Pion production and absorption in heavy-ion collisions

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Based on the isospin dependent transport model IBUU, the pion production and its absorption are thoroughly studied in the central collision of Au + Au at a beam energy of 400 MeV/nucleon. It is found that the pions are firstly produced by the hard ∆ decay at the average density around 1.75ρ0, whereas about 18% of them are absorbed absolutely in the subsequent inelastic collisions. For the free pions observed, more than half of them have been scattered for one or more times before they are free from matter. And the more scattering numbers of pions, the higher the momentum they possess. These pions, due to longer time of their existence in high density nuclear matter, carry more information about the symmetry energy of the nuclear matter at high densities.

I. INTRODUCTION

Heavy-ion collisions (HIC) offer a unique possibility to study bulk properties of hot and compressed nuclear matter. One of the main goals of such study is to determine the density-dependence of the symmetry energy (SE) at high densities. The symmetry energy plays essential roles in understanding a number of physical phenomena and processes. However it can not be directly measured in experiments and has to be extracted from observables which are sensitive to the symmetry energy.

Pion production is a dominant feature in heavy-ion collisions at intermediate energies. In the collisions near the pion-production threshold the nuclear matter can be compressed up to about 2 times normal density before it expands again. During the compression stage the pions are produced mainly from the decay of the ∆ resonances. Therefore the pion production is considered to be important for extracting the information of the properties of the nuclear matter at high densities. Since the charged pion ratio in heavy-ion collisions was first suggested in Ref. 13, pion production has attracted much attention in pinning down the density dependence of the symmetry energy. During the last decade, a lot of pion observables have been proposed as promising probes, such as the collective pion flows, the cone-azimuthal emission of charged pions, etc. However, comparing the theoretical results of the π−/π+ ratio with the experimental data, analyses came to rather conflicting conclusions on the stiffness of the nuclear symmetry energy at high densities.

In the reactions near the threshold of pion-production the pions are mainly the decay products of the ∆ resonances. However most of the them can be absorbed into ∆ resonances because of the inelastic πN collisions, and then may decay into pions frequently, i.e. scattering process. The detail of the scattering process plays an important role for extracting the information of the symmetry energy at different densities. At the same time, it is commonly known that pions observed are produced at high densities in heavy-ion collisions. The quantitative study of the density at which pion produces, as well as the pion absorption or its scattering, is seldom reported yet. To further extract the information of compressed matter by pionic observables, it is necessary to perform a detailed analysis of the pions production and its absorption in HIC.

However, the information of the pion absorption or its scattering process can not be extracted in experiments so far but can be obtained in theoretical calculations. In this paper, a detailed statistical investigation of all the inelastic collisions in the reaction of Au + Au at 400 MeV/nucleon beam energy is performed. All the inelastic collisions are recorded and investigated statistically. Further more, the charged pions at the freeze out are categorized by their production and re-scattering processes. The analysis shows the pions are indeed produced at high densities. Moreover about 60% of the pions observed have re-absorption and re-decay processes after they were produced first time, and due to longer time of their existence in high density nuclear matter, they carry more substantial information of the high-density behavior of the symmetry energy than those without any scattering process, which are produced from hard ∆ decay directly and free from matter immediately.

II. THE THEORETICAL MODEL

In the past decade the the isospin-dependent Boltzmann-Uehling- Uhlenbeck transport model (IBUU) have been very successful in describing the dynamical evolution of nucleons in phase space, as well as the reaction dynamics of heavy-ion collisions. The present IBUU transport model originating from the IBUU04 model can describe the time evolution of the single-
The parameter values are

\[
\frac{\partial f}{\partial t} + \nabla_\vec{p} E - \nabla \vec{r} f = I_e,
\]

where \( I_e \) is the collision item and \( f(\vec{r}, \vec{p}, t) \) is the phase-space distribution function which denotes the probability of finding a particle at time \( t \) with momentum \( \vec{p} \) at position \( \vec{r} \). \( E \) denotes the total energy, ie, \( E = E_{\text{kin}} + U \). \( U \) is the mean-field potential of the single particle and can be expressed as

\[
U(\rho, \delta, \vec{p}, \tau) = A_u(x) \frac{\rho}{\rho_0} + A_l(x) \frac{\rho}{\rho_0} + B \left( \frac{\rho}{\rho_0} \right)^{\sigma} (1 - \frac{x}{\rho_0}) - 8 \pi \frac{B}{\sigma + 1} \frac{\rho^{-1}}{\rho_0^2} \delta \rho^	au + 2 C_{\tau, \tau} \rho_0 \int d^3 \vec{p} \frac{f_{\tau}(\vec{r}, \vec{p}')} {1 + (\vec{p} - \vec{p}')^2 / \Lambda^2} + 2 C_{\tau', \tau} \rho_0 \int d^3 \vec{p}' \frac{f_{\tau'}(\vec{r}, \vec{p}')} {1 + (\vec{p} - \vec{p}')^2 / \Lambda^2},
\]

where \( \tau, \tau' = \pm 1/2 \) denote the neutron and the proton, respectively. The variable \( x \) denote the stiffness of the symmetry energy. Varying the \( x \), one can get different forms of the symmetry energy predicted by various symmetry energy nuclear models without changing any property of symmetric nuclear matter and the value of symmetry energy at normal density \( \rho_0 \). The parameters \( A_u(x), A_l(x) \) are \( x \) dependent and defined as

\[
A_u(x) = -95.98 - \frac{2B}{\sigma + 1} x,
\]

\[
A_l(x) = -120.57 + \frac{2B}{\sigma + 1} x.
\]

The parameter values are \( B = 106.35 \) MeV, \( \sigma = 4/3 \). \( \Lambda = p_F^0 \) is the nucleon Fermi momentum in symmetric nuclear matter, \( C_{\tau, \tau} = -103.4 \) MeV and \( C_{\tau', \tau} = -11.7 \) MeV. The \( C_{\tau, \tau'} \) and \( C_{\tau', \tau} \) terms are the momentum-dependent interactions of a nucleon with unlike and like nucleons in the surrounding nuclear matter. With this potential we can get binding energy -16 MeV and incompressibility 211 MeV for symmetric nuclear matter and the symmetry energy 31.5 MeV at saturation density. The resonance \( \Delta \) potential is given by

\[
U_{\Delta}^- = U_n,
\]

\[
U_{\Delta}^0 = \frac{2}{3} U_n + \frac{1}{3} U_p,
\]

\[
U_{\Delta}^+ = \frac{1}{3} U_n + \frac{2}{3} U_p,
\]

\[
U_{\Delta}^{++} = U_p.
\]

In the present work, pions are produced via the decay of \( \Delta \) resonance. Near the pion-production threshold, the inelastic reaction channels as follows are taken into account

\[
NN \rightarrow N \Delta(\text{hard} \Delta \text{production}),
\]

\[
N \Delta \rightarrow NN(\Delta \text{absorption}),
\]

\[
\Delta \rightarrow N \pi(\Delta \text{decay}),
\]

\[
N \pi \rightarrow \Delta(\text{soft} \Delta \text{production}).
\]

The free inelastic isospin decomposition cross sections are

\[
\sigma_{pp \rightarrow n \Delta^{++}} = \sigma_{nn \rightarrow p \Delta^-} = \sigma_{10} + \frac{1}{2} \sigma_{11},
\]

\[
\sigma_{pp \rightarrow p \Delta^+} = \sigma_{nn \rightarrow n \Delta^0} = \frac{3}{2} \sigma_{11},
\]

\[
\sigma_{np \rightarrow n \Delta^0} = \sigma_{np \rightarrow n \Delta^+} = \frac{1}{2} \sigma_{11} + \frac{1}{4} \sigma_{10}
\]

The \( \sigma_{II'} \) can be parametrized by

\[
\sigma_{II'}(\sqrt{s}) = \frac{\pi (hc)^2}{2p^2} \alpha(pF_0)^2 \left| m_0^2 \Gamma^2(q/q_0)^3 - m_0^2 \Gamma_0^2 \right|^2.
\]

Here the \( I \) and \( I' \) are the initial state and final state isospins of two nucleons, for explicit details and parameters, see ref.\[35\]. The cross section for the two-body free inverse reaction can be described by the modified detailed balance,

\[
\sigma_{NN \rightarrow \Delta \pi} = \frac{m_\Delta^2 \Gamma_\Delta^2 \Sigma_{NN \rightarrow \Delta \pi}} {2(1 + \delta)p_i} \int \sqrt{s - m_{NN}} \frac{dm_{\Delta} \Delta(p_{\Delta})} {2\pi},
\]

where \( p_f \) and \( p_i \) are the nucleon center of mass momenta in the \( NN \) and \( N \Delta \) channels, respectively. \( P(m_{\Delta}) \) is the mass function of the \( \Delta \) produced in \( NN \) collision and can be defined according to a modified Breit-Wigner function\[36\],

\[
P(m_{\Delta}) = \frac{p_f m_{\Delta} \times 4 m_{\Delta}^2 \Gamma_{\Delta}} {m_{\Delta}^2 - m_{\Delta 0}^2 + m_{\Delta 0}^2},
\]

where \( m_{\Delta 0} \) denotes the centroid of the resonance and \( \Gamma_{\Delta} \) is the width of the resonance \( \Delta \). Assuming the \( \Delta \) be produced isotropically in the nucleon-nucleon center of mass, and the decay of \( \Delta \rightarrow \pi N \) with an isotropic angular distribution in the \( \Delta \) rest frame, the width of \( \Delta \) resonance can be given in a simplicistic fashion

\[
\Gamma_{\Delta} = \frac{0.47 \alpha^3} {m_\pi^2 [1 + 0.6(q/m_\pi)^2]^3}.
\]

The \( q \) is the pion momentum in the \( \Delta \) rest frame and defined as

\[
q = \sqrt{(m_{\Delta}^2 - m_n^2 + m_p^2)^2 - 4m_{\Delta}^2},
\]

The decay of the resonance into the nucleon and the pion is carried out by the Monte Carlo method for each time step \( dt \) in our calculation, with the probability as

\[
P_{\text{decay}} = 1 - \exp(-dt \Gamma_{\Delta}/\hbar).
\]
The meson-baryon interactions in our calculations are treated via the formation of baryon resonances, and the Breit-Wigner form of resonance formation can be modified as:

\[
\sigma_{\pi+N} = \sigma_{\max}(\frac{q_0}{q})^2 \frac{\frac{1}{3} \Gamma^2}{(m_\Delta - m_\Delta^0)^2 + \frac{1}{3} \Gamma^2},
\]  

where \( q_0 \) represents the pion momentum at the centroid \( m_\Delta = 1.232 \text{ GeV} \) of the resonance mass distribution. The maximum cross sections are given by:

\[
\begin{align*}
\sigma_{\pi^+p-\Delta^+} = \sigma_{\max}^{-n-\Delta^-} &= 200 \text{ mb}, \\
\sigma_{\pi^-p-\Delta^0} = \sigma_{\max}^{+n-\Delta^+} &= 66.67 \text{ mb}, \\
\sigma_{\pi^0p-\Delta^+} = \sigma_{\max}^0n-\Delta^0 &= 133.33 \text{ mb}.
\end{align*}
\]  

III. RESULTS AND DISCUSSIONS

Transport theories have been very successful in describing the reaction dynamics of heavy-ion collisions. Due to their strong interaction with the nuclear environment pionic observables at the freeze out are the result of complex creation and rescattering processes. In order to obtain detail information of the pion production and its absorption, we study all the inelastic collisions in the central collision of \( Au + Au \) at a beam energy of 400 MeV/nucleon within the frame work of IBUU. Firstly we investigate the numbers of different inelastic reactions and the densities at which they take place, which is shown in Fig. 1. By comparing the solid line and the dash line, it can be seen that only 33% of hard \( \Delta \) can decay into pions and the rest of them are subsequently absorbed into nucleon without any decay. Furthermore, due to the low production threshold, pions are reabsorbed and re-produced quite frequently. About 18% of the pions from hard \( \Delta \) will be absorbed thoroughly, and the rest are to be free particles ultimately, but probably having one or more scattering process (\( \pi N \to \Delta \to \pi N \)) before they are detected.

In Fig. 1 we can also see that the reaction \( NN \to N\Delta \) takes place at the average density around \( 1.75 \rho_0 \), and in the almost same range of the density, the hard \( \Delta \) decay into pions. Most of the pions from hard \( \Delta \) decay will be scattered in the evolution. Due to the scattering process, the \( \Delta \) decays into free pions at the freeze-out take place in a wide density range. Nevertheless, the average density is up to \( 1.5 \rho_0 \). Therefore it is reasonable for pion production to probe the properties of the nuclear matter at high densities.

Fig. 2 shows the evolution of the inelastic collision number in the reaction. As common known, the \( NN \) inelastic collisions and the hard \( \Delta \) decay take place in the early stage. The hard \( \Delta \) mostly produces at the time of about \( 15 fm/c \), and after after 30 \( fm/c \), there are almost no new hard \( \Delta \) produced. It can also be estimated Fig. 2 that the absorptivity of the pions is about 18%.

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{fig1.png}
\caption{Local density distributions of the number of the inelastic reaction in the central collision of \( Au + Au \) at a beam energy of 400 MeV/nucleon.}
\end{figure}

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{fig2.png}
\caption{Time evolution of the number of the inelastic reaction in the central collision of \( Au + Au \) at a beam energy of 400 MeV/nucleon.}
\end{figure}

In the following, we focus on the charge pions at the freeze out because of their advantage in the detector acceptance. With the analysis of the complex collision history, we classified the free charge pions, according to the \( \pi N \) rescattering number in their history. Fraction of charged free pions originating from specific rescattering process are plotted in Fig. 3. Here the abscissas are the cycle-index of the \( \pi N \) scattering of the collision history for the detectable pions. For example, 0 on the abscissa is
corresponding the pions without any absorption process after they are produced from hard $\Delta$ decay, 1 corresponding the pions have been $\pi N$ scattered for one time, i.e. by the channel $NN \rightarrow N\Delta \rightarrow NN\pi \rightarrow N\Delta \rightarrow NN\pi_{free}$, and so on.

It can been seen that most of the free pions have been absorbed into $\Delta$ and then re-decay into pions after they are firstly produced by hard $\Delta$ decay. Our calculation shows less than 40% of the detectable negative pions are seen to freeze out as soon as they are produced by hard $\Delta$ decay, without any $\pi N$ scattering. It can been also seen that at least 5% of the negative pions have been scattered for at least five times before they are detected.

For positive pions, the fraction of those without $\pi N$ scattering process is about 44%, larger than that of the negative ones, as a result of the Coulomb potential from protons, i.e., negative pions are attracted to while positive ones are repelled away.

We next investigate the pion creation history, and focus on the two processes. One is the first formation, i.e. the decay of the hard $\Delta$ into pion, another is last formation, i.e. the $\Delta$ decay into free pion at the freeze-out. Fig. 3 shows the average density at which the first and last creation take place, for different categories of free pion which is categorised in Fig. 3. As shown the average density of the first formation for all categories is apparently above the normal nuclear density. The pions with more scattering processes are mostly produced from hard $\Delta$ decay at higher density. The average density of the last formation of course is much lower than that of the first formation. Nevertheless, it is still above the normal nuclear density for most of categories. The average densities of the formations of $\pi^+$ are little higher than that of $\pi^-$ because of the Coulomb potential.

Fig. 4 shows the average time of the first and the last formations of the pion, versus the different categories of pion. One can see that for the categories of the pion with less scatterings, their first formations take place later. It is not surprising for the fact that the pions which created from $\Delta$ decay earlier have more probabilities to be scattered, and the whole scattering process lasts in a larger time range.

In Fig. 5 we investigate the average transverse momen-
FIG. 6: The average transverse momentum of the different types of free pions categorized by their scattering numbers, in the central collision of \( Au + Au \) at a beam energy of 400 MeV/nucleon.

FIG. 7: The average scattering number in the history of the free pions as a function of the transverse momentum, in the central collision of \( Au + Au \) at a beam energy of 400 MeV/nucleon.

FIG. 8: The \( \pi^-/\pi^+ \) ratio versus the different types of free pions categorized by their scattering numbers in the central collision of \( Au + Au \) at a beam energy of 400 MeV/nucleon with different symmetry energies.

The average transverse momentum of the different categories of free pions. One can see that negative pions with more scattering processes in general have higher transverse momenta. It also can been seen the average momentum of the positive pions is obviously higher than that of the negative pions for each category, due to the coulomb potential.

We also calculated the average scattering number as shown in Fig. 7 as a function of pion transverse momentum \( p_t \). The results show the pions with higher transverse momentum mostly have more scattering processes, which is consistent with that shown in Fig. 6. It should be noticed that in the transverse momentum spectrum at small \( p_t \leq 50 \text{MeV} \), the average scattering number is closed to zero, indicating most of these pions have never been scattered since they were produced from hard \( \Delta \) decay.

Fig. 8 shows the \( \pi^-/\pi^+ \) ratio for different categories of free pions, with soft and stiff symmetry energies, respectively. We can find that the ratios increase with increasing scattering number, in both situations. Because of the coulomb potential, negative pions have more probabilities to be absorbed and scattered than positive ones. It should also be noticed the effects of the symmetry energy are apparent for most categories. However for zero category, i.e. these pions without any scattering process, the effect appear to be negligible. While with increasing scattering number, the ratio shows more sensitive to the symmetry energy. In particular for the pions with five scattering processes, the effect reach to as much as more than 15%. The difference of the sensitivity for different categories is reasonable. The pions without more scattering processes, are created by hard \( \Delta \) decay and freeze out to the detector directly. Whereas for those pions with more scattering processes, their first formations take place earlier and last formations take place later. Therefore, the time of they exit in high density matter is longer, which enhancing the sensitivity to the stiffness of nuclear symmetry energy at high densities. This implies that the effect of the symmetry energy is governed not only by the density where the pions are originated but also by the time that they spent in high density region.
during their whole formation processes.

IV. CONCLUSIONS

In conclusion, based on the framework of the transport model IBUU, we investigate the central collision of $^{197}\text{Au} + ^{197}\text{Au}$ at a beam energy of 400 MeV/nucleon. We analyzed all the inelastic collisions to extract the information of the pion production and its absorption. The statistical investigation shows the pions are firstly produced by the hard $\Delta$ decay at the average density around $1.75\rho_0$. However about 18% of them are absorbed absolutely in the subsequent inelastic collisions, thus can not be observed in the experiment. From categorizing the free pions by their scattering processes, it is found that most of the free pions have been scattered for one or more times. The pions with more scattering numbers have existed in high-density region for longer time. As a result those pions carry more information of the high-density region and exhibit significant sensitivity to the symmetry energy.

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