Elements of bases of mechanics of wood-polymer composites

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Abstracts. Disclosed is a solution of a particular case of a problem of polycomponent motion within a poly speed model, which is reduced to setting conditions of joint motion of components and determining intercomponent thermal-force interaction. The study of construction of the basis of mechanics of wood-polymer composites from phenomenological positions of mechanics of heterogeneous media is presented by construction of equations of laws of mass and momentum preservation for two component media and their closure by a viscous-elastic-plastic rheological model of the composite material for development of the basis of mechanics of wood-polymer composites from phenomenological positions of mechanics of heterogeneous media. Contributing to improvement of production of structured composite for FDM-printing technologies due to production of composites on the basis of thermopolymers with filling of them with ground wood. Raw materials can be wastes of logging and wood processing industries, the product is a composite with high filling of wood with a wide field of application in different sectors of national economy and industry. The equations are closed by a viscous-elastic-plastic rheological model of the composite material and by matching the equations of inseparability and the displacement strain vector.

1. Introduction

The problem of polycomponent motion within a poly speed model is reduced to setting conditions for the joint motion of the components and determining the intercomponent thermal-force interaction. Where the inertial effects of the relative motion of the components are not essential, a representation of the quasigomogenicity of the medium, as an averaged superposition of the components of the heterogeneous medium, can be used. Currently, FDM printing based on polymer materials is the most widely used in the production of consumer goods, so the creation of cheap composite materials for them is relevant all over the world. The most efficient solution is the production of composites based on thermopolymers with filling them with crushed wood. Raw materials for filling are wastes of logging and wood processing industries, agriculture, household, etc. As a result of high structural compatibility, component [1-5] becomes possible to produce high wood-filled composites which can be used in construction (40% wood-ground filling), design (80%), volume modeling (90%), etc. [5]. Structured composite in the form of thread for FDM-printing is produced by means of extrusion technology, in which operations are carried out from mixing of components to production of wood-polymer thread with viscous-elastic-plastic rheology.

2. Methods and Materials

From the position of solid medium mechanics [6-8] the wood-polymer composite represents two component heterogeneous continuum. Unlike a single-component medium, in a multi-component,
each component is characterized by specific volume, density, velocity, etc., the averaged superposition of which allows to build a dynamic picture of the movement of the heterogeneous continuum as a whole.

For a wood-polymer composite, averaged equations for components can be written:

- share of occupied volumes
  \[ m_1 + m_2 = 1. \]  
  where, \( m_1, m_2 \) are respective fractions of polymer and crushed wood volumes;

- density
  \[ \rho = \rho_1 \cdot m_1 + \rho_2 \cdot m_2, \]  
  or
  \[ \rho = \rho_1 \cdot m_1 + \rho_2 \cdot m_2 \cdot m_2^*, \]  
  where \( \rho_1, \rho_2, \rho_2^* \) - according to density of polymer, wood and woody substance, \( m_2^* \) - porosity of wood; Composite consists of non-compressible polymer and compressible wood material, so (3) rewrite as
  \[ \rho = \rho_1 \cdot (m_1 + \rho_2 \cdot m_2 \cdot m_2^*) = \rho_1 \cdot M_1, \]  
  - speeds by movement quantity
  \[ V = \frac{\rho_1 \cdot m_1 \cdot V_1 + \rho_2 \cdot m_2 \cdot V_2}{\rho}, \]  
  where, \( V_1, V_2 \) is the velocity of the component in rectangular coordinates
  \[ V_1 = i \cdot u_1 + j \cdot v_1 + k \cdot w_1, \]  
  and
  \[ V_2 = i \cdot u_2 + j \cdot v_2 + k \cdot w_2, \]  
  where, \( i, j, k \) are unit orths along the rectangular coordinates \( x, y, z \).

3. Results and Discussion

The mechanics of the wood-polymer composite are built on the basis of the laws of mass and momentum preservation for each component and their dynamic superposition.

The mass conservation equation in the form of the inseparability equation for each component takes the form
\[ \partial \rho_{i*,} / \partial t + \text{div} \rho_{i*,} \cdot V_1 = 0, \]  
or
\[ \partial \rho_{i*,} / \partial t + \partial \rho_{i*,} \cdot u_1 / \partial x + \partial \rho_{i*,} \cdot v_1 / \partial y + \partial \rho_{i*,} \cdot w_1 / \partial z = 0, \]  
and
\[ \partial \rho_{2*,} / \partial t + \text{div} \rho_{2*,} \cdot V_2 = 0, \]  
or
\[ \partial \rho_{2*,} / \partial t + \partial \rho_{2*,} \cdot u_2 / \partial x + \partial \rho_{2*,} \cdot v_2 / \partial y + \partial \rho_{2*,} \cdot w_2 / \partial z = 0, \]  
where, \( \rho_{i*} = \rho_1 \cdot m_1, \rho_{2*} = \rho_2 \cdot m_2, t - \text{time.} \)

In the extrusion formation of the composite structure, it can be assumed that the relative velocities of the component are small (\( V_1 = V_2 = V \)), so that dynamic interaction of the component can be neglected, and under these conditions the continuity equation for the composite as a whole can be written
\[ \partial \rho / \partial t + \text{div} \rho \cdot V = 0, \]  
or
\[ \partial \rho / \partial t + \partial \rho u / \partial x + \partial \rho v / \partial y + \partial \rho w / \partial z = 0. \]
Taking into account (3.1) equation (7) takes the form

\[ \frac{\partial M}{\partial t} + \frac{\partial M u}{\partial x} + \frac{\partial M v}{\partial y} + \frac{\partial M w}{\partial z} = 0. \] (12.1)

Representing the time of the 4th virtual rectangular coordinate \( L \), the continuity equation (7, a) is written as

\[ \frac{\partial M f}{\partial t} + \frac{\partial M u}{\partial x} + \frac{\partial M v}{\partial y} + \frac{\partial M w}{\partial z} = 0. \] (13)

where, \( L = f \cdot \tau \), \( \tau \) – virtual rate that translates time into a spatial coordinate.

Introduction of the generalized speed

\[ M U = M \cdot (i \cdot u + j \cdot v + k \cdot w + o \cdot f ), \] (14)

The inseparability equation (8) can be written as

\[ \text{div } M U = 0, \] (15)

where, \( o \) – unit orth along the \( L \) coordinate.

Equation of pulse preservation for each component under condition of neglect of volumetric forces and convection shall be recorded as [9]

\[
\begin{align*}
\rho \frac{\partial u_1}{\partial t} &= (\partial \sigma_{x_1} / \partial x + \partial \tau_{x_1 y_1} / \partial y + \partial \tau_{x_1 z_1} / \partial z), \\
\rho \frac{\partial v_1}{\partial t} &= (\partial \tau_{y_1 x_1} / \partial x + \partial \sigma_{y_1} / \partial y + \partial \tau_{y_1 z_1} / \partial z), \\
\rho \frac{\partial w_1}{\partial t} &= (\partial \tau_{z_1 x_1} / \partial x + \partial \tau_{z_1 y_1} / \partial y + \partial \sigma_{z_1} / \partial z),
\end{align*}
\] (16)

and

\[
\begin{align*}
\rho \frac{\partial u_2}{\partial t} &= (\partial \sigma_{x_2} / \partial x + \partial \tau_{x_2 y_2} / \partial y + \partial \tau_{x_2 z_2} / \partial z), \\
\rho \frac{\partial v_2}{\partial t} &= (\partial \tau_{y_2 x_2} / \partial x + \partial \sigma_{y_2} / \partial y + \partial \tau_{y_2 z_2} / \partial z), \\
\rho \frac{\partial w_2}{\partial t} &= (\partial \tau_{z_2 x_2} / \partial x + \partial \tau_{z_2 y_2} / \partial y + \partial \sigma_{z_2} / \partial z),
\end{align*}
\] (17)

Here \( \sigma \) – normal stress (lower index indicates axis parallel to which stress acts), \( \tau \) – tangential stress (lower two indices: the first indicates to which axis the elementary site in question is perpendicular, and the second - axis parallel to which tangential stress acts).

Under the assumption made earlier of the small relative velocities of the components, the equation of momentum conservation law for the composite takes the form

\[
\begin{align*}
\rho \frac{\partial u_1}{\partial t} &= (\partial \sigma_{x_1} / \partial x + \partial \tau_{x_1 y_1} / \partial y + \partial \tau_{x_1 z_1} / \partial z) + \\
&+ (\partial \sigma_{x_2} / \partial x + \partial \tau_{x_2 y_2} / \partial y + \partial \tau_{x_2 z_2} / \partial z) = (\partial \sigma_{x_1} / \partial x + \partial \tau_{x_1 y_1} / \partial y + \partial \tau_{x_1 z_1} / \partial z), \\
\rho \frac{\partial v_1}{\partial t} &= (\partial \tau_{y_1 x_1} / \partial x + \partial \sigma_{y_1} / \partial y + \partial \tau_{y_1 z_1} / \partial z) + \\
&+ (\partial \tau_{y_2 x_2} / \partial x + \partial \sigma_{y_2} / \partial y + \partial \tau_{y_2 z_2} / \partial z) = (\partial \tau_{y_1 x_1} / \partial x + \partial \sigma_{y_1} / \partial y + \partial \tau_{y_1 z_1} / \partial z), \\
\rho \frac{\partial w_1}{\partial t} &= (\partial \tau_{z_1 x_1} / \partial x + \partial \tau_{z_1 y_1} / \partial y + \partial \sigma_{z_1} / \partial z) + \\
&+ (\partial \tau_{z_2 x_2} / \partial x + \partial \tau_{z_2 y_2} / \partial y + \partial \sigma_{z_2} / \partial z) = (\partial \tau_{z_1 x_1} / \partial x + \partial \tau_{z_1 y_1} / \partial y + \partial \sigma_{z_1} / \partial z),
\end{align*}
\] (18)

The deformation state of the continuum is determined by the relative linear change of volume caused by normal stresses and angular deformations caused by the action of tangential stresses [9]. Record the relative volume change due to normal stresses
\[
\frac{d(\varepsilon V)}{\varepsilon V} = e = \varepsilon_x + \varepsilon_y + \varepsilon_z + \varepsilon_L,
\]
(19)

- shift vector
\[
s = m \cdot (i \cdot a + j \cdot b + k \cdot c + o \cdot g),
\]
(20)

- shift vector speed
\[
\frac{ds}{dt} = \frac{d(i \cdot a + j \cdot b + k \cdot c + o \cdot g)}{dt},
\]
(21)

where, \(e\) – elongation (the subscript indicates that the corresponding axis is parallel).

Submissions (19) and (20) should
\[
e = \text{div} s = \frac{\partial a}{\partial x} + \frac{\partial b}{\partial y} + \frac{\partial c}{\partial z} + \frac{\partial g}{\partial L},
\]
(22)

where, \(\varepsilon_x = \frac{\partial a}{\partial x}, \varepsilon_y = \frac{\partial b}{\partial y}, \varepsilon_z = \frac{\partial c}{\partial z}, \varepsilon_L = \frac{\partial g}{\partial L}.

Record average value of relative volumetric change of composite material
\[
e_0 = \frac{\varepsilon_x + \varepsilon_y + \varepsilon_z + \varepsilon_L}{4}.
\]
(23)

When the composite material moves at component speeds (9), we will match the speed of the mixing vector (16)
\[
U = M \cdot (i \cdot a + j \cdot v + k \cdot w + o \cdot f) = \frac{ds}{dt},
\]
(24)

therefore
\[
Mu = \frac{da}{dt}, Mv = \frac{db}{dt}, Mw = \frac{dc}{dt}, Mf = \frac{dg}{dt},
\]
(25)

The value of the displacement vector components is obtained by integration
\[
a = \int Mu \ dt - h_x, b = \int Mv \ dt - h_y, c = \int Mw \ dt - h_z, g = \int Mf \ dt - h_L,
\]
(26)

Then you can record the relationship between strain parameters and motion speeds
where, \( \gamma \) – shear angle (the lower two indices indicate the change in the angle between the respective axes).

Based on the structural compatibility of rheological models of polymer and wood materials for voltage tensor closure, we adopt a linear viscous-elastic-plastic rheological model for composite material [10]:

- for normal tension

\[
\sigma = \sigma^* + t^* \cdot d \sigma^* / d t = E \varepsilon + \mu^* \dot{\varepsilon} / d t + K^*.
\]

- for tangent tension

\[
\tau = \tau^* + t^* \cdot d \tau^* / d t = G \gamma + \mu \cdot d \gamma / d t + K,
\]

where, \( t^* \) – relaxation time, \( \mu \) – viscosity, \( K \) – plasticity, \( E \) – modulus of elasticity, \( G \) – shear modulus.

Taking into account submissions (27), (28) and (29), we obtain

\[
\begin{align*}
\sigma_x &= -p + E(\int Mu \, dt) + \mu^* \eta_x + K^*, \\
\sigma_y &= -p + E(\int Mv \, dt) + \mu^* \eta_y + K^*, \\
\sigma_z &= -p + E(\int Mw \, dt) + \mu^* \eta_z + K^*, \\
\tau_{xy} &= G(\int \eta_{xy} \, dt + \mu \eta_{xy}) + K, \\
\tau_{yz} &= G(\int \eta_{yz} \, dt + \mu \eta_{yz}) + K, \\
\tau_{zx} &= G(\int \eta_{zx} \, dt + \mu \eta_{zx}) + K,
\end{align*}
\]

where, pressure \( p = E \cdot e_0 \).
Equations of motion of viscous-elastic-plastic material of wood-polymer composite take the form:

\[
P^* \partial u / \partial t = -\partial p / \partial x + \partial / \partial t \left[ E \int \eta_x dt + \mu \eta_x + K \right] + \partial / \partial z \left[ G \int \eta_x dt + \mu \eta_x + K \right]
\]
\[
+ \partial / \partial z \left[ G \int \eta_y dt + \mu \eta_y + K \right] + \partial / \partial y \left[ E \int \eta_y dt + \mu \eta_y + K \right]
\]
\[
+ \partial / \partial y \left[ G \int \eta_y dt + \mu \eta_y + K \right] + \partial / \partial x \left[ E \int \eta_z dt + \mu \eta_z + K \right]
\]
\[
+ \partial / \partial x \left[ G \int \eta_z dt + \mu \eta_z + K \right] + \partial / \partial t \left[ G \int \eta_z dt + \mu \eta_z + K \right]
\]
\[
(31)
\]

\[
P^* \partial v / \partial t = -\partial p / \partial y + \partial / \partial t \left[ E \int \eta_y dt + \mu \eta_y + K \right] + \partial / \partial z \left[ G \int \eta_y dt + \mu \eta_y + K \right]
\]
\[
+ \partial / \partial z \left[ G \int \eta_x dt + \mu \eta_x + K \right] + \partial / \partial x \left[ G \int \eta_x dt + \mu \eta_x + K \right]
\]
\[
+ \partial / \partial x \left[ E \int \eta_z dt + \mu \eta_z + K \right] + \partial / \partial y \left[ E \int \eta_z dt + \mu \eta_z + K \right]
\]
\[
(32)
\]

\[
P^* \partial w / \partial t = -\partial p / \partial z + \partial / \partial t \left[ G \int \eta_z dt + \mu \eta_z + K \right] + \partial / \partial x \left[ G \int \eta_z dt + \mu \eta_z + K \right]
\]
\[
+ \partial / \partial x \left[ G \int \eta_y dt + \mu \eta_y + K \right] + \partial / \partial y \left[ G \int \eta_y dt + \mu \eta_y + K \right]
\]
\[
+ \partial / \partial y \left[ G \int \eta_x dt + \mu \eta_x + K \right] + \partial / \partial t \left[ E \int \eta_x dt + \mu \eta_x + K \right]
\]
\[
(33)
\]

4. Conclusion
The study on the development of the basis of mechanics of wood-polymer composites from the phenomenological position of mechanics of heterogeneous media will contribute to the improvement of production of structured composite for FDM - printing technologies.

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