Search for exotic spin-dependent interactions using polarized helium

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We investigate the sensitivity of a search for exotic spin-dependent interactions using polarized $^3$He with unpolarized mass of BGO and polarized mass of DyIG. The spin-dependent interactions between the polarized nuclear spins of $^3$He and the mass induces an effective magnetic field, causing the frequency shift to the precession of $^3$He. The signal of the $^3$He nuclear spins can be measured using optically pumped magnetometers which is the most sensitive noncryogenic magnetic-field sensor. A careful design using gradiometers or co-magnetometers can help to reduce the systematics and explore different spin-dependent interactions. An alternative approach is the resonance method proposed by Arvanitaki and Geraci [Phys. Rev. Lett. 113, 161801 (2014)]. The mass movement is modulated at the frequency equal to the Larmor frequency of $^3$He. The effective magnetic field induced by spin-dependent interactions can rotate the $^3$He nuclear spins. For a given ultra-long relaxation time of polarized $^3$He spins, this resonance method could be sensitive to the magnetic field much below $1 \, \text{fT}$. If the resonance method works, the sensitivity of the spin-dependent interactions can be significantly improved over existing experiments, and new limits on the coupling strengths can be set in the interaction range below $10^{-11} \, \text{m}$.

Since the discovery of intrinsic spin [1], exotic spin-dependent interactions between fermions apart from the magnetic interaction have been considered. Moody and Wilczek [2] considered new macroscopic interactions existing between spin-polarized and spin-unpolarized particles through a spin-0 boson. Later, Dobrescu and Mocioiu [3] extended this idea by including the relative velocity dependence in the non-relativistic limit between two interacting particles, mediated by a spin-1 boson. Recently, these exotic spin-dependent interactions have attracted people’s attention because they are observables in many mysteries in fundamental physics. For example, the axion or axion-like-particles (ALPs) [5] may also explain the cold dark matter [6]. Several theoretical motivations including string theory [7], hierarchy problem [8], dark energy [9], unparticles [10], dark photons [11] also predict the existence of new spin-0 or spin-1 bosons. A review article describing the recent theoretical progress in this field can be found in Ref. [14].

There are fifteen possible exotic spin-dependent interactions described in Ref. [3], which have been revisited in a convenient format [13]. The contact terms have also been studied for the superficial singularity [15, 16]. In this paper we still use the fifteen interaction formats for easier experiment setups adopting the numbering scheme in Ref. [3]. In a system of two particles (particle 1, 2 can be electrons, neutrons or protons, etc.) with spin 1 ($\hat{s}_1$) and 2 ($\hat{s}_2$), and mass 1 ($m_1$) and 2 ($m_2$) respectively, their relative distance and relative velocity are $\vec{r}$ and $\vec{v}$. These interactions can be grouped as one-spin-dependent interactions, spin-spin-dependent interactions and spin-velocity-dependent interactions. Group one can be presented as:

$$V_{4+5} = -f_4 + 5 \frac{\hbar^2}{8\pi m_1 c} \left[ \hat{s}_1 \cdot (\vec{v} \times \hat{r}) \right] \left( \frac{1}{\lambda r} + \frac{1}{r^2} \right) e^{-r/\lambda},$$

$$V_{9+10} = f_9 + 10 \frac{\hbar^2}{8\pi m_1} \hat{s}_1 \cdot \hat{r} \left( \frac{1}{\lambda r} + \frac{1}{r^2} \right) e^{-r/\lambda},$$

$$V_{12+13} = f_{12} + 13 \frac{\hbar^2}{8\pi} \hat{s}_1 \cdot \hat{v} \left( \frac{1}{r} \right) e^{-r/\lambda}. \quad (1)$$

Group two are:

$$V_2 = f_2 \frac{\hbar c}{4\pi} \left( \hat{s}_1 \cdot \hat{s}_2 \right) \left( \frac{1}{r} \right) e^{-r/\lambda},$$

$$V_3 = f_3 \frac{\hbar^3}{4\pi m_1 m_2 c} \left[ \hat{s}_1 \cdot \hat{s}_2 \right] \left( \frac{1}{\lambda r^2} + \frac{1}{r^2} \right) - \hat{s}_1 \cdot \hat{r} \hat{s}_2 \cdot \hat{r} \left( \frac{1}{\lambda r^2} + \frac{3}{\lambda r^2} + \frac{3}{r^2} \right) e^{-r/\lambda}, \quad (5)$$

$$V_{11} = -f_{11} \frac{\hbar^2}{4\pi m_\mu} \hat{s}_1 \times \hat{s}_2 \cdot \hat{r} \left( \frac{1}{\lambda r} + \frac{1}{r^2} \right) e^{-r/\lambda}, \quad (6)$$

and the remaining six are:

$$V_{6+7} = -f_6 + 7 \frac{\hbar^2}{4\pi m_\mu c} \times \left[ \hat{s}_1 \cdot (\vec{v} \times \hat{r}) \right] \left( \frac{1}{\lambda r} + \frac{1}{r^2} \right) e^{-r/\lambda}, \quad (7)$$

$$V_8 = f_8 \frac{\hbar}{4\pi c} \left( \hat{s}_1 \cdot \vec{v} \right) \hat{s}_2 \cdot \vec{v} \left( \frac{1}{r} \right) e^{-r/\lambda}, \quad (8)$$

$$V_{14} = f_{14} \frac{\hbar}{4\pi} \left( \hat{s}_1 \cdot \hat{s}_2 \right) \hat{r} \left( \frac{1}{r} \right) e^{-r/\lambda}, \quad (9)$$

$$V_{15} = -f_{15} \frac{\hbar^3}{8\pi m_1 m_2 c^2} \times \left[ \hat{s}_1 \cdot (\vec{v} \times \hat{r}) \right] \left( \hat{s}_2 \cdot \hat{r} + \hat{s}_1 \cdot \hat{r} \right) \left( \hat{v} \times \hat{r} \right) \times \left( \frac{1}{\lambda^2 r} + \frac{3}{\lambda r^2} + \frac{3}{r^2} \right) e^{-r/\lambda}, \quad (10)$$

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$V_{16} = -f_{10} \frac{\hbar^2}{8\pi m_{\mu} c^2} \times \left\{ (\hat{\sigma}_1 \cdot (\vec{v} \times \hat{r})) (\hat{\sigma}_2 \cdot \vec{v}) + (\hat{\sigma}_1 \cdot \vec{v}) (\hat{\sigma}_2 \cdot (\vec{v} \times \hat{r})) \right\} \times \left( \frac{1}{\lambda r} + \frac{1}{r^2} \right) e^{-r/\lambda}$ \hfill (11)

where $m_{\mu}$ is the reduced mass of $m_1$ and $m_2$, and $\lambda$ is the interaction length. $f_i$’s are the coupling strengths that we are measuring, which can be the combination of scalar, pseudoscalar, vector and axial-vector coupling \[14, 16\]. All spin-dependent interactions have the form of the potential as $\hat{\sigma}_1 \cdot \hat{A}$ where $\hat{A}$ is a field induced by the spin-dependent interactions, which is similar to the system of a spin in a magnetic field, $\hat{\sigma}_1 \cdot \vec{B}$ \[15\]. Therefore, this effective field $\hat{A}$ can affect a spin as a magnetic field.

Several experimental methods on various spin-dependent interactions over a broad interaction range have been conducted, including spectroscopy, torsion-pendulum, magnetometry, parity nonconservation and electric dipole moment experiments \[14, 16\]. However, most experimental searches are still related to static spin-dependent interactions including $V_2, V_3, V_{9+10}, V_{11}$. For the velocity-dependent spin interactions, Yan and Snow used neutron beams to study the spin-velocity-dependent interaction ($V_{12+13}$) \[19\] and later Yan et al. used the relaxation of polarized $^3\text{He}$ to explore the same interaction \[20\]. Adelberger and Wagner also set new constraints by combining different experimental limits \[21\]. Piegsa and Pignol used Ramsey’s technique of separated oscillatory fields with a cold neutron beam to investigate $V_{4+5}$ \[22\]. Hunter et al. first applied polarized geoelectrons to search for long-range spin-spin interactions \[23\] and later expanded the idea to the velocity-dependent interactions \[24\]. Ji et al. proposed to use K-$^3\text{He}$ spin-exchange-relaxation-free (SERF) comagnetometers with SmCo$_5$ spin sources \[25\] to search for exotic spin-dependent interactions and later Ji et al used K-Rb SERF comagnetometers with SmCo$_5$ spin sources to set new limits on spin-spin-velocity-dependent interactions \[26\]. Leslie et al. proposed to search exotic spin-dependent interactions with rare earth iron garnet test masses (dysprosium iron garnet(DyIG)) \[15\] while the paramagnetic insulator, gadolinium gallium garnet (GGG), also has potential for spin-dependent interactions \[27\].

In Ref. \[18\], Rb SERF magnetometers were first proposed to search for all fifteen spin-dependent interactions of electron spins with unpolarized mass (bismuth germanate, BGO) and polarized mass (DyIG) and later, the same group published new constraints of $V_{4+5}$ \[28\] and $V_{12+13}$ \[29\]. In this paper, we consider spin-exchange optical pumping (SEOP) polarized $^3\text{He}$ target \[30\] for all fifteen spin-dependent interactions of the nuclear spins with unpolarized nucleons in unpolarized mass (BGO) and electron spins in polarized mass (DyIG) \[18\]. Meanwhile, Arvanitaki and Geraci proposed the resonance method \[31\] having potential to significantly improve the current constraints, which will be also discussed in this paper.

TABLE I. Geometry of experiment for each interactions for the frequency method. $\hat{\sigma}_1, \hat{\sigma}_2, \vec{v}, \vec{B}_{\text{eff}}$ for different interactions.

| interaction | $\hat{\sigma}_1$ | $\hat{\sigma}_2$ | $\vec{v}$ position | $\vec{B}_{\text{eff}}$ | $\delta r$(mm) |
|-------------|-----------------|-----------------|-------------------|-----------------|----------------|
| $V_{4+5}$   | $\hat{z}$       | $0$             | $\phi$           | $z$             | $\hat{z}$      | $1$            |
| $V_{9+10}$  | $\hat{0}$       | $\hat{z}$       | $z$              | $\hat{z}$       | $1$            |
| $V_{12+13}$ | $\hat{0}$       | $\hat{0}$       | $z$              | $\hat{z}$       | $6$            |
| $V_2$       | $\hat{z}$       | $\hat{z}$       | $z$              | $\hat{z}$       | $1$            |
| $V_3$       | $\hat{z}$       | $\hat{z}$       | $\hat{z}$        | $z$             | $1$            |
| $V_{11}$    | $\hat{x}$       | $\hat{y}$       | $\hat{z}$        | $\hat{z}$       | $1$            |
| $V_{6+7}$   | $\hat{z}$       | $\hat{z}$       | $\hat{z}$        | $z$             | $15$           |
| $V_6$       | $\hat{z}$       | $\hat{z}$       | $z$              | $\hat{z}$       | $15$           |
| $V_{14}$    | $\hat{x}$       | $\hat{y}$       | $\hat{z}$        | $\hat{z}$       | $15$           |
| $V_{15}$    | $\hat{z}$       | $\hat{z}$       | $\phi$           | $z$             | $10$           |
| $V_{16}$    | $\hat{x}$       | $\hat{y}$       | $\hat{z}$        | $z$             | $10$           |

A typical SEOP $^3\text{He}$ cell has an optical pumping chamber and a target chamber connected by a glass tube as shown in Fig. 1 \[30\]. The top piece is a spherical pumping chamber having a comparable volume of the target chamber. The bottom piece is a cylinder target chamber with the radius of 1 cm and the length of 2 cm. The reasonable cell wall thickness is about 1 mm while a thinner wall about the order of 100 $\mu$m is possible \[32\]. The target chamber is at the center of the holding field $B_0\hat{z}$ with correction coils in order to optimize the relaxation time. We plan to use BGO as the unpolarized mass for its high nuclear density and small magnetic effects \[33\], and DyIG as the polarized mass which has shown the property of near-zero magnetization at the critical temperature around 220–240K \[15\]. The BGO mass is a cube of 2 cm used in Ref. \[28\] and Ref. \[29\]. The DyIG is a cylinder with the radius of 0.4 cm and the length 0.2
In the scenario of the frequency method, the spin-dependent interactions act as an effective magnetic field shifting the Larmor frequency. Two recent experiments are considered in order to design the experiment using polarized \(^3\)He and estimate the sensitivity. Chu et al. applied the method of gradiometers, using two pickup coils to measure different parts of the \(^3\)He target, which have different effects from the spin-dependent interactions. In the environment without magnetic shielding, their frequency sensitivity is about \(10^{-5}\) Hz, corresponding to \(3 \times 10^{-13}\) T for \(^3\)He with the gyromagnetic ratio \(\gamma_3 = (2\pi) \times 32.4\) MHz/T. Tullney et al. applied the Xe-\(^3\)He comagnetometer in the magnetically shielded room, measuring the precession of two atomic species, which have different effects from the spin-dependent interactions. They can reach the sensitivity of \(10^{-9}\) Hz, corresponding to \(3 \times 10^{-17}\) T for \(^3\)He. The practical sensitivity is probably between these two values if using a magnetically shielded room. Table I shows the experimental setup using the frequency method. The \(\hat{\sigma}_1\) means the spin orientation of the \(^3\)He along the holding field. The \(\hat{\sigma}_2\) means the spin orientation of the polarized mass. The \(\vec{v}\) is the velocity direction of the mass, where \(\phi\) means the rotation around the \(\hat{z}\)-axis, \(\hat{z}\) means the movement along the \(\hat{z}\)-axis, vice versa. The position means the position of the mass relative to the \(^3\)He target, where we assume the mass is always at the side to the \(\hat{z}\)-axis in this scenario. \(\vec{B}_{\text{eff}}\) means the effective magnetic field direction which is always along holding field direction. \(\delta r\) means the relative minimum distance from the mass to the \(^3\)He target. \(\delta r = 1\) mm means the unpolarized mass can touch the target while \(\delta = 5\) mm means the unpolarized mass needs to have a small distance for the velocity-dependent effect. \(\delta r = 10\) mm means an additional thermal insulator thickness for the polarized mass can be reduced. Although at the critical temperature the polarized mass DyIG has zero magnetization, the temperature fluctuation could induce additional magnetic noise. Additional magnetic shields may be necessary to reduce the magnetic effect.

We also consider the resonance method in the room temperature environment in order to simplify the experimental apparatus as a pathfinder for the low temperature experiment. The gradiometer method can be considered, while the comagnetometer method unlikely works because of different resonance frequencies between the two atomic species. The spin-dependent interactions are modulated at the frequency equal to the Larmor frequency, \(\omega_N = \gamma_3 B_0\). The induced effective magnetic field \(B_{\text{eff}}(t) \approx B_{\text{eff}} \cos(\omega_N t)\) acts as a linear oscillatory magnetic field perpendicular to the holding field, which can rotate spins. The time-varying perpendicular magnetization \(M_x\) of \(^3\)He is considered in response to the effective magnetic field as

\[
M_x(t) \approx \frac{1}{2} p n_s \mu_3 \gamma_3 B_{\text{eff}} T_2 (e^{-t/T_1} - e^{-t/T_2}) \cos(\omega_N t)
\]

where \(p\) is the polarization fraction of \(^3\)He spins, \(n_s\) is the spin density in the sample, and \(\mu_3 = -2.12 \times \mu_N\) is the nuclear magnetic moment of \(^3\)He, \(T_1\) and \(T_2\) are the longitudinal and transverse relaxation time \((T_1 \gg T_2)\) and the measurement time \(t \approx T_2\). The polarization is close to 1. The spin density is equal to the particle density, which can be calculated using \(PV = nRT\) with \(P = 1\) atm, \(V = 1\) cm\(^3\) and \(T = 300\) K. The reasonable \(T_2\) is about 1000 seconds while it can reach up to 53 hours. The magnetization noise is \(\sqrt{\frac{M_N^2}{V}} = \sqrt{\hbar \gamma_3 n_s \mu_3 T_2/2V} \approx 2\) fT/\(\mu_0\). Here we propose using optical pumping magnetometers to measure \(M_x(t)\) and its sensitivity of the magnetization is about 1 fT, which is about the same level of the magnetization noise. If the signal is still \(\sim 1\) fT after 1000 seconds of measurement, this implies the oscillatory magnetic field upper limit is about \(5 \times 10^{-20}\) T. Therefore, the sensitivity of the effective magnetic field is about \(5 \times 10^{-20}\) T or \(2.5 \times 10^{-22}\) T for \(T_2 = 1000\) s or \(T_2 = 53\) h respectively.

| TABLE II. Parameter values in Eq. (12) |
|----------------------------------------|
| Parameters    | Symbol | Value       |
| polarization  | \(p\) | 1           |
| spin density of \(^3\)He | \(n_s\) | \(2.4 \times 10^{19}\) cm\(^-3\) |
| nuclear magnetic moment of \(^3\)He | \(\mu_3\) | \(-1.07 \times 10^{-26}\) J/T |
| gyromagnetic ratio of \(^3\)He | \(\gamma_3\) | \(2.03 \times 10^8\) Hz/T |
| transverse relaxation time | \(T_2\) | 1000 s or 53 h |
| BGO nucleon density | \(\rho_{BGO}\) | \(4.29 \times 10^{30}\) m\(^-3\) |
| DyIG spin density | \(\rho_{DyIG}\) | \(1 \times 10^{26}\) m\(^-3\) |

| TABLE III. Geometry of experiment for each interaction for the resonance method. \(\hat{\sigma}_1\), \(\hat{\sigma}_2\), and \(\vec{v}\) for different interactions. |
|---------------------------------|
| interaction | \(\hat{\sigma}_1\) | \(\hat{\sigma}_2\) | \(\vec{v}\) | position | \(B_{\text{eff}}\) | \(\delta r\) (mm) |
| \(V_{4+5}\) | \(\hat{x}\) | 0 | \(\hat{\phi}\) | \(\hat{z}\) | \(\hat{z}\) | 1 |
| \(V_{4+10}\) | \(\hat{x}\) | 0 | \(\hat{\phi}\) | \(\hat{z}\) | \(\hat{z}\) | 1 |
| \(V_{12+13}\) | \(\hat{x}\) | 0 | \(\hat{\phi}\) | \(\hat{z}\) | \(\hat{z}\) | 6 |
| \(V_2\) | \(\hat{x}\) | \(\hat{z}\) | \(\hat{z}\) | \(\hat{z}\) | \(\hat{z}\) | 1 |
| \(V_3\) | \(\hat{x}\) | \(\hat{z}\) | \(\hat{z}\) | \(\hat{z}\) | \(\hat{z}\) | 1 |
| \(V_{11}\) | \(\hat{z}\) | \(\hat{z}\) | \(\hat{y}\) | \(\hat{z}\) | \(\hat{x}\) | 1 |
| \(V_{6+7}\) | \(\hat{x}\) | \(\hat{z}\) | \(\hat{z}\) | \(\hat{z}\) | \(\hat{z}\) | 15 |
| \(V_8\) | \(\hat{x}\) | \(\hat{z}\) | \(\hat{z}\) | \(\hat{z}\) | \(\hat{z}\) | 15 |
| \(V_{14}\) | \(\hat{z}\) | \(\hat{z}\) | \(\hat{z}\) | \(\hat{z}\) | \(\hat{z}\) | 15 |
| \(V_{15}\) | \(\hat{x}\) | \(\hat{z}\) | \(\hat{z}\) | \(\hat{z}\) | \(\hat{z}\) | 10 |
| \(V_{16}\) | \(\hat{z}\) | \(\hat{y}\) | \(\hat{y}\) | \(\hat{y}\) | \(\hat{y}\) | 10 |
Table [11] shows the experimental setup using the resonance method. The definition of each parameter is the same as Table 1. $\vec{B}_{\text{eff}}$ means the effective magnetic field direction which is always perpendicular to the holding field direction. For the static spin-dependent interaction such as $V_{9+10}$, $V_2$, $V_3$, and $V_{11}$, the effective magnetic field can be modulated by the distance $r(t) = r(1 + \cos(\omega_N t)) + \delta r$ so that $B_{\text{eff}}(t) \approx B_{\text{eff}}(1 + \cos(\omega_N t))$. Because of $B_{\text{eff}} \ll B_0$, $\omega_N \approx \gamma_3 B_0$. Figure [2] shows the example of $V_{9+10}$. The effective magnetic field is proportional to $A \equiv (\frac{\lambda}{\gamma_3} + \frac{1}{2\gamma_3}) e^{-r/\lambda}$, showing spikes with the frequency equal to $\omega_N$. The components of high frequencies larger than $\omega_N$ can be neglected when considering the effective oscillatory magnetic field, which should be considered when estimating the sensitivity to the effective magnetic field in Eq. [12].

For the spin-velocity-dependent interactions such as $V_{9+10}$, $V_{2+3+13}$, $V_{6+7}$, $V_{14}$ and $V_{15}$, it is straightforward to modulate the velocity such as $v(t) = v_0 \cos(\omega_N t)$ so that $B_{\text{eff}}(t) \approx B_{\text{eff}} \cos(\omega_N t)$. Figure [2] shows the example of $V_{12+13}$ using the same modulation as in Fig. [2]. The effective magnetic field is proportional to $A \equiv (\frac{\lambda}{\gamma_3} + \frac{1}{2\gamma_3}) e^{-r/\lambda}$, showing spikes with the frequency equal to $\omega_N$. For the velocity-velocity-spin-dependent interactions such as $V_8$ and $V_{16}$, the effective oscillatory field at the resonance of $\omega_N$ can be still generated by the modulation of the velocity as shown in Fig. [4] using the same modulation in Fig. [2]. However, the fast Fourier transform (FFT) of $A(t)$ implies that the effective magnetic field is weaker for the velocity-velocity-spin-dependent interactions.

In Fig. [3] [7], we simply consider the magnetic sensitivity of $3 \times 10^{-13}$, $3 \times 10^{-17}$, $5 \times 10^{-20}$, and $2.5 \times 10^{-22}$ T for each interaction. In $V_{4+5}$, the only limit for the nuclear spin-dependent interaction was done using neutron beams with Ramsey's technique of separated oscillating fields [22]. We expect to improve the constraint with the frequency method or the resonance method. In $V_{9+10}$, most experiments have worked on this interaction [32] [33] [36] [12] and the resonance method at room temperature has a significant opportunity to improve the current limit and even overcome the astrophysics constraints [43]. In $V_{2+3+13}$, the resonance method can also significantly improve the current constraints using neutron beams [19] the relaxation of polarized $^3$He spin relaxation [20], and the combination of different experiments can be also used to derive the limits [21]. For spin-spin-dependent interactions, we only consider nuclear-electron interactions using polarized nuclear spins of $^3$He and polarized electron spins of DyIG. In $V_3$, the only constraint was done using $^{88}$Be$^+$ ion stored in Penning ion trap with polarized electron spins of a magnet [26] and the resonance method should improve the constraints. In $V_2$, $V_{11}$, $V_{6+7}$, $V_{14}$, $V_{15}$, $V_{16}$, the only constraints were all done using the $^{199}$Hg-Cs comagnetometer with polarized electron spins of Earth [23] [24] for possible long-range interactions and there is no constraint in the region of 1 cm for the nuclear-spin-electron-spin-dependent interactions. Using polarized $^3$He target could be the first one to explore this mass interaction length region with the frequency method or the resonance method.

In conclusion, we estimate the sensitivity using a polarized $^3$He target with the frequency method and the resonance method [31] to search for the exotic spin-dependent interactions for polarized nucleons. Our calculations of the projected experimental sensitivity showed that the experiments are sensitive to the interaction range of $10^{-2}$ to $10^{-4}$ m. The resonance method especially has a significant potential to improve the current constraints.

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FIG. 2. The distance between the mass and the target is \( r(t) = 0.01(1 + \cos(\omega_N t)) + 0.001 \) and the velocity is \( v(t) = 0.01\omega_N \sin(\omega_N t) \) where \( \omega_N = 2\pi f_3 = 2\pi \times 1 \) Hz. If \( \lambda = 0.01 \) m, the effective magnetic field \( B_{\text{eff}} \) due to \( V_{9+10} \) is proportional to \( A \equiv \left( \frac{1}{\lambda^2} + \frac{1}{\lambda^2} \right) e^{-r/\lambda} \) which is roughly a function of \( \sin(\omega_N t) \). The fast Fourier transform of \( A \) is FFT(\( A \)) showing the component of \( \omega_N \).

FIG. 3. If \( \lambda = 0.01 \) m, the effective magnetic field \( B_{\text{eff}} \) due to \( V_{12+13} \) is proportional to \( A \equiv v(\frac{1}{r})e^{-r/\lambda} \) which is roughly a function of \( \sin(\omega_N t) \). The fast Fourier transform of \( A \) is FFT(\( A \)) showing the component of \( \omega_N \).

FIG. 4. If \( \lambda = 0.01 \) m, the effective magnetic field \( B_{\text{eff}} \) due to \( V_8 \) is proportional to \( A \equiv v^2(\frac{1}{r})e^{-r/\lambda} \) which is roughly a function of \( \sin(\omega_N t) \). The fast Fourier transform of \( A \) is FFT(\( A \)) showing the component of \( \omega_N \).
FIG. 5. The limits and the sensitivity estimation of $V_{4+5}$, $V_{9+10}$, $V_{12+13}$ and $V_{12+13}$.

FIG. 6. The limits and the sensitivity estimation of $V_2$, $V_3$, $V_{11}$.
FIG. 7. The limits and the sensitivity estimation of $V_{6+7}$ [24, 25], $V_8$ [24], $V_{14}$ [24, 25], $V_{15}$ [24, 25] and $V_{16}$ [24, 25].