Numerical Analysis of the Influence of Superconductor and Magnet Material Parameters on the Dynamic Stability of Maglev Systems

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Abstract. In previous research, a two-dimensional numerical model based on Newton’s second law and Maxwell’s equations was developed to analyze the dynamic behavior of superconducting maglev systems. Studies showed that vibration and drift phenomena will occur in the vertical and lateral directions triggered by external disturbances, resulting in a change in the levitation point and even instability. In this paper, a numerical analysis is further carried out to alleviate these situations. The dependence of the drifts of vertical and lateral vibration centers of the levitated body upon system material parameters, such as the magnetization of permanent magnets and the critical current density of superconductor, is studied in terms of a power law model. In addition, the influence of the configuration of permanent magnets on the dynamic stability is analyzed.

1. Introduction
The magnetic levitation realized by superconductors (SCs) and permanent magnets (PMs) has resulted in a wide range of applications including industrial levitation devices with the advantages of self-stability with no active control and no energy consumption. Among these devices, there is a great interest in maglev systems where bulk high-temperature SCs attain the levitation of vehicles above PM guideways with longitudinal geometry, such as linear magnetic bearings, launching and transportation systems [1, 2], such that, due to the huge potential for application, improving their levitation performance becomes particularly important. Some experiments have reported that the levitation force exhibits time-dependent relaxation, and the downward drift of vibration center of levitated body will occur under external disturbances. The thermally activated flux motion, an intrinsic feature of the SC, is regarded as the primary cause of these nonlinear behaviors. Meanwhile, numerous numerical studies have tried to understand the dynamic levitation by considering the flux motion by means of different numerical methods such as the T-method, control volume method, and finite element method [1-3]. Many of these studies mainly focus on the disturbance-related levitation of the maglev by assuming the SC moving along some virtual displacement. Such displacement is not necessarily equal to the one followed by the system after a perturbation of equilibrium. However, from the perspective of dynamics, the motion trajectory of the SC subjected to external disturbances should generally be expected to be a complex spatial curve under the combined action of gravity and magnetic forces. Based on Newton’s second law and the A-V formulation, recently, we have developed a two-dimensional numerical model...
to analyze the dynamics of maglevs and found that vibration and drift phenomena will occur in the lateral and vertical directions triggered by external disturbances. However, there is not a complete numerical study that analyzes how the dynamic stability of these maglevs depends on the system material parameters or the guideway configuration. In this paper, we use the same model to further study the dynamic behavior of several typical maglev systems. The aim of this paper is to reveal the influence of the SC and PM properties, such as magnetization, critical current density, and configuration, on the dynamic stability and establish some general guidelines on resisting external disturbances.

2. Model Description

We consider maglev systems consisting of a bulk high-temperature SC and a guideway formed by different sets of PMs, as shown in figure 1. The PM configurations are distinguished as follows: PM2 and PM2L stand for two magnets with vertical and antiparallel horizontal magnetization, respectively; PM3L denotes three magnets with vertical magnetization in the central magnet and antiparallel horizontal magnetization in the lateral ones; and PM3N indicates three magnets with antiparallel vertical magnetization. In these systems, the SC and PM are infinitely long in the z direction and have rectangular cross-sections of \( a_{SC} \times b_{SC} \) and \( a_{PM} \times b_{PM} \) in the x and y directions, respectively. The SC has a critical current density \( J_c \) and the PMs have a magnetization \( M \) with a horizontal interval \( a \).

![Figure 1. Sketch of the studied maglev systems.](image)

We assume that the SC is cooled at very large height above the PMs, and then it is forced downwards with a small velocity \( 10^{-4} \text{ m/s} \) to a static equilibrium height \( h_0 \), i.e., the working height. Once reaching such a height, the dynamic motion of the SC is triggered by an external disturbance. Using Newton’s second law of motion, the equations of lateral and vertical movements of the SC can be expressed as

\[
mw'' - F_x = 0 \quad (1)
\]

\[
 mh'' + mg - F_y = 0 \quad (2)
\]

and the initial conditions are taken into account by
\(t=0: \ w=0, \ h=h_0, \ w'=v_x=v_0\cos(\theta), \ h'=v_y=v_0\sin(\theta)\) \hspace{1cm} (3)

where \(m\) is the mass per unit length of the SC, \(v_0\) is the disturbance-induced velocity with a tilt angle \(\theta\), \(g\) is the gravitational acceleration, and \(F_x\) and \(F_y\) are the horizontal (guidance) and vertical (levitation) magnetic forces per unit length on the SC, respectively, traditionally calculated by Lorentz’\’s equation (3).

In general, the Maxwell’s equations are used to describe macroscopic electromagnetic phenomena. By adopting the quasi-static approximation of the Maxwell’s equations, the following formulation is obtained.

\[
E = -A' - \nabla V \hspace{1cm} (4)
\]

\[
\nabla^2 A = -\mu_0 J \hspace{1cm} (5)
\]

where \(\mu_0\), \(V\), \(E\), \(A\) and \(J\) are the permeability, electric potential, electric field, vector potential and current density, respectively. For the SC, it has a highly nonlinear current-voltage characteristic and the material constitutive relation between \(E\) and \(J\) can be expressed with a power-law as follows.

\[
E = E_c (J/J_c)^n \hspace{1cm} (6)
\]

where \(E_c\) is the critical electric field and \(n\) is the creep exponent. It is obvious that once the levitation position is changed, the SC will undergo a magnetic field variation and then its induced currents redistribute. Thus, the magnetic forces also adjust themselves to the change of the magnetic field and induced currents. On the other hand, the adjustment of the magnetic forces causes a change in the levitation position. That is to say, the dynamics of the maglev system is coupled with superconducting phenomena. Recently, we have proposed an iterative method to solve similar coupling problems. More details on the numerical procedure to deal with equations (1)-(6), including the theoretical validation, can be found in Refs. [4] and [5].

| Table 1. Parameter values used in the numerical simulation [3-5]. |
|---------------------------------------------------------------|
| Creep exponent \(n\) | 21 |
| Critical electric field \(E_c\) | \(10^{-4}\) V/m |
| Disturbance-induced velocity \(v_0\) | 0.01 m/s |
| Tilt angle of disturbance-induced velocity \(\theta\) | 45° |
| Characteristic critical current density \(J_0\) | \(3.7 \times 10^6\) A/m² |
| Characteristic magnetization \(M_0\) | \(7.95 \times 10^5\) A/m |
| Geometry of superconductor \(a_{SC}/b_{SC} = a\) | 0.05 m |
| Geometry of permanent magnets \(a_{PM}/b_{PM} = a\) | 0.05 m |
| Working height of superconductor \(h_0\) | 0.5a = 0.025 m |

3. Results and Discussion

Figure 2 displays the calculated field lines and current profiles for PM2L with different \(J_c\) and \(M\) at the working height. One sees that when the SC is moved vertically downward, flux vortices created by the PMs will mainly penetrate into the bottom region of the SC. Such penetrated region will be reduced with increasing \(J_c\), which leads to an increase in the induced currents and further in the levitation force (see figures 2(a), (b) and (c)). Besides, an increase in \(M\) can attain an intensification of the magnetic field and flux gradient above the guideway, which results in larger field-penetrated region and induced currents and thereby bigger levitation force (see figures 2(a), (d) and (e)).
Figure 2. Field lines and current profiles for PM2L with different \( J_c \) and \( M \) at the working height.

Figure 3 shows the dynamic response in PM2L with different \( J_c \) and \( M \) under the external disturbance. In figure 3, one sees that after being disturbed, the SC will be simultaneously vibrated in the \( x \) and \( y \) directions, and the vertical and lateral vibration centers gradually drift along their vibration directions. Since flux vortices move in and out during vibration, resulting in energy dissipation, the vibration amplitudes in both \( x \) and \( y \) directions decrease with time little by little. The results of figures 3(a) and (b) show that with the increase in \( J_c \), the drift of vertical vibration center is monotonously decreased. This is due to that increasing \( J_c \) can lead to an increase in the values of the induced currents in the field-penetrated region. From figures 3(c) and (d), one sees that for the case of larger \( M \), the PMs have a stronger magnetic field and a higher flux gradient at their upper surfaces, tending to produce larger magnetic forces. In the \( y \) direction, due to the hysteresis effect, the levitation force will rapidly decay with time. Thus, the drift of vertical vibration center shows a monotonic increase with increasing \( M \).

Figure 4 shows the dependence of the SC’s dynamic response on the guideway configuration at \( M=M_0 \) and \( J_c=J_0 \). It is seen that for configurations with two PMs, PM2L is a good choice for application as it obtains a large levitation force and a small drift for vertical vibration center, although it exhibits a slightly larger drift for lateral vibration center relative to PM2. However, for a guideway composed of three PMs, the levitation force provided by PM3L is much larger than that provided by PM3N. However, the capacity of resisting external disturbance for the latter is superior to that for the former. Thus, one
needs to make a tradeoff between the levitation force and dynamic stability when selecting such two configurations.

![Figure 4](image)

**Figure 4.** Time responses of levitation height, lateral displacement, levitation and guidance forces for the SC in the four studied systems with $J_c=J_0$ and $M=M_0$.

**Summary**

We present a systematic study of the dependence of the vibration and drift phenomena triggered by external disturbances in typical maglev systems composed of a bulk SC and a guideway formed by different sets of PMs upon the system material parameters. Results show that although the levitation force can be increased by modifying system material parameters, this may entail a reduction in the stability. For example, increasing the SC’s critical current density leads to a decrease in the drift of vertical vibration center but an increase in that of the lateral vibration center. Similarly, increasing the PMs’ magnetization causes the drift of vibration center to decrease in the lateral direction but that to increase in the vertical direction. Moreover, we find that for a guideway formed by two or three PMs, the configuration PM3N appears to be the best choice in a maglev application since it provides a large levitation force with good dynamic stability.

**References**

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