On strongly regular graph with parameters 
(65, 32, 15, 16)

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Abstract

We construct a strongly regular graph with the parameters (65, 32, 15, 16). The idea is to search for an adjacency matrix that consists of circulant blocks. Equations with such matrices can be reduced to congruences with polynomials matrices of smaller orders. We can consider these congruences over different moduli for a more efficient computational approach.

1 Introduction

A strongly regular graph (SRG) with parameters \((v, k, \lambda, \mu)\) is an undirected graph with \(v\) vertices, each one has \(k\) edges and every two adjacent or non-adjacent vertices have \(\lambda\) or \(\mu\) common neighbours respectively. A special subset of SRG are conference graphs with parameters \(k = \frac{v-1}{2}, \lambda = \frac{v-5}{4}\) and \(\mu = \frac{v-1}{4}\). Conference graphs are related to symmetric conference matrices, i.e., matrices \(C\) with 0 on the diagonal and \(\pm 1\) off the diagonal such that \(C^2 = (n - 1)I_n\), where \(n\) is the order of \(C\) and \(I_n\) is the identity matrix of order \(n\).

An existence of a SRG for given parameters is a question of great interest. There are several necessary conditions, however, for many parameters it is unknown whether there is such a SRG. All feasible sets with \(v \leq 1300\) with constructions for known graphs are tracked at Andries Brouwer’s website [1]. Until recently, the smallest unknown case was a conference graph with the parameters (65, 32, 15, 16).

Denote \(J_n\) a matrix of order \(n\) consisting of ones, \(c_n(x) = 1 + x + x^2 + \ldots + x^{n-1}\). It is well known that if \(A\) is an adjacency matrix of a SRG with the parameters \((v, k, \lambda, \mu)\), it must satisfy the equations

\[
A^2 + (\mu - \lambda)A = (k - \mu)I_n + \mu J_n, \quad AJ_n = kJ_n,
\]

where \(n = v\). Also denote \(M_n(R)\) a ring of matrices of order \(n\) over a ring \(R\).
2 Circulant matrices and matrix blocks

Suppose we are trying to find some solution of a matrix equation, but iterating over all possible matrices would take too much time. We can try to search for a matrix with some specific structure, for example, we can suppose it consists of circulant blocks. In this section we’ll see how to represent such matrices as polynomial matrices of a smaller order. It might be easier to search for solutions having this specific form, but there is no guarantee that such a solution would exist, even if there is a solution of the original equation.

Suppose we have two circulant matrices $B, C \in M_m(\mathbb{Z})$, i.e. $B_{ij} = b_{(i-j) \mod m}$ and $C_{ij} = c_{(i-j) \mod m}$. There is a one-to-one correspondence between such matrices and polynomials in $\mathbb{Z}[x] / (x^m - 1)$, namely, $B$ and $C$ correspond to $b(x) = \sum_{i=0}^{m-1} b_i x^i$ and $c(x) = \sum_{i=0}^{m-1} c_i x^i$ respectively. Note that the product $BC$ is also a circulant matrix with the corresponding polynomial $b(x)c(x) \pmod{x^m - 1}$.

Now, if a matrix $B \in M_{lm}(\mathbb{Z})$ can be split into $l \times l$ blocks, where each block $B_{ij} \in M_m(\mathbb{Z})$ is circulant, we consider the matrix $B(x) \in M_l(\mathbb{Z}[x] / (x^m - 1))$ consisting of the polynomials $b_{ij}(x)$ corresponding to these blocks $B_{ij}$. If another matrix $C \in M_{lm}(\mathbb{Z})$ has the same structure and corresponds to $C(x) \in M_l(\mathbb{Z}[x] / (x^m - 1))$, their product $BC$ also has the same structure, i.e., consists of $l \times l$ circulant blocks, and its corresponding polynomial matrix is $B(x) \cdot C(x) \pmod{x^m - 1}$.

For example, consider the adjacency matrix $B$ of the Petersen graph, a SRG with the parameters $(10, 3, 0, 1)$.

\[
\begin{pmatrix}
0 & 1 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 \\
1 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
0 & 1 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 1 & 0 \\
1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 \\
1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 \\
0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 1 \\
0 & 0 & 0 & 1 & 0 & 1 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 & 1 & 1 & 0 & 0
\end{pmatrix}
\]

Note that $B^2 + B = 2I_{10} + J_{10}$, and that $B$ consists of 4 blocks $5 \times 5$, each one is a circulant matrix corresponding to the polynomials $x + x^4$, 1, 1, and $x^2 + x^3$ respectively. So the matrix $B$ can be ‘compacted’ to

\[
B(x) = \begin{pmatrix}
x + x^4 & 1 \\
1 & x^2 + x^3
\end{pmatrix}
\]

such that $B^2(x) + B(x) \equiv 2I_2 + c_0(x)J_2 \pmod{x^5 - 1}$.

Note that if a ‘compacted’ matrix can be split into circulant blocks too, we can repeat this procedure, using a different polynomial variable. To illustrate this, consider the Hoffman–Singleton graph, a SRG with the parameters...
Knowing that $A_f$ consists of zeros and ones and $B$ be expanded to the larger matrix $BJ = M = 50I_{50}$ such that $B = 6I_{50} + 5J_{50}$. Since $B$ is also a symmetric matrix consisting of zeros and ones and $BJ_{50} = 7J_{50}$, it is the adjacency matrix of a SRG with the parameters $(50, 7, 0, 1)$.

So if we a searching for some solution of some matrix equation, say, $A^2 + pA = qI_n + rJ_n$, where $n = ml$, we can try to search for a matrix consisting of $l \times l$ circulant blocks, and to reduce this matrix equation to $A^2(x) + pA(x) = qI_1 + r c_m(x)J_1$ (mod $x^m - 1$). We can also consider this congruence modulo some factors of $x^m - 1$, for example, $x - 1$.

Of course if we want $A$ to consist only of zeros and ones, polynomial coefficients of $A(x)$ must also be zeros and ones. Furthermore, if we are looking for a symmetric matrix, $A(x)$ must satisfy the congruence $A^T(x) = A(x^{m-1})$ (mod $x^m - 1$), in other words, $A_{ji}(x) = A_{ij}(x^{m-1})$ (mod $x^m - 1$).

3 Searching for $srg(65, 32, 15, 16)$

Now let’s search for a SRG with the parameters $(65, 32, 15, 16)$. Its adjacency matrix $A \in M_{65}(\mathbb{Z})$ must satisfy the equations $A^2 + A = 16I_{65} + 16J_{65}$ and $AJ_{65} = 32J_{65}$, according to $I$.

Keeping the previous section in mind, we can suppose that $A$ can be split into $5 \times 5$ circulant blocks; or in $13 \times 13$ circulant blocks; or that the whole $A$ is circulant. Unfortunately, neither approach works.

In the first case $A$ is reduced to a matrix $A_5(x) \in M_5(\mathbb{Z}[x]/(x^{13} - 1))$ such that

$$A_5^2(x) + A_5(x) \equiv 16I_5 + 16c_{13}(x)J_5 \pmod{x^{13} - 1}.$$

By taking $x = \zeta_{13} = e^{2\pi i/13}$, we get $A_5^2(\zeta_{13}) + A_5(\zeta_{13}) = 16I_5$. The eigenvalues of $A_5(\zeta_{13})$ are the solutions of the equation $\lambda^2 + \lambda = 16$, i.e., $-1 \pm \sqrt{65}$. Thus $tr(A_5(\zeta_{13})) = k \frac{1+\sqrt{65}}{2} + (5 - k) \frac{1-\sqrt{65}}{2}$, where $0 \leq k \leq 5$, and for any such $k$ $tr(A_5(\zeta_{13})) \in \mathbb{Q}(\sqrt{65}) \setminus \mathbb{Q}$. On the other hand, $tr(A_5(x)) \in \mathbb{Z}[x]$, so $tr(A_5(\zeta_{13})) \in \mathbb{Z}[\zeta_{13}]$. However, one can easily check that $\sqrt{65} \notin \mathbb{Q}(\zeta_{13})$.

Similarly, $A$ can’t consist of $13 \times 13$ circulant blocks, as $\sqrt{65} \notin \mathbb{Q}(\zeta_5)$. If $A$ itself is circulant, it corresponds to a polynomial $a(x) \in \mathbb{Z}[x]/(x^{65} - 1)$, where $a^2(x) + a(x) \equiv 16 + 16c_{65}(x) \pmod{x^{65} - 1}$. Once again we can take $x = \zeta_5$ and conclude there is no such matrix.

Instead, let’s try the following modification. The first row of $A$ has 32 ones, so without loss of generality we may assume $A_{1j} = 1$ for $2 \leq j \leq 33$ and

$$(50, 7, 0, 1).$$ For brevity we won’t write its adjacency matrix here, but note it can be compacted to

$$B(x, y) = \begin{pmatrix} x + x^4 & 1 + x(y + y^4) + x^4(y^2 + y^3) \\ 1 + x^4(y + y^4) + x(y^2 + y^3) & x^2 + x^3 \end{pmatrix}$$

such that $B^2(x, y) + B(x, y) \equiv 6I_2 + c_5(x)c_5(y)J_2 \pmod{x^5 - 1, y^5 - 1}$. Taking its $y$-coefficients, we can expand it to the larger matrix $B(x) \in M_{10}(\mathbb{Z}[x]/(x^5 - 1))$ such that $B^2(x) + B(x) \equiv 6I_{10} + c_5(x)J_{10} \pmod{x^5 - 1}$. $B(x)$ in turn can be expanded to $B \in M_{50}(\mathbb{Z})$ such that $B^2 + B = 6I_{50} + J_{50}$. Since $B$ is also a symmetric matrix consisting of zeros and ones and $BJ_{50} = 7J_{50}$, it is the adjacency matrix of a SRG with the parameters $(50, 7, 0, 1)$.
$A_{ij} = 0$ for $34 \leq j \leq 65$. Denote $B = (1 \ 1 \ \ldots \ 100 \ \ldots \ 0) = (A_{ij})$, $2 \leq j \leq 65$, $C = (A_{ij}) \in M_{64}(\mathbb{Z})$, $i, j \geq 2$. Then the matrix $A$ can be written as

$$\begin{pmatrix} 0 & B \\ B^T & C \end{pmatrix}.$$ 

The equality $A^2 + A = 16I_{65} + 16J_{65}$ holds iff the following 3 equalities hold:

$$\begin{cases} 0^2 + BB^T + 0 = 32 \\ 0B + BC + B = 16J_{1,64} \\ B^TB + C^2 + C = 16I_{64} + 16J_{64} \end{cases} \quad (2)$$

Here $J_{1,64}$ is the matrix of size $1 \times 64$ consisting of ones.

The first equality of (2) obviously holds. The second is equivalent to

$$\sum_{i=1}^{32} C_{ij} = \begin{cases} 15, & j \leq 32, \\ 16, & j \geq 33. \end{cases} \quad (3)$$

Since we also need $AJ_{65} = 32J_{65}$, it follows from (3) that

$$\sum_{i=33}^{65} C_{ij} = 16 \ \forall j \quad (4)$$

Denote $H_{64} = 16J_{64} - B^TB$, the matrix consisting of 4 blocks $15J_{32}$, $16J_{32}$, $16J_{32}$, and $16J_{32}$. Also denote $H_{2n} \in M_{2n}(\mathbb{Z})$ the matrix consisting of 4 blocks $15J_{n}$, $16J_{n}$, $16J_{n}$, and $16J_{n}$.

The last equation of (2) can be rewritten as

$$C^2 + C = 16I_{64} + H_{64},$$

and now we’ll apply the method described in the previous section to this equation. Suppose $C$ consists of $4 \times 4$ circulant blocks of order 16, i.e. it can be ‘compacted’ to $C(x) \in M_4(\mathbb{Z}[x]/(x^{16} - 1))$ such that

$$C^2(x) + C(x) \equiv 16I_4 + c_{16}(x)H_4 \pmod{x^{16} - 1}. \quad (5)$$

Let’s take this congruence modulo $x - 1$ first:

$$C^2(1) + C(1) = 16I_4 + 16H_4$$

As all polynomial coefficients of $C(x)$ must be zeros or ones, elements of $C(1)$ must be non-negative integers. By a simple brute-force one can find there are just four such matrices, two of them are

$$C(1) = \begin{pmatrix} 7 & 8 & 6 & 10 \\ 8 & 7 & 10 & 6 \\ 6 & 10 & 8 & 8 \\ 10 & 6 & 8 & 8 \end{pmatrix}, \begin{pmatrix} 9 & 6 & 7 & 9 \\ 6 & 9 & 9 & 7 \\ 7 & 9 & 6 & 10 \\ 9 & 7 & 10 & 6 \end{pmatrix} \quad (6)$$
and the other two are obtained from these two by permuting the third and the fourth rows and columns, so we won’t consider them. Note that if we successfully expand such matrices into $C \in M_{94}(\mathbb{Z})$, the equalities 5 and 1 will hold.

Next, we consider the congruence 6 modulo $x + 1$.

$$C^2(-1) + C(-1) = 16I_4.$$ 

If $f(x) \in \mathbb{Z}[x]$ is any polynomial with non-negative coefficients and $f(1) = a$ is known, $f(-1)$ must be a value from $-a$ to $a$ of the same parity, i.e. $f(-1) \in \{-a, -a + 2, -a + 4, \ldots, a - 2, a\}$. So if we know the matrix $C(1)$, we can search for the matrix $C(-1) \equiv C(1) \pmod{2}$ with coefficients less or equal to those of $C(1)$ in absolute values. Using a brute-force approach, we find that each matrix in 6 produces 32 possible matrices $C(-1)$; we can keep only 10 of them for the first and 10 for the second, the rest will be their permutations. For brevity we won’t list them all, just one of them obtained from the second matrix 6:

$$C(-1) = \begin{pmatrix} 1 & -2 & -3 & -1 \\
-2 & 1 & -1 & -3 \\
-3 & -1 & -2 & 2 \\
-1 & -3 & 2 & -2 \end{pmatrix}$$ (7)

Next, we consider the congruence 6 modulo $x^2 + 1$, trying to find possible values of $C(i)$. 

$$C^2(i) + C(i) = 16I_4.$$ 

Again, if $f(x) \in \mathbb{Z}[x]$ has non-negative coefficients and we know $f(1) = a$ and $f(-1) = b \equiv a \pmod{2}$, possible values of $f(i)$ have the real part in $\{-\frac{a+b}{2}, -\frac{a+b}{2} + 2, \ldots, \frac{a+b}{2} - 2, \frac{a+b}{2}\}$ and the imaginary part in $\{-\frac{a-b}{2}, -\frac{a-b}{2} + 2, \ldots, \frac{a-b}{2} - 2, \frac{a-b}{2}\}$. Again, we use brute-force to find possible matrices $C(i)$, given $C(1)$ and $C(-1)$. Recall that in order for the final matrix $C$ to be symmetric, we need $C^T(x) \equiv C(x^{15}) \pmod{x^{16} - 1}$, therefore $C(i) = C^T(-i)$. It turns out there are 1422 possible values of $C(i)$ for the first matrix in 6 and 1224 for the second. One of $C(i)$ corresponding to 7 is

$$C(i) = \begin{pmatrix} 1 & -2 & -i & -3i \\
-2 & 1 & -3i & -i \\
i & 3i & -2 & 2 \\
3i & i & 2 & -2 \end{pmatrix}$$ (8)

Similarly, given $C(1)$, $C(-1)$ and $C(i)$, we find possible matrices $C(\zeta_8)$, where $\zeta_8 = \frac{-1 + \sqrt{2(1+i)}}{2}$, such that

$$C^2(\zeta_8) + C(\zeta_8) = 16I_4$$

and $C^T(\zeta_8) = C(-\zeta_8^3)$. One of possible values corresponding to 7 and 8 is

$$C(\zeta_8) = \begin{pmatrix} -1 - 2\zeta_8 + 2\zeta_8^3 & 2\zeta_8^2 & -1 + \zeta_8^2 - \zeta_8^3 & 1 - \zeta_8^2 - \zeta_8^3 \\
-2\zeta_8^2 & -1 + 2\zeta_8 - 2\zeta_8^3 & -\zeta_8 & -2\zeta_8^2 \\
1 + \zeta_8 - \zeta_8^2 & \zeta_8^3 & 2\zeta_8 - 2\zeta_8^3 & -2\zeta_8^2 \\
-\zeta_8^3 & 1 + \zeta_8 + \zeta_8^2 & 2\zeta_8^2 & -2\zeta_8 + 2\zeta_8^3 \end{pmatrix}$$
Given \( C(1), C(-1), C(i) \) and \( C(\zeta_8) \) (the values of \( C(x) \) modulo \( x - 1, x + 1, x^2 + 1 \) and \( x^4 + 1 \)), we can calculate \( C(x) \) (mod \( x^8 - 1 \)):

\[
C_{11}(x) = 1 + x^2 + 2x^3 + 2x^4 + 2x^5 + x^6 \pmod{x^8 - 1},
\]
\[
C_{12}(x) = x + 2x^2 + x^3 + x^5 + x^7 \pmod{x^8 - 1},
\]
\[
C_{13}(x) = x + x^2 + x^3 + x^4 + x^5 + 2x^7 \pmod{x^8 - 1},
\]
\[
C_{14}(x) = 1 + x + x^2 + 2x^3 + x^4 + x^6 + 2x^7 \pmod{x^8 - 1},
\]
\[
C_{22}(x) = 1 + 2x + x^2 + 2x^4 + x^5 + 2x^7 \pmod{x^8 - 1},
\]
\[
C_{23}(x) = 1 + x^2 + 2x^3 + x^4 + x^5 + x^6 + 2x^7 \pmod{x^8 - 1},
\]
\[
C_{24}(x) = 1 + x + x^3 + x^5 + x^6 + 2x^7 \pmod{x^8 - 1},
\]
\[
C_{33}(x) = 2x + x^2 + x^6 + 2x^7 \pmod{x^8 - 1},
\]
\[
C_{34}(x) = 2 + x + x^3 + 2x^4 + x^5 + 2x^9 + x^7 \pmod{x^8 - 1},
\]
\[
C_{44}(x) = x^2 + 2x^3 + 2x^5 + x^6 \pmod{x^8 - 1}.
\]

(The rest \( C_{ij}(x) \) can be calculated from \( C_{ij}(x) \equiv C_{ji}(x^7) \) (mod \( x^8 - 1 \)).) Note that this \( C(x) \) satisfies the congruence

\[
C^2(x) + C(x) \equiv 16I_4 + c_{16}(x)H_4 \pmod{x^8 - 1}.
\]

The next step would be to lift the last congruence to \( (\text{mod } x^{16} - 1) \), or, equivalently, given \( C(1), C(-1), C(i) \) and \( C(\zeta_8) \) to find possible values of \( C(\zeta_{16}) \), \( \zeta_{16} = e^{2\pi i/16} \). Unfortunately, a brute-force doesn’t yield any such matrices. Another approach could be to introduce a new variable \( y \) and try to find a matrix \( C(x, y) \) such that

\[
C^2(x, y) + C(x, y) \equiv 16I_4 + c_8(x)c_2(y)H_4 \pmod{x^8 - 1, y^2 - 1}
\]

We could then expand \( C(x, y) \) into \( C(x) \in M_8(\mathbb{Z}[x]/(x^8 - 1)) \) and then into \( C \in M_{64}(\mathbb{Z}) \). However, a brute-force doesn’t find any such matrices either.

## 4 Last step

Consider again the calculated matrices \( C(x) \) (mod \( x^8 - 1 \)) such that \( C^2(x) + C(x) \equiv 16I_4 + 2c_8(x)H_4 \) (mod \( x^8 - 1 \)) (one of them is \( \text{[9]} \)). Let’s expand them into symmetric matrices \( D \in M_{32}(\mathbb{Z}) \) consisting of \( 4 \times 4 \) circulant blocks of order 8 such that

\[
D^2 + D = 16I_{32} + 2H_{32}.
\]

Suppose now that \( C \) consists of \( 32 \times 32 \) circulant blocks of order 2, therefore it can be compacted into \( \tilde{C}(x) \in M_{32}(\mathbb{Z}[x]/(x^2 - 1)) \) such that

\[
\tilde{C}^2(x) + \tilde{C}(x) \equiv 16I_{32} + c_2(x)H_{32} \pmod{x^2 - 1}
\]

Furthermore, suppose that \( \tilde{C}(1) \) is one of the matrices \( D \) we just found.

Denote \( E = \tilde{C}(0) \), so that \( \tilde{C}(x) = E + (D - E)x \). Then \( \text{[11]} \) is equivalent to the two same congruences modulo \( x - 1 \) and \( x + 1 \), where the first one holds because of \( \text{[10]} \) and the second is

\[
(2E - D)^2 + (2E - D) = 16I_{32}.
\]
After simple transformations we obtain

\[ 2E^2 - ED - DE + E - D + H_{32} = 0. \]  

(12)

We can take this equation modulo 2 to get rid of the square term \(E^2\):

\[ ED + DE + E + D + H_{32} \equiv 0 \pmod{2}. \]  

(13)

This is a linear equation with respect to \(E\). We can solve it as a system of linear equations of \(E_{ij}\).

There are several requirements for the final matrix \(C\), they impose restrictions on \(E\). First, \(C\) must be symmetric, so must be \(E\). Second, all elements of \(C\) must be 0 or 1, so must be the elements of \(E\) and \(D - E\) (and therefore, finding \(E\) (mod 2) is enough to find \(E\)). Thus, \(D_{ij} = 0\) implies \(E_{ij} = 0\), \(D_{ij} = 2\) means \(E_{ij} = 1\), and for \(D_{ij} = 1\) \(E_{ij}\) can be either 0 or 1. Finally, \(C\) can have only zeros on the main diagonal because it is an adjacency matrix of a graph, thus \(E_{ii} = 0\). We can iterate over all solutions of (13) with such restrictions and check if they also satisfy (12).

Unfortunately, there are too many solutions, so it may be infeasible to try them all. For example, for the matrix \(D\) constructed from (9) the solutions of (13) form an affine space of dimension 62, giving \(2^{62} \approx 4.6 \cdot 10^{18}\) possible matrices to check. So we need to reduce the set of solutions somehow.

Note that if we multiply some rows and columns of \(C(x)\) with the same indices by \(x \pmod{x^2 - 1}\), the congruence (11) will still hold. In the matrix \(C\) that would mean a permutation of some pairs of rows and columns, or in the final graph it would be a permutation of some vertices pairs, and in the matrix \(E\) that corresponds to adding 1 (mod 2) to some rows and columns. Therefore, if for every row \(i\) we denote \(g(i) = \min\{j: D_{ij} = 1\}\), we may assume without loss of generality that \(E_{i,g(i)} = 0\) if \(1 \leq g(i) < i\). This additional restriction can reduce the dimension of solutions of (13). For example, for the matrix \(D\) obtained from (9) there would be just \(2^{32} \approx 4.3 \cdot 10^9\) solutions, so it is now feasible on a modern computer to iterate over all of them, checking whether (12) holds.

And luckily one of such matrices indeed satisfies this equation.
From this matrix and from $D$ we can construct $C(x) \in M_{32}(\mathbb{Z}[x]/(x^2 - 1))$, then we can expand $C(x)$ to $C \in M_{64}(\mathbb{Z})$. Then by adding a row and a column of $B$ (32 ones followed by 32 zeros) we finally obtain the adjacent matrix $A$ of a SRG with the parameters $(65, 32, 15, 16)$, we also write this matrix below. One can easily check that it consists only of zeros and ones, that it is symmetric, that each row has 32 ones, and each two rows $i, j$ have 15 or 16 common ones, depending on $A_{ij}$.

Note that we can also construct a symmetric conference matrix of order 66 from it by replacing all 1 with $-1$, then all 0 with 1, keeping zeros on the main diagonal and adding a row and a column of ones. A square of such matrix is equal to $65I_{66}$.

One can also compute the automorphism group of this SRG. It consists of
32 elements, thus the graph is not vertex-transitive. In fact, its vertex orbits are \( \{1\}, \{2, 3, \ldots, 33\}, \) and \( \{34, 35, \ldots, 65\} \).

Here is the adjacency matrix of the SRG:

```
011111111111111111111111111111111100000000000000000000000000
1010011111111111010001001010101110110111010011101
1100101111111000001011000100010101001010010011011001110011
100100011111101100010001100100110011010101101011101110101011
1011001111111010010011111100010011011010101101101101101101101
10100011111111110100011111100010011011010101101101101101101101
11100011111110100010011111100010011011010101101101101101101101
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