Totality of Subquantum Nonlocal Correlations

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Abstract

In a series of previous papers we developed a purely field model of microphenomena, so called prequantum classical statistical field theory (PCSFT). This model not only reproduces important probabilistic predictions of QM including correlations for entangled systems, but it also gives a possibility to go beyond quantum mechanics (QM), i.e., to make predictions of phenomena which could be observed at the subquantum level. In this paper we discuss one of such predictions – existence of nonlocal correlations between prequantum random fields corresponding to all quantum systems. (And by PCSFT quantum systems are represented by classical Gaussian random fields and quantum observables by quadratic forms of these fields.) The source of these correlations is the common background field. Thus all prequantum random fields are “entangled”, but in the sense of classical signal theory. On one hand, PCSFT demystifies quantum nonlocality by reducing it to nonlocal classical correlations based on the common random background. On the other hand, it demonstrates total generality of such correlations. They exist even for distinguishable quantum systems in factorizable states (by PCSFT terminology – for Gaussian random fields with covariance operators corresponding to factorizable quantum states).
1 Introduction

Tremendous development of quantum technologies provides new intriguing possibilities for tests of foundations of QM and even generates expectations to test predictions of prequantum models, i.e., models describing microphenomena at the subquantum level and reproducing QM as emerging theory. In a series of previous papers [1] I developed a purely field model of microphenomena, so called prequantum classical statistical field theory (PCSFT). By this theory quantum systems are represented by classical Gaussian random fields. Correlations of quantum observables $A_1$ and $A_2$ on a composite system $S = (S_1, S_2)$ are represented as correlations of quadratic forms, $f_{A_1}(\phi_1), f_{A_2}(\phi_2)$, of components of the prequantum random field $\omega \rightarrow \phi(\omega) = (\phi_1(\omega), \phi_2(\omega))$ representing $S$ at the subquantum level. (Here $\omega$ is a random parameter.) PCSFT not only reproduces important probabilistic predictions of QM including correlations for entangled systems, but it gives a possibility to go beyond QM, i.e., to make predictions on phenomena which could be observed at the subquantum level. In particular, our prequantum model predicts not only correlations of functionals (quadratic forms) of the prequantum field, $\phi \rightarrow f(\phi)$, corresponding to quantum observables, but even correlations between components of the prequantum field corresponding to the subsystems $S_1$ and $S_2$ of $S$. These correlations are always nonzero, even for random fields corresponding to quantum systems in factorizable states, $\Psi = \Psi_1 \otimes \Psi_2$. We investigate this property of the prequantum field in very detail and found that the situation is very tricky from the probabilistic viewpoint. Although components of the prequantum field are always correlated, all functionals of this field corresponding to quantum observables for $S_1$ and $S_2$ are never correlated (in the case of factorizable states). Thus this effect, nonzero correlation between prequantum random fields representing e.g. proton and electron which had been prepared independently e.g. at Växjö and Moscow, cannot be found by using “ordinary quantum observables”. New more delicate measurement procedures have to be designed. In aforementioned example we took proton and electron to emphasize that correlations on the subquantum level have no direct relation to the quantum entanglement for indistinguishable quantum systems, e.g., a pair of electrons. Moreover, PCSFT predicts correlations even between prequantum random fields corresponding to bosons and fermions (even in the case of factorizable states). What is a source of subquantum...
correlations? Why are prequantum random fields corresponding to all quantum particles correlated?

PCSFT is heavily based on the assumption of the presence of a sufficiently strong background field, cf. with stochastic electrodynamics. It is impossible to introduce a positively defined covariance operator and hence to construct a classical random field representation for quantum systems without such a field. We can speculate that the common background field is responsible for totality of correlations at the subquantum level. The situation is very special from the probabilistic viewpoint. For some quantum states, entangled states of QM, it is in principle impossible to separate the background field from so to say the intrinsic field of e.g. an electron. The latter field does not exist as a classical random field (this is again the problem of positive definiteness of the covariance operator). For entangled states, the contribution of the background field can be separated from the “intrinsic contribution” only on the level of averages: the quantum average is obtained as the difference between the average with respect the “total prequantum signal” and the average with respect to the background signal (a kind of calibration procedure), see (5), (6).

In this situation of totality of mutual correlations it is natural to consider a prequantum grand field. Random fields corresponding to quantum systems are simply random signals generated by such a grand field. Hence, this work can be considered as a step towards classical unified field theory. However, this is a very preliminary step.

We start with a brief review of our previous results about classical representation of quantum correlations, see [1] for details. Then we modify the previously developed formalism to obtain a symmetric coupling between quantum and classical covariances. Then we show that quadratic forms of Gaussian signals corresponding to quantum systems in factorizable states are not correlated. Finally, we present a criterium to distinguish prequantum random fields corresponding to entangled quantum systems from fields corresponding to disentangled ones.

To simplify considerations, we will study quantum systems with finite-dimensional Hilbert spaces. Moreover, we consider a toy-model with the real Hilbert state space.
2 Classical representation of quantum correlations

Take a Hilbert space \( H \) as the space of states of classical random fields. In classical signal theory \( H = L_2(\mathbb{R}^3) \). To escape from mathematical problems we work in the finite-dimensional case. However, in the appendix we work (on the physical level of rigorousness) with \( H = L_2(\mathbb{R}^3) \).

Consider a probability distribution \( P \) on \( H \) having zero average (it means that \( \int_H y dP(\phi) = 0 \) for any \( y \in M \)) and the covariance operator \( D \):

\[
(Dy_1, y_2) = \int_H (y_1, \phi)(y_2, \phi)dP(\phi), \quad y_1, y_2 \in M. \tag{1}
\]

The \( P \) can be considered as the probability distribution of an \( H \)-valued random variable – random field (signal) (the terminology which matches better the case \( H = L_2(\mathbb{R}^3) \)). We remark that a covariance operator does not determine the random signal uniquely. However, in the Gaussian case each \( D \) determines uniquely the Gaussian measure with zero mean value.

Let \( H = \mathbb{C}^n \) and \( \phi(\omega) = (\phi_1(\omega), \ldots, \phi_n(\omega)) \), then zero average condition is reduced to \( E\phi_k \equiv \int_\Omega \phi_k(\omega)dP(\omega) = 0, k = 1, \ldots, n \), where \( E \) is the operation of classical mathematical expectation; the covariance matrix \( D = (d_{kl}) \), where \( d_{kl} = E\phi_k \phi_l \equiv \int_\Omega \phi_k(\omega)\phi_l(\omega)dP(\omega) \). We also recall that the dispersion of the random variable \( \phi \) is given by

\[
\sigma_\phi^2 = E\|\phi(\omega) - E\phi(\omega)\|^2 = \sum_{k=1}^n E|\phi_k(\omega) - E\phi_k(\omega)|^2.
\]

In the case of zero average we simply have

\[
\sigma_\phi^2 = E\|\phi(\omega)\|^2 = \sum_{k=1}^n E|\phi_k(\omega)|^2.
\]

2.1 Operator representation of wave function of composite system

In this section we show that the wave function of a composite system has an operator representation which is useful in coupling quantum and classical correlations at the subquantum level, see [1].
Let $H$ be a real Hilbert space. We denote the space of self-adjoint operators acting in $H$ by the symbol $\mathcal{L}_s(H)$. Since in this paper we consider only the finite dimensional real case, this space can be realized as the space of all symmetric matrices.

Let $H_1$ and $H_2$ be two real (finite dimensional) Hilbert spaces. We put $H = H_1 \otimes H_2$. Any vector $\Psi \in H$ can be represented in the form $\Psi = \sum_{j=1}^{m} \psi_j \otimes \chi_j$, $\psi_j \in H_1$, $\chi_j \in H_2$, and it determines a linear operator from $H_2$ to $H_1$

$$\hat{\Psi}\phi = \sum_{j=1}^{m} (\phi, \chi_j) \psi_j, \phi \in H_2.$$ Of course, $\hat{\Psi}\hat{\Psi}^* : H_1 \to H_1$ and $\hat{\Psi}^*\hat{\Psi} : H_2 \to H_2$ and these operators are self-adjoint and positively defined. Consider operator $\rho = \Psi \otimes \Psi : H_1 \otimes H_2 \to H_1 \otimes H_2$ and the operators $\rho^{(1)} \equiv \text{Tr}_{H_2}\rho$ and $\rho^{(2)} \equiv \text{Tr}_{H_1}\rho$. If the vector $\Psi$ is normalized by 1, then $\rho$ is the density operator corresponding to the pure state $\Psi$ and the operators $\rho^{(i)} \equiv \rho^{S_i}, i = 1, 2,$ are the reduced density operators. These density operators describe quantum states of subsystems $S_i, i = 1, 2,$ of a composite quantum system $S = (S_1, S_2)$. For any $\Psi \in H_1 \otimes H_2$, the following equalities hold:

$$\rho^{(1)} = \hat{\Psi}\hat{\Psi}^*, \rho^{(2)} = \hat{\Psi}^*\hat{\Psi}.$$ For any pair of operators $\hat{A}_j \in \mathcal{L}_s(H_j), j = 1, 2,$ the following equality holds [11]:

$$\text{Tr}\hat{\Psi}\hat{A}_2\hat{\Psi}^*\hat{A}_1 = \langle \hat{A}_1 \otimes \hat{A}_2 \rangle_\Psi = \langle \hat{A}_1 \otimes \hat{A}_2 \Psi, \Psi \rangle.$$ (2)

It will play a fundamental role in representation of quantum correlations as classical correlations of quadratic forms of the prequantum random field.

Let the state vectors of systems $S_1$ and $S_2$ belong to Hilbert spaces $H_1$ and $H_2$, respectively. Then by QM the state vector $\Psi$ of the composite system $S = (S_1, S_2)$ belongs to $H = H_1 \otimes H_2$. We remark that the interpretation of the state vector $\Psi \in H$ of a composite system is not as straightforward as for a single system. It is known that, in general, a pure state $\Psi$ of a composite system does not determine pure states for its components. This viewpoint matches well our approach. We shall interpret a normalized vector $\Psi \in H$ not as the state vector of a concrete composite system $S = (S_1, S_2)$, but as one of blocks of the covariance operator for the prequantum random field $\omega \to \phi(\omega) = (\phi_1(\omega), \phi_2(\omega))$ describing $S = (S_1, S_2)$. 

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2.2 From classical to quantum correlations

Let $\phi_1$ and $\phi_2$ be two random vectors, in Hilbert spaces $H_1$ and $H_2$, respectively. Consider Cartesian product of these Hilbert spaces: $\mathbf{H} = H_1 \times H_2$ (don’t mix with $H = H_1 \otimes H_2$) and the random vector $\omega \to \phi(\omega) = (\phi_1(\omega), \phi_2(\omega)) \in \mathbf{H}$ such that: a) its expectation $E\phi = 0$; b) its dispersion $\sigma^2(\phi) = E||\phi||^2 < \infty$. Take its covariance operator $D$ which is determined by the symmetric (positive) bilinear form: $(Du, v) = E(u, \phi)(v, \phi)$, where vectors $u, v \in \mathbf{H}$. This operator has the block structure $D = \left( \begin{array}{cc} D_{11} & D_{12} \\ D_{21} & D_{22} \end{array} \right)$, where $D_{ii} : H_i \to H_i$, $D_{ij} : H_j \to H_i$.

Let $\hat{A}_i \in \mathcal{L}_s(H_i), i = 1, 2$. It determines the quadratic function on the Hilbert space $H_i : f_{A_i}(\phi_i) = (\hat{A}_i \phi_i, \phi_i)$. Such quadratic functionals are prequantum physical variables corresponding to quantum observables.

For any Gaussian random vector $\phi = (\phi_1, \phi_2)$ having zero average and any pair of operators $\hat{A}_i \in \mathcal{L}_s(H_i), i = 1, 2$, the following equality takes place \[1\]
\[
\langle f_{A_1}, f_{A_2} \rangle_\phi \equiv Ef_{A_1}(\phi_1)f_{A_2}(\phi_2) = (\text{Tr}D_{11}\hat{A}_1)(\text{Tr}D_{22}\hat{A}_2) + 2\text{Tr}D_{12}\hat{A}_2D_{21}\hat{A}_1. \tag{3}
\]
We also remark that \[2\] $\text{Tr}D_{3i}\hat{A}_i = Ef_{A_i}(\phi_i), i = 1, 2$. Thus $Ef_{A_1}f_{A_2} = Ef_{A_1}Ef_{A_2} + 2\text{Tr}D_{12}\hat{A}_2D_{21}\hat{A}_1$. Now take an arbitrary pure state of a composite system $\mathcal{S} = (S_1, S_2)$, a normalized vector $\Psi \in H$. Consider a Gaussian vector random field such that $D_{12} = \hat{\Psi}$. By operator equality \[2\] the last summand in the right-hand side of \[3\] is equal to the QM-average. Hence, we obtain \[2\]
\[
\frac{1}{2}E(f_{A_1} - Ef_{A_1})(f_{A_2} - Ef_{A_2}) = (\hat{A}_1 \otimes \hat{A}_2\Psi, \Psi) \equiv (\hat{A}_1 \otimes \hat{A}_2)\Psi, \text{ or, for the covariance of two classical random vectors } f_{A_1}, f_{A_2}, \text{ we have:}
\]
\[
\frac{1}{2}\text{cov}(f_{A_1}, f_{A_2}) = \langle \hat{A}_1 \otimes \hat{A}_2 \rangle_\Psi. \tag{4}
\]
This formula was derived in \[1\]. One of problems is its asymmetry: classical covariance on one side, i.e., centered correlation, but non-centered quantum correlation on another side. We shall obtain symmetric representation. However, first we have to discuss a more fundamental problem.

Operators $D_{ii}$ are responsible for averages of random variables $\omega \to f(\phi_i(\omega))$, i.e., depending only on one of components of the vector random field $\phi$. In particular, $Ef_{A_1}(\phi_i) = \text{Tr}D_{ii}\hat{A}_i.$
We shall construct such a random field that these averages will match those given by QM. It is natural to take the covariance operator $\hat{D}_\Psi = \begin{pmatrix} \hat{\Psi}\hat{\Psi}^* & \hat{\Psi} \\ \hat{\Psi}^* & \hat{\Psi}^*\Psi \end{pmatrix}$. However, in general this operator is not positively defined and, hence, it cannot serve as a covariance operator. In [1] it was proposed to modify aforementioned operator and consider $D_\Psi = \begin{pmatrix} \hat{\Psi}\hat{\Psi}^* + \epsilon I & \hat{\Psi} \\ \hat{\Psi}^* & \hat{\Psi}^*\Psi + \epsilon I \end{pmatrix}$, where $\epsilon > 0$ is sufficiently large.

We remark that white noise is a Gaussian random variable with zero average and the unit covariance operator $I$. Thus additional terms in diagonal blocks are related to the white noise background. The situation is tricky: in general it is impossible (in the classical mathematical model) to separate this noisy background from a random prequantum field. We cannot consider a random field with the covariance operator $D_\Psi$ as the sum of two signals, e.g., an electron signal and the background signal. For some states (entangled states), the matrix with $\epsilon = 0$ is not positively defined. We discuss this point in more detail:

Suppose now that $\phi(\omega)$ is a random vector with the covariance operator $D_\Psi$. Then

$$\langle \hat{A}_1 \rangle_\Psi = Ef_A_1(\phi_1(\omega)) - \epsilon \text{Tr}\hat{A}_1,$$  \hspace{1cm} (5)

$$\langle \hat{A}_2 \rangle_\Psi = Ef_A_2(\phi_2(\omega)) - \epsilon \text{Tr}\hat{A}_2.$$  \hspace{1cm} (6)

These relations for averages and relation (4) for the correlation provide coupling between theory of classical Gaussian signals (in the finite-dimensional case simply theory of Gaussian random variables) and QM. Quantum statistical quantities can be obtained from corresponding quantities for classical random field. One may say that “irreducible quantum randomness” is reduced to randomness of classical prequantum fields. However, the situation is more complicated. The equalities (5), (6) imply that quantum averages are obtained as the shift-type renormalizations of averages with respect to classical random fields. The shift corresponds to subtraction of the contribution of the background field. Thus quantum averages are not simply classical averages. They are obtained as the result of renormalization with respect of the background field.
3 Modification of correspondence between quantum observables and classical variables

Although equality (4) establishes the coupling between classical correlations of random signals and quantum correlations, it is not completely satisfactory from the purely probabilistic viewpoint. On the left-hand side of (4) we have the classical covariation, \( \text{cov}(f_{A_1}, f_{A_2}) \), but on the right-hand side we have just the quantum average of the correlation observable \( \hat{A}_1 \otimes \hat{A}_2 \). We want to modify the correspondence between quantum and classical models to obtain a symmetric relation between classical and quantum covariances. We recall that the latter is given by \( \text{cov}(\hat{A}_1, \hat{A}_2) \equiv \langle \hat{A}_1 \otimes \hat{A}_2 \rangle - \langle \hat{A}_1 \rangle \langle \hat{A}_2 \rangle \).

We set
\[
\hat{A}_0 = \hat{A} - \langle A \rangle I, \quad i = 1, 2,
\]
which implies \( \langle \hat{A}_0 \rangle = 0 \) and
\[
\text{cov}(\hat{A}_01, \hat{A}_02) = \langle \hat{A}_01 \otimes \hat{A}_02 \rangle = \text{cov}(\hat{A}_1, \hat{A}_2). \tag{8}
\]

Let us modify the correspondence between classical and quantum variables, see section 2.2. Instead of the formerly used correspondence \( \hat{A} \rightarrow f_A(\phi) = (\hat{A}_\phi, \phi) \), we introduce a new map from the quantum model to the classical prequantum model
\[
\hat{A}_0 \rightarrow f_{A_0}(\phi) = (\hat{A}_0\phi, \phi), \quad \hat{A}_0 = \hat{A} - \langle A \rangle I. \tag{9}
\]

By using (4) for \( \hat{A}_{0i} \) instead of \( \hat{A}_i \), we obtain
\[
\text{cov}(f_{A_{01}}, f_{A_{02}}) = \langle \hat{A}_{01} \otimes \hat{A}_{02} \rangle.
\]
Thus
\[
\frac{1}{2} \text{cov}(f_{A_{01}}, f_{A_{02}}) = \text{cov}(\hat{A}_{01}, \hat{A}_{02}). \tag{10}
\]

We remark that in (1) the factor 2 in front of the quantum correlation disappears in the complex case \([?]\). So, in the complex case the correspondence becomes really symmetric. (We proceed in the real Hilbert space, since in the complex case the basis operator equality \([?]\) is more complicated and its presentation is based on more complicated operator theory.)
3.1 Independence of components of prequantum random fields corresponding to factorizable quantum states

Consider now a factorizable quantum state $\Psi = \Psi_1 \otimes \Psi_2$, where $\Psi_i \in H_i, i = 1, 2$. Then

$$\langle \hat{A}_{01} \otimes \hat{A}_{02} \rangle = (\hat{A}_{01} \Psi_1, \Psi_1)(\hat{A}_{02} \Psi_2, \Psi_2) = 0.$$  

Hence,

$$\text{cov} (\hat{A}_1, \hat{A}_2) = \text{cov} (\hat{A}_{01}, \hat{A}_{02}) = 0.$$  \hspace{1cm} (11)

Thus by (10),

$$\text{cov} (f_{A_{01}}, f_{A_{02}}) = 0.$$  

Factorization of a pure quantum state $\Psi$ implies that, for any two quantum observables $\hat{A}_i$ on the subsystems $S_i, i = 1, 2$, of a composite system $S = (S_1, S_2)$, the corresponding prequantum variables, $f_{A_0i}(\phi_i)$, where $\hat{A}_0i = \hat{A}_i - \langle A_i \rangle I$, are uncorrelated.

3.2 Totality of correlations at the subquantum level

We remark that, although for a factorizable quantum state all prequantum physical variables corresponding to quantum observables are uncorrelated, see previous section, components $\phi_1(\omega)$ and $\phi_2(\omega)$ of the prequantum field are always correlated. The covariance operator $D_\Psi$ always has nonzero off-diagonal block $D_{12} = \hat{\Psi}$. Thus we can find other prequantum physical variables, nonquadratic functionals of the prequantum field, which are nontrivially correlated. Roughly speaking the presently used class of observables is too restricted to find this totality of correlations at the subquantum level. Not only prequantum fields corresponding to entangled quantum systems are correlated, but even prequantum fields corresponding to distinguishable quantum systems which have been prepared independently at huge distance from each other. Thus subquantum nonlocality is even more general than quantum one. However, the former is purely classical nonlocality of correlations of random fields which are coupled through the common background field.
4 Distinguishing property of “entangled prequantum fields”

Thus all prequantum random fields are correlated. Can one distinguish random fields corresponding to entangled quantum states from random fields corresponding to factorizable states? One of distinguishing features of “entangled prequantum fields” is impossibility to separate the “intrinsic field” of a quantum system from the background field. The intrinsic field does not exist as classical random field. This is again the problem of positive definiteness of the covariance operator. We remark that the operator \( \tilde{D}_\Psi = \begin{pmatrix} \tilde{\Psi}^* & \tilde{\Psi} \\ \tilde{\Psi}^* & \tilde{\Psi} \end{pmatrix} \) is positively defined iff the quantum state \( \tilde{\Psi} \) is factorizable, \( \Psi = \Psi_1 \otimes \Psi_2 \). The step from factorizability to positive definiteness is trivial. For \( \phi = (\phi_1, \phi_2) \in \mathbf{H} \), we have

\[
(\tilde{D}_\Psi \phi, \phi) = ||\tilde{\Psi}^* \phi_1||^2 + 2(\tilde{\Psi}^* \phi_1, \phi_2) + ||\tilde{\Psi} \phi_2||^2 = \\
(\Psi_1, \phi_1)^2 + 2(\Psi_1, \phi_1)(\Psi_2, \phi_2) + (\Psi_2, \phi_2)^2 \geq 0.
\]

Suppose now that \( \Psi \) is not factorizable. Consider its Schmidt decomposition \( \Psi = \sum_i \alpha_i e_i^{(1)} \otimes e_i^{(2)}, \) where \( \{e_i^{(1)}\} \) and \( \{e_i^{(2)}\} \) are orthonormal systems in \( H_1 \) and \( H_2 \), respectively. We shall explore the following property of Schmidt decomposition: it contains just one summand if and only if \( \Psi \) is factorizable. Consider coordinates \( x_i = (\phi_1, e_i^{(1)}), y_i = (\phi_2, e_i^{(2)}). \) Then \( (\tilde{D}_\Psi \phi, \phi) = \sum_i (\alpha_i^2 x_i^2 + 2\alpha_i x_i y_i + \alpha_i^2 y_i^2). \) In the case of entanglement all coefficients \( \alpha_i < 1. \) Set \( x_i = y_i = 0, \) for \( i > 1. \) Then \( (\tilde{D}_\Psi \phi, \phi) = \alpha_1^2 x_1^2 + 2\alpha_1 x_1 y_1 + \alpha_1^2 y_1^2. \) This quadratic form is not positively defined.

In the case of a prequantum random field corresponding to a factorizable quantum state the operator \( D_\Psi = \tilde{D}_\Psi + \epsilon I \) and the first summand is positively defined. Consider Gaussian random fields \( \phi_\Psi, \hat{\phi}_\Psi, \eta \) with zero mean values and the covariance operators \( D_\Psi, \tilde{D}_\Psi, \epsilon I. \) Suppose that the “intrinsic field” of a system \( \hat{\phi}_\Psi \) and the background field \( \eta \) are independent. Then \( \phi_\Psi \) can be represented as \( \phi_\Psi(\omega) = \hat{\phi}_\Psi(\omega) + \eta(\omega), \) where \( \omega \) is a random parameter. Thus in the absence of entanglement the “intrinsic field” of a system can be distilled from the background. This is impossible for entangled states.

**Conclusion.** At the subquantum level entanglement is an exhibition of fundamental nonseparability from the background field.
References

[1] Khrennikov A., J. Phys. A 38 (2005) 9051; Phys. Lett. A 357 (2006) 171; Ibid 372 (2008)6588; EPL 88 (2010) 40005; Ibid 90 (2010) 40004.