Mathematical Conceptual Understanding about Geometry of 8th Grade Students in Classroom Using Lesson Study and Open Approach with The Geometer’s Sketchpad

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Abstract. This research aims to analyze the Mathematical Conceptual Understanding (MCU) about Geometry in classroom using Lesson Study and Open Approach [11] with the Geometer’s Sketchpad (GSP). It used qualitative research methodology which focuses on protocol analysis through analytic description. The target group was 9 (divided into 3 groups) of 8th grade students from Kaeng Khro Wittaya School, Kaeng Khro District, Chaiyaphum Province, 2016 Academic Year. The research tools were lesson plan of Figures and Geometry Shapes with GSP, Video recorder and field note. The data included students’ problem solving protocols, students’ interviewing protocols, collaboratively lesson plan and reflection protocols, and the students’ written works. Data were analyzed by APOS framework [7]. The results were found that: 1) Posing Open-ended problems, GSP program was used for surveying details and conditions of problems. Students have the MCU in the action level, the students were able to follow the instructions, 2) Students’ self-learning, students applied the surveying details and conditions from GSP program in the 1st step to solve their mathematical problems and could have various ideas. Students have the MCU in the process level, they can draw the connection in the problematic problems as well as describe that idea in descriptive manner. They can also present and elaborate their ideas, 3) Whole class discussion and comparison, GSP program was used for expanding and comparing the students’ ideas. Students have the MCU in the object level, The students can make conclusion as well as give definitions, principles qualities to make the connection or solve the problems, and 4) Summarization through connecting students’ mathematical ideas emerged in the classroom, GSP Program was used to connect and summarize the students’ mathematical idea. Students have the MCU in the object and schema levels, the students are able to make connection of their current lesson with the other lesson such as geometry and so on.

1. Introduction

In present days, the classroom in Thailand faced several issues such as the learning methods that focus only on academic achievement. The students are evaluated only by their examination marks, and the process of learning is overlooked completely. It is apparent that in most classrooms, a teacher plays a role as a key person. Learning methods that those teachers has used depend on spoon-feeding, or the technique that teachers provide knowledge to students such as lecture, demonstration from teachers with the use of tools or description [12]. This is similar to Maxwell, Daniel and Peerasit [3]. stated that
the main problem of Thai education system is that teachers depend too much on rote-learning method and evaluate students by marking how much content they can memorize without training them to have critical thinking, analyze or conduct research in scientific manners. The consequences of this are included learner’s lack of understanding and poor development of emotion quotient (E.Q.) and social quotient (S.Q.). As a result, students tend to be passive. They learn quietly and obey teacher’s instruction easily without asking, arguing, giving their opinions or using their creativity. This is much different from the classroom taught in Japan as it encourages students to use their creativity, be themselves as much as possible and dare to think differently. The issues found in Thai classroom seem to be the result of spoon-feeding and rote-learning method without teaching learners to think, analyze or study with a sense of self-reliance [17]. Programme for International Student Assessment (PISA), an international academic evaluation, included open-ended questions that required students to reflect their thought after reading as answers. The marks are determined by the reason given for answers. If this type is used to evaluate the students in Thailand, it is very likely that it would indicate that Thai education is in the serious crisis. Therefore, it can be inferred that the tools we used in classroom, such as teaching methods, are problematic [13].

One of the solutions to these problems is to change the pattern of activities in classroom. For example, a teacher can encourage students to using their ideas to solve problems by themselves, and they should pay attention to study their behaviours while the students are solving problems (Steen, 1990 cited in [12, 13]), therefore, applied and adapted Lesson Study with the co-operation of the teachers. The outstanding aspects of Lesson Study are included gradual and consistent development as well as the alteration of classroom by integrating Open Approach, or the use of open-ended problems in the mathematical activities. Inprasitha [10] adapted some processes of Lesson Study, originated in Japan, and concluded it into 3 steps which are 1) weekly collaborative planning 2) weekly collaborative observing 3) weekly collaborative reflecting. The methods in integrating Open Approach to the mathematics classroom can be divided into 4 steps: posing open-ended problems, students’ self-learning, whole class discussion and comparison, summarization through connecting students’ mathematical ideas emerged in the classroom [10]. The Open Approach-centered learning patterns aim for preparing learners with variety of open-ended questions to respond to different requirements, interests or mathematical thinking ability of each individual [21].

Lesson Study is applied to study the motivation behind the use of new innovation of The Geometer’s Sketchpad Program. It is found that Geometer’s Sketchpad used in the group of mathematics teachers through the Lesson Study and co-operation. The data accumulated from teaching plan arrangement and the interview with each participant reveals that classroom study help teachers to learn new skills efficiently. Students also positively responded to the study environment of The Geometer’s Sketchpad Program. The outcome shows that the studying in classroom by using The Geometer’s Sketchpad Program in mathematics classroom tend to be successful and efficient [17]. By using The Geometer’s Sketchpad Program to design learning activities used in mathematics classroom, the changes are found in the activities that used The Geometer’s Sketchpad in classroom study group. It is found that the gradual improvement in skills and credibility of the participants in the classroom.

By applying Open Approach with The Geometer’s Sketchpad-centered teaching plan in classroom, it reflected the variety of mathematical thinking such as surveying, reasoning, investigating and diversity in solving problems [15]. The skill that citizens in the 21st century should develop is learning and invention skills such as creative thinking, inventing, critics thinking, problem solving, communication, co-operation and technological skills [3].

As Thai education is under the consistent development, the use of technology in learning will become an important factor and a significant alternative choice that will be beneficial to classroom arrangement. It is expected to improve education as well as teaching lesson arrangement to be more efficient, and this will give direct impact to children and alter Thailand to be a society of learning [23]. Furthermore, technology is very important to mathematics study, and it affects the advance teaching and learning [20]. The impact of using technology in mathematics study is technology’s efficiency in emphasizing on practicing to improve making conclusion, structure connecting and problem solving
skills [3]. It can be interpreted that technology can be used for assisting in teaching and enhancing student’s ability in learning. With the appropriate use of technology, students are enabled to improve their knowledge and understanding in mathematics as technology can be used to support in investigation of students in any aspects of mathematics studies and enhance learners to see the importance of actions, consequences, reasons and solving existing problems [20]. However, Technology is no genie, and it cannot replace a teacher. A teacher is the one who choose to use or design problematic situations or learning activities so that it would be possible to use technology to its fullest potential [20]. Therefore, the two things that teachers might need to make decisions on the use of technology in the classroom are to select the programs according to the range of importance according to Laborde [19] and secondly, to design problematic situations [8].

The Geometer’s Sketchpad (GSP) is the software program for creating and investigating dynamics models of mathematical objects, diagrams and graphs. GSP is an efficient tool for creation, demonstration and investigation, especially for geometry. For example, The Geometer’s Sketchpad can be used to investigate geometric shapes. The Institute for the Promotion of Teaching Science and Technology (IPST) has been encouraging the use of this program as a new tool in Thai mathematics study to improve the level of mathematics study. Teachers can use The Geometer’s Sketchpad to assist the teaching activities because this program has moving pictures that make it easy to explain complicated content. Also, this program encourages self-learning as a student can practice this program by himself or herself. Last but not least, The Geometer’s Sketchpad is designed for the use of creativity, survey and mathematical content analysis [25].

Geometry is the very useful content for mathematical improvement such as inductive reasoning, reductive reasoning, fact proof, estimation as well as categorization and definition of geometric shapes [20]. The understanding of geometry is one of the essential skills in studying mathematics. However, most of students usually reflected their understandings toward geometry with negative attitude such as ‘I can’t do it’, ‘it isn’t important’, ‘it’s tedious’, or even ‘it’s too challenging’ [22]. The best way to study mathematics is to understand it. Therefore, the main objective is in advance mathematics learning, students should deal with it by themselves [18]. Understanding is the fundamental aspect of learning, and it is a pattern of learning that should be involved with problem solving [9]. Hence, the object of many mathematical studies in hope for academic achievement is to encourage students to study mathematics with understandings [9]. In studying mathematics, understanding is the most essential and significant aspect that motivates students to study with their fullest potential. Teaching with Open Approach could provide teachers solutions and ‘How to’ directions to understand student’s mathematical concept more closely, and this is similar to the concept developed by Inprasitha [13]. In studying mathematics understanding after the use of The Geometer’s Sketchpad as an assisting tool, it is found that the program can provide opportunities in thinking and solving problems more efficiently, and this would prepare them for the advance study [1].

With these reasons, it is readily apparent that students usually have problems with learning geometry because they find it difficult to understand. Hence, technology can be used to help them understand the lesson better. The researchers aim for using The Geometer’s Sketchpad to create problematic situations or teaching activities. However, the limitation of the technology is seen, so the researchers apply the innovation of Lesson Study and Open Approach as well [12]. They also analyze students’ by based on APOS theoretical framework [7].

2. Research Objectives
This research aimed to analyze the level of student’s understanding in geometry taught in classroom, being applied Lesson Study, Open Approach and The Geometer’s Sketchpad.
3. Definition of Technical Terms

3.1. Mathematical Conceptual Understanding (MCU):
This term means an insight or action in each content, and students would show their understanding through responding, the action of problem solving, expressing their ideas, role-play, reasoning, expressing through words, gestures, photos, or symbols as well as connecting and concluding ideas or give the mathematical definitions that are meaningful to oneself. According to Dubinsky & McDonald’s APOS theatrical framework, mathematical conceptual understanding can be categorized into 4 types:

1. Action Level: this means student’s ability to respond to external stimulation. The students in each are required to adhere to conditions or instructions in order. Each step needs to be done before they can do the next one. Therefore, students who have mathematical conceptual understanding in action level would be able to arrange their actions with both physical and conceptual objects in given order. For example, the students can answer the questions given by teachers, or they can perform orders in problematic situations.

2. Process Level: this is defined as student’s ability in elaborating, reflecting, or thinking about the processes retrospectively. They do not need to elaborate the actions taken in each step, but this occurs from the understanding their actions gained from doing repetitively. The students can have pictures in mind to predict the outcome or make conclusion. For instance, the students can draw relationship occurred in problem solving as well as descriptive narration of different relations. The students can explain idea of their own group without showing repetitive actions.

3. Object Level: this is defined as an ability to understand the whole picture level as though everything is the same object. The process arrangement could alter the outcome of the process, and it provides the new advance level concept. The students improve their mathematical conceptual understanding in process level so that they can compare their group ideas with other ideas emerged in the classroom. They can find similarities, differences, explain formula or make conclusion of the ideas they have found. They can also give examples to make it easier to understand and connect the different ideas such as ideas from their own group and other groups. For example, the students can make conclusion, give definition or qualities to explain or solve problems.

4. Schema Level: this is an ability to connect between experiences or new processes with the existing cognitive structure, and it might be connected with the previous separated process to be a part of the cognitive structure. The students would be able to realize their concepts and see the connections of different concepts occurred in the classroom by comparing the pair of concepts or connect the different ideas such as ideas from their own group and other groups to make conclusion of the ideas that meaningful to oneself. For example, the students can connect the content emerged from the horizontal relationship, regarding geometric shapes and other topics.

3.2. Classroom using Lesson Study and Open Approach:
This is the classroom that integrated Lesson Study and Open Approach to create a classroom that concentrate on soling problem with a sense of self-reliance. The classroom is applied with Lesson Study and Open Approach according to Inprasitha [11-13].

3.3. Lesson Study
It can be defined as an approach in working together between teachers, researchers and researching assistants. Three of them worked together to design lesson plan and apply it to the classroom. The classroom would be observed and reflected by the team. According to Inprasitha [11], the process of this method can be seen as following details:
1. **Collaborative Plan**: In this step, the lesson study team analyzes and designs open-ended problems by using The Geometer’s Sketchpad and the content in KEIRINKAN’s mathematics textbook, the sixth chapter: geometric figures and shapes. They also predict concept and design teaching media by using The Geometer’s Sketchpad.

2. **Collaborative Observing**: In this step, classroom study team implemented their lesson plan in the classroom. Researchers and researching assistants observed the classrooms by concentrating student’s ideas in solving problems to analyze their mathematical conceptual understanding.

3. **Collaborative Reflecting**: This step is to reflect after using the lesson plan in the classroom, the team worked together to reflect the appropriation of the use of The Geometer’s Sketchpad in the classroom, student’s ideas and their mathematic understanding emerged in the classroom. They also worked to reflect the instructional problems and the way to improve the lesson plan or lesson in the future.

3.4. **Open Approach**: This means the method in providing classes that focuses on solving mathematical problems by using open-ended problem, provides opportunities for students to solve problems by themselves and encourages mathematical understandings. According to Inprasitha [11], there are 4 steps as following,

1. Posing open-ended problem: the teacher poses open-ended problem by using The Geometer’s Sketchpad to the students in the classroom.

2. Students’ self-learning: the students solve the problems by themselves or solve them with the colleagues in the same group. In each group, there are three members. While students were solving problems, there were no interventions from the teachers.

3. Whole class discussion and comparison: students in each group present their group’s ideas to the class and discuss them with the classmates. The teachers attempt to elaborate the students’ ideas in each group as well as the ideas in The Geometer’s Sketchpad so that students can understand the ideas from the colleagues in the class. The teachers also provided other students the chances to ask questions.

4. Summarizing by connecting students’ mathematical ideas emerged in the classroom: in this step, the teachers used The Geometer’s Sketchpad to make the conclusion of the students’ ideas. Both teachers and students worked together in making conclusion of concepts and important topics and connected them together as a conclusion of the lesson.

3.5. **The Geometer’s Sketchpad program**: This software is created to use for investigation and analysis of various aspects of mathematics content. We can use dynamics geometry to create mathematical models that can react in different ways, ranging from basic research of shapes and figures as well as complicated, dynamic and advance drawing. This can be used to enhance the understandings towards geometry, and it also can be used as a learning tool as it can give figurative information about how to create geometric shapes. The outstanding aspect of this program is dynamics. A user can create figure or geometric shapes with the tools in the program as well as adapting information or pictures to see the changes of geometric shapes, make prediction for learning and having concluding ideas.

3.6. **Geometry**: The basic knowledge about dot, line, plane and types of 2-D and 3-D shapes, experiences in seeing and drawing line, angle, triangle and other shapes. The students can imagine the creation of shapes and figures by using what they have seen in daily life. For lower secondary level, geometry depends on the introduction of basic information of geometry. The students can investigate its relationship by
measuring, observing, comparing the geometric alterations and the categorization of geometric objects.

4. Research Methodology

4.1. The Context of Classroom in the Study
This research relies on the context of The Project of Development of Student's Mathematics Higher-Order Thinking at North East of Thailand. This project is developed by Center for Research in Mathematics Education, Khon Kaen University. The data is collected from Kang Kraw Wittaya School. This school has been adhering to the project by using Lesson Study and Open Approach. The directors participated in the meeting of the network to commit the co-operation MOU of the project on July 3-4, 2016 at Education Faculty, Khon Kaen University. The project is launched on July 4, 2016. This project had been prepared in advance since June 22, 2016 by the project of Development of Teachers' Formative Assessment by Using Lesson Study and Open Approach which is supported by Centre of Excellence in Mathematics (CEM) in part of mathematics education, Khon Kaen University and University of North Texas America.

4.2. Target Group
The target group of this research is the students in Kang Kraw Wittaya School, Chaiyapoom, from class M 2/1, by focusing on their second semester 2016. These students have been taught with Open Approach since they were in Matthayom 1 (or grade 7). The researchers start to observe the class in the first semester, 2016 by focusing on the students’ behaviors in solving problems and ideas. They also make students get used to having them in class. The selection is drawn by choosing the students who can work together in sharing opinions while solving open-ended problem. The researchers chose 9 students and divided them into 3 groups, and they remained in the same group for the rest of research process.

4.3. Research Instruments
1) Collecting data tools are included the lesson plans by using The Geometer’s Sketchpad, fieldnote, worksheets, voice recorder, video recorder and camera.
2) Analyzing data tool are included Dubinsky & McDonald [7]’s APOS theoretical framework, protocol recorded from voice recorder and video recorder, protocol recorded form interviewing and student’s written works.

5. Data Collection
The researchers and lesson study team studied the general context of Kang Kraw Wittaya School, especially in the class that they want to study. They worked together to make lesson plan, which is applied Lesson Study, Open Approach and The Geometer’s Sketchpad. Also, their study included observing the behaviors of the target group and attempting to make students accustomed to problem solving. While collecting data, researchers planned together with the lesson study team, the researching assistants recorded audio, video and photos. The researchers and their assistants also observe mathematical ideas of students, problem solving through observation, the chance given for discussion between teachers and students. While studying, the researchers and the assistants could not intervene student’s way of thinking.

6. Data Analysis
Qualitative manner is applied to this research. The protocol is made by transferring audio records into texts. It is analyzed together with target students' written works, fieldnote and interviewing records. The data is analyzed by the framework of APOS Theory [7]. The mathematical conceptual understanding can be divided into 4 levels which are 1) Action 2) Process 3) Object and 4) Schema. The example of data analysis can be seen as the following.
6.1. **Step 1 Posing Open-Ended Problems.**  
In this step, GSP program is used to pose the problems and explore problem’s details and conditions. It can be inferred that students have mathematical conceptual understanding in the action level.

![Figure 1](image.png)  
**Figure 1.** The steps of posing open-ended problem, regarding lesson unit 3: shapes rotation

6.1.1 **Action Level:** mathematical conceptual understandings in action level can be seen when the students can solve problem situation by adhering to conditions or instructions given systematically, and this can be seen in the situation below.

The teacher posed open-ended problem by using The Geometer’s Sketchpad, like “when I rotate this figure around l-line, what do you think it would happen”. The students were expected to gather and observe it together. They were initially instructed so that they can experiment what solutions can be used to solve the given situation. This can be as in the following protocol:

| Item   | Teacher | Question                                           |
|--------|---------|---------------------------------------------------|
| Item 2 | Teacher | When I rotate this rectangle around l-line, what do you think it would happen? |
| Item 3 | Nice    | Cylinder.                                         |
| Item 11 | Teacher | And, if I rotate this right triangle?              |
| Item 12 | Gear    | Cone.                                             |
| Item 15 | Teacher | What happens if I rotate this half circle around l-line? |
| Item 16 | Gear    | Sphere.                                           |
| Item 17 | Teacher | Someone said that it is a sphere. Are you sure? Well, let’s think that it rotates around l-line. |
| Item 18 | Ryo     | A ball.                                           |
| Item 19 | Teacher | A ball? How do we call this ball?                 |
| Item 20 | Smile   | Sphere.                                           |

According to Item 2-16, while posing opening-ended problems, the teacher used Geometer’s Sketchpad. The students in group 1 can responded the problems, like if plane figures (2D) are rotated, what solid shapes (3D), they would turn into. This can be inferred that students have mathematical conceptual understanding in **action level**.
6.2. **Step 2: Students ’Self-Learning :**
In this step, it can be seen that students can use what they have observed from using GSP program in the first step to solve mathematical problems, and this brought them several ideas. Thus, this illustrates that the students have mathematical conceptual understanding in process level.

**Figure 2.** the steps of student’s self-learning, regarding lesson unit 3: shapes rotation

6.2.1 **Process Level:** The students can develop their understandings from action level to process level. They can draw relationship in the problem situations by drawing picture or writing messages and geometric symbols. This can be seen in the step of student’s self-learning, and it can be seen in the following details.

In this step, the students solve problems by themselves. They gathered and discussed about the given problem situations, and they drew solid figures by rotating figures around l-line, as it can be seen in the following protocol:

**Table 2.** An example protocol of students’ mathematical conceptual understanding in process level

| Item   | Action | Description                                                                                                                                 |
|--------|--------|------------------------------------------------------------------------------------------------------------------------------------------|
| Item 31| Smile  | Rotate it around l-line, right?                                                                                                          |
| Item 32| Nice   | I get it now. From this point, if we rotate it, it would turn into a sphere because it has constant radius.                                 |
| Item 33| Smile  | Rotate around l-line, and it becomes a sphere around l-line.                                                                              |
| Item 34| Gear   | Let’s identify each side.                                                                                                                |
| Item 35| Nice   | The first figure is a square. We will name each side a, b, c, and d. For b-side, it is next to l-line. When rotating a-side and c-side around l-line, it will turn into the cross-section of a circle. |
| Item 36| Smile  | What’s about D-side? D-side become the lateral surface.                                                                                   |
| Item 37| Nice   | Well, it is an area of lateral surface of a cylinder.                                                                                      |
| Item 38| Smile  | For the other figures, we can write them similar to this.                                                                                |
Figure 3. demonstrates problem solving and writing of students the steps of students’ self-learning, regarding lesson unit 3: shapes rotation

According to Item 31-37 (the protocol of Group 2), it is inferred that the students can explain the figures that can be when rotating the figures around l-line. For example, they can explain that a figure can turn into a circle because it has constant radius. They can also identify each side and give conditions for each side of the given figure so that they can explain the compositions that can make a plane figure become solid figures. The above protocol shows that the students have mathematical conceptual understanding in the process level.

6.3.  Step 3: Whole Class Discussion and Comparison:
In this step, GSP program is used to extend and compare student’s mathematical ideas occurred in the classroom. This illustrates that students have mathematical conceptual understanding in the object level.

Figure 4. The use of The Geometer’s sketchpad for the lesson unit 3: problematic plane in the step of class discussion and comparison
6.3.1 **Object Level**: In this level, the students can tell the compositions of all solid shapes that created by rotating plane around l-line once, they can give definitions and principles in creating solid shapes. This happens in the learning activities, in the process of connecting students’ mathematical ideas.

In the step of whole class discussion and comparison, the students in the Group 2 shared ideas after solving the problems by themselves. They tried to find a way to explain this in the different way from other colleagues. They explained ideas and gave examples as the following protocols:

**Table 3: An example protocol of students’ mathematical conceptual understanding in object level**

| Item   | Character | Protocol Description                                                                                                                                                                                                 |
|--------|-----------|----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| Item 65 | Smile     | When a rectangle rotates around l-line, it will become a cylinder. We know this by seeing the distance here. For this figure, if it is rotated, it will become a circle. This is because of a distance here. The length of each rotation is equal, so it becomes a circle. |
| Item 66 | Nice      | We use l-line as a center of rotation. For c-side and a-side, when we rotate them by using l-line as a center, we’ll get a circle that have c-side and a-side as radius of the circle. It became the cross-section of a cylinder D. For this side, if we rotate it, it will become the lateral surface of cylinder. The height of cylinder is determined by the height of the length side that rotates around l-line and becomes a cylinder. |
| Item 67 | Smile     | If this triangle rotates around l-line, it will become a cone. We know it by seeing the base and the length of the base. When it rotates, we’ll get a cross-section of a cone in a shape of a circle. The length of the base is the radius of a circle or a cross-section of a cone. The reason why at the end of cone is sharp is that there is no distance from l-line. This is different from the first figure as that the lengths of that figure are equal, so there is no angle. |
| Item 68 | Teacher   | Can’t you explain more about no angle?                                                                                                                                                                                  |
| Item 69 | Gear      | If there is a length from l-line, it’ll be a circle, but if it isn’t, it’ll be a sharp-ended angle.                                                                                                                                 |
| Item 70 | Smile     | Let’s see the next figure. When the half-circle rotates around l-line, it’ll become a sphere. It’s because there is no distance between the top or the base of half-circle from l-line. In other words, there isn’t an angle. But, there are distance between the other points and l-line, so it’ll become a sphere. |
Figure 5. the steps of whole class discussion and comparison, regarding lesson unit 3: shapes rotation

According to protocol Item 65-70, the students presented the compositions that they thought would create solid shapes. They said that l-line is the center of the rotation and determined the conditions so that they could tell what parts in the solid shapes those lines are. For example, a-line and c-line are the radius of a circle and they make a cross-section of cylinder. D-side or the edge of the given figure is the side surface of solid shapes. Therefore, it can be interpreted that those students have mathematical conceptual understanding in object level.

6.4. Step 4: Summarizing by Connecting Students’ Mathematical Ideas Emerged in the Classroom

In the previous step, GSP program is used to extend and compare mathematical ideas that emerged in the classroom. It can be seen that the students have mathematical conceptual understanding in Schema level.

Figure 6. Step 4: Summarizing by Connecting Students’ Mathematical Ideas Emerged in the Classroom
**Table 4.** An example protocol of students' mathematical conceptual understanding in schema level

| Item | Role | Statement |
|------|------|-----------|
| 122  | Teacher | Everyone can explain it very clearly. However, I’d like each of you to try to see the connection between things. For example, a part of triangle does what function in transforming it into a solid shapes. |
| 132  | Gear | After rotating, the area of this figure will create an area of a shape. |
| 133  | Teacher | Wow. That’s incredible. No one has mentioned that. If we rotating the area of a figure, what would happen? |
| 134  | Nice | Volume. |
| 135  | Teacher | Kim said that it’s an area of a shape, but Nice called it volume. So, what should we call it? |
| 136  | Gear | If we rotate an area of a figure, it’ll become an area of a shape, or what we call volume. |

According to Item 136, “If we rotate an area of a figure, it’ll become an area of a shape, or what we call volume”, this illustrates that the students in Group 3 can connect the knowledge of area of a figure and volume of a solid shapes. This can be inferred that these students have mathematical understanding in the **level of Schema**.

**7. Results of Study**

According to the analysis of student’s mathematical conceptual understanding about geometry, it is found that all of three groups have mathematical conceptual understanding according to APOS Theory (Dubinsky & McDonald, 2001). It can be seen that after being taught in the class that applied Lesson Study, Open Approach and The Geometer’s Sketchpad, students have mathematical conceptual understanding in four levels.

The data is analyzed according to Inprasitha’s Open Approach (2011), and it is also analyzed by Dubinsky & McDonald’s APOS Theory (2001). As a result, student’s mathematical conceptual understanding after being taught in the classroom that used Lesson Study, Open Approach and The Geometer's Sketchpad can be summarized as shown the following table.

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Figure 7. Demonstrates student's writing and using GSP in the steps of summarizing by connecting students’ mathematical thinking emerged in the classroom, regarding lesson unit 3: shapes rotation.
Table 5: The summary of levels of students’ mathematical conceptual understanding in geometry taught in classroom using Lesson Study and Open Approach with The Geometer’s Sketchpad

| Lesson Plans | Steps of Open Approach | Levels of Mathematical Conceptual Understanding |
|--------------|------------------------|-----------------------------------------------|
|              |                        | Action | Process | Object | Schema               |
|              |                        | Group 1 | Group 2 | Group 3 | Group 1 | Group 2 | Group 3 |
| 1            | Posing open-ended problems | ✓      | ✓      | ✓      |                     |
|              | Students’ self-learning | ✓      | ✓      | ✓      |                     |
|              | Whole class discussion and comparison | ✓  | ✓  | ✓  |                     |
|              | Summarization through connecting students’ mathematical ideas emerged in the classroom | ✓  | ✓  | ✓  |                     |
| 2            | Posing open-ended problems | ✓      | ✓      | ✓      |                     |
|              | Students’ self-learning | ✓      | ✓      | ✓      |                     |
|              | Whole class discussion and comparison | ✓  | ✓  | ✓  |                     |
|              | Summarization through connecting students’ mathematical ideas emerged in the classroom | ✓  |     |     |                     |
| 3            | Posing open-ended problems | ✓      | ✓      | ✓      |                     |
|              | Students’ self-learning | ✓      | ✓      | ✓      |                     |
|              | Whole class discussion and comparison | ✓  | ✓  | ✓  |                     |
|              | Summarization through connecting students’ mathematical ideas emerged in the classroom | ✓  | ✓  | ✓  |                     |
| 4            | Posing open-ended problems | ✓      | ✓      | ✓      |                     |
|              | Students’ self-learning | ✓      | ✓      | ✓      |                     |
|              | Whole class discussion and comparison | ✓  | ✓  | ✓  |                     |
|              | Summarization through connecting students’ mathematical ideas emerged in the classroom | ✓  |     |     |                     |
| 5            | Posing open-ended problems | ✓      | ✓      | ✓      |                     |
|              | Students’ self-learning | ✓      | ✓      | ✓      |                     |
|              | Whole class discussion and comparison | ✓  | ✓  | ✓  |                     |
|              | Summarization through connecting students’ mathematical ideas emerged in the classroom | ✓  |     |     |                     |

According to table 4, it is readily apparent that in the step of posing open-ended problems, all of three target groups demonstrated their mathematical conceptual understanding in the action level. In the step of students’ self-learning, it is found that the students have mathematical conceptual understanding in the process level. In the step of whole class discussion and comparison, it is found that the students have mathematical conceptual understanding in the object level. In the step of summarizing by connecting students’ mathematical ideas emerged in the classroom, mathematical conceptual understanding in the schema level can be found only in some groups of the students.
9. Discussion
According to the research result, it can be seen that the classroom that has been applying Lesson Study, Open Approach and The Geometer’s Sketchpad (GSP), the students were enabled to have mathematical conceptual understanding (MCU) in 4 levels: (1) MCU in Action Level can be seen when problem were posed. GSP program is used to pose problems and students investigate the details and conditions of problem situation. The students were able to follow the instructions (2) MCU in process level can be seen in the step of student’s self-learning. The students can apply what they had observed form using GSP in the first step to solve problems by themselves, and it also brought them have multiple ideas. They can draw the connection of actions in problem solving and describe their ideas in descriptive manner. They can also present and elaborate their ideas. (3) MCU in object level can be seen in the step of discussion and comparison in class. GSP was used to extend and compare mathematical ideas of each student in the classroom. The students can make conclusion as well as give definitions, principles from solving the problems. (4) MCU in schema level can be witnessed during the step of summarizing by connecting students’ mathematical ideas emerged in the classroom. By using GSP, the students can make connection of their new knowledge with the previous knowledge and also the other knowledge of geometry.

10. Recommendation
10.1. Recommendation for Implementation of Research Findings
   1. Teachers can use the research findings as the guideline for planning the lessons with the use of Lesson Study, Open Approach and The Geometer’s Sketchpad so that they can encourage students to have mathematical conceptual understanding.
   2. Teachers can use the research findings as guidance of technology usage in the problems solving classroom, and they can also use technology as cognitive technology.

10.2. Recommendation for Further Research
   1. to extend the study field by using other program such as GeoGebra, Cabri 3D and so on, and to apply them in the classroom together with Lesson Study and Open Approach by choosing the program that is appropriate with the contents.
   2. the learners should use the program to solve the problems by themselves. This will improve their ability in using technology. They would be more skillful in choosing, using and processing technology which are the skills expected according to the 5th item in Fundamental National Curriculum (2008).

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