DIBARYONS IN NUCLEAR MATTER

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Abstract

The possibility for occurrence of a Bose condensate of dibaryons in nuclear matter is investigated within the framework of the Walecka model in the mean-field approximation. Constraints for the $\omega$- and $\sigma$-meson coupling constants with dibaryons following from the requirement of stability of dibaryon matter against compression are derived and the effect of $\sigma$- and $\pi$-meson exchange current contributions to the $\sigma$-dibaryon coupling constant is discussed. The mean-field solutions of the model are constructed. The effective nucleon mass $m^*_N$ vanishes when the density of dibaryons approaches a critical value $\rho_{D^*_V}^{c,max} \approx 0.15$ fm$^{-3}$. The Green’s functions of the equilibrium binary mixture of nucleons and dibaryons are constructed by solving the Gorkov-Dyson system of equations in the no-loop approximation. We find that when the square of the sound velocity is positive, the dispersion laws for all elementary excitations of the system are real functions. This indicates stability of the ground state of the heterophase nucleon-dibaryon mixture. In the model considered, production of dibaryons becomes energetically favorable at higher densities as compared to estimates based on a model of non-interacting nucleons and dibaryons.
1 INTRODUCTION

In 1977 R.Jaffe predicted [1] the existence of the loosely bound dihyperon, $H$, with a mass just below the $\Lambda\Lambda$ threshold. The $H$ particle, if it exists decays only weakly. The exciting prospect to observe the long-lived $H$ particle stimulated considerable theoretical and experimental activity. Calculations of the $H$-particle mass in the bag models [2, 3], the constituent quark model [4, 5], the Skyrme model [6], on the lattice [7], and in other models [8] showed that the existence of a dihyperon near the $\Lambda\Lambda$-threshold is plausible. The searches were proposed to examine the $H$ particle production in proton-proton [9, 10], proton-nucleus [11, 12], nucleus-nucleus [12] collisions, via a $(K^-, K^+)$ reaction on a nuclear target [13], and through a strong decay of a $\Xi^-$-atom system [14]. Weak decays of the $H$ have also been studied [15]. The experiments [9, 16, 17] did not give a positive sign for existence of the $H$ particle, however, a weak decay of the $H$ produced in the $p - C$ reaction has been reported [18]. The existence of the $H$ particle remains an open question which must eventually be settled by experiment. The candidates for double-lambda hypernucleus whose existence constrain the binding energy of the $H$ have been observed [19].

The non-strange dibaryons with exotic quantum numbers, which have a small width due to zero coupling to the $NN$-channel, are promising candidates for experimental searches [3, 20, 21]. The lowest-lying isospin $T = 0$ dibaryons with quantum numbers $J^P = 1^-(^1P_1), 3^-(^1F_3), 1^+(^3S_1, ^3D_1), 2^+(^3D_2)$, etc. couple to the $NN$-channel. They have large widths and are difficult to observe experimentally. On the other hand, dibaryons made up of exotic quark clusters, for example, a $q^4$ and a $q^2$ cluster with relative orbital angular momentum $L = 1$ may have unusual quantum numbers $T = 0, J^P = 0^-$. The data on pion double charge exchange (DCE) reactions on nuclei [22]-[24] exhibit a peculiar energy dependence at an incident total pion energy of 190 MeV, which can be interpreted [25] as evidence for the existence of a narrow $d'$ dibaryon with quantum numbers $T = 0, J^p = 0^-$ and the total resonance energy of 2063 MeV. Recent experiments at TRIUMPF (Vancouver) and CELSIUS (Uppsala) seem to support the existence of the $d'$ dibaryon [50].

A method for searching narrow, exotic dibaryon resonances in the double proton-proton bremsstrahlung reaction is discussed in Ref. [27] and some indications for a $d_1(1920)$ dibaryon in this reaction have recently been found [28].

Dibaryons can be formed in nuclear matter. The properties of nuclear matter with admixture of multiquark clusters are discussed by Baldin et al. [29] and Chizhov et al. [30]. In these papers, the interaction between nucleons and multiquark clusters is included through a van der Waals volume correction. The occurrence of a heterophase state of
nucleons and 6-quark clusters is found to be energetically favorable in a wide region of temperatures and densities. A model for nuclear matter with an admixture of dibaryons with the short-range nuclear forces approximated by a \( \delta \)-function-like pseudopotential is discussed by Mrowczynski \[31\]. Nuclear matter with a Bose condensate of dibaryons belongs to a class of heterophase substances whose properties are reviewed by Shumovskii and Yukalov \[32\].

The occurrence of a dibaryon Bose condensate in nuclear matter results in a softening of the equation of state (EOS) of nuclear matter. In the ideal gas approximation, the incompressibility of the heterophase nucleon-dibaryon matter vanishes. Occurrence of a dibaryon Bose condensate in interiors of neutron stars decreases the maximum masses of neutron stars \[33\]. The effect of the strongly interacting \( H \)-particle on the structure of massive neutron stars is investigated by Tamagaki \[34\] and Olinto \textit{et al.} \[35\]. The \( H \)-particle interaction with nucleons and the \( HH \)-interaction are studied in the non-relativistic quark cluster model \[5\], \[36\]-\[39\].

In a recent paper \[40\] an exactly solvable model for a one-dimensional Fermi-system of fermions interacting through a potential leading to a resonance in the two-fermion channel is constructed. This model takes the Pauli principle for fermions and the composite nature of the two-fermion resonances into account. The behavior of the system with increasing density can be interpreted in terms of a Bose condensation of two-fermion resonances. A Bose condensate of dibaryons can presumably be formed at higher densities when relativistic effects become important. In order to describe such a system, one should go beyond the non-relativistic many-body framework. The relativistic field-theoretical Walecka model \[41, 43\] is known to be very successful in describing properties of infinite nuclear matter and of ordinary nuclei throughout the periodic table. It constitutes the basis for the quantum hadrodynamics (QHD) approach for studying nuclear phenomena.

In this paper we study the effect of narrow dibaryon resonances on nuclear matter in the framework of the Walecka model. The Lagrangian of the model contains nucleons interacting through \( \omega \)- and \( \sigma \)-meson exchanges. We add to the Lagrangian dibaryons interacting with nucleons and with each other via the exchange of \( \omega \)- and \( \sigma \)-mesons.

The outline of the paper is as follows. In the next Sect., we start with a discussion of the properties of heterophase nucleon-dibaryon matter in the ideal gas approximation. We give simple qualitative estimates at what densities (chemical potentials) and for which dibaryon masses a dibaryon Bose condensate may occur. The softening of the EOS for neutron matter and its consequences for the gravitational stability of neutron stars is discussed. In Sect. 3 an extension of the Walecka model including dibaryon fields in the Lagrangian is presented. We discuss in detail the constraints for the meson-dibaryon...
coupling constants which follow from the requirement of stability of dibaryon matter against compression. We also calculate the numerical values for these coupling constants in the additive model and estimate the size of $\sigma$- and $\pi$-meson exchange current corrections to the $\sigma$-dibaryon coupling constant.

In Sect. 4, the mean-field (MF) solutions to the extended Walecka model are constructed. We show that the self-consistency equation for the effective nucleon mass in presence of the dibaryon component can formally be reduced to an analogous equation of the standard Walecka model. The MF solutions are examined from the point of view of the Hugenholtz-Van Hove theorem [44] which serves as a check for the internal consistency. It is shown that the thermodynamic pressure coincides with the hydrostatic pressure. The numerical results show a softening of the EOS for nuclear matter with a dibaryon component. In Sect. 5, we construct Green’s functions of the heterophase nucleon-dibaryon matter by solving a system of the Gorkov-Dyson equations in the no-loop approximation. We find that when square of the sound velocity is positive, the dispersion laws for elementary excitations are real functions, indicating the stability of the heterophase ground state of nucleon-dibaryon matter.

2 IDEAL GAS APPROXIMATION

In the ideal gas approximation, the physical picture of dibaryon condensation is very simple. Suppose we fill a box with neutrons (see Fig.1). Due to the Pauli principle, the neutrons occupy successively higher energy levels. This process is continued until the chemical potential of two neutrons on top of the Fermi sphere is larger than the dibaryon mass. When the Fermi energy of the neutrons becomes larger, it becomes energetically favorable for the neutrons to form dibaryons. The critical density for dibaryon formation is determined by the mass of the lightest dibaryon. Above the critical density, the chemical potential of the nucleons $\mu_n$ is frozen at the value $\mu_n^{\text{max}} = m_D/2$ where $m_D$ is the dibaryon mass. This equation is a consequence of the chemical equilibrium with respect to transitions $nn \leftrightarrow D$. Dibaryons are Bose particles, so they are accumulated in the ground state and form a Bose condensate. Dibaryons have zero velocities, therefore they do not collide with the boundary and do not contribute to the pressure. Because the Fermi energy of the neutrons is frozen, the pressure does not increase with the density (see Fig. 2). Consequently, nuclear matter loses its elasticity and its incompressibility vanishes. Nuclear matter with such properties cannot protect neutron stars against gravitational compression and subsequent collapse.

Obviously, a dibaryon Bose condensate does not exist in ordinary nuclei. The equality
\( \mu_n = \mu_p \) follows from the charge symmetry of nuclei. Assuming that the shell model potential for dibaryons is twice as deep as the one for nucleons, we conclude that the masses of dibaryons coupled to the NN-channel should be greater than

\[
m_D > 2\mu_N = 2(m_N + \varepsilon_F) = 1.96 \text{ GeV} \tag{2.1}
\]

where \( \varepsilon_F = 40 \text{ MeV} \) is the Fermi energy of nucleons in nuclei. For example, the \( d' \) dibaryon \[22]-[24] is coupled to the \( NN\pi \) channel only. In the medium, the reaction \( pd' \leftrightarrow nnp \) is possible. In nuclei the equilibrium condition for the chemical potential has the form \( \mu_D + \mu_p = 2\mu_n + \mu_p \). Since \( \mu_p = \mu_n = \mu_N \), we arrive at the same constraint (2.1).

In chemical equilibrium with respect to the \( \beta \)-decays \( p + e \leftrightarrow n + \nu_e \), the chemical potentials for neutrons and protons in neutron matter satisfy the equality: \( \mu_n = \mu_p + \mu_e \), so that dibaryons coupled to the \( nn-, np-, \) and \( pp- \) channels actually occur at slightly different densities; and dibaryon condensation starts if dibaryon masses are smaller, respectively, than \( 2\mu_n \), \( 2\mu_n - \mu_e \), and \( 2\mu_n - 2\mu_e \). In case of the \( d' \), the critical value for the neutron chemical potential is determined by the condition \( m_{d'} = 2\mu_n - \mu_e \). The electron chemical potential \( \mu_e \) is positive. The \( H \) particle is coupled through a double weak decay to the \( NN \) channel so that the \( H \) particle occurs at \( m_H = 2\mu_n \).

Nonrelativistically, the density \( \rho \) and pressure \( p \) distributions inside of neutron stars are described by Euler’s equation

\[
\rho \frac{\partial \mathbf{v}}{\partial t} + \rho (\mathbf{v} \cdot \nabla) \mathbf{v} = -\nabla p - \rho \nabla \Phi
\]

where \( \Phi \) is the gravitational potential and \( \mathbf{v} \) is the velocity of the neutrons. If a dibaryon Bose condensate is formed, the pressure in the internal region of the neutron star should be constant (the dotted horizontal line on Fig.2), and therefore \( \nabla p = 0 \). Gauss’s law implies \( \int d\mathbf{S} \cdot \nabla \Phi = 4\pi G M(r) \), where \( M(r) \) is the mass inside of a sphere of radius \( r \). It follows that \( \nabla \Phi \neq 0 \). From Euler’s equation, we get \( \mathbf{v} \neq 0 \), i.e. there is no static solution. We thus conclude that in the ideal gas approximation there are no stable solutions if a dibaryon Bose condensate is formed and neutron stars are gravitationally unstable. These physical arguments are also valid in general relativity where the neutron stars are described by the Oppenheimer-Volkoff equation \[33\] and are qualitatively correct beyond the ideal gas approximation \[35\]. Therefore there is an interesting connection between the masses of the lightest dibaryons and the upper limit for masses of neutron stars.

The masses of several neutron stars are reliably determined to be above \( 1.4M_\odot \). In the tensor interaction model (a stiff model) and in the Reid model (a soft model; for a review of these nuclear matter models see e.g. \[15\]), the neutron chemical potential in the center of a mass \( 1.4M_\odot \) neutron star can be evaluated to be 1090 MeV and 1015 MeV,
respectively. The requirement that there be no Bose condensate of dibaryons coupled to the NN-channel (like the $H$-particle) inside neutron stars gives

$$m_D > 2.18 \text{ GeV} \quad \text{and} \quad m_D > 2.03 \text{ GeV}. \quad (2.2)$$

These numbers are in the range of present experimental searches for dibaryons. They are valid, however, only when the interaction of dibaryons is neglected.

The interaction of dibaryons with neutrons and with each other increases the pressure. The equation of state of neutron matter becomes stiffer, yielding stability of neutron stars in some interval of densities [33]. If dibaryons are formed in a first order phase transition (such a scenario is discussed by Tamagaki [34] for the $H$-particles), neutron stars become unstable at a critical density provided that the jump $\Delta \rho$ of the density in the phase transition point is sufficiently large [37, 48]. In Fig. 2 $\Delta \rho = \infty$. In Newtonian gravity, the criterion is given by $\Delta \rho > (3/2)\rho$.

3 DIBARYON EXTENSION OF THE WALECKA MODEL

Many successful phenomenological applications of QHD demonstrate that the interactions of hadrons at large and intermediate distances can be adequately described in terms of hadronic degrees of freedom. Many observables are not sensitive to the contributions from very short distances. In QHD, the effects of retardation and causality are rigorously taken into account.

Parameters of the Walecka model are fixed by fitting the properties of nuclear matter at the saturation density. Once the model parameters are fixed, other consequences can be extracted without any further assumptions. The inclusion of dibaryons to the model entails several uncertainties due to the lack of reliable information on dibaryon masses and coupling constants. However, many conclusions can be drawn on quite general grounds without knowing precise values for the newly added parameters.

3.1 Lagrangian of the model

We consider an extension of the Walecka model by including dibaryon fields to the Lagrangian density

$$L = \bar{\Psi}(i\partial_\mu\gamma_\mu - m_N - g_\sigma\sigma - g_\omega\omega_\mu\gamma_\mu)\Psi + \frac{1}{2}(\partial_\mu\sigma)^2 - \frac{1}{2}m_\omega^2\sigma^2 - \frac{1}{4}F_{\mu\nu}^2 + \frac{1}{2}m_\omega^2\omega_\mu^2 +$$

$$(\partial_\mu - ih_\omega\omega_\mu)\varphi^*(\partial_\mu + ih_\omega\omega_\mu)\varphi - (m_D + h_\sigma\sigma)^2\varphi^*\varphi + \mathcal{L}_c. \quad (3.1)$$
Here, $\Psi$ is the nucleon field, $\omega_\mu$ and $\sigma$ are the $\omega$- and $\sigma$-meson fields, $F_{\mu\nu} = \partial_\nu \omega_\mu - \partial_\mu \omega_\nu$ is the field strength tensor of the vector field; $\varphi$ is the dibaryon field, for which we assume that it is an isoscalar-scalar (or isoscalar-pseudoscalar) field. This assumption includes the interesting cases of the $H$-particle and the $d'$-dibaryon. The values $m_\omega$ and $m_\sigma$ are the $\omega$- and $\sigma$-meson masses and $g_\omega$, $g_\sigma$, $h_\omega$, $h_\sigma$ are the coupling constants of the $\omega$- and $\sigma$-mesons with nucleons ($g$) and dibaryons ($h$).

The term $\mathcal{L}_c$ describes the conversion of dibaryons into nucleons. The $H$-particle is coupled to the $NN$-channel through a double weak decay, so that $\mathcal{L}_c = O(G_F^2)$. For the non-strange $d_1$ and the $d'$ dibaryon, we neglect possible virtual transitions e.g. to the $NN\sigma$ channel. The on-shell couplings for these dibaryons are small. The exotic $d_1$ dibaryon decays to the $NN\gamma$-channel only, and therefore $\mathcal{L}_c = O(\alpha)$. The $d'$ dibaryon decays to the $NN\pi$ channel. Due to the Adler’s self-consistency condition \[13\] $\mathcal{L}_c \propto \partial_\mu \pi$.

In the MF approximation $\partial_\mu \pi = 0$, and the term $\mathcal{L}_c$ does not modify the MF equations. In what follows we set $\mathcal{L}_c = 0$. The effect of a small term $\mathcal{L}_c$ reduces to providing a chemical equilibrium with respect to transitions between dibaryons and nucleons.

The field equations corresponding to the Lagrangian have the form

\[
(i \partial_\mu \gamma_\mu - m_N - g_\sigma \sigma - g_\omega \omega_\mu \gamma_\mu) \Psi = 0, \tag{3.2}
\]

\[
(-\Box - m_\sigma^2) \sigma = g_\sigma \bar{\Psi} \Psi + 2h_\sigma (m_D + h_\sigma \sigma) \varphi^* \varphi, \tag{3.3}
\]

\[
((-\Box - m_\omega^2)g_{\mu\nu} - \partial_\mu \partial_\nu) \omega_\nu = g_\omega \bar{\Psi} \gamma_\mu \Psi + h_\omega \varphi^* i \leftrightarrow \partial_\mu \varphi - 2h_\omega^2 \omega_\mu \varphi^* \varphi, \tag{3.4}
\]

\[
((\partial_\mu + ih_\omega \omega_\mu)^2 + (m_D + h_\sigma \sigma)^2) \varphi = 0. \tag{3.5}
\]

The field operators can be expanded into $c$-number- and operator parts:

\[
\sigma = \sigma_c + \hat{\sigma},
\]

\[
\omega_\mu = g_\mu \delta \omega_c + \hat{\omega}_\mu,
\]

\[
\varphi = \varphi_c + \hat{\varphi},
\]

\[
\varphi^* = \varphi_c^* + \hat{\varphi}^*.
\]

The $c$-number parts of the fields $A = \sigma, \omega_\mu, \varphi$, and $\varphi^*$ are defined as expectation values $A_c = \langle A \rangle$ over the ground state of the system. The average values of the operator parts are zero by definition: $\langle \hat{A} \rangle = 0$.

The $\sigma$-meson mean field determines the effective nucleon and dibaryon masses in the medium

\[
m_N^* = m_N + g_\sigma \sigma_c, \tag{3.7}
\]

\[
m_D^* = m_D + h_\sigma \sigma_c. \tag{3.8}
\]
The nucleon and dibaryon currents have the form
\[ j^N_\mu = \bar{\Psi} \gamma_\mu \Psi, \quad (3.9) \]
\[ j^D_\mu = \varphi^* i \overset{\leftrightarrow}{\partial_\mu} \varphi - 2 h_\omega \omega_\mu \varphi^* \varphi. \quad (3.10) \]
The baryon number current is defined by
\[ j^B_\mu = j^N_\mu + 2 j^D_\mu. \quad (3.11) \]
The \( \omega \)-field is coupled to the current
\[ j^\omega_\mu = g_\omega j^N_\mu + h_\omega j^D_\mu. \quad (3.12) \]
The nucleon vector and scalar densities are defined by expectation values
\[ \rho_{NV} = \langle \bar{\Psi} \gamma_0 \Psi \rangle, \quad (3.13) \]
\[ \rho_{NS} = \langle \bar{\Psi} \Psi \rangle. \quad (3.14) \]
The scalar density of the dibaryon condensate is defined by
\[ \rho^c_{DS} = |\langle \varphi(0) \rangle|^2. \quad (3.15) \]
The time evolution of the condensate \( \varphi \)-field is determined by the chemical potential \( \mu_D \) of dibaryons
\[ \varphi_c(t) = e^{-i \mu_D t} \sqrt{\rho^c_{DS}}. \quad (3.16) \]
It is useful to separate the contribution of the \( \omega \)-meson mean field to the chemical potential energy of dibaryons
\[ \mu_D = \mu^*_D + h_\omega \omega_c. \quad (3.17) \]
The dibaryon number density is according to Eq.(3.10) given by
\[ \rho^c_{DV} = 2 \mu^*_D \rho^c_{DS}. \quad (3.18) \]
The possibility of existence of a Bose condensate of dibaryons depends on values of the coupling constants of dibaryons with the \( \omega \)- and \( \sigma \)-mesons.

### 3.2 Stability of dibaryon matter against compression

The \( \omega \)- and \( \sigma \)- meson coupling constants \( h_\omega \) and \( h_\sigma \) enter the Yukawa potential for two dibaryons
\[ V(r) = \frac{h^2_\omega}{4\pi} \frac{e^{-m_\omega r}}{r} - \frac{h^2_\sigma}{4\pi} \frac{e^{-m_\sigma r}}{r}. \quad (3.19) \]
The interaction energy for dibaryons in the condensate for a constant density distribution \( \rho_D(x) = \rho_D \) is equal to

\[
W = \frac{1}{2} \int dx_1 dx_2 \rho_D(x_1) \rho_D(x_2) V(|x_1 - x_2|) = 2\pi N_D \rho_D \left( \frac{h_\omega^2}{4\pi m_\omega^2} - \frac{h_\sigma^2}{4\pi m_\sigma^2} \right) \tag{3.20}
\]

where \( N_D \) is the total number of dibaryons. The integral (3.20) is linear in density. A negative \( W \) would imply instability of the system against compression. The value \( W \) is positive and the system is stable for

\[
\frac{h_\omega^2}{4\pi m_\omega^2} > \frac{h_\sigma^2}{4\pi m_\sigma^2}. \tag{3.21}
\]

In a nonrelativistic theory for interacting bosons and in the model considered (see Sect.4), the requirement of stability is equivalent to the requirement of a negative value for the boson forward scattering amplitude and/or a positive value of square of the sound velocity \( (a_s^2 > 0) \). In the Born approximation, the forward scattering amplitude is, as in Eq.(3.20), proportional to the volume integral of the potential.

The \( H \)-particle interactions were studied in the non-relativistic quark cluster model which is successful in describing the \( NN \)-phase shifts. The calculation of the interaction integral (3.20) with the adiabatic \( HH \)-potential gives a negative number, so that the \( H \)-dibaryon matter is probably unstable against compression. The coupling constants of the mesons with the \( H \)-particle can be fixed by fitting the depth and the position of the minimum of the \( HH \)-potential to give \( h_\omega^2 = 603.7 \) and \( h_\sigma^2 = 279.2 \). These values, while yielding a negative \( W \), probably overestimate the repulsion between \( H \)-particles at small distances, and therefore underestimate \(|W|\). The meson-dibaryon coupling constants for the case of the \( d_1 \) and \( d' \) dibaryons are presently unknown.

### 3.3 Coupling constants in the additive model

In the additive picture, mesons interact with the constituents of the dibaryon (Fig.3). For non-strange dibaryons coupled to the \( NN \)-channel, the \( \sigma \) - and \( \omega \)-meson couplings are in the nonrelativistic approximation simply twice the corresponding meson-nucleon coupling constants: \( h_\omega = 2g_\omega \) and \( h_\sigma = 2g_\sigma \). The scalar charge is, however, suppressed by the Lorentz factor. This effect decreases the value of \( h_\sigma \). Note that the magnitudes of the meson coupling constants for the \( H \)-particle, extracted from the adiabatic \( HH \)-potential are consistent with the additive estimates: \( h_\omega/(2g_\omega) = 0.89 \) and \( h_\omega/(2g_\sigma) = 0.80 \).

For the standard set of parameters of the Walecka model, \( m_\sigma = 520 \text{ MeV}, g_\omega^2 = 190.4, \) and \( g_\sigma^2 = 109.6 \), the inequality (3.21)

\[
98.85\left(\frac{h_\omega}{2g_\omega}\right)^2\text{GeV}^{-2} > 129.0\left(\frac{h_\sigma}{2g_\sigma}\right)^2\text{GeV}^{-2} \tag{3.22}
\]
is not satisfied. The precision of the additive estimates is, however, not better than 30% of the central values. Exchange current contributions which violate the additivity are discussed below.

The violation of the inequality (3.21) for the additive estimates of the meson-dibaryon couplings is not accidental. For homophase nuclear matter, \( g_\omega \) should be greater than \( g_\sigma \) in order to get sufficient repulsion between nucleons at small distances. To reproduce the properties of nuclear matter at the saturation density, the following inequality should hold

\[
\frac{g_\omega^2}{4\pi m_\omega^2} < \frac{g_\sigma^2}{4\pi m_\sigma^2}. \tag{3.23}
\]

At small densities, the inequality (3.23) provides a negative value for the interaction integral (3.20) between the nucleons, resulting to the local instability of nuclear matter against compression. When the density increases, the relativistic effects become important. The scalar density of nucleons increases slower than the vector density, since it is suppressed by the Lorentz factor \( <1/\gamma > \). This finally leads to an equilibrium at the saturation density of nuclear matter.

For exotic dibaryons like the \( d' \), one should take into account the presence of a pion in the resonance wave function. The \( \omega \)-meson decays into three pions. It is not coupled to the pion in the \( d' \) wave function (see Fig.3), so \( h_\omega = 2g_\omega \).

The \( \sigma - \pi \) cubic couplings are described by the effective Lagrangian density

\[
\Delta \mathcal{L} = -\frac{\kappa}{6} \sigma^3 - \frac{\kappa'}{6} \sigma \pi^2 \tag{3.24}
\]

where \( \pi \) is the pion field. The cubic terms generate three-body forces between nucleons. Phenomenological fits to the bulk nuclear properties give \[42\]

\[
\frac{\kappa}{m_N} = 0.9 \div 5.3. \tag{3.25}
\]

The lower and upper values correspond, respectively, to a small negative and a large positive term \( \lambda \sigma^4 \) in the effective Lagrangian.

The non-linearities of the linear sigma-model are qualitatively different \[25\]

\[
\frac{\kappa}{m_N} = \frac{\kappa'}{m_N} = -3g_\sigma \frac{m_\sigma^2 - m_\pi^2}{m_N^2} = -8.9. \tag{3.26}
\]

In the additive model for \( d' \), we get

\[
\frac{h_\sigma}{2g_\sigma} = 1 + \frac{\kappa'}{24g_\sigma m_\pi} = 1 + 0.027 \frac{\kappa'}{m_N}. \tag{3.27}
\]

The last term describes the contribution from the first of the diagrams in Fig.3(b). For \(|\kappa'/m_N| < 10\) the correction to the additive value is smaller than 30%. The sign, however, is not defined.
The Brown-Rho scaling \[50\] for non-strange hadron masses is reproduced at the tree level for \( h_\sigma = (m_D/m_N)g_\sigma \). In such a case,
\[
\frac{m_N^*}{m_N} = \frac{m_D^*}{m_D}.
\] (3.28)
Since \( m_D > 2m_N \), Brown-Rho scaling gives \( h_\sigma = (m_D/m_N)g_\sigma > 2g_\sigma \).

### 3.4 Exchange current contributions to the coupling constants

The exchange current contributions to the \( \sigma \)-dibaryon coupling constants shown in Fig.4 can be extracted from the Lagrangian (3.24). The \( \sigma \)-meson field generated by a pointlike source of charge \( h_\sigma \) is determined from the equation
\[
(\Delta - m_\sigma^2)\sigma(x) = h_\sigma \delta(x).
\] (3.29)

The sign of the right hand side of the equation is chosen such as to yield a negative (attractive) \( \sigma \)-meson field for a positive \( h_\sigma \). It is useful to compare Eq.(3.29) with the static limit of the equation of motion for the \( \sigma \)-meson field determined by the Lagrangian \( L + \Delta L \) with no dibaryon component:
\[
(\Delta - m_\sigma^2)\sigma(x) = g_\sigma \sum_{k=1}^{2} \bar{\Psi}_k(x)\Psi_k(x) + \frac{\kappa}{2} \sigma^2(x) + \frac{\kappa'}{6}\pi^2(x).
\] (3.30)

The constant \( h_\sigma \) measures the scalar charge generating the \( \sigma \)-meson field around dibaryons, so that we can write
\[
h_\sigma = \int dx [g_\sigma \sum_{k=1}^{2} \bar{\Psi}_k(x)\Psi_k(x) + \frac{\kappa}{2} \sigma^2(x) + \frac{\kappa'}{6}\pi^2(x)].
\] (3.31)

Nonrelativistically, the \( \sigma \)-meson field created by nucleons located at points \( x_k \) is
\[
\sigma(x) = -\sum_{k=1}^{2} g_\sigma \frac{e^{-m_\sigma |x-x_k|}}{4\pi |x-x_k|}.
\] (3.32)

Substituting this expression into Eq.(3.31) and omitting diagonal terms, we get for the exchange-current contribution an expression
\[
\Delta h_\sigma (D)_{\sigma-MEC} = \frac{\kappa}{16\pi m_\sigma} g_\sigma^2 < e^{-m_\sigma |x_1-x_2|}>.
\] (3.33)

where \( x_1 \) and \( x_2 \) are coordinates of the two nucleons. The exponential term should be averaged over the dibaryon wave function.

To give an order-of-magnitude estimate, we use for the relative wave function of two nucleons Hulthen wave functions
\[
\psi(r) = \sqrt{\frac{\mu\nu(\mu+\nu)}{2\pi(\nu-\mu)^2 r}} (e^{-\mu r} - e^{-\nu r}).
\] (3.34)
with \( \mu = 1 \text{ fm}^{-1} \) and \( \nu = 4\mu \). The correction (3.33) can be estimated to be

\[
\frac{\Delta h_{\sigma}(D)^{\sigma-MEC}}{2g_{\sigma}} = 0.040 \frac{\kappa}{m_N}.
\]

The \( d' \)-dibaryon contains a constituent \( \pi \)-meson. The \( \sigma \)-meson field created by the pion is given by

\[
\sigma(x) = -\frac{\kappa'}{12m_{\pi}} \frac{e^{-m_\sigma|x-x_3|}}{4\pi|x-x_3|} \tag{3.35}
\]

where \( x_3 \) is the pion coordinate. The total change in the \( \sigma \)-meson coupling with the \( d' \) equals

\[
\Delta h_{\sigma}(d')^{\sigma-MEC} = \frac{\kappa}{16\pi m_{\sigma}} g_{\sigma} \left[ g_{\sigma} < e^{-m_\sigma|x_1-x_2|} > + \frac{\kappa'}{6m_{\pi}} < e^{-m_\sigma|x_1-x_3|} > \right]. \tag{3.36}
\]

Assuming that the two-body probability densities are given by the square of the wave function (3.34), we get

\[
\frac{\Delta h_{\sigma}(d')^{\sigma-MEC}}{2g_{\sigma}} = 0.040 \frac{\kappa}{m_N} \left( 1 + 0.108 \frac{\kappa'}{m_N} \right).
\]

The contribution of the pion exchange currents can be evaluated in a similar way. The pion field created by nucleons has the form

\[
\pi(x) = \sum_{k=1}^{2} i \frac{g_{\sigma}}{2m_N} \tau_k (\sigma_k \cdot \nabla) \frac{e^{-m_\sigma|x-x_k|}}{4\pi|x-x_k|} \tag{3.37}
\]

For dibaryons consisting of two nucleons and also for the \( d' \), the exchange pion current contributions are given by

\[
\Delta h_{\sigma}(D,d')^{\pi-MEC} = \frac{\kappa'}{144\pi} \left( \frac{g_{\sigma}}{2m_N} \right)^2 (\tau_1 \cdot \tau_2)(\sigma_1 \cdot \sigma_2) \frac{\langle \left( \frac{2}{|x_1-x_2|} - m_{\pi} \right) e^{-m_\sigma|x_1-x_2|} \rangle}{(\tau_1 \cdot \tau_2)(\sigma_1 \cdot \sigma_2)} \tag{3.38}
\]

The evaluation of this expression using the wave function (3.34) gives

\[
\frac{\Delta h_{\sigma}(d')^{\pi-MEC}}{2g_{\sigma}} = 0.0015 \frac{\kappa'}{m_N} (\tau_1 \cdot \tau_2)(\sigma_1 \cdot \sigma_2).
\]

The pion exchange current correction to the \( \sigma \)-dibaryon coupling is therefore significantly smaller than the corresponding sigma exchange current corrections.

In view of the estimates (3.25) and (3.26) the exchange current corrections cannot exceed 30\%-40\% of the additive values. At present, the coupling constants of the mesons with dibaryons are not known with sufficient precision to draw any definite conclusion concerning the stability of dibaryon matter.

The \( H \)-particle, however, is studied better than other dibaryons. In what follows, we use a realistic \( HH \)-interaction, based on the quark cluster model \[3], [36]-[39].
4 THE MEAN-FIELD SOLUTIONS

The MF solutions of the Walecka model are asymptotically exact in the high density limit\textsuperscript{[42]}. They serve as a starting point for the calculation of corrections for finite densities within the QHD approach.

Neglecting the operator parts of the meson fields in Eqs.(3.2)-(3.5), we get the following expressions for the meson mean fields

$$\omega_c = \frac{g_\omega \rho_{NV} + h_\omega 2h_\omega^2 \mu_D^*}{m_\omega^2},$$

$$\sigma_c = \frac{-g_\sigma \rho_{NS} + h_\sigma 2h_\omega^2 \mu_D^*}{m_\sigma^2},$$

(4.1)

(4.2)

The effective masses of mesons in a heterophase nucleon-dibaryon matter are given by

$$\tilde{m}_\sigma^2 = m_\sigma^2 + 2h_\sigma^2 \rho_D^c,$$

$$\tilde{m}_\omega^2 = m_\omega^2 + 2h_\omega^2 \rho_D^c.$$

(4.3)

(4.4)

The mechanism responsible for the change of meson masses is essentially the Higgs mechanism.

Substituting expression (3.16) into Eq.(3.5), we get

$$\mu_D^* = m_D^*.$$}

(4.5)

The nucleon and dibaryon chemical potentials have the form

$$\mu_N = E_F^* + g_\omega \omega_c,$$

$$\mu_D = m_D^* + h_\omega \omega_c,$$

(4.6)

(4.7)

where $E_F^* = \sqrt{m_N^2 + k_F^2}$ is the Fermi energy of nucleons with the effective mass $m_N^*$.

4.1 The self-consistency equation

The nucleon vector and scalar densities are given by

$$\rho_{NV} = \gamma \int \frac{dk}{(2\pi)^3} \theta(k_F - |k|),$$

$$\rho_{NS} = \gamma \int \frac{dk}{(2\pi)^3} \frac{m_N^*}{E_F^*(k)} \theta(k_F - |k|).$$

(4.8)

(4.9)

The statistical factor $\gamma = 4$ (2) for nuclear (neutron) matter.

The self-consistency equation has the form

$$m_N^* = m_N - \frac{g_\sigma}{m_\sigma^2} (g_\sigma \rho_{NS} + h_\sigma \rho_D^c).$$

(4.10)
It follows from Eqs. (3.7), (3.18), (4.2), and (4.5). The total baryon number density equals $\rho_{TV} = \rho_{NV} + 2\rho_{DV}$. Eq.(4.10) can be transformed to a form equivalent to the self-consistency equation of the standard Walecka model without dibaryons:

$$m_N^* = \tilde{m}_N - \frac{g_\sigma^2}{m_\sigma^2} \rho_{NS}, \quad \text{(4.11)}$$

where

$$\tilde{m}_N = m_N \frac{\rho_{c,\text{max}}^c - \rho_{c,\text{max}}^d}{\rho_{DV}^c} \quad \text{(4.12)}$$

$$\rho_{c,\text{max}}^c = \frac{m_N m_\sigma^2}{g_\sigma h_\sigma} = 0.1507 \left(\frac{2g_\sigma}{h_\sigma}\right) \text{fm}^{-3}. \quad \text{(4.13)}$$

If the densities $\rho_{TV}$ and $\rho_{DV}^c$ are fixed, equation (4.11) allows to find the effective nucleon mass $m_N^*$. Solutions to Eq.(4.11) exist for an arbitrary total density $\rho_{TV}$ when the value $\tilde{m}_N$ is positive. This is the case for $\rho_{DV}^c < \rho_{c,\text{max}}^c$. Note that the dibaryon mass does not enter Eq.(4.11) directly.

In Figs.5 (a,b) we show for $h_\omega = 2g_\omega$ the critical density for occurrence of a Bose condensate of dibaryons in (a) symmetric nuclear matter and (b) neutron matter as a function of the $\sigma$-dibaryon coupling constant. The critical density is determined from equation $2\mu_N - \mu_D = 0$. In the region $a_\sigma^2 > 0$ the requirement (3.21) is fulfilled. The dibaryon components of the heterophase nuclear- and neutron-dibaryon matter are stable against compression.

For $2m_N \leq m_D \leq 1.89$ GeV, we start at zero density from heterophase nuclear-dibaryon matter (a). With increasing density, the matter transforms into homophase nuclear matter and then again back to heterophase nuclear-dibaryon matter. For $m_D > 1.89$ GeV, we start at zero density from homophase nuclear matter which converts with increasing density (at $\rho_{TV} > \rho_0$ for $h_\sigma/(2g_\sigma) < 0.8754$) into heterophase nuclear-dibaryon matter.

When the $\sigma$-meson coupling with dibaryons decreases, the effective dibaryon mass $m_D^*$ increases (see Eq.(3.8)) and dibaryon formation is therefore suppressed. We thus conclude that the curves in Figs.5 (a,b) should have a negative slope for a transition from homophase to heterophase matter and a positive slope for a transition from heterophase to homophase matter. The region of instability for neutron matter is greater because at the same density we have higher values of the neutron chemical potentials and therefore more favorable conditions for the production of dibaryons.

We show the results for $d_1$-dibaryon with possible quantum numbers $J^p = 1^+$. For such dibaryons, the MF equations derived above for the scalar (pseudoscalar) dibaryons $J^p = 0^\pm$ remain the same. In case of the $J^p = 1^\pm$ dibaryons, the dibaryon part of the
Lagrangian (3.1) can be written in the form

\[ \mathcal{L}_D = -D^*_\mu \varphi^*_\nu D\mu \varphi_\nu + (m_D + h_\sigma \sigma)^2 \varphi^*_\mu \varphi_\mu + \lambda D^*_\mu \varphi^*_\nu + \lambda^* D\mu \varphi_\nu \]

where \( D\mu = \partial\mu + ih_\omega \omega_\mu \) and \( \lambda \) and \( \lambda^* \) are Lagrange multipliers. The field equations are

\[
\begin{align*}
(D^2 + (m_D + h_\sigma \sigma)^2) \varphi_\mu - D\mu \lambda &= 0, \\
D\mu \varphi_\mu &= 0.
\end{align*}
\]

The last equation removes the unphysical timelike component of the dibaryon vector field in the co-moving frame of the particle. It is equivalent to the requirement \( u_\mu \varphi_\mu = 0 \) where \( u_\mu \) is a four-vector velocity of the particle.

The condensate field evolves like

\[ < \varphi_\mu(t) > = (0, \mathbf{e}) \sqrt{\rho_{DS}} e^{-i\mu_D t} \]

where \( \mathbf{e} \) is a unit vector. The field equations give \( \lambda = \lambda^* = 0 \) and result in Eqs.(4.1) and (4.2). The energy and pressure have the form (4.22) and (4.23) of the \( J^p = 0^\pi \) dibaryons.

In Fig.6 we classify possible behaviors of the difference \( 2\mu_N - \mu_D \) between the chemical potentials of nucleons and dibaryons with growth of the dibaryon fraction.

It is clear that when the difference is positive and the dibaryon density is zero, \( \rho^c_{DV} = 0 \), production of dibaryons is energetically favorable. In such a case, the fraction of dibaryons increases. The state \( \rho^c_{DV} = 0 \) is therefore unstable. If the difference \( 2\mu_N - \mu_D \) is negative and the substance consists of dibaryons only, production of nucleons is energetically favorable. This state is unstable. If the difference \( 2\mu_N - \mu_D \) is zero, but increases with the dibaryon fraction, small fluctuations bring the substance away from the equilibrium. Such a state is unstable also.

The system is stable in the following three cases.

(i) Homophase nuclear and neutron matter:

\[
2\mu_N - \mu_D < 0, \\
\rho^c_{DV} = 0.
\]

(ii) Homophase dibaryon matter:

\[
2\mu_N - \mu_D > 0, \\
2\rho^c_{DV} = \rho_{TV}.
\]

(iii) Heterophase nucleon-dibaryon matter:

\[
2\mu_N - \mu_D = 0, \\
\frac{d(2\mu_N - \mu_D)}{d\rho^c_{DV}} |_{\rho_{TV}} < 0.
\]
In the first case there are no dibaryons, in the second case there are no nucleons, and in the third case we have a heterophase mixture of nucleons and dibaryons. Small fluctuations around the state $2 \mu_N - \mu_D = 0$ bring the substance back to the equilibrium point. Eqs. (4.16) and (4.17) therefore describe a stable equilibrium.

In Fig. 7 (a) we show the nucleon effective mass $m^*_N$ versus the dibaryon fraction $2 \rho_{DV}/\rho_{TV}$ in heterophase nuclear matter and in heterophase neutron-dibaryon matter for the coupling constants $h_\omega = 2g_\omega$ and $h_\sigma/(2g_\sigma) = 0.8$. At the same total baryon number density, neutron matter is more relativistic. The scalar charge density of neutron matter is thus lower, the $\sigma$-meson mean field is smaller, and therefore the effective nucleon mass is greater. On the plot, the dotted lines corresponding to neutron matter lie above the solid lines corresponding to symmetric nuclear matter.

When the dibaryon vector density $\rho_{DV}$ approaches its maximum value $\rho_{DV}^{c,\text{max}}$, the effective nucleon mass vanishes. This effect can be interpreted as follows. Two nucleons on the top of the Fermi sphere have energy $2E_F^*$. In chemical equilibrium with respect to transitions $NN \leftrightarrow D$, the relation $2E_F^* = m_D^*$ holds. In transitions $NN \leftrightarrow D$ the baryon vector charge does not change. However, the scalar charge changes. For two nucleons the scalar charge equals $2g_\sigma \frac{m_N^*}{E_F^*}$, whereas a dibaryon in the condensate has scalar charge $h_\sigma$. When the system is nonrelativistic, formation of new dibaryons is accompanied by a decrease of the scalar charge density, since $2g_\sigma \frac{m_N^*}{E_F^*} \approx 2g_\sigma > 1.6g_\sigma = h_\sigma$. This phenomenon is reflected in the slight increase of the effective nucleon mass with the dibaryon fraction at low total baryon number densities. When the density is high, the system becomes relativistic, and so the Lorentz factor $\frac{m_N^*}{E_F^*}$ comes into play. The scalar charge of the two nucleons is small, whereas the dibaryon scalar charge is large. As a result, the scalar charge density increases, the scalar mean field increases, and the effective nucleon mass decreases with the dibaryon fraction. In the standard Walecka model, the effective nucleon mass vanishes at infinite density. In the model considered, the effective nucleon mass vanishes when the density of dibaryons approaches the value $\rho_{DV}^{c,\text{max}}$. Note that the behavior of the effective nucleon mass $m_N^*$ with growing dibaryon fraction, shown in Fig. 7 (a), does not depend on the vacuum value of the dibaryon mass, because the dibaryon mass does not enter the self-consistency equation (4.11).

In Fig. 7 (b) we show the difference for the chemical potentials versus the dibaryon fraction for $m_D = 1.96$ GeV. Since the vacuum dibaryon mass does not enter the self-consistency equation (4.11) and enters linearly in the difference $2\mu_N - \mu_D$ (see Eqs. (3.8) and (4.6), (4.7)), the results for other dibaryon masses can be obtained simply by vertical parallel displacements of the curves. The results for the $m_D = 2.08$ GeV ($d'$ dibaryon) can be obtained e.g. by a 100 MeV negative shift, etc.
The MF solutions exist at all densities \( \rho_{TV} \) for sufficiently small densities of dibaryons, \( \rho_{DV}^c < \rho_{DV}^{c,\text{max}} \). It means that we can always investigate the stability of homophase nuclear and neutron matter with respect to formation of a dibaryon Bose condensate. When the total density \( \rho_{TV} \) is very high, the dibaryon production is energetically favorable. The MF solutions disappear, however, before the system reaches equilibrium.

4.2 Thermodynamic consistency checks for the mean-field solutions

The canonical energy-momentum tensor corresponding to the Lagrangian density (2.1) can be written in the form

\[
T_{\mu\nu} = T_{\mu\nu}^N + T_{\mu\nu}^\sigma + T_{\mu\nu}^\omega + T_{\mu\nu}^D
\]

where

\[
T_{\mu\nu}^N = \bar{\Psi} i\gamma_{\mu} \partial_{\nu} \Psi,
\]

\[
T_{\mu\nu}^\sigma = -\frac{1}{2} g_{\mu\nu} (\partial_\tau \sigma \partial_\tau \sigma - m_\sigma^2 \sigma^2) + \partial_{\mu} \sigma \partial_{\nu} \sigma,
\]

\[
T_{\mu\nu}^\omega = \frac{1}{2} g_{\mu\nu} (\partial_\tau \omega_\lambda \partial_\tau \omega_\lambda - m_\omega^2 \omega_\tau \omega_\tau) - \partial_{\mu} \omega_\tau \partial_{\nu} \omega_\tau
\]

\[
T_{\mu\nu}^D = 2 \partial_{\mu} \phi^* \partial_{\nu} \phi - h_\omega \omega_\mu \phi^* i \leftrightarrow \partial_{\nu} \phi.
\]

The energy density \( \varepsilon = \langle T_{00} \rangle \) given by average value of the \( T_{00} \) component has the form

\[
\varepsilon = \gamma \int_0^{k_F} \frac{dk}{(2\pi)^3} (E^*(k) + g_\omega \omega_c) + (m_D^* + h_\omega \omega_c) \rho_{DV}^c + \frac{1}{2} m_\sigma^2 \sigma^2 - \frac{1}{2} m_\omega^2 \omega^2.
\]

Here, \( E^*(k) + g_\omega \omega_c \) is the total energy of a nucleon with momentum \( k \), and \( m_D^* + h_\omega \omega_c \) is the total energy of a dibaryon in the ground state (with zero total momentum) in the external \( \omega \)-meson mean field. The last two terms are the contributions of the classical \( \omega \)- and \( \sigma \)-meson fields to the energy density.

The hydrostatic pressure \( p = -\frac{1}{3} \langle T_{ii} \rangle \) has the form

\[
p = \frac{\gamma}{3} \int_0^{k_F} \frac{dk}{(2\pi)^3} \frac{k^2}{E^*(k)} - \frac{1}{2} m_\sigma^2 \sigma^2 + \frac{1}{2} m_\omega^2 \omega^2.
\]

Because dibaryons in the condensate are at rest they do not contribute to the pressure.

In agreement with the general requirements

\[
\mu_N = \frac{\partial \varepsilon}{\partial \rho_{NV}},
\]

\[
\mu_D = \frac{\partial \varepsilon}{\partial \rho_{DV}^c}.
\]
The pressure can be calculated in two different ways: from Eq.(4.23) and from the thermodynamic relation

\[ p = -\varepsilon + \mu_N \rho_N + \mu_D \rho_D^c. \]  

(4.26)

It is not difficult to check that the thermodynamic pressure (4.26) coincides with the hydrostatic pressure (4.23).

The Hugenholtz-Van Hove theorem [44] (HV) requires that the energy of fermions at the Fermi surface and the average energy of a physical system at zero pressure (at the saturation density) are equal. This theorem is useful for checking the internal consistency of approximations. The mean-field theory and the relativistic Hartree approximation of the standard Walecka model obey this theorem.

The vacuum dibaryon mass enters our model as a free parameter. The formal assumption that the dibaryon condensate occurs at the saturation density does not lead to any contradictions. In chemical equilibrium with respect to the reaction \( NN \leftrightarrow D \), the relations (4.16) are fulfilled. In agreement with the HV theorem, for \( p = 0 \) Eqs.(4.6) and (4.26) give

\[ \mu_N = E_F^* + g \omega \omega_c = \frac{\varepsilon}{\rho_{TV}}. \]  

(4.27)

In Figs. 8 and 9 we show the energy per nucleon and the pressure versus the total baryon number density for some possible dibaryons. It is useful to compare Figs.2 and 9. In Fig.2, we schematically show the behavior of the pressure as a function of the density in the ideal gas approximation. In Fig.9, the pressure is calculated for the dibaryon extension of the Walecka model. The effect of vanishing incompressibility in the ideal gas model (the horizontal dotted line in Fig.2) is displayed in Fig.9 as a softening of the EOS of the heterophase nuclear (dashed line in Fig.9) and neutron-dibaryon matter (dotted line in Fig.9). The occurrence of \( H \)-dibaryons provides a possible mechanism for the formation of strange matter.

Note that the pressure of the heterophase system of nucleons and dibaryons obeys the basic inequality of statistical mechanics [52]

\[ \frac{\partial p}{\partial \rho_{TV}} \geq 0. \]  

(4.28)

4.3 The concept of equilibrium of heterophase substances for \( \mathcal{L}_c = 0 \) and \( \mathcal{L}_c \neq 0 \).

We discuss here the effect of a small term \( \mathcal{L}_c \) describing transitions between dibaryons and nucleons.

In homophase substances, the energy density is a function of the total baryon number density \( \rho_{TV} \) only,
\[ \varepsilon = \varepsilon(\rho_{TV}). \]

In a heterophase substance, the energy is a function of additional parameters \( \xi_i \)

\[ \varepsilon = \varepsilon(\rho_{TV}, \xi_i). \]

The parameters \( \xi_i \) characterize components of the heterophase substance. The equilibrium state is determined by the conditions

\[ \frac{\partial \varepsilon(\rho_{TV}, \xi_i)}{\partial \xi_k} = 0 \quad (k = 1, 2, \ldots). \quad (4.29) \]

Eigenvalues of the matrix

\[ \frac{\partial^2 \varepsilon(\rho_{TV}, \xi_i)}{\partial \xi_k \partial \xi_l} \quad (4.30) \]

should be positive to guarantee a minimum of the energy.

In our case, the state is specified by the nucleon chemical potential \( \mu_N \), the dibaryon chemical potential \( \mu_D \), and by the expectation value of the dibaryon field \( \rho_{DS} \). The nucleon chemical potential \( \mu_N \) is a free parameter. The dibaryon chemical potential \( \mu_D \) determines the evolution of the condensate part of the dibaryon field (Eq.(3.16)).

The value \( \mu_D \) is fixed by the Hugenholtz-Pines condition [51]. In the MF approximation, \( \mu_D^* = m_D^* \). There remain two free parameters: \( \mu_N \) and \( \rho_{DS}^* \). The total density is a sum of the two terms \( \rho_{TV} = \rho_{NV} + 2\rho_{DV}^* \). This expression is also valid for \( \mathcal{L}_c \neq 0 \) (if there are no derivatives in \( \mathcal{L}_c \)).

We can choose as independent parameters the values \( \rho_{NV} \) and \( \rho_{DV}^* \). It is not necessary to require that \( \rho_{NV} \) and \( \rho_{DV}^* \) be timelike components of two conserved currents. It is sufficient that they characterize uniquely the phases of the binary mixture. Equations (4.29) give then

\[ 2 \frac{\partial \varepsilon(\rho_{NV}, \rho_{DV}^*)}{\partial \rho_{NV}} = \frac{\partial \varepsilon(\rho_{NV}, \rho_{DV}^*)}{\partial \rho_{DV}^*}. \quad (4.31) \]

One can use expression (4.22) for the energy density to verify that Eq.(4.16) follows from Eq.(4.31). This is a consequence of Eqs.(4.24) and (4.25). The matrix (4.30) is positively definite when Eq.(4.17) is fulfilled.

For every conserved current one can introduce an independent chemical potential. For \( \mathcal{L}_c = 0 \), the parameters \( \mu_N \) and \( \mu_D \) have the meaning of the chemical potentials corresponding to two conserved nucleon and dibaryon currents. For \( \mathcal{L}_c \neq 0 \), there is only one conserved baryon current and only one baryon chemical potential.

In the both cases (\( \mathcal{L}_c = 0 \) and \( \mathcal{L}_c \neq 0 \)), the equilibrium is determined by Eq.(4.31).

For \( \mathcal{L}_c = 0 \), this equation reduces to Eq.(4.16). It can be naturally interpreted in terms of the chemical equilibrium between nucleon and dibaryon phases.
For $L_c \neq 0$, the values $\rho_{NV}$ and $\rho_{DV}$ are no longer timelike components of conserved currents. They play the role of formal parameters $\xi_i$ characterizing the nucleon and dibaryon phases. In such a case, the condition (4.31) cannot be interpreted in terms of a chemical equilibrium, since we cannot determine individual contributions of nucleons and dibaryons to the total baryon number of the system.

For $L_c \neq 0$, one should write additional terms $O(L_c)$ on the right hand side of Eqs.(4.16) and (4.17). For narrow dibaryons which we discuss here, the corrections to these equations are small. For example, in case of the $H$-particle the corrections are of order $10^{-10}$.

5 GREEN’S FUNCTIONS AND ELEMENTARY EXCITATIONS

In the MF approximation, all dibaryons are in the Bose condensate. It is known that interaction between Bose particles brings some fraction of bosons out of the condensate. In the nonrelativistic approximation, the density of bosons which are not in the condensate increases with the total density faster than the density of bosons in the condensate \[46\].

The bosons that are not in the condensate have finite momenta and they contribute to the pressure. One expects that the dibaryons which are not in the condensate shift the critical density for disappearance of the effective nucleon mass to higher values, because the dibaryon scalar charge is suppressed by the Lorentz factor. The contribution of dibaryons out of the condensate to the energy density and pressure can be calculated using the diagram technique developed for Bose systems by Belyaev \[53\]. To go beyond the MF approximation, it is necessary to determine the Green’s functions of the system.

5.1 Solutions of the Gorkov-Dyson equations

To eliminate the time dependence from the condensate parts of the $\varphi$-fields, we pass to the $\mu$-representation for dibaryons. This can be done by the substitution $\varphi \rightarrow \varphi e^{i\mu D t}$ and $\varphi^* \rightarrow \varphi^* e^{-i\mu D t}$.

The Green’s functions are defined by

$$iG_{\alpha\beta}(x' - x) = \langle T\bar{\Psi}_\alpha(x')\Psi_\beta(x) \rangle,$$  \hspace{1cm} (5.1)

$$iD^{AB}(x' - x) = D^{AB}(x-x') = \langle T\hat{A}(x')\hat{B}(x) \rangle,$$  \hspace{1cm} (5.2)

with $A, B = \sigma, \omega, \varphi, \varphi^*$. In momentum space

$$D^{AB}(k) = D^{BA}(-k).$$  \hspace{1cm} (5.3)
The $\sigma$- and $\omega$-vertices with dibaryons are depicted in Fig.10. The crosses on the
dibaryon double lines denote the appearance or disappearance of dibaryons from the Bose
condensate. In Fig.11 we graphically show a representation of Eqs.(4.1) and (4.2). The
set of these diagrams can be summed up either to modify the meson propagators (the
bold dashed lines) or the meson vertices with dibaryons. The dressed meson-dibaryon
vertices are determined by the diagrams shown in Fig.12.

It is useful to distinguish three kinds of propagators. The bare ones denoted by thin
lines, the mean field ones denoted by thick lines, and the complete ones denoted by blobs
with outgoing thick lines. The MF propagators are determined by the effective nucleon
and dibaryon masses (3.7) and (3.8) and by the effective masses of the mesons (4.3) and
(4.4). The graphical representations for the MF propagators are shown in Fig.13.

The total Green’s functions can be determined self-consistently by solving a system of
Gorkov-Dyson equations. As an example we derive an equation for the $\sigma$-meson propa-
gator. Let us multiply Eq.(3.3) by $\hat{\sigma}$, take the time-ordered product of the equation, and
find the average value of the equation over the ground state

$$(-\Box - m_\sigma^2) < T\sigma(1)\sigma(2) > = \delta^4(1, 2) + g_\sigma < T\bar{\Psi}(1)\Psi(1)\sigma(2) >$$
$$+ 2h_\sigma < T(m_D + h_\sigma(1))\varphi(1)\varphi(1)\sigma(2) > .$$

Taking into account the second-order terms with respect to the operator fields, we obtain

$$(-\Box - m_\sigma^2) < T\sigma(1)\sigma(2) > = \delta^4(1, 2) + 2h_\sigma^2 \rho_{DS} < T\sigma(1)\sigma(2) >$$
$$+ (2m_\sigma^2 h_\sigma \sqrt{\rho_{DS}})(< T\varphi(1)\sigma(2) > + < T\varphi^*(1)\sigma(2) >).$$

The second term on the right hand side redefines the mass of the $\sigma$-meson (see Fig.13 and
Eq.(4.3)). In the momentum representation, Eq.(5.5) takes the form

$$D^{\sigma\sigma}(k) = \tilde{D}^{\sigma\sigma}(k) + \tilde{D}^{\sigma\sigma}(k)2m_D^2 h_\sigma \sqrt{\rho_{DS}}(D^{\sigma\sigma}(k) + D^{\sigma^*\sigma}(k)),$$

where

$$\tilde{D}^{\sigma\sigma}(k) = \frac{1}{k^2 - \bar{m}_\sigma^2} .$$

is the $\sigma$-meson MF propagator.

The equations for other Green’s functions can be obtained in similar way to give

$$D^{\sigma\omega}_{\mu}(k) = \tilde{D}^{\sigma\omega}_{\mu}(k)2m_D^2 h_\sigma \sqrt{\rho_{DS}}(D^{\sigma\omega}_{\mu}(k) + D^{\sigma^*\omega}_{\mu}(k)),$$

$$D^{\sigma\varphi}(k) = \tilde{D}^{\sigma\varphi}(k)2m_D^2 h_\sigma \sqrt{\rho_{DS}}(D^{\sigma\varphi}(k) + D^{\sigma^*\varphi}(k)),$$

$$D^{\sigma\varphi^*}(k) = \tilde{D}^{\sigma\varphi^*}(k)2m_D^2 h_\sigma \sqrt{\rho_{DS}}(D^{\sigma\varphi^*}(k) + D^{\sigma^*\varphi^*}(k)),$$

$$D^{\omega\omega}_{\mu\nu}(k) = \tilde{D}^{\omega\omega}_{\mu\nu}(k)h_\omega \sqrt{\rho_{DS}}(2\mu_D^2 + k_\tau)D^{\omega\omega}_{\mu\nu}(k)$$
$$(2\mu_D^2 - k_\tau)D^{\omega\omega}_{\mu\nu}(k) ,$$
\[ D_{\mu}^{\phi\phi}(k) = \tilde{D}_{\mu}^{\phi\phi}(k) h_\omega \sqrt{\rho_{DS}}[(2\mu_D^* + k)\tau D_\tau^{\phi\phi}(k) + (2\mu_D^* - k)\tau D_\tau^{\phi\phi}(k)], \]  
\tag{5.12}

\[ D_{\mu}^{\phi\phi^*}(k) = \tilde{D}_{\mu}^{\phi\phi^*}(k) h_\omega \sqrt{\rho_{DS}}[(2\mu_D^* + k)\tau D_\tau^{\phi\phi^*}(k) + (2\mu_D^* - k)\tau D_\tau^{\phi\phi^*}(k)], \]  
\tag{5.13}

\[ D^{\phi\phi^*}(k) = \tilde{D}^{\phi\phi^*}(k) + D^{\phi\phi^*}(k)[h_\omega \sqrt{\rho_{DS}}(2\mu_D^* + k)\tau D_\tau^{\phi\phi^*}(k) + 2m_D^* h_\sigma \sqrt{\rho_{DS}} D^{\sigma\phi}(k)], \]  
\tag{5.14}

\[ D^{\phi\phi^*}(k) = \tilde{D}^{\phi\phi^*}(k)[h_\omega \sqrt{\rho_{DS}}(2\mu_D^* + k)\tau D_\tau^{\phi\phi^*}(k) + 2m_D^* h_\sigma \sqrt{\rho_{DS}} D^{\sigma\phi}(k)] + 2m_D^* h_\sigma \sqrt{\rho_{DS}} D^{\sigma\phi}(k), \]  
\tag{5.15}

\[ D^{\phi\phi^*}(k) = \tilde{D}^{\phi\phi^*}(k)[h_\omega \sqrt{\rho_{DS}}(2\mu_D^* - k)\tau D_\tau^{\phi\phi^*}(k) + 2m_D^* h_\sigma \sqrt{\rho_{DS}} D^{\sigma\phi}(k)], \]  
\tag{5.16}

Here

\[ \tilde{D}_{\mu\nu}^{\phi\phi}(k) = \frac{-g_{\mu\nu} + k_{\mu}k_{\nu}/\tilde{m}_\omega^2}{k^2 - \tilde{m}_\omega^2}, \]  
\tag{5.17}

\[ \tilde{D}^{\phi\phi^*}(k) = \frac{1}{(k + \mu_D^2)^2 - m_D^2} \]  
\tag{5.18}

are the \(\omega\)-meson and dibaryon MF propagators.

The system of equations (5.6) and (5.8)-(5.16) is shown graphically in Fig.14. It admits an explicit solution. The propagators \(D^{\phi\phi^*}(k)\) and \(D^{\phi\phi^*}(k)\) are expressible in terms of the propagators \(D^{\phi\phi^*}(k)\) and \(D^{\phi\phi^*}(k)\) and analogous propagators for the \(\omega\)-meson. These propagators in turn are expressible through the propagators \(D^{\phi\phi^*}(k)\) and \(D^{\phi\phi^*}(k)\).

The system of two equations for dibaryon Green’s functions

\[ D^{\phi\phi^*}(k) = \tilde{D}^{\phi\phi^*}(k) + \tilde{D}^{\phi\phi^*}(k)\Sigma^{\phi\phi^*}(k) D^{\phi\phi^*}(k) + \tilde{D}^{\phi\phi^*}(k)\Sigma^{\phi\phi^*}(k) D^{\phi\phi^*}(k), \]  
\tag{5.19}

\[ D^{\phi\phi^*}(k) = \tilde{D}^{\phi\phi^*}(k)\Sigma^{\phi\phi^*}(k) D^{\phi\phi^*}(k) + \tilde{D}^{\phi\phi^*}(k)\Sigma^{\phi\phi^*}(k) D^{\phi\phi^*}(k), \]  
\tag{5.19}

where

\[ \Sigma^{\phi\phi^*}(k) = \Sigma^{\phi\phi^*}(-k) = (h_\omega \sqrt{\rho_{DS}})^2(2\mu_D^* + k)\mu D_{\mu\nu}^{\phi\phi}(k)(2\mu_D^* + k)_\nu + (2m_D^* h_\sigma \sqrt{\rho_{DS}})^2 D^{\sigma\phi}(k), \]  
\tag{5.20}

\[ \Sigma^{\phi\phi^*}(k) = \Sigma^{\phi\phi^*}(-k) = (h_\omega \sqrt{\rho_{DS}})^2(2\mu_D^* + k)\mu D_{\mu\nu}^{\phi\phi}(k)(2\mu_D^* + k)_\nu + (2m_D^* h_\sigma \sqrt{\rho_{DS}})^2 D^{\sigma\phi}(k), \]  
\tag{5.20}

constitutes therefore a closed system of equations. These equations are shown graphically in Fig.15. They are identical to the Gorkov equations in the theory of superconductivity [16]. The relativistic version of these equations for the SU(2) color quark matter is discussed in Ref. [54]. The system has solutions

\[ D^{\phi\phi^*}(k) = \frac{\tilde{D}^{\phi\phi^*}(k) - \Sigma^{\phi\phi^*}(k)}{\tilde{D}^{\phi\phi^*}(k) - \Sigma^{\phi\phi^*}(k)(\tilde{D}^{\phi\phi^*}(k) - \Sigma^{\phi\phi^*}(k)) - \Sigma^{\phi\phi^*}(k)\Sigma^{\phi\phi^*}(k)}, \]  
\tag{5.21}
\[
D^{\varphi^*\varphi^*}(k) = \frac{\Sigma^{\varphi^*\varphi^*}(k)}{(\bar{D}^{\varphi^*}(k)^{-1} - \Sigma^{\varphi^*}(k))(\bar{D}^{\varphi^*\varphi^*}(k)^{-1} - \Sigma^{\varphi^*\varphi^*}(k)) - \Sigma^{\varphi^*\varphi^*}(k)\Sigma^{\varphi\varphi}(k)}. \tag{5.22}
\]

From expression
\[
D^{\varphi\varphi}(k) = \bar{D}^{\varphi^*}(k)\Sigma^{\varphi^*}(k)D^{\varphi^*\varphi}(k) + \bar{D}^{\varphi^*\varphi}(k)\Sigma^{\varphi\varphi}(k)D^{\varphi^*\varphi^*}(k) \tag{5.23}
\]
we find also
\[
D^{\varphi\varphi}(k) = \frac{\Sigma^{\varphi\varphi}(k)}{(\bar{D}^{\varphi^*}(k)^{-1} - \Sigma^{\varphi^*}(k))(\bar{D}^{\varphi^*\varphi}(k)^{-1} - \Sigma^{\varphi^*}\varphi^*(k)) - \Sigma^{\varphi^*}\varphi^*(k)\Sigma^{\varphi\varphi}(k)}. \tag{5.24}
\]

The Green’s functions for other particles are expressible in terms of the constructed dibaryon Green’s functions.

### 5.2 Dispersion laws for elementary excitations

The mass operators \(\Sigma^{\varphi^*\varphi^*}(k)\) has the form
\[
\Sigma^{\varphi^*\varphi^*}(k) = (h\sqrt{\rho_{DS}})^2 \left[ \frac{4\mu_D^2}{m_{\omega}^2} \frac{\omega^2 - m_{\omega}^2}{\omega^2 - k^2 - m_{\omega}^2} + \frac{4\mu_D^*\omega}{m_{\omega}^2} + \frac{\omega^2 - k^2}{m_{\omega}^2} \right].
\]
\[
+ (h\sigma\sqrt{\rho_{DS}})^2 \frac{4m_{\sigma}^2}{\omega^2 - k^2 - m_{\sigma}^2}. \tag{5.25}
\]

Denoting terms appearing in this expression, successively, by \(a, b, c,\) and \(d,\) one can write the following representations for the mass operators: \(\Sigma^{\varphi^*}(k) = a + b + c + d,\)
\(\Sigma^{\varphi^*}(k) = a - b + c + d,\) and \(\Sigma^{\varphi^*\varphi^*}(k) = a - c + d.\)

The chemical potential of the system is determined from the relation
\[
\mu_D^2 - m_D^2 = \Sigma^{\varphi^*\varphi^*}(0) - \Sigma^{\varphi\varphi}(0) \tag{5.26}
\]
which constitutes a relativistic extension of the non-relativistic relation derived by Hugenholtz and Pines [51] for Bose systems. For \(k = 0, b = c = 0,\) and the right hand side of Eq.(5.26) vanishes. Therefore, the MF relation (4.5) remains valid. A relation of such a kind is necessary to get a pole in the dibaryon Green’s functions at \(\omega = k = 0\) i.e. for the existence of sound in the medium.

The spectrum of elementary excitations of the system is determined by zeros of the inverse Green’s functions:
\[
(\bar{D}^{\varphi^*}(k)^{-1} - \Sigma^{\varphi^*}(k))(\bar{D}^{\varphi^*\varphi}(k)^{-1} - \Sigma^{\varphi^*}\varphi^*(k)) - \Sigma^{\varphi^*\varphi^*}(k)\Sigma^{\varphi\varphi}(k) = 0. \tag{5.27}
\]

To eliminate poles coming from the meson propagators, we multiply the denominator of the Green’s functions by \((k^2 - \bar{m}_\omega^2)(k^2 - \bar{m}_\sigma^2)/m_{\omega}^2\). We then get an equivalent 4th order polynomial with respect to \(\omega^2:\)
\[
\sum_{n=0}^{4} \omega^{2n}c_n = (k^2 - \bar{m}_\omega^2)(k^2 - \bar{m}_\sigma^2)m_{\omega}^2 \times [(\bar{D}^{\varphi^*}(k)^{-1} - \Sigma^{\varphi^*}(k))(\bar{D}^{\varphi^*\varphi}(k)^{-1} - \Sigma^{\varphi^*}\varphi^*(k)) - \Sigma^{\varphi^*\varphi^*}(k)\Sigma^{\varphi\varphi}(k)] \tag{5.28}
\]
with coefficients

\[ c_0 = k^8 + k^6(\tilde{m}_\omega^2 + \tilde{m}_\sigma^2) + k^4(\tilde{m}_\omega^2\tilde{m}_\sigma^2 + 8\mu_D^2h_\omega^2\rho_{DS} - 8\mu_D^2h_\sigma^2\rho_{DS}^2) + 8k^2(\tilde{m}_\sigma^2\mu_D^2h_\omega^2\rho_{DS} - \tilde{m}_\sigma^2\mu_D^2h_\sigma^2\rho_{DS}^2), \]

\[ c_1 = -4k^6 - k^4(3\tilde{m}_\omega^2 + 3\tilde{m}_\sigma^2 + 4\mu_D^2) - 2k^2(\tilde{m}_\omega^2\tilde{m}_\sigma^2 + 2\tilde{m}_\omega^2\mu_D^2 + 2\tilde{m}_\sigma^2\mu_D^2 + 4\mu_D^2h_\omega^2\rho_{DS} - 8\mu_D^2h_\sigma^2\rho_{DS}^2) - 4\tilde{m}_\omega^2\tilde{m}_\sigma^2\mu_D^2 + 8\tilde{m}_\omega^2\mu_D^2 h_\sigma^2\rho_{DS}^2, \]

\[ c_2 = 6k^4 + k^2(3\tilde{m}_\omega^2 + 3\tilde{m}_\sigma^2 + 8\mu_D^2) + \tilde{m}_\omega^2\tilde{m}_\sigma^2 + 4\tilde{m}_\omega^2\mu_D^2 + 4\tilde{m}_\sigma^2\mu_D^2 - 8\mu_D^2h_\sigma^2\rho_{DS}^2, \]

\[ c_3 = -4k^2 - \tilde{m}_\omega^2 - \tilde{m}_\sigma^2 - 4\mu_D^2, \]

\[ c_4 = 1. \]

The polynomial (5.28) determines four different excitations, two of dibaryon type (particles and antiparticles, the first excitation is sound) and two of \(\sigma\) - and \(\omega\) - meson types. These four poles occur in all Green’s functions because of the \(\sigma\)-\(\omega\)-\(\varphi\)-\(\varphi^*\) mixing. Such a mixing occurs because the \(\sigma\)- and \(\omega\)-mesons can be absorbed by the dibaryons in the condensate as a result of which the dibaryons leave the condensate and propagate as normal particles. The mixing describes also processes with creation of dibaryon-antidibaryon pairs with subsequent absorption of dibaryons by condensate and propagation of antidibaryons.

One can verify that the Green’s function \(D^{\sigma\sigma}(k)\) has no pole at \(k^2 = \tilde{m}_\sigma^2\). The poles of \(D^{\sigma\sigma}(k)\) coincide with the poles of the dibaryon Green’s functions.

The second term in the \(\omega\)-meson propagator in Eqs.(5.11) is factorizable and there is no \(g_{\mu\nu}\) tensor structure. Therefore, the pole at \(k^2 = \tilde{m}_\omega^2\) in the first term cannot be cancelled by the second term, as in case of the \(\sigma\)-meson. The \(\omega\)-meson in heterophase nucleon-dibaryon matter has therefore two branches of excitations.

The velocity of sound \(a_s\) can be found from Eq.(5.27) by keeping terms of order \(O(k^2)\) and \(O(\omega^2)\). In this limit only the sound mode \(\omega = \omega_s(k)\) survives. We get

\[ a_s^2 = \left(\frac{\partial\omega_s(k)}{\partial k}\right)^2_{k=0} = \frac{\alpha}{1 + \alpha} \]

with

\[ \alpha = 2\rho_{DS} \frac{m_\omega^2}{m_\sigma^2} \left( \frac{h_\omega^2}{m_\omega^2} - \frac{h_\sigma^2}{m_\sigma^2} \right). \]

When the condition for stability (3.21) is fulfilled, the value \(a_s^2\) is positive and less than unity (i.e. less than the velocity of light).

The numerical analysis of Eq.(5.27) shows that there are no complex or negative \(\omega^2\) when the inequality (3.21) is satisfied. This result implies the stability of the ground state of the system for \(a_s^2 > 0\). If the poles occur symmetrically at \(\pm \omega_s(k)\), the denominator of
the dibaryon Green’s function has no zeros on the imaginary axis of the complex $\omega$-plane. One can check that after a Wick rotation, the denominator really becomes positive for all values of $\omega$ and $k$.

The group velocities of all four excitations are less than the velocity of light in absolute values. This is quite natural, because the model is relativistically invariant. It the limit $k \to \infty$ we can keep the leading terms in $k^2$ in Eq.(5.28). The polynomial (5.29) can then be summed up to $(\omega^2 - k^2)^4$. The dispersion laws for all four types of excitations behave asymptotically as $\omega^2 \sim k^2$, and the group velocities approach unity in the limit of large $k^2$.

6 DISCUSSIONS AND CONCLUSIONS

The qualitative estimates based on a model of non-interacting nucleons and dibaryons show that in normal nuclear matter a dibaryon Bose condensate does not exist provided the inequality $m_D > 1.96$ GeV is fulfilled. A more accurate estimate can be made on the basis of the relativistic MF model. From the requirement of absence of a dibaryon Bose condensate for $\rho_{TV} \leq \rho_0$, where $\rho_0 = 0.15$ fm$^{-3}$ is the saturation density for nuclear matter, we get for $h_\omega = 2g_\omega$ a constraint

$$m_D > 1.89 \text{ GeV}.$$ 

This constraint is valid provided that dibaryon matter is stable against compression. It follows that the $d_1$-resonance with a mass $m_D = 1.92$ GeV does not affect the properties of ordinary nuclei.

It would be interesting to check astrophysical data for the presence of a dibaryon condensate in the interiors of massive neutron stars as well as possible signatures of their instability caused by dibaryons. From the existence of massive neutron stars, one can put a lower limit on the masses of dibaryons. The estimates (2.2) and the results based on a more realistic model including interactions between dibaryons [35] show that such a constraint can be physically significant. Conversely, the experimental discovery of dibaryons will have important astrophysical implications.

Phase transitions of nuclear matter to strange matter [55, 56] have been widely discussed in the literature (for a review see [57]). Dense nuclear matter with a dibaryon Bose condensate can exist as an intermediate state below the quark-gluon phase transition. This is the case when dibaryon matter is stable against compression. If dibaryon matter is unstable against compression, the creation of dibaryons could be a possible mechanism for the phase transition to quark matter.
The soft core of the $HH$-interaction \[39\] is responsible for the relatively low value of the critical density for formation of $H$-dibaryons in nuclear and neutron matter and for the possible instability of $H$-matter. The energetically favorable compression of $H$-matter will eventually lead to the formation of absolutely stable strange matter. Possible astrophysical examples are bursters and roentgen pulsars which accret matter from companion stars. This leads to an increase of the mass and central density of these neutron stars. Once the density exceeds the critical density, $H$-particles can be created, leading to the formation of strange matter. The neutron star converts then to a strange star \[58, 59, 60].

The experimental observation of a Bose condensate of dibaryons in heavy-ion collisions would be of great importance for understanding physics of nuclear matter at supranuclear densities. In the center-of-mass frame of the condensate a large fraction of dibaryons has zero velocities. When the density decreases, dibaryons in the condensate decay to their specific channels. Experimentalists would observe in every collision events with the same invariant mass $m_D$ and the same total momentum. An excess of such events can be considered as a possible signature for the formation of a dibaryon condensate in heavy-ion collisions.

The contribution of dibaryons out of the condensate to pressure and energy density can be calculated in relativistic Hartree approximation only. We constructed Green’s functions of the system. The one-loop calculation of the EOS for heterophase nucleon-dibaryon matter will be given elsewhere \[61\].

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Fig.1. A schematic representation for the process of dibaryon Bose condensation in neutron matter in the ideal gas approximation. When the chemical potential of nucleons exceeds \( m_D/2 \), production of dibaryons becomes energetically favorable. Dibaryons are Bose particles. They are accumulated in the ground state of the system with zero momentum and form a Bose condensate.

Fig.2. Schematic behavior of the pressure as a function of the density in the ideal gas approximation. Above the critical density for formation of the dibaryon Bose condensate (indicated by the arrow), the neutron chemical potential \( \mu_n \) is frozen at a value \( m_D/2 \) and the neutron density is fixed, but the total baryon number density is still increasing along the dashed line due to the dibaryon formation. Dibaryons are in the ground state and do not contribute to the pressure. Therefore the pressure for the binary mixture remains constant (horizontal dotted line). The solid line gives the EOS for homophase neutron matter.

Fig.3. Diagrams contributing to the coupling constants of the \( \omega \)- and \( \sigma \)-mesons with dibaryons in the additive model. The dibaryon \( D \) couples strongly to the \( NN \)-channel (a), the \( d' \)-dibaryon decays into the \( \pi NN \)-channel (b). The \( \omega \)-meson is coupled to nucleons (a, b), the \( \sigma \)-meson is coupled to nucleons and the \( \pi \)-meson in the \( d' \) (b).

Fig.4. The \( \sigma \)- and \( \pi \)-meson exchange current contributions to the \( \sigma \)-meson coupling constants with the dibaryon \( D \) coupled strongly to the \( NN \)-channel (a), and with the \( d' \)-dibaryon decaying into the \( \pi NN \)-channel (b).

Fig.5 (a,b). The critical density for occurrence of a Bose condensate of dibaryons in nuclear (a) and neutron (b) matter versus the \( \sigma \)-meson coupling constant \( h_\sigma \) for \( m_D = 1.88 \text{ GeV} \) (= 2\( m_N \); the long dashed curve No. 1), 1.96 GeV (the solid curve No. 2), etc. with a step 80 MeV. The results for the \( d_1(1920) \) and \( d'(2060) \) dibaryons are shown (the dashed curves). Dibaryon matter is stable against compression when the square of the sound velocity \( a_s \) is positive. This is the case for \( h_\sigma/(2g_\sigma) < 0.8754 \). The value \( \rho_0 = 0.15 \text{ fm}^{-3} \) is the saturation density for nuclear matter. The occurrence of \( H \)-dibaryons in nuclear and neutron matter is denoted by the crosses.

Fig.6. Possibilities for the behavior of the difference \( 2\mu_N - \mu_D \) between the two-nucleon and the dibaryon chemical potential versus the dibaryon fraction \( 2\rho_{DV}/\rho_{TV} \). Figure (a) shows unstable equilibrium states. Figure (b) shows stable equilibrium states.

Fig.7 (a,b) The effective nucleon mass \( m_N^* \) in GeV versus the dibaryon fraction \( 2\rho_{DV}/\rho_{TV} \) in heterophase matter (a). The results do not depend on the dibaryon mass. The difference \( 2\mu_N - \mu_D \) between the two nucleon chemical potentials and the dibaryon chemical potential versus the dibaryon fraction \( 2\rho_{DV}/\rho_{TV} \) (b). The results are given for
total baryon densities 1, 2, 3, 4, 5, and 6 times the saturation density $\rho_0$. The normal homophase matter is stable when $2\mu_N - \mu_D < 0$ and $\rho^c_{DV} = 0$. An intersection of a curve with a negative slope with the horizontal line $2\mu_N - \mu_D = 0$ indicates occurrence of a stable equilibrium in heterophase matter. Two such states for nuclear and neutron matter, occurring at $\rho_{TV} = 3\rho_0$ and $\rho_{TV} = 2\rho_0$, are denoted by the arrows. The results are given for $m_D = 1.96$ GeV. The dibaryon mass does not enter the self-consistency condition (4.11) and enters linearly in the difference $2\mu_N - \mu_D$, and the curves for other dibaryon masses can be obtained by vertical parallel displacements. The results for $m_D = 2.06$ GeV ($d'$-dibaryon) can be obtained e.g. by a 100 MeV negative shift, etc. The solid lines stand for nuclear ($\gamma = 4$) matter, the dashed lines stand for neutron ($\gamma = 2$) matter.

Fig. 8. The energy per nucleon in homophase nuclear and neutron matter (solid lines) and in heterophase matter (dashed and dotted lines) versus the total baryon number density $\rho_{TV} = \rho_{NV} + 2\rho^c_{DV}$ for $d_1(1920)$ and $d'(2060)$ dibaryons using $h_\omega = 2g_\omega$ and $h_\sigma/(2g_\sigma) = 0.8$. The dibaryon Bose condensation decreases the energy of the ground states. The occurrence of $H$-particles in nuclear and neutron matter is shown. It results in the formation of strange matter. The value $\rho_0$ is the saturation density of nuclear matter.

Fig. 9. Equation of state for homophase nuclear and neutron matter (solid lines) and for heterophase nuclear- and neutron-dibaryon matter (dashed and dotted lines) versus the total baryon number density $\rho_{TV}$ for $d_1(1920)$ and $d'(2060)$ dibaryons at $h_\omega = 2g_\omega$ and $h_\sigma/(2g_\sigma) = 0.8$. Dibaryon Bose condensation at high densities softens the EOS for nuclear and neutron matter. The critical density for the occurrence of $H$-particles in nuclear and neutron matter is indicated.

Fig. 10. There are two kinds of vertices corresponding to interactions of the $\omega$- and $\sigma$-mesons with dibaryons and a vertex describing creation and absorption of dibaryons by the condensate.

Fig. 11. Pictorial representation of the series for the $\omega$- and $\sigma$-mesons mean fields (Eqs. (4.1) and (4.2)). The diagrams can be summed up to produce (i) the dressed meson MF propagators without modification of the meson vertices or (ii) the dressed meson MF vertices with dibaryons without modification of the meson MF propagators and the meson-nucleon MF vertices.

Fig. 12. The dressed $\omega$- and $\sigma$-meson MF vertices with dibaryons.

Fig. 13. The Dyson equations for the MF propagators of the $\sigma$- and $\omega$-mesons, nucleons, and dibaryons. Thin lines define the bare propagators, thick lines define the MF propagators. The dashed lines, the solid lines, and the double solid lines describe, respectively, the meson propagators, the nucleon propagator, and the dibaryon propagator.
Fig.14. Pictorial representation of the Gorkov-Dyson equations for the complete Green's functions in the heterophase nucleon-dibaryon matter. Note the correspondence between the diagrams and the equations in the text: (a)- Eqs.(5.6) and (5.11), (b)- Eq.(5.8), (c)- Eqs.(5.9) and (5.12), (d)- Eqs.(5.10) and (5.13), (e)- Eq.(5.14), (f)- Eq.(5.16), and (g)-Eq.(5.15).

Fig.15. Pictorial representation of the system of two coupled equations (5.19) for the normal and anomalous dibaryon Green’s functions.