New fermions and a vector-like third generation

in $SU(3)_c \otimes SU(3)_L \otimes U(1)_N$ models

Vicente Pleitez

Instituto de Física Teórica

Universidade Estadual Paulista

Rua Pamplona, 145

01405-900– São Paulo, SP

Brazil

Abstract

We study two 3-3-1 models with i) five (four) charge $2/3$ ($-1/3$) quarks and, ii) four (five) charge $2/3$ ($-1/3$) quarks and a vector-like third generation. Possibilities beyond these models are also briefly considered.

PACS numbers: 12.60.-i; 12.60.Cn; 12.10.Dm
I. INTRODUCTION

Nowadays it seems that in some sense the third generation may be different from the other ones: although a heavy top quark [1] can still be barely accommodated in the Standard Model it can bring some unexpected features to the mass spectrum problem, also, possibly the bottom quark couples to the $Z^0$ with a strength which is different from the strength of the $d$ and $s$ quarks [2]. The properties of the tau lepton and its neutrino can still bring up surprises [3]. On the other hand, if the cross section $\sigma(p\bar{p} \rightarrow t\bar{t} + X)$ obtained by the CDF Collaboration [1] is in fact higher than the prediction of quantum chromodynamics, this may be a signature of new quarks.

In the model based on the gauge symmetry $SU(3)_c \otimes SU(3)_L \otimes U(1)_N$ of Ref. [4], the effective $SU(2) \otimes U(1)$ model coincides with the usual electroweak one. The three families belong to left-handed doublets and right-handed singlets of $SU(2)$. Hence, all of them have, at leading order, the same couplings to the $W^\pm$ and $Z^0$ bosons. Also the extra quarks in that model have exotic $5/3$ and $-4/3$ charges. The lepton sector is exactly the same of the Standard Model (SM). Although this model coincides at low energies with the usual electroweak model, it explains the fundamental questions: i) the family number and, ii) why $\sin^2\theta_W < 1/4$ is observed. Therefore, it is possible from the last constraint, to compute an upper limit to the mass scale of the $SU(3)$ breaking of about 3 TeV [5]. This turns the 3-3-1 model an interesting possibility for the physics beyond the Standard Model, in particular if future experiences confirm in more detail an $SU(2) \otimes U(1)$ model (for instance if the $Z \rightarrow b\bar{b}$ decay and several $Z$-pole asymmetries confirm the value expected in the model) and no new quarks with charge $2/3$ and $-1/3$ were found.

However, if in the future new quarks were observed having the same charge than the quarks already known or, if the third generation turns to be in fact different from the other two generations (say, with different interactions), it will be necessary to consider modifications of the original 3-3-1 model. For instance, a model with five charge $-1/3$ and four charge $2/3$ quarks has already been considered in Ref. [6]. There are, however, other
representation contents: four charge $-1/3$ and five charge $2/3$ quarks or, by the choice of
the third generation in a vector-like representation of the electroweak symmetry. We will
give below an example of such a sort of model which is an extension of one of the models
proposed some years ago in Ref. [7].

Once we are convinced that theories based on the 3-3-1 gauge symmetry are interesting
possibilities for the physics at the TeV range, we must study how the basic ideas of this sort
of models can be generalized.

In (almost) all these models, which we recall that are indistinguishable from the Standard
Model at low energies, in order to cancel anomalies the number of families ($N_f$) must be
divisible by the number of color degrees of freedom (3). Hence the simplest alternative is
$3 = N_f$. By denoting $N_q$ and $N_l$ the number of quark and lepton families, respectively, we
will see that the relation $N_q = N_l = N_f = 3$ is a particular feature of 3-$m$-1 models. When
$n \neq 3$, $N_q$ and $N_l$ are still related to each other but it is not necessary that $N_q = N_l$ in order
to have anomaly cancellation.

In this work we also want to generalize these sort of models in several ways. Firstly,
by expanding the color degrees of freedom ($n$) and the electroweak sector ($m$), i.e., we will
consider models based on the gauge symmetry

$$SU(n)_c \otimes SU(m)_L \otimes U(1)_N.$$  \hfill (1.1)

In most of these extensions the anomaly cancellation occurs among all generations together,
and not generation per generation. However, if a 3-3-1 model has the third generation not
anomalous, in its extension also it will be so.

We will use the criterion that the values for $m$ in Eq. (1.1) are determined by the leptonic
sector. It means that if each generation is treated separately, $SU(4)$ is the highest symmetry
group to be considered in the electroweak sector. Thus, there is no room for $SU(5)_L \times U(1)_N$
if we restrict ourselves to the case of leptons with charges $\pm 1, 0$.

In the color sector, for simplicity, in addition to the usual case of $n = 3$, we will comment
the cases $n = 4, 5$. These extensions have been considered in the context of the $SU(2)_L \otimes$
Next, models with left-right symmetry and/or with horizontal symmetry are also considered. We discuss too a $SU(6)$ grand unified theory in which one of these models may be embedded.

We must stress that all extensions of 3-3-1 models have flavor changing neutral currents (FCNC). However, up to now in all models of this kind which have been considered in detail, it was always possible to have, in the sector of the model which coincides with the observed one, natural conservation of flavor in the neutral currents. Hence, FCNC effects are restricted to the exotic sectors of the models. The only exception is model B below since in this case there are right-handed currents coupled to the $Z^0$ which do not conserve flavor but they involve arbitrary right-handed mixing matrix. Since all extensions of the 3-3-1 model that we will consider here have an $SU(3)_L$ subgroup, we think that the suppression of the FCNC in the observed part of the particle spectrum is a general feature of this kind of models. This is far from being an obvious fact but it was showed in several 3-3-1 models and recently in the 3-4-1 case. For details see Refs. [6,10,11].

This work is organized as follows. In Sec.II we consider two new possibilities of 3-3-1 models. In Sec.III we consider models with $n = 3$, $m = 3, 4$ (Sec.III A). There we will also discuss the cases for $n = 4, 5$ (Sec.III B). In Sec.IV we give general features of the extensions with left-right symmetry (Sec.IV A) and with horizontal symmetries (Sec.IV B). We also consider (Sec.IV C) possible embedding in $SU(6)$. The last section is devoted to our conclusions.

II. TWO 3-3-1 MODELS

Here, we will treat two interesting possibilities of 3-3-1 models with the electric charge operator defined as $2Q = \lambda_3 + \lambda_8/\sqrt{3} + 2N$. Both models have the same gauge boson sector, they differ slightly in the scalar sector but they are quite different in the fermion sector. One of the models (model A) has five charge $2/3$ quarks and four charge $-1/3$ ones;
the other model (model B) has four charge $2/3$ and five charge $-1/3$ quarks and the third generation in a vector-like representation of $SU(3)$. Model B is an extension with three quark generations of one of the models put forward in Ref. [7]. In model A anomalies cancel out only among all generations with each generation being anomalous. In model B only the third generation is not anomalous.

### A. Model A

All leptons generations transform as triplets of $SU(3)$

$$
\Psi_{aL} = \begin{pmatrix}
\nu_a \\
l_a^- \\
E_a^-
\end{pmatrix}_L \sim (3, -2/3), \quad a = e, \mu, \tau;
$$

(2.1)

while quarks transform as follows

$$
Q_{iL} = \begin{pmatrix}
  \nu_i' \\
  u_i \\
  d_i
\end{pmatrix}_L \sim (3^*, 1/3), \quad i = 1, 2; \quad Q_{3L} = \begin{pmatrix}
  w_3 \\
  d_3
\end{pmatrix}_L \sim (3, 0),
$$

(2.2)

and all charged right-handed fields in singlets. Neutrinos remain massless as long as no right-handed components are introduced. We have omitted the color index.

In the quark sector the phenomenological states in Eqs. (2.2) are linear combinations of the mass eigenstates $(u, c, t, t', t'')$ and $(d, s, b, b')$ for the charge $2/3$ and charge $-1/3$ sectors, respectively. Three of the left-handed charge $2/3$ $(1/3)$ quark fields are part of $SU(2)$ doublets $u_{1,2,3}$ $(d_{1,2,3})$. The other fields $u'_{1,2}$ and $d_4$ are in singlets of $SU(2)$.

Let us introduce three triplets of Higgs bosons

$$
\eta = \begin{pmatrix}
\eta_2^+ \\
\eta_1^+ \\
\eta_0^-
\end{pmatrix} \sim (3^*, 2/3), \quad \sigma = \begin{pmatrix}
\sigma_2^0 \\
\sigma_1^0 \\
\sigma^-
\end{pmatrix} \sim (3^*, -1/3),
$$

(2.3)

and a third one $\sigma'$ transforming like $\sigma$. 5
The most general quark Yukawa couplings are

\[ - \mathcal{L}_Y = \sum_{i\alpha} A_{i\alpha} \bar{Q}_i L D_{\alpha R} + \sum_{i\beta} [B_{i\beta} \bar{Q}_i L \sigma + B'_{i\beta} \bar{Q}_i L \sigma' \eta] U_{\beta R} \]
\[ + \sum_{\alpha} [E_{3\alpha} \bar{Q}_{3L} \sigma^* + E'_{3\alpha} \bar{Q}_{3L} \sigma'' \eta^*] D_{\alpha R} + \sum_{\beta} F_{3\beta} \bar{Q}_{3L} U_{\beta R} \eta^* + \text{H.c.} \quad (2.4) \]

where \( i = 1, 2, \alpha = 1, 2, 3, 4, \beta = 1, 2, 3, 4, 5 \) and we have chosen the basis \( D_{\alpha R} = d_{1,2,3,4R} \), \( U_{\beta R} = u_{1,2,3R}, u'_{1,2R} \); with \( \eta^*, \sigma^* \) the respective antitriplets and we have omitted \( SU(3) \) indices.

Let us assume the following vacuum expectation values (VEVs):

\[ \langle \sigma_1^0 \rangle \neq 0, \langle \sigma_2^0 \rangle = 0, \langle \sigma_1^* \rangle = 0, \langle \sigma_2^* \rangle \neq 0; \quad (2.5) \]

and we also assume that the mass scale characteristic of the \( SU(3) \) symmetry is rather high:

\[ \langle \sigma_2^0 \rangle \gg \langle \sigma_1^0 \rangle, \langle \eta^0 \rangle. \quad (2.6) \]

Before going on, let us consider the neutral currents coupled to \( Z^0 \). We must determine which fields have the same couplings than in the \( SU(2) \otimes U(1) \) effective theory i.e., when the condition in Eq. (2.6) is satisfied. The photon field is given by

\[ A_\mu = s_W (W_\mu^3 + \frac{1}{\sqrt{3}} W_\mu^8) + \frac{1}{\sqrt{3}} (3 - 4s_W^2) \frac{1}{2} B_\mu. \quad (2.7a) \]

The massive neutral bosons are

\[ Z_\mu = c_W W_\mu^3 - \frac{1}{\sqrt{3}} \tan \theta_W [s_W W_\mu^8 + (3 - 4s_W^2) \frac{1}{2} B_\mu], \quad (2.7b) \]

which correspond to the usual \( Z^0 \) and the heavier one

\[ Z'_\mu = \frac{1}{\sqrt{3}c_W} [-(3 - 4s_W^2) \frac{1}{2} W_\mu^8 + s_W B_\mu] \quad (2.7c) \]

where \( s_W \equiv \sin \theta_W, c_W \equiv \cos \theta_W \) and \( \theta_W \) is the usual weak mixing angle. From the electric charge definition we get

\[ \frac{g'^2}{g^2} = \frac{3s_W^2}{3 - 4s_W^2}, \quad (2.8) \]
hence, $\sin^2 \theta_W < 3/4$ \footnote{The ratio $\sin^2 \theta_W$ is less than $3/4$ in the standard model.}. 

The neutral current interactions of fermions ($\psi_i$) can be written as usual

$$ \mathcal{L}^{NC} = -\frac{g}{2 \cos \theta_W} \left[ \sum_i L_i \bar{\psi}_i L \gamma^\mu \psi_i L + R_i \bar{\psi}_i R \gamma^\mu \psi_i R \right] Z_\mu $$

$$ = -\frac{g}{2 \cos \theta_W} \sum_i \bar{\psi}_i \gamma^\mu (g_V^i - \gamma^5 g_A^i) \psi_i Z_\mu, \quad (2.9) $$

where $g_V^i \equiv \frac{1}{2}(L_i + R_i)$ and $g_A^i \equiv \frac{1}{2}(L_i - R_i)$. We obtain for the charge $-1/3$ sector

$$ L_{d_1} = L_{d_2} = L_{d_3} = -1 + \frac{2}{3} s_W^2, \quad L_{d_4} = \frac{2}{3} s_W^2, \quad (2.10a) $$

$$ R_{d_1} = R_{d_2} = R_{d_3} = R_{d_4} = \frac{2}{3} s_W^2. \quad (2.10b) $$

and,

$$ g_{d_1}^d = g_{d_2}^d = g_{d_3}^d = -\frac{1}{2} + \frac{2}{3} s_W^2, \quad g_{d_4}^d = \frac{2}{3} s_W^2; \quad (2.10c) $$

$$ g_{d_1}^i = g_{d_2}^i = g_{d_3}^i = -\frac{1}{2}, \quad g_{d_4}^i = 0. \quad (2.10d) $$

Similarly, for the charge $2/3$ sector

$$ L_{u_1} = L_{u_2} = L_{u_3} = 1 - \frac{4}{3} s_W^2, \quad L_{u_4} = L_{u_4}' = -\frac{4}{3} s_W^2, \quad (2.11a) $$

$$ R_{u_1} = R_{u_2} = R_{u_3} = R_{u_4}' = R_{u_4} = -\frac{4}{3} s_W^2, \quad (2.11b) $$

and

$$ g_{u_1}^u = g_{u_2}^u = g_{u_3}^u = \frac{1}{2} - \frac{4}{3} s_W^2, \quad g_{u_4}^u = g_{u_4}' = -\frac{4}{3} s_W^2; \quad (2.11c) $$

$$ g_{u_1}^i = g_{u_2}^i = g_{u_3}^i = g_{u_4}^i = g_{u_4}' = g_{u_4} = 0. \quad (2.11d) $$

From Eqs. (2.10) we see that $d_1, d_2$ and $d_3$ have the same couplings to the $Z^0$ as the $d, s, b$ quarks in the standard electroweak model, but $d_4$ has a pure vector coupling. In the charge $2/3$ sector we observe from Eqs. (2.11) that $u_1, u_2$ and $u_3$ have the same couplings of the
usual \( u, c, t \) quarks in the Standard Model, while \( u'_1, u'_2 \) have pure vector couplings to the \( Z^0 \). The mass eigenstates will be denoted by \( u, c, t, t', t'' \) and \( d, s, b, b' \). Hence, if we avoid a general mixing in the mass matrix we will implement a GIM mechanism \([12]\) in the model.

Finally, for leptons we have

\[
L_{\nu_a} = 1, \quad L_{l_a} = -1 + 2 s_W^2, \quad L_{E_a} = 2 s_W^2, \quad (2.12a)
\]

\[
R_{\nu_a} = 0, \quad R_{l_a} = R_{E_a} = 2 s_W^2. \quad (2.12b)
\]

or,

\[
g_{\nu a}^\nu = \frac{1}{2}, \quad g_{l a}^l = -\frac{1}{2} + 2 s_W^2, \quad g_{E a}^{E a} = 2 s_W^2, \quad (2.12c)
\]

\[
g_{A a}^\nu = \frac{1}{2}, \quad g_{l a}^l = -\frac{1}{2}, \quad g_{A a}^E = 0. \quad (2.12d)
\]

We see that neutrinos and \( e^-, \mu^-, \tau^- \) have the same couplings to the \( Z^0 \) than in the \( SU(2) \otimes U(1) \) model. The heavy leptons \( E_a \) have vector-like couplings. Lepton couplings with the \( Z^0 \) conserve flavor in each sector: \( \nu_a, l_a^- \) and \( E_a^- \). This is not a surprise since lepton generations are treated democratically.

Knowing the neutral current couplings, given in Eqs. \((2.10)\) and \((2.11)\), in order to avoid a general mixing in the mass matrices we will introduce the following discrete symmetries:

\[
d_{1,2,3R} \rightarrow d_{1,2,3R}; \quad d_{4R} \rightarrow -d_{4R}, \quad u_{1,2,3R} \rightarrow u_{1,2,3R}; \quad u'_{1,2R} \rightarrow -u'_{1,2R}, \quad (2.13a)
\]

\[
Q_{1,2,3L} \rightarrow Q_{1,2,3L}, \quad \eta, \sigma \rightarrow \eta, \sigma, \quad \sigma' \rightarrow -\sigma'. \quad (2.13b)
\]

For leptons the discrete symmetries are

\[
\Psi_{aL} \rightarrow \Psi_{aL}, \quad l_a^- \rightarrow l_a^-, \quad E_a^- \rightarrow -E_a^- . \quad (2.14)
\]

The quark mass terms have the form
\[ \mathcal{D}_{\alpha L} M^D_{\alpha \alpha'} \mathcal{D}_{\alpha' R}, \quad \mathcal{U}_{\beta L} M^U_{\beta \beta'} \mathcal{U}_{\beta' R}. \]  

(2.15)

With the discrete symmetries in Eq. (2.13) the mass matrices become

\[ M^U = \begin{pmatrix} M^U_1 & 0 \\ 0 & M^U_2 \end{pmatrix}, \]  

(2.16)

for the charge \( 2/3 \) sector, and

\[ M^D = \begin{pmatrix} M^D_1 & 0 \\ 0 & 1 \end{pmatrix}, \]  

(2.17)

for the charge \(-1/3\) one. \( M^U_1 \) and \( M^D_1 \) are arbitrary \( 3 \times 3 \) matrices in the basis \( u_1, u_2, u_3 \) and \( d_1, d_2, d_3 \), respectively; \( M^U_1 \) is an arbitrary \( 2 \times 2 \) matrix in the basis \( u'_1, u'_2 \). The unit matrix in \( M^D \) means that \( d_4 \) does not mix with the other quarks of charge \(-1/3\). The mass matrices in Eqs. (2.15)–(2.17) can be diagonalized. In terms of the mass eigenstates they become

\[ \mathcal{D}_L \tilde{M}^D \mathcal{D}_R, \quad \mathcal{U}_L \tilde{M}^U \mathcal{U}_R \]  

(2.18)

where \( \tilde{M}^U = \text{diag}(m_u, m_c, m_t, m'_t, m'_t) \), and \( \tilde{M}^D = \text{diag}(m_d, m_s, m_b) \) and \( U \) and \( D \) denote \( (u, c, t, t', t'') \) and \( (d, s, b, b') \), respectively. Note that in fact, the mixing occurs among \( u_1, u_2, u_3; u_4, u_5 \) and \( d_1, d_2, d_3 \) while \( d_4 \) does not mix and it is in fact the \( b' \) quark.

The mass matrices in Eqs. (2.16) and (2.17) are diagonalized by performing the transformations

\[ \mathcal{U}_L = V^U_L V_L, \quad \mathcal{D}_L = V^D_L D_L, \quad \mathcal{D}_R = V^D_R D_R, \]  

(2.19)

with

\[ V^U_L = \begin{pmatrix} V^U_{1L} & 0 \\ 0 & V^U_{2L} \end{pmatrix}, \quad V^U_R = \begin{pmatrix} V^U_{1R} & 0 \\ 0 & V^U_{2R} \end{pmatrix}, \]  

(2.20a)

\[ V^D_L = \begin{pmatrix} V^D_{1L} & 0 \\ 0 & 1 \end{pmatrix}, \quad V^D_R = \begin{pmatrix} V^D_{1R} & 0 \\ 0 & 1 \end{pmatrix}. \]  

(2.20b)
Next, let us consider the interactions in the quark sector. We have the currents

\[
\mathcal{L}_q = -\frac{g}{\sqrt{2}} \left( \bar{u}_{iL} \gamma^\mu u_{iL} X^0_\mu - \bar{u}_{iL} \gamma^\mu d_{iL} V^+_{\mu} + \bar{u}_{iL} \gamma^\mu d_{iL} W^+_{\mu} \right. \\
\left. + \bar{u}_{3L} \gamma^\mu d_{3L} W^+_{\mu} + \bar{u}_{3L} \gamma^\mu d_{4L} V^+_{\mu} + \bar{d}_{3L} \gamma^\mu d_{4L} X^0_{\mu} \right) + H.c.
\] (2.21)

In particular, the interaction with the $W^+$ boson can be written as usual with the mixing matrix defined as

\[
V_{KM} = V_{UL} V_{DL}^\dagger.
\]

On the other hand we have the currents coupled to the $V^+$

\[
\mathcal{L}_{qV}^{CC} = -\frac{g}{\sqrt{2}} (\bar{u}_1 \bar{u}_2 \bar{u}_3 \bar{u}_2')_L \Delta \gamma^\mu \begin{pmatrix} d_1 \\ d_2 \\ d_3 \\ d_4 \\ 0 \end{pmatrix}_L V^+_{\mu} + H.c.,
\] (2.22)

where $\Delta$ is a $5 \times 5$ matrix with $-\Delta_{41} = -\Delta_{52} = \Delta_{34} = 1$ and all other elements vanish. In terms of the mass eigenstates we can write Eq. (2.22) as

\[
\mathcal{L}_{qV}^{CC} = -\frac{g}{\sqrt{2}} (\bar{u} \bar{c} \bar{t} \bar{t}' \bar{t}'')_L K \gamma^\mu \begin{pmatrix} d \\ s \\ b \\ b' \\ 0 \end{pmatrix}_L V^+_{\mu} + H.c.,
\] (2.23)

where $K$ is defined as

\[
K \equiv V_{UL}^U \Delta V_{DL}^D,
\] (2.24)

being $V_{UL}^U, V_{DL}^D$ the matrices in Eqs. (2.20a) and (2.20b). In these sort of models it is not interesting to define $V_{UL}^U$ as being the unit matrix as it is usually done in the $SU(2) \otimes U(1)$ model. This is because this matrix appears also in the neutral currents with the extra $Z^0$ present in the model [10]. So we do not assume that the charge $2/3$ mass eigenstates appear
unmixed. There are similar interactions with the $X^0$ boson but in this case there are mixture involving the matrices $V^U_{1L}$ and $V^U_{2L}$ in the charge $2/3$ sector and $V^D_{1L}$ in the charge $-1/3$ one. We will return to this point later.

In the lepton sector, neutrinos and the usual charged leptons have the same couplings to the $W^+$ boson as in the $SU(2) \otimes U(1)$ model,

$$\mathcal{L}_l = -\frac{g}{2} \sum_a \left[ \bar{\nu}_{aL} \gamma^\mu l_a^+ W^+_{\mu} + \bar{\nu}_{aL} \gamma^\mu E_{aL} V^+_{\mu} + \bar{l}_{aL} \gamma^\mu E_{aL} X_{\mu} \right] + H.c., \quad (2.25)$$

The charged leptons get a mass via the interaction with the $\sigma^*$ and $\sigma'^*$ scalars. With discrete symmetries in Eq. (2.14) the Yukawa interactions are

$$-\mathcal{L}_{Y} = \sum_{ab} \bar{\Psi}_{aL} \left[ h_{ab} l_{bR} \sigma^{*} + h'_{ab} E_{bR} \sigma'^{*} \right] + H.c., \quad (2.26)$$

$h_{ab}, h'_{ab}$ are arbitrary $3 \times 3$ matrices and neutrinos remain massless if right-handed neutrinos are not introduced. We can define the neutrino fields in such a way that there is not mixing in the $W^+$–interactions but there are mixings in the $V^+, X^0$ ones.

In the lepton sector we have no mixing among $l_a^-$ and $E_a^-$ in the mass matrix since the discrete symmetries in (2.14) forbid it. So there is not flavor violation in Higgs-boson couplings.

**B. Model B**

This is an extended version of the model proposed some years ago by Georgi and Pais [7]. Here we have to consider a third quark generation since today there is evidence of the existence of a $t$ quark [8]. As we will see later, the phenomenology of this model is rather different from that of Georgi and Pais’s model in both sectors, quarks and leptons.

Let us first consider leptons. This is the same of Ref. [7] with four antitriplets $(3^*, -1/3)$

\[
\begin{align*}
\Psi_{eL} : & \begin{pmatrix} \nu_e^c \\ \nu_e \end{pmatrix} , \quad \Psi_{\mu L} : \begin{pmatrix} \nu_\mu^c \\ \nu_\mu \end{pmatrix} , \quad \Psi_{\tau L} : \begin{pmatrix} \nu_\tau^c \\ \nu_\tau \end{pmatrix} , \quad \Psi_{TL} : \begin{pmatrix} \nu_T^c \\ \nu_T \end{pmatrix} , \quad (2.27a)
\end{align*}
\]
and two other antitriples with \((3^*, 2/3)\)

\[
\Psi'_e L : \begin{pmatrix} e^+ \\ \tau^+ \\ N_1^0 \end{pmatrix}_L, \quad \Psi'_\mu L : \begin{pmatrix} \mu^+ \\ T^+ \end{pmatrix}_L
\]

and two neutral singlets \((N_{1L}^0)^c, (N_{2L}^0)^c\).

The quark fields of the first two generations (suppressing the color indices) are in two left-handed triplets \((3, 0)\)

\[
Q_{iL} = \begin{pmatrix} u_i \\ d_i \\ d_i' \end{pmatrix}_L, \quad i = 1, 2
\]

and the right-handed components in singlets \(u_iR \sim (1, 2/3)\) and \(d_iR, d_i'R \sim (1, -1/3)\). Finally, the third quark generation transforms in a vector-like representation

\[
Q_{3L} = \begin{pmatrix} u_4 \\ u_3 \\ d_3 \end{pmatrix}_L \sim (3^*, 1/3), \quad Q_{3R} = \begin{pmatrix} u_4 \\ u_3 \\ d_3 \end{pmatrix}_R \sim (3^*, 1/3).
\]

The scalar fields are those in Eq. \((2.3)\) but now we can introduce a singlet neutral scalar \(\phi\) with VEV \(\langle \phi \rangle \neq 0\).

As we said before, the gauge bosons are the same of model A. In particular, the neutral ones are given in Eqs. \((2.7)\). Thus, in this model we get the neutral current couplings defined in Eq. \((2.3)\),

\[
L_{d_1} = L_{d_2} = L_{d_3} = -1 + \frac{2}{3} s_W^2, \quad L_{d_1} = L_{d_2} = \frac{2}{3} s_W^2,
\]

\[
R_{d_1} = R_{d_2} = \frac{2}{3} \sin^2 \theta_W, \quad R_{d_3} = -1 + \frac{2}{3} s_W^2, \quad R_{d_1} = R_{d_2} = \frac{2}{3} s_W^2,
\]

or,

\[
g_{V}^{d_1} = g_{V}^{d_2} = g_{V}^{d_3} = \frac{1}{2} + \frac{2}{3} s_W^2, \quad g_{V}^{d_1} = g_{V}^{d_2} = \frac{2}{3} s_W^2,
\]
\[ g^d_1 = g^d_2 = -\frac{1}{2}, \quad g^d_3 = g^d'_3 = g^d_4 = 0, \quad (2.30d) \]

for the charge \(-1/3\) quarks. For the charge \(2/3\) sector we have

\[ L_{u_1} = L_{u_2} = L_{u_3} = 1 - \frac{4}{3} s^2_W, \quad L_{u_4} = -\frac{4}{3} s^2_W, \quad (2.31a) \]

\[ R_{u_1} = R_{u_2} = -\frac{4}{3} s^2_W, \quad R_{u_3} = 1 - \frac{4}{3} s^2_W, \quad R_{u_4} = \frac{4}{3} s^2_W; \quad (2.31b) \]

or

\[ g^{u_1}_v = g^{u_2}_v = g^{u_3}_v = \frac{1}{2} - \frac{4}{3} s^2_W, \quad g^{u_4}_v = -\frac{4}{3} s^2_W, \quad (2.32a) \]

\[ g^u_1 = g^u_2 = \frac{1}{2}, \quad g^{u_3}_u = g^{u_4}_u = 0. \quad (2.32b) \]

In this model, for leptons we have

\[ L_{\nu_a} = 1, \quad L_{l_a} = -1 + 2 s^2_W, \quad L_{N_1} = L_{N_2} = -1, \quad (2.33a) \]

\[ R_{\nu_a} = 0, \quad R_e = R_\mu = 2 s^2_W, \quad R_\tau = R_T = -1 + 2 s^2_W, \quad R_{N_1} = R_{N_2} = 0. \quad (2.33b) \]

where \(\nu_a = \nu_e, \nu_\mu, \nu_\tau, \nu_T\) and \(l_a = e, \mu, \tau, T\). We see that neutrinos, electron and muon have the same couplings than in the Standard Model, \(N_i\) have right-handed couplings, and the lepton \(\tau\) and \(T\) have both only vector couplings:

\[ g^\nu_\nu = \frac{1}{2}, \quad g^e_e = g^\mu_\mu = \frac{1}{2} + 2 s^2_W, \quad g^\tau_\tau = g^T_T = -1 + 2 s^2_W, \quad g^{N_1}_V = g^{N_2}_V = -\frac{1}{2}; \quad (2.33c) \]

\[ g^{\nu_a}_A = \frac{1}{2}, \quad g^e_A = g^\mu_A = 0, \quad g^\tau_A = g^T_A = 0, \quad g^{N_1}_A = g^{N_2}_A = -\frac{1}{2}. \quad (2.33d) \]

Next, we will introduce the following discrete symmetries

\[ d_{1,2,3R} \to d_{1,2,3R}, \quad d'_{1,2R} \to -d'_{1,2R}, \quad u_{1,2,3R} \to u_{1,2,3R}, \quad u_4R \to -u_4R; \quad (2.34a) \]

\[ Q_{1,2L}, Q_3 \to Q_{1,2L}, Q_3; \quad \eta, \sigma \phi \to \eta, \sigma, \phi; \quad \sigma' \to -\sigma'. \quad (2.34b) \]
\[
\Psi_{eL}, \Psi_{\mu L} \to \Psi_{eL}, \Psi_{\mu L}, \quad \Psi_{\tau L}, \Psi_{TL}, \Psi'_{eL}, \Psi'_{\mu L} \to -\Psi_{\tau}, -\Psi_{T}, -\Psi'_{eL}, -\Psi'_{\mu L}, \quad (2.34c)
\]

\[
N_{1R}, N_{2R} \to -N_{1R}, -N_{2R}. \quad (2.34d)
\]

As we said before, the Higgs multiplets are also the same that in the previous model given in Eq. (2.3) but we can also introduce the singlet \( \phi \). Hence, the most general Yukawa couplings compatible with the symmetries in Eq. (2.34) are

\[
- \mathcal{L}_{qY} = \sum_{ij} \bar{Q}_{ij}^L [A_{ij} u_{jR} \eta^* + B_{ij}^l Q_{ij}^L d_{jR} \sigma^*] + \sum_{i\alpha} B_{i\alpha}^l \bar{Q}_{i\alpha}^L d_{i\alpha R} \sigma^* + \lambda \bar{Q}_{3L} Q_{3R} \phi \\
+ \sum_j \bar{Q}_{3L} [E_{3j} u_{jR} \sigma + D_{3j} d_{jR} \eta] + E' \bar{Q}_{3L} u_{4R} \sigma' + H.c. \quad (2.35)
\]

Where \( i, j = 1, 2, 3 \), \( \alpha = 1, 2 \) and \( \eta^*, \sigma^* \) are the respective antitriplets. (The \( \lambda \) matrices used in this work are defined in the Appendix of Ref. [6].)

From Eq. (2.35) we obtain mass matrices like in Eqs. (2.16) and (2.17) but now a 4 \( \times \) 4 matrix for the charge 2/3 sector and a 5 \( \times \) 5 one for the charge \(-1/3\) sector. Instead of the unitary matrices in Eqs. (2.20b) and (2.20a) we get

\[
V_L^D = \begin{pmatrix}
V_{1L}^D & 0 \\
0 & V_{2L}^D
\end{pmatrix}, \quad V_R^D = \begin{pmatrix}
V_{1R}^D & 0 \\
0 & V_{2R}^D
\end{pmatrix} \quad (2.36a)
\]

\[
V_L^U = \begin{pmatrix}
V_{1L}^U & 0 \\
0 & 1
\end{pmatrix}, \quad V_R^U = \begin{pmatrix}
V_{1R}^U & 0 \\
0 & 1
\end{pmatrix} \quad (2.36b)
\]

where \( V_{1L,R}^D, V_{1L,R}^U \) are unitary 3 \( \times \) 3 matrices and \( V_{2L,R}^D \) are 2 \( \times \) 2 ones.

Hence, we get a 2 \( \times \) 2 mass matrix for \( d''_\alpha \) quarks depending only on \( \langle \sigma^0_2 \rangle \); 3 \( \times \) 3 mass matrices for \( d'_i \)’s and \( u'_i \)’s. Both matrices have contributions from the three Higgs fields \( \eta, \sigma \) and \( \phi \). The fourth charge 2/3 quark, \( u_4 \), get a mass \( m_{e'} = \lambda \langle \phi \rangle + E' \langle \sigma^0_2 \rangle \).

From the mass matrices coming from Eq. (2.35) we obtain mixing among each of the three sectors \( u_i, d_i \) and \( d''_i \), but \( u_4 \) does not mix at all. Thus, \( u_i \) have the respective mass eigenstates \( u, c, t \). In the charge \(-1/3\) sector the mixing occurs among \( d_i \) with the mass eigenstates
denoted as usual $d, s, b$ and among the $d_i'$ sector with the respective mass eigenstates denoted by $s', b'$.

Finally, let us consider the lepton-scalar couplings. The discrete symmetry in Eqs. (2.34c) and (2.34d) allows the following Yukawa interactions

$$- \mathcal{L}_{Y} = \sum_{ab} \epsilon \Gamma_{ab} (\Psi_{aL})^c \Psi_{bL}^c \sigma + \sum_{db} \epsilon' \Gamma'_{db} (\Psi_{dL})^c \Psi_{bL}^c \sigma' + \sum_{bi} H_b \Psi_{bL}^c N_{iR}^0 \eta + H.c.$$  (2.37)

where $a, b = e, \mu; d = \tau, T; i = 1, 2; \Psi^c$ is the charge conjugated field and $\epsilon$ is the completely antisymmetric $SU(3)$ tensor. Recall that the scalar fields must obey the discrete symmetry in Eq. (2.34d).

Charged and non-hermitian neutral currents are

$$\mathcal{L}_q = -\frac{g}{\sqrt{2}} \left[ \sum_i \left( \bar{u}_{iL} \gamma^\mu d_{iL} W_\mu^+ + \bar{u}_{iL} \gamma^\mu d_{iL}^\prime V_\mu^+ + \bar{d}_{iL} \gamma^\mu d_{iL}^\prime X_\mu^0 \right) + \bar{u}_{4L} \gamma^\mu u_{3L} X_\mu^0 - \bar{u}_{4L} \gamma^\mu d_{3L} V_\mu^+ 
+ \bar{u}_{3L} \gamma^\mu d_{3L} W_\mu^+ + \bar{u}_{4R} \gamma^\mu u_{3R} X_\mu^0 - \bar{u}_{4R} \gamma^\mu d_{3R} V_\mu^+ + \bar{u}_{3R} \gamma^\mu d_{3R} W_\mu^+ \right] + H.c.$$  (2.38)

for quarks, and

$$\mathcal{L}_l = \frac{g}{\sqrt{2}} \left[ \sum_a \left( \bar{e}_{aL} \gamma^\mu \nu_{aL} X_\mu^0 - \bar{e}_{aL} \gamma^\mu \nu_{aL}^\prime V_\mu^+ + \bar{\nu}_{aL} \gamma^\mu \nu_{aL}^\prime W_\mu^+ \right) + \bar{e}_{L}^+ \gamma^\mu \tau_{L}^0 X_\mu^0 - \bar{e}_{L}^+ \gamma^\mu N_{1L}^0 V_\mu^+ 
+ \bar{\tau}_{L}^+ \gamma^\mu N_{1L}^0 W_\mu^+ + \bar{\mu}_{L}^+ \gamma^\mu T_{L}^0 X_\mu^0 - \bar{\mu}_{L}^+ \gamma^\mu N_{2L}^0 V_\mu^+ + \bar{T}_{L}^+ \gamma^\mu N_{2L}^0 W_\mu^+ \right] + H.c.$$  (2.39)

for leptons. As long as neutrinos remain massless there is not mixing in the charged currents coupled to $W^+$. However, there is mixing in the currents coupled to $V^+$ and $X^0$. Notice that the right-handed currents coupled to $W^+$ involve the charged leptons $\tau$ and $T$ and the heavy neutral fermions $N_{1,2}$.

After neutrinos getting mass through radiative corrections, mixing will appear in the interactions with $W^+$. Recall that the scalar $\eta_2^+$ is an $SU(2)$ singlet and there are three $SU(2)$ doublets in the scalar multiplets given in Eq. (2.3), hence the Zee mechanism for generating neutrino masses may be implemented in this models [13].

The charged currents coupled to the $W^+$ boson can be written, in the quark sector, as

$$\mathcal{L}_{CC}^{qW} = -\frac{g}{\sqrt{2}} (\bar{u}_1 \bar{u}_2 \bar{u}_3) L \gamma^\mu \begin{pmatrix} d_1 \\ d_2 \\ d_3 \end{pmatrix} W_\mu^+ - \frac{g}{\sqrt{2}} \bar{u}_{3R} \gamma^\mu d_{3R} W_\mu^+ + H.c.$$  (2.40)
The left-handed currents in Eq. (2.40) can be parametrized in terms of mass eigenstates and Kobayashi-Maskawa mixing matrix, $V_{KM} = V_L^U V_D^L$. The right-handed current in Eq. (2.40) can be written in terms of the mass eigenstates as follows

$$L_{qWR}^{CC} = -\frac{g}{\sqrt{2}} (\bar{u} \bar{c} \bar{t} R) R V_U^U \gamma^\mu \Delta V_R^D \begin{pmatrix} d \\ s \\ b \end{pmatrix} R W_R^+ + H.c. \quad (2.41)$$

where $\Delta = \text{diag}(0, 0, 1)$. The other matrices $V_U^U, V_D^L$ also survive in the interactions involving the bosons $V^+$ and $X^0$.

Notice that, besides the neutral currents coupled to the $Z^0$ given in Eq.(2.9) we have additional right-handed couplings $[\bar{u}_3 R \gamma^\mu u_3 R - \bar{d}_3 R \gamma^\mu d_3 R] Z_\mu^0$, or, written in terms of the mass eigenstates

$$L^{nNC} = L^{NC} + L^{NC}', \quad (2.42)$$

with $L^{NC}$ being parametrized like in Eq. (2.3) and the coefficients $L, R$s or $V, A$s being defined as in the Standard Model, and with

$$L_{U}^{NC} = -\frac{g}{2\cos\theta_W} (\bar{u} \bar{c} \bar{t} R) R \gamma^\mu U_R^U U_R^U \Delta U_R^U \begin{pmatrix} u \\ c \\ t \end{pmatrix} R Z_\mu^0, \quad (2.43)$$

for the charge 2/3 sector, and

$$L_{D}^{NC} = +\frac{g}{2\cos\theta_W} (\bar{d} \bar{s} \bar{b} R) R \gamma^\mu U_R^D U_R^D \Delta U_R^D \begin{pmatrix} d \\ s \\ b \end{pmatrix} R Z_\mu^0, \quad (2.44)$$

for the charge $-1/3$ one. There are similar currents coupled to the $V^+, X^0, Z^0$ bosons. Although Eqs. (2.43) and (2.44) are FCNC, all flavors have the same dependence on the weak mixing angle. This is not the case of the neutral currents coupled to $Z^0$, here each flavor has a different dependence on that angle. In the leptonic sector we have the GIM
mechanism at tree level in the neutral currents coupled to the $Z^0$ and $Z'$ (as long as the symmetries in Eqs. (2.34c) and (2.34d) are preserved) but there are FCNC in the couplings to the $X^0$. Mixing between $e \leftrightarrow \mu$ and $\tau \leftrightarrow T$ appear in the current coupled to $X^0$ and those coupled to $V^+$ induce transitions $l_a \leftrightarrow N_i$ which are sensible on the Cabibbo-like mixing in the charged leptons.

Unlike the model of Ref. [7], in the present model all Higgs bosons couple to quarks even the scalar with a large VEV. Another difference between our model B and that of Ref. [8] is that in the latter one, there is a mixing between $d,s$ and between $b,b'$. In our case the mixing is as usual among $d,s,b$, but $b'$ has no mixing with other charge $-1/3$ quarks, at least as long as the discrete symmetries in Eq. (2.34) are preserved. In fact, the mass matrices in model B are different from those of the model of Ref. [8]. We can see these discrete symmetries only as an indication of which ones are the dominant mixings. Eventually, we could allow them to be broken.

Notice also that in both models, A and B, the extra quarks are very heavy since they get mass through the larger VEV $\langle \sigma^0 \rangle$.

We can also build a model in which there are two quark generations transforming as $(3^*,1/3)_L$ and one as $(3,0)_{L+R}$. In this case there are two leptonic antitriplets $(3,1/3)$ and four ones transforming as $(3,-2/3)$. In this case it is necessary, however, to include right-handed charged leptons in singlets.

III. MODELS WITH EXTENDED COLOR AND ELECTROWEAK SECTORS

A. Models with extended electroweak sector

First, let us consider $n = 3$ models. When $m = 2,3$ we have the Standard Model and the 3-3-1 models respectively. Next, there is a 3-4-1 model in which the electric charge operator is defined as

$$Q = \frac{1}{2} \left( \lambda_3 - \frac{1}{\sqrt{3}} \lambda_8 - \frac{2}{3} \sqrt{6} \lambda_{15} \right) + N,$$  (3.1)
where the $\lambda$-matrices are \[14\],

\[\lambda_3 = \text{diag}(1, -1, 0, 0), \quad \lambda_8 = \left(\frac{1}{\sqrt{3}}\right) \text{diag}(1, 1, -2, 0), \quad \lambda_{15} = \left(\frac{1}{\sqrt{6}}\right) \text{diag}(1, 1, 1, -3).\]

Leptons transform as $(1, 4, 0)$, two of the three quark families, say $Q_{iL}$, $i = 1, 2$, transform as $(3, 4^*, -1/3)$, and one family, $Q_{3L}$, transforms as $(3, 4, +2/3)$

\[
\psi_{aL} = \begin{pmatrix} \nu_a \\ l_a \\ \nu_a^c \\ l_a^c \end{pmatrix}_L, \quad Q_{iL} = \begin{pmatrix} j_i \\ d_i' \\ u_i \end{pmatrix}_L, \quad Q_{3L} = \begin{pmatrix} u_3 \\ d_3 \\ u_3' \\ J \end{pmatrix}_L, \quad (3.2)
\]

where $u_3'$ and $J$ are new quarks with charge $+2/3$ and $+5/3$ respectively; $j_i$ and $d_i'$, $i = 1, 2$ are new quarks with charge $-4/3$ and $-1/3$ respectively. We remind that in Eq. (3.2) all fields are still symmetry eigenstates. Right-handed quarks transform as singlets under $SU(4)$.

A model with $SU(4)_L$ symmetry and leptons transforming as in Eq. (3.2) was proposed by Voloshin some years ago \[15\]. In this context it can be possible to understand the existence of neutrinos with large magnetic moment and small mass. However in Ref. \[15\] it was not considered the quark sector.

Quark masses are generated by introducing the following Higgs $SU(4)_L \otimes U(1)_N$ multiplets: $\chi \sim (4, -1), \rho \sim (4, +1), \eta$ and $\eta' \sim (4, 0)$.

In order to obtain massive charged leptons it is necessary to introduce a $(10^*, 0)$ Higgs multiplet, because the lepton mass term transforms as $\bar{\psi}_L' \psi_L \sim (6_A \oplus 10_S)$. The $6_A$ will leave some leptons massless and some others mass degenerate. Therefore we will choose $H = 10_S$. Neutrinos remain massless at least at tree level but the charged leptons gain mass. The corresponding VEVs are $\langle \eta \rangle = (v, 0, 0, 0)$, $\langle \rho \rangle = (0, w, 0, 0)$, $\langle \eta' \rangle = (0, 0, v', 0)$, $\langle \chi \rangle = (0, 0, 0, u)$, and $\langle H \rangle_{12} = v''$ for the decuplet. In this way we have that the symmetry breaking of the $SU(4)_L \otimes U(1)_N$ group down to $SU(3)_L \otimes U(1)_{N'}$ is induced by the $\chi$ Higgs. The $SU(3)_L \otimes U(1)_{N'}$ symmetry is broken down into $U(1)_{em}$ by the $\rho, \eta, \eta'$ and $H$ Higgs. As in the models of Sec.$\|$, it is necessary to introduce some discrete symmetries which
ensure that the Higgs fields give a quark mass matrix in the charge $-1/3$ and $2/3$ sectors of the direct sum form in order to avoid general mixing among quarks of the same charge. In this case the quark mass matrices can be diagonalized with unitary matrices which are themselves direct sum of unitary matrices.

In fact, we have the symmetry breaking pattern, including the $SU(3)$ of color,

$$
SU(3)_c \otimes SU(4)_L \otimes U(1)_N \nonumber \\
\downarrow \langle \chi \rangle \\
SU(3)_c \otimes SU(3)_L \otimes U(1)_{N'} \nonumber \\
\downarrow \langle \eta' \rangle \\
SU(3)_c \otimes SU(2)_L \otimes U(1)_{N''} \nonumber \\
\downarrow \langle x \rangle \\
SU(3)_c \otimes U(1)_{em}
$$

where $\langle x \rangle$ means $\langle \rho \rangle, \langle \eta \rangle, \langle H \rangle$.[11]

The electroweak gauge bosons of this theory consist of a $15$ $W_i^\mu, i = 1, ..., 15$ associated with $SU(4)_L$ and a singlet $B_\mu$ associated with $U(1)_N$.

There are four neutral bosons: a massless $\gamma$ and three massive ones: $Z, Z', Z''$. The lightest one, say the $Z$, corresponds to the Weinberg-Salam-Glashow neutral boson. Assuming the approximation $u \gg v' \gg v, v'', w$ the extra neutral bosons, say $Z', Z''$, have masses which depend mainly on $u, v'$.

Concerning the charged vector bosons, as in the model of Ref. [4] there are doubly charged vector bosons and there are doublets of $SU(2) (X^+_\mu, X^0)$ and $(\bar{X}^0_\mu, X^-_\mu)$ which produce interactions like $\bar{\nu}_{aL} \gamma^\mu l_{aL} X^+_\mu$ and $\bar{\nu}_{aL} \gamma^\mu \nu_{aL} X^0_\mu$, as in model I of Ref. [6]. We have also the $V_{1,2}^\pm$ vector bosons with interactions like $\bar{\ell}_{aL} \gamma^\mu \nu_{aL} V_{1}^+\mu$ and $\bar{\ell}_{aL} \gamma^\mu \nu_{aL} V_{2}^+\mu$. All charged currents, including that ones coupled with quarks, are given in Ref. [11].
B. $n = 4, 5$ models

Let us consider now $n = 4, 5$ models. Although the $SU(3)_c$ gauge symmetry is the best candidate for the theory of the strong interactions, there is no fundamental reason why the colored gauge group must be $SU(3)_c$. In fact, it is possible to consider other Lie groups. In general we have the possibilities $SU(n)$, $n \geq 3$ [10].

In particular, models in which quarks transform under the fundamental representations of $SU(4)_c$ and $SU(5)_c$ were considered in Refs. [8] and [9], respectively, in the context of the SM. These models preserve the experimental consistency of the SM at low energies. For instance, in the $SU(5)_c \otimes SU(m)_L \otimes U(1)_N$ model a Higgs field transforming as the 10 representation of $SU(5)_c$ breaks the symmetry as follows [11]

$$SU(5)_c \otimes SU(m)_L \otimes U(1)_N \downarrow \langle 10 \rangle$$

$$SU(3)_c \otimes SU(2)' \otimes SU(m)_L \otimes U(1)_N.$$  \hspace{1cm} (3.4)

Later the electroweak symmetry will be broken and the remaining symmetry will be $SU(3)_c \otimes SU(2)' \otimes U(1)_{em}$ as in the models with $m = 3, 4$ considered above. Notice that, due to the relation between the color degrees of freedom and the number of families, it is necessary to introduce four and five leptonic families for $n = 4$ and $n = 5$ respectively if we assume that the number of quark families is still three. In general we have $N_l = |n(n_1 - n_2)|$ where $n_1$ and $n_2$ are the number of quark multiplets transforming as $m$ and $m^*$ respectively and $N_q = n_1 + n_2$. If $n_1 > n_2$ leptons must transform as $m^*$ and if $n_1 < n_2$ leptons are assigned to $m$. It is still possible to have $N_q = N_l$. Assuming this condition (and $n_1 > n_2$), for the case of even $n$ i.e., $n = 2p$, $p \geq 2$ we have $n_1/n_2 = (2p + 1)/(2p - 1)$; and for odd $n$ i.e., $n = 2p + 1$, $p \geq 1$ we get $n_1/n_2 = (p + 1)/p$. For $n = 4$ the condition $N_q = N_l$ is satisfied if $n_1/n_2 = 5/3$. Analogously, for $n = 5$ we have $n_1/n_2 = 3/2$. It means that, if we let the number of quark families to be equal to the number of the lepton families, the minimal number of families is eight for $n = 4$ and five for $n = 5$.

On the other hand, if we maintain $N_q = 3$ it is necessary, as we said before, to introduce
new lepton families. Let us denote these additional families by \((N_i, E_i, E^c_i)\) with \(i = 1\) for \(n = 4\); or \(i = 1, 2\) when \(n = 5\). The new charged leptons must be heavy enough in order to keep consistency with phenomenology. Since the right-handed neutrinos, transforming as singlets under the gauge group, do not contribute to the anomaly, their number is not constrained by the requirement of obtaining an anomaly free theory. Hence, we can introduce an arbitrary number of such fields. When these singlets are added, the \(Z^0\) invisible width is always smaller than the prediction of the minimal SM. In fact it has been shown that in this case \([17]\)

\[
\Gamma(Z \to \text{neutrinos}) \leq N_l \Gamma^0,
\]

where \(N_l\) is the number of left-handed lepton families and \(\Gamma^0\) is the standard width for one massless neutrino. Hence, it will be always possible to choose the neutrinos’s mixing angles and masses in such a way that the theoretical value in (3.5) be consistent with the experimental one \([18,19]\).

IV. OTHER POSSIBLE EXTENSIONS

Other possibilities are models with left-right symmetry in the electroweak sector \(SU(n)_c \otimes SU(m)_L \otimes SU(m)_R \otimes U(1)_N\) and also models with horizontal symmetries \(G_H\) i.e., \(SU(n)_c \otimes SU(m)_L \otimes U(1)_N \otimes G_H\).

A. Left-right symmetries

In models with left-right symmetry the \(V - A\) structure of weak interactions is related to the mass difference between the left- and right-handed gauge bosons, \(W^\pm_L\) and \(W^\pm_R\), respectively, as a result of the spontaneous symmetry breaking \([20]\).

This sort of models are easily implemented in the 3-3-1 context by adding a new charged lepton \(E\). For instance, in models with left-handed leptons transforming as \((\nu_a, l_a^-, E^+_a)^T_L\) the right-handed triplet is \((\nu_a, l_a^-, E^+_a)^T_R\). In the quark sector, the left-handed components are as
in Ref. [4] and similarly the right-handed components, in such a way that anomalies cancel in each chiral sector. Explicitly, the charge operator is defined as

\[ Q = I_{3L} + I_{3R} + \frac{Y}{2} \]  

(4.1)

where \( I_{3L(3R)} \) and \( Y/2 \) are of the form \( (1/2)\lambda_3 \) and \( -(\sqrt{3}/2)\lambda_8 + N1 \), respectively, for the model of Refs. [4]. The Higgs multiplet \((3, 3^*, 0)\) and its conjugate give mass to all fermions but in order to complete the symmetry breaking it is necessary to add more Higgs multiplets.

**B. Horizontal Symmetries**

Particle mixture occurs in the Standard Model among particles which are equivalent concerning their position in the gauge multiplets. It was noted some years ago that it is possible to determine the weak mixing angles in terms of the quarks masses, provided we assume that all equivalent multiplets of the vertical gauge symmetry transform in the same way under horizontal symmetries. Therefore, the three families are put into a single representation of the horizontal group [21].

That is, in the context of the SM the gauge symmetry in the horizontal direction was considered as a transformation among the left-handed doublets and among right-handed singlets. At first sight, horizontal symmetries are less interesting in the context of 3-3-1 models since quark generations transform in a different way under SU(3)_L \( \otimes \) U(1)_N.

Apparently, the only possibility is the horizontal \( G_H = SU(2)_H \) symmetry. In this case there are no additional conditions for canceling gauge anomalies since \( SU(2) \) is a safe group. For instance, with \( n = 3, m = 3, 4 \), the three quark generations transform, in both left- and right- handed sectors, in the following way: two of them as a doublet and the third one as a singlet under \( SU(2)_H \) [22,23]. The same is valid for leptons but in this case the three lepton triplets can transform as the adjoint representation as well.

The horizontal gauge bosons and the extra Higgs bosons have to be heavier than the \( W \) bosons or very weakly coupled to the usual fermions in order to suppress appropriately flavor changing neutral transitions in both, quark and lepton sectors.
C. Embedding in $SU(6)$

There are also the grand unified extensions of all the possibilities we have treated above. The group $SU(3)_c \otimes SU(3)_L \otimes U(1)_N$ has rank 5 and it is a subgroup of $SU(6)$. In the last group, it has been shown that the anomalies, $A$, of $15$ and $6^*$ are such that $A(15) = -2A(6^*)$ \cite{24}. Then, pairs of $15$ and $15^*$; $6^*$ and $6$ \cite{23} and, finally one $15$ and two $6^*$ are the smallest anomaly free irreducible representations in $SU(6)$. On the other hand, the representation $20$ is safe.

Just as an example, let us consider the $SU(6)$ symmetry which is a possible unified theory for model B. Using the notation of Ref. \cite{26}, in the entry $(a, b)_f(N)$, $a$ is an irreducible representation of $SU(3)_c$ and $b$ is an irreducible representation of $SU(3)_L$. The subindex $f$ means, in an obvious notation, the respective fields of the model and the second parenthesis contains the value of the $U(1)_N$ generator when acting on the states in the $(a, b)$.

In model B there are 66 degrees of freedom. Left-handed leptons and three of the right-handed $d$-type quarks are in $6^*$:

$$6^*_j = (3^*, 1)d_{jL}^c(\pm 1/3) + (1, 3^*)_{jL}(-1/3), \quad (4.2)$$

where $d_{jL}^c = d_{1L}^c, d_{2L}^c, d_{c1L}^c, d_{2cL}^c; \psi_{jL} = \psi_{eL}, \psi_{\mu L}, \psi_{\tau L}, \psi_{TL}$ (see Eq. (2.27a)). Two quark generations transforming as $(3, 3, 0)$ and the right-handed $u$-type quarks are in two $15$

$$15_{Q_{iL}} = (3^*, 1)u_{iL}^c(-2/3) + (1, 3^*)\psi_{iL}^c(2/3) + (3, 3)_{Q_{iL}}(0), \quad (4.3)$$

where $u_{iL}^c = u_{1L}^c, u_{2L}^c; \psi_{iL}^c = \psi_{eL}^c, \psi_{\mu L}^c$ (see Eq. (2.27b)). The left-handed and right-handed quarks of the third generation are in one $20$

$$20_{Q_{iL}} = (1, 1)_{N_{1L}^c}(0) + (1, 1)_{N_{2L}^c}(0) + (3, 3^*)_{Q_{iL}}(+1/3) + (3^*, 3)_{Q_{iL}^c}(-1/3), \quad (4.4)$$

with $N_{iL}, i = 1, 2$ the neutral leptons which are singlets of the 3-3-1 symmetry. Thus, we have an anomaly free theory with the fields of the first two generations in $6^*$ and $15$. The third generation is not anomalous.
Let us consider the prediction of the weak mixing angle, \( \sin \theta_W \). In \( SU(N) \) theories we have

\[
\sin^2 \theta_W = \frac{\sum_a (t_{3a})^2}{\sum_a (Q_a)^2},
\]

(4.5)

where \( t_3 \) is the third component of the weak isospin, \( Q \) is the electric charge and the sum extends over all fields in a given representation. Hence, in \( SU(6) \) we have the prediction that \( \sin^2 \theta_W = 3/8 \). This is the same value of the \( SU(5) \) model [27]. It is easy to verify that all representations \( 6^*, 15 \) and \( 20 \) in Eqs. (4.2)-(4.4) give the same answer as it must be. On the other hand, we recall that in models A and B it holds that \( \sin^2 \theta_W < 3/4 \) [3]. Thus, the theory has a Landau pole when \( \sin^2 \theta_W = 3/4 \). The theory might be, before getting this pole, unified in an \( SU(6) \) model.

However, it is not a trivial issue to show that the unification in \( SU(6) \) may actually occur [28]. This is so, because in 3-3-1 the couplings \( \alpha_c \) and \( \alpha_{3L} \) have \( \beta_c > \beta_{3L} \).

Since the model have new particles, we may have to consider mass threshold corrections for the \( \beta \)-functions, since the new particles could have masses below the unification energy scale, or even, we may not assume the decoupling theorem. We recall that in the Standard Model with two Higgs doublets the decoupling theorem [29] must not be necessarily valid, since there are physical effects proportional to \( m^2_{\text{Higgs}} \) [30]. Hence, it could be interesting to study the way in which the masses of the extra Higgs and exotic quarks in the model become large, as it has been done in the standard model scenario for an extended Higgs sector [30] or for the mass difference between fermions of a multiplet [31]. It means that there is no “grand desert” if 3-3-1 models are realized in nature.

How can we study the embedding of the SM in 3-3-1? The last model has fields which do not exist in the minimal SM, but which are present in the same multiplet of 3-3-1 with the known quarks. For instance, the quarks \( J \)'s have to be added to the SM transforming as \( (3, 1, Q_J) \) under the 3-2-1 factors. The scalar and vector boson sectors of the SM have also to be extended with new fields. Hence, we must add scalar fields transforming as (i) four singlets \( (1, 1, Y_S) \): one with \( Y_S = 0 \), one with \( Y_S = 1 \) and two with \( Y_S = 2 \), (ii) four doublets
(1, 2, Y_D): one with \( Y_D = -3 \) and three with \( Y_D = 1 \); finally, (iii) one triplet \((1, 3, -2)\). It is also necessary to add extra vector bosons \((U^{++}, V^+)\) which transform as \((1, 2, 3)\). For this reason we believe that 3-3-1 models are not just an embedding of the SM but an alternative to describe the same interactions.

V. CONCLUSIONS

The 3-3-1 symmetry is in fact an interesting extension of the standard model. It gives answers to some questions put forward by the later model and new physics could arise at not too high energies, say in the TeV range.

In the previous sections we have examined two 3-3-1 models, both of them with extra heavy quarks and leptons, and also possible extensions.

What we want to do now is to discuss briefly some possible phenomenological consequences concerning models A and B discussed in Secs. II A and II B, respectively.

i) In the Higgs sector, by using the gauge invariance it is not possible to choose all VEVs to be real. Hence, there is CP violation via scalars exchange. Since the quark mass matrices receive contributions from two VEVs, there are also FCNC in the Higgs bosons couplings but their effects could be suppressed by fine tuning among some parameters \([6]\) or by heavy scalars.

ii) In model A, the left-handed quark mixing matrices \(V_L^U\) and \(V_L^D\), defined in Eqs. (2.19) or (2.20), survive in the Lagrangian. See for instance, Eqs. (2.23) and (2.24). Mixings are also different in the interactions with \(X_0^\mu\) and with \(V_\mu^-\), as can be seen from Eq. (2.21). This induces new sources of CP violation since there are phases in these interactions which cannot be absorbed. This also happens in model B. However, in this case even the right-handed quark mixing matrices, \(V_R^U\) and \(V_R^D\), survive. We recall that in the Standard Model although the matrices \(V_{L,R}^{U,D}\) are needed, after the diagonalization of the quark mass matrices the only place in the Lagrangian where these matrices appear is in charged currents coupled to the \(W^+\) and only in the form \(V_L^{U,D}V_L^{D,U}\). In this case, \(V_L^D\) is identified with the usual Cabibbo-
Kobayashi-Maskawa mixing matrix by choosing $V^U_L = 1$. Since this matrix does not appear in other places of the Lagrangian, this choice is enough. This is not the case for all 3-3-1 models [11].

**iii)** It is very well known that almost all $Z^0$-pole observables are in agreement with the Standard Model predictions [18]. There are, however, two of these observables which seem not to agree with the model’s predictions:

a) the first one concerns the heavy quark production rates $R_f = \Gamma(Z^0 \to f \bar{f})/\Gamma(Z \to \text{hadrons})$, which have been measured for $c$ and $b$ quarks. Considering $R_c$ as the SM prediction ($R_c \approx 0.171$), one has $R_b = 0.2192 \pm 0.0018$ which is about $2\sigma$ discrepancy with respect to the expected value, $R_b = 0.2156 \pm 0.0006$.

b) The second one, is the value of the left-right asymmetry $A^{0e}_{LR}$ obtained by LEP measurements of the forward-backward asymmetry. It corresponds to $\sin^2 \theta_W = 0.2321 \pm 0.0005$ [32] while SLD left-right asymmetry measurement implies $\sin^2 \theta_W = 0.2292 \pm 0.0010$ [33]. This results are in conflict with one another at about two standard deviations. If confirmed, this could indicate new physics coupled in a different way to the third generation. For instance: 1) extended gauge structures with extra neutral bosons, like the $Z'$; 2) extra fermions like $t', b'$ or even of heavy leptons as $E^-; 3)$ non-standard Higgs particles, and 4) new heavy particles loop effects like exotic leptons, quarks or supersymmetric particles.

It is possible that this will be an indication of new physics generating a vertex correction to the $Z$ coupling or by new box contributions, however the most exciting possibility is a new physics at tree level.

All 3-3-1 models have some of these requirements and no doubt they deserve to be study. In particular model B has the flavor changing right-handed current in Eq. (2.44). In fact, it has been pointed out recently that the discrepancy between both measurements can be reconciled if a new neutral gauge boson $Z^0$ nearly degenerate with the $Z^0$ do exist [34]. This new neutral gauge boson may also be responsible for the observed value of $R_b$.

Notice that according to Eqs. (2.33) and (2.44), there are right-handed $u \leftrightarrow c \leftrightarrow t$ and $d \leftrightarrow s \leftrightarrow t$ transitions mediated by the $Z^0$ at tree level. In the charge $-1/3$ sector, the
$K_L - K_S$ mass difference constrains only the matrix elements $(V_R^D)_{3d}(V_R^D)_{3s}$. In order to determine the other elements of the matrix $V_R^D$ it is necessary to study in detail $B$ decays. A similar situation occurs in the charge 2/3 sector.

On the other hand, all 3-3-1 models have both $Z'$ and extra fermions. In particular there are heavy leptons in the models that we have considered above \[33\]. In these models it could be necessary to take into account all new fields present in the model. Constraints coming from the neutral $K$ mass difference would not necessarily imply a heavy $Z'$ since we can obtain consistency with the observed value of this mass difference by choosing appropriately some of the matrix elements of $V_L^D$. The later ones are different from the mixing angles appearing in the observables $R_b$ and $A_{LR}$. The couplings defined in Eq. (2.9) for the case of the $Z'$ are all flavor violating \[3] and as we have extra mixing matrices in these models it is possible that a global analysis of all data will show compatibility among the low energy processes like $m_{K_L} - m_{K_S}$ mass difference and the $Z$-peak observables. Exotic fermions can also give contributions to the $Z \rightarrow b\bar{b}$ through loop effects.

iv) Some time ago it was pointed out that since the left-handiness of the $b$ quark has not been tested experimentally this quark may decay through, in the extreme case, purely right-handed couplings to the $c$ and $u$ quarks \[36,37\]. A test of the chirality of the $b$ quark is the decay of polarized $\Lambda_b$ baryons. These ideas were worked out in the context of an $SU(2)_L \otimes SU(2)_R \otimes U(1)$ model. In such a model the smallness of the $b$ to $c$ coupling is due not to the value of the corresponding mixing angle but to the small value of the right-handed Fermi constant $G_{FR}$, and the right-handed $W_R$ boson must be light since \[38\]

$$
\frac{G_{FR}}{G_{FL}} \approx \frac{1}{\sqrt{2}} \left( \frac{g_{R}^2}{M_{W_R}^2} \right) \left( \frac{g_{L}^2}{M_{W_L}^2} \right) \approx V_{bc} \approx 0.04. \tag{5.1}
$$

In model B (Sec. II B), an intermediate situation is realized. In Eq. (2.40) the charged left-handed currents are the usual ones. However, there are also right-handed currents coupled to the $W^+$ boson with the same strength $G_F$ but it depends on some of the right-handed couplings $V_R^U$ and $V_R^D$ appearing in Eq. (2.41). Hence, the constraint in Eq. (5.1) implies only that $V_{3cR}^U V_{3bR}^D < 0.04$. 

27
Notice that the left-handed coupling of the $b$ quark to the $c$ and $u$ quarks are the same of the Standard Model (See Eq. 2.40). However, there are contributions to the semileptonic $b$-decays in which a) a right-handed $b$-to-$c(u)$ current couples to a left-handed lepton current (see Eq. (2.39) and (2.41)); b) a left-handed $b$-to-$c(u)$ current couples to a right-handed lepton current and, c) both currents are right-handed. The cases b) and c) involve the heavy lepton sector: $\bar{T} c L \gamma^\mu N_{1L} = -\bar{N}_{1R} \gamma^\mu T R W_\mu^+$ or $\bar{T} c L \gamma^\mu N_{2L} = -\bar{N}_{2R} \gamma^\mu T R W_\mu^+$, and can be suppressed if $N_i$ and $T$ are heavy.

Analyses of the $B_d^0 - B_c^0$ and $B_s^0 - B_d^0$ mixings must be done in our context too. The dominant contributions in our model come from two-$t$-quarks box diagrams as in the Standard Model. This involve other matrix elements of $V_{DL}$. Hence, as we said before, in our models it would be necessary to make a global analysis involving $Z$-pole observables, CP violation, semileptonic $B$ decays and other processes in order to fit the several parameters appearing in it.

v) These models predict new processes in which the initial states have the same electric charge as $ff \to W^-V^-$. This type of processes have only recently begun to be studied [40,41]. Also in some extensions of these models, with spontaneous and/or explicit breaking of $L + B$ symmetry, it is possible to have kaon decays with $|\Delta L| = 2$, like $K^+ \to \pi^-\mu^+\mu^+, \pi^-\mu^+e^+$ Similarly in $D$ and $B$ mesons decays. Experimental data imply $B(K^+ \to \pi^-\mu^+\mu^+) < 1.5 \times 10^{-4}$ [40]. The process $e^-e^- \to W^-W^-$ which also could occur in some extensions of the 3-3-1 models has been recently investigated in other context [41].

In summary, none of these models is severely constrained at low energies. For instance, in the leptonic sector both of them are consistent with the existence of three light neutrinos [18]. Notice that in model B, the massless neutrinos (at tree level) $\nu_e, \nu_\mu$ do not mix with the heavy neutral fermions $N^0_i$ because of the symmetries in Eqs. (2.34c) and (2.34d). Mixing occurs only among $\nu_\tau, \nu_T$ and $N^0_{iL}$. Thus it is not necessary to assume that $H_4 (\eta^0) \gg \Gamma (\sigma^0_1)$ in Eq. (2.37). Neutrinos will get mass through radiative corrections and some of their properties as the magnetic moments will be studied elsewhere.

Another interesting feature of this kind of models is that they include some extensions
of the Higgs sector which have been considered in the context of the $SU(2) \otimes U(1)$ theory: more doublets, single and doubly charged singlets, triplets, etc.

The supersymmetric version of the model of Ref. [6] has been considered in Ref. [42].

ACKNOWLEDGMENTS

I would like to thank the Conselho Nacional de Desenvolvimento Científico e Tecnológico (CNPq) for partial financial support.
REFERENCES

[1] F. Abe et al. (CDF Collaboration), Phys. Rev. Lett. 73, 225 (1994) and Phys. Rev. D 50, 2956 (1994).

[2] J. L. Rosner, Status of the Standard Model, Preprint EFI-94-38/August 1994 (unpublished).

[3] A. J. Weinstein and R. Stroynowsky, Ann. Rev. Nucl. Part. Phys. 43, 457 (1994)

[4] F. Pisano and V. Pleitez, Phys. Rev. D46, 410(1992); P. H. Frampton, Phys. Rev. Lett. 69, 2889(1992); R. Foot, O.F. Hernandez, F. Pisano and V. Pleitez, Phys. Rev. D47, 4158(1993).

[5] D. Ng, Phys. Rev. D 49, 4805 (1994).

[6] J.C. Montero, F. Pisano and V. Pleitez, Phys. Rev. D. 47, 2918(1993) and references therein.

[7] H. Georgi and A. Pais, Phys. Rev. D 19, 2746 (1979).

[8] R. Foot, Phys. Rev. D 40, 3136 (1989).

[9] R. Foot and O. F. Hernandez, Phys. Rev. D 41, 2283 (1990); ibid D 42, 948 (1990).

[10] D. Dumm, F. Pisano and V. Pleitez, Mod. Phys. Lett. A9, 1609 (1994).

[11] V. Pleitez, $SU(n)_c \otimes SU(m)_L \otimes U(1)_N$ generalizations of the standard model, Preprint IFT-P.010/93, hep-ph/9302287 (unpublished). Details of the model are in F. Pisano and V. Pleitez, $SU(4) \otimes U(1)_N$ model for electroweak interactions, preprint IFT-P.003/94 (submitted for publications). This model was proposed also recently by R. Foot, H. N. Long and T. A. Tran, Phys. Rev. D 50, R34 (1994).

[12] S. Glashow, J. Iliopoulos and L. Maiani, Phys. Rev. D 2, 1285 (1970).

[13] A. Zee, Phys. Lett. B93, 389 (1980).
[14] D. Amati et al., Nuovo Cimento. 34, 1732(1964).

[15] M.B. Voloshin, Sov. J. Nucl. Phys. 48, 512(1988).

[16] S. Okubo, Phys. Rev. D 16, 3535(1977).

[17] C. Jarlskog, Nucl. Phys. A518, 129(1990); C. Jarlskog Phys. Lett. B241, 579(1990).

[18] Review of Particle Properties, L. Montanet et al., Phys. Rev. D 50, 1173 (1994).

[19] C.O. Escobar, O.L.G. Peres, V. Pleitez and R. Zukanovich Funchal, Phys. Rev. D47, R1747(1993).

[20] G. Senjanovic and R.N. Mohapatra, Phys. Rev. D 12, 1502(1975); G. Senjanovic, Nucl. Phys. B153, 334(1979).

[21] F. Wilczek and A. Zee, Phys. Rev. Lett. 42, 421 (1979).

[22] This possibility has been suggested in the context of the standard electroweak model by R. Foot, G.C. Joshi, H. Lew and R.R. Volkas, Phys. Lett. B 226, 318 (1989).

[23] The cancellation of global anomalies requires an even number of $SU(2)_H$ doublets. Thus it is necessary to add right-handed neutrinos. See D.S. Shaw and R.R. Volkas, Phys. Rev. D 47, 241 (1993).

[24] J. Banks and H. Georgi, Phys. Rev. D 14, 1159(1976).

[25] H. Georgi and S. L. Glashow, Phys. Rev. D 6, 429(1972).

[26] R. Slansky, Phys. Rep. C79, 1(1981).

[27] H. Georgi and S. L. Glashow, Phys. Rev. Lett. 32, 438 (1974); H. Georgi, H. R. Quinn and S. Weinberg, Phys. Rev. Lett. 33, 451 (1974).

[28] Unified theories based on $SU(15)$ with leptons $(l, l^c, \nu)$ and $SU(16)$ with leptons $(\nu, l, l^c, \nu^c)$ belonging to the same multiplet were considered by S.L. Adler, Phys. Lett. 225B, 143(1989) and J.C. Pati, A. Salam and J. Strathdee, Nucl. Phys. B185,
445(1981), respectively. Notwithstanding, in these models the $SU(2)$ subgroup rotating the charged fermion to their antiparticles includes both lepton and quark sectors.

[29] T.W. Appelquist and J. Carazzone, Phys. Rev. D11, 2856(1975).

[30] D. Toussaint, Phys. Rev. D 18, 1626(1978).

[31] M. Veltman, Nucl. Phys. B123, 89(1977).

[32] J. Erler, The SLD Asymmetry in View of thr LEP Results, preprint UPR-0619T, June 1994, and references therein,

[33] K. Abe et al., (SDL Collaboration), Phys. Rev. Lett. 73, 25 (1994).

[34] F. Caravaglios and G. G. Ross, Reconciling the LEP and SLAC measurements of $\sin^2\theta_W$, preprint CERN-TH.7474/94.

[35] See also V. Pleitez and M.D. Tonasse, Phys. Rev. D 48, 2353 (1993).

[36] M. Gronau and S. Wakaizumi, Phys. Rev. Lett. 68, 1814 (1992); Phys. Rev. D 47, 1262 (1993).

[37] J. F. Amudson, J. L. Rosner, M. Worah and M. B. Wise, Phys. Rev. D 47, 1260 (1993).

[38] R. N. Mohapatra and S. Nussinov, Phys. Lett. B339, 101 (1994).

[39] P.H Frampton, J.T. Liu and B. Charles Rasco, SSC phenomenology of the 3-3-1 model of flavor, hep-ph/9304294.

[40] L.S. Littenberg and R.E. Shrock, Phys. Rev. Lett. 68, 443(1992).

[41] C.A. Heusch and P. Minkowski, Nucl. Phys. B416, 3 (1994).

[42] T. V. Duaong and E. Ma, Phys. Lett. B316, 307 (1993).