Cosmogenic neutrinos and signals of TeV gravity in air showers and neutrino telescopes

J.I. Illana, M. Masip, and D. Meloni

Centro Andaluz de Física de Partículas Elementales (CAFPE)
and
Departamento de Física Teórica y del Cosmos
Universidad de Granada, E-18071, Granada, Spain

jillana@ugr.es, masip@ugr.es, meloni@ugr.es

Abstract

The existence of extra dimensions allows the possibility that the fundamental scale of gravity is at the TeV. If that is the case, gravity could dominate the interactions of ultra-high energy cosmic rays. In particular, the production of microscopic black holes by cosmogenic neutrinos has been estimated in a number of papers. We consider here gravity-mediated interactions at larger distances, where they can be calculated in the eikonal approximation. We show that for the expected flux of cosmogenic neutrinos these elastic processes give a stronger signal than black hole production in neutrino telescopes. Taking the bounds on the higher dimensional Planck mass $M_D (D = 4+n)$ from current air shower experiments, for $n = 2 \ (6)$ elastic collisions could produce up to 118 \ (34) events per year at IceCube. On the other hand, the absence of any signal would imply a bound of $M_D \gtrsim 5 \text{ TeV}$.


**Introduction.** We observe extensive air showers produced when a cosmic ray from outer space hits a nucleon in the upper atmosphere. The observed events have energies of up to $10^{11}$ GeV, and their profile and distribution are consistent with a primary proton of extragalactic origin. In their way to the Earth these protons would interact with the CMB photons and produce pions:

$$p + \gamma_{2.7K} \rightarrow \Delta^+ \rightarrow n + \pi^+ (p + \pi^0).$$

The flux of cosmogenic neutrinos is created in the decay of the charged pions, and it will appear correlated with observable fluxes of nucleons and photons.

Cosmogenic neutrinos are of great interest as probes of new TeV physics because of two generic reasons. First, they provide large center of mass energies. Second, the relative effect of new physics on the weakly interacting neutrinos is larger than on quarks or charged leptons, making it easier to see deviations. The signals of new physics could be detected in deeply penetrating air showers and neutrino telescopes.

In particular, in models with extra dimensions and the fundamental Planck scale at the TeV the gravitational interactions are unsuppressed in the transplanckian regime. The possibility of black hole (BH) formation by cosmogenic neutrinos has been discussed in several papers. Here we will study the gravitational interaction at larger distances, where it can be calculated using the eikonal approximation. This approximation involves linearized gravity and is not affected by the uncertainties in the cross section for BH formation. After discussing the bounds on the gravitational scale from air shower experiments, we will show that these elastic processes are more frequent than BH formation in neutrino telescopes. At large impact parameters the neutrino interacts, loses a small fraction of energy and keeps going. Telescopes could detect these processes because they are sensitive to events of energy three or four orders of magnitude below the typical energy (around $10^9$ GeV) of the cosmogenic neutrinos.

**TeV gravity.** The simplest picture of TeV gravity includes only two free parameters: the value of the higher dimensional Planck scale $M_D$, and the number $n$ of compact dimensions where gravity propagates. A third parameter, the (common) length $2\pi R$ of the $n$ dimensions, could be deduced from the 4d Newton constant:

$$G_D = (2\pi R)^n G_N = \frac{(2\pi)^{n-1}}{4M_D^{n+2}}.$$  \hspace{1cm} (2)

At processes below $M_D$ the model-independent signature of extra dimensions is graviton emission. The amount of energy radiated would be proportional to the accessible phase space or, in the Kaluza-Klein (KK) picture, to the number of KK modes of mass below the center of
mass energy. In this type of experiments for a given $n$ one sets bounds on $R$ and then deduces the limits on $M_D$. From collider experiments one obtains $M_D \geq 1.4 \ (1.0)$ TeV for $n = 2 \ (\geq 3)$ [11], whereas from SN1987A the bounds go up to 22 TeV for $n = 2$ [12]. One should keep in mind, however, that the gravitons emitted in the supernova explosion have a KK mass below $\approx 50$ MeV. The simple picture could be modified above this energy, for example, with four more dimensions at $R' \sim (100 \text{ GeV})^{-1}$, which would bring the fundamental scale of gravity down to 1 TeV without affecting the physics in the supernova.

The bounds obtained from transplanckian collisions are complementary in the sense that given $n$ they are a direct probe of $M_D$, and $R$ is then adjusted in order to reproduce $G_N$. At energies above $M_D$ and impact parameters smaller than $R$ the collision is a pure higher-dimensional process independent of the compactification details that fix the value of the effective Newton constant. The transplanckian collision does not see that the extra dimensions are compact, they could be taken infinite with no effect on the cross section.

**Neutrino-nucleon cross section.** The TeV gravity model should be embedded in a string theory, which would relate $M_D$ with the string scale $M_S$. In the simplest set-up [14] the standard model (SM) fields (open strings) would be attached to a 4d brane, whereas gravity (closed strings) would propagate in the whole $Dd$ space. In this case

$$M_D^{n+2} = \frac{8\pi}{g^2} M_S^{n+2},$$

with $g$ the string coupling. The transplanckian regime corresponds then to energies above the string scale, where any tree-level amplitude becomes very soft. In the ultraviolet string amplitudes go to zero exponentially at fixed angle and, basically, only the forward (long distance) contribution of the graviton survives. This is precisely the regime where the eikonal approximation is valid.

Let us consider the elastic collision of a neutrino and a parton that exchange $D$-dimensional gravitons (see [5] [10] for details). The eikonal amplitude $A_{\text{eik}}(s, t)$ resums the infinite set of ladder and cross-ladder diagrams. It is reliable as far as the momentum carried by the gravitons is smaller than the center of mass energy or, in terms of the fraction of energy $y = (E_\nu - E'_\nu)/E_\nu$ lost by the incoming neutrino, if $y = -t/s \ll 1$ ($s$ and $t$ refer to the Mandelstam parameters at the parton level). In this limit the amplitude is independent of the spin of the colliding particles. Essentially, $A_{\text{eik}}$ is the exponentiation of the Born amplitude in impact parameter space:

$$A_{\text{eik}}(s, t) = \frac{2s}{t} \int d^2 b \ e^{i b \cdot \chi} \left( e^{i \chi(s, b)} - 1 \right),$$

where $\chi(s, b)$ is the eikonal phase and $b$ spans the (bidimensional) impact parameter space. $\chi(s, b)$ can be deduced from the Fourier transform to impact parameter space of $A_{\text{Born}}(s, t)$.  

3
Our Born amplitude comes from the $t$-channel exchange of a higher dimensional graviton:

$$A_{\text{Born}} = -\frac{s^2}{M_{D}^{n+2}} \int \frac{d^n q_T}{t - q_T^2},$$

where the integral over momentum $q_T$ along the extra dimensions (equivalent to the sum over KK modes) gives an UV divergence. The *magic* of the eikonal amplitude is that it will be well defined despite we obtain it from an UV dependent Born amplitude: the contributions from large $q_T$ introduce corrections to the phase $\chi(s,b)$ only at small $b$ ($\approx 1/q_T$), but this small $b$ region gives a negligible contribution to $A_{\text{eik}}$ in the transplanckian regime.

From $A_{\text{eik}}$ one obtains $\chi(s,b) = (b_c/b)^n$, with

$$b_c^n = \frac{(4\pi)^{n-1}}{2} \Gamma\left(\frac{n}{2}\right) \frac{s}{M_{D}^{n+2}}.$$  

The amplitude in Eq. (1) can then be written as $A_{\text{eik}}(s,q) = 4\pi s b_c^2 F_n(b_c q)$, where

$$F_n(y) = -i \int_0^\infty dx \ x J_0(xy) \left(e^{ix-1} - 1\right),$$

$q = \sqrt{-t}$, and the integration variable is $x = b/b_c$. For $q < b_c^{-1}$ this integral is dominated by impact parameters around $b_c$, and for $q > b_c^{-1}$ by a saddle point at $b_s$. As $q$ (or $y = q^2/s$) grows nonlinear corrections (H diagrams) become important [10]. For $-t/s \approx 1$ $b_s$ approaches [5] the Schwarzschild radius $R_S$ of the system:

$$R_S = \left(\frac{2n \pi^{n+3}}{n+2} \Gamma\left(\frac{n+3}{2}\right) \frac{s}{M_{D}^{n+4}}\right)^{\frac{1}{(n+1)}}.$$ 

At $b \leq R_S$ one expects an inelastic collision, with a significant emission of gravitons, and black hole (BH) formation. The latter possibility has been considered in several analyses [3]-[8], where is it also shown that a number of factors (angular momentum, charge, geometry of the trapped surface, radiation before the collapse) make a quantitative estimate difficult. In particular, the higher curvature corrections discussed in [15] could affect the evolution of the collision after the horizon has formed, making the simple picture of single BH production and subsequent Hawking evaporation unlikely.

The differential cross section $d\sigma_{\text{eik}}/dy$ grows as $y$ decreases [5]. For example, taking $M_D = 1$ TeV and $E_\nu = 10^{10}$ GeV, for $n = 2$ (6) it is a factor of 265 (62) larger at $y = 10^{-3}$ than at $y = 0.1$. The small $y$ region corresponds to long distance processes where the neutrino interacts with a parton and transfers only a small fraction of its energy. This region is less important for a larger number of extra dimensions, since then gravity *dilutes* faster and becomes weaker at long distances. On the other hand, values of $y$ close to 1 mean
shorter distance interactions. For example, we obtain that for \( y = 0.5 \) a 52% of the \( \nu N \) eikonal cross section comes from impact parameters \( b < R_S \) for \( n = 2 \) (or a 71% for \( n = 6 \)). We will then separate two types of transplanckian (\( s > M_D^2 \) at the parton level) processes:

(i) Inelastic processes where the neutrino interacts with a parton at distances \( b \leq R_S \). The cross section for these processes, \( \sigma_{BH} = \pi R_S^2 \), would include BH formation and hard scatterings with important graviton emission where the neutrino loses most of its initial energy.

(ii) Elastic processes where the neutrino transfers to the parton a small fraction \( y \) of its energy (we take \( y_{max} = 0.2 \)), and keeps going. We use the eikonal approximation to describe these processes. They are dominated by impact parameter distances larger than \( R_S \) and thus non-linear effects and graviton emission are expected to be small.

**Air showers from cosmogenic neutrinos.** The flux of cosmogenic neutrinos depends on the production rate of primary nucleons. It will appear correlated with proton and photon fluxes that should be consistent, respectively, with the number of ultrahigh energy events at AGASA and HiRes [1] and with the diffuse \( \gamma \)-ray background measured by EGRET [16]. We will consider two neutrino fluxes described in [17]. The first one saturates the observations by EGRET, whereas for the second one the correlated flux of \( \gamma \)-rays contribute only a 20% to the data, with the nucleon flux normalized in both cases to AGASA/HiRes. The higher flux predicts 820 down-going neutrinos of each flavor with energy between \( 10^8 \) GeV and \( 10^{11} \) GeV per year and km\(^2\), versus 370 for the lower one.

AGASA and Fly’s Eye are able to detect efficiently penetrating air showers of energies above \( \approx 10^{10} \) GeV [4]. When a cosmogenic neutrino of energy \( E_\nu \gtrsim 10^{10} \) GeV enters the atmosphere it can experience the two types of processes described above. Short distance collisions may produce a BH, whose thermal evaporation would start an air shower of energy (for negligible graviton emission) up to \( 0.8 E_\nu \). On the other hand, if the neutrino suffers a long distance collision it will transfer to the parton a fraction \( y \) of energy up to \( y_{max} = 0.2 \). These processes, well described by the eikonal approximation, start hadronic showers of energy \( y E_\nu \).

We have calculated the combined number of events at AGASA and Fly’s Eye as a function of \( M_D \) (we take the exposures to penetrating showers from [4]). Within the SM one expects 0.03 (0.009 for the lower flux) deeply penetrating showers started by a neutrino, with a background of 1.72 events from hadronic showers. In these experiments 1 event passes all the cuts, which implies [4] an upper bound of 3.5 neutrino events at 95% CL. For the higher flux and \( n = 2 \) we obtain 3.5 events (2.1 BH and 1.4 elastic) if \( M_D = 1.0 \) TeV, whereas for \( n = 6 \) we have 2.6 BH plus 0.9 eikonal events if \( M_D = 1.5 \) TeV. Neglecting the short distance
collisions, for \( n = 2 \) we obtain 3.5 elastic events for \( M_D = 0.7 \) (0.9) TeV. For the lower flux, the 3.5 events limit would be obtained (from elastic processes only) for \( M_D = 0.4 \) TeV if \( n = 2 \) or for \( M_D = 0.5 \) TeV if \( n = 6 \).

**TeV gravity events at IceCube.** IceCube \(^{18}\) is a large scale \((\text{km}^3)\) neutrino telescope currently under construction in the Antarctic ice. Its center is at a depth of 1.8 km, which implies that if \( \sigma_{\nu N} \leq 0.01 \) mb neutrinos can reach it vertically with no previous interactions, whereas if \( \sigma_{\nu N} \leq 0.0001 \) mb they could also reach it horizontally after crossing 150 km of ice. The detector is sensitive to hadronic showers of energy \( E_{sh} > 500 \) TeV.

To be definite, let us consider a cosmogenic neutrino of energy \( E_\nu = 10^{10} \) GeV for \( M_D = 1 \) TeV and \( n = 2 \) (6). The probability that the neutrino survives to reach the detector from a zenith angle \( \theta_z \) is

\[
P_{\text{surv}} = \exp[-X(\theta_z) \sigma N_A],
\]

where \( X(\theta_z) \approx \rho_{\text{ice}} L(\theta_z) \) is the column density of material \((L(\theta_z) \text{ is the length of the column in ice})\) in its way to the detector, and \( \sigma = \sigma_{\text{BH}} + \sigma_{\text{SM}} \) is the inelastic cross section (we do not include \( \sigma_{\text{eik}} \) because the elastic processes with \( y < 0.2 \) introduce a negligible distortion in the energy of the neutrinos that reach the detector). For \( \cos \theta_z \approx 0.11 \) (0.45) the length \( L(\theta_z) \) is equal to its mean free path \( L = 1/\rho_{\text{ice}} \sigma N_A \), therefore the neutrino should typically reach the detector from smaller angles. Within the SM this angle would go up to \( \cos \theta_z \approx -0.03 \). Once in the detector, the probability that the neutrino experiences a short distance interaction is given by

\[
P_{\text{BH}}^{\text{int}} = 1 - \exp[-L \rho_{\text{ice}} \sigma_{\text{BH}} N_A],
\]

where \( L \approx 1 \) km is the linear dimension of the detector. We obtain values of \( P_{\text{BH}}^{\text{int}} \) from 0.06 for \( n = 2 \) to 0.2 for \( n = 6 \). On the other hand, the probability \( P_{\text{eik}}^{\text{int}} \) that the neutrino interacts elastically, loses a fraction of energy between \( y_{\text{min}} = (500 \) TeV\)/\( E_\nu = 5 \times 10^{-5} \) and \( y_{\text{max}} = 0.2 \), and starts an observable hadronic shower, can be read from the expression above just by changing \( \sigma_{\text{BH}} \) by \( d\sigma_{\text{eik}}/dy \) integrated between \( 5 \times 10^{-5} \) and 0.2. We obtain that \( P_{\text{eik}}^{\text{int}} \) goes from 0.4 for \( n = 2 \) to 0.3 for \( n = 6 \). These probabilities only change a 1% if we take \( y_{\text{max}}/2 \) or \( 2y_{\text{max}} \). Within the SM the probability that the neutrino starts a shower inside the detector is just \( P_{\text{int}}^{\text{SM}} = 0.002 \).

Given a cosmogenic neutrino flux \( \Phi_\nu \), the number \( N_{sh} \) of shower events at IceCube can be estimated as

\[
N_{sh} = \sum_i 2\pi A T \int \, d\cos \theta_z \int \, dE_\nu \frac{d \Phi_\nu}{dE_\nu} P_{\text{surv}} P_{\text{int}},
\]

where the sum goes over the three neutrino and antineutrino species, \( A \approx 1 \) km\(^2\) is the detector’s cross sectional area with respect to the \( \nu \) flux, and \( T \) is the observation time. For
Figure 1: Number of (eikonal and SM) shower events per year at IceCube for the two cosmogenic neutrino fluxes.

the higher (lower) flux, within the SM we obtain 1.4 (0.5) hadronic or electromagnetic events (muons and taus do not shower) per year above 500 TeV.

We now consider values of $M_D$ above the bounds obtained from the absence of penetrating air showers and calculate the number of events per year at IceCube. Taking the higher flux, for $n = 2$ (6) we obtain a maximum of 118 (34) elastic events versus just 20 (24) short distance events. The elastic events correspond to soft processes with $y < 0.2$. In particular, for $n = 2$, 95 of the 118 showers have an energy below $10^8$ GeV, versus just 0.08 of the 20 showers from short distance interactions. For $n = 6$, 18 of the 34 eikonal events are below $10^8$ GeV.

In Fig. 1 we plot the number of elastic events per year as a function of $M_D$ for the two cosmogenic fluxes. We find that IceCube could detect TeV gravity effects above the SM background for $M_D$ up to approximately 5 TeV. The characteristic signature would be always a hadronic shower, as charged leptons are never produced in the gravitational interaction starting the shower.

**Discussion.** Cosmogenic neutrinos interact with the terrestrial nucleons at center of mass energies $\sqrt{2m_N E_\nu} \approx 10^5$ GeV, so they could be used as probes of new TeV physics. In particular, the possibility of BH formation in models with extra dimensions has been entertained by several groups. These analyses are based on a geometric cross section that
assumes single BH production whenever the neutrino and the parton interact at impact parameters smaller than $R_S$.

The problem with this estimate is that, despite the large energy of cosmogenic neutrinos, the $\nu N$ cross section is dominated by the small $x$ region, and most of the BHs produced in penetrating air showers or neutrino telescopes would be very light, with masses just above $M_D$. These light BHs would be very sensitive to effects like graviton emission during the collapse, higher curvature corrections, or non-thermal effects in the evaporation, which add uncertainty to the estimate.

In this paper we have analyzed a different type of signal. It is produced when the neutrino interacts elastically with the parton at typical distances larger than $R_S$ and transfers a small fraction $y$ of its energy. The process is properly described by the eikonal approximation, and its distinct experimental signature would be a hadronic shower of energy $y E_\nu$. Electromagnetic showers would never be produced in the initial $\nu N$ interaction.

We have computed the number of penetrating air showers from soft ($y \leq 0.2$) elastic processes and from short distance ($b < R_S$) processes, and have found that the expected number of events at AGASA and Fly’s Eye from these two types of processes is similar. We obtain bounds on $M_D$ that are below the ones from SN1987A for $n = 2$, but are similar to the limits from collider experiments for any value of $n$.

In contrast, we have shown that these elastic interactions provide a clear and model-independent signal of TeV gravity that would dominate over BH production in neutrino telescopes. The reason is that telescopes are sensitive to showers of energies up to four orders of magnitude below the average energy of cosmogenic neutrinos. The excess of hadronic showers at IceCube could be observed for $M_D$ up to 5 TeV.

We would like to thank Eduardo Battaner and Roberto Emparan for valuable conversations. This work has been supported by MCYT (FPA2003-09298-C02-01) and Junta de Andalucía (FQM-101). J.I.I. and D.M. (M.M.) acknowledge financial support from the European Community’s Human Potential Programme HPRN-CT-2000-00149 (HPRN-CT-2000-00152).

**References**

[1] L. Anchordoqui, T. Paul, S. Reucroft and J. Swain, Int. J. Mod. Phys. A 18 (2003) 2229.
[2] N. Arkani-Hamed, S. Dimopoulos and G. Dvali, Phys. Lett. B 429 (1998) 263; I. Antoniadis, N. Arkani-Hamed, S. Dimopoulos and G. Dvali, Phys. Lett. B 436 (1998) 257.

[3] P. C. Argyres, S. Dimopoulos and J. March-Russell, Phys. Lett. B 441 (1998) 96; R. Emparan, G. T. Horowitz and R. C. Myers, Phys. Rev. Lett. 85 (2000) 499; D. M. Eardley and S. B. Giddings, Phys. Rev. D 66 (2002) 044011. S. Dimopoulos and G. Landsberg, Phys. Rev. Lett. 87 (2001) 161602; S. B. Giddings and S. Thomas, Phys. Rev. D 65 (2002) 056010.

[4] J. L. Feng and A. D. Shapere, Phys. Rev. Lett. 88 (2002) 021303; L. A. Anchordoqui, J. L. Feng, H. Goldberg and A. D. Shapere, Phys. Rev. D 65 (2002) 124027; Phys. Rev. D 66 (2002) 103002; Phys. Rev. D 68 (2003) 104025.

[5] R. Emparan, M. Masip and R. Rattazzi, Phys. Rev. D 65 (2002) 064023; M. Masip, [arXiv:hep-ph/0210143](http://arxiv.org/abs/hep-ph/0210143)

[6] A. Ringwald and H. Tu, Phys. Lett. B 525 (2002) 135; M. Kowalski, A. Ringwald and H. Tu, Phys. Lett. B 529 (2002) 1; S. I. Dutta, M. H. Reno and I. Sarcevic, Phys. Rev. D 66 (2002) 033002.

[7] J. Álvarez-Muniz, J. L. Feng, F. Halzen, T. Han and D. Hooper, Phys. Rev. D 65 (2002) 124015.

[8] E. J. Ahn, M. Cavaglia and A. V. Olinto, [arXiv:hep-ph/0312249](http://arxiv.org/abs/hep-ph/0312249)

[9] G. ’t Hooft, Phys. Lett. B 198 (1987) 61; I. J. Muzinich and M. Soldate, Phys. Rev. D 37 (1988) 359; D. Amati, M. Ciafaloni and G. Veneziano, Phys. Lett. B 197 (1987) 81; D. Kabat and M. Ortiz, Nucl. Phys. B 388 (1992) 570.

[10] G. F. Giudice, R. Rattazzi and J. D. Wells, Nucl. Phys. B 630 (2002) 293.

[11] E. A. Mirabelli, M. Perelstein and M. E. Peskin, Phys. Rev. Lett. 82 (1999) 2236; G. F. Giudice, R. Rattazzi and J. D. Wells, Nucl. Phys. B 544 (1999) 3; for a review, see F. Feruglio, [arXiv:hep-ph/0401033](http://arxiv.org/abs/hep-ph/0401033)

[12] S. Cullen and M. Perelstein, Phys. Rev. Lett. 83 (1999) 268.

[13] S. Hannestad and G. G. Raffelt, Phys. Rev. D 67 (2003) 125008 [Erratum-ibid. D 69 (2004) 029901].
[14] S. Cullen, M. Perelstein and M. E. Peskin, Phys. Rev. D 62 (2000) 055012; F. Cornet, J. I. Illana and M. Masip, Phys. Rev. Lett. 86 (2001) 4235.

[15] V. S. Rychkov, arXiv:hep-ph/0401116

[16] P. Sreekumar et al., Astrophys. J. 494 (1998) 523.

[17] D. V. Semikoz and G. Sigl, JCAP 0404 (2004) 003.

[18] J. Ahrens [IceCube Collaboration], arXiv:astro-ph/0305196; see also http://icecube.wis.edu/