Cosmological Theories From
$SO(2, 2)/SO(2) \times SO(1, 1)$

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Abstract

We herein set forth intrinsically four-dimensional string solutions and analyze some of its properties. The solutions are constructed as gauged WZW models of the coset $SO(2, 2)/SO(2) \times SO(1, 1)$. We recover backgrounds having metric and antisymmetric tensors, dilaton fields and two electromagnetic fields. The theories describe anisotropically expanding and static universes for some time values.

1 Introduction

Recently constructing exact string solutions on gravitational backgrounds has interested many string theorists, the main motivation of which has been that these backgrounds can shed light on different areas of gravitation such as the nature of singularities and black hole physics. Most of the found solutions are constructed as the product of two theories. One element that is most frequently included in these constructions is the two-dimensional, dilatonic black hole. A remarkable step was taken by Johnson

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who, by means of heterotic coset models [2], was able to move away from the
typical solution constituted by the two-dimensional dilatonic black hole in
the radial sector and a $SU(2)/U(1)$ theory in the angular sector. Heterotic
coset models provide a natural opportunity to introduce twists between the
radial and angular sectors. The use of heterotic coset models gives rise to
the possibility of employing anomalously gauged WZW models to construct
a consistent theory. The fact that there is a unique gauge extension of the
WZW model having a classical anomaly depending only on the gauge field
[3] is fundamentally important in the construction of heterotic coset models.
This anomaly cancelled against the chiral fermionic anomalies. In this pa-
per we will use heterotic coset model techniques to construct solutions that
are intrinsically four-dimensional, meaning the coset is not the product of
lower dimensional cosets. This could potentially play a fundamental role in
studying the nature of singularities. These solutions reveal very interesting
features from the gravitational viewpoint. Intrinsically four-dimensional so-
lutions constructed from vector gauged WZW models have been presented
[5], but without any evident symmetry, and so their gravitational interpre-
tation is not clear. A complete list of cosets having a single time coordinate
and up to ten dimensions has been presented in [6] based on equivalences
between gauged WZW cosets and ordinary coset models. In this paper we
focus on one four-dimensional coset that has a single time coordinate. The
classification presented [6] does not give any information on the structure of
the background in which the string propagates. The only relevant informa-
tion gained from the classification is the dimension of the space and that it
has a single time coordinate. Here we investigate the concrete structure of
the background corresponding to $SO(2, 2)/SO(2) \times SO(1, 1)$ and show that
for some values of the time coordinate this coset describes anisotropically
expanding and static universes.

2 The $SO(2, 2)/SO(2) \times SO(1, 1)$ as a vector
gauged WZW model

The key construction of this section is the coset $SO(2, 2)/SO(2) \times SO(1, 1)$
as it is defined by the diagonal vector gauging of WZW models. The action
of the diagonal vector gauging of WZW model is a particular case, corre-
spontaneously to $A^R = A^L$, of the following the action:

\[
\begin{align*}
I(g, A^R, A^L) &= kI(g) + kI_A \\
I(g) &= \frac{1}{4\pi} \int d^2 \Sigma \text{Tr} \partial z g \partial z g^{-1} \\
&+ \frac{1}{6\pi} \varepsilon^{ijk} \int_{B, \partial B = \Sigma} d^3 y g^{-1} \partial_i g g^{-1} \partial_j g g^{-1} \partial_k g \\
I_A &= \frac{1}{2\pi} \int d^2 \Sigma \text{Tr} [A^R_{\bar{z}} g^{-1} \partial_z g - A^L_{\bar{z}} \partial_z g g^{-1} \\
&+ \frac{1}{2} (A^R_{\bar{z}} A^R_{\bar{z}} + A^L_{\bar{z}} A^L_{\bar{z}})] .
\end{align*}
\]

(1)

We parametrize the group element $g \in SO(2, 2)$ as in [3], i.e., $g = h\tau$, where $h \in SO(2, 1)$ and $\tau \in SO(2, 2)/SO(2, 1)$. The matrix $\tau$ has the form:

\[
\tau = \begin{pmatrix}
\frac{b}{(b + 1)} & \frac{(b + 1)\hat{X}^\nu}{(b + 1)} \\
-(b + 1)\hat{X}_\mu & \frac{\eta^\nu_\mu}{(b + 1)} - \frac{(b + 1)\hat{X}_\mu \hat{X}^\nu}{(b + 1)}
\end{pmatrix},
\]

here $\hat{X}^\mu$ is the arrow $(x_1, -x_2, -x_3)$, $b = (1 - x^2)/(1 + x^2)$ and the indices are contracted with the Minkowski metric $\eta^\mu_\nu = diag(1, -1, -1)$. The matrix $h$ is as follows:

\[
h = \begin{pmatrix}
1 & 0 \\
0 & h^\nu_\mu
\end{pmatrix},
\]

where $h^\nu_\mu = [(1 + a_\mu^\nu)(1 - a_\mu^\nu)]^{-1}$, with $a^\mu_\nu = -a_\mu^\nu$. We consider the gauge transformation generated by

\[
T^0 = \frac{1}{2} \begin{pmatrix}
0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0
\end{pmatrix},
\]
We will impose gauge conditions on $h$, namely $a_{12} = a_{23} = 0$. A convenient substitution is $\cosh r = (1 + a_{13}^2)/(1 - a_{13}^2)$, which is possible for $|a_{13}| < 1$. The kinetic term of the WZW model can be represented in a more compact form after the following change of variables which diagonalizes the part of the metric corresponding to the coset element $\tau$:

$$
\begin{align*}
    x_1 &= t \cosh \theta, \\
    x_2 &= t \sinh \theta \cos \psi, \\
    x_3 &= t \sinh \theta \sin \psi.
\end{align*}
$$

In these new coordinates we obtain

$$
T^1 = \frac{1}{2} \begin{pmatrix}
    0 & 0 & 0 & 0 \\
    0 & 0 & 0 & 0 \\
    0 & 0 & 0 & 1 \\
    0 & 0 & -1 & 0
\end{pmatrix}.
$$

(2)

The Wess–Zumino term is by construction a closed three-form. According to the Poincare lemma it can be rewritten locally as a two-form. For the case under consideration we have:

$$
\begin{align*}
    Tr \partial_{\mu} g \partial^{\mu} r^{-1} &= \frac{8}{(1 + t^2)^2} \partial_{\mu} t \partial^{\mu} t \\
    &- \frac{8t^2}{1 + t^2} \left( \partial_{\mu} \theta \partial^{\mu} \theta + \sinh^2 \theta \partial_{\mu} \psi \partial^{\mu} \psi + \sin \psi \partial_{\mu} \theta \partial^{\mu} r \\
    &+ \frac{2}{2} \cos \psi \partial_{\mu} \psi \partial^{\mu} r \right) - 2 \partial_{\mu} r \partial^{\mu} r.
\end{align*}
$$

(4)

The currents of the model, which are defined as $J^a_{\bar{z}} = Tr T^a \partial_z g g^{-1}$ and $J^a_z = Tr T^a g^{-1} \partial_z g$ with $a = 0, 1$, are:

$$
\epsilon^{\mu\nu} b^a_{ij} \partial_{\mu} x_i \partial_{\nu} x_j = \epsilon^{\mu\nu} \frac{2t^2}{1 + t^2} (2 \sin \psi \partial_{\mu} \theta \partial_{\nu} r + \cos \psi \sinh 2 \theta \partial_{\mu} \psi \partial_{\nu} r) 
$$

(5)
\[
J^0_z = \frac{t^2}{1 + t^2} (4 \cos \psi \cosh \theta \partial_z \theta \\
- 4 \sinh \theta (\sin \psi \cosh \theta \cosh r - \sinh r \sinh \theta) \partial_z \psi)
\]

\[
J^1_z = \frac{t^2}{1 + t^2} (4 \cos \psi \sinh \theta \partial_z \theta \\
- 4 \sinh \theta (\sin \psi \cosh \theta \sinh r - \cosh r \sinh \theta) \partial_z \psi)
\]

\[
J^0_z = \frac{t^2}{1 + t^2} (2 \sinh^2 \theta \sin \psi \partial_z r \\
- 4 \cos \psi \partial_z \theta + 2 \sin \psi \sinh 2 \theta \partial_z \psi)
\]

\[
J^1_z = \frac{t^2}{1 + t^2} (2 \sinh 2 \theta \cos \psi \partial_z r + 4 \sinh^2 \theta \partial_z \psi).
\]

\(6\)

According to general practice we now exclude gauge field in order to read off the low energy content of the theory. The result is

\[
I(g, A(g)) = kI(g) - \frac{k}{2\pi} \int_{\Sigma} d^2 z. J^a_z (M^{ab})^{-1} J^b_z,
\]

\(7\)

where \(M^{ab} = Tr(T^a T^b - T^a g T^b g^{-1})\). The values of the inverse matrix are

\[
(M^{-1})_{00} = \frac{2t^2}{\Delta(1 + t^2)} (\cosh 2 \theta \cosh r \\
+ \sin \psi \sinh 2 \theta \sinh r - 1 + \frac{2}{t^2} \sinh^2 (r/2))
\]

\[
(M^{-1})_{01} = -\frac{2t^2}{\Delta(1 + t^2)} (\cosh 2 \theta (\sinh r + \sin \psi \cosh r) + \frac{1}{t^2} \sinh r)
\]

\[
(M^{-1})_{10} = -\frac{2t^2}{\Delta(1 + t^2)} (\sin \psi (\sinh 2 \theta \cosh r + \sin \psi \cosh 2 \theta \sinh r) + \cos^2 \psi
\]

\[
- \frac{1}{t^2} \sinh r)
\]

\[
(M^{-1})_{11} = \frac{2t^2}{\Delta(1 + t^2)} (\sin \psi (\sinh 2 \theta \sinh r + \sin \psi \cosh 2 \theta \cosh r)
\]

\[
+ \cosh r \cos^2 \psi + 1 - \frac{2}{t^2} \sinh^2 (r/2))
\]

\(8\)
where $\Delta = \det M^{ab}$. The proper integration of the gauge field in the large $k$ (one-loop) approximation includes shifting the dilaton field as follow:

$$
\hat{\Phi} = \Phi_0 + \ln \det M^{ab} \\
= \Phi_0 + \ln \left( \frac{t^2}{1 + t^2} \cos^2 \psi \sinh^2 \theta \cosh r + \frac{2}{1 + t^2} \sinh^2 (r/2) \\
+ \frac{t^2(t^2 - 1)}{(1 + t^2)^2} \cos^2 \psi \sin^2 \theta \right)
$$

(9)

The whole low energy background of this theory consists as expected of a metric and antisymmetric tensors as well as a dilaton field. In general it does not have any evident symmetry. A very interesting feature of this solution is that although for $t = 0$ the metric has an initial singularity the dilaton does not blow up at this point and equals $\hat{\Phi} = 2 \ln \sinh (r/2)$. For small values of $t$ the metric is described by eq. (4), which we rewrite as

$$
ds^2 = dt^2 - \frac{1}{4} dr^2 - t^2(d\theta^2 + \sinh^2 \theta d\psi^2) \\
- t^2(\sin \psi d\theta dr + \frac{\sinh \theta}{2} \cos \psi d\psi dr).
$$

(10)

The first line describes a Kantowski-Sachs metric with negative curvature, the second line is a modification. This metric represents an expanding universe since the volume element

$$
\sqrt{-g} = \frac{1}{2} t^2 \sinh \theta
$$

constantly increases in the range of $t$ under consideration. The separation between two observers is $t \Delta \theta$ if only their $\theta$ coordinates differ and $\Delta r$ if only their $r$ coordinates differ. Thus, the distances measured only in the $\theta$ and $\psi$ directions expand on a rate proportional to $t$ while in the $r$ direction do not expand at all. We can conclude that the model describes an anisotropically expanding universe with the metric being a modification of the Kantowski-Sachs metric with negative curvature. It can also be checked that for large values of $t$, i.e. $t >> 1$, the metric tensor has the form:

$$
ds^2 = \frac{1}{t^4} dt^2 - h_{ij} dx^i dx^j,
$$

(11)
where \( h_{ij} \) is independent of \( t \) and must be read off from eqs. (4,6-8) in the large \( t \) limit, and \( x^i = (r, \theta, \psi) \). After the substitution \( T = 1/t \) the metric becomes
\[
ds^2 = dT^2 - h_{ij} dx^i dx^j,
\]
which is evidently a static Universe. We can conclude that the low energy limit of the analyzed conformal field theory is a universe that has an initial singularity and then expands to a static limit. For intermediate values of \( t \) we are not able to give an exact cosmological interpretation of the theory. To consider this bosonic solution as part of a heterotic string we have to follow the Kazama–Suzuki construction and add left and right fermions whose anomalies cancel among themselves. In the next section we will construct generalized solutions of this result.

3 Heterotic Coset Model techniques

3.1 A Background with Electromagnetic fields

In this subsection we present the simplest background that can be obtained using heterotic coset model techniques. In fact, this is a cousin of the monopole theory of Giddings Polchinski and Strominger \[7\] as presented by Johnson \[4\]. We start with the right gauging of the WZW model, which can be obtained from (1) taking \( A^L = 0 \) and \( A^R = A \):
\[
I_{RZWZW}(g, A) = kI(g) + \frac{k}{2\pi} \int d^2z Tr A_z g^{-1} \partial_z g - \frac{k}{4\pi} \int d^2z Tr A_z A_z.
\]
Under the transformations
\[
g \rightarrow gh \quad \delta g = -gu \quad \delta A_a = -D_a u = -\partial_a u - [A_a, u],
\]
the action changes to
\[
\delta I(g, A) = \frac{k}{4\pi} \int d^2z u(\partial_z A_z - \partial_z A_z).
\]
In the heterotic coset model this classical anomaly cancels against the one-loop anomalies produced by the fermions. The anomaly of the right fermions
is fixed by supersymmetry since we require supersymmetry in the right sector. The number of right fermions is $\text{Dim}(G) - \text{Dim}(H) = 4$, they are all minimally coupled to $A_z$ and produce an anomaly of the form:

$$\delta I_F^R = \frac{1}{4\pi} \int \frac{d^2z}{\Sigma} Tr u F_{z\bar{z}}. \tag{14}$$

The left moving fermions play the role of current algebra, are minimally coupled to $A_z$ with coupling constant $Q$ and produce an anomaly:

$$\delta I_F^L = -\frac{Q^2}{4\pi} \int \frac{d^2z}{\Sigma} Tr u F_{z\bar{z}}. \tag{15}$$

The anomaly cancelation condition is

$$k = Q^2 - 1$$

Reaching the low energy limit is not as easy as in the previous section, the problem is that now the level $k$ is related to the coupling $Q$. This implies that the fermions affect the metric in the same order as the gauge fields. To solve this puzzle and be able to integrate out the gauge field we need a classically gauge invariant action. This can be achieve by bosonizing the fermions. The bosonization of the fermions is not trivial. Keeping in mind that taking care of the anomalies is our main concern, it will be enough to find a bosonic action with the same anomaly as the fermion system. An anomalously gauged WZW model at level $k = 1$ exhibits the desired anomaly properties. Nevertheless, following this direction we find that reading off the metric is very complicated. We can proceed as \ref{eq:14} and introduce a bosonic action yielding the same anomaly,

$$I_B = \frac{1}{4\pi} \int \frac{d^2z}{\Sigma} (\partial_\pm \phi^a - Q_\pm A_\pm^a)(\partial_\pm \phi^a - Q_\pm A_\pm^a) - Q_\pm \phi^a F_{a z\bar{z}}, \tag{16}$$

here $Q_\pm = Q \pm 1$, $a = 0, 1$. This action under the transformations $\delta \phi^a = Q_+ u^a$ and $\delta A_\pm^a = \partial_\pm u^a$ yields the desired anomaly. The index $a$ is contracted using the metric $\eta^{ab} = Tr T^a T^b = 1/2 diag(1, -1)$. The total action that we get is:
\[ I_{total} = kI(g) + \frac{1}{4\pi} \int \frac{d^2z}{\Sigma} (\partial_z \phi^a \partial_z \phi^a) \]

\[ + \ A_2^a(2kJ_2^a - (Q_+ - Q_-)\partial_z \phi^a) \]

\[ - \ A_2^a(Q_+ + Q_-)\partial_z \phi^a \]

\[ + \ A_2^aA_2^a(-k + Q_2^a) \]. \tag{17} \]

Since the gauge field is nondynamical and enters the action quadratically we can integrate it obtaining

\[ I_{total} = kI(g) + \frac{1}{4\pi} \int \frac{d^2z}{\Sigma} (\partial_z \phi^a \partial_z \phi^a) \]

\[ - \ \frac{(Q_+ + Q_-)}{k - Q_2^a} \partial_z \phi^a (2kJ_2^a - (Q_+ - Q_-)\phi^a) \]. \tag{18} \]

¿From this action it is still not clear the way the metric should be affected by the integration of the gauge field, to see this explicitly we have to prepare the action for refermionization or at least present it in a suitable form. We have to build a convenient term of the form \( \tilde{D}_+ \phi \tilde{D}_- \phi \), forming this term uniquely determines the metric and the gauge field. We rewrite the total action as:

\[ I_{total} = kI(g) + \frac{Q_2^a - Q_2^a}{4\pi(k - Q_2^a)} \int \frac{d^2z}{\Sigma} (\partial_z \phi^a \partial_z \phi^a) \]

\[ + \ \frac{1}{4\pi} \int \frac{d^2z}{\Sigma} \left( (\partial_z \phi^a - \frac{k(Q_+ + Q_-)}{k - Q_2^a} J_2^a) (\partial_z \phi^a - \frac{k(Q_+ + Q_-)}{k - Q_2^a} I_2^a) \right) \]

\[ - \ \frac{k(Q_+ + Q_-)}{k - Q_2^a} (J_2^a \partial_z \phi - \partial_z \phi I_2^a) \]

\[ - \ \frac{k^2(Q_+ + Q_-)^2}{(k - Q_2^a)^2} \left( J_2^a I_2^a \right) \]. \tag{19} \]

where \( I_2^a = TrT^a g^{-1} \partial_z g \). The last term is the one that affects the part of the metric defined by \( kI(g) \), the third term tells us that the fermions will now be in the presence of an abelian field. The final background is:
\[ ds^2 = \frac{1}{(1 + t^2)^2} dt^2 - \frac{t^2}{1 + t^2} h_{ij} dx^i dx^j - \frac{t^4}{(1 + t^2)^2} \gamma_{ij} dx^i dx^j - 2dr^2 \]

\[ b_{\theta r} = \frac{4t^2}{1 + t^2} \sin \psi \quad b_{\psi r} = \frac{2t^2}{1 + t^2} \sinh 2\theta \cos \psi \]

\[ A^0_\psi = \frac{2t^2}{1 + t^2} \sinh^2 \theta \sin 2\psi \quad A^1_\psi = \frac{2t^2}{1 + t^2} \sin 2\theta \cos \psi \]

\[ A^0_\theta = -\frac{4t^2}{1 + t^2} \cos \psi \quad A^1_\theta = 0 \]

\[ A^0_\psi = \frac{2t^2}{1 + t^2} \sin 2\theta \sin \psi \quad A^1_\psi = \frac{4t^2}{1 + t^2} \sinh^2 \theta \]

\[ A^0_\theta = 0 \quad A^1_\theta = 0 \]

\[ \Phi = \Phi_0 + \ln(-k + Q^2_+) \]. (20)

As we see the dilaton field is trivially shifted by a constant, the expression for the antisymmetric tensor is entirely determined by \( I(g) \). As in the previous section for small values of \( t \), we have an anisotropically expanding universe and for large values of \( t \) we have a static universe. The major difference with the vector gauge WZW of the previous section is the presence of two sets of abelian gauge fields. Both electromagnetic fields depend on time. As a function of time the electric and magnetic fields are zero for \( t = 0 \) and for large values of \( t \) the electric field tends to zero while the magnetic field goes to a constant. One feature of this solution is that the value of the charge is fixed by the choice of \( k \). In the next subsection we will see that heterotic coset model techniques provide an opportunity to avoid this restriction and construct solutions whose charge is arbitrary in the sense that it is not constrained by the choice of the level \( k \).

### 3.2 A background with arbitrary charge

The next generalization of the presented construction consists of including a parameter \( \delta \) to move away for the restricting condition on \( Q \). We now start with an anomalously gauged WZW action eq. (1) for which \( t_R = T^a \) and \( t_L = \delta T^a \) with \( a = 0, 1 \). The fermionic action is the same as in the previous subsection and the anomaly cancellation condition now reads as
\[ k(1 - \delta^2) = Q^2 - 1 \] (21)

Again we bosonize the fermionic sector to properly integrate out the gauge field. The total action is now of the form:

\[
I_{total} = kI(g) + \frac{1}{4\pi} \int d^2 z \partial_\Sigma \phi^a \partial_\Sigma \phi^a
+ \frac{1}{4\pi} \int d^2 z \left( A^a_z \right) \left( 2kJ^a_z - (Q_+ - Q_-) \partial_\Sigma \phi^a \right)
- A^a_z \left( 2kJ^a_z + (Q_+ + Q_-) \partial_\Sigma \phi^a \right)
- A^a_z A^b_z \left( \eta^{ab} (k(1 + \delta^2) - Q^2_{+}) - 2k\delta Tr T^a g T^b g^{-1} \right). \tag{22}
\]

Integrating out the gauge field and accommodating the action as in the previous subsection we obtain:

\[
I_{total} = kI(g) + \frac{1}{4\pi} \int d^2 z \left( \partial_\Sigma \phi^a \partial_\Sigma \phi^b \Lambda^{ab} \right)
- 4k^2 J^a_z \Lambda^{ab} J^b_z
+ \frac{1}{4\pi} \int d^2 z \left( \left( \partial_\Sigma \phi^a - 2kQ J^a_z \Lambda^{ab} \right) + 2k I^b_z \Lambda^{ba} \right)
\times \left( \partial_\Sigma \phi^a - 2kQ I^a_z \Lambda^{ab} + 2k J^a_z \Lambda^{ba} \right)
- k(Q_+ + Q_-) \Lambda^{ab} \left( J^a_z \partial_\Sigma \phi^b - I^a_z \partial_\Sigma \phi^b \right)
- k(Q_+ - Q_-) \Lambda^{ba} \left( J^a_z \partial_\Sigma \phi^b - I^a_z \partial_\Sigma \phi^b \right)
- (k(Q_+ + Q_-))^2 J^a_z I^a_z \Lambda^{ai} \Lambda^{aj}
- (k(Q_+ - Q_-))^2 I^a_z J^a_z \Lambda^{ai} \Lambda^{aj}
- k^2 \left( Q^2_+ - Q^2_- \right) \left( I^a_z J^a_z \Lambda^{ai} \Lambda^{aj} + I^a_z I^a_z \Lambda^{ai} \Lambda^{aj} \right), \tag{23}
\]

where \( I^a_z = Tr \partial_\Sigma gg^{-1} \) and \( \Lambda^{ab} \) is the inverse matrix to \( M^{ab} = \eta^{ab} (k(1 + \delta^2) - Q^2_{+}) - 2k\delta Tr T^a g T^b g^{-1} \). The dilaton is shifted by

\[ \tilde{\Phi} = \Phi_0 + \ln det M^{ab}. \]

As opposed to the previous subsection the dilaton shift is not trivial. Reading off the background is essentially the same as in the previous subsection.
The general result is very complicated and so we outline the relevant characteristics of this solution. For $\delta = 0$ the background is reduced to the one of the previous subsection and for $\delta = 1$ we recover the results of the previous section for the anomaly free gauged WZW model. From here we can already conjecture that this is another cosmological theory for some values of $\delta$ and in the limits of large and small $t$. For arbitrary values of $\delta$ one can check that the metric indeed behaves the same as the other presented backgrounds. The two electromagnetic fields present the same asymptotical behavior in time as in the previous subsection, but do not depend on the time coordinate as simply as in the previous case.

4 Conclusions

We have constructed some intrinsically four-dimensional heterotic string solutions. The low energy limit of the first solution (vector gauged WZW model) includes metric and antisymmetric tensors and dilaton field, this solution describes an anisotropically expanding universe for small values of $t$ and a static universe for large values of $t$; it also has an initial singularity but it is remarkable that the dilaton does not blow up at this singularity. The other solutions, apart from these characteristics possess two sets of electromagnetic fields, in one case with a fixed value of the charge and in the other with arbitrary charge. We conjecture that the charged solutions are possibly extremal solutions since we actually have very few parameters; in the first case all the parameters are fixed, in the second we only have one free parameter, the charge $Q$ (or $\delta$). An interesting unanswered question is to what extend these solutions can be obtained as low energy limits of higher dimensional theories. Given that for large values of $t$ the metric describes a static universe and has one timelike killing vector it seems interesting to relate the properties of these solutions to the symmetries of the low energy string effective action and determine which of them can be obtained starting from other simpler known solutions.

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