Towards the Identification of Simple Mechanisms
Describing the Development of Settlements

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Abstract: The rapid increase of settlement structures demands the development of suitable models for their description. In this context, different approaches and works are known. An increasing amount of data leads to more and more complicated models. In this work an alternative approach is proposed, which uses the knowledge from the modeling of physical processes with the help of differential equations, in order to represent phenomena of the pattern formation in settlement systems.

We propose a method to investigate the properties of settlement dynamics using the spatial and temporal changes in time resolved population density patterns.

Starting from the more common finding in the literature that settlements often arrange themselves in a regular manner, we examine four fast-growing regions in the Global South using our methodology. Although no clear mechanisms could be identified in the approach presented so far, the workflow presented here creates the possibility of a new view on pattern formation processes to be studied geographically.

I. INTRODUCTION

The human habitat are settlements. They are highly interconnected and fast growing systems. Especially in the Global South, the fraction of population living in urban settlement structures is growing exponentially [1, 2]. This trend poses one of the main challenges of our world [3] as the rising population in such structures is in need of vital infrastructure [4] simultaneously affecting climatic developments [5, 6]. Consequently, it is crucial to understand underlying process of urbanization and anticipate the emergence of settlement structures. The field of urban studies has developed a large variety of models usually trying to achieve accuracy with complexity [7] and large data volumes [8].

Often, these approaches try to represent a multitude of interactions between people with different goals among themselves and with their environment, which leads to a description of cities as complex systems. According to these kind of models the development of cities can only be explained, by modeling the socio-economic-technic system as precise as possible.

In this letter, we propose an alternative approach: We ask, if the development of settlements could be the outcome of some simple rules and how those kind of rules could be detected.

We start with a phenomenological consideration of the settlement arrangement. Considering the arrangement of $N$ settlements within a region of area $A$, this arrangement can take three basic types (see Fig.1).

They may be (i) clustered, suggesting mechanisms of attraction in the settlement process. They may be (ii) randomly distributed in $A$ and thus the product of a random process, and they may be (iii) regularly distributed, indicating repulsive forces in this settlement process.

A simple theoretical approach on the development of settlement structures specifically aiming towards the settlement distribution was already formulated by Hudson [9]. Hudson linked the emergence of this structures to the amount of variables in a niche space $\langle E \rangle$ containing $n$ independent environmental variables (e.g. climatic, cultural, economical or technical). Here three stages were defined that can be differentiated by a specific distribution as the amount and values of environmental variables change in a given physical biotope space by the arrival of human beings: First the initial colonization of an uninhabited area $A$ takes place in which settlers tend to create buildings in close proximity to each other resulting in clusters of settlements. After the initial state is reached and the settlers are able to obtain sufficient supply to survive, the second transformation occurs called spread. Here, settlers aim on populating the empty space and creating farms on their own which results in a random distribution of farms with differently sized plots of land. This transformation ends when all spaces are populated leading to the last transformation called competition. As different farms have varying economical success, all farmers try to increase their outcome by competing with other farmers. The economically successful settlers are able to suppress weaker actors and even take over their property.

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FIG. 1. Possible settlement arrangements following the phenomenological interpretation as point processes.
leading to a regular distribution of equally sized ranches in A. Apart from theory, all three arrangement of settlements can be found in distributions of settlements in agriculturally dominated regions [10][13], suggesting that settlements could follow certain mechanisms, be they attractive, random or repulsive processes. An particularly interesting feature of these regions is the existence of regularities in settlement distributions. Here, Henn et al. [10] proposed that this ”can be used to develop mathematical models that can describe the pattern formation of urban structures by models of non-equilibrium thermodynamics” .

The concept of linking spatial distribution with specific driving mechanisms is mostly prominent in the field of plant and animal ecology [14][17]. Usually simple mechanisms are considered and a limited number of environmental or behavioral aspects is taken into account. Here one might think: How can such simplistic approaches possibly describe the complex mechanisms of urbanization? This question is satisfied, yet assumes that complexity has to be met with at least equally complex models that not necessarily yield to favorable results. Moreover, simplicity does not necessarily exclude complexity of results as May [18] demonstrated: Even simple models containing ordinary or partial differential equations can exhibit very complex dynamical behavior. An example for such models are the well-known Turing instabilities described by reaction-diffusion (RD) equations. They have been proposed by Turing in his ground breaking publication [19] and used to model the creation of patterns on animal skins [20] or in social interaction of ants [21]. Beyond its simplicity, the RD model is widely used to describe spontaneous pattern formation in thermodynamic systems. The occurrence of patterns is contradicting the second law of thermodynamics stating that closed systems have to maintain or increase their entropy over time. Therefore pattern formation is a feature of open, dissipative systems [22]. Furthermore, patterns produced from RD models are able to produce patterns with features similar in sizes and regular in spacing [23].

Both aspects of RD models point towards the applicability of such a simple pattern forming mechanism on settlement structures, as cities and sociological structures are open systems that gain in structure over time. A possible physical interpretation for this was given by Pelz et al. [24] where the formation of informal settlements is depicted by economically driven social interactions.

If we combine the findings so far, we have found features of settlements structures that lead to the assumption that process are responsible for their emergence and we introduced a simple model providing rules for this emergence. In order to substantiate this assumption, we could now follow the common approach of validation: We extract a satellite image of a settlement structure of interest and vary the parameters of our model until the modeled structure matches the data.

However, as Kondo and Miura have shown the observation of singular patterns is not sufficient to produce evidence for such an assumption [25]. They state:

Skeptics rightly point out that just because there is water, it does not mean there are waves. (...) No matter how vividly or faithfully a mathematical simulation might replicate an actual (...) pattern, this alone does not constitute proof that the simulated state reflects the reality.

In consequence an investigation of static patterns and deduction of model parameters does not lead to substantial results. To overcome this, their proposed solution is to compare the dynamical properties of a simulated pattern to the emergence of real structures.

Therefore, the aim of this work is to present a fundamental methodology on how underlying, driving mechanisms of settlement pattern formation can be identified. All materials used and the methodology is described in detail in section II, including the relevant data sets used for examination, the identification approach, used models and an exemplary workflow. Subsequently, results are presented, exhibiting a possible analysis approach (section III), followed by a discussion and summary of our approach (section IV).

II. MATERIALS AND METHODS

The arising question on how the suggested approach can be executed is especially challenging if the scale of the targeted patterns is taken into consideration. Kondo and Miura [25] show the presence of RD mechanisms at skin patterns of a zebra fish, lying in the order of
TABLE I. Investigated regions with their geopolitical location and attributes

| Region                  | Country or Countries | Attributes                                                                 |
|-------------------------|----------------------|-----------------------------------------------------------------------------|
| Punjab                  | India                | northwest India, border region with Pakistan, agricultural region called    |
|                         |                      | Granary of India                                                            |
| Nile delta              | Egypt                | north Egypt, densely populated, fast growing agricultural region            |
| Kano State              | Nigeria, Niger       | north Nigeria, border region to Niger, agricultural region in one of the    |
|                         |                      | fastest growing economies                                                   |
| Province Buenos Aires   | Argentina            | northwest to Buenos Aires, agricultural region, in the area of tension      |
|                         |                      | between an industrialized state and a state of the Global South            |

$10^{-3} \text{ m} \text{ to} 10^{-2} \text{ m}$, by removing part of the skin’s black pigments and observing the reappearance of the pattern. In contrast, settlement structures range between $10^1 \text{ m}$ and $10^4 \text{ m}$ in scale $10^{-2} \text{ m}$, thus comparable experiments can not be performed. Therefore, we use time resolved population density distribution data of four different regions and apply a method proposed by Rudy et al. [27]. It allows to identify possible mechanisms in the form of partial differential equations leading to regular patterns.

A. Data selection

A variety of time resolved satellite data sets is accessible, for a comprehensive summary see Thomson et al. [28]. The amount of such data sets has increased significantly during the last 10 years.

Here two aspects regarding the data are significant: Firstly, the more points in time are available the better. Secondly, higher spatial resolution is to be preferred. Furthermore, an important role is taken by the data type; multiple authors [10, 11, 24] use built-up data of settlements in a binary or discrete format. The value 1 in a pixel represents built-up area, a value of 0 the contrary. Considering the patterns that are created by the aforementioned mechanisms there is a discrepancy. Most of the pattern forming mechanisms describe concentration distributions of different chemical species and do not create discrete patterns [20, 23]. To address this circumstance, population density data has been chosen, since a spatially resolved population is a concentration of inhabitants. In Egypt regularity can be found for $5 \times 10^4$ inhabitants and in Argentina for $4 \times 10^3$ inhabitants. Additionally we found that settlement patterns behave quasi statically not undergoing significant changes over time and in object size ranges.

B. Regularity

As we identified regularity to be a major indicator for the existence of underlying pattern forming mechanisms, we analyzed the selected regions by abstracting them as point patterns which can be categorized in respect of their distribution. Commonly, three different kinds of distributions are distinguished: clustered, regular and random distributions [30, 31]. Examination of the regions shows the dependence of pattern distributions from time and size of the investigated structures. In contrast to [10] or [11] not the Average Nearest Neighbor index is used [30] but the pair correlation function (PCF) [31]. The PCF is than repeatedly applied on the derived point pattern of a region for each year, while the minimal size (here population per cluster) of a settlement is increased successively, resembling a wavelet analysis. With the dependence of pattern distribution from time and size can be visualized in a contour diagram. A detailed description of this analysis, the derivation of system parameters and conclusions for infrastructural decision making can be found in [13].

The important deduction from [13] is that regular patterns can be found ubiquitously within the investigated regions. For region of India, regularity can be found while analyzing all objects up to a population of $10^4$ and above $10^5$ inhabitants. In Egypt regularity can be found for objects larger than $10^5$ inhabitants and in Argentina for objects larger than $10^3$ up to $4 \cdot 10^3$ inhabitants. Additionally we found that settlement patterns behave quasi statically not undergoing significant changes over time and in object size ranges.

C. Methodology

The subsequent question is how the found regular structures can be analyzed to reveal the underlying governing mechanisms. At this point a method not yet applied to settlement data is used which was proposed by Rudy et al. [27] based on the idea of Brunton et al. [33].
FIG. 3. Workflow of our proposed methodology aiming to unveil underlying pattern forming mechanisms by utilizing spatio-temporal satellite imagery of population density data.

The method called *PDE Find* uses time resolved series of spatial images and is able to extract the underlying equations using regression algorithms.

1. Identification Approach

To apply this methodology, we assume that the underlying partial differential equation for a quantity $u$ is of the following form:

$$u_t = N(u, u_x, u_y, ..., x, \xi).$$

The time derivative of $u$ is a function of the quantity itself, its spatial derivatives and a set of parameters $\xi$. Regarding RD equations this form is given when the equations are formulated with e.g. the Schnakenberg kinetic \[34\] with two chemical species $u$ and $v$:

$$u_t = \Delta u + \gamma (a - u + u^2 v),$$

$$v_t = d \Delta v + \gamma (b - u^2 v).$$

The parameters are $d$ the dimensionless diffusion coefficient and $\gamma$ an area parameter \[20\]. As can be seen, the equations Eqs. (2) and (3) are linear combinations of the quantities, their derivatives and combinations up to third order. This property is inherit to partial differential equations of the form in Eq. (1). Assuming that the underlying PDE is not known, similarly to our case, the equation can be written as a general polynomial:

$$u_t = \xi_1 + \xi_2 u + \xi_3 u^2 + \xi_4 u_x + \xi_5 u_{xx} + ...$$

This equation can be reformulated as a row vector containing all combinations and derivatives of the quantity, called the term library and a parameter vector $\xi$ containing all parameters:

$$u_t = \begin{pmatrix} 1 & u & u^2 & u_x & u_{xx} & ... \end{pmatrix} \cdot \xi$$

(5)

The values of each term can be calculated from a single shot at a given point in time, from which the parameters in the $\xi$ vector can be determined. This can be extended to all available time points forming a linear system of equations with the unknown $\xi$:

$$\begin{pmatrix} u_t \\ \vdots \end{pmatrix} = \begin{pmatrix} 1 & u & u^2 & u_x & u_{xx} & ... \end{pmatrix} \cdot \begin{pmatrix} \xi \end{pmatrix}$$

(6)

$$u_t = \Theta \cdot \xi$$

(7)

This system poses an optimization problem for values of $\xi$ and can be solved by using regression models together with a parsimonious approach assuming that the vector is sparse (for more detailed information on the regression algorithm see \[27\]). Additionally, in order to reduce computational expense, not all points of each image are utilized to calculate the parameter vector. As has been shown, a rather small percentage of points can be used to achieve results of near equal quality \[27, 35\]. Thus, the size of the linear system can be reduced significantly.
From the proposed Schnakenberg kinetic, we are able to create term libraries for the model, either in general formulation, where all single terms are included, or in a compressed way, where operators replace single term formulations. To do this, combinations up to third order are created (see Table II).

2. Investigation procedure

The last point of the method concerns the applied procedure to investigate the governing mechanisms. For this, as the data is not optimal and noise free, it has to be prepared resulting in a set of original data and a set where the data is normalized. Together with the separation of term libraries in a compressed and not compressed form, four investigation modes can be derived: Mode 0 using original data as well as individual terms, mode 1 with normalized data and individual terms and in the same manner modes 2, compressed library with original data, and 3, compressed library and normalized data.

The further investigation takes place in the form of a convergence study: To examine how sensitive the calculation is towards the amount of used points, the fraction of total points is successively increased from 0.34 % up to ca. 20 % in 24 steps. By doing this, we are able to determine if there are point ranges where the resulting model is either under fitted, over fitted or showing a convergence towards a certain amount of terms. Moreover, if convergence occurs we can ascertain what term combinations are predominant. The PDE Find algorithm provides the fitting error of each least square interpolation for each data point, i.e. when the internal algorithm is converging, which can be used to assess the quality of results.

D. Workflow

Finally, the method can be summarized as follows (see Fig. 3): First, a time and spatially resolved referenced data set is required involving as high as possible time and spatial resolutions. Second, we have to create term libraries of mechanisms assumed to be responsible for the emergence of a given pattern. They have to be a partial differential equation in the form of Eq. (1). Thereafter, the data set can be passed on to the PDE Find algorithm, whereby the amount of points used for calculation is increased successively. By evaluating the results, we are able to holistically assess the role of simple pattern forming mechanisms in the emergence of settlement structures.

III. RESULTS

As we apply our methodology on the aforementioned data sets of regions of the Global South, we are able
to answer two questions: First, is convergence towards a certain amount of combinations observable? Second, are there combinations that occur more frequently than others?

A. Analysis

The results obtained from the investigation can be seen in Fig. 4. Here, all data modes are assigned to columns and the \( u \) and \( v \) components of the RD mechanism to lines. The regions differ in color applied to the graphs, showing the amount of detected combinations of the term library found in the population density data sets when the fraction of applied points is increased.

When inspecting the figure, we find that the amount of combinations identified reaches values either in close proximity to the maximum of combinations or one or few term combinations independently of the mode or mechanism considered. Slight deviations can be found when analyzing the result of Argentina. In case of the compressed mode for Argentina, the amount of identified combinations is lower than for the other regions as it ranges between 60 and 80% for the majority of plots. Despite this, it can be generally stated that no convergence towards a certain amount of term combinations can be identified. This is generally valid for all chemical species and for all regions, showing that no reaction-diffusion mechanism in the form proposed by Schnakenberg can be identified nor indicates the occurrence of other common reaction-diffusion kinetics.

Furthermore, this finding is supported when the error of the internal least square algorithm is considered (see Fig. 5). The figure depicts the respective error of the algorithm for each region and mode. The absence of all or specific graphs in different diagrams is due to the not converging internal algorithm not allowing to determine an error. When errors are calculated, it becomes apparent that with increasing number of points, the error is increasing accordingly. This development can be interpreted as a systematic error of the calculation leading to the conclusion that either one or several systemically relevant terms or term combinations are neglected within the composed term libraries.

Apparently, no mechanism can be detected directly from the data. Therefore, one additional secondary measure can be taken into account: the occurrence of specific combinations. The results can be seen in Fig. 6 where following the structure of the previous figures, the normalized occurrence in the analyzed 24 point fractions of specific combinations is displayed. This allows to determine whether we are able to detect trends in occurring combinations belonging to the postulated mechanism. We find that in most cases the combinations determined for each region do appear consistently at each point fraction used, independently of region or mode. This means that identification of combinations though not leading to a mechanism exhibits repeatability. Nevertheless, as have shown the previous analysis, most combinations appear at every data point. Even if considering results with fewer combinations, the resulting combinations do not lead to a specific pattern forming mechanism, as the most common terms are \( u, u^2 \) and \( u^3 \) (for the \( u \) component, also true for the \( v \) component). By this, we are able to confirm earlier findings and answer the second question.

FIG. 5. Residual or approximation error of the internal least square algorithm of PDE Find for each region and mode. The absence of several graphs results from the lack of convergence of the least square algorithm not allowing to determine an error.
B. Validation

To round up the investigation and to conclusively disclose that potentially non common RD equations could be formulated by our method, we are validating our results. For this, we simulate the found equations starting from the real structures found in 1975 with the simple method found in [36] modified for our purposes. After 10 seconds, where one second corresponds to one year, we are able to compare the resulting structures. In Fig. 7, an exemplary validation for Argentina at 14.87% point fraction is presented using the following equations:

\begin{align}
    u_t &= -3.74u^3, \\
    v_t &= -0.75v - 1.4v^2 + 1.54v^3.
\end{align}

Not only do the equations not resemble a known RD equation but obviously do lead to a diverging population density distribution from the real. As Fig. 7 shows the overall amount of population and it’s density is decreasing, i.e. the density peaks are damped. This is trivially due to the negative sign of highest order terms and generally high order terms. The overall dampening of concentration in all investigated regions can be seen for all determined equations at every data point, supporting the conclusions from the analysis. Nevertheless, with this exemplary application of our methodology we are able to thoroughly and holistically analyze large scale, man made structures of urbanization and provide comprehensive proof or in this case refutation to our formulated assumptions towards the emergence of settlement structures.

IV. DISCUSSION AND SUMMARY

While formulating such an optimistic view on our results and the possibilities to find the potential existence of simple mechanisms in urbanization, we are aware of the limitations of our analysis. All aspects of our idea, meaning the modeling, data selection or the choice of algorithm, can be put into question.

Regarding the model, we assume that two-component reaction-diffusion equations are governing within the system. This does not have to be true as there are several other pattern forming mechanisms based on one component (e.g. the Swift-Hohenberg equation [37] or Cahn-Hilliard equation [38]) or more than two components (e.g. see [39]) that are able to exhibit a desired behavior. An other, trivial possibility is that no simple mechanism is active and the concept presented in [24], extended to rural settlements in the fashion of [8], is not applicable and linking spatial parameters to specific socioeconomic behavior is faulty.

In respect of data, the choice of sufficiently accurate spatially and temporarily resolved population densities on a global scale is crucial. Here, the currently used and existing data sets do not satisfy the desired or required attributes - Rudy [27], as well as [33] or [35] have investigated noise-free highly time resolved data. This gap in data from satellite imagery is slowly closing and the produced data is increasing in quality over the last years [28]. To name one possible data set under construction, the World Pop data set [40] particularly as the top-down constrained method, where census data is linked to built-up structures, is promising. At the moment of submission, this data sets contains no temporal data (only for...
FIG. 7. Changes of region of Argentina over the course of 10 years: Top the changes simulated with the found model (see Eqs. (8 and 9), bottom the real changes between 1975 and 1985. On the right the changes are displayed with green depicting gains and red loses in population densities, with absolute loses of density.

year 2020) and therefore can not be used.

Lastly, the PDE-Find algorithm and it’s use of linear regression in order to determine parameters can be considered critically. Not only does the algorithm assume that a process is clearly detectable with only a fraction of all points included, but also assumes that the parameters are time-independent. The last aspect was addressed by the authors themselves (see [41]) and could improve the overall results. Furthermore, in the case of sparse spatial and temporal data the use of Bayesian statistics can offer an advantageous solution: As Zao et al. [42] has shown, it enables identification of parameters despite noisy or missing data. A disadvantage of Bayesian statistics is the required repeated extrapolation of partial differential equations making it computationally expensive [43].

Albeit the opportunities for improvement in all main aspects of the method, the essence of the approach remains in place: We were able to provide a repeatable, comparable, holistic and novel approach for investigating the role of simple mechanisms in the creation of complex structures. In our case, the a clear identification of RD mechanisms within population density data was so far fruitless. Nevertheless, the presented workflow can be used to provide proof for formulated approaches in urbanization like in [24] or [8] and change how we perceive large scale processes within populations and urbanization. Through it’s modularity in each aspect, we can extend our approach to other large scale patterns observed in e.g. biological and ecological systems, while utilizing different models, data sets or algorithms. Moreover, it can be key to resolve an ongoing dispute over pattern formation in animal and plant ecosystems, namely if seemingly regular patterns result from self-organization with scale dependent positive and negative feedbacks or by social interaction and competition of different players, or a combination of both [15].

Appendix A: Region Information

Here we attach the geographical data of the regions used to demonstrate the workflow of our method, see Table III. The coordinates are given in decimal degrees in reference system WGS84.

| Region     | West   | South  | East   | North  |
|------------|--------|--------|--------|--------|
| India      | 75.3855| 28.8265| 77.4804| 30.6380|
| Egypt      | 29.8650| 29.9671| 32.1010| 31.8803|
| Argentina  | -63.1349| -34.0166| -60.8382| -32.1041|
| Nigeria    | 7.3774 | 11.1881| 9.3336 | 13.1056|

Appendix B: Index List of Combinations

In Appendix B we attached the index list in Table IV of all combinations created from the libraries in Table II and used in Fig. 6. The combinations are split in the compressed and not compressed libraries. The number 1 in the first line indicates parts of $\xi$ that are numbers not factors of the respective combinations.
TABLE IV. Index list of combinations with c. – compressed and n.c. – not compressed.

| Index | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 |
|-------|---|---|---|---|---|---|---|---|---|----|----|----|----|----|----|
| n.c.  | u_{xx} | u_{yy} | v_{xx} | v_{yy} | v | u | u' | u'' | u'' | u'' | u'' | u'' | u'' | u'' |
| c.    | ∆u | ∆v | u | u'' | u'' | u'' | u'' | u'' | u'' | u'' | u'' | u'' | u'' | u'' |

| Index | 16 | 17 | 18 | 19 | 20 | 21 | 22 | 23 | 24 | 25 | 26 | 27 | 28 | 29 | 30 |
|-------|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|
| n.c.  | u_{xx} | v_{xx} | u_{xx} | v_{xx} | u_{xx} | v_{xx} | u_{xx} | v_{xx} | u_{xx} | v_{xx} | u_{xx} | v_{xx} | u_{xx} | v_{xx} |
| c.    | u_{xx}u_{xx} | u_{xx}u_{xx} | u_{xx}u_{xx} | u_{xx}u_{xx} | u_{xx}u_{xx} | u_{xx}u_{xx} | u_{xx}u_{xx} | u_{xx}u_{xx} | u_{xx}u_{xx} | u_{xx}u_{xx} | u_{xx}u_{xx} | u_{xx}u_{xx} | u_{xx}u_{xx} |

| Index | 31 | 32 | 33 | 34 | 35 | 36 | 37 | 38 | 39 | 40 | 41 | 42 | 43 | 44 | 45 |
|-------|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|
| c.    | u_{xx} | v_{yy} | u_{yy} | v_{xx} | u_{xx} | v_{xx} | u_{xx} | v_{xx} | u_{xx} | v_{xx} | u_{xx} | v_{xx} | u_{xx} | v_{xx} |

| Index | 46 | 47 | 48 | 49 | 50 |
|-------|----|----|----|----|----|
| c.    | u_{yy} | v_{yy} | u_{yy} | v_{yy} | u_{yy} | v_{yy} |

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