Infrared Properties of Hadronic Structure of Nucleon in Neutron Beta Decays to Order $O(\alpha/\pi)$ in Standard $V-A$ Effective Theory with QED and Linear Sigma Model of Strong Low–Energy Interactions

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Within the standard $V-A$ theory of weak interactions, Quantum Electrodynamics (QED) and the linear $\sigma$–model (LoM) of strong low–energy hadronic interactions we analyse infrared properties of hadronic structure of the neutron and proton in the neutron $\beta^–$–decays to leading order in the large nucleon mass expansion. We confirm validity and high confidence level of contributions of hadronic structure of the nucleon to the radiative corrections, calculated by Sirlin (Phys. Rev. 164, 1767 (1967)) to leading order in the large nucleon mass expansion. At the level of order $10^{-5}$ relative to Sirlin’s infrared divergent contribution to the neutron radiative $\beta^–$–decay (inner bremsstrahlung) we find an infrared divergent contribution, induced by hadronic structure of the nucleon through the one–pion–pole exchange, to the rate of the neutron lifetime from the neutron radiative $\beta^–$–decay, which should be cancelled by contributions of virtual photon exchanges to the neutron $\beta^–$–decay. Following Ivanov et al. 1805.09702 [hep-ph] we argue that a consistent analysis of such a cancellation may be carried out well in the combined quantum field theory including the Standard Electroweak Model (SEM) and the LoM of strong low–energy interactions, where the effective $V-A$ hadron–lepton current–current vertex is caused by the $W^–$–electroweak–boson exchange.

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It is well–known that the neutron radiative $\beta^–$–decay $n \rightarrow p + e^- + \bar{\nu}_e + \gamma$ (inner bremsstrahlung) plays an important role in the cancellation of infrared divergences, caused by virtual photon exchanges, to the neutron $\beta^–$–decay to order $O(\alpha/\pi)$ [1]–[24], where $\alpha$ is the fine–structure constant [25]. As a physical process the neutron radiative $\beta^–$–decay has been investigated theoretically to order $O(\alpha/\pi)$ in [26]–[28] (see also [21]) and to order $O(\alpha^2/\pi^2)$ in [24]–[28], respectively, and experimentally in [32]–[34]. Recently [35] we have analysed gauge properties of hadronic structure of the nucleon in the neutron $\beta^–$–decays within the combined quantum field theory including the standard $V-A$ effective theory of weak interactions [36]–[38], Quantum Electrodynamics (QED) and the linear $\sigma$–model (the LoM) of strong low–energy interactions [39]–[48], which is renormalizable [10, 13, 15, 17] and in the infinite limit of the scalar $\sigma$–meson mass $m_\sigma \rightarrow \infty$ reproduces the results of the current algebra [11, 12]. We have shown that in the limit $m_\sigma \rightarrow \infty$, to leading order in the large nucleon mass expansion and after renormalization such a combined quantum field theory

FIG. 1: The Feynman diagrams, describing the amplitude of the neutron $\beta^–$–decay, defined in the limit $m_\sigma \rightarrow \infty$, to leading order in the large nucleon mass expansion and after renormalization in the LoM.

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where the contributions of strong low–energy interactions are presented by the axial coupling constant $g_A$ and the one–pion–pole exchange, and the gauge invariant amplitude of the neutron radiative $\beta^-$–decay, where the contributions of strong low–energy interactions are presented in terms of the axial coupling constant $g_A$ and one–pion– and two–pion–pole exchanges. The Feynman diagrams of the amplitude of the neutron $\beta^-$–decay, calculated to leading order in the large nucleon mass expansion, are shown in Fig. 4. The amplitude of the neutron $\beta^-$–decay is defined by

$$M(n \to p e^- \bar{\nu}_e) = -G_V \langle p|\bar{K}_p, \sigma_p\rangle |J^+_\mu(0)|n(\bar{K}_n, \sigma_n)\right\rangle \left[\tilde{u}_e(\bar{K}_e, \sigma_e)\gamma^\mu(1 - \gamma^5)v_\nu(\bar{K}_\nu, +\frac{1}{2})\right],$$ (1)

where $G_V = G_F V_{ud}/\sqrt{2}$ is the vector weak coupling constant, and $G_F$ and $V_{ud}$ are the Fermi weak coupling constant and the matrix element of the Cabibbo–Kobayashi–Maskawa (CKM) mixing matrix, respectively, and $\tilde{u}_e\gamma^\mu(1 - \gamma^5)v_\nu$ is the matrix element of the leptonic $V - A$ current. The matrix element of the hadronic $n \to p$ transition calculated in the limit $m_N \to \infty$, to leading order in the large nucleon mass expansion and after renormalization, takes the form

$$\langle p|\bar{K}_p, \sigma_p\rangle |J^+_\mu(0)|n(\bar{K}_n, \sigma_n)\right\rangle = \tilde{u}_p(\bar{K}_p, \sigma_p) \left\{\gamma^\mu(1 - g_A\gamma^5) + \frac{\kappa}{2m_N} i\sigma_{\mu\nu}q^\nu - \frac{2g_A m_N}{m_N^2 - q^2} q^\mu q^5\right\} u_n(\bar{K}_n, \sigma_n),$$ (2)

where the contributions of strong low–energy interactions are presented by the axial coupling constant $g_A$ and the one–pion–pole exchange. The contribution of the one–pion–pole exchange is necessary for local conservation of the axial hadronic current in the limit $m_N \to 0$ [38, 39].

The one–pion–pole exchange contribution appears also in the current algebra approach [38] (see also [51]). The term with the Lorentz structure $i\sigma_{\mu\nu}q^\nu/2m_N$, where $m_N$ is the nucleon mass, describes the contribution of the weak magnetism [38, 39] with the isovector anomalous magnetic moment of the nucleon $\kappa$.

The Feynman diagrams of the amplitude of the neutron radiative $\beta^-$–decay are shown in Fig. 4. The analytical expression of the amplitude of the neutron radiative $\beta^-$–decay, defined by the Feynman diagrams in Fig. 4, is given by

$$M(n \to p e^- \bar{\nu}_e) = eG_V \times \left\{ \left[\tilde{u}_p(\bar{K}_p, \sigma_p)\gamma^\mu(1 - g_A\gamma^5)u_n(\bar{K}_n, \sigma_n)\right]\left[\tilde{u}_e(\bar{K}_e, \sigma_e)\frac{1}{2k_e \cdot k} Q_{e,\lambda}(1 - \gamma^5)v_\nu(\bar{K}_\nu, +\frac{1}{2})\right] \right\} \left[\tilde{u}_p(\bar{K}_p, \sigma_p)Q_{p,\lambda}\frac{1}{2k_p \cdot k}\gamma^\mu(1 - g_A\gamma^5)u_n(\bar{K}_n, \sigma_n)\right]\left[\tilde{u}_e(\bar{K}_e, \sigma_e)Q_{e,\lambda}\gamma^\mu(1 - \gamma^5)v_\nu(\bar{K}_\nu, +\frac{1}{2})\right] + \frac{2g_A m_N(q - k)\mu}{m_N^2 - (q - k)^2 - i0} \left[\tilde{u}_p(\bar{K}_p, \sigma_p)\gamma^5 u_n(\bar{K}_n, \sigma_n)\right]\left[\tilde{u}_e(\bar{K}_e, \sigma_e)Q_{e,\lambda}\frac{1}{2k_e \cdot k}\gamma^\mu(1 - \gamma^5)v_\nu(\bar{K}_\nu, +\frac{1}{2})\right]$$

FIG. 2: The Feynman diagrams the amplitude of the neutron radiative $\beta^-$–decay within the combined quantum field theory including the standard $V - A$ effective theory of weak interactions, QED and the LoM of strong low–energy interactions in the limit $m_N \to \infty$, to leading order in the nucleon mass expansion and after renormalization.
where $F \leq 14 \text{ keV}$ by Czarnecki mass regularization and takes the form \[21\] shown in \[35\] the contributions of the sum of the Feynman diagrams in Fig. 1a and Fig. 2b and the sum of Fig. 2c - coupling constant $g$. The contribution of the first two terms in Eq. (4) to the rate of the neutron radiative $\beta$-decay has been calculated in \[1\] - \[6, 18\] (see also and \[21\]) within finite –photon interaction. The contribution of the first two terms in Eq. (4) to the rate of the neutron radiative $\beta$-decay, taking to leading order in the large nucleon mass expansion with photon from the energy region $\omega_{\text{min}} \leq \omega \leq \omega_{\text{max}}$, has been calculated in \[21, 26, 28\]. It takes the form \[21, 26, 28\]

\[\lambda_{\beta\gamma}(\omega_{\text{max}}, \omega_{\text{min}}) = \frac{\alpha}{\pi} \left(1 + 3g_A^2\right) \frac{|G_V|^2}{\pi^3} \int_{\omega_{\text{min}}}^{\omega_{\text{max}}} d\omega \int_{m_e}^{E_0 - \omega} dE_e \sqrt{E_e^2 - m_e^2} E_e F(E_e, Z = 1) (E_0 - E_e - \omega)^2 \times \left\{ \left( 1 + \frac{\omega}{E_e} + \frac{\omega^2}{2E_e^2} \right) \left[ \ln \left( 1 + \frac{\beta}{1 - \beta} \right) - 2 \right] + \frac{\omega^2}{E_e^2} \right\}, \]

where $F(E_e, Z = 1)$ is the well-known relativistic Fermi function of the Coulomb proton-electron final-state interaction, $E_e$ and $\beta = \sqrt{E_e^2 - m_e^2}/E_e$ are the energy and velocity of the decay electron, $E_0 = (m_n^2 - m_e^2 + m_e^2)/2m_n = 1.2927 \text{ MeV}$ is the end-point energy of the electron-energy spectrum \[21\]. In the experimental energy region $14 \text{ keV} \leq \omega \leq 782 \text{ keV}$ \[34\] the rate Eq. (4) defines the branching ratio $B_{\beta\gamma} = 3.94 \times 10^{-3}$, calculated for the neutron lifetime $\tau_n = 879.6(1.1) \text{ s}$ calculated in \[21\] at $g_A = 1.2750(9)$ \[54\] (see also \[52\]). Such a branching ratio does not contradict the experimental value $B_{\beta\gamma} = 3.35(16) \times 10^{-3}$ \[54\] within two standard deviations. The values of the neutron lifetime $\tau_n = 879.6(1.1) \text{ s}$ and axial coupling constant $g_A = 1.2750(9)$ agree well with recent values of the neutron lifetime $\tau_n^{\text{favored}} = 879.4(6) \text{ s}$ and axial coupling constant $g_A^{\text{favored}} = 1.2755(11)$, which were recommended by Czarnecki et al. \[56\] as favored.

The contribution of these two terms, taken to leading order in the large nucleon mass expansion, to the radiative corrections to the rate of the neutron $\beta$-decay has been calculated in \[1, 4, 18\] (see also and \[21\]) within finite–photon mass regularization and takes the form \[21\]

\[\lambda_{\beta\gamma}(E_0, \mu) = \frac{\alpha}{\pi} \left(1 + 3g_A^2\right) \frac{|G_V|^2}{\pi^3} \int_{m_e}^{E_0} dE_e \sqrt{E_e^2 - m_e^2} E_e F(E_e, Z = 1) (E_0 - E_e)^2 \times \left\{ \left[ 2 \ln \left( \frac{2(E_0 - E_e)}{\mu} \right) \right] - 3 + \frac{2}{3} \frac{E_0 - E_e}{E_e} \left( 1 + \frac{E_0 - E_e}{8E_e} \right) \right\} \left[ \frac{1}{2\beta} \ln \left( \frac{1 + \beta}{1 - \beta} \right) - 1 \right] + 1 + \frac{1}{12} \frac{(E_0 - E_e)^2}{E_e^2} \times \left( \ln \left( \frac{1 + \beta}{1 - \beta} \right) \right) - \frac{1}{4\beta} \ln ^2 \left( \frac{1 + \beta}{1 - \beta} \right) - \frac{1}{4\beta} \ln ^2 \left( \frac{1 + \beta}{1 - \beta} \right) - \frac{1}{4\beta} \ln ^2 \left( \frac{1 + \beta}{1 - \beta} \right)

\right\},

where $\ln(z)$ is the Polylogarithmic function and $\mu$ is an infinitesimal photon mass, which should be taken in the limit $\mu \to 0$ \[1, 2, 3\] \[18\] (see also \[21\]). The rate of the neutron $\beta$-decay $n \to p + e^- + \bar{\nu}_e$, taking into account radiative corrections, caused by virtual photon exchanges, calculated within the finite–photon mass $\mu$ regularization, is given by (see Appendix D of \[21\])

\[\lambda_{\beta}(E_0, \mu) = (1 + 3g_A^2) \frac{|G_V|^2}{\pi^3} \int_{m_e}^{E_0} dE_e \sqrt{E_e^2 - m_e^2} E_e F(E_e, Z = 1) (E_0 - E_e)^2 \left\{ 1 + \frac{\alpha}{\pi} \left(2\ln \left( \frac{\mu}{m_e} \right) \right) \right\}.

\]
\[
\frac{1}{2\beta} \ln\left(\frac{1 + \beta}{1 - \beta}\right) - 1 + \frac{3}{2} \ln\left(\frac{m_e}{m_N}\right) - \frac{11}{8} - \frac{1}{\beta} \text{Li}_2\left(\frac{2\beta}{1 + \beta}\right) - \frac{1}{4\beta} \ln^2\left(\frac{1 + \beta}{1 - \beta}\right) + \frac{\beta}{2} \ln\left(\frac{1 + \beta}{1 - \beta}\right) \right). \quad (8)
\]

Summing up the contributions of the rates \(\lambda_\beta(E_0, \mu)\) and \(\lambda_{\beta^\gamma}(E_0, \mu)\) and taking the limit \(\mu \to 0\) we arrive at the rate of the neutron decay

\[
\lambda_n(E_0) = (1 + 3g_\lambda^2) \frac{|G_V|^2}{\pi^3} \int_{m_e}^{E_0} dE_e \sqrt{E_e^2 - m_e^2} E_e F(E_e, Z = 1) (E_0 - E_e)^2 \left(1 + \frac{\alpha}{\pi} \bar{g}_n(E_e)\right), \quad (9)
\]

where the function \(\bar{g}_n(E_e)\), defining the radiative corrections to the neutron lifetime, is equal to \(\bar{g}_n\) (see also Appendix D of [21]).

\[
\bar{g}_n(E_e) = \frac{3}{2} \ln\left(\frac{m_e}{m_N}\right) - \frac{3}{8} + 2 \left[\frac{1}{2\beta} \ln\left(\frac{1 + \beta}{1 - \beta}\right) - 1\right] \left[\ln\left(\frac{2(E_0 - E_e)}{m_e}\right) - \frac{3}{2} + \frac{1}{\beta} \frac{E_0 - E_e}{E_e}\right]
- \frac{2}{\beta} \text{Li}_2\left(\frac{2\beta}{1 + \beta}\right) + \frac{1}{\beta} \left[\ln\left(\frac{1 + \beta}{1 - \beta}\right) + 1\right] \left(\ln\left(\frac{1 + \beta}{1 - \beta}\right) + 1\right) \left(1 - \frac{1}{12}\right) \left(\frac{E_0 - E_e}{E_e}\right)^2 - \ln\left(\frac{1 + \beta}{1 - \beta}\right). \quad (10)
\]

The contribution of the electroweak–boson exchanges together with QCD corrections has been calculated by Czarnecki et al. [16] (see also [13]). This defines the the radiative corrections to the neutron lifetime, which are described by the function \(g_n(E_e) = \bar{g}_n(E_e) + C_{\text{QCD}}\), where \(C_{\text{QCD}} = 10.249\) (see Appendix D of [21]) is caused by electroweak–boson exchanges and QCD corrections [16].

The infrared divergent contribution of hadronic structure of the nucleon to the rate of the neutron radiative \(\beta^\gamma\)-decay, induced by the term \(\sum_{\lambda=1,2} e_\lambda^2(k) \cdot k_c (e_\lambda(k) \cdot k_c)/\langle k_c \cdot k\rangle\) in the rate of the neutron radiative \(\beta^\gamma\)-decay [21, 28, 51], is equal to

\[
\lambda_{\beta^\gamma}^{(\text{h.s.})}(E_0, \mu) = \frac{\alpha}{\pi} (1 + 3g_\lambda^2) \frac{|G_V|^2}{\pi^3} \int_{m_e}^{E_0} dE_e \sqrt{E_e^2 - m_e^2} E_e F(E_e, Z = 1) (E_0 - E_e)^2 \Delta \bar{g}_n^{(\text{h.s.})}(E_e, \mu). \quad (11)
\]

Thus, the contribution of hadronic structure of the nucleon to the radiative corrections of order \(O(\alpha/\pi)\) to the neutron lifetime is equal to

\[
\Delta \bar{g}_n^{(\text{h.s.})}(E_e) = - \frac{2g_\lambda^2}{1 + 3g_\lambda^2} \frac{m_e^2}{m_N^2} \left[2\ln\left(\frac{2(E_0 - E_e)}{\mu}\right) + \ln\left(\frac{1 + \beta}{1 - \beta}\right) + 1\right] + \frac{1}{\beta} \ln\left(\frac{1 + \beta}{1 - \beta}\right) - \frac{1}{4\beta} \ln^2\left(\frac{1 + \beta}{1 - \beta}\right) \right). \quad (12)
\]

Relative to Sirlin’s infrared divergent contribution Eq. (12) the order of this correction is of about \(10^{-5}\). This confirms validity and high confidence level of the contribution of hadronic structure of the nucleon to the radiative corrections of the neutron lifetime, calculated by Sirlin [3] (see Eq. (10)), who proved a factorization of strong low–energy and electromagnetic interactions to leading order in the large nucleon mass expansion and, practically, dealt with structureless neutron and proton.

The main problem of the infrared divergent correction Eq. (12) can be related only to a cancellation of such a divergence by the contribution of virtual photon exchanges, that should be similar to cancellation of the infrared divergence in Eq. (11) by the infrared divergence in Eq. (8). An impossibility to cancel such an infrared divergent correction Eq. (12), calculated to leading order in the large nucleon mass expansion, by virtual photon exchanges in the neutron \(\beta^\gamma\)-decay should testify a certain inconsistency of the approach. Following [33] we may argue that a problem of possible non–cancellation of the infrared divergence Eq. (12) by virtual photon exchanges in the neutron \(\beta^\gamma\)-decay can be related to the use of the standard \(V-A\) effective theory of weak interactions. The point is that the effective \(V-A\) vertex of nucleon–lepton current–current interaction, accounting for both baryonic and mesonic currents coupled to leptonic current, is not the vertex of the combined quantum field theory including QED and LeM or any other theory of strong low–energy interactions. For correct account for contributions of hadronic structure of the nucleon to radiative corrections to the neutron \(\beta^\gamma\)-decays one has to use a combined quantum field theory including the Standard Electroweak Model (SEM) and a theory of strong low–energy interactions (e.g. the LeM). However, first of all in our forthcoming publication we are planning to perform an analysis of cancellation of the divergent correction Eq. (14) by virtual photon exchanges in the neutron \(\beta^\gamma\)-decay within the combined quantum field theory described in [33] and discussed in this paper.

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