Measuring mode indices of a partially coherent vortex beam with HBT type experiment

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It is known that the cross-correlation function (CCF) of a partially coherent vortex (PCV) beam shows a robust link with the radial and azimuthal mode indices. However, the previous proposals are difficult to measure the CCF in practical system, especially in the case of astronomical objects. In this letter, we demonstrate experimentally that the Hanbury Brown and Twiss effect can be used to measure the mode indices of the original vortex beam and investigate the relationship between the spatial coherent width and the characterization of CCF of a PCV beam. The technique we exploit is quite efficient and robust, and it may be useful in the field of free space communication and astronomy which are related to the photon’s orbital angular momentum.

In 1992, L. Allen et al.¹ pointed out that beam with spiral phase distribution of \( \exp(il\varphi) \) carries an orbital angular momentum (OAM) of \( l\hbar \), where \( l \) is an integer which denotes the azimuthal mode index (topological charge), and \( \varphi \) is the azimuthal coordinate. Beam with this kind of phase structure is also called vortex beam. Vortex beam has been widely studied in the last two decades and found a lot of applications, such as optical tweezers², spiral phase contrast microscopy³. Since the values of \( l \) is theoretically unlimited, OAM states construct an infinite dimensional Hilbert space, and it exhibits great potential for applications in the field of quantum information process⁴,⁵, free-space information transfer and communications⁶–⁸. Despite the extensive applications, determining the topological charge \( l \) of OAM state remains an intriguing problem, and a lot of methods has been proposed. Such as Mach-Zehnder interferometer⁹, diffraction pattern with specific masks¹⁰–¹³, image reformatting¹⁴, intensity analysis¹⁵,¹⁶.

Recently, more and more researches have focused on the partially coherent vortex (PCV) beam¹⁷–³¹. In 2004, Palacios et al. firstly verified that a robust phase singularity exists in the spatial coherence function when a vortex is presented in the original beam. Meanwhile, in Refs.²¹–²³, the authors revealed the linkage between the mode indices and the cross correlation function (CCF) of a PCV beam, and they also discussed the spatial coherence on determining the mode indices of a PCV beam.

Also, there has been interest in the use of spatial modes, such beams carrying OAM, as an additional degree of freedom to increase the available information bandwidth for free-space communication. Because a large Hilbert space is helpful to improve the security of cryptographic keys transmitted with a quantum key distribution system.³³. Since the important application of free space communication, the effects of propagation through random aberrations (atmospheric turbulence) on coherence for single-photon communication systems based on orbital angular momentum states are also investigated³⁴,³⁵. However, determining the mode indices of a PCV beam is still difficult. In Refs.¹⁷,²³, the authors experimentally used a wavefront folding interferometer to measure the mode indices of a PCV beam. When the size of PCV beam is large compared to the measuring instrument, their scheme is difficult to be used to measure the cross-correlation function. Meanwhile, the interference pattern in their results is complex and the visibility is low, which is difficult to characterize the mode indices from the experimental results. In this letter, we experimentally prove that the mode indices of a PCV beam can be measured efficiently through a Hanbury Brown and Twiss (HBT) type experiment which was firstly used in the field of astronomy³⁷,³⁸.

An incoherent or partially coherent source both tend to emit photons together (bunching) with enhanced photon number fluctuations relative to classical expectations (super-Poisson statistics). This kind of bunching effect was firstly measured in the experiment of observing the second-order temporal and spatial intensity correlations of star light by Hanbury Brown and Twiss³⁷,³⁸. It was believed that the nontrivial HBT correlation is caused by the measured intensity fluctuations of the thermal light. In the HBT experiment, the second-order correlation function is expressed as

\[
G^{(2)}(r_1, r_2) = \langle I(r_1)I(r_2) \rangle = \langle I(r_1) \rangle \langle I(r_2) \rangle + |\Gamma(r_1, r_2)|^2, \tag{1}
\]

where \( \langle ... \rangle \) denotes the ensemble average, \( I(r_1) \) and \( I(r_2) \) are the intensity at spatial point \( r_1 \) and \( r_2 \), respectively. \( \Gamma(r_1, r_2) \) is the mutual coherence function (MCF) of the light source. Since the second-order correlation function in Eq. (1) can be easily obtained by the coincidence measurement of two detectors, we may get the mode indices of PCV beam from the measured magnitude square of MCF, \( |\Gamma(r_1, r_2)|^2 \).

Laguerre-Gaussian (LG) modes are circularly symmetric and structurally stable solutions of the paraxial wave...
which are imprint on the of the generated beam cor-
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(b) and (c) display the example computer-
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to the value of azimuthal mode index \( l \) of the LG mode.
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Since the direct measurement of the CCF is troublesome,
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will discuss and measure the \( |\Gamma'_{c}(\vec{r}', -\vec{r}')|^2 \)
in the following. As shown in Fig. 1, the orange-solid lines rep-
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represent the magnitude square of the CCF, \( |\Gamma_{c}(\vec{r}', -\vec{r}')|^2 \),
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which have the same number of dislocation rings as the
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original far field CCF. It means that the HBT effect of
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the partially coherent beam could be used to reveal the
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features of the CCF indirectly.

A partially coherent LG beam can be generated by
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propagating the coherent LG beam through a rotating
ground glass disk (GGD). It is well known that
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the statistical distribution\(^{39}\) of the generated beam corre-
sponds to a Gaussian-Schell correlator
\( C(\vec{r}_1, \vec{r}_2) = \exp[-|\vec{r}_1 - \vec{r}_2|^2/\sigma_g^2] \),
where \( \sigma_g \) represents the transverse spatial coherence width.
Combining Eq. (2) with
\( C(\vec{r}_1, \vec{r}_2) \), the MCF of the partially coherent LG at source
plane \( z = 0 \) can be expressed as\(^{33}\)
\[
\Gamma(\vec{r}_1', \vec{r}_2') \propto C(\vec{r}_1', \vec{r}_2')(\frac{\vec{r}_1' \cdot \vec{r}_2'}{w_0^2})^l L_p(|\vec{r}_1'|/w_0) L_p(|\vec{r}_2'|/w_0) \\
\times \exp\left(-\frac{r_1'^2}{w_0^2} + i\lambda \vec{r}_1' \cdot \vec{r}_2' - \frac{k}{z} \cdot \vec{r}_1' - \vec{r}_2' \cdot \vec{r}_2'\right),
\]
After propagating a distance \( z \) in the far field, the MCF

\( \Gamma'_{c}(\vec{r}', -\vec{r}') \), shows a closed relationship with the
topological charge of the original vortex beam, and it has
been proved that the CCF will maintain the dislocation
rings. The blue-dash line in Figs. 1 (a)-(c) show some
simulation results of the far field one dimensional CCF
for mode indices \( p = 0, l = 1, 2, 3 \) LG beams. In the sim-
ulation, the waist width to spatial coherence width ratio
is set as \( w_0/\sigma_g = 0.5 \). It is straightforward to find that
the number of dislocation rings in the \( \Gamma_{c}(\vec{r}', -\vec{r}') \) is equal

\( \frac{1}{\sqrt{2\pi z^2}} \int \int \int \int d\vec{r}_1 d\vec{r}_2 \Gamma(\vec{r}_1', \vec{r}_2') \\
\times \exp\left[-i\frac{k}{z} (\vec{r}_1' \cdot \vec{r}_1 - \vec{r}_2' \cdot \vec{r}_2)\right],
\]
where \( k \) is the wave number. As we known, the far field
CCF, \( \Gamma_{c}(\vec{r}', -\vec{r}') \), shows a closed relationship with the
topological charge of the original vortex beam, and it has
been proved that the CCF will maintain the dislocation
rings. The blue-dash line in Figs. 1 (a)-(c) show some
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equation. The electromagnetic field amplitude of LG
 modes in \( z = 0 \) is given by
\[
u_p(r, \varphi, 0) \propto (\frac{r}{w_0})^l L_p(\frac{2r^2}{w_0^2}) e^{-r^2/w_0^2} e^{i\varphi},
\]
where \( r \) and \( \varphi \) are the radial and azimuthal coordinates,
respectively. \( p \) is the radial mode index, and \( l \) is the az-
imuthal mode index which describes the phase structure.
\( w_0 \) denotes the waist width and \( L_p(\cdots) \) is the generalised
Laguerre polynomial.
is a 1040 × 1392 array of 6.45 × 6.45 µm² pixels, and the measurement is made with an exposure time of 0.2 ms to guarantee that the acquired images are temporally incoherent. The field on the plane of the CCD can be regarded as the partially coherent LG beam. Instead of using two point detectors for the coincidence measurement of the second-order correlation function in Eq. (1), a CCD is used here to recorded the intensity pattern and then analyse the second-order correlation on computer\textsuperscript{41}.

![Image](https://via.placeholder.com/150)

**FIG. 3.** Magnitude square of cross correlation function for the far-field partially coherent Laguerre-Gaussian \((p = 0, l = 1, 2, 3)\) modes. (a)-(c) and (d)-(f) are simulated and experimental results, respectively. The results correspond to waist width \(w_0 = 0.5\sigma_g\).

To acquire the \(|\Gamma_c'(\vec{r}', -\vec{r}'')|^2\) of the partially coherent LG beam with the setup in Fig. 2 (a), we record \(N = 5000\) frames of images in each experiment. According to definition of intensity correlation in Eq. (1), the recorded data for the magnitude square of CCF are processed as follows

\[
|\Gamma_c'(\vec{r}', -\vec{r}'')|^2 = \frac{1}{N} \sum_{i=1}^{N} I_i(\vec{r}')I_i(-\vec{r}'') - \frac{1}{N^2} \sum_{i=1}^{N} I_i(\vec{r}') \sum_{i=1}^{N} I_i(-\vec{r}''), \tag{5}
\]

where \(I_i(\vec{r})\) is the \(i\)th frame of intensity distribution on the CCD plane and \(\vec{r}'\) denotes the pixel coordinate of the CCD. Before recording the data for \(|\Gamma_c'(\vec{r}', -\vec{r}'')|^2\), the CCD is placed beforehand on the plane of the SLM in Fig. 2 (a) to measure spatial coherent width \(\sigma_g\).

**FIG. 4.** Magnitude square of the cross correlation function for the partially coherent Laguerre-Gaussian \((p = 0, l = 1)\) mode with different waist width to spatial coherence width ratio. (a)-(d) and (e)-(h) are corresponding to simulated and experimental results, respectively.

The experimental patterns agree well with the theoretical distributions. From Fig. 4 we can see that the bright spot in the center of the \(|\Gamma_c'(\vec{r}', -\vec{r}'')|^2\) will gradually diminish in radius as the increase of the spatial coherent width, and eventually vanish in the situation of a coherent LG beam. On the contrary, the outside rings also will be too blurry to be invisible as the spatial coherence of the beam become quite low. The high and low coherence cases are shown in Figs. 4 (c) and (h), respectively. In both of these two situations, it might be difficult to count the number of dislocation rings for low contrast ratio.

![Image](https://via.placeholder.com/150)

**FIG. 5.** Experimental results of magnitude square of the cross correlation function for the higher-order partially coherent Laguerre-Gaussian modes. Their waist width to spatial coherence width ratios are \(w_0/\sigma_g = 0.5\). To show the picture clearly, we reduce the contrast of the picture by setting the values of \(|\Gamma_c'(\vec{r}', -\vec{r}'')|^2\) to 0.3 times the maximum \(|\Gamma_c'(\vec{r}', -\vec{r}'')|^2\) value.

\[|\Gamma_c'(\vec{r}', -\vec{r}'')|^2 > 0.3 \times \max\{|\Gamma_c'(\vec{r}', -\vec{r}'')|^2\}.\]

In Ref.\textsuperscript{23}, the authors investigated the linkage between the number of dislocation rings \(\Omega\) in the far-field cross...
correlation function and the mode indices of a partially coherent LG beam, and they showed that $\Omega = 2p + |l|$. Their experiments demonstrated that the spatial correlation singularity still exist even for a non-vortex ($l = 0$) partially coherent beam if the radial index $p$ is non-zero. Likewise, we experimentally prove the relationship $\Omega = 2p + |l|$ for some higher-order partially coherent LG beams with waist width to spatial coherence width ratios to be $w_0/\sigma_p = 0.5$, and the results are shown in Fig. 5 which have high visibility. Figs. (a) and (d) show the $|\Gamma'_c(\vec{r}', -\vec{r})|^2$ of non-vortex partially coherent LG beam with radial index $p = 1$ and $p = 2$, respectively. It is obvious that the spatial correlation singularity still exists and the number of dislocation rings is $2p$. We also measured the $|\Gamma'_c(\vec{r}', -\vec{r})|^2$ of higher-order partially coherent LG modes which are shown in Figs. (b), (e), (f). A desirable results in the experiment are achieved, and it is easy to count the number of dislocation rings as $\Omega = 2p + |l|$.

In conclusion, we experimentally show the relationship between the number of dislocation rings of the magnitude square of the CCF and the radial and azimuthal mode indices ($p$, $l$) of a PCV beam through a HBT type experiment, and our results prove that the number of dislocation rings is identical to $2p + |l|$. We also investigate the effect of spatial coherent width on determining the mode indices of the PCV beams. Our method offers a powerful tool to investigate the character of the MCF of a PCV beam and determine the mode indices of the PCV beam, which is efficient and robust in practical measurement system because only the HBT type setup is used to get the second-order intensity correlation. It has potential applications in the field of free space communication and astronomical research with the photon’s OAM degree of freedom, especially in the situation with non-ignorable effect of atmospheric turbulence scattering medium.

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