Magnetic spectrum of the two-dimensional antiferromagnet La$_2$CoO$_4$ studied by inelastic neutron scattering

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We report measurements of the magnetic excitation spectrum of the layered antiferromagnet La$_2$CoO$_4$ by time-of-flight neutron inelastic scattering. In the energy range probed in our experiments (0–250 meV) the magnetic spectrum consists of spin-wave modes with strong in-plane dispersion extending up to 60 meV, and a nearly dispersionless peak at 190 meV. The spin-wave modes exhibit a small (~1 meV) dispersion along the magnetic zone boundary. We show that the magnetic spectrum can be described very well by a model of a Heisenberg antiferromagnet that includes the full spin and orbital degrees of freedom of Co$^{2+}$ in an axially distorted crystal field. The collective magnetic dynamics are found to be controlled by dominant nearest-neighbor exchange interactions, strong XY-like single-ion anisotropy and a substantial unquenched orbital angular momentum.

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I. INTRODUCTION

The evolution from antiferromagnetism to high-temperature superconductivity with carrier doping of the layered copper oxides has inspired a vast literature on the electronic, structural, dynamical, and chemical properties of related materials. It has become clear from these studies that superconductivity and commensurate antiferromagnetic order are only two out of many different competing ordering tendencies found in systems of strongly interacting electrons.

Among other forms of order found in doped Mott insulators are nematic phases characterized by unidirectional density-wave states involving combined charge and spin order. Such ‘striped’ phases were first identified many years ago in hole-doped (La,Nd)$_2$CuO$_4$ (Ref. 2) and La$_2$NiO$_4$ (Refs. 3–6), but their significance for high-temperature superconductivity has been the subject of a continuing debate. Although much of the focus has been on the cuprates, the nickelates have contributed to this debate on account of their relatively well correlated and stable stripe order which is amenable to experimental investigation. One drawback, however, is that holes localized on Ni$^{3+}$ ions in hole-doped La$_2$NiO$_4$ carry a spin which can interact magnetically both with other spins in the charge stripes and with the surrounding antiferromagnetic matrix of Ni$^{2+}$. The influence of these interacting magnetic degrees of freedom on the properties of stripes in nickelates has yet to be fully evaluated, but spin correlations associated with both Ni sites have been observed7 and there remain some unexplained features in the spin excitation spectra.8

Recently, evidence has been presented for the existence of stripe phases in the layered cobaltate system La$_{2-x}$Sr$_x$CoO$_4$ (Ref. 9), which is isostructural with hole-doped La$_2$CuO$_4$ and La$_2$NiO$_4$. Neutron diffraction measurements on half-doped (x=0.5) La$_2$CoO$_4$ show clear evidence for a charge ordering of Co$^{2+}$ and Co$^{3+}$ ions in a checkerboard pattern below $T_{co}$ $\approx$ 825 K, with an antiferromagnetic ordering transition at a much lower temperature $T_N$ $\sim$ 60 K.10–12 The antiferromagnetic order in La$_{2-x}$Sr$_x$CoO$_4$ in the range 0.3 $< x < 0.6$ was observed to be periodically modulated with the modulation wavevector nearly proportional to $x$,9 a characteristic of the stripe phase found in La$_{2-x}$Sr$_x$NiO$_4$. The Sr-doped cobaltate has an advantage over the nickelates in that for the compositions in which striperlike magnetic correlations occur there is evidence to suggest that the Co$^{3+}$ ions adopt the low spin ($S=0$) state at low temperatures and are therefore not magnetically active.13–15 Hence, the layered cobaltates offer the chance to investigate the fundamental interactions and excited states of an ordered stripe phase in which the doped holes do not possess low-energy spin degrees of freedom.

Attempts to understand the electronic phases in La$_{2-x}$Sr$_x$CoO$_4$ will require some basic knowledge of the parent antiferromagnet La$_2$CoO$_4$. Although the crystal structure and magnetic order of La$_2$CoO$_4$ have been studied in detail,16 no measurements of the magnetic excitation spectrum have been published before now.

In this paper we present high-resolution measurements of the magnetic spectrum of La$_2$CoO$_4$ by neutron inelastic scattering. The methodology relates closely to that employed in a recent investigation carried out on the half-doped cobaltate La$_{1.5}$Sr$_{0.5}$CoO$_4$ (Ref. 15). The data presented here on La$_2$CoO$_4$ extend up to 250 meV in energy and reveal spin-wave-like excitations with a bandwidth of 60 meV. At much higher energies, there are excitations of a localized character. A good description of the data is achieved with a spin-wave model for a quasi-two-dimensional antiferromagnet that includes the full spin and orbital degrees of freedom of the Co$^{2+}$ ions. The results show that La$_2$CoO$_4$ has dominant nearest-neighbor exchange interactions, although a weak dispersion along the zone boundary indicates that more distant interactions or nonlinear terms in the Hamiltonian are not negligible. The anisotropy is strongly XY-like but there is also a weak in-plane anisotropy.
In the absence of interlayer coupling all these structural phases: \( \text{LT T} \), space group \( 14/mmm \); \( \text{LT L} \), space group \( Cmca \); \( \text{LT T} \), space group \( P4_2/nmc \). The structural transition temperatures for \( \text{La}_2\text{CoO}_4 \) are \( T_1 \approx 900 \text{ K} \) (Ref. 9) and \( T_2 = 120–135 \text{ K} \) (Ref. 16). The latter is reported to be first order. Throughout this paper we shall use the conventional \( 14/mmm \) unit cell as a basis for the reciprocal lattice. The low-temperature lattice constants referred to this cell are \( a = b = 3.91 \text{ Å} \) and \( c = 12.6 \text{ Å} \). The true unit cell has in-plane dimensions which are \( \sqrt{2} \times \sqrt{2} \) larger than those of the \( 14/mmm \) pseudocell—see Fig. 1(a).

The transition to magnetic order occurs at \( T_N = 275 \text{ K} \), and a magnetic reorientation occurs at \( T_H \) coincident with the \( \text{LTO–LTT} \) structural transition. The antiferromagnetic structure has an ordering wave vector \( q_{m} = (0.5, 0.5, 0) \), with ordered moments lying in the \( \text{CoO}_2 \) plane. Assuming collinear order, the difference between the magnetic structures in the \( \text{LTT} \) and \( \text{LTO} \) phases is that in the \( \text{LTT} \) phase the moments are parallel to \( q_{m} \) whereas in the \( \text{LTO} \) phase they are perpendicular to \( q_{m} \). The distinction between these structures depends on the relationship between adjacent layers. Another possibility is that the structure is collinear within the layers but the moment direction rotates by \( \pm 90^\circ \) from one layer to the next.16 In the absence of interlayer coupling all these structures have the same energy. As we did not observe any evidence in the excitation spectrum for interlayer coupling we will treat the magnetic order as two-dimensional. Figure 1(a) shows the in-plane magnetic order with the moments arbitrarily chosen to point along the horizontal axis.

III. EXPERIMENTAL DETAILS

A single-crystal sample of \( \text{La}_2\text{CoO}_4 \) with a mass of approximately 5 g was grown in Oxford by the optical floating-zone method. Polycrystalline \( \text{La}_2\text{CoO}_4 \) was prepared from high-purity (>99.99%) \( \text{La}_2\text{O}_3 \) and \( \text{Co}_3\text{O}_4 \) by solid-state reaction. Stoichiometric amounts of the oxides were mixed and reacted at 1050 °C for 48 h under a flowing atmosphere of \( \text{CO}/\text{CO}_2 \) mixed in the ratio 1:1. A reducing atmosphere is needed to avoid the formation of \( \text{LaCoO}_3 \). The powder was reground and sintered at 1100 °C in a flow of argon for a further 48 h. No impurity phases could be detected in the product by x-ray powder diffraction. The \( \text{La}_2\text{CoO}_4 \) powder was isostatically pressed into rods of diameter 12 mm and length 120 mm. The rods were sintered in an argon atmosphere at 1250 °C for 24 h. Crystal growth was carried out in a four-mirror image furnace (Crystal Systems Corporation) in flowing argon at a growth speed of approximately 2 mm hr\(^{-1}\) with counter-rotation of the feed and seed rods at 25 rpm.

Crystals grown by this method contain an excess of oxygen. To achieve stoichiometry the as-grown crystal was annealed at 850 °C for 72 h in flowing \( \text{CO}/\text{CO}_2 \) (1:10 ratio). A fragment of the annealed crystal was ground to a powder and subjected to a thermogravimetric analysis. From the measured weight loss we determined the oxygen non-stoichiometry to be \( \delta = 0.03 \pm 0.02 \). This suggests that the crystal is close to the ideal stoichiometry, if anything slightly oxygen-deficient.

Magnetization measurements were performed with a superconducting quantum interference device (SQUID) magnetometer (Quantum Design) on a small crystal cut from the same rod as the neutron scattering crystal. Measurements were made by the dc method with a measuring field of strength 1000 Oe (\( \mu_0H = 0.1 \text{ T} \)) applied along the crystallographic c axis. Zero-field-cooled (ZFC) data were recorded on warming after the sample had been initially cooled from 350 K in zero applied field, and field-cooled (FC) data were recorded while cooling the sample from 350 K in the measuring field.

Unpolarized-neutron inelastic scattering measurements were performed on the direct-geometry chopper spectrometer MAPS at the ISIS facility.17 Neutron time-of-flight instruments with large position sensitive detector arrays such as MAPS allow sampling of vast regions of \( (Q, \omega) \) space simultaneously, where \( Q \) and \( \omega \) are, respectively, the wave vector and energy transferred from the neutron to the sample. This is very advantageous in studies where the excitation spectrum is required throughout the Brillouin zone.

In preparation for the neutron measurements the \( \text{La}_2\text{CoO}_4 \) crystal was sealed in a thin-walled aluminum can containing helium exchange gas and aligned with the c axis parallel to the direction of the incident-neutron beam. Cooling was provided by a closed-cycle refrigerator. Data were collected with incident-neutron energies of 51, 86, 111, 152, and 303 meV. The energy resolution was typically 5% of the incident energy (full width at half maximum) at zero energy transfer, decreasing slightly with increasing energy transfer. Under the chosen experimental conditions the wave vector resolution is largely determined by the divergence of the incident-neutron beam which is approximately 0.5°. Spectra from \( \text{La}_2\text{CuO}_4 \) were recorded at several temperatures between 6 and 300 K. Separate measurements of a standard vanadium sample were made at each incident energy to normalize the spectra and place them on an absolute intensity scale.

FIG. 1. (Color online) (a) In-plane magnetic structure of \( \text{La}_2\text{CoO}_4 \). The dashed square shows the conventional \( 14/mmm \) unit cell of the HTT phase, and the filled square represents the magnetic unit cell, which coincides with the \( \sqrt{2} \times \sqrt{2} \) chemical unit cell of the LTT phase. The exchange interactions used to model the magnetic spectrum are indicated. (b) Diagram of the reciprocal space lattice corresponding to the \( 14/mmm \) cell. The filled square indicates the magnetic Brillouin zone centered on \( (0.5,0.5) \). The dashed lines show the path through reciprocal space along high-symmetry directions used for detailed analysis of the magnetic excitation spectrum—see Fig. 5.
deviation of \( \sigma_T \). This function was found to give a good description of diffraction data near \( T_N \) in Ref. 16. The parameters obtained from our data were \( (T_N) = 275.5(5) \, K \), \( \sigma_T = 1(1) \, K \), and \( \beta = 0.15(1) \). The transition temperatures measured on our sample are consistent with previously reported values of \( T_N = 275 \, K \) and \( T_F = 135 \, K \) for a nominally stoichiometric crystal.\(^{16}\) Although there is some discussion in the literature about the precise composition of \( \text{La}_2\text{CoO}_4 \) prepared under different conditions,\(^{16,19-21}\) we can at the very least be confident that our crystal is close in composition to the one used in Ref. 16.

We note that the FC and ZFC susceptibility curves separate below 350 K, which is not expected in the paramagnetic phase. This indicates that the sample contains a small amount of ferromagnetic impurity. The FC–ZFC separation was not observed in the as-grown crystal. The most probable explanation is that a tiny amount of elemental Co was formed during the CO/CO\(_2\) annealing step. This is consistent with the slight oxygen deficiency found from the thermogravimetric analysis. As there is no unexplained secondary signal in our neutron scattering spectra this impurity must be present in very small quantities so is of no consequence to our neutron results, but it does mean that the susceptibility curves shown in Fig. 2 contain a background signal in additional to the signal from pure \( \text{La}_2\text{CoO}_4 \).

We now turn to the neutron scattering spectra. Figure 3 provides an overview of the data collected at 6 K. Panels (a)–(c) are constant-energy slices at three different energies, and panels (g)–(i) are energy–Q slices to illustrate the magnetic dispersion. The spectrum is dominated by a spin-wave-like conical dispersion which rises from the in-plane antiferromagnetic ordering wavevector \( \textbf{q}_m = (0.5, 0.5) \) and equivalent positions [the M-points of the square-lattice Brillouin zone—see Fig. 1(b)]. This mode has a gap of approximately 10 meV at the M-point and rises to a maximum energy of 60 meV at the Σ-point on the Brillouin zone boundary. A much weaker branch, displaying an upwards dispersion with a minimum energy at M of 46 meV, corresponds to the first mode translated by \( \textbf{q}_m \). The large splitting of the modes at M shows that the anisotropy is strongly XY-like. The lower and upper modes correspond to in-plane and out-of-plane fluctuations, respectively. Figure 3(i) shows data up to the maximum energy explored in our experiment. This reveals only one other significant feature—a band of scattering in a narrow range of energies close to 190 meV.

We note that scattering from phonons is much weaker than magnetic scattering in the range of Q studied.

To give a different impression of the data we present in Fig. 4 examples of constant-Q cuts taken through the data volumes measured with incident energies \( E_i = 51, 111 \) and 303 meV. To extract the magnetic dispersion in a form suitable for fitting to a model we performed a large number of such constant-energy cuts at wave vectors along the reciprocal-space paths indicated in Fig. 1(b). The peaks in these plus some additional constant-wave-vector cuts were fitted with Gaussian functions on a linear background. The peak centers determined this way are plotted along the high symmetry directions in Fig. 5.

An interesting behavior is observed along the magnetic zone boundary: the energy of the magnon branch is not con-

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**Fig. 2.** (Color online) Main figure: temperature dependence of the (1.5,1.5,1) magnetic Bragg peak of \( \text{La}_2\text{CoO}_4 \) showing the onset of magnetic order at \( T_N \approx 276 \, K \). The solid line shows a power law fit to data with a Gaussian distribution of Néel temperatures. Inset: field-cooled (FC) and zero-field-cooled (ZFC) magnetic susceptibility of \( \text{La}_2\text{CoO}_4 \) recorded with a measuring field of 1000 Oe applied parallel to the \( c \) axis. The magnetic ordering transition and the LTO–LTT structural transition are indicated by arrows.

For presentation and analysis, the neutron data were transformed from raw time-of-flight spectra into an intensity map as a function of \( Q \) and \( h\omega \). With a fixed sample orientation, only three out of the four components of \( (Q_x, Q_y, Q_z) \) are independent. We chose the two in-plane wave vector components \( (Q_x, Q_y) = (h, k) \times 2\pi/a \) and energy as the independent variables, which means that the out-of-plane wave vector \( Q_z = l \times 2\pi/c \) varies implicitly with energy transfer. For a two-dimensional scattering system, however, there is no dispersion in the out-of-plane direction and the gradual variation of scattering intensity with \( Q_z \) can be included in a model (and was done so in this work). The justification for treating \( \text{La}_2\text{CoO}_4 \) as a two-dimensional magnetic system is that the magnetic spectra show no discernible periodic modulation in intensity with \( Q_z \) (i.e., with \( h\omega \)).

In order to quantify the magnetic dispersion we performed a series of constant-energy and constant-wave-vector cuts through the data volume along high-symmetry directions using the MSlice software.\(^{18}\) Before performing these cuts, data at symmetry-equivalent wavevectors were averaged to improve the signal.

**IV. RESULTS**

The temperature dependence of the FC and ZFC susceptibility (\( \chi = M/H \)) is shown in Fig. 2 (inset). Both curves show a change in slope at approximately 276 K consistent with the antiferromagnetic transition, and sharp anomalies at 124 K close to the temperature \( T_F \) at which the LTO–LTT structural transition is expected. The onset of antiferromagnetism at \( T_N = 276 \, K \) is confirmed by the temperature dependence of the neutron diffraction intensity recorded at the magnetic Bragg peak position (1.5,1.5,1), shown in Fig. 2. To estimate \( T_N \) we fitted the data to a power law \( I \propto (1 - T/T_N)^{\beta} \), assuming a Gaussian distribution of Néel temperatures about the mean value \( (T_N) \) with standard
FIG. 3. (Color online) Measured and simulated magnetic spectra of La$_2$CoO$_4$ at 6 K. Panels (a)–(c) show intensity maps averaged over 2 meV energy ranges centered on different energies as indicated, with the corresponding calculated spectra shown in (d)–(f). The magnetic dispersion along two high-symmetry directions is displayed in (g)–(i), with corresponding simulations in (j)–(l). Data in (a)–(h) were measured with an incident-neutron energy $E_i$ = 86 meV, while (i) was measured with $E_i$ = 303 meV. The units of intensity indicated by the colorbars are mb sr$^{-1}$ meV$^{-1}$ f.u.$^{-1}$. The simulated spectra are calculated from the spin-orbital spin-wave model described in the text.

stant but varies by approximately 1.5 meV. As discussed below, this is significant because a dispersion along the zone boundary indicates a need to go beyond a linear spin-wave model with nearest-neighbor interactions only. To emphasize this effect, we show in Fig. 6(a) the energy and (b) the integrated intensity of the magnon peak along the entire length of a zone boundary (XΣX). The maximum in the dispersion at Σ is seen to coincide with a minimum in its intensity. Because the dispersion surface forms a ridge along the zone boundary care was taken to select an appropriately-sized box in $Q$ over which to average the data so as to avoid systematic errors from the curvature of the dispersion surface while at the same time having good enough statistics to extract the peak energies and integrated intensities. Figure 6(c) shows energy cuts taken at an X-point and a Σ-point to illustrate the difference between the magnon peaks at the zone corner and zone edge.

V. ANALYSIS AND DISCUSSION

Magnetism in Co$^{2+}$ compounds such as La$_2$CoO$_4$ is generally influenced to a significant degree by unquenched orbital angular momentum which is responsible for, among other things, the strong anisotropy in the susceptibility observed in many such compounds. In a recent study of the magnetic excitations in the half-doped cobaltate La$_{1.5}$Sr$_{0.5}$CoO$_4$, which is also an antiferromagnet, we developed a model to describe the magnetic spectrum including both the spin and orbital angular momentum of the Co$^{2+}$ in the high-spin configuration ($3d^{7}, S=3/2, L=3$). The model is an advance over conventional (spin-only) spin-wave theory in that it includes level-mixing within the $2S+1L$ term caused by the ligand and exchange fields, and hence the parameters that describe the single-ion anisotropy and exchange interactions are physically realistic. As far as the magnetic spectrum is concerned, the admixture of basis states means that excitations to levels above the first excited single-ion level can propagate and can be observed by neutron scattering. More-
over, the orbital component of the single-ion states needs to be included for an accurate calculation of the neutron cross section.

The model employs the Hamiltonian

$$\mathcal{H} = \sum_{(jk)} J_{jk} \mathbf{S}_j \cdot \mathbf{S}_k + \sum_j \left[ \sum_{L_m} B^m_{ij} O^m_{ij}(\mathbf{L}_j) + \lambda \mathbf{L}_j \cdot \mathbf{S}_j + \mathbf{H}^{\text{ex}} \cdot \mathbf{S}_j \right].$$

(1)

The first term describes an isotropic Heisenberg exchange interaction between pairs of $S=3/2$ spins. For La$_2$CoO$_4$ we include only the nearest-neighbor and next-nearest-neighbor exchange interactions $J_1$, $J_2$, and $J_3$, as defined in Fig. 1. The remaining terms in (1) are single-ion terms. The first of these represents the crystal (ligand) field acting on the Co$^{2+}$ ions. The $O^m_i$ are Stevens operator-equivalents with $B^m_i$ the corresponding crystal-field parameters. The axially distorted octahedral crystal field from the neighboring O$^{2-}$ ions is described by the operators $O^0_{2z}$, $O^4_{2z}$, and $O^2_{2z}$. We kept the same values for the parameters $B^0_{2z}$ and $B^2_{2z}$ as found for La$_{1.5}$Sr$_{0.5}$CoO$_4$ in Ref. 15: $B^0_{2z} = -1.35$ meV and $B^2_{2z} = -8.00$ meV. These are estimated from a point-charge calculation and scaled to match the cubic crystal field splitting observed in CoO.$^{22}$ The parameter $B^0_{2z}$ controls the out-of-plane anisotropy and was adjusted to obtain a good fit to the magnetic spectrum. Its final value (see below) differs from that deduced for La$_{1.5}$Sr$_{0.5}$CoO$_4$ by only ~10%. The term $\lambda \mathbf{L}_j \cdot \mathbf{S}_j$ is the spin-orbit coupling. The coupling constant $\lambda = -18.7$ meV used here has been deduced from reflectivity measurements of CoO by optical spectroscopy.$^{22}$ The final term $\mathbf{H}^{\text{ex}} \cdot \mathbf{S}_j$ represents a small uniaxial anisotropy which defines the in-plane orientation of the moments and produces a spin gap at the $\Gamma$-point (and, equivalently, the M-point). We chose the moments to lie along the $x$ axis, and to achieve this the anisotropy field $\mathbf{H}^{\text{ex}}$ points along $\pm x$ on one of the antiferromagnetic sublattices and along $-x$ on the other.

The partial differential scattering cross section depends on the response functions $S^{\alpha\alpha}(Q, \omega)$ describing $\alpha\alpha$ magnetic correlations. In the dipole approximation the relation is\textsuperscript{23}

$$\frac{k_i}{k_f} \frac{d^2\sigma}{d\Omega dE_f} = \left( \frac{\gamma_0}{2} \right)^2 \eta^2(Q) e^{-2W} \sum_a (1 - \tilde{Q}_a^2) S^{\alpha\alpha}(Q, \omega),$$

(2)

where

$$S^{\alpha\alpha}(Q, \omega) = \sum_j |\langle j | M^{\alpha\alpha}(Q) | 0 \rangle|^2 \delta(\omega - \omega_j(Q)).$$

(3)

Here, $k_i$ and $k_f$ are initial and final neutron wave vectors, $(\gamma_0/2)^2 = 72.8$ mb, $f(Q)$ is the dipole magnetic form factor of Co$^{2+}$, $e^{-2W}$ is the Debye-Waller factor which is close to unity at low temperatures, and $\tilde{Q} = Q_a/|Q|$ is the $\alpha$ component of a unit vector in the direction of $Q$. The response function (per La$_2$CoO$_4$ f.u.) described in Eq. (3) takes into account both the spin and orbital magnetization $\mathbf{M} = -(\mathbf{L} + 2S)$ in the transition matrix element connecting the ground state to an excited mode $j$. The procedure to diagonalize the Hamiltonian (1) to obtain the dispersion and response func-

![FIG. 5. (Color online) The lower figure shows the dispersion of the magnetic excitations of La$_2$CoO$_4$ along high symmetry directions in the 2D Brillouin zone defined in Fig. 1. Open circles are points extracted from cuts through the measured data volume. The lines show the dispersion of the modes calculated with the many-level spin-wave model described in the text. The upper figure shows the response functions $S^{\alpha\alpha}$ for each mode calculated from the many-level model. The normalization of the response functions is per formula unit of La$_2$CoO$_4$.](image)

![FIG. 6. (Color online) Dispersion of the magnon peak along the magnetic zone boundary in La$_2$CoO$_4$. Variation of (a) the peak position and (b) the integrated intensity of the magnon peak in constant-Q cuts. (c) Constant-Q cuts at $Q = (0.75, 0.75)$ and $(0.5, 1)$ fitted with Gaussian lineshapes. The data are from the run with incident energy 111 meV and sample temperature 6 K.](image)
tions of the magnetic modes is described in detail in Ref. 15.

The parameters of the model were refined from a fit to the measured dispersion carried out by a simulated-annealing al-
gorithm. Because of the computer time required to diagonal-
ize the Hamiltonian for the complete set of $2 \times (2L+1)/(2S +1)-1)=54$ excited states (twice the number of single-ion excited states because we have two magnetic sublattices) we
restricted the number of observables included in the fit to just
enough to represent all the important features of the data,
including the high-energy signal at $\sim 190$ meV. The param-
eters varied in the fit were $B_1^2$, $J_1$, $J_2$, and $H^*$.

The best fit was achieved with parameters $B_1^2=14.6(1)$ meV, $J^*
=9.69(2)$ meV, $J_1=0.14(2)$ meV, $J_2=0.43(1)$ meV, and
$H^*=0.66(6)$ meV. The calculated dispersion and response
functions of the magnetic modes are shown in Fig. 5 together
with the full set of data points for the lowest energy modes
determined from the measurements. The agreement is seen to
be very good. The fit indicates that the next-nearest-neighbor
exchange constants $J_1$ and $J_2$ are very small but not zero. As
a test, we repeated the fit with $J_1$ and $J_2$ fixed to zero and
found that the quality of best fit worsened, as indicated by the
standard goodness-of-fit parameter $\chi^2$ per degree of free-
dom which increased from 4.5 to 11.1. Therefore, we believe
that the obtained values of $J_1$ and $J_2$, though small, are sig-
nificant.

To further visualize and assess the model we calculated in-
tensity maps and cuts to simulate those obtained from the ex-
periment. Figures 3 and 4 show the simulations alongside the
corresponding experimental data. The quantity plotted is
$(k/k_0)dB/\partial \Omega \partial \phi E_2$ per formula unit (f.u.), i.e., the par-
tial differential cross section multiplied by a factor $k/k_0$ as defined in Eq. (2). The dipole magnetic form factor of $\text{Co}^{2+}$ and the
$Q$ orientation factor that determines the weighting of the dif-
ferent response functions are included in the simulated spectra.
The simulations also take into account a number of other ex-
perimental factors: (i) we averaged over a 50:50 mixture of equiva-
lent magnetic domains in which the or-
dered moments point along the $x$ and $y$ axes, respectively;
(ii) the spectra are broadened in energy and wave vector by
the estimated resolution of the MAPS spectrometer (see Sec.
III); (iii) we included an estimate of the absorption and self-
shielding of the neutron beam by the sample, which reduces the
intensity by a factor of typically $0.65-0.80$ on the incident-neutron energy and $\hbar \omega$. An additional scale fac-
tor of 0.4 was applied uniformly to all calculated spectra in
order to match the measured absolute scattering intensity.

The simulations show that the model provides a very good
description of the entire observed spectrum of $\text{La}_2\text{CoO}_4$. The relative intensities of the magnetic excitations are
reproduced to within 10–20 %, including the band of
scattering at $\sim 190$ meV, which from Fig. 5 is seen to origi-
nate from a mode with longitudinal ($xx$) character together with
some less-intense transverse modes. Magnetic excitations are
also present in the model at around $\sim 115$ and
$\sim 165$ meV but are predicted to carry negligible spectral
weight and are not observed—see Fig. 4(c). The additional
scale factor of 0.4 needed to match the absolute intensity is
similar to that required for $\text{La}_2\text{Sr}_2\text{CuO}_4$ (Ref. 15). It is
accounted for partly by the size of the ordered moment. The
observed ordered moment is $2.9\mu_B$ (Ref. 16) whereas the
ordered moment in the (ionic) model is $4.1\mu_B$. The differ-
ence between observed and calculated moments may be an
effect of covalency, which would also modify the magnetic
form factor relative to the free ion form factor in such a way
that could cause an additional reduction in intensity, as re-
cently found in a cuprate chain compound.24

For reference, we also compared the low-energy part of the
spectrum ($\hbar \omega < 60$ meV) with standard linear spin-wave
theory for an effective spin-$1/2$ antiferromagnet, which ne-
glects the orbital component of the modes. We used the
same model as described in Ref. 15 in which the magnetic
anisotropy is described by anisotropic nearest-neighbor ex-
change interactions $J_1=J(1+\epsilon)$, $J_2=J$, and
$J_2=J(1-\delta)$. The parameters $\epsilon$ and $\delta$ control the in-plane and out-of-plane
anisotropy, respectively. The more distant interactions $J_1$
and $J_2$ were included too, but because they are relatively
small we treated these as isotropic. We found that the
lower energy modes can be well described by this model.

In fact, an equally good description of the data (as reflected
in the value of $\chi^2$) could be found with sets of param-
eters in which $J_1$ and $J_2$ are both positive or both
positive: (i) $J=9.89(1)$ meV, $J_1=0.04(1)$ meV,
$J_2=0.13(1)$ meV, $\epsilon=0.013(1)$, $\delta=0.283(4)$, or
(ii) $J=8.30(6)$ meV, $J_1=-0.35(2)$ meV, $J_2=-0.63(3)$ meV,
$\epsilon=0.024(1)$, $\delta=0.383(5)$. By contrast, the spin-orbital many-
level model clearly favors the case with $J_1$ and $J_2$ both posi-
tive. The spin-orbital model can discriminate the two cases
because of the inclusion of the higher excited levels. Only
the parameter set with $J_1$ and $J_2$ both positive fits the low
energy modes ($E<60$ meV) and reproduces the peak in the
spectrum at $\sim 190$ meV and absence of any other measur-
able peaks between 60 and 250 meV. Another drawback of
the effective spin-$1/2$ linear spin-wave model is that the in-
tensities are not accurately described because of the neglect of
the orbital degrees of freedom.

It is interesting to compare the magnetic spectrum of
$\text{La}_2\text{CuO}_4$ with that of other two-dimensional, square-lattice,
antiferromagnetic insulators, particularly in relation to the
anomalous dispersion along the zone boundary. By “anoma-
lous,” we mean that the zone-boundary dispersion cannot be
described within the framework of an antiferromagnetic
spin-wave model in the linear approximation with only
nearest-neighbor interactions. Inclusion of (i) interactions
with more distant neighbors, or (ii) terms beyond the linear
approximation, are two ways in which a zone-boundary
dispersion can be obtained. Other layered antiferromag-
nets which exhibit zone-boundary dispersion include
$\text{La}_2\text{CuO}_4$ (Refs. 25 and 26), $\text{Sr}_2\text{CuO}_2\text{Cl}_2$ (Ref. 27) and
$\text{Cu(DCOO)}_2 \cdot 4\text{D}_2\text{O}$ (CFTD, Refs. 28 and 29). These are all
highly two-dimensional, $S=1/2$ Heisenberg antiferromagnets
with almost isotropic interactions, and it is thought that the
zone-boundary dispersion is caused by nonlinear terms in the
nearest-neighbor Heisenberg model. For example, in
$\text{La}_2\text{CuO}_4$ a model with a four-spin ring exchange was
employed,25 and for CFTD a resonating-valence-bond model
describing entangled spin-dimer states was proposed to ex-
plain the data.29 Interestingly, the behavior along the zone
boundary is different in these two materials: in $\text{La}_2\text{CuO}_4$
both the energy and intensity are higher at X than at $\Sigma$
whereas in CFTD both the energy and intensity are higher at
Finally, we consider the temperature dependence of the magnetic spectrum. Figures 7(a)–7(c) show maps of the magnetic scattering measured at T=6, 150 and 300 K, and Fig. 7(d) displays constant-Q cuts at the magnetic zone center for the same temperatures. On increasing the temperature from 6 to 150 K the 11 meV peak increases in intensity due to the increasing thermal population but remains at the same energy, while the 46 meV peak broadens and shifts to lower energy [Fig. 7(d) inset]. Although La$_2$CoO$_4$ undergoes a first-order phase transition coincident with a magnetic reorientation at $T_s \approx 125$ K, the in-plane lattice parameters in the LTO phase differ only slightly from those in the LTT phase, and the change in the magnetic structure only affects the stacking along the c axis. It is not surprising, therefore, that the transition does not significantly affect the magnetic spectrum. At T=300 K, the spectrum has become quasielastic and there are no longer any sharp inelastic peaks. This indicates the absence of long-range magnetic correlations for $T>T_N$.

**VI. CONCLUSION**

We have measured the excitation spectrum of single-crystal La$_2$CoO$_4$, an excellent realization of a two-dimensional Heisenberg antiferromagnet. We have combined the experimental results with numerical simulations to achieve a very good description of the magnetic spectrum throughout the entire Brillouin zone, up to an energy of 250 meV. The magnetic anisotropy is strongly XY-like, but a small uniaxial anisotropy is present which will make the low-temperature magnetic properties Ising-like. An anomalous dispersion along the antiferromagnetic zone boundary is observed and can be reproduced by including exchange interactions beyond the nearest neighbors but which could also be a manifestation of quantum fluctuations in a nearest-neighbor model.

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