The Chiral Magnetic Effect:
Measuring event-by-event $P$- and $CP$-violation with heavy ion-collisions

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Gluon field configurations with nonzero topological charge induce $P$- and $CP$-odd effects. Such configurations are likely to be produced during heavy ion collisions. In this article, I will argue that in the intense (electromagnetic) magnetic field produced in non-central heavy ion collisions, topological charge creates an electromagnetic current in the direction of the magnetic field. This is the Chiral Magnetic Effect. It leads to separation of positive from negative charge along the direction of the magnetic field. I will point out that this effect can be investigated experimentally with a charge correlation study and will refer to interesting data from the STAR collaboration.

1 Introduction

The goal of the study of heavy ion collisions is to understand the properties of nuclear matter under extreme circumstances. The fundamental theory that describes the relevant dynamics during such collisions is Quantum Chromodynamics (QCD). QCD contains really a lot of intriguing features. One of them is its connection to topology. Another highlight is quantum mechanical symmetry breaking (also called anomaly). And the last example I want to mention here is the existence of parity ($P$) and charge-parity ($CP$)-odd effects. In this article I will discuss these examples in more detail and show that they are intricately linked to each other. Since heavy ion collisions probe QCD, these effects should somehow play a role during the collisions. The question which then immediately pops up is: how big is this role and how can we observe it. I will try to answer these two questions to some extent in this article.

2 Topology in QCD

The vacuum (i.e. state with lowest energy) of a non-Abelian field theory like QCD has non-trivial structure. It turns out that there are an infinite number of different vacua that all can be characterized by an integer winding number $n_W$. This winding number is a topological invariant, that means that a smooth deformation of the vacuum state while staying in the ground state cannot change $n_W$. One needs energy to change $n_W$, therefore all vacua classified according to their winding number are separated by a potential barrier. A gauge field configuration with nonzero topological charge interpolates between these distinct vacua and hence probes the potential. The topological charge $Q$ is given by $Q = \frac{2}{\pi^2} \int d^4x F^a_{\mu\nu} \tilde{F}^{a\mu\nu}$, here $F^a_{\mu\nu}$ is the gluon field-strength tensor and $\tilde{F}^{a\mu\nu} = \frac{1}{2} \epsilon^{\mu\nu\rho\sigma} F^a_{\rho\sigma}$ its dual, with the complete antisymmetric tensor $\epsilon^{0123} = 1$. In general $Q$ is not quantized, but if the gluon field configuration starts from a vacuum
at $t = -\infty$ and ends in a vacuum at $t = \infty$, one can show that $Q$ is equal to the change in winding number and hence an integer, i.e. $Q = n_W(t = \infty) - n_W(t = -\infty)$.

One possibility of interpolating between two different vacua is by quantum tunneling through this potential barrier. The relevant configurations in this case are called instantons and the tunneling rate (which is equal to the Euclidean topological susceptibility $\chi_E = \langle Q^2 \rangle / V$) is proportional to $\exp(-2|Q|/\alpha_s)^3$. At low energies and temperatures this rate is sizable, from the Witten-Veneziano relation it follows that $\chi_E \approx (180 \text{ MeV})^4$. At very high energies where perturbation theory is valid, this rate becomes very small so that in that case instantons can safely be neglected. The instantons are even more suppressed at high temperatures due to screening. But then also a new possibility appears which is traversing over the barrier in real-time. The relevant configurations are sphalerons (originally discussed for weak-interactions, but also existent in QCD), and the rate (real-time topological susceptibility) is at high temperatures much less suppressed (since it does not invoke tunneling) and proportional to $\alpha_s^5 T^4$ with a large prefactor. This rate means that one could expect of order several transitions per fm$^{-3}$ per fm/c in the deconfined phase. In strongly coupled supersymmetric Yang-Mills theory the sphaleron rate is sizable too, as was found by applying the AdS/CFT correspondence. The discussed rates are for thermalized isotropic systems. To obtain the rate of production of topological charge in heavy ion collisions one should also take into account the collision geometry and the fact that equilibrium might not have been achieved. This can change these estimates, especially just after the collision when the quark gluon plasma has not yet been formed.

3 Axial anomaly

Hadrons which are build out of quarks are the QCD states measured in the detectors. So in order to find experimental evidence for topological charge it is necessary to understand how topological charge deals with quarks. While in the limit of vanishing quark masses axial U(1) is a symmetry of the QCD Lagrangian, it is broken by quantum effects. This quantum mechanical symmetry breaking (or axial anomaly) gives rise to an exact identity. In the limit of zero quark mass this identity reads for each flavor separately \( \Delta N_5 \equiv \Delta(N_R - N_L) = -2Q \) where $N_{R,L}$ denotes the number of quarks minus antiquarks with right/left-handed chirality. The quantity $\Delta N_5$ is the change in chirality in time. A particle with right-handed chirality has right-handed helicity, while an anti-particle with right-handed chirality has left-handed helicity. Right (left)-handed helicity means spin and momentum (anti)-parallel. The difference $N_R - N_L$ can also be read as the total number of quarks plus antiquarks with right-handed helicity minus the total number of quarks plus antiquarks with left-handed helicity.

So topological charge induces chirality by the axial anomaly. Here we see the three intriguing features mentioned in the introduction coming together. Let us dive a little deeper into the relation between topological charge and $\mathcal{P}$- and $\mathcal{CP}$-violation.

One could add the so-called $\theta$ term (to be precise $\theta Q$) to the QCD action. Such term gives rise to direct $\mathcal{P}$- and $\mathcal{CP}$-violation. Measurements of the electric dipole moment of the neutron constrain $|\theta|$ to be smaller than of order $10^{-10}$. Since $\theta$ couples to topological charge in the action, at nonzero $\theta$, $\mathcal{P}$ and $\mathcal{CP}$ are broken due to gluon fields with nonzero topological charge. In the case that $\theta = 0$ the probability to generate either a gluon configuration with positive or negative topological charge is equal so that $\mathcal{P}$ and $\mathcal{CP}$ are unbroken.

Instantons and sphalerons are objects with a certain size and life-time (the size of the sphaleron is limited by the magnetic screening length $1/\alpha_s T$). Therefore in the matter produced in heavy ion collisions many of such configurations will be generated at different points in space and time with different values of topological charge. Since this is essentially a random processes, in each event a net topological charge can be generated globally. Only if one averages over many events one should find that $\langle Q \rangle = 0$ (if $\theta = 0$). From the anomaly we then know how
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Now that we have seen that topological charge induces chirality, let us see how one could measure this nonzero chirality. When two heavy ions collide with nonzero impact parameter, a (electromagnetic) magnetic field of enormous magnitude is created in the direction of angular momentum of the collision (at 0.2 fm/c after the collision it is for moderate impact parameters of order $10^3 \sim 10^4$ MeV$^2$ corresponding to $10^{17}$ G [13]). In a background magnetic field, the quarks can gain energy by aligning their magnetic moments along the magnetic field. Positively charged quarks/antiquarks with right-handed helicity have positive magnetic moment and will tend to align their spin parallel to the magnetic field. Since right-handed helicity means that spin and momentum are parallel, also the momentum will be pointing parallel to the magnetic field. Negatively charged quarks/antiquarks with right-handed helicity have negative magnetic moment, and for the same reasons tend to point their momentum anti-parallel to the magnetic field. The particles and antiparticles with left-handed helicity will move in the opposite direction. Therefore if a nonzero chirality is present in a background magnetic field, an electromagnetic current will be induced in the direction of the magnetic field. This is the so-called Chiral Magnetic effect [11,12,14,15]. I have illustrated this effect in Fig. 1.

For extremely large magnetic fields so that the quarks are fully polarized one can quickly convince oneself (use Fig. 1) that the total induced current is equal to $J = \sum_f |q_f| N_5 = -2 \sum_f |q_f| Q$, where $f$ denotes a sum over light flavors and $q_f$ is the charge of one particular flavor. For smaller magnetic fields, we have computed induced current in a more general setting. For constant and homogeneous magnetic fields the size of the current density is determined by the electromagnetic axial anomaly and equal to [15] $j = \sum_f q_f^2 \mu_5 B/(2\pi^2)$, see also [16]. Here $\mu_5$ denotes the chiral chemical potential, which is used to describe a system with nonzero chirality. The chiral chemical potential can be expressed in terms of $N_5$ by taking a derivative of the thermodynamic potential with respect to $\mu_5$, $n_5 = -\partial \Omega/\partial \mu_5$. In this way we were able to reproduce the large magnetic field result. For smaller magnetic fields ($B$) and large temperatures ($T$) we obtained for the total current [15] (here $\mu$ denotes quark chemical potential)

$$J = -\frac{3}{\pi^2} \frac{Q}{T^2 + \mu^2/\pi^2} B \sum_f q_f^2.$$ (1)
In heavy ion collisions this current leads to separation of charge along the direction of angular momentum, which is perpendicular to the reaction plane. This leads to nontrivial correlations between the azimuthal angles (the angle between the particle and the reaction plane) of charged particles. To predict the behavior of these correlations one should compute from Eq. [1] how much charge is separated, fold it with the rate of topological charge production and the time-dependent magnetic field, and integrate over the reaction volume.\textsuperscript{13,14}

An observable that measures these correlations was proposed by Voloshin\textsuperscript{17} and preliminary data of the STAR collaboration on these correlations have been presented at the Quark Matter conference.\textsuperscript{18} The very interesting data suggests that charge is separated perpendicular to the reaction plane. In order to establish observation of the Chiral Magnetic effect is important to obtain accurate predictions on for example the impact parameter and beam energy dependence, confront these predictions with experimental results, and rule out other possible explanations.

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