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Phase transition free regions in the Ising model via the Kac-Ward operator.  
(English)  
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Summary: We provide an upper bound on the spectral radius of the Kac-Ward transition matrix for a general planar graph. Combined with the Kac-Ward formula for the partition function of the planar Ising model, this allows us to identify regions in the complex plane where the free energy density limits are analytic functions of the inverse temperature. The bound turns out to be optimal in the case of isoradial graphs, i.e., it yields criticality of the self-dual Z-invariant coupling constants.

MSC:  
82B20 Lattice systems (Ising, dimer, Potts, etc.) and systems on graphs arising in equilibrium statistical mechanics  
82B27 Critical phenomena in equilibrium statistical mechanics

Keywords:  
Kac-Ward transition matrix; Kac-Ward formula; Ising model

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