The Normal Neutrino Mass Hierarchy is Exactly What We Need!

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The preference of the normal neutrino mass hierarchy from the recent cosmological constraints and the global fits of neutrino oscillation experiments does not seem like a wise choice at first glance since it obscures the neutrinoless double beta decay and hence the Majorana nature of neutrinos. Contrary to this naive expectation, we point out that the actual situation is the opposite. Choosing the normal mass hierarchy opens up the possibility of determining the solar octant and simultaneously measuring the two Majorana CP phases. If the neutrino mass hierarchy is normal, the funnel region would completely disappear if the solar mixing angle lives in the higher octant. With a typical $\mathcal{O}(\text{meV})$ sensitivity on the effective mass $|m_{ee}|$, the neutrinoless double beta decay experiment can tell if the funnel region really exists and hence if the solar octant takes the higher octant. With the sensitivity further improved to sub-meV, the two Majorana CP phases can be simultaneously determined. We need the normal neutrino mass hierarchy with very good reasons.

Introduction – The neutrino oscillation [1] is the first clear indication of new physics beyond the Standard Model (SM) of particle physics [2], although whether it is the genuine neutrino mass term or some environmental effect [3–6] is not that straightforward. In the last 20 years, various neutrino experiments have made impressive processes by measuring the neutrino mixing angles and the two mass eigenvalue splittings [7]. The neutrino oscillation (mixing and mass splitting) patterns are coherently weaved, to be wise after the event. It seems there is some intelligent design of neutrino parameters [8, 9]: 1) The solar splitting $\Delta m^2_{\odot} \equiv \Delta m^2_{21}$ is at the right scale to have MSW resonance; 2) The solar angle $\theta_s \equiv \theta_{12}$ takes the right choice to have large enough oscillation ($\sim 0.8$) at KamLAND; 3) The atmospheric splitting $\Delta m^2_{\text{atm}} \equiv \Delta m^2_{31}$ allows full oscillation in the middle range of possible distances that atmospheric neutrinos travel to the detector; 4) The atmospheric angle $\theta_{\text{atm}} \equiv \theta_{23}$ is big enough so that oscillations could be easily seen; 5) The reactor angle $\theta_r \equiv \theta_{13}$ is small enough so as not to confuse interpretation of the above measurements but still large enough to allow the leptonic CP phase and mass hierarchy (MH) measurements. The latest results from T2K and NOvA indicates a maximal Dirac CP phase around $\delta_D \sim -\pi/2$, which is also a good sign.

The only exception comes from the neutrino MH. According to the latest global fit [7] and cosmological constraint [10], the normal hierarchy (NH) is preferred. This is especially not understandable, in contrast to the coherent picture of mixing angles and mass splittings as described above. With NH, the neutrinoless double beta ($0\nu 2\beta$) decay has sizable chance to fall into the funnel region [11] and hence become invisible even for Majorana neutrinos. Even if the effective mass $|m_{ee}|$ is not inside the funnel region, it is still much more difficult to experimentally see the $0\nu 2\beta$ decay with NH. A naive expectation is that the inverted hierarchy (IH) is a better choice for the sake of observing the $0\nu 2\beta$ decay. Why making it difficult for the Majorana nature of neutrinos after paving the way of measuring the oscillation patterns? Especially, the Majorana nature has the strongest theoretical motivations than the oscillation patterns. While the mixing angles and mass splittings are essentially model parameters [12], the Majorana nature is physically well motivated by the seesaw mechanisms [13–16], leptogenesis [17], and anomaly [18]. If there is some intelligent design behind the established the oscillation patterns, it is hard to imagine the $0\nu 2\beta$ decay for measuring the Majorana nature is left unattended. Choosing the NH is hence considered as “God’s Mistake” [9].

This naive expectation is not necessarily true and we provide two arguments. Choosing the NH can induce much more interesting phenomenological consequences and opens unique ways of measuring unknown parameters, including both the solar octant and the two Majorana CP phases. Although NH makes the $0\nu 2\beta$ decay experiments more difficult, the compensations are still quite valuable.

Solar Octant – In the presence of the vector type non-standard interactions (NSI), the solar octant becomes obscured by the degeneracy with the neutrino MH, the Dirac CP phase, and for high energy experiments the $\epsilon_{ee}$ element from the vector NSI [19]. To make it clear, it is better to parametrize the neutrino mixing matrix as $V_{\nu} = U_{23}(\theta_a)U_{13}(\theta_r)U_{12}(\theta_s, \delta_D)$ and the Hamiltonian as

$$\mathcal{H} = \frac{V_{\nu} D^2_{\nu}}{2E_{\nu}} + V_{ee} \begin{pmatrix} 1 + \epsilon_{ee} & 0 \\ 0 & 0 \end{pmatrix},$$

where $D_{\nu} \equiv \text{diag}\{-\frac{1}{2}\Delta m^2_{\odot}, \frac{1}{2}\Delta m^2_{\text{atm}}, \Delta m^2_{\odot} - \frac{1}{2}\Delta m^2_{\text{atm}}\}$ is the diagonal mass matrix. Note that parametrizing the Dirac CP phase $\delta_D$ in the 1–2 mixing $U_{12}(\theta_s, \delta_D)$ is equivalent to the conventional parametrization [2] in the 1–3 mixing, up to rephasing matrices on both sides of $V_{\nu}$. For simplicity, only the $\epsilon_{ee}$ element from the vector NSI is considered since the others are not relevant here.
The vacuum term of (1) receives a minus sign under the change, \( \sin \theta_s \leftrightarrow \cos \theta_s \), \( \delta_D \rightarrow \pi - \delta_D \), and \( \Delta m^2_\alpha \rightarrow -\Delta m^2_\alpha + \Delta m^2_\beta \). For the matter potential term, the minus sign comes from \( \epsilon_{ee} \rightarrow -2 - \epsilon_{ee} \). This degeneracy appears in all neutrino oscillation experiments. Even at low energy where the effect of the vector NSI is suppressed and hence the matter potential term can be simply omitted, the degeneracy still remains between the solar octant, the Dirac CP phase, and the neutrino MH. Without breaking this degeneracy, the solar mixing angle can have two solutions in the lower or higher octant, respectively.

So determining whether the solar mixing angle is in the lower octant (LO), \( \theta_s < \frac{\pi}{4} \), or in the higher octant (HO), \( \theta_s > \frac{\pi}{4} \), is very important for the measuring the Dirac CP phase and the neutrino MH. Since the degeneracy is universal in all neutrino oscillation experiments, independent measurement is necessary. Although neutrino scattering data can help to break the degeneracy to some extent [19, 20], it can only apply for heavy enough mediator and hence is not conclusive. In this paper, we try to find another way of measuring the solar octant with the \( 0\nu 2\beta \) decay.

The octant transformation, \( c_s \leftrightarrow s_s \) where \( (c_s, s_s) \equiv (\cos \theta_s, \sin \theta_s) \), is actually equivalent to \( m_1 \leftrightarrow m_2 \). The effective mass \( m_{ee} \) for the \( 0\nu 2\beta \) decay is,

\[
m_{ee} = c^2_e c^2_s m_1 e^{i \delta_{11}} + c^2_e s^2_s m_2 + s^2_e m_3 e^{i \delta_{33}},
\]

where \( \delta_{11} \equiv \delta_{21} - \delta_D \) is a combination of the Majorana CP phase \( \delta_{21} \) and the Dirac one \( \delta_D \). Note that this form is the same as the conventional parametrization with two complex phases attached to the \( m_1 \) and \( m_3 \) terms. Although the \( m_2 \) term has no complex phase, it plays an equal role as the \( m_1 \) term since both are vectors on the complex plane. This can become transparent by simply rotating the phase \( e^{i \delta_{33}} \) away from the \( m_3 \) term, rendering both the \( m_1 \) and \( m_2 \) terms complex. Since the two Majorana CP phases \( \delta_{11} \) are unknown and can take any values, the effective mass \( |m_{ee}| \) distribution is invariant under the combined switch \( c^2_s m_1 \leftrightarrow s^2_s m_2 \). The effect of \( c_s \leftrightarrow s_s \) is the same as \( m_1 \leftrightarrow m_2 \). A direct consequence is that, if \( m_1 \approx m_2 \), the octant transformation \( c_s \leftrightarrow s_s \) would leave no significant consequence in the \( 0\nu 2\beta \) decay.

Since the two Majorana CP phases are completely free, the transformation of the Dirac CP phase, \( \delta_D \rightarrow \pi - \delta_D \), can be easily absorbed into its Majorana counterparts. To see the effect of switching the solar octants, the two mass eigenvalues have to be non-degenerate. Similarly, this observation also applies for the beta decay where the key parameter is \( m_\beta \equiv m_1^2 + m_2^2 \equiv m_1^2 + m_3^2 \). The Fig. 1 shows the ratio of \( m_1/m_2 \) as a function of the lightest mass, \( m_0 = m_1 \) for NH and \( m_0 = m_3 \) for IH. Since the atmospheric mass splitting is much larger than the solar one, \( \Delta m^2_\alpha/\Delta m^2_\beta \approx 3\% \ll 1 \), \( m_1 \) and \( m_2 \) are almost degenerate across the whole parameter space for IH. In contrast, they can be non-degenerate for NH. With \( m_1 \lesssim 40 \text{ meV} \), there is apparent deviation from being degenerate. The smaller \( m_1 \), the bigger the deviation.

As expected, there is no visible difference between the solar octants for IH while for NH the effect is sizable, as shown in Fig. 2. For IH, the predictions with LO and HO almost completely overlap with each other. So we show only one case in green color and labeled as “IH”. For NH, the prediction with LO (in red color and labeled as “NH-LO”) is totally different from the one with HO (in blue color and labeled as “NH-HO”). Especially, the funnel region for NH-LO completely disappears for NH-HO. Instead, the effective mass \( |m_{ee}| \) is bounded from below across the whole parameter range.

According to the geometrical picture [23], the lower and upper limits are completely determined by the length of the three complex vectors, \( L^1 \equiv c^2_e c^2_s m_1, L^2 \equiv c^2_e s^2_s m_2 \), and \( L_3 \equiv s^2_e m_3 \). With \( m_1 \) and \( m_2 \) switched, namely \( L^1 \equiv s^2_e c^2_s m_1 \) and \( L^2 \equiv c^2_e s^2_s m_2 \) for HO, the situation becomes totally different from the
FIG. 3. The relevant boundary parameters for NH-HO.

LO case. For convenience, we use only the LO value for the solar angle, $\theta_s < \pi/4$, globally. As shown in Fig. 3, $L_{2}^{\text{HO}} > L_{2}^{\text{HO}} + L_{3}$ holds for the whole parameter space. Consequently, the lower limit of the effective mass is always $|m_{ee}|_{\text{NH-HO min}} = L_{2}^{\text{HO}} - L_{2}^{\text{HO}} - L_{3}$. Most importantly, $L_{2}^{\text{HO}}$ never crosses $L_{1}^{\text{HO}} + L_{3}$ since switching $c_s \approx \sqrt{2/3}$ with $s_s \approx \sqrt{1/3}$ significantly amplifies $L_{2}^{\text{HO}}$ while suppressing $L_{1}^{\text{HO}}$. This is especially true for small $m_1$ and hence small $m_1/m_2$. Although $L_3$ contains the largest mass eigenvalue $m_3$, the suppression $s_s^2$ is too significant and renders $L_3$ too small to compensate the difference between $L_{1}^{\text{HO}}$ and $L_{2}^{\text{HO}}$, hence $L_{2}^{\text{HO}} > L_{1}^{\text{HO}} + L_{3}$ always holds. For comparison, the boundary parameters for NH-LO can be found in the Fig. 10b of [11].

Although the lower boundary for the effective mass $|m_{ee}|$ with NH-HO is established, the prediction can still receive significant uncertainty from the neutrino oscillation parameters for both lower and upper boundaries, shown as dashed curves for the $3\sigma$ variations in Fig. 2. As argued in similar situations [11, 24, 25], the largest variation comes from the uncertainties in the solar angle $\theta_s$. This is the place where the intermediate baseline reactor neutrino experiment JUNO [26] can help. The precision measurement on the solar angle $\theta_s$ comes from the slow oscillation modulated by the smaller mass splitting $\Delta m_2^2$ [25, 27],

$$P_{ee} = 1 - \cos^2 \theta_s \sin^2 2\theta_s \sin^2 \Delta_s + \cdots, \quad (3)$$

where $\Delta_s \equiv \Delta m_2^2 L/4E_{\nu}$ while $\cdots$ stands for the higher frequency modes modulated by the larger atmospheric mass splitting $\Delta m_3^2$ and its variation $\Delta m_3^2 - \Delta m_2^2$. The above (3) clearly indicates that the constraint on the solar angle is in the form of $\sin^2 2\theta_s = 4c_s^2 s_s^2$, instead of the individual $c_s$ or $s_s$. The simulations found that $\sin^2 2\theta_s$ can be measured with 0.54% precision [26–28], from which the uncertainty of the individual $s_s^2$ can be extracted as,

$$\delta s_s^2 = 2c_s s_s \delta \theta_s = \frac{c_s^2 s_s^2}{c_s^2 - s_s^2} \delta \sin^2 2\theta_s \sin^2 2\theta_s. \quad (4)$$

The left-hand side is invariant under the octant transformation $c_s \leftrightarrow s_s$, regardless of an overall minus sign. Since the coefficient $2c_s s_s$ of the solar angle variation $\delta \theta_s$ is also invariant under the octant transformation, the absolute uncertainty of the solar angle is not affected, no matter which octant it rests in. The JUNO experimental precision on the solar angle is quite robust against the solar oscillation degeneracy and we can directly use the simulated precision from the JUNO Collaboration [26].

The filled regions in Fig. 2 show the $3\sigma$ range after taking JUNO into account. Adding JUNO significantly reduces the uncertainty in the predicted effective mass, which already seems significant in a log scale plot. Especially, in the vanishing mass limit, $m_1 \rightarrow 0$, the two regions of NH-LO and NH-HO overlap with each other with current global fit values of the oscillation parameters and separate from each other after combining the projected JUNO result. Since the lightest mass eigenvalue $m_1$ is negligible in this range, the upper limit for NH-LO, $|m_{ee}|^{\text{NH-LO max}} = L_{2}^{\text{LO}} + L_{3}$, and the lower limit for NH-HO, $|m_{ee}|^{\text{NH-HO min}} = L_{2}^{\text{HO}} - L_{3}$ are fully determined by the $m_2$ and $m_3$ terms. The difference between these two limits is, $L_{2}^{\text{HO}} - L_{2}^{\text{LO}} - 2L_{3} = c_s^2 (c_s^2 - s_s^2) m_2 - 2s_s^2 m_3$. Since the ratio of the coefficients $2s_s^2/|c_s^2 (c_s^2 - s_s^2)| \approx 6s_s^2 \approx 13.4\%$ is smaller than $m_2/m_3 \approx \sqrt{\Delta m_2^2/\Delta m_3^2} \approx 17.1\%$, $|m_{ee}|^{\text{NH-HO}} - |m_{ee}|^{\text{NH-LO}}$ is always positive. To allow overlap of the NH-LO and NH-HO regions, the solar angle should take a value

$$\cos 2\theta_s = \frac{2s_s^2 \sqrt{\Delta m_2^2}}{c_s^2 \sqrt{\Delta m_3^2}} \approx 26.8\% \quad \Rightarrow \quad \theta_s \approx 37.2^\circ, \quad (5)$$

which is already more than $4\sigma$ away from the current experimental best fit value [7]. In other words, even considering the fact that the best fit value of $\sin^2 2\theta_s$ from JUNO could vary, it is highly impossible the NH-LO and NH-HO regions can overlap. Having $s_s^2 \approx 1/3$ so that the missing solar neutrino measurements have consistently measured $1/3$ of the predicted flux is not the only coincidence. The solar angle not being too large so that the $0\nu 2\beta$ decay can optimize the chance for distinguishing
the solar octant degeneracy adds one more argument to the advertised intelligent design of neutrino parameters [8, 9].

Since the JUNO experiment can essentially reduce the uncertainty from oscillation parameters to almost zero, the remaining uncertainty comes from the $0\nu\beta\beta$ decay measurement itself, including the effective mass sensitivity $\sigma_{|m_{ee}|^2}$ and its central value $|m_{ee}|$, as well as the uncertainty of the cosmological constraint on the neutrino mass sum, $\sigma_{\text{sum}}$. For both, we assume Gaussian distribution with central value at zero unless stated otherwise. The Fig. 4 shows the probability for NH under the joint constraints of the $0\nu\beta\beta$ decay and the cosmological observation. For $\sqrt{|m_{ee}|^2} > 50$ meV, the sensitivity mainly comes from the cosmological constraint and otherwise from the $0\nu\beta\beta$ decay. Around $\sqrt{|m_{ee}|^2} \sim 50$ meV, the two mass hierarchies can already be distinguished with sensitivity $P_{\text{NH}} \equiv \mathcal{L}_{\text{NH}}/(\mathcal{L}_{\text{NH}} + \mathcal{L}_{\text{IH}}) \approx 0.7$, with $\mathcal{L}_{\text{NH}}/\mathcal{L}_{\text{IH}}$ being the likelihood for NH/IH. After establishing the (normal) neutrino mass hierarchy, if $\sqrt{|m_{ee}|^2}$ further improves to the meV scale, it is possible to distinguish the two solar octants for $|m_{ee}| \lesssim 4$ meV as shown in Fig. 5. According to Fig. 2, the lowest point of the lower boundary for NH-HO is $|m_{ee}| = 3.2$ meV at $m_1 = 5.3$ meV without JUNO or $|m_{ee}| = 3.8$ meV at $m_1 = 4.5$ meV with JUNO, lower than 4 meV [29, 30]. However, the realistic measurement has no clear cut. As long as the $0\nu\beta\beta$ sensitivity $\sqrt{|m_{ee}|^2}$ goes below 10 meV, the preference for NH-LO over NH-HO already starts to arise. Then, without observing the $0\nu\beta\beta$ decay, the NH-HO solution can be excluded, with external input for the Majorana nature of neutrinos [31–35].

The Two Majorana CP Phases – If the $0\nu\beta\beta$ decay sensitivity further improves to the sub-meV scale, it is then possible to simultaneously determine the two Majorana CP phases [11, 36]. The basic logic is that the three complex vectors in (2) form a closed Majorana triangle if the effective mass $|m_{ee}|$ vanishes. Once the lengths $L_i$ of its three sides are known, its three inner angles can be uniquely determined as functions of $L_i$. Two of the three inner angles are actually the two Majorana CP phases as defined in (2).

Observing the $0\nu\beta\beta$ decay indicates a nonzero effective mass $|m_{ee}|$, corresponding to only one degree of freedom. Then only one combination of the two Majorana CP phases can be determined or constrained. But a vanishing effective mass, $|m_{ee}| = 0$, gives two independent constraints, $m_{ee} = 0$ or more explicitly, $\Re(m_{ee}) = 0 = \Im(m_{ee})$, where $\Re$ and $\Im$ extract the real and imaginary components, respectively. Two constraints can resolve two degrees of freedom, explaining why the two Majorana CP phases can be simultaneously determined. The same situation can happen for the more realistic case with some upper limit, $|m_{ee}| \leq U$, which can convert to two independent upper limits, $\Re(m_{ee}) \leq U$ and $\Im(m_{ee}) \leq U$. The two Majorana CP phases can then be determined/constrained within some contour. Again, the JUNO experiment [26] can play an important role by reducing the experimental uncertainties from the oscillation parameters to almost zero.

This simultaneous determination of the two Majorana CP phases can only happen when the effective mass $|m_{ee}|$ falls into the funnel region and hence only for NH. With IH, one physical degree of freedom would become invisible forever, which is a big loss for physics search. In contrast, NH makes it possible to measure all physical variables without losing any information.

It seems that the vanishing $|m_{ee}|$ is a disappointing future for the $0\nu\beta\beta$ decay experiments, which is not necessarily true either. The prospect of simultaneously determining the two Majorana CP phases provides a continuous motivation for improving the experimental sensitivity. Either we can verify the Majorana nature or measure the two Majorana CP phases. Both are physically important. To some extend, the $0\nu\beta\beta$ decay has no-loss future.

Conclusion – We envision the future prospect of neutrino mass hierarchy and its role in the $0\nu\beta\beta$ decay. The NH is not the seemingly boring option or “God’s Mistake”, but can lead to much more vivid landscape. First, with $O(10 \text{ meV})$ sensitivity on the effective mass $|m_{ee}|$, the $0\nu\beta\beta$ decay measurement can distinguish NH from IH. Second, if the sensitivity further improves to $O(\text{meV})$, the $0\nu\beta\beta$ decay measurement can distinguish the solar octants. Different from the NH-LO option that has a funnel region in the effective mass distribution, the effective mass of the NH-HO option is bounded from below, $|m_{ee}| \geq 3.2 (3.8) \text{ meV}$ without (with) input from JUNO. The solar angle is at the right value to separate the NH-LO region from the NH-HO one with vanishing lightest mass $m_1$. Finally, if the sensitivity even further improves to sub-meV, NH allows the two Majorana CP phases to be simultaneously determined in the absence of the $0\nu\beta\beta$ decay signal, observing all physical degrees
of freedom. During this adventure, the input on solar angle from JUNO and the input on the Majorana nature from independent measurements are necessary. The rich mine in the $0\nu2\beta$ decay is just starting to appear and the global fit preference of NH is not a nightmare, but an inspiring herald of a new era. The normal neutrino mass hierarchy is exactly what we need!

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