Deriving particle physics from quantum gravity: a plan

Tom Banks

Department of Physics and SCIPP
University of California, Santa Cruz, CA 95064
E-mail: banks@scipp.ucsc.edu
and
Department of Physics and NHETC, Rutgers University
Piscataway, NJ 08540

Abstract

I give a short review of the holographic approach to quantum gravity, with emphasis on its application to deriving the properties of elementary particles.
1 Introduction

The holographic approach to the quantum theory of gravity is an attempt to generalize existing string models of gravity in order to describe cosmology and real world particle physics, including supersymmetry breaking. Its basic starting point is a strong form of the Fischler-Susskind-Bousso covariant entropy conjecture, which identifies the area in Planck units of the holographic screen of a causal diamond, with four times the logarithm of the dimension of the Hilbert space describing all possible observations in that diamond. When combined with the usual connection between causally separated regions and commuting operator algebras, it provides a way to encode all of the data in a Lorentzian geometry in terms of purely quantum mechanical concepts.

A quantum space-time is a collection of Hilbert spaces with a set of causal relations between them. Each Hilbert space is associated with a causal diamond. A common tensor factor of two Hilbert spaces (a unitary map from a tensor factor of one to a tensor factor of the other) is identified with the maximal area causal diamond in the space-time overlap between the two diamonds. A pattern of overlaps determines the geometry. The Hamiltonian dynamics within each Hilbert space is constrained by conditions of compatibility with that on overlapping spaces. It may be that all such compatible nets of quantum systems, with the property that Hilbert spaces become very large in the future, generate Lorentzian geometries satisfying Einsteins’ equations coupled to some sort of matter.

In this holographic description of quantum gravity, the space-time metric is not a fluctuating quantum variable. That is to say, the causal structure and conformal structure of the emergent space-time, come from exact features of the quantum theory: tensor inclusions and Hilbert space dimensions. In this view the space-time metric should be thought of as a collective variable, somewhat analogous to invariants in large $N$ expansions. In perturbation theory, the variable satisfies some kind of classical equations, and instanton solutions of these equations can even be used to compute tunneling probabilities. However there is no sense in which the theory can be viewed non-perturbatively as a quantization of these equations or a path integral over fluctuating configurations of the metric.

\footnote{We will use the notation $G^{-\frac{1}{2}} = M_P \approx 10^{19}$ GeV = $\sqrt{8\pi\hbar G}$, and units where $\hbar = c = 1$.}
1.1 The Dense Black Hole Fluid

There is only one solved example which supports the conjecture that this sort of quantum system gives rise to an emergent space-time[1], and it corresponds to a flat FRW cosmology, with equation of state $p = \rho$, which saturates the covariant entropy bound at all times. It is called the dense black hole fluid (DBHF). Heuristic arguments[2] show that local low entropy defects in the DBHF can evolve as more ordinary cosmologies, but must evolve to an asymptotic de Sitter (dS) universe in the future, with cosmological constant (c.c.) determined by a cosmological initial condition. It is related by the holographic principle to the dimension of the Hilbert space of the subsystem that began in a low entropy state.

I will include a brief presentation of the DBHF here. To describe the Hilbert space of a general observer in a Big Bang cosmology, we start from the observation that in the classical geometry, the observer’s particle horizon shrinks to zero near the Big Bang, and expands with the universe (we will not be describing Big Crunch cosmologies here). The holographic connection between area and entropy, and the translation of causality into commutation of operators says that we should describe this quantum mechanically as a sequence of Hilbert spaces, $\mathcal{H}_n$, with $\mathcal{H}_n = \mathcal{H}_{n-1} \otimes \mathcal{P}$. Since our present discussion will lead to an open subluminally expanding universe, $n$ will eventually go to $\infty$ and we call the limiting Hilbert space $\mathcal{H}_\infty$. The single pixel Hilbert space $\mathcal{P}$ describes all of the degrees of freedom that can be observed on a holographic screen in the non-compact dimensions of space-time[2]. In a later section I will argue that it is a representation of a super-algebra, which encodes the geometry of the compact dimensions. As a consequence of the holographic principle, that geometry must have only a finite dimensional function algebra. A natural framework to describe it is finite dimensional non-commutative geometry, also called fuzzy geometry. In $\dim \mathcal{P}$ is one quarter of the area of the smallest pixel of holographic screen, measured in units of the $d$ dimensional Planck scale. $d$ is the number of non-compact dimensions.

The dynamics of our fiducial observer is described by a sequence of unitary transformations $U(k)$, all operating in the full space $\mathcal{H}_\infty$. If we postulate that $U(k) = U_{in}(k) \otimes U_{out}(k)$ where $U_{in}(k)$ operates in $\mathcal{H}_k$, then we incorporate the notion of particle horizon into our holographic cosmology: the degrees of freedom in $\mathcal{H}_k$ do not interact with the rest of the universe until time $k + 1$, when "the particle horizon has grown large enough to incorporate more degrees of

---

2It is worth noting that in the holographic context, a future asymptotically dS universe has a non-compact spatial topology. The maximal causal diamond of an observer in such a space-time has a null boundary, whose holographic screen has finite area.
freedom”.

All of experimental physics, throughout the history of the universe, can be viewed as “the observations of a single observer”, but we like our theoretical structures to incorporate multiple observers, related by consistency conditions. In the present framework we do this by introducing a lattice, which encodes the topology of the non-compact dimensions of space. This topology does not change with time. We will let $x$ symbolize points on the spatial lattice and $\mathcal{H}_n(x)$ be the Hilbert space at time $n$ of the observer whose world line is labeled by $x$. From the bulk space-time point of view, we are in a synchronous coordinate system, defined by equal area time slicing: the area of the holographic screen of the causal diamond stretching from the observer’s current position back to the Big Bang, is the same at every point on the lattice. It should be noted that the integer $n$ does not directly count the time, but rather the area of the causal diamond. In a wide variety of space-time geometries, $n$ is related to the proper time coordinate of “natural” observers by $n \sim t^{d-2}$.

For the present discussion, we will ignore compact dimensions. Anticipating the discussion of section 2, we conclude that the pixel Hilbert space, $\mathcal{P}$, is the irreducible representation of

$$[S_a, S_b]_+ = \delta_{ab},$$

where $S_a$ are the real components of the transverse $(d-2$ dimensional) spinor.

To complete the definition of a general quantum cosmology we must specify the overlap Hilbert space $\mathcal{O}_n(x, y)$, which is a tensor factor of both $\mathcal{H}_n(x)$ and $\mathcal{H}_n(y)$, for each pair of points on the lattice. By definition (of nearest neighbor), for nearest neighbor points we have $\mathcal{O}_n(x, x_{\text{n.n.}}) = \mathcal{P}$. Furthermore, the overlap must decrease in dimension as the minimum number of lattice steps between the two points increases. We will see that it is this decrease which defines the geometrical distance function on the lattice.

Geometrically, the overlap Hilbert space is associated with the largest area causal diamond that fits in the intersection between the causal diamonds represented by $\mathcal{H}_n(x)$ and $\mathcal{H}_n(y)$. The geometrical interpretation is only supposed to make sense when all Hilbert space dimensions are large. It’s clear that the causal structure and conformal factor of the emergent space-time geometry are completely fixed by the quantum rules. Geometry does not fluctuate in this model of quantum gravity. It is an emergent classical variable, morally similar to the singlet variables of large $N$ matrix models.

There is a very strong set of constraints on the quantum dynamics of a holographic cosmology. Each sequence of overlap Hilbert spaces $\mathcal{O}_n(x, y)$ has two sequences of unitary trans-
formations induced on it from the dynamics on the individual observer spaces. These two sets of transformations must be unitarily related

\[ U(n, x; x, y) = W(n, x, y)U(n, y; x, y)W(n, x, y). \]

The definition of the DBHF dynamics is motivated by the observation of Fischler and Susskind\textsuperscript{[3]} that a flat FRW universe with equation of state \( p = \rho \) can saturate the covariant entropy bound at all times. We thus search for a random Hamiltonian on the individual observer’s Hilbert space. Begin by choosing \((U(n) \equiv e^{iH(n)})\)

\[ H(n) = \sum S_a(K)A(n; K, M)S_a(M), \]

where \( A(n; K, M) \) is a random \( n \times n \) antisymmetric matrix chosen (independently at each \( n \)) from the Gaussian ensemble. It is well known that as \( n \to \infty \), \( H(n) \), as a consequence of the Wigner semi-circle law, approaches a Hamiltonian for free massless relativistic 1+1 dimensional fermions, with a cutoff. If we now add a random polynomial of order \( \geq 6 \) in the \( S_a(K) \) to \( H(n) \), the large \( n \) behavior is unchanged, away from the cutoff scale, because of the standard 1 + 1 dimensional renormalization group flows. The thermodynamic quantities obey scaling laws in the limit, because the fixed point is scale invariant. Random changes in \( A(n, K, M) \) as \( n \) is varied, guarantee that the system will explore its full Hilbert space, given a generic initial state. The covariant entropy bound is thus saturated.

Define the overlap Hilbert spaces by \( O_n(x, y) = \mathcal{H}_{n-d(x,y)} \), where \( d(x, y) \) is the minimum number of lattice steps between the points. When the subscript becomes negative \( O \) vanishes. We can define a causal boundary between two points at any “time” \( n \), by looking at the locus of points beyond which the overlap vanishes. For large \( n \), this occurs at large values of \( d(x, y) \), and on any regular lattice with the topology of flat space the locus approaches a sphere. Thus, for large \( n \) the system has an emergent spherical symmetry.

We can satisfy the infinite set of dynamical consistency conditions by insisting that the random Hamiltonian seen by each observer at a given time is independent of \( x \) and letting the dynamics in the overlaps be exactly the dynamics seen by an observer at the earlier time, whose causal diamond has the same size as the overlap. This is probably the only way to satisfy the consistency conditions given the random dynamics for an individual observer.

We have thus proven that the geometry that emerges from this system is homogeneous and isotropic. The fact that it is flat follows from the scale invariance of the large \( n \) dynamics.
The flat FRW metric, with single component equation of state, has a conformal Killing vector corresponding to simultaneous scaling of space and time coordinates, with different exponents:

\[ ds^2 = -dt^2 + t^a d(x)^2, \]

\[ t \to \lambda t \quad x \to \lambda^{1-\frac{a}{2}} x. \]

We identify this with the emergent quantum scale invariance and learn that in this particular case (unlike, for example a gas of photons) the conformal isometry is in fact a symmetry of the full quantum system. Negatively curved FRW space, which is compatible with the topology of our lattice, would not have such a scaling symmetry.

The fact that the equation of state of this system is \( p = \rho \) already follows from the fact that its thermodynamics is that of a conformal field theory in \( 1 + 1 \) dimensions. We define the space time energy density of the FRW model to be the thermal expectation value of the \( 1 + 1 \) dimensional energy density. There are a variety of other checks of the flatness, homogeneity, and isotropy of the emergent geometry, and of its equation of state. These have been explained in \[1\].

The DBHF is not a nice place to visit, and you couldn’t live there if you tried. All the degrees of freedom within a horizon volume are in equilibrium at all times. Heuristically, the universe consists at all times of a single black hole which fills the particle horizon. Fischler and I proposed to model a more realistic cosmology by imagining a sprinkling of low entropy regions at the time of the Big Bang, which form a defect in the DBHF. This defect has a fixed size and shape in the geometry defined by the DBHF. The size and shape are determined by maximizing the initial entropy subject to the constraint that a more or less normal region of the universe survives into the future. We do not yet know much about the shape, except that it cannot be even approximately spherical. The size is a cosmological initial condition, which is fixed only by insisting on properties of the asymptotic future state. We argue that this initial condition determines the c.c. The point is that the quantum meaning of the size of the initial defect is just the logarithm of the number of quantum states associated with the initial defect region. This number is finite. Given the model of quantum dS space in section 3, as a quantum system with a finite number of states, it seems clear that these two numbers are the same.

A simple calculation in GR confirms this identification. Consider a sphere of normal FRW universe, embedded in the DBHF and ask whether the causal diamond of a normal observer can last indefinitely. The Israel junction condition shows that the answer is NO. The coordinate radius of the normal sphere must shrink. Heuristically this is because the pressure of the DBHF
at fixed entropy is larger than that of the normal region. On the other hand, if the universe is future asymptotically dS, then the causal diamond of a maximal observer is a marginally trapped null surface of fixed area. We can join it onto the event horizon of a black hole of equal area, embedded in the $p = \rho$ FRW background, satisfying the Israel condition. Thus, holographic cosmology predicts that the normal region of the universe must asymptote to dS space, with a c.c. determined by cosmological initial conditions. Since the DBHF is infinite, we can choose initial conditions in which it is seeded with an arbitrary finite number of normal regions, each with its own c.c. This is a multiverse in which the regions with different physics never talk to each other, but it nonetheless provides a model in which the c.c. is subject to environmental selection constraints.

The DBHF defect cosmology evolves to a matter dominated FRW universe. On the equal area time slices, regions of normal universe grow in volume relative to those regions which behave like the DBHF. Even if the defect was a small fraction of the initial volume, the eventual picture is that of a normal universe seeded with small regions of DBHF, each of which will just become a black hole in the normal universe. Note however that this picture is self consistent only if the resulting distribution of black hole masses, velocities and positions is close to a homogeneous isotropic fluid. Otherwise, black holes will collide and grow and the system collapses back to the DBHF. Thus, in this model, the universe undergoes a transition from a DBHF to a dilute, nearly homogeneous and isotropic black hole gas.

Holographic cosmology thus provides an explanation for the homogeneity, isotropy, flatness and low entropy of the initial normal universe, without invoking inflation\[^3\]. It cannot however explain two salient properties of the Cosmic Microwave Background: the long range, visible universe spanning, correlations between fluctuations, and the fact that these fluctuations enter the horizon oscillating in phase. The latter fact is responsible for the acoustic peaks that are observed in the data.

One must thus postulate that the particle physics of the normal universe contains a field with the properties of an inflaton. Note that the initial conditions imposed by the DBHF explain why the initial inflaton field is homogeneous over the whole region which evolves to our current horizon. If it were not homogeneous at the time when the universe is dominated by the dilute black hole gas, black holes would again collide and cause collapse to the DBHF.

There are two sources of fluctuation in the inflaton energy density after inflation. The

\[^3\]In fact, I believe, with Penrose, that the inflationary explanation of homogeneity, isotropy and low initial entropy is not valid.
first comes from initial conditions. Locally, the transition from the dilute black hole gas to an inflationary era comes about when the energy densities of the two become equal. Thus, fluctuations in the initial black hole density, which must be small but need not be zero, are imprinted on the evolution of the inflaton field. In addition, we have the usual quantum fluctuations of the inflaton. One can show that the former lead to a slightly blue tilted or at best exactly scale invariant spectrum\(^4\), while the latter are famously red-tilted. The data seem to favor quantum fluctuations as the origin of the fluctuations in the CMB, but it is not ruled out that there is some contribution from the initial black hole density fluctuations, on the scales important for structure formation.

Note that the number of e-folds of inflation that are required to explain the CMB data is of order 20, considerably fewer than one postulates in conventional inflationary cosmology. This alleviates the problem of fine tuning of the inflaton potential. It is also likely that the scale of inflation is much lower than in conventional models. The universe must go through the DBHF phase, and the dilute black hole gas phase, before inflation begins.

This ends our brief recapitulation of holographic cosmology. We next turn to study the variables of the holographic quantum theory.

2 SUSY and the holographic screens

The variables of quantum gravity are related to the classical concept of the orientation of a pixel on the holographic screen of a causal diamond. For a finite area causal diamond, the holographic principle implies a UV cutoff on the number of functions describing the geometry of the screen. We can implement this by replacing the algebra of functions on the screen by a finite dimensional associative algebra. It turns out that in order to describe particles, we should choose this to be a non-commutative matrix algebra. A pixel simply means a single element of a basis of this algebra\(^5\). Following the rules of non-commutative geometry, vector bundles over the screen are then rectangular matrices.

The orientation of pixels on the screen is described by a section of the spinor bundle over the screen. Indeed, the screen is a leaf of a foliation of the null boundary of the causal diamond. That is to say, there is a null direction penetrating each pixel of the screen. The Cartan-Penrose

---

\(^4\)The spectrum of fluctuations in the black hole density is exactly scale invariant over a finite range of scales, and zero outside that range. The blue tilt comes from evolution of the inflaton field.

\(^5\)We will use the phrase *localized pixel* for the more conventional geometrical notion of pixel - a basis of the algebra satisfying \(f_if_j \approx \delta_{ij}\), in the large area limit.
\[ \bar{\psi} \gamma^{\mu} \psi \left( \gamma_{\mu} \right)^{\alpha}_{\beta} \psi^\beta = 0, \]
defines a null vector in space-time and a \(d - 2\) plane transverse to it. The components \(\bar{\psi} \gamma^{\mu_1 \cdots \mu_k} \psi\), with \(k \geq 2\) all lie in this plane. The local Lorentz and scaling symmetries of the CP equation are gauge symmetries (the scaling symmetry is broken to a \(Z_2\) gauge symmetry, which we identify with \((-1)^F\), by the quantization rules below). In the limit of a large screen, a localized pixel uniquely determines a null direction up to an ingoing-outgoing ambiguity. Given that null direction, the solution of the CP equation is just a null plane spinor, \(S_a\) a spinor under the transverse rotation group. Thus, the screen orientation variables live in the spinor bundle over the screen.

If we are describing a \(d\) dimensional infinite space, with asymptotic \(SO(d - 1)\) symmetry, the most general commutation relations compatible with the holographic principle, for single pixel variables is

\[ [S_a, S_b]_+ = \delta_{ab}, \]

where for simplicity we have chosen a dimension where the transverse spinor representation is real. SUSY aficionadas will recognize this as the commutation relations for the spin degrees of freedom of a \(d\) dimensional massless supermultiplet - the quantized screen orientations on a pixel have a space of states identical to the spin states of a massless superparticle. For multiple pixels, we should expect the independent degrees of freedom to commute, but we can use the \(Z_2\) gauge symmetry to write the quantum algebra as

\[ [S_a(m), S_b(n)]_+ = \delta_{ab} \delta_{mn}, \]

where we recall that the labels \(m, n\) refer to a basis in the spinor bundle over the fuzzy screen, which is a space of rectangular matrices.

### 2.1 Holographic compactification

In the limit of an infinite, rotationally invariant holographic screen, these commutation relations lead to the space of states of a massless super-particle\(^4\)\(^5\). However, if the number of non-compact dimensions is \(d < 11\) they do not automatically lead to super-gravitons\(^6\). For \(d = 4\), in the fully \(SO(3)\) invariant formulation of \(^5\) they lead only to massless chiral multiplets.\(^7\)

\(^4\)If \(d > 11\) they lead to particle spectra that are compatible only with a trivial S-matrix.
String theory suggests an obvious solution to this problem, namely the introduction of compact dimensions. For compactifications to $d = 4$ from 11 dimensions (or 10 with Type II spinors), the spinor bundle is described by variables $(\Psi^I)_i^A, (\Psi^I)^j_B$, where $I, J$ are 8 component spinor indices. The $N \times N + 1$ and $N + 1 \times N$ matrices $\Psi$ and $\Psi^\dagger$ are the two spinor bundles over the fuzzy two-sphere.

The anti-commutation relations are

$$[(\Psi^I)_i^A, (\Psi^I)^j_B]^+ = \delta_i^j \delta_A^B M^{IJ},$$

and the geometry of the compactification is encoded in the super-algebra formed by the bosonic generators $M^{IJ}$ and the pixel orientation variables $\Psi$ and $\Psi^\dagger$. Indeed $M^{IJ}$ are in the bi-spinor over the compact space, which is a sum of $p$-form charges. From experience in string theory, we expect to write each $p$-form charge as a sum over wrapped brane charges associated with cycles. These will generically form an abelian bosonic algebra, but on singular manifolds, when some cycles shrink to zero size it can become non-abelian. For $N \to \infty$, we expect to be able to achieve 4 dimensional super-Poincare invariant compactifications. Partial SUSY breaking will be encoded in the fact that the bosonic generators $M^{IJ}$ do not commute with all of the $\Psi^I$. Holographic compactifications can be classified by finding all such algebras whose $N \to \infty$ spectrum contains the supergravity multiplet for some super-Poincare algebra in four dimensions.

One of the salient features of this approach to compactification is that it is manifestly invariant under string-dualities. A well known feature of such dualities is that the SUSY algebra is invariant, and only its interpretation in terms of branes wrapped on cycles of a large smooth manifold, Kaluza-Klein momenta, or purely internal symmetries, changes from one duality frame to another.

The actual geometry of the internal manifold is determined by the explicit matrix representation of the pixel super-algebra, which must, by the holographic principle, be finite dimensional. This means that arbitrarily large internal manifolds can only arise by some sort of correlated limit as $N \to \infty$. In particular, for dS space, where the total entropy is finite, the size of the

\[\text{From the holographic point of view, AdS space-times are quite different from either asymptotically flat, or dS space. In the latter two cases, causal diamonds with finite proper time separation between past and future tips, all have finite holographic area. In AdS space, the area becomes infinite in finite proper time, and this is the reason that the conformal boundary of space-time is time-like and its quantum dynamics is described by a quantum field theory. In dS or asymptotically flat space, the dynamical object for local observers is the scattering matrix\(\text{[6]}\) (which has small ambiguities in the dS case), mapping the past and future boundaries of the causal diamond onto each other.}\]
internal manifold is bounded. We will discuss this in more detail in the next subsection. More generally, the discreteness of finite dimensional representation theory assures us that all moduli of the theory are frozen in dS space. Continuous moduli spaces are artifacts of the infinite entropy asymptotically flat limit.

Implicitly, we have assumed that the algebra of functions on the internal manifold is finite dimensional and it is likely to be non-commutative. In the large volume limit, a remnant of non-commutativity would show up as a Poisson structure on the manifold. Many of the standard compactification manifolds of string theory have such a structure. Many of them are compact Kahler manifolds, and one could find sequences of matrix algebras approximating \( C^\infty(M) \), by the methods of geometric/deformation quantization. The seven manifolds that appear in compactification of M-theory to four dimensions also have natural Poisson structures. For example, Horava-Witten compactification is a Calabi-Yau bundle over an interval, and there is a natural Poisson structure induced by the Kahler form of the Calabi-Yau manifolds. Manifolds of \( G_2 \) holonomy are less well understood, but the construction of non-compact manifolds of this type can usually be understood as the lift of D6 branes in Type IIA string theory on a non-compact Calabi-Yau 3-fold, so the 7 manifold is a circle bundle over the Calabi-Yau 3-fold, and there is again a natural Poisson structure.

### 2.2 Matrices and particles

Suppose that we have found a sequence of matrix algebras \( A_n \), of \( n \times n \) matrices, which converges, in an appropriate sense, to the algebra \( \mathcal{F} \) of functions (with some continuity or smoothness property) on the holographic screen at infinity for a space-time of the form \( M^{1,d-1} \times X \). We can consider the block matrices

\[
\begin{pmatrix}
M_{k_1} & \cdots & 0 \\
\vdots & \ddots & \vdots \\
0 & \cdots & M_{k_p}
\end{pmatrix},
\]

where \( M_k \) is a \( k \times k \) matrix in \( A_k \). We can view this as a subalgebra of all \( N \times N \) matrices with \( N = \sum k_i \).

Taking the limit where all \( k_i \) go to infinity, at fixed ratio, we can obtain not just single super-particle states, but an entire Fock space. In addition, the (in the limit continuous) block sizes \( k_i \), supply the magnitude of the null momenta of these particles, which was missing in
our kinematic description above. The permutation gauge symmetry of particle statistics is a natural consequence of the unitary equivalence of different bases in the full $\sum k_i \times \sum k_i$ space of matrices. Combining this with our $(-1)^F$ gauge symmetry, we get the correct connection between spin and statistics.

Technical details of the limit, which lead to Lorentz invariance, involve a famous infinite dimensional operator algebra, discovered by Murray and von Neumann. These details can be found in [4]. This construction raises the question of what to think about the rest of the matrix degrees of freedom, apart from the diagonal blocks. We will see that the quantum theory of dS space has an answer to this question. They represent particles propagating in horizon volumes causally disconnected from “our own”.

3 Quantum interpretation of semi-classical properties of dS space

One should take seriously those aspects of quantum gravity in dS space that can be reliably described in the semi-classical approximation. Among these are

- The dS vacuum is actually a density matrix with high entropy equal to $\pi (R M_P)^2$, where $R$ is the dS radius, related to the c.c. by $R^{-1} = \frac{3 m_P}{\Lambda^{1/2}}$. According to the strong form of the holographic principle advocated above, this entropy is the logarithm of the dimension of the Hilbert space describing stable dS space.

- There is an operator we will call $P_0$, which converges to the Minkowski Hamiltonian in a particular Lorentz frame (that of a given static observer in dS space) in the limit $R M_P \to \infty$. The mass parameter in dS black hole solutions (see below) is interpreted as the eigenvalue of this operator. With respect to this operator, the density matrix is a thermal state, with a unique temperature, $T = \frac{1}{2 \pi R}$. The instanton calculation of [7] shows that the probability of nucleating black holes of small mass is also thermal, with the same temperature.

- Although one often claims that the dS group converges to the Poincare group through Wigner-Inonue contraction, a proper GR interpretation of generators in terms of their action on the boundary of a causal diamond, contradicts this. Near the future cosmological horizon ($v \to 0$ below) the dS metric looks like

$$ds^2 = R^2 (-dudv + d\Omega^2),$$
and the generator of static time translation is $\propto (u \partial_u - v \partial_v)$, while the Minkowski metric near future null infinity is

$$ds^2 = \frac{-dudv + d\Omega^2}{v^2},$$

and the time translation is $\propto \partial_u$. This suggests that the quantum theory of dS space has another Hamiltonian, in addition to $P_0$.

- **Instanton transitions between two dS minima** of an effective potential always exist, and satisfy a principle of detailed balance

$$P_{1 \rightarrow 2} = P_{2 \rightarrow 1} e^{-(\Delta S)_{12}},$$

appropriate for a system at infinite temperature (free energy equals entropy times temperature). For transitions to negative c.c. Big Crunches, the instanton calculation gives a probability $\sim e^{-\Delta S}$, if one interprets the entropy of the Crunch state as one quarter of the area of the maximal causal diamond of an observer in the crunching region, but only for potentials which are *above the great divide*\(^8\). If, for any given dS minimum of a potential, one shifts the energy down to zero, one can ask whether the resulting Minkowski solution has a positive energy theorem. The space of potentials that, after such a shift, have a static domain wall between the Minkowski solution and an AdS solution in the negative minimum, is co-dimension one in the space of all potentials. On one side of this great divide there is a positive energy theorem, and a dS solution gotten by shifting the potential upwards will have the $e^{-\Delta S}$ behavior. We interpret this as saying that only potentials above the divide can appear in the low energy effective field theory of stable dS space.

- **Schwarzschild black hole solutions**\(^9\) in dS space have the form

$$ds^2 = -dt^2 f(r) + \frac{dr^2}{f(r)} + r^2 d\Omega^2,$$

where $f(r) = 1 - \frac{2M}{M_P r} - \frac{\kappa^2}{R^2}$. These have two horizons with

$$R^2 = R_+^2 + R_-^2 + R_+ R_-,$$

$$2M = M^2 \frac{R_+ R_- (R_+ + R_-)}{R^2}.$$

Note that the sum of the areas of the two horizons is less than the vacuum area and decreases with increasing $M$, reaching a minimum at the maximum mass black hole

$$M_N = \frac{M^2 \sqrt{R}}{3 \sqrt{3}}.$$

\(^8\)As long as the potential at the top of the barrier is not too flat. There are reasons to believe that this degree of flatness is not consistent with quantum gravity\[^8\].

\(^9\)The conclusions we will draw here also hold for charged black holes. We have not yet modeled spinning black holes in the quantum theory.
• The maximum entropy in a dS horizon volume, which can be described by quantum field theory, without invoking black holes of order the horizon scale, is $\sim (RM_P)^{3/2}$.

### 3.1 The quantum theory of stable dS space

We now sketch a quantum theory of dS space based on the pixel variables introduced above and consistent with all of this semi-classical “data”. The variables consist of $N(N + 1)$ copies of the single pixel algebra, so the entropy, to be identified with $\pi(RM_P)^2$ in the large $N$ limit is $N^2 \ln D_P$, where $D_P$ is the dimension of representation of the single pixel algebra.

On the other hand, by standard Kaluza-Klein arguments the entropy per pixel is given by

$$\ln D_P = \left(\frac{m_P}{m_d}\right)^2,$$

where $m_d$ is the higher dimensional Planck mass. If, following Witten [10], we identify this with the scale of coupling unification, then

$$\ln D_P \sim 10^4.$$

According to our general discussion, particle states of the system will arise by restricting attention to block diagonal matrices, of the form

$$\begin{pmatrix}
\Psi_{k_1} & \cdots & 0 \\
\vdots & \ddots & \vdots \\
0 & \cdots & \Psi_{k_p}
\end{pmatrix}.$$

If we want to maximize the entropy in such particle states, we distribute it evenly among the different particles. We would like to have a lot of particles, but each one should have a lot of angular momentum states, so that we can localize it on the sphere and justify the name particle. The compromise is to have $N^{1/2}$ blocks of size $N^{1/2} \times N^{1/2}$. Since $N \propto (RM_P)$ this gives an entropy scaling for particles in a single horizon volume identical to that derived in our semi-classical argument.

In this context there is an obvious interpretation of the other degrees of freedom in the matrix variables. Indeed they can be organized into $N^{1/2}$ bands, each consisting of a collection of blocks identical to the ones we have associated with particles in a single horizon volume. The semi-classical theory of dS space seems, at very early and late times, to accommodate an infinite
number of copies of the particle degrees of freedom in a single horizon volume. Here we see that there is an organization of the Hilbert space into $N^{1/2}$ such copies. Since $N \sim M_P R$, this agrees with the semi-classical limit in which $M_P \to \infty$ with the dS radius fixed in centimeters. We will see later that this decomposition breaks down when we begin to consider black hole states of the system.

In order to explain the gross properties of semi-classical dS space, we introduce a Hamiltonian $H$, which may be viewed as the evolution operator with respect to the static time coordinate. As noted this should be different from, and should not commute with, the Hamiltonian $P_0$ which is the sum of localized particle Hamiltonians in each horizon volume. $H$ is the Hamiltonian whose dynamics is probed by the instanton computations we have described above.

We model it as a Hamiltonian with a random spectrum, obeying the bound $||H|| \leq cT$. By random spectrum, I mean that time evolution under $H$ obeys the rules of statistical mechanics: for any initial state, the time dependent expectation values of a large class of observables will evolve rapidly to thermal equilibrium, with a temperature determined by the initial expectation value of $H$. For a random choice of initial state, the temperature will be infinite, with probability 1. If we interpret the dS vacuum state, following the holographic principle, as the infinite temperature ensemble, then we can understand the gross features of the probabilities of Coleman-DeLucia tunneling events in dS space, assuming the low energy effective theory is above the divide.

To explain the fact that same density matrix is thermal, with a finite temperature, for the Hamiltonian $P_0$ we need only postulate a relationship between the eigenvalues of $P_0$ and the degeneracies of the corresponding eigenspaces. This corresponds to a formula

$$P_0 = \sum E^I e_I,$$

where $e_I$ are commuting orthogonal projectors satisfying

$$\text{Tr} e_I \sim e^{\pi (R M_P)^2 - \frac{p_I}{T}},$$

where the $\sim$ sign means equality to leading order in $\frac{p_I}{m_N}$. $m_N = \frac{m^2_P R}{3\sqrt{3}}$, is the mass of the maximal, Nariai, black hole in dS space. $P_0$ is an operator defined only for states localized in a single horizon volume. That is, we view the full Hilbert space of the stable dS theory as a

---

10This statement should be understood in the sense of scattering theory. All localized states in dS space decay to the vacuum by emission of particles, and the particles are non-interacting when they are far apart, so one can calculate the energy just by summing up the kinetic energies of particles.
tensor product of a space of states localized in a single horizon value, with its complementary tensor factor. \( P_0 \) acts only in the localized factor i.e. it acts as the unit operator in the complementary factor. We know that the maximal entropy density matrix describable by field theory in the single horizon has localized entropy of order \((RM_P)^{3/2}\). For the maximal size black hole, the entropy (counting both black hole and cosmological horizons) approaches \( \frac{2}{3}(RM_P)^2 \). This indicates that all localized states have entropy much smaller than that of the dS vacuum and that entropy decreases with increasing \( P_0 \). The maximal eigenvalue of \( P_0 \) is the Nariai black hole mass. The formula above incorporates both the predictions of field theory, of a thermal distribution at the dS temperature for particle states, and the instanton calculation of [7], which indicates a thermal probability at the same temperature for black hole nucleation.

There is some semi-classical evidence for this connection between the energy and entropy of eigenstates of \( P_0 \). For eigenvalues much larger than the Planck scale, but much smaller than the Nariai mass, the black hole entropy formula in dS space gives

\[
S(M) = S_{dS} - \frac{M}{2\pi R},
\]

which is just the energy/entropy connection we have postulated above.

To connect the evolution under the Hamiltonian \( H \) (which produces the dS ground state density matrix \( \rho = e^{-S} \), ) to that of \( P_0 \), we have to postulate commutation relations between these two operators. These are motivated by the geometrical picture of the action of the static dS and static Minkowski Killing vectors on the boundaries of the corresponding causal diamonds (Item 3 in our list of the properties of dS space)

\[
[(u\partial_u - v\partial_v), \partial_u] = \partial_u.
\]

In the quantum theory we postulate

\[
[H, P_0] = M_P^2 g(\frac{P_0}{M_P^2 R}),
\]

where \( g(x) \) is a bounded function, linear in \( x \) for small \( x \), and with vanishing trace. This coincides with a properly normalized version of the Killing vector commutation relation in the subspace with \( P_0 \) eigenvalues \( \ll \) the Nariai mass.

The spectrum of \( H \) is highly degenerate, with level spacings as small as \( \frac{1}{R}e^{-\pi(RM_P)^2} \). The maximum spacing is of order \( 1/R \), which gives evolution on a time scale roughly the current age of the universe. \( P_0 \) breaks the degeneracy and evolves the system on much shorter time scales. The commutation relations guarantee that the low lying eigenstates of \( P_0 \) will be relatively
stable under $H$ evolution. Other approximate quantum numbers could give rise to stability on even longer time scales. For example, although charged particles centered at the origin of the static coordinate system are not stable, the mechanism of decay is nucleation of a particle of opposite charge, which has probability $e^{-2\pi R m_c}$, where $m_c$ is the mass of the lightest charged particle. One would conjecture that to reproduce this behavior all that is necessary is that the Poincare invariant $(R M_P) \rightarrow \infty$ limit have a massless $U(1)$ gauge boson in its spectrum.

### 3.2 Black holes and pixels

We will now show that the same variables that describe particle excitations of dS space can describe the properties of Schwarzschild-de Sitter black holes. Recall the equations relating the two horizon radii

$$R^2 = R_+^2 + R_-^2 + R_+ R_-,$$

$$2M = M_P^2 \frac{R_+ R_- (R_+ + R_-)}{R^2}.$$

Recall also that

$$\pi (R M_P)^2 = \ln D_P N^2$$

and define

$$\pi (R_{\pm} M_P)^2 = \ln D_P N_{\pm}^2$$

Choose $N_-$ to be an integer and $N_+$ to be the greatest integer in the solution of

$$N^2 = N_+^2 + N_-^2 + N_+ N_-$$

We define the ensemble of black hole states to be the states satisfying

$$\psi_i^A |BH\rangle = 0$$

for $1 \leq i \leq N_-$, $1 \leq A \leq N_+$. The entropy of this ensemble is, for large $N$ and $N_-$ the same as that of the corresponding black hole. Note that we had to choose a basis for the matrices in order to define this ensemble. As with particle states, this corresponds to choosing the horizon volume in which the black hole sits. However, for general large $N_-$, of order $N$, there will be no multiple black hole states.

The fermion number operator has a large expectation value and small statistical fluctuations in this ensemble. Using the above formula for the black hole mass one can write a function
of the number operator whose statistical expectation value $= M$ in this ensemble, thus realizing the black hole spectrum in this quantum mechanical model. The precise expression can be found in [3], and is not particularly illuminating or edifying.

What we see from these formulae is that the same variables can describe both black holes and particles, and that states with too many or too energetic particles will inevitably look like black holes.

4 Constraints on the low energy effective action

In the previous sections, I’ve outlined a quantum theory of stable dS space and indicated how to make it compatible with all of the semi-classical data we have about the nature of quantum dS space. Now we will proceed to discuss the constraints on low energy particle physics which follow from this picture. The first of these is that we should view the c.c. as a tunable input parameter, rather than a calculable quantity. Indeed, one might hope that it is the only tunable parameter in the model. There is no real argument for this extreme position, but there certainly are arguments that stable dS models are a lot fewer and further between than other classes of string models, like supersymmetric AdS models and models of asymptotically flat space.

The mere fact that the c.c. is tunable means that for small c.c. we should be able to model the dynamics of localizable states by a low energy effective Lagrangian density. As usual in string theory we view the construction of this Lagrangian as an act of theoretical phenomenology. The underlying theory prescribes some properties (which in perturbative string theory, but only in perturbative string theory, correspond to a systematic calculation of all observables in an expansion in some dimensionless parameter), which must be reproduced by the Lagrangian. In our case, these properties include the value of the c.c. and the fact that the limiting Lagrangian when $RM_P \rightarrow \infty$ must describe a consistent coupling to some super-Poincare gravity multiplet. As a consequence, the model must have a dS solution, with a c.c. that can be tuned to zero.

As far as I know, this implies that we have $3+1$ non-compact dimensions with $N = 1$ SUSY. Furthermore, since SUSY is a global limit of a gauge symmetry, its breaking must occur through the super-Higgs mechanism.

Generic SUGRA Lagrangians and generic supersymmetric stationary points, have negative

\footnote{It is certain that there are no super-Poincare invariant gravitational scattering matrices in $1+1$ dimensions, and likely that the same is true in $2+1$ dimensions. Higher dimensional supergravities coming from unitary theories, and higher $\mathcal{N}$ supergravity in four dimensions do not appear to have dS solutions.}
rather than vanishing c.c. A natural way to account for the vanishing of the c.c. in the supersymmetric limit is to postulate that the low energy Lagrangian in this limit has a discrete R symmetry $\mathbb{Z}_k$ with $k \geq 3$. This ensures the vanishing of the superpotential at supersymmetric, R-symmetric stationary points. The terms that must be added to the Lagrangian when $\Lambda > 0$ explicitly break this R-symmetry, since they must in particular include a constant $W_0$ which will tune the c.c. to $\Lambda$ when the scale of SUSY breaking takes on its correct value (which we'll discuss in a moment). When these terms are discarded the Lagrangian must preserve SUSY, and their addition is what triggers SUSY breaking.

The underlying model also suggests a scale for the gravitino mass. We have seen that single particle excitations of the model arise from fermionic matrices of size $K$, where maximum entropy multi-particle states have $^{12}K \sim N^{1/2} \sim \pi^{1/4} (\frac{m_D}{m_P} R M_P)^{1/2}$. There is a precise action of the rotation group on these states, but the maximum spherical harmonic in their wave functions has $L \sim N \sim 10^{60}$. Lorentz boosts act on the two sphere at infinity as the coset of rotations in its $SO(1,3)$ conformal group. The breaking of Lorentz invariance due to the restriction to only $10^{60}$ spherical harmonics is too small to be measurable by current experiments.

On the other hand, it is reasonable to suppose that the breaking of SUSY is governed by $N^{-1/2}$, the natural expansion parameter of this system. That is, the commutator of the SUSY generators with the Hamiltonian will be of order $N^{-1/2}$ in Planck units. If we choose $m_D$ to be the unification scale, $\sim 2 \times 10^{16}$ GeV, we get the estimate for the gravitino mass $^{13}$

$$m_{3/2} = 10 K \Lambda^{1/4} = K (10)^{1/4} 10^{-2} \text{eV}.$$ 

A similar estimate is obtained from the following hand waving argument. Our underlying model implies that the gravitino mass vanishes in the limit $R M_P \to \infty$. Assume that the gravitino is the lightest R-charged particle in the theory. Then, the terms in the effective Lagrangian that lead to the breaking of SUSY must arise from interactions between the gravitino and states localized on the dS horizon. Those states cannot be modeled properly by effective field theory. A better model is Landau levels on a two sphere, with an entropy given by the dS entropy. I use this analogy because there is a basis of Landau level states which is localized on the sphere, and we can talk about the entropy of states in a given area $A$. Assuming that the gravitino mass vanishes like a power of $RM_P$ in the limit that this parameter goes to infinity,

---

$^{12}$Here we have used the relation between the total entropy and the dS radius in four dimensional Planck units, as well as the KK estimate of the number of degrees of freedom per four dimensional pixel. $m_D$ is the higher dimensional Planck scale.

$^{13}$In the super-Higgs phase, the super-partner of any particle state is that state plus a longitudinal gravitino.
we can write a term in the effective Lagrangian coming from a gravitino going out to infinity as (to leading exponential order)

$$\delta \mathcal{L} \sim e^{-2m_{3/2} R} \sum |\langle 3/2|V|n \rangle|^2.$$ 

$V$ is a “vertex operator” coupling the gravitino to the degrees of freedom on the horizon. Note that there are no large energy denominators in this formula. The horizon dynamics all takes place over a much longer time scale than the duration of the gravitino’s sojourn on the horizon. Indeed, since the gravitino is massive and the horizon is null, it can only be viewed as propagating near the horizon for a proper time of order $m_{3/2}^{-1}$. Free quantum particles perform random walks and the natural step size for this walk on the horizon is the Planck scale. Thus, one expects the horizon area covered by the gravitino to be $\frac{1}{m_{3/2} M_P}$, and the number of states for which the matrix element is non-zero to be of order $e^{m_{3/2}/2}$, where $b$ is one of those proverbial constants of order one.

In order for the expression $e^{-2m_{3/2} R + \frac{b}{m_{3/2}} M_P}$ to be neither exponentially small or large in the large $R$ limit, $m_{3/2}$ must converge to $\frac{1}{2} M_P (R M_P)^{-1/2}$, which is the $N^{-1/2}$ scaling we postulated above. Note that the conventional “no fine tuning” prediction of SUGRA is $m_{3/2} \sim R^{-1}$. This is the right scaling in AdS space, but would lead to an inconsistent, exponentially large correction from interactions with the boundary. Of course, this scaling is already inconsistent with our underlying model. A gravitino at rest with such a mass would, according to our formalism have only one or a few spherical harmonics in its wave function on the sphere, and would never behave as a localized particle.

### 4.1 Particle physics

The low SUSY breaking scale implied by the above arguments rules out almost all extant models of SUSY breaking. In particular, gravitationally coupled hidden sector models are ruled out, as well as all gauge mediated models with weakly interacting messengers. The same is true for anomaly mediation, gaugino mediation, semi-direct mediation, etc. In all of these models, the observed lower bounds on super-partner masses are inconsistent with our estimate for the gravitino mass. Extra factors of $10 \sim 10^3$ in the latter estimate could make CSB compatible with some of the low energy mechanisms, but one must also be careful to avoid the window of gravitino masses ruled out by cosmology, which is something like 20 eV to 1 GeV. This rules out $F$ terms between $2 \times 10^6$ GeV $\sqrt{F} < 10^9$ GeV. The nominal value of $\sqrt{F}$ in CSB is $\sqrt{F} \sim 10^4$.  

20
Even $2 \times 10^5$ GeV is a low scale for most conventional gauge mediated models.

Before starting to put in detailed phenomenological constraints, let me note that the simplest model consistent with the general ideas of CSB has a low energy sector consisting of a single chiral superfield, $G$, with zero charge under the fundamental discrete R-symmetry. The R-breaking terms consist of a general superpotential, which is a function of $G/m_P$ with the linear term adjusted to give the CSB formula for the gravitino mass and the constant tuned to set the c.c. equal to its underlying value $\Lambda$. Another important constraint is that the decay probability to any lower minimum of the potential (there is likely to be a supersymmetric minimum with negative c.c.) vanish as $e^{-\pi (R M_P)^2}$ when $\Lambda$ is taken to zero. In the language of [9], the potential must be above the Great Divide. This involves order 1 adjustments in the dimensionless expansion coefficients in $W(G/m_P)$.

As soon as we insist on standard model gauge symmetries, things become complicated. First note that pure non-abelian gauge groups are not allowed, we must have matter fields in fairly large representations (not necessarily irreducible). Non-abelian SUSY gauge theories with small representations tend to break all R symmetries and are not consistent with a super-Poincare invariant $\Lambda = 0$ model. For example, the standard model gauge group with one generation is not consistent with CSB at $\Lambda = 0$. QCD with two flavors would generate an $R$-violating superpotential, and does not give a super-Poincare invariant space-time. We will see that phenomenological considerations put stringent upper and lower bounds on the number of low energy fields charged under the standard model.

Indeed, having introduced the standard model, we must enforce the experimental lower bounds on the chargino and gluino masses. In terms of the Goldstino superfield $G$, these will come from a term of the form

$$\beta_i \frac{G}{M} (W^i) \alpha^2,$$

with the $\beta_i$ dimensionless constants of order 1. Recalling that, according to the CSB relation between the gravitino mass and the c.c., $F_G/m_P = m_{3/2} \sim 1.75 \times 10^{-11}$ GeV, the bounds on the chargino mass imply that

$$M \leq \frac{\beta_2}{1.7} \text{ TeV},$$

where we have used the strongest possible chargino bound (> 160 GeV from the Tevatron)\footnote{This bound is valid in a class of models, but may not be general. The model independent bound is $\sim 105$.}. There is also an experimental lower bound of order 300 GeV on the gluino mass. In the model.
we eventually settle on, there is another parameter, which can give a variety of values for the gluino mass, below the vanilla gauge mediated prediction

\[ m_{1/2}^{(3)} = \frac{\beta_3 \alpha_3}{\beta_2 \alpha_2} m_{1/2}^{(2)}. \]

We note that the parameters \( \beta_i \) lend an additional uncertainty to these estimates.

We learn two things from this calculation: there must be new states, charged under the standard model with mass of scale \( \sim M \), and \( M \) is quite low. Note that we cannot push the chargino mass much higher than its experimental lower bound, because that would bring down the scale \( M \), and contradict the experimental absence of these new states. Thus, models based on CSB predict light chargino states, close to the current experimental bounds. The particular model we choose, the Pyramid Scheme, also predicts a gluino mass somewhere below the “vanilla gauge mediation” value quoted above.

Note also that the gravitino is very light, and consequently cosmologically safe. It is however, the LSP and the NLSP (which is likely to be either the bino or a right handed slepton) is coupled to it strongly. Conventional SUSY dark matter is not obtained in this model. We will discuss dark matter in the context of the Pyramid Scheme below.

When combined with the requirement of coupling unification, the low scale \( M \) for additional matter with standard model quantum numbers, puts strong constraints on the nature of the hidden sector whose dynamics becomes strong at the scale \( M \). In order to preserve one loop unification predictions in a natural manner, the hidden sector matter must be in complete multiplets of the unified group. On the other hand, if the hidden sector group is large, this extra matter will produce Landau poles below the unification scale, destroying the prediction of unification.

To this constraint, we must add the fact that the strong dynamics of many hidden sector models is phenomenologically unacceptable, or violates the principles of CSB. The latter is true for example if the hidden sector theory preserves SUSY, but not an R symmetry. The phenomenological problems are connected to the existence of light pseudo-Nambu-Goldstone bosons (PNGB) in the hidden sector.

While an exhaustive search has not been carried out, the only model that has been found, which is compatible with unification in \( SU(5) \) or some larger simple group, is the Pentagon model of [12]. However, this model has Landau poles below the unification scale, and contains a PNGB whose interactions with electrons lead red giant stars to cool too rapidly[14]. In addition, discussions of this model have concentrated on vacuum states that are meta-stable in
the field theory approximation, neglecting gravitational effects. We will argue below that no such model can truly represent the physics of CSB.

Before turning to that discussion we introduce the most promising model of CSB that has so far been found, the Pyramid Scheme[13]. The basic idea is to replace unification in a simple group by Glashow’s trinification, the semi-direct product of $SU_1(3) \times SU_2(3) \times SU_3(3) \rtimes Z_3$, where the $Z_3$ permutes the three copies of $SU(3)$, cyclically. The indices on the different $SU(3)$ groups (almost) indicate which subgroup of the standard model gauge group is embedded in which unified $SU(3)$. The exception is the $U(1)$ of weak hypercharge, which is a linear combination of the generator in $SU_2(3)$ that commutes with the standard model $SU(2)$, and a generator in $SU_1(3)$. At the unification scale, the matter content of the theory consists of 3 copies of the equilateral triangular quiver consisting of $(3, \bar{3}, 1)$ + cyclic permutations. This can be thought of as the 27 of $E_6$, decomposed under the trinification subgroup. Another, probably more interesting way of looking at it, which even gets the number of generations correct, is to view this as 3 $D3$ branes at a $Z_3$ orbifold singularity in Calabi-Yau compactification of Type IIB string theory. We must arrange the GUT scale model so that below the unification scale, only the 15 chiral fields of an MSSM generation survive. This can probably be achieved with Wilson lines in the compact dimensions, but a complete stringy construction of the Pyramid Scheme is not yet available. The standard model Higgs fields can arise in a variety of ways.

Now we can introduce a hidden sector gauge group $G$, which extends the quiver to a three sided pyramid. The fields that connect the apex of the pyramid to its base are in the

$$(R, 3, 1, 1) + (\bar{R}, 3, 1, 1) + \text{cyclic permutations.}$$

If the representation $R$ of $G$ is small enough, we will have one loop unification with no Landau poles. We will make the choice of the group $G$ and representation $R$ in order to ensure that the low energy dynamics of the model reproduces the behavior expected from a quantum theory of dS space with a finite number of states. We pause to review that behavior in the next subsection.

4.2 SUSY breaking and tunneling

We have already argued that the low energy effective Lagrangian must be 4 dimensional $\mathcal{N} = 1$ SUGRA, with a dS minimum. It must contain parameters enabling us to tune both the SUSY breaking scale and the c.c. to zero, with the relation

$$m_{3/2} = 10K\Lambda^{1/4}.$$
In the limit of vanishing c.c., it must have a super-Poincare invariant solution with a discrete complex R symmetry, or perhaps a compact moduli space of such solutions\textsuperscript{15}.

A further restriction comes from considering Coleman-De Luca tunneling from the dS minimum to other, lower minima of the potential. In particular, one must generally expect a SUGRA Lagrangian with a dS minimum, to have other solutions where all SUSY order parameters vanish. Generally, there will be CDL tunneling from the vicinity of the dS minimum to the vicinity of the supersymmetric one. If, as is typical, the supersymmetric minimum has negative vacuum energy, classical evolution after tunneling leads to a Big Crunch space-time. The covariant entropy bound then implies that no observer in the post-tunneling space-time can see more than a finite, usually small, amount of entropy.

Such a transition makes sense in a finite theory of stable dS space, if the tunneling probability is of order

$$P \sim e^{-\pi (R M P)^2}$$

as $R M P \to \infty$. This is the probability one expects for a Heisenberg/Poincare recurrence in a finite system with a number of states equal to the exponential of the dS entropy. It is a catastrophe analogous to all of the air collecting in the corner of a room, rather than a true instability. In \textsuperscript{9} my collaborators and I did an analysis of the conditions in low energy effective field theory, which guarantee this kind of behavior. The space of all possible potentials, modulo an additive constant is divided in two, separated by a co-dimension one “Great Divide”. A precise definition of the divide is obtained by adding a constant to the potential, to bring the dS minimum in question down to zero potential energy.

The Great Divide is the submanifold of the space of potentials, on which there exists a static domain wall solution, connecting the zero energy minimum to the negative minimum, which has the highest CDL transition probability when $\Lambda \neq 0$. On one side of the Divide, the zero energy minimum has a positive energy theorem and no CDL instability. On the other side, it decays. On the side with the positive energy theorem, we can now restore the positive c.c. and show that the CDL probability for dS space to decay is that of a recurrence in a finite system.

It is important to emphasize that the Divide is defined by the particular one parameter deformation of the potential by an additive constant. In particular, any system for which SUSY

\textsuperscript{15}In all known string theory motivated models of SUSY breaking - SUSY is asymptotically restored in non-compact directions of moduli space, and the Coleman De Luca tunneling amplitudes from the dS minimum to the non-compact region, do not obey the requirement that the dS minimum represents a stable system with a finite number of states. We review this requirement below.
is restored as the c.c. is tuned to zero will have a tunneling probability that behaves like
\[ e^{-\pi (RM)^2}, \]
as \( RM_P \to \infty \). However, if \( M \ll M_P \) then this is much larger than the probability of a recurrence. In particular, this criterion rules out all models of SUSY breaking in which the dS minimum is meta-stable in the \( M_P \to \infty \), like the models of [15]. In such models, the potential has the form
\[ \mu^4 v(\phi/M), \]
where \( \mu \) and \( M \) are \( \ll m_P \). In that case one can show [9] that the CDL instanton is essentially the false vacuum dS sphere, with a small polar cap cut out, in which the non-gravitational [16] instanton is inserted. The action difference, which defines the transition probability, is of order \( (\frac{\mu}{M})^4 \). The probability indeed vanishes exponentially in the supersymmetric limit \( \mu \to 0 \), but it is much larger than that of a recurrence in a finite system with total entropy of order \( (\frac{m_P}{\mu})^4 \).

If we now combine the general features of CSB with an old result of Nelson and Seiberg [17], we come to a rather interesting conclusion. We have argued that, when gravity is neglected, a system consistent with CSB has a low gravitino mass and must break SUSY spontaneously, once we add the appropriate R-violating terms to the Lagrangian. Nelson and Seiberg argued that, in such an R-violating system, spontaneous SUSY breaking was only possible for non-generic Lagrangians. The requirement that a Lagrangian be generic, \( i.e. \) that it contain all terms compatible with symmetries, with values determined roughly by dimensional analysis, is a consequence of the renormalization group. Experience shows that this is what one obtains when integrating out degrees of freedom, even when those are just the higher momentum modes of the fields in the low energy theory. It is even true in conventional string theory.

However, in the context of CSB the R violating terms in the Lagrangian have a very different origin. In our hand waving argument, they come from Feynman graphs where a single gravitino interacts with non-field theoretic degrees of freedom on the horizon. It seems plausible that RG genericity might not hold for these terms. Indeed, if one simply insists that the effective Lagrangian reproduce the basic property of the underlying quantum theory of dS space, \( viz. \) that it be a finite system with a stable high entropy density matrix describing the dS ground state, our argument two paragraphs above shows that the Lagrangian cannot be generic.

From a practical point of view, it is fairly easy to write down a version of the Pyramid Scheme, which realizes these principles. We introduce a different singlet \( S_i \) for each of the trianons. The R preserving superpotential is
\[ W = W_{\text{std}} + \sum \alpha_i S_i H_u H_d + \sum \beta_i S_i \text{Tr} T_i \bar{T}_i, \]

where \( W_{\text{std}} \) contains the Yukawa couplings of the MSSM, and some of the coefficients \( \alpha_i \) and/or \( \beta_i \) might vanish in the R-symmetry limit. Other polynomials in the \( S_i \) are also forbidden by R-symmetry. We then add the non-generic R-violating superpotential,

\[ \delta W = W_0 + \sum M_i^2 S_i + \sum m_i \text{Tr} T_i \bar{T}_i. \]

In general, the \( S_i \) might have different R charges, and some of the \( \beta_i \) might be zero. Indeed, it turns out that the phenomenological requirement that the R symmetry forbid dimension four and five operators that violate \( B \) and \( L \) forces some of the \( \beta_i \) to vanish. A more fundamental reason for this to be the case is that if the \( M_i \) are nonzero, and one of the \( \beta_i \) vanishes, then the model has no SUSY preserving vacuum in the \( m_P \to \infty \) limit. It can thus serve as the low energy effective field theory of a finite quantum theory of dS space.

### 4.3 Dark matter and axions in the shade of the Pyramid

There are a variety of scenarios for Dark Matter in the Pyramid Scheme. The dynamics of the model is controlled by the ratios of the trianon masses \( m_i \) to scales determined by the gauge coupling at the apex of the pyramid. We consider \( m_{1,3} \) to be in the 10s of TeV range, and imagine that the gauge coupling at the unification scale is strong enough that the confinement scale \( \Lambda_3 \) of the \( N_F = N_C = 3 \) gauge theory below these masses is a few TeV\textsuperscript{16}. The 1, 3 trianon numbers are then conserved quantum numbers in the low energy field theory and reasonable primordial asymmetries can guarantee that the lightest particle carrying these quantum numbers is a dark matter candidate. If this is the origin of dark matter, then searches for dark matter annihilation signals will be fruitless. Alternatively, even without asymmetries, non-thermal production of these \textit{pyrma-baryons} can produce the right dark matter density. In the latter case, dark matter will annihilate primarily into the pseudo-Nambu Goldstone boson associated with spontaneous breakdown of the baryonic quantum number associated with the second trianon, whose mass is of order the confinement scale. The mass of this PNGB is of order an MeV, so it decays into leptons and photons only. This might be of use in explaining the results of the PAMELA and Fermi observations of excesses in the cosmic abundance of leptons. However, it seems likely that there are also astro-physical explanations for these excesses.

\textsuperscript{16}This appears to lead to a Landau pole below the unification scale, which might indicate that \( SU(3) \) must be the Higgsed remnant of a larger gauge group.
Briefly then, the hidden sector of the Pyramid scheme seems rich enough to produce a dark matter candidate in various ways. Alternatively one might imagine that the QCD axion is the dark matter. Here, an interesting issue arises. The terms in the low energy Lagrangian of the Pyramid Scheme, which preserve the discrete R symmetry, are such that one can simultaneously eliminate all phases in Yukawa couplings except that of Cabibbo-Kobayashi and Maskawa, as well as the $\theta$ parameters of both QCD and the strong group at the apex of the Pyramid. The Lagrangian is CP invariant, up to the CKM phase. In the field basis where all other couplings are real, the terms that violate discrete R symmetry are generically complex. Recall however that these terms are non-generic. One might try to argue, that the combination of IR CP conservation, and the thermal nature of the states on the horizon, which are putatively responsible for generating the R violating terms, implies that all CP violation in these terms should be proportional to the Jarlskog invariant. This could solve the strong CP problem in a novel way, without the need for a QCD axion, vanishing up quark mass, or Nelson-Barr mass matrix.

Carpenter et al. [19] have recently studied the QCD axion solution of the strong CP problem in the context of gauge mediated SUSY breaking. They found that the idea was strongly constrained and the constraints are not satisfied in the Pyramid Scheme. Furthermore, in order to make their models “natural”, these authors had to invoke a landscape of states with a variety of values for low energy parameters, with axion dark matter an a priori selection parameter. There are many ways in which these considerations conflict with the underlying idea of CSB. Thus, axion dark matter does not appear to be a likely prediction of the Pyramid Scheme.

We will not delve further into the properties of the Pyramid Scheme. Its phenomenology is quite interesting, and has been outlined in [13] but a number of questions require more of a solution of the strongly coupled gauge theory than is currently available.

5 Executive summary

The holographic approach to space-time geometry views geometry as a collective description of a network of interlocking quantum systems. At the end of the day, processes accessible to a single observer can all be described by an ordinary quantum Hamiltonian in a single Hilbert space, but the form of the Hamiltonian is constrained by the picture of overlapping systems with partial information. The only full mathematical solution of these constraints is the dense
black hole fluid (DBHF) cosmology of [1].

The quantum variables of holographic space-time are the orientations of pixels on the holographic screen of a causal diamond. In the limit of large diamonds they are best thought of in terms of the degrees of freedom of relativistic super-particles, penetrating the screen. This is the way that particle physics emerges from the formalism, and it always emerges together with (approximate) supersymmetry. The commutation relations of these variables define the non-commutative geometry of compact dimensions. The details have only been worked out for maximally supersymmetric compactifications. The automatic emergence of supersymmetry, and the unification of particle statistics with the non-commutative geometry of finite area holographic screens, are two of the most interesting aspects of the formalism. The description of particles in terms of block diagonal matrix algebras, raises the question of what the off-block diagonal matrix elements mean. In our quantum theory of de Sitter space, they represent degrees of freedom outside the horizon of any given observer.

The DBHF cosmology does not actually contain any observers\(^\text{17}\). All of its degrees of freedom are in constant equilibrium with each other. An heuristic description of a more realistic cosmology can be based on the idea of low entropy defects in the DBHF. This is a partial explanation of why our universe began in a low entropy state, so that it could exhibit a second law of thermodynamics. The most probable initial conditions for such a cosmology is one in which the defect involves the smallest number of degrees of freedom that is compatible with whatever \textit{a priori} environmental constraints one wishes to impose. If we apply the Israel junction condition to the asymptotic future state of the defect, embedded in the DBHF, we conclude that it must approach a dS space, with the largest c.c. compatible with the \textit{a priori} constraints.

The theory of particle physics is then equivalent to the quantum theory of dS space, with a fixed, small, value of the c.c. That theory must approach an S-matrix theory for a super-Poincare invariant, discrete R-symmetric, model. The SUSY breaking scale is related to the c.c. by the formula \( m_{3/2} = KA^{1/4} \), where the current best estimate of \( K \) is \( o(10) \). For very small \( A \) this situation must be described approximately by effective field theory, and this implies that the dS space is four dimensional. This is the only dimension in which SUGRA has dS solutions. A further constraint is that the CDL tunneling probability from the dS minimum to any negative energy density region of the effective potential, must behave as \( e^{-\pi(RM_P)^2} \), in the limit as \( RM_P \to \infty \). This rules out models of SUSY breaking based on flat space field theory at

\(^{17}\)Observer $\equiv$ isolated quantum system with many quasi-classical operators.
meta-stable states. When we combine old results of Nelson and Seiberg with the R symmetry properties of CSB, we come to the conclusion that the low energy Lagrangian cannot be the most generic form consistent with some set of symmetries.

In the framework of CSB, the R invariant part of the Lagrangian is that of a super-Poincare invariant S-matrix theory, much like ordinary string theory. Experience shows that, in both asymptotically flat and AdS string theory, such Lagrangians satisfy the usual rules of genericity, familiar from the renormalization group in quantum field theory. In dS space on the other hand, the R violating parts of the Lagrangian, come from interactions between particle-like degrees of freedom in a given horizon volume, and the degrees of freedom on the horizon. They all come from diagrams where a single gravitino line goes out to the horizon and returns. Higher order diagrams are exponentially suppressed as the c.c. goes to zero. There is thus no reason to assume that the terms in the R breaking Lagrangian are generic.

It has turned out to be extraordinarily difficult to construct a low energy Lagrangian consistent with all of these constraints, with the rudiments of low energy phenomenology, and with gauge coupling unification. This has led to a more or less unique model, dubbed The Pyramid Scheme, since it has a pyramidal/tetrahedral quiver diagram. Aspects of this model should be testable at the LHC.

There are two big lacunae in the framework for particle physics outlined here. The first is the classification of compactifications of the holographic formalism, which lead to minimal SUSY in four dimensions. One would like to know what constraints this puts on the four dimensional gauge group. Recall that conventional string theory has produced no examples of exactly $N = 1$ super-Poincare invariant models in four dimensions, with a compact moduli space. In the holographic formalism it is clear that for finite c.c. there are no continuous moduli (everything is determined by the representation theory of compact finite dimensional super-algebras), so things should be even more constrained.

The other unsolved problem is to write down dynamical equations for the scattering matrix. This is of course not a new problem. Perturbative string theory gives us an asymptotic series for the scattering matrix, but, except for those special cases where Matrix Theory is applicable ($\leq 4$ out of 11 compactified dimensions) we do not have a non-perturbative formulation of the theory. In no case do we have a non-perturbative formulation of models of quantum gravity in asymptotically flat space, which manifests all of the symmetries of the system.

In ancient times, the analytic S-matrix program attempted to find a set of equations that
completely determined the scattering matrix without referring to the bulk of space-time. This program failed because the high energy behavior of amplitudes was not under control. In a perturbative context, with a finite number of stable particles, the analytic S-matrix rules seem to lead back to quantum field theory, with the usual issues of renormalization replaced by the question of subtractions in dispersion relations. Recently, we have seen the development of new rules for direct computation of the scattering matrix, particularly in maximally supersymmetric models\cite{18}. In my opinion, the best way forward is to try to work out the analogous formalism for an S-matrix for eleven dimensional SUGRA. This model has the maximal space-time symmetry and simplest particle content, among all possible super-Poincare invariant S matrices including gravitons. I would guess that once we have a completely non-perturbative formulation of this model in hand, it will be relatively straightforward to generalize it to compactified models with less SUSY. The combination of these dynamical equations and the kinematic classification of quantum theories of dS space, which we described above, will give us the tools to construct a real theory of the world we live in, and to understand the extent to which “God had no choice in its creation”.

On a more pedestrian level, there are a number of issues with the low energy Pyramid Scheme, which remain to be worked out. Primary among these is the development of tools for calculating the spectrum of superpartners, and their interactions. The model will stand or fall on its comparison with LHC data, but we need more precise predictions in order to make this comparison. The issue of Landau poles in the Pyramid gauge coupling also needs work. In \cite{13} we suggested embedding $SU_P(3)$ in $SU_P(4)$, but did not provide a dynamical implementation of the Higgs mechanism. Another avenue for resolving this problem is suggested by the vanishing one loop beta function of SUSY QCD with 9 flavors and 3 colors. Perhaps the multidimensional RG flows in the space of this gauge coupling and the Yukawa couplings, could lead to some sort of quasi fixed point, which eliminates the Landau pole below the unification scale. Preliminary investigation of this scenario has not had promising results, but more work remains.

Finally, it would be interesting to find a connection between the supersymmetric limit of the Pyramid Scheme, and D-brane constructions in string theory. In this connection, M. Cvetic has pointed out to me that 3 $D3$ branes at the $Z_3$ orbifold, provide a local model of the base of the Pyramid, including the 3 generations of standard model matter. It is of interest to see if one can obtain a local model of the full Pyramid by adding D7 branes to the mix.
6 Acknowledgments

I would like to thank Willy Fischler and Tomeu Fiol for important input into the program described in this note. This research was supported in part by DOE grant number DE-FG03-92ER40689.
References

[1] T. Banks, W. Fischler and L. Mannelli, “Microscopic quantum mechanics of the p = rho universe,” Phys. Rev. D 71, 123514 (2005) [arXiv:hep-th/0408076].

[2] T. Banks and W. Fischler, “An holographic cosmology,” arXiv:hep-th/0111142. T. Banks and W. Fischler, “Holographic cosmology 3.0,” Phys. Scripta T117, 56 (2005) [arXiv:hep-th/0310288].

[3] W. Fischler and L. Susskind, “Holography and cosmology,” arXiv:hep-th/9806039.

[4] T. Banks, “II(infinity) factors and M-theory in asymptotically flat space-time,” arXiv:hep-th/0607007.

[5] T. Banks, B. Fiol and A. Morisse, “Towards a quantum theory of de Sitter space,” JHEP 0612, 004 (2006) [arXiv:hep-th/0609062].

[6] T. Banks and W. Fischler, “M-theory observables for cosmological space-times,” arXiv:hep-th/0102077.

[7] P. H. Ginsparg and M. J. Perry, “Semiclassical Perdurance Of De Sitter Space,” Nucl. Phys. B 222, 245 (1983).

R. Bousso and S. W. Hawking, “Primordial black holes: Pair creation, Lorentzian condition, and evaporation,” Int. J. Theor. Phys. 38, 1227 (1999).

[8] T. Banks, M. Dine, P. J. Fox and E. Gorbatov, “On the possibility of large axion decay constants,” JCAP 0306, 001 (2003) [arXiv:hep-th/0303252]. N. Arkani-Hamed, L. Motl, A. Nicolis and C. Vafa, “The string landscape, black holes and gravity as the weakest force,” JHEP 0706, 060 (2007) [arXiv:hep-th/0601001].

[9] A. Aguirre, T. Banks and M. Johnson, “Regulating eternal inflation. II: The great divide,” JHEP 0608, 065 (2006) [arXiv:hep-th/0603107].

[10] E. Witten, “Strong Coupling Expansion Of Calabi-Yau Compactification,” Nucl. Phys. B 471, 135 (1996) [arXiv:hep-th/9602070].

[11] M.V. Berry, Regular and irregular semiclassical wavefunctions, J. Phys A 10, 2083 (1977); J.M. Deutsch, Quantum statistical mechanics in a closed system Phys. Rev. A 43, 2046 (1991); M. Srednicki, Chaos and quantum thermalization Phys. Rev. E 50, 888 (1994) [arXiv:cond-mat/9403051]; M. Srednicki, Thermal fluctuations in quantized
chaotic systems J. Phys. A 29, L75 (1996) [arXiv:chao-dyn/9511001; M. Srednicki, The approach to thermal equilibrium in quantized chaotic systems J. Phys. A 32, 1163 (1999) [arXiv:cond-mat/9809360]. See the last of these for a more extensive list of references.

[12] T. Banks, “Cosmological supersymmetry breaking and the power of the pentagon: A model of low energy particle physics,” arXiv:hep-ph/0510159 T. Banks, “Remodeling the pentagon after the events of 2/23/06,” arXiv:hep-ph/0606313.

[13] T. Banks and J. F. Fortin, “A Pyramid Scheme for Particle Physics,” JHEP 0907, 046 (2009) [arXiv:0901.3578 [hep-ph]]. T. Banks and J. F. Fortin, “Tunneling Constraints on Effective Theories of Stable de Sitter Space,” arXiv:0906.3714 [hep-th].

[14] T. Banks and H. E. Haber, “Note on the pseudo-Nambu-Goldstone Boson of Meta-stable SUSY Violation,” arXiv:0908.2004 [hep-ph].

[15] K. A. Intriligator, N. Seiberg and D. Shih, “Dynamical SUSY breaking in meta-stable vacua,” JHEP 0604, 021 (2006) [arXiv:hep-th/0602239].

[16] S. R. Coleman, “The Fate Of The False Vacuum. 1. Semiclassical Theory,” Phys. Rev. D 15, 2929 (1977) [Erratum-ibid. D 16, 1248 (1977)].

[17] A. E. Nelson and N. Seiberg, “R symmetry breaking versus supersymmetry breaking,” Nucl. Phys. B 416, 46 (1994) [arXiv:hep-ph/9309299].

[18] F. Cachazo, P. Svrcek and E. Witten, “MHV vertices and tree amplitudes in gauge theory,” JHEP 0409, 006 (2004) [arXiv:hep-th/0403047]. F. Cachazo, P. Svrcek and E. Witten, “Twistor space structure of one-loop amplitudes in gauge theory,” JHEP 0410, 074 (2004) [arXiv:hep-th/0406177]. R. Britto, F. Cachazo, B. Feng and E. Witten, “Direct Proof Of Tree-Level Recursion Relation In Yang-Mills Theory,” Phys. Rev. Lett. 94, 181602 (2005) [arXiv:hep-th/0501052]. N. Arkani-Hamed, F. Cachazo and J. Kaplan, “What is the Simplest Quantum Field Theory?,” arXiv:0808.1446 [hep-th]. N. Arkani-Hamed, F. Cachazo, C. Cheung and J. Kaplan, “The S-Matrix in Twistor Space,” arXiv:0903.2110 [hep-th]. N. Arkani-Hamed, F. Cachazo, C. Cheung and J. Kaplan, “A Duality For The S Matrix,” arXiv:0907.5418 [hep-th].

[19] L. M. Carpenter, M. Dine, G. Festuccia and L. Ubaldi, “Axions in Gauge Mediation,” arXiv:0906.5015 [hep-th].

L. M. Carpenter, M. Dine and G. Festuccia, “Dynamics of the Peccei Quinn Scale,” arXiv:0906.1273 [hep-th].