LETTER

Parking and the visual perception of space

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Abstract. Using measured data we demonstrate that the statistical properties of the distances between parked cars and the distances between birds perching on a power line are identical. We show that this observation is easily explained by the fact that birds and humans use the same mechanism of distance estimation. We give a simple mathematical model of this phenomenon.

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Everyone knows that parking a car in a city center is problematic. The number of available places is limited and they have to be shared between too many interested parties. Birds face the same problem when a flock tries to perch on an electric power line. The common factor in these situations is the need to place oneself safely between two parties limiting the available space from the left and from the right—we will call it ‘the parking problem’ in the following text. This leads necessarily to an interaction preventing collisions between the participants and making the parking maneuver not random. It should therefore not be a surprise that the ‘random car parking model’ introduced by Renyi [1] (see also [2] and [3] for review) does not describe real parking data correctly [4, 5]. The exact character of the ‘parking’ interaction, however, is unknown and hardly describable in physical terms. It is clear that it is primarily triggered by the brain and then mediated through the muscles (by pressing the accelerator/brake pedal (cars) or by flapping the wings (birds)). Though the particular locomotive activity is different, the basic neuroregulatory mechanism is similar in both cases. The complex interplay between the individuals participating in the parking process leaves footprints in the statistics of the nett distances (spacings) between the neighbors. (The nett distance is the distance between the front and tail bumpers of the two adjoined cars or between the wings of two birds perching side-by-side.) This spacing can be measured. The interesting point is that the obtained probability distribution does not depend on the kind of parked object—it seems to be the same for cars as well as for birds.

To be more specific: the subjects of our study are cars parked parallel to the kerb in a city center and starling flocks perching on electric power lines. (For the dynamics of a starling flock see [6] and [7].) Our aim is to present a simple theory that combines psychophysical knowledge with a mathematical model and is able to explain the observed universality of the spacing statistics. The psychophysical part is based on the visual perception of space. This perception mechanism is evolutionarily very old and is shared by many species ranging from birds to mammals. In humans it involves automatic processing without conscious control. We will use the perception mechanism to understand the statistical properties of the distances between the neighboring competitors in a situation when the available space is limited. To illustrate the approach we focus on the spacing distribution between cars parked in parallel. Let us assume that the street segment used for parking has a length $L$ and is free of any kind of obstruction. Drivers are free to park their car anywhere provided they find an empty space to do it. We suppose also for simplicity that all cars have the same length $l_0$. Since many cars are cruising for parking there are no free parking lots and a car can be parked only when another parked car leaves. To simplify the formulation of the problem and to avoid troubling with the boundary effects we assume that the street is very long: $L \gg l_0$. Denoting by $D_k$ the spacing between cars $k$ and $k + 1$ we get

$$\sum_{k=1}^{N} D_k = L - Nl_0$$

(1)

where $N$ is the number of parked cars which is assumed to be constant for a given street (all lots are occupied). The simplex (1) makes the spacings $D_k$ statistically dependent. However, for a street that is long enough this constraint does not play any part and we will ignore it. Since the parked cars gradually leave the street and are replaced by new cars parking in the vacant lots the distances $D_k$ undergo continual changes. This process

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eliminates the particular spacing dependence resulting from the fact that if an individual car parks in a lot and leaves more space in the front then less space is left at its back. But the cars delimiting the parking lot from the front and from the back will also be replaced in course of the process. This finally eliminates the back–front asymmetry and the two spacings become statistically independent.

The car spacing distribution is obtained as a steady solution of this parking–leaving process. Suppose that in one time step only one car can leave the street. (We intentionally omit the situations when two and more neighboring cars leave simultaneously.) The related distance mapping goes as follows: in the first step a randomly chosen car leaves the street and the two adjoined lots merge into a single one. In the second step a new car parks in this empty space and splits it again into two smaller lots. The splitting is random with certain preference reflecting the parking maneuver. When a car leaves the neighboring spacings $D_n$ and $D_{n+1}$ merge into a single lot of a length $D$:

$$D = D_n + D_{n+1} + l_0.$$  \hspace{1cm} (2)

A newly parked car splits $D$ again into two spacings $\tilde{D}_n, \tilde{D}_{n+1}$:

$$\tilde{D}_n = a(D - l_0) \quad \tilde{D}_{n+1} = (1 - a)(D - l_0)$$  \hspace{1cm} (3)

where $a \in (0, 1)$ is a random variable with a probability density $q(a)$. We assume that all drivers have the same habits, i.e. they share the same $q(a)$. The meaning of the variable $a$ is straightforward. For $a = 0$ the car parks immediately in front of the car delimiting the parking lot from the left without leaving any empty space. For $a = 1/2$ it parks exactly in the center of the lot $D$ and for $a = 1$ it stops behind the car on the right. Combining (2) and (3) finally gives the distance mapping:

$$\tilde{D}_n = a(D_n + D_{n+1}) \quad \tilde{D}_{n+1} = (1 - a)(D_n + D_{n+1}).$$  \hspace{1cm} (4)

The car length $l_0$ drops out. For the various choices of $n$ the mappings (4) are regarded as statistically independent.

We suppose that the distribution $q(a)$ known. Since the process eliminates the back–front asymmetry we assume the distribution $q(a)$ to be symmetric: $q(a) = q(1 - a)$. For $q(a)$ given we search for a distribution $p(D)$ of $D_n$ that is invariant under the transform (4). In other words we look for $p(D)$ solving the equation

$$D \triangleq a(D + D')$$  \hspace{1cm} (5)

where $D'$ is a statistically independent copy of the variable $D$ and the symbol $\triangleq$ means that the left- and right-hand sides of (5) have identical statistical properties. Distributional equations of this type are mathematically well studied. In particular one can prove that for a given distribution $q(a)$ there is a unique distribution $p(D)$ that favors the equation (5) (see for instance [8] and [9]). The choice of $q(a)$ reflects the interaction between the cars. To simplify further considerations we restrict $q(a)$ to the class of standard $\beta$ distributions. This restriction enables us to solve equation (5) explicitly. It is a direct consequence of the following statement [10]:

**Statement:** Let $D_1, D_2$ and $a$ be independent random variables with the distributions: $D_1 \sim \Gamma(g_1, 1)$, $D_2 \sim \Gamma(g_2, 1)$ and $a \sim \beta(g_1, g_2)$. Then $a(D_1 + D_2) \sim \Gamma(a_1, 1)$.

The symbol $\sim$ means that the related random variable has the specified probability density. $\Gamma(g, 1), \beta(g_1, g_2)$ denote the standard gamma and beta distributions respectively.
Taking \( g_1 = g_2 = g \) we see that the choice \( q(a) = \beta(g, g, a) \) leads to an explicit solution of (5): \( p(D) = \Gamma(g, 1, D) \) where \( g \) is a free parameter. We will show that the psychophysical arguments set the parameter to \( g = 3 \).

The point is that for small \( D \) the behavior of \( p(D) \) reflects the capability of the driver/bird to estimate small distances to avoid collisions during the ranging process. The distance perception is, however, a complex task and there are several cues for it. In humans some of them are monocular (linear perspective, monocular movement parallax, etc), others oculomotor (accommodation convergence) and finally binocular (i.e. based on stereopsis). All of them work simultaneously and are reliable under different conditions (see [11] for more details). For the ranging, however, the crucial information is not the distance itself but the estimated time to collision with the neighbor. Even more generally: a faultless avoidance of collisions with the surrounding objects is important for the survival of the given species. This is why a special mechanism for evaluating collision time was developed early in evolution and is now shared with the majority of species. In the psychophysical literature the estimated collision time is usually denoted as \( \tau \). It was shown by Lee in a seminal paper [12] that \( \tau \) is derived as the relative rate of expansion of the retinal image of the object. Let \( \theta(t) \) be the instantaneous angular size of the observed obstacle at time \( t \). Then the estimated time to contact with this object is (according to Lee) obtained as [12]

\[
\tau = \frac{\theta}{d\theta/dt}.
\]

(Note that \( \tau \) gives the contact time without explicitly knowing the mutual velocity, the size of the object and its distance.) Behavioral experiments have indicated that \( \tau \) is indeed controlling such actions like contacting surfaces by birds and mammals (see [13]–[15]). Moreover studies have also provided abundant evidence that \( \tau \) is processed by a specialized neural mechanism in the retina and subsequently in the brain [16]. The approved hypothesis is that the estimated time to contact evaluated by (6) is the main informative variable for collision-free motion in a traffic flow (see [17] for a review). It was demonstrated recently that pedestrians are trying to keep a constant value of \( \tau \) while moving in a dense stream (see [18]).

To be more specific, let us have an obstacle of width \( W \) placed at a distance \( D(t) \). Its angular size \( \theta \) equals \( \theta(t) = 2 \arctan(W/2D(t)) \) and we get

\[
\tau(t) = - \frac{W^2 + 4D(t)^2}{2W(dD(t)/dt)} \arctan\left(\frac{W}{2D(t)}\right).
\]

For \( D \gg W \) and a constant approach speed \( v = -dD/dt \) the estimated time to contact simply equals the physical arrival time: \( \tau = D/v \). For small distances, however, \( \tau \approx D^2 \) and the estimated time to contact decreases quadratically with distance. Our hypothesis is that the courage to exploit small distances is directly proportional to the estimated time to contact. This means in particular that for small \( \tau \) (i.e. when a collision is impending) movement into further proximity is stopped. This hypothesis is supported by the observations in traffic streams (see [18]). Applied to the parking maneuver it postulates that the probability of driving a car very close to another car is proportional to \( \tau \) and hence to \( D^2 \). For \( q(a) = \beta(g, g, a) \) the solution of (5) is \( p(D) = \Gamma(g, 1, D) \) and the small \( D \) behavior sets \( g = 3 \). Normalizing the mean spacing to 1 the parking clearance
The probability density $p(D)$ evaluated for the measured data (crosses) is compared with the function $2.3D^2$ (full line). A log–log scale is used.

Figure 1. The probability density $p(D)$ evaluated for the measured data (crosses) is compared with the function $2.3D^2$ (full line). A log–log scale is used.

distribution reads

$$p(D) = \Gamma(3, 1/3, D) = \frac{2\tau}{3}D^2e^{-3D}.$$  \hfill (8)

The $\tau$-mechanism works, so to speak, in the background, i.e. without being consciously realized, and it is identical for humans and for birds. This has a simple and measurable consequence: the spacing distribution for the parked cars and for birds perching on a power line should be the same. And exactly this will be demonstrated with the help of the measured data.

Before continuing several remarks are necessary. First of all: the solution of (5) depends smoothly on $q(a)$. The model supposes a symmetric distribution: $q(a) = q(1-a)$ and the $\tau$ hypothesis sets its behavior for small $a$ to $q(a) \approx a^2$. These two facts already establish the basic properties of $p(D)$. So using another $q(a)$ (i.e. not exactly the $\beta$ distribution) does not change the resulting $p(D)$ significantly. The explicit formula for $p(D)$ will, however, be lost and the equation (5) has to be solved numerically.

The second remarks concern the car parking. A car is not a part of the driver’s body. It is—so to speak—an extension of it. In this sense, to drive a car resembles using a tool. The known fact is that tool usage change the somatosensory representation of the body and the tool is included in the body schema. The perception changes as well. For instance the use of a mechanical grabber changes the perception of hand length (see [19] for more details). In this sense the car is incorporated into the driver’s body and he or she can literally ‘feel’ the distance between the car bumper and an obstacle.

The last remark concerns the behavior of (7). A simple plot of this function shows that the quadratic behavior ($\tau \approx D^2$) is valid up to $D \approx W$ and then it quickly changes to a linear relation $\tau \approx D$. So for cars with $W \approx 1.7m$ the quadratic distance estimation of $\tau$ is valid up to nearly 2 m, i.e. well beyond the front end of the car. In addition the perception changes caused by the inclusion of the car into the driver’s body schema ‘subtract’ the front and end parts of the car and the quadratic behavior is extended beyond the car’s outlines.

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Anyhow: the last two remarks are to certain extent speculative and have to be checked experimentally. To examine the validity of the $\tau$ hypothesis we use the distances between cars stopping on a crossing equipped with traffic lights. If the light is red the cars form a queue. We assume that the drivers stop independently and in a distance controlled by $\tau$. According to the hypothesis the queue clearance probability density should be proportional to $\tau$ and behave quadratically on small distances, otherwise the red light data cannot to be mixed with the solution of equation (5). There is no clearance reshuffling inside the queue—once stopped the car clearance does not change.

We photographed cars queuing in the front of a stoplight. The photographs were all taken from the same spot and at the same time of day. The clearances were obtained by photograph digitalization. Altogether we extracted 1000 car distances from one particular crossing in the city of Hradec Králové (Czech Republic) and evaluated the corresponding probability density. The result is plotted in figure 1. The clearance probability density indeed displays a quadratic behavior. Similar measurements have also been done at several crossings in Prague and in Beijing (see [22, 23]). There are also some direct measurements done under laboratory conditions (see [20, 21]).

Let us now return to the clearance measured for cars parked in parallel and for birds perching on a power line. In both cases the ‘parking segment’ was full, i.e. there was no free space for an additional participant. In addition the segments under consideration were long and contained a large number of objects. So the constraint (1) can be omitted and the solvable variant of the model gives $p(D) = \Gamma(3, 1/3, D)$. We measured the bumper to bumper distances between the cars parked in the center of Hradec Králové. The street was located in a place with a large parking demand and usually without any free parking lots and it was free of any dividing elements (sideways, marks etc). Altogether we measured
700 spacings under this condition. In the case of the birds we photographed flocks of starlings resting on power lines during their flight to the south. The chosen line was ‘full’. Other starlings from the flock were forced to use the space vacated by another starling or to use another line to perch. The bird-to-bird distances were obtained by photograph digitalization—altogether 1000 spacings. Both data sets were scaled to the mean distance equal to 1 and the results plotted in figure 2. The probability distributions resulting from the two data sets seems to be identical and in a good agreement with the model’s prediction. This fact is interesting since the ‘hardware’ used is completely different. The underlying psychophysical mechanism, however, is identical. (For the experimental results concerning the relevance of \( \tau \) for the space perception of pigeons see [24].)

To summarize we have demonstrated that the clearance statistics obtained for parked cars and perching birds are very similar. This observation can be understood as a consequence of a universal and inborn distance-controlling mechanism based on the estimated time to contact \( \tau \).

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