Probing the Majorana neutrino CP phases and masses in neutrino-antineutrino conversion

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Abstract. We propose a new strategy for detecting the CP-violating phases and the effective mass of muon Majorana neutrinos by measuring observables associated with neutrino-antineutrino oscillations in π± decays. Within the generic framework of quantum field theory, we compute the non-factorizable probability for producing a pair of same-charged muons in π± decays as a distinctive signature of νµ − ¯νµ oscillations. We show that an intense neutrino beam through a long baseline experiment is favored for probing the Majorana phases. Using the neutrino-antineutrino oscillation probability reported by MINOS collaboration, a new stringent bound on the effective muon-neutrino mass is derived.

1. Introduction.
As is well known, the Majorana nature of neutrinos can be established via the observation of ΔL = 2 processes [1]. The parameter characterizing the rate of such transitions, the effective neutrino mass ⟨mν⟩ ≡ ∑li U2 li mνi, involves a combination of neutrino masses, mixings and phases. Although it is clear that a combination of neutrino oscillation and nuclear beta decay experiments currently provide strong constraints on neutrino masses, having direct constraints on effective Majorana masses would be useful to have a more complete picture of Majorana neutrinos. Actually, it turns out that the only way to access the values of Majorana phases is through observables associated to ΔL = 2 transitions [2].

Measurements of the effective electron-neutrino mass in the neutrinoless double beta decay (0νββ) experiments can not restrict the two Majorana CP violating phases present in the PMNS mixing matrix. This may be expected since in (0νββ) one measures the lifetime of the decay of two neutrons in a nucleus into two protons and two electrons, which is a CP conserving quantity [3]. On the other hand, direct bounds on other effective neutrino mass parameters ⟨mν⟩ from present experimental data are very poor. Currently, the strongest bound for the muon-neutrino case from the K+ → π−μ+μ+ branching fraction [4] is only |⟨mµν⟩| ≤ 0.04 TeV [5], which leads to a negligible constraint on the neutrino masses and CP violating phases.
Here we propose a mechanism, based on neutrino-antineutrino oscillation [6, 7, 8, 9], to derive a strong bound on the effective mass of the muon-neutrino, using the results reported by the MINOS collaboration [10]. In addition, it provides a method for detecting the Majorana neutrino CP violating phases through measuring the CP asymmetry of the $\pi^\pm$ decay. It is worth noting that the probability of a process associated to neutrino oscillation is usually assumed to be factorized into three independent parts: the production process, the oscillation probability and the detection cross section. Here, we adopt the S-matrix amplitude method described in [11], in order to avoid the usual factorization scheme.

2. Neutrino-Antineutrino Oscillation

Let us start by considering a positive charged pion which decays into a virtual neutrino at the space-time location $(x,t)$ together with a positive charged muon. After propagating, the neutrino can be converted into an antineutrino which produces a positive charged muon at the point $(x',t')$ when it interacts with a target, as shown in Fig. 1. The non-observation of neutrino-antineutrino conversion can be used to set bounds on the effective Majorana mass of the muon neutrino.

For definiteness, we illustrate this process with the production of the neutrino in $\pi^+$ decay and its later detection via its weak interaction with a target nucleon $N$

$$\pi^+(p_1) \rightarrow \nu^+(p_2) + \bar{\nu}_\mu(p) + N(p_N) \rightarrow N'(p_{N'}) + \mu^+(p_{\mu})$$

where the superscript $s(d)$ refers to the virtual neutrino (antineutrino) at the source (detection) vertex. This is a $|\Delta L| = 2$ process.

If one ignores other flavors, the time evolution of the $\nu_\mu - \bar{\nu}_\mu$ system would be analogous to that of the $K^0 - \overline{K^0}$ or $B^0 - \overline{B^0}$ systems. Instead, we prefer to use the formalism developed in Ref.[11], where the whole reaction includes the production and detection processes of neutrinos. The decay amplitude becomes:

$$T_{\nu_\mu - \bar{\nu}_\mu}(t) = (2\pi)^4 \delta^4(p_1 - p_\nu + p_{N'} + p_2 - p_1)(G_FV_{ud})^2(J_{N,N'})\mu_f \nu_i \times \sum_i \bar{v}_\mu(p_i)\gamma^\mu(1 + \gamma_5)\phi_i \nu(p_2) \times U_{\mu i} U_{\nu i}^* (m_\nu) e^{-it\Delta E_{\nu i}}$$

(1)

where the relation $\nu_k = \sum U_{k\alpha}\nu_\alpha$ between flavor $k$ and mass $\alpha$ neutrino eigenstates has been used, $f_\pi = 130.4$ MeV is the $\pi^\pm$ decay constant, and $J_{N,N'}$ parametrizes the interaction with the nucleon. For simplicity, one assumes that

$$(J_{N,N'})_\mu = \pi_{N'}(p_{N'})\gamma_\mu [g_V(q^2) + g_A(q^2)\gamma_5]u_N(p_N)$$

(2)

where we keep only the contributions of leading vector $g_V(q^2)$ and axial-vector $g_A(q^2)$ form factors, with $q = p_{N'} - p_N$.

If we neglect terms of $O(m_\mu/m_{N,N'})$, one obtains

$$\left|T_{\nu_\mu - \bar{\nu}_\mu}(t)\right|^2 = (2\pi)^4 \delta^4(p_1 - p_\nu + p_{N'} + p_2 - p_1)(G_FV_{ud})^4 \left|f_\pi\right|^2 64(g_A - 1)^2 m_Nm_\mu^2 \times \sum_i U_{\mu i} U_{\mu j}^* U_{\nu i} U_{\nu j}^* e^{-it\Delta E_{\nu j}} \left\{ \frac{m_\mu m_\nu}{4E_{\nu i}E_{\nu j}} (E_2 - E_p) \left[ 1 - \frac{m_N}{(E_2 - E_p)} G(g_A) \right] \right\} p_1 \cdot p_2$$

$$- 2m_N F(g_A) \left[ E_2 - E_1 \left( 1 + \frac{m_N^2}{m_\mu^2} \right) - \frac{1}{2} (m_\mu^2 - m_\nu^2) \right]$$

(3)

where $E_2(E_1), E_p$ are, respectively, the initial (final) muon and the pion energies and $\Delta E_{\nu j} = E_{\nu i} - E_{\nu j}$. The functions $F(g_A)$ and $G(g_A)$ are given by: $F(g_A) = \frac{g_A^2 + 1}{(g_A - 1)^2}$, $G(g_A) = \frac{g_A^2 + 1}{g_A - 1}$. One
can easily check that Eq. (3) is not factorizable into (production) × (propagation) × (detection) subprocesses due to the terms proportional to $p_1 \cdot p_2 = E_1 E_2 - |p_1||p_2| \cos \alpha$, where $\alpha$ is the angle between the directions of $\mu^+$ particles. This is an important difference with respect to the case of neutrino-neutrino ($\Delta L = 0$) oscillations where it was shown in Ref. [11] that the S-matrix formalism reproduces the hypothesis of factorization of the probabilities.

We shall neglect the $q^2$-dependence of the nucleon form factors (namely, we take $g_V = g_V(q^2 = 0) = 1$ and $g_A = g_A(q^2 = 0) \approx -1.27$ [2]). As is well known [12], the cross section of charged current neutrino-nucleon quasielastic scattering is sensitive to the $q^2$-dependence of these form factors. However, as long as we confine to the CP rate asymmetry for neutrino → antineutrino oscillations (see below) we expect that the effects of the momentum-transfer dependence of $g_V, A$ will partially cancel in the ratio of oscillation rates. Thus, after integration over kinematical variables, it is possible to write the rate of the complete process as

$$\Gamma_{\nu_\mu - \bar{\nu}_\mu} = \sum_i U_{\mu i}^2 \frac{m_{\mu i}}{2E_{\nu_i}} e^{itE_{\nu_i}} |F(M, \phi)|^2,$$  
(4)

where $F(M, \phi)$ denotes the kinematical function.

There are two interesting limits for this process. At very short times, (as in short-baseline neutrino experiments), $\Gamma_{\nu_\mu - \bar{\nu}_\mu} \simeq \frac{(m_{\mu i})^2}{E_{\nu_i}^2} \times F(M, \phi)$, where $m_{\mu i}$ is the effective Majorana mass for the muon neutrino. However in the long time limit (long-baseline neutrino experiments) the oscillation terms cannot be neglected. In the limit of $\theta_{13} = 0$, the Majorana phases $\alpha_{1,2}$ are the only sources of CP violation and hence

$$a_{CP} \simeq \tan[2(\alpha_2 - \alpha_3)] \sin \gamma \quad \text{where} \quad \gamma = \frac{\Delta m_{23}^2 L(\text{km})}{2E_{\nu}(\text{GeV})}$$
(5)

Thus, in the case of LBL neutrino experiment like MINOS where the distance $L$ is given by $L = 735$ km and the energy $E_{\nu}$ is typically around $2 - 3$ GeV, one finds that $\sin \gamma \sim O(1)$ [13],[14]. Thus, measuring CP asymmetry will be unavoidable indication for large CP violating Majorana phases.

3. Application to MINOS results on neutrino-antineutrino oscillations

Recently, MINOS [14] has measured the spectrum of $\nu_\mu$ events which are missing after travelling 735 km. It is these missing events which are the potential source of $\bar{\nu}_\mu$ appearance. In their preliminary analysis, they were able to put a limit on the fraction of muon neutrinos transition to muon anti-neutrinos [10]: $P(\nu_\mu \rightarrow \bar{\nu}_\mu) < 0.026$ (90\% c.l.). Assuming CPT, this limit can be written as $a_{\nu_{\mu} - \bar{\nu}_{\mu}} < 0.026$.

In the limit of ultrarelativistic neutrinos, $E_{\nu_i} \simeq E_{\nu_i}(1 + m_{\nu_i}^2/2E_{\nu})$ and keeping the leading terms in the $m_{\nu_i}/E_{\nu}$ terms, we get

$$\left| \sum_i U_{\mu i}^2 m_{\nu i} e^{itE_{\nu_i}} \right|^2 \lesssim 0.001 \times E_{\nu}^2$$
(6)

To illustrate the usefulness of this relation, let us consider the general case of 3 generations. In this case, one finds

$$0.001E_{\nu}^2 \gtrsim \left| \langle m_{\mu \mu} \rangle \right|^2 - 4 \sum_{i>j} \text{Re} \left(U_{\mu i}^2 U_{\mu j}^{*2}\right) m_{\nu_i} m_{\nu_j} \sin^2 \frac{\Delta m_{ij} L}{4E_{\nu}}$$
$$- 2 \sum_{i>j} \text{Im} \left(U_{\mu i}^2 U_{\mu j}^{*2}\right) m_{\nu_i} m_{\nu_j} \sin \frac{\Delta m_{ij} L}{2E_{\nu}}$$
(7)
Figure 1. Region of the parametric space \((\alpha, \beta)\) for which the \(A(\alpha, \beta)\) coefficient is positive. The light and dark gray zones correspond to an inverted and normal hierarchy schemes respectively.

Assuming that the only phases that appear in the neutrino mixing matrix are the Majorana phases, it is possible to get a bound on the effective muon-neutrino Majorana mass, only depending on the values of the Majorana phases as the oscillation terms cannot be neglected. In such a case, Eq.(7) can be written as:

\[
0.001 E_\nu^2 \gtrsim |\langle m_{\mu\mu} \rangle|^2 + 4 \sin \left( \frac{\gamma_{32}}{2} \right) m_{\nu_2} m_{\nu_3} |U_{\mu_2}^2 U_{\mu_3}^*|^2 \sin \left( 2\alpha_2 - 2\alpha_3 \pm \frac{\gamma_{32}}{2} \right) \\
- 4 \sin \left( \frac{\gamma_{21}}{2} \right) m_{\nu_2} m_{\nu_1} |U_{\mu_2}^2 U_{\mu_1}^*|^2 \sin \left( \alpha_1 + \frac{\gamma_{21}}{2} \right) \\
\pm 4 \sin \left( \frac{\gamma_{31}}{2} \right) m_{\nu_1} m_{\nu_3} |U_{\mu_1}^2 U_{\mu_3}^*|^2 \sin \left( \alpha_2 \pm \frac{\gamma_{31}}{2} \right) \tag{8}
\]

where \(\gamma_{ij} = \frac{\Delta m_{ij}^2 (\text{km})}{4 E_\nu (\text{GeV})}\), and the positive and negative signs refer to normal and inverted hierarchies, respectively. We can further neglect \(\sin \gamma_{21} \approx 0\) such that \(m_{\nu_1} \approx m_{\nu_2}\) at first order in \(\mathcal{O}(\frac{\Delta m_{32}^2}{4 E_\nu})\) for the fixed experimental parameters in MINOS, then Eq.(8) can be written as

\[
0.001 \times E_\nu^2 \gtrsim |\langle m_{\mu\mu} \rangle|^2 + A(\alpha_1, \alpha_2) m_{\nu_2} m_{\nu_3} \gtrsim |\langle m_{\mu\mu} \rangle|^2 \tag{9}
\]

where the coefficient \(A(\alpha_1, \alpha_2)\) is a function of the Majorana phases. In Fig. (1) we show the regions in which \(A(\alpha_1, \alpha_2) > 0\) is satisfied for both cases of normal or inverted hierarchies, and therefore we can find a stringent bound on the effective Majorana mass.

Thus, using \(E_\nu \approx 2\ \text{GeV}\), one gets the following bound on \(|\langle m_{\mu\mu} \rangle|\)

\[64\ \text{MeV}\]

Over the excluded regions it is not possible to get a conservative bound on \(|\langle m_{\mu\mu} \rangle|\) without making extra assumptions on the neutrino mass matrix, however Eq.(7) can be used to bound Majorana parameters (masses and phases) which appear in \(|\langle m_{\mu\mu} \rangle|\). As an example, if \(0 \leq 2(\alpha_2 - \alpha_3) \leq \pi - \gamma/2\) and assuming that the effective muon-neutrino Majorana mass is dominated by \(m_{\nu_2}\) and \(m_{\nu_3}\) (the two flavour limit case), it is still possible to get the following conservative bound:

\[109\ \text{MeV}\]
A bound on the effective Majorana mass of the muon neutrino, which is independent of the mass hierarchies and Majorana phases, can be obtained using the fluxes of $\nu_\mu$ and $\bar{\nu}_\mu$ measured with the near detector of the MINOS experiment [15]. Since the near detector is located $L=1.04\, \text{km}$ away from the target and for neutrino energies above 1 GeV, all oscillatory terms in Eq. (5) are equal to 1. Under the assumption that the excess of $\bar{\nu}_\mu$ events arises from $\nu_\mu \to \bar{\nu}_\mu$ transitions we get (note the muon-neutrino and muon-antineutrino total cross sections induced by charged currents are flat for neutrino energies above 2 GeV [2]):

$$|\langle m_{\mu\mu} \rangle|^2 \frac{\int dE_\nu}{\int dE_\nu} \leq \frac{\int (\Phi^{\text{Obs}}_{\nu_\mu}(E_\nu) - \Phi^{\text{MC}}_{\bar{\nu}_\mu}(E_\nu))dE_\nu}{\int \Phi^{\text{Obs}}_{\bar{\nu}_\mu}(E_\nu)dE_\nu}. \quad (10)$$

where $\Phi^{\text{Obs(MC)}}_{\nu,\bar{\nu}}$ denote the observed(expected) fluxes. Using the expected and measured integrated fluxes by the MINOS collaboration for the energy region $5 \leq E_\nu \leq 50$ GeV, we get the following bound: $|\langle m_{\mu\mu} \rangle| \leq 2.7 \, \text{GeV}$, which looks rather poor compared to the value reported above.

4. Conclusions

The production of leptons with same charges at the production and detection vertices of neutrinos will be a clear manifestation of $|\Delta L| = 2$ processes. One interesting result is that the time evolution probability of the whole process is not factorizable into production, oscillation and detection probabilities, as is the case in neutrino oscillations [11]. We find that, for very short times of propagation of neutrinos, the observation of $\mu^+\mu^+$ events would lead to a direct bound on the effective mass of muon Majorana neutrinos. In the case of long-baseline neutrino experiments, the CP rate asymmetry for production of $\mu^+\mu^+/\mu^-\mu^-$ events would lead to direct bounds on the difference of CP-violating Majorana phases. Finally, using the current bound on muon neutrino-antineutrino oscillations reported by the MINOS Collaboration we are able to set the bound $(m_{\mu\mu}) = 64 \, \text{MeV}$, which is several orders of magnitude below current bounds reported in the literature.

Future results from MINOS are expected from the analysis of twice the data set used to get the bound reported so far [10] and quoted in Eq. (12) above. Since current uncertainties in the observed and expected number of $\bar{\nu}_\mu$ events are dominated by statistical errors [10], we could expect only a slight improvement by a factor $1/\sqrt{2}$ on the effective Majorana mass of the muon neutrino. Neutrino factories may improve this bound by more than one order of magnitude.

As a consequence of these results, neutrino experiments aiming to measure neutrino-antineutrino oscillations with different short- and long-baseline setups can be useful to get direct and complementary constraints on the masses and phases of Majorana neutrinos.

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