The Lucernic Behaviour of Gravity

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Abstract In this work it is proposed a scientific line of reasoning called Lucernics which aims to describe natural phenomena in terms of properties of light. It also carries out an analysis about how gravity could be intermediated by virtual particles similar to photons, namely guardions. It also proposes that massive bodies are sinks of guardions. When a massive body absorbs guardions from vacuum fluctuations, dark guardions are emitted. A dark guardion is a virtual particle which carries momentum in a direction opposite to its displacement. The time-derivative of this momentum describes a possible attractive force between massive particles which emit them. This derived force was compared to Newtonian gravitational force and the caliber constant obtained was equal to Planck’s mass. This same study was done for electrostatic force, obtaining Planck’s charge as a caliber constant. Newton’s gravitational constant and Einstein’s field equation were decomposed, and it also proposes a Lucernic Field Equation. The phenomena that this equation could describe will be discussed in future works.

Keywords Plank units · gravitational constant · relative strength · radiant tensor

1 Introduction

Gravitation is not described by The Standard Model of Particle Physics. This model encompasses weak, strong and electromagnetic interactions. The fundamental particles are classified into two categories: fermions and bosons [1]. Fermions are field generating particles, present half-integer spin, follow Fermi-Dirac statistics [2] and obey Pauli’s exclusion principle. They are represented by leptons and quarks. Bosons, however, are intermediate particles of integer spin which follow Bose-Einstein statistics and do not obey Pauli’s exclusion principle [1]. Gluons are the strong force intermediate particles, particles Z, W^+ and W^- are the intermediates of weak interaction and photons are responsible for electromagnetic interaction [1]. However, The Standard Model does not present evidence of an intermediate particle for gravitation interaction.

In The Standard Model, which boson would present the closest characteristics to a gravitational interaction intermediate particle?

Certainly, it would not be the gluons nor the heavy bosons, as they both act only within very short distances in the order of magnitude of the atomic nucleus. A gravitational interaction intermediate particle must be capable of traveling long distances through the observable universe. The only intermediate particle in The Standard Model able to do so is the photon. This (OBSERVATION 1) and further ones relating gravity and light are listed as follow:

Observation 1 As with the gravitational field, light is able to travel long distances through the observable universe [3];

Observation 2 The intensity of the gravitational field, like intensity of light, decreases with the squared inverse of the distance from puntual or spherically symmetric sources [4];

Observation 3 Perturbations in the gravitational field propagate in the speed of light [5];

Observation 4 A ray of light can be deflected by a gravitational field [6].
The intensity of the gravitational field described by Newton’s gravitation decreases with the squared inverse of the distance from punctual or spherically symmetric sources [4] (OBSERVATION 2). After the discovery of gravitational waves, it was observed that they travel at the speed of light [5] (OBSERVATION 3). Light suffers a deflection described by General Relativity Theory [6] while traveling near an intense gravitational field, which is generated by a massive body (OBSERVATION 4).

However, problems arise when we try to treat gravity as a luminous phenomenon:

**Problem 1** Two luminous masses tend to repel each other due to a pressure exerted by photons [7,8]. This does not represent the behavior of gravitational force, which is attractive;

**Problem 2** The gravitational field presents a high capacity of penetration in materials and, unlike electromagnetic field, does not seem to depend on the properties of the medium such as electric conductivity, electric permittivity and magnetic permeability;

**Problem 3** The relative strength of the gravitational field (10^{-38}) is much lower than that relative strength of the electromagnetic field (10^{-2}) [1];

**Problem 4** It is understood by General Relativity (GR) that gravity is an inertial force that results from the time-space warp in the vicinity of an energy concentration such as a massive body. This dismisses the necessity of intermediate particles to describe gravitation [12].

Regarding Problem 1, it is a consensus that photons can transmit momentum to the barionic matter, as it happens with solar sails [7] and with optic levitation [8]. However, in both these approaches, photons promote repulsive interactions between the bodies involved. It is common sense that photons can intermediate an attractive interaction between two bodies if these bodies present an excess of electrically opposite charges. But how could photons intermediate an attractive interaction between electrically neutral bodies? If particles like photons are responsible for gravitational interaction, could there also be a repulsive version of gravity? Could it also explain the accelerated expansion of the universe attributed to dark energy [9,10]?

Problem 2 refers to the independence of the gravitational field from the properties of the medium. For example, the electromagnetic field depends on electric conductivity, electric permittivity and magnetic permeability. Light can be reflected, refracted and also be extinguished depending on those characteristics. The gravitational field is, apparently, not affected by variations of those parameters like electromagnetic waves are. However, precise measurements of gravitational constant by two different methods, namely time-of-swing (TOS) and angular-acceleration-feedback (AAF), indicate values slightly different outside the error margins [11]. Previous measurements with lead and other materials also indicate slightly different values for G as compiled in [11] while comparing TOS and AAF with classical methods.

Problem 3 refers to the relative strength of the gravitation (10^{-38}) and of electromagnetism (1/137) [1]. How could gravitational field be related to electromagnetic field if the former is around 36 orders of magnitude weaker than the later?

As of Problem 4, in literature, it is accepted that the best theory to describe gravitation is the General Relativity (GR) [12], which has been through many tests and dispenses the participation of intermediate particles.

However, GR presents inconsistencies with Quantum Mechanics (QM). GR is a deterministic theory in which time-space is a continuum and described by the metrics with the use of Riemannian Geometry in a non-linear formalism, while QM is, intrinsically, a probabilistic theory in which at least space is quantized in the surroundings of a source of potential described in a linear formalism in terms of a Hilbert space with eigenvalues and eigenvectors [13].

It is likely that a more general theory of light could play the role of a unifying theory between Electromagnetism, General Relativity and Quantum Mechanics, given that photons are an entity common to these three areas of physics. Schrödinger believed that electromagnetism, the theory of light, occupied a privileged hierarchical position in the structure of science, like theory of measurements. In [14], he interpreted the time-space warp as the vacuum refraction variable index in the vicinity of a massive body. However, electromagnetism is not sufficient to describe all the properties of light, once it does not take into account the dual wave-particle nature. Quantum Mechanics takes this dual nature into account, but is not compatible with General Relativity.

This work then aims to develop a more general theory of light, namely Lucernics, presented in Chapter 2.

General Relativity describe Newton’s gravitation for weak fields and low speeds. Newton’s gravity theory estimates half the value given by General Relativity for the deflection of a ray of light traveling close to a star, as observed in the 1919 eclipse in Sobral, Brazil, and in Príncipe Island [6]. In [15] there is an analysis of the individual contributions of the time and space in the deflection of the light of a star traveling near a massive body such as the Sun. In [15], it first assumes space as being curved and time as being flat. Then,
the deflection of light due the flat space and curved time. The result was that in the vicinities of the star the time curve contributed with 0.8725" and the space curve contributed with another 0.8725", and the resulting deflection is 1.75".

First, we interpret in this work that Newton’s gravity is equivalent to space’s warp only, not to that of time. This means that we shall consider time as being absolute in order to reproduce the law of universal gravitation.

We also propose that all force is equivalent to a force derived from the collision with particles. We refer to this equivalent force as lucernic interaction.

As will be demonstrated in Chapter 2, comparing the law of gravitation and electrostatic law with lucernic interactions leads to two gauge constants, which are Planck’s mass and Planck’s charge. Planck units are named as such for having being popularized by Max Planck. However, similar studies were carried out previously by [16,17].

These units were obtained originally in literature through dimensional analysis between physical constants [17], and lacked a consistent theoretical foundation. In the Lucernic Theory, as it will be presented in Chapter 2, these constants can be determined as being gauge constants.

In Chapter 3, we discuss important implications in the definition of Newton’s gravitational constant and present another form of Einstein’s field equation. However, the consequences of this Lucernic Field Equation is out of the scope of this work and shall be discussed in future works.

2 Lucernics

In this section we formalize the basic notions of the Lucernic Theory by means of four definitions and two postulates. In addition, we assume two propositions from which we describe gravitation. Lucernics establishes the following definitions:

**Definition 1** Lucernics, which derives from the latin *lucerna*, which means lamp, and from the greek *dynamikós*, which means force, is a science branch that aims to describe natural phenomena in partial terms of the properties of light.

**Definition 2** We call a phenomenon lucernic if we describe it partially by means of some properties of light.

**Definition 3** Guardion (\(\bar{g}\)) is a generic boson which is stable and does not have rest mass. It represents intermediate particles for electromagnetic and possibly gravitational interactions (\(G\)).

**Definition 4** A dark guardion (\(\bar{g}\)) is a virtual particle emitted by a massive body as it absorbs a guardion from vacuum fluctuations. This particle then represents a guardion which travels in opposite direction. It has the same properties of the absorbed guardion, but this particle carries a momentum with a direction opposite to its displacement.

Definition 1 formalizes the concept of Lucernics and characterizes it as a science that aims to describe natural phenomena in partial terms of the properties of light.

Definition 2 characterizes a phenomenon as lucernic if this phenomenon can be understood at least partially in terms of some property of light.

The properties of light referred to in Definitions 1 and 2 are, essentially, physical quantities such as speed, frequency, wavelength, momentum, energy and luminosity. However, the phenomena cannot always be explained simply in terms of photons, and therefore, in Definition 3, we use the generic term guardion to refer to the intermediate particles of electromagnetism or gravitation.

The Lucernic Theory introduces two postulates or principles.

I Similarity principle

*The gravitational and electromagnetic interactions are intermediated by particles that are similar as to their lifetime, speed of propagation in vacuum and divergence.*

II Reciprocity principle

*A force can be though of as a time-space warp, and the time-space warp can be though of as a force.*

Principle I emphasizes the similarities between intermediate particles of electromagnetic interactions and, possibly, gravitational. This principle was based on Observations 1 – 3. The lifetime of the intermediate particles of electromagnetic and gravitational interaction must be considerably large. This implies that both particles shall have enough time to travel great lengths across the universe. Another common characteristic between these particles is that both travel at the speed of light. According to Restrict Relativity, this implies in null rest masses. At last, we emphasize in the Principle of Similarity that the fields generated by the particles obey to Gauss divergence theorem. Thus, we use the term guardion to refer to a generic particle that represent electromagnetic or gravitational intermediate particles.

Principle II was based on Observation 4, admitting that there must be a link between Electromagnetism and General Relativity. Principle II is the same GR's
equivalence principle, but it emphasizes that this equivalence is reciprocal, i.e., a force can be thought of as a time-space warp, and a time-space warp can be thought of as a force.

Thus, we should consider gravity as a force derived from the collision of particles that do not oppose the principles of GR as questioned in Problem 4 presented in the Introduction.

As to Problems 1, 2 and 3, we assume some propositions as true so that we can model a process equivalent to gravitation from lucernic concepts. Consider the following model propositions:

**Proposition 1** A massive body is a sink of \( \hat{g} \) and an isotropic source of \( \hat{v} \).

**Proposition 2** Radiation \( \hat{g} \) has high penetration power, given that only a very small fraction \( \alpha \ll 1 \) of its momentum \( \hat{p} \) is transmitted to neutrons and/or protons.

Proposition 1 was taken to work around Problem 1, where luminous sources promote a repulsive interaction between each other, and not attractive, like gravity. Admitting that massive bodies are a sink of \( \hat{g} \)-particles can solve this problem. It is intuitive to imagine sinks attracting each other as two hose-ends draining water from a reservoir.

However, in order to formalize Proposition 1, first consider a massive body \( A \) isolated, as illustrated in Fig. 1. Massive body \( A \) has a spherical symmetry and absorbs guardions \( \hat{g} \) with wave vector \( \hat{k} \) in the direction \( -\hat{r} \). These guardions are supplied by vacuum fluctuations, leaving energy deficits called dark guardions \( (\hat{g}) \). The dark guardions propagate themselves at the speed of light as if they were virtual particles moving away from \( A \) with wave vector \( \hat{k} \).

![Fig. 1](image)

Fig. 1 A massive body \( A \) functions a sink of \( \hat{g} \) guardions. These \( \hat{g} \) guardions propagate themselves with wave vector \( \hat{k} \) towards the inside of \( A \). We assume that these guardions are supplied by vacuum fluctuations leaving energy deficits called *dark guardions* \( (\hat{g}) \). The dark guardions propagate themselves at the speed of light as if they were virtual particles moving away from \( A \) with wave vector \( \hat{k} \).

- b) it can be reemitted in longer wavelengths in the shape of thermal energy.
- c) it can supply the energy irradiated by electrons accelerated in its orbitals, preventing them from falling into the atomic nucleus, possibly explaining the stability of the atom.
- d) it can be converted into weak interaction heavy bosons.
- e) it can be converted into dark matter and/or dark energy.
- f) it can also be dissipated in extra dimensions.

These possibilities are speculative, but from the principle of conservation of energy. Momentum must also be conserved, as the absorption of guardions and the radiation of *dark guardions* was considered isotropic as per Proposition 1. For example, in the case of an isolated body as shown in Fig. 1, for each guardion absorbed in the surface of \( A \) there is a guardion absorbed in the diametrically opposite. This shows that if body \( A \) is isolated, the absorption of guardions does not alter its momentum.

Now, consider a second massive body \( B \) close to \( A \) as depicted in Fig. 2. Initially, consider \( B \) as identical to body \( A \) and that there is no gravitational force between them. When a dark guardion \( \hat{g} \) emitted by \( A \) reaches another massive body \( B \), a very small fraction of the momentum \(-\hat{a}\hat{p}\) is transferred to \( B \) in the direction of \( A \). In the same manner, a dark guardion emitted by \( B \) when it collides with \( A \) transfer a very small fraction \( \hat{a}\hat{p} \) of its momentum to \( A \) in the direction of \( B \). If this transfer of momentum repeats at every period \( \tau \), we can derive an attractive force between \( A \) and \( B \).

As for Problem 2, about attenuation, reflection and refraction of the electromagnetic field in different media, a property that the gravitational field apparently does not have, we take Preposition 2 as true.
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2.1 Derivation of the lucernic interaction

We assume that the momentum carried by a guardion is given by the Brooglie relation and expressed as:

$$\vec{p} = h\vec{k}. \quad (1)$$

Where $\vec{p}$ is the momentum carried by the guardion, $h$ is the reduced Planck’s constant, $\vec{k}$ is the wavenumber vector associated to guardion $\hat{g}$ which propagates to the left in Fig. 1. The dark guardion $\hat{g}$ is a signal that propagates itself to the right with wavenumber vector $\vec{k} = -\vec{k}'. \quad (2)$

Thus, the momentum carried by a dark guardion would be given by

$$\vec{p} = -h\vec{k}' = -2\pi \frac{h}{\lambda} \hat{r}. \quad (3)$$

As it can be seen in Fig. 2, an intermediate particle $\hat{g'}$, during a collision to a massive body $B$, transfers a fraction of its momentum $\alpha\vec{p}$ to $B$.

If $n$ particles $\hat{g'}$ collide with particle $B$ per time unit, the time-derivative of the resulting force at $B$ ($\vec{F} = \Sigma\vec{F}_B$) is given by

$$\vec{F} = -\alpha \frac{\partial(p_n)}{\partial t} \hat{r} = -2\pi \alpha \frac{h}{\lambda} \frac{\partial n}{\partial t} \hat{r}, \quad (4)$$

where $\lambda = \frac{2\pi}{\hat{r}}$ is the radiation wavelength $\hat{g'}$. We admit that the wavelength does not vary in time, and therefore $\lambda$ is constant in Eq. 4. $\hat{r}$ is the unit-vector in the radial direction as illustrated in Fig. 2.

Admiting a monochromatic emission, and that each wavelength carries a particle $\hat{g'}$ for a period $\tau$, the number of particles $\hat{g'}$ per time-unit is related to frequency $f$ as

$$\frac{\partial n}{\partial t} = \frac{1}{\tau} = f. \quad (5)$$

Combining Eqs. 4 and 5,

$$\vec{F} = -\alpha \frac{hf}{\lambda} \hat{r}. \quad (6)$$

In order to make the following expressions, we define

$$\alpha_g \equiv 2\pi\alpha. \quad (7)$$

The frequency and wavelength are related by

$$f = \frac{v}{\lambda}, \quad (8)$$

where $v$ is the intermediate particle’s speed of propagation, in the case of gravitational waves and electromagnetic waves in the classical vacuum is equal to the speed of light $c$. Thus,

$$\vec{F} = -\alpha_g \frac{hc}{\lambda^2} \hat{r}. \quad (9)$$

Eq. 9 gives the force derived from the momentum transferred by guardions or dark guardions responsible for the lucernic interaction. In the case of Fig. 2, both bodies $A$ and $B$ are identical. We also estimate this force without considering the divergence of radiation. The number of guardions at a given distance from an isotropic source depends on the inverse of the squared distance and can be estimated by Gauss’s Divergence Theorem.

In the following sections, we generalize the lucernic interaction for bodies of differenc mass. We also consider the divergence of the fields of guardions and dark guardions. We compare the lucernic interaction to Newton’s gravitational interaction described in Sec. 2.2 and to the electrostatic or coulomb interaction described in Sec. 2.3.

2.2 Newtonian Gravitational Manifestation

We can obtain the acceleration $\vec{g}$ of a particle $B$ of mass $m_g$ from Newton’s Second Law

$$\vec{g} = \frac{\vec{F}}{m_g}. \quad (10)$$
Replacing Eq. 9 in 10,
\[ \vec{g} = -\alpha_g \frac{\hbar c}{m_g \lambda^2} \hat{r} . \]  
(11)

The acceleration field is being generated by and isotropic source on body A. Thus, we can apply Gauss’s Divergence Theorem to find the acceleration field flux.

The flux of the acceleration field crossing a gaussian surface \( S \) is given by
\[ \oint_S \vec{g} \cdot d\vec{S} = \alpha_g \frac{\hbar c}{m_g \lambda^2} 4\pi r_g^2 , \]  
(12)
where \( r_g \) is the radius of a spherical gaussian surface. In this case, the flux does not depend on the size nor the shape of the gaussian surface. Thus we can chose a spherical surface of radius \( r_g \) such that \( r_g = \lambda \).

Eq. 13 gives the flux of the acceleration field generated by a massive body \( A \) which consists of a single particle. The gaussian flux must be directly proportional to the number of source particles inside the gaussian surface. For a body consisting of \( N \) particles,
\[ \oint_S \vec{g} \cdot d\vec{S} = 4\pi \alpha_g \frac{\hbar c}{m_g} N . \]  
(13)

The number of particles \( N \) can be written in terms of the mass \( M \) of a body as
\[ N = \frac{M}{m_g} , \]  
(15)
where \( m_g \) is a mass to be gauged. We hope that this Newtonian approach is valid for \( r \gg \lambda \). We also hope it is valid for \( N \gg 1 \), which implies in \( M \gg m_g \). In this work, we do not discuss the cases of short distances and low masses in detail.

Substituting Eq. 15 in Eq. 14,
\[ \oint_S \vec{g} \cdot d\vec{S} = 4\pi \alpha_g \frac{\hbar c}{m_g^2} M . \]  
(16)

The Gauss Law for classical gravitational field is given by
\[ \oint_S \vec{g} \cdot d\vec{S} = 4\pi G M , \]  
(17)
where \( G \) ("G" armorial\(^1\)) is the lucernic gravitational constant. For now, we make use of the lucernic \( G \) and later we introduce the case for which it is equal to Newton’s gravitational constant \( G_N \).

Equations 16 and 17 are equal if
\[ G = \alpha_g \frac{\hbar c}{m_g^2} . \]  
(18)

According to Eq. 18, knowing that \( \hbar \) and \( c \) are exact constants per definition [19], the lucernic gravitational constant can vary with \( \alpha_g \) and \( m_g \). This could explain the variations of the different values of the gravitational constant as observed by [11].

For \( \alpha_g = 1 \) and \( G = G_N \),
\[ G_N = \frac{\hbar c}{m_g^2} . \]  
(19)

Isolating mass in Eq. 19,
\[ m_g = \sqrt{\frac{\hbar c}{G_N}} = m_r , \]  
(20)
where \( m_r = 2.174 \times 10^{-8} \) kg = 1.220890(14) \times 10^{18} \text{ GeV}/c^2 [19] is known as Planck’s mass, and has been originally obtained in literature through dimensional analysis. However, we obtained this mass as a gauge constant for the case \( \alpha_g = 1 \). This can mean that Planck’s mass has the larger coupling fraction \( \alpha_g \) with a radiation \( \hat{g} \) or \( g \) which we interpret that probably pervades the universe. This radiation would have a wavelength related to the de Broglie wavelength of a Planck’s mass. As per Appendix A, this wavelength would be \( \ell_p = 1.616 \times 255(18) \times 10^{-35} \text{ m} \), known as Planck’s wavelength, which corresponds to Planck’s frequency (in the order of 10^{43} \text{ Hz}) [19]. Even though this frequency corresponds to a very high energy, we suppose the baryonic matter interacts with a small fraction of this energy. Therefore, this energy would be very discrete, manifesting itself only as gravity. Maybe a repulsive version of the lucernic interaction with \( \hat{g} \) particles could correspond to dark energy.

We estimated the gravitational coupling fraction \( \alpha_G \) on protons and neutrons for \( G = G_N \) and \( m_g \approx u \) in Eq. 18. Where \( G_N \) is the Newtonian gravitation constant and \( u \) is the atomic mass unit,
\[ \alpha_G = \frac{G_N u^2}{\hbar c} . \]  
(21)

Using the constants adopted by the Committee on Data for Science and Technology (CODATA) [19] as per Appendix A, we obtain a coupling fraction in the order of magnitude of \( \alpha_G \approx 10^{-38} \), which is a very small value, as expected. This order of magnitude is similar to the relative strength between the coupling constants of the gravitational interaction and the strong nuclear interaction. If we admit that the strong force can also be generalized by the formalism presented, some mechanism should make the proton and the neutron reach

\(^1\) The armorial alphabet is a popular alphabet in northeast of Brazil created by dramatist and fiction writer Ariano Suassuna.
the maximum coupling fraction \( (\alpha_g = 1) \) in the atomic nucleus scale. Thus, the strong force would be a kind of strong gravity, which would be a lucernic phenomenon. However, this interpretation should be better analyzed in future works.

In this work we do not discuss in details about the strong force, atomic models and dark energy. However, it was necessary to give a possible interpretation for this hypothetical radiation \( \gamma \) in complement to the theoretical development presented.

However, if we admit that a fraction of the coupling is actually equal to the gravitational relative strength, and if the gauge mass were equal to the atomic mass unit, we could use Eq. 18 to obtain

\[
G_N = \alpha_g \frac{\hbar c}{\mu g} . \tag{22}
\]

Thus, we write the newtonian gravitional constant in terms of relative strength between strong nuclear and gravitational interactions, from the reduced Planck’s constant, from the speed of light and from the atomic mass unit.

2.3 Coulomb Electrostatic Manifestations

Consider now that bodies \( A \) and \( B \) from Fig. 2 are sources of electric charge. From the Third Law of Thermodynamics and from the uncertainty principle, bodies \( A \) and \( B \) cannot be rigorously static. Every body tends to present some type of vibration. Thus, the electrostatic field would be just a macroscopic approximation which neglects the intrinsic vibration of the bodies. This equivalent electrostatic field is classically defined as

\[
\vec{E} = \frac{\vec{F}}{q_g} , \tag{23}
\]

where \( q_g \) is an arbitrary electric charge.

Rigorously, all charges tend to keep in a vibration state which is capable of generating guardions, in this case, represented by photons. In the Lucernic Theory interpretation, the force derived from the collision with guardions is given by Eq. 9.

Substituting Eq. 9 into Eq. 23,

\[
\vec{E} = \frac{\alpha_g \hbar c}{\lambda^2 q_g} \hat{r} , \tag{24}
\]

The flux of this field generated by a liquid charge \((Q = Nq_g)\) inside of a gaussian spherical surface with area \((4\pi \lambda^2)\) would be given by

\[
\oint_S \vec{E} \cdot d\vec{S} = 4\pi \frac{\alpha_g \hbar c}{q_g} Q , \tag{25}
\]

As the flux does not depend on the shape or the size of the surface, Eq. 25 is valid for any gaussian surface.

In its integral form, Gauss’s Law is given by

\[
\int_S \vec{E} \cdot d\vec{S} = \frac{Q}{\epsilon} . \tag{26}
\]

Comparing Eqs. 25 and 26, we have

\[
4\pi \frac{\alpha_g \hbar c}{q_g^2} = \frac{1}{\epsilon} . \tag{27}
\]

Isolating \( q_g \),

\[
q_g = 4\pi \epsilon \alpha_g \hbar c . \tag{28}
\]

For \( \alpha_g = 1 \) and \( \epsilon = \epsilon_o \) in free space we obtain Planck’s charge given in literature by

\[
q_g = \sqrt{4\pi \epsilon_o \hbar c} = q_{\nu} , \tag{29}
\]

where \( q_{\nu} = 1.875546 \times 10^{-18} \) C.

The elementary charge can be obtained for a coupling fraction equal to fine structure constant \((\alpha_{EM})\) i.e.

\[
e = \sqrt{4\pi \epsilon_o \alpha_{EM} \hbar c} . \tag{30}
\]

We can confirm this, isolating \( \alpha_{EM} \) in Eq. 30.

\[
\alpha_{EM} = \frac{e^2}{4\pi \epsilon_o \hbar c} . \tag{31}
\]

Thereby, the fraction \( \alpha_g \) between the \( \gamma \) radiation and the elementary charge is equal to the fine structure constant \((\alpha_{EM} \approx 1/137)\).

From Planck’s mass and Planck’s charge, it is possible to dedude other Planck units as per Appendix A.

3 Decomposition of the Field Equation

The weak equivalence principle of GR establishes that gravity’s acceleration is a consequence of the time-space warp due to a concentration of energy. This warp is described by Einstein’s field equation [12] as

\[
G_{\mu\nu} = \frac{8\pi G_N}{c^4} T_{\mu\nu} , \tag{32}
\]

where \( G_{\mu\nu} \) is Einstein’s tensor, \( G_N \) is Newton’s gravitational constant, \( c \) is the speed of light in the classical vacuum and \( T_{\mu\nu} \) is the stress-energy tensor [12]. For a perfect dust, the stress-energy tensor can be given in Cartesian coordinates as [20]

\[
T_{\mu\nu} = \rho u_{\mu} u_{\nu} , \tag{33}
\]

where \( \rho \) is the total rest mass density and \( u_{\mu} \) or \( u_{\nu} \) are the four-velocity.
In Principle 2, Lucernic Theory proposes that the equivalence between force and time-space warp is a reciprocal relation. Thus, we suppose that time-space warp could be equivalent to a force derived from the collision with guardians or dark guardians.

In order to promote a lucernic description of gravity in accordance with General Relativity, we shall decompose Einstein’s field equation, as it will be described in Sec. 3.1.

3.1 Lucernic Field Equation

Newton’s Gravitational Constant in Eq. 32 can be described in terms of reduced Planck constant $h$, speed of light $c$ and Planck mass $m_p$, as in Eq. 19. Then, substituting Eq. 19 in Eq. 32, we obtain

$$G_{\mu\nu} = \frac{8\pi hc}{c^4 m_p^2} T_{\mu\nu} .$$

It can be observed that the denominator in Eq. 34 is equal to Planck’s Energy squared $(E_P^2)$ presented in Equation 62 from Appendix A.5, i.e.

$$G_{\mu\nu} = \frac{8\pi hc}{E_P^2} T_{\mu\nu} .$$

Multiplying Eq. 35 by $E_P^2$, and knowing that $h = \frac{\hbar}{2\pi}$,

$$G_{\mu\nu} \frac{E_P}{2} = hc \frac{T_{\mu\nu}}{E_P/2} .$$

The product of Einstein’s tensor with half of Planck’s energy will be defined as

$$\mathcal{E}_{\mu\nu} \equiv G_{\mu\nu} \frac{E_P}{2} \text{ or }$$

$$\mathcal{E}_{\mu\nu} \equiv R_{\mu\nu} \frac{E_P}{2} - \frac{1}{2} g_{\mu\nu} R \frac{E_P}{2},$$

where $\mathcal{E}_{\mu\nu}$ (“R” armorial) will be denominated radiant tensor, and the dimensional unit of its elements is energy per area $(J.m^{-2})$. $R_{\mu\nu}$ is Ricci’s tensor, and $R$ is Ricci’s scalar.

The ratio of the stress-energy tensor by half of Planck’s energy will be defined as concentration/refraction tensor, $\mathcal{J}_{\mu\nu}$ ("N" armorial), i.e.

$$\mathcal{J}_{\mu\nu} \equiv \frac{T_{\mu\nu}}{E_P/2} .$$

Substituting Eqs. 33 and 62 in 39,

$$\mathcal{J}_{\mu\nu} = \frac{\rho u_\mu u_\nu}{m_p c^2/2} .$$

We define a concentration $\rho$ obtained by the ration of the mass density $\rho = \rho(m)$ by half of Planck’s mass, that is

$$\rho = \frac{\rho(m)}{m_p/2} .$$

We can also describe the concentration $\rho$ in terms of the ration of a charge density $\rho = \rho(q)$ by half of Planck’s mass, i.e.

$$\rho = \frac{\rho(q)}{q_r/2} .$$

The symbol $\rho$ can be written in TEX using the command $\\rho \\cdot \\rho$, and it originates in the agglutination of the letters $\rho$ and $\rho$. This symbol was chosen because the letter $\rho$ is frequently associated to the number of particles per unit of volume or refraction index. The letter $\rho$ is frequently used to represent the density of particles in movement, as it is the case of electric current density.

The relativistic fraction will be defined as the ratio between the speed of light $c$ by the four-velocity component $u_\mu$ or $u_\nu$ of the particles, that is

$$n_\mu \equiv \frac{c}{u_\mu} ,$$

A second order relativistic fraction tensor will be defined as

$$n_{\mu\nu} \equiv n_\mu n_\nu = \frac{c^2}{u_\mu u_\nu} .

For \mu and \nu equal to 0, u_\mu = u_\nu = c and for \nu and \mu more than 0, u_1, u_2 and u_3 represent the four-velocity components. Thereby, the second order relativistic fraction tensor is expressed as

$$n_{\mu\nu} = \left( \begin{array}{ccc}
1 & n_1 & n_2 \\
1 & 1 & n_3 \\
n_2 & n_1 & 1 \\
n_3 & n_1 & n_2 \\
n_2 & n_1 & n_3 \\
n_3 & n_1 & n_2
\end{array} \right).$$

Substituting Eq. 44 in Eq. 40, the tensor $\mathcal{J}_{\mu\nu}$ can be expressed as

$$\mathcal{J}_{\mu\nu} = \rho / n_{\mu\nu} .$$

As $n_{\mu\nu}$ is dimensionless and $\rho$ has dimension $m^{-3}$, the elements of $\mathcal{J}_{\mu\nu}$ have dimensional unit of concentration.

Substituting Eqs. 37 and 39 into Eq. 36 we obtain the Lucernic Field Equation expressed as

$$\mathcal{E}_{\mu\nu} = hc \mathcal{J}_{\mu\nu} .$$

Eq. 47 is expressed in terms of the main constants of Quantum Mechanics (Planck’s constant, $\hbar = 6.626 \times 10^{-34}$ J.s) [19] and of the Relativity Theory...
(speed of light in vacuum, \( c = 299 792 458 \) m/s) [19]. From the Lucernic Field Equation it is possible to obtain Einstein’s Field Equation through the reverse process. Thus, Eq. 47 generalizes all gravitational phenomena described by the works of Newton and Einstein. It is possible that many of the phenomena in Physics can be described by Eq. 47, although this analysis is beyond the scope of this work, which is restricted to the lucernic behavior of gravity.

4 Final considerations

Given that the gravitational field is so similar to the electromagnetic field, the former must present some dependency on the medium, even if very slightly. This would explain why the gravitational constant has such a high uncertainty and also why different methods for measuring \( G \) produce values that are different even outside the error margins.

The presented approach for gravitation as a lucernic phenomenon implies in some predictions that can be tested.

1 According to the General Relativity Theory, variations of the gravitational field can make atomic clocks lose synchronism. However, if the possible intermediate particles of gravitational interaction exist and are similar to photons, we speculate by Lucernic Theory that the photons can also interfere in the relativity of time. Thus, if an atomic watch without electromagnetic shielding is submitted to some electromagnetic, maybe some loss of synchronism in relation to an atomic clock not affected by radiation will occur. From an optimistic perspective to the Lucernic Theory, maybe radio waves can already cause some lagging or leading effect on atomic clocks. However, the effect of x-rays and of gamma radiation also must be verified.

2 The electromagnetic theory points out that the electrostatic field inside of conductors tend to be null or very attenuated. If the gravitational field is intermediated by particles similar to photons, it is possible that some material in the universe is capable of at least attenuating the gravitational field. We propose to analyze if superconductor materials could have this property. Thus, we propose to verify if a loss of synchronism between an atomic clock inside and another one outside of a superconductor shielding.

If implications 1 and 2 are observed experimentally, it would be possible to enhance Lucernic Theory, bringing a new field of study concerned with the comprehension of nature, specially with gravity. Otherwise, Lucernic Theory should be modified or disproved.

5 Conclusions

In this work, we proposed a line of scientific thought which aims to describe natural phenomena in terms of the properties of light. This line of thought was named Lucernics. However, in this specific work, we aimed to describe the lucernic behavior of gravity by describing it as a force derived from the collision of virtual particles named guardians. A guardian is thereby a generic term to describe a boson which has no mass and is stable to represent the intermediate particles of electromagnetism and possibly gravitation. We also coined the term dark guardian to represent a signal emitted by a massive body while absorbing a guardian. A dark guardian works as a particle that carries a momentum in the direction opposite to its displacement.

We defined a lucernic phenomenon as a phenomenon partially described in terms of the properties of light. We also defined a lucernic interaction as an equivalent force which is derived from the collision with guardians.

We raised four observations that promote a lucernic description of gravity: Observation 1, the gravitational field, like light, is able to travel large distances across the universe; Observation 2, the gravitational field intensity, like the intensity of light, is inversely proportional to the squared distance from punctual or spherically symmetric sources; Observation 3, perturbations in the gravitational field propagate themselves at the speed of light; Observation 4, a ray of light can be deflected by a gravitational field.

We also present some problems regarding the description of gravity as a lucernic phenomenon. Problem 1: two luminous masses tend to repeal each other due to the collision of photons, and not to attract each other; Problem 2: the gravitational field apparently is not attenuated by any medium, while the intensity of light depends on the properties of the medium; Problem 3: the gravitational relative strength is approximately 36 orders of magnitude weaker than electromagnetic relative strength; Problem 4: the most accepted theory to describe gravitation is General Relativity, which agrees with a variety of experiments and waives the participation of intermediate particles.

In order to deal with the aforementioned problems, we made some assumptions. To deal with Problem 1, we assumed Proposition 1, which stated that every massive body is considered a sink of guardians and consequently an isotropic source of dark guardians. Thereby, from a lucernic point of view, two bodies attract each other because they absorb energy (guardians) from time-space like two lose ends of hoses draining water from a reservoir. We list some speculative possibilities of what happens to the energy absorbed by a massive body: a) part
of this energy can be converted in nuclear potential energy; b) part of the energy can be reemitted to the medium in larger wavelengths in the form of thermal radiation; c) part of the energy can be reutilized by the electrons for maintaining themselves in their orbitals, promoting stability to the atom; d) part of the energy can be converted into heavy bosons of weak interaction; e) part of the energy can be converted in dark matter and/or dark energy, and f) part of the energy can be dissipated in extra dimensions. We did not verify these speculative possibilities and they shall be analysed in detail in future works.

To deal with Problems 2 and 3, we assume Proposition 2, in which we consider that only a small fraction of the momentum carried by dark guardions is transmitted to neutrons and protons. Thus, most of the dark guardions is crosses matter easily and only a small fraction of the momentum per time unit is transmitted, producing the effect of a small force equivalent to gravitational force. This would imply that the gravitational field should present a small dependency on the properties of the medium. This is consistent with the different measurements for the gravitational constant presented in literature.

As for Problem 4, in order to elaborate a description of the lucernic behaviour of gravity consistent with General Relativity, we assumed Principle 2. This principle emphasizes that the weak equivalence principle is reciprocal, i.e., a force could be though of as a time-space warp and a time-space warp could be though of as a force. With this assumption, we derived a force that originates from the collision with guardions and compared it to Newton’s gravitational force and with Coulomb’s electrostatic force.

While comparing lucernic interaction with newtonian gravitational force was obtained Planck masses as gauge constants. While comparing lucernic interaction with coulombian electrostatic force, we obtained Planck’s charge as gauge constant. Thus, two Planck units were obtained through the theoretic foundations proposed in this work. This represents an alternative theoretical method for the dimensional analysis used in literature to obtain Planck’s mass and charge. We also deduced the gravitational and electromagnetic relative strength.

We also deduced an expression for the Newtonian gravitational constant in terms of gravitational relative strength ($\alpha_G$), of Planck’s reduced constant ($\hbar$), of the speed of light in the classic vacuum ($c$) and of the atomic mass unit ($u$).

In accordance with the Principle of Reciprocity (Principle 2), once that the Newtonian gravitational constant can be decomposed, we proposed to also decompose Einstein’s Field Equation in order to obtain the Lucernic Field Equation. This equation is capable of generalizing Einstein’s Field Equation by reversing the composition process, but other implications of this equation to describe other phenomena in Physics were not discussed in this work, which is limited to gravitational phenomena.

We identified two possible implications of the proposed theory which are not predicted by Electromagnetism, General Relativity or Quantum Mechanics. If the gravitational field can alter the timing in atomic clocks, Lucernics predicts that maybe electromagnetic radiation could also cause loss of synchronism in such clocks. Another prediction from Lucernics is that, just like electromagnetic waves can be blocked by conductor materials, if the gravitational field is intermediated by photon-like particles, it is possible that the gravitational field can be at least attenuated by superconductor materials. These possibilities should be investigated through experimental procedures.

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Conflict of interest

The authors declare that they have no conflict of interest.

Appendix

A Planck Units Identities

In this appendix we present some of the identities related to Planck units. The values of the constants used are based on the Committee on Data for Science and Technology (CODATA) [19]. Table 1 presents the constants used and its respective values and units according to the International System of Units (SI). The constants used were Planck’s reduced constant ($\hbar$), the speed of light in vacuum ($c$), the Newtonian gravitational constant ($G_N$) and the atomic mass unit ($u$).

A.1 Electromagnetic coupling constant

The fine-structure constant characterizes the magnitude of the electromagnetic interaction. It is also known as the fine-
Table 1 Constants used according to CODATA [19]

| Constant | Value                     | unit       |
|----------|---------------------------|------------|
| $\hbar$  | $1.054 571 817 \times 10^{-34}$ | J s        |
| $c$      | 299 792 458               | m s$^{-1}$ |
| $G_N$    | $6.674 30(15) \times 10^{-11}$ | m$^3$kg$^{-1}$s$^{-2}$ |
| $u$      | $1.660 539 066(50) \times 10^{-27}$ | kg        |

structure constant or electromagnetic relative strength. It is expressed as

$$\alpha_{EM} = \frac{e^2}{4\pi \epsilon_0 \hbar c}.$$  

The denominator in Eq. 48 is equal to Planck’s charge squared, according to Eq. 29. Thereby:

$$\alpha_{EM} = \frac{e^2}{\lambda_c^2}.$$  

This results in $\alpha = 7.297 352 5693(11) \times 10^{-3}$ [19].

A.2 Gravitational coupling constant

Similar to electromagnetic coupling constant there is the gravitational coupling constant. We can obtain this constant equation like Eq. 49 using Lucernic Theory combining the Eqs. 19 and 21, i.e.

$$\alpha_G = \frac{u^2}{m_p^2}.$$  

In Sec. 2.1 we identified the gravitational coupling constant as being the gravitational relative strength, which is the ratio of the gravitational force per strong nuclear force between two protons.

A.3 Schwarzschild-Planck radius

Planck’s mass is the mass for which both Quantum Mechanics and General Relativity are relevant. From Eq. 20 we can write

$$m_p^2 = \frac{\hbar c}{G_N},$$  

$$m_p G_N = \frac{\hbar c}{m_p}.$$  

Multiplying Eq. 52 by $2/c^2$,

$$2G_N m_p = \frac{2\hbar}{m_p c},$$  

$$r_{S,P} = \frac{\lambda_{c,P}}{\pi} = 2\lambda_{c,P},$$

where $r_{S,P}$ is the Schwarzschild’s radius for Planck’s mass, $\lambda_c$ is the Compton wavelength associated to Planck’s mass and

$$\lambda_{c,P} \equiv \frac{\lambda_c}{2\pi} = \frac{\hbar}{m_pc}$$

is the reduced Compton wavelength for this same Planck’s mass.

A.4 Planck’s wavelength

Planck’s wavelength ($\ell_p$) is equal the reduced Compton wavelength given by Eq. 55, which corresponds to half of Schwarzschild’s radius, i.e.

$$\ell_p = \frac{\lambda_{c,P}}{2},$$  

$$\ell_p = \frac{\hbar}{m_p c}.$$  

Substituting Eq. 20 into 57,

$$\ell_p = \sqrt{\frac{\hbar G}{c}}.$$  

Then, we obtain Planck’s wavelength found in literature:

$$\ell_p = \sqrt{\frac{\hbar G}{c}}.$$  

This results in $\ell_p = 1.616 255(18) \times 10^{-35}$ m [19].

From Equations 59, we can write the Newtonian gravitational constant in terms of Planck’s wavelength, Planck’s constant and the speed of light as

$$G_N = \frac{c^3}{\ell^2_p}.$$  

We can also write the Newtonian gravitational constant in terms of Planck’s mass and of Planck’s wavelength. For that we divide Eq. 58 by Eq. 20 and isolate $G_N$, obtaining:

$$G_N = \frac{c^2}{m_p \ell_p}.$$  

A.5 Planck’s energy

Planck’s energy can be defined as the rest energy of Planck’s mass, that is

$$E_p \equiv m_p c^2.$$  

Which results in $E_p = 1.220 890(14) \times 10^{19}$ GeV [19].

Planck’s energy can also be described in terms of Planck’s charge as

$$E_p \equiv q_p \phi_p,$$  

and of Planck’s frequency or wavelength

$$E_p = \hbar f_p = \frac{\hbar c}{\ell_p}.$$  

A.6 Planck’s electric potential

Comparing Eqs. 62 and 63 we realise that the speed of light squared relates to Planck’s electric potential in the same way as the mass relates to the charge. Thus, we obtain Planck’s potential ($\phi_p$) as

$$\phi_p \equiv \frac{m_p c^2}{q_p}.$$
A.7 Planck’s Frequency

Isolating frequency in Eq. 64:

\[ f_p = \frac{E_p}{h}, \tag{66} \]
\[ f_p = \frac{m_p c^2}{h}, \tag{67} \]
\[ f_p = \frac{c}{\ell_p}. \tag{68} \]

This frequency would be in the order of magnitude of $10^{43}$ Hz.

A.8 Planck’s Time

Planck’s time is defined as the time light takes to travel a distance equal to Planck’s wavelength in vacuum, that is:

\[ t_P = \frac{\ell_p}{c}, \tag{69} \]
\[ t_P = \frac{h}{m_p c^2}, \tag{70} \]
\[ t_P = \frac{h}{E_p}, \tag{71} \]
\[ t_P = \frac{1}{f_p}. \tag{72} \]

That results in $t_P = 5.391 \times 10^{-44}$ s [19].

The reader should note that Planck’s time is equal to the inverse of Planck’s frequency.

A.9 Planck’s Power

Planck’s power is defined as the ratio between Planck’s energy and Planck’s time, that is

\[ P_p = \frac{E_p}{t_p}. \tag{73} \]

Substituting Eqs. 62 and 71 into Eq. 73, we obtain Planck’s power in terms of speed of light and Newton’s gravitational constant.

\[ P_p = \frac{(m_p c^2)^2}{h}, \tag{74} \]
\[ P_p = \frac{\left(\frac{4\pi}{h c^2}\right)^2}{h}, \tag{75} \]
\[ P_p = \frac{c^5}{G_N}. \tag{76} \]

That gives us $P_p \approx 3.62831 \times 10^{32}$ W. and is consistent with [21].

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Figures

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Figure 1

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Figure 2

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