Stability and symmetry breaking in a three-Higgs-doublet model with lepton family symmetry $O(2) \otimes \mathbb{Z}_2$

M. Maniatis$^1$, D. Mehta$^{2,3}$ and Carlos M. Reyes$^1$.

$^1$Departamento de Ciencias Básicas, Universidad del Bío Bío, Casilla 447, Chillán, Chile.
$^2$Department of Applied and Computational Mathematics and Statistics, University of Notre Dame, Notre Dame, IN 46556, USA.
$^3$Centre for the Subatomic Structure of Matter, Department of Physics, School of Physical Sciences, University of Adelaide, Adelaide, South Australia 5005, Australia.

With a view on explaining the current neutrino data, an extension of the Standard Model with three Higgs-boson doublets has been proposed. Imposing an $O(2) \otimes \mathbb{Z}_2$ family symmetry, a neutrino mixing matrix with $\theta_{23} = \pi/4$ and $\theta_{13} = 0$ appears in a natural way. Even though these values for the mixing matrix do not follow the recent experimental constraints, they are nevertheless a good approximation. We study the Higgs potential of this model in detail. We apply recent methods which allow for the study of any three-Higgs-boson doublet model. It turns out that for a variety of parameters the potential is stable, has the correct electroweak symmetry-breaking, and has vacuum-expectation values corresponding to the electroweak precision data.

1. THE $O(2) \otimes \mathbb{Z}_2$ MODEL

The experimental neutrino mixing data show that the neutrino mixing is very different from the quark mixing. In the usual parametrization of the neutrino mixing matrix (see for instance [1]), experimental data suggests that the angle $\theta_{13}$ is small (but nonzero), and $\theta_{23}$ close to $\pi/4$ [2]. A lot of effort is spent to find a mechanism for this behavior (for some attempts we refer to [3–5]). It appears quite natural to impose instead of one Higgs-boson doublet three Higgs-boson doublets and by an appropriate symmetry generate this mixing matrix.

Here we want to study in detail the Higgs potential of a three Higgs-boson doublet model which imposes a $O(2) \otimes \mathbb{Z}_2$ symmetry. Let us closely follow the motivation of [6]. The starting point is a neutrino mass matrix which is symmetric in the generations two and three,

$$M_\nu = \begin{pmatrix} x & y & y \\ y & z & w \\ y & w & z \end{pmatrix}. \quad (1.1)$$

This mass matrix may be diagonalized as usual, that is, $U^T M_\nu U = \text{diag}(m_{\nu_1}, m_{\nu_2}, m_{\nu_3})$, where $U$ is the neutrino mixing matrix and $m_{\nu_1}, m_{\nu_2}, m_{\nu_3}$ the neutrino masses. Expressing the mixing matrix $U$ in terms of the usual parametrization [1], we get in particular, $\theta_{13} = 0$ and $\theta_{23} = \pi/4$. Even though the experimental results are not in exact agreement with these values, in particular $\theta_{13}$ is nonzero, they at least appear to be approximately fulfilled. The mass matrix (1.1) may be generated by the introduction of three Higgs-boson doublets $\varphi_i$, $i = 1, 2, 3$, and a symmetry $O(2) \otimes \mathbb{Z}_2 \cong \mathbb{Z}_2' \times \mathbb{Z}_2 \otimes U(1)$, where all elementary particles are assigned to an appropriate transformation behavior on this symmetry. The reflection symmetry $\mathbb{Z}_2'$ is responsible for the $\mu-\tau$ symmetry of (1.1):

$$\mathbb{Z}_2' : \quad D_{\mu L} \leftrightarrow D_{\tau L}, \quad \mu_R \leftrightarrow \tau_R, \quad \nu_{\mu R} \leftrightarrow \nu_{\tau R}, \quad \phi_1 \leftrightarrow \phi_2 \quad (1.2)$$

Here $D_{\mu L}$ and $D_{\tau L}$ denote the left-handed $SU(2)$ lepton doublets, $\nu_{\mu R}$ and $\nu_{\tau R}$ the right-handed neutrinos, and all remaining fields transform trivially under the $\mathbb{Z}_2$ symmetry. The $\mathbb{Z}_2$ symmetry is given by a sign change,

$$\mathbb{Z}_2 : \quad \nu_{e R} \rightarrow -\nu_{e R}, \quad \nu_{\mu R} \rightarrow -\nu_{\mu R}, \quad \nu_{\tau R} \rightarrow -\nu_{\tau R}, \quad e_R \rightarrow -e_R, \quad \varphi_3 \rightarrow -\varphi_3, \quad (1.3)$$

$^*$E-mail: MManiatis@ubiobio.cl
$^1$E-mail: dmehta@nd.edu
$^2$E-mail: creyes@ubiobio.cl
with $\nu_{eR}, \nu_{\mu R}, \nu_{\tau R}$ the right-handed neutrinos, $e_R$ the right-handed electron, and all other fields unchanged under this $\mathbb{Z}_2$ symmetry. Eventually the assignment with respect to the phase symmetry $U(1)$ is

$$U(1): \quad X \rightarrow e^{i\theta}X,$$

with $X$ one of the fields on the right-hand side of the table transforming as $X \rightarrow e^{i\theta}X$ with corresponding phase $\theta$ given explicitly in the table. All other fields transform trivially.

By virtue of these symmetries – besides the electroweak $SU(2)_L \times U(1)_Y$ symmetry – there appear in particular the invariant Yukawa couplings

$$\mathcal{L}_Y = -y_d (\bar{D}_{\mu L} \varphi_1 \mu_R + \bar{D}_{\tau L} \varphi_2 \tau_R) + h.c.$$

The most general potential for the three Higgs-boson doublets $\varphi_1, \varphi_2, \varphi_3$ reads

$$V_{O(2) \times \mathbb{Z}_2} = \mu_0 \varphi_3^\dagger \varphi_1^2 + \mu_{12} \left( \varphi_1^\dagger \varphi_1 + \varphi_2^\dagger \varphi_2 \right) + \mu_m \left( \varphi_1^\dagger \varphi_2 + \varphi_2^\dagger \varphi_1 \right) + a_1 (\varphi_3^\dagger \varphi_3)^2 + a_2 \varphi_3^\dagger \varphi_3 \left( \varphi_1^\dagger \varphi_1 + \varphi_2^\dagger \varphi_2 \right) + a_3 \left( \varphi_3^\dagger \varphi_1 \cdot \varphi_1^\dagger \varphi_3 \varphi_3^\dagger \varphi_3 \varphi_2^\dagger \varphi_2 \right) + a_4 \varphi_3^\dagger \varphi_1 \cdot \varphi_3^\dagger \varphi_3 \varphi_2^\dagger \varphi_2 + a_5 (\varphi_1^\dagger \varphi_1)^2 + (\varphi_2^\dagger \varphi_2)^2) + a_6 \varphi_1^\dagger \varphi_1 \cdot \varphi_2^\dagger \varphi_2 + a_7 \varphi_1^\dagger \varphi_1 \cdot \varphi_2^\dagger \varphi_2 + a_8 \varphi_1^\dagger \varphi_1 \cdot \varphi_2^\dagger \varphi_2.$$

where the term $\mu_m (\varphi_1^\dagger \varphi_2 + \varphi_2^\dagger \varphi_1)$ breaks the $U(1)$ symmetry (1.4) explicitly but softly, since this is a quadratic term. For a non-vanishing parameter $\mu_m$ in this way additional Goldstone bosons are avoided, which otherwise would appear by spontaneous symmetry breaking of the $U(1)$ symmetry. This potential has nine real parameters and one complex parameter $a_4$, corresponding to eleven real parameters in total.

Now we want to discuss stability, stationarity, and electroweak symmetry breaking of this model. Of course only a model with a stable potential, having the correct electroweak symmetry breaking behavior and the correct vacuum-expectation values is physically acceptable. These obvious constrains restrict the parameter space of the potential. Here, we focus on the Higgs potential and not on any further experimental limits. For instance, the expressions for the oblique parameters $S, T, U$ are available for any nHDM [7].

Even though the potential (1.6) appears to be rather involved we will see that it is indeed accessible in the bilinear approach [8,10]. Based on these bilinears, gauge degrees of freedom are avoided systematically. Moreover, the corresponding equations for stability and stationarity simplify, in particular the degree of systems of equations is lowered. Recently, the bilinear approach for the study of stability, stationarity, and electroweak symmetry breaking has been extended to the study of any 3HDM [11], which we now briefly review.

The scalar products of the type $\varphi_i^\dagger \varphi_j$, $i, j \in \{1, 2, 3\}$, in the potential (1.6) may be arranged in a $3 \times 3$ matrix

$$K = \begin{pmatrix} \varphi_1^\dagger \varphi_1 & \varphi_1^\dagger \varphi_2 & \varphi_1^\dagger \varphi_3 \\ \varphi_2^\dagger \varphi_1 & \varphi_2^\dagger \varphi_2 & \varphi_2^\dagger \varphi_3 \\ \varphi_3^\dagger \varphi_1 & \varphi_3^\dagger \varphi_2 & \varphi_3^\dagger \varphi_3 \end{pmatrix}.$$

By the introduction of the bilinears,

$$K_\alpha = K_\alpha^* = \text{tr}(K \lambda_\alpha), \quad \alpha = 0, \ldots, 8$$

with $\lambda_\alpha$ the $3 \times 3$ Gell-Mann matrices, the following replacements can be made in the potential,

$$\varphi_1^\dagger \varphi_1 = \frac{K_0}{\sqrt{6}} + \frac{K_3}{2} + \frac{K_8}{2\sqrt{3}}, \quad \varphi_1^\dagger \varphi_2 = \frac{1}{2} (K_1 + iK_2), \quad \varphi_1^\dagger \varphi_3 = \frac{1}{2} (K_4 + iK_3),$$

$$\varphi_2^\dagger \varphi_2 = \frac{K_0}{\sqrt{6}} - \frac{K_3}{2} + \frac{K_8}{2\sqrt{3}}, \quad \varphi_2^\dagger \varphi_3 = \frac{1}{2} (K_6 + iK_7), \quad \varphi_3^\dagger \varphi_3 = \frac{K_0}{\sqrt{6}} - \frac{K_8}{\sqrt{3}}.$$

Comparing the potential, written in terms of bilinears with the general form of the potential,

$$V = \xi_\alpha K_\alpha + K_\alpha \bar{E}_{\alpha \beta} K_\beta,$$

(1.10)
with \( \alpha, \beta = (0, \ldots, 8) \) we find the new parameters

\[
(\xi_\alpha) = \left( \frac{1}{\sqrt{6}} (\mu_0 + 2\mu_{12}), \mu_m, 0, 0, 0, 0, 0, \frac{1}{\sqrt{3}} (\mu_{12} - \mu_0) \right)^T,
\]

\[
\left( \tilde{E}_{\alpha\beta} \right) = \frac{1}{4} \left( \begin{array}{cccccccc}
\frac{2}{3}(a_1 + 2a_2 + 2a_5 + a_6) & 0 & 0 & 0 & 0 & 0 & 0 & \frac{\sqrt{8}}{2}(-a_1 - \frac{a_2}{2} + a_5 + \frac{a_6}{2}) \\
0 & a_7 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & a_7 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 2a_5 - a_6 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & a_3 & 0 & \text{Re}(a_4) & \text{Im}(a_4) \\
0 & 0 & 0 & 0 & 0 & a_3 & \text{Im}(a_4) & -\text{Re}(a_4) \\
0 & 0 & 0 & 0 & \text{Re}(a_4) & \text{Im}(a_4) & a_3 & 0 \\
0 & 0 & 0 & 0 & \text{Im}(a_4) & -\text{Re}(a_4) & a_3 & 0 \\
\end{array} \right)
\]

(1.11)

Obviously, all parameters are real in terms of bilinears. We note that there is a one-to-one correspondence between the Higgs-boson doublets and the bilinear matrix \( \tilde{K} = 1/2K_\alpha \lambda_\alpha \) with rank smaller or equal to two - except for irrelevant gauge degrees of freedom; see [9].

Supposed the potential is bounded from below, at the global minimum, or the degenerate minima, the gradient of the potential has to vanish. The corresponding equations may be used to fix some of the parameters. In order to obtain these equations we start with the parametrization of the three Higgs-boson doublets with all vacuum-expectation values and neutral fields real and upper complex charged fields,

\[
\varphi_i = \left( \frac{1}{\sqrt{2}} (\phi_i^R + i\phi_i^I) \right), \quad i = 1, 2, 3.
\]

(1.12)

The derivatives of the potential (1.16), inserting (1.12) with respect to the expansion fields \( \phi_i^R, \phi_i^I, \phi_i^+, \) and \( \phi_i^- \equiv (\phi_i^+)^\dagger \) at the vacuum, that is, for vanishing expansion fields give:

\[
\mu_0 = -\frac{1}{2} (a_2 + a_3)(v_1^2 + v_2^2) - a_1 v_3^2 - \text{Re}(a_4) v_1 v_2,
\]

\[
\mu_{12} = -\frac{1}{2} (a_2 + a_3)v_3^2 - a_5(v_1^2 + v_2^2),
\]

\[
\mu_m = \frac{1}{2} (2a_5 - a_6 + a_7)v_1 v_2 - \text{Re}(a_4)v_3^2,
\]

\[
\text{Im}(a_4) \cdot v_2v_3^2 = 0,
\]

\[
\text{Im}(a_4) \cdot v_1v_3^2 = 0,
\]

\[
\text{Im}(a_4) \cdot v_1 v_2 v_3 = 0.
\]

(1.13)

For positive vacuum-expectation values, the last equation immediately dictates that \( a_4 \) has to be real. Eventually, by means of the equations (1.13) the quadratic parameters \( \mu_0, \mu_{12}, \mu_m \) may be expressed by the quartic parameters and the three vacuum-expectation values \( v_1, v_2, v_3 \). Further, the vacuum-expectation values are restricted, with view on the Yukawa couplings (1.15), that is, its ratio of the vacuum-expectation values \( v_1 \) and \( v_2 \) has to be \( v_1/v_2 = m_\mu/m_\tau \) at tree level accuracy. In addition, the vacuum-expectation value \( v \equiv \sqrt{v_1^2 + v_2^2 + v_3^2} \approx 246 \text{ GeV} \) is given by the electroweak precision data. Therefore, all quadratic parameters follow from the quartic parameters and one vacuum-expectation value, say \( v_3 \). Therefore, it appears reasonable to start with the following set of parameters,

\[
a_1, a_2, a_3, \text{Re}(a_4), a_5, a_6, a_7, v_3, v \approx 246 \text{ GeV}, \quad v_1/v_2 = m_\mu/m_\tau.
\]

(1.14)

Note that the tadpole conditions (1.13) only ensure that there is at least one stationary solution. By no means this guarantees that the corresponding potential is stable and has a global minimum with the correct partially broken electroweak symmetry. As we shall see in the next section, we fix the quadratic parameters by more specific stationarity equations.

2. STABILITY AND ELECTROWEAK SYMMETRY BREAKING IN THE \( O(2) \otimes \mathbb{Z}_2 \) MODEL

In this section we analyze the potential of the \( O(2) \otimes \mathbb{Z}_2 \) model while varying two of its parameters. As discussed above we could apply the tadpole conditions in order to fix the quadratic parameters. Alternatively, we may substitute
the original parameters of the potential by the requirement that there is a stationary solution of the potential with rank 1 of the matrix \( K \) (see [11] for details). Fixing the quadratic parameters in this way has the advantage that this guarantees that the corresponding stationary solutions have the correct electroweak symmetry breaking. Of course, these solutions give not necessarily the global minimum. The stationary equations are given by (2.5) below. Starting with the parameters (1.14) we fix the quadratic parameters in this way.

Quantitatively, we choose the quartic parameters motivated by the central point given in [6] with a variation of the two parameters \( a_1 \in [0, 5] \) and \( a_2 \in [-3, 3] \) in steps of 0.2:

\[
\begin{align*}
  a_1 &\in [0, 5], \quad a_2 \in [-3, 3], \quad a_3 = -5, \quad a_4 = -0.05, \quad a_5 = 1.5, \quad a_6 = 2, \quad a_7 = 3, \quad v_3 = v/\sqrt{2}.
\end{align*}
\]

(2.1)

The central point in particular passes the electroweak precision observables – for details see [6]. We discard initial parameter sets which do not have a real solution for the quadratic mass parameters \( \mu_0, \mu_{12}, \mu_m \). These cases are denoted by the little squares (black) in Fig. 1 depending on the variation of \( a_1 \) and \( a_2 \).

Firstly, we study stability of the potential. Therefore we separate the potential into the quadratic and quartic parts, \( V = K_0 J_2 + K_0^2 J_4 \), with \( J_2 \) and \( J_4 \) given by [11]

\[
\begin{align*}
  J_2(\mathbf{k}) &= \frac{\mu_0 + 2\mu_{12}}{\sqrt{6}} + \left( \frac{\mu_{12} - \mu_0}{\sqrt{3}} \right) k_8 + \mu_m k_1, \\
  J_4(\mathbf{k}) &= \frac{1}{6} \left( a_1 + 2a_2 + 2a_5 + a_6 \right) + \frac{1}{3\sqrt{2}} (-2a_1 - a_2 + 2a_5 + a_6) k_8 + \frac{a_7}{4} (k_1^2 + k_2^2) + \frac{1}{4} (2a_5 - a_6) k_3^2 \\
  &\quad + \frac{a_3}{4} (k_4^2 + k_5^2 + k_6^2 + k_7^2) + \frac{\text{Re}(a_4)}{2} (k_4 k_6 - k_5 k_7) + \frac{\text{Im}(a_4)}{2} (k_4 k_7 + k_5 k_6) + \frac{1}{12} (4a_1 - 4a_2 + 2a_5 + a_6) k_9^2,
\end{align*}
\]

(2.2)

where the vector \( \mathbf{k} = (K_a/K_0), \ a = 1, \ldots, 8 \) is defined for \( K_0 \neq 0 \).

The stationary points of \( J_4(\mathbf{k}) \) corresponding to a matrix \( \mathbf{K} \) with rank 2 are obtained from

\[
\begin{align*}
  \nabla_{k_1, \ldots, k_8} \left[ J_4(\mathbf{k}) - u \left( \det(\sqrt{2/3} \mathbf{1}_3 + \mathbf{k}_a \lambda_a) \right) \right] &= 0, \\
  \det(\sqrt{2/3} \mathbf{1}_3 + \mathbf{k}_a \lambda_a) &= 0, \\
  2 - k^2 &= 0, \tag{2.3}
\end{align*}
\]

and the stationary points corresponding to a matrix \( \mathbf{K} \) with rank 1 are obtained from

\[
\begin{align*}
  \nabla_{k_1, \ldots, k_8} \left[ J_4(\mathbf{k}) - u_1 \left( \det(\sqrt{2/3} \mathbf{1}_3 + \mathbf{k}_a \lambda_a) \right) \right] - u_2 (2 - k^2) &= 0, \\
  \det(\sqrt{2/3} \mathbf{1}_3 + \mathbf{k}_a \lambda_a) &= 0, \\
  2 - k^2 &= 0. \tag{2.4}
\end{align*}
\]

Any real solutions \( \mathbf{k} \) of the systems of polynomial equations (2.3) and (2.4) with \( J_4(\mathbf{k}) > 0 \) or at least \( J_4(\mathbf{k}) = 0 \) but then in addition \( J_2(\mathbf{k}) \geq 0 \) gives a stable potential. In other words, if there is for a given initial parameter set one solution with \( J_4(\mathbf{k}) < 0 \) or \( J_4(\mathbf{k}) = 0 \) but in addition \( J_2(\mathbf{k}) < 0 \) the potential is unstable. Ignoring the inequality for the moment, the first system consists of nine polynomial equations in nine variables, and the second system consists of ten polynomial equations in ten variables. The variables of these sets of equations are the eight components of the vector \( \mathbf{k} \) and one \( (u) \) and two Lagrange multipliers \( (u_1, u_2) \), respectively. The unstable cases for the variation of parameters (2.1) are denoted by the larger full disks (blue) in Fig. 1. For all other values of parameters the potential is stable.

Let us note that the quartic parameters \( a_1 \) and \( a_2 \) appear as coefficients of \( (\varphi_1^2 \varphi_3)^2 \) and \( \varphi_1^4 \varphi_3 (\varphi_1^2 \varphi_1 + \varphi_2^2 \varphi_2) \), respectively, in the potential (1.10). Since the quadratic parameters are fixed by the rank 1 solutions, we typically encounter them to be negative in order to get a non-trivial vacuum at the origin. Therefore it is clear that the potential is unstable for small valued of \( a_1 \) and too negative values for \( a_2 \).

Having determined parameter sets giving a stable potential we proceed by the study of the stationary points. We systematically will look for all stationary points of the potential. The stationary points giving the lowest potential value are the global minima. To this end we have to solve the following systems of polynomial equations, corresponding...
to solutions which break electroweak symmetry partially (conserving the electromagnetic $U(1)_{em}$ symmetry), and solutions which break the electroweak symmetry completely.

The stationary solutions with partial electroweak symmetry breaking, corresponding to stationarity matrices $K = K_\alpha \lambda_\alpha/2$ of rank 1 are obtained from

$$\nabla_{K_0} \cdots K_8 \left[ V(K_0, \ldots, K_8) - u_1 (2K_0^2 - K_a K_a) - u_2 \det(K) \right] = 0,$$

$$2K_0^2 - K_a K_a = 0,$$

$$\det(K) = 0,$$

$$K_0 > 0.$$  \hfill (2.5)

The stationary solutions with full electroweak symmetry breaking, corresponding to stationarity matrices $K = K_\alpha \lambda_\alpha/2$ of rank 2 are obtained from

$$\nabla_{K_0} \cdots K_8 \left[ V(K_0, \ldots, K_8) - u \det(K) \right] = 0,$$

$$2K_0^2 - K_a K_a > 0,$$

$$\det(K) = 0,$$

$$K_0 > 0.$$  \hfill (2.6)

Here $u$, respectively $u_1$ and $u_2$ are Lagrange multipliers. The first system consists of 11 polynomial equations in 11 variables, and the second system consists of 10 equations in 10 variables, ignoring the inequalities. In addition there is always a solution for a vanishing potential, corresponding to an unbroken electroweak symmetry.

The global minimum, that is, the vacuum, is given by the stationary point with the deepest potential value. In case this solution originates from the set (2.5) we can directly calculate the vacuum-expectation value of this minimum, which, by the electroweak precision data has to be

$$v_0 = \sqrt{\sqrt{6} K_0} = 246 \text{ GeV}.$$  \hfill (2.7)

In case there is a stationary solution originating from the set (2.5) and there is no lower stationary point and finally the vacuum-expectation-value fulfills (2.7) we have detected a viable global minimum. These cases are marked by little (green) dots in Fig. 1. In case the deepest potential value does not come from this set or the vacuum-expectation-value is not in accordance with (2.7) this parameter point is denoted by a circle (red) in Fig. 1.

As we can see by the scattering of points, it is not trivial to adjust the parameters accordingly. The pattern of valid points appears very sensitive to the parameter values. This of course is a consequence of the rather involved potential (1.6). However, by the proposed methods as presented in [11] it is shown that in this model there are valid parameter values which pass the different theoretical constraints.

Eventually, let us remark on the technical aspects to solve the rather involved systems of equations - on the one hand for the study of stability (2.3), (2.4), and on the other hand for the study of stationarity (2.5), (2.6). We apply for all the polynomial systems of equations the homotopy continuation approach as implemented in the PHCpack package [12] (for a brief introduction of the homotopy continuation method see for instance [13]). As required by the bilinear approach, only real solutions are acceptable, therefore we discard all non-real solutions. Practically, we treat a solution as real if the imaginary part of each of the variables has an amount smaller than 0.001. For every real solution we check the corresponding inequalities.

3. CONCLUSIONS

The $O(2) \otimes \mathbb{Z}_2$ model [6] introduces three Higgs-boson doublets accompanied by an appropriate assignment of the elementary particles to irreducible representations. In this way a neutrino mass matrix is generated which corresponds to mixing angles which are close to the experimental measurements. However, even though the symmetry restricts the model, the Higgs potential appears to be rather involved. Nevertheless, the recently introduced methods to study any three-Higgs doublet model [11] were applied to study the potential in detail. We have investigated stability, the stationary points, and electroweak symmetry breaking of the Higgs potential by solving the corresponding stationary equations employing numerical polynomial homotopy continuation. The method numerically finds all the isolated complex solutions out of which we can extract the physical real solutions straightforwardly. We scanned over a range
FIG. 1: Stability and stationarity solutions of the 3HDM Higgs potential, varying the two quartic parameters $a_1$ and $a_2$ of the potential (1.6). The other quartic parameters are set to $a_3 = -5$, $a_4 = -0.05$, $a_5 = 1.5$, $a_6 = 2$, $a_7 = 3$. The quadratic parameters are fixed by the stationarity equations (2.5). In case there is no solution of these equations this is marked by little squares (black). The larger full circles (blue) show points where the potential is unstable. The open circles (red) show parameters where no correct electroweak symmetry breaking appears. Finally, the small dots (green) have a viable global minimum corresponding to the correct vacuum-expectation value and are physically acceptable.

of values of the potential parameters. As expected, for too low values of the quartic parameters an unstable potential is encountered. For parameter values, corresponding to a stable potential, the global minimum was detected. Our study reveals that in the range of investigated parameters there are values corresponding to a stable global minimum with correct electroweak symmetry breaking and a vacuum-expectation values in accordance with the electroweak precision data.
Acknowledgement

We would like to thank Luis Lavoura and Walter Grimus very much for valuable comments. DM was supported by a DARPA Young Faculty Award and an Australian Research Council DECRA fellowship. CR and MM were supported partly by the Chilean research project FONDECYT, with project numbers 1140781, respectively 1140568.

[1] K. A. Olive et al. [Particle Data Group Collaboration], Chin. Phys. C 38, 090001 (2014).
[2] M. C. Gonzalez-Garcia, M. Maltoni, J. Salvado and T. Schwetz, JHEP 1212, 123 (2012) [arXiv:1209.3023 [hep-ph]].
[3] D. Atwood, S. Bar-Shalom and A. Soni, Phys. Lett. B 635, 112 (2006) [hep-ph/0502234].
[4] E. Ma, Mod. Phys. Lett. A 25, 2215 (2010) [arXiv:0908.3165 [hep-ph]].
[5] R. Gonzalez Felipe, H. Serodio and J. P. Silva, Phys. Rev. D 88, no. 1, 015015 (2013) [arXiv:1304.3468 [hep-ph]].
[6] W. Grimus, L. Lavoura and D. Neubauer, JHEP 0807, 051 (2008) [arXiv:0805.1175 [hep-ph]].
[7] W. Grimus, L. Lavoura, O. M. Ogreid and P. Osland, J. Phys. G 35, 075001 (2008) [arXiv:0711.4022 [hep-ph]].
[8] F. Nagel, “New aspects of gauge-boson couplings and the Higgs sector,” PhD-thesis, Heidelberg University (2004).
[9] M. Maniatis, A. von Manteuffel, O. Nachtmann and F. Nagel, “Stability and symmetry breaking in the general two-Higgs-doublet model,” Eur. Phys. J. C 48, 805 (2006) [hep-ph/0605184].
[10] C. C. Nishi, Phys. Rev. D 74, 036003 (2006) [Erratum-ibid. D 76, 119901 (2007)] [hep-ph/0605153].
[11] M. Maniatis and O. Nachtmann, JHEP 1502, 058 (2015) [arXiv:1408.6833 [hep-ph]].
[12] J. Verschelde, “Algorithm 795: PHCpack: A General Purpose Solver for Polynomial Systems by Homotopy Continuation,” ACM Transactions on Mathematical Software, Vol 25, 2 (1999).
[13] M. Maniatis and D. Mehta, Eur. Phys. J. Plus 127, 91 (2012) [arXiv:1203.0409 [hep-ph]].