The spin Hall effect in a quantum gas

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Electronic properties such as current flow are generally independent of the electron’s spin angular momentum, an internal degree of freedom possessed by quantum particles. The spin Hall effect, first proposed 40 years ago, is an unusual class of phenomena in which flowing particles experience orthogonally directed, spin-dependent forces—analogous to the conventional Lorentz force that gives the Hall effect, but opposite in sign for two spin states. Spin Hall effects have been observed for electrons flowing in spin–orbit–coupled materials such as GaAs and InGaAs (refs 2, 3) and for laser light traversing dielectric junctions. Here we observe the spin Hall effect in a quantum-degenerate Bose gas, and use the resulting spin-dependent Lorentz forces to realize a cold-atom spin transistor. By engineering a spatially inhomogeneous spin–orbit coupling field for our quantum gas, we explicitly introduce and measure the requisite spin-dependent Lorentz forces, finding them to actuate analogue to the archetypal semiconductor spintronic device, the Datta–Das spin transistor9,10.

The spin Hall effect (SHE) is generated by spin-dependent forces orthogonal to a particle’s motion—akin to the Lorentz force—that can act on electronic2,3,11 photons4 or, as here, neutral atoms. Each of these has an internal, or ‘spin’, degree of freedom (a generalization of conventional quantum mechanical spin) that can be either up or down, creating a spin-1/2 (or pseudospin-1/2) system. In materials, microscopic spin–orbit coupling (SOC) induces the SHE in one of two primary ways: by means of an intrinsic mechanism driven directly by SOC13 or by means of an extrinsic mechanism that additionally requires scattering from impurities14,15. The motion of spins in systems with a SHE is strikingly similar to the motion of charges in an external magnetic field, but with equal and opposite effective Lorentz forces for each of the two spin states. Thus, just as the Lorentz force gives rise to the Hall effect for charged particles, spin-dependent Lorentz forces (SDLFs) generate SHEs.

In the Hamiltonian description of quantum mechanics, forces are described in terms of associated potentials. For example, a magnetic field \( \mathbf{B} = \nabla \times \mathbf{A} \) is generated from a vector potential \( \mathbf{A} \) that enters into the Hamiltonian \( \hat{H} = (\hat{p} - q_0 \hat{A})^2 / 2m \) with canonical momentum \( \hat{p} \), charge \( q_0 \) and mass \( m \) (‘hats’ on variables indicate quantum mechanical operators acting on continuous degrees of freedom). We engineered a vector potential that depends on an effective spin degree of freedom with opposite sign for the two effective spin states. This can create a SDLF and a SHE when the spins move perpendicularly to the resulting spin-dependent \( \mathbf{B} \).

More formally, this vector potential can be expressed as a vector of 2 × 2 matrices, leading to a relationship between the generalized vector potential \( q_0 \hat{A} \rightarrow \mathbf{A} \) and generalized magnetic field \( q_0 \mathbf{B} \rightarrow \mathbf{B} \) (ref. 14; ‘checks’ on variables indicate quantum mechanical operators acting in pseudo-spin space):

\[
\mathbf{B} = \nabla \times \mathbf{A} - \frac{i}{\hbar} \mathbf{A} \times \mathbf{A}
\]  

(1)

The Heisenberg equations of motion show that \( \mathbf{B} \) is the generalized magnetic field in a spin-dependent Lorentz force law (Methods). The first term in equation (1) is analogous to the conventional magnetic field, and the second term is non-zero only when the vector components of \( \mathbf{A} \) do not all commute, that is, when \( \mathbf{A} \) is non-Abelian. The generalized Lorentz force for the two spin states can be equal and opposite, for example when \( \mathbf{B} = B_0 \hat{\sigma}_y \mathbf{e}_z \), where \( B_0 \) describes the field’s magnitude; \( \hat{\sigma}_1 \), \( \hat{\sigma}_2 \) and \( \hat{\sigma}_3 \) are the 2 × 2 Pauli matrices; and \( \mathbf{e}_x \), \( \mathbf{e}_y \) and \( \mathbf{e}_z \) are the three Cartesian unit vectors.

Two different classes of vector potentials (unrelated by gauge transformations) lead to this magnetic field, one resulting from each term in equation (1). For example, in two-dimensional material systems, almost every possible form of linear SOC—combinations of linear Dresselhaus16 or Rashba16 couplings—is equivalent to a spatially uniform non-Abelian vector potential with \(-i(\mathbf{A} \times \mathbf{A})/\hbar \hat{\sigma}_3 \mathbf{e}_z \) (Methods and ref. 17). In contrast, we engineered a spin–orbit-coupled Hamiltonian with a spatially dependent, Abelian vector potential that produces \( \nabla \times \mathbf{A} \propto \hat{\sigma}_3 \mathbf{e}_z \).

In this letter, we demonstrate spin Hall effects in atoms. A spin–orbit-coupled Bose–Einstein condensate is prepared in two spatially separated wells of opposite spin, illustrated in Figure 1a. A modified Raman laser coupling \( \omega + \delta \omega \) at \( \delta \omega \rightarrow \delta \omega \rightarrow \delta \omega \) propagating along \( \mathbf{e}_z \) couples two states in the \( f = \frac{1}{2} \) ground–state manifold of \(^{87}\)Rb. Dynamic control of an optical trapping beam propagating along \( \mathbf{e}_y \) allowed the BEC to be moved along \( \mathbf{e}_y \), giving a time- and position-dependent Raman coupling. The Raman coupling altered the free-particle dispersion along \( \mathbf{e}_y \), creating double wells25 in quasimomentum \( q \). The modified dispersions, \( E(q) \), shown for the three different \( y \) positions (i, ii and iii) marked in a. We associate states near the minimum of each well with dressed spins, and identify the location of the minima with a vector potential \( \mathbf{A} \).

Figure 1 | Schematic of experimental set-up.

a, Raman beams with frequencies \( \omega \) and \( \omega + \delta \omega \) propagating along \( \mathbf{e}_z \) coupled two states in the \( f = \frac{1}{2} \) ground–state manifold of \(^{87}\)Rb. Dynamic control of an optical trapping beam propagating along \( \mathbf{e}_y \) allowed the BEC to be moved along \( \mathbf{e}_y \), giving a time- and position-dependent Raman coupling. The Raman coupling altered the free-particle dispersion along \( \mathbf{e}_y \), creating double wells25 in quasimomentum \( q \). b, The modified dispersions, \( E(q) \), shown for the three different \( y \) positions (i, ii and iii) marked in a. We associate states near the minimum of each well with dressed spins, and identify the location of the minima with a vector potential \( \mathbf{A} \).

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The relationship between these two distinct vector potentials is unusual. Although the equations of motion describe the same SDLF leading to an intrinsic SHE, the associated energy spectra are different (for example, in the two-dimensional material systems discussed above, $[B, H] \neq 0$, implying that $B$ is time dependent in the Heisenberg picture). However, both can give rise to time-reversal-invariant topological insulators. The case with a spatially uniform vector potential mirrors the typical situation in materials where the intrinsic SOC leads to topological band structure. The case with a spatially dependent vector potential leads to the simplest conceptual example of a topological insulator: two superimposed quantum Hall systems with equal but opposite magnetic fields (a single quantum Hall system is a topological insulator, but with broken time-reversal symmetry). Both types of vector potential exhibit the quantum SHE (QSHE) leading to topological insulators. Those resulting from spatially dependent vector potentials are a direct extension of the quantum gas SHE demonstrated in this work but are impractical in material systems (Methods and Supplementary Information).

We realized the SHE with ultracold atoms following the proposal of ref. 19 by subjecting pseudospin-1/2 $^{87}$Rb Bose–Einstein condensates (BECs) to spin- and space-dependent vector potentials. Two laser beams (which we will refer to as ‘Raman lasers’) with wavelength $\lambda$, propagating in opposite directions parallel to $\mathbf{e}_x$, coupled the $|j = 1, m_p = 0, -1\rangle = |\uparrow, \downarrow\rangle$ spin states comprising our pseudospin-1/2 system (in analogy to the spin-1/2 electron) with strength $\Omega$ (Fig. 1a). The wavelength determines the single-photon recoil energy, $E_R = \hbar^2 k_R^2 / 2m$, momentum, $\hbar k_R = 2\pi\hbar / \lambda$, and velocity, $v_R = \hbar k_R / m$, where $m$ is the mass of a $^{87}$Rb atom and $2\pi\hbar$ is Planck’s constant. In this configuration, the Hamiltonian describing motion along $\mathbf{e}_x$ includes an effective SOC term $^{21-24}$, altering the dispersion relation as shown in Fig. 1b. Thus modified, the dispersion relation of these laser-dressed atoms features two degenerate wells, each displaced from zero by an amount $A = k_R \left[1 - (\hbar \Omega / 4E_R)^2\right]^{1/2}$ for $\hbar \Omega < 4E_R$ (Methods Summary). Particles with momenta near these minima can be thought of as dressed spin states $|\uparrow\rangle, |\downarrow\rangle$ (which we will colloquially refer to as spin states when no ambiguity is possible) in the presence of a vector potential $A = A \delta_3 \mathbf{e}_x$. Given that $\Omega$ depends on the intensity of the Raman lasers, $A$ has the spatial dependence of the Raman lasers’ Gaussian intensity profile.

The spatial dependence of $A$ gives rise to a SHE in our quantum gas $^{20,23}$. To probe the mechanism underlying the SHE, we abruptly changed $A$ and observed spin-dependent shearing of the atomic cloud (Fig. 2). We then observed—for a time-independent $A$—the resulting SHE using two techniques: we propelled atoms in either state $|\uparrow\rangle$ or state $|\downarrow\rangle$ along $\mathbf{e}_x$ and detected a spin-dependent Lorentz-like response along $\pm \mathbf{e}_x$ (Fig. 3); and, using a mixture of both dressed spin states, we used the SDLF to realize a spin transistor (Fig. 4).
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beams, travelling along |1⟩, |1⟩ or mixtures thereof, confined in a crossed-beam optical dipole trap with typical axial frequencies \(\omega_z/2\pi \approx 35 \text{ Hz}\), \(\omega_x/2\pi \approx 35 \text{ Hz}\) and \(\omega_y/2\pi \approx 100 \text{ Hz}\). The \(\lambda = 790.13\text{-nm} \) Raman laser beams, travelling along \(\pm \hat{e}_x\), had 170-μm waists (1/e^2 radius, where \(\epsilon\) is Euler’s number). We moved the BECs along \(\hat{e}_x\), sampling this inhomogeneous Raman laser profile, by displacing the appropriate trap beam. At any given initial \(y\) position, \(y_0\), we then adiabatically turned on the Raman lasers in 150 ms, Raman-dressing the BEC and transforming our initial spin states into their dressed counterparts, at rest

![Image](https://example.com/image.png)

**Figure 4** | Spin Hall currents. a, Calculated spin current versus potential gradient, \(V\), and coupling strength, \(\Omega\). The two cuts (black lines) show the parameters at which measurements of \(J_y\) were made. b, Spin current, \(J_y\), versus \(V\). We note that \(d\Omega/dy = -115 \mu\text{m} = 2.3(2)\text{E}_R\). The central solid curve is a fit of our model to the data, with fitted value \(k_Q = 2.05\text{E}_R\). The remaining curves are the modelled response at \(d\Omega = 1\text{E}_R\) (lower magnitude) and 2.5\text{E}_R (higher magnitude). Inset: FET drain–source current, \(I_{DS}\), versus drain–source voltage, \(V_{DS}\), for three different gate–source voltages, \(V_{GS}\) = 1.5, 2.4 and 3.5 V above threshold. c, Dependence of spin current on coupling strength at a fixed trap displacement from \(y_0 = -122\mu\text{m}\) to \(y_f = -95\mu\text{m}\) with \(V = 272/\hbar\Omega_{\text{rel}}\). The solid curve is the fit of our model to the data (Methods). The scatter in the data is reflective of typical uncertainties.

These experiments began with BECs of \(5 \times 10^6\) atoms prepared in |1⟩, |1⟩ or mixtures thereof, confined in a crossed-beam optical dipole trap with typical axial frequencies \(\omega_z/2\pi \approx 35 \text{ Hz}\), \(\omega_x/2\pi \approx 35 \text{ Hz}\) and \(\omega_y/2\pi \approx 100 \text{ Hz}\). The \(\lambda = 790.13\text{-nm} \) Raman laser beams, travelling along \(\pm \hat{e}_x\), had 170-μm waists (1/e^2 radius, where \(\epsilon\) is Euler’s number). We moved the BECs along \(\hat{e}_x\), sampling this inhomogeneous Raman laser profile, by displacing the appropriate trap beam. At any given initial \(y\) position, \(y_0\), we then adiabatically turned on the Raman lasers in 150 ms, Raman-dressing the BEC and transforming our initial spin states into their dressed counterparts, at rest

![Image](https://example.com/image.png)

We explored the spin and space dependence of the vector potential \(A(y)\) by observing the response of the BECs to abrupt temporal changes in \(A\). When \(A\) depended on \(y\), these changes sheared the BECs’ density distribution. We prepared spin-polarized BECs at a variable position, \(y_0\) (Methods). Each Raman-dressed BEC therefore sampled a range of Raman coupling strengths across its 40-μm diameter (Fig. 2a). When the Raman lasers were abruptly turned off, the initially motionless BEC experienced a spin-dependent ‘electric’ force, \(-\partial A/\partial t\), resulting from a time-changing vector potential along \(\hat{e}_r\) (ref. 28). We probed this system by switching off the dipole trap and the Raman beams in less than 1 μs and absorption-imaging the atoms after a 30-ms time of flight (TOF; its duration was common to all of our measurements).

Because \(A(y)\) depended on both spin and \(y\) position, we observed a spin- and \(y_0\)-dependent shear in the density distribution (Fig. 2b) after TOF, described by \(n(x,y,z) \propto 1 - (x/R_x)^2 - (y/R_y)^2 - (z/R_z)^2 - s_{xy}xy/R_xR_y - s_{yx}yx/R_yR_x\), where \(R_x, R_y\) and \(R_z\) are the BEC’s Thomas–Fermi radii. The spatial dependence of \(A\) is quantified by the shear coefficient, \(s_{xy}\), which is obtained by fitting this distribution (integrated along \(\hat{e}_x\)) to the TOF BEC density distribution. The spin-dependent nature of the vector potential is evident in the opposite sign of the shear for each spin (Fig. 2b–f) and in that the magnitude of the shear coefficient follows the local derivative of the vector potential at the centre of the BEC.

We first observed the SHE using spin-polarized BECs. This would be atypical in condensed-matter systems, where both spins are usually present. After preparing a spin-polarized BEC at a position, \(y_0\), between \(y_{\text{min}} = -135\mu\text{m}\) and \(y_{\text{max}} = -95\mu\text{m}\) (grey shaded region in Fig. 2a, a region over which the SDLF was both reasonably large and uniform), we abruptly displaced the centre of the harmonic trap to \(y_f = y_{\text{max}}\) or \(y_f = y_{\text{min}}\). This displacement can formally be understood as resulting from an applied potential with gradient \(V\). The atoms accelerated to a final \(y\) momentum \(\hbar k_y\) in \(\sim 7\text{ ms}\) (one-quarter of the \(\epsilon_r\)-trap period).

During this time, the SDLF accelerated the atoms perpendicular to their instantaneous momentum, resulting in a final \(x\)-momentum \(\hbar k_x\). By waiting this quarter-period after trap displacement, we ensured that the atoms always arrived at \(y_f\) (regardless of the choice of \(y_0\)). Subsequently, the trap was turned off abruptly (in less than 1 μs), the Raman lasers were turned off slowly by comparison with dressed-state bandgaps (\(\sim 500\mu\text{s}\)), and the atoms were imaged after TOF to determine their final momentum (Fig. 3b). With this turn-off procedure, the atoms experienced a force \(-\partial A/\partial t\) along \(\hat{e}_r\) (independent of \(y_0\)) that shifted the final centre-of-mass position after TOF from the observed position of atoms released in the absence of \(A\) (Methods). We calibrated this zero-momentum TOF position by detecting atoms released from rest at \(y_f\).

Each spin-polarized BEC acquired a momentum along \(\epsilon_r\) that was directed oppositely for the two spins and related to its final momentum along \(\epsilon_r\), demonstrating an intrinsic SHE. We modelled the dynamics of each spin (Fig. 3b, solid curves, and Methods) by solving the Heisenberg equations of motion. Because our atoms remain in the lowest-energy band plotted in Fig. 1b, the Heisenberg equations of motion reduce to classical dynamics subject to the spin–orbit-coupled dispersion shown in Fig. 1b. The model predicts both \(K_x\) and \(K_y\) as functions of initial and final trap displacement. We leave \(\Omega\) as a fitting parameter, the value of which is within 15% of our calibrated value. The results of this model are plotted along with the data in Fig. 3b.

Next we realized the SHE in a configuration analogous to solid systems by using mixtures of both spins. In the presence of both spins, we define average spin and particle current densities \((\langle j_s \rangle = \langle j_x \rangle + \langle j_y \rangle)\) and \((\langle j_p \rangle = \langle j_y \rangle - \langle j_x \rangle)\), where the average current density for spin \(i\) (either \(\uparrow\) or \(\downarrow\)) is \((\langle j_i \rangle = (1/V) \int n_i(r)v(r)\; dr)\), with density \(n\), velocity \(v\) and in situ BEC volume \(V\). An equal current of each spin moving in the same direction corresponds to a pure particle current, and an equal current of each spin moving in opposite directions gives a pure spin current.

This third class of experiments started with BECs in a mixture of both spins (Methods). We generated a pure particle current using the trap displacement technique described above. As before, the system evolved under the SDLF for \(\sim 7\text{ ms}\) after which time the atoms were released from the trap and the Raman lasers adiabatically turned off (Methods). Each TOF image contained information about both dressed spin states, allowing us to determine the spin and particle currents simultaneously. We modelled the resulting spin current along \(\hat{e}_r\) as a function of coupling strength, \(\Omega\), and potential gradient, \(V\). The system’s spin response, \((\langle j_s \rangle = \langle j_x \rangle \cdot \epsilon_r)\), is shown in Fig. 4a. By varying one parameter at a time (Fig. 4a, black lines), we measured the spin current as a function of \(V\) (Fig. 4b) and as a function of \(\Omega\) (Fig. 4c). In both cases, the experiment agrees with our model.

Despite the existence of the SHE in all spin–orbit–coupled metals and semiconductors, the technology for studying the SHE was developed only recently. Soon afterwards, the SHE was exploited to develop spintronic devices\(^1\). In this spirit, our experiment describes an externally actuated ‘atomtronic’ bipolar spin transistor\(^8,10\), where \(\Omega\) plays the part of the transistor’s gate voltage and the potential gradient, \(V\), is analogous to the drain–source voltage. The spin current turns on abruptly at \(\hbar\Omega \approx \text{1E}_R\) (Fig. 4c), with a final spin current set by the potential gradient. For a given Raman coupling (‘gate voltage’),
however, the spin current turns on smoothly with positive or negative potential gradient (Fig. 4b). This similarity between our system and a field-effect transistor (FET) is further highlighted in Fig. 4b, where the three black curves modelling our system’s response at three different Raman coupling strengths are compared with the characteristic response of a FET’s drain–source current as a function of drain–source voltage at three different gate–source voltages.

In atomic systems, other techniques can separate particles according to spin, such as the well-known Stern–Gerlach effect. Our technique complements these, because the spin-dependent force depends not on the atoms’ positions (as in the Stern–Gerlach effect), but on their velocities. For example, a device with a finite SHE interaction region will deflect an incoming atomic beam by an amount independent of the velocity with which the atoms enter the region; although an increase in initial velocity decreases the interaction time, the perpendicularly force increases (for interaction times much less than 2π/mB). For devices using the Stern–Gerlach effect, the deflection depends only on the interaction time, which changes with initial velocity. A spin transistor might operate using either our SDLF or a Stern–Gerlach-type force, but in each case its behaviour will be quite different. For example, using our transistor as the input and output beam splitter in Mach–Zehnder-type inertial sensors could yield coherent adiabatic momentum splitting that is independent of the atoms’ longitudinal velocity profile.

We have demonstrated an intrinsic SHE in a quantum gas using a precisely engineered spin- and space-dependent vector potential. Systems such as this—with the experimental parameters available at present—are candidates for ac gravity gravimeters, when applied to dilute clouds where interaction effects are negligible. In addition, time-reversal-invariant topological insulators manifest the QSHE. Using present technologies, our method for producing the SHE could produce the QSHE in an ultracold gas of fermionic 40K (Methods and Supplementary Information). Despite the technical challenges, the simplicity of our setup—two atomic spin states and two oppositely directed lasers—makes our approach an appealing means for achieving the QSHE in similar parameter regimes. It may be possible to realize exotic, interacting topological insulators using Bose gases.

METHODS SUMMARY

System preparation. A bias magnetic field of B0 = 2.1 mT lifted the degeneracy of the f = 1, mF = 0, ±1 spin states in the electronic ground-state manifold of 87Rb, leading to an energy splitting, ΔE = 2πk × 15 MHz, between |mF = −1⟩ and |mF = 0⟩ that matched the hδ0 energy difference between the Raman laser beams’ photons, where δ0 is the frequency difference between the Raman beams. Owing to the large bias field, the |mF = +1⟩ spin state was detuned from Raman resonance by 17.8B0 and was inactive in our experiments.

In the limit of zero Raman coupling, each dressed spin continuously connects to a bare spin with quasi-momentum h|q| = hkB0. To load a specific dressed spin, we started with a BEC in |mF = −1⟩, |mF = 0⟩ or a mixture of those states and turned on the Raman lasers in 150 ms. During experiments on spin-polarized BECs, we avoided any undesired population of the other dressed spin state by applying a detuning hδ = ΔE − δ0 = 0.15B0 during the ramp up of δ and then shifting to resonance (δ = 0) with a 1-ma ramp of B0. An acousto-optic modulator shifted the position of the dipole trap beam propagating along z, allowing controlled optical manipulation of the atomic sample along z, and any associated references are available in the online version of the paper.

Dressed states. The single particle properties of our system are well described by the Hamiltonian:

$$\hat{H} = \frac{\hbar^2}{2m}(\frac{\partial^2}{\partial z^2} + \frac{\partial^2}{\partial h^2}) + \frac{\hbar \Omega}{2} \sigma_1 - \frac{\hbar^2 k_{\perp}}{m} \sigma_3 + E_\perp \hat{1}$$

for resonant Raman coupling, as we use here. The eigenergies with

$$E_{\pm}(q) = \frac{\hbar^2}{2m} \left( \frac{k_{\perp}^2}{q} \pm \frac{\hbar k_{\parallel}}{m} \right)$$

define a pair of effective dispersion relations, the lower of which, E−(q), is plotted for k⊥ = k∥ = 0 in Fig. 1b for a selection of coupling strengths.

Full Methods and any associated references are available in the online version of the paper.

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Supplementary Information is available in the online version of the paper.

Author Contributions M.C.B. led the data-taking effort, in which all co-authors and any associated references are available in the online version of the paper.

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METHODS

System preparation. A $B_0 = 3.1$-mT bias magnetic field lifted the degeneracy of the $|f = 1, m_f = 0, \pm 1\rangle$ spin states in the electronic ground-state manifold of $^{87}\text{Rb}$, leading to an energy-level splitting, $\Delta E = 2\hbar \Omega_{\text{RF}} \times 15 \text{ MHz}$, between $|m_F = -1\rangle$ and $|m_F = +1\rangle$ that matched the $\hbar \omega_0$ energy difference between the Raman laser beams' photons, where $\omega_0$ is the frequency difference between the Raman beams. Owing to the large bias field, the $|m_F = +1\rangle$ spin state was detunoned from Raman resonance by $17.8 \hbar \omega_0$ and was inactive in our experiments.

In the limit of zero Raman coupling, each dressed spin continuously connects to a bare spin with quasimomentum $\hbar q = \hbar k_0$. To load a specific dressed spin, we started with a BEC in $|m_F = -1\rangle$, $|m_F = 0\rangle$ or a mixture of those states, and turned on the Raman lasers in 150 ms. During experiments on spin-polarized BECs, we avoided any undesired population of the other dressed spin by applying a detuning $\hbar \omega = \Delta E - \hbar \omega_0 = 0.15 \hbar \omega_0$ during the ramp up of $\Omega$ then by shifting to resonance ($\hbar \omega = 0$) with a 1-ms ramp of $B_0$. An acousto-optic modulator shifted the position of the dipole trap beam propagating along $\hat{e}_z$, allowing controlled translation of the atomic sample along $\hat{e}_z$.

Dressed states. The single particle properties of our system are well-described by the Hamiltonian

$$H = \frac{\hbar^2}{2m} (\frac{q^2}{2} + \frac{k_x^2 + k_y^2}{2} + \hat{e}_z) + \frac{\hbar \Omega}{2} \hat{\sigma}_x + \frac{\hbar^2 \omega_0 q}{m} \hat{\sigma}_x + \hbar \omega_0 \hat{1}$$

for resonant coupling, as we use here. The eigenenergies

$$E_{\pm}(q) = \pm \sqrt{\frac{\hbar^2 \omega_0 q}{m} + \frac{\hbar^2 \Omega^2}{2}}$$

define a pair of effective dispersion relations, the lower of which, $E_-(q)$, is plotted for $k_z = 0$ in Fig. 1b for a selection of coupling strengths.

Quantum SHE. Our technique for producing the SHE can be extended to realize the QSHE in two-dimensional ultracold Fermi gases. A simple example system that exhibits the QSHE can be constructed by overlapping two integer quantum Hall systems with filling factors of $\nu = 1$ and opposite magnetic field, the second of which implies that they have opposite Chern numbers$^{18}$. Although this construct—two separate, spatially overlapped electron systems that experience opposite magnetic field—is artificial, the QSHE can arise from an equal mixture of spins experiencing strong opposite, spin-dependent ‘magnetic’ fields.

To understand intuitively how this might work, we consider our effective pseudospin Hamiltonian in two dimensions for $\hbar^2 < 4\hbar \omega_0$ (ignoring the optical confinement, the scalar light shift from the Raman lasers and the zero-energy shift from the Raman dressing):

$$H = \frac{1}{2m} (\begin{pmatrix} \hat{p} - A \hat{\sigma} \omega \hat{e}_x \end{pmatrix})^2$$

Here $\hbar$ is the $2 \times 2$ identity matrix, $A = \hbar k_0 [1 - (\hbar \Omega / 4 \hbar \omega_0)]^{1/2}$ is the magnitude of the Raman-laser-induced vector potential, $A = A \sigma_x\hat{e}_x$ is the matrix-valued vector potential, $\hat{p}$ is the canonical momentum and $m^*$ is the effective mass tensor. Here, pseudospin is a good quantum number and the system can be thought of as two independent systems that respond oppositely to temporal and spatial gradients in $A$. By introducing a large, non-zero $V \times A$, each spin taken separately could be driven to the integer quantum Hall regime, thereby creating a QSHE in a system composed of an equal mixture of both spins.

Our specific proposal to extend our work and realize the QSHE uses $^{87}\text{Rb}$ confined in a quasi-two-dimensional geometry in the $\hat{e}_z-\hat{e}_x$ plane. Two Raman lasers counterpropagating along $\hat{e}_z$ couple together two magnetic sublevels in the $|f = 0, \pm 1\rangle$ ground-state manifold. Tailoring the Raman lasers (using a spatial light modulator$^{21-24}$, for instance) to have a position-dependent coupling $d(t) = 4\hbar \omega_0 \sqrt{\frac{l_x^2 + l_y^2}{L_y}}$ for $y(0, L_y)$ along $\hat{e}_x$ produces a linearly varying $A$. Each pseudospin experiences an oppositely directed, uniform, synthetic magnetic field with cyclotron frequency $\omega_c = \hbar k_0 m^*/\hbar$, for $y(0, L_y)$.

To reach the QSHE regime, the thermal energy scale, $k_0 T$, the Fermi energy, $\hat{e}_F$, and the cyclotron energy, $\hbar \omega_c$, must satisfy $k_0^2 T = \hbar \omega_c^2 / 4 \hbar \omega_0$ (so that the Fermi energy falls in the gap between the ground and first Landau levels). Here, $k_0$ is Boltzmann’s constant and $T$ is the temperature. The cyclotron frequency therefore sets the energy scales necessary to produce a QSHE. For realistic system sizes of $5$–$10 \mu m$, the cyclotron frequency is $\lesssim 100$ Hz. In Supplementary Information, we make this argument rigorous for our actual experimental configuration.

Notes on Fig. 2. The Raman coupling strength in Fig. 2a was measured as described in refs 23, 35. For the data in Fig. 2b–f, the aspect ratio of the BEC was adjusted from its typical cylindrical symmetry to be 50% longer along $\hat{e}_x$ than along $\hat{e}_y$, by adjusting the optical trap, and the atom number was maintained at $>10^5$.

Measurement and analysis. To measure the atoms’ momenta, the optical confinement was turned off abruptly while the Raman lasers’ intensity was linearly ramped to zero in 0.5–1 ms. This procedure transferred each dressed spin to a bare spin moving with an $x$ momentum equal to its quasimomentum $q$ and a $y$ momentum equal to its in-trap $y$ momentum, $K_y$. A magnetic field gradient applied for a few milliseconds during the 30–60 ms TOF separated the two bare spins along $\hat{e}_y$ through the Stern–Gerlach effect. After this separation, we measured the atomic density distribution and obtained its mean position. To determine the atoms’ in situ momenta, we referenced the measured TOF positions to the TOF positions observed for atoms under the same experimental conditions, but at rest. For example, when the trap was suddenly displaced as in Fig. 3 or 4, the reference position was determined by adiabatically dressing the atoms at the final trap position and measuring the TOF position. Subtracting the TOF position of the abruptly displaced atoms from the reference TOF position allowed us to determine the in-trap momentum.

This measurement of the momenta contained two contributions that biased the TOF positions away from the actual momenta. If the atoms do not reach their equilibrium position in the trap before TOF begins, our subtraction procedure does not yield the actual velocity, because this initial displacement is interpreted as momentum after TOF. According to our simulations, this resulted in a systematic underestimation of the momentum along $\hat{e}_y$ and $\hat{e}_z$. In addition, to compensate gravity during displacement of the optical trap, the overall intensity of the optical trapping beams was increased by 25% at the same time as the position of the optical trap was changed. Owing to the competition between the optical trap and the near-linear spatial dependence of the energy minimum of the Raman-dressed bands, this power increase shifted the equilibrium position of the atoms along $\hat{e}_x$, even in the absence of an optical trap displacement. We measured the equilibrium position of our atoms by increasing the power of the optical trap for ~7 ms without displacing it, leading to a small difference in our measured zero momentum from the actual zero momentum. These effects, which result in a momentum correction of up to 20%, were all included in our simulations.

Small fluctuations in our laboratory magnetic bias field lift the energy degeneracy of the two pseudospin states, leading to fluctuations in the pseudospin population distribution. When working with a mixture of pseudospins, we discarded any measurement for which the population of one spin state was greater than 150% of the other, resulting in up to 60% of the data from each sequence being omitted from analysis. In addition, when both dressed spins were used together, there was an initial spatial segregation of the spins owing to a repulsive interaction between them$^{11,25,26}$. Although the in situ spatial distribution of the spins was modified before the experiment began, this interaction energy did not significantly affect our momentum measurements, because the in situ displacement was small compared with the typical TOF displacement caused by the momentum signal.

Simulations. Because transitions between the dressed–spin bands are energetically suppressed owing to the large energy gap between bands (compared with the energy of the dynamics), the Heisenberg equations of motion for our system were the same as Hamilton’s classical equations of motion in the lowest band. In our simulation, the classical calculation included the modified position-dependent dispersion relation along $\hat{e}_x$ (Fig. 1b), the scalar potential from the Raman beams, the scalar potential from the optical dipole trap and the gravitational potential. The dispersion relation was calculated by diagonalizing our system’s spin–orbit-coupled Hamiltonian$^{27}$ and retaining only the lowest energy band. It is the position-dependent modified dispersion relation that drives the observed SHE. The solutions to Hamilton’s coupled differential equations yielded values for the position and momentum (or quasimomentum) in all three spatial directions as functions of time. For a given dressed spin, the simulated mechanical momentum $K_y$ was the difference between $\phi(t)$ and the location of the minimum of the dispersion curve associated with that dressed spin. Our model does not predict values of $K_y > 1.2 k_0 T$, that were observed in the experiment, but this can be explained by deviations of our optical trap from the ideal Gaussian beams used in our model.

Linear Dresselhaus and Rashba SOC as a vector potential. Consider the Rashba and linear Dresselhaus SOC Hamiltonians in two dimensions$^{28}$:

$$H_{\text{RH}} = \frac{\hbar}{m^*} (\hat{p}_x \hat{\sigma}_z - \hat{p}_z \hat{\sigma}_x)$$

and

$$H_{\text{LDS}} = \frac{\hbar}{m^*} (\hat{p}_x \hat{\sigma}_z - \hat{p}_z \hat{\sigma}_x)$$

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Here $\hat{p}_i$ is the momentum along the $i\text{e} = \{e_x, e_y, e_z\}$ spatial direction and $\alpha$ and $\beta$ are the respective strengths of the Rashba and Dresselhaus SOCs. The total Hamiltonian containing both of these terms

$$\hat{H}_\text{SOC} = \frac{\hat{p}_i^2}{2m} + \hat{H}_\text{D} + \hat{H}_\text{R}$$

can be expressed as

$$\hat{H}_\text{SOC} = \frac{1}{2 m} \left( \hat{p} - \hat{\mathbf{A}} \right)^2 - \frac{m}{\hbar^2} \hat{1} (\alpha^2 + \beta^2)$$

with

$$\hat{\mathbf{A}} = -\frac{m}{\hbar} \left( \sigma_1 \sigma_2 - \beta \sigma_1, \beta \sigma_2 - \alpha \sigma_1, 0 \right)$$

(2)

The generalized magnetic field from this vector potential is

$$\hat{\mathbf{B}} = \nabla \times \hat{\mathbf{A}} - \frac{\hbar}{m} \hat{\mathbf{A}} \times \hat{\mathbf{A}} = \frac{2m^2}{\hbar^2} (\alpha^2 - \beta^2) \sigma_3 e_z$$

(3)

**Lorentz force.** A generalized magnetic field defined by equation (3) gives a generalized Lorentz force law. Following ref. 37, we start with a Hamiltonian

$$\hat{H} = \frac{1}{2 m} (\hat{p} - \hat{\mathbf{A}})^2$$

containing a non-Abelian vector potential in three spatial dimensions with a finite number of internal degrees of freedom. The Heisenberg equation of motion for the position $\hat{x}$ is

$$\frac{d\hat{x}}{dt} = \frac{1}{\hbar} [\hat{x}, \hat{H}] = \frac{1}{m} (\hat{p}_x - \hat{A}_x) = \frac{1}{m} \hat{p}_x$$

We identify $\hat{p}_x$ as the particle’s mechanical momentum. The commutator $[\hat{H}, \hat{p}_x] = i\hbar \epsilon_{ijk} \hat{B}_j$, or $\hat{\mathbf{B}} = \nabla \times \hat{\mathbf{A}} - (\hbar / m) \hat{\mathbf{A}} \times \hat{\mathbf{A}}$, defines the generalized magnetic field ($\epsilon_{ijk}$ is the Levi-Civita symbol). For Abelian vector potentials, the different components of $\hat{\mathbf{A}}$ all commute, and this expression for $\hat{\mathbf{B}}$ reduces to the familiar $\hat{\mathbf{B}} = \nabla \times \hat{\mathbf{A}}$.

We derive the Lorentz force law starting with the Heisenberg equation of motion for the mechanical momentum:

$$\frac{d\hat{p}_x}{dt} = \frac{1}{\hbar} \left[ \hat{p}_x, \hat{H} \right]$$

For an individual component

$$[\hat{H}, \hat{p}_x] = \frac{1}{2m} \left( \hat{B}_y \hat{p}_z - \hat{B}_z \hat{p}_y + \hat{H}_y \hat{B}_z - \hat{H}_z \hat{B}_y \right)$$

which is the $x$th component of the symmetrized Lorentz force law

$$F = m \frac{d^2 \hat{x}}{dt^2} = \frac{1}{2} \left( \frac{d\hat{x}}{dt} \times \hat{\mathbf{B}} - \hat{\mathbf{B}} \times \frac{d\hat{x}}{dt} \right)$$

Because the $\hat{\mathbf{B}}$ field from linear combinations of Rashba and Dresselhaus SOCs (equation (3)) and the $\hat{\mathbf{B}}$ from our experiment are both proportional to $\hat{\sigma}_3$, the equations of motion for the mechanical momentum in the two cases are the same. However, for the vector potential in equation (2), $\hat{\mathbf{B}}$ does not commute with the Hamiltonian, leading to an additional Heisenberg equation of motion for $\hat{\mathbf{B}}$ which must be included. Despite this additional complexity, the SDFL generates the SHE in both situations.

**Gauge invariance.** The magnetic field defined by equation (3) is not gauge invariant. The definition of gauge transformations is generalized in any discussion of non-Abelian vector potentials. For the SU(2) symmetry group, a gauge transform is a position-dependent unitary rotation in spin space $^{13,14}$

$$\psi \rightarrow V(\hat{x})\psi$$

with

$$V(\hat{x}) = \exp \left[ i \alpha (\hat{x} \times \hat{\sigma}) \right]$$

where $\alpha$ is an arbitrary vector of functions of $\hat{x}$ and $\hat{\sigma}$ is the vector of $2 \times 2$ Pauli matrices including the identity. Under this gauge transformation, the Lagrangian must remain unchanged, requiring the magnetic field to transform according to

$$\hat{\mathbf{B}} \rightarrow V(\hat{x}) \hat{\mathbf{B}} V(\hat{x})$$

Despite the lack of gauge invariance of the magnetic field, an Abelian magnetic field cannot be gauge transformed to a non-Abelian field.

This definition for gauge transforms can be generalized to a gauge with generators from any continuous symmetry group by replacing $\hat{\sigma}$ with a vector of the generators of the symmetry group. For instance, in the case of a scalar vector potential from classical electrodynamics with U(1) symmetry, the generator of the symmetry group is a scalar, and the gauge transformation becomes the familiar position-dependent phase.

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