The Kormendy relation for early-type galaxies*

Dependence on the magnitude range

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ABSTRACT

Aims. Previous studies have indicated that faint and bright early-type galaxies (ETGs) present different coefficients and dispersions for their Kormendy relation (KR). A recently published paper states that the intrinsic dispersion of the KR depends on the magnitude range within which the galaxies are contained, therefore we investigate here whether the magnitude range also has an influence on the values of the coefficients of the KR: α (zero point) and β (slope). If the values of the KR coefficients depend on the magnitude range, and this fact is not considered when performing comparisons of different galaxy samples, the differences which might be found may be misinterpreted.

Methods. We perform an analysis of the KR coefficients for 4 samples of galaxies, which contain an approximate total of 9400 ETGs in a relatively wide magnitude range (ΔM ~ 6 mag). We calculate the values of the KR coefficients in two ways: i) we consider the faintest galaxies in each sample and we progressively increase the width of the magnitude interval by inclusion of the brighter galaxies (increasing magnitude intervals); and ii) we consider narrow magnitude intervals of the same width (ΔM = 1.0 mag) over the whole magnitude spectrum available (narrow magnitude intervals). We also perform simulations of the distribution of galaxies in the log(r_e) – ⟨μ_e⟩ plane and compare the KR defined by the simulations with that obtained from the real galaxy samples.

Results. The main results we find are as follows: i) in both increasing and narrow magnitude intervals the KR coefficients change systematically as we consider brighter galaxies; ii) non-parametric tests show that the fluctuations in the values of slope of the KR are not products of chance variations and that there is evidence of an underlying trend; and iii) this trend suggest a maximum of the slope around absolute magnitude M_B ~ −18 ± 1.

Conclusions. We conclude that the values of the KR coefficients depend on the width of the magnitude range and the brightness of galaxies within the magnitude range. This dependence is due to the fact that the distribution of galaxies in the log(r_e) – ⟨μ_e⟩ plane depends on luminosity and that this distribution is not symmetrical, that is, the geometric shape of the distribution of galaxies in the log(r_e) – ⟨μ_e⟩ plane plays an important role in the determination of the values of the coefficients of the KR.

Key words. Galaxy: fundamental parameters – galaxies: elliptical and lenticular, cD

1. Introduction

It is well known that the structural parameters of ordinary ETGs follow the Fundamental Plane (FP) relation (Djorgovskv & Davis 1987; Dressler et al. 1987). The FP relation is usually expressed as a correlation among the logarithm of the effective radius (log r_e), effective mean surface brightness ⟨μ_e⟩ and logarithm of the central velocity dispersion (log σ_0), and is expressed mathematically with the following equation:

$$\log(r_e) = a \log(σ_0) + b \langle μ_e \rangle + c.$$  (1)

The FP relation is a direct consequence of the dynamical equilibrium condition (virial theorem) and of the regular behaviour of both the mass-luminosity ratio and of the galactic structure of the ETGs along all the luminosity range. Due to its small intrinsic dispersion (~0.1 dex in r_e and σ_0 and ~0.1 mag in ⟨μ_e⟩) the FP is considered a powerful tool in measuring galactic distances and also in studies of galaxy formation and evolution (Kjærgaard et al. 1993; Jorgensen et al. 1996, 1999; Kelson et al. 1997).

A physically significant projection of the FP is the correlation between log(r_e) and ⟨μ_e⟩, known as the Kormendy relation (KR):

$$⟨μ_e⟩ = α + β \log(r_e).$$  (2)

Several studies have demonstrated that the ETGs in clusters define the KR with an intrinsic dispersion of approximately 0.4 mag in ⟨μ_e⟩ (Hamabe & Kormendy 1987; Hoessel et al. 1987; Sandage & Peremulter 1991; Sandage & Lubin 2001; La Barbera et al. 2003). Some authors contend that the high intrinsic dispersion reported is due mainly to the fact that the KR does not consider the third parameter (the velocity dispersion of the FP) (Ziegler et al. 1999). The value of this dispersion is slightly increased by the measurement errors as well as the systematic errors due to the photometry calibration and also due to the corrections introduced for different biases (zero point and color transformation, K-correction and reddening).

Recent studies have shown that low-density-environment ETGs and isolated ETGs also follow the KR with the same coefficients and intrinsic dispersion as do ETGs in clusters (Reda et al. 2004; Nigoche-Netro et al. 2007). On the other hand, previous studies have demonstrated that ETGs (plus bulges of spiral...
The galaxies form two distinct families on the structural parameters plane; i.e. the family of bright ETGs ($M_B \leq -18$) that follows the KR and the family of dwarf ETGs ($M_B > -18$) with more dispersed and heterogeneous properties (their effective parameters range over a wide interval for the same total luminosity) (Kormendy 1985; Capaccioli et al. 1992). Subsequently Graham & Guzmán (2003) showed that the difference between the bright elliptical (bright E) and the dwarf elliptical (dE) galaxies is not real and that there is a continuous structural relation between both classes. They say that the different behaviour presented by the bright E and the dE galaxies in the log($r_e$)–($\mu_e$) plane and the variations in the value of the slope of the relation do not imply a different formation mechanism, rather it may be interpreted as a systematic change in the shape of the light-profile with galactic magnitude ($M$). This result, together with the fact that the intrinsic dispersion of the KR depends on the magnitude range (Nigoche-Netro et al. 2007; Nigoche-Netro 2007) prompted us to study the behaviour of the coefficients of the KR as functions of the magnitude range. If indeed the values of the coefficients of the KR depend on the magnitude range within which the galaxies are contained, and this fact is not considered when performing comparisons of galaxy samples such as the dependence of the KR on the environment, on redshift or on wavelength, the differences which might be found may be misinterpreted.

In this paper, we present a compilation of 4 samples of ETGs which contain approximately 9400 galaxies and cover a relatively wide magnitude range ($\Delta M \sim 6$ mag). Using these data we analyze the behaviour of the coefficients of the KR with respect to several characteristics of the magnitude range. We also present simulations of the distribution of the galaxies on the log($r_e$)–($\mu_e$) plane. These simulations allow us to reproduce in a reasonable manner the results we obtain from the real samples of galaxies.

This paper is organized as follows. In Sect. 2, we present the different samples used in the analysis of the KR. Section 3 describes the fitting method used in calculating the KR coefficients, as well as the behaviour of the $\beta$ coefficient with respect to the absolute magnitude range. Section 4 presents simulations of the distribution of the galaxies on the log($r_e$)–($\mu_e$) plane and finally in Sect. 5 the conclusions are presented.

### 2. The samples

We use a Sloan Digital Sky Survey (SDSS) sample of 8666 ETGs (Bernardi et al. 2003a) in filters $g^*$, $r^*$, $i^*$ and $z^*$ (total absolute magnitude range $-18 \geq M_B > -24.1$ and its equivalent in other filters) as well as a sample of 626 ETGs in the Johnson $V$ filter ($-16 \geq M_V > -22$) from 7 Abell clusters (WINGS project, Fasano et al. 2002; Varela 2004), a sample in filter Gunn $r$ ($-17 \geq M_{Gr} > -24$) with 196 ETGs from the Coma cluster (Jorgensen et al. 1995; Milvang-Jensen 1997; Jorgensen 1999; Aguerri et al. 2005) and a 54 ETGs sample in the filter Gunn $r$ ($-18 \geq M_{Gr} > -22$) from the Hydra cluster (Milvang-Jensen 1997). In the 7 Abell clusters sample we include the following clusters: A147, A168, A193, A2457, A2589, A2593, and A2626. All the samples are redshift-homogeneous (the galaxies are contained within a narrow redshift interval), except for the SDSS sample which cover a relatively ample redshift interval (0.01 $\leq z \leq 0.3$). This sample is magnitude-limited (Bernardi et al. 2003b). Besides, within large volumes there could be evolution effects of the parameters of the galaxies. So, in order to have a representative sample of the universe in a given volume without any evolution effects it is important to consider narrow redshift intervals. Bernardi et al. (2003b) recommend $\Delta z = 0.04$. This value comes from the sizes of the largest structures in the universe seen in numerical simulations of the cold dark matter family of models (Colberg et al. 2000). On the other hand, it is also well known that the SDSS photometry underestimates the luminosity of the brightest objects in crowded fields (Bernardi et al. 2007). To probe the possible evolution effects and the photometric bias of the brightest galaxies, we have built a subsample from the SDSS in the redshift interval 0.04 $\leq z \leq 0.08$. This subsample has 1670 galaxies in each filter and covers a magnitude range ($\Delta M$) $\sim$ 4 mag ($-18.5 \geq M_{Gr} > -22.9$ and its equivalent in other filters). This subsample will be referred to, from now on, as the homogeneous sample from the SDSS.

In Table 1 we present relevant information (number of galaxies, magnitude range, redshift and type of photometric profile) for the samples of galaxies we use in this paper. The magnitude range information is given in relation to the different filters used (from the literature) and also the approximate range for the $B$-magnitude (calculated by us). The transformation to the $B$ filter was accomplished by use of the following equations: $B = Gunn\,r = 1.15$ (Milvang-Jensen 1997), $B - V = 0.92$ (Michard 2000) and $B - g* = 0.5$ (Fukugita et al. 1996).

All the samples consider the photometric parameters log($r_e$) and ($\mu_e$) corrected for different biases, such as seeing (Saglia et al. 1993), galactic extinction (Schlegel et al. 1998), K correction (Jorgensen et al. 1992; Bruzual & Charlot 2003) and cosmological dimming (Jorgensen et al. 1995). These parameters as well as their uncertainties were taken directly from the different papers cited above, except for the parameters for the Abell samples which constitute a private communication from the WINGS project team (Fasano et al. 2002; Varela 2004), and those for Coma (data from Aguerri et al. 2005). In this last case, Aguerri et al. (2005) give information for effective surface brightness ($\mu_e$) instead of effective mean surface brightness ($\langle \mu_e \rangle$), therefore, we transform these data following the procedure described by Graham & Driver (2005). Additionally, to be consistent, we review carefully that the photometric parameters

| Sample | $N$ | Magnitude range | Approximate magnitude range in the $B$-filter | $z$ | TP |
|--------|-----|-----------------|---------------------------------------------|-----|----|
| 7 Abell clusters ($V$ filter) | 626 | $-16.0 \geq M_V > -22.0$ | $-15.1 \geq M_B > -21.1$ | 0.048 | S |
| Coma cluster (Gunn $r$ filter) | 196 | $-17.0 \geq M_{Gr} > -24.0$ | $-15.9 \geq M_B > -22.9$ | 0.024 | $S, dV$ |
| Hydra cluster (Gunn $r$ filter) | 54 | $-18.0 \geq M_{Gr} > -22.0$ | $-16.9 \geq M_B > -20.9$ | 0.014 | $dV$ |
| SDSS ($g^*$ filter) | 8666 | $-18.0 \geq M_{Gr} > -24.1$ | $-17.5 \geq M_B > -23.6$ | $\leq 0.3$ | $dV$ |
| SDSS ($r^*$ filter) | 8666 | $-18.6 \geq M_{Gr} > -24.7$ | $-17.5 \geq M_B > -23.6$ | $\leq 0.3$ | $dV$ |
| SDSS ($i^*$ filter) | 8666 | $-19.0 \geq M_{Gr} > -25.1$ | $-17.5 \geq M_B > -23.6$ | $\leq 0.3$ | $dV$ |
| SDSS ($z^*$ filter) | 8666 | $-19.3 \geq M_{Gr} > -25.3$ | $-17.5 \geq M_B > -23.6$ | $\leq 0.3$ | $dV$ |
and their errors were obtained by the same methods for all samples.

It is important to note that the photometric parameters of the faint and bright ETGs in the Abell sample as well as the faint ETGs in the Coma sample were obtained using Sérsic profile-fits, whereas for the bright ETGs from the other samples (which for the SDSS sample consists of the entire sample), these parameters were obtained using de Vaucouleurs $r^{1/4}$ profile-fits (see Table 1). In the literature we find that the use of one profile or the other may affect the estimations of the photometric parameters (Caon et al. 1993; Fritz et al. 2005); this in turn will also affect the estimations of the KR parameters (Ziegler et al. 1999; Kelson et al. 2000), however, we also find that the de Vaucouleurs profile-fits represent a good approximation to the Sérsic profile-fits for bright galaxies (Prugniel & Siemen 1997). In Sect. 3.3 it will be shown that the use of the de Vaucouleurs profiles as approximations to the Sérsic profiles for bright ETGs does not produce important biases on the results obtained in this paper.

Finally, an important characteristic of S0 galaxies is that, in general, the structural properties of their bulges show approximately the same homogeneity as those of E galaxies. Due to this fact, and because KR uses effective parameters, we use the photometric parameters from the whole galaxy in the case of bright Es, and from the bulge in the case of bright S0s. For dwarf galaxies we follow the same procedure: information from the whole galaxy for the dEs and from the bulge for the dS0s.

3. Kormendy relation

3.1. Calculation of the Kormendy relation

The estimation of the KR coefficients may be severely affected by the fitting method and by the choice of dependent variable.
The biases may be larger if there are measurement errors in the variables, if these errors are correlated and/or if there is intrinsic dispersion. The Bivariate Correlated Errors and Intrinsic Scatter bisector (BCES$_{bin}$) fit (Isobe et al. 1990; Akritas & Bershady 1996) is a statistical model that takes into account the different sources of bias mentioned above, which are precisely those that affect our samples. In this work we use the BCES$_{bin}$ method for the determination of the KR coefficients.

From the photometric parameters of the different galaxy samples, we calculate the coefficients of the KR in different magnitude ranges, the uncertainties of the coefficients, the correlation coefficient (Pearson Statistics) and the intrinsic dispersion of the KR (subtracting in quadrature from the intrinsic dispersion in $⟨μ⟩_e$ the dispersion of the residues due to the measurement errors of $⟨μ⟩_e$ and $log(r_e)$) (La Barbera et al. 2003). According to La Barbera et al. (2003), for the calculation of the intrinsic dispersion it is necessary to have the measurement errors in $⟨μ⟩_e$ and $log(r_e)$, these errors come directly from the papers from which we take the galaxy samples (see Sect. 2), while the errors in the KR coefficients were calculated by us following Akritas & Bershady (1996) in 1σ intervals.

The KR coefficients were calculated both in increasing magnitude intervals, as well as in narrow magnitude intervals. This allows us, among other things, to characterize the behavior of the KR coefficients with respect to the width of the magnitude range and the brightness of galaxies within the magnitude range, so the results obtained may be utilized as a reference for other studies in which different magnitude ranges are used. In Appendices A and B we present the results for the values of the coefficients of the KR for the different samples of galaxies and for different magnitude intervals.

Both the figures in the main section of this paper as well as the tables in Appendix A contain the photometric information given in the filters in which the samples were originally observed, however for comparison purposes of the values of the KR coefficients for different samples, we present in Appendix B the relevant figures (Figs. 1, 2 and 7) and the corresponding tables with the photometric information given in the $B$-filter reference frame.

3.2. Behaviour of the Kormendy relation coefficients with respect to absolute magnitude range

From the analysis of the data for our different samples we notice that the intrinsic dispersion of the KR ($σ_{KR}$), as clearly stated by Nigroche-Netro et al. (2007), changes appreciably each time we include brighter galaxies in the samples (upper magnitude cut-off), that is to say when we use increasing magnitude intervals (see Tables A.1 and B.1). We also notice that the correlation coefficient ($R$) for each fit diminishes considerably. Apart from these changes, we can also see changes in the coefficients of the KR and that these changes are greater than the associated errors for most of the cases (differences in the β coefficient may be as great as 48%). The distribution of the β coefficient may be seen in Fig. 1 (see also Fig. B.1). On the other hand, if we perform the KR analysis considering first the brightest galaxies in each sample and we progressively include the fainter galaxies (lower magnitude cut-off) (see Tables A.2 and B.2), the behaviour of the KR parameters is similar to that described above: both the intrinsic dispersion and the KR coefficient-values change systematically as we increase the width of the magnitude interval.

We also perform an analysis of the data using magnitude intervals that are narrow (see Tables A.3 and B.3), that is, considering galaxy samples in magnitude intervals of the same width and progressively brighter. For this case we can also see changes in the coefficients of the KR, however, the changes are less pronounced but still larger than the associated errors for most of the cases (differences in the β coefficients may be as great as 48%). We also find that the intrinsic dispersion of the KR is relatively low in all cases and that the correlation coefficient is, on the average, superior to 0.9. The variation of the β coefficient when we consider narrow magnitude intervals may be seen in Fig. 2 (see also Fig. B.2) where a dependence between the β coefficient and absolute magnitude appears to be hinted at (it is interesting to note that this distribution, in the case of the Coma and Abell clusters, seems to reach a maximum at $M_B \sim -18 \pm 1$). However, it may be possible for the β coefficient to be constant ($β = 5$) and that the differences we find are the result of statistical fluctuations.

The need for β to be constant and equal to 5 comes from the definition of absolute magnitude in terms of effective radius and effective mean surface brightness, that is:

$$M = ⟨μ⟩_e - 5 \log (R_e) - 2.5 \log (2\pi) - 5 \log (D) + 5$$

where $R_e$ [arcsec] and $D$ [pc] represent the effective radius and the distance to the object in question respectively and $⟨μ⟩_e$ [mag/arcsec$^2$] is the effective mean surface brightness.

If we consider the effective radius in kiloparsecs ($r_e$) and a constant magnitude, we obtain the KR as follows:

$$⟨μ⟩_e = α + 5 \log (r_e).$$

Which implies that

$$β = 5.$$  

However, the observational data for β seem to move away from the expected value. In order to clarify this point, it is necessary to apply non-parametric tests to make certain that the fluctuations in the value of the slope are not products of chance variations.

3.3. Hypothesis tests for the evaluation of β data

There are several non-parametric methods for the evaluation of sets of data (Bendat & Piersol 1966). One of the most popular is the chi-square test. This test measures the discrepancy between an observed probability density and a theoretical probability density (i.e. a normal distribution). Another important

| Sample (increasing magnitude intervals) | Test 1 | Test 2 | Test 3 |
|----------------------------------------|-------|-------|-------|
| Abell                                  | 94%   | 81%   |       |
| Coma                                   | 99%   | 89%   |       |
| Abell+Coma+Hydra                       | 99%   | 95%   |       |
| SDSS in each filter                    | 99%   | 96%   |       |
| SDSS sum of 4 filters                  | 99%   | 96%   | 90%   |
| Abell+Coma+Hydra+SDSS sum of 4 filters | 99%   | 96%   | 95%   |

| Sample (1 mag interval)                |       |       |       |
|----------------------------------------|-------|-------|-------|
| Abell                                  | 84%   | 81%   |       |
| Coma                                   | 99%   | 89%   |       |
| Abell+Coma+Hydra                       | 99%   | 95%   |       |
| SDSS in each filter                    | 99%   | 96%   |       |
| SDSS sum of 4 filters                  | 99%   | 96%   | 90%   |
| Abell+Coma+Hydra+SDSS sum of 4 filters | 99%   | 96%   | 95%   |
test, which does not a priori assume a specific distribution, is that known as a run test. There is one particularly interesting non-parametric test (mean value test), which helps to determine whether the data in question are distributed at random around a given value or whether this distribution is not a random one. For full details of the hypothesis tests see Appendix C.

In Table 2 we show the results of the application of the tests to the different galaxy samples in increasing and narrow magnitude intervals. The null hypothesis of the mean value test (test 1) is that \( \beta \) has a normal distribution and that its mean value is 5 (we consider the data to have a measurement error equal to 10\%), the null hypothesis of the run test (test 2) is that there is not an underlying trend in the \( \beta \) data and finally the null hypothesis of the chi-square test (test 3) is that the \( \beta \) data are random and that they follow a normal distribution. The percentages given in Table 2 refer to the confidence level with which we can reject the null hypothesis.

From Table 2 we can see that, on average, the null hypothesis may be rejected with a level of confidence of 95\%. This implies that there are strong reasons to believe that the mean value of \( \beta \) is not 5, that there is an underlying trend in the values of \( \beta \) and that the distribution of these values is not normal.

From the previous results there is a question that arises: why is there an underlying trend in the values of \( \beta \) when we consider increasing and narrow magnitude intervals? A possible answer to this question is that the change in the value of the slope might be due to the fact that the distribution of the galaxies on the \( \log(r_e) - (\mu)_e \) plane depends on the luminosity (Fig. 3, see further details in Varela 2004; D’Onofrio et al. 2006; Nigoche-Netro et al. 2007). It could also be due to the geometrical shape of the galaxy distribution on this plane. If the distribution of galaxies takes a rectangular shape, fitting a straight line to these data will produce different results from a straight-line fit to a galaxy distribution that takes a triangular shape or any other shape. In the following section we present simulations of the galaxy distribution on the \( \log(r_e) - (\mu)_e \) plane that elucidate clearly the effects that the shape of the distribution of galaxies on this plane has over the values of the coefficients of the KR.

Here, it is important to note that the behaviour of the parameters of the KR in the bright regime is similar for all the samples, in other words, the use of the de Vaucouleurs profiles as an approximation of the Sérsic profiles (for bright galaxies in some samples) does not appreciably affect the behaviour of the parameters of the KR (see Fig. B.2 and Table B.3). For example, if we compare the data from the brightest part (\( M_B \lesssim -18 \)) of the Abell sample (Sérsic profile) with the rest of our samples (de Vaucouleurs profile) we find that the average difference between the \( \beta \) coefficients is 9\%, which is the size of the errors. On the other hand, when we compare the behaviour of the \( \beta \) coefficient for the heterogeneous SDSS sample and the homogeneous SDSS sample (Figs. 1 and 2) we note that this behaviour is similar for both samples except for the brightest part; however, it is precisely for these galaxies for which the photometry bias could be more pronounced. So it is not possible to say that the differences found are produced by evolution effects.
4. Simulations of the galaxy distribution on the log($r_e$) – $\langle \mu \rangle_e$ plane

To investigate whether the dependence of the KR coefficients on the magnitude range is due to the geometric shape of the distribution of galaxies on the log($r_e$) – $\langle \mu \rangle_e$ plane (geometrical effect), we perform simulations for each of our samples of galaxies (Fig. 4). The simulations consist of giving values to log($r_e$) in a similar radius range as that of the sample in question, fixing the faintest zero-point ($\alpha_0$) of this sample (it is equivalent to fixing a magnitude) and fixing a slope of 5 (expected slope considering the definition of total absolute magnitude). Then, we take the same range of log($r_e$), an $\alpha_0$ slightly brighter (we take increments of 0.1 mag) and the same slope of 5, and so on until we cover the whole range of brightness of the sample in question. The resulting data distribution consists of parallel lines of slope 5 which shift to brighter magnitude. Once the data are generated, we simulate the observed depopulation effect on the upper region of the galaxy distribution (Fig. 5). This depopulation is known as the exclusion zone (Bender et al. 1992) and may be characterized by a straight line (Line of Avoidance or LOA) that has a slope approximately equal to 2.7 (D’Onofrio et al. 2006). Since we do not know the way in which galaxy locations in the lower part of the log($r_e$) – $\langle \mu \rangle_e$ plane behave, we consider the following 3 cases in the simulation of this region:

- **case 1:** it consists of setting a fixed limit-radius for all the magnitude intervals, so that the galaxies are contained within a triangle, one of whose sides is parallel to the $\langle \mu \rangle_e$ axis;
- **case 2:** in this case we consider that the lower region of the diagram is limited by a 2.7-slope straight line, that is, there is an identical exclusion zone in this part of the diagram as that observed in the upper part. Therefore the galaxies appear to be contained within a parallelogram and their distribution appears to be symmetric with respect to an axis that contains the barycenter of the galaxy distribution and is parallel to the log($r_e$) axis; we shall call this axis from now on the $X_{\text{Bright}}$ axis. The galaxy distribution is symmetrical under a reflection with respect to the $X_{\text{Bright}}$ axis and a 180° rotation with respect to a perpendicular line to the $X_{\text{Bright}}$ axis that contains the barycenter of the galaxy distribution (see Fig. 6);
- **case 3:** we consider the lower region as limited in brightness, so galaxies are contained within a triangle with one of its sides parallel to the log($r_e$) axis.

The results of the analysis of the KR show that when we consider increasing magnitude intervals (Tables A.4 and B.4), there are slope changes in all the cases, and these changes are always larger than the errors. We also find that the geometric shape of the distribution of galaxies changes each time we include brighter galaxies in the samples (Fig. 5). On the other hand, when we consider narrow 1-mag intervals (Tables A.5 and B.5), there are also slope changes (except for case 2), however, the changes are less pronounced but still larger than the errors and if the magnitude intervals are progressively narrower, then the changes diminish considerably, becoming even closer to the value of 5, just as it occurs for the real samples. Finally, if the magnitude interval is equal to 0.1 mag then the slope is exactly equal to 5 since that is the way we define the samples.

It is important to mention that when we consider narrow magnitude intervals we are able to reproduce, in a reasonable manner, the $\beta$ coefficient variations and the underlying trend found for the real samples (except for case 2) (see Figs. 7 and B.3). We also find that this trend seems to have a maximum around absolute magnitude $M_B \sim -18 \pm 1$. Finally, we find that the geometric shape of the distribution of galaxies changes systematically when we consider brighter magnitude intervals (except for case 2) (Fig. 5). We must remind the reader that Case 2 corresponds to a symmetrical galaxy distribution over the log($r_e$) – $\langle \mu \rangle_e$ plane.

Moreover, we find that both the zero point ($\alpha$) and the intrinsic dispersion ($\sigma_{\text{KR}}$) of KR change systematically when we consider brighter galaxies. This latter result confirms the dependence of the intrinsic dispersion on the magnitude range as reported in Nigoche-Netro et al. (2007).
changes are made, the more pronounced will be the changes in the values of the KR coefficients.

5. Conclusions

We have compiled 4 samples of ETGs with information for their photometric parameters \( \log(r_e) \) and \( \langle \mu \rangle_e \). These samples contain a total of \( \sim 9400 \) galaxies in a relatively wide magnitude range \((\Delta M_I \sim 6 \text{ mag})\). From the values of their photometric parameters, we have analysed the behaviour of the coefficients and intrinsic dispersion of the KR with respect to several characteristics of the magnitude range within which the galaxies are contained. The results from this study are as follows:

- We find that when we include in the samples galaxies that become progressively brighter (increasing magnitude intervals) or if we consider galaxy samples in progressively brighter fixed-width magnitude intervals (narrow magnitude intervals), the KR coefficients change and these changes are larger than the associated errors for most of the cases. We also find that the distribution of the values of the \( \beta \) coefficient in narrow magnitude intervals might have a maximum at \( M_B \approx -18 \pm 1 \). We perform non-parametric tests on the \( \beta \) coefficient data and they indicate that the variations are real and that there is evidence of an underlying trend, that is, there is evidence that the \( \beta \) coefficient changes systematically when we consider brighter galaxies.

- We perform simulations of the different samples of galaxies under study. The results of the analysis of the variation of the KR coefficients, both in increasing and in narrow magnitude intervals, show that the coefficients depend on the width and brightness of the magnitude range and that this dependence is the result of a geometrical effect due to the fact that:
  - the distribution of galaxies on the \( \log(r_e) - \langle \mu \rangle_e \) plane depends on luminosity; and
  - the geometric shape of the distribution of the galaxies on this plane is not symmetrical (see Sect. 4, for full details).

- Simulations confirm the fact that the intrinsic dispersion of the KR depends on the magnitude range, as asserted in Nigoche-Netro et al. (2007).

From the previously mentioned results, we stress that if the magnitude range is not taken into consideration when performing comparisons of galaxy samples, such as the dependence of the KR on the environment, on redshift or on wavelength, the differences which might be found may be misinterpreted.

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### Appendix A: The Kormendy relation. Dependence on the magnitude range. 

The information presented here corresponds to the original photometry of the different galaxy samples.

#### Table A.1. KR coefficients for the different galaxy samples in increasing intervals of magnitude (upper magnitude cut-off). MI is the total absolute magnitude interval within which the galaxies are distributed, $N$ is the number of galaxies in the magnitude interval, $\alpha_{\text{Bis}}$ is the zero point of KR, $\beta_{\text{Bis}}$ is the slope of KR, $\sigma_{\text{KR}}$ is the intrinsic dispersion of KR and $R$ is the correlation coefficient of the fit (Pearson Statistics).

| MI | $N$ | $\alpha_{\text{Bis}}$ | $\beta_{\text{Bis}}$ | $\sigma_{\text{KR}}$ | $R$ |
|----|-----|----------------------|----------------------|----------------------|-----|
|    |     |                      |                      |                      |     |
| Abell (V) | | | | | |
| $-16 \geq M_V > -17$ | 136 | $21.997 \pm 0.063$ | $4.733 \pm 0.268$ | $0.201$ | $0.933$ |
| $-16 \geq M_V > -18$ | 284 | $21.232 \pm 0.075$ | $5.949 \pm 0.274$ | $0.488$ | $0.834$ |
| $-16 \geq M_V > -19$ | 394 | $20.917 \pm 0.116$ | $5.721 \pm 0.411$ | $0.719$ | $0.715$ |
| $-16 \geq M_V > -20$ | 519 | $20.574 \pm 0.190$ | $5.216 \pm 0.630$ | $1.031$ | $0.504$ |
| $-16 \geq M_V > -21$ | 593 | $20.967 \pm 0.216$ | $5.102 \pm 0.673$ | $1.143$ | $0.326$ |
| $-16 \geq M_V > -22$ | 626 | $21.229 \pm 0.155$ | $2.070 \pm 0.445$ | $1.143$ | $0.251$ |
|    |     |                      |                      |                      |     |
| Coma (Gunn r) | | | | | |
| $-17 \geq M_{G-r} > -18$ | 17 | $20.019 \pm 0.138$ | $2.537 \pm 0.276$ | $0.536$ | $0.802$ |
| $-17 \geq M_{G-r} > -19$ | 68 | $19.377 \pm 0.111$ | $3.266 \pm 0.313$ | $0.692$ | $0.705$ |
| $-17 \geq M_{G-r} > -20$ | 82 | $19.302 \pm 0.095$ | $3.281 \pm 0.229$ | $0.667$ | $0.769$ |
| $-17 \geq M_{G-r} > -21$ | 109 | $19.135 \pm 0.093$ | $3.210 \pm 0.219$ | $0.656$ | $0.743$ |
| $-17 \geq M_{G-r} > -22$ | 169 | $18.917 \pm 0.090$ | $2.982 \pm 0.167$ | $0.674$ | $0.725$ |
|    |     |                      |                      |                      |     |
| Hydra (Gunn r) | | | | | |
| $-18 \geq M_{G-r} > -19$ | 4 | $19.722 \pm 0.024$ | $5.037 \pm 0.062$ | $0.077$ | $0.996$ |
| $-18 \geq M_{G-r} > -20$ | 15 | $18.964 \pm 0.122$ | $5.721 \pm 0.158$ | $0.378$ | $0.961$ |
| $-18 \geq M_{G-r} > -21$ | 42 | $18.295 \pm 0.151$ | $5.277 \pm 0.213$ | $0.685$ | $0.885$ |
| $-18 \geq M_{G-r} > -22$ | 54 | $18.068 \pm 0.170$ | $5.097 \pm 0.256$ | $0.824$ | $0.813$ |
|    |     |                      |                      |                      |     |
| SDSS (g+) | | | | | |
| $-18.5 \geq M_r > -19$ | 3 | $18.332 \pm 0.070$ | $5.098 \pm 0.727$ | $0.170$ | $0.853$ |
| $-18.5 \geq M_r > -20$ | 26 | $18.392 \pm 0.040$ | $4.808 \pm 0.100$ | $0.128$ | $0.982$ |
| $-18.5 \geq M_r > -21$ | 517 | $18.978 \pm 0.027$ | $4.726 \pm 0.057$ | $0.297$ | $0.900$ |
| $-18.5 \geq M_r > -22$ | 2856 | $18.686 \pm 0.020$ | $3.909 \pm 0.033$ | $0.362$ | $0.825$ |
| $-18.5 \geq M_r > -23$ | 6687 | $18.790 \pm 0.016$ | $3.057 \pm 0.021$ | $0.406$ | $0.763$ |
| $-18.5 \geq M_r > -24$ | 8616 | $18.949 \pm 0.014$ | $2.641 \pm 0.016$ | $0.425$ | $0.744$ |
| $-18.5 \geq M_r > -25$ | 8661 | $18.989 \pm 0.013$ | $2.572 \pm 0.015$ | $0.425$ | $0.749$ |
|    |     |                      |                      |                      |     |
| SDSS (i+) | | | | | |
| $-19 \geq M_r > -20$ | 31 | $17.458 \pm 0.059$ | $4.717 \pm 0.209$ | $0.219$ | $0.937$ |
| $-19 \geq M_r > -21$ | 505 | $18.033 \pm 0.027$ | $4.540 \pm 0.062$ | $0.311$ | $0.894$ |
| $-19 \geq M_r > -22$ | 2805 | $17.605 \pm 0.020$ | $3.941 \pm 0.036$ | $0.371$ | $0.816$ |
| $-19 \geq M_r > -23$ | 6645 | $17.663 \pm 0.015$ | $3.047 \pm 0.022$ | $0.406$ | $0.741$ |
| $-19 \geq M_r > -24$ | 8512 | $17.808 \pm 0.013$ | $2.598 \pm 0.013$ | $0.422$ | $0.716$ |
| $-19 \geq M_r > -25$ | 8664 | $17.843 \pm 0.012$ | $2.529 \pm 0.016$ | $0.422$ | $0.723$ |
|    |     |                      |                      |                      |     |
| SDSS (z+) | | | | | |
| $-19.5 \geq M_r > -20$ | 15 | $17.331 \pm 0.038$ | $4.482 \pm 0.104$ | $0.116$ | $0.984$ |
| $-19.5 \geq M_r > -21$ | 210 | $17.940 \pm 0.035$ | $4.742 \pm 0.085$ | $0.299$ | $0.908$ |
| $-19.5 \geq M_r > -22$ | 1799 | $17.312 \pm 0.022$ | $4.259 \pm 0.042$ | $0.358$ | $0.850$ |
| $-19.5 \geq M_r > -23$ | 5440 | $17.264 \pm 0.017$ | $3.309 \pm 0.025$ | $0.400$ | $0.765$ |
| $-19.5 \geq M_r > -24$ | 8277 | $17.437 \pm 0.013$ | $2.680 \pm 0.017$ | $0.414$ | $0.738$ |
| $-19.5 \geq M_r > -25$ | 8665 | $17.504 \pm 0.012$ | $2.543 \pm 0.015$ | $0.413$ | $0.748$ |
|    |     |                      |                      |                      |     |
| $-19.5 \geq M_r > -25.3$ | 8665 | $17.507 \pm 0.012$ | $2.537 \pm 0.015$ | $0.413$ | $0.750$ |
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Table A.2. KR coefficients for the different galaxy samples in increasing intervals of magnitude (lower magnitude cut-off). MI is the total absolute magnitude interval within which the galaxies are distributed, \(N\) is the number of galaxies in the magnitude interval, \(\sigma_{\text{fit}}\) is the zero point of KR, \(\beta_{\text{fit}}\) is the slope of KR, \(\sigma_{\text{KR}}\) is the intrinsic dispersion of KR and \(R\) is the correlation coefficient of the fit (Pearson Statistics).

| MI   | \(N\) | \(\alpha_{\text{fit}}\) | \(\beta_{\text{fit}}\) | \(\sigma_{\text{KR}}\) | \(R\) |
|------|------|----------------|----------------|----------------|-----|
|      | Abell (V) | Hydra (Gunn r) |      | Coma (Gunn r) | SDSS (g*) | SDSS (r*) | SDSS (i*) | SDSS (z*) |
|      |      |      |      |      |      |      |      |      |      |      |      |
| \(-21 \geq M_V > -22\) | 33 | 18.246 ± 0.071 | 3.830 ± 0.072 | 0.146 ± 0.972 | \(-23 \geq M_V > -24\) | 16.813 ± 0.024 | 3.569 ± 0.211 | 0.243 ± 0.894 | 14.993 ± 0.086 | 4.118 ± 0.072 | 0.161 ± 0.951 | 18.246 ± 0.071 | 3.830 ± 0.072 | 0.146 ± 0.972 |
| \(-20 \geq M_V > -22\) | 107 | 18.797 ± 0.052 | 3.594 ± 0.081 | 0.355 ± 0.902 | \(-22 \geq M_V > -24\) | 17.251 ± 0.126 | 3.592 ± 0.188 | 0.285 ± 0.912 | 17.251 ± 0.126 | 3.592 ± 0.188 | 0.285 ± 0.912 | 18.797 ± 0.052 | 3.594 ± 0.081 | 0.355 ± 0.902 |
| \(-19 \geq M_V > -22\) | 232 | 19.571 ± 0.054 | 3.346 ± 0.098 | 0.501 ± 0.791 | \(-21 \geq M_V > -24\) | 17.492 ± 0.107 | 3.937 ± 0.193 | 0.483 ± 0.829 | 19.571 ± 0.054 | 3.346 ± 0.098 | 0.501 ± 0.791 | 19.571 ± 0.054 | 3.346 ± 0.098 | 0.501 ± 0.791 |
| \(-18 \geq M_V > -22\) | 342 | 19.757 ± 0.111 | 3.464 ± 0.170 | 0.756 ± 0.599 | \(-20 \geq M_V > -24\) | 17.900 ± 0.090 | 3.562 ± 0.162 | 0.521 ± 0.786 | 19.757 ± 0.111 | 3.464 ± 0.170 | 0.756 ± 0.599 | 19.757 ± 0.111 | 3.464 ± 0.170 | 0.756 ± 0.599 |
| \(-17 \geq M_V > -22\) | 490 | 20.551 ± 0.147 | 2.751 ± 0.392 | 0.947 ± 0.373 | \(-19 \geq M_V > -24\) | 18.197 ± 0.073 | 3.195 ± 0.119 | 0.550 ± 0.796 | 20.551 ± 0.147 | 2.751 ± 0.392 | 0.947 ± 0.373 | 20.551 ± 0.147 | 2.751 ± 0.392 | 0.947 ± 0.373 |
| \(-16 \geq M_V > -22\) | 626 | 21.229 ± 0.155 | 2.070 ± 0.445 | 1.143 ± 0.251 | \(-18 \geq M_V > -24\) | 18.628 ± 0.080 | 2.832 ± 0.137 | 0.654 ± 0.713 | 21.229 ± 0.155 | 2.070 ± 0.445 | 1.143 ± 0.251 | 21.229 ± 0.155 | 2.070 ± 0.445 | 1.143 ± 0.251 |
| ... | ... | ... | ... | ... | \(-17 \geq M_V > -24\) | 196 | 18.837 ± 0.089 | 2.598 ± 0.158 | 0.708 ± 0.651 | 18.837 ± 0.089 | 2.598 ± 0.158 | 0.708 ± 0.651 | 18.837 ± 0.089 | 2.598 ± 0.158 | 0.708 ± 0.651 |

|      |      |      |      |      |      |      |      |
|      |      |      |      |      |      |      |      |      |      |      |      |      |      |      |
Table A.3. KR coefficients for the different galaxy samples in narrow 1 mag intervals. MI is the total absolute magnitude interval within which the galaxies are distributed, N is the number of galaxies in the magnitude interval, $\alpha_{\text{Hii}}$ is the zero point of KR, $\beta_{\text{Hii}}$ is the slope of KR, $\sigma_{\text{KR}}$ is the intrinsic dispersion of KR and R is the correlation coefficient of the fit (Pearson Statistics).

| MI | N  | $\alpha_{\text{Hii}}$ | $\beta_{\text{Hii}}$ | $\sigma_{\text{KR}}$ | R  |
|----|----|----------------------|----------------------|----------------------|----|
| Abell (V) | | | | | |
| $-16.0 \geq M_V > -17.0$ | 136 | 21.997 ± 0.063 | 4.733 ± 0.268 | 0.201 | 0.933 |
| $-17.0 \geq M_V > -18.0$ | 148 | 21.009 ± 0.051 | 5.276 ± 0.193 | 0.275 | 0.924 |
| $-18.0 \geq M_V > -19.0$ | 110 | 20.223 ± 0.082 | 4.886 ± 0.319 | 0.310 | 0.918 |
| $-19.0 \geq M_V > -20.0$ | 125 | 19.128 ± 0.044 | 4.983 ± 0.093 | 0.287 | 0.929 |
| $-20.0 \geq M_V > -21.0$ | 74 | 18.310 ± 0.070 | 4.791 ± 0.120 | 0.289 | 0.926 |
| $-21.0 \geq M_V > -22.0$ | 33 | 18.246 ± 0.071 | 3.830 ± 0.072 | 0.146 | 0.941 |
| Hydra (Gunn r) | | | | | |
| $-18.0 \geq M_r > -19.0$ | 4 | 19.722 ± 0.024 | 5.037 ± 0.062 | 0.077 | 0.996 |
| $-19.0 \geq M_r > -20.0$ | 11 | 18.994 ± 0.115 | 5.255 ± 0.167 | 0.312 | 0.968 |
| $-20.0 \geq M_r > -21.0$ | 27 | 17.902 ± 0.063 | 5.127 ± 0.078 | 0.237 | 0.987 |
| $-21.0 \geq M_r > -22.0$ | 12 | 17.142 ± 0.112 | 4.821 ± 0.148 | 0.240 | 0.974 |
| SDSS ($g^*$) | | | | | |
| $-18.5 \geq M_g > -19.0$ | 26 | 18.392 ± 0.040 | 4.808 ± 0.100 | 0.128 | 0.982 |
| $-19.0 \geq M_g > -20.0$ | 491 | 18.922 ± 0.021 | 4.760 ± 0.044 | 0.238 | 0.936 |
| $-20.0 \geq M_g > -21.0$ | 2339 | 18.254 ± 0.013 | 4.463 ± 0.021 | 0.249 | 0.918 |
| $-21.0 \geq M_g > -22.0$ | 3831 | 17.590 ± 0.014 | 4.277 ± 0.017 | 0.245 | 0.912 |
| $-22.0 \geq M_g > -23.0$ | 1829 | 16.874 ± 0.023 | 4.279 ± 0.023 | 0.223 | 0.927 |
| $-23.0 \geq M_g > -24.0$ | 145 | 16.386 ± 0.086 | 4.118 ± 0.072 | 0.161 | 0.951 |
| SDSS ($r^*$) | | | | | |
| $-18.5 \geq M_r > -19.0$ | 3 | | | | |
| $-19.0 \geq M_r > -20.0$ | 74 | 18.646 ± 0.035 | 4.966 ± 0.091 | 0.220 | 0.948 |
| $-20.0 \geq M_r > -21.0$ | 937 | 18.036 ± 0.014 | 4.619 ± 0.028 | 0.233 | 0.937 |
| $-21.0 \geq M_r > -22.0$ | 3043 | 17.366 ± 0.011 | 4.342 ± 0.018 | 0.244 | 0.914 |
| $-22.0 \geq M_r > -23.0$ | 3589 | 16.624 ± 0.014 | 4.320 ± 0.017 | 0.242 | 0.905 |
| $-23.0 \geq M_r > -24.0$ | 984 | 16.016 ± 0.027 | 4.224 ± 0.028 | 0.199 | 0.934 |
| $-24.0 \geq M_r > -24.7$ | 36 | 15.474 ± 0.162 | 4.162 ± 0.132 | 0.151 | 0.953 |
| SDSS ($i^*$) | | | | | |
| $-19.0 \geq M_i > -20.0$ | 31 | 17.458 ± 0.059 | 4.717 ± 0.209 | 0.219 | 0.937 |
| $-20.0 \geq M_i > -21.0$ | 474 | 17.926 ± 0.018 | 4.713 ± 0.039 | 0.235 | 0.941 |
| $-21.0 \geq M_i > -22.0$ | 2300 | 17.213 ± 0.012 | 4.481 ± 0.021 | 0.249 | 0.918 |
| $-22.0 \geq M_i > -23.0$ | 3840 | 16.572 ± 0.012 | 4.250 ± 0.016 | 0.241 | 0.904 |
| $-23.0 \geq M_i > -24.0$ | 1867 | 15.877 ± 0.020 | 4.239 ± 0.023 | 0.219 | 0.918 |
| $-24.0 \geq M_i > -25.0$ | 152 | 15.212 ± 0.068 | 4.240 ± 0.062 | 0.160 | 0.956 |
| SDSS ($z^*$) | | | | | |
| $-19.5 \geq M_z > -20.0$ | 15 | 17.331 ± 0.038 | 4.482 ± 0.104 | 0.116 | 0.984 |
| $-20.0 \geq M_z > -21.0$ | 195 | 17.859 ± 0.024 | 4.838 ± 0.056 | 0.229 | 0.986 |
| $-21.0 \geq M_z > -22.0$ | 1589 | 17.065 ± 0.013 | 4.610 ± 0.024 | 0.246 | 0.980 |
| $-22.0 \geq M_z > -23.0$ | 3641 | 16.475 ± 0.011 | 4.213 ± 0.015 | 0.236 | 0.980 |
| $-23.0 \geq M_z > -24.0$ | 2837 | 15.831 ± 0.015 | 4.142 ± 0.018 | 0.223 | 0.977 |
| $-24.0 \geq M_z > -25.0$ | 381 | 15.308 ± 0.035 | 4.027 ± 0.033 | 0.158 | 0.973 |
Table A.4. KR coefficients for the SDSS simulation in $r^*$ filter. Increasing magnitude intervals. MI is the total absolute magnitude interval within which the galaxies are distributed, $N$ is the number of galaxies in the magnitude interval, $\alpha_{\text{Bis}}$ is the zero point of KR, $\beta_{\text{Bis}}$ is the slope of KR, $\sigma_{\text{KR}}$ is the intrinsic dispersion of KR and $R$ is the correlation coefficient of the fit (Pearson Statistics).

| MI    | $N$ | $\alpha_{\text{Bis}}$ | $\beta_{\text{Bis}}$ | $\sigma_{\text{KR}}$ | $R$  |
|-------|-----|------------------------|-----------------------|-----------------------|------|
| Case 1 |
| $-20 \geq M_r > -21$ | 455 | 18.587 ± 0.015 4.971 ± 0.011 0.285 0.995 |
| $-20 \geq M_r > -22$ | 810 | 18.232 ± 0.022 4.865 ± 0.017 0.555 0.977 |
| $-20 \geq M_r > -23$ | 1065 | 18.029 ± 0.025 4.673 ± 0.022 0.785 0.945 |
| $-20 \geq M_r > -24$ | 1220 | 17.977 ± 0.025 4.442 ± 0.027 0.953 0.907 |
| $-20 \geq M_r > -25$ | 1275 | 17.995 ± 0.025 4.305 ± 0.031 1.030 0.885 |
| Case 2 |
| $-20 \geq M_r > -21$ | 140 | 18.533 ± 0.020 4.356 ± 0.032 0.234 0.968 |
| $-20 \geq M_r > -22$ | 280 | 18.313 ± 0.020 3.604 ± 0.023 0.330 0.953 |
| $-20 \geq M_r > -23$ | 420 | 18.270 ± 0.019 3.213 ± 0.018 0.365 0.961 |
| $-20 \geq M_r > -24$ | 560 | 18.277 ± 0.018 3.014 ± 0.013 0.381 0.971 |
| $-20 \geq M_r > -25$ | 676 | 18.311 ± 0.017 2.872 ± 0.011 0.388 0.976 |

Table A.5. KR coefficients for the SDSS simulation for $r^*$ filter. Narrow 1 mag intervals. MI is the total absolute magnitude interval within which the galaxies are distributed, $N$ is the number of galaxies in the magnitude interval, $\alpha_{\text{Bis}}$ is the zero point of KR, $\beta_{\text{Bis}}$ is the slope of KR, $\sigma_{\text{KR}}$ is the intrinsic dispersion of KR and $R$ is the correlation coefficient of the fit (Pearson Statistics).

| MI    | $N$ | $\alpha_{\text{Bis}}$ | $\beta_{\text{Bis}}$ | $\sigma_{\text{KR}}$ | $R$  |
|-------|-----|------------------------|-----------------------|-----------------------|------|
| Case 1 |
| $-20 \geq M_r > -21$ | 455 | 18.587 ± 0.015 4.971 ± 0.011 0.285 0.995 |
| $-21 \geq M_r > -22$ | 355 | 17.614 ± 0.020 4.953 ± 0.016 0.284 0.992 |
| $-22 \geq M_r > -23$ | 255 | 16.677 ± 0.031 4.912 ± 0.025 0.280 0.985 |
| $-23 \geq M_r > -24$ | 155 | 15.878 ± 0.060 4.786 ± 0.048 0.270 0.963 |
| $-24 \geq M_r > -25$ | 55  | 15.727 ± 0.200 4.305 ± 0.147 0.212 0.885 |
| Case 2 |
| $-20 \geq M_r > -21$ | 140 | 18.533 ± 0.020 4.356 ± 0.032 0.234 0.968 |
| $-21 \geq M_r > -22$ | 140 | 17.810 ± 0.024 4.356 ± 0.032 0.234 0.968 |
| $-22 \geq M_r > -23$ | 140 | 17.087 ± 0.033 4.356 ± 0.032 0.234 0.968 |
| $-23 \geq M_r > -24$ | 140 | 16.364 ± 0.045 4.356 ± 0.032 0.234 0.968 |
| $-24 \geq M_r > -25$ | 140 | 15.788 ± 0.069 4.356 ± 0.032 0.234 0.968 |
| Case 3 |
| $-20 \geq M_r > -21$ | 245 | 18.602 ± 0.018 4.819 ± 0.025 0.275 0.984 |
| $-21 \geq M_r > -22$ | 194 | 17.709 ± 0.024 4.735 ± 0.032 0.269 0.976 |
| $-22 \geq M_r > -23$ | 140 | 16.984 ± 0.037 4.508 ± 0.043 0.253 0.960 |
| $-23 \geq M_r > -24$ | 85  | 16.697 ± 0.068 4.028 ± 0.065 0.218 0.925 |
| $-24 \geq M_r > -25$ | 35  | 17.391 ± 0.168 3.097 ± 0.119 0.140 0.891 |
Appendix B: Figures and Tables of the behaviour of the KR coefficients with respect to absolute magnitude range. The information presented here corresponds to photometry in the $B$ filter.

**Fig. B.1.** Equivalent to Fig. 1 with photometric information in the $B$-filter. Variation of the KR slope ($\beta$) for the different samples of galaxies in increasing magnitude intervals (see Table B.1). Diamonds represent the SDSS homogeneous sample.

**Fig. B.2.** Equivalent to Fig. 2 with photometric information in the $B$-filter. Variation of the KR slope ($\beta$) for the different samples of galaxies. Each point represents a 1 mag interval (see Table B.3). Diamonds represent the SDSS homogeneous sample.
Fig. B.3. Equivalent to Fig. 7 with photometric information in the $B$-filter. Variation of the KR slope ($\beta$) for one of the SDSS simulations. Each point represents a 1 mag interval (see Table B.5).

Table B.1. Equivalent to Table A.1 with photometric information in the $B$-filter. KR coefficients for the different galaxy samples in increasing intervals of magnitude (upper magnitude cut-off). MI is the total absolute magnitude interval within which the galaxies are distributed, $N$ is the number of galaxies in the magnitude interval, $\alpha_{\text{bin}}$ is the zero point of KR, $\beta_{\text{bin}}$ is the slope of KR, $\sigma_{\text{KR}}$ is the intrinsic dispersion of KR and $R$ is the correlation coefficient of the fit (Pearson Statistics).

| MI | $N$ | $\alpha_{\text{bin}}$ | $\beta_{\text{bin}}$ | $\sigma_{\text{KR}}$ | $R$ |
|----|-----|-----------------------|----------------------|---------------------|-----|
| Abell (V) |
| $-15.1 \geq M_B > -16.1$ | 136 | $22.917 \pm 0.063$ | $4.733 \pm 0.268$ | $0.201 \pm 0.933$ |
| $-15.1 \geq M_B > -17.1$ | 284 | $22.152 \pm 0.075$ | $5.949 \pm 0.274$ | $0.488 \pm 0.834$ |
| $-15.1 \geq M_B > -18.1$ | 394 | $21.837 \pm 0.116$ | $5.721 \pm 0.411$ | $0.719 \pm 0.715$ |
| $-15.1 \geq M_B > -19.1$ | 519 | $21.494 \pm 0.190$ | $5.216 \pm 0.630$ | $1.031 \pm 0.504$ |
| $-15.1 \geq M_B > -20.1$ | 593 | $21.887 \pm 0.216$ | $3.102 \pm 0.673$ | $1.143 \pm 0.326$ |
| $-15.1 \geq M_B > -21.1$ | 626 | $22.149 \pm 0.155$ | $2.070 \pm 0.445$ | $1.143 \pm 0.251$ |
| Coma (Gunn r) |
| $-15.9 \geq M_B > -16.9$ | 17 | $21.169 \pm 0.138$ | $2.537 \pm 0.276$ | $0.536 \pm 0.802$ |
| $-15.9 \geq M_B > -17.9$ | 68 | $20.527 \pm 0.111$ | $3.266 \pm 0.313$ | $0.692 \pm 0.705$ |
| $-15.9 \geq M_B > -18.9$ | 82 | $20.452 \pm 0.095$ | $3.281 \pm 0.229$ | $0.667 \pm 0.769$ |
| $-15.9 \geq M_B > -19.9$ | 109 | $20.285 \pm 0.093$ | $3.210 \pm 0.219$ | $0.656 \pm 0.743$ |
| $-15.9 \geq M_B > -20.9$ | 169 | $20.067 \pm 0.090$ | $2.982 \pm 0.167$ | $0.674 \pm 0.725$ |
| $-15.9 \geq M_B > -21.9$ | 192 | $19.974 \pm 0.092$ | $2.687 \pm 0.173$ | $0.711 \pm 0.645$ |
| $-15.9 \geq M_B > -22.9$ | 196 | $19.987 \pm 0.089$ | $2.598 \pm 0.158$ | $0.708 \pm 0.651$ |
| Hydra (Gunn r) |
| $-16.9 \geq M_B > -17.9$ | 4 | $20.872 \pm 0.024$ | $5.037 \pm 0.062$ | $0.077 \pm 0.996$ |
| $-16.9 \geq M_B > -18.9$ | 15 | $20.114 \pm 0.122$ | $5.721 \pm 0.158$ | $0.378 \pm 0.961$ |
| $-16.9 \geq M_B > -19.9$ | 42 | $19.445 \pm 0.151$ | $5.277 \pm 0.213$ | $0.685 \pm 0.885$ |
| $-16.9 \geq M_B > -20.9$ | 54 | $19.218 \pm 0.170$ | $5.097 \pm 0.256$ | $0.824 \pm 0.813$ |
| SDSS ($g^+$) |
| $-18.0 \geq M_B > -18.5$ | 26 | $18.892 \pm 0.040$ | $4.808 \pm 0.100$ | $0.128 \pm 0.982$ |
| $-18.0 \geq M_B > -19.5$ | 517 | $19.478 \pm 0.027$ | $4.726 \pm 0.057$ | $0.297 \pm 0.900$ |
| $-18.0 \geq M_B > -20.5$ | 2856 | $19.186 \pm 0.020$ | $3.909 \pm 0.033$ | $0.362 \pm 0.825$ |
| $-18.0 \geq M_B > -21.5$ | 6687 | $19.290 \pm 0.016$ | $3.057 \pm 0.021$ | $0.406 \pm 0.763$ |
| $-18.0 \geq M_B > -22.5$ | 8516 | $19.449 \pm 0.014$ | $2.641 \pm 0.016$ | $0.425 \pm 0.744$ |
| $-18.0 \geq M_B > -23.5$ | 8661 | $19.489 \pm 0.013$ | $2.572 \pm 0.015$ | $0.425 \pm 0.749$ |
| SDSS ($i^+$) |
| $-17.5 \geq M_B > -18.5$ | 31 | $18.958 \pm 0.059$ | $4.717 \pm 0.209$ | $0.219 \pm 0.937$ |
| $-17.5 \geq M_B > -19.5$ | 505 | $19.533 \pm 0.027$ | $4.540 \pm 0.062$ | $0.311 \pm 0.894$ |
| $-17.5 \geq M_B > -20.5$ | 2805 | $19.105 \pm 0.020$ | $3.941 \pm 0.036$ | $0.371 \pm 0.816$ |
| $-17.5 \geq M_B > -21.5$ | 6645 | $19.163 \pm 0.016$ | $3.047 \pm 0.022$ | $0.406 \pm 0.741$ |
| $-17.5 \geq M_B > -22.5$ | 8512 | $19.308 \pm 0.013$ | $2.598 \pm 0.013$ | $0.422 \pm 0.716$ |
| $-17.5 \geq M_B > -23.5$ | 8664 | $19.343 \pm 0.012$ | $2.529 \pm 0.016$ | $0.422 \pm 0.723$ |

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Table B.2. Equivalent to Table A.2 with photometric information in the $B$-filter. KR coefficients for the different galaxy samples in increasing intervals of magnitude (lower magnitude cut-off). MI is the total absolute magnitude interval within which the galaxies are distributed, $N$ is the number of galaxies in the magnitude interval, $\alpha_{\text{br}}$ is the zero point of KR, $\beta_{\text{br}}$ is the slope of KR, $\sigma_{\text{KR}}$ is the intrinsic dispersion of KR and $R$ is the correlation coefficient of the fit (Pearson Statistics).

| MI $M_B >$ | $N$ | $\alpha_{\text{br}}$ | $\beta_{\text{br}}$ | $\sigma_{\text{KR}}$ | $R$ |
|----------|-----|-------------------|-------------------|-------------------|-----|
| Abell (V) |     |                   |                   |                   |     |
| $-20.1 \geq M_B > -21.1$ | 33 | 19.166 ± 0.071 | 3.830 ± 0.072 | 0.146 0.972 |     |
| $-19.1 \geq M_B > -21.1$ | 107 | 19.717 ± 0.052 | 3.594 ± 0.081 | 0.355 0.902 |     |
| $-18.1 \geq M_B > -21.1$ | 232 | 20.291 ± 0.054 | 3.346 ± 0.098 | 0.501 0.791 |     |
| $-17.1 \geq M_B > -21.1$ | 342 | 20.677 ± 0.111 | 3.464 ± 0.270 | 0.756 0.599 |     |
| $-16.1 \geq M_B > -21.1$ | 490 | 21.471 ± 0.147 | 2.751 ± 0.392 | 0.947 0.373 |     |
| $-15.1 \geq M_B > -21.1$ | 626 | 22.149 ± 0.155 | 2.070 ± 0.445 | 1.143 0.251 |     |
| Coma (Gunn r) |     |                   |                   |                   |     |
| $-21.9 \geq M_B > -22.9$ | 4 | 17.963 ± 0.234 | 3.569 ± 0.211 | 0.243 0.894 |     |
| $-20.9 \geq M_B > -22.9$ | 36 | 18.401 ± 0.126 | 3.592 ± 0.188 | 0.285 0.912 |     |
| $-19.9 \geq M_B > -22.9$ | 87 | 18.642 ± 0.107 | 3.937 ± 0.193 | 0.483 0.829 |     |
| $-18.9 \geq M_B > -22.9$ | 114 | 19.050 ± 0.090 | 3.562 ± 0.162 | 0.521 0.786 |     |
| $-17.9 \geq M_B > -22.9$ | 128 | 19.347 ± 0.073 | 3.195 ± 0.119 | 0.550 0.796 |     |
| $-16.9 \geq M_B > -22.9$ | 179 | 19.778 ± 0.080 | 2.832 ± 0.137 | 0.654 0.713 |     |
| Hydra (Gunn r) |     |                   |                   |                   |     |
| $-19.9 \geq M_B > -20.9$ | 12 | 18.292 ± 0.112 | 4.821 ± 0.148 | 0.240 0.974 |     |
| $-18.9 \geq M_B > -20.9$ | 39 | 18.819 ± 0.095 | 5.019 ± 0.144 | 0.490 0.934 |     |
| $-17.9 \geq M_B > -20.9$ | 50 | 19.242 ± 0.157 | 4.803 ± 0.228 | 0.705 0.854 |     |
| $-16.9 \geq M_B > -20.9$ | 54 | 19.218 ± 0.170 | 5.097 ± 0.256 | 0.824 0.813 |     |
| SDSS ($q^*$) |     |                   |                   |                   |     |
| $-22.5 \geq M_B > -23.5$ | 145 | 15.493 ± 0.086 | 4.118 ± 0.072 | 0.161 0.951 |     |
| $-21.5 \geq M_B > -23.5$ | 1974 | 17.745 ± 0.028 | 3.867 ± 0.028 | 0.261 0.902 |     |
| $-20.5 \geq M_B > -23.5$ | 5805 | 18.697 ± 0.016 | 3.272 ± 0.019 | 0.347 0.826 |     |
| $-19.5 \geq M_B > -23.5$ | 8144 | 19.293 ± 0.012 | 2.765 ± 0.015 | 0.400 0.776 |     |
| $-18.5 \geq M_B > -23.5$ | 8635 | 19.472 ± 0.012 | 2.589 ± 0.015 | 0.421 0.754 |     |
| $-17.0 \geq M_B > -23.5$ | 8661 | 19.489 ± 0.013 | 2.572 ± 0.015 | 0.425 0.749 |     |
| SDSS ($r^*$) |     |                   |                   |                   |     |
| $-22.5 \geq M_B > -23.5$ | 152 | 15.319 ± 0.068 | 4.240 ± 0.062 | 0.160 0.956 |     |
| $-21.5 \geq M_B > -23.5$ | 2019 | 17.745 ± 0.024 | 3.791 ± 0.027 | 0.252 0.896 |     |
| $-20.5 \geq M_B > -23.5$ | 5859 | 18.636 ± 0.014 | 3.194 ± 0.018 | 0.337 0.811 |     |
| $-19.5 \geq M_B > -23.5$ | 8159 | 19.161 ± 0.011 | 2.721 ± 0.015 | 0.394 0.757 |     |
| $-18.5 \geq M_B > -23.5$ | 8633 | 19.321 ± 0.012 | 2.555 ± 0.015 | 0.418 0.729 |     |
| $-17.5 \geq M_B > -23.5$ | 8664 | 19.343 ± 0.012 | 2.529 ± 0.016 | 0.422 0.723 |     |
Table B.3. Equivalent to Table A.3 with photometric information in the $B$-filter. KR coefficients for the different galaxy samples in narrow 1 mag intervals. MI is the total absolute magnitude interval within which the galaxies are distributed, $N$ is the number of galaxies in the magnitude interval, $\sigma_{\text{int}}$ is the zero point of KR, $\beta_{\text{int}}$ is the slope of KR, $\sigma_{\text{KR}}$ is the intrinsic dispersion of KR and $R$ is the correlation coefficient of the fit (Pearson Statistics).

| MI         | $N$ | $\alpha_{\text{int}}$ | $\beta_{\text{int}}$ | $\sigma_{\text{KR}}$ | $R$  |
|------------|-----|------------------------|------------------------|------------------------|------|
| Abell (V)  |     |                        |                        |                        |      |
| $-15.1 \geq M_B > -16.1$ | 136 | 22.917 ± 0.063          | 4.733 ± 0.268          | 0.201 ± 0.933          |      |
|            |     |                        |                        |                        |      |
| Coma (Gunn r) |     |                        |                        |                        |      |
| $-15.9 \geq M_B > -16.9$ | 17  | 21.169 ± 0.138          | 2.537 ± 0.276          | 0.536 ± 0.802          |      |

| Hydra (Gunn r) |     |                        |                        |                        |      |
| $-16.9 \geq M_B > -17.9$ | 4   | 20.872 ± 0.024          | 5.037 ± 0.062          | 0.077 ± 0.996          |      |
|            |     |                        |                        |                        |      |
| SDSS ($g^*$) |     |                        |                        |                        |      |
| $-18.0 \geq M_B > -18.5$ | 26  | 18.892 ± 0.040          | 4.808 ± 0.100          | 0.128 ± 0.982          |      |
|            |     |                        |                        |                        |      |
| SDSS ($i^*$) |     |                        |                        |                        |      |
| $-17.4 \geq M_B > -17.9$ | 3   | ...                    | ...                    | ...                    |      |
Table B.4. Equivalent to Table A.4 with photometric information in the $B$-filter. KR coefficients for the SDSS simulation. Increasing magnitude intervals. MI is the total absolute magnitude interval within which the galaxies are distributed, $N$ is the number of galaxies in the magnitude interval, $\alpha_{\text{B}0}$ is the zero point of KR, $\beta_{\text{B}0}$ is the slope of KR, $\sigma_{\text{KR}}$ is the intrinsic dispersion of KR and $R$ is the correlation coefficient of the fit (Pearson Statistics).

| MI   | $N$  | $\alpha_{\text{B}0}$  | $\beta_{\text{B}0}$ | $\sigma_{\text{KR}}$ | $R$  |
|------|------|------------------------|----------------------|-----------------------|------|
| Case 1 |      |                        |                      |                       |      |
| $-18.9 \geq M_B > -19.9$ | 455 | 19.687 ± 0.015 4.971 ± 0.011 0.285 0.995 | | | |
| $-18.9 \geq M_B > -20.9$ | 810 | 19.332 ± 0.022 4.865 ± 0.017 0.555 0.977 | | | |
| $-18.9 \geq M_B > -21.9$ | 1065 | 19.129 ± 0.025 4.673 ± 0.022 0.785 0.945 | | | |
| $-18.9 \geq M_B > -22.9$ | 1220 | 19.077 ± 0.025 4.442 ± 0.027 0.953 0.907 | | | |
| $-18.9 \geq M_B > -23.9$ | 1275 | 19.095 ± 0.025 4.305 ± 0.031 1.030 0.885 | | | |
| Case 2 |      |                        |                      |                       |      |
| $-18.9 \geq M_B > -19.9$ | 140 | 19.633 ± 0.020 4.356 ± 0.032 0.234 0.968 | | | |
| $-18.9 \geq M_B > -20.9$ | 280 | 19.413 ± 0.020 3.604 ± 0.023 0.330 0.953 | | | |
| $-18.9 \geq M_B > -21.9$ | 420 | 19.370 ± 0.019 3.213 ± 0.018 0.365 0.961 | | | |
| $-18.9 \geq M_B > -22.9$ | 560 | 19.377 ± 0.018 3.014 ± 0.013 0.381 0.971 | | | |
| $-18.9 \geq M_B > -23.9$ | 676 | 19.411 ± 0.017 2.872 ± 0.011 0.388 0.976 | | | |

Table B.5. Equivalent to Table A.5 with photometric information in the $B$-filter. KR coefficients for the SDSS simulation. Narrow 1 mag intervals. MI is the total absolute magnitude interval within which the galaxies are distributed, $N$ is the number of galaxies in the magnitude interval, $\alpha_{\text{B}0}$ is the zero point of KR, $\beta_{\text{B}0}$ is the slope of KR, $\sigma_{\text{KR}}$ is the intrinsic dispersion of KR and $R$ is the correlation coefficient of the fit (Pearson Statistics).

| MI   | $N$  | $\alpha_{\text{B}0}$  | $\beta_{\text{B}0}$ | $\sigma_{\text{KR}}$ | $R$  |
|------|------|------------------------|----------------------|-----------------------|------|
| Case 1 |      |                        |                      |                       |      |
| $-18.9 \geq M_B > -19.9$ | 455 | 19.702 ± 0.018 4.819 ± 0.025 0.275 0.984 | | | |
| $-18.9 \geq M_B > -20.9$ | 439 | 19.460 ± 0.022 4.305 ± 0.027 0.478 0.943 | | | |
| $-18.9 \geq M_B > -21.9$ | 579 | 19.438 ± 0.023 3.708 ± 0.028 0.585 0.906 | | | |
| $-18.9 \geq M_B > -22.9$ | 664 | 19.512 ± 0.026 3.268 ± 0.029 0.627 0.889 | | | |
| $-18.9 \geq M_B > -23.9$ | 669 | 19.574 ± 0.027 3.053 ± 0.028 0.640 0.887 | | | |
| Case 2 |      |                        |                      |                       |      |
| $-18.9 \geq M_B > -19.9$ | 140 | 19.633 ± 0.020 4.356 ± 0.032 0.234 0.968 | | | |
| $-18.9 \geq M_B > -20.9$ | 140 | 18.910 ± 0.024 4.356 ± 0.032 0.234 0.968 | | | |
| $-18.9 \geq M_B > -21.9$ | 140 | 18.187 ± 0.033 4.356 ± 0.032 0.234 0.968 | | | |
| $-18.9 \geq M_B > -22.9$ | 140 | 17.464 ± 0.045 4.356 ± 0.032 0.234 0.968 | | | |
| $-18.9 \geq M_B > -23.9$ | 140 | 16.888 ± 0.069 4.356 ± 0.032 0.234 0.968 | | | |

Case 3

| MI   | $N$  | $\alpha_{\text{B}0}$  | $\beta_{\text{B}0}$ | $\sigma_{\text{KR}}$ | $R$  |
|------|------|------------------------|----------------------|-----------------------|------|
| $-18.9 \geq M_B > -19.9$ | 245 | 19.702 ± 0.018 4.819 ± 0.025 0.275 0.984 | | | |
| $-18.9 \geq M_B > -20.9$ | 194 | 18.809 ± 0.024 4.735 ± 0.032 0.269 0.976 | | | |
| $-20.9 \geq M_B > -21.9$ | 140 | 18.084 ± 0.037 4.508 ± 0.043 0.253 0.960 | | | |
| $-21.9 \geq M_B > -22.9$ | 85  | 17.797 ± 0.066 4.028 ± 0.065 0.218 0.925 | | | |
| $-22.9 \geq M_B > -23.9$ | 35  | 18.491 ± 0.168 3.097 ± 0.119 0.140 0.891 | | | |
Appendix C: Hypothesis tests

C.1. Mean value test

Consider the case where some estimator \( \hat{\Phi} \) is computed from a sample of \( N \) independent observations of a random variable \( x(k) \). Assume that there is reason to believe that the true parameter \( \Phi \) being estimated has a specific value \( \Phi_0 \). Now, even if \( \Phi = \Phi_0 \), the sample value \( \hat{\Phi} \) will probably not be exactly equal to \( \Phi_0 \) because of the sampling variability associated with \( \hat{\Phi} \). Hence the following questions arise. If it is hypothesized that \( \Phi = \Phi_0 \), how much difference between \( \hat{\Phi} \) and \( \Phi_0 \) must occur before the hypothesis should be rejected as being invalid? This question can be answered in statistical terms by considering the probability of any noted difference between \( \hat{\Phi} \) and \( \Phi_0 \) based upon the sampling distribution for \( \Phi \). If the probability of a given difference is not small, the difference would be accepted as a normal statistical variability and the hypothesis that \( \Phi = \Phi_0 \) would be accepted.

To clarify the general technique, assume that \( \hat{\Phi} \) has a probability density function of \( p(\Phi) \). Now, if a hypothesis that \( \Phi = \Phi_0 \) is true, then \( p(\Phi) \) would have a mean value of \( \Phi_0 \). The probability that \( \Phi \) would fall below the lower value \( \Phi_{1-\alpha/2} \) is:

\[
\text{Prob}[\Phi \leq \Phi_{1-\alpha/2}] = \int_{-\infty}^{\Phi_{1-\alpha/2}} p(\Phi) \, d\Phi = \frac{\alpha}{2}.
\]

(C.1)

The probability that \( \Phi \) would fall above the upper value \( \Phi_{\alpha/2} \) is:

\[
\text{Prob}[\Phi > \Phi_{\alpha/2}] = \int_{\Phi_{\alpha/2}}^{\infty} p(\Phi) \, d\Phi = \frac{\alpha}{2}.
\]

(C.2)

Hence the probability that \( \Phi \) would be outside the range between \( \Phi_{1-\alpha/2} \) and \( \Phi_{\alpha/2} \) is \( \alpha \). Let the observations be grouped into \( K \) class intervals and computing the expected frequency for each interval assuming \( p(x) = p_0(x) \), compute \( X^2 \) as indicated in Eq. (C.3). Since any deviation of \( p(x) \) from \( p_0(x) \) will cause a \( X^2 \) increase, a one-sided test is used. The region of acceptance is:

\[
X^2 \leq \chi^2_{\alpha}.
\]

(C.4)

If the sample value of \( X^2 \) is greater than \( \chi^2_{\alpha} \), the hypothesis that \( \Phi = \Phi_0 \) is rejected at the \( \alpha \) level of significance. If \( X^2 \) is less than or equal to \( \chi^2_{\alpha} \), the hypothesis is accepted.

C.3. Run test

The run-test is a statistical non-parametric test which does not presuppose a specific distribution for the data which will be evaluated. A run is defined as a sequence of identical observations that are followed or preceded by a different observation or no observation at all. The number of runs which occur in a sequence of observations gives an indication as to whether or not results are independent random observations of the same random variable.

A run-test is used when there is reason to believe that the data present an underlying trend; that is, the probability of the same event changes from one observation to the next. The mean value and variance of the number of runs in a sequence that contains \( N \) observations is given by:

\[
\mu_r = \frac{N}{2} + 1,
\]

(C.5)

\[
\sigma_r^2 = \frac{N(N-2)}{4(N-1)}.
\]

(C.6)

The hypothesis can be tested at any desired level of significance \( \alpha \) by comparing the number of observed runs \( r \) to the number of runs that are contained within the interval \((r_{n,1-\alpha/2}, r_{n,\alpha/2})\) where \( n = \frac{N}{2} \). This number is calculated from the
equation that represents the distribution of the number of runs in a sequence of \( N \) observations; therefore, if we suppose that there is no underlying trend in the data we analyse, and we test this hypothesis with a level of significance \( \alpha \), then, \( r \) in our data must satisfy the following condition \( r_{n,1-\alpha/2} \leq r \leq r_{n,\alpha/2} \); if so we say that the data present no underlying trend with a significance level of \( 1 - \alpha \) or that the significance level of the underlying trend is equal to \( \alpha \).