Global Phase Diagram of a Dirty Weyl liquid and Emergent Superuniversality

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Pursuing complimentary field-theoretic and numerical methods, we here paint the global phase diagram of a three-dimensional dirty Weyl system. The generalized Harris criterion, augmented by a perturbative renormalization-group (RG) analysis shows that weak disorder is an irrelevant perturbation at the Weyl semimetal (WSM)-insulator quantum critical point (QCP). But, a metallic phase sets in through a quantum phase transition (QPT) at strong disorder across a multicritical point (MCP). The field theoretic predictions for the correlation length exponent \( \nu = 2 \) and dynamic scaling exponent \( z = 5/4 \) at this MCP are in good agreement with the ones extracted numerically, yielding \( \nu = 1.98 \pm 0.05 \) and \( z = 1.23 \pm 0.05 \), from the scaling of the average density of states (DOS). Deep inside the WSM phase, generic disorder is also an irrelevant perturbation, while a metallic phase appears at strong disorder through a QPT. We here demonstrate that in the presence of generic, but strong disorder the WSM-metal QPT is ultimately always characterized by the exponents \( \nu = 1 \) and \( z = 3/2 \) (to one-loop order), originating from intra-node or chiral symmetric (e.g., regular and axial potential) disorder. We here anchor such emergent chiral superuniversality through complimentary RG calculations, controlled via \( \epsilon \)-expansions, and numerical analysis of average DOS across WSM-metal QPT. In addition, we also discuss a subsequent QPT (at even stronger disorder) of a Weyl metal into an Anderson insulator by numerically computing the typical DOS at zero energy. The scaling behavior of various physical observables, such as residue of quasiparticle pole, dynamic conductivity, specific heat, Gruneisen ratio, inside various phases as well as across various QPTs in the global phase diagram of a dirty Weyl liquid are discussed.

I. INTRODUCTION

The complex energy landscape of electronic quantum-mechanical states in a solid state compound, commonly known as band structure, can often display band touching at isolated points in the Brillouin zone, which can be either accidental or symmetry protected [1–9]. In the vicinity of such diabolic points, the low energy excitations can often be described as quasi-relativistic Dirac or Weyl fermions [10–12], which may provide an ideal platform for condensed matter realization of various peculiar phenomena, such as the chiral anomaly, Casimir effect, and axionic electrodynamics [13–15]. Recently, three dimensional Weyl semimetals (WSMs) have attracted a lot of interest due to the growing evidence of their material realization [16–23].

A WSM, the prime example of a gapless topological phase of matter, is constituted by so called Weyl nodes that in the reciprocal space (Brillouin zone) act as the source and sinks of Abelian Berry curvature, and thus always appear in pairs [26]. In a nutshell, the Abelian Berry flux enclosed by the system determines the integer topological invariant of a WSM and the degeneracy of topologically protected surface Fermi arc. A question of fundamental and practical importance in this context concerns the stability of such gapless topological phase against impurities or disorder, inevitably present in real materials. Combining complimentary field theoretic renormalization group (RG) calculations and a numerical analysis of the average density of states (DOS), we here...
study the role of randomness in various regimes of the phase diagram of a Weyl system to arrive at the *global phase diagram*, schematically illustrated in Fig. 1.

A WSM can be constructed by appropriately stacking two-dimensional layers of quantum anomalous Hall insulator (QAHI) in the momentum space along the $k_z$ direction, for example. Thus, by construction a WSM inherits the two dimensional integer topological invariant of constituting layers of QAHI, and the *momentum space skyrmion number* of QAHI jumps by an integer amount across two Weyl nodes. As a result, the Weyl nodes serve as the sources and sinks for Abelian Berry curvature, and across two Weyl nodes. As a result, the Weyl nodes serve as the sources and sinks for Abelian Berry curvature, and in a clean system WSM is sandwiched between a topological Chern and a trivial insulating phase, as shown in Fig. 1. In an effective tight-binding model a WSM-insulator quantum phase transition (QPT) can be tuned by changing the effective hopping in the $k_z$ direction, as demonstrated in Sec. III. In this work we first assess the stability of such clean semimetal-insulator quantum critical point (QCP) in the presence of generic randomness. In this regard we come to the following conclusions.

1. By generalizing the Harris criterion [27], we find that WSM-insulator QCP is stable against sufficiently weak, but otherwise generic disorder (see Sec. III). Such an outcome is further substantiated from the scaling analysis of disorder couplings, suggesting that any disorder is an irrelevant perturbation at such a clean QCP.

2. Subscribing to an appropriate controlled $\epsilon$-expansion (see Sec. III), we demonstrate that a *multi-critical point* (MCP) emerges at stronger disorder, where the WSM, a band insulator (either Chern or trivial) and a metallic phase meet. The critical semimetal residing at the phase boundary between a WSM and an insulator then becomes unstable toward the formation of a compressible metal through such a MCP. The exponents capturing the instability of critical excitations toward the onset of a metal are: (a) correlation length exponent (CLE) $\nu = 2$, and (b) dynamic scaling exponent (DSE) $z = 5/4$ to the leading order in the $\epsilon$-expansion. These two exponents also determine the scaling behavior of physical observables across the anisotropic critical semimetal-metal QPT.

3. By following the scaling of DOS along the phase boundary between the WSM and insulator with increasing randomness in the system, we numerically extract $\nu$ and $z$ at the MCP across the critical semimetal-metal QPT [see Fig. 2]. Numerically extracted values of these two exponents are $\nu = 1.98 \pm 0.05$ and $z = 1.23 \pm 0.05$ [see Sec. III D], which are in excellent agreement with our prediction from the leading order $\epsilon$-expansion.

We now turn our focus on the WSM phase. The study of disorder effects in topological phases of matter has recently attracted a lot of attention, leading to a surge of analytical [28] [49] and numerical [50] [63] works. In particular, the focus has been concentrated on massless Dirac critical point separating two topologically distinct insulators (electrical or thermal), as well as inside a Dirac and Weyl semimetal phases. Even though the effects of generic disorders have been studied to some extent theoretically [30] [36] [41] [43], most of the numerical works solely focused on random charge impurities (for exception see Refs. [53] [56]). By now there is both analytical and numerical evidence that chemical potential disorder when strong enough drives a QPT from the WSM to a diffusive metal, leaving its imprint on different observables, e.g., average DOS, specific heat and conductivity. Deep inside the WSM phase, the system possesses various *emergent symmetries* (see Table III), such as a *continuous global chiral U(1) symmetry* that is tied with the translational symmetry of a clean noninteracting WSM in the continuum limit [64]. In the absence of both inversion and time-reversal symmetries, the simplest realization of a WSM with only two Weyl nodes is susceptible to *sixteen* possible sources of elastic scattering, displayed in Table III. They can be grouped in *eight* classes, among which only four preserve the emergent global chiral symmetry (intra-node scattering), while the remaining ones directly mix two Weyl nodes with opposite (left and right) chiralities.
Numerical Analysis | Field Theory
--- | ---
Disorder | $W_c$ | $z$ | $\nu$ | $z$ | $\nu$
--- | --- | --- | --- | --- | ---
Potential | $1.65 \pm 0.05$ | $1.45 \pm 0.05$ | $0.99 \pm 0.05$ | $3/2$ | $1$
Axial | $2.60 \pm 0.05$ | $1.47 \pm 0.06$ | $1.06 \pm 0.06$ | $3/2$ | $1$
Magnetic | $1.80 \pm 0.05$ | $1.51 \pm 0.06$ | $1.05 \pm 0.06$ | $3/2$ | $1$
Current | $1.65 \pm 0.05$ | $1.48 \pm 0.06$ | $0.99 \pm 0.06$ | $3/2$ | $1$

Table I: Comparison of numerically extracted values of dynamic scaling exponent ($z$) and correlation length exponent ($\nu$) across the WSM-metal QPT [takes place at $W = W_c$], with the ones obtained from the leading order $\epsilon$-expansions using field theoretic techniques. All four disorders preserve continuous global chiral symmetry of a WSM. This comparison strongly suggests that a WSM-metal transition driven by a CSP disorder is insensitive to the nature of elastic scatterers, thus motivating an emergent chiral superuniversality class of the QPTs, as detailed in Sec. V. For details of the data analysis see Sec. VI and Figs. 14 and 15.

(internode scattering) $^1$. As we demonstrate in the paper, such characterization of disorders based on the chiral symmetry allows us to classify the WSM-metal QPTs in the presence of generic disorder.

To motivate our theoretical analysis, we now discuss the possible microscopic origin of disorders in the Weyl materials. Furthermore, knowing this in future may facilitate a control over randomness in experiments on these materials. For example, chemical potential disorder can be controlled by modifying the concentration of random impurities. Random asymmetric shifts of chemical potential between the left and right chiral Weyl cones correspond to the axial potential disorder. Therefore, in an inversion asymmetric WSM such disorder is always present. Magnetic disorder is yet another type of chiral symmetry preserving (CSP) disorder, and the strength of random magnetic scatterers can be efficiently tuned by systematically injecting magnetic ions in the system $^2$. In contrast, all chiral symmetry breaking (CSB) disorders cause mixing of two Weyl nodes and in an effective model for WSMs they stem from various types of random bond disorder that also cause random fluctuation of band-width (see Appendix D). Therefore, strength of CSB disorder may be tuned by applying inhomogeneous pressure (hydrostatic or chemical) in the Weyl materials. Since the WSMs are found in strong spin-orbit coupled materials, a random spin-orbit coupling can be achieved when hopping (hybridization) between two orbitals with opposite parity acquires random spatial modulation. Yet another CSB but vector-like type of disorder is a random axial Zeeman coupling. Its source is the different $g$-factor of two hybridizing bands that touch at the Weyl point $^3$. Therefore, when magnetic impurities are injected in the system such disorder is naturally introduced, and depending on the relative strength of the $g$-factor in different bands, one can access regular (intranode) or axial (internode) random magnetic coupling. Finally, two different types of CSB mass disorders that tend to gap out the Weyl points are represented by random charge- or spin-density-wave order, depending on the microscopic details $^3$. These disorders correspond to random scalar and pseudo-scalar mass in the field theory language. Due to their presence, Weyl nodes are gapped out in each disorder configuration, but the sign of the gap is random from realization to realization, and in the thermodynamic limit the nodes remain gapless. To the best of our knowledge, it is currently unknown how to tune the strength of all individual sources of elastic scattering in real Weyl materials. Nevertheless, we elucidate how all possible disorders can be obtained from a simple effective tight-binding model on a cubic lattice for a WSM with two nodes (see Appendix D), allowing us to numerically investigate the effects of generic disorder in this system.

Here we address the stability of a disordered WSM (i) in the field-theoretical framework by using two different renormalization-group (RG) schemes: (a) an $\epsilon_m$-expansion about a critical disorder distribution, where $\epsilon_m = 1 - m$, with the Gaussian white noise distribution realized as $m \to 0$, and (b) $\epsilon_d = 2 - d$-expansion about $d_l = 2$, the lower-critical spatial dimension for WSM-metal QPT, and (ii) lattice-based numerical evaluation of average DOS by using the kernel polynomial method (KPM) $^5$ in the presence of generic chiral symmetric disorder [see Fig. 3] as well as non-chiral disorder [see Fig. 4]. Comparisons between the field theoretic predictions and numerical findings for all chiral disorders are given in Table II. Our central results can be summarized as follows.

1. From the scaling analysis of disorder couplings, we show in Sec. $^4$ that all types of disorder (both CSP and CSB) are irrelevant perturbations in a WSM. Such outcome is also substantiated numerically (see

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$^1$ Throughout this paper, we will use chiral-symmetric and internode disorder synonymously. We also will use chiral-symmetry breaking and inter-node disorder synonymously.

$^2$ We assume here that magnetic impurities are dilute, so that we do not have to worry about Kondo effect.

$^3$ Comparisons between the field theoretic predictions and numerical findings for all chiral disorders are given in Table II. Our central results can be summarized as follows.
Figs. 3 and 4, depicting that DOS scales as $\rho(E) \sim |E|^2$ for small energy $|E|$, when generic disorder is sufficiently weak.

2. We show in Sec. V that irrespectively of the details of two distinct $\epsilon$-expansions, in the presence of a CSP disorder, the WSM-metal QPT takes place through either a QCP (when either potential or axial potential disorder is present) or a line of QCPs (when both types of scalar disorder are present simultaneously), characterized by critical exponents

$$z = 1 + \frac{\epsilon}{2}, \quad \nu^{-1} = \epsilon,$$

obtained from the leading order in $\epsilon$-expansions, where $\epsilon = \epsilon_m$ or $\epsilon_d$, and $\epsilon = 1$ corresponds to the physical situation. Therefore, irrespective of the nature of elastic scatterers, the universality class of the WSM-metal QPT in the presence of a CSP disorder is unique, and we name such universality class chiral superuniversality.

3. In Sec. VI we carry out a thorough numerical analysis of DOS in the presence of all four CSP disorders, obtained by using KPM from a lattice model (see Fig. 3). Within the numerical accuracy we find that $z \approx 1.5$ and $\nu \approx 1$ across possible CSP disorder driven WSM-metal QPTs (see Fig. 15 and also Table I). Thus numerically extracted values of critical exponents are in excellent agreement with superuniversality.
Figure 4: Scaling of numerically computed average density of states, obtained by using the kernel polynomial method [69] in a dirty Weyl semimetal in the presence of four individual internode or non-chiral elastic scatterers: (a) spin-orbit (represented by temporal component of tensor), (b) axial magnetic (represented by spatial component of tensor), (c) scalar mass and (d) pseudo-scalar mass [see Table III for definition and field theoretic nomenclature]. All non-chiral disorders drive WSM-metal QPTs, around which \( W = W_c \) the ADOS scales roughly as \( \varrho(E) \sim |E| \), indicating \( z \approx 1.5 \) even in the presence of strong internode scattering [see Table II for more precise values of various exponents across the WSM-metal QPT]. Thus in corroboration with Fig. 3 the scaling of ADOS in the presence of impurities of arbitrary nature is strongly suggestive of a robust superuniversality across the WSM-metal QPT. For field-theoretic analysis of non-chiral disorders see Sec. VII.

4. In Sec. VII we show that the CSB disorder can also drive a WSM-metal QPT through either an isolated QCP or a line of QCPs. Irrespective of the actual details of an \( \epsilon \)-expansion scheme, the values of the critical exponents at the QCP or line of QCPs in the presence of such disorder are in a stark contrast to the ones reported in Eq. (1), and typically \( z > d \). In particular, the DSE varies continuously across the line of QCPs supported by a strong CSB disorder. On the other hand, \( \nu^{-1} = \epsilon \) to the leading order in an \( \epsilon \) expansion, irrespective of the RG scheme.

5. Since \( z > d \) (always), the CSP disorder as well as the higher gradient terms (inevitably present in a lattice model) become relevant at the CSB disorder driven QCPs separating a WSM from a metallic phase. Consequently, in lattice-based simulations the WSM-metal QPT is expected to ultimately be controlled by the QCPs associated with CSP disorder. We anchor this outcome by numerically computing the DOS in the presence of all four internode scattering [see Fig. 4] and find that across WSM-metal QPTs, driven by any CSB disorder \( z \approx 1.5 \) and \( \nu \approx 1 \) [see Table III]. Therefore, generic disorder driven WSM-metal QPT offers rather a sparse example of superuniversality, characterized by the critical exponents \( z = 3/2 \) and \( \nu = 1 \) [see Eq. (1)].

6. In Sec. VIII we show that various experimentally measurable quantities, such as average DOS, dynamic conductivity, specific heat and Gr"uneisen ra-
tio, exhibit distinct scaling behavior in terms of CLE and DSE in different phases of a dirty WSM. As such, they may be useful to distinguish types of disorder in a WSM. Most importantly, distinct scaling of observables can allow to pin the onset of various phases in real materials.

We point out that the notion of superuniversality is realized rather sparsely in condensed matter systems. Most prominent examples in this regard include the quantum Hall plateau transitions and one-dimensional disordered superconducting wires. Therefore, dirty Weyl semimetal represents, to the best of our knowledge, the only example of a three-dimensional system exhibiting superuniversality.

It is worth mentioning that for sufficiently strong disorder the metallic phase in a Weyl system undergoes a second continuous QPT into an Anderson insulating phase [28, 53, 74]. In Sec. IX, we address the metal-insulator Anderson transition (AT), but only in the presence of random charge impurities. Our central achievements regarding the fate of the AT in strongly disordered Weyl metal are the following:

1. We show that a Weyl metal undergoes a second transition at stronger disorder into an Anderson insulator (AI) phase. By numerically computing the typical density of states (TDOS) at zero energy \( g_\ell(0) \) we show that \( g_\ell(0) \) vanishes smoothly across the Weyl metal-AI QPT, while displaying critical and single-paramter scaling. In particular, \( g_\ell(0) \) is pinned at zero in the WSM and AI phases, while it is finite inside the entire metallic phase. By contrast, the average DOS at zero energy \( \rho(0) \) remains finite in the metallic as well as AI phases, while being zero only in the weakly disordered WSM. Otherwise, \( \rho(0) \) decreases smoothly and monotonically across the Weyl metal-AI QCP.

2. We demonstrate that TDOS at zero energy displays single-parameter scaling across both (a) WSM-metal and (b) metal-AI QPTs. Specifically the order-parameter exponent for \( g_\ell(0) \), \( \beta_\ell \), defined as \( g_\ell(0) \sim |\delta|^{\beta_\ell} \), where \( \delta \) defines the reduced distance from transition point, is \( \beta_\ell = 1.80 \pm 0.20 \) across the WSM-metal QPT (which is different from the one for the average DOS at zero energy for which \( \beta_\rho = 1.50 \pm 0.05 \)).

3. We show that inside the metallic phase the mobility edge, separating the localized states from the extended ones reside at finite energy. With increasing strength of disorder the mobility edge slides down to smaller energy and across the AT the entire energy width is occupied by localized states.

The rest of the paper is organized as follows. In Sec. II we introduce a simple tight-binding model for a Weyl system and discuss possible phases and the phase transitions in the clean limit. In Sec. III we demonstrate the effects of generic disorder near the clean WSM-insulator QCP, and perturbatively address the effects of strong disorder. In Sec. IV we set up the theoretical framework for addressing the role of randomness deep inside the WSM phase, and introduce the notion of \( \epsilon_m \) and \( \epsilon_d \) expansions for perturbative treatment of disorder. This section is rather technical and readers familiar with the formalism or interested in physical outcomes may wish to skip it. We devote Sec. V to the effects of CSP disorder and promote the notion of chiral superuniversality. Detailed numerical analysis of the scaling of DOS is presented in Sec. VI. Effects of CSB disorder are discussed in Sec VII and scaling of various physical observables, such as DOS, specific heat, conductivity, etc., across the WSM-metal QPT is discussed in Sec. VIII. We discuss the Anderson transition of the metallic phase at stronger disorder in Sec. IX. Concluding remarks and a summary of our main findings are presented in Sec. X. Some additional technical details have been relegated to the Appendices.

II. LATTICE MODEL FOR WEYL SYSTEM

Let us begin the discussion with a lattice realization of chiral Weyl fermions in a three-dimensional cubic lattice. Even though in most of the commonly known Weyl materials, such as the binary alloys TaAs and NbP, Weyl fermions emerge from complex band structures in non-centrosymmetric lattices, their salient features can be captured from a simple tight-binding model

\[
H = \sum_k \psi_k^\dagger [N(k) \cdot \sigma] \psi_k. \tag{2}
\]

The two-component spinor is defined as \( \psi_k^\dagger = (c_{k,\uparrow}, c_{k,\downarrow}) \), where \( c_{k,s} \) is the fermionic annihilation operator with momentum \( k \) and spin/pseudospin projection \( s = \uparrow, \downarrow \), and \( \sigma \)s are standard Pauli matrices. We here choose

\[
N_3(k) = t_z \cos(k_z a) - m_z + t_0 [2 - \cos(k_x a) - \cos(k_y a)], \tag{3}
\]

where \( a \) is the lattice spacing. The first term gives rise to two isolated Weyl nodes along the \( k_z \) axis at \( k_z = \pm k_z^0 \), where

\[
\cos(k_z^0 a) = \frac{t_0}{t_z} \left[ \frac{m_z}{t_0} + \cos(k_x a) + \cos(k_y a) - 2 \right], \tag{4}
\]

with the following choice of pseudospin vectors

\[
N_1(k) = t \sin(k_x a), \quad N_2(k) = t \sin(k_y a). \tag{5}
\]

The second term in Eq. (3), namely \( N_3(k) \), plays the role of a momentum dependent Wilson mass [57, 68]. We subscribe to this tight-binding model in Secs. III, IV, VI and IX to numerically study the effects of randomness in various regimes of a dirty Weyl system.
The projection of the Weyl nodes in the WSM phase are always located along the \( k_z \) direction. Respectively WSM\(_{1,2,3} \) supports one, two and one pair of Weyl nodes. The transition among these phases.

With the above chosen form of the Wilson mass only a single pair of Weyl nodes is realized at \( k^0 = (0, 0, \pm k^0_z) \), when \( m_z \leq 1 \), where \( m_z = m_z/t_0 \). For \( 1 \leq m_z \leq 2 \), two pairs of Weyl nodes are found at \( (0, \pi, \pm k^0_x) \) and \( (0, 0, \pm k^0_\theta) \). When \( m_z \geq 2 \), again a single pair of Weyl points is realized at \( (\pi, \pi, \pm k^0) \). Notice that when \( t_z < 1 \) three distinct realizations of WSM phase are separated by intervening insulating phases, representing three dimensional Chern insulator, while the insulating phases realized for \( m_z < -1 \) and \( m_z > 4 \) are topologically trivial. Hence, by tuning \( m_z \) one can drive the system through WSM-insulator QPT as long as \( t_z < 1 \), where \( t_z = t_z/t_0 \). However, for \( t_z > 1 \) the noninteracting model supports only trivial insulating phases when \( m_z/t_z < -1 \) or \( (m_z - 4)/t_z > 1 \). Direct and continuous transitions between two different WSMs take place without an intervening insulating phase when \( t_z > 1 \). Therefore, our proposed model in Eqs. (5) and (6) supports translationally active topological phases with band-inversions at non-\( \Gamma \) points in the Brillouin zone \([9, 75]\) and the resulting phase diagram is shown in Fig. 5.

For the sake of simplicity, we hereafter only consider the parameter regime \(-1 < m_z < 1 \) and \( t_z \leq 1 \), so that only a single pair of Weyl fermions is realized at \( k^0 = (0, 0, \pm \cos^{-1} (m_z/t_z)) \). In the vicinity of these two points the Weyl quasiparticles can be identified as left and right chiral fermions, respectively. A WSM can be found when \( |m_z/t| \leq 1 \) and the system becomes an insulator for \( |m_z/t| > 1 \). Even though we here restrict our analysis within the aforementioned parameter regime, this analysis can be generalized to study the semimetal-insulator QPTs in various other regimes shown in Fig. 5.

Within this parameter regime, to capture the Weyl semimetal-insulator QPT which occurs along the line \( t_z/m_z = 1 \), we expand the tight-binding model around the \( \Gamma = (0, 0, 0) \) point of the Brillouin zone to arrive at the effective low energy Hamiltonian

\[
\hat{H}_Q(\Delta) = v (\sigma_1 k_x + \sigma_2 k_y) + \sigma_3 (b k_z^2 - \Delta),
\]

where \( v = t_0 \) is the Fermi velocity in the \( xy \) plane and \( b = t_z a^2/2 \) bears the dimension of inverse mass. For \( \Delta = t_z - m_z < 0 \) the system becomes an insulator (Chern or trivial). On the other hand, when \( \Delta > 0 \), the lattice model describes a WSM. The QPT in this clean model between these two phases takes place at \( \Delta = 0 \). Hence, \( \Delta \) plays the role of a tuning parameter across the WSM-insulator QPT. The QCP separating these two phases is described by an anisotropic semimetal, captured by the Hamiltonian \( H_Q(0) \) in Eq. (6), that in turn also determines the universality class of the transition. Notice that the expansion of the lattice Hamiltonian [see Eq. (5)] also yields terms \( \sim k_x^2 \) and \( \sim k_y^2 \) and higher order (from the Wilson mass), which are, however, irrelevant in the RG sense, and therefore do not affect the critical theory for the WSM-insulator QPT. Hence, we omit these higher gradient terms for now. We will discuss the paramount importance of such higher gradient terms close to the CSB disorder driven WSM-metal QPT in Sec. VII. Next we address the stability of this quantum critical semimetal against disorder in the system using scaling theory and RG analysis.

### III. EFFECTS OF DISORDER ON SEMIMETAL-INSULATOR TRANSITION

The imaginary time (\( \tau \)) action associated with the low energy Hamiltonian [see Eq. (6)] reads as

\[
S_0 = \int d\tau d^2 x \psi ^\dagger (\partial_\tau - i v \partial_j \sigma_j - \sigma_3 (b \partial_z^2 + \Delta)) \psi.
\]
In the proximity to the Weyl semimetal-insulator QPT, the system can be susceptible to both random charge and random magnetic impurities, and their effect can be captured by the Euclidean action
\[
S_D = \int d\tau d^2 x \psi^\dagger \left[ V_0(x) \sigma_0 + V_1(x) (\sigma_1 + \sigma_2) \right] \psi + V_2(x) \sigma_3 \psi,
\]
where \( V_j(x) \) are random variables. The effect of random charge impurities is captured by \( V_0(x) \), while \( V_1(x) \) and \( V_2(x) \) represent random magnetic impurities with the magnetic moment residing in the easy or \( xy \) plane and in the \( z \) direction (denoted here by \( x_3 \) for notational clarity), respectively, which allow due to the anisotropy of the Hamiltonian [see Eq. (6)]. All types of disorder are assumed to be characterized by Gaussian white noise distributions.

The scale invariance of the noninteracting action [see Eq. (7)] mandates the following scaling ansatz: \( \tau \to \tau', (x,y) \to e^l (x,y) \) and \( x_3 \to e^{l/2} x_3 \), followed by the rescaling of the field operator \( \psi \to e^{-5l/4} \psi \), where \( l \) is the logarithm of running RG scale. The scaling dimension of the tuning parameter \( \Delta \) is then given by \( [\Delta] = 1 \), implying that \( \Delta \) is a relevant perturbation at the WSM-insulator QCP, located at \( \Delta = 0 \). The scaling dimension of the tuning parameter \( \Delta \) plays the role of the correlation length exponent (\( \nu \)) at this QCP, implying \( \nu = 1 \). In the presence of disorder, as we show in Appendix A, the Harris stability criterion [27] can be generalized for the WSM-insulator QCP with the quantum-critical theory of the form given by Eq. (9), but in a system with the topological or monopole charge \( c \) [see Eq. (A1)]. The generalized Harris criterion then suggests that WSM-insulator QCP in clean system remains stable against sufficiently weak disorder only if
\[
\nu > \frac{2}{d_s}, \quad \text{with} \quad \frac{2}{d_s} = \frac{4c}{(4 + c)},
\]
and \( d_s \) as the effective spatial dimensionality of the system under the coarse graining procedure. At the WSM-insulator QCP \( \nu = 1 \), and the critical excitations residing at \( \Delta = 0 \) are therefore stable against weak disorder when \( c = 1 \) (regular WSM). We next analyze the effects of disorder on the WSM-insulator QCP using a RG approach. The same outcome can be arrived at from the computation of inverse scattering life-time (1/\( \tau \)) within the framework of self-consistent Born approximation [see Appendix I].

A. Perturbative RG analysis

After performing the disorder averaging in the action [see Eq. (5)] within the replica formalism, we arrive at the replicated Euclidean action

\[
\tilde{S} = \int d\tau d^2 x \psi^\dagger_a [\partial_\tau - iv (\partial_x \sigma_1 + \partial_y \sigma_2) + \sigma_3 (-i \nu b_0 \partial_\tau - \Delta)] \psi_a - \int d\tau d\tau' d^2 x \psi^\dagger_j \left[ \frac{\Delta_0}{2} (\psi^\dagger_j \psi_j)_{(x,\tau)} + \frac{\Delta_2}{2} (\psi^\dagger_j \sigma_2 \psi_j)_{(x,\tau)} \right],
\]

where \( a, b \) are replica indices. Notice that here we have replaced \( k_2^2 \to k_2^3 \), with \( n \) as an even integer so that such deformation of spectrum does not change the symmetry of the system. We will show that such deformation of the quasiparticle spectrum allows us to control the perturbative RG calculation in terms of disorder coupling. The above imaginary-time action \( \tilde{S} \) remains invariant under the space-time scaling \( (x,y) \to e^l (x,y) \), \( x_3 \to e^{l/n} x_3 \) and \( \tau \to e^{l/2} \tau \). At the bare level the scale invariance of the free part of the action requires the field renormalization factor \( Z_\psi = e^{-(2 + 1/n)l} \) and \( \psi \to Z_\psi^{-1/2} \psi \). From this scaling analysis we immediately find that the scaling dimension of disorder couplings is \( [\Delta_j] = -1/n \), for \( j = 0, \perp, z \). Therefore, at the WSM-insulator QCP, characterized by \( n = 2 \), disorder is an irrelevant perturbation, in accordance with the prediction from the generalized Harris criterion, implying the

\[
\beta_X = -X (\Delta_0 + 2\Delta_\perp + \Delta_z) = (1-z)X,
\]

\[
\beta_\Delta = \Delta [1 + \Delta_0 - 2\Delta_\perp + \Delta_z],
\]

stability of this QCP against sufficiently weak randomness. Note that disorder couplings are marginal in a hypothetical limit \( n \to \infty \), for which the system effectively becomes a two-dimensional Weyl semimetal. Therefore, perturbative analysis in the presence of generic disorder is controlled via an \( \epsilon_n \)-expansion, where \( \epsilon_n = 1/n \), about \( n \to \infty \), following the spirit of \( \epsilon \)-expansions about upper or lower critical dimension [76] and infinite monopole charge [77,78].

Upon integrating out the fast Fourier modes within the momentum shell \( \Lambda e^{-l} < k_\perp < \Lambda \), where \( k_\perp = \sqrt{k_2^2 + k_2^3} \), \( 0 < k_2^2 < \infty \) and accounting for perturbative corrections to one-loop order (see Fig. 6), we arrive at the following flow equations

\[
\beta_X = -X (\Delta_0 + 2\Delta_\perp + \Delta_z) = (1-z)X,
\]

\[
\beta_\Delta = \Delta [1 + \Delta_0 - 2\Delta_\perp + \Delta_z],
\]
chemical potential

scalar mass

axial potential

spatial tensor

current
	pseudo-scalar mass

temporal tensor

Table III: Various types of disorder represented by fermionic bilinears \((j = 1, 2, 3)\), together with their symmetries under pseudo time-reversal \((T)\), parity \((P)\), continuous chiral rotation \((U_c)\) and charge-conjugation \((C)\). The disorder couplings are represented by \(\Delta_N\) and \(\Sigma_{\mu\nu} = [\gamma_\mu, \gamma_\nu]/(2i)\). Note that true time-reversal symmetry in WSM is already broken, the pseudo time-reversal symmetry \(T\) is generated by an anti-unitary operator \(\gamma_0\gamma_2 K\), where \(K\) is complex conjugation, such that \(T^2 = -1\) (The true time-reversal operator is \(\gamma_1\gamma_0 K\)). The parity operator is \(P = \gamma_0\), while the charge-conjugation operator is \(C = \gamma_2\). The continuous chiral symmetry \((U_c)\) is generated by \(\gamma_5\), the generator of translational symmetry in the continuum limit in a clean Weyl semimetal [64]. The Hermitian \(\gamma\) matrices satisfy standard anti-commutation relation \([\gamma_\mu, \gamma_\nu] = 2i\delta_{\mu\nu}\) for \(\mu, \nu = 0, 1, 2, 3, 5\), and for explicit representation of \(\gamma\)-matrices see Sec. [IV]A. Here \(\gamma\) and \(\hat{\gamma}\) signify even and odd under a symmetry operation, respectively. With a slightly different tight-binding model, where \(N_f(k) = t\cos(k)\) and \(N^{(3)}_f(k) = \left[\sin(k1\alpha) + \sin(k2\alpha) - 2\sin(k3\alpha)\right]\) [see Eq. [9]], the axial current corresponds to magnetization, temporal and spatial tensors to spin-orbit and axial magnetization, respectively. However, such microscopic details do not alter any physical outcome.

are \(\nu^{-1} = \epsilon_n\) determining the relevance of disorder coupling \(\Delta_0\), which drives the anisotropic critical semimetal [described by \(H_Q(0)\)] into a diffusive metallic phase, and \(\nu^{-1} = 1\) that determines the relevance of the tuning parameter \(\Delta\), controlling the WSM-insulator transition. The DSE for critical semimetal-metal QPT is \(z = 1 + \frac{\epsilon_n}{2}\). Therefore, for a three-dimensional anisotropic critical semimetal-metal QPT, setting \(\epsilon_n = 1/2\), the critical exponents are \(\nu_M = 2\) and \(z = 1.25\).

The RG flow and the resulting phase diagrams are shown in Fig. 7(a) and 7(b) respectively. At the multicritical point the average DOS scales as \(g(E) \sim |E|^{d/\nu - 1} \approx |E|^{2}\) to one-loop order. Beyond the critical strength of disorder system becomes a metal where the average DOS at zero energy \([g(0)]\) is finite and the order parameter exponent \(\beta = (d - z)\nu = 3.5\) determines the scaling of \(g(0)\) according to \(g(0) \sim \delta^2 = \delta^{3.5}\) in the metallic phase, where \(\delta = (\Delta_0 - \Delta_0^*)/\Delta_0^*\) is the reduced disorder coupling from the critical one at \(\Delta_0 = \Delta_0^*\). Next we numerically demonstrate (a) stability of WSM-insulator QCP at weak disorder, (b) emergence of a metallic phase through a MCP at finite disorder coupling that masks the direct transition between WSM and insulator by numerically computing the average DOS using the kernel

\[
\begin{align*}
\Delta_0 &= -\epsilon_n \Delta_0 + 2\Delta_0 \left(\Delta_0 + 2\Delta_\perp + \Delta_z\right) \\
\beta\Delta_\perp &= -\epsilon_n \Delta_\perp + 2\Delta_0 \Delta_\perp \\
\beta\Delta_z &= -\epsilon_n \Delta_z + 2\Delta_3 \left(2\Delta_\perp - \Delta_0 - \Delta_z\right) + 4\Delta_0 \Delta_\perp,
\end{align*}
\]

in terms of dimensionless parameters

\[
\hat{\Delta} = \frac{\Delta}{v \Delta}, \quad \hat{\Delta}_j = \Delta_j \left[\frac{\Lambda j}{(2\pi)^2 b_n^2 v^2 - \epsilon_n}\right],
\]

for \(X = v, b_n, j = 0, \perp, z\), \(\beta_Q = dq/Q\) is the \(\beta\)-function for the running parameter \(Q\), and for brevity we omit the hat notation in Eq. (11). In the above flow equations, we have kept only the leading divergent contribution that survives as \(n \to \infty\). Inclusion of subleading divergences yields only nonuniversal corrections, as shown in Appendix [E]. The \(\beta\)-function for in-plane Fermi velocity \((v)\) and \(b_n\) leads to a scale dependent DSE

\[
z(I) = 1 + (\Delta_0 + 2\Delta_\perp + \Delta_z) (I).
\]
Figure 8: Analysis of average density of states (DOS) in various regimes along the black dashed line shown in Fig. 2 (left). Recall the black dashed line for weak disorder defines the phase boundary between the WSM and insulator, while when extended into the metallic phase [the red shaded regime in Fig. 2 (left)] captures the instability of critical excitations residing at the WSM-insulator QCP toward the formation of a metallic phase. (a) Scaling of average DOS at zero energy \( \langle \rho(0) \rangle \) along the blacked dashed line as a function of increasing disorder (\( W \)), showing that \( \rho(0) \) remains pinned at zero up to a critical strength of disorder \( W_c = 1.20 \pm 0.05 \). (b) Scaling of average DOS at finite energy \( \langle \rho(E) \rangle \) around the multi-critical point residing in the two dimensional coupling constant space (\( m_z, W \)), indicating the dynamic scaling exponent for critical excitation-metal QPT is \( z = 1.23 \pm 0.05 \). (c) Scaling of \( \rho(0) \) along the black dashed line inside the metallic phase indicating that correlation length exponent for critical excitation-metal QPT is \( \nu = 1.98 \pm 0.05 \). Details of the data analysis are presented in Sec. III B.

B. Scaling of density of states near WSM-insulator QCP: Numerical demonstration of the MCP

Before we discuss the scaling behavior of the average DOS along the WSM-insulator phase boundary and inside the metallic phase, setting in through the instability of critical semimetallic phase, let us point out some crucial subtle issues associated with such analysis. Note that the average DOS of the critical semimetal [described by \( \hat{H}_Q(0) \) in Eq. (6)] vanishes as \( \rho(E) \sim |E|^{3/2} \), while that in the WSM phase vanishes as \( \rho(E) \sim |E|^2 \). But, in the insulating phase average DOS displays hard gap. Based on scaling analysis we expect WSM, insulator and the critical semimetal to be stable against sufficiently weak disorder. We exploit these characteristic features to pin the WSM-insulator phase boundary for weak disorder. On the other hand, for stronger disorder onset of a metallic phase can be identified from the existence of finite average DOS at zero energy. Following these diagnostic tools we arrive at the phase diagram of a Weyl materials residing in the close proximity to the WSM-insulator QPT; see Fig. 2 (left). We are ultimately interested in exposing the existence of a MCP in the \((m_z, t_z)\) plane [the red dot in Fig. 7(a)] which has two relevant directions. One of them controls critical semimetal-metal QPT, while the other one drives WSM-insulator QPT. Since we consider the former transition, our focus will be restricted on the black dashed line shown in Fig. 2.

More specifically, we here compute the average DOS by employing the kernel polynomial method [69] starting with the tight-binding model, introduced in Eqs. (2), (3), and (6), and staying in the close vicinity of \( m_z/t_0 = 0.5 \) and \( t_z/t_0 = 0.5 \) (see the phase diagram in Fig. 3). The tight-binding model is implemented on a cubic lattice with periodic boundary conditions in all three directions and the linear dimensionality of the system in each direction is \( L = 140 \). Even though average DOS is a self-averaged quantity, we perform average over 20 random disorder realization to minimize the residual statistical fluctuations, compute 4096 Chebyshev moments and take trace over 12 random vector to obtain average DOS. For the sake of simplicity we here account for only random charge impurities. Potential disorder is distributed uniformly and randomly within the range \([-W, W]\). The scaling of average DOS can be derived in the following way.

Since we are following only one relevant direction associated with the MCP, effectively it can be treated as a simple QCP across which various physical observables (such as average DOS) display single parameter scaling. Note that total number of states \( N(E, L) \) in a \( d \)-dimensional system of linear dimension \( L \), below the energy \( E \) is proportional to \( L^d \), and in general is a function of two dimensionless parameters \( L/\xi \) and \( E/E_0 \). Here, \( \xi \sim \delta^{-\nu} \) is the correlation length that diverges at the

---

3 We also note that following the same spirit of RG analysis, controlled via “band-flattening” can also be applied to address the effect of randomness deep inside the WSM phase. We, however, relegate that discussion to Appendix G.
QCP, located at $\delta = 0$, where $\delta = \frac{W - W_c}{W}$ is the reduced distance from the QCP, located at $W = W_c$. Consequently, the correlation energy, defined as $E_0 \sim \delta^{\nu z}$ vanishes as the QCP is approached from either side of the transition \cite{70}. Following the standard formalism of scaling theory we then can write

$$N(E, L) = \left( \frac{L}{\xi} \right)^{d} G \left( \frac{E}{\delta \nu z}, \frac{L}{\delta - \nu} \right),$$

(13)

where $G$ is an universal but unknown scaling function. Therefore, from the definition of average DOS $\varrho(E, L) = L^{-d} dN(E, L)/dE$ we arrive at the following scaling form

$$\varrho(E, L) = \delta^{\nu (d-z)} F \left( |E|\delta^{-\nu z}, \delta L^{1/\nu} \right),$$

(14)

where $F$ is yet another universal, but typically unknown scaling function. However, we can access the behavior of the scaling function in different regimes along the black dashed line shown in Figs. 2 (left), which we exploit to compute critical exponents characterizing the critical semimetal-metal QPT across the MCP. In the final step we have used the fact that average DOS remains particle-hole symmetric, but on average. Note we will use exactly the same scaling function deep inside the WSM phase in the presence of generic disorder, discussed in Sec. VI.

First of all, notice that average DOS $\varrho(0)$ is pinned to zero along the phase boundary between the WSM and insulator for weak enough disorder, as shown in Fig. S(a). Therefore, critical semimetal separating these two phases remains stable against weak disorder and the nature of the WSM-insulator direct transition remains unchanged for weak enough randomness. However, beyond a critical strength of disorder, $W_c \approx 1.20 \pm 0.05$, $\varrho(0)$ becomes finite and metallicity sets in through the MCP, see Figs. 2 (left) and S(a). Beyond this point there exists no direct transition between the WSM and an insulator. Also note that for $W < W_c$, $\varrho(E) \sim |E|^{1.5}$ as shown in Fig. 2 (right).

Now we consider very close proximity to the MCP, located at $W = W_c$ along the disorder axis. At this MCP average DOS becomes independent of $\delta$, yielding $F(x) \sim x^{\frac{d}{2} - 1}$. By comparing $\varrho(E)$ with $E$, obtain the DSE associated with critical semimetal-metal QPT to be $z = 1.23 \pm 0.05$, see Fig. S(b).

Next we move into the metallic phase, but continue to follow the black dashed line from Fig. 2 (left). In the metallic phase $\varrho(0)$ becomes finite [see Fig. S(a)]. Thus to the leading order $F(x) \sim x^{d}$ and consequently $\varrho(0) \sim \delta^{(d-z)\nu}$. With the prior notion of $z = 1.23 \pm 0.05$, now by comparing $\varrho(0)$ vs. $\delta$ we obtain the CLE at the MCP associated with the critical semimetal-metal QPT to be $\nu = 1.98 \pm 0.05$, as shown in Fig. S(c).

Therefore, numerically extracted values of two critical exponents, namely $\nu$ and $z$, at the MCP associated with the critical semimetal-metal QPT match extremely well with the field theoretic prediction obtained from an $\varepsilon_m$-expansion introduced in this work, which allows to control the RG calculation by tuning the flatness of the quasiparticle dispersion along the $k_z$ direction: a controlled ascent from two spatial dimension.

We now discuss two different types of data collapses across the disorder-driven MCP. The results are shown in Fig. 9. First we focus on the largest system with $L = 140$. From Eq. (14), upon neglecting the finite size effects, we compare $\varrho(E)\delta^{-(d-z)\nu}$ vs. $|E|\delta^{-\nu z}$ along the black line from Fig. 2 (left). With numerically obtained values of $\nu$ and $z$ we find that all data nicely collapse onto two branches (corresponding to the anisotropic semimetal and metallic sides of the QPT), which meet in the critical regime, as shown in Fig. 9(a). Next we compare the average DOS at zero energy in the metallic phase, namely $\varrho(0)L^{d-z}$ vs. $L^{1/\nu}$\delta in systems of different sizes ($L$), as shown in Fig. 9(b). We also obtain excellent finite-size data collapse for a wide range of system sizes using already numerically extracted values of $\nu$ and $z$. Therefore, field-theoretic predictions and numerical findings across the disorder-driven MCP are in good agreement with each other. Next we address the effects of disorder inside the WSM phase by pursuing complimentary field theoretic and numeric approaches.

**IV. DIRTY WEYL SEMIMETAL: MODEL AND SCALING ANALYSIS**

In this section, we set up the field theoretical framework to analyze the role of disorder when the system is deep inside the WSM phase. We will introduce the notion of two different $\varepsilon$-expansions: (a) an $\varepsilon_m$-expansion about a critical disorder distribution, where $\varepsilon_m = 1 - m$ with
Gaussian white noise distribution recovered as \( m \to 0 \); (b) an \( \epsilon_d \)-expansion, with Gaussian white noise distribution from outset, about the lower critical dimension \( d_c = 2 \) for WSM-metal QPT, where \( \epsilon_d = d - 2 \), and therefore for three spatial dimensions \( \epsilon_d = 1 \).

### A. Hamiltonian and action

The effective low energy description of WSM can be obtained by expanding the lattice Hamiltonian [see Eq. (3)] around the Weyl nodes located at \( \mathbf{K}^0 = (0,0,\pm k_0^0) \), with \( k_0^0 = \cos^{-1}(\frac{m}{2}) \). The resulting low energy Hamiltonian reads

\[
H_W = \tau_0 \otimes v(k_x \sigma_1 + k_y \sigma_2) + \tau_3 \otimes \sigma_3 v z k_z, \tag{15}
\]

where \( v = ta \), \( v_z = \frac{a}{\sqrt{T^2 - m_z^2}} \), and the momentum is measured from the Weyl nodes. For simplicity we hereafter take the Fermi velocity to be isotropic, \( v = v_z \), so that the low energy Hamiltonian becomes rotationally symmetric. Upon performing a unitary rotation with \( U = \sigma_0 \otimes \sigma_3 \), the above Hamiltonian assumes a quasirelativistic form \( H_W = i \gamma_0 \gamma_j v k_j \), where \( \gamma_0 = \tau_1 \otimes \sigma_0 \), \( \gamma_j = \tau_2 \otimes \sigma_j \) for \( j = 1,2,3 \) are mutually anti-commuting \( 4 \times 4 \) Hermitian matrices, and summation over repeated spatial indices is assumed. To close the Clifford algebra of five mutually anticommuting matrices we define \( \gamma_5 = \tau_3 \otimes \sigma_0 \). Two sets of Pauli matrices \( \sigma_\mu \) and \( \tau_\nu \) respectively operate on spin/pseudospin and valley or chiral (left and right) indices. The low energy effective Hamiltonian enjoys variety of emergent discrete and continuous symmetries. The above Hamiltonian is invariant under a pseudo-time-reversal symmetry, generated by antiunitary operator \( T = \gamma_0 \gamma_2 K \), where \( K \) is the complex conjugation, a charge conjugation symmetry, generated by \( C = \gamma_2 \), and parity or inversion symmetry generated by \( P = \gamma_0 \). Furthermore, the Hamiltonian [see Eq. (15)] also possesses a global chiral \( U(1) \) symmetry, generated by \( \gamma_0 \), which in the low energy limit corresponds to the generator of translational symmetry \( \delta \).

To incorporate the effects of disorder we consider the following minimal continuum action for a dirty WSM

\[
S = \int d^d x d\tau \left[ \bar{\Psi} (\gamma_0 \partial_\tau + v \gamma_j \partial_j) \Psi - \varphi_N(\bar{\Psi} N \Psi) \right], \tag{16}
\]

with \( x \) as \( d \)-dimensional spatial coordinates, the four-component spinor \( \Psi^\dagger = (u^\dagger_{\tau,\uparrow}, u^\dagger_{\tau,\downarrow}, \bar{u}_{\tau,-\uparrow}, \bar{u}_{\tau,-\downarrow}) \), where \( u_{\tau,\sigma} \) is the fermionic creation operator near the Weyl point at \( \tau \mathbf{k}^0 \) for \( \tau = \pm \) (left/right) and with spin \( \sigma = \uparrow, \downarrow \), while \( \bar{\Psi} = \Psi^\dagger \gamma_0 \), as usual. Various disorder fields \( \varphi_N \), coupled to the fermion bilinears, are realized with different choices of \( 4 \times 4 \) matrices, \( N \), as shown in Table III.

Notice that the matrices associated with four types of disorder anticommute with \( \gamma_5 \) and represent chiral symmetric disorder, while for the other four types of disorder \( [N, \gamma_5] = 0 \) and the corresponding disorder vertex breaks the \( U(1) \) chiral symmetry. As we demonstrate in this paper, this global chiral symmetry plays a fundamental role in classifying the disorder-driven WSM-metal QPTs.

### B. \( \epsilon_m \) expansion in three dimensions

We assume that the disorder field obeys the distribution \[38,79\]

\[
\langle \varphi_N(x) \varphi_N(y) \rangle = \Delta_N \frac{1}{|x - y|^{d-m}}, \tag{17}
\]

or in the momentum space

\[
\langle \varphi_N(q) \varphi_N(0) \rangle = \tilde{\Delta}_N \frac{1}{|q|^m}, \tag{18}
\]

and the limit \( m \to 0 \) corresponds to the Gaussian white noise distribution, which we are ultimately interested in. This form of the white noise distribution stems from the following representation of the \( d \)-dimensional \( \delta \)-function \[43\]

\[
\delta^{(d)}(x - y) = \lim_{m \to 0} \frac{\Gamma\left(\frac{d-m}{2}\right)}{2^{m-1} \pi^{d/2} \Gamma(m/2)} \frac{1}{|x - y|^{d-m}}. \tag{19}
\]

We now carry out the scaling analysis of the continuum action for a WSM given by Eq. (16). The scaling dimensions of the momentum and frequency are \([d] = 1\), and \([\omega] = z\). The form of the Euclidean action [see Eq. (16)] then implies that the engineering scaling dimension of the fermionic field \( [\Psi] = d/2 \) and \([v] = z - 1\), while the scaling dimension of the disorder field is \([\varphi_N] = z + \eta_{\varphi N}\), since the engineering dimension of the disorder field is equal to the DSE \( z \) for any choice of \( N \), and \( \eta_{\varphi N} \) is its anomalous dimension. Eq. (17) then yields

\[
[\Delta_N] = 2(z + \eta_{\varphi N}) - d + m. \tag{20}
\]

Due to linearly dispersing low-energy quasiparticles, a WSM corresponds to \( z = 1 \) fixed point, and in \( d = 3 \) the engineering dimension of the disorder strength is \([\Delta_N] = m - 1\). A first implication of this result is that the white noise disorder, \( m = 0 \), is irrelevant close to the WSM ground state in \( d = 3 \). Second, for \( m = 1 \), the disorder is marginal and we use that to introduce the deviation from this value as an expansion parameter \( \epsilon_m = 1 - m \).

The \( \beta \)-function (infrared) for the disorder coupling \( \Delta_N \) in the \( \epsilon_m \) expansion is given in terms of its scaling dimension in Eq. (20), yielding

\[
\beta_{\Delta_N} = \Delta_N [-\epsilon_m + 2(z - 1) + 2\eta_{\varphi N}], \tag{21}
\]

in \( d = 3 \). Therefore, to obtain the explicit form of this \( \beta \)-function in terms of the disorder couplings, we have to compute the DSE and the anomalous dimension of the disorder field. The former is obtained from the fermion self-energy with the diagram shown in Fig. 10(a), while
where \( \epsilon_m = m - 1 \) expansion. Evaluations of these two diagrams are shown in Appendix C. Here, solid (dashed) lines represent fermion (disorder) fields.

The latter is found from the vertex diagram in Fig. 10(b). Evaluation of these two diagrams has been carried out using field-theoretic method (see Appendix C). Alternatively, one may choose to integrate out the fast modes within the momentum shell \( \Lambda e^{-1} < k < \Lambda \), with \( \Lambda \) as an ultraviolet cutoff in the momentum, to arrive at the RG flow equations for \( \Delta_N \).

### 1. Self-energy and dynamic scaling exponent

We first show the computation of the self-energy diagram, shown in Fig. 10(a), yielding the dynamical exponent and the anomalous dimension for the fermion field within the regularization scheme defined by the parameter \( \epsilon_m = 1 - m \), the deviation from the critical disorder distribution. All the integrals are therefore performed in \( d = 3 \). The divergent part of the integral appears as a pole \( \sim 1/\epsilon_m \), analogously to the case of the dimensional regularization where the deviation from the upper or lower critical space-time dimension plays the role of an expansion parameter. To find renormalization constants, we use minimal subtraction, i.e. we keep only divergent part appearing in the corresponding diagrams.

The action [see Eq. (16)] without the disorder yields the inverse free fermion propagator \( G_0^{-1}(i\omega, k) = i(\gamma_0\omega + \nu_0\gamma_j k_j) \), with \( \nu_0 \) as the bare Fermi velocity. Taking into account the self-energy correction, the inverse dressed fermion propagator is

\[
G^{-1}(i\omega, k) = G_0^{-1}(i\omega, k) + \Sigma(i\omega, k),
\]

with \( \Sigma(i\omega, k) \) as the self-energy. After accounting for all possible disorders, we arrive at the following compact expression for the self-energy (see Appendix C for details)

\[
\Sigma(i\omega, k) = i\nu_0\omega \left( \frac{f_1(\Delta_j)}{\epsilon_m} \right) + i\nu_0\gamma_j k_j \left( \frac{f_2(\Delta_j)}{3\epsilon_m} \right),
\]

where

\[
f_1(\Delta_j) = \Delta V + \Delta A + 3\Delta M + 3\Delta C + 3\Delta_{SO} + 3\Delta_{AM},
\]

\[
f_2(\Delta_j) = -\Delta V - \Delta A + 2\Delta M + 3\Delta_{SO} + 3\Delta_{AM}.
\]

Figure 11: (a) The RG flow diagram and (b) the phase diagram in the \( \Delta_V - \Delta_A \) plane, for \( \epsilon_m = 1 \), obtained from Eq. (35). Here \( \Delta_V \) and \( \Delta_A \) are respectively the strength of potential and axial potential disorder. The red line in (a) corresponds to the line of quantum critical points [see Eq. (39)] that in turn defines the phase boundary between the Weyl semimetal and metallic phases, as shown in panel (b). A similar flow and phase diagram is obtained from the RG calculation performed within the framework of an \( \epsilon_d \) expansion [see Eq. (48)].

\[
+ \Delta_S + \Delta_{PS},
\]

\[
f_2(\Delta_j) = -\Delta V - \Delta A + \Delta M + \Delta C - \Delta_{SO} - \Delta_{AM} + \Delta_S + \Delta_{PS},
\]

with \( \hat{\Delta}_j = \Delta_j k^m/(2\pi^2 v^2) \) as the dimensionless disorder strength, and for brevity we here drop the hat symbol in the final expression. From the above expression of the self-energy, together with the renormalization condition \( G^{-1}(\omega, k) = Z_V(i\gamma_0\omega + Z_v\nu_0\gamma_j k_j) \), with \( v \) as the renormalized Fermi velocity, we arrive at the expression for the fermion-field renormalization \( (Z_V) \) and velocity renormalization \( (Z_v) \)

\[
Z_V = 1 + \frac{f_1(\Delta_j)}{\epsilon_m}, \quad Z_v = 1 - \frac{1}{\epsilon_m} \left[ f_1(\Delta_j) - \frac{f_2(\Delta_j)}{3} \right].
\]

This equation then yields the anomalous dimension for the fermion field

\[
\eta\bar{\nu} = -\sum_j \frac{d\ln Z_V}{d\Delta_j} \beta_{\Delta_j}.
\]

Furthermore, the renormalization factor \( Z_v \) enters the renormalization condition for the Fermi velocity \( Z_v v = \nu_0 \). Using Eq. (23), together with \( \beta_{\Delta_N} = -\epsilon_m \Delta_N + O(\Delta_j^2) \), we find

\[
\beta_v = -\frac{1}{3} v \left[ 3f_1(\Delta_j) - f_2(\Delta_j) \right].
\]

Finally, the \( \beta \)-function of the Fermi velocity is \( \beta_v = (1 - z)v \), which together with Eq. (28) determines the DSE

\[
z = 1 + \frac{1}{3} \left[ 3f_1(\Delta_j) - f_2(\Delta_j) \right].
\]
2. Vertex correction: Anomalous dimension of disorder field

We now turn to the vertex correction due to the disorder, shown in Fig. 10(b), which yields the anomalous dimension of the disorder field. As shown in Appendix C, the vertex represented by the matrix \( N \) receives the correction of the form

\[
V_N(k) = \sum_M [M \gamma_j N \gamma_j M] \Delta_M M \beta \frac{3 \epsilon_m}{\beta \Delta_j}.
\]  

The corresponding renormalization condition that determines the renormalization constant \( Z_{\varphi N} \) for the disorder field reads

\[
Z_{\varphi N} N + V_N = N,
\]

with \( Z_{\varphi} \) given by Eq. (26). The above condition in turn yields the anomalous dimension of the disorder field as

\[
\eta_{\varphi N} = - \sum_j \frac{d \ln Z_{\varphi N}}{d \Delta_j} \beta \Delta_j,
\]

which we then use to write the explicit form of the \( \beta \)-function Eq. (21) in terms of the disorder couplings.

C. \( \epsilon_d \)-expansion about \( d = 2 \)

Alternatively, one may take the Gaussian white noise distribution in Eq. (17) with \( m \to 0 \) from the outset. In that case, the engineering dimension of the disorder coupling is equal to \( 2 - d \), since \( z = 1 \) in a clean WSM. Therefore, \( d = 2 \) is the lower critical dimension in the problem and we can use \( \epsilon_d = d - 2 \) as an expansion parameter, following the spirit of \( \epsilon \)-expansion [30, 36, 37, 40, 41, 44, 55, 76]. In this scheme, after performing the disorder averaging using the replica method, the imaginary time action assumes a similar form of Eq (10).

Within the framework of the \( \epsilon_d \) expansion only the temporal (frequency-dependent) component of self energy acquires a disorder-dependent correction to the leading order. The self-energy correction due to disorder reads as

\[
\Sigma(i\omega, k) = i\gamma_0 \omega \left( \frac{f_1(\Delta_j)}{\epsilon_d} \right),
\]

with the function \( f_1(\Delta_j) \) given by Eq. (24), and \( \Delta_j \Lambda^d/(2\pi v^2) \to \Delta_j \). This result is obtained from Eq. (C1) with \( d = 2 + \epsilon_d \) and \( m = 0 \). As a result, the field renormalization factor \( Z_{\varphi} = 1 + f_1(\Delta_j)/\epsilon_d \) and the velocity renormalization factor is \( Z_v = 1 - f_1(\Delta_j)/\epsilon_d \). Using the renormalization condition \( Z_v v = v_0 \), together with \( \beta \Delta_N = -\epsilon_m \Delta_N + O(\Delta_j^2) \), we obtain the leading order RG flow equation for the Fermi velocity

\[
\beta_v = v(1 - z) = -v f_1(\Delta_j),
\]

which yields a scale dependent dynamic exponent \( z = 1 + f_1(\Delta_j) \). The seemingly different expressions for the flow equation and DSE in these two schemes stems from underlying different methodology of capturing the ultraviolet divergences of various diagrams. However, such details do not alter any physical outcome.

V. CHIRAL SYMMETRIC OR INTRA-NODE DISORDER

We first focus on chiral-symmetric disorders. For a single pair of Weyl fermions there are four such disorders, namely chemical potential, axial potential, current and axial current disorders, as shown in Table (11). With appropriate lattice model axial current disorder corresponds to magnetic impurities and from here onward we use this terminology. We will address the effect of weak and strong chiral symmetric disorder using both \( \epsilon_m \) and \( \epsilon_d \) expansions.

A. \( \epsilon_m \) expansion

Let us first analyze this problem pursuing the \( \epsilon_m \) expansion. Using Eqs. (21), (29), (31) and (32), we obtain the following RG flow equations for the coupling constants to the leading order in \( \epsilon_m \),

\[
\begin{align*}
\beta_{\Delta_V} &= \Delta_V \left[ -\epsilon_m + \frac{8}{3} (\Delta_V + \Delta_A) + \frac{16}{3} (\Delta_C + \Delta_M) \right], \\
\beta_{\Delta_A} &= \Delta_A \left[ -\epsilon_m + \frac{8}{3} (\Delta_V + \Delta_A) + \frac{16}{3} (\Delta_C + \Delta_M) \right], \\
\beta_{\Delta_M} &= \epsilon_m \Delta_M, \quad \beta_{\Delta_C} = -\epsilon_m \Delta_C.
\end{align*}
\]  

Figure 12: (a) The renormalization group flow diagram and (b) corresponding phase diagram in the \( \Delta_X - \Delta_Y \) plane, where \( X = V, A \) and \( Y = C, M \) obtained from Eq. (35). In these planes there is only one QCP at \( \Delta_X = 3 \epsilon_m/8, \Delta_Y = 0 \) (the red dot). The phase boundary between the Weyl semimetal and metal in panel (b) is determined by the irrelevant direction, shown by blue dotted line in panel (a).
shown in Fig. 12(b). Importantly, the QPT separating irrelevant
mined by the

and CLE are respectively given by

in Fig. 11(b). At each point of this line of QCPs the DSE

responding phase diagram in the $\Delta_A - \Delta_Y$ plane is shown in Fig. 11(a). The line of QCPs also de-

where the quantities with subscript “$\ast$” represent their

values with subscript “s” represent their critical values for WSM-metal QPT. The RG flow in this

plane is shown in Fig. 11(a). The line of QCPs also de-

determines the WSM-metal phase boundary, and the corre-

sponding phase diagram in the $\Delta_V - \Delta_A$ plane is shown

in Fig. 11(b). At each point of this line of QCPs the DSE

and CLE are respectively given by



\begin{align}
\Delta_{V,s} + \Delta_{A,s} &= \frac{3}{8} \epsilon_m, \\
z &= 1 + \frac{\epsilon_m}{2}, \quad \nu^{-1} = \epsilon_m.
\end{align}

where

Therefore, for the Gaussian white noise distribution, real-

alized for $\epsilon_m = 1$, we obtain $z = 3/2$ and $\nu = 1$ from the

leading order $\epsilon_m$-expansion. If the bare value of either the

chemical potential or axial potential disorder strength is

zero, the quantum-critical behavior is governed by the

QP corresponding to the disorder of a nonvanishing

critical point at $\Delta_A = \epsilon_d/2, \Delta_Y = 0$ (the red dot).

The RG flow equations in the $\Delta_Y - \Delta_A$ plane is shown

in Fig. 12(b). Importantlty, the QPT separating the metallic and the semimetallic phase in any

plane is governed by the QCP located at $\Delta_{X,s} = 3\epsilon_m/8$.

The phase boundary between these two phases is deter-

mined by the irrelevant direction at this QCP. Therefore, across the entire WSM-metal phase boundary in these

planes the universality class of the QPT is identical and

characterized by $z = 1 + \epsilon_m/2$ and $\nu^{-1} = \epsilon_m$.

B. $\epsilon_d$ expansion

The RG flow equations for the chiral symmetric disorder coupling constants within the framework of $\epsilon_d$-

expansion are

\begin{align}
\beta_{\Delta_A} &= \Delta_A \left[-\epsilon_d + 2F_+(\Delta_A)\right] + 8\Delta_M \Delta_C \\
\beta_{\Delta_M} &= \Delta_M \left[-\epsilon_d + \frac{2}{3}F_-(\Delta_M)\right] + \frac{8}{3} (\Delta_C \Delta_V + \Delta_A \Delta_M) \\
\beta_{\Delta_C} &= \Delta_C \left[-\epsilon_d + \frac{2}{3}F_-(\Delta_C)\right] + \frac{8}{3} (\Delta_C \Delta_V + \Delta_A \Delta_M),
\end{align}

where $F_\pm(\Delta) = (\Delta_V + \Delta_A) \pm (\Delta_C + \Delta_M)$. These couple
d flow equations also support only a line of QCPs in the

$\Delta_V - \Delta_A$ plane, as we previously found from Eq. (35)

using $\epsilon_m$-expansion, now determined by

\begin{align}
\Delta_{V,s} + \Delta_{A,s} &= \frac{\epsilon_d}{2},
\end{align}

similar to the one in Eq. (36). The critical exponents at each point of such line of QCPs are $z = 1 + \epsilon_d/2$ and $\nu^{-1} = \epsilon_d$. Therefore, in three spatial dimensions $\epsilon_d = 1$ and we find $z = 3/2$ and $\nu = 1$.

The RG flow diagram and the corresponding phase diagram are similar to the ones shown in Figs. 11(a) and 11(b). Only the location of the line of QCPs and the phase boundary shift in a nonuniversal fashion. The differences in the flow equations (36) and (38), arise from two diagrams shown in Fig. 10(c) and (d), which produce ultraviolet divergent contributions, but only within the $\epsilon_d$ expansion scheme. In the presence of only potential disorder we find $z = 3/2$ and $\nu = 1$.

Notice that if we start with only magnetic or current disorder, the axial disorder gets generated from Feynman diagrams (c) and (d) in Fig. 10. Thus, to close the RG flow equations, we need to account for $\Delta_A$ coupling from the outset, and the resulting RG flow equations read

\begin{align}
\beta_{\Delta_A} &= \Delta_A \left[-\epsilon_d + 2(\Delta_A + 3\Delta_Y)\right] + 4\Delta_Y^2 \\
\beta_{\Delta_Y} &= \Delta_Y \left[-\epsilon_d + \frac{2}{3}(\Delta_Y - \Delta_A)\right] + \frac{8}{3} \Delta_A \Delta_Y,
\end{align}

for $Y = M, C$. The above set of coupled RG flow equations supports only one QCP, located at $\Delta_{A,s} = \epsilon_d/2, \Delta_Y = 0$. The RG flow and the resulting phase diagrams are shown in Figs. 13(a) and (b). Hence, in the presence of magnetic and current disorder the transition to the metallic phase is controlled by the QCP due to axial disorder. If we also take into account the presence of potential disorder, then such a semimetal-metal QPT takes place through one of the points residing on the line of QCPs in the $\Delta_V - \Delta_A$ plane, depending on the bare relative strength of these two disorder couplings.
C. Chiral superuniversality

From the discussion in previous two subsections, we can conclude that in the presence of chiral-symmetric disorder in a WSM, the semimetal-metal QPT takes place either through a QCP or a line of QCPs. The location of the line of QCPs and the resulting phase boundaries are nonuniversal and thus dependent on the RG scheme. However, the universal quantum critical behavior with chiral symmetric disorder couplings is insensitive of these details, at least to the leading order in the expansion parameter, and all QPTs in the four-dimensional hyperplane of disorder coupling constants, are characterized by an identical set of critical exponents, namely $z = 1 + \epsilon/2$ and $\nu^{-1} = \epsilon$, with $\epsilon = 1$. Therefore, emergent quantum critical behavior for strong chiral-symmetric disorder stands as a rare example of superuniversality, and we name it chiral superuniversality. Next we demonstrate emergence of such superuniversality across WSM-metal QPT by numerically analyzing the scaling of average DOS in the presence of generic chiral symmetric disorder.

VI. NUMERICAL DEMONSTRATION OF CHIRAL SUPERUNIVERSALITY

Motivated by the field-theoretic prediction of emergent chiral superuniversality across the WSM-metal QPTs driven by CSP disorder, next we numerically investigate the scaling of average DOS across such QPTs. Since $g(0)$ vanishes and is finite in the WSM and metallic phases, respectively, it can be promoted as a bona fide order parameter across the WSM-metal QPT. In addition, such analysis endows an opportunity to extract the critical exponents for the transition nonperturbatively and, at the same time, test the validity of the proposed scenario for chiral superuniversality. The WSM phase is realized from the tight-binding model, defined through Eqs. (3) and (5), which we implement on a cubic lattice of linear dimension $L = 220$ in each direction. Note $g(0)$ is pinned to zero up to a critical strength of disorder $W_c$, and then it becomes finite, indicating the onset of a metallic phase. The value of $W_c$ for a particular disorder is quoted in each panel [and also in Table I and II]. The numerical methodology is described in details in Sec. VI.
Figure 15: Scaling analysis of average density of states (ADOS) in various regimes of the phase diagram of a dirty WSM for all four possible intranode scatterings; plots from top to bottom rows correspond to potential ($\Delta_V$), axial current ($\Delta_A$) and current ($\Delta_C$) disorders. In a certain lattice model axial current disorder also correspond to magnetic impurities, see Table [III]. First column shows the scaling of ADOS $\rho(E)$ vs. $E$ around the critical strength of disorder ($W = W_c$). The second column depicts the scaling of ADOS at zero energy $\rho(0)$ vs. $\delta$, the reduced distance from the critical disorder defined as $\delta = \frac{W - W_c}{W_c}$. In the third column we display $\rho(E)\delta^{-(d-z)\nu} \propto |E|^{(d-z)\nu}$ for weak ($W < W_c$) and strong ($W > W_c$) disorder and $|E| \ll t(\sim 1)$. All data collapse onto two branches. The top branch represents the metallic phase, while the lower one WSM. Note that these two branches meet at large values of $|E|^{(d-z)\nu}$, corresponding to the quantum critical regime. All data in first three columns are obtained from a system of linear dimension $L = 220$. The finite size data collapse inside the metallic phase is shown in the forth column, where we compare $\rho(0)L^{d-z-\psi}$ vs. $\delta L^{1/\nu}$ for $100 \leq L \leq 220$. Notice that all data collapse onto one branch for small to moderate values of $\delta L^{1/\nu}$, with the numerically extracted values of the critical exponents $z$ and $\nu$, quoted in the figure and summarized in Table [II]. The quality of the data collapse progressively worsens for larger values of $\delta L^{1/\nu}$ due to the existence of a second QPT of a three-dimensional dirty Weyl metal into the Anderson insulating phase, discussed in Sec. [IX]. Scaling of ADOS and data analysis are discussed in details in Sec. [VI].

Figure 15: Scaling analysis of average density of states (ADOS) in various regimes of the phase diagram of a dirty WSM for all four possible intranode scatterings; plots from top to bottom rows correspond to potential ($\Delta_V$), axial current ($\Delta_A$) and current ($\Delta_C$) disorders. In a certain lattice model axial current disorder also correspond to magnetic impurities, see Table [III]. First column shows the scaling of ADOS $\rho(E)$ vs. $E$ around the critical strength of disorder ($W = W_c$). The second column depicts the scaling of ADOS at zero energy $\rho(0)$ vs. $\delta$, the reduced distance from the critical disorder defined as $\delta = \frac{W - W_c}{W_c}$. In the third column we display $\rho(E)\delta^{-(d-z)\nu} \propto |E|^{(d-z)\nu}$ for weak ($W < W_c$) and strong ($W > W_c$) disorder and $|E| \ll t(\sim 1)$. All data collapse onto two branches. The top branch represents the metallic phase, while the lower one WSM. Note that these two branches meet at large values of $|E|^{(d-z)\nu}$, corresponding to the quantum critical regime. All data in first three columns are obtained from a system of linear dimension $L = 220$. The finite size data collapse inside the metallic phase is shown in the forth column, where we compare $\rho(0)L^{d-z-\psi}$ vs. $\delta L^{1/\nu}$ for $100 \leq L \leq 220$. Notice that all data collapse onto one branch for small to moderate values of $\delta L^{1/\nu}$, with the numerically extracted values of the critical exponents $z$ and $\nu$, quoted in the figure and summarized in Table [II]. The quality of the data collapse progressively worsens for larger values of $\delta L^{1/\nu}$ due to the existence of a second QPT of a three-dimensional dirty Weyl metal into the Anderson insulating phase, discussed in Sec. [IX]. Scaling of ADOS and data analysis are discussed in details in Sec. [VI].
Figure 16: Scaling analysis of numerically extracted average density of states in various regimes of the phase diagram of a dirty Weyl semimetal, but in the presence of inter-node scattering; each column is identical to the corresponding one in Fig. 15. The plots from top to bottom rows correspond to temporal component tensor (ΔSO), spatial component of tensor (ΔAM), scalar (ΔS) and pseudo scalar (ΔPS) mass disorder [see Table III for definitions]. In a specific lattice model the former two sources of elastic scattering respectively correspond to random spin-orbit coupling and random axial-magnetization. Methods of analysis and system size are identical to the ones in Fig. 15. Final results of our analysis are quoted in Table IV.

A. Numerical analysis with random intra-node scatterers or chiral-symmetric disorder

We begin the discussion on the effects of randomness on WSM by focusing on the intra-node or chiral symmetric disorder. Let us first focus on the quantum critical regime and for now we assume that the system size is sufficiently large so that we can neglect the L-dependence in Eq. (14). In this regime the scaling function must be independent of δ, dictating $F(x) \sim x^{\frac{d-1}{2}}$. Therefore, when $W = W_c$ we compare $\rho(E)$ vs. $E^{\frac{d-1}{2}}$ and extract the DSE $z$. Such analysis for all four possible CSP disorders is shown in the first column of Fig. 15 and numerically extracted values of $z$ are quoted in Table IV. Within the numerical accuracy, we always find $z \approx 1.5$ in excellent agreement with the field-theoretic result, obtained from the leading order $\epsilon$ expansions.

Next we proceed to the metallic side of the transition, where average DOS at zero energy becomes finite. From the scaling function in Eq. (14), we obtain $\rho(0) \sim \delta^{(d-2)/\nu}$. Thus by comparing $\rho(0)$ vs. $\delta$, we extract the CLE $\nu$, using already obtained value of the DSE $z$, as shown in the second column of Fig. 15. The numerically found CLE is also quoted in Table IV and within numerical accuracy $\nu \approx 1$ always, irrespective of the nature of CSP disorder. Once again we find an excellent agreement of numerically extracted values of $\nu$ with the one obtained.
from the leading order $\epsilon$-expansions. These two results strongly support the picture of chiral superuniversality.

To test the quality of our numerical analysis we search for two types of data collapse. First, we compare $\varrho(E)|\delta|^{-\nu(d-z)}$ vs. $|\delta|^{-\nu}|E|$, motivated by the scaling form of average DOS, displayed in Eq. (14). Using numerically obtained values of $\nu$ and $z$, we find that for energies much smaller than the bandwidth ($|E| \ll 1$), all data collapse onto one separate branch for all four disorders, as shown in the third column of Fig. 13. While the top branch corresponds to the metallic phase, the lower one stems from the WSM phase and eventually these two branches meet in the quantum critical regime.

Finally, we demonstrate a finite size data collapse for $\varrho(0)$ for different system sizes ($L$) by focusing on the metallic side of the transition. Setting $E = 0$ in Eq. (14), we obtain $\varrho(0) = L^{z-d}F(0, \delta L^{1/\nu})$. Hence, we compare $\varrho(0)L^{d-z}$ vs. $\delta L^{1/\nu}$ and find an excellent data collapse for $100 < L < 220$, using numerically obtained values of $\nu$ and $z$ for all four disorders, as shown in the fourth column of Fig. 13. The data collapse becomes systematically worse for large values of $\delta$ or stronger disorder due to the existence of a second transition that takes the system from a metallic phase to an Anderson insulator. Therefore, our thorough numerical analysis provides a valuable and unprecedented insight into the nature of the WSM-metal QPTs driven by generic chiral symmetric disorder, and staunchly supports the proposal of an emergent chiral superuniversality across such QPTs.

B. Numerical analysis with random inter-node scatterers or non-chiral disorder

Motivated by the intriguing possibility of realizing an emergent superuniversality we further seek to examine its robustness in the presence of inter-node scattering (also referred as non-chiral disorder). In the simplest version of a Weyl semimetal comprised of only two Weyl nodes there are four sources of internode scattering, highlighted in Table III and their lattice realization is shown in Appendix D. We rely on the scaling of average DOS in the presence of non-chiral disorder as well, and all the parameters and numerical strategies are identical to the ones pursued for chiral symmetric (intranode) disorder. The analyses of average DOS in various regimes of the phase diagram of disordered WSM are performed in the same fashion. The locations of WSM-metal QPT are shown in Fig. 14 (lower row), and numerically extracted values of two critical exponents $\nu$ and $z$ are reported in Table III. The details of the data analysis are displayed in Fig. 16.

Within the numerical accuracy we find that the WSM-metal QPT driven by CSB disorder is also characterized by $\nu \approx 1$ and $z \approx 1.5$. Therefore, the chiral superuniversality appears to be generic in a dirty WSM, and the WSM-metal QPTs belong to the same universality class, irrespective of the nature of impurities. Hence, there is a substantial evidence supporting emergence of superuniversality across WSM-metal QPT. Such an intriguing outcome further motivates us to understand the effect of internode scattering in a WSM from a field theoretic point of view, which we present in the following section by carrying out two different $\epsilon$-expansions, described in Secs. IV B and IV C.

VII. CHIRAL SYMMETRY BREAKING OR INTER-NODE DISORDER

In a WSM constituted by a single pair of Weyl nodes, there are four CSB disorders, namely temporal and spatial components of a tensor disorder, which in a suitable lattice model respectively represents spin-orbit and axial magnetic disorder, as well as scalar and pseudoscalar mass disorder, see Table III. We will address the effects of weak and strong CSB disorder by using both $\epsilon_m$ and $\epsilon_d$ expansions.

A. $\epsilon_m$ expansion

Within the framework of an $\epsilon_m$ expansion the RG flow equations to one-loop order read as

\[ \beta_{\Delta_{SO}} = \Delta_{SO} \left[ -\epsilon_m + \frac{4}{3} (\Delta_{AM} - \Delta_{S}) \right], \]

\[ \beta_{\Delta_{AM}} = \Delta_{AM} \left[ -\epsilon_m + \frac{4}{3} (\Delta_{SO} - \Delta_{P}) \right], \]

\[ \beta_{\Delta_{S}} = \Delta_{S} \left[ -\epsilon_m + \frac{4}{3} (5\Delta_{SO} - 4\Delta_{AM} - 2\Delta_{S} + \Delta_{PS}) \right], \]

\[ \beta_{\Delta_{PS}} = \Delta_{PS} \left[ -\epsilon_m + \frac{4}{3} (4\Delta_{AM} - 4\Delta_{SO} - 2\Delta_{PS} + \Delta_{S}) \right]. \]
where $\Delta = \Delta_1$ different in the sense that while the DSE $z = 1 + \epsilon_m/2$, with $\epsilon = \epsilon_m$ or $\epsilon_d$, is fixed along the entire line of QCPs in the $\Delta_V - \Delta_A$ plane, it varies continuously along the line of QCPs in the $\Delta_V^+ - \Delta_S^-$ plane according to

$$z = 1 + \frac{2}{3} \left[ 5 \Delta_{V,s}^+ + \Delta_{S,s}^+ \right] = 1 + 5 \epsilon_m + 4 \Delta_{S,s}^+,$$

(44)

where the quantity with subscript “$s$” denote the critical value for WSM-metal transition. Such continuously varying DSE leaves its signature in critical scaling of various physical observables, as we discuss below, and qualitatively mimics the picture of Kosterlitz-Thouless transition. Notice that the end point of such line of QCPs on the $\Delta_V^+$ axis reside in the $\Delta_S - \Delta_{AM}$ plane at $\Delta_S = \Delta_{AM} = 3 \epsilon_m/4$, and the RG flow in this plane is shown in Fig. 18(a). The phase diagram of a dirty WSM containing only spin-orbit and axial magnetic disorder in this plane is shown in Fig. 18(b), with $z = 1 + 5 \epsilon_m$, which is directly obtained from Eq. (44) by setting $\Delta_S^+ = 0$. It is worth pointing out that in the $\Delta_S - \Delta_{AM}$ plane the phase boundary between the WSM and metallic phase is set by the irrelevant parameter associated with the QCP, while such QCP percolates through $\Delta_V^+ - \Delta_S^-$ plane in the form of a line of QCPs, it is determined by the relevant direction at each point on the line of QCPs.

**B. $\epsilon_d$ expansion**

Next let us address the effects of CSB disorder within the framework of an $\epsilon_d$ expansion. In this method the RG flow equations become very complicated due to the ultraviolet divergent contribution arising from the class of the Feynman diagrams shown in Fig. 6 (c) and (d), and it is challenging to decode the emergent quantum-critical phenomena. Thus we attempt to unearth critical properties by focusing on various coupling constant subspaces that remain closed under the RG, at least to the leading order. Let us first focus on spin-orbit or axial

\[ \beta_{\Delta_V^+} = -\epsilon_m \Delta_V^+ - \frac{2}{3} \left[ \frac{1}{2} \Delta_V^+ \right]^2 - \Delta_V^+ \Delta_S^+ - \Delta_V^+ \Delta_S^- \]

\[ \beta_{\Delta_V^-} = -\epsilon_m \Delta_V^- - \frac{2}{3} \left[ \Delta_V^- \Delta_S^+ + \Delta_V^+ \Delta_S^- \right] \]

\[ \beta_{\Delta_S^+} = -\epsilon_m \Delta_S^+ - \frac{2}{3} \left[ \Delta_S^+ \Delta_V^+ + 3 (\Delta_S^-)^2 - \Delta_V^+ \Delta_S^- - 9 \Delta_V^- \Delta_S^- \right] \]

\[ \beta_{\Delta_S^-} = -\epsilon_m \Delta_S^- - \frac{2}{3} \left[ \Delta_S^- \Delta_V^+ - 4 \Delta_S^+ \Delta_S^- - 9 \Delta_S^+ \Delta_V^- \right] \]

where $\Delta_V^\pm = \Delta_{SO} \pm \Delta_{AM}$, $\Delta_S^\pm = \Delta_S \pm \Delta_{PS}$. The above set of RG flow equations supports a line of QCPs determined by the equation

$$\Delta_{V,s}^+ = \Delta_{S,s}^+ + \frac{3 \epsilon_m}{2}, \quad \Delta_{V,s}^- = 0, \quad \Delta_{S,s}^- = 0.$$
magnetic disorder. The RG flow equations read
\[
\beta_{\Delta_X} = -\epsilon_d \Delta_X - \frac{2}{3} \Delta_X^2 + 2 \Delta_X \Delta_A,
\]
\[
\beta_{\Delta_A} = -\epsilon_d \Delta_A + 2 \Delta_A^2 - 6 \Delta_A \Delta_X + 4 \Delta_X^2,
\]
where $X = SO, AM$. Notice that even though the bare theory contains only spin-orbit or axial magnetic disorders, the CSP axial disorder gets generated and in order to keep the RG flow equations closed we need to include the latter from the outset. The coupled flow equations support one QCP, located at $\Delta_{X,*} = 9\epsilon_d/10, \Delta_{A,*} = 6\epsilon_d/5$ \cite{30, 31}. The RG flow diagram is shown in Fig. 19(a) and the resulting phase diagram is displayed in Fig. 19(b). Note that QCP obtained in the absence of the CSB disorders, located at $\Delta_{A,*} = \epsilon_d/2$ now becomes unstable in the presence of either spin-orbit or axial magnetic disorder, and a new QCP results from the competition between these two disorders, as mentioned above. This outcome although bears contrast with our previously reported results obtained from $\epsilon_m$ expansion, still shows some qualitative similarities, as we argue below. Notice that the DSE and CLE at the new QCP, shown in Fig. 19(a) are respectively given by
\[
z = 1 + \frac{9}{2} \epsilon_d, \quad \nu^{-1} = \epsilon_d.
\]
As a result the mean DOS at the QCP diverges as $\rho(E) \sim |E|^{-5/11}$ for $\epsilon_d = 1$ or $d = 3$, since $z > d$. Hence, both $\epsilon$-expansions give rise to diverging DOS at the QCP controlled via spin orbit and axial magnetic disorder. Although the calculated values of DSE depend on the RG scheme, to the leading order they do not differ significantly, $z = 6$ for $\epsilon_m = 1$, and $z = 11/2$ for $\epsilon_d = 1$, while $\nu = 1$, is independent of the RG scheme.

C. Mass disorder

We now discuss the role of mass disorder in WSMs. It should be noted that a WSM can be susceptible to two different types of mass disorder (a) scalar mass disorder and (b) pseudo-scalar mass disorder. Both of them break the chiral symmetry, but can be rotated into each other by the generator of the chiral symmetry $\gamma_5$. The flow equation for mass disorder within the framework of an $\epsilon$ expansion reads as
\[
\beta_{\Delta_X} = -\epsilon_j \Delta_X - \tilde{\alpha}_j \Delta_X^2,
\]
\[
\beta_{\Delta_A} = -\epsilon_d \Delta_A + 2 \Delta_A^2 - 6 \Delta_A \Delta_X + 4 \Delta_X^2,
\]
for $X = S, PS$, where $\Delta_\epsilon = \Delta_A - \Delta_X$, $\tilde{\alpha}_m = 8/3$, $\tilde{\alpha}_d = 2$, and respectively $j = m, d$ corresponds to $\epsilon_m$ and $\epsilon_d$ expansions, respectively. Hence, by itself scalar or pseudoscalar mass disorder does not drive any WSM-metal QCP, at least within the leading order in $\epsilon$-expansions. In this regard both $\epsilon_m$ and $\epsilon_d$ expansions yield an identical result.

Finally, we discuss yet another interesting aspect of mass disorder, when it coexists with the axial one. The flow equations in the presence of these two disorders are
\[
\beta_{\Delta_A} = -\Delta_A [\epsilon_j - \tilde{\alpha}_j \Delta_\epsilon], \quad \beta_{\Delta_X} = -\Delta_X [\epsilon_j + \tilde{\alpha}_j \Delta_\epsilon],
\]

D. Why chiral superuniversality is so robust?

Leaving aside the interesting possibilities of realizing such as line of QCPs with continuously varying critical exponents, perhaps the most urgent issue to be addressed is the following: Why does the disorder-driven WSM-metal QPT always display same universality class, characterized by $\nu \approx 1$ and $z \approx 1.5$?

Answer to question in presence of intra-node or chiral-symmetric disorders has already been provided in Sec. \textbf{V}.\textbf{B}.\textbf{2} Note that scaling dimension of any disorder coupling in a $d$-dimensional WSM is $[\Delta_\epsilon] = 2z - d$. But at all CSB disorder driven QCPs controlling the WSM-metal QPT, $z > d$ irrespective of the RG methodology. Therefore, even though the bare values of CSP disorders in lattice-based simulations are set to be zero, as done in the ones discussed in Sec. \textbf{VIB}, they do get generated as we approach the Weyl points through the coarse graining procedure. Ultimately the CSP disorder becomes relevant at CSB disorder driven WSM-metal QCPs. As a result, the dirty system even though initially tends to flow toward the QCPs with $z > d$, described in this section, it flows back toward the chiral symmetric QCP or line of QCPs shown in Fig. 11(a). This is the reason why the WSM-metal QPTs are always characterized by CLE $\nu \approx 1$ and DSE $z \approx 1.5$ (within numerical accuracy), the characteristics of the proposed chiral superuniversality. The above argument is very generic and does not depend on the number of Weyl nodes. Therefore, in any lattice system, we expect WSM-metal QPT to always belong to the chiral superuniversality. This outcome can be anchored from the RG calculation in the presence of all eight possible disorder couplings (since in strong disorder regime all disorders get generated even if the bare coupling for some specific channel is set to be zero), as
shown in Appendix [E] within the framework of both $\epsilon_m$ and $\epsilon_d$ expansions. Such analysis confirms that only the line of QCPs, defined through Eq. [36] or Eq. [39], and shown in Fig. [11(a)] ultimately controls the quantum-critical behavior. This strongly supports the above argument in favor of chiral superuniversality under generic circumstances.

The specific tight-binding model we subscribe in this work [see Sec. II] also contains Wilson mass that bears higher gradient terms, such that $\tau_3 b_\perp (k_x^2 + k_y^2)$, with $b_\perp = t_0 a^2/2$. The scaling dimension of such operator is $[b_\perp] = -z - 2$. Hence, the higher gradient terms are irrelevant at clean WSM fixed point ($[b_\perp] = -1$) as well as at the chiral symmetric line of QCPs ($[b_\perp] = -1/2$), but becomes relevant at pure CSB disorder-driven QCPs (since $z > d > 2$). This is also the reason why chiral superuniversality is such a generic and utmost stable situation.

Nevertheless, we believe pure CSB disorder driven QCPs (with $z > d$) can in principle be realized in a numerical simulation performed in momentum space, where forward/intraneode/CSP scattering processes can be suppressed deliberately and higher gradient terms can be avoided completely. Such an analysis is an interesting exercise of a pure academic interest, and we leave it for a future investigation.

VIII. QUANTUM CRITICAL SCALING OF PHYSICAL OBSERVABLES

As demonstrated in the previous two sections that QPT from a WSM to a diffusive metal can be driven by different types of elastic scatterers, and the critical exponents are remarkably independent of the actual nature of randomness. We here highlight how these exponents can affect the scaling behavior of measurable quantities as the Weyl material undergoes this QPT.

4 We note that the quality of data collapses for CSB disorders, shown in Fig. [16] is slightly less pronounced than those for CSP disorder, displayed in Fig. [13] which can qualitatively be understood in the following way. In the presence of only inter-node scatterers system first tends to flow toward the line of QCPs set by purely CSB disorder, discussed early in this section. Only when disorder gets sufficiently strong the intra-node disorder becomes relevant and the system starts flowing toward the line of QCPs discussed in Sec. [V]. The system then gets stuck in the crossover regime dominated by CSP disorder, and consequently the data collapse (involving finite energy states) becomes slightly less prominent. To achieve equally good quality data collapse even in the presence of CSP disorder we therefore need to subscribe to larger systems, which can be numerically challenging.

5 In spite of the emergent superuniversality, the putative line of QCPs driven by CSB disorders with continuously varying DSE $z > d$ may leave its imprint on the physical observables in the crossover regime before the CSP disorders take over and ultimately the system flows toward the chiral symmetric quantum-critical line with $z = 3/2$ and $\nu = 1$. In that sense the physical observables we discuss in this section can also distinguish between different types of disorder (inter-node vs intra-node).

Residue of quasiparticle pole: As the WSM-metal QCP is approached from the semimetallic phase, the residue of quasiparticle pole vanishes and beyond the critical strength of disorder Weyl fermions cease to exist as sharp quasiparticle excitations, similar to the situation for two-dimensional Dirac fermion Mott insulator QPT in the presence of a strong Hubbard interaction [81][82]. The residue of quasiparticle pole ($\Psi$) vanishes according to

$$Z \sim \left(\frac{\Delta_\ast - \Delta^*}{\Delta_\ast}\right)^{\nu \eta_\Psi} \equiv \delta^{\nu \eta_\Psi},$$

where $\eta_\Psi$ is the fermionic anomalous dimension at the critical point located at the disorder strength $\Delta = \Delta_\ast$. Within the framework of an $\epsilon_d$ expansion $\eta_\Psi = 0$ to the leading order in $\epsilon_d$ and one needs to account for two-loop diagrams to obtain finite $\eta_\Psi$. In contrast, in the $\epsilon_m$ expansion we obtain nontrivial fermionic anomalous dimension even to the one-loop order, and $\eta_\Psi \sim \epsilon$, as shown in Eq. [27]. Therefore, at the WSM-metal QCP, the quasiparticle spectrum displays a branch-cut and the critical point represents a strongly coupled non-Fermi liquid. Alternatively, the residue of quasiparticle pole plays the role of a bona fide order parameter on the semimetallic side. It is worth mentioning that the disappearance of residue of quasiparticle pole has recently been tracked in quantum Monte Carlo simulations for Hubbard model in two-dimensional honeycomb lattice [82], and we can expect that future numerical work can verify our proposed scaling form in Eq. (51) across the disorder driven WSM-metal QPTs. The Fermi velocity scales as $v \sim |\delta|^\nu(z^{-1})$, and since $z > 1$ at the QCP or the quantum-critical line, the Fermi velocity vanishes at the transition to the metallic phase.

Average density of states: The most widely studied physical quantity in numerical simulations across the WSM-metal QPT is the average DOS [51][53][55][56][58][59][60][62]. Since throughout the paper we have already extensively used the average DOS to characterize phases, for the sake of completeness we here review only its salient features. We can infer the scaling form of the average DOS in the thermodynamic limit $L \to \infty$ in different phases by using its scaling function [see Eq. (14)]. In the quantum critical regime $\varrho(E)$ should be independent of $\delta$, yielding $\varrho(E) \sim E^{d/z-1}$. Inside the WSM phase, the average DOS scales as $\varrho_W(E) \sim \delta^{1-(d-z)/\nu} |E|^2$. In the metallic phase average DOS at zero energy is finite and scales as $\varrho(0) \sim \delta^{d(z-1)}$. From the quoted values of DSE and CLE, it is straightforward to find the scaling of average DOS in these three regimes of the phase diagram in a dirty WSM, which we have used in the numerical analysis of this observable in the previous sections.
• **Conductivity:** The optical conductivity ($\sigma$) at $T = 0$ can as well serve as an order parameter across the WSM-metal QPT, and assumes the following scaling ansatz for frequency ($\Omega$) much smaller than the bandwidth $W$:

$$\sigma(\Omega) = \delta^{d(2-d)} G \left( \Omega \delta^{-\nu} \right), \quad (52)$$

where $G$ is yet another unknown universal scaling function. This scaling form remains operative even at finite temperature as long as $\Omega \gg T$, i.e., in the collision dominated regime. In the collision dominated regime at $T \gg \Omega$, the dc conductivity also assumes a similar scaling form as in Eq. $(52)$, upon replacing the frequency ($\Omega$) by temperature ($T$). In the WSM side of the transition, the OC vanishes linearly with $\Omega$ and scales as $\sigma_W(\Omega) \sim \delta^{d(1-z)(d-2)} \Omega^{d-2}$. Inside the critical regime, the OC scales as $\sigma_Q(\Omega) \sim \Omega^{d-2/2 - z}$. In the presence of strong CSP disorder $z = 3/2$, and the optical conductivity inside the quantum critical regime thus vanishes as $\sigma_Q(\Omega) \sim \Omega^{2/3}$. Since for non-chiral disorder the DSE is typically much bigger than in the presence of chiral symmetric one, the optical conductivity vanishes with a weaker power as $\Omega \rightarrow 0$ when the system is still dominated by CSB disorder before CSP disorder takes over. Hence, in this regime the system becomes more metallic in the presence of CSB disorder than with only CSP disorder.

Inside the metallic phase, the optical conductivity vanishes with a linear dependence for three dimensional Weyl fermion. Inside the critical regime, the specific heat, analogous to the conductivity, displays distinct power-law behavior in three different phases of a dirty WSM.

- **Grüneisen parameter:** Yet another directly measurable quantity is the Grüneisen parameter, defined as $\gamma = \alpha / C_P$, where $\alpha$ is the thermal expansion parameter, and $C_P$ is the specific heat measured at constant pressure. The Grüneisen ratio in the WSM phase $\gamma_W \sim T^{-4}$, while inside the critical regime $\gamma_Q \sim T^{-1/2}$.

- **Specific heat:** The specific heat ($C_V$) also displays distinct scaling behavior in three regimes of the phase diagram of a dirty WSM. The scaling of specific heat at temperature much smaller than bandwidth follows the ansatz $[35]$

$$C_V(T) = \frac{T^{d/z}}{v^d} \mathcal{H} \left( T^{d-2dz} \right), \quad (53)$$

where $\mathcal{H}$ is also an unknown universal scaling function. In the WSM phase, $\mathcal{H}(x) \sim x^{d(2-d)/z}$ and the specific heat scales as $C_V \sim \delta^{d(1-d)/2} T^d$, so that we recover $T^3$ dependence for three dimensional Weyl fermion. Inside the metallic phase, $\mathcal{H}(x) \sim x^{1-d/z}$, yielding $C_V \sim \delta^{d(2-d)/2} T$ and we obtain $T$-linear specific heat, similar to the situation in Fermi liquids. By contrast, inside the critical regime $H(x) \sim x^0$, yielding $C_V \sim T^{3/2}$. Therefore, the specific heat, analogous to the conductivity, displays distinct power-law dependence on temperature inside the quantum critical regime depending on the dominant source of disorder, while its scaling inside the WSM and metallic phases is insensitive to the nature of random impurities. Hence, the scaling of specific heat can be used to extract the extent of the critical regime and crossover boundaries among different phases of a dirty Weyl system at finite temperature $[55]$.  

- **Mean-free path:** The quasiparticle mean-free path ($\mathcal{L}$) also follows the critical scaling

$$[\mathcal{L}(E)]^{-1} = \delta^{d/2} J \left( E^{d-\nu} \right), \quad (54)$$

where $J$ is a universal, but unknown scaling function, with energy much smaller than bandwidth. At the QCP ($\delta = 0$) the mean-free path should be independent of $\delta$, implying $J(x) \sim x^{-1/2}$. Therefore, inside the quantum critical fan, the mean-free path at zero energy diverges as $\mathcal{L}(E) \sim E^{-1/2}$. In the metallic phase, $J(x) \sim x^0$ as $x \rightarrow 0$, leading to finite mean-free path at zero energy, and $\mathcal{L}(0) \sim \delta^{-\nu}$. On the other hand, in the WSM phase, the mean-free path $\mathcal{L}_W(E) \sim \delta^{d(1-d)} E^{-1}$, as $E \rightarrow 0$. Since at all disorder driven QCPs $z > 1$, $\mathcal{L}_W(E)$ decreases with increasing disorder, indicating propensity toward the onset of a metallicity in the system.

Fascinating scaling behavior can also be observed for the magnetic Grüneisen ratio, defined as $\Gamma_H = (\partial M / \partial T)_H / C_H$, where $M \propto H$ is magnetization, $C_H$ is the molar specific heat, and $H$ is the magnetic field strength. In the presence of sufficiently weak randomness when Landau quantization is sharp ($\omega_c \ll 1$), where $\omega_c$ is cyclotron frequency and $\tau$ is scattering lifetime) and dominates over the Zeeman coupling, leading to $\Gamma_H \sim T^{-4/2}$. On the other hand, in the presence of strong elastic scattering when $\omega_c \tau \ll 1$ the Landau levels are sufficiently broadened and the dominant energy scale is set by Zeeman coupling, yielding $\Gamma_H \sim T^{-2}$. Therefore, for a fixed weak magnetic field, as the strength of impurities is gradually increased, the magnetic Grüneisen ratio should display a smooth crossover from $T^{-4}$ to $T^{-2}$ dependence. Note that such a crossover will take place even before the system enters the quantum critical regime and will persist in the metallic regime as well, since elastic scattering is strong in these two phases.

**IX. ANDERSON TRANSITION**

As a final topic, we discuss the Anderson transition (AT) of a disordered diffusive Weyl metal at stronger strength of disorder. For the sake of simplicity we here focus only on the effects of random charge impurities. Possible AT in the presence of all other disorder is left
for a future investigation. To study the AT we compare three different types of DOS, namely average DOS \( \langle \rho_a(E) \rangle \), local DOS (LDOS) \( \langle \rho_L(E) \rangle \) and typical DOS (TDOS) \( \langle \rho_t(E) \rangle \), respectively defined as \[69, 74]\n
\[
\rho_a(E) = \langle \frac{1}{2L^3} \sum_{i=1}^{L^3} \sum_{\alpha=1}^{2} \delta (E - E_{i,\alpha}) \rangle, \quad (55)
\]
\[
\rho_L^{i,\alpha}(E) = \sum_{k,\beta} |\langle k, \beta | i, \alpha \rangle|^2 \delta (E - E_{k,\beta}), \quad (56)
\]
\[
\rho_t(E) = \exp \left[ \frac{1}{2N_s} \sum_{j=1}^{N_s} \sum_{\alpha=1}^{2} \left\langle \log \rho_L^{i,\alpha}(E) \right\rangle \right]. \quad (57)
\]

Here \( L^3 \) is the system size, |i,\alpha\rangle is the eigenstate with site index i and orbital index \( \alpha (= 1, 2) \) at energy \( E_{i,\alpha} \). As previously discussed, average DOS is a self-averaging quantity so to minimize statistical fluctuations we only extract the disorder-averaged smoothed data, which we carry out by computing \( N_m = 1024 \) Chebyshev moments and performing disorder average over 20 random disorder realizations. On the other hand, LDOS and TDOS are not self-averaging quantities. Therefore, numerical extraction of TDOS is extremely demanding for which we compute \( N_m = 8192 \) moments and perform disorder average over 100 random disorder realization to construct the TDOS. To further suppress statistical fluctuations in TDOS we average over a small cube of size \( N_s = L_s^3 \ll L^3 \), and we here take \( L_s = 4 \). Such averaging is justified since translational symmetry gets restored after disorder averaging has been performed.

The scaling of average DOS and TDOS over a wide range of disorder strength is shown in Fig. 20(a). Note that in the WSM phase both average DOS and TDOS at zero energy are pinned to zero, which then become finite across the WSM-metal QPT at \( W_{c,1} = 1.65 \pm 0.05 \). Therefore, either average DOS or TDOS can be identified as a bona fide order-parameter to pin the WSM-metal QCP. Respectively these two quantities scale as

\[
\rho_a(0) \sim \left( \frac{W - W_{c,1}}{W_{c,1}} \right)^{\beta_a}, \quad \rho_t(0) \sim \left( \frac{W - W_{c,1}}{W_{c,1}} \right)^{\beta_t}, \quad (58)
\]

near the WSM-metal QCP, with

\[
\beta_a = 1.50 \pm 0.05, \quad \beta_t = 1.80 \pm 0.20, \quad (59)
\]

as shown in Fig. 20(b). Even though the numerical error-bar for \( \beta_t \) is quite large, in general, we expect it to be different from \( \beta_a \), as their difference, \( \Delta \beta = \beta_t - \beta_a \), is intimately tied with the multifractal dimension of the wavefunction across a disorder-driven QPT \[83, 87\]. However, more precise determination of \( \beta_t \) requires additional extensive numerical simulation. Therefore, we leave this issue as a subject for a future investigation.

Inside the compressible diffusive metallic phase these two quantities increase monotonically and follow each other up to a moderate strength of disorder \( W_s \approx 3.5 \). Upon further increasing strength of disorder the TDOS smoothly vanishes around \( W_{c,2} = 9.30 \pm 0.25 \). Therefore, a metal-insulator transition (MIT) takes place at \( W_{c,2} = 9.30 \) in the \( N_m \to \infty \) limit. In Fig. 21(a) we present the scaling of TDOS with the number of Chebyshev moments \( N_m \). We explicitly compute TDOS from moderate to strong disorder regime \( 6 \leq W \leq 10 \), in the close vicinity of the AT, for \( N_m = 2048, 4096 \) and 8192. From the scaling of \( \rho_t(0) \) vs. \( N_m \) we conclude that AT (identified with \( \rho_t(0) \to 0 \)) takes place around \( W_{c,2} = 9.30 \) in the \( N_m \to \infty \) limit. Therefore, we can conclude that a three-dimensional diffusive Weyl metal is a stable phase of matter for moderately strong disorder, which ultimately undergoes a QPT.
focus on this quantity in the strong disorder regime

In particular, the mobility edge defines the boundary be-

tween the extended and localized states, and we here

focus on this quantity in the strong disorder regime

$W \geq 2 > W_{c,1}$. The results are shown in Fig. 22(b).

For weak disorder the mobility edge resides at high-energy,

indicating the metallic nature of a moderately dirty Weyl

system. However, the mobility edge progressively slides
down toward smaller energy with increasing randomness

in the system. Finally, across the AT the mobility edge
comes down to zero energy, indicating that all states in-

side the Anderson insulator are localized. Notice that

the shape of the mobility edge is quite distinct in a Weyl

metal in comparison to its counterpart in conventional

metal $SS$, which however can solely be attributed to the

linear dispersion of Weyl quasiparticles in the clean sys-

In this paper we have studied the role of generic disor-
der in a Weyl semimetal, by considering its simplest re-
alization, comprised of only two Weyl nodes. When the

system resides in the proximity of semimetal-insulator

quantum phase transition, the generalized Harris crite-

rion suggests that such critical point is stable in the pres-

cence of weak but generic disorder. By contrast, a mul-

ticritical point appears in the phase diagram for strong

disorder, where the Weyl semimetal, an insulator and a

metallic phase meet. Within the framework of an appro-

priate $\epsilon$-expansion we show that the critical exponents

at such multicritical point are (i) dynamic scaling expo-

nent $z = 1 + \epsilon_n/2$, and (ii) correlation length exponent

$\nu = 1/\epsilon_n$ that controls the relevance of disorder coupling,

where $\epsilon_n = 1/2$ for physical system. These findings are

in good agreement with the ones obtained numerically,
yielding $\nu = 1.98 \pm 0.05$ and $z = 1.23 \pm 0.05$.

On the other hand, when the system is deep inside the

Weyl semimetal phase, we have shown that the continu-

ous global chiral $U(1)$ symmetry plays a fundamental rule

in classifying the disorder-driven Weyl semimetal-metal

quantum phase transitions. The simplest realization of a

Weyl semimetal is susceptible to eight types of disorder,
among which only four preserve such chiral symmetry.

Using two different $\epsilon$-expansions, we have shown that
the chiral symmetric disorder driven semimetal-metal tran-
sition takes place through either a quantum critical point
or a line of quantum critical points. Irrespective of de-
tails, the critical exponents are always $z = 1 + \epsilon/2$ and
$\nu = \epsilon^{-1}$, and $\epsilon = 1$ corresponds to the physical situa-
tion. Such unique set of exponents in the presence of
generic chiral symmetric disorder gives birth to an emer-
gent chiral superuniversality across the Weyl semimetal-
metal quantum phase transition.

Furthermore, we have performed a thorough numerical
analysis of average density of states in Weyl semimetals
with chiral symmetric disorder. The emergence of chiral
superuniversality has been demonstrated through numeri-

cal analysis of average density of states near zero energy.

We show that for any such disorder Weyl semimetal un-
dergoes a continuous quantum phase transition into a
diffusive metallic phase. Within the numerical accuracy,
we find that across this transition $z \approx 1.5$ and $\nu \approx 1$, in excellent agreement with our field theoretic predictions obtained from leading order $\epsilon$-expansions (see Table I for comparison). The quality as well as reliability of our numerical analysis has been anchored through two completely different types of high-quality data collapses, shown in Fig. 15, in the entire phase diagram of a dirty Weyl semimetal for all possible chiral disorder.

For chiral symmetry breaking disorder, the Weyl semimetal-metal quantum phase transition also takes place through a critical point or a line of critical points, but the critical exponents are significantly different from the ones reported in the presence of chiral disorder. Even though the critical exponents across such semimetal-metal transition turn out to be slightly dependent on the renormalization group scheme, we always find $z > d$ and $\nu = 1/\epsilon$ from leading order $\epsilon$-expansions. Consequently, all chiral symmetric or intra-node disorder (as well as higher gradient terms that are inevitably present in a lattice) become relevant at such putative line of critical points. As a result, inter-node disorder driven semimetal-metal phase transition is ultimately always governed by the chiral symmetric disorder, yielding $\nu \approx 1$ and $z \approx 3/2$, characteristic of chiral superuniversality. We anchor these outcomes by numerically extracting the scaling of average density of states in the presence of inter-node disorder, and the results are shown in Table I and Figs. 4 and 16.

Even though we promoted such classification scheme in a Weyl semimetal with only two nodes, our prescription can easily be generalized to Weyl systems with multiple flavors, as well as topological Dirac semimetals with bonafide time-reversal symmetry that has recently been found in Cd$_2$As$_3$ [89] and Na$_3$Bi [90] and the ones at the quantum critical point residing between two topologically distinct insulating vacua.

We here mention that $\epsilon_d$ expansion can be problematic beyond the leading order in $\epsilon_d$, since the contribution from diagrams (c) and (d) in Fig. 6 and their higher-loop cousins are typically ultraviolet divergent and one looses the order by order control over the perturbative calculation [40, 44]. Such a class of diagrams is, however, ultraviolet finite and thus does not contribute to renormalization group flow equations in the $\epsilon_m$ expansion scheme. We, therefore, strongly believe that higher order perturbation theory can only be reliable in the $\epsilon_m$ expansion scheme, and our results can serve as an ideal testbed to investigate the reliability of these two schemes.

In addition to the Weyl semimetal-metal quantum phase transition, we also establish that a compressible Weyl metal undergoes a a subsequent transition at stronger disorder into a Anderson insulator. We track the typical density of states to pin the onset of such insulating phase that only accommodates localized states. In particular, we show that across the Weyl metal-insulator transition the typical density of states at zero energy ($g_\theta(0)$) smoothly vanishes, and thus serving as bonafide order-parameter, while the average density of states remains non-critical across this transition. In addition, we also find that $g_\theta(0)$ remains pinned in the Weyl semimetal phase and becomes finite in the metallic phase. Therefore, typical density of states at zero energy serves as a unified order-parameter across all possible disorder-driven quantum phase transition in a Weyl semimetal.

Finally, we comment on some non-perturbative effects of disorder in Weyl semimetals, such as puddles [91], Lishifetz tail [92], and rare-region states and Griffiths physics [33, 60]. Puddles are inevitable in real materials as there are always density fluctuations that locally shift the chemical potential away from the Weyl nodes, while maintaining the overall charge neutrality of the system. In addition, presence of disorder can also support quasi-localized rare states at zero-energy even for sub-critical strength of disorder. Although such effects are important and interesting, they do not affect the quantum critical behavior. This is so because rare and critical excitations appear to be decoupled from each other. Also a recent numerical work has demonstrated that such non-perturbative effects can be systematically suppressed with a suitable choice of the distribution of disorder [62]. Therefore, these effects do not alter any physical outcome we reported in this paper.

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Appendix A: Generalized Harris criterion at WSM-insulator QCP

In this Appendix, we present a generalization of the Harris criterion applicable near the clean WSM-insulator QCP. Let us first consider a generalized version of the Hamiltonian from Eq. 6 describing the gapless excitations residing at general WSM-insulator QCP [77]

$$\hat{H}_G(k, \Delta) = \alpha c [\sigma_1 k_\perp^2 \cos(c \phi_k) + \sigma_2 k_\perp \sin(c \phi_k)] + \sigma_3 (b k_3^2 - \Delta),$$

(A1)

where $k_\perp = \sqrt{k_x^2 + k_y^2}$ and $\phi_k = \tan^{-1}(k_y/k_x)$. The above Hamiltonain for any value of $\Delta$ possesses the same symmetry, but describes two distinct phases: (i) a band
insulator for $\Delta < 0$ and (ii) WSM for $\Delta > 0$, with $c$ representing the monopole charge of the Weyl nodes. Respectively for $c = 1, 2$ and 3, single, double and triple WSMs are realized in a crystalline environment \[93–95\]. The effective dimensionality ($d_\ast$) of such critical semimetallic phase can be found from the corresponding imaginary time Euclidean action

$$S^\ast = \int d\tau d^2x \psi^\dagger \left[ \partial_\tau + \hat{H}_Q(k \rightarrow -i\nabla, \Delta) \right] \psi,$$

(A2)

where $\psi$ is a two component spinor, describing the critical excitations residing at the WSM-insulator QCP. All parameters, such as $\alpha$, and $b$, remain invariant under the rescaling of space-time (imaginary) co-ordinates according to $\tau \rightarrow e^{\Delta} \tau, (x, y) \rightarrow e^{1/c}(x, y), x_3 \rightarrow e^{1/2}x_3$, when accompanied by the field normalization $\psi \rightarrow Z^\ast_\psi \psi$, where $Z^\ast_\psi = \exp \left[ -\left( \frac{1}{c} + \frac{1}{2} \right) l \right]$ $\exp[-d_\ast l]$. The spatial measure $d^2x \rightarrow e^{l} d^2x$, where $d_\ast = \left( \frac{1}{c} + \frac{1}{2} \right)$ is the effective dimensionality of the system under the rescaling of spatial coordinates. Note that $\Delta$ in Eq. (A1) is the tuning (relevant) parameter at the WSM-insulator QCP, and the scaling dimension of $\Delta$, denoted by $[\Delta]$, is tied with the CLE ($\nu$) at this QCP, according to $\nu^{-1} = [\Delta] = 1$. The stability of the clean WSM-insulator QCP against mass disorder [denoted by $V_z(x)$ in Eq. (8)] can be assessed from the generalized Harris criterion, suggesting that such QCP is stable against mass disorder when

$$\nu > \frac{2}{d_\ast} = \frac{4c}{4+c}. \quad (A3)$$

Therefore, only the single ($c = 1$) WSM-insulator QCP is stable against sufficiently weak mass/bond disorder. Furthermore, the stability of the WSM-insulator QCP in the presence of generic disorder, which appears similar to $V_z(x)$ in Eq. (8), can be established from the generalized Harris criterion (A3). Hence, a single WSM-insulator QCP is guaranteed to be stable against generic disorder. In this regard a comment is due. Our derivation of generalized Harris criterion differs from the original one in Ref. [27], where $d_\ast$ is replaced by the physical dimensionality of the system ($d$) and the CLE $\nu$ varies depending on the nature of the phase transition. On the other hand, within the framework of anisotropic scaling of spatial co-ordinates we always find $\nu = 1$, but actual spatial dimension gets replaced by an effective dimensionality of the system ($d_\ast$) under the process of coarse graining. We believe that these two methods are complimentary to each other.

Appendix B: RG analysis near WSM-insulator QCP

In this Appendix, we provide technical details of the RG calculations near the WSM-insulator QPT with disorder. First, we show the effects of subleading divergences in the RG flow equations within the $\epsilon_n$ expansion introduced in Sec. [III] and its consequences [see Sec. (B1)]. Next we display the perturbative analysis of disorder near the WSM-insulator in an expansion about the lower critical dimension of the theory [see Sec. (B2)].

1. $\epsilon_n$ expansion

Within the framework of $\epsilon_n$ expansion, discussed in Sec. [III] after integrating out the fast Fourier modes within the Wilsonian shell $\Lambda e^{-l} < k_\perp < \Lambda$ and $0 < k_\perp < \infty$ and accounting for subleading ultraviolet divergences, the RG flow equations read

$$\beta_X = -X \left( \Delta_0 + 2\Delta_\perp + \Delta_3 \right) [h_1(n) + h_2(n)] = (1 - z)X, \quad \beta_\Delta = \Delta + [\Delta [h_1(n) + h_2(n)] - h_3(n)] \left( \Delta_0 - 2\Delta_\perp + \Delta_3 \right)$$

$$\beta_{\Delta_0} = -\epsilon_n \Delta_0 + 2\Delta_0 \left( \Delta_0 + 2\Delta_\perp + \Delta_3 \right) [h_1(n) + h_2(n)] + 4\Delta_\perp \Delta_3 h_1(n) + 4h_2(n) \left[ \delta_{n,2m} \Delta_0 \Delta_3 + \delta_{n,2m+1}(\Delta_3^2 + \Delta_0^2) \right]$$

$$\beta_{\Delta_\perp} = -\epsilon_n \Delta_\perp + 2\Delta_\perp \left( \Delta_\perp - \Delta_0 \right) h_2(n) + 2\Delta_\perp \Delta_3 h_1(n) + 4h_2(n) \left[ \delta_{n,2m} \Delta_\perp + \delta_{n,2m+1}(\Delta_0) \right]$$

$$\beta_{\Delta_3} = -\epsilon_n \Delta_3 + 2\Delta_3 \left( 2\Delta_\perp - \Delta_0 - \Delta_3 \right) [h_1(n) - h_2(n)] + 4\Delta_\perp \Delta_3 h_1(n) + 2h_2(n) \delta_{n,2m} (\Delta_3^2 + \Delta_0^2 + \Delta_\perp^2), \quad (B1)$$

where $\delta_{n,m}$ is the Kronecker delta function, $n, m$ are integers and $X = v, b$. Functions $h_i(n), i = 1, 2, 3,$ are defined as

$$h_1(n) = \frac{\pi(2n - 1) \csc \left( \frac{\pi}{2n} \right)}{4n^2} = 1 - \frac{1}{2n} + \mathcal{O} \left( n^{-3} \right)$$

$$h_2(n) = \frac{\pi \csc \left( \frac{\pi}{2n} \right)}{4n^2} = \frac{1}{2n} + \mathcal{O} \left( n^{-2} \right). \quad (B2)$$

Therefore, as $n \rightarrow \infty$ contribution only from $h_1(n)$ survives and for any finite $n, h_2(n)$ and $h_3(n)$ give rise to subleading divergences. The RG flow equations obtained by keeping only the leading divergence are shown in Eq. (11) of the main text. In order to reliably extract the critical exponents, in particular the DSE $z$ at the multicritical point shown as a red dot in Fig[7] we should only keep...
the leading order \((n \to \infty)\) contribution. At least to the leading order in \(\epsilon_n\) expansion, inclusion of subleading divergences [arising from \(h_2(n)\) and \(h_3(n)\)] does not alter the value of CLE, and we always find \(\nu^{-1} = \epsilon_n = 1/2\).

2. \(\epsilon'_d\) expansion about lower critical dimension \(d_l = \frac{5}{2}\)

In this section we demonstrate the role of disorder in the vicinity of WSM-insulator QPT perturbatively using an \(\epsilon'_d\) expansion near the lower critical dimension \(d_l = 5/2\) in the theory, where \(\epsilon'_d = d - 5/2\). As we will see the outcomes are qualitatively the same as in the \(\epsilon_n\) regularization scheme. The exact values of the critical exponents are, however, different from the ones announced in Sec. III, although only slightly so, at least to the one-loop order. Upon integrating the fast modes to the one-loop order. Upon integrating the fast modes within the shell \(E_c e^{-l} < \sqrt{v^2 k^2 + b_r^2} < E_c\), where \(E_c\) is the ultraviolet energy cutoff for critical excitations residing the WSM-insulator QCP, we arrive at the following flow equations to the leading order in \(\epsilon'_d\) expansion

\[
\begin{align*}
\beta_X &= -5X (\Delta_0 + 2\Delta_\perp + \Delta_3) = (1-z)X, \\
\beta_\Delta &= \Delta + (\Delta - 1) [\Delta_0 - 2\Delta_\perp + \Delta_3], \\
\beta_{\Delta_0} &= -\epsilon'_d \Delta_0 + 10 \Delta_0 (\Delta_0 + 2\Delta_\perp + \Delta_3) - 16\Delta_3 \Delta_\perp, \\
\beta_{\Delta_\perp} &= -\epsilon'_d \Delta_\perp + 2\Delta_\perp (\Delta_3 - \Delta_0) + 4\Delta_3 (\Delta_\perp - 2\Delta_0), \\
\beta_{\Delta_3} &= -\epsilon'_d \Delta_3 + 6\Delta_3 (2\Delta_\perp - \Delta_0 - \Delta_3) \nonumber \\
&\quad + 4 (\Delta_0^2 + \Delta^2 + \Delta_\perp \Delta_3 - 4\Delta_0 \Delta_\perp + 2\Delta_3^2). \quad (B3)
\end{align*}
\]

for \(X = v, b\), after defining the dimensionless disorder coupling constant as \(\Delta_j \alpha \to \Delta_j\) for \(j = 0, \perp, z\), where \(\alpha = E_c^{1/2}/(2\pi^2 v^2 b^{1/2})\) and \(\Delta/E_c \to \Delta\). Then, the \(\beta\)-function for \(v, b\) in the presence of disorder yields a scale dependent dynamic scaling exponent

\[
z(l) = 1 + 5 [\Delta_0 + 2\Delta_\perp + \Delta_3] (l). \quad (B4)
\]

The coupled RG flow equations from Eq. (B3) also support only two fixed points: (i) \((\Delta, \Delta_0, \Delta_\perp, \Delta_3) = (0, 0, 0, 0)\), representing the WSM-insulator QCP in the clean limit (the blue dot in Fig. 23), and (ii) \((\Delta, \Delta_0, \Delta_\perp, \Delta_3) = (\frac{0.058}{1-0.058\epsilon_d}, 0.058\epsilon_d, 0.000\epsilon_d, 0.02\epsilon_d)\) representing a multicritical point. The critical exponents at this multicritical point for the anisotropic critical semimetal-metal transition are

\[
\nu^{-1} = \epsilon'_d, \quad z = 1 + 0.48\epsilon'_d, \quad (B5)
\]

which is extremely close to the ones reported in Sec. III for \(\epsilon'_d = 1/2\). Therefore, both methods produce qualitatively similar results near WSM-insulator QPT, and the obtained critical exponents for anisotropic semimetal-metal transition are extremely close to each other, at least to the leading order. The resulting phase diagram is shown in Fig. 23.

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Figure 23: The phase diagram of a dirty Wey material residing in the close proximity to WSM-insulator QPT, obtained by solving the RG flow equations (B3). Here, \(\Delta\) is the tuning parameter for WSM-insulator transition in the clean system and \(\Delta_0\) is the strength of random charge impurities.

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**Appendix C: Details of \(\epsilon_m\) expansion**

In this appendix we display the detailed analysis of various one-loop diagrams, shown in Fig. 10 within the framework of an \(\epsilon_m\) expansion.

1. **Self-energy**

Let us first consider the self energy diagram in Fig. 10 (a). The expression for the self-energy reads

\[
\Sigma(i\omega, \mathbf{k}) = \sum_N \int \frac{d^d q}{(2\pi)^d} N G_0(i\omega, \mathbf{k} - \mathbf{q}) \frac{\Delta_N}{q^m} \equiv \sum_N \Sigma_N(i\omega, \mathbf{k}), \quad (C1)
\]

with \(d = 3\), the summation is taken over all eight types of disorder (see Table III) and \(q \equiv |\mathbf{q}|\).

The contribution from one-loop self-energy diagram from the disorder represented by the matrix \(N\) reads

\[
\Sigma_N(i\omega, \mathbf{k}) = -i\Delta_N \int \frac{d^d q}{(2\pi)^d} \frac{N \gamma_0 \omega + \gamma_j (k - q)_j}{[\omega^2 + v^2 (\mathbf{k} - \mathbf{q})^2]^{m/2}} q^m. \quad (C2)
\]

We will evaluate the temporal and spatial components of the self-energy diagram separately. Let us first set \(\mathbf{k} = 0\), for which

\[
\Sigma_N(i\omega, 0) = \Delta_N (-i\omega) \frac{N \gamma_0 N}{v^3 m} \int \frac{d^3 q}{(2\pi)^3} \frac{1}{(w^2 + q^2) q^m} \nonumber \\
= \Delta_N (-i\omega) \frac{N \gamma_0 N}{v^3 m} \frac{\Gamma(1 + \frac{m}{2})}{\Gamma(m/2)}. \quad (C3)
\]

\[
\times \int_0^1 dx x^{m-1} \int \frac{d^3 q}{(2\pi)^3} \frac{1}{[q^2 + (1 - x)\omega^2]^{1 + \frac{m}{2}}},
\]
where $x$ is the Feynman parameter. Upon completing the integrals over $q$ and $x$, and setting $m = 1 - \epsilon$ (for brevity, we use here shorthand notation $\epsilon_m \to \epsilon$) we obtain

$$\Sigma_N(i\omega, 0) = [iN\gamma_0\omega N] \left( \frac{\Delta_N}{2\pi^2 v^2} \right) \frac{1}{\epsilon} + \mathcal{O}(1). \quad \text{(C4)}$$

Next we set $\omega = 0$ and the spatial component of self-energy correction is then given by

$$\Sigma_N(0, k) = \Delta_N \left[ -iN\gamma_j N \right] \frac{1}{v^3-m} \int \frac{d^3q}{(2\pi)^3} \frac{(k-q)_j}{\epsilon_m} q^m$$

$$= \Delta_N \left[ -iN\gamma_j N \right] \left[ \frac{\Gamma(1 + \frac{m}{2})}{\Gamma(m/2)} \right] \int_0^1 dx x \frac{q^m}{x^m} \cdot (k-q_j)$$

$$\times \int \frac{d^3q}{(2\pi)^3} \left[ q^2 - 2(1-x)q \cdot k + (1-x)k^2 \right]^{1+\frac{m}{2}}. \quad \text{(C5)}$$

After shifting the momentum variable according to $q - (1-x)k \to q$ and setting $m = 1 - \epsilon$, we obtain

$$\Sigma_N(0, k) = [iN\gamma_j k_j N] \left( \frac{\Delta_N}{2\pi^2 v^2} \right) \frac{k^\epsilon}{3\epsilon} + \mathcal{O}(1). \quad \text{(C6)}$$

Hence, the total self energy correction reads

$$\Sigma_N(i\omega, 0) = iN \left[ \gamma_0\omega + \frac{1}{3} \gamma_j k_j \right] N\Delta_N \frac{1}{\epsilon} + \mathcal{O}(1), \quad \text{(C7)}$$

where we have redefined $\Delta_N k^\epsilon/(2\pi^2 v^2) \to \Delta_N$, which is Eq. (23) in the main text.

### 2. Vertex

The vertex correction for the disorder vertex shown in Fig. 10(b) with the matrix $N$ reads

$$V_N(k) = \sum_M \int \frac{d^3q}{(2\pi)^3} MG_0(0, k-q)NG_0(0, k-q)M \frac{\Delta_M}{q^m}, \quad \text{(C8)}$$

where we kept only one external momentum as an infrared regulator. The last expression can be compactly written as

$$V_N(k) = -\sum_M [M\gamma_j N\gamma_i M] \frac{\Delta_M}{v^2} I_{jl}(k), \quad \text{(C9)}$$

where

$$I_{jl}(k) = \int \frac{d^3q}{(2\pi)^3} \frac{(k-q)_j (k-q)_l}{(q-k)^4 q^m}. \quad \text{(C10)}$$

We now present the evaluation of the above integral

$$I_{jl} = \int \frac{d^3q}{(2\pi)^3} \frac{(k-q)_j (k-q)_l}{(q-k)^4 q^m} = \frac{\Gamma(1 + \frac{m}{2})}{\Gamma(m/2)} \int_0^1 dx x^{1-x} \frac{\epsilon}{2^{\frac{m}{2}}}.$$  \hspace{1cm} \text{(C11)}$$

After shifting the momentum variable as $q - xk \to q$, we obtain

$$I_{jl} = \frac{\Gamma(1 + \frac{m}{2})}{\Gamma(m/2)} \int_0^1 dx x(1-x) \frac{\epsilon}{2^{\frac{m}{2}} - 1} \quad \text{(C12)}$$

After taking $m = 1 - \epsilon$, since only the $q$–dependent part in the numerator of the integrand yields a divergent contribution. We use the last expression to obtain Eq. (30) in the main text.

### Appendix D: Lattice realization of generic disorder in Weyl semimetal

In this appendix, we demonstrate the lattice realization of sixteen possible fermionic bilinears (shown in Table III) from the two band tight-binding model, displayed in Eqs. (5) and (3). By the virtue of the chosen tight-binding Hamiltonian, we obtain

- Since two Weyl nodes are located on the $k_z$ axis at $\pm k_0^2 = \pm \pi/(2a)$, any fermionic bilinear odd under the exchange of two Weyl nodes, can be realized by adding $h = \sum_k \Psi_k^\dagger \sin(k_z a) \sigma_j \Psi_k$ to the tight binding model, where $j = 0, 1, 2, 3$. Such perturbation corresponds to an imaginary hopping along the $z$ direction, and does not renormalize the band width.

- Any fermionic bilinear that couples two Weyl nodes, which therefore necessarily breaks translational symmetry, can be realized through a periodic and commensurate modulation of the nearest-neighbor hopping amplitude, but only along the $z$ direction.

With these two construction principles we can realize all sixteen fermion bilinears by adding the following terms to the tight-binding Hamiltonian.

1. Regular chemical potential:

$$\sum_r \Psi_r^\dagger V(r) \sigma_0 \Psi_r.$$

2. Axial chemical potential:

$$\sum_r \Psi_r^\dagger \left[ \frac{iV(r)}{2} \sigma_0 \right] \Psi_{r+e_z} + H.c.,$$
In this Appendix we present the coupled RG flow equations for eight disorder couplings shown in Table III obtained within the framework of $\epsilon_m$-expansion [defined in Sec. IV.B] and $\epsilon_d$-expansion [defined in Sec. IV.C]. We show that under generic circumstances on the line of QCPs, defined in Eq. (36) [obtained from $\epsilon_m$-expansion] or Eq. (39) [obtained from $\epsilon_d$-expansion], in the $(\Delta_V, \Delta_A)$ plane (two chiral symmetric disorders) is the legitimate solution, which provides a strong justification for the chiral superuniversality across generic disorder driven WSM-metal QPT, qualitatively discussed in Sec. VIII.D.

1. RG flow equations from $\epsilon_m$ expansion

The leading order coupled RG flow equations in the presence of all eight disorder couplings within the framework of an $\epsilon_m$-expansion read as

$$\beta_{\Delta V} = \Delta V \left[ -\epsilon_m + \frac{4}{3} \left( 2\Delta A + 5\Delta_{AM} + 4\Delta C + 4\Delta M + \Delta_{PS} + \Delta S + 5\Delta_{SO} + 2\Delta V \right) \right],$$

$$\beta_{\Delta A} = \Delta A \left[ -\epsilon_m + \frac{8}{3} \left( \Delta A - 2\Delta_{AM} + 2\Delta_c + 2\Delta M - \Delta_{PS} - \Delta S - 2\Delta_{SO} + \Delta V \right) \right],$$

$$\beta_{\Delta M} = \Delta M \left[ -\epsilon_m + \frac{4}{3} \left( \Delta_{AM} - \Delta_{PS} - \Delta S + \Delta_{SO} \right) \right],$$

$$\beta_{\Delta C} = -\epsilon_m \Delta C,$$

$$\beta_{\Delta_{SO}} = \Delta_{SO} \left[ -\epsilon_m + \frac{4}{3} \left( \Delta_{AM} - \Delta M - \Delta S + \Delta V \right) \right],$$

$$\beta_{\Delta_{AM}} = \Delta_{AM} \left[ -\epsilon_m - \frac{4}{3} \left( \Delta M + \Delta_{PS} - \Delta_{SO} - \Delta V \right) \right],$$

$$\beta_{\Delta S} = \Delta S \left[ -\epsilon_m + \frac{4}{3} \left( 2\Delta A - 4\Delta_{AM} + 4\Delta C - 5\Delta M + \Delta_{PS} - 2\Delta S + 5\Delta_{SO} - \Delta V \right) \right],$$

$$\beta_{\Delta_{PS}} = \Delta_{PS} \left[ -\epsilon_m + \frac{4}{3} \left( 2\Delta A + 5\Delta_{AM} + 4\Delta C - 5\Delta M - 2\Delta_{PS} + \Delta S - 4\Delta_{SO} - \Delta V \right) \right].$$

The above set of coupled flow equations only supports a line of QCPs, given by $\nu^{-1} = \epsilon_m$ and $z = 1 + \epsilon_m/2$ in three dimensions. Therefore, for Gaussian white
noise distribution ($\epsilon_m = 1$), we obtain $\nu = 1$ and $z = 3/2$. This outcome strongly supports the proposed emergent superuniversality across the WSM-metal QPT, driven by arbitrary disorder.

2. RG flow equations from $\epsilon_d$ expansion

The coupled RG flow equations for eight symmetry allowed disorder couplings to the leading order in the $\epsilon_d$-expansion read as

$$
\beta_{\Delta_V} = -\epsilon_d \Delta_V + 2 \Delta_V [\Delta_A + 3 \Delta_{AM} + 3 \Delta_C + 3 \Delta_M + \Delta_{PS} + \Delta_S + 3 \Delta_{SO} + \Delta_V] \\
+ 4 (2 \Delta_C \Delta_M + \Delta_{AM} \Delta_{PS} + \Delta_S \Delta_{SO}), \\
\beta_{\Delta_A} = -\epsilon_d \Delta_A + 2 \Delta_A (\Delta_A - 3 \Delta_{AM} + 3 \Delta_C + 3 \Delta_M - \Delta_{PS} - \Delta_S - 3 \Delta_{SO} + \Delta_V) \\
+ 4 (\Delta_{AM}^2 + \Delta_C^2 + \Delta_M^2 + \Delta_{SO}^2), \\
\beta_{\Delta_M} = -\epsilon_d \Delta_M + \frac{2}{3} \Delta_M (-\Delta_A + \Delta_{AM} + \Delta_C + \Delta_M - \Delta_{PS} - \Delta_S + \Delta_{SO} - \Delta_V) \\
+ \frac{4}{3} (2 \Delta_A \Delta_M + 7 \Delta_{AM} \Delta_{SO} + 2 \Delta_C \Delta_V + \Delta_{PS} \Delta_S), \\
\beta_{\Delta_C} = -\Delta_C + \frac{2}{3} \Delta_C (-\Delta_A - \Delta_{AM} + \Delta_C + \Delta_M + \Delta_{PS} + \Delta_S - \Delta_{SO} - \Delta_V) \\
+ \frac{8}{3} (\Delta_A \Delta_C + \Delta_{AM} \Delta_S + \Delta_M \Delta_V + \Delta_{PS} \Delta_{SO}), \\
\beta_{\Delta_{SO}} = \epsilon_d \Delta_{SO} - \frac{2}{3} \Delta_{SO} (\Delta_A - \Delta_{AM} - \Delta_C + \Delta_M - \Delta_{PS} + \Delta_S + \Delta_{SO} - \Delta_V) \\
+ \frac{4}{3} (2 \Delta_A \Delta_{SO} + 7 \Delta_{AM} \Delta_M + 2 \Delta_C \Delta_{PS} + \Delta_S \Delta_V), \\
\beta_{\Delta_{AM}} = -\epsilon_d \Delta_{AM} - \frac{2}{3} \Delta_{AM} (\Delta_A + \Delta_{AM} - \Delta_C + \Delta_M + \Delta_{PS} - \Delta_S - \Delta_{SO} - \Delta_V) \\
+ \frac{4}{3} (2 \Delta_A \Delta_{AM} + 2 \Delta_C \Delta_S + 7 \Delta_M \Delta_{SO} + \Delta_{PS} \Delta_V), \\
\beta_{\Delta_S} = -\epsilon_d \Delta_S + 2 \Delta_S (\Delta_A - 3 \Delta_{AM} + 3 \Delta_C - 3 \Delta_M + \Delta_{PS} - \Delta_S + 3 \Delta_{SO} - \Delta_V) \\
+ 4 (2 \Delta_{AM} \Delta_C + \Delta_M \Delta_{SO} + \Delta_{PS} \Delta_V), \\
\beta_{\Delta_{PS}} = -\epsilon_d \Delta_{PS} + 2 \Delta_{PS} (\Delta_A + 3 \Delta_{AM} + 3 \Delta_C - 3 \Delta_M - \Delta_{PS} + \Delta_S - 3 \Delta_{SO} - \Delta_V) \\
+ 4 (\Delta_{AM} \Delta_{PS} + 2 \Delta_C \Delta_{SO} + \Delta_M \Delta_S). \\
$$

The above set of coupled flow equations supports only a line of QCPs, given by Eq. (39), in the $\Delta_V - \Delta_A$ plane, shown in Fig. 11. Along the entire line of QPCs, the exponents are $\nu^{-1} = \epsilon_d$ and $z = 1 + \epsilon_d/2$ (to the leading order in $\epsilon_d$). Therefore, in a three-dimensional WSM ($\epsilon_d = 1$) the semimetal-metal QPT driven by arbitrary disorder potential is always characterized by $\nu = 1$ and $z = 3/2$, thus strongly supporting the proposed emergent chiral superuniversality.

Appendix F: Alternative derivation of correction to optical conductivity

Direct computation of the correction to the OC due to arbitrary disorder by using the Kubo formula has already been presented in Ref. [42]. Specifically, we compute the disorder driven correction to the current-current correlation function (involving computation of two-loop diagrams) and then via analytic continuation we found the OC at frequency $\Omega$ in a weakly disordered WSM to be

$$
\sigma(\Omega) = \frac{N e_0^2 \Omega}{12 \hbar^2} \left[1 + \frac{\Delta_V \Lambda}{\pi^2 v^2}\right] \equiv \sigma_0(\Omega) \left[1 + \frac{\Delta_V \Lambda}{\pi^2 v^2}\right], \quad (F1)
$$

where $N$ is the number of Weyl nodes, $e_0$ is the electron charge in vacuum [see Eq. (3) of Ref. [42]]. For concreteness, we here restrict ourselves to potential disorder or random charge impurities ($\Delta_V$), possessing Gaussian white noise distribution in three dimensions. In the absence of disorder ($\Delta_V = 0$), we recover the OC in a clean WSM, $\sigma_0(\Omega)$. [50, 52, 54]. We now present an alternative derivation of the same expression.

The OC is given by

$$
\sigma(\Omega) = \lim_{\Omega \to 0} \frac{1}{\Omega} \int d^D x \ e^{i \Omega x_0} \langle j_x(0) j_x(0) \rangle_R \\
= Z^2 \left[ \lim_{\Omega \to 0} \frac{1}{\Omega} \int d^D x \ e^{i \Omega x_0} \langle j_x(0) j_x(0) \rangle_0 \right], \\
= Z^2 \left( \frac{N e_0^2 \Omega}{12 \hbar^2} \right), \quad (F2)
$$

where $Z = \left[1 + \Delta_V \Lambda / (2 \pi^2 v^2)\right]$ is the field renormaliz-
tion factor, as presented in Sec. IV for $\epsilon_d = 1$. The same expression for the field-renormalization factor can directly be obtained by integrating over the entire Weyl-band with $0 \leq |k| \leq \Lambda$, which is legitimate since we are interested in the OC of a weakly disordered WSM for which sharp quasiparticle excitations persists all the way down to zero energy or momentum. Upon substituting $Z_\psi$ in the above expression we immediately recover Eq. (F1).

**Appendix G: $\epsilon_n$-expansion for WSM-metal QPT**

We devote the final appendix of the paper to address yet another controlled route to address the effects of disorder deep inside the WSM phase. Without any loss of generality we can express the Weyl Hamiltonian as

$$H_W = v_\perp \sum_{j=1,2} i\gamma_0 \gamma_j k_j + v_3 i\gamma_0 \gamma_3 k_3,$$  \hspace{1cm} (G1)

and so far we have considered $v_\perp = v_3 = v$. Following the spirit of “band-flattening” method, demonstrated in Sec. IIIA we deform the above Hamiltonian to

$$H_W \rightarrow H_W^n = v_\perp \sum_{j=1,2} i\gamma_0 \gamma_j k_j + C_n i\gamma_0 \gamma_3 k_3^n,$$  \hspace{1cm} (G2)

with the restriction that $n$ can now only take odd integer values, so that all symmetry properties of a WSM remain unaffected. The DOS of such a deformed system is $\rho(E) \sim |E|^{1+1/n}$. Notice in the limit $n \rightarrow \infty$ the DOS scales linearly with $E$, and disorder then become a marginal variable (outcome from a self-consistent Born calculation). Such special limit represents a two-dimensional Weyl system (since quasiparticles do not possess any dispersion along $k_\perp$). Otherwise, following the same steps of coarse-graining we find that the scaling dimension of disorder couplings after performing the disorder-averaging using the replica formalism is $[\Delta_j] = -1/n$. Therefore, we can perform a controlled RG calculation about $n \rightarrow \infty$ limit, following the spirit of an $\epsilon_n$-expansion, with $\epsilon_n = 1/n$, since $[\Delta_j] = -\epsilon_n$. For physically relevant case $\epsilon_n = 1$. Otherwise, the steps are identical to the ones presented in Sec. IIIA and the relevant Feynman diagrams are already shown in Fig. 6. For the sake of simplicity, we here focus only on the potential disorder. A detailed RG analysis within the framework of the $\epsilon_n$-expansion in the presence of generic eight disorder is left for a future investigation. The leading order RG calculation yields the following flow equations

$$\beta_X = -\Delta_V H_0(n) X = (1 - z) X,$$

$$\beta_{\Delta_V} = \Delta_V [ -\epsilon_n + \Delta_V H_0(n)],$$  \hspace{1cm} (G3)

where $X = v_\perp, C_n, \Delta_V = 2\Delta_V \Lambda^n / [(2\pi)^2 C^{\epsilon_n} v_\perp^{2-\epsilon_n}]$ is the dimensionless disorder coupling and for brevity we have dropped the ‘hat’ notation in the last set of equations. The function $H_0(n)$ reads as

$$H_0(n) = 1 + \frac{\pi^2}{24} \frac{1}{n^2} + O \left( \frac{1}{n^4} \right).$$  \hspace{1cm} (G4)

Therefore, $H_0(n)$ is a well controlled function of $1/n$. Keeping the leading order term in $H_0(n)$, the RG equations becomes

$$\beta_X = -\Delta_V X = (1 - z) X,$$

$$\beta_{\Delta_V} = \Delta_V [ -\epsilon_n + \Delta_V].$$  \hspace{1cm} (G5)

The DSE from the first equation reads as $z = 1 + \Delta_V$. The second equation supports only two fixed points: (i) the one at $\Delta_V = 0$ represents the stable WSM phase, while (ii) the unstable fixed point at $\Delta_V = \epsilon_n/2$ represents the WSM-metal QCP. The DSE and the CLE at this fixed point are respectively

$$z = 1 + \frac{\epsilon_n}{2}, \quad \nu^{-1} = \epsilon_n.$$  \hspace{1cm} (G6)

Therefore, for physically relevant case of simple WSM ($\epsilon_n = 1$), we obtain $z = 3/2$ and $\nu = 1$, same as the ones obtained from $\epsilon_m$ and $\epsilon_d$ expansions, declared in Sec. V. Note that even if we chose to keep the entire function $H_0(n)$ in the RG flow equations, we obtain the same set of critical exponents.

**Appendix H: Self-consistent Born approximation at WSM-insulator QCP**

In this Appendix, we present the computation of the inverse scattering life-time $(1/\tau_s)$ within the framework of self-consistent Born approximation, in the presence of disorder. In this formalism the $\tau_s$ is computed from the following self-consistent equation

$$\int_0^{E_\Lambda} dE \frac{\rho(E)}{(\hbar/\tau_s)^2 + E^2} = \frac{1}{W_c},$$  \hspace{1cm} (H1)

where $E_\Lambda$ is the ultraviolet energy cut-off up to which critical excitations separating a WSM and an insulator possess anisotropic dispersion, captured by $H_Q(0)$ in Eq. (6). Since at the WSM-insulator QCP, the average DOS scales as $\rho(E) \sim |E|^{3/2}$ the right-hand side of the above equation displays ultraviolet divergence $\sim E_\Lambda^{1/2}$. Such divergence can be regulated by introducing a parameter

$$\frac{1}{W_c} = \int_0^{E_\Lambda} dE \frac{\rho(E)}{E^2},$$  \hspace{1cm} (H2)

where $W_c$ corresponds to the critical strength of disorder for the instability of ballistic critical fermions. The above gap equation can then be casted as

$$\delta = \int_0^{E_\Lambda} dE \rho(E) \left[ \frac{1}{E^2} - \frac{1}{(\hbar/\tau_s)^2 + E^2} \right].$$  \hspace{1cm} (H3)
where $\delta = W - W_c/(WW_c)$ measures the reduced disorder strength from the critical one ($W = W_c$). After regularizing the ultraviolet divergence we can take the limit $E_\alpha \to \infty$ without encountering any divergence. The self-consistent solution of the scattering life-time is then obtained from the following universal scaling form

$$\sqrt{\frac{\tau_s}{\tau_s}} = \sqrt{\frac{\pi}{\delta}},$$

which immediately implies that $\tau_s^{-1}$ is finite only when $\delta > 0$ or $W > W_c$, and for $W < W_c$ we get $\tau_s^{-1} = 0$. Therefore, critical fermions separating a WSM and an insulator retain its ballistic nature up to a critical strength of disorder $W_c \sim E^1_{\chi}/2$. Only for strong disorder $W > W_c$ a metallic phase emerges where $\tau_s^{-1}$ is finite. Therefore, conclusion from self-consistent Born approximation is in qualitative agreement with our results found by field theoretic RG analysis and numerical calculation, presented in Sec. III.

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