Closed-Loop Two-Stage Stochastic Optimization of Offshore Wind Farm Collection System

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Abstract. A two-stage stochastic optimization model for the design of the closed-loop cable layout of an Offshore Wind Farm (OWF) is presented. The model consists on a Mixed Integer Linear Program (MILP) with scenario numeration incorporation to account for both wind power and cable failures stochasticity. The objective function supports simultaneous optimization of: (i) Initial investment (network topology and cable sizing), (ii) Total electrical power losses costs, and (iii) Reliability costs due to energy curtailment from cables failures. The mathematical program is embedded in an iterative framework called PCI (Progressive Contingency Incorporation) in order to simplify the problem while providing the global optimum. The applicability of the method is demonstrated by tackling a real-world instance. Results show the functionality of the tool in quantifying the economic profitability when applying stochastic optimization compared to a deterministic approach, given certain values of failure parameters.

1. Introduction

Offshore Wind Farms (OWFs) are shaping up as one of the main drivers towards the transition to carbon-neutral power systems. Ambitious targets set by the European Commission see offshore wind power reaching 450 GW by 2050 [1]. Offshore electrical systems have a sizeable weight in the capital investments, reaching 15% of the total initial expenses [2], with the power cables being backbone component of the whole chain in the Balance of Plants (BoP). Furthermore, power cables can be single points of failure, leading to strongly undesired contingency [3]. Shallow waters, buried depth, seabed terrain movements [4], and electro-thermal stress, are differential factors in the context of OWFs, giving rise to higher failure rates of OWF submarine cables compared to those reported by CIGRE [5] [6].

OWF export cables are generally built with redundancy, as the high voltage levels and long distances increase the failure probability. Likewise, cables for collection systems may also be arranged to provide greater levels of reliability, typically resulting in a closed-loop topology. However, tailored-made models to design collection system with a closed-loop structure, using global optimization, integrated with analytical methods for reliability assessment, are not readily available in the scientific literature. Radial topology, i.e., without electrical redundancy (trees according to graph theory [7]) has been the most common subject of study in literature in this context, and currently represents the most frequent choice by OWFs developers. However, with the increase in the OWFs capacities and the trend of moving towards subsidy-free operating regimes, quantification of economic suitability for closed-loop or radial topologies are becoming essential.

Radial topology for OWF collection systems has its own mathematical entity, and it is classified in computational complexity as a NP-Hard problem. Thus, scalability is the main challenge, as state-of-the-art OWFs are in the order of hundreds of Wind Turbines (WTs). Mathematical models are
proposed and solved through global optimization solvers in [8], [9], [10], [11], and [12]. Nevertheless, a
deterministic approach is followed given the assumption of no cables failures along the project’s lifetime. Contrarily, studies adopting stochastic techniques are available in [13], [14], and [15]. A Mixed Integer Quadratic Program is presented in [13], which aims to analyze the suitability of having redundancy for system components subject to failure, by solving entirely the stochastic program including all possible contingencies. Model reduction implementing a Mixed Integer Linear Program along with Progressive Contingency Incorporation (PCI), and decomposition strategies is performed in [14] and [15], proving the ability to decrease computational resources while solving to optimality small-scale OWFs. The latter works provide remarkable advances on stochastic optimization supporting several wind power and cables failure scenarios. Nonetheless, the inclusion of practical engineering constraints such as non-crossing of cables, closed-loop topological network, and others are missing in these works. This gap is covered in this manuscript, where a MILP program based computational tool is presented, supporting decision makers during the design stage of OWFs. An algorithmic framework is developed targeting further computational simplification, supporting an objective function combining simultaneously initial investment, total electrical power losses, and energy curtailment due to cables failures. A recourse problem is solved to assess the benefits of stochastic optimization compared to the deterministic counterpart.

This paper is structured as follows: In Section 2 the optimization model is explained in details; followed by Section 3 where the algorithmic framework is explained. Finally, a case study is performed in Section 4 and the work is finalized with the conclusions in Section 5.

2. Optimization model

2.1. Graph and model representation

The aim of the optimization is to design a closed-loop cable layout of the collection system for an OWF, i.e., to interconnect through power cables the n_w WT's to the available OSS, while providing a redundant power evacuation route. Let N_{w} = \{2, \cdots, 1 + n_w\}. Consequently, N = 1 \cup N_{w}.

The Euclidean norm between the positions of the points i and j, is defined as d_{ij}. These inputs gather in a weighted undirected graph G(N, E, D), being N the vertex set, E the set of available edges arranged as a pair-set, and D the set of associated euclidean norms for each element [ij] \in E, where i \in N \land j \in N. In general, G(N, E, D) is a complete undirected graph, or it may be bounded by defining uniquely those edges connecting the \upsilon closest WTs to each WT, and by the \sigma edges directly reaching the OSS (i = 1).

Likewise, let T be a predefined list of available cable types, and U be the set of cables capacities sorted in non-decreasing order as in T, being measured in Amperes (A), such as u_t is the capacity of cable t \in T.

Furthermore, each cable type t \in T has a cost per unit of length, c_c, in such a way that u_t and c_c describe a positive correlation. The set of metric capital expenditures is defined as C_c. Similarly the set of metric installation costs is defined by C_p.

The parametric graph representation of the problem is continued with its modelling and formulation. The model is able to optimize simultaneously for Investment (cables’ capital and installation costs), Electrical
Losses (in a conservative and approximated fashion), and Reliability (cost of energy curtailment). The problem is formulated as a stochastic optimization program, modelled with two stages. See Figure 1. The first-stage is for investment decisions, while the second-stage is for operational aspects. Uncertainty is represented by means of a scenario tree (ϒ), expressing simultaneously how the stochasticity is developing over time, the different states of the random parameters, and the definition of the non-anticipative decisions in the present. The set of wind power generation scenarios is Ω, while the representative system states are K. The nominal generation scenario is ωn, and the base system state (κu) represents the case of no failures. The base case is therefore represented by the scenario {ωn, κu}.

A wind power generation scenario ω has associated a duration time τω (in hours), and power magnitude ζω (in p.u.), and each system state k, a system probability ψk, calculated using a discrete Markov model to define the cable’ complementary states: available, and unavailable. In the same way, given the low failure rates of these components a N-1 criterion must be considered in each system state [16]. The first-stage variables are the binary variables xij,t, and yij; where xij,t is equal to one if active edge [ij] (yij = 1) uses cable type t ∈ T. The second-stage variables are the continuous variables Iω,kij, δω,kij, and δω,kij. The electrical current in edge [ij] in wind power generation scenario ω ∈ Ω, and system state k ∈ K is represented by Iω,kij. While the voltage angle at each WT busbar is θω,kij. The curtailed current at wind turbine j in wind power generation scenario ω ∈ Ω, and system state k ∈ K is δω,kij. Note that δω,kij is bounded by the current generated at j in the same scenario, Iω,j, where Iω,j = Pnζω,k√3/Vn, being Pn the nominal power of an individual WT, and Vn the line-to-line nominal voltage of the system.

2.2. Cost coefficients and objective function

2.2.1. Neglecting total electrical power losses The objective function in this case consists of a simultaneous valuation of the total initial investment plus reliability. The investment is intuitively computed as the sum of cables costs installed in each edge [ij]; on the other hand, reliability is quantified through the estimation of the economic losses due to cables failures, as the result of undispatched current from each WT. In this way, the objection function is formalized as:

\[
\min \sum_{[ij] \in E} \sum_{t \in T} (c_{ct} + c_{pe}) \cdot d_{ij} \cdot x_{ij,t} + c_{ce} \cdot \sum_{i \in N_w} \sum_{\omega \in \Omega} \sum_{k \in K} \tau_{ij} \cdot \psi_k \cdot \delta_{ij} \cdot k
\]

Where cce is the cost of energy in €/Ah. The sum of system states probabilities must be equal to one, \( \sum_{k \in K} \psi_k = 1 \), given the mutually exclusive nature of the considered events (only up to one cable is subject to fail, N-1 criterion). Finally, the system probability for each state \( \psi_k \) is considered equal to the linked cable failure probability. This implies that the availability probability of the other installed cables is considered to be equal to one [13], representing a conservative approach.

2.2.2. Considering total electrical power losses Total electrical power losses are non-linear in function of the current, cable type, and total length [7]. The designer must try to find a proper balance between modelling fidelity and program complexity. A pre-processing strategy is proposed in this manuscript in order to incorporate this factor into the objective function.

\[
f_t = \frac{\sqrt{3} \cdot V_n \cdot u_t}{P_n \cdot 1000} \quad \forall t \in T
\]
The set of cables capacities in terms of number of supportable WTs is defined in (2). Let the new cable type set be:

\[
T' = \left\{ 1, 2, \ldots, f_1, f_1 + 1, \ldots, f_2, f_2 + 1, \ldots, f_{|T|} \cdots f_{|T|} \right\}
\]  

(3)

This implies that \( T' \) is the discretized form of the maximum capacity \( U = \max U \). Note that this is translated into the creation of additional variables \( x_{ij,t'} : t' \in T' \). Likely, if the floor function in (2) is replaced by a decimal round down function, and \( T' \) is also discretized using same decimal steps, then the number of variables will increase accordingly, to the benefit of gaining in accuracy for the cables capacities.

In \( T' \) is contained the non-dominated cable sub-types from \( T \); this means that each cable sub-type \( t' \in T' \) is related to a cable type \( t \in T \), inheriting physical properties such as total metric cost \( (c_{e_t} + c_{p_t}) \), metric electrical resistance \( (R_t) \), and metric electrical reactance \( (X_t) \); see (3) where this relation is presented graphically. Acknowledging that the investment cost of a cable \( t \) exceeds the electrical power losses costs, then the selected cable sub-type to connect \( n \) WTs will always be the cheapest (smallest) cable with sufficient capacity, rather than a bigger one with lower electrical power losses as the electrical resistance decreases with size.

As a consequence of the aforementioned, let a new cables capacities set be:

\[
U' = \{1, 2, \ldots, f_1, f_1 + 1, \ldots, f_2, f_2 + 1, \ldots, f_{|T|} \cdots 1, \ldots, f_{|T|} \}
\]

(4)

Let the functions \( f(t'), g(t'), \) and \( h(t') \) obtain the total metric cost, metric electrical resistance, and metric electrical reactance for cable sub-type \( t' \), respectively, which are inherited from a cable type \( t \). Wherefore, the objective function for simultaneous optimization of investment, electrical losses, and reliability is:

\[
\begin{align*}
\min & \sum_{[ij] \in E} \sum_{t' \in T'} \left[ f(t') + 3 \cdot 1.5 \cdot g(t') \cdot \left( \frac{c_{p_t}}{\sqrt{3} \cdot V_n} \cdot 1000 \right) \cdot \sum_{\omega \in \Omega} \left( u_{t'}^\omega \cdot \zeta^\omega \right)^2 \cdot \tau^\omega \right] \cdot d_{ij} \cdot x_{ij,t'} \\
\text{subject to} & \\
& h(t'). \text{ Pre-processing for total electrical power losses} \\
& \text{Reliability}
\end{align*}
\]

(5)

The factor \((3 \cdot 1.5)\) in (5) accounts the joule, screen and armouring losses for the three-phase system. The whole term for total electrical power losses \((h(t'))\) is calculated for each \( t' \in T' \), beforehand launching the MILP program into the external solver. Therefore, the objective function is a linear weighting of the desired targets: investment, electrical losses, and reliability.

As discussed previously, one of the task of the designer is to balance out modelling fidelity and program complexity. The objective function in (5) is a linear function, thus the following simplifications are assumed: (i) integer discretization in (3) which restricts the capacity of cables, and may cause overestimation of electrical losses. This can be diminished by decimal round down, and by increasing discretization steps in (4) at expense of incrementing the number of variables correspondingly. (ii) Neglection of system states (cables failures) apart of the base state (no failures); however, this is the
state with highest probability. (iii) Power flow estimation in a conservative fashion, i.e., overestimating the incoming power flow by neglecting the total power losses downstream. All those simplifications may impact the final layout, however their conservative nature means rather over-designing than impacting the robustness.

2.3. Constraints
The first-stage constraints are:

$$
\sum_{t \in T} x_{ij,t} = y_{ij} \ \forall [ij] \in E \ \vee \sum_{t' \in T'} x_{ij,t'} = y_{ij} \ \forall [ij] \in E
$$

(6)

In case edge edge $[ij]$ is active in the solution, then one and only one cable type $t \in T$ or $t' \in T'$ must be chosen (6). Note that in case total electrical power losses are considered, then the cable types set is $T'$, otherwise $T$; same logic for $U-U'$, $t-t'$, and $u_t-u'_t$. This applies for the forthcoming mathematical expressions.

$$
\sum_{j \in N \backslash i} y_{ij} = 2 \ \forall l \in N_w : l = i \lor l = j
$$

(7)

A closed-loop (sunflower petals) collection system topology is forced through (7).

$$
\sum_{i \in N \backslash j} y_{ij} \leq \phi_j = 1 \ \forall \omega \in \Omega \ \forall k \in K
$$

(8)

Limiting the number of feeders (upper limit of $\phi$ feeders) connected to the OSS is carried out by means of (8).

$$
y_{ij} + y_{uv} \leq 1 \ \forall \{[ij],[uv]\} \in \chi
$$

(9)

The set $\chi$ stores pairs of edges $\{[ij],[uv]\}$, which are crossing each other. Excluding crossing edges in the solution is ensured by the simultaneous application of the linear inequalities (6) and (9). The no-crossing cables restriction is a practical requirement in order to avoid hot-spots, and potential single-points of failure caused by overlapping cables [8]. Constraint (9) exhaustively lists all combinations of crossings edges. The constraints in (6) ensure that no active arcs are crossing or overlapping between each other. These constraints thus link the variables $y_{ij}$ and $x_{ij,t}$.

The second-stage constraints are:

$$
\sum_{i \in N \backslash \omega} \sum_{k \in K} \sum_{j \notin i} I_{\omega,k}^{\omega,j} - I_{\omega,j}^{\omega,k} + \delta_{\omega,j} = I_{\omega,j} \ \forall j \in N_w \ \forall \omega \in \Omega \ \forall k \in K
$$

(10)

The flow conservation, which also avoids disconnected solutions, is considered by means of one linear equality per wind turbine as per (10).

The tender constraints, to link first and second stage constraints, are:

$$
I_{ij}^{\omega,k} - \frac{1000 \cdot V_n \cdot (\theta_{ij}^{\omega,k} - \theta_{ji}^{\omega,k})}{\sqrt{3} \cdot X_t \cdot d_{ij}} - M \cdot (1-x_{ij,t}) - M \cdot r_{ij} \leq 0 \ \forall [ij] \in E \ \forall \omega \in \Omega \ \forall k \in K
$$

(11)

$$
-I_{ij}^{\omega,k} + \frac{1000 \cdot V_n \cdot (\theta_{ij}^{\omega,k} - \theta_{ji}^{\omega,k})}{\sqrt{3} \cdot X_t \cdot d_{ij}} - M \cdot (1-x_{ij,t}) - M \cdot r_{ij} \leq 0 \ \forall [ij] \in E \ \forall \omega \in \Omega \ \forall k \in K
$$

(12)

DC power flow is abided with (11) and (12), where $r_{ij}^{\omega}$ is a parameter equal to one if edge $[ij]$ is failed, or zero if otherwise, $X_t$ is the metric electrical reactance of cable $t$ (in case of inclusion of total electrical
power losses, let $X_{i'} = h(t')$, and $M$ is a big enough number to guarantee feasibility for those inactive or failed components.

$$
\sum_{t \in T} u_t \cdot x_{ij,t} \cdot (1 - r_{ij}^k) \geq I_{ij}^{\omega,k} \quad \forall [ij] \in E \quad \forall \omega \in \Omega \quad \forall k \in K
$$ (13)

$$
\sum_{t \in T} -u_t \cdot x_{ij,t} \cdot (1 - r_{ij}^k) \leq I_{ij}^{\omega,k} \quad \forall [ij] \in E \quad \forall \omega \in \Omega \quad \forall k \in K
$$ (14)

The cables capacities are not exceeded by including the bilateral constraints (13) and (14). The current $I_{ij}^{\omega,k}$ may circulate either from $i$ to $j$ or viceversa.

Finally, Constraints (15) to (19) define the nature of the formulation by the variables definition, a MILP program.

$$
x_{ij,t} \in \{0, 1\} \quad \forall t \in T \quad \forall [ij] \in E
$$ (15)

$$
y_{ij} \in \{0, 1\} \quad \forall [ij] \in E
$$ (16)

$$
-0.1 \leq \theta_i^{\omega,k} \leq 0.1 \quad \forall i \in N \quad \forall \omega \in \Omega \quad \forall k \in K
$$ (17)

$$
-U \leq I_{ij}^{\omega,k} \leq U \quad \forall [ij] \in E \quad \forall \omega \in \Omega \quad \forall k \in K
$$ (18)

$$
0 \leq \delta_i^{\omega,k} \leq I_{ij}^{\omega,k} \quad \forall i \in N_w \quad \forall \omega \in \Omega \quad \forall k \in K
$$ (19)

2.4. The stochastic optimization program

To summarize, the formulation of the MILP program consists of the objective function (1) or (5), and the constraints defined in (6) - (19). Let this stochastic optimization program be $P_{\Omega,K}$.

3. Optimization framework

Since the two-stage variables scale-up exponentially as a function of the scenario tree size, the representative systems states must be limited [14]. The basic version of the stochastic program presented in Section 2 encompasses the full set $E$; each element $[ij]$ gives place to a system state $k$ to form the system states set $K$.

Nevertheless, the actual selected edges in a solution (i.e. a feasible point satisfying the optimality criteria) is only a subset $E' \subset E$; let the complement set $E''$ contains the unused elements from $E$, and let define the subset $E''' \subset E''$. Hereafter, it is proved that any representative system states set containing at least the scenarios linked to $E'$ ($K_{E'} = \Phi(E')$), is necessary and sufficient to obtain the optimum in $P_{\Omega,K}$.

Let the necessary and sufficient set $K'$ encompass:

$$
K' = k_o \cup K_{E'} \cup K_{E'''}
$$ (20)

Where $K_{E'''}$ is the system states linked to the subset of unused edges $E'''$.

**Axiom 1** The second-stage variables linked to unused elements are equal to the base system state

$$
\forall i \in N_w \quad \forall k \in K_{E'''} \quad \forall \omega \in \Omega, \quad \delta_i^{\omega,k} = \delta_i^{\omega,k_o}
$$

An intuitive proposition is reflected in Axiom 1. The curtailed currents in the system state of unused edges are the same than in the base system state. This basically means that the failures of unused elements will not deteriorate the operation of the system.

From (1) it follows:

$$
\sum_{[ij] \in E} \sum_{t \in T} (c_{c_i} + c_{p_i}) \cdot d_{ij} \cdot x_{ij,t} + c_{c_p} \cdot \sum_{i \in N_w} \sum_{\omega \in \Omega} \sum_{k \in K \setminus \{k_o\}} \tau^{\omega} \cdot \psi^k \cdot \delta_i^{\omega,k} + c_{c_p} \cdot \sum_{i \in N_w} \sum_{\omega \in \Omega} \tau^{\omega} \cdot \left(1 - \sum_{k \in K \setminus \{k_o\}} \psi^k \cdot \delta_i^{\omega,k_o}\right)
$$ (21)
Expression (21) with (20) becomes:
\[
\sum_{i,j \in E} \sum_{e \in T} (c_{ij} + c_{pi}) \cdot d_{ij} \cdot x_{ij,t} + c_{cp} \cdot \sum_{i \in \mathcal{N}_w} \sum_{\omega \in \Omega} \sum_{k \in K_{E'}^{\omega}} \tau^\omega \cdot \psi_k^\omega \cdot \delta_k^\omega +
\]
\[
c_{cp} \cdot \sum_{i \in \mathcal{N}_w} \sum_{\omega \in \Omega} \sum_{k \in K_{E'}^{\omega}} \tau^\omega \cdot \psi_k^\omega \cdot \delta_k^\omega +
\]
Expression (22) with Axiom 1 becomes:
\[
\sum_{i,j \in E} \sum_{e \in T} (c_{ij} + c_{pi}) \cdot d_{ij} \cdot x_{ij,t} + c_{cp} \cdot \sum_{i \in \mathcal{N}_w} \sum_{\omega \in \Omega} \sum_{k \in K_{E'}^{\omega}} \tau^\omega \cdot \psi_k^\omega \cdot \delta_k^\omega +
\]
\[
c_{cp} \cdot \sum_{i \in \mathcal{N}_w} \sum_{\omega \in \Omega} \sum_{k \in K_{E'}^{\omega}} \tau^\omega \cdot \psi_k^\omega \cdot \delta_k^\omega
\]
Expression (23) is analogous to (21) but with \(K' \setminus \{k_o\} = K_{E'}\). This proves that any set \(K'\) containing at least the system states associated to all selected edges is sufficient and necessary to find the global optimum of the full problem \(P^{\Omega,K}\). Conversely, any instantiation for which \(K' \subset K_{E'}\) would lead to an underestimation of operational costs, ultimately causing falling into suboptimal. The proof also applies when including total electrical power losses (3).

This contingency structure opens the door for a Progressive Contingency Incorporation (PCI) strategy, aiming to find a proper set \(K'\) following an iterative approach. Algorithm 1 is presenting the PCI implementation. In the first line a deterministic instance of the full problem is tackled. This means considering uniquely the scenario \(\{\omega_i, k_o\}\). For this problem a valid assumption is to consider zero curtailed power. After this, the active edges of interest corresponding to the first-stage optimization variables are stored as \(E'\), along with the obtained solution variables in \(X_{ws}\) (where \(X_d\) and \(Y_d\) contains the solution sets corresponding to \(x_{ij,t}\) and \(y_{ij}\) for the deterministic case, respectively). As no previous iteration has been conducted, cumulative solution variables are unavailable (\(E'_0\)). Since the second-stage

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**Algorithm 1:** The main algorithm

considering uniquely the scenario \(\{\omega_i, k_o\}\). For this problem a valid assumption is to consider zero curtailed power. After this, the active edges of interest corresponding to the first-stage optimization variables are stored as \(E'\), along with the obtained solution variables in \(X_{ws}\) (where \(X_d\) and \(Y_d\) contains the solution sets corresponding to \(x_{ij,t}\) and \(y_{ij}\) for the deterministic case, respectively). As no previous iteration has been conducted, cumulative solution variables are unavailable (\(E'_0\)). Since the second-stage
variables express contingency scenarios of the components delimited by the first-stage variables, the tree $\Phi$ uniquely considers the failure states associated to those components. For the case presented in Algorithm [1] solely those feeders which satisfy the reliability level $r_c$, are subject to fail. Parameter $r_c$ defines the degree of connection towards the OSS, so for example, $r_c = 1$ brings along the main feeders (rooted at $i = 1$), and $r_c = 2$ includes the last ones together with the feeders connected to the main ones, and so on for $r_c > 2$. By means of those parameters, the model can be further relaxed for large instances.

The Progressive Contingency Incorporation routine is started at line 4. The opening step is to intersect the current active edges set $E^i$, and the cumulative set $E''_\omega$. If the intersection set is equal to the current active edges $E^i$, then the process is terminated, otherwise more iterations are attempted. For the former case, the algorithm is stopped, with solution $[X, Y]$; for the latter case, the iterative process is continued to the subsequent iteration $\kappa$. Trivially, for $\kappa = 1, A = \emptyset$. Therefore, in line 9 the union set is obtained to update $E''_\omega$. A new instance of the main problem is solved in line 10, using the initial point $X_{\text{res}}$ (warm-start point), while considering the full wind power generation scenarios indicated by the user $\Omega$, and the system states related to edges cumulatively installed in all iterations, $K' = \Phi(E''_\omega)$.

When the Algorithm [1] converges, the scenario criterion is met: obtention of the proper set $K'$; meaning that all representative systems states have been already considered.

4. Case study
The applicability of the method explained in Section 3 is illustrated using Ormonde OWF [17] as case study. The experiments have been carried out on an Intel Core i7-6600U CPU running at 2.50 GHz and with 16 GB of RAM. The chosen solver is IBM ILOG CPLEX Optimization Studio V12.7.1 [18]. The main input parameters are shown in Table 1. A Mean Time To Repair (MTTR) per failure of 30 days (720 hours) is considered in this study [4]. The objective function (1) is applied in this case study as the most critical for OWFs medium voltage cables under operation in [4]. Early stage in offshore projects maturity and the consequent scarcity of available data cast uncertainty over the accuracy of this parameter, implying that more critical situations can be faced in future projects. Each MTBF value as the main aim is to present the model’s performance and quantitative comparison versus a deterministic version.

Table 1. Data inputs

| $P_n$ | $V_n$ | Life | $c_{pu}$ | MTTR | $U$ | $C_e + C_p$ | $n_w$ | $v$ | $\sigma$ | $\phi$ | $r_c$ |
|-------|-------|------|----------|-------|-----|-------------|-------|----|--------|-------|------|
| 5 MW  | 33 kV | 30 y | 2.86 $\frac{\text{Ah}}{\text{kWh}}$ | 720 h | $[530,655,775]$ A | $[450,510,570]$ k€/km | 30 | 6 | 10 | 4 | 1 |

The power magnitudes are $\varsigma^1 = 1 (\omega_n), \varsigma^2 = 0.5, \varsigma^3 = 0.2, \text{ and } \varsigma^4 = 0$. The duration times account for the project’s lifetime and considering 8760 hours per year: $\tau^1 = 65700$ hours ($\omega_n$), $\tau^2 = 91980$ hours, $\tau^3 = 91980$ hours, and $\tau^3 = 13140$ hours. A cable failure probability is calculated as per $\psi^k = \frac{MTTR}{MTTR + MTBF}$, with $d_{ij}$ being the edge length where the component is installed. The results for this case study are obtained by varying the MTBF from 10 to 178, with the latter value being listed as the most critical for OWFs medium voltage cables under operation in [4]. Early stage in offshore projects maturity and the consequent scarcity of available data cast uncertainty over the accuracy of this parameter, implying that more critical situations can be faced in future projects. Each MTBF value represents a different stochastic problem meaning that the model is run individually.

Quantitative assessment for the comparison between the output of the stochastic model ($[X_d, Y_d]$), and the output of the deterministic model ($[X, Y]$), is carried out. For the latter, a recourse problem is tackled $Q([X_d, Y_d])$, defined as minimization of the expected costs (reliability costs) given the scenario tree ($\Phi$) obtained from the wind power generation scenarios $\Omega$, and the system states linked to $Y_d$. For all the launched instances of MTBF an optimality gap of 0% has been set up, and a reliability level of $r_c = 1$ is assumed; this means that only those feeders connected directly to the OSS are subject to failure (the main feeders).

In-detail cable layout results for a MTBF of 10 are shown in Figure 2. Due to the rather straightforward layout of the wind farm, the main difference between the deterministic and stochastic technique is the use of larger cables on the connections close to the OSS. For OWFs with more wind turbines and/or more irregular layout, the changes would likely expand to the connections between the wind turbines. The economic comparison between deterministic and probabilistic model (see Figure 3(a)), indicates that...
for MTBF values inferior to 30, the stochastic model provides a cheaper system in overall terms; this is achieved as reliability costs are lower at expense of more costly cable infrastructure (in this particular case only due to cable sizing). The values in Figure 3(a) are expressed as the percentage difference between the deterministic and stochastic model relative to the latter.

Conversely, beyond MTBF=30, the failures probabilities drop considerably, resulting in an equal
weight for each cost unit (initial investment, electrical losses costs, and reliability costs) in the objective function of the deterministic model. This basically means that the reliability costs become trivial, and hence the focus is merely on the investment costs reduction. To compare the overall costs, the deterministic layout solution is run with the scenarios analyzed in the stochastic.

For large enough values of \( r_c \), for instance such that all edges are covered, one could expect that the MTBF break-even point would move further right, that is, stochastic programming would provide overall more economic benefits for less frequent failures rates.

Regarding computing time, in Figure 3(b) it is noticeable that the stochastic model for each MTBF instance is more complex computationally, as for instance, for a value of 10 the computing time is 530 times larger than the deterministic version. Likewise, the larger the MTBF, the more simplified the model becomes as the reliability costs share are decreasing. The deterministic cases on the other hand are independent to the failures rate and converge in couple of seconds.

5. Conclusions

The increase of OWFs size and complexity leads to an increased focus on cost effective, efficient and reliable design of electrical systems for power integration. Cable failures are becoming critical as their impact over availability is crucial, while their probability of occurrence may be determinant in collection system design. A stochastic model to quantify the economic suitability of building closed-loop collection system for OWFs is introduced in this manuscript. The objective function allows for a simultaneous consideration of initial investment, total electrical power losses costs, and reliability costs. The focus of this work has been to describe the proposed model, while presenting its application in a case study.

Results of this article point out that in function of failure parameters, network topologies with redundant power corridors may bring along significant cost benefits. The stochastic model presents a complex mathematical structure impacting considerably the required computational resources. Lastly, the contingency structure of the problem has been exploited analytically in order to simplify its complexity. Future work will present the application of this method for comparing quantitatively different network topologies, such as closed-loop and radial systems.

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