THE DYNAMICS OF STAR STREAM GAPS

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ABSTRACT
A massive object crossing a narrow stream of stars orbiting in the halo of the galaxy induces velocity changes both along and transverse to the stream that can lead to the development of a visible gap. For a stream narrow relative to its orbital radius, the stream crossing time is sufficiently short that the impact approximation can be used to derive the changes in angular momenta and radial actions along the star stream. The epicyclic approximation is used to calculate the evolution of the density of the stream as it orbits around in a galactic potential. Analytic expressions are available for a point mass, however, the general expressions are easily numerically evaluated for perturbing objects with arbitrary density profiles. With a simple allowance for the velocity dispersion of the stream, moderately warm streams can be modeled. The predicted evolution agrees well with the outcomes of simulations of stellar streams for streams with widths up to 1% of the orbital radius of the stream. The angular momentum distribution within the stream shears out gaps with time, further reducing the visibility of streams, although the size of the shear effect requires more detailed simulations that account for the creation of the stream. An illustrative model indicates that shear will set a lower limit of a few times the stream width for the length of gaps that persist. In general, the equations are useful for dynamical insights into the development of stream gaps and their measurement.

Key words: dark matter – Galaxy: kinematics and dynamics – Galaxy: structure

1. INTRODUCTION
Star streams are created as the gravitational field of a galaxy tidally disrupts a dwarf galaxy or globular star cluster orbiting in the halo of the galaxy. Streams will exhibit a rich range of features along their length as a result of variations in the gravitational field. A progenitor in any non-circular and, depending on mass, decaying orbit will experience varying tidal fields around the orbit, leading to an increased mass-loss rate near perigalacticon (Dehnen et al. 2004). As the unbound stream stars orbit, they bunch up at apogalacticon and spread out at perigalacticon (Johnston 1998; Johnston et al. 2001). Stars leave the progenitor near the inner and outer Lagrange points where unbound stars are roughly collimated into a radial outward stream, which leads to regular cycloidal density variations near the beginning of the stream (Küpper et al. 2008, 2010, 2012). Finally, within an LCDM dark matter halo, there are predicted to be large numbers of dark matter sub-halos (Diemand et al. 2007; Springel et al. 2008; Stadel et al. 2009) with \( n(M) \, dM \propto M^{-1.9} \, dM \). The sub-halos heat the stream ( Ibata et al. 2002; Johnston et al. 2002; Carlberg 2009) and induce density variations and gaps when they cross a star stream (Siegal-Gaskins & Valluri 2008; Yoon et al. 2011; Carlberg 2012).

The dynamics leading to gap creation have the attractive property that the individual cross section depends on their radius, \( R \propto M^{0.43} \), which means that the cross section for gap creation integrated over the sub-halo population, \( \int n(M) \, dM \), is dominated by the smallest masses that can create a visible gap. The dominance of the low mass members of the population is useful in discriminating between cold and some forms of warm dark matter, where warm dark matter has very few sub-halos below \( \sim 10^{6-7} \, M_\odot \) for a 1 keV dark matter particle ( Barkana et al. 2001; Bode et al. 2001; Benson et al. 2013; Angulo et al. 2013; Schneider et al. 2013). Gaps are purely a statistical measure of the dark matter sub-halo population, which is extremely valuable, but a gap does not uniquely point to the generating object, which would be valuable in directly establishing the existence of essentially completely dark sub-halos.

In this paper, we develop general expressions for the changes in velocities as a result of the passage of a massive object. We re-derive the result of Yoon et al. (2011) for the velocity change along the stream and present the result perpendicular to the stream. We use the general expressions for an arbitrary mass density profile to derive the changes for a stream on a galactic orbit and the resulting evolution of density along the stream. Adding a dispersion in angular momentum and radial action allows for the finite width of streams. The predictions are then compared with velocity changes and gaps in simulations.

2. RESPONSE OF THE STREAM STARS TO AN ENCOUNTER
We make a few assumptions to simplify the analysis of the encounter between a massive object and a stream of stars. First, we assume that the encounter is sufficiently fast and weak that the changes in the star stream velocities are small relative to their orbital velocities, i.e., the impact approximation. Second, we do the basic analysis for a zero initial width stream. However, we will show that the results are applicable to streams of finite width with random velocities and shear velocities. Third, we assume that the orbit of the stream is circular in an axisymmetric potential. In a non-circular orbit, there will be an additional systematic compression and extension of the stream and the features within it around the orbit.

2.1. Velocity Changes of the Stars
The epicyclic approximation expands about an orbit of constant angular momentum, \( J_\phi \), where \( \phi \) is the angle around a circular orbit. The local coordinate system for our analysis is shown in Figure 1. A Cartesian frame with the \( x \) coordinate is aligned in the radial direction (Binney & Tremaine 2008) measured from the center of the progenitor. The stream then moves in the positive \( y \) direction. Stars with the same angular momentum share a common guiding center radius, and hence
A common rate of angular rotation and orbital period, but differences in radial velocity lead to different size epicycles around the guiding center. Stars with larger angular momenta have a larger guiding center radius where the rate of angular rotation is lower, so stars with guiding centers initially aligned in angle but at different x values will gradually shear with respect to each other as they move forward along the y axis at different rates, as illustrated in Figure 1.

The section of the stream that interacts most significantly with a passing sub-halo is located within a few times the sub-halo’s scale radius (Carlberg 2012). Sub-halos of mass $10^8 M_\odot$ have a scale radius of about 0.8 kpc so the interaction region for a sub-halo orbiting in the halo beyond the disk is normally sufficiently small relative to the orbital radius that the initial response to the perturbation can be calculated as if the stream was a straight line. The coordinate distances between the stream stars moving at a uniform velocity $V_\xi$ and a sub-halo moving at velocity $[V_x, V_y, V_z]$, which crosses $y = 0$ at time $t = 0$, is

$$d(y, t) = [b_x + V_x t, y + (V_y - V_x) t, b_z + V_z t].$$

where $y$ is the $t = 0$ location of stars along the stream and the closest approach distance $b$ is resolved into its x and z components. A perturbing mass with a spherically symmetric gravitational potential $\Phi(r)$ induces a net velocity change along the stream of

$$\Delta v(y) = -\int_{-\infty}^{\infty} \nabla \Phi([d(y, t)]) dt.$$  

Equation (2) is straightforward to numerically integrate for most radially symmetric density profiles.

To illustrate the behavior of these equations, the velocity changes of Equation (2) can be analytically integrated for a perturbing point mass, M. We define the velocity of the point mass relative to the stream, $v_1 = v_\| - V_\xi$ and orient the x–z frame so that $v_\perp$ is the velocity toward the stream, $\sqrt{V_x^2 + V_z^2}$. The impact parameter is $b = \sqrt{b_x^2 + b_z^2}$.

The substitutions allow the integral of Equation (2) for the component of the velocity change parallel to the stream motion for the point mass to be written as

$$\Delta v_\parallel(y) = \int_{-\infty}^{\infty} \frac{-GM (v_1 t + y)}{v_\perp^2 t^2 + (v_1 t + y)^2 + b_\perp^2}^{1/2} dt.$$

Integrating Equation (3) gives the change in the $v_1$ component of the stream stars as

$$\Delta v_\parallel(y) = \frac{-2GM v_\perp^2 y}{v(v^2 b_x^2 + v_\perp^2 y^2)}.$$ (4)

where $v = \sqrt{v_\perp^2 + v_\parallel^2}$ is the speed of the perturbing mass relative to the stream stars. This equation has been previously derived in Yoon et al. (2011) with a slightly different notation.

Perpendicular to the stream, the velocity change is

$$\Delta v_\perp(y) = \int_{-\infty}^{\infty} \frac{-GM v_\perp t}{v_\perp^2 t^2 + (v_1 t + y)^2 + b_\perp^2}^{1/2} dt,$$

which integrates to

$$\Delta v_\perp(y) = \frac{2GM v_\perp y}{v(v^2 b_x^2 + v_\perp^2 y^2)}.$$ (6)

Equation (6) is non-zero along the stream only in the special case that $v_1 = 0$.

It is interesting to note that the ratio of Equations (4) and (6) is

$$\frac{\Delta v_\perp(y)}{\Delta v_\parallel(y)} = -\frac{v_\parallel}{v_\perp}.$$ (7)

The ratio of the two velocity perturbations does not depend on distance along the stream or impact parameter, only on the velocity of the perturbing mass relative to the stream. We note that since $v_\parallel$ is the relative velocity in the stream direction of the perturber and the stream stars, $v_\parallel$ can be small and even change sign.

2.2. Orbital Changes and Stream Gaps

The stream is orbiting within a galaxy, hence the response of the stream to the perturbing mass needs to be placed within the framework of orbital dynamics. For simplicity, we continue to assume that the stream is on a circular orbit at a radius $X_0$. The relation between the linear coordinate $y$ and the $t = 0$ angular coordinates, $\phi(0)$, is simply $y = \phi(0)X_0$. The component of the velocity changes parallel to the motion of the stream along the stream, $\Delta v_\parallel(y)$, represent changes in the angular momentum, $J_\phi$, of the stream stars. For small ellipticity orbits, the epicyclic approximation gives the angular momentum as $J_\phi = XX_0$, where $X$ is the guiding center radius and $v_\parallel(X)$ is the circular velocity. The guiding center rotates forward at a uniform rate $\Omega(X)$. The epicycle is centered on the guiding center, with the star traveling around the elliptical epicycle in the opposite direction as the mean angular rotation at a rate $\kappa(X)$ (Binney & Tremaine 2008). After the encounter with a sub-halo, the angular momentum along the stream is

$$J_\phi(y) = v_\parallel(X_0)X_0 + \Delta v_\parallel(y)X_0.$$ (8)
where $X_0$ is the pre-encounter guiding center radius. The angular momentum changes lead to new guiding centers along the stream:

$$X = X_0 \frac{v_c(X_0) + \Delta v_\parallel(y)}{v_c(X)}.$$  \hspace{1cm} (9)

Consequently, after the encounter, the stars move at angular rates that are a function of their new guiding centers, $\Omega = v_c / X$,

$$\Omega(y) = \frac{v_c}{X_0} \left[ 1 + \frac{\Delta v_\parallel(y)}{v_c} \right]^{-1},$$  \hspace{1cm} (10)

where we have assumed for simplicity that $v_c$ is locally constant, although the result can be generalized to any rotation curve shape using a linear expansion. An acceleration forward leads to the stars rotating more slowly and vice-versa, which opens up a gap in the star stream centered on $y = 0$, the crossing point of the sub-halo.

### 2.2.1. The Gap Density Profile

The length and approximate density profile of a gap are readily calculated from the perturbed motion of the stars. The linear density along the stream is $\rho \equiv X_0^{-1} \frac{dn}{d\phi}$. After the encounter, the stars in the stream are at angles $\phi(t) = \phi + \Omega(X_0) t$. Therefore, the differential in the density equation becomes

$$d\rho(t) = \left(1 + X_0 \frac{d\Omega(y)}{dy} t \right) \frac{dn}{d\phi}. \hspace{1cm} (11)$$

Using Equation (10) in Equation (11), the linear density along the stream becomes

$$\rho(y, t) = \rho_0 \left[ 1 - \left(1 + \frac{\Delta v_\parallel(y)}{v_c} \right)^{-2} \frac{\Delta v_\parallel(y)}{dy} t \right]. \hspace{1cm} (12)$$

In principle, one could allow for streams on non-circular orbits by including the variation of $\Omega(X)$.

Equation (12) gives the density as a function of time, labeled with the initial $y$ values. The gap has its greatest depth at $y = 0$, where the derivative of Equation (4) in Equation (12) is most negative for reasonable perturbing mass profiles. The density in the gap approaches zero asymptotically as $t^{-1}$. On either side of the gap where $d\Delta v_\parallel(y)/dy > 0$, Equation (12) will fail for sufficiently large $t$ values that the expression in brackets goes through zero.

As the guiding centers change in response to the angular momentum changes, the mean orbital angle at which the density applies is modified from the pre-encounter $\phi(t) = y / X_0 + \Omega_0 t$ to

$$\phi(y, t) = \frac{y}{X_0} + \Omega(y) t,$$  \hspace{1cm} (13)

where we use $\Omega(y)$ from Equation (10). Equation (13) ignores the epicyclic oscillations of individual stars, on the basis that this oscillating term will normally average to zero in a realistic stream that contains random motions. Equation (13) is the angle relative to the current location of the middle of the gap. Subtracting the gap center angular rotation of $v_c / X_0$ from the $\Omega(y)$ of Equation (10) removes the mean motion to give $\Delta \phi(y, t) = \phi(y, t) - v_c / X_0$, which, when inserted into Equation (13), gives

$$\Delta \phi(y, t) = \frac{y}{X_0} - \frac{v_c}{X_0} \frac{\Delta v_\parallel(y)}{v_c} \left[ 1 + \frac{\Delta v_\parallel(y)}{v_c} \right]^{-1} t.$$  \hspace{1cm} (14)

### 2.2.2. Changes of Epicyclic Motions

We are also interested in the changes in the sizes of the epicycles. For an epicycle of radial extent $a$, $x(t) = a \cos(\kappa t + \psi)$ and the velocity $v(t) = -a \kappa \sin(\kappa t + \psi)$, where $a$ and $\psi$ are the amplitude and orbital phase, respectively, of the epicycle. In the direction of the mean motion, the epicyclic velocity is $y(t) = -2\Omega a \cos(\kappa t + \psi)$ (Binney & Tremaine 2008). The epicycle is an ellipse, with axes, $y/x$, in the ratio $2\Omega/\kappa$.

An encounter with an initially completely cold stream will induce a systematic epicyclic motion through the combined effects of the velocity changes along and perpendicular to the stream. The radial motion immediately after the encounter is $\Delta v_\perp(y)$. Along the stream, there is a velocity change of $\Delta v_\parallel(y)$ to which we need to add the effective velocity change that results from the change to the new guiding center, Equation (9). The change of mean guiding center leads to an angular velocity difference between the initial orbits and the post-encounter orbit that is simply the initial angular velocity, $v_c / X_0$, minus the new angular rotation rate along the stream, $\Omega(y)$, of Equation (10). Expanding Equation (10) to first order and multiplying by the radius to obtain a linear velocity, we find a second velocity change factor of $\Delta v_\parallel(y)$. Therefore, to first order, the new velocity along the stream, in the frame of the new guiding center, is $2\Delta v_\parallel(y)$. Since $v_c / k = a \sin(\kappa t + \psi)$ and $v_c/(2\Omega) = a \cos(\kappa t + \psi)$, the resulting added epicycle has a size in terms of the stream perturbations of

$$a(y) = \sqrt{\frac{\Delta v_\parallel^2(y)}{k^2} + \frac{\Delta v_\parallel^2(y)}{\Omega^2}}.$$

The initial epicyclic phase, $\psi$, as a function of the stream position of this induced epicycle can be derived from the ratio of the two velocities at $t = 0$:

$$\tan(\psi(y)) = \frac{2\Omega \Delta v_\parallel(y)}{k \Delta v_\parallel(y)}.$$

In a warm stream, the pre-encounter epicycles will have random phases to which the sub-halo induced epicycle will add, leading to a spread in change in epicyclic size. Because the added epicyclic phase is coherent along the stream, the stars will oscillate in and out together, with the stream varying from nearly straight to a sideways reversed “S” shape.
3. COMPARISON WITH IDEALIZED SIMULATIONS

3.1. Simulation Setup

Running some simple gravitational simulations of a star stream orbiting in a galactic potential that encounters a sub-halo with an extended mass profile tests the accuracy and the generality of the analytic predictions. The galactic potential is an Navarro–Frenk–White halo (Navarro et al. 1997) with the peak of the rotation curve at 30 kpc where the circular velocity is 210 km s$^{-1}$, a rough match to the dark halo of the Milky Way (Springel et al. 2008), although those details are not important for this illustrative calculation. The radius and mass define a characteristic mass of $3.077 	imes 10^{11} M_{\odot}$, which together give a set of characteristic scales to normalize the numerical calculation units.

We place 100,000 particles on an arc of length 1 rad on a circular orbit at 30 kpc with velocities equal to the local circular velocity. The arc is positioned so that a perturbing sub-halo will cross the middle of the stream. Restricting our attention to circular orbits is not a significant limitation for the relatively small size of the gaps that sub-halos induce, although the gap size and density will expand and contract to conserve angular momentum if the stream is on a non-circular orbit. One version of the stream is completely cold with all particles having the same angular momentum, no radial velocities, and no initial width about the stream centerline. To understand the extension to warm streams with a finite width, we also set up streams with a Gaussian distribution of epicycles about the same initial guiding centers. All the particles have the same angular momentum to first order, so there is no shear in the stream. This fact allows direct comparison with the model predictions above, but shear will need to be taken into account for comparison with real streams. The fact that streams exist tells us there must be a spread in guiding centers in order to spread the debris along the orbit. In the warm simulations, the epicycles are sufficiently small, generally less than 1% of the orbital radius, that the asymmetric drift correction to the mean orbital velocity, which is a second order effect, is ignored.

A sub-halo with a Hernquist mass profile and a mass of $10^6 M_{\odot}$ is sent on a straight line orbit toward the stream at a constant velocity of $1.2 v_c$, unless otherwise specified. The simulation is initiated with the sub-halo at a distance of one stream radius away from where it will cross the stream at $y = 0$ at time zero. The predictions for $\Delta v_{\parallel}(y)$ and $\Delta v_{\perp}(y)$ are worked out with the integrals of Equation (2). The results are only very weakly dependent on the precise angle with respect to the orbital plane with which the sub-halo encounters the stream, but these differences are minimal.

We consider the response of completely cold and warm streams on circular orbits to a sub-halo of mass $10^6 M_{\odot}$ with a Hernquist mass profile having a scale radius of 110 pc. For most encounters, the impact parameter is set to 0.0001 of the orbital radius, that is, 3 pc, which is effectively a direct hit on the stream. The mass of $10^6 M_{\odot}$ is chosen on the basis of being a sufficiently small mass that the linear approximations in the analysis should in principle give a good description of the dynamics. A significantly smaller sub-halo mass would be a size scale smaller than the narrowest streams and is of less practical interest since the gap may be too shallow to be readily visible. The steep LCDM mass spectrum of sub-halos, $N(M) \propto M^{-1.9}$, means that there be should be large numbers of sub-halos at these low masses ensuring that real streams will have many encounters with masses in this range and that these encounters dominate those with objects that contain visible baryons.

Two streams were generated: the first, perfectly cold and the second, warm. The completely cold stream is well matched to the assumptions of our analysis. The warm stream is normally created with a width of 70 pc, the same as the projected width of the GD-1 star stream (Carlberr & Grillmair 2013), or 0.23% of the orbital radius and is still very thin. A yet wider stream will be used to see where the approximations of our analysis begin to lead to significant quantitative errors.

3.2. Position and Velocity Changes in the Stream

Figure 2 shows the change in guiding center, $\Delta X = X - X_0$, for the cold stream along with the predicted relation of Equation (9), integrated for the Hernquist potential used for the sub-halo in the simulation. The prediction is virtually indistinguishable from the simulation points. Figure 3 shows the same prediction against the post-encounter guiding centers calculated from the angular momentum values in a 70 pc wide stream. In this case, there is a spread of values around the
prediction, which we attribute to the fact that in a warm stream the individual stars have a range of velocities relative to the sub-halo, which leads to a range of relative $v_\parallel$ and $v_\perp$ values for individual particles. For yet warmer streams (not shown), there is even more scatter and the angular momentum changes can even reverse sign, as anticipated in our analytic results for the point mass object in Equation (7).

The change in the sizes of the epicycles, $\Delta a = a - a_0$, where $a$ and $a_0$ are the post-encounter and initial epicyclic sizes, respectively, are compared with the prediction of Equation (17) for the cold stream (where $a_0 = 0$) in Figure 4 showing that the prediction is quite good. The height of the two peaks in the $\Delta a$ distribution in the simulation are slightly asymmetric, whereas the prediction is completely symmetric about the center point.

To identify the source of the asymmetry, some variants on the basic simulation were done. The plot of Figure 4 uses a sub-halo started at twice the orbital radius and was shot inward. We repeated the simulation with the sub-halo shot outward from the center, finding that the asymmetry reverses and become stronger.

The time evolution of $\Delta a$ shows that before the sub-halo arrives at the stream, $\Delta a$ rises with time approximately linearly with distance along the stream and is strongest in the leading part of the stream, where the sub-halo was closest as it approached the crossing point. Starting the sub-halo closer to the stream reduces the size of the pre-crossing $\Delta a$ and the final asymmetry. Larger mass sub-halos do not lead to proportionally larger asymmetries.

We conclude that the approximation of a straight line stream for the calculation of the velocity perturbations, which are then translated into the motions of a stream on a circular orbit, does not capture all of the dynamics, but the approximation remains good to a few percent for the cases studied.

For the warm stream, the changes in epicycle sizes, $\Delta a$, are displayed in Figure 5. The perturbation-induced epicycles are adding to pre-existing epicycles with random phases, hence there will be a spread from completely in-phase addition to out of phase subtraction. Figure 5 shows that the predicted relation and its negative largely bound the outcomes.

The evolution in time of the $xy$ locations of the particles in a cold stream simulation, effectively equivalent to the guiding centers of a warm stream, is shown in Figure 6. For this example, we use a sub-halo of mass $3 \times 10^6$, which increases the change in shape of the perturbed region with time. The stream starts with the shape of Figure 2, however, with time the outer part falls behind the inner part. The characteristic sideways reversed “S” shape gradually becomes closer to a saw-tooth shaped. Note that the $x$ scale is greatly magnified relative to the $y$ scale. The stream is shown every 0.8 units of time (about 1/8 of an orbital period) for 100 times. Line segments are drawn between a subset of the simulation points.

### 3.3. Gaps in Streams

The development of the gap in the density of a stream according to Equations (12) and (13) is compared with a cold stream stimulation in Figure 7. The predictions do a very good job of describing the shape of the gap and its growth with time.

Warm streams have epicyclic motions that blur the density out along the stream; these motions are not yet included in our cold stream analysis. Figure 8 shows the outcome of the same small impact parameter encounter as in Figure 7, but now on a warm
Figure 7. Development of a density gap in a nearly completely cold stream vs. the angular coordinate after an encounter with a \(10^6 M_*\) sub-halo at dimensionless times 20, 40, 60, and 80, where the rotation period is \(2\pi\). The dotted lines show the predicted relation at different times.

Figure 8. Same as Figure 7, except for a warm stream of width 70 pc. The dashed line shows the effect of adding in the epicyclic smoothing of the stream width along the stream. The cold stream prediction (dotted line) does not do a very good job. Therefore, we modify the prediction by simply convolving the predicted density distribution with a Gaussian. The spread \(\sigma\) of the Gaussian is related to the FWHM of the stream multiplied by \(1.414^2/2.355\), where the factor of 1.414 allows for the fact that the epicycles along the stream are larger than their perpendicular size in the ratio \(2\Omega/\kappa\), which is \(\sqrt{2}\) for our locally flat rotation curve. The factor of 2.355 converts from FWHM to a Gaussian width.

We compare the gap measurements to the predictions for a 70 pc wide stream in Figure 8. The cold stream prediction (dotted line) gives a deeper and sharper gap than is found in the simulation (solid line). The velocity dispersion allowance does quite a good job predicting the shape of the gap (dashed line). Figure 9 shows a stream of width 150 pc where the width-corrected prediction of the overall shape remains qualitatively correct, but a significant asymmetry is developing and the predicted depth is greater than measured. Not shown is a 300 pc wide stream, 1% of the orbital radius, which continues the trend of increasing discrepancy from our predictions with increasing stream width. The most significant error is that the depth is measured to be 0.5–0.6 of the mean level, whereas the predictions go nearly to 100% depth. The prediction continues to provide a useful estimate of gap width, but does not give an accurate density profile.

3.4. The Effect of Shear in Streams

The individual stars in stellar streams are drawn out from their progenitor objects with a range of angular momenta and radial velocities (Johnston 1998; Helmi & White 1999; Eyre & Binney 2011), which together fix the width of the stream. Both the angular momentum and the radial action, \(J_r = \kappa a^2\), are conserved quantities in an axisymmetric potential, which applies once the stars are away from the progenitor object. Progenitors that lose stars through a fairly well-collimated outflow through the Lagrange points will have streams that are largely \(J_r\) dominated, with relatively little shear (Küpper et al. 2008, 2012; Eyre & Binney 2011).

If there is a significant range of angular momentum in the stream, then it will cause gaps to shear with time so that any one-dimensional measure will see a decreased gap density once the tilt of the gap exceeds the gap width. The spreading of the gap into a tilted stripe along the stream and the consequent reduction of the depth at any location is straightforward to calculate. However, a useful prediction needs to have some way to estimate the range of angular momenta that are likely to be present. Such a prediction requires full \(n\)-body simulations that take into account the details of how the progenitor dissolves as it orbits in the potential of its host galaxy, which is beyond the scope of this paper. Yoon et al. (2011) provide a specific example in their Figures 7 and 9.

A simple calculation shows when shear becomes an important effect. For illustration, we assume that in a 60 pc wide stream half of the width at the origin of the stream is due to the angular momentum spread of the stream (Eyre & Binney 2011). In this case, the range of angular momentum relative to the mean is 0.001 at a 30 kpc orbital radius. Since small gaps are blurred out by the velocity dispersion in the stream, we consider the evolution of a gap having a length twice the width of the stream, 120 pc, 0.004 times the orbital radius. The stream shear in this case will reduce the density contrast in this gap to about half its initial depth in the time it takes the low and high end of the
angular momentum in the stream to rotate apart by about twice the initial width of the gap. That time is two times the initial gap width divided by the fractional shear or $2 \times 0.004/0.001$, which is 8 units of time or about 1.3 rotation periods, which at 30 kpc is about 1 Gyr. The range of angular momentum spread diminishes linearly with distance down the stream, so at half the stream length the gap would take 2 Gyr to diminish to half its depth. Wider gaps would blur out over times directly proportionally to their length, so a gap 10 times the stream width half way down the stream would last 10 Gyr. The simulations of dissolving globular clusters by Küpper et al. (2008, 2012) suggest that the angular momentum range in those cases may be so small that angular momentum smearing is not a significant effect. We emphasize that the range of angular momenta relative to the radial action in the stars lost through tidal lobes has not been extensively studied and the question of how both shear and substantially elliptical stream orbits affect the visibility of gaps remains to be resolved.

4. DISCUSSION AND CONCLUSIONS

This paper provides a first order dynamical analysis of the velocity changes and subsequent development of a gap in a star stream after a massive object passes. A gap with a length comparable to the stream width will be blurred out through the subsequent orbital motions within the stream. Gaps that survive need to be two to five times the stream width, depending on details of the orbit and how stars are unbound from the progenitor object. If shear creates half the stream width, a gap with a length twice the width at half the distance down the stream from the progenitor will blur out in about 2 Gyr. Gaps will have more time to be formed further down the stream and survive longer. The theory provides useful quantitative predictions for the resulting development of gaps in a stream, provided that the stream width is less than about 1% of its mean orbital radius. The numerical model can be used to make predictions of gap shapes in a variety of orbital situations. One insight is that although the single shape approach to gap-filtering developed in Carlberg et al. (2012) and Carlberg & Grillmair (2013) is a reasonable approximation, the density profile gaps vary widely in the sharpness of their shoulders.

The limitation of this study is that we study circular stream orbits and the details of the angular momentum, radial action, and phase angle distributions of stars once unbound from the progenitor system are not yet sufficiently well understood in general situations to be incorporated into the model. Those details have an important influence on the real space appearance of streams and are the focus of ongoing studies.

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