Finite $N_c$ Results for $F/D$ Ratios of the Baryon Vertices and $I = J$ Rule

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Abstract: We calculate the $F/D$ ratios of spin-nonflip baryon vertex for an arbitrary number of color degrees of freedom $N_c$ both in the non-relativistic quark model with the $SU(6)$ spin-flavor symmetry and in the chiral soliton model with $SU(3)$ flavor symmetry. We find that the spin-nonflip $F/D$ ratio tends to $-1$ in the limit of $N_c \to \infty$. We show that this leading value $F/D = -1$ of spin-nonflip baryon vertex in the $1/N_c$ expansion corresponds to the isoscalar dominance while the well known leading value $F/D = 1/3$ of the spin-flip vertex corresponds to the isovector dominance. We discuss origins of the dominance of isovector in spin-flip and isoscalar in spin-nonflip baryon vertices, referred to as the $I = J$ rule.

In terms of the matrix elements of the operator which transform as the generator $\lambda^8$ of the $SU(3)$ symmetry we derive the model independent isoscalar formula for baryon vertices and apply this to the mass formula and the isoscalar part of the baryon magnetic moments. The same Okubo-Gell-Mann mass relation and its refined relation among the octet baryons as the one for the case $N_c = 3$ is derived model independently for arbitrary color degrees of freedom $N_c$. Contrary to $\lambda^8$, the Coleman-Glashow mass relation which is derived from an operator that transform as $\lambda^3$ of $SU(3)$ symmetry only hold for $N_c = 3$.

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1 Introduction

The $1/N_c$ expansion was proposed as a non-perturbative approach to QCD by ’t Hooft in 1974 \cite{1}. He has shown that the Feynman diagrams which are relevant at the leading order of the $1/N_c$ expansion are the planar diagrams without internal quark loops.

Based on this $1/N_c$ expansion Witten suggested that in the large $N_c$ limit a baryon looks like soliton\cite{2}. From this viewpoint the Skyrme’s conjecture that baryons are solitons of the nonlinear chiral Lagrangian for the chiral fields has been revived in early 1980’s and has succeeded in describing the baryon sector from the meson sector at least semi-quantitatively\cite{3}.

The non-relativistic quark model(NRQM), on the other hand, has successfully described the hadron phenomena in various aspects and played a crucial role in the way of establishing QCD. However the NRQM description of the low energy hadron physics has not been derived from QCD and looks to have different origin compared with the chiral soliton model(CSM).

The $1/N_c$ expansion method has been considered to be a qualitative method to study QCD. Recent extensive studies of the consistency conditions \cite{4} \cite{5} \cite{6} show that the $1/N_c$ expansion method of QCD is useful for the analysis of NRQM and CSM from model independent viewpoint.

In the previous paper\cite{7} we have calculated the $F/D$ ratios of spin-flip baryon vertices, denoted hereafter as $F_-/D_-$, both in the NRQM and the CSM for arbitrary color degrees of freedom $N_c$. The value of $F_-/D_-$ tends to $1/3$ in the large $N_c$ limit and the physical meaning of this value was nothing but the isovector dominance for the spin-flip baryon vertex.

In this paper we study the spin-nonflip baryon vertices for arbitrary $N_c$ both in the NRQM and the CSM and calculate the $F/D$ ratio of the spin-nonflip baryon vertices, denoted as $F_+/D_+$. The $F/D$ ratios are considered to reflect characteristics of QCD that do not depend on details of the special effective models of QCD. We discuss the obtained results and compare with those of the spin-flip baryon vertices.
vertices from the viewpoint of $1/N_c$ expansion and consistency conditions.

In §2, we define the baryon state for arbitrary $N_c$. In §3 and §4, we calculate the $F/D$ ratios of spin-nonflip baryon vertex for arbitrary $N_c$ both in the NRQM and the CSM. In §5, we discuss the implications of $F_-/D_- = 1/3$ and $F_+/D_+ = -1$ and the convergence problem of $1/N_c$ expansion. In §6, we derive the isoscalar formula for arbitrary $N_c$. We also comment on the mass formula which was derived by Dashen, Jenkins and Manohar[5]. We show there that the same Okubo-Gell-Mann mass relation and its refined relation among the octet baryons as the one for the case $N_c = 3$ is derived model independently for arbitrary $N_c$ while the Coleman-Glashow mass relation is derived only for $N_c = 3$.

2 The Large $N_c$ Baryon States

In order to study the properties of baryons in the $SU(N_c)$ symmetric QCD with arbitrary $N_c$ we have to introduce the extended baryon state which is totally antisymmetric color singlet state of the $SU(N_c)$ symmetry. In the case of two flavors with the $SU(2)$ symmetry the extensions from both NRQM and CSM are simple, because the spin wave functions are the same as those of flavor giving the $I = J$ structure

\[ I = J = \frac{1}{2}, \frac{3}{2}, \frac{5}{2}, \ldots, \frac{N_c}{2}, \ldots \]  

(1)

This is a consequence of the fact that spin-flavor states of the ground state baryon are totally symmetric in NRQM and is similar to that of the hedgehog Ansatz in CSM, i.e. $I = J = 1/2, 3/2, \ldots, N_c/2, \ldots$. In both models with flavor $SU(2)$, the baryon ground state with spin $1/2$ belong to the isospin $1/2$ state.

In the case of three or more flavors there are some ambiguities in extending the baryons for large $N_c$ and we have to introduce some unphysical members of the $SU(N_f)$ multiplet. However, we will show that the following special choice of extension for the large $N_c$ baryon is appropriate to obtain the correct large $N_c$ behavior of various baryon matrix elements.

The ground state spin $1/2$ baryons for arbitrary $N_c$ belong to $(1 + k)(3 + k)$ dimensional representation of flavor $SU(3)$ symmetry because of total symmetry in the spin-flavor states where $k = (N_c - 1)/2$ ($k = 0, 1, 2, \ldots$). This representation
is specified by the Young diagram with the first row of length \( k + 1 \) and the second row of length \( k \) and the root diagram shown in Fig. 1. The usual octet baryons are located at the top region of this root diagram. The flavor wave functions are represented by the tensors with one superscript and \( k \) subscripts and the usual octet baryons are given by

\[
\begin{align*}
p &: B^1_{3,\ldots,3} = 1 & n &: B^2_{3,\ldots,3} = 1 \\
\Sigma^+ &: B^1_{23,\ldots,3} = \frac{1}{\sqrt{k}} & \Sigma^- &: B^2_{13,\ldots,3} = -\frac{1}{\sqrt{k}} \\
\Sigma^0 &: B^1_{13,\ldots,3} = -B^2_{23,\ldots,3} = -\frac{1}{\sqrt{2k}} \\
\Lambda &: B^1_{13,\ldots,3} = B^2_{23,\ldots,3} = \frac{1}{\sqrt{4 + 2k}} & B^3_{3,\ldots,3} = -\frac{2}{\sqrt{4 + 2k}} \\
\Xi^0 &: B^3_{23,\ldots,3} = -\frac{3}{2}B^2_{223,\ldots,3} = -3B^1_{123,\ldots,3} = \sqrt{\frac{3}{k(k+2)}} \\
\Xi^- &: B^3_{13,\ldots,3} = -\frac{3}{2}B^1_{113,\ldots,3} = -3B^1_{123,\ldots,3} = \sqrt{\frac{3}{k(k+2)}}
\end{align*}
\]

For this definition of baryon states, the hypercharge extended to arbitrary \( N_c \) is given by

\[
Y = \frac{N_c B}{3} + S,
\]

where \( B \) is the baryon number and \( S \) is the strangeness which reduces to \( Y = B + S \) in the physical case with \( N_c = 3 \).

We use the wave function (2) of proton and neutron also for the case of flavor \( SU(2) \) symmetry, where the subscript 3 means antisymmetric pair of \( u \) and \( d \) quarks.

By use of the above baryon states we calculate the matrix elements \( < B_f | O^{ai} | B_i > \) and \( < B_f | \lambda^a \sigma^i | B_i > \) where \( a = 0, 1, \ldots, 8 \) and \( i = 0, 1, 2, 3 \). We denote by \( \lambda^0 \) the unit matrix of flavor \( SU(3) \) and \( \sigma^0 \) is the unit matrix of spin space representing spin-nonflip. These matrix elements correspond to the various physical quantities of baryons as the mass, the magnetic moment, the axial coupling constant etc.

1. \( < B | \lambda^0 \sigma^0 | B > \) the mass \( m_B \)
2. \( < B | Q \sigma^i | B > \) with \( Q = \lambda^3/2 + \lambda^8/2\sqrt{3} \) the magnetic moment \( \mu_B \),
(3) \( < B | \lambda^a \sigma^i | B > \cdots \) the axial vector coupling constant \( g_A \),

(4) \( < B_f | \lambda^{4+i5} \sigma^i | B_i > \cdots \) where \( SU(3) \) matrix \( \lambda^{4+i5} \) changes \( s \) quark to \( u \) quark and corresponds to raising operator of the \( V \)-spin \( V^+ \) and contributes to the semi-leptonic decay of hyperons.

(5) \( < B_f | \lambda^{6+i7} \sigma^0 | B_i > \cdots \) where \( SU(3) \) matrix \( \lambda^{6+i7} \) changes \( s \) quark to \( d \)-quark and corresponds to the raising operator of the \( U \)-spin, \( U^+ \) and contributes to the non-leptonic decay of hyperons.

In the case of the \( SU(3) \) symmetry the most general flavor octet vertex constructed from spin 1/2 baryon is represented as a sum of two independent terms as follows:

\[
< B_f | O^a | B_i > = M \text{tr}(\bar{B}_f \lambda^a B_i) + N \text{tr}(\bar{B}_f B_i \lambda^a) \\
= F \text{tr}(\lambda^a [\bar{B}_f, B_i]) + D \text{tr}(\lambda^a \{\bar{B}_f, B_i\}),
\]

for the baryon vertex, where \( \lambda^a \) is a flavor octet matrix with \( a = 1, \ldots, 8 \). This \( F/D \) ratio will be denoted by \( F_+/D_+ \). For the spin-flip vertex \( < B_f | O^{ai} | B_i > \) we have a similar expression and the \( F/D \) ratio is denoted by \( F_-/D_- \). The \( F/D \) ratios are useful to observe the large \( N_c \)-dependence of QCD independently of details of the specific effective model of QCD.

The operators \( O^a_0 \) and \( O^{ai} \) with \( i = 1, 2, 3 \) transform as \( (8, 1) \) and \( (8, 3) \), respectively under \( (SU(3)_f, SU(2)_s) \) symmetric transformations.

In the case of the \( SU(2) \) flavor symmetry we also introduce the concept of \( F/D \) ratios given by (1) as in the case of \( SU(3) \). Furthermore we assume that the isospin-nonflip baryon vertex corresponds to the generator \( \lambda^8 \) of the \( SU(3) \).

3 The \( F/D \) Ratios from \( SU(4) \) and \( SU(6) \) NRQMs

In the NRQMs with the \( SU(4) \) and \( SU(6) \) symmetries the spin 1/2 baryons are given by the completely symmetric representation with respect to spin and flavor which are represented by the Young diagrams with the first row of length \( k + 1 \) and the second row of length \( k \) where \( k \) is given by \( N_c = 2k + 1 \) (\( k = 0, 1, 2, \ldots \)). Their
dimensions are $6H_{Nc}$ and $4H_{Nc}$ for $SU(4)$ and $SU(6)$ symmetry, where $nH_r$ is a repeated combination $nH_r = n+r-1C_r = (n + r - 1)!/r!(n - 1)!$.

In order to calculate the $F/D$ ratio of spin-nonflip baryon vertices in the case of two flavors, we consider the vertex which transform as the charge operator $Q = (\lambda^3 + \lambda^8/\sqrt{3})/2$ under the $SU(3)$ transformation and reduce to $Q = (\tau^3 + 1)/2$ for arbitrary $N_c$.

The vertex is given by

$$Q_p = Q_F + \frac{1}{3}Q_D,$$ (10)

$$Q_n = -\frac{2}{3}Q_D,$$ (11)

where $Q_F$ and $Q_D$ are the $F$ and $D$ type contributions to the operator $Q$. In the case of the $SU(4)$ symmetric NRQM with two flavors the isospin-flip or isovector and isospin-nonflip or isoscalar baryon vertices are given by

$$Q_{SU(4)}^{I=1} = \left( \begin{array}{ccc} 15 & 4H_{Nc} & 4H_{Nc}^* \\ \lambda^3\sigma^0 & B & B \end{array} \right) c,$$ (12)

$$Q_{SU(4)}^{I=0} = \left( \begin{array}{ccc} 15 & 4H_{Nc} & 4H_{Nc}^* \\ \sigma^0 & B & B \end{array} \right) c,$$ (13)

where $c$ is an unknown constant in the leading order of $1/N_c$.

It is noted here that in the $SU(4)$ symmetric NRQM, both of the isoscalar part and isovector part of the spin-flip vertex are generators which belong to the same $SU(4)$ supermultiplet $15$.

In the $SU(3)$ case we consider the $S$-wave non-leptonic hyperon decay to study the spin-nonflip baryon vertex $< B_f | \lambda^{4+i5} \sigma^0 | B_i >$. In general the effective Hamiltonian of hyperon decay transforms as an element of$(SU(3)_{L} \times SU(3)_{R}) = (8,1)$ and $(27,1)$. In this paper we assume the octet dominance or $\Delta I = 1/2$ enhancement[8].

The baryon vertex of the non-leptonic decay $B_i \rightarrow B_f + \pi^-$ is given by

$$\mathcal{V}(B_i \rightarrow B_f + \pi^-) = < B_f | [I^+, U^+] | B_i >$$

$$= < B_f | V^+ | B_i >,$$ (14)

where $I^+ = I^1 + iI^2$, $U^+ = U^1 + iU^2$ and $V^+ = V^1 + iV^2$ are the raising operators of isospin, $U$-spin and $V$-spin, respectively in the $SU(3)$ flavor space. In terms of
the $F_+/D_+$ ratio defined by (9) the $S$-wave decay vertices of $\Sigma^- \to n + \pi^-$ and $\Xi^- \to \Lambda + \pi^-$ for arbitrary $N_c$ are expressed as

\[
\mathcal{V}(\Sigma^- \to n + \pi^-) = \frac{1}{k}(F_+ - D_+),
\]
\[
\mathcal{V}(\Xi^- \to \Lambda + \pi^-) = \frac{1}{k + 2} \sqrt{\frac{3}{2k}} \{ (2k + 1)F_+ - D_+ \}.
\]

(15)

(16)

In the $SU(6)$ NRQM, the baryon vertex of the non-leptonic hyperon decay is given by

\[
\mathcal{V}(B_i \to B_f + \pi^-)_{SU(6)} = \left( \frac{35}{\lambda^{(4+i5)} \sigma^0} \right) \frac{6H_{N_c}}{B} \frac{6H_{N_c}^*}{B} c'.
\]

(17)

Here the isoscalar part and isovector part of the spin-nonflip vertex belong to the same supermultiplet 35 of $SU(6)$.

In the $SU(6)$ NRQM the $S$-wave decay vertices of $\Sigma^- \to n + \pi^-$ and $\Xi^- \to \Lambda + \pi^-$ for arbitrary $N_c$ are given by

\[
\mathcal{V}(\Sigma^- \to n + \pi^-)_{SU(6)} = 2c',
\]
\[
\mathcal{V}(\Xi^- \to \Lambda + \pi^-)_{SU(6)} = \sqrt{6k} c',
\]

(18)

(19)

where $c'$ is an unknown constant in the leading order of $1/N_c$.

In the $SU(2)$ case we can obtain the $F/D$ ratio of the spin-nonflip baryon vertex for the “baryon” with spin 1/2 of arbitrary $N_c$ by comparing (10) and (11) with (12) and (13). In the $SU(3)$ case, we obtain the same $F/D$ ratio from (15), (16), (18) and (19). The obtained result is

\[
\left( \frac{F_+}{D_+} \right)_{SU(4),SU(6)} = -\frac{N_c + 1}{N_c - 3}.
\]

(20)

The same value of $F/D$ ratio for the $SU(4)$ NRQM with that of the $SU(6)$ model is due to the fact that the wave functions for nucleons are the same in $SU(4)$ and $SU(6)$ NRQM contrary to the case of CSM. The same result can also be derived by the algebraic method. It is noted that in the case of NRQMs with $SU(4)$ and $SU(6)$ symmetries the amplitude of non-leptonic hyperon decay depends on the way of extrapolation to the large $N_c$ baryon. However the $F/D$ ratio does not depend on the way of extrapolation to large $N_c$ baryons.
In the $SU(4)$ and $SU(6)$ NRQMs the $F_+/D_+$ ratio tends to $-1$ in the limit of $N_c \to \infty$. For $N_c = 1$ the $F_+/D_+$ ratio becomes 1 reflecting the fact that the baryons are quarks themselves. For $N_c = 3$ the ratio becomes $\infty$ reflecting the fact that the quark number is conserved implying the pure $F$-type amplitude[9].

Next we turn to the $F_-/D_-$ ratio of the spin-flip baryon vertex for arbitrary $N_c$[7][10]. In the $SU(4)$ NRQM the magnetic moment of baryon $B$ is given by

$$\langle \mu_B \rangle_{SU(4)} = \left( \frac{15}{\sigma^3 Q} \frac{4H_{Nc}}{B} \frac{4H^*_{Nc}}{B} \right) \mu $$

where $\sigma^3$ represents spin up or down and $Q = (\tau^3 + 1)/2$ the charge of the nucleon and $\mu$ is an unknown constant to the leading order in $1/N_c$. It is noted here that in the $SU(4)$ NRQM, both of the isoscalar part and isovector part of the spin-flip vertex are generators which belong to the same $SU(4)$ supermultiplet $15$.

Similarly, in the $SU(6)$ NRQM case the magnetic moment of spin $1/2$ baryon $B$ is given by

$$\langle \mu_B \rangle_{SU(6)} = \left( \frac{35}{\sigma^3 Q} \frac{6H_{Nc}}{B} \frac{6H^*_{Nc}}{B} \right) \mu ,$$

where $Q = \lambda^3/2 + \lambda^8/2\sqrt{3}$ is the charge operator. The isoscalar part and isovector part of the spin-flip vertex belong to the same supermultiplet $35$ of $SU(6)$.

Both of the $SU(4)$ and $SU(6)$ NRQMs give the same magnetic moment of nucleons for arbitrary $N_c$:

$$\langle \mu_p \rangle_{SU(4),SU(6)} = (k + 2)\mu, \tag{23}$$
$$\langle \mu_n \rangle_{SU(4),SU(6)} = -(k + 1)\mu. \tag{24}$$

In terms of the $F/D$ ratio the magnetic moments of nucleons are given by

$$\mu_p = \mu_F + \frac{1}{3}\mu_D, \tag{25}$$
$$\mu_n = -\frac{2}{3}\mu_D, \tag{26}$$

where $\mu_D$ and $\mu_F$ are the $D$ and $F$ type contributions to the magnetic moment of baryons. Comparing (23) and (24) with (25) and (26) we obtain the $F/D$ ratio of spin-flip baryon vertex for the “baryon” with spin $1/2$ and arbitrary $N_c$. The
obtained result is
\[
\left( \frac{F_-}{D_-} \right)_{SU(4),SU(6)} = \frac{N_c + 5}{3(N_c + 1)}.
\]

In the \( SU(4) \) and \( SU(6) \) NRQMs the \( F_-/D_- \) ratio tends to 1/3 in the limit of \( N_c \to \infty \). For \( N_c = 1 \) the \( F/D \) ratio becomes 1 reflecting the fact that the baryons are quarks themselves. For physical baryon with \( N_c = 3 \) the \( F/D \) ratio takes the familiar value 2/3 of the \( SU(6) \) NRQM\[9\].

Here we point out that there is a relation between two \( F/D \) ratios of spin-nonflip and spin-flip baryon vertices in the NRQM\[11\] independent of \( N_c \);
\[
(4f_+ + 1)(4f_- - 1) = 3,
\]
where \( f_\pm = F_\pm/(F_\pm + D_\pm) \).

4 The \( F/D \) Ratios in \( SU(2) \) and \( SU(3) \) CSMs

In the \( SU(2) \) CSM the spin 1/2 baryon state is represented by the elements of \( SU(2) \) matrix in the fundamental representation of \( SU(2) \) which is independent of color degrees of freedom \( N_c \).

In \( SU(2) \) CSM the isoscalar part and the isovector part of currents for the baryon have distinct origins. That is, the isovector part is the space integral of the time component of conserved isovector current which is the Noether current reflecting the symmetry of the chiral Lagrangian and is given by
\[
J^\mu = f_\pi^2 \text{tr}[(\partial_\mu U U^\dagger)\Omega] + \frac{i}{8\epsilon^2} \text{tr}\{[\partial_\nu U U^\dagger, \Omega][\partial^\mu U^\dagger, \partial^\alpha U^\dagger]\},
\]
where \( \Omega \) is the generator of \( SU(2) \) flavor symmetry.

On the other hand the isoscalar part comes from the space integral of the time component of the baryon number current which is topologically conserved;
\[
B^\mu = \frac{\epsilon^{\mu\nu\alpha\beta}}{24\pi^2} \text{tr}[(U^\dagger \partial_\nu U)(U^\dagger \partial_\alpha U)(U^\dagger \partial_\beta U)].
\]

Thus there is no direct relation between the isovector current \( J^\mu \) and the isoscalar part \( B^\mu \) \[\text{[12]}\].
In order to calculate the $F/D$ ratio of spin-nonflip baryon vertices we consider the charge operator $Q$ of nucleon. The vertex is given by

$$Q_{SU(2)\ CSM} = \left( \begin{array}{ccc} 1 & 2 & 2 \\ \tau^0 & B & \bar{B} \\ \sigma^0 & B & \bar{B} \end{array} \right) c + \cdots. \quad (31)$$

The ellipsis in (31) denotes contributions from time derivative of dynamical variables of the spin-isospin rotation of the chiral soliton where the isoscalar part of magnetic moment given by the topological or baryon number current is contained.

$$Q_{p,\ SU(2)\ CSM} = c + \cdots, \quad (32)$$

$$Q_{n,\ SU(2)\ CSM} = c + \cdots. \quad (33)$$

Thus the $F/D$ ratio in the $SU(2)$ CSM is

$$\left( \frac{F_+}{D_+} \right)_{SU(2)\ CSM} = -1 + \cdots, \quad (34)$$

On the other hand in the NRQM the isovector and isoscalar parts of the spin-flip baryon vertex are space integrals of space component of Noether current and topological current, respectively.

Therefore the magnetic moment of the spin $1/2$ baryon is given by

$$(\mu_B)_{SU(2)\ CSM} = \left( \begin{array}{ccc} 3 & 2 & 2 \\ \tau^3 & B & \bar{B} \end{array} \right) \left( \begin{array}{ccc} 3 & 2 & 2 \\ \sigma^3 & B & \bar{B} \end{array} \right) \mu^{I=1} + \cdots, \quad (35)$$

where $\tau^3/2 = Q^{I=1}$ is the isovector part of charge. The ellipsis in (35) denotes contributions from the time derivative of dynamical variables where the isoscalar part are contained. The magnetic moments of the nucleons are

$$(\mu_p)_{SU(2)\ CSM} = \frac{1}{2} \mu^{I=1} + \cdots, \quad (36)$$

$$(\mu_n)_{SU(2)\ CSM} = -\frac{1}{2} \mu^{I=1} + \cdots. \quad (37)$$

From these results and (23) and (26) we obtain the $F_-/D_-$ ratio of spin-flip baryon vertex in the $SU(2)$ CSM as

$$\left( \frac{F_-}{D_-} \right)_{SU(2)\ CSM} = \frac{1}{3} + \cdots. \quad (38)$$
In the $SU(3)$ CSM the spin 1/2 baryon states are represented by the $SU(3)$ matrix elements of the $(1, k) = (1 + k)(3 + k)$ dimensional representation where $k = (N_c - 1)/2$. $(1, k)$ denotes the representation of $SU(3)$ with the Young diagram which has the first row of length $k + 1$ and the second row of length $k$. In the case $N_c = 3$ this is the octet or the regular representation of $SU(3)$.

The baryon vertex of $S$-wave non-leptonic hyperon decay in the $SU(3)$ CSM is given by

$$V(B_i \to B_f + \pi^-)_{SU(3)}^{CSM} = \sum_n \left( \frac{8}{\lambda^{4+5}} \begin{pmatrix} 1 & 1, k \end{pmatrix}_B \begin{pmatrix} 1, k \end{pmatrix}_B^* \right) \left( \frac{8}{\sigma^0} \begin{pmatrix} 1, k \end{pmatrix}_B \begin{pmatrix} 1, k \end{pmatrix}_B^* \right) c' + \cdots, \quad (39)$$

where the summation over $n$ means two orthogonal states of baryons of $(1, k)^*$ representation and $c'$ is a constant. The ellipsis denote corrections from the time derivative of the $SU(3)$ matrix valued dynamical variable $A(t)$ describing the “rotations” in spin-flavor space and contains the higher order term of $1/N_c$ expansion.

The $S$-wave decay vertices of $\Sigma^- \to n + \pi^-$ and $\Xi^- \to \Lambda + \pi^-$ for arbitrary $N_c$ are expressed as

$$V(\Sigma^- \to n + \pi^-)_{SU(3)}^{CSM} = 2(k + 3)c' + \cdots, \quad (40)$$

$$V(\Xi^- \to \Lambda + \pi^-)_{SU(3)}^{CSM} = \sqrt{6k}c' + \cdots. \quad (41)$$

From these results and (10) and (11) we obtain the $F+/D_+$ ratio of spin-nonflip baryon vertex in the $SU(3)$ CSM

$$\left( \frac{F_+}{D_+} \right)_{SU(3)}^{CSM} = -\frac{N_c^2 + 4N_c - 1}{N_c^2 + 4N_c - 9} + \cdots. \quad (42)$$

The $F_+/D_+$ ratio tends to $-1$ in the limit $N_c \to \infty$ and it is 1 and $-5/3$ for $N_c = 1$ and $N_c = 3$, respectively.

In Fig. 2 we compare the $D_+/F_+$ ratios in the $SU(6)$ NRQM and the ratio in the $SU(3)$ CSM. Here $D_+/F_+$ ratios are shown instead of $F_+/D_+$ in order to avoid singular behaviors of the $F_+/D_+$ which appear at $N_c = 3$ and $-2 + \sqrt{13}$ in the NRQM and CSM, respectively. Two ratios coincide at $N_c = 1$ and $N_c \to \infty$ tending to $-1$. The experimental value of the $F/D$ ratio for spin-nonflip baryon
vertex \((F_+/D_+)_\text{exp} = -3.0 \pm 0.2\) or \((D_+/F_+)_\text{exp} = -0.33 \pm 0.02\) lies also between those of NRQM and CSM.

Next we turn to the \(F_-/D_-\) ratio of spin-flip vertex. In the \(SU(3)\) chiral soliton model differently from the \(SU(2)\) CSM, the magnetic moment of spin 1/2 baryon \(B\) is given by

\[
(\mu_B)_{SU(3)\, CSM} = \sum_n \left( \frac{8}{Q} \begin{pmatrix} 1 \end{pmatrix}_B \begin{pmatrix} k \end{pmatrix}_B \right) \left( \frac{8}{\sigma^3} \begin{pmatrix} 1 \end{pmatrix}_B \begin{pmatrix} k \end{pmatrix}_B \right)^* \mu + \cdots \tag{43}
\]

where the summation over \(n\) means two orthogonal states of baryons of \((1,k)\) representation and the ellipsis denote corrections from the time derivative of the \(SU(3)\) matrix valued dynamical variable \(A(t)\) describing the “rotations” in spin-flavor space which contain the higher order term of \(1/N_c\) expansion. The results for the magnetic moments are

\[
(\mu_p)_{SU(3)\, CSM} = \frac{k + 3}{3(k + 4)} \mu + \cdots, \tag{44}
\]

\[
(\mu_n)_{SU(3)\, CSM} = \frac{k^2 + 5k + 3}{3(k + 2)(k + 4)} \mu + \cdots. \tag{45}
\]

There is a difference in the magnetic moments of nucleons in the chiral soliton models between flavor 2 and 3 contrary to the NRQM. This comes from the fact that the nucleon states belong to the fundamental representation \(2\) in the \(SU(2)\) soliton irrespective of \(N_c\) while in the \(SU(3)\) case the states belong to the regular representation \((1,k)\).

From these results we obtain the \(F/D\) ratio of spin-flip baryon vertex in the \(SU(3)\) CSM

\[
\left( \frac{F_-}{D_-} \right)_{SU(3)\, CSM} = \frac{N_c^2 + 8N_c + 27}{3(N_c^2 + 8N_c + 3)} + \cdots. \tag{46}
\]

In the \(SU(2)\) and \(SU(3)\) CSMs the \(F/D\) ratio becomes 1/3 if we take the limit \(N_c \to \infty\). For \(N_c = 1\), the \(F/D\) ratio becomes 1 and for \(N_c = 3\) it is \(5/9\).

In Fig. 3 we compare the \(F/D\) ratios of spin-flip baryon vertex in the \(SU(6)\) NRQM and in the \(SU(3)\) CSM. Two ratios coincide at \(N_c = 1\) and \(N_c \to \infty\). The experimental value of the \(F/D\) ratio for spin-flip baryon vertex \((F_-/D_-)_\text{exp} = 0.58 \pm 0.04\) lies between the two lines nearer to that of \(SU(3)\) chiral soliton.
In the case of CSM the relation between $F/D$ ratios of spin-nonflip and spin-flip baryon vertices given by (28) which holds in the NRQM is not satisfied.

5 The $1/N_c$ Expansion of $F/D$ Ratios in the NRQM and the CSM and the $I=J$ rule

From the previous calculation, we find the value of $F/D$ ratios of both spin-flip and nonflip baryon vertices for arbitrary $N_c$. In order to investigate the physical meaning of the obtained $F/D$ ratios of baryon vertices, we expand the $F/D$ ratios assuming $N_c$ is large as follows,

\[
\left( \frac{F_+}{D_+} \right)_{SU(4),SU(6)} = - \frac{N_c + 1}{N_c - 3} = -1 - \frac{4}{N_c} + \frac{12}{N_c^2} + \cdots , \tag{47}
\]

\[
\left( \frac{F_+}{D_+} \right)_{SU(3)_{CSM}} = - \frac{N_c^2 + 4N_c - 1}{N_c^2 + 4N_c - 9} = -1 + \frac{0}{N_c} - \frac{8}{N_c^2} + \cdots , \tag{48}
\]

\[
\left( \frac{F_+}{D_+} \right)_{SU(2)_{CSM}} = -1 + \cdots . \tag{49}
\]

Similarly,

\[
\left( \frac{F_-}{D_-} \right)_{SU(4),SU(6)} = \frac{N_c + 5}{3(N_c + 1)} = \frac{1}{3} + \frac{4}{3N_c} - \frac{4}{3N_c^2} + \cdots , \tag{50}
\]

\[
\left( \frac{F_-}{D_-} \right)_{SU(3)_{CSM}} = \frac{N_c^2 + 8N_c + 27}{3(N_c^2 + 8N_c + 3)} = \frac{1}{3} + \frac{0}{N_c} + \frac{8}{N_c^2} + \cdots , \tag{51}
\]

\[
\left( \frac{F_-}{D_-} \right)_{SU(2)_{CSM}} = \frac{1}{3} + \cdots . \tag{52}
\]

The $1/N_c$ expansions of $F_+/D_+$ for both the NRQM and CSM do not converge at $N_c = 3$. The expansion (47) in the NRQM converges for $N_c > 3$ while the expansion (48) in the CSM does at $N_c > 2 + \sqrt{13} = 5.6 \cdots$. On the other hand the expansion (50) in the NRQM converges for $N_c > 1$, but the expansion (51) in the CSM converges at $N_c > 4 + \sqrt{13} = 7.6 \cdots$.

First we consider the physical meaning of the limiting value $-1$ and $1/3$ for the $F/D$ ratios and introduce a “charge” which represents both for spin-flip and
I = 0

J = 0
\[ \bar{\mu}(O(N_c)) \]

J = 1
\[ \bar{\mu}(O(N_c^0)) \]

I = 1

J = 0
\[ \bar{\mu}(O(N_c^0)) \]

J = 1
\[ \bar{\mu}(O(N_c)) \]

Table 1: the $I = J$ rule for the charge operator in the chiral soliton model

| $I$ = $J$ | $I = 0$ | $I = 1$ |
|-----------|---------|---------|
| $J = 0$   | $\bar{\mu}(O(N_c))$ | $\bar{\mu}(O(N_c^0))$ |
| $J = 1$   | $\bar{\mu}(O(N_c^0))$ | $\bar{\mu}(O(N_c))$ |

nonflip nucleon vertices with arbitrary $N_c$

\[ < B \mid \bar{Q} \mid B > = \bar{\mu}_B, \]  

(53)

where $B$ is $p$ or $n$. By the use of wave functions of nucleon (2) the isovector and isoscalar parts are given by

\[ \bar{\mu}^{I=1} = \bar{\mu}_p - \bar{\mu}_n = (F + D)\mu \]  

(54)

\[ \bar{\mu}^{I=0} = \bar{\mu}_p + \bar{\mu}_n = (F - \frac{1}{3}D)\mu \]  

(55)

In the case of spin-nonflip baryon vertex, if we substitute the $1/N_c$ expansions of $F_+/D_+$ we find $\bar{\mu}^{I=1}$ is $O(N_c^0)$ and $\bar{\mu}^{I=0}$ is $O(N_c)$. On the other hand in the case of spin-flip baryon vertex, we obtain the result that $\bar{\mu}^{I=1}$ is $O(N_c)$ and $\bar{\mu}^{I=0}$ is $O(N_c^0)$. Namely $I = J$ part of the baryon vertex is enhanced compared to $I \neq J$ part as is summarized in Table 1. The $I = J$ enhancement is a consequence of the large $N_c$ counting rules for spin-nonflip and spin-flip baryon vertices.

Generally the isovector part is suppressed by $1/N_c$ in comparison to the isoscalar part of the spin-nonflip vertex while the isoscalar part is suppressed by $1/N_c$ in comparison to the isovector part of the spin-flip vertex. Therefore we can neglect the isoscalar part of spin-flip baryon vertex and the isovector part of spin-nonflip baryon vertex in the large $N_c$ limit. Conversely the limiting value $F/D = -1$ in the spin-nonflip vertex and $F/D = 1/3$ in the spin-flip vertex are derived if we assume the $I = J$ enhancement.

The $I = J$ enhancement is seen in the low energy hadron phenomena. For instance the tensor coupling of $g_{\rho NN}$, which is spin-flip vertex, is larger than the vector coupling of $g_{\rho NN}$, which is spin-nonflip vertex, and the vector coupling of $g_{\omega NN}$ is larger than the tensor coupling of $g_{\omega NN}$ as displayed in Table 2. This is a consequence of $1/N_c$ expansion [15].
Table 2: the $I = J$ rule for $g_{\rho \pi \pi}$ and $g_{\omega \pi \pi}$ coupling constant

|       | Vector  | Tensor    |
|-------|---------|-----------|
| $g_{\rho \pi \pi}$ | $O(1/\sqrt{N_c})$ | $O(\sqrt{N_c})$ |
| $g_{\omega \pi \pi}$ | $O(\sqrt{N_c})$ | $O(1/\sqrt{N_c})$ |

Table 3: the $I = J$ rule for the charge operator

|       | $I = 0$          | $I = 1$          |
|-------|------------------|------------------|
| $J = 0$ | $B^0(O(N_c))$  | $V^0(O(N_c^0))$  |
| $J = 1$ | $B^1(O(N_c^0))$ | $V^1(O(N_c))$    |

In the $SU(2)$ CSM this $I = J$ rule is explained by the fact that the isovector part of spin-nonflip baryon vertex comes from the time component of the Noether current which is $O(N_c^0)$ and the isoscalar part comes from that of the topological current which is $O(N_c)$. On the other hand the isovector part of spin-flip baryon vertex comes from the space component of Noether current $J^5_{\mu}$ which is $O(N_c)$ and the isoscalar part comes from that of topological current which is $O(N_c^0)$ (See Table 3).

We note here that the $1/N_c$ correction to $F/D$ ratio has two different origins. One is the $1/N_c$ correction from the baryon wave function and the other is from the $1/N_c$ correction of the dynamical quantities. In the $SU(4)$ symmetric model the $1/N_c$ correction comes only from the baryon wave functions and not from the vertex operators. Contrary to the $SU(4)$ symmetric NRQM, in the $SU(2)$ CSM the $1/N_c$ corrections come from the dynamical variables (time derivative of spin-isospin rotation matrix $A(t)$) not from the baryon wave functions.

Next we comment on the $1/N_c$ correction in the $SU(3)$ CSM. We calculated the $F/D$ ratios from the nonleptonic hyperon decay and the magnetic moment. There are, however, additional contributions to the nonleptonic hyperon decay vertices and the magnetic moments from the correction due to the soliton rotation in the spin-isospin space. By taking these effects into account, we obtain the additional contributions to the $F/D$ ratios which are suppressed by $1/N_c$. Therefore there
exist the 1/N_c corrections even if we consider the SU(3) CSM.

Let’s consider the reason why the I = J rule works in the SU(2) CSM. In the SU(2) CSM the spin and the isospin operator are

\[ I^a = i \lambda \text{tr}(A^i \tau^a \dot{A}), \quad J^i = -i \lambda \text{tr}(\sigma^i A^i \dot{A}), \]

where \( \lambda \) is a moment of inertia of the order \( O(N_c) \) and \( \dot{A} \) is a time derivative of \( A \).

If spin and isospin of baryon state are of \( O(N_c^0) \), then from (56) and (57) we find that \( \dot{A} \) is \( O(1/N_c) \). The space component of Noether current (29) and the time component of baryon number current (30) are suppressed compared to the time component of Noether current and the space component of baryon number current i.e. the \( I = J \) rule (I \( \neq \) J suppression) is satisfied.

But if spin and isospin of baryon state are of \( O(N_c) \), then \( \dot{A} \) is \( O(N_c^0) \). The space component of Noether current and the time component of baryon number current are not suppressed compared to the time component of Noether current and the space component of baryon number current i.e. the \( I = J \) rule breaks down (I \( \neq \) J enhancement).

It is desirable of course we need to understand the \( I = J \) rule from quarks and gluons i.e. from QCD.

6 The model independent analysis of the F/D ratios in the limit \( N_c \rightarrow \infty \)

In this section we study the physical meaning of the F/D ratios in the limit \( N_c \rightarrow \infty \) in more detail. The matrix element of the diagonal operator \( H^8 \) can be expressed in general according to the Wigner-Eckart theorem[16] as follows;

\[ \langle B_f \mid H^8 \mid B_i \rangle = a Y + b\{I(I+1) - \frac{Y^2}{4} - \frac{F^a F_a}{3}\}, \]

where \( F^a \) is the SU(3) generator and \( F^a F_a \) is the Casimir operator of the SU(3) and

\[ F^a F_a = \frac{(k+2)^2}{3}, \]
for the \((1, k)\) representation of \(SU(3)\).

On the other hand we can calculate the matrix element of the diagonal operator \(H^8\) using \(F\) and \(D\) and baryon states given by (2) \(\sim\) (7).

\[
< N | H^8 | N > = -F + \frac{1}{3}D 
\]

\[
< \Sigma | H^8 | \Sigma > = -F + \frac{1}{3}D + \frac{1}{k}(F - D)
\]

\[
< \Lambda | H^8 | \Lambda > = -F + \frac{1}{3}D + \frac{2}{k + 2}(F + D - \frac{1}{k}(F - D))
\]

\[
< \Xi | H^8 | \Xi > = -F + \frac{1}{3}D + \frac{3}{k + 2}(F + D - \frac{1}{k}(F - D))
\]

The general expression that satisfy these equations is

\[
< B_f | H^8 | B_i > = -F + \frac{1}{3}D + \frac{2}{k}(F - D)K + \frac{1}{k + 2}(F + D - \frac{1}{k}(F - D))\left\{I(I + 1) - (K + \frac{1}{2})(K + \frac{3}{2})\right\}
\]

where \(K = -S/2, S\) being the strangeness.

From (60) \(\sim\) (63) and (64) we find the following structure among the matrix elements of flavor \(SU(3)\) generator \(\lambda^8\). The first part of (64) gives the same contribution \(-F + D/3\) to all states. The second part proportional to \(F - D\) which is of the order \(O(1/N_c)\) gives increasing contributions with \(K\). The third part which includes both \(O(1/N_c)\) term proportional to \(F + D\) and \(O(1/N_c^2)\) term proportional to \(F - D\) appears only for the states located on the inner triangle of the root diagram because of the factor \(\{(I + 1) - (K + 1/2)(K + 3/2)\}\).

Using the definition \(Y = N_cB/3 + S\), we can rewrite the above expression as

\[
< B_f | H^8 | B_i > = -\frac{1}{6k(k + 2)}\{(k + 5)(2k + 1)F + (k - 1)(2k + 5)D\}Y
\]

\[
-\frac{1}{k + 2}(F + D - \frac{1}{k}(F - D))\left\{I(I + 1) - \frac{Y^2}{4} - \frac{(k + 2)^2}{9}\right\}
\]

Here we note that we have to define the hypercharge as \(Y = N_cB/3 + S\) which is consistent with the extended baryon states expressed by (2) \(\sim\) (4). The strangeness
$S$ and the baryon number $B$ are the quantities of order $O(1)$, thus the extended hypercharge $Y$ is of order $O(N_c)$ so that the center of the root diagram for baryon multiplet in the $(1, k)$ representation is located at the origin of the $(I^3, Y)$ plane.

On the other hand if we define the hypercharge as $Y = B + S$, we cannot obtain the result that is consistent with the Wigner-Eckart Theorem.

The generator $F^a$ of $SU(3)$ flavor symmetry is represented in terms of $(1 + k)(3 + k) \times (1 + k)(3 + k)$ matrix, the dimension of spin 1/2 baryon. The general expressions of these matrix elements contain the Casimir operator.

If $H^8$ transforms as $(SU(3)_f, SU(2)_s) = (8, 1)$, then $F_+ / D_+ = -1 + O(1/N_c)$. With this $F_+ / D_+$ ratio of the mass formula for the baryons, the mass difference of baryons is given by

$$\Delta m_B = N_c a + bK + \frac{c}{N_c} \{ I(I + 1) - (K + \frac{1}{2})(K + \frac{3}{2}) \}, \quad (66)$$

where $a$, $b$ and $c$ are constants independent of $N_c$ and isospin. We find that the terms which depend on the isospin and the strangeness are suppressed to the order $O(1/N_c)$. A similar formula is also derived in Ref.[5]

On the other hand if $H^8$ transforms as $(SU(3)_f, SU(2)_s) = (8, 3)$, then $F_- / D_- = 1/3$. We can apply this formula to the isoscalar part of the magnetic moment. Then we obtain

$$\mu_B^{I=0} = a' + b'K + c' \{ I(I + 1) - (K + \frac{1}{2})(K + \frac{3}{2}) \} \quad (67)$$

where $a'$, $b'$ and $c'$ are constants independent of $N_c$.

The isospin and strangeness dependence survives in this formula even if we take the limit $N_c \to \infty$.

The isospin and strangeness dependent terms are not necessarily to suppressed at the lowest order in $1/N_c$ expansion.[6][17][18][19].

From (60) $\sim$ (63), we can derive the model independent and $N_c$ independent relations.

The model independent relations are the Okubo-Gell-Mann mass relation

$$3\Lambda + \Sigma = 2(N + \Xi), \quad (68)$$

and the refined Okubo-Gell-Mann relation[20]

$$3\Lambda + \Sigma^+ + \Sigma^- - \Sigma^0 = p + n + \Xi^0 + \Xi^-, \quad (69)$$
which is more accurately satisfied by experimental masses are $N_c$-dependent. These relations are not trivial, since the multiplet belonging to spin 1/2 baryon is no longer an octet representation of $SU(3)$.

Actually the Coleman-Glashow mass relation

$$\Sigma^+ - \Sigma^- = p - n + \Xi^0 - \Xi^-$$

(70)

holds only for $N_c = 3$. In the case of Coleman-Glashow mass relation the relevant operator is $H^3$ which transforms as generator $\lambda^3$. The diagonal matrix elements of operator $H^3$ for the octet baryons are expressed as

$$< p | H^3 | p > = F + D$$

(71)

$$< n | H^3 | n > = -(F + D)$$

(72)

$$< \Sigma^+ | H^3 | \Sigma^+ > = F + D + \frac{1}{k}(F - D)$$

(73)

$$< \Sigma^0 | H^3 | \Sigma^0 > = 0,$$

$$< \Sigma^- | H^3 | \Sigma^- > = -(F + D) - \frac{1}{k}(F - D)$$

(74)

$$< \Lambda | H^3 | \Lambda > = 0,$$

$$< \Xi^0 | H^3 | \Xi^0 >$$

$$= -\frac{1}{3}(F + D) - \frac{2}{3k}(F - D) - \frac{1}{k+2}\{-3F + D - \frac{3}{k}(F - D)\}$$

(75)

$$< \Xi^- | H^3 | \Xi^- >$$

$$= \frac{1}{3}(F + D) + \frac{2}{3k}(F - D) + \frac{1}{k+2}\{-3F + D - \frac{3}{k}(F - D)\}.$$ 

(76)

As noted in the case of the matrix elements of generator $\lambda^8$ here we also find a systematic structure in the matrix elements of generator $\lambda^3$. From these matrix elements $< B_f | H^3 | B_i >$ it is seen that the Coleman-Glashow relation is derived only for $N_c = 3$.

Comparing these matrix elements of $H^3$ to those of $H^8$ we find special sets of the $F$ and $D$ terms which correspond to the $F/D$ ratios that appear in the order $O(1), O(1/N_c)$ and $O(1/N_c^2)$ of $1/N_c$ expansion. The general structure of the matrix elements of the flavor generators will be discussed elsewhere.
7 Summary

In the preceding sections we have calculated the $F/D$ ratios of spin-nonflip and spin-flip baryon vertices for arbitrary number of color degrees of freedom $N_c$ both in the non-relativistic quark model with the $SU(6)$ spin-flavor symmetry and in the chiral soliton model with $SU(3)$ flavor symmetry. We find that the spin-nonflip $F/D$ ratio of baryon vertex($F_\uparrow/D_\uparrow$) goes to $-1$ in the limit of $N_c \to \infty$, while that of spinflip vertex($F_\downarrow/D_\downarrow$) goes to $1/3$. We have shown that the leading value $F/D = -1$ of spin-nonflip baryon vertex in the $1/N_c$ expansion corresponds to the isoscalar dominance while the leading value $F/D = 1/3$ of the spin-flip vertex obtained in our previous paper corresponds to the isovector dominance, representing the $I = J$ rule for baryon vertices.

We have derived the model independent result by using the NRQM and the CSM. These results can be interpreted as the $I = J$ rule for baryon vertex.

We have explained the reason why the $I = J$ rule is satisfied. In the $SU(2)$ CSM, both of the isoscalar part of spin-flip vertex and the isovector part of spin-nonflip vertex contain the time-derivative of dynamical variable for the spin-isospin rotation $\dot{A}$, while the isovector part of spin-flip and the isoscalar part of spin-nonflip do not contain $\dot{A}$. This is the content of the $I = J$ rule.

In terms of the matrix elements which transform as the generators $\lambda_8$ and $\lambda_3$ of the $SU(3)$ symmetry we derive the model independent isoscalar and isovector formula for baryon vertices and apply these to the mass formulae and the isoscalar and isovector parts of the baryon magnetic moments. We obtain the same Okubo-Gell-Mann mass relation (68) and the refined relation (69) among the octet baryons as the one for the case $N_c = 3$, derived model independently for arbitrary color degrees of freedom $N_c$. In the case of $\lambda_3$ of $SU(3)$ symmetry the Coleman-Glashow mass relation is derived only for $N_c = 3$.

The matrix elements of generators $\lambda_8$ and $\lambda_3$ have a systematic structure. The leading part in the $1/N_c$ expansion gives common contributions to all states of baryon. The second part, which is of the order $O(1/N_c)$ gives contributions which increase as we go towards the bottom of the root diagram of baryon states. The third part appears only for states lying on the inner triangle of the root diagram.
We will discuss the general structure of the matrix elements of the flavor generators elsewhere.

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Figure Captions

Fig. 1. The root diagram for the \((k+1)(k+3)\) dimensional representation of baryon states. The “octet” baryons are located in the upper part of the root diagram.

Fig. 2. \(N_c\)-dependence of \(F_-/D_-\) ratios of spin-flip baryon vertex in the NRQM and CSM. The large \(N_c\) limiting value \(1/3\) and the experimental value \(0.58 \pm 0.04\) which lies between the curves of NRQM and CSM.

Fig. 3. \(N_c\)-dependence of \(D_+/F_+\) ratios of spin-nonflip baryon vertex in the NRQM and CSM. Here \(D/F\) ratios are shown instead of \(F/D\) in order to avoid singular behaviors of the \(F/D\) which appear at \(N_c = 3\) and \(-2 + \sqrt{13}\) in the NRQM and CSM, respectively. The large \(N_c\) limiting value \(-1\) and the experimental value \(-0.33 \pm 0.02\) which lies also between the curves of NRQM and CSM are shown.
Fig. 1
Fig. 2

Fig. 3