Chiral nodes and oscillations in the Josephson current in Weyl semimetals

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The separation of the Weyl nodes in a broken time-reversal symmetric Weyl semimetal leads to helical quasi-particle excitations at the Weyl nodes, which, when coupled with overall spin conservation allows only inter-nodal transport at the junction of the Weyl semimetal with a superconductor. This leads to an unusual periodic oscillation in the Josephson current as a function of $k_0 L$, where $L$ is the length of the Weyl semimetal and $2k_0$ is the inter-nodal distance. This oscillation is robust and should be experimentally measurable, providing a direct path to confirming the existence of chiral nodes in the Weyl semimetal.

PACS numbers: 74.45.+c, 74.50.+r, 73.21.-b

Introduction.— Weyl semimetals (WSM), which have received much interest recently due to their non-trivial transport characteristics, are 3D topological systems where conduction and valence bands touch at two or more ‘Weyl’ points.1–4 According to a no-go theorem,5 gapless Weyl nodes in a WSM appear as pairs in momentum space with each of the nodes having a definite ‘chirality’, a quantum number that depends on the Berry flux enclosed by a closed surface around the node. Gauss law prevents the annihilation of the nodes unless two of them with opposite chirality are brought together, which provides the ‘topological’ protection of the Weyl nodes.6

A WSM phase requires broken time-reversal and/or inversion symmetry and a growing number of systems has been put forward which realize the WSM phase.7–11

The separation of the chiral nodes, allows charge pumping between the nodes in the presence of parallel electric and magnetic fields, as a consequence of the chiral anomaly12,13 and this has led to detailed studies of transport in Weyl semi-metals in several recent papers.13–32

In this paper we study the current in a simple Josephson junction setup, depicted in Fig. 1(a). The helical quasi-particle excitations at the Weyl nodes, due to the overall spin conserving processes at a WSM-superconductor (SC) junction, allow only inter-nodal transport.33 Further, we show, unlike in a normal metal-SC interface, the inter-nodal ‘normal’ (electron to electron) reflection process in a WSM-SC interface is not suppressed even for energies close to Fermi-energy, due to the broken time-reversal symmetry separating the Weyl nodes. The Josephson current, flowing through the bound levels formed by multiple inter-nodal ‘normal’ and Andreev (electron to hole) processes in a SC-WSM-SC system, consequently, acquires a specific periodicity as a function of the length of the WSM which depends only on the separation of the Weyl nodes in the momentum space (see Fig. 1(b)). We argue that both of these features are robust because they are not only bulk effects, but they are also protected by the robustness of the Weyl nodes. We also discuss the feasibility of experimental observations of this transition in our system, which can confirm the presence of chiral nodes in WSM.

This oscillation in the Josephson current and the resulting changes of sign of the critical current at arbitrary values of $\phi$ (or the 0-π transition) is an inherent property of the SC-ferromagnet-SC junction,33,34 and has also been experimentally observed.35 Since our model also explicitly violates time-reversal invariance, our results show quite a strong similarity with the Josephson current in similar systems36 as well as in semiconductor nanowires with Zeeman coupling.37

Model and geometry.— We consider the geometry as shown in Fig. 1(a) with the superconductors at $z < 0$.
and \( z > L \) and the Weyl semimetal (WSM) in the region \( 0 < z < L \). We model the WSM starting from the standard Hamiltonian describing a 3D TI in the Bi\(_2\)Se\(_3\) family\(^{11,12}\) regularized on a simple cubic lattice and adding a time-reversal breaking perturbation \( b_z \) to access the WSM phase\(^{13}\) -

\[
H_0 = \epsilon_k \tau_x - \lambda_z \sin k_z \tau_y - \lambda r_z (\sigma_x \sin k_y - \sigma_y \sin k_x) + b_z \sigma_z. \tag{1}
\]

Here \( \epsilon_k = \epsilon - 2t \sum_i \cos k_i \) is the kinetic energy, \( \tau (\sigma) \) represent the orbital (spin) degrees of freedom and \( \lambda, \lambda_z \) are the strengths of the spin-orbit coupling. In the limit \( \lambda_z \ll M \ll b_z \), (where \( M \) is defined as \( \epsilon - b_0 \)), this simplifies to a two-band model for a WSM, where, in the absence of the spin-orbit coupling, \( \lambda \), the bands have opposite spin\(^{13,14}\). The chiral fermion excitations around the two Weyl nodes (which we choose to be at \((0,0,\pm k_0)\), where \( tk_0^2 = b_z - M \)) are described by the Hamiltonian

\[
H_{\text{WSM}} = \epsilon_k \sigma_z - \mu_W + \lambda (k_x \sigma_x + k_y \sigma_y) \tag{2}
\]

with \( \epsilon_k = (k^2/2m_W)((k_x^2 + k_y^2 + k_z^2 - k_0^2)/2m_S - \mu_S) \). \( m_S \) is the effective mass. \( \phi_j \) is the superconducting phase of the \( j \)-th superconductor. For the left and right superconductors, \( j = L, R \). The parameter \( \mu_S \) depends on the details of the superconducting material. In the numerical results shown, we consider \( \mu_S \gg \Delta \), which is the realistic limit. Also, for simplicity, we consider \( m_S \approx m_W \).

WSM-SC junction.—The solutions of Eq. (2) in the Nambu-Gor’kov space are now 4 component spinors. For incident energy \( E \), the right-moving solutions with the wavefunctions proportional to \( e^{i\phi} k_z^\nu \) for electrons and \( e^{-i\phi} k_z^\nu \) for holes can be written in the basis of the two bands \( \nu = \pm \), with

\[
k_z^{\nu} = \sqrt{k_0^2 - \nu^2 (2m_w/\hbar^2) \sqrt{(\mu_W + (-)E)^2 - (\lambda \nu)^2}}.
\]

\( p = \sqrt{k_x^2 + k_y^2} \). The left-moving solutions can be written similarly with \( k_z^{\nu} \rightarrow -k_z^{\nu} \). For the case of a WSM-SC junction, the WSM and the superconducting wavefunctions on the two sides of the junction can be matched at the junction by requiring the continuity of the wavefunction and its first derivative\(^{15}\). This leads to the net reflection matrix \( R^j \) from the WSM-SC junction, which connects the left and right-moving solutions,

\[
R^j = \begin{pmatrix}
r_{ee}^j & r_{eh}^j \\
r_{he}^j & r_{hh}^j
\end{pmatrix}, \tag{4}
\]

where the ‘normal’ reflection matrices \( r_{ee}^j (r_{hh}^j) \), and the Andreev reflection matrices \( r_{eh}^j (r_{he}^j) \) denote, respectively, electron to electron (hole to hole) and hole to electron (electron to hole) processes at the interface with the \( j \)-th superconductor.
FIG. 3. The variation of the bound levels (solutions of Eq. (6)) near the chemical potential with the length $L$ of the WSM for various values of $\theta$, where $\theta = 2k_0L \mod(2\pi)$. The parameters used are the same as in Fig. 1(b).

For the purpose of physical interpretation, let us take the case of near-normal incidence ($k_0 \gg p$) of an electron, where the reflection matrices reduce to the form:

$$r_{ee}^j = \begin{pmatrix} \chi^+ & 0 \\ 0 & \chi^- \end{pmatrix}, \quad r_{he}^j = e^{-i\phi_j} \begin{pmatrix} 0 & \eta^+ \\ \eta^- & 0 \end{pmatrix}. \tag{5}$$

In this simplified form it is immediately clear that both the reflection and the Andreev reflection change the chirality (see also Fig. 2(a)) and can only take place from one node to another because of the chiral nature of the nodes. We plot the probabilities of normal and the Andreev reflection in Fig. 2(b). We note that even at energies close to the Fermi energy, normal reflection is not suppressed. The existence of the new momentum scale $k_0 \neq k_F$, introduced by breaking the time-reversal symmetry, allows the incident electron momentum to be different from the Fermi momentum of the superconductor. This leads to the non vanishing of normal reflection.$^{23,24}$

In contrast, note that for a topological insulator in 3D, the bulk is gapped and the non-trivial transport in junctions with superconductors is purely due to the surface states, where, the surface states consist of a Dirac metal with an odd number of nodes whose fermions have their spins aligned with the direction of motion (spin-momentum locking). This leads to completely different physics for a topological insulator-superconductor junction.$^{25,26}$ Two dimensional graphene, on the other hand, is metallic and the transport is through the bulk. However, in graphene, although there are two Dirac nodes (valleys), each of the nodes has fermions of both chiralities and the resulting process at the superconducting interface is purely intra-nodal Andreev reflection.$^{27,28}$

Bound levels in the SC-WSM-SC geometry.—Multiple reflections at the WSM-SC boundaries lead to bound electronic levels in the SC-WSM-SC geometry. But as discussed above, normal reflection amplitudes are not small at the WSM-SC interface and, in general, there is no simple way of summing up the amplitudes between the two superconductors to obtain the resonance condition when both Andreev and normal reflection amplitudes are non-zero. For the case of near normal incidence, however, the problem simplifies and the bound levels $E_b$ can be found by solving

$$\det \begin{bmatrix} I_{4 \times 4} - R_L M R_M \end{bmatrix} \bigg|_{E = E_b} = 0, \tag{6}$$

where $M$ is the matrix which accounts for the phase the electron/hole acquires while moving from one junction to another. We note that for $k_0 \gg p$, Eq. (6) can still be used for approximate solutions. Writing

$$R_L M R_M = \begin{pmatrix} T_{ee} & T_{eh} \\ T_{he} & T_{hh} \end{pmatrix}, \tag{7}$$

in the limit of near normal incidence with $k_0^2/2m_W$ much larger than incident energy $E$ and $\mu_S$ much larger than pairing potential $\Delta$, the $T$ matrices have the simplified
form (with $m_s = m_W$)

$$T_{ee} = \left( \begin{array}{cc} \alpha^+ & 0 \\ 0 & \alpha^- \end{array} \right), \quad T_{he} = \left( \begin{array}{cc} 0 & \beta^+ \\ \beta^- & 0 \end{array} \right),$$ (8)

with $\alpha^+ \approx e^{\pm 2\pi k_0 L} (1 + 4iE\delta)$, 
$\beta^+ \approx \pm e^{\mp 2\pi k_0 L} 2i(1 + e^{-i\phi})\Delta$.

where $\delta = \sqrt{2mW\mu_s/k_0 \Omega}$ and $\phi = \phi_R - \phi_L$. Also $T_{hh} = T_{ee}(E \rightarrow -E)$, $T_{eh} = T_{he}(E \rightarrow -E)$. This immediately shows the periodicities of the $T$ matrices, $T(\phi) = T(\phi + 2\pi)$ and $T(2k_0 L) = T(2k_0 L \rightarrow 2k_0 L + 2\pi)$, which implies that the bound levels $E_b$, the solutions of Eq. [10] also inherit the same periodicities in $\phi$ and $2k_0 L$. This additional periodicity of the levels with period $(\pi/k_0)$ in length appears as a consequence of the inter-nodal normal and the Andreev reflections. The periodicities of $E_b$ in the difference of the superconducting phases $\phi$ and in $2k_0 L$, in the limit of $k_0 \gg p$ is shown in Fig. [3]. This is our central result.

**Periodic oscillations in the Josephson current.**—The Josephson current for the system with the total Hamiltonian $H$ is written as $J_{\text{J}} = \frac{2e}{\hbar} \int \frac{\partial \mu}{\partial \phi}$, where the average is taken over the states of the system. For the non-interacting system, where the length $L$ is much smaller than the coherence length in superconductors, the Josephson current flows through the bound levels (neglecting the continuum contribution) and can be estimated as

$$J(\mu_W) = \frac{2e}{\hbar} \sum_b \frac{\partial E_b}{\partial \phi} f(E_b - \mu_W),$$ (9)

where $f$ is the Fermi-distribution function. Apart from the $2\pi$ periodicity of the Josephson current in $\phi$, as the bound levels $E_b$ are periodic in $L$ with the periodicity of $\pi/k_0$, the Josephson current also inherits the same periodicity. This periodicity is shown explicitly in Fig. [1(b)] for the case when $p = 0$.

The periodic dependence in $L$ can also be written as an approximate periodicity in $k_0$ with a period of $\pi/L$. For large values of $L$, the rapid oscillations of $E_b$ with a small variation of $k_0$ outweighs any other dependence on $k_0$ and the periodicity is almost exact. The Josephson current as a function of both $k_0$ and $L$ is also shown in Fig. [4], where the locus of constant current approximately follows $\theta = 2k_0 L \mod(2\pi)$. This is another of our main results.

**Lattice simulation.**—To analyse the case for non-normal incident angle $p \neq 0$, we compute the Josephson current from the lattice version of Eq. [1] through its Green’s function $\Sigma_i$. The Green’s function of the WSM, $g(\omega) = \{[\omega + i\delta](I - H_0)^{-1}$, is coupled with two superconductors $i = L, R$ through the on-site self-energy $\Sigma_i$.

$$\Sigma_i(\omega) = -\frac{i}{\sqrt{\Delta^2 - \omega^2}}(I_\sigma^r + \pi^r)[\omega I_\zeta - \Delta e^{i\phi} I_\zeta^r]I_\sigma^r.$$ (10)

where $\zeta$ acts on the particle-hole degree of freedom in the Nambu basis and $\Sigma$ is defined only on the sites in contact with the $i^{th}$ superconductor. $\tilde{t}$ characterises the tunnelling between the superconductor and the WSM. Then writing the full Green’s function as $G(\omega) = (g^{-1}(\omega) - \Sigma_L(\omega) - \Sigma_R(\omega))^{-1}$ we compute the Josephson current $J(\omega)$. We find that the oscillations of current remain intact and this result has been summarized in Fig. [5]. The bound levels $E_b$ can also be found approximately for $p \ll k_0$ using Eq. [6], and the corresponding Josephson current also shows that the oscillation with $k_0 L$ remains intact.

**Feasibility of experimental realization.**—In the predicted WSM material TaAs, chiral node pairs (formed by breaking inversion symmetry) are separated in momentum space by a distance $\sim 0.02\AA^{-1}$. Assuming standard electron mass, the relevant energy scale is about a milli electronvolt, which only becomes larger if the effective mass is smaller. Combining this with the fact that large momentum scattering (from $-k_0$ to $k_0$) is needed to break the topological protection of the chiral nodes, helical excitations in WSM are expected to be robust against disorder in a relatively clean sample. The periodicity of the Josephson current as well as the bound levels that we have discussed are, in principle, observable in tunneling.
experiments. The periodic variation of the bound-levels can also be probed in Andreev spectroscopy. For a typical sample, the length scales for such periodic variations would be of the order of few tens of nanometers. The separation of the Weyl nodes can also be tuned by adjusting the magnetic doping to observe periodicities with the separation of the Weyl nodes.

The effect of having many Weyl nodes complicates the theoretical modeling and presents a weakness in our proposal. But, as long as the transport takes place along a pair of Weyl nodes, a similar periodicity in the Josephson current is expected.

**Summary and conclusion.** —To summarize, we have shown explicitly, employing a simple model of the WSM, the occurrence of inter-nodal reflection processes at an SC interface due to spin conservation. This gives rise to an unusual periodicity in the bound state spectra and consequently in the Josephson current that depends only on the separation of the two Weyl nodes and the size of the sample. This provides a direct path for possible observations of the manifestation of inter-nodal Andreev reflection in Weyl semimetals.

In closing, we sketch some problems for future studies. Apart from transport signatures of the chiral anomaly in WSM, the appearance of surface states, and the consequent Fermi arc dispersion is another remarkable feature of the time-reversal broken WSM. Their transport characteristics in the Josephson current would be interesting to study. Finally, quantitative investigations of the effects of disorder and interactions on transport in the WSM are also left for future studies.

A. K. was supported in part by the NSF through Grant No. DMR-1350663.

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Clearly, $b_y$ lifts the spin degeneracy and shifts the bands up or down in energy while $\lambda$ mixes the uppermost and lowermost bands and the two bands in between. If the chemical potential is not large, the lowest energy excitations are only in the two bands close to zero energy. Limiting ourselves to the lowest energy excitations, we get the effective 2-band model

$$
\begin{pmatrix}
    M + b_z + \alpha k_y^2 + tp^2 & 0 \\
    0 & M - b_z + \alpha k_y^2 + tp^2 \\
    \lambda(k_y - ik_x) & \lambda(k_y + ik_x)
\end{pmatrix}.
$$

(13)

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$$
\begin{pmatrix}
    t(\alpha' k_y^2 + p^2 - k_y^2)W & \lambda(k_y - ik_x) \\
    \lambda(k_y + ik_x) & -t(\alpha' k_y^2 + p^2 - k_y^2)
\end{pmatrix},
$$

(14)

where $tk_y^2 = b_z - M$ (assuming $b_z > M$) and $\alpha' = \alpha/t = 1 + \lambda_y^2/2Mt \approx 1$. This is the Hamiltonian that we use in the main text.

### APPENDIX

#### A. Model Hamiltonian for the WSM

In this section we derive Eq. (2) near the Weyl nodes in Eq. (1) of the main text. Starting with parent Hamiltonian Eq. (1):

$$
H(k) = \epsilon_k \tau_x - \lambda_z \sin k_y \tau_y - \lambda \tau_z (\sigma_x \sin k_y - \sigma_y \sin k_x) + b_z \sigma_z,
$$

(11)

we define a mass term $M = \epsilon - 6t$, so that, around the $\Gamma$ point

$$
\epsilon_k = M + 2t \left( 3 - \sum_i \cos k_i \right) \approx M + t \sum_i k_i^2.
$$

(12)

For $\lambda = 0 = b_z$ the Hamiltonian (about $\Gamma$ point) is $(M + t|k|^2)\tau_x - \lambda_z k_z \tau_y$, giving two doubly degenerate bands with the dispersion

$$
\approx \pm \sqrt{M^2 + k_y^2(2Mt + \lambda_z^2) + 2p^2(Mt)},
$$

where $p^2 = k_x^2 + k_y^2$. Assuming that the mass is large enough, we can write

$$
\pm \sqrt{M^2 + k_y^2(2Mt + \lambda_z^2) + 2p^2(Mt)} \approx \pm (M + \alpha k_z^2 + tp^2),
$$

where $\alpha = t + \lambda_z^2/2M$. This quadratic dispersion with a non-zero gap $(2M)$ at the $\Gamma$ point is the starting point in Ref. [33]. Motivated by this, we define a unitary matrix $U$ such that

$$
U^\dagger [(M + t|k|^2)\tau_x - \lambda_z k_z \tau_y] U = (M + \alpha k_z^2 + tp^2)\tau_z.
$$

In this new basis the full Hamiltonian $U^\dagger H(k)U$ is

$$
\begin{pmatrix}
    0 & \lambda(k_y + ik_x) \\
    \lambda(k_y - ik_x) & 0 \\
    -M + b_z - \alpha k_z^2 - tp^2 & 0 \\
    0 & -M - b_z - \alpha k_z^2 - tp^2
\end{pmatrix}.
$$

(13)

#### B. Solving the WSM-SC Interface

In this section, we provide the details for the derivation of the reflection matrix in a WSM-SC system. Following Ref. 35 (Uchida et al.), the wavefunction of energy $E_i$ in the WSM is given by the following solutions of Eq. (1)
of the main text in the Nambu-Gor’kov space (with the Hamiltonian in the hole space written as $-H_{WSM}(-k)$):

$$
\psi_{WSM}(z) = \sum_{\sigma = \pm} \left\{ \mathcal{E}^\sigma \left( a_{R\sigma}^* e^{i k_z z} + a_{L\sigma}^* e^{-i k_z z} \right) + \mathcal{H}^\sigma \left( b_{R\sigma}^* e^{-i k_z z} + b_{L\sigma}^* e^{i k_z z} \right) \right\}, \quad (15)
$$

where $\sigma$ is the band index, $a(b)$ denotes the electron (hole) amplitude and $L(R)$ denotes the left (right) moving solution. $\mathcal{E}^\sigma(\mathcal{H}^\sigma)$ are normalized eigenvectors, which are non-zero in electron (hole) sector of the Hamiltonian. In each sector $\mathcal{E}(\mathcal{H})^+ \propto (f_e(h), (-\lambda)^+)^T$, and $\mathcal{E}(\mathcal{H})^- \propto ((-\lambda)^-, f_e(h))^T$, with $f_e(h) = \mu_W + (-\lambda E_i + \sqrt{\mu_W^2 + (-\lambda E_i)^2})$, $\lambda = \lambda(k_x + i k_y)$.

In the superconductor, the solutions of Eq. (2) of the main text are:

$$
\psi_{SC}(z) = \begin{pmatrix} u e^{-i q e z} + v d e^{-i q h z} \\ -u c e^{i q e z} + v d e^{i q h z} \end{pmatrix},
$$

where, with $\Omega = \sqrt{\Delta^2 - E_i^2}$,

$$
u(v) = \sqrt{(E_i + (-\lambda t))^2/2E_i},$$

and $q_e$ and $-q_h$ are, respectively, the outgoing electron and hole momenta in the superconductor, defined as (with Fermi momentum $k_F$)

$$
q_{e(h)} = \sqrt{k_F^2 - p^2} + (-2 m s \Omega)/\hbar^2.
$$

The boundary conditions at $z = 0$ are given by the continuity of the wavefunction and its derivative:

$$
\psi_{WSM}(0) = \psi_{SC}(0),
$$

$$
m_S \begin{pmatrix} \sigma_z & 0 \\ 0 & \sigma_z \end{pmatrix} \partial_z \psi_{WSM}(z) |_{z=0} = m_W \partial_z \psi_{SC}(z) |_{z=0},
$$

with $\sigma_z$ being the Pauli matrix. By solving them one gets the reflection matrices,

$$
\begin{pmatrix} a_L^+ \\ a_L^- \\ b_L^+ \\ b_L^- \end{pmatrix} = \begin{pmatrix} r_{ee} & r_{eh} \\ r_{he} & r_{hh} \end{pmatrix} \begin{pmatrix} a_R^+ \\ a_R^- \\ b_R^+ \\ b_R^- \end{pmatrix}, \quad (16)
$$

C. Non-vanishing normal reflection

For the simplest case, if we take the only incident amplitude to be nonzero as $a_L^+ = 1$ on the left side of Eq. (16), we can show that the amplitude of normal reflection is

$$
a_R^+ = \frac{u c e^{i q_e z} + v d e^{-i q_h z}}{2 \left( 1 - \frac{q_e}{k_e} \right) + \frac{v d}{2 \left( 1 + \frac{q_h}{k_h} \right)}}
$$

with $u = v = 1$. (17)

In a normal metal (with normal incidence), $q_e \approx q_h \approx k_F \approx k_F$ when the incident energy is small, so that the amplitude of the normal reflection vanishes. However, in the WSM, $q_e \approx q_h \approx k_F$ but $k_F \neq k_F$ and hence, for the WSM, the normal reflection is not suppressed in general at small incident energies.

D. Bound state spectrum

The bound state spectrum for the SC-WSM-SC geometry can be found by using the reflection matrix at a single WSM-SC interface defined in Eq (S6). At near normal incidence, the bound states are given by the zeroes of the determinant

$$
\det \left[ I_{4 \times 4} - R^L M R^R M \right] |_{E = E_b} = 0, \quad (18)
$$

The $R^L$ and $R^R$ matrices are the reflection matrices at SC-WSM and WSM-SC interfaces respectively and the $M$ matrix is the phase picked up by the electrons and holes in the WSM region (of length $L$). At near normal incidence, we can compute the matrices analytically and write

$$
R^L M R^R M = \begin{pmatrix} T_{ee} & T_{eh} \\ T_{he} & T_{hh} \end{pmatrix},
$$

where $(\phi = \phi_R - \phi_L$),

$$
T_{ee} = \begin{pmatrix} \alpha_1^+ + \alpha_2^- & 0 \\ 0 & \alpha_1^- + \alpha_2^+ \end{pmatrix},
$$

$$
T_{he} = \begin{pmatrix} \beta_0^+ (e^{-i \phi} \beta_1^- + \beta_2^+ ) \\ 0 \end{pmatrix}.
$$

\[
\alpha_1^+ = \frac{4 e^{i(k_x^e + k_x^h)L} e^{-i \phi} k_z^e k_z^h (q_e + q_h)^2 u^2 v^2}{[(k_z^e + q_e)(k_z^h - q_h) u^2 - (k_z^h + q_h)(k_z^e - q_e) v^2]^2},
\]

\[
\alpha_2^+ = e^{2ik_z^e/L} \left[ (k_z^e+q_e)(k_z^h+q_h)u^2+(k_z^h-q_e)(k_z^e+q_h)v^2 \right]^2, \quad (19)
\]

\[
\alpha_1^- = \frac{4 e^{i(k_x^e - k_x^h)L} e^{-i \phi} k_z^e k_z^h (q_e - q_h)^2 u^2 v^2}{[(k_z^e + q_e)(k_z^h + q_h) u^2 - (k_z^h - q_e)(k_z^e - q_h) v^2]^2}.
\]
immediately shows the periodicities of the momenta $k^\pm$ the transverse momenta $k^T$. The expressions can be simplified in the limit $\mu^T$ of the main text.

The zero-temperature Josephson current at a fixed superconducting phase difference $\phi = \pi/2$ with the length of the WSM, $L$, keeping $k_0$ the same for transverse momenta $k_x, k_y = 0$ in the blue (dashed) curve, while having the transverse momenta $k_x = 0.1, k_y = 0$ in the red (solid) curve. Other parameters are the same as in Fig. 1(b) of the main text.

and $T_{hh} = T_{ee}(E \rightarrow - E), T_{eh} = T_{ee}(E \rightarrow - E)$. This immediately shows the periodicities of the $T$ matrices, $T(\phi) = T(\phi + 2\pi)$ and $T(2k_0L) \approx T(2k_0L + 2\pi)$ considering $E \ll k^2_T/m_w$, i.e. $k^\pm_0 \approx k_0$. The expressions can be simplified in the limit $\mu^T \gg \Delta$ and $k^2_0/2m_w \gg E$ (the incident energy) and are given in the main text.

E. Variation of Bound levels for $p \neq 0$

We show a plot of the zero-temperature Josephson current with varying length of the WSM in Fig. 6 for transverse momentum $p \neq 0$ (but still one order of magnitude smaller than $k_0$). For the approximate determination of the bound levels $E_0$, we assume that Eq. (6) of the main text still remains valid in this range of parameters. The figure shows that the periodicity with $k_0L$ is intact, which implies the robustness of the periodicity in non-normal but small-angle incidence of electron.

F. Green’s function formalism of Josephson current

Following Ref. [50] if $N_i$ represents the number operator of the $i$th site in the metallic system (WSM), then $-\epsilon N_i$ can be written as the sum of the current flowing from the $(i - 1)$th site to the $i$th site and from the $i$th site to the $(i + 1)$th site, i.e. $N_i = (1/e)(J_{i, i-1} + J_{i+1, i})$. Each of these terms represents the Josephson current flowing in the system and is independent of the site $i$ for a large enough system. To elaborate, let us write the lattice Hamiltonian for the WSM in the following form (in the basis of spin operator $\sigma_z$ with eigenvalues $\sigma$ and parity operator $\tau_z$ with eigenvalues $\tau$):

$$H = H^1 + H^2 = \sum_{i, \sigma, \sigma', \tau, \tau'} a^\dagger_{i, \sigma, \sigma', \tau, \tau'} h_{i, \sigma, \sigma', \tau, \tau'}(1) + \sum_{i, \sigma, \sigma', \tau, \tau'} a^\dagger_{i, \sigma, \sigma', \tau, \tau'} h_{i, \sigma, \sigma', \tau, \tau'}(2)$$

Then first term does not contribute to the current. In our case the second term of the Hamiltonian is (assuming translation invariance in the directions perpendicular to that of the flow of Josephson current)

$$H^2 = a^\dagger_{i}(t)(-t_{\text{WSM}} \tau_x + i\lambda_\gamma \sigma_0 a_j + \text{h.c.})$$

And the current from the $i$th to the $(i + 1)$th site as

$$J_i(t) = -\frac{i\epsilon}{\hbar} \sum_{\sigma, \tau} (t_{\text{WSM}} - \tau \lambda_\gamma) \times$$

$$\left( \langle a^\dagger_{i+1, \sigma, \tau}(t)a_{i+1, \sigma, \tau}(t) \rangle - \langle a^\dagger_{i, \sigma, \tau}(t)a_{i, \sigma, \tau}(t) \rangle \right)$$

where $\bar{\tau} = -\tau$. The above averages can be written in terms of the Green’s function as

$$G_{i, \sigma, \tau}(t, t') = \frac{\langle a^\dagger_{i, \sigma, \tau}(t')a_{i, \sigma, \tau}(t) \rangle - \langle a^\dagger_{i, \sigma, \tau}(t')a_{i, \sigma, \tau}(t) \rangle - \langle a^\dagger_{i, \sigma, \tau}(t')a^\dagger_{i, \sigma, \tau}(t) \rangle - \langle a_{i, \sigma, \tau}(t')a_{i, \sigma, \tau}(t) \rangle}{\langle a^\dagger_{i, \sigma, \tau}(t')a_{i, \sigma, \tau}(t) \rangle \langle a^\dagger_{i, \sigma, \tau}(t')a^\dagger_{i, \sigma, \tau}(t) \rangle}$$

In the absence of any applied voltages the correlation function $G^{+\pm}(\omega)$ is given by $G^{+\pm}(\omega) = f(\omega)|G_A(\omega) - G_R(\omega)|$. $f(\omega)$ is the Fermi function and $G_A/R$ are the
advanced and retarded Green’s functions.