On modelling inelastic deformation of permeable rocks with accounting time-dependent effects

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Abstract. A model has been developed for describing stress state and filtration of rocks that include creep-like effects. The model describes elastic-inelastic transition, and further deformation in inelastic region. The key point consists in accounting for anisotropy of elastic, ultimate and filtration properties. The model is particularized for the properties of rocks of one of oil deposits of Russia.

Studying ultimate and rheological properties is an important problem aimed at ensuring safety and efficient development of hydro-carbonate deposits.

Before reaching yield stresses permeable rocks are adequately described by equations of poroelasticity [1, 2] that may be written for small strains as follows

\[
\begin{align*}
\sigma_{ij,i} &= 0, \\
\sigma_{ij} &= \sigma_{ij} + \alpha_p p \delta_{ij}, \\
q_i &= -\kappa_{ij} p_j, \\
q_{i,i} &= 0, \\
\sigma_{ij} &= \Lambda_{ijkl} \varepsilon_{kl}^E, \\
\varepsilon_{ij}^E &= \varepsilon_{ij}^T = \frac{1}{2} (u_{i,j} + u_{j,i}).
\end{align*}
\] (1)

Equations (1) together with boundary conditions for mechanical and filtration values form a closed system. Here \(\sigma_{ij}\), \(\sigma_{ij}\) are components of the total and effective (acting on rock skeleton) stresses; \(\varepsilon_{ij}^E\), \(\varepsilon_{ij}^T\) are components tensors of total and elastic; \(u_i\) are components of displacement vector; \(p\) is pore pressure; \(\Lambda_{ijkl}\) are components of tensor of elasticity; \(\kappa_{ij}\) are components of tensor of permeability; \(\delta_{ij}\) is unite Kroniker’s tensor; \(0 \leq \alpha_p \leq 1\) is Biot’s coefficient, characterizing influence of pore pressure on stress redistribution and depending on pore structure. For well permeable rocks \(\alpha_p\) is approaching to unity [3], so in practice \(\alpha_p = 1\) may be set.

Permeability is considered as an experimentally determined function of the maximal achieved intensity of shear stresses [4–6]

\[
\kappa_{mn} = \kappa_{mn}(s_i), \quad s_i = \sqrt{\frac{2}{3}(s_{jk} - \frac{1}{3}s_{ii}\delta_{jk})(s_{jk} - \frac{1}{3}s_{ii}\delta_{jk})}.
\] (2)
Due to this dependency the system of equations of deformation and filtration becomes coupled. For transversally isotropic media permeability tensor is characterized by two principle values corresponding to permeability in the plane of isotropy (plane of bedding) and in the normal direction.

On reaching elastic-plastic transition limit the last equation (1) should be replaced with the following

\[ \varepsilon^T_{ij} = \varepsilon^F_{ij} + \varepsilon^P_{ij} + \varepsilon^C_{ij} = \frac{1}{2}(u_{i,j} + u_{j,i}). \]  

Here \( \varepsilon^P_{ij} \) and \( \varepsilon^C_{ij} \) are inelastic strains that are decomposed into time-independent (plastic) and time-dependent parts. To describe time-independent part of inelastic deformation a non-associate law of plastic flaw \([4–7]\) will be used, which is a generalization of anisotropic Hill’s \([8]\) and models \([9, 10]\). According to that law during active loading the affix of stresses lay on the yield surface

\[ F = \sqrt{G^0_{(23)}(s_{22} - s_{33})^2 + G^0_{(13)}(s_{11} - s_{33})^2 + G^0_{(12)}(s_{11} - s_{22})^2 + 2L^0_{(23)}s^2_{23} + 2L^0_{(13)}s^2_{31} + 2L^0_{(12)}s^2_{12}} \]

\[ + (B^0_{(1)}s_{11} + B^0_{(2)}s_{22} + B^0_{(3)}s_{33}) - A(k). \]

Here \( A(k) \) is a reduced yield limit; \( G^0_{(ij)}, L^0_{(ij)}, B^0_{(ij)} \) are material parameters determining its strength anisotropy. Isotropic hardening is governed by changing of parameter, determined by the work of plastic strains

\[ dk = s_{ij} d\varepsilon^p_{ij}. \]

During active loading further increase of stresses is accompanied by growth of inelastic strains describing by a law of plastic flaw, according to which increments of plastic strains are proportional to partial derivatives of plastic potential \( Q \) over components of stress tensor

\[ d\varepsilon^p_{ij} = d\lambda \frac{\partial Q}{\partial \sigma_{ij}}, \]

where \( d\lambda \) is a plastic multiplier determined by belonging the affix of stresses to the yield surface

\[ d\lambda = \frac{(\partial F/\partial s_{ij})ds_{ij}}{H(\partial Q/\partial s_{mn})s_{mn}}. \]

Here \( H \equiv -\partial F/\partial k \) is a material characteristic making sense as the inverse of plasticity modulus \( H = E_p^{-1} \).

Using yield function as plastic potential \( Q = F \) (associate law) allows an elegant derivation of governing equations describing adequately the observed stress-strain dependencies in metals, however using this theory for rocks leads to strong overestimating of volumetric strains. Thus non-associate laws \( Q \neq F \) have been introduced \([11]\) to describe inelastic deformation of rocks and concretes. Earlier a concept of dilatancy \([12, 13]\) have been introduced, according to which the change in volumetric strain is proportional to the change in intensity of inelastic shear strains, rather than to the change in volumetric stress. Adopting this concept immediately results in violating the associate law for volumetric part of inelastic strains. However “deviatoric” associativity preserves. Therefore plastic potential is supposed to have the same form as the yield surface, differing by the factors at the linear terms determining the influence of the normal stresses

\[ Q = \sqrt{G^0_{(23)}(s_{22} - s_{33})^2 + G^0_{(13)}(s_{11} - s_{33})^2 + G^0_{(12)}(s_{11} - s_{22})^2 + 2L^0_{(23)}s^2_{23} + 2L^0_{(13)}s^2_{31} + 2L^0_{(12)}s^2_{12}} \]

\[ + (B^1_{(1)}s_{11} + B^1_{(2)}s_{22} + B^1_{(3)}s_{33}) - A(k) \]
For small dilatancy (which is the case for large number of rocks under investigation) it is sufficient to set

\[ B^1(i) = B^0(i) - B_0, \quad B_0 = \frac{1}{3} \sum_{j=1}^{3} B^0(j). \] (9)

A lot of rheological and creep models have been suggested for describing time-dependent inelastic behavior including theories of ageing, flaw, hardening [14–17]. The majority of these theories are constructed analogously to theories of plasticity, in which functions involved (plastic potential, yield stress) become implicitly or explicitly depending on time. Unfortunately all these theories are based on speculative assumptions and involve large number of parameters to be determined. As was mentioned [15], due to lack of sufficient reliable required experimental data there is usually no possibility to choose the most proper variant, and the choice is usually determined by convenience and simplicity. Dealing with rocks all these problems becomes even more actual. According to [15, 16] creep strains are determined in terms of a potential, which is similar to plasticity potential (8).

According to experimental results for studied rocks two regimes of creep deformation may be distinguished: restricted (decaying) creep occurring under stress state below some critical value, and unrestricted (non-decaying) creep occurring under the stresses above this value. For the first case the following law for inelastic deformation may be accepted

\[ \varepsilon = \left[ \frac{1}{E_{pi}} + \left( \frac{1}{E_{p0}} - \frac{1}{E_{pi}} \right) \exp \left( \frac{-t}{\tau_0} \right) \right] \sigma = \frac{\sigma}{E_p}. \] (10)

Here \( \tau_0 \) is a characteristic relaxation time; \( E_{p0}, E_{pi} \) are values of simultaneous and long term plasticity moduli. All three values are model parameters that may generally depend on stress-strain state history, but at first approximation may be considered as constants.

In case of unrestricted creep rather fast failure of samples were observed under increasing strain rate. Such a behavior may still be described by expression similar to (10) with adding an additional term corresponding to growing deformation under stresses above the critical ones

\[ \frac{1}{E_p} = \frac{1}{E_{pi}} + \left( \frac{1}{E_{p0}} - \frac{1}{E_{pi}} \right) \exp \left( -\frac{t}{\tau_u} \right) + \frac{\theta(\sigma_i)}{E_{pu}} \left[ \exp \left( \frac{t}{\tau_u} \right) - 1 \right], \quad \theta(\sigma_i) = \begin{cases} 0, & \sigma_i < \sigma_{cr}^i, \\
1, & \sigma_i \geq \sigma_{cr}^i. \end{cases} \] (11)

Here \( E_{pu}, \tau_u \) are modulus and characteristic time corresponding to unrestricted creep; \( \sigma_{cr}^i \) is the stress corresponding to creep transition to unrestricted stage. Note that choice of types of functions involved in (10), (11) is rather arbitrary, especially for the function describing unrestricted creep. It is followed from the above that for given \( F, Q, H \) equations (5)–(7) together with (1), (2) form a closed system.

In all experiments the observed transition from restricted to unrestricted creep was rather sharp. Characteristic time was of the order of minutes. Therefore modeling of mechanical behavior of rocks in near well zones for time intervals peculiar for technological processes (of the order of hours) may be carried out for \( t \to \infty \), i.e. by using long term plasticity modulus. For that kind of processes the description is simplified strongly due to accounting creep deformation together with plastic deformation, the value of long term plasticity modulus being used for \( H = E_{pi}^{-1} \) in (7).

Figures 1 and 2 demonstrate experimental (solid lines) and modeled (markers) dependencies of creep deformation on time for two samples of reservoirs of one of the oil deposits of Russia. Creep strains correspond to the direction of reducing stresses.

The following experimental data were used.

**Sample P5-4.** The sample was tested according to loading program under constant first invariant of stress tensor. During each (out of three) step the maximal principle compressive
stress increased (56.2, 57.6, 59.5 MPa), the second principle stress remained constant (33 MPa), the third principle stress was decreased at the same extend as the increase of the first principle stress (9.8, 8.4, 6.5 MPa). The loading program is depicted on figure 3.

The critical stress of elastic-inelastic transition was $\sigma_{cr} = 19$ MPa. For each branch value of stress intensity $\sigma_i$ was calculated as the difference between the stress invariant and the critical value $\sigma_{cr}$. Parameters of restricted creep involved in (11), obtained by fitting experimental results are the following: $E_p0 = 5.2$ GPa; $\tau_0 = 48$ s; $E_{pi} = 5.13 - 0.39\sigma_i$ GPa; parameters of unrestricted creep are $E_{pu} = 67$ GPa; $\tau_u = 19$ s.

Sample P7-6. The sample was tested according to loading program under different first
invariant of stress tensor. During each (out of three) step the maximal principle compressive stress increased (50.6, 53.3, 55.5, 57.8, 59.9 MPa), the second principle stress remained constant (13.2 MPa), the third principle stress was decreased at less extent comparing to increase of the first principle stress (9.1, 8.4, 7.9, 7.3, 6.9 MPa). The loading program is depicted on figure 3b. Such a loading program corresponded to lateral stresses of 0.4 of the vertical rock pressure.

For each branch value of stress intensity $\sigma_i$ was calculated as the difference between the stress invariant and the critical value corresponding to the current first invariant. Parameters of restricted creep are $E_p = 5.2$ GPa; $\tau_0 = 48$ s; $E_{pi} = 4.6 - 0.67\sigma_i$ GPa. The dependence is valid for relatively small values of intensity of shear only (figure 2). Faster growth of measured comparing to calculated strains for higher stress intensity is also noticeable. The latter is probably due to starting of unrestricted creep.

Note that parameters immediate plasticity modulus, $E_p$, and characteristic time, $\tau_0$, appeared the same for both samples and not depending on stress state. Value of delayed plasticity modulus, $E_{pi}$, appeared depending on the level of stress and different for the tested samples. The linear dependence was observed for the first sample, while for the second sample the linearity is observed only for initial part of loading, i.e. for small values of the intensity of shear stresses. The difference may be due to discrepancy of the samples properties, but more likely, due to influence of the first invariant of stresses, which was different in these tests.

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