Theory and phenomenology of generalized parton distributions: a brief overview.

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Abstract

The generalized parton distributions are non-perturbative objects, which encode information on long distance dynamics in a number of exclusive processes. They are hybrids of conventional parton densities, distribution amplitudes and hadron form factors. We give a brief review of theoretical developments in understanding of their properties, higher order perturbative effects, power corrections, and experimental observables where they are accessible.

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1 Exclusive QCD: from form factors to generalized parton distributions

The main issue of hadronic physics is the determination of the hadron’s wave function, Ψ, which encodes a fundamental information on its structure and dynamics of elementary constituents forming the hadron. The cleanest indirect access to the former is achieved in lepton-hadron experiments, which do not suffer from difficulties intrinsic to hadron-hadron reactions due to initial and final state strong interaction. Chronologically, these are electromagnetic form factors extracted from the elastic lepton scattering off hadrons ℓN → ℓN’, which provided charge and magnetization distributions within the nucleon. In a microscopic picture the form factor arises in the parametrization of an off-forward matrix element of the (local) quark electromagnetic current and is related to the wave functions by the Drell-Yan-West overlap formula

\[ F(\Delta^2) = \langle P_2 | \bar{\psi}(0) \Gamma \psi(0) | P_1 \rangle \sim \int dx \int dk_\perp \Psi^*(x, k_\perp + (1 - x) \Delta_\perp) \Psi(x, k_\perp). \]  

(1)

Here \( x \) is the longitudinal momentum fraction of the struck parton in the nucleon and \( k_\perp/k_\perp + (1 - x) \Delta_\perp \) being its transverse momentum before/after the interaction with the probe. \( \Gamma \) stands for an appropriate Dirac matrix matching the quantum numbers of the external states.

The measurements of \( F(\Delta^2) \) gave a first insight into the composite substructure of hadrons, which was unraveled in its full depth in the pioneering deeply inelastic experiments \( \ell N \rightarrow \ell' X \) at
dynamics and presently this can mostly be judged from experiment only) is decomposed as the Fourier transform (Ft) of the hadronic part of these processes at large momentum transfer $\gamma$ (DVCS), naive parton model that expresses the cross section in terms of a probability density $\psi$ function in a variety of hard meson production, $\gamma$ fields, quark/gluon operators sandwiched between the states with different momenta, e.g. for quark Similarly, to all previous characteristics, they are defined as Fourier transforms of non-local characteristics that contain exhaustive information on the nucleon wave function. This was triggered hard scattering subprocess, while $\Psi$ function in a variety of hard meson production, $\gamma$ production of the latter in the final state, e.g. for quark and similarly for gluons. Altogether, at leading twist level there are three quark and three gluon twist-two operators. The parametrization of e.g. the non-local vector current $\bar{\psi}\gamma_{\mu}\gamma_{5}\psi$ involves two leading twist functions $H$ and $E$ analogous to Dirac and Pauli form factors. Similarly, in the parity odd sector we have $\bar{H}$ and $\bar{E}$, which correspond to the axial-vector and pseudoscalar form factors, etc. While being known for a while, GPDs have attracted an essential attention only recently, after it was realized that they can shed some light onto the spin content of the nucleon. Namely, the second moment ($J_1$’s sum rule) of $H + E$ is related to the total orbital angular momentum dynamics, $J$, carried by partons in the nucleon, $\lim_{\Delta \to 0} \int dx x(H + E) = \frac{1}{2} J$.

The cleanest reaction that gives access to GPDs is the deeply virtual Compton scattering (DVCS), $\gamma^* (q + \frac{1}{2}) N \left( \frac{\Delta}{2} \right) \rightarrow \gamma (q - \frac{1}{2}) N \left( \frac{\Delta'}{2} \right)$. In addition to this, they enter as a soft function in a variety of hard meson production, $\gamma^* N \rightarrow MN'$, and diffractive processes. In QCD the Fourier transform (Ft) of the hadronic part of these processes at large momentum transfer (the answer to ‘How large is it really?’) heavily depends on the underlying strong interaction dynamics and presently this can mostly be judged from experiment only) is decomposed as

\[
\langle N' | T\{j_{\mu}(z)j_{\nu}(0)\} | N \rangle \overset{\text{Ft}}{=} T_{\mu\nu} \int_{-1}^{1} dx C(x, \xi) A(x, \eta) + T_{\mu\nu}^3 \int_{-1}^{1} dx C^3(x, \xi) A^3(x, \eta) + O(Q^{-2}),
\]

\[
\langle N' | M | N \rangle \overset{\text{Ft}}{=} T_{\mu} \int_{-1}^{1} dy \phi(y) \alpha_s C(y, x, \xi) A(x, \eta) + O(Q^{-1}),
\]

for DVCS and exclusive meson production amplitudes, respectively. Here $C$ stands for the hard scattering subprocess, while $A$ and $\phi$ stand for GPDs and meson distribution amplitudes
adding an independent function from it and respects the polynomiality condition alluded to above. An alternative solution given and ∆ the j and was found in and ∆ for the generalized Compton amplitude. In Ref. the parametrization in terms of spectral, or double distribution (DD), function reads

\[ \left\{ \begin{array}{c} A \\ \tilde{H} \end{array} \right\}(x, \eta, \Delta^2) = \int_{-1}^{1} dy \int_{-1+|y|}^{1-|y|} dz \delta(y + \eta z - x) \left\{ \frac{x F}{\tilde{F}} \right\}(y, z, \Delta^2). \]  

(7)

While the last definition coincides with the original one introduced by, the former differs from it and respects the polynomiality condition alluded to above. An alternative solution given in Ref. keeps of for A the representation in terms of DDs of the second line and adding an independent function \( D(x/\eta) \) to it so that this term produces the missing \( \eta^2 \)-term in the \( j \)th moment. The inverse transformation of DDs in terms of GPDs has been derived in Ref. and was found in to be in one-to-one correspondence with the Radon transformation.

Let us consider a particular example of GPDs. One assumes a factorizable ansatz of \( (x, \eta) \) and \( \Delta^2 \) dependence: \( \tilde{H}(x, \eta, \Delta^2) = \tilde{F}(\Delta^2)\tilde{H}(x, \eta) \), with \( \tilde{H}(x, \eta) \) expressed in terms of a double distribution \( \tilde{F}(y, z) \) that is modeled as

\[ \tilde{F}(y, z) = \{ \Delta q(y)\theta(y) - \Delta \bar{q}(-y)\theta(-y) \} \pi(|y|, z), \quad \pi(y, z) = \frac{3}{4} \frac{(1 - y)^2 - z^2}{(1 - y)^3}, \]  

(8)

and obeys the \( \tilde{F}(y, -z) = \tilde{F}(y, z) \) symmetry. For illustration purposes let us discuss the isospectral combination. Assuming an \( SU(2) \) symmetric sea \( \Delta q^{(3)}(y) = \Delta u(y) - \Delta d(y) = 0 \), we have \( \Delta q^{(3)}(y) = \Delta u_{val}(y) - \Delta d_{val}(y) \) and model it by a semi-realistic ansatz \( \Delta q^{(3)}(y) = \frac{\Gamma(1-n)}{\Gamma(4)(1-n)} \frac{(1-y)^3}{y^3} \) and \( g_A = 1.26 \). This allows to get a simple analytical representation

\[ \frac{\tilde{H}^{(3)}(x, \eta, \Delta^2)}{g_A F(\Delta^2)} = \frac{(1 - \frac{2}{\eta})}{\eta^3} \left\{ \theta(x > -\eta) \left( \frac{x + \eta}{1 + \eta} \right)^{2-n} \left( \eta^2 - x + (2 - n)\eta(1 - x) \right) - (\eta \rightarrow -\eta) \right\}, \]  

(9)

where a dipole parametrization \( F(\Delta^2) = (1 - \Delta^2/m_A^2)^{-2} \) with \( m_A^2 = 0.9 \text{ GeV}^2 \) is used. One sees that \( \tilde{H}^{(3)}(x, \eta) \) vanishes for \( x < -\eta \). Other models arise from computations in the framework of the bag and the chiral quark soliton model or based on the overlap representation.

3 Power suppressed corrections

The leading twist-two predictions \( \tilde{H}(x, \eta, \Delta^2) \) (with \( A^3 = 0 \)) and \( \tilde{F}(y, z) \) are affected by a number of corrections, with power suppressed effects in \( Q \) being one of them. They have been addressed in detail recently for the generalized Compton amplitude. In Ref. using a QCD improved parton picture of Ellis et al., it was shown for the example of a scalar target that the twist-three contributions induce gauge restoring pieces (lacking in the twist-two approximation) in the Lorentz structure of the Compton amplitude. A complete OPE-based analysis has been done in where
generalized Wandzura-Wilczek relations have been found. The twist-three GPDs are expressed by these relations in terms of twist-two ones as well as interaction dependent antiquark-gluon-quark correlations, 
\[ A_3 = W \otimes A + \langle \bar{\psi} G \psi \rangle. \]
In Refs. 17, 18, it was observed that for the DVCS kinematics, \( \eta \approx -\xi \), the Wandzura-Wilczek relations alluded to above develop a singularity, however, the latter does not show up in the cross section in which the twist-three functions enter in a specific combination. In an earlier attempt 19 a generalized Wandzura-Wilczek relation has been derived by neglecting all twist-three operators, and is, therefore, erroneous.

Since the cross sections for the reactions in question are peaked at low momentum transfer, mass corrections may play a vital role in the confrontation of theoretical predictions with experimental measurements. As a first step towards this direction we have resummed the target mass corrections, \( \sum_k c_k (M^2/Q^2)^k \), stemming from the trace terms of the twist-two operators. The same can be done in the twist-three sector and will provide effects of the same order. The twist-four sector awaits its unraveling since the renormalon analysis demonstrated a potentially sizable effect coming from the latter already for the moderate momentum transfer 20.

4 Perturbative corrections

Leading order calculations in QCD are known to be unreliable and they are strongly affected by perturbative corrections. For the cases at hand, the coefficient function is a series in coupling
\[ C = C_0 + \frac{\alpha_s}{2\pi} C_1 + \cdots. \] (10)

The one-loop correction \( C_1 \) to the handbag approximation of the generalized Compton scattering amplitudes have been derived in 21, 22, 23. For the hard meson production with quark dominated parton subprocess, \( C_1 \) has been recently extracted in Ref. 24 making use of the known result for the pion form factor. The analyses have shown that they can produce a very large modification of the leading order (LO) predictions 25, 24.

Yet another source of corrections stems from the evolution of GPDs (and DAs), which is governed (at twist-two level) by the equation (with \( \eta = 1 \) for DAs)
\[ \frac{d}{d\ln Q^2} A(x, \eta) = \int_{-1}^{1} dy K(x, y, \eta) A(y, \eta), \quad \text{with} \quad K = \frac{\alpha_s}{2\pi} K_0 + \left( \frac{\alpha_s}{2\pi} \right)^2 K_1 + \cdots. \] (11)

The one loop kernels for all channels have been computed by Lipatov et al. in Ref. 20 and recalculated by a number of groups in the last few years of XXth century (see 2 for a complete list of references). The two-loop quantities have become available first in the form of local anomalous dimensions from Refs. 25, 26, 27 and later have been converted into the momentum fraction form in 30 by means of an extensive use of conformal 28 and supersymmetric 31 Ward identities. Note that the flavour nonsinglet kernel has been known for quite a while from 32, 33, 34.

The evolution effects have been studied by methods based on the orthogonal polynomial reconstruction 35,11, extended in 36 to NLO accuracy, and the direct numerical integration 37. The resulting two-loop effects have been found to alter the LO evolution in a percentage range.

5 Observables

In the electroproduction process of a real photon the DVCS signal is strongly contaminated by the Bethe-Heitler (BH) process, \( \sigma \propto |DVCS + BH|^2 \). However, individual terms in this sum are distinguished by peculiar dependencies on the lepton charge, lepton \( \lambda \) and hadron spin, and the azimuthal angle \( \varphi \) of the outgoing photon. The best quantity to access the GPDs is, of course, the DVCS-BH interference since the GPDs enter linearly in it. Its precise experimental
extraction is only possible on machines having lepton beams of both charges, \( \sigma^+ - \sigma^- \propto \text{DVCS-BH} \). This allows a clean separation of Fourier components w.r.t. azimuthal angle, \( \text{DVCS-BH} \propto \sum_m [c_m \cos(m\varphi) + \lambda_m \sin(m\varphi)] \). For a lepton beam with a given charge, other observables (like diverse spin asymmetries, which isolate the DVCS-BH interference at leading power in \( 1/Q^2 \)-expansion) will be contaminated by (sizable) power suppressed (higher twist) effects. For instance, in the cross section \( \sigma \), the coefficient of \( \cos/\sin(\varphi) \) stemming from the twist-two DVCS-BH will get corrected by \( |\text{DVCS}|^2 \) at twist-three level, etc. Note however, that the twist-three effects do not induce the same angular dependence in DVCS-BH and \( |\text{DVCS}|^2 \), separately, which is already generated by leading twist contributions, i.e. in DVCS-BH they produce \( \cos/\sin(2\varphi) \) only (for a numerical study see\[13\]), so that the coefficient of \( \cos/\sin(\varphi) \) will be modified at twist-four level\[2\]. The experimental isolation of different Fourier components can be done by forming appropriate azimuthal asymmetries or weighting data with corresponding angular functions \( \cos/\sin(m\varphi) \). As distinguished from DIS, in DVCS one can access tensor gluons, which are not contaminated by quarks at twist-two. They show up at one-loop order\[3\] and generate a specific \( \cos/\sin(3\varphi) \) azimuthal angle dependence in the cross section\[11\]. Provided we have separated DVCS-BH interference alone, the single spin asymmetries make it possible to extract the imaginary part of the DVCS amplitude and thus give access to the shape (at LO in \( \alpha_s \) in complete analogy to DIS) of GPDs on the diagonal \( x = \xi \), e.g.\[11\]:

\[
\frac{d\sigma^- - d\sigma^+}{d\varphi} = \frac{\alpha^2_\text{em} y^2 (2 - x_B)(2 - y)}{\sqrt{1 - y Q^2} \Delta^2} |\Delta_\perp| \sin \varphi \text{Im}\left\{ F_1 H + \frac{x_B}{2 - x_B} G_M \bar{H} - \frac{\Delta^2}{4 M^2} F_2 \bar{E}\right\},
\]

where \( A = C \otimes A \), see Eq. (3), \( F_1, F_2 \) and \( G_M = F_1 + F_2 \) are Dirac, Pauli and magnetic Sachs form factors, and the transverse momentum squared is \( \Delta^2 = ((1 - x_B)\Delta^2 + x_B^2 M^2)/(1 - x_B/2)^2 \).

In hard exclusive meson production one measures products of off-forward amplitudes, e.g. the asymmetry of the \( \pi^+ \)-production off a transversely polarized proton reads\[14\]:

\[
\frac{d\sigma^- - d\sigma_\parallel}{d\Delta^2 d\varphi} = -\alpha^2_\text{em} \frac{4\pi f^2}{9 Q^6} \frac{x_B^3}{2 - x_B} |\Delta_\perp| M \sin \varphi \text{Im}\left\{ \tilde{H}^{(3)\ast} \tilde{E}^{(3)}\right\},
\]

with \( (\tilde{H}, \tilde{E}) = \phi \otimes \tilde{C} \otimes (H, E) \), see Eq. (12). Cross sections for production of other meson species can be found in\[13\]. Cross sections for production of other meson species can be found in\[13\]. Cross sections for production of other meson species can be found in\[13\]. Cross sections for production of other meson species can be found in\[13\]. Note that quark transversity GPD does not show up in the meson production due to preservation of the chiral symmetry in perturbation theory\[12\]. The current experimental situation is discussed in\[2\] and\[2\] for HERMES and JLab settings, respectively.

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