Equation Systems Derived from the Analysis of Porticoed Structures: Graphic Resolution Methods and Comparative Study

Agustin Gregorio Lacort 1
1 University of the Basque Country, Donostia, Spain
agustingregorio.lacort@ehu.eus

Abstract. This paper conducts first-order theoretical and linear analyses of a building structure using a classical equilibrium method. The structure comprises a single storey and is made up of ordinary equal porticoes connected by a one-way floor slab. The equation systems for the floor slab and the porticoes are resolved with different visual procedures described previously, which were drawn up to facilitate the manual analysis of these structures. These procedures exactly reproduce in graphic form the operations of Cholesky’s and Gauss’ procedures, the last one deducted by another method inspired in Cross. Some are designed to resolve generic equation systems and others to resolve systems derived from specific types. In this paper, the drawings are made freehand and the results are accurate enough because they are obtained on surfaces for which it is not possible to commit major errors. The graphic procedures used are classified according to suitability, by comparing the exactness of their results and the number of lines required in each case. It is observed that the application time i) varies from one method to another, even though the mathematical procedures do not change, and ii) could be shortened when a graphic method adapted to the relevant type is used. It is also observed that a consistent image of all the operations makes it easier to simplify them and monitor any errors committed. This study also suggests that other graphic methods could be drawn up that would enable different types of building structure to be analysed exactly in just a few operations.

1. Introduction

Currently there is interest in developing simplified manual methods for pre-dimensioning and estimating internal efforts in building structures consisting of few elements. In this field, we have developed a semi-graphical, exact procedure [1] inspired by the Cross method for analysing models formed with beams in the elastic regime and in First Order Theory without numerically resolving systems of equations. The procedure interpreted each deformation as a sum of primary deflections, each calculated immediately and produced by a load state called “primary state” (figures 1a, b). In each primary state, an active action operated that caused the movement of one or various nodes and various restrictions that prevented the rest of the nodal movements. Considering that the exterior loads were a sum of primary states, the method proposed an exact manner for determining them from other primary states ω with unitary active actions. The ω states were represented in a graph G (figure 1c). Each vertex represented the nodal movement due to a ω state and each arch the relationship between two nodal movements. The procedure graphically transformed G into another graph G’ that depended on the order of eliminating the unknowns. The deflection was determined with G’ (figure 1d) “ascending” in the graph with the nodal loads up to the last unknown eliminated, “descending” in the same way...
and summing the results. It was seen that $G$ (figure 1c) also showed the model’s stiffness matrix, as (1) of figure 1a, and that its transformation in $G'$ coincided with the act of factorising that matrix with the Gauss method [2].

$$\frac{1}{\theta'_A} = \frac{1}{\theta'_B}$$

Two graphical versions of [2] were proposed. The first [3] was prepared for analysing continuous beams and certain pergolas only, and the second [4] for analysing any type. In both versions, the information in $G$ was represented with segments of areas $\psi$ formed by boxes of unitary sides. Dashed lines were drawn in those that depended on figures 1e, f, g and that visually reproduced the operations in [2]. It was seen that the perimeter of the areas $\psi$ prevented any major errors from being committed when the drawing was made freehand. Thus, with freehand drawing the calculation speed was increased at the expense of only a minimal error that did not depend on the procedure but rather on the tool used. In addition, another similar graphical procedure was developed [5] that reproduced the operations of the Cholesky’s method.

The objective of this paper is to classify the procedures [3], [4] and [5] according to their suitability, visually and numerically comparing the precision of their results and the application time when used to analyse a joist freehand and the worst portico in the model (figure 2e). In the approach, the analyses were repeated considering different drawing sizes and without taking into account certain strategies in the now classical literature to resolve systems of equations optimally [6] because they could be too laborious. The following describes the analysis model.

2. Model

The model consists of four equal porticos connected by a one-way slab consisting of joists spaced every 70 cm. The movements of the porticos in $OX$ are restricted by cables, such as $A-I$. Considering the hypothesis of manual analysis and ignoring torsions, the nodal movements are inter-related to form systems of equations that have been represented as graphs. There are two graphs, $G_1$ and $G_2$, per portico (figures 2a, b), where $G_2$ is a sub-graph of a $G_1$ associated with the entire structure (figure 2c). There is also a $G_3$ per joist (figure 2d). The inertias $I$ of the model depend on each section and the sign criteria are shown in figure 2f. The actions considered for the analysis are two horizontal ones (figure 2e) and one vertical $q$ applied on the entire roof with a value of 4.5 kN/m². The nodal actions used to define the nodal movements are the same in all the analyses and have been determined numerically.
3. Graphical analyses

3.1 Of a joist

3.1.1 Analysis 1. The order of elimination of unknowns in figure 3a was considered and the method proposed in [3] was used with boxes with 4 cm side. Figure 3b shows the lines that transform G_4 into G'_4: the values of the vertices in G'_4 have been obtained in the area I; segments K have been obtained in II and the Ks have been placed in area III on diagonals that determine a generic deflection. In figure 3c the nodal rotations have been calculated in the form of segments as a function of 1/EI, “ascending” by box IIIa, “descending” by IIIb and summing the results. The procedure was not repeated with smaller boxes because the drawings of areas I and II would have been difficult.

3.1.2 Analysis 2. The order of elimination is the same, using the method in [4] and considering a stepped area ψ and cells with 3 cm side. Lines drawn in figure 3d are equivalent to those in figure 3b, and figures 3e,f show those for those of areas IIIa and IIIb, respectively.

3.1.3 Analysis 3 (figures 3h-j). The order of elimination in figure 3g has been considered and the previous method used with the same box size.

3.1.4 Analyses 4 and 5 (figures 3 l,m). The box sizes are 2 and 1 cm, respectively.

3.1.5 Observations. i) The method in [3] seems less suitable than that in [4] since it needs more mnemonics and is limited to certain types. It also requires more lines to be drawn in smaller spaces. Perhaps because of this, the results do not improve on those obtained with the other analyses (Table 1). ii) The area of figures 3h-j seems more suitable than that of figures 3d-f because it has fewer empty boxes and fewer lines; however, greater drawing precision is needed in box θ^2. iii) The results of analyses 2-5 indicate that when the box size is reduced, the error committed tends to increase and the drawing time to decrease.
Figure 3. Geometrical analysis of a joist: 1) With graph $G'$, analysis 1: prior drawing $b$) and deflection $c$; analysis 2: prior drawing $d$) and deflection $e$, $f$; 2) With graph $G'$, analysis 3: prior drawing $h$) and deflection $i$, $j$; $l$) deflection of analysis 4; $m$) deflection of analysis 5.
Table 1. Joist: results of analyses

|     | $\theta_E^0$ | $\theta_F^0$ | $\theta_G^0$ | $\theta_H^0$ | $m_{EF}^v$ (mKn) | $v_{EF}^v$ (Kn) |
|-----|--------------|--------------|--------------|--------------|------------------|---------------|
| Exact | 3.06         | 2.26         | -0.30        | -7.96        | 4.19             | 1.99          |
| Nº 1 | 2.58         | 3.55         | 0.00         | -8.40        | 4.33             | 2.30          |
| Nº 2 | 3.50         | 2.25         | -1.25        | -7.50        | 4.62             | 2.15          |
| Nº 3 | 3.64         | 2.80         | -0.52        | -7.50        | 5.04             | 2.41          |
| Nº 4 | 3.78         | 2.94         | -0.63        | -7.98        | 5.25             | 2.53          |
| Nº 5 | 3.36         | 2.52         | -1.25        | -7.56        | 4.62             | 2.2           |

$\theta$ is in function of $1/\text{EI}$

3.2 Of the worst portico considering $G_1$

3.2.1 Analyses 6, 7 and 8. Undertaken with the procedure in [4] using boxes with sides of 3, 2 and 1 cm, respectively. As happened with analyses 3, 4 and 5, no important differences between the deflections were seen visually (figures 4d,e,f).

3.2.2 Analyses 9, 10 and 11. Undertaken with the procedure in [5] using the above cell sizes, respectively. To make drawing easier, the stiffness matrix and nodal loads values were divided by $\alpha = 4$. This increased the values of the $G_1$ vertices by $\alpha$ and divided those of the arches by $\alpha$. Figure 4g is similar to figure 4b and the results (figures 4i,l,j) are similar to the ones in figures 4d,e,f.

3.2.3 Observations. i) In all cases, the order of the elimination of unknowns in figure 4a was considered. ii) The results were obtained as a function of $F$ of the pillars and it was always deduced that there is no $\delta^*$ because this movement without the presence of the cable is positive (figures 4c,h,k). iii) The analyses with [5] must be made with more care because they seem more sensitive to errors (Table 2), perhaps because they require more lines to define the deflection. iv) The error committed in the efforts tends to increase as the size of the boxes decreases. v) The mnemonic rules used in figure 4b seem more suitable than those used in figure 4g because they are simpler to apply even though they generate more lines. vi) The procedure in [4] seems more precise than that of [5] (Table 2).

Table 2. Portico: results of the analyses considering $G_1$

|     | $\theta_A^0$ | $\theta_B^0$ | $\delta_A^0$ | $m_{EF}^V$ | $v_{CA}^0$ |
|-----|--------------|--------------|--------------|------------|------------|
| Exact | 32.66        | -34.84       | 0.00         | 21.77      | 21.77      |
| Nº 6 | 30.87        | -42.10       | " "         | 20.58      | 20.58      |
| Nº 7 | 33.68        | -37.90       | " "         | 22.45      | 22.45      |
| Nº 8 | 37.84        | -42.05       | " "         | 25.22      | 25.22      |
| Nº 9 | 36.57        | -42.20       | " "         | 24.38      | 24.38      |
| Nº 10| 35.78        | -37.90       | " "         | 25.65      | 25.65      |
| Nº 11| 29.47        | -33.68       | " "         | 19.64      | 19.64      |

$\theta$, $\delta$ are in function of $1/\text{EI}$
3.3 Of the worst portico considering $G_2$

3.3.1 Analyses 12, 13 and 14. Solved by applying [4]. To make drawing easier in all three cases, the values of the stiffness matrix and those of the nodal loads have been multiplied by $\alpha = 2$. This transforms figure 2b into figure 5g. Figure 5a shows the lines that transform $G_2$ into $G_2'$ considering boxes with sides of 3 cm (analysis 12) and figure 5b gives the deflection introducing the nodal actions multiplied by 2. Comparing it with figures 5d,c (analysis 13 and 14) made with box sides 2 and 1 cm long, respectively, no important differences are seen (Table 3).

Figure 4. Geometrical analyses of the portico considering $G_1$: a) Graph $G'_1$; analysis 6; b) prior drawing and $c,d$) deflection; analysis 7; e) deflection of analysis 7; f) deflection of analysis 8; analysis 9; g) prior drawing and h,i) deflection; k,l) deflection of analysis 10; j) deflection of analysis 11.
3.3.2 Analyses 15 and 16. Undertaken according to [3] modifying figure 2b as in cases 9, 10 and 11 but considering $\alpha = 2$. The box lengths are 3 and 2 cm respectively and the resulting deflections are similar (figures 5h). Boxes with $L = 1$ cm were not used because they couldn’t be drawn with precision.
3.3.3 Observations. i) The results depend on \( I_x \) of the pillars. ii) The segments that modify the vertices of the movements and the arch that relates them must be obtained with great precision because they must be multiplied by the number of porticos. iii) The method in [4] is easier to draw than that in [5] because the lines are better distributed in the drawing area.

Table 3. Portico: results of the analyses considering G2

|   | \( \delta_i^z \) | \( \delta_z^\theta \) | \( \theta_i^x \) | \( \theta_z^x \) | \( \theta_i^y \) | \( \theta_z^y \) |
|---|-----------------|-----------------|-----------|-----------|-----------|-----------|
| Exact | 22.12 | 48.61 | 4.30 | 4.38 | 10.95 | 17.98 |
| N° 12 | 23.00 | 37.00 | 4.00 | 8.00 | 9.00 | 17.00 |
| N° 13 | 32.00 | 48.00 | 6.00 | 8.00 | 18.00 | 20.00 |
| N° 14 | 16.00 | 46.00 | 4.00 | 5.00 | 8.00 | 16.00 |
| N° 15 | 13.50 | 46.5 | 4.50 | 5.00 | 6.00 | 12.75 |
| N° 16 | 15.50 | 15.5 | 9.00 | 7.50 | 7.50 | 16.6 |

\( \theta, \delta \) are in function of \( 1/EI \)

3.4 Common observations

- The least operative procedure seems to be that of [3] and the methods in [4] and [5] may be considered similar.
- Not enough examples were made to arrive at definitive conclusions.
- It seems that procedure [5] i) has fewer lines than [4] but they must be made in more areas; ii) it is difficult to draw when the areas are small.
- The application time varies according to the method although the mathematical procedure does not change.
- A unitary image of all the operations facilitates their simplification, controlling the error committed.

4. Conclusions

- Based on this research, the possibility emerges of calculating certain hyperstatic structures commonly found in building graphically and almost exactly by freehand.
- Possible lines of research are also pointed to in the developing of further graphic methods that can analyse other types of structure directly and accurately.

Acknowledgments

I would like to thank my friend José Múgica, Manager of Tradutecnia, Peter Rodwell and Chris Pellow for the help that they have provided in translating this manuscript into English.

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