Neutrino Masses in $SO(10)$ Theories

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Abstract

We review the status of a class of gauge unified models based on $SO(10)$ group and discuss the main phenomenological implications of these models in particular for neutrino masses.

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1 Introduction

The standard model is a very successful theory which describes strong and electro-weak interactions with the gauge group $G_{SM} \equiv SU(3)_c \otimes SU(2)_L \otimes U(1)_Y$ and with the left-handed fermions of each family classified in the multiplets

$$(1,1,1) + \left(3,2,-\frac{1}{6}\right) + \left(\bar{3},1,-\frac{2}{3}\right) + \left(1,2,-\frac{1}{2}\right) + \left(3,1,\frac{1}{3}\right),$$

and the scalar Higgses in the representation

$$\left(1,2,\frac{1}{2}\right) + \left(1,2,-\frac{1}{2}\right).$$

However, there is no explanation for the quantization of the electric charge, $Q$, and of the weak hypercharge, $Y$, and there are too many multiplets, 5, for each family.

A strong indication in favour of unification with a simple larger gauge group comes from the values of the trace of $Y$ and of the square of the generators of $G_{SM}$ for the multiplets of one family,

| Multiplet            | $Tr(Y)$ | $Tr(Y^2)$ | $Tr(T_3^2)$ | $Tr(F_3^2)$ |
|----------------------|---------|-----------|-------------|-------------|
| $(3,2,-\frac{1}{6})$ | 1       | $\frac{1}{6}$ | $\frac{3}{2}$ | 1           |
| $(\bar{3},1,-\frac{2}{3})$ | -2     | $\frac{4}{3}$ | 0           | $\frac{1}{2}$ |
| $(1,1,1)$            | 1       | 1         | 0           | 0           |
| $(1,2,-\frac{1}{2})$ | -1      | $\frac{1}{2}$ | $\frac{1}{2}$ | 0           |
| $(\bar{3},1,\frac{1}{3})$ | 1      | $\frac{1}{3}$ | 0           | $\frac{1}{2}$ |

In fact, the sum on all the multiplets of $Tr(Y)$ vanishes, as is expected for a generator of a simple group, and the sums for $Tr(T_3^2)$ and $Tr(F_3^2)$ take the same values. Moreover, if we combine the first three multiplets and the last two, we have $Tr(Y) = 0$ and equal values for $Tr(T_3^2)$ and $Tr(F_3^2)$ and, in both cases, $3/5$ smaller than the contribution to $Tr(Y^2)$. All these facts strongly suggest to classify the two sets of multiplets in the 10 and $\bar{5}$ representations of $SU(5)$, respectively. The gauge group of the standard model, $G_{SM}$, is a maximal subgroup of $SU(5)$ and, by taking the Higgses in the adjoint (24) and fundamental ($5 + \bar{5}$) representations of $SU(5)$, it is possible to achieve the desired breaking pattern,

$$SU(5) \to G_{SM} \to SU(3)_c \otimes U(1)_Y,$$
and the right quantization for the electric charge and the weak hypercharge.

Another indication concerns the coupling constants of the standard model, which are in the relation $g_3(M_Z) > g_2(M_Z) > g_1(M_Z)$ and almost meet at a higher scale \[3\]. However, only the meeting point of $g_3$ and $g_2$ is sufficiently high ($> 1.6 \cdot 10^{15} \text{GeV}$) to comply with the experimental lower limit on proton lifetime, $\tau_{\text{exp}}^{\text{p} \rightarrow \text{e}^+ + \pi_0} > 9 \cdot 10^{32}$ years.

Concerning the ability of $SO(10)$ in improving the predictions of minimal $SU(5)$, it is worth to recall that there are independent motivations \[4\] to consider it, rather than $SU(5)$, as the unification gauge group:

- The fermions of each family can be classified in one Irreducible Representation (IR) of $SO(10)$, the spinorial 16, with a $SU(5)$ content of $10 + \bar{5} + 1$. In this respect, the singlet can be identified as a left-handed antineutrino.

- The accidental cancellation between the opposite anomalies of 10 and $\bar{5}$ representations of $SU(5)$ is a general property of $SO(10)$ group, which depends by the absence of a third order Casimir operator.

- $SO(10)$ contains $SU(5)$ as well $SO(6) \otimes SO(4) \sim SU(4)_P SU(2)_L \otimes SU(2)_R$, first introduced by Pati and Salam \[5\], which is very elegant in classifying the left-handed fermions of a family in the $(4, 2, 1) + (\bar{4}, 2, 1)$ representation, in agreement with the hadron-lepton universality of the charged weak current.

Going back to the problem of unification of the standard model constants, to prevent conflict with experiment the evolution of $g_1$ should be modified in such a way to cross the meeting point of $g_2$ and $g_3$. In $SO(10)$ unified theories, where $Y = T_3^R + (B - L)/2$ (\(B\) and $L$ are the baryonic and leptonic numbers, respectively), this may be easily achieved considering an intermediate symmetry group $G'$ larger than $G_{SM}$ and containing $SU(2)_R$ and/or $SU(4)$.

### 2 The spontaneous symmetry breaking of $SO(10)$

An intermediate symmetry between $SO(10)$ and $G_{SM}$ is generally expected, since the smallest IR’s of $SO(10)$ (with the only exception of the spino-vector 144 \[6\]) have $G_{SM}$-singlets with a symmetry larger than $G_{SM}$. One therefore expects as intermediate symmetry the little group of the Higgs with the highest Vacuum Expectation Value (VEV). The spinor (16) and the bispinor (126) representations have $G_{SM}$-singlets invariant under
Table 1: In the breaking of $SO(10)$, the following intermediate symmetry groups lead to phenomenologically interesting predictions.

| $G''$ | Higgs direction | Representation |
|-------|-----------------|----------------|
| $SU(4)_{PS} \times D$ | $2(S_{11} + ... + S_{06}) - 3(S_{77} + ... + S_{00})$ | 54 |
| $SU(4)_{PS}$ | $\Phi_T \equiv \Phi_{7890}$ | 210 |
| $SU(3)_c \times U(1)_{B-L} \times D$ | $\Phi_L \equiv \Phi_{1234} + \Phi_{1256} + \Phi_{3456}$ | 210 |
| $SU(3)_c \otimes U(1)_{B-L}^L$ | $\cos \theta \Phi_L + \sin \theta \Phi_T$ | 210 |

Table 2: Values of the unification and intermediate scales, $M_X$ and $M_R$, for the four patterns of breaking of $SO(10)$ considered in the text.

| $G''$ | $M_X$ | $M_R$ |
|-------|-------|-------|
| $SU(4)_{PS} \times D$ | 0.6 | 460 |
| $SU(3)_c \otimes U(1)_{B-L} \times D$ | 1.6 | 0.7 |
| $SU(4)_{PS}$ | 4.7 | 2.8 |
| $SU(3)_c \otimes U(1)_{B-L}$ | 9.5 | 0.067 |

$SU(5)$. Besides these, one is able to identify the four interesting cases of Table 1, with the intermediate symmetry $G' = G'' \otimes SU(2)_L \otimes SU(2)_R$. In Table 1 $D$ is a discrete symmetry exchanging the left-handed and right-handed $SU(2)$’s and we adopt the following convention for the 10 representation of $SO(10)$: the indices 1...6 correspond to $SO(6) \sim SU(4)$ and 7...0 to $SO(4) \sim SU(2) \otimes SU(2)$.

With a model where one of the Higgses just described takes the highest VEV and the 126 representation breaks $G'$ into $G_{SM}$ (the 16 would give too small Majorana masses for the right-handed neutrinos), one can predict the scale of $SO(10)$ breaking, $M_X$, and the one of $G'$ breaking, $M_R$, in terms of the values of the gauge couplings at the scale $M_Z$, for which we take $\sin^2 \theta_W(M_Z) = 0.2315 \pm 0.0002$, $\alpha_s(M_Z) = 0.120 \pm 0.005$, $\frac{1}{\alpha}(M_Z) = 127.9 \pm 0.09$. In the analysis we assume the extended survival hypothesis, which states that the Higgs scalars acquire their masses at the highest possible scale whenever this is not forbidden by symmetries, and find the values shown in Table 2.
3 Phenomenology of SO(10) GUT’s

In SO(10) the theoretical value of the proton lifetime is

$$\tau_{p \rightarrow e^+ + \pi_0} = (1.1 - 1.4) \cdot 10^{32} \left(\frac{M_X}{10^{15}\text{GeV}}\right)^4 \text{years},$$

so that the experimental lower limit, $$\tau_{p \rightarrow e^+ + \pi_0} > 9 \cdot 10^{32} \text{years},$$ excludes the intermediate symmetries containing $$D$$. $$M_R$$ is related, via the see-saw mechanism, to the masses of the left-handed neutrinos,

$$m_{\nu_{iL}} = \left(\frac{m_\tau}{m_b}\right)^2 \frac{m_{u_i}}{M_R} \frac{g_{2R}}{f_i},$$

where $$f_i$$ is the Yukawa coupling of the scalars of the 126 to the $$i$$-th family.

With respect to $$SU(5)$$, for which proton decay is the only typical new phenomenon, $$SO(10)$$ has other possible signatures, one of which is neutron-antineutron oscillation. In fact, minimal $$SU(5)$$ has $$B - L$$ as a global symmetry, while in $$SO(10)$$ this is a generator and must be spontaneously broken, since there is no massless boson coupled to its associated current. Very brilliant experiments have reached the lower limit of $$0.86 \cdot 10^8 \text{sec} \ (90\% \ CL)$$ for the $$n - \bar{n}$$ time of oscillation. Indeed, in the 126 representation there are scalars with the proper quantum numbers to mediate $$n - \bar{n}$$ transitions. However, since the exchange of three of them is needed, they should not be larger than $$\sim 10^{4.5} \text{GeV}$$ to provide an oscillation time at reach of experimental detection. The lower limit found by Baldo-Ceolin et al. allowed to prove that a longer $$n - \bar{n}$$ oscillation would have negligible effects on the evolution of neutron stars. This happens because the effect would also be dumped by a quantum Zeno effect, which would make the energy loss for the oscillation and subsequent annihilation dependent on the volume of the neutron star and not on the density.

4 $$SO(10)$$ and neutrino masses

A relevant difference between $$SO(10)$$ and $$SU(5)$$ concerns neutrino masses. In $$SU(5)$$, like in the standard model, one does not need right-handed neutrinos. It is possible to build Majorana masses for the left-handed neutrinos by coupling them to $$I = 1$$ Higgses, which are also not necessary in the standard model. In $$SU(5)$$, by classifying the electro-weak Higgses in a $$5 + \bar{5}$$, one gets the equality of the mass matrices for charged leptons and
Table 3: Values of the combination \( \frac{m_\tau}{m_b} m_t \) in Eq. (6), corresponding to the Majorana mass of right-handed neutrinos. All the quantities are measured in GeV.

| \( M_R \) | \( \frac{m_\tau}{m_b} m_t \) | \( \frac{m_\tau}{m_b} m_c \) |
|---------|-----------------|-----------------|
| \( 2.5 \cdot 10^{11} \) | 4 | 1 |
| \( 2.5 \cdot 10^{13} \) | 40 | 10 |
| \( 2.5 \cdot 10^{15} \) | 400 | 100 |
| Experimental value | \( \sim 70 \) | \( \sim 0.5 \) |

\(-1/3\) quarks at the highest scale. This prediction is modified by the renormalization group equations (RGE) into \( \frac{m_\tau}{m_b} \sim 3 \) at lower scales, in qualitative agreement with experiment. By classifying the electro-weak Higgses in the 10 representation of \( SO(10) \) (5 + \( \bar{5} \) under \( SU(5) \)), one gets the equality of the Dirac neutrino and 2/3 quark mass matrices at the highest scale, which would be a disaster, were not for the see-saw mechanism \([12]\), which transforms that prediction into the intriguing one that neutrino masses are much smaller than the masses of the other fermions.

Neutrino masses and mixings are now advocated to explain the anomalies in atmospheric and solar neutrinos with square mass differences \( 3.5 \cdot 10^{-3} eV^2 \) and \( 2.5 \cdot 10^{-5} eV^2 \) (for the MSW solution \([15]\)), respectively. This corresponds to the Dirac masses in the last two column of Table 3, depending on the Majorana mass of the right-handed neutrinos (with the simplifying assumption to classify the Higgs doublets in the 10 representation of \( SO(10) \)). The orders of magnitude comply well with the value advocated for solar neutrinos in the case of the the model with intermediate symmetry \( SU(4)_{PS} \otimes SU(2)_L \otimes SU(2)_R \) \([16]\), while for atmospheric neutrinos a larger Majorana mass, \( \sim 10^{13} GeV \), would be preferred.

It is worth reminding that the effective \( \nu_L \) mass matrix is given by

\[
M_D^T M_M^{-1} M_D, \tag{7}
\]

and is expected to give a large \( \nu_\mu - \nu_\tau \) mixing, while the quark mixing angles in the CKM matrix \([17]\) are small. We have the option of giving Majorana masses to the right-handed neutrinos either by the \( \Delta |B - L| = 2 \) \( SU(5) \) singlet of the 126 or by the \( \Delta |B - L| = 1 \) \( SU(5) \) singlet of the 16. In the last case one gets only higher loop contribution to the Majorana masses, which imply that the VEV of the 16 should be some order of magnitude larger than \( M_M \).
As we have seen, the model with $SU(4)_{PS} \otimes SU(2)_L \otimes SU(2)_R$ intermediate symmetry gives a Majorana mass for the right-handed neutrinos in agreement with the value advocated for the MSW explanation of solar neutrinos' anomaly. However, to give the mass requested for the atmospheric neutrinos, one has to assume that the highest Dirac mass of the neutrinos is $\sim 8 \text{GeV}$; this corresponds, by evolving the mass with the RGE at the higher scales, to a value an order of magnitude smaller than the top mass, while the hypothesis of classifying the Higgs doublets in the 10 representation of $SO(10)$ would imply equal values for them. This is not so unreasonable, since we know that one needs other representations to avoid the prediction of an equal mass at the highest scale for the strange quark and the muon and that the neutrino mixing is different from the quark one.

5 SUSY $SO(10)$ models

Besides the non-SUSY models described in the previous sections, it is worth studying whether SUSY $SO(10)$ models can provide a higher value for the Majorana masses of right-handed neutrinos. For a long time model builders of SUSY gauge unified theories have been stressing that, with SUSY breaking at $\text{TeV}$ scale, as is needed to protect the electro-weak scale, the RGE are modified in such a way that the three gauge coupling constants meet at a sufficiently high scale to comply with the lower limit on proton lifetime. In that framework it is not necessary to go beyond minimal $SU(5)$ as in non-SUSY models. Let us nevertheless study the expectations for right-handed neutrino Majorana masses in SUSY $SO(10)$ models.

In SUSY unified theories one has less freedom in building the Higgs potential, which implies that it is more difficult to achieve the desired pattern of symmetry breaking and, conversely, more meaningful the construction of consistent models. In fact, the potential consists of two parts, both of which non-negative,

\[ \sum_{\alpha} |D_\alpha|^2 = \sum_{\alpha} g_\alpha^2 |\Phi|Q_\alpha|\Phi|^2, \]
\[ \sum_{\alpha} |F_a|^2 = \sum_{\alpha} \left| \frac{\delta F}{\delta \Phi_a} \right|^2, \]

where $Q_\alpha$ are the gauge group charges and the superpotential $F$ is an invariant function of $\Phi$ of degree $\leq 3$. There is a complementarity between $D_\alpha$ and $F_a$, since a necessary and sufficient condition for a field $\Phi_a$ to give $D_\alpha = 0$ is the existence of at least an invariant
function of $\Phi_a$, $G$, such that

$$\left( \frac{\delta G}{\delta \Phi_a} \right)_{\Phi = <\Phi>} = k <\Phi_a^* >,$$

with $k \neq 0$ \[8\].

To get non-trivial zeros for the SUSY potential, one a) may exclude some terms in $F$ (this can seem unnatural, but some discrete symmetry may help), b) consider only invariant functions $G$ with degree $\geq 4$, or c) (the most elegant possibility) with more than one invariant of degree $\leq 3$ obeying the condition in Eq. (9).

By studying the SUSY extensions of $SO(10)$ models previously considered, with the Higgses responsible for the spontaneous symmetry breaking of $SO(10)$ in the $16 + 45$ or in the $126 + 54$, respectively, it is easy to see that, in both cases, one needs at least an additional 16 representation to have vanishing $Q_a$.

6 Conclusion

The increasing evidence in favour of neutrino masses and mixings is a serious hint for $SO(10)$ unification, which provides all the elements, left-handed antineutrinos and very high Majorana masses for them, for a successful see-saw mechanism. This is a strong encouragement for the construction of a consistent $SO(10)$ theory. It is fair to stress that the most convincing facts in favour of physics beyond the standard model come from neutrino oscillations first proposed by Pontecorvo and successfully developed by him in collaboration with Gribov and Bilenky \[19\]. Also the radiochemical method to detect solar neutrinos, successfully applied in the Homestake, Galles and Sage experiments, was invented more than fifty years ago by him \[20\].

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