An Incentive Mechanism for Building a Secure Blockchain-based Industrial Internet of Things

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Abstract—The world-changing blockchain technique provides a novel method to establish a secure, trusted, and decentralized system for solving the security and personal privacy problems in Industrial Internet of Things (IIoT) applications. The mining process in blockchain requires miners to solve a proof-of-work puzzle, which requires high computational power. However, the lightweight IIoT devices cannot directly participate in the mining process due to the limitation of power and computational resources. The edge computing service makes it possible for IIoT devices to act as followers. We analyze the existence and uniqueness of the Stackelberg equilibrium, and propose an efficient algorithm to compute the Stackelberg equilibrium point. Furthermore, we evaluate the performance of our algorithm through extensive simulations, and analyze the strategies of blockchain platform and IIoT devices under different situations.

Index Terms—Industrial internet of things, blockchain, cloud mining, incentive mechanism, Stackelberg game.

I. INTRODUCTION

Currently, Internet of Things (IoT) has attracted more and more attention in many areas, such as smart cities, agriculture, health care, industry, etc. It is estimated that the total number of connected IoT devices will be 50 billion by the end of 2020 [1]. Specifically, the widespread application of the IoT has stimulated the evolution of factories to the fourth industrial revolution (Industry 4.0) [2]. Industrial IoT (IIoT) provides interconnection to smart factories by connecting different types of industrial machines and devices, which helps to realize the intelligent manufacturing. To deal with the huge number of IIoT devices, a traditional centralized architecture is applied to provide services for IIoT devices, where IIoT devices are connected to a cloud server through the internet. With the rapid growth of the number of IIoT devices and the performance requirement of the IIoT applications, however, the traditional centralized IIoT architecture faces many challenges, such as security, personal privacy, bandwidth constraint, and service delay [3]. To avoid these issues, some works introduce decentralized peer-to-peer (P2P) architectures for IIoT applications [4]–[6], where each device can exchange information or trade directly with other devices without a third-party organization. However, these P2P architectures still face with security and personal privacy issues.

In the past few years, blockchain, as a world-changing technology, has shown its excellent properties in many fields [7]–[10]. Blockchain is a decentralized public ledger that stores data in a list of blocks, these blocks are linked using cryptography, each block stores the hash value of the previous block. No central server is required for maintaining the blockchain, instead, all the blocks are copied and shared by each user in the blockchain network, and the blockchain is maintained by multiple participants in the P2P network. Blockchain provides a novel technology that helps establish a secure, trusted, and decentralized system for solving the security and personal privacy problems in IIoT applications. To generate a new block, the participants (miners) of the blockchain need to solve a proof-of-work (PoW) puzzle [11], which is hard to be solved but easy to be validated. The one who first solve the puzzle has the right to packet a new block (the process is called mining), and will get a reward from the platform. Nevertheless, the lightweight IIoT devices cannot directly participate in the mining process due to the huge computational resources requirement.

The edge computing architecture makes it possible for IIoT applications to establish a blockchain network, where IIoT devices can offload the computational tasks to edge servers [12]–[14]. Specifically, incentivized by the reward from the platform for packeting a new block, each IIoT device will purchase a certain amount of computational resources (such as CPU and GPU) from edge servers to participate in the mining process for maximizing its own profit. Some existing works [15], [16] study the pricing problem between resource provider (edge server) and miners (IIoT devices), i.e., the resource provider offer a price to maximize its own profit, and miners decide their demand for computational resource to maximize their payoffs. However, these works didn’t pay attention to the safety of the blockchain network. Generally, the blockchain platform will dynamically adjust the threshold value of the hash puzzle to stabilize the generating speed of

This work was supported by the National Natural Science Foundation of China (Grant NO.11671400, 61972404, 61672524). (Corresponding author: Deying Li.)
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the new blocks. That is, if the total computational power of all miners is small, the platform will give a large hash threshold value, otherwise, it will give a small hash threshold value. The total computational power of all miners will directly affect the safety of the blockchain network. An attacker who wants to tamper with the context in a block of the blockchain needs to solve the hash puzzle faster than the current whole network. Thus it’s more harder for the attacker to change the context of blocks if all miners provide more computational power in the mining process.

In this paper, therefore, we study the incentive mechanism of the blockchain platform to motivate IIoT devices to purchase more computational resources to participate in the mining process, thus that a more secure blockchain network can be established for IIoT. The main contributions of this paper are listed as follows.

- We design an incentive mechanism for the IIoT blockchain platform where the blockchain platform provides a certain reward to participating IIoT devices, to attract IIoT devices to purchase more computational power from edge servers to participate in the mining process, thereby building a more secure blockchain network.
- We analyze the relationship between the security of the blockchain network and the total computational power of the entire network, and give the probability that an attacker can successfully tamper with the blockchain.
- We formulate the interaction between blockchain platform and miners as a two-stage Stackelberg game. We analyze the existence and uniqueness of the Stackelberg equilibrium, and propose an efficient algorithm to compute the Stackelberg equilibrium point.
- We conduct extensive simulations to evaluate the performance of our proposed algorithm, and we analyze the strategies of platform and miners in different situations. Our work is helpful for the IIoT blockchain platform to set a reasonable reward pricing strategy to maximize its utility, which is closely related to the security of the blockchain network.

The remainder of this paper is structured as follows. In Section II, we introduce the related works of this paper. In Section III, we describe the system model and analyze the blockchain security that motivated our problem, and then we formulate our problem as a two-stage Stackelberg game. In Section IV, we analyze the existence and uniqueness of the Stackelberg equilibrium, and give the best strategies for miners and blockchain platform. We conduct extensive performance evaluations in Section V. And finally, we conclude this paper in Section VI.

II. RELATED WORKS

Recently, there are numerous works concentrate on the IoT blockchain networks. Huang et al. [17] build a redactable consortium blockchain based on the first threshold chameleon hash and accountable and sanitizable chameleon signature schemes, which is efficient for lightweight IIoT devices to operate. Fu et al. [18] propose an integrated architecture of cooperative computing to support the demand of computing power in IoT blockchain networks, their goal is to maximize the system energy efficiency. Xu et al. [19] propose a blockchain-enabled computation offloading method to ensure data integrity for IoT in mobile edge computing. To overcome the huge operational overhead in energy trading market in the industrial IoT, which is resulted by the frequent transactions, Hou et al. [20] investigate the local electricity storage for the blockchain-based energy trading in the IIoT. Their solutions can achieve a good tradeoff between credit utility and operational overhead. The authors in [21] integrate technologies such as service-oriented architecture, public key infrastructure and enablers for Service Selection with blockchain technology, and propose a new blockchain-based architecture for service interoperation in IoT, which ensures data validity and guarantees the trust of quality of service attributes for service selection. Xu et al. [22] propose a blockchain-based fair nonrepudiation network computing service provisioning scheme for IIoT, in which the blockchain is used as a service publication proxy and an evidence recorder. Their solutions overcome the drawbacks of traditional IIoT systems, where malicious services or clients may cheat others due to their own interests.

Moreover, there are a series of works study the blockchain from the aspect of auction or game theory. Yao et al. [16] model the resource management and pricing problem as a Stackelberg game, and they design a multiagent reinforcement learning algorithm to search the near-optimal policy. Li et al. [23] propose a secure energy trading system for industrial IoT, in which they use Stackelberg game theory design an optimal pricing strategy scheme for energy-coin loans to maximize the profits of credit banks. Jiao et al. [24] propose an auction-based market model for the trading between the cloud computing service provider and miners. Their purpose is to efficiently allocate computing resources to maximize the social welfare. Wang et al. [25] propose a blockchain and double auction mechanism-based decentralized electricity transaction mode for microgrids, to achieve secure and quick electricity transactions. Xiong et al. [26] formulate the interaction between the cloud providers and miners as a Stackelberg game, and apply backward induction to analyze the equilibria in each sub-game.
III. SYSTEM MODEL AND PROBLEM FORMULATION

In this section, we first introduce the edge computing system of the IIoT about blockchain network, we then analyze the security of the blockchain network, and finally, we formulate the incentive problem between blockchain platform and miners.

A. System Model

In the blockchain network, the core problem is to achieve distributed consensus. Satoshi Nakamoto proposed the PoW consensus protocol in 2008 which is used for Bitcoin [11]. In the PoW consensus, the users who want generate a new block need to solve a hash puzzle, which is very costly to be addressed but easy for others to verify. This process is called mining, and these users are termed as miners. These miners compete with each other to solve a hash puzzle, the one who first solve the hash puzzle has the right to generate a new block and will get a reward from the blockchain platform.

For the IIoT blockchain network, the lightweight IIoT devices cannot directly participate the mining process due to the limited computational capability. Incentivized by the reward from the blockchain platform, IIoT devices will purchase computation resource from edge servers, each edge server offers its own unit price for computational resource. As shown in Fig. 1, the blockchain network is maintained by all of the IIoT devices. As for the mining process, each miner will offload its computational task to the edge server to compete with others, as shown in Fig. 2. The probability of each miner winning the competition depends on the amount of computational resource it purchased. All the miners purchase computational resource with the goal of maximizing their own profits.

B. Blockchain Security

Blockchain is a list of blocks that are linked by block hash value, each block record a set of transactions. More specifically, a block contains two parts: block content and block header. The block content is the details of transactions information, which records all the inputs and outputs of each transaction. The block header consists of the previous block hash value, which is used as a cryptographic link that creates the chain, a version number that used for tracking for software or protocol updates, a timestamp that records the time at which the block is generated, a Merkle tree root of all the transactions, a hash threshold value that records the current mining difficulty, and a nonce, which is used for solving the PoW puzzle.

The blockchain starts with a genesis block which is given by the blockchain platform, all subsequent blocks will put some previously generated block’s hash value into their block header. Miners compete with each other to solve a hash puzzle, the one who first solve the puzzle has the right to generate a new block, and new blocks will be added behind the genesis block. Forks may happen when multiple miners solve the hash puzzle at the same time, thus each user maintains the blocks in the form of a block tree [29]. According to the longest chain principle [11], each user will choose the longest branch in the block tree as the current blockchain, as shown in the left part of Fig. 3.

For a blockchain miner, to get the right to attach a new block to the chain, it needs to solve a hash function (PoW puzzle), that is, it needs to find a nonce and record it in the block header such that the hash value of the block header is less than the hash threshold. As the hash function has no back door, the only way to find such a nonce is to run many hash operations. The difficulty of the PoW puzzle is determined by the given hash threshold value, which is a 256-bit binary number that starts with a certain number of consecutive zeros (difficulty). For example, if the hash threshold starts with 60 consecutive zeros, the probability of finding the correct nonce by performing a hash operation is $2^{-60}$, which means that it takes an average of $2^{60}$ hash operations to solve the PoW puzzle. To stabilize the growth rate of the blockchain, the platform will dynamically adjust the difficulty of the PoW puzzle based on the total computational power of the whole blockchain network. Take
the bitcoin blockchain as an example, the difficulty of PoW puzzle will be updated every 2 weeks to ensure that it takes 10 minutes (on average) to add a new block to the blockchain.

Assume that an attacker wants to tamper with the context of a block, the change of the context will change the hash value of the block, so the attacker needs to find a new nonce to solve the PoW puzzle of this block. Moreover, as each block contains the hash value of the previous block, the change of a block context will change all the subsequent blocks, so the attacker should find the nonce for every subsequent block. In fact, the attacker needs to fork a new branch, and start a block mining race against other miners of the network. The attacker successfully tamper with the blockchain once the attacker wins the mining race against other miners of the network. The attacker needs to fork a new branch, and start a block mining race against other miners of the network. The attacker should find the nonce for every subsequent block. In fact, the attacker needs to fork a new branch, and start a block mining race against other miners of the network. The attacker successfully tamper with the blockchain once the attacker wins the mining race against other miners of the network.
assume that the blockchain will generate an average of $N$ new blocks per day. Then the expected reward of miner $s_i$ in a day is $p_iRN$, and its cost is $\lambda_i\mu_i$. We use $P_i$ to represent the expected profit of miner $s_i$ in one day, $P_i$ can be calculated as follows,

$$P_i = p_iRN - \lambda_i\mu_i. \tag{5}$$

Both blockchain platform and miners will dynamically adjust their strategies to get the maximum profit. We model the interaction between blockchain platform and miners as a two-stage Stackelberg game. In the upper stage, the blockchain platform sets the reward to incentivize miners participate the mining process. In the lower stage, miners decide the optimal amount of computational power they purchase. We formulate the optimization problems for the blockchain platform and miners as follows.

We first introduce the lower stage of the game. Given the reward $R$ of the blockchain platform and other miners’ strategies $\mu_{-i}$, where $\mu_{-i} = \{\mu_1, \mu_2, \ldots, \mu_{i-1}, \mu_{i+1}, \ldots, \mu_n\}$. The miner $s_i$ decides the amount of computational power $\mu_i$ it purchased to maximize its own profit. This sub-game problem can be written as follows.

**Problem 1.** miners’ sub-game.

$$\max_{\mu_i} \quad P_i(\mu_i|\mu_{-i}, R)$$

s.t. \hspace{0.5cm} \mu_i \geq 0

For the upper stage of the game, the blockchain platform will dynamically adjust the reward $R$ to maximize its utility. As defined in equation (3), the utility of the blockchain platform is directly related to the strategies $\mu$ of miners, where $\mu = \{\mu_1, \mu_2, \ldots, \mu_n\}$, and the reward $R$. This sub-game can be formulated as follows.

**Problem 2.** blockchain platform’s sub-game.

$$\max_{R} \quad U(R, \mu)$$

s.t. \hspace{0.5cm} 0 \leq R \leq B

Problem 1 and Problem 2 together form a Stackelberg game. The object of the game is to find a Stackelberg equilibrium point where neither the leader (blockchain platform) nor the followers (miners) want to change their strategies. In this paper, the Stackelberg equilibrium can be defined as follows.

**Definition 1.** Let $\mu^*$ and $R^*$ be the optimal strategies of miners and blockchain platform, respectively, where $\mu^* = \{\mu_1^*, \mu_2^*, \ldots, \mu_n^*\}$. Then, the point $(\mu^*, R^*)$ is the Stackelberg equilibrium point if it satisfies the following two conditions,

$$U(R^*, \mu^*) \geq U(R, \mu^*), \forall 0 \leq R \leq B, \tag{6}$$

and

$$P_i(\mu_i|\mu_{-i}^*, R^*) \geq P_i(\mu_i|\mu_{-i}^*, R^*), \forall i, \forall \mu_i \geq 0, \tag{7}$$

where $\mu_{-i}^* = \mu^* \setminus \{\mu_i^*\}$.

IV. GAME ANALYSIS FOR THE INCENTIVE MECHANISM

In this section, we analyze the existence and the uniqueness of the Stackelberg equilibrium of our proposed Stackelberg game. We first analyze the lower stage of the game, in which we aim to find the Nash equilibrium for the miners’ sub-game. Based on the analysis of the lower stage, we then analyze the utility maximization of the blockchain platforms sub-game in the upper stage.

A. Analysis of the miners’ sub-game

After the blockchain platform set the reward $R$ for miners, all of the miner will dynamically adjust their strategies to get the maximum profits until reach a Nash equilibrium. In the following, we will prove that the Nash equilibrium point exists in the miners’ sub-game through a theorem.

**Theorem 1.** The Nash equilibrium point exists in the miners’ sub-game.

**Proof.** For the miners’ sub-game, the object function $P_i(\cdot)$ is defined in $[0, \infty)$. From equation (5), we can know that $\mu_i \leq \frac{RN}{\lambda_i}$, otherwise, the profit of miner $s_i$ will be negative. Thus $\mu_i$ is continuously chosen in $[0, \frac{RN}{\lambda_i}]$, which is a non-empty, convex and compact subset of the Euclidean space. Next, we calculate the first order and second order derivatives of function $P_i(\cdot)$.

$$\frac{\partial P_i}{\partial \mu_i} = RN \sum_{s_j \in S \setminus \{s_i\}} \mu_j - \mu_i - \lambda_i, \tag{8}$$

$$\frac{\partial^2 P_i}{\partial \mu_i^2} = 2RN \sum_{s_j \in S \setminus \{s_i\}} \mu_j - \mu_i - \lambda_i \leq 0. \tag{9}$$

Therefore, $P_i(\cdot)$ is a strictly concave function, and we then conclude that the Nash equilibrium point exists in the miners’ sub-game. \hfill \Box

As $P_i(\cdot)$ is a strictly concave function with $\mu_i$, given the reward $R$ of the blockchain platform and other miners’ strategies $\mu_{-i}$, miner $s_i$ has a unique best strategy $u_i$, and it can be achieved when the first order derivative of $P_i(\cdot)$ equals to 0, i.e.,

$$\frac{\partial P_i}{\partial \mu_i} = RN \sum_{s_j \in S \setminus \{s_i\}} \mu_j - \mu_i - \lambda_i = 0. \tag{10}$$

Solving equation (10), we have $\mu_i = \sqrt{\frac{RN \sum_{s_j \in S \setminus \{s_i\}} \mu_j}{\lambda_i}}$, or

$$\sum_{s_j \in S \setminus \{s_i\}} \mu_j \leq \lambda_i, \tag{11}$$

where $\mu_{-i} = \mu^\ast \setminus \{\mu^*_i\}$.

**Corollary 1.** Given the optimal strategies of miners $\mu^* = \{\mu^*_1, \mu^*_2, \ldots, \mu^*_n\}$, for any $\mu^*_i, \mu^*_j \in \mu^*$, if $\lambda_i \leq \lambda_j$, then $\mu^*_i \geq \mu^*_j$. 

Proof. We prove this corollary by contradiction. Suppose that there exist two miners’ strategies \( \mu^*_i, \mu^*_j \in \mu^* \), where \( \lambda_i \leq \lambda_j \) and \( \mu^*_i > \mu^*_j > 0 \). As \( \mu^*_j > 0 \), according to equations (10) and (11), we can easily know that \( \frac{\partial P_i}{\partial \mu_j} \big|_{\mu^*_j} = 0 \).

Similarly, If \( \mu^*_i > 0 \), we have \( \frac{\partial P_i}{\partial \mu_i} \big|_{\mu^*_i} = 0 \). If \( \mu^*_i = 0 \), according to equation (11), we have \( \sum_{k \in S \setminus \{i\}} \mu^*_k \leq \lambda_i \), substituting it in to equation (10), we have \( \frac{\partial P_i}{\partial \mu_i} \big|_{\mu^*_i} \leq 0 \).

In summary, we know that \( \frac{\partial P_i}{\partial \mu_i} \big|_{\mu^*_i} \leq 0 \). Therefore, the corollary holds.

Sort the miners in ascending order of \( \lambda_i \), for clarity, miners are renumbered and still denoted by \( S = \{s_1, s_2, \ldots , s_n\} \). According to Corollary 1, we have \( \mu_1 \geq \mu_2 \geq \cdots \geq \mu_n \geq 0 \).

We assume that the the first \( q \) miners have a non-zero strategy, i.e., \( \mu_q > 0 \), \( \mu_{q+1} = 0 \). We let \( S_q = \{s_1, s_2, \ldots , s_q\} \). It’s obvious that \( \sum_{s_j \in S} \mu_j = \sum_{s_j \in S_q} \mu_j \), and we have the following corollary.

Corollary 2. Let \( q \) be the number of miners that have a non-zero strategy in a Nash equilibrium, then we have \( q \geq 2 \).

Proof. Firstly, it’s clear that \( q > 0 \), otherwise, according to equation (5), any miner \( s_i \) which purchase computational power with the amount in \( (0, \frac{RN}{\lambda_1}) \) can get more profit. Then we consider the case that \( q = 1 \). Let \( s_1 \) be the miner who has a positive strategy, and its current best strategy is to purchase \( \mu_1 \) amount of computational power. According to equation (5), \( s_1 \)’s current profit is \( RN - \lambda_1 \mu_1 \). However, \( s_1 \) can increase its profit by continuously reducing \( \mu_1 \) to 0, which indicates that miners hadn’t reach a Nash equilibrium point. Therefore, the corollary holds.

Summing up equation (10) with \( i = 1, 2, \ldots , q \), we have
\[
\frac{RN(q-1)}{\sum_{s_j \in S} \mu_j} - \sum_{s_j \in S} \lambda_i = 0.
\] (13)

Thus we have
\[
\sum_{s_j \in S} \mu_j = \frac{RN(q-1)}{\sum_{s_j \in S} \lambda_i}.
\] (14)

Substituting equation (14) into equation (10), we obtain
\[
\mu_i = \frac{RN(q-1)}{\sum_{s_j \in S} \lambda_j} \left( 1 - \frac{(q-1)\lambda_i}{\sum_{s_j \in S} \lambda_j} \right).
\] (15)

As \( \mu_q > 0 \), and we have proven that \( q \geq 2 \), we then have
\[
1 - \frac{(q-1)\lambda_q}{\sum_{s_j \in S} \lambda_j} > 0, \text{ i.e., } \lambda_q < \frac{\sum_{s_j \in S} \lambda_j}{q-2}.
\]

Algorithm 1 Calculate Nash equilibrium for miners.

1. Sort miners in ascending order of \( \lambda \), and renumber miners, i.e., \( \lambda_1 \leq \lambda_2 \leq \cdots \leq \lambda_n \).
2. \( S' = \{s_1, s_2\} \), \( q = 2 \)
3. while \( q < n \) and \( \lambda_{q+1} < \frac{\sum_{s_j \in S'} \lambda_j}{|S'|-1} \)
4. \( S' \leftarrow S' \cup \{s_{q+1}\} \)
5. \( q \leftarrow q + 1 \)
6. end while
7. for \( i = 1; i \leq n; i + + \) do
8. if \( s_i \in S' \) then
9. \( \mu_i = \frac{RN(q-1)}{\sum_{s_j \in S'} \lambda_j} \left( 1 - \frac{(q-1)\lambda_i}{\sum_{s_j \in S'} \lambda_j} \right) \)
10. else
11. \( \mu_i = 0 \)
12. end if
13. end for
14. return \( \mu^* = \{\mu_1^*, \mu_2^*, \ldots , \mu_n^*\} \)

Based on the above analysis, we design the following algorithm to find the Nash equilibrium point for the miners’ sub-game.

In the following, we first prove the strategies produced by Algorithm 1 is a Nash equilibrium for the miners’ sub-game, then we prove that the Nash equilibrium is unique.

Theorem 2. The strategies produced by Algorithm 1 is a Nash equilibrium for the miners’ sub-game.

Proof. For miners in \( S' \), their strategies are calculated by equation (15), it’s clear that these miners get the current best strategies as the first order of \( P_i(\cdot) \) equals to 0 for \( s_i \in S' \). To prove the theorem, we only need to prove that for any miner \( s_j \in S \setminus S' \), its current best strategy is 0. According to the description of Algorithm 1, we have
\[
\lambda_j \geq \frac{\sum_{s_j \in S \setminus S'} \lambda_j}{|S'|-1}, \forall s_j \in S \setminus S'.
\] (16)

From equation (14), we know that \( \sum_{s_j \in S'} \lambda_j = \frac{RN(|S'|-1)}{\sum_{s_j \in S \setminus S'} \mu_i} \), substituting it into equation (16), we obtain that \( \lambda_j \) for any \( s_j \in S \setminus S' \). As \( s_j \notin S' \), we have \( \mu_i = \sum_{s_j \in S'} \mu_i \). Therefore, \( \frac{RN}{\sum_{s_j \in S \setminus S'} \lambda_j} \leq \lambda_j \) for any \( s_j \in S \setminus S' \). According to equation (11), we know that the current best strategy for any miner \( s_j \in S \setminus S' \) is 0. Thus the theorem holds.

The following corollary helps us to prove the uniqueness of the Nash equilibrium for the miners’ sub-game.

Corollary 3. Given any Nash equilibrium \( \mu^{ne} = \{\mu_{1}^{ne}, \mu_{2}^{ne}, \ldots , \mu_{n}^{ne}\} \) for the miners’ sub-game, let \( S_h \) be the set of miners with a non-zero strategy, then we have \( S_h = S' \), where \( S' \) is got by Algorithm 1 and \(|S'| = p \).

Proof. Assume that miners have been sorted in ascending order of \( \lambda_i \). According to Corollary 1, we know that \( \mu_{1}^{ne} \geq \mu_{2}^{ne} \geq \cdots \geq \mu_{n}^{ne} \geq 0 \). Suppose that \( S_h = \{s_1, s_2, \ldots , s_h\} \), i.e. \(|S_h| = h \). To prove \( S_h = S' \), we only need to prove that \( h = p \). If \( h > p \), then \( s_{p+1} \in S_h \), from description
of Algorithm 1 we have \( \lambda_{p+1} \geq \sum_{s_i \in S'} \lambda_i \geq \sum_{s_i \in S_h} \lambda_i \), substituting this into equation (15), we obtain that \( \mu_{h+1}^\nu \geq 0 \), which contradicts \( s_{h+1} \in S_h \). If \( h < p \), then we have \( \mu_{h+1} = 0 \) and \( \lambda_{h+1} < \sum_{s_i \in S_h} \lambda_i \), according to equation (10), the first order derivative of \( P_{h+1}' \) with respect to \( \mu_{h+1} \) when \( \mu_{h+1} = \mu_{h+1}^\nu = 0 \) is

\[
\frac{\partial P_{h+1}}{\partial \mu_{h+1}}(0, \mu_{h+1}^\nu) = R N \sum_{s_j \in S_h} \mu_j - \frac{\lambda_{h+1}}{h} - \lambda_{h+1},
\]

where \( \mu_{h+1}^\nu(s_{h+1}) = \mu_{h+1}^\nu \setminus \{ \mu_{h+1}^\nu \} \).

According to equation (14), we have \( \sum_{s_j \in S_h} \mu_j = \frac{R N (h+1)}{\sum_{s_j \in S_h} \lambda_j} \), substituting it into equation (17), we have that

\[
\frac{\partial P_{h+1}}{\partial \mu_{h+1}}(0, \mu_{h+1}^\nu) = \frac{\sum_{s_j \in S_h} \lambda_j}{h} - \lambda_{h+1}.
\]

As \( \lambda_{h+1} < \sum_{s_j \in S_h} \lambda_j = \frac{\sum_{s_j \in S_h} \lambda_j}{h-1} \), we know that \( \frac{\partial P_{h+1}}{\partial \mu_{h+1}}(0, \mu_{h+1}^\nu) > 0 \), which implies that miner \( s_{h+1} \) can improve its profit by increasing its strategy \( \mu_{h+1}^\nu \). This contradicts that \( \mu_{h+1}^\nu \) is an Nash equilibrium. Therefore, we have \( h = p \), and thus the corollary holds.

**Theorem 3.** The miners’ sub-game has a unique Nash equilibrium point.

**Proof.** According to Corollary 3, the miners’ sub-game can be seen as a game among miners in \( S' \), as for any miner \( s_i \in S' \), we always have \( \mu_i = 0 \) in a Nash equilibrium. Therefore, we only need to prove that the sub-game of miners in \( S' \) has a unique Nash equilibrium point.

As the strategy of each miner in \( S' \) is positive, and the profit \( P_i(\cdot) \) for any \( s_i \in S' \) is a concave function according to equation (9), each miner will get its best strategy when the first order derivative of \( P_i(\cdot) \) equals to 0. As shown in equations (13)–(15), we get unique solutions by solving the set of functions that the first order derivative of \( P_i(\cdot) \) equals to 0 for each \( s_i \in S' \). Therefore, the miners’ sub-game among miners in \( S' \) has a unique Nash equilibrium, and thus we can conclude that Theorem 3 holds.

**B. Analysis of the blockchain platform’s sub-game**

According to the analysis in Section IV-A, for any value of reward \( R \) given by the blockchain platform, there always exists a unique Nash equilibrium for the miners. Therefore, given a value of \( R \), the blockchain platform has a unique utility, and it can maximize its utility by setting an optimal \( R \).

Substituting the result of Algorithm 1 into equation (3) and combining equation (14), we have

\[
U = \alpha \left[ \sigma \left( \beta \cdot \sum_{s_i \in S'} \mu_i^* \right) - \frac{1}{2} \right] - R
\]

where \( X = \frac{N(q-1)}{\sum_{s_i \in S'} \lambda_i} \).

**Theorem 4.** There exists a unique Stackelberg equilibrium \( (\mu^*, R^*) \) in our proposed Stackelberg game, where \( \mu^* \) and \( R^* \) are optimal strategies for miners and blockchain platform.

**Proof.** As the definition of the blockchain platform’s sub-game, the utility function \( U(\cdot) \) is defined with \( R \in [0, B] \). We then calculate the first order and second order derivatives of \( U(\cdot) \) with respect to \( R \),

\[
\frac{\partial U}{\partial R} = \alpha \beta X \sigma(\beta XR)(1 - \sigma(\beta XR)) - 1,
\]

\[
\frac{\partial^2 U}{\partial R^2} = \alpha \beta^2 X^2 \sigma'(\beta XR)(1 - \sigma(\beta XR))(1 - 2\sigma(\beta XR)).
\]

As \( \beta XR \geq 0 \), the range of the sigmoid function \( \sigma(\beta XR) \) is \([\frac{1}{2}, 1]) \), and then we have \( 1 - \sigma(\beta XR) > 0 \) and \( 1 - 2\sigma(\beta XR) < 0 \). Thus \( \frac{\partial^2 U}{\partial R^2} \leq 0 \) holds. Therefore the utility function \( U(\cdot) \) is strictly concave with \( R \in [0, B] \). It means that a unique \( R^* \) can be found to maximize \( U(\cdot) \). Combined with Theorem 3, we conclude that there exists a unique Stackelberg equilibrium in our proposed Stackelberg game.

The maximization of \( U(\cdot) \) is achieved either at the extreme point where the first order derivative of \( U(\cdot) \) equals to 0, or at the boundary of domain area (i.e., \( R = 0 \) or \( B \)). If \( \alpha \beta X < 4 \), we have \( \frac{\partial U}{\partial R} < 0 \), \( \forall R \in [0, B] \), then \( U(\cdot) \) is a decreasing function in \([0, B]\), and the best strategy of the blockchain platform is \( R^* = 0 \). If \( \alpha \beta X \geq 4 \), by solving the equation \( \frac{\partial U}{\partial R} = 0 \), we have \( \sigma(\beta XR) = \sqrt{\frac{1}{4} - \frac{1}{\alpha \beta X}} + \frac{1}{2} \), and thus the best strategy of the blockchain platform is

\[
R^* = \min \left\{ \frac{1}{\alpha X} \log \left( \frac{\frac{1}{4} + \frac{1}{\alpha \beta X}}{\frac{1}{4} + \frac{1}{\alpha \beta X}} \right), B \right\}.
\]

In summary, we have

\[
R^* = \begin{cases} 
0, & \text{if } \alpha \beta X < 4, \\
\min \left\{ \frac{1}{\alpha X} \log \left( \frac{\frac{1}{4} + \frac{1}{\alpha \beta X}}{\frac{1}{4} + \frac{1}{\alpha \beta X}} \right), B \right\}, & \text{otherwise.}
\end{cases}
\]

**V. PERFORMANCE EVALUATION**

In this section, we conduct extensive simulations to evaluate the performance of our proposed incentive mechanism for blockchain-based internet of things.

**A. Experimental settings**

In our experiments, we set the basic parameters of our problem as follows. We assume there are totally 1000 IoT devices that are interested in participating in the blockchain mining process, i.e., \(|S| = 1000\). The unit price \( \lambda_i \) of each miner purchasing computational power from edge servers is uniformly range from 100 to 105. For the blockchain platform utility model, \( \alpha \) is set to be 10000, \( \beta \) is set to be 0.001, and \( B \) is set to be 2000. We assume that it takes an average of 10 minutes for the blockchain platform to generate a new block, and thus it will generate an average of 144 new blocks per
day, i.e., \( N \) is set to be 144. Unless otherwise stated, the above parameters will be set as default settings. Each value in figures in this section is the average of 100 runs.

### B. Results and Analyses

1) **Number of participating miners (\(|S'||)\**: As described in Algorithm 1 only the miners in \( S' \) will purchase computational power to participate in the mining process, then we study the impact of unit price of computational power and number of IIoT devices on \(|S'||\).

In Fig. 5 the minimum unit price of computational power is set to be 100, and \(|S|\) is set to be 1000. The difference of unit price represents the range of \( \lambda_i \) for each miner. For example, when the difference of unit price is set to be 2%, then \( \lambda_i \) is randomly chosen in \([100, 102]\) for each miner. From Fig. 5 we can see that \(|S'||\) decreases when the difference of unit price of computational power increases. This is because when the difference of unit price is large, \( \lambda_i \) of each miner will become diverse, and thus there will be more miners violate the condition of Algorithm 1. We can also see that the effect of the difference of the unit price of computational power is very significant, even when the difference of unit price is as small as 1%, there are only about 44.8% miners participate the blockchain mining process.

In Fig. 6 we fix the difference of unit price of computational power at 5%, and study how the number of IIoT devices affect \(|S'||\). It can be seen that the growth of \(|S'||\) does not have a linear relationship with the number of IIoT devices. When \(|S|\) = 500, there are about 28.5% miners participate in the blockchain mining process, and when \(|S|\) increased to 5000, there are only about 9% miners participate the blockchain mining process.

2) **Effect of unit price \( \lambda_i \) on utilities and strategies**: We fix the difference of unit price of computational power at 5%, and study how \( \lambda_i \) of each miner affect the utilities and strategies of blockchain platform and miners.

Fig. 7 shows that when the unit price of computation power increases, the blockchain platform needs to improve the reward to achieve maximum utility until the maximum value of the reward is reached. And miners tend to purchase less computa-
that all miners have been sorted in ascending order of \( \lambda \), the reason is that when \( |S| \) increases from 500 to 1000, \( \lambda_i \) of each miner becomes more tight, and then the difference between its unit price of computational power and that of other miners in front of it decreases. So miner \( s_{100} \) become more competitive and thus can get more profits. In detail, when \( |S| = 500 \), \( \lambda_1 = 100.011, \lambda_{50} = 100.504 \) and \( \lambda_{100} = 100.992 \); when \( |S| = 1000 \), \( \lambda_1 = 100.004, \lambda_{50} = 100.247 \) and \( \lambda_{100} = 100.496 \). We can also see that the unit price of computational power will significantly affect the profits of miners, especially when the number of participating miners is small. For example, when \( |S| = 500 \), the profit of miner \( s_1 \) is 2.5 times the profit of the miner \( s_{50} \) and 10.6 times the profit of miner \( s_{100} \), while the unit price of the computational power of the miners \( s_{50} \) and \( s_{100} \) is 0.49% and 0.98% larger than that of \( s_1 \), respectively.

VI. CONCLUSION

In this paper, we design an incentive mechanism for IIoT blockchain network that can be used to motivate IIoT devices to purchase more computational resources from edge servers to participate in the blockchain mining process, thus that a secure blockchain network can be established. We analyze the relationship between the security of the blockchain network and the total computational power of the entire network, and give the probability that an attacker can successfully tamper with the blockchain. We model the interaction between blockchain platforms and IIoT devices as a two-stage Stackelberg game, in which the leader, i.e., the blockchain platform, first sets theStackelberg equilibrium point. We also conduct extensive simulations to evaluate the performance of our designs. Our work is helpful for the IIoT blockchain platform to set a reasonable reward to build a secure blockchain network.

REFERENCES

[1] A. Nordrum, “Popular internet of things forecast of 50 billion devices by 2020 is outdated,” https://spectrum.ieee.org/tech-talk/telecom/internet/popular-internet-of-things-forecast-of-50-billion-devices-by-2020-is-outdated, last accessed 17 Mar 2020. [Online].
[2] F. Shrouf, J. Ordieres, and G. Miragliotta, “Smart factories in industry 4.0: A review of the concept and of energy management approaches in production based on the internet of things paradigm,” in 2014 IEEE international conference on industrial engineering and engineering management. IEEE, 2014, pp. 697–701.
[3] M. Rehman, N. Iavaid, M. Awais, M. Imran, and N. Naseer, “Cloud based secure service providing for iots using blockchain,” in IEEE Global Communications Conference (GLOBECOM 2019), 2019.
[4] S. Krco, D. Cleary, and D. Parker, “P2p mobile sensor networks,” in Proceedings of the 38th Annual Hawaii International Conference on System Sciences. IEEE, 2005, pp. 324c–324c.
[5] R. Mietz, S. Groppe, O. Kleine, D. Bimschas, S. Fischer, K. Römer, and D. Pristerer, “A p2p semantic query framework for the internet of things,” PIK-Praxis der Informationsverarbeitung und Kommunikation, vol. 36, no. 2, pp. 73–79, 2013.
X. Xu, X. Zhang, H. Gao, Y. Xue, L. Qi, and W. Dou, “Become: A blockchain-powered crowdsourcing method with privacy preservation in mobile environment,” IEEE Transactions on Computational Social Systems, vol. 6, no. 6, pp. 1407–1419, 2019.

Q. Yang, R. Lu, C. Rong, Y. Challal, M. Laurent, and S. Wang, “Guest editorial: the convergence of blockchain and iot: Opportunities, challenges and solutions,” IEEE Internet of Things Journal, vol. 6, no. 3, pp. 4556–4560, 2019.

J. Wan, J. Li, M. Imran, and D. Li, “A blockchain-based solution for enhancing security and privacy in smart factory,” IEEE Transactions on Industrial Informatics, vol. 15, no. 6, pp. 3652–3660, 2019.

S. Nakamoto, “Bitcoin: A peer-to-peer electronic cash system,” Manubot, Tech. Rep., 2019.

M. Liu, F. R. Yu, Y. Teng, V. C. Leung, and M. Song, “Computation offloading and content caching in wireless blockchain networks with mobile edge computing,” IEEE Transactions on Vehicular Technology, vol. 67, no. 11, pp. 11008–11021, 2018.

S. Guo, X. Hu, S. Guo, X. Qiu, and F. Qi, “Blockchain meets edge computing: A distributed and trusted authentication system,” IEEE Transactions on Industrial Informatics, 2019.

M. Liu, F. R. Yu, Y. Teng, V. C. Leung, and M. Song, “Distributed resource allocation in blockchain-based video streaming systems with mobile edge computing,” IEEE Transactions on Wireless Communications, vol. 18, no. 1, pp. 695–708, 2018.

Z. Xiong, Y. Zhang, D. Niyato, P. Wang, and Z. Han, “When mobile blockchain meets edge computing,” IEEE Communications Magazine, vol. 56, no. 8, pp. 33–39, 2018.

H. Yao, T. Mai, J. Wang, Z. Ji, C. Jiang, and Y. Qian, “Resource trading in blockchain-based industrial internet of things,” IEEE Transactions on Industrial Informatics, vol. 15, no. 6, pp. 3602–3609, 2019.

K. Huang, X. Zhang, Y. Mu, X. Wang, G. Yang, X. Du, F. Rezaeibagha, Q. Xia, and M. Guizani, “Building redactale consortium blockchain for industrial internet-of-things,” IEEE Transactions on Industrial Informatics, vol. 15, no. 6, pp. 3670–3679, 2019.

S. Fu, Q. Fan, Y. Tang, H. Zhang, X. Jian, and X. Zeng, “Cooperative computing in integrated blockchain based internet of things,” IEEE Internet of Things Journal, 2019.

X. Xu, X. Zhang, H. Gao, Y. Xue, L. Qi, and W. Dou, “Become: Blockchain-enabled computation offloading for iot in mobile edge computing,” IEEE Transactions on Industrial Informatics, 2019.

W. Hou, L. Guo, and Z. Ning, “Local electricity storage for blockchain-based energy trading in industrial internet of things,” IEEE Transactions on Industrial Informatics, vol. 15, no. 6, pp. 3610–3619, 2019.

W. Viriyasitavat, L. Da Xu, Z. Bi, and A. Sapsomboon, “New blockchain-based architecture for service interoperations in internet of things,” IEEE Transactions on Computational Social Systems, vol. 6, no. 4, pp. 739–748, 2019.

Y. Xu, J. Ren, G. Wang, C. Zhang, J. Yang, and Y. Zhang, “A blockchain-based nonrepudiation network computing service scheme for industrial iot,” IEEE Transactions on Industrial Informatics, vol. 15, no. 6, pp. 3632–3641, 2019.

Z. Li, J. Kang, R. Yu, D. Ye, Q. Deng, and Y. Zhang, “Consortium blockchain for secure energy trading in industrial internet of things,” IEEE transactions on industrial informatics, vol. 14, no. 8, pp. 3690–3700, 2017.

Y. Jiao, P. Wang, D. Niyato, and K. Suankaewmanee, “Auction mechanisms in cloud/fog computing resource allocation for public blockchain networks,” IEEE Transactions on Parallel and Distributed Systems, vol. 30, no. 9, pp. 1975–1989, 2019.

J. Wang, Q. Wang, N. Zhou, and Y. Chi, “A novel electricity transaction mode of microgrids based on blockchain and continuous double auction,” Energies, vol. 10, no. 12, p. 1971, 2017.

Z. Xiong, S. Feng, W. Wang, D. Niyato, P. Wang, and Z. Han, “Cloud/fog computing resource management and pricing for blockchain networks,” IEEE Internet of Things Journal, vol. 6, no. 3, pp. 4585–4600, 2019.

S. Biswas, K. Sharif, F. Li, B. Nour, and Y. Wang, “A scalable blockchain framework for secure transactions in iot,” IEEE Internet of Things Journal, vol. 6, no. 3, pp. 4650–4659, 2018.