Do two flavour oscillations explain both KamLAND Data and the Solar Neutrino Spectrum?

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Abstract

The recent measurement of $\Delta_{\text{sol}}$ by the KamLAND experiment with very small errors, makes definitive predictions for the energy dependence of the solar neutrino survival probability $P_{ee}$. We fix $\Delta_{\text{sol}}$ to be the KamLAND best fit value of $8 \times 10^{-5}$ eV$^2$ and study the energy dependence of $P_{ee}$ for solar neutrinos, in the framework of two flavour oscillations and also of three flavour oscillations. For the case of two flavour oscillations, $P_{ee}$ has a measurable slope in the $5 - 8$ MeV range but the solar spectrum measurements in this range find $P_{ee}$ to be flat. The predicted values of $P_{ee}$, even for the best fit value of $\theta_{\text{sol}}$, differ by 2 to 3 $\sigma$ from the Super-K measured values in each of the three energy bins of the $5 - 8$ MeV range. If future measurements of solar neutrinos by Super-K and SNO find a flat spectrum with reduced error bars (by a factor of 2), it will imply that two flavour oscillations can no longer explain both KamLAND data and the solar spectrum. However a flat solar neutrino spectrum and the $\Delta_{\text{sol}}$ measured by KamLAND can be reconciled in a three flavour oscillation framework with a moderate value of $\theta_{13} \simeq 13^\circ$. 
I. INTRODUCTION

Recently the KamLAND experiment has measured $\Delta_{sol}$ with great accuracy [1]. Their fit to the data, assuming neutrino oscillations, gives the result:

$$\Delta_{sol} = (8.0 \pm 0.5) \times 10^{-5} \text{eV}^2. \quad (1)$$

The error in this measurement, already as low as 6%, can be further improved with more data. The accuracy of KamLAND in determining the mixing angle $\theta_{sol}$ is not so good. The reason, for this great disparity in the accuracies of the two oscillation parameters, is the following: The anti-neutrino survival probability, for two flavour oscillations, is given by

$$P_{\bar{e}\bar{e}} = 1 - \sin^2 2\theta_{sol} \sin^2 (1.27\Delta_{sol}L/E). \quad (2)$$

This expression has minima when the energy takes the following values, $E_p = 2.54\Delta_{sol}L/(2p + 1)\pi$, where $p$ is an integer. Thus the position of the minimum is dictated by $\Delta_{sol}$ and the depth of the minimum is dictated by $\theta_{sol}$. The inverse beta-decay spectrum of positrons, measured by KamLAND as a function of the anti-neutrino energy, is a product of the spectrum in the case of no oscillations (which is a smooth function) and the above survival probability. The minima of the survival probability lead to kinks (or distortions) in the measured spectrum. Given the baseline of KamLAND and the energy range of the anti-neutrinos only the minimum corresponding to $p = 1$ is relevant. The position of the kink produced by this minimum can be measured with great accuracy which leads to $\Delta_{sol}$ being determined with great accuracy. The amount of distortion in the kink can be measured only with a limited accuracy which means that $\theta_{sol}$ can be determined only with a limited accuracy. Detailed statistical calculations also lead to the same conclusion [2].

Given the accurate measurement of $\Delta_{sol}$ by KamLAND, it is worth raising the question: What is its impact on the solar neutrino problem? In the $\Delta_{sol} - \theta_{sol}$ plane, the allowed contours of global solar data and that of KamLAND data have a significant overlap [3]. This overlap region is essentially what is obtained if an analysis of all solar neutrino and KamLAND data is performed [1, 3, 6-8]. However, in this paper, we address the following question: Given $\Delta_{sol} = 8 \times 10^{-5} \text{eV}^2$, what predictions do we get for the solar neutrino spectrum? We first analyze the neutrino survival probability ($P_{ee}$) of the Large Mixing Angle (LMA) solution of the solar neutrino problem assuming two flavour oscillations. We study
the energy dependence of $P_{ee}$ in detail as a function of the mixing angle $\theta_{sol}$ and show how these details can be tested in the future by precise data from Super-K and SNO experiments. We then demonstrate that a potential conflict with future spectral measurements can be resolved by reanalyzing $P_{ee}$ in a three flavour framework and by invoking a moderate value of $\theta_{13}$. Varying $\Delta_{sol}$ within the range allowed by KamLAND, does not change our conclusions.

II. TWO FLAVOUR ANALYSIS

In this section we assume that only two mass eigenstates participate in the oscillations of electron neutrinos and those of their anti-neutrinos. This is equivalent to setting $\theta_{13} = 0$ in three flavour oscillations and we have

$$\Delta_{sol} = \Delta_{21} \text{ and } \theta_{sol} = \theta_{12}. $$

We also assume CPT invariance and take the mass-square difference measured by KamLAND to be $\Delta_{sol}$. The electron neutrino survival probability, in the case of LMA solution of the solar neutrino problem, is given by

$$P_{ee} = \frac{1 + \cos 2\theta_{12} \cos 2\theta_{12}}{2},$$

(3)

where $\theta_{12}$ is the matter dependent value of $\theta_{12}$ at the solar core and is given by,

$$\cos 2\theta_{12} = \frac{\Delta_{21} \cos 2\theta_{12} - A_c}{\Delta_{21}^c}. $$

(4)

Here $A_c$(in eV$^2$) = $10^{-5}$ $E$ (in MeV) is the Wolfenstein term in the core of the sun and $\Delta_{21}^c$ is the corresponding mass-squared difference. It is given by

$$\Delta_{21}^c = \sqrt{(\Delta_{21} \cos 2\theta_{12} - A_c)^2 + (\Delta_{21} \sin 2\theta_{12})^2}. $$

(5)

Analysis of solar neutrino data restricts the $3\sigma$ range of $\theta_{12}$ to be $27^\circ - 41^\circ$, with the best fit at $34^\circ$.

The data from various gallium experiments requires that the average $P_{ee}$ for low energy ($E < 1$ MeV) should be about 0.54 and Super-K spectrum data requires that for high energy ($E > 5$) MeV, $P_{ee}$ should be essentially constant with a value of about 0.35. To see the variation of $P_{ee}$ with respect to energy, we differentiate it and obtain

$$\frac{dP_{ee}}{dE} \text{ (per MeV)} = \frac{\cos 2\theta_{12} \sin^2 2\theta_{12} 10^{-5}}{\Delta_{21}^c} \frac{10^{-5}}{2\Delta_{21} \left[(\cos 2\theta_{12} - A_c/\Delta_{21})^2 + (\sin 2\theta_{12})^2\right]^{3/2}}. $$

(6)

(7)
We now point out the two essential features of Eq. (7).

- The slope of $P_{ee}$ is steepest at the energy when $A^e \approx \Delta_{21} \cos 2\theta_{12}$. This energy of steepest slope is given by $E_{ss} \approx 8 \cos 2\theta_{12}$, where we have used the value of $\Delta_{21}$ from KamLAND.

- In the neighbourhood of $E_{ss}$, the slope achieves its maximum value of $\cos 2\theta_{12}/(16 \sin 2\theta_{12})$. This shows that smaller the value of the vacuum mixing angle greater will be the slope of $P_{ee}$ in the neighbourhood of $E_{ss}$.

\hspace{1cm}

**FIG. 1:** $P_{ee}$ vs $E$ for the energy range 5-15 MeV for $\Delta_{21} = 8 \times 10^{-5}$ eV$^2$ and for $\theta_{12} = 27^\circ$, $34^\circ$ and $41^\circ$. Data points with 1σ error bars are obtained using the Super-K data from [15].

In figure 1, we have plotted $P_{ee}$ vs $E$, for three values of $\theta_{12}$, which are the best fit value and the lower and the upper 3σ bounds obtained by global analysis. As stated earlier, the value of $\Delta_{21}$ is fixed to be the KamLAND best fit value $8 \times 10^{-5}$ eV$^2$. To facilitate the comparison between LMA predictions and the data, the spectrum data from Super-K [15] are also shown in the figure.

We observe the following important features of each of these plots.
1. For a low value of $\theta_{12} = 27^\circ$, $P_{ee}$ decreases sharply from 0.49 at 5 MeV to 0.36 at 8 MeV and further decreases to 0.26 at 14 MeV. This is in conflict with the essentially flat spectrum observed by Super-K and SNO for $E > 5$ MeV [15, 16]. For this value of $\theta_{12}$, the energy region of the steepest slope in $P_{ee}$ is centered around $E_{ss} \approx 5$ MeV. Hence there is a sharp drop in the probability in this range. The average value of $P_{ee}$ for this curve is close to the average $P_{ee}$ measured by Super-K but the sharp drop in the $5 - 8$ MeV region means that the curve misses the points at both low and at high energies.

2. For a high value of $\theta_{12} = 41^\circ$, we do have a flat spectrum, but the average value of $P_{ee}$ is 0.45, which is considerably higher than 0.35 measured by Super-K and SNO [15, 16]. For this value of $\theta_{12}$, the energy region of the steepest slope in $P_{ee}$ is centered around $E_{ss} \approx 1.1$ MeV, so the energy range $5 - 8$ MeV is reasonably away from this point. The flat spectrum in this case is the consequence of $\theta_{12}$ being close to maximal mixing, i.e $45^\circ$, in which case matter effects play no role and where $P_{ee}$ is predicted to be close to 0.5 for the whole energy range.

3. For the best fit value of $\theta_{12} = 34^\circ$, $P_{ee}$ changes from 0.45 at 5 MeV to 0.4 at 8 MeV and further decreases to 0.35 at 14 MeV. For this value of $\theta_{12}$, the energy region of the steepest slope in $P_{ee}$ is centered around $E_{ss} \approx 3$ MeV. The energy range $5 - 8$ MeV is a little away from this point but well within the region where $dP_{ee}/dE$ (c.f. Eq. 7) is not negligible.

Thus we find that low values of $\theta_{12}$ give the correct average value of $P_{ee}$ in the energy range visible at Super-K but predict a steep slope. High values of $\theta_{12}$ have flat spectrum but the average value of $P_{ee}$ is much higher than that measured by Super-K. The best fit value is a presently acceptable compromise between the two different pulls on $\theta_{12}$ generated by the two different features of $P_{ee}$, i.e. its average value and its slope. But even the best fit curve suffers from two objections when compared to the data.

1. The predicted values of $P_{ee}$ in the energy range $5 - 8$ MeV are 2 to $3\sigma$ too high compared to the data.

2. The predicted values show slope in the $5 - 8$ MeV range which is not observed in the data.
These features are also reflected in table 1, where the present Super-K data [15] and the two flavour LMA predictions for $P_{ee}$ for the best fit value of $\theta_{12}$ are shown. If future data from Super-K and SNO reduce the error bars in these bins by a factor 2, and if the central values of the measurements do not change, then it must be concluded that two flavour oscillations are inadequate to explain both KamLAND data and the solar neutrino spectrum.

| Energy Range (MeV) | $(\text{Obs/SSM})_{SK}$ | $P_{ee}(\text{expt})$ | LMA-$P_{ee}$ $\theta_{12} = 34^\circ$ |
|-------------------|--------------------------|----------------------|----------------------------------------|
| 5 – 5.5           | 0.467 ± 0.04 ± 0.017     | 0.36 ± 0.05 ± 0.018  | 0.446                                  |
| 5.5 – 6.5         | 0.458 ± 0.014 ± 0.007    | 0.35 ± 0.017 ± 0.008 | 0.429                                  |
| 6.5 – 8           | 0.476 ± 0.008 ± 0.006    | 0.37 ± 0.01 ± 0.007  | 0.407                                  |
| 8 – 9.5           | 0.460 ± 0.009 ± 0.006    | 0.35 ± 0.01 ± 0.007  | 0.385                                  |
| 9.5 – 11.5        | 0.463 ± 0.01 ± 0.006     | 0.356 ± 0.012 ± 0.007 | 0.367                                 |
| 11.5 – 13.5       | 0.462 ± 0.017 ± 0.006    | 0.354 ± 0.02 ± 0.007 | 0.352                                  |
| 13.5 – 16         | 0.567 ± 0.039 ± 0.008    | 0.48 ± 0.047 ± 0.01  | 0.342                                  |
| 16 – 20           | 0.555 ± 0.146 ± 0.008    | 0.466 ± 0.175 ± 0.01 | 0.332                                  |

TABLE I: $P_{ee}$ extracted from Super-K spectral data vs the values predicted by the LMA solution. All error bars are 1 $\sigma$.

### III. THREE FLAVOUR OSCILLATIONS

We now consider three flavour oscillations with non-zero $\theta_{13}$. We are concerned only with $P_{ee}$ and $P_{\bar{e}\bar{e}}$ which are independent of the mixing angle $\theta_{23}$ and CP-phase $\delta$, but do depend on $\theta_{13}$. Because $\Delta_{31} \gg \Delta_{21}$, oscillations due to $\Delta_{31}$ are averaged out at KamLAND and we have

$$P_{\bar{e}\bar{e}} = 1 - \frac{1}{2} \sin^2 2\theta_{13} - \cos^2 \theta_{13} \sin^2 2\theta_{12} \sin^2 (1.27 \Delta_{21} L/E).$$

(8)

For this expression of $P_{\bar{e}\bar{e}}$ also, the position of the kink in the positron spectrum depends only on $\Delta_{21}$ and is independent of $\theta_{12}$ and $\theta_{13}$. Hence the value of $\Delta_{21}$ determined by the spectral distortion data of KamLAND, even in the case of three flavour oscillations, is the same as in the case of two flavour oscillations and has the same accuracy.
$P_{ee}$ for solar neutrinos for the LMA solution is given by [9, 17]

$$
P_{ee} = \cos^2 \theta_{13} \cos^2 \theta_{13}^c \left( \cos^2 \theta_{12} \cos^2 \theta_{12}^c + \sin^2 \theta_{12} \sin^2 \theta_{12}^c \right) + \sin^2 \theta_{13} \sin^2 \theta_{13}^c,
$$  \hspace{1cm} (9)

where $\theta_{12}^c$ is given by Eq. (4) and Eq. (5), with the modification that $A_c$ in those equations should be replaced by $A_c \cos^2 \theta_{13}$. Matter dependence of $\theta_{13}$ is given by

$$
\sin \theta_{13}^c = \sin \theta_{13} \left[ 1 + \frac{A_c}{\Delta_{31}} \cos^2 \theta_{13} \right]; \cos \theta_{13}^c = \cos \theta_{13} \left[ 1 - \frac{A_c}{\Delta_{31}} \sin^2 \theta_{13} \right].
$$  \hspace{1cm} (10)

Here $\Delta_{31} = \Delta_{atm} = 2 \times 10^{-3} \text{ eV}^2 \gg A_c$ for the whole energy range of solar neutrinos. Hence in Eq. (9) $\theta_{13}^c$ can be replaced by $\theta_{13}$ and $P_{ee}$, of three flavour oscillations, simplifies to:

$$
P_{ee} = \cos^4 \theta_{13} \left( \cos^2 \theta_{12} \cos^2 \theta_{12}^c + \sin^2 \theta_{12} \sin^2 \theta_{12}^c \right) + \sin^4 \theta_{13}.
$$  \hspace{1cm} (11)

Thus the solar neutrino survival probability in three flavour oscillations depends only on one additional parameter $\theta_{13}$.

Analysis of only solar neutrino data allows $\theta_{13} \leq 40^\circ$ [18] and analysis of only atmospheric neutrino data allows $\theta_{13} \leq 30^\circ$ [19, 20, 21]. It is the CHOOZ experiment, which in combination with the $\Delta_{31}$ measurement from atmospheric neutrino data which sets a stringent upper bound on $\theta_{13}$ [22]. Present lower bound on $\Delta_{31}$ from Super-K data is $1.5 \times 10^{-3} \text{ eV}^2$ [23]. For this small a value of $\Delta_{31}$, CHOOZ sets an upper bound of about $15^\circ$ [24].

The potential conflict between the LMA predictions for $\Delta_{21} = 8 \times 10^{-5} \text{ eV}^2$ and the Super-K measurement of the solar spectrum can be naturally resolved in a three flavour oscillation framework. In the three flavour case, the energy range where $A_c \cos^2 \theta_{13} \approx \Delta_{21} \cos 2 \theta_{12}$ is centered around the energy $E_{ss} = 8 \cos 2 \theta_{12} / \cos^2 \theta_{13}$. Thus, for a given $\theta_{12}$, the strongest distortion is shifted to slightly higher energies as compared to the two flavour case. For even moderate values of $\theta_{13}$, $\cos^4 \theta_{13}$ differs appreciably from 1. Hence, in three flavour oscillations, both $P_{ee}$ and its slope become smaller, thus mitigating the two discrepancies between the solar spectrum data and LMA predictions of two flavour oscillations [25].

First we take a look at low energies, where $E \leq 1 \text{ MeV}$. In this energy range the matter term $A_c \ll \Delta_{21}$ and we have $\theta_{12}^c \simeq \theta_{12}$. Then Eq. (11) simplifies to:

$$
\langle P_{ee} \rangle = \cos^4 \theta_{13} \left( \cos^2 \theta_{12} + \sin^4 \theta_{12} \right) + \sin^4 \theta_{13}
$$

$$
= \cos^4 \theta_{13} \left( 1 - \frac{1}{2} \sin^2 2 \theta_{12} \right) + \sin^4 \theta_{13}
$$

$$
= 1 - \frac{1}{2} \sin^2 2 \theta_{13} - \frac{1}{2} \sin^2 2 \theta_{12} \cos^4 \theta_{13}.
$$  \hspace{1cm} (12)
If we set $\theta_{13} = 0^\circ$, Eq. (12) gives us the usual low energy form for the LMA solution. From the gallium experiments we have $P_{ee}^l \simeq 0.54 \pm 0.045$. In pure two flavour oscillations, this data pulls $\theta_{12}$ towards $\pi/4$. However, we see from Eq. (12) that a moderate value of $\theta_{13}$ allows $\theta_{12}$ to differ appreciably from $\pi/4$.

To study the behaviour at high energies it is convenient to recast Eq. (9) as,

$$
\langle P_{ee} \rangle = \cos^4 \theta_{13} \left( \frac{1}{2} + \frac{1}{2} \cos 2\theta_{12} \cos 2\theta_{12} \right) + \sin^4 \theta_{13}.
$$

(13)

There is no immediate simplification of the above formula because for the solar neutrino energy range the Wolfenstein term is never much larger than $\Delta_{21}$. Hence we use the full expression and study its dependence on $\theta_{13}$ for the best fit value of $\theta_{12}$, by means of graphs of $P_{ee}$ vs $E$.

![Graph](image.png)

**FIG. 2:** $P_{ee}$ vs $E$ for the energy range 5-15 MeV for $\Delta_{21} = 8 \times 10^{-5}$ eV$^2$, $\theta_{12} = 34^\circ$ and $\theta_{13} = 0^\circ, 13^\circ, 20^\circ$. For comparison, Super-K data [15] with 1$\sigma$ error bars also are included.

In Fig. (2), for $\theta_{12} = 34^\circ$, we plot $P_{ee}$ for different values of $\theta_{13}$ along with the graph for $\theta_{13} = 0^\circ$. Note that the effect of even moderate values of $\theta_{13}$ is to pull the predictions for $P_{ee}$ towards the data, thus leading to a better agreement.
IV. CONCLUSION

Recent KamLAND data has unambiguously picked the parameters in the LMA region as the solution to the solar neutrino problem, confirming the expectations of global analyses of solar neutrino data. However, for the large value of $\Delta_{21}$ obtained by KamLAND, there is a discrepancy between the two flavour predictions of $P_{ee}$ and the Super-K spectrum data for all allowed values of $\theta_{12}$. For low values of $\theta_{12}$ the discrepancy is in the slope of $P_{ee}$ in the range $5 - 8$ MeV and for larger values of $\theta_{12}$ the discrepancy is in the average value of $P_{ee}$ in the energy range $5 - 15$ MeV. This discrepancy can turn into a full fledged conflict if the error bars on the solar neutrino spectrum data can be reduced by a factor of 2. This conflict between the large value of $\Delta_{21}$ and the Super-K spectrum can be resolved in a three flavour oscillation analysis for moderate values of $\theta_{13}$. A non zero value of $\theta_{13}$ of about $13^\circ$ can both reduce the average value of $P_{ee}$ and give a flatter spectrum and the conflict with the spectrum measurement can be resolved. An equivalent and perhaps more striking way of looking at this analysis is that a careful study of solar neutrino spectrum gives us an handle on $\theta_{13}$ independent of the reactor constraints.

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