Designs of faded-example to increase problem solving skills and procedural fluency in algebraic division

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Abstract. This study aims to review on literature of faded example, its design and instructional implication to increase problem solving skills and procedural fluency. As students required to apply problem solving skills and promote procedural fluency when solving mathematics task. A role of learning instructions is essential when facilitating students to develop those abilities effectively. In addition, Cognitive Load Theory (CLT) suggests learning instruction should be developed in manner of which suitable to human cognitive architecture where working memory is limited when processing information and knowledge was stored as a schema in long-term memory. Faded-example, an instruction based on this theory proposed a smooth mechanism when solving several tasks as well as applying the most effective procedure. Faded-example begins with a fully guided-task (worked example) and then gradually reduces step-by-step solutions within problems become a fully no guidance task (problem-solving). Theoretically, this strategy may help students to foster their problem-solving skill and procedural fluency step-by-step. Besides, design of faded example (forward and backward procedures) was provided. Also, the implementation of faded example on algebraic division learning consist of four procedures: preliminary, introductory, acquisition and test.

1. Introduction

Cognitive Load Theory (CLT) suggest that learning instruction should be develop by taking account of human cognitive architecture in order to facilitate better learning outcome [1], [2]. The most essential part of human cognitive architecture, which frequently discussed are working memory and long-term memory. Working memory refers to a cognitive system that process and manipulate every single information when undergoing such cognitive tasks [3]. Working memory is limited in capacity which can only handle around seven information in a moment [4]. Moreover, information in working memory might be vanished if no longer processed or associated to existed knowledge [5]. Other part, long-term memory functionate as a very large storage where the information was developed as schemas or knowledge [1].

Further, working memory limitations may have consequences on instructional design. CLT assumed that when the information was encoded in our cognitive system using learning instructional without any consideration on working memory limitation as mention earlier, a problem namely cognitive load may arise [2]. This cognitive load could be categorized into three kinds: intrinsic, extraneous and germane cognitive load. Intrinsic cognitive load is imposed by the nature of information or subject-domain which learned in classroom. Intrinsic cognitive load is closely related to element interactivity. Higher element interactivity could imposed higher intrinsic cognitive load, vice
versa [1]. Otherwise, the load imposes by the manner in which the information presented or design of instruction is called extraneous cognitive load. Moreover, germane cognitive load is defined as cognitive resources that provide support for the information acquisition process [1], [2], [6].

One of difficult materials in mathematics is algebraic division. This material is a part of algebraic operation topic. Algebraic division may consist of higher degree of element interactivity. It might be categorized as complex problems. For instance, ‘calculate $x^2 - 5x - 14$ is divided by $x + 2$’. According to this task, no less than eleven information which related and need to process simultaneously to calculate the solution. It still does not include the procedures and operations applied later. In this condition, this task may impose higher intrinsic cognitive load. Nevertheless, intrinsic cognitive load can not be reduced, unlike extraneous cognitive load which is manageable [1]. Thus, the role of teachers is as a means to develop instructional design which facilitate student to learn algebraic division effectively and efficiently.

1.1. Problem solving skill
Problem solving as an activity means that students engage in a task which the solution cannot be directly determine (higher level problems) [7]. In mathematics classroom, problem solving activity is widely known as the center of learning and teaching process [8]. Meanwhile, problem solving activity is closely related to student’s prior knowledge. Prior knowledge is a primary element to understand the problem which impose higher possibility of generating best procedure [9].

Related to this activity, an ability to solve higher level problem called as problem solving skills. NCTM [8] stated that problem solving skills should be the main focus of mathematics learning. Mathematics classroom should provide students with an orientation in problem situation that it could improve problem solving skills [10]. Further, Kilpatrick [11] distinguished five methods to acquire problem solving skills: osmosis (action-oriented and implicit), memorisation (particular strategy-task), imitation (expert imitation), cooperation (groups) and reflection (action-oriented and explicit). Thus, we could also used Polya [12] problem solving steps as the indicator of problem solving skills: ability to understand the problem, ability of planning a procedure to solve the problem, ability to carry out the plan and ability to check the solution.

1.2. Procedural fluency
Procedural fluency is an ability to solve problem using efficient, accurate and correct procedures [13]. Expert students (good procedural fluency) could find effective and efficient procedures through their cognitive thinking process. Otherwise, no procedural fluency may create difficulty when solving problem and constructing the relevant conceptual knowledge [14]. There are three aspects of procedural fluency: students know the correct procedure, they understand the proper situation (how and when) to apply the procedure, students are able to carry out the procedure effectively and accurately [15].

Hiebert and Lefevre [16] state that concept understanding that are not supported by procedural fluency could increase student’s intuition about a concept but unable to completely solve the problem. On the other hand, a procedural fluency with less conceptual understanding may develop student’s proficiency in manipulating symbols with no understanding of symbols. In this condition, students may solve the problem without any understanding of what they are doing. Therefore, conceptual understanding should be equipped with procedural fluency in order to become an expert problem solver.

1.3. Faded example
The main idea of faded example is based on an integration of worked example and problem-solving instructions. It aims to create a smooth transition from complete guidance instruction to fully no guidance instruction [17]. Besides, fading is related with fewer unproductive learning events which increase learning outcomes [18]. Ozcan [19] and Renkl et al., [17] suggest that faded example is recommended as a learning instruction because its effective and easy to implement in classroom.

Faded example was presented in manner of which the first task is a worked example. The presentation of second task is an example form which one single step is omitted (fading). Thus, the
number of omitted steps is increased until the last task has no more steps left or problem solving [20]. In this way, there are two kinds of faded example: forward and backward fading procedure [17], [20]. For instance, four tasks with each task required three solution steps are presented. First task of either forward or backward fading procedure is a worked example. On second task, forward fading hides the first step where backward fading hides the step 3. On third task, the step 1 and 2 of forward fading are hided, whereas the step 2 and 3 of backward fading are hided. On fourth task, all steps are hided thus it becomes a fully problem-solving task.

Backward fading procedure is more favorable than forward fading procedure [17], [20]. At least, there are three arguments to explain the superiority of backward fading procedure: delay of problem-solving, delay of feedback and problem-specific information as support (overview see Renkl et al., [17]). Further, Backward fading with self-explanation prompt is better than backward fading only on near and far transfer [18]–[21].

In level of expertise area, low level of prior knowledge students was more benefitted from worked example, in other hand, high level of prior knowledge was more favorable when using problem solving [22]. It means that the early skill acquisition may well developed using worked example, while problem solving was superior in the later skill acquisition [20]. However, skill acquisition can be distinguished into three phases: early, intermediate and late phase [23]. Early phase is defined as encoding activity of students without applying the acquired knowledge (i.e. study of material). Intermediate phase is a retrieve activity of students in order to solve problems. They should apply the existed knowledge to when solving related problems. Later phase is defined as automate activity where students has mastered the targeted knowledge or skills. They tend to practice which increase speed and accuracy.

From this perspective, Renkl and Atkinson [20] argued example-problem pair which largely used in example-based learning has no clear explanation about the transition mechanism from early to later stage of skill acquisition. Thus, knowledge construction is not an automatic process by studying examples or solving problems. In addition, how to elaborate the theoretical status of examples and problems as their functions shift over respectively on different phases of skill acquisition was also vague.

As alternative, Renkl et al., [17] develop a combination of worked example and problem solving in such a way to answer those questions. Interestingly, Renkl and Atkinson [20] found that faded example could facilitate the skills acquisition from early to later stage better than worked example-problem solving pair. A study also found that low level of prior knowledge is more favorable when using slow fading procedure, while, intermediate level of prior knowledge is more favorable when using medium and fast fading procedure [24].

Learning using faded example, students may acquire procedural skills gradually. It shifted from example-analogical base processing to rule-base processing cognitive activity [25]. In addition, faded example provides correct procedure to students as it presents problem solving steps thoroughly and in detail. Fade example steps are developed by considering the task characteristic and concepts or principles which used in problem solving. Besides, description of steps may also give understanding to student about how to carry out the procedure effectively and accurately and apply it to other tasks. Taking together, faded example may facilitate student’s problem-solving skill acquisition in intermediate phase condition. Therefore, procedural fluency which related to problem solving activity may also well-developed.

2. Methods
2.1. Developing faded example
In order to foster develop their problem-solving skills and procedural fluency. Faded example should be designed according to the key aspect of those competencies. In this section, we discuss two kinds of fading procedures on algebraic division. We provided seven symmetrical tasks for forward fading by considering the number of steps required to solve the task completely. The number of tasks and steps of backward fading was developed similarly with forward fading unless the working mechanism of fading procedure.
The first task of either forward and backward fading is fully guided-task (worked example). In forward fading, the second task faded step 1, the third task faded step 1 and 2, the fourth task faded step 1 – 3, until the seventh task faded all the steps where the task become a fully problem solving (no guidance). Unlike forward fading, backward fading presented the second task with the last step (step 6) was faded. Thus, the third task, step 5 and 6 were faded, the fourth task, step 4 – 6 were faded. Therefore, the seventh task has no guidance as in the seventh task of forward fading.

The step and solution were presented in a pair to minimize the unproductive activity when students learn the particular procedure. Also, coloration was used to make student focus on the procedure of manipulating the basic operation algebra. Design of instruction which may impose cognitive load was also reduced.

### 2.2. Forward and backward fading type

The fading instruction of algebraic division was followed (Figure 1).

| Task | Steps | Procedure | Description |
|------|-------|-----------|-------------|
| 1    | 1     | \(x^2\)  | \(x^2\) divided by \(x\) equal to \(x\) |
|      | 2     | \(x^2 - 3x\) | \(x\) is multiplied by \(x\) equal to \(x^2\) |
|      | 3     | \(-18\) | \(-18\) is subtracted by \(-18\) equals to \(0\) |

Sample of faded example was shown on Table 1.

### Table 1. Sample of forward and backward faded example

| Type | Forward fading | Backward fading |
|------|----------------|-----------------|
|      | Procedure      | Description     |
| 1    | \(x-6\) \(x^2 - 3x - 18\) | \(x\) is divided by \(x\) equal to \(x\) |
|      | \(x\) \(x^2 - 3x\) | \(x\) is multiplied by \(x\) equal to \(x^2\) |
|      | \(-18\) | \(-18\) is subtracted by \(-18\) equals to \(0\) |

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Figure 1. Design of forward and backward faded example
3. Result and Discussion

3.1. Learning schemas

Knowledge was constructed in human mind as schema. It was defined as a relation between the information which possessed (prior knowledge) and the material being studied [26]. The more association between that information, the more knowledge has been constructed. Therefore, teacher’s role in our classroom should be a means to help students develop their schemas with foster as much as possible association during learning. Teachers may require to consider students prior knowledge and how to create an effective ways in manner of which those existed knowledge relate to the initial learning. Further, adequate prior knowledge could impose better learning, while, the lack of prior knowledge may create failed schema acquisition.

As this topic is a part of basic operation of algebra, so that others properties such as addition, subtraction and multiplication of algebraic form was essential. Students are required to employ those concepts easily before studying algebraic division. Therefore, the lack of those concepts may impose negatively to algebraic division learning. Structured schemas illustrate those association was shown on Table 2.

| Table 2. Structured schemas on algebraic division learning |
|------------------------------------------------------------|
| Prior knowledge                                           | Initial learning                          |
| Basic operation on algebra                                 | Targeted concept:                        |
| • Addition (e.g add $3x + 2x$)                             | Division on algebra                      |
| • Subtraction (e.g subtract $4x - x$)                      | (e.g divide $x^2 - 4$ by $x - 2$)        |
| • Multiplication (e.g multiply $x$ by $2x$)                |                                           |

3.2. Implementation of faded example

The implementation of faded example was also critical. Implementation should consider students condition as well as the instructional implication in classroom. Students conditions cover at least their cognitive aspect such as students’ prior knowledge. Also, instructional implication which associated with faded example.

There are four phases may be used as learning procedure: preliminary, introductory, acquisition and test. Preliminary and introductory phase aimed to familiarize the material to students, give learning context, motivate students and encourage them as well as activate relevant prior knowledge. Acquisition phase aimed to facilitate students to master the topic (algebraic division). Moreover, it also used as a tool to develop students problem solving skills and their procedural fluency. Test phase is aimed to measure to students’ level of success in learning. Further, the description of the learning procedure was followed (Table 3).

| Table 3. Structured schemas on algebraic division learning |
|------------------------------------------------------------|
| Phase | Description |
|-------|-------------|
| Preliminary | Teacher provides stimulus to activate student’s prior knowledge relate to algebraic division. Teacher may use explanatory instruction or quiz which aims to retrieve student’s existing knowledge related the required concepts. Teacher should evoke student’s motivation such as giving compliment and encouragement as well as providing the purpose and benefit of learning. |
| Introductory | Teacher should provide clear explanation about the concept of algebraic division. Demonstration the problem-solving process may required to give students an image about what kind of activity they will be carried out. Worked example may help student with insufficient knowledge base. |
| Acquisition | Students work using faded example instruction. Students work step by step, task per task using faded example. Teacher may help students with their work. Student may explain the result in front of class. Teacher give feedback to fix the possible misconception, errors, and to highlight the learning conclusion. |
| Test | Test was conducted to measure student’s ability. In this case, teacher may provide a test which measured problem solving skills and procedural fluency. The result of test could be use as assessment and evaluation process in order to improve the quality of learning. |
4. Conclusion
Design of faded example could facilitate student to study algebraic division effectively. Instead of asking student to solve problems directly with no guidance at all or limited, it’s better to provide faded example to encourage them from fully guidance instruction (worked example) to inquiry with no guidance instruction (problem solving). Worked example provide knowledge aid which may be used as a scaffolding for novice learner. Further, the reduction of given steps which contribute to the improvement of level solving activity could foster student’s problem-solving skills. Thus, the presentation of steps and descriptions of faded example may increase students’ problem-solving skills. In other word, students acquire problem solving skill and procedural fluency gradually.

Therefore, this study suggests four phases of learning procedure: preliminary, introductory, acquisition and test. Besides, students’ conditions and schema construction were necessarily to consider. However, further research to examine the effect of faded example on problem solving skill and procedural fluency are required.

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