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Controlling the dynamics of cloud cavitation bubbles through acoustic feedback

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Cloud cavitation causes nontrivial energy concentration and acoustic shielding in liquid, and its control is a long-standing challenge due to complex dynamics of bubble clouds. We present a framework to study closed-loop control of cavitation through acoustic feedback. While previous approaches used empirical thresholding, we employ model-based state estimation of coherent bubble dynamics based on theory and high-performance computing. Using a pulsed ultrasound setup, we demonstrate set-point control of the pulse repetition frequency (PRF) to modulate acoustic cavitation near a solid target over $O(100)$ s. We identify a quasi-equilibrium correlation between PRF and the bubble dynamics, and an optimal PRF to minimize acoustic shielding of the target. This framework can be readily scaled up by enhanced acoustic sensing and computational power.

I. INTRODUCTION

Control of cloud cavitation - nucleation of bubble clusters due to rapid fall of the local pressure in liquid - is a long-standing challenge for optimization of medical and hydraulic systems, as well as sonoluminescence and sonochemistry [1–10]. For example, in extracorporeal ultrasound (US) therapy, the tensile component of high-intensity focused ultrasound (HIFU) can nucleate cavitation bubbles in the human body. These bubbles violently oscillate and collapse at a submicrosecond time scale to cause damage in surrounding materials [8, 11–13] as well as acoustic shielding of the targets [14, 15]. The intensity of cavitation can largely fluctuate due to non-equilibrium, stochastic nature of nucleation events, depending on the applied pressure fields [16]. Once nucleated, bubble clouds can persist and proliferate by subsequent waves that arrive before dissolution [17–19]. These fascinating bubble dynamics have been quantified through advanced experiments [10, 20, 21], although direct observation is limited to specialized setups. In practice, far-field, bubble-scattered acoustic signals are the only observable quantities. For US-induced acoustic cavitation, open-loop control of the US waveform has been explored to favorably trigger violent collapse of bubble clouds to enhance cavitation erosion [22]. Closed-loop control has been utilized to excite stationary cavitation by modulating incident US waves, such that a stable acoustic feedback is maintained [23–28]. These systems, however, rely on empirical thresholds and lack a quantifiable state estimation of cavitation, as the bubble dynamics are not modeled in the feedback loop. This limitation motivates us to pursue model-based feedback control, not only to stabilize cavitation but also to modulate cavitation based on a quantitative state estimation.

Modeling of cloud cavitation has been extensively explored in the past decades [29]. A recent effort has identified a scaling parameter that dictates the coherent dynamics of spherical bubble clouds in ultrasound fields, the dynamic cloud interaction parameter: $B_D = N < R_b > / R_c$, where $N$ and $R_b$ are the total number and the average radius of bubbles in the cloud, $R_c$ is the radius of the cloud, and $<>$ denotes the time-average during periodic oscillations [30]. The parameter characterizes the structure of bubble clouds and the bubble-induced acoustic fields. In the limit of linear oscillations of bubbles, a static form of such a parameter can be obtained using a mean-field theory [31]. $B_D$ was derived from the first-principle hydrodynamic many-body theory and extends to nonlinear dynamics of cavitation bubbles that grow far from equilibrium. In experiments, correlations have been identified between the energy state of bubble clouds and the bubble-scattered acoustic waves [15, 30]. These results are promising for real-time estimation of bubble dynamics through acoustic measurements [32]. In this work, we design a framework to study model-based closed-loop control of cloud cavitation through acoustic feedback, and demonstrate its implementation in a pulsed US system.

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II. METHODS

A. Theory and setup

Figure 1 shows the schematic of the control and the US setup. In the setup, pulses of a US wave are generated by a transducer and focused on a cylindrical target made of epoxy resin, with the base diameter and the length of 6.25 and 10 mm in water. Each pulse contains 10 cycles of a sinusoidal wave packet with a frequency of 340 kHz and a peak focal amplitude of 7.0 MPa. Following our previous study, the water is degassed to realize an \( \text{O}_2 \) level of 65% saturation [15]. The \( \text{O}_2 \) level is positively correlated with the intensity of cavitation since the concentration of non-condensible gas in water controls the number of nuclei that are cavitated, as well as the violence of cavitation collapse and post-collapse bubble dissolution rate. Although the present setup is focused on the specific state of water for demonstration, the control framework is in principle not limited by the concentration of non-condensible gas. During the passage of the wave, a layer of cavitation bubbles is nucleated on the proximal surface of the target to cause energy shielding. The acoustic waves scattered by the bubbles are measured using an array transducer. Further technical details of this setup (without control) are described elsewhere [15]. The controller varies the Pulse-Repetition-Frequency (PRF); the intensities of both cavitation and the bubble-scattered acoustics are positively correlated with PRF (fig 1b,c); high PRF indicates a short pulse interval. With a short interval, fewer bubbles dissolve before the next pulse arrives. The pulse causes growth and violent collapse of the bubbles, leading to proliferation of daughter bubbles that serve as nuclei in the subsequent pulse (see supplementary information). For the dense cloud (fig 1c), the bubbles in the cloud most proximal to the transducer experience greater acoustic pressure due to shielding, and are excited more than the distal ones. A similar anisotropic structure was previously observed in isolated, spherical bubble clouds [30]. The measurement is used to estimate the energy that is transmitted across the bubble clouds into the target. Based on the offset of the estimation from a set point, a proportional-integral (PI) controller varies PRF, but with a
constant pressure amplitude, to modulate cavitation and to achieve the set point value. Various types of controller can be instead used. Using the setup, we experimentally demonstrate real-time control of cavitation during $O(100)$ s of pulsed US radiation and identify an empirical correlation between PRF and the bubble dynamics, and an optimal PRF to minimize the shielding by cavitation.

The acoustic signals measured at each array element are projected onto a correlation-based imaging functional $F$ [33], which is used as an estimator input. $F$ is a normalized measure of the amplitude of the coherent acoustic scattering from the target region: $F = I/I_0$, where

$$ I = \text{MAX}_{i}[\sum_{j,l} C_T(\tau(z, x_j) + \tau(z, x_l), x_j, x_l)], $$

and $C_T$ is the cross-correlation of the signals:

$$ C_T(\tau, x_j, x_l) = \frac{1}{T} \int_0^T u(t, x_j)u(t + \tau, x_l)dt, $$

where $u(t, x_j)$ is the signal at the $j$-th array at $x_j$ at time $t$, $z$ is the coordinate of the domain, and $T$ is the time horizon of the process. $\tau(z, x)$ is the acoustic travel time from $z$ to $x$. $I_0$ is the reference value obtained with a case without bubbles.

The estimator outputs a scalar variable $E$, which is defined as the energy transmitted across bubbles into the target, normalized by the reference value obtained without bubbles. This choice of the input and output parameters is based on a first-principle model for the dynamics of bubble clouds. The kinetic energy of incompressible potential flow induced by oscillations of interacting spherical bubbles can be expressed as

$$ K = 2\pi \rho_l \sum_{i=1}^N \left[ R_i^2 \dot{R}_i^2 + \sum_{j \neq i} \frac{R_i^2 R_j^2 \dot{R}_i \dot{R}_j}{r_{ij}} \right] + (H.O.T), $$

where $R_i$, $\dot{R}_i$, and $r_{ij}$ are the radius and the radial velocity of bubble $i$, and distance between the centers of bubbles $i$ and $j$, respectively. For a cloud with $N \gg 1$, the kinetic energy can be approximated as

$$ K \approx 2\pi \rho_l N < R^3 \dot{R}^2 > (1 + B_d), $$

where $B_d = N < R > /L$. $L$ is the length-scale of the inter-bubble interaction. $B_d$ is therefore a measure of the relative contribution of the coherence among bubbles. Hydrodynamically, $B_d$ corresponds to the scale of the added-inertia of a bubble cloud. For a spherical bubble cloud, $L \sim R_e$ and $B_d$ corresponds to the aforementioned dynamic interaction parameter. For the present bubbly layer, $L \sim R_t$, where $R_t$ is the radius of the base of the target. Since the bubble cloud is the only component that dynamically alters the energy state of the system, both $E$ and $F$ are expected to be scaled by $B_d : B_d = N < R > /R_t$, and thus these variables are correlated, regardless of $N$ and $< R >$.

**B. CFD-based estimator**

Based on this physical insight, we conduct numerical experiments to obtain the quantitative correlation between $E$ and $F$. A coupled Eulerian-Lagrangian method was used for the simulation [34]. In the method, we formulate the dynamics of the multi-component mixture using the compressible, multi-component, Navier-Stokes equation. We model the cylindrical target as an elastic solid with zero shear modulus with a density of 1200 kg m$^{-3}$ and a longitudinal sound speed of 2440 m s$^{-1}$. The coupled dynamics of the target and the surrounding water are modeled using an interface capturing method [35, 36]. For modeling the dynamics of bubble cloud excited in an ultrasound field, we use an Eulerian-Lagrangian method. The method was previously derived and validated in detail [37], and applied to parametric simulation of the dynamics of spherical bubble clouds excited by burst waves [15]. We provide a brief summary of the method here. In the method, we describe the dynamics of bubbly-mixture using volume-averaged equations of motion [38–41]:

$$ \frac{\partial \mathbf{p}}{\partial t} + \nabla \cdot (\mathbf{p} \mathbf{u}) = 0, $$

$$ \frac{\partial (\mathbf{p} \mathbf{u})}{\partial t} + \nabla \cdot (\mathbf{p} \mathbf{u} \otimes \mathbf{u} + \mathbf{p} \mathbf{I} - \mathbf{T}) = 0, $$

$$ \frac{\partial \mathbf{E}}{\partial t} + \nabla \cdot ((\mathbf{E} + \mathbf{p}) \mathbf{u} - \mathbf{T} \cdot \mathbf{u}) = 0, $$
where \( \rho \) is the density, \( \mathbf{u} = (u, v, w)^T \) is the velocity, \( p \) is the pressure and \( E \) is the total energy, respectively. \( \bar{\cdot} \) denotes the volume averaging operator that acts on arbitrary field variables \( \bar{\cdot} \): \( \bar{\cdot} = (1 - \beta)(\cdot)_l + \beta(\cdot)_g \), where \( \beta \in [0, 1) \) is the volume fraction of gas (void fraction), and subscripts \( l \) and \( g \) denote the liquid and gas phase, respectively. \( \mathcal{T} \) is the effective viscous stress tensor of the mixture, that we approximate as that of the liquid phase: \( \mathcal{T} \approx \mathcal{T}_l \). We invoke two approximations valid at the limit of low void fraction: the density of the mixture is approximated by that of the liquid: \( \bar{\rho} \approx (1 - \beta)\rho_l \); the slip velocity between the two phases is zero: \( \bar{u} \approx u_l = u_g \).

Equations (5-7) can then be rewritten as conservation equations in terms of the mass, momentum and energy of the liquid with source terms, as an inhomogeneous hyperbolic system:

\[
\frac{\partial \mathbf{q}_l}{\partial t} + \nabla \cdot \mathbf{f}(\mathbf{q}_l) = \mathbf{g}(\mathbf{q}_l, \beta, \beta'),
\]

where

\[
\mathbf{q}_l = [\rho_l, \rho_l \mathbf{u}_l, E_l]^T,
\]

\[
\mathbf{f} = [\rho_l \mathbf{u}_l, \rho_l \mathbf{u}_l \otimes \mathbf{u}_l + p\mathcal{I} - \mathcal{T}_l, (E_l + p)\mathbf{u}_l - \mathcal{T}_l \cdot \mathbf{u}_l]^T,
\]

and

\[
\mathbf{g} = \frac{1}{1 - \beta} \frac{d\beta}{dt} \mathbf{q}_l - \frac{\beta}{1 - \beta} \nabla \cdot (\mathbf{f} - \mathbf{u}_l \mathbf{q}_l).
\]

For a thermodynamic closure for the liquid, we employ stiffened gas equation of state:

\[
p = (\gamma - 1)\rho \varepsilon - \gamma \rho \pi_\infty,
\]

where \( \varepsilon \) is the internal energy of liquid, \( \gamma \) is the specific heat ratio, and \( \pi_\infty \) is the stiffness, respectively. In the present study we use \( \gamma = 7.1 \) and \( \pi_\infty = 3.06 \times 10^8 \) Pa for water.

To model the gas phase, we employ a Lagrangian point-bubble approach, in that the gas is treated as spherical, radially oscillating cavities consisted of a non-condensible air and liquid vapor. In the simulation, we assume that the finite number of bubbles are initially distributed near the focal target. This condition models the experimental setup in that pre-existing nuclei of non-condensible gas mixed with vapor, many of which are excited in the previous pulse and remain until the subsequent pulses arrive. Note that in clean, pure liquid, vapour bubbles can grow without such pre-existing nuclei (homogeneous nucleation). In this ideal conditions, the nucleation can be directly modeled [42–45]. The center of \( n \)th bubble (\( n \in \mathbb{Z} : n \in [1, N] \)), with a radius of \( R_n \) and a radial velocity of \( \dot{R}_n \), is initially defined at the coordinate \( \mathbf{x}_n \) and tracked as Lagrangian points during simulations. To define the continuous field of the void fraction in the mixture at coordinate \( \mathbf{x} \), we smear the volume of bubble using a regularization kernel \( \delta \):

\[
\beta(\mathbf{x}) = \sum_{n=1}^{N} V_n(R_n) \delta(d_n),
\]

where \( V_n \) is the volume of bubble \( n \), \( V_n = 4\pi/3R_n^3 \), and \( d_n \) is the distance of the coordinate \( \mathbf{x} \) from the center of the bubble, \( d_n = |\mathbf{x} - \mathbf{x}_n| \). Throughout the present study, we use a second-order Gaussian function for \( \delta \) with a kernel width of 1.1\( \Delta \), where \( \Delta \) is the grid width. For numerical representation, we discretize equation (8) on an axi-symmetric grid and spatially integrate using a fifth-order finite volume WENO scheme [36]. A 4th/5th order Runge-Kutta-Cash-Karp (RKCK) algorithm [46] is employed for time integration of solutions.

To model the dynamics of volumetric oscillations of bubbles, we employ the Keller-Miksis equation [47]:

\[
\left( R_n \left( 1 - \frac{\dot{R}_n}{c} \right) \right) \ddot{R}_n + \frac{3}{2} \dot{R}_n^2 \left( 1 - \frac{\dot{R}_n}{3c} \right) = \frac{p_n - p_\infty}{\rho} \left( 1 + \frac{\dot{R}_n}{c} \right) + \frac{R_n \dot{p}_n}{\rho c},
\]

where \( p_n \) is the pressure at the bubble wall:

\[
p_n = p_{Bn} - \frac{4\mu_l \dot{R}_n}{R_n} - \frac{2\sigma}{R_n}.
\]

\( p_{Bn} \) is the pressure inside the bubble, \( \sigma \) is the surface tension, and \( p_\infty \) is the component of the pressure that forces the radial oscillations of the bubble. We use a reduced-order model to account for heat and mass transfer across the bubble-liquid interface [48].
The grid size is uniform with a radial and axial width of $\Delta = 100 \ \mu\text{m}$ in the target-wave interaction region. We track the radial evolution of the bubbles at the sub-grid scale and resolve the bubble-scattered acoustics on the grid. The parameters are chosen as discussed below.

This numerical setup is designed to fully mimic the physical setup. In the initial condition, Lagrangian bubble nuclei with a uniform radius of 10 $\mu\text{m}$ are randomly distributed in the cylindrical region on the proximal base of the target. A previous study identified that the dynamics of cavitation bubble clouds are insensitive to the polydispersity of the nuclei, when excited by ultrasound waves in regimes similar to those considered in the present study [30]. Oscillations of the bubbles are tracked as solutions of the Keller-Miksis equation. The US wave and bubble-scattered pressure waves are computed as solutions of the compressible Navier-Stokes equations on a structured Eulerian grid with a sufficiently high resolution (see supplemental information for further details of the method). Twenty cases were simulated during the passage and scattering of a single US pulse, with various values of the bubble’s number density, $n$, and the thickness of the bubbly layer, $h$, within ranges of $0 \leq h \leq 1.0 \ \text{mm}$ and $0 \leq n \leq 9.6 \ \text{mm}^{-3}$, respectively.

Figure 2a and c show the snapshots of the bubbles during the passage of the US with $(n, h) = (1.2, 0.25)$ and $(9.6, 1.0)$, at the same instance as the experimental images (fig. 6b,c). Figure 3b and d show the contours of the maximum pressure on the cross-plane throughout the same simulations, $p_{\text{max}}$, respectively. The morphology of the numerical bubble clouds is similar to that in the experimental images. The anisotropic structure is clear in the dense numerical cloud. In the pressure contours, the region with a high maximum pressure ($p_{\text{max}} > 6 \ \text{MPa}$) is widely distributed in the proximal interior of the target with the dilute bubbles, while the maximum pressure is nominally small ($p_{\text{max}} < 4 \ \text{MPa}$) in the target with the dense cloud. These results indicate that the US wave penetrates into the target across the small bubble cloud, while a large portion of the wave energy is scattered by the large cloud. In the latter contour, clear vertical bands of high pressure are observed in the proximal liquid, which can be explained by the interference of the reflected and incoming parts of the wave.

To quantify the anisotropy, we use the normalized moment of kinetic energy, $\mu_K$, defined as

$$\mu_K = \frac{\sum_{i=1}^{N} K_i (x_i - x_c)}{\sum_{i=1}^{N} K_i h},$$

(16)

where $K_i$ is the kinetic energy of incompressible liquid induced by the oscillations of $i$th bubble: $K_i = 2\pi p R_i^3 \bar{R}^2$, where $x_i$ is the coordinate of bubble $i$ along the acoustic axis, and $x_c$ is the center of the cloud: $x_c = \frac{\sum_{i=1}^{N} x_i}{N}$. Negative $\mu_K$ indicates the spatial bias of active bubbles in the proximal side of the cloud. A similar moment was previously used to characterize the energy state of spherical bubble clouds [30]. Figure 3a-c respectively show correlations of $\mu_K$, $E$, and $F$ against $B_d$, obtained from the simulation. Data points are collapsed well along a single curve, indicating invariance to the number of bubbles. $\mu_K$ and $E$ decrease, while $F$ increases with increasing $B_d$. $E$ and $F$ become invariant for $B_d > 4$, indicating that the energy shielding is saturated in this regime. Figure 3d shows correlations between $E$ and $F$. The data points are well collapsed on a single curve. Note that the $E - F$ correlation is not
FIG. 3: (a-c) Correlations of $\mu_k$, $E$, and $F$ against $B_d$, respectively. The bubble’s number density $n$ mm$^{-3}$ and the thickness of the bubble cloud $0 < h < 1.0$ mm are varied. (d) Correlation of $E$ and $F$. The fitting function serves as the state estimator shown in fig. 1a.

guaranteed to be linear due to the non-linear dynamics of bubbles. The numerical simulation is therefore critical to obtain this correlation. Notice also that the correlation is non-monotonic for $F < 1$: the reference state without bubbles ($n = 0$) is placed at $(F, E) = (1, 1)$. This anomaly can be explained by the breakdown of the scaling with $B_d$, for small $N$.

These analyses indicate that the bubble dynamics are dictated by the interaction parameter, and the acoustic wave and the energy transmission are monotonically correlated, as predicted by the theory. With the increase in this parameter, the anisotropy and the scattering are enhanced, while the energy transmission is decreased. From a macroscopic point of view, the greatest portion of the acoustic energy is scattered by only the surface bubbles when clouds are thick and/or dense, while otherwise a large portion of the acoustic energy is transmitted and all bubbles oscillate in a similar manner regardless of their locations. We apply nonlinear regression to data in fig. 3d to obtain a fitting function. This function serves as the estimator that uniquely outputs $E$ against a real-time input of $F$, for $F > 1$. Due to the data-driven nature of the estimator, we do not expect that the estimation is accurate for bubble clouds whose state is far from that considered in the simulation database. Conversely, for accurate estimation, it is desirable to have prior knowledge about the expected state of cavitation when constructing the database, for instance through preliminary experiments.

III. CONTROL DEMONSTRATION

Figure 4a-c respectively show evolution of the energy transmission outputted by the estimator, that of the imaging functional, and that of PRF outputted by the controller during US radiation, with four distinct values of the set point: $E_s = [0.2, 0.3, 0.4, 0.5]$. The feedback rate was $f_P/5$ Hz, where $f_P$ is PRF, and $K_P = 20$ and $K_I = 20$ were used for the proportional and integral gains of the controller. At around $t = 100$ s, both $E$ and $F$ reach their steady states and then oscillate around constant values. The values of $E$ at the steady states correspond to the set point values, for all cases. Similarly, PRF evolves with constant oscillations around its stationary values for $t > 100$ s, in the range of $20 < f_P < 70$ Hz. Note that without control, cavitation bubbles intermittently proliferate over the
FIG. 4: Evolution of (a) $E$, (b) $F$, and (c) PRF during feedback control with set point values of $E_s = [0.2, 0.3, 0.4, 0.5]$. Dotted lines in (a) denote set point values.

timescale considered in the present study, and this phenomena is not even reproducible trial by trial. Such proliferating
cavitation is not included in the numerical database and therefore outside of the range of the present estimator (fig.
3d). Although it is known that the increase in PRF can enhance the intensity of acoustic cavitation [17–19, 49], there
exists no quantitative measure to uniquely specify the bubble dynamics given PRF, due to this high nonstationarity.
With the real-time control, cavitation can, for the first time, achieve quasi-equilibrium over pulses. We are thus
motivated to correlate PRF (controller output) and the transmitted energy (estimator output) in these states.

Figure 5a shows the correlation. For $f_P < 5$ Hz, we did not observe bubbles and $F < 1$. In this range, we set $E$ to
unity. For $f_P > 5$ Hz, the energy monotonically decreases, indicating the enhancement of cavitation with increasing
$f_P$. In order to further characterize the correlation between PRF and the energy transmission, we define the effective
rate of energy delivery to the target excluding the portion scattered by the bubbly layer, Effective-PRF: $f_{EP} = Ef_P$.

Interestingly, $f_{EP}$ has a convex profile and takes a peak value of 14.6 Hz at $f_{PC} = 35$ Hz (Fig. 5b), which can be
interpreted as the optimal PRF for energy transmission. For $f_P < 5$ Hz, $E = 1$ and $f_P = f_{EP}$ there is no effect of
cavitation. For $f_P < 5$ Hz,

$$\frac{\partial f_{EP}}{\partial f_P} = E + f_P \frac{\partial E}{\partial f_P} \approx E + \gamma f_P,$$

where $\gamma$ is a constant that approximates the slope of $E$: $\partial E/\partial f_P \approx \gamma < 0$. $\partial^2 f_{EP}/\partial f_P^2 \approx 2\gamma < 0$, indicating
FIG. 5: (a) The energy transmitted into the target, $E$, as a function of actual PRF. Both variables are averaged over the period of feedback control during $100 < t < 300$ s. (b) The effective PRF as a function of actual PRF, in terms of the mean values during feedback control. The dotted line denotes linear reference with $E = 1$.

IV. CONCLUSION

In conclusion, we have designed a framework for model-based closed-loop control of cavitation through acoustic feedback, by using a data-driven state estimator. In our demonstration using a US system, set-point control of PRF modulated cloud cavitation near a solid target as designated, and identified an optimal PRF that can minimize the cavitation-induced energy shielding. Although the nucleation and dissolution of bubbles are not explicitly modeled, the control system can implicitly maintain and quantify the balance between those phenomena in dynamic equilibrium, through correlating estimation and control. This system is the first demonstration of optimization of cavitation by means of controlling PRF. The system can be particularly useful for optimizing energy transfer to a target in HIFU-based medical therapies. For different applications, the control framework can permit other sets of estimation and control parameters. For example, one can consider estimating the maximum pressure due to bubble cloud collapse and controlling the US amplitude, to modulate cavitation damage of a target. The accuracy of state estimation and control depends on the quality of numerical data and acoustic measurements. Therefore, the framework can be scaled up with enhanced computational power and precision acoustic sensing.

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Appendix A: Response of cavitation to a step change in the Pulse Repetition Frequency

In this section we present supplementary data on the response of cavitation to the Pulse-Repetition-Frequency (PRF) during a short-time exposure in our setup. Without control, we send 480 pulses with the same waveform, following that used in the main manuscript, but a time-varying PRF: $f_P = 200$ Hz from the 1st through the 160th pulse; $f_P = 10$ Hz from the 181st through the 320th pulse; $f_P = 200$ Hz thereafter. During the passage of each pulse, we capture a high-speed image of cavitation bubbles and concurrently use the estimator to obtain the energy transmission $E$. The high-speed camera was triggered 35 µs after the head of each pulse arrives at the target, at which the bubble cloud is expected to reach its maximum volume during the passage of the pulse. This trigger timing is consistent with both experimental and numerical images presented in this study (fig. 1b and c, fig 2, and supplemental fig. 1a). In the images, we compute the total area occupied by bubbles within the rectangular region on the proximal
surface of the target (solid line in supplemental fig 1.a), $A$ mm$^2$.

![Image of cavitation bubbles during the passage of a US pulse with $f_P = 200$ Hz without control.](a)

**Fig. 6**: (a) Representative image of cavitation bubbles during the passage of a US pulse with $f_P = 200$ Hz without control. (b) Evolution of the area of bubbles in the rectangular region in fig. 1a. (b) Corresponding evolution of the transmitted energy outputted by the estimator.

Fig. 6 shows results. Both $A$ and $E$ show distinct behaviors against the two values of PRF. While $f_P = 200$ Hz, the area and the imaging functional largely oscillate around $A \approx 0.5$ and $E \approx 0.4$, respectively. While $f_P = 10$ Hz, they oscillate with smaller amplitudes around $A \approx 0.1$ and $E \approx 0.6$, respectively. As mentioned in the main manuscript, at this high PRF without control, the bubbles grow not only near the target, but also far from the target (e.g. bubbles outside the rectangular region in supplemental fig. 1a). The observation of such outlying bubbles were not statistically repeatable, and not modeled in the numerical experiment. The estimator is not designed to capture the effect of them, and the large-amplitude oscillations of $E$ here may not provide quantitative information. Nevertheless, the instant changes of both $A$ and $E$ following the step changes of PRF indicates the strong positive dependency of the intensity of cavitation on PRF.

Note again that, during a longer time scale of exposure without control in our setup, the response of the bubbles can become non-stationary and non-repeatable due likely to the intermittent growth and proliferation of the outlying bubbles far from the target.

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