Heavy Meson Hyperfine Splittings:
A Puzzle for Heavy Quark Chiral Perturbation Theory*

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Abstract

We show that there is a large discrepancy between the expected light flavor dependence of the heavy pseudoscalar–vector mass splittings and the measured values. We demonstrate that the one-loop calculation is unreliable. Moreover, agreement with experiment requires the leading dependence on SU(3) symmetry breaking to be nearly cancelled, so that the heavy quark mass dependence is unknown and the expected dependence on the light quark mass is not realized.

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1 Introduction

Much attention has been devoted to the heavy quark chiral effective theory [1]. The idea is to incorporate both heavy quark and chiral symmetry into an effective theory which can describe heavy meson interactions with low momentum pions. In addition to tree level predictions of the form factors, such a theory yields a way to predict or at least estimate the size of SU(3) violating effects.

However, SU(3) violating predictions have not yet been experimentally tested. In this paper, we use the heavy quark chiral effective theory to estimate and calculate the SU(3) violating parameter

$$\Delta_H \equiv (m_{H_s} - m_{H}) - (m_{H_{ns}} - m_{H_{ns}})$$

where $ns$ stands for nonstrange. We argue that the estimate one obtains based on a naive expansion in SU(3) violation is a significant overestimate. This means that at this point there is no experimental evidence that an expansion in the strange quark mass works for heavy quark systems.

We explicitly calculate the leading log contribution to verify the existence of large contributions to $\Delta_H$ which disagree with the measurements. We show furthermore that the subleading term (in powers of $m_s$) contributes as large an amount as the leading term. This demonstrates that the procedure of retaining only the one–loop contribution in chiral perturbation theory is inconsistent. But the result that the expansion in SU(3) violation has not worked contradicts what would be naively expected from any model which incorporates SU(3) violating light quark masses.

Rosner and Wise [2] recently considered this same parameter, $\Delta_H$. They enumerated the operators which are responsible for distinguishing the various heavy meson masses. They left the coefficients arbitrary and fit to existing data on heavy quark masses. They then used their assumed dependence on heavy quark and SU(3) violating parameters to predict $\Delta_B = (m_c/m_b)\Delta_D$. They concluded that the
photons which are emitted in $B_s^*$ decay and $B^*$ decay should have energies which agree to within an MeV.

In their paper they observed that the operator which contributes to $\Delta_H$ has a small coefficient. We claim that this small coefficient indicates the operator analysis has failed, so one does not in fact know the leading dependence on heavy quark parameters. Because the near cancellation of $\Delta_D$ could involve higher order terms in the $1/m_c$ expansion, the prediction of $\Delta_B$ is not reliable.

We reach two conclusions. First, calculations including only one-loop contributions in the heavy quark chiral effective theory are not reliable, since the tree level contribution should be comparable. Second, there is an interesting physical puzzle as to why heavy quark chiral perturbation theory does not give the correct result, even at the order of magnitude level.

In this letter, we first describe the experimental situation, and review the operator analysis of ref. [2]. We estimate the result that would be expected on dimensional grounds in the heavy quark chiral lagrangian. The following section contains a one-loop calculation in which we obtain a chiral log term in accordance with our estimate. We discuss possible implications of this result.

2 Experimental Situation and Operator Analysis

Much is known about the heavy pseudoscalar-vector meson mass splittings [3, 4, 5]:

\begin{align*}
  m_{D^{*+}} - m_{D^+} & = 140.64 \pm 0.08 \pm 0.06 \text{ MeV} \\
  m_{D_s^*} - m_{D_s} & = 141.5 \pm 1.9 \text{ MeV} \\
  m_{B^*} - m_B & = 45.4 \pm 1.0 \text{ MeV} \ (\text{or}) \ 46.2 \pm 0.3 \pm 0.8 \text{ MeV} \\
  m_{B_s^*} - m_{B_s} & = 47.0 \pm 2.6 \text{ MeV}
\end{align*}
It should be observed that the values in the first two lines and the last two lines are very similar. The differences $\Delta_H$ are only a few percent of the SU(3)–symmetric splittings:

\[
\begin{align*}
\Delta_D &= 0.9 \pm 1.9 \text{ MeV} \\
\Delta_B &= 1.2 \pm 2.7 \text{ MeV}.
\end{align*}
\]

The extremely small sizes of these differences are particularly surprising when compared to what one expects on the basis of a simple operator analysis, as we now show.

Consider in the chiral heavy quark theory the operators which contribute to the spin splittings in eqs. (1–4) at leading order in the light quark mass matrix, $m_q$, and the inverse heavy quark mass matrix, $m_Q^{-1}$. If the operator which contributes to the SU(3) symmetric splittings is

\[
O_1 = \lambda\text{Tr}[\tilde{H}_a^i \sigma^{\mu\nu} H_j^a \sigma_{\mu\nu}](m_Q^{-1})_i^j,
\]

one would expect the operator which contributes to the chiral symmetry breaking differences, $\Delta_D$ and $\Delta_B$, to be

\[
O_2 = \lambda'\text{Tr}[\tilde{H}_a^i \sigma^{\mu\nu} H_j^b \sigma_{\mu\nu}](m_Q^{-1})_i^j \frac{(m_q)_a^b}{\Lambda_{\text{CSB}}},
\]

where the scale of chiral suppression is set by naive application of the chiral lagrangian dimensional factors. Here $H_a^i$ is the field of a heavy meson containing a heavy quark of flavor $i$ and a light antiquark of flavor $a$. (Note the value of the light quark masses should be the same as those taken from fitting the pions, kaons, and nucleons. There is an arbitrary strong interaction constant relating the heavy meson mass splitting to these quark masses, so we cannot reliably extract the values without a more comprehensive fit.)

However, if this were the case, one would expect the spin splitting in the $D_s$ system to differ from that in the $D^+$ system by about $0.15 \cdot 141 \text{ MeV} \approx 20 \text{ MeV}$. As can be seen from the measured value of $\Delta_D$, this is a significant overestimate, off by about an order of magnitude. (By comparing the spin-splitting of heavy mesons containing
an strange quark with the splitting for heavy mesons containing a down quark (so that $\Delta_H$ measures $V$-spin breaking), we avoid the small electromagnetic splittings. Electromagnetic interactions contribute to $(D^{*0} - D^0) - (D^{*+} - D^+)$ which has been measured to be $1.48 \pm 0.09 \pm 0.05$ [3] and is in accordance with expectations.) Notice that this difficulty in understanding the dependence of the heavy meson spin splittings on the strange quark mass is in contrast to our experience with light baryons, whose spin splittings are well described by a nonrelativistic quark model where the magnetic moments of the quarks are inversely proportional to their masses.

One might assume that the operator responsible for SU(3) dependence of the spin–dependent mass splittings is suppressed. We show in the next section that this would then be inconsistent at the one loop level. We calculate an explicit contribution to the splitting that agrees well with the above estimate. In fact, if one kept only the chiral log correction, as has been done in various papers on chiral heavy quark theory, one calculates a difference of about 15 MeV for the $D$ system. With subleading terms included, the predicted value is even larger.

3 One-Loop Calculation of $\Delta_H$

The method of calculation is by now standard. We assume the heavy meson effective lagrangian, given by

$$\mathcal{L} = -iTr[H_a^i v_\mu \partial^\mu H_b^i] + \frac{i}{2}Tr[H_a^i H_b^j v_\mu (\xi^\dagger \partial^\mu \xi + \xi \partial^\mu \xi^\dagger)^a_b$$

$$+ \frac{ig}{2}Tr[H_a^i H_b^j \gamma_\mu \gamma_5] (\xi^\dagger \partial^\mu \xi - \xi \partial^\mu \xi^\dagger)^a_b - \rho Tr[H_a^i H_b^j] (m_q)^a_b$$

$$+ \lambda Tr[H_a^i \sigma^{\mu\nu} H_j^a \sigma_{\mu\nu}] (m_Q^{-1})^j_i.$$ (9)

Here $\xi = \exp(i\pi^\alpha T^\alpha / f_\pi)$ and $v$ is the heavy quark velocity. We have kept the leading chiral symmetry breaking contribution to the masses of the heavy mesons explicit in the lagrangian. In practice, we use the experimental value of the strange-nonstrange heavy meson mass difference to fix $\rho$, which automatically incorporates the correct leading
contribution to the mass differences due to SU(3) symmetry breaking. We have also explicitly included the leading heavy quark symmetry breaking operator, $O_1$, which is suppressed by $1/m_Q$. This appears as an explicit vertex in the calculation.

We calculate two types of diagrams. In the first, Figure 1, we insert the spin dependent operator $O_1$, and have a pseudogoldstone boson emitted and absorbed through the axial coupling. In the second, Figure 2, we calculate the SU(3) wave function renormalization which also contributes at the same order, given the existing spin splitting, and contributes exactly $-3$ times the amount of the graph in Figure 1 (to all orders in $m_{H_s} - m_{H_{ns}}$). The result is

$$\Delta^0_H \equiv \frac{(m_{H_s} - m_{H_{ns}}) - (m_{H_{ns}} - m_{H_{ns}})}{m_{H_s} - m_{H_{ns}}} = \frac{g^2}{16\pi^2} \left\{ -\frac{3 m^2_\pi}{2 f^2_\pi} \ln \frac{m^2_\pi}{\Lambda^2_{CSB}} + \frac{m^2_K}{f^2_K} \ln \frac{m^2_K}{\Lambda^2_{CSB}} + \frac{1}{2} \frac{m^2_\eta}{f^2_\eta} \ln \frac{m^2_\eta}{\Lambda^2_{CSB}} \right\}. \quad (10)$$

Applying this result to the $D$ system yields a mass splitting of $\Delta^0_D = -15$ MeV if we take $g^2 = 0.5$. In the $B$ system $\Delta^0_B = -5.1$ MeV.

To check the consistency of the calculation as an expansion in the chiral symmetry breaking parameter, $m_s$, we also calculate the $m_{\frac{3}{2}}$ contribution which results from a linearly divergent loop integral. It is

$$\Delta^1_H \equiv \frac{g^2}{16\pi^2} \{ -6\pi m_K (m_{H_s} - m_{H_{ns}}) \}, \quad (11)$$

where $m_{H_s} - m_{H_{ns}}$ is the heavy-quark symmetric strange-nonstrange heavy meson mass splitting which is fit to be $99.5 \pm 0.6$ MeV for the $D$ system and found to be $80-130$ MeV for the $B$ system. This contribution is given by the difference of an $H_s$ meson self-energy graph with an intermediate $H$ meson and an $H$ meson self-energy graph with an intermediate $H_s$ meson. The strange-nonstrange mass splitting contributes with opposite signs in these two graphs so that in the difference of the two graphs these terms add constructively. In the contributions that are zeroth order (i.e., $\Delta^0_H$) and second order in
there are cancellations between the $H_s$ and $H$ meson self-energy graphs. So while the term second order in $m_{H_s} - m_{H_{ns}}$ turns out to be negligible, the term linear in $m_{H_s} - m_{H_{ns}}$ is larger than the zeroth order term, $\Delta^0_H$, contributing $\Delta^1_D = -32$ MeV in the $D$ system and $\Delta^1_B = -11$ MeV in the $B$ system (using the central value for $m_{B_s} - m_{B_{ns}}$). Note that these contributions reinforce the $\Delta^0_H$ contributions found above and give $\Delta_D = -47$ MeV and $\Delta_B = -16$ MeV. There are also extra finite pieces zeroth order in $m_{H_s} - m_{H_{ns}}$ that are quadratic in the pseudogoldstone masses. These are also comparable in size to the log terms in $\Delta^0_H$. The large size of each of the non-log terms shows that retaining only log terms is not a reasonable approximation.

This calculation demonstrates that the parameter $\Delta_H$ will not scale linearly with the strange quark mass, since terms proportional to $m_s$ and $m_s^{(3/2)}$ were of comparable importance. Although not manifest in this calculation, which was only done to order $1/m_Q$, we demonstrate that straightforward estimates of the size of terms which are higher order in inverse powers of the heavy quark mass are also far from negligible.

This can be seen by an operator analysis analogous to that in the second section, but including higher order operators. For example, the operator

$$O_3 = \lambda^{''}\text{Tr}[]\bar{H}_a^i\sigma^{\mu\nu}H_j^b\sigma_{\mu\nu}](m_Q^{-2})^{-1}_i(m_Q^2)^j \frac{(m_Q^2)^g}{\Lambda_{CSB}}$$

should contribute about $1.5$ MeV to $\Delta_D$, which is still larger than the measured value.

In fact, a one–loop estimate of the contribution to $\Delta_D$ at order $1/m_Q^2$ is even larger. To study the $1/m_Q^2$ terms, we could insert into the heavy quark line in Figure 1 or 2 the two-derivative piece of the heavy meson kinetic term, $\frac{1}{2}\text{Tr}[]\bar{H}_a^i\partial^2H_j^a](m_Q^{-1})^i_j$, or we could simply insert the spin-splitting operator, $O_1$, a second time. We would expect contributions to $\Delta_H/(m_{H^*} - m_H)$ of order $m_K^3/(m_Q\Lambda_{CSB}^2)$ and $m_K(m_{H^*} - m_H)/\Lambda_{CSB}^2$ respectively. This is actually larger than the
contribution from $O_3$ above. For the $D$ system, these terms each contribute of order 10 MeV to $\Delta_D$.

Clearly we cannot account for the very small size of $\Delta_D$ if we truncate its expansion in powers of $1/m_c$ at the first term, proportional to $1/m_c$. All we know is that there is a conspiracy between a large number of terms generated at tree and loop level, all of which individually would generate a large contribution to $\Delta_c$, but whose sum is small. Therefore, we cannot assume $\Delta_B = (m_c/m_b)\Delta_D$. Furthermore, because we do not know the role of the higher order terms in the cancellation which produces a small value of $\Delta_D$, we cannot necessarily conclude that $\Delta_B$ is small, although preliminary measurements do give a small value.

### 4 Discussion

This result is clearly disturbing. The picture of the heavy meson based on leading SU(3) and heavy quark symmetric physics broken at order $\Lambda_{QCD}/m_Q$ and $m_s/\Lambda_{CSB}$ does not predict the correct size of $\Delta_H$, assuming the existing data is correct. This behavior is peculiar from any viewpoint, independent of the heavy quark effective theory, since one would naively expect a fairly large effect due to the fact that the magnetic moment of the strange constituent quark is less than that of the non–strange counterpart by a significant amount. For example, using a quark model, Close in 1979 [7] predicted $m_{D_s^*}-m_{D_s} \approx \frac{2}{3}(m_{D^*}-m_D)$.

We also evaluated using the bag model the change in color magnetostatic energy for a strange quark mass of 200 MeV (a small value from the standpoint of the bag model, since the net mass contributed to the meson is then only about 100 MeV). The strange heavy meson magnetostatic energy in this model was also about $2/3$ of the corresponding value for a nonstrange heavy meson. However, it is interesting to note that an estimate by Godfrey and Isgur [8] predicted $\Delta_D \approx -10$ MeV.

This disagreement between the expected and measured dependence
on the strange quark mass is clearly a puzzle. From the standpoint of the expansion in chiral symmetry breaking and the inverse of the heavy quark mass, it would be attributable to a cancellation among tree and loop contributions. The role of higher order terms in $1/m_Q$ will be put to the test when the photon energies for the transitions $B_s^* \to B_s \gamma$ and $B_s^{*0} \to B^0 \gamma$ are more accurately determined. A photon energy larger than an MeV will demonstrate the importance of higher order terms in the inverse of the heavy quark mass; a small value would however be inconclusive, since it could arise from a cancellation between tree and loop contributions to $\Delta_D$ at any given order in $1/m_c$.

If there is an accidental cancellation in the one measured quantity computed with heavy quark chiral perturbation theory, it is possible this can happen elsewhere. It would be difficult to determine which predictions are reliable. One might hope this calculation is somehow distinct from others which have been done. However this is difficult to reconcile with the fact that wave function renormalization alone would in itself generate a large effect.

The small value of $\Delta_H$ might signal something fundamental about the heavy meson, indicating that the operator analysis is not the best description. For example, the authors of ref. [2] suggested that the SU(3) breaking strong hyperfine splitting due to the change in chromomagnetic moments of the light quarks is cancelled by a change in the wave function of the heavy meson. It would be interesting to test other predictions of the heavy quark theory. For example, the transition magnetic moment of the heavy meson [10], the flavor dependence of $f_H$ and $B_H$ calculated in ref. [1] or of the Isgur–Wise function [11] would all be interesting measurements, if they can be done. These measurements would answer the following questions: 1) Do the one–loop calculations give the correct result? 2) Are there other parameters whose values do not have the expected dependence on light quark mass? and 3) If there is a model in which the wave function at zero interquark separation cancels the dependence on the
quark mass of the magnetic moment, does it give correct predictions for these other quantities? These would not only settle the issue of whether the discrepancy between the values of $\Delta_H$ obtained from the expected chiral expansion and from experiment was purely accidental, but could also test the flavor dependence of the wave function of the heavy meson.

We conclude that the discrepancy between the expected and measured values of $\Delta_H$ is a very interesting puzzle. If the data in the $D$ and $B$ systems is correct, we might have an interesting probe of heavy mesons at hand. On the other hand, if there is simply an accidental cancellation between large terms, it would be worth investigating if this happens in other measurable parameters as well. It would be useful to supplement this measurement with other measurements of SU(3) violating effects to test the validity of possible proposed forms of light quark flavor dependence.

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Figure Captions

Figure 1. Diagram with an insertion of the spin dependent operator, $O_1$. The dotted line is a pseudogoldstone boson (which can be strange or nonstrange) and the solid line is the heavy meson (which can be strange or nonstrange, spin one or spin zero).

Figure 2. Wave function renormalization. Notation same as for Figure 1.