\[ \tau \text{ lepton mass measurement at Belle II} \]

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Abstract

The reconstruction of tau-pair production, $e^+e^- \rightarrow \tau^+\tau^-$, from the subsequent 3-prong ($\tau^+ \rightarrow \pi^+\pi^-\pi^+\bar{\nu}_\tau$) and 1-prong ($\tau^- \rightarrow \ell^-\bar{\nu}_\ell\nu_\tau$, $\tau^- \rightarrow h^-\nu_\tau$ or $\tau^- \rightarrow \pi^-\pi^0\nu_\tau$) decays, is presented using 8.8 fb$^{-1}$ of $e^+e^-$ collision data of Belle II at the center-of-mass energy $\sqrt{s} = m_{\Upsilon(4S)}$. The pseudomass technique developed by the ARGUS experiment is used to measure the $\tau$-lepton mass $m_\tau$ in the 3-prong $\tau^+ \rightarrow \pi^+\pi^-\pi^+\bar{\nu}_\tau$ decay, resulting in $m_\tau = 1777.28 \pm 0.75$ (stat.) $\pm 0.33$ (sys.) MeV/c$^2$. 
I. INTRODUCTION

Precise measurements of the lepton properties provide stringent tests of the Standard Model (SM) and accurate determinations of its parameters. For example, the SM predicts unambiguous and simple relationships among the lepton lifetime, mass, and leptonic branching fractions. Therefore, their experimental determination to the highest possible precision is essential; deviations from the predictions at any level could signal the presence of physics beyond our present understanding.

Figure 1 illustrates the test of the SM prediction of the relation among the τ leptonic branching fractions, \( B_{\tau \ell} = B(\tau^- \to \ell^- \bar{\nu}_\ell \nu_\tau) \) with \( \ell = e^- \) or \( \mu^- \); the τ lifetime \( \tau_\tau \); the τ mass \( m_\tau \); and the respective muon parameters:

\[
B_{\tau \ell}^{SM} = B_{\mu \ell} \frac{\tau_\tau}{\tau_\mu} \frac{m_\tau^5}{m_\mu^5} f_{\tau \ell} \frac{r_\gamma}{r_\mu} \frac{r_\tau}{r_\ell} .
\]  

Here \( f_{\ell \nu} = f \left( \frac{m_\nu^2}{m_\ell^2} \right) \equiv f(x) = 1 - 8x + 8x^2 - x^4 - 12x^2 \ln x \), with \( r_\gamma \) and \( r_\mu \) being the weak and electromagnetic radiative corrections. This relation is very sensitive to the value of the τ mass.

Figure 1. Test of the SM prediction of the relation between the τ leptonic branching fractions and the τ lifetime and mass. \( B_{\tau e} \) denotes the statistical average of \( B_{\tau e} = B(\tau^- \to e^- \bar{\nu}_e \nu_\tau) \) and the \( B_{\tau e} \) SM prediction from the \( B_{\tau \mu} \) measurement \( B_{\tau e} = B_{\tau \mu} \cdot (f_{\tau e}/f_{\tau \mu}) \). The yellow band represents the uncertainty from the τ lifetime. The plot is from Ref. [1].

The τ leptonic branching fractions and the τ lifetime are known up to a relative precision of 0.2%, far from the impressive accuracy achieved for the μ lifetime. Comparing the global average of the \( e, \mu \) and τ masses reported by the PDG [2],

\[
m_e = (0.5109989461 \pm 0.00000000031) \text{ MeV}/c^2, \]  
\[
m_\mu = (105.6583745 \pm 0.0000024) \text{ MeV}/c^2, \]  
\[
m_\tau = (1776.86 \pm 0.12) \text{ MeV}/c^2, \]

\( \tau_\tau \) and \( \tau_\mu \) are the statistical averages of the lifetime measurements of the τ and μ lifetimes, respectively.
it is clear that the value of $m_\tau$ is also far less precise. The average of $m_\tau$ is dominated by the result from the BES III experiment [3] that makes an energy scan of $\sigma(e^+e^- \rightarrow \tau^+\tau^-)$ around the $\tau^+\tau^-$ production threshold, followed by the measurements reported by the Belle [4] and BaBar [5] collaborations in which the $\tau$ pseudomass spectrum from $\tau^- \rightarrow \pi^-\pi^+\pi^-\nu_\tau$ decay is used applying the ARGUS method [6]. Though less precise, the pseudomass endpoint method allows us to test CPT conservation by measuring the masses of $\tau^-$ and $\tau^+$ individually.

In this work, preliminary results on the reconstruction of $\tau$-pair production, $e^+e^- \rightarrow \tau^+\tau^-$, from the subsequent 3-prong ($\tau^+ \rightarrow \pi^+\pi^-\pi^+\nu_\tau$) and 1-prong ($\tau^- \rightarrow \ell^-\bar{\nu}_\ell\nu_\tau$, $\tau^- \rightarrow h^-\nu_\tau$ or $\tau^- \rightarrow \pi^-\pi^0\nu_\tau$) decays\footnote{Charge-conjugate modes are implied in this study.} and the $\tau$ lepton mass measurement are presented.

II. EVENT SELECTION

The analysis presented here is based on 8.8 fb$^{-1}$ of data accumulated during 2019 at the $\Upsilon(4S)$ resonance ($\sqrt{s} = 10.58$ GeV) with the Belle II detector at the SuperKEKB asymmetric-energy $e^+e^-$ collider [7]. The Belle II detector consists of several subdetectors arranged around the beam pipe in a cylindrical structure. A superconducting solenoid, situated outside of the calorimeter, provides a 1.5 T magnetic field. Subdetectors relevant to this analysis are briefly described here; a description of the full detector is given in Ref. [8]. The innermost subdetector is the vertex detector (VXD), which includes two layers of silicon pixels and four outer layers of silicon strips. Charged-particle tracking is done using the VXD and a large helium-based small-cell central drift chamber (CDC). The electromagnetic calorimeter (ECL) consists of a barrel and two endcaps made of CsI(Tl) crystals. The $z$-axis of the laboratory frame is along the detector solenoidal axis in the direction of the electron beam. Events are selected by the hardware trigger; no further software trigger selection is applied.

The Belle II experiment records data in a quite severe beam background environment. Due to the short lifetime of the $\tau$ lepton, its decay products are expected to originate close to the interaction point (IP). Thus, the $\tau$-pair candidate events are selected by requiring only four charged tracks, with zero net charge, originating from a narrow window close to the IP. The beam-induced photon background has typically low energy and can be largely reduced by imposing a cut on the minimum energy of the photons.

The $\pi^0 \rightarrow \gamma\gamma$ candidates are reconstructed from photon candidates with $17^\circ < \theta_\gamma < 150^\circ$ and an energy threshold of 100 MeV in the invariant mass window $0.115$ GeV/c$^2 < M_{\gamma\gamma} < 0.152$ GeV/c$^2$.

In the $e^+e^- \rightarrow \tau^+\tau^-$ centre-of-mass system (CMS), both $\tau$ leptons are back-to-back and their decay products are well separated in two opposite hemispheres defined by the plane perpendicular to the thrust axis [9, 10]. The thrust axis $\hat{n}_{\text{thrust}}$ is defined such that the value $V_{\text{thrust}}$

$$V_{\text{thrust}} = \sum_i \frac{|\vec{p}_i^{\text{CMS}} \cdot \hat{n}_{\text{thrust}}|}{\sum_i |\vec{p}_i^{\text{CMS}}|}$$

is maximised. Here, $\vec{p}_i^{\text{CMS}}$ is the CMS momentum of each charged particle, $\pi^0$ or photon. A pion mass hypothesis is used for all charged tracks. Given the vector $\hat{n}_{\text{thrust}},$ one
hemisphere is expected to contain the products of 3-prong $\tau$ decay, while the other one the products of 1-prong $\tau$ decay.

The identification of charged particles is based on $E_{\text{ECL}}/P_{\text{lab}}$, where $E_{\text{ECL}}$ is the energy deposit in the ECL, while $P_{\text{lab}}$ is the momentum of the particle measured in the tracking systems (VXD+CDC). For the reconstruction of the 3-prong decay, pions are required to satisfy the condition, $E_{\text{ECL}}/P_{\text{lab}} < 0.8$. This requirement enhances the selection of $\tau^+ \rightarrow \pi^+\pi^-\pi^+\bar{\nu}_\tau$ in 3-prong decays. The $\pi^0$ are vetoed on the 3-prong side, in order to reduce the background contamination from $\tau^+ \rightarrow \pi^+\pi^-\pi^+\pi^0\bar{\nu}_\tau$ decays. The 1-prong side is expected to contain one charged track and at most one neutral pion.

There are processes other than the $\tau$-pair production that may satisfy selection criteria, such as $e^+e^- \rightarrow Y(4S) \rightarrow BB$, $e^+e^- \rightarrow q\bar{q}$ (with $q = u, d, s, c$), $e^+e^- \rightarrow \ell^+\ell^-\gamma$ (with $\ell = e, \mu$) and two-photon processes $e^+e^- \rightarrow e^+e^-\ell^+\ell^-$ and $e^+e^- \rightarrow e^+e^- q\bar{q}$. In order to reduce the background contamination from $e^+e^- \rightarrow q\bar{q}$ processes, any event with a photon having $E > 200$ MeV that is not the daughter of a $\pi^0$ is rejected. To suppress other background sources, differences in the distributions of the thrust value and the total visible energy of event in the CMS are used, where the energy of the pions is calculated from their momentum and mass. A signal-to-background optimization leads to $0.9 \leq V_{\text{thrust}} \leq 0.99$ and the visible energy in CMS, $E_{\text{visible}}^{\text{CMS}}$, within $[2.5, 10.2]$ GeV. After the trigger efficiency correction, the data and Monte Carlo agreement is within a few percent, as shown in Fig. 2.

After the selection is applied, there remain about 150k events of $e^+e^- \rightarrow \tau^+\tau^-$ events with the subsequent 3-prong ($\tau^+ \rightarrow \pi^+\pi^-\pi^+\bar{\nu}_\tau$) and 1-prong ($\tau^- \rightarrow \ell^-\bar{\nu}_\ell\nu_\tau$, $\tau^- \rightarrow h^-\nu_\tau$ or $\tau^- \rightarrow \pi^-\pi^0\nu_\tau$) decays. The invariant mass of the three tracks on the 3-prong side ($M_{3\pi}$) is shown in Fig. 3. The efficiency of reconstructing the signal events is 16.6%; the purity of the sample 84.5%. The background contamination in the $M_{3\pi}$ distribution arises mainly from $\tau$ decays other than $\tau^+ \rightarrow \pi^+\pi^-\pi^+\bar{\nu}_\tau$. The largest contribution is from $\tau^+ \rightarrow \pi^+\pi^-\pi^0\nu_\tau$ decays where the $\pi^0$ is not reconstructed, followed by the contribution from events with $K-\pi$ misidentification. This is due to the fact that no pion identification is used in the analysis.

III. MEASUREMENT OF THE $\tau$ LEPTON MASS

The analysis procedure is blinded while establishing the technique, selection criteria and evaluation of the systematic uncertainties. The $\tau$-lepton mass measurement is performed following the pseudomass method developed by the ARGUS collaboration [6]. The pseudomass is defined as

$$M_{\text{min}} = \sqrt{M_{3\pi}^2 + 2(E_{\text{beam}} - E_{3\pi})(E_{3\pi} - P_{3\pi})} \leq m_\tau,$$

(6)

where $E_{\text{beam}}$ is the energy of one of the beams in CMS, and $M_{3\pi}$, $E_{3\pi}$, $P_{3\pi}$ stand for the invariant mass, energy and momentum, respectively, of the three pion system in CMS. In the absence of initial (ISR) and final (FSR) state radiations, and a perfect measurement of the four-momentum of the hadronic system, the distribution of $M_{\text{min}}$ extends up to and has a sharp edge at the mass of the $\tau$ lepton, $m_\tau$. The ISR/FSR and the detector resolution smear the endpoint and result in a large tail in the pseudomass distribution, as seen on the left panel of Fig. 4. The mass of the $\tau$ lepton $m_\tau$ is then measured by determining the position of the endpoint.
The distributions of $V_{\text{thrust}}$ (top) and visible energy in CMS (bottom) in data and Monte Carlo after the selection and trigger correction. The light green dashed curve shows the simulated distribution of the $e^+e^- \rightarrow \tau^+\tau^-$ process with subsequent 3-prong ($\tau^+ \rightarrow \pi^+\pi^-\pi^0\bar{\nu}_\tau$) and 1-prong ($\tau^- \rightarrow \ell^-\bar{\nu}_\ell\nu_\tau$, $\tau^- \rightarrow h^-\nu_\tau$ or $\tau^- \rightarrow \pi^-\pi^0\nu_\tau$) decays, while the dark green dotted distribution corresponds to the background contribution from other $\tau$ decay modes. The background contamination from $q\bar{q}$ process is shown with dash-dotted magenta curves. The sum of all Monte Carlo contributions and the corresponding total statistical uncertainties are shown by the solid blue curve and the grey hatched area, respectively. The Monte Carlo distributions are rescaled to a luminosity of $8.8\text{fb}^{-1}$ of data and reweighted according to the trigger efficiency measured in data. The bottom panel of each plot shows the ratio of the data and total MC prediction.

Figure 2. The distributions of $V_{\text{thrust}}$ (top) and visible energy in CMS (bottom) in data and Monte Carlo after the selection and trigger correction. The light green dashed curve shows the simulated distribution of the $e^+e^- \rightarrow \tau^+\tau^-$ process with subsequent 3-prong ($\tau^+ \rightarrow \pi^+\pi^-\pi^0\bar{\nu}_\tau$) and 1-prong ($\tau^- \rightarrow \ell^-\bar{\nu}_\ell\nu_\tau$, $\tau^- \rightarrow h^-\nu_\tau$ or $\tau^- \rightarrow \pi^-\pi^0\nu_\tau$) decays, while the dark green dotted distribution corresponds to the background contribution from other $\tau$ decay modes. The background contamination from $q\bar{q}$ process is shown with dash-dotted magenta curves. The sum of all Monte Carlo contributions and the corresponding total statistical uncertainties are shown by the solid blue curve and the grey hatched area, respectively. The Monte Carlo distributions are rescaled to a luminosity of $8.8\text{fb}^{-1}$ of data and reweighted according to the trigger efficiency measured in data. The bottom panel of each plot shows the ratio of the data and total MC prediction.

The $M_{\text{min}}$ distribution from $1.70\text{GeV}/c^2$ to $1.85\text{GeV}/c^2$ is of particular interest for
Figure 3. The invariant mass $M_{3\pi}$ of the three tracks on the 3-prong side after the selection criteria. The description of the plot is the same as that of Fig. 2.

eventing the $\tau$ mass. Unlike the invariant mass distribution where the main background contamination is due to misidentified $\tau^+ \to \pi^+\pi^-\pi^+\bar{\nu}_\tau$ decays, in the region of interest of the pseudomass distribution all background contributions are negligible except for $e^+e^- \to q\bar{q}$. While $\tau^+ \to \pi^+\pi^-\pi^+\bar{\nu}_\tau$ decays show the endpoint behaviour, the background processes in the selected region have a featureless $M_{\text{min}}$ distribution.

An empirical probability density function (p.d.f.)

$$F(M, \vec{P}) = (P_3 + P_4 M) \cdot \tan^{-1}[(M - P_1)/P_2] + P_5 M + 1,$$

is used to estimate the $\tau$ lepton mass from the $e^+e^- \to (\tau^+ \to \pi^+\pi^-\pi^+\bar{\nu}_\tau)(\tau^- \to e^-, \mu^-, \pi^-, \pi^0\pi^0)$ Monte Carlo sample [11, 12], in which the parameter $P_1$ is an estimator of the $\tau$ lepton mass. The fit results in

$$P_1 = 1777.72 \pm 0.17 \text{ MeV}/c^2.$$  

At the generator level, $m_\tau$ is set to 1777 MeV/$c^2$; thus, the $P_1$ parameter shows a bias in the estimation of the $\tau$ mass as observed in previous measurements of the $\tau$ mass using the pseudomass method [4–6]. The average bias of $P_1$ is estimated to be $0.72 \pm 0.12 \text{ MeV}/c^2$ independently of the generated $\tau$ mass, by performing fits to Monte Carlo samples that were generated using $m_\tau$ values shifted with respect to the nominal values.

IV. SYSTEMATIC UNCERTAINTIES

The impact of various systematic sources of uncertainties on the $\tau$ mass measurement has been estimated. Table I summarizes the systematic uncertainties; the sources are described in detail below.
Figure 4. Distribution of the pseudomass for $e^+e^-\rightarrow \tau^+\tau^-$ process with subsequent 3-prong ($\tau^+\rightarrow \pi^+\pi^-\pi^+\bar{\nu}_\tau$) and 1-prong ($\tau^-\rightarrow \ell^-\bar{\nu}_\ell\nu_{\tau}$, $\tau^-\rightarrow h^-\nu_{\tau}$ or $\tau^-\rightarrow \pi^-\pi^0\nu_{\tau}$) decays in the entire range (up), and in the range 1.70 to 1.85 GeV/$c^2$ (bottom). The description of the plot is the same as that of Fig. 2.

- **Momentum shift due to the B-field map:** The leading source of uncertainty for this measurement comes from a momentum scale factor of $0.056^{+0.051}_{-0.042}$% that is introduced to compensate for the imperfections of the magnetic-field map used during the reprocessing of data. The scale factor is measured according to an observed shift in the invariant mass of $D^0$ in data vs the PDG [13] value. The central value for the scale factor is used to correct the momenta of the tracks in data, and the average of the impact due to the up and down variations is used in Monte Carlo to estimate the associated systematic uncertainty of 0.29 MeV/$c^2$. 

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| Systematic uncertainty                          | MeV/c^2   |
|------------------------------------------------|-----------|
| Momentum shift due to the B-field map          | 0.29      |
| Estimator bias                                | 0.12      |
| Choice of p.d.f.                               | 0.08      |
| Fit window                                    | 0.04      |
| Beam energy shifts                            | 0.03      |
| Mass dependence of bias                       | 0.02      |
| Trigger efficiency                            | ≤ 0.01    |
| Initial parameters                            | ≤ 0.01    |
| Background processes                          | ≤ 0.01    |
| Tracking efficiency                           | ≤ 0.01    |

Table I. Summary of systematic uncertainties.

- **Estimator bias:** The limited size of the samples used in determining the fit bias results in an uncertainty of 0.12 MeV/c^2.

- **Dependence on the choice of p.d.f.:** Two alternate functions,

\[
F_1(M, \vec{P}) = (P_3 + P_4 M) \cdot \frac{M - P_1}{\sqrt{P_2 + (M - P_1)^2}} + P_3 M + 1, \tag{9}
\]

\[
F_2(M, \vec{P}) = (P_3 + P_4 M) \cdot \frac{-1}{1 + \exp \left(\frac{(M - P_1)}{P_2}\right)} + P_3 M + 1, \tag{10}
\]

are used for estimating the fit bias. The RMS of the corrected \(m_\tau\) values corresponding to these alternative methods versus the nominal one is calculated to be 0.08 MeV/c^2.

- **Choice of the fit window:** The importance of the \(M_{\text{min}}\) window used for the fit procedure is tested by varying the lower and upper edges of the window separately and repeating the fit bias estimation for each case. The weighted average of the differences between the true mass and the corrected \(m_\tau\) values corresponding to each window is 0.04 MeV/c^2.

- **Beam energy shifts:** The calculation of the pseudomass variable relies on an accurate knowledge of the beam energy. The energy of the beam is measured by using the beam-energy-constrained mass (\(M_{\text{bc}}\)) of fully reconstructed neutral and charged B decays for various data taking periods with statistical uncertainties of up to 0.19 MeV. This uncertainty is then propagated to the \(\tau\) mass measurement by taking advantage of additional Monte Carlo samples with beam energies shifted with respect to the nominal beam energy value. The measurement procedure is applied to each Monte Carlo sample to estimate the \(\tau\) mass as a function of the beam energy shift. This yields a systematic uncertainty of 0.03 MeV/c^2. Additional systematic uncertainties in the measurement of the beam energy shift are not currently estimated. Once these uncertainties are available, they will be propagated to the \(\tau\) mass measurement as well.
• **Dependence of the fit bias on the true mass:** Allowing for the fit bias to vary as a function of the true mass results in a difference of 0.02 MeV/c² in the corrected mass with respect to the method described in Section III.

• **Trigger efficiency:** The trigger efficiency is measured to be an essentially constant value of 80% in the pseudomass region used for the $m_\tau$ measurement. The impact of this efficiency is seen to be negligible by repeating the fit on the signal Monte Carlo sample with a reweighting of the data by a linear parameterisation of the trigger efficiency as a function of $M_{\text{min}}$.

• **Model for background processes:** The distribution of background processes in the pseudomass window of interest is featureless. The fit procedure on the signal Monte Carlo sample is repeated by adding the background processes to the fit. The result of fit is insensitive to the presence of these background events.

• **Decay model of $a_1(1260)$:** To test the dependence of the fit result on the decay model of $\tau^+ \rightarrow \pi^+\pi^-\pi^+\bar{\nu}_\tau$, the nominal fit is performed on the generated Monte Carlo sample with a phase space decay of the $a_1(1260)$. The shape of the $M_{\text{min}}$ distribution in the fit window is found to be independent of the $a_1(1260)$ decay model and therefore the size of this systematic is expected to be small. However, due to the large statistical uncertainties of the alternative decay scenario, at this point, a quantitative estimate of the size of this systematic uncertainty cannot be made. Within the uncertainty, the result of the fit on the alternative decay model is observed to be consistent with that of the nominal decay model [14].

• **Tracking efficiency:** As a test, the track reconstruction efficiency is artificially reduced by $\sim 1\%$ in Monte Carlo to match that in data. This reduction of the efficiency, however, has not impact on the result of the fit.

V. RESULTS

After unblinding the data, the fit procedure is performed on the data. Figure 5 shows the result of the fit, indicating a $P_1$ estimator value of $1778.00 \pm 0.75$ MeV/c². Using the correction factor obtained in section III and the systematic uncertainties described in section IV, the mass of the $\tau$ lepton is measured as:

$$m_\tau = 1777.28 \pm 0.75 \text{ (stat.)} \pm 0.33 \text{ (sys.) MeV/c}^2$$  \hspace{1cm} (11)

VI. CONCLUSIONS

In summary, the mass of the $\tau$ lepton has been measured using the pseudomass method in a blinded analysis procedure. Using the $\tau^+ \rightarrow \pi^+\pi^-\pi^+\bar{\nu}_\tau$ decays in $8.8\,\text{fb}^{-1}$ of data, the mass of the $\tau$ lepton has been found to be $m_\tau = 1777.28 \pm 0.75 \text{ (stat.)} \pm 0.33 \text{ (sys.) MeV/c}^2$. This measurement is in good agreement with the current world average [2]. Figure 6 shows this result compared to other recent $\tau$ mass measurements.
Figure 5. The pseudomass ($M_{\text{min}}$) distribution in the data sample (black points) and the results of the fit (blue line).

Figure 6. The comparison of the $\tau$ mass measurements obtained in this analysis (in blue text) with the PDG average and measurements from various experiments. The green and blue bands indicate the statistical and total uncertainties, respectively.

The leading source of systematic uncertainty is the momentum scale factor, which is expected to be reduced in the near future. With the present level of the systematic
uncertainties, the Belle II $\tau$ mass measurement is expected to be statistically dominated until around $50 \text{ fb}^{-1}$ of data. After improvements in the momentum scale factor systematic uncertainty, a scenario with a total systematic uncertainty of $0.15 \text{ MeV/c}^2$ is foreseen, and about $200 \text{ fb}^{-1}$ of data would be needed to become systematically dominated, as illustrated in Fig. 7. The systematic uncertainties can be reduced further by increasing the Monte Carlo statistics in the estimation of the bias, which is currently the second-largest systematic uncertainty. Thus, a better systematic precision is expected in the future.

Figure 7. Projection of the statistical uncertainty as a function of luminosity for the $\tau$ mass measurement. The black dot represents the statistical uncertainty in this measurement and the red triangles mark the projected statistical uncertainties for 50, 100 and 200 $\text{ fb}^{-1}$.

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