Hadron Production in Ultra-relativistic Nuclear Collisions and the QCD Phase Diagram: an Update

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Abstract

We summarize our current understanding of the connection between the QCD phase line and the chemical freeze-out curve as deduced from thermal analyses of yields of particles produced in central collisions between relativistic nuclei.

Quantum chromodynamics (QCD), the theory of strong interactions, contains, as a key prediction, a new phase of matter, the quark-gluon plasma (QGP). This partonic QGP was the main ingredient (in addition to leptons and neutrinos) of the matter in the early universe until about 10 µs after the big bang. Around that time the QGP was transformed into hadronic matter through the QCD phase transition. In general, strongly interacting matter in equilibrium is characterized by two quantities, the temperature $T$ and the baryon chemical potential $\mu_B$ or equivalently, the (net) baryon density $n_B$.

How does this phase transition come about? Due to asymptotic freedom, the strong coupling constant $\alpha_s$ runs, i.e. diminishes with increasing energy scale, implying that interactions among strongly interacting particles will get weaker as $T$ or $\mu_B$ (or both) are increased. One may ask at what energy scale will there be new physics such as deconfinement for strongly interacting matter in equilibrium? The only two scales relevant appear to be $\Lambda_{QCD} \approx 200$ MeV along the temperature axis and the nucleon mass $m_N$ along the $\mu_B$ axis. From phenomenological considerations and based on ever more accurate solutions of QCD on a discrete space-time lattice (‘lattice-QCD’) convincing arguments were put forward to demonstrate that strongly interacting matter undergoes indeed a (phase) transition from a dense hadronic medium where all constituents (hadrons) are confined to a deconfined plasma of interacting quarks and gluons. This implies the existence of a line in the ($T - \mu_B$) plane, the QCD phase boundary, anchored by parametrically critical parameters $T_c(\mu_B = 0) \approx \Lambda_{QCD}$ and $(\mu_B)_c(T = 0) \approx m_N$. For recent detailed reviews on the QCD phase diagram, where many of the above short arguments are exposed in detail and the state of theory and relevant experiments is summarized, see [1, 2].
This article is an attempt to update a paper on the subject which we wrote 15 years ago [3] in honor of Gerry Brown’s 70th birthday. While we have (with one exception [4]) never published together with Gerry, his constant probing and provocative questions on the QGP and related areas have, during nearly 2 decades of close scientific interactions, deeply influenced our thinking on the subject. The experimental basis for making a connection between data from ultra-relativistic nuclear collisions and the QCD phase boundary is dramatically improved after 15 years of intense research on the subject. Given all this we consider it an honor and an opportunity to be able to summarize our most recent thinking on the subject as part of a Festschrift on the occasion of Gerry’s 85th birthday.

Strongly interacting matter at high temperature and baryon density is produced in collisions between atomic nuclei at ultra-relativistic energies. Depending on the center-of-mass energy reached in the collisions the temperature and baryon density can be ‘tuned’ systematically, as discussed in detail below. A sequence of such experiments conducted over the past 25 years over a wide range of energies has provided convincing evidence for a new state of matter, see, e.g., [5, 6, 7, 8, 9, 10, 11, 12]. In November 2010, first results from Pb–Pb collisions at the Large Hadron Collider at CERN have provided a first glimpse into fireballs with ultra-high temperatures (approaching 1 GeV) and opened a new era in quark matter research [13, 14, 15, 16, 17, 18].

The range of temperatures and densities reached in these experiments is clearly of the order of or (for RHIC and LHC experiments) significantly exceeding the typical values discussed above. For a review and summary of the evidence, see [19]. Much harder has been to assess the degree of equilibration reached in the collisions and to provide direct evidence on the QCD phase boundary. Below we will argue that the most direct information on equilibration and the QCD phase boundary available to-date is obtained from analyses of the multiplicities of hadrons produced in central collisions between ultra-relativistic nuclei.

These data have been analyzed independently by various groups, using hadron resonance gas models with chemical freeze-out as a key ingredient. In this resonance gas approach, described in detail in [20], it is assumed that the final hadron yields result from the decay of fully equilibrated hadronic matter, comprising the full QCD hadron mass spectrum - and thereby implicitly containing all of QCD. The measured hadrons, those not decaying further by the strong interaction, are then obtained by summing up all strong decay products of the hadron mass spectrum with their corresponding thermal weight. By assuming that the measured hadrons are generated at a common surface at which all particles simultaneously decouple, values of the baryon chemical potential, $\mu_B$, and temperature, $T$, on this surface, the chemical freeze-out surface, are extracted. Fitting these two parameters, $\mu_B$ and $T$, together with the volume parameter gives values for the particle abundances which are in very good agreement with experiment [21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34], for an overall summary see [20].

These results clearly demonstrate that, at chemical freeze-out, a very high degree of equilibration is reached in central nucleus-nucleus collisions, implying that matter in the proper sense with thermodynamic properties has been formed at the latest at this stage of the collision. We note that the system most likely is in or very close to equilibrium much earlier in the collision, as demonstrated by, among other things, hydrodynamic analysis of...
Figure 1: The temperature and baryon chemical potential of Statistical Model fits to hadrochemical abundances as a function of center of mass energy per nucleon pair for collisions of heavy nuclei (Figure taken from [33, 34]).

azimuthal anisotropies (‘elliptic flow’). For a brief summary see [12, 19].

The resulting values of $\mu_B$ and $T$ are shown in Fig. 1 as functions of center-of-mass energy per nucleon pair. We note first a very important outcome of these investigations: near 10 GeV center of mass energy, the temperature saturates with increasing beam energy, reaching an asymptotic value of about 160 MeV, while the baryon chemical potential decreases smoothly. This temperature saturation concept can be tested thoroughly when the newly taken data at the much higher energy of $\sqrt{s_{NN}} = 2.76$ TeV from the LHC are analyzed.

All these data points have been obtained by analyzing hadron yields, implying the existence of a limiting temperature to which a hadron ‘gas’ can be heated, as has been conjectured more than 40 years ago by Hagedorn [35]. The constancy of $T$ above $\sqrt{s_{NN}} = 10$ GeV indicates that a boundary has been reached. Is this the QCD phase boundary? Could there be hadronic matter hotter than is observed here? Since the data analyzed here comprise the full hadronic spectrum, this would imply the existence of a hot and dense hadronic medium between the QCD phase boundary and the chemical freeze-out curve that leaves no trace in hadronic observables. As we will discuss below, this would contradict the existence of chemical freeze-out outlined above.

Plotting these temperature-chemical potential pairs for all available energies results in a phase diagram-like picture as is illustrated in Fig. 2. In the $\mu_B$ region from 800 to 400 MeV, as $T$ increases from 50 to 150 MeV, the experimental points rise approximately linearly. In contrast, below $\mu_B \simeq 400$ MeV, the temperature is approximately constant, $T \simeq 160$ MeV. The highest collision energies studied to date at RHIC are those for which $\mu_B \simeq 25$ MeV. Soon an experimental point at $\mu_B \simeq 1$ MeV will be available from the recent campaign at the LHC. Also shown on this plot are lines of fixed energy per particle and fixed entropy.
Figure 2: The decoupling temperatures and chemical potentials extracted by Statistical Model fits to experimental data. The freeze-out points are from Refs. \cite{28} and \cite{33,36,37}. The open points are obtained from fits to data at mid-rapidity, the full-points refer to 4$\pi$ data. The inverse triangle at $T = 0$ indicates the position of normal nuclear matter at $T = 0$. The lines are different model calculations to provide a phenomenological understanding of the freeze-out curve \cite{32,38,39}. The shaded lines are drawn to indicate different regimes in this diagram. Figure taken from \cite{40}

density per $T^3$ as well as a line of hadron percolation \cite{32,38,39}.

These experimental results can be compared to the phase boundary computed on the lattice \cite{41,42}. Numerical simulations in lattice QCD can be performed at nonzero temperature only for small values of $\mu_B$, typically $\mu_B < T$, without running into the 'sign' problem. At $\mu_B = 0$, these simulations indicate that there is no true phase transition from hadronic matter to a quark-gluon plasma, but rather a very rapid rise in the energy density at a temperature $T_c$ of $155 - 175$ MeV within current systematic errors, in very close agreement with the chemical freeze-out temperatures determined for small values of $\mu_B$.

Further, studies using the lattice technique imply that $T_c$ decreases very little as $\mu_B$ increases, at least for moderate values of $\mu_B$. A very recent study \cite{43} provides a quantitative estimate of the curvature of the phase boundary near $\mu_B = 0$. By comparison to the curvature of the line of fixed energy/particle \cite{39} which has been constructed to get a phenomenological understanding of the complete chemical freeze-out curve, these authors argue that there is a significant difference between the data on chemical freeze-out and the lattice predictions, implying in their view the existence of a dense hadronic phase between chemical freeze-out and the QCD phase boundary.
A precise determination of the curvature of the chemical freeze-out curve can, however, only be obtained by analyzing the data themselves, not a curve such as the line of fixed energy/particle or any of the other lines shown in Fig. We have recently investigated the curvature of the chemical freeze-out curve in the region $\mu_B < 250$ MeV. The results are presented in Fig. The chemical freeze-out points are taken from. In addition, we show in Fig. also fit results from the STAR collaboration, where only pions, kaons and protons are analyzed to determine the freeze-out curve. This leads to a slightly lower chemical freeze-out temperature, but we believe that its energy (or $\mu_B$) dependence contains the correct information. Independent of which chemical freeze-out parameters are used, the resulting description using the parabolic fit function as in (long dashes) and the same function with twice the slope parameter (short dashes) are in agreement with the data, within the (still significant systematic) uncertainties, as shown in Fig. For reference we include, in Fig. the line of constant $E/N$. From the available data one cannot construct evidence for a dense hadronic phase between chemical freeze-out and the phase boundary.

![Figure 3: Comparison of the experimental $\mu_B$ dependence of the chemical freeze-out temperature with recent lattice predictions. The STAR chemical freeze-out values are from, the other values are from. For more discussion see text.](image)

With the parametrizations of $T$ and $\mu_B$ from Fig. one can compute the energy dependence of the production yields of various hadrons relative to pions, shown in Fig. Important for our purposes is the observation that there are peaks in the abundances of strange to non-strange particles at center of mass energies near 10 GeV, i.e. where the temperature reaches its limiting value. In particular, the $K^+/\pi^+$ and $\Lambda/\pi$ ratios exhibit rather pronounced maxima there. We further note that in the region near 10 GeV, there is also a minimum in the chemical freeze-out volume, obtained from the Statistical Model fit to particle yields, as well as in the thermal freeze-out volume obtained from the
Hanbury-Brown and Twiss (HBT) radii of the fireball [47]. The energy dependence of the chemical freeze-out volume is shown in Fig. 5. Included in this figure is the most recent point at LHC energy from ALICE [13] obtained by analyzing the pseudo-rapidity density of charged particles for central Pb–Pb collisions.

Figure 4: Energy dependence of hadron yields relative to pions. The points are experimental data from various experiments. Lines are results of the Statistical Model calculations. The Figure is taken from [33, 34]).

These experimental observations have long resisted interpretation in terms of a transition between hadronic matter and a quark-gluon plasma[1]. The general structures observed in the data are well reproduced only by the most recent model calculations [33]. There, it is shown that these structures arise due to the interplay between the limit in hadronic temperature (see Fig. 1), in our interpretation due to the QCD phase transition, and the rapid decrease of $\mu_B$ with increasing energy. The minimum near a center of mass energy of 10 GeV and the

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1We note the interpretation given in [48], obtained within a schematic 1st order phase transition model.
rise towards higher energies in the thermal freeze-out volume was explained as due to the increasing meson to baryon ratio in this region combined with a meson-meson cross section which is significantly smaller than the pion-nucleon cross section. The chemical freeze-out volume depicted in Fig. 5 behaves similarly, with a minimum at an energy close to where the chemical freeze-out temperature saturates. At the highest (LHC) energy the chemical freeze-out volume reaches, for Pb–Pb central collisions, a value of about 4 times, the thermal freeze-out volume about 7 times that of a Pb nucleus. The strong volume increase between RHIC and LHC energy, where neither $T$ nor $\mu_B$ are expected to change much, provides a strong argument that chemical freeze-out takes place at a fixed density. These findings further strengthen the connection between hadron gas and quark-gluon plasma.

The same chemical freeze-out mechanism also governs the production of light nuclei and their anti-particles and even complex and exotic objects such as light hypernuclei and their anti-particles \[49, 50, 51\]. This comes at first glance as a surprise to many, as the binding energy of these objects is typically 1-2 orders of magnitude smaller than the chemical freeze-out temperature of $T \approx 160$ MeV. Can it be that such loosely bound objects are messengers of the phase transition between hadronic matter and the quark-gluon plasma? Entropy conservation comes to the rescue! After the phase transition the hot fireball expands adiabatically and the overall entropy as well as the entropy/baryon are conserved quantities. As was realized already more than 30 years ago \[52\] the assembly yield of complex nuclei from a gas of hot nucleons is a measure of the entropy/baryon, and hence can be used to diagnose the hot and dense phase of the collision. The answer to the above question is therefore strongly affirmative: the yield of composite objects such as light nuclei and hyper-nuclei (and their anti-particles) produced in central nucleus-nucleus collisions at ultra-relativistic energy is a direct measure of the entropy/baryon at chemical freeze-out, and hence, of the QCD phase transition.

At RHIC energies, chemical freeze-out was shown \[53\] to take place very close (within less than about 10 MeV) to the phase boundary, driven by multi-particle collisions in the high density regime of, and by the the rapid density change across the phase transition. Further it is argued that freeze-out ends when the system is fully hadronized, i.e. at comparatively low density in the hadronic phase. Without a phase transition, freeze-out in a purely hadronic medium takes place over a considerable time and temperature range, as has been recently demonstrated from very general considerations \[54\]. This would necessarily lead to different freeze-out parameters for each hadron species due to widely different hadronic cross sections. This is not observed. Conversely, the rapid change of density with temperature near the phase transition makes the system insensitive to the different cross sections. We conclude that the observed (nearly) simultaneous chemical freeze-out of all hadrons is inconsistent with hypothesis of a dense hadronic phase between the chemical freeze-out line and the phase boundary, at least for relatively small values of $\mu_B < 250$ MeV.

We believe, however, that the above argument is generic \[53\]: to ensure simultaneous freeze-out (within a very small interval in temperature and chemical potential) of all hadrons, the freeze-out curve has to be very close to a line with a rapid density change, significantly faster than can be obtained within the scenario of an expanding hadronic medium. An immediate consequence of this would be that the chemical freeze-out curve delineates phase
boundaries, not only for small values of $\mu_B$ but everywhere.

But what provides the phase boundary for large values of $\mu_B$, where the deconfinement transition seems far away, at least if one follows the guidance from lattice QCD calculations? In line with Gerry Brown’s daring attitude concerning new scientific directions we have recently speculated [40] that the transition from hadronic to quarkyonic matter might provide the missing link. Another, related possibility might be that, at high baryon density, the lines corresponding to the chiral and deconfinement phase transition split, leading to a region of chirally symmetric but still confined matter in the QCD phase diagram.

Whatever the case may be, it is clear that the phase diagram at high baryon density is difficult to explore, both experimentally and theoretically. Speculations have been put forward about the possible existence of a critical endpoint [55] or a triple point [10] in the QCD phase diagram. However, despite significant efforts, no clear signals have been discovered to-date [56, 57] of either critical points or, more generally, of dense phases beyond chemical freeze-out. This research on the high baryon density region of the phase diagram remains a challenge for the future and will have to be largely driven by the running campaign at RHIC [57] and the planned new experiments at FAIR [58] and DUBNA [59].

We have provided strong evidence that, for not too large values of the baryo-chemical potential $\mu_B < 250$ MeV, the experimentally observed chemical freeze-out curve closely coincides with the QCD phase boundary between the hadronic world and the quark-gluon plasma. This implies a direct connection between results from experiments with ultra-relativistic nuclear collisions and a fundamental prediction of QCD concerning the phase structure of strongly interacting matter. For large values of $\mu_B$, i.e. large net baryon densities, we can
presently only speculate, but in Gerry Brown fashion have put forward an argument that also there the chemical freeze-out curve is driven by a phase transition.

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References

[1] P. Braun-Munzinger and J. Wambach, Rev. Mod. Phys. 81, 1031 (2009).
[2] K. Fukushima and T. Hatsuda, Rept. Prog. Phys. 74, 014001 (2011); arXiv:1005.4814 [hep-ph].
[3] P. Braun-Munzinger and J. Stachel, Nucl. Phys. A 606, 320 (1996); arXiv:nucl-th/9606017.
[4] G.E. Brown, J. Stachel, and G. Welke, Phys. Lett. B 253, 19 (1991).
[5] H. J. Specht, Nucl. Phys. A 698, 341c (2002).
[6] U. W. Heinz and M. Jacob, arXiv:nucl-th/0002042.
[7] J. Stachel, Nucl. Phys. A 654, 119c (1999).
[8] I. Arsene et al. [BRAHMS Collaboration], Nucl. Phys. A 757, 1 (2005).
[9] B. B. Back et al. [PHOBOS Collaboration], Nucl. Phys. A 757, 28 (2005).
[10] J. Adams et al. [STAR Collaboration], Nucl. Phys. A 757, 102 (2005).
[11] K. Adcox et al. [PHENIX Collaboration], Nucl. Phys. A 757, 184 (2005).
[12] P. Braun-Munzinger and J. Stachel, Nature 448, 302 (2007).
[13] K. Aamodt et al. [ALICE Collaboration], Phys. Rev. Lett., in print, arXiv:1011.3916 [nucl-ex].
[14] K. Aamodt et al. [ALICE Collaboration], Phys. Rev. Lett., in print, arXiv:1011.3914 [nucl-ex].
[15] K. Aamodt et al. [ALICE Collaboration], Phys. Lett. B, in print, arXiv:1012.1004 [nucl-ex].
[16] K. Aamodt et al. [ALICE Collaboration], arXiv:1012.1657 [nucl-ex].
[17] K. Aamodt et al. [ALICE Collaboration], arXiv:1012.4035 [nucl-ex].
[18] G. Aad et al. [ATLAS Collaboration], Phys. Rev. Lett. 105 (2010) 252203.
[19] M. Gyulassy and L. McLerran, Nucl. Phys. A 750, 30 (2005) [arXiv:nucl-th/0405013].

[20] P. Braun-Munzinger, K. Redlich, and J. Stachel, in Quark-Gluon Plasma 3, Eds. R.C. Hwa and X.N. Wang, (World Scientific Publishing, 2004) 491; nucl-th/0304013.

[21] P. Braun-Munzinger, J. Stachel, J. P. Wessels, and N. Xu, Phys. Lett. B 344, 43 (1995).

[22] P. Braun-Munzinger, J. Stachel, J. P. Wessels, and N. Xu, Phys. Lett. B 365, 1 (1996).

[23] J. Cleymans and K. Redlich, Phys. Rev. C 60, 054908 (1999).

[24] U. W. Heinz, Nucl. Phys. A 661, 140 (1999).

[25] J. L. Letessier and J. Rafelski, Int. J. Mod. Phys. E 9, 107 (2000).

[26] P. Braun-Munzinger, D. Magestro, K. Redlich, and J. Stachel, Phys. Lett. B 518, 41 (2001).

[27] W. Florkowski, W. Broniowski, and M. Michalec, Acta Phys. Polon. B 33, 761 (2002).

[28] F. Becattini, J. Manninen, and M. Gaździcki, Phys. Rev. C 73, 044905 (2006).

[29] F. Becattini et al., Phys. Rev. C 64, 024901 (2001).

[30] J. Cleymans, H. Oeschler, K. Redlich, and S. Wheaton, Eur. Phys. J. A 29, 119 (2006)

[31] A. Andronic, P. Braun-Munzinger, and J. Stachel, Nucl. Phys. A 772, 167 (2006).

[32] J. Cleymans, H. Oeschler, K. Redlich, and S. Wheaton, Phys. Rev. C 73, 034905 (2006).

[33] A. Andronic, P. Braun-Munzinger, and J. Stachel, Phys. Lett. B 673, 142 (2009); Erratum, ibid. B 678, 516 (2009).

[34] A. Andronic, P. Braun-Munzinger, and J. Stachel, Acta Phys. Polon. B 40, 1005 (2009).

[35] R. Hagedorn, Nuovo Cim. Suppl. 3, 147 (1965); Nuovo Cim. A 56, 1027 (1968).

[36] J. Manninen and F. Becattini, Phys. Rev. C 78, 054901 (2008).

[37] J. Cleymans et al., Phys. Rev. C 59, 1663 (1999).

[38] V. Magas and H. Satz, Eur. Phys. J. C 32, 115 (2003).

[39] J. Cleymans and K. Redlich, Phys. Rev. Lett. 81, 5284 (1998).

[40] A. Andronic et al., Nucl. Phys. A 837, 65 (2010); [arXiv:0911.4806 [hep-ph]].

[41] C. DeTar and U. M. Heller, Eur. Phys. J. A 41, 405 (2009).

[42] Y. Aoki, Z. Fodor, S. D. Katz, and K. K. Szabo, Phys. Lett. B 643, 46 (2006); M. Cheng et al., Phys. Rev. D 77, 014511 (2008); A. Bazavov et al., Phys. Rev. D 80, 014504 (2009).
[43] F. Karsch et al., arXiv:1011.3130 [hep-lat].

[44] A. Andronic, P. Braun-Munzinger and J. Stachel, manuscript in preparation.

[45] B. I. Abelev, et al., [STAR Collaboration], Phys. Rev. C79 034909 (2009).

[46] V. D. Toneev and A. S. Parvan, J. Phys. G 31, 583 (2005).

[47] D. Adamova et al. [CERES Collaboration], Phys. Rev. Lett. 90, 022301 (2003).

[48] M. Gazdzicki and M. I. Gorenstein, Acta Phys. Polon. B 30, 2705 (1999).

[49] P. Braun-Munzinger and J. Stachel, J. Phys. G 21, L17 (1995).

[50] P. Braun-Munzinger and J. Stachel, J. Phys. G 28, 1971 (2002), nucl-th/0112051.

[51] A. Andronic, P. Braun-Munzinger, J. Stachel and H. Stocker, Phys. Lett. B submitted Oct. 15, 2010, arXiv:1010.2995 [nucl-th].

[52] P. J. Siemens and J. I. Kapusta, Phys. Rev. Lett. 43, 1486 (1979).

[53] P. Braun-Munzinger, J. Stachel, and C. Wetterich, Phys. Lett. B 596, 61 (2004).

[54] J. Knoll, Nucl. Phys. A 821 235 (2009).

[55] M. A. Stephanov, K. Rajagopal, and E. V. Shuryak, Phys. Rev. Lett. 81, 4816 (1998); Phys. Rev. D 60, 114028 (1999).

[56] The CBM Physics Book: Compressed Baryonic Matter in Laboratory Experiments, B. Friman, C. Hoehne, J. Knoll, S. Leupold, J. Randrup, R. Rapp, P. Senger, editors, Lecture Notes In Physics, (Springer, Heidelberg, 2010).

[57] M. M. Aggarwal et al. [STAR Collaboration], Phys. Rev. Lett. 105, 022302 (2010); arXiv:1004.4959 [nucl-ex].

[58] J. Steinheimer, H. Stöcker, I. Augustin, A. Andronic, T. Saito and P. Senger, Prog. Part. Nucl. Phys. 62, 313 (2009).

[59] A. N. Sissakian and A. S. Sorin [NICA Collaboration], J. Phys. G 36, 064069 (2009).