MULTIDIMENSIONAL ANALOGUES OF BOHR’S THEOREM
ON POWER SERIES

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Abstract. Generalizing the classical result of Bohr, we show that if an \(n\)-variable power series converges in \(n\)-circular bounded complete domain \(D\) and its sum has modulus less than 1, then the sum of the maximum of the modulii of the terms is less than 1 in the homothetic domain \(r \cdot D\), where \(r = 1 - \sqrt{2}/3\). This constant is near to the best one for the domain \(D = \{z : |z_1| + \ldots + |z_n| < 1\}\).

1. Preliminaries

The following formulation of the Bohr’s result [5] (due to the work of M. Riesz, I. Schur and F. Wiener) is known.

Theorem 1. If a power series

\[
\sum_{k=0}^{\infty} c_k z^k
\]

converges in the unit disk and its sum has modulus less than 1, then

\[
\sum_{k=0}^{\infty} |c_k z^k| < 1
\]

in the disk \(\{z : |z| < 1/3\}\). Moreover, the constant 1/3 cannot be improved.

Recently H.P. Boas and D. Khavinson obtained some multidimensional generalizations of this result ([4], see also for more references there).

Denote by \(K_n\) the largest number such that if the series

\[
\sum_{\alpha} c_\alpha z^\alpha
\]

converges in the unit polydisk \(U_1 = \{z : |z_j| < 1, j = 1, \ldots, n\}\) and the estimate

\[
|\sum_{\alpha} c_\alpha z^\alpha| < 1
\]

is valid there, then

\[
\sum_{\alpha} |c_\alpha z^\alpha| < 1
\]
holds in $K_n \cdot U_1$; here $\alpha = (\alpha_1, ..., \alpha_n)$, all $\alpha_j$ are non-negative integers, $z = (z_1, ..., z_n)$, $z^n = z_1^{\alpha_1} \cdot ... \cdot z_n^{\alpha_n}$.

**Theorem 2** (Boas, Khavinson). It is true for $n > 1$ that

$$\frac{1}{3 \sqrt{n}} < K_n < \frac{2 \log n}{\sqrt{n}}.$$  \hfill (6)

**Theorem 3** (Boas, Khavinson). Let the series (3) converge in a complete $n$-circular domain (Reinhardt domain) $D$ and (4) holds in $D$. Then (5) is true in the homothetic domain $K_n \cdot D$.

Notice that Remark 1 from [4], in fact, contains a result stronger than the left part of the inequality (6), namely

$$K_n > \left\{ \begin{array}{ll}
\frac{2}{3 \sqrt{n}} & \text{for } n > 1, \\
\frac{1}{2 \sqrt{n}} & \text{for large enough } n.
\end{array} \right.$$  \hfill (7)

2. **Main results**

We consider some other multidimensional variations of Bohr’s problem. Denote by $B_n(D)$ the largest number $r$ such that if the series (3) converges in a complete $n$-circular bounded domain $D$ and (4) holds in it, then

$$\sum_{\alpha} \sup_{D_r} | c_{\alpha} z^\alpha | < 1,$$

where $D_r = r \cdot D$ is the homothetic transformation of $D$. If $D = U_1$, then $B_n(D) = K_n$. We point out that our consideration is also a natural generalization of Bohr’s theorem (Theorem 1), because it was shown in [2] that any power series (3), converging in $D$, converges also in the sense of the left part of (7) for all $r$, $0 < r < 1$.

**Theorem 4.** The inequality

$$1 - \sqrt{\frac{2}{3}} < B_n(D)$$

is true for any complete, bounded $n$-circular domain $D$.

This estimate can be improved for concrete domains.

**Theorem 5.** For the unit ball $D^1 = \{ z : |z_1|^2 + ... + |z_n|^2 < 1 \}$ the following estimate is true:

$$B_n(D^1) > \left\{ \begin{array}{ll}
\frac{2}{3n} & \text{for } n > 1, \\
\frac{1}{zn} & \text{for large enough } n.
\end{array} \right.$$  \hfill (8)

**Theorem 6.** For the unit hypercone $D^\circ = \{ z : |z_1| + ... + |z_n| < 1 \}$ the following inequality holds:

$$B_n(D^\circ) < 0.446663 \frac{1}{n}.$$  \hfill (9)

**Corollary.**

$$1 - \sqrt{\frac{2}{3}} < B_n(D^\circ) < 0.446663 \frac{1}{n}.$$  \hfill (10)
Remark 1. The asymptotic equality
\[ 1 - \sqrt{2/3} = \frac{\log 3/2}{n} + O\left(\frac{1}{n^2}\right) \]
is true, where \( \log 3/2 \approx 0.405465 \). Denoting by \( B_- \) the left part of (10) and by \( B_+ \) the right one, we get
\[ 1 < \lim_{n \to \infty} \sup \frac{B_+}{B_-} < 1.10 \, \text{,} \]

We now turn our attention to a related problem. Denote by \( L_n(D) \) the biggest number \( r \) such that if the series (3) converges in the hypercone \( D \) and (4) holds in it, then
\[ \sum_{\alpha} \|c_\alpha z^\alpha\|_{L^1(\partial D_r)} < 1, \]
where the \( L^1 \)-norm is considered with respect to the measure \( \mu_r \). The measure \( \mu_r \) is the image of the measure
\[ d\mu = \frac{(n-1)!}{(2\pi i)^n} d|z_1| \cdots |z_n-1| |\frac{dz_1}{z_1} \cdots \frac{dz_n}{z_n}| \]
by the homothetic transformation \( z \to rz \). Usually the measure \( d\mu \) is used for calculating the Szegő kernel for \( D \) (see [3]); notice that \( \mu(\partial D) = 1 \). For \( n = 1 \) this problem coincides with Bohr’s problem. The analogous problem for the polydisk \( U_1 \) (with \( L^1 \)-norm on its Shilov boundary with respect to usual Lebesgue measure on it) is equivalent to the problem considered in Theorem 2.

**Theorem 7.** For the hypercone \( D \) the following estimates are true:
\[ \frac{1}{3e^{1/3}} < L_n(D) \leq \frac{1}{3}. \]

Surprisingly, the estimates in (12) do not depend on \( n \). This is different from the results of Theorems 2–6.

We consider now a variation of the multidimensional Bohr problem, dealing with the expansions into a series of homogeneous polynomials, which is also a natural generalization of the power series expansion.

Let \( Q \) be a complete circular domain (Cartan’s domain) centered at \( 0 \in Q \). Then any function \( f(z) \), holomorphic in \( Q \), can be expanded into the series
\[ f(z) = \sum_{k=0}^{\infty} P_k(z), \]
where \( P_k(z) \) is a homogeneous polynomial of degree \( k \) for every \( k \in \mathbb{N} \).

**Theorem 8.** If the series (13) converges in the domain \( Q \) and the estimate \( |f(z)| < 1 \) holds in it, then
\[ \sum_{k=0}^{\infty} |P_k(z)| < 1 \]
in the homothetic domain \( \frac{1}{r}Q \). Moreover, if \( Q \) is convex, then \( 1/3 \) is the best possible constant.

Next we consider for the hypercone \( D \) the same problem, which was considered in Theorems 2–3 for \( U_1 \). Denote by \( K_n(D) \) the largest number such that if series (3) converges in \( D \) and estimate (4) is valid there, then (5) holds in \( K_n(D) \cdot D \).
Theorem 9. For the hypercone $D^o$ the following estimates are true:

$$\frac{1}{3e^{1/3}} < K_n(D^o) \leq 1/3.$$

Moreover, if $z \notin 1/3 \cdot D^o$, then there exists a series of the form (3) such that it converges in $D^o$ and the estimate (4) is valid there, but (5) fails at the point $z$.

3. Proofs

Proof of Theorem 4. The following generalization of Cauchy inequalities was considered in [2]: if (4) holds in $D$, then

$$|c_\alpha| \leq \frac{1}{d_\alpha(D)},$$

where $d_\alpha(D) = \max_D |z^\alpha|$. Using Wiener’s method, it is easy to strengthen the estimates (15):

$$|c_\alpha| \leq (1 - |c_0|^2)^{\frac{1}{2}} \frac{1}{d_\alpha(D)}$$

for $|\alpha| = \alpha_1 + \ldots + \alpha_n > 1$. We do not present the proof of (16), because it repeats the proof for polydisk $U_1$ (see [4]), but deals with inequalities (15) instead of Cauchy inequalities. If (4) holds in $D$, then, applying (16), we get

$$\sum_{\alpha} \sup_{D_r} |c_\alpha z^\alpha| = \sum_{\alpha} |c_\alpha| d_\alpha(D_r)$$

$$\leq |c_0| + (1 - |c_0|^2) \sum_{|\alpha|=1}^{\infty} \frac{d_\alpha(D_r)}{d_\alpha(D)}$$

$$= |c_0| + (1 - |c_0|^2) \sum_{k=1}^{\infty} \left( \frac{n+k-1}{k} \right) r^k$$

$$= |c_0| + (1 - |c_0|^2) \left[ \frac{1}{(1-r)^n} - 1 \right].$$

Now if

$$\frac{1}{(1-r)^n} - 1 \leq \frac{1}{2}.$$ 

then

$$\sum_{\alpha} \sup_{D_r} |c_\alpha z^\alpha| \leq |c_0| + (1 - |c_0|^2)^{\frac{1}{2}} \frac{1}{2} = 1 - \frac{1}{2} (1 - |c_0|^2) < 1.$$ 

The condition (17) means that (7) is true if $r \leq 1 - \sqrt{\frac{2}{3}}$.

Proof of Theorem 5. Consider Borel probability measure on $\partial D^1$, which is invariant under all unitary transformations of $\mathbb{C}^n$:

$$d\mu = \frac{(n-1)!}{(2\pi i)^n} d |z_1|^2 \wedge \cdots \wedge d |z_{n-1}|^2 \wedge \frac{dz_1}{z_1} \wedge \cdots \wedge \frac{dz_n}{z_n}.$$

The monomials $z^\alpha$ are orthogonal with respect to integration by $\mu$ and

$$\int_{\partial D^1} |z^{2\alpha}| d\mu = \frac{\alpha_1! \cdots \alpha_n!(n-1)!}{(|\alpha| + n - 1)!}. $$

Next, we repeat the proof of Theorem 2 from [4] using Wiener’s method, but integrating on the sphere $\partial D^3$ with respect to the measure $\mu$ instead of integrating on the unit torus as in [4]. Thus, we obtain
\[
\sum_{|\alpha|=k} |c_\alpha|^2 \frac{\alpha_1! \cdots \alpha_n!(n-1)!}{(|\alpha|+n-1)!} \leq (1-|c_0|^2)^2.
\]

Furthermore, notice that from the Schwarz Lemma for the ball $D^3$ it follows (again by using Wiener’s method from [4]) that
\[
\sum_{|\alpha|=1} |c_\alpha|^2 \leq (1-|c_0|^2)^2.
\]

Then, recalling that
\[
d_\alpha(D^3) = \sqrt{\frac{\alpha_1^{\alpha_1} \cdots \alpha_n^{\alpha_n}}{|\alpha|^{\overline{\alpha}}}},
\]
where $0^0 = 1$, we get
\[
\sum_{\alpha} \sup_{D^3} |c_\alpha z^\alpha| = |c_0| + \left( \sum_{|\alpha|=1} |c_\alpha| \right) r + \sum_{k=2}^{\infty} \sum_{|\alpha|=k} |c_0| \sqrt{\frac{\alpha_1^{\alpha_1} \cdots \alpha_n^{\alpha_n}}{|\alpha|^{\overline{\alpha}}}} r^k
\leq |c_0| \left( 1 - |c_0|^2 \right) r \sqrt{n} + (1-|c_0|^2) \sum_{k=2}^{\infty} \left( \sum_{|\alpha|=k} \frac{(k+n-1)!}{k^n \alpha_1^{\alpha_1} \cdots \alpha_n^{\alpha_n}} \right)^{1/2} r^k
\leq |c_0| \left( 1 - |c_0|^2 \right) r \sqrt{n} + (1-|c_0|^2) \sum_{k=2}^{\infty} \left( \frac{k+n-1}{k^{n-1}} \right)^{1/2} \sum_{|\alpha|=k} \frac{k!}{\alpha_1! \cdots \alpha_n!} (r \sqrt{n})^k
= |c_0| \left( 1 - |c_0|^2 \right) r \sqrt{n} + (1-|c_0|^2) \sum_{k=2}^{\infty} \sqrt{k+n-1} (r \sqrt{n})^k.
\]

It remains now to apply the estimate, obtained in Remark 1 from [4].

Proof of Theorem 6. Consider the function
\[
f_a(z) = \frac{1+a}{2} \frac{1-(z_1+\cdots+z_n)}{1-a(z_1+\cdots+z_n)} = \sum_{\alpha} c_\alpha z^\alpha,
\]
where $0 < a < 1$. Then $|f_a(z)| < 1$ in hypercone $D^3$ and
\[
\sum_{\alpha} |c_\alpha z^\alpha| = \frac{1+a}{2} + \frac{1-a^2}{2} \sum_{k=1}^{\infty} a^{k-1} \sum_{|\alpha|=k} k! \frac{1}{\alpha_1! \cdots \alpha_n!} |z^\alpha|.
\]

Since
\[
d_\alpha(D^3) = \sqrt{\frac{\alpha_1^{\alpha_1} \cdots \alpha_n^{\alpha_n}}{|\alpha|^{\overline{\alpha}}}},
\]
it follows that

\[ \sum_{\alpha} c_{\alpha} |d_{\alpha}(D^n) = \frac{1 + a}{2} + \frac{1 - a^2}{2} \sum_{k=1}^{\infty} \sum_{|\alpha|=k} a^{k-1} k! \frac{\alpha_1! \cdots \alpha_n!}{\alpha_1! \cdots \alpha_n!} r^k \]

\[ > \frac{1 + a}{2} + \frac{1 - a^2}{2} \sum_{k=1}^{\infty} a^{k-1} k! \left( \sum_{|\alpha|=k} \frac{1}{\alpha_1! \cdots \alpha_n!} \right) r^k \]

\[ = \frac{1 + a}{2} + \frac{1 - a^2}{2a} \sum_{k=1}^{\infty} \frac{(anr)^k}{k^k} \geq 1, \]

if

\[ \sum_{k=1}^{\infty} \frac{(anr)^k}{k^k} \geq \frac{a}{1 + a}. \]

Let \( x_0(a) \) be the root of the equation

\[ \sum_{k=1}^{\infty} \frac{x^k}{k^k} = \frac{a}{1 + a}; \]

then, if \( anr \geq x_0(a) \), (7) fails for that \( r \) and \( D = D^n \). Now, considering \( a \to 1 \), we obtain that (7) is not true for \( D = D_0 \) if \( r \geq \frac{1}{n0} \), where \( x_0 = x_0(1) \). Notice that \( x_0 \) is a root of the equation

\[ \sum_{k=1}^{\infty} \frac{x^k}{k^k} = 1/2; \]

hence \( B_n(D^n) \leq x_0/n \). Using the program “Mathematica 3.0” [7], we estimated \( x_0 \) from above. We obtained that the equation

\[ \sum_{k=1}^{p} \frac{x^k}{k^k} = 1/2 \]

has a root 0.446662 (where the last decimal digit is precise) if \( p \) runs from 5 till 25. So, the equation (20) has a root less than 0.446663 if \( 5 \leq p \leq 25 \); hence this estimate is true for \( x_0 \).

**Proof of Theorem 7.** Notice that by analogy with (18) it is true that

\[ \int_{\partial D^n} |z^\alpha| \, d\mu_r = \frac{\alpha_1! \cdots \alpha_n!(n-1)!}{(\alpha_1 + n - 1)!} r^{\alpha}. \]

Using (16), we get

\[ \sum_{\alpha} ||c_{\alpha} z^{\alpha}||_{L^1(\partial D^n)} \]

\[ \leq |c_0| + (1 - |c_0|^2) \sum_{k=1}^{\infty} \sum_{|\alpha|=k} \frac{k^k (n-1)!}{(n+k-1)!} \frac{\alpha_1! \cdots \alpha_n!}{\alpha_1! \cdots \alpha_n!} r^{\alpha} \]

\[ < |c_0| + (1 - |c_0|^2) \sum_{k=1}^{\infty} \frac{k^k (n-1)!}{(n+k-1)!} \sum_{|\alpha|=k} r^{\alpha} \]

\[ = |c_0| + (1 - |c_0|^2) \sum_{k=1}^{\infty} \frac{k^k}{k!} r^k. \]
Denote by \( r_0 \) the root of the equation
\[
\sum_{k=1}^{\infty} \frac{k^k}{k!} x^k = \frac{1}{2}.
\]

Then (11) holds for \( r = r_0 \); therefore \( L_n(D^a) \geq r_0 \). The identity
\[
\sum_{k=1}^{\infty} \frac{k^k}{k!} x^k e^{-kx} = -1 + \frac{1}{1-x}
\]
holds for \( x \) in a neighborhood of 0. This can be verified by computing the Maclaurin series coefficients of the left-hand side and observing that they reduce to the value 1 by a standard theorem on sums of binomial coefficients. Now put \( x = 1/3 \) in equation (22) to recognize the solution of equation (21).

Finally, consider, as in the proof of Theorem 6, the function \( f_a(z) \) for which
\[
\sum_{\alpha} \| c_{\alpha} z^\alpha \|_{L^1(\partial D^a)} = \frac{1 + a^2}{2} + \frac{1 - a^2}{2} \sum_{k=1}^{\infty} \frac{k!(a-1)!a^{k-1}}{(n+k-1)!} \sum_{|\alpha|=k} r_{|\alpha|} \geq 1,
\]
if
\[
\sum_{k=1}^{\infty} (ar)^k \geq \frac{a}{1+a}.
\]
When \( a \to 1 \), we get that the inequality (11) fails if \( r > 1/3 \).

**Proof of Theorem 8.** In each section of the domain \( Q \) by a complex line
\[
\alpha = \{ z : z_j = a_j t, \quad j = 1, \ldots, n; \quad t \in \mathbb{C} \}
\]
the series turns into the power series by \( t \)
\[
f(at) = \sum_{k=0}^{\infty} P_k(a) t^k
\]
and, in addition, \( |f(at)| < 1 \). By Theorem 1
\[
\sum_{k=0}^{\infty} |P_k(a)t^k| < 1
\]
in the section \( \alpha \cap \left( \frac{1}{3} \cdot Q \right) \). But it is just (14), since \( \alpha \) is an arbitrary complex line passing through the origin. Conversely, let the domain \( Q \) be convex; then \( Q \) is an intersection of half-spaces
\[
Q = \bigcap_{a \in J} \{ z : Re(a_1 z_1 + \cdots + a_n z_n) < 1 \}
\]
for some \( J \). Since \( Q \) is circular, we obtain
\[
Q = \bigcap_{a \in J} \{ z : a_1 z_1 + \cdots + a_n z_n < 1 \}.
\]

It is sufficient now to show that the constant 1/3 cannot be improved for each domain \( P_a = \{ z : |a_1 z_1 + \cdots + a_n z_n| < 1 \} \). From Theorem 1 it follows that for any \( r > 1/3 \) there exists a function \( f(z) \), represented by (1) and such that \( |f(z)| < 1 \) in
the unit disk, but (2) fails in the disk \( \{ z : |z| < r \} \). To complete the proof we use the functions \( f(a_1 z_1 + \ldots + a_n z_n) \).

**Proof of Theorem 9.** If \( z \in r \cdot D^o \), then from (16) it follows

\[
\sum_{\alpha} |c_{\alpha} z^\alpha| \leq |c_0| + (1 - |c_0|^2) \sum_{|\alpha|=1}^\infty \frac{|\alpha|!}{\alpha_1! \ldots \alpha_n!} |z|^\alpha |z^\alpha| = |c_0| + (1 - |c_0|^2) \sum_{|\alpha|=1}^\infty \frac{|\alpha|!}{\alpha_1! \ldots \alpha_n!} |z|^\alpha |z^\alpha|
\]

\[
< |c_0| + (1 - |c_0|^2) \sum_{k=1}^\infty k^k \frac{r^k}{k!}.
\]

Thus (5) holds for \( r > r_0 \), where \( r_0 \) is the root of equation (21).

Suppose \( z \not\in 1/3 \cdot D^o \); then we have for the function \( f_\alpha(z) \) (which was used in the proof of Theorem 6)

\[
\sum_{\alpha} |c_{\alpha} z^\alpha| = \frac{1 + a}{2} + \frac{1 - a^2}{2} \sum_{k=1}^\infty a^{k-1}(|z_1| + \ldots + |z_n|)^k \geq 1
\]

if

\[
\sum_{k=1}^\infty a^k(|z_1| + \ldots + |z_n|)^k \geq \frac{a}{1 + a}.
\]

Hence inequality (5) fails as \( a \to 1 \).

4. **Final remarks**

**Remark 2.** In the proof of Theorem 5 we used the facts that in equality (18) the domain in consideration is the ball \( D^1 \) and that the estimates \( d_\alpha(D^1) \leq 1 \) are valid. Therefore, an analogous theorem holds for any complete \( n \)-circular domain \( D \) if \( D^1 \subset D \subset U_1 \). But for the hypercone \( D^o \) this is not true (see Theorem 6).

**Remark 3.** In the proof of Theorem 6 was used the following quite rough inequality:

\[
\sum_{|\alpha|=k} \frac{\alpha_1^{\alpha_1} \ldots \alpha_n^{\alpha_n}}{\alpha_1! \ldots \alpha_n!} \geq \sum_{|\alpha|=k} \frac{1}{\alpha_1! \ldots \alpha_u!}.
\]

Therefore, in fact, the estimate from above in the theorem might be decreased. For example, if \( n = 2 \) (10) implies \( B_2(D^o) < 0.223332 \), but using a PC one can show that \( B_2(D^o) < 0.191373 \). Hence, \( 0.183502 < B_2(D^o) < 0.191373 \).

**Remark 4.** Comparing Theorem 2 and the corollary we can see that \( B_n(D) \) depends essentially on the domain \( D \) since \( B_n(U_1) = K_n \). It seems that it is more natural to consider not a single number in the problem considered in Theorem 2 and Theorem 3, but the largest subdomain \( D_B \) of \( D \) such that (5) holds. From [4] it follows, for example, that \( U_1B \) contains the ball \( \frac{1}{3} \cdot D^1 \), and from Theorem 9 it follows that \( D^o_B \subset 1/3 \cdot D^o \).

**Remark 5.** Notice that there exist such unbounded \( n \)-circular domains \( D \), for which the problems defined by the conditions (5) and (7) are equivalent. Consider, for example, the domains \( D = \{ z : |z|^\beta < c \} \), where \( \beta \) is a multiindex with coprime components. Bounded holomorphic functions in such domains depend only on one
variable $z^\beta$ and the exact value of Bohr’s radius equals $(\frac{1}{\pi})^{1/|\beta|}$. Thus there exist $n$-circular domains with Bohr’s radius arbitrarily close to 1. Therefore it is impossible to remove the assumption about convexity in Theorem 8.

**Remark 6.** Unlike Theorems 1–7, in Theorem 8 the series (13) is not a basis expansion. In [1] it was shown that there exists a basis in the space of all holomorphic functions in $Q$, consisting of homogeneous polynomials $P_{k,m}(z)$, where $k$ is the degree of the polynomial and $m = 1, \ldots, \binom{k + n - 1}{k}$. It is reasonable to consider Bohr’s problem for such basis expansions, but there are no results yet in this direction. This question is a particular case of a more general problem in Bohr spirit for expansions by an arbitrary basis in a domain $D \subset \mathbb{C}^n$ (under some restrictions on the basis, because, for example, the basis $\{1, (z - 1)/2, z^2, z^3, \ldots\}$ in the unit disk has no Bohr’s constant).

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