Adaptive Passivity-Based Pose Tracking Control of Cable-Driven Parallel Robots for Multiple Attitude Parameterizations

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Abstract—This article presents a pose tracking controller for a six degree-of-freedom (DOF) overconstrained cable-driven parallel robot (CDPR). The proposed control method uses an adaptive feedforward-based controller to establish a passive input–output mapping for the CDPR. This is used alongside a linear time-invariant (LTI) strictly positive real (SPR) feedback controller to guarantee robust closed-loop input–output stability and asymptotic pose trajectory tracking via the passivity theorem. A novelty of the proposed controller is its formulation for use with a range of payload attitude parameterizations, including any unconstrained attitude parameterization, the quaternion, or the direction cosine matrix (DCM). The performance and robustness of the proposed controller is demonstrated through numerical simulations of a CDPR with rigid and flexible cables models. The results show that making use of a multiplicative computation of the pose error, such as when the quaternion or DCM is used within the control law, results in better performance compared to the use of linearized Euler-angle parameterization often used for control of CDPR.

Index Terms—Adaptive control, attitude parameterizations, cable-driven parallel robots (CDPRs), motion control, robust control.

I. INTRODUCTION

OVERCONSTRAINED cable-driven parallel robots (CDPRs) are a class of parallel robots that make use of a redundant set of tensile cable forces to actuate an end-effector or payload. CDPRs typically feature large workspaces and are capable of relatively high payload accelerations due to their low inertia compared to traditional parallel and serial robotic manipulators. Accurate and robust pose (position and attitude/orientation) control of the CDPR’s payload or end-effector is challenging, as overconstrained CDPRs are redundantly actuated, which requires a force distribution algorithm (see [1] for a summary of commonly used methods), and they can have highly uncertain dynamics (e.g., payload with uncertain inertia or flexible/sagging cables). Uncertainty in the CDPR’s dynamics can be accounted for with adaptive control techniques, which are often coupled to a specific form of a feedback controller (e.g., a constant-gain proportional-derivative controller) and a specific representation of the payload’s attitude (e.g., an Euler-angle sequence) [2], [3], [4], [5], [6].

Passivity-based control is capable of providing guarantees of robust closed-loop input–output stability for large ranges of system uncertainty and has been widely implemented on serial robotic manipulators for joint-space trajectory tracking [7], [8]. Passivity-based control has recently been extended to the robust control of parallel robots [9], [10], [11], and in particular, CDPRs [12], [13], [14], [15], [16], [17]. For example, a robust adaptive passivity-based control method for single degree-of-freedom (DOF) CDPRs capable of tracking desired payload trajectories in the presence of model uncertainty and flexible cables was presented in [15]. Early work on the passivity-based control of CDPRs focused on suspended CDPRs with the same number of cables as payload DOFs [12] or relied on having twice as many cables as payload DOFs in the overconstrained case [13], [14], [15]. The work of [11], [16], and [17] demonstrated that passivity-based task-space translational [11], [16] and pose [17] control of CDPRs can be decoupled from the choice of control allocation method, which greatly expanded its applicability to realistic CDPR configurations that typically feature one or two more cables than payload DOFs.

There are a number of applications outside of CDPRs that have gravitated toward the use of attitude parameterizations other than Euler angles (e.g., quaternions, rotation matrix/direction cosine matrix (DCM)/SO(3), and modified Rodrigues parameters (MRPs)) to perform pose tracking. Examples include the pose tracking of unmanned aerial vehicles (UAVs) [18], [19], [20], [21], autonomous underwater vehicles (AUVs) [22], [23], and spacecraft [24], [25], [26], [27], [28]. The benefits of using these attitude parameterization in a feedback control law typically include improved efficiency and robustness due to the explicit manner in which they consider the nonlinear multiplicative nature of attitude kinematics. Unfortunately, the CDPR research community has not yet taken advantage of these benefits, as virtually all CDPR pose regulation and tracking controllers in the literature make use of Euler angles to compute the attitude portion of the control law. Moreover, in the area of task-space CDPR control, which is the focus of this article, state-of-the-art approaches define attitude errors in an additive sense by subtracting the Euler angles of the CDPR payload from the desired payload Euler angles.
angles [29], [30], [31], [32], [33]. By considering additive Euler-angle-based attitude errors, the control law implicitly features a small-angle or linearization assumption, which can directly impact tracking performance and closed-loop stability. Exceptions to the use of additive Euler angles include the DCM-based controller for pose regulation in [17] and the rotation-vector-based controller used for visual serving control in [34]. It is worth noting that joint-space control of CDPRs is also commonly used [2], [4], [5], and [35], where feedback is performed locally on each of the CDPR’s winches to track desired cable lengths associated with a desired payload pose. As joint-space control relies primarily on inverse kinematics and does not require the formation of an attitude error, it is outside the scope of comparison considered in this article.

Although the state-of-the-art additive Euler-angle-based controllers in [29], [30], [31], [32], and [33] have been shown to work in practice, it is unnecessary to restrict CDPR pose control to this one choice of attitude parameterization, especially when the rotation matrix or DCM describing the attitude of the CDPR payload is typically available though the forward kinematics is needed to operate the CDPR. In addition, advances in nonlinear pose estimation has led to methods that directly estimate the rotation matrix/DCM [36], quaternion [37], or more generally any attitude parameterization [38] associated with a CDPR payload, making these quantities readily available for feedback control. Considering the prevalence of literature on the use of other attitude parameterizations for pose feedback control in other applications, it is a surprise that there is such an omission in the control of CDPRs.

This article attempts to fill this void within the CDPR research community by presenting an adaptive passivity-based CDPR pose tracking controller that uses the passivity theorem to guarantee closed-loop input–output stability and asymptotic tracking of a desired payload pose trajectory, where various attitude parameterizations of the payload attitude can be used. The proposed controller takes inspiration from multiple sources, including passivity-based adaptive controllers designed for CDPRs [6], redundantly actuated flexible manipulators [39], and spacecraft [28]. The novel contributions of the proposed controller compared to other adaptive CDPR controllers in the literature, including [2], [3], [4], [5], and [6], is: 1) its ability to make use of any unconstrained attitude parameterization, the quaternion, or the DCM when computing the pose tracking error and 2) its ease of use with any input-strictly passive (ISP) or strictly positive real (SPR) feedback controller. The first contribution has the potential to lead to a more homogeneous CDPR operation framework, where the same attitude parameterization can be used both for kinematics and motion control. At a minimum, the proposed control method provides the CDPR operator with a choice as to which attitude parameterization they desire to use for feedback, which, to the best of the knowledge of the authors, is a limitation in the CDPR literature, where Euler-angle sequences are almost exclusively used for control (exceptions include a DCM-based pose-regulation controller that was used in [17] and a rotation-vector-based controller that was implemented in [34]). This contribution brings all of the benefits associated with the use of quaternion- and DCM-based feedback control with multiplicative attitude error to the application of CDPRs, which traditionally has been lacking. Moreover, by also including an unconstrained attitude parameterization formulation in our proposed control formulation, we provide a road map regarding how to correctly implement an Euler-angle-based pose tracking controller without relying on a small-angle or linearization assumption.

The second contribution related to the use of an ISP or SPR controller has practical benefits, as the design of the feedback controller can be decoupled from the closed-loop stability analysis and practical control designs, such as controllers that are capable of filtering out high-frequency measurement noise, can be implemented. This is a unique benefit provided by the passivity-based approach employed in this work. In contrast with other rigid-body pose tracking controllers in the literature (e.g., [19], [20], [21], [22], [23], [24], [25], [26], [27]) whose closed-loop stability is typically proven for a specific controller form (e.g., a proportional-derivative controller), the proposed passivity-based approach allows for any ISP or SPR feedback controller to be implemented with the guarantee of closed-loop input–output stability. This opens up an entire class of potential feedback controllers, providing additional degrees of design freedom to the control engineer. In this sense, the proposed pose tracking controller may provide interesting capabilities in other applications beyond CDPRs.

The form of the proposed passivity-based adaptive controller used in this article is motivated by the classic work of [7] and [40], which focuses on joint-space control of robotic manipulators. An extension of passivity theory to task-space pose control of a flexible robotic manipulator using unconstrained attitude parameterizations is found in [39]. Another useful contribution to the field is found in the passivity-based attitude control of a spacecraft with quaternions described in [28]. The novelty of the proposed controller compared to the theory developed in [39] includes extending its use to the quaternion and DCM, as well as its application and validation on a CDPR. The quaternion-based spacecraft attitude controller in [28] is modified for use in CDPR pose tracking to yield the proposed quaternion-based method. The work in this article is also an extension of the preliminary study on passivity-based pose regulation of a CDPR in [17], which assumed perfect knowledge of the CDPR dynamics, did not provide any mathematical guarantees of pose tracking error convergence, and was limited to the use of the DCM to represent the attitude of the CDPR’s payload. The control method proposed in this article removes these restrictions and assumptions found in [17], while additionally assessing the robustness of the proposed method in a numerical simulations of a CDPR with flexible cables.

The remainder of this article proceeds as follows. Important preliminaries, including notation, theorems, and a description of the CDPR kinematics and dynamics are presented in Section II. Section III presents the proposed adaptive passivity-based control formulation using unconstrained attitude parameterizations, the quaternion, and the DCM. Section IV includes numerical simulations of the proposed controller implemented on a six DOF CDPR that is chosen
to either have fully rigid cables or flexible cables. Concluding remarks in Section V.

II. PRELIMINARIES

Notation and theorems used throughout the article are presented in this section, followed by an overview of the CDPR kinematics and dynamics considered in this work.

A. Notation, Definitions, and Theorems

For this article, the identity matrix and a matrix of zeros are respectively written as $I$ and $0$. All identity matrices are $I \in \mathbb{R}^{3 \times 3}$ unless indicated otherwise and the dimensions of $0$ are only specified when not readily discernible from context. Matrices are represented in bold (e.g., $A \in \mathbb{R}^{n \times m}$). Positive definite matrices are represented by $A = A^T > 0$. The cross operator, $(\cdot) \times : \mathbb{R}^3 \rightarrow so(3)$, is defined as

$$v^\times = \begin{bmatrix} 0 & -v_3 & v_2 \\ v_3 & 0 & -v_1 \\ -v_2 & v_1 & 0 \end{bmatrix}$$

where $v^T = [v_1 \ v_2 \ v_3]$ and $so(3) = \{S \in \mathbb{R}^{3 \times 3} | S + S^T = 0 \}$. The reverse or uncross operator, $(\cdot)^\vee : so(3) \rightarrow \mathbb{R}^3$, is defined as $A^\vee = [a_1 \ a_2 \ a_3]^T$, where $A = -A^T = (A^\vee)^\times$. The antisymmetric projection operator, $\mathcal{P}(\cdot) : \mathbb{R}^{3 \times 3} \rightarrow so(3)$, projects a matrix $\Xi \in \mathbb{R}^{3 \times 3}$ to the set of antisymmetric matrices, where $\mathcal{P}(\Xi) = (1/2)(\Xi - \Xi^T)$. For $v \in \mathbb{R}^3$ and $\Xi \in \mathbb{R}^{3 \times 3}$, it follows that [41]:

$$\frac{1}{2} \text{tr}(v^\times \Xi) = -v^T\mathcal{P}(\Xi)^\vee.$$

Another useful cross operator identity is given by [42]

$$v^\times A + A^T v^\times = ((\text{tr}(A)I - A)v^\times)$$

where $v \in \mathbb{R}^3$ and $A \in \mathbb{R}^{3 \times 3}$. The signal $y(t)$ satisfies $y \in \mathcal{L}_2$ if $\|y\|_2^2 = \int_0^\infty y^T(t)y(t)dt < \infty$. The signal $y(t)$ satisfies $y \in \mathcal{L}_{2\epsilon}$ if $y_T \in \mathcal{L}_2$ for all $T \in \mathbb{R}_{\geq 0}$, where $y_T(t) = y(t)$ for $0 \leq t \leq T$ and $y_T(t) = 0$ for $T < t$.

The attitude of reference frame $F_p$ relative to reference frame $F_a$ is described by the DCM $C_{pa}$, which is a member of the special orthogonal group $SO(3)$, where $SO(3) = \{C \in \mathbb{R}^{3 \times 3} | C^T C = I, \det(C) = 1 \}$. The DCM $C_{pa}$ is related to the rotation matrix, $R$, that rotates frame $F_a$ to $F_p$ by $C_{pa} = R^T$. Parameterizations of the DCM are represented in this article as $\mathbf{q}_{pa} \in \mathbb{R}^n$, examples of which include an Euler-angle sequence ($\mathbf{q}_{pa} \in \mathbb{R}^3$), the quaternion ($\mathbf{q}_{pa} \in \mathbb{R}^4$), or even the columns of the DCM ($\mathbf{q}_{pa} \in \mathbb{R}^3$). Poisson’s equation relates the angular velocity to the time derivative of the DCM as $C_{pa} = -\omega^pa^TC_{pa}$, where $\omega^pa$ is the angular velocity of $F_p$ relative to $F_a$ resolved in $F_p$. The attitude parameterization rates are related to angular velocity by $\omega^pa = S(\mathbf{q}_{pa})\dot{\mathbf{q}}_{pa}$, where $S(\mathbf{q}_{pa})$ is a mapping matrix whose contents depends on the choice of attitude parameterization [43], [44].

Definition 1 (Passivity [8]): The input–output mapping $u \mapsto y$ associated with the operator $G : \mathcal{L}_{2\epsilon} \rightarrow \mathcal{L}_{2\epsilon}$, where $y = G(u)$, is ISP if for all $u \in \mathcal{L}_{2\epsilon}$ and $T \in \mathbb{R}_{\geq 0}$ there exist $\delta \in \mathbb{R}_{> 0}$ and $\beta \in \mathbb{R}$ such that

$$\int_0^T y^T(t)u(t)dt \geq \delta \|u_T\|_2^2 + \beta.$$  

If (3) is satisfied with $\delta = 0$, then $u \mapsto y$ is passive. The scalar $\beta$ is a constant related to initial conditions.

One of the main benefits of demonstrating that a particular system is passive is that it enables the use of passivity-based control techniques. Specifically, the passivity theorem guarantees closed-loop stability properties of a passive plant in negative feedback with any controller that falls within a specific passivity class [8, p. 358]. This principle guides the development of the proposed passivity-based controller in this work and is used to prove closed-loop input–output stability in Section III-C.

B. CDPR Kinematics and Dynamics

Consider an overconstrained CDPR with $m$ rigid cables actuated by winches and connected to a rigid-body payload, where $m > 6$, as shown in Fig. 1. The CDPR’s equation of motion are derived in [2], [32], [45], and [46] using a Newton–Euler formulation with the assumption of rigid and taut cables, resulting in

$$\mathbf{M}(\rho)\ddot{\mathbf{v}} + \mathbf{D}(\rho, \mathbf{v})\dot{\mathbf{v}} + \mathbf{g}(\rho) = \mathbf{\Pi}^T(\rho)\tau$$

where $\rho^T = [\mathbf{r}^T \ \mathbf{q}_{pa}^T]$ represents the payload’s pose, $\mathbf{r} \in \mathbb{R}^3$ is the position of the payload’s center of mass relative to a point at the origin of an inertial frame $F_a$ resolved in $F_p$, and $\mathbf{q}_{pa} \in \mathbb{R}^n$ is an attitude parameterization of the payload-fixed reference frame $F_p$ relative to $F_a$. The augmented payload velocity is given by $\mathbf{v}^T = [\dot{\mathbf{r}}^T \ \mathbf{\omega}^pa^T]$, where $\mathbf{\omega}^pa \in \mathbb{R}^3$ is the angular velocity of $F_p$ relative to $F_a$ resolved in $F_p$. The torques applied by the winches are denoted as $\tau^T = [\tau_1 \cdots \tau_m]$. The remaining terms in (4) are the mass matrix $\mathbf{M}(\rho) = \mathbf{M}^T(\rho) > 0$; the nonlinear term $\mathbf{D}(\rho, \mathbf{v})$, which contains centrifugal and Coriolis forces; and the gravitational term $\mathbf{g}(\rho)$. Furthermore, it is known that $\mathbf{M}(\rho) - 2\mathbf{D}(\rho, \mathbf{v})$ is skew-symmetric [32]. The winch torques are distributed through the wrench matrix $\mathbf{\Pi}(\rho) \in \mathbb{R}^{m \times 6}$, which is uniquely defined through inverse velocity kinematics and is full rank when the payload remains
within its wrench-feasible workspace. The dynamic model presented in (4) does not consider cable flexibility, cable sag, pulley kinematics, or winch actuator dynamics, all of which can be important effects to model depending on the properties of a given CDPR. An overview of the literature focused on modeling these effects and discussions on when these effects are nonnegligible can be found in [1, Ch. 6]. The control formulation in Section III assumes that the pose of the CDPR payload is known at each instance in time, which is the case for forward kinematics and winch encoder measurements [38], [47], or a Kalman filter that makes use of multiple sensor measurements [36], [48], [49]. However, it is shown in the numerical simulations performed in Section IV with flexible cables that imperfect pose measurements are tolerable.

III. CONTROL FORMULATION AND PASSIVITY AND STABILITY ANALYSES

The pose tracking controller is presented in this section, where the objective is to ensure that the CDPR payload is known at each instance in time, which can be obtained in practice through a vision system [37], forward kinematics, winch encoder measurements [38], or a Kalman filter that makes use of multiple sensor measurements [36], [48], [49]. However, it is shown in the numerical simulations performed in Section IV with flexible cables that imperfect pose measurements are tolerable.

The adaptive feedforward-based control input is derived by first considering a desired feedforward-based input where the objective is to ensure that the control wrench to be applied to the payload and its first three elements are a force resolved in the wrench-feasible workspace. The dynamic model of the CDPR payload is known at each instance in time, which is the case for forward kinematics and winch encoder measurements [38], [47], or a Kalman filter that makes use of multiple sensor measurements [36], [48], [49]. However, it is shown in the numerical simulations performed in Section IV with flexible cables that imperfect pose measurements are tolerable.

The proposed control input is described by

\[ f = \Pi^T(\rho) \tau \]

where \( f \in \mathbb{R}^6 \) is the control wrench to be applied to the payload (i.e., its first three elements are a force resolved in \( F_a \) and its last three elements are a torque resolved in \( F_p \)). See [1] for a summary of a force distribution methods that can be used to determine the torques \( \tau \) that generate the desired control wrench. Force distribution is not a contribution of this work and the following analysis and results are valid for any method, provided \( f = \Pi^T(\rho) \tau \).

The proposed control input is described by

\[ f = f_{ff} + f_{fb} \]

where \( f_{ff} \) is an adaptive feedforward-based input and \( f_{fb} \) is a feedback input. The remainder of this section outlines the proposed adaptive feedforward-based and feedback control inputs, formulations of the pose tracking errors for different attitude parameterizations, along with proofs of passivity and closed-loop trajectory tracking convergence.

A. Adaptive Feedforward-Based Control

The adaptive feedforward-based control input is derived by first considering a desired feedforward-based input

\[ f_d = \begin{bmatrix} m_p \mathbf{1} & 0 \\ \mathbf{0} & \mathbf{I}_p \end{bmatrix} \dot{v}_r + \begin{bmatrix} 0 & 0 & \omega_{pa} & \mathbf{0} \\ \mathbf{0} & \mathbf{I}_p \end{bmatrix} v_r + \begin{bmatrix} m_p g \mathbf{1}_3 \\ \mathbf{0} \end{bmatrix} \]

where \( m_p \in \mathbb{R} \) and \( \mathbf{I}_p = \mathbf{I}_p^T \in \mathbb{R}^{3 \times 3} \) represent the constant mass and inertia of the payload, and \( \mathbf{1}_3 = [0 \ 0 \ 1] \). Note that (6) is equivalent to (4), with the assumption that the dynamics of the CDPR are dominated by those of its rigid-body payload and the replacement of \( \nu \) by \( \nu_r \). It has been shown that many CDPRs, including those with flexible cables [14], [16], have dynamics that are dominated by the rigid-body payload due to the payload having significantly more mass and inertia compared to that of the cables and winches.

\[ \nu_r = \nu_d - PA\dot{p} \]

where \( A = A^T > 0 \) is a control gain, and the terms \( \dot{p} \in \mathbb{R}^6 \) and \( \nu_d \in \mathbb{R}^6 \) are the pose tracking error and desired augmented velocity, respectively, which are defined in Section III-B for different choices of attitude parameterizations, along with the matrix \( P \in \mathbb{R}^{6 \times 6} \). The variable \( \nu_r \) is related to the virtual reference trajectory in [40], but is designed in a distinct manner to accommodate various attitude parameterizations, as outlined in Section III-B. Although (6) includes the desired trajectory, the presence of \( \dot{p} \) introduces feedback within \( f_d \), which is why the term “feedforward-based control input” is used. The feedforward-based input in (6) can be alternatively written as

\[ f_d = Wa \]

where \( a^T = [m_p \ i_{11} \ i_{22} \ i_{33} \ i_{12} \ i_{13} \ i_{23}] \), \( I_{ij} \) represent the six unique entries of \( I_p \), and \( W = (\partial f_d / \partial a) \). The term \( W \) is a function of \( \nu_r \), \( \dot{\nu}_r \), and \( \omega_{pa} \), while \( a \) is equivalent to the minimal parameter formulation developed in [40].

In practice, the entries of \( a \) are not known exactly, so instead an estimate of \( \hat{a} \) is employed, which is denoted as \( \hat{a} \). The adaptive control input in (5) is defined as

\[ f_{ff} = W\hat{a} \]

where \( \hat{a} \) evolves through the adaptive update law

\[ \dot{\hat{a}} = -YW^T\nu_r \]

and \( Y = Y^T > 0 \) is a constant used to adjust the adaptation rate [39]. Subtracting (6) from (4), assuming the dynamics of the CDPR in (4) are dominated by those of its rigid-body payload, and substituting the expressions for the control inputs (5) and (9) yields the error dynamics

\[ M(\rho)\nu_r + D(\rho, \nu)v_r = f - f_d = W\hat{a} + f_{fb} \]

where \( \hat{a} = \hat{a} - a \) and

\[ \dot{v}_r = v - \nu_r = v - (\nu_d - PA\dot{p}) = \dot{v} + P\dot{a} \]

\[ \dot{v} = v - \nu_d. \] Note that since \( a \) is constant, \( \dot{\hat{a}} = \dot{\hat{a}}. \)

B. Feedback Variable Formulation With Various Parameterizations of Payload Attitude

One of the main contributions of this article is extending the control formulation of [39] to accommodate quaternion and SO(3) (rotation matrix/DCM) attitude parameterizations. This extension relies on the derivation of a suitable filtered error system output for each parameterization that yields a passive input–output mapping and fits within the control formulation of [39].

The filtered error output is of the form

\[ s = \dot{p} + A\dot{p} \]
where \( \Lambda = \Lambda^T > 0 \) is a proportional-like control gain that is also featured in the definition of \( v \), in (7). In order to demonstrate a passive input–output mapping, closed-loop input–output stability, and convergence of the pose tracking error as shown in Section III-C, it is required that \( \bar{v}_r = v - v_r = \bar{P}s \), where \( v_r \) is a function of \( v_d, P \), and \( \bar{P} \).

The remainder of this section focuses on determining suitable choices of \( P, \bar{P}, \) and \( v_d \) that ensures this property is satisfied for different choices of attitude parameterizations.

1) Unconstrained Attitude Parameterizations:

Unconstrained attitude parameterizations, such as Euler-angle sequences, the rotation vector, and MRPs, are made up of three parameters that are free to evolve in time without any constraints, but suffer from singularities at one or more attitudes. For example, the 3-2-1 Euler-angle sequence, the rotation vector, and MRPs, are made up for different choices of attitude parameterizations.

Lemma 1: Consider an unconstrained attitude parameterization \( q^{pa} \in \mathbb{R}^3 \). The definitions

\[
P = \begin{bmatrix}
1 \\
0 \\
S(q^{pa})
\end{bmatrix}
\]

\( v_d = P \left[ \dot{r}_d \right] \)

\( \bar{P} = \left[ q^{pa} - q^{da} \right] \)

where \( \dot{r} = r - r_d \), ensure that \( \bar{v}_r = Ps \).

Proof: Substituting (17) into (13) and multiplying by (15) results

\[
\bar{P}s = P \left[ \left( \frac{\dot{r}}{\delta \epsilon} \right) \right] + A \bar{P}
\]

Multiplying out the first term in (18) and using the fact that \( \omega^{pa} = S(q^{pa})q^{pa} \) yields

\[
P \left[ \dot{q}^{pa} \right] = S(q^{pa})q^{pa} = q^{pa} \omega^{pa} = v.
\]

Substituting (19) into (18) and using (12) and (17) gives \( \bar{P}s = \bar{v} + PA\bar{P} = \bar{v}_r \).

As in [39], the definition of \( v_d \) involves evaluating \( S(q^{pa}) \) with the payload attitude and not the desired attitude.

2) Quaternion: The quaternion \( q^{pa} = \left[ e^T \eta \right] \) is composed of the vector portion \( e \in \mathbb{R}^3 \) and scalar part \( \eta \in \mathbb{R} \), which satisfy the constraint \( q^{pa}q^{pa} = e^T \eta + \eta^2 = 1 \). The quaternion error is defined in [28] and [43] as

\[
\delta q = \left[ \begin{array}{l}
\delta \epsilon \\
\delta \eta
\end{array} \right] = \left[ \begin{array}{l}
\eta_1 - \epsilon_x \\
-\epsilon_y \\
\eta_2 - \epsilon_z
\end{array} \right]
\]

where \( q^{da} = \left[ e^T \eta_d \right] \) is the desired quaternion. The desired angular velocity is defined as a function of the rate of the desired quaternion as \( \omega^{da} = 2 \left[ \eta_1 - \epsilon_x - \epsilon_y \right] q^{da} \).

Lemma 2: Consider the quaternion attitude parameterization \( q^{pa} \in \mathbb{R}^4 \). The definitions

\[
P = \begin{bmatrix}
1 \\
0 \\
2(\delta \eta_1 + \delta \epsilon_x)^{-1}
\end{bmatrix}
\]

\( v_d = \left[ \begin{array}{l}
\dot{r}_d \\
\omega^{da} + 2(\delta \eta_1 + \delta \epsilon_x)^{-1}\omega^{da} \delta \epsilon
\end{array} \right]
\]

\( \bar{P} = \left[ \begin{array}{l}
\dot{r} \\
\delta \epsilon
\end{array} \right]
\]

ensure that \( \bar{v}_r = Ps \), where \( \delta \epsilon \) and \( \delta \eta \) are defined in (20).

Proof: Substituting (25) into (13) and multiplying by (21) results in

\[
\bar{P}s = P \left[ \left( \frac{\dot{r}}{\delta \epsilon} \right) \right] + A \bar{P}
\]

The term \( \delta \epsilon \) can be expanded using the property

\[
\delta \epsilon = -\omega^{da} \delta \epsilon + \frac{1}{2}(\delta \eta_1 + \delta \epsilon_x)\left( \omega^{pa} - \omega^{da} \right)
\]

whose derivation can be found in [43]. Substituting (25) into (24) and making use of (22) yields

\[
\bar{P}s = \left[ \omega^{pa} - \omega^{da} - 2(\delta \eta_1 + \delta \epsilon_x)^{-1}\omega^{da} \delta \epsilon \right] + A \bar{P}
\]

\[
= \bar{v} + PA\bar{P} = \bar{v}_r.
\]

The inverse of the matrix \( (\delta \eta_1 + \delta \epsilon_x) \) that is used to define \( P \) exists provided \( \delta \eta \neq 0 \). This singularity is avoided as long as \( F_p \) and \( F_d \) are within a \( \pm \pi \) rad rotation of each other, which is to be expected for overconstrained CDPRs.

3) SO(3) (the DCM): The DCM can be used directly with the antisymmetric projection operator to form an attitude error and satisfy the desired property.

Lemma 3: Consider an SO(3) description of attitude with the DCM \( C_{pa} \in SO(3) \). The definitions

\[
P = \begin{bmatrix}
1 \\
0 \\
-2(\text{tr}(C_{pd})I - C_{pd})^{-1}
\end{bmatrix}
\]

\( v_d = \left[ \begin{array}{l}
\dot{r}_d \\
\omega^{da}
\end{array} \right]
\]

\( \bar{P} = \left[ \begin{array}{l}
\left( \text{P}(C_{pd}) \right) V
\end{array} \right]
\]

ensure that \( \bar{v}_r = Ps \), where \( C_{pd} = C_{pa}C_{da}^T \) and \( C_{da} \) represents the desired payload attitude.

Proof: Substituting (28) into (13) and multiplying by (28) gives

\[
\bar{P}s = P \left[ \left( \frac{\dot{r}}{\delta \epsilon} \right) \right] + A \bar{P}
\]
Poisson’s equation, \( \dot{C}_{pd} = -\omega^{pd*}C_{pd} \), and the identities in (1) and (2) are used to compute
\[
\frac{d}{dt} \left( P(C_{pd}) V \right) = -\frac{1}{2} \left( \omega^{x}C_{pd} + C^{T}_{pd}\omega^{pd*} \right) V
\]  
\[
= -\frac{1}{2} \left( \delta \left( (tr(C_{pd})I - C_{pd}) \omega^{pd} \right) \right) V
\]  
\[
= -\frac{1}{2} \left( tr(C_{pd})I - C_{pd} \right) \omega^{pd} \tag{30}
\]
where \( \omega^{pd} = \omega^{pa} - \omega^{da} \). Substituting (30) into (29) and using (27) yields
\[
Ps = \begin{bmatrix} \dot{r} \\ \dot{\omega} \end{bmatrix} + P\dot{\lambda}P = \dot{v} + P\dot{\lambda}P = \ddot{v}_{r}.
\]

The inverse of \((tr(C_{pd})I - C_{pd})\) in the definition of \(P\) exists as long as \(tr(C_{pd}) \neq 1\). Similar to the case of the quaternion, this singularity is avoided as long as \( F_{f} \) and \( F_{d} \) are within a \( \pm \pi/2 \) rad rotation of each other, which is large enough to account for the wrench-feasible workspaces of most overconstrained CDPRs.

It is worth noting that the term \( W \) in the adaptive feedforward-based control input of (9) relies on the computation of \( \dot{v}_{r} = \dot{v}_{d} - (P\dot{\lambda}P + P\dot{\lambda}P) \), which requires an expression for \( P \). For the case of unconstrained attitude parameterizations, such as a 3-2-1 Euler-angle sequence, this involves simply taking the time derivative of \( S(q^{pa}) \). For the quaternion, this computation is more involved, where \( P \) is solved for using the matrix product rule \((d/dt)(A^{-1}) = -A^{-1}AA^{-1}\) to obtain
\[
\dot{P} = \text{diag}(0, -2(\delta\eta I + \delta e^{x} - 1)(\delta\eta I + \delta e^{x})(\delta\eta I + \delta e^{x} - 1)^{-1})
\]
where \( \delta\eta \) and \( \delta e \) are found by differentiating (20) with respect to time. A similar procedure is used to compute \( P \) when using the SO(3) description of attitude, where
\[
\dot{P} = \text{diag}(0, -2\Gamma (tr(C_{pd})I - C_{pd}) \Gamma)
\]  
\[
= \text{diag}(0, -2\Gamma (tr(\omega^{x}C_{pd})I + \omega^{x}C_{pd}) \Gamma).
\]
\( \Gamma = (tr(C_{pd})I - C_{pd})^{-1} \), and Poisson’s equation is used to simplify the expression for \( C_{pd} \).

Additionally, it should be acknowledged that attitude singularities are typically not a concern for the majority of CDPRs due to the singularities being outside the wrench-feasible workspace.

C. Passivity and Closed-Loop Stability Analyses

The central results of this article demonstrating that the proposed control strategy renders the plant passive and ensures closed-loop input–output stability via the passivity theorem are presented in the following theorem and corollary.

\textbf{Theorem 1:} Consider a CDPR with error dynamics defined in (11) and the adaptive feedforward-based control input of (5) with the adaptive update law in (10). Assuming that the dynamics of the CDPR are dominated by its rigid-body payload, the input–output mapping \( \bar{f}_{fb} \mapsto s \) is passive, where \( \bar{f}_{fb} = P^{T}\bar{f}_{fb} \).

\textbf{Proof:} Define the nonnegative function
\[
V_{1} = \frac{1}{2} \dot{v}_{r}^{T}M\ddot{v}_{r} + \frac{1}{2} \dot{a}^{T}Y^{-1}a
\]
where \( M = M^{T} > 0 \) and \( \Gamma = \Gamma^{T} > 0 \). Taking the derivative of \( V_{1} \), substituting in the adaptive update law and (11) results in
\[
\dot{V}_{1} = \dot{v}_{r}^{T}(M\ddot{v}_{r} + f_{fb}) + \frac{1}{2} \dot{a}^{T}(M - 2D)\ddot{a} - \dot{a}^{T}w^{T}\ddot{v}_{r}
\]  
\[
= \ddot{v}_{r}^{T}f_{fb} = (Ps)^{T}f_{fb} = s^{T}f_{fb} \tag{31}
\]
Integrating (31) from \( t = 0 \) to \( t = T \), where \( T \in \mathbb{R}_{t} \) gives
\[
\int_{0}^{T} s^{T}f_{fb}dt = V_{1}(T) - V_{1}(0) \geq -V_{1}(0)
\]
which proves the mapping \( \bar{f}_{fb} \mapsto s \) is passive.

\textbf{Corollary 1:} The closed-loop system involving the CDPR with error dynamics defined in (11), the adaptive feedforward-based control input of (5), and an ISP negative feedback controller (or alternatively a linear time-invariant (LTI) SPR negative feedback controller) is input–output stable (i.e., \( s \in L_{2} \)).

\textbf{Proof:} Knowing that the input–output mapping \( \bar{r}_{fb} \mapsto s \) is passive and an ISP controller is implemented in a negative feedback connection with this mapping (see Fig. 2), the passivity theorem guarantees that \( s \in L_{2} \) [8, p. 358]. In the case of an SPR feedback controller, Theorem 8.10 in [50, p. 219] can be used to obtain the same result.

Corollary 1 guarantees closed-loop input–output stability with the use of any ISP or SPR feedback controller. This result does not rely on exact knowledge of the parameters of the CDPR’s dynamics, and thus, robust closed-loop input–output stability is guaranteed. However, it is worth noting that robustness to pose estimation error is not guaranteed and falls beyond the scope of this work. The closed-loop system is also robust to certain types of disturbance wrenches. Specifically, the passivity theorem [8, p. 358] guarantees that if \( s \in L_{2} \) in the presence of an additive disturbance wrench acting at the same location as \( \bar{f}_{fb} \) in Fig. 2, provided that this disturbance is in \( L_{2} \). Since \( P^{-T} \) is always bounded, this is equivalent to accounting for all disturbance signals in \( L_{2} \) that enter the system as a wrench on the system (at the same location as \( f \) in Fig. 2). Moreover, any constant disturbance forces acting on the system in the vertical direction will be accounted through

![Fig. 2. Block diagram of a CDPR with an adaptive feedforward-based control input and pretension and force distribution satisfying \( f = \Pi^{T}(\rho)r \).](image)
adaptation of $\hat{m}_p$. Although not considered in this work, other disturbance wrenches can be accounted for through their inclusion in (6) and the adaptive parameter $\hat{a}$. Additionally, it is worth noting that although passivity-based control can often provide robustness to large degrees of model uncertainty, as shown by the robustness of the proposed controller to inexact knowledge of the CDPR mass and inertia parameter, performance of the closed-loop system is not guaranteed and could possibly be worse than other model-based controllers. This is not a surprise, as such a tradeoff between robustness and performance is a fundamental aspect of control theory. The authors believe that for the application of CDPRs, the proposed controller provides essential robustness to the large amounts of uncertainty present in practice when CDPRs manipulate large, uncertain payloads.

Although there are a number of ISP controllers that can be used to ensure closed-loop input–output stability, an SPR controller with transfer matrix $G_r(s) = C_r(sI - A_r)^{-1}B_r$ is considered in this article, where $A_r$, $B_r$, and $C_r$ are matrices that define a state-space realization of the SPR controller. The SPR property of $G_r(s)$ ensures that there exist $P_c = P_c^T > 0$ and $Q_c = Q_c^T > 0$ such that [8, p. 93]

$$P_cA_c + A_c^T P_c = -Q_c, \quad P_cB_c = C_c^T.$$  

The feedback control input is then chosen as $f_{fb} = -P^{-T}y_c$, which results in $f_{fb} = P_c^T f_{fb} = -P^T(P^{-T}y_c) = -y_c$, where $y_c(s) = G_r(s)s(s)$.

In addition to proving that the proposed controller guarantees robust closed-loop input–output stability, it is desired to demonstrate that the closed-loop system achieves asymptotic convergence of the pose tracking error. This is addressed in the following theorem.

**Theorem 2:** The control law in (5) and (9), where $f_{fb} = -P^{-T}y_c$ and $y_c$ is the output of an SPR controller with input $s$, ensures asymptotic convergence of the pose tracking error (i.e., $\hat{p} \to 0$ and $\hat{v} \to 0$ as $t \to \infty$), when applied to the CDPR with dynamics given by (4).

**Proof:** From Corollary 1, it is known that $s \in L_2$. Rearranging (13) yields $\hat{p} = -A\hat{p} + s$, which is an asymptotically stable LTI system whose input is in $L_2$. This results in $\hat{p} \in L_2 \cap L_\infty$, $\dot{\hat{p}} \in L_2$, and $\hat{p} \to 0$ as $t \to \infty$ [8, p. 269]. To prove that $\hat{v} \to 0$ as $t \to \infty$, define the non-negative function

$$V_2 = V_1 + x_c^T P_c x_c,$$

where $P_c = P_c^T > 0$. Making use of the SPR property of the feedback controller, the time derivative of $V_2$ is

$$\dot{V}_2 = -s^T y_c + x_c^T (P_c A_c + A_c^T P_c) x_c + x_c^T P_c B_c s$$

$$\leq -s^T y_c - x_c^T Q_c x_c + x_c^T C_c^T s$$

$$\leq -s^T y_c - \lambda_{\min}(Q_c) x_c^T x_c + y_c^T s$$

$$\leq -\lambda_{\min}(Q_c) x_c^T x_c \leq 0. \quad (32)$$

Integrating (32) from $t = 0$ to $t = T$ results in $V_2(T) \leq V_2(0)$, which proves that $\{\hat{v}, \hat{a}, x_c\} \in L_\infty$. Through the relationship $s = P^{-1} \hat{p}_r$, where $P^{-1}$ is bounded, it is known that $s \in L_\infty$. This also results in $\hat{p} \in L_\infty$, since $\hat{p} = -A\hat{p} + s$. Assuming that $v_d \in L_\infty$ and $P$ is bounded, $\hat{p} \in L_\infty$ ensures that $v_r \in L_\infty$ through (7). Taking the time derivative of (7) yields $\dot{v}_r = v_d - (P\hat{p} + PA\hat{p})$. Assuming that $\hat{v}_r \in L_\infty$ and $P$ is bounded, $\{\hat{p}, \hat{p}\} \in L_\infty$ ensures $\dot{v}_r \in L_\infty$. With $v_r, v_r, \tilde{a}, x_c \in L_\infty$, $f = f - f_2 = W_{\alpha} - C_v x_c \in L_\infty$. Through the error dynamics of (11), $\{v_r, f - f_2\} \in L_\infty$ results in $\dot{v}_r \in L_\infty$. Knowing that $s \in L_2$, the relationship $\tilde{v}_r = Ps$ leads to $\hat{v}_r \in L_2$. Barbalat’s lemma can be used to prove $v_r \to 0$ as $t \to \infty$, since $\dot{v}_r \in L_2$, and $\dot{v}_r \in L_\infty$ [8, p. 657]. It then follows that $\tilde{v} = \tilde{v}_r - PA\hat{p}$ and both $\tilde{v}, 0 \to 0$ as $t \to \infty$. $\square$

Theorem 2 demonstrates that the proposed control law ensures that $\tilde{v} \to 0$ and $\hat{p} \to 0$ as $t \to \infty$. This results in the position in the CDPR’s payload satisfying $r \to r_d$ as $t \to \infty$. The interpretation of $\tilde{v} \to 0$ as $t \to \infty$ depends on the chosen attitude parameterization: $q^{pa} \to q^{da}$ for unconstrained attitude parameterizations, $\delta e \to 0$ (equivalent to $q^{pa} \to \pm q^{da}$) for the quaternion, and $C_{pa} \to C_{da}$ for SO(3), all of which describe asymptotic convergence of the attitude of the CDPR’s payload to the desired attitude. Note that as in [7], [39], and [40], there is no guarantee that $\hat{a} \to a$ as $t \to \infty$, since this would require the feedforward-based input be constructed using the exact model of the plant and there to be a persistence of excitation. Instead, the control formulation is set up such that $\hat{a}$ evolves in a manner that only guarantees asymptotic tracking of the desired payload pose.

**IV. CDPR NUMERICAL EXAMPLE**

Consider a six DOF CDPR with $m = 8$ cables and a rigid-body payload, as shown in Fig. 1, with numerical parameters provided in Table I. The locations of the eight stationary winches and the attachment points of the cables on the rigid-body payload are given in Table II. A crossed-cable configuration similar to the IPAnema 2 setup described in [1, p. 319] is used, which results in a relatively large wrench-feasible translational and rotational workspace while avoiding cable collisions. The numerical simulation is fashioned from the Lagrangian-based dynamic model developed in [15] for flexible cables whose mass and stiffness properties vary with the length of the cable, and is extended to accommodate a six DOF, eight-cable CDPR. A first set of simulations is performed with cables modeled as rigid straight lines, where the elastic coordinates of the model from [15] are constrained to be zero (i.e., no elastic deformation can occur). The second set of simulations models elastic deformation of the cables in the axial and two transverse directions with the Rayleigh–Ritz method in [15]. This second set of simulations is included to assess the robustness of the proposed controllers in the presence of cable flexibility, which is not directly accounted for in the passivity and stability analyses of Section III-C. Both numerical models include cable mass and allow the cables to transmit forces only when under tension. An aramid cable with properties listed in Table I is used. In the case with flexible cables, the pose of the payload used by the controller is computed through forward kinematics [44] using only the
rigid rotation of the winches in order to simulate a realistic implementation scenario and demonstrate robustness to imperfect knowledge of the payload pose. All pose tracking errors in the result plots are of the actual payload pose, computed using the deformed cables.

The desired payload position and attitude trajectories are defined as

\[
r_d = 0.1 \begin{bmatrix} \cos(0.6\pi t) \\ \sin(0.6\pi t) \\ \cos(0.6\pi t) + 4.65 \end{bmatrix} \text{ m}
\]

\[
q^{da} = 20 \begin{bmatrix} \cos(0.4\pi t - \pi/2) \\ \cos(0.4\pi t - \pi/4) \\ \cos(0.4\pi t) \end{bmatrix} \text{ deg}
\]

where \(q^{da}\) is described in terms of a 3-2-1 Euler-angle sequence. Note that while an Euler-angle sequence is used here to define the desired attitude trajectory, any attitude parameterization can be used for this purpose and converted to the attitude parameterization chosen for the controller. The trajectory is chosen such that the position and attitude range is large and rich enough for the adaptive portion of the controller to mature quickly, while still remaining inside the wrench-feasible workspace of the chosen cable-winch configuration without any interference between the cables. The desired translational trajectory in shown in Fig. 1, which is purposefully kept rather small to allow for a larger attitude trajectory. This tradeoff between the translational and attitude workspace of a CDPR is described in [1, p. 218].

Numerical simulations are performed with the proposed adaptive control law using various payload attitude parameterizations, including a 3-2-1 Euler angle sequence, SO(3) (the DCM), the quaternion, the rotation vector, and MRPs. As a comparison, two simplifications of the proposed controller with a 3-2-1 Euler angle sequence are also tested in simulation, where small Euler angles are assumed (i.e., \(\omega^{pa} \approx \dot{q}^{pa}\)) in either the feedback controller and the adaptive feedforward-based controller (denoted as Simplified Euler) or only the feedback controller (denoted as Simplified FB Euler). These simplifications are based on the state-of-the-art approach in CDPRs that assumes small-angle/linearized Euler angles, an example of which is found in [46].

The negative feedback controller is implemented as \(f_{fb} = -P^T y_c\), where \(y_c\) is the output of an SPR controller with feedthrough and input \(s\). A variety of methods can be used to design an SPR controller (e.g., see [51]) and in this work a simple first-order low-pass filter

\[
y_c(s) = K_d \text{diag} \left[ \frac{\omega_c}{s + \omega_c}, \ldots, \frac{\omega_c}{s + \omega_c} \right] s(s)
\]

where \(K_d = K_d^T > 0\) is the derivative gain, \(\omega_c = 2\pi \text{ rad/s}\) is the chosen cut-off frequency. The inertia entries of \(\hat{A}\) are all initially set to zero, while the payload mass is assumed to be approximately known and therefore \(m_p\) is initialized within 20% of the true payload mass. The control parameters used for the rigid cable simulation are \(\Lambda = 10 \cdot 1\), \(\Upsilon = 5 \cdot 1\), \(K_d = \text{diag}(K_{d,v}, K_{d,w})\), \(K_{d,v} = 125 \cdot 1\), and \(K_{d,w} = 16(2/3) \cdot 1\). Note that \(\Lambda \in \mathbb{R}^{6 \times 6}\) and \(\Upsilon \in \mathbb{R}^{7 \times 7}\). For the quaternion-based controller, \(K_d\) is doubled to ensure the control gain is the same for small angles across all attitude parameterizations and a fair performance comparison can be made (i.e., \(\delta \approx (1/2)q^{pu}\), where \(q^{pu}\) is an unconstrained attitude parameterization). The control gains when simulating the CDPR with flexible cables are reduced to increase robustness to the unmodeled dynamics. Specifically, the terms \(K_d\) and \(\Lambda\) are reduced by a factor of 5 and 2, respectively. This is a common strategy used when controlling the motion of a CDPR with flexible cables [13].

The control wrench, \(f\), is distributed to the winch torques, \(\tau\), using the improved closed-form solution from [1] as

\[
\tau = \tau_{pt} + U^T(\theta) \left( f - P^T(\rho) \tau_{pt} \right)
\]

where \(\tau_{pt} = \text{diag}(r_1, \ldots, r_8) t_{pt}\) is a pretension torque, \(r_i\) is the radius of the \(i\)th winch, \(t_{pt} \in \mathbb{R}^8\) contains the desired pretension in the eight cables, and \(U(\theta)\) is a pseudoinverse of \(P(\rho)\). A pretension value of 59 N is used for each cable, with the goal of ensuring that the cable tensions are greater than 7.9 N and less than 3937 N, which is equivalent to the winch
torques being greater than 0.2 N·m and less than 100 N·m. If at a particular instance in time, a cable tension exceeds the allowed range, the algorithm sets the cable tension to the limiting value and recomputes (33) with the row associated with that particular cable removed.

Simulation results are presented in Figs. 3 through 8, including detailed results for the case of flexible cables and the SO(3)-based controllers in Figs. 3–5. Specifically, Fig. 3 features the desired payload pose and the closed-loop response of the payload pose, where \( r^T(0) = [0 0 0.465] \) m and the initial payload attitude is associated with a 3-2-1 Euler angle sequence with all angles equal to \(-15\) deg. The CDPR’s winch torques as a function of time are included in Fig. 4 to demonstrate that positive cable tensions are maintained as shown by the dark red line representing cable 7 that meets the floor of 0.2 N·m without any impact on pose tracking performance. Fig. 5 includes the estimated system parameters \( \hat{\mathbf{a}} \) as a function of time. Although the estimated parameters \( \hat{\mathbf{a}} \) do not exactly match the true system parameters \( \mathbf{a} \), this is to be expected, as the adaptation of \( \hat{\mathbf{a}} \) is designed only to guarantee that the tracking error converges to zero asymptotically and also this simulation is carried out on the CDPR with flexible cables whose dynamics do not exactly match those in (4). The fact that \( \hat{\mathbf{a}} \) converges rather closely to the true values of \( \mathbf{a} \) (e.g., \( m_p \to 6.5 \) kg as \( t \to \infty \), while \( m_p = 6.75 \) kg) demonstrates robustness of the controller despite the large initial errors in \( \hat{\mathbf{a}} \) and significant model uncertainty.

The complete set of simulated controllers is compared by computing the angle portion of an axis-angle parameterization associated with the attitude tracking error. The resulting error angle is plotted versus time in Fig. 6(a). To further quantify the differences in attitude tracking errors, the root mean square (rms) value of the error angle is shown in Fig. 6(b) for the seven controllers and is separated into the rms error of the transient response during the first 2 s of the simulation and the steady-state response after the first 2 s of the simulation. What is worth noting is the much higher transient error in the first 2 s for the simplified Euler and simplified feedback Euler results, which is due to the errors in their linearized approximations, which manifests as control inefficiency when attempting to correct for larger angle errors. Once the tracking errors become smaller, this linearized approximation is more accurate and the difference between the simplified Euler/simplified feedback Euler and the other controllers is minimal, resulting in similar steady-state performance.

This is also evident in Fig. 6(b), where the rms attitude tracking errors are largest for these controllers after the first 2 s. For a visual comparison, the pose tracking errors versus time are included for the simplified Euler, Euler, and SO(3)-based controllers with rigid cables in Fig. 7 and flexible cables in Fig. 8. Quick convergence of the tracking error is seen with rigid cables in Fig. 7 and reasonably small...
Fig. 7. Payload pose tracking errors in the rigid-cable simulations with (a) simplified Euler-angle-based controller, (b) correctly implemented Euler-angle-based controller, and (c) SO(3)-based controller. For visualization purposes, the attitude errors are plotted using a 3-2-1 Euler-angle sequence (denoted $\tilde{\theta}_1$, $\tilde{\theta}_2$, and $\tilde{\theta}_3$). The position errors in the three axes of $F_a$ are denoted as $\tilde{r}_x$, $\tilde{r}_y$, and $\tilde{r}_z$.

Fig. 8. Payload pose tracking errors in the flexible-cable simulations with (a) simplified Euler-angle-based controller, (b) correctly implemented Euler-angle-based controller, and (c) SO(3)-based controller. For visualization purposes, the attitude errors are plotted using a 3-2-1 Euler-angle sequence ($\tilde{\theta}_1$, $\tilde{\theta}_2$, and $\tilde{\theta}_3$). The position errors in the three axes of $F_a$ are denoted as $\tilde{r}_x$, $\tilde{r}_y$, and $\tilde{r}_z$.

Tracking error is present with the flexible cables in Fig. 8, which demonstrates the robustness of the proposed controller. The simplified Euler controller features larger oscillations in tracking errors compared to both the Euler-angle and SO(3)-based controllers.

V. Conclusion

This article presented an adaptive passivity-based CDPR pose tracking controller for various attitude parameterizations. The benefit of performing CDPR pose tracking with carefully defined attitude errors was demonstrated, where a state-of-the-art linearized Euler-angle parameterization was shown to yield inferior tracking results compared to other attitude parameterizations, such as (correctly defined) Euler angles, SO(3), quaternions, and the rotation vector. Closed-loop asymptotic convergence of the pose tracking error was proven and shown to be robust to parameter uncertainty through nonlinear stability analysis and also in simulation with a CDPR that featured unmodeled and uncertain flexible cable dynamics. The work of this article serves as a step in motivating the CDPR control community to move on from describing attitude errors in terms of linearized Euler angles, and consider using other attitude parameterizations (e.g., the quaternion or DCM/rotation matrix) that are now commonly leveraged in other robotic applications.

Future work will focus on experimental implementation of the proposed control law on multiple trajectories and explicit consideration of flexible cables in the controller formulation and stability analysis.

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