TWO-POINT CORRELATION FUNCTION OF HIGH-REDSHIFT OBJECTS ON A LIGHT CONE:
EFFECT OF THE LINEAR REDSHIFT-SPACE DISTORTION

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ABSTRACT

A theoretical formulation for the two-point correlation function on a light cone is developed in redshift space. On the basis of the previous work by Yamamoto & Suto, in which a theoretical formula for the two-point correlation function on a light cone has been developed in real space, we extend it to the formula in redshift space by taking the peculiar velocity of the sources into account. A simple expression for the two-point correlation function is derived. We briefly discuss QSO correlation functions on a light cone adopting a simple model of the sources.

Subject headings: cosmology: theory — dark matter — galaxies: distances and redshifts — large-scale structure of universe — space vehicles

1. INTRODUCTION

The clustering of high-redshift objects is one of the current topics in the fields of observational cosmology and astrophysics. High-redshift objects of \( z \gtrsim 1 \) are becoming fairly common, and evidences of the clustering nature of such cosmic objects are reported in the various observational bands, e.g., X-ray-selected active galactic nuclei (AGNs) (Carrera et al. 1998), the FIRST survey (Cress et al. 1996; Magliocchetti et al. 1998), high-redshift galaxies (Steidel et al. 1998; Giavalisco et al. 1998), and QSO surveys (Croom & Shanks 1996; Boyle et al. 1998). The statistics of these high-redshift objects are increasing, and we will be able to discuss the clustering at a quantitative level precisely in the near future. From a theoretical point of view, the most important subject is to clarify the physical process of the formation history of these objects. The standard theoretical framework for the cosmic structure formation is based on the cold dark matter (CDM) model with Gaussian initial density fluctuations. The clustering nature of the high-redshift objects provides us with many kinds of tests for the theoretical models (e.g., Peacock 1998; Jing & Suto 1998).

When analyzing the clustering nature of the high-redshift objects at a quantitative level, we must take the light-cone effect into account properly. Namely, such cosmological observations are feasible only on the light-cone hypersurface defined by the current observer. Then, the effect of the time evolution of the sources, i.e., the luminosity function, the clustering amplitude, and the bias, contaminate observational data. Thus this light-cone effect is especially important in discussing the three-dimensional two-point correlation function of the high-redshift objects. Some aspects of the light-cone effect have been discussed (Matarrese et al. 1997; Matsubara, Suto, & Szapudi 1997; Nakamura, Matsubara, & Suto 1998; de Laix & Starkman 1998). Recently one of the authors (K. Y.) & Suto developed a formulation for the two-point correlation function for the high-redshift objects defined on the light-cone hypersurface (Yamamoto & Suto 1999, hereafter Paper I). The expression for the two-point correlation function on the light cone was derived in a rigorous manner starting from first principles corresponding to the conventional pair-count analysis. This investigation is very important because it gives a rigorous relation between an observational data processing and a theoretical prediction as to the two-point correlation function on a light cone for the first time. However, this investigation is restricted to the formula in real space, though observational maps of the high-redshift objects are obtained in redshift space.

It is well known that the peculiar velocity of sources distorts their distribution in redshift space (e.g., Davis & Peebles 1983; Kaiser 1987; Hamilton 1998). And this effect has been discussed as a probe of cosmological density parameters (e.g., Szalay, Matsubara, & Landy 1998; Nakamura et al. 1998; Matsubara & Suto 1996; Hamilton & Culhane 1996; Heavens & Taylor 1995; Suto et al. 1999). In the previous paper (Paper I) the effect of the redshift-space distortion due to the peculiar velocity of the sources is not taken into account because it was formulated in real space. From a practical point of view, the formula in redshift space must be developed. The purpose of the present paper is to develop such a theoretical formula for the two-point correlation function on a light cone by taking the redshift-space distortion due to the peculiar velocity into account.

The paper is organized as follows: In § 2 we develop a formulation for the two-point correlation function on the light-cone hypersurface in redshift space in order to incorporate the linear redshift-space distortion. The expression for the two-point correlation function is presented in a rather simple form by using appropriate approximations. The main result is equation (32). As a demonstration of the usefulness of our formalism, we apply the formula to QSO correlation functions adopting a simple model of source distribution and cosmological models. The validity of the plane-parallel, or distant observer, approximation for the correlation function of high-redshift objects is also discussed. Section 4 is devoted to discussion and conclusion. Throughout this paper we use units in which the velocity of light \( c \) is unity.

2. TWO-POINT CORRELATION FUNCTION IN REDSHIFT SPACE

In this section we develop a theoretical formulation for the two-point correlation function on a light-cone hypersurface in redshift space by taking the peculiar motion of sources into account. In the present paper, we focus on the spatially flat
Friedmann-Lemaître universe, whose line element is expressed in terms of the conformal time \( \eta \) as
\[
ds^2 = a^2(\eta)(-d\eta^2 + d\chi^2 + \chi^2 d\Omega_\Lambda^2).
\]
(1)

Here the scale factor is normalized to be unity at present, i.e., \( a(\eta_0) = 1 \). The Friedmann equation is
\[
\left( \frac{\dot{a}}{a} \right)^2 = H_0^2 \left( \frac{\Omega_\Lambda}{a^3} + \frac{\Omega_0}{a^3} \right),
\]
(2)
where \( \Omega_\Lambda = 1 - \Omega_0 \), the dot denotes \( \eta \) differentiation, and \( H_0 \) is the Hubble constant \( H_0 = 100 \ h \ km \ s^{-1} \ Mpc^{-1} \).

Since our fiducial observer is located at the origin of the coordinates \( (\eta = \eta_0, \chi = 0) \), an object at \( \chi \) and \( \eta \) on the light-cone hypersurface of the observer satisfies a simple relation \( \eta = \eta_0 - \chi \). Then the (real-space) position of the source on the light-cone hypersurface is specified by \( (\chi, \gamma) \), where \( \gamma \) is a unit directional vector. In order to avoid confusion, we introduce the radial coordinate \( r \) instead of \( \chi \), and we denote the metric of the three-dimensional real space on which the observable sources are distributed as follows:
\[
ds_{LC}^2 = dr^2 + r^2 d\Omega_\Lambda^2.
\]
(3)

Denoting the comoving number density of observed objects at a conformal time \( \eta \) and at a position \( (\chi, \gamma) \) by \( n(\eta, \chi, \gamma) \), then the corresponding number density projected onto the space (eq. [3]) is obtained by
\[
n^{LC}(r, \gamma) = n(\eta, \chi, \gamma)|_{\eta = \eta_0 - r, \chi = \gamma}.
\]
(4)

Introducing the mean observed (comoving) number density \( n_0(\eta) \) at time \( \eta \) and the density fluctuation of luminous objects \( \Delta(\eta, \chi, \gamma) \), we write
\[
n(\eta, \chi, \gamma) = n_0(\eta)[1 + \Delta(\eta, \chi, \gamma)];
\]
(5)
then equation (4) is rewritten as
\[
n^{LC}(r, \gamma) = n_0(\eta)[1 + \Delta(\eta, \chi, \gamma)]|_{\eta = \eta_0 - r, \chi = \gamma}.
\]
(6)

Note that the mean observed number density \( n_0(\eta) \) is different from the mean number density of the objects \( \bar{n}(\eta) \) at \( \eta \) by a factor of the selection function \( \phi(\eta) \), which depends on the luminosity function of the objects and thus the magnitude limit of the survey; for instance, \( n_0(\eta) = \bar{n}(\eta)\phi(\eta) \).

In a similar way, if we know the peculiar velocity field, the corresponding quantity projected onto the space (eq. [3]) is obtained by
\[
v^{LC}(r, \gamma) = v_s(\eta, \chi, \gamma)|_{\eta = \eta_0 - r, \chi = \gamma},
\]
(7)
where \( v_s(\eta, \chi, \gamma) \) is the CDM velocity field. Here we assume that the peculiar velocity field of luminous objects agrees with the CDM velocity field.

In Appendix A we summarize equations for the linear perturbation theory in a CDM-dominated universe. Thus the linearized CDM density perturbation can be solved completely. However, the evolution of the source density fluctuations cannot be solved completely since the bias mechanism is not well understood at present, unfortunately. Then we must assume a model for the bias that connects the CDM density perturbations and the source number density fluctuations. In the present paper we assume a scale-dependent bias model:
\[
b(k; \eta) = \frac{\Delta_{k,0}(\eta)}{\delta_{k,0}(\eta)}.
\]
(8)

where \( \Delta_{k,0}(\eta) \) and \( \delta_{k,0}(\eta) \) are the Fourier coefficients for the source number density fluctuation and the CDM density fluctuation, respectively (see also Appendix A).

The next task is to describe the relation between real space and redshift space, since we consider the distribution of sources in redshift space. First we consider how the peculiar velocity of a source distorts the estimation of the distance to the source. Let us assume that a source at redshift \( z \) [at a position \( (r, \gamma) \) in real space] is moving with a peculiar velocity \( v_s \). The observed photon frequency \( v_{obs} \) and the emitted photon frequency \( v_{emit} \) are related as
\[
v_{obs} = \frac{v_{emit}}{1 + z}(1 - \gamma \cdot v_s).
\]
(9)

From this equation, we find the shift in the apparent redshift due to the peculiar velocity as
\[
\delta z = (1 + z)(\gamma \cdot v_s).
\]
(10)

From the Friedmann equation (eq. [2]), we have
\[
\delta \eta = \frac{1}{H_0 \sqrt{\Omega_0 + \Omega_\Lambda a^3}}.
\]
(11)
Combining equations (10) and (11), we obtain the apparent shift in the comoving coordinate due to the peculiar velocity as

$$\delta r = -\delta \eta = \frac{\mathcal{P}(\eta)}{H_0} \gamma \cdot \mathbf{v}_{\eta \rightarrow \eta_0 - \mathbf{r}, \mathbf{x} - \mathbf{r}} \tag{12}$$

where we defined

$$\mathcal{P}(\eta) = \frac{a(\eta)^{1/2}}{\sqrt{\Omega_m + \Omega_\Lambda a(\eta)^4}} \tag{13}$$

We introduce the variable \(s\) to denote the radial coordinate in redshift space. Then a position in redshift space is specified by \((s, \gamma)\), while a position in real space is specified by \((r, \gamma)\). The relation between the redshift position and the real position is

$$s = r + \delta r \tag{14}$$

where \(\delta r\) is specified by equation (12). The conservation of the number of sources gives (Hamilton 1998)

$$n_i(s, \gamma)ds d\Omega_r = n^{LC}(r, \gamma)r^2 dr d\Omega_r \tag{15}$$

where \(n_i(s, \gamma)\) denotes the number density in redshift space and \(n^{LC}(r, \gamma)\) denotes the number density in real space. These two equations (14) and (15) specify the relation between redshift space and real space.

Now let us consider the two-point correlation function in redshift space. We start from the following ensemble estimator for the two-point correlation function:

$$\mathcal{X}(R) = \frac{1}{V^{LC}} \int \frac{d\Omega_r}{4\pi} \int ds_1 s_1^2 d\Omega_{s_1} \int ds_2 s_2^2 d\Omega_{s_2} n_i(s_1, \gamma_1)n_i(s_2, \gamma_2)\delta^3(s_1 - s_2 - R), \tag{16}$$

where \(s_1 = (s_1, \gamma_1)\) and \(s_2 = (s_2, \gamma_2)\) and \(R = |R|\), \(\hat{R} = R/R\), and \(V^{LC}\) is the comoving survey volume of the data catalog:

$$V^{LC} = \int_{s_{\min}}^{s_{\max}} s^2 ds \int d\Omega_r = \frac{4\pi}{3} (s_{\max}^3 - s_{\min}^3), \tag{17}$$

with \(s_{\max} = s(z_{\max})\) and \(s_{\min} = s(z_{\min})\) being the boundaries of the survey volume. Equation (16) is a natural extension of the estimator for the two-point correlation function in the real space (see also Paper I).

By using equations (14) and (15), we rewrite equation (16) in terms of the variables in real space:

$$\mathcal{X}(R) = \frac{1}{V^{LC}} \int \frac{d\Omega_r}{4\pi} \int dr_1 r_1^2 d\Omega_{r_1} \int dr_2 r_2^2 d\Omega_{r_2} n^{LC}(r_1, \gamma_1)n^{LC}(r_2, \gamma_2)\delta^3(x_1 + \delta x_1 - x_2 - \delta x_2 - R), \tag{18}$$

where \(x_1 + \delta x_1 = (r_1 + \delta r_1, \gamma_1)\), \(x_2 + \delta x_2 = (r_2 + \delta r_2, \gamma_2)\), and \(\delta r_1\) and \(\delta r_2\) are given by equation (12). Then we approximate as

$$\delta^3(x_1 + \delta x_1 - x_2 - \delta x_2 - R) \simeq \left(1 + \delta x_1 \cdot \frac{\partial}{\partial x_1} \right) \left(1 + \delta x_2 \cdot \frac{\partial}{\partial x_2} \right) \delta^3(x_1 - x_2 - R), \tag{19}$$

where we can write \(\delta \mathbf{x} \cdot \partial / \partial \mathbf{x} = \delta r \cdot \partial / \partial r\) since only the radial component of \(\delta \mathbf{x}\) has a nonzero value. By using this approximation and equation (6) we derive the following equation from (18):

$$\mathcal{X}(R) = \frac{1}{V^{LC}} \int \frac{d\Omega_r}{4\pi} \int dr_1 r_1^2 d\Omega_{r_1} \int dr_2 r_2^2 d\Omega_{r_2} \int dr_3 r_3^2 d\Omega_{r_3} n^{LC}(r_1)n^{LC}(r_2)n^{LC}(r_3) \sum_{i=1}^{2} \left[1 + \Delta(r_i, \gamma_i) + \delta r_i \frac{\partial}{\partial r_i} \right] \delta^3(x_1 - x_2 - R), \tag{20}$$

where we used the notations

$$n^{LC}(r) = n_0(\eta)|_{\eta \rightarrow \eta_0 - r}, \Delta(r, \gamma) = \Delta(\eta, \chi, \gamma)|_{\eta \rightarrow \eta_0 - r, \mathbf{x} \rightarrow \mathbf{r}}, \tag{21}$$

and \(\delta r_i\) is understood as

$$\delta r_i = \frac{\mathcal{P}(\eta)}{H_0} \gamma \cdot \mathbf{v}|_{\eta \rightarrow \eta_0 - r_i, \mathbf{x} \rightarrow \mathbf{r}_i}, \tag{22}$$

where \(i = 1, 2\).

Next we consider the ensemble average of the ensemble estimator \(\mathcal{X}(R)\). Since \(\delta r_1\) and \(\delta r_2\) are the order of linear perturbation, then the ensemble average is written as

$$\langle \mathcal{X}(R) \rangle = \mathcal{X}(R) + \mathcal{X}(R), \tag{23}$$

where

$$\mathcal{X}(R) = \frac{1}{V^{LC}} \int \frac{d\Omega_r}{4\pi} \int dr_1 r_1^2 d\Omega_{r_1} \int dr_2 r_2^2 d\Omega_{r_2} \int dr_3 r_3^2 d\Omega_{r_3} n^{LC}(r_1)n^{LC}(r_2)\delta^3(x_1 - x_2 - R), \tag{24}$$
where $b$ and $P$ are defined as and $\cos \theta$ should be replaced by after operating the differentiation with respect to $r_1$ and $r_2$.

In Appendix B we present the explicit calculations for $\Psi(R)$. According to the result, equation (25) reduces to the following form within the linear theory of perturbation:

$$
\Psi(R) = \frac{1}{\sqrt{\Lambda c}} \frac{1}{\pi R} \int_0^R dr_1 dr_2 r_1 r_2 n^0_C(r_1)n^0_C(r_2)D_1(\eta_0 - r_1)D_1(\eta_0 - r_2)
$$

and

$$
\Psi_s(R) = \frac{1}{\sqrt{\Lambda c}} \frac{1}{\pi R} \int_{s_{\min}}^{s_{\max}} d(r_1) r_1^2 \int_{s_{\min}}^{s_{\max}} d(r_2) r_2^2 \int d\Omega_{r_1} \int d\Omega_{r_2} n^0_C(r_1)n^0_C(r_2)D_1(\eta_0 - r_1)D_1(\eta_0 - r_2)
$$

where

$$
\Psi_s(R) = \frac{1}{\sqrt{\Lambda c}} \frac{1}{\pi R} \int_{s_{\min}}^{s_{\max}} d(r_1) r_1^2 \int_{s_{\min}}^{s_{\max}} d(r_2) r_2^2 \int d\Omega_{r_1} \int d\Omega_{r_2} n^0_C(r_1)n^0_C(r_2)D_1(\eta_0 - r_1)D_1(\eta_0 - r_2)
$$

and $P(k)$ is the CDM power spectrum at present, $D_1(\eta)$ is the linear growth rate normalized to be unity at present, $f(\eta)$ is defined as $f(\eta) = d \ln D_1(\eta)/d \ln a(\eta)$, and $\cos \theta$ should be replaced by $\cos \theta = (r_1^2 + r_2^2 - 2r_1 r_2 \cos \theta)/2r_1 r_2$ after operating the differentiation with respect to $r_1$ and $r_2$.

Omitting the second term in the derivative (eq. [27]), equation (26) reduces to the simple form in the case $R \ll r_{\min}$ and $R \ll r_{\max}$ (see Appendix B):

$$
\Psi(R) \approx \frac{4\pi}{\sqrt{\Lambda c}} \int_{r_{\min}}^{r_{\max}} drr^2 n^0_C(r)^2 \frac{1}{2\pi^2} \int k^2 dk P(k)[b(k; \eta_0 - r) - k^2 \varphi(r)] j_0(k \sqrt{r_1^2 + r_2^2 - 2r_1 r_2 \cos \theta})
$$

where $b(k; \eta)$ is defined by

$$
b(k; \eta) = \frac{f(\eta)}{b(k; \eta)} = \frac{1}{b(k; \eta)} \frac{d \ln D_1(\eta)}{d \ln a(\eta)}
$$

and we assumed $s_{\max} = r_{\max}$ ($s_{\min} = r_{\min}$).

We can derive the following equation from a similar calculation in the above equation (see also Paper I):

$$
\Psi(R) \approx \frac{4\pi}{\sqrt{\Lambda c}} \int_{r_{\min}}^{r_{\max}} drr^2 n^0_C(r)^2
$$

Following Paper I, we define the two-point correlation function on the light-cone hypersurface:

$$
\xi_{s,LC}(R) = \frac{\langle \Psi_s(R) \rangle - \Psi(R)}{\Psi(R)}
$$

Substituting equations (28) and (30) into equation (31), we have

$$
\xi_{s,LC}(R) \approx \left[ \int_{r_{\min}}^{r_{\max}} drr^2 n^0_C(r)^2 \right]^{-1} \int_{r_{\min}}^{r_{\max}} drr^2 n^0_C(r)^2 \frac{1}{2\pi^2} \int k^2 dk P(k)[b(k; \eta_0 - r^2) - k^2 \varphi(r)] j_0(k \sqrt{r_1^2 + r_2^2 - 2r_1 r_2 \cos \theta})
$$

This is the final expression for the two-point correlation function on the light-cone hypersurface in which the linear redshift-space distortion is taken into account. Comparing this result with $\xi_{s,LC}(R)$ in Paper I, the terms in proportion to $\beta(k; \eta)$ are the new terms which represent the effect of the linear redshift-space distortion.

3. A SIMPLE DEMONSTRATION

In this section we apply the formula developed in the previous section to the QSO two-point correlation function. Evidence for the spatial correlation in the QSO distribution has been reported (Croom & Shanks 1996; Boyle et al. 1998); however, it seems difficult to draw definite cosmological conclusions from a comparison with the currently available data. Therefore, we only demonstrate the usefulness of our formalism by calculating the QSO two-point correlation function based on a simplified model for the distribution and the bias model. As for the bias, we here consider the scale-independent bias model by Fry (1996):

$$
b(\eta) = 1 - \frac{1}{D_1(\eta)} (b_0 - 1),
$$
where \( b_0 \) is a constant parameter. Note that the bias \( b(\eta) \) at high redshift becomes larger as \( b_0 \) becomes larger. Here we also assume that the sources are distributed in the range \( 0.3 \leq z \leq 3 \) with a constant number density, i.e., \( n_0 = \text{const} \). This model may be oversimplified; however, we have checked that the qualitative features have not been changed even when adopting more realistic models in Paper I.

In Figures 1 and 2 we show the two-point correlation function \( \xi^{LC}(R) \) and other mass correlation functions for comparison. We show the case for the standard cold dark matter (SCDM) model in Figure 1, in which we adopted \( \Omega_0 = 1, \Omega_\Lambda = 0, \) and \( h = 0.5 \), and the CDM density power spectrum normalized as \( \sigma_8 = 0.56 \) (Kitayama & Suto 1997). The case for the cosmological model with a cosmological constant (CDM model) is shown in Figure 2, in which \( \Omega_0 = 0.3, \Omega_\Lambda = 0.7, h = 0.7, \) and \( \sigma_8 = 1.0 \) are adopted. Figures 2a–2c, show the correlation functions on a light cone. The solid line shows \( \xi^{LC}(R) \) of equation (32). The dashed line shows the case when neglecting the effect of the redshift-space distortion by setting \( \beta = 0 \) in equation (32). On the other hand, Figure 2d shows the linear and nonlinear mass two-point correlation functions defined on a constant time hypersurface \( z = 0, 1, \) and \( 2 \).

From these figures it is apparent that the larger bias at high redshift derives the larger amplitude of the correlation function on a light cone. Furthermore the effect of the redshift-space distortion always amplifies the correlation function from comparing the solid line and the dashed line in Figures 2a–2c, as expected. However, the relative difference between the solid line and the dashed line becomes smaller as the bias becomes large and more effective. This is an expected feature because \( \beta \)-factor becomes smaller as the bias becomes larger (see eq. [29]).

We have also calculated the correlation function by adopting the exact expression equation (B20) instead of equation (28). The difference is less than 1% for \( R \leq 100 \, h^{-1} \, \text{Mpc} \) and is negligible. Thus equation (32) is a well-approximated formula and is a useful expression for the correlation function for high-redshift objects. Equation (32) is easily understood in an intuitive manner. Namely, the linear power spectrum in redshift space is amplified by \( (1 + \beta k_p)^2 \) over its unredshifted counterpart \( P(k) \) in the plane-parallel approximation, where \( \beta \) is defined in the same way as in equation (29) and \( k_p = c \cdot k/k \) (e.g., Kaiser 1987; Hamilton 1998). This formula implies that the angle-averaged redshift power spectrum is amplified by the factor, \( (1 + 2\beta/) \)

![Figure 1](image_url)

**Fig. 1.** Absolute values of the two-point correlation function for QSOs on a light cone and the linear and nonlinear mass two-point correlation functions in the standard CDM model, where we adopted \( \Omega_0 = 1, \Omega_\Lambda = 0, h = 0.5, \) and the CDM density power spectrum normalized as \( \sigma_8 = 0.56 \) (Kitayama & Suto 1997). The parameter \( b_0 \) for the bias model is adopted as \( a) b_0 = 1, \) \( b) b_0 = 1.5, \) and \( c) b_0 = 2. \) Here we assumed that the sources are distributed in the range \( 0.3 \leq z \leq 3 \) with a constant number density. In panels (a)–(c), the solid line shows our \( \xi^{LC}(R) \), and the dashed line shows the case when the redshift-space distortion is neglected by setting \( \beta = 0 \); panel (d) shows linear (lower curves) and nonlinear (upper curves) mass correlation functions by Peacock & Dodds (1996) defined on constant-time hypersurfaces \( z = 0, 1, \) and \( 2 \).
3 + \beta^2/5), over the unredshifted power spectrum. Thus equation (32) is the expected formula obtained by multiplying the factor \((1 + 2\beta/3 + \beta^2/5)\) at each cosmological time over unredshifted counterpart \(P(k)\) in the correlation function in real space. In this sense equation (32) is based on the plane-parallel approximation.\(^1\) And our investigation shows that the use of the plane-parallel approximation is valid for the (angle-averaged) correlation function of high-redshift objects on a light cone.

Because we plotted absolute values of the two-point correlation functions in Figures 1 and 2, we can read that the solid line and the dashed line show the anticorrelation at the large separation \(R \gtrsim \text{a few } 10 h^{-1}\text{Mpc}\) in the SCDM model (see Fig. 1). Equation (32) implies that the zero point of the correlation function is invariant even when the redshift space distortion is taken into account, as long as the bias does not depend on the scale \(k\). The critical correlation length, where the correlation changes to the anticorrelation, is given by

\[
R = \frac{16.6 \ h^{-1} \text{Mpc}}{\Omega_b h \exp \left[ -\Omega_b - \sqrt{2\Omega_b/\Omega_b} \right]},
\]

where we assumed the Harrison-Zeldovich initial density power spectrum and used the fitting formula for the transfer function (Bardeen et al. 1986; Sugiyama 1995). This critical correlation length may be observed in the upcoming 2dF and SDSS QSO surveys and may be tested for the cosmological models and the theoretical models of bias.

4. SUMMARY AND DISCUSSION

In this paper we have developed a theoretical formulation for the two-point correlation function for high-redshift objects on a light cone in redshift space. Our formula has been developed by extending the previous work (Paper I) to the formula in redshift space. We have started our formulation from considering the ensemble estimator of the two-point correlation function in redshift space, and then we have calculated the ensemble average of the estimator. Thus our formula has been derived in a rigorous manner starting from first principles corresponding to the conventional pair-count analysis. The calculation was cumbersome; however, a rather simple expression (eq. [32]) has been derived.

\(^1\) We thank T. Matsubara for his comment that our formula reproduces the formula based on the plane-parallel approximation.
We have demonstrated the effect of the redshift-space distortion by showing the QSO two-point correlation function adopting a very simple model of the source distribution and the bias, though it seems premature to draw definite cosmological conclusions from comparison with currently available data. As discussed below, our model adopted in this paper may be oversimplified in order to compare with a real data sample. Nevertheless, our investigation is instructive, and we have shown how the redshift-space distortion affects the correlation function for high-redshift objects on a light cone (§ 3). Our investigation shows that the redshift-space distortion becomes a small effect for time-varying bias models that have large values at high redshift. The validity of the plane-parallel approximation is also shown for the correlation function of high-redshift objects on a light cone.

There remain uncertainties in making precise theoretical predictions. First, we did not attempt to examine possible bias models other than the model by Fry (1996). However, theoretical investigations for the time- and scale-dependent bias are just beginning (Fry 1996; Mo & White 1996; Dekel & Lahav 1999; Tegmark & Peebles 1998; Taruya, Koyama, & Soda 1999). Conversely, the clustering of high-redshift objects will be a good tool to test the bias models. Second, we did not consider the realistic model for the time evolution of number density in calculating the QSO two-point correlation function. Concerning this point, a finite solution will be obtained in upcoming 2dF and SDSS QSO surveys. Third, we have considered only the realistic model for the time evolution of number density in calculating the QSO two-point correlation function. Concerning this point, a finite solution will be obtained in upcoming 2dF and SDSS QSO surveys. Third, we have considered only the clustering of high-redshift objects will be a good tool to test the bias models. Second, we did not consider the effect in redshift space has not been well understood especially for high-redshift objects, and so it must be investigated in future work. Numerical approaches would probably be needed for that purpose.

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APPENDIX A

REVIEW OF THE LINEAR THEORY OF THE CDM DENSITY PERTURBATIONS

In this Appendix we summarize equations for the linear theory of the CDM density perturbations and explain the notations that are used in the present paper. The linearized CDM density perturbation in the CDM dominated universe obeys the following equations:

\[ \dot{\delta}_c + v^i_c \partial_i \delta_c = 0 , \]  
\[ \dot{\delta}_c + \frac{\dot{a}}{a} \delta_c + \Psi^{(i)} = 0 , \]  
\[ \Psi^{(i)} = 4\pi G \rho a^2 \delta_c = \frac{3\Omega_m H_0^2}{2a} \delta_c , \]

where \( \delta_c \) is the CDM density contrast, \( v^i_c \) is the CDM velocity field, and \( \Psi \) is the gravitational potential, which follows the gravitational Poisson equation (A3), and \( \partial_i \) denotes the covariant derivative on three-dimensional space.

As we are interested in the scalar perturbation, we expand the CDM density contrast \( \delta_c \) and the velocity field \( v^i_c \) in terms of the scalar harmonics as follows (e.g., Kodama & Sasaki 1984):

\[ \delta_c(\eta, \chi, \gamma) = \int_0^\infty dk \sum_{l,m} \delta^{(0)}_{l,m}(\eta) Y_{l,m}(\chi, \gamma) , \]  
\[ v^i_c(\eta, \chi, \gamma) = \int_0^\infty dk \sum_{l,m} \hat{v}^{(i)}_{l,m}(\eta) Y^i_{l,m}(\chi, \gamma) , \]

where \( Y_{l,m} \) are the normalized scalar harmonics:

\[ Y_{l,m}(\Omega_{\chi}, \gamma) = X^i_{l}(\chi) Y_{l,m}(\Omega_{\gamma}) , \]

with

\[ X^i_{l}(\chi) = \frac{\sqrt{2}}{\sqrt{\pi}} j_{l}(k\chi) , \]

\( Y_{l,m}(\Omega_{\chi}, \gamma) \) and \( j_{l}(k\chi) \) are the spherical harmonics and the spherical Bessel function, respectively; \( k \) denotes the eigenvalue of the eigenvalue: \( \delta^{(0)}_{l,m} = -k^2 \delta^{(0)}_{l,m} \), and \( \hat{v}^{(i)}_{l,m} \) is defined as

\[ \hat{v}^{(i)}_{l,m}(\chi, \gamma) = -\frac{1}{k} Y^i_{l,m}(\chi, \gamma) X^i_{l}(\gamma) . \]
From the linearized perturbation equations (A1)–(A3), we have
\[ \delta_{klm}^{(0)} + kv_{klm} = 0, \quad (A9) \]
\[ \dot{v}_{klm} + \frac{\dot{a}}{a} v_{klm} - k\Psi_{klm} = 0, \quad (A10) \]
\[ k^2\Psi_{klm} = -\frac{3\Omega_0 H_0^2}{2a} \delta_{klm}^{(c)}, \quad (A11) \]
where \( \Psi_{klm} \) is the Fourier coefficient defined in the same way as in equation (A4). Combining these equations, we have
\[ \delta_{klm}^{(c)} + \frac{\dot{a}}{a} \delta_{klm}^{(c)} - \frac{3}{2} \Omega_0 H_0^2 \delta_{klm}^{(c)} = 0. \quad (A12) \]
In the Friedmann-Lemaître universe, the growing mode solution is well known:
\[ \delta_{klm}^{(c)}(\eta) = \delta_{klm}(\eta_0) D_1(a), \quad (A13) \]
with
\[ D_1(a) = \frac{\Omega_0}{a^2 + \Omega_0} \left[ \int_0^a \frac{da}{\Omega_0 + a^2(1 - \Omega_0)} \right]^{3/2}. \quad (A14) \]
Here \( A \) is a constant to be determined so that \( D_1 \) is unity at present. From equations (A9) and (A5), we finally have
\[ v_l^{(c)}(\eta, \chi, \gamma) = \int_0^\infty \frac{dk}{k} \sum_{l,m} \frac{\delta_{klm}^{(c)}(\eta)}{k^2} \Psi_{klm}(\chi, \gamma)^{1/1}. \quad (A15) \]

\section*{APPENDIX B

\textbf{CALCULATION OF \( \mathcal{W}(R) \)}}

In this Appendix, we present an explicit calculation of \( \mathcal{W}(R) \):
\[ \mathcal{W}(R) = \frac{1}{V^{1/2}} \int \frac{d\Omega_k}{4\pi} \int dr_1 r_1^2 \int d\Omega_{r_1} \int dr_2 r_2^2 \int d\Omega_{r_2} n_0^{LC}(r_1)n_0^{LC}(r_2) \times \left[ \Delta(r_1, r_2) + \delta r_1 \frac{\partial}{\partial r_1} \left[ \Delta(r_2) + \delta r_2 \frac{\partial}{\partial r_2} \right] \right] \delta^{(3)}(x_1 - x_2 - R). \quad (B1) \]
Here \( \Delta(r, \gamma) \) and \( \delta r(r, \gamma) \) are explicitly written as
\[ \Delta(r, \gamma) = \int_0^\infty dk \sum_{l,m} \delta_{klm}^{(c)}(\eta_0) b(k; \eta_0 - r) D_1(\eta_0 - r)\Psi_{klm}(r, \gamma), \quad (B2) \]
\[ \delta r(r, \gamma) = \frac{\partial \Delta(r, \gamma)}{\partial \eta_0} \times \int_0^\infty dk \sum_{l,m} \delta_{klm}^{(c)}(\eta_0) f(\eta_0 - r) D_1(\eta_0 - r)k^{-2}\Psi_{klm}(r, \gamma)^{1/1}, \quad (B3) \]
where we defined
\[ f(\eta) = \frac{\partial \Delta(\eta)}{\partial \eta_0} D_1(\eta) \left[ \frac{1}{D_1(\eta)} \right] = \frac{d \ln D_1(\eta)}{d \ln a(\eta)}, \quad (B4) \]
and we used equations (2), (12), (8), (A13), and (A15). Substituting equations (B2) and (B3) into equation (B1), we obtain
\[ \mathcal{W}(R) = \frac{1}{V^{1/2}} \int \frac{d\Omega_k}{4\pi} \int dr_1 r_1^2 \int d\Omega_{r_1} \int dr_2 r_2^2 \int d\Omega_{r_2} n_0^{LC}(r_1)n_0^{LC}(r_2) D_1(\eta_0 - r_1) D_1(\eta_0 - r_2) \times \int dk_1 \sum_{l_1,m_1} \int dk_2 \sum_{l_2,m_2} \langle \delta_{klm_1}(\eta_0) \delta_{klm_2}^{(c)}(\eta_0) \rangle Y_{l_1,m_1}(\Omega_{r_1}) Y_{l_2,m_2}(\Omega_{r_2}) \times \delta^{(3)}(x_1 - x_2 - R). \quad (B5) \]
In addition, we use the relations

\[ \delta^{(3)}(x_1 - x_2 - R) = \frac{1}{(2\pi)^3} \int d^3k \ e^{-ik \cdot (x_1 - x_2 - R)}, \]  

\[ e^{-ik \cdot x} = 4\pi \sum_{m=-1}^{1} (-i)^{j(k \mid x \mid)} Y_{lm}(\Omega_k) Y_{lm}^\ast(\Omega_k); \]

then equation (B5) becomes

\[ \mathcal{I}(R) = \frac{1}{V_L \omega_c} \int \frac{d\Omega_k}{4\pi} \int d_1 r_1 x_1 \int d_2 r_2 x_2 \int d\Omega_{\Omega_1} \int d\Omega_{\Omega_2} n_0^{LC}(r_1)n_0^{LC}(r_2)D_1(\eta_0 - r_1)D_1(\eta_0 - r_2) \]

\[ \times \int dk_1 \sum_{l_1,m_1} \int dk_2 \sum_{l_2,m_2} \langle \delta_{l_1 l_2 m_1 m_2}(\eta_0) \rangle Y_{l_1 m_1}(\Omega_{\Omega_1}) Y_{l_2 m_2}(\Omega_{\Omega_2}) \]

\[ \times \int \frac{2}{L_2 M_2} \sum_{i=1} \left[ b(k; \eta_0 - r_1)X_{k}^j(r_1) + f(\eta_0 - r_1)k^{-2} X_{k}^j(r_1) \right] \frac{\partial}{\partial r_1} \int \frac{d^3k 4\pi \sum_{l_1 M_1} (-i)^{j} j_{l_1}(kr_1) Y_{l_1 M_1}(\Omega_k) Y_{l_1 M_1}^\ast(\Omega_{\Omega_1})}{d^3k 4\pi \sum_{l_2 M_2} (-i)^{j} j_{l_2}(kr_2) Y_{l_2 M_2}(\Omega_k) Y_{l_2 M_2}^\ast(\Omega_{\Omega_2})} \]  

\[ \times 4\pi \sum_{l_3 M_3} (i)^{l_3} j_{l_3}(kr_3) Y_{l_3 M_3}(\Omega_k) Y_{l_3 M_3}^\ast(\Omega_{\Omega_2}), \]

where \( k = |k| \) and \( \mathcal{K} = k/|k| \). Integrating over \( \Omega_{\Omega_1}, \Omega_{\Omega_2}, \) and \( \Omega_k \) yields

\[ \mathcal{I}(R) = \frac{1}{V_L \omega_c} \int d_1 r_1 x_1 \int d_2 r_2 x_2 n_0^{LC}(r_1)n_0^{LC}(r_2)D_1(\eta_0 - r_1)D_1(\eta_0 - r_2) \int dk_1 \sum_{l_1 m_1} \int dk_2 \sum_{l_2 m_2} \langle \delta_{l_1 l_2 m_1 m_2}(\eta_0) \rangle \]

\[ \times \int \frac{2}{L_2 M_2} \sum_{i=1} \left[ b(k; \eta_0 - r_1)X_{k}^j(r_1) + f(\eta_0 - r_1)k^{-2} X_{k}^j(r_1) \right] \frac{\partial}{\partial r_1} \int \frac{d^3k 4\pi \sum_{l_1 M_1} (-i)^{j} j_{l_1}(kr_1) Y_{l_1 M_1}(\Omega_k) Y_{l_1 M_1}^\ast(\Omega_{\Omega_1})}{d^3k 4\pi \sum_{l_2 M_2} (-i)^{j} j_{l_2}(kr_2) Y_{l_2 M_2}(\Omega_k) Y_{l_2 M_2}^\ast(\Omega_{\Omega_2})} \]  

\[ \times 4\pi \sum_{l_3 M_3} (i)^{l_3} j_{l_3}(kr_3) Y_{l_3 M_3}(\Omega_k) Y_{l_3 M_3}^\ast(\Omega_{\Omega_2}), \]

Further integration over \( \Omega_k \) gives

\[ \mathcal{I}(R) = \frac{1}{V_L \omega_c} \int d_1 r_1 x_1 \int d_2 r_2 x_2 n_0^{LC}(r_1)n_0^{LC}(r_2)D_1(\eta_0 - r_1)D_1(\eta_0 - r_2) \int dk_1 \int dk_2 \sum_{l} (2l + 1)P(l)P(\langle k_1 - k_2 \rangle) \]

\[ \times \int \frac{2}{L_2 M_2} \sum_{i=1} \left[ b(k; \eta_0 - r_1)X_{k}^j(r_1) + f(\eta_0 - r_1)k^{-2} X_{k}^j(r_1) \right] \frac{\partial}{\partial r_1} \int \frac{d^3k 4\pi \sum_{l_1 M_1} (-i)^{j} j_{l_1}(kr_1) Y_{l_1 M_1}(\Omega_k) Y_{l_1 M_1}^\ast(\Omega_{\Omega_1})}{d^3k 4\pi \sum_{l_2 M_2} (-i)^{j} j_{l_2}(kr_2) Y_{l_2 M_2}(\Omega_k) Y_{l_2 M_2}^\ast(\Omega_{\Omega_2})} \]  

\[ \times 4\pi \sum_{l_3 M_3} (i)^{l_3} j_{l_3}(kr_3) Y_{l_3 M_3}(\Omega_k) Y_{l_3 M_3}^\ast(\Omega_{\Omega_2}), \]

where we used the relation of the Gaussian random field in linear theory:

\[ \langle \delta_{l_1 l_2 m_1 m_2}(\eta_0) \rangle = P(l)P(\langle k_1 - k_2 \rangle) \delta_{l_1 l_2} \delta_{m_1 m_2}, \]

and \( \delta_{l_1 l_2} \) and \( \delta_{m_1 m_2} \) are the Kronecker delta. Integration by parts yields

\[ \mathcal{I}(R) = \frac{1}{V_L \omega_c} \int d_1 r_1 x_1 \int d_2 r_2 x_2 n_0^{LC}(r_1)n_0^{LC}(r_2)D_1(\eta_0 - r_1)D_1(\eta_0 - r_2) \]

\[ \times \int dk_1 k_{1}^2 P(k_1) \sum_{l} (2l + 1)P(l) \int d^3k d^3j j_{l_1}(kr_1) j_{l_2}(kr_2) j_{l_3}(kr_3) \sum_{i=1} \left[ b(k; \eta_0 - r_1) - k_{1}^{-2} \mathcal{D}_{r_1} \right] j_{l_1}(kr_1), \]

where \( \mathcal{D}_{r} \) denotes the region \( |r_1 - r_2| \leq R \leq r_1 + r_2 \).

By using the additional theorem for the spherical Bessel function,

\[ \sum_{l} (2l + 1)P(l)P(\theta) j_{l_1}(kr_1) j_{l_2}(kr_2) j_{l_3}(kr_3) = j_0(k \sqrt{r_1^2 + r_2^2 - 2r_1 r_2 \cos \theta}), \]
we can write

$$\Psi(R) = \frac{1}{V_{\text{LC}}} \frac{1}{\pi R} \int_{0}^{R} \int_{\varphi} d\varphi \, dr_{1} \, dr_{2} \, r_{1} n_{0}^{L}(r_{1}) n_{0}^{L}(r_{2}) D_{1}(\eta_{0} - r_{1}) D_{1}(\eta_{0} - r_{2}) \times \left[ \int dk^{2} P(k) \sum_{i=1}^{2} \left[ b(k; \eta_{0} - r_{i}) - k^{-2} \partial \varphi_{i} \right] j_{0}(k \sqrt{r_{1}^{2} + r_{2}^{2} - 2r_{1} r_{2} \cos \theta}) \right],$$

where $\cos \theta$ is replaced by $\cos \theta = (r_{1}^{2} + r_{2}^{2} - R^{2})/2r_{1} r_{2}$ after operating the differentiations by $r_{1}$ and $r_{2}$.

Introducing the notation $z = (r_{1}^{2} + r_{2}^{2} - 2r_{1} r_{2} \cos \theta)^{1/2}$, we can show the formulæ

$$k^{-2} \frac{\partial^{2}}{\partial r_{1}^{2}} j_{0}(kz) = \frac{j_{2}(kz)}{z^{2}} (r_{1} - r_{2} \cos \theta)^{2} - \frac{j_{2}(kz)}{kz^{3}} (r_{1} - r_{2} \cos \theta)^{2}$$

and

$$k^{-4} \frac{\partial^{2}}{\partial r_{1}^{2}} \frac{\partial^{2}}{\partial r_{2}^{2}} j_{0}(kz) = \frac{j_{4}(kz)}{z^{4}} (r_{1} - r_{2} \cos \theta)^{2} (r_{2} - r_{1} \cos \theta)^{2} - \frac{j_{4}(kz)}{kz^{5}} (r_{1} - r_{2} \cos \theta)^{2}$$

$$- 4 \cos \theta (r_{1} - r_{2} \cos \theta)(r_{2} - r_{1} \cos \theta) + (r_{2} - r_{1} \cos \theta)^{2} + \frac{j_{4}(kz)}{(kz)^{5}} (2\cos^{2} \theta + 1).$$

Using these formulæ and omitting the second term in the derivative, i.e., $\partial \varphi \approx f(\eta_{0} - r) \partial^{2} / \partial r^{2}$, we have

$$\Psi(R) \approx \frac{1}{V_{\text{LC}}} \frac{1}{\pi R} \int_{0}^{R} \int_{\varphi} d\varphi \, dr_{1} \, dr_{2} \, r_{1} n_{0}^{L}(r_{1}) n_{0}^{L}(r_{2}) \left[ \int dk^{2} P(k) \sum_{i=1}^{2} \left[ b(k; \eta_{0} - r_{i}) D_{1}(\eta_{0} - r_{i}) \right] \times \left[ j_{0}(kz) + \beta(k; \eta_{0} - r_{2}) I(R; r_{1}, r_{2}) + \beta(k; \eta_{0} - r_{1}) I(R; r_{2}, r_{1}) + \beta(k; \eta_{0} - r_{1}) \beta(k; \eta_{0} - r_{2}) J(R; r_{1}, r_{2}) \right] \right],$$

where $\beta(k; \eta)$ is defined by equation (29), and $I(R; r_{1}, r_{2})$ and $J(R; r_{1}, r_{2})$ are defined by

$$I(R; r_{1}, r_{2}) = \frac{j_{2}(kR)}{kR} - \frac{j_{2}(kR)}{R^{2}} \left( \frac{R^{2} + r_{1}^{2} - r_{2}^{2}}{2r_{1}} \right)^{2},$$

and

$$\frac{j_{2}(kR)}{kR} \left[ 2 \left( \frac{r_{1}^{2} + r_{2}^{2} - R^{2}}{2r_{1} r_{2}} \right)^{2} + \frac{1}{R^{2}} \right] + \frac{j_{2}(kR)}{R^{4}} \left( \frac{R^{2} + r_{1}^{2} - r_{2}^{2}}{2r_{1}} \right)^{2} \left( \frac{R^{2} + r_{1}^{2} - r_{2}^{2}}{2r_{2}} \right)^{2} - \frac{j_{2}(kR)}{kR^{3}} \left[ \left( \frac{R^{2} + r_{1}^{2} - r_{2}^{2}}{2r_{1}} \right)^{2} + \left( \frac{R^{2} + r_{2}^{2} - r_{1}^{2}}{2r_{2}} \right)^{2} - \frac{R^{2} + r_{1}^{2} - r_{2}^{2}}{r_{1}} \frac{R^{2} + r_{2}^{2} - r_{1}^{2}}{r_{2}} \right],$$

respectively.

Since we are generally interested in the case of $R \ll r_{\text{max}}$, we can use the approximation

$$\int_{0}^{R} \int_{\varphi} d\varphi \, dr_{2} \approx \int_{0}^{r_{\text{max}}} dr_{1} \int_{-R}^{R} dx,$$

where we introduced $x = r_{2} - r_{1}$. By expanding $I(R; r_{1}, r_{2})$ and $J(R; r_{1}, r_{2})$ in terms of $x$, we have

$$I(R; r_{1}, r_{2}) = \frac{j_{2}(kR)}{kR} - \frac{j_{2}(kR)}{R^{2}} x^{2},$$

and

$$J(R; r_{1}, r_{2}) = 3 \left( \frac{j_{3}(kR)}{kR} - \frac{6 j_{3}(kR)}{R^{3}} x^{2} + \frac{j_{4}(kR)}{R^{4}} x^{4} \right).$$

Integration by $x$ leads to the final expression:

$$\Psi(R) \approx \frac{4\pi}{V_{\text{LC}}} \int_{0}^{r_{\text{max}}} \frac{drr_{2} n_{0}^{L}(r_{2})}{2r_{2}} \int k^{2} dk P(k) \left[ b(k; \eta_{0} - r) D_{1}(\eta_{0} - r) \right] \left[ 1 + \frac{2}{3} \beta(k; \eta_{0} - r) + \frac{1}{5} \beta(k; \eta_{0} - r) \right] j_{0}(kR).$$

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