A Lévy Flight Based Firefly Algorithm for Multilevel Thresholding Image Segmentation

Suping Liu, Yi Wang*

College of Software Engineering, Guangdong university of science and technology, Dongguan, Guangdong 532000, China

*Corresponding author e-mail:1471836256@gdust.edu.cn

Abstract. Traditional exhaustive search methods have high time complexity in solving multilevel threshold problems. To address this problem, a Levy flight based firefly algorithm called LFA is proposed. Otsu is regard as its objective function. A series of standard test images were used to evaluate the performance of the algorithm. The Peak signal to noise ratio(PSNR) and Structural similarity index (SSIM) are utilized to evaluate the segmented image. The experimental results show that the LFA is better than FA for multilevel thresholding image segmentation.

Keywords: Firefly algorithm; Levy flight; multilevel thresholding; Otsu.

1. Introduction

Image segmentation has always been a very important step in computer vision, different types of image segmentation methods are not the same [1]. According to the segmentation of image classification can be divided into: color image segmentation and gray image segmentation, gray image segmentation has been a hot research issue. Threshold image segmentation is a very important method in gray image segmentation. According to the number of thresholds of image segmentation is usually divided into single threshold image segmentation and multilevel threshold image segmentation. Single threshold image segmentation can only adapt to the segmentation of a single object in the image. If there are multiple objects in the image, a single threshold can not be used for image segmentation effectively. However, the time complexity of the traditional exhaustive search method for multi-level threshold is very high, which is an NP-hard optimization problem [2].

The proposal and application of the optimization algorithm provide a new way to solve the multilevel threshold problem. To solve the multi-level threshold optimization problem, many optimization algorithms have been proposed such as bat algorithm (BA) [3], artificial bee colony (ABC) [4], flower pollination algorithm (FPA) [5], firefly algorithm (FA) [6], gravitational search algorithm (GSA) and so on [7]. The FA algorithm has a simple structure and few parameter Settings, which has attracted the attention of many researchers. However, FA needs to be further improved when solving complex optimization problems. To address this problem, this paper proposes an FA algorithm based on Levy flight.

The paper is organized as follows. Section 2 presents the Otsu for multilevel thresholding. Section 3 presents the proposed FLA. Experimental results and analysis are discussed in Section 4. Conclusion is drawn in Section 5.
2. Otsu for multilevel thresholding

There are many kinds of multilevel threshold image segmentation methods, and the basic idea is to use a certain index to measure the quality of the segmented image. The basic methods include entropy to measure, or the maximum inter-class variance method. Then the multilevel threshold problem is transformed into the problem of solving the maximum value of the function, so the optimization algorithm is usually used to solve the problem. Otsu based image thresholding is initially proposed in 1979. At the beginning, it is mainly used to solve the single threshold of gray image. Later, it was gradually extended to the solution of multilevel threshold. For a single threshold problem, if an image can be divided into two classes such as A0 and A1 (background and objects) by a threshold at a level “t.” The class A0 encloses the gray levels in the range 0 to t-1 and class A1 encloses the gray levels from t to L-1. The probability distributions for the gray levels A0 and A1 can be expressed as[8]:

\[ A_0 = \frac{p_0}{\omega_0(t)} ... \frac{p_{i-1}}{\omega_0(t)}, \quad A_1 = \frac{p_i}{\omega_1(t)} ... \frac{p_{L-1}}{\omega_1(t)} \]  

(1)

Where, \( p_i \) is the gray level probability, \( \omega_0(t) = \sum_{i=0}^{t-1} p_i, \omega_1(t) = \sum_{i=t}^{L-1} p_i, \) and \( L = 256 \). And, the mean levels \( \mu_0 \) and \( \mu_1 \) for \( C_0 \) and \( C_1 \) can be measured by

\[ \mu_r = \frac{\sum_i i p_i}{\omega_r(t)}, \quad \mu_i = \frac{\sum_i i p_i}{\omega_i(t)} \]  

(2)

The mean intensity(\( \mu(t) \)) of the entire image can be described as

\[ u_r = \omega_0 \mu_0 + \omega_1 \mu_1, \quad \omega_0 + \omega_1 = 1 \]  

(3)

The objective function for the bi-level thresholding problem can be defined as

\[ F^{\text{opt}} = \arg \max (\delta_r + \delta_i) \]  

(4)

Where, \( \sigma_r = \omega_0 (u_r - u_0)^2 \) and \( \sigma_i = \omega_1 (u_i - u_r)^2 \).

The bi-level thresholding can be extended to multilevel thresholding problem by increase the various “m” values as follows. Let us consider that there are “m” thresholds \( t_1, t_2, ..., t_m \), which divide the image into “m” classes: \( A_0 \) with gray levels in the range 0 to \( t_1 \), \( A_1 \) with enclosed gray levels in the range \( t_1 \) to \( t_2-1 \), ..., and \( A_m \) with gray levels from \( t_m \) to \( L-1 \). The objective function for the multilevel thresholding problem can be expressed as

\[ F^{\text{opt}} = \arg \max (\delta_1 + \delta_2 + ... + \sigma_n) \]  

(5)

Where, \( \sigma_r = \omega_0 (u_r - u_0)^2, \sigma_i = \omega_1 (u_i - u_r)^2, ..., \sigma_n = \omega_n (u_n - u_r)^2 \).

3. The proposed algorithm

3.1. Firefly algorithm

The firefly algorithm simulates the behavior of fireflies in nature looking for companions through their own light. Fireflies rely on the intensity of their own light to attract other companions. The stronger the light, the stronger the attraction. The light intensity \( I(r) \) can be defined as:

\[ I(r) = I_0 e^{-\gamma r^2} \]  

(6)

where \( I_0 \) represents the initial light intensity, \( r \) is the distance between two fireflies, and \( \gamma \) is a fixed light absorption coefficient. The attractiveness \( \beta(r) \) based on the light intensity, that can be expressed as:

\[ \beta(r) = \beta_0 e^{-\gamma r^2} \]  

(7)

where \( \beta_0 \) is the attractiveness at \( r = 0 \).

The attractiveness \( \beta \) depends on the distance \( r \), the distance between any two fireflies \( i \) and \( j \) at \( X_i \) and \( X_j \) can be computed according to the Euclidian distance:

\[ r_{ij} = \| X_i - X_j \| = \sqrt{\sum_k (x_{i,k} - x_{j,k})^2} \]  

(8)

where \( x_{i,k} \) is the k-th element of the i-th firefly and \( d \) is the dimension of the problem. Each firefly \( i \) moves toward to firefly \( j \), as follow:
\[ X_i = X_j + \beta_i e^{-\delta_i} (X_j - X_i) + \alpha \left( \text{rand} - \frac{1}{2} \right) \]  

where, \( \alpha \) is called step factor, \( \text{rand} \) represents uniformly distributed random number within \([0, 1]\).

3.2. Firefly algorithm based on Lévy flight

Lévy flight is a random flight process with variable step length, many birds in nature follow the Lévy flight. In Lévy flight, the flight span and the length between two successive changes in direction are drawn from a probability distribution. Usually, Lévy flight process often obeys Lévy distribution, but Lévy distribution is difficult to achieve. Mantegna proposed a method to imitate the Lévy distribution, in which two Gaussian distribution functions were used to realize the Lévy flight process, the rand step size \( s \) is defined as [9]:

\[ \text{Le}vy(s) = \frac{\mu}{|\Gamma(1/\lambda)|} \]  

where, \( \lambda \) is a real parameter, \( \mu, \nu \) follow the Gaussian distribution, which are defined as:

\[ u \sim N(0, \delta_u) \]  
\[ v \sim N(0, \delta_v) \]  

where, \( \delta_u=1, \delta_v \) can be calculated by:

\[ \delta = \left[ \Gamma(1+\lambda) \sin(\pi \lambda / 2) / \lambda \Gamma(1+\lambda / 2)2^{(\lambda-1)/2} \right] \]  

where, \( \Gamma \) is gamma function. If \( \lambda = 1.5 \) , and steps=600, the Lévy flight is shown in Fig.1.

So the Eq(9) can be written by the following formula:

\[ X_i = X_j + \beta_i e^{-\delta_i} (X_j - X_i) + \alpha \cdot \text{sign} \left( \text{rand} - \frac{1}{2} \right) \odot \text{Le}vy \]  

\[ \text{Fig.1 Lévy flight process} \]
4. Experiments results and analysis

The proposed method are evaluated under a set of benchmark images. The test images of size 512*512 along with their histograms are exhibited in Fig. 2. The parameter settings of the LFA are shown below: Population size is 20, the maximum number of iterations sets to 200, besides $\gamma=1$, $\lambda=1.5$, $\alpha=0.1$. For evaluating the performance of algorithms, significant parameters included are as follows [10]:

**Peak signal to noise ratio:**

$$PNSR = 20\log_{10}\left(\frac{255}{RMSE}\right)\ (dB)$$

where $RMSE$ is the root mean-squared error, defined as:

$$RMSE = \sqrt{\frac{\sum_{i=1}^{N}\sum_{j=1}^{M}(i(i, j)-\bar{i}(i, j))^2}{MN}}$$

Here $I$ and $\bar{I}$ are original and segmented images of size $M*N$, respectively.

**Structural similarity index:**

$$SSIM(C, I) = \frac{(2\mu_{C}+\mu_{I})(2\delta_{C,I}+C)}{\left(\mu_{C}^2+\mu_{I}^2+C\right)\left(\delta_{C}^2+\delta_{I}^2+C\right)}$$

where

$$S_c(x) = S_m(x)S_g(x)$$

$$S_m(x) = \frac{2PC(x)PC(y) + T_1}{PC(x)^2 + PC(y)^2 + T_1}$$

$$S_g(x) = \frac{2G(x)G(y) + T_2}{G(x)^2 + G(y)^2 + T_2}$$

$$PC(x) = \max\{|PC(x), PC(y)|$$

**Fig. 2** The test images and their histograms

4.1. **Comparison of PSNR and SSIM computed by LFA and FA**

In order to verify the performance of the method, Four test images are used to implement this experiment. The performance metrics for checking the effectiveness of the method are chosen as the PSNR and SSIM. Table 1 shows the experimental results. It can be seen from the analysis of the experimental results, the larger the number of threshold values, the larger the PSNR and SSIM values,
the better the image segmentation effect. Besides, LFA algorithm is better than standard FA algorithm in PSNR and SSIM. The overall results show that the improved FA algorithm is more accurate.

Table 1. The PSNR and SSIM experimental results

| Test images | Levels | LFA PSNR | SSIM | FA PSNR | SSIM |
|-------------|--------|----------|------|---------|------|
| Barbara     | 2      | 20.6160  | 0.5235| 20.3514 | 0.5124 |
|             | 3      | 23.9804  | 0.5969| 23.5682 | 0.5866 |
|             | 4      | 25.3816  | 0.6498| 25.1243 | 0.6293 |
|             | 5      | 26.3556  | 0.6859| 26.1521 | 0.6789 |
| Birds       | 2      | 22.1082  | 0.4996| 22.0863 | 0.4768 |
|             | 3      | 23.3212  | 0.5141| 23.1242 | 0.5012 |
|             | 4      | 24.1217  | 0.5530| 23.9867 | 0.5369 |
|             | 5      | 26.5580  | 0.5755| 26.3598 | 0.5463 |
| Fish        | 2      | 19.7769  | 0.4056| 19.5867 | 0.3896 |
|             | 3      | 21.1223  | 0.5145| 20.0321 | 0.5012 |
|             | 4      | 23.1837  | 0.5904| 22.3689 | 0.5864 |
|             | 5      | 25.1203  | 0.6509| 24.8695 | 0.6359 |
| Owl         | 2      | 20.4508  | 0.4921| 20.2365 | 0.4685 |
|             | 3      | 24.0431  | 0.6029| 23.9861 | 0.5862 |
|             | 4      | 25.6168  | 0.6619| 25.4263 | 0.6489 |
|             | 5      | 27.2381  | 0.7222| 27.0124 | 0.7123 |

5. Conclusion
In this paper, A FA algorithm based on Levy flight is proposed to solve multilevel threshold, PSNR and SSIM are used to evaluate the performance of the proposed algorithm. The experimental results show that the proposed LFA algorithm can effectively solve multi-level threshold problems, and its accuracy is higher than that of the FA algorithm. The proposed LFA algorithm is more conducive to multi-level image segmentation

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