HIGHER-ORDER CORRECTIONS TO BFKL EVOLUTION FROM $t$-CHANNEL UNITARITY

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Abstract

Using reggeon diagrams as a partial implementation of $t$-channel unitarity, $O(g^4)$ corrections to the BFKL evolution equation have been obtained. We describe the spectrum and holomorphic factorization properties of the resulting scale-invariant kernel. For a gauge theory, $t$-channel unitarity can be studied directly in the complex $j$-plane by implementing Ward identity constraints together with the group structure of reggeon interactions. We discuss how both the $O(g^2)$ BFKL kernel and the $O(g^4)$ corrections can then be derived.

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Currently the most familiar application of the BFKL equation is to the evolution of parton distributions at small-x. The solution \( F(x, k^2) \sim x^{1-\alpha_0} \), with \( \alpha_0 \sim 1.5 \), is well-known. In the non-forward direction the equation becomes a “reggeon Bethe-Salpeter equation” i.e.

\[
F(\omega, k, q - k) = \tilde{F} + \frac{1}{(2\pi)^3} \int \frac{d^2k'}{(k')^2(q' - q)^2} \Gamma_2(\omega, k', q - k') \\
\times \tilde{K}(k, k', q) F(\omega, k', q - k')
\]

(0.1)

\( \omega (= j - 1) \) is conjugate to \( ln[x^2] \). \( K \) is a 2-2 reggeon interaction and \( \Gamma_2 \) is a two-reggeon propagator. To obtain higher-order corrections we must study higher-order reggeon interactions. We can do this directly via high-energy “s-channel” unitarity calculations. Alternatively we can sew reggeon amplitudes together in the \( j \)-plane via “t-channel” unitarity.

Initially we suggested sewing reggeon amplitudes together by using reggeon diagrams. With a “nonsense zero” in the three-reggeon vertex, many reggeon singularities in the diagrams are cancelled, leaving only particle singularities generating leading and next-to-leading order reggeon interactions. We obtained a leading-order 2-4 kernel and the \( O(g^4) \) higher-order 2-2 kernel \( K^{4n} \), written in terms of transverse momentum diagrams in Fig. 1

Fig. 1 The diagrammatic representation of \( K^{4n} \).

\( K^{4n} \) is infra-red finite and satisfies the Ward identity constraints of gauge invariance.

The last diagram of Fig. 1 is the most difficult to evaluate since it contains the box diagram

\[
I_4(p_1, p_2, p_3, p_4, m^2) = \int d^2p \sum_{i=1}^{4} \frac{1}{[(p - p_i)^2 - m^2]}
\]

We evaluated \( I_4 \) as a sum of logarithms, i.e.

\[
I_4 = \sum_{j < k} A_{jk} F_{jk}
\]

(0.2)
where the $A_{jk}$ are “tree-diagrams” obtained by putting internal lines $j$ and $k$ on-shell and

$$F_{jk} = \frac{i\pi}{\lambda^{1/2}(p^2_{jk}, m^2, m^2)} \Log \left[ \frac{p^2_{jk} - 2m^2 - \lambda^{1/2}(p^2_{jk}, m^2, m^2)}{p^2_{jk} - 2m^2 + \lambda^{1/2}(p^2_{jk}, m^2, m^2)} \right]$$  \hspace{1cm} (0.3)

with

$$p_{jk} = (p_j - p_k)^2.$$  \hspace{1cm} (0.4)

In the forward direction the $A_{jk}$ simplify considerably and if $K_{BFKL}$ is the leading-order BFKL kernel we find we can write

$$K^{(4n)}(k, k') = \frac{1}{4}(K_{BFKL})^2 + \mathcal{K}_2$$  \hspace{1cm} (0.5)

where

$$\mathcal{K}_2 = \frac{1}{2\pi^2} \frac{k^2 k'^2 (k^2 - k'^2)}{(k + k')^2 (k - k')^2} \Log \left[ \frac{k^2}{k'^2} \right]$$  \hspace{1cm} (0.6)

$\mathcal{K}_2$ has a number of attractive properties. It is separately infra-red finite and has a spectrum very reminiscent of the leading-order kernel. We anticipate that eventually it will be established as the forward component of a well-defined conformally invariant $O(g^4)$ contribution to the BFKL kernel.

Using the orthogonal eigenfunctions

$$\phi_{\nu,n}(k) = (k^2)^{1/2 + i\nu} e^{i n \theta}$$  \hspace{1cm} (0.7)

the eigenvalues of $\mathcal{K}_2$ are

$$\Lambda(\nu, n) = -\frac{1 + (-1)^n}{8\pi} \left( \beta'\left(\frac{|n| + 1}{2} + i\nu \right) + \beta'\left(\frac{|n| + 1}{2} - i\nu \right) \right).$$  \hspace{1cm} (0.8)

where

$$\beta'(x) = \frac{1}{4} \left( \psi'\left(\frac{x + 1}{2} \right) - \psi'\left(\frac{x}{2} \right) \right)$$  \hspace{1cm} (0.9)

with

$$\psi'(z) = \sum_{r=0}^{\infty} \frac{1}{(r + z)^2}.$$  \hspace{1cm} (0.10)
Note that

$$\Lambda(-\nu, -n) - \Lambda(\nu, n) \sim \sum_{t=-n/4}^{n/4-1} \left[ \frac{1}{(t + \frac{3}{4} - \nu)^2} - \frac{1}{(t + \frac{1}{4} - \nu)^2} \right]$$

\[ + \frac{1}{(t + \frac{3}{4} + \nu)^2} - \frac{1}{(t + \frac{1}{4} + \nu)^2} \]  

\[ = 0 \]  

for \( n \) even. As a result we can write

$$\Lambda(\nu, n) = G[m(1 - m)] + G[\bar{m}(1 - \bar{m})]$$  

where

\[ m = \frac{1}{2} + i\nu + \frac{n}{2} \quad \text{and} \quad \bar{m} = \frac{1}{2} + i\nu - \frac{n}{2} \]  

are conformal weights. This is the property of holomorphic factorization which generally accompanies conformal invariance.

Moving on to numerical results, we note that \( \Lambda(0, 0) \) is the leading eigenvalue. With a simple reggeon diagram normalization, the correction to \( \alpha_0 \) is given by

$$\frac{9g^4}{16\pi^3}\Lambda(0, 0) = -\frac{9g^4}{32\pi^4}\beta'(1/2)$$

\[ \sim -16.3\frac{\alpha_s^2}{\pi^2} \]  

\[ \sim -0.06 \]  

However, \( K^{4n}(q, k, k') \) contains disconnected diagrams (the first term in Fig. 1) which can not be interpreted in terms of reggeization effects. Eliminating these diagrams, while retaining scale-invariance, leads uniquely to

$$\bar{K}^{(4)} = K^{(4n)} - [K_{BFKL}]^2$$  

as a consistent scale-invariant \( O(g^4) \) kernel. In this case the modification of \( \alpha_0 \) is \( (\chi(0, 0) \) is the \( O(g^2) \) eigenvalue)

$$\frac{9g^4}{16\pi^4}\left(-3[\chi(0, 0)]^2 - \Lambda(0, 0)\right)$$

\[ \sim -68\frac{\alpha_s^2}{\pi^2} \]  

\[ \sim -0.25 \]

3
indicating that higher-order corrections may give a substantial negative correction to the leading-order BFKL result.

Unfortunately, several questions related to the significance of the numerical results are left unanswered by the reggeon diagram construction. In particular,

- what is the justification for the reduction to transverse momentum diagrams?
- How do scales enter and what is the significance of conformal invariance?

A related but more fundamental approach to the derivation of reggeon interactions, which potentially can answer such questions, is provided by the analytic continuation of multiparticle unitarity equations in the $j$-plane. This is a powerful formalism extensively based on multiparticle dispersion theory. A full description of the application to gauge theories can be found in [3]. The essential elements are

- Gauge invariance is input via the Ward identity constraint that reggeon interaction vertices vanish when any reggeon transverse momentum goes to zero.
- The “nonsense” zero/pole structure required by general analyticity properties is imposed.
- The group structure is input via the triple reggeon vertex.
- $t$-channel unitarity is used to determine both $j$-plane Regge cut discontinuities and particle threshold discontinuities due to “nonsense” states.
- The $j$-plane and $t$-plane discontinuities are expanded around $j = 1$ and in powers of $g^2$.

The most important element is the nonsense state particle discontinuities. Unitarity dictates that $j$-plane reggeon discontinuities are given by transverse momentum integrals. For particle discontinuities this is the case only when special kinematic circumstances determine that only nonsenses states are involved. When this happens, expansion around $j = 1$ (the equivalent of expanding in powers of logs in momentum space), and in powers of $g^2$, straightforwardly gives interactions etc. in terms of transverse momentum diagrams. The $O(g^2)$ contribution to the trajectory function arises from the two-particle discontinuity of the reggeon propagator. The 2-2 reggeon interaction, i.e. the BFKL kernel, is extracted from the nonsense-state discontinuities of the two reggeon propagator Green function illustrated in Fig. 2.
The $O(g^2)$ kernel is unambiguously derived from the three-particle discontinuity as $q^2 \to 0$ since only nonsense states are involved. Contributions to the $O(g^4)$ kernel that we have discussed above are from the four-particle state. More elaborate kinematic constraints are necessary for the reduction to transverse momentum diagrams and a number of qualifications of the reggeon diagram results emerge.

The most attractive part of the $O(g^4)$ kernel, $K_2$, can be derived directly when both $q^2 \to 0$ and either $k^2 \to 0$, or $k'^2 \to 0$, apart from an overall normalization factor. The existence of all but the first term in Fig. 1 follows once the existence of a 1-3 reggeon vertex is assumed. (We have noted above that the first term in Fig. 1 is removed by the introduction of the square of the leading-order kernel). The reduction to transverse momentum integrals is again only valid for $q^2, k^2 \to 0$, or $q^2, k'^2 \to 0$. These results are consistent with those obtained by Kirschner\cite{4} from the $s$-channel multi-Regge effective lagrangian formalism. It seems possible that the scale-dependence can be built up by adding internal logarithms to the transverse momentum integrals while maintaining the Ward identity constraints.

Finally we note that both $t$-channel unitarity and the multi-Regge effective lagrangian imply that the introduction of scales will modify the normalization and significantly modify the kernel at large $q^2, k^2, k'^2$. When the full $O(\alpha_s^2)$ kernel is calculated we hope the comparison will show how the reggeon diagram formalism can usefully approximate yet higher-order contributions.

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