temperature evolution, pion spectra, and HBT radii for these different collision systems with the aim of both testing the model against experimental data where available and providing model predictions for the design of future experimental studies. In addition, we also show results for SONIC for central p+p collisions at $\sqrt{s} = 7$ TeV energy, even though evidence for forming an equilibrated quark–gluon plasma in these systems is currently lacking.

A fundamental question regarding the quark–gluon plasma is at what temperature and what scale a strong coupling description is most appropriate versus weak coupling. Near-inviscid hydrodynamic modeling indicates strong coupling, though the exact sensitivity of final state hadrons to the temperature dependence of $\eta/s$ is currently under investigation. There are experimental observables when compared to model calculations that are potential indicators of stronger coupling at temperatures near the transition point. Inclusion of this strongest coupling near the transition is proposed to help reconcile the full suite of jet quenching observables including the anisotropy in mid-central collisions [4, 5], the larger than expected $v_2$ and $v_3$ of direct photons [6] and heavy quark observables [7]. An important motivation for the sPHENIX upgrade [8] is to answer the question regarding the underlying nature of the quark–gluon plasma near the point of strongest coupling. A key question is whether high statistics data sets in Au+Au collisions at $\sqrt{s} = 200$ GeV at RHIC and Pb+Pb collisions at $\sqrt{s} = 2.76$ TeV at the LHC are substantially augmented by hard process observables at lower RHIC energies and with different nuclear geometries for emphasizing emission and parton quenching interactions closer or further away from this transition temperature. In this work, we explore the temperature evolution of different systems and provide access to the space–time snapshots for utilization in jet quenching, photon emission, and heavy quark diffusion calculations.

II. METHODOLOGY

We model heavy-ion collisions by using a super hybrid model which we call SONIC which combines pre-equilibrium flow with hydrodynamics and a late-stage hadronic afterburner. Introducing the radius $r = \sqrt{x^2 + y^2}$, the different nuclei are modeled by employing an overlap function

$$T_A(r) = \epsilon_0 \int_{-\infty}^{\infty} dz \left[ 1 + e^{-(r^2+z^2-R^2)/a} \right]$$

with $R, a$ the charge radius and skin depth parameters listed in Table I. $\epsilon_0$ is an overall normalization constant that controls the total final multiplicity. The pre-equilibrium flow has been calculated numerically assuming an infinite number of colors and infinite coupling for central (and smooth) Pb-Pb collisions at $\sqrt{s} = 2.76$ TeV in [1]. We reanalyzed the results from Ref. [1], finding that after the system has thermalized, the velocity is consistent with the early-time analytic result derived in [9] up to an overall factor of two (see Figure 1). Therefore, in the following we will employ a pre-equilibrium radial flow velocity

$$v^\tau(r, \tau) = -\frac{T_A^2}{3} \partial_r T_A^2(r),$$

where $\tau = \sqrt{T^2 - z^2}$, see Fig. 1. Using Eq. (2) and an initial energy density profile given by (cf.9)

$$\epsilon(\tau, r) = T_A^2(r),$$

we start the hydrodynamic evolution at a time $\tau_{sw}$. Following the observations in Ref. [1][10], $\tau_{sw}$ has to be large enough such that a local rest-frame can be defined as ($\tau_{sw} \gtrsim 0.35$ fm/c) and before non-linear effects prohibit the use of Eq. (2) ($\tau_{sw} \lesssim 0.6$ fm/c). Using the energy density from Eq. (3), the flow profile from Eq. (2), and setting the initial shear and bulk stresses to zero, we can solve the subsequent system evolution using the relativistic viscous hydrodynamics solver VH2+1 [12], version 1.7. The fluid shear viscosity over entropy ratio is set to $\eta/s = 0.08$ and the bulk viscosity over entropy ratio is set to $\eta/s = 0.01$. The equation of state used is that from Ref. [11] which is consistent with lattice QCD data [13] at vanishing baryon density and matches a hadron resonance gas at low
temperatures. We monitor the isothermal hypersurface defined by \( T_S = 170 \) MeV throughout the system evolution until the last fluid cell has cooled below \( T_S \).

From the information about fluid temperature, velocity and dissipative stress components we generate hadrons with masses up to 2.2 GeV and follow their rescattering dynamics using the hadron cascade code B3D. We generate 5000 B3D events for each hydro event. Once the particles have stopped interacting we collect information about the particle spectra and report total charged multiplicity \( dN_{ch} dy \), the mean pion transverse momentum \( <p_T> \) and the pion HBT radii \( R_{out}, R_{side}, R_{long} \).

With the pre-equilibrium flow given by Eq. (2), and adjusting \( \epsilon_0 \) so that total multiplicity is constant, we find that the final particle \( <p_T> \) and the extracted HBT radii are insensitive to the choice of \( \tau_{sw} \), just as in the full gauge/gravity+hydro+cascade calculation (cf. Ref. [1]). Thus, \( \tau_{sw} \) is not a relevant parameter of SONIC. This leaves a total of 6 relevant parameters for the system evolution: three numbers \( (R,a,T_S) \) and three functions (the temperature dependent ratios \( \eta/s, \zeta/s \) and the equation of state). Note that \( \epsilon_0 \) is fixed by requiring the final charged multiplicity to match experimental data, wherever it is available [15–17], cf. Tab. II. For C+C and Al+Al collisions at \( \sqrt{s} = 200 \) GeV, we employ the formula

\[
\frac{dN_{ch}}{dy} = \left[ \alpha(\sqrt{s}) N_{coll} + \frac{1 - \alpha(\sqrt{s})}{2} N_{part} \right] \frac{dN_{pp}}{dy},
\]

where \( \frac{dN_{pp}}{dy} \) is the charged multiplicity for nucleon-nucleon collisions at a given collision energy \( \sqrt{s} \), and \( \alpha(\sqrt{s} = 200 \text{GeV}) = 0.13 \) (cf. Ref. [18]).

We note that a full 2+1 dimensional simulation of a single symmetric nuclear collision can be executed on a modern desktop in a matter of minutes, which makes SONIC a viable tool to investigate collisions having granular initial

![Figure 1](image-url) FIG. 1. Comparison between the pre-equilibrium radial flow velocity obtained for Pb+Pb collisions at \( \sqrt{s} = 2.76 \) TeV using a full numerical relativity simulation [1] (“exact”) and the model equation (2).

| Isotope | \( \sqrt{s} \) [GeV] | \( R \) [fm] | \( a \) [fm] | \( T_0(\tau = 0.5 \text{fm/c}) \) [MeV] |
|---------|----------------|-------------|-------------|-----------------|
| p-1     | 7000           | -           | 0.4         | 390             |
| C-12    | 200            | 2.355       | 0.522       | 238             |
| Al-27   | 200            | 3.061       | 0.519       | 287             |
| Cu-63   | 62.4           | 4.163       | 0.606       | 300             |
| Cu-63   | 200            | 4.163       | 0.606       | 327             |
| Au-197  | 62.4           | 6.380       | 0.535       | 325             |
| Au-197  | 200            | 6.380       | 0.535       | 370             |
| Pb-208  | 2760           | 6.624       | 0.549       | 470             |

TABLE I. Model parameters for different collision systems. For all systems we use \( T_S = 170 \) MeV, \( \eta/s = 0.08, \zeta/s = 0.01, \) and QCD equation of state at zero baryon density [11]. Parameters \( R,a \) correspond to Eq. (1) except for p-1 where \( a \) denotes the width of a Gaussian, i.e. \( T_A(r) = \epsilon_0 \int dz e^{-r^2/2a^2} \).
Central Temperature Evolution

![Graph showing temperature evolution vs. proper time for different collision systems and energies.]

FIG. 2. Temperature evolution as a function of proper time at the center of the fireball ($r = 0$) for different collision systems and different collision energies. Full lines denote evolution within hydrodynamics ($T > T_S$), dashed lines denote hadron gas regime ($T < T_S$). For reference, also $p + p$ collisions at $\sqrt{s} = 7$ TeV are shown, even though this system may not equilibrate at all. The “kink” at 2 fm/c in the temperature evolution in the $p + p$ system around $T = T_S$ is due to the center $r = 0$ being cooler than the surrounding matter.

conditions on an event-by-event basis in the future.

III. RESULTS

Our results for the multiplicity and mean pion transverse momentum in the different systems are reported in Table II alongside with experimental results where available. Since the experimental multiplicity is used to fix one of the model parameters ($\epsilon_0$), only the pion $<p_T>$ is a non-trivial model output. Comparing experimental measurements of $<p_T>$ with model output from SONIC, we find that with model parameter choices $\eta/s = 0.08, \zeta/s = 0.01, T_S = 170$ MeV and a QCD equation of state, there is good agreement with experimental data for all collision systems at all collision energies.

For reference, we also show SONIC runs for $p+p$ collisions at $\sqrt{s} = 7$ TeV collision energy, even though this system may not form an equilibrated state of matter (and thus SONIC would not be applicable in this case). Note that, nevertheless, the $<p_T>$ value for $p+p$ collisions is not too far from the experimental value, which may just be a reflection of the fact that transverse flow is not an indicator of system equilibration (cf.[1]).

It should be noted that our current implementation of SONIC does not properly reproduce the experimentally measured proton spectra because the number of protons and antiprotons is too high. The reason for this has been identified to be the missing implementation of baryon–antibaryon annihilation [25, 26]. For this reason, we currently are unable to report physically viable results for baryons.

The time evolution for the temperature in the center of the fireball ($r = 0$ fm) is reported in Fig. 2, where we distinguish between the evolution spent in the hydrodynamic phase ($T > T_S$) and the hadron gas phase at low temperature. Shown in Fig. 3 is the radial velocity profile for the different collision systems at $\tau = 2$ fm/c inside the hydrodynamic phase. Not surprisingly, larger systems tend to build up smaller radial flow and tend to live longer than smaller systems. However, possibly interesting features for temperature evolution between different systems may also be identified in Fig. 3. For instance, note that Fig. 3 implies that the central temperature evolution in Au+Au collisions at $\sqrt{s} = 62.4$ GeV starts out close to the Cu+Cu $\sqrt{s} = 200$ GeV results, but then eventually approaches
Radial Velocity Profile at $\tau = 2$ fm/c

FIG. 3. Velocity profile at $\tau = 2$ fm/c for the different collision systems ($\tau = 1.9$ fm/c for p+p). The velocity profiles for the Pb+Pb and Au+Au systems are similar because the systems have similar geometry and the final-observed larger radial flow at higher $\sqrt{s}$ is simply due to the longer lifetime of the Pb+Pb system.

| Isotope   | $\sqrt{s_{NN}}$ [GeV] | $N_{part}$ | $N_{coll}$ | $dN_{ch}/d\eta$ | $\langle p_T \rangle$ [MeV] | Comments                  |
|-----------|------------------------|------------|------------|------------------|-----------------------------|---------------------------|
| p-1       | 7000                   | -          | -          | 7                | 599                         | for $p_T > 0.15$ GeV      |
| p-1 (exp.)| 7000                   | -          | -          | 6                | $622\pm21$                 | [15, 19], min.-bias       |
| C-12      | 200                    | 17         | 19         | 21               | 396                         |                           |
| Al-27     | 200                    | 45         | 70         | 68               | 415                         |                           |
| Cu-63 (th.)| 62.4                   | 111        | 227        | 144              | 403                         |                           |
| Cu-63 (exp.)| 62.4                  | 106 $\pm$ 3| 162 $\pm$ 13| $138\pm10$       | $379\pm20$                 | [16, 20, 21]             |
| Cu-63 (th.)| 200                    | 113        | 227        | 193              | 421                         |                           |
| Cu-63 (exp.)| 200                   | 108 $\pm$ 4| 189 $\pm$ 14| $198\pm15$       | $420\pm20$                 | [16, 20, 21]             |
| Au-197 (th.)| 62.4                   | 375        | 1173       | 436              | 391                         |                           |
| Au-197 (exp.)| 62.4                  | 356 $\pm$ 11| -          | $472\pm36$      | $405\pm11.0$               | [16, 22]                 |
| Au-197 (th.)| 200                    | 378        | 1173       | 677              | 424                         |                           |
| Au-197 (exp.)| 200                   | 361 $\pm$ 11| 1065 $\pm$ 105| $691\pm52$       | $453\pm33$                 | [16, 23]                 |
| Pb-208 (th.)| 2760                   | 399        | 1217       | 1635             | 503                         |                           |
| Pb-208 (exp.)| 2760                  | 382 $\pm$ 27| -          | $1584\pm80$     | $517\pm19$                 | [17, 24]                 |

TABLE II. Details for collision systems compared to experimental data. $<p_T>$ is for pion transverse momentum except for p+p collisions where report $<p_T>$ for $\pi, K, p$. We use $\frac{dN_{ch}}{d\eta} = 1.1 \frac{dN_{pi}}{d\eta}$ to convert model multiplicity to pseudorapidity distribution.

As noted in Ref. [28], it is possible to resolve this ‘puzzle’ by a combination of different ingredients, notably pre-
equilibrium flow, viscosity, and a QCD-like equation of state. Since all of these ingredients are naturally incorporated in SONIC, it is gratifying to observe that the HBT puzzle is no longer a puzzle but rather a (small) discrepancy in some of the data-model comparison.

In Figure 4, we show the pion transverse momentum spectra for the different collision systems. As remarked above, we do find that with constant values of $\eta/s = 0.08, \zeta/s = 0.01$ and a QCD equation of state, SONIC provides a good overall description of the available experimental data. Note that the discrepancy in the pion spectra for Pb+Pb collisions at $p_T > 1.5$ GeV was not observed in Ref. [1]. The reason is that in Ref. [1], the actual calculation erroneously used a model parameter value of $R = 6.48$ fm instead of $R = 6.62$ fm (cf. Tab. 1) for Pb. Once correcting for this error, we do find slightly less transverse flow in Pb+Pb collisions, leading to the discrepancy observed in Fig. 4. However, it is expected that implementing more realistic granular initial conditions will lead to higher transverse flow velocities. This could help to improve the description of experimental data at $p_T > 1.5$ GeV in SONIC in the future.

IV. CONCLUSIONS

We have presented SONIC, a new super hybrid model for heavy-ion collisions that combines pre-equilibrium flow, viscous hydrodynamics, and hadronic cascade dynamics into one package. SONIC was used to simulate boost-invariant, central symmetric collisions of smooth nuclei (Pb,Au,Cu,Al,C) at energies ranging from $\sqrt{s} = 62.4$ GeV to $\sqrt{s} = 2.76$ TeV. We found that for a QCD equation of state and a choice of QCD viscosity over entropy ratios of $\eta/s = 0.08, \zeta/s = 0.01$, the particle spectra and pion HBT radii were in reasonable agreement with available experimental data. We also made predictions for pion mean transverse momentum and HBT radii for C+C and Al+Al collisions at $\sqrt{s} = 200$ GeV. The 2+1 dimensional space–time evolution of the temperature obtained with SONIC are publicly available [27] in order to be of use in future studies of jet energy loss or photon emission.

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FIG. 5. Pion HBT radii for the different collision systems. Shown are model results (SONIC) and experimental results where available. For p+p collisions, our numerical method to calculate HBT radii is breaking down, so we only report partial results.
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