Top Radiative Corrections in Non-minimal Standard Models

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Abstract

We derive the one-loop effective action induced by a heavy top in models with an extended Higgs sector. We use the effective action to analyze the top corrections to the $\rho$ parameter and to the Higgs-gauge boson couplings. We show that in models with $\rho \neq 1$ at tree-level, one does not lose generally the bound on $m_t$ from the $\rho$ parameter.
1. Introduction

Recent precision measurements at LEP allow us to strongly constrain the top mass in the standard model (SM)\(^{[1]}\). These constraints are obtained by analyzing the radiative corrections induced by the top quark to measurable quantities. From the \(\rho\) parameter, \(\rho = m_W^2/(m_Z^2 \cos^2 \theta_W)\), we get the strongest constraint on \(m_t\), since the top radiative corrections to \(\rho\) grow quadratically with the top mass.

In the minimal SM in which the Higgs sector consists of one Higgs doublet, the value of \(\rho\) at tree level, \(\rho_{\text{tree}}\), is equal to unity so radiative corrections must be finite. Nevertheless, when an extended Higgs sector is considered (non-minimal SMs), one can have \(\rho_{\text{tree}} \neq 1\). Since the experimental value of \(\rho\) is very close to unity, one expects that, in such non-minimal SMs, a simultaneous expansion in \((\rho_{\text{tree}} - 1)\) and \(g^2 m_t^2/m_W^2\) can be carried out such that the top corrections to \(\rho\) are the same as that in the SM. It has been recently claimed\(^{[2]}\), however, that such an expansion is meaningless, i.e., the limit \(\rho_{\text{tree}} \to 1\) is not continuous. It has been argued that in these models \(\rho\) is a free parameter, so it cannot be computed, but must be extracted from the experiments. The explicit calculation of the top corrections to \(\rho\) was carried out in ref. [2] and it was claimed not to be finite. It implies that one loses the bounds on \(m_t\).

In this paper we show using two different methods that in non-minimal standard models the radiative corrections to \(\rho\) are finite and meaningful, even for large values of \(\rho_{\text{tree}} - 1\). In the particular model considered in ref. [2], we find that the bound on \(m_t\) is as strong as that in the SM. This has also been stressed in refs. [1,3]. In section 2, we compute the top corrections to \(\rho\) following the effective action approach\(^{[4]}\). Such an approach is suited to computing radiative corrections to relations that depend on the vacuum expectation values (VEVs) of the scalar fields\(^*\). Neither tadpole diagrams nor counterterms for the VEVs of the scalars need to be considered, since the one-loop effective action is computed in the sym-

\* See ref. [5], for an example in which the effective potential is used to compute the top radiative corrections to the Higgs mass in the minimal supersymmetric model.
metric phase – before the electroweak symmetry breaking (ESB). Furthermore, the effective action approach allows one to relate the top corrections of different low-energy processes. In section 3, we reinforce our statement by computing the top corrections to $\rho$ following the usual counterterm approach.

2. Effective action approach

The effective action, $\Gamma(\phi)$, is defined as the generator of the one particle irreducible (1PI) $n$-point Green’s functions, $\Gamma^{(n)}$,  

$$\Gamma(\phi) = \sum_n \frac{1}{n!} \int d^4x_1 \cdots d^4x_n \Gamma^{(n)}(x_1, \cdots, x_n)\phi(x_1)\cdots\phi(x_n).$$

An alternative expansion of the effective action is in powers of momentum about the point where all external momenta vanish,

$$\Gamma(\phi) = \int d^4x [ - V(\phi) + \frac{Z(\phi)}{2} \partial_\mu \phi \partial^\mu \phi + \cdots ] ,$$

where $V(\phi)$ is the so-called effective potential

$$V(\phi) = - \sum_n \frac{1}{n!} \Gamma^{(n)}(p_i = 0) \phi^n .$$

Let us now consider the model of ref. [2]. The Higgs sector consists of a Higgs doublet with $Y = 1$ and a real Higgs triplet with $Y = 0$,

$$\Phi = \begin{pmatrix} \phi^+ \\ \frac{1}{\sqrt{2}}(\phi + iG^0) \end{pmatrix} \quad \text{and} \quad \Sigma = \begin{pmatrix} \Sigma_+ \\ \Sigma_0 \\ \Sigma_- \end{pmatrix} ,$$

respectively. Our phase convention is such that $\Sigma_- \equiv -(\Sigma_+)^*$. We want to analyze the one-loop effects of a heavy top quark. Since our model is $SU(2)_L \times U(1)_Y$
invariant, the one-loop effective action before the ESB is given by (following an expansion as in eq. (2))

\[
\Gamma = \int d^4x \left\{ -V(\Phi, \Sigma) + [1 + A(\Phi^\dagger \Phi)](D_\mu \Phi)^\dagger (D^\mu \Phi) + \frac{1}{2} (D_\mu \Sigma)^\dagger (D^\mu \Sigma) \\
+ B(\Phi^\dagger \Phi)(\Phi^\dagger D_\mu \Phi)((D^\mu \Phi)^\dagger \Phi) + \frac{1}{2} C(\Phi^\dagger \Phi)[(\Phi^\dagger D_\mu \Phi)(\Phi^\dagger D^\mu \Phi) + h.c.] + \cdots \right\},
\]

(5)

where we have only kept terms with a maximum of two covariant derivatives, which are the only terms relevant to our analysis. Note that the operators \(A(\Phi^\dagger \Phi), B(\Phi^\dagger \Phi)\) and \(C(\Phi^\dagger \Phi)\) that arise at one-loop level only depend on \(\Phi\) because \(\Sigma\) does not couple to the quarks. When the neutral Higgs develop VEVs, \(\langle \phi \rangle \equiv v\) and \(\langle \Sigma_0 \rangle \equiv v_3\), the operators in eq. (5) induce mass terms for the gauge bosons. The last three terms in eq. (5) contribute differently to the \(W\) and \(Z\) masses, \(i.e.,\) they break the custodial \(SU(2)\) symmetry\(^7\). The Higgs triplet kinetic term only contributes to the \(W\) mass, while \(B(\Phi^\dagger \Phi)(\Phi^\dagger D_\mu \Phi)((D^\mu \Phi)^\dagger \Phi)\) and \(C(\Phi^\dagger \Phi)(\Phi^\dagger D_\mu \Phi)(\Phi^\dagger D^\mu \Phi)\) contribute only to the \(Z\) mass\(^4\). Notice that these two terms are finite since they correspond to operators of dimension higher than 4. The first two terms in eq. (5), however, are not finite. The effective potential \(V(\Phi, \Sigma)\) can be renormalized following ref. [6]. The kinetic term for the Higgs doublet can be made finite by a field redefinition

\[
\Phi \rightarrow (1 - A)^{1/2} \left| \left. \Phi = \langle \Phi \rangle \right. \right. \Phi.
\]

(6)

After the rescaling (6) and the renormalization of the effective potential, the one-loop effective action is finite.

As a function of the neutral Higgs and gauge bosons, the effective action (5) before the redefinition (6) is given by

\[
\Gamma = \int d^4x \left\{ -V(\phi, \Sigma_0) + \frac{Z(\phi^2)}{2} \partial_\mu \phi \partial^\mu \phi + \frac{1}{2} \partial_\mu \Sigma_0 \partial^\mu \Sigma_0 \\
+ \Pi_W(\Sigma_0^2, \phi^2)W_\mu W^\mu + \frac{1}{2} \Pi_Z(\phi^2)Z_\mu Z^\mu \right\},
\]

(7)
where

\[ Z(\phi^2) = 1 + A(\phi^2) + \frac{\phi^2}{2} [B(\phi^2) + C(\phi^2)] , \]

\[ \Pi_W(\Sigma_0^2, \phi^2) = g^2 \Sigma_0^2 + \frac{g^2 \phi^2}{4} [1 + A(\phi^2)] , \]

\[ \Pi_Z(\phi^2) = \frac{g^2 \phi^2}{4 \cos^2 \theta_W} [1 + A(\phi^2)] + \frac{g^2 \phi^4}{8 \cos^2 \theta_W} [B(\phi^2) - C(\phi^2)] . \]

The \( \Pi_W \) and \( \Pi_Z \) Green’s functions can be easily calculated. They correspond to the 1PI Green’s functions with two external \( W \) or \( Z \) and arbitrary number of external \( \phi \). To one top-bottom loop order, they are given by

\[ \Pi_W = g^2 \Sigma_0^2 + \frac{g^2 \phi^2}{4} + \frac{g^2 N_c}{32 \pi^2} \left[ \sum_{i=t,b} m_i^2 \left( \Delta - \ln \frac{m_i^2}{\mu^2} + \frac{1}{2} \right) - \frac{m_t^2 m_b^2}{m_t^2 - m_b^2} \ln \frac{m_t^2}{m_b^2} \right] , \]

\[ \Pi_Z = \frac{g^2 \phi^2}{4 \cos^2 \theta_W} + \frac{g^2 N_c}{32 \pi^2 \cos^2 \theta_W} \left[ \sum_{i=t,b} m_i^2 \left( \Delta - \ln \frac{m_i^2}{\mu^2} \right) \right] , \]

where

\[ m_{t,b} = \frac{h_{t,b}}{\sqrt{2} \phi} , \]

\( N_c \) is the colour number \((N_c = 3 \text{ for quarks})\), \( \mu \) is the renormalization constant and \( \Delta = \ln 4 \pi - \gamma + 1/\epsilon \), where \( \gamma \) is the Euler constant and \( \epsilon = (4 - n)/2 \) with \( n \) being the space-time dimension. From eqs. (8) and (9), we can extract \( A(\phi^2) \) and \( [B(\phi^2) - C(\phi^2)] \). We now rescale the neutral Higgs doublet as in eq. (6), and we obtain

\[ \Pi_W = g^2 \Sigma_0^2 + \frac{g^2 \phi^2}{4} , \]

\[ \Pi_Z = \frac{g^2 \phi^2}{4 \cos^2 \theta_W} - \frac{m_W^2}{\cos^2 \theta_W} \Delta \rho_{tb} , \]

\[ \text{To obtain the explicit form of } B(\phi^2) \text{ and } C(\phi^2) \text{ we need to calculate } Z(\phi^2) \text{. Nevertheless, only the difference } [B(\phi^2) - C(\phi^2)] \text{ is relevant to our analysis.} \]
\[ \Delta \rho_{tb} \equiv -\frac{g^2 \phi^4}{8m_W^2}[B(\phi^2) - C(\phi^2)] = \frac{g^2 N_c}{32\pi^2 m_W^2} \left[ \frac{1}{2}(m_t^2 + m_b^2) - \frac{m_t^2 m_b^2}{m_t^2 - m_b^2} \ln \frac{m_t^2}{m_b^2} \right]. \]

(12)

Thus, the \( \rho \) parameter is given by

\[ \rho = \rho_0(1 + \rho_0 \Delta \rho_{tb}), \]

(13)

with

\[ \rho_0 = 1 + \frac{4v_3^2}{v_2^2}. \]

(14)

In eq. (14) \( v \) and \( v_3 \) are renormalized quantities (the values of \( \phi \) and \( \Sigma_0 \) that minimize the renormalized effective potential), so the radiative corrections to the \( \rho \) parameter are finite and meaningful. In our particular non-minimal SM, we have, from the experimental value of the \( \rho \) parameter\(^\dagger\), \( \rho = 1.005 \pm 0.0024 \), a stronger upper bound on \( m_t \) than that in the SM since both contributions (from the Higgs triplet and the top) are positive. In the limit \( v_3 \to 0 \), we get the SM prediction.

We can write eq. (14) as a function of only \( v_3 \) using the relation

\[ \frac{G_F}{\sqrt{2}} = \frac{g^2}{8\Pi_W(\Sigma_0^2, \phi^2)} \bigg|_{\text{VEV}} = \frac{1}{2[v^2 + 4v_3^2]}, \]

(15)

where \( G_F \) is the Fermi constant measured from the \( \mu \)-decay. Explicitly,

\[ \rho_0 = 1 + 4\sqrt{2}G_Fv_3^2, \]

(16)

that implies

\[ v_3 < 7.8 \text{ GeV}. \]

(17)

In models with a non-minimal Higgs sector, large radiative corrections can also be induced by Higgs bosons\(^\S\). In the model (4), however, we have noted that,\(^\dagger\)

\(^\dagger\) Since the leading top contributions [eq. (9)] do not depend on the energy scale, \( \rho \) can be extracted from experiments at the \( m_Z \) scale or from low-energy experiments such as neutrino scattering. Our value of \( \rho \) is taken from ref. [1].
neglecting terms of $O(v_3/v) \sim 3 \cdot 10^{-2}$, the Higgs sector has an approximate global $SU(2)$ custodial symmetry under which $\Sigma$ transforms as a triplet. It follows that Higgs corrections to $\rho$ are very small and the bound (17) holds. It is important to note that eq. (13) is a result valid for any non-minimal SM. In a general case,

$$\rho_0 = \frac{\sum_i (T^2_i - T^2_{3i} + T_i) |\langle \phi_i \rangle|^2}{\sum_i 2T^2_{3i} |\langle \phi_i \rangle|^2},$$  \hspace{1cm} (18)$$

where $T_i$ and $T_{3i}$ are the total and third component of the weak isospin of $\phi_i$. As is well known \cite{9}, for a SM with an additional complex Higgs triplet with $Y = 2$, $\chi$, we have $\rho_0 = 1 - 4\sqrt{2}G_F \langle \chi \rangle^2$ for small values of $\langle \chi \rangle$. Then, a partial cancellation can take place between the terms $4\sqrt{2}G_F \langle \chi \rangle^2$ and $\Delta \rho_{tb}$ so that a larger $m_t$ is allowed in this model. For a very heavy top, however, a non-perturbative calculation of $\rho$ is necessary. Such a calculation was carried out in ref. \cite{10} using a $1/N_c$ expansion.

The Higgs effective potential, once renormalized, depends on $m_t$. Then, if $v_3$ is obtained from the minimization conditions of the effective potential, $v_3$ will depend on $m_t$. One would expect

$$v^2_3(m_t) = v^2_3(m_t = 0) + \Delta,$$  \hspace{1cm} (19)$$

where $\Delta$ is of $O(g^2m^2_t)$ or even of $O(g^2m^4_t/m^2_W)$, \textit{i.e.}, the smallness of $v_3$ is not stable under radiative corrections of a heavy top. It is easy to see, however, that this cannot be the case. Consider the most general Higgs potential \cite{9}

$$V(\Sigma_0, \phi) = a_1\Sigma_0^2 + a_2\Sigma_4 + a_3\Sigma_0^2\phi^2 + a_4\Sigma_0\phi^2 + V(\phi).$$  \hspace{1cm} (20)$$

From the minimization condition of eq. (20), we have

$$v_3(m_t = 0) \simeq \frac{-a_4v^2}{2[a_1 + a_3v^2]},$$  \hspace{1cm} (21)$$

where $v_3$ has been assumed to be small. Because $\Sigma$ does not couple to the top, there is no vertex correction to $a_i$. The only correction arises from the redefinition
of the Higgs doublet (6). Thus,
\[
V^{1-loop}(\Sigma_0, \phi) = a_1 \Sigma_0^2 + a_2 \Sigma_0^4 + a_3 (1 + \Delta) \Sigma_0^2 \phi^2 + a_4 (1 + \Delta) \Sigma_0 \phi^2 + V(\phi), \quad (22)
\]
with \(\Delta \sim \mathcal{O}(g^2 m_t^2)\). The explicit form of \(\Delta\) depends on how we renormalize the effective potential, \(i.e.,\) the definitions of the renormalized \(a_3\) and \(a_4\). From eqs. (21) and (22), we have
\[
v_3^2(m_t) = v_3^2(m_t = 0)[1 + \frac{a_1}{a_1 + a_3 v} \Delta]. \quad (23)
\]
Therefore, \(v_3\) has a weak dependence on the top mass and on the renormalization prescription of the effective potential.

The one-loop effective action (5) gives us more information than the top-bottom corrections to the gauge boson masses. From eq. (5) one can also obtain the one-loop Higgs-gauge boson couplings. In the case of a neutral Higgs, the \(\phi^n W W (\phi^n Z Z)\) coupling is given by the \(n\)th derivative of \(\Pi^W (\Pi^Z)\) respect to \(\phi\) at \(\phi = v\). For example, the one-loop \(\phi Z Z\) vertex is given by
\[
\Gamma_{\phi Z Z} = \frac{\partial \Pi^Z}{\partial \phi} \bigg|_{\phi = v} = \frac{g^2 v}{2 \cos^2 \theta_W} + \frac{g^2 N_c}{16 \pi^2 v \cos^2 \theta_W} \left[ \sum_{i=t,b} m_i^2 \left( \Delta - \ln \frac{m_i^2}{\mu^2} - 1 \right) \right], \quad (24)
\]
in agreement with the explicit one-loop calculation \cite{12}. In the model (4), the \(H^+ W Z\) coupling can also be obtained from eq. (5). The \(H^+\) is the orthogonal state to the charged Goldstone boson, \(i.e.,\)
\[
H^+ = -\sin \beta \phi^+ + \cos \beta \Sigma_+, \quad (25)
\]
where \(\tan \beta = 2v_3/v\). Eqs. (4)–(6), (12) and (25) yield
\[
\mathcal{L}_{H^+ W Z} = \left[ \frac{g^2 v \sin \beta}{2 \cos \theta_W} - \frac{g m_W^2 \sin \beta}{m_Z \cos^2 \theta_W} \Delta_{tb} \right] H^+ W_\mu Z^\mu. \quad (26)
\]
Note that the \(H^+ W Z\) vertex at tree-level (first term of eq. (26)) is very small because it is proportional to \(\sin \beta \sim v_3/v\). Such a proportionality to \(\sin \beta\) is
maintained at one-loop level (second term of eq. (26)), so top corrections to $H^+ W Z$ are also small. In models with two Higgs doublets, $\Phi_\alpha = (\phi_\alpha^+, \frac{1}{\sqrt{2}}[\phi_\alpha + iI_\alpha])^T \alpha = 1, 2$ (such as the minimal supersymmetric model), the $H^+ W Z$ coupling is zero at tree-level\textsuperscript{[13]} but can be induced to one-loop order\textsuperscript{[14]}. In these models, if we now neglect $m_b$, the one-loop effective action is given by eq. (5) replacing $\Phi$ by $\Phi_2$, the Higgs doublet that couples to the top, and $\Sigma$ by $\Phi_1$. The $H^+$ is given by

\begin{equation}
H^+ = -\sin \beta \phi_1^+ + \cos \beta \phi_2^+,
\end{equation}

where now $\tan \beta = \langle \phi_2 \rangle / \langle \phi_1 \rangle$. From eqs. (5), (6), (12) and (27) we obtain

\begin{equation}
\mathcal{L}_{H^+ W Z} = -\frac{g^2 \phi_2^3}{4 \cos \theta_W} [B(\phi_2^2) - C(\phi_2^2)] |_{\phi_2 = \langle \phi_2 \rangle} \phi_2^+ W Kenmu^\mu = \frac{g^3 N c m_W^2 \cot \beta}{64 \pi^2 m_W \cos \theta_W} H^+ W Kenmu^\mu.
\end{equation}

Notice that the $H^+ W Z$ vertex arises only from the custodial breaking terms of eq. (5)\textsuperscript{[15]}.

3. Counterterm approach

Let us for simplicity assume $g' = 0$. In this case, the gauge sector of the SM depends on only two independent parameters $g$ and $v$. The conditions that we choose to fix the two counterterms $\delta g$ and $\delta v$ are:

a) We define the $Z$ mass to be the physical mass, \textit{i.e.}, $m_Z^{2, phy} \equiv m_Z^2 = g^2 v^2 / 4$. It follows that

\begin{equation}
\delta m_Z = \frac{1}{2} [v^2 g \delta g + g^2 v \delta v] = -A_Z,
\end{equation}

where $A_Z$ is the coefficient of $g^{\mu \nu}$ in the vacuum polarization tensor of the $Z$ (it corresponds to $\Pi_Z (\phi^2 = v^2) - m_Z^2$ in eq. (9)).

b) We identify $\frac{g^3}{\sqrt{2}} \equiv g^2 / 8 m_W^2$, which implies

\begin{equation}
v \delta v = -\frac{2}{g^2} A_W.
\end{equation}

If we now add a Higgs triplet to the SM, a new parameter, $v_3$, is introduced in the model. We fix $\delta v_3$ following the renormalization prescription of the Higgs sector.
of ref. [16], *i.e.*, the renormalized \( v_3 \) is defined to be the true VEV of \( \Sigma_0 \) at one-loop. Neglecting the mixing between the Higgs doublet and the triplet, which is of \( \mathcal{O}(v_3/v) \), one finds \( \delta v_3 = 0 \). Thus, eqs. (29) and (30) still hold, and the physical \( W \) mass is given by

\[
m_{W}^{2}\vert_{phy} = m_{Z}^{2} + \delta m_{Z}^{2} + g^{2} v_{3}^{2} + 2v_{3}^{2} g \delta g + A_{W} \\
= m_{Z}^{2} \left\{ 1 + \frac{A_{W} - A_{Z}}{m_{Z}^{2}} \right\} + 4v_{3}^{2} \left[ 1 + \frac{A_{W} - A_{Z}}{m_{Z}^{2}} \right], \tag{31}
\]

and eq. (13) is recovered. As was noted in section 2, a change in the renormalization prescription of the effective potential (or a change in the input data) implies a shift \( v_{3}^{2} \rightarrow v_{3}^{2}[1 + \Delta] \) with \( \Delta \sim \mathcal{O}(g^{2} m_{t}^{2}) \) and then a negligible change in eq. (31).

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