Probing Neutrino Hierarchy and Chirality via Wakes

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The relic neutrinos are expected to acquire a bulk relative velocity with respect to the dark matter at low redshifts, and downstream of dark matter halos neutrino wakes are expected to develop. We propose a method of measuring the neutrino mass based on this mechanism. The neutrino wake will cause a dipole distortion of the galaxy-galaxy lensing pattern. This effect could be detected by combining upcoming lensing surveys, e.g. the LSST and Euclid surveys with a low redshift galaxy survey or a 21cm intensity mapping survey which can map the neutrino flow field. The data obtained with LSST and Euclid should enable us to make positive detection if the three neutrino masses are Quasi-Degenerate, and a future high precision 21cm lensing survey would allow the normal hierarchy and inverted hierarchy cases to be distinguished, and even the right handed Dirac neutrinos may be detectable.

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Introduction.—The squared mass differences of the three neutrino species have been measured from neutrino oscillation experiments, but the individual masses are still unknown. Based on the mass square differences, the neutrino masses may form the normal hierarchy \((m_1 \sim m_2 \ll m_3)\), inverted hierarchy \((m_3 \ll m_1 \sim m_2)\), or quasi-degenerate \((m_1 \sim m_2 \sim m_3)\) (c.f. §14 of [1]). Determining the hierarchy is very important in modern physics [2]. The suppression of matter power spectrum on small scale provides a way to measure the sum of the neutrino masses [2, 4], complementary to particle physics experiments, and in some cases the combination of these two approach may allow the determination of the mass hierarchy [3].

Recently, it was shown that a bulk relative velocity field between neutrinos and cold dark matter (CDM) exists, with coherent flows over several Mpc [6]. As neutrinos flow over dark matter halos, they are gravitationally focused into a wake. This downstream excess could then be observed through gravitational lensing. This wake is unique to neutrinos as the CDM-baryon relative velocity is far too small to mimic the effect. The full three-dimensional relative velocity field can be computed from the galaxy density field using linear perturbation theory with good accuracy. By exploiting the asymmetry of relative velocity, the neutrino wakes can be isolated and used to determine the neutrino masses.

Besides the neutrino masses, the very nature of the neutrinos, i.e. whether they are Majorana or Dirac particles, are still unknown. If neutrinos are Majorana particles, the lepton number conservation law is broken, and this may induce neutrinoless \(\beta\)-decay, but so far such decays have not been detected despite of many search efforts [1]. This question may be answered if the Big Bang relic neutrinos can be detected with tritium capture experiment [8], for the total capture rate in the Majorana case could be twice as large as that of the Dirac case [3]. However, detecting the relic neutrinos is not easy, and furthermore, this effect is confounded by the fact that the local overdensity of neutrinos due to gravitational clustering is unknown. Here we note that for Dirac neutrinos there are right-handed neutrinos \((\nu_R)\) distinct from the left handed ones \((\nu_L)\), and they could be produced during the Big Bang. In many beyond-the-Standard-Model theories, the left-right symmetry is restored at high energy, then in the very early universe there would be a thermalized \(\nu_R\) background in addition to the \(\nu_L\) one. The primordial \(\nu_R\) s would decouple before the \(\nu_L\) , and evolve as a separate relic background. At later time, these neutrinos would propagate in mass eigenstates, with both right and left (re-generated from Yukawa coupling) components, but their temperature and number density would be distinct from the primordial \(\nu_L\) background which decoupled much later. Since the relative velocity with respect to CDM depends on the initial velocity dispersion [6], the relative velocity field of the \(\nu_R\) and the resulting wakes would also be different, thus enabling a new way to answer this important question.¹

In this Letter we discuss how these neutrino wakes are produced by non-linear CDM halos and compute the expected size and signal of the wakes, and investigate the

¹ In the see-saw models of neutrino mass [11], the light \(\nu_L\) are primarily Majorana particles. There are also right handed neutrinos in such models, but they are very heavy and should have decayed in the early Universe.
observability with upcoming surveys. Moreover, we also delineate the evolution of the $\nu_R$ for the case of Dirac neutrinos, and consider its detection with this method.

**Neutrino wakes and Lensing Signal.**—At high redshift individual neutrinos move at large velocities, but the bulk flow of neutrinos with respect to the CDM is small. Because of their large velocities, neutrinos traverse large distances and experience the inhomogeneous gravitational potentials at different places, as a result a bulk relative velocity field between the neutrinos and dark matter particles is induced. The relative velocity field can be computed from the primordial density perturbations and its variance is given by

$$\langle \nu_{vc}^2 \rangle = \int \frac{dk}{k} \Delta^2(k) \left[ \frac{\theta_{\nu}(k, z) - \theta_{c}(k, z)}{k} \right]^2,$$

where $\theta \equiv \nabla \cdot v$ is the velocity divergence transfer function, and $\Delta^2(k)$ is the primordial curvature perturbation spectrum [3]. This variance was computed for $\nu_L$ in [6] where it was found to be comparable with the thermal relative velocity at late times. The $\nu_L$ relative velocity power spectrum is shown in Fig. 1 for the case of a single 0.05 eV neutrino. The relative velocity fields are well correlated at large scales [3]. The coherency scale, which is defined here as the distance where the correlation function drops to half its maximum value, is $R = 14.5$ Mpc/$h$ for $\nu_L$ at $z = 0.3$. We take $\sqrt{(\nu_{L,c})} = 418$ km/s as the typical relative velocity. The one dimensional neutrino velocity dispersion is $\sigma(z) = 2.077 T_{\nu}(z)/m_\nu \simeq 2716$ km/s for $\nu_L$.

We now compute the neutrino wake induced by a dark matter halo with a given mass $M$ via linear response theory. We approximate the neutrinos’ distribution as Maxwell-Boltzmann, which is sufficiently accurate for the relevant densities and temperatures. Assuming neutrinos flow over the dark matter halo coherently with velocity $v_{vc} = \sqrt{\nu L / m_\nu}$, the density contrast at point $r$ is given by [11]:

$$\delta_r = \frac{r_B}{r} \exp \left( - \frac{v_{vc}^2 \sin^2 \theta}{2\sigma^2} \right) \left[ 1 + \text{erf} \left( \frac{v_{vc} \cos \theta}{\sqrt{2}\sigma} \right) \right],$$

where $r_B = GM/\sigma^2$ is the Bondi radius, $\theta$ is the angle between the relative velocity and $r$ and $X = v_{vc}/(\sqrt{2}\sigma)$. The accuracy of the approximate form in the second line is better than 95% for the cases we consider.

With Eq. (2) in hand we can now compute the expected lensing signal from this wake. The geometry is shown in Fig. 2. Taking the $z$-axis as the direction of our line of sight (l.o.s.) and the relative velocity $v_{vc}$ to lie in the $x - z$ plane at angle $\theta$ from the $z$-axis, then the induced density contrast at any point $r \equiv (x, y, z)$ can be determined from Eq. (2) via coordinate transformations.

Here we also define the polar coordinates $(\varpi, \phi)$ on the lens $(x - y)$ plane, $x = \varpi \cos \phi, y = \varpi \sin \phi$, for later use.

The perturbed surface density is obtained by integration along the l.o.s., $\Sigma_{\nu}(x, y) = \rho_0 \int^{L/2}_{-L/2} dz \delta_{\nu}(x, y, z)$, where $\rho_0$ is the unperturbed neutrino mass density and $L$ is the effective coherent scale of the neutrino-CDM relative flow field, defined as $(4/3)\pi R^3 = L^3$. We find $L = 23.4$ Mpc/$h$ for $\nu_L$ at $z = 0.3$ for $m_\nu = 0.05$eV. Using
Eq. (2), we obtain
\[
\frac{\Sigma_\nu}{\rho_\nu} = (1 - X^2 \sin^2 \theta) \ln \left[ \frac{1 + \sin \eta}{1 - \sin \eta} \right] + \frac{4 \eta X}{\sqrt{\pi}} \sin \theta \cos \phi \\
+ X^2 (3 \sin^2 \theta - 2) \sin \eta - X^2 (\cos^2 \theta - 1) \sin \eta \cos 2\phi,
\]
where \( \eta = \arctan(L/2\pi) \).

The contribution to the weak lensing convergence is given by \( \kappa(\varpi, \phi) = \frac{\Sigma_\nu(\varpi, \phi)}{\Sigma_{\text{crit}}}, \) where \( \Sigma_{\text{crit}} = \frac{c^2}{4 \pi G D_d^2 D_s^2} \) is the critical surface density, \( z_d \) is the redshift of the halo, and \( D_s, D_d, D_{ds} \) are the comoving distances between the observer and source, the observer and lens, and the lens and source respectively. As our fiducial model, we assume the background galaxies are located at \( z = 0.8 \), which is about the median redshift of galaxies in the Large Synoptic Survey Telescope (LSST) survey \([12]\) and the Euclid survey \([13]\, [14]\). The lensing geometric factor \( D_d D_{ds}/(1 + z_d) \) is maximized for lenses located at about the middle point between the source and observer in comoving distance, in this case \( z = 0.3 \). The lensing effect is significant over the center part of the path, when the lens is not too closer to either of the two ends. We define \( D_{\text{eff}} \) as the comoving length along our l.o.s. where the weight \( D_d D_{ds}/(1 + z_d) \) is larger than half of its maximum value. For the fiducial case here, \( D_{\text{eff}} = 1357 \text{Mpc}/h \).

We can make a multipole expansion for \( \kappa \):
\[
\kappa(\varpi, \phi) = \kappa_0(\varpi) + \kappa_1(\varpi) \cos \phi + \kappa_2(\varpi) \cos 2\phi.
\]

In Fig. 3 we plot the contribution to the different components of convergence field from a single halo \( \kappa_{0a}, \kappa_{1a} \) and \( \kappa_{2a} \), at \( z = 0.3 \) for \( m_\nu = 0.05 \text{ eV} \) in the case that the relative velocity is perpendicular to the l.o.s., i.e. \( \theta = \pi/2 \). Here we take \( M = 10^{13} M_\odot \), which is the typical mass of a halo. Although the effect is very small, except for the neutrino wake lensing, no other effect is known to produce the dipole term \( \kappa_{1a} \). The halo density field itself may have a dipole; however, its contribution can be removed by correlating with the relative velocity field direction: \( \xi(r) = \langle \delta_b(x) \delta_b(x + r) \sin \theta \rangle \), as it is antisymmetric. We have verified this is the case by N-body simulations, without the neutrinos the measured lensing signal is indeed consistent with zero within the numerical accuracy.

**Principle of Observation.**—In a velocity coherent volume \( V = L^3 \), each halo produces a wake in the same direction and add constructively to the dipole term (cos \( \phi \) has the same sign). For simple estimate, the number of halos of typical mass in such a volume is \( \bar{n} V \), where \( \bar{n} = \rho_m/M \) and \( \rho_m \) is the matter density at the corresponding redshift. Thus we can multiply the response density by \( \bar{n} V \), or in more sophisticated treatment calculate the integrated response using the Press-Schechter model.

We can reconstruct the relative velocity field from the large scale structure survey data \( \delta_g \), in Fourier space the velocity is given by
\[
\nu_{\text{rel}}(k) = \frac{i \mathbf{k} \cdot \theta_g(k) - \theta_v(k) \delta_g(k)}{T_v(k) b},
\]
where \( \theta_v \) and \( \theta_g \) are the velocity divergence transfer functions, \( T_v \) is the density transfer function and \( b \) is a linear bias. The lensing dipole can be calculated with respect to the direction of velocity, so that the neutrino wakes add coherently. However, as the neutrino mass is unknown, we would have to generate the relative velocity fields for many neutrino masses and determine the neutrino mass as the best fit to \( \kappa_1 \sim \bar{n} V \kappa_{1a} \).

This procedure is complicated by the fact that along each l.o.s., in the lensing effective length, there are on average \( D_{\text{eff}}/L \) coherent relative velocity volumes. Each of these contribute to the dipole term, but the dipole directions are randomly oriented with respect to one another. If the contribution to \( \kappa_1 \) from each of these coherent volumes can be measured individually, one may construct a stacked \( \kappa_1 \), which will total as \( D_{\text{eff}}/L \) as discussed previously. However, it is perhaps too difficult to measure \( \kappa_1 \) for individual velocity coherent volumes. Instead, the total \( \kappa_1 \) along each l.o.s. may be observed and the dipoles from each velocity coherent volume will now add incoherently. Despite this complication, we can still predict the net dipole resulting from this incoherent superposition and fit the result. The only change to our previous discussion is that \( \kappa_1 \) grows as \( \bar{n} V \sqrt{D_{\text{eff}}/L} \).

**Right handed neutrinos.**—If the neutrinos are Dirac, the primordial right handed neutrinos have the same mass as the left handed ones, but with different temperature, so that its distribution is also different. We assume that the \( \nu_{\text{RH}} \) were fully thermalized during the
early universe but decoupled before any other SM particle. The temperature of these neutrinos can be computed by ensuring conservation of entropy density \( g_{S}^{\nu R}T_{\nu R}^{3} = \frac{g_{S}^{\nu R}T_{\nu R}^{3}}{\nu_{R}} \), where \( g_{S}^{\nu R} \) is the effective degree of freedom when \( \nu_{R} \) decoupled. If the Standard Model is applicable at such energy scales except for the addition of right hand neutrinos, we have \( g_{S}^{\nu R} = 106.75 \), and \( g_{S}^{\nu R} \approx g_{S}^{SM} \approx 3 \) (fermions) \( \times 3 \) (number of species) \( \times 2 \) (particle and anti-particle). Currently, \( g_{S} = 3.91 \) but again a factor of \( \frac{7}{8} \times 6 \times \frac{T_{\nu R}}{T_{\gamma}} \) needs to be added. This leaves

\[
\left( \frac{T_{\nu R}}{T_{\gamma}} \right)^{3} = \frac{106.75 + \frac{7}{8} \times 6 \times \frac{T_{\nu R}}{T_{\gamma}}}{3.91 + \frac{7}{8} \times 6 \times \frac{T_{\nu R}}{T_{\gamma}}} \]

so the extrapolated right hand neutrino temperature is \( T_{\nu R} = \left( \frac{7}{8} \right)^{1/3} \nu_{R} \approx 0.905 \) K. With \( \nu_{R} \) at this temperature, the number of relativistic degrees of freedom during the nucleosynthesis era is \( N_{\text{eff}} = 3.04 \times (1 + (\frac{T_{\nu R}}{T_{\nu}})^{4}) \approx 3.18 \). This is consistent with the current big bang nucleosynthesis (BBN) constraints \([14]\). Note that the \( \nu_{R} \) density might be greater if its production mechanism is non-thermal, but the BBN constraints its density to be not much higher than the thermal value. To model \( \nu_{R} \), we use the CLASS code \([16]\) with neutrinos of the same mass but a lowered temperature. The results presented here only depend on the \( \nu_{R} \) being thermalized and their current temperature.

We can compute the variance of \( \nu_{R} \), the procedure is similar to the \( \nu_{L} \), except that \( \nu_{R} \) decoupled earlier, have lower temperature, i.e. velocity dispersion, and so their relative velocities are different from that of \( \nu_{L} \). The \( \nu_{L} \) and \( \nu_{R} \) relative velocity spectrum is shown in Fig. [5]. The coherency scale is \( R = 10.0 \) Mpc/\( h \) for \( \nu_{R} \) at \( z = 0.3 \). In this model we take \( \sqrt{\nu_{\nu_{R}}(z)} = 373 \) km/s as typical relative velocities. The one dimensional neutrino velocity dispersions are \( \sigma(z) = 2.077T_{\nu_{L}}(z)/m_{\nu_{R}} \approx 1263 \) km/s for \( \nu_{R} \) at \( z = 0.3 \). The direction of the flow deviates from that of \( \nu_{L} \), the angle \( \langle \cos \theta \rangle = \int \Delta_{\xi} \delta_{\nu_{L}} \delta_{\nu_{R}} / k^{3} dk / \int \Delta_{\xi} \delta_{\nu_{L}}^{2} / k^{2} d ln k \int \Delta_{\xi} \delta_{\nu_{R}}^{2} / k^{2} d ln k \), and we find \( \theta \approx 20^\circ \).

The right handed neutrino wake direction differs from the left handed wake by an angle of typically 20 degrees. This direction is computable, and differs from patch to patch. The signal is a small effect on the amplitude of the wake, which is degenerate with measurement systematics and small variations in the left handed neutrino mass. Instead, we use the difference in direction, and only consider the right handed wake component orthogonal to the left handed wake to be observable, and adjust sensitivity estimates correspondingly.

The Observability.—For forecasting purposes, we consider a combined LSST+Euclid dataset as well as a future 21cm lensing survey. The LSST will survey about 3 billion galaxies and the expected error for \( \kappa \) is about 0.28/\( \sqrt{N_{\text{eff}}} \approx 5.2 \times 10^{-6} \), where \( N_{\text{eff}} = 2.88 \times 10^{9} \). The Euclid survey gives a similar expected error. Here for simple estimates we will neglect the overlap between these two surveys and assume that they provide independent data set. By combining these data, we can reach a precision of \( \sigma_{\kappa} = 5.2 \times 10^{-6}/\sqrt{2} = 3.68 \times 10^{-6} \). Except for the minimal mass case \((m_{\nu_{R}} \approx 0.05 \) eV\)), it will allow determination of the neutrino mass and hierarchy.

Ultimately, extremely high precision measurements can be achieved with 21cm lensing observations such as the one proposed in \([17]\). For such a survey, the error on \( \kappa \) is \((k_{\max}/k_{\min})^{1.5} \). For a survey with \( k_{\max} = 1.47 \times 10^{2} h / \)Mpc and \( k_{\min} = 1.47 \times 10^{-3} h / \)Mpc, the expected error is about \( 3.16 \times 10^{-8} \), which is far smaller than the signals. With such future 21cm lensing surveys, we can measure neutrino masses with unprecedented precision, and may even determine whether the neutrinos are Majorana or Dirac particles.

Based on the measurement of neutrino mass differences from the neutrino oscillation experiments, three possible hierarchies are being considered\((c.f. \S 14.8 \) of \([1]\)): (i) the Normal Hierarchy (NH) case, with \( m_{1} \approx m_{2} \approx 0, m_{3} \approx 0.05 \) eV; (ii) the Inverted Hierarchy case (IH), with \( m_{1} \approx m_{3} \approx 0.05 \) eV; (iii) the Quasi-Degenerate case (QD), with \( m_{1} \approx m_{2} \approx m_{3} \approx 0.1 \) eV. In the IH and NH cases, we can neglect the contribution of the lighter neutrinos, so that the NH is equivalent to a single neutrino with mass of 0.05 eV, while IH is equivalent to two 0.05 eV neutrinos. The QD case is almost equivalent to three 0.1 eV neutrinos.

In Figure 4 we plot the expected dipole term \( \kappa_{1} = \int \kappa_{1}(\theta) d\theta / \int d\theta \) for \( \nu_{L} \) (left panel), and \( \nu_{R} \) (right panel) for these three cases. The dipole signal for \( \nu_{R} \) is suppressed by a \( \cos 20^\circ \) since we only consider the wake component geometrically orthogonal to \( \nu_{L} \). We also plot the the measurement errors for the LSST+Euclid lensing survey and the future 21cm lensing survey described above. As can be seen, the LSST+Euclid data should be able
to positively detect the QD case. With the future 21cm lensing survey, the NH and IH cases can be distinguished, and even the $\nu R$ may be detectable.

Conclusion.—We have computed the density contrast of neutrino wakes produced by the relative motions of neutrinos and dark matter. We have estimated the observability of these wakes via gravitational lensing and shown that it may be possible to observe both the mass hierarchy as well as right handed neutrinos (should they exist). The wake directions differ by a distinctive angular signature.

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