THE ELECTROWEAK CHIRAL LAGRANGIAN AND NEW PRECISION MEASUREMENTS

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March 26, 2022

Abstract

A revised and complete list of the electroweak chiral lagrangian operators up to dimension-four is provided. The connection of these operators to the $S$, $T$ and $U$ parameters and the parameters describing the triple gauge boson vertices $WW\gamma$ and $WWZ$ is made, and the size of these parameters from new heavy physics is estimated using a one flavor-doublet model of heavy fermions. The coefficients of the chiral lagrangian operators are also computed in this model.

1 Introduction

If electroweak symmetry breaking is driven by new strong interactions at TeV energies, the electroweak chiral Lagrangian provides the most economical description of electroweak physics below this scale [1, 2, 3]. Deviations from the standard (but Higgsless) theory can be parameterized in terms of a low energy expansion, consisting of operators of increasing dimension. All deviations consistent with the $SU(2)_L \times U(1)$ symmetry are describable in this way, with the level of accuracy depending on the the order to which one goes in the low energy expansion.

The rapidly improving precision of electroweak measurements [4] and the coming of LEP-II and possibly even higher energy $e^+e^-$ colliders have focused new attention on the chiral lagrangian approach [5-12]. The purpose of this paper is to
contribute to this program in three ways. First, we provide a revised and complete list of all the chiral lagrangian operators up to a certain order in the low energy expansion. This list will include both CP-invariant and CP-violating operators. We then provide a dictionary connecting these operators to the measured $S$, $T$ and $U$ parameters and to the parameters describing the $W^+W^-\gamma$ and $W^+W^-Z$ vertices. Finally, we report the results of a one-loop estimate of these operators arising from the presence of new fermions such as might be present in technicolor type theories. The computation will be restricted to the CP conserving operators.

2 The Chiral Lagrangian

The terms in the chiral lagrangian must respect the (spontaneously broken) $SU(2)_L \times U(1)$ gauge symmetry. Experiment demands that the Higgs sector also approximately respect a larger, $SU(2)_L \times SU(2)_C$ symmetry, though the $SU(2)_C$ custodial symmetry is broken by the Yukawa couplings and the $U(1)$ gauge couplings. The possibility of an even larger global symmetry of the Higgs sector, leading to the presence of pseudo-Goldstone bosons, will not be considered here. The chiral lagrangian is thus constructed using the dimensionless unitary unimodular matrix field $U(x)$, which transforms under $SU(2)_L \times SU(2)_C$ as $(2,2)$. The covariant derivative of $U(x)$ is:

$$D_\mu U = \partial_\mu U + ig_\pi \bar{\tau} \cdot \vec{W}_\mu U - ig' U T_3 B_\mu. \tag{1}$$

In constructing the most general chiral $SU(2)_L \times U(1)_Y$ invariant effective lagrangian order by order in the energy expansion, it is convenient to define the
basic building blocks which are $SU(2)_L$ covariant and $U(1)_Y$ invariant as follows:

$$T \equiv U\tau_3U^\dagger, \quad V_\mu \equiv (D_\mu U)U^\dagger \quad (2)$$

$$W_{\mu\nu} \equiv \partial_\mu W_\nu - \partial_\nu W_\mu + ig[W_\mu, W_\nu] \quad (3)$$

where $T, V_\mu$ and $W_{\mu\nu}$ have dimensions zero, one, and two respectively.

The familiar pieces of the chiral lagrangian, that emerge for example from the $M_H \to \infty$ limit of the linear theory at tree level, are:

$$\mathcal{L}_0 \equiv \frac{1}{4}f^2Tr[(D_\mu U)^\dagger(D^\mu U)] - \frac{1}{4}B_{\mu\nu}B^{\mu\nu} - \frac{1}{2}TrW_{\mu\nu}W^{\mu\nu}, \quad (4)$$

where $f \simeq 250\text{GeV}$ is the symmetry breaking scale, and $B_{\mu\nu} \equiv \partial_\mu B_\nu - \partial_\nu B_\mu$. The first term has dimension two, while the second two (kinetic energy) terms have dimension four. The gauge couplings to the quarks and leptons must also be added to Eq. 4. The Yukawa couplings of the quarks and leptons to the symmetry breaking sector will be neglected here.

There is one, additional dimension-two operator allowed by the $SU(2)_L \times U(1)$ symmetry [2]:

$$\mathcal{L}_1' \equiv \frac{1}{4}\beta_1g^2f^2[Tr(TV_\mu)]^2. \quad (5)$$

This term, which does not emerge from the $M_H \to \infty$ limit of the renormalizable theory at tree level, violates the $SU(2)_C$ custodial symmetry even in the absence of the gauge couplings. It is the low energy description of whatever custodial-symmetry breaking physics exists, and has been integrated out, at energies above roughly $\Lambda_\chi \equiv 4\pi f \simeq 3\text{TeV}$. In technicolor theories, this breaking arises from the
extended technicolor interactions, typically at scales well above $\Lambda_\chi$. Used at tree level, $\mathcal{L}_1'$ contributes directly to the deviation of the $\rho$ parameter from unity.

At the dimension-four level, there are a variety of new operators that can be written down. Making use of the equations of motion, and first restricting attention to CP-invariant operators, the list can be reduced to eleven independent terms:

\begin{align}
\mathcal{L}_1 &\equiv \frac{1}{2} \alpha_1 g' g T_r (T W_{\mu\nu}) & \mathcal{L}_2 &\equiv \frac{1}{2} i \alpha_2 g' B_{\mu\nu} T_r (T [V^\mu, V^\nu]) \\
\mathcal{L}_3 &\equiv i \alpha_3 g T_r (W_{\mu\nu} [V^\mu, V^\nu]) & \mathcal{L}_4 &\equiv \alpha_4 [T_r (V^\mu V^\nu)]^2 \\
\mathcal{L}_5 &\equiv \alpha_5 [T_r (V^\mu V^\nu)]^2 & \mathcal{L}_6 &\equiv \alpha_6 T_r (V^\mu V^\nu) T_r (T V^\mu) T_r (T V^\nu) \\
\mathcal{L}_7 &\equiv \alpha_7 T_r (V^\mu V^\nu) T_r (T V^\mu) T_r (T V^\nu) & \mathcal{L}_8 &\equiv \frac{1}{4} \alpha_8 g^2 [T_r (T W_{\mu\nu})]^2 \\
\mathcal{L}_9 &\equiv \frac{1}{2} i \alpha_9 g T_r (T W_{\mu\nu}) T_r (T [V^\mu, V^\nu]) & \mathcal{L}_{10} &\equiv \frac{1}{2} \alpha_{10} [T_r (T V^\mu) T_r (T V^\nu)]^2 \\
\mathcal{L}_{11} &\equiv \alpha_{11} g \epsilon^{\mu\nu\rho\lambda} T_r (T V^\mu) T_r (V^\nu W_{\rho\lambda})
\end{align}

The first ten terms were written down by Longhitano [2]. They have been reconsidered recently by several authors [9, 12]. The operator $\mathcal{L}_{11}$ is new [13] and it completes the list of all CP invariant operators up to dimension four (see Appendix A for more details). $\mathcal{L}_{11}$ corresponds to a CP-conserving, but $C$ and $P$ violating, term in the general parameterization of the triple gauge boson vertex. It will be considered further in Section 4. We use the convention $\epsilon_{0123} = -\epsilon^{0123} = 1$.

Longhitano’s list [2] also contains CP-violating, dimension-four operators. The full list of such operators, after making use of the equations of motion, contains five terms in addition to those written down by Longhitano. They are all listed in Appendix A. In this paper, detailed considerations will be restricted to the CP invariant operators.
3 Oblique Corrections

Since experimental work is so far restricted to energies below the W-pair threshold, the only operators in the above list that have been directly constrained experimentally are those that contribute to the gauge boson two-point functions. In addition to $\mathcal{L}_0$, they are $\mathcal{L}_1$, $\mathcal{L}_1$ and $\mathcal{L}_8$, and they can be directly related to the $S$, $T$ and $U$ parameters introduced by Peskin and Takeuchi [14]. By setting the Goldstone boson fields to zero in these operators (“going to unitary gauge”), one finds

$$S \equiv -16\pi \frac{d}{dq^2} \Pi_{3B}(q^2)|_{q^2=0} = -16\pi \alpha_1,$$  \hspace{1cm} (7)

$$\alpha T \equiv \frac{e^2}{c^2 s^2 m_W^2} (\Pi_{11}(0) - \Pi_{33}(0)) = 2g^2 \beta_1,$$  \hspace{1cm} (8)

$$U \equiv 16\pi \frac{d}{dq^2} [\Pi_{11}(q^2) - \Pi_{33}(q^2)]|_{q^2=0} = -16\pi \alpha_8.$$  \hspace{1cm} (9)

The $\Delta \rho (\equiv \rho - 1)$ parameter is related to $T$ by $\Delta \rho_{new} = \Delta \rho - \Delta \rho_{SM} = \alpha T$, where $\Delta \rho_{SM}$ is the contribution arising from standard model corrections.

4 The Triple Gauge Vertex

The next generation of $e^+e^-$ colliders will operate above the W pair production threshold, and will therefore be able to directly measure the triple gauge vertices (TGV’s). The most general polynomial structure of the TGV has been derived [13] by imposing Lorentz invariance and on-shell conditions for the $W^+$ and $W^-$. The corresponding effective lagrangian for this vertex is:

$$\frac{\mathcal{L}_{WWW}}{g_{WWW}} = ig_1 V (W^+ \mu W^- \nu V^\nu - W^\mu W^\nu V^\nu) + i\kappa V W^+ W^- V^\mu V^\nu$$

$$+ \frac{i\lambda V}{\Lambda^2} W^{-\mu} W^\nu V^\mu + g_4 W^+ W^- (\partial^\mu V^\nu + \partial^\nu V^\mu)$$
+ g_\beta^\gamma \epsilon^{\mu\nu\rho\lambda} [ W_\mu^+(\partial_\rho W_\nu^-) - (\partial_\rho W_\mu^+) W_\nu^- ] V_\lambda + i \tilde{\kappa}_V W_\mu^+ W_\nu^- \tilde{V}^{\mu\nu} + \frac{i \tilde{\lambda}_V}{\Lambda^2} W_{\mu\nu}^+ W^- \rho \tilde{V}^{\rho\mu}, \tag{10}

where $V = \gamma$ or $Z$, $W_{\mu\nu}^\pm = \partial_\mu W_{\nu}^\pm - \partial_\nu W_{\mu}^\pm$, $V_{\mu\nu} = \partial_\mu V_\nu - \partial_\nu V_\mu$, and $\tilde{V}_{\mu\nu} = \frac{1}{2} \epsilon^{\mu\nu\rho\lambda} V^{\rho\lambda}$. The coupling constants $g_{WW\gamma}$ and $g_{WWZ}$ are given by $g_{WW\gamma} = -e$ and $g_{WWZ} = -e \xi$, where $e$ is the renormalized electric charge, and $c \equiv \cos \theta_w|_Z$ and $s \equiv \sin \theta_w|_Z$ are the “Z-standard” definition of the weak mixing angle [16]:

$$s^2 c^2 \equiv \frac{\pi \alpha}{\sqrt{2} G F m_Z^2},$$

the value of $\alpha$ in this definition is $\alpha^{-1}(m_Z^2) = 128.80 \pm 0.12$ [14].

This effective lagrangian contains two terms of dimension six. Unlike Ref.[15], we have used the scale $\Lambda_\chi \equiv 4\pi f$ to define the corresponding dimensionless parameters $\lambda_V$ and $\tilde{\lambda}_V$ in these terms. This is the natural thing to do if this effective lagrangian is assumed to arise from integrating out only high energy physics, above the scale $\Lambda_\chi$. That is what is being done here, since we want to make direct contact between this effective lagrangian and the terms in the chiral lagrangian in Section 2.

The above dimension-six terms then correspond to dimension-six operators in the chiral lagrangian, which are higher order in the low energy expansion, (suppressed by a factor of $O(\frac{1}{\Lambda_\chi^2})$ relative to the dimension-four operators). These dimension-six terms will be neglected from here on.
Using the convention displayed in the diagram above, the Feynman rules for the triple gauge vertices, described by the above effective lagrangian, can be written down:

\[
\Gamma^\mu_\nu^\rho(p, q, k) = g^V_1(p - q)^\rho g^{\mu\nu} + (g^V_1 + \kappa^V)(k^\mu g^{\rho\nu} - k^\nu g^{\rho\mu}) \\
+ ig^V_4(k^\mu g^{\rho\nu} + k^\nu g^{\rho\mu}) + ig^V_5 \varepsilon^{\mu\nu\lambda\rho}(p - q)\lambda \\
- \tilde{\kappa}^V \varepsilon^{\rho\mu\lambda\kappa} k_\lambda
\]

where \( k = p + q \) by conservation of momenta, and terms corresponding to dimension-six operators have been ignored. Note that the first two terms in the above expression are \( C \) and \( P \) invariant (thus \( CP \) invariant), the \( g^V_5 \) term is \( CP \) invariant but \( C \) and \( P \) violating, and that the other two terms are \( CP \) violating.

To make the connection between the TGV parameters and the general chiral lagrangian of Section 2, we restrict attention to the \( CP \) conserving sector. We also use the convention of defining the parameters of the TGV’s to include the effects of corrections to the \( W, Z \) and \( \gamma \) propagators and effects of the \( \gamma Z \) mixing, coming from physics above \( \Lambda_\chi \). Thus these parameters are related to those in the chiral lagrangian through the latter’s contribution to both gauge boson three-point and two-point functions. Using the “Z-standard” definition of the renormalized weak
mixing angle $\theta_{w|Z}$ and by going to the unitary gauge, one finds\cite{17}: 

$$
g^Z_1 - 1 = \frac{1}{s^2(c^2 - s^2)}e^2 \beta_1 + \frac{1}{c^2(c^2 - s^2)}e^2 \alpha_1 + \frac{1}{s^2 c^2} e^2 \alpha_3$$

$$
g^\gamma_1 - 1 = 0$$

$$
\kappa_Z - 1 = \frac{1}{s^2(c^2 - s^2)}e^2 \beta_1 + \frac{1}{c^2(c^2 - s^2)}e^2 \alpha_1 + \frac{1}{c^2} e^2(\alpha_1 - \alpha_2) + \frac{1}{s^2} e^2(\alpha_3 - \alpha_8 + \alpha_9)$$

$$
\kappa_\gamma - 1 = \frac{1}{s^2} e^2(-\alpha_1 + \alpha_2 + \alpha_3 - \alpha_8 + \alpha_9)$$

$$
g^Z_5 = \frac{1}{s^2 c^2} e^2 \alpha_{11},$$

$$
g^\gamma_5 = 0,$$  \hspace{1cm} (13)

where $s \equiv \sin \theta_{w|Z}$, $c \equiv \cos \theta_{w|Z}$.

The first two terms in the expressions for $g^Z_1 - 1$ and $\kappa_Z - 1$ are the contributions from the gauge boson two-point functions of the chiral lagrangian. The $\beta_1$ dependence of the first term is a consequence of using the “$Z$-standard” definition $\theta_{w|Z}$, whose renormalization depends on the $Z$ mass renormalization induced by the $\beta_1$ term in the chiral lagrangian. The other terms in $g^Z_1 - 1$ and $\kappa_Z - 1$ come from the gauge boson three-point function contribution of the chiral lagrangian. The parameters $\kappa_\gamma - 1$ and $g^Z_5$ have contributions only from the three-point functions of the chiral lagrangian. Note that $g^\gamma_1$ measures the electric charge of the $W$ in unit of $e$ and that the vanishing of $g^\gamma_5$ is a consequence of $U(1)_{em}$ gauge invariance.

The right hand side of Eq.(13) measures deviations from the standard-model tree-level predictions, coming from new high energy physics. The $\alpha$ parameters are expected to be of order $\frac{1}{16\pi^2}$, or smaller if they arise only from weak-isospin breaking effects. This expectation will be born out in the model computations in the next section. In order to isolate these effects experimentally, one-loop radiative corrections within the standard model (arising from momentum scales less than
Λχ must also be included. These corrections will not be considered in this paper.

Notice that the $g_5^Z$ term is a weak-isospin breaking operator and vanishes in any weak-isospin symmetric theory. Thus it is not related to the Wess-Zumino-Witten anomaly \[18\], which is not dependent on weak-isospin symmetry breaking in the new fermionic sector. Since we will restrict attention to the low energy effects of an anomaly free heavy fermion sector, we will not need to worry about the WZW term.

5 One-Loop Estimates

The electroweak chiral lagrangian operators have been connected to the parameters $(S, T$ and $U)$ that describe the “oblique” corrections coming from new physics above $Λχ$ (Section 3), and to terms in the TGV that will be directly probed at LEP-II (Section 4). We next estimate the size of these parameters arising from physics above $Λχ$ in a simple model consisting of one flavor-doublet of heavy fermions $U$ and $D$. By assigning both $U$ and $D$ to the fundamental representation of an $SU(N)$ group, this model can be viewed as a simplified version of a technicolor theory with the technicolor interactions neglected.

A small weak-isospin asymmetry arising from the mass splitting in the fermion doublet is also included. The masses of U- and D-type fermions are denoted by $m_U$ and $m_D$ respectively, and the weak-isospin asymmetry parameter is defined by $\delta \equiv \frac{m_U - m_D}{m_U + m_D}$. The electric charges of the U- and D-type fermions are set to be $+\frac{1}{2}$ and $-\frac{1}{2}$ respectively by anomaly cancellation condition for the new fermionic sector.

The computation of the $S$, $T$ and $U$ parameters has been done before (see
for example [14]). The results are:

\[ S = \frac{N}{6\pi} (1 - Y \ln \frac{m_U^2}{m_D^2}) = \frac{N}{6\pi} \]  

(14)

\[ \alpha T \simeq \frac{Ne^2}{48\pi^2 s_s c_c} \frac{(\Delta m)^2}{m_Z^2} \]  

(15)

\[ U \simeq \frac{8N}{15\pi} \delta^2 \]  

(16)

where \( \Delta m \equiv m_U - m_D \), and the hypercharge \( Y \) for each left-handed \( U \) and \( D \) doublet is zero for anomaly cancellation. Note that since the \( U \) and \( D \) are the source of electroweak symmetry breaking, \( f \simeq 250 \text{GeV} \) can be expressed in terms of \( m_U \) and \( m_D \). In our simple model, the one-technifermion-loop expression for \( f \) is \( f^2 = \frac{N}{4\pi^2} m^2 \ln \frac{\Lambda^2}{m^2} \), where \( m \simeq m_U \simeq m_D \) and where \( \Lambda \) is an ultraviolet cutoff. Of course, in a real technicolor theory, the technifermion masses will be soft, having values of order \( m_U \) and \( m_D \) at scales of order \( \Lambda_\chi \), and falling with increasing momentum. The integrals will then be cut off in the ultraviolet at momenta of order \( \Lambda_\chi \), and the \( \ln \frac{\Lambda^2}{m^2} \) will be replaced by a factor of order unity.

We turn next to the triple gauge boson vertices. To simplify the computations, we consider the process \( e^+ e^- \to W^+ W^- \), with the \( W \)’s on mass-shell, that will be studied at LEP-II. Recall that the new fermion masses (of order \( \Lambda_\chi \) ) are much larger than the center of mass energy of the reaction and the \( W \) mass.

Dimensional regularization and on-shell renormalization are used throughout the computation. As for \( \gamma_5 \), we employ in \( n \) dimensions the definition [19]:

\[ \gamma_5 \gamma_\sigma + \gamma_\sigma \gamma_5 = 0 \quad \gamma_5^2 = 1 \]  

(17)

where \( \sigma = 0, 1, \cdots, n - 1 \), and the prescription:

\[ tr \gamma_5 \gamma_\mu \gamma_\omega \gamma_\rho \gamma_\lambda = 4i\epsilon_{\mu\nu\rho\lambda} \]  

(18)
for \( \mu, \nu, \rho, \lambda = 0, 1, 2, 3 \). The resulting ambiguity associated with the trace of one \( \gamma_5 \) and six or more \( \gamma_\sigma \) matrices is resolved by demanding that the \( \gamma_\sigma \)'s are not to be anticommutted with \( \gamma_5 \) inside the trace.

Fig. 2. The (direct) one-loop contributions to the \( WWV \) (\( V = \gamma \) or \( Z \)) vertex function coming from the heavy fermion doublet \( U \) and \( D \). The electric charge assignments are \( Q_U = +\frac{1}{2} \) and \( Q_D = -\frac{1}{2} \), for anomaly cancellation.

The \( WW\gamma \) vertex will be considered first. The two Feynman diagrams contributing directly to the three-point function are shown in Fig. 2. Indirect contributions coming from the gauge boson two-point functions must also be included. Keeping only terms linear in the external momenta (the higher order terms are suppressed at least by inverse square powers of \( \Lambda_\chi \) ) and performing the electric charge and mass renormalization at the one-loop level, we find, to order \( \delta^2 \),

\[
\Gamma_{\gamma}^{\mu\nu\rho}(p, q, p + q) = \]

\[
\left[ -g_{\rho\mu}(2p + q)_\nu + g_{\rho\nu}(p + 2q)_\mu + g_{\mu\nu}(p - q)_\rho \right] \cdot (1 - \frac{Ne^2}{96\pi^2s^2})
- \frac{Ne^2}{96\pi^2s^2} [g_{\rho\mu}(p - q)_\nu + g_{\rho\nu}(p - q)_\mu - g_{\mu\nu}(p - q)_\rho]
+ \frac{Ne^2}{96\pi^2s^2} \frac{4}{5} \delta^2 [g_{\rho\nu}(p + q)_\mu - g_{\rho\mu}(p + q)_\nu] \tag{19}
\]

where \( \delta = \frac{m_U - m_D}{m_U + m_D} \).
By using the on-shell condition for the W’s, the above expression can be rewritten in the form of Eq. (12), and the values for the WWγ vertex parameters can be extracted:

\[ g_1^\gamma - 1 = 0 \quad \kappa_\gamma - 1 = -\frac{Ne^2}{96\pi^2 s^2} \left( 1 - \frac{4}{5} \delta^2 \right) \quad g_5^\gamma = 0. \quad (20) \]

The vanishing of \( g_1^\gamma - 1 \) and \( g_5^\gamma \) are consistent with the chiral lagrangian analysis of Section 4.

The one-loop computation for the WWZ vertex can be similarly done. Keeping only terms linear in the external momenta, performing the “Z charge”, mass and weak mixing angle renormalization and using the “Z-standard” definition of the weak mixing angle \( \theta_w|_Z \), it is found that,

\[
\Gamma_{WWZ}^{\mu\nu\rho}(p,q,p+q) = \]

\[
\left[ -g_{\rho\mu}(2p+q)_\nu + g_{\rho\nu}(p+2q)_\mu + g_{\mu\nu}(p-q)_\rho \right] \cdot (1 - \frac{Ne^2}{96\pi^2 s^2} \left( \frac{1}{s^2(c^2-s^2)} + \frac{1}{96\pi^2 s^2 c^2(c^2-s^2)} \frac{(\Delta m)^2}{m_Z^2} \right) ) + \frac{Ne^2}{96\pi^2 s^2} \left( g_{\rho\mu}(p-q)_\nu + g_{\rho\nu}(p-q)_\mu - g_{\mu\nu}(p-q)_\rho \right) \cdot \left( 1 - \frac{1}{s^2} \right) + \frac{Ne^2}{96\pi^2 s^2} \frac{4}{5} \delta^2 \|g_{\rho\mu}(p+q)_\mu - g_{\rho\nu}(p+q)_\nu\| + \frac{-iNe^2}{96\pi^2 s^2} c_{\alpha\nu\rho\mu}(p-q)^\alpha (\delta + \mathcal{O}(\delta^3)), \quad (21) \]

where terms of higher order than \( \delta^2 \) have been dropped.

By using the on-shell condition for the W’s, the above result can be put into the form of Eq. (12). The one-loop values for the WWZ vertex parameters are:

\[
g_1^{WZ} - 1 = -\frac{Ne^2}{96\pi^2 s^2} \cdot \left( \frac{1}{s^2(c^2-s^2)} - \frac{1}{s^2 c^2(c^2-s^2)} \frac{(\Delta m)^2}{m_Z^2} \right) + \frac{Ne^2}{96\pi^2 s^2 c^2 s^2} \frac{2}{5} \delta^2 \]

\[
\kappa_{WZ} - 1 = -\frac{Ne^2}{96\pi^2 s^2 c^2 s^2} + \frac{Ne^2}{96\pi^2 s^2 c^2 s^2} \frac{(\Delta m)^2}{m_Z^2} + \frac{Ne^2}{96\pi^2 s^2 c^2 s^2} \frac{4}{5} \delta^2 \]

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\[ g_5^Z = \frac{Ne^2}{96\pi^2 s^2 c^2} \left( \delta + \mathcal{O}(\delta^3) \right) \] (22)

Recall that the \( g_5^Z \) term is \( CP \) conserving but parity and charge conjugation violating. This term is proportional to the axial coupling of the neutral gauge boson to the fermions, and is odd under the interchange of the masses of the \( U \) and \( D \) fermions. This results in a linear dependence on \( \Delta m \equiv m_U - m_D \) for \( \Delta m \ll m_U \approx m_D \).

All the \( CP \)-conserving chiral lagrangian coefficients that enter the gauge boson two- and three-point functions can now be determined from Eqs. (7), (8), (9) and (13). To order \( \delta^2 \), they are given by:

\[
\begin{align*}
\beta_1 & \simeq \frac{N}{96\pi^2 c^2} \frac{(\Delta m)^2}{m_Z^2} \\
\alpha_1 & = -\frac{N}{96\pi^2} \\
\alpha_2 & = -\frac{N}{96\pi^2} \\
\alpha_3 & = -\frac{N}{96\pi^2} \left(1 - \frac{2}{5} \delta^2\right) \\
\alpha_8 & = -\frac{N}{96\pi^2} \frac{16}{5} \delta^2 \\
\alpha_9 & = -\frac{N}{96\pi^2} \frac{14}{5} \delta^2 \\
\alpha_{11} & = \frac{N}{96\pi^2} (\delta + \mathcal{O}(\delta^3))
\end{align*}
\] (23-29)

Note that \( \alpha_{11} \) is linear in the mass difference of the \( U \) and \( D \) fermions for a small weak-isospin asymmetry. We stress that these results apply only to our simple model in which technicolor interactions are neglected. They will be modified by the strong interaction of a realistic technicolor theory.

All the \( \alpha \) parameters are finite dimensionless constants, not suppressed by powers of \( \frac{1}{m^2} \), with \( m \approx m_U \approx m_D \). This is consistent with the fact that they are

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the coefficients of dimension-four operators, in which the heavy physics does not
decouple.

The size of the TGV parameters coming from physics above $\Lambda_\chi$ can be estimated in this simple model by assigning values for $N$ and the masses. We choose:

$$N = 4, \quad m_U \simeq m_D \simeq 1.5\text{TeV},$$

(30)
together with $\Delta m \equiv m_U - m_D \ll \frac{m_U + m_D}{2}$. It is then found that

$$\kappa_\gamma - 1 \simeq -1.8 \times 10^{-3}.$$

The values of $g_{1}^Z - 1$ and $\kappa_{Z} - 1$ depend on the size of $\beta_1$, and therefore on the mass splitting $\Delta m$ between the U and D fermions. The $g_{5}^Z$ parameter is proportional to $\Delta m$. Their values are tabulated below for various values of $\Delta m$.

Table 1. The size of the TGV parameters ($g_{1}^Z - 1$, $\kappa_{Z} - 1$ and $g_{5}^Z$) in the simple model being considered here. The results are for $N = 4$ and $m_U \simeq m_D \simeq 1.5\text{TeV}$, with various values of $\Delta m$. These results will be modified by the strong interactions of a realistic technicolor theory.

| $\Delta m$ (GeV) | $\Delta \rho_{\text{new}}$ ($\times 10^{-2}$) | $g_{1}^Z - 1$ ($\times 10^{-3}$) | $\kappa_{Z} - 1$ ($\times 10^{-3}$) | $g_{5}^Z$ ($\times 10^{-4}$) |
|------------------|------------------|------------------|------------------|------------------|
| 150              | 1.25             | 10.6             | -2.3             | 8.3              | 10.6             | -1.8             | 8.8              | 1.0              |
| 100              | 0.56             | 4.2              | -2.3             | 1.9              | 4.2              | -1.8             | 2.4              | 0.7              |
| 44               | 0.11             | 0.0              | -2.3             | -2.3             | 0.0              | -1.8             | -1.8             | 0.3              |
| 0.0              | 0.0              | -1.0             | -2.3             | -3.3             | -1.0             | -1.8             | -2.8             | 0.0              |

We have separated the gauge boson two-point function contribution from the gauge boson three-point function contribution in $g_{1}^Z - 1$ and $\kappa_{Z} - 1$ in order to see explicitly the dependence on $\beta_1$ (or $\Delta \rho_{\text{new}}$). Recall that the gauge boson two-point
function contributions are the first two terms in the expressions for $g_1^Z - 1$ and $\kappa_Z - 1$ in Eq. (13).

It can be seen from the above table that for a large mass splitting between the U and D fermions, the gauge boson two-point function contribution dominates and is opposite in sign to the gauge boson three-point function contribution. The overall signs for $g_1^Z - 1$ and $\kappa_Z - 1$ are both positive. With the mass difference about 45GeV, the gauge boson two-point function contribution vanishes and only the gauge boson three-point function contribution survives. For an even smaller mass splitting, the gauge boson two-point function contributes with the same sign as the gauge boson three-point function.

Notice that the typical size of both $g_1^Z - 1$ and $\kappa_Z - 1$ in this simple model, coming from physics above $\Lambda_\chi$, is $\mathcal{O}(\pm 10^{-3})$ for a small to moderate mass splitting, and that the size of $\kappa_\gamma - 1$ is $\mathcal{O}(-10^{-3})$. In models with more than one flavor-doublet (such as the one family technicolor model), the size of the TGV parameters could be as large as $\mathcal{O}(10^{-2})$. Assuming the strong technicolor interactions do not substantially change these results, the estimates indicate that deviations of the size of the TGV parameters from standard model predictions could possibly be seen in a 500 GeV $e^+e^-$ linear collider, but are too small to be seen in LEP-II experiments [6].

6 Conclusions

The $S$, $T$ and $U$ parameters introduced by Peskin and Takeuchi give a complete description of the oblique radiative corrections from physics above $\Lambda_\chi$. As the energies of forthcoming experiments move above the W pair production threshold, new phe-
nomenological parameters describing the TGV’s become relevant. The electroweak chiral lagrangian provides a comprehensive description of all these effects, coming from physics above $\Lambda_\chi$.

All the electroweak chiral lagrangian operators up to dimension four have been included here, including one new CP invariant operator that has not been considered previously in the literature. This operator corresponds to the $g_5^Z$ term in the general parameterization of the TGV and gives a contribution linear in the mass difference of the heavy fermion doublet in the model used in our computation. This is in contrast to the quadratic dependence of $\beta_1$, $\alpha_8$ and $\alpha_9$ on the mass splitting. On the other hand, as a consequence of $U(1)_{em}$ gauge invariance, there is no corresponding chiral lagrangian operator for $g_5^\gamma$.

The one-loop computation involving one flavor-doublet of heavy fermions (U and D) that carry technicolor number $N$ (but with the technicolor interactions neglected) indicates that the size of the TGV parameters $g_1^Z - 1$, $\kappa_Z - 1$ and $\kappa_\gamma - 1$ is of order $10^{-3}$ for $N = 4$. In models with more than one flavor-doublet, they could be of order $10^{-2}$. Assuming these results, with technicolor interactions neglected, are reliable estimates, they suggest that the deviations from standard model predictions for the TGV parameters are within the reach of a 500GeV $e^+e^-$ linear collider, but beyond the precision of LEP-II. The size of $g_5^Z$ is of order $10^{-4}$ or smaller in the one flavor-doublet model with $N = 4$, depending on the splitting of the masses of the heavy fermion doublet.

All the CP violating operators of dimension-four have also been listed in the appendix. These include five new operators that had not been listed. In most models, CP violating effects are small, and we have not considered these effects in
this paper.

Acknowledgments

We would like thank W. Marciano, S. Rey, J. Terning and G. Triantaphyllou for helpful discussions. We are especially grateful to Martin Einhorn for a critical reading of the manuscript and for several helpful suggestions. This research is supported in part by a grant from the Texas National Research Laboratory Commission.

Appendix A

All chiral lagrangian operators up to dimension-four satisfying $SU(2)_L \times U(1)_Y$ gauge invariance, both CP conserving and CP violating, are constructed in this appendix. By invoking the basic building blocks $T$, $V_\mu$ and $W_{\mu\nu}$ defined at the beginning of Section 2, the following list of operators with increasing dimension can be written down straightforwardly:

| Dimension 0 | $T$               |
|-------------|-------------------|
| Dimension 1 | $D_\mu T$         |
|             | $V_\mu$           |
| Dimension 2 | $D_\mu D_\nu T$   |
|             | $D_\mu V_\nu$     |
|             | $W_{\mu\nu}$      |
| Dimension 3 | $D_\mu D_\nu D_\rho T$ |
|             | $D_\mu D_\nu V_\rho$ |
|             | $D_\mu W_{\nu\rho}$ |
| Dimension 4 | $D_\mu D_\nu D_\rho D_\lambda T$ |
|             | $D_\mu D_\nu D_\rho V_\lambda$ |
|             | $D_\mu D_\nu W_{\rho\lambda}$ |

The left covariant derivative $D_\mu$ appearing in the above list is defined as:

$$D_\mu \mathcal{O} \equiv \partial_\mu \mathcal{O} + ig[W_\mu, \mathcal{O}] \tag{A.1}$$

where $\mathcal{O}$ is any $SU(2)_L$ covariant and $U(1)_Y$ invariant operator.

However, the above operators are not all independent because of the following
identities:

\[ \mathcal{D}_\mu T = [V_\mu, T] \quad [\mathcal{D}_\mu, \mathcal{D}_\nu] \mathcal{O} = [W_{\mu\nu}, \mathcal{O}] \quad (A.2) \]

\[ \mathcal{D}_\mu V_\nu - \mathcal{D}_\nu V_\mu = igW_{\mu\nu} - ig'B_{\mu\nu}T + [V_\mu, V_\nu] \quad (A.3) \]

For the purpose of applying the chiral effective lagrangian, the equations of motion for the operator fields should also be properly taken into account. The equation of motion for the \( V(x) \) field can be found to be:

\[ \mathcal{D}_\mu V^\mu \simeq 0 \quad (A.4) \]

where the approximation is valid as long as the mass scale of the external fermions lies well below the electroweak symmetry breaking scale \( f \simeq 250GeV \), which is true for the process \( e^+e^- \rightarrow W^+W^- \).

With the above identities and the equation of motion, operators of the form \( \mathcal{D}_\mu \mathcal{D}_\nu \cdots \mathcal{D}_\rho T \) and \( \mathcal{D}_\mu \mathcal{D}_\nu \cdots \mathcal{D}_\rho V_\lambda \) can be eliminated in favor of \( T, V_\mu, W_{\mu\nu} \) and \( B_{\mu\nu} \). While operators of the form \( \mathcal{D}_\mu \cdots \mathcal{D}_\nu W_{\rho\lambda} \) can be gotten rid of via integration by parts. We are left with only operators containing no \( \mathcal{D}_\mu \) in them. After making use of the trace identities of Pauli matrices \( \tau_i \) and the fact that \( T, V_\mu \) and \( W_{\mu\nu} \) all can be expressed in terms of the linear combinations of \( \tau_i \), only two independent trace structures exist:

\[ Tr(O_1O_2)Tr(O_3O_4)\cdots Tr(O_{2n-1}O_{2n}) \quad (A.5) \]
\[ Tr(O_1O_2)Tr(O_3O_4)\cdots Tr(O_{2n-1}O_{2n})Tr(O_{2n+1}O_{2n+2}O_{2n+3}) \quad (A.6) \]

where \( \mathcal{O} = T, V_\mu \) or \( W_{\mu\nu} \). With \( T, V_\mu, W_{\mu\nu} \) and \( B_{\mu\nu} \), the chiral lagrangian operators of dimensions two and four can be written down systematically. The \( CP \) invariant
ones are given in section 2, the $CP$ noninvariant ones are:

\begin{align}
\mathcal{L}_{12} & \equiv \alpha_{12} g Tr(TV_\mu) Tr(V_\nu W^{\mu\nu}) \\
\mathcal{L}_{13} & \equiv \alpha_{13} g g' \epsilon^{\mu\nu\rho\sigma} B_{\mu\nu} Tr(TW_{\rho\sigma}) \\
\mathcal{L}_{14} & \equiv i \alpha_{14} g' \epsilon^{\mu\nu\rho\sigma} B_{\mu\nu} Tr(T[V_\rho, V_\sigma]) \\
\mathcal{L}_{15} & \equiv i \alpha_{15} g \epsilon^{\mu\nu\rho\sigma} Tr(W_{\mu\nu}[V_\rho, V_\sigma]) \\
\mathcal{L}_{16} & \equiv \alpha_{16} g^2 \epsilon^{\mu\nu\rho\sigma} Tr(TW_{\mu\nu}) Tr(TW_{\rho\sigma}) \\
\mathcal{L}_{17} & \equiv i \alpha_{17} g \epsilon^{\mu\nu\rho\sigma} Tr(TW_{\mu\nu}) Tr(T[V_\rho, V_\sigma]) \\
\mathcal{L}_{18} & \equiv \alpha_{18} g^2 \epsilon^{\mu\nu\rho\sigma} B_{\mu\nu} B_{\rho\sigma} \\
\mathcal{L}_{19} & \equiv \alpha_{19} g^2 \epsilon^{\mu\nu\rho\sigma} Tr(W_{\mu\nu} W_{\rho\sigma})
\end{align}

Note that $\mathcal{L}_{12}$ above corresponds to $\mathcal{L}_{14}$ in Longhitano’s list. And $\mathcal{L}_{13}$ through $\mathcal{L}_{17}$ have not been listed before.
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