Dark matter and dark energy: cosmology of spacetime with surface tension

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Abstract. A mechanical model was introduced at a prior conference for describing spacetime with surface tension. It was shown that continuum wave mechanics governing micro-perturbations of spacetime itself provide an alternate geometric formulation for quantum mechanics. At a second conference, the model was extended to include gravity. In this presentation, the surface tension model of spacetime is applied to cosmology. It is shown that the model can be arranged to exhibit components resembling dark matter and dark energy.

The dark matter component of the model is used to predict stellar velocity and compared with rotation curves for 15 galaxies. By adjusting mass-to-light ratios to best-fit predicted rotational velocity at extreme distances, the model is shown to also match the initial slope and overall shape of measured rotation curves. Mass-to-light ratios from this approach are much lower than previously thought. Total luminosities of the subject galaxies are shown to be proportionate to the square of their best-fit galactic masses. When this proportion is reinserted into the model, the Tully-Fisher relation is derived.

Dark energy components of the model are applied to describe universal expansion. A non-linear Hubble-Lemaître function is found with asymptotic separation velocity of 3c thereby matching observations. Dark matter and dark energy are postulated to be the cosmological manifestations of surface tension of spacetime.

1. Introduction

Many physicists believe that gravity and quantum mechanics eventually will be unified under a single theory. Earlier works on unification aimed to “geometrize” the electromagnetic field. Some approaches connected an additional linear form to the metric (Weyl), expanded the dimensionality of space (Kaluza), suggested an asymmetric Ricci Tensor (Eddington), added an antisymmetric tensor to the metric (Bach, Einstein), and replaced the metric by a 4-bein field (Einstein) [1]. More recent approaches to unification include string theory and M-theory [2,3], stochastic mechanics [4], and relativistic quantum electrodynamics [5,6,7], and many others.

The approach being evaluated here began by trying to answer a simple question, how would nature construct small particles from the geometry of spacetime itself? The proposed answer draws upon known concepts from physical chemistry to introduce surface tension to spacetime [8-12]. From everyday experiences, we know that surface tension has a predominant role in the physics of small
objects. In a similar way, it was argued in [8-12] that incorporating surface tension into general relativity may help build a bridge to quantum mechanics.

The proposed introduction of surface tension to spacetime [8-12] can be summarized by the following arguments: 1.) thermodynamics requires negative terms in the spatial diagonal of the stress energy tensor, and 2.) both gravitational and high-energy fields can be combined into one geometry by replacing the traditional gravitational constant by an anisotropic invertible coupling tensor. Making these changes to general relativity allows for the creation of various small particles and quantizes spacetime according to Planck’s constant. The wave equations for these systems resemble basic field equations in quantum mechanics wherein probability waves are replaced by strain and displacement fields in the fabric of spacetime itself.

The question addressed in this paper is how the proposed changes to general relativity would affect cosmology. Simple algebraic manipulation of surface tension tensor equations produces a form of general relativity that resembles two of the most widely accepted and yet unexplained aspects of mainstream cosmology, dark matter and dark energy. Many are working to explain dark matter and dark energy phenomena through modifications to the equations of general relativity (see for example [13-17]). Surface tension of spacetime provides an alternative explanation for dark matter and dark energy.

This paper summarizes the surface tension model of spacetime [8-12] including the underlying thermodynamics and the postulated stress energy tensor with negative spatial terms. The model is then used to predict galaxy rotation curves. The component of the model resembling dark matter differs from ordinary mass in that it has a scalar potential energy field and yields flat velocity curves at astronomical distances. The model is applied to derive the Tully-Fisher relation and a non-linear Hubble-Lemaitre universe expansion function.

2. Geometry

A coordinate system is created by an observer in order to record the position of objects, particles, or events as well as their mass, velocity, electric charge, magnetic field, and other properties relative to the observer. A coordinate system is a record of position in spacetime relative to a single clock. Making an observation creates a map between the observer and different points or events in a manifold. Other observers create different maps of the same manifold relative to their own coordinate system with their own clock. Relativity teaches that the coordinate systems of all observers are Lorentz transformations of the same manifold. Said another way, coordinate frames at different velocities are rotations by an imaginary angle of the same tensor equations.

It is well known that observers disagree on simultaneity of events. For this reason, the word “simultaneous” has become near synonymous with misunderstanding relativity. This bias is so great that the simultaneity of an observer (coordinate system with one clock) with events and objects in its present time is approached with skepticism. However, the simultaneity of energy, matter, and events at an instant of time IS the very nature of a coordinate system comoving with a single clock. The concept of one clock per observer is important to this work, because it is the simultaneity of energy in any arbitrary reference frame that manifestly causes spacetime surface tension.
Let us define an observer’s 4-dimensional coordinate system by $x^\mu \in \mathfrak{M}$, where $x^\mu = (x^0, x^1, x^2, x^3)$ and $x^0 = f(t)$. A plot of matter and energy as “seen” by an arbitrary observer is depicted in Figure 1. Each point represents some concentration of matter or energy like an atom, dust grain, star, or galaxy – depending on scale. Each point may be moving at a different velocity within the coordinate system. Matter and energy are dispersed through the observer’s space and confined to an infinitesimally thin instant of time as read on the observer’s clock upon making an observation.

Figure 1. Four-Dimensional Depiction of Matter and Energy as “Seen” by an Arbitrary Observer

One of the three surfaces in time and two coordinates of space from Figure 1 is depicted in Figure 2. According to general relativity, the observer’s coordinate system would have some curvature caused by the density and distribution of mass energy. Thus, the surface in Figure 2 is shown to be undulating and irregular.

Figure 2. Spatial 3-Surface Exhibiting Curvature due to Energy Contained Therein
3. Surface tension

A fundamental principle of physical chemistry is that a surface of energy must have surface tension [18]. Physical chemists [18] use a very simple mechanical analogy to explain this principle. Consider the mechanical model in Figure 3. The model consists of a wire frame window with one moveable side and some density of stored surface energy, $\mathcal{Q}$, dispersed across the opening. In thermodynamic terms, the window represents quantum fields in the present that can affect the probability of future states. The fact that these states are organized in an increment of coordinate time is a higher order (lower entropy) than complete temporal dispersion (loss of causality, faster than light travel, future states affecting the present). The energy required to maintain this lower entropy is stored surface energy. The work done on the system by opening the window some discrete amount is given by,

$$Work = \mathcal{Q}dA$$

which for this system with one moveable side can be written,

$$Work = \mathcal{Q}Ldx$$

where $dA = Ldx$ represents the additional surface area. Now, given the definition of work is force times length, it appears that $\mathcal{Q}$ in the second formulation is a tension force per unit length or *surface tension*. Thus, surface tension and surface energy are mechanically equivalent concepts in the physical chemistry of surfaces. Whenever there is energy organized onto a surface, the surface must have surface tension. This postulate holds regardless of dimensionality of the surface. It holds even for a spatial 3-surface at an instant of time. In relativity, a three volume is a three surface in four dimensions. The fact that matter and energy are organized onto a surface of time for the arbitrary observer and not dispersed in time means that spacetime must have surface tension.

![Surface Energy Stretched Across a Wire Frame Window with One Moveable Side](image)

**Figure 3.** Surface Energy Stretched Across a Wire Frame Window with One Moveable Side

The simple model used by physical chemists for a two-dimensional surface can be mathematically adapted to a 3-surface in relativity. The approach taken is to borrow techniques from continuum mechanics which build a set of coordinate-independent tensor equations to describe the configuration manifold, stress energy states, and the rate of deformation of the energy 3-surface. An infinitesimal element (a discrete point) of 3-surface from Figure 2 is enlarged in Figure 4. The infinitesimal element’s frame (also called material coordinates or tangent space) can be described by coordinates, $y^\nu$. The frame
of the infinitesimal point is mapped to the observer’s coordinates, \( x^\nu \), by the four velocity, \( u \), with components, \( u^\nu = \frac{\partial x^\nu}{\partial s} \), where \( s \) is the spacetime separation between observer and the point (particle or event).

![Figure 4. Infinitesimal Element of Spacetime with Energy Density, \( P \), and Surface Tension, \( \varrho \)](image-url)

The stress energy tensor for a point in a spacetime continuum in the principle frame (tangent space) at the infinitesimal element point is,

\[
T_{\mu\nu} = \begin{bmatrix}
    dP & 0 & 0 & 0 \\
    0 & -\varrho & 0 & 0 \\
    0 & 0 & -\varrho & 0 \\
    0 & 0 & 0 & -\varrho
\end{bmatrix} \quad (1)
\]

where \( dP \) is differential temporal pressure (mass-energy). Surface tension, \( \varrho \), is a tensor field, \( \varrho_{ij} \), whose components may vary. However, in the case of spatially flat fluid space, where torsion and shear stresses (off diagonal spatial terms) are zero, then surface tension, \( \varrho \), is a constant. The spatially flat fluid space with uniform surface tension is assumed herein to be consistent with usual assumptions of physical chemistry of surfaces, unless otherwise noted.

The total energy, \( W \), of a three-surface is,

\[
W = \iiint_\sigma -\varrho \, d\sigma \quad (2)
\]

From the definition of work, stored energy of a three-volume (temporally infinitesimal four-volume) is,

\[
W = \iiint_V dPdV \quad (3)
\]

When combined together through conservation of work and energy, (2) and (3) become,
According to the divergence theorem,
\[ \oint_{\sigma} \varpi d\sigma = \oint_{V} (\nabla \cdot \varpi) dV \quad (5) \]
which means by direct comparison of (4) and (5),
\[ \nabla \cdot \varpi = -dP \quad (6) \]
Differential temporal pressure (mass energy) is the spatial divergence of surface tension.

This line of logic is somewhat analogous to the treatment of corpuscular, capillary, and meniscus geometry in physical chemistry of surfaces. An example of corpuscular geometry is shown in Figure 5. For two-dimensional curved surfaces, surface tension acts against differential surface pressure, \(dP\).

\[ \varpi 2\pi R = dP \frac{\pi R^2}{2} \quad 2-D \text{ Surface in 3-Space} \]
\[ \frac{2\varpi}{R} = dP \]

![Figure 5. Corpuscular Analog of the Divergence Theorem in Physical Chemistry of Surfaces](image)

From this analogy, one can intuitively derive a similar relationship for spatial three-surfaces intrinsic in time,

\[ \varpi 4\pi R^2 = dP \frac{4}{3} \pi R^3 \quad 3-D \text{ Surface in 4-Space} \]
\[ \frac{3\varpi}{R} = dP \quad (7) \]

For the laboratory observer, the time-time term of the stress energy tensor differs from the spatial diagonal terms by \(c^2\). Thus, in laboratory time, (7) is properly written as
\[ \frac{3\varpi'}{R} = \frac{dP'}{c^2} \quad (8) \]
Note that the foregoing corpuscular analogy is not intended to imply cosmological bubbles in spacetime. What is intended is to show a graphical representation of the relationship between two-dimensional curvature and differential pressure as used by physical chemists in order to help the reader understand the leap to a curved three-surface with differential temporal pressure (mass-energy) in spacetime. It is not necessary for spacetime to form a closed 3-sphere for (8) to be true. Equations (7) and (8) are true for any spatially polar symmetric curvature where $R$ is the radius of curvature.

4. Dark matter and dark energy

Simply by adding and subtracting $\mathcal{Q}$ from the time-time term, the stress-energy tensor with surface tension can be rewritten as,

$$T_{\mu\nu} = \begin{bmatrix} dP + \mathcal{Q} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} - \mathcal{Q}g_{\mu\nu}$$ (9)

In this form, surface tension appears as an additional mass term, $+\mathcal{Q}$, and a negative cosmological constant, $-\mathcal{Q}g_{\mu\nu}$. In cosmology applications, the additional mass term could be interpreted as “dark matter” and the negative cosmological constant as “dark energy”. Ordinary luminous matter, $dP$, and “dark matter”, $+\mathcal{Q}$, are interrelated by (6) and (8) of the previous section. A correspondence between luminous matter and dark matter is to be expected. Eloquent arguments for such a correspondence are given in [19].

5. Orbital velocities

In this section, the gravitational field around a galaxy or other massive astronomical object is derived following similar steps as Albert Einstein’s famous ‘Newton’s Theory as a First Approximation’ [20] with the extra rearranged surface tension “dark energy” term, $+\mathcal{Q}$, included. Suppose that the metric of spacetime at some distance, $R$, from the center of a galaxy differs from flat by a small amount due to a gravitational field generated exclusively by matter in the finite region of the galaxy and its corresponding surface tension. If the velocities of stars within and around the galaxy are small as compared with the speed of light, then spatial components of general relativity become insignificant and the gravitational field is quasi-static. With these assumptions, the general relativity equivalent of Newton’s law of gravitation around a galaxy with surface tension becomes,

$$\frac{1}{c^2} \frac{d^2x_\tau}{dt^2} = -\frac{1}{2} \frac{\partial g_{00}}{\partial x_\tau}$$ (10)

$$\nabla^2 g_{00} = \frac{8\pi G}{c^2} (dP' + \mathcal{Q}')$$

where $\tau = 1,2,3$ and differential temporal pressure, $dP'$, is in units of mass density. Given the relationship (8), the second equation above is replaced by,
∇^2 g_{00} = \frac{8\pi G}{c^2} \left( dP' + \frac{r dP'}{3c^2} \right) \tag{11}

From Gauss’ law,

\oint_V \nabla^2 g_{00} \, dV = \oint_V (\nabla \cdot \nabla g_{00}) \, dV = \iint_{\sigma} \nabla g_{00} \cdot d\sigma \tag{12}

Substituting (10) and (11) into (12) yields,

\frac{8\pi G}{c^2} \int_0^\pi \int_0^{2\pi} \int_0^R \left( dP' + \frac{r dP'}{3c^2} \right) r^2 Sin\varphi d\varphi d\theta dr = -\int_0^\pi \int_0^{2\pi} \frac{2}{c^2} \frac{d^2 x_t}{dt^2} R^2 Sin\varphi d\varphi d\theta \tag{13}

\frac{8\pi G}{c^2} 4\pi \int_0^R \left( dP' r^2 + \frac{dP'}{3c^2} r^3 \right) dr = -\frac{8\pi}{c^2} \frac{d^2 x_t}{dt^2} R^2

G 4\pi \left( \frac{dP'}{3} \frac{1}{R} + \frac{dP'}{12c^2} \frac{1}{R^2} \right) = -\frac{d^2 x_t}{dt^2}

Let mass density, \(dP'\), equal galaxy mass contained in a sphere of radius \(R\) divided by volume of that sphere (e.g. \(dP'=\frac{3M}{4\pi R^3}\)) and solve (13) to obtain,

G 4\pi \left( \frac{3M}{12\pi R^2} + \frac{3M}{48\pi R c^2} \right) = -\frac{d^2 x_t}{dt^2}

which reduces to Newton’s equation for gravitational acceleration with surface tension,

\begin{equation}
-G \left( \frac{M}{R^2} + \frac{M}{4R c^2} \right) = \frac{d^2 x_t}{dt^2} \tag{14}
\end{equation}

Note that the second term on the left side of (14) is dimensionless. See the Unit Transformation section for more information on units.

The orbital velocity of stars around a galaxy, \(v\), is related to gravitational acceleration by,

\begin{equation}
\frac{v^2}{R} = -\frac{d^2 x_t}{dt^2} \tag{15}
\end{equation}

Hence, from (14) and (15) orbital velocity is given by,

\begin{equation}
v = \sqrt{GM \left( \frac{1}{R} + \frac{1}{4c^2} \right)}
\end{equation}

If a galaxy or orbiting system has an axis of rotation at an angle, \(i\), relative to the observer’s line of sight, then a correction must be made to the velocity. The corrected orbital velocity is given by,
\[
v_r = \sin i \sqrt{\frac{G M}{R} + \frac{1}{4c^2}} \quad (16)
\]

The parameter \(i\) is called inclination. When a galaxy is at an inclination of 0 degrees, the observer’s line of sight is directly in-line with the axis of rotation, the galaxy is being viewed “head-on”, none of the orbital velocity is towards the observer, and no Doppler velocity difference is measured. When a galaxy is at an inclination of 90 degrees, the axis of rotation is perpendicular to the line of sight, the galaxy is being viewed “on-edge”, and Doppler shift measurements provide the correct orbital velocity.

6. Galaxy rotation curves

To test the validity of the surface tension model, the predicted orbital velocity from (16) is compared with astronomical measurements for fifteen example galaxies. Basic properties of these galaxies are shown in Table 1. For this work, orbital velocities from astronomical measurement of Doppler shifts were obtained from [21] for fourteen galaxies and from [22] for M31. Inclination angle, distance from Earth, and size were gathered from various readily available sources. Solar luminosity was determined by integration of surface brightness over galaxy aperture. Solar mass and mass-to-light ratio were determined by best-fit of the surface tension model as described here.

Table 1. Properties of Subject Galaxies

| Galaxy | Constellation | Inclination Angle, \(i\) deg | Distance from Earth, \(d\) My  | Size, \(L\) L\(_{\odot}\) | Solar Luminosities* \(L/\odot\) | Solar Masses** \(M/\odot\) | Mass/Light Ratio \(ML/\odot\) |
|--------|---------------|-------------------------------|-----------------------------|----------------|-----------------|----------------|----------------|
| NGC 2903 | Leo | 66 | 30.6 | 100 kly | 2.5E+10 | 1.1E+08 | 0.42% |
| NGC 3198, Herschel 146 | Ursa Major | 72 | 47 | 114 kly | 1.7E+10 | 6.9E+07 | 0.41% |
| NGC 2403, Caldwell 7 | Camelopardalis | 55 | 8 | 50 kly | 2.7E+09 | 4.2E+07 | 1.59% |
| NGC 925 | Triangulum | 50 | 30.3 | 92 kly | 2.3E+10 | 1.1E+08 | 0.47% |
| NGC 2941 | Ursa Major | 69 | 46 | 108 kly | 1.2E+11 | 2.3E+08 | 0.20% |
| NGC 2976 | Ursa Major | 54 | 11.6 | 20 kly | 3.2E+09 | 4.0E+07 | 1.23% |
| NGC 3031, M81, Bodes | Ursa Major | 59 | 11.74 | 90 kly | 4.7E+10 | 1.6E+08 | 0.33% |
| NGC 3521, Woolly | Leo | 69 | 26 | 117 kly | 3.9E+10 | 1.5E+08 | 0.39% |
| NGC 4736, M94 | Canes Venatici | 44 | 16.0 | 49 kly | 3.0E+10 | 1.4E+08 | 0.45% |
| NGC 4799A, DDO 154 | Coma Berenices | 70 | 14 | 12 kly | 1.2E+08 | 5.7E+06 | 4.80% |
| NGC 5055, M63, Sunflower | Canes Venatici | 51 | 29.3 | 107 kly | 6.8E+10 | 1.6E+08 | 0.24% |
| NGC 6946, Caldwell 12, Fireworks | Cepheus | 35 | 25.2 | 74 kly | 2.6E+10 | 1.5E+08 | 0.57% |
| NGC 7331, Caldwell 30 | Pegasus | 77 | 40 | 120 kly | 1.1E+11 | 1.9E+08 | 0.17% |
| NGC 224, M31, Andromeda | Andromeda | 77 | 2.5 | 220 kly | 3.2E+10 | 1.6E+08 | 0.50% |
| NGC 7793 | Sculptor | 43 | 12.7 | 34 kly | 3.5E+09 | 5.6E+07 | 1.61% |

*Determined by integration of surface brightness
**Determined by best-fit of surface tension model

Stellar orbital velocity (16) depends on the luminous matter density distribution. Current estimates of luminous matter density found in many cosmology references are biased, because they were generated in-part by curve fitting galaxy rotation curves to Newton’s gravity [13]. To avoid any bias, surface brightness measurements were used to directly estimate luminous matter density as a function of radial
distance from the galaxy center. Total solar luminosities from integration of surface brightness across each galaxy are shown in Table 1.

Surface brightness data for the galaxies under consideration were found in [23] and [24]. An example of one of the surface brightness profiles from [23] for the galaxy NGC 2903 is reproduced in Figure 6. The various dashed and solid lines in the top chart represent surface brightness measured in J, H, and K bands and 2MASS. The bottom chart shows assumed stellar mass-to-light ratio. The thin line represents assumed mass-to-light ratio based on galaxy formation models in J-K bands and the thick line is from 3.6 μm emission data scaled down by the diet Salpeter correction. The diet Salpeter correction is biased, because it is derived in part from curve fitting to match Newton’s gravity. In this work, surface brightness measurements in the J band were used along with the unscaled J-K band assumed stellar mass-to-light ratios. The J band was selected, because it is where the spectrum of the sun and various galaxies overlap with greatest brightness. For the M31 galaxy, J band surface brightness is not presented in [24] so the I band was used as it is the next closest band.

Figure 6. Surface Brightness of NGC2903 [23]
The ratio of flux density, \( F \), coming from a discrete area of galaxy relative to the flux density, \( F_0 \), that would be measured if a single star with luminosity of the Sun were located at the same distance from Earth, \( \frac{F}{F_0} \), provides the number density, \( n \), of equivalent Suns in the galaxy at that location. Flux density is quantified in astrophysics by surface brightness on a log base 10 scale and is typically given in units of magnitudes per square arcsecond. The ratio, \( \frac{F}{F_0} \), can be obtained from surface brightness in the following way,

\[
\mu - \mu_0 = -2.5 \log_{10} \left( \frac{F}{\alpha^2} \right) + 2.5 \log_{10} \left( \frac{F_0}{\alpha^2} \right) = -2.5 \log_{10} \left( \frac{F}{F_0} \right)
\]

\[
\mu - \mu_0 = -2.5 \log_{10}(n) \quad (17)
\]

where \( \mu \) is the total measured surface brightness of an area, and \( \mu_0 \) is the reference surface brightness of that same area if it were illuminated by a single star equivalent to the Sun. The visual angle, \( \alpha \), defining the discrete area of sky cancels from the ratio. Reference surface brightness, \( \mu_0 \), is found from,

\[
\mu_0 = -2.5 \log_{10} \left( \frac{F_0}{\alpha^2} \right) = -2.5 \log_{10} \left( \frac{L_\odot}{4\pi d^2} \frac{\pi^2}{D^2} \frac{180^2 60^4}{4\pi} \right)
\]

\[
\mu_0 = -2.5 \log_{10} \left( \frac{L_\odot}{4\pi D^2} \right) - 2.5 \log_{10} \left( \frac{\pi^2}{180^2 60^4} \right)
\]

\[
\mu_0 = -2.5 \log_{10} \left( \frac{L_\odot}{4\pi D^2} \right) + 26.6 \quad (18)
\]

where \( L_\odot \) is solar luminosity, \( d \) is the distance to the galaxy, \( D \) is the length of galaxy subtended by the angle \( \alpha \), and the ratio \( \frac{d^2}{\alpha^2} \) is the small angle approximation of \( \alpha^2 \), since \( \frac{d}{\alpha} = \tan \alpha \approx \alpha \) in radians. The constants \( \pi \), 180, and 60 convert the square angle in radians to square arcseconds. The expression can be further simplified by introducing the absolute magnitude of the Sun, \( M_\odot \), which is a measure of the flux density from the Sun if it were placed at a distance of 10 pc from the earth.

\[
M_0 = -2.5 \log_{10} \left( \frac{L_\odot}{4\pi 10^2} \right) \quad (19)
\]

Adding and subtracting the absolute magnitude (19) from (18), yields

\[
\mu_0 = -2.5 \log_{10} \left( \frac{L_\odot}{4\pi D^2} \right) + 26.6 + M_0 + 2.5 \log_{10} \left( \frac{L_\odot}{4\pi 10^2} \right)
\]

\[
\mu_0 = -2.5 \log_{10} \left( \frac{4\pi 10^2}{4\pi D^2} \right) + 26.6 + M_0
\]

\[
\mu_0 = -2.5 \log_{10} \left( \frac{10^2}{D^2} \right) + 26.6 + M_0
\]
which, for $D^2=1 \text{ pc}^2$, becomes simply,

$$\mu_0 = 21.6 + M_0 \quad (20)$$

Thus, (17) can be rearranged with (20) to find the number density, $n$, of equivalent Suns in a $1 \text{ pc}^2$ area of galaxy. An equivalent Sun is defined as a star with luminosity equivalent to the Sun. Absolute magnitude, $M_0$, varies by light frequency band width and should be matched to the surface brightness measurement. Based on [25] and a supplemental internet search, the absolute magnitude of the Sun in this study is taken to be 3.87 in the J-band and 4.11 in the I-band. In this analysis, extinction of light from absorption and scattering by gas and dust has been ignored. Failure to account for light extinction could result in an underestimation of both total luminosity and mass.

The subtotal gravitational luminous mass, $M$, located within a distance, $R$, from the center of a galaxy can be approximated by integrating the number density as a function of radial distance, $n(r)$, thus,

$$M(R) = 2\pi M_\odot C \int_0^R Y(r) n(r) r dr \quad \text{for} \ r \leq r_b, \quad C = 1$$
$$\text{for} \ r > r_b, \quad C = \cos i \quad (21)$$

where $Y(r)$ is the assumed mass-to-light ratio from galaxy formation models as reported in [23] and [24], $r_b$ is radius of the bulge, $i$ is inclination angle, $C$ is a factor to account for inclination angle of the disc, and $M_\odot$ is solar mass. This integration was performed for each galaxy by estimating number density from surface brightness for discrete radii, multiplying by the discrete area of sky located within these radii, and subtotaling the result to obtain the total mass, $M(R)$, contained within distance $R$ from the center of the galaxy. The resulting discretized mass values, $M(R)$, were then inserted into (16) to obtain predicted orbital velocity with surface tension.

The initial fit to measured rotational velocity data was poor. The shape of the fit is intriguing, but (16) overpredicts orbital velocity by a factor of 7 or more when the overall mass-to-light ratios of the galaxies are assumed equal to the Sun. In order to obtain a better fit, the number density function, $n(r)$, was diminished by a single fitting factor for each galaxy. The effect of this adjustment is to reduce the overall mass-to-light ratio for each galaxy until a best-fit is obtained. Results of this best-fit are shown in Figures 7 through 21. Predicted stellar velocities from the model are shown by the solid white line. Measured galaxy rotation curves from [21] and [22] are shown by the yellow dots. The dashed line near the bottom of each chart is the orbital velocity predicted by Newton with the fitted number density. As can be seen, fitting the surface tension model to the rotational velocity curves by way of adjusting the mass-to-light ratio results in predicted velocity curves that generally match the initial slope, overall shape, and distant flatness of measured velocity curves.

In essence, the single fitting parameter applied to the surface brightness of each galaxy sets the mass-to-light ratio of that galaxy. The final mass-to-light ratios obtained from the best-fit for the rotation of each galaxy to the surface tension model are shown in Table 1. Mass-to-light ratios vary between 0.0017 and 0.048. These values are much lower than previous predictions based on Newtonian gravitational models. If correct, these mass-to-light ratios would suggest each galaxy contains a large concentration of high temperature main series stars.
Figure 7. Predicted and Measured Galaxy Rotation Curves for NGC2903

Figure 8. Predicted and Measured Galaxy Rotation Curves for NGC3198
Figure 9. Predicted and Measured Galaxy Rotation Curves for M31

Figure 10. Predicted and Measured Galaxy Rotation Curves for DDO154
Figure 11. Predicted and Measured Galaxy Rotation Curves for NGC4736

Figure 12. Predicted and Measured Galaxy Rotation Curves for NGC925
Figure 13. Predicted and Measured Galaxy Rotation Curves for NGC2841

Figure 14. Predicted and Measured Galaxy Rotation Curves for NGC2976
Figure 15. Predicted and Measured Galaxy Rotation Curves for M81

Figure 16. Predicted and Measured Galaxy Rotation Curves for NGC5055
Figure 17. Predicted and Measured Galaxy Rotation Curves for NGC7793

Figure 18. Predicted and Measured Galaxy Rotation Curves for NGC2403
Figure 19. Predicted and Measured Galaxy Rotation Curves for NGC3521

Figure 20. Predicted and Measured Galaxy Rotation Curves for NGC6946
Astronomical observations of galaxies made over great distances look backward in time. Plotting some property of galaxies as a function of distance from the observer provides a chronological history of galaxy formation and an indication if that property is part of the natural evolution of galaxies. Best-fit mass-to-light ratios are plotted against distance from Earth in Figure 22. As can be seen by the degree of scatter, shallow slope, and the low $R^2$ value of a power function regression, there appears to be minimal correlation between best-fit mass-to-light ratios and distance from Earth. The slight downward trend in mass-to-light ratio with distance could indicate higher temperature main sequence stars are more prevalent in the early history of galaxies. This graph does not take into account light extinction, which also should be a function of observation distance. Light extinction could be a factor in the trend of data in Figure 22.

Attempts were made to evaluate the effects of other parameters on resulting best-fit mass-to-light ratios. Mass-to-light ratios were plotted against inclination angle and background brightness, but no strong correlations were found. The strongest correlations were found between best-fit mass-to-light ratio and the square root of solar luminosity as shown in Figure 23, and also between logarithm of luminosity and logarithm of mass as shown in Figure 24. Both of these figures show different representations of the same underlying phenomenon.

If the surface tension model is correct, then there is a statistically strong relationship ($0.956 > R^2 > 0.966$) between total mass and total luminosity of galaxies. Based on the data considered here, the total light energy produced by all stars in a galaxy is approximately proportionate to the square of galactic luminous mass. A similar relationship exists for low mass main sequence stars where the light energy is related to stellar mass to the power of 2.3. The stellar mass-to-light relationship is well accepted and has been justified using thermodynamics. It makes intuitive sense that there should be a
thermodynamic reason why galaxies would form in such a way to exhibit a similar mass-to-light relationship. Research into a thermodynamic argument for this relationship was beyond the scope of the present work and remains an area of continued study.

**Figure 22.** Effect of Distance from Earth on Best-Fit Mass-to-light Ratio

**Figure 23.** Effect of Total Luminosity on Best-Fit Mass-to-light Ratio
Figure 24. Mass–Luminosity Relation for Galaxies with Surface Tension

The low mass-to-light ratios predicted by the best-fit of the surface tension model define a precise relationship between luminosity and galactic mass. The high statistical reliability of the relationship is an indication of the validity of the model. However, it could also indicate a systematic error in calculations. Until this work is confirmed over a wider variety of rotation velocity data, the predictions should be approached with caution.

7. Tully-Fisher relation

The surface tension model predicts orbital velocities approach a constant at large distance from galactic centers. This asymptotic velocity is a function of galactic luminous mass only. In this section, the model will be compared to the Tully-Fisher relation.

With some algebraic manipulation, the inverse relationship between mass-to-light ratio and square root of total luminosity from statistical regression shown in Figure 23 can be re-written as,

$$M = 568 \, M_\odot \left( \frac{L}{L_\odot} \right)^{1/2}$$  (22)

According to (16), the rotational velocity at large distances ($R \to \infty$) as predicted by surface tension is given by,
where effects of galaxy inclination angle have been ignored. Equation (23) has units of distance and defines the proper distance relative to a unit increment of proper time. Substitution of (22) into (23) yields,

\[ v_{r \rightarrow \infty} = \frac{12 \sqrt{G M_\odot}}{c} \left( \frac{L}{L_\odot} \right)^{1/4} \]  

As can be seen in (24), rotational velocities at large distances from the center of galaxies are proportionate to the total luminosity of the galaxy to the \( \frac{1}{4} \) power. Equation (24) has the mathematical form of the well-known empirical Tully-Fisher relation.

What is being shown here is that if the total luminosity of galaxies is a function of the square of total galactic mass as in Figure 23, then the surface tension model matches galaxy rotation curves and also produces the Tully-Fisher relation. One could also state the opposite, if the Tully-Fisher relation is true and is represented in the rotation curve data of the 15 galaxies studied here, then the surface tension model can only match these data if there is a relationship between total luminosity and the square of total galactic mass.

8. Hubble-Lemaitre universe expansion

In this section, the model is evaluated with regard to the “dark energy” or negative cosmological constant. The Hubble-Lemaitre parameter is often used in cosmology to define the rate of expansion of the universe. A corresponding parameter can be derived using the surface tension model.

If space is homogeneous-isotropic and its time dependence is contained in the spatial components of the metric, then one can define, in the usual way, a time-dependent length scale factor, \( a(t) dx \), as the spatial diagonal components of the metric tensor, \( g_{ii} \), thus,

\[ a^2(t) = g_{ii} \]  

From [8], the linearized metric tensor (infinitesimal strain theory applied to spacetime continuum mechanics) is given by,

\[ g_{ii} = 1 - 2D_{ii} ds \]  

where \( D_{ii} \) is the rate of deformation tensor, which was shown in [9] to be related to the Einstein curvature tensor (in 3+1 space), and \( ds \) is a differential increment of proper distance. As described in [8] and [9], the spacetime continuum mechanics equivalent of Einstein’s equation of general relativity is the constitutive equality between stress-energy and rate of deformation whose spatial terms are given by,

\[ T_{ii} \frac{8\pi G}{c^2} = D_{ii} \]  

For readers familiar with [9], see the Unit Transformation section for an important note about the missing anisotropic elastic coupling tensor. Combining (25), (26), (27) together with spatial terms in the surface tension stress-energy tensor (1) and the conservation energy relation between mass and surface tension (7) yields,

\[
a^2(t) = 1 - 2T_{ii} \frac{8\pi G}{c^2} \, ds
\]

\[
a^2(t) = 1 + \frac{16\pi G}{c^2} Q \, ds
\]

\[
a^2(t) = 1 + \frac{16\pi G dP R}{3c^2} \, ds
\]

\[
a(t) = \sqrt{1 + \frac{16\pi G dP R}{3c^2} ds}
\]

Note that (7) was used in lieu of (8) since the expansion of space involves only the spatial components of the tensors. More discussion is given under the section Unit Transformations. The time derivative of the scale factor is given by,

\[
\dot{a}(t) = \frac{16\pi G dP R}{3c^2} \frac{ds}{dt}
\]

The Hubble-Lemaitre parameter, \( H_o \), is found from taking the time derivative of the scale factor divided by the scale factor,

\[
H_o = \frac{\dot{a}(t)}{a(t)}
\]

By inserting (28) and (29) into this equation, one finds when \( \frac{ds}{dt} = c \),

\[
H_o = \frac{16\pi G dP R}{3c} \left(1 + \frac{16\pi G dP R}{3c^2} ds\right)
\]

which when \( dP = \frac{M}{\frac{4}{3} \pi R^3} \) can be rearranged as,

\[
H_o = \frac{c \left(\frac{c^2 R^2}{4G M} + ds\right)}{M}
\]

For reasons that will be explained in the next section on Unit Transformation, the spacetime separation, \( ds \), for spatial components within equations of general relativity should be taken as \( \frac{R}{\frac{4}{3}} \) and the Hubble-Lemaitre parameter becomes,
\[ H_o = \frac{3c/R}{(3c^2 R / 4GM + 1)} \]  

where \( R \) is the distance from the observer to the galaxy, and \( M \) is galactic mass. For objects that are very massive compared to their distance from Earth, (30) reduces to \( H_o = 3c/R \) which suggests a constant general relativistic velocity of separation approaching 3c. This result is not unlike modern cosmological models described in [26] and [27] and replicated in Figure 25. The model in Figure 25 that most closely matches observational data is the FRW model shown by the bold line with gray background \( (\Omega_M, \Omega_X) = (0.3, 0.7) \) approaches an asymptote centered at a velocity of 3c.

![Figure 25. Velocity as a Function of Redshift for Various Models and Approximations [27]](image)

9. Unit transformation

Units, units, units! For anyone keeping track of units, this work is entirely bothersome. Several equations, particularly those regarding the rotational velocity as governed by surface tension, appear to have incorrect units. This section will discuss various unit transformations and pitfalls when working in natural laboratory units.

Work in general relativity is generally performed in units where \( c=1 \). Consider the proper 4-dimensional coordinate system of \( x^\mu \in \mathbb{M} \), where \( x^\mu = (x^0, x^1, x^2, x^3) \) and \( x^0 = ct \). In order for the laboratory observer to unravel the units of this coordinate system so that they may use a common chronometer, the time-time term of the stress energy tensor should be multiplied by \( c^2 \). The time-space components in the first column and top row of the stress energy tensor should be multiplied by \( c \) and the spatial components by \( 1 \). Thus, a proper conversion of units is accomplished by the coordinate transformation,
\[ T'_{\mu\nu}(x'^0=t) = (C_{\mu\nu}) \circ (T_{\mu\nu}(x^0=ct)) \]

where \( T_{\mu\nu}(x^0=ct) \) is the stress energy tensor in \( c=1 \) units, \( T'_{\mu\nu}(x'^0=t) \) is the prime stress energy tensor in laboratory units, and

\[
C_{\mu\nu} = \begin{bmatrix}
    c^2 & c & c & c \\
    c & 1 & 1 & 1 \\
    c & 1 & 1 & 1 \\
    c & 1 & 1 & 1 \\
\end{bmatrix}
\]

is the coordinate transformation. In this notation the symbol \( \circ \) indicates the Hadamard product which is a term-by-term or point-wise product. To those accustomed to Einstein notation, this type of coordinate transformation might appear inappropriate, because there is an imbalance of dependent indices. When taking the Hadamard product, Einstein summation is not implied. This transformation is legitimate in relativity as the speed of light, and hence the basis of transformation, is constant for all reference frames. Parentheses are customarily used along with \( \circ \) to indicate the Hadamard product.

The transformation \( C_{\mu\nu} \) is symmetric. None of the terms in \( C_{\mu\nu} \) are equal to zero, so it is invertible by Hadamard rules and the product \( (C_{\mu\nu}) \circ (C_{\mu\nu})^{-1} \) is equal to the Hadamard identity matrix,

\[
\begin{bmatrix}
1 & 1 & 1 & 1 \\
1 & 1 & 1 & 1 \\
1 & 1 & 1 & 1 \\
1 & 1 & 1 & 1 \\
\end{bmatrix}
\]

Applying this transformation to both sides of the equation of general relativity modifies units of \( T_{\mu\nu} \) and the geometric curvature tensor and is a consistent way to switch back and forth between dimensional forms for \( c=1 \) and \( c=c \). Due to the nature of this transformation, it is important when working on components of general relativity in mass and laboratory time to apply \( c^2 \) for the time-time component and 1 for spatial components. Thus, when working out matters of gravity and dark matter in laboratory time, (8) is used to relate surface tension to mass for the time-time component. Whereas, when working with energy at \( c=1 \) time or for the spatial components at all times utilize (7) to define surface tension. It is for this reason that (7) is used in the analysis of space scaling, universal expansion, and dark energy.

As explained in [28], the natural units of the stress-energy tensor are derived from the four-momentum. A depiction of this is shown in the tensor below for the time-time component and first space-space component.

\[
T_{0,0..1,1} = \begin{bmatrix}
\Delta p/\Delta x_0 \\
\Delta x_1 \Delta x_2 \Delta x_3 \\
\Delta p/\Delta x_1 \\
\Delta x_2 \Delta x_3 \Delta x_0 \\
\end{bmatrix} \rightarrow \begin{bmatrix}
k g \\
\frac{m^2 s^2}{m^2 s^2} \\
\frac{kg}{m^2 s^2} \\
\frac{kg}{m^2 s^2} \\
\end{bmatrix}
\]

When integrated over a spacetime volume and multiplied by \( G \), this form of the stress-energy tensor provides an invariant dimensionless curvature. To solve problems in general relativity, it is preferred
to work with a stress-energy tensor with units of mass density kg/m³. This is accomplished by multiplying each term by the spacetime increment between observations, \( ds = 1 \) unit, divided by the speed of light squared, \( \frac{ds}{c^2} = 1 \). This conversion is often hidden in units of \( c=1 \).

This scalar transformation has units of \( s^7/m \); the same units as the conversion from \( \varphi \) to \( dP \) given by \( R/3c^2(\varphi^2/m) \). In this study, negative surface tension components along the spatial diagonal are shown to play an important role in cosmology. The work energy balance between ordinary matter and surface tension (8) shows that the transformation required to work in ordinary units of mass density is anisotropic such that

\[
T'_{0.0.1.1} \rightarrow \begin{bmatrix}
\frac{kg}{m^2s^2} & \cdots & \frac{kg}{m^2s^2} \\
\cdots & \frac{R}{3c^2} & \cdots \\
\frac{kg}{m^2s^2} & \cdots & \frac{kg}{m^2s^2}
\end{bmatrix} \rightarrow \begin{bmatrix}
\frac{kg}{m^3} \\
\frac{kg}{m^3} \\
\frac{kg}{m^3}
\end{bmatrix}
\]

which when integrated over ordinary spatial volume and multiplied by \( G/c^2 \) provides curvature in units of \( 1/m^2 \). The scalar \( R/3 \) is a kind of time dilation for spatial terms. When moving spatial components of surface tension into ordinary units of mass density and bringing them into the time-time term as pseudo-dark matter in (9), this dilation adjusts the metric scale of the spacetime increment thus, \( ds=R/3 \).

In previous work [10-12], an anisotropic coupling tensor between mass-energy and curvature was suggested to combine equations for gravitational attraction and a geometric form of quantum mechanics in general relativity. In this prior work, it was suggested that quantum fluctuations exist within spacetime itself. The anisotropy arose when attempting to marry these diverse energy fields. It was realized after derivation of the relationship between surface tension and ordinary mass in (2)-(8) that such an anisotropic coupling tensor is not necessary. One need only analyze a quantized particle in terms of (4) which is rewritten below,

\[
\oint \oint \sigma - \varphi d\sigma = \oint \oint P dV
\]

The right side of the foregoing equation can be simply replaced by particle mass, \( M \). If a quantized particle occupies the smallest imaginable volume of Planck radius, \( \lambda_p \), then the integrated surface area in the left side of the equation can be replaced by the surface area of a sphere of Planck radius, \( 4\pi\lambda_p^2 \), giving forth,

\[
-\varphi = \frac{M}{4\pi\lambda_p^2} \quad (31)
\]

This is the exact anisotropy identified in [10-12] necessary to provide geometric waves in spacetime analogous with quantum mechanics using only Einstein’s general relativity having Newton’s scalar coupling constant. By introducing quantized surface tension defined by (31), the coupling between negative spatial surface tension terms and curvature becomes \( \frac{8\pi G}{4\pi\lambda_p^2c^2} = \frac{2c}{h} \), the correct Planck-ian units for geometric quantum mechanics when working with point masses.
10. Discussion

Overall mass-to-light ratios from best-fit of the theoretical surface tension model to the rotational velocity data suggests a statistically significant relationship between mass and luminosity. The statistical significance arose from 15 different galaxies despite a wide range in overall mass, distance from Earth, total luminosity, and structure of the test subjects. This relationship suggests the surface tension model has some merit and deserves further study. A reliable statistical best-fit relation adds credence to the model predictions. However, thermodynamic or other justification for these low mass-to-light ratios is unknown at this time and needs to be determined. If correct, the model predictions of mass-to-light ratio redefine galaxy mass and evolution.

Another possible explanation for the low mass-to-light ratios is that there is a systemic mathematical error in the derivation or in the application of the model. For example, the correction of 1/100 is exactly equal to \((1/10 \text{ pc})^2\), the standard distance used to calculate absolute magnitude. If the surface brightness data obtained from [23] and [24] were somehow corrected for absolute magnitude instead of apparent magnitude, then such a correction could bring mass-to-light ratios closer to previously accepted values. Of course, another explanation is that the surface tension model itself is incorrect and that the best-fit obtained here is some sort of accident. More work and the modeling of many more galaxies is needed to find the answer.

It should be noted that the proposed model is affected to a small extent by referenced distance to galaxy, referenced absolute solar magnitude, and surface brightness band variations. Corrections were made for inclination. Redshift received consideration, but preliminary calculations suggested none of these parameters could explain the systematic variance between model predictions and measured rotational curves. Background surface brightness of more distant celestial objects located beyond the subject galaxies was evaluated. Accounting for background brightness seems to make predicted velocity curves trend downward as in Figures 15 and 20. Taking into account background surface brightness helps explain the trailing shape of some of the measured rotation curves, but here again does not explain the systematic variance between model predictions and measurements. It is possible that reference surface brightness should be attenuated by scattering and/or absorption phenomena but that would increase calculated mass and worsen the correlation.

One of the most encouraging aspects of the fit between the model and measured galaxy rotation curves is the shape of the predicted curves. The addition of hypothetical dark matter to Newton gravity allows one to correct the shape of Newtonian rotation curves so that they do not drop off at \(1/r^{1/2}\), but no amount or configuration of dark matter can explain the discrepancy in the initial slope of rotation curves. Figure 26 is a duplication of Figure 7 except with overall mass-to-light ratio set to 1.0. As can be seen in the figure, Newton gravity not only does not match rotation curves at large distances, it overpredicts the initial slope of the rotation curve. The same can be said for almost all of the rotation data examined herein. The surface tension model not only corrects rotation curves at large radii, it also tends to flatten the initial rotation curve resulting in an overall better fit.

Surface brightness measurements vary in literature. For example, [25] provides much different brightness values as compared with the referenced work [22]. More work is needed to comb through the vast amount of literature on surface brightness and velocity rotation curves to verify the surface tension model. If dark matter were a correct theory, then the velocity should fall-off with Newton’s
gravity at further reaches of a galaxy where dark matter dissipates. Yet, this fall-off in rotational velocity is not observed. Rotational velocity seems to be constant at large distances from galactic centers.

Figure 26. Rotation Curves for NGC 2903 with Mass-to-Light Ratio of 1.0

11. Conclusions
The model of spacetime with surface tension acting in the place of “dark matter” correlates well with the shape of measured rotational velocity curves for the 15 galaxies in this investigation. The model also correlates well in magnitude to measured rotational velocity if a systematic statistically reliable correction is made to the overall mass-to-light ratio. The systematic nature of this correction gives merit to the model, but suggests either the current understanding of galaxy formation (e.g. considerable concentration of high temperature main sequence stars) or a consistent misinterpretation of the data was made (e.g. correction for absolute versus apparent magnitude).

The surface tension model was applied to generate a relationship between distant orbital stellar velocity and overall luminosity similar to the Tully-Fisher relation. It also was applied to generate a non-linear Hubble-Lemaître universal expansion parameter that has some features matching previous FRW models and observations. These two facts are a testimonial to the versatility of the surface tension model in cosmology which demands further study.

The apparent gravitation induced by surface tension is insignificant at solar system scales, because it varies with the speed of light squared. However, at galactic scales, surface tension would govern the motion of stars. The contribution of Newton’s gravity is minimal for luminous matter within and around galaxies in comparison with surface tension. If this simple theory is correct, it would
drastically change the current understanding of galaxy mass, density, evolution, and mechanisms of formation.

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