CONFORMAL HIGGS, OR TECHNI-DILATON
- COMPOSITE HIGGS NEAR CONFORMALITY

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In contrast to the folklore that Technicolor (TC) is a “Higgs less theory”, we shall discuss
existence of a composite Higgs boson, Techni-Dilaton (TD), a pseudo-Nambu-Goldstone
boson of the scale invariance in the Scale-invariant/Walking/Conformal TC (SWC TC)
which generates a large anomalous dimension $\gamma_m \simeq 1$ in a wide region from the dynamical
mass $m = \mathcal{O}(\text{TeV})$ of the techni-fermion all the way up to the intrinsic scale $\Lambda_{\text{TC}}$ of the
SWC TC (analogue of $\Lambda_{\text{QCD}}$), where $\Lambda_{\text{TC}}$ is taken typically as the scale of the Extended
TC scale $\Lambda_{\text{ETC}}$: $\Lambda_{\text{TC}} \approx \Lambda_{\text{ETC}} \sim 10^3 \text{ TeV} \gg m$. All the techni-hadrons have mass on
the same order $\mathcal{O}(m)$, which in SWC TC is extremely smaller than the intrinsic scale
$\Lambda_{\text{TC}} \approx \Lambda_{\text{ETC}}$, in sharp contrast to QCD where both are of the same order. The mass of
TD arises from the non-perturbative scale anomaly associated with the techni-fermion
mass generation and is typically 500-600 GeV, even smaller than other techni-hadrons
of the same order of $\mathcal{O}(m)$, in another contrast to QCD which is believed to have no
scalar $\bar{q}q$ bound state lighter than other hadrons. We discuss the TD mass in various
methods, Gauged NJL model via ladder Schwinger-Dyson (SD) equation, straightforward
calculations in the ladder SD/ Bethe-Salpeter equation, and the holographic approach
including techni-gluon condensate. The TD may be discovered in LHC.

Keywords: Walking Technicolor, Scale Invariance, Conformal Symmetry, Techni-Dilaton,
Fixed Point, Composite Higgs, Large Anomalous Dimension, Holographic Gauge Theory

1. Introduction

Toshihide Maskawa is famous for 2008 Nobel prize-winning paper with Makoto
Kobayashi on CP violation but did also fundamental contributions particularly to
the SCGT, the topics of this workshop: Back in 1974 he found with Hideo Nakajima¹
that spontaneous chiral symmetry breaking (S\chiSB) solution does exists for and only
for the strong gauge coupling, with the critical coupling of order 1, based on the
ladder Schwinger-Dyson (SD) equation with non-running (scale-invariant) coupling,
namely the walking gauge dynamics what is called today. This turned out to be the
origin of SCGT activities toward understanding the Origin of Mass. The present
workshop SCGT 09 was held in honor of his 70th birthday on February 7, 2010
and the 35th anniversary of his crucial contributions to SCGT. I will later explain

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impact of Maskawa-Nakajima solution on the conformal gauge dynamics.

The Origin of Mass is the most urgent issue of the particle physics today and is to be resolved at the LHC experiments. In the standard model (SM), all masses are attributed to a single parameter of the vacuum expectation value (VEV), $\langle H \rangle$, of the hypothetical elementary particle, the Higgs boson. The VEV simply picks up the mass scale of the input parameter $M_0$ which is tuned to be tachyonic ($M_0^2 < 0$) in such a way as to tune $\langle H \rangle \approx 246 \text{ GeV}$ (“naturalness problem”). As such SM does not explain the Origin of Mass.

Technicolor (TC)$^2$ is an attractive idea to account for the Origin of Mass without introducing ad hoc Higgs boson and tachyonic mass parameter: The mass arises dynamically from the condensate of the techni-fermion and the anti techni-fermion pair $\langle \bar{T}T \rangle$ which is triggered by the attractive gauge forces between the pair analogously to the quark-antiquark condensate $\langle \bar{q}q \rangle$ in QCD. For the TC with $SU(N_{TC})$ gauge symmetry and $N_f$ flavors ($N_f/2$ weak doublets) of techni-fermions, the techni-pion decay constant $F_\pi = \langle H \rangle / \sqrt{N_{TC}}$ corresponds to the pion decay constant $f_\pi \approx 93 \text{ MeV}$ in QCD, and hence the TC may be a scale-up of QCD by the factor $F_\pi / f_\pi \approx 2650 / \sqrt{N_f/2}$. Then the mass scale of the condensate $\Lambda_\chi = (\langle \bar{T}T \rangle / N_{TC})^{1/3}$ as the Origin of Mass may be estimated as

$$\Lambda_\chi \approx \left( \frac{-\langle \bar{q}q \rangle}{N_c} \right)^{1/3} \cdot \frac{F_\pi / \sqrt{N_{TC}}}{f_\pi / \sqrt{N_c}} \approx 450 \text{ GeV} \cdot \left( \frac{N_c / N_{TC}}{N_f / 2} \right)^{1/2},$$

where we have used a typical value $(-\langle \bar{q}q \rangle)^{1/3} \approx 250 \text{ MeV}$ ($N_c = 3$).

The dynamically generated mass scale of the condensate $\Lambda_\chi$, or the dynamical mass of the techni-fermion, $m (\sim \Lambda_\chi \sim F_\pi)$, in fact picks up the intrinsic mass scale $\Lambda_{TC}$ of the theory (analogue of $\Lambda_{QCD}$ in QCD) already generated by the scale anomaly through quantum effects (“dimensional transmutation”) in the gauge theory which is scale-invariant at classical level (for massless flavors):

$$\Lambda_{TC} = \mu \cdot \exp \left( - \int^{\alpha(\mu)}_{\alpha(0)} \frac{d\alpha}{\beta(\alpha)} \right) = \Lambda_0 \cdot \exp \left( - \int^{\alpha(\Lambda_0)}_{\alpha(0)} \frac{d\alpha}{\beta(\alpha)} \right),$$

where the running of the coupling constant $\alpha(\mu)$, with non-vanishing beta function $\beta(\alpha) \equiv \mu \frac{d\alpha(\mu)}{d\mu} \neq 0$, is a manifestation of the scale anomaly and $\Lambda_0$ is a fundamental scale like Planck scale. Note that $\Lambda_{TC}$ is independent of the renormalization point $\mu$, $\frac{d\alpha_{TC}}{d\mu} = 0$, and can largely be separated from $\Lambda_0$ through logarithmic running (“naturalness”). Thus the Origin of Mass is eventually the quantum effect in this picture: In the simple scale-up of QCD we would have

$$\text{Naturalness (QCD scale up)} : \quad m \sim \Lambda_\chi \sim \Lambda_{TC} \ll \Lambda_0.$$

The original version of TC, just a simple scale-up of QCD, however, is plagued by the notorious problems: Excessive flavor-changing neutral currents (FCNCs), and excessive oblique corrections of $O(1)$ to the Peskin-Takeuchi $S$ parameter compared with the typical experimental bound about 0.1.
The FCNC problem was resolved long time ago by the TC based on the near conformal gauge dynamics with $\gamma_m \approx 1$ initially dubbed “scale-invariant TC” and then “walking TC”, with almost non-running (conformal) gauge coupling, based on the pioneering work by Maskawa and Nakajima who discovered non-zero critical coupling, $\alpha_{\text{cr}}(\neq 0)$, for the $S\chi SB$ to occur. We may call it “Scale-invariant/Walking/Conformal TC” (SWC TC) (For reviews see Ref.6).

In addition to solving the FCNC problem, the theory made a definite prediction of “Techni-dilaton (TD)”, a pseudo Nambu-Goldstone (NG) boson of the spontaneous breaking of the (approximate) scale invariance of the theory. This will be the main topics of this talk in the light of modern version of SWC TC.

The modern version of SWC TC is based on the Caswell-Banks-Zaks (CBZ) infrared (IR) fixed point, $\alpha^* = \alpha^*(N_f, N_{TC})$, which appears at two-loop beta function for the number of massless flavors $N_f(<11 N_{TC}/2)$ larger than a certain number $N_f^* (\gg N_{TC})$. See Fig. 1 and later discussions. Due to the IR fixed point the coupling is almost non-running (“walking”) all the way up to the intrinsic scale $\Lambda_{\text{TC}}$ which is generated by the the scale anomaly associated with the (two-loop) running of the coupling analogously to QCD scale-up in Eq. (2). For $\mu > \Lambda_{\text{TC}}$ (Region I of Fig. 1) the coupling no longer walks and runs similarly to that of QCD. When we set $\alpha^*$ slightly larger than $\alpha_{\text{cr}}$, we have a condensate or the dynamical mass of the techni-fermion $m (\sim \Lambda_\chi) $, much smaller than the intrinsic scale of the theory $m \ll \Lambda_{\text{TC}}$. The CBZ-IR fixed point $\alpha^*$ actually disappears (then becoming would-be IR fixed point) at the scale $\mu \lesssim m$ where the techni-fermions have acquired the mass $m$ and get decoupled from the beta function for $\mu < m$ (Region III in Fig. 1). Nevertheless, the coupling is still walking due to the remnant of the CBZ-IR fixed point conformality in a wide region $m < \mu < \Lambda_{\text{TC}}$ (Region II in Fig. 1). Thus the symmetry responsible for the natural hierarchy $m \sim \Lambda_\chi \ll \Lambda_{\text{TC}}$ is the (approximate) conformal symmetry, while the naturalness for the hierarchy $\Lambda_{\text{TC}} \ll \Lambda_0$ is the same as that of QCD scale-up in Eq. (3):

\[
\text{Naturalness (SWC TC)} : \quad \Lambda_\chi \ll \Lambda_{\text{TC}} \ll \Lambda_0. \quad (4)
\]

The theory acts like the SWC-TC: It develops a large anomalous dimension $\gamma_m \approx 1$ for the almost non-running coupling in the Region II. Here $\Lambda_{\text{TC}}$ plays a
role of cutoff $\Lambda$ identified with the ETC scale: $\Lambda_{TC} = \Lambda = \Lambda_{ETC}$.

Moreover, there also exists a possibility\textsuperscript{11,12} that the $S$ parameter can be reduced in the case of SWC-TC.

In this talk I will argue\textsuperscript{13} that in contrast to the simple QCD scale-up which is widely believed to have no composite Higgs particle (“higgsless”), a salient feature of SWC TC is the conformality which manifests itself by the appearance of a composite Higgs boson (“conformal Higgs”) as the Techni-dilaton (TD)\textsuperscript{4} with mass relatively lighter than other techni-hadrons: $M_{TD} < M_{\rho}, M_{a_1} \cdots = O(\Lambda_\chi) \ll \Lambda_{TC} = \Lambda_{ETC}$, where $M_{\rho}, M_{a_1} \cdots$ denote the mass of techni-$\rho$, techni-$a_1$, etc. This is contrasted to the QCD dynamics where there are no scalar bound states lighter than others. Note that there is no idealized limit where the TD becomes exactly massless to be a true NG boson, in sharp contrast to the chiral symmetry breaking. Scale symmetry is always broken explicitly as well as spontaneously\textsuperscript{b}.

For the phenomenological purpose, I will argue through several different calculations\textsuperscript{13,15,16} that the techni-dilaton mass in the typical SWC TC models will be in the range (see the footnote below Eq. (30), however):

$$m_{TD} = 500 - 600 \text{ GeV},$$

which is definitely larger than the SM Higgs bound but still within the discovery region of the LHC experiments.

2. Scale-invariant/Walking/Conformal Technicolor

Let us briefly review the SWC TC.

The FCNC problem is related with the mass generation of quarks/leptons mass. In order to communicate the techni-fermion condensate to the quarks/leptons masses $m_{q/l}$, we would need interactions between the quarks/leptons and the techni-fermions which are typically introduced through Extended TC (ETC)\textsuperscript{17} with much higher scale $\Lambda_{ETC}(\gg \Lambda_\chi)$: $m_{q/l} \sim \frac{1}{\Lambda_{ETC}} \langle \bar{T}T \rangle \Lambda_{ETC}$, where $\langle \bar{T}T \rangle \Lambda_{ETC}$ is the condensate measured at the scale of $\Lambda_{ETC}$. (We here do not refer to the origin of the mass scale $\Lambda_{ETC}$ which should also be of dynamical origin such as the tumbling.) Since the newly introduced ETC interactions characterized by the same scale $\Lambda_{ETC}$ should induce extra FCNC’s, we should impose a constraint $\Lambda_{ETC} > 10^6 \text{ GeV}$ in order to avoid the excessive FCNC’s (typically involving $s$ quark). If we assume a

\textsuperscript{a} Preliminary discussions on the revival of the techni-dilaton\textsuperscript{4} were given in several talks\textsuperscript{14}.

\textsuperscript{b} The straightforward calculations near the conformal edge indicated\textsuperscript{16} that there is no isolated massless spectrum: $M_{TD}/F_{\pi}, M_{TD}/M_{\rho}, \cdots \to \text{const.} \neq 0$ even in the limit of $\alpha_s \to \alpha_c (N_f \to N_c^{\text{crit}})$ where $F_{\pi}/\Lambda_{TC}, M_{TD}/\Lambda_{TC}, M_{\rho}/\Lambda_{TC}, \cdots \to 0$. In the case of holographic TD,\textsuperscript{13} this fact is realized in a different manner: Although there apparently exists an isolated massless spectrum, $M_{TD}/F_{\pi} \to 0$ while $M_{\rho}/F_{\pi}, M_{a_1}/F_{\pi} \to \text{const.} \neq 0$, the decay constant of the TD diverges $F_{TD}/F_{\pi} \to \infty$ in that limit and hence it gets decoupled. See later discussions.

\textsuperscript{c} The same can be done in a composite model where quarks/leptons and techni-fermions are composites on the same footing.\textsuperscript{18}
simple QCD scale up, \(\langle \bar{T}T \rangle_{\Lambda_{ETC}} \approx \langle \bar{T}T \rangle_{\Lambda} = -N_{TC} \cdot \Lambda_{\chi}^3\), we would have
\[
m_{q/l} \sim \frac{\Lambda_{\chi}^3}{\Lambda_{ETC}} \cdot N_{TC} < 0.1 \text{ MeV} \cdot N_{TC} \left( \frac{N_c/N_{TC}}{N_d} \right)^{3/2},
\]
which implies that the typical mass (s-quark mass) would be roughly \(10^{-3}\) smaller than the reality. We would desperately need \(10^3\) times enhancement.

This was actually realized dynamically by the TC based on the near conformal gauge dynamics,\(^4,5\) based on the Maskawa-Nakajima solution\(^1\) of the (scale-invariant) ladder Schwinger-Dyson (SD) equation for fermion full propagator \(S_F(p)\) parameterized as
\[
iS_F^{-1}(p) = A(p^2)/p - B(p^2)
\]
with non-running (conformal, an ideal limit of the “walking”) gauge coupling, \(\alpha(Q) \equiv \alpha = \text{constant}, \) with \(Q^2 \equiv -p^2 > 0.\)

\[\text{(See Fig. 2)}\]

Fig. 2. Graphical expression of the SD equation in the ladder approximation.

Maskawa and Nakajima discovered that the \(S_{\chi SB}\) can only take place for strong coupling \(\alpha > \alpha_{cr} = O(1),\) non-zero critical coupling\(^6\). The critical value reads\(^20\)
\[
\alpha_{cr} = (\pi/3) \cdot 2N_{TC}/(N_{TC}^2 - 1) \quad (7)
\]
in the \(SU(N_{TC})\) gauge theory, where \(C_2(F)\) is the quadratic Casimir of the techni-fermion representation of the TC. The asymptotic form of the Maskawa-Nakajima \(S_{\chi SB}\) solution of the fermion mass function \(\Sigma(Q) = B(p^2)/A(p^2)\) in Landau gauge \((A(p^2) \equiv 1)\) reads\(^1,20\)
\[
\Sigma(Q) \sim 1/Q \quad (Q \gg \Lambda_{\chi}).
\]

We then proposed a “Scale-invariant TC”\(^4\), based on the observation that Eq.(8) implies a special value of the anomalous dimension
\[
\gamma_m = -\Lambda \frac{\partial \ln Z_m}{\partial \Lambda} = 1,
\]
to be compared with the operator product expansion (OPE), \(\Sigma(Q) \sim 1/Q^2 \cdot (Q/\Lambda_{\chi})^{\gamma_m}.\) Accordingly, we had an enhanced condensate \(\langle \bar{T}T \rangle_{\Lambda_{ETC}} = Z_m^{-1} \cdot \langle \bar{T}T \rangle_{\Lambda} \approx -N_{TC}(\Lambda_{ETC}/\Lambda_{\chi})^3,\) with the (inverse) mass renormalization constant being \(Z_m^{-1} = (\Lambda_{ETC}/\Lambda_{\chi})^{\gamma_m} \simeq \Lambda_{ETC}/\Lambda_{\chi} \simeq 10^3,\) which in fact yields the desired enhancement. We actually obtained a different formula than Eq.(6):\(^4\)
\[
m_{q/l} \sim \frac{\Lambda_{\chi}^3}{\Lambda_{ETC}} \cdot N_{TC} \cdot \Lambda_{\chi}. \quad \text{(10)}
\]

\(^{6}\) Earlier works\(^19\) in the ladder SD equation with non-running coupling all confused explicit breaking solution with the SSB solution and thus implied \(\alpha_{cr} = 0.\)
The model was formulated in terms of the Renormalization Group Equation (RGE) a la Miransky for the Maskawa-Nakajima solution $\Sigma(m) = m$ which takes the form:

$$\Lambda_{\chi} \sim m \sim 4\Lambda \exp \left(-\frac{\pi}{\sqrt{\alpha/\alpha_{\text{cr}}} - 1}\right),$$  \hspace{1cm} (11)

where $\Lambda$ is the cutoff of the SD equation. This has an essential singularity often called “Miransky scaling” and implies the non-perturbative beta function having a multiple zero:

$$\beta(\alpha)_{\text{NP}} = \Lambda \frac{\partial \alpha(\Lambda)}{\partial \Lambda} = -\frac{2}{3c_2(F)} \left(\frac{\alpha}{\alpha_{\text{cr}}} - 1\right)^{3/2},$$  \hspace{1cm} (12)

with the critical coupling $\alpha_{\text{cr}}$ identified with a nontrivial ultraviolet (UV) stable fixed point $\alpha = \alpha(\Lambda) \rightarrow \alpha_{\text{cr}}$ as $\Lambda/m \rightarrow \infty$.

Subsequently, similar enhancement effects of the condensate were also studied within the same framework of the ladder SD equation, without use of the RGE concepts of anomalous dimension and fixed point, rather emphasizing the asymptotic freedom of the TC theories with slowly-running (walking) coupling which was implemented into the ladder SD equation (“improved ladder SD equation”).

Today the Scale-invariant/Walking/Conformal TC (SWC TC) is simply characterized by near conformal property with $\gamma_m \simeq 1$ (For a review see Ref.\textsuperscript{6}). Such a theory should have an almost non-running and strong gauge coupling (larger than a certain non-zero critical coupling for $S_{\chi_{SB}}$) to be realized either at UV fixed point or IR fixed point, or both (“fusion” of the IR and UV fixed points), as was characterized by “Conformal Phase Transition (CPT)”\textsuperscript{9}.

The essential feature of the above is precisely what happens in the modern version\textsuperscript{7–9} of the SWC TC based on the CBZ IR fixed point\textsuperscript{10} of the large $N_f$ QCD, the QCD-like theory with many flavors $N_f (\gg N_{TC})$ of massless techni-fermions. See Fig. 1. The two-loop beta function is given by

$$\beta(\alpha) = \mu \frac{d\alpha(\mu)}{d\mu} = -b\alpha^2(\mu) - c\alpha^3(\mu),$$

where $b = (11N_{TC} - 2N_f)/(6\pi)$, $c = [34N_{TC}^2 - 10N_fN_{TC} - 3N_f(N_{TC}^2 - 1)/(24\pi^2)]$. When $b > 0$ and $c < 0$, i.e., $N_f^* < N_f < \frac{11}{2}N_{TC} (N_f^* \simeq 8.05$ for $N_{TC} = 3)$, there exists an IR fixed point (CBZ IR fixed point) at $\alpha = \alpha_*$, $\beta(\alpha_*) = 0$, where

$$\alpha_* = \alpha_*(N_{TC}, N_f) = -b/c.$$

Note that $\alpha_* = \alpha_*(N_f, N_{TC}) \rightarrow 0$ as $N_f \rightarrow 11N_{TC}/2$ ($b \rightarrow 0$) and hence there exists a certain range $N_f^* < N_f < 11N_{TC}/2$ (“Conformal Window”) satisfying $\alpha_* < \alpha_{\text{cr}}$, where the gauge coupling $\alpha(\mu) (< \alpha_*)$ gets so weak that attractive forces are no longer strong enough to trigger the $S_{\chi_{SB}}$ as was demonstrated by Maskawa-

\textsuperscript{4} Simple zero of the beta function, $\beta(\alpha) \sim (\alpha - \alpha_{\text{cr}})^1$, never reproduces the essential singularity scaling, as is evident from Eq.(2).

\textsuperscript{5} For SWC TC based on higher representation/other gauge groups see, e.g., Ref.\textsuperscript{22}
Nakajima.\textsuperscript{1} \(N_f^{ct}\) such that \(\alpha_s(N_{TC}, N_f^{ct}) = \alpha_{cr}\) may be evaluated by using the value of \(\alpha_{cr}\) from the ladder SD equation Eq.(7).\textsuperscript{8} \(N_f^{ct} \approx 4N_{TC} (= 12\) for \(N_{TC} = 3\))\textsuperscript{4}

Here we are interested in the S\(\chi\)SB phase slightly off the conformal window, \(0 < \alpha^* - \alpha_{cr} \ll 1\) (\(N_f \approx N_f^{ct}\)). We may use the same equation as the ladder SD equation with \(\alpha(\mu) \approx \text{const.} = \alpha_s\), yielding the same form as Eq.(11)\textsuperscript{8}:

\[
m \sim 4\Lambda_{TC} \exp\left(-\pi/\sqrt{\alpha_s/\alpha_{cr} - 1}\right) \ll \Lambda_{TC} \quad (\alpha_s \approx \alpha_{cr}),
\]

where the cutoff \(\Lambda\) was identified with \(\Lambda_{ETC}\) (= \(\Lambda_{TC}\)). We also have the same result as Eqs. (8),(9):

\[
\Sigma(Q) \sim 1/Q, \quad \gamma_m \approx 1.
\]

Hence it acts like SWC TC. Incidentally, Eq.(14) implies a \textit{multiple zero} at \(\alpha_s = \alpha_{cr}\) in a non-perturbative beta function for \(\alpha_s = \alpha_s(\Lambda)\) similar to Eq.(12), which would suggest “running” of the IR fixed point \(\alpha_s\) with its UV fixed point \(\alpha_{cr}\) in the limit \(\Lambda_{TC}/m \rightarrow \infty\).

The actual running of the coupling largely based on two-loop perturbation is already depicted in Fig. 1. The critical coupling \(\alpha_{cr}\) can be regarded as the UV fixed point viewed from the IR part of the Region II (\(m < \mu < \mu_{cr}\), with \(\mu_{cr}\) such that \(\alpha(\mu_{cr}) = \alpha_{cr}\)), while it is regarded as the IR fixed point from the UV part of the Region II (\(\Lambda_{TC} > \mu > \mu_{cr}\)), with the Region II regarded as \textit{the fusion of the IR and UV fixed points} in the idealized limit of non-running (perturbative) coupling in Region II (or \(\Lambda_{TC}/m \rightarrow \infty\)). Although the perturbative (two-loop) beta function has a \textit{simple zero}, which \textit{never corresponds to the essential singularity scaling} as we noted before, the coupling near \(\alpha_{cr}\) should be sensitive to the non-perturbative effects in such a way that the beta function looks like the \textit{multiple zero} as in Eq. (12) from both sides, corresponding to the \textit{essential singularity scaling} as in Eq. (11).

This should be tested by the fully non-perturbative studies like lattice simulations.

A possible phase diagram (Fig 3 of Ref.\textsuperscript{9}) of the large \(N_f\) QCD on the lattice is also waiting for the test by simulations.

### 3. Conformal Phase Transition\textsuperscript{9,14}

Such an \textit{essential singularity} scaling law like Eq.(11),(14), or equivalently the \textit{multiple zero} of the non-perturbative beta function, characterizes an unusual phase transition, what we called “\textit{Conformal Phase Transition (CPT)}”, where the Ginzburg-Landau effective theory breaks down.\textsuperscript{9} Although it is a second order (continuous) phase transition where the order parameter \(m (\alpha_s > \alpha_{cr})\) is continuously changed to \(m = 0\) in the symmetric phase (conformal window, \(\alpha_s < \alpha_{cr}\), the spectra do not, i.e., while there exist light composite particles whose mass vanishes at the critical

\textsuperscript{8} The value should not be taken seriously, since \(\alpha_s = \alpha_{cr}\) is of \(O(1)\) and the perturbative estimate of \(\alpha_s\) is not so reliable there, although the chiral symmetry restoration in large \(N_f\) QCD has been supported by many other arguments, most notably the lattice QCD simulations,\textsuperscript{23,24} which however suggest diverse results as to \(N_f^{ct}\); See e.g.,\textsuperscript{25} for recent results.
point when approached from the side of the SSB phase, no isolated light particles do not exist in the conformal window, recently dubbed “unparticle”. This reflects the feature of the conformal symmetry in the conformal window. In fact explicit computations show no light (composite) spectra in the conformal window, in sharp contrast to the SSB phase where light composite spectra do exist with mass of order $O(m)$ which vanishes as we approach the conformal window $N_f \to N_f^{cr}$.  

The essence of CPT was illustrated by a simpler model, 2-dimensional Gross-Neveu Model. This is the $D \to 2$ limit of the $D$-dimensional Gross-Neveu model ($2 < D < 4$) which has the beta function and the anomalous dimension:

$$\beta(g) = -2g(g - g_*) \quad \gamma_m = 2g,$$  

where $g = g_*(\equiv D/2 - 1) = g_{cr}$ and $g = 0$ are respectively the UV and IR fixed points of the dimensionless four-fermion coupling, $g$, properly normalized (as $g_* = 1$ for the $D = 4$ NJL model). There exist light composites $\pi, \sigma$ near the UV fixed point (phase boundary) $g \simeq g_*$ in both sides of symmetric ($0 < g < g_*$) and SSB ($g > g_*$) phases as in the NJL model.  

Now we consider $D \to 2$ ($g_* \to 0$) where we have a well-known effective potential: $V(\sigma, \pi) \sim (1/g - 1)/\rho^2 + \rho^2 \ln(\rho^2/\Lambda^2)$, or $\partial^2 V/\partial \rho^2|_{\rho=0} = -\infty$, where $\rho^2 = \pi^2 + \sigma^2$. This implies breakdown of the Ginzburg-Landau theory which distinguishes the SSB phases as in the NJL model.

Now look at the SWC TC as modeled by the large $N_f$ QCD: When the walking coupling $\alpha(Q) \simeq \alpha_*$ is close to the critical coupling, $\alpha_* \simeq \alpha_{cr}$, we should include the induced four-fermion interaction, $(G/2)[(\bar{\psi}\psi)^2 + (\bar{\psi}i\gamma_5\psi)^2]$, which becomes relevant operator due to the anomalous dimension $\gamma_m = 1$, and the system becomes “gauged Nambu-Jona-Lasinio” model whose solution in the full parameter space was obtained in Ref.30.

Thus we may regard the SWC TC as the gauged Nambu-Jona-Lasinio (NJL) model. It was found that SSB solution exists for the parameter space $g > g_{cr}(+) = (1 + \sqrt{1 - \alpha_*/\alpha_{cr}})2/4$ ($\alpha_* < \alpha_{cr}$) as well as the region $\alpha_* > \alpha_{cr}$, where the dimensionless four-fermion coupling $g \equiv G \Lambda^2 (N_f/4\pi^2)$ is normalized as $g = 1$ for $\alpha_* = 0$ (pure NJL model without gauge interaction). Based on the solution (including the
running coupling case, the RGE flow in \((\alpha, g)\) space was found to be along the line of \(\alpha = \alpha_s\) (\(\alpha\) does not run), on which the four-fermion coupling \(g\) runs, with the beta function and anomalous dimension given by \(^{28,32,33}\)

\[
\beta(g) = -2(g - g_{(+)})(g - g_{(-)}), \quad \gamma_m = 2g + \alpha_s/(2\alpha_{cr})
\]

(18)

where \(g = g_{(\pm)} \equiv (1 \pm \sqrt{1 - \alpha_s/\alpha_{cr}})^2/4\) are regarded as the UV/IR fixed points (fixed lines) for \(\alpha_s \leq \alpha_{cr}\). The above anomalous dimension takes the values: \(\gamma_m = 1 + \sqrt{1 - \alpha_s/\alpha_{cr}}\) \(^{31}\) at the UV fixed line while \(\gamma_m = 1 - \sqrt{1 - \alpha_s/\alpha_{cr}}\) at the IR fixed line. Light composite spectra only exist near the UV fixed line (phase boundary) \(g \geq g_{(+)}\) in both SSB \((g > g_{(+)}\)) and symmetric \((g < g_{(+)}\)) phases as in NJL model. Thus it follows that as \(\alpha_s \to \alpha_{cr}\) Eq. (18) takes the form

\[
\beta(g) = -2(g - g_s)^2, \quad \gamma_m|_{g=g_s} = 1, \quad (\alpha_s = \alpha_{cr}),
\]

(19)

with \(g_{(\pm)} \to 1/4 \equiv g_s\), and hence we again got a multiple zero and fusion of UV and IR fixed lines \(^{28,32,33}\) which corresponds to the essential singularity; \(^{30}\) \(m^2 \sim \Lambda^2 \exp(-1/(g - g_s))\). A similar observation was also made recently. \(^{34}\)

In passing, it should be stressed that the anomalous dimension never changes discontinuously across the phase boundary as is seen from Eq.(16) and Eq.(18) \(^{27,28,32}\).

The scale anomaly in this case is given by \(^{9}\)

\[
\langle \partial^\mu D_\mu \rangle = (\theta^0_\mu) = 4(\theta^0_0) = \frac{\beta(g)}{g} \cdot \frac{G}{2} \langle (\bar{\psi}\psi)^2 + (\bar{\psi}i\gamma_5\psi)^2 \rangle
\]

\[
\simeq -m^4 \cdot (4N_fN_{TC}/\pi^4) = O(\Lambda^4),
\]

(20)

where the second line was from the explicit computation\(^{35}\) of the vacuum energy \(\langle \theta^0_0 \rangle\) in the limit \(\Lambda/m \to \infty\) \((g \to g_s)\) at \(\alpha \equiv \alpha_{cr}\) (The result coincides with the one for \(\alpha \to \alpha_{cr}\) with \(g \equiv 0\), see Eq.(24),\(^{36}\)). Again there is a composite state this time having mass \(^{15}\)

\[
M_\sigma = \sqrt{2m},
\]

(21)

as \(g \to g_s + 0\), while there are no composites \(|M|^2 \sim \Lambda^2 \exp(-1/(g - g_s)) \to \infty\) for \(g \to g_s - 0\). Eq.(21) is compared with \(M_\sigma = 2m\) in the pure NJL case with \(\alpha \equiv 0\). This slightly lighter scalar may be identified with the techni-dilaton in the SWC TC. I will come back to this later.

The absence of the composites in the symmetric phase \(g < g_s\) may be understood as in the 2-dimensional Gross-Neveu model for \(g < 0\), namely the repulsive four-fermion interactions: From the analysis of the RG flow, it was argued\(^{32}\) that the IR fixed line \(g = g_{(-)}\) is due to the induced four-fermion interaction by the walking TC dynamics itself, while deviation from that line, \(g - g_{(-)}\), is due to the additional four-fermion interactions, repulsive \((g < g_{(-)}\)) and attractive \((g > g_{(-)}\)), from UV

\(^{b}\) The beta function in Eq.(12) may be regarded as an artificial one keeping \(g \equiv \text{const.}\) which is not along the renormalized trajectory in the extended parameter space \((\alpha, g)\).
dynamics other than the TC (i.e., ETC). It is clear that no light composites exist for repulsive four-fermion interaction \( g < g(-) \), which becomes \( g < g_\ast \) at \( \alpha_\ast = \alpha_\text{ct} \).

4. S Parameter Constraint

Now we come to the next problem of TC, so-called \( S, T, U \) parameters\(^3\) measuring possible new physics in terms of the deviation of the LEP precision experiments from the SM. In particular, \( S \) parameter excludes the TC as a simple scale-up of QCD which yields \( S = (N_f/2) \cdot \hat{S} \) with \( \hat{S}_{\text{QCD}} = 0.32 \pm 0.04 \). For a typical ETC model with one-family TC, \( N_f = 8, \) we would get \( S = O(1) \) which is much larger than the experiments \( S < 0.1 \). This is the reason why many people believe that the TC is dead. However, since the simple scale-up of QCD was already ruled out by the FCNC as was discussed before, the real problem is whether or not the walking/conformal TC which solved the FCNC problem is also consistent with the \( S \) parameter constraint above. There have been many arguments\(^{11,12}\) that the \( S \) parameter value could be reduced in the walking/conformal TC than in the simple scale-up of QCD. Recently such a reduction has also been argued\(^{37,38}\) in a version of the holographic QCD\(^39\) deformed to the walking/conformal TC by tuning a parameter to simulate the large anomalous dimension \( \gamma_m \simeq 1 \).

Here we present the most straightforward computation of the \( S \) parameter for the large \( N_f \) QCD, based on the SD equation and (inhomogeneous) Bethe-Salpeter (BS) equation in the ladder approximation.\(^{12}\) The \( S \) parameter \( S = (N_f/2)\hat{S} \) is defined by the slope of the the current correlators \( \Pi_{VV}(Q^2) \) at \( Q^2 = 0 \):

\[
\hat{S} = -4\pi \frac{d}{dQ^2} \left[ \Pi_{VV}(Q^2) - \Pi_{AA}(Q^2) \right] \bigg|_{Q^2=0},
\]

where \( \delta^{ab} (q_\mu q_\nu/q^2 - g_{\mu\nu}) \Pi_{JJ}(q^2) = \mathcal{F} \cdot \mathcal{T} \cdot i(0|T J_\mu^a(x) J_\nu^b(0)|0), \) \( J_\mu^a(x) = V_\mu^a(x), A_\mu^a(x), \) with \( F_\pi^2 = \Pi_{VV}(0) - \Pi_{AA}(0) \). The current correlators are obtained by closing the fermion legs of the BS amplitudes \( \chi^{(J)}_{\mu}(p,q) \sim \mathcal{F} \cdot \mathcal{T} \cdot (0|T \psi(r/2) \bar{\psi}(-r/2) J_\mu(x) |0), \) which is determined by the ladder BS equation (Fig.3). Solving the BS equation with the fermion propagator given as the solution of the ladder SD equation, we can evaluate the \( \Pi_{VV}(Q^2) - \Pi_{AA}(Q^2) \) numerically. From this result we may read its slope at \( Q^2 = 0 \) to get \( \hat{S} \).

The results show definitely smaller values of \( \hat{S} \) than that in the ordinary QCD and moreover there is a tendency of the \( \hat{S} \) getting reduced when approaching the
conformal window $\alpha_\ast \sim \alpha_{\text{CT}} \left( N_f / N_f^* \right)$. However, due to technical limitation of the present computation getting very close to the conformal window, the reduction does not seem to be so dramatic as the walking TC being enough to be consistent with the experimental constraints. It is highly desirable to extend the computation further close to the conformal window.

Another approach to this problem is the deformation of the holographic QCD by the anomalous dimension. The reduction of $S$ parameter in the SWC TC has been argued in a version of the hard-wall type bottom up holographic QCD\textsuperscript{39} deformed to the SWCTC by tuning a parameter to simulate the large anomalous dimension $\gamma_m \simeq 1.37$. We examined\textsuperscript{38} such a possibility paying attention to the renormalization point dependence of the condensate. We explicitly calculated the $S$ parameter in entire parameter space of the holographic SWC TC. We here take a set of $F_\pi/M_\rho$ and $\gamma_m$. We find that $S > 0$ and it monotonically decreases to zero in accord with the previous results\textsuperscript{37}. However, our result turned out fairly independent of the value of the anomalous dimension $\gamma_m$, yielding no particular suppression solely by tuning the anomalous dimension large, $S \sim B(F_\pi/M_\rho)^2 \rightarrow 0$ as $F_\pi/M_\rho \rightarrow 0$, with $B \simeq 27(32)$ for $\gamma_m \simeq 1(0)$, in sharp contrast to the previous claim\textsuperscript{37}. Although $B$ contains full contributions from the infinite tower of the vector/axial-vector Kaluza-Klein modes (gauge bosons of hidden local symmetries)\textsuperscript{40} of the 5-dimensional gauge bosons, the resultant value of $B$ turned out close to $B \simeq 4\pi a \simeq 8\pi$ of the single $\rho$ meson dominance, where $a \simeq 2$ is the parameter of the hidden local symmetry only for the $\rho$ meson.\textsuperscript{40} This implies that as far as the pure TC dynamics (without ETC dynamics, etc.) is concerned, an obvious way to dynamically reduce $S$ parameter is to tune $F_\pi/M_\rho$ very small, namely techni-$\rho$ mass very large to several TeV region. (See, however, footnote below Eq.(30).)

We would need more dynamical information other than the holographic recipe, since the parameter corresponding to $F_\pi/M_\rho$ as well as the scale parameter is a pure input in all the holographic models, whether bottom up or top down approach, in contrast to the underlying gauge theory which has only a single parameter, a scale parameter like $\Lambda_{\text{QCD}}$.

Curiously enough, when we calculate $F_\pi/M_\rho$ from the SD and the homogeneous BS equations\textsuperscript{16} and $S$ from the SD and the inhomogeneous BS equation\textsuperscript{12} both in the straightforward calculation in the ladder approximation, a set of the calculated values of $(F_\pi/M_\rho, S)$ lies on the line of the holographic result.\textsuperscript{38}

5. Techni-dilaton

Now we come to the discussions of Techni-dilaton (TD). Existence of two largely separated scales, $\Lambda_\chi \sim m$ and $\Lambda_{\text{TC}}$ such that $\Lambda_\chi \ll \Lambda_{\text{TC}}$, is the most important feature of SWC-TC, in sharp contrast to the ordinary QCD with small number of flavors (in the chiral limit) where all the mass parameters like dynamical mass of quarks are of order of the single scale parameter of the theory $\Lambda_{\text{QCD}}$, $m \sim \Lambda_\chi \sim \Lambda_{\text{QCD}}$. See Fig. 1. The intrinsic scale $\Lambda_{\text{TC}}$ is related with the scale anomaly
corresponding to the *perturbative* running effects of the coupling, with the ordinary

\[ \langle \partial^\mu D_\mu \rangle = \frac{\beta(\alpha)}{4\alpha^2} \langle \alpha G_{\mu\nu}^2 \rangle = \mathcal{O}(\Lambda_{TC}^4), \]  

(23)

which implies that all the techni-glue balls have mass of \( \mathcal{O}(\Lambda_{TC}) \).

On the other hand, the scale \( \Lambda_\chi \) is related with totally different scale anomaly
due to the dynamical generation of \( m (\sim \Lambda_\chi) \) which does exist even in the idealized
case with non-running coupling \( \alpha(\mu) \equiv \alpha(> \alpha_{cr}) \) such as the Maskawa-Nakajima
solution,\(^1\) as was discussed some time ago.\(^{36}\) Such an idealized case well simulates the

\[ \text{dynamics of Region II of Fig. 1,} \quad \gamma_m \approx 1 \quad \text{and} \quad m \ll \Lambda_{TC} \],

(24)

in the numerical calculations,\(^{16}\) with the *perturbative* coupling constant in Region
II being almost constant slightly larger than \( \alpha_{cr}, \alpha_{cr} < \alpha(\mu)< \alpha_* \), for a wide

infrared region. The coupling \( \alpha \equiv \alpha_* \) in the “idealized Region II” actually runs *non-perturbatively*
according to the essential-singularity scaling (Miransky scaling\(^{21}\)) of mass generation, Eq.(11), with the

\[ \text{non-perturbative} \quad \beta_{NP}(\alpha), \quad \text{Eq.(12)}, \]

having a *multiple zero* at \( \alpha = \alpha_{cr} \). Then the *non-perturbative* scale anomaly reads\(^9\)

\[ \frac{\langle \partial^\mu D_\mu \rangle_{NP}}{4\alpha^2} \cdot \frac{\langle \alpha G_{\mu\nu}^2 \rangle_{NP}}{m^4} = -\frac{4N_fN_{TC}}{\pi^4} = \mathcal{O}(\Lambda_\chi^4), \]

(24)

where \( \langle \cdots \rangle_{NP} \) is the quantity with the perturbative contributions subtracted;\(^{36}\)

\[ \langle \cdots \rangle_{NP} \equiv \langle \cdots \rangle - \langle \cdots \rangle_{\text{perturbative}}, \quad \text{Eq.(24) coincides with Eq.(20)} \]

and \( \langle \partial^\mu D_\mu \rangle_{NP}/\Lambda_{TC}^4 \)

vanishes with \( \langle \partial^\mu D_\mu \rangle_{NP}/m^4 \rightarrow \text{const.} \neq 0, \) when we approach the conformal window
from the broken phase \( \alpha_* \prec \alpha_{cr} \) (\( m/\Lambda_{TC} \rightarrow 0 \)). All the techni-fermion bound states
have mass of order of \( m \),\(^{41}\) while there are no light bound states in the symmetric
phase (conformal window) \( \alpha_* < \alpha_{cr} \), a characteristic feature of the conformal phase
transition.\(^9\) The TD is associated with the latter scale anomaly and should have
mass on order of \( m(\ll \Lambda_{TC}) \).

**5.1. Calculation from Gauged NJL model in the ladder SD equation\(^{15}\)**

More concretely, the mass of TD or scalar bound state in the SWC-TC was esti-
mated in various methods: The first method\(^{15}\) was based on the the ladder SD
equation for the gauged NJL model which well simulates\(^8,9\) the conformal phase transition in the large \( N_f \) QCD. The result was already given by Eq.(21):

\[ M_{TD} \simeq \sqrt{2}m. \]  

(25)

\(^1\)In terms of the gauged NJL model mentioned in Section 3 this is the expression of the scale anomaly for \( \alpha \rightarrow \alpha_{cr} \) with \( g = \text{const.} = g_* \), in contrast to Eq.(20) for \( g \rightarrow g_* \) with \( \alpha = \text{const.} = \alpha_* = \alpha_{cr} \). Both yield the same vacuum energy and hence the same scale anomaly.
5.2. Straightforward Calculation from Ladder SD and BS equations

Also a straightforward calculation of mass of TD, the scalar bound state was made in the vicinity of the CBZ-IR fixed point in the large $N_f$ QCD, based on the coupled use of the ladder SD equation and (homogeneous) BS equation lacking the first term in Fig. 5. All the bound states masses are $M = O(m)$ and $M/\Lambda_{TC} \to 0$, when approaching the conformal window $\alpha_s \to \alpha_{cr} (N_f \to N_f^0)$ such that $m/\Lambda_{TC} \to 0$. Near the conformal window ($N_f \gg N_f^0$) the calculated values are $M_{\rho}/F_\pi \simeq 11, M_{\omega}/F_\pi \simeq 12$ (near degenerate). On the other hand, the scalar mass sharply drops near the the conformal window, $M_{TD}/F_\pi \gtrsim 4$, or

$$M_{TD} \lesssim 1.5m \simeq 500 \text{GeV} \ (26)$$

in the case of the one-family TC model with $F_\pi \simeq 125 \text{GeV}$.

5.3. Holographic Techni-dilaton

Recently, we have calculated mass of TD in an extension of the previous paper on the hard-wall-type bottom-up holographic SWC-TC by including effects of (techni-)gluon condensation parameterized as

$$\Gamma \equiv \left( \frac{\left(\frac{\frac{1}{2}(\alpha g_{\mu\nu}^2)/F_\pi^2)}{\left(\frac{\frac{1}{2}(\alpha g_{\mu\nu}^2)/f_\pi^2\right)}_\text{QCD}} \right)}{1/4} \right) \ (28)$$

through the bulk flavor/chiral-singlet scalar field $\Phi_X$, in addition to the conventional bulk scalar field $\Phi$ dual to the chiral condensate.

The five-dimensional action is given by

$$S_5 = \int d^4x \int_\mu^{\ell} dz \sqrt{-g} \frac{1}{g_5^2} e^{\varphi_X(z)} \left( -\frac{1}{4} \text{Tr} \left[ L_{MN} L^{MN} + R_{MN} R^{MN} \right] \right.$$

$$\left. + \text{Tr} \left[ D_M \Phi \right] D^M \Phi - m_\Phi^2 \Phi^\dagger \Phi \right) + \frac{1}{2} \partial M \Phi_X \partial^M \Phi_X \ ) \ (29)$$

where the anti-de-Sitter space (AdS$_5$) with the curvature radius $L$ of AdS$_5$ is described by the metric $ds^2 = g_{MN} dx^M dx^N = (L/z)^2 \left( \eta_{\mu\nu} dx^\mu dx^\nu - dz^2 \right)$ with $\eta_{\mu\nu} = \text{diag}[1, -1, -1, -1], g = \text{det}[g_{MN}] = -(L/z)^4, g_5$ denotes the gauge coupling in five-dimension and $c$ is the dimensionless coupling constant, and $L_M(R_M) = L_5^2(R_M)T^a/10$ with the generators of $SU(N_f)$ are normalized by $\text{Tr}[T^a T^b] = \delta^{ab}; \ L(R)_{MN} = \partial_M L(R)_N - \partial_N L(R)_M - i[L(R)_M, L(R)_N].$ The covariant derivative acting on $\Phi$ is defined as $D_M \Phi = \partial_M \Phi + iL_M \Phi - i\Phi R_M.$
The TD, a flavor-singlet scalar bound state of techni-fermion and anti-technifermion, will be identified with the lowest KK mode coming from the bulk scalar field $\Phi$, not $\Phi_X$. Thanks to the additional explicit bulk scalar field $\Phi_X$, we naturally improve the matching with the OPE of the underlying theory (QCD and SWC-TC) for current correlators so as to reproduce gluonic $1/Q^4$ term, which is clearly distinguished from the same $1/Q^4$ terms from chiral condensate in the case of SWC-TC with $\gamma_m \simeq 1$. Our model with $\gamma_m = 0$ and $N_f = 3$ well reproduces the real-life QCD.

It is rather straightforward\cite{37,39} to compute masses of the techni-$\rho$ meson ($M_\rho$), the techni-$a_1$ meson ($M_{a_1}$), while for that of the TD, flavor-singlet scalar meson ($M_{TD}$), we would need additional IR potential with quartic coupling $\lambda$ to stabilize the $S\chi$SB vacuum.\cite{42} Such an IR potential might be regarded as generated by techni-fermion loop effects and we naturally expect $\lambda \sim N_{TC}/(4\pi)^2$. The $S$ parameter was also calculated through the current correlators by the standard way. We found general tendency of the dependence of the meson masses relative to $F_\pi$, $(M_\rho/F_\pi, M_{a_1}/F_\pi, M_{TD}/F_\pi)$ on $\gamma_m$, $S$, and $\Gamma$.

We find a characteristic feature of the techni-dilaton mass related to the conformality of SWC-TC: For fixed $S$ and $\gamma_m$, absolute values of $(M_\rho/F_\pi)$ and $(M_{a_1}/F_\pi)$ are not sensitive to $\Gamma$, although they get degenerate for large $\Gamma$. On the contrary, $(M_{TD}/F_\pi)$ substantially decreases as $\Gamma$ increases. Actually, in the formal limit $\Gamma \to \infty$ we would have $(M_{TD}/F_\pi) \to 0$ (This is contrast to the straightforward computation through ladder SD and BS equations mentioned before\cite{1}). For fixed $S$ and $\Gamma$, again $(M_\rho/F_\pi)$ and $(M_{a_1}/F_\pi)$ are not sensitive to $\gamma_m$, while $(M_{TD}/F_\pi)$ substantially decreases as $\gamma_m$ increases.

Particularly for the case of $\gamma_m = 1$, we study the dependence of the $S$ parameter on $(M_\rho/F_\pi)$ for typical values of $\Gamma$. It is shown that the techni-gluon contribution reduces the value of $S$ about 10% in the region of $S \lesssim 0.1$, although the general tendency is similar to the previous paper\cite{38} without techni-gluon condensation: $\tilde{S}$ decreases to zero monotonically with respect to $(F_\pi/M_\rho)$. This implies $(M_\rho/F_\pi)$ necessarily increases when $\tilde{S}$ is required to be smaller.

To be more concrete, we consider a couple of typical models of SWC-TC with $\gamma_m \simeq 1$ and $N_{TC} = 2, 3, 4$ based on the CBZ-IRFP in the large $N_f$ QCD. Using the non-perturbative conformal anomaly Eq.(23) together with the non-perturbative beta function Eq.(12) and Eq.(11), we find a concrete relation between $\Gamma$ and $(\Lambda_{ETC}/F_\pi)$: In the case of $N_{TC} = 3$ ($N_f = 4\, N_{TC}$) and $S \simeq 0.1$, we have $\Gamma \simeq 7$ for $(\Lambda_{ETC}/F_\pi) = 10^4\text{--}10^5$ (required by the FCNC constraint). Thanks to the large anomalous dimension $\gamma_m \simeq 1$ and large techni-gluon condensation $\Gamma \simeq 7$, we obtain a relatively light techni-dilaton with mass

$$M_{TD} \simeq 600 \text{ GeV}$$

(30) compared with $M_\rho \simeq M_{a_1} \simeq 3.8 \text{ TeV}$ (almost degenerate). Eq. (30) is consistent with the perturbative unitarity of $W_L W_L$ scattering even for large $M_\rho, M_{a_1}$. Note that largeness of $M_\rho$ and $M_{a_1}$ is essentially determined by the requirement of $S = 0.1$.
fairly independently of techni-gluon condensation.\footnote{The calculated \(S\) parameter here was from the TC dynamics alone and could be drastically changed by incorporating contributions from the generation mechanism of the mass of SM fermions such as the strong ETC dynamics. For instance, the fermion delocalization\footnote{This does not mean existence of true (exactly massless) NG boson of the broken scale symmetry, since such a would-be NG boson gets decoupled in our case: The decay constant of techni-dilaton \(F_{TD}\) would diverge, \(F_{TD}/F_{\pi} \to \infty\), in that limit through the PCDC relation \(F_{TD}^2 = -4\langle \theta^\mu \rangle^2 / M_{TD}^2 \sim m^4 / M_{TD}^2 \sim F_{\pi}^2 / M_{TD}^2\). The scale symmetry is broken explicitly as well as spontaneously.} in the Higgsless models as a possible analogue of certain ETC effects in fact can cancel large positive \(S\) arising from the 5-dimensional gauge sector which corresponds to the pure TC dynamics. If it is the case in the explicit ETC model, then the overall mass scale of techni-hadrons including TD would be much lower than the above estimate down to, say, 300 GeV.}

The essential reason for the large \(\Gamma\) is due to the existence of the wide conformal region \(F_\pi(\sim m) < \mu < \Lambda_{ETC} / F_\pi = 10^4 - 10^5\), which yields the smallness of the beta function (see Eq.\,(12) and Eq.\,(11)\footnote{The calculated \(S\) parameter here was from the TC dynamics alone and could be drastically changed by incorporating contributions from the generation mechanism of the mass of SM fermions such as the strong ETC dynamics. For instance, the fermion delocalization\footnote{This does not mean existence of true (exactly massless) NG boson of the broken scale symmetry, since such a would-be NG boson gets decoupled in our case: The decay constant of techni-dilaton \(F_{TD}\) would diverge, \(F_{TD}/F_{\pi} \to \infty\), in that limit through the PCDC relation \(F_{TD}^2 = -4\langle \theta^\mu \rangle^2 / M_{TD}^2 \sim m^4 / M_{TD}^2 \sim F_{\pi}^2 / M_{TD}^2\). The scale symmetry is broken explicitly as well as spontaneously.} in the Higgsless models as a possible analogue of certain ETC effects in fact can cancel large positive \(S\) arising from the 5-dimensional gauge sector which corresponds to the pure TC dynamics. If it is the case in the explicit ETC model, then the overall mass scale of techni-hadrons including TD would be much lower than the above estimate down to, say, 300 GeV.}) and hence amplifies the techni-gluon condensation compared with the ordinary QCD with \(\Gamma = 1\). Actually, in the idealized (phenomenologically uninteresting) limit \(\Lambda_{ETC} / F_\pi \to \infty\), we would have \(\Gamma \to \infty\), which in turn would imply \(M_{TD}/F_\pi \to 0\) as mentioned above.\footnote{The calculated \(S\) parameter here was from the TC dynamics alone and could be drastically changed by incorporating contributions from the generation mechanism of the mass of SM fermions such as the strong ETC dynamics. For instance, the fermion delocalization\footnote{This does not mean existence of true (exactly massless) NG boson of the broken scale symmetry, since such a would-be NG boson gets decoupled in our case: The decay constant of techni-dilaton \(F_{TD}\) would diverge, \(F_{TD}/F_{\pi} \to \infty\), in that limit through the PCDC relation \(F_{TD}^2 = -4\langle \theta^\mu \rangle^2 / M_{TD}^2 \sim m^4 / M_{TD}^2 \sim F_{\pi}^2 / M_{TD}^2\). The scale symmetry is broken explicitly as well as spontaneously.} in the Higgsless models as a possible analogue of certain ETC effects in fact can cancel large positive \(S\) arising from the 5-dimensional gauge sector which corresponds to the pure TC dynamics. If it is the case in the explicit ETC model, then the overall mass scale of techni-hadrons including TD would be much lower than the above estimate down to, say, 300 GeV.}\footnote{The calculated \(S\) parameter here was from the TC dynamics alone and could be drastically changed by incorporating contributions from the generation mechanism of the mass of SM fermions such as the strong ETC dynamics. For instance, the fermion delocalization\footnote{This does not mean existence of true (exactly massless) NG boson of the broken scale symmetry, since such a would-be NG boson gets decoupled in our case: The decay constant of techni-dilaton \(F_{TD}\) would diverge, \(F_{TD}/F_{\pi} \to \infty\), in that limit through the PCDC relation \(F_{TD}^2 = -4\langle \theta^\mu \rangle^2 / M_{TD}^2 \sim m^4 / M_{TD}^2 \sim F_{\pi}^2 / M_{TD}^2\). The scale symmetry is broken explicitly as well as spontaneously.} in the Higgsless models as a possible analogue of certain ETC effects in fact can cancel large positive \(S\) arising from the 5-dimensional gauge sector which corresponds to the pure TC dynamics. If it is the case in the explicit ETC model, then the overall mass scale of techni-hadrons including TD would be much lower than the above estimate down to, say, 300 GeV.}

To conclude, various methods predicted the mass of the techni-dilaton (“conformal Higgs”) in the range of \(M_{TD} \sim 500 - 600\) GeV, which is within reach of LHC discovery. Since the SWC TC models are strong coupling theories and the ladder approximation/holographic calculations would be no more than a qualitative hint, more reliable calculations are certainly needed, including the lattice simulations, before drawing a definite conclusion about the physics predictions. Besides the phase diagram including the TC-induced/ETC-driven four-fermion couplings on the lattice, more reliable calculations such as the spectra as well as anomalous dimensions, non-perturbative beta functions, \(S\) parameter, etc. are highly desired.

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