The influence of profile geometric parameters on characteristics of rotor-gearing compressor

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Abstract. Dry (without lubrication) claw compressors have a promising future because they feature both advantages of piston and rotary compressors. The rotor profiles have sophisticated form including several curves of similar type, combined so, that the line of contact is always parallel to the rotor axis. The effect of the main geometric parameters of the profile (\(R\), \(\alpha_R\) and \(r\)) on the rotor-swept area value is considered. The research was carried out by the developed computer program intended for selection of the optimum variant of a ratio of profile geometrical parameters. In this paper the results of theoretical studies of influence of profile geometric parameters on integrated characteristics of rotor-gear compressor are given. Optimal relations between parameters are given, defining geometry of gas distribution elements.

1. Object
Oil-free claw compressors are very promising since they combine advantages of piston pumps and those of rotary compressors. In a claw compressor, the relative length of rotors may be shorter than in other types of rotor machines. With that optimal tip speeds along the rotor periphery may be obtained at a lower speed of rotation. A larger diameter allows greater ratios of suction port areas and working space volumes, thus reducing gas-dynamic losses at a higher speed. Rotor profiles are of complex shape and are so designed that the contact line is always parallel to rotor axes [1].

The rotor profile shall meet the process requirements, that is shall allow easy machining [2]. That is why profiles are made with mainly straight sections and arcs of a circle are preferred. As a result, using similar single-tooth rotors with a straight back of the tooth would be efficient in the compressor design.

2. Profile geometry
A profile with a straight back of a tooth shall be developed basing on the following parameters (see figure 1): pitch radius \(r\), relative radius of addendum circle \(R = R/\alpha_R\) and angular tooth thickness over addendum circle \(\alpha_R\) [3].

The profile consists of seven curves fitting one another in such a manner that almost backlash-free mesh of rotors is obtained during their rotation. Analytical description of curves looks like a system of parametric equations. To derive curve equations, mutual arrangement of profile sections was taken into account, with these sections matched (see figure 1). Equations in their general presentation were expressed via the parameters that determine profile geometry \(\bar{R}\), \(\alpha_R\) and \(r\):
- **AB curve** – original straight section described by equations (1)

\[
\begin{align*}
X &= r \cdot \cos\left[-\left(\arccos\left(\frac{1}{R}\right) + \alpha_R\right)\right] - r \cdot \tan \varphi \cdot \sin\left[-\left(\arccos\left(\frac{1}{R}\right) + \alpha_R\right)\right], \\
Y &= r \cdot \sin\left[-\left(\arccos\left(\frac{1}{R}\right) + \alpha_R\right)\right] + r \cdot \tan \varphi \cdot \cos\left[-\left(\arccos\left(\frac{1}{R}\right) + \alpha_R\right)\right],
\end{align*}
\]

\(0' < \varphi < \arccos\left(\frac{1}{R}\right)\);

- **BC curve** – an arc of a circle described by equations (2)

\[
\begin{align*}
X &= r \cdot R \cdot \cos \varphi \cdot \cos(-\alpha_R) - r \cdot R \cdot \sin \varphi \cdot \sin(-\alpha_R), \\
Y &= r \cdot R \cdot \cos \varphi \cdot \sin(-\alpha_R) + r \cdot R \cdot \sin \varphi \cdot \cos(-\alpha_R),
\end{align*}
\]

\(0' < \varphi < \alpha_R\);

- **CD curve** – epicycloidal oblong section described by equations (3)

\[
\begin{align*}
X &= 2 \cdot r \cdot \cos \varphi - r \cdot R \cdot \cos(2\varphi), \\
Y &= 2 \cdot r \cdot \sin \varphi - r \cdot R \cdot \sin(2\varphi),
\end{align*}
\]

\(\arccos\left(\frac{1}{R}\right) > \varphi > 0'\);

- **DE curve** – an arc of a circle described by equations (4)

\[
\begin{align*}
X &= r \cdot (2 - R) \cdot \cos \varphi, \\
Y &= r \cdot (2 - R) \cdot \sin \varphi,
\end{align*}
\]

\(0' < \varphi < \alpha_R\);

- **EF curve** – epicycloidal oblong section described by equations (5)

\[
\begin{align*}
X &= [2 \cdot r \cdot \cos \varphi - r \cdot R \cdot \cos(2\varphi)] \cdot \cos \alpha_R + [2 \cdot r \cdot \sin \varphi - r \cdot R \cdot \sin(2\varphi)] \cdot \sin \alpha_R, \\
Y &= [2 \cdot r \cdot \cos \varphi - r \cdot R \cdot \cos(2\varphi)] \cdot \sin \alpha_R - [2 \cdot r \cdot \sin \varphi - r \cdot R \cdot \sin(2\varphi)] \cdot \cos \alpha_R,
\end{align*}
\]

\(0' < \varphi < \varphi^*\);

**Figure 1.** Profile with a straight back of a tooth.
- **FG curve** – curvilinear section described by equations (6)

\[
X = r \cdot \left[ 2 \cdot (1 - \tan^2 \varphi) \left( 1 - \tan^2 \varphi \right) - (1 - 2 \cdot \tan^2 \varphi) \right] \cdot \cos \left( \arccos \left( \frac{1}{R} \right) + \alpha_R \right) + \\
Y = r \cdot \left[ 2 \cdot (1 - \tan^2 \varphi) \left( 1 - \tan^2 \varphi \right) - (1 - 2 \cdot \tan^2 \varphi) \right] \cdot \sin \left( \arccos \left( \frac{1}{R} \right) + \alpha_R \right) - \\
\varphi^* > \varphi > 0;
\]

- **GA curve** – an arc of a circle described by equations (7)

\[
X = r \cdot \cos \varphi \cdot \cos \left[ - \left( \arccos \left( \frac{1}{R} \right) + \alpha_R \right) \right] - r \cdot \sin \varphi \cdot \sin \left[ - \left( \arccos \left( \frac{1}{R} \right) + \alpha_R \right) \right],
\]

\[
Y = r \cdot \cos \varphi \cdot \sin \left[ - \left( \arccos \left( \frac{1}{R} \right) + \alpha_R \right) \right] + r \cdot \sin \varphi \cdot \cos \left[ - \left( \arccos \left( \frac{1}{R} \right) + \alpha_R \right) \right],
\]

\[
2 \cdot \left( \arccos \left( \frac{1}{R} \right) + \alpha_R \right) - 360^\circ < \varphi < 0^\circ;
\]

A cross point of **EF** and **FG** curves determines values of \( \varphi^* \). A cross point of **FG** and **GA** curves determines values of \( \varphi^{**} \). These angles are derived by the numerical technique.

Gas pressure and temperature in the working chamber at a random moment of time, is determined by dependence of the chamber volume on the angle of rotation of rotors. Since rotors are straight-line, of gas compression in the compressor be dependence of the working chamber end area \( S(\varphi) \).

The current value \( S(\varphi) \) is determined by the profile geometry and depends on the rotor-swept area, which is the difference of the standard cross-sectional area of the housing bore and standard cross-sectional area of rotors. The rotor-swept area may be derived by equation (8)

\[
S = 2 \cdot \left( r \cdot \sqrt{R} \right)^2 \cdot \left[ \pi - \arccos \left( \frac{1}{R} \right) + 0.5 \cdot \sin \left( 2 \cdot \arccos \left( \frac{1}{R} \right) \right) \right] - 2 \cdot S_1, \tag{8}
\]

The study of profile geometry influence on the compressor performance employed an algorithm that allows calculating theoretical profile coordinates at any physically sound values of \( \bar{R} \), \( \alpha_R \) and \( r \) for a random angle of rotor rotation, mutual arrangement of rotors relative each other, standard cross-sectional area of rotors and rotor-swept area.

The computer program developed allowed to obtain and approximate dependences of the rotor-swept area value on each of the three profile parameters (\( \bar{R} \), \( \alpha_R \) and \( r \)) with the other parameters remaining unchanged. The profile with \( r = 40 \) mm, \( \bar{R} = \sqrt{2} \), \( \alpha_R = 20^\circ \) (\( S_0 \)) was assumed as a reference variant. Results are shown in figures 2 – 4.

Results of theoretical calculations shown in Figures 2-4 demonstrate that dependence of the rotor-swept area on pitch radius \( r \) is virtually parabolic. Dependence of the rotor-swept area on relative depth and angular thickness \( \alpha_R \) of a tooth were derived in a relative form as compared to the reference profile variant and are virtually linear. After approximating the given dependencies, a simplified equation (9) is obtained for calculating the rotor-swept area

\[
S = 5.3 \cdot r^2 \cdot (2.758 \cdot \bar{R} - 2.895) \cdot \left( 1.023 - 0.406 \cdot \frac{\alpha_R^*}{360^\circ} \right), \tag{9}
\]
A value of the rotor-swept area found by the proposed formula is used to calculate the current value of the working chamber end cross-section area $S(\phi)$. To determine the working chamber volume, it is enough to multiply the calculated area by the rotor working length.

Figure 5 shows calculation dependence $V/V_{\text{max}}(\phi)$ for the profile with parameters $R = \sqrt{2}$, $\alpha_r = 20^\circ$. Working chamber maximum volume $V_{\text{max}}$ is determined as a product of the rotor working length and the rotor-swept area.
Suction and delivery ports are located on end covers and are so designed that rotation of one rotor causes suction ports to open and close, and rotation of the other rotor causes delivery ports to open and close.

Knowledge of the working chamber volume law of variation allows determining a delivery port angle of opening $\varphi_3$ (see figure 6)

$$V(\varphi_3)/V_{\text{max}} = \left( \frac{p_1}{p_3} \right)^{\frac{1}{n}},$$

where $p_1$ – gas pressure in the working chamber at the starting moment of geometric compression; $p_3$ – gas pressure in the working chamber at the moment of delivery port opening.

Delivery port closing angle $\varphi_4$ (see figure 6) is selected subject to less flow-overs and working chamber leak-tightness. This angle determines the entrapped volume value which leads to indicated power growth.

**Figure 5.** Dependence $V/V_{\text{max}}(\varphi)$ for the profile with parameters $\bar{R} = \sqrt{2}$, $\alpha_r=20^\circ$.

**Figure 6.** Angles suction port and closing port.
Suction port opening angle $\varphi_1$ (see figure 6) is selected subject to provision of the processable suction port section, which is well developed along the entire depth of the space. Angular length of the suction port influences the relative value of gas flow-overs and the rate of underutilization of the working chamber volume, which determines suction port closing angle $\varphi_2$ (see figure 6).

Arrangement of suction and delivery ports that determines $\varphi_1$, $\varphi_2$ and $\varphi_4$ is justified geometrically. A theoretical outline of gas distribution ports results from crossing of opening and closing profile edges. An actual outline of gas distribution ports has edges easier in manufacturing and slightly different sizes, which has almost no influence on compressor performance because narrow and low-efficient parts of gas distribution ports are cut off. Gas flow-overs from the compression chamber may be decreased by changing the relative height of the top edge of gas distribution ports $\rho/r$ (see figure 6).

3. Calculation method
To study the influence of profile geometry ratios and other determinants of gas- and thermodynamics of the compression process on the claw compressor performance, a program to calculate integral characteristics was developed to select the optimum ratio of geometry parameters of the profile and gas distribution items.

The thermodynamic calculation was performed according to the procedure of Saint Petersburg Polytechnic University, designed for calculation of the claw compressor [4]. The procedure is based on compressor efficiency estimation with the use of the coefficient of efficiency [5]:

$$\lambda_e = \lambda_p \cdot \lambda_t \cdot \left(1 - \nu_u - \nu_l - \nu_{el} \right),$$

where $\lambda_p$ is pressure losses at flow through the suction port.

Pressure factor is a factor taking into account pressure losses at gas flow through the suction port, which is determined by equation (12)

$$\lambda_p = 1 - \frac{\Delta p}{p_s},$$

where $\Delta p$ is pressure losses at flow through the suction port.

Temperature factor $\lambda_t$ considers the increase of suction gas temperature, i.e. $\lambda_t = T_f/T_i$, where $T_i$ is suction temperature, $T_f$ is the temperature in the working cavity at the end of the suction process. A rise of temperature leads to a decrease of the intake gas density and, as a consequence, a decrease of capacity.

Utilization factor $\nu_u$ shows how much of the theoretically possible volume is used in the suction process, i.e. $\nu_u = 1 - S_s/S$, where $S_s$ the end area of the chamber at the time of closing the suction port.

Leakage factor $\nu_l$ considers gas leakage from the high-pressure cavities in the low-pressure cavity. Leaks reduce the volume of the suction gas, as they themselves occupy part of the suction cavity volume and heat the intake gas.

External leakage factor $\nu_{el}$ considers the volume of gas that flows from the compressor's working cavity to the environment.

Inflows and outflows were calculated by the S. E. Zakharchenko method [4]. Mass rates of inflows and outflows at each fixed angle of rotor rotation are found from the formula (flows from pressure space $p_2$ to pressure space $p_1$):

$$m_\theta = \mu \cdot \delta \cdot l \cdot \left[ \rho_1^* \cdot p_1 \cdot \left( \xi^2 - 1 \right) \right]^{\frac{1}{2}},$$

where $\rho_1^* = p_1^*/(R \cdot T_2)$; $\xi = p_2/p_1$; $\Sigma = \left[ b/(2 \cdot \delta) \right] \cdot [1 + \delta/l]$.

The flow value from equation (13) is found by the iteration method. The first step specifies the zero approximation $m_{\theta 0}$. In the second step, according to the value of $m_{\theta 0}$ found, the number $Re_0$ is calculated from equation (14).
\[ \text{Re}_0 = \frac{2\dot{m}_h}{\mu_\nu (\delta + L)}, \]  

(14)

where \( \mu_\nu \) is dynamic viscosity of gas.

Depending on Re, the coefficients of friction are found, and the value found is substituted in (13). Thus, a first approximation of the mass flow rate \( \dot{m}_h \) is found. Substituting \( \dot{m}_h \) into equation (14), is found a new value of Re and friction coefficient. The calculation is repeated until the condition \( |\dot{m}_{h+i} - \dot{m}_{h}| / \dot{m}_{h} \geq \Delta \), where \( \Delta \) is given accuracy.

A value of the coefficient of local resistances depends on the gap shape: for gaps with abrupt narrowing and expansion of flow \( \xi = 2.5 \); for gaps with smooth narrowing and expansion of flow \( \xi = 1.418 \).

A value of the roughness coefficient is derived from empirical dependences: for \( \text{Re} < 1.200 \) by equation \( \lambda_\nu = 189.2 \cdot \text{Re}^{-1.127} \); for \( \text{Re} \geq 1.200 - \lambda_\nu = 3.6 \cdot \text{Re}^{-0.566} \).

4. Calculated data representation

Using the developed computer program, let's estimate the profile geometry influence on performance of a single-stage compressor with the following parameters: 2 m\(^3\)/min capacity, 0.25 MPa final pressure, 3,000 rpm synchronous speed of rotors.

There are two ways to design the claw compressor: 1) by determining the optimal working length of the rotor and addendum geometry, with the rotor profile pitch radius remaining unchanged; 2) by determining the optimal working length of the rotor and addendum geometry, with the housing bore radius remaining unchanged. The selected design method does not influence dependence patterns of efficiency-related parameters of compressor performance. The developed program allows calculation for both design methods. The profile geometry influence on compressor performance designed by the first method is given below. The profile pitch radius was assumed to be \( r = 55 \text{ mm} \) from design considerations.

Figures 7-12 shows results of the study. The profile geometry determines the rotor working length required for the capacity specified at the selected speed of compressor rotors (see figure 7). Herewith, profile parameters and rotor working length determine the geometry of end and longitudinal gaps where the air overflows thus influencing the compressor efficiency.

![Figure 7](image-url)  
**Figure 7.** Axial working length of rotors depending on relative addendum and angular tooth thickness.
Figure 8. Utilization factor of the steam space volume.

Figure 9. Relative value of internal inflows: a) depending on the gap type; b) total.

Figure 10. The rate of underutilization of the steam space volume, depending on the axial length of the suction port ($\alpha_0=20^\circ$).
5. Conclusion
Claw compressors belong to the rotating machinery class where the prevailing determinant of the performance efficiency is gas flows through gaps. Reducing gas flows from one working space to another and to the environment is an urgent task for rotary compressors. Capacity and power losses are directly dependent on amount of gas flowing through gaps.

The developed program allows estimating the claw compressor performance at various geometry parameters of the rotor profile and for different design solutions. The program is intended for calculation of the rotor profile geometry and working space volume, as well as integral characteristics of the
compressor under design. Since the compressor analysis performed has demonstrated that the compressor capacity is influenced by internal inflows through gaps, the first task to accomplish is to minimize the end clearance from the suction port end. According to the analysis of the suction port length, decrease thereof leads to drop in the coefficient of efficiency.

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List of symbols and abbreviations

| Symbols | Description | Unit |
|---------|-------------|------|
| b       | reduced length of gas travel in a gap | m |
| l       | reduced length of a gap | m |
| n       | temperature indicator of compression polytrope based on terminal parameters | - |
| p       | pressure | N m⁻² |
| R       | gas constant | m² s⁻³ K⁻¹ |
| Sᵣ      | standard cross-sectional area of the rotor | m² |
| V       | volume | m³ |
| Σ       | coefficient of form | - |
| δ       | reduced width of a gap | m |
| λᵣ      | roughness coefficient | - |
| λₚ      | pressure factor | - |
| λₜ      | temperature factor | - |
| μ       | flow rate factor | - |
| ξ       | coefficient of local resistances | - |
| νᵤ      | utilization factor | - |
| ν₁      | leakage factor | - |
| νₑᵣ     | external leakage factor | - |
| ρ       | density | kg m⁻³ |

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