Fock-Bargmann Representation of the Distorted
Heisenberg Algebra

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Abstract

The dynamical algebra associated to a family of Isospectral Oscillator Hamiltonians, named Distorted Heisenberg Algebra because its dependence on a distortion parameter $w \geq 0$, has been recently studied. The connection of this algebra with the Hilbert space of entire analytic functions of growth $(1/2, 2)$ is analyzed.

Key-Words: Coherent states, Heisenberg algebra

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1 Introduction

In 1980, Abraham and Moses found a general class of 1-dimensional potentials isospectral to the oscillator one by means of the Gelfand-Levitan formalism \[1\]. An elegant way to construct the same class, used by Mielnik \[2\], consists in the application of a variant of the standard factorization method. The connection between the Darboux transformation and this generalized factorization has been recently discussed \[3, 4, 5\]. The Mielnik construction is suitable to easily identify a pair of annihilation and creation operators for the isospectral oscillator Hamiltonians \( \{ A, A^\dagger \} \), which are adjoint to each other, although their commutator is not the identity \[2\]. Departing from the previous operators, a different pair, \( \{ A, B^\dagger \} \), was constructed such that the commutator is the identity but \((A)^\dagger \neq B^\dagger \) \[3\]. Recently, a third choice was done \[6\], where the annihilation \( C_w \) and creation \( C_w^\dagger \) operators are adjoint to each other and commute to the identity on a subspace of the state space, imitating then the behavior of the usual annihilation and creation operators for the harmonic oscillator, \( i.e. \), the Heisenberg-Weyl algebra. It turns out that \( C_w \) and \( C_w^\dagger \) depend on a parameter \( w \geq 0 \). The appearance of this parameter is important because it leads to the Heisenberg-Weyl algebra for some of its particular values. Moreover, the coherent states constructed from these operators reach the standard form of the harmonic oscillator coherent states for such \( w \)-values. In the general case \((w\) arbitrary\), these operators have an algebraic structure very similar to the harmonic oscillator one.

The Fock-Bargmann representation of the Heisenberg-Weyl algebra is widely used in physics and mathematics \[8\]. The first application was made in quantum field theory, where the operators \( \bar{z} \) and \( \frac{\partial}{\partial \bar{z}} \) represent the creation and annihilation of bosons. Recently, this algebra has been considered also in the study of tensor bosons, arising in composite object theories, which according of their symmetry properties have been classified as symmetric or antisymmetric \[9\].

The resemblance between the operator pair \( \{ C_w, C_w^\dagger \} \) and the Heisenberg-Weyl corresponding one \( \{ a, a^\dagger \} \), suggests the following question, which we will try to answer in this paper: which are the properties of the operators \( C_w \) and \( C_w^\dagger \) when they act on a space of entire analytic functions? In particular, we will consider just one of the two kinds of coherent states obtained by Fernández et al \[7\], and we will show that the corresponding Fock-Bargmann space \( \mathcal{F}_w \) contains entire analytic functions of growth \((1/2, 2)\). The realization of \( C_w \) and \( C_w^\dagger \) on \( \mathcal{F}_w \) is a multiplication by \( \bar{z} \) and a derivation with respect to \( \bar{z} \) plus a new term dependent on the distortion parameter respectively. We discuss also the specific example with \( w = 1 \), and we show that the usual harmonic oscillator case is recovered on \( \mathcal{F}_1 \).
2 Distorted Heisenberg algebra revisited

Let us consider an infinite discrete set of orthonormal state vectors in the Hilbert space $H$, \{ $|\theta_n\rangle$, $n = 0, 1, 2, ...$ \}. This is a basis set in $H$ related to the standard harmonic oscillator basis \{ $|\psi_n\rangle$, $n = 0, 1, 2, ...$ \} by \cite{2}:

\[
b|\theta_n\rangle = \sqrt{n} |\psi_{n-1}\rangle, \quad b^\dagger |\psi_n\rangle = \sqrt{n+1} |\theta_{n+1}\rangle.
\]

The $|\theta_n\rangle$'s, $n = 0, 1, 2, ...$, satisfy the following eigenvalues equation

\[
H_\lambda |\theta_n\rangle = E^{(\lambda)}_n |\theta_n\rangle
\]

where $E^{(\lambda)}_n = n + 1/2$, and $H_\lambda = b^\dagger b + 1/2$ is the Hamiltonian isospectral to the harmonic oscillator Hamiltonian $H = a^\dagger a + 1/2 = aa^\dagger - 1/2$ obtained through the generalized factorization method of Mielnik \cite{2}. The explicit expression of $H_\lambda$ in the coordinate representation is

\[
H_\lambda = -\frac{1}{2} \frac{d^2}{dx^2} + \frac{x^2}{2} - \frac{d}{dx} \left[ \frac{e^{-x^2}}{\lambda + \int_0^x e^{-y^2}dy} \right].
\]

Notice that the factorizing operators $b^\dagger$ and $b$ are not the raising and lowering operators for the basis \{ $|\theta_n\rangle$ \}. According to Fernández et al \cite{7}, the raising and lowering operators similar to those of the harmonic oscillator, and leading to the distorted Heisenberg-Weyl algebra, are built by means of the condition:

\[
[C, C^\dagger] = I \quad \text{on} \quad H_s \subset H.
\]

where $H_s$ is a subspace of the Hilbert space $H$.

It turns out that these operators depend on a parameter $w \geq 0$ and take the form \cite{7}:

\[
C_w = b^\dagger \frac{1}{N + 1} \sqrt{\frac{N + w}{N + 2}} a b, \quad C^\dagger_w = b^\dagger a^\dagger \frac{1}{N + 1} \sqrt{\frac{N + w}{N + 2}} b.
\]

Let’s remark that since $b$, $b^\dagger$, $a$, $a^\dagger$ are first order differential operators, then $C_w$ and $C^\dagger_w$ are differential operators of order greater than three (indeed they are infinite order differential operators). Their action on the basis \{ $|\theta_n\rangle$ \} is given by:

\[
C_w |\theta_n\rangle = (1 - \delta_{n,0} - \delta_{n,1}) \sqrt{n-2+w} |\theta_{n-1}\rangle, \quad \text{(2.1)}
\]

\[
C^\dagger_w |\theta_n\rangle = (1 - \delta_{n,0}) \sqrt{n-1+w} |\theta_{n+1}\rangle,
\]

\[
I_w |\theta_n\rangle = [1 - \delta_{n,0} + \delta_{n,1}(w-1)] |\theta_n\rangle,
\]
where $I_w \equiv [C_w, C^\dagger_w]$. From this action, it is clear that for $w > 0$ the set of operators\
$\{C_w, C^\dagger_w, I_w\}$ enables one to decompose the Hilbert space $\mathcal{H}$ as a direct sum of two invariant\
subspaces, $\mathcal{H} = \mathcal{H}_0 \oplus \mathcal{H}_r$, where $\mathcal{H}_0$ is spanned by $|\theta_0\rangle$ and $\mathcal{H}_r$ by $\{|\theta_n\rangle, n \geq 1\}$. These two\
subspaces induce irreducible representations of $C_w, C^\dagger_w$, and $I_w$. The usual representation of the Heisenberg algebra is recovered on $\mathcal{H}_r$ for $w = 1$. On the other hand, for $w = 0$ $\mathcal{H}$ decomposes (under the action of $\{C_0, C^\dagger_0, I_0\}$) as the direct sum of three invariant\
subspaces $\mathcal{H} = \mathcal{H}_0 \oplus \mathcal{H}_1 \oplus \mathcal{H}_s$, where $\mathcal{H}_1$ is generated by $|\theta_1\rangle$ and $\mathcal{H}_s$ by $\{|\theta_n\rangle, n \geq 2\}$, and the usual Heisenberg algebra representation is recovered once again on $\mathcal{H}_s$. In this\
paper we will work the case $w > 0$, and when we consider a particular case we will analyze just the one with $w = 1$.

We can propose the new operator $N_w = C^\dagger_w C_w$, analogous to the standard number operator $N = a^\dagger a$, which relevant representation arises when we consider its action on vectors $|\psi\rangle \in \mathcal{H}_r$. The standard number operator representation is recovered on $\mathcal{H}_r$ by taking $w = 1$ in $N_w$ and relabeling the eigenstates of $H_\lambda$ as $|\phi_n\rangle \equiv |\theta_{n+1}\rangle$. However, the dependence of $N_w$ as a function of the Hamiltonian $H_\lambda$ is not obvious (in the general case $w > 0$). A nonlinear dependence is probably in the context of the recent generalized Fock treatment [10], but it is quite involved to determine it precisely. This is due to the fact that the order of $C_w$ and $C^\dagger_w$ is in general infinity while the one of $H_\lambda$ is finite (two).

The normalized ground state of $H_\lambda$, $|\theta_0\rangle$, can be introduced by means of the requirements $C_w |\theta_0\rangle = C^\dagger_w |\theta_0\rangle = 0$. The solution to these equations in the coordinate representation is given by [3]:

$$\theta_0(x) \propto \frac{e^{-x^2/2}}{\lambda + \int_0^\infty e^{-y^2}dy}.$$  

where $\lambda \in \mathbb{R}, |\lambda| > \sqrt{\pi}/2$. Hence $|\theta_0\rangle$ is orthogonal to all $|\theta_n\rangle \in \mathcal{H}_r$ [2]. Because $C_w |\theta_1\rangle = 0$ (see 2.1), the eigenvalue $z = 0$ of $C_w$ is doubly degenerated. As a consequence, the state $|\theta_0\rangle$ is disconnected of the space $\mathcal{H}_r$ and $|\theta_1\rangle$ is the state playing the role of the extremal state for the distorted Heisenberg-Weyl algebra. Thus, the operator $C^\dagger_w$ can be used to construct the basis of the state space $\mathcal{H}_r$ from its repeated action on $|\theta_1\rangle$:

$$|\theta_n\rangle = \sqrt{\frac{\Gamma(w)}{\Gamma(w+n-1)}} (C^\dagger_w)^{n-1} |\theta_1\rangle, \quad w \neq 0.$$  

A similar situation has been recently reported by Spiridonov for systems and creation and annihilation operators different to the ones used in this paper (see [4], section VII). Spiridonov also looks for the coherent states as eigenstates of the corresponding annihilation operator, and they could be found through the solution of some third order differential
equations ((7.10) and (7.13) of [4]). Unfortunately, be means of such a method one cannot find easily and directly those coherent states; however, it can be done for the isospectral Hamiltonians we are dealing with [6, 7]. This is one of the advantages of the coherent states we will present in the next section.

As a final point of this section, the resolution of the identity in terms of the basis \{ |\theta_n\rangle, n \geq 0 \} is the standard one

\[ I = \sum_{n=0}^{\infty} |\theta_n\rangle \langle \theta_n| . \]

It let us to expand any state vector \( |h\rangle \in \mathcal{H} \) as:

\[ |h\rangle = \sum_{n=0}^{\infty} a_n |\theta_n\rangle; \quad a_n \equiv \langle \theta_n | h \rangle. \]

3 Distorted Heisenberg algebra Coherent States

Recently, a family of coherent states has been constructed as eigenstates of the annihilation operator, \( C_w |z, w\rangle = z |z, w\rangle \); they are called here \( w \)-coherent states. Their explicit form in terms of \( |\theta_n\rangle \) is given by [7]:

\[ |z, w\rangle = \sqrt{\frac{\Gamma(w)}{\pi^{1/2}}} \sum_{n=0}^{\infty} \frac{z^n}{\Gamma(w+n)} |\theta_{n+1}\rangle, \quad z = re^{i\varphi} \] (3.1)

where \( _1F_1(a, b; x) \) is a hypergeometric function. The ket \( |\theta_0\rangle \), by construction, is also a coherent state. The set \{ \( |\theta_0\rangle, |z, w\rangle \) \} is complete in \( \mathcal{H} \), with an identity resolution

\[ I = |\theta_0\rangle \langle \theta_0| + \int |z, w\rangle \langle z, w| d\mu(z, w) \] (3.2)

where \( d\mu(z, w) \) is the measure written as:

\[ d\mu(r, w) = \frac{1}{\pi \Gamma(w)} \frac{1F_1(1, w; r^2)}{1F_1(1, w; r^2)} e^{-r^2} r^{2(w-1)} \] (3.3)

As usual, this new basis set is not orthogonal because the inner product between two different coherent states

\[ \langle z, w|z', w\rangle = \frac{_{1F_1(1, w; \bar{z}z')}}{_{1F_1(1, w; r^2)}_{1F_1(1, w; r^2)}} \] (3.4)

is in general, different from zero. Any vector state \( |h\rangle \in \mathcal{H} \) is now expanded as

\[ |h\rangle = h_0 |\theta_0\rangle + \int \tilde{h}(z, \bar{z}, w)|z, w\rangle d\mu(z, w), \] (3.5)
where \( h_0 \equiv \langle \theta_0|h \rangle \),
\[
\tilde{h}(z, \bar{z}, w) \equiv \langle z, w|h \rangle = \frac{h_w(\bar{z})}{\sqrt{1 F_1(1, w; r^2)}},
\] (3.6)
and
\[
h_w(\bar{z}) \equiv \sum_{n=0}^{\infty} \sqrt{\frac{\Gamma(w)}{\Gamma(w+n)}} \bar{z}^n \langle \theta_{n+1}|h \rangle.
\] (3.7)

In this representation, the inner product between \(|f\rangle \in \mathcal{H}\) and \(|g\rangle \in \mathcal{H}\) is:
\[
\langle f|g \rangle = \bar{f}_0 g_0 + \int \bar{f}_w(\bar{z})g_w(\bar{z})d\mu(\bar{z}, w),
\] (3.8)
where
\[
d\sigma_w(z) = \frac{e^{-r^2} r^{2(w-1)}}{\pi \Gamma(w)} r \, dr \, d\varphi.
\] (3.9)

In the special case, \( f = g \) we obtain:
\[
||f||^2 = |f_0|^2 + \int |f_w(\bar{z})|^2 d\sigma_w(z) \geq 0
\] (3.10)

4 The Hilbert space of Entire Functions.

4.1 Basic properties of \( \mathcal{F}_w \).

The goal of this section is to characterize any vector state by an entire analytic function. Such a realization of the Hilbert space \( \mathcal{H} \) is called the Fock-Bargmann representation [11, 12], and it can be obtained through any coherent state system. Here, we will consider the coherent states \(|z, w\rangle\) and the Hilbert space \( \mathcal{H}_r \subset \mathcal{H} \).

Let us introduce the Hilbert space \( \mathcal{F}_w \), which elements are all functions of the form (3.7). For each \( w \), the inner product is defined by:
\[
(f, g)_w \equiv \int \bar{f}_w(\bar{z})g_w(\bar{z})d\sigma_w(z)
\] (4.1)
where the integral is extended over all \( C \). In particular, for \( f = g \) we require (see 3.10)
\[
(f, f)_w = \int |f_w(\bar{z})|^2 d\sigma_w(z) < \infty.
\] (4.2)

It can be easily seen that \( f_w(z) \) is an entire analytic function (see 3.7). We want to study in more detail the properties of \( f_w(z) \). With this aim, let us analyze equation (3.6).
Because $|z,w\rangle$ is normalized, the Schwarz inequality gives: $|\langle z,w|f\rangle| \leq ||f||$, $|f\rangle \in \mathcal{H}_r$, and one obtains:

$$|f_w(z)| \leq \sqrt{F_1(1,w,r^2)} \ ||f||.$$  \hfill (4.3)

Hence, the behavior of $|f_w(z)|$ at infinity is the same as the one of $\sqrt{F_1(1,w,r^2)}$. As it has been shown, the generalized hypergeometric function

$$pF_q(a_1,\ldots,a_p,b_1,\ldots,b_q,z^s) \equiv \frac{\Gamma(b_1)\cdots\Gamma(b_q)}{\Gamma(a_1)\cdots\Gamma(a_p)} \sum_{n=0}^{\infty} \frac{\Gamma(a_1+n)\cdots\Gamma(a_p+n)}{\Gamma(b_1+n)\cdots\Gamma(b_q+n)} \frac{z^n}{n!}$$

is an entire analytic function (exponential type) of order $\rho_s$ and type $\sigma_s$, i.e., growth $(\sigma_s,\rho_s)$. The relation between $\rho_s$ and $\sigma_s$ is given by $\sigma_s = s/\rho_s = 1+q-p$. In the particular case of $F_1(1,w,r^2)$ we find $\rho_s = 2$ and $\sigma_s = 1$. Then, because:

$$|\sqrt{pF_q(a_i,b_j;z^s)}| = |\sqrt{pF_q(a_i,b_j;z^s)}| \leq \exp(\frac{\sigma_s r^{\rho_s}}{2})$$ \hfill (4.5)

one obtains that $\sqrt{F_1(1,w,r^2)}$ has $\rho = 2$ and $\sigma = 1/2$, i.e., growth $(1/2,2)$. This behavior at infinity is equal to the corresponding one for the harmonic oscillator case, which shows that the $w$ coherent states are “good” to generate the Fock-Bargmann representation of $\mathcal{H}_r$. Thus, we have realized the Hilbert space $\mathcal{H}_r$ as a space $\mathcal{F}_w$ of entire analytic functions $f_w(z)$ of the form (3.7) and satisfying condition (4.2).

The orthonormal basis $\{ |\theta_n\rangle, n \geq 1 \}$ in $\mathcal{F}_w$ has a simple form

$$\theta_n(z) \equiv \sqrt{F_1(1,w,r^2)} \langle z,w|\theta_n \rangle = \bar{z}^{(n-1)} \frac{\Gamma(w)}{\Gamma(w+n-1)}. \hfill (4.6)$$

This is the simplest orthonormal set of vectors in $\mathcal{F}_w$. The functions $h_w(z)$ are then written as:

$$h_w(z) = \sum_{n=0}^{\infty} A_{n+1} \theta_{n+1}(z); \quad A_{n+1} \equiv \langle \theta_{n+1}|h\rangle, \hfill (4.7)$$

and the corresponding representation of the coherent state $|\alpha,w\rangle$, $\alpha \in \mathbb{C}$ is:

$$\alpha_w(z) = \frac{\sqrt{F_1(1,w,r^2)}}{\sqrt{F_1(1,w,|\alpha|^2)}} \langle z,w|\alpha\rangle \hfill (4.8)$$

Notice that (4.6) and (4.8) reproduce the usual harmonic oscillator case when we take $w = 1$. 

6
4.2 Principal vectors $e_a$, and the reproducing kernel

Equation (4.3) is quite typical for a Hilbert space of analytic functions, and the term $\sqrt{1F_1(1,w,r^2)}$ arises due to the set of coherent states taken into account. In this section we will derive some results independent of the function $1F_1(1,w,r^2)$ and valid for all possible spaces $F$.

Let us consider the elements $e_a$ of $F_w$ (for every $a \in \mathbb{C}$) given by:

$$e_a(z) \equiv \sum_{n=0}^{\infty} \theta_{n+1}(z)\overline{\theta}_{n+1}(a) = 1F_1(1,w,z\overline{a}).$$

Using these vectors, called the principal vectors of $F_w$, we can introduce a bounded linear functional:

$$f_w(a) = (e_a, f)_w.$$ (4.10)

The integral form reads:

$$f_w(a) = \int 1F_1(1,w,a\overline{z})f_w(z)d\sigma_w(z).$$ (4.11)

Then, $1F_1(1,w,a\overline{z})$ is the reproducing kernel for $F_w$. Conversely, (4.10) implies (4.3) with $||e_a|| = 1F_1(1,w,|a|^2)$.

The connection between the reproducing kernel and the coherent states as represented in the $F_w$ space is given by (4.8). This could be seen also by expressing (4.11) in terms of the $w$ coherent states $|z,w\rangle \in H_r$ because:

$$f_w(\overline{a}) = \sqrt{1F_1(1,w,|a|^2)}(a,w|f)$$

$$= \int \frac{1F_1(1,w,a\overline{z})}{\sqrt{1F_1(1,w,r^2)}}(z,w|f)d\mu(z,w)$$

$$= \int 1F_1(1,w,z\overline{a})f_w(\overline{z})d\sigma_w(z).$$

From this, we get the proportionality between the reproducing kernel and the inner product of two $w$ coherent states $|a,w\rangle$ and $|z,w\rangle$ both in $H_r$.

Finally, the set of vectors $e_a$ is complete, i.e. their finite linear combinations are dense in $F_w$, because the only vector orthogonal to all of them is $f = 0$, as follows immediately from (4.10).

4.3 Realization of the distorted Heisenberg Algebra in $F_w$

Let us represent now the generators of the distorted algebra in $F_w$, which is a subspace of the space of entire functions of growth $(1/2,2)$. At this point, it is convenient to introduce
the unnormalized $w$ coherent states $|z, w\rangle_e = \sqrt{1_F}(1, w, r^2)|z, w\rangle$. In terms of these the function $h_w(\bar{z})$ can be written as:

$$h_w(\bar{z}) = e\langle z, w| h\rangle.$$

The action of $C_w^\dagger$ on $h_w(\bar{z})$ arises from the inner product of $|h\rangle$ with the adjoint of $C_w|z, w\rangle_e = z|z, w\rangle_e$:

$$C_w^\dagger h_w(\bar{z}) \equiv e\langle z, w| C_w^\dagger |h\rangle = \bar{z}h_w(\bar{z}).$$

(4.12)

The action of $C_w$ on $h_w(\bar{z})$ is as well obtained from the inner product of $|h\rangle$ with the adjoint of

$$C_w^\dagger h_w(\bar{z}) = \bar{z}h_w(\bar{z}).$$

(4.13)

We get then:

$$C_w h_w(\bar{z}) = \frac{\partial}{\partial \bar{z}} h_w(\bar{z}) + \frac{(w - 1)}{\bar{z}} (h_w(\bar{z}) - h_1),$$

(4.14)

where $h_1 \equiv \langle \theta_1| h\rangle$, $\bar{z} \neq 0$. Finally, the action of $I_w$ on $h(\bar{z})$ is given by:

$$I_w h_w(\bar{z}) = (w - 1)h_1 + h_w(\bar{z}).$$

(4.15)

At this point we see the great resemblance, almost equality, of equations (4.12, 4.14, 4.15), with the corresponding ones for the harmonic oscillator case. The difference rests on the dependence on $w > 0$. When $w = 1$, however, (4.12, 4.14, 4.15) are the same as those for the usual harmonic oscillator in the Segal-Bargmann space. There is an isomorphism then between $\mathcal{F}_w$ and $\mathcal{H}_r$ for all $w > 0$. Notice that it is also possible to find an isomorphism between $\mathcal{F}_w$ and a subspace of $\mathcal{H}$ for $w = 0$, but in this case it will be $\mathcal{H}_s$.

We would like to show, finally, that the coherent states $\alpha_w(z) \in \mathcal{F}_w$, expressed in (4.8) in terms of $|z, w\rangle \in \mathcal{H}_r$, can be obtained through a standard procedure \[6, 7\] as eigenfunctions of $C_w$ in $\mathcal{F}_w$:

$$C_w \alpha_w(\bar{z}) = \alpha \alpha_w(\bar{z}), \quad \alpha \in \mathbb{C}.$$  

(4.16)

Let’s take

$$\alpha_w(\bar{z}) = \sum_{n=1}^{\infty} a_n \theta_n(\bar{z}).$$

(4.17)
Using (4.14), with \( h_w(\bar{z}) = \alpha_w(\bar{z}) \), and (4.16) one obtains the coefficients \( a_n \). In the case \( w \neq 0 \) we get

\[
a_{n+1} = \alpha^n \sqrt{\frac{\Gamma(w)}{\Gamma(w+n)}} a_1, \quad n = 1, 2, \ldots
\]

(4.18)

Then, by imposing the normalization, with \( a_1 \geq 0 \), we obtain

\[
\alpha_w(\bar{z}) = \frac{1_F(1, w, \alpha \bar{z})}{\sqrt{1_F(1, w, |\alpha|^2)}}
\]

which, obviously coincides with (4.8). Notice that the case \( w = 1 \) is straightforwardly obtained of the previous formula:

\[
\alpha_1(\bar{z}) = e^{\left( -\frac{1}{2} |\alpha|^2 + \alpha \bar{z} \right)}
\]

This is the usual representation for the harmonic oscillator coherent state in the Segal-Bargmann space.

All these facts show that \( C_w \) and \( C_w^\dagger \), adjoint to each other in \( \mathcal{H}_r \), possess the same properties on \( \mathcal{F}_w \). Hence, the eigenstates of \( C_w \) in \( \mathcal{H}_r \) correspond to eigenfunctions of \( C_w \) in \( \mathcal{F}_w \) and vice versa.

Remarks

The \( w \) coherent states \( |z, w\rangle \) and the operators \( C_w, C_w^\dagger \) and \( I_w \), admit a simplest representation in the space of analytic functions \( \mathcal{F}_w \) compared with the representation in \( \mathcal{H}_r \). The usual harmonic oscillator representation is achieved when \( w = 0 \) or \( w = 1 \). The fact that \( C_w \) and \( C_w^\dagger \) are adjoint to each other on \( \mathcal{H}_r \) is preserved on \( \mathcal{F}_w \) because the isomorphism between \( \mathcal{F}_w \) and \( \mathcal{H}_r \). This representation of the distorted Heisenberg algebra on \( \mathcal{F}_w \) as far as we know, has not been explored previously. It could be important in the analysis of the appropriate displacement operator \( D_w \) to perform the Perelomov construction of the coherent states for the isospectral oscillator Hamiltonians \( H_\lambda \).

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