Reparametrization Invariance of Perturbatively Defined Path Integrals.

II. Integrating Products of Distributions

H. Kleinert* and A. Chervyakov†
Freie Universität Berlin
Institut f"ur Theoretische Physik
Arnimallee14, D-14195 Berlin

We show how to perform integrals over products of distributions in coordinate space such as to reproduce the results of momentum space Feynman integrals in dimensional regularization. This ensures the invariance of path integrals under coordinate transformations. The integrals are performed by expressing the propagators in $1 - \varepsilon$ dimensions in terms of modified Bessel functions.

I. INTRODUCTION

While the Schrödinger equation and time-sliced path integrals, as defined in the textbook [1], are invariant under coordinate transformations by construction, this invariance is completely nontrivial in perturbatively defined path integrals in curvilinear coordinates. The reason for this is that the Feynman integrals appearing in the perturbation expansion are multiple temporal integrals over highly singular integrands, which are products of generalized functions [for general discussion, see [2]]. In the presently existing theory of distributions, generalized functions can only be combined linearly and by a convolution, but products of them are still forbidden calling for regularization [3]. Thus, if we want to describe quantum mechanics in curvilinear coordinates by perturbative defined path integrals, we must extend the theory of distributions to allow for integrals over products of generalized functions. This will extend the linear space of distributions to a group.

In a previous note [4] we have shown that invariance can be achieved by defining path integrals perturbatively as a limit $D \to 1$ of a $D$-dimensional functional integral. The perturbative calculations in [4] were performed in momentum space, where Feynman integrals in a continuous number of dimensions $D$ are known from the prescriptions of 't Hooft and M. Veltman [5]. For many applications, however, momentum space calculations are rather clumsy, for instance if one wants to find the effective action of a field system in curvilinear coordinates, where the kinetic term depends on the dynamic variable. Then one needs rules for performing temporal integrals over Wick contractions of local fields.

*E-mail: kleinert@physik.fu-berlin.de
†On leave from LCTA, JINR, Dubna, Russia E-mail: chervyak@physik.fu-berlin.de

II. PROBLEM WITH COORDINATE TRANSFORMATIONS

Recall the origin of the difficulties with coordinate transformations in path integrals. Let $q(\tau)$ be the euclidean coordinates of a quantum-mechanical point particle of unit mass in a harmonic potential $m^2q^2/2$ as a function of the imaginary time $\tau = -it$. Under a coordinate transformation $q(\tau) \to \phi(\tau)$ defined by $q(\tau) = f(\phi(\tau)) = \phi(\tau) + \sum_{n=1}^\infty a_n\phi^n(\tau)$, the kinetic term $\dot{\phi}^2(\tau)/2$ goes over into $\dot{\phi}^2(\tau)f^2(\phi(\tau))/2$. If the path integral over $\phi(\tau)$ is performed perturbatively, the expansion terms contain temporal integrals over Wick contractions which, after suitable partial integrations, are products of the following basic correlation functions

$$\Delta(\tau - \tau') \equiv \langle \phi(\tau)\phi(\tau') \rangle = \cdots,$$

$$\partial_\tau\Delta(\tau - \tau') \equiv \langle \dot{\phi}(\tau)\phi(\tau') \rangle = \cdots,$$

$$\partial_\tau\partial_\tau'\Delta(\tau - \tau') \equiv \langle \dot{\phi}(\tau)\dot{\phi}(\tau') \rangle = \cdots.\quad(3)$$

The right-hand sides define the line symbols to be used in Feynman diagrams for the interaction terms.

Explicitly, the first correlation function (4) reads

$$\Delta(\tau, \tau') = \frac{1}{2m}e^{-m|\tau - \tau'|}.\quad(4)$$

The second correlation function (5) has a discontinuity

$$\partial_\tau\Delta(\tau, \tau') = -\frac{1}{2}e(\tau - \tau')e^{-m|\tau - \tau'|},\quad(5)$$

where

$$e(\tau - \tau') \equiv 2\int_{-\infty}^\tau d\tau''\delta(\tau'' - \tau').\quad(6)$$

is a distribution which has a jump at $\tau = \tau'$. The third correlation function (7) contains a $\delta$-function:

$$\partial_\tau\partial_\tau'\Delta(\tau, \tau') = \delta(\tau - \tau') - \frac{m}{2}e^{-m|\tau - \tau'|}.\quad(7)$$

The temporal integrals in $\tau$-space over products of such distributions are undefined.

In our previous paper [6] we have shown that it is possible to define unique perturbation expansions which lead to a reparametrization invariant theory. This is done by extending the path integral to a $D$-dimensional functional integral, and by performing the perturbation expansion in $D$-dimensional momentum space, with a limit $D \to 1$ taken at the end.
In this note we want to set up an extension of distribution theory which allows us to define the same theory by doing the integrals over products of the correlation functions in τ-space. This requires regularizing the correlation functions in such a way that all τ-integrals yield the same results as the momentum integrals in the limit $D \to 1$. The result is a reparametrization invariant perturbative definition of path integrals via Feynman integrals in τ-space.

### III. MODEL SYSTEM

As in our momentum space treatment in Ref. [1], we shall prove the reparametrization invariance for a simple but typical system, consisting of a point particle in a box of size $d$. We shall do this by performing explicit calculation up to three loops.

The ground-state energy of this system is exactly known, $E = \pi^2/2d^2$. In a recent paper we have shown that this energy can be obtained perturbatively to any desired accuracy by performing perturbation expansions in powers of $g \equiv \pi^2/d^2$, and taking the limit $g \to \infty$ of this series [1], following the strong-coupling theory developed in Ref. [1].

In the model system, the three-loop expansion for the ground state energy reads

$$E = \frac{m}{2} - g \oint + \frac{9}{2} g^2 \oint$$

where $m$ regularizes the Feynman integral in the infrared. The ground state energy is obtained from the limit $m \to \infty$.

After the coordinate transformation of the free-particle action in the path integral by $g(\tau) = f(\phi(\tau)) = \varphi - g\phi^3/3 + g^2\phi^5/5 - \cdots$, we find for the energy the following graphical expansion [1]:

$$E = \frac{m}{2} - g \oint + \frac{9}{2} g^2 \oint$$

where $m$ regularizes the Feynman integral in the infrared. The ground state energy is obtained from the limit $m \to \infty$.

After the coordinate transformation of the free-particle action in the path integral by $g(\tau) = f(\phi(\tau)) = \varphi - g\phi^3/3 + g^2\phi^5/5 - \cdots$, we find for the energy the following graphical expansion [1]:

$$E = \frac{m}{2} - g \oint + \frac{9}{2} g^2 \oint$$

The path integral is extended to an associated functional integral in a $D$-dimensional coordinate space $x$, with coordinates $x_\mu \equiv (x_1, x_2, x_3, \ldots)$, by replacing $\phi^2(\tau)$ in the kinetic term by $(\partial_\mu \phi(x))^2$, with $\partial_\mu = \partial/\partial x_\mu$. In the extended coordinate space, the correlation function [1] becomes

$$\Delta(x) = \int \frac{d^Dk}{(2\pi)^D} e^{ikx}.$$  

In writing down the expansion [1] we have ignored all diagrams coming from the Jacobian of the coordinate transformation, since these carry a prefactor $\delta^{(D)}(0)$ which, according to a basic rule of 'tHooft and Veltman [1], vanishes in dimensional regularization.

In our calculations, we shall encounter generalized $\delta$-functions, which are multiple derivatives of the ordinary $\delta$-function:

$$\delta^{(D)}_{\mu_1 \ldots \mu_n}(x) \equiv \partial_{\mu_1 \ldots \mu_n} \delta^{(D)}(x) = \int d^Dk (ik)_{\mu_1} \ldots (ik)_{\mu_n} e^{ikx},$$

with $\partial_{\mu_1 \ldots \mu_n} \equiv \partial_{\mu_1} \ldots \partial_{\mu_n}$, and with $d^Dk \equiv d^Dk/(2\pi)^D$. In dimensional regularization, these vanish at the origin as well:

$$\delta^{(D)}_{\mu_1 \ldots \mu_n}(0) = \int d^Dk (ik)_{\mu_1} \ldots (ik)_{\mu_n} = 0,$$

thus establish the contact with our previous momentum space discussion [1].

### IV. FEYNMAN DIAGRAMS AND DISTRIBUTIONS

We shall set up rules for calculating the singular expansion terms in (2) which will yield the following values:

\begin{align*}
\oint & = -\Delta(0) \Delta_{\mu\mu}(0) = -\frac{1}{4}, \\
\oint & = -\Delta^2(0) \Delta_{\mu\mu}(0) = -\frac{1}{8m}, \\
\oint & = \Delta^2(0) \int d^Dx \Delta^2_{\mu\nu}(x) = \Delta^2(0) \int d^Dx \Delta^2_{\mu\mu}(x) = - (1 + D/2) m^2 \Delta^3(0) = - \frac{3}{16m}, \\
\oint & = \Delta^2_{\mu\mu}(0) \int d^Dx \Delta^2(x) = (1 - D/2) m^2 \Delta^3(0) = \frac{1}{16m}, \\
\oint & = -\Delta(0) \Delta_{\mu\nu}(0) \int d^Dx \Delta^2_{\mu\nu}(x) = - (D/2) m^2 \Delta^3(0) = - \frac{1}{16m}, \\
\oint & = \int d^Dx \Delta^2(x) \Delta^2_{\mu\nu}(x) = \frac{m^2}{3(3D - 4)} \left[ (8 - 7D) \Delta^3(0) + (D + 4) m^2 \int d^Dx \Delta^4(x) \right] = - \frac{3}{32m}, \\
\oint & = \int d^Dx \Delta(x) \Delta_{\mu}(x) \Delta_{\nu}(x) \Delta_{\mu\nu}(x) = \frac{m^2}{6(3D - 4)} \left[ (5D - 8) \Delta^3(0) \right].
\end{align*}
\[ -2(D - 4)m^2 \int d^D x \Delta^4(x) = \frac{1}{D-1} - \frac{1}{32m} \tag{19} \]

where \( \Delta^4(x) = \int d^D x \Delta^6(x) \Delta^2(x) \Delta^2(0) + (D + 4)m^2 \int d^D x \Delta^4(x) = \frac{1}{D-1} - \frac{1}{32m} \tag{20} \)

Summing up all contributions, we recover (8), thus confirming the invariance of the perturbatively defined path integral under coordinate transformations.

V. BESSEL FUNCTION REPRESENTATION

In \( D = 1 - \varepsilon \) dimensions, the distributions in the Feynman integrals can be expressed in terms of modified Bessel functions \( K_{\nu}(z) \). The basic correlation function in \( D \)-dimension is obtained from the integral in Eq. (17) as

\[ \Delta(x) = c_D z^{1-D/2} K_{1-D/2}(z), \tag{21} \]

where \( z \equiv m|x| \) is reduced length of \( x_\mu \), with the usual euclidean norm \( |x| = \sqrt{x_1^2 + \ldots + x_D^2} \), and \( K_{1-D/2}(z) \) is the modified Bessel function. The constant factor in front is \( c_D = m^{D-2}/(2\pi)^{D/2} \). In one dimension, the correlation function (21) reduced to (3). The short-distance properties of the correlation functions is governed by the small-\( z \) behavior of Bessel function at origin (4):

\[ K_{\nu}(z) \approx \frac{1}{2} \Gamma(\nu)(z/2)^{-\nu}, \text{ Re } \nu > 0. \tag{22} \]

In the application to path integrals, we set the dimension equal to \( D = 1 - \varepsilon \) with a small positive \( \varepsilon \), whose limit \( \varepsilon \to 0 \) will yield the desired results in \( D = 1 \) dimension. In this regime, Eq. (22) shows that the correlation function (21) is regular at the origin:

\[ \Delta(0) = \frac{m^{D-2}}{(4\pi)^{D/2}} \Gamma \left( 1 - \frac{D}{2} \right) = \frac{1}{D-1} = \frac{1}{2m}. \tag{23} \]

The first derivative of the correlation function (21), which is the \( D \)-dimensional extension of time derivative (3), reads

\[ \Delta_{\mu}(x) = -c_D z^{1-D/2} K_{D/2}(z) \partial_{\mu} z, \tag{24} \]

where \( \partial_{\mu} z = m x_\mu/|x| \). By Eq. (22), this is regular at the origin for \( \varepsilon > 0 \), such that the antisymmetry \( \Delta_{\mu}(-x) = -\Delta_{\mu}(x) \) makes \( \Delta_{\mu}(0) = 0 \).

In contrast to these two correlation functions, the second derivative

\[ \Delta_{\mu\nu}(x) = \Delta(x) (\partial_\mu z)(\partial_\nu z) + \frac{c_D}{D-2} z^{D/2} K_{D/2}(z) \partial_{\mu\nu} z^{2-D} \tag{25} \]

is singular at short distance. The singularity comes from the second term in (25):

\[ \partial_{\mu\nu} z^{2-D} = (2 - D) m^{2-D} \left( \frac{\delta_{\mu\nu} - D x_\mu x_\nu}{|x|^2} \right), \tag{26} \]

which is a distribution that is ambiguous at origin, and defined up to the addition of a \( \delta^{(D)}(x) \)-function. It is regularized in the same way as the divergence in the Fourier representation Eq. (12). Contracting the indices \( \mu \) and \( \nu \) in Eq. (26), we obtain

\[ \partial^2 z^{2-D} = (2 - D) m^{2-D} S_D \delta^{(D)}(x), \tag{27} \]

where \( S_D = 2\pi^{D/2}/\Gamma(D/2) \) is the surface of a sphere in \( D \) dimension. Contracted Eq. (27)

\[ \Delta_{\mu\nu}(x) = m^2 \Delta(x) - c_D m^{2-D} S_D \frac{1}{2} \Gamma(D/2) \Gamma(2D/2) \delta^{(D)}(x) \]

\[ = m^2 \Delta(x) - \delta^{(D)}(x), \tag{28} \]

coincides with the definition of the correlation function by the inhomogeneous field equation

\[ (-\partial^2_{\mu} + m^2) g(x) = \delta^{(D)}(x). \tag{29} \]

From Eqs. (12) and (28) we see that

\[ \Delta_{\mu\nu}(0) = m^2 \Delta(0) = \frac{m}{D-1}. \tag{30} \]

which determines the Feynman diagrams (13) and (14).

A further relation between distributions is found from the derivative

\[ \partial_{\mu} \Delta_{\mu\nu}(x) = \partial_{\mu} \left[ -\delta^{(D)}(x) + m^2 \Delta(x) \right] + m S_D \left[ \Delta(x)|x|^{D-1}(\partial_{\nu} z) \right] \delta^{(D)}(x) = \partial_{\mu} \Delta_{\lambda\lambda}(x). \tag{31} \]

Let us now turn to the calculation of Feynman integrals (13)–(20) over products of distributions.

VI. INTEGRALS OVER TWO DISTRIBUTIONS

Consider now the integrals over products of two such distributions. If an integrand \( f(|x|) \) depends only on \(|x|\), we may perform the integrals over the directions of the vectors

\[ \int d^D x f(x) = S_D \int_0^\infty dr r^{D-1} f(r), \quad r \equiv |x|. \tag{32} \]

Thus we can calculate directly (3):

\[ \int d^D x \Delta^2(x) = m^{-D} c_D^2 S_D \int_0^\infty dz z K_{1-D/2}^2(z) \]

\[ = m^{-D} c_D^2 S_D \frac{1}{2} (1 - D/2) \Gamma(1 - D/2) \Gamma(D/2) \]

\[ = \frac{2 - D}{2m^2} \Delta(0). \tag{33} \]
and
\[ \int d^D x \Delta^2_\mu(x) = m^{2-D} c_D^3 S_D \int_0^\infty dz z K^2_{D/2}(z) \]
\[ = m^{2-D} c_D^2 S_D \frac{1}{2} \Gamma (1 + D/2) \Gamma (1 - D/2) = \frac{D}{2} \Delta(0), \quad (34) \]
which together with (33) explains the values given for the Feynman integrals in Eqs. (16) and (17).

Note that due to the relation (3):
\[ K_{D/2}(z) = -z^{D/2-1} \frac{d}{dz} \left[ z^{1-D/2} K_{1-D/2}(z) \right], \quad (35) \]
the integral over \( z \) in Eq. (34) can also be performed by parts, yielding
\[ \int d^D x \Delta^2_\mu(x) \]
\[ = -m^{2-D} c_D^2 S_D \left( z^{D/2} K_{D/2} \right) \left[ z^{1-D/2} K_{1-D/2} \right] \bigg|_0^\infty \]
\[ - m^2 \int d^D x \Delta^2(x) = \Delta(0) - m^2 \int d^D x \Delta^2(x). \quad (36) \]

The upper limit on the right-hand side does not contribute because of the exponentially fast decrease of the Bessel function at infinity (3).

Using the explicit representation (33), we calculate similarly the integral
\[ \int d^D x \Delta^2_\mu(x) = \int d^D x \Delta^2_\nu(x) \quad (37) \]
\[ = m^4 \int d^D x \Delta^2(x) - m^{4-D} c_D^3 \Gamma (D/2) \Gamma (1 - D/2) S_D \]
\[ = m^4 \int d^D x \Delta^2(x) - 2m^2 \Delta(0) = -(1 + D/2) m^2 \Delta(0). \]

The first equality follows from two partial integrations and Eq. (31). In the last equality we have used (33). We have omitted an integral containing modified Bessel functions
\[ (D - 1) \left[ \int_0^\infty dz K_{D/2}(z) K_{1-D/2}(z) + \right. \]
\[ \left. + \frac{D}{2} \int_0^\infty dz z^{-1} K^2_{D/2}(z) \right] \quad (38) \]
since this vanishes in one dimensions as follows:
\[ -\frac{\pi}{4} \Gamma (1 - \varepsilon/2) [\Gamma (\varepsilon/2) + \Gamma (-\varepsilon/2)] \varepsilon^2 \Gamma (\varepsilon) \varepsilon \to 0. \]

The result (37) explains the value stated for the Feynman integral in Eq. (15).

The results (33), (34), (35) can be used to derive a fundamental rule that the integral over the square of the \( \delta \)-function vanishes. Indeed, solving the inhomogenous field equation (38) for \( \delta(D)(x) \), and squaring it, we obtain
\[ \int d^D x \left[ \delta^2(x) \right]^2 = m^4 \int d^D x \Delta^2(x) \]
\[ + 2m^2 \int d^D x \Delta^2_\mu(x) + \int d^D x \Delta^2_\nu(x) = 0. \quad (39) \]
Thus we may formally calculate
\[ \int d^D x \delta^2(D) = \delta^2(0) = 0, \quad (40) \]
pretending that one of the two \( \delta \)-functions is an admissible test function of ordinary distribution theory.

VII. INTEGRALS OVER FOUR DISTRIBUTIONS

We now calculate the integrals over products of four distributions required for the diagrams (18)–(20). These integrals are straightforward in \( D = 1 \) dimension, as long as they are unique. Only ambiguous cases require a calculation in \( D = 1 - \varepsilon \) dimension, with the limit \( \varepsilon \to 0 \) taken at the end.

A unique case is
\[ \int d^D x \Delta^4(x) = c_D^4 m^{-D} S_D \int_0^\infty dz z^{3-D} K^4_{1-D/2}(z) \]
\[ \approx c_D^4 m^{-1} S_D \frac{\pi^2}{2} \Gamma (\frac{3}{2} - \frac{D}{2}) \Gamma (D) = \frac{1}{32 m^3}. \quad (41) \]

Similarly, we calculate
\[ \int d^D x \Delta^2(x) \Delta^2_\mu(x) \]
\[ = m^{2-D} c_D^3 S_D \int_0^\infty dz z^{3-D} K^2_{D/2}(z) K_{1-D/2}(z) \]
\[ = \frac{1}{3} \int d^D x \Delta^4(x) \Delta^2(x) \Delta^2_\mu(x) \]
\[ = \frac{1}{3} \Delta^3(0) - m^2 \int d^D x \Delta^4(x) \]
\[ = \frac{1}{3} \left[ \Delta^3(0) - m^2 \int d^D x \Delta^4(x) \right] = \frac{1}{32 m^3}. \quad (42) \]

Using the expressions (24) and (25), we find for the integral in \( D = 1 - \varepsilon \) dimensions
\[ \int d^D x \Delta(x) \Delta_\mu(x) \Delta_\nu(x) \]
\[ = m^2 \int d^D x \Delta^2(x) \Delta^2_\mu(x) + I_D, \quad (43) \]
where \( I_D \) denotes the singular integral
\[ I_D = (D - 1) m^{4-D} c_D^4 S_D \]
\[ \times \int_0^\infty dz z^{2-D} K_{1-D/2}(z) K^3_{D/2}(z). \quad (44) \]

In spite of the prefactor \( D - 1 \), this has a nontrivial limit for \( D \to 1 \), the zero being compensated by a pole from
Indeed, using the Bessel expressions (24) and (28), we substitute (46) into (44), we obtain the finite value
\[ K \approx \frac{1}{2} \left( 1 \mp i \right) \Gamma \left( \frac{1}{2} \mp i \right) \gamma \left( \frac{1}{2} \right) \mp \int_0^\infty dt (cosh t)^{-2 \nu} cos(z sinh t). \] (45)

In one dimension where \( \nu = 1/2 \), this becomes simply
\[ K_{1/2}(z) = \sqrt{\pi/2} e^{-z}. \] For \( \nu = D/2 \) and \( \nu = 1 - D/2 \) written as \( \nu = (1 \mp \epsilon)/2 \), it is approximately equal to
\[ K_{(1 \mp \epsilon)/2}(z) = \frac{\pi}{2} \left( z/2 \right)^{(1 \pm \epsilon)/2} \Gamma \left( 1 \mp \frac{\epsilon}{2} \right) \times \int_0^\infty dt (cosh t)^{-1} \ln(cosh t) cos(z sinh t), \] (46)

where the \( t \)-integral is regular at \( z = 0 \). After substituting (46) into (44), we obtain the finite value
\[ I_D \approx -\left( m^{4-D} c^D_s S_D \right) \epsilon \times \frac{\pi^2}{4} \Gamma \left( 1 + \epsilon/2 \right) \Gamma \left( 1 - \epsilon/2 \right) \times 2^{-5\epsilon} \Gamma(2\epsilon) \]
\[ \approx -\left( \frac{1}{2m\pi^\epsilon} \right) \frac{\pi^2}{8} = -\frac{1}{16\pi^\epsilon}. \] (47)

The prefactor \( D - 1 = -\epsilon \) in (44) has been canceled by the pole in \( \Gamma(2\epsilon) \).

The nontrivial nature of the integral \( I_D \) was first observed in another form in the momentum space calculations of Ref. [1], where \( I_D \) appeared in the integral
\[ \int d^D x \Delta^2(x) [\Delta^2_{\mu\nu}(x) - \Delta^2_{\lambda\lambda}(x)] = -2I_D, \] (48)

Indeed, using the Bessel expressions (24) and (28), we find
\[ \int d^D x \Delta^2(x) [\Delta^2_{\mu\nu}(x) - \Delta^2_{\lambda\lambda}(x)] = -(D - 1)m^{4-D} \]
\[ \times c^D_s S_D \int_0^\infty dz \left( z^{1-D/2} K_{1-D/2} \right) \frac{2}{dz} K^2_{D/2}, \] (49)

and a partial integration
\[ \int_0^\infty dz \left( z^{1-D/2} K_{1-D/2} \right) \frac{2}{dz} K^2_{D/2} \]
\[ = 2 \int_0^\infty dz z^{2-D} K_{1-D/2} K^3_{D/2}, \] (50)

establishes contact with the integral (44) for \( I_D \).

Knowing \( I_D \), we also determine, after integrations by parts, the integral
\[ \int d^D x \Delta^2_{\mu}(x) \Delta^2_{\nu}(x) = -3m^2 \int d^D x \Delta^2(x) \Delta^2_{\mu}(x) - 2I_D. \] (51)

It remains to calculate one more unproblematic integral over four distributions:
\[ \int d^D x \Delta^2(x) \Delta^2_{\lambda}(x) = \left[ -2m^2 \Delta^3(0) + m^4 \int d^D x \Delta^4_{\lambda}(x) \right]_{D=1} = -\frac{7}{32}. \] (52)

Combining this with (47) and (51) we find the the Feynman diagram (18). The combination of (14) and (51) with (17) and (42), finally, yields the diagrams (19) and (21), respectively.

**VIII. SUMMARY**

In this note we have shown that with appropriate Bessel function representations, we can evaluate integrals over products of distributions in configuration space which reproduce the results of dimensional regularization. This ensures the invariance of perturbatively defined path integral under coordinate transformations observed in [1].

[1] H. Kleinert, Path Integrals in Quantum Mechanics, Statistics, and Polymer Physics, World Scientific, Singapore, 1995 (www.physik.fu-berlin.de/~kleinert/b3).

[2] N.N. Bogoliubov and D.V. Shirkov, Introduction to the Theory of Quantized Fields, Interscience, New York, 1959.

[3] L. Schwartz, Theorie des distributions, Vols.I-II, Hermann & Cie, Paris, 1950-51; I. M. Gel’fand, G. E. Shilov, Generalized functions, Vols.I-II, Academic Press, New York-London, 1964-68.

[4] H. Kleinert and A. Chervyakov, Phys. Lett. B 464, 257 (1999) [hep-th/9906154].

[5] G. ’t Hooft and M. Veltman, Nucl. Phys. B 44, 189 (1972).

[6] H. Kleinert, A. Chervyakov, B. Hamprecht, Phys. Lett. A 260, 182 (1999) [cond-mat/9906241].

[7] H. Kleinert, Phys. Rev. D 57, 2264 (1998) (www.physik.fu-berlin.de/~kleinert/257); Addendum: ibid. D 58, 1077 (1998) [cond-mat/9803268].

[8] W. Magnus, F. Oberhettinger, R. P. Soni, Formulas and Theorems for the Special Functions of Mathematical Physics, Springer-Verlag, Berlin, Heidelberg, New York, 1966.

[9] H. Bateman and A. Erdelyi, Higher transcendental functions, v.2, McGraw-Hill Book Company, Inc., 1953.

[10] I.S. Gradshteyn and I.M. Ryzhik, Table of Integrals, Series, and Products, Academic Press, New York, London, 1965.

[11] See Ref. [1], p. 85, or Ref. [4], p. 83, Formula 27.

[12] See Ref. [10], Formulas 3.511.1 and 3.521.2.