Semiclassical zero temperature black holes in spherically reduced theories

C. Barbachoux\textsuperscript{a} and A. Fabbri\textsuperscript{b} \\ \textsuperscript{(a)}LRM-CNRS, Université Pierre et Marie Curie, ERGA \ 4 place Jussieu, 75005 Paris, France \textsuperscript{(b)}Dipartimento di Fisica dell’Università di Bologna and INFN sezione di Bologna, \ Via Irnerio 46, 40126 Bologna, Italy

Abstract

We numerically integrate the semiclassical equations of motion for spherically symmetric Einstein-Maxwell theory with a dilaton coupled scalar field and look for zero temperature configurations. The solution we find is studied in detail close to the horizon and comparison is made with the corresponding one in the minimally coupled case.

\textsuperscript{1}Email: barba@ccr.jussieu.fr \textsuperscript{2}Email: fabbria@bo.infn.it
1 Introduction

The most attractive feature of zero temperature black holes is that they are the natural candidates as end-state of the evaporation process. Indeed, they represent the ideal setting where one can address the various issues connected to the quantum evolution of the black holes, such as for instance the problem of information loss (see e.g. [1]).

In spherically symmetric Einstein-Maxwell theory the only solution with this property is the extremal Reissner-Nordström (RN) black hole. Turning to the semi-classical theory, quantum corrections induced by the vacuum expectation value of the stress energy tensor due to matter fields modify the spacetime geometry and it is very important to check whether the resulting solution has still zero temperature or not. Perturbative corrections $O(\bar{h})$ to the classical geometry evaluated close to the horizon in four dimensions do not appear to answer unambiguously the above question [2], [3]. It is clear that more information would come only if one knew the exact analytical solution to the semiclassical eqs. of motion. For the simple case of spherically reduced Einstein-Maxwell theory coupled with 2D minimal scalar fields Trivedi [4] was able to prove the existence of zero temperature solutions which reduce, as $\bar{h} \to 0$, to the extreme RN black hole. He also showed that although the energy density measured by an infalling observer close to the horizon diverges for the classical solution, the semiclassical configuration is regular there (only a mild singularity emerges in the second derivative of the scalar curvature). The drawback of this analysis is that, due to the special type of matter fields used, these results do not have an obvious four dimensional interpretation. In order to improve this study, we consider here a more realistic 2D model that recently has received a lot of attention. We employ a 2D conformal scalar field nonminimally coupled to the dilaton field, which classically corresponds to the $s$-wave sector of a 4D minimal scalar field (this model was first studied in [5]). We will perform a numerical integration of the semiclassical equations of motion and show good evidence that zero temperature black holes exist in this theory. In particular, we inspect in detail the spacetime geometry in the region close to the horizon and compare with the results one gets by numerical integration of the minimally coupled case.

The outline of this article is the following. In Section 2 we briefly review the spherically reduced Einstein-Maxwell theory and its zero-temperature solution, the extreme RN black hole. In Section 3 the matter model we shall use will be introduced and the expression of the $\langle T_{ab} \rangle$ in the extreme RN background derived. In Section 4 we numerically solve the backreaction equations and finally section 6 contains a discussion of our results and a comparison with the case analysed in [4].
2 Einstein-Maxwell theory in $D=2$

Let us start with Einstein-Maxwell theory in four dimensions

$$S = S_G + S_{EM},$$

(1)

where $S_G$ is the Einstein-Hilbert action

$$S_G = \frac{1}{8\pi} \int d^4x \sqrt{-g(4)} R(4),$$

(2)

and $S_{EM}$ denotes the action associated to the electromagnetic field

$$S_{EM} = -\frac{1}{8\pi} \int d^4x \sqrt{-g(4)} F^2,$$

(3)

$R(4)$ is the 4-dimensional scalar curvature and $F^2$ the strength of the electromagnetic field $F_{\mu\nu}$. Assuming spherical symmetry, the 4D metric can be written

$$ds^2 = g_{ab}(2) dx^a dx^b + e^{-2\phi(x_a)} d\Omega^2,$$

(4)

where $g_{ab}(x^a)$ ($a, b = 1, 2$) is the 2-dimensional metric in the $(r-t)$-plane, $\phi(x_a)$ the dilaton and $d\Omega^2 = d\theta^2 + \sin^2 \theta d\phi^2$ the line element of the unit two-sphere. Dimensional reduction of the Einstein-Hilbert action (2) can be performed by integrating over the angles $\theta$ and $\phi$

$$S_G^{(2)} = \frac{1}{2} \int d^2x \sqrt{-g^{(2)}} e^{-2\phi} \left( R^{(2)} + 2(\nabla \phi)^2 + 2e^{2\phi} \right).$$

(5)

Proceeding similarly and considering $F_{\mu\nu} = F_{\mu\nu}(x^a)$ the Maxwell action becomes

$$S_{EM}^{(2)} = -\frac{1}{2} \int d^2x \sqrt{-g^{(2)}} e^{-2\phi} \bar{F}^2,$$

(6)

where $\bar{F}^2$ represents the field strength of a 2-dimensional gauge field. Black hole solutions of the theory defined by

$$S^{(2)} = S_G^{(2)} + S_{EM}^{(2)}$$

(7)

are given by the Reissner-Nordström solution

$$ds^2 = -f(r) dt^2 + \frac{1}{f(r)} dr^2, \quad e^{-2\phi} = r^2,$$

(8)

with

$$f(r) = 1 - \frac{2M}{r} + \frac{Q^2}{r^2}.$$  

(9)

\footnote{We use units where $\hbar = G = c = k_B = 1$}
The parameter $M$ is the ADM mass and $Q$ the electric charge (the corresponding field strength is $F_{rt} = \frac{Q}{r^2}$). The equation $f = 0$ has two solutions for $M > |Q|$ given by $r_\pm = M \pm \sqrt{M^2 - Q^2}$. $r_+ \equiv r_h$ and $r_-$ are, respectively, the event horizon and the inner horizon. The Hawking temperature is

$$T_H = \frac{\sqrt{M^2 - Q^2}}{2\pi r_h^2}. \quad (10)$$

Vanishing of $T_H$, i.e. $M = |Q|$, defines the extremal configuration for which $r_+ = r_- \equiv r_h = M$.

## 3 Matter fields

In order to inquire on the existence of zero-temperature solutions in the semiclassical theory we must couple $S^{(2)}$ in eq. (7) to quantized free matter fields. In [4] it was considered a 2D minimally coupled scalar field described by the classical action

$$S_M = -\frac{1}{4} \int d^2x \sqrt{-g^{(2)}} (\nabla \tilde{f})^2. \quad (11)$$

which, after quantization, yields the well-known Polyakov effective action [8]

$$S_{\text{eff}} = -\frac{1}{96\pi} \int d^2x \sqrt{-g^{(2)}} R^{(2)} \frac{1}{\Box} R^{(2)}. \quad (12)$$

This action can be formally obtained by functional integration of the trace anomaly

$$\langle T \rangle = \frac{R^{(2)}}{24\pi}. \quad (13)$$

As it was mentioned in [4], due to the 2d origin of this field the results one gets using $S_{\text{tot}} = S^{(2)} + S_M + S_{\text{eff}}$ do not have an obvious four dimensional interpretation. To start with, we shall instead consider a 4D minimally coupled scalar field

$$S_M^{(4)} = -\frac{1}{16\pi} \int d^4x \sqrt{-g^{(4)}} (\nabla \tilde{f})^2. \quad (14)$$

In a spherically symmetric spacetime, the matter fields can be expanded into spherical harmonics, the $s$-wave sector $\tilde{f}$ of the scalar field $f$ depending only on $t$ and $r$. For the $s$-wave field $\tilde{f}$ dimensional reduction gives the 2D action:

$$S_M^{(2)} = -\frac{1}{4} \int d^2x \sqrt{-g^{(2)}} e^{-2\phi} (\nabla \tilde{f})^2. \quad (15)$$

\footnote{Usually, in order to make physical sense of the semiclassical approximation one considers $N$ matter fields and consider the large $N$ limit while keeping $N\hbar$ fixed. In this way the quantum corrections due to the other fields can be neglected.}
Comparison with the scalar field in (11) shows that the field \( \tilde{f} \), though still 2D conformal, has acquired a nontrivial coupling with the dilaton field \( \phi \). The corresponding trace anomaly has additional \( \phi \)-dependent terms [5]

\[
\langle T \rangle = \frac{1}{24\pi} \left( R^{(2)} - 6(\nabla \phi)^2 + 6 \Box \phi \right). 
\] (16)

Performing a functional integration of this expression we get the following effective action [5], [7], [8]

\[
S_{\text{eff}}^{(2)} = -\frac{1}{2\pi} \int d^2x \sqrt{-g^{(2)}} \left[ \frac{1}{48} R^{(2)} \frac{1}{\Box} R^{(2)} - \frac{1}{4} (\nabla \phi)^2 \frac{1}{\Box} R^{(2)} + \frac{1}{4} \phi R^{(2)} \right]. 
\] (17)

where the first nonlocal term is the same as in (12). It is important to point out that unlike (12) this effective action is not exact. Unphysical results obtained for the evaporation of Schwarzschild black holes [5], [8] suggest that, at least at finite temperature, \( S_{\text{eff}}^{(2)} \) must be modified by the addition of conformally invariant (local and nonlocal) terms [9], [10]. Considering instead zero-temperature configurations \( S_{\text{eff}}^{(2)} \) gives physically meaningful results [10].

In conformal gauge

\[
ds^2 = -f \, du \, dv
\] (18)

this action becomes local (i.e. \( \frac{1}{48} R^{(2)} = -\ln f \)) and for static configurations

\[
f = f(r)
\] (19)

where

\[
u = t - r_*, \quad v = t + r_*, \quad r_* = \int \frac{dr}{f(r)}
\] (20)

the components of the 2D stress energy tensor read

\[
\langle T^{(2)}_{uu} \rangle = \langle T^{(2)}_{vv} \rangle = \frac{1}{96\pi} \left[ f f'' - \frac{1}{2} (f')^2 \right]
\] + \frac{1}{64\pi} f^2 \left[ \left( \frac{k'}{k} \right)^2 \ln f - \left( \frac{k'}{k} \right)^2 + 2 \frac{k''}{k} \right],
\] (21)

\[
\langle T^{(2)}_{uv} \rangle = \frac{1}{96\pi} f f'' + \frac{1}{32\pi} f \left[ f' \frac{k'}{k} + f \frac{k''}{k} - \frac{1}{2} f \left( \frac{k'}{k} \right)^2 \right],
\] (22)

where the notation

\[
k = e^{-2\phi}
\] (23)

has been introduced and the prime denotes derivative with respect to \( r \). Considering now the dependence on the dilaton field, another relation can be deduced by functional differentiation of the effective action (17) with respect to the dilaton

\[
\frac{1}{\sqrt{-g^{(2)}}} \frac{\delta S_{\text{eff}}^{(2)}}{\delta \phi} = \frac{1}{4\pi} \left[ \left( f' \frac{k'}{k} - f \left( \frac{k'}{k} \right)^2 + f \frac{k''}{k} \right) \ln f + f \frac{k'}{k} - f'' \right].
\] (24)
This term is specific to the effective action considered and does not appear in the Polyakov theory. In a 4D viewpoint it is related to the tangential pressure \( \langle P \rangle = \langle T^\theta_\theta \rangle \) through the relation \((\ref{eq:24}), \ref{eq:25})

\[
\langle P \rangle = \frac{1}{8\pi e^{-2\phi} \sqrt{-g^{(2)}}} \frac{\delta S^{(2)}_{\text{eff}}}{\delta \phi},
\]

(25)

For the particular case of the extremal Reissner-Nordström black hole \( f = (1 - M/r)^2 \), \( k = r^2 \) we obtain the following results (see also \([11]\))

\[
\langle T^{(2)}_{uu} \rangle = \langle T^{(2)}_{vv} \rangle = -\frac{1}{48\pi} \frac{M}{r^3} \left[ 2 - 3 \frac{M}{r} \right] + \frac{1}{16\pi r^2} f^2 \ln f,
\]

(26)

\[
\langle T^{(2)}_{uv} \rangle = -\frac{1}{16\pi r^4} \left[ 1 - \frac{M}{r} \right] \left[ 1 - 3 \frac{M}{r} \right] \ln f + \frac{M}{16\pi r^5} \left( 4 - 5 \frac{M}{r} \right).
\]

(27)

where the first term on the r.h.s of these equations comes from the Polyakov contribution to the effective action. Also, from Eqs. \((\ref{eq:24})\) and \((\ref{eq:25})\) we obtain the 4D tangential pressure:

\[
\langle P \rangle = -\frac{1}{16\pi^2 r^4} \left[ 1 - \frac{M}{r} \right] \left[ 1 - 3 \frac{M}{r} \right] \ln f + \frac{M}{16\pi^2 r^5} \left( 4 - 5 \frac{M}{r} \right).
\]

(28)

Another important physical quantity is

\[
F = \frac{(T^r_r - T^t_t)}{f} = \frac{4 \langle T_{uu} \rangle}{f^2},
\]

(29)

which is proportional to the energy density measured by an infalling observer \([13]\). Equation \((\ref{eq:24})\) leads to

\[
F = -\frac{1}{6\pi r^2} \frac{M}{r - M} + \frac{1}{2\pi r^2} \ln \left| 1 - \frac{M}{r} \right|.
\]

(30)

As in the minimally coupled case \( F \) diverges when \( r \to M \). The term \( \sim 1/(r - M) \) is the same as that found in the Polyakov theory \([4]\), but despite its presence it was shown in \([4]\) that the corresponding semiclassical zero-temperature solution is regular at the horizon (only a mild divergence is present in the second derivative of the scalar curvature \( R \)). In our case, in addition to this term there appears a subleading logarithmic divergence \( \sim \ln(r - M) \), which is present also in the analytic approximations in four dimensions proposed in \([12]\), as well as a nontrivial pressure \( \langle P \rangle \) eq. \((\ref{eq:28})\). As stressed in the first of Refs. \([2]\), the divergence of \( F \) on the horizon of the classical extreme black hole makes the perturbative expansion in powers of \( \hbar \) to break down there. The calculations performed in \([3]\) are motivated by the fact that \( F \) has been proven to be finite at \( r = M \) numerically in \( D=4 \) \([13]\), but due to the result \((\ref{eq:30})\) reliable near horizon calculations for zero temperature black holes performed using the effective action \([17]\) must be nonperturbative in \( \hbar \). Similarly, the \( O(\hbar) \) results presented in \([4]\) (obtained by considering near-extreme black holes in the near horizon region) do not appear to have much physical meaning.
4 Backreaction

We now come to the main question addressed in this paper: do self-consistent zero-temperature black holes exist in the semiclassical theory? For this purpose, we need first of all to write down the semiclassical Einstein equations, which can be derived by differentiation of the action \( S_{\text{tot}}^{(2)} = S^{(2)} + S_{M}^{(2)} + S_{\text{eff}}^{(2)} \) (see eqs. (7), (15) and (17)) with respect to the 2d metric \( g_{ab} \) and the dilaton field \( \phi \). In conformal gauge (18) and considering static configurations (19), (20) the relevant expressions concerning the matter part of the action have been derived in (21), (22) and (24). The corresponding quantities coming from the gravity and electromagnetic actions (where \( F_{2} = -\frac{2}{\kappa^{2}} Q_{2} \)) can be easily obtained by differentiation of (5) and (6).

The \( uu \) (or \( vv \)) constraint reads

\[
0 = f'^2 k'' - \frac{1}{2} \left( f' \frac{k'}{k} \right)^2 k + \xi \left( f'' f - \frac{1}{2} (f')^2 \right) + 3\xi \left[ \frac{1}{2} \left( f' \frac{k'}{k} \right)^2 \ln f - \frac{1}{2} \left( f' \frac{k'}{k} \right)^2 + 3f^2 k'' \right], \tag{31}
\]

where the coefficient \( \xi = \frac{\hbar}{12\pi} \) has been introduced (we have reintroduced \( \hbar \) in the formulas in order to make the distinction between classical and quantum terms more clear). The equation obtained by varying the trace of the metric (i.e. \( g_{uv} \)) reads

\[
0 = -2 + f' k' + f k'' + 2 \frac{Q^2}{k} + \xi f'' + 3\xi \left[ f' \frac{k'}{k} + f k'' - \frac{1}{2} f \left( \frac{k'}{k} \right)^2 \right]. \tag{32}
\]

Finally, differentiation with respect to \( \phi \) gives

\[
0 = f'' - \frac{1}{2} f \left( \frac{k'}{k} \right)^2 + f' \frac{k'}{k} + f k'' - 2 \frac{Q^2}{k^2} - \frac{2\pi}{2\xi} \left[ f' \frac{k'}{k} + f'' - f \left( \frac{k'}{k} \right)^2 + f k'' \right]. \tag{33}
\]

For \( \xi = 0 \) (31), (32) and (33) are the classical eqs. of motion, for which the only zero-temperature configuration is the extremal Reissner-Nordstrom black hole \( f = \left( 1 - \frac{M}{r} \right)^2, k = r^2 \). In the quantum terms, we have separated the ones multiplying \( \xi \), coming to the Polyakov contribution to the effective action and present also in (3), and those proportional to \( 3\xi \) representing the additional contributions in the effective action \( S_{\text{eff}}^{(2)} \) (Eq. (17)).

As the three previous equations involve only two independent functions \( f(r) \) and \( k(r) \), one is redundant. Indeed, they are related through the Bianchi identities combined with the “nonconservation” eqs. for the matter part [8]

\[
\nabla_{a} \left( T_{b}^{(2)a} \right) + 8\pi e^{-2\phi} \langle P \rangle \nabla_{b}\phi = 0. \tag{34}
\]
Also, as these non-linear differential equations involve the second order derivatives of \( f \) and \( k \), two boundary conditions on these functions are required to determine them uniquely. For a zero temperature black hole, natural boundary conditions can be imposed on the function \( f \) at the horizon. First of all, \( f \) has to vanish there. Moreover, in the gauge used the temperature of the black hole takes the simple expression

\[
T_H = \frac{\kappa}{2\pi},
\]

where

\[
\kappa = \left. \frac{1}{2} f' \right|_{r=r_h}
\]

is the surface gravity and \( r_h \) denotes the radius of the horizon. So \( T_H = 0 \) means \( f' = 0 \) at \( r = r_h \).

In the Polyakov case, starting from these boundary conditions Trivedi [4] has found the form of the exact solution (nonperturbative in \( \hbar \)) close to the horizon as a (non-analytic) expansion in powers of the coordinate distance from the horizon \( r - r_h \).

In our case, however, the terms proportional to \( \ln f \) complicate exceedingly this analysis and seem to prevent an expansion in closed form of the solution for small values of \( r - r_h \) analogous to that proposed in [4].

A numerical resolution has then been undertaken and the boundary conditions have been imposed at infinity [15], where the solution is to a good approximation the extreme RN. In this region apart from a finite but very small renormalization of the classical mass (following [3] it is \( M_R/Q = 1 + O(\frac{\xi^2}{Q^4}) \)) the first quantum corrections to the spacetime metric are of the order \( O(1/r^3) \).

To start with, we have introduced dimensionless variables and functions in the differential equations that we have to integrate:

\[
x = r/Q, \quad \tilde{k}(x) = k(r)/Q^2, \quad \tilde{f}(x) = f(r).
\]

This means choosing the black hole charge to be the natural unit of length.

Our numerical integrations have been performed using the \( uu \)-constraint equation (31) and the \( \phi \)-equation (33) for the value \( \frac{\xi}{Q^2} = 10^{-5} \). All along our calculations, the solutions of these equations have been checked to be compatible with eq. (32) as well with a precision less than \( 10^{-7} \).

In order to probe the accuracy of our numerical simulations, we have first considered the integration of the semiclassical eqs. for the minimally coupled case (i.e. discarding the terms proportional to \( 3\xi \) in Equations (31), (32) and (33)) and compared the numerical results with the form of the exact solution close to the horizon given by Trivedi [4]. We find that, for a value of the horizon \( x_P = 1.0229 \) (in units where \( Q = 1 \)), i.e. with a deviation of about 2,3\% from the classical value, the functions \( f \) and \( f' \) behave like those in [4] with a precision of about \( 5 \times 10^{-4} \) when \( x \to x_P \) and that \( k \) is accurate with a precision \( 5 \times 10^{-5} \) and \( k' \) with the accuracy \( 2 \times 10^{-4} \). We can then expect the global precision of our simulation to be at least about \( 2 \times 10^{-2}\% \).
Considering now the nonminimally coupled case, we have integrated Equations (31) and (33). The results of this simulation are illustrated by the plots on the left of Figs. 1-4 where the functions $f$ and $k$ and their first derivatives have been shown for $x$ varying from the horizon $x_D = 1,0378$ to 5 (in units where $Q = 1$). To facilitate the comparison, the same functions in the minimal case (the plots on the right of Figs. 1-4) have been reported for $x$ varying from $x_P = 1,0229$ to 5.

5 Discussion and conclusions

Our numerical simulations presented in Figs. 1-4 and the comparison with the corresponding solution of the Polyakov theory appear to give good evidence that zero-temperature configurations exist in this theory. To get some insights from these results, we have compared the numerical values close to the horizon of these solutions with those obtained in the minimally coupled case. It turns out that the differences between the values of the functions $f$ and $f'$ for the two theories are less than $8 \cdot 10^{-4}$ (as deduced previously, the numerical precision is about $2 \cdot 10^{-4}$). The same reasoning applies to the function $k$ with a difference less than $8 \cdot 10^{-5}$. The first noticeable difference between the two theories appears at the first order derivative of the function $k$: the value of $k'$ on the horizon for our model is estimated at $-18,094$ compared to $+2,075$ for the Polyakov case. Going further in the derivatives of the function $f$ we have that the third derivative of $f$ blows up at the horizon for the nonminimally coupled field compared to the divergence of the fourth derivative in the minimal case. Moreover, it is interesting to stress that the value of the coordinate $x$ at the horizon $x_D = 1,0378$ (in units where $Q = 1$) differs from $x_P = 1,0229$ of about 1.5% (up to a numerical precision of about $2 \cdot 10^{-2}$).

Curvature invariants can be easily calculated starting from the results presented here. The 2D Ricci scalar $R^{(2)} = -f''$ is finite on the horizon, as it is shown in the plot of Fig. 5. Moreover, finiteness of $k$ and $k'$ are enough to prove that also the corresponding four dimensional scalar curvature $R^{(4)}$ is regular as the horizon is approached. A similar conclusion, despite the divergence of $F$ at the horizon of the extreme RN black hole, was found in the minimally coupled case [4]. In our case, since the leading term in eq. (34) is the same as in the Polyakov theory, it is reasonable that the main conclusion about the regularity of the geometry at the horizon is unchanged. The difference with respect to the case analysed in [4] is that the “mild” singularity appearing at the horizon in the second derivative of the curvature close to the horizon ($\sim f''''$) is replaced by a “stronger” divergence in its first derivative (i.e. $f'''$). This is due to the logarithmic divergence $\ln(f)$ appearing in eq. (30) as well as in eqs. (31) and (33).

In conclusion, by numerical integration of the two-dimensional semiclassical equations of motion for the case of spherically reduced Einstein-Maxwell theory and a scalar field nonminimally coupled to the dilaton we have found solutions describing zero-temperature black holes. Similarities and differences with respect to
the simpler minimally coupled case have been studied in detail. Due to the intrinsic four dimensional nature of the matter field used, our results could be relevant in order to address the same issue in the physical world $D = 4$. Finally, following Refs. an interesting extension of this work would be to check whether the solution found here does indeed represent the end-point of the evaporation process (for the minimally coupled case this problem has been addressed both in the near-horizon approximation [16] and in the whole spacetime numerically [17]).

Acknowledgements

We would like to thank R. Balbinot and P. Nicolini for useful discussions and for collaboration at an earlier stage of this work. C.B. was supported by the Italian Minister of Foreign Affairs.
Figure 1: Plot of the function $f$ for our model (left) and the corresponding $f$ for the minimally coupled case (right). We have set $Q = 1$.

Figure 2: Comparison of the values of $f'$ for the two theories.
Figure 3: Plots of the function $k$.

Figure 4: Plots of the functions $k'$.
Figure 5: Values of $f'' = -R^{(2)}$ for our model.
References

[1] A. Strominger and S.P. Trivedi, *Phys. Rev.* D **48** (1993), 5778; D.A. Lowe and M. O’Laughlin, *Phys. Rev.* D **48** (1993), 3735; A. Fabbri, D.J. Navarro and J. Navarro-Salas, Low-energy scattering of extremal black holes by neutral matter, [hep-th/0110294](http://arxiv.org/abs/hep-th/0110294).

[2] P.R. Anderson, W.A. Hiscock and B.E. Taylor, *Phys. Rev. Lett.* **85** (2000), 2438; *Phys. Rev. Lett.* **87** (2001), 029002; Zero and near-zero temperature black holes in semiclassical gravity, [gr-qc/0102111](http://arxiv.org/abs/gr-qc/0102111).

[3] D.A. Lowe, *Phys. Rev. Lett.* **87** (2001), 029001.

[4] S.P. Trivedi, *Phys.Rev.* D **47** (1993), 4233.

[5] V. Mukhanov, A. Wipf and A. Zelnikov, *Phys. Lett.* B **332** (1994), 283.

[6] A.M. Polyakov, Phys.Lett. B**103**, 207, 1981.

[7] W. Kummer and D.V. Vassilevich, *Ann. Phys.* **8** (1999), 801; S. Nojiri and S.D. Odintsov, *Int. J. of Modern Physics* A **16** (2001), 1015.

[8] R. Balbinot and A. Fabbri, *Phys. Rev.* D **59**, 044031, 1999.

[9] Y.V. Gusev and A.I. Zelnikov, *Phys. Rev.* D **61** (2000), 084010.

[10] R. Balbinot and A. Fabbri, 2D black holes and effective actions, [hep-th/0012140](http://arxiv.org/abs/hep-th/0012140).

[11] M. Buric and V. Radovanovic, *Class. and Quant. Grav.* **16** (1999), 3937.

[12] V.P. Frolov and A.I. Zelnikov, *Phys. Rev.* D **35** (1987), 3031; P.R. Anderson, W.A. Hiscock and D.A. Samuel, *Phys. Rev.* D **51** (1995), 4337.

[13] P.R. Anderson, W.A. Hiscock and D.J. Loranz, *Phys. Rev. Lett.* **74** (1995), 4365.

[14] A.J.M. Medved, Reissner-Nordström near extremality from a Jackiw-Teitelboim perspective, [hep-th/0111091](http://arxiv.org/abs/hep-th/0111091).

[15] D.A. Lowe, *Phys.Rev.* D **47** (1993), 2446.

[16] A. Fabbri, D.J. Navarro and J. Navarro-Salas, *Phys. Rev. Lett.* **85** (2000), 2434; *Nucl. Phys.* B **595** (2001), 381.

[17] E. Sorkin and T. Piran, *Phys. Rev.* D **63** (2001), 124024.