New Aspects on a Seesaw Mass Matrix Model

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Abstract

Recent development of the universal seesaw mass matrix model is reviewed. The model was proposed in order to explain why quark and lepton masses are so small compared with the electroweak scale \( \Lambda_L = \langle \phi_L^0 \rangle = 174 \) GeV. However, the recently observed top-quark mass \( m_t \approx 180 \) GeV seems to make an objection against the seesaw mass picture. For this problem, it has recently pointed out that the seesaw mass matrix model is rather favorable to the fact \( m_t \sim \Lambda_L \) if we consider the model with \( \det M_F = 0 \) for up-quark sector, where \( M_F \) is a \( 3 \times 3 \) mass matrix of hypothetical heavy fermions \( F \). The model can give a natural explanation why only top-quark acquire the mass of the order of \( \Lambda_L \). The model with \( \det M_U = 0 \) offers abundant new physics to us (e.g., the fourth up-quark \( t' \), FCNC, and so on).

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1. Introduction

One of the most challenging problems in the particle physics is to give a unified understanding of quark and lepton masses and mixings, of course, including the neutrino sector. For this purpose, many models have been proposed [1].

In such a model-building, our interests are as follows: Why is $m_t$ so extremely larger than $m_b$ in the third family in spite of $m_u \sim m_d$ in the first family? Why is only $m_t$ of the order of $\Lambda_L$ (electroweak scale)? Related to these topics, the recent development of the universal seesaw mass matrix model [2] and its special example are reviewed.

As well-known, a would-be seesaw mass matrix for $(f, F)$ is expressed as

$$M = \begin{pmatrix} 0 & m_L \\ m_R & M_F \end{pmatrix} = m_0 \begin{pmatrix} 0 & Z_L \\ \kappa Z_R & \lambda Y_f \end{pmatrix}, \quad (1.1)$$

where $f = u, d, \nu, e$ are the conventional quarks and leptons, $F = U, D, N, E$ are hypothetical heavy fermions, and they belong to $f_L = (2, 1), f_R = (1, 2), F_L = (1, 1)$ and $F_R = (1, 1)$ of SU(2)$_L \times$SU(2)$_R$. The matrices $Z_L, Z_R$ and $Y_f$ are of the order one. For the case $\lambda \gg \kappa \gg 1$, the mass matrix (1.1) leads to the well-known seesaw expression

$$M_f \simeq m_L M_F^{-1} m_R. \quad (1.2)$$

The mechanism was first proposed [3] in order to answer the question of why neutrino masses are so invisibly small. Then, in order to understand that the observed quark and lepton masses are considerably smaller than the electroweak scale, the mechanism was applied to the quarks [2].

However, the observation of the top quark of 1994 [4] aroused a question: Can the observed fact $m_t \simeq 180$ GeV $\sim \Lambda_L = O(m_L)$ be accommodated to the universal seesaw mass matrix scenario? Because $m_t \sim O(m_L)$ means $M_F^{-1} m_R \sim O(1)$.

For this question, a recent study gives the answer "Yes": Yes, we can do [5,6] by putting an additional constraint

$$\det M_F = 0. \quad (1.3)$$

on the up-quark sector ($F = U$). In the next section, we will review the mass generation scenario on the basis of the universal seesaw mass matrix model with the constraint (1.3).

In Sec. 3, we discuss an abnormal structure of the quark mixing matrices and flavor changing neutral currents (FCNC) effects. In Sec. 4, we review a model with specific forms of the matrices $Z_L, Z_R$ and $Y_f$, the so-called "democratic seesaw mass matrix model" [5]. In Sec. 5, we give a short review of an application to the neutrino mass matrix. Finally,
Sec. 6 is devoted to the concluding remarks.

2. Energy scales and fermion masses

For convenience, we take the diagonal basis of the matrix $M_F$. Then, the condition (1.3) means that the heavy fermion mass matrix $M_F$ in the up-quark sector is given by

$$M_U = \lambda m_0 \begin{pmatrix} O(1) & 0 & 0 \\ 0 & O(1) & 0 \\ 0 & 0 & 0 \end{pmatrix},$$

(2.1)

although the other heavy fermion mass matrices $M_F$ ($F \neq U$) are given by

$$M_F = \lambda m_0 \begin{pmatrix} O(1) & 0 & 0 \\ 0 & O(1) & 0 \\ 0 & 0 & O(1) \end{pmatrix}, \quad (F \neq U).$$

(2.2)

Note that for the third up-quark the seesaw mechanism does not work (see Fig. 1).

\[
\text{det}M_F \neq 0 \implies \text{Seesaw Mass} \quad \quad \text{det}M_F = 0 \implies \text{Non-Seesaw Mass}
\]

Therefore, the mass generation at each energy scale is as follows. First, at the energy scale $\mu = \Lambda_S$, the heavy fermions $F$, except for $U_3$, acquire the masses of the order of $\Lambda_S$. Second, at the energy scale $\mu = \Lambda_L$, the SU(2)$_R$ symmetry is broken, and the fermion $u_{R3}$ generates a mass term of the order of $\Lambda_L$ by pairing with $U_{L3}$. Finally, at $\mu = \Lambda_L$, the SU(2)$_L$ symmetry is broken, and the fermion $u_{L3}$ generates a mass term of the order $\Lambda_L$.
by pairing with \( U_{R3} \). The other fermions \( f \) acquire the well-known seesaw masses (1.2).

The scenario is summarized in Table 1.

| Energy scale | d- & e-sectors | u-sector (\( i \neq 3 \)) |
|--------------|----------------|----------------------------|
| At \( \mu = \Lambda_S \sim \lambda m_0 \) | \( m(F_L, F_R) \sim \Lambda_S \) | \( m(U_{Li}, U_{Ri}) \sim \Lambda_S \) |
| At \( \mu = \Lambda_R \sim \kappa m_0 \) | \( m(u_{R3}, U_{L3}) \sim \Lambda_R \) | |
| At \( \mu = \Lambda_L \sim m_0 \) | \( m(f_L, f_R) \sim \frac{\Lambda_L \Lambda_R}{\Lambda_S} \) | \( m(u_{L3}, U_{R3}) \sim \Lambda_L \) |

Thus, we can understand why only top quark \( t \) acquires the mass \( m_t \sim O(m_L) \).

Next, we discuss the neutrino mass generation. At present, we have two scenarios.

One (Scenario A) is a trivial extension of the present model: we introduce a further large energy scale \( \Lambda_{SS} \) in addition to \( \Lambda_S \), and we assume that \( M_F \sim \Lambda_S \) (\( F = U, D, E \)), while \( M_N \sim \Lambda_{SS} \) (\( \Lambda_{SS} \gg \Lambda_S \)).

Another scenario (Scenario B) is more attractive because we does not introduce an additional energy scale. The neutral heavy leptons are singlets of \( SU(2)_L \times SU(2)_R \) and they do not have \( U(1) \)-charge. Therefore, it is likely that they acquire Majorana masses \( M_M \) together with the Dirac masses \( M_D \equiv M_N \) at \( \mu = \Lambda_S \). Then, the conventional light neutrino masses \( m_\nu \) are given with the order of

\[
m_\nu \sim \frac{\Lambda_L^2}{\Lambda_S^2} = \frac{1}{\kappa} \frac{\Lambda_L \Lambda_R}{\Lambda_S}.
\]

In order to explain the smallness of \( m_\nu \), the model [7,8] requires that the scale \( \Lambda_R \) must be extremely larger than \( \Lambda_L \).
On the other hand, the scenario A allows a case with a lower value of $\Lambda_R$. Then, we may expect abundant new physics effects as we discuss later.

The neutrino mass generation scenarios are summarized in Table 2.

### 3. Abnormal structure of $U_R^u$ and FCNC

The most excited features of the present model is an abnormal structure of the right-handed fermion mixing matrix $U_R^u$ [9].

For the down-quark sector, where the seesaw expression (1.2) is valid, the mixing matrices $U_L^d$ and $U_R^d$ are given by

$$U_L^d = \begin{pmatrix} A_d & \frac{1}{\lambda} C_d \\ \frac{1}{\lambda} C'_d & B_d \end{pmatrix}, \quad U_R^d \approx \begin{pmatrix} A_d^* & \frac{\kappa}{\lambda} C'_d \\ \frac{\kappa}{\lambda} C''_d & B_d \end{pmatrix},$$  \hspace{1cm} (3.1)$$

where $A, B, C \sim O(1)$. However, in contrast with the down-quark sector, for the up-quark sector, where the seesaw expression is not valid any more, the mixing matrices $U_L^u$ and $U_R^u$ are given by

$$U_L^u = \begin{pmatrix} * & * & * & \sim \frac{1}{\lambda} & \sim \frac{1}{\lambda} & \sim \frac{1}{\lambda} \\ * & * & * & \sim \frac{1}{\lambda} & \sim \frac{1}{\lambda} & \sim \frac{1}{\lambda} \\ * & * & * & \sim \frac{1}{\lambda} & \sim \frac{1}{\lambda} & \sim \frac{1}{\lambda} \\ \sim \frac{\kappa}{\lambda} & \sim \frac{\kappa}{\lambda} & \sim \frac{\kappa}{\lambda} & * & * & * \\ \sim \frac{\kappa}{\lambda} & \sim \frac{\kappa}{\lambda} & \sim \frac{\kappa}{\lambda} & * & * & * \\ \sim \frac{\kappa}{\lambda} & \sim \frac{\kappa}{\lambda} & \sim \frac{\kappa}{\lambda} & * & * & * \end{pmatrix},$$  \hspace{1cm} (3.2)$$

$$U_R^u = \begin{pmatrix} * & * & * & \sim \frac{\kappa}{\lambda} & \sim \frac{\kappa}{\lambda} & \sim \frac{\kappa}{\lambda} \\ * & * & * & \sim \frac{\kappa}{\lambda} & \sim \frac{\kappa}{\lambda} & \sim \frac{\kappa}{\lambda} \\ \sim \frac{\kappa}{\lambda} & \sim \frac{\kappa}{\lambda} & \sim \frac{\kappa}{\lambda} & * & * & * \\ \sim \frac{\kappa}{\lambda} & \sim \frac{\kappa}{\lambda} & \sim \frac{\kappa}{\lambda} & * & * & * \\ \sim \frac{\kappa}{\lambda} & \sim \frac{\kappa}{\lambda} & \sim \frac{\kappa}{\lambda} & * & * & * \end{pmatrix},$$  \hspace{1cm} (3.3)$$

where the symbol * denotes numerical factors of $O(1)$. Note that the right-handed up-quark mixing matrix $U_R^u$ has a peculiar structure as if the third and fourth rows of $U_R^u$ are exchanged each other in contrast to $U_L^u$.

This is understood from the following expression of the Hermitian matrixes $H_L$ and $H_R$: On the diagonal basis of $M_F$, the Hermitian matrices for the up-quark sector are
given by

\[
H_L^u = MM^\dagger = m_0^2 \begin{pmatrix}
Z_L Z_L^\dagger & \lambda Z_L Y_u^\dagger \\
\lambda Y_u Z_L^\dagger & \lambda^2 Y_u Y_u^\dagger + \kappa^2 Z_R Z_R^\dagger
\end{pmatrix}
\]

\[
= m_0^2 \begin{pmatrix}
0 & \lambda \lambda \\
0 & \lambda^2 \lambda \\
\lambda \lambda & \kappa^2 \kappa \\
\lambda^2 \lambda & \kappa^2 \kappa^2
\end{pmatrix}
\]

\[
H_R^u = M^\dagger M = m_0^2 \begin{pmatrix}
\kappa^2 Z_R Z_R^\dagger & \kappa \lambda Z_R^\dagger Y_u \\
\kappa \lambda Y_u Z_R^\dagger & \lambda^2 Y_u Y_u^\dagger + \kappa^2 Z_L Z_L^\dagger
\end{pmatrix}
\]

\[
= m_0^2 \begin{pmatrix}
\kappa^2 \kappa^2 \kappa^2 & 0 & 0 & \kappa \kappa \kappa \\
0 & \kappa^2 \kappa^2 \kappa^2 & 0 & \kappa \kappa \kappa \\
0 & 0 & 0 & \kappa \kappa \kappa \\
\kappa \kappa \kappa & \kappa \kappa \kappa & \kappa \kappa \kappa & \kappa \kappa \kappa
\end{pmatrix}
\]

(3.4)

That is, in the present model, the roles of \(u_{3R}\) and \(U_{1R}\) are exchanged each other in \(H_R^u\).

This means that the mass-partners are given by

\[
\begin{align*}
u \simeq u_1 &= (u_{1L}, u_{1R}), & d \simeq d_1 &= (d_{1L}, d_{1R}), \\
c \simeq u_2 &= (u_{2L}, u_{2R}), & s \simeq d_2 &= (d_{2L}, d_{2R}), \\
t \simeq u_3 &= (u_{3L}, U_{1R}), & b \simeq d_3 &= (d_{3L}, d_{3R}), \\
t' \simeq u_4 &= (U_{1L}, u_{3R}), & D \simeq d_4 &= (D_{1L}, D_{1R}), \\
C \simeq u_5 &= (U_{2L}, U_{2R}), & S \simeq d_5 &= (D_{2L}, D_{2R}), \\
T \simeq u_6 &= (U_{3L}, U_{3R}), & B \simeq d_6 &= (D_{3L}, D_{3R}),
\end{align*}
\]

(3.5)

where, for convenience, the numbering of the heavy up-quarks \(U\) has been changed from the definition based on (2.1).

As seen in (3.5), for a model with a low value of \(\Lambda_R\) (for example, \(\kappa \sim 10\)), we may expect [9] a single production of \(t'\) with \(m_{t'} \simeq \kappa m_t \sim\) a few TeV, through the exchange of \(W_R: d + u \to t' + d\), i.e., \(p + p \to t' + \bar{X}\) at LHC.

In the present model, the FCNC effects induced by the abnormal structure of the mixing matrix appear. The magnitudes are proportional to the factor

\[
\xi^f = U_{fF} U_{fF}^\dagger,
\]

(3.6)

where

\[
U = \begin{pmatrix}
U_{ff} & U_{fF} \\
U_{Ff} & U_{FF}
\end{pmatrix}.
\]

(3.7)

We can obtain sizable values of \(|(\xi^u_R)_{ic}|\) and \(|(\xi^u_R)_{iu}|\), although the other factors are invisibly small, e.g., \(|(\xi^u_R)_{ij}| \sim (\kappa/\lambda)^2\), \(|(\xi^u_R)_{ij}| \sim (1/\lambda)^2\). Therefore, if \(\kappa \sim 10\), the FCNC effects
appear visibly in the modes related to top-quark. Then, for example, we may expect the following single-top-production: $e^- + p \rightarrow e^- + t + X$ at HERA, $e^- + e^+ \rightarrow t + \tau$ at JLC, and so on.

The numerical results for a model with a specific matrix form can be found in Ref.[9].

4. Democratic seesaw mass matrix model

So far, we have not assumed explicit structures of the matrices $Z_L$, $Z_R$ and $Y_f$. Here, in order to give a realistic numerical example, we put the following working hypotheses [5]:

(i) The matrices $Z_L$ and $Z_R$, which are universal for quarks and leptons, have the same structure:

$$Z_L = Z_R \equiv Z = \text{diag}(z_1, z_2, z_3), \quad (4.1)$$

with $z_1^2 + z_2^2 + z_3^2 = 1$, where, for convenience, we have taken a basis on which the matrix $Z$ is diagonal.

(ii) The matrices $Y_f$, which have structures dependent on the fermion sector $f = u, d, \nu, e$, take a simple form $[(\text{unit matrix})+(\text{a rank one matrix})]$:

$$Y_f = 1 + 3b_fX. \quad (4.2)$$

(iii) The rank one matrix is given by a democratic form

$$X = \frac{1}{3} \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}, \quad (4.3)$$

on the family-basis where the matrix $Z$ is diagonal.

(iv) In order to fix the parameters $z_i$, we tentatively take $b_e = 0$ for the charged lepton sector, so that the parameters $z_i$ are given by

$$\frac{z_1}{\sqrt{m_\text{e}}} = \frac{z_2}{\sqrt{m_\mu}} = \frac{z_3}{\sqrt{m_\tau}} = \frac{1}{\sqrt{m_\text{e} + m_\mu + m_\tau}}. \quad (4.4)$$

The mass spectra are essentially characterized by the parameter $b_f$. The fermion masses $m^f_i$ versus $b_f$ are illustrated in Fig. 2. At $b_f = 0$, the charged lepton masses have been used as input values for the parameters $z_i$. Note that at $b_f = -1/3$, the third fermion mass takes a maximal value, which is independent of $\kappa/\lambda$. Also note that at $b_f = -1/2$ and $b_f = -1$, two fermion masses degenerate.
We take $b_u = -1/3$ for up-quark sector, because, at $b_u = -1/3$, we can obtain the maximal top-quark mass enhancement (see Fig. 2)

$$m_t \simeq \frac{1}{\sqrt{3}} m_0,$$

and a successful relation

$$\frac{m_u}{m_c} \simeq \frac{3}{4} \frac{m_e}{m_\mu},$$

independently of the value of $\kappa/\lambda$.

The value of $\kappa/\lambda$ is determine from the observed ratio $m_c/m_t$ as $\kappa/\lambda = 0.0198$. Considering the successful relation

$$\frac{m_d m_s}{m_b^2} \simeq 4 \frac{m_e m_\mu}{m_\tau^2},$$

for $b_d \simeq -1$, we seek for the best fit point of $b_d = -e^{i\beta_d}$ ($\beta_d^2 \ll 1$). The observed ratio $m_d/m_s$ fixes the value $\beta_d$ as $\beta_d = 18^\circ$. Then we can obtain the reasonable quark mass ratios [4], not only $m_i^u/m_j^u$, $m_i^d/m_j^d$, but also $m_i^u/m_j^d$:

$$m_u = 0.000234 \text{ GeV}, \quad m_c = 0.610 \text{ GeV}, \quad m_t = 0.181 \text{ GeV},$$

$$m_d = 0.000475 \text{ GeV}, \quad m_s = 0.0923 \text{ GeV}, \quad m_b = 3.01 \text{ GeV}. $$

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Fig. 2. Masses $m_i$ ($i = 1, 2, \cdots, 6$) versus $b_f$ for the case $\kappa = 10$ and $\kappa/\lambda = 0.02$. The solid and broken lines represent the cases arg$b_f = 0$ and arg$b_f = 18^\circ$, respectively. The figure was quoted from Ref. [12].
Here, we have taken \((m_0\kappa/\lambda)_q/(m_0\kappa/\lambda)_e = 3.02\) in order to fit the observed quark mass values at \(\mu = m_Z\) [10]

\[
m_u = 0.000233\ \text{GeV}, \quad m_c = 0.677\ \text{GeV}, \quad m_t = 0.181\ \text{GeV}, \\
+0.000042 \quad +0.056 \quad \pm 13 \\
m_d = 0.000469\ \text{GeV}, \quad m_s = 0.0934\ \text{GeV}, \quad m_b = 3.00\ \text{GeV}, \\
+0.000060 \quad +0.0118 \quad \pm 0.056 \quad -0.061 \\
\]

We also obtain the reasonable values of the Cabibbo-Kobayashi-Maskawa (CKM) [11] matrix parameters:

\[
|V_{us}| = 0.220, \quad |V_{cb}| = 0.0598, \\
|V_{ub}| = 0.00330, \quad |V_{td}| = 0.0155. \\
\]

(The value of \(|V_{cb}|\) is somewhat larger than the observed value. For the improvement of the numerical value, see Ref.[12].)"

So far, we have not mentioned why we call the present model (4.2) “democratic” seesaw mass matrix model. As far as the masses are concerned, the model with the democratic form

\[
M_F = \lambda m_0 \begin{pmatrix}
1 & 0 & 0 \\
0 & 1 & 0 \\
0 & 0 & 1 \\
\end{pmatrix} + b_f \begin{pmatrix}
1 & 1 & 1 \\
1 & 1 & 1 \\
1 & 1 & 1 \\
\end{pmatrix}
\]

is equivalent to the model with the diagonal form

\[
M_F = \lambda m_0 \begin{pmatrix}
1 & 0 & 0 \\
0 & 1 & 0 \\
0 & 0 & 1 + 3b_f \\
\end{pmatrix}.
\]

However, for the prediction of the CKM matrix parameters, the models show different features: In the former model where the matrix \(Z\) is diagonal, phases \(\delta_f^i\) are brought into the model as

\[
H_{mass} = y_L v_L \sum_i z_i \left( e^{i\delta^u_i} u_{Li} u_{Ri} + e^{i\delta^d_i} d_{Li} d_{Ri} \right) + \cdots
\]

i.e.,

\[
Z_u = P(\delta_u)\ Z, \quad Z_d = P(\delta_d)\ Z, \\
P(\delta) = \text{diag}(e^{i\delta_1}, e^{i\delta_2}, e^{i\delta_3}).
\]
Then, the observed CKM matrix parameters was successfully given [5] when we took

\[ P(\delta_u)P(\delta_d)^\dagger = \text{diag}(1, 1, -1) \quad (4.16) \]

On the other hand, if we want the similar results for the latter case, we need a complicated form of the matrices \( Z_u \) and \( Z_d \):

\[ Z_u = AP(\delta_u)ZA^T, \quad Z_d = AP(\delta_d)ZA^T, \quad (4.17) \]

where

\[ A = \begin{pmatrix}
1 & -1 & 0 \\
\frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} & 0 \\
\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{6}} & -\frac{2}{\sqrt{6}} \\
\frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} \\
\end{pmatrix}. \quad (4.18) \]

Because of the simplicity of the former model, we consider that the democratic basis of \( M_F \) has a deep meaning.

5. Application to the neutrino mass matrix

The model can readily give a large mixing between two neutrino states by taking \( b_\nu \simeq -1/2 \) or \( b_\nu \simeq -1 \) as anticipated from Fig. 2. For example, the choice of \( b_\nu \simeq -1/2 \) gives

\[ m_1^\nu \ll m_2^\nu \simeq m_3^\nu, \quad (5.1) \]

and

\[ U_L \simeq \begin{pmatrix}
1 & \frac{1}{\sqrt{2}} \left( \sqrt{m_\mu} - \sqrt{m_\tau} \right) & \frac{1}{\sqrt{2}} \left( \sqrt{m_\mu} + \sqrt{m_\tau} \right) \\
-\sqrt{m_e/m_\mu} & \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\
-\sqrt{m_e/m_\tau} & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\
\end{pmatrix}. \quad (5.2) \]

This is favorable to the large mixing picture suggested by the atmospheric neutrino data [13]. However, in order to give the simultaneous explanation of the atmospheric and solar neutrino [14] data, we need a further study.

Examples based on the scenario A and scenario B are found in Refs. [15] and [8], respectively.

6. Concluding remarks

In conclusion, we have pointed out the following features of the seesaw mass matrix model:
(i) The seesaw mass matrix with the form $M_F = [ (\text{unit matrix}) + (\text{rank-one matrix}) ]$ and $\det M_U = 0$ can naturally understand the observed facts $m_t \gg m_b$ in spite of $m_u \sim m_d$ and why $m_t \sim \Lambda_W$.

(ii) The democratic seesaw mass matrix model with the input $b_e = 0$ can give reasonable quark mass ratios and CKM matrix by taking $b_u = -1/3$ and $b_d = -e^{i18^\circ}$, and a large neutrino mixing $\nu_\mu - \nu_\tau$ by taking $b_\nu \simeq -1/2$.

(iii) The model will provide new physics in abundance if $\Lambda_R \sim \text{a few TeV}$: we can expect observations of the fourth up-quark $t'$ with $m_{t'} \sim \text{a few TeV}$ and FCNC effects due to the abnormal structure of $U^R_R$.

However, this model is still in its beginning stages and there are many future tasks:

(i) How do we understand the fermions $f$ and $F$? Many ideas have been proposed for the unified understanding of the quarks and leptons $f$, while in such a unification model there are no seats which should be assigned to the fermions $F_L$ and $F_R$.

For this question, for example, we can understand that the fermions $(f_L, F_R)$ belong to 16 of a unification symmetry $\text{SO(10)}_L$ and $(f_R, F_L)$ belong to 16 of another unification symmetry $\text{SO(10)}_R$, and the symmetries $\text{SO(10)}_L \times \text{SO(10)}_R$ are broken as follows:

$\text{SO(10)}_L \times \text{SO(10)}_R$

\[ \Downarrow \Lambda_{\text{GUT}} \]

$[\text{SU}(3) \times \text{SU}(2) \times \text{U}(1)]_L \times [\text{SU}(3) \times \text{SU}(2) \times \text{U}(1)]_R$

\[ \Downarrow \Lambda_S \]

because of $\langle F_L F_R \rangle$

$\text{SU}(3)_c \times \text{SU}(2)_L \times \text{SU}(2)_R \times \text{U}(1)_Y$

\[ \Downarrow \Lambda_R \]

$\text{SU}(3)_c \times \text{SU}(2)_L \times \text{U}(1)_Y$

\[ \Downarrow \Lambda_L \]

$\text{SU}(3)_c \times \text{U}(1)_{\text{em}}$

However, regrettably, we found [16] that the numerical results of the evolution of the gauge coupling constants conflicts with the observed low energy data.

(ii) How do we understand the structure of $Z$? In the present stage, the values of $z_i$ have been given by hand, i.e., by taking $b_e = 0$. For an attempt to understand the structure of $Z$, for example, see Ref. [17].
(iii) How do we understand the structure of $M_F$, especially, the parameter $b_f$? For example, there is a correlation between the parameter $b_f$ and electric charge $Q$:

\[
Q_\nu = 0, \quad Q_e = -1, \quad Q_d = -1/3, \quad Q_u = 2/3, \quad b_\nu = ?, \quad b_e = 0, \quad b_d \simeq -1, \quad b_u = -1/3,
\]

Is it accidental? At present, we have no idea.

We hope that many people direct their attention to the universal seesaw mass matrix model and thereby a great development of the quark and lepton physics will be promoted along the line suggested by present model.

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