Simulation of ocean waves in coastal areas using the shallow-water equation

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Abstract. This study simulates shallow water waves using the Navier-Stokes equation. This simulation uses the MatLab application, especially Quickersim with 2-dimensional output. Mesh in simulation is made using Gmsh. Research about shallow water has an essential role in studying the characteristics of ocean waves. The depth of the sea influences this characteristic. Data obtained from this simulation is in the wave height and velocity positions at any time. The limitations in the data collected are not comparable with the experimental results because there are no experimental Navier-Stokes simulations, but these simulation results have shown the phenomenon of seawater movement. In future work, the results of this study can be used to analyse its application in tsunami waves.

1. Introduction

Generally, the geometry of most problems in real life of surface water flows in rivers, lakes, estuaries, and coastal water bodies are irregular and nonlinearity [1]. So, the application of mathematical models with structured mesh is quite limited, and the accuracy could not ensure as the mesh is not dense enough [2]. Triangular cells, to gear best the natural coastal lines, were firstly used in the Finite Element Method (FEM) and Finite Difference Method (FDM) for shallow water simulations. In the last decade, combined with unstructured mesh, especially the triangular cells, the FVM has been widely used for it has a clear physical meaning and computational conservation [1]. Besides FEM, FDM, and FVM, Nuraiman (2015) made shallow water simulation using the lattice Boltzmann method (LBM) was originated from Boltzmann’s kinetic theory of gases. The basic idea is that the fluid can be imagined containing a lot of particles [3].

The research on water wave in shallow water is one of the main topics in water wave dynamics and bears on the hydrodynamics and ocean dynamics [4]. The term of shallow water is used to describe a body of water in which the boundaries are close to the ship only in the vertical direction. When the water becomes shallow, the resistance of a ship moving through it will become greater [5]. The mechanical property of the water wave between deep water and shallow water is different. The variety of water depth may cause refraction, diffraction, and shallow shoaling effect of a water wave in terms of dynamic mechanism [4].
The problem for wave height distribution research in shallow water is to establish the random statistical model. Based on Stokes wave, we have developed a statistical distribution of nonlinear random water wave height with the wave steepness as the control parameter and also derived a nonlinear distribution of wave surface elevation with the wave steepness and the factor of shallow water as the control parameters. Similarly, these two parameters are important in the distribution of wave height in shallow water. The new wave height distribution in shallow water enriches the traditionally statistical waves’ research and is promising in the future practical applications [6]. According to reference, the regimes of the fluid motion can be roughly divided into that based on the shallow water theory \((h/l < 0.1)\), that with an intermediate water depth \((0.1 \leq h / l \leq 0.2 - 0.25)\) and that under finite water depth conditions \((0.2 - 0.25 < h / l < 1.0)\) [7], [8].

2. Method

2.1. Shallow water method

The model of shallow water flow simulation using either FEM, FDM, or FVM, a mesh is needed. Some of the mesh that can be used are unstructured quadtree rectangular mesh and triangular quadratic mesh shown in Fig. 1 and Fig. 2. In addition to the mesh for modeling, the shallow water simulation can use LBM, the lattice shown in Fig. 3.

![Figure 1. Control volume quadtree rectangular mesh [1].](image1)

![Figure 2. A quadratic triangle with the three edge nodes that purposefully do not lie at the mid-points of the edges. A1, A2, and A3 called corner vertices, and A4, A5, and A6 are called edge nodes [9].](image2)

![Figure 3. Illustration of the computational domain of LBM using D2Q9 lattice pattern [3,10].](image3)
2.2. **Simulation equation**

The mathematical equations describing the 2D shallow water model, however, can be derived from the Navier–Stokes equations as the basic equations of fluid flow [11]. First, we note that a solution of the full Navier–Stokes equations, including the effects of viscosity [12]. In order using this simplification domain of phenomenon that we want to simulate has to be significantly smaller in the vertical direction.

\[
\frac{\partial h}{\partial t} + \frac{\partial}{\partial t} \left( (H + h)u \right) + \frac{\partial}{\partial x} \left( (H + h)v \right) = 0,
\]

(1)

\[
\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} - fv = -g \frac{\partial h}{\partial x} - bu + \nu \left( \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \right),
\]

(2)

\[
\frac{\partial v}{\partial t} + u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} + fu = -g \frac{\partial h}{\partial y} - bv + \nu \left( \frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} \right),
\]

(3)

Where:

- \( h \): Height of free surface (solution field)
- \( v \): Kinematic viscosity
- \( H \): Mean height of free surface (constant)
- \( u \): Velocity in the x-direction
- \( f \): Coriolis coefficient
- \( v \): Velocity in the y-direction
- \( g \): Acceleration due to gravity
- \( h \): Height deviation of the horizontal pressure surface from its mean height
- \( b \): Viscous drag coefficient

It is often the case that the terms quadratic in \( u \) and \( v \), which represent the effect of bulk advection, are small compared to the other words this is called geostrophic balance. Assuming that the wave height is very small compared to the mean height \( (h \ll H) \), we have (without lateral viscous forces):

\[
\frac{\partial h}{\partial t} + H \left( \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} \right) = 0,
\]

(4)

\[
\frac{\partial u}{\partial t} - fv = -g \frac{\partial h}{\partial x} - bu,
\]

(5)

\[
\frac{\partial v}{\partial t} + fu = -g \frac{\partial h}{\partial y} - bv,
\]

(6)

The value of the Coriolis coefficient for selected situations shown in Table 1 [13].

| Minimum | Maximum | Average |
|---------|---------|---------|
| River valley, Over flooded | 1.50 | 2.00 | 1.75 |
| River under ice cover | 1.20 | 2.00 | 1.50 |
| Natural streams and torrents | 1.15 | 1.50 | 1.30 |
| Regular channels, flumes, spillways | 1.10 | 1.20 | 1.15 |

In this simulation, we use the channel river valley and over flooded with average Coriolis Coefficient, 1.75.
3. Result and discussion

3.1. Mesh

The solving of 2D geometry represented the same with a sketch of the oblique hydraulic jump by surface shown in Fig. 4 [2]. Because this simulation uses FEM, in this research, we made a mesh quadratic triangular mesh with 3723 total nodes shown in Fig. 5. By using the FEM method, the correction of boundary errors for all types of mesh elements can be minimized; it is necessary to make the high-quality mesh [11]. The selection of quadratic triangular mesh because this type has better dense than quadtree rectangular mesh.

![Figure 4. 2D Geometry build using Gmsh.](image1)

![Figure 5. Mesh with quadratic triangular cells build using Gmsh.](image2)

3.2. Simulation equation

This simulation was running by using MATLAB and QuickerSim Apps. The initials conditions are $b = 0.3; f = 1.75; g = 10; H = 0.4$. When the computation develops with $dt = 0.005$ second and total step = 1050. The equation used is $h = 2*\exp (-((x-2.5)^2 + (y-0.5)^2)/0.25)$.

![Figure 6. Initial condition of simulation (step = 1).](image3)

![Figure 7. Shallow water simulation (step = 100).](image4)
Figure 8. Initial solution of simulation (step = 200).

Figure 9. Shallow water simulation (step = 300).

Figure 10. Initial solution of simulation (step = 400).

Figure 11. Shallow water simulation (step = 500).

Figure 12. Initial solution of simulation (step = 600).

Figure 13. Shallow water simulation (step = 700).
Figure 14. Initial solution of simulation (step = 800).

Figure 15. Shallow water simulation (step = 900).

Figure 16. Initial solution of simulation (step = 1000).

Figure 6-16 shown the initial simulation (step =1) and shallow water simulation in step 100, 200, 300, 400, 500, 600, 700, 800, 900, and 1000. The method presented here is only the beginning of work carried out to develop a shallow water simulation using the Navier-Stokes equation. There are no experimental data Navier-Stokes simulations for comparison. The lack of data makes some doubts on the validity of the hypothesis modeling, prediction, and various applications [9]. Although the method presented has not been exploited and but the simulation results have shown the phenomenon of seawater movement.

4. Conclusion
The aspect in question is that waves that radiate from the deep sea to the coast will change shape due to changes in the depth of the sea. If the wave moves closer to the beach, the wave movement at the bottom that borders the sea floor will slow down. The friction between the sea floor and the water; meanwhile, the top of the wave on the surface of the water will continue to accelerate. The more the wave moves towards the beach, the crest of the wave will get sharper, and the valley will be flatter.

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