Particle Mixing and CP-Violation

Fayyazuddin

National Centre for Physics and Physics department, Quaid-i-Azam University, 45320
Islamabad, Pakistan
E-mail: fayyazuddins@gmail.com

Abstract: In this review, the $X^0 - \overline{X}^0$ mixing ($X^0 = B^0, B_s^0, K^0$) and its implication for CP violation in the standard model are discussed. Both direct and mixing induced CP violation for $K^0(\overline{K}^0)$, $B^0(\overline{B}^0)$ and $B_s^0(\overline{B}_s^0)$ are reviewed.

Keywords: CP violation, Particle Mixing, Flavor Physics
1. Introduction

Symmetries have played an important role in particle physics. In quantum mechanics a symmetry is associated with a group of transformations under which a Lagrangian remains invariant. Symmetries limit the possible terms in a Lagrangian and are associated with conservation laws. Here we will be concerned with the role of discrete symmetries: Space Reflection (Parity) $P: \vec{x} \to -\vec{x}$, Time Reversal $T: t \to -t$ and Charge Conjugation $C$: particle $\to$ antiparticle.

Quantum Electrodynamics (QED) and Quantum Chromodynamics (QCD) respect all these symmetries. Also, all Lorentz invariant local quantum field theories are $CPT$ invariant. However, in weak interactions $C$ and $P$ are maximally violated.

First indication of parity violation was revealed in the decay of a particle with spin parity $J^P = 0^-$, called $K$-meson into two modes $K^0 \to \pi^+\pi^-$ (parity violating), and $K^0 \to \pi^+\pi^-\pi^0$(parity conserving).

Lee and Yang in 1956, suggested that there is no experimental evidence for parity conservation in weak interaction. They suggested number of experiments to test the validity of space reflection invariance in weak decays. One way to test this is to measure the helicity of outgoing muon in the decay:

$$\pi^+ \to \mu^+ + \nu_\mu$$
The helicity of muon comes out to be negative, showing that parity conservation does not hold in this decay. In the rest frame of the pion, since $\mu^+$ comes out with negative helicity, the neutrino must also come out with negative helicity because of the spin conservation. Thus confirming the fact that neutrino is left handed.

$$\pi^+ \rightarrow \mu^+(-) + \nu_\mu$$

Under charge conjugation,

$$\pi^+ C \rightarrow \pi^- \quad \mu^+ C \rightarrow \mu^- \quad \nu_\mu C \rightarrow \bar{\nu}_\mu$$

Helicity $H = \frac{\vec{p} \cdot \vec{p}}{|\vec{p}|}$ under $C$ and $P$ transforms as,

$$H \overset{C}{\rightarrow} H, \quad H \overset{P}{\rightarrow} -H$$

Invariance under $C$ gives,

$$\Gamma_{\pi^+ \rightarrow \mu^+(-)\nu_\mu} = \Gamma_{\pi^- \rightarrow \mu^-(+)\bar{\nu}_\mu}$$

Experimentally,

$$\Gamma_{\pi^+ \rightarrow \mu^+(-)\nu_\mu} \gg \Gamma_{\pi^- \rightarrow \mu^-(+)\bar{\nu}_\mu}$$

showing that $C$ is also violated in weak interactions. However, under $CP$,

$$\Gamma_{\pi^+ \rightarrow \mu^+(-)\nu_\mu} \overset{CP}{\rightarrow} \Gamma_{\pi^- \rightarrow \mu^-(+)\bar{\nu}_\mu}$$

which is seen experimentally. Thus, $CP$ conservation holds for this decay.

The $CP$ violation in weak interaction is not universal, does not embrace all weak processes unlike $C$ and $P$ violation. The $C$ and $P$ violation is incorporated in the basic structure of theory by assigning the left-handed and the right-handed fermions to doublet and singlet representations of the electroweak group $SU_L(2) \times U_Y(1)$

$$\psi_q = \begin{pmatrix} u_i \\ d'_i \\ \bar{s}'_i \\ \bar{b}'_i \end{pmatrix}_L; \quad Y = \frac{1}{3}$$

$$u_{iR} : \quad Y = \frac{4}{3}$$

$$d_{iR} : \quad Y = -\frac{2}{3}$$

$$\psi_l = \begin{pmatrix} \nu_{e^-} \\ \bar{e}_i \end{pmatrix}; \quad Y = -1$$

$$e_{iR} : \quad Y = 2$$

Here $i$ is the generation index. The weak eigenstates $d'$, $s'$ and $b'$ are different from the mass eigenstates $d$, $s$ and $b$. They are related to each other by a unitarity transformation,

$$\begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} = V \begin{pmatrix} d \\ s \\ b \end{pmatrix}$$
where $V$ is called the CKM matrix.

$$V = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$

$$\simeq \begin{pmatrix} 1 - \frac{1}{2} \lambda^2 & \lambda & A \lambda^3 (\rho - i \eta) \\ -\lambda & 1 - \frac{1}{2} \lambda^2 & A \lambda^2 \\ A \lambda^3 (1 - \rho - i \eta) & -A \lambda^2 & 1 \end{pmatrix} + O(\lambda^4), \quad \lambda = 0.22$$

(1.2)

With three generations of quarks, there is one independent weak phase as reflected with non zero value of $\eta$. The unitarity of $V$, $[\text{Fig.1}] \ V V^\dagger = 1$ gives

$$V_{ud} V_{ub} + V_{cb} V_{cd} + V_{td} V_{tb} = 0$$

(1.3)

The second line in equation (1.2) expresses $V$ in terms of Wolfenstein parametrization. Thus,

$$V_{cb} = A \lambda^2$$
$$V_{ub} = |V_{ub}| e^{-i \gamma}$$
$$V_{td} = |V_{td}| e^{-i \beta}$$

where,

$$\tan \gamma = \frac{\eta}{\bar{\rho}} = \frac{\bar{\eta}}{\bar{\rho}}, \quad \tan \beta = \frac{\bar{\eta}}{1 - \bar{\rho}}, \quad \bar{\rho} = \rho(1 - \frac{\lambda^2}{2}), \quad \bar{\eta} = \eta(1 - \frac{\lambda^2}{2}).$$

The weak angles $\beta$ and $\gamma$ play a leading role in $CP$ violation. However these weak angles are in $V_{ub}$ and $V_{td}$, which connect the first generation with the third generation. Hence the role of $\beta$ and $\gamma$ in $K$ and $D$ decays is perepheral as both $K$ and $D$ are bound states of the first and second generation quarks.

The current in the standard model: $\bar{\Psi}_i \gamma^\mu (1 - \gamma^5) \Psi_j$, under $CP$ and time reversal transforms as

$$\bar{\Psi}_i \gamma^\mu (1 - \gamma^5) \Psi_j \xrightarrow{CP} -\eta (\mu) \bar{\Psi}_j \gamma^\mu (1 - \gamma^5) \Psi_i \xrightarrow{T} \eta (\mu) \bar{\Psi}_i \gamma^\mu (1 - \gamma^5) \Psi_j$$

where,

$$\eta (\mu) = \begin{cases} +1, & \text{if } \mu = 0 \\ -1, & \text{if } \mu = 1, 2, 3 \end{cases}$$

The Lagrangian,

$$\mathcal{L}_W = \bar{\Psi}_i \gamma^\mu (1 - \gamma^5) \Psi_j W_\mu^+ + \text{h.c.} \xrightarrow{CP} \bar{\Psi}_j \gamma^\mu (1 - \gamma^5) \Psi_i W_\mu^- + \text{h.c.} \xrightarrow{T} \bar{\Psi}_i \gamma^\mu (1 - \gamma^5) \Psi_j W_\mu^- + \text{h.c.}$$
where

$$W^\pm \xrightarrow{CP} -\eta(\mu) W^\mp, \ W^\pm \xrightarrow{T} \eta(\mu) W^\pm$$

Note that under $CP$, $\bar{\Psi}_i \gamma^\mu (1 - \gamma^5) \Psi_j$ goes over to its Hermitian conjugate. The flavor changing part of the current is the charged current and contains the CKM matrix. The flavor non changing part of the current is the neutral current, $CP$ violation is not possible in weak processes involving neutral currents. Similarly in the process involving the lepton current $\bar{\Psi}_i \gamma^\mu (1 - \gamma^5) \Psi_j^I$, $CP$ is conserved.

The following comments are in order. The quarks are basic constituent of hadrons. Each quark has a definite charge, definite mass and definite flavor. In a weak process, hadronic flavor changes. Hence the weak eigen states can be a mixture of mass eigenstates. With three generations of quarks, the weak eigenstates are related to the mass eigenstates by CKM matrix. The CKM matrix also takes care of the experimental fact, the suppression of weak processes from one generation to other.

The mismatch between the weak and mass eigenstates, involve the weak phase in the CKM matrix. This can be a source of $CP$-violation in flavor changing weak processes in the standard model.

In the standard model, the lepton number and the baryon number are conserved $\tau_e > 4.6 \times 10^{26} \text{ yr}$, $\tau_p > 10^{31} \text{ yr}$. However for leptons, there is another conservation law: the lepton number for each generation is separately conserved i.e. not only $\Delta L = 0$, but $\Delta L_e = 0$, $\Delta L_\mu = 0$, $\Delta L_\tau = 0$. For purely leptonic process, the limit on flavor changing processes $\mu \rightarrow e\gamma$, $\tau \rightarrow \mu(e)\gamma$ is $\Gamma(\mu \rightarrow e\gamma)/\Gamma(\mu \rightarrow all) < 1.2 \times 10^{-11}$. Hence for charged leptons, there is a stringent limit on flavor changing processes $\Delta L_e \neq 0$, $\Delta L_\mu \neq 0$ and $\Delta L_\tau \neq 0$. Thus for charged leptons, there is no difference between weak and mass eigenstates. However for neutrios separate conservation of lepton number is not required, as a consequence neutrino mixing analogous to CKM quark mixing is possible. This results in neutrino oscillations. Neutrinos are stable particles, the neutrino mixing plays no role in $CP$ violation. The $CP$ violation in lepton sector will violate lepton number. Hence $CP$ violation in lepton sector falls in the same catagory as $CP$ violation required for baryogenesis along with $\Delta B \neq 0$. 
Under charge conjugation, a particle is transformed to its antiparticle. Since in the standard model, the electric charge, baryon and lepton number are conserved; hence for $CP$ eigenstates formed from the states with $Q = 0$, $B = 0$, $L = 0$, $\Delta B = 0 = \Delta L$. However for neutrinos, and neutral baryons, $CP$ eigenstates would have $\Delta B = 2$ and $\Delta L = 2$, hence not allowed in the standard model. Thus only $CP$-eigenstates for neutral mesons viz $X^0 \equiv (K^0, D^0, B^0, B^0_s)$ are allowed. Now under $CP$:

$$|X^0\rangle \xrightarrow{CP} \eta_X^{CP} \bar{X}^0\rangle$$

where $\eta_X^{CP}$ is the $CP$-phase. We select $\eta_X^{CP} = -1$, with this convention, the $CP$-eigenstates are

$$|X^0_{1,2}\rangle = \frac{1}{\sqrt{2}} \left[ |X^0\rangle \mp |\bar{X}^0\rangle \right], \quad CP = \pm 1$$

In the weak interaction, both hypercharge and isospin are violated, so only $CP$-eigenstates can be mass eigenstates when weak interaction Hamiltonian is included in the Hamiltonian. When weak interaction is switched off; the mass eigenstates are $|X^0\rangle$ and $|\bar{X}^0\rangle$, with same mass and same lifetime, a consequence of CPT theorem. To summarize

- For $X^0 - \bar{X}^0$ complex ($X^0 = K^0, B^0, B^0_s$); the mass matrix is not diagonal in $|X^0\rangle$ and $|\bar{X}^0\rangle$ basis.
- However, assuming $CP$ conservation, the $CP$ eigenstates $|X^0_{1}\rangle$ and $|X^0_{2}\rangle$ can be mass eigenstates and hence mass matrix is diagonal in this basis.
- The two sets of states are related to each other by superposition principle of quantum mechanics.
- This gives rise to quantum mechanical interference so that even if we start with a state $|X^0\rangle$, the time evolution of this state can generate $|\bar{X}^0\rangle$. This is a source of mixing induced $CP$ violation.
- However, both in $K^0 - \bar{K}^0$ and $B^0 - \bar{B}^0$ complex, the mass eigenstates $|K^0_{S}\rangle, |K^0_{L}\rangle$ and $|B^0_{S}\rangle, |B^0_{L}\rangle$ are not $CP$ eigenstates.
- In the case of $K^0 - \bar{K}^0$ complex,

$$|K^0_{S}\rangle = |K^0_{1}\rangle + \epsilon |K^0_{2}\rangle$$

$$|K^0_{L}\rangle = |K^0_{2}\rangle + \epsilon |K^0_{1}\rangle$$

there is a small admixture of wrong $CP$ state characterized by small parameter $\epsilon$, which gives rise to the $CP$ violating decay $K^0_L \rightarrow \pi^+\pi^-$. This was the first $CP$ violating decay observed experimentally.
- For $D^0 - \bar{D}^0$ complex, there is no mismatch between $CP$ eigenstates $|D^0_{1}\rangle, |D^0_{2}\rangle$ and mass eigen states; $|D^0\rangle, |\bar{D}^0\rangle$ are bound states of $1^{st}$ and $2^{nd}$ generations of quarks-antiquarks.
• For $B^0 - \overline{B}^0$ complex, the mismatch between mass eigenstates and $CP$ eigenstates $|B^0_L\rangle$ and $|B^0_H\rangle$ is given by the phase factor $e^{2i\phi_M}$ where the phase factor $\phi_M = -\beta$ in the standard model viz. one of the angle in the CKM matrix.

$$|B^0_L\rangle = \frac{1}{\sqrt{2}} \left[ |B^0\rangle - e^{2i\phi_M} |\overline{B}^0\rangle \right]$$

$$|B^0_H\rangle = \frac{1}{\sqrt{2}} \left[ |B^0\rangle + e^{2i\phi_M} |\overline{B}^0\rangle \right]$$

• For $B^0_s - \overline{B}^0_s$, there is no mismatch between $CP$ eigenstates $|B^0_{1s}\rangle$ and $|B^0_{2s}\rangle$ and the mass eigenstates. With three generations of quarks, no extra phase is available to generate mismatch between mass and $CP$ eigensates for $B^0_s - \overline{B}^0_s$ complex.

• The quantum mechanical interference gives rise to non zero mass difference $\Delta m_K$, $\Delta m_B$ and $\Delta m_{B_s}$ between mass eigenstates. The mixing induced $CP$ violation involves these mass differences.

2. $CPT$ and $CP$ invariance

It is instructive to discuss the restrictions imposed by $CPT$ invariance. $CPT$ invariance implies,

$$\text{out} \langle f | \mathcal{L} | X \rangle = \text{out} \left\langle f \right| (CPT)^{-1} \mathcal{L} CPT \left| X \right\rangle$$

$$= \eta_T^X \eta_T^f \text{in} \left\langle \tilde{f} \right| (CPT)^{-1} \mathcal{L} (CPT)^{-1} \left| \tilde{X} \right\rangle^*$$

$$= \eta_T^X \eta_T^f \left\langle \tilde{X} \right| (CPT)^{-1} \mathcal{L} (CPT) \left| \tilde{f} \right\rangle \text{in}$$

where $\tilde{\cdot}$ means momentum and spin of the final state are reversed; $\tilde{\cdot}$ may be dropped. Further, we may choose the $CP$ phase such that

$$CP |X\rangle = - |\tilde{X}\rangle \quad (2.1)$$

$$CP |f\rangle = \eta^C_P |\tilde{f}\rangle \quad (2.2)$$

Thus we have

$$\text{out} \langle f | \mathcal{L} | X \rangle = \eta_f \left\langle \overline{X} \right| \mathcal{L} |\tilde{f}\rangle \text{in} \quad (2.3)$$

where

$$\eta_f = -\eta^C_P \eta_T^X \eta_T^f \quad (2.4)$$

Hence on using

$$|f\rangle_{\text{in}} = S_f |f\rangle_{\text{out}} = \exp(2i\delta_f) |f\rangle_{\text{in}} \quad (2.5)$$

we get

$$\text{out} \langle f | \mathcal{L} | X \rangle = \eta_f e^{2i\delta_f} \left\langle \overline{X} \right| \mathcal{L} |\tilde{f}\rangle_{\text{out}} \quad (2.6)$$

$$= \eta_f e^{2i\delta_f} \eta_T^f \left\langle \tilde{f} \right| \mathcal{L} |\tilde{X}\rangle^* \quad (2.7)$$
Hence finally we have
\[ \bar{A}_f = \eta_f e^{2i\delta_f} A_f^* \quad (2.8) \]
If CP-invariance holds, then,
\[ \text{out} \langle f|\mathcal{L}|X\rangle = \text{out} \langle \bar{f}|\mathcal{L}|\bar{X}\rangle \Rightarrow \bar{A}_f = A_f. \quad (2.9) \]
Thus, the necessary condition for CP-violation is that the decay amplitude \( A \) should be complex. In view of our discussion, under CP an operator \( O(\vec{x},t) \) is replaced by,
\[ O(\vec{x},t) \rightarrow O(\vec{x},t) \quad (2.10) \]
and under time reversal
\[ O(\vec{x},t) \rightarrow O(\vec{x},-t) \quad (2.11) \]
The effective Lagrangian has the structure \( (\mathcal{L}^\dagger = \mathcal{L}) \),
\[ \mathcal{L} = aO + a^*O^\dagger \quad (2.12) \]
CPT gives
\[ \bar{A}_f = \eta_f e^{2i\delta_f} A_f^* = \eta_f e^{-i\phi} e^{i\delta_f} |A_f| \]
For direct CP violation, at least two amplitudes with different weak phases are required
\[ A_f = A_{1f} + A_{2f} \quad (2.13) \]
CPT gives
\[ \bar{A}_f = e^{2i\delta_f} A_{1f}^* + e^{2i\delta_f} A_{2f}^* \]
\[ A_{if} = e^{i\phi} e^{i\delta_f} |A_{if}| \]
where \( (\delta_{1f}, \delta_{2f}), (\phi_1, \phi_2) \) are strong final state phases and the weak phases respectively. Thus the direct CP violation is given by
\[ A_{CP} = \frac{\Gamma(\vec{X} \rightarrow \vec{f}) - \Gamma(\vec{X} \rightarrow f)}{\Gamma(\vec{X} \rightarrow \vec{f}) + \Gamma(\vec{X} \rightarrow f)} = \frac{2 |A_{1f}| |A_{2f}| \sin \phi \sin \delta_f}{|A_{1f}|^2 + |A_{2f}|^2 + 2 |A_{1f}| |A_{2f}| \cos \phi \cos \delta_f} \quad (2.14) \]
where \( \delta_f = \delta_{2f} - \delta_{1f}, \phi = \phi_1 - \phi_2 \). Hence the necessary condition for non zero direct CP violation is \( \delta_f \neq 0 \) and \( \phi \neq 0 \). The weak phase may be a consequence of phase in CKM matrix.
3. Particle Mixing

In \(|X^0⟩ - |\bar{X}^0⟩\) basis,

\[
|\psi(t)⟩ = a(t)|X^0⟩ + \bar{a}(t)|\bar{X}^0⟩
\]

\[
\frac{idt}{dt}|\psi(t)⟩ = M|\psi(t)⟩
\]

The mass matrix \(M\) is not diagonal and is given by,

\[
M = m - \frac{i}{2}\Gamma = \begin{pmatrix}
    m_{11} - \frac{i}{2}\Gamma_{11} & m_{12} - \frac{i}{2}\Gamma_{12} \\
    m_{21} - \frac{i}{2}\Gamma_{21} & m_{22} - \frac{i}{2}\Gamma_{22}
\end{pmatrix}
\]

(3.1)

Hermiticity of matrices \(m_{\alpha\alpha'}\) and \(\Gamma_{\alpha\alpha'}\) gives \((\alpha = \alpha' = 1, 2)\),

\[
(m)_{\alpha\alpha'} = (m^\dagger)_{\alpha'\alpha} = (m^*)_{\alpha'\alpha}, \quad \Gamma_{\alpha\alpha'} = \Gamma^*_{\alpha'\alpha}
\]

\[
m_{21} = m^*_{12} \quad \Gamma_{21} = \Gamma^*_{12}
\]

(3.2)

CPT invariance gives,

\[
\langle X^0 | M | X^0 \rangle = \langle \bar{X}^0 | M | \bar{X}^0 \rangle
\]

\[
m_{11} = m_{22}, \quad \Gamma_{11} = \Gamma_{22}
\]

\[
\langle \bar{X}^0 | M | X^0 \rangle = \langle X^0 | M | X^0 \rangle: \text{identity}
\]

(3.3)

Diagonalization of mass matrix \(M\) in eq. (3.1) gives,

\[
m_{11} - \frac{i}{2}\Gamma_{11} - pq = m_1 - \frac{i}{2}\Gamma_1
\]

\[
m_{11} - \frac{i}{2}\Gamma_{11} + pq = m_2 - \frac{i}{2}\Gamma_2
\]

(3.4)

where,

\[
p^2 = m_{12} - \frac{i}{2}\Gamma_{12}, \quad q^2 = m^*_{12} - \frac{i}{2}\Gamma^*_{12}
\]

(3.5)

The eigenstates are given by,

\[
|X_{1,2}⟩ = \frac{1}{\sqrt{|p|^2 + |q|^2}} [p|X^0⟩ \mp q|\bar{X}^0⟩]
\]

(3.6)

From Eq (3.4), taking the real and imaginary parts, we have

\[
m_1 = m_{11} - \text{Re}pq
\]

\[
m_2 = m_{11} - \text{Re}pq
\]

\[
\Gamma_1 = \Gamma_{11} + 2\text{Im}pq
\]

\[
\Gamma_2 = \Gamma_{11} - 2\text{Im}pq
\]
Thus finally we have

\[ \Delta m = m_2 - m_1 = 2Re \rho \]
\[ m = \frac{m_1 + m_2}{2} = m_{11} \]
\[ \Delta \Gamma = \Gamma_2 - \Gamma_1 = -4Im \rho \]
\[ \Gamma = \frac{\Gamma_1 + \Gamma_2}{2} = \Gamma_{11} \]  

(3.7)

Let us define

\[ q/p = \frac{1 - \epsilon}{1 + \epsilon} \]
\[ = \sqrt{\frac{m_{12} - \frac{i}{2} \Gamma_{12}}{m_{12} - \frac{i}{2} \Gamma_{12}}} \]  

(3.8)

It follows that CP-violation is determined by the parameter

\[ \epsilon = \frac{p - q}{p + q} \]  

(3.9)

Now \( |X_1\rangle \) and \( |X_2\rangle \) are mass eigenstates. They form a complete set (in units \( \hbar = c = 1 \)),

\[ |\psi(t)\rangle = a(t) |X_1\rangle + b(t) |X_2\rangle \]
\[ i \frac{d|\psi(t)\rangle}{dt} = \left( \begin{array}{cc} m_1 - \frac{i}{2} \Gamma_1 & 0 \\ 0 & m_2 - \frac{i}{2} \Gamma_2 \end{array} \right) |\psi(t)\rangle. \]  

(3.10)

The solution is,

\[ a(t) = a(0) \exp \left( -im_1 t - \frac{1}{2} \Gamma_1 t \right) \]
\[ b(t) = b(0) \exp \left( -im_2 t - \frac{1}{2} \Gamma_2 t \right) \]

Suppose we start with the state \( |X^0\rangle \), i.e.,

\[ |\psi(0)\rangle = |X^0\rangle \]

After time \( t \)

\[ |\psi(t)\rangle = \frac{\sqrt{|p|^2 + |q|^2}}{2p} \left[ \exp \left( -im_1 t - \frac{1}{2} \Gamma_1 t \right) |X_1\rangle + \exp \left( -im_2 t - \frac{1}{2} \Gamma_2 t \right) |X_2\rangle \right] \]
\[ = \frac{1}{2} \left\{ \left[ \exp \left( -im_1 t - \frac{1}{2} \Gamma_1 t \right) + \exp \left( -im_2 t - \frac{1}{2} \Gamma_2 t \right) \right] |X^0\rangle \right. \]
\[ \left. - \frac{q}{p} \left[ \exp \left( -im_1 t - \frac{1}{2} \Gamma_1 t \right) - \exp \left( -im_2 t - \frac{1}{2} \Gamma_2 t \right) \right] |\bar{X}^0\rangle \right\} \]  

(3.11)

Eq (3.11) clearly shows the particle mixing. Similarly if we start with \( |\bar{X}^0\rangle \) we get after time \( t \)

\[ |\psi(t)\rangle = \frac{1}{2} \left\{ \frac{p}{q} \left[ \exp \left( -im_1 t - \frac{1}{2} \Gamma_1 t \right) - \exp \left( -im_2 t - \frac{1}{2} \Gamma_2 t \right) \right] |X^0\rangle \right. \]
\[ \left. - \left[ \exp \left( -im_1 t - \frac{1}{2} \Gamma_1 t \right) + \exp \left( -im_2 t - \frac{1}{2} \Gamma_2 t \right) \right] |\bar{X}^0\rangle \right\} \]  

(3.12)
From Eqs. (3.11) and (3.12), we can determine $X^0$ and $\bar{X}^0$ mixing. It is clear that if we start with $X^0$, then at time $t$, the probability of finding the particles $X^0$ or $\bar{X}^0$ is given by [using Eq (3.11)]

$$\left|\langle X^0 | \psi(t) \rangle \right|^2 = \frac{1}{4} \left[ e^{-\Gamma_1 t} + e^{-\Gamma_2 t} + 2e^{-\Gamma t} \cos \Delta m t \right]$$

(3.13)

$$\left|\langle \bar{X}^0 | \psi(t) \rangle \right|^2 = \frac{1}{4} \left| \frac{1 - \epsilon}{1 + \epsilon} \right|^2 e^{-\Gamma_1 t} + e^{-\Gamma_2 t} - 2e^{-\Gamma t} \cos \Delta m t$$

(3.14)

We define the mixing parameter $r$ as

$$r = \frac{\int_0^T \left| \langle X^0 | \psi(t) \rangle \right|^2 dt}{\int_0^T \left| \langle X^0 | \psi(t) \rangle \right|^2 dt}$$

(3.15)

where $T$ is a sufficiently long time. In the limit $T \to \infty$, using Eqs (3.13) and (3.14), we get

$$r = \left| \frac{1 - \epsilon}{1 + \epsilon} \right|^2 \frac{x^2 + y^2}{2 + x^2 - y^2}$$

(3.16)

where $x = \frac{\Delta m}{\Gamma}$ and $y = \frac{\Delta \Gamma}{\Gamma}$. If we start with $\bar{X}^0$, we can use Eq (3.12) then

$$\bar{r} = \frac{\int_0^T \left| \langle X^0 | \psi(t) \rangle \right|^2 dt}{\int_0^T \left| \langle X^0 | \psi(t) \rangle \right|^2 dt} \xrightarrow{T \to \infty} \frac{1 + \epsilon}{1 - \epsilon} \left| \frac{x^2 + y^2}{2 + x^2 - y^2} \right|$$

(3.17)

When $CP$-violation effects are neglected, then

$$r = \bar{r} = \frac{x^2 + y^2}{2 + x^2 - y^2}$$

(3.18)

The asymmetry parameter $a$

$$a = \frac{\bar{r} - r}{\bar{r} + r} = \frac{4 \text{Re} \epsilon}{1 + |\epsilon|^2}$$

(3.19)

is a measure of $CP$-violation. We define another parameter which is also a measure of particle mixing. Let $\chi$ be the probability $X^0 \to \bar{X}^0$, then

$$\chi = \int_0^T \left| \langle \bar{X}^0 | \psi(t) \rangle \right|^2 dt$$

$$1 - \chi = \int_0^T \left| \langle X^0 | \psi(t) \rangle \right|^2 dt$$

Thus

$$r = \frac{\chi}{1 - \chi}, \chi = \frac{r}{1 + r}$$

(3.20)

Similarly, we get

$$\bar{r} = \frac{\bar{\chi}}{1 - \bar{\chi}}, \bar{\chi} = \frac{\bar{r}}{1 + \bar{r}}$$

(3.21)
We note from definitions, \( x = \Delta m / \Gamma \), \( y = \Delta \Gamma / \Gamma \)

\[
0 \leq x^2 \leq \infty \\
0 \leq y^2 \leq 1
\]

obviously

\[
0 \leq r \leq 1
\]

### 4. \( K^0 - \bar{K}^0 \) Complex and \( CP \)-Violation in \( K \)-Decay

Since,

\[
CP (\pi^+ \pi^-) = (-1)^l (-1)^l = 1
\]

therefore, it is clear that,

\[
K_1^0 \rightarrow \pi^+ \pi^-
\]

is allowed by \( CP \) conservation.

However, experimentally it was found that long lived \( K_2^0 \) also decay to \( \pi^+ \pi^- \) but with very small probability. Small \( CP \) non conservation can be taken into account by defining,

\[
|K_S\rangle = |K_1^0\rangle + \varepsilon |K_2^0\rangle \\
|K_L\rangle = |K_2^0\rangle + \varepsilon |K_1^0\rangle
\]  

where \( \varepsilon \) is a small number. Thus \( CP \) non conservation manifests itself by the ratio:

\[
\eta_{+-} = \frac{A(K_L \rightarrow \pi^+ \pi^-)}{A(K_S \rightarrow \pi^+ \pi^-)} = \varepsilon
\]

\[
|\eta_{+-}| \approx (2.286 \pm 0.017) \times 10^{-3}
\]

Now \( CP \) non conservation implies,

\[
m_{12} \neq m_{12}^*, \quad \Gamma_{12} \neq \Gamma_{12}^*
\]  

Since \( CP \) violation is a small effect, therefore,

\[
\text{Im}m_{12} \ll \text{Re}m_{12} \quad \text{Im}\Gamma_{12} \ll \text{Re}\Gamma_{12}.
\]

Further, if \( CP \)- violation arises from mass matrix, then,

\[
\Gamma_{12} = \Gamma_{12}^*
\]

For small \( \varepsilon \)

\[
\frac{q^2}{p^2} = 1 - 4\varepsilon \\
4\varepsilon = \frac{p^2 - q^2}{p^2}
\]
Now
\[ p^2, q^2 \approx \left( \text{Re} m_{12} - \frac{i}{2} \Gamma_{12} \right) \left[ 1 \pm \frac{i \text{Im} m_{12}}{\text{Re} m_{12} - \frac{i}{2} \Gamma_{12}} \right] \]

Hence
\[ e = \frac{i \text{Im} m_{12}}{2 \left( \text{Re} m_{12} - \frac{i}{2} \Gamma_{12} \right)} = \frac{i \text{Im} m_{12}}{(m_2 - m_1) - i (\Gamma_2 - \Gamma_1)} \quad (4.7) \]

Then from Eq. (3.7) up to first order, we get,
\[ \Delta m = m_2 - m_1 \rightarrow m_{KL} - m_{KS} \]
\[ = 2 \text{Re} m_{12} \]
\[ \Delta \Gamma = \Gamma_2 - \Gamma_1 = \Gamma_L - \Gamma_S = 2 \Gamma_{12} \quad (4.8) \]

Now,
\[ \Delta m = m_L - m_S \]
\[ \Delta \Gamma = \Gamma_L - \Gamma_S \]
\[ \Gamma_S = \frac{\hbar}{\tau_S} = 7.367 \times 10^{-12} \text{ MeV}, \]
\[ \tau_S = (0.8935 \pm 0.0008) \times 10^{-10} \text{ s} \]
\[ \Gamma_L = \frac{\hbar}{\tau_L} = 1.273 \times 10^{-14} \text{ MeV}, \]
\[ \tau_L = (5.17 \pm 0.04) \times 10^{-8} \text{ s} \]
\[ \Delta \Gamma \simeq -\Gamma_S \]
\[ m_L = m + \frac{1}{2} \Delta m \]
\[ m_S = m - \frac{1}{2} \Delta m \quad (4.9) \]

Hence from Eq. (3.11),
\[ |\psi(t)\rangle \approx \frac{1}{2} e^{\frac{i}{2} \Delta m t} \left\{ \left[ e^{\frac{1}{2} \Gamma_S t} e^{\frac{i}{2} \Delta m t} + e^{-\frac{i}{2} \Delta m t} \right] |K^0\rangle \right\} - \left\{ \left[ e^{\frac{1}{2} \Gamma_S t} e^{\frac{i}{2} \Delta m t} - e^{-\frac{i}{2} \Delta m t} \right] |\bar{K}^0\rangle \right\} \quad (4.10) \]

Therefore, probability of finding $\bar{K}^0$ at time $t$ (recall that we started with $K^0$),
\[ P (K^0 \rightarrow \bar{K}^0, t) = |\langle \bar{K}^0 | \psi(t) \rangle|^2 \]
\[ = \frac{1}{4} \left( 1 + e^{-\Gamma_S t} - 2 e^{-\frac{1}{2} \Gamma_S t} \cos(\Delta m t) \right) \]
\[ = \frac{1}{4} \left( 1 + e^{-t/\tau_S} - 2 e^{-\frac{1}{2} t/\tau_S} \cos(\Delta m t) \right) \quad (4.11) \]
If kaons were stable \((\tau_S \to \infty)\), then,

\[
P(K^0 \to \bar{K}^0, t) = \frac{1}{2} [1 - \cos(\Delta m t)] \tag{4.12}
\]

which shows that a state produced as pure \(Y = 1\) state at \(t = 0\) continuously oscillates between \(Y = 1\) and \(Y = -1\) state with frequency \(\omega = \frac{\Delta m}{\hbar}\) and period of oscillation,

\[
\tau = \frac{2\pi}{(\Delta m / \hbar)}. \tag{4.13}
\]

Kaons, however, decay and their oscillations are damped. By measuring the period of oscillation, \(\Delta m\) can be determined.

\[
\Delta m = m_L - m_S = (3.483 \pm 0.006) \times 10^{-12} \text{ MeV.} \tag{4.14}
\]

\[
= 0.47 \Gamma_S \tag{4.15}
\]

Such a small number is measured as a consequence of superposition principle of quantum mechanics.

Coming back to \(CP\)-violation,

\[
\varepsilon = \frac{i \text{Im} m_{12}}{\Delta m - i \Delta \Gamma / 2} \quad \varepsilon = |\varepsilon| e^{i \phi_\varepsilon} \tag{4.16}
\]

\[
\tan \phi_\varepsilon = -2 \Delta m / \Delta \Gamma = \Delta m / \Gamma_S - \Gamma_L \approx 2 \times 0.474 \Gamma_S / 0.998 \Gamma_S \Rightarrow \phi_\varepsilon = (43.51 \pm 0.05)^\circ \tag{4.17}
\]

\[
|\varepsilon| = (2.228 \pm 0.011) \times 10^{-3} \tag{4.18}
\]

So far we have considered \(CP\)-violation due to mixing in the mass matrix. It is important to detect the \(CP\)-violation in the decay amplitude if any. This is done by looking for a difference between \(CP\)-violation for the final \(\pi^0\pi^0\) and \(\pi^+\pi^-\) states. Now due to Bose statistics, the two pions can be either in \(I = 0\) or \(I = 2\) states. Using Clebsch-Gordon (CG) coefficients,

\[
A(K^0 \to \pi^+\pi^-) = \frac{1}{\sqrt{3}} \left[ \sqrt{2} A_0 e^{i\delta_0} + A_2 e^{i\delta_2} \right] \tag{4.19}
\]

\[
A(K^0 \to \pi^0\pi^0) = \frac{1}{\sqrt{3}} \left[ A_0 e^{i\delta_0} - \sqrt{2} A_2 e^{i\delta_2} \right]
\]

Now \(CPT\)-invariance gives,

\[
A(\bar{K}^0 \to \pi^+\pi^-) = \frac{1}{\sqrt{3}} \left[ \sqrt{2} A_0^* e^{i\delta_0} + A_2^* e^{i\delta_2} \right] \tag{4.20}
\]

\[
A(\bar{K}^0 \to \pi^0\pi^0) = \frac{1}{\sqrt{3}} \left[ A_0^* e^{i\delta_0} - \sqrt{2} A_2^* e^{i\delta_2} \right]
\]

The dominant decay amplitude is \(A_0\) due to \(\Delta I = 1/2\) rule, \(|A_2/A_0| \simeq 1/22\). Using the Wu–Yang phase convention, we can take \(A_0\) to be real. Neglecting terms of order \(\varepsilon \text{Re} A_2^* / A_0\)
and $\varepsilon \text{Im} \frac{A_2}{A_0}$, we get,

$$
\begin{align*}
\eta_{+-} &\equiv |\eta_{+-}| e^{i\phi_{+-}} \simeq \varepsilon + \varepsilon' \\
\eta_{00} &\equiv |\eta_{00}| e^{i\phi_{00}} \simeq \varepsilon - 2\varepsilon'
\end{align*}
$$

(4.21)

where,

$$
\varepsilon' = \frac{i}{\sqrt{2}} e^{i(\delta_2 - \delta_0)} \text{Im} \frac{A_2}{A_0}
$$

(4.22)

Clearly $\varepsilon'$ measures the CP-violation in the decay amplitude, since $CP$-invariance implies $A_2$ to be real.

After 35 years of experiments at Fermilab and CERN, results have converged on a definitive non-zero result for $\varepsilon'$,

$$
R = \left| \frac{\eta_{00}}{\eta_{+-}} \right|^2 = \frac{|\varepsilon - 2\varepsilon'|^2}{|\varepsilon + \varepsilon'|}, \quad \varepsilon' \ll \varepsilon
$$

$$
\text{Re} \left( \varepsilon'/\varepsilon \right) = 1 - \frac{R}{6} \simeq 1 - 6 \text{Re} \left( \varepsilon'/\varepsilon \right)
$$

(4.23)

$$
(1.65 \pm 0.26) \times 10^{-3}.
$$

(4.24)

$$
\sin(\phi_{00} - \phi_{+-}) = 3 \text{Re} (\varepsilon'/\varepsilon) \tan(\phi_\varepsilon - \phi_{\varepsilon'})
$$

(4.25)

This is an evidence that although $\varepsilon'$ is a very small, but $CP$-violation does occur in the decay amplitude. Further we note from Eq. (4.22),

$$
\phi_{\varepsilon'} = \delta_2 - \delta_0 + \frac{\pi}{2} \approx 42.3 \pm 15^0
$$

where numerical value is based on an analysis of $\pi\pi$ scattering. The experimental values of $CP$-violation parameters are as follows

$$
|\eta_{+-}| = (2.233 \pm 0.010) \times 10^{-3}
$$

$$
|\eta_{00}| = (2.222 \pm 0.010) \times 10^{-3}
$$

(4.26)

$$
\phi_{+-} = (43.4 \pm 0.007)^\circ
$$

$$
\phi_{00} = (43.7 \pm 0.008)^\circ
$$

(4.27)

$CPT$ invariance predicts (Cf Eq. 4.25)

$$
\phi_{00} - \phi_{+-} \approx 3 \text{Re} (\varepsilon'/\varepsilon) (\phi_\varepsilon - \phi_{\varepsilon'}) \approx 0
$$

(4.28)
We now discuss the CP-asymmetry in leptonic decays of kaon.

\[
\frac{\Delta S}{\Delta Q} = 1 \\
K^+ \to \pi^0 + l^+ + \nu_l \\
K^0 \to \pi^- + l^+ + \nu_l = f \\
\bar{K}^0 \to \pi^+ + l^- + \bar{\nu}_l = f^* \text{ CPT} \\
\frac{\Delta S}{\Delta Q} = -1 \\
K^0 \to \pi^+ + l^- + \bar{\nu}_l = g^* \\
\bar{K}^0 \to \pi^- + l^+ + \nu_l = g \text{ CPT}
\]

\[
A(K_L^0 \to \pi^- + l^+ + \nu_l) = \frac{1}{\sqrt{2}} [(1 + \epsilon) f + (1 - \epsilon) g] \\
A(K_L^0 \to \pi^+ + l^- + \bar{\nu}_l) = \frac{1}{\sqrt{2}} [(1 + \epsilon) g^* + (1 - \epsilon) f^*]
\]

The CP-asymmetry parameter \(\delta_l\):

\[
\delta_l = \frac{\Gamma(K_L^0 \to \pi^- + l^+ + \nu_l) - \Gamma(K_L^0 \to \pi^+ + l^- + \bar{\nu}_l) \Gamma(K_L^0 \to \pi^- + l^+ + \nu_l) + \Gamma(K_L^0 \to \pi^+ + l^- + \bar{\nu}_l)}{2 \text{Re}[|f|^2 - |g|^2] + (f g^* + f^* g) + O(\epsilon^2)}
\]

(4.29)

In the standard model \(\frac{\Delta S}{\Delta Q} = -1\) transitions are not allowed, thus \(g = 0\). Hence

\[
\delta_l \approx 2 \text{Re} \epsilon = (3.32 \pm 0.06) \times 10^{-3} \text{[Expt. value]}
\]

From Eq. (4.16), we get

\[
2 \text{Re} \epsilon = 2 |\epsilon| \cos \phi_{\epsilon}
\]

which gives on using experimental values for \(|\epsilon|\) and \(\phi_{\epsilon}\)

\[
2 \text{Re} \epsilon = (3.23 \pm 0.02) \times 10^{-3}
\]

in agreement with the experimental value for \(\delta_l\). The experimental value of \(\delta_l\) shows the internal consistency of the standard model and the CPT invariance.

Finally we discuss CP-asymmetries for \(K \to 3\pi\) decays. The decays

\[
K^+ \to \pi^+ \pi^0, \pi^+ \pi^-
\]

\[
K^0 \to \pi^+ \pi^- , \pi^0 \pi^0, \pi^0 \pi^0, \pi^0 \pi^0
\]

are parity conserving decays i.e. the parity of the final state is \(-1\). Now the C-parity of \(\pi^0\) and \((\pi^+ \pi^-)_\nu\) are given by

\[
C(\pi^0) = 1, \quad C(\pi^+ \pi^-) = (-1)^\nu
\]
and $G$-parity of pion is $-1$. Thus

\[ CP|\pi^0\pi^0\pi^0> = -|\pi^0\pi^0\pi^0> \]

\[ CP|\pi^+\pi^-\pi^0> = (-1)^{l'+1}|\pi^+\pi^-\pi^0> \]

Hence $CP$-conservation implies

\[ K^0_2 \rightarrow \pi^0\pi^0\pi^0 \text{ allowed. } \]
\[ K^0_1 \rightarrow \pi^0\pi^0\pi^0 \text{ is forbidden. } \]
\[ K^0_1 \rightarrow \pi^+\pi^-\pi^0 \text{ allowed if } l' \text{ is odd. } \]
\[ K^0_2 \rightarrow \pi^+\pi^-\pi^0 \text{ allowed if } l' \text{ is even. } \]

Now $G$-parity of three pions $\pi^+\pi^-\pi^0$:

\[ G = C(-1)^{I} = (-1)^{l'+I} = -1 \]

Hence $l' = \text{even, } I(\text{odd}); \quad I = 1, 3$
\[ l' = \text{odd, } I(\text{even}); \quad I = 0, 2 \]

Only $l' = 0$ decays are favored as the decays for $l' > 0$ are highly suppressed due to centrifugal barrier. Hence $K^0 \rightarrow \pi^+\pi^-\pi^0$ is highly suppressed. Thus we have to take into account $I = 1, 3$ amplitudes viz $a_1$ and $a_3$. $I = 3$ contribution is expected to be suppressed as it requires $\Delta I = \frac{5}{2}$ transition.

Hence $CP$-asymmetries of $K^0 \rightarrow 3\pi$ decays are given by

\[ \eta_{000} = \frac{A(K_S \rightarrow \pi^0\pi^0\pi^0)}{A(K_L \rightarrow \pi^0\pi^0\pi^0)} = \frac{(a_1 - a_3) + \epsilon(a_1 + a_3)}{(a_1 + a_3) + \epsilon(a_1 - a_3)} = \frac{[i\text{Re}a_1 + \epsilon\text{Im}a_1]}{\epsilon\text{Re}a_1 + i\text{Im}a_1} \]

\[ \eta_{l'0} = \frac{A(K_S \rightarrow \pi^+\pi^-\pi^0)}{A(K_L \rightarrow \pi^+\pi^-\pi^0)} \approx \epsilon + i\frac{\text{Im}a_1}{\text{Re}a_1} \]

\[ \eta_{l'-0} = \approx \epsilon + i\frac{\text{Im}a_1}{\text{Re}a_1} = \eta_{000} \]

5. $B^0 - \bar{B}^0$ Complex

For $B^0_q$ ($q=d$ or $s$) we show below that both $m_{12}$ and $\Gamma_{12}$ have the same phase. Thus,

\[ m_{12} = |m_{12}|e^{-2i\phi_M} \]
\[ \Gamma_{12} = |\Gamma_{12}|e^{-2i\phi_M} \]

\[ |\Gamma_{12}| \ll |m_{12}| \]
\[ p^2 = e^{-2i\phi_M} \left[ |m_{12}| - i |\Gamma_{12}| \right] \approx |m_{12}|e^{-2i\phi_M} \]
\[ q^2 = e^{+2i\phi_M} \left[ |m_{12}| - i |\Gamma_{12}| \right] \approx |m_{12}|e^{2i\phi_M} \]

\[ q/p = e^{-2i\phi_M} \]
\[ 2pq = 2|m_{12}| = (m_2 - m_1) - \frac{i}{2}(\Gamma_2 - \Gamma_1) \]
\[ \Rightarrow \Delta m_B = (m_2 - m_1) = 2|m_{12}| \]
\[ \Delta \Gamma = \Gamma_2 - \Gamma_1 = 0 \]

\[ \approx \epsilon + i\frac{\text{Im}a_1}{\text{Re}a_1} = \eta_{000} \]
For $B_d : \phi_M = -\beta$
For $B_s : \phi_M = 0$

The above equations follow from the fact that,

$$m_{12} - i\Gamma_{12} = \langle \bar{B}_0 q | H_{\text{eff}}^{\Delta B - 2} | B_0^0 \rangle$$

$H_{\text{eff}}^{\Delta B - 2}$ induces particle-antiparticle transition. For $B_0^0 \rightarrow \bar{B}_0^0$, $H_{\text{eff}}^{\Delta B - 2}$ arises from the box diagram as shown in Fig. 2, where the dominant contribution comes out from the $t$–quark.

Thus,

$$m_{12} \propto (V_{tb})^2 (V_{tq}^*)^2 m_t^2$$

Now,

$$\Gamma_{12} \propto \sum_f \langle B_0^0 | H_W | f \rangle \langle f | H_W | B_0^0 \rangle$$

where the sum is over all the final states which contribute to both $B_0^0$ and $\bar{B}_0^0$ decays. Thus,

$$\Gamma_{12} \propto (V_{cb} V_{cq}^* + V_{ub} V_{uq}^*)^2 m_b^2 \propto (V_{tb})^2 (V_{tq}^*)^2 m_b^2$$

Hence we have the result that,

$$\frac{\Gamma_{12}}{|m_{12}|} \sim \frac{m_b^2}{m_t^2}$$

Figure 2: Box Diagrams
Now $B_d^0 \to \bar{B}_d^0$ transition:

$$(V_{tb})^2 (V_{td}^*)^2 = A^2 \lambda^6 \left[(1 + \rho)^2 + \eta^2\right] e^{2i\beta}$$

Hence,

$$m_{12} = |m_{12}| e^{2i\beta}, \quad \Gamma_{12} = |\Gamma_{12}| e^{2i\beta}, \quad \phi_M = -\beta$$

On the other hand, $B_s^0 \to \bar{B}_s^0$ transition:

$$(V_{tb})^2 (V_{ts}^*)^2 = |V_{ts}|^2 \approx A^2 \lambda^4$$

$$m_{12} = |m_{12}|, \quad \Gamma_{12} = |\Gamma_{12}| \quad (5.5)$$

$$\phi_M = 0 \quad (5.6)$$

Also we have,

$$\frac{\Delta m_{B_s}}{\Delta m_{B_d}} = \frac{|m_{12}|_s}{|m_{12}|_d} = \frac{1}{\lambda^2 \left[(1 - \rho)^2 + \eta^2\right]} \xi \approx 34 \xi \quad (5.7)$$

where $\xi$ is $SU(3)$ breaking parameter. The numerical value is obtained using the experimental values $\lambda = 0.225$, $\rho = 0.132$, $\eta = 0.341$.

Hence it follows from Eqs. (3.6), (5.2) and (5.3) the mass eigenstates $B_L^0$ and $B_H^0$ can be written as:

$$|B_L^0\rangle = \frac{1}{\sqrt{2}} \left[ |B^0\rangle - e^{2i\phi_M} |\bar{B}^0\rangle \right] \quad CP = +1, \phi_M \to 0 \quad (5.9)$$

$$|B_H^0\rangle = \frac{1}{\sqrt{2}} \left[ |B^0\rangle + e^{2i\phi_M} |\bar{B}^0\rangle \right] \quad CP = -1, \phi_M \to 0 \quad (5.10)$$

In this case, $CP$ violation occurs due to phase factor $e^{2i\phi_M}$ in the mass matrix.

Now one gets (from Eq. (3.11)), using Eqs.(5.4), (5.9) and (5.10),

$$|B^0(t)\rangle = e^{-i\Gamma t} e^{-\frac{1}{2} \Delta m t} \left\{ \cos \left(\frac{\Delta m}{2} t\right) |B^0\rangle - i e^{+2i\phi_M} \sin \left(\frac{\Delta m}{2} t\right) |\bar{B}^0\rangle \right\} \quad (5.11)$$

Similarly we get,

$$|\bar{B}^0(t)\rangle = -e^{-i\Gamma t} e^{-\frac{1}{2} \Delta m t} \left\{ \cos \left(\frac{\Delta m}{2} t\right) |\bar{B}^0\rangle - i e^{-2i\phi_M} \sin \left(\frac{\Delta m}{2} t\right) |B^0\rangle \right\} \quad (5.12)$$
Suppose we start with $B^0$ viz $|B^0(0)\rangle = |B^0\rangle$, the probabilities of finding $\bar{B}^0$ and $B^0$ at time $t$ is given by,

$$P(B^0 \rightarrow \bar{B}^0, t) = |\langle \bar{B}^0 | B^0(t) \rangle|^2 = \frac{1}{2} e^{-\Gamma t} (1 - \cos(\Delta m) t)$$

$$P(B^0 \rightarrow B^0, t) = |\langle B^0 | B^0(t) \rangle|^2 = \frac{1}{2} e^{-\Gamma t} (1 + \cos(\Delta m) t)$$

These are equations of a damped harmonic oscillator, the angular frequency of which is,

$$\omega = \frac{\Delta m}{\hbar}$$

Now the mixing parameter,

$$r = \frac{\int_{0}^{T} |\langle \bar{B}^0 | B^0(t) \rangle|^2 dt}{\int_{0}^{T} |\langle B^0 | B^0(t) \rangle|^2 dt} = \frac{\chi}{1 - \chi}$$

$$\xrightarrow{T \rightarrow \infty} \frac{(\Delta m/\Gamma)^2}{2 + (\Delta m/\Gamma)^2} = \frac{x^2}{2 + x^2} \quad (5.13)$$

Experimentally, for $B^0_d$ and $B^0_s$,

$$\Delta m_{B^0_d} = (0.507 \pm 0.005) \times 10^{12} \text{hs}^{-1} = (3.337 \pm 0.033) \times 10^{-10}\text{MeV}$$

$$\tau_{B^0_d} = (1.525 \pm 0.009) \times 10^{-12}\text{s} \quad (5.14a)$$

$$\Delta m_{B^0_s} = (17.77 \pm 0.12) \times 10^{12} \text{hs}^{-1} = (117 \pm 0.8) \times 10^{-10}\text{MeV}$$

$$\tau_{B^0_s} = (1.472 \pm 0.0024) \times 10^{-12}\text{s} \quad (5.14b)$$

$$x_d = \left(\frac{\Delta m_{B^0_d}}{\Gamma_{B^0_d}}\right) = 0.77 \pm 0.008 \quad (5.14c)$$

$$x_s = \left(\frac{\Delta m_{B^0_s}}{\Gamma_{B^0_s}}\right) = 26.2 \pm 0.5 \quad (5.14d)$$

Non zero values of $x_d$ and $x_s$ clearly show mixing between $B_q$, $B_q(q = s, d)$. The large value of the $x_s$ compared to $x_d$ is in conformity with Eq. (5.8).

From Eq. (5.11) and (5.12), the decay amplitudes for,

$$B^0(t) \rightarrow f \quad A_f(t) = \langle f | H_w | B^0(t) \rangle$$

$$\bar{B}^0(t) \rightarrow \bar{f} \quad \bar{A}_f(t) = \langle \bar{f} | H_w | B^0(t) \rangle \quad (5.15)$$

are given by,

$$A_f(t) = e^{-imt} e^{-\frac{1}{2} \Gamma t} \left\{ \cos \left(\frac{\Delta m}{2} t\right) A_f - ie^{2i\phi_M} \sin \left(\frac{\Delta m}{2} t\right) \bar{A}_f \right\} \quad (5.16)$$

$$\bar{A}_f(t) = e^{-imt} e^{-\frac{1}{2} \Gamma t} \left\{ \cos \left(\frac{\Delta m}{2} t\right) \bar{A}_f - ie^{-2i\phi_M} \sin \left(\frac{\Delta m}{2} t\right) A_f \right\} \quad (5.17)$$
From Eqs. (5.16) and (5.17), we get for the decay rates,

\[
\Gamma_f(t) = e^{-\Gamma t} \left[ \frac{1}{2} \left( |A_f|^2 + |\bar{A}_f|^2 \right) + \frac{1}{2} \left( |A_f|^2 - |\bar{A}_f|^2 \right) \cos \Delta mt \right]
- \frac{i}{2} \left( 2i \text{Im} e^{2i\phi M} A_f^* \bar{A}_f \right) \sin \Delta mt
\]
\[+ \frac{1}{2} \left( |A_f|^2 - |\bar{A}_f|^2 \right) \cos \Delta mt \]
\[
\bar{\Gamma}_f(t) = e^{-\Gamma t} \left[ \frac{1}{2} \left( |\bar{A}_f|^2 + |A_f|^2 \right) - \frac{1}{2} \left( |A_f|^2 - |\bar{A}_f|^2 \right) \cos \Delta mt \right]
+ \frac{i}{2} \left( 2i \text{Im} e^{2i\phi M} A_f^* \bar{A}_f \right) \sin \Delta mt
\]
\[+ \frac{1}{2} \left( |A_f|^2 - |\bar{A}_f|^2 \right) \cos \Delta mt \]
\]

(5.18)
(5.19)

For \(\bar{\Gamma}_f\) and \(\bar{\Gamma}_f\) change \(f \to \bar{f}\) and \(\bar{f} \to f\) in \(\Gamma_f\) and \(\bar{\Gamma}_f\) respectively. As a simple application of the above equations, consider the semi-leptonic decays of \(B^0\),

\(B^0 \to l^+ \nu X^- : f\) for example \(X^- = D^-\)
\(\bar{B}^0 \to l^- \bar{\nu} X^+ : \bar{f}\) for example \(X^+ = D^+\)

In the standard model, \(\bar{B}^0\) decay into \(l^+ \nu X^-\) and \(B^0\) decay into \(l^- \bar{\nu} X^+\) is forbidden. Thus,

\[\bar{A}_f = 0, \quad A_f = 0\]
\[\Gamma_f(t) = e^{-\Gamma t} \frac{1}{2} |A_f|^2 (1 + \cos \Delta mt)\]
\[\bar{\Gamma}_f(t) = e^{-\Gamma t} \frac{1}{2} |\bar{A}_f|^2 (1 - \cos \Delta mt), \quad |\bar{A}_f| = |A_f|\]

(5.20)

Hence,

\[\delta = \frac{\int_0^\infty \Gamma_f(t) dt}{\int_0^\infty \bar{\Gamma}_f(t) dt} = \frac{x_d^2}{2 + x_d^2} = r_d\]

(5.21)

Non zero value of \(\delta\) would indicate mixing. If, however, \(\bar{A}_f \neq 0\) and \(A_f \neq 0\) due to some exotic mechanism, then \(\delta \neq 0\) even without mixing. Thus

\[\frac{\Gamma(\mu^- X^+)}{\Gamma(\mu^+ X^-) + \Gamma(\mu^- X^+)} = \frac{r_d}{1 + r_d} = \chi_d\]

(5.22)

which gives,

\[x_d = 0.723 \pm 0.032\]

in agreement with \(x_d\) given in Eq (5.14c).

6. \(CP\)-Violation in \(B\)-Decays

6.1 Case I

In this section, we discuss the \(CP\)-violation for \(B \to f, \bar{f}\) where

\[|\bar{f}| = CP(f) = |f|\]
For this case we get, from Eqs. (5.16) and (5.17),

$$A_f(t) = \frac{\Gamma_f(t) - \bar{\Gamma}_f(t)}{\Gamma_f(t) + \bar{\Gamma}_f(t)} = \cos(\Delta mt) \left( |A_f|^2 - |\bar{A}_f|^2 \right)$$

$$-i \sin(\Delta mt) \left( e^{2i\phi M} A_f^* \bar{A}_f - e^{-2i\phi M} A_f \bar{A}_f^* \right) / \left( |A_f|^2 + |\bar{A}_f|^2 \right)$$

$$= \cos(\Delta mt) C_f + \sin(\Delta mt) S_f \quad (6.1)$$

where,

$$C_f = \frac{1 - |\bar{A}_f|^2}{1 + |\bar{A}_f|^2} / |A_f|^2 = \frac{1 - |\lambda|^2}{1 + |\lambda|^2} \quad \lambda = \frac{\bar{A}_f}{A_f} \quad (6.2)$$

This is the direct $CP$ violation and,

$$S_f = \frac{2\text{Im} \left( e^{2i\phi M} \lambda \right)}{1 + |\lambda|^2} \quad (6.3)$$

is the mixing induced $CP$-violation.

If the decay proceeds through a single diagram (for example tree graph), $\bar{A}_f/A_f$ is given by $CPT$,

$$\lambda = \frac{\bar{A}_f}{A_f} = e^{i(\phi + \delta_f)} = e^{2i\phi} \quad (6.5)$$

where $\phi$ is the weak phase in the decay amplitude. Hence from Eq. (6.1), we obtain,

$$A_f(t) = \sin(\Delta mt) \sin(2\phi_M + 2\phi) \quad (6.6)$$

In particular for the decay (Fig 3),

$$B^0 \to J/\psi K_s, \quad \phi = 0$$
we obtain,
\[ A_{\psi K_s} (t) = \sin (2\phi_M) \sin (\Delta m t) = -\sin 2\beta \sin (\Delta m t) \] (6.7)

and,
\[ A_{\psi K_s} = \frac{\int_0^\infty \left[ \Gamma_f (t) - \bar{\Gamma}_f (t) \right] dt}{\int_0^\infty \left[ \Gamma_f (t) + \bar{\Gamma}_f (t) \right] dt} \]
\[ A_{\psi K_s} = -\sin (2\beta) \frac{(\Delta m / \Gamma)}{1 + (\Delta m / \Gamma)^2} \] (6.8)

Experiment : \( \left( \frac{\Delta m}{\Gamma} \right)_{B^0} = 0.776 \pm 0.008 \) (6.9)

\( A_{\psi K_s} \) has been experimentally measured. It gives,
\[ \sin 2\beta = 0.678 \pm 0.025 \]

Corresponding to the decay \( B^0 \to J/\psi K_s \), we have the decay \( B_s^0 \to J/\psi \phi \). Thus for this decay
\[ A_{J/\psi \phi}^{(t)} = -\sin 2\beta_s \sin (\Delta m_{B_s} t) \] (6.10)
\[ A_{J/\psi \phi} = -\sin 2\beta_s \frac{(\Delta m_{B_s} / \Gamma_s)}{1 + (\Delta m_{B_s} / \Gamma_s)^2} \] (6.11)

In the standard model, \( \beta_s = 0 \), \( A_{J/\psi \phi} = 0 \). This is an example of \( CP \)-violation in the mass matrix. Non zero value of \( A_{J/\psi \phi} \) will be a signal for physics beyond standard model. We now discuss the direct \( CP \)-violation.

Direct \( CP \)-violation in \( B \) decays involves the weak phase in the decay amplitude. The reason for this being that necessary condition for direct \( CP \)-violation is that decay amplitude should be complex as discussed in section 1. But this is not sufficient because in the limit of no final state interactions, the direct \( CP \)-violation in \( B \to f, \bar{B} \to \bar{f} \) decay vanishes. To illustrate this point, we discuss the decays \( \bar{B}^0 \to \pi^+ \pi^- \). The main contribution to this decay is from tree graph (see Fig. 4a); But this decay can also proceed via the penguin diagram (see Fig. 4b).

The contribution of penguin diagram can be written as
\[ P = V_{ub} V_{ud}^* f (u) + V_{cb} V_{cd}^* f (c) + V_{tb} V_{td}^* f (t) \] (6.12)
where \( f (u), f (c) \) and \( f (t) \) denote the contributions of \( u, c \) and \( t \) quarks in the loop. Now using the unitarity equation \([1,3]\), we can rewrite Eq. (6.12) as,
\[ P_c = V_{ub} V_{ud}^* (f (u) - f (t)) + V_{cb} V_{cd}^* (f (c) - f (t)) \]
or \[ P_t = V_{ub} V_{ud}^* (f (u) - f (c)) + V_{tb} V_{td}^* (f (t) - f (c)) \] (6.13)

Due to loop integration \( P \) is suppressed relative to \( T \) but still its contribution is not negligible. For the decay \( \bar{B}^0 \to f (f = \pi^+ \pi^-) \) the decay amplitude is given by
\[ A_f = A (\bar{B}^0 \to \pi^+ \pi^-) = |T| e^{i(-\gamma + \delta_T)} + |P| e^{i(\phi + \delta_P)} \] (6.14)
where $\delta_T$ and $\delta_P$ are strong interaction phases which have been taken out $\phi$ is the weak phase in Penguin graph. $CPT$ invariance gives,

$$ A_f \equiv A(B^0 \to \pi^+\pi^-) = |T| e^{-i(-\gamma - \delta_T)} + |P| e^{-i(\phi - \delta_P)}. \quad (6.15) $$

Direct $CP$–violation asymmetry is given by,

$$ A_{CP} = \frac{-\Gamma(B^0 \to \pi^+\pi^-) + \Gamma(\bar{B}^0 \to \pi^+\pi^-)}{\Gamma(B^0 \to \pi^+\pi^-) + \Gamma(\bar{B}^0 \to \pi^+\pi^-)} $$

$$ = \frac{1 - |\lambda|^2}{1 + |\lambda|^2} = -C_{\pi\pi} \quad (6.16) $$

For the second choice,

$$ \phi = \beta, \quad F_{CKM} = \frac{|V_{tb}|}{|V_{td}|} \approx \sqrt{1 - \bar{\rho}^2 + \eta^2} \sqrt{\bar{\rho}^2 + \eta^2}, \quad r = \frac{R_t}{R_b} |T| $$

$$ A_{\pi^+\pi^-} = |T| e^{i\gamma} e^{i\delta_T} [1 - r e^{i(\alpha + \delta)}] \quad (6.18) $$

$$ \delta = \delta_P - \delta_T \quad (6.19) $$

Hence we get, from Eqs (6.2), (6.3), (6.4) and (6.18)

$$ C_{\pi^+\pi^-} = -A_{CP} = \frac{2r \sin \delta \sin \alpha}{1 - 2r \cos \delta \cos \alpha + r^2} \quad (6.20) $$

$$ S_{\pi^+\pi^-} = \frac{2r \cos \delta \sin \alpha}{1 - 2r \cos \delta \cos \alpha + r^2} \approx \sin 2\alpha + 2r \cos \delta \sin \alpha \cos 2\alpha \quad (6.21) $$

From Eqs. (6.3), (6.17) and (6.18), we can express $S_{\pi^+\pi^-}$ in the form

$$ S_{\pi^+\pi^-} = \sqrt{1 - C_{\pi^+\pi^-}^2 - \frac{1}{|\lambda|^2} \text{Im}[e^{-2i\beta} \lambda]} $$

$$ = \sqrt{1 - C_{\pi^+\pi^-}^2 - \text{Im}[e^{-2i\beta} e^{-2i\gamma} e^{i\delta_{+-}}]} $$

$$ = \sqrt{1 - C_{\pi^+\pi^-}^2 - \text{Im}[2\alpha + \delta_{+-}]} \quad (6.21) $$

where we have put

$$ \lambda = \frac{|T|}{A_f}, \quad e^{2i\gamma} e^{i\delta_{+-}} = \frac{|T|^2}{|A_f|^2} e^{2i\gamma} e^{i\delta_{+-}} \quad (6.22a) $$

For $B^0 \to \pi^0\pi^0, B^+ \to \pi^+\pi^0$ the decay amplitudes are given by,

$$ A_{00} = A(B^0 \to \pi^0\pi^0) = \frac{1}{\sqrt{2}} |T| e^{i\delta_T} e^{i\gamma} \left[-r C e^{i\delta_C T} - r e^{i(\alpha + \delta)}\right] $$

$$ A_{+0} = A(B^+ \to \pi^+\pi^0) = \frac{1}{\sqrt{2}} |T| e^{i\delta_T} e^{i\gamma} \left[1 + r C e^{i\delta_C T}\right] $$

$$ r_C = \frac{|C|}{|T|}, \quad \delta_C T = \delta_C - \delta_T \quad (6.23) $$
Thus

\[ A_{+0}^{CP} = 0 \]

\[ C_{\pi^0\pi^0} = -A_{00}^{CP} = \frac{-2r/r_C \sin(\delta - \delta_{CT}) \sin \alpha}{1 + r^2/r_C^2 + 2r/r_C \cos(\delta - \delta_{CT}) \cos \alpha} \]  

(6.24)

\[ S_{\pi^0\pi^0} = \frac{\sin 2\alpha + 2r/r_C \cos(\delta - \delta_{CT}) \sin \alpha}{1 + r^2/r_C^2 + 2r/r_C \cos(\delta - \delta_{CT}) \cos \alpha} \]  

(6.25)

Experimental values for CP-asymmetries are

\[ C_{\pi^+\pi^-} = 0.38 \pm 0.17, \quad S_{\pi^+\pi^-} = -0.61 \pm 0.08 \]

For \( B^0(\bar{B}^0) \to K^+\pi^-(K^-\pi^+) \) decay \( CP|f) = |\bar{f} \rangle \neq |f \rangle \). For this case only direct CP-violation is possible. It is easy to see that the decay amplitude can be expressed in the following form

\[ A(\bar{B}^0 \to K^-\pi^+) = T + P = |P|e^{i\delta_P}[1 + r e^{i(\gamma - \delta)}] \]

\[ A(B^0 \to K^+\pi^-) = |P|e^{i\delta_P}[1 + r e^{i(\gamma - \delta)}] \]
where

\[ \delta = \delta_P - \delta_T \]
\[ r = \frac{|V_{ub}| |V_{us}| |T|}{|V_{cb}| |V_{cs}| |P|} \]

Hence

\[ A_{CP}(K^+\pi^-) = \frac{-2r \sin \gamma \sin \delta}{1 + 2r \cos \gamma \cos \delta + r^2} = -0.089 \pm 0.013 \text{ (Expt value)} \]

Finally it is convenient to write, from Eqs.(5.18) and (5.19), the decay rates in the following form,

\[ \left[ \Gamma_f(t) - \bar{\Gamma}_f(t) \right] + \left[ \bar{\Gamma}_f(t) - \Gamma_f(t) \right] = \frac{\cos \Delta m t \left[ (|A_f|^2 - |\bar{A}_f|^2) + (|A_{f'}|^2 - |\bar{A}_{f'}|^2) \right]}{1 + 2r \cos \gamma \cos \delta + r^2} \]
\[ + 2 \sin \Delta m t \left[ \text{Im} \left( e^{2i\phi_M A_f^* \bar{A}_f} \right) + \text{Im} \left( e^{2i\phi_M A_{f'}^* \bar{A}_{f'}} \right) \right] \]

(6.26)

\[ \left[ \Gamma_f(t) + \bar{\Gamma}_f(t) \right] - \left[ \bar{\Gamma}_f(t) + \Gamma_f(t) \right] = \frac{\cos \Delta m t \left[ (|A_f|^2 + |\bar{A}_f|^2) - (|A_{f'}|^2 + |\bar{A}_{f'}|^2) \right]}{1 + 2r \cos \gamma \cos \delta + r^2} \]
\[ + 2 \sin \Delta m t \left[ \text{Im} \left( e^{2i\phi_M A_f^* \bar{A}_f} \right) - \text{Im} \left( e^{2i\phi_M A_{f'}^* \bar{A}_{f'}} \right) \right] \]

(6.27)

We end this section with the following remarks. The CP asymmetries in the hadronic decays of $B$, $B_s$ and $K$ mesons involve strong final state phases. The strong interactions effects at the quark level are taken care of by perturbative QCD in terms of Wilson coefficients. The CKM matrix which connects the weak eigenstates with mass eigenstates is another aspect of strong interactions at quark level. In the case of semi leptonic decays, the long distance strong interaction effects manifest themselves in the form factors of final states after hadronization. Likewise the strong interaction final state phases are long distance effects. These phase shifts essentially arise in terms of S-matrix which changes an 'in' state into an 'out' state viz.

\[ |f\rangle_{\text{in}} = S|f\rangle_{\text{out}} = e^{2i\delta_f}|f\rangle_{\text{out}} \]

(6.28)

In fact, the CPT invariance of weak interaction Lagrangian gives for the weak decay $B(\bar{B}) \to f(\bar{f})$

\[ \bar{A}_f \equiv_{\text{out}} \langle f | L_w | \bar{B} \rangle = \eta_f e^{2i\delta_f} A_{f}^* \]

(6.29)

It is difficult to reliably estimate the final state strong phase shifts. It involves the hadronic dynamics. However, using isospin, C-invariance of S-matrix and unitarity of S-matrix, we can relate these phases. In this regard, the decays $B^0 \to f, \bar{f}$ described by two independent single amplitudes $A_f$ and $A_{f'}$ discussed in section 6.2 and the decays described by the weak amplitudes $A_f \neq A_{f'}$, described in section 6.3 are of interest.
The $C$ invariance of S-matrix viz. $S_f = S_f$ would imply

$$
\delta_f = \delta'_f, \quad \delta_{1f} = \delta_{1f}, \quad \delta_{2f} = \delta_{2f}
$$

In the above decays, $b$ is converted into $b \rightarrow c(u) + \bar{u} + d$. In particular, for the tree graph, the configuration is such that $\bar{u}$ and $d$ essentially go together into color singlet states while the third quark $c(u)$ recoiling; there is a significant probability that system will hadronize as a two body final state. Thus at least for the tree amplitude

$$
\delta_f^T = \delta'_f^T \approx 0.
$$

### 6.2 Case II

In this section we first consider the case in which single weak amplitudes $A_f$ and $A'_f$ with different weak phases describe the decays:

$$
A_f = \langle f | L W | B^0 \rangle = e^{i \phi} F_f
$$

$$
A'_f = A_f = \langle f^* | L' W | B^0 \rangle = e^{i \phi'} F'_f
$$

(CPT)* gives,

$$
A_f = \langle f | L W | B^0 \rangle = e^{2i \delta_f} A_f^*
$$

$$
A'_f = A_f = \langle f^* | L' W | B^0 \rangle = e^{2i \delta'} A_f^*
$$

For these decays, only mixing induced $CP$-asymmetries are possible. Note $\delta_f$ and $\delta'_f$ are strong phases; $\phi$ and $\phi'$ weak phases. The states $|f > and |f'> are C-conjugate of each other such as states $D^{(*)-\pi^+}(D^{(*)+\pi^-})$, $D_s^{(*)-\bar{K}^+}(D_s^{(*)+\bar{K}^-})$, $D^-\rho^+(D^+\rho^-)$

For this case

$$
A(t) = \frac{2 |F_f| |F'_f|}{|F_f|^2 + |F'_f|^2} \sin \Delta mt \sin(2\phi_M - \phi - \phi') \cos(\delta_f - \delta_f')
$$

$$
\mathcal{F}(t) = \frac{|F_f|^2 - |F'_f|^2}{|F_f|^2 + |F'_f|^2} \cos \Delta mt
$$

$$
- \frac{2 |F_f||F'_f|}{|F_f|^2 + |F'_f|^2} \sin \Delta mt \sin(2\phi_M - \phi - \phi) \cos(\delta_f - \delta_f')
$$

We now apply the above formula to $B \rightarrow \pi D$ and $B_s \rightarrow K D_s$ decays. For these decays,

$$
\phi = 0, \quad \phi' = \gamma
$$

$$
\phi_M = \begin{cases} 
-\beta, & \text{for } B^0 \\
-\beta_s, & \text{for } B_s^0
\end{cases}
$$
Thus
\[ A_f = \langle D^- \pi^+ | \mathcal{L}_W | B^0 \rangle = F_f \]
\[ A'_f = \langle D^+ \pi^- | \mathcal{L}_W' | B^0 \rangle = e^{i\gamma} F'_f \]
\[ A_{f_s} = \langle K^+ D_s^- | \mathcal{L}_W | B^0_s \rangle = F_{f_s} \]
\[ A'_{f_s} = \langle K^- D_s^+ | \mathcal{L}_W' | B^0_s \rangle = e^{i\gamma} F'_{f_s} \]

Note that the effective Lagrangians for decays \((q = d, s)\) are given by,
\[
\mathcal{L}_W = V_{cb} V_{uq}^{*} \bar{q} \gamma^\mu (1 - \gamma_5) u \bar{c} \gamma^\mu (1 - \gamma_5) b \\
\mathcal{L}_W' = V_{ub} V_{cq}^{*} \bar{q} \gamma^\mu (1 - \gamma_5) c \bar{u} \gamma^\mu (1 - \gamma_5) b
\] (6.34a)
\[
\mathcal{L}_W = V_{ub} V_{cq}^{*} \bar{q} \gamma^\mu (1 - \gamma_5) c \bar{u} \gamma^\mu (1 - \gamma_5) b \\
\mathcal{L}_W' = V_{cb} V_{uq}^{*} \bar{q} \gamma^\mu (1 - \gamma_5) u \bar{c} \gamma^\mu (1 - \gamma_5) b
\] (6.34b)

respectively. In the Wolfenstein parametrization of CKM matrix,
\[
\frac{|V_{ub}| |V_{cq}|}{|V_{cb}| |V_{uq}|} = (\lambda^2, 1) R_b, \quad q = d, s
\] (6.35)

Define,
\[
r = \lambda^2 R_b \frac{|F'_f|}{F_f}, \quad r_s = R_b \frac{|F'_f|}{F_f}
\]

Thus, we get from Eqs. (6.32) and (6.33) for \(B^0\) decays, (replacing \(\frac{|F'_f|}{F_f}\) by \(r\)),
\[
A(t) = -\frac{2r}{1 + r^2} \sin \Delta m_B t \sin (2\beta + \gamma) \cos (\delta_f - \delta'_f)
\] (6.36)
\[
F(t) = \frac{1 - r^2}{1 + r^2} \cos \Delta m_B t - \frac{2r}{1 + r^2} \sin \Delta m_B t \cos (\beta + \gamma) \sin (\delta_f - \delta'_f)
\] (6.37)

For the decays,
\[
\bar{B}^0_s (B^0_s) \rightarrow K^- D_s^+ (K^+ D_s^-) \\
\bar{B}^0_s (B^0_s) \rightarrow K^+ D_s^- (K^- D_s^+)
\]
we get,
\[
A_s(t) = -\frac{2r_s}{1 + r_s^2} \sin(\Delta m_{B_s} t) \sin (2\beta_s + \gamma) \cos (\delta_{f_s} - \delta'_{f_s})
\] (6.38)
\[
F_s(t) = \frac{1 - r_s^2}{1 + r_s^2} \cos \Delta m_{B_s} t - \frac{2r_s}{1 + r_s^2} \sin \Delta m_{B_s} t \cos (\beta_s + \gamma) \sin (\delta_{f_s} - \delta'_{f_s})
\] (6.39)

We note that for time integrated CP-asymmetry,
\[
A_s = -\frac{2r_s}{1 + r_s^2} \frac{\Delta m_{B_s}/\Gamma_s}{1 + (\Delta m_{B_s}/\Gamma_s)^2} \sin (2\beta_s + \gamma) \cos (\delta_{f_s} - \delta'_{f_s})
\] (6.40)
The $CP$–asymmetry $A_\pm(t)$ or $A_\pm$ involves two experimentally unknown parameters $\sin(2\beta_s + \gamma)$ and $\Delta m_{B_s}$. Both these parameters are of importance in order to test the unitarity of $CKM$ matrix viz whether $CKM$ matrix is a sole source of $CP$–violation in the processes in which $CP$–violation has been observed.

From Eqs. (6.36) and (6.37), we note that $CP$–asymmetries:

$$-\frac{S_+ + S_-}{2} = \frac{2r}{1 + r^2} \sin(2\beta + \gamma) \cos(\delta_f - \delta'_f)$$

$$-\frac{S_+ - S_-}{2} = \frac{2r}{1 + r^2} \cos(2\beta + \gamma) \sin(\delta_f - \delta'_f)$$

involve the weak phase $2\beta + \gamma$ and strong phase $\delta_f - \delta'_f$. For $B_s^0$, replace $r \rightarrow r_s$, $\delta_f \rightarrow \delta_{fs}$, $\delta'_f = \delta'_f$ and $\beta$ by $\beta_s$. In the standard model $\beta_s = 0$. The decays $B \rightarrow \pi D, \pi D^*, \rho D$ and $B_s \rightarrow K D_s, \pi D_s, K^* D_s$ are described by tree amplitudes (see Fig 5). For tree graphs, we assume factorization, factorization gives for the decay

$$B \rightarrow D^{(*)+}\pi^-$$

$$|\bar{F}^{s}| = |\bar{T}_f| = G^{s}[f_{\pi}(m_B^2 - m_D^2)f_{B-D}^{0}(m_\pi^2), 2f_{\pi_m}n_{B}^{0}A_{B-D}^{0}(m_\pi^2), (6.47)]$$

$$|\bar{F}^{s}| = |\bar{T}_f| = G^{s}[f_{D}(m_B^2 - m_\pi^2)f_{B-D}^{0}(m_D^2), 2f_{D^*}m_D^{0}f_{B-D}^{0}(m_D^2), (6.48)]$$

$$G = \frac{G_F}{\sqrt{2}}|V_{ud}|, |V_{cb}|a_1, \quad G' = \frac{G_F}{\sqrt{2}}|V_{cd}| |V_{ub}|$$

Equation (6.44) is consistent with experimental values. Factorization gives for the decay $B \rightarrow D^{(*)+}\pi^-$:

$$f_0(t) = \frac{\sqrt{m_B m_D}}{m_B + m_D} (1 + \omega) h_0(\omega)$$

$$A_0(t) = \frac{m_B + m_D}{2\sqrt{m_B m_D}} h_A(\omega)$$

$$A(t) = \frac{\sqrt{m_B m_{D^*}}}{m_B + m_{D^*}} (1 + \omega) h_A(\omega)$$

$$t = m_B^2 + m_D^2 - 2m_B m_D^*$$
we get
\[ h_0(\omega_{\text{max}}) = 0.51 \pm 0.03, \quad h_{A_0}(\omega_{\text{max}}^*) = 0.54 \pm 0.03 \] (6.50)
to be compared with the value
\[ |h_{A_1}(\omega_{\text{max}}^*)| = 0.52 \pm 0.03 \] (6.51)

obtained from the analysis of semi-leptonic decay $B^0 \rightarrow D^{(*)+} l^- \bar{\nu}_l$. The agreement between the wo values is remarkable. Hence the factorization assumption for $B^0 \rightarrow \pi D^{(*)}$ decays is experimentally on solid footing and is in agreement with HQET.

From Eqs. (6.47) and (6.48), we obtain
\[
 r = \lambda^2 R_b \frac{|T_f'|}{|T_f|} \\
 = \lambda^2 R_b \left[ \frac{f_D(m_B^2 - m_s^2)f_{f_0}^{B-\pi}(m_{D_s}^2)}{f_{f_s}(m_B^2 - m_s^2)f_{f_0}^{B-\pi}(m_{D_s}^2)} \frac{f_{D^*}f_{f_0}^{B-\pi}(m_{D_{s*}}^2)}{f_{f_s}A_{f_0}^{B-\pi}(m_{D_{s*}}^2)} \right] 
\] (6.52)

Using the value of $r$ obtained from (6.52) one gets
\[
 - \left( \frac{S_+ + S_-}{2} \right)_{D^*\pi} = 2(0.017 \pm 0.003) \sin(2\beta + \gamma) 
\] (6.53)
Using the experimental value of the $CP$ asymmetry for $B^0 \rightarrow D^*\pi$ decay which has the least error, one gets the following bounds

\[
\sin(2\beta + \gamma) > 0.69 \quad (6.54)
\]

\[
44^\circ \leq (2\beta + \gamma) \leq 90^\circ \quad (6.55)
\]

or \[
90^\circ \leq (2\beta + \gamma) \leq 136^\circ \quad (6.56)
\]

Selecting the second solution, and using $2\beta \approx 43^\circ$, we get

\[
\gamma = (70 \pm 23)^\circ \quad (6.57)
\]

To end this section, we discuss the decays $\bar{B}_s^0 \rightarrow D^{(*)+}K^-$, $D^{(*)+}K^-$ are for which no experimental data are available. However using factorization and SU(3) one gets the following branching ratios

\[
\frac{\Gamma(\bar{B}_s^0 \rightarrow D^{(*)+}K^-)}{\Gamma_{\bar{B}_s^0}} = (1.94 \pm 0.07) \times 10^{-4}[(1.96 \pm 0.07) \times 10^{-4}] \quad (6.58)
\]

and

\[
-\left( \frac{S_+ + S_-}{2} \right)_{D^{(*)}K} = (0.41 \pm 0.08) \sin(2\beta_s + \gamma) \quad (6.59)
\]

In the standard model with three generations of quarks $\beta_s = 0$. This asymmetry is of spacial interest to test the physics beyond standard model. The experimental results for these decays will be relevant not only for the standard model but also for physics beyond standard model.

**6.3 Case III: $A_f \neq A_f$**

\[
A_f = \langle f | L_W | B^0 \rangle = \left[ e^{i\phi_1} F_{1f} + e^{i\phi_2} F_{2f} \right]
\]

\[
A_f = \langle \bar{f} | L_W | B^0 \rangle = \left[ e^{i\phi_1} F_{1f} + e^{i\phi_2} F_{2f} \right]
\]

Examples:

$B^0 \rightarrow \rho^- \pi^+(f): A_f \quad B^0 \rightarrow \rho^+ \pi^-(\bar{f}): A_f$

$B_s^0 \rightarrow K^+K^- \quad B_s^0 \rightarrow K^{*+}K^-$

$CPT$ gives,

\[
\bar{A}_{f,\bar{f}} = \sum_i \left[ e^{-i\phi_i} e^{2i\delta_{ij} F_{ij}^*} \right]
\]

For these decays, subtracting and adding Eqs. (6.26) and (6.27), we get,

\[
\frac{\Gamma_f(t) - \bar{\Gamma}_f(t)}{\Gamma_f(t) + \bar{\Gamma}_f(t)} = C_f \cos \Delta m t + S_f \sin \Delta m t \quad (6.60)
\]

\[
= (C + \Delta C) \cos \Delta m t + (S + \Delta S) \sin \Delta m t
\]
\[
\frac{\Gamma_f(t) - \bar{\Gamma}_f(t)}{\Gamma_f(t) + \bar{\Gamma}_f(t)} = C_f \cos \Delta mt + S_f \sin \Delta mt
\]
\[
= (C - \Delta C) \cos \Delta mt + (S - \Delta S) \sin \Delta mt
\]  \hspace{1cm} (6.61)

For these decays, the decay amplitudes can be written in terms of tree amplitudes \(e^{i\phi_T T_f}\) and the penguin amplitude \(e^{i\phi_P P_f}\). We confine to decays:

\[
B^0 \rightarrow \rho^- \pi^+ : A_f; \quad B^0 \rightarrow \rho^+ \pi^- : \bar{A}_f; \quad \phi_T = \gamma, \phi_P = -\beta \]  \hspace{1cm} (6.62)

Hence for \(B^0 \rightarrow \rho^- \pi^+, B^0 \rightarrow \rho^+ \pi^-, B^0 \rightarrow \rho^+ \pi^-, B^0 \rightarrow \rho^- \pi^+,\) we have

\[
A_f = |T_f| e^{-i\gamma} e^{i\delta_f} [1 - r_f e^{i(\alpha + \delta_f)}] \\
\bar{A}_f = |T_f| e^{-i\gamma} e^{i\delta_f} [1 - r_f e^{i(\alpha + \delta_f)}] \\
\text{where} \quad r_{f,\bar{f}} = \frac{|V_{tb}| |V_{td}| |P_{f,\bar{f}}|}{|V_{ub}| |V_{ud}| |T_{f,\bar{f}}|} = \frac{R_e |P_{f,\bar{f}}|}{R_b |T_{f,\bar{f}}|} \]  \hspace{1cm} (6.63)

In order to take into account the contributions of tree and penguin diagram, we introduce the angles \(\alpha_{eff}^{f,\bar{f}}\), defined as follows

\[
e^{i\beta} A_{f,\bar{f}} = |A_{f,\bar{f}}| e^{-i\alpha_{eff}^{f,\bar{f}}} \\
e^{-i\beta} \bar{A}_{f,\bar{f}} = |\bar{A}_{f,\bar{f}}| e^{i\alpha_{eff}^{f,\bar{f}}} \]  \hspace{1cm} (6.65)

With this definition, we separate out tree and penguin contributions:

\[
e^{i\beta} A_{f,\bar{f}} - e^{-i\beta} \bar{A}_{f,\bar{f}} = |A_{f,\bar{f}}| e^{-i\alpha_{eff}^{f,\bar{f}}} - |\bar{A}_{f,\bar{f}}| e^{i\alpha_{eff}^{f,\bar{f}}} \\
= 2i T_{f,\bar{f}} \sin \alpha \]  \hspace{1cm} (6.66)

\[
e^{i(\alpha + \beta)} A_{f,\bar{f}} - e^{-i(\alpha + \beta)} \bar{A}_{f,\bar{f}} = |A_{f,\bar{f}}| e^{-i(\alpha_{eff}^{f,\bar{f}} - \alpha)} - |\bar{A}_{f,\bar{f}}| e^{i(\alpha_{eff}^{f,\bar{f}} - \alpha)} \\
= (2i T_{f,\bar{f}} \sin \alpha) r_{f,\bar{f}} e^{i\delta_{f,\bar{f}}} \\
= 2i P_{f,\bar{f}} \sin \alpha \]  \hspace{1cm} (6.67)

From Eqs. (6.66) and (6.67), we get

\[
R_{f,\bar{f}} \left[ 1 - \sqrt{1 - A_{eff}^{f,\bar{f}}^2 \cos(2\alpha_{eff}^{f,\bar{f}})} \right] = 2 |T_{f,\bar{f}}|^2 \sin^2 \alpha \]  \hspace{1cm} (6.68)

\[
r_{f,\bar{f}}^2 = \frac{1 - \sqrt{1 - A_{eff}^{f,\bar{f}}^2 \cos(2\alpha_{eff}^{f,\bar{f}} - 2\alpha)}}{1 - \sqrt{1 - A_{eff}^{f,\bar{f}}^2 \cos 2\alpha_{eff}^{f,\bar{f}}}} \]  \hspace{1cm} (6.69)

\[
r_{f,\bar{f}} \cos \delta_{f,\bar{f}} = \frac{\cos \alpha - \sqrt{1 - A_{eff}^{f,\bar{f}}^2 \cos(2\alpha_{eff}^{f,\bar{f}} - \alpha)}}{1 - \sqrt{1 - A_{eff}^{f,\bar{f}}^2 \cos 2\alpha_{eff}^{f,\bar{f}}}} \]  \hspace{1cm} (6.70)

\[
r_{f,\bar{f}} \sin \delta_{f,\bar{f}} = \frac{-A_{eff}^{f,\bar{f}} \sin \alpha}{1 - \sqrt{1 - A_{eff}^{f,\bar{f}}^2 \cos 2\alpha_{eff}^{f,\bar{f}}}} \]  \hspace{1cm} (6.71)

\[
S_{f,\bar{f}} = \sqrt{1 - C_{eff}^{f,\bar{f}}^2 \sin(2\alpha_{eff}^{f,\bar{f}} \mp \delta)} \]  \hspace{1cm} (6.72)
where the phase $\delta$ is defined as
\[
\delta_f = \frac{|\hat{A}_f|}{|A_f|} \hat{A}_f e^{i\delta} \tag{6.73}
\]

Thus one sees it is convenient to analyse these decays in terms of $\alpha_{\text{eff}}^{f,\bar{f}}$. From Eq. (6.66), we get

\[
\sin 2\delta_{f,\bar{f}}^T = -A_{CP}^{f,\bar{f}} \sin 2\alpha_{\text{eff}}^{f,\bar{f}} \tag{6.74}
\]

\[
\cos 2\delta_{f,\bar{f}}^T = \sqrt{1 - A_{CP}^{f,\bar{f}}^2} \cos 2\alpha_{\text{eff}}^{f,\bar{f}} \tag{6.75}
\]

Now factorization implies
\[
\delta_{f}^T = 0 = \delta_{\bar{f}}^T \tag{6.76}
\]

Thus in the limit $\delta_{f,\bar{f}}^T \to 0$, we get from Eqs. (6.74)

\[
\cos 2\alpha_{\text{eff}}^{f,\bar{f}} = -1, \quad \alpha_{\text{eff}}^{f,\bar{f}} = 90^\circ \tag{6.77}
\]

and from Eqs. (6.70), (6.71) and (6.77)

\[
r_{f,\bar{f}} \cos \delta_{f,\bar{f}} = \cos \alpha \tag{6.78}
\]

\[
r_{f,\bar{f}} \sin \delta_{f,\bar{f}} = \frac{-A_{CP}^{f,\bar{f}} \sin \alpha}{\sqrt{1 - A_{CP}^{f,\bar{f}}^2}} \tag{6.79}
\]

\[
r_{f,\bar{f}}^2 = \frac{1 + \sqrt{1 - A_{CP}^{f,\bar{f}}^2} \cos 2\alpha}{1 + \sqrt{1 - A_{CP}^{f,\bar{f}}^2}} \tag{6.80}
\]

Finally the $CP$ asymmetries in the limit $\delta_{f,\bar{f}}^T \to 0$

\[
S_f = S + \Delta S = -\sqrt{1 - C_f^2} \cos \delta \tag{6.81}
\]

\[
S_{\bar{f}} = S - \Delta S = \sqrt{1 - C_{\bar{f}}^2} \cos \delta \tag{6.82}
\]

For $B^0(\bar{B}^0) \to \rho^-\pi^+, \rho^+\pi^-(\rho^+\pi^-, \rho^-\pi^+, \pi^+)$ decays the experimental results are

\[
\Gamma = R_f + R_{\bar{f}} = (22.8 \pm 2.5) \times 10^{-6} \tag{6.83}
\]

\[
A_{CP}^f = -0.16 \pm 0.23, \quad A_{CP}^{\bar{f}} = 0.08 \pm 0.12 \tag{6.84}
\]

\[
C = 0.01 \pm 0.14, \quad \Delta C = 0.37 \pm 0.08 \tag{6.85}
\]

\[
S = 0.01 \pm 0.09, \quad \Delta S = -0.05 \pm 0.10 \tag{6.86}
\]

With above values, it is hard to draw any reliable conclusion.
7. CP-Violation in Hadronic Weak Decays of Baryons

So far we have discussed the CP violation in $K^0 - \bar{K}^0$, $B^0_q - \bar{B}^0_q$ systems. There is a need to study CP violation outside these systems. The hadronic weak decays of baryons and antibaryons provide another framework to study CP violation.

The hadronic weak decays

$$N(p) \rightarrow N(p') + \pi(q)$$

is described by the amplitude

$$M_f = \overline{u}(p') [A - \gamma_5 B] u(p) \sim \chi^\dagger [a_s + a_p \sigma . n] \chi \quad (7.1)$$

(Note here we have designated a baryon by N, not to confuse with a B-meson and $\pi$ is any pseudoscalar meson).

Under charge conjugation ($C$):

$$u(p) \rightarrow C \overline{v}^T(p), \quad C = i \gamma^0 \gamma^2$$

Under space reflection ($P$):

$$u(r)(p) \rightarrow u(r)(-p) = \gamma_0 u(p)$$

Under time reversal ($T$):

$$u(r)(p) \rightarrow u^*(r)(-p) = B u(r)(p), \quad B = \gamma^1 \gamma^3$$

Thus, under these transformations

$$M_f \rightarrow - \overline{v}(p') [A + \gamma_5 B] v(p') = \overline{M}_T \sim \chi^\dagger (-a_s + a_p \sigma . n)$$

$$M_f \rightarrow \overline{v}(p') [A^* - \gamma_5 B^*] u(p)$$

$$M_f \rightarrow - \overline{v}(p') [A^* + \gamma_5 B^*] v(p') = \overline{M}_T \quad (7.2)$$

When final state interactions are taken into account, the partial wave amplitudes $a_s$ and $a_p$ acquire strong final state phases $e^{i\delta_f}$ and $e^{i\delta_f}$ respectively. Thus with final state interactions

$$\langle f | H | B \rangle \rightarrow \langle f | H | B \rangle^* = e^{2i\delta_f} \langle f | H | B \rangle^*$$

Hence under CP and CPT

$$M_f \rightarrow \overline{M}_T = \chi^\dagger [-|a_s| e^{i\delta_f} + |a_p| e^{i\delta_f} \sigma . n] \chi \quad (7.3a)$$

$$M_f \rightarrow - \overline{v}(p') [e^{2i\delta_f} A^* - \gamma_5 e^{2i\delta_f} B^*] v(p)$$

$$= \overline{M}_T \sim \chi^\dagger [-e^{2i\delta_f} a_s^* + e^{2i\delta_f} a_p^* \sigma . n] \chi \quad (7.3b)$$

Hence from Eq (7.3a), we conclude that CP symmetry gives

$$\Gamma = \Gamma, \quad \overline{\alpha} = \alpha, \quad \overline{\beta} = \beta \quad (7.4)$$
From Eqs (7.3a) and (7.3b), we note that both CP and CPT invariance give the same result given in Eq (7.4), unless the S-wave amplitude A and P-wave amplitude B have different weak phases. Hence to leading order, CP-odd observables

\[ \delta \Gamma = \frac{\Gamma - \Gamma'}{\Gamma + \Gamma'} \]

\[ \delta \alpha = \frac{\alpha + \alpha'}{\alpha - \alpha'} \]

\[ \delta \beta = \frac{\beta + \beta'}{\beta - \beta'} \]

are non zero only if the above condition is satisfied.

The decays of \( B(\bar{B}) \) mesons to baryon-antibaryon pair \( N_1 \bar{N}_2 (\bar{N}_1 N_2) \) and subsequent decays of \( N_2, \bar{N}_2 \) or \( (N_1, \bar{N}_1) \) to a lighter hyperon (antihyperon) plus a meson also provide a means to study CP-odd observables as for example in the process,

\[ e^- e^+ \rightarrow B, \bar{B} \rightarrow N_1 \bar{N}_2 \rightarrow N_1 \bar{N}_2' \bar{\pi}, \quad \bar{N}_1 N_2 \rightarrow \bar{N}_1 N_2' \bar{\pi} \]

The decay \( B \rightarrow N_1 \bar{N}_2(f) \) is described by the matrix element,

\[ M_f = F_q e^{i\phi} [\bar{u}(p_1)(A_f + \gamma_5 B_f)v(p_2)] \]

where \( B \rightarrow \bar{N}_1 N_2(f) \) is described by the matrix elements

\[ M'_f = F'_q e^{i\phi'} [\bar{u}(p_2)(A'_f + \gamma_5 B'_f)v(p_1)] \]

where \( F_q \) is a constant containing CKM factor, \( \phi \) is the weak phase. The amplitude \( A_f \) and \( B_f \) are in general complex in the sense that they incorporate the final state phases \( \delta_f^p \) and \( \delta_f^s \) and they may also contain weak phases \( \phi_s \) and \( \phi_p \). Note that \( A_f \) is the parity violating amplitude (p-wave) whereas \( B_f \) is parity conserving amplitude (s-wave). The CPT invariance gives the matrix elements for the decay \( \bar{B} \rightarrow \bar{N}_1 N_2(\bar{f}) \) :

\[ \bar{M}_f = F_q e^{-i\phi} [\bar{u}(p_2)(-A_f^* e^{2i\delta_f^p} + \gamma_5 B_f^* e^{2i\delta_f^s})v(p_1)] \]

if the decays are described by a single matrix element \( M_f \). If \( \phi_s = 0 = \phi_p \) then CPT and CP invariance give the same predictions viz

\[ \bar{\Gamma}_f = \Gamma_f, \quad \bar{\alpha}_f = -\alpha_f, \quad \bar{\beta}_f = -\beta_f, \quad \bar{\gamma}_f = \gamma_f \] (7.8)

In order to test these predictions, consider for example the decay

\[ B^0_d \rightarrow p\Lambda_c \rightarrow p\bar{p}K^0 \]

\[ \bar{B}^0_d \rightarrow \bar{p}\Lambda^+ \rightarrow \bar{p}pK^0 \] (7.9)

By analysing the final states \( p\bar{p}K^0, \bar{p}pK^0 \) one may test \( \bar{\alpha}_f = -\alpha_f \) for the chamed hyperon (antihyperon) decays.
8. Conclusion

1. Discrete symmetries are not universal both \( C \) and \( P \) are violated in weak interaction but respected by electromagnetic and strong interaction. Violation of \( C \) and \( P \) are incorporated in the basic structure of weak interaction by assigning left-handed fermions to the doublet and right-handed fermions to the singlet representation of electroweak unification group.

2. Unlike \( C \) and \( P \) violation, \( CP \) violation does not embrace all weak processes. \( CP \) violation is observed in the semi-leptonic and weak hadronic decays of mesons.

3. Effective weak interaction Lagrangian in the standard model can accommodate \( CP \) violation to mismatch between mass eigenstates and \( CP \) eigenstates and/or mismatch between weak eigenstates and mass eigenstates at quark level. The mixing induced \( CP \) violation involves the mass difference \( \Delta m_B \) and \( \Delta m_{B_s} \).

4. There is no evidence of \( CP \) violation in lepton sector and in processes involving neutral currents. The effective weak interaction Lagrangian of the standard model cannot accommodate \( CP \) violation in these sectors. Any experimental observation of \( CP \) violation in these sectors would indicate, physics beyond the standard model.

5. There is no evidence of \( CP \) violation in \( D^0 - \bar{D}^0 \) complex. \( D^0 \) and \( \bar{D}^0 \) being bound states of first and second generation quark and anti-quark; no weak phase in CKM matrix is available to generate \( CP \) violation in \( D^0 - \bar{D}^0 \) complex. Any observation of \( CP \) violation in these sector would indicate, physics beyond the standard model.

6. With three generations of quarks, with one phase no extra phase is available to generate the mismatch between \( CP \) and mass eigenstates for \( B_s^0 - \bar{B}_s^0 \) complex. The mixing induced \( CP \) asymmetries for the decays \( B_s^0 \to J/\psi \phi \) and \( B_s^0 \to K^+ D_s^+(s^-) (K^- D_s^{(*)+}) \)

\[
A_{J/\psi \phi} = -\sin 2\beta_s \frac{\Delta m_{B_s}/\Gamma_s}{1 + (\Delta m_{B_s}/\Gamma_s)^2} = 0, \beta_s = 0 \text{ in standard model}
\]

\[
-\left( \frac{S_+ + S_-}{2} \right)_{D_s^{(*)}K} \propto \sin(2\beta_s + \gamma) = \sin \gamma, \text{ in standard model}
\]

The experimental determination of these \( CP \) asymmetries in future experiments when enough data on \( B_s^0 \) decays would be available will be crucial for any extension of the standard model from three generations of fermions to four generations of fermions.

Finally for baryon genesis, both \( C \) and \( CP \) violation are required. How the \( CP \) violation in meson sector is related to \( CP \) violation required for baryongenesis? There is no answer to this question yet.

For a review see for instance refs (1-5)
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