Mind the Gap

The Space between Coincidence and Colocation

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Received: 22 December 2020 / Accepted: 31 May 2021 / Published online: 15 June 2021
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Abstract
In debates about the metaphysics of material objects examples of colocated objects are commonly taken to be examples of coincidence too. But the argument that colocation is best understood as involving coincidence is never spelled out. This paper shows under what conditions colocation entails coincidence and argues that the entailment depends on a principle that actually rules out certain forms of colocation. This undermines the argument from colocation to coincidence.

Keywords Mereology · Parthood · Colocation · Coincidence

1 Introduction
Many philosophers are attracted to The Standard View according to which an artefact is distinct from its matter [1, 10, 12, 23, 26–28, 32, 39, 43, 46]. Since the artefact and its matter have the same location at exactly the same moment in time, this view implies that some objects are colocated. Those who accept colocation will be called ‘colocationists’. Colocationists commonly also hold that distinct objects can be made of the same parts. This amounts to a denial of the extensionality of proper parthood, the principle which states that sameness of proper parts is sufficient for identity. I will use ‘coincidentalists’ as a name for those who accept that objects can coincide.

Most coincidentalists are colocationists. This is not too surprising. If distinct objects can be made of the same parts, it seems plausible these objects are also colocated. However, one could be a coincidentalist while denying colocation; for
example, by holding that in all cases where two objects coincide they have multilo-
cated parts that compose distinct objects, one fusion of those parts is located at one
region, whilst the other fusion is located at another region. For example, Michael
Burke [4] and Michael Rea [36] deny colocation but allow for distinct objects made
of the same parts, although not the same parts at the same time. If we take differ-
ent times to be different spacetime regions, their view is an example of coincidence
without colocation. Another possible example of coincidence without colocation, as
suggested by Kit Fine [12, p. 198], is a loaf of bread and the bread of which it is
made: the loaf is also (weakly) located at the air pockets in the bread, but the bread is
not. (Thanks to a reviewer for this journal for drawing my attention to this example.)

Conversely, most colocationists are coincidentalists. Notable exceptions are bundle
theorists who hold that an object is a fusion of colocated properties, be they
tropes or universals; and certain Neo-Aristotelians who defend hylomorphic theo-
ries according to which an object like a statue has a formal part that its matter lacks
(more about this below). But by far the majority of colocationists are coincidentalists.
So much so that colocation and coincidence are often conflated or that colocation is
taken to entail coincidence. For example, Karen Bennett writes that ‘the puzzle of
colocation can be framed in mereological terms. The question is whether a mereolog-
ical principle called uniqueness or extensionality is true—can the same parts compose
more than one thing?’ [3, p. 45]. Similarly, we find remarks such as ‘they deny that
two numerically distinct physical objects could be “wholly co-located”. That is, they
deny that two distinct physical objects could be composed of exactly the same parts
at some level of decomposition’ [31, p. 38]; and ‘cases of coloclated objects are cases
of part sharing’ [45, p. 625]. (See also [9, p. 310], [40, pp. 498–99], [39, p. 399], and
[43, pp. 117 and 248]).

So although colocation and coincidence are different things—as has been explicit-
ly stated before by Fine [12, p. 198] and Achille Varzi [48, pp. 118–119]—
colocation is commonly taken to entail coincidence. But the argument for this is never
given. My aim is to map the exact terrain in logical space by showing under what
conditions colocation and coincidence are equivalent. I will also ask whether there
are reasonable exit points on the road from colocation to coincidence. In particular
I will show that the argument from colocation to coincidence uses, unsurprisingly,
a principle that Neo-Aristotelians could object to. But it also uses another principle
that, quite surprisingly, is objectionable from the perspective of a colocationalist. It
turns out that examples of colocation are examples of coincidence only if one accepts
a principle that bans certain forms of colocation.

Here’s the plan. Section 2 introduces some definitions and principles concern-
ing coincidence, colocation, and related notions. I then present two arguments from
coincidence to colocation in Section 3; and, more significantly, an argument from
colocation to coincidence in Section 4. Since examples of colocation are commonly
taken as examples of coincidence too, the rest of the paper discusses the effectiveness
of this argument from colocation to coincidence. In particular, Section 5 discusses a
key principle in the argument and explains why it is not colocation-friendly. I con-
clude that whatever reasons one has for accepting colocation, they do not transfer to
reasons for accepting coincidence, too. To the contrary, a friend of colocation would
deny a crucial principle needed in the derivation from colocation to coincidence.
2 Locations and Extensional Mereology

We start with purely mereological definitions and principles, i.e. those concerning the part–whole relation. The mereological theory presented here is extensional because it identifies entities with the same overlappers or the same parts. Coincident objects are thus ruled out. This set-up is deliberate since, first, for the arguments from coincidence to colocation we will have to suppose that regions of space form a model of extensional mereology (but in those cases we suppose very little about the mereology of objects) and, second, the argument from colocation to coincidence below will be a *reductio ad absurdum*, making it easier to see under which conditions colocation is compatible with extensionality.

Since we care about the mereological structure of both objects and regions of space the following definitions and principles are meant to apply to both. Later we distinguish between variables ranging over objects and variables ranging over regions.

From the primitive ‘is part of’, formalised as $Pxy$, we define the following notions

\[
PPxy = df Pxy \land \neg Pyx \quad \text{(Proper Parthood)}
\]

\[
Oxy = df \exists z (Pzx \land Pzy) \quad \text{(Overlap)}
\]

\[
Ax = df \neg \exists y PPxy \quad \text{(Atom)}
\]

\[
Fu(z, \varphi x) = df \forall x (\varphi x \rightarrow Pxz) \land \forall y (Pyz \rightarrow \exists w (Owy \land \varphi w)) \quad \text{(Fusion)}
\]

(Where ‘$z$’ and ‘$y$’ do not occur free in ‘$\varphi x$’)

These definitions are quite straightforward. (To avoid unwanted connotations I will often use the word ‘simple’ instead of ‘atom’; also, by ‘composite’ I mean anything that is not simple).

The following principles tell us that parthood is a partial order; that if everything that is part of $y$ overlaps $x$, then $y$ is part of $x$; and that for any non-empty condition $\varphi$, there is a fusion of the entities satisfying $\varphi$.

\[
\forall x Pxx \quad \text{(Reflexivity)}
\]

\[
\forall x \forall y \forall z ((Pxy \land Pyz) \rightarrow Pxz) \quad \text{(Transitivity)}
\]

\[
\forall x \forall y ((Pxy \land Pyx) \rightarrow x = y) \quad \text{(Antisymmetry)}
\]

\[
\forall x \forall y (\neg Pxy \rightarrow \exists z (Pyz \land \neg Ozx)) \quad \text{(Strong Supplementation)}
\]

\[
\exists x \varphi x \rightarrow \exists z Fu(z, \varphi x) \quad \text{(Unrestricted Composition)}
\]

The first four principles form an axiomatic basis for Extensional Mereology (EM); adding Unrestricted Composition as an axiom schema turns this into General Extensional Mereology (GEM) or ‘Classical Mereology’ (the nomenclature I use follows [49] to which the reader is also directed for more information on mereology). Whether the fusion operation is unrestricted is a controversial matter, so our discussion will focus on EM. However, in the Appendix A we show that there are colocation-friendly models that satisfy GEM.

(Note also that due to Antisymmetry the above definition of proper parthood is equivalent to the definition of ‘non-identical proper part’, i.e. ‘$x$ is part of $y$ and $x \neq y$’. Later we will consider a mereology without Antisymmetry where these two notions of proper parthood come apart).
We can now make two notions of coincidence precise.

\[ OC_{xy} =_{df} x \neq y \land \forall z (Ozx \leftrightarrow Ozy) \]  
\( \text{(O-Coincidence)} \)

\[ PC_{xy} =_{df} \neg Ax \land \neg Ay \land x \neq y \land \forall z (PPzx \leftrightarrow PPzy) \]  
\( \text{(PP-Coincidence)} \)

Two objects O-coincide if they are numerically distinct but overlap all the same objects. Two objects PP-coincide if they are numerically distinct composites that have the same proper parts. Coincidence can be made precise in purely mereological terms which provides further evidence, if that were needed, that coincidence and colocation are distinct notions. In EM, if \( x \) and \( y \) overlap the same things, or are composite and have the same proper parts, then \( x = y \) [43, pp. 28ff]. So EM is incompatible with either notion of coincidence.

To make colocation and neighbouring notions precise we start with ‘is exactly located at’ as a primitive two-place predicate. One should understand the relation of exact location as holding between an object and a region if and only if the object has the same shape and size, and stands in all the same spatial relations to other entities, as the region. The relation is symbolised as ‘\( L_{xr} \)’, where \( x \) is a variable taking objects as values while \( r \) takes regions as values. (From hereon variables \( x, y, z \) take objects as their values while \( r, s, t \) take regions. Moreover, I assume substantivalism about space: regions of space are entities in addition to objects that can occupy such regions. I think this assumption is inessential: everything I say would still hold in a relationist framework, assuming that a relationist can make sense of colocation. But the assumption greatly streamlines the presentation and formulation of the relevant principles. I will assume the falsity of supersubstantivalism—the view according to which an object is identical with its exact region. Supposing supersubstantivalism would end the discussion prematurely since, together with the transitivity of identity, it rules out colocation).

We can make colocation precise as a case where distinct objects have the exact same location:

\[ CL_{xy} =_{df} \exists r (L_{xr} \land L_{yr} \land x \neq y) \]  
\( \text{(Colocation)} \)

A model is colocation-friendly if there are objects in the model satisfying this definition. Note that we can distinguish three forms of colocation: a case where both objects are simple, where both objects are composite, and a mixed case where a composite object is colocated with a simple.

We then also define ‘is weakly located at’, ‘is an extended simple’, and ‘is an unextended composite’ as follows:

\[ WL_{xr} =_{df} \exists s (L_{xs} \land Ors) \]  
\( \text{(Weak Location)} \)

\[ ESx =_{df} Ax \land \exists r (L_{xr} \land \neg Ar) \]  
\( \text{(Extended Simple)} \)

\[ UCx =_{df} \neg Ax \land \exists r (L_{xr} \land Ar) \]  
\( \text{(Unextended Composite)} \)

There is no standard or classical theory of location and the reader is directed to [6, 18, 35] for more elaborate discussions. The real philosophical meat concerns the way the mereological structure of an object combines with the mereological structure of space. There are six principles relevant to our discussion. The first three play a role in all the arguments that we will present, while each of the others matters only to one argument.
The first is Expansivity and states that \( y \) is exactly located at a region that has \( r \) as a part if a part of \( y \) is exactly located at \( r \). This ensures a whole is located wherever its parts are located. The second principle, Functionality, ensures that a located object has at most one exact location. The idea that every object has a location is expressed by Totality, the third principle. These three principles are used in all the arguments in Sections 3 and 4. The fourth principle, Arbitrary Partition, states that every subregion of an object’s exact location is the exact location of a part of that object. Delegation says that parts of the location of a composite object are weakly occupied by proper parts of the composite object. These two principles matter to the arguments in Section 3. No Interpenetration is the final principle and states that objects overlap if they are located at overlapping regions. This principle matters for the argument in Section 4 and is discussed extensively in Section 5. Formally:

\[
\forall x \forall y \forall r ((Pxy \land Lxr) \rightarrow \exists s (Lys \land Prs)) \quad \text{(Expansivity)}
\]
\[
\forall x \forall r \forall s ((Lxr \land Lxs) \rightarrow r = s) \quad \text{(Functionality)}
\]
\[
\forall x \exists r Lxr \quad \text{(Totality)}
\]
\[
\forall x \forall r \forall s ((Lxr \land Psr) \rightarrow \exists y (Pyx \land Lys)) \quad \text{(Arbitrary Partition)}
\]
\[
\forall x \forall r \forall y \forall s ((Lxr \land Lys \land Ors) \rightarrow Ox y) \quad \text{(No Interpenetration)}
\]

(A short note about the origins of these principles. To the best of my knowledge Expansivity comes from [35, p. 213]. There is a similar principle, sometimes given the same name [18, Sec. 3], which seems to come from Casati and Varzi [6, p. 122] and which they call ‘L.3’. This latter principle appears as ‘Strong Expansivity’ in the next section. Functionality also comes from Casati and Varzi and is so named because it ensures ‘\( L \) is a functional relation’ [6, p. 121]. Totality appears in [6, p. 126] as L.12; I call it ‘Totality’ because it ensures \( L \) is a total functional relation whenever it is a functional relation. Arbitrary Partition appears as L.5 in [6, p. 122] and seems to be baptised by [35, p. 211]. Delegation and No Interpenetration come from [18, Sec. 3]. A principle very similar to Delegation, and with the same name, can be found in [16, p. 130]. [18, Sec. 4] gives No Interpenetration, which corresponds to [47, p. 209]’s 2\( \rho b \) if one assumes Functionality).

In the next section I will show two ways in which one can argue from coincidence to colocation using some of the above principles of location. There I also point to some ways in which the arguments can be resisted. In the section after that I discuss the other direction: how to argue from colocation to coincidence. Since it is quite popular to regard examples of colocation as examples of coincidence too, I devote Section 5 to discussing the weakest link in the argument from colocation to coincidence.

### 3 Two Arguments from Coincidence to Colocation

I have distinguished two notions of coincidence: O-Coincidence and PP-Coincidence. To derive colocation from coincidence it matters which form of coincidence we assume. (I would like to thank a reviewer for this journal for asking...
me to discuss both cases.) If parthood is transitive and we are only considering composite objects, PP-Coincidence entails O-Coincidence. (To see this, suppose some \( z \) overlaps with \( x \) and let \( x \) and \( y \) be PP-Coincident (the case where \( z \) instead overlaps \( y \) is similar). If \( z \) overlaps with \( x \) because \( z \) is a proper part of \( x \) or because a proper part of \( x \) is also part of \( z \), then \( z \) also overlaps \( y \) because \( x \) and \( y \) have the same proper parts. So suppose instead that \( x \) is part of \( z \). Then since \( x \) is composite it has a proper part, \( w \), which is also a proper part of \( y \). By Transitivity, \( w \) is part of \( z \), so \( z \) overlaps \( y \).) So let us start with O-Coincidence.

We can derive colocation from O-Coincidence if we assume that the principles of EM hold for the mereological structure of space (but, of course, not for the mereological structure of objects) and that Arbitrary Partition, Expansivity, Functionality, and Totality hold.

To streamline the discussion it is worth noting some consequences of the above principles. First, Expansivity and Functionality together entail Strong Expansivity:

\[
\forall x \forall y \forall r \forall s ((Pxy \land Lxr \land Lys) \rightarrow Prs) \quad \text{(Strong Expansivity)}
\]

To see this, suppose that \( x \) is part of \( y \) and located at \( r \) and \( y \) is located at \( s \). Since \( x \) is part of \( y \) and located at \( r \), Expansivity gives a region \( t \) such that \( r \) is part of \( t \) and \( y \) is exactly located at \( t \). By Functionality \( t = s \), hence \( r \) is part of \( s \). (As noted, the nomenclature is not uniform here: Casati and Varzi call Strong Expansivity ‘L.3’ [6, p. 122] and [18, Sec. 3] calls it ‘Expansivity’. Parsons [35, p. 224] prefers calling it ‘Weak Expansivity’ because it allows for non-located objects that have parts that are located while Expansivity rules this out. Below I explain why I prefer calling it ‘Strong Expansivity’ instead).

Strong Expansivity together with Totality gives the following principle:

\[
\forall x \forall y (Oxy \rightarrow \forall r \forall s ((Lxr \land Lys) \rightarrow Ors)) \quad \text{(Sharing Space)}
\]

(Strong Expansivity)

To see this, suppose that \( x \) is part of \( y \) and located at \( r \) and \( y \) is located at \( s \). Since \( x \) is part of \( y \) and located at \( r \), Expansivity gives a region \( t \) such that \( r \) is part of \( t \) and \( y \) is exactly located at \( t \). By Functionality \( t = s \), hence \( r \) is part of \( s \). (As noted, the nomenclature is not uniform here: Casati and Varzi call Strong Expansivity ‘L.3’ [6, p. 122] and [18, Sec. 3] calls it ‘Expansivity’. Parsons [35, p. 224] prefers calling it ‘Weak Expansivity’ because it allows for non-located objects that have parts that are located while Expansivity rules this out. Below I explain why I prefer calling it ‘Strong Expansivity’ instead).

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\]

(Strong Expansivity)

To see this, suppose in accordance with the antecedent that \( x \) and \( y \) are O-Coincident and located at \( r_1 \) and \( r_2 \) respectively (by Totality they must each have at least one location). Since \( z \) is part of \( x \) and these objects are located at \( r_3 \) and \( r_1 \) respectively, \( r_3 \) is part of \( r_1 \) by Strong Expansivity. Similar reasoning will show that \( r_3 \) is part of \( r_2 \). So \( r_1 \) and \( r_2 \) overlap.) (This principle is similar to (2.6) from [5, p. 127].)

Now, note that in EM entities that overlap have a product, so the following holds for regions of space:

\[
\forall r_1 \forall r_2 (Or_{r_1}r_2 \rightarrow \exists s \forall t (P_{ts} \leftrightarrow (P_{tr_1} \land P_{tr_2}))) \quad \text{(Product)}
\]

From Sharing Space and Arbitrary Partition we can then derive the following principle:

\[
\forall x \forall y \forall r \forall s ((\forall z (Ozx \leftrightarrow Ozy) \land Lxr \land Lys) \rightarrow \forall t (O_{ts} \leftrightarrow O_{tr})) \quad \text{(T1)}
\]

(Strong Expansivity)

To see this, suppose in accordance with the antecedent that \( x \) and \( y \) are O-Coincident and located at \( r_1 \) and \( r_2 \) respectively. And suppose for contradiction that either some region \( t_1 \) overlaps \( r_1 \) but not \( r_2 \) or some region overlaps \( r_2 \) but not \( r_1 \). These two cases are symmetrical so we only consider the first case. Since \( t_1 \) and \( r_1 \) overlap they have a product, \( r_3 \). From Arbitrary Partition and since \( r_3 \) is part of \( r_1 \) and \( x \) is located at \( r_1 \) we get that \( x \) has a part, \( z \), exactly located at \( r_3 \). Since \( z \) overlaps...
x and since x and y overlap all the same things, y overlaps z; and thus, by Sharing Space, r2 overlaps with r3. Let t2 be a part of r2 and r3 share. Thus t2 is part of r3 and r3 is part of t1; hence r2 and t1 overlap at t2 by Transitivity—contrary to our supposition that r2 and t1 did not overlap.

Since we are supposing that regions of space form a model of EM, and since in EM having the same overlappers is sufficient for identity [43, pp. 28ff], the consequent of T1 directly leads to the desired conclusion that r = s and thus that x and y are colocated if they are O-Coincident. The above argument would also work for PP-Coincidence if parthood is transitive because, as noted above, composites that are PP-Coincident are also O-Coincident.

But there is another route from PP-Coincidence to colocation worth noting. Suppose we have Expansivity, Functionality, Totality, and Delegation instead of Arbitrary Partition. We can then reason from PP-Coincidence to colocation as follows. Suppose x is coincident with y, that x is exactly located at r1, and has proper part, z, exactly located at s. (All these things have a location due to Totality.) Since x is coincident with y, z is also a proper part of y, and thus part of y. So by Expansivity, there is a region, r2 such that s is part of r2 and y is exactly located at r2. By classical logic r1 = r2 or r1 ≠ r2. In the first case we get that x and y are colocated so we are done. So suppose instead that r1 ≠ r2.

Again by classical logic, r1 is part of r2 or r1 is not part of r2. Suppose it is, then r1 is a proper part of r2, and r2 is thus not part of r1. By Strong Supplementation there is a t that is part of r2 and does not overlap r1. Since t is part of r2 and y is exactly located at r2 there is a proper part of y, z2, that is weakly located at t (by Delegation). By the definition of weak location z2 is exactly located at some region t2 which overlaps t. Since x PP-coincides with y, z2 is a proper part of x. So by Expansivity and Functionality, t2 is part of r1. Since t overlaps t2 and t2 is part of r1, we get via Transitivity that t overlaps with r1. Contradiction.

If, on the other hand, r1 is not part of r2, we get via Strong Supplementation that there is a t such that t is part of r1 and t does not overlap r2. The reasoning is now basically the same as in the previous case, except that x is swapped with y, and r1 is swapped with r2. In either case we get a contradiction.

These two arguments from coincidence to colocation are not as philosophically significant as the argument from colocation to coincidence in the next section because most coincidentalists are happy to be colocationists too. However, as noted in the introduction, there are some exceptions which warrants a brief discussion of possible ways to resist the above two arguments.

Some philosophers, such as [4] and [36], accept coincidence because they think the same collection of objects may compose different wholes at different times. But they deny colocation and hold that no collection of objects composes two different wholes at one and the same time. They would probably resist both arguments for colocation by denying Functionality, i.e. by accepting some form of multi-location. This is well-motivated since they already think that, for example, the marble particles compose a piece of clay at t1 but those very same particles compose a statue at t2—and at no time do the particles compose both a piece of clay and a statue. So it makes sense they would deny Functionality because each of the clay particles is already, according
to them, a counterexample to it (i.e., each of them is multi-located at (spacetime) regions $t_1$ and $t_2$).

Burke and Rea should probably also deny Strong Expansivity since that principle implies that all the locations of an object are parts of each other. Thus if parthood is antisymmetric, Strong Expansivity entails Functionality. (In this respect Expansivity is thus weaker than Strong Expansivity: Expansivity does not imply that the locations of an object are all parts of each other. As noted above, [35, p. 224] prefers to call Strong Expansivity ‘Weak Expansivity’ since it still allows for non-located objects that have located parts. But it seems to me that Totality is often, implicitly, assumed in discussions about location, which would already rule out such cases. The fact that Strong Expansivity entails Functionality if parthood is antisymmetric seems to me a more relevant consideration—although I realise Parsons would probably disagree since he takes Functionality as non-negotiable [35, pp, 219ff]).

Now, a different possible example of coincidence without colocation is Fine’s loaf of bread and the bread it is made from [12, p. 198]. Denying Functionality seems unmotivated in this case because the example does not appeal to multi-location. In this case it is thus not immediately obvious which principle to deny to block both arguments from coincidence to colocation. Fine thinks the loaf and the bread have the same parts but not the exact same location because, it seems, only one of them is (weakly) located at the pockets of air in the bread. So either only one of them is weakly located at those pockets of air without having any parts there or only one of them is not weakly located at those pockets of air despite having parts there. (The air pockets might be, for example, negative parts of the loaf and the bread [6, Ch. 8].) I guess Fine would go for the first option, in which case he would deny Delegation and Arbitrary Partition. But one could also go for the second option and deny Expansivity.

4 The Argument from Colocation to Coincidence

There are thus two ways in which one could derive colocation from coincidence. But it is more common to travel the road in the other direction: many philosophers take examples of colocation to be examples of coincidence too. And we can indeed derive coincidence from a case of colocation given Expansivity, Functionality, Totality, and No Interpenetration. (Let’s call this argument ‘the argument from colocation’).

To see this, suppose for contradiction that there is no coincidence, i.e. that we have an extensional mereology such as EM. (To be precise, we need to assume a mereology that satisfies Antisymmetry, Transitivity, and Strong Supplementation.)

Consider first a case where $x$ and $y$ are simple and colocated at $r$. By definition $r$ overlaps itself. So since $x$ and $y$ are located at an overlapping region, by No Interpenetration $x$ overlaps $y$ and they must thus share a part. But since they are simple they can only share a part by being part of each other. Hence by Antisymmetry $x = y$—contradiction.

So suppose instead that $x$ is composite and colocated with $y$ at $r$. Since $x$ and $y$ are colocated $x \neq y$. Thus by extensionality there is an object $z$ that is a proper part of one but not of the other. Say $z$ is a proper part of $x$ but not of $y$ (the argument is
basically the same if instead \( z \) is a proper part of \( y \). Since \( z \) is not a proper part of \( y \), either (i) \( z \) is not part of \( y \) or (ii) \( z = y \).

In case (i) we have by Strong Supplementation a \( w \) that is part of \( z \) but does not overlap \( y \). By Totality \( w \) is located at some region \( s \). \( w \) is part of \( x \) since parthood is transitive. By Expansivity \( s \) is part of a region where \( x \) is located, which, by Functionality, means that \( x \) is part of, and thus overlaps with, \( r \). Since \( y \) is located at \( r \), \( w \) is located at \( s \), and \( s \) overlaps with \( r \), by No Interpenetration we have that \( y \) overlaps \( w \)—contradiction.

In case (ii) \( y \) is a proper part of \( x \), so \( x \) is not part of \( y \). Hence, by Strong Supplementation there is a \( w \) that is part of \( x \) but does not overlap \( y \). By Totality \( w \) is located at some region \( s \). (From hereon the argument is the same as the second half of case (i)).

In either case we end up with a contradiction, hence colocation is impossible in an extensional mereology given Expansivity, Functionality, No Interpenetration, and Totality. Many colocationists take this to be a reason to deny extensionality by denying Strong Supplementation or Antisymmetry. But could we give up one of the other principles instead?

Expansivity is very plausible to anyone who thinks there is some correlation between an object’s location and the location of its parts. It would be bad news for extensional mereologists if their position can be made compatible with colocation only by denying Expansivity. So I take it to be non-negotiable in this context. Functionality is not so plausible for it rules out certain forms of multilocation that plenty of philosophers consider possible or even actual [2, 20, 29]. In particular, many would deny Functionality because they think enduring objects are multilocated objects, which is particularly interesting since many coincidentalists are also endurantists. These philosophers thus have a hard time arguing for coincidence on the basis of colocation exactly because that argument depends on a principle they consider false.

However, I do not consider dropping Functionality here for two reasons. First, to properly evaluate Functionality we have to consider time and persistence, which would bring us too far afield and may not get to the heart of the matter since colocation seems possible in a timeless universe. Second, as far as I know no one denies Functionality in order to have an extensional mereology compatible with colocation. And I am not sure what such a defence would look like: what is it about sharing a single location that forces an object to have another location too? So I leave Functionality as it is, for sake of the argument.

That leaves No Interpenetration and Totality; dropping either suffices to make EM colocation-friendly. The next section is entirely devoted to No Interpenetration. Here I will just say a few words about the other option. Denying Totality amounts to accepting objects that lack an exact location. Since weak location is defined in terms of exact location, such objects are not located anywhere. (But note, as an anonymous reviewer pointed out, that if we had started with ‘weak location’ as primitive instead, we might deny Totality while still doing justice to the intuition behind it—‘everything needs to be somewhere’—by holding that everything has a weak location, but not everything has an exact location. Denying Totality (in its official formulation) might then be easier to swallow for some. In the remainder, however, we stick to our official set-up with ‘exact location’ as primitive).
Anyone who thinks that objects can have at least two kinds of parts may be attracted to dropping Totality—at least as long as one is willing to hold that one kind of part is not (exactly) located in space. This non-located parts approach may be particularly attractive to philosophers who hold that some objects have both material and formal parts (assuming formal parts are non-located). Such views are sometimes called ‘neo-Aristotelian’ or ‘hylomorphic’ theories [13, 26] and all hold that an object is more than its matter and this ‘more’ is often called ‘form’. Neo-Aristotelian views are not often combined with an extensional mereology but the combination of EM, Expansivity, Functionality, and No Interpenetration is perfectly consistent as shown in the Appendix A.

In the remainder I ignore the option of dropping Totality because we have far stronger reasons to deny No Interpenetration instead. Any friend of colocation should be sceptical of No Interpenetration, irrespective of whether one accepts Totality.

5 No Interpenetration as an Anti-Colocation Principle

The strength of the argument from colocation thus depends, partly, on the plausibility of No Interpenetration. This section argues that No Interpenetration blocks certain forms of colocation even with only very weak mereological assumptions in place. The argument from colocation to coincidence from the previous section is thus weakened by the fact that it only works if one denies certain forms of colocation. I start this section by briefly mentioning some general reasons for and against No Interpenetration and I argue that the reasons for No Interpenetration cannot be used to back up the argument from colocation to coincidence. I then argue that colocationists in particular should be suspicious of No Interpenetration because it blocks certain forms of colocation even in very weak mereological theories. I conclude the section by briefly considering how No Interpenetration could be amended to develop a more plausible version of the argument from colocation to coincidence.

Some philosophers will have no problems with No Interpenetration. For example, supersubstantivalists are trivially committed to No Interpenetration but, as noted above, no colocationist can accept supersubstantivalism on pain of contradiction. A related view holds that an object coincides with its exact location [17, 19]. As Cody Gilmore [18, supplement] points out, this view also entails No Interpenetration. However, defending the argument from colocation on the basis of this form of substantivalism is circular: coincidence is established by supposing coincidence. (Note that, as far as I know, defenders of this type of substantivalism do not commit this fallacy since they already presuppose coincidence between objects instead of arguing for it on the basis of colocation.) So, as far as I can see, there is no straightforward argument for No Interpenetration that the colocationist can use in their argument from colocation.

Furthermore, there are plenty of reasons to opt for the ‘interpenetration approach’ (nomen est omen). The position could be associated with L. A. Paul’s [33, 34] theory according to which, for example, a statue and its matter are colocated yet not coincident. They are not coincident because they have different properties and, according to Paul, properties are literally parts of objects. (Paul, however, would probably deny
Functionality as well because she thinks properties are multilocated [34, p. 632]. The interpenetration approach is also found in the systems developed by Nelson Goodman [15] since he accepts individuals can be ‘together’ (colocated) but denies coincidence. Indeed, banning coincident objects is the hallmark of his version of nominalism [14].

One need not endorse either Paul’s or Goodman’s theory to be attracted to the interpenetration approach. No Interpenetration has been criticised for various reasons. On some interpretations of particle physics bosons violate No Interpenetration [21, pp. 55–56]. (On other interpretations bosons are compatible with No Interpenetration [38, p. 140].) And No Interpenetration commits one to brute metaphysical necessities because it is incompatible with plausible principles of recombination [30, 37, 41].

There is no need to review the arguments for interpenetration here (but see [18] for an overview) because in the present context there is a more direct complaint we can put forward. No Interpenetration bans certain forms of colocation even in very weak mereological theories. Hence anyone who thinks colocation is possible should be suspicious of it. I take this to be the fundamental problem with the argument from colocation: in order to use that argument one should suppose that some forms of colocation are impossible.

To see this note first that Antisymmetry and No Interpenetration block colocated simples. No Interpenetration entails that colocated simples overlap. But since they are simple they cannot overlap unless they are identical (by Antisymmetry). (This was the first part of the argument from Section 4.) Thus the colocationist who accepts No Interpenetration and Antisymmetry must hold that simples cannot be colocated. But this is an unmotivated position. Why should simples crowd each other out while composites can share an address?

One might respond that composites can be colocated because they have the same proper parts while simples cannot be colocated precisely because they do not have any proper parts. But, first, it is not at all obvious that colocated objects have the same parts. For example, the head of a statue is definitely a part of the statue but, pre-theoretically, it would not count as part of the marble (see [1, p. 81], [48] and [50] inter alia). Second, this response makes the argument for coincidence circular. To derive coincidence from colocation we need No Interpenetration but this response on behalf of No Interpenetration assumes coincidence.

A better response would be to keep No Interpenetration and allow for colocated simples by denying Antisymmetry; the latter counts as a light extensionality principle for it identifies entities that are parts of each other. There may thus be a neat symmetry: strong extensionality principles are denied to allow for colocated composites and Antisymmetry is denied to allow for colocated simples. Indeed, Judith Jarvis Thomson [46] and A. J. Cotnoir [7] explicitly deny Antisymmetry and hold that cases of colocation are cases of mutual parthood. (Note that this is a minority view: most colocationists hold that, say, a statue has its matter as a part, but the statue is not part of the matter. Mutual parthood theorists instead hold that ‘colocated objects (...) are mutual parts’ [8, p. 963]—see also [46, p. 155]). But mutual parthood theories only get a colocationist so far. For even with Antisymmetry gone, some form of colocation is still blocked.
In particular, No Interpenetration together with Functionality, Totality, and Expansivity does not allow for the colocation of a simple with a composite if we suppose some supplementation principle for parthood. Besides Strong Supplementation (stated in Section 2), there are two other main candidates to express the idea that a composite object has at least two parts:

\[ \forall x \forall y (P_{xy} \rightarrow \exists z (P_{zy} \land \neg O_{zx})) \]  
(Weak Supplementation)

\[ \forall x \forall y (P_{xy} \rightarrow \exists z \exists w (P_{zy} \land P_{wy} \land \neg O_{zw})) \]  
(Quasi-Supplementation)

(If parthood is transitive and reflexive, Weak Supplementation entails Antisymmetry [49, Sec. 3.1]. So, in order to consider a consistent scenario, we will not assume that parthood is transitive. Note also that Weak Supplementation entails Quasi-Supplementation given our definition of proper parthood: just let \( w \) be \( x \) in the consequent of Quasi-Supplementation).

Now, any of the supplementation principles leads to a contradiction when we assume that a composite is colocated with a simple and No Interpenetration, Functionality, Expansivity, and Totality hold.

To see this, suppose \( x \) is simple and colocated at region \( r_1 \) with composite \( y \). Since \( y \) is composite, it has a proper part \( z \). We then argue as follows, depending on which supplementation principle is accepted.

(Case 1) If one accepts Strong Supplementation, we reason as follows. Since \( z \) is a proper part of \( y \), \( y \) is not part of \( z \), hence by Strong Supplementation, there is a \( w \) such that \( w \) is part of \( y \) and does not overlap \( z \). By Totality and Functionality \( z \) and \( w \) have unique locations \( r_2 \) and \( r_3 \), respectively. By Expansivity, both \( r_2 \) and \( r_3 \) are parts of some location of \( y \), and this location of \( y \) has to be \( r_1 \) because of Functionality. So, by No Interpenetration, both \( z \) and \( w \) overlap \( x \). Since \( x \) is simple, it can only overlap with \( z \) and \( w \) if it is part of \( z \) and \( w \). So \( z \) and \( w \) do overlap, contrary to our supposition.

(Case 2) If one accepts Weak Supplementation, we reason as follows. Since \( z \) is a proper part of \( y \), there is a \( w \) such that \( w \) is part of \( y \) and does not overlap \( z \). By Totality and Functionality \( z \) and \( w \) have unique locations \( r_2 \) and \( r_3 \), respectively. (The reasoning is now the same as in the previous case).

(Case 3) Finally, if one accepts Quasi-Supplementation, we reason as follows. Since \( z \) is a proper part of \( y \), there is a \( v \) and a \( w \) such that \( v \) is part of \( y \), \( w \) is part of \( y \), and \( v \) does not overlap with \( w \). (The reasoning is now basically the same as in the previous case, but with \( v \) and \( w \) rather than \( w \) and \( z \)).

In either case we end up with a contradiction. Hence, No Interpenetration, Expansivity, Functionality, and Totality block the possibility of a simple object being colocated with a composite object if we suppose some supplementation principle. So even if we deny Antisymmetry, No Interpenetration still rules out some form of colocation. (Note that the above argument is valid in very weak mereological systems for the arguments only suppose that parthood is supplemented in some way.) Hence, a colocationist is better off denying No Interpenetration because only the denial of No Interpenetration allows for all forms of colocation.

So colocation only entails coincidence if one explicitly denies some forms of colocation. Of course, some colocationists might think that these other forms of
colocation should be denied in any case—maybe because extended simples and unextended composites are impossible and thus never colocated with anything. Still, no one has argued for coincidence on the basis of colocation and a denial of extended simples and unextended composites. And such an argument for coincidence is vulnerable: anyone who is comfortable with the possibility of either extended simples or unextended composites would not be convinced that colocation should be understood as a form of coincidence.

Still, No Interpenetration is not completely implausible and, as a reviewer for this journal suggested, one might defend a restricted version of it that holds in cases like the statue and its marble but not in others. Ideally, such a restriction would also exclude the problematic case of a simple being colocated with a composite. Whether this is dialectically effective depends on the formulation of this restriction. For example, it would be question-begging to hold that No Interpenetration holds for all those cases that are commonly taken as examples of both colocation and coincidence. But if one could find something these standard puzzles have in common—and is not had by any of the counterexamples to No Interpenetration—then one could defend a (principled) restricted version of No Interpenetration. Such a restricted version might then be used to reinstate the argument from colocation to coincidence from Section 4.

The good news for the argument from colocation to coincidence is thus that there is, in theory, a way to save it: a principled restriction for No Interpenetration that applies to all and only the ‘good’ cases would suffice. The bad news is that it hard to see what this restriction would look like. No Interpenetration gets its initial plausibility from cases where we have composite objects that are qualitatively similar. For example, semi-detached houses are located at overlapping regions not because two walls are colocated but rather because the houses overlap, they share a single wall. We could generalise from this example and say that No Interpenetration holds for objects of the same kind or category. (Never mind what kinds or categories are exactly or how to individuate them.) This restriction to objects of the same kind could exclude the problematic case of a composite that is colocated with a simple by holding that simple and composite are different ontological kinds.

However, the problem with this move is that the statue and its marble are often said to be of different kinds. Indeed, this is precisely the reason why, for example, [51] thinks such entities can be colocated. So this proposed restriction actually excludes the very examples that it was meant to cover in order to argue from colocation to coincidence. (Note that it is contentious whether friends of colocation can restrict colocation to objects of different kinds, see [11, 22, 24, 42, 44]).

Perhaps we should instead focus on the fact that semi-detached houses as well as statues and pieces of marble are composite objects and thus restrict No Interpenetration to composites. This would again exclude the case of a composite colocated with a simple. But this restriction is not without problems either. It would raise the question why only composites need to overlap in order to be colocated? If simples can be colocated without overlapping, it seems composites should be able to do so, too. And if simples cannot be colocated we return to a question raised earlier in this section: why do simples crowd each other out while composites made from such simples do not?
Of course, showing that two ways of restricting No Interpenetration raise further questions does not mean a good restriction cannot be given. It does show that it is a challenge. No Interpenetration gets its plausibility from certain examples but it is unclear whether these have enough in common with, say, the statue and its marble such that No Interpenetration can be convincingly used to argue from colocation to coincidence. It is up to those who hold that colocation is best understood as involving coincidence to meet this challenge. (See also [25] for a more general discussion about different ways of being colocated).

6 Conclusion

Colocation and coincidence are often conflated, which would be less of a problem if they entail each other under highly plausible assumptions. In this paper I have shown that there are ways to derive one from the other. If one accepts Expansivity, Functionality, Totality and either Delegation or Arbitrary Partition, then composite objects with the same proper parts will also have the same exact location. And if we assume Expansivity, Functionality, Totality, and No Interpenetration, then objects with the same exact location have the same proper parts or overlap the same things. This last argument is particularly significant given that many philosophers think colocation is actual or at the very least metaphysically possible. If the argument were convincing it would present a strong case against extensional mereologies: such mereologies would not be able to handle colocation since they leave no room for coincidence.

However, the derivation from colocation to coincidence is unconvincing and may be blocked by denying No Interpenetration. Any friend of colocation should be suspicious of No Interpenetration because this principle bans certain forms of colocation in the presence of rather plausible additional principles. If we assume that parthood is antisymmetric, No Interpenetration bans colocated simples. And if we assume Quasi-Supplementation, Weak Supplementation, or Strong Supplementation, then No Interpenetration together with Expansivity, Functionality, and Totality, bans the colocation of a composite object with a simple. So No Interpenetration is an anticolocation principle and constitutes the most problematic part of any argument that aims to show that colocation should be understood in terms of coincidence. This weakens the case for coincidence.

Of course, I have not given any positive reasons for denying coincidence, i.e. for accepting an extensional mereology. Such a case has to be made on another occasion. This paper merely defends extensionality from arguments that start with an example of colocation and aim to derive coincidence. The gap between colocation and coincidence is wider than it may have seemed and the bridge between them looks frail—reasonable people may decide not to cross it.

Appendix A: Colocation models

There are at least two ways in which an extensional mereologist can allow for colocation: the non-located parts approach and the interpenetration approach. More
sophisticated philosophical interpretations of these two approaches are not at issue here. Instead I will present a few Hasse diagrams that illustrate how general extensional mereology can have colocation-friendly models.

A GEM non-located parts model is represented in Fig. 1, where full lines going upwards represent proper parthood relations between objects (its transitive closure is omitted) and dotted arrows represent the relation of exact location. So x and y₁ are colocated at s, a region composed of r₁ and r₂. Note that z₃ is a non-located atom, which means that the composites y₂ and y₃ are also each colocated with one of their atomic parts because their other atomic part is non-located.

Fig. 2 GEM with interpenetration
A model of GEM that allows for interpenetration is represented in Fig. 2. Note that $y_3$ is colocated with each of its parts, because all its parts are colocated. One can easily check that these figures satisfy all the relevant principles.

Acknowledgements Parts of this paper were presented at the ILLC in Amsterdam on 28 June 2019 and, online, at the Philosophy Department in Aarhus on 22 November 2019. I would like to thank the organisers Arianna Betti and Peter Hawke, and Johanna Seibt, respectively, and the audiences at those events for their discussion. Special thanks to Claudio Calosi and David Liggins for comments on previous versions of this paper. Thanks also to various anonymous reviewers and especially the reviewer for this journal for their comments. And, finally, many thanks to the Swedish Research Council for funding my research (international postdoc grant number 2017-06160_3).

Funding Open access funding provided by Lund University.

Conflicts of Interest The author declares that he has no conflict of interest.

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