Super-Penrose process for extremal charged white holes

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We consider collision of two particles 1 and 2 near the horizon of the extremal Reissner-Nordström (RN) black hole that produce two other particles 3 and 4. There exists such a scenario that both new particles fall in a black hole. One of them emerges from the white hole horizon in the asymptotically flat region, the other one oscillates between turning points. However, the unbounded energies $E$ at infinity (super-Penrose process - SPP) turn out to be impossible for any finite angular momenta $L_{3,4}$. In this sense, the situation for such a white hole scenarios is opposite to the black hole ones, where the SPP is found earlier to be possible for the RN metric even for all $L_i = 0$. However, if $L_{3,4}$ themselves are unbounded, the SPP does exist for white holes.

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I. INTRODUCTION

In last decade, great interest is provoked in high energy collisions of particles in strong gravitational field. This happened after findings in [1], where it was shown that the energy in the center of mass $E_{c.m.}$ can grow unbounded if collision takes place near the horizon of the extremal Kerr black hole (this is called the BSW effect, after the name of its authors). Later, many papers appeared in which a list of potential sources of high energy collisions was extended. In particular, it includes white holes. One of scenarios consists in collision in our

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universe between particles one of which falls into a black hole while the second one emerges from a white hole \[2, 3\]. Recently, another scenario was considered for the extremal Kerr metric in which collision happens near the black hole but, afterwards, particles cross the horizon, leave our universe and appear in a white hole region \[4\]. As a result, high energy fluxes can be registered in another universe. Or, vice versa, if the process started in some another world, this leads to high energy particles that are, in principle, can be detected in our Universe, so this would possibly give explanation of astrophysically relevant high energy processes.

Our aim is to consider a charged counterpart of \[4\]. By definition, if \(E\) is unbounded, one can speak about the super-Penrose process (SPP). Instead of rotating black holes, we consider a more simple case of the Reissner-Nordström (RN) black-white hole. It was shown earlier that in such a metric, the counterpart of the effect found in \[1\] also occurs \[5\]. Moreover, it turned out that for the RN metric the SPP is also possible \[6\] in contrast to the case of rotating black holes \[7\]. Thus there are two mutually complimentary cases, each of which deserves special attention - rotating and static charged black-white holes. In the present case we consider just the second option. We elucidate, whether or not the SPP is possible for such white holes.

Some reservations and additional arguments concerning our motivation are in order. There are strong factors that testify in favour of the instability of white holes (see Sec. 15 of \[8\]). However, the nontrivial structure of the RN space-time that includes white hole regions \[9\] follows from the theory anyway, so the complete theory of the BSW effect should include in consideration the corresponding version of this effect. Moreover, white holes can reveal themselves as windows from other worlds through which energy can flow into ours \[10, 11\]. In this context, a new potential source of ultra high energy can be just one more manifestations of white holes instability thus deserving to be studied. Also, we would like to draw attention to the following detail. After the paper \[1\] the interest to more earlier works on high energy collisions near the horizon was revived \[12, 13\]. Meanwhile, the head-on collisions considered in \[13\] (see eq. 2.57 there) correspond just to white holes.
II. BASIC EQUATIONS

Let us consider the metric

\[ ds^2 = -dt^2 N^2 + \frac{dr^2}{N^2} + r^2 d\omega^2, \]  

(1)

where \( d\omega^2 = d\theta^2 + \sin^2 \theta d\phi^2 \). We will deal with the extremal RN metric for which

\[ N = 1 - \frac{r_+}{r}, \]  

(2)

where \( r_+ = Q = M \) is the horizon radius, \( Q \) being the electric charge of a black hole, \( M \) its mass. We use the system of units in which fundamental constants \( G = c = 1 \).

For a particle with the electric charge \( q \) and the mass \( m \) the equations of motion within the plane \( \theta = \frac{\pi}{2} \) read

\[ m\dot{t} = \frac{X}{N^2}, \]  

(3)

\[ m\dot{r} = \sigma P, \quad \sigma = \pm 1, \]  

(4)

\[ P \equiv \sqrt{X^2 - \tilde{m}^2 N^2}, \quad \tilde{m}^2 \equiv m^2 + \frac{L^2}{r^2}, \]  

(5)

\[ m\dot{\phi} = \frac{L}{r^2}. \]  

(6)

Here

\[ X = E - q\varphi, \]  

(7)

\( E \) is the energy, \( \varphi = \frac{r_+}{r} = 1 - N \) is the electric potential of the extremal RN black hole, \( L \) being the angular momentum, \( \sigma = \pm 1 \), dot denotes derivative with respect to the proper time \( \tau \). The forward-in-time condition requires

\[ X \geq 0. \]  

(8)

Let us consider the following scenario. Particles 1 and 2 fall from infinity, collide in point \( r = r_c \) close to the horizon and create particles 3 and 4. Thus \( \sigma_1 = \sigma_2 = -1 \). The conservations of the electric charge, energy and radial momentum in the point of collision give us

\[ X_0 \equiv X_1 + X_2 = X_3 + X_4, \]  

(9)

\[ q_0 \equiv q_1 + q_2 = q_3 + q_4, \]  

(10)

\[ L_0 \equiv L_1 + L_2 = L_3 + L_4, \]  

(11)
\[ E_0 \equiv E_1 + E_2 = E_3 + E_4, \quad (12) \]
\[ -P_1 - P_2 = \sigma_3 P_3 + \sigma_4 P_4. \quad (13) \]

We will use the standard classification. If \( X_H = 0 \) (subscript "H" means that a corresponding quantity is taken on the horizon), a particle is called critical. If \( X_H \neq 0 \) is separated from zero, it is called usual. If \( X_H \neq 0 \) but is very small (of the order \( N_c \equiv N(r_c) \)), it is called near-critical. As for the extremal RN black hole, \( \varphi(r_+) = 1 \), the criticality condition reads \( E = q \).

For the critical particle,
\[ X = EN, \quad (14) \]
\[ P = N\sqrt{E^2 - \tilde{m}^2}. \quad (15) \]

If \( N \ll 1 \), we have for the usual particle,
\[ X = E - q + qN, \quad (16) \]
\[ P = X + O(N^2). \quad (17) \]

In what follows, we are interested in high-energy processes, so we assume that particle 1 is critical and particle 2 is usual. This choice guarantees that the energy in the center of mass frame \( E_{\text{c.m.}} \) is unbounded \[5\].

III. TYPES OF SCENARIO

We consider collision near the horizon, so \( r_c \approx r_+ \). Assuming that all masses and angular momenta are finite and taking the limit \( r_c \to r_+ \), one can infer from (9) and (13) that particles 3 and 4 cannot be both usual. Let particle 3 be near-critical and particle 4 be usual. It is convenient to write for a near-critical particle
\[ q = E(1 + \delta), \quad (18) \]
where \( \delta \ll 1 \). As it is substantiated in \[6\], it makes sense to take \( \delta \) of the order \( N_c \), so
\[ \delta = C_1 N_c + C_2 N_c^2 + \ldots. \quad (19) \]
Actually, the terms of \( N_c^2 \) and higher can be neglected. Then, we have for such a particle
\[ X = NE(1 - C_1) + O(N^2), \]  
\[ P = N\sqrt{E^2(1 - C_1)^2 - \tilde{m}^2} + O(N^2). \]  

Then, we can classify the scenarios of collision by means of two parameters. If immediately after collision a particle moves inward, the scenario is called IN, if it moves outward, the scenario is called OUT. And, depending on the sign of \( C_1 \), we write + or −. As a result, we have 4 possible cases OUT+, OUT−, IN+, IN−. The first three of them were already analyzed in [6]. What remains to be seen is the property of scenario IN−. It was rejected in [6] since it corresponds to fall of both particles into a black hole. However, it is this scenario that is of interest to us now since it ensures the energy transfer to a white hole region (see details below).

For our scenario IN− we have \( \sigma_3 = -1 \). Then, it follows from (9) and (13) for \( r_c \) close to \( r_+ \) that \( \sigma_4 = -1 \). In other words, two particle collide near the black hole horizon and enter the inner region. Now, we are going to elucidate, whether or not in this process \( E_3 > 0 \) can be unbounded. If yes, \( E_4 = E_0 - E_3 \) is negative and unbounded for any finite \( E_0 \) in (12). The properties of corresponding trajectories are described below.

**IV. DYNAMICS OF COLLISION**

It follows from (14) - (17) and (20), (21) that the conservation of the radial momentum (13) with \( \sigma_3 = \sigma_4 = -1 \) can be rewritten in the form similar to that used in [6]:

\[ F = -\sqrt{E_3^2(1 - C_1)^2 - \tilde{m}^2}, \]  

where

\[ F \equiv A + E_3(C_1 - 1), \]  

\[ A \equiv E_1 - \sqrt{E_1^2 - \tilde{m}^2}. \]  

Taking the square of (22), we obtain

\[ C_1 = 1 - \frac{\tilde{m}^2 + A^2}{2AE_3}, \]  

\[ F = \frac{A^2 - \tilde{m}^2}{2A}. \]
As, for our scenario IN, $C_1 < 0$, we immediately obtain that

$$E_3 < \frac{\tilde{m}_3^2 + A^2}{2A}. \quad (27)$$

We see that $E_3$ is bounded from the above, so SPP in the white hole region is impossible. From another hand, the condition $F < 0$ that follows from (22), gives us a lower bound on the effective mass, $\tilde{m}_3 > A$.

The result about impossibility of the SPP retains its validity if, instead of the given process we consider its Schnittmann analogue [14], when the critical particle 1 comes from the horizon, so $\sigma_1 = +1$. The only changes is that $A = E_1 + \sqrt{E_1^2 - m_1^2}$ instead of (24).

V. RAPIDLY ROTATING PARTICLES

In the above consideration, it was assumed that $L_3$ is bounded. Then, because of finiteness of the total angular momentum $L_0$ (11), the quantity $L_4$ is bounded as well. Meanwhile, there is a separate question: is it possible to achieve large $E_3$ due to large $L_3$? If yes, restriction (27) becomes irrelevant. For a scenario of such a type, one has to take into account large $L_{3,4}$ from the very beginning, already in $P_{3,4}$. A new picture, qualitatively different from the one considered above, arises if

$$L_3 = \frac{l_3}{\sqrt{N_c}}, \quad L_4 = L_0 - \frac{l_3}{\sqrt{N_c}}, \quad (28)$$

where $l_3$ does not contain small parameters. In this case, the analysis of eq. (13) should be carried out anew. Then, taking the limit of $N_c \to 0$ and equating the terms of the zeroth order with respect to $N_c$ in eq. (13), we obtain that $\sigma_3 = \sigma_4 = -1$. For particle 1 we can use (14), (15), for particle 2 it is sufficient to take (17). For particles 3, 4 the centrifugal terms with $L_{3,4}^2$ in $P_{3,4}$ give the correction that should be taken into account:

$$P_{3,4} \approx \sqrt{X_{3,4}^2 - N_c \frac{l_{3,4}^2}{r_+^2}} \approx X_3 - N_c \frac{l_{3,4}^2}{2X_3 r_+^2}. \quad (29)$$

Collecting all terms of the order $N_c$ and, one can obtain from the conservation laws (9), (13) that

$$A = \frac{l_3^2}{2r_+^2} \left( \frac{1}{X_3} + \frac{1}{X_4} \right). \quad (30)$$

Taking into account (9) one more time, we obtain the final expression

$$(X_{3,4})_c = \frac{X_0}{2} (1 \pm \sqrt{1 - b}), \quad (31)$$
\[ b = \frac{2l_3^2}{r_+^2 X_0 A}. \]  

(32)

It is implied that \( b < 1 \). Then,

\[ E_3 = (X_3)_c + q_3 \varphi(r_+) \approx (X_3)_c + q_3. \]  

(33)

In doing so, there is no bound like (27) at all. This is because both particles 3 and 4 are usual, so the conditions \( P_{3,4}^2 > 0 \) are satisfied automatically since \( X = O(1) \) and \( N_c L = O(\sqrt{N_c}) \to 0 \) in (5) and there are no additional constraints. Both particles fall in a black hole. Thus we can have formally unbounded \( E_3 \) provided \( q_3 \) is also unbounded, to keep \( X_3 \) finite. Actually, there are no unbounded \( q \) in nature (\(|q| < Z_e |e|\), where \( Z_e \approx 170 \), \( e \) being the electron charge) that restricts the value of \( E_3 \) in a way similarly to what takes place in black hole scenarios \([6]\) (see also Sec. V in \([15]\) for discussion of macroscopic charged bodies). But \( E_3 \) is sufficiently large anyway, according to (33).

**A. Trajectories with \( E > 0 \) beyond a black hole horizon**

The expressions (31) are valid near the point of collision. To gain some energy due to particle 3 in the asymptotically flat region in the white hole zone, we need (i) the turning point that prevents a particle from falling in the singularity, (ii) the absence of the turning point outside the next horizon, for \( r > r_+ \). To simplify formulas, let us consider the case when \( m_3 = 0 \) or is negligible. Condition (i) is satisfied automatically, if \( E_3 > q_3 \) that is indeed valid according to (33). Then, the location of the turning point \( r_0 < r_+ \) is

\[ \frac{r_+}{r_0} = \frac{1}{2} \left( 1 - \frac{q_3 r_+}{L_3} \right) + \sqrt{\frac{1}{4} \left( 1 - \frac{q_3 r_+}{L_3} \right)^2 + \frac{E r_+}{L_3}} > 1. \]  

(34)

Condition (ii) is satisfied, if

\[ q_3 > \frac{L_3}{r_+}. \]  

(35)

If \( L_3 \) obeys (28), we can take

\[ q_3 = \frac{\alpha}{r_+ \sqrt{N_c}}, \]  

(36)

with

\[ \alpha > l_3. \]  

(37)

Then,

\[ E_3 \approx (X_3)_c + \frac{\alpha}{r_+ \sqrt{N_c}}, \]  

(38)

can be made as large as we like due to sufficiently small \( N_c \). Thus the SPP does exist.
B. Trajectories with $E < 0$

Particle 3 in the scenario under discussion has large $E_3 > 0$, so particle 4 has large $E_4 < 0$. It follows from (8) that now $q_4 = -|q_4|$, $X_4 = |q_4| \frac{r_+}{r_+ - |E_4|}$. It is clear that such a particle cannot escape to infinity since this would violate (8). It oscillates between turning points. They can be found from the condition $P_4 = 0$. If $m_4 = 0$ or is negligible, the corresponding equation is solved in a compact form,

$$
\frac{r_+}{(r_0)_{out}} = \frac{1}{2} \left( 1 - \frac{|q|r_+}{|L|} \right) + \sqrt{\frac{1}{4} \left( 1 - \frac{|q|r_+}{|L|} \right)^2 + \frac{|E|r_+}{|L|}}
$$

outside the horizon, $(r_0)_{out} > r_+$. For shortness, we omit subscript ”4”.

The generalized ergoregion [16] lies at $E = 0$, (35) for particle 4, and small but nonzero mass,

$$
\frac{r_{\text{erg}}}{r_+} \approx \sqrt{\frac{q^2 - \frac{L_4^2}{r_+^2}}{m}} \gg 1,
$$

We have taken into account that $L_4 = L_0 - L_3 \approx -L_3$ and eq. (35).

Thus $(r_0)_{in} < r_{\text{erg}}$ and the turning point lies inside the ergoregion, as it should be. There is no coincidence with eq. (12) of [16], since it corresponds to $L = 0$, whereas we consider the opposite case $m \ll \frac{|L|}{r_+}$.

Inside the black hole horizon

$$
\frac{r_+}{(r_0)_{in}} = \frac{1}{2} \left( 1 + \frac{|q|r_+}{|L|} \right) + \sqrt{\frac{1}{4} \left( 1 + \frac{|q|r_+}{|L|} \right)^2 - \frac{|E|r_+}{|L|}}.
$$

One can check that the conditions (35) and $|E_4| = -E_4 = |q_4| - X_4 < |q_4|$ do guarantee that $(r_0)_{out} > r_+$, $(r_0)_{in} < r_+$, so the picture is self-consistent.

One can also introduce the notion of the ergoregion inside the horizon, there the situation is opposite, $r_{\text{erg}} < (r_0)_{in} < r_+$ where now

$$
\frac{r_+}{r_{\text{erg}}} = 1 + \frac{|q|r_+}{|L|}.
$$

Thus the particle in question crosses the black hole horizon $r_+$, enters the white hole region, bounces in the turning point $(r_0)_{in}$ and moves to larger radii, crosses the new horizon $r_+$, bounces in the point $(r_0)_{out}$, falls inside the horizon $r_+$ again, etc. Earlier, it was pointed out in [17] that in the Kerr metric a particle with $E < 0$ cannot remain in the outer region and necessarily dives inside the horizon, where it either falls in a singularity or extends to an
infinite affine distance, remaining inside the original horizon. We see that both cases with particles with negative energies are similar in this sense.

VI. CONCLUSIONS

Thus we showed that particle collision on our side on the horizon do not lead to unbounded energies in the white hole region, if all parameters of original particles (masses, angular momenta, charges) are finite. In this sense, for the RN metric the black and white hole scenarios under discussion are complementary to each other. There exists the SPP for the black hole case [6] but there is no such a process for the white hole case. From another hand, there are special subcases, when a particles created in collision have unbounded angular momenta. For them, the SPP does indeed exist. In doing so, the electric charge should be also large (formally unbounded). Thus white holes can be indeed sources of ultrahigh energy fluxes in our universe created in the other ones but with reservation that the corresponding matter or radiation should be rapidly rotating.

The next problem is to extend the present approach to rotating white holes. This needs separate treatment.

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