On the zero crossing of the three-gluon vertex

A. Athanodorus a, D. Binosi b, Ph. Boucaud c, F. De Soto d, J. Papavassiliou e, J. Rodríguez-Quintero f, S. Zafeiropoulos g

aDepartment of Physics, University of Cyprus, POB 20537, 1678 Nicosia, Cyprus
bEuropean Centre for Theoretical Studies in Nuclear Physics and Related Areas (ECT*) and Fondazione Bruno Kessler, Villa Tambosi, Strada delle Tabarelle 286, I-38050 Villazzano (TN), Italy.
cLaboratoire de Physique Théorique (UMR8627), CNRS, Univ. Paris-Sud, Université Paris-Saclay, 91405 Orsay, France
dDpto. Sistemas Físicos, Químicos y Naturales, Univ. Pablo de Olavide, 41013 Sevilla, Spain
eDepartment of Theoretical Physics and IFIC, University of Valencia-CSIC, E-46100, Valencia, Spain.
fDepartment of Integrated Sciences; University of Huelva, E-21071 Huelva, Spain.
gInstitut für Theoretische Physik, Goethe-Universität Frankfurt, Max-von-Laue-Str. 1, 60438 Frankfurt am Main, Germany

Abstract
We report on new results on the infrared behaviour of the three-gluon vertex in quenched Quantum Chromodynamics, obtained from large-volume lattice simulations. The main focus of our study is the appearance of the characteristic infrared feature known as ‘zero crossing’, the origin of which is intimately connected with the nonperturbative masslessness of the Faddeev-Popov ghost. The appearance of this effect is clearly visible in one of the two kinematic configurations analyzed, and its theoretical origin is discussed in the framework of Schwinger-Dyson equations. The effective coupling in the momentum subtraction scheme that corresponds to the three-gluon vertex is constructed, revealing the vanishing of the effective interaction at the exact location of the zero crossing.

Keywords: Lattice simulations, three-gluon vertex, zero crossing, Schwinger-Dyson equations

1. Introduction
One notable aspect of the ongoing intense exploration of the infrared (IR) sector of Quantum Chromodynamics (QCD) has been the detailed scrutiny of the fundamental Green’s functions of the theory using large-volume lattice simulations [1–8], together with a variety of continuum approaches [9–30]. Even though off-shell Green’s functions are not physical quantities, given their explicit dependence on the gauge-fixing parameter and the renormalization scheme, they encode valuable information on fundamental nonperturbative phenomena such as confinement, chiral symmetry breaking, and dynamical mass generation, and constitute the basic building blocks of symmetry-preserving formalisms that aim at a veracious description of hadron phenomenology [14, 31–38].

The most important findings of the aforementioned studies are related with the two-point sector of the theory. Specifically, it is now firmly established that, in the Landau gauge, the gluon propagator, \( \Delta(q^2) \), reaches a finite value in the deep IR, whilst the ghost propagator, \( D(q^2) \), remains massless, but with an IR finite dressing function, \( F(q^2) \) [note that \( D(q^2) = F(q^2)/q^2 \)]. This characteristic behavior has led to the critical reappraisal of previously established theoretical viewpoints, and has sparked a systematic effort towards a ‘top-down’ derivation of the ingredients that enter in the dynamical equations describing the properties of mesons [38].

The aforementioned results may be explained in a rather natural way within the framework of the Schwinger-Dyson equations (SDEs), by invoking the concept of a dynamically generated gluon mass [39]. The self-consistent picture that emerges may be succinctly summarized by saying that [40] (a) the gluon acquires an effective mass through a subtle realization of the Schwinger mechanism, the implementation of which hinges on the dynamical formation of longitudinally-coupled poles, and (b) the ghost is transparent to this mechanism, and remains massless; however, its dressing function is protected by the gluon mass, that tames any possible IR divergence and enforces its finiteness at the origin.

This profound difference in the IR behaviour between gluons and ghosts induces characteristic effects to other Green’s functions [but also to \( \Delta(q^2) \)], essentially due to the inequivalence between loops containing ‘massive’ gluons or massless ghosts [41]. Specifically, while the former are ‘protected’ by the gluon mass, \( m \), yielding IR finite contributions of the type \( \log(q^2 + m^2) \), the latter are ‘unprotected’, yielding (potentially) IR divergent terms of the type \( \log q^2 \).

In the case of \( \Delta^{-1}(q^2) \), the ghost-loop contained in its self-energy generates a term \( q^2 \log q^2 \), and therefore remains IR finite; however, the first derivative of \( \Delta^{-1}(q^2) \) diverges at the origin, precisely as an unprotected logarithm.

The corresponding effect on the three-gluon vertex is particularly striking. Specifically, in certain special kinematic limits, some of the vertex form factors are dominated in the IR by the gluon mass, \( m \), yielding IR finite contributions of the type \( \log(q^2 + m^2) \), and therefore remains IR finite; however, the first derivative of \( \Delta^{-1}(q^2) \) diverges at the origin, precisely as an unprotected logarithm.
Three-gluon vertex, renormalization, and effective charge. The connected three-gluon vertex is defined as the correlation function \( \langle q + r + p = 0 \rangle \)

\[
\mathcal{G}_{\text{tree}}^{abc}(q, r, p) = \langle Q_a^b(q) Q_r^c(r) Q_p^d(p) \rangle = f_{abc} \mathcal{G}_{\text{tree}}(q, r, p),
\]

where the sub (super) indices represent Lorentz (color) indices and the average \( \langle \cdot \rangle \) indicates functional integration over the gauge space. In terms of the usual 1-particle irreducible (1-PI) function, one has

\[
\mathcal{G}_{\text{tree}}(q, r, p) = g \Gamma_{\alpha \beta \gamma}^{\mu \nu}(q, r, p) \Delta_{\alpha \gamma}(q) \Delta_{\beta \gamma}(r) \Delta_{\mu \nu}(p),
\]

with \( g \) the strong coupling constant. In the Landau gauge, the transversality of the gluon propagator, i.e.,

\[
\Delta_{\mu \nu}(q) = \langle Q_{\mu}(q) \bar{Q}_{\nu}(-q) \rangle = \delta_{\mu \nu} \Delta(p^2) W_{\mu \nu}(p),
\]

where \( W_{\mu \nu}(q) = \delta_{\mu \nu} - q_{\mu} q_{\nu}/q^2 \), implies directly that \( \mathcal{G} \) is totally transverse: \( q \cdot \mathcal{G} = r \cdot \mathcal{G} = p \cdot \mathcal{G} = 0 \).

In what follows we will consider two special momenta configurations. The first one is the so-called symmetric configuration, in which \( q^2 = r^2 = p^2 \) and \( q = r = p = -q^2/2 \); in this case, there are only two totally transverse tensors, namely

\[
\lambda_{\text{tree}}^{(a)}(q, r, p) = \Gamma_{\alpha \beta \gamma}^{\mu \nu}(q, r, p) \Delta_{\alpha \gamma}(q) \Delta_{\mu \nu}(p),
\]

\[
\lambda_{\text{asym}}^{(a)}(q, r, p) = (q - r)_\mu (p - q)_\nu (q - r)_\gamma / r^2,
\]

where \( \Gamma_{\text{tree}}^{(a)} \) is the usual tree-level vertex. Indicating with \( \Gamma_{\text{sym}} \) and \( \Gamma_{\text{asym}} \) the corresponding form factors in the decomposition of \( \mathcal{G} \) (respectively, \( \Gamma \)) in this momentum configuration, Eq. (2) implies the relation

\[
T_{\text{sym}}(q^2) = g \Gamma_{\text{sym}}^{(s)}(q^2) \Delta(q^2),
\]

\[
S_{\text{sym}}(q^2) = g \Gamma_{\text{asym}}^{(s)}(q^2) \Delta(q^2).
\]

In particular, the \( T_{\text{sym}} \) form factor can be projected out through

\[
T_{\text{sym}}(q^2) = \frac{W_{\text{tree}}(q, r, p) \mathcal{G}_{\text{tree}}(q, r, p)}{W_{\text{tree}}(q, r, p) W_{\text{tree}}(q, r, p)_{\text{sym}}},
\]

with \( W = \lambda_{\text{tree}} + \lambda_{\text{asym}} / 2 \).

The second configuration we will study, which will be called ‘asymmetric’ in what follows, is defined by taking the \( q \to 0 \) limit, while imposing at the same time the condition \( r^2 = -p^2 \cdot r \). In this configuration \( \lambda_{\text{sym}} \sim r_\alpha r_\beta r_\gamma \) becomes totally longitudinal, and the only transverse tensor one can construct is obtained by the \( q \to 0 \) limit of \( \lambda_{\text{asym}} \) (obviously omitting the \( q \) projector), i.e.,

\[
T_{\text{asym}}(r^2) = g \Gamma_{\text{asym}}^{(r)}(r^2) \Delta(0) \Delta^2(r^2),
\]

where now \( W = \lambda_{\text{tree}}^{(r)} \).

All the quantities defined so far are bare, and a dependence on the regularization cut-off must be implicitly understood. Within a given renormalization procedure, the renormalized Green’s functions are calculated in terms of the renormalized fields \( Q_R = Z_Q^{1/2} Q \), so that

\[
\Delta_{\text{sym}}(q^2; \mu^2) = Z_Q^{-1}(\mu^2) \Delta(q^2),
\]

\[
T_{\text{sym}}(q^2; \mu^2) = Z_Q^{-3/2}(\mu^2) T_{\text{sym}}(q^2),
\]

and similarly for the asymmetric configuration. Within the MOM scheme that we will employ, one then requires that all the Green’s functions take their tree-level expression at the subtraction point, namely

\[
\Delta_{\text{sym}}(q^2; \mu^2) = Z_Q^{-1}(\mu^2) \Delta(q^2),
\]

\[
T_{\text{sym}}(q^2; \mu^2) = Z_Q^{-3/2}(\mu^2) T_{\text{sym}}(q^2) / g_{\text{sym}}(q^2) / q^6.
\]

The first equation yields the renormalization constant \( Z_Q \) as a function of the bare propagator, which when substituted into the second equation provides a renormalization group invariant definition of the three-gluon MOM running coupling \([42, 43]:\)

\[
\frac{\Gamma_{\text{sym}}(q^2)}{[\Delta(q^2)]^{1/2}} = q^3 T_{\text{sym}}(q^2; \mu^2) / g_{\text{sym}}(q^2). \]

In the asymmetric configuration the relation is slightly different, as in this case one has

\[
T_{\text{asym}}(r^2; \mu^2) = Z_Q^{-3/2}(r^2) T_{\text{asym}}(r^2) = \Delta_{\text{asym}}(0; q^2) g_{\text{asym}}(r^2) / r^4,
\]

implies

\[
g_{\text{asym}}(r^2) = r^3 T_{\text{asym}}(r^2) / [\Delta(r^2)]^{1/2} \Delta(0; q^2) / [\Delta(r^2; \mu^2)]^{1/2} \Delta(0; \mu^2).
\]
Finally, in both cases the above equations yield for the 1-PI form factors the relation
\[
g'_i(\mu^2) \Gamma^i_{\Delta R}(l^2; \mu^2) = \frac{g'_i(l^2)}{(l^2 \Delta(l^2; \mu^2))^{1/2}}, \tag{14}
\]

where \(i\) indicates either the symmetric or the asymmetric momentum configuration, and, correspondingly, \(l^2 = q^2, r^2\).

This latter result is of special interest because it establishes a connection between the three-gluon MOM running coupling, which many lattice and continuum studies have paid attention to, and the vertex function of the amputated three-gluon Green’s function, a fundamental ingredient within the tower of (truncated) SDEs addressing non-perturbative QCD phenomena. In fact, these quantities are related only by the gluon propagator \(\Delta\), which, after the intensive studies of the past decade, is very well understood and accurately known.

3. Lattice set-up and results. The lattice set-up used for our simulations is that of [44], where quenched SU(3) configurations at several large volumes and different bare couplings \(\beta\) were obtained employing the tree-level Symanzik gauge action. In particular, we use 220 configurations at \(\beta = 4.20\) for a hypercubic lattice of length \(L = 32\) (corresponding to a physical volume of 4.5\(^4\) fm\(^4\)) and 900 configurations at \(\beta = 3.90\) for a \(L = 64\) lattice (physical volume 15.6\(^4\) fm\(^4\)). The data extracted from these new gauge configurations have been supplemented with the one derived from the old configurations of [45], obtained using the Wilson gauge action at several \(\beta\) (ranging from 5.6 to 6.0), lattices (from \(L = 24\) to \(L = 32\)) and physical volumes (from 2.4\(^4\) to 5.9\(^4\) fm\(^4\)).

In Fig. 1 we plot the form factor \(T_R\) renormalized at \(\mu = 4.3\) GeV for both the symmetric (left panel) and asymmetric (right panel) momentum configuration. In the symmetric case \(T^\text{sym}_R\) displays a zero crossing located in the IR region around 0.1–0.2 GeV, after which the data seems to indicate that some sort of divergent behavior manifests itself. In the asymmetric case the situation looks less clear as data are noisier, as a result of forcing one momentum to vanish.

4. SDE analysis. In [41] it was shown that the nonperturbative ghost loop diagram contributing to the SDE of \(\Delta(q^2)\) is the source of certain noteworthy effects, the underlying origin of which is the masslessness of the propagators circulating in this particular loop. Specifically, employing a nonperturbative Ansatz for the gluon-ghost vertex that satisfies the correct STI, the leading IR contribution, denoted by \(\Pi_i(q^2)\), is given by [41]
\[
\Pi_i(q^2) = \frac{g^2C_4}{6} q^2 F(q^2) \int k^2 \frac{F(k^2)}{k^2(k + q)^2}, \tag{15}
\]

where \(C_4\) is the Casimir eigenvalue in the adjoint representation, and \(\int_k \equiv \int^{\mu'}/(2\pi)^d \int d^dk\) is the dimensional regularization measure, with \(d = 4 - \epsilon\) and \(\mu\) is the ‘t Hooft mass; evidently, in the limit \(q^2 \to 0\), the above expressions behave like \(q^2 \log q^2/\mu^2\). Even though this particular term does not interfere with the finiteness of \(\Delta(q^2)\), its presence induces two main effects: (i) \(\Delta(q^2)\) displays a mild maximum at some relatively low value of \(q^2\), and (ii) the first derivative of \(\Delta^{-1}(q^2)\) diverges logarithmically at \(q^2 = 0\). The form of the renormalized gluon propagator that emerges from the complete SDE analysis may be accurately parametrized in the IR by the expression
\[
\Delta^{-1}_R(q^2; \mu^2) = q^2 \left[ a + b \log \frac{q^2 + m^2}{\mu^2} + c \log \frac{q^2}{\mu^2} \right] + m^2, \tag{16}
\]

with \(a, b, c,\) and \(m^2\) suitable parameters, which captures explicitly the two aforementioned effects. Note that \(\Delta^{-1}_R(0; \mu^2) = m^2\), and that the “protected” logarithms stem from gluonic loops.

Higher order \(n\)-point functions \((n > 2)\) are also affected in not-\(\mu\)table ways by the presence of ghost loops in their diagrammatic expansion.\(^1\) If the external legs correspond to background gluons (as was the case in [41]), the leading IR behavior of projectors such as (6) and/or (8) is proportional to the

\(^1\)We refer to ghost loops that exist already at the one-loop level. Ghost loops nested within gluon loops do not produce particular effects, because the additional integrations over virtual momenta soften the IR divergence.
The band indicates the variation of the fit between $L = 72$ and $L = 96$, $L = 80$ being somewhere in between. Notice also that after the dashed vertical line the scale becomes linear, to expose the propagator behavior at the origin.

derivative of the inverse gluon propagator [41], by virtue of the Abelian STIs. Thus, eventually, a logarithmic divergence appears, which drives the aforementioned projectors from positive to (infinitely) negative values, causing invariably the appearance of a zero crossing. Use of the ‘background quantum’ identities [46, 47], which relate background Green’s functions with quantum ones, reveals that the same behavior is expected for quantum external legs, modulo a (finite) function determined by the ghost-gluon dynamics [41]. The exact position of the zero crossing is difficult to estimate, because it depends on the details of all finite contributions that are ‘competing’ against the logarithm coming from the ghost loop; however, it is clear that the tendency, in general, is to appear in the deep IR.

We emphasize that independent analyses within the SDE formalism, employing a variety of techniques and truncation schemes, have confirmed these claims in the three-point [49–51] and the four-point [52, 53] gluon sector. In addition, unquenching techniques, such as those developed in [54, 55], coupled with the lattice results of [7], show that the presence of light quarks slightly modifies the behavior of the gluon and ghost two-point sector only at the quantitative level; therefore, the expected IR pattern “zero crossing plus logarithmic IR divergence” for $n$-point gluon Green’s functions seems to constitute a robust prediction for QCD.

In particular, for the form factors under scrutiny, one expects the (configuration independent) IR behavior

$$\Gamma_{T,R}(\ell^2;\mu^2) \approx F(0;\mu^2) \frac{\partial}{\partial \ell^2} \Delta_{c1}(\ell^2;\mu^2),$$

(17)

where $F(0) \approx 2.9$ at $\mu = 4.3$ GeV [5].

To see if indeed the lattice data conform to the expected behavior, we start by estimating the propagator’s parameters $a$, $b$, $c$ and $m^2$ by fitting the lattice data of [5]. The results are shown in Fig. 2, with the parameter values obtained for the available data sets listed in Table 1. In what follows we will not distinguish between these different fits; rather we will use a single curve with bands representing its ‘uncertainty’.

At this point we can use Eq. (17) and the relation (5) to determine the expected IR behavior of the connected form factors $T^{\text{sym}}$, and $T^{\text{asym}}$, and compare with the data. The results are shown in Fig. 3. While it is evident that in the symmetric case a good description of the IR data is achieved, in the asymmetric case the positive excess in the data coupled to the large errors make it more difficult to discern the low momentum behavior of $T^{\text{asym}}$ and $T^{\text{sym}}$.

There is an interesting conclusion one might draw from the behavior of these form factors. As discussed in detail in [44], when quantum fluctuations can be either neglected or suppressed, gluon correlation functions appear to be dominated by a semiclassical background described in terms of a multi-instanton solution. In particular, specializing to the symmetric configuration, one has in this case

$$g^{\text{sym}}(\mu^2)T^{\text{sym}}_{T,R}(q^2;\mu^2) \approx \frac{2}{9nq^2 \left[\Delta(p^2;\mu^2)\right]^3},$$

(18)

where $n = 7.7$ fm$^{-4}$ is the instanton density in the semiclassical background. The resulting curve is shown by the dashed line in the lower left panel of Fig. 3. Then, we see that while the approximation (18) appears to be justified for momenta roughly below $q \sim 1$ GeV [44, 45], it fails in the deep IR region, around $q \sim 0.2$–0.3 GeV. This can be understood once we notice that at such low momenta (where the zero crossing takes place), the dynamics is entirely dominated by massless ghosts; plainly, this is a quantum effect that cannot be captured within the framework of a semiclassical approach.

Next, using Eqs. (11) and (13), we can construct the effective coupling $\alpha'(\ell^2) = g^2(\ell^2)/4\pi$ both from the lattice data and the determined IR behavior. In particular, the $\alpha'$ derived from the three-gluon vertex is proportional to the square of the form factor $\Gamma_T$, and displays a striking behavior; $\alpha'$ is forced to vanish at the zero crossing, and then ‘bounces’ back to positive values, as can be clearly seen in Fig. 4. According to this result, the part of the amplitude ‘gluon + gluon $\rightarrow$ gluon + gluon’ that

| Parameter | $L = 72$ | $L = 80$ | $L = 96$ |
|-----------|---------|---------|---------|
| $a$       | -0.471  | -0.151  | -1.146  |
| $b$       | -0.546  | -0.458  | -0.922  |
| $c$       | 0.362   | 0.352   | 0.546   |
| $m^2$     | 0.151   | 0.154   | 0.157   |

Table 1: Best fit parameters for the IR propagator (16) obtained using the SU(3) data of [5] for $\beta = 5.7$ and $L = 72$, 80 and 96 lattices.
is mediated by the (fully dressed) one-gluon exchange diagram vanishes at some special IR momentum; to be sure, this is not true for the entire physical amplitude, since additional diagrams (such as ‘boxes’) will furnish nonvanishing contributions.

4. Conclusions. We have presented new lattice results for the three-gluon vertex form factor $T_R$ proportional to the tree-level tensor structure. The data were obtained from large 4-dimensional volumes configurations generated for an SU(3) Yang-Mills theory gauge fixed in the Landau gauge, and the form factor evaluated in the so-called symmetric and asymmetric momentum configurations. The IR behavior of $T$ was then scrutinized in detail and contrasted with (model independent) SDE predictions finding good agreement. In doing so, we have discussed, the failure of a semiclassical picture based on instantons due to the quantum effects associated to massless ghost loops.

It is tempting to speculate that the behavior seen in the deep IR in the asymmetric case is not entirely due to statistical fluctuations. In fact, it has been shown in [57] that in this momentum configuration the 1-PI form factor receives contributions from quantities describing the appearance of (longitudinally coupled) massless poles in the fundamental vertices of the theory. When such poles are present, in fact, the (Abelian) STIs acquires new terms that account for both the IR finiteness of the gluon propagator and the deformation of Eq. (17), which would now read [57]

$$\Gamma^{\text{asy}}_{T,R}(r^2; \mu^2) \simeq F(0; \mu^2) \left[ \frac{\partial}{\partial r^2} \Delta^{-1}_R(r^2; \mu^2) - C'_i(r^2; \mu^2) \right],$$

where $C'_i$ corresponds, modulo a numerical factor, to the wave function amplitude associated to the formation of the massless pole. The shape of $C'_i$ and the corresponding qualitative modification induced to the form factor has been sketched in [57]; the expected signal is a positive excess in the IR region which is very similar to what the data in Fig. 3 show.

Acknowledgements. The research of J.P. and J. R-Q is supported by the Spanish MINECO under grant FPA2014-53631-C2-1-P and FPA2014-53631-C2-2-P and SEV-2014-0398, and Gener-
Figure 4: (color online) Comparison between the lattice results for the three gluon effective coupling and the SDE prediction in the symmetric (left) and asymmetric (right) configuration. Notice that on the y axis scale switch from logarithmic to linear at the location of the dashed gray line (and then back to logarithmic for y < 0); while this choice exaggerates the error bars, it has the advantage of exposing the vanishing of the coupling at a non vanishing momentum value.

alitat Valenciana under grant Prometeo II/2014/066. S. Z. acknowledges support by the Alexander von Humboldt foundation. We thank K. Cichy, M. Creutz, O. Pene, O. Philipsen, M. Teper, J. Verbaarschot for fruitful discussions. Numerical computations have used resources of CINES and GENCI-IDRIS as well as resources at the IN2P3 computing facility in Lyon.

References

[1] A. Cucchieri, A. Maas, T. Mendes, Phys.Rev. D74 (2006) 014503.
[2] A. Cucchieri, A. Maas, T. Mendes, Phys.Rev. D77 (2008) 094510.
[3] A. Cucchieri, T. Mendes, PoS LAT2007 (2007) 297.
[4] A. Cucchieri, T. Mendes, PoS QCD-TN19 (2009) 026.
[5] I. Bogolubsky, E. Ilgenfritz, M. MueUer-Preussker, A. Sternbeck, Phys. Lett. B676 (2009) 69–73.
[6] O. Oliveira, P. Silva, PoS LAT2009 (2009) 226.
[7] A. Ayala, A. Bashir, D. Binosi, M. Cristoforetti, J. Rodriguez-Quintero, Phys. Rev. D86 (2012) 074512.
[8] A. G. Duarte, O. Oliveira, P. J. Silva, arXiv:1605.00594 [hep-lat].
[9] A. C. Aguilar, D. Binosi, J. Papavassiliou, Phys. Rev. D78 (2008) 025010.
[10] P. Boucaud, et al., JHEP 06 (2008) 099.
[11] C. S. Fischer, A. Maas, J. M. Pawlowski, Annals Phys. 324 (2009) 2408–2437.
[12] J. Rodriguez-Quintero, JHEP 1101 (2011) 105.
[13] M. Pennington, D. Wilson, Phys. Rev. D84 (2011) 119901.
[14] P. Maris, C. D. Roberts, Int.J.Mod.Phys. E12 (2003) 297–365.
[15] A. C. Aguilar, A. A. Natale, JHEP 08 (2004) 057.
[16] P. Boucaud, et al., hep-ph/0507104.
[17] C. S. Fischer, J. Phys. G32 (2006) R253–R291.
[18] K.-I. Kondo, Phys. Rev. D74 (2006) 125003.
[19] D. Binosi, J. Papavassiliou, Phys. Rev. D77 (2008) 061702.
[20] D. Binosi, J. Papavassiliou, JHEP 0811 (2008) 063.
[21] P. Boucaud, J. P. Leroy, A. Le Yaouanc, A. Y. Lokhtov, J. Micheli, O. Pene, J. Rodriguez-Quintero, C. Roiesnel, JHEP 03 (2007) 076.
[22] D. Dudal, S. P. Sorrella, N. Vandersickel, H. Verschelde, Phys. Rev. D77 (2008) 071501.
[23] D. Dudal, J. A. Gracey, S. P. Sorrella, N. Vandersickel, H. Verschelde, Phys. Rev. D78 (2008) 064047.
[24] K.-I. Kondo, Phys.Rev. D84 (2011) 061702.
[25] A. P. Szczepaniak, E. S. Swanson, Phys. Rev. D65 (2002) 025012.
[26] A. P. Szczepaniak, Phys. Rev. D69 (2004) 074031.
[27] D. Epple, H. Reinhardt, W. Schleifenbaum, A. Szczepaniak, Phys. Rev. D77 (2008) 085007.
[28] A. P. Szczepaniak, H. H. Matevosyan, Phys. Rev. D81 (2010) 094007.

[29] P. Watson, H. Reinhardt, Phys.Rev. D80 (2009) 045010.
[30] P. Watson, H. Reinhardt, Phys.Rev. D85 (2012) 025014.
[31] L. Chang, C. D. Roberts, Phys. Rev. Lett. 103 (2009) 081601.
[32] L. Chang, C. D. Roberts, P. C. Tandy, Chin.J.Phys. 49 (2011) 955–1004.
[33] S.-x. Qin, L. Chang, Y.-x. Liu, C. D. Roberts, D. J. Wilson, Phys. Rev. C84 (2011) 042202.
[34] S.-x. Qin, L. Chang, Y.-x. Liu, C. D. Roberts, D. J. Wilson, Phys. Rev. C85 (2012) 035202.
[35] A. Bashir, L. Chang, I. C. Cloet, B. El-Bennich, Y.-X. Liu, et al., Commun.Theor.Phys. 58 (2012) 79–134.
[36] G. Eichmann, Prog. Part. Nucl. Phys. 67 (2012) 234–238.
[37] I. C. Cloet, C. D. Roberts, Prog. Part. Nucl. Phys. 77 (2014) 1–69.
[38] D. Binosi, L. Chang, J. Papavassiliou, C. D. Roberts, Phys.Lett. B742 (2015) 183–188.
[39] J. M. Cornwall, Phys. Rev. D26 (1982) 1453.
[40] D. Binosi, J. Papavassiliou, Phys. Rept. 479 (2009) 1–152.
[41] A. C. Aguilar, D. Binosi, D. Ibate, J. Papavassiliou, Phys. Rev. D89 (2014) 085008.
[42] B. Allès, D. Henty, H. Panagopoulos, C. Parrinello, C. Pitteri, D. G. Richards, Nucl. Phys. B502 (1997) 325–342.
[43] P. Boucaud, J. P. Leroy, J. Micheli, O. Pene, C. Roiesnel, JHEP 10 (1998) 017.
[44] A. Athenodorou, P. Boucaud, F. De Soto, J. Rodriguez-Quintero, S. Zafeiropoulos, arXiv:1604.08887 [hep-ph].
[45] P. Boucaud, F. De Soto, A. Le Yaouanc, J. P. Leroy, J. Micheli, H. Moutarde, O. Pene, J. Rodriguez-Quintero, JHEP 04 (2003) 005.
[46] P. A. Grassi, T. Hurth, M. Steinhauser, Annals Phys. 288 (2001) 197–248.
[47] D. Binosi, J. Papavassiliou, Phys.Rev. D66 (2002) 025024.
[48] P. J. Green, B. W. Silverman, Nonparametric Regression and Generalized Linear Models: A roughness penalty approach, volume 58 of Monographs on Statistics & Applied Probability, Chapman & Hall, 1994.
[49] A. Blum, M. Q. Huber, M. Mitter, L. von Smekal, Phys.Rev. D89 (2014) 061703.
[50] G. Eichmann, R. Williams, R. Alkofer, M. Vujinovic, Phys.Rev. D89 (2014) 105014.
[51] A. K. Cyrol, L. Fister, M. Mitter, J. M. Pawlowski, N. Strodthoff, arXiv:1605.01856 [hep-ph].
[52] D. Binosi, D. Ibate, J. Papavassiliou, JHEP 1409 (2014) 059.
[53] A. K. Cyrol, M. Q. Huber, L. von Smekal, Eur. Phys. J. C75 (2015) 102.
[54] A. C. Aguilar, D. Binosi, J. Papavassiliou, Phys.Rev. D86 (2012) 014032.
[55] A. C. Aguilar, D. Binosi, J. Papavassiliou, Phys.Rev. D88 (2013) 074010.
[56] C. T. Figueiredo, A. C. Aguilar, Effects of divergent ghost loops in the presence of dynamical quarks, http://sites.if.unicamp.br/qcd-tnt/files/2015/08/figueiredo.pdf, 2015.
[57] A. C. Aguilar, D. Binosi, C. T. Figueiredo, J. Papavassiliou, arXiv:1604.08456 [hep-ph].