Trajectory control with continuous thrust applied to a rendezvous maneuver

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Abstract. A rendezvous mission can be divided into the following phases: launch, phasing, far range rendezvous, close range rendezvous and mating (docking or berthing). This paper aims to present a close range rendezvous with closed loop controlled straight line trajectory. The approaching is executed on V-bar axis. A PID controller and continuous thrust are used to eliminate the residual errors in the trajectory. A comparative study about the linear and nonlinear dynamics is performed and the results showed that the linear equations become inaccurate insofar as the chaser moves away from the target.

1. Introduction
Rendezvous and docking or berthing (RVD/B) technology are key elements in mission such as assembly in orbit of larger units, re-supply of orbital platforms and stations, exchange of crew in orbital stations, repair of spacecraft in orbit, retrieval of spacecraft, and others. The RVD/B process includes several orbital maneuvers and trajectory control. On 16 March 1966, took place the first rendezvous and docking between two spacecrafts, and the first automatic RVD took place only on 30 October 1967. A rendezvous mission can be divided into a number of phases: launch, phasing, far range rendezvous, close range rendezvous and mating.

This paper will focus on close range rendezvous phase, which can be divided into two sub-phases: a preparatory phase leading to the final approach corridor, often called closing, and a final approach phase leading to the mating conditions. The objectives of the closing phase are the reduction of the range to the target and the achievement of conditions allowing the acquisition of the final approach corridor. Different acquisition strategies for V-bar and R-bar approaches can be executed. The objective of the final approach phase is to achieve docking or berthing capture conditions in terms of positions and velocities and of relative attitude and angular rates. The straight line or the quasi-straight line trajectory are used in this maneuver phase. The guidance, navigation and control (GNC) system of the chaser must following the direction of the target docking axes. To meet this requirement, the relative attitude between the docking ports of the two spacecrafts should be measured and taken to zero at the end of the maneuver. For observability and safe reasons a cone-shaped approach corridor will usually be defined, within which the approach trajectory has to remain. This corridor has a half cone angle of 10-15 deg [1].

A comparison between a linear dynamics model - take into account a circular orbit - and a nonlinear dynamics model - for an orbit with eccentricity of 0.1 - was done by [2]. Although his
main intention was present a new method for applying the Hill equations to the elliptical orbits, this comparative study is a valuable reference. So, the purposes of this paper are the following: analyze the relative motion for several initial conditions; find out the difference between the linear and nonlinear spacecraft dynamics considering the same orbital eccentricity; and to perform a close range rendezvous with closed loop controlled straight line trajectory on V-bar axis. The Section 2 shows the equations of the control system. The simulations and results are shown in Section 3. Lastly, a conclusion is given in Section 4.

2. Mathematical modeling
To perform a close range rendezvous it was developed a trajectory control system. The control system has the function of correcting the errors in the chaser trajectory with respect to a reference trajectory. It was adopted a PID controller and continuous thruster as actuator. The basic diagram of the closed loop control system is presented in Figure 1.

![Figure 1. Block diagram of the control system.](image)

It was considered a constant approaching velocity for calculation of the reference trajectory. The **Guidance** function is given by,

\[ x_r(t) = x_0 - v_x t \]  

where \( x_r \) is the reference position; \( x_0 \) is the initial distance of the chaser from the target; \( v_x \) is the approaching velocity on the V-bar axis; and \( t \) is the time.

Several types of controllers can be used in a control system, according to their application, such as nonlinear controller, LQR, LQG, PID, PID-fuzzy, \( H_{\infty} \), etc. Most industrial controllers are Proportional-Integral-Derivative (PID) due to its flexibility, low cost and robustness. For applications in RVD/B maneuvers, **PID controllers** usually meets the needs and its control law is given by [3]

\[ c(t) = K_p e(t) + K_i \int e(t)dt + K_d \frac{de(t)}{dt} \]  

where \( e(t) \) is the state error; \( K_p \), \( K_i \) and \( K_d \) are the proportional, integral and derivative gains, respectively.

Actuators are devices that operate on the spacecraft in order to maintain its orbit and attitude or change their orientation and orbit through maneuvers. **Thrusters** are the only actuators that can be used both to orbital maneuvers as for attitude maneuvers, but the thrust magnitude is different for each purpose. Spacecraft propulsion systems are divided into three categories: cold gas, chemical (solid and liquid), and electrical. The basic equation of propulsion holds for all kinds of propellants. A rocket engine develops its thrust \( (F) \) by expelling propellant (such as gas molecular or ions) at a high exhaust velocity \( (V_e) \) relative to the satellite body. The amount of thrust can be calculated as follow [4],
\[ F = V_e \frac{dm}{dt} + A_e [P_e - P_a] = V_{ef} \frac{dm}{dt} \] (3)

where \( P_e \) and \( P_a \) are the gas and ambient pressures, respectively; \( V_e \) is the exhaust velocity; \( V_{ef} \) is the effective exhaust velocity of the expelled mass with respect to the satellite; \( \frac{dm}{dt} \) is the mass flow rate of the propellant; and \( A_e \) denotes the area of the nozzle exit.

During the close range rendezvous the chaser is a few tens of meters from the target and therewith the relative navigation assumes greater importance. The following considerations are taken in the development of equations: the spacecraft is treated as a mass point; the chaser motion is disturbed by the effects of spherical earths gravitational fields and the thruster forces. Consider \( X, Y \) and \( Z \) as axis of the inertial frame; \( x, y \) and \( z \) are the axis of the target centered local orbital frame; \( r_c \) and \( r_t \) is vector radius of the chaser and target, respectively, as shown in Figure 2. In the rendezvous literature, the \( x \) axis, \( y \) axis and \( z \) axis is called by \( V \)-bar, \( H \)-bar and \( R \)-bar, respectively.

**Figure 2.** Local orbital frame and inertial frame.

The relative acceleration \( \ddot{s} \) is

\[ \ddot{s} = \ddot{r}_c - \ddot{r}_t \] (4)

The general equations for motion under the influence of a central force are written as [1]

\[ \ddot{r}_t = -\frac{\mu}{r_t^3} \]  
\[ \ddot{r}_c = -\frac{\mu}{r_c^3} + f_u \] (5)

where \( \mu \) is the earth’s gravitational constant and \( f_u \) is the specific force vector of control. The equations 4 and 5 represent the nonlinear dynamics of the system.

The target centered local orbital frame is a non-inertial frame and it is rotating around the central body with an orbital rate \( \omega \). It is common to express the relative motion by linear time varying differential equations, as shown in [1],
\[ \ddot{x} - 2\omega \dot{z} = f_x \]
\[ \ddot{y} + \omega^2 y = f_y \]
\[ \ddot{z} + 2\omega \dot{x} - 3\omega^2 z = f_z \]  

(6)

where \( \omega \) is the orbital angular velocity that for the special case of circular orbits is considered constant and can be expressed by,

\[ \omega^2 = \frac{\mu}{r_i^3} \]  

(7)

and \( f_x \), \( f_y \) and \( f_z \) are the components of the specific force vector of control.

The Equations 6 represent the linear dynamics of the relative motion and are known as Hill equations. The derivation of these equations starts from the linearization of the nonlinear equations and can be consulted in [1] and [5].

3. Results

This section will present the results about the relative motion between two spacecrafts. Two study cases were prepared: the first shows the relative motion dynamic for different initial conditions of the chaser vehicle, a comparison of the nonlinear and linear dynamics is performed in order to identify the error originating from the linearization and the second case illustrates a rendezvous maneuver considering closed loop controlled straight line trajectory. All results are presented in the target centered local orbital frame.

It was used a simulation step of 1 second. The initial keplerian elements of the target vehicle are shown in Table 1.

| Parameter                  | Value |
|----------------------------|-------|
| Altitude (km)              | 400   |
| Eccentricity               | 0     |
| Inclination (degrees)      | 30    |
| RAAN (degrees)             | 0     |
| Perigee argument (degrees) | 0     |
| Mean anomaly (degrees)     | 10    |

3.1. Case 1: Analysis of the relative motion dynamics

In this first case, the relative motion of the linear and nonlinear dynamics was evaluated for two orbits of the target. The initial condition of the chaser is shown in the legend of each figure. The first situation is considered with the chaser starting at 10 m away from the target in the direction of the R-bar axis, the relative motion can be seen in the Figure 3. It can be observed that after two orbits, the chaser is approximately 750 m away from the target. In order to evaluate the error from the linearization, the Figure 4 presents the difference between the models, considering the initial conditions of the Figure 3, as a function of target distance. Exceptionally in this graph, the stopping criterion was changed to four orbits of the target. It
is noted that the Hill equations lose their accuracy since the distance between the spacecrafts increases, reaching an error of the order of 1 m in 1500 km from the target.

Consider that the spacecrafts are coupled and the chaser is subjected to a impulse of magnitude of 0.01 m/s in the direction of +V-bar, i.e., a impulsive $\Delta V$ in the tangential direction to the orbit. The resulting movement is depicted in Figure 5. Now the same impulse is applied, however, in the radial direction to the orbit, i.e., toward +R-bar. The relative dynamic behavior, in this case, can be seen in Figure 6. The resulting trajectory is an ellipse, that is, the spacecraft returns to the initial position each orbital revolution. From the Figures 5 and 6 we can conclude that the displacements due to a radial $\Delta V$ are much smaller than those due to a tangential $\Delta V$ of the same size.

The relative motion, considering the initial condition of 10 m in the H-bar axis, as a function of time, is presented in the Figure 7. We can highlight the sinusoidal behavior of the motion when the vehicle starts from a point outside the orbital plane.

3.2. Case 2: Close range rendezvous with closed loop controlled straight line trajectory

In this second case, it was considered a trajectory control system to perform a close range rendezvous in the V-bar axis. The chaser is 100 m from the target in the V-bar direction and it is wanted to perform the rendezvous with a constant velocity of 0.05 m/s. The Figure 8 shows the transitory response with the linear and nonlinear dynamics. The movement in each axis as function of time is presented in Figure 9. It is observed that in the V-bar axis, the vehicle has met the target at a constant velocity, as desired; movement on the H-bar axis is not occurred because there was not application of forces outside the orbital plane, and there was a deviation in the R-bar axis due to coupling between the R-bar and V-bar axis.
### 4. Conclusions

This paper presented a study about rendezvous maneuver. In the first case, free drift motion and impulsive maneuvers were analyzed for several initial conditions. The divergence between the linear and nonlinear spacecraft dynamics were also investigated. The second case showed the application of the trajectory control system in a close range rendezvous. The first results indicated that an specific motion can be yielded for each initial condition, like a sinusoidal, radial or cyclic behavior. Because of the linearization, the Hill equations become inaccurate insofar as the chaser moves away from the target. So, for long distances the nonlinear model is most advisable. Although we have different transitory responses, the trajectory control system met the requirements and executed the close range rendezvous with both models.

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