Cosmological Baryon Asymmetry in Supersymmetric Standard Models and Heavy Particle Effects

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Abstract

Cosmological baryon asymmetry $B$ is studied in supersymmetric standard models, assuming the electroweak reprocessing of $B$ and $L$. Only when the soft supersymmetry breaking is taken into account, $B$ is proportional to the primordial $B - L$ in the supersymmetric standard models. The ratio $B/(B - L)$ is found to be about one percent less than the nonsupersymmetric case. Even if the primordial $B - L$ vanishes, scalar-leptons can be more efficient than leptons to generate $B$ provided that mixing angles $\theta$ among scalar leptons satisfy $|\theta| < 10^{-8} (T/\text{GeV})^{1/2}$.

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Explaining the cosmological baryon asymmetry in the evolutionary history of the universe is one of the most fascinating problems in cosmology and particle theory [1]. The electroweak $B + L$ anomaly interactions ($B$ for baryon number density, $L$ for lepton number density) provide an efficient mechanism to transmute between $B$ and $L$ [2–4]. Since they are rapid enough to maintain thermal equilibrium at least well above the temperature for the electroweak phase transition, they may wash away (or at least alter) the primordial baryon asymmetry which has been generated in the early universe. It has been difficult to produce sufficient baryon asymmetry at the electroweak phase transition. Since $B - L$ suffers no anomaly, it is conserved in this electroweak reprocessing of baryon asymmetry. We assume that $B$ is obtained from the reprocessing of the primordial $B - L$. A primordial lepton number $L$ has been proposed as a source of the baryon asymmetry through the electroweak reprocessing [5]. The resulting ratio $B/(B - L)$ has been determined in the standard model considering chemical equilibrium and conservation laws [6].

Recent low energy data [7] strongly support supersymmetric grand unified theories which have been proposed to solve the gauge hierarchy problem [8]. The effective supersymmetry breaking scale is expected to be around the electroweak mass scale. Therefore we are led to consider that supersymmetric particles may still be regarded as light (not decoupled) at the lowest temperature $T_0$ for the electroweak reprocessing. Then we need to reconsider the chemical equilibrium conditions taking account of new particles (supersymmetric particles) and new reactions (supersymmetric interactions). Electroweak reprocessing in supersymmetric models has been discussed by many groups in various contexts [9], [10].

The purpose of this paper is to study the chemical equilibrium conditions in the electroweak baryon number reprocessing for supersymmetric standard models (SSM) and to examine the threshold effect of possible heavy particles, namely supersymmetric particles, top quarks and Higgs particles. Assuming a nonvanishing primordial $B - L$, we obtain a nonvanishing $B$ and $L$ in supersymmetric standard models. Since there are many more species of particles in supersymmetric models than in nonsupersymmetric models, $B$ depends on the primordial amount of supersymmetric particles and is not proportional to $B - L$. However, $B/(B - L)$ becomes a constant once soft supersymmetry breaking is taken into account. If the lowest effective temperature $T_0$ for the electroweak reprocessing is higher than the electroweak phase transition temperature $T_c$, we find a theorem: $B/(B - L)$ for the supersymmetric models with $N_h$ pairs of Higgs doublets becomes identical to $B/(B - L)$ for the nonsupersymmetric models with $N_h$ Higgs doublets (half as many Higgs as the supersymmetric case). If $T_0 < T_c$, $B/(B - L)$ for the supersymmetric models turns out to be about one percent less than that for the nonsupersymmetric models. We also consider threshold effects for heavy particles, namely supersymmetric particles, top quarks and Higgs particles, in view of their unknown masses.
We find that the ratio $B/(B - L)$ is about five percent less than the case of the standard model if the top quark mass is much higher than $T_0$. As a heavy particle effect, we also study an alternative mechanism of baryon asymmetry operative even if primordial $B - L$ vanishes. We find that a mass difference of scalar-leptons produces a nonvanishing baryon asymmetry, provided that the lepton number for each generation is separately conserved during the cosmological evolution. We examine the most popular mechanism for the supersymmetry breaking, namely the model with the hidden sector embedded in supergravity [11]. This model conserves the lepton number for each generation to a sufficient accuracy. Unfortunately mass differences among lepton generations turn out to be too small. Let us stress that we do not need the assumption of separate lepton number conservation for each generation except for this last mechanism due to scalar lepton mass differences.

After we have written up this work, we became aware of a recent paper which contains results partially overlapping with ours and is complementary to our work [12]. They studied electroweak reprocessing in supersymmetric models aiming at constraining the possible $B$ or $L$ violating interactions.

Supersymmetric standard models contain supermultiplets for gauge bosons $g, W, B$ of the $SU(3)_c \times SU(2)_L \times U(1)_Y$ and for $N_h$ pairs of Higgs doublets $\phi_1$ and $\phi_2$. The remaining particle content is $N$ generations of quarks and leptons and their superpartners. The masses of up-type quarks (down-type quarks and leptons) are given by the Yukawa coupling with $\phi_1$ ($\phi_2$). The particle content of the supersymmetric standard models is shown in Tab. 1, where the ordinary particles (those present in the nonsupersymmetric standard models) are listed in the left column and their superpartners in the right column.

First, we consider the fully supersymmetric case (neglecting soft supersymmetry breaking) and neglect threshold effects for massive particles (all the particles are then assumed to be ultra-relativistic). We assign chemical potential $\mu_X$ for a particle $X$ and have altogether $21 + 6N$ chemical potentials as listed in Tab. 1.

When an interaction is rapid enough, it enforces an equilibrium relation among the chemical potentials. The $W^\pm$ interactions with $SU(2)_L$ doublet fields provide the following relations between the chemical potentials within the doublet:

$$\mu(I_3 = -1/2) = \mu(I_3 = 1/2) + \mu_W,$$

where the $I_3 = -1/2$ and $I_3 = 1/2$ doublet pairs correspond to $(d_L, u_L), (\tilde{d}_L, \tilde{u}_L), (e_L, \nu_L), (\tilde{e}_L, \tilde{\nu}_L), (\phi^0_1, \phi^+_1), (\phi^0_1, \phi^+_1), (\phi^0_2, \phi^+_2)$ and $(\tilde{\phi}^-_2, \tilde{\phi}^0_2)$, respectively. Neutral Higgs boson interactions with quarks and charged leptons give

$$\mu_{u_R} = \mu_{u_L} + \mu_{10}, \quad \mu_{d_R} = \mu_{d_L} + \mu_{20}, \quad \mu_{4R} = \mu_{4L} + \mu_{20}.$$

Neutral gaugino interactions lead to the following relations between the chemical
potentials of the superpartners.

\[
\begin{align*}
\mu_{\tilde{\nu}_0} & = \mu_0 = \mu_{\tilde{B}}, \\
\mu_{\tilde{\nu}_L} & = \mu_W + \mu_{\tilde{B}}, \\
\mu_{\tilde{\nu}_R} & = \mu_W - \mu_{\tilde{B}}, \\
\mu_{\tilde{\nu}_H} & = \mu_{\tilde{W}} + \mu_{\tilde{B}}, \\
\mu_{\tilde{\nu}_L} & = \mu_{u_L} - \mu_{\tilde{B}}, \\
\mu_{\tilde{\nu}_R} & = \mu_{d_L} + \mu_{\tilde{B}}, \\
\mu_{\tilde{\nu}_H} & = \mu_{d_R} - \mu_{\tilde{B}}, \\
\mu_{\tilde{\nu}_H} & = \mu_{u_L} + \mu_{\tilde{B}}, \\
\mu_{\tilde{\nu}_R} & = \mu_{d_R} - \mu_{\tilde{B}}, \\
\mu_{\tilde{\nu}_H} & = \mu_{u_L} - \mu_{\tilde{B}}, \\
\mu_{\tilde{\nu}_R} & = \mu_{d_R} - \mu_{\tilde{B}}.
\end{align*}
\]

(3)

Higgsino mass terms provide the mixing between \(\tilde{\phi}_1\) and \(-i\sigma_2\tilde{\phi}'_2\) implying \(\mu_{\tilde{\nu}_0} = -\mu_{\tilde{\nu}_0}, \mu_{\tilde{\nu}_+} = -\mu_{\tilde{\nu}_+}\). Therefore we obtain by using eq. (3)

\[
\mu_{\tilde{B}} = \frac{1}{2}(\mu_{10} + \mu_{20}).
\]

(4)

No more new relations are obtained from other interactions in the supersymmetric models. The above relations allow us to express all the chemical potentials in terms of \(4 + N\) independent chemical potentials, say, \(\mu_W, \mu_{\tilde{B}} = (\mu_{10} + \mu_{20})/2, \mu_0 := (\mu_{10} - \mu_{20})/2, \mu_{u_L}\) and \(\mu_i\).

The difference of number density of particle \(n_+\) and that of antiparticle \(n_-\) with mass \(m\) at an equilibrium temperature \(T\) is given for a small chemical potential \(\mu\)

\[
n_+ - n_- = g \frac{T^3}{\pi^2} \frac{\mu}{T} \int_0^\infty dx \frac{x^2 \exp \left\{-\sqrt{x^2 + (\frac{m}{T})^2}\right\}}{\left[\theta + \exp \left\{-\sqrt{x^2 + (\frac{m}{T})^2}\right\}\right]^2}
\]

\[
=: f_{b,f}(m/T) \times (n_+ - n_-)|_{m=0},
\]

(5)

where \(g\) is the number of internal degrees of freedom and \(\theta = -1\) for bosons and \(\theta = +1\) for fermions. The particle asymmetry becomes in the ultra-relativistic limit \(m \ll T\)

\[
(n_+ - n_-)|_{m=0} = \begin{cases} 
\frac{gT^2}{3\pi}\left(\frac{\mu}{T}\right) & \text{(massless boson)}, \\
\frac{gT^2}{9\pi^3}\left(\frac{\mu}{T}\right)^3 & \text{(massless fermion)}.
\end{cases}
\]

(6)

Let us call the ratio \(f_{b,f}(f_{b,f})\) between the particle asymmetry of the massive case and that of the massless case as threshold functions for bosons (fermions). They are functions of \(m/T\) with \(f_{b,f}(0) = 1\) and \(f_{b,f}(\infty) = 0\) as shown in Fig. 1.

If we are interested in the total lepton number, but not in lepton numbers of each generation separately, we need only the sum of lepton chemical potentials \(\mu = \sum_i \mu_i\) to express baryon, lepton and charge densities. Using eqs. (6) and (1)–(4),

\[
B = \frac{T^2}{6} \left[3 \cdot 2N(2\mu_{u_L} + \mu_W + \mu_{\tilde{B}})\right],
\]

(7)

\* Our equations eqs. (6)–(8) disagree slightly from those in [9]. Even if we discard the Higgsino mass term, our formulas do not reduce to theirs.
\[
L = \frac{T^2}{6} [3(3\mu + 2N\mu_W - N\mu_0) + 5N\mu_B],
\]

\[
Q = \frac{T^2}{6} [3 \cdot 2 \{N\mu_{uL} - \mu - (2N + 1 + N_h)\mu_W + (2N + N_h)\mu_0\}].
\]

When a Higgsino mass term is present, there are no other global continuous symmetries than \(B\) and \(L_i\). Neglecting the mixed gravitational anomaly [9], we only have the electroweak anomaly, which implies \(N(\mu_{uL} + 2\mu_{dL}) + \sum_i \mu_i = 0\), or, from eq. (1),

\[
3N\mu_{uL} + 2N\mu_W + \mu = 0.
\]

We require \(Q = 0\) in our universe. Above the electroweak phase transition temperature \(T_c\), the unbroken SU(2)\(_L\) symmetry implies \(\mu_W = 0\). Then we find

\[
B = \frac{T^2}{6} [3 \cdot 2N(2\mu_{uL} + \mu_B)], \quad B - L = \frac{T^2}{6} \left[3N\frac{22N + 13N_h}{2N + N_h}\mu_{uL} + N\mu_B \right].
\]

Let us note that \(B - L = 0\) implies \(B + L \propto \mu_B \neq 0\) in general. Namely we can have nonvanishing \(B + L\) due to the primordial supersymmetric particles even when the primordial \(B - L\) vanishes. This situation in the fully supersymmetric models is quite different from the standard model.

We now incorporate soft supersymmetry breaking. The following soft supersymmetry breaking terms provide nontrivial relations among chemical potentials:

1. Higgs mass mixing terms:

\[
B(i\sigma_2\phi_1)^T\phi_2 + \text{H.c.},
\]

2. gaugino mass terms:

\[
-\frac{1}{2}m_\tilde{W}(-\tilde{W}^+\tilde{W}^+ + \tilde{W}^+\tilde{W}^- + \tilde{W}^-\tilde{W}^0 - \tilde{W}^0\tilde{W}^0) - \frac{1}{2}m_\tilde{B}\tilde{B}\tilde{B} - \frac{1}{2}m_\tilde{g}\tilde{g}\tilde{g},
\]

3. scalar tri-linear terms:

\[
- A_u \tilde{u}_R^T(i\sigma_2\phi_1)^T\tilde{q}_L - A_d \tilde{d}_R^T(-i\sigma_2\phi_2)^T\tilde{q}_L - A_e \tilde{e}_R^T(-i\sigma_2\phi_2)^T\tilde{l}_L + \text{H.c.}
\]

They imply the following relations in addition to those in the fully supersymmetric case:

\[
\mu_\tilde{B} = \frac{1}{2}(\mu_{10} + \mu_{20}) = 0, \quad \mu_{10} = -\mu_{20} = \mu_0.
\]

Although we have suppressed the generation indices in eq. (14), the soft supersymmetry breaking can give an arbitrary pattern of generation mixing in general.
There are also quadratic terms which mix among scalar particles. Therefore we do not assume separate lepton number conservation for each generation except at the end of this work. Since the soft supersymmetry breaking terms should be present, we take these relations into account from now on. Thus we have altogether \(3 + N\) independent chemical potentials.

Gauge interactions, Yukawa interactions and their supersymmetric extensions are in thermal equilibrium well below \(T_c\). However, we should take account of threshold effects of heavy particles, since we expect the supersymmetric particles, top and Higgs particles to be heavy.

Now, using eq. (3), we can examine the threshold effect of heavy particles. For simplicity, we approximate all the standard model particles to be massless, all the scalar-quarks and scalar-leptons to have a common mass \(m_{\text{SB}}\) and all the charginos to have a common mass \(m_{\text{SF}}\). Let us define

\[
f_{\text{SB}} = f_b \left( \frac{m_{\text{SB}}}{T_0} \right), \quad f_{\text{SF}} = f_t \left( \frac{m_{\text{SF}}}{T_0} \right),
\]

as threshold functions defined in eq. (5) for supersymmetric bosons and fermions, respectively. Using eqs. (1)–(4) and (16), we find

\[
B = \frac{T_0^2}{6} \left[ N \left\{ \mu_{u_L} + \mu_{u_R} + \mu_{d_L} + \mu_{d_R} + 2f_{\text{SB}} \left( \mu_{\bar{u}_L} + \mu_{\bar{u}_R} + \mu_{\bar{d}_L} + \mu_{\bar{d}_R} \right) \right\} \right]
\]

\[
= \frac{T_0^2}{6} 2(1 + 2f_{\text{SB}})N(2\mu_{u_L} + \mu_W), \quad (17)
\]

\[
L = \frac{T_0^2}{6} \left[ \sum_i (\mu_i + \mu_{iL} + \mu_{iR}) + 2f_{\text{SB}} \sum_i (\mu_i + \mu_{\bar{iL}} + \mu_{\bar{iR}}) \right]
\]

\[
= \frac{T_0^2}{6} (1 + 2f_{\text{SB}})(3\mu + 2N\mu_W - N\mu_0), \quad (18)
\]

\[
Q = \frac{T_0^2}{6} \left[ 2N(\mu_{u_L} + \mu_{u_R}) - N(\mu_{d_L} + \mu_{d_R}) - \sum_i (\mu_{iL} + \mu_{iR})
\right.
\]

\[
- 4\mu_W + 2N h(\mu_{1+} - \mu_{2-}) + f_{\text{SF}} \left\{ -\mu_{\bar{W}_L} - \mu_{\bar{W}_R} + N_h \left( \mu_{\bar{1}+} - \mu_{\bar{2}2} \right) \right\}
\]

\[
+ f_{\text{SB}} \left\{ 4N (\mu_{\bar{u}_L} + \mu_{\bar{u}_R}) - 2N (\mu_{\bar{d}_L} + \mu_{\bar{d}_R}) - 2 \sum_i (\mu_{\bar{i}L} + \mu_{\bar{i}R}) \right\} \]
\]

\[
= \frac{T_0^2}{6} \left\{ 2(1 + 2f_{\text{SB}})(N\mu_{u_L} - \mu) - 2 \left\{ 2(1 + 2f_{\text{SB}})N + (2 + f_{\text{SF}})(1 + N_h) \right\} \mu_W
\right.
\]

\[
+ 2 \left\{ 2(1 + 2f_{\text{SB}})N + (2 + f_{\text{SF}})N_h \right\} \mu_0 \} \right\} \mu_W
\]

\[
= \frac{T_0^2}{6} \left\{ 2(1 + 2f_{\text{SB}})(N\mu_{u_L} - \mu) - 2 \left\{ 2(1 + 2f_{\text{SB}})N + (2 + f_{\text{SF}})(1 + N_h) \right\} \mu_W
\right.
\]

\[
+ 2 \left\{ 2(1 + 2f_{\text{SB}})N + (2 + f_{\text{SF}})N_h \right\} \mu_0 \} \right\} \mu_W.
\]

We have thus obtained a formula which interpolates between the nonsupersymmetric standard models (at the large mass limit for supersymmetric particles \(f_{\text{SB, SF}} \to 0\)) and the fully supersymmetric models (at the small mass limit \(f_{\text{SB, SF}} \to 1\)). One
should note that we have only four independent combinations of chemical potentials to describe the above quantities, since we are only interested in total lepton number rather than lepton number for each generation.

Let us consider the case where the electroweak reprocessing ceases to be effective above the critical temperature \( T_c \) for the electroweak phase transition. Above the critical temperature \( T_0 > T_c \), we should impose \( \mu_W = 0 \) and \( Q = 0 \). Taking account of the relation (1) due to the soft supersymmetry breaking terms, we can express \( B \) and \( L \) in terms of a single chemical potential as in the standard model case:

\[
B = \frac{T_0^2}{6} 4(1 + 2f_{Sb}) N \mu_{ul}, \quad B - L = \frac{T_0^2}{6} (1 + 2f_{Sb}) N \frac{22N + 13SN_h}{2N + SN_h} \mu_{ul},
\]
where we use the following ratio that is convenient to express the effect of supersymmetric particles:

\[
S = \frac{2 + f_{sf}}{1 + 2f_{Sb}}.
\]

From these, we obtain the following relation:

\[
B = \frac{8N + 4SN_h}{22N + 13SN_h} (B - L).
\]

The fully supersymmetric case is obtained by \( f_{Sb,f} \rightarrow 1 \). On the other hand, the nonsupersymmetric model case results if supersymmetric particles are heavy \( f_{Sb,f} \rightarrow 0 \). It is interesting to observe that these two limits are different only by the coefficient \( S \) in front of \( N \). The \( S \) for the supersymmetric case becomes half of the \( S \) for the nonsupersymmetric case. The physical reason behind this fact is the following. The coefficient in front of \( N \) is essentially counting the degrees of freedom for quarks and leptons, whereas that in front of \( N_h \) is essentially counting the degree of freedom of Higgs particles. As shown in eq. (6), bosons contribute for this coefficient twice as much as fermions. In the supersymmetric case, we have superpartners of quarks and leptons which are bosons, and superpartners of Higgs particles which are fermions. Therefore the scalar-quarks and scalar-leptons multiply the coefficient in front of \( N \) by \( 1 + 2 = 3 \), whereas Higgsinos multiply the coefficient in front of \( N_h \) by \( 1 + 1/2 = 3/2 \). Therefore the relative importance of Higgs-Higgsino contributions to \( B/(B - L) \) in the supersymmetric case becomes half as much as the Higgs contributions in the nonsupersymmetric case. As a corollary, we find the following theorem: the supersymmetric standard model with \( N_h \) pairs of Higgs-Higgsino supermultiplets gives identical \( B/(B - L) \) to the standard model with only \( N_h \) Higgs particles (half as many Higgs particles compared to supersymmetric case) above the electroweak phase transition temperature.

Let us now turn to the case where the lowest effective temperature for the electroweak reprocessing is below the critical temperature \( T_c \) for the electroweak phase
transition. Below the critical temperature $T_0 < T_c$, $Q = 0$ but $\mu_W \neq 0$. On the other hand, $\mu_0$ must vanish because of the vacuum condensation of neutral Higgs bosons. By using eq. (10), we can again express $B$ and $L$ in terms of a single chemical potential and we obtain the following relation using $S$ defined in eq. (22):

$$B = \frac{8N + 4S(N_h + 1)}{24N + 13S(N_h + 1)}(B - L).$$

(24)

For the minimal supersymmetric standard model with three generations ($N = 3$) and one pair of Higgs doublet ($N_h = 1$) by setting $f_{Sb,t} = 1$, we obtain

$$B = \frac{16}{49}(B - L) = 0.3265 \cdots \times (B - L).$$

(25)

For the minimal standard model with three generations and one Higgs doublet [6],

$$B = \frac{12}{37}(B - L) = 0.3243 \cdots \times (B - L).$$

(26)

Supersymmetry gives a correction of about one percent for $B/(B - L)$.

The top-quark mass $m_t$ or the charged Higgs mass $m_\phi$ may be large or comparable to the lowest effective temperature $T_0$ for the electroweak anomaly interaction. Therefore we should also consider the threshold effect of a heavy top-quark. Let us take the number of generations to be three ($N = 3$) and the number of pairs of Higgs doublets to be one ($N_h = 1$). The neutral physical Higgs particle does not contribute to $B, L, Q$. However, charged Higgs particles do contribute and they may be heavy too. Therefore we introduce the threshold functions for the top $f_t$ and charged Higgs particles $f_\phi$:

$$f_t := f_t \left( \frac{m_t}{T_0} \right), \quad f_\phi := f_b \left( \frac{m_\phi}{T_0} \right).$$

(27)

Imposing $\mu_0 = Q = 0$ and using eq. (10), we obtain

$$\frac{B}{B - L} = \frac{2(5 + 12f_{Sb,t} + f_t)(9 + 12f_{Sb,t} + 2f_{St} + f_\phi)}{291 + 6(171 + 141f_{Sb,t} + 26f_{St} + 12f_t + 13f_\phi)f_{Sb,t} + 2(37 + f_t)f_{St} + 2(21 + f_\phi)f_t + 37f_\phi}.$$  

(28)

The most significant threshold effect is obtained when the top quark mass is important. Supposing $f_{Sb,t} = f_\phi = 0$ in particular, we obtain the standard model case where only the top-quark mass may not be negligible:

$$B = \frac{6(5 + f_t)}{97 + 14f_t}(B - L) \to \frac{30}{97}(B - L) = 0.309 \cdots \times (B - L),$$

(29)
where the top quark mass is much larger than the lowest effective temperature \( T_0 \) for the electroweak reprocessing. We see that a heavy top quark gives a \( B/(B-L) \) that is about five percent less than that for the light top quark case, given in eq. (26). We can see also the effect of the charged Higgs from eq. (28). If the standard model with one Higgs doublet is modified by adding an extra Higgs doublet (\( f_{Sb,t} = 0, f_\phi = f_t = 1 \) in eq. (28)), the \( B/(B-L) \) decreases about 0.5%.

It has been noted \[3\], \[12\] that the nonperturbative electroweak reprocessing of baryon and lepton numbers actually gives conserved quantities \( l_i = L_i - B/N \) for each generation separately as implied by eq. (10), where \( L_i \) is the lepton number of the \( i \)-th generation. This conservation law offers a possibility that a baryon asymmetry may be generated even if the primordial \( B-L \) vanishes, provided masses and the primordial \( l_i \) differ for leptons of different generations. We find that this mechanism can be more significant for scalar-leptons, since bosons have stronger temperature dependence than fermions \( (f_b \approx 1 - \frac{3}{2} \frac{m_\pi}{T}; f_f \approx 1 - \frac{3}{2} (\frac{m_\pi}{T})^2 ) \) for \( m \ll T \) as shown in Fig. 1.

From now on, we assume the separate conservation of \( l_i \) during the cosmological evolution and examine the consequences of the electroweak transmutation of \( l_i \) into \( B \). It is convenient to use the \( N \) conserved quantities \( l_i \) instead of the \( N \) chemical potentials \( \mu_i \). Three other chemical potentials \( \mu_W, \mu_0, \mu_{uL} \), which remain from the 3 + \( N \) chemical potentials, can be determined by \( Q = 0, \mu_W = 0 (\mu_0 = 0) \) for \( T_0 > T_c \) \( (T_0 < T_c) \) and by the electroweak selection rule eq. (10). Let us denote the threshold function of a scalar-lepton of \( i \)-th generation as \( f_i \). For simplicity, we assume \( l_N \neq 0, L-B \equiv \sum l_i = 0 \). As a typical example, we study the mechanism by assuming that the scalar-tau-lepton is heavier than the scalar-quarks and the rest of scalar-leptons: \( f_1 = \cdots = f_{N-1} = f_{Sb} \).

In the case of \( T_0 > T_c \), we obtain the baryon number in terms of \( f_{Sb} \) and \( f_N \) using \( S \) defined in eq. (22):

\[
-\frac{B}{l_N} = \frac{8}{3} \alpha \left\{ 5N + 3SN_h + \frac{2(f_{Sb} - f_N)}{1+2f_{Sb}} \right\},
\]

where we define \( \alpha = \frac{f_{Sb} - f_N}{1+2f_{Sb}} \). For an illustration, we assume \( N = 3, N_h = 1 \) and that supersymmetric particles are light except the scalar-tau-lepton: \( f_{Sb} = f_{Sf} = 1 \). The ratio \(-B/l_3\) is shown in Fig. 3 as a function of the mass of the scalar-tau-lepton divided by the temperature \( m_\tau/T_0 \).

In the case of \( T_0 < T_c \), we obtain the baryon number in terms of \( f_{Sb} \) and \( f_N \):

\[
-\frac{B}{l_N} = \frac{4}{3} \alpha \left\{ 11N + 6SN_h(N_h + 1) + \frac{8(f_{Sb} - f_N)}{1+2f_{Sb}} \right\},
\]

\[
\cdot \cdot \cdot = \frac{4}{3} \alpha \left\{ 11N + 6SN_h(N_h + 1) + \frac{8(f_{Sb} - f_N)}{1+2f_{Sb}} \right\},
\]

\[
\cdot \cdot \cdot = \frac{4}{3} \alpha \left\{ 11N + 6SN_h(N_h + 1) + \frac{8(f_{Sb} - f_N)}{1+2f_{Sb}} \right\}.
\]
We assume $N = 3$, $N_h = 1$ and that the scalar-tau-lepton and charginos are heavy ($f_3 = f_{S_3} = 0$). In Fig. 3, $-B/l_3$ is shown as a function of the mass of the other scalar supersymmetric particles divided by the temperature $m_{S_3}/T_0$. We see that this mechanism is very efficient: it can convert about half of the primordial $l_3$ to $B$.

In general we can obtain sufficient mass differences among scalar leptons, since the soft supersymmetry breaking allows arbitrary differences of the order of the electroweak mass scale. However, the separate conservation of $l_i$ during the cosmological evolution imposes a stringent limit on the possible mixing among generations in order for our mechanism to be viable. To estimate the limit for mixing angles $\theta$, we take a typical generation changing interaction:

$$\tilde{\nu}_\tau + W \rightarrow \tilde{\nu}_\mu + W,$$

where the scalar-tau-neutrino interacts with $W$ boson and is converted into the scalar-mu-neutrino. The conversion rate $\Gamma$ is given by

$$\Gamma \cong n_W \sigma v \sim \frac{(\theta g^2)^2 T^5}{(T^2 + M^2)^2} \sim \theta^2 g^4 T,$$

where $n_W$ is the number density of $W$, $\sigma$ is the cross section for (32), $v$ is the velocity of scalar-tau-neutrino, $\theta$ is the mixing angle between scalar-tau-neutrino and scalar-mu-neutrino, and $M$ is the mass of the scalar-lepton. The primordial $l_i$ may be maintained during the cosmological evolution, if the conversion rate $\Gamma$ is much less than the expansion rate $H := \frac{1}{R} \frac{dR}{dt} \cong g_s^{1/2} T^2 / m_{Pl} \sim 10^{-18} (T/\text{GeV})^2$ where $g_s \sim 10^2$ is the total effective number of degrees of freedom. Using $g^2 \sim 10^{-1}$ we find

$$\theta \ll 10^{-8} (T/\text{GeV})^{1/2}. \quad (34)$$

This bound is quite stringent. For instance, $\theta \ll 10^{-6} (10^{-7})$ for $T \sim 10 \text{ TeV} (100 \text{ GeV})$.

We have studied the supergravity model as the most popular model for the supersymmetry breaking. The spontaneous breaking occurs only in the hidden sector, and is transmitted by the supergravity to the observable sector [11]. We find that the mixing among lepton generations is small enough to satisfy (34). However, the mass differences among scalar leptons of different generations turn out to be small even if the seesaw mechanism due to large Majorana mass for right-handed neutrino is introduced. Therefore we see that we need to consider another model for supersymmetry breaking in order for our mechanism using separate $l_i$ conservation to be effective.

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**SM particles** | **chemical potentials** | **superpartners** | **chemical potentials**
---|---|---|---
$(u_i)_{L}$ | $\mu_{uL}$ | $(\tilde{u}_i)_{L}$ | $\mu_{\tilde{u}L}$
$(d_i)_{L}$ | $\mu_{dL}$ | $(\tilde{d}_i)_{L}$ | $\mu_{\tilde{d}L}$
$(\nu_i)_{L}$ | $\nu_i$ | $(\tilde{\nu}_i)_{L}$ | $\tilde{\nu}_i$
$e_i$ | $\nu_i$ | $(\tilde{e}_i)_{L}$ | $\tilde{\nu}_i$
$u_{iR}$ | $\mu_{uR}$ | $\tilde{u}_{iR}$ | $\mu_{\tilde{u}R}$
$d_{iR}$ | $\mu_{dR}$ | $\tilde{d}_{iR}$ | $\mu_{\tilde{d}R}$
$e_{iR}$ | $\mu_{eR}$ | $\tilde{e}_{iR}$ | $\mu_{\tilde{e}R}$
$W^−$ | $\mu_W$ | $W^−_L$ | $\mu_{\tilde{W}_L}$
$W^0$ | $\mu_{W^0}$ | $W^0_L$ | $\mu_{\tilde{W}^0}$
$B^0$ | $\mu_B = 0$ | $B^0_L$ | $\mu_{\tilde{B}}$
$g$ | $\mu_g = 0$ | $g_L$ | $\mu_{\tilde{g}}$
$(\phi^+_{1i})_{L}$ | $\mu_{1^+}$ | $(\phi^+_{1i})_{L}$ | $\mu_{1^+}$
$(\phi^0_{1i})_{L}$ | $\mu_{10}$ | $(\phi^0_{1i})_{L}$ | $\mu_{10}$
$(\phi^+_{2i})_{L}$ | $\mu_{2^+}$ | $(\phi^+_{2i})_{L}$ | $\mu_{2^+}$
$(\phi^0_{2i})_{L}$ | $\mu_{2^−}$ | $(\phi^0_{2i})_{L}$ | $\mu_{2^−}$

Table 1: Particles and chemical potentials of SSM.

Figure 1: Threshold functions for bosons $f_b$ and fermions $f_f$ as functions of the mass $m$ divided by the temperature $T$. Solid line shows $f_f$ and dashed line shows $f_b$. 
Figure 2: The ratio of the baryon number to the primordial lepton number $-B/l_3$ as a function of $m_{\tau}/T_0$ for $T_0 > T_c$.

Figure 3: The ratio of the baryon number to the primordial lepton number $-B/l_3$ as a function of $m_{S_b}/T_0$ for $T_0 < T_c$. 
This figure "fig1-1.png" is available in "png" format from:

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This figure "fig1-2.png" is available in "png" format from:

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