On the Data Fight Between Cities and Mobility Providers

Guillermo Baltra*, Basileal Imana†, Wuxuan Jiang‡ and Aleksandra Korolova§
University of Southern California
Email: *baltra@usc.edu, †imana@usc.edu, ‡wuxuanji@usc.edu, §korolova@usc.edu

Abstract—E-Scooters are changing transportation habits. In an attempt to oversee scooter usage, the Los Angeles Department of Transportation has put forth a specification that requests detailed data on scooter usage from scooter companies. In this work, we first argue that L.A.’s data request for using a new specification is not warranted as proposed use cases can be met by already existing specifications. Second, we show that even the existing specification, that requires companies to publish real-time data of parked scooters, puts the privacy of individuals using the scooters at risk. We then propose an algorithm that enables formal privacy and utility guarantees when publishing parked scooters data, allowing city authorities to meet their use cases while preserving riders’ privacy.

I. INTRODUCTION

E-scooters are changing transportation habits. At the same time, their increased usage is bringing new challenges such as clogged sidewalks and danger of injury to both riders and pedestrians. In an effort to oversee and regulate scooter companies and scooter use, Los Angeles proposed the Mobility Data Specification (MDS) [1], which requires the scooter companies to provide real-time data of scooter locations and a historical view of all scooter operations.

The requirement to provide data according to MDS has not been positively received by dockless scooter companies and privacy advocates. The scooter companies argue that such data gives city governments an ability to track individual trips, which may have negative privacy implications for individuals [2]. Advocacy groups, such as the Electronic Frontier Foundation and the Center for Democracy and Technology, have also expressed concerns about rider privacy [3] [4] and requested information on how the Los Angeles Department of Transportation (LADOT) intends to aggregate, de-identify, and anonymize MDS data if it is to be made public. Overall, the question of the data that is necessary for the scooter companies to provide for the cities to oversee scooter usage, while at the same time protecting privacy of riders, is highly contested.

Several cities in the US have implemented ad-hoc mechanisms to anonymize scooter data when sharing it publicly. For instance, Louisville, KY [5] aggregates GPS locations at three decimals for latitude and longitude, which equates to aggregation at approximately a block level. Austin, TX [6] replaces GPS coordinates with census tract IDs, and Minneapolis, MN [7] follows a similar approach but replaces GPS coordinates with street centerline IDs. Yet, these approaches fail to protect privacy [8].

Contributions: In this work we make three contributions to the debate around the scooter data sharing.

• First, we argue that Los Angeles’s (LA) proposal for the new MDS specification is not warranted [IV]. We do this by showing that LA’s proposed data use cases can be met using data that is already being published as part of an existing specification called GBFS (which was proposed in 2015 and is already implemented by most scooter companies). We focus on two of LA’s main use cases: determining the number of scooters operating in a region and determining the distribution of scooters across neighborhoods.

• Second, we show that the currently publicly available GBFS data puts individual rider privacy at risk [V]. We demonstrate that this data is susceptible to a trip reconstruction attack, contrary to the claim by GBFS’s creators that the data does not violate privacy of riders and their trips [9].

• Third, we recommend changes to the MDS and GBFS specifications by proposing an algorithm for publishing scooter data with formal privacy and utility guarantees, which could allow city authorities to meet their use cases while preserving riders’ privacy [VI]. We provide a preliminary analyses on the trade-offs between privacy guarantees and possible utility goals of city authorities.

II. BACKGROUND: MDS AND GBFS

LADOT proposed MDS [1] in November 2019 as a standardized approach for mobility service providers to give trip and event level information to the city and for the city to regulate mobility service providers. The proposed use cases include enforcing device cap limits, determining when scooters are parked in illegal areas, ensuring equitable deployment of devices across neighborhoods, and informing the city planners’ future capital investments and infrastructure planning [1].

This proposal contrasts with an existing General Bikeshare Feed Specification (GBFS) [1], which has already been implemented by most mobility companies [10]. GBFS specification is designed to provide the status of a micro-mobility system at the moment (e.g., the number and locations of scooters that are currently unused). Its goal is to meet “data needs for oversight and planning” while “protecting traveler privacy.” [9].

The specification for GBFS [9] requires service providers to publish certain data feeds as JSON files. Out of the 4 data feeds that are required, the one that is relevant for our
privacy discussion is free_bike_status.json, which provides a near real-time information on the absolute location of all currently parked scooters. The feed is updated every TTL seconds. Although no specific value is recommended in the specification, 60 or 300 seconds is used as the TTL in the datasets we analyze.

MDS, in addition to requiring the implementation of GBFS, also requires providing a historical view of all trips and events related to the operation of scooters. The API specification for MDS has three components, called Provider, Agency, and Policy; each one with a different use case. Provider API is to be implemented by mobility service providers, and gives a historical view of all past trips and events on demand. Agency API is implemented by regulatory agencies to provide endpoints that service providers must call to upload events in near real-time (such as a trip starting or ending), and to provide telemetry data (such as scooter location, speed, and heading). Policy API is to be implemented by regulatory agencies to enable them to publish machine-readable policies (such as localized caps, excluded zones and speed restrictions), and update the policies as needed. The Provider and Agency APIs provide very granular trip- and event-level information to the city authorities. The requirements of these two APIs are the ones that have raised privacy concerns in MDS.

III. DATA COLLECTION AND ETHICS

For our experiments, we collected data from four scooter companies (Spin, Bolt, Lyft and Bird) that publish GBFS data feeds for their scooters in Los Angeles. For each company, we identify the URL for its free_bike_status.json data feed, and scrape this URL every TTL seconds. Spin, Bolt, and Bird use a TTL value of 60 seconds, while Lyft uses 300 seconds. All results presented are based on data we collected over 10 days between 2019-11-09 and 2019-11-18.

Our goal is to show that there are privacy risks in implementing the GBFS and MDS APIs as specified, and propose a solution that works for both the scooter companies and the regulatory agencies. Thus, we limit our work to the minimum data collection and analysis necessary to show that the privacy risks for data published under GBFS are not merely theoretical possibilities but tangible risks. Consequently, we refrain from associating trips to specific individuals except for trips we took ourselves.

IV. USING GBFS TO MEET USE CASES

Here we show that GBFS already provides information needed to meet the goals of LA’s two main use cases: determining the number of scooters in operation and their distribution across neighborhoods. We focus on these two use cases out of the 11 listed by [1] because they form the basis for many of the other use cases such as enforcing device caps, guiding future capital investments based on distribution of scooter usage and informing policy making. Studying other

use cases such as enforcing parking regulations is left as an area of future work [1].

A. Use Case 1: Determining the Number of Scooters

The first use case for which the City of Los Angeles requests MDS data is to determine how many scooters are operating [1]. This helps the city enforce device cap limits per company for a given area. We show that the GBFS specification already includes data that can be used to closely estimate the total number of scooters in operation.

Fig. 1 shows a time series of the number of parked scooters for four companies: Bird, Lyft, Bolt, and Spin. The peaks in these plots, which represent times of inactivity, can be used to estimate the total number of scooters in operation.

LADOT permits each company to run 3,000 scooters, with an additional 2,500 allowed in disadvantaged communities [1]. Based on Fig. 1, Spin, Lyft, and Bolt are well below the 3,000 limit. For Bird, one needs to look at the distribution of scooters to see whether the scooters beyond the 3,000 count are deployed in disadvantaged communities (the next use case).

B. Use Case 2: Determining Scooter Distribution

The other use case for MDS data that we analyze is the distribution of scooters within LA neighborhoods. The goal of the city could be to verify that scooters are operating only in allowed regions or to verify scooters are being deployed equitably across neighborhoods. The city of LA incentivizes scooter companies to not only target tourist areas but also to provide their services to less privileged city areas.

We claim the City of Los Angeles already has enough data available in GBFS to meet this use case. We demonstrate this by counting the number of parked scooters in LA neighborhoods for a single GBFS data snapshot. To

1It would be interesting to examine whether the precision provided by GPS coordinates is sufficient for use cases such as enforcing parking rules, regardless of privacy.
determine neighborhood limits, we use shapefiles provided by the City [12].

In [Fig. 2] we show Bird’s scooter distribution on 2019-11-12 at 2:54am. Bird was the first scooter provider in LA and the one with the largest number of available scooters in the city. We see three major areas where Bird favors deploying scooters: Santa Monica - UCLA campus, LA Downtown - USC campus, and Hollywood. That is, scooters are more prevalent in areas frequented by tourists and students, and less common in poorer areas. Therefore, this figure shows that GBFS gives exactly the information needed for the use case of determining scooter distribution across neighborhoods.

V. ATTACKS ON GBFS

We next show how the access to data in GBFS violates the privacy of scooter users. We demonstrate this by reconstructing trips using parked scooter data and clustering the trips to identify interesting trips, which we then further investigate.

A. Reconstructing Trips

We use parked scooter data in GBFS to reconstruct trips, showing that GBFS, despite publishing much less information about scooters than MDS, does not protect the privacy of scooter users.

We check the location of all parked scooters on a minute by minute basis. If the location of a scooter changes, we record the scooter’s last location and the last time it was this location as the start location and start time of a trip, respectively. We record the scooter’s new location and the first time it was this location as the end location and end time of the trip, respectively.

This process provides us a dataset of trips, including potentially “fake trips” that may be due to a scooter being relocated for maintenance or charging purposes. After looking at the overall distribution of trip distances and durations, we filtered out such potentially fake trips using a trip distance threshold of 100 meters minimum (to filter out slight relocations of a scooter) and a time duration threshold of 1 hour maximum (to filter out relocations due to charging or maintenance).

We looked for specific trips as a means to validate our trip reconstruction. To accomplish this without violating other people’s privacy, we took scooter rides ourselves, and recorded the time and location for both start and end points. We were able to match the time and location of trips we took with a specific record in the trip dataset, which we reconstructed using the method described above. We are able to do this simply because we have information regarding the time and location our trips start and end at. This approach, in addition to validating our reconstruction, demonstrates the risk of identifying trips by other individuals when side information is available. Studying how different sources of side information, which prior works have shown are often easily available [13], [14], can be used to match trips with individuals is an interesting area for future work.

B. Identifying Visits to Interesting Destinations

Certain habits and preferences, although legal, are considered personal and private. GBFS data allows any member of the public access to data related to these habits. We show how we use clustering on the reconstructed trips to identify popular destinations and learn about trips to specific places.

1) Detecting Interesting Spots by Clustering: We use clustering to identify hotspots. The top clusters of parked scooters indicate popular places to start and end a scooter ride. Such statistics may not contain much private information if the hotspot is an area such as a popular shopping destination, which would obviously bring many scooters. Tracking an individual scooter may be hard in these areas. However, if the hotspots are near houses or other establishments, they may help us infer the habits of individual users.

A basic attack begins by associating the start and end locations of trips. Trips may end in a parking hotspot in a public area or at a specific house or establishment. By finding a cluster of trips ending at a person’s home, we can check the start and end locations of those trips to find out where the person is going to or coming from. Moreover, this data includes timestamps, thus we can infer more information about these trips. End points can be more sensitive than start points, as people might need to walk a certain distance to find a scooter before starting a trip, but they typically end a trip at their exact destination.

We use the standard k-means clustering algorithm on the start locations. By increasing the target number of clusters, we can make each cluster quite small and concentrate it in a narrow geographic area, which we can use to identify particular places. [Fig. 3] shows the distribution of cluster sizes when we have 100 clusters. Since large clusters may be popular public parking areas, we concentrate on smaller clusters with about 10 trips, which may contain more interesting information. In the 10 days of collected data from a company with a small

More sophisticated techniques can be used to filter out fake trips but they are not necessary for our proof-of-concept demonstration.
number of scooters, such as Spin, it is reasonable to assume that a person takes a ride every day. Cluster 54, with size 8, centering in \([34.02019, -118.27633]\), is located directly next to a marijuana dispensary. Below, we will explore trips to this location.

2) Case Study: A Cluster Outside a Marijuana Dispensary: Based on our clustering (see section V-B1), we identified a marijuana dispensary near a university campus, located within a 2km distance of the university campus and student housing areas. The lack of other stores or apartments on the same block as the dispensary, helps to identify the marijuana store as the likely ride destination of nearby parked scooters.

In Fig. 4 we show Spin trips starting and ending close to the dispensary between 2019-11-11 and -12. We focused on two locations, i.e., (a) and (b), which are the end of a trip followed by the start of a second trip a few minutes later. More specifically, for (a) the trip ends at 2019-11-11 11:32am, and resumes 5 minutes later. For (b) the trip ends at 2019-11-12 3:08pm and resumes 44 minutes later. In case (a), the time difference between the end of the first trip and the start of the second trip provides sufficient time to make a purchase, but is short enough to suggest the same rider.

For ethical reasons, we do not check where the trip originates before a rider reaching the store, and where it ends after a scooter user leaving the store, although doing so is possible using the data. Moreover, going beyond checking the starting location of a trip, one can potentially correlate the trip start and end points with specific riders using auxiliary data such as public records for inhabitants of particular addresses. One could also gain more confidence in the inferred trips and their correlation to specific individuals through a longitudinal study focusing on recurrent trips. In other words, a malicious adversary could use the GBFS data to learn about personal habits of particular individuals using the scooters.

C. Attacks without using scooter IDs

The trip reconstruction described in section V-A assumes a scooter’s ID does not change over time, which is true for the Spin dataset we used. Some companies, such as Bird, change their scooter IDs frequently, despite the GBFS specifications requiring IDs to be consistent over time. We explain why the changes in scooter IDs may not be sufficient to prevent privacy violations. We give a high-level description but leave exploration of possible attacks as an area the future work.

The simplest method of attack is to find a scooter that is isolated from other scooters either in time (it is the only scooter that moved) or distance (it is in an isolated area). We hypothesize this scenario can be generalized to the case of multiple scooters. By using the general distribution of duration and distance of scooter trips, we can build a probabilistic model and compute possible matchings to track scooters over time. The ground truth can be obtained from some datasets whose IDs do not change. For example, we can train the model on Spin data and then use it to predict trips on Bird data.

In summary, if locations of scooters are not reported in a privacy-preserving manner, even without using stable IDs, this type of attacks may succeed. That is why, in the next section, we recommend using techniques with formal privacy guarantees to report noisy, rather than precise locations, which would increase the difficulty of privacy-violating inferences.

VI. G-E-O-I-N-D-I-ST-I-N-G-U-I-S-H-A-B-I-L-I-T-Y A-S A P-O-T-E-N-T-I-A-L S-O-L-U-T-I-O-N

In this section we discuss one way to define location privacy and a mechanism through which we can achieve this type of privacy. We apply this mechanism a restricted set of MDS’s use cases and evaluate its performance. In addition, we provide suggestions for improving GBFS and MDS.

A. Definition and Mechanism

A basic technique to protect location privacy is to report a noisy location. Intuitively, if two locations are close to each other, it can be difficult to distinguish between them during the observation. Geo-Indistinguishability (G-I) [15] is a popular location privacy definition that provides protection to such location pairs. Formally, for any two locations \(x\) and \(x'\) with
a maximum distance $R$, the observation $S$ of them is $(\epsilon, R)$ indistinguishable if the following condition is satisfied:

$$\frac{\Pr(S|x)}{\Pr(S|x')} \leq e^{\epsilon R}, \forall R > 0, \forall S, \forall x, x' : d(x, x') \leq R$$

The definition is parameterized by the radius $R$ and the privacy loss parameter $\epsilon$. Note that $\epsilon$ contains the unit of measurement, while the $\epsilon$ in the traditional differential privacy definition [16] does not. In this work, we choose kilometer as the unit of measurement. For example, if we choose $R = 0.2$, $\epsilon = \frac{\ln 4}{R} = 5 \ln 4$, that means for any two locations $x$ and $x'$ which are within 0.2km, the probabilities of them having the same observation $S$, $\Pr(S|x)$ and $\Pr(S|x')$, the ratio of the two probabilities is upper bounded by 4. Namely, $\Pr(S|x) \leq 4 \Pr(S|x')$ and $\Pr(S|x') \leq 4 \Pr(S|x)$.

Algorithm [1] shows the basic framework of a mechanism that achieves G-I. For implementation details, see Appendix A.

**Algorithm 1: Geo-Indistinguishability [15]**

**Input**: $\epsilon$, $R$, location $(\text{lat}, \text{lon})$

**Output**: Noisy location $(\text{lat}', \text{lon}')$

1. Sample angle $\theta$ and length $r$ according to $\epsilon$, $R$.
2. Move the location $(\text{lat}, \text{lon})$ by $(\theta, r)$.
3. Output $(\text{lat}', \text{lon}')$.

**B. Utility Analysis**

In this section we define utility and show that for reasonable privacy parameters, a city can allow scooter companies to use G-I and still satisfy its goals with only a moderate increase in measurement error. We consider use cases discussed in section [IV] (1) total number of scooters within city limits, and (2) scooter distribution by neighborhood, and evaluate these use cases using publicly available scooter location data with G-I.

We measure utility as the difference between the true number of scooters within a bounded region and the perturbed number of scooters within that region due to noisy location reporting. A smaller difference reflects a better utility. Our expectation is to see a utility drop as privacy guarantees are increased. For example, more congested neighborhoods losing scooters to lesser populated ones (use case 2), or scooters leaving the overall city limits (use case 1).

To evaluate the impact of G-I on use case 1, we perturb parked scooter locations for different values of radius $R$, varying it from 0 to 1 km in increments of 0.05 km. The repeat the procedure 100 times for each value of $R$, and record the average number of scooters leaving LA boundaries. Fig. 5 (left) shows the number of Bird scooters outside city boundaries using a data snapshot corresponding to 2019-11-12 02:54am. These scooters randomly moved to neighboring cities, or even into the ocean. The figure shows that, as expected, the privacy guarantees (and hence the noise) increase, the number of scooters incorrectly placed increases in an upward linear trend.

To evaluate the impact of G-I on use case 2, we consider the average number of scooters lost per neighborhood; in this case 108 neighborhoods. As Fig. 5 (right) shows, neighborhood loss follows a similar upward trend as in use case 2, where adding more noise results in higher error.

In the context of densely populated cities such as LA, a radius $R$ that is reasonably large and covers multiple blocks can help obfuscate specific destinations. We pick 0.25 km as the radius for our privacy protection, which is comparable to two and a half blocks, and analyze how the algorithm performs with this parameter. With $\epsilon = \frac{\ln 6}{R} = 4 \ln 6$, in Fig. 6 we see a total loss of 115 scooters (3%) (left), and an average of 4 scooters lost per neighborhood (right). This suggests that for reasonable privacy parameters the utility loss is not catastrophic.

In addition to the above empirical analysis, we theoretically calculate the utility provided by the values we chose for $R$ and $\epsilon$. We plot in Fig. 6 the probability of a sampled noisy location being within a certain distance from the true location. The cumulative function $F_\epsilon$ is given by the following equation [15]:

$$F_\epsilon(x) = 1 - (1 + cx)e^{-c\epsilon x}$$

As the figure shows, about 50% of the noisy locations will land within 250m of the true locations, while 99% of the noisy locations will land within a 1 km distance. This implies that in expectation, the parameters we picked in the mechanism do not greatly change the true locations.

**C. How should GBFS and MDS change?**

Having demonstrated an approach towards publishing GBFS data with privacy guarantees and reasonable utility loss, we recommend the debate between scooter companies and the cities be resolved by using rigorous state-of-art techniques for privacy-preserving data publishing. GBFS data feeds could use this mechanism to report the noisy locations of scooters, and MDS requirements could be updated to remove the trip and event level data requested in Provider and Agency APIs. The Policy API does not require the service providers to publish any data, so we do not recommend any changes to the Policy API. Both sides of the debate, and, most importantly, the riders, would benefit if scooter companies would be willing to invest resources in implementing privacy-preserving mechanisms, and cities would be open to considering performing their analyses on privacy-protected data.

**VII. RELATED WORK**

Privacy challenges of sharing location data have been well-documented in the literature. For instance, Culnane et al. [8] show the vulnerabilities of Melbourne’s public transportation by identifying co-travelers and complete strangers when considering the uniqueness of each record within the dataset. Other datasets, such as the New York City taxi cab dataset [13] and geolocation data gathered by apps from mobile devices [14], have been shown to have similar privacy risks. In addition, Bassolas et al. [17] show that sharing precise user locations may carry privacy risks.
In the last two decades, researchers have demonstrated approaches for mitigating the privacy risks of location data sharing [15], [17], [18]. For example, the recent work of Bassolas et al. [17] proposed a framework for analyzing the hierarchical mobility structure of cities. In order to greatly reduce privacy risks, their data of trip flows are anonymized and aggregated. Primault et al. [18] provide a detailed survey of the existing location privacy protection mechanisms.

In this work, we based our proof-of-concept solution approach on Geo-Indistinguishability [15], which is a definition that aims to relax the notion of differential privacy for location-specific use cases.

VIII. CONCLUSION

Our work demonstrates that the data fight between cities and mobility data providers could benefit from adoption of state-of-the-art privacy-preserving techniques. The adoption of such techniques would require flexibility both from the side of scooter companies and the cities. However, the result would be increased privacy for the riders while providing cities the ability to achieve their regulatory goals.

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REFERENCES

[1] Open Mobility Foundation. (2019) Mobility Data Specification. [Online]. Available: https://github.com/openmobilityfoundation/mobility-data-specification
[2] D. Zipper. (2019) Cities can see where you are taking that scooter. [Online]. Available: https://slate.com/business/2019/04/scooter-data-cities-mds-uber-lyft-los-angeles.html
[3] The Los Angeles Department of Transportation ride tracking pilot is out of control. [Online]. Available: https://www.eff.org/deeplinks/2019/04/los-angeles-department-transportations-ride-tracking-pilot-out-control
[4] Comments to LADOT on privacy & security concerns for data sharing for dockless mobility. [Online]. Available: https://cdt.org/insights/comments-to-ladot-on-privacy-security-concerns-for-data-sharing-for-dockless-mobility/
[5] Louisville dockless vehicles data. [Online]. Available: https://data.louisvilleky.gov/dataset/dockless-vehicles
[6] Austin shared micromobility vehicle trips. [Online]. Available: https://data.austintexas.gov/Transportation-and-Mobility/Shared-Micromobility-Vehicle-Trips/7d8e-dm7r/data
[7] Motorized foot scooter trips, minneapolis. [Online]. Available: http://opendata.minneapolismn.gov/datasets/motorized-foot-scooter-trips-
august-2019
[8] C. Culnane, B. I. P. Rubinstein, and V. Teague, “Stop the open data bus, we want to get off,” 2019. [Online]. Available: https://arxiv.org/pdf/1908.05004.pdf
[9] North American Bikeshare Association, “General bikeshare feed specification,” 2019. [Online]. Available: https://github.com/NABSA/gbfs
[10] North American Bike-share Association, “Systems implementing GBFS,” 2019. [Online]. Available: https://github.com/NABSA/gbfs/blob/master/systems.csv
[11] LADOT. (2019) Dockless mobility program. [Online]. Available: https://ladot.io/programs/dockless/
APPENDIX

A. Geo-indistinguishability

In this section we talk about the details of generating the noisy location. The privacy preserving algorithm \[1\] randomly selects the angle \( \theta \) uniformly. Then length \( r \) is drawn from Laplace distribution. \( \theta \) tells which direction we should go and \( r \) represents the distance in that direction. With the pair \((\theta, r)\), we move from the original location and obtain a noisy location. In total, this pair is sampled from zero-centered polar Laplacian.

\[
D_{\epsilon}(r, \theta) = \frac{\epsilon^2}{2\pi} r e^{-\epsilon r}
\]

According to the analysis in \[15\], algorithm \[1\] using the above sampling technique preserves \((\epsilon, R)\) Geo-Indistinguishability.

We sample \( r \) from the above distribution and sample \( \theta \) uniformly from \([0, 2\pi]\). Since we are working on geo-coordinates, we need to use the following formula to compute the noisy location (assuming \( R_{\text{earth}} = 6378.1 \text{ km} \)). Given latitude \( x_{\text{lat}} \) and longitude \( x_{\text{lon}} \) for a location, the noisy latitude \( x'_{\text{lat}} \) and longitude \( x'_{\text{lon}} \) can be computed by:

\[
x'_{\text{lat}} = \arcsin(\sin x_{\text{lat}} \cos \frac{r}{R_{\text{earth}}} + \cos x_{\text{lat}} \sin \frac{r}{R_{\text{earth}}} \cos \theta)
\]
\[
x'_{\text{lon}} = x_{\text{lon}} + \arctan(\cos \frac{r}{R_{\text{earth}}} - \sin x_{\text{lat}} \sin x'_{\text{lat}} \sin \theta \sin \frac{r}{R_{\text{earth}}} \cos x_{\text{lat}})
\]