Geomagnetic inverse modeling to estimate the volume of magnetization in areas with topographic variations

Eko J Wahyudi
Institut Teknologi Bandung
ekojw@gf.itb.ac.id

Abstract. A three-dimensional geomagnetic modeling is proposed to obtain information about a magnetization over topographic area from magnetic total force from land data survey. The model is divided into three-dimensional mesh grid of rectangular prisms that cover a finite depth below topographic elevation. The model parameter is assigned to each prism describing magnetization. A set of linear equations was formulated in terms of volume magnetization for each prism. The inverse calculation is obtained using the solution of over-determined problems. In this inverse calculation over three-dimensional mesh grid, the weighted matrix will be used to determine distribution of volume magnetization in the subsurface and keep the zero value of volume magnetization above the surface. A computer program has been tested with artificial topographic data and applied to digital elevation model of Bandung. The result of the inversion shows the comparison between synthetic model and inverse model to provide several analysis of algorithm performance. The high magnetization value can be characterize using inverse model up to 77% of volume area with average success recovery of model parameter value up to 56%.

1. Introduction
Qualitative interpretation of geomagnetic data through inversion modeling is generally used to obtain a general distribution of the magnetic susceptibility of the study area. Several developments for computational inversion modeling with synthetic data simulation as shown by several publications [1, 2, 3, 4] are able to provide a good reconstruction of subsurface conditions. In the early stages of work, this research is part of laboratory work related to geomagnetic data modeling which aims to obtain an estimate of the volume of magnetization distributed in the discretization of rectangular prisms below the surface. Inversion modeling calculations were developed to perform topographic evaluations of the geomagnetic data observed near the surface (ground survey). Furthermore, the focus of this work is to analyze the performance of the algorithm based on the comparison of the synthetic model and the inversion calculation model.

2. Modeling Computation
The discretization beneath the surface can be approximated by a fairly simple set of elements in a form that represents the model of a three-dimensional magnetic body [5, 6, 7, 8]. For practical purposes, the subsurface discretization of a body can be divided into an array of small volume elements in the form of a rectangular prism. Each prism is arranged in a direction parallel to the x-axis, y-axis, and z-axis. Each volume element is a dipole having a uniform magnetization value \( M \) in terms of its prism volume.
(dV = dx dy dz). Each volume element can then add up its effects cumulatively to a set of coordinates at the geomagnetic observation point.

The geomagnetic anomaly of a prism also involves a regional field whose direction is assumed parallel to $\mathbf{F} = (\bar{F}_x, \bar{F}_y, \bar{F}_z)$. As described in the textbook [9] magnetization inside a boundary of volume elements $x_1 \leq x \leq x_2, y_1 \leq y \leq y_2, z_1 \leq z \leq \infty$ is as follows:

$$M = M(m_x + jM_y + kM_z). \quad (1)$$

The subroutine [9] provides an explanation related to mathematical equations that can be used in the numerical calculation of the total-field anomaly of a prism with an upper limit at $z_1$ and a lower limit at $\infty$. The observed total-field anomaly ($\Delta T$) at the station coordinates $(0,0,0)$ is as follows:

$$\Delta T = C_m M \left[ \left(\frac{M_y \bar{F}_x + M_z \bar{F}_y}{2}\right) \log\left(\frac{r - x}{r + x}\right) + \left(\frac{M_x \bar{F}_y + M_z \bar{F}_x}{2}\right) \log\left(\frac{r - y}{r + y}\right) \right] - \left(\frac{M_x \bar{F}_y + M_z \bar{F}_x}{2}\right) \log(r + z_1) - M_x \bar{F}_x \tan^{-1}\left(\frac{xy}{r^2 + rz_1 + z_1^2}\right) - M_y \bar{F}_y \tan^{-1}\left(\frac{xy}{r^2 + rz_1 - x^2}\right) + M_z \bar{F}_z \tan^{-1}\left(\frac{xy}{r z_1}\right) \right] \bigg|_{x=x_1, y=y_1}^{x=x_2, y=y_2}, \quad (2)$$

where $r^2 = x^2 + y^2 + z^2$ and $C_m$ is proportionality constant (in SI units $C_m = \frac{\mu_0}{4\pi} = 10^{-7}$ H/m). The equation (2) can be evaluated two times with:

(a) value of $z_1 = z_a$ as upper limit of the prism and $M = M_o$.

(b) value of $z_1 = z_a + dz = z_b$ as lower limit of the prism and $M = -M_o$.

Based on the principle of superposition (illustration is shown in Figure 1) will produce a dipole field of a prism with magnetization $M_o$ inside the boundary of the volume element $x_1 \leq x \leq x_2, y_1 \leq y \leq y_2, z_1 \leq z \leq z_b$.

The form of linear matrix notation from the prism by accommodating $M_o$ and $-M_o$ for a set number of data ($d = [d_1, d_2, ... d_P] = [(\Delta T_{xa} - \Delta T_{xb});]$ over a set of discretized number of model parameters ($m = [m_1] = [m_1, m_2, ..., m_Q] = [(M_o)];$) then the system of linear equations from the forward calculation can be arranged in more detail as follows:

$$\begin{bmatrix}
(\Delta T_{xa} - \Delta T_{xb})_1 \\
(\Delta T_{xa} - \Delta T_{xb})_2 \\
\vdots \\
(\Delta T_{xa} - \Delta T_{xb})_N
\end{bmatrix} = \begin{bmatrix}
G_{11} & G_{12} & \cdots & G_{1Q} \\
G_{21} & G_{22} & \cdots & G_{2Q} \\
\vdots & \vdots & \ddots & \vdots \\
G_{P1} & G_{P2} & \cdots & G_{PQ}
\end{bmatrix} \begin{bmatrix}
(M_o)_1 \\
(M_o)_2 \\
\vdots \\
(M_o)_Q
\end{bmatrix}. \quad (3)$$

The model parameters in equation (3) show the corresponding physical properties of the geomagnetic method, namely magnetization ($M_o$) with uniform value for each element of the prism volume. Based on equations (2) and (3) it shows that the dipole field depends linearly on the value of its physical property (magnetization). A more commonly known physical property (namely magnetic susceptibility) can be derived from the conversion of the magnetization value ($M_o$) at each discretization position of the model parameter ($j = 1, 2, \ldots Q$). The more general and simpler form of matrix notation of equation (3) is as follows:

$$d = Gm. \quad (4)$$
where $\mathbf{G} = G_{ij}$ is the Kernel matrix with dimension $P \times Q$. Figure 2 shows a plot illustration of the horizontal slice of a synthetic model at a certain depth with a representation of colored pixels to represent variations in the model parameter values.

**Figure 1.** An illustration of the calculation with equation (2) which is evaluated at $z_1 = \bar{z}_1$ and $z_1 = \bar{z}_2$ to produce a rectangular prism shaped volume element.

**Figure 2.** Illustration of the plot (view on the X-Y plane) of the discretization model showing single square (pixel) as a rectangular prism representation of uniform dimensions $(dx \times dy \times dz)$ for the volume of magnetization shown by the color scale.

Generally, the linear inverse problem for equation (4) is solved to get the estimated value of a model parameter $(\hat{m} = \{(M_o)_{ij}\})$ which is divided into several rectangular prisms. Observational data input $(\mathbf{d})$ used in the computation for the solution of inversion problem (a minimum-norm solution). The inverse problem is under-determined with the dimensions $P < Q$ or in other words the number of observation data matrix $(\mathbf{d}^{obs})$ are smaller than the number of model parameter matrix $(\hat{m} = \{(M_o)_{ij}\})$. The computational scheme in this study also involves the initial information as the initial model in the default form of an uniform model parameter matrix value $(\hat{m}^{k=0} = 0)$. The miss-fit data $(\phi^k)$ for $k$-iteration calculated using root-mean-square-error (RMSE) as follows:

$$\phi^k = \left(\frac{\sum_{i=1}^{P} (d_{ij}^{obs} - G_{ij}\hat{m}_j^k)^2}{P}\right)^{1/2}$$

(5)
For observational data measured on a rounded topography, the mesh grid is designed to involve sufficient size with at least the upper boundary of the mesh grid covering the maximum height of the topography in the study area. Topographic evaluation in a mesh grid for discretization of model parameters needs to be involved in the iterative scheme. Each mesh grid coordinate as the center point of the rectangular prism volume in the vertical direction is compared to the coordinates of the observation station which is located on its vertical component. For each mesh grid coordinate whose position is above the coordinates of the observation station on its vertical component, the change in the model solution in the $k$-iteration ($\Delta m^k$) is given a zero multiplier factor, while for other mesh grid coordinates is given one multiplier. The model parameter values ($\hat{m}^{k+1}$) which are updated on every $k$-iteration are evaluated based on the bounds model for the minimum and maximum limits of the magnetization value.

The minimum-norm solution which is updated iteratively in the $k$-iteration is calculated with the following equations:

$$\hat{m}^{k+1} = \hat{m}^k + \Delta m^k,$$

$$\Delta m^k = G^T D [DGG^T D + \varepsilon I]^{-1} D (d_{obs} - G\hat{m}^k).$$

Equations (6) and (7) are use superscripts $k$ and $T$, which describe the $k$-iteration and transpose matrix respectively. In equation (7) the symbol $\varepsilon$ is a damping factor and $I$ is the identity matrix. Mendonca and Silva [10] used a normalized matrix $D$ with diagonal matrix elements as follows:

$$D_{ii} = \left( \sum_{j=1}^{Q} G_{ij}^2 \right)^{-1/2}.$$

3. Modeling Results

Inversion algorithm testing is carried out with a synthetic model with a mesh grid size which is a set of rectangular prisms totaling $(30 \times 30 \times 25)$ with topographic variations using DEM data in the Bandung Basin (www.tides.big.go.id/DEMNAS downloaded at 24 April 2019). Topographic variations that range from 500 to 2000 meters above sea level, are modeled inside the mesh grid in a three-dimensional direction with the upper boundary of the mesh grid at an elevation of 1450 meters above sea level to the lower boundary of the mesh grid at an elevation of 1050 meters below sea level (Figure 3). In this study, calculations with forward modeling were carried out for two simulation schemes of the distribution of the observation points from the ground survey and airborne survey. The position for the plane, which is a simulation of a synthetic airborne path, is carried out at an elevation of 1600 m above sea level (the illustration is shown in Figure 4A). The position of the observation point for the ground survey simulation is close to the topographic elevation (Figure 4B). The synthetic data from the ground survey and airborne survey simulations are shown in Figures 4C and 4D, respectively. Progress of iterative calculation shown Figure 5 with the changes of RMSE.
**Figure 3.** Synthetic model with mesh grid that cover DEM in the Bandung Basin.

**Figure 4.** Illustration of design and synthetic data at the forward calculation stage is shown by (A) synthetic model mesh grid illustration, (B) topographic discretization, (C) synthetic data for ground survey simulation, and (D) synthetic data for airborne survey simulation.
Figure 5. RMSE vs iteration number from two synthetic data (ground and airborne survey).

Comparison model shown in Figure 6 with the example of vertical slice of the mesh grid in X=788000 (South to North section). The topographic shown with black dash-line and the parameter model value of magnetization shown using color scale. Comparison synthetic model (Figure 6A) with the inverse model (Figure 6B, 6C, and 6D) showing an agreement the image of high magnetization body location. The test inverse model (Figure 6B) is compute using free-value of model parameter (bound-less), while the inverse model (Figure 6C and 6D) are compute using boundary of parameter model 0 to 5 A/m. Qualitatively, the capability of the iterative calculation can be used to characterize high magnetization in the subsurface. From this study (using ground and airborne survey simulation) the value of parameter model from the inverse model successfully characterize 77.01% and 63.63% synthetic model (of high magnetization value). Average success of recovery synthetic value of parameter model are 42.18% and 56.01% for ground and airborne survey simulation, respectively.

Figure 6. Example of vertical slice (X=788000): A) synthetic model, B) test inverse model, C) inverse model using ground data simulation, and D) inverse model using airborne data simulation.

4. Conclusion
The algorithm of inverse modeling using topographic variation can be used to estimate the solution of three dimensional distribution volume magnetization in the subsurface. From this synthetic study, the numerical example of high magnetization value can be characterize using inverse model up to 77% of volume area with average success recovery of model parameter value up to 56%.
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