Vibration synchronization intelligent control with trajectory tracking

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Abstract. Based on the dynamic model of the double-mass vibrating mechanical system, the adaptive control theory is introduced to study the motor control synchronization problem in the vibrating mechanical system. First, the dynamic model of the dual-mass nonlinear vibration system is studied, and the overall relationship between the motor motion and the vibration system is analyzed. Then, a dual-motor adaptive control strategy is designed, so that the parameters in the control system can be intelligently adjusted with the synchronization state of the motors, and the synchronization stability and motion trajectory between the motors are studied. Lyapunov stability criterion is used to verify the stability of the synchronous control system. Finally, MATLAB/Simulink is used to simulate the effectiveness of the dual-motor adaptive control system, which proves that the system has good robustness. Through research, it is found that the dual-motor vibration synchronization adaptive control strategy can keep the motors in the dual-mass nonlinear vibration system with good synchronization effects, and the system has good robustness. The research content can provide a certain theoretical reference for the research and design of multi-motor synchronous machinery.

1. Introduction

The self-synchronization theory of multiple motors is widely used in vibrating mechanical systems, and has made a huge contribution to the development of human society and industry. However, with the needs of industrial development, the volume of some self-synchronizing machinery has become huge[1]. In order to meet the conditions of mechanical self-synchronization, the structure of the machinery has become more and more complex and precise, making the repair and maintenance of the equipment more difficult[2,3]. Therefore, domestic and foreign experts and scholars have introduced control synchronization theory into the design of self-synchronizing mechanical systems, hoping to achieve good synchronization effects while simplifying the mechanical structure.

In the past two decades, domestic and foreign scholars have published a large number of research documents related to control synchronization. Tomizuka et al. introduced adaptive feedforward control theory in synchronous motion[4]. Miklos et al. used an adaptive sliding film control algorithm when studying the synchronization and speed tracking of multiple motors[5]. Li Xiaohao et al. studied the synovial membrane approaching law control synchronization of the single-mass nonlinear vibration system[6]. However, in these studies, few scholars have specifically studied the synchronization control strategy of multiple motors in a dual-mass vibration system.

Based on the model reference adaptive theory, this paper associates the motors in the multi-motor vibration system and designs a synchronous control strategy. Then, the stability of the system is proved...
by the Lyapunov stability theory, and the vibration synchronization effect of the control system is verified by modeling and simulation. Therefore, the vibration mechanical system using the synchronization control strategy in this paper has the advantages of simple structure, easy installation, and good vibration synchronization effect, which provides a new idea for the synchronization of multi-motor vibration control.

2. The dynamic model of the double-mass vibration system

The dual-mass vibration mechanical system in this article is mainly composed of two upper and lower masses, two symmetrically distributed motor excitation motors, a spring and a damper connecting the upper and lower masses, and a spring and damper connecting the base and the mass 2. The dynamic model of the vibration system is shown in Figure 1.

According to the principle of Lagrange mechanics, the differential equation of motion of the two-mass vibration system is established as follows:

\[
\begin{align*}
    m_1 \ddot{x}_1 + c_1 (\dot{x}_2 - \dot{x}_1) + k_1 (x_2 - x_1) &= 0 \\
    m_2 \ddot{x}_2 + c_2 (\dot{x}_2 - \dot{x}_1) + k_1 (x_2 - x_1) + k_2 x_2 + c_2 \dot{x}_2 &= \\
    J_{\phi_1} \ddot{\phi}_1 - T_{m1} - T_{f1} - c_1 \dot{\phi}_1 - m_1 r_1^2 \ddot{\phi}_1 + m_0 r_1 \ddot{\phi}_1 + m_0 r_2 \ddot{\phi}_2 + m_0 r_2 g \cos \phi_2 &= \\
    J_{\phi_2} \ddot{\phi}_2 - T_{m2} - T_{f2} - c_2 \dot{\phi}_2 - m_2 r_2^2 \ddot{\phi}_2 + m_0 r_2 \ddot{\phi}_1 + \ddot{\phi}_2 &=
\end{align*}
\]

In this equation, \( x_1, \dot{x}_1, \ddot{x}_1 \) are the vibration displacement, the velocity and the acceleration of the vibrating body 1 in the vertical direction, respectively. \( x_2, \dot{x}_2, \ddot{x}_2 \) are the vibration displacement, the velocity and the acceleration of the vibrating body 2 in the vertical direction, respectively. \( m_1 \) is the total mass of the vibrating body 1, \( m_2 \) is the total mass of the vibrating body 2 and the total mass \( m_2 \) is composed of two components, the mass of the vibrating body 2 and the mass of two eccentric rotors on two excited-motors \( m_{01} \) and \( m_{02} \). \( m_{01} \) and \( m_{02} \) are the mass of two eccentric rotors on two excited motors, respectively. \( r_i \) \((i=1,2)\) is the radius of the eccentric rotor around \( O_i \) \((i=1,2)\). \( \phi_i \) \((i=1,2)\) is the angular phase of the eccentric rotor \( i \), respectively. \( \dot{\phi}_i \) \((i=1,2)\) is the angular velocity of the eccentric rotor \( i \), respectively. \( \ddot{\phi}_i \) \((i=1,2)\) are the angular acceleration of the eccentric rotor \( i \), respectively. \( c_1 \) is the linear damping between the vibrating body 1 and the vibrating body 2 and \( k_1 \) is the linear stiffness between the vibrating body 1 and the vibrating body 2. \( c_2 \) is the linear damping of the vibrating body 2 and \( k_2 \) is the linear stiffness of the vibrating body 2. \( c_{20} \) \((i=1,2)\) is the rotating damping of the excited motor \( i \). \( J_{\phi_i} \) \((i=1,2)\) is the moment of the inertia of eccentric block \( i \). \( T_{mi} \) \((i=1,2)\) is the electromagnetic torque on the excited-motor \( i \). \( T_{fi} \) \((i=1,2)\) is the friction torque on the excited-motor \( i \). \( g \) is the acceleration of gravity.
3. System control synchronization scheme

3.1. Control model design
In order to facilitate the derivation and simulation calculation of the control synchronization theory in the dual-mass vibration system, the parameters of the two motors on the vibrating body are kept consistent, and the midpoint of the O1O2 connection is completely symmetrical. Based on the reference model adaptive theory, this paper studies the control synchronization strategy of the dual-mass vibration system. Take the motor 2 on the left as a reference model, control the motor 1 on the right, and let the motor 1 synchronously track the motion state of the motor 2, so that the two motors can achieve real-time synchronization effect. Taking the speed of the two motors as the reference object and the control object, there is a control equation:

\[ \dot{\theta} + a_1 \dot{\theta} + a_2 \theta = a u \]

\[ \dot{\theta}_m + b_1 \dot{\theta}_m + b_2 \theta_m = b r \]  

Define the error signal as:

\[ e = \theta_m - \theta \]

Therefore, the error dynamic equation is defined as:

\[ \dot{e} + b_1 \dot{e} + b_2 e = b_r - a u + (a_1 - b_1) \theta + (a_2 - b_2) \dot{\theta} \]

Definition \( e = [e \ e^T] \), Then the error state equation is as follows:

\[ \dot{e} = [\dot{e} \ e^T] = A \dot{e} - \begin{bmatrix} 0 \\ \Delta \end{bmatrix} u + \begin{bmatrix} 0 \\ \Delta \end{bmatrix} \]

In this equation, \( A = \begin{bmatrix} 0 & 1 \\ -b_1 & -b_2 \end{bmatrix} \), \( \Delta = b_r + (a_1 - b_1) \theta + (a_2 - b_2) \dot{\theta} \)

3.2. Design of adaptive control law
Through the design of b1 and b2, the eigenvalue of matrix A has a negative real part, exist:

\[ A^T P + PA = -Q \]  

Take the PD form to define the control item as:

\[ \dot{e} = p_{21} e + p_{22} \dot{e} \]

\[ \begin{bmatrix} p_{11} & p_{12} \\ p_{21} & p_{22} \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 0 & 1 \end{bmatrix} P \]

In this equation, The form design control law of pre-feedback plus PD feedback is:

\[ u = k_0 r + k_1 \theta + k_2 \dot{\theta} \]

Substituting formula (8) into formula (4), we get:

\[ \dot{e} + b_1 \dot{e} + b_2 e = b_r - a (k_0 r + k_1 \theta + k_2 \dot{\theta}) + (a_1 - b_1) \theta + (a_2 - b_2) \dot{\theta} \]

\[ = (b - ak_0) r + (a_2 - b_2 - ak_2) \theta + (a_1 - b_1) \dot{\theta} \]  

In formula (9), in order to ensure that \( e \) approaches 0 and realize the adaptive control of unknown parameters \( a, a_1 \) and \( a_2 \), the lyapunov function is designed as:

\[ V = \frac{1}{2} e^T Q e - \frac{1}{2 \lambda_a} a \]

\[ V = \frac{1}{2} (b - ak_0) e + \frac{1}{2 \lambda_a} (a_2 - b_2 - ak_2) e + \frac{1}{2 \lambda_a} \]

Derived from the above formula, we can finally get:

\[ \dot{V} = -\frac{1}{2} e^T \dot{Q} e + \left( \dot{e} r - k_0 \right) \frac{1}{\lambda_a} (b - ak_0) + \left( \dot{e} \theta - k_1 \right) \frac{1}{\lambda_a} (a_2 - b_2 - ak_2) + \left( \dot{e} \theta - k_2 \frac{1}{\lambda_2} \right) (a_1 - b_1 - ak_2) \]
Therefore, the design adaptive law is: 
\[ \dot{k}_1 = \lambda_1 \dot{e} \theta \quad \dot{k}_2 = \lambda_2 \dot{e} \theta \]

The synchronous control scheme of the dual-mass vibration system designed according to the model reference adaptive control law is shown in Figure 2.

![Figure 2 Adaptive control synchronization scheme of the double-mass vibration system](image)

**4. Simulation verification and analysis**

After the theoretical derivation, this article uses MATLAB software to simulate the synchronization control. The main parameters in the simulation are as follows: 
- \( m_1 = 89 \text{kg}, \ m_2 = 56 \text{kg} \), 
- \( k_1 = 6500000 \text{N/m} \), 
- \( k_2 = 4000000 \text{N/m} \), 
- \( c_1 = 100 \text{Nm} \cdot \text{s/rad} \), 
- \( c_2 = 100 \text{Nm} \cdot \text{s/rad} \), 
- \( m_{01} = 3.5 \text{kg} \), 
- \( m_{02} = 3.5 \text{kg} \), 
- \( r_1 = r_2 = 0.08 \text{m} \), 
- \( c_{01} = 0.01 \text{Nm} \cdot \text{s/rad} \), 
- \( c_{02} = 0.01 \text{Nm} \cdot \text{s/rad} \), 
- \( J_{01} = 0.01 \text{kg} \cdot \text{m}^2 \), 
- \( J_{02} = 0.01 \text{kg} \cdot \text{m}^2 \).

The simulation result is shown in Fig. 3, and it can be seen from Fig. 3(A) that the speeds of the two motors are not synchronized initially, and the speed of motor 1 is slightly behind that of motor 2. However, during operation, the rotation speed of the motor 1 catches up with the motor 2 in a short period of time, and remains consistent with the motor 2 in the subsequent movement. Figure 3(B) shows the parameter change of the control motor 1. In the initial stage, because the motion states of the two motors are not synchronized, the control parameters fluctuate significantly, so that the speed of the motor 1 is consistent with the motor 2 in a short time. After the speeds of the two motors are the same, the input parameters of the control motor 1 remain stable due to the better synchronization state of the motor movement. Figure 3(C) shows the changes of the three parameters of the adaptive law \( k_1 k_2 \) and \( k_3 \) in the control system during the simulation process. When the rotational speeds of the two motors are not synchronized, the adaptive law changes greatly, so that the input signal of the motor 1 is changed, so as to adjust the rotational speed of the motor 1 so that the vibration system changes to a synchronized state. After the two motors reach the synchronization state, the adaptive law is still adjusting itself in order to ensure the steady state of the control system. After finding the most suitable parameters during the adjustment process, the three adaptive law parameters are maintained at the optimal values.

![Simulation results](image)

(A) motor track tracking and tracking error
5. Conclusions

(1) Based on the establishment of the dynamic model of the dual-mass vibration system, based on the adaptive control theory, the control and synchronization strategy of the dual-mass vibration system is designed. First, according to the structure of the double-mass vibration system, the article is established. Then, the article designs and derives the adaptive control synchronization control strategy to realize the synchronization control of the two motors. Finally, based on the Lyapunov stability theory, a system stability control system is established. This control strategy has universal applicability to the synchronous control of the dual-motor mechanical vibration system.

(2) The simulation results prove the effectiveness of the control strategy here for the synchronous control of the dual-motor vibration system. This control strategy can provide a theoretical direction for the application and design of multi-vibration motors.

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