Probing $\bar{u}/\bar{d}$ Asymmetry in the Proton via 
$W$ and $Z$ Production

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Abstract

The sensitivity of $W$ and $Z$ production at RHIC to the possible $\bar{u}/\bar{d}$ asymmetry in the proton is studied. The ratios of the $W^+$ over $W^-$ production cross sections in $p+p$ collision, as well as the ratios of the $W^+$ and $Z$ production cross sections for $p+p$ over $p+d$ collisions, are shown to be sensitive to this asymmetry. Predictions of various theoretical models for these ratios are presented.
The validity of the Gottfried sum rule (GSR) [1], proposed more than 25 years ago, was recently tested by the New Muon Collaboration (NMC) in a muon deep inelastic scattering (DIS) experiment [2]. The Gottfried sum is given as

$$S_G = \int_0^1 dx \frac{[F_{2}^{\mu p}(x) - F_{2}^{\mu n}(x)]}{x} = \frac{1}{3} + \frac{2}{3} \int_0^1 dx [\bar{u}(x) - \bar{d}(x)].$$  \hspace{1cm} (1)

The second term in Eq. (1) vanishes if the antiquark distributions in the proton are SU(2) flavor symmetric, and the Gottfried sum rule, $S_G = 1/3$, is obtained. Based on their measurements of muon DIS on hydrogen and deuterium targets and an estimate of the contributions from the unmeasured small $x$-region, the NMC found $S_G = 0.235 \pm 0.026$, significantly different from the value of $1/3$ predicted by the Gottfried sum rule.

A number of theoretical models [3-13] have been proposed to account for the apparent violation of the GSR. Some models [3,4] assume that the valence quark distributions in the proton are sufficiently singular at small $x$ such that a large contribution to the Gottfried sum occurs at a region not probed by the NMC experiment. Therefore, the GSR is not violated and the assumption of $\bar{u}/\bar{d}$ symmetry in the proton remains valid. Other theoretical models [5-13] accept the NMC result as an evidence that the $\bar{u}$ and $\bar{d}$ distributions in the proton are different. Empirical expressions for $\bar{d}(x) - \bar{u}(x)$ have been proposed [3,4,13] and several recent sets of parton distribution functions [14,15] explicitly allow $\bar{u}/\bar{d}$ asymmetry to accommodate the NMC result. The origin of the enhancement of $\bar{d}$ over $\bar{u}$ in the proton has been attributed to pion cloud [5,7,8,12], diquark clustering in the nucleon [10], as well as Pauli-blocking effect [11].

Eq. (1) shows that GSR is sensitive only to the integral of $\bar{u} - \bar{d}$. To provide additional inputs to the possible $\bar{u}/\bar{d}$ asymmetry in the proton, it is important to measure the $\bar{u}/\bar{d}$ ratio as a function of $x$. The proton-induced Drell-Yan process can probe the $\bar{u}/\bar{d}$ asymmetry [6,16,17]. Indeed, the E772 Drell-Yan data have been compared with predictions from various $\bar{u}/\bar{d}$ asymmetric models [18]. More recently, the NA51 collaboration has measured the ratio of Drell-Yan cross sections on hydrogen versus deuterium targets and the result indicates a large asymmetry of $\bar{u}/\bar{d} = 0.51 \pm 0.04 \pm 0.05$ at $x = 0.18$ [19]. Another Drell-Yan experiment covering a wider $x$ range ($0.05 < x < 0.3$) has also been proposed [20].

In addition to the DIS and the Drell-Yan processes, there are other interactions sensitive to the sea-quark distributions in the nucleon. One example is the proton-induced $J/\psi$ and $\Upsilon$ production at a kinematic region where the cross section is dominated by the quark-antiquark annihilation [21]. The opportunity to study $p+p$ and $p+{A}$ collisions at the future...
heavy ion collider, RHIC, suggests yet another process, namely the production of $W$ and $Z$ bosons, which is sensitive to the $\bar{u}/\bar{d}$ asymmetry. In this paper, the sensitivity of $W$ and $Z$ production to the sea quark distribution in the nucleon is studied.

The differential cross section for $W^+$ production in hadron-hadron collision can be written as [22]

$$
\frac{d\sigma}{dx_F}(W^+) = K \frac{\sqrt{2\pi}}{3} G_F \left( \frac{x_1 x_2}{x_1 + x_2} \right) \left\{ \cos^2 \theta_c \left[ u(x_1) \bar{d}(x_2) + \bar{d}(x_1) u(x_2) \right] + \sin^2 \theta_c \left[ u(x_1) \bar{s}(x_2) + \bar{s}(x_1) u(x_2) \right] \right\},
$$

(2)

where $u(x), d(x)$, and $s(x)$ signify the up, down, and strange quark distribution functions in the hadrons. $x_1, x_2$ are the fractional momenta carried by the partons in the colliding hadron pair and $x_F = x_1 - x_2$. $G_F$ is Fermi coupling constant and $\theta_c$ is the Cabbibo angle. The factor $K$ takes into account the contributions from first-order QCD corrections [22]

$$
K \approx 1 + \frac{8\pi}{9} \alpha_s(Q^2).
$$

(3)

At the $W$ mass scale, $\alpha_s \simeq 0.1158$ and $K \approx 1.323$. This indicates that higher-order QCD processes are relatively unimportant for $W$ production. An analogous expression for $W^-$ production cross section is given as

$$
\frac{d\sigma}{dx_F}(W^-) = K \frac{\sqrt{2\pi}}{3} G_F \left( \frac{x_1 x_2}{x_1 + x_2} \right) \left\{ \cos^2 \theta_c \left[ \bar{u}(x_1) d(x_2) + d(x_1) \bar{u}(x_2) \right] + \sin^2 \theta_c \left[ \bar{u}(x_1) s(x_2) + s(x_1) \bar{u}(x_2) \right] \right\},
$$

(4)

It is straightforward to show from Eqs. (2) and (4) that, for $p+\bar{p}$ collisions, the total cross section for $W^+$ production is identical to that for $W^-$ production. Figure 1 shows the $W$ production cross sections measured at the SPS [23] and Tevatron [24] energies. Calculations using Eq. (2) and the MRSD- structure functions reproduce the magnitude and the energy dependence of $W$ production cross sections well.

An interesting quantity to be considered is the ratio of the differential cross sections for $W^+$ and $W^-$ production. If one ignores the much smaller contribution from the strange quarks, this ratio can be written as

$$
R(x_F) \equiv \frac{\frac{d\sigma}{dx_F}(W^+)}{\frac{d\sigma}{dx_F}(W^-)} = \frac{u(x_1) \bar{d}(x_2) + \bar{d}(x_1) u(x_2)}{\bar{u}(x_1) d(x_2) + d(x_1) \bar{u}(x_2)}.
$$

(5)

For $p+p$ collision, it is evident that $R(x_F)$ is symmetric with respect to $x_F = 0$, namely, $R(x_F) = R(-x_F)$. For existing $p+\bar{p}$ collider experiments at the SPS and Tevatron, $R(x_F)$ is...
predominantly sensitive to the valence quark distributions in the proton. This can be readily seen by examining \( R(x_F) \) at \( x_F >> 0 \), namely,

\[
R(x_F >> 0)(p + \overline{p} \text{ collision}) = \frac{u(x_1)d(x_2) + \overline{d}(x_1)\overline{u}(x_2)}{\overline{u}(x_1)d(x_2) + d(x_1)\overline{u}(x_2)} \approx \frac{u(x_1) d(x_2)}{d(x_1) \overline{u}(x_2)}. \tag{6}
\]

The terms containing \( \overline{u}(x_1) \) and \( \overline{d}(x_1) \) were dropped since the sea quark distributions are negligible at large \( x_1 \). One might also note that \( R \) is equal to 1 at \( x_F = 0 \). The recent CDF measurement [25] on the lepton asymmetry in \( W \) production has indeed provided an accurate determination of the \( u/d \) valence quark distributions at \( x \approx 0.1 \) [26].

The situation is very different for \( p + p \) collision, where \( R(x_F) \) becomes sensitive to the sea quark distributions in the proton. Again, it is useful to consider the kinematic region \( x_F >> 0 \), where

\[
R(x_F >> 0)(p + p \text{ collision}) = \frac{u(x_1)d(x_2) + \overline{d}(x_1)u(x_2)}{\overline{u}(x_1)d(x_2) + d(x_1)\overline{u}(x_2)} \approx \frac{u(x_1) \overline{d}(x_2)}{d(x_1) \overline{u}(x_2)}. \tag{7}
\]

At \( x_F = 0 \), where \( x_1 = x_2 = x \), one obtains

\[
R(x_F = 0)(p + p \text{ collision}) = \frac{u(x)\overline{d}(x) + \overline{d}(x)u(x)}{\overline{u}(x)d(x) + d(x)\overline{u}(x)} = \frac{u(x) \overline{d}(x)}{d(x) \overline{u}(x)}. \tag{8}
\]

As the \( u(x)/d(x) \) ratios are already well determined from the CDF data, a measurement of \( R(x_F) \) in \( p + p \) collision gives an accurate determination of the ratio \( \overline{d}(x)/\overline{u}(x) \).

Figure 2(a) shows the predictions of \( R(x_F) \) for \( p + p \) collision at \( \sqrt{s} = 500 \text{ GeV} \). Five different structure function sets together with the full expressions for \( W^+, W^- \) production cross sections given by Eqs. (2) and (4) have been used in the calculations. The solid curve corresponds to the \( \overline{u}/\overline{d} \) symmetric DO1.1 structure functions [27], while the dashed and dotted curves are results for the \( \overline{u}/\overline{d} \) asymmetric structure function sets MRSD-’ and CTEQ2pM, respectively. These two structure function sets were obtained from recent global fits to Drell-Yan and DIS data including the NMC result [14,15,28]. Finally, the calculations using the parameterization for \( \overline{d}(x) - \overline{u}(x) \) given by Ellis and Stirling [6] and Eichten et al. [13] are also shown in Figure 2(a). The parameterization for \( \overline{d}(x) - \overline{u}(x) \) were given at \( Q^2 \) of 4 GeV\(^2\), and it was evolved to the \( W \) mass by using the Altarelli-Parisi equation for flavor non-singlet structure functions [29].

Figure 2(a) shows that the measurement of \( R(x_F) \) for \( W \) production in \( p + p \) collision at RHIC could provide a sensitive test of various models. In contrast, \( R(x_F) \) for \( p + \overline{p} \) collision is insensitive to the \( \overline{u}/\overline{d} \) asymmetry. This is illustrated in Figure 2(b), where calculations using various structure function sets are shown.
Another observable sensitive to the $\bar{u}/\bar{d}$ asymmetry is the ratio of $W^+$ production cross sections for $p+p$ and $p+d$ collisions, which could also be studied at RHIC. The ratio $R'(x_F)$ is defined as
\begin{equation}
R'(x_F) \equiv 2 \frac{d\sigma}{dx_F}(p+p \rightarrow W^+) \approx \frac{u(x_1)d(x_2) + \bar{d}(x_1)u(x_2)}{u(x_1)[\bar{u}(x_1) + d(x_2)] + d(x_1)[u(x_2) + d(x_2)]}.
\end{equation}
It is interesting to note that at large $x_F$, $R'(x_F)$ is probing the $\bar{u}/\bar{d}$ ratio, while $R'(x_F)$ is sensitive to $u/d$ at very negative $x_F$. More specifically,
\begin{equation}
R'(x_F) \approx 1 + \left(\frac{d(x_2) - \bar{u}(x_2)}{d(x_2) + \bar{u}(x_2)}\right) \text{ at } x_F >> 0;
\end{equation}
\begin{equation}
R'(x_F) \approx 1 + \left(\frac{u(x_2) - d(x_2)}{u(x_2) + d(x_2)}\right) \text{ at } x_F << 0.
\end{equation}
Figure 3 shows the predictions of $R'(x_F)$ for various models using the full expression for $W^+$ production given by Eq. (2). The divergence of the various curves at large $x_F$ reflects the different prescriptions of $\bar{u}/\bar{d}$ for the various models. The predictions converge at negative $x_F$, showing that the various models have very similar parameterizations for the well determined valence quark distributions.

Finally, the $Z^0$ production could also provide information on the $\bar{u}/\bar{d}$ asymmetry in the proton. The $Z^0$ differential cross section is given as
\begin{equation}
\frac{d\sigma}{dx_F}(Z^0) = K \frac{\pi}{3\sqrt{2}} G_F \left(\frac{x_1 x_2}{x_1 + x_2}\right) \left\{ \left(1 - \frac{8}{3} \chi_w + \frac{32}{9} \chi_w^2\right) [u(x_1)\bar{u}(x_2) + \bar{u}(x_1)u(x_2)] + \left(1 - \frac{4}{3} \chi_w + \frac{8}{9} \chi_w^2\right) [d(x_1)\bar{d}(x_2) + \bar{d}(x_1)d(x_2) + s(x_1)s(x_2)] \right\}.
\end{equation}
$\chi_w$ denotes $\sin^2 \theta_w$, where $\theta_w$ is the Weinberg angle. The $K$-factor at the $Z^0$ mass scale is equal to 1.317, according to Eq. (3). Using Eq. (11) and the MRSD' structure function, the prediction for $Z^0$ production in $p+p$ collisions is shown in Figure 1. The $Z^0$ production cross sections measured at UA2 and CDF are well reproduced.

For $p+p$ collision at large $x_F$, the $u(x_1)\bar{u}(x_2)$ term in Eq. (11) gives the dominant contribution. The ratio of $Z^0$ production cross sections in $p+p$ and $p+d$ collisions can be written as
\begin{equation}
R''(x_F) \equiv 2 \frac{d\sigma}{dx_F}(p+p \rightarrow Z^0) \approx \frac{\bar{d}(x_2) - \bar{u}(x_2)}{d(x_2) + \bar{u}(x_2)}.
\end{equation}
Figure 4 shows the predictions of $R''(x_F)$ for various models. For the $\bar{u}/\bar{d}$ symmetric model, DO1.1, $R''(x_F)$ approaches one for $x_F \geq 0.1$. For the $\bar{u}/\bar{d}$ asymmetric models, the values of $R''(x_F)$ at large $x_F$ directly reflect the amount of $\bar{u}/\bar{d}$ asymmetry.
In conclusion, $W$ and $Z$ production at RHIC would offer an independent means to examine the possible $\bar{u}/\bar{d}$ asymmetry in the proton. Measurements of the cross section ratios of $W^+$ and $W^-$ production in $p + p$ collisions, as well as the $W^+ (Z^0)$ cross section ratios in $p + p$ and $p + d$ collisions, would provide a sensitive test of various theoretical models on the sea quark distributions of the proton. While the Drell-Yan and quarkonium production experiments would provide important information on the $\bar{u}/\bar{d}$ asymmetry at low $Q^2$, the $W$ and $Z$ production experiments offer the unique opportunity of probing the possible asymmetry at very high $Q^2$. 
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FIG. 1. Cross section times the electron decay branching ratio for W and Z production from UA2 [23] and CDF [24] collaborations. The curves are predictions using Eqs. (2) and (11) and the structure function MRSD′ [25].
FIG. 2. Predictions of $R(x_F)$ using Eqs. (2) and (4) and various structure functions for a) $p+p$ collisions, and b) $p+\bar{p}$ collisions at $S^{1/2} = 500$ GeV.
FIG. 3. Predictions of $R'(x_F)$ using Eq. (2) and various structure functions for $W^+$ production at $S^{1/2} = 500$ GeV.
FIG. 4. Predictions of $R''(x_F)$ using Eq. (11) and various structure functions for $Z^0$ production at $S^{1/2} = 500$ GeV.