$Z \to b\bar{b}$ in non-minimal Universal Extra Dimensional Model

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Abstract

We calculate the effective $Zbb$ coupling at one loop level, in the framework of non-minimal Universal Extra Dimensional (nmUED) model. Non-minimality in the Universal Extra Dimensional (UED) framework is realized by adding kinetic terms with arbitrary coefficients to the action at the boundary points of the extra space like dimension. A recent estimation of the Standard Model (SM) contribution to $Zbb$ coupling at two loop level, points to a $1.2\sigma$ discrepancy between the experimental data and SM estimate. We compare our calculation with the difference between the SM prediction and experimental estimation of the above coupling and constrain the parameter space of nmUED. We also review the limit on compactification radius of UED in view of the new theoretical estimation of SM contribution to $Zbb$ coupling. For suitable choice of BLKT parameters, 95% C.L. lower limit on $R^{-1}$ comes out to be in the ballpark of a TeV in the framework of nmUED; while in UED, the lower limit on $R^{-1}$ is 505 GeV which is a significant improvement over an earlier estimate.

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1 Introduction

Extra Dimensional theories can offer unique solutions to many long standing puzzles of Standard Model (SM) such as gauge coupling unifications [1] and fermion mass hierarchy [2]. Most importantly they can provide a Dark Matter candidate of the universe [3]. In this article, we are interested in a particular incarnation of extra dimensional theory referred as Universal Extra Dimensional Model (UED) where all the SM fields can propagate in $4 + 1$ dimensional space-time, the extra dimension (say, $y$) being compactified on a circle ($S^1$) of radius $R$ [4]. The five dimensional action consists of the same fields of SM and would respect the same $SU(3)_c \times SU(2)_L \times U(1)_Y$ gauge symmetry also. $R^{-1}$ is the typical energy scale at which the four dimensional effective theory would start to show up the dynamics of Kaluza-Klein (KK) excitations of SM fields. The masses of KK- modes are $m^2_n = m^2 + \frac{n^2}{R^2}$; where $n$ is an integer, called KK-number which corresponds to the discretized momentum in the compactified dimension, $y$. $m$ is any mass parameter that has been attributed to the respective five dimensional field. The $n = 0$ mode fields in the effective theory could be identified with the SM particles.

To generate the correct structure of chiral fermions in SM, one needs to impose some extra symmetry on the action called orbifolding which is nothing but a discrete $Z_2$ symmetry : $y \rightarrow -y$. The fields which have zero mode are chosen to be even under this $Z_2$ symmetry. There are KK-excitations of other fields which are odd under this transformation. Consequently they cannot have

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any zero modes. The space of \( y \) is called \( S^1/Z_2 \) orbifold with effective domain of \( y \) being from 0 to \( \pi R \). These two boundary points will be called the fixed points of the orbifold.

The KK mass spectrum in UED are highly degenerate, and radiative corrections to KK masses lift this degeneracy \([5,6]\). Radiative corrections to masses include finite bulk corrections originated from compactification and boundary corrections due to orbifolding. Boundary corrections to masses have logarithmic dependence on the unknown cut-off scale \( \Lambda \). Furthermore they are localized at the boundary points. In minimal UED (mUED), the boundary terms are considered to be vanishing at the cut-off scale \( \Lambda \). This is of course a very special assumption and a more general scenario where this assumption has been relaxed is called non-minimal UED (nmUED) \([7]\). We will be interested in a particular non-minimal scenario in which kinetic terms involving fields are added to the five dimensional action, at the boundary points. The coefficients of boundary-localized kinetic terms (BLKT) can be chosen as free parameters and experimental data can be used to constrain these.

The various phenomenological aspects of nmUED have been discussed in \([8,9]\). In particular, studies have been made to constrain the non-minimality parameters from the perspective of electroweak observables \([8]\), S, T and U parameters \([10]\), relic density \([11]\) and from LHC experiment \([12]\).

The precision electroweak variables like \( \rho \) parameter, \( R_b \) ( \( Z \) boson decay width to a pair of \( b \) quarks normalized to total hadronic decay width), \( A_{FB} \) (forward-backward asymmetry of \( b \) quarks at \( Z \) pole) always have played the role of guiding light in search of new physics. Incidentally, all of these electroweak precision variables are very much sensitive to the radiative corrections and these quantum corrections themselves are amplified by the large top quark mass. In the same spirit, we would like to investigate how one of the precisely known electroweak variable \( R_b \) could constrain the nmUED parameter space.

Estimation of radiative corrections to \( Zb\bar{b} \) vertex in the UED framework has been done previously in Ref. \([13,14]\). However, introduction of non-minimality through BLKTs would shift the masses of the KK-excitations in a non-trivial manner from their respective UED values. Moreover, some of the couplings involving KK-excitations, in nmUED, are also being modified by some factors which are nontrivial functions of BLKT parameters. So our calculation would not be a straight forward rescaling of earlier calculations of \( Zb\bar{b} \) vertex done in the context of UED \([13,14]\). To our knowledge, this is the first effort to estimate the radiative correction to \( Zb\bar{b} \) interaction in non-minimal UED framework.

The plan of the paper is the following. In the next section, we will derive the necessary interactions and vertices in the framework of nmUED with a brief introduction of the model. In section 3, we will present the calculational details. Section 4 will be devoted to the numerical results including the constraint on the parameter space of nmUED and also a review of \( Zb\bar{b} \) constraints in UED. Finally in section 5 we summarize our results and observations.

### 2 A brief review of masses and couplings in nmUED

In this section we will very briefly review the non-minimal Universal Extra Dimensional Model keeping in mind the necessary masses and couplings which will be used in our calculations of effective \( Zb\bar{b} \) coupling. A more detailed account of the model will be found in Ref. \([7–9]\).

We start our discussion with boundary-localized kinetic terms (BLKT) for fermions. The resulting action in five dimension is given by

\[
S_{\text{quark}} = \int d^4x \int_0^{\pi R} dy \left[ \overline{Q} i\Gamma^M D_M Q + r_f \{ \delta(y) + \delta(y - \pi R) \} \overline{Q} i\gamma^\mu D_\mu P_L Q \right]
\]
where the four component five dimensional fields can be expressed in terms of two component chiral spinors and their Kaluza-Klein excitations as:

\[ Q_{t,b}(x,y) = \sum_{n=0}^{\infty} \left( \begin{array}{c} Q_{t,bL}^{n}(x) f_{L}^{n}(y) \\ Q_{t,bR}^{n}(x) g_{L}^{n}(y) \end{array} \right), \]

\[ U(x,y) = \sum_{n=0}^{\infty} \left( \begin{array}{c} U_{t}^{n}(x) f_{R}^{n}(y) \\ U_{b}^{n}(x) g_{R}^{n}(y) \end{array} \right), \]

\[ D(x,y) = \sum_{n=0}^{\infty} \left( \begin{array}{c} D_{L}^{n}(x) f_{R}^{n}(y) \\ D_{R}^{n}(x) g_{R}^{n}(y) \end{array} \right). \]

In the above expression (and also in the following) \( M, N = 0, 1, 2, 3, 4 \) are the five dimensional Lorentz indices, with the metric convention \( g_{MN} \equiv \text{diag}(+,-,-,-,-) \). The covariant derivative is defined as \( D_{M} \equiv \partial_{M} - \bar{\gamma} W_{M}^{a} T^{a} - \bar{\gamma}' B_{M} Y \), where \( \bar{\gamma} \) and \( \bar{\gamma}' \) are the corresponding five dimensional gauge coupling constants of \( SU(2)_L \) and \( U(1)_Y \) respectively. \( T^{a} \) and \( Y \) are the corresponding generators. \( \Gamma_{M} \) are the representations of \( 4+1 \)-dimensional Clifford algebra with \( \Gamma_{\mu} = \gamma_{\mu}; \Gamma_{4} = i\gamma_{5} \). Since we are dealing with only third generation quark, the compact form of doublet is given as \( Q = (Q_{t}, Q_{b})^{T} \).

Upon compactification and orbifolding, this would give rise to the left-handed doublet consisting of \( t_{L}^{0} \) and \( b_{R}^{0} \). \( U \) and \( D \) are the four component fields in five dimension from which \( t_{L}^{n} \) and \( b_{R}^{n} \) would emerge in four dimensional effective theory. The \( y \) dependent wave functions with appropriate boundary conditions are given by

\[ f_{L}^{n} = g_{R}^{n} = N_{Qn} \left\{ \begin{array}{ll} \cos(M_{Qn} (y - \pi R/2)) & \text{for } n \text{ even}, \\ -\sin(M_{Qn} (y - \pi R/2)) & \text{for } n \text{ odd}, \end{array} \right. \]

(3)

and

\[ g_{L}^{n} = f_{R}^{n} = N_{Qn} \left\{ \begin{array}{ll} \sin(M_{Qn} (y - \pi R/2)) & \text{for } n \text{ even}, \\ \cos(M_{Qn} (y - \pi R/2)) & \text{for } n \text{ odd}, \end{array} \right. \]

(4)

with

\[ C_{Qn} = \cos \left( \frac{M_{Qn} \pi R}{2} \right), \quad S_{Qn} = \sin \left( \frac{M_{Qn} \pi R}{2} \right). \]

(5)

These wave functions satisfy the orthonormality conditions

\[ \int dy [1 + r_{f}\{\delta(y) + \delta(y - \pi R)\}] k^{n}(y) k^{m}(y) = \delta^{nm} = \int dy l^{n}(y) l^{m}(y), \]

(6)

where \( k^{n}(y) \) can be \( f_{L}^{n} \) or \( g_{R}^{n} \) and \( l^{n}(y) \) corresponds to \( g_{L}^{n} \) or \( f_{R}^{n} \). From the above condition,

\[ N_{Qn} = \sqrt{\frac{2}{\pi R}} \left[ \frac{1}{\sqrt{1 + \frac{r_{f}^{2} M_{Qn}^{2}}{4} + \frac{r_{f}^{2}}{\pi R}}} \right]. \]

(7)
The mass $M_{Q_n}$ of the $n$th KK-mode is no longer equal to $n/R$ as in UED, rather it satisfies the following transcendental equations

$$r_f M_{Q_n} = \begin{cases} 
-2 \tan \left( \frac{M_{Q_n} \pi R}{2} \right) & \text{for } n \text{ even}, \\
2 \cot \left( \frac{M_{Q_n} \pi R}{2} \right) & \text{for } n \text{ odd.} 
\end{cases} \tag{8}$$

It is evident that, for the zero modes ($n = 0$), $M_{Q_n}$ vanishes identically. The other required couplings of the theory must be supplemented by the action of gauge fields, Higgs field and the Yukawa interactions between Higgs and the fermions:

$$S_A = \int d^4x \int_0^{\pi R} dy \left[ -\frac{1}{4} F^{MN\alpha} F^\alpha_{MN} - \frac{r_g}{4} \{ \delta(y) + \delta(y - \pi R) \} F^{\mu\nu\alpha} F^\alpha_{\mu\nu} - \frac{1}{4} B^{MN} B_{MN} - \frac{r_g}{4} \{ \delta(y) + \delta(y - \pi R) \} B^{\mu\nu} B_{\mu\nu} \right], \tag{9}$$

$$S_\phi = \int d^4x \int_0^{\pi R} dy \left[ \left( D^M \Phi \right)^\dagger \left( D_M \Phi \right) + r_\phi \{ \delta(y) + \delta(y - \pi R) \} \left( D^\mu \Phi \right)^\dagger \left( D_\mu \Phi \right) \right], \tag{10}$$

$$S_Y = -\int d^4x \int_0^{\pi R} dy \left[ \tilde{y}_t \bar{Q} \phi U + \tilde{y}_b \bar{Q} \phi D + r_y \{ \delta(y) + \delta(y - \pi R) \} \{ \tilde{y}_t \bar{Q} \phi U_R + \tilde{y}_b \bar{Q} \phi D_R \} + \text{h.c.} \right]. \tag{11}$$

In the above, $F^{\alpha}_{MN} \equiv (\partial_M W^\alpha_N - \partial_N W^\alpha_M + \tilde{g} f^{abc} W^b_M W^c_N)$ is the field strength associated with the SU(2)$_L$ gauge group ($a$ is the SU(2) gauge index) and, $B_{MN} = \partial_M B_N - \partial_N B_M$ is that of the U(1)$_Y$ group. $\Phi$ and $\Phi^c (\equiv i\tau^2 \Phi^*)$ are the standard Higgs doublet and its charge conjugated field; $r_\phi$, $r_g$ and $r_y$ are the BLKT parameters for scalar field, gauge fields and Yukawa interactions respectively. Five dimensional gauge couplings $\tilde{g}$ and $\tilde{g}'$ are connected to their four dimensional counterparts via the following relation:

$$g \ (g') = \frac{\tilde{g} \ (\tilde{g}')}{\sqrt{r_g + \pi R}}.$$

In the limit, $r_g = r_\phi^3$ (which we will be using throughout in our analysis), gauge and scalar fields have the same $y$ dependent profile as $f_L$ and $g_R$ given in eq.3 and their KK-excitations have masses $M_{gn} (= M_{\phi_n})$ which follow the same transcendental equations given in eq.8 with $r_f$ replaced by $r_g (= r_\phi)$. $\tilde{y}_t$ and $\tilde{y}_b$ denote the Yukawa interactions strengths for the third generations in five dimensional theory.

Finally, one must note down the gauge fixing action, which is very crucial for the calculation at our dispense, as we would proceed with our calculation in 't-Hooft Feynman gauge. Following Ref. [15] one can have,

$$S_{GF}^W = -\frac{1}{\xi_y} \int d^4x \int_0^{\pi R} \left| \partial_\mu W^{\mu+} + \xi_y (\partial_5 W^{5+} - i M_W \phi^+ \{ 1 + r_\phi (\delta(y) + \delta(y - \pi R)) \} \right|^2,$$

\[^3\text{In general when Higgs and gauge BLKTs are unequal, the differential equation governing the dynamics of gauge profile in } y \text{ direction contains a term proportional to } r_\phi \text{ due to breakdown of electroweak symmetry [15] and solutions will be different from those given in eq.3.}\]
In the above, $M_W$ is the $W$-boson mass and $\xi_y$ is related to physical gauge fixing parameter $\xi$ (taking values 1 in Feynman gauge and 0 in Landau gauge) via

$$\xi = \xi_y \left\{ 1 + r_\phi \left[ \delta(y) + \delta(y - \pi R) \right] \right\}. \quad (13)$$

Before delving into the interactions needed for the calculation, let us spend some time discussing the physical eigenstates which are the outcome of mixing of some of the states originally present in the four dimensional effective theory. These are quite similar but not exactly the same as in UED. So we have decided to make a dedicated discussion on this issue. There are two such cases relevant for our calculation. Let us first focus on the mixing in the quark sector. This mixing is driven by the Yukawa coupling and thus is only important and relevant for the top quarks.

Substituting the modal expansions for fermions given in eq. 2, in actions given in eq. 1 and eq. 11 one can easily find the bilinear terms involving the doublet and singlet states of the quarks. In $n$th KK-level, mass matrix reads as

$$\left( \bar{Q}^{(m)}_{tL} \begin{array}{c} \mathcal{U}^{(m)}_L \end{array} \right) \left( \begin{array}{cc} -M_{Qn} \delta_{mn} & m_t \mathcal{I}^{mn} \\ m_t \mathcal{I}^{mn} & M_{Qn} \delta_{mn} \end{array} \right) \left( \begin{array}{c} Q^{(n)}_{tL} \\ U^{(n)}_R \end{array} \right) + h.c., \quad (14)$$

where $M_{Qn}$ are the solutions of transcendental equations as in eq. 8. $\mathcal{I}^{mn}$ is an overlap integral of the form

$$\int_0^{\pi R} \left[ 1 + r_y \delta(y) + r_y \delta(y - \pi R) \right] f^{m}_L(y) g^{n}_R(y) \, dy.$$ 

This integral is in general, non-zero for both $n = m$ and $n \neq m$. The second case would lead to the (KK-)mode mixing among the quark of a particular flavour. However, the choice $r_y = r_f$ would make this integral equal to 1 (when $m = n$) or 0 ($m \neq n$). So by choosing equal fermion and Yukawa BLKTs one could easily avoid the mode mixing and end up in a simpler form of the fermion mixing matrix without any loss of generality. In the following we will stick to the choice of equal $r_y$ and $r_f$.

One can note that the strength (off-diagonal terms) of the mixing is proportional to quark mass (denoted by $m_t$ here), hence the mixing only important for top quark (and we will denote top-quark mass by $m_t$ in the following). The resulting matrix can be diagonalized by the following unitary transformations for the left- and right-handed fields respectively:

$$\mathcal{U}^{(n)}_L = \begin{pmatrix} -\cos \alpha_n & \sin \alpha_n \\ \sin \alpha_n & \cos \alpha_n \end{pmatrix}, \quad \mathcal{U}^{(n)}_R = \begin{pmatrix} \cos \alpha_n & -\sin \alpha_n \\ \sin \alpha_n & \cos \alpha_n \end{pmatrix}, \quad (15)$$

where $\alpha_n = \frac{1}{2} \tan^{-1} \left( \frac{m_t}{M_{Qn}} \right)$ is the mixing angle. The relations between gauge eigenstates $Q^{(n)}_t$ and $U^{(n)}$ and mass eigenstates $Q^{(n)}_t$ and $U^{(n)}$ are given by,

$$Q^{(n)}_{tL/R} = \mp \cos \alpha_n Q^{(n)}_{tL/R} + \sin \alpha_n U^{(n)}_{L/R}, \quad (16)$$

$$U^{(n)}_{L/R} = \pm \sin \alpha_n Q^{(n)}_{tL/R} + \cos \alpha_n U^{(n)}_{L/R}, \quad (17)$$

where the mass eigenstates share the same mass eigenvalue $m_{Q^{(n)}_t} = m_{U^{(n)}} = \sqrt{m_t^2 + M_{Qn}^2} = M_u$ (say).
Four dimensional effective Lagrangian would also contain bilinear terms involving the KK-excitations (starting from KK-level \( n = 1 \) and above) of 5th components of \( W^\pm \) (\( Z \)) bosons and the KK-excitations of \( \phi^\pm \) (\( \chi^0 \)) of the Higgs doublet field \([16]\). In the following we note down the bilinear terms involving the KK-modes of \( W^5_5 \) and \( \phi^\pm \), which are relevant for our calculation. Using eqs.9,10, and eq.12 one can write in \( R_\xi \) gauge,

\[
\mathcal{L}_{\phi^\pm n^\pm} = - \left( \left( W^5_5 - \phi^{(n)\pm} \right) \left( M^2_W + \xi M^2_{\Phi^\pm n} - i(1 - \xi) M_W M_{\Phi^\pm n} M^2_{\Phi^\pm n} + \xi M^2_W \right) \left( W^5_5 + \phi^{(n)\pm} \right) \right). \tag{18}
\]

The above mass matrix upon diagonalization would lead to a tower of charged Goldstone bosons (with mass square \( \xi (M^2_{\Phi^\pm n} + M^2_W) \)),

\[
G^{(n)\pm}(\pm) = \frac{1}{M_{W_n}} \left( M_{\Phi n} W^{\pm 5(n)} \mp i M_W \phi^{(n)} \right),
\]

and the physical charged Higgs pair (with mass square \( M^2_{\Phi^\pm n} + M^2_W \)):

\[
H^{(n)\pm}(\pm) = \frac{1}{M_{W_n}} \left( M_{\Phi n} \phi^{(n)} \mp i M_W W^{\pm 5(n)} \right).
\]

So the fields \( W^{(n)\pm} \), \( G^{(n)\pm} \) and \( H^{(n)\pm} \) share the common mass eigenvalue \( M_{W_n} \equiv \sqrt{M^2_{\Phi^\pm n} + M^2_W} \) in 't-Hooft Feynman gauge (\( \xi = 1 \)). These combinations of charged Higgs and Goldstone ensure the vanishing coupling of \( \gamma H^{n\pm} W^{\mu\pm} \) as it should be with a doublet Higgs at our dispense.

The necessary interactions involving \( Z \) boson, fermions and scalars in four dimensional effective theory can be derived from the above action by simply inserting the appropriate \( y \) dependent profile for the respective five dimensional fields and then integrating out the extra direction. In contrast to mUED, where the \( y \) dependent profiles are either \( \sin(\frac{ny}{R}) \) or \( \cos(\frac{ny}{R}) \), some of the couplings in nmUED are hallmarkled by the presence of few overlap integrals of the form :

\[
I = \int_0^{\pi R} dy f_\alpha^m(y) f_\beta^m(y) f_\rho^p(y) \tag{19}
\]

Here, the greek indices refer to the kind of fields involved in the coupling while the roman indices refer to the KK-level of respective fields.

At this end, let us pay some attention to the overlap integrals \( I_{mn}^{1m} \) and \( I_{mn}^{2m} \) which are relevant for our calculations appearing in the interactions listed in appendix A. \( I_{mn}^{1m} \) and \( I_{mn}^{2m} \) are the following overlap integrals:

\[
I_{mn}^{1m} = \int_0^{\pi R} dy \left[ 1 + r_f \{ \delta(y) + \delta(y - \pi R) \} \right] f^{(m)} Q_{L} f^{(n)} \phi \left( f^{(0)} \right), \tag{20}
\]

\[
I_{mn}^{2m} = \int_0^{\pi R} dy f^{(m)} Q_{R} f^{(n)} \left( f^{(0)} \right). \tag{21}
\]

These integrals are non-zero when \( n + m \) is even. The integrals and the interactions among KK-states with odd \( n + m \) identically vanish due to a conserved KK-parity. Even in the former case, the integrals are of the order 1 when \( n = m \). When \( m \) differs from \( n \) (in the case of even \( n + m \)) values of the integrals diminish generally by an order of magnitude than the \( m = n \) case\(^4\). Keeping this

\(^{4}\)As for example, when \( r_f = 1 \) and \( r_\phi = 2 \): \( I_{11}^{11} = 0.82, I_{12}^{12} = 0.88, I_{13}^{13} = 0.92, I_{14}^{14} = 0.94, I_{15}^{15} = 0.96, I_{11}^{31} = 0.01, I_{11}^{51} = 0.004, I_{12}^{12} = 0.02, I_{13}^{13} = 0.03, I_{14}^{14} = 0.03; I_{22}^{22} \sim I_{23}^{33} \sim I_{24}^{44} \sim I_{25}^{55} = 0.99 \) and \( I_{32}^{22} = 0.07, I_{33}^{23} = 0.02, I_{34}^{24} = 0.08. \)
in mind we will be only considering the interactions with \( n = m \) neglecting the other sub dominant contributions coming from interactions in which \( n \neq m \). The expressions for the integrals (upon integrating over \( y \)) are given in appendix A along with the necessary Feynman rules.

3 Calculation for Radiative Correction to \( Zb\bar{b} \) vertex:

We are now all set to discuss the details of the calculation leading to the corrections of \( Zb\bar{b} \) vertex in the framework of nmUED. However, as a preamble we will first briefly discuss the meaning of \( R_b \) and its correlation to \( Zb\bar{b} \) coupling in the SM. The tree level \( Zb\bar{b} \) coupling, in SM, can be defined as

\[
\frac{g}{\cos \theta_W} \bar{b}^0 \gamma^\mu (g_L^0 P_L + g_R^0 P_R) b^0 Z_\mu^0,
\]

where \( Z_\mu^0 \) and \( b^0 \)'s are SM fields, \( P_{R,L} = (1 \pm \gamma_5)/2 \) are the right- and left-chirality projectors respectively and

\[
g_L^0 = -\frac{1}{2} + \frac{1}{3} \sin^2 \theta_W, \quad g_R^0 = \frac{1}{3} \sin^2 \theta_W.
\]

Any higher order quantum corrections either from SM or from new physics (NP) can be incorporated uniformly as the modification to this tree level couplings given as

\[
g_L = g_L^0 + \delta g_L^{SM} + \delta g_L^{NP}, \quad g_R = g_R^0 + \delta g_R^{SM} + \delta g_R^{NP},
\]

where \( \delta g_{L/R}^{SM} \) are the radiative corrections from SM and \( \delta g_{L/R}^{NP} \) are that of NP [13]. These corrections can modify the \( Z \) decay width to \( b \) quarks normalized to the total hadronic decay width of \( Z \), defined by a dimensionless variable,

\[
R_b \equiv \frac{\Gamma(Z \to b\bar{b})}{\Gamma(Z \to \text{hadrons})}.
\]

We will only be considering the effects of third generation quarks. Normally, at one loop order (SM & also in NPs) only \( g_L \) receives corrections proportional to \( m_t^2 \), and \( g_R \) receives corrections proportional to \( m_b^2 \) (due to the difference in couplings between two chiralities) where \( m_t \) (\( m_b \)) is the zero mode top (bottom) quark mass. We have neglected the \( b \) mass in our calculation and thus a shift \( \delta g_L^{NP} \) translates into a shift in \( R_b \) given by,

\[
\delta R_b = 2R_b(1 - R_b) \frac{\hat{g}_L}{\hat{g}_L^2 + \hat{g}_R^2} \delta g_L^{NP},
\]

with \( \hat{g}_L \) and \( \hat{g}_R \) given by

\[
\hat{g}_L^b = \sqrt{\rho_b} (-\frac{1}{2} + \kappa_b \frac{1}{3} \sin^2 \theta_W), \quad \hat{g}_R^b = \frac{1}{3} \sqrt{\rho_b \kappa_b} \sin^2 \theta_W,
\]

after incorporating the SM electroweak corrections only [17]. Here, \( \rho_b = 0.9869 \) and \( \kappa_b = 1.0067 \).
In general, the $g_L^{NP}$ is calculable in a given framework while $R_b$ is an experimentally measurable quantity. Thus eq.28 can be used to constrain the parameters of the model. We would exactly like to do this exercise in the framework of nmUED in the following.

Since we have neglected the interactions involving KK-states with unequal KK-numbers in an interaction vertex, the number of diagrams contributing to radiative corrections of the $Zb\bar{b}$ vertex in nmUED are same as that of minimal UED. Fig.1 shows the Feynman diagrams involving KK-excitations of top quarks, charged Higgs/Goldstone bosons in the loop. The contribution coming from the diagrams of Fig.1 is dominant for the presence of Yukawa coupling which is proportional to $m_t$. In our calculations, we have considered momentum of each external leg to be zero and have neglected the $b$ quark mass. The results of each diagram, for $n$th KK-mode, can be expressed in terms of a single function, $f^n(r_n, r'_n, M')$, defined as,

$$i\mathcal{M}^{(n)} = i\frac{g}{\cos \theta_W} u(p_1, s_1) f^n(r_n, r'_n, M') \gamma^\mu P_L v(p_2, s_2) \epsilon^\mu(q) ,$$

where $r_n \equiv m_t^2/M_Q^n$, $r'_n \equiv M_W^2/M_Q^n$, $M' \equiv M_{\Phi^n}/M_Q^n$.

The amplitudes of different diagrams of Fig.1 (evaluated in 't-Hooft- Feynman gauge) are given by,

$$f^n_{1(a)}(r_n, r'_n, M') = \frac{\beta}{(4\pi)^2} \frac{g^2}{4} \left\{ \frac{4}{3} \sin^2 \theta_W \left( I_2^2 + I_1^2 \frac{m_t^2}{M_W^2}\right) + I_3^2 \left( \cos^4 \alpha_n + \sin^4 \alpha_n \right) + 2I_1^2 \frac{m_t^2}{M_W^2} \sin^2 \alpha_n \cos^2 \alpha_n \right\} \left[ \delta_n - 1 + \{5(r_n + 1)^2 + 3(r'_n + M')^2 \right\}$$

Figure 1: Loop involving KK-mode of scalar and fermion propagators.
is obtained by adding $b$ in the renormalization of charge (of vertex must be the same to any correction proportional to $\sin^2 \alpha_n$ among themselves. Here, any correction proportional to $\sin^2 \alpha_n$ is given by the following expression:

$$f_{1(b)}^n(r_n, r'_n, M') = \frac{\beta}{(4\pi)^2} \frac{g^2}{4} \left( 2 I_2^2 \sin^2 \alpha_n \cos^2 \alpha_n - 2 I_2^2 m_t^2 \sin^2 \alpha_n \cos^2 \alpha_n \right) \left[ \delta_n - 1 + \{-3(r_n+1)^2+3(r_n'+M')^2 \right.
-2(1+r_n)^2 \ln(1+r_n) - 2(M'+r_n')^2 \ln(M'+r_n')
+8(1+r_n)(M'+r_n') \ln(1+r_n)
-4(1+r_n)(M'+r_n') \ln(M'+r_n') \right]/2\{(r_n+1) - (M'+r_n')^2 \right], \quad (30)
$$

$$f_{1(c+d)}^n(r_n, r'_n, M') = \frac{\beta}{(4\pi)^2} \frac{g^2}{4} \left( -1 + 2 \sin^2 \theta_W \right) \left( I_2^2 + I_1^2 \frac{m_t^2}{M_W^2} \right) \left[ \delta_n + \{3(r_n+1)^2+(r_n'+M')^2 \right.
-4(r_n+1)(r_n'+M') - 2(1+r_n)^2 \ln(1+r_n)
-2(M'+r_n')^2 \ln(M'+r_n') + 4(1+r_n)(M'+r_n') \ln(M'+r_n') \right]/2\{(r_n+1) - (M'+r_n')^2 \right], \quad (31)
$$

$$f_{1(e+f)}^n(r_n, r'_n, M') = \frac{\beta}{(4\pi)^2} \frac{g^2}{4} \left( 1 - \frac{2}{3} \sin^2 \theta_W \right) \left( I_2^2 + I_1^2 \frac{m_t^2}{M_W^2} \right) \left[ \delta_n + \{3(r_n+1)^2+(r_n'+M')^2 \right.
-4(r_n+1)(r_n'+M') - 2(1+r_n)^2 \ln(1+r_n)
-2(M'+r_n')^2 \ln(M'+r_n') + 4(1+r_n)(M'+r_n') \ln(M'+r_n') \right]/2\{(r_n+1) - (M'+r_n')^2 \right], \quad (32)
$$

where $\delta_n \equiv 2/\epsilon - \gamma + \log(4\pi) + \log(\mu^2/M_{Qn}^2)$, and $\mu$ is the 't-Hooft mass scale; $\beta \equiv \frac{\pi R^2 r_n^2}{\pi R^2 r_n^2}$. $I_1$ and $I_2$ stand for the overlap integrals given in equations (20) and (21) respectively for $n = m$. The amplitude of diagrams of (e) and (f) are multiplied by a factor of $\frac{1}{2}$ which comes from the usual convention of contributing one-half of this correction into self-energy and the other half in the wave function renormalization. Total amplitude ($i\mathcal{M}_{1(n)}$) of the diagrams in Fig.1 is obtained by adding the individual amplitudes for each diagram and is given by the following expression:

$$i\mathcal{M}_{1(n)} = \frac{i}{(4\pi)^2} \frac{g^3}{2 \cos \theta_W} \pi(p_1, s_1) \frac{r_n^\beta}{\{(r_n+1) - (M'+r_n')\}^2} \left( -I_2^2 + I_1^2 \frac{m_t^2}{M_W^2} \right) \left[ (1+r_n) - (M'+r_n') + (M'+r_n') \ln \left( \frac{M'+r_n'}{1+r_n} \right) \right] \gamma_\mu P_L V(p_2, s_2) \epsilon_\mu(q). \quad (34)
$$

From the above expression, it is evident that terms proportional to $\delta_n$, as well as $\sin^2 \theta_W$ cancel among themselves. Here, any correction proportional to $\sin^2 \theta_W$ in $Z\bar{b}b$ coupling must be reflected in the renormalization of charge (of $b$ quark). This implies that any finite renormalization to $\gamma\bar{b}b$ vertex must be the same to any correction proportional to $\sin^2 \theta_W$ in $Z\bar{b}b$ vertex. We have explicitly
checked that both of these corrections coming from the diagrams of same topology depicted in Fig.1 identically vanishes.

There is a second set of diagrams contributing to effective $Zb\bar{b}$ interaction mainly arising from the KK-excitations of $W$ bosons and quarks. These are sub dominant with respect to the contributions coming from Fig.1.

In the following we present the amplitudes of all the individual diagrams given in Fig.2:

$$f_{2(a)}^n(r_n, r'_n, M') = \frac{I_2^2 \beta}{(4\pi)^2} \frac{g^2}{2} \left[ \frac{-4}{3} \sin^2 \theta_W + \cos^4 \alpha_n + \sin^4 \alpha_n \right] \left[ \delta_n - 2 + \{(5(r_n + 1)^2 + 3(r'_n + M')^2) - 8(r_n + 1)(r'_n + M') - 2(1 + r_n)^2 \ln(1 + r_n) - 2(M' + r'_n)^2 \ln(M' + r'_n) + 4(1 + r_n)(M' + r'_n) \ln(M' + r'_n) \right] / 2 \{(r_n + 1) - (M' + r'_n)\}^2, \tag{35}$$

$$f_{2(b)}^n(r_n, r'_n, M') = \frac{I_2^2 \beta}{(4\pi)^2} \frac{g^2}{2} \left[ 2 \sin^2 \alpha_n \cos^2 \alpha_n \right] \left[ \delta_n - 2 + \{-3(r_n + 1)^2 + 3(r'_n + M')^2 - 2(1 + r_n)^2 \ln(1 + r_n) - 2(M' + r'_n)^2 \ln(M' + r'_n) + 8(1 + r_n)(M' + r'_n) \ln(1 + r_n) - 4(1 + r_n)(M' + r'_n) \ln(M' + r'_n) \right]$$
Therefore for each mode, correction in $g_L$:

$$
\delta g_L^{(n)NP} = \Sigma_i f_i^{(n)}(r_n, r'_n, M') = \frac{\sqrt{2} G_F m_i^2}{16\pi^2} F_{nm\text{UED}}^{(n)}(r_n, r'_n, M'),
$$

where

$$
F_{nm\text{UED}}^{(n)}(r_n, r'_n, M') = \frac{2r_n \beta}{[(1 + r_n) - (r'_n + M')]^2} \left[ I_1^2 \left( 1 - \frac{2M_W^2}{m_i^2} \right) - I_2^2 \frac{M_W^2}{m_i^2} \right]
$$
\[ \times \left\{ (1 + r_n) - (r_n' + M') + (r_n' + M') \ln \left( \frac{r_n' + M'}{1 + r_n} \right) \right\} \\
+ \frac{4M_W^2}{m_t^2} I_t^2 \left\{ -(1 + r_n) + (r_n' + M') + (1 + r_n) \ln \left( \frac{1 + r_n}{r_n' + M'} \right) \right\}. \] (43)

Total new physics contribution \( \delta g_{L}^{NP} \) (and similarly \( F_{nmUED}^{(n)} \)) can be obtained by summing \( \delta g_{L}^{NP} \) over KK-modes \( (n) \). It can be checked that the new physics contribution \( \delta g_{L}^{NP} \) and hence \( F_{nmUED}^{(n)} \) goes to zero when \( R^{-1} \to \infty \), as expected in a decoupling theory.

4 Results

Let us begin the discussions of the results with the present status of experimental and theoretical estimation of \( Zb\bar{b} \) coupling. Following the Gfitter Collaboration [19] and an improved estimation [20] of \( R_b \) incorporating higher order effects in the framework of SM, the experimental and theoretical (SM) values are

\[ R_b^{exp} = 0.21629 \pm 0.00066 \quad \text{and} \quad R_b^{SM} = 0.21550 \pm 0.00003. \]

Above results implies a 1.2 standard deviation discrepancy between the experimental value of \( R_b \) and its SM estimate. Eq.42, 43 in conjunction with Eq.28 could be used to translate this 1.2\( \sigma \) discrepancy on \( R_b \) to the possible allowed range for \( F_{nmUED} \): \(-0.3165 \pm 0.2647 \). One can now easily use this to constrain the model parameters of nmUED.

4.1 Relook at the bound on \( R^{-1} \) in mUED from \( R_b \)

Before we present our main results in the framework of nmUED, we would like to give a look at the limit on the \( R^{-1} \), in case of UED, keeping in mind the new estimate of SM radiative corrections to the \( Zb\bar{b} \) vertex at two loop level [20]. One can easily retrieve the UED contributions to \( \delta g_{L}^{NP} \) by simply setting the BLKT parameters to zero. In this limit, overlap integrals (\( I_1 \) and \( I_2 \)) used in the couplings become one and the \( M_{Qn}, M_{gn} \) and \( M_{bn} \) all become equal to \( \frac{2}{n} \) in \( n \)th KK-level; the ratios \( \beta, M' \) will be unity and our expressions completely agree with those given in Ref. [14]. One can define a function \( F_{UED}^{(n)}(r_{1n}, r_{1n}') \) in the same spirit following Eq.43:

\[ F_{UED}^{(n)}(r_{1n}, r_{1n}') = \frac{2r_{1n}}{(r_{1n} - r_{1n}')^2} \left\{ \frac{1 - 3M_W^2}{m_t^2} \right\} \left\{ (r_{1n} - r_{1n}') + (1 + r_{1n}') \ln \left( \frac{1 + r_{1n}'}{1 + r_{1n}} \right) \right\} \\
+ \frac{4M_W^2}{m_t^2} \left\{ r_{1n}' - r_{1n} + (1 + r_{1n}) \ln \left( \frac{1 + r_{1n}}{1 + r_{1n}'} \right) \right\}. \] (44)

Here, \( r_{1n} \equiv m_t^2/m_n^2 \), \( r_{1n}' \equiv M_W^2/m_n^2 \) and \( m_n = \frac{2}{n} \).

In Fig.3, we plot \( F_{UED} \) with \( R^{-1} \), the only free parameter in the model after summing the contributions \( (F_{UED}^{(n)}) \) coming from 5 KK-levels starting from \( n = 1 \). This has been done in the view of recently discovered Higgs mass and its implication on the cut-off scale of UED [21]. The horizontal line in Fig.3 represents the 95\% C.L. upper limit on the value of \( F_{UED} \) calculated from difference between the experimental value of \( R_b \) and its theoretical (SM) estimate \( (F_{UED} : -0.3165 \pm 0.2647) \). So the intersection of the horizontal line with the line showing the variation of \( F_{UED} \) would lead us to the present lower bound on \( R^{-1} \) from \( R_b \). It clearly points that at 95\% C.L. \( R^{-1} \) must be greater...
than 505 GeV, which is a substantial improvement over the earlier limit which was 300 GeV [13]. If we ignore the correlation between Higgs mass and cut-off scale of UED, then one could sum up to 20-40 levels. This would slightly push up the magnitude of $F_{\text{UED}}$ which in turn results into a higher value of lower limit of $R^{-1}$. However, this limit is still not competitive to the bound derived from experimental data on SM Higgs production and its subsequent decay to $WW$ [22].

4.2 Possible bounds on $nmUED$ from $R_b$

Now we are ready to discuss the main results of our analysis. Contribution to $F_{nmUED}$, coming from each KK-level are already listed in the previous section. One has to sum over the KK-levels to get the total contribution. We have taken into consideration the first 5 levels into the summation. And we have explicitly checked that taking 20 levels into the summation, would not change the results significantly.

In Fig.4, we have presented the variation of $F_{nmUED}$ as a function of $R^{-1}$ for some representative values of the scaled BLKT parameters $R_{\phi}$ ($\frac{c}{R}$) and $R_{f}$ ($\frac{t}{R}$). One common feature that comes out from all of the plots, namely the monotonic decrement of $F_{nmUED}$ with increasing $R^{-1}$, showing the decoupling nature of the new physics under our consideration. Panels (a and b) show the dependence of $F_{nmUED}$ on $R_{f}$ keeping the value of $R_{\phi}$ fixed to 1.5 and 4.5 respectively. While in the lower panels of Fig.4, we have presented how $F_{nmUED}$ changes with varying $R_{\phi}$ with two fixed values of $R_{f}$ namely 1.5 (c) and 4.5 (d) respectively.

From the figures it is evident that $R_{\phi}$ and $R_{f}$ have more or less same effects on $F_{nmUED}$ and $\delta g_{L}^{NP}$. While the effect of $R_{\phi}$ is somewhat modest, $F_{nmUED}$ is being more sensitive to any change of $R_{f}$. Consequently, by increasing the BLKT parameters one could enhance the radiative effects on the effective $Zbb$ coupling. It is quite clear that presence of BLKTs could significantly shift the lower bound on $R^{-1}$ compared to the one derived in the context of UED (previous subsection) from the

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5 This is due to the fact that experimental data from LHC on Higgs boson production and subsequent decay to $WW$ is more consistent to the SM than $R_b$ in which there is 1.2σ new physics window.
Figure 4: Variation of $F_{\text{nmUED}}$ with $R^{-1}$ for different values of BLKT parameters. The horizontal line represents the 95 % C.L. upper limit of the value of $F_{\text{nmUED}}$ calculated from difference between the experimental value of $R_b$ and its theoretical (SM) estimate.

consideration of $R_b$. As for example, for $R_\phi = 1.5$ and $R_f = 9$, the 95 % C.L. lower bound on $R^{-1}$ is the ballpark of a TeV. This limit comes down to 652 Gev for $R_f = 1$.

Finally in Fig.5, we present the allowed parameter space in $R_\phi - R_f$ plane for several values of $R^{-1}$. We plot the contours of constant $F_{\text{nmUED}}$ which corresponds to 95 % C.L. upper limit. Thus a region right to a particular line is ruled out from the consideration of $R_b$ according to our analysis. The near vertical nature of the contours at lower values of $R_f$ points out to the modest dependence of $F_{\text{nmUED}}$ on $R_\phi$ already exhibited in Fig.4(a) and (b). It has been already pointed out that with higher values of BLKT parameters $R_f$ and $R_\phi$, $F_{\text{nmUED}}$ is being increased in magnitude. So as we go towards right with increasing $R_f$ and fixed $R_\phi$ for a particular value of $R^{-1}$, $F_{\text{nmUED}}$ would increase. Furthermore the higher value of $R^{-1}$ decreases $F_{\text{nmUED}}$ showing the decoupling nature of new physics. Thus the increment of $F_{\text{nmUED}}$ (with $R_f$) has been nullified by the higher values of $R^{-1}$ corresponding to different lines. Thus to compensate one must tune $R^{-1}$ to a comparatively higher values.
Figure 5: Contours of constant $F_{\text{nmUED}}$ corresponding to 95\% C.L. upper limit in $R_\phi - R_f$ plane. Different lines represent different values of $R^{-1}$. Region right to a particular contour is being ruled out at 95\% C.L. from the consideration of $R_b$ for a given value of $R^{-1}$ (in GeV) on each contour.

\section{Conclusion}

In summary, we have estimated the one loop contribution to $Zb\bar{b}$ vertex in the framework of a Universal Extra Dimensional Model where kinetic terms are added to the fixed points of the extra space like dimension. These boundary-localized terms, with their coefficients as free parameters, parametrize the quantum corrections to the masses of the KK-excitations and their mutual interactions. We have calculated the interactions necessary for our calculation. Some of these interactions are very similar to those in UED. However, some of the interactions are modified in comparison to their UED counterparts by some overlap integrals (in extra dimension) involving the extra dimensional profiles of the fields present in the interaction vertex.

The effect of BLKTs on the masses of KK-modes and their interactions can be summarized as the following. For a given $R^{-1}$, increasing BLKT parameter would drive the respective masses to lower values. Strength of an interaction does not have such a simple dependence on the BLKT parameters. We have derived all the necessary interactions involving the KK-excitations of top quarks, W bosons, charged Higgs and Goldstone bosons in the framework of nmUED with the assumption of equal gauge and Higgs BLKT along with equal fermion and Yukawa BLKTs. Gauge and Higgs BLKTs have been chosen to be equal to avoid the unnecessary complication created by the presence of $r_\phi$ in equation of motion of gauge fields. While unequal fermion and Yukawa BLKT would lead to the KK-mode mixing in the definition of physical states of KK-excitations of top quarks. So without loss of any generality and to avoid unnecessary complications we have assumed equal fermion and Yukawa BLKTs.

In general, coupling of a $b$ quark to $Z$ boson involves both the left- and right-chiral projectors. However, quantum corrections which go into the coefficient of the left-chiral projector are proportional to $m_t^2$ while the $m_b^2$ proportional terms go into the coefficient of the right-chiral projector. We have done the calculation in the limit where $m_b \to 0$. There are two main classes of Feynman diagrams contributing to $\delta g_L^{NP}$, (the contribution to $Zb\bar{b}$ vertex in nmUED framework) in 't-Hooft Feynman gauge. First set of diagrams listed in Fig.1, captures the dominant contribution (because of Yukawa coupling which is proportional to $m_t$) coming from the participation of KK-excitations of top quarks.
and charged Higgs boson/Goldstones in the loops. The remaining set consists of contribution mainly coming from the KK-excitations of $W$ bosons and top quarks inside the loops. These diagrams are listed in Fig. 2.

The explicit expressions for the contributions coming from each of the diagrams are listed in the section 3. Sum of the contributions to $\delta g^\text{NP}_L$ from the diagrams in Fig. 1 is finite and independent of $\sin^2 \theta_W$. While the second set of diagrams needs to be regularized, after summing up, it is still ultraviolet divergent and also contains a term which grows with $R^{-1}$. We have used a regularization scheme following Ref. [14, 18], upon which the total contribution becomes finite and also becomes independent of $\sin^2 \theta_W$.

A recent theoretical estimation of $Z \bar{b} b$ vertex in the framework of SM at two loop level has squeezed the window for new physics that might be operating at TeV scale. The experimentally measured value of $R_b$ differs from the SM prediction at 1.2 $\sigma$ level. We have used the experimental data and the recent results from the SM on $R_b$, to constrain the parameter space of non-minimal Universal Extra Dimensional Model. We have relooked into the UED by setting the BLKT parameters to zero in our calculation. The resulting expressions can be used to put bound on $R^{-1}$ in UED model using the same experimental data and the SM estimations of $R_b$. It has been found that $R^{-1}$ in UED model should be greater than 505 GeV at 95 % C.L.

Next we focus into our main result. Comparing the numerical estimation of $F_{nmUED}$ with the difference between experimental data and SM estimation we have constrained the parameters in nmUED. First we look into the limits on $R^{-1}$. Both the BLKT parameters have positive effects on $F_{nmUED}$. This function is very sensitive to any change in $R_f$ while the effect of $R_\phi$ is very mild. The bottom line is that both the BLKT parameters can push the allowed value of $R^{-1}$ to higher values. Depending on magnitude of BLKT parameters $R_\phi$ and $R_f$ (which we have chosen to be positive), lower limit on $R^{-1}$ could be in the ballpark of a TeV. Finally, we plot contours of constant $F_{nmUED}$ having the 95 % C.L upper limit value for different value of $R^{-1}$ in $R_\phi - R_f$ plane. As for a fixed value of $R^{-1}$ i.e. for a fixed curve the value of the function $F_{nmUED}$ increases with increase of $R_f$ the left side of that curve represents the allowed region of this function for respective $R^{-1}$.

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**APPENDIX A**

In the following Feynman rules, all momenta and fields are assumed to flow into the vertices.
\[ W_n^{\mu \pm} \quad : \frac{ig}{\sqrt{2}} \gamma_\mu C_L P_L \]

\[ W^{n+} Q_t^m b_L^0 : C_L = -I_1 \sqrt{\beta} \cos \alpha_n, \]
\[ W^{n+} U^m b_L^0 : C_L = I_1 \sqrt{\beta} \sin \alpha_n, \]
\[ W^{n-} b_L^0 Q_t^m : C_L = -I_1 \sqrt{\beta} \cos \alpha_n, \]
\[ W^{n-} b_L^0 U^m : C_L = I_1 \sqrt{\beta} \sin \alpha_n. \]

\[ H^{n \pm} G^{n \pm} \quad : \frac{q}{\sqrt{2M_W}} C_{L/R} P_{L/R} \]

\[ H^{n+} Q_t^m b_L^0 : C_L = -i \sqrt{\beta} \left( I_1 \frac{m_t M_{\Phi n}}{M_W} \sin \alpha_n - I_2 M_W \cos \alpha_n \right), \]
\[ H^{n-} b_L^0 Q_t^m : C_R = -i \sqrt{\beta} \left( I_1 \frac{m_t M_{\Phi n}}{M_W} \sin \alpha_n - I_2 M_W \cos \alpha_n \right), \]
\[ G^{n+} Q_t^m b_L^0 : C_L = \sqrt{\beta} \left( I_1 m_t \sin \alpha_n + I_2 M_{\Phi n} \cos \alpha_n \right), \]
\[ G^{n-} b_L^0 Q_t^m : C_R = -\sqrt{\beta} \left( I_1 m_t \sin \alpha_n + I_2 M_{\Phi n} \cos \alpha_n \right), \]
\[ H^{n+} U^m b_L^0 : C_L = i \sqrt{\beta} \left( I_1 \frac{m_t M_{\Phi n}}{M_W} \cos \alpha_n + I_2 M_W \sin \alpha_n \right), \]
\[ H^{n-} b_L^0 U^m : C_R = i \sqrt{\beta} \left( I_1 \frac{m_t M_{\Phi n}}{M_W} \cos \alpha_n + I_2 M_W \sin \alpha_n \right), \]
\[ G^{n+} U^m b_L^0 : C_L = -\sqrt{\beta} \left( I_1 m_t \cos \alpha_n - I_2 M_{\Phi n} \sin \alpha_n \right), \]
\[ G^{n-} b_L^0 U^m : C_R = \sqrt{\beta} \left( I_1 m_t \cos \alpha_n - I_2 M_{\Phi n} \sin \alpha_n \right). \]

\[ Z_\mu^0 \quad : ig \cos \theta_W \left\{ (p_1 - p_2)^\mu g^{\alpha \beta} + (p_2 - q)^\alpha g^{\beta \mu} + (q - p_1)^\beta g^{\alpha \mu} \right\} \]
\[ Z_\mu^n : \frac{i g}{6 \cos \theta_W} \gamma^\mu (C_L P_L + C_R P_R) \]

\[ Z^0 Q_t^m Q_t^m : \begin{cases} 
C_L = -4 \sin^2 \theta_W + 3 \cos^2 \alpha_n \\
C_R = -4 \sin^2 \theta_W + 3 \cos^2 \alpha_n
\end{cases}, \quad Z^0 U^m U^m : \begin{cases} 
C_L = -4 \sin^2 \theta_W + 3 \sin^2 \alpha_n \\
C_R = -4 \sin^2 \theta_W + 3 \sin^2 \alpha_n
\end{cases}, \quad Z^0 Q_t^m U^m : \begin{cases} 
C_L = -3 \sin \alpha_n \cos \alpha_n \\
C_R = -3 \sin \alpha_n \cos \alpha_n
\end{cases}, \quad Z^0 U^m Q_t^m : \begin{cases} 
C_L = -3 \sin \alpha_n \cos \alpha_n \\
C_R = -3 \sin \alpha_n \cos \alpha_n
\end{cases} \]

\[ Z_\mu^n : \frac{g}{2 \cos \theta_W M_W^2} (p_1 - p_2)^\mu C \]

\[ Z^0 H^+ H^- : C = i \{ (-1 + 2 \sin^2 \theta_W) M_W^2 \phi_n - 2 \cos^2 \theta_W M_W^2 \}, \quad Z^0 G^+ G^- : C = i \{ (-1 + 2 \sin^2 \theta_W) M_W^2 \phi_n - 2 \cos^2 \theta_W M_W^2 \}, \quad Z^0 H^- G^+ : C = -M_W \phi_n, \quad Z^0 G^- H^+ : C = M_W \phi_n. \]

\[ Z_\mu^n : \frac{g_\mu^\nu}{\cos \theta_W M_W^2} C \]

\[ Z^0 W^\pm : C = (M_W^2 \sin^2 \theta_W + M_\phi^2 \cos^2 \theta_W), \quad Z^0 W^- H^+ : C = -i M_\phi M_W, \quad Z^0 W^+ H^- : C = -i M_\phi M_W. \]
$I_1$ and $I_2$ have the following form:

\[
I_1 = \frac{2}{\pi R} \left[ \frac{1}{\sqrt{1 + \frac{r^2 M_n^2}{4} + \frac{r_f}{\pi R}}} \right] \left[ \frac{1}{\sqrt{1 + \frac{r^2 M_n^2}{4} + \frac{r_\phi}{\pi R}}} \right] \frac{M_n^2 \left(-r_f + r_\phi\right)}{\left(M_n^2 - M_\Phi^2\right)} ,
\]

\[
I_2 = \frac{2}{\pi R} \left[ \frac{1}{\sqrt{1 + \frac{r^2 M_n^2}{4} + \frac{r_f}{\pi R}}} \right] \left[ \frac{1}{\sqrt{1 + \frac{r^2 M_n^2}{4} + \frac{r_\phi}{\pi R}}} \right] \frac{M_n M_n \left(-r_f + r_\phi\right)}{\left(M_n^2 - M_\Phi^2\right)} .
\]