QCD Effective Field Theories for Heavy Quarkonium

Nora Brambilla

Dipartimento di Fisica dell’Università di Milano and INFN

Abstract. QCD nonrelativistic effective field theories (NREFT) are the modern and most suitable frame to describe heavy quarkonium properties. Here I summarize few relevant concepts and some of the interesting physical applications (spectrum, decays, production) of NREFT.

INTRODUCTION

The study of heavy quark-antiquark bound states touches upon several important physics areas, within and beyond the Standard Model of Particle Physics. These multi-scale systems probe all the energy regimes of QCD, from the hard region, where an expansion in the coupling constant is legitimate, to the low energy region, where nonperturbative effects dominate. Heavy quark-antiquark states are thus an ideal and to some extent unique laboratory where our understanding of nonperturbative QCD and its interplay with perturbative QCD may be tested in a controlled framework. This has been so historically, since quarkonium has been deeply related to the development of QCD, and it is so even more today for the two following main reasons. The first reason is that in the last few years a wealth of new experimental results has become available. The diversity, quantity and accuracy of the data currently being collected is impressive and includes: data on quarkonium formation from BES at BEPC, E835 at Fermilab, KEDR (upgraded) at VEPP-4M, and CLEO-III, CLEO-c; clean samples of charmonia produced in B-decays, in photon-photon fusion and in initial state radiation, from the B-meson factory experiments, BaBar at SLAC and Belle at KEK, including the unexpected observation of large amounts of associated \((c\bar{c})(c\bar{c})\) production; the CDF and D0 experiments at Fermilab measuring heavy quarkonia production from gluon-gluon fusion in \(p\bar{p}\) annihilations at 2 TeV; ZEUS and H1, at DESY, studying charmonia production in photon-gluon fusion; PHENIX and STAR, at RHIC, and NA60, at CERN, studying charmonia production, and suppression, in heavy-ion collisions. In the near future, even larger data samples are expected from the BES-III upgraded experiment, while the B factories and the Fermilab Tevatron will continue to supply valuable data for several years. Later on, new facilities will become operational (LHC at CERN, Panda at GSI, much higher luminosity B factories at KEK, a Linear Collider, etc.) offering fantastic challenges and opportunities in this field. A comprehensive review of the experimental and theoretical status of heavy quarkonium physics may be found in the Cern Yellow Report prepared by the Quarkonium Working Group [1]. See also the experimental review talks [2] at this conference.
The second reason is the remarkable theoretical progress of the last few years. Effective field theories (EFT), such as Nonrelativistic QCD (NRQCD) [3, 4], provided new tools and definite predictions concerning, for instance, heavy quarkonium production and decays. New effective field theories for heavy quarkonium, as potential NRQCD (pNRQCD)[7, 8] and velocity NRQCD (vNRQCD)[5], have been recently developed and are producing a wealth of new results. The lattice implementation of such effective theories has been partially carried out and many more results with drastically reduced systematic uncertainties are expected in the near future. An extensive review of the latest development in nonrelativistic EFTs can be found in [9].

Therefore, on one hand the progress in our understanding of NREFTs makes it possible to move beyond phenomenological models and to provide a systematic description from QCD of all aspects of heavy-quarkonium physics. On the other hand, the recent progress in the measurement of several heavy-quarkonium observables makes it meaningful to address the problem of their precise theoretical determination. In this situation heavy quarkonium becomes a very special and relevant system to advance our understanding of strong interaction and our control of some parameters of the Standard Model.

In the following we present the main conceptual ideas and simplifications underlying the EFT framework, a brief summary of the main ingredients of NRQCD and pNRQCD and list several applications to the phenomenology of quarkonium, referring to the original publications for the details.

**NONRELATIVISTIC EFFECTIVE FIELD THEORIES**

**Nonrelativistic bound states**

Nonrelativistic bound states are characterized by a small relative velocity $v$ (in the centre-of-mass frame) of the particles inside the bound system. The bound state dynamics has quite distinctive features with respect to the scattering case. If one considers the simpler example of a QED nonrelativistic bound system like positronium (or hydrogen), then $v \simeq \alpha \ll 1$. The fine structure constant $\alpha$ being small in QED, physical observables may be evaluated in perturbation theory. However, the bound state is nonperturbative and in this case one needs to resum infinite sets of Feynman diagrams. Due to the fact that the bound state lives close to threshold, i.e. $v \simeq \alpha$, the bound state pole (at leading order) is obtained by the resummation of all the Coulomb ladder photon contributions, $\simeq \left(\frac{\alpha}{\sqrt{\pi}}\right)^n$, in the Feynman diagram series (cf. Fig.1). As a consequence by solving the Schrödinger equation $(\frac{p^2}{2m} + V)\phi = E\phi$, with $V$ a Coulomb potential, one generates the dynamical scales of the momentum transfer $p \simeq m\alpha$ and of the bound state kinetic energy $E = \frac{p^2}{m} \simeq m\alpha^2$ (for some review see [6]). Such scales manifest themselves in the scalings of positronium radial ($\sim m\alpha^2$), fine and hyperfine splittings ($\sim m\alpha^4$). This is different from a pure scattering calculation where no scales involving $\alpha$ are generated.

**Quarkonium scales**

Heavy quarkonium provides a non-relativistic system potentially very similar to a QED bound state, with the difference that the low-energy scales will be sensitive to
FIGURE 1. QED series of diagram defining the $e^+e^-$ interaction. The resummation of the series of all ladder photon diagrams (of the type of the two first diagrams): $\alpha (1 + \frac{\alpha}{\Lambda} + \ldots)$ (with $\alpha \sim v$) gives the (leading order) Coulomb bound state energy pole.

$\Lambda_{\text{QCD}}$. Therefore the description of hadrons containing two heavy quarks is a rather challenging problem, which adds to the complications of the bound state in field theory those coming from a nonperturbative QCD low-energy dynamics. A simplification is provided by the nonrelativistic nature of heavy quarkonium, suggested by the large mass of the heavy quarks and manifest in the spectrum pattern. Quarkonium is thus characterized by three energy scales, hierarchically ordered by the quark velocity $v \ll 1$: the mass $m$ (hard scale), the momentum transfer $mv$ (soft scale), which is proportional to the inverse of the typical size of the system $r$, and the binding energy $mv^2$ (ultrasoft scale), which is proportional to the inverse of the typical time of the system. In bottomonium $v^2 \sim 0.1$, in charmonium $v^2 \sim 0.3$. In perturbation theory $v \sim \alpha_s$. Feynman diagrams will get contributions from all momentum regions associated with the scales. Since these momentum regions depend on $\alpha_s$, each Feynman diagram contributes to a given observable with a series in $\alpha_s$ and a non trivial counting. For energy scales close to $\Lambda_{\text{QCD}}$, the scale at which nonperturbative effects become dominant, perturbation theory breaks down and one has to rely on nonperturbative methods. Regardless of this, the non-relativistic hierarchy $m \gg mv \gg mv^2$ will persist also below the $\Lambda_{\text{QCD}}$ threshold.

The wide span of energy scales involved makes also a lattice calculation in full QCD extremely challenging since one needs a space-time grid that is large compared to the largest length of the problem, $1/mv^2$, and a lattice spacing that is small compared to the smallest one, $1/m$. To simulate, for instance, a $b\bar{b}$ state where $m/mv^2 \sim 10$, one needs lattices as large as $100^4$, which are extremely time computing demanding.

We may, however, take advantage of the existence of a hierarchy of scales by substituting QCD with simpler but equivalent EFTs. A hierarchy of EFTs may be constructed by systematically integrating out modes associated to energy scales not relevant for the quarkonium system. Such integration is made in a matching procedure that enforces the complete equivalence between QCD and the EFT at a given order of the expansion in $v$ and achieves a factorization between the high energy and the low energy contributions.

**What is an EFT?**

Let $H$ be a system described by a fundamental Lagrangian $\mathcal{L}$. Suppose that $H$ has two characteristic physical scales: $\Lambda \gg \lambda$. The EFT Lagrangian, $\mathcal{L}_{\text{EFT}}$, suitable to describe $H$ at scales lower than $\Lambda$ is then defined by: a cut off $\Lambda \gg \mu \gg \lambda$; some degrees of freedom that exist at scales lower than $\mu$. $\mathcal{L}_{\text{EFT}}$ involves all the operators $O_n$ that may be built from the effective degrees of freedom and are consistent with the symmetries of
\[ \mathcal{L}_{\text{EFT}} = \sum_n c_n(\Lambda, \mu) \frac{O_n(\mu, \lambda)}{\Lambda^n} \]

- Once the scale \( \mu \) has been run down to \( \lambda \), \( \langle O_n \rangle \sim \lambda^n \), so that the EFT is organized as an expansion in the small parameter \( \lambda / \Lambda \).
- The EFT is renormalizable order by order in \( \lambda / \Lambda \).
- The matching coefficients \( c_n(\Lambda, \mu) \) encode the non-analytic behaviour in \( \Lambda \). They are calculated by imposing that \( \mathcal{L}_{\text{EFT}} \) and \( \mathcal{L} \) describe the same physics at any finite order in the expansion: this is called matching procedure.
- If \( \Lambda \gg \Lambda_{\text{QCD}} \) then the \( c_n(\Lambda, \mu) \) may be calculated in perturbation theory.

The EFT has a well defined power counting in a small parameter so that the expansion is systematic. In QCD the EFT approach makes it possible to achieve a rigorous factorization between the high-energy dynamics encoded into matching coefficients calculable in perturbation theory and the nonperturbative QCD dynamics encoded into few well-defined nonperturbative operator matrix elements to be fitted on the data or calculated on the lattice or in QCD vacuum models. Thus, several model independent QCD predictions become possible. EFTs have the additional advantage that they are often close to popular phenomenological models to which experimental results are usually compared to, the main difference being a good control on the systematic errors.

**NRQCD**

Integrating out degrees of freedom of energy \( m \), which for heavy quarks can be done perturbatively, leads to NRQCD [3, 4]. In the language of the previous section this is the EFT that follows from QCD when \( \Lambda = m \). The matching is perturbative. The Lagrangian is organized as an expansion in \( v \) and \( \alpha_s(m) \):

\[ \mathcal{L}_{\text{NRQCD}} = \sum_n c_n(m, \mu) \times O_n(\mu, mv, mv^2, \Lambda_{\text{QCD}}) / m^n. \]

The Wilson coefficients \( c_n \) are series in \( \alpha_s \) and encode the ultraviolet physics that has been integrated out from QCD. The operators \( O_n \) describe the low-energy dynamics and are counted in powers of \( v \). However since two scales, the soft and the ultrasoft, still remain dynamical, NRQCD does not have an unambiguous power counting in \( v \). The operators bilinear in the fermion (or in the antifermion) fields are the same that can be obtained from a standard Foldy-Wouthuysen transformation [38] (however corrected with the matching coefficients). In this framework the imaginary part of the NRQCD Hamiltonian, i.e. the imaginary part of the Wilson coefficients of the 4-fermion operators, is responsible for heavy quarkonium annihilations. Being this a field theory, the heavy quarkonium Fock state is given by a series of terms in which the leading one is a \( Q \bar{Q} \) in a color singlet state and the first correction, suppressed in \( v \), comes from a \( Q \bar{Q} \) in an octet state with a gluon.

**Applications of NRQCD**

**Spectrum.** The NRQCD Lagrangian is well suited for lattice calculations. The quark propagators are the nonrelativistic ones and since we have integrated out the scale of
the mass, the lattice step used in the simulation may be a factor \(1/\nu\) bigger. Lattice evaluation of heavy systems like bottomonium become thus feasible. The latest results for the spectra (quenched and unquenched) are given e.g. in [10]. The radial splittings are accurate up to \(O(\alpha_s \nu^2)\) while fine and hyperfine splittings are accurate only up to \(O(\alpha_s)\), due to the fact that only tree level matching coefficients have been used. A calculation of the NRQCD matching coefficients in the lattice regularization scheme is still missing and would be relevant to improve the precision of the lattice data.

Decays. NRQCD gives a factorization formula for heavy quarkonium inclusive decay widths into light hadrons and electromagnetic decays involving four-fermion matching coefficients and matrix elements of four fermion operators [4]. Singlet operator expectation values may be easily related to the square of the quarkonium wave functions (or derivatives of it) at the origin. Octet contributions remain as nonperturbative matrix elements of operators evaluated over the quarkonium states. In some situations the octet contributions may not be suppressed and become as relevant as the singlet contributions in the NRQCD power counting. In particular octet contributions may reabsorb the dependence on the infrared cut-off of the Wilson coefficients, solving the problem that arised in the color singlet potential model. Systematic improvements are possible, either by calculating higher-order corrections in the coupling constant or by adding higher-order operators. If one goes on in the expansion in \(\nu\), that seems to be necessary for charmonium, the number of involved nonperturbative matrix elements of octet operators increases in such a way that limits the prediction power [11]. In the matching coefficients large contributions in the perturbative series coming from bubble-chain diagrams may need to be resummed [12].

Production. Before the advent of NRQCD, colour singlet production and colour singlet fragmentation underestimated the data on prompt quarkonium production at Fermilab by about an order of magnitude indicating that additional fragmentation contributions were missing [13]. The missing contribution is precisely the gluon fragmentation into colour-octet \(3S_1\) charm quark pairs. The probability to form a \(J/\psi\) particle from a pointlike \(c\bar{c}\) pair in a colour octet \(3S_1\) state is given by a NRQCD nonperturbative matrix element which is suppressed by \(\nu^4\) with respect to to the leading singlet term but is enhanced by two powers of \(\alpha_s\) in the short distance matching coefficient for producing colour-octet quark pairs. When one introduces the leading colour-octet contributions, then the data of CDF can be reproduced. Still remains a puzzle the behaviour of the polarization at high \(p_T\) (cf. the production chapter in [1]).

PNRQCD

In NRQCD the role of the potential and the quantum mechanical nature of the problem are not yet maximally exploited. A higher degree of simplification may be achieved building another EFT where only the ultrasoft degrees of freedom remain dynamical. pNRQCD [7, 8, 9] is the EFT for heavy quarkonium that follows from NRQCD when \(\Lambda = 1/r \sim mv\). We may distinguish two situations: 1) weakly coupled pNRQCD when \(mv \gg \Lambda_{QCD}\), where the matching from NRQCD to pNRQCD may be performed in perturbation theory 2) strongly coupled pNRQCD when \(mv \sim \Lambda_{QCD}\), where the matching has to be nonperturbative. Recalling that \(r^{-1} \sim mv\), these two situations correspond to systems with inverse typical radius smaller than or of the same order as \(\Lambda_{QCD}\).
Weakly coupled pNRQCD

The effective degrees of freedom are: low energy $Q\bar{Q}$ states (that can be decomposed into a singlet and an octet field under colour transformations) with energy of order $\Lambda_{\text{QCD}}, mv^2$ and momentum $p$ of order $mv$, plus ultrasoft gluons with energy and momentum of order $\Lambda_{\text{QCD}}, mv^2$. All the gluon fields are multipole expanded (i.e. expanded in $r$). The Lagrangian is then given by terms of the type

$$\frac{c_k(m, \mu)}{m^k} \times V_n(r\mu', r\mu) \times O_n(\mu', mv^2, \Lambda_{\text{QCD}}) r^n.$$  \hspace{1cm} (3)

where the potential matching coefficients $V_n$ encode the non-analytic behaviour in $r$. At leading order in the multipole expansion, the singlet sector of the Lagrangian gives rise to equations of motion of the Schrödinger type. We point out that: each term in the pNRQCD Lagrangian has a definite power counting and there is a systematic procedure to calculate corrections in $\nu$ to physical observables; higher order perturbative (bound state) calculations in this framework become much simpler \[15, 16\]. In particular the EFT can be used for a very efficient resummation of the large logs (typically logs of the ratio of energy and momentum scales) using the renormalization group (RG) adapted to the case of correlated scales \[17\]; retardation (or non-potential) effects start at the NLO in the multipole expansion and are systematically encoded inside pNRQCD. They are typically related to nonperturbative effects \[8, 9\]; Poincaré invariance is not lost, but shows up in some exact relations among the matching coefficients \[14\]. We emphasize that inside the EFT framework the renormalon subtraction may be implemented systematically. The renormalon subtraction allows us to obtain convergent perturbative series and to unambiguously define power corrections\[9\]. This is one of the main reasons for the success of the several applications listed below (for any details see the quoted references).

Applications of weakly coupled pNRQCD

QCD Singlet Static potential. The singlet and octet potentials are well defined objects to be calculated in the perturbative matching. In \[16\] a determination of the singlet potential at three loops leading log has been obtained and correspondingly also a determination of $\alpha_V$ showing how this quantity starts to depend on the infrared behaviour of the theory at three loops. The perturbative calculation of the static potential at (almost) three loops and with the RG improvement has been compared to the lattice calculation of the potential and found in good agreement up to about 0.35 fm \[29\].

$b$ and $c$ masses.

Heavy quarkonium is one of the most suitable systems to extract a precise determination of the mass of the heavy quarks $b$ and $c$. Perturbative determinations of the $\Upsilon(1S)$ and $J/\psi$ masses have been used to extract the $b$ and $c$ masses. The main uncertainty in these determinations comes from nonperturbative contributions (local and nonlocal condensates \[8\]) together with possible effects due to subleading renormalons. Table \[1\] shows some recent determinations. For a discussion about the errors and the differences among the given results see \[9\]. A recent analysis performed by the QWG \[1\] and based on all the previous determinations existing in the literature indicates that the mass extraction from heavy quarkonium involves an error of about 50 MeV both in the bottom...
Perturbative quarkonium spectrum.

$B_c$ mass. Table 2 shows some recent determinations of the $B_c$ mass in perturbation theory at NNLO accuracy compared with a recent lattice study [28] and the value of the CDF experimental $B_c$ mass. This would support the assumption that nonperturbative contributions to the quarkonium ground state are of the same magnitude as NNLO or even NNNLO corrections, which would be consistent with a $m v^2 \gtrsim \Lambda_{QCD}$ power counting. Hyperfine splittings. $c\bar{c}$, $b\bar{b}$, $B_c$ ground state hyperfine splittings have been recently calculated at NLL in [31]. The prediction for $\eta_b$ mass is $M(\eta_b) = 9421 \pm 10 \text{(th)} \pm 9 \text{(δα_s)} \text{MeV}$. The logs resummation seems to be important. If the experimental error in future measurements of $M(\eta_b)$ will not exceed few MeV, the bottomonium hyperfine separation will become a competitive source of $\alpha_s(M_Z)$ with an estimated accuracy of ±0.003. Higher bottomonium resonances have been investigated in the framework of perturbative QCD most recently in [18, 19, 21]. The surprising result of these studies is that some gross features of the lowest part of the bottomonium spectrum, like the approximate equal spacing of the radial levels, are reproduced by a perturbative calculation that implements the leading-order renormalon cancellation. If this is coincidental or reflects the (quasi-)Coulombic nature of the states will be decided by further studies. A recent NLO calculation of the $1P$ bottomonium fine splittings has been performed in [32]. It seems to indicate either the existence of large NLL/NNLO corrections (as it happens in the hyperfine splittings of the $1S$ levels) or sizeable nonperturbative corrections.

Radiative transitions (M1,E1). A theory of M1 and E1 transitions in heavy quarkonium has been recently formulated using pNRQCD [33]. This may shed some light on recent CLEO results on radiative M1 transitions in the $\eta_b$ search that have ruled out several models.

Seminclusive radiative decays of $\Upsilon(1S)$. In [35] the end-point region of the photon spectrum in semi-inclusive radiative decays of heavy quarkonium has been discussed using Soft-Collinear Effective Theory and pNRQCD. Including the octet contributions a good understanding of the experimental data is obtained.

Top-antitop production near threshold at ILC. In [37] the total cross section for top quark pair production close to threshold in $e^+e^-$ annihilation is investigated at NNLL in vNRQCD. The summation of logarithms leads to a convergent expansion for the normalization of the cross section, and small residual scale dependence. This makes precise extractions of the strong coupling, the top mass and the top width feasible.

Determinations of $\alpha_s$. Heavy quarkonia leptonic and non-leptonic inclusive decays rates may provide means to extract $\alpha_s$. The present PDG determination of $\alpha_s$ from heavy quarkonium pulls down the global $\alpha_s$ average noticeably, due to an error that has been largely underestimated [1]. Using the latest development in the calculation of relativistic corrections and in the treatment of perturbative series in $\alpha_s$ it will be possible to obtain a more appropriate determination of $\alpha_s$ from heavy quarkonium.

Gluelump spectrum. In pNRQCD [8, 34] the full structure of the gluelump spectrum has been studied, obtaining model independent predictions on the shape, the pattern, the degeneracy and the multiplet structure of the hybrid static energies for small $Q\bar{Q}$ distances that well match and interpret the existing lattice data.
Properties of baryons made of two or three heavy quarks. Recently the SELEX experiment has detected first signals from three-body bound states made of two heavy quarks and a light one. These systems are theoretically quite interesting due to the interplay of HQET and NRQCD in the construction of a suitable EFT. Triggered by these experimental data in [36] EFT Lagrangians describing $QQQ$ states and $QQq$ states have been constructed and some model independent predictions on the spectra (hyperfine separations) have been obtained.

### Table 1.

| Ref. | order   | $m_b^{\overline{MS}}(m_b^{\overline{MS}})$ (GeV) |
|------|---------|-----------------------------------------------|
| [22] | NNLO    | 4.24 ± 0.09                                  |
| [23] | NNLO    | 4.21 ± 0.09                                  |
| [24] | NNLO    | 4.210 ± 0.090 ± 0.025                        |
| [19] | NNLO    | 4.190 ± 0.020 ± 0.025                        |
| [27] | NNNLO   | 4.349 ± 0.070                                |
| [25] | NNNLO   | 4.20 ± 0.04                                 |
| [26] | NNNLO   | 4.241 ± 0.070                               |

### Table 2.

Different perturbative determinations of the $B_c$ mass compared with the experimental value and a recent lattice determination.

| State | [30] (expt) | [28] (lattice) | [20] (NNLO) | [18] (NNLO) | [19] (NNLO) |
|-------|-------------|----------------|-------------|-------------|-------------|
| $1^1S_0$ | 6287 ± 4.8 ± 1.1 | 6304 ± 12$^{+12}_{-0}$ | 6326(29)     | 6324(22)     | 6307(17)     |

**Strongly coupled pNRQCD**

In this case the matching to pNRQCD is nonperturbative [39]. In the situation where the other degrees of freedom (like those associated with heavy-light meson pair threshold production and heavy hybrids) develop a mass gap of order $\Lambda_{QCD}$, the quarkonium singlet field $S$ remains as the only low energy dynamical degree of freedom in the pNRQCD Lagrangian (if no ultrasoft pions are considered), which reads [39, 40, 8, 9]:

$$L_{pNRQCD} = \text{Tr} \left\{ S^{\dagger} \left( i\partial_0 - \frac{P^2}{2m} - V_S(r) \right) S \right\}.$$  \hspace{1cm} (4)

In this regime we recover the quark potential singlet model from pNRQCD. The matching potential $V_S$ (static and relativistic corrections) is nonperturbative: the real part controls the spectrum and the imaginary part controls the inclusive decays. The potential is calculated in the nonperturbative matching procedure between NRQCD and pNRQCD [39, 9]. The great advantages of this approach include: factorization of hard (in the NRQCD matching coefficients) and soft scales (contained in Wilson loops or nonlocal gluon correlators); the fact that the low energy objects are only glue dependent: this opens a window to confinement investigations, on the lattice [41] or in QCD vacuum.
models [38]; the existence of a clear power counting indicating leading and subleading
terms in quantum-mechanical perturbation theory; the fact that the quantum mechanical
divergences (like the ones coming from iterations of spin delta potentials) are absorbed
by NRQCD matching coefficients; the definitive disappearance of fake problems like the
“Lorentz structure of the potentials”; the fact that one no longer needs to repeat a lattice
evaluation for each quarkonium state, but gets in one step the full potential (which, in
turn, inserted inside the Schrödinger equation, will produce all the quarkonium masses).
The calculations involve only QCD parameters (at some scale and in some scheme).

**Applications of strongly coupled pNRQCD**

**Nonperturbative potentials and Spectrum.** The final result for the potential (static and
relativistic corrections) appears factorized in a part containing the high energy dynamics
(and calculable in perturbation theory) which is inherited from the NRQCD matching
coefficients, and a part containing the low energy dynamics given in terms of Wilson
loops and chromo-electric and chromo-magnetic insertions in the Wilson loop [39]. The
expression obtained for the potential is the QCD expression, in particular all the pertur-
bative contributions to the potential at the hard scale are correctly taken into account.
This solves the problem of consistency with perturbative one-loop calculations that was
previously encountered in the Wilson loop approach. Moreover, further contributions, in-
cluding a $1/m$ nonperturbative potential, appear with respect to the Wilson loop original
results [38, 42]. The full expressions for the potentials is given in [39]. Comprehensive
phenomenological applications of these full results are still missing.

**Decays.** The inclusive quarkonium decay widths in pNRQCD can be factorized with
respect to the wave function (or its derivatives) calculated in zero, which is suggestive of
the early potential models results: $\Gamma(H \to LH) = F(\alpha_s, \Lambda_{QCD}) \cdot |\psi(0)|^2$. Similar expres-
sions hold for the electromagnetic decays. However, the coefficient $F$ depends here both
on $\alpha_s$ and $\Lambda_{QCD}$. In particular all NRQCD matrix elements, including the octet ones, can
be expressed through pNRQCD as products of universal nonperturbative factors by the
squares of the quarkonium wave functions (or derivatives of it) at the origin. The non-
perturbative factors are typically integral of nonlocal electric or magnetic correlators and
thus depending on the glue but not on the quarkonium state [40]. Typically $F$ contains
both the NRQCD matching coefficients at the hard scale $m$ and the nonperturbative cor-
relators at the low energy scale $\Lambda_{QCD}$. The nonperturbative correlators, being state inde-
pendent, are in a smaller number than the nonperturbative NRQCD matrix elements and
thus the predictive power is greatly increased in going from NRQCD to pNRQCD. In
[40] the inclusive decay widths into light hadrons, photons and lepton pairs of all S-wave
and P-wave states (under threshold) have been calculated up to $O(m^3 \times (\Lambda_{QCD}^2/m^2, v^2))$
and $O(mv^5)$. A large reduction in the number of unknown nonperturbative parameters
has been achieved and, therefore, after having fixed the nonperturbative parameters on
charmonium decays, new model-independent QCD predictions have been obtained for
the bottomonium decay widths [40].

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