Optimization of open-hole variable stiffness composite plates under tensile loading using curved continuous fibers

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Abstract. This paper describes optimization of variable stiffness composite plates (VSCPs) with open-holes under tensile loading. Curved continuous fibers are used to optimize distributions of fiber orientation and the fiber volume fraction during the optimization for the VSCPs. The modeling of VSCPs is performed by means of fiber trajectories, which are aligned in the direction of maximum principal stress. The progressive failure of the VSCPs is modeled by a material property degradation method, taking into account the variable distributions of material properties to predict the ultimate load. It is demonstrated that the ultimate load for the composite plates is increased using the change in a reinforcement structure and a transition from unidirectional to curvilinear reinforcement of fibers. Thus, application of VSCPs with curved fibers compared with unidirectional composites results in more effective use of composite materials in various fields of industry.

1. Introduction
Fiber reinforced polymers (FRPs) are used in various fields of industry especially where it is necessary to minimize weight due to the high specific stiffness and the high specific strength of composite materials. The most common composite structures are laminates consisting of unidirectional (UD) layers. The laminates are optimized by variations of fiber orientations of the UD layers to maximize the efficiency of FRPs. To optimize the laminates, fiber orientations are aligned in the direction of loads. However, the optimization is effective only if there are not stress concentrations.

To improve performance of composite structures with stress concentrations, shape optimization [1] and local reinforcements [2] are used. However, since stiffness in UD FRPs within a layer is constant, the variation of design variables during the optimizations is limited. To change stiffness locally within a layer of laminates and enlarge design variables, fiber orientation and the fiber volume fraction can be varied along the layer coordinates. Similar composite structures are observed in nature, namely in wood where curvilinear fibers are curved in the vicinity of a knot. There are different design methods to model composite structures with curved fibers. For example, fiber orientation can be aligned in the direction of fluid streamlines [3], load paths [4, 5] and maximum principal stress [6-11]. Besides, fiber orientations can be varied depending on the objective function during optimization [12-16].

There are various advanced manufacturing technologies such as automated fiber placement, tailored fiber placement [2, 10] and 3D printing [11, 17-27], which are used to make composite structures with curved fibers. It is shown in tests [4, 5, 9-11, 24-27] that the change in a composite structure from UD to curvilinear reinforcement of fibers leads to increases in the efficiency of FRPs.
Thus, it is possible to design and make FPRs with greater efficiency using curved fibers compared to standard UD composite structures.

2. Definition of the problem

Variable stiffness composite plates (VSCPs) with holes were modeled to estimate the effect of curved fibers on the ultimate load of the VSCPs. Dimensions of the plates are shown in figure 1a. Two types of reinforcements were considered for the plates, namely, curvilinear reinforcement and unidirectional reinforcement. Curvilinear reinforcement was modeled in the VSCPs where curved fiber trajectories are aligned in the direction of the maximum principal stress. Fiber orientation and the fiber volume fraction are changed along coordinates of the VSCPs depending on the distribution of fiber trajectories. The maximum fiber volume fraction of the VSCPs is 57.7%. Fiber orientation in the direction of the x-axis (figure 1) and the fiber volume fraction ($V_f$) of 57.7% were used to model the UD plates. The fourth part of the plates, namely the left and top part of the plates ($0 \leq x \leq L/2$ and $0 \leq y \leq W/2$), were only modeled since the plates are symmetrical in the geometry, material and boundary conditions. The boundary conditions for the plates correspond to the tensile test where loading is directed along the x-axis. All types of the considered plates with different dimensions and reinforcements are given in table 1.

![Figure 1. Dimensions (a) and distributions of fiber trajectories (b), fiber orientations [°] (c) and the fiber volume fraction [%] (d) for the plate. The distributions correspond to C-3.](image)

| Table 1. Labels, dimensions and reinforcement structures of composite plates. |
|------------------|-----------|-----------|------------------|
| Label  | $D$ (mm) | $L$ (mm) | $W$ (mm) | Reinforcement of composite structures |
| C-1    | 6        | 60       | 9      | Curvilinear reinforcement of fibers |
| C-2    | 6        | 60       | 15     | Curvilinear reinforcement of fibers |
| C-3    | 6        | 60       | 24     | Curvilinear reinforcement of fibers |
| C-4    | 6        | 60       | 36     | Curvilinear reinforcement of fibers |
| UD-1   | 6        | 60       | 9      | Unidirectional reinforcement of fibers |
| UD-2   | 6        | 60       | 15     | Unidirectional reinforcement of fibers |
| UD-3   | 6        | 60       | 24     | Unidirectional reinforcement of fibers |
| UD-4   | 6        | 60       | 36     | Unidirectional reinforcement of fibers |

To model the composite structures, carbon FRP of IM7/8552 is used. Properties of carbon fiber of IM7 and matrix of 8552, as well as ultimate strengths of the IM7/8552 with $V_f = 57.7\%$, are given in
tables 2 and 3, where $E_1$ and $E_2$ are Young’s moduli in the longitudinal and transverse fiber direction, $G_{12}$ is shear modulus in the 1–2 plane, $\mu_{12}$ is the Poisson’s ratio, and $\varepsilon_T$ is ultimate tensile strain. $X$ and $Y$ are longitudinal strength and transverse strength, respectively. In this case, the subscripts $T$ and $C$ for $X$ and $Y$ denote tension and compression. $S_{12}$ and $S_{23}$ are the shear strengths in the 1–2 and 2–3 planes.

### Table 2. Material properties of IM7 fiber and 8552 matrix [8].

| Material properties | $E_1$ (GPa) | $E_2$ (GPa) | $G_{12}$ (GPa) | $\mu_{12}$ | $X_T$ (MPa) | $\varepsilon_T$ (%) |
|---------------------|-------------|-------------|----------------|------------|-------------|---------------------|
| Fiber               | 276         | 19.5        | 70             | 0.28       | 5654        | 1.9                 |
| Matrix              | 4.76        | 4.76        | 1.74           | 0.37       | 121         | 1.7                 |

### Table 3. Ultimate strengths for the IM7/8552 with $V_f = 57.7\%$ [8].

| Ultimate strengths | $X_T$ (MPa) | $X_C$ (MPa) | $Y_T$ (MPa) | $Y_C$ (MPa) | $S_{12}$ (MPa) | $S_{23}$ (MPa) |
|-------------------|------------|------------|------------|------------|----------------|----------------|
| Values            | 2524       | 1690       | 63.4       | 285.7      | 101.1          | 107.6          |

3. Methods

3.1. Modeling method of VSCPs

Initially, an isotropic material ($E = 200$ GPa, $\mu = 0.3$) is used to model the VSCPs and compute stress. On the basis of the stress fields, fiber trajectories are modeled, which are aligned in the direction of the maximum principal stress [28]. Then variable material properties along coordinates of the plates are simulated by the finite element method (FEM). Each element of the VSCPs is assigned to its own material properties depending on the distribution of fiber trajectories. Fiber orientation of each element is aligned in the direction of the maximum principal stress. The change in distance between fiber trajectories is modeled by the variable $V_f$, where the maximum $V_f$ is located in the zones with the minimum distance between fiber trajectories. The $V_f$ decreases with increasing distance between fiber trajectories. Material properties depend on fiber orientation and the fiber volume fraction. The modeling of variable distributions of fiber orientations and the fiber volume fraction, as well as the change in material properties, are described in more detail in [8].

After material properties are assigned to the VSCPs, the distribution of fiber trajectories is changed due to redistribution of stresses. Thus, the modeling process is iterative, and it is completed when the effective stress concentration factor ($K'_T$) will not exceed 1% between iterations. The calculation of the $K'_T$ is performed by equation (1) where $\sigma_1$ and $V_f$ are stress in the fiber direction and the fiber volume fraction, and $\sigma_0$ and $V_{f0}$ are the averaged values of $\sigma_1$ and $V_f$ along the left side of the plates. Distributions of fiber trajectories, fiber orientations and the fiber volume fraction are shown in figure 1b, 1c and 1d for the VSCP with label of C-3 after finishing the simulation.

$$K'_T = (\sigma_1 \cdot V_{f0})/(\sigma_0 \cdot V_f)$$

3.2. Progressive failure modeling of VSCPs

To model the progressive failure of the composite structures, a material property degradation method (MPDM) is used. According to the MPDM, if failure occurs, material properties of FRPs are decreased with increasing load in the zone with the failure. To fix the beginning of failure, the Hashin criterion is used. The decrease of material properties is performed by the degradation factors [29, 30]. If the zone of fiber failure crosses from the hole to the free edge of the plate, total failure of the composite structures occurs.

Since material properties are changed in the VSCPs, ultimate strengths are assigned to each element in accordance with the fiber volume fraction. To calculate the ultimate tensile strength in the
fiber direction, component properties of the IM7/8552 (table 2) and the modified rule of mixtures [31] are used. Other ultimate strengths are constant and they are not changed depending on the fiber volume fraction. The progressive failure modeling is described in more detail in [8].

4. Results and discussion
The stress concentration factor and the effective stress concentration factor are calculated for all types of plates and shown in figure 2. It is worth noting that since $V_f = V_f^0$ for the UD plates, $K_t = K_t'$ for these plates. The effective stress concentration factor significantly decreases with the change in composite structure and a transition from UD reinforcement to curvilinear reinforcement.

![Figure 2. The stress concentration factor ($K_t$) and the effective stress concentration factor ($K_t'$) for all types of plates.](image.png)

Transverse stresses and shear stress are shown in figure 3. The stresses are given for loading when tensile stress is 1 MPa. As seen from figure 3, transverse stresses are close to each other for the same geometry of plates, but shear stress for the VSCPs is always lower than it is for the UD plates. Since fiber trajectories are aligned in the direction of the maximum principal stress, in fact, shear stress is minimized. The difference in shear stress is especially noticeable between UD-4 and C-4 where shear stress decreases by 8.8 times using the transition from the UD-4 plate to the C-4 plate.

![Figure 3. Minimum transverse stress ($\sigma_{2min}$) (MPa), maximum transverse stress ($\sigma_{2max}$) (MPa) and shear stress ($\tau_{12}$) (MPa).](image.png)
Tensile stress values (MPa) are shown in figure 4, at which the failures of the matrix (MF) and fiber (FF) begin as well as the proportional limit (PL) being reached for the composite plates with different reinforcements. The results show that load with the initial fiber failure increases by about 4 times if the reinforcement structure is changed from UD fibers to curvilinear fibers for the plates. Since the failures are achieved at higher load for the VSCPs than for the UD plates, the VSCPs can be exploited with greater efficiency and safety. To understand how much the efficiency can be increased, the proportional limit is used. It has been established in tests [32, 33] that the number of the acoustic emission counts grows only after exceeding the proportional limit on the stress–displacement curves. When the load is higher than the proportionality limit, irreversible damage of the composite structures already begins. Thus, it is necessary to exploit composite structures up to the proportional limit to ensure their safe use. The proportional limits and the stress–displacement curves (for C-3 and UD-3) are shown in figures 4 and 5, respectively.

![Figure 4](image1.png)

**Figure 4.** Failure of the matrix (MF) and fiber (FF) as well as the proportional limit (PL) for different reinforcement structures.

As seen from figure 4, the proportional limit for the VSCPs is higher by about 3 times compared to the UD plates. The maximum values of the stress–displacement curves (ultimate loads) are given in table 4. Thus, the results show that the change in reinforcement structure significantly increases the efficiency of composites.

![Figure 5](image2.png)

**Figure 5.** The stress–displacement curves for C-3 and UD-3.
Table 4. Ultimate loads for all types of plates.

| Label | UD-1 | C-1 | UD-2 | C-2 | UD-3 | C-3 | UD-4 | C-4 |
|-------|------|-----|------|-----|------|-----|------|-----|
| Ultimate loads (MPa) | 569  | 559 | 827  | 1238| 1130 | 1509| 1535 | 1748|

5. Conclusions

Different composite structures with both UD fibers and curved fibers were modeled. A material property degradation method was used to model the progressive failure and calculate the proportional limits and the ultimate loads for all types of considered plates. As the results showed, the proportional limit of composites significantly increases with a transition from UD composite structures to curvilinear composite structures. Composites with curved fibers can be made by advanced manufacturing technologies where fiber orientation is controlled. Thus, composites can be used with even greater efficiency, which will lead to safer and more reliable application of composites in various industries.

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