\(D^0\) Mixing at Belle

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We report the recent two results of \(D^0\overline{D^0}\) mixing studies at Belle in \(D^0 \rightarrow K^+K^-/\pi^+\pi^-\) and \(D^0 \rightarrow K^0_\pi^+\pi^-\) decays. The former measures the relative difference of the lifetimes \(y_{CP}\), giving the evidence of \(D^0\overline{D^0}\) mixing; the latter measures the \(D^0\) mixing parameters \(x\) and \(y\).

1. Introduction

Mixing phenomenon, i.e. the oscillation of a neutral meson into its corresponding anti-meson as a function of time, has been observed in the \(K_0^0, B^0\), and most recently \(B^0_s\) systems. This process is also possible in the \(D\)-meson system, but has not previously been observed.

Mixing in heavy flavor systems such as that of \(B^0\) and \(B^0_s\) is governed by the short-distance box diagram. However, in the \(D^0\) system this diagram is both GIM-suppressed and doubly-Cabibbo-suppressed relative to the amplitude dominating the decay width, and thus the short-distance rate is very small. Consequently, \(D^0\overline{D^0}\) mixing is expected to be dominated by long-distance processes that are difficult to calculate; theoretical estimates for the mixing parameters \(x = (m_1 - m_2)/\Gamma\) and \(y = (\Gamma_1 - \Gamma_2)/2\Gamma\) range over two-three orders of magnitude [1]. Here, \(m_1, m_2, \Gamma_1, \Gamma_2\) are the masses (decay widths) of the mass eigenstates \(|D_{1,2}\rangle = p|D^0\rangle \pm q|\overline{D^0}\rangle\), and \(\Gamma = (\Gamma_1 + \Gamma_2)/2\). The parameters \(p\) and \(q\) are complex coefficients satisfying \(|p|^2 + |q|^2 = 1\).

The general experimental method identifies the flavor of the neutral meson when produced by reconstructing the decay \(D^0 \rightarrow K^\pm \pi^\mp\) or \(D^0 \rightarrow K^0_\pi^\pm\) [2]; the charge of the accompanying pion identifies the \(D\) flavor. Because the energy release in \(D^0\) decays is nearly 6 MeV, the background is largely suppressed. The \(D^0\) decay time \((t)\) is calculated via \((l/p) \times m_{\pi}\), where \(l\) is the distance between the \(D^0\) and \(D^0\) decay vertices and \(p\) is the \(D^0\) momentum. The \(D^*\) vertex position is taken to be the intersection of the \(D^0\) momentum with the beamspot profile. To reject \(D^*\) decays originating from \(B\) decays, one requires \(p_{D^*} > 2.5\) GeV, which is the kinematic endpoint.

2. \(CP\)-eigenstates \(K^+K^-\) and \(\pi^+\pi^-\)

We have studied the decays to \(CP\) eigenstates \(D^0 \rightarrow K^+K^-\) and \(D^0 \rightarrow \pi^+\pi^-\); treating the decay-time distributions as exponential, we measured the quantity

\[y_{CP} = \frac{\tau_{K^-}\pi^+}{\tau_{K^+}\pi^-} - 1,\]

where \(\tau_{K^-}\pi^+\) and \(\tau_{K^+}\pi^-\) are the lifetimes of \(D^0 \rightarrow K^-\pi^+\) and \(D^0 \rightarrow K^+\pi^-\) (or \(D^0 \rightarrow \pi^+\pi^-\)) decays. It can be shown that \(y_{CP} = y\cos \phi - x\sin \phi\) [3], where \(A_M\) parameterizes \(CPV\) in mixing and \(\phi\) is a weak phase. If \(CP\) is conserved, \(A_M = \phi = 0\) and \(y_{CP} = y\). This method has been used by numerous experiments to constrain \(y_{CP}\) [4]. Our measurement, based on \(540\) fb\(^{-1}\) data, yields a nonzero value of \(y_{CP}\) with \(>3\sigma\) significance [5]. We also searched for \(CPV\) by measuring the quantity

\[A_\Gamma = \frac{\tau(\overline{D^0} \rightarrow K^-\pi^+)}{\tau(\overline{D^0} \rightarrow K^+\pi^-)} + \tau(D^0 \rightarrow K^+\pi^-);\]

this observable equals \(A_\Gamma = \frac{1}{2}A_M y\cos \phi - x\sin \phi\) [3].

We reconstruct \(D^{*+} \rightarrow D^0\pi^+\) decays and \(D^0 \rightarrow K^-\pi^+, K^-\pi^+,\) and \(\pi^+\pi^-\). Candidate \(D^0\) mesons are selected using two kinematic observables: the invariant mass of the \(D^0\) decay products, \(M\), and the energy release in the \(D^{*+}\) decay, \(Q = (M_{D^*} - M - m_{\pi^+})c^2\). According to Monte Carlo (MC) simulated distributions of \(t\), \(M\), and \(Q\), background events fall into four categories: (1) combinatorial, with zero apparent lifetime; (2) true \(D^0\) mesons combined with random slow pions (this has the same apparent lifetime as the signal) (3) \(D^0\) decays to three or more particles, and (4) other charm hadron decays. The apparent lifetime of the latter two categories is 10-30\% larger than \(\tau_{D^0}\).

For the lifetime measurements, we select the events satisfying \(|\Delta M|/\sigma_M < 2.3, |Q - 5.9\) MeV| < 0.80 MeV and \(\sigma_I < 370\) fs, where \(\Delta M \equiv M - m_{\pi}\), and \(\sigma_I\) is the decay time uncertainties calculated event-by-event. The invariant mass resolution \(\sigma_M\) varies from 5.5-6.8 MeV/c\(^2\), depending on the decay channel. The selection criteria are chosen to minimize the expected statistical error on \(y_{CP}\) using the MC. We find \(111 \times 10^4\ K^+K^-\), \(1.22 \times 10^6\ K^-\pi^+\) and \(49 \times 10^3\ \pi^+\pi^-\) signal events, with purities of 98\%, 99\%, and 92\% respectively.

The relative lifetime difference \(y_{CP}\) is determined by performing a simultaneous binned maximum likelihood fit to the \(D^0 \rightarrow K^+K^-\), \(D^0 \rightarrow K^+\pi^-\), \(D^0 \rightarrow \pi^+\pi^-\) decay time distributions. Each distribution is assumed to be a sum of signal and background contributions, with the signal contribution being a convolution of an exponential and a detector resolution
The analysis also measures decay time distributions, variation of selection criteria, and statistical precision. We measure $K^-$ decay time. The fitted lifetime of $D^0$ times. The effect is visible in Fig. 1d, which plots the normalized distribution of the decay time uncertainties $\sigma_t$. The analysis was conducted to correspond to a Gaussian resolution term of width $\sigma_t$, with a weight given by the fraction $f_i$ of events in that bin. However, the distribution of “pulls”, i.e. the normalized residuals $(t_{\text{rec}} - t_{\text{gen}})/\sigma_t$ (where $t_{\text{rec}}$ and $t_{\text{gen}}$ are reconstructed and generated decay times), is not well-described by a Gaussian. We found that this distribution can be fitted with a sum of three Gaussians of different widths $\sigma_i$ and fractions $w_i$, constrained to the same mean. Therefore, we choose the parameterization

$$R(t-t') = \sum_{i=1}^{n} f_i \sum_{i=1}^{3} w_k G(t-t'; \sigma_{ik}, t_0),$$

with $\sigma_{ik} = s_k \sigma_i^{\text{pull}}$, where the $s_k$ are three scale factors introduced to account for differences between the simulated and real $\sigma_i^{\text{pull}}$, and $t_0$ allows for a (common) offset of the Gaussian terms from zero.

The background $B(t)$ is parameterized assuming two lifetime components: an exponential and a $\delta$ function, each convolved with corresponding resolution functions as parameterized by Eq. (4). Separate $B(t)$ parameters for each final state are determined by fits to the $t$ distributions of events in $M$ sidebands. The MC is used to select the sideband region that best reproduces the time distribution of decay events in the signal region.

Fitting the $K^-\pi^+$, $K^+K^-$, and $\pi^+\pi^-$ decay time distributions fits $y_{\text{CP}} = (1.31 \pm 0.32 \pm 0.25)\%$, which deviates from zero by 3.2$\sigma$. The systematic error is dominated by uncertainty in the background decay time distribution, variation of selection criteria, and the assumption that $t_0$ is equal for all three final states. The analysis also measures

$$A_\Gamma = (0.01 \pm 0.30 \pm 0.15)\%,$$

which is consistent with zero (no CPV). The sources of systematic error for $A_\Gamma$ are similar to those for $y_{\text{CP}}$.

Figure 1: Projections of the decay-time fit superimposed on the data for $D^0 \rightarrow K^+K^-$, $D^0 \rightarrow K^-\pi^+$, and $D^0 \rightarrow \pi^+\pi^-$ decays. The hatched area represents the background contribution. Plot (d) shows the ratio of decay-time distributions for $D^0 \rightarrow (K^+K^- + \pi^+\pi^-)$ and $D^0 \rightarrow K^-\pi^+$; the solid line is a fit to the points.

3. Dalitz Plot Analysis of $D^0 \rightarrow K^0_S \pi^+\pi^-$

The time dependence of the Dalitz plot for $D^0 \rightarrow K^0_S \pi^+\pi^-$ decays is sensitive to mixing parameters $x$ and $y$ without ambiguity due to strong phases. For a particular point in the Dalitz plot $(m_1^2, m_2^2)$, where $m_+ \equiv m(K^0_S \pi^+)$ and $m_- \equiv m(K^0_S \pi^-)$, the overall decay amplitude is

$$A_{D^0}(m_1^2, m_2^2) \frac{e_1(t) + e_2(t)}{2} + \left( \frac{2}{p} \right) A_{D^0}(m_1^2, m_2^2) \frac{e_1(t) - e_2(t)}{2},$$

where $e_{(1,2)}(t) = e^{-(m_1^2 + m_2^2)t}$. The first term represents the (time-dependent) amplitude for $D^0 \rightarrow K^0_S \pi^+\pi^-$, and the second term represents the amplitude for $D^0 \rightarrow \overline{D}^0 \rightarrow K^0_S \pi^+\pi^-$. Taking the modulus squared of Eq. (7) gives the decay rate or, equivalently, the density of points $\rho(m_1^2, m_2^2; t)$. The result contains terms proportional to $\cos(y \Gamma t)$, $\cos(x \Gamma t)$, and $\sin(x \Gamma t)$, and thus fitting the time-dependence of $\rho(m_1^2, m_2^2; t)$ determines $x$ and $y$. This method was developed by CLEO [7].

To use Eq. (7) requires choosing a model for the decay amplitudes $A_{D^0}(m_1^2, m_2^2)$. This is usually taken to be the “isobar model” [8], and thus, in addition to $x$ and $y$, one also fits for the magnitudes and phases of various intermediate states. Specifically, $A_{D^0}(m_1^2, m_2^2) = \sum_j a_j e^{i \delta_j} A_j$, where $\delta_j$ is a
strong phase, $A_j$ is the product of a relativistic Breit-Wigner function and Blatt-Weiskopf form factors, and the parameter $j$ runs over all intermediate states. This sum includes possible scalar resonances and, typically, a constant non-resonant term. For no direct CPV, $A_{T^0}(m^2, m^2_\pi^*) = A_{D^0}(m^2, m^2_\pi^*)$; otherwise, one must consider separate decay parameters $(a_j, \delta_j)$ for $D^0$ decays and $(\bar{a}_j, \bar{\delta}_j)$ for $\bar{D}^0$ decays.

We have fit a large $D^0 \rightarrow K^0_S \pi^+ \pi^-$ sample selected from 540 fb$^{-1}$ of data [9]. The analysis proceeds in two steps. First, signal and background yields are determined from a two-dimensional fit to variables $M(K\pi\pi)$ and $Q = M(K\pi\pi) - M(K\pi\pi) - m_{\pi^+}$. Within a signal region $|M(K\pi\pi) - m_{D^0}| < 15$ MeV/c$^2$ and $|Q - 5.9$ MeV/$c^2| < 1.0$ MeV (corresponding to 3σ in resolution), there are 534,000 signal candidates with 95% purity. These events are fit for $x$ and $y$: the (unbinned ML) fit variables are $m^2_{\pi}$, $m^2_\pi$, and the decay time $t$. Most of the background is combinatoric, i.e., the $D^0$ candidate results from a random combination of tracks. The decay-time distribution of this background is modeled as the sum of a delta function and an exponential function convolved with a Gaussian resolution function, and all parameters are determined from fitting events in the sideband 30 MeV/c$^2 < |M(K\pi\pi) - m_{D^0}| < 55$ MeV/c$^2$.

The results from two separate fits are listed in Table I. In the first fit CP conservation is assumed, i.e., $q/p = 1$ and $A_{D^0}(m^2, m^2_\pi^*) = A_{D^0}(m^2, m^2_\pi^*)$. The free parameters are $x, y, \tau_{D^0}$, some timing resolution function parameters, and decay model parameters $(a_j, \delta_j)$. The results for the latter are listed in Table II. The results for $x$ and $y$ indicate that $x$ is positive, about 2σ from zero. Projections of the fit are shown in Fig. 2. The fit also yields $\tau_{D^0} = (409.9 \pm 1.0)$ fs, which is consistent with the PDG value [6] (and actually has greater statistical precision).

| Table I | Fit results and 95% C.L. intervals for $x$ and $y$, from analysis of $D^0 \rightarrow K^0_S \pi^+ \pi^-$ decays. The errors are statistical, experimental systematic, and decay-model systematic, respectively. |
|---------|---------------------------------------------------------------|
| Fit     | Param. | Result | 95% C.L. inter. |
| No      | $x$ (%) | $0.80 \pm 0.29^{+0.09}_{-0.07}$ | $(0.0, 1.6)$ |
| CPV     | $q/p$ (%) | $0.33 \pm 0.24^{+0.08}_{-0.12}$ | $(0.34, 0.96)$ |
| $y$ (%) | $0.81 \pm 0.30^{+0.16}_{-0.14}$ | $|y| < 1.6$ |
| $q/p$ (%) | $0.37 \pm 0.25^{+0.07}_{-0.13}$ | $|q/p| < 0.99$ |
| $|q/p|$ | $0.86 \pm 0.29^{+0.06}_{-0.03}$ | $|q/p| < 0.99$ |
| $\phi$ (°) | $-14^{+16}_{-8}^{+13}_{-3}$ | $|\phi| < 0.99$ |

For the second fit, CPV is allowed and the $D^0$ and $\bar{D}^0$ samples are considered separately. This introduces additional parameters $|q/p|$, Arg($q/p$) = $\phi$, and $(\bar{a}_j, \bar{\delta}_j)$. The fit gives two equivalent solutions, $(x, y, \phi)$ and $(-x, -y, 0)$. Aside from this pos-

| Table II | Fit results for $D^0 \rightarrow K^0_S \pi^+ \pi^-$ Dalitz plot parameters. The errors are statistical only. The fit fraction is defined as the ratio of the integral $\int |\sum_{a} c_{a}^e \bar{A}_{a}(m^2, m^2_\pi^*)|^2 dm^2 dm^2_\pi^*$ to $\int |\sum_{a} c_{a} A_{a}(m^2, m^2_\pi^*)|^2 dm^2 dm^2_\pi^*$. |
|---------|-----------------|-----------------|-----------------|-
| Resonance | Amplitude | Phase (deg) | Fit fraction |
| $K^*(892)^-$ | $1.629 \pm 0.006$ | $134.3 \pm 0.3$ | $0.6227$ |
| $K^*_0(1430)^-$ | $2.12 \pm 0.02$ | $-0.9 \pm 0.8$ | $0.0724$ |
| $K^*(1410)^-$ | $0.87 \pm 0.02$ | $-47.3 \pm 1.2$ | $0.0133$ |
| $K^*(1410)^+$ | $0.65 \pm 0.03$ | $111 \pm 4$ | $0.0048$ |
| $K^*(1680)^-$ | $0.60 \pm 0.25$ | $147 \pm 29$ | $0.002$ |
| $K^*(892)^+$ | $0.152 \pm 0.003$ | $-37.5 \pm 1.3$ | $0.0054$ |
| $K^*_0(1430)^+$ | $0.541 \pm 0.019$ | $91.8 \pm 2.1$ | $0.0047$ |
| $K^*_0(1430)^*$ | $0.276 \pm 0.013$ | $-106 \pm 3$ | $0.0013$ |
| $K^*(1410)^+$ | $0.33 \pm 0.02$ | $-102 \pm 4$ | $0.0013$ |
| $K^*(1680)^+$ | $0.73 \pm 0.16$ | $103 \pm 11$ | $0.0004$ |

Figure 2: Projection of the unbinned ML fit superimposed on the data for $D^0 \rightarrow K^0_S \pi^+ \pi^-$ decays. In (d), the hatched area represents the combinatorial background contribution, and the lower plot shows the ratio of decay-time distributions for events in the $K^*(892)^+$ and $K^*(892)^-$ regions, where sensitivity to $(x, y)$ is highest.
possible sign change, the effect upon $x$ and $y$ is small, and the results for $|q/p|$ and $\phi$ are consistent with no CPV. The sets of Dalitz parameters $(a_r, \delta_r)$ and $(\bar{a}_r, \bar{\delta}_r)$ are consistent with each other, indicating no direct CPV. Taking $a_j = \bar{a}_j$ and $\delta_j = \bar{\delta}_j$ (i.e., no direct CPV) and repeating the fit gives $|q/p| = 0.95 +0.22$ and $\phi = (-2.10)^\circ$.

The dominant systematic errors are from the time dependence of the Dalitz plot background, and the effect of the $p_D^-$ momentum cut used to reject $D^*$'s originating from $B$ decays. The default fit includes $\pi\pi$ scalar resonances $\sigma_1$ and $\sigma_2$: when evaluating systematic errors, the fit is repeated without any $\pi\pi$ scalar resonances using $K$-matrix formalism [10]. The influence upon $x$ and $y$ is small and included as a systematic error.

The 95% C.L. contour for $(x, y)$ is plotted in Fig. 3. The contour is obtained from the locus of points where $-2 \ln L$ rises by 5.99 units from the minimum value; the distance of the points from the origin is subsequently rescaled to include systematic uncertainty. We note that for the CPV-allowed case, the reflections of the contours through the origin are also allowed regions.

Figure 3: 95% C.L. contours for $(x, y)$: dotted (solid) is statistical (statistical plus systematic) contour for no CPV; dashed-dotted (dashed) is statistical (statistical plus systematic) contour allowing for CPV. The point is the best-fit value for no CPV.

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