The modelling of detached chip area at square shafts machining with rack tools

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Abstract. The paper deals with analyzing, by 3D modeling, the shape and the area of the chips detached during the machining of shafts with square cross section, by slotting with a rack tool. The proposed modeling requires at first to determine the tool profile. This step is accomplished by finding the equations of the surfaces family described by the tool relative to workpiece, and by using the enwrapping condition, written according to Minimum distance theorem. The generating process is modeled then, on the base of CATIA environment facilities, aiming to find, in graphical manner, the variation law of the detached chip. Solving this problem could be a primary level of analysis needed for designing the hobbing tool used for machining the same type of part. A numerical and graphical application of the proposed modeling algorithm is also included. The final goal of the research is to deliver the necessary information for smoothing the addressed machining process by modifying the kinematics of the generating process.

1. Introduction

The detached chips modelling in the existing literature concerns almost exclusively the detaching mechanism, e.g. [1, 2]. Only few researches approached the modelling of the cutting schemes specific to cutting tools which generate by enwrapping surfaces delimited by polyhedral peripheric surfaces (shafts with square or hexagonal section, gears with involute or arc of circle profile etc.), following to reveal the unevenness of the machining process, no matter of the type of tool that is used (rack tool, pinion cutter, hob mill) [3]. This modelling can be used for finding the laws governing the addressed machining process from energetic point of view, further enabling the development of technical solutions aiming to the smoothening of the process.

Existing solutions for smoothening the machining process concern the constructive modification of the cutting tool (be it rack tool, pinion cutter or hob mill), such as the area of the chips detached by successive teeth of the tool, during the process of generating by rolling, becomes more uniform. Thus, the cutting force magnitude also becomes more uniform, leading to process smoothening from energetical point of view. Examples of such constructive modification for the rack tool are suggested in [4], while [5] presents a new type of hob mill.

The development of graphical soft packages (e.g. CATIA) are underpinning a new approach regarding the problems in connection to the graphical analysis of the cutting schemes associated to the process of generating by rolling polyhedral surfaces, like the shafts with square or hexagonal section, with tools having finite number of teeth (rack tool case).

This paper addresses the problem of identifying the cutting scheme and the variation of the detached chips area, in the mentioned case (square shaft / rack tool) in order to enable the modification of the
speed of circular feed motion, seen as a new type of approach in solving the problem of machining process smoothening from energetic point of view. Problem solution was found by using the graphical modelling of the machining process, in CATIA environment, according to the specific of the rolling process when generating with the rack tool, expressed through both the fundamental theorems [6] and the complementary theorems [7] of enwrapped surfaces.

In what concerns paper structure, next section presents the finding of the analytical model of the rack tool used for generating square shafts, and the third section deals with the graphical model of the tool. The fourth section is dedicated to the modelling of the detached chips area, while the last is for conclusion.

2. The tool tooth analytical model

Figure 1 presents the cross section of the square shaft, the rolling centrodes, associated to the workpiece and to the cutting tool (whose profile is needed to be found) and the involved reference systems:

- \(\text{xyz}\), meaning a global system, immobile, having its \(z\) axis coincident to workpiece rotation axis;
- \(\text{XYZ}\) – local system, mobile, attached to workpiece centrode, \(C_1\) (circle of \(R_r\) radius), initially overlapped to \(\text{xyz}\) system;
- \(\xi\eta\zeta\) – local system, mobile, associated to rack-tool centrode, \(C_2\) (straight line) and having \(\eta\) axis along \(C_2\).

The rolling motions, executed by the two centrodes, are:

- \(I\), meaning the rotation motion of \(C_1\) centrode around \(z\)-axis, described by the angular parameter \(\varphi\);
- \(II\) – the translation motion of \(C_2\), described by the linear parameter \(\lambda\).

The rolling condition expresses the dependence between the parameters from above:

\[
\lambda = R_r \cdot \varphi.
\]  

Note: The value of rolling radius \(R_r\) must be chosen such as the normal in any point of the generated profile intersects the centrode \(C_1\) [6]. Hereby, there is a minimum value of \(R_r\). In the addressed problem, the selected value of \(R_r\) is the radius of the circle circumscribed to the square, while the minimum value is the radius of the circle inscribed in the square.

The parametric equations of the generated profile, meaning the coordinates of the generic point belonging to square profile, written into \(XYZ\) system, are:
\[ X = -a; \]
\[ Y = u; \]
\[ Z = v. \]

(2)

In equations (2), \( a \) means half of the square side length, while \( u \) and \( v \) are two independent parameters. Due to the specific of the profiling problem, it can be further considered as a plane problem. In this case, only the first two equations from (2) are needed, with \( u \in [0, a] \).

The expressing of the generating process kinematics means to find the equations of the relative motion between the rack-tool and the workpiece. In this purpose, the equations of the absolute motions must be written, at first:

\[ x = \omega_3^T(\varphi) \cdot X, \]

(3)

meaning the equation of workpiece rotation, and

\[ x = \xi + A, \text{ with } A = \begin{pmatrix} -R_r \\ -\lambda \end{pmatrix}, \]

(4)

meaning the equation of rack-tool translation. In equation (3), \( \omega_3 \) is the well-known matrix of coordinates transform at rotation around \( z \)-axis.

The equations of the relative motion can be found then, by reducing \( x \) between equations (3) and (4):

\[ \xi = \omega_3^T(\varphi) \cdot X - A. \]

(5)

The motion (5) represents the motion of the current point from XYZ space relative to the rack-tool system, \( \zeta \eta \zeta \). If here \( X \) means the parametric equations (2) of the generated profile, then after developing and calculus we obtain the equations of the family of profiles \( \Sigma \) referred to \( \zeta \eta \zeta \) system:

\[ (\Sigma)_{\varphi} \left\{ \begin{array}{l}
\xi = -a \cdot \cos \varphi - u \cdot \sin \varphi + R_r; \\
\eta = -a \cdot \sin \varphi + u \cos \varphi + R_r \cdot \varphi.
\end{array} \right. \]

(6)

The rack-tool profile results as envelop of \( (\Sigma)_{\varphi} \) family, by associating to equations (6) the enwrapping condition, here determined according to Minimum distance method [7]. In order to find it, the gearing pole (the point of tangency between \( C_1 \) and \( C_2 \) centrodes, see figure 1) is identified at first as:

\[ P \left\{ \begin{array}{l}
\xi_p = 0; \\
\eta_p = R_r \cdot \varphi,
\end{array} \right. \]

(7)

corresponding to a generic position in the rolling process.

Then, the distance between the gearing pole and the current point of the profiles family from \( \zeta \eta \) space, is calculated as:

\[ d = \sqrt{(\xi - \xi_p)^2 + (\eta - \eta_p)^2}. \]

(8)

Finally, according to minimum distance theorem, the condition of minimum is imposed to distance \( d \) by annulling the derivative of its expression against \( \varphi \) leads to the relation:

\[ (\xi - \xi_p) \cdot \xi'_u + (\eta - \eta_p) \cdot \eta'_u = 0. \]

(9)

In the last relation, \( \xi'_u \) and \( \eta'_u \) represent the derivatives of \( \xi = \zeta(u, \varphi) \) and \( \eta = \eta(u, \varphi) \) functions, calculated with (6), against \( u \):

\[ \xi'_u = -\sin \varphi \text{ and } \eta'_u = \cos \varphi. \]

(10)

After replacing (6), (7) and (10) in (9) and calculus, the final form of the enveloping condition results:

\[ \varphi = \sin^{-1}\left(\frac{u}{R_r}\right). \]

(11)

The ensemble formed by the equations (6) and the condition (11) determine the profile of the rack tool tooth, or, in other words, the analytical model of tool tooth.
3. The rack tool graphical model

The finding by graphical modelling in CATIA of variation law for detached chip area, during the machining of a square shaft by slotting with a rack tool, firstly requires to build the graphical 3D model of the rack tool. For a better understanding, the algorithm developed in this purpose is explained when applied for solving a given problem, which addresses the case of machining a square shaft having \( a = 20 \) mm, starting from a cylindrical piece of raw material with radius \( R = R_r = a \cdot \sqrt{2} \) mm.

The coordinates for a number of points \( N_p = 101 \), defining the rack tool profile necessary for generating half of square side, were calculated with relations (6) and (11) from previous section, by discretizing \( u \) variation interval (which is \([0, a]\)) in 100 sub-intervals. Some of these coordinates are sampled in table 1.

| Point c.r.t. no. | \( u \) [mm] | \( \phi \) [rad] | \( \xi \) [mm] | \( \eta \) [mm] |
|------------------|----------------|----------------|---------------|---------------|
| 1                | 0              | 0              | 8.2842        | 0             |
| 2                | 0.2000         | 0.0070         | 8.2833        | 0.2585        |
| 3                | 0.4000         | 0.0141         | 8.2806        | 0.5171        |
| 4                | 0.6000         | 0.0212         | 8.2760        | 0.7756        |
| 5                | 0.8000         | 0.0282         | 8.2696        | 1.0341        |
| 6                | 1              | 0.0353         | 8.2614        | 1.2924        |
| 7                | 1.2000         | 0.0424         | 8.2513        | 1.5507        |
| 8                | 1.4000         | 0.0495         | 8.2394        | 1.8089        |
| 9                | 1.6000         | 0.0565         | 8.2257        | 2.0669        |
| 10               | 1.8000         | 0.0636         | 8.2102        | 2.3247        |
| 46               | 9              | 0.3238         | 6.4600        | 11.3275       |
| 47               | 9.2000         | 0.3312         | 6.3793        | 11.5648       |
| 48               | 9.4000         | 0.3387         | 6.2970        | 11.8011       |
| 49               | 9.6000         | 0.3462         | 6.2131        | 12.0364       |
| 50               | 9.8000         | 0.3538         | 6.1276        | 12.2708       |

The points are represented by importing their coordinates in Sketcher module and joined then into a spline curve, hence resulting half of the 2-D profile of a tool tooth, represented in figure 2.

![Figure 2. Tool tooth semi-profile.](image)

The tool tooth semi-profile is mirrored to its left extremity, the mirror-curve being joined to the initial one, hence resulting the full tooth profile.
The rack tool is designed as having three teeth, so its profile can be obtained as an array with three items, identical to tooth profile above drawn (figure 3).

Figure 3. Profile of the modelled rack-tool.

A close profile is created then, starting from the active profile of rack tool teeth, in order to enable the extruding of tool 3-D model, with the help of Pad tool, from Part design module. The resulted model is presented in figure 4.

Figure 4. Rack tool 3-D model.

4. The modelling of the detached chips area
Because both entities involved in the generating process have to execute specific motions, the modelling of the detached chip area in the graphical environment CATIA must be performed by concomitantly running two sessions, dedicated to create one, the tool model and the other – the workpiece model.

The tool model building was already presented in previous section. In tool session, an incremental translation motion of $\Delta \lambda$ parameter is applied to the tool model, along $C_2$ centrode.

The workpiece model has the shape of a disc, whose diameter is equal to the diagonal of the square section of the shaft. In workpiece session, the workpiece executes an incremental rotation motion of $\Delta \phi$ angular parameter, around the centre of $C_1$ centrode.

The two models are positioned such as the two centrodes are tangent. The rolling condition (1) is constantly imposed between the two motion increments. More specific, a workpiece rotation is sequenced in 120 steps, so $\Delta \phi = \pi/60$ rad, while $\Delta \lambda = \pi \cdot \sqrt{2}/3 = 1.481$ mm. At each tool / workpiece repositioning, the tool model is imported and overlapped to the workpiece model, the common surface giving the detached chip shape and area (figure 5). The option Remove of the modelling tool Operation enables the removing form workpiece model of the common surface (the chip). If the workpiece area is
measured before and after this action (with Measure item tool from Measure toolbar, which works very accurately), the area of the detached chip results by difference. The values of the successively detached chips can be calculated by repetitively applying the presented procedure (figure 6). At the present stage of the research, the iterations are performed manually. The future implementation of the modelling method will require a piece of software in order to automate its application.

After applying the presented procedure for measuring the detached chip area at each of the 120-tool double-strokes needed for entirely generating the square shaft, the values for all detached chips areas (some of them sampled in table 2) resulted. The variation of detached chips area $A_c$ during the machining process, presented in figure 7, has been drawn on this base.

**Table 2.** Detached chips area (excerpt).

| Dbl. stroke | $\varphi$ [deg] | $A_c$ [mm$^2$] | Dbl. stroke | $\varphi$ [deg] | $A_c$ [mm$^2$] | Dbl. stroke | $\varphi$ [deg] | $A_c$ [mm$^2$] |
|-------------|----------------|--------------|-------------|----------------|--------------|-------------|----------------|--------------|
| 1           | 3              | 4.834        | 51          | 153           | 10.986       | 101         | 303           | 3.111        |
| 2           | 6              | 4.240        | 52          | 156           | 10.496       | 102         | 306           | 3.046        |
| 3           | 9              | 3.669        | 53          | 159           | 9.951        | 103         | 309           | 2.847        |
| 4           | 12             | 3.127        | 54          | 162           | 9.361        | 104         | 312           | 2.591        |
| 5           | 15             | 2.620        | 55          | 165           | 8.740        | 105         | 315           | 2.329        |
| 6           | 18             | 2.156        | 56          | 168           | 8.097        | 106         | 318           | 2.062        |
| 7           | 21             | 1.735        | 57          | 171           | 7.440        | 107         | 321           | 1.814        |
| 8           | 24             | 1.362        | 58          | 174           | 6.777        | 108         | 324           | 1.579        |
| 9           | 27             | 3.403        | 59          | 177           | 6.116        | 109         | 327           | 1.359        |
| 10          | 30             | 4.126        | 60          | 180           | 5.461        | 110         | 330           | 1.151        |
| 11          | 33             | 8.718        | 61          | 183           | 4.833        | 111         | 333           | 0.958        |
| 12          | 36             | 10.163       | 62          | 186           | 4.239        | 112         | 336           | 0.777        |
| 13          | 39             | 11.077       | 63          | 189           | 3.669        | 113         | 339           | 0.609        |
| 14          | 42             | 11.659       | 64          | 192           | 3.126        | 114         | 342           | 0.456        |
| 15          | 45             | 12.006       | 65          | 195           | 2.621        | 115         | 345           | 0.322        |
| 16          | 48             | 12.176       | 66          | 198           | 2.155        | 116         | 348           | 0.212        |
| 17          | 51             | 12.192       | 67          | 201           | 1.735        | 117         | 351           | 0.126        |
| 18          | 54             | 12.037       | 68          | 204           | 1.360        | 118         | 354           | 0.062        |
| 19          | 57             | 11.765       | 69          | 207           | 3.403        | 119         | 357           | 0.021        |
| 20          | 60             | 11.414       | 70          | 210           | 4.126        | 120         | 360           | 0.000        |
5. Conclusion
The subject addressed in this paper is the modelling of the chips detached chip area, during the machining of a square shaft with the rack tool. In this purpose, the analytical model of tool tooth has been found at first. Then, on the base of the deducted equations for tool active profile, the tool 3-D model has been built in CATIA graphical environment. Finally, by using CATIA facilities, the detached chips areas have been found in the case of machining a square shaft having the side length of 40 mm.

The results obtained within the performed simulation enable to draw some conclusions:
- The CATIA graphical application is feasible in finding the shape and the area of detached chip;
- The detached chip area has a periodic variation, exception making the zones corresponding to rack tool engagement and disengagement;
- The detached chip area shows a significant variation (comprised between 1.36 and 12.19 mm$^2$), which sustains the opportunity of implementing a solution for cutting process smoothing.

The target of future researches is to design an experimental stand for actually validating the proposed solution for the diminishing the cutting force unevenness (the use of a variable circular feed), whose first step is the modelling of the detached chip area.

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