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Economic Aspects of Building Energy Audit

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Abstract

Within the practice of construction economics, cost-benefit audits are carried out by proprietary audits with the intention of reporting the adequacy of any action and decision taken, meeting planned objective of a project or by efficiency audit which requires a more concise and restrictive investigation (like energy optimization) for its reporting. The efficiency audit system is most appropriate for energy utilization and performance investigation since it seeks to compare actual level of energy uses as against planned targets. This economic audit system of building energy requires that information about the energy designs are collected by means of management information system (MIS), reestablishing the data collected, comparing potential energy financial parameters with actuals, establishing the possible causes of variance. This is often justified or validated by such techniques as budgeted energy cost variance analysis, present value depreciation method, profit variance analysis, and cash flow and financial criteria analysis.

Keywords: building energy, energy audit, thermal properties, cost variance analysis, profit variance analysis

1. Introduction

The basic idea that heat is a form of energy that flows from one point to another as a result of difference in temperatures though governed by the laws of thermodynamics suggests that it can take a pattern of distribution in space according to the medium it travels through. Going through the space by way of transfer and interacting with the bodies it comes in contact with is a function of the phase and provided that there is no such change in phase, the heat required to raise the temperature of a mass of a building element \( m \) by a temperature \( T \) is associated by Eq. (1):

\[
Q = mct \quad c = \text{being the specific heat capacity of the element} \quad (1)
\]

where \( m \) = mass of building element; \( t \) = temperature of building element.

At the point of temperature, increase Eq. (2) heat is absorbed or evolved when there is a phase change such that:

\[
Q = ml, l = \text{being the specific latent heat} \quad (2)
\]

On the contrary, the absorption or evolution of heat causes heat loss to its surrounding, such that a reversal process of that nature makes heat to be loss at a
certain rate with a typical building space. The rate of such heat loss brings about the cooling of the environment or space which is proportional to the excess temperature of the building space over the external temperature of its surroundings based on forced convection when the excess temperature is small.

2. Thermal properties in building

Within a building space, heat is distributed or transferred by three fundamental methods which names are:

2.1 Conduction

This method of heat distribution or transfer in building spaces is a resultant effect of kinetic energy transfers at molecular level in any of the three states of matter (solids, liquids and gases). Conduction method of heat distribution or transfer in buildings is naturally validated to flow in the direction of tapering temperature. Such conductive behavior of heat loss in buildings is attractively noticeable in opaque walls during the winter [1]. There are experimental agreements between thermal and electrical conduction pattern in solids. The earliest formal knowledge of heat conduction law through a medium of either solid, liquid or gas was idealized by Joseph Fourier who postulated the law of heat conduction transfer method in the early part of the nineteenth century [2]. We take a congruency of how heat is conveyed through building elements (materials) or its space to be analogous to heat conduction in the building. Firstly, Fourier stated based on experimental verification for a steady conduction that the rate of heat transfer in any medium (inclusive of building elements) by conduction \( Q \) is proportional to the temperature difference and the heat flow area impacted by the heat in such a way that the heat conduction rate \( Q \) is inversely proportional to the distance through which the conduction traveled [1, 2]. Clearly in mathematical expression Eq. (3), Fourier meant that

\[
Q = -KA \frac{dT}{dx}
\]  

with \( K \) = thermal conductivity of the materials (W/(m.k)); \( A \) = area through which heat flow occurred; and \( \frac{dT}{dx} \) = temperature gradient at any point in \( x \) been the space through which the heat flow.

The minus sing indicating a flow from higher point to a lower point. For a recourse to building walls made of brick, block, fiber, paneled steel with known wall thickness, conductivity of heat from outer skin to inner skin can be estimated by:

\[
Q = KA \frac{T_1 - T_2}{\Delta x}
\]  

with \( k \) = thermal conductivity (W/(m.k)); \( T_1 \) = outer/higher temperature; \( T_2 \) = inner/lower temperature; \( A \) = area through which conduction flowed; and \( \Delta x \) = thickness of materials in which the conduction occurred.

The property of heat causing the differential between outer and inner temperature value is occasioned by the material’s resistance to heat which is obtained when the above expression is having the conductivity \( k \) related to the area \( A \) as Eq. (5):

\[
Q = \frac{T_1 - T_2}{\Delta x/KA} R = \frac{\Delta x}{KA} \text{ as unit thermal resistance}
\]
In practice, the term is commonly referred to as $R$-value such that Eq. (6)

$$R_{th} = \frac{\Delta x}{k} = AR$$

Another pseudo form of measuring thermal conductance in materials is the $U$-value which is expressed as the reciprocal of the $R$ value:

$$U = \frac{1}{R_{th}}$$

The possibility that a building wall is layered with different materials and different geometries suggests that Fourier laws cannot be restricted to single layer with uniform thermal resistance [1]. HVAC systems carries different insulating and piping materials which calls for Eq. (8) cylindrical examination of Fourier law of steady heat conduction in cylindrical coordinates [3]. Under such consideration,

$$Q = \frac{T_1 - T_2}{ln(r_i/r_o)/2\pi Kl}$$

with $r_i =$ outer radius; $r_o =$ inner radius; $L =$ pipe length; and $K =$ thermal conductivity.

As a general rule, to the effect of other geometries, a shape factor ($S$) is introduced Eq. (9) to accommodate any derived shape for the measurement of heat loss of pipes in buried walls conveying hot fluid as:

$$Q = KS\Delta T = KS (T_1 - T_n)$$

Usually shape factors are derivatives of components meant in design to restrain losses which are either isothermal cylinder Eq. (10) with $S$-value of

$$S = \frac{2\pi L}{Cosh^{-1}(D/r)}$$

where $L \gg r$; and $D =$ buried depth in semi-infinite medium Eqs. (11) and (12).

$$S = \begin{cases} \frac{2\pi L}{ln (2D/r)} & L \gg r \\ \frac{2\pi L}{ln \left(1 - \frac{ln (D/r)}{ln (r)}\right)} & D \gg r \end{cases}$$

There can also be conduction between two isothermal cylinder Eq. (13) buried in infinite medium with $S$-value of

$$S = \frac{2\pi L}{Cosh^{-1}\left(\frac{D^2}{\sqrt{r_1r_2}}\right)}$$

It can also take the form of conduction through two composite rectangular plane sections with edge section of two adjoining walls having combined $k$-value and inner/outer surface uniform temperatures $S$-value of Eq. (14)

$$s = \frac{aL}{\Delta x} + \frac{bL}{\Delta x} + 0.54$$
2.2 Convention

Unlike heat conduction where the transference of heat is through a body (solid) without visible motion of any part of the body to the naked eyes, convention is a method of heat transfer in fluid by the movement of the fluid itself [4]. Heat convention primarily takes two forms:

a. **Natural (or free)** heat convention is when the motion of the fluid is due solely to the presence of the hot body in it giving rise to temperature with a resultant mediums’ density gradient causing the fluid to move under the control or restriction of gravity.

b. **Forced** heat convention is a process where heat is transferred with relative motion between the hot body and the fluid maintained by some external agency such as draught, making the relative velocity to contribute to the gravity current negligibly.

Free convention particularly results in density differences in the fluid, occasioned by contact with the surface originating the heat transfer [5]. Free conventions are evident in gentle air circulation in rooms due to solar-warmed windows or walls. On the other hand, forced convention occurs from the effect of an external force. Beside gravity to the problem, fluid moves past a warmer or cooler surface in obedience to Newton’s laws of cooling. Under the two considerations, fluid velocities in free convention are considerably lower than fluid velocities in forced convention. Efficiency of heat transferred is a direct consequence of greater mechanical energy consumed in forced flow situation [1, 2]. Forced convention is seen applicable in the heat transfer process from heating and cooling coils. Convention is majorly responsible for cooling of buildings making it a common mode of heat transfers in buildings (Tables 1–3).

As a fall out of Newton’s law of cooling which simply states that the rate at which heat is transferred by convention is proportional to the temperature difference and the heat transfer area. Mathematically, Newton by the law Eq. (15) is expressed as:

\[
Q = h_{con} A (T_s - T_f) = h_{con} A (\Delta T)
\]  

(15)

with \(h_{con}\) = convention coefficient (W/m².k)); \(A\) = surface area through which convention occurs; \(T_s\) = surface temperature; and \(T_f\) = fluid temperature, away from wall.

| Material      | Specific Mass (kg/m³) | Thermal Conductivity W/m²K |
|---------------|-----------------------|---------------------------|
| brick         | 1600-1900             | 0.6-0.7                   |
| marble        | 2553                  | 2.2                       |
| Gravel Concrete | 2300-2500             | 1.7                       |
| Light Concrete | 1600-1900             | 0.7-0.9                   |
| Glass         | 2500                  | 0.8                       |
| Gypsum        | 1300                  | 0.5                       |
| Hardwood      | 800                   | 0.17                      |
| Softwood      | 550                   | 0.14                      |
| Plywood       | 700                   | 0.17                      |
| Floor Tiles   | 2000                  | 1.5                       |
| Asphalt       | 2100                  | 0.7                       |
| Linoleum      | 1200                  | 0.17                      |

Table 1a.  
*Thermal conductivity of common building materials (a, b, c).*
The best theoretical approach for analyzing heat convention is attained by parameters of dimensional analysis using mass, length, time, and temperature as focal dimensions Eq. (16). For dynamically similar bodies, natural convention is measured by

$$h_{con} = \frac{l^3 g \alpha P T}{C_1 \eta \lambda p}$$

This expression contains three dimensionless groups which include the Nusselt number $h_{con}$, the Grashof or free convention number $l^3 g \alpha P T$, and the Prandtl number $C_1 \eta \lambda p$ where $f_1$ and $f_2$ is assumed to be dependent on the shapes of the dynamic bodies involved which serves as equivalents of shape factors ($s$) in conduction mode. When the convention is not free, i.e., forced, the analyzing equation Eq. (16) takes the form of Eq. (17)

$$h_{con} = \frac{l^2 P T}{C_1 \eta \lambda p}$$

Since it is a forced convention, this expression omits the free component (Grashof) and introduces the Reynolds number to the expression $l^2 P T$. With all the numbers having reference tables, it makes it easy for HVAC designers to measure.
A corresponding derivation for $R$ value and thermal resistance exists for convention methods of transfer that serves for both forced and free conventions Eq. (18) as

$$ R = \frac{1}{h_{\text{con}} A} \tag{18} $$

With a heat transfer function of $Q = \frac{\Delta T}{R}$ (19)

Resistance to thermal effusion under convention with $R_{th}$ value and the associated $U$-value are given by Eqs. (20) and (21)
Heat flows outside of buildings have been a source of heating in the inside of buildings. It is well a good preemptive move to determine heat flows outside of buildings which naturally contributes to the heating in the entire building envelope [1, 3, 5]. Most external heat flows in buildings with forced convention flows are usually regarded as turbulent, but usually again take the form of laminar or

\[ R_{th} = \frac{1}{U h_{con}} \quad (20) \]

\[ U \equiv \frac{1}{R_{th}} = h_{con} \quad (21) \]
turbulent flow when the convention currents are free [1–3]. Keeping units of measurements in SI units, there are experimental proofs that with air temperature between 19 and 21°C for interior walls and window surfaces having confluence, with exterior surfaces, laminar free convention of air from internal surfaces is given as Eq. (22)

\[ h_{con} = 1.4232 \left( \frac{\Delta T \sin \beta}{L} \right)^{1/4} \]  

wherein \( \Delta T \) equates the temperature difference as \( (T_s - T_f) \), with \( L \) as the length of the horizontal framing member on a vertical stud and titling surface in the direction of the buoyed-driven flow caused by the convention. \( \beta \) is the surface tilt angle with acute properties (30°–90°) and the flow condition that \( L^3 \Delta T < 1.0 \).

If \( L^3 \Delta T < 1 \), a turbulent flow occurs for which turbulent free convention from a tilted surface in air gives Eq. (23)

\[ h_{con} = 1.3131 \left( \Delta T \sin \beta \right)^{3/4} \]  

For horizontal members’ particularly horizontal pipes and cylindrical members in air, laminar free flow convention is estimated from Eq. (24)

\[ h_{con} = 1.3131 \left( \Delta T / D \right)^{3/4} \]  

with \( D \) as the cylinder’s outer diameter. Both turbulent and laminar flows have the same standards for test and measurement with those of tilted members and an adjustment for \( L \) with \( D \). Notwithstanding, building elements with cylindrical components in air have their turbulent free flow convention computed from Eq. (25)

\[ h_{con} = 1.2401 \left( \Delta T \right)^{3/4} \]  

However, structural members or surfaces say flat roots having complete 100% exposure horizontally to solar warming without recourse to solar angle have their laminar free flow convention coefficient estimated from Eq. (26)

\[ h_{con} = 1.3203 \left( \Delta T / L \right)^{3/4} \]  

with \( L \) as the average uniform length of the horizontal surface. The flow condition for the above expression is also true for humid or cold surfaces in reversed contact with the sun as obtainable in the surface of a plane skylight in roof tops. Warm surfaces in direct exposure with solar light have their turbulent free convection coefficient computed from Eq. (27) turbulent flow as

\[ h_{con} = 1.5214 \left( \Delta T \right)^{3/4} \]  

warmed surfaces not having direct surface exposures have their laminar convection coefficient reduced owing to stable stratification condition.

2.3 Radiation

This is the process whereby radiant heat energy is transferred from one point to another. It belongs to the class of electromagnetic spectrum between higher and radio waves with a range of wavelength between 740 and 0.3 mm approximately.
Heat radiations, being electromagnetic in nature, are originated by thermal movement of particles in matter. At temperatures higher than absolute zero, all matter sends out thermal radiation. The dynamical behavior or movements of particles results in charge acceleration that produces electromagnetic radiation. Thermal emitting bodies at any temperature consist of a wide range of frequencies [3]. Most radiating bodies have dominant frequencies which shift to higher frequencies as the temperature of the source increases. In most thermal radiation situations, the total amount of radiation for all frequencies increases sharply as the temperature rises at a rate of $T^4$, with ‘$T$’ as the absolute temperature of the body [6, 7]. Consequently, the rate of the electromagnetic radiation emitted at a certain frequency is proportional to the amount of absorption that it would experience by the source [3, 6].

Estimation of heat transferred by radiation is called net radiative heat transfer, which is the heat transferred from one surface to another, being the heat leaving the first surface for the other and subtracting the heat arriving from the second surface.

For radiating black bodies, the radiation rate from Surface A to Surface B is given in Eq. (28):

$$Q_{1-2} = A_1E_{b1}F_{1-2} - A_2E_{b2}F_{2-1}$$  \hspace{1cm} (28)

where $A$ is surface area, $E_{b1}$ is energy flux, $F_{1-2}$ is the view factor originating from surface 1 to surface 2.

Since the two surfaces exchange their heat loses, the reciprocity rule holds for the view factors as $A_1F_{1-2} = A_2F_{2-1}$ with heat flux emissive power of black body given as $E_b = \sigma T^4$ then $Q_{1-2} = A_1F_{1-2}(T_1^4 - T_2^4)$ where ‘$\sigma$’ is the Stefan-Boltzmann constant and $T$ is absolute temperature. Should the value of $Q$ give negative value; it suggests that net heat transfer is from surface 2 to surface 1.

As a departure for black bodies, two gray surfaces retaining an enclosure have their heat transfer rate given in Eq. (29):

$$Q = \frac{\sigma(T_1^4 - T_2^4)}{c_1 + \frac{1}{A_1F_{1-2}} + \frac{1}{A_2F_{2-1}}}$$  \hspace{1cm} (29)

where $c_1$ and $c_2$ are the emissivity of the surfaces and $\epsilon = \frac{E}{E_b}$ is the actual emissive power over the emissive power of black body.

Theoretically, the Stefan-Boltzmann law as stated in Eq. 28 above governs radiation emission of a blackbody (ideal radiator). Besides the emissivity of materials, other indices used for computing the rate of radiation heat transfer from surfaces includes absorptivity ($\alpha$), transmissivity ($\tau$) and reflectivity ($\rho$) [3, 5, 8]. All radiating surfaces have these three properties Eq. (30) related by the law of conservation of energy as:

$$\alpha + \tau + \rho = 1.$$  \hspace{1cm} (30)

However, this relation depends on the nature of the wave-length been radiated which is verifiably true for single wavelengths and gray surfaces. For wavelengths whose range are over the three properties are calculated as same.

Temperature absorption questions through building elements with dark boundaries have been given extensive analysis in the works of [7] by integro-differential means in Eq. (31)

$$N_T^* \frac{d^2\theta(\tau)}{d\tau^2} = n^2(\tau)\theta^4(\tau) - 1/2 \left[ \beta(\tau)E_2(\tau) + \beta(\tau^*)E_2(\tau^* - \tau) + \int_0^{\tau^*} n^2(\tau^*)E_1(|\tau - \tau^|)\theta(\tau^*)d\tau^* \right]$$  \hspace{1cm} (31)
Noting the boundary condition to be

\[ \theta(0) = \theta_2, \theta \tau^n = 1.0 \]

whereas, [7] showed estimation of the absorbed heat with dimensionless temperature value which has equally been shown to be of value expressed in Eq. (32)

\[ \theta \tau^n = G(\tau) = \frac{1}{2N r_0} \int_0^\tau n^2(\tau) \left\{ -E_3([\tau - \tau^n]) + E_3(\tau^n) + \frac{\tau}{\tau^n} [E_3(\tau^n - \tau) - E_3(\tau^n)] \right\} d\tau^n \]  
\[ \text{(32)} \]

Annotated by Eq. (33)

\[ G(\tau) = \frac{1}{2N r_0} \left( \beta(0) \left[ -E_4(\tau) + \frac{\tau}{\tau^n} E_4(\tau^n) + \frac{1}{3} \left( 1 - \frac{\tau}{\tau^n} \right) \right] \right. \\
+ \beta(\tau^n) \left[ \left( 1 - \frac{\tau}{\tau^n} E_4(\tau^n) - E_4(\tau^n - \tau) + \frac{1}{3} \frac{\tau}{\tau^n} \right) \right] \\
+ 2N_r \left\{ \theta(0) + \frac{\tau}{\tau^n} \left[ \theta(\tau^n) - \theta(0)1 \right] \right\} \]  
\[ \text{(33)} \]

In near real life situation, determination of temperature profiles in buildings have not been successful with closed-form solutions but with numerical methods Eq. (34) to obtain the total building heat flux through its elements as

\[ q_t = \frac{k_c}{L} (T_1 - T_2) + 2\alpha \left( T_2^4 \left[ E_3 r^n - \frac{1}{2} E_4 r^n - \frac{1}{3} \frac{1}{r^n} \right] + T_1^4 \left[ -E_3(\tau^n - \tau) \right] \right) \\
+ \frac{1}{2} \left[ E_3(\tau^n - \tau) - E_3(\tau) \right] T^4(\tau) d\tau + \sigma T^4 + 2\sigma T_2^4 E_3(\tau^n) \\
- 2\sigma \left[ n^2(\tau^n) E_2(\tau^n - \tau) (T^4 \tau) d\tau \right] \]  
\[ \text{(34)} \]

The radiative and conductive fluxes are closely outlined by the terms of Eq. 34 in such a way that the first two terms of Eq. 34 are suggestive of conductive flux, while the last three terms are radiative [8, 9]. Upon the combination of both integrals, Eq. 34 becomes Eq. (35)

\[ q_t = \frac{k_c}{L} (T_1 - T_2) + 2\alpha \left( T_2^4 \left[ \frac{1}{2} E_3 r^n - \frac{1}{3} \frac{1}{r^n} \right] + T_1^4 \left[ -\frac{1}{2} E_4 r^n + \frac{1}{3} \frac{1}{r^n} \right] \right) \\
+ \left[ n^2(\tau^n) \frac{1}{2} [E_3(\tau^n - \tau) - E_3(\tau)] (T^4 \tau) d\tau \right] \]  
\[ \text{(35)} \]

with such algebraic treatment, Eq. 33 can as well be treated with integral calculus to give Eq. (36)
\[ \frac{d\theta}{d\tau} = (1/2N_r) [\beta(0) |E_3(\tau) + E_4(\tau^\circ)/\tau^\circ - 1/3\tau^\circ|] \\
+ \left( \frac{\beta(\tau^\circ)}{\tau} \right) \left[ -\frac{E_4(\tau^\circ)}{\tau} - E_3(\tau^\circ - \tau) + 1/3\tau^\circ \right] + (2N_r/\tau^\circ)[\theta(\tau^\circ) - \theta(0)] \\
+ \int_0^\tau \! n^2(\tau^\circ) \{E_2(|\tau - \tau^\circ|) + (1/\tau^\circ)|E_3(\tau^\circ - \tau) - E_3(\tau^\circ)|\} \theta^4(\tau^\circ) d\tau \\
\]

By securitizing Eq. 36, the steep behavior or temperature gradient of the absorption can be inferred from the dimensionless gradient \( \beta \) which satisfies the Schwartz inequality in Eq. (37)

\[ \beta = \frac{d\left(\frac{\sigma T - T}{T^4 - T^4} \right)}{d(\tau)} \geq 1 \quad \tau = 0 \]  

In recent times, many simulation techniques have been developed to determine temperature profiles and heat fluxes in building, but many of which are interactive in nature.

This brings us to the absorptivity and emissivity of gray surfaces which under the Kirchoff’s identity are equal, been

\[ \alpha = \epsilon \]

Even for non-gray surfaces at a stipulated wavelength, drawing a clue from [5] experiment that the mean free path of a photon \( \left( \frac{1}{E} \right) \) passing through an object with thickness, \( L \) is related by the formula

\[ \tau^\circ = \frac{L}{1/E} - EL \ll 1 \]

This is premised on the notion that the radiant heat flux is not tempered by the material and provided the conductive radiative mechanisms are not acting with each other, the building element will experience a total heat flux Eq. (38) equal to

\[ q_t = q_c + q_r \]  

where \( q_t \) = total heat flux; \( q_r \) = radiative heat flux; \( q_c \) = conductive heat flux—by Fourier Law.

Such that planar elements with thickness \( L \), having uniform properties and unidirectional steady state heat flow have their values computed from Eq. (39)

\[ q_r = k_c \frac{(T_1 - T_2)}{L} \]  

And radiant heat flux with two infinite parallel plates with temperatures at \( T_1 \), and having \( T_2 \) and emissivities \( \epsilon_1 \) and \( \epsilon_2 \) computed by Eq. (40)

\[ q_r = \frac{\sigma(T_1^4 - T_2^4)}{\frac{1}{\epsilon_1} + \frac{1}{\epsilon_2} - 1} \]  

with reference to Eq. (38), \( q_t \), and by substitution, reduces to Eqs. (41) and (42)
\[ q_t = k_c \frac{(T_1 - T_2)}{L} + \frac{\sigma (T_1^4 - T_2^4)}{\frac{1}{\varepsilon_1} + \frac{1}{\varepsilon_2} - 1} \]  

(41)

Since \( \varepsilon_1 = \varepsilon_2 = 1 \) for black plates, \( q_t \) becomes

\[ q_t = k_c \frac{(T_1 - T_2)}{L} + \sigma (T_1^4 - T_2^4) \]  

(42)

Besides [5] investigation for the optically thin limit case, the limiting case for the optical thickness limit was investigated by [10] for elements that are large compared to the mean free path of the photon causing the radiation, giving rise to conductive heat transfer process. By experiment, from [10]

\[ \tau^o = \frac{L}{1/E} = EL \gg 1 \]

So that radiant heat flux arising from radiant energy Eq. (43) becomes

\[ q_t = -k_r \frac{dT}{dx} \]  

(43)

And by combining the conductive and radiative heat transfers of the element, the total heat flux for the building element at a steady state for uni-directional heat flow as Eq. (44) becomes

\[ q_t = k_c \frac{T_1 - T_2}{L} + 4\sigma \frac{(T_1^4 - T_2^4)}{3aL} \]  

(44)

where \( k_r = \) radiative conductivity of a gray medium \( \approx \frac{16\sigma a T^3}{\pi} \)

\[ k_{app} = k_c + k_r \]

\[ q_t = -k_{eff} \frac{dT}{dx} \]

With this totality conduction, apparent thermal conductivity is obtained by the relationship

\[ k_{app} = \frac{q_t L}{(T_1 - T_2)} \]

With particular reference to the thickness (\( L \)) of the material, the conductivity through the optical element at constant temperatures of the plate as Eq. (45) becomes:

\[ k_{app} = k_c + \frac{\sigma (T_1^4 - T_2^4)}{(T_1 - T_2) L} \]  

(45)

Experiments have shown that the upper limit of the apparent thermal conductivity of the material greatly depends on the absorption coefficient of the material’s thickness and extreme absorption coefficients. With these two conditions, \( k_{app} \) becomes Eq. (46)
Discussing conductivity and heat radiation of building elements with respect to the element’s thickness as it affects the apparent thermal properties of insulation has its credit due to [8]. The basic concept of [8] idea is that by the very nature of insulation, conduction and radiation does not occur and their individual heat fluxes are sums. Going by [8] theorem, at heat radiation equilibrium, radiant heat flux between two infinite parallel plates separated by a di-heat gray material or medium at temperatures \( T_1 \) and \( T_2 \) as given in Eq. (47)

\[
q_r = \frac{\sigma (T_1^4 - T_2^4)}{1 + \left( \frac{1}{\epsilon_1} + \frac{1}{\epsilon_2} \right) - 2Q}
\]  

(47)

As stated in heat transfer literature, several methods exist for treating the effect of thickness on apparent thermal properties of insulation. But exact solution is found in the approach of [9] with the condition that \( T^o \gg 1 \); then

\[
Q = \frac{4/3}{\tau^o + \gamma}
\]

where \( \gamma = 1.4209 \).

Appropriate approximation to this problem is found in the \textit{exponential-kernel} as

\[
Q = \frac{4/3}{\tau^o + \frac{4}{3}}
\]

While that of [8] is consistent with the \textit{exponential-kernel} approximation, Rennex only introduced a factor in the approximation value of Eq. (48) by proposing that

\[
Q = \frac{4/3}{\tau^o + \frac{4}{3} \text{[factor]}}
\]

(48)

Factor = 1 + 0.0657 tanh (27°) while addressing the [9] \( Q \)-value estimation. A further theory on the effect of elements thickness on the apparent thermal conductivity with the assumption that radiative and conductive heat fluxes do not interact and premised on the computation that total heat flux is equal to the sum of the individual fluxes, [8] obtained the value of \( K_{app} \) by substitutions in Eq. (49)

\[
K_{app} = k_c + \frac{4\sigma T_m^3}{\frac{4}{3} - 1 + \frac{4}{3} \tau^o + 0.0657} \frac{L}{L}
\]

\[
K_{app} = k_c + \frac{4\sigma T_m^3}{\frac{4}{3} - 1 + \frac{4}{3} \tau^o} \frac{L}{L}
\]  

(49)

Due to Rennex, Eq. (50) we have

\[
K_{app} = \frac{4\sigma T_m^3}{\frac{4}{3} - 1 + \frac{4}{3} \tau^o + 0.0657 \tan (2\tau^o)} \frac{L}{L}
\]

(50)

Provided \( \epsilon_1 = \epsilon_2 = \epsilon \) and \( \frac{(T_1^4 - T_2^4)}{(T_1^4 - T_2^4)} \approx 4T_m^3 \).
Thickness has its effects on the conductivity of building elements demands [9, 10]. On the whole, optical thickness of elements increases the materials thermal conductivity by asymptotic expansion which tends to a limiting value in such a way that apparent thermal resistance has a linear dependence on element’s thickness as the element’s thickness approaches infinity. In the same vein, apparent thermal resistivity of the element is equal to the apparent thermal resistance divided by the elements thickness [6, 9, 10].

3. Building energy analysis

Building energy analysis in the twenty first century is becoming a prerequisite for building comfort design and its universal acceptability as a practice is not unconnected to energy crises with the advent of increased global warming. Consequently building energy analysis is required right from the project conceptual/scheme design stage to accommodate the integration of options and alternatives towards maximizing energy uses (see Figures 1-5) [1, 11]. This practice has some economic consideration with recourse to profit maximization of the project. In Europe, scheme designs must accommodate energy analysis report as a statutory requirement for building development by authorities with respect to various legislation backing it up [11]. Before now, building energy analysis is time consuming for experts, but in recent scheme development such arduous process have been responded to with Building Information Model (BIM) energy appraisals with the attendant aim of gaining project time and cost savings. Issues of time and cost in construction projects are economic indices of estimating project success and performance with varying degrees of their weights in a project. Most times, both variables are often contesting like lunar eclipse to avoid project escalations. The use of BIM in building energy analysis, strongly assist experts in avoiding complicated laborious calculations. Most common tool which software based in evaluating or analyzing building analysis are AUTODESK ECOTECT™, AUTODESK Green Building Studio™ Integrated Environment Solutions (IES) Virtual Environment and Revit™ [11].

To a large extent, energy uses and building performance largely depends on the envelop properties of the building since it attempts to balance energy transfer.
process between the internal and external environment. If properly handled after the scheme designs and documents are approved, energy analysis of the building envelope will give a clear cut direction for optimization and systems sizing towards energy efficiency and thermal comfort [3, 11]. Such building energy analysis distributes energy performances based on calculations from data.

The idea behind building energy analysis is to economically allocate annual energy budgets, economic optimization of energy, evaluating complicate with
statutory energy standards and assessment of alternative components, systems and subsystems designs (see Figures 3 and 4). Notwithstanding the benefits of building energy analysis, there are normative issues bothering on guidelines and standards required in carrying out this processes as required by the European Union building energy performance guidelines who stipulated the methodology for carrying out building energy performance calculations [3]. The methodology requires a comprehensive analysis on heating installations, air-conditioning installation, positioning and orientation of building, natural ventilation, internal climate conditions, passive solar systems and solar protection, thermal characteristics of the building, Built-in lightning installation. It is often required that energy audit is conducted at post occupancy stage with the aim of addressing the deficits between initial design values to actual values at occupancy [3, 11, 12].

There are legislations amongst European nations addressing issues that are intrinsically related to ethical standards and practices. With reference to the above legislation, building energy analysis must as a matter of uniformity of practice, accommodate the following input data as requisite of an energy analysis process. These includes, utility rates, weather data, building orientation, thermal properties of building elements, building geometry and anthropometrics, building orientation in space, building energy load, and heating, ventilation and air-conditioning system [11–13].

The process of energy analysis runs building performance simulations with the aim of optimizing energy efficiency and too promotes carbon zeroing estimated in initial design stage (see Figures 4 and 5). The economics of this process is well validated in its time and cost effectiveness in achieving high performance buildings. The general procedure for building energy analysis using AUTODESK Revit™ as copiously stated by Jangalve et al. [3] is that

1. Input BIM data for analysis
   i. Project information
   ii. Energy settings
   iii. Materials
2. Input rooms/spaces/zones
   i. Define space limits

3. Define analysis information
   i. Reports
   ii. Schedule data
   iii. Details

4. Run heating and cooling load analysis

5. Export to gbxml for Autodesk GBS

6. Run or perform energy simulation

A typical output of building energy analysis illustrated in Jangalve et al., is shown in the accompanying graphical illustration in terms of CO₂ emission within the building arising from energy consumption (see Figures 3 and 4).

Figure 6.
Annual wind rose (frequency distribution) annual wind rose and humidity.

Figure 7.
Monthly fuel consumption.
In all, the use of energy simulation software to do analysis of building energy analysis is econometrically efficient in reporting the above listed evaluation outcome supports designers in actualizing building envelope properties and building energy requirements (Figures 6 and 7).

4. Building energy audit

Attempts towards optimizing energy use in buildings stems from building energy analysis which in itself have been discussed extensively in previous section (Figures 1–8). Now for the possible of energy improvements, savings and optimization in a building which has its toot in building energy audit as a process requires the adaptation of agreeable and validated reference climates and energy simulation procedures [13, 14]. Such procedures look out for priority of energy uses in buildings. Energy audit as tool is used to benchmark where and how energy is being used in buildings. This is done by intensifying opportunities and providing solutions towards energy savings and economic cost too. Principal to this solution is energy data management suitable or compactable with appropriate energy saving technologies. It may also be in the form of structural improvements and systems modernization towards conservation of energy. In the process of energy auditing, the unified Lider Calener tool becomes indispensable to the process of auditing since it processing of results outcome allows for alternative response priorities to be earmarked [11, 15]. In economic sense, this process will serve as a guideline to developers and energy managers in taking decisions towards necessary reforms that will lead to substantial savings and enhanced payback period. In construction economics, the idea of conducting energy audit so as to save substantially ion energy is gaining sufficient space in construction literature and amongst developers, since the tool presents them the opportunity of making decisions on savings on energy consumption pattern with respect to economic indices of payback terms of the property investment [4]. The energy audit process seeks to advance and canvass improvement in the extent to which an energy budget is used for the intended purpose and cultivate energy savings towards offsetting greenhouse gas emission. It is important that energy auditing on the thermal properties of building envelopes should as a matter of economic science be conducted to give a closer direction of energy use to bring about decision bothering on the reasonability of a proposed investment [3, 15].

In as much as thermal comfort is important for occupants and must be paramount in design considerations, investors are usually worried about the cost

Figure 8.
Annual wind rose and humidity.
implication of such design been incorporate to meeting occupants satisfaction, yet must be comparatively admissible to be economically worthy to invest in. In the auditing process, thermal properties within the building envelope are assessed to ascertain the losses and gains that occur using the unified Lider Calener tool towards selecting the building element that will result to improved energy savings [16]. The decision to invest as an economic yardstick is to some extent dependent on the energy audit outcome which as a matter of necessity must take into cognizance the auditing method by considering time, speed, technical know-how, cost, sensitivity, accuracy, reproductively and ease of use. It is well articulated in building energy literature that buildings consume nearly one-third of the energy used in the United States [17]. This is not different if not more in most European nations. The operational cost of most buildings consequently absorbs at least 30% of operating cost. With this hindsight in the mind of developers, a constructability balance between economic gain and occupants comfort hangs on a balance that requires an economic assessment of cost-benefit appraisal by energy auditing. Particularly, building energy audit tends to reduce greenhouse gas emission and air pollution if properly done. It also addresses the air quality, lighting quality and occupants’ satisfaction. It significantly lower electrical, natural gas, steam, water and sewer cost on the long run [13, 15]. Therefore, it becomes absolutely imperative that a knowledge bank of energy footprint of cities is established so as to identify the gaps of opportunities to savings in energy use and costs. The footprint repository will provide the necessary guidance for investors to making cost-benefit decisions, for the now and in the near future on energy saving alternatives and strategies [11, 13, 16].

5. Energy audit procedure

Before 2002, energy audit in the construction industry was not popular as what was available that somewhat looks like it was the European Union directive on energy efficiency of buildings, which was the 2002/91/EC, European Directive. In a later amendment of that document which was the 2006/32/EC, item 18 of the explanatory note mandates all member states to guarantee the availability of energy audit as statutory requirement and obligations in building construction projects. Subsequently in 2010 in 2010/31/EU specified the issuance of certificate of energy audit to the property owner as a mark of building energy efficiency certification. Further to the provision of the 2010 document, the 2012 version made it obligatory and a routine of every 4 years activity for large companies with the attendant energy savings obtained in the period under review and to be inclusive of non-SME not later than December of 2015. In addition to the provisions of those directives, it stipulates that energy audit must show detailed calculation and proposed measure by furnishing clear information towards mitigating potential losses [11, 13, 15]. Since a rightly performed energy audit spells out the value of gain or loss at each energy point over a certain period, it will in time to come become a fundamental tool or document for showing compliance when energy intervention measures in a building that may lead to certain levels off savings in energy consumptions and reducing CO₂ emissions into the atmosphere are proposed. The efforts emanating from the accompanying legislations above are efforts geared towards minimizing non-renewable primary energy. Windows in buildings have been reported to be great sources of about 20–40% cause of energy losses in buildings, such that it must be taken into account when proposing energy-saving measures. Response measures towards the inhibition of energy uses via condensations must be considered from
the view of avoiding a later investigation that could hamper the smooth running of thermal envelop elements [12, 15].

According to the New Jersey Energy Audit Guidance, there are three types of building energy audit which exists, namely

1. Computer simulation audit

This method is used to predict building systems performance taking external factors like weather into consideration. It is suitable for complicated buildings systems and facilities.

2. Walk-through audit

This method of building audit uses visual inspection of a building's energy system and review of its energy data usage. It is a further referenced by way of comparison to industry's normative average. This form of auditing determines if a further comprehensive audit is required. It is at best informative.

3. Standard audit

This method of building auditing is used to assess all equipment and the associated operational systems and generate more elaborate calculations of energy use. It primarily identifies areas of potential technical improvements and makes recommendations based on their projected energy and cost savings.

A typical energy audit procedure is enumerated hereunder.

• Collection of building data.

• Introduction of technical data into the HULC simulation program.

• Energy simulation of the building in its current state.

• Quantification of losses and gains through the different elements of the thermal envelop.

• Pre-selection of action measures based on the previous results.

• Simulation of possible refurbishments.

• Quantification of savings in each of the refurbishments proposed.

• Economic calculation and calculation of paybacks.

• Selection of final measures, based on the previous results.

Though the procedure is not limited to the arrangement stated above since it varies from practice to practice and from country to country, most European nations now have legislation and practice procedure. In Spain, for example, the Ministry of Development and the Ministry of Industry, Energy and Tourism freely issues the Unified Tool Lider Calener energy audit simulation program to practitioners to comply with national regulatory framework on energy performance of buildings for the purpose of energy certification. Typically, energy audit process is shown in the flow chart below:
Energy audit flow chart
Source: South African Energy Department.
According to South African Energy Department, the key steps in the energy audit, after the initial client meetings and historical data analysis,

1. **Conduct a walk-through inspection**—to assess the general level of repair, housekeeping, and operational practices that have a bearing on energy efficiency, and to flag situations that have merit for further assessment as the audit is implemented, walk-through inspections will also be carried out to verify the findings of other analysis steps, as indicated in the flow chart;

2. **Analyze energy consumption and costs**—collect, organize, summarize and analyze historical energy billings and the tariffs that apply to them;

3. **Compare energy performance**—determine energy use indices and compare them internally from one period to another, one facility to a similar one within your portfolio, one system to a similar one; or externally to measures of good practice within your industry;

4. **Establish the audit mandate**—secure commitment from management and define expectations and outcomes of the detailed audit;

5. **Establish the audit scope**—define the energy consuming system to be audited.

6. **Profile energy use patterns**—determine the time relationships of energy use, as in the electricity demand profile;
7. **Inventory energy use**—prepare a list of all energy consuming loads in the audit area, and quantify their consumption and demand characteristics;

8. **Identify energy management opportunities**—including operational and technological measures to reduce energy waste.

9. **Assess the benefits**—quantify the level of energy and cost savings, along with any co-benefits.

10. **Report for action**—report the audit findings and communicate as required for implementation.

Each step involves a number of tasks that are described in the following sections. As suggested by the flow chart, several of the steps may result in the identification of potential EMOs.

### 6. Economics of building energy audit

Patel [4] did an extensive investigation on this subject matter. Patel [4] work noted that the auditor of the completed project has to be very careful in carrying out the audit. He must follow some procedure so that full justice is done to the work. Some points related to the energy audit procedure are described below.

#### 6.1 Collection of appropriate information

The starting point for collecting post audit information is the project completion report. Energy audit generally compares the projected data with the accounting data collected through the regular MIS. The MIS needs to be geared up so that the projected cash flow from the original capital budget can be compared with the actual cash flows realized during the period elapsed before the energy audit of the project has started. Another point that needs to be kept in mind is that the auditor needs to collect total cost figures. Incremental cash flow figures are readily available for green-filled projects but it is not so easy for the projects in an existing plant. The data in the latter case need to be appropriately dealt with to arrive at the incremental cost figures due to [4].

#### 6.2 Recasting the data

Collected data or budgeted data should be recast before they are compared. The significant time gap and a host of factors, which were not considered at the budgeting stage, would warrant the recasting of data. For example, inflation should be adjusted before comparison is made. Sales mix difference due to external factors also should be taken into account. In the absence of adjustment for those “external” factors, the quality of audit would suffer. Inflation adjustment is subsequently into results.

#### 6.3 Comparison of projected financial parameters with actual

This is the next important step in the post completion audit procedure. There are four techniques available for the comparison of actual with the projected financial parameters. The comparison is the starting point from which the real audit begins. Only comparable data is compared. Adjustments are first done for inflation and external factors before comparison is carried out under any method. Methods
described later on are not mutually exclusive. More than one method may be applied for comparison if there is such a requirement. A broad level ROI or NPV comparison can be done initially, followed by detailed cost variance or cash flow variance analysis. Comparison is a step-by-step approach so that causes are identified systematically with minimum cost, time and energy [4].

\[
ROI = \frac{Net \ income}{Cost \ of \ Investment} = \frac{Investment \ gain}{Investment \ base} = \frac{Gain \ from \ investment - cost \ of \ investment}{Cost \ of \ Investment} = \frac{Return(Benefit)}{Investment \ (cost)}
\]

6.4 Establish the possible causes of variance

Once the variance figures are calculated, if they are significant, the possible causes for the same are explored. An auditor goes by exceptions from there he tries to reach the root causes of deviations. This process of investigation can be effective only if an auditor possesses skills of inquisitiveness and skills of persuasion and negotiation [4]. A summary report of the energy audit findings should also be prepared.

6.5 Final recommendations

Once the causes are ascertained, the post completion auditor can give his recommendations based on which the manager may take decisions for cash flow forecasting to reinvest or abandon the ongoing project. Hopefully, after the post completion audit, the cash flow prediction and project evaluation become more accurate [4].

7. Building energy audit economic techniques

There are three techniques of building energy audit economics, namely

a. Cost variance analysis,

b. Profit variance analysis

c. Cash flow and financial criteria analysis and

d. Present value depreciation technique.

7.1 Cost variance analysis

In this method, only the project cost (actual and estimated) is studied and the revenue aspect is not included in the audit. This approach is adopted when the energy audit is conducted during the execution or just after the completion of the project.
CV \[\text{Earned Value (EV)} - \text{Actual Cost (AC)}\]
EV = % of worth completed × Budgeted cost
AC = what has been spent on the project

7.2 Profit variance analysis

In this method, plant-wise profit analysis is carried out by the auditor and the estimated gain adjusted with the inflationary effect is compared with actual. An important point to note here is that even if the aggregates of gains (realized and estimated) are the same there can be wide variations for individual projects, indicating the need for further investigations.

7.3 Cash flow and financial criteria analysis

This method is developed around four schedules described below. These schedules can provide the management with the information it needs to find engineering, operational and administrative costing faults of past projects.

a. Profit variance analysis schedule: this schedule is prepared for the calculation of profit variance between projected and the actual project results. The information for the “projected” column is obtained from the approved capital expenditure request. The information for the “actual” column is obtained from regular accounting sources. Supplementary schedules are required to itemize and explain the basis of calculation of revenues, costs and expenses need to be given.

b. Cash flow and financial criteria variance analysis schedule: this is used to illustrate project cash flow and return variances between the projected and actual results. The approved capital expenditure request is again used to provide information for the “projected” column and regular accounting sources for the “actual” column.

c. Project cash flow schedule (projected and actual): these are used to show the projected and actual cash flows of the project. They illustrate the timing of cash flows to compute payback and to provide the net period cash flow information required for the IRR calculation. Each cash flow entry is made according to the time it was projected to be incurred or was actually incurred. The period cash flows are for individual quarters whereas the cumulative cash flows represent all cash flow for the project. The payback point is reached when the cumulative net cash flow equals zero.

d. Supplementary schedules: the supplementary schedules provide explanations for the significant variances.

7.4 Present value depreciation technique

Discounting factor technique give only a single value of the NPV which is for the whole life of the project. The IRR is the average return during the life. But at the time of conducting the energy audit, the major part of the project life is not completed. Then how can we compare the actual with the total net present value or average internal rate of return? A uniform annual series cannot be considered because it is an average figure and the project need not offer and NPV at a constant rate over its life. The concept of e present value of depreciation is used in some techniques for the calculation of the year-wise NPV and IRR. Present value
Depreciation is defined as the decline in the present value of the expected future cash flow during the year using the IRR as the discount rate. Two models, namely the IRR model and the NPV model, are suggested under the technique of present value depreciation in Eq. (51).

\[
NPV = \sum_{t=1}^{n} \frac{CF_t}{(1 - K)^t} - C
\]  

(51)

where \( CF_t = \) cash flow in period \( t; K = \) discount rate; \( C = \) initial outlay

8. Conclusions

There are numerous literatures that have dealt with this subject of building energy audit. This particular text emphasizes economic assessment moving from a theoretical review of the subject of building energy analysis with respect to thermal conditions of the building envelope. The process of obtaining data for building energy audit was spelt out from the review of prevalent building information modeling software which were analyzed to obtain actual and project energy loads. The point of divergence of the two measures were econometrically reviewed to ascertain financial control mechanism, providing information for future capital expenditure decision, impacts on proposals for capital investments.

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Conflict of interest

This text has no conflict of interest declaration.

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