Modeling the volatility of gross domestic product based on the volatility of money supply, inflation, interest rate, and exchange rate using the fractional cointegration model

A Abdullah, H Kuswanto, D D Prastyo, and Suhartono

Department of Statistics, Faculty of Mathematics, Computing and Data Science- Institut Teknologi Sepuluh Nopember (ITS) Indonesia

Email: heri_k@statistika.its.ac.id

Abstract. Economic growth is a quantitative index to determine the level of economic progress that can be measured by gross domestic product (GDP). Various literatures aim to solve problems regarding patterns of economic growth as well as about the determinants wherein the analysis volatility models are used. Some indicators of economic growth are money supply, inflation, interest rates, and exchange rates. Economic variables are recorded as time series data with different periods, and hence temporal aggregation is required to analyze. Economic time series data showing the existence of long memory and structural breaks, which leads to the difficulty on explaining the data only with a general model. Therefore, both long memory models and structural break are proposed to be taken into account in the model. In this analysis, the residual from a linear combination of economic variables follows a long memory process indicating a cointegration relationship between these variables. This paper models GDP volatility based on money supply, inflation, interest rate and exchange rate volatility in Indonesia, Philippines, Malaysia, and Singapore using the Fractional Cointegration model with long memory and structural break. The analysis shows that structural break is only found in the volatility of GDP of Indonesia, Malaysia, and Singapore while the nature of long memory is found in almost all economic variables’ volatility. Furthermore, we found the cointegration model, stationary and persistent noise in all countries.

1. Introduction
Economic growth is a classic problem both for developed and developing countries. Economic growth is a quantitative index that measures the level of economic progress of a country as well as its important factors. Therefore, problems related to growth patterns and understanding the determinants of economic growth are fundamental problems in various theoretical and empirical studies. Calculating the GDP of the country concern is one of the ways for measuring whether economic growth is high or low. Achieving high and stable economic growth is one of the objectives of macroeconomic policy. The adopted policies generally place several important indicators, one of which is money supply, inflation, interest rates, and exchange rates. Some previous studies have proven that money supply, inflation, interest rates, and exchange rates can influence economic growth. A research conducted by [1] discussed the relationship between Gross Domestic Product (GDP) variables, Money Supply, Inflation, Interest Rate, and Exchange Rate in the Laos using the Error
Correlation Model. This study concludes that an increase in exchange rate can cause an increase in GDP, otherwise an increase in money supply, inflation, and interest rate can cause a decrease in GDP. These conclusions are used to improve the economy by adjusting policies. In order to obtain information about changes in value on economic variables, the volatility model can be used in the analysis. Many economic data are recorded at different frequencies, due to the costs of collecting or measuring variables that can vary greatly. Most economic variables are collected on a monthly, quarterly or annual basis. Temporal aggregation is the most common method for obtaining the same frequency which can be done by taking the average monthly data into quarterly data in the corresponding month. Temporal aggregation in this way will minimize the risk of losing information in the data.

It is known that many of the economic time series data indicate long memory or long-term dependency. As shown by [2] and [3] that the volatility of most financial time series data shows strong persistence and can be described as a long memory process. This includes money supply data, inflation, interest rates, exchange rates, and gross domestic products. A work done by [4] shows that there can be a lack of clarity between long memory and Markov Switching. In economic data, it can also be seen that not only contains long memory but there are also changes in fluctuations or trends. These considerations lead to combining long memory and structural break into modeling. Research conducted by [5] shows that the existence of structural breaks can have influence on the long memory. In analyzing the long memory relationship between economic variables, it can be seen that the residuals of the linear combination of these variables can also follow the long memory process indicating a cointegration relationship on these variables. Research conducted by [6] uses fractional integration and fractional cointegration approaches in analyzing daily stock market, industrial average, and exchange rate data where fractional cointegration approach needs to be done to determine the relationship of variables that have long memory. Instead, in the structural break model, the authors [7] proposed the Markov Switching model. Research conducted by [8] uses the Regime Switching Long Memory Model to cover long memory and structural break simultaneously in data. A study by [9] discussed the relationship between macroeconomic variables and the stock market using fractional cointegration with the structural break long memory model.

2. Materials and Methods

In this study, an analysis is performed on the GDP volatility model based on the volatility of the money supply, inflation, interest rates, and exchange rates simultaneously using the fractional cointegration model with structural break and long memory.

2.1 Markov Switching

Markov switching model is a good and proved model for describing the nonlinear dynamic especially for financial time series. Nonlinearities in time series caused by the discrete shifts between the regimes where the shifts are breaks in the mean of the process. The Markov switching model with k -regime and AR model of order p and transition probability can be written as

\[ y_t = \mu_{s_t} + \phi_{s_t}y_{t-1} + \ldots + \phi_{p,s_t}y_{t-p} + \sigma_{s_t}a_t, \]

where \( a_t \sim N(0, \sigma_{s_t}^2) \), \( \mu_{s_t}, \phi_{s_t}, \) and \( \sigma_{s_t} \) are parameters under the corresponding state \( S_t \). The probability that state \( i \) will be followed by state \( j \) can be shown by the transition probability \( P_{ij} \).

2.2 Long Memory

Long memory can be interpreted that observation are still strongly correlated up to very long lead. If the long memory parameter of a time series \( Y_t, t=1,2,\ldots,N \) is \( d \in (0,0.5) \), it indicates that \( Y_t \) is a stationary long memory process. Furthermore, \( Y_t \) can also be indicated as a long memory process if it
has a correlation function that decays hyperbole. If \( d \in (-0.5, 0) \) the process is said to be short memory process and it is anti-persistent, while for \( d \in (0, 1) \) the process is said to be nonstationary long memory process but it is mean-reverting.

2.3 Fractional Cointegration

Fractional Cointegration (FCI) is a model where there are several series, \( y_t \) and \( x_u \) with \( i = 1, 2, \ldots, p \), which is a \( I(d) \) process so that the linear combination of the series \( y_t \) and \( x_u \) can be written as

\[
y_t = \beta_1 y_{t-1} + \beta_2 y_{t-2} + \ldots + \beta_p y_{t-p} + u_t.
\]

If \( u_t \) is a \( I(d-b) \) process with \( b > 0 \) and \( 0 < b < d \) then it is called FCI, that shows \( y_t \) and \( x_u \) has long memory process in a period that is not far apart. The FCI modeling is done by estimating the cointegration parameters and long memory parameter of \( u_t \) which is the residual of the linear combination of \( y_t \) and \( x_t \).

2.4 Cointegration Model with Regime Switching and Long Memory

Regime switching long memory model allows the process in each regime to behave as a specific ARFIMA process with a certain degree of integration \( d \). Consider the following general model of regime switching long memory as

\[
(1 - B)^{d_{i_*}} y_t = \mu_{i_*} + \phi_{i_*} y_{t-1} + \ldots + \phi_{i, i_{-p}} y_{t-p} + \sigma_{i_*} a_t
\]

where \( a_t \sim \text{nid}(0, \sigma^2_{i_*}) \) and \( S_t \) specifies the regime. Suppose that we have a set of \( p \)-variables \( x_{t_1}, x_{t_2}, \ldots, x_{t_p} \) that each \( x_t \) is a \( I(b) \) process, and there is a \( \beta_1, \beta_2, \ldots, \beta_p \) such that \( u_t = (y_t - \beta_1 x_{t_1} - \beta_2 x_{t_2} - \ldots - \beta_p x_{t_p}) \) is \( I(d_{i_*} - b) \) process with \( d_{i_*} > b \). This means that the equation (2) makes sense because \( y_t \) and \( x_{t_1}, x_{t_2}, \ldots, x_{t_p} \) do not drift too far apart from each other over time.

The data analyzed in this research are quarterly GDP growth (%), monthly money supply (billion USD), monthly inflation (%), monthly short term interest rate (%), and monthly exchange rate (national currency/USD) of Indonesia, Philippines, Malaysia, and Singapore from January 1998 - March 2019.

The analysis in this research was carried out with the steps described as follows:

- Modeling each variable with the ARMA model and calculating the data volatility
- Aggregating the volatility of inflation, money supply, interest rates, and exchange rates
- Testing the mean changes in GDP volatility using the Cumulative Sum (CUSUM) Test
- Modeling GDP volatility using the Markov switching model
- Estimating the long memory parameters on data volatility in each regime
- Forming a cointegration model and calculating the residual cointegration models in each regime
- Estimating the long memory parameters in the residual of the cointegration model in each regime
- Interpreting the model that has been developed

3. Main Results

We start the analysis by performing the descriptive statistic of the GDP growth. Table 1 presents some basic information about the analyzed data.

| Tab. 1. Descriptive Statistic of GDP Growth |
|------------------------------------------|
| Indonesia | Philippines | Malaysia | Singapore |
| Mean | 4.288 | 4.969 | 4.545 | 4.930 |
The values presented in Table 1 give some information about the distribution of the series. Based on Table 1, it can be seen that GDP growth in all countries is almost the same. Besides, GDP growth in Singapore has the highest diversity (more volatile) while GDP growth in the Philippines has the lowest diversity tend to be stationary with low volatility. Furthermore, from the skewness value, it can be concluded that the GDP grow positively over time following non-normal distribution, excepts in Singapore. Meanwhile from the kurtosis values it can be concluded that GDP growth tends to be homogeneous.

We further analyze the volatility of all series. The volatility has been obtained by fitting ARMA models to all variables, selected based on the criteria of the best ARMA models and the smallest estimated AIC. Figure 1 below depicts the time series plots of the GDP Growth volatility.

Figure 1 shows that there is heterogeneity in GDP growth volatility, especially in Singapore. This conclusion is also proved by the results of the Lagrange Multiplier (LM) test. Next, a CUSUM test is performed on GDP growth volatility data to find out the change in mean. The results of the LM tests and the CUSUM test are presented in Table 2.

**Table 2.** LM Test and CUSUM Test

| Country   | LM Test               | CUSUM Test               |
|-----------|-----------------------|--------------------------|
|           | p-value               | conclusion               | p-value | conclusion               |
| Indonesia | 0.0000                 | Volatility is heterogeneous | 0.0025  | There is change in mean  |
|           | 0.4852                 |                          |         |                          |
|           | 0.9952                 |                          |         |                          |
|           | 1.0000                 |                          |         |                          |
|           | 1.0000                 |                          |         |                          |
| Philippines | 0.0000               | Volatility is heterogeneous | 0.5500  | There is no change in mean |
|           | 0.0000                 |                          |         |                          |
|           | 0.0003                 |                          |         |                          |
|           | 0.1040                 |                          |         |                          |
|           | 0.7859                 |                          |         |                          |
|           | 0.9936                 |                          |         |                          |
| Malaysia  | 0.0000                 | Volatility is heterogeneous | 0.0460  | There is change in mean  |
|           | 0.3155                 |                          |         |                          |
|           | 0.9635                 |                          |         |                          |
|           | 0.9997                 |                          |         |                          |
|           | 1.0000                 |                          |         |                          |
|           | 1.0000                 |                          |         |                          |
| Singapore | 0.0108                 | Volatility is heterogeneous | 0.0020  | There is change in mean  |
|           |                       |                          |         |                          |
0.6659
0.9955
1.0000
1.0000
1.0000

Based on the LM test, it can be concluded that there is heterogeneity in the volatility of GDP growth in all countries. Next, under 5% level of significance, the CUSUM test rejects the null hypothesis for GDP growth volatility in Indonesia, Malaysia, and Singapore. It concludes that there is a change in mean in volatility. Meanwhile, the CUSUM test accepts the null hypothesis for GDP growth volatility in the Philippines and informs that there is no change in mean in volatility.

Furthermore, Markov Switching modeling will be carried out on GDP growth volatility in all countries. Now, we proceed with the Markov Switching analysis with two regimes, which represent the condition of expansion and recession. In the first step, the maximum likelihood estimation in [7] is applied to estimate the transition probabilities. The parameters of the model are estimated as well. Modeling using the Markov Switching model on GDP volatility is done with three alleged models namely the MS-AR (1), MS-AR (2), and MS-AR (3) models. The model is chosen based on the smallest estimated AIC. All estimated parameters are reported in Table 3.

**Table 3. Markov Switching Parameters**

| Parameter | Indonesia | Philippines | Malaysia | Singapore |
|-----------|-----------|-------------|---------|-----------|
|           | MS-AR(2)  | MS-AR(2)    | MS-AR(1) | MS-AR(3)  |
| $\mu_1$   | 0.1693    | 10.7056     | 0.2955   | 0.6839    |
| $\mu_2$   | 9.1172    | 0.6209      | 10.6091  | 4.9803    |
| $\phi_{1,1}$ | 0.1008    | -0.1640     | 0.0596   | 0.1929    |
| $\phi_{1,2}$ | -0.1133   | 0.0002      | -0.1178  | 0.1778    |
| $\phi_{2,1}$ | 0.1431    | -0.0405     | -0.1369  |           |
| $\phi_{2,2}$ | -0.1684   | 0.0309      | 0.0218   |           |
| $\phi_{3,1}$ |           |             | 0.1179   |           |
| $\phi_{3,2}$ |           |             | 0.7230   |           |
| $p_{1,1}$  | 0.8766    | 0.3218      | 0.7760   | 0.8112    |
| $p_{2,2}$  | 0.7480    | 0.9055      | 0.6693   | 0.8605    |

Based on estimation results in Table 3, the Markov Switching models can be written as in Table 4.

**Table 4. Markov Switching Models**

| Country | Model | $P$ |
|---------|-------|-----|
| Indonesia | $y_t = \begin{cases} 0.1693 + 0.1008 y_{t-1} + 0.1434 y_{t-2} + a_{t,1} & \text{for regime 1} \\ 9.1172 - 0.1133 y_{t-1} - 0.1684 y_{t-2} + a_{t,2} & \text{for regime 2} \end{cases}$ | $P = [0.8766, 0.2520]$ |
| Philippines | $y_t = \begin{cases} 10.7056 - 0.1640 y_{t-1} - 0.0405 y_{t-2} + a_{t,1} & \text{for regime 1} \\ 0.6209 + 0.0002 y_{t-1} + 0.0309 y_{t-2} + a_{t,2} & \text{for regime 2} \end{cases}$ | $P = [0.3218, 0.0945, 0.6782, 0.9055]$ |
| Malaysia | $y_t = \begin{cases} 0.2955 + 0.0596 y_{t-1} + a_{t,1} & \text{for regime 1} \\ 10.6091 - 0.1178 y_{t-1} + a_{t,2} & \text{for regime 2} \end{cases}$ | $P = [0.7780, 0.3307, 0.2220, 0.6693]$ |
| Singapore | $y_t = \begin{cases} 0.6839 + 0.1929 y_{t-1} - 0.1369 y_{t-2} + 0.1179 y_{t-3} + a_{t,1} & \text{for regime 1} \\ 4.9803 + 0.1778 y_{t-1} + 0.0218 y_{t-2} + 0.7230 y_{t-3} + a_{t,2} & \text{for regime 2} \end{cases}$ | $P = [0.8112, 0.1395, 0.1888, 0.8605]$ |
The structural changes in GDP volatility refer to two states or regimes, namely the recession state in regime 1 and the expansion state in regime 2. The volatility that has been divided into these two regimes has different dependency level. Except for Philippines, regimes 2 has a higher dependency compared to regime 1 which is shown by a larger intercept of the model. This means that every change in GDP with the same nominal will affect more volatility of GDP in regime 2 than when it was in regime 1. A persistent condition indicated by the high value of the estimated transition probability \(( p_e > 0.5 )\) is found in all countries except the Philippines in regime 1. These conclusions describe the economic conditions that tend to be consistent in certain conditions. The long memory parameter is estimated using the exact local Whittle estimation applied in [10]. Table 5 lists the result of parameter estimation.

**Table 5. Long Memory Parameter \(( d )\)**

| Country  | Regime | GDP      | Money Supply | Inflation | Interest Rate | Exchange Rate |
|----------|--------|----------|--------------|-----------|---------------|---------------|
| Indonesia| 1      | 0.3120   | 0.5963       | 0.9974    | 0.2938        | 0.1749        |
|          | 2      | 0.4034   | 0.7511       | 0.9331    | 0.5122        | 0.1553        |
| Philippines| 1   | 0.5046   | -0.4999      | -0.0162   | 0.5309        | -0.0095       |
|          | 2      | 0.2244   | 0.5434       | 0.5153    | 0.7903        | 0.3455        |
| Malaysia | 1      | 0.4944   | 0.7513       | -0.2064   | 0.1085        | 0.1980        |
|          | 2      | 0.5361   | 0.7610       | 0.3008    | 0.4146        | 0.1382        |
| Singapore| 1      | 0.3763   | 0.1206       | 0.6064    | 0.1397        | 0.8575        |
|          | 2      | 0.7493   | 0.3322       | 0.4340    | 0.6023        | 0.5779        |

Based on the estimation as reported in Table 5, it can be seen there are variables that are not stationary \(( d > 0.5 )\) in both regime 1 and regime 2. The final analysis is to estimate the cointegration parameter and the long memory parameter of the residual obtained from the cointegration estimate in each regime. The cointegration parameter is estimated using a fully modified narrow band least squares estimation applied in [11]. The estimated cointegration parameters and estimated long memory parameters for residual are reported in table 6.

**Table 6. Cointegration Parameter**

| Country  | Regime | Variable | \( \beta_1 \) | \( \beta_2 \) | \( \beta_3 \) | \( \beta_4 \) | \( d \) |
|----------|--------|----------|----------------|----------------|----------------|----------------|------|
| Indonesia| 1      | Money Supply | 0.49 \times 10^{-6} | -0.1823 | 2.2496 | 0.32 \times 10^{4} | 0.4234 |
|          | 2      |           | -0.54 \times 10^{-5} | -1.6814 | 4.7572 | -0.54 \times 10^{-5} | 0.0896 |
| Philippines| 1    | Inflation   | 0.0781 | -22.9188 | -15.7440 | 1.6360 | 0.3771 |
|          | 2      |            | 0.0061 | 41.4761 | -26.7004 | 8.3578 | 0.1479 |
| Malaysia | 1      | Interest Rate | -0.0327 | 1.2871 | 28.0826 | -122.5699 | 0.3004 |
|          | 2      |             | -0.0046 | 128.5876 | 43.7091 | -120.8222 | 0.1510 |
| Singapore| 1      | Exchange Rate | -0.7571 | 33.8756 | 137.5434 | -0.7477 | 0.1979 |
|          | 2      | Residual    | 0.0155 | 35.0394 | 5.9160 | -0.0178 | 0.3731 |

Based on estimation results in Table 6, the Cointegration models can be written as in Table 7. It can be seen that the residuals of the cointegration model follow a stationary long memory process. Therefore it can be concluded that there is cointegration on economic variables in all countries. We can also conclude that the volatility of GDP, money supply, inflation, interest rate, and exchange rate of all countries shows equilibrium in the long run and is a stationary process of the same degree. It can also
be interpreted that for each short-term period all variables tend to adjust to each other to reach their long-term equilibrium point.

**Table 7. Cointegration Models**

| Country    | Model                                                                 | Residual |
|------------|----------------------------------------------------------------------|----------|
| Indonesia  | $y_t = \begin{cases} (0.49 \times 10^{-6}) x_{1t} - 0.1823 x_{2t} + 2.2496 x_{3t} + (0.32 \times 10^{-4}) x_{4t} + u_{1t} \\ - (0.54 \times 10^{-5}) x_{1t} - 1.6814 x_{2t} + 4.7572 x_{3t} - (0.54 \times 10^{-5}) x_{4t} + u_{2t} \end{cases}$ | $u_{1t} = (1-B)^{-0.4234} e_{1t}$, $u_{2t} = (1-B)^{-0.8968} e_{2t}$ |
| Philippines | $y_t = \begin{cases} 0.0781 x_{1t} - 22.9188 x_{2t} - 15.7440 x_{3t} + 1.6360 x_{4t} + u_{1t} \\ 0.0061 x_{1t} + 41.4761 x_{2t} - 26.7004 x_{3t} + 8.3578 x_{4t} + u_{2t} \end{cases}$ | $u_{1t} = (1-B)^{-0.3771} e_{1t}$, $u_{2t} = (1-B)^{-0.1479} e_{2t}$ |
| Malaysia   | $y_t = \begin{cases} -0.0327 x_{1t} + 1.2871 x_{2t} + 28.0826 x_{3t} - 122.5699 x_{4t} + u_{1t} \\ -0.0046 x_{1t} + 128.5876 x_{2t} + 43.7091 x_{3t} - 120.8222 x_{4t} + u_{2t} \end{cases}$ | $u_{1t} = (1-B)^{-0.3004} e_{1t}$, $u_{2t} = (1-B)^{-0.1510} e_{2t}$ |
| Singapore  | $y_t = \begin{cases} -0.7571 x_{1t} + 33.8756 x_{2t} + 137.5434 x_{3t} - 0.7477 x_{4t} + u_{1t} \\ 0.0155 x_{1t} + 35.0939 x_{2t} + 5.9160 x_{3t} - 0.0178 x_{4t} + u_{2t} \end{cases}$ | $u_{1t} = (1-B)^{-0.1979} e_{1t}$, $u_{2t} = (1-B)^{-0.3794} e_{2t}$ |

**4. Conclusion**

This research shows that there are structural breaks in GDP volatility in Indonesia, Philippines, Malaysia, and Singapore. The long memory is detected exist in the volatility of GDP, money supply, inflation, interest rate, and exchange rate in each regime proved by the value of the estimated integrated parameter. This evidence informs that several volatilities are non stationary. Furthermore, the cointegration model of all volatilities in all countries in each regime produces stationary residuals. These empirical results conclude that there is cointegration among all volatilities of GDP, money supply, inflation, interest rate, and exchange rate in all countries studied in this research in each regime. This cointegration model with structural break and long memory can be used as an alternative in modeling volatility of economic variables, which may lead to the proper forecasting results and guide to the minimum risk in the practice. Therefore, given the results of this research there are many outstanding issues that we leave for future research. First, it would also be interesting to analyze how these results would carry over the forecast. Furthermore, the results of this paper provide a better reference for the policy-makers controlling the volatility of GDP Growth of the country as a longterm plan.

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**References**

[1] Srithilat K and Sun G 2017 The impact of monetary policy on economic development: Evidence from Lao PDR *Global Journal of Human-Social Science* 17(2) 9-15
[2] Charfeddine L 2014 True or spurious long memory in volatility: Further evidence on the energy futures markets *Energy Policy* 71
[3] Borup D and Jakobsen J S 2019 Capturing volatility persistence: a dynamically complete realized EGARCH-MIDAS model *Quantitative Finance* 19(11) 1-17
[4] Diebold F and Inoue A 2001 Long memory and regime switching *Journal of Econometrics* 105 131-159
[5] Charfeddine L and Guegan D 2012 Breaks or long memory behavior: An empirical investigation *Physica A: Statistical Mechanics and its Applications* 391(21) 5712-26
[6] Aloy M et al 2010 Fractional integration and cointegration in stock prices and exchange rates HAL

[7] Hamilton J D 1989 A new approach to the economic analysis of nonstationary time series and the business cycle *Econometrica* **57** 357-384

[8] Kuswanto H and Salamah M 2009 Regime switching long memory model for german stock returns *European Journal of Economics, Finance and Administrative Sciences*

[9] Beltratti A and Morana C 2006 Breaks and persistency: macroeconomic causes of stock market volatility *Journal of Econometrics* **131** 151-177

[10] Shimotsu K and Phillips P C B 2005 Exact local whittle estimation of fractional integration *The Annals of Statistics* **33** 1890-1933

[11] Nielsen and Frederiksen 2011 Fully modified narrow-band least squares estimation of weak fractional cointegration *The Econometrics Journal* **14** 77-120