Nuclear Structure and Reaction Properties of Ne, Mg and Si Isotopes with RMF Densities

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We have studied nuclear structure and reaction properties of Ne, Mg and Si isotopes, using relativistic mean field densities, in the frame work of Glauber model. Total reaction cross section $\sigma_R$ for Ne isotopes on $^{12}$C target have been calculated at incident energy 240 MeV. The results are compared with the experimental data and with the recent theoretical study [W. Horiuchi et al., Phys. Rev. C, 86, 024614 (2012)]. Study of $\sigma_R$ using deformed densities have shown a good agreement with the data. We have also predicted total reaction cross section $\sigma_R$ for Ne, Mg and Si isotopes as projectiles and $^{12}$C as target at different incident energies.

Keywords: Relativistic Mean Field Theory, Charge distribution, Nucleon distributions and halo features, low and intermediate energy

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1. Introduction

The development of accelerator technique for Radioactive Ion Beams (RIBs) help to study numerous experimental as well as theoretical measurements for nuclei far from $\beta$ - stability line. Experimental methods and theoretical analysis have been widely used to collect information about the nuclear size, valence nucleon distribution and halo structure. The measurement of various cross sections like reaction cross section $\sigma_R$, nucleon removal cross section $\sigma_{-1n}$ and longitudinal momentum distribution $P||$ are some of the established tools for exploring unstable nuclei. Island of inversion

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(IOI) is one of the most important current subjects in nuclear physics. This was first applied by Warburton to a region of very neutron-rich nuclei from $^{30}$Ne to $^{34}$Mg.\textsuperscript{1} Discovery of the halo structure is another important progress of research on unstable nuclei. A halo structure of $^{31}$Ne was reported by the experiment on the one neutron removal reaction.\textsuperscript{2} Experimentally this is the heaviest halo nucleus. The formation of halo in a nucleus near the drip-line is due to the very small binding of the valence particles. The quadrupole deformation of the halo is determined by the structure of the weakly bound valence orbital and it does not depend on the shape of the core.\textsuperscript{3} On the other hand existence of two nucleon halo is most unlikely in a deformed nucleus.\textsuperscript{4} It is shown by Nunes with a variety of 3-body NN tensor force which goes beyond the usual pairing in Hartree-Fock-Bogoliubov (HFB) and the coupling due to core deformation/polarization significantly reduce the formation of 3-body Borromean systems. In a recent work,\textsuperscript{5} halo phenomena in deformed nuclei are analysed within deformed Relativistic Hartree Bogoliubov (RHB) theory and their finding in weakly bound $^{44}$Mg nucleus indicates a decoupling of the halo orbitals from the deformed core agreeing with the conclusion of Ref.\textsuperscript{3}.

To develop consistent nuclear reaction systematics along with the nuclear structure, several theoretical models have been a matter of wide interest. In this context, the relativistic mean field (RMF) or the effective field theory motivated RMF (E-RMF) models provide the internal structure or sub-structure information of the nuclei through density distributions,\textsuperscript{6} which are used as input while calculating the observables in conjunction with Glauber model.\textsuperscript{7,8,9,10,11} A systematic study of various nuclear reaction cross sections, such as the total nuclear reaction cross sections, differential elastic cross sections etc. enables us to know the nuclear structure of unstable nuclei in detail. This will also help in studying the formation of neutron-rich nuclei that are surrounded by a high pressure or temperature. In the present paper, our aim is to calculate the bulk properties, such as binding energy (BE), root mean square charge radius $r_{ch}$, and quadrupole deformation parameter $\beta_2$ for Ne, Mg and Si isotopes in the RMF and E-RMF formalisms. Then we analyze the total nuclear reaction cross section $\sigma_R$ for the scattering of $^{20-32}$Ne from a $^{12}$C target at 240 MeV/nucleon by using the densities obtained from the RMF formalisms in the frame work of Glauber model. We have also predicted total reaction cross sections for Ne, Mg and Si cases at different incident energies.

The paper is designed as follows: The RMF/E-RMF formalisms and the reaction mechanism in the framework of Glauber model are explained briefly in Section II. The results obtained from our calculations are discussed in Section III. In this Section we intend to study the applicability of Glauber model in the context of both stable and drip-line nuclei. Finally, a brief summary and concluding remarks are given in the last Section IV.
2. Theoretical Framework

The successful applications of RMF both in finite and infinite nuclear systems make more popular the formalism in the present decades. The RMF model has been extended to the Relativistic Hartree-Bogoliubov (RHB) and density functional approach to study the static and dynamic aspects of exotic nuclear structure [13,14]. The use of RMF formalism for finite nuclei as well as infinite nuclear matter are well documented and details can be found in [15-21,22,23] respectively.

The working expressions for the density profile and other related quantities are available in [7,8,9,11,15,16,17,20,21,22,23]. The details to calculate $\sigma_R$ using Glauber approach has been given by R. J. Glauber [24]. This model is based on the independent, individual nucleon-nucleon ($NN$) collisions along the eikonal [25]. It has been used extensively to explain the observed total nuclear reaction cross-sections for various systems at high energies. The standard Glauber form for the total reaction cross-sections at high energies is expressed as [24,26]

$$\sigma_R = 2\pi \int_0^\infty b [1 - T(b)] db , \quad (1)$$

where $T(b)$ is the transparency function with impact parameter $b$. The function $T(b)$ is calculated in the overlap region between the projectile and the target assuming that the interaction is formed from a single $NN$ collision. It is given by

$$T(b) = \exp \left[ - \sum_{i,j} \mathcal{F}_{ij} \int d\vec{s} \rho_{ti}(\vec{s}) \rho_{tj}(\vec{s}) \left( |\vec{b} - \vec{s}| \right) \right] . \quad (2)$$

The summation indices $i$ and $j$ run over proton and neutron and subscript $p$ and $t$ refers to projectile and target, respectively. The experimental nucleon-nucleon reaction cross-section $\mathcal{F}_{ij} \equiv \sigma_{ij}$ varies with energy. The $z$-integrated densities $\mathcal{P}(\omega)$ are defined as

$$\mathcal{P}(\omega) = \int_{-\infty}^{\infty} \rho \left( \sqrt{\omega^2 + z^2} \right) dz , \quad (3)$$

with $\omega^2 = x^2 + y^2$. The argument of $T(b)$ in Eq. (2) is $|\vec{b} - \vec{s}|$, which stands for the impact parameter between the $i^{th}$ and $j^{th}$ nucleons.

The original Glauber model was designed for high energy approximation. However, it was found to work reasonably well for both the nucleus-nucleus reaction and the differential elastic scattering cross-sections over a broad energy range [24,27]. To include the low energy effects of $NN$ interaction, the Glauber model is modified to take care of the finite range effects in the profile function and Coulomb modified trajectories [28-30]. The modified $T(b)$ is given by [28,30]

$$T(b) = \exp \left[ - \int_{t}^{p} \sum_{i,j} \Gamma_{ij} (\vec{b} - \vec{s} + \vec{t}) \right] \mathcal{P}_{pi}(\vec{t}) \mathcal{P}_{tj}(\vec{s}) d\vec{s} d\vec{t} . \quad (4)$$
The profile function $\Gamma_{ij}(b_{eff})$ is defined as
\begin{equation}
\Gamma_{ij}(b_{eff}) = 1 - \frac{i\alpha_{NN}}{2\pi\beta_{NN}^2}\sigma_{ij}\exp\left(-\frac{b_{eff}^2}{2\beta_{NN}^2}\right),
\end{equation}
with $b_{eff} = |\vec{b} - \vec{s} + \vec{t}|$, $\vec{b}$ is the impact parameter, $\vec{s}$ and $\vec{t}$ are the dummy variables for integration over the $z$-integrated target and projectile densities. The parameters $\sigma_{NN}$, $\alpha_{NN}$, and $\beta_{NN}$ are usually case-dependent (proton-proton, neutron-neutron or proton-neutron), but we have used the appropriate average values from Refs. 26, 31, 32, 33, 34. It is worth mentioning that the result in Glauber model is sensitive to the in-medium NN cross-section with proper treatment of the input densities 35 and also depends on the accuracy of the profile function.

At intermediate energies, medium effects can be taken into account on nucleon-nucleon cross-sections. In NN scattering the basic input is the NN elastic t-matrix. This t-matrix is modified to take into account nuclear medium effects in both projectile and target. A. Bertulani et. al 36 have shown that the nucleon knockout reactions involving halo nuclei are more sensitive to medium modifications compared to normal nuclei. The deformed or spherical nuclear densities obtained from the RMF model are fitted to a sum of two Gaussian functions with suitable co-efficients $c_i$ and ranges $a_i$ chosen for the respective nuclei which is expressed as
\begin{equation}
\rho(r) = \sum_{i=1}^{2} c_i\exp[-a_i r^2].
\end{equation}
The deformed intrinsic RMF densities are converted to its spherical equivalent using this equation which is consistent with the Glauber theory applied in the laboratory frame 25. Then, the Glauber model is used to calculate the total reaction cross-section for both the stable and unstable nuclei considered in the present study. In Refs. 9, 10, 25, 29, 30 it is shown that the Glauber model can be used for relatively low energy even at 25, 30 and 85 MeV/nucleons. Although it is a better prescription to take deformation into account directly through the transparency function [Eqn. (4)], but to our knowledge no such scheme is available in this model. In our earlier calculations 7, 8, 9, 11 we have used the present approach to take deformation into account where the results are quite encouraging and show clear deformation effect.

It is to be noted that similar methodology is also adopted by some other authors 30. Also, it is important to note that the densities for halo-nuclei have long tails which generally are not reproduced quantitatively by harmonic oscillator expansion in a mean field formalism.

3. Calculations and Results

We obtain the field equations for nucleons and mesons from the RMF and E-RMF Lagrangian. For the deformed case (RMF only), these equations are solved by expanding the upper and lower components of the Dirac spinners and the boson fields
in an axially deformed harmonic oscillator basis. The set of coupled equations are solved numerically by a self-consistent iteration method taking different inputs of the initial deformation $\beta_0$. For spherical densities, we follow the numerical procedure of Refs. [21,22] for both RMF and E-RMF models. The constant gap BCS pairing is used to add the pairing effects for open shell nuclei. In the present calculations, we have dealt reaction studies for nuclei Ne, Mg and Si with C target. All these nuclei are in the lower region of the mass table, where the contribution of pairing effect is minimal even for open shell nuclei [38]. We also understand that pairing plays a crucial role for open shell nuclei for relatively heavier mass region. If one use the conventional pairing gaps similar to $\Delta = 11.2/\sqrt{A}$ MeV, then BCS treatment of pairing is not reliable. However, using small pairing gap near the dripline [22,39], this error can be minimised. We have used this scheme in our earlier calculations [40] and able to reproduce the results with data till the dripline whenever available. The centre-of-mass motion (c.m.) energy correction is estimated by the usual harmonic oscillator formula $E_{c.m.} = \frac{3}{4}(41A^{-1/3})$.

Since the main input in the Glauber model estimation is the RMF or E-RMF densities, it is important to have information about these quantities. We have plotted the spherical $\rho_p$ and $\rho_n$ for both proton and neutron distributions of Ne isotopes in Figure 1 using RMF (NL3*) parameter set [18,19]. As expected, we find the values of $\rho_n$ and $\rho_p$ are almost similar for $^{20}$Ne which can be seen from Figure 1. Extension of $\rho_n$ is much more than $\rho_p$ for rest of the nuclei. It is maximum for $^{32}$Ne in Neon isotopic chain, because of high neutron to proton ratio for these cases. The axially deformed density for the halo case $^{31}$Ne is shown in Figure 2. The z-axis
In Ref. 41, it is noticed that $^{31}\text{Ne}$ possesses a $3\alpha-$ cluster with a tetrahedral configuration. The structure of this neutron-rich $^{31}\text{Ne}$ has a prolate ground state deformation. The density plot shows that the central part of the nucleus is a compact core, which is surrounded by a thin layer of nucleons.

We calculate the bulk properties, such as binding energy (BE), root mean square charge radius $r_{ch}$, and quadrupole deformation parameter $\beta_2$ for the neutron rich $^{18-32}\text{Ne}$, $^{24-34}\text{Mg}$ and $^{26-36}\text{Si}$ isotopes in the RMF and E-RMF formalisms. The calculated nuclear structure results are compared with T. Sumi et al. 42 and available experimental data 43, 44, 45 in Tables 1, 2 and 3. It is clear that our results agree remarkably well with the data. For example, the RMF binding energy for $^{18}\text{Ne}$ is 131.8 MeV and 135.3 MeV from E-RMF where as the experimental value

![Image of density distribution for $^{31}\text{Ne}$ with RMF (NL3*) parameter set. The width and height of the boxes are 8 fm each with the uniform contour spacing of 0.01 fm$^{-3}$.](image-url)

is chosen as the symmetry axis, the density is evaluated in the $z\rho$ plane, where $\sqrt{x^2 + y^2} = \rho$. In Ref. 41, it is noticed that $^{31}\text{Ne}$ possesses a $3\alpha-$ cluster with a tetrahedral configuration. The structure of this neutron-rich $^{31}\text{Ne}$ has a prolate ground state deformation. The density plot shows that the central part of the nucleus is a compact core, which is surrounded by a thin layer of nucleons.

![Image of density distribution for $^{31}\text{Ne}$ with RMF (NL3*) parameter set. The width and height of the boxes are 8 fm each with the uniform contour spacing of 0.01 fm$^{-3}$.](image-url)
Table 1. Calculated results for binding energy (BE), root mean square charge radius $r_{ch}$, and quadrupole deformation parameter $\beta_2$ for the neutron rich $^{18-32}\text{Ne}$ isotopes using RMF and E-RMF densities obtained from NL3* and G2 parameter sets respectively. The available experimental data are given for the comparison. BE is in MeV and $r_{ch}$ in fm.

| Nucleus   | BE  | $r_{ch}$ | $\beta_2$ | RMF | E-RMF | Expt. | RMF | E-RMF | Expt. | RMF | Expt. |
|-----------|-----|----------|-----------|-----|-------|-------|-----|-------|-------|-----|-------|
| $^{18}\text{Ne}$ | 131.8 | 135.3 | 132.1 | 2.963 | 3.055 | 2.972 | 0.238 | 0.68(3) |
| $^{20}\text{Ne}$ | 156.7 | 156.6 | 160.6 | 2.972 | 2.986 | 3.00 | 0.537 | 0.479 | 0.70(20) |
| $^{22}\text{Ne}$ | 175.7 | 174.2 | 177.8 | 2.94 | 2.903 | 2.954 | 0.502 | 0.400 | 0.564(4) |
| $^{24}\text{Ne}$ | 189.1 | 190.2 | 191.8 | 2.88 | 2.879 | 2.903 | −0.259 | 0.258 | 0.41(5) |
| $^{26}\text{Ne}$ | 200.0 | 202.7 | 201.5 | 2.926 | 2.886 | 2.927 | 0.277 | 0.221 | 0.39(3) |
| $^{28}\text{Ne}$ | 208.3 | 211.7 | 206.9 | 2.965 | 2.925 | 2.963 | 0.225 | 0.526 | 0.36(3) |
| $^{30}\text{Ne}$ | 215.2 | 218.2 | 211.3 | 2.992 | 2.982 | 3.00 | 0.246 | 0.400 | 0.49(17) |
| $^{31}\text{Ne}$ | 216.3 | 220.0 | 211.6 | 3.027 | 2.974 | 3.027 | 0.244 | 0.422 |
| $^{32}\text{Ne}$ | 218.7 | 221.2 | 213.2 | 3.069 | 2.982 | 3.069 | 0.369 | 0.335 |

Table 2. Same as table 1 but for $^{22-34}\text{Mg}$ isotopes.

| Nucleus   | BE  | $r_{ch}$ | $\beta_2$ | RMF | E-RMF | Expt. | RMF | E-RMF | Expt. | RMF | Expt. |
|-----------|-----|----------|-----------|-----|-------|-------|-----|-------|-------|-----|-------|
| $^{22}\text{Mg}$ | 166.42 | 165.63 | 168.58 | 3.092 | 3.142 | 3.142 | 0.5128 | 0.65(12) |
| $^{24}\text{Mg}$ | 194.31 | 189.44 | 198.26 | 3.043 | 3.037 | 3.057 | 0.4874 | 0.613(14) |
| $^{26}\text{Mg}$ | 212.54 | 211.20 | 216.68 | 2.978 | 2.982 | 3.033 | 0.2728 | 0.484(6) |
| $^{28}\text{Mg}$ | 228.76 | 228.45 | 231.63 | 3.048 | 3.011 | 3.011 | 0.3447 | 0.484(20) |
| $^{30}\text{Mg}$ | 240.51 | 241.68 | 241.64 | 3.062 | 3.042 | 3.042 | 0.2406 | 0.41(3) |
| $^{32}\text{Mg}$ | 250.59 | 252.69 | 249.81 | 3.090 | 3.076 | 3.076 | 0.1190 | 0.51(5) |
| $^{34}\text{Mg}$ | 257.39 | 259.47 | 256.48 | 3.150 | 3.091 | 3.091 | 0.3432 | 0.55(6) |
| $^{36}\text{Mg}$ | 264.13 | 264.08 | 260.80 | 3.198 | 3.102 | 3.102 | 0.4344 |

Table 3. Same as table 1 but for $^{26-36}\text{Si}$ isotopes.

| Nucleus   | BE  | $r_{ch}$ | $\beta_2$ | RMF | E-RMF | Expt. | RMF | E-RMF | Expt. | RMF | Expt. |
|-----------|-----|----------|-----------|-----|-------|-------|-----|-------|-------|-----|-------|
| $^{26}\text{Si}$ | 200.86 | 202.84 | 206.04 | 3.118 | 3.136 | 3.136 | −0.2800 | 0.444(21) |
| $^{28}\text{Si}$ | 232.13 | 230.54 | 236.54 | 3.122 | 3.065 | 3.122 | −0.3308 | 0.412(4) |
| $^{30}\text{Si}$ | 250.58 | 251.55 | 255.62 | 3.070 | 3.09 | 3.133 | 0.1481 | 0.33(22) |
| $^{32}\text{Si}$ | 268.45 | 269.25 | 271.41 | 3.137 | 3.116 | 3.116 | −0.2007 | 0.26(4) |
| $^{34}\text{Si}$ | 284.45 | 285.05 | 283.43 | 3.147 | 3.152 | 3.152 | 0.0005 | 0.18(4) |
| $^{36}\text{Si}$ | 291.57 | 295.59 | 292.03 | 3.186 | 3.166 | 3.166 | −0.1616 | 0.25(4) |
Fig. 3. Calculated reaction cross sections for scattering of Ne isotopes on $^{12}$C target at 240 MeV/nucleon with experimental data.

Fig. 4. Comparison of our results with W. Horiuchi et al., Phys. Rev. C, 86, 024614 (2012) and experimental data at 240 MeV/nucleon for scattering of $^{20-32}$Ne isotopes on $^{12}$C target.

considerably from the data $^{46,47,48}$ which are quoted in this figure. For example, in case of $^{28}$Ne$+^{12}$C, the observed value of $\sigma_R$ is $1273 \pm 11$ MeV as compared to the estimated results of $1440$ MeV with NL3* parametrization.

In this context, it is interesting enough to see deformation effect on $\sigma_R$. We
repeat the calculations for $\sigma_R$ with the deformed densities (RMF only) as input in the Glauber model. We obtained spherical equivalent of the axially deformed densities using equations (3) and (6) following the prescription of Refs. The NL3* parameter set for this purpose is used and the results are presented in Figures 3(b) and 3(c). The parameter set NL3* is reasonably a good set for
these neutron-rich nuclei. It shows that most of the $\sigma_R$ matches quite well with the experimental data [14-18] and still halo case does not agree in Figure 3(b). In Figure 3(c), we have taken core + one neutron case and found a remarkable agreement with the data. In figure 4, we have made a comparison of our results with W. Horiuchi et al. [49] as well as with the data for the scattering of $^{20-32}$Ne isotopes on $^{12}$C target at 240 MeV/Nucleon. It is found that our results are little bit
higher for relatively lower mass nuclei but there is better fitting for neutron rich side. Summarising the whole discussion on reaction cross sections, in general, one can say that the spherical density used from RMF (NL3*) fails to reproduce the data. When we use the deformed densities to evaluate the total nuclear reaction cross section, the predicted $\sigma_R$ matches reasonably well with the experimental measurement. In Figures 5, 6 and 7, we have presented the $\sigma_R$ with various incident energies for Ne, Mg and Si isotopes as projectiles and $^{12}$C as target using the deformed NL3* densities in the Glauber model calculation. A comparison of our results with W. Horiuchi et al. [49] for the scattering of Mg and Si isotopes on $^{12}$C− target at 240 MeV/Nucleon is also given in Figure 8. It is observed a similar trend as reported in [49].

4. Summary and Conclusion

In summary, the binding energy, charge radius and quadrupole deformation parameter for the neutron-rich $^{18-32}$Ne, $^{24-34}$Mg and $^{26-36}$Si isotopes have been calculated using RMF (NL3*) and E-RMF (G2) formalisms. Using the RMF densities, the reaction cross sections $\sigma_R$ are evaluated in the Glauber model. The $\sigma_R$ are in good agreement with the experiments and also with the recent study by W. Horiuchi et al. [49] when we consider the deformation effect in the densities. It is also concluded in the present paper that deformation for total nuclear reaction cross section is important for stable nuclei as projectiles which reproduces the experimental data reasonably well. In $^{31}$Ne halo case, the deformed core + one neutron shows better agreement with the data.

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