LANDAU AND GRÜSS TYPE INEQUALITIES FOR INNER PRODUCT TYPE INTEGRAL TRANSFORMERS IN NORM IDEALS

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Abstract. For a probability measure $\mu$ and for square integrable fields $(A_t)$ and $(B_t)$ ($t \in \Omega$) of commuting normal operators we prove Landau type inequality

$$\left\| \int_{\Omega} A_tX B_t d\mu(t) - \int_{\Omega} A_t d\mu(t)X \int_{\Omega} B_t d\mu(t) \right\| \leq \left( \int_{\Omega} |A_t|^2 d\mu(t) - \left( \int_{\Omega} A_t d\mu(t) \right)^2 \right)^{1/2} \times \left( \int_{\Omega} |B_t|^2 d\mu(t) - \left( \int_{\Omega} B_t d\mu(t) \right)^2 \right)^{1/2}$$

for all $X \in \mathcal{B}(H)$ and for all unitarily invariant norms $\| \cdot \|$.

For Schatten $p$-norms similar inequalities are given for arbitrary double square integrable fields. Also, for all bounded self-adjoint fields satisfying $C \leq A_t \leq D$ and $E \leq B_t \leq F$ for all $t \in \Omega$ and some bounded self-adjoint operators $C,D,E$ and $F$, and for all $X \in \mathfrak{C}_{\|\cdot\|}(H)$ we prove Grüss type inequality

$$\left\| \int_{\Omega} A_tX B_t d\mu(t) - \int_{\Omega} A_t d\mu(t)X \int_{\Omega} B_t d\mu(t) \right\| \leq \frac{\|D-C\| \cdot \|F-E\|}{4} \cdot \|X\|.$$ 

More general results for arbitrary bounded fields are also given.

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