Dynamical Transverse Meissner Effect and Transition in Moving Bose Glass

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We study moving periodic structures in presence of correlated disorder using renormalisation group. We find that the effect of disorder persists at all velocities resulting at zero temperature in a Moving Bose Glass phase with transverse pinning. At non zero temperature we find two distinct moving glass phases. We predict a sharp transition, as velocity increases between the Moving Bose Glass, where the transverse Meissner effect persists in the direction transverse to motion and a Correlated Moving Glass at high velocity, where it disappears. Experimental consequences for vortex lattices and charge density wave (CDW) systems are discussed.

The interplay between elastic or plastic properties of periodic systems and quenched disorder, relevant to many experimental systems, has been proposed to lead to several static glass phases with complex ground states, pinning barriers and slow creep dynamics. Experiments on vortex lattices support a transition upon increase of point disorder between a dislocation-free quasi-ordered Bragg glass (BrG) and an amorphous vortex glass VG. Correlated disorder, e.g. heavy ion columnar tracks, leads to a stronger glass, termed the Bose glass (BoG) by analogy to localized 2D bosons. The hallmark of the static BoG, compared to an amorphous VG, is the transverse Meissner effect (TME), the flux lines being localized along the columns, the equilibrium response to a perpendicular field vanishes below a threshold \( H_{\perp}^c (T) \). Although there is experimental evidence for a liquid to BoG transition at \( T_{BoG} \) with anomalous angular dependences of \( T_{BoG} \) and transport, only recently have attempts been made to observe the TME directly.

The dynamical states for moving periodic structures in presence of point disorder have been investigated recently. The naive expectation is that a fast moving system averages out disorder, resulting only in an increase of the effective temperature. It was found instead that the residual periodicity transverse to motion still sees static disorder, and glassy features survive, such as transverse pinning, leading to a moving glass (MG) state. The system was shown to be well described by the MG model which involves only transverse displacements. Rows of a vortex lattice driven by a Lorentz force should thus flow along well defined static channels, with barriers to transverse motion and a transverse depinning threshold \( F_{\perp}^c (v) \). This was confirmed in numerical simulations. Bitter decoration experiments and STM imaging show vortex lattices moving along their principal axis and forming stable channel structures either fully coupled (no dislocations) at large velocity \( v \) or decoupled (with smectic order) at lower \( v \), as found in simulations. Transverse barriers could also explain the anomalously small Hall effect in Wigner solids. Finally, CDW systems in a steady current were observed to exhibit a depinning threshold under an electric field applied along their periodicity direction transverse to carrier motion. It provides a direct measurement of the transverse critical force \( F_{\perp}^c (v) \), observed to decay exponentially with \( v \) in \( d = 3 \), as predicted.

These studies having focused on point disorder it is natural to ask whether lattices moving in presence of correlated disorder also retain some glassy features of the static BoG. One question is whether the transverse periodicity results in a persistence of the TME when the field is applied transverse to motion. Additional physics compared to point disorder is expected since the localization effect of the columns (reducing thermal wandering) competes with heating by motion. Probing these effects as well as vortex correlations is experimentally accessible.

In this Letter we investigate moving lattices in presence of correlated disorder. We describe the system within the MG model which involves only transverse displacements. Using renormalization group (RG) we find that the effect of weak disorder persists at all \( v \) resulting at \( T = 0 \) in a moving Bose glass phase (MBoG) with transverse pinning and TME. At \( T > 0 \) we find two distinct moving glass phases. Our model exhibits a sharp transition, as velocity increases, between the MBoG where the TME persists and a higher velocity phase, the Correlated Moving Glass (CMG), where it disappears. While in the CMG the thermal fluctuations grow logarithmically with distance, in the MBoG they remain bounded as the channels remain localized. The properties of these moving phases and transition are summarized in Table 1 and should be testable in vortex and CDW systems.

| phase     | Response | \( u_{\text{stat}} \) | \( u_{\text{th}} \) |
|-----------|----------|---------------------|-----------------|
| MBoG      | TME      | \( x^{1/4} \)      | \( x^{1/4} \)  |
| \( T = 0 \) \( v > 0 \) or \( v < v_c(T) \) |          | bounded             | \( u_{\text{th}} \rightarrow l_\perp \) |
| transition \( v_c^\perp \) | \( H_{\perp}^c \rightarrow \text{cst} \) | \( l_\perp \rightarrow \text{cst} \) |
| transition \( v_c^\perp \) | \( c_{44} \sim \eta \sim \epsilon^{1/(v-v_c)} \) | delocalized       |
| CMG       | no TME   | \( x^{1/4} \)      | \( y^{1/2} \)  |
| \( T > 0, v > v_c(T) \) |          | logarithmic         | \( l_\perp = +\infty \) |

Table I: Characteristics of the two moving phases

Since heavy ions tracks act as strong pinning centers, except at higher \( T \) or fields, it is also important here, as
in the case of point disorder, to investigate the effect of dislocations. At large enough $v$ the effect of disorder is strongly reduced and the system should recover a large degree of topological order. Point disorder studies in 2D suggest by analogy to straight lines in columnar disorder that transverse periodicity (even in the presence of few dislocations) survives down to relatively low velocity. Hence, we expect the transverse MG model based on the channel structure to be good starting point.

We consider a lattice moving over a substrate with weak gaussian disorder correlated along $z$. The velocity $v$ is along a principal lattice direction $x$. The MG model only assumes topological order in the direction transverse to motion $y$ and consists in the equation of motion for the component of the displacement field along $y$, denoted $u_{rt}$, $r = (x, y, z)$. It describes both (i) fully elastic flow, coupled channels (weak disorder or large $v$) (ii) flow with phase slips (dislocations) occuring between channels (stronger disorder or intermediate $v$) and reads:

$$
(\eta \partial_t + v \partial_x - c_x \partial^2_x - c_y \partial^2_y - c_z \partial^2_z)u_{rt} = F[x, y, u_{rt}] + \zeta_{rt}
$$

(1)

with $\langle \zeta_{rt} \zeta_{r't'} \rangle = 2\eta T \delta^d(r - r')\delta(t - t')$ and the correlator of the static pinning force along $y$ is

$$
\tilde{F}[x, y, u] F[x', y, u'] = \delta(x - x') \delta^{d_h}(y - y') \Delta(u - u').
$$

The bare $\Delta(u)$, defined in [11], is of range $l_T$ and has the lattice period $a$. We denote the spatial dimension of $y$ by $d_y$, the case of physical interest being $d = 2 + d_y = 3$. The bare friction $\eta$ is absorbed in $v$. The $c_i$ can be estimated for a flux lattice with the field along $z$ [22].

Adding a transverse field $H^y$ along $y$ amounts to add in a surface force $\langle u_{rt} \rangle = \delta(z - z_L/2) - \delta(z + z_L/2), H^y \propto H^y$.

Let us first analyze the effect of disorder using first order perturbation theory. Correlations split into static disorder-induced displacements and thermal displacements $\tilde{\langle} u_{rt} \tilde{\rangle}^2 = \delta(\omega) \Delta(z) \tilde{C}_{stat}(\omega) + \tilde{C}_{th}(q, \omega)$. The static displacements are $z$ independent and identical to the case of point disorder in $d - 1$ dimensions:

$$
\langle (u_{rt} - u_{0rt})^2 \rangle \simeq \Delta(0) \int_q (c_x q_x^2 + c_y q_y^2)^2 + v^2 q_z^2
$$

(2)

They become unbounded for $d < d_{uc} = 4$ ($d_{uc} = 3$ for point disorder). As in [13] simple perturbation theory breaks down beyond a dynamical Larkin length $R_{L}(v)$ (at which $u_{stat} \sim r_T$) which interpolates between the static Larkin length $R_{L}(v = 0) \sim (1/\Delta(0))^{1/(5-d)}$ and the large $v$ estimate (from [3]) $R_{L}(v) \sim (c_y v^2)/(\Delta(0))^{1/(4-d)}$.

A characteristic feature of the static BoG is the reduction of thermal displacements by disorder (while they are unaffected by point disorder) resulting in the upward shift of the melting line [4]. To analyze the competition between these localization effects and heating by motion we compute equal time thermal displacements at low $T$:

$$
C_{th}(q, t = 0) = (1 + |\Delta''(0)| G_v(q)) T/(c_x q_x^2)
$$

(3)

We find [14] that $G_v(q)$ changes sign as a function of $v$: it is negative (reduced thermal displacements) for $v < v^*(q)$ and positive (heating by motion) when $v > v^*(q)$. Setting $1/q_T$ to be the Larkin length raises the possibility of a dynamical transition at $v_c$ where heating by motion [23] wins over the localization by the columns.

To describe the system beyond the Larkin length we now use the RG on the dynamical field theory associated to (3). Reducing the cutoff $\Lambda = \Lambda e^{-l}$ for $q_{y}$ we get [9]:

$$
\partial_t \ln c_x = \partial_t \ln h - \frac{\Delta''(0)}{1 + \tilde{v}^2 e^{2l}}
$$

(4)

$$
\partial_t \ln \tilde{T} = -d_y + \Delta''(0) \frac{\tilde{v}}{1 + \tilde{v}^2 e^{2l}} \tilde{T}(u) + \tilde{T} \Delta''(u)
$$

(5)

$$
\partial_t \Delta(u) = (2 - d_y + \frac{1}{1 + \tilde{v}^2 e^{2l}}) \Delta(u) + \Delta''(u) \Delta(u) - \frac{\Delta'(u)^2}{1 + \tilde{v}^2 e^{2l}}
$$

(6)

c_x, c_y, v$ are not renormalized, $\tilde{v} = v/(2\Lambda \sqrt{h} e^{2l})$.

Let us first analyze the effect of disorder using first order perturbation theory. The bare $\Delta(u)$, defined in [11], is of range $l_T$ and has the lattice period $a$. Since $\tilde{\Delta}(0) \sim (\Lambda e^{-l} - \tilde{v} e^{2l})$ and the reduced temperature [22] and $\Delta(u) = S \Lambda e^{-d_1} \tilde{\Delta}(u)/4 c_y \sqrt{1 + \tilde{v}^2 e^{2l}} = e^{-d_2}$.

These equations reveal three phases and a transition as follows:

**Static Bose Gas:** at $v = 0$ [14] describe dynamically an elastic version of the BoG similar to the one studied at equilibrium in [14]. The model exhibits analytically many of the properties of the BoG induced by strong columnar defects e.g. the TME [5]. At $T = 0$ the ground state is $z$-independent, thus the problem is identical to point disorder in $x, y$ space with identical RG equations. Beyond the Larkin length $R_{c} \sim a e^{l_e}$, $\tilde{\Delta}(u)$ develops a cusp $\tilde{\Delta}'(0) \to -\infty$ giving rise to a depinning threshold. From [3] the tilt modulus $\tilde{c}_z$ diverges at $R_{c}$ implying a vanishing linear response to $H_{\perp}$ and leading to the TME [5]. The $T = 0$ fixed point reads [22] to $\tilde{T} = \tilde{T}_c \Lambda l_{1} e^{-l} / \tilde{c}_z e^{l}$ the reduced temperature [22] and $\tilde{\Delta}(u) = S \Lambda e^{-5} \tilde{\Delta}(u)/4 c_y \sqrt{1 + \tilde{v}^2 e^{2l}}$. These equations reveal three phases and a phase transition as follows:

**Moving Bose Glass:** in the moving system, at $T = 0$ for any $v$ and at $T > 0$ for $v < v_c(T)$ defined below, the RG flows to a $T = 0$ fixed point, which we call the MBoG. The RG equation [3] at $\tilde{T} = 0$ resembles the one for the MG with point disorder in $d - 1$ dimensions [14]. To $\tilde{O}(\epsilon = 5 - d)$, $\tilde{\Delta}(q, \omega) = \tilde{O}(\epsilon) - c_y r_T \tilde{\Delta}(a - u)$ (for $0 < u < a$). At $T > 0$ we observe [23] from our RG equations a remarkable feature compared to point disorder: the effective temperature $\tilde{T} = 0$ at a finite length scale $R_{loc} > R_{c}$. This is consistent with localization effects, expected in the BoG, setting in beyond $R_{loc}$. The result is that $\tilde{\Delta}(u)$ develops a cusp $\tilde{\Delta}'(0) \to -\infty$ giving rise to a depinning threshold $T_{c}(v) \sim c_y r_T / (R_{c}(v)^2)$ it can be checked by adding a small transverse force and is manifest from the divergence of the relaxation time $\eta_{s} \sim R_{c}(v) (from
in the MBoG, the from the surface inside the bulk. Thus the TME persists as a function of \( v < v_c \) (known to exhibit BoG and TME at \( v > v_c \)).

Localizations and heating leads to a sharp transition in (1) \( \nu \), as a function of \( v \), consistent with the above first order estimate. Our RG shows that this competition between localization effects survives in the MBoG as well. Therefore the field cannot penetrate the penetration length along \( z \) is \( R_z^e(v) = \sqrt{c_z/c_y R_y^v(v)} \).

Positional correlations in the MBoG are found to be \( \nu \), consistent with the above first order estimate. Our RG shows that this competition between localization effects survives in the MBoG as well. Therefore the field cannot penetrate the penetration length along \( z \) is \( R_z^e(v) = \sqrt{c_z/c_y R_y^v(v)} \).

We now study the stability of this phase to temperature. As long as \( \tilde{T}_l > 0 \), \( \Delta_j(u) \) is analytic (\( \Delta_j''(0) < 0 \)). Remarkably, the coefficient of \( \Delta_j''(0) \) in (3) changes sign as a function of \( v \). Thus \( \tilde{T}_l \) decreases to small \( v \) and increases at large \( v \), consistent with the above first order estimate. Our RG shows that this competition between localization and heating leads to a sharp transition at \( v = v_c(T) \). Indeed, as depicted in fig. 1 for \( v < v_c \), \( \tilde{T}_l \) decreases to \( \tilde{T}_l = 0 \) at a finite scale \( R_{loc}(v) = \alpha e^{\nu_{loc}} \) and remains exactly zero thereafter. Thus all properties beyond \( R_{loc} \) are governed by the T = 0 MBoG fixed point. For \( v > v_c \), heating wins (\( \tilde{T}_l \) never vanishes), driving the system to the CMG phase described below.

Thus the MBoG \( T = 0 \) fixed point is stable to temperature for \( v < v_c \). This is specific to correlated disorder and is in contrast with the MG with point disorder. We surmise that, as in the static BoG, barriers also diverge in the MBoG, leading to transverse creep, which would be interesting to check numerically. Localization effects survive in the MBoG as seen from equal time connected fluctuations along \( z \), \( L_z^2 = \lim_{t \to -\infty} \langle (u_{xyz})^2 - (u_{xyz}(t))^2 \rangle_c = \int T \tilde{T}_l \), which are bounded since the integral only runs up to \( \tilde{T}_l \). To confirm the persistence of Bose Glass features in moving systems we also studied numerically the \( d_{y^c} = 0 \) version of (3) (known to exhibit BoG and TME at \( v = 0 \) in equilibrium). Results are consistent with a TME at \( v, T > 0 \).

**Correlated Moving Glass:** as shown in fig. 1 for \( T > 0 \), \( v > v_c(T) \), the RG flows to a finite temperature fixed point (perturbative in \( \mathcal{O}(\epsilon = 4 - d) \)) corresponding to a novel dynamical phase, the CMG. It is characterized by \( \tilde{T}^* \sim c_{y^c}^2 t_6^{1/16} \) and by an analytic fixed point \( R_{loc} \) for \( \Delta_j(u) - \Delta_j(0) \). A correlated random force \( \Delta_j(0) \sim e^{\nu^c} \) is also generated. This behavior is analogous, but not identical to the MG fixed point \( \tilde{T}_l \) with point disorder, marginal in \( d = 3 \) while the CMG is well below its upper critical dimension (\( \epsilon = 1 \)). Contrarily to the BoG and the MBoG, no cusp singularity occurs. From (4) one hence finds that the system responds linearly to transverse forces and tilting fields, with finite renormalized coefficients \( \eta_R \) and \( \epsilon_R^c \). However, since at \( T = 0 \) the system instead flows to the MBoG fixed point, one does expect strongly non linear transverse \( I - \nu \) and \( B^y \) characteristics and diverging \( \eta_R^c(0) \) and \( \epsilon_R^c(T) \) as \( T \to 0 \). Due to the correlated random force, the static roughness of the channels is \( u \sim y^{1/2} \sim x^{1/4} \) as in the MBoG (with a smaller amplitude). Thermal displacements however, while bounded in the MBoG, grow logarithmically in the CMG, as \( \langle (u(y) - u(0))^2 \rangle_c \approx 2T^* \ln y \). Hence the channels are thermally broadened, with slow decay of \( \langle e^{2\nu^c y^c} \rangle_c \sim y^{-\eta} \), \( \eta \approx 0.48 \) in \( d = 3 \), in contrast with the finite-width channels of the MG.

**FIG. 1.** Flow of the effective temperature \( \tilde{T}_l \). Dotted line: separatix between the MBoG \( v < v_c \) and the CMG \( v > v_c \). Inset: responses and \( v, T \) phase diagram of the MG model.

**FIG. 2.** Schematic phase diagrams for (a) weak disorder (b) strong disorder, in \( v, T \) plane.

**Dynamical transition:** although for \( T = 0 \) the moving system is always in the MBoG phase, for \( T > 0 \) a sharp transition occurs at a critical velocity \( v_c(T) \) between the two above phases. The separatrix in the RG flow (fig. 1) is parabolic. Thus the transition occurs when the MBoG length \( R_{loc}(T, v) = R_0(v) \equiv \sqrt{2c_x c_y} / v \), the (here \( T \)-independent) length in the CMG at which the flow of \( \tilde{T}_l \) reverts. At low \( T \), \( R_{loc}(T, v) \approx R_{loc}(0^+, v) = R_y^v(v) \) and thus \( \eta_R v_c \sim 1.3 \sqrt{c_x c_y} / R_0 \) for weak disorder whereas it saturates at \( \eta_R v_c \sim \sqrt{c_x c_y} / a \) for stronger disorder. For lattices with \( c_{y^c} \ll c_{1,1}, c_{b,1} \) (23) yields \( c_y \sim c_{y^c} \) near \( v_c \) and \( \eta_R v_c \sim c_{y^c} \max(R_c, a) \) for weak disorder \((R_c > a, F_c = c_{y^c} t_6^f / R_y^v)\), one gets \( \eta_R v_c / F_c \sim R_c / \gamma_f \). The parabolic shape of the separatix leads in the CMG to renormalized tilt modulus and friction diverging as \( c_y^R \sim \eta^R \sim e^{\nu^c} \), \( \alpha = \frac{1}{4} \) for \( v \to v_c^+ \). It appears as an ordering transition along \( z \) with a characteristic length \( R_y^v \) along \( y \) remaining finite while the
length \( R^z \sim e^{c_{\text{mix}} R^y} \) diverges as \( R^z \sim e^{-\frac{c_{\text{mix}}}{\sqrt{R^y}}} \). However, coming from the MBoG, \( l_1 \) jumps discontinuously to \( l_2 = +\infty \) in the CMG, and thus the transition is of mixed discontinuous-continuous character.

Since the MG model relies on the transverse order it describes the system for \( v > v_t \) at which channels appear. For \( v > v_{\text{top}} \) these channels recouple and the lattice recovers a good amount of topological order \[22\]. Thus the dynamical transition should be observable (fig. 2 a) for \( v > v_t \) (weaker disorder) while it is rounded by plastic effects for \( v < v_t \) (stronger disorder). Lindemann estimates of \( v_{\text{top}} \) can be obtained \[19\] as in \[11\] (for \( c_{\text{mix}} \ll c_1, v_t \sim v_{\text{top}} \)). We consider columnar defects of spacing \( d \), of strength \( u_0 \equiv U_0(T)/\epsilon_0 \), with \( r_f = l_1(T)(= b_0 \text{ at low } T) \), using \[17\]. When the decay length of translational order \( R_a \) in the static BoG is such that \( R_a > a \), i.e \( d/a > u_0(r_f/a)^{3/2} \), the MBoG and the transition at \( \eta_0 v_c \sim c_a/a^3 \) should be observable. For stronger disorder the CMG exists (fig. 2 b) for \( \eta_0 v > \eta_0 v_t \sim U_0(r_f/a^2) \). These should be compared to \( F_c \sim U_0/(r_f a^2) \), the single vortex pinning longitudinal threshold.

The plastic limit at stronger disorder can be investigated extending the creep arguments of \[1\]. We find that the penetration of \( H^y \) results from (super)kinks nucleated at the \( \pm \) surfaces with a bias along \( \pm y \) and propagating inside the bulk. For not very thick samples they produce a small \( B^y \) of order the creep velocity. In thick samples the response is determined by their competition with (super)kinks nucleated in the bulk unaffected by \( H^y \), which propagate to the surface. At \( T = 0, f > f_c \) the persistence of anomalous tilt response depends on whether trajectories of a point in 2D attract on average \[30\]. Thus the absence of \( H^y \) penetration in the MG model studied here results from interactions responsible for the transversely ordered channel structure. Surface kinks along \( y \) will also be nucleated at \( T > 0 \) but in the MBoG they should not penetrate deep in the bulk.

Additional weak point disorder (known to destabilize the \( d = 3 \) plastic BoG \[23\]) transforms the CMG (above a large length) into a new fixed point \[14\] with mixed features: the static roughness of the channels along \( x,y \) grows as in the CMG but is logarithmic along \( z \) (delocalization) with finite thermal width. There is a \( T = 0 \) transverse critical force but no TME. At smaller \( v \) the MBoG exhibits a greater stability to point disorder.

To conclude, some effects of correlated disorder on moving periodic structures (e.g. the existence of two distinct moving phases) were found to differ radically from those due to point disorder. In particular we found a \( T > 0 \) moving phase (MBoG) with strong glassy features.

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[18] via the mapping \( J_x \sim v, E_y \sim f_y \), Bose glass effects, TME and transverse depinning should also be observable in CDW with correlated disorder using surface fields.
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[21] The heating by motion measured by the “shaking temperature” of \[1\] which measures static displacements.
[22] \( C_l = \frac{S_l}{2\pi} \sinh \sqrt{\frac{c_{\text{mix}}}{c_y} \Delta^y} \) depends weakly on \( l, \Omega \) is the cutoff along \( z \) and \( S_{\text{dy}} \) is the unit sphere area over \((2\pi)^d\).
[23] As in \[22\] \( \Delta_l(0) - \Delta_l(u) = T_l(u)/(c_y/\Omega) \) satisfies \[4\] with \((1+g)^2 + g - x q^2 = 2 \) which has a well behaved solution. Thus \( \Delta_l(0) \) diverges as \(-\chi^2/\Omega c_y^2 \) and \[4\] yields the result.
[24] with \( R_c \equiv 2 \sqrt{c_v c_y}/v, y = R_v(c_v)/R_v, x = R_v(c_v)/R_v, \) one has \((y^2 - 1)/1 + y^2 = 3x^2/2 - 1 \).
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[26] since there \( T_l \sim \sqrt{T_l(t)} \).
[27] As discussed in \[14\] for 2D point disorder, these velocities do not determine whether true order exist at all scales.
[28] One estimates \[4\] \( c_x = c_{11}, c_y = c_{00} \) and \( c_y = c_{11} \) (large \( v \)), \( c_y(q) = c_{11} - \frac{c_{11} - c_{00}}{c_{11} - c_{00} + c_3} \frac{q}{q_0} \) (smaller \( v \)).
[29] Using \[4\] and a numerical study of \[4\] which shows that \(-J_l \Delta^y(0) \) is not singular near the transition.
[30] while true for an egg carton potential \( V = f(u_x) + g(u_y) \) it is non trivial in general.