Managing the reliability of the tubing string in impulse non-stationary flooding

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Abstract. Impulse non-stationary flooding requires managing the processes influencing reliability and durability of the tubing string. In this case, it is necessary to determine the medium displacement for known dimensional parameters and stress fields, while taking into account the dynamic solution for the low-frequency limit. The results obtained from the analysis are compared against the measurements performed by various authors. Assuming there is a certain frequency of injected fluid oscillation or pressure pulses, one could expect that the radial movement is in equilibrium with the pressure existing at any given period of time. As a result, expressions describing the behavior of impulse waves propagating along the pipe wall were obtained. The reflected waves appear in all cases: whether the change in speed is caused by a change in the pipe wall thickness, Young modulus or shear rigidity of the environment. Changes in density or bulk modulus of the injected fluid or in the radius of the wellbore also cause reflected waves; all the waves may be described by a single equation.

1. Introduction

When studying the processes accompanying the movement of pressure impulses through the column of injected impulse fluid from the wellhead to the formation, it is necessary to use a simplification that the wave length is significantly larger than the wellbore diameter [1, 2]. Such conditions hold in all the cases that arise when analyzing non-stationary injection of fluid [3]. In this case, it is necessary to determine the medium displacement for known dimensional parameters and stress fields, while taking into account the dynamic solution for the low-frequency limit. The results obtained from the analysis will be then compared against the measurements performed by various authors. It should be noted, that the obtained results would allow approximating the low-frequency wave range, thus allowing determining the required parameters with sufficient accuracy in cases where exact values cannot be obtained.

2. Results and Discussion

In case of a thin pipe shown in Figure 1, sonic pressure and axial displacement of particles are seen as functions of only one coordinate and time \( p(z, t) \) and \( u_z(z, t) \) [4]. Elastic recoil leads to some radial displacement, but the gradients of radial pressure accompanying it are too small to change the piston-like movement along the pipe. This movement depends on the pressure gradient in the fluid that may be determined by equalizing the force to the mass with some accounts for acceleration for a certain length of the injected impulse fluid column [4]:

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\[
\frac{\partial p}{\partial z} \Delta z = -\rho \left( \pi b^2 \Delta z \right) \frac{\partial^2 u_z}{\partial z^2},
\]

(1)

where \( b \) is the wellbore radius, m; \( \rho \) is fluid density, kg/m\(^3\).

It is possible to find an additional relation between the pressure and displacement from the fact that the injected fluid is characterized with the bulk modulus \( B \) in accordance with the equation \( p = -B (\Delta V/V) \). As the pressure grows, the elementary length changes (Figure 1).

**Figure 1.** Changes in the elementary volume of the injected fluid in the well.

Volume change consists of two components: the first one \(-\pi b^2 (\partial u_z/\partial z) \Delta z\), is determined by the axial displacement, while the second one \(-2\pi b^2 \Delta u_r \) is determined by the radial expansion of the wellbore wall. Dividing the sum of these components by the volume \( \pi b^2 \Delta z \) gives the ratio between the pressure and the displacement:

\[
\frac{P}{B} = \left( \frac{\partial u_z}{\partial z} + \frac{2u_r}{b} \right).
\]

(2)

This equation may be transformed into an expression that includes only pressure and displacement [5]. Assuming there are low-frequency or slowly-changing pressure pulses, one could expect that the radial movement is in equilibrium with the pressure existing at any given moment of time. The same assumption of low frequencies allows believing that at a distance of several wellbore diameters in the axial direction, the pressure is largely uniform. Considering these limitations, one may expect that the ratio between pressure and radial expansion of the wellbore wall is going to be adequately described with the static elasticity theory. Lamb [6] has obtained the radial displacement due to pressure inside a thick-walled pipe with the interior radius \( b \) and outside radius \( a \) and is characterized with Young modulus \( E \) and Poisson’s ratio \( \nu \). From the ratios obtained by Lamb, the following expression may be developed:

\[
\frac{u_r}{b} = \frac{P}{E} \left[ \frac{(1+\nu)(a^2+b^2)-2\nu b^2}{a^2-b^2} \right] = \frac{P}{2M},
\]

(3)

where \( u_r \) is a radial deformation of the pipe, mm; \( M \) is the plane deformation modulus, Pa.

Substituting this equation into the expression (2) allows obtaining the desired ratio between the pressure and the axial displacement:

\[
\frac{P}{B} = \frac{1}{b} \frac{\partial u_z}{\partial z}.
\]

(4)

Differentiating the equation (4) with respect to \( z \) and substituting it into the expression (1), we get the wave equation that describes the movement of the injected fluid column in a thick-walled pipe:

\[
\frac{\partial^2 u_z}{\partial z^2} = \frac{1}{b} \frac{\partial^2 u_z}{\partial t^2}.
\]

(5)

It follows that the column of the injected fluid in the thick-walled pipe is capable of sustaining impulses of any waveform \( f(t-z/C_T) \) or \( g(t+z/C_T) \), propagating in any direction without dispersion or attenuation. The tube wave velocity in the thick-walled pipe [14] is

\[
C_T = \left( \frac{1/\rho + 1/M}{b} \right)^{-1/2},
\]

(6)

where
\[ M = \frac{\rho(a^2 - b^2)}{2[(1+v)(a^2 + b^2) - 2v b^2]} \]

For a thin-walled pipe with a thickness of \( h = (a - b) \), the values of its interior and outside radii are very close, while the value of \( M \) tends to the value of \( Eh/2b \). In this case, the wave propagation speed is determined with

\[ C_T = \left[ \rho \left( \frac{1}{B} + \frac{1}{\frac{Eh}{2b}} \right) \right] \frac{1}{2}. \] (7)

For a thick-walled pipe, its outside radius \( a \) is significantly larger than its interior radius \( b \), while the value of \( M \) tends to the modulus of shear rigidity \( \mu \). The following expression is used to determine the pressure wave propagation speed

\[ C_T = \left[ \rho \left( \frac{1}{B} + \frac{1}{M} \right) \right] \frac{1}{2}. \] (8)

It is of interest to obtain the ratio between the pressure and particle displacement in the fluid. If the particle displacement is seen as an impulse passing in the positive direction, then \( u_z = f(t - z/C_T) \), and from formulas (4) and (6) it follows that

\[ p = \rho C_T f'(t - \frac{z}{C_T}). \] (9)

Where the prime mark designates differentiation with respect to a variable substituting the bracketed expression. Then, we determine the particle speed \( \nu = f \left( t - \frac{z}{C_T} \right) \).

The research data corresponded to conditions where the wellbore is surrounded with a uniform medium [7]. Let us then consider the conditions of the pressure wave propagation through a boundary between two elastic semi-spaces (Figure 2).

![Figure 2](image)

**Figure 2.** Wave direction diagram in the vicinity of the interface between two different layers of rock.

In this case, two pressure waves arise: reflected and transient. Then, we may represent the equations (taking into account continuous nature of pressure and the particle displacement speed) \( p_1 + p_n = p_T \) and \( p_1/\rho C_{T1} = p_R/\rho C_{T1} = p_T/\rho C_{T2} \). According to that,

\[ p_1 = f(t - \frac{z}{C_{T1}}), \quad p_R = \frac{\rho C_{T2} - \rho C_{T1}}{\rho C_{T2} + \rho C_{T1}} f(t + \frac{z}{C_{T1}}), \quad p_R = \frac{2\rho C_{T2}}{\rho C_{T2} + \rho C_{T1}} f(t - \frac{z}{C_{T2}}). \] (10)

Borehole waves observed at such boundary are shown in Figure 3.
Figure 3. Oscillogram of fluid pressure waves near the interlayer boundary (a) and the same oscillogram with a 30x amplification (b): 1 – a wave advancing through the rock; 2 – a wave propagating through the fluid; 3 – reflection from the interlayer boundary; 4 – bottomhole reflection; 5 – impulse at a depth of 30 m; 6 – impulse at a depth of 60 m.

At this segment (for our purpose, at a boundary between chalk and slate) wave recordings for the ratio $p_R/p_1 = 0.15$ [8] are presented. If we substitute the known values of $C_{T1}$ (770 m/s) and $C_{T2}$ (1140 m/s) into the equation (10), we get the value of 0.19. Such a correspondence may be deemed quite satisfactory, as no corrections were introduced to account for pressure wave attenuation when propagating from the pressure transmitters to the boundary and back.

The description of the impulse waves at the boundary is fundamentally an approximation. On the one hand, the radial displacement in both mediums at the same pressure of the injected fluid is different; besides, continuity of the radial displacement is one of the limiting condition at the contact boundary between the two layers. At a distance of several wellbore radii above or below the boundary, the radial displacement values can be quite satisfactory described with the equation (3). Between two values, there is a smooth transition over the interval, which is, as it was previously assumed, small; thus, this transition may be substituted with a step change at the boundary. Actual conditions at the boundary shall necessarily include interaction with the body waves; in particular, the impulse wave passing through the boundary shall cause radiation of some shear and longitudinal waves into the medium. If the wellbore diameter is significantly smaller than the shortest wave of interest, this effect may be neglected.

It is evident, that the formulas (10) describe the waves propagating along the pipe wall. The reflected waves appear in all cases: whether the change in speed is caused by a change in the pipe wall thickness, Young modulus or shear rigidity of the environment. Changes in density or bulk modulus of the injected fluid or in the radius of the wellbore also cause reflected waves; all the waves may be described by a single equation. Figure 4 illustrates the abrupt changes in one or several such parameters.
Riggs [9] gives measured values of speed at 1320 m/s in a cased wellbore and 895 m/s in a nearby uncased borehole, which are in agreement with the expressions above. From his values for shear wave speed (771 m/s) and rock density (= 2000 kg/m$^3$), the value of the shear modulus was calculated $\mu = 1.15 \cdot 10^9$ Pa. Riggs reported that the impulse waves in an isolated tube would have the speed of 1280 m/s. This speaks to the fact that $Eh/2b$ has the value of $5.5 \cdot 10^9$ Pa, which is true for a steel pipe with a diameter of 12.7 cm and a wall thickness of 0.3 cm. These constants, being substituted into (8), provide two values of speed in agreement with the measurement results. Thus, in our reasoning, we are going to assume that the permeable formation matrix is rigid. Less supported is another assumption according to which the matrix material is incompressible. At low frequencies and within several diameters, the oscillation pressure changes very insignificantly through the wellbore. In this case, the short element of the wall behaves approximately as if the pressure is independent of the axial distance. Both elastic expansion and mean direction of flow passing through the pores are radial. Wall impedance developed for the radial displacement will be applicable for low frequencies.

3. Experimental part
Evaluation of phase velocity and wave attenuation for four porous rocks took into account the values given in the table [10]. The wellbore fluid and the fluid in the void space was assumed as water with the bulk modulus of elasticity of $B = 2.2 \cdot 10^9$ Pa and viscosity $\eta = 0.00001$ Pa s, while the wellbore radius was taken as 10 cm.

| Rock type | Shear modulus, $\mu \cdot 10^9$ Pa | Displacement potential, $\Phi$ | Permeability, $\kappa \cdot 10^{-7}$ m$^2$ |
|-----------|-----------------------------------|-------------------------------|----------------------------------|
| A         | 1.40                              | 0.30                          | 10.000                           |
| B         | 1.40                              | 0.30                          | 1000                             |
| C         | 4.90                              | 0.21                          | 300                              |
| D         | 2.83                              | 0.10                          | 100                              |

Figure 5 shows the graphs of the phase velocity and impulse wave attenuation in fluid as functions of oscillation frequency.
Figure 5. Phase velocity and attenuation of impulse waves for four rock types from the Table.

From Figure 5 it is evident that the phase velocity decreases at frequencies below 100 Hz, while attenuation is very high.

4. Conclusions
The obtained wave equation describing the behavior of the injected fluid column in a thick-walled pipe shows that the injected fluid column in a thick-walled pipe is able to support impulses of any waveform propagating in any direction without dispersion or attenuation.

Additionally, expressions describing the behavior of impulse waves propagating along the pipe wall were obtained. The reflected waves appear in all cases: whether the change in speed is caused by a change in the pipe wall thickness, Young modulus or shear rigidity of the environment. Changes in density or bulk modulus of the injected fluid or in the radius of the wellbore also causes reflected waves; all the waves may be described by a single equation.

Evaluation of phase velocity and wave attenuation for four porous rocks shows that the phase velocity decreases, while the attenuation is very high at frequencies below 100 Hz.

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