Determining the WIMP Mass from Direct Dark Matter Detection Data

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First Annual School of EU Network “UniverseNet” – The Origin of the Universe
September 27, 2007

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based on arXiv:0707.0488 [astro-ph]
Reconstructing the velocity distribution function of WIMPs
Deriving $f_1(v)$ from the scattering spectrum
Reconstructing $f_1(v)$ from experimental data

Determining the WIMP mass

Summary
Deriving $f_1(v)$ from the scattering spectrum

- Differential rate for elastic WIMP-nucleus scattering

$$ \frac{dR}{dQ} = A F^2(Q) \int_{v_{\text{min}}}^{\infty} \left[ \frac{f_1(v)}{v} \right] dv $$

Here

$$ v_{\text{min}} = \alpha \sqrt{Q} $$

is the minimal incoming velocity of incident WIMPs that can deposit the recoil energy $Q$ in the detector.

$$ A \equiv \frac{\rho_0 \sigma_0}{2 m_\chi m_r^2} \quad \alpha \equiv \sqrt{\frac{m_N}{2 m_r^2}} \quad m_r = \frac{m_\chi m_N}{m_\chi + m_N} $$

$\rho_0$: WIMP density near the Earth

$\sigma_0$: total cross section ignoring the form factor suppression

$F(Q)$: elastic nuclear form factor
Deriving $f_1(v)$ from the scattering spectrum

- Normalized one-dimensional velocity distribution function

$$f_1(v) = \mathcal{N} \left\{ -2Q \cdot \frac{d}{dQ} \left[ \frac{1}{F^2(Q)} \left( \frac{dR}{dQ} \right) \right] \right\}_{Q=v^2/\alpha^2}$$

$$\mathcal{N} = \frac{2}{\alpha} \left\{ \int_{0}^{\infty} \frac{1}{\sqrt{Q}} \left[ \frac{1}{F^2(Q)} \left( \frac{dR}{dQ} \right) \right] dQ \right\}^{-1}$$

- Moments of the velocity distribution function

$$\langle v^n \rangle = \mathcal{N}(Q_{\text{thre}}) \left( \frac{\alpha^{n+1}}{2} \right) \left[ \frac{2 Q_{\text{thre}}^{(n+1)/2}}{F^2(Q_{\text{thre}})} \left( \frac{dR}{dQ} \right) \right]_{Q=Q_{\text{thre}}} + (n + 1) I_n(Q_{\text{thre}})$$

$$\mathcal{N}(Q_{\text{thre}}) = \frac{2}{\alpha} \left[ \frac{2 Q_{\text{thre}}^{1/2}}{F^2(Q_{\text{thre}})} \left( \frac{dR}{dQ} \right) \right]_{Q=Q_{\text{thre}}}^{-1} + I_0(Q_{\text{thre}})$$

$$I_n(Q_{\text{thre}}) = \int_{Q_{\text{thre}}}^{\infty} Q^{(n-1)/2} \left[ \frac{1}{F^2(Q)} \left( \frac{dR}{dQ} \right) \right] dQ$$

[M. Drees and C. L. Shan, JCAP 0706, 011]
Reconstructing $f_1(v)$ from experimental data

- Experimental data

$$Q_n - \frac{b_n}{2} \leq Q_{n,i} \leq Q_n + \frac{b_n}{2} \quad i = 1, 2, \cdots, N_n, \quad n = 1, 2, \cdots, B$$

- Theoretically predicted scattering spectrum
Reconstructing $f_1(v)$ from experimental data

- Ansatz: in the $n$th $Q$-bin

\[
\left( \frac{dR}{dQ} \right)_n \equiv \left( \frac{dR}{dQ} \right)_{Q \approx Q_n} = \tilde{r}_n e^{\kappa_n (Q - Q_n)} \equiv r_n e^{\kappa_n (Q - Q_{s,n})}
\]

\[
\tilde{r}_n \equiv \left( \frac{dR}{dQ} \right)_{Q = Q_n}
\]

\[
r_n \equiv \frac{N_n}{b_n}
\]

- Recoil spectrum at $Q = Q_n$

\[
\tilde{r}_n = \frac{N_n}{b_n} \left( \frac{\kappa_n}{\sinh \kappa_n} \right)
\]

\[
\kappa_n \equiv \left( \frac{b_n}{2} \right) k_n
\]

- Logarithmic slope and shifted point in the $n$th $Q$-bin

\[
\overline{Q}_n - Q_n = \frac{b_n}{2} \left( \coth \kappa_n - \frac{1}{\kappa_n} \right)
\]

\[
\overline{Q}_n = \frac{1}{N_n} \sum_{i=1}^{N_n} Q_{n,i}
\]

\[
Q_{s,n} = Q_n + \frac{1}{\kappa_n} \ln \left( \frac{\sinh \kappa_n}{\kappa_n} \right)
\]
Reconstructing $f_1(v)$ from experimental data

- Reconstructing the one-dimensional velocity distribution

$$f_{1,r}(v_s,\mu) = \mathcal{N} \left[ \frac{2Q_s,\mu r_\mu}{F^2(Q_s,\mu)} \right] \left[ \frac{d}{dQ} \ln F^2(Q) \right]_{Q=Q_s,\mu} - k_\mu$$

$$v_{s,\mu} = \alpha \sqrt{Q_{s,\mu}}$$

$$\mathcal{N} = \frac{2}{\alpha} \left[ \sum_a \frac{1}{\sqrt{Q_a} F^2(Q_a)} \right]^{-1}$$

- Determining the moments of the velocity distribution

$$\langle v^n \rangle = \alpha^n \left[ \frac{2Q^{1/2}_{\text{thre}} r_{\text{thre}}}{F^2(Q_{\text{thre}})} + I_0 \right]^{-1} \left[ \frac{2Q^{(n+1)/2}_{\text{thre}} r_{\text{thre}}}{F^2(Q_{\text{thre}})} + (n + 1)I_n \right]$$

$$I_n = \sum_a \frac{Q_a^{(n-1)/2}}{F^2(Q_a)}$$

$$r_{\text{thre}} = \left( \frac{dR}{dQ} \right)_{Q=Q_{\text{thre}}}$$

[M. Drees and C. L. Shan, JCAP 0706, 011]

C. L. Shan, Universität Bonn

Determining the WIMP Mass from DDMD Data
Determining the WIMP mass

- Using two different target nuclei

\[ \langle v^n \rangle = \alpha^n_X \left[ \frac{(n+1)I_{n,X}}{I_{0,X}} \right] = \alpha^n_Y \left[ \frac{(n+1)I_{n,Y}}{I_{0,Y}} \right] \]
Determining the WIMP mass

- Using two different target nuclei
  \[ \langle v^n \rangle = \alpha_X^n \left( \frac{(n + 1)I_{n,X}}{I_{0,X}} \right) = \alpha_Y^n \left( \frac{(n + 1)I_{n,Y}}{I_{0,Y}} \right) \]

- WIMP mass
  \[ m_\chi = \frac{\sqrt{m_X m_Y} - m_X R_n}{R_n - \sqrt{m_X/m_Y}} \quad R_n \equiv \frac{\alpha_Y}{\alpha_X} = \left( \frac{I_{n,X}}{I_{0,X}} \cdot \frac{I_{0,Y}}{I_{n,Y}} \right)^{1/n} \quad (n \neq 0, -1) \]

- 1-\sigma statistical error
  \[ \sigma(m_\chi) = \frac{R_n \sqrt{m_X/m_Y} |m_X - m_Y|}{\left( R_n - \sqrt{m_X/m_Y} \right)^2} \]
  \[ \times \frac{1}{|n|} \left[ \frac{\sigma^2 (I_{n,X})}{I_{n,X}^2} + \frac{\sigma^2 (I_{0,X})}{I_{0,X}^2} - \frac{2 \text{cov} (I_{0,X}, I_{n,X})}{I_{0,X} I_{n,X}} + (X \rightarrow Y) \right]^{1/2} \]

[C. L. Shan, arXiv:0707.0488]
Determining the WIMP mass

- 1-σ statistical error for different combinations
  \((1 - 200 \text{ keV}, n = 1, 25 + 25 \text{ events})\)

\[\Delta m_{\chi_{\text{in}}} \text{[GeV]} \]

\[\Delta m_{\chi_{\text{out}}} \text{[GeV]} \]

\(\text{Q}_\text{max} = 200 \text{ keV}, \text{Q}_\text{min} = 1 \text{ keV}, n = 1, 25 + 25 \text{ events}\)

[C. L. Shan, arXiv:0707.0488]
Determining the WIMP mass

- Reproduced WIMP mass
  \((1 - 200 \text{ keV}, n = 1, ^{76}\text{Ge} + ^{28}\text{Si}, 25 + 25 \text{ events})\)

\(Q_{\text{max}} = 200 \text{ keV}, Q_{\text{min}} = 1 \text{ keV}, n = 1, 25 + 25 \text{ events}, \text{Ge-76 + Si-28}\)

[C. L. Shan, arXiv:0707.0488]
Determining the WIMP mass

- Reproduced WIMP mass
  \((1 - 200 \text{ keV}, n = 1, ^{76}\text{Ge} + ^{28}\text{Si}, 250 + 250 \text{ events})\)

\[ Q_{\text{max}} = 200 \text{ keV}, \; Q_{\text{min}} = 1 \text{ keV}, \; n = 1, \; 250 + 250 \text{ events}, \; ^{76}\text{Ge} + ^{28}\text{Si} \]

[C. L. Shan, arXiv:0707.0488]
Determining the WIMP mass

- With $Q_{\text{thre}} > 0$

$$
\mathcal{R}_n(Q_{\text{thre}}) = \left[ \frac{2Q_{\text{thre},X}^{(n+1)/2}r_{\text{thre},X} + (n+1)I_{n,X}F_X^2(Q_{\text{thre},X})}{2Q_{\text{thre},X}^{1/2}r_{\text{thre},X} + I_0,X F_X^2(Q_{\text{thre},X})} \right]^{1/n} (X \rightarrow Y)^{-1}
$$

- Choosing $n = -1$

$$
\mathcal{R}_{-1}(Q_{\text{thre}}) = \frac{r_{\text{thre},Y}}{r_{\text{thre},X}} \left[ \frac{2Q_{\text{thre},X}^{1/2}r_{\text{thre},X} + I_0,X F_X^2(Q_{\text{thre},X})}{2Q_{\text{thre},Y}^{1/2}r_{\text{thre},Y} + I_0,Y F_Y^2(Q_{\text{thre},Y})} \right]^2
$$

$$
\sigma(\mathcal{R}_{-1}) = \mathcal{R}_{-1} \left\{ \left[ \frac{I_0,X F_X^2(Q_{\text{thre},X})}{2Q_{\text{thre},X}^{1/2}r_{\text{thre},X} + I_0,X F_X^2(Q_{\text{thre},X})} \right]^2 \times \left[ \frac{\sigma^2(r_{\text{thre},X})}{r_{\text{thre},X}^2} + \frac{\sigma^2(I_0,X)}{I_{0,X}^2} - \frac{2\text{cov}(r_{\text{thre},X},I_0,X)}{r_{\text{thre},X} I_{0,X}} \right] + (X \rightarrow Y) \right\}^{1/2}
$$

[C. L. Shan, arXiv:0707.0488]
Summary

- By using experimental data with different detector materials we can determine the WIMP mass.

- The larger the mass difference between two target nuclei, the smaller the statistical error will be.

- Our method is model-independent and needs only measured recoil energies.

- With 200 keV maximal measuring energy and 25 events from each experiment, we can already extract meaningful information about the WIMP mass.