Fusion modeling approach for novel plasma sources

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Abstract. The physics involved in the coupling, propagation and absorption of RF helicon waves (electronic whistler) in low temperature Helicon plasma sources is investigated by solving the 3D Maxwell-Vlasov model equations using a WKB asymptotic expansion. The reduced set of equations is formally Hamiltonian and allows for the reconstruction of the wave front of the propagating wave, monitoring along the calculation that the WKB expansion remains satisfied. This method can be fruitfully employed in a new investigation of the power deposition mechanisms involved in common Helicon low temperature plasma sources when a general confinement magnetic field configuration is allowed, unveiling new physical insight in the wave propagation and absorption phenomena and stimulating further research for the design of innovative and more efficient low temperature plasma sources. A brief overview of this methodology and its capabilities has been presented in this paper.

1. Introduction

Analytical and numerical Ray-Tracing techniques, which are based upon the Wentzel-Kramers-Brillouin-Jeffreys (WKBJ) approximation and assume the wave pattern as a superposition of noninteracting modes, have been extensively employed to provide deep insight on wave propagation and absorption and to carry out heating modeling studies in reactor relevant Tokamak devices. These techniques have been proven essential for simulation of Lower Hybrid (LH) and Electron Cyclotron Heating (ECH) and Current Drive (CD), for which the wavelengths are much smaller than the machine dimensions, and they have been also used for the simulation of Ion Cyclotron Resonance Heating (ICRH) and Neoclassical Tearing Modes (NTM) stabilization. The application of these methods has been considered for the study of whistler mode propagation in linearly stratified plasma in magnetosheath and plasma sheet regions of the Earth’s magnetosphere, the analysis of whistlers reflected by the magnetosphere as a source of plasmaspheric hiss and the conversion of whistler waves to lower hybrid waves in the presence of short-scale field-aligned density irregularities (striations) in Earth’s low-altitude ionosphere.

In Ray-Tracing techniques the wavevector satisfies the local dispersion relation, and the wave amplitude must not change much over a wavelength. This can fail in a number of situations, such as in the presence of strong wave focussing or reflection, in mode conversion regions, and inside...
evanescent regions. However, Ray-Tracing has been proven a powerful modeling tool, providing much insight on wave propagation, especially for the simulation of wave related phenomena where associated wavelengths are much smaller than the machine dimensions and finite difference or finite element discretisations would require huge meshes, forbidding the treatment of 2D and 3D problems. Dependences of the plasma density and temperature, the frequency, the parallel wave number spectrum, and the antenna location can be taken into account as well. Maxwell’s equations are usually treated as Boundary Value Problem (BVP), whereas the propagation described by means of the Ray-Tracing approach is reduced to an ensemble of independent initial value problems, whose initial conditions give the starting wave vector and power density at each point of a reference surface of the propagating wave. The required initial conditions for Ray-Tracing are usually determined by a Full-Wave solution of the field radiated by a finite-sized antenna but generally these conditions are obtained by means of auxiliary models describing the antenna or waveguide launcher and its surroundings.

Moreover, not only the Full-Wave solution can be used to provide the Ray-Tracing technique with the necessary initial conditions, but also the combined Ray-Tracing/Full-Wave approach has been applied\[10], \[19] when tunneling, mode conversion or local power deposition due to damping phenomena are important. Specifically, whenever a wave reaches a confluence region a transmitted wave, a reflected wave and a mode converted wave are excited; at each encounter of a ray with the confluence layer, the full wave model predicts the power redistribution and a set of sub-rays is relaunched.

Furthermore, Ray-Tracing has been coupled to the Fokker-Planck equation; considering the modeling of LH waves in the JET tokamak\[11], the LH waves propagation solved by means of Ray-Tracing has been combined to a solver of the Fokker-Planck equation for the evaluation of the electron distribution function self-consistently\[18] with the wave absorption.

Thanks to this treatment of electromagnetic propagation, Ray-Tracing techniques offer a fresh and new approach to study the wave propagation phenomena and power deposition mechanisms involved in efficient Helicon low temperature plasma sources, which are gaining more and more interest in a broad range of applications.

The physics of wave propagation and absorption involved in these plasma sources has been studied in many theoretical and experimental works so far, but only for confinement magnetostatic field considered constant and uniform along the axis of the plasma cylinder. This last assumption can be completely removed by the Ray-Tracing approach, unveiling new physical insight in the power deposition mechanisms. Moreover, such a technique so used in Tokamak plasma reactors could bring some heating modeling concepts coming from plasma fusion scenarios and exploit them in low temperature plasma devices to design and build innovative plasma sources.

2. Plasma source physical model

2.1. Low temperature plasma source applications

Plasmas are currently of considerable interest for a large variety of applications including semiconductor manufacturing, spacecraft propulsion, material surface modification, material processing, basic and applied laboratory research. Low-pressure discharges, whose ionized mechanism is related to helicon waves, are usually called Helicon plasma sources and have been studied for their possible use in industrial etching or surface treatment due to their high density and their unusually high ionization efficiency: the achieved densities of plasma (up to $10^{19}$ m$^{-3}$) are almost an order of magnitude higher than in other discharges (at comparable pressures and input powers). As a matter of fact, Helicon plasma sources have been recognized to be much more efficient rather than capacitive and inductive sources in generating dense plasmas \[12],\[13],\[14],\[15] for low temperature industrial application; their plasma potential is as low as in Electron Cyclotron Resonant (ECR) sources but it needs a magnetic field much
lower (below <0.1 T) than ECR and other high efficiency wave-heated industrial sources. A DC confinement magnetic field along the device axis is necessary for all these sources to be operative, increasing the cost and complexity unlike other plasma sources; however, electromagnets can be replaced by Permanent Magnets (PM), reducing cost and complexity issues, but this could lead to a magnetic field configuration that is no longer perfectly aligned with the axis inside the source region, deeply influencing the wave propagation and power deposition mechanisms. All these features become particularly severe for the development of a plasma source for an electric thruster for space application; these and many other aspects have been considered in the European project HPH.com[8] (Helicon plasma hydrazine combined micro) which aims to develop a space plasma thruster based on helicon plasma sources working in the radiofrequency regime at low-power (<100 W of RF power).

2.2. Helicon Plasma Model

As far as Helicon plasma sources are concerned, a plasma cylinder of radius \( r_a \), surrounded by an antenna placed at radius \( r_b \), and enclosed inside a conducting tube of radius \( r_c \) are to be considered; Fig. 1 shows an axial section of the geometry. Fields are forced by an RF oscillating current given by different antenna configurations; Fig. 2 and Fig. 3 show two common antennas used in Helicon experiments. A magnetostatic field \( B_0 \) is directed along the \( z \) axis, \( B_0 = B_0 \hat{z} \), perpendicular to the \((r, \theta)\) plane.

These sources are able to sustain a steady-state magnetized plasma at low pressure, by means of the propagation and absorption of radiofrequency electromagnetic waves in the range of \( \omega_{ci} \ll \omega_b \ll \omega \ll \omega_{ce} \), where \( \omega_{ci} \) and \( \omega_{ce} \) are the ion and electron cyclotron angular frequencies, and \( \omega_b \) is the lower hybrid frequency.

In this range of frequencies two coupled plasma waves can propagate inside the magnetized plasma column, namely the Helicon and Trivelpiece-Gould (TG) waves [16]. The first wave has a long transverse wavelength \( \lambda_1 \), the second a short one \( \lambda_2 \). The values of the two wavelengths \( \lambda_{1,2} \) is expressed by the dispersion relation [16]:

\[
\lambda_{1,2} \left( n_\parallel \right) = 2\pi \left[ n_\parallel \frac{\omega_{ce}^2}{4c^2} \left( 1 \mp \sqrt{1 - 4 \left( \frac{\omega_{pe}}{\omega_{ce}} \right)^2 \frac{1}{n_\parallel^2} - \frac{\omega_{pe}^2}{\omega_{ce}^2} n_\parallel^2} \right)^2 - n_\parallel \frac{\omega_{pe}^2}{c^2} \right]^{-1/2},
\]

where \( \lambda_{1,2} \) is the transverse wavelength of the Helicon and TG modes respectively, \( \omega_{pe} \) is the plasma frequency, \( \omega_{ce} \) is the electron cyclotron frequency, \( \omega \) is the antenna angular frequency, \( c \) is the speed of light, and \( n_\parallel \) is the parallel wave number, defined as \( n_\parallel = k_z/k_0 \), where \( k_0 = \omega/c \) is the vacuum wave number.

Once that source geometry (in terms of \( r_a, r_b \) and \( r_c \)), antenna type, working frequency \( f \) and plasma characteristics (in terms of gas specie, neutral pressure \( p_n \), DC magnetic field \( B_0 \), plasma
density \( n_0 \) are given, Eq. 1 gives all the wave modes that can propagate and deposite power inside the plasma column. The power deposition due to collisional or collisionless damping and for all the accessible propagative modes can be evaluated by the following equation:

\[
P = \frac{1}{2} \int_{V_{\text{plasma}}} \mathbf{E}^* \cdot \mathbf{J}_{\text{plasma}} dV = \frac{|I_0|^2}{2} (R_p + jX_p),
\]

where \( P \) is the power coupled to the plasma by the antenna, \( \mathbf{E}^* \) is the conjugate electric field inside the plasma, \( \mathbf{J}_{\text{plasma}} \) is the plasma current density, \( I_0 \) is the antenna current, \( R_p \) and \( X_p \) are the electrical resistance and reactance, respectively. In the following, we show how the power deposition can be influenced by plasma parameters in a homogeneous Helicon source [9].

**Figure 4.** Electrical impedance as a function of the plasma parameters for five \((r_a = 1.0 \cdot 10^{-2} \text{ m, } r_a = 2.0 \cdot 10^{-2} \text{ m, } r_a = 3.0 \cdot 10^{-2} \text{ m, } r_a = 4.0 \cdot 10^{-2} \text{ m, } r_a = 5.0 \cdot 10^{-2} \text{ m})\) homogeneous magnetized plasma sources : (a),(d) \( B_0 = 0.05 \text{ T, } n_0 = 5 \cdot 10^{18} \text{ m}^{-3}, p_n = 2.0 \text{ Pa}, \) (b),(e) \( B_0 = 0.05 \text{ T, } f = 10 \text{ MHz, } p_n = 2.0 \text{ Pa}, \) (c),(f) \( f = 10 \text{ MHz, } n_0 = 5 \cdot 10^{18} \text{ m}^{-3}, p_n = 2.0 \text{ Pa}. \) Parameters are: \( m = 0 \text{ antenna, } I_0 = 1.0 \text{ A}. \)

The power deposition mechanism can be influenced whenever a more realistic plasma density profile is considered, slightly changing the plasma electrical impedance; but still the behaviour - as a function of the working frequency, the DC confinement magnetic field and the plasma density - showed in Fig. 4 for a uniform source can be considered representative. Once we remove the hypothesis that the DC magnetic field is aligned along the axial direction of the plasma cylinder, and a general confinement magnetic field is allowed, a completely new picture in the wave propagation and wave absorption phenomena arises. This analysis can be done by means of the Ray-Tracing technique, which can at the same time unveil new physical evidence in the power deposition mechanisms and exploit some heating modeling concepts coming from plasma fusion scenarios in order to design innovative and more efficient low temperature plasma sources.
3. WKB methods for 3D Maxwell equation
The physics involved in the coupling, propagation and absorption of RF helicon waves (electronic whistler) in low temperature plasma sources is investigated by solving the 3D Maxwell-Vlasov model equations using a WKB asymptotic expansion in \( \delta_0 = (\omega a) / c \) (expansion parameter), where \( a \) is the plasma radius of the plasma cylinder, and \( c \) is the speed of light. We solve numerically the reduced set of the WKB equations for the wave phase and for the power damping. In this way we deduce the propagation characteristics of the whistler wave (e.g. mode conversion on the Trivelpiece-Gould branch) and the absorption level of the wave (e.g. power deposition profiles), in the quasi-homogeneous plasma approximation.

The Ray-Tracing approach, which uses the asymptotic technique (WKB) for the solution of Maxwell wave equations, describes the wave propagation in a similar way to the geometric optics where light propagation is described in terms of rays. The reduced set of equations, after the WKB treatment at the lowest order in the expansion parameter, describes a bundle of trajectories in the phase space \((r, k)\), and allows for the reconstruction of the wave front of the propagating wave. These equations are formally Hamiltonian.

In the past, this technique has been used extensively in the laboratory plasma physics to study the propagation and absorption of electromagnetic waves in magnetized plasmas [2]. The equations written in cylindrical geometry are:

\[
\frac{dx}{d\tau} = \delta_0^{-1} \frac{\partial \mathcal{H}}{\partial n_x}; \quad \frac{d\theta}{d\tau} = \delta_0^{-1} \frac{\partial \mathcal{H}}{\partial m}; \quad \frac{dz}{d\tau} = \delta_0^{-1} \frac{\partial \mathcal{H}}{\partial n_z};
\]

\[
\frac{dn_x}{d\tau} = -\delta_0^{-1} \frac{\partial \mathcal{H}_S}{\partial n_x} + \frac{m}{\partial m} \frac{\partial \mathcal{H}}{\partial n_x} + n_z \frac{\partial \mathcal{H}}{\partial n_z};
\]

\[
\frac{dS_0}{d\tau} = n_x \frac{\partial \mathcal{H}}{\partial n_x} + m \frac{\partial \mathcal{H}}{\partial m} + n_z \frac{\partial \mathcal{H}}{\partial n_z};
\]

\[
\frac{dP_{\text{power}}}{dt} = -2\Gamma_{\text{new}}(r, k, \omega) P_{\text{power}}
\]

where \( r/a = (x, \theta, z) \) is the normalized space coordinate, and \( kc/\omega = (n_r, m, n_z) \) is the wave number. Here \( \omega \) is the wave frequency, \( S_0 \) is the wave front surface, \( P_{\text{power}} \) is the power carried by the wave along the trajectory, and \( \Gamma_{\text{new}} \) is the damping coefficient related to the anti-hermitian part of the Stix dielectric tensor and which accounts for Electron Landau Damping (ELD) and the collisional damping. Note that \( \mathcal{H} \) is:

\[
\mathcal{H}(r, n_\parallel, n_\perp) \equiv A(r, n_\parallel) n_\perp^4 + B(r, n_\parallel) n_\perp^2 + C(r, n_\parallel),
\]

where

\[
A(r, n_\parallel) = S
\]

\[
B(r, n_\parallel) = \left[ (n_\parallel^2 - S) (P + S) + D^2 \right]
\]

\[
C(r, n_\parallel) = P \left[ (n_\parallel^2 - S)^2 - D^2 \right]
\]

and \( S, D, P \) are the usual Stix elements of the dielectric tensor, with \( n_\parallel = \hat{b}_r n_r + \hat{b}_z n_z \), \( n_\perp^2 = n_z^2 + (\delta_0^{-1} m/x)^2 + n_r^2 - n_\parallel^2 \). Eq. 4 gives the dispersion relation for the cold electromagnetic electron whistler wave when \( \mathcal{H} = 0 \). The structure of the external magnetic field \( \hat{b}_r = B_r/B, \hat{b}_z = B_z/B \) can be accounted exactly in the definition of the parallel and perpendicular wave-number. This is important because this kind of high frequencies wave (at short wavelength)
are influenced by the structure of the medium (the density and the surrounding magnetic field) during the propagation and their absorption.

It is worth to recall that the validity of the WKB hypothesis relies on the assumption that: i) the wavelength of the propagating mode is much less of the characteristic scale length $L$: $\lambda \ll L = \left( \frac{1}{A_0} \frac{da}{dr} \right)^{-1}$ where $A_0$ is the electric field amplitude, ii) the curvature radius of the wavefront is much greater than the wavelength $k \propto (\nabla \cdot \nabla S_0) \gg \lambda$ (absence of diffraction effects in the wave propagation).

We show now the numerical solution of the Ray-Tracing equation system in the case of whistler applied to the HPH.com facility[8] just with few examples, referring for a more exhaustive analysis of all the features of the propagation, conversion, and absorption of the wave to a subsequent paper[17]. The plasma parameters we have used in the simulation are: i) plasma radius $a = 10$ cm, length of the cylinder $L = 20$ cm, applied frequency $f = 13.56$ MHz, magnetic field on axis $B = 250$ Gauss, density at the plasma center $n_{center} = 10^{12}$ cm$^{-3}$, density at the edge $n_{edge} = 5 \times 10^{11}$ cm$^{-3}$, and 2D profile: $n_e(r, z) = C e^{-(r/a)^2-(z/L)^2} = C e^{x^2-\hat{z}^2}$, central temperature $T_{central} = 10$ eV, edge temperature $T_{edge} = 1$ eV, argon plasma with neutral density $n_{neutral} = 10^{13}$ cm$^{-3}$, the equivalent neutral pressure is $p = 3$ mTorr, poloidal wave-number $m = 0$, and $30 < n_z < 90$. In Fig. 5 we have plotted the dispersion branches of the dispersion relation Eq. 4 for a typical situation with constant confinement magnetic field and the density profile $n_e(x) = Ce^{-x^2}$. Fig. 5 shows the perpendicular wavelength (in cm) vs the $z$-component of the wave number $n_z$ of the launched wave at several radial positions from the edge ($x = 1$) to the center ($x = 0$). Notice that the perpendicular wavelength of the slow branch of the dispersion (Trivelpiece-Gould branch) is less than 1 cm while for the fast branch (Whistler-Helicon branch) the wavelength remains of the order of the device dimensions (10 cm) for a large fraction of the parallel refraction index $26 \leq n_z \leq 56$. At higher value of $n_z > 60$, the perpendicular wavelength of the fast branch increases rapidly with an asymptotic behavior for several radial position so that cut-off occurs for each of these radial positions; this behavior causes the breakdown of the WKB asymptotic expansion.

Notice in Fig. 6 the presence of the various component of the confinement magnetic field and the 2D character of the density profile, which induce a radial reflection around half radius ($x/a \sim 0.5$).
Moreover, Figs. 7-8 show the very strong variation of the axial wave-number and an axial resonance after a radial reflection, which occurs at $x \sim 0.5$ (half radius). It is important to recall that in a simple 1D geometry with a constant confinement magnetic field $B_z$ and density profile depending only on the adimensional radial coordinate $x$, the axial wave-number remained constant inside the plasma. This feature of the wave propagation in a complex 2D or 3D geometry is accounted very well by using this asymptotic technique, without integrating explicitly the Maxwell-Vlasov or Maxwell-Newton system, which is a very expensive task still when considering a small portion of plasma. Obviously it is necessary to pay attention and monitor along the calculation that the WKB expansion remains satisfied. A detailed comparison of the WKB reduced analysis and full Maxwell-Vlasov integration is under consideration[17].

4. Conclusion
In this paper, we presented a brief overview of Ray-Tracing technique for the analysis and design of innovative and more efficient low temperature plasma sources. This technique is based upon the WKB approximation and it has been extensively employed to provide new insight on wave propagation and absorption for simulation of LH, ECH, CD and ICRH to carry out heating modeling studies in fusion plasmas. More importantly, Ray-Tracing is still of high interest in heating scenarios even when mode conversion or local power deposition due to damping phenomena are important and a coupled solution with a Full-Wave method is necessary. We used Ray-Tracing technique to study the physics involved in the coupling, propagation and absorption of RF helicon waves (electronic whistler) in low temperature plasma sources; specifically, we solved numerically the 3D Maxwell-Vlasov equations using a WKB asymptotic expansion for the propagation characteristics of the whistler wave, and the absorption level of the wave. Though the physics of wave propagation and absorption involved in these kind of sources has been studied by many theoretical and experimental works, the axial confinement magnetostatic field hypothesis has been completely removed by the Ray-Tracing approach, unveiling new physical insight in the power deposition mechanisms and giving the chance to use heating modeling concepts coming from plasma fusion scenarios and exploit them in low temperature plasma devices to design and build innovative plasma sources.
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