Coherent backscattering from an ensemble of Mie-particles immersed in a magneto-active medium

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Abstract. We show that coherent backscattering of light from a disordered sample in a magnetic field can exhibit unusual features. The helicity retention in scattering from particles of the sample underlies these features. In the magnetic field parallel to the sample surface, the peak of coherent backscattering of circularly polarized light shifts from the backward direction by an angle proportional to the magnetic field strength. For linearly polarized light the coherent backscattering cone splits into two ones. The results obtained correlate well with the available experimental data.

1. Introduction
The effect of weak localization is a result of interference of waves propagating along the same trajectory towards each other. It is directly related to the T-invariance of the propagation process [1, 2]. The presence of a magnetic field violates the T-symmetry and leads to the appearance of an additional phase of the wave. If the phase shift arising due to the magnetic field is random (for example, due to multiple scattering), then the interference of the waves is suppressed. In electron transport through metals with impurities, this reveals itself in decrease of the interference correction to conductivity when a magnetic field is applied (see, e.g., Refs. [3, 4]). In reflection of electrons from a disordered medium, the dephasing in a magnetic field results in blunting the coherent backscattering peak [5]. A similar effect can be observed in coherent backscattering of polarized light from magneto-active samples containing small (Rayleigh) scatterers [6] (see also Refs. [7, 8]) where interaction of light with magnetic field is due to the Faraday effect. In this case, the peak of coherent backscattering is smoothed, and its amplitude decreases. The effects predicted in Refs. [6–8] were observed experimentally in Refs. [9, 10].

Unusual features of coherent backscattering in a magnetic field which contradict Refs. [6–10] were observed in a later experiment [11]. In particular, two maxima in the angular profile of coherent backscattering were observed upon scattering of linearly polarized light. For circularly polarized light, the peak of coherent backscattering was shifted from the exact backward direction.

The difference between the experimental data of Refs. [9, 10] and Ref. [11] can be explained by different depolarization rates of circularly polarized waves in the samples used in Refs. [9, 10] and Ref. [11], respectively. The matter is that the phase shifts of the left-handed and right-handed circularly polarized waves resulting from the Faraday effect are opposite in sign. If the helicity changes rapidly upon collisions (at paths of the order of the mean free path l, as in
the case of Rayleigh scatterers [6–9]), then the electromagnetic wave propagating in a magnetic field acquires a random phase consisting of a large number of shifts of different signs. On the contrary, if the helicity retains, then the wave phase is the sum of shifts of the same sign and depends only on the distance between the incoming and outgoing points of the wave path at the surface of the medium. The helicity retention (or, more precisely, its slow attenuation) can be observed upon scattering by relatively large Mie-particles (such a sample was used in Ref. [11]), and, especially, by small particles in the vicinity of the first Kerker point [12,13].

In a magnetic field, the Green function that describes wave propagation between two scattering events has the form

$$G_{ik}(r, r') = \frac{e^{ik|r-r'|}}{4\pi |r-r'|} \left[ P_{ik}^{(+)}(\mathbf{n}) \exp\left(-\frac{i k_0}{2} \cdot \mathbf{h} (r - r')\right) + P_{ik}^{(-)}(\mathbf{n}) \exp\left(\frac{i k_0}{2} \cdot \mathbf{h} (r - r')\right) \right]$$  \hspace{1cm} (1)

where $k = k_0 + i/(2l)$, $k_0$ is the wavenumber, $l$ is the mean free path, $P_{ik}^{(\pm)}(\mathbf{n}) = \delta_{ik} - n_i n_k \pm i \varepsilon_{ikj} n_j$ are the projection operators on the right-handed and left-handed circularly polarized components of the field, $\mathbf{n} = (r - r')/|r - r'|$. The vector $\mathbf{h}$ appearing in Eq.(1) is the gyration vector, $\mathbf{h} = V \mathbf{H}/k_0$, where $V$ is the Verdet constant and $\mathbf{H}$ is the magnetic field strength. Therefore, in the limiting case of retaining circular polarization, interference of the waves in multiple scattering occurs independently for the left-handed and right-handed circularly polarized components. The shift between the phases of the waves propagating towards each other along the same path, can be written as

$$\Delta \varphi (r, r') = q (r - r') \pm k_0 \int_{\mathcal{L}(r', r)} \mathbf{h} \, d\mathbf{r}$$  \hspace{1cm} (2)

where $q$ is the projection of the wave vector $\mathbf{k}_f$ of the backscattered wave on the medium surface (i.e., the plane $(x, y)$). The first term appearing in the right-hand-side of Eq.(2) is the phase shift for the scalar case [1,2]. The second term is due to the Faraday effect. The sign before the
second term is determined by the polarization of the wave, clockwise or counterclockwise (i.e., by the helicity). In a uniform magnetic field, Eq.(2) can be transformed as follows

$$\Delta \varphi (\mathbf{r}, \mathbf{r}') = (\mathbf{q} \pm k_0 \mathbf{h}) (\mathbf{r} - \mathbf{r}')$$  \hspace{1cm} (3)$$

The interference contribution to the intensity is obtained by summing over all wave trajectories [1] and, in accordance with Eq.(3), is equal to

$$J^{(c)}_{ij} = J^{(c)}_{\text{scalar}} (\mathbf{q} \pm k_0 \mathbf{h})$$  \hspace{1cm} (4)$$

where $J^{(c)}_{\text{scalar}} (\mathbf{q})$ is the interference contribution to the backscattering intensity calculated in the scalar approximation with no magnetic field [1, 2, 14]. As follows from Eq.(4), when depolarization is neglected, the magnetic field does not lead to suppression of interference, but results in displacement of the coherent backscattering peak by the angle $\Delta \vartheta = \pm h$ from the backward direction. Similar effect was found in electron backscattering for a nonzero average magnetic flux through the contour formed by electron trajectories [5].

For linearly polarized light that can be represented as superposition of two fields with opposite circular polarizations, the intensity of coherent backscattering is determined by the relation

$$J^{(c)}_L = \frac{1}{2} \left( J^{(c)}_{\text{scalar}} (\mathbf{q} + k_0 \mathbf{h}) + J^{(c)}_{\text{scalar}} (\mathbf{q} - k_0 \mathbf{h}) \right)$$  \hspace{1cm} (5)$$

According to Eq.(5), two maxima at $\vartheta = \pm h$ should be observed in the angular profile of the intensity. This effect was found in the experiment [11].

Below, we derive Eqs.(4) and (5) using calculations based on the diffusion approximation for wave transport and the assumption that the helicity is retained upon scattering.

2. Basic relations

Let a plane electromagnetic wave with the wave vector $\mathbf{k}_i = (0, 0, k_0)$ be incident on a magneto-active medium along the inward normal to the surface. We assume that there is no absorption in the medium. As is known (see, e.g., Ref. [6]), the coherent contribution to the backscattering intensity is determined by the sum of the maximally crossed diagrams $\hat{C}$. The value of $\hat{C}$ is expressed in terms of the sum of the ladder diagrams $\hat{\Gamma}$ as follows [6] (see Fig. 2):

$$C_{ij,kl} (\mathbf{r}, \mathbf{r}_1 | \mathbf{r}', \mathbf{r}'_1) = \Gamma_{ik,lj} (\mathbf{r}, \mathbf{r}_1 | \mathbf{r}', \mathbf{r}_1)$$  \hspace{1cm} (6)$$

Using the relation between the sum of the maximally crossed diagrams in the coordinate representation and the angular distribution of backscattered waves (see, e.g., Ref. [14]), we can write the coherent contribution to backscattering intensity in the form

$$J_{ij,kl}^{(c)} (\vartheta) = \int_0^\infty \int_0^\infty dz z' \mathcal{G}_{ij,kl} (z | \mathbf{q}) \mathcal{G}^*_{j',l'} (z' | \mathbf{q}) \mathcal{G}_{i',k'} (z | 0) \mathcal{G}^*_{l',l} (z' | 0)$$  \hspace{1cm} (7)$$
where $\mathbf{q} = k_0 \mathbf{\hat{q}}$, $\mathbf{\hat{q}}$ is the angle of deflection from the backward direction (see Fig. 1). The matrix function $\hat{G}$ entering into (7) is equal to

$$
\hat{G}_{ik} (z \mid \mathbf{q}) = \left[ P_{ik}^{(+)} (\mathbf{n}) \cdot \exp (iz \mathbf{\hat{z}}_\pm) + P_{ik}^{(-)} (\mathbf{n}) \cdot \exp (iz \mathbf{\hat{z}}_\mp) \right] (8)
$$

where $\mathbf{n} = \mathbf{k}/k_0$, $\mathbf{\hat{z}}_\pm = k_0 \pm k_0 h z / 2 + i/(2 l_{tr})$, $l_{tr}$ is the transport mean free path. Equation (7) assumes that the function $\hat{\Gamma}$ is calculated in the diffusion approximation.

Under the assumption of circular polarization memory (or the helicity retention), the transport of a vector electromagnetic field can be described in the approximation of two modes. One of the modes is the scalar intensity (the first Stokes parameter). Another mode is the basic mode of circular polarization (the fourth Stokes parameter). The contribution of the modes, which are described by other Stokes parameters and correspond to the transfer of linear polarization, decays on scales of the order of the transport mean free path $l_{tr}$ [13, 15, 16]. In the diffusion approximation, this contribution is small and can not be taken into account. Therefore, under the condition of circular polarization memory, the function $\hat{\Gamma}(z, z' \mid \mathbf{q})$ can be represented as the sum

$$
\Gamma_{il, km} (z, z' \mid \mathbf{q}) = P_{il}^{(+)} (\mathbf{n}) \Gamma_{++} (z, z' \mid \mathbf{q}) P_{km}^{(-)} (\mathbf{n}) - P_{il}^{(-)} (\mathbf{n}) \Gamma_{--} (z, z' \mid \mathbf{q}) P_{km}^{(+)} (\mathbf{n}) (9)
$$

The quantities $\Gamma_{\pm \pm}$ entering into Eq.(9) describe the propagation of the left-handed ($I - V)/2$ and right-handed ($I + V)/2$ polarized modes ($I$ and $V$ are the first and fourth Stokes parameters).

Under the condition of circular polarization memory, the modes $\Gamma_{++}$ and $\Gamma_{--}$ do not interact with each other and obey separate diffusion equations

$$
\left\{ 2 \frac{\partial^2}{\partial z^2} - \gamma_\pm^2 \right\} \Gamma_{\pm \pm} (z, z' \mid \mathbf{q}) = - \frac{3}{l_{tr}} \cdot \delta (z - z') \cdot \gamma_\pm = |\mathbf{q} \pm k_0 h| (10)
$$

The functions $\Gamma_{\pm \pm} (z, z' \mid \mathbf{q})$ are subject to the symmetry condition

$$
\Gamma_{\pm \pm} (z, z' \mid \mathbf{q}) = \Gamma_{\pm \pm} (-z', z \mid \mathbf{q}) (11)
$$

The boundary condition for Eq.(10) has the standard form

$$
\Gamma_{\pm \pm} (z = -z_0, z' \mid \mathbf{q}) = 0 (12)
$$

where $z_0$ is the extrapolated length (see, e.g., Ref. [15]). We can derive Eq.(10) from the results of Ref. [14], if taking into account of the magnetic field in the Green function (see Eq.(1)).

The calculations with Eq.(7) result in the following formula for the coherent contribution to the Stokes parameters:

$$
J_{\alpha \beta}^{(c)} (\mathbf{\hat{q}}) = \frac{1}{2} \left( \frac{I^{(c)} (\mathbf{\hat{q}}) + Q^{(c)} (\mathbf{\hat{q}})}{U^{(c)} (\mathbf{\hat{q}}) - i \cdot V^{(c)} (\mathbf{\hat{q}})} - i \cdot V^{(c)} (\mathbf{\hat{q}}) \right) =
\rho_{\alpha \beta}^{(c)+} \cdot J_{scalar}^{(c)} (\mathbf{q} + k_0 \mathbf{h}) + \rho_{\alpha \beta}^{(c)-} \cdot J_{scalar}^{(c)} (\mathbf{q} - k_0 \mathbf{h}) (13)
$$

where

$$
J_{scalar}^{(c)} (\mathbf{q}) = \frac{3}{8\pi} \cdot \frac{1}{(1 + q l_{tr})^2} \left[ 1 + \frac{1}{q l_{tr}} (1 - e^{-2 q z_0}) \right] (14)
$$

is the coherent contribution to backscattering intensity with no magnetic field (see, e.g., Refs. [1, 2]). The quantities $I^{(c)}$, $Q^{(c)}$, $U^{(c)}$ and $V^{(c)}$ appearing in Eq.(13) are the coherent contributions to the corresponding Stokes parameters, the matrices $\rho_{\alpha \beta}^{(c)\pm}$ are defined by the relation

$$
\rho_{\alpha \beta}^{(c)\pm} = \frac{1}{4} \cdot \left( f^{(0)} \pm V^{(0)} \right) \cdot \left( \frac{1}{1 \pm i} \right) (15)
$$
According to Eq.(13), the angular profile of coherent backscattering in the magnetic field is determined by the superposition of the angular distributions \( J_{\text{scalar}}^{(c)} \) shifted relative to the direction exactly backward by \( \pm h \). In the absence of the magnetic field (\( h = 0 \)), the Stokes parameters (13) coincide with those found in Ref. [14] if we put \( I = V \) (i.e., under the condition of circular polarization memory).

In the diffusion regime, a magnetic field does not affect the Stokes parameters of incoherently scattered radiation [6]. In the vicinity of the exact backward direction, they are determined by the relation (see, also, Ref. [14])

\[
J_{\text{in}}^{(c)} = \frac{3}{8\pi} \cdot \left( 1 + \frac{2z_0}{l_{tr}} \right) \cdot \begin{pmatrix} I^{(0)} & iV^{(0)} \\ -iV^{(0)} & I^{(0)} \end{pmatrix}
\]

The ratio \( J_{\alpha\beta}^{(c)}/J_{\alpha\beta}^{(in)} \) characterizes the coherent enhancement of light backscattering.

### 3. Discussion

Let us analyze the qualitative features of the angular profile of intensity in the cone of coherent backscattering. Consider, for simplicity, the limit \( z_0 = 0 \). In this case, the expression (13) for the Stokes parameters takes the form:

\[
J_{\alpha\beta}^{(c)} (\vartheta) = \frac{3}{8\pi} \cdot \left[ \frac{\rho_{\alpha\beta}^{(c)+}}{(1 + k_0l_{tr}|\vartheta + h|)^2} + \frac{\rho_{\alpha\beta}^{(c)-}}{(1 + k_0l_{tr}|\vartheta - h|)^2} \right]
\]

If the angular profile of coherent backscattering is measured in the plane perpendicular to the magnetic field (\( \vartheta \perp h, |\vartheta \pm h| = \sqrt{\vartheta^2 + h^2} \)), then it does not depend on the polarization of the incident radiation,

\[
J_{\alpha\beta}^{(c)} (\vartheta) = \frac{3}{16\pi} \cdot \left( 1 - 2k_0l_{tr}\sqrt{\vartheta^2 + h^2} \right) \cdot \begin{pmatrix} I^{(0)} & iV^{(0)} \\ -iV^{(0)} & I^{(0)} \end{pmatrix}
\]

According to Eq.(18), the peak of coherent backscattering has a "parabolic" shape in the plane \( \vartheta \perp h \).

If the angular profile of coherent backscattering is measured in the plane parallel to the magnetic field (\( \vartheta \parallel h \)), the situation is different. For linearly polarized light \( (I^{(0)} = Q^{(0)} = 1, U^{(0)} = V^{(0)} = 0) \), the intensity of coherent backscattering is determined by the expression

\[
J_{||,\perp}^{(c)} (\vartheta) = \frac{3}{16\pi} \cdot \left[ \frac{1}{(1 + k_0l_{tr}|\vartheta \pm h|)^2} + \frac{1}{(1 + k_0l_{tr}|\vartheta - h|)^2} \right]
\]

Under the condition of circular polarization memory, the co- and cross-polarized components of backscattering intensity prove to be coincident with each other [14]. As follows from Eq.(19) (see also Introduction), two maxima should be observed in the coherent backscattering cone at deviation angles \( \vartheta_{\max} = \pm h \) (see Fig. 3). In backscattering of circularly polarized light \( (I^{(0)} = 1, V^{(0)} = \pm 1, Q^{(0)} = U^{(0)} = 0) \), the polarization of radiation in the coherent backscattering cone coincides with the polarization of the incident one [14]. The intensity of the corresponding component \( J_{\parallel,\perp}^{(c)} \) is equal to

\[
J_{\parallel,\perp}^{(c)} (\vartheta) = \frac{3}{16\pi} \cdot \frac{1}{(1 + k_0l_{tr}|\vartheta \pm h|)^2}
\]

In this case, the backscattering peak shifts from the direction exactly backward with increasing the magnetic field strength (see Fig. 3).
**Figure 3.** Angular dependence of the backscattering enhancement factor \( \eta = 1 + \frac{I^{(c)}_\parallel}{I^{(in)}_\parallel} \) for linearly polarized light. \((h = 0 \text{ solid black curve, } h = 0.5, \vartheta \parallel h, \text{ dashed red curve; } h = 0.5, \vartheta \perp h, \text{ dotted blue curve}).

**Figure 4.** Angular dependence of the backscattering enhancement factor \( \eta = 1 + \frac{I^{(c)}_\parallel}{I^{(in)}_\parallel} \) for circularly polarized light \((h = 0 \text{ solid black curve, } h = 0.25, \text{ dotted blue curve, } h = 0.5, \text{ dashed red curve, } \vartheta \parallel h).\)

In multiple scattering of light by an ensemble of Rayleigh particles, the angular profile of coherent backscattering does not depend on the relative orientation of the vectors \( h \) and \( \vartheta \) and is always characterized by the peak of a "parabolic" shape [6, 7].

As is known (see, e.g., Refs. [1, 17]), a triangular peak in the angular distribution of coherent backscattering indicates that the paths of light rays in the medium along which wave interference occurs are unlimited. According to the results obtained above, under the condition of circular polarization memory, the triangular peak is retained in the presence of a magnetic field. Thus, the magnetic field itself does not violate the coherence of multiply scattered waves. For the multiple scattering process reversed in time, simultaneously with replacing \( t \) by \( -t \), it is necessary to change the sign of the magnetic field. In a uniform field, the phase shift due to the magnetic field depends only on the distance between the incoming and outgoing points of the rays (see Fig. 1) and does not depend on the specific shape of trajectories in the medium (see Eq. (3)). Under the conditions considered, the symmetry relation \( J^{(c)}_\vartheta(\vartheta, h) = J^{(c)}_\vartheta(-\vartheta, -h) \) is true. Destruction of wave interference can occur only if the helicity of the waves randomly changes when scattered by particles of the medium.

Two maxima in the angular profile of coherent backscattering of linearly polarized light (see Eq.(19)) and shift of the coherent backscattering peak of circularly polarized light were experimentally discovered in Ref. [11]. The sample of the medium (magneto-active glass with air bubbles) used in Ref. [11] is characterized by a visible effect of circular polarization memory: the depolarization mean free path \( l_{dep} \) (for definition, see Refs. [14–16]) of circularly polarized light is noticeably greater than the transport mean free path. Therefore, to explain the results of Ref. [11] we can, as a first approximation, neglect the attenuation of circular polarization and take advantage of the relations presented above (depolarization can be further included as an effective absorption [14]). The results obtained in Refs. [6–8, 18] do not take into account the circular polarization memory effect and, therefore, can not explain the experimental data of Ref. [11].
4. Conclusions
We have studied coherent backscattering of light in a magnetic field under the condition of circular polarization memory. In the magnetic field parallel to the surface, the peak of coherent backscattering of circularly polarized light has been shown to shift from the direction exactly backward. For linearly polarized light, the coherent backscattering cone splits into two components. Two peaks in the angular profile of backscattering intensity, are spaced symmetrically relative to the direction exactly backward. The deviation of the peaks from the backward direction is proportional to the magnetic field strength. The results obtained show how to explain the experimental data of Ref. [11] based on the diffusion theory with account of the circular polarization memory effect.

The unusual polarization features of coherent backscattering of electromagnetic waves in a magnetic field, along with the optical Hall effect (see, e.g., [19, 20]), are examples of photonic mesoscopic phenomena that have no analogues in electron transport.

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