CORRELATION BETWEEN THE ISOTROPIC ENERGY AND THE PEAK ENERGY AT ZERO FLUENCE FOR THE INDIVIDUAL PULSES OF GAMMA-RAY BURSTS: TOWARD A UNIVERSAL PHYSICAL CORRELATION FOR THE PROMPT EMISSION

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ABSTRACT

We find a strong correlation between the peak energy at zero fluence ($E_{\text{peak,0}}$) and the isotropic energy ($E_{\gamma,\text{iso}}$) of the 22 pulses of nine gamma-ray bursts (GRBs) detected by the Fermi satellite. The correlation holds for the individual pulses of each GRB, which shows the reality of the correlation. The derived correlation (Spearman correlation coefficient, $r$, which is 0.96) is much stronger compared to the correlations using $E_{\text{peak}}$ (in place of $E_{\text{peak,0}}$) determined from the time-integrated spectrum ($r=0.8$), the time-resolved spectrum without accounting for broad pulse structures ($r=0.37$), or the pulsewise spectrum ($r=0.89$). Though the improvement in the $E_{\text{peak}}$–$E_{\gamma,\text{iso}}$ relation (the Amati relation) for a pulsewise analysis is known earlier, this is the first time a parameter derived from a joint spectral and timing fit to the data is shown to improve the correlation. We suggest that $E_{\text{peak,0}}$, rather than $E_{\text{peak}}$, is intrinsic to a GRB pulse and a natural choice as the parameter in pulsewise correlation studies.

Key words: gamma-ray burst: general – methods: data analysis – methods: observational

1. INTRODUCTION

Gamma-ray bursts (GRBs) are the most luminous events in the universe, predominantly observed in the hard X-ray to gamma-ray energies. An individual GRB appears as a flash of gamma-ray event lasting for seconds and then continuously shifts toward the lower energy bands—all the way to radio wavelength (see, for example, Mészáros 2006). In the initial phase of high-energy emission, known as the prompt emission, it undergoes significant spectral evolution. Over the years, researchers have developed empirical models to describe the time-integrated spectrum, the light curve, and the spectral evolution. It has been shown that various model parameters correlate with the energy-related physical parameters of the full GRB. For example, the parameter peak energy of the $\nu F_{\nu}$ spectra ($E_{\text{peak}}$) correlates with the isotropic energy ($E_{\gamma,\text{iso}}$) (Amati et al. 2002), isotropic peak luminosity ($L_{\text{iso}}$) (Schaefer 2003; Yonetoku et al. 2004), and collimation-corrected energy ($E_{\gamma}$) (Ghirlanda et al. 2004). Similar correlations hold in the time domain, e.g., spectral delay ($\tau_{\text{lag}}$)–$L_{\text{iso}}$ (Norris et al. 2000), variability ($\nu$)–$L_{\text{iso}}$ (Fenimore & Ramirez-Ruiz 2000), and rise time ($\tau_{\text{rise}}$)–$L_{\text{iso}}$ (Schaefer 2007). The objective of such correlation studies is twofold: first, to use GRB as a standard candle of cosmology, alongside the commonly used standard candle, the supernova, and second, to understand the GRB physics itself. GRBs can be observed at a much higher redshift (as high as $z>6$) than supernova ($z\leq1.7$). With the advent of the Fermi satellite, we now have the record holder—GRB 090423 ($z=8.2$; see Ghirlanda et al. 2010). In spite of this tremendous advantage over supernovae, GRBs have one serious drawback. Unlike supernovae, whose mechanism is very well understood and supported by numerous observations, the physics of GRBs is not understood properly. Hence, it is interesting to investigate empirical models that can predict the correct energetics of GRBs. The predictions of the theoretical model should conform to those of a data-driven empirical model, and this whole process gives a strong constraint on the possible physics of GRBs.

Pulse analysis of GRB prompt emission has recently received considerable attention. It has the potential to unravel physical processes responsible for the observed correlations and help in standardizing the energy budget so that GRBs can be used as precise cosmological distance indicators (Hakkila et al. 2009). In the prompt emission phase, a GRB varies so greatly in time and energy domains that no two bursts have the same temporal and spectral characteristics (Norris et al. 1996). This situation simplifies if the whole GRB event is considered as an ensemble of temporally separated pulses of self-similar shapes (Nemiroff 2000), generated nearly simultaneously in a wide range of energy bands (Norris et al. 2005). Hence, each of the pulses can be modeled separately, and then the whole GRB event can be reproduced by shifting and adding these pulses. One of the strongest constraints pulse analysis gives is that the pulses of a GRB, despite having separate sets of parameters and different energetics, have the same redshift. Hence, each pulse can be used as a distance indicator and must conform with each other. It has been shown that all the correlations studied for the full GRB event hold well, sometimes even better, if the inherent pulse property of GRBs is taken into account. Krimm et al. (2009), for example, showed that the $E_{\text{peak}}$–$E_{\gamma,\text{iso}}$ correlation derived from pulsewise analysis is consistent with the time-integrated analysis. Hakkila et al. (2008) studied $\tau_{\text{lag}}$–$L_{\text{iso}}$ correlation and conclusively showed that the spectral lags are pulse properties rather than burst properties.

These kinds of correlations are, however, empirical in nature, and they may not have any physical significance (Band & Preece 2005; Butler et al. 2007, 2009). Moreover, they do not use the full information available in the sense that the spectral and timing parameters are considered independent of each other. In such correlation studies, the information of time evolution of the spectral parameters and the energy dependence of the timing parameters are lost. Hence, the derived spectral and timing parameters are average quantities. Liang & Kargatis (1996) showed that one of the spectral parameters, namely, $E_{\text{peak}}$, follows a time evolution law. Kocevski & Liang (2003) used this interdependence to study the evolution of $E_{\text{peak}}$ and found that $E_{\text{peak}}$ exponentially decreases with the fluence of an individual pulse in a GRB. Recently, Basak & Rao (2012) developed a new method for the complete empirical description of the individual pulses of GRB prompt emission, simultaneously in the time and
energy domains, based on well-established empirical formulae. The method conclusively shows that the two new parameters of the Liang & Kargatis (1996) model, namely, the peak energy at zero fluence ($E_{\text{peak},0}$) and the characteristic fluence ($\phi_0$), can be used to predict the timing parameters, namely, spectral delay and width. In other words, these two parameters are more intrinsic to the GRB pulses than the average spectral and timing parameters, and the other pulse characteristics can be derived from them. In this paper, we demonstrate that pulsewise analysis gives a better correlation than both the time-integrated analysis and time-resolved analysis, which do not take broad pulses into account. We also show that $E_{\text{peak},0}$ instead of $E_{\text{peak}}$ gives even better correlation and is the correct choice of parameter in correlation studies. The structure of the paper is as follows: In Section 2, we briefly describe the method, and in Section 3 we give the data analysis and results. Major conclusions are discussed in Section 4.

2. SIMULTANEOUS TIMING AND SPECTRAL DESCRIPTION OF A GRB PULSE

Pulse analysis of GRBs essentially involves extracting the constituent substructures based on various empirical time description of pulses (Norris et al. 1996; Nemiroff 2000; Hakikla et al. 2009; Hakikla & Preece 2011), e.g., the fast rise exponential decay model (Kocevski et al. 2003) and the Norris model (Norris et al. 2005). Though this method is useful in extracting isolated and slightly overlapping pulses, it cannot presently be used to extract “heavily overlapping” and low signal-to-noise pulses (Hakkila & Preece 2011). The energy spectrum of a GRB is popularly described by an empirical model given by Band et al. (1993). The same model can also be used for individual pulses. Choosing the Norris model for the time domain and the Band model for the energy domain, and employing the time evolution of the peak energy ($E_{\text{peak}}$) of the Band model, as proposed by Liang & Kargatis (1996), Basak & Rao (2012) developed a technique to determine the model parameters ($E_{\text{peak},0}$ and $\phi_0$).

Basak & Rao (2012) analyzed the Swift/BAT and Fermi/GBM data of the brightest GRB in the Fermi era—GRB 090618. They measured the global parameters of the individual pulses of this GRB, namely, the low-energy photon index ($\alpha$), the high-energy photon index ($\beta$) using the Band model (Band et al. 1993) for the time-integrated spectral data, and the characteristic timescales ($t_1$ and $t_2$) by fitting the energy-integrated light curve with the Norris model (Norris et al. 2005). These global parameters were then used to generate the XSPEC table model with the parameters $E_{\text{peak},0}$ and $\phi_0$ as variables. From spectral fitting in XSPEC, $E_{\text{peak},0}$ and $\phi_0$ were determined.

The data analysis for the global parameters (see Basak & Rao 2012) showed improvement due to the inclusion of the Swift/BAT along with the Fermi/GBM. The simultaneous spectral fitting, however, showed some systematic errors in the overlapping energy regions. Hence, they used only Swift/BAT data for the simultaneous timing and spectral description of the GRB pulses. The typical value of the peak energy ($E_{\text{peak}}$) of GRBs (and the individual pulses) is $\sim 300$ keV, and we expect still higher values of the parameter, $E_{\text{peak},0}$. The Swift/BAT energy range ($15$–$150$ keV; Barthelmy et al. 2005) is inadequate for accurate measurement of these parameters in many instances. Hence, the values derived by Basak & Rao (2012) show large uncertainties (typically $50$–$100$ keV). The Gamma-ray Burst Monitor (GBM) on board the Fermi satellite provides an unprecedented energy range ($\sim 8$ keV to $\sim 40$ MeV) with adequate sensitivity (Meegan 2009; Meegan et al. 2009) for constraining the values of $E_{\text{peak}}$ and $E_{\text{peak},0}$. Hence, in the present analysis, we have used Fermi/GBM data. We also have carefully chosen the time interval of each pulse, avoiding contamination from the other pulses. For example, in GRB 090618 (see Table 1), we have chosen time intervals $61$–$76$, $76$–$95$, and $106$–$126$ s for the pulses 2, 3, and 4, respectively (compared with the time divisions of Basak & Rao 2012: $50$–$77$, $77$–$100$, and $100$–$130$ s, respectively). We have included the precursor burst of this GRB (Pulse 1) in the present analysis.

3. DATA ANALYSIS AND RESULTS

In our analysis, we have chosen the set of GRBs that was considered for time-resolved spectral analysis by Ghirlanda et al. (2010). This set contains 12 long GRBs with known redshift, detected by Fermi/GBM through the end of 2009 July. Among these, three GRBs (GRB 080905, GRB 080928, and GRB 081007) are very weak bursts and could be fit only with a single power law with unconstrained peak energy. Hence, we have used the data for the remaining nine GRBs and their individual pulses (a total of 22 pulses). Table 1 contains the full list of our sample with the name, measured redshift, and pulses of GRBs in the first, second, and third columns, respectively.

In a given GRB, we select those pulses that have broader width compared to the rapid spikes in light curve. While selecting such pulses, we carefully avoid those pulses and the portions of the pulses that have too much overlap with others. Fishman & Meegan (1995), for example, have discussed various categories of light curves: (1) single pulse; (2) smooth, either single or multiple, well-defined peaks; (3) distinct, well-separated episodes of emission; and (4) very erratic, chaotic, and spiky bursts. More than one such category of pulses can show up in a single GRB. In such cases, the fourth category gives rise to many overlapping pulses in some parts, while the other parts of the same GRB might be dominated by the second or third category of pulses. The temporal regions populated by the fourth category do not allow a unique measurement of model parameters of a pulse, if each pulse has to have an independent set of parameters. Hence, we avoid such regions and take only clear portions of a burst.

The broad pulses selected in our analysis are listed in Table 1 (also see Ghirlanda et al. 2010; Figure 2), with the appropriate start time ($t_1$) and stop time ($t_2$) shown in Columns 4 and 5. Ghirlanda et al. (2010) conducted a time-resolved analysis, whereas we have done a pulsewise analysis. In the former case, the GRB light curve is arbitrarily divided into a large number of bins, and the evolution of spectra is examined. Krimm et al. (2009) showed that the correlation between the peak energy and the isotropic energy of broad GRB pulses improves from that of the whole GRBs. In our analysis, we follow the same approach and select the time cut according to the broad pulse structure. Some portions of the light curves are neglected, because they are either dominated by the fourth category of pulses or have count rates too low to accurately determine model parameters. GRB 080810 and GRB 080916C have low count rates after $30$ s and $55$ s, respectively. GRB 090323 contains multiple overlapping spikes between the $30$–$59$ s and $75$–$135$ s regions which do not contain any broad pulse structures (also count rate is low in $75$–$135$ s region). GRB 090328 has two overlapping spikes between $20$ and $26$ s (taken as $20$–$24$ s and $24$–$26$ s in Ghirlanda et al. 2010) and is hence neglected.

We essentially use the method described in Basak & Rao (2012). The global parameters ($\alpha$, $\beta$, $t_1$, and $t_2$) are determined.
to generate a three-dimensional pulse model for a set of $E_{\text{peak},0}$ and $\phi_0$. The time-integrated spectra for these sets of values give a two-parameter XSPEC table model. We perform $\chi^2$ minimization of the spectral fit of the data with this model to determine the best-fit values of the model parameters and their nominal 90% confidence level errors ($\Delta \chi^2 = 2.7$). Also, the normalization, which is a free parameter in the model, is determined by XSPEC.

The results of our spectral analysis are shown in Table 1. The observer frame peak energy ($E_{\text{peak}}$) calculated from the Band model, the zero fluence peak energy ($E_{\text{peak},0}$), and the measured nominal 90% confidence level errors for each are shown in Columns 6 and 8. In order to compare the improvement in correlation of these parameters with $E_{\gamma,\text{iso}}$ (Column 10 of Table 1), we perform a linear fit of the form $\log(y) = K + \delta \log(x)$ using the technique of joint likelihood for the coefficients $K$ and $\delta$ and the intrinsic scatter ($\sigma_{\text{int}}$) (D’Agostini 2005; Wang et al. 2011). Here, $y = E_{\text{peak}}$ (full and pulsewise study) and $E_{\text{peak},0}$ are in units of 100 keV, and $x = E_{\gamma,\text{iso}}$ is in units of $10^{52}$ erg.

In Figure 1, we have plotted the source peak energy ($E_{\text{peak}}$) as a function of $E_{\gamma,\text{iso}}$ for the 22 individual pulses of the nine GRBs (stars). For comparison, in the same figure we have plotted the time-integrated values for the full GRB (filled boxes), based on the data given in Ghirlanda et al. (2010). For illustrative purposes, a scatter plot of the values for time-resolved spectral analysis is also given in the figure (small circles). Note that $L_{\text{iso}}$ is the more appropriate parameter for a time-resolved spectral analysis, but to compare with our results we have converted the flux given by Ghirlanda et al. (2010) to $E_{\gamma,\text{iso}}$ using the multiplication factor of the time bin. The straight line shown in the figure is the log($E_{\text{peak}}$) = $K + \delta \log(E_{\gamma,\text{iso}}$) fit to the pulsewise data using the joint likelihood method. The time-integrated data is fit by the same technique. The Spearman correlation coefficient ($r$), the probability that the correlation occurred by chance ($P$), the parameters for the linear fit, and the intrinsic

![Figure 1](image-url)

Figure 1. Time-integrated, time-resolved (not accounting for broad pulse structure), and pulsewise $E_{\text{peak}}$ as a function of the isotropic energy ($E_{\gamma,\text{iso}}$) for the nine Fermi GRBs with measured redshift. The large filled boxes denote the time-integrated points while small circles denote the time-resolved points (GRBs are chosen from Ghirlanda et al. 2010). The stars denote the pulsewise $E_{\text{peak}}$ as determined by the Band model (present work).
shows a poor correlation (of 4 scatter in the data are given in Table 2. For comparison, the
values reported by Ghirlanda et al. (2010) for the time-integrated
values for 10 GRBs are also given in the table. A comparison of
correlation and $P$ for the time-integrated ($0.80$ and $9.60 \times 10^{-3}$,
respectively) and pulsewise ($0.89$ and $2.95 \times 10^{-8}$, respectively)
analyses shows that there is an improvement in the correlation
and the reality of the correlation (i.e., lower $P$) in the latter case,
favoring pulsewise analysis. Intrinsic scatter of the data ($\sigma_{\text{int}}$) per
point is also reduced ($0.225/9$ to 0.244/22). This improvement
in the correlation of $E_{\text{peak}}-E_{\gamma,\text{iso}}$ for the pulsewise analysis is
known in earlier works. Krimm et al. (2009), for example, used
a sample of Swift–Suzaku GRBs, which gave the Spearman
correlation as 0.74 with a chance probability of $7.58 \times 10^{-5}$ for
a sample of 22 GRBs. This correlation improves, for a set of
59 pulses of these GRBs, to 0.80 with a chance probability of
$5.32 \times 10^{-14}$.

The time-resolved study not accounting for broad pulses
shows a very poor correlation ($r = 0.37$; see Table 2). This is
expected because, in such time divisions, broad pulse structure
is not considered. Even if the spikes are considered as pulses, they
overlap in any such small time division and are hence unusable
to determine $E_{\text{peak}}$, which can be uniquely associated with a
pulse. Time divisions taken for a broad pulse, on the other hand,
have the facility to study the evolution of $E_{\text{peak}}$ and uniquely determine the $E_{\text{peak,0}}$ in that pulse. In our analysis, we ignore
these rapidly varying spikes and concentrate on the broad pulse
structures.

In Figure 2, we show $E_{\text{peak,0}}$ as a function of $E_{\gamma,\text{iso}}$ along
with a straight line fit. A comparison of Figure 1 with Figure 2
immediately shows that the new parameter $E_{\text{peak,0}}$ has a better
relation in the $E_{\text{peak,0}}-E_{\gamma,\text{iso}}$ plane compared to the pulsewise
$E_{\text{peak}}-E_{\gamma,\text{iso}}$ analysis, which already showed improvement in
terms of correlation, $P$, and $\sigma_{\text{int}}$ compared to the time-integrated
analysis. All these values for the $E_{\text{peak,0}}-E_{\gamma,\text{iso}}$ fitting are shown
in the last row of Table 2. The correlation coefficient ($r = 0.96$)
is significantly higher than that of $E_{\text{peak}}-E_{\gamma,\text{iso}}$, whether time-
integrated (0.80), time-resolved (0.37), or pulsewise (0.89).
The chance probability also decreases. As a comparison with
results found in previous works, we quote some of the
results presented by Krimm et al. (2009): The original Amati
(Amati 2006) catalog of 39 bursts has $r = 0.87$ with a chance
probability of $4.72 \times 10^{-13}$. A sample of all the long bursts (91)
shows a poor correlation ($r = 0.76$) with a chance probability
of $4.72 \times 10^{-18}$. Krimm et al.’s (2009) sample of 59 GRB
pulses has a correlation of 0.80 with a chance probability
of $5.32 \times 10^{-14}$. Our analysis shows a clear improvement in

the correlations, whether time-integrated, time-resolved (not
accounting for broad pulse), or pulsewise analysis.

It has long been debated whether a correlation such as
$E_{\text{peak}}-E_{\gamma,\text{iso}}$ might be a result of observational selection effects.
In particular, the $E_{\text{peak}}$ values are stretched due to the multi-
plication factor ($1 + z$), making the correlation appear stronger
(Nakar & Piran 2005; Band & Preece 2005; Butler et al. 2007,
2009; Shahmoradi & Nemiroff 2009, 2011). However, there are other
researchers who argue in favor of the reality of these cor-
relations (Ghirlanda et al. 2008; Nava et al. 2008; Krimm et al.
2009; Amati et al. 2009; Ghirlanda et al. 2010). If these cor-
relations hold within a GRB as for a sample of many GRBs,
then we can conclude that the correlations are indeed physical.
In Figure 2, we have marked the data points for GRB 090424
(stars) and GRB 090618 (filled triangle), for which we analyze
the highest number (four) of pulses. The data show that the
pulses of the same GRB follow the same $E_{\text{peak,0}}-E_{\gamma,\text{iso}}$ correla-
tion. The fact that this correlation is tighter than the $E_{\text{peak}}-E_{\gamma,\text{iso}}$
iteration signifies that $E_{\text{peak,0}}$ is more intrinsic of a GRB pulse
than $E_{\text{peak}}$; hence, this parameter should be used for such cor-
relation studies. It is interesting to note that the precursor of GRB 0901618 also follows the same correlation. GRB 090328
is also marked (filled squares). First, two pulses of this GRB
(with higher $E_{\text{peak,0}}$ values) show particularly high deviation.

| Method | $r$ | $P$ | $K$ | $\delta$ | $\sigma_{\text{int}}$ | $\chi^2_\text{red}$ (dof) |
|--------|----|----|----|--------|----------------|-----------------|
| I      | 0.80 | 0.0096 | 0.166 $\pm$ 0.080 | 0.473 $\pm$ 0.048 | 0.225 $\pm$ 0.067 | 0.64 (7) |
| II     | ... | 0.004 | 0.162 $\pm$ 0.085 | 0.476 $\pm$ 0.079 | ... | ... |
| III    | 0.37 | 0.0095 | ... | ... | ... | ... |
| IV     | 0.89 | 2.95 $\times 10^{-8}$ | 0.289 $\pm$ 0.055 | 0.516 $\pm$ 0.049 | 0.244 $\pm$ 0.048 | 0.56 (20) |
| V      | 0.96 | 1.60 $\times 10^{-12}$ | 0.640 $\pm$ 0.050 | 0.555 $\pm$ 0.050 | 0.291 $\pm$ 0.039 | 1.04 (20) |

Notes. The methods are— I: time-integrated study (present work) for nine GRBs; II: time-integrated study for 10 GRBs (quoted from
Ghirlanda et al. 2010); III: time-resolved study not accounting for broad pulse structures (calculated from Ghirlanda et al. 2010),
IV: pulsewise study (present work); V: pulsewise $E_{\text{peak,0}}-E_{\gamma,\text{iso}}$ correlation. The Spearman correlation coefficient ($r$), chance
probability ($P$), and the parameters of linear fit ($K$, $\delta$, and $\sigma_{\text{int}}$) are reported here.
An examination of the $E_{\text{peak}} - E_{\gamma, \text{iso}}$ values of these two pulses (Figure 1) also shows a much higher deviation. Interestingly, the third pulse of this GRB is consistent with the trend. It was already known that the first pulses of GRBs tend to be harder and thus deviate from the correlation (Krimm et al. 2009; Ghirlanda et al. 2010). However, this deviation is much smaller compared to the pulsewise $E_{\text{peak}}$ in the $E_{\text{peak}} - E_{\gamma, \text{iso}}$ correlation.

4. DISCUSSION AND CONCLUSIONS

The strong correlation between the luminosity and the parameter obtained from a joint spectral and timing fit for the individual pulses of GRBs clearly indicates that the basic radiation/emission process is similar in diverse GRBs, and the dispersion in the other parameters such as Lorentz factor, beaming angle, etc., are quite minimal. An examination of the data in Table 1 and Figure 2 shows that the dispersion in the correlation for different pulses of a GRB is of the same magnitude as the dispersion between GRBs. Hence, it is worthwhile to investigate the radiation/emission process in operation by making a direct model fit to the data. This would help us in finding the fundamental parameter/variable responsible for the correlation. If such an exercise brings down the dispersion in the relation, it may be possible to use GRBs as a distance indicator for cosmological purposes. It is also interesting to note that the parameters derived for the “precursor” in GRB 090618 are consistent with the global correlation.

Asano & Mészáros (2011) performed simulations of the spectral and temporal evolution of GRBs using internal dissipation models and concluded that the models reproduce the Band spectra and the generic time evolution. Dado et al. (2007) used the master formula based on the “cannonball” model and explained the various correlations observed for the prompt emission of the GRBs. The present work demonstrates that it is possible to fit the data with a comprehensive set of formulae describing the temporal and spectral evolution of the bursts. Hence, it should be possible to directly fit the data with the model predictions and derive the fundamental quantities responsible for the universal correlation. Any model contains basic physical assumptions along with other details. A direct fit to the data taking individual pulses should segregate the basics from the details. For example, in the “cannonball” model (Dado et al. 2007), the Lorentz factor and the viewing angle of a GRB determine most of the properties of a GRB pulse. Since the viewing angle would be the same for the different pulses of a given GRB, a direct fit to the spectral and temporal profile can have the additional constraint of the constancy of this parameter.

Further improvements for the method described here (see also Basak & Rao 2012) could be made by iteratively including color corrections to the light curve and also re-confirming the $E_{\text{peak}}$ evolution formula. It would also be interesting to repeat this exercise for short GRBs and X-ray flares, which would give further clues to the emission mechanisms responsible for the correlation presented here.

The choice of isotropic energy ($E_{\gamma, \text{iso}}$) over the other two physical quantities, namely, isotropic peak luminosity ($L_{\text{iso}}$) and collimation-corrected energy ($E_{\gamma}$), can be justified as follows: Peak luminosity is measured based on the assumption that the spectral shape at the peak is the same as the average shape, which is not very physical (Wang et al. 2011). On the other hand, $E_{\gamma}$ could have been a better choice, as the collimation effect is corrected. In practice, the beaming angle ($\theta_j$) is very ill determined (e.g., see the weak constraints in the measured $\theta_j$ by Goldstein et al. 2011). It is evident from Table 2 that the intrinsic scatter does not improve in the new analysis. This might happen due to some inherent assumptions of our model, particularly the hard-to-soft evolution and/or the uncertainty in the measured redshift. A close inspection of Figure 2 reveals that the pulses of the same GRB are scattered on the same side of the correlation line. This might be due to the fact that the correct energy budget is the collimation corrected energy and not the isotropic energy. Analysis of 14 pulses of six GRBs (for which $\theta_j$ values could be collected) shows a correlation of 0.91. We believe that this correlation might improve with increasing accuracy of the measured $\theta_j$.

To summarize, we have used the method of joint timing and spectral description of GRB pulses and have found that $E_{\text{peak}}$ is a fundamental parameter in the pulse description.

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ERRATUM: “CORRELATION BETWEEN THE ISOTROPIC ENERGY AND THE PEAK ENERGY AT ZERO FLUENCE FOR THE INDIVIDUAL PULSES OF GAMMA-RAY BURSTS: TOWARD A UNIVERSAL PHYSICAL CORRELATION FOR THE PROMPT EMISSION” (2012, ApJ, 749, 132)

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In the published version of the paper, the values quoted for correlation study correspond to the Pearson correlation ($r$). In the paper, however, it was erroneously mentioned as the Spearman correlation.

Please read “Pearson” instead of “Spearman” in the following places.
1. Page 1, Abstract, line 3;
2. Page 3, Column 2, line 5;
3. Page 4, Table 2 (Notes), line 3.

The value reported by Krimm et al. (2009) is Spearman rank correlation ($\rho$).

The advantage of the Pearson correlation is that it calculates the correlation coefficient between the variables, $x$ and $y$, with the exact values of the variables. The disadvantage is that the derived coefficient is calculated without invoking the knowledge of the individual distribution of $x$ and $y$. Using the Spearman rank correlation, one can overcome this difficulty, as it assigns ranks to the variables and hence the distribution becomes uniform over the number of data points. The disadvantage of this description is that it does not take the actual values of the variables into account, but uses the ranks assigned to them. Hence, both these correlations have some advantages over the other. However, if we demand that the $x$ and $y$ values, which are the isotropic energy and peak energy, are equally likely to occur in the whole range, then the Pearson correlation may be preferred over the Spearman correlation.

In this erratum, we include the Spearman rank correlation coefficient ($\rho$). The following is the new version of Table 2 of the published paper (Basak & Rao 2012). If the Spearman correlation is the better description, then the method IV and V are comparable (see Table 2). However, pulsewise study is always preferred over a time-integrated study or time-resolved study for Amati correlation.

### Table 2

Statistical Analysis for the Correlations of $E_{\text{peak}}$ (I to IV) or $E_{\text{peak},0}$ (V) with $E_{\gamma,\text{iso}}$ and Parameters for the Linear Fit for the Nine Fermi GRBs and Their Individual Pulses

| Method | $r$ ($\rho$) | $P$ ($P_{\rho}$) | $K$ | $\delta$ | $\sigma_{\text{int}}$ | $\chi^2_{\text{red}}$(dof) |
|--------|-------------|-----------------|-----|---------|-----------------|---------------------------|
| I      | 0.80        | 0.0096          | 0.166 ± 0.080 | 0.473 ± 0.048 | 0.225 ± 0.067 | 0.64 (7) |
|        | (0.75)      | (0.0199)        | ...          | ...          | ...             | ...                       |
| II     | 0.37        | 0.0095          | 0.162 ± 0.085 | 0.476 ± 0.079 | ...             | 0.47 (8) |
|        | (0.486)     | (0.0003)        | ...          | ...          | ...             | ...                       |
| IV     | 0.89        | $2.95 \times 10^{-8}$ | 0.289 ± 0.055 | 0.516 ± 0.049 | 0.244 ± 0.048 | 0.56 (20) |
|        | (0.88)      | (4.57 \times 10^{-8}) | ...          | ...          | ...             | ...                       |
| V      | 0.96        | $1.60 \times 10^{-12}$ | 0.640 ± 0.050 | 0.555 ± 0.050 | 0.291 ± 0.039 | 1.04 (20) |
|        | (0.87)      | (1.43 \times 10^{-7}) | ...          | ...          | ...             | ...                       |

Notes. The methods are as follows. (I) Time-integrated study (present work) for nine GRBs. (II) Time-integrated study for 10 GRBs (quoted from Ghirlanda et al. 2010). (III) Time-resolved study not accounting for broad pulse structures (calculated from Ghirlanda et al. 2010). (IV) Pulsewise study (present work). (V) Pulsewise $E_{\text{peak},0}$–$E_{\gamma,\text{iso}}$ correlation. The Pearson correlation coefficient ($r$), the Spearman rank correlation coefficient ($\rho$), chance probability ($P$) of both these correlations, and the parameters of linear fit ($K$, $\delta$, and $\sigma_{\text{int}}$) are reported here.

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