A free wake vortex lattice model for vertical axis wind turbines: Modeling, verification and validation

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Abstract. Since the 1970s several research activities had been carried out on developing aerodynamic models for Vertical Axis Wind Turbines (VAWTs). In order to design large VAWTs of MW scale, more accurate aerodynamic calculation is required to predict their aero-elastic behaviours. In this paper, a 3D free wake vortex lattice model for VAWTs is developed, verified and validated. Comparisons to the experimental results show that the 3D free wake vortex lattice model developed is capable of making an accurate prediction of the general performance and the instantaneous aerodynamic forces on the blades. The comparison between momentum method and the vortex lattice model shows that free wake vortex models are needed for detailed loads calculation and for calculating highly loaded rotors.

1. Introduction
Vertical axis wind turbines have recently regained interest in the wind energy community. The main interest comes from the possible application of VAWTs in adverse wind conditions, where large wind direction fluctuations are present, such as on the rooftops of high buildings in urban regions. But the prediction of the behaviour of VAWTs is more complex than their horizontal axis counterparts.

During the 1970s, Sandia National Laboratories carried out a number of projects focused on Darrieus eggbeater-type rotors [1] including both analytical and experimental studies. The aerodynamic analysis and modeling of VAWTs are challenging because the induction and flows field are unsteady, the occurrence of blade-vortex interaction and dynamic stall make aerodynamic modeling and computation complex. Modeling of VAWTs follows three different approaches: First, the momentum method, which is based on equating the forces on the rotor blades to the rate of change of momentum in the streamwise direction through the rotor. Second, the vortex method, which is based upon vortex representations of the blades and their wakes. Third, the Computational Fluid Dynamic (CFD) methods, which are based on solving Reynolds Averaged Navier-Stokes (RANS) equations numerically. The momentum method needs less computation time but the CFD method is more accurate. The vortex method offers a good balance between the momentum method and the more accurate but more computationally expensive CFD method.

The double multiple streamtubes model (DMST), which is based on Blade Element Momentum (BEM) theory developed by Paraschivoiu [2], is one of the momentum methods. It predicts general performance quite well and is computationally efficient under conditions where...
the rotor tip-speed ratios are low. This makes BEM an useful practical method for design. However, it breaks down for large tip-speed ratios because in these cases the momentum theory becomes invalid, a situation which deteriorates with increasing rotor solidity. To calculate the forces under this condition and to obtain the forces more accurately with relatively less computationally expense than CFD, a vortex model is needed. In the following sections, the free wake vortex lattice method is described, implemented and validated.

2. Free wake vortex lattice model implementation

Free wake vortex methods have been well established for modeling the aerodynamics of lifting bodies and they cover a wide range of approaches. The particular type of the vortex wake method implemented in this paper is the vortex filament method [3], which is also known as the vortex line or vortex lattice method. The method assumes that the flow field is incompressible everywhere and inviscid everywhere except in the thin layer of fluid in the wake of the lifting bodies. The effect of the viscous wake on the flow field is modeled by a number of discrete vortex lattices, the strengths of which are determined from the loads variation of the lifting bodies. Many text books have provided a detailed explanation of vortex lattice method so only an overview of the particular approach implemented in this paper is provided here.

2.1. Vortex lattice wake

In this section, the free wake vortex lattice model for H-type of VAWTs, shown in figure 1, is described. Figure 2 shows a three-dimensional vortex lattice system associated with a single blade element $i$ at time $j$. In the figure, $\Gamma_{b_{i,j}}$ is the circulation of the bound vortex of blade element $i$ at time $j$, $\Gamma_{s_{i,j-1}}$ is the circulation of the shed vortex of blade element $i$ at time $j - 1$ and $\Gamma_{t_{i,j}}$ is the circulation of the trailing vortex of blade element $i$ at time $j$. The end of each vortex filament is indicated by the “vortex markers” where the velocity of the vortex filaments is calculated at each time step.

Bound vortices ($\Gamma_{b_{i,j}}$), in figure 2, are attached to the quarter-chord of the blade elements and move with the blade elements. The strength of each bound vortex is governed by the Kutta-Joukowsky theorem [4] as

$$L = \rho V_{rel} \Gamma_b$$

(1)

Where, $L$ is the lift per unit span on a blade element and $\Gamma_b$ is the strength (or circulation) of the bound vortex of the blade element. $\rho$ is the air density, and $V_{rel}$ is the local relative flow velocity in the plane of the airfoil section, which is perpendicular to the span of the blade. In this model,
the blade element is replaced by a bound vortex filament called a “lifting line”. According to the airfoil theory [5], the lift can also be formulated in terms of the two-dimensional sectional lift coefficient ($C_l$), the chord length ($c$) and the relative velocity ($V_{rel}$) as

$$ L = \frac{1}{2} \rho c C_l V_{rel}^2 \quad (2) $$

Combining equations 1 and 2, the strength of the bound vortex can be expressed as

$$ \Gamma_b = \frac{1}{2} \rho c C_l V_{rel} \quad (3) $$

Equation 3 provides the relationship between the strength of the bound vortex and the induced velocity at the blade element because the airfoil lift coefficient and the relative velocity ($V_{rel}$) are function of the induced velocity. It can be noticed that the effects of aerodynamic stall are automatically included by using this method.

The relationship between the strength of the bound vortex of blade element $i$ at time step $j$ and the strength of the shed vortex from this element is given by the Kelvin’s law, which requires that the circulation around any closed curve remains constant over time. It satisfies the following equation

$$ \Gamma_{s_{i,j-1}} = \Gamma_{b_{i,j-1}} - \Gamma_{b_{i,j}} \quad (4) $$

Where, $\Gamma_{s_{i,j-1}}$ is the strength of the shed vortex at element $i$ and time step $j$. In reality, shed vortices are released continuously at the trailing edge. In this paper, the newest shed vortices were placed at 25% from the current location of the trailing edge to the location of the trailing edge at the previous time step and the other shed vortices are transported by the calculated velocity.

According to Helmholtz’s theorem [4], a vortex line must not end in the fluid field and it also remains constant. Therefore, in order to satisfy Helmholtz’s theorems, a trailing vortex must emit from the blade at each blade element where the strength of each discrete bound vortex changes to allow for the varying circulation along the span. The strength of a trailing vortex is the difference between the strengths of the bound vortices from where it emits. It can be seen in
the figure 2, the strength of a trailing vortex released between sections \(i\) and \(i-1\) at time-step \(j\) is given by the following equation:

\[
\Gamma_{i,j} = \Gamma_{bi,j} - \Gamma_{bi-1,j}
\]  

(5)

2.2. Biot-Savart law and viscous core model

The velocity at an arbitrary point \(P\) induced by a single vortex filament is calculated by the Biot-Savart law, and it can be re-formulated into equation 6 by using the notation defined in figure 3.

\[
V_p = \frac{\Gamma}{4\pi h} \left( \frac{\vec{r}_0 \cdot \vec{r}_1}{||\vec{r}_0|| ||\vec{r}_1||} - \frac{\vec{r}_0 \cdot \vec{r}_2}{||\vec{r}_0|| ||\vec{r}_2||} \right) \frac{\vec{r}_1 \times \vec{r}_2}{||\vec{r}_1 \times \vec{r}_2||}
\]  

(6)

Where, \(h\) is the perpendicular distance from the arbitrary point to the vortex filament. \(\vec{r}_0\), \(\vec{r}_1\) and \(\vec{r}_2\) follow the direction defined in the figure 3. In the equation 6, when \(h\) approaches zero, the induced velocity becomes infinite which is inconsistent with the physical reality. In reality, viscous effects are encountered that reduce the velocity to zero at the center of the vortices. To model this behaviour, the Vatistas viscous core model is implemented to account for viscous effects of the vortex core. Then, equation 6 can be written as the following equation to include the viscous effect.

\[
V_p = \frac{\Gamma}{4\pi \sqrt{r_c^2 + h^2}} \left( \frac{\vec{r}_0 \cdot \vec{r}_1}{||\vec{r}_0|| ||\vec{r}_1||} - \frac{\vec{r}_0 \cdot \vec{r}_2}{||\vec{r}_0|| ||\vec{r}_2||} \right) \frac{\vec{r}_1 \times \vec{r}_2}{||\vec{r}_1 \times \vec{r}_2||}
\]  

(7)

Here, \(r_c\) is the viscous core radius, equal to approximately 5-10% of the chord length of the blade.

The total velocity at each “vortex marker” at each time step is calculated by summing the induced velocity calculated from all shed vortex filaments and trailing vortex filaments using equation 7 and adding the free stream velocity. During every time step the positions of all vortex filaments are updated with Adams-Bashforth integration method given by the following equation.

\[
\vec{X}_t = \vec{X}_{t-1} + \frac{3\Delta t}{2} V_t - \frac{\Delta t}{2} V_{t-1}
\]  

(8)

In this equation, \(\vec{X}_t\) and \(\vec{X}_{t-1}\) are position of each “vortex marker” at current and previous time step. \(V_t\) and \(V_{t-1}\) are velocity of each “vortex marker” at current and previous time step.

2.3. Blade loads and rotor performance

Figure 4 shows the rotor and blade element coordinate system used in this implementation. The direction of the normal force coefficient \((F_n)\) and the tangential force coefficient \((F_t)\) are shown in the figure. \(F_n\) and \(F_t\) can be expressed in terms of lift and drag coefficients \((C_l, C_d)\) and the relative velocity \((V_{rel})\). In this implementation, the spanwise force and the pitching moment are neglected.

\[
F_n = \left( \frac{V_{rel}}{V_\infty} \right)^2 (C_l \cos \alpha + C_d \sin \alpha)
\]

\[
F_t = \left( \frac{V_{rel}}{V_\infty} \right)^2 (-C_l \sin \alpha + C_d \cos \alpha)
\]  

(9)

In equation 9, \(\alpha\) is the angle-of-attack at the blade element. \(V_\infty\) is the free stream velocity. When \(F_t\) is calculated, the torque coefficient of one blade element is calculated as

\[
T_e = \frac{I_e CF_t}{A}
\]  

(10)
Where, \( A \) is the total frontal area of the rotor. \( l_e \) is the length of the blade element. \( C \) is the chord length. Then, the average power coefficient \((Cp)\) for the entire rotor during one revolution is given as

\[
Cp = \frac{1}{NI} \sum_{j=1}^{NI} \sum_{i=1}^{NE} (T_e \lambda)
\]

(11)

Where, \( NI \) is the number of time steps per revolution and \( \lambda \) is the tip-speed ratio.

Figure 4. Rotor and blade element coordinate system

3. Verification and validation

3.1. Water tank experiment description

Most aerodynamic codes for VAWTs are validated against the \( Cp - \lambda \) curve, and very little experiment data is available in the form of detailed blade forces. A widely referenced experiment that does include the detailed blade force measurements is the Strickland’s water tank study \([6],[7]\), which measured the blade forces of a H-type vertical axis hydraulic turbine at \( \lambda = 2.5, 5, 7.5 \). It also provides the photographs of steam lines to indicate the wake structure. In the experiment, the rotor consists of 1, 2, or 3 straight blades with a chord length of 0.0914 meter. The profile is the NACA0012 airfoil. The rotation speed \((\omega)\) is fixed at 0.457 m/s for an overall Reynolds number of 40 000. The rotor diameter is 1.22 meter. 0.91 meter of each blade is submerged in the water, with another 0.15 meter of water from the blade tip to the bottom of the tank. The towing speed varies from 0.061 m/s to 0.183 m/s in order to achieve the different tip-speed ratios \((\lambda = 7.5, 5, 2.5)\). In the following sections, the verification and validation of simulation results based on this experiment are shown. For each simulation, the duration is chosen to be ten revolutions in order to reach the converged periodic solution. Figure 5 shows the convergency investigation. It can be shown that for different tip-speed ratios, the solution converges after about 6 to 8 revolutions.

3.2. Verification

In this section, the power coefficient curve of the H-type hydraulic turbine mentioned above is predicted by DMST model, VDART2 \([7]\) and the 3D free wake vortex lattice code (FWVLM3D)
developed in this work. The comparison results are shown in figure 6. The absolute error between DMST model and FWVLM3D is also plotted in figure 6. It can be seen from the absolute error that the agreement between DMST theory and the vortex lattice code is quite good at tip-speed ratio less than 5.0 for this two bladed rotor. At higher tip-speed ratios (λ ≥ 5.0), the absolute error is increasing except at tip-speed ratio equal 6.0. The comparison between VDART2 and FWVLM3D is also shown in this figure. The power coefficient predicted by VDART2 is only available at three tip-speed ratios (λ = 2.5, 5.0, 7.5) due to the limited data from literature. The agreement between the VDART2 and FWVLM3D is also good except at tip-speed ratio equal to 7.5.

3.3. Validation and discussion
The blade force coefficients are shown in figure 7 and 8 for the three test cases which were run. From the figure 7 and 8 the effect of rotor solidity, which is a progressive retardation of the flow in the downstream area, can be captured by the code but it is over predicted. The results show very good agreement between the 3D free wake vortex lattice code and water tank measurement at tip-speed ratio(λ = 5.0) for the blade normal force coefficients whereas the tangential force coefficients don’t agree very well. In general, the tangential force coefficient
is one order of magnitude smaller than the normal force coefficient and, therefore, is more susceptible to extraneous noise during experiment. From the comparison it can be seen that in the downstream region the difference between the predicted $F_n$, $F_t$ and experiment results is bigger than in the upstream region. This is because the vortices shed from the blade in the upstream region interact with the blade in the downstream region. That is to say, the blades in the downstream region will enter into the wake of their own and other blades. This increases the aerodynamic complexity and makes the aerodynamic simulation difficult.

![Figure 7. Blade normal force coefficient at $\lambda = 5.0$, $Re = 40000$](image1)

![Figure 8. Blade tangential force coefficient at $\lambda = 5.0$, $Re = 40000$](image2)

### 4. Conclusion and future work

#### 4.1. Conclusion

The rotor power coefficient predicted by the 3D free wake vortex lattice model is in good agreement with the DMST model except at high tip-speed ratio where the momentum theory is invalid. Comparisons to the experiment results show that the 3D free wake vortex lattice model developed is capable of making an accurate prediction of the instantaneous aerodynamic forces on the blades with respect to the azimuthal position. The effect of rotor solidity, a progressive retardation of the flow in the downstream region, is over predicted by the vortex lattice code. Future investigation is needed for improving the accuracy in the downstream area.
4.2. Future work
As it is concluded, the 3D free wake vortex lattice method models the physics of VAWTs more closely than the blade element momentum theory and is more accurate. The disadvantage of 3D free wake vortex lattice method is that it is very computationally expensive, which is the main barrier for applying this method to the engineering design codes for vertical axis wind turbines. The future tasks are to develop a high performance free wake vortex lattice code by using effective, fast algorithms and paralleled computation technique.

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