Modelling of Mechanical Coupling for Piezoelectric Energy Harvester Adapted to Low-Frequency Vibration

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Abstract. In our previous work, we have developed a mechanical coupling for energy harvester from vibration source. This energy harvester uses piezoelectric with additional cantilever beam and permanent magnets. Our work proposed alternative scheme of mechanical coupling for tune the vibration input into resonant frequency of piezoelectric. Based on the experiment, correlation between the length of cantilever beam and the output power also evaluated. In this paper, we try to modelling our work into mathematical model and apply it to some case study. For example application, we apply our energy harvester system to generate electrical energy to enlighten the street. The human footsteps can be used as vibration source to generate electrical energy.

1. Introduction
Harvesting vibration energy in several recent years has become a trend. The main purpose is how to take advantage of daily life activity as sources of electrical energy. The example of sources are vibration on some surface when stepped by human footstep, machine vibration, bridge vibration, or ship vibration caused by sea wave. Many ways are used to convert those vibrations into electrical energy. This conversion mechanism is commonly known as transduction. There are three famous transduction mechanisms to pursue energy scavenging, i.e. electrostatic, electromagnetic, and piezoelectric [1]. Electrostatic transduction is electrical energy that generate by the movement of two capacitive plates which separated by air medium, vacuum, or dielectrics. Principle of electromagnetic transduction is using Faraday Law by moving coil relatively to or from permanent magnetor or otherwise [2, 3]. Different from the two of transduction mechanism, piezoelectric transduction is unique. Piezoelectric can generate electrical energy by changing strain on piezoelectric material. Vibration on the surface of piezoelectric caused strain alteration. Piezoelectric transduction has advantages compared first two transduction methods. There are large power densities and ease of application [4].

Every piezoelectric have resonance frequency that can optimize power output. Therefore, recent research is focused to wider resonance bandwidth. Material, mass, dimension, or the thickness of the piezoelectric structure is main factor that determine of resonant frequency. Not all researchers have to dig some new from all main factors to determine suitably resonant frequency. Some researcher just
have utilize some product that produced by some piezoelectric vendor. The main problem for this kind of researcher is how to tune frequency of the vibration source with resonant frequency of the piezoelectric [5]. When the piezoelectric reaches the resonant frequency, it can produce maximum output power. Variety techniques are determined, such as integrate piezoelectric with cantilever in many configurations, combine two piezoelectric with blast technique, combine some piezoelectric into array that have different resonant frequencies, or integrate piezoelectric with combination of cantilever-permanent magnet [1, 5-8].

The last technique was developed with many efforts. Our previous work is included with that last technique. Additional cantilever and pairs of permanent magnet was utilized to reach maximum power from piezoelectric. Based on empirical data, the maximum power could be reached when the length of additional cantilever beam is twice of the length of the piezoelectric dimension, although there was use of some constraint on that case. When it gives impulse input, the power reaches 12.54 mW [5]. The next project is how to apply the previous result to the real application. Before reaching that goal, numerical simulation could be an important guidance to determine the suitable output power. Vibration source in this simulation comes from surface that is trampled by human walking activity.

2. Theoretical background

2.1. Introduction

Device of vibration energy harvester will harvest energy from low-level ambient energy, below 10 Hz for the human movement and above 30 Hz for vibration on the engine [10]. On the market, several types of piezoelectric for vibration energy harvester have a resonant frequency range between 30-1500 Hz [9, 10]. There is no problem when the vibration source has same frequency with piezoelectric. But, when it below the resonant frequency piezoelectric, it will cause piezoelectric can not generate output power optimally. Therefore, it requires some way to solve that problem.

Mechanical coupling is a system that used to amplify low vibration frequency in order to tune with resonant frequency of piezoelectric. Many techniques used in mechanical coupling. But one of the easier ways is to utilize permanent magnet and additional cantilever [1, 5]. Attractive magnetic force and repulsive force will shift natural frequency of cantilever compared with unapplied additional permanents magnets. The permanent magnets also alter the effective stiffness of the cantilever [1]. The permanent magnets and additional cantilever also would generate vibrations a lot more and longer than without it [5].

![Figure 1. Schematic of mechanical coupling for energy harvesting device.](image-url)
Four permanent magnets are used. Two magnets are fixed at the free end of the cantilever beam, while the other one magnet is fixed to the enclosure of the device at the bottom, and the last one placed at the top that can move with vertical direction as depicted in figure 1. The magnets are placed such that attractive and repulsive magnetic forces can be applied on each side of the beam. The function of spring is to make top magnet movable. When top magnet is pushed up, it can force to move the magnets on the beam down. The oscillations of the spring can makes vibrations on the cantilever a lot more and longer than without use of the spring. Figure 2 depicted the design of the mechanical coupling for energy harvesting device. When the pusher is pushed by an applied force, the cantilever beam has to move-down. After the force removed, i.e. no longer stepped by human, the cantilever beam will oscillate and the system will produce electricity.

![Figure 2. Design of the mechanical coupling for energy harvesting device.](image)

Spacers at top and bottom side are preventing the cantilever beam adheres the permanent magnets. The cantilever and piezoelectric are fixed on a clamp that can be vertically displaced using a screw mechanism.

2.2. Magnetic force impact

Cylindrical magnets are used to apply attractive magnetic force and repulsive force on the mechanical coupling system. The magnetic force between any two cylindrical magnets is given as

\[
F_{\text{mag}}(d) = \frac{B_r^2 A_m}{\pi \mu_0 l^2} \left( \frac{l}{d^2} + \frac{1}{(d + 2l)^2} + \frac{1}{(d + l)^2} \right),
\]

where \(B_r\) is the residual flux density of the magnet, \(A_m\) is the common area between the magnets, \(l\) is the length of the magnet, \(r\) is the radius of the magnet, \(d\) is the distance between the magnets, and \(\mu_0\) is the permeability of the intervening medium. Applying magnetic force will alter the effective stiffness of the beam and it will be changing on resonant frequency of the beam [1].

2.3. Equivalent model for a mechanical coupling

Designed mechanical coupling consists of cantilever, permanent magnets and piezoelectric. Piezoelectric is mounted under cantilever. The setup description is depicted in figure 2. From the information in figures 1 and 2, the mathematical model can be developed as depicted in figure 3.
where $F(t)$ is force function that applied to the system, $M_k$ is mass of cantilever, $k_k$ is elasticity coefficient of cantilever, $b_k$ is damping coefficient of cantilever, $k_{mag}$ is elasticity coefficient of permanent magnet, $x_k$ is the displacement position of the cantilever, $M_{pzt}$ is mass of piezoelectric, $k_{pzt}$ is elasticity coefficient of piezoelectric, $b_{pzt}$ damping coefficient of piezoelectric, and $x_{pzt}$ is the displacement position of the piezoelectric, $\Theta$ is effective piezoelectric coefficient, $C_p$ capacitance of piezoelectric, $I(t)$ is current function through the circuit, $V_p(t)$ is the voltage across the piezoelectric element, and $V_c(t)$ is the voltage across the rectifier and filter circuit.

The governing equations of the piezoelectric vibrator can be described by following equations

\[
F(t) - k_{eff} \left( x_k + x_{pzt} \right) - b_k \left( \dot{x}_k + \dot{x}_{pzt} \right) = M_k \ddot{x}_k ,
\]

\[
k_{eff} \left( x_k + x_{pzt} \right) - b_k \left( \dot{x}_k + \dot{x}_{pzt} \right) - k_{pzt} x_{pzt} - b_{pzt} \dot{x}_{pzt} = M_{pzt} \ddot{x}_{pzt} + \Theta V_p(t) ,
\]

\[
\Theta \dot{x}_{pzt} + C_p \dot{V}_p(t) = -I(t) ,
\]

where $k_{eff}$ is $k_{mag} + k_k$. A mechanical coupling system is part of a harvesting energy system from vibration energy. Ideally, the system has no damping factor, so when given the instantaneous vibration input, a mechanical coupling system will continually generate vibration for piezoelectric. But in fact, damping factor will always appear in any system, including mechanical coupling. This can be caused by friction between materials, material with the air, properties of materials used, and theoretically there is applied the law of conservation of energy. Although there is no ideal condition, but in this research will be determine how to maintain the vibrations as long as possible when no more vibration input.

Influencing parameters of the equation of mechanical coupling is $M_k$, $k_k$, $b_k$, $k_{mag}$, $M_{pzt}$, $b_{pzt}$, $k_{pzt}$, $\Theta$, dan $C_p$. For $M_{pzt}$, $k_{pzt}$, $b_{pzt}$, $\Theta$, dan $C_p$ are default parameters from piezoelectric, so those parameters can not be changed nor sought for optimal value. This is because we only use one type of piezoelectric that is Vulture V21BL. So that the parameters that can be searched or altered for the purposes of the experiment is $M_k$, $k_k$, $b_k$, $k_{mag}$. Thus the material type, length, width and thickness of the cantilever will change the value of $M_k$, $k_k$, $b_k$. In this research, the type of material, width, and thickness of the cantilever are constant, so only a cantilever length which will be altered for the purpose of experimentation and data collection. $k_{mag}$ from magnet position is also constant.

The associated values of the variables as used in the prototype described later in this paper are listed in table 1 [12].

![Figure 3. Equivalent model of mechanical coupling for energy harvester.](image)
Table 1. Variable descriptions of mechanical coupling.

| Symbol | Description                              | Value  | Unit  |
|--------|------------------------------------------|--------|-------|
| F(t)   | Forcing function that applied to the system |        | N     |
| M_k    | Mass of cantilever                       | 225    | g     |
| k_k    | Effective stiffness of cantilever         | 5623.06| N/mm  |
| b_k    | Damping coefficient of cantilever         | 0.015  | N/mm  |
| x_k    | Displacement position of the cantilever   |        | mm    |
| k_mag  | Effective stiffness of magnet             | 4234.08| N/mm  |
| M_pzt  | Mass of piezoelectric                    | 2.92   | g     |
| b_pzt  | Damping coefficient of piezoelectric      | 0.09   | N/mm  |
| k_pzt  | Effective stiffness of piezoelectric      | 18013.53| N/mm |
| x_pzt  | Displacement position of the piezoelectric|        | mm    |
| Θ      | Effective piezoelectric coefficient      | 390 × 10^{-12}| C/N |
| k_eff  | Effective stiffness of beam               | 12390.47| N/mm |
| C_p    | Capacitance                              | 43 × 10^{-9}| F    |
| I(t)   | The current flowing into the specified circuit |  | A    |
| V_p(t)| The voltage across the piezoelectric element |  | V    |
| V_c(t)| The voltage across the rectifier and filter circuit |  | V    |

In this simulations F(t), x_k, x_pzt, I(t), V_p(t), and V_c(t) are functions of time t, which could be obtained by solving equations (2) – (4).

2.4. Changing the value of natural frequency

In our prototype, changing the M_k, k_k, b_k will affect the value of the natural frequency of the cantilever \( \omega_n \) and damping factor of the cantilever \( \zeta \). If \( \omega_n = \sqrt{k/m} \), \( \zeta = b/2\sqrt{km} \), and \( k = 3E/l^3 \), then

\[
k \approx \frac{3Em}{l}, \tag{5}
\]

\[
\omega_n = \frac{3E}{\sqrt{l}}, \tag{6}
\]

\[
\zeta = \frac{b\sqrt{l}}{2\sqrt{3Em^2}}. \tag{7}
\]

A critical factor when designing the mechanical coupling for energy harvester is how to keep the natural frequency of the cantilever can match the natural frequency of the piezoelectric. And also has a low damping coefficient, so the cantilever has a high total deviation. From the equation 6 and 7 show
that to get $\omega_n$ or natural frequency of the cantilever can match the natural frequency of the piezoelectric is by changing for the length of cantilever. And to get low value of $\varsigma$ or damping factor in order to cantilever has continually oscillate is by increasing the value of $l$. The role of the permanent magnet in this system is to reduce the damping.

### Table 2. Variable descriptions of changing the value of the natural frequency.

| Symbol | Description                              | Value  | Unit    |
|--------|------------------------------------------|--------|---------|
| $\omega_n$ | Natural frequency of the cantilever        | 299    | rad/sec |
| $\varsigma$ | Damping factor                          | 4.42 x 10^{-8} |       |
| $E$    | Young’s modulus [13]                      | 189 x 10^9 | N/mm^2  |
| $l$    | The length of cantilever                  | 100    | mm      |

2.5. Output power

Power output from rectifier and filter circuit can obtained from

$$P(t) = V_c(t) I(t),$$

where $I(t)$ and $V_c(t)$ are to be found.

3. Simulation with human footsteps as vibration source

3.1. Vibration source

Vibration on a surface that caused by human footsteps is used in our simulation. The purpose is for calculate how many foot step or mechanical coupling needed to produce desired electrical energy. When a footstep contacts a surface, it will trigger the mechanical coupling to generate vibrations that needed. The explanation of the process of collecting human footsteps is depict in figure 4

![Figure 4. Collecting process of human footsteps as vibration source.](image)

Supposed that a human has mass $M_h$ dan he start stepping on the pusher at time $t_i$ and leave it at time $t_f$ then $F(t)$ can be proposed as
\[ F(t) = u(t-t_0)\left[1-u(t-t_f)\right] M_0 g, \]  

(9)

where \( u(t-a) \) is a step function

\[ u(t-a) = \begin{cases} 0, & t < a, \\ 1, & t \geq a. \end{cases} \]  

(10)

If there are some humans stepping on the pusher, that series of equation (9) are required. Each human will have its own \( t_i \) and \( t_f \).

**Figure 5.** Single mechanical coupling application.

Single unit of energy harvesting system has schematic is shown in figure 5, while multiple units of the same system are shown in figure 6. Details how the schematic will be implemented in the real system are not in the scope of this work, which are subject to be discussed in future work.

**Figure 6.** Multi-mechanical coupling application.

3.2. Finite difference method

In order to solve equations (2) – (4) following finite difference relations

\[ \ddot{x} = \frac{x(t+\Delta t) - x(t)}{\Delta t} \]  

(11)

and

\[ \dddot{x} = \frac{x(t+\Delta t) - 2x(t) + x(t-\Delta t)}{(\Delta t)^2} \]  

(12)

are required.

4. Results and discussion

Implementation of equations (11) and (12) in equations (2) – (4) will lead to following numerical equations
\[(M_2 - \Delta b_2) x_2(t + \Delta t) + \Delta t b_2 x_{pa}(t + \Delta t) = \]
\[(\Delta t)^2 F(t) + \left[\Delta t b_2 - (\Delta t)^2 k_{eff}\right] x_{pa}(t) + \left[2M_2 + \Delta t b_2 - (\Delta t)^2 k_{eff}\right] x_2(t) - M_2 x_2(t - \Delta t),\]  \hspace{1cm} (13)
\[
\Delta t b_2 x_2(t + \Delta t) + \left[\Delta t b_2 + \Delta t (b_2 + b_{pa})\right] x_{pa}(t) = \]
\[\left[2M_2 + \Delta t (b_2 + b_{pa}) + (\Delta t)^2 (k_{eff} - k_{pa})\right] x_{pa}(t) - M_{pa} x_{pa}(t - \Delta t), \] \hspace{1cm} (14)
\[
\Delta t b_2 \hat{x}_2(t + \Delta t) + \left[\Delta t b_2 + \Delta t k_{eff}\right] \hat{x}_2(t) - M_{pa} \hat{x}_{pa}(t - \Delta t), \]
\[
\Theta \hat{x}_{pa} + C_p \hat{V}_p(t) = -I(t), \] \hspace{1cm} (15)
\[(\Delta t)^2 F(t) - \Delta t k_{eff} \hat{x}_2(t) - b_2 \hat{x}_2(t + \Delta t) - x_2(t) + x_{pa}(t + \Delta t) - x_{pa}(t) = \]
\[M_2 \left[\hat{x}_2(t + \Delta t) - 2 \hat{x}_2(t) + x_2(t - \Delta t)\right] \] \hspace{1cm} (16)
\[
\Delta t k_{eff} \hat{x}_2(t + \Delta t) + \hat{x}_2(t + \Delta t) - x_2(t) + x_{pa}(t + \Delta t) - x_{pa}(t) = \]
\[-\Delta t k_{eff} \hat{x}_{pa} - \Delta t b_2 \hat{x}_{pa}(t + \Delta t) - x_{pa}(t) = \]
\[M_p \hat{x}_{pa}(t + \Delta t) - 2 \hat{x}_{pa}(t) + x_{pa}(t - \Delta t) = \]
\[
\Theta \hat{x}_{pa}(t + \Delta t) - x_{pa}(t) + C_p \hat{V}_p(t) - \Theta \hat{V}_p(t) = -I(t). \] \hspace{1cm} (17)

5. Summary
Numerical model of mechanical coupling for harvesting energy using piezoelectric at low frequency vibration has been formulated. The numerical scheme has not been tested.

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