Calculation of road clothes by elastic deflection criteria taking into account damage to asphalt concrete

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Abstract. In the article the analysis of methods of calculating non-rigid road pavements according to the criterion of elastic deflection. The possibility of using different solutions (Burmister, Korsunsky, Ivanov, etc.) to calculate the deflection of layers of monolithic and granular materials is evaluated. Based on this analysis, the formulas for calculating the equivalent modulus of elasticity of a two-layer system, the upper layer of which is made of monolithic or granular material, are recommended. The authors made the modification of formulas for calculating the deflection of asphalt concrete layers and the equivalent modulus of elasticity on their surface. The modification was made by introducing into the calculated dependences a scalar quantity, called damage. To calculate the damage, the principle of equivalence of deformations is recommended, in which the damage is determined using the relations of elastic modules of the damaged and solid body. To calculate the elastic modulus of the damaged body, a functional dependence on the number of loads was obtained.

1. Introduction

The calculation of pavement by the elastic deflection criterion is one of the three conditions for the designing pavement for strength. This criterion allows calculating the thickness of each layer of pavement so that its deflection \( l \) does not exceed the limit value of \( l_{\text{lim}} \). According to the requirements of Russian standards, the strength of the pavement is estimated by the coefficient of strength \( K_{\text{str}} \).

When calculating the pavement by the elastic deflection criterion, the strength coefficient is determined by one of two formulas:

\[
K_{\text{str}} = \frac{l_{\text{lim}}}{l}; \quad \text{or} \quad K_{\text{str}} = \frac{E_{\text{com[required]}}}{E_{\text{com}}},
\]

where \( E_{\text{com}} \) – calculated total modulus of elasticity of construction, determined at the design load, MPa; \( E_{\text{com[required]}} \) – required total modulus of elasticity of the structure, determined at the design load, MPa.

The calculation of the pavement can be performed by determining the elastic deflection or calculated total modulus of elasticity of construction. When calculating the pavement according to the standard ODN 218.046-01, the calculated total modulus of elasticity of the structure is determined. As the required value of the modulus of elasticity of construction, a parameter called the minimum required total modulus of elasticity of construction \( E_{\text{min}} \) is used. The calculation criterion is written in the form:
the required strength coefficient of the pavement on the criterion of elastic deflection, taken depending on the required level of reliability and type of pavement.

The value of the minimum required total modulus of elasticity is calculated by the formula:

\[ E_{\text{min}} = 98.65 \cdot \left( \lg \sum N - c \right) \],

where \( \Sigma N \_p \) - total calculated number of load applications over the life of the pavement; \( c \) - empirical parameter taken equal to the calculated axle load: 100 kN - 3.55; 110 kN - 3.25; 130 kN - 3.05.

From the analysis of dependence (3) it follows that it is intended to take into account the influence of fatigue processes on construction strength. At the same time, with increasing load on the axis, the parameter \( c \) decreases. This leads to the fact that for the same value of \( \Sigma N \), the value of the \( E_{\text{min}} \) module will be the greater, the greater the load on the axis. Note that the use of formula (3) is the advantage of the calculation, but this dependency does not take into account the fatigue of the layer material. For example, for asphalt concrete pavements and bases the intensity of the development of fatigue processes depends primarily on the value of the stresses that occur at the most dangerous point of the layer, as well as on the number of repeated loads. The value of the stresses depends on both the load and the stiffness of the base underlying the layer in question. The more pliable the base, that is, the smaller the total modulus of elasticity of the base, the greater the value of stresses arising in the layer under consideration. Therefore, a more accurate and justify solution would be one in which the fatigue function refers to the material of the layer in question, rather than to the entire pavement.

Given this observation, the authors will attempt to introduce into known solutions a fatigue function that takes into account the accumulation of damage in the asphalt concrete layer. To perform this modification, it is necessary to analyze the solutions of the problem of the deflection of a layered half-space under the action of a load distributed over a round platform.

Solutions designed to calculate the deflection of pavement can be classified by the number of layers in the structure. Solutions to the problem of deflection of two-layer, three-layer and multi-layer systems are known [1,2]. The second factor in the classification of solutions are the conditions on the contact of layers. The contact of layers can be perfectly smooth interface, but can be considered as perfectly rough interface [3,4]. The conditions on the contact of the layers are taken into account when solving the problem of bending of the layered system. When applying the methods of the theory of elasticity, the boundary conditions of the solved differential equations depend on the conditions on the contact of the layers. Solutions of the problem of the deflection of the layered system under different contact conditions are given in the works [1,2], G.M. Dormon [5], Y.H. Huang [6], A. Jones [7], D. Milovic [8], K.R. Peattie [9], E.L. Skok and Finn F.N. [10], K. Ueshita and G.G. Meyerhof [11], A.C. Whiffin and N.W. Lister [12] and other specialists. Such solutions are designed to determine the deflection of the pavement, they are given in the form of accurate mathematical formulas or an approximate method based on the use of auxiliary graphs and tables. As an example of exact mathematical solutions in table 1 dependences of D. M. Burmister [1] and specialists of Russia are given.

Considering the solutions of table 1, we note that they are obtained using different starting ideas. The solutions of D. Burmister and M.B. Korsunsky are based on the theory of bending of thin plates. Decision of N.N. Ivanov performed in the framework of the axisymmetric problem of calculating the draft of a layer of finite thickness on an elastic half-space. These two groups of solutions have their advantages and disadvantages. For example, the solutions of D. Burmister and M. B. Korsunsky give the upper layer of a two-layer system the ability to work on the bending. This is true for layers of monolithic materials, which have a thickness-to-width ratio characteristic of thin plates. Layers of granular materials do not work on bending. It is known from the Mohr-Coulomb criterion that the resistance to all-round stretching is the product of the cohesion and the cotangent of the angle of internal friction. In granular materials and sandy soils cohesion is small, so these materials are not able...
to work on the bending. The decision of N.N. Ivanov obtained from the preconditions of the actions in the layer of normal stress the vertical compression. This is true for granular materials and soils, but this assumption cannot be accepted for layers of monolithic materials that are thin plates.

Therefore, we can conclude that it is expedient to use in the calculations a combined solution that combines the approaches of N.N. Ivanov and M.B. Korsunsky.

### Table 1. Formulas for calculating the deflection of a two-layer system.

| Author of decision | Mathematical dependence |
|--------------------|-------------------------|
| D.M. Burmister     | $l = I_0 \cdot m \cdot r \cdot \left[ \frac{1 + m \cdot h - 0.5 \cdot L - 0.5 \cdot K \cdot (1 + 2 \cdot m \cdot h) \cdot \exp(-m \cdot h)}{1 - (L + K + 4 \cdot K \cdot m^2 \cdot h^2) + K \cdot L \cdot \exp(-4 \cdot m \cdot h)} + \frac{K \cdot L \cdot (1 - 0.5 \cdot h - 0.5 \cdot K \cdot (1 + 2 \cdot m \cdot h) \cdot \exp(-3 \cdot m \cdot h))}{1 - (L + K + 4 \cdot K \cdot m^2 \cdot h^2) + K \cdot L \cdot \exp(-4 \cdot m \cdot h)} \right]$; |
|                    | $K = \frac{1 - \frac{E_2 \cdot (1 + \mu_1)}{E_1 \cdot (1 + \mu_2)}}{1 + \frac{E_2 \cdot (1 + \mu_1)}{E_1 \cdot (1 + \mu_2)}} \cdot \left[ \frac{3 - 4 \cdot \mu_2}{n + (3 - 4 \cdot \mu_2)} \right]$; |
|                    | At $\frac{h_{eq}}{D} \geq 0.5$ ; $l = \frac{p \cdot K \cdot D \cdot (1 - \mu_{med}^2)}{E_2} \left( 1 - \frac{2}{\pi} \cdot \text{arctg} ^2 \frac{h_{eq}}{D} \right)$; |
|                    | At $\frac{h_{eq}}{D} < 0.5$ ; $l = \frac{p \cdot K \cdot D \cdot (1 - \mu_{med}^2)}{E_2} \left( 1 - \frac{2}{\pi} \cdot \text{arctg} ^3 \frac{h_{eq}}{D} \right)$, |
| M.B. Korsunsky     | $h_{eq} = 1.1 \cdot h \cdot \left( \frac{E_1 \cdot (1 - \mu_2^2)}{E_2 \cdot (1 - \mu_1^2)} \right)^{1/3}$; |
|                    | $l = \frac{\int_0^D \sigma(z) \cdot dz}{E_{col(y)}} = \frac{p}{E_1} \int_0^D \frac{p \cdot dz}{1 + \frac{h_{eq}}{D}}$; $h_{eq} = h_1 \left( \frac{E_1}{E_2} \right)^{1/n}$; |
| N.N. Ivanov        | $l = \frac{p \cdot D}{a \cdot \sqrt[3]{E_2}} \left( \frac{\pi}{2} - \left[ \frac{E_2}{E_1} \right]^{4/3} \cdot \text{arctg} \left( \frac{h_{eq}}{D} \right)^{1/2} \left( \frac{E_1}{E_2} \right)^{1/3} \right)$, |

where $I_0$ – Bessel function of the first kind, zero order; $m$ – dimensionless parameter; $r$ – radial distance from the center of loaded area to the point, $m$; $K$ and $L$ – functions, depending parameters of layers materials (elastic modulus and Poisson's ratios); $h$ – thickness of the upper layer of the two-layer system; $E_1$ and $E_2$ – elastic modulus of materials of the upper and lower layers, if underlying layer is a layered half-space, then $E_2$ can be found as the total modulus of elasticity of the layers located below the considered layer, Pa; $\mu_1$ and $\mu_2$ – Poisson's ratios of materials of the upper and lower layers of a two-layer system; $p$ – pressure of the wheel on the coating (assumed to be 600 kPa or 800 kPa); $D$ – diameter of the circle on which the load is distributed, m; $h_{eq}$ – equivalent thickness, m; $\mu_{med}$ – average value of the Poisson's ratio (taken 0.3); $\sigma_a(z)$ – function of the change in
vertical normal stress at the layer depth, Pa; \( n \) – coefficient taking into account the conditions on contact layers; \( a \) – parameter that takes into account the type of pavement.

2. Materials and Methods
In mechanics, fatigue processes are taken into account by the introduction of special functions that allow to take into account the effect of damage accumulated in the material on its strength and deformability parameters [13-18]. For long-acting loads, fatigue functions are a function of time, which takes into account the value of the stresses. When exposed to cyclic loads fatigue function takes into account the number of applications of the load and the value of the stresses. Asphalt concrete pavement perceives repeatedly applied loads. This gives reason to look for a fatigue function that takes into account the total number of applied design loads and the value of the stress arising from the design load.

Based on fatigue theories [13], we note that two measures called continuity \( C \) (introduced by L.M. Kachanov) and damage (introduced by Yu.N. Rabotnov) are used to account for the value of damage. These measures are related to each other, and their relationship is expressed by formulas [14,15]

\[
D + C = 1; \quad D = 1 - C; \quad C = 1 - D.
\]

Given the dependencies (4), it is sufficient to enter only one of these two measures into the calculations, for example, damage, which is determined by the formula [13, 19]:

\[
D = \frac{A - A_{\text{eff}}}{A} \quad \sigma_{ij} = \frac{\sigma_{ij}}{1 - D},
\]

where \( A \) – is the overall cross-section area, m\(^2\); \( A_{\text{eff}} \) – is the effective cross-section area, m\(^2\); \( \sigma_{ij} \) – components of the stress tensor, Pa; \( \sigma_{ij} \) – components of the effective stress tensor, Pa.

In works [14,15,19] it is shown that the second dependence of formulas (5) allows to enter parameter \( D \) in criteria of durability of continuous bodies, modifying them so that there is a possibility to consider damages.

In this paper it is necessary to obtain the functional dependence \( D \) on the number of loads and stresses. Note that for the calculation of the damage \( D \) when exposed to repeated loads use different principles of summation of damages: linear (principle of the Palmgren-Miner), bilinear (principle Manson) or nonlinear (principle of the Richart-Newmark) [20-22].

It is now known that at a certain level of stress road materials, including asphalt, are deformed non-linearly. Due to the non-linear nature of the deformation, the damage in the asphalt accumulated is also not linear [15, 18]. To find a functional dependence that relates the damage \( D \) to the number of applied loads, consider one of the principles of determining the parameter \( D \) [13]. This method of calculating the parameter \( D \) is called the principle of equivalence of deformations of the damaged and solid body. The properties of the damaged material are characterized by the elastic modulus \( E_{1D} \), and the properties of the intact material are characterized by the elastic modulus \( E_1 \). The damage is calculated by the formula:

\[
D = 1 - \frac{E_{1D}}{E_1}.
\]

Suppose that in the process of impact by repeated loads the negative increment of the modulus of elasticity from the load with number \( n \) is described by the power-law equation

\[
\Delta E_{1n} = -b \cdot n^c \frac{dv}{dx},
\]

where \( b \) and \( c \) – parameters that take into account the level of stress state and physical characteristics of asphalt concrete (mixture composition, temperature, degree of compaction, etc.)
\[ E_{DN} = E_1 \left( 1 - b \cdot \frac{N}{n} \right) \]  \quad (8)

For the case \( c \neq 1 \) we obtain:

\[ E_{DN} = E_1 \left( 1 - b \cdot \frac{N}{n} \right) = E_1 \left( 1 - b \cdot \frac{N^{c-1}}{c+1} \right) \]  \quad (9)

If \( c = -1 \), then as a result of integration the law of accumulation

\[ E_{DN} = E_1 \left( 1 - b \cdot \frac{1}{N} \right) = E_1 \cdot (1 - b \cdot \ln N) \]  \quad (10)

Substituting dependencies (9) and (10) into formula (6), we obtain

\[ D = b \cdot \frac{N^{c-1} - 1}{c+1}; \quad D = b \cdot \ln N \]  \quad (11)

Substituting dependencies (9) and (10) into formula (6), we obtain

\[ E_{DN} = E_1 \left( 1 - b \cdot \frac{N^{c-1} - 1}{c+1} \right) \] or \[ E_{DN} = E_1 \cdot (1 - b \cdot \ln N) \]  \quad (12)

Formulas (12) can be applied in the calculations of road structures by the elastic deflection criterion.

3. Results

As a modified calculation of the pavement, we consider the improvement of generally accepted solutions obtained by N.N. Ivanov and M.B. Korsunsky, to calculate the equivalent modulus of elasticity of a two-layer system. Calculation of the elastic modulus of a two-layer system with an upper layer made of a material working on compression (for example, from a granular material) must be calculated by the formula N.N. Ivanov. This formula is used in its original form; no damage parameter is entered into it.

\[ E_{\text{com}} = E_2 \cdot \left( 1 - 2 \cdot \frac{E_2}{E_1} \cdot \frac{h}{2} \cdot \arctg \frac{h}{2} \cdot \left( \frac{E_1}{E_2} \right)^{\frac{1}{2}} \right)^{-1} \]  \quad (13)

As a modulus \( E_2 \) take the modulus of elasticity of the soil or equivalent modulus on the surface of the layers below the calculated layer having a thickness \( h \) and modulus \( E_1 \).

Instead of formula (13), an alternative dependence can be derived. For this purpose in integral equations N. N. Ivanov, presented in table 1, it is enough to replace the functional dependence of the stress change in depth. In addition, the influence of other stress tensor components on the deformation can be taken into account in the integral equations. For such accounting, you can use the functions obtained in the works [23–26].

The elastic modulus of a two-layer system with an upper layer of bending material is calculated by the formula.
In formula (14), the elastic modulus of the damaged material is calculated by one of the formulas (12). The purpose of the calculation remains the same, it is necessary to ensure the implementation of criterion (2). The calculation is performed according to the scheme from the bottom up.

4. Discussion
According to the results of the research, it can be argued that:

- Determination of the modulus of elasticity of asphalt concrete, taking into account the scalar value D, which considers growth of damage in asphalt concrete and representing a function of the number of loads, allows, in comparison with the known solutions, more justified to calculate the thickness of asphalt concrete layers from the standpoint of fatigue theory.
- Determination the value of the total modulus of elasticity of the layers of granular materials located below the calculated asphalt concrete layer, according to the formula N.N. Ivanov more accurately takes into account the stress state of such materials.

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