Optical Depth of the Cosmic Microwave Background and Reionization of the Intergalactic Medium

J. Michael Shull & Aparna Venkatesan

University of Colorado, Department of Astrophysical & Planetary Sciences, CASA, 389-UCB, Boulder, CO 80309

mshull@casa.colorado.edu, avenkatesan@usfca.edu

ABSTRACT

We examine the constraints on the epoch of reionization (redshift $z_r$) set by recent WMAP-3 observations of $\tau_e = 0.09 \pm 0.03$, the electron-scattering optical depth of the cosmic microwave background (CMB), combined with models of high-redshift galaxy and black hole formation. Standard interpretation begins with the computed optical depth, $\tau_e = 0.042 \pm 0.003$, for a fully ionized medium out to $z = 6.1 \pm 0.15$, including ionized helium, which recombines at $z \approx 3$. At $z > z_r$, one must also consider scattering off electrons produced by from early black holes (X-ray pre-ionization) and from residual electrons left from incomplete recombination. Inaccuracies in computing the ionization history, $x_e(z)$ add systematic sources of uncertainty in $\tau_e$. The required scattering at $z > z_r$ can be used to constrain the ionizing contributions of “first light” sources. In high-z galaxies, the star-formation efficiency, the rate of ionizing photon production, and the photon escape fraction are limited to producing no more than $\Delta \tau_e \leq 0.03 \pm 0.03$. The contribution of minihalo star formation and black-hole X-ray preionization at $z = 10–20$ are suppressed by factors of 5–10 compared to recent models. Both the CMB optical depth and H I (Ly$\alpha$) absorption in quasar spectra are consistent with an H I reionization epoch at $z_r \approx 6$ providing $\sim 50\%$ of the total $\tau_e$ at $z \leq z_r$, preceded by a partially ionized medium at $z \approx 6–20$.

Subject headings: cosmology: theory — cosmic microwave background — intergalactic medium

1Now at Department of Physics, 2130 Fulton St., University of San Francisco, San Francisco, CA 94117
1. INTRODUCTION

In recent years, an enormous amount of exciting cosmological data have appeared, accompanied by theoretical statements about early galaxy formation and the first massive stars. Many of these statements were reactions to first-year (WMAP-1) results (Kogut et al. 2003; Spergel et al. 2003) from the Wilkinson Microwave Anisotropy Probe (WMAP). These papers inferred a high optical depth to the cosmic microwave background (CMB) and suggested early reionization of the intergalactic medium (IGM). Other conclusions came from simplified models and assumptions about the stellar initial mass function (IMF), atomic/molecular physics, radiative processes, and prescriptions for star formation rates and escape of photoionizing radiation from protogalaxies.

In this paper, we focus on the reionization epoch, defined as the redshift \( z_r \) when the IGM becomes nearly fully ionized over most of its volume (Gnedin 2000, 2004). Our knowledge about reionization comes primarily from two types of observations: H I (Ly\( \alpha \)) absorption in the IGM and optical depth of the CMB. Spectroscopic studies of the “Gunn-Peterson” (Ly\( \alpha \)) absorption toward high-redshift quasars and galaxies imply that H I reionization occurred not far beyond \( z \sim 6 \) (Becker et al. 2001; Fan et al. 2002, 2006), and that He II reionization occurred at \( z \sim 3 \) (Kriss et al. 2001; Shull et al. 2004; Zheng et al. 2004). Third-year data from WMAP (Spergel et al. 2006; Page et al. 2006) suggest that reionization might occur at \( z \approx 10 \), although large uncertainties remain in modeling of the CMB optical depth.

The WMAP and Ly\( \alpha \) absorption results are not necessarily inconsistent, since they probe small amounts of ionized and neutral gas, respectively. In addition, both the H I absorbers and ionized filaments in the “cosmic web” (Cen & Ostriker 1999) are highly structured at redshifts \( z < 10 \) and affect the optical depths in Ly\( \alpha \). For example, in order to effectively absorb all the Ly\( \alpha \) radiation at \( z \approx 6 \) requires a volume-averaged neutral fraction of just \( x_{HI} \approx 4 \times 10^{-4} \) (Fan et al. 2006). Simulations of the reionization process (Gnedin 2004; Gnedin & Fan 2006) show that the transition from neutral to ionized is extended in time between \( z = 5 - 10 \). The first stage (pre-overlap) involves the development and expansion of the first isolated ionizing sources. The second stage marks the overlap of the ionization fronts and the disappearance of the last vestiges of low-density neutral gas. Finally, in the post-overlap stage, the remaining high-density gas is photoionized. The final drop in neutral fraction and increase in photon mean free path occur quite rapidly, over \( \Delta z \approx 0.3 \).

Initial interpretations of the high WMAP-1 optical depth were based on models of “sudden reionization”, neglecting scattering from a partially ionized IGM at \( z > z_r \). The revised optical depths, \( \tau_e = (0.09 - 0.10) \pm 0.03 \) (Spergel et al. 2006; Page et al. 2006) also assume a single step to complete ionization \( (x_e = 1 \text{ at redshift } z_r) \), although Spergel et al. (2006) explored a two-step ionization history, resulting in a broader distribution of \( \tau_e \). However, the \( \chi^2 \) curves
for two-step ionization provide little constraint on the redshift of reionization, or on an IGM with low partial ionization fractions, \( x_e \ll 0.1 \), as we discuss in § 3. Figure 3 of Spergel et al. (2006) shows that the parameters \( x_e \) and \( z_r \) are somewhat degenerate, each with a long tail in the likelihood curves. This emphasizes the possible importance of contributions to \( \tau_e \) from ionizing UV photons at \( z > z_r \), from early massive stars and X-rays from accreting black holes (Ricotti & Ostriker 2004; Begelman, Volonteri, & Rees 2006).

Models of the extended recombination epoch (Seager, Sasselov, & Scott 2000) predict a partially ionized medium at high redshifts, owing to residual electrons left from incomplete recombination. These electrons produce additional scattering, \( \Delta \tau_e \approx 0.06 \left( z \approx 10^{-700} \right) \). These effects must be computed in CMB radiation transfer codes such as CMBFAST and RECFAST, but only a portion of this scattering affects the large angular scales (\( \ell \leq 10 \)) where WMAP detects a polarization signal. X-ray preionization can also produce CMB optical depths \( \tau_e \geq 0.01 \) (Venkatesan, Giroux, & Shull 2001, hereafter VGS01; Ricotti, Ostriker, & Gnedin 2005).

In § 2.1, we show that a fully ionized IGM from \( z = 0 \) to the reionization epoch at \( z_r = 6.1 \pm 0.15 \) (Gnedin & Fan 2006), produces optical depth, \( \tau_e \approx 0.042 \pm 0.003 \), nearly half the WMAP-3 value. Therefore, the high-redshift ionizing sources are limited to producing an additional optical depth, \( \Delta \tau_e \leq 0.03 \pm 0.03 \). In § 3, we discuss the resulting constraints on the amount of star formation and X-ray activity at \( z \gtrsim 7 \) and limits on star formation in mini-halos. The IGM probably has a complex reionization history, with periods of extended reionization for H I and He II (Venkatesan, Tumlinson, & Shull 2003; Cen 2003; Wyithe & Loeb 2003; Hui & Haiman 2003). Because \( \tau_e \) measures the integrated column density of electrons, there are many possible scenarios consistent with the current (WMAP-3) level of CMB data.

\section*{2. OPTICAL DEPTH TO ELECTRON SCATTERING}

\subsection*{2.1. Analytic Calculation of Optical Depth}

To elucidate the dependence of CMB optical depth on the epoch of reionization (redshift \( z_r \)), we integrate the electron scattering optical depth, \( \tau_e(z_r) \), for a homogeneous, fully ionized medium out to \( z_r \). For instantaneous, complete ionization at redshift \( z_r \), we calculate \( \tau_e \) as the integral of \( n_e \sigma_T dl \), the electron density times the Thomson cross section along proper length,

\[
\tau_e(z_r) = \int_0^{z_r} n_e \sigma_T (1 + z)^{-1} \left[ \frac{c}{H(z)} \right] dz .
\]

We adopt a standard ΛCDM cosmology, in which \( (dl/dz) = c(dt/dz) = (1 + z)^{-1} [c/H(z)] \), where \( H(z) = H_0 [\Omega_m (1 + z)^3 + \Omega_\Lambda]^{1/2} \) and \( \Omega_m + \Omega_\Lambda = 1 \) (no curvature). The densities of
hydrogen, helium, and electrons are written \( n_H = [(1-Y) \rho_{cr}/m_H](1+z)^3 \), \( n_{He} = yn_H \), and \( n_e = n_H(1+y) \), if helium is singly ionized. We also add the small \( (\tau_e \approx 0.0020) \) contribution from doubly-ionized helium (He III) at redshifts \( z \leq 3 \) (Shull et al. 2004). We assume a helium mass fraction by mass \( Y = 0.244 \) and define \( y = (Y/4)/(1-Y) \approx 0.0807 \) (He fraction by number). The critical density is \( \rho_{cr} = (1.879 \times 10^{-29} \text{ g cm}^{-3})h^2 \) where \( h = (H_0/100 \text{ km s}^{-1} \text{ Mpc}^{-1}) \). The above integral can be done analytically:

\[
\tau_e(z_r) = \left(\frac{c}{H_0}\right) \left(\frac{2\Omega_b}{3\Omega_m}\right) \left[\frac{\rho_{cr}(1-Y)(1+y)\sigma_T}{m_H}\right] \left\{\Omega_m(1+z_r)^3 + \Omega_\Lambda\right\}^{1/2} - 1 ,
\]

where \( (1-Y)(1+y) = (1-3Y/4) \). For large redshifts, \( \Omega_m(1+z)^3 \gg \Omega_\Lambda \), and the integral simplifies to

\[
\tau_e(z_r) \approx \left(\frac{c}{H_0}\right) \left(\frac{2\Omega_b}{3\Omega_m}\right) \left[\frac{\rho_{cr}(1-3Y/4)\sigma_T}{m_H}\right] (1 + z_r)^{3/2} \approx (0.0533) \left[\frac{(1 + z_r)}{8}\right]^{3/2} .
\]

Here, we substituted the WMAP-3 parameters (Spergel et al. 2006): \( \Omega_b h^2 = 0.0223 \pm 0.0009 \), \( \Omega_m h^2 = 0.127 \pm 0.007 \), and \( h = 0.73 \pm 0.03 \), and adopted a primordial helium abundance \( Y_p = 0.244 \pm 0.002 \) (Izotov & Thuan 1998; Olive & Skillman 2001). A recent paper (Peimbert et al. 2007) suggests a revised \( Y_p = 0.2474 \pm 0.0028 \) based on new He I atomic data.

From the approximate expression (eq. 3), we see that \( \tau_e \propto (\rho_{cr}\Omega_b\Omega_m^{1/2}H_0^{-1}) \). Thus, \( \tau_e \) is nearly independent of the Hubble constant, since \( \rho_{cr} \propto h^2 \) while the combined parameters, \( \Omega_b h^2 \) and \( \Omega_m h^2 \), are inferred from D/H, CMB, and galaxy dynamics. The scaling with \( h \) cancels to lowest order; a slight dependence remains from the small \( \Omega_\Lambda \) term in equation (2).

If we invert the approximate equation (3), we can estimate the primary reionization redshift, \( (1+z_r) \approx (6.6)[\tau_e(z_r)/0.04]^{2/3} \), where we scaled to the value, \( \tau_e = 0.04 \), expected for full ionization back to \( z_r \approx 6 \). As discussed in § 2.2, this is approximately the WMAP-3 value of optical depth \( (\tau_e \approx 0.09) \) reduced by \( \Delta \tau_e \approx 0.05 \). This extra scattering may arise from high-z star formation, X-ray preionization, and residual electrons left after incomplete recombination. The latter electrons are computed to have fractional ionization \( x_e \approx (0.5 - 3.0) \times 10^{-3} \) between \( z = 10-700 \) (Seager et al. 2000). Inaccuracies in computing their contribution therefore add systematic uncertainty to the CMB-derived value of \( \tau_e \). Partial ionization may also arise from the first stars (Venkatesan, Tumlinson, & Shull 2003, hereafter VTS03) and from penetrating X-rays produced by early black holes (VGS01; Ricotti & Ostriker 2004, 2005). These additional ionization sources contribute electron scattering that must be subtracted from the WMAP-3 values. They will lower the reionization redshift, \( z_r = 10.7_{-2.3}^{+2.7} \), derived (Spergel et al. 2006) in the absence of partial ionization at \( z > z_r \), and they may bring the WMAP-3 and Gunn-Peterson results into agreement for the epoch of complete reionization.

In our calculations, described in § 3, we make several key assumptions. First, we assume a fully ionized IGM out to \( z_r \approx 6 \), accounting for both \( \text{H}^+ \) and ionized helium. (Helium
contributes 8% to $\tau_e$, assuming He II at $z > 3$ and an additional $\tau_e \approx 0.002$ for He III at $z \leq 3$). Second, we investigate the effects of IGM partial ionization at $z > z_r$. Finally, in computing the contribution of residual electrons at high redshifts, we adopt the concordance parameters from the WMAP-3 data set. The CMB optical depth is formally only a 3$\sigma$ result, which may change, as WMAP produces better determinations of the matter density, $\Omega_m$, and the parameters, $\sigma_8$ and $n_s$, that govern small-scale power. Both $\sigma_8$ and $n$ have well-known degeneracies with $\tau_e$ in CMB parameter extraction (Spergel et al. 2006). Therefore, their derived values may change in future CMB data analyses, especially as the constraints on $\tau_e$ continue to evolve. In addition, inaccuracies in the incomplete recombination epoch and residual ionization history, $x_e(z)$, add uncertainties to the CMB radiative transfer, the damping of $\ell$-modes, and the polarization signal used to derive an overall $\tau_e$.

2.2. Residual Electrons in the IGM

We now discuss the contribution of residual electrons in the IGM following the recombination epoch at $z \approx 1000$. Scattering from these electrons is significant and is normally accounted for in CMB transport codes such as CMBFAST (Seljak & Zaldarriaga 1996) through the post-recombination IGM ionization history, $x_e(z)$. However, a number of past papers are vague on how the ionization history is treated, which has led to confusion in how much residual optical depth and power-damping has been subtracted from the CMB signal. Modern calculations of how the IGM became neutral have been done by Seager et al. (2000), although their code (RECFAST) continues to be modified to deal with subtle effects of the recombination epoch and the atomic physics of hydrogen ($2s \rightarrow 1s$) two-photon transitions (W. Wong & D. Scott, private communication).

To illustrate the potential effects of high-$z$ residual electrons, we have used numbers from Figure 2 of Seager et al. (2000), the top-panel model, which assumed a cosmology with $\Omega_{\text{tot}} = 1$, $\Omega_b = 0.05$, $h = 0.5$, $Y = 0.24$, and $T_{\text{CMB}} = 2.728$ K. At low redshifts, $z \approx z_r$, just before reionization, they find a residual electron fraction $x_{e,0} \approx 10^{-3.3}$. We fitted their curve for log $x_e$ out to $z \approx 500$ to the formula:

$$x_e(z) = x_{e,0} \ 10^{0.001(1+z)} \approx (5 \times 10^{-4}) \ \exp[\alpha(1+z)],$$

where $\alpha \approx 2.303 \times 10^{-3}$. More recent recombination calculations (W. Y. Wong & D. Scott, private communication) using WMAP-3 parameters ($\Omega_b = 0.04$, $h = 0.73$, $\Omega_m = 0.24$, $\Omega_{\Lambda} = 0.76$, $Y = 0.244$) find somewhat lower values, $x_{e,0} \approx 10^{-3.67}$ with $\alpha \approx 2.12 \times 10^{-3}$. We attribute the lower $x_{e,0}$ to the faster recombination rates arising from their higher assumed baryon density, $\Omega_b h^2 = 0.0213$, compared to $\Omega_b h^2 = 0.0125$ in Seager et al. (2000).
To compute the electron-scattering of the CMB from these “frozen-out” electrons, we use the same integrated optical depth formula (equation 1), in the high-\(z\) limit, where \(\frac{dv}{dz} = (1 + z)^{-1}[c/H(z)] \approx (c/H_0)\Omega_m^{-1/2}(1 + z)^{-5/2}\). We integrate over the residual-electron history, from \(z_r \approx 7\) back to a final redshift \(z_f \gg z_r\), to find

\[
(\Delta \tau_e)_{\text{res}} = \left(\frac{c}{H_0}\right) \left[\frac{\rho_{\text{ix}}(1 - Y)\sigma_T\Omega_b}{\Omega_m^{1/2} m_H}\right] \int_{z_r}^{z_f} (1 + z)^{1/2} x_e(z) \, dz
\]

\[
= (3.27 \times 10^{-3}) x_{e,0} \int_{(1+z_r)}^{(1+z_f)} u^{1/2} \exp(\alpha u) \, du \, .
\] (5)

A rough estimate to the residual scattering comes from setting \(\alpha = 0\) and adopting a constant ionized fraction \(x_e(z)\),

\[
(\Delta \tau_e)_{\text{res}} \approx (2.36 \times 10^{-3})[(1 + z_f)^{3/2} - (1 + z_r)^{3/2}] \langle x_e \rangle \, .
\] (6)

This estimate gives \(\tau_e = 0.044\) for \(z_r = 6\), \(z_f = 700\), and \(\langle x_e \rangle \approx 10^{-3}\). More precise values of \(\tau_e\) can be derived from the exact integral (eq. 5) by expanding the exponential as a sum and adopting the limit \(z_f \gg z_r\),

\[
(\Delta \tau_e)_{\text{res}} = (0.0184) \left[\frac{(1 + z_f)}{501}\right]^{3/2} \sum_{n=0}^{\infty} \frac{[\alpha(1 + z_f)]^n}{n!} \, .
\] (7)

From \(z_r = 7\) out to various final redshifts, we find cumulative optical depths: \((\Delta \tau_e)_{\text{res}} = 0.0041\) (from \(7 < z < 200\)), \((\Delta \tau_e)_{\text{res}} = 0.0088\) (from \(7 < z < 300\)), \((\Delta \tau_e)_{\text{res}} = 0.0157\) (from \(7 < z < 400\)), and \((\Delta \tau_e)_{\text{res}} = 0.0256\) (from \(7 < z < 500\)). For \(z > 500\), the approximate formula (eq. 4) underestimates \(x_e\), but one can integrate the appropriate curves (Seager et al. 2000) using piecewise-continuous linear fits. Between \(500 < z < 600\), we find \(x_e = (2.94 \times 10^{-4})\exp[0.003224(1 + z)]\), and for \(600 < z < 700\), \(x_e = (1.28 \times 10^{-4})\exp[0.004606(1 + z)]\). Integration then yields additional contributions of \(\Delta \tau_e \approx 0.0135\) for \(z = 500–600\) and \(\Delta \tau_e \approx 0.021\) for \(z = 600–700\). These calculations therefore give a total optical depth in residual electrons \(\tau_e \approx 0.06\) back to \(z = 700\). These electrons have maximum influence on angular scales with harmonic \(\ell_{\text{max}} \approx 2z^{1/2} \approx 20–50\) (Zaldarriaga 1997). At higher redshifts, \(x_e\) rises to \(10^{-2.1}\) at \(z = 800\) and to \(10^{-1.1}\) at \(z = 1000\), where the CMB source function will affect the “free-streaming” assumption used in CMBFAST (Seljak & Zaldarriaga 1996).

3. IMPLICATIONS FOR REIONIZATION MODELS

The WMAP-3 measurements of fluctuations in temperature (\(T\)) and polarization (\(E\)) have been interpreted to estimate total electron-scattering optical depths of \(\tau_e = (0.09–0.10) \pm 0.03\).
The central values, $\tau_e = 0.09$ (Spergel et al. 2006) and $\tau_e = 0.10$ (Page et al. 2006) come, respectively, from computing the likelihood function for the six-parameter fit to all WMAP data (TT, TE, EE) and for just the EE data as a function of $\tau_e$. Because of the challenges in translating a single parameter ($\tau_e$) into a reionization history, $x_e(z)$, it is important to recognize the sizable error bars on $\tau_e$. At the 68% confidence level, $\tau_e$ could range from “low values” (0.06–0.07) up to values as high as 0.12–0.13.

In § 2.1, we showed that $\sim 50\%$ of this $\tau_e$ can be accounted for by a fully ionized IGM at $z \leq z_r$. Recent Gunn-Peterson observations of 19 quasars between $5.7 < z < 6.4$ (Fan et al. 2006; Gnedin & Fan 2006) are consistent with a reionization epoch of $z_r = 6.1 \pm 0.15$. According to equation (2), this produces $\tau_e = 0.042 \pm 0.003$, where our error propagation includes relative uncertainties in $z_r$ (2.5%), $\Omega_m h^2$ (8.4%), and $\Omega_b h^2$ (4.0%). Residual post-recombination electrons produce a substantial optical depth from $z \approx 10$ back to $z \approx 700$, which uniformly damps all angular scales. However, their effect on the TE and EE power is considerably less on large angular scales ($\ell \leq 10$). Thus, we can characterize a portion of the WMAP-3 observed optical depth, $\tau_e \approx 0.09 \pm 0.03$, through known sources of ionization. The “visible ionized universe” out to $z_r = 6.1$ accounts for $\tau_e = 0.042$, while high-$z$ partial ionization could contribute anywhere from $\tau_e \approx 0.01$ to 0.06. We therefore assume that an additional optical depth, $\Delta \tau_e \leq 0.03 \pm 0.03$, can be attributed to star formation and early black hole accretion at $z > z_r$.

Our calculations represent an important change in the derivation of $z_r$ from $\tau_e$, suggesting that the amount and efficiency of high-$z$ star formation need to be suppressed. This suggestion is ironic, since WMAP-1 data initially found a high $\tau_e = 0.17 \pm 0.04$ (Spergel et al. 2003) implying a surprisingly large redshift for early reionization, ranging from $11 < z_r < 30$ at 95% confidence (Kogut et al. 2003). These results precipitated many investigations of star formation at $z = 10 – 30$, some of which invoked anomalous mass functions, very massive stars (VMS, with $M > 140 M_\odot$), and an increased ionizing efficiency from zero-metallicity stars (VTS03; Wyithe & Loeb 2003; Cen 2003; Ciardi, Ferrara, & White 2003; Sokasian et al. 2003, 2004). Tumlinson, Venkatesan, & Shull (TVS04) disputed the hypothesis that the first stars had to be VMS. They showed that an IMF dominated by $10–100 M_\odot$ stars can produce the same ionizing photon budget as VMS, generate CMB optical depths of 9–14%, and still be consistent with nucleosynthetic evidence from extremely metal-poor halo stars (Umeda & Nomoto 2003; Tumlinson 2006; Venkatesan 2006).

Although the IGM recombination history, $x_e(z)$, is included in calculations of CMBFAST and in CMB parameter estimation, the best-fit values of $\tau_e$ from WMAP-3 and earlier CMB experiments have been attributed exclusively to the contribution from the first stars and/or black holes at $z \leq 20$. The contributions from post-recombination electrons ($20 < z < 1100$)
have not always been subtracted from the data. This post-recombination contribution was relatively small in some earlier models of reionization (Zaldarriaga 1997; Tegmark & Silk 1995) that explored optical depths of $\tau_e = 0.5 - 1.0$ and suggested reionization epochs up to $z_r \sim 100$. However, with current data indicating late reionization, it becomes particularly important to consider contributions to $\tau_e$ prior to the first sources of light.

The new WMAP-3 results find a lower $\tau_e$, but they also suggest less small-scale power available for reionizing sources, owing to lower normalization parameters, $\sigma_8 \approx 0.74^{+0.05}_{-0.06}$ and $\Omega_m h^2 \approx 0.127^{+0.007}_{-0.013}$. This reduction is somewhat offset by the reduction in spectrum tilt from $n_s = 0.99 \pm 0.04$ (WMAP-1) to $n_s = 0.951^{+0.015}_{-0.019}$ (WMAP-3). Alvarez et al. (2006) argue, from the lower values of $\tau_e$ and $\sigma_8$, that both WMAP-3 and WMAP-1 data require similar (high) stellar ionizing efficiencies. Haiman & Holder (2003) use the lower $\tau_e$ to suggest that massive star formation was suppressed in minihalos. Our results on a lower $\Delta \tau_e$ make these requirements even more stringent, as we now quantify.

Semi-analytic and numerical models of reionization (Ricotti, Gnedin, & Shull 2002a,b; VTS03, Haiman & Holder 2003) show that the efficiency of ionizing photon injection into the IGM can be parameterized by the “triple product”, $N_\gamma f_* f_{\text{esc}}$. Here, $f_*$ represents the star-formation efficiency (the fraction of a halo’s baryons that go into stars), $N_\gamma$ is the number of ionizing photons produced per baryon of star formation, and $f_{\text{esc}}$ is the fraction of these ionizing photons that escape from the halo into the IGM. We can now use our calculations to constrain the amount of high-$z$ star formation through the product of these three parameters, henceforth referred to as the “efficiency”. For the ionization history in equation (5), we set $x_e(z) = N_\gamma f_* f_{\text{esc}} c_L(z) f_b(z)$, where $c_L(z)$ is the space-averaged baryon clumping factor of ionized hydrogen, $c_L \equiv \langle n_{\text{HI}}^2 \rangle / \langle n_{\text{HII}} \rangle^2$. We assume that $c_L$ is the same for H II and He III. The factor $f_b(z)$ is the fraction of baryons in collapsed halos, computed through the Press-Schechter formalism (as in VTS03) for the cosmological parameters from WMAP-3. We assume that $N_\gamma f_* f_{\text{esc}}$ is constant with redshift.

There is surely some dependence of each of these parameters on the halo mass and environment (Haiman & Bryan 2006; Ricotti & Shull 2000). Since we have already parameterized the intrahalo recombinations through $f_{\text{esc}}$, we account for the loss of ionizing photons on IGM scales through $c_L$ in two forms: (1) a power-law form with slope $\beta = -2$ from the semi-analytic work of Haiman & Bryan (2006); and (2) the numerical simulations of Kohler, Gnedin & Hamilton (2006), using their case C (overdensity $\delta \sim 1$ for the large-scale IGM) for $C_R$, the recombination clumping factor corresponding to our definition of $c_L$. In the latter case, the clumping factor is almost constant ($c_L \approx 6$) until the very end of reionization. Together, these two different cases provide bounds on the range of possible values.

With these assumptions, we can use equation (5) and the allowed additional optical depth,
$\Delta \tau_e \leq 0.03 \pm 0.03$, to constrain the ionizing efficiency of the first stars. In Figure 1, we plot the efficiency as a function of $\Delta \tau_e$, for star formation in Ly$\alpha$-cooled halos (virial temperature $T_{\text{vir}} \geq 10^4$ K) and in H$_2$-cooled minihalos ($T_{\text{vir}} \geq 10^3$ K). We consider the two clumping factors from Kohler et al. (2006) and Haiman & Bryan (2006). For $f_\star = f_{\text{esc}} = 0.1$, we indicate the efficiencies corresponding to two cases of interest, assuming a Salpeter initial mass function (IMF): (1) $N_\gamma = 60,000$ for a metal-free IMF ($10^{10} - 140 M_\odot$) that agrees with both CMB and nucleosynthetic data; and (2) $N_\gamma = 4000$ for a present-day IMF ($1-100 M_\odot$). Note that $N_\gamma = 34,000$ is consistent with the WMAP-3 inference, $\tau_e \sim 0.1$, if the photons arise from zero-metal stars (Tumlinson 2006). These values of $N_\gamma$ were derived (TVS04) from the lifetime-integrated ionizing photon production from various stellar populations and IMFs and used as inputs in cosmological reionization models.

Figure 1 shows that fairly modest efficiencies of massive star formation are consistent with limiting the ionizing contribution of minihalos to $\Delta \tau_e \leq 0.03 - 0.06$. This is consistent with suppression of star formation and ionizing photon production in mini-halos (Haiman & Bryan 2006). Much larger efficiencies, close to those of metal-free stellar populations, are required for larger halos. Interestingly, it seems to make little difference what form is assumed for the clumping factor, $c_L$, in constraining the ionizing efficiency of the first stars. This is largely a statistical effect arising from the insensitivity of the average optical depth, or electron column density, averaged over many beams passing through a clumpy medium.

The CMB optical depth can also constrain the level of X-ray preionization from high-redshift black holes. Ricotti et al. (2005) were able to produce large optical depths, $\tau_e \approx 0.17$, using accreting high-z black holes with substantial soft X-ray fluxes. Their three simulations (labeled M-PIS, M-SN1, M-SN2) produced hydrogen preionization fractions $x_e = 0.1 - 0.6$ between $z = 15$ and $z = 10$, with large co-moving rates of star formation, $(0.001 - 0.1) M_\odot \text{Mpc}^{-3} \text{yr}^{-1}$, and baryon fractions, $\omega_{\text{BH}} \approx 10^{-6} - 10^{-5}$ accreted onto black holes. Any significant contribution to $\tau_e$ from X-ray preionization requires $x_e \geq 0.1$. Therefore, the lower value of $\tau_e$ from WMAP-3 reduces the allowed X-ray preionization and black-hole accretion rates significantly compared to these models. Ricotti et al. (2005) demonstrated that high optical depths ($\tau_e \approx 0.17$) could be achieved by black-hole X-rays. In their models, the IGM at $z > z_r$ was highly ionized ($x_e \gg 0.01$). In this limit, there were few X-ray secondary electrons and most of the X-ray energy went into heating the ionized medium. By contrast, the WMAP-3 optical depth suggests that the IGM was much less ionized at $z > z_r$.

Relating the effects of X-ray ionization from early black holes to a $\Delta \tau_e$ and a related X-ray production efficiency factor is less straightforward compared to the star formation case, for the following reasons. First, unlike ionization by UV photons, X-ray ionization is non-equilibrium in nature and the timescales for X-ray photoionization at any epoch prior to $z = 6$ typically
exceeds the Hubble time at those epochs (VGS01). Therefore, a one-to-one correspondence between X-ray production at an epoch, and the average efficiency of halos at that epoch is more difficult to establish relative to the UV photon case. In addition, X-ray ionization (whether from stars or black holes), at least initially when $x_e < 0.1–0.2$, will be dominated by secondary ionizations from X-ray-ionized helium electrons rather than from direct photoionization. This may therefore constrain the physical conditions in the IGM (e.g., the level of He ionization) rather than those in the parent halo, when we attempt to translate a $\Delta \tau_e$ into an X-ray ionization efficiency. Thus, it may be difficult to make precise inferences about the black hole density and accretion history from $\tau_e$.

We define an efficiency parameter, $\epsilon_X$, for X-rays analogous to the previous case for massive star formation. Here, $\epsilon_X$ is the product of the average fraction of baryons in black holes (in halos at $z \gtrsim 7$) and the number of X-ray photons produced (per baryon accreted onto such black holes). We assume that the clumping factor ($c_L$) and escape fraction ($f_{esc}$) are roughly unity for X-rays, given their high penetrating power relative to UV photons. Thus, we define $x_e(z) = \epsilon_X f_b(z)$ for X-rays, where electrons come from $H^+$, $He^+$, and $He^{+2}$. We assume that each X-ray photon produces $\sim 12$ hydrogen ionizations, primarily through secondary ionizations from X-ray photoelectrons (VGS01). Figure 2 shows the allowed additional optical depth, analogous to the constraints of Figure 1, for X-ray efficiency in both Ly$\alpha$-cooled halos and minihalos. A comparison with Figure 1 reveals that X-rays are capable of much higher ionization efficiency relative to Pop III or Pop II star formation.

In summary, we have shown that the revised (WMAP-3) values of CMB optical depth, $\tau_e = 0.09 \pm 0.03$, lead to a more constrained picture of early reionization of the IGM. Approximately half of the observed $\tau_e$ comes from a fully ionized IGM back to $z_r = 6.1 \pm 0.15$. The remainder probably arises from the first massive stars and from accretion onto early black holes at $z > z_r$. Some of the observed $\tau_e$ may come from scattering from residual electrons left from recombination; inaccuracies in computing this ionization history add systematic uncertainty to the CMB inferred signal. We have assumed extra scattering, $\Delta \tau_e = 0.03 \pm 0.03$ at $z > z_r$, and used this to constrain the efficiencies for production and escape of ionizing photons, both from the first massive stars (Figure 1) and early black holes (Figure 2).

In both cases, the picture is of a partially ionized IGM at redshifts $z = 6 - 20$. For X-ray pre-ionization by early black holes, equation (6) can be used to provide an estimate of the effects of partial reionization. Between redshifts $z_2 \approx 20$ and $z_1 \approx 6$, an IGM with ionized fraction $x_{e,0} = 0.1$ (Ricotti et al. 2005) would produce $(\Delta \tau_e) = (0.018)(x_{e,0}/0.1)$, which is a significant contribution to the observed $\tau_e = 0.09 \pm 0.03$. Ricotti et al. (2005) suggested a large $x_e \approx 0.1 – 0.6$ and pushed their black-hole space densities and accretion rates to large values in order to reach the WMAP-1 estimates of $\tau_e = 0.17$. Because such large values of $\tau_e$
are no longer required, the black hole densities and IGM ionization fractions are likely to be considerably less.

All these constraints depend heavily on uncertain parameterizations of the efficiency of star formation and ionizing photon production. However, with more precise measurements of CMB optical depth from future missions, there is hope that more stringent constraints on high-z star formation and black-hole accretion will be possible.

Acknowledgements

We are grateful to David Spergel, Licia Verde, Rachel Bean, and Nick Gnedin for useful discussions regarding the interpretation of WMAP data and numerical simulations. We thank Douglas Scott and Wan Yan Wong for providing their calculations of recombination history. This research at the University of Colorado was supported by astrophysical theory grants from NASA (NAG5-7262) and NSF (AST02-06042).
REFERENCES

Alvarez, M., Shapiro, P. R., Ahn, K., & Iliev, I. 2006, ApJ, 644, L101
Becker, R., et al. 2001, AJ, 122, 2850
Begelman, M. C., Volonteri, M., & Rees, M. J. 2006, MNRAS, 370, 289
Cen, R. 2003, ApJ, 591, L5
Cen, R., & Ostriker, J. P. 1999, ApJ, 519, L109
Ciardi, B., Ferrara, A., White, S. D. M. 2003, MNRAS, 344, L7
Fan, X., et al. 2002, AJ, 123, 1247
Fan, X., et al. 2006, AJ, 132, 117
Gnedin, N. Y. 2000, ApJ, 535, 530
Gnedin, N. Y. 2004, ApJ, 610, 9
Gnedin, N. Y., & Fan, X. 2006, ApJ, 648, 1
Gnedin, N. Y., & Ostriker, J. P. 1997, ApJ, 486, 581
Haiman, Z., & Holder, G. P. 2003, ApJ, 595, 1
Haiman, Z., & Bryan, G. L. 2006, ApJ, 650, 7
Heger, A., & Woosley, S. E. 2002, ApJ, 567, 532
Izotov, Y. I., & Thuan, T. 1998 ApJ, 500, 188
Hui, L., & Haiman, Z. 2003, ApJ, 596, 9
Kogut, A., et al. 2003, ApJS, 148, 161
Kohler, K., Gnedin, N. Y., & Hamilton, A. J. S. 2006, ApJ, submitted, astro-ph/0511627
Kriss, G. A., et al. 2001, Science, 293, 1112
Olive, K. A., & Skillman, E. D. 2001, New Astronomy, 3, 119
Page, L., et al. 2006, ApJ, submitted, astro-ph/0603450
Peimbert, M., Luridiana, V., & Peimebrt, A. ApJ, submitted (astro-ph/0701580)
Ricotti, M., & Shull, J. M. 2000, ApJ, 542, 548
Ricotti, M., Gnedin, N. Y., & Shull, J. M. 2002a, ApJ, 575, 33
Ricotti, M., Gnedin, N. Y., & Shull, J. M. 2002b, ApJ, 575, 49
Ricotti, M., & Ostriker, J. P. 2004, MNRAS, 350, 539
Ricotti, M., Ostriker, J. P., & Gnedin, N. Y. 2005, MNRAS, 357, 207
Seager, S., Sasselov, D., & Scott, D. 2000, ApJS, 128, 407
Seljak, U., & Zaldarriaga, M. 1996, ApJ, 469, 437
Shull, J. M., Tumlinson, J., Giroux, M. L., Kriss, G. A., & Reimers, D. 2004, ApJ, 600, 570
Sokasian, A., Abel, T., Hernquist, L., & Springel, V. 2003, MNRAS, 344, 607
Sokasian, A., Yoshida, N., Abel, T., Hernquist, L., & Springel, V. 2004, MNRAS, 350, 47
Spergel, D. N., et al. 2003, ApJS, 148, 175
Spergel, D. N., et al. 2006, ApJ, submitted, astro-ph/0603449
Tegmark, M., Silk, J., & Blanchard, A. 1994, ApJ, 420, 484
Tegmark, M., & Silk, J. 1995, ApJ, 441, 458
Tumlinson, J. 2006, ApJ, 641, 1
Tumlinson, J., Venkatesan, A., & Shull, J. M. 2004, ApJ, 612, 602 (TVS04)
Umeda, H., & Nomoto, K. 2003, Nature, 422, 871
Venkatesan, A. 2006, ApJ, 641, L81
Venkatesan, A., Giroux, M. L., & Shull, J. M. 2001, ApJ, 563, 1 (VGS01)
Venkatesan, A., Tumlinson, J., & Shull, J. M. 2003, ApJ, 584, 621 (VTS03)
White, M., Scott, D., & Silk, J. 1994, ARA&A, 32, 319
Wyithe, S., & Loeb, A. 2003, ApJ, 586, 693
Zaldarriaga, M. 1997. Phys. Rev. D, 55, 1822
Zheng, W., et al. 2004, ApJ, 605, 631

This preprint was prepared with the AAS LATEX macros v5.2.
Fig. 1.— Efficiency factor, $N_\gamma f_\star f_{\text{esc}}$, for production and escape of photoionizing radiation vs. allowed additional optical depth from first stars, $\Delta \tau_e$ at $z > z_r$. Efficiency is defined as in Haiman & Bryan (2006) for star formation in Ly$\alpha$-cooled halos ($T_{\text{vir}} \geq 10^4$ K) and in H$_2$-cooled minihalos ($T_{\text{vir}} \geq 10^3$ K). Solid and dashed curves correspond to clumping factors from Kohler et al. (2006) and Haiman & Bryan (2006) respectively. Horizontal solid lines correspond to the efficiencies (TVS04) from a metal-free (10–140 $M_\odot$) Pop III IMF and a standard Salpeter (1-100 $M_\odot$) Pop II IMF, with $f_\star = f_{\text{esc}} = 0.1$. See text for discussion.
Fig. 2.— Required X-ray production factor, $\epsilon_X$, versus electron-scattering optical depth, $\Delta \tau_e$, at redshifts $z > z_r$. This factor is taken to be the baryon fraction in black holes times the photons per accreted baryon (see § 3 for details) and controls the rate of producing escaping X-rays from halos with virial temperatures of $10^3$ K and $10^4$ K. The electron ionized fraction is related to the total baryon fraction by $x_e(z) = \epsilon_X f_b(z)$. 