A fast and effective MIP-based heuristic for a selective and periodic inventory routing problem in reverse logistics

Leopoldo E. Cárdenas-Barrón * Rafael A. Melo †

April 10, 2020

Abstract

We consider an NP-hard selective and periodic inventory routing problem (SPIRP) in a waste vegetable oil collection environment. This SPIRP arises in the context of reverse logistics where a biodiesel company has daily requirements of oil to be used as raw material in its production process. These requirements can be fulfilled by using the available inventory, collecting waste vegetable oil or purchasing virgin oil. The problem consists in determining a period (cyclic) planning for the collection and purchasing of oil such that the total collection, inventory and purchasing costs are minimized, while meeting the company’s oil requirements and all the operational constraints. We propose a MIP-based heuristic which solves a relaxed model without routing, constructs routes taking into account the relaxation’s solution and then improves these routes by solving the capacitated vehicle routing problem associated to each period. Following this approach, an a posteriori performance guarantee is ensured, as the approach provides both a lower bound and a feasible solution. The performed computational experiments show that the MIP-based heuristic is very fast and effective as it is able to encounter near optimal solutions with low gaps within seconds, improving several of the best known results using just a fraction of the time spent by a state-of-the-art heuristic. A remarkable fact is that the proposed MIP-based heuristic improves over the best known results for all the large instances available in the literature.

Keywords: reverse logistics; inventory routing; mixed integer programming; heuristics; sustainability.

1 Introduction

Nowadays, several industries have an increasing concern about the implementation of green actions in order to collaborate with the sustainability of the environment. For this reason, Lin, Choy, Ho, Chung, and Lam (2014) argued that green logistics is a big challenge that all companies must consider in their agendas. Reverse logistics (Dekker, Fleischmann, Inderfurth, & van Wassenhove, 2013) is an important area of study that deals with sustainable supply chains, as it includes the application of methods which take into consideration, among other characteristics, the reuse of recyclable material.

The inventory routing problem (IRP) is a variant of the vehicle routing problem (VRP) which integrates inventory and vehicle routing decisions in a sole formulation. It is important to mention that the IRP models numerous distribution and collection situations that occur in practice. Good surveys were presented in Andersson, Hoff, Christiansen, Hasle, and Løkketangen (2010) and Coelho, Cordeau, and Laporte (2014) with the aim of having a good comprehension of the IRP problem. On the one hand, Andersson et al. (2010) dealt with the IRP’s applications. On the other hand, Coelho et al. (2014) studied IRP methodological aspects. Coelho and Laporte (2013) solved the following three variants of the IRP by applying a branch-and-cut approach:

*Department of Industrial and Systems Engineering, School of Engineering and Sciences, Tecnológico de Monterrey, E. Garza Sada 2501 Sur, C.P. 64849, Monterrey, Nuevo León, Mexico. (lecarden@tec.mx)

†Universidade Federal da Bahia, Departamento de Ciência da Computação, Computational Intelligence and Optimization Research Lab (CInO), Salvador, Brazil. (melo@dcc.ufba.br)
algorithm: the multi-vehicle IRP considering both homogeneous and heterogeneous fleet, the IRP with trans-
shipment options, and the IRP with added consistency features. In a subsequent paper, Coelho and Laporte
(2014) built an exact mathematical formulation by implementing valid inequalities for some types of IRPs.
Mjirda, Jarboui, Macedo, Hanafi, and Mladenovic (2014) considered a multi-product IRP and tackled it us-
ing a variable neighborhood search. Soysal, Bloemhof-Ruwaard, Haijema, and van der Vorst (2015) formu-
lated a multi-period IRP for perishable products considering that demand is uncertain and the work of
Soysal, Bloemhof-Ruwaard, Haijema, and van der Vorst (2018) introduced a green IRP for perishable items.
Cordeau, Lagana, Musmanno, and Vocaturo (2017) considered the multi-product IRP and developed a three-
stage heuristic algorithm taking into account a decomposition of the seller’s decision process. Raa (2015)
proposed a family of cyclic IRP. The aforementioned papers are related to forward logistics. However, some
authors also considered IRPs in reverse logistics. In this direction, Aksen, Kaya, Salman, and Akça (2012)
formulated an IRP for a reverse logistics situation where a biodiesel company requires oil to be used as raw
material, which can be obtained as both virgin oil and waste vegetable oil, in its production process. Thus,
the collection of waste vegetable oil is a significant reverse operation to be performed with the aims of having
economic gains and improving environmental sustainability.

In fact, Aksen et al. (2012) proposed a selective and periodic inventory routing problem (SPIRP) for
a waste vegetable oil collection, which is the problem we consider in our work. In a subsequent pa-
per, Aksen, Kaya, Salman, and Tüncel (2014) presented an adaptive large neighborhood search algorithm
(ALNS) to solve the SPIRP. Recently, Cárdenas-Barrón, González-Velarde, Treviño-Garza, and Garza-Nuño-
éz (2019) developed a reduce and optimize approach (ROA) heuristic for this SPIRP. The ROA defines a reduced
feasible region based on the original problem to be optimized. The ROA heuristic was able to outperform
the ALNS of Aksen et al. (2014) for most of the benchmark instances of the SPIRP, at the expense of
computational time.

The main contribution of this paper is the proposal of a fast and effective MIP-based heuristic for
the SPIRP. The proposed approach can provide near-optimal solutions whose optimality gaps are close to
those obtained with a state-of-the-art approach, which consumes high computational time. As a matter
of fact, computational experiments have proven the effectiveness of our approach, as new best results were
encountered for several of the considered instances. To the best of our knowledge, this was a main open
research area for the considered SPIRP.

The remainder of this paper is structured as follows. Section 2 formally defines and describes a mathemat-
ical formulation of the SPIRP. Section 3 presents a new MIP-based heuristic for the SPIRP which basically
solves a relaxation without routing variables and constraints, builds routes based on the relaxation’s solution
and then improves the routes using an approach for the capacitated vehicle routing problem (CVRP). Section
4 presents the experiments carried out to evaluate the performance of the proposed MIP-based heuristic.
Finally, section 5 gives some concluding remarks and future research avenues.

# 2 Problem definition and mixed integer programming formulation

The selective and periodic inventory routing problem (SPIRP) can be formally defined as follows. There
exist a set \( IC = \{1, \ldots, n\} \) of waste oil collection points (collection nodes) and one biodiesel production
facility (depot node). The collection network can be seen as a complete directed graph in which there is an
arc \((i, j)\) between every pair of nodes with an associated distance \(d_{ij}\). A fixed cyclic planning horizon is
defined with a predetermined set of periods \( T = \{1, \ldots, \tau\} \) in each cycle. At each collection node \( i \in IC\),
a waste oil accumulation rate \( a_{it}\) (in liters) at period \( t \in T\) occurs. If a collection vehicle visits a collection
node \( i \in IC\) at period \( t \in T\), then the whole waste oil stored up in period \( t\) must be collected. This means
that partial collection is not permitted. The biodiesel company requires \( r_t\) liters of oil for each period \( t \in T\)
of the planning horizon and needs to determine, for each period, the amount to of virgin oil to be purchased
as well as the set of collection routes taking into consideration its oil requirements and the constraint that
only one vehicle must be allocated to each route. Additional characteristics of the SPIRP are as follows.
There is a fixed capacity \( Q\) for each collection vehicle. The oil necessities of the biodiesel company can be
covered from collected waste oil, purchased virgin oil, inventory on hand, or with a combination of these
three sources. The costs involved in the SPIRP are: traveling cost $c$ per unit distance traveled, operating cost $v$ per vehicle per period, holding cost $h$ per liter of waste oil per period, and purchase cost $p$ per liter of virgin vegetable oil. The main goal is to decide how much virgin vegetable oil to buy and how much waste vegetable oil to gather from a set of collection points. Moreover, it is necessary to take decisions related to which collection nodes to include in the collection program, how many vehicles are needed and their specific routes to define a periodic collection schedule that must be repeated in each cycle in such a manner that the total cost is minimized subject to the production requirements and vehicle capacity constraints.

2.1 Mixed integer programming formulation

In this section, we describe the formulation proposed in Aksen et al. (2012). Let the index sets be:

- $I = \{0, 1, \ldots, n\}$;
- $IC = \{1, \ldots, n\}$;
- $T = \{1, \ldots, \tau\}$.

Consider the parameters:

- $c$: traveling cost per unit distance;
- $d_{ij}$: distance from node $i$ to node $j$;
- $a_{it}$: amount of waste oil accumulated at node $i$ in period $t$;
- $r_t$: oil requirement in period $t$;
- $h$: per liter inventory holding cost at the depot;
- $v$: per vehicle fixed operating cost;
- $p$: per liter purchase cost of virgin vegetable oil;
- $Q$: vehicle capacity in liters.

Define the decision variables:

- $X_{ijt}$: binary variable which indicates whether arc $(i, j)$ is traversed by a vehicle in period $t$;
- $Y_{it}$: binary variable which indicates whether node $i$ is visited in period $t$;
- $Z_i$: binary variable which indicates whether node $i$ is visited at all during the planning cycle;
- $F_{ijt}$: the amount of waste oil traversing arc $(i, j)$ in period $t$;
- $W_{it}$: the amount of waste oil collected from node $i$ in period $t$;
- $I_{it}$: the amount of wast oil held as stock in node $i$ at the end of period $t$;
- $I_{i0}$: amount of waste oil in storage at the beginning of the planning cycle;
- $S_t$: amount of virgin oil purchased in period $t$. 

3
The problem can thus be formulated as the following mixed integer linear program:

\[
    z_{IR} = \min \sum_{i \in I} \sum_{j \in I} \sum_{t \in T} d_{ij} x_{ijt} + v \sum_{i \in IC} \sum_{t \in T} x_{0it} + h \sum_{t \in T} I_{0t} + p \sum_{t \in T} S_t \tag{1}
\]

\[(IR) \sum_{j \in I} F_{ijt} - \sum_{j \neq i} F_{ijt} = W_{it}, \quad \text{for } i \in IC, \ t \in T, \tag{2}\]

\[F_{ijt} \leq (Q - a_{ijt}) x_{ijt}, \quad \text{for } i \in I, \ j \in I, \ t \in T, \ i \neq j, \tag{3}\]

\[F_{ijt} \leq Q - W_{jt}, \quad \text{for } i \in I, \ j \in IC, \ t \in T, \ i \neq j, \tag{4}\]

\[F_{ijt} \geq W_{it} - A_i(1 - x_{ijt}), \quad \text{for } i \in IC, \ j \in I, \ t \in T, \ i \neq j, \tag{5}\]

\[
    \sum_{j \in I} x_{ijt} = Y_{it}, \quad \text{for } i \in IC, \ t \in T, \tag{6}\]

\[
    \sum_{j \neq i} x_{ijt} = Y_{it}, \quad \text{for } i \in IC, \ t \in T, \tag{7}\]

\[
    \sum_{i \in IC} x_{0it} = \sum_{i \in IC} X_{0it}, \quad \text{for } t \in T, \tag{8}\]

\[W_{it} \leq A_i Y_{it}, \quad \text{for } i \in IC, \ t \in T, \tag{9}\]

\[I_{it} \leq A_i(1 - Y_{it}), \quad \text{for } i \in IC, \ t \in T, \tag{10}\]

\[I_{it} = I_{it-1} + a_{it} Z_i - W_{it}, \quad \text{for } i \in IC, \ t \in T, \tag{11}\]

\[I_{i0} = I_{iT}, \quad \text{for } i \in I, \tag{12}\]

\[I_{0t} = I_{0t-1} + \sum_{i \in IC} W_{it} + S_t - r_t, \quad \text{for } t \in T, \tag{13}\]

\[Z_i \leq \sum_{t \in T} Y_{it}, \quad \text{for } i \in IC, \tag{14}\]

\[Z_i \geq Y_{it}, \quad \text{for } i \in IC, \ t \in T, \tag{15}\]

\[X_{ijt} \in \{0, 1\}, \quad \text{for } i \in I, \ j \in I, \ t \in T, \ i \neq j, \tag{16}\]

\[Y_{it} \in \{0, 1\}, \quad \text{for } i \in IC, \ t \in T, \tag{17}\]

\[Z_i \in \{0, 1\}, \quad \text{for } i \in IC, \tag{18}\]

\[F_{ijt} \geq 0, \quad \text{for } i \in I, \ j \in I, \ t \in T, \ i \neq j, \tag{19}\]

\[W_{it} \geq 0, \quad \text{for } i \in IC, \ t \in T, \tag{20}\]

\[I_{it} \geq 0, \quad \text{for } i \in I, \ t \in \{0\} \cup T, \tag{21}\]

\[S_t \geq 0, \quad \text{for } t \in T. \tag{22}\]

The objective function (1) minimizes the total traveling, vehicle usage, inventory and purchase costs. Constraints (2) are flow balance constraints for each node \(i \in IC\) and guarantee that \(W_{it}\) liters of oil are collected in the node whenever this amount is nonzero. Constraints (3)- (5) define upper and lower bounds on the flow variables. Constraints (6) and (7) ensure each collection node is visited exactly once whenever it is chosen to be visited. Constraints (8) guarantee that all vehicles leaving the depot in a given period return. Constraints (9) imply that oil can only be collected from node \(i \in IC\) if it is selected to be visited. Constraints (10) enforce that no inventory remains in collection node \(i \in IC\) at the end of a given period if it is visited, i.e., all the oil available in storage must be collected. Constraints (11) are inventory balance constraints for each collection node \(i \in IC\). Constraints (12) ensure that the initial and end inventories are the same. Constraints (13) are inventory balance constraints for the depot. Constraints (14) and (15) set the \(Z_i\) variables to zero if node \(i \in IC\) is never visited and to one otherwise. Constraints (16)-(22) define integrality and nonnegativity requirements on the variables.
3 MIP-based heuristic

In this section, we describe a MIP-based heuristic for the selective and periodic inventory routing problem. The approach consists of three main steps, which are: (a) solving a relaxation without routing of the problem, (b) constructing routes based on the solution of the relaxation, and (c) improving the routes for each period using a MIP-search. The approach follows a similar structure to the method proposed in Montagne, Gamache, and Gendreau (2014) for a related problem, but it differentiates greatly in the way the routes are built and in the improvement of these constructed routes. In what follows, Section 3.1 describes the relaxation without routing. Section 3.2 demonstrates how the routes are constructed based on the solution of the relaxation, using heuristics for the traveling salesman problem (TSP) (Applegate, Bixby, Chvatal, & Cook, 2006). Section 3.3 explains the MIP-search for routing improvements, which is based on an exact approach for the capacitated vehicle routing problem (CVRP) (Laporte, 2009).

3.1 Relaxation without routing

The relaxation without routing described in this section is very similar to the one proposed in Aksen et al. (2014), where the authors have shown that such relaxation could provide very strong bounds in short computational times. Basically, the routing related variables and constraints are removed from formulation IR and are replaced by new variables and constraints which guarantee a lower bound on the total routing cost. Define the additional decision variables

- \( V_t \): number of vehicles needed to transport the collected oil in period \( t \in T \);
- \( R_t \): slack capacity to fulfill vehicle with fractional usage in period \( t \in T \).

The relaxation without routing can thus be formulated as

\[
\begin{align*}
\min c \left[ \sum_{i \in IC} (d_{0i} + d_{it}) \frac{W_{it}}{Q} + \left( \min_{i \in IC} \{d_{0i}\} + \min_{i \in IC} \{d_{it}\} \right) R_t \right] + v \sum_{t \in T} V_t + h \sum_{t \in T} I_{0t} + p \sum_{t \in T} S_t
\end{align*}
\]

\[
(I_{IRR})
\]

\[
\begin{align*}
V_t = \sum_{i \in IC} \frac{W_{it}}{Q} + R_t, & \quad \text{for } t \in T, \\
W_{it} \leq Q Y_{it}, & \quad \text{for } i \in IC, \ t \in T, \\
W_{it} \leq A_i Y_{it}, & \quad \text{for } i \in IC, \ t \in T, \\
I_t \leq A_i (1 - Y_{it}), & \quad \text{for } i \in IC, \ t \in T, \\
I_t = I_{t-1} + a_{it} Z_i - W_{it}, & \quad \text{for } i \in IC, \ t \in T, \\
I_{0t} = I_{0, t-1}, & \quad \text{for } i \in I, \\
I_{0t} = I_{0, t-1} + \sum_{i \in IC} W_{it} + S_t - r_t, & \quad \text{for } i \in IC, \ t \in T, \\
Z_i \leq \sum_{t \in T} Y_{it}, & \quad \text{for } i \in IC, \\
Z_i \geq Y_{it}, & \quad \text{for } i \in IC, \ t \in T \\
Y_{it} \in \{0, 1\}, & \quad \text{for } i \in IC, \ t \in T, \\
Z_i \in \{0, 1\}, & \quad \text{for } i \in IC, \\
V_t \in \{0, 1\}, & \quad \text{for } t \in T, \\
W_{it} \geq 0, & \quad \text{for } i \in IC, \ t \in T, \\
I_t \geq 0, & \quad \text{for } i \in I, \ t \in \{0\} \cup T, \\
S_t \geq 0, & \quad \text{for } t \in T, \\
R_t \in [0, 1], & \quad \text{for } t \in T.
\end{align*}
\]

(27)
The objective function (23) minimizes a lower bound on the routing cost plus the inventory and purchase costs. The lower bound can be interpreted as follows. The first term implies that the fraction of the vehicle used to collect waste oil from a given collection node must at least travel from the depot to that node and return back. The second term ensures that the unused capacity of the vehicle must at least travel to and return from the collection nodes which are the closest to and from the depot. Constraints (24) set the number of vehicles in each period to the sum of fractional vehicles plus the slack capacity. Constraints (25) ensure that the total amount collected in a node in a given period does not exceed the capacity of the vehicle. Constraints (26) and (27) ensure the integrality and nonnegativity requirements on the $V$ and $R$ variables.

Additionally, although not needed in the formulation, we observed that inequalities
\[ \sum_{i \in IC} W_{it} \leq QV_t, \quad \text{for } t \in T, \] (28)
could help reducing the time to solve several relaxations to optimality using the MIP solver.

### 3.2 Routes construction

The proposed heuristic to construct routes attempts to build a solution with as few vehicles as possible, starting from a partial solution obtained from solving the relaxation without routing (IRR) described in section 3.1. The main idea is that the collection nodes to be visited are partitioned into subsets not exceeding the capacity of the vehicles, which are later routed using greedy heuristics for the traveling salesman problem (TSP) (Applegate et al., 2006). The pseudocode of the approach is described in Algorithm 1 and takes as input a partial solution $\mathcal{S}$ obtained from a feasible solution to the relaxation without routing.

Algorithm 1 builds routes for each time period independently in the for each loop of lines 1-12. For each time period $t \in T$, the approach firstly initializes the set $\mathcal{B}$ of subsets of collection nodes in line 2 and the collection nodes to be visited $\mathcal{I}$ in line 3. The while loop of lines 4-7 heuristically partitions the collection nodes to be visited in an attempt to minimize the number of subsets in the partition. While there are collection nodes which are not selected for a subset (line 4), a new subset is constructed with those maximizing the usage of the vehicle capacity by solving a knapsack problem. Consider $\alpha_i$ to be a binary variable representing whether collection node $i$ is selected. This knapsack problem is formulated as
\[
 z_{KN}(\mathcal{I}, \hat{W}) = \max \sum_{i \in \mathcal{I}} \hat{W}_{it} \alpha_i \] (29)
\[
 [ KN(\mathcal{I}, \hat{W}) ] \sum_{i \in \mathcal{I}} \hat{W}_{it} \alpha_i \leq Q, \] (30)
\[
 \alpha_i \in \{0, 1\}, \quad \text{for } i \in \mathcal{I}. \] (31)

The objective function (29) maximizes the volume of the selected collection nodes. Constraints (30) guarantee that the capacity of the vehicle is not exceeded while constraints (31) ensure the integrality of the variables. A subset $\mathcal{B}$ is built by solving this knapsack problem in line 8 using dynamic programming by the auxiliary procedure DP-KNAPSACK $(\mathcal{I}, \hat{W})$. Subset $\mathcal{B}$ is thus inserted into $\mathcal{B}$ in line 9 and the collection nodes in $\mathcal{B}$ are removed from $\mathcal{I}$ in line 10.

Next, in case the partition $\mathcal{B}$ contains more subsets (or parts) than a certain lower bound (line 11), the approach tries to further reduce the number of subsets by exactly solving a bin packing problem (lines 12-13). Considering binary variables $\gamma_b$ to define whether bin $b$ is used and $\beta_{ib}$ to indicate whether collection node
is put into bin \( b \), this bin packing problem is formulated as

\[
    z_{BP(I, \hat{W})} = \min \sum_{b \in B} \gamma_b
    \quad \text{(32)}
\]

\[
    [BP(I, \hat{W})] \sum_{i \in I} \hat{W}_{it} \beta_{ib} \leq Q \gamma_b,
    \quad \text{(33)}
\]

\[
    \gamma_b \geq \gamma_{b'}, \quad \text{for } b < b',
    \quad \text{(34)}
\]

\[
    \beta_{ib} \in \{0, 1\}, \quad \text{for } i \in I, b \in B,
    \quad \text{(35)}
\]

\[
    \gamma_b \in \{0, 1\}, \quad \text{for } b \in B.
    \quad \text{(36)}
\]

The objective function (32) minimizes the number of used bins. Constraints (33) ensure the capacity of each bin is not exceeded. Constraints (34) are symmetry breaking constraints. Finally, constraints (35) and (36) enforce the integrality requirements on the variables.

In the following, routes are built for each subset individually by heuristically tackling the corresponding traveling salesman problem (lines 11-12). This is performed by using fast heuristics for the TSP, namely, nearest neighbor and farthest insertion. These two greedy heuristics are executed and the best obtained tour amongst the two of them is selected to compose the solution.

Finally, the complete solution is returned in line 13.

Algorithm 1: ROUTES-CONSTRUCTION (\( S \))

1. foreach \( t \in T \) do
2. \( B \leftarrow \emptyset \);
3. \( I \leftarrow \{ i \in IC | S.\hat{Y}_{it} = 1 \} \);
4. while \( I \neq \emptyset \) do
5. Construct subset \( B \) by solving DP-KNAPSACK (\( I, S.\hat{W} \));
6. Insert subset \( B \) into \( B \);
7. \( I \leftarrow I \setminus B \);
8. if \( |B| > \left\lceil \frac{\sum_{i \in IC} S.\hat{W}_{it}}{Q} \right\rceil \) then
9. \( I \leftarrow \{ i \in IC | S.\hat{Y}_{it} = 1 \} \);
10. \( B \leftarrow \text{partition constructed from the solution of the bin packing } BP (I, S.\hat{W}) \text{ solved using a MIP solver;} \)
11. foreach \( B \in B \) do
12. Build route for \( S \) in period \( t \) visiting collections in \( B \) using a greedy heuristic for the TSP;
13. return complete feasible solution \( S \);

3.3 MIP-search

The proposed MIP-search is a postprocessing procedure which tries to improve the solution by refining the routes traveling to the collection nodes to be visited in a given period. This is achieved by solving the associated capacitated vehicle routing problems (CVRPs) for each period. In order to achieve this goal, a one-commodity flow mixed integer programming formulation (Gavish & Graves, 1978) is considered for each of these CVRPs. Variables \( x_{ij} \) and \( f_{ij} \), are the single period counterparts of variables \( X_{ijt} \) and \( F_{ijt} \). Such
CVRP is thus formulated as

\[
\begin{align*}
\text{z}_{\text{CVRP}}(t, I_t, \hat{W}, lb_t) &= \min \ c \sum_{i \in I_t} \sum_{j \in I_t, j \neq i} d_{ij} x_{ij} + v \sum_{i \in I_t} x_{0i} \\
[ \text{CVRP}(t, I_t, \hat{W}, lb_t) ] &= \sum_{j \in I_t, j \neq i} f_{ij} - \sum_{j \in I_t, j \neq i} f_{ji} = \hat{W}_{it}, \quad \text{for } i \in I_t, \\
f_{ij} &\leq (Q - W_{jt}) x_{ij}, \quad \text{for } i \in I, j \in I, i \neq j, \\
f_{ij} &\geq \hat{W}_{it} x_{ij}, \quad \text{for } i \in I_t, j \in I, i \neq j, \\
\sum_{j \in I_t, j \neq i} x_{ji} &= 1, \quad \text{for } i \in I_t, \\
\sum_{j \in I_t, j \neq i} x_{ij} &= 1, \quad \text{for } i \in I_t, \\
\sum_{i \in I_t} x_{0i} &= x_{0i}, \\
\sum_{i \in I_t} x_{0i} &\geq lb_t, \\
x_{ij} &\in \{0, 1\}, \quad \text{for } i \in I_t \cup \{0\}, j \in I_t \cup \{0\}, i \neq j, \\
f_{ij} &\geq 0, \quad \text{for } i \in I_t \cup \{0\}, j \in I_t \cup \{0\}, i \neq j.
\end{align*}
\]

The objective function (37) minimizes the total traveling and vehicle usage costs. Constraints (38) are balance constraints for node \(i\) and guarantee \(\hat{W}_{it}\) liters of oil are collected in node \(i\). Constraints (39) and (40) link the \(f\) and \(x\) variables, and also define upper and lower bounds on the flow variables. Constraints (41) and (42) ensure each collection point is visited exactly once. Constraints (43) guarantee that all vehicles leaving the depot must return. Constraints (44) ensures the number of used vehicles is greater or equal than a lower bound \(lb_t\), which is defined by the feasible solution obtained with Algorithm 1. Constraints (45) and (46) impose integrality and nonnegativity requirements on the variables, respectively.

### 3.4 The complete MIP-based heuristic

The pseudocode of the MIP-based heuristic is described in Algorithm 2. Firstly, the relaxation without routing is solved in line 1 using a MIP solver to obtain partial solutions, i.e., without determined routes. In this step, all the solutions encountered by the solver during the search process are stored, and an elite set of solutions containing those which are not more than \(\delta\)% worse than the best one are returned at the end of the execution. The idea behind selecting an elite set of solutions and not simply the best one is to permit variability in the complete constructed solutions. After that, routes are constructed for each of these elite partial solutions in lines 2-3. In the following, a set of elite complete solutions is determined with the \(k\) best complete solutions (line 4). In the following, a set of elite complete solutions is determined with the \(k\) best complete solutions (line 4). The best encountered solution is returned in line 7.

**Algorithm 2: MIP-BASED-HEURISTIC**

1. \(S \leftarrow \) subset of elite partial solutions obtained by solving IRR using a MIP solver;
2. foreach \(S \in S\) do
3. \(\downarrow \) \(S \leftarrow \) complete solution obtained after executing ROUTES-CONSTRUCTION(\(S\));
4. \(S' \leftarrow \) subset of elite complete solutions in \(S\);
5. foreach \(S \in S'\) do
6. \(\downarrow \) \(S \leftarrow \) improved solution after executing the MIP-search over \(S\) for each period \(t \in T\);
7. return \(\arg\min_{S \in S} \{z_I(S)\}\);
4 Computational experiments

This section reports the computational experiments conducted to assess the performance of the proposed MIP-based heuristic. All computational experiments were carried out on a standard Dell Inspiron 15 laptop, running under Ubuntu GNU/Linux, with an Intel(R) Core(TM) i7-7500U CPU @ 2.70GHz processor and 8GB of RAM. The algorithms were coded in Julia v1.4.0, using JuMP v0.18.6. The formulations were solved using Gurobi 8.0.1 with the standard configurations, except the relative optimality tolerance gap which was set to $10^{-6}$. A time limit of 60 seconds was imposed for every execution of the MIP solver.

4.1 Benchmark instances

The considered benchmark instance set is composed of three benchmark sets used in the literature. In all the considered instances, the cyclic planning horizon consists of seven days.

The first benchmark set was introduced in Aksen et al. (2012), containing 36 small instances with 25 collection nodes. There are two types of vehicles, namely: Fiat Doblo Cargo Maxi (Dob) and Fiat Fiorino Cargo (Fio). Dob has a capacity ($Q$) of 920 liters, operating cost ($v$) of 110 monetary units per day and per km traveling cost ($c$) of 0.22 monetary units. Fio has a capacity ($Q$) of 550 liters, operating cost ($v$) of 90 monetary units per day and per km traveling cost ($c$) of 0.19 monetary units. The waste vegetable oil inventory cost ($h$) is 0.02 monetary units per liter per day. Virgin vegetable oil prices ($p$) lie in \{0.25, 0.50, 1.25\} monetary units per liter. Daily accumulation rates are either low (30 liters per day) or high (60 liters per day). Three levels of oil requirement are established for each value of accumulation rate. For accumulation rate 30, waste requirements can be low (600 liters), medium (700 liters) or high (900 liters). For accumulation rate 60, waste requirements can be low (1200 liters), medium (1500 liters) or high (1800 liters).

The second benchmark set was proposed in Aksen et al. (2014), containing 54 small and medium instances based on a real-world case study with 20, 25, 30, 35, 40, 50, 60, 80, and 100 collection nodes. There is a single type of vehicle with capacity ($Q$) of 550 liters. The operating cost of a vehicle ($v$) is 90 monetary units per day while the per km traveling cost ($c$) corresponds to 0.24 monetary units. The waste vegetable oil inventory cost ($h$) is 0.02 monetary units per liter per day, while the virgin oil purchasing price ($p$) lie in \{2.5, 3.5\} monetary units per liter. Three levels of oil requirement are established as low, medium, and high. The instances are identified as ($P_1$n-$P_2$r-$P_3$p), where $P_1$ denotes the number of collection nodes, $P_2$ gives the per day oil requirement and $P_3$ represents the virgin oil purchasing price.

The third benchmark set was introduced in Cárdenas-Barrón et al. (2019), containing 24 large instances with 120, 160, 200, and 300 collection nodes. These instances were generated based on those proposed in Aksen et al. (2014).

4.2 Considered approaches and settings

The following approaches are considered in the comparisons:

- MIP-based heuristic in which the MIP-search is not executed (MH);
- MIP-based heuristic with MIP-search (MH$^+$);
- Adaptive large neighborhood search (ALNS) (Aksen et al., 2014);
- Reduce and optimize approach (ROA) (Cárdenas-Barrón et al., 2019).

For MH and MH$^+$, the subset of elite partial solutions obtained by solving IRR (Algorithm 2, line 1) is composed of those whose values lie within $\delta = 5\%$ of the best encountered one. For MH$^+$, the subset of elite solutions in $S$ (Algorithm 2, line 3) is composed of simply the best solution, i.e. $k = 1$, as the goal was to have an approach which could deliver good quality results within low computational times.

The values presented for ALNS and ROA correspond to those reported in their corresponding works. ALNS was coded in Java, and the computational experiments were executed on a workstation running under...
a 64-bit Windows 7 Professional Service Pack 1 operating system, with an Intel Xeon E5-2643 3.30 GHz Quad-Core processor, 32 GB RAM. The running times of ALNS are reported in our tables. ROA was coded in C++ using Microsoft Visual Studio 2008, and the computational experiments were executed on an HP Elite 8300, running under a 64-bit Windows 7 operating system, with an Intel Core i5-3470M CPU @ 3.30 GHz processor, 8 GB RAM, and CPLEX Interactive Optimizer 12.5.1.0 used to solve the formulations. The running time of each execution of ROA was 36 minutes (2160 seconds), and thus we do not report this information in our tables.

4.3 Results

Table 1 presents the results obtained with MH, MH$^+$ and ROA for the instances from the first benchmark set. The first column identifies the instance. The next six columns report, for MH and MH$^+$, the best solution value, the running time and the optimality gap obtained with each of the approaches, calculated as $100 \times (ub - lb)/ub$, where $ub$ is the encountered heuristic value and $lb$ is the bound obtained by the relaxation without routing. The last two columns give the best solution value and the optimality gap obtained with ROA. We remark that Aksen et al. (2014) did not report results for ALNS on this instance set. The results show that MH could obtain a low average gap (3.3%) within a very short average computational time (19.7 seconds). MH$^+$ could further improve the results obtained by MH, while keeping the average running time below 40.0 seconds. Note that although ROA obtains a lower average gap (1.6%), these results were obtained using a much larger computational time of 2160.0 seconds on average, which represents nearly 110× the average running time of MH. Furthermore, new best results were obtained for 16.7% of the instances.

Tables 2 and 3 show the results obtained with MH, MH$^+$, ALNS and ROA for the second benchmark set. The first column identifies the instance. The next nine columns report, for MH, MH$^+$ and ALNS, the best solution value, the running time and the optimality gap obtained with each of the approaches. The last two columns give the best solution value and the optimality gap obtained with ROA. We remark that there are two lines with a problem in the values reported in (Aksen et al., 2014) for ALNS, which are marked with an ‘∗’, as their reported values are below the bound provided by the relaxation without routing. These tables show the effectiveness of our approach. Table 2 shows that, for the small instances, MH found an average gap of 3.0% within an average time of 16.9 seconds, and MH$^+$ could further reduce this average gap to 2.7% within an average time of 25.9 seconds. Both results improve over that obtained by ALNS, and are very close to the one obtained by ROA which uses a much larger computational time of 2160 seconds. Besides, MH and MH$^+$ improved the result of ALNS for, respectively, 53.3% and 66.7% of the instances. These improvements over the results of ROA were, respectively, 23.3% and 26.7%. Table 3 shows that the improvements achieved by MH and MH$^+$ over ALNS are even more evident, as better solutions were obtained for, respectively, 75.0% and 91.7% of the instances. It is remarkable that the average gap obtained by MH$^+$ is much lower than that of ALNS and is very close to that of ROA, but spending just a fraction of the time on average. Furthermore, new best solutions were encountered for 25.9% of these instances.

Table 4 presents the results obtained with MH, MH$^+$ and ROA for the instances from the third benchmark set. The columns are the same as in Table 1. The results show the impressive results of our heuristics, as they could improve the best known result for all the instances. MH was able to obtain an average gap of 1.6% within 90.9 seconds on average, while MH$^+$ could reduce this gap to 1.4% within 469.6 seconds on average. These average gaps are much lower than the 5.9% average gap obtained by ROA using an average time of 2160 seconds.
### Table 1: Results for MH, MH$^+$ and ROA on instances from the first benchmark set.

| Instances          | MH | MH$^+$ | ROA |
|--------------------|----|--------|-----|
|                    | $z_{MH}$ | $t_{MH}$ | $g_{MH}$ | $z_{MH}^+$ | $t_{MH}^+$ | $g_{MH}^+$ | $z_{ROA}$ | $g_{ROA}$ |
| Fio-30acc-LOW-025  | 867.4 | 14.6   | 1.6  | 866.3 | 15.3 | 1.5 | 865.7 | 1.5  |
| Fio-30acc-MED-025  | 1099.6 | 17.2   | 1.5  | 1098.6 | 18.0 | 1.4 | 1094.6 | 1.1  |
| Fio-30acc-HIGH-025 | 1348.3 | 15.3   | 2.6  | 1345.0 | 22.3 | 2.4 | 1331.5 | 1.4  |
| Fio-60acc-LOW-025  | 1851.7 | 21.8   | 7.8  | 1847.6 | 33.8 | 7.6 | 1748.9 | 2.4  |
| Fio-60acc-MED-025  | 2273.8 | 16.1   | 6.1  | 2258.8 | 31.0 | 5.4 | 2233.8 | 4.4  |
| Fio-60acc-HIGH-025 | 2874.7 | 13.7   | 8.5  | 2869.2 | 17.2 | 8.3 | 2713.3 | 3.0  |
| Dob-30acc-LOW-025  | 701.1 | 15.3   | 1.1  | 699.2 | 16.1 | 0.8 | 701.5 | 1.2  |
| Dob-30acc-MED-025  | 856.7 | 13.9   | 2.1  | 855.1 | 14.8 | 1.9 | 847.4 | 1.1  |
| Dob-30acc-HIGH-025 | 1024.0 | 14.5   | 2.1  | 1022.5 | 15.7 | 1.9 | 1019.0 | 1.6  |
| Dob-60acc-LOW-025  | 1299.7 | 52.8   | 2.1  | 1297.9 | 53.1 | 1.9 | 1289.9 | 2.4  |
| Dob-60acc-MED-025  | 1636.8 | 15.5   | 2.3  | 1632.5 | 19.7 | 2.0 | 1647.1 | 2.9  |
| Dob-60acc-HIGH-025 | 2144.5 | 17.6   | 2.5  | 2140.0 | 21.5 | 2.3 | 2133.2 | 2.0  |
| Fio-30acc-LOW-050  | 907.4 | 31.9   | 1.2  | 907.0 | 32.2 | 1.1 | 902.4 | 0.6  |
| Fio-30acc-MED-050  | 1140.0 | 15.7   | 1.8  | 1135.1 | 16.8 | 1.3 | 1127.9 | 0.7  |
| Fio-30acc-HIGH-050 | 1549.5 | 14.8   | 1.7  | 1546.6 | 16.3 | 1.6 | 1540.2 | 2.4  |
| Fio-60acc-LOW-050  | 998.3 | 85.6   | 2.1  | 995.7 | 134.2 | 12.1 | 989.6 | 3.0  |
| Fio-60acc-MED-050  | 1236.1 | 15.1   | 8.0  | 1232.1 | 146.4 | 7.7 | 1226.2 | 3.9  |
| Fio-60acc-HIGH-050 | 3247.2 | 239.8  | 2.4  | 3243.2 | 239.8 | 2.4 | 3221.8 | 2.8  |
| Dob-30acc-LOW-050  | 975.9 | 18.8   | 1.2  | 970.8 | 15.6 | 1.1 | 971.2 | 0.5  |
| Dob-30acc-MED-050  | 853.9 | 15.4   | 1.8  | 853.1 | 16.2 | 1.7 | 845.5 | 0.8  |
| Dob-30acc-HIGH-050 | 1219.3 | 14.0   | 1.8  | 1219.3 | 14.9 | 1.8 | 1218.0 | 1.6  |
| Dob-60acc-LOW-050  | 1302.0 | 37.8   | 1.3  | 1302.0 | 38.4 | 1.3 | 1302.0 | 4.9  |
| Dob-60acc-MED-050  | 1692.7 | 15.9   | 3.5  | 1687.5 | 31.6 | 3.1 | 1647.7 | 0.9  |
| Dob-60acc-HIGH-050 | 2814.2 | 14.0   | 5.8  | 2814.2 | 14.0 | 5.8 | 2657.0 | 0.2  |
| Fio-30acc-LOW-125  | 911.3 | 33.8   | 1.6  | 907.6 | 34.6 | 1.2 | 902.5 | 0.7  |
| Fio-30acc-MED-125  | 1136.6 | 14.5   | 1.5  | 1132.8 | 16.0 | 1.1 | 1125.8 | 0.5  |
| Fio-30acc-HIGH-125 | 2064.7 | 13.0   | 2.3  | 2054.9 | 15.2 | 1.8 | 2025.4 | 0.4  |
| Fio-60acc-LOW-125  | 1897.7 | 19.1   | 6.6  | 1892.7 | 24.8 | 6.4 | 1781.0 | 0.5  |
| Fio-60acc-MED-125  | 2390.5 | 16.1   | 8.4  | 2382.6 | 165.7 | 8.1 | 2242.5 | 2.4  |
| Fio-60acc-HIGH-125 | 4972.4 | 15.5   | 3.8  | 4967.6 | 16.8 | 3.7 | 4841.2 | 1.2  |
| Dob-30acc-LOW-125  | 707.0 | 15.9   | 1.4  | 706.3 | 16.8 | 1.3 | 703.4 | 0.9  |
| Dob-30acc-MED-125  | 853.7 | 15.3   | 1.8  | 852.8 | 16.0 | 1.7 | 845.6 | 0.9  |
| Dob-30acc-HIGH-125 | 1765.1 | 13.1   | 1.9  | 1764.4 | 13.9 | 1.9 | 1740.3 | 0.6  |
| Dob-60acc-LOW-125  | 1367.6 | 27.8   | 1.0  | 1364.9 | 28.7 | 0.8 | 1361.5 | 0.5  |
| Dob-60acc-MED-125  | 1680.8 | 14.3   | 2.7  | 1675.2 | 15.0 | 2.3 | 1654.4 | 1.1  |
| Dob-60acc-HIGH-125 | 4390.6 | 12.8   | 3.7  | 4383.5 | 14.0 | 3.5 | 4233.2 | 0.1  |

Averages: 19.7, 3.3, 38.1, 3.1, 1.6

| $z_{ROA}$ | $g_{ROA}$ |
|-----------|-----------|
| 16.7      | 16.7      |

## Concluding remarks

In this paper, we tackled the selective and periodic inventory routing problem (SPIRP) for waste vegetable oil collection by applying a fast and effective MIP-based heuristic. This relevant problem in the context of reverse logistics was an open area for research due to the fact that the best performing solution procedure available in the literature consumes a lot of time to provide solutions.

Computational experiments have demonstrated that our newly proposed MIP-based heuristic is very fast and effective when providing near optimal solutions for the considered instances, achieving considerable low gaps within seconds. The approach has the benefit of providing an *a posteriori* quality guarantee, as it contains the resolution of a valid relaxation of the problem in its mechanism. Besides, the MIP-based heuristic was able to obtain high quality solutions, better than those found by a state-of-the-art solution.
Table 2: Results for MH, MH⁺, ALNS and ROA on the small instances from the second benchmark set.

| Instances      | MH  | MH⁺ | ALNS | ROA |
|----------------|-----|-----|------|-----|
|                | \(z_{MH}\) | \(t_{MH}\) | \(gap_{MH}\) | \(z_{MH⁺}\) | \(t_{MH⁺}\) | \(gap_{MH⁺}\) | \(z_{ALNS}\) | \(t_{ALNS}\) | \(gap_{ALNS}\) | \(z_{ROA}\) | \(gap_{ROA}\) |
| 20n-270r-2.5p  | 474.1 | 13.9 | 0.8 | 474.0 | 14.6 | 0.7 | 480.1 | 3.2 | 2.0 | 475.1 | 1.0 |
| 20n-410r-2.5p  | 714.2 | 15.1 | 1.8 | 713.7 | 15.8 | 1.7 | 711.8 | 2.8 | 1.5 | 711.7 | 1.4 |
| 20n-540r-2.5p  | 856.7 | 12.7 | 2.7 | 856.7 | 13.5 | 2.7 | 830.6 | 8.6 | -0.3 | 841.2 | 0.9 |
| 20n-270r-3.5p  | 474.9 | 13.5 | 0.9 | 474.6 | 14.3 | 0.9 | 479.9 | 3.9 | 2.0 | 475.1 | 1.0 |
| 20n-410r-3.5p  | 713.5 | 14.4 | 1.7 | 713.3 | 15.1 | 1.7 | 712.0 | 3.7 | 1.5 | 708.8 | 1.0 |
| 20n-540r-3.5p  | 868.0 | 12.8 | 2.7 | 868.0 | 13.5 | 2.7 | 830.6 | 9.8 | -1.7 | 852.3 | 0.9 |
| 25n-320r-2.5p  | 593.1 | 14.2 | 1.4 | 593.0 | 15.0 | 1.4 | 593.2 | 4.7 | 1.4 | 592.2 | 1.2 |
| 25n-480r-2.5p  | 820.9 | 15.3 | 2.5 | 820.0 | 16.2 | 2.4 | 813.0 | 7.1 | 1.5 | 808.8 | 1.0 |
| 25n-640r-2.5p  | 1103.6 | 15.5 | 4.2 | 1102.0 | 16.4 | 4.0 | 1074.1 | 25.4 | 1.5 | 1071.3 | 1.3 |
| 25n-320r-3.5p  | 595.3 | 14.9 | 1.7 | 594.4 | 15.8 | 1.6 | 594.6 | 5.1 | 1.6 | 593.8 | 1.5 |
| 25n-480r-3.5p  | 818.7 | 14.6 | 2.1 | 815.3 | 15.4 | 1.8 | 820.2 | 6.7 | 2.4 | 809.5 | 1.1 |
| 25n-640r-3.5p  | 1093.0 | 15.2 | 3.0 | 1089.3 | 16.0 | 2.7 | 1083.4 | 35.3 | 2.2 | 1073.5 | 1.3 |
| 30n-420r-2.5p  | 709.6 | 21.4 | 2.1 | 708.8 | 22.1 | 2.0 | 709.4 | 12.1 | 2.1 | 709.0 | 2.1 |
| 30n-630r-2.5p  | 998.6 | 16.8 | 3.0 | 996.1 | 17.6 | 3.0 | 998.4 | 15.2 | 4.0 | 1050.6 | 7.8 |
| 30n-840r-2.5p  | 1344.3 | 14.5 | 3.9 | 1341.1 | 15.5 | 3.7 | 1420.4 | 224.4 | 9.0 | 1323.4 | 2.4 |
| 30n-420r-3.5p  | 713.5 | 25.3 | 2.7 | 712.7 | 26.1 | 2.6 | 713.5 | 14.0 | 2.7 | 710.3 | 2.2 |
| 30n-630r-3.5p  | 1009.1 | 18.4 | 3.0 | 1007.7 | 19.3 | 2.9 | 1066.3 | 14.1 | 8.2 | 1053.5 | 7.1 |
| 30n-840r-3.5p  | 1353.7 | 17.3 | 4.3 | 1345.8 | 27.8 | 3.8 | 1421.5 | 170.9 | 8.9 | 1326.0 | 2.3 |
| 35n-480r-2.5p  | 812.0 | 15.9 | 3.4 | 810.6 | 16.7 | 3.2 | 809.0 | 6.3 | 3.0 | 799.5 | 1.8 |
| 35n-710r-2.5p  | 1143.3 | 24.5 | 3.6 | 1139.5 | 65.4 | 3.3 | 1178.3 | 13.7 | 6.4 | 1166.5 | 5.5 |
| 35n-950r-2.5p  | 1557.8 | 15.3 | 3.4 | 1551.1 | 16.2 | 3.0 | 1546.6 | 94.1 | 3.2 | 1520.7 | 1.0 |
| 35n-480r-3.5p  | 812.6 | 16.9 | 3.4 | 811.9 | 17.8 | 3.3 | 810.8 | 8.7 | 3.2 | 796.4 | 1.5 |
| 35n-710r-3.5p  | 1175.5 | 16.1 | 4.5 | 1164.5 | 90.0 | 3.6 | 1180.5 | 10.9 | 4.9 | 1158.7 | 3.1 |
| 35n-950r-3.5p  | 1565.4 | 18.0 | 3.7 | 1542.6 | 20.2 | 2.3 | 1581.2 | 79.6 | 4.7 | 1525.2 | 1.2 |
| 40n-550r-2.5p  | 819.4 | 16.0 | 3.5 | 817.8 | 16.9 | 3.5 | 824.4 | 7.5 | 5.2 | 831.8 | 5.1 |
| 40n-820r-2.5p  | 1692.9 | 26.5 | 3.4 | 1692.9 | 30.8 | 2.6 | 1634.5 | 123.3 | 5.1 | 1280.6 | 1.1 |
| 40n-1090r-2.5p | 1692.9 | 13.4 | 4.7 | 1672.2 | 63.9 | 3.5 | 1673.4 | 141.1 | 3.5 | 1641.6 | 1.7 |
| 40n-550r-3.5p  | 823.5 | 17.8 | 4.2 | 820.7 | 18.8 | 3.9 | 835.8 | 9.5 | 5.9 | 844.3 | 6.6 |
| 40n-820r-3.5p  | 1341.7 | 26.4 | 3.6 | 1305.7 | 30.1 | 3.0 | 1373.5 | 11.8 | 7.8 | 1287.7 | 1.6 |
| 40n-1090r-3.5p | 1697.5 | 13.7 | 4.8 | 1674.9 | 18.2 | 3.5 | 1671.5 | 239.8 | 3.3 | 1650.7 | 2.1 |

Averages:<br>
\(< z_{ALNS} \) (%)<br>53.3<br>\(< z_{ROA} \) (%)<br>23.3
Table 3: Results for MH, MH+, ALNS and ROA on the medium instances from the second benchmark set.

| Instances            | MH       |          |          | MH+      |          |          | ALNS     |          |          | ROA      |          |
|----------------------|----------|----------|----------|----------|----------|----------|----------|----------|----------|----------|----------|
|                      | $z_{MH}$ | $time_{MH}$ | $gap_{MH}$ | $z_{MH+}$ | $time_{MH+}$ | $gap_{MH+}$ | $z_{ALNS}$ | $time_{ALNS}$ | $gap_{ALNS}$ | $z_{ROA}$ | $gap_{ROA}$ |
| 50n-650r-2.5p        | 1076.9   | 20.2     | 4.1      | 1073.8   | 21.0     | 3.8      | 1069.3   | 10.2     | 3.4      | 1055.5   | 2.1      |
|                      | 1453.7   | 17.1     | 4.6      | 1436.2   | 89.2     | 3.5      | 1447.1   | 19.2     | 4.2      | 1396.0   | 0.7      |
| 50n-300r-2.5p        | 2057.8   | 21.5     | 3.9      | 2032.3   | 72.7     | 2.7      | 2129.9   | 268.4    | 7.1      | 2011.1   | 1.7      |
| 50n-650r-3.5p        | 1066.4   | 26.4     | 3.1      | 1062.5   | 26.7     | 2.8      | 1070.2   | 12.4     | 3.5      | 1054.6   | 2.0      |
| 50n-910r-3.5p        | 1443.1   | 18.8     | 3.9      | 1431.1   | 29.6     | 3.1      | 1489.6   | 29.7     | 6.9      | 1405.2   | 1.3      |
| 50n-1300r-3.5p       | 2059.2   | 21.3     | 4.0      | 2033.1   | 29.1     | 2.7      | 2134.2   | 291.4    | 7.3      | 2025.9   | 2.4      |
| 50n-650r-3.5p        | 1290.1   | 29.0     | 3.6      | 1283.1   | 30.1     | 3.1      | 1320.8   | 17.4     | 5.9      | 1265.0   | 1.7      |
| 50n-910r-3.5p        | 1885.6   | 18.0     | 3.3      | 1872.0   | 21.6     | 2.6      | 1936.5   | 48.6     | 5.9      | 1851.8   | 1.6      |
| 50n-1300r-3.5p       | 2513.9   | 20.0     | 3.8      | 2490.7   | 27.0     | 2.9      | 2544.9   | 432.4    | 5.0      | 2460.0   | 1.7      |
| 50n-650r-3.5p        | 1286.4   | 28.6     | 3.3      | 1281.3   | 30.2     | 3.0      | 1326.5   | 19.3     | 6.3      | 1264.1   | 1.6      |
| 50n-910r-3.5p        | 1910.3   | 16.3     | 4.6      | 1878.6   | 25.3     | 3.0      | 1954.1   | 55.1     | 6.7      | 1857.3   | 1.8      |
| 50n-1300r-3.5p       | 2523.9   | 19.7     | 3.9      | 2488.9   | 31.8     | 2.5      | 2562.6   | 395.0    | 5.3      | 2471.9   | 1.8      |
| 50n-650r-3.5p        | 1634.0   | 36.0     | 4.2      | 1622.4   | 38.4     | 3.6      | 1722.8   | 59.5     | 9.2      | 1599.8   | 2.2      |
| 50n-910r-3.5p        | 2461.4   | 28.5     | 3.6      | 2435.9   | 37.8     | 2.6      | 2535.4   | 173.2    | 6.4      | 2462.7   | 3.7      |
| 50n-1300r-3.5p       | 2519.9   | 13.4     | 5.5      | 2490.7   | 225.3    | 3.5      | 2540.5   | 979.5    | 5.0      | 3361.1   | 1.8      |
| 50n-650r-3.5p        | 1632.9   | 42.7     | 4.2      | 1617.8   | 45.5     | 3.3      | 1732.8   | 53.0     | 9.7      | 1598.5   | 2.1      |
| 50n-910r-3.5p        | 2457.6   | 24.2     | 3.5      | 2429.5   | 34.3     | 2.4      | 2581.9   | 198.7    | 8.1      | 2434.5   | 2.6      |
| 50n-1300r-3.5p       | 3532.9   | 17.2     | 5.5      | 3458.3   | 211.5    | 3.4      | 3512.1   | 1038.3   | 4.9      | 3419.2   | 2.3      |
| 50n-650r-3.5p        | 2024.0   | 44.5     | 5.1      | 2005.1   | 161.7    | 4.2      | 2083.5   | 89.1     | 7.8      | 2063.3   | 6.9      |
| 50n-910r-3.5p        | 3082.0   | 43.8     | 4.4      | 3048.2   | 358.8    | 3.3      | 3149.7   | 256.3    | 6.4      | 3149.5   | 6.4      |
| 50n-1300r-3.5p       | 4313.7   | 19.7     | 7.0      | 4297.3   | 440.7    | 6.7      | 4293.2   | 3492.3   | 6.6      | 4234.8   | 5.3      |
| 50n-650r-3.5p        | 2921.0   | 50.5     | 5.0      | 1994.2   | 190.7    | 3.7      | 2149.1   | 77.4     | 10.7     | 2060.2   | 6.8      |
| 50n-910r-3.5p        | 3094.8   | 47.0     | 4.8      | 3060.7   | 327.5    | 3.7      | 3233.7   | 295.1    | 8.9      | 3164.8   | 6.9      |
| 50n-1300r-3.5p       | 4335.5   | 20.5     | 6.7      | 4327.4   | 441.6    | 6.5      | 4333.3   | 3294.8   | 6.6      | 4253.6   | 4.9      |
| Averages             |          |          |          |          |          |          |          |          |          |          |          |
| z_{ALNS} (%)        | 57.0     | 4.4      |          |          |          |          |          |          |          |          |          |
| z_{ROA} (%)         | 20.0     |          |          |          |          |          |          |          |          |          |          |

Table 4: Results for MH, MH+ and ROA on the very large instances from the third benchmark set.

| Instances | zMH | MH | gapMH | zMH+ | MH+ | gapMH+ | ROA | gapROA |
|-----------|-----|-----|-------|------|-----|-------|-----|--------|
| 120n-1410r-2.5p | 1686.2 | 73.4 | 1.4 | 1681.6 | 343.9 | 1.1 | 1766.7 | 5.9 |
| 120n-2030r-2.5p | 2424.2 | 48.3 | 1.4 | 2416.6 | 325.9 | 1.1 | 2565.6 | 6.9 |
| 120n-2820r-2.5p | 3404.0 | 82.9 | 2.8 | 3393.2 | 504.7 | 2.5 | 3530.4 | 6.3 |
| 120n-1410r-3.5p | 1687.3 | 63.5 | 1.1 | 1683.5 | 294.7 | 1.2 | 1763.9 | 5.7 |
| 120n-2030r-3.5p | 2426.0 | 59.9 | 1.5 | 2420.0 | 434.7 | 1.3 | 2569.1 | 7.0 |
| 160n-1810r-2.5p | 3525.7 | 78.8 | 1.5 | 3514.3 | 401.7 | 1.2 | 3702.8 | 6.3 |
| 160n-2930r-2.5p | 4039.6 | 91.9 | 3.0 | 4028.5 | 513.1 | 1.8 | 4157.9 | 5.7 |
| 160n-3320r-2.5p | 2216.0 | 73.2 | 1.3 | 2209.2 | 277.5 | 1.0 | 2235.8 | 2.2 |
| 160n-2930r-3.5p | 3518.1 | 149.1 | 1.3 | 3509.7 | 495.1 | 1.1 | 3624.4 | 4.2 |
| 160n-3320r-3.5p | 3983.8 | 93.2 | 1.6 | 3977.8 | 460.0 | 1.5 | 4066.1 | 3.6 |
| 200n-2110r-2.5p | 2505.7 | 74.5 | 1.3 | 2498.9 | 451.9 | 1.0 | 2575.1 | 3.9 |
| 200n-3430r-2.5p | 4065.5 | 60.7 | 1.2 | 4064.4 | 481.6 | 1.1 | 4257.9 | 5.6 |
| 200n-4120r-2.5p | 4899.6 | 154.3 | 1.3 | 4891.9 | 575.2 | 1.2 | 5168.3 | 6.5 |
| 200n-2110r-3.5p | 2507.8 | 75.1 | 1.5 | 2505.0 | 496.0 | 1.2 | 2575.3 | 3.9 |
| 200n-3430r-3.5p | 4069.5 | 68.3 | 1.2 | 4061.4 | 489.9 | 1.0 | 4250.4 | 5.4 |
| 200n-4120r-3.5p 8 | 4907.2 | 93.5 | 1.5 | 4901.4 | 514.4 | 1.4 | 5087.7 | 5.0 |
| 300n-2510r-2.5p 9 | 3018.1 | 76.1 | 1.3 | 3013.5 | 497.1 | 1.1 | 3207.5 | 7.1 |
| 300n-4030r-2.5p | 5014.6 | 152.2 | 4.0 | 5004.3 | 574.0 | 3.8 | 5262.3 | 8.5 |
| 300n-4520r-2.5p | 5433.6 | 96.2 | 1.0 | 5430.8 | 517.5 | 1.0 | 5865.7 | 8.3 |
| 300n-2510r-3.5p | 3025.2 | 74.9 | 1.5 | 3025.0 | 495.8 | 1.5 | 3197.9 | 6.8 |
| 300n-4030r-3.5p | 4861.7 | 134.8 | 0.9 | 4856.5 | 555.7 | 0.8 | 5258.7 | 8.4 |
| 300n-4520r-3.5p | 5443.0 | 159.1 | 1.2 | 5437.6 | 580.0 | 1.1 | 5973.3 | 10.0 |
| Averages | | | | | 90.9 | 1.6 | 469.6 | 1.4 |

< zROA (%) | 100.0 | 100.0 | 5.9

The SPIRP considered in this paper contains a single depot and a single type of product/oil. Thus, an immediate extension to be considered is to reformulate and solve this problem in a multi-depot environment. Also, it would be interesting to study the situation in which there are multiple types of product/oil with one or multiple depots. These are some research opportunities that could be explored in the near future.

Acknowledgments: Work of Rafael A. Melo was supported by the State of Bahia Research Foundation (FAPESB) and the Brazilian National Council for Scientific and Technological Development (CNPq).

References

Aksen, D., Kaya, O., Salman, F. S., & Akça, Y. (2012). Selective and periodic inventory routing problem for waste vegetable oil collection. *Optimization letters, 6*(6), 1063–1080.

Aksen, D., Kaya, O., Salman, F. S., & Tüncel, O. (2014). An adaptive large neighborhood search algorithm for a selective and periodic inventory routing problem. *European Journal of Operational Research, 239*(2), 413 - 426.

Andersson, H., Hoff, A., Christiansen, M., Hasle, G., & Løkketangen, A. (2010). Industrial aspects and literature survey: Combined inventory management and routing. *Computers & Operations Research, 37*(9), 1515–1536.
Applegate, D. L., Bixby, R. E., Chvatal, V., & Cook, W. J. (2006). *The traveling salesman problem: A computational study*. Princeton, New Jersey: Princeton University Press.

Coelho, L. C., Cordeau, J.-F., & Laporte, G. (2014). Thirty years of inventory routing. *Transportation Science, 48*(1), 1–19.

Coelho, L. C., & Laporte, G. (2013). The exact solution of several classes of inventory-routing problems. *Computers & Operations Research, 40*(2), 558–565.

Coelho, L. C., & Laporte, G. (2014). Improved solutions for inventory-routing problems through valid inequalities and input ordering. *International Journal of Production Economics, 155*, 391–397.

Cordeau, J.-F., Laganà, D., Musmanno, R., & Vocaturo, F. (2015). A decomposition-based heuristic for the multiple-product inventory-routing problem. *Computers & Operations Research, 55*, 153–166.

Cárdenas-Barrón, L. E., González-Velarde, J. L., Treviño-Garza, G., & Garza-Nuñez, D. (2019). Heuristic algorithm based on reduce and optimize approach for a selective and periodic inventory routing problem in a waste vegetable oil collection environment. *International Journal of Production Economics, 211*, 44 - 59.

Dekker, R., Fleischmann, M., Inderfurth, K., & van Wassenhove, L. (Eds.). (2013). *Reverse logistics: quantitative models for closed-loop supply chains*. Springer Science & Business Media.

Gavish, B., & Graves, S. C. (1978). *The travelling salesman problem and related problems* (Working Paper). Cambridge, MA: Operations Research Center, Massachusetts Institute of Technology.

Laporte, G. (2009). Fifty years of vehicle routing. *Transportation Science, 43*(4), 408–416.

Lin, C., Choy, K. L., Ho, G. T., Chung, S. H., & Lam, H. (2014). Survey of green vehicle routing problem: past and future trends. *Expert Systems with Applications, 41*(4), 1118–1138.

Mjirda, A., Jarboui, B., Macedo, R., Hanafi, S., & Mladenović, N. (2014). A two phase variable neighborhood search for the multi-product inventory routing problem. *Computers & Operations Research, 52*, 291–299.

Montagné, R., Gamache, M., & Gendreau, M. (2019). A shortest path-based algorithm for the inventory routing problem of waste vegetable oil collection. *Journal of the Operational Research Society, 70*(6), 986-997.

Raa, B. (2015). Fleet optimization for cyclic inventory routing problems. *International Journal of Production Economics, 160*, 172–181.

Soysal, M., Bloemhof-Ruwaard, J. M., Haijema, R., & van der Vorst, J. G. (2015). Modeling an inventory routing problem for perishable products with environmental considerations and demand uncertainty. *International Journal of Production Economics, 164*, 118–133.

Soysal, M., Bloemhof-Ruwaard, J. M., Haijema, R., & van der Vorst, J. G. (2018). Modeling a green inventory routing problem for perishable products with horizontal collaboration. *Computers & Operations Research, 89*, 168–182.