DYNAMICAL NATURE OF THE NUCLEAR PSEUDOSPIN 
AND ITS ISOSPIN ASYMMETRY

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Pseudospin symmetry in nuclei is investigated by solving the Dirac equation with Woods-Saxon scalar and vector radial potentials. We relate the pseudospin interaction with a pseudospin-orbit term in a Schrödinger-like equation for the lower component of the Dirac spinor. We show that this term gives a large contribution to the energy splittings of pseudospin partners, so that the near pseudospin degeneracy arises from a significant cancellation among the different terms in that equation. This is a manifestation of the dynamical character of this symmetry in the nucleus. We analyze the isospin dependence of the pseudospin symmetry in a nuclear isotope chain by including a vector-isovector potential \( V_\rho \) and a Coulomb potential and conclude that \( V_\rho \) gives the main contribution to the observed pseudospin isospin asymmetry.

1 Introduction

Pseudospin symmetry was introduced in the late 60’s to account for a degeneracy in single-particle nucleon levels of certain nuclei, with quantum numbers \((n, \ell, j = \ell + 1/2)\) and \((n - 1, \ell + 2, j = \ell + 3/2)\). These levels have the same quantum number \( \bar{\ell} = \ell + 1 \), called “pseudo” orbital angular momentum, and a “pseudo” spin quantum number, \( \bar{s} = 1/2 \). In general, for a \( n\ell j \) state, we have \( \bar{\ell} = \ell \pm 1 \) if \( j = \ell \pm 1/2 \). Pseudospin symmetry is exact when doublets with \( j = \bar{\ell} \pm \bar{s} \) are degenerate. Since then the origin of pseudospin symmetry in nuclei has been a subject of intense theoretical research.

Recently, Ginocchio uncovered the relativistic character of the symmetry, noting that the pseudo-orbital angular momentum is the orbital angular momentum of the lower component of the Dirac spinor. This is an exact SU(2) symmetry for the Dirac Hamiltonian with an attractive scalar potential, \( S \), and a repulsive vector potential \( V \), when they have the same magnitude, i.e., \( \Sigma = S+V = 0 \). This would explain why there are quasi-degenerate pseudospin doublets in nuclei, since in relativistic mean field (RMF) models nuclear sat-
uration arises from an extensive cancellation between a large attractive scalar potential and a large repulsive vector potential. However, since $\Sigma$ acts as a binding potential, one cannot set $\Sigma = 0$ in the nucleus, because then there would not exist any bound states. Other authors have studied the role of pseudospin-orbit coupling in pseudospin symmetry.

2 Dynamical pseudospin symmetry and isospin asymmetry

Pseudospin symmetry is related to an invariance under a SU(2) transformation of the Dirac Hamiltonian with central scalar $S$ and vector $V$ potentials

$$H = \alpha \cdot p + \beta(m + S) + V,$$

whose generators are $S_i = s_i \frac{1}{2}(1 - \beta) + \tilde{s}_i \frac{1}{2}(1 + \beta)$ where $\tilde{s}_i = \frac{\sigma \cdot p}{p} s_i$, $s_i = \sigma_i / 2$. The commutator of $S_i$ with (1) is zero if $\Sigma = 0$ or $\frac{d \Sigma}{dr} = 0$.

The Dirac equation $H \Psi = \epsilon \Psi$ can be written as a pair of second-order differential equations for the upper and lower components. Setting $\Delta = V - S$, $V$ and $S$ being radial functions, and defining $\epsilon = E + m$, we have

$$p^2 \psi_+ = \frac{\Delta'}{E + 2m - \Delta} \left( \frac{\partial}{\partial r} - \frac{1}{r} \sigma \cdot L \right) \psi_+ + (E + 2m - \Delta)(E - \Sigma) \psi_+,$$  
$$p^2 \psi_- = \frac{\Sigma'}{E - \Sigma} \left( \frac{\partial}{\partial r} - \frac{1}{r} \sigma \cdot L \right) \psi_- + (E + 2m - \Delta)(E - \Sigma) \psi_-,$$

where $\psi_\pm = [(1 \pm \beta)/2] \psi$ are the upper and lower components and the primes denote derivatives with respect to $r$. The $\sigma \cdot L$ term in (3) is the pseudospin-orbit term.

We solve the Dirac equation with the Hamiltonian (1) with scalar and vector potentials with Woods-Saxon shape: $U(r) = U_0 / (1 + \exp[(r - R)/a])$. We fitted the parameters of this potential to the neutron spectra of $^{208}$Pb, obtaining the values $R = 7$ fm, $\Delta_0 = 650$ MeV, $\Sigma_0 = -66$ MeV and $a = 0.6$ fm. We then varied the diffuseness, radius and $\Sigma_0$ separately, keeping all the other parameters fixed and showed that the splittings of the pseudospin doublets vary notably, namely they decrease as $|\Sigma_0|$ decreases and diffuseness increases, sometimes reversing sign. The details can be found in ref. 7.

The effect of the pseudospin-orbit term on pseudospin energy splittings can be assessed by writing (3) as a Schroedinger-like equation, dividing it by an energy- and $r$-dependent effective mass $m^* = (E + 2m - \Delta)/2$ and taking the expectation value of each term relative to $\psi_-:

$$\left\langle \frac{p^2}{2m^*} \right\rangle + \langle V_{PSO} \rangle + \langle V_D \rangle + \langle \Sigma \rangle = E,$$
where

\[
\langle \frac{p^2}{2m^*} \rangle = \int \Psi_+^\dagger \frac{p^2}{2m^*} \Psi_- \, d^3r / \int \Psi_+^\dagger \Psi_- \, d^3r , \tag{5}
\]

\[
\langle V_{\text{PSO}} \rangle = P \int \Psi_+^\dagger \frac{1}{2m^*} \sum' \frac{1}{E - \Sigma} \sigma \cdot L \Psi_- \, d^3r / \int \Psi_+^\dagger \Psi_- \, d^3r , \tag{6}
\]

\[
\langle V_D \rangle = -P \int \Psi_+^\dagger \frac{1}{2m^*} \sum' \frac{\partial \Psi_-}{\partial r} \, d^3r / \int \Psi_+^\dagger \Psi_- \, d^3r , \tag{7}
\]

\[
\langle \Sigma \rangle = \int \Psi_+^\dagger \Sigma \Psi_- \, d^3r / \int \Psi_+^\dagger \Psi_- \, d^3r , \tag{8}
\]

where ‘P’ denotes the principal value of the integral. These terms can be identified, respectively, as a kinetic term, a pseudospin-orbit term, a potential term related to what is sometimes called Darwin term and the mean value of the Σ potential with respect to the lower component, Ψ_−.

Figure 1. Differences for the energy terms in Eq. (4) for (1i\textsubscript{11/2}, 2g\textsubscript{9/2}), (2f\textsubscript{5/2}, 3p\textsubscript{3/2}) and (1h\textsubscript{9/2}, 2f\textsubscript{7/2}) pseudospin partners. Figure 2. Effect of Coulomb and ρ potentials V\textsubscript{c} and V\textsubscript{ρ} in the splitting of the (2d\textsubscript{5/2}, 1g\textsubscript{7/2}) pseudospin partners in a Sn isotopic chain.

In Fig. 1 are plotted the differences of the terms in Eq. (4) between each member of the three topmost pseudospin partners in \(^{208}\text{Pb}\), (1i\textsubscript{11/2}, 2g\textsubscript{9/2}), (2f\textsubscript{5/2}, 3p\textsubscript{3/2}) and (1h\textsubscript{9/2}, 2f\textsubscript{7/2}). One sees that the contribution of \(V_{\text{PSO}}\) for the pseudospin energy splittings is larger than the splittings themselves and has the opposite sign of the kinetic and \(\langle \Sigma \rangle\) terms. For all doublets there is a significant cancellation of \(\langle V_{\text{PSO}} \rangle\) with the kinetic and \(\langle \Sigma \rangle\) terms. A similar analysis was made in ref. \(^\text{6}\), concluding that the pseudospin-orbit potential is non-perturbative. We also found that there is a clear correlation between \(\langle V_{\text{PSO}} \rangle\) and the pseudospin energy splitting when the diffusivity and the depth \(\Sigma_0\) are varied \(^\text{7}\).
All these findings point to the conclusion that pseudospin symmetry has a dynamical nature, since it is very much dependent on the shape of the $\Sigma$ nuclear mean field potential and results from a strong cancellation among the several contributing terms to the energy splittings.

This pseudospin dependence on the shape of the $\Sigma$ potential allows us to explain why the proton and neutron levels have different pseudospin splittings.

In RMF models, the $\rho$ meson interaction modifies the vector potential to $V = V_\omega + V_\rho = V_\omega \pm \frac{\rho}{\rho_0}$, where the plus sign refers to protons and the minus sign refers to neutrons, and $\rho_0$ is the time component of the $\rho$ field which is proportional to $Z - N$. The magnitude of the $\rho$ potential is around 4-8 MeV. For heavy nuclei, $\rho_0 < 0$ and therefore $V_p < V_n$ which implies $|\Sigma_n| < |\Sigma_p|$. This means that, for neutron-rich nuclei, the pseudospin symmetry is favored for neutrons. Notice, however, that the Coulomb potential, which has the opposite sign of $V_\rho$ for protons, also affects proton spectra. Both of these effects can be seen in Figure 2, where is plotted the effect of $V_\rho$ and the Coulomb potential on the energy splitting of the $(2d_{5/2}, 1g_{7/2})$ pseudospin partners along the Sn isotopic chain from $A = 130$ to $A = 170$. We see that the potential $V_\rho$ gives the main contribution for the observed isospin asymmetry.

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