A brief review of \(R\)-parity-violating couplings

Gautam Bhattacharyya

Dipartimento di Fisica, Università di Pisa and INFN, Sezione di Pisa, I-56126 Pisa, Italy

Abstract.
I review the upper limits on the \(R\)-parity-violating (\(\bar{R}\)) Yukawa couplings from indirect searches. Some limits have been updated using recent data.

1. Introduction

In supersymmetric theories ‘\(R\)-parity’ is a discrete symmetry under which all Standard Model (SM) particles are even while their superpartners are odd. It is defined as \(R = (-1)^{(3B+L+2S)}\), where \(S\) is the spin, \(B\) is the baryon-number and \(L\) is the lepton-number of the particle \(\bar{B} \bar{L}\). An exact \(R\) implies that superparticles could be produced only in pairs and the lightest supersymmetric particle (LSP) is stable. However, \(B\)- and \(L\)-conservations are not ensured by gauge invariance and therefore it is worthwhile to investigate what happens when \(R\)-parity is violated. In this talk, I concentrate on explicit \(R\)-parity violation \(\bar{R}\). Notice that in the Minimal Supersymmetric Standard Model (MSSM), the gauge quantum numbers of the Higgs superfield \(H_d\) (responsible for the generation of down-type quark masses) are the same as those of the \(SU(2)\)-doublet lepton superfield. So if \(L\) is not a good quantum number, the latter can replace the former in the Yukawa superpotential. If \(B\)-conservation is not assumed, no theoretical consideration prevents one from constructing a term involving three \(SU(2)\)-singlet quark superfields. These give rise to a \(R\) superpotential:

\[
W_R = \frac{1}{2} \lambda_{ijk} L_i L_j E^c_k + \lambda'_{ijk} L_i Q_j D^c_k + \frac{1}{2} \mu''_{ijk} U^c_i D^c_j D^c_k + \mu_i L_i H_u, \tag{1}
\]

1 E-mail: gautam@ibmth.difi.unipi.it  \[\text{[Invited Talk presented at “Beyond the Desert”, Castle Ringberg, Tegernsee, Germany, 8-14 June 1997.]}\]
where \( L_i \) and \( Q_i \) are SU(2)-doublet lepton and quark superfields respectively; \( E^c_i, U^c_i, D^c_i \) are SU(2)-singlet charged lepton, up- and down-quark superfields respectively; \( H_u \) is the Higgs superfield which is responsible for the generation of up-type quark masses; \( \lambda'_{ijk} \) and \( \lambda_{ijk} \) types are \( L \)-violating while \( \lambda''_{ijk} \) types are \( B \)-violating Yukawa couplings. \( \lambda_{ijk} \) is antisymmetric under the interchange of the first two generation indices, while \( \lambda''_{ijk} \) is antisymmetric under the interchange of the last two. Thus there could be 27 \( \lambda' \)-type and 9 each of \( \lambda \) and \( \lambda'' \)-type couplings. Hence including the 3 additional bilinear \( \mu \)-terms \( (\mu_i) \), there are 48 additional parameters in the theory.

2. Indirect limits

2.1. Proton stability

Non-observation of proton decay places very strong bounds on the simultaneous presence of both \( \lambda' \) and \( \lambda'' \) couplings. The combinations involving the lighter generations are most tightly constrained: \( \lambda'_{11k} \lambda''_{11k} \leq 10^{-22} \) for \( k = 2, 3 \) and for \( \tilde{m} = 100 \text{ GeV} \) \([4]\). Detailed analyses have been presented in \([4,5]\). It has been shown in \([6]\) that any flavour combination of the product \( \lambda' \lambda'' \leq 10^{-10} \) for \( \tilde{m} = 100 \text{ GeV} \).

2.2. \( n-\bar{n} \) oscillation

Goity and Sher \([7]\) have put a (model independent) limit \( \lambda''_{113} \leq 10^{-4} - 10^{-5} \) for \( m_t = 100 \text{ GeV} \) from the consideration of electroweak box graph induced \( n-\bar{n} \) oscillation. The corresponding limit on \( \lambda''_{112} \) is diluted by a relative factor of \( m^2_t/m^2_b \). However the best constraint on \( \lambda''_{112} \) comes from the consideration of double nucleon decay into two kaons and the bound is estimated to be \( \leq 10^{-6} - 10^{-7} \) \([7]\).

2.3. \( \nu_e \)-Majorana mass

An approximate expression for \( \nu_e \)-Majorana mass induced by an appropriate \( \lambda \) (or \( \lambda' \)), via self-energy type diagrams, is

\[
\delta m_{\nu_e} \approx \frac{\lambda^2 N_c}{8\pi^2} \frac{1}{\tilde{m}^2} M_{\text{SUSY}} m^2.
\]

In the numerator of the RHS of eq. (2), one power of \( m \) (the fermion mass in the loop) appears due to chirality-flip in an internal line. The left-right sfermion mixing has been assumed to be \( M_{\text{SUSY}} \). Requiring \( \delta m_{\nu_e} \leq 5 \) eV and assuming \( M_{\text{SUSY}} = \tilde{m} \), the \( \lambda_{133} \)-induced interaction with \( \tau \bar{\tau} \) loops \( (N_c = 1) \) yields the constraint \( (1\sigma) \ \lambda_{133} \leq 0.003 \) for \( m_{\nu_e} = 100 \text{ GeV} \) \([8]\). The \( \lambda'_{333} \)-induced diagrams with \( b\bar{b} \) loops \( (N_c = 3) \) leads to \( \lambda'_{333} \leq 0.0007 \) for \( m_{\nu_e} = 100 \text{ GeV} \) \([8]\).
2.4. Neutrinoless double beta decay

It has been known for a long time that neutrinoless double beta decay \((\beta\beta)_{0\nu}\) is a sensitive probe of \(L\)-violating processes. In \(R\) scenario, the process \(dd \rightarrow uue^{-}\) is mediated by \(\tilde{e}\) and \(\tilde{N}\) (neutralino) or by \(\tilde{q}\) and \(\tilde{g}\), yielding \(\lambda'_{111} \leq 0.00035\) for a squark and gluino mass of 100 GeV [10,11]. The particular combinations of nuclear matrix elements that lead to bounds on \(\lambda'_{111}\) do not significantly suffer from the uncertainties of model approximations in those calculations [11]. A bound on the product coupling \(\lambda'_{113}\lambda'_{131} \leq 3 \times 10^{-8}\) has been placed from the consideration of the diagrams involving the exchange of one \(W\) boson and one scalar boson [12].

2.5. Charged-current universality

Universality of the lepton and quark couplings to the \(W\)-boson is violated by the presence of \(\lambda\) and \(\lambda'\)-type couplings. The scalar-mediated new interactions could be written in the same \((V - A) \otimes (V - A)\) structure as the \(W\)-exchanged SM graph. The experimental value of \(V_{ud}\) is related to \(V_{ud}^{SM}\) by

\[
|V_{ud}^{\text{exp}}|^2 \simeq |V_{ud}^{SM}|^2 \left[ 1 + \frac{2r'_{11k}(d_R^k)}{V_{ud}} - 2r_{12k}(e_R^k) \right],
\]

where,

\[
r_{ijk}(l) = \left( \frac{M_W^2}{g^2} \right) \left( \frac{\lambda_{ijk}^2}{m_l^2} \right),
\]

and \(r'_{ijk}\) is defined using \(\lambda'_{ijk}\) analogously. Assuming the presence of only one \(R\) coupling at a time, one obtains, for a common \(m = 100\) GeV, \(\lambda_{12k} \leq 0.05\) (1σ) and \(\lambda'_{11k} \leq 0.02\) (2σ), for each \(k\) [13,14].

2.6. \(e-\mu-\tau\) universality

In the presence of \(\lambda'\)-type interaction, the ratio \(R_\pi \equiv \Gamma(\pi \rightarrow e\nu)/\Gamma(\pi \rightarrow \mu\nu)\) takes the form

\[
R_\pi = R_{\pi}^{SM} \left[ 1 + \frac{2}{V_{ud}} \left\{ r'_{11k}(d_R^k) - r'_{21k}(e_R^k) \right\} \right].
\]

A comparison with experimental results yields, for a common mass \(m = 100\) GeV and at 1σ, \(\lambda'_{11k} \leq 0.05\) and \(\lambda'_{21k} \leq 0.09\), for each \(k\), assuming only one coupling at a time [13].

Similarly, from the consideration of \(\Gamma(\tau \rightarrow e\nu\bar{\nu})/\Gamma(\tau \rightarrow \mu\nu\bar{\nu})\), one obtains, \(\lambda_{13k} \leq 0.06\) and \(\lambda_{23k} \leq 0.06\), for each \(k\), at 1σ and for \(m = 100\) GeV [13,14].
2.7. $\nu_e - e$ scattering

The neutrino-electron scattering cross section at low energies are given by

$$
\sigma(\nu_e e) = \frac{G_F^2 s}{\pi} (g_L^2 + \frac{1}{3} g_R^2),
$$
$$
\sigma(\bar{\nu}_e e) = \frac{G_F^2 s}{\pi} (\frac{1}{3} g_L^2 + g_R^2);
$$

where in the presence of $R$ interactions ($x_W \equiv \sin^2 \theta_W$)

$$
g_L = x_W - \frac{1}{2} - (\frac{1}{2} + x_W) r_{12k}(\bar{e}_R^k),
$$
$$
g_R = x_W + r_{121}(\bar{e}_L^1) + r_{231}(\bar{e}_L^2) - x_W r_{12k}(\bar{e}_R^k).
$$

The upper limits (at 1σ) are $\lambda_{12k} \leq 0.34$, $\lambda_{121} \leq 0.29$ and $\lambda_{231} \leq 0.26$ for $\bar{m} = 100$ GeV [13].

2.8. Atomic parity violation (APV)

The parity-violating part of the Hamiltonian of the electron-hadron interaction is

$$
H = \frac{G_F}{\sqrt{2}} (C_{11} \bar{e} \gamma_\mu \gamma_5 e \bar{q} \gamma_\mu q + C_{21} \bar{e} \gamma_\mu \gamma_5 q \bar{q}),
$$

where $i$ runs over the $u$- and $d$-quarks. The $R$ interactions modify $C_{11}$ and $C_{21}$ in the following way:

$$
C_{1u} = -\frac{1}{2} + \frac{4}{3} x_W - r_{11k}(d_R^k) + \frac{1}{2} - \frac{4}{3} x_W r_{12k}(e_R^k),
$$
$$
C_{2u} = -\frac{1}{2} + 2 x_W - r_{11k}(d_R^k) + \frac{1}{2} - 2 x_W r_{12k}(e_R^k),
$$
$$
C_{1d} = \frac{1}{2} - \frac{2}{3} x_W + r_{1j1}(\bar{q}_L^j) - \frac{1}{2} - \frac{2}{3} x_W r_{12k}(e_R^k),
$$
$$
C_{2d} = \frac{1}{2} - 2 x_W - r_{1j1}(\bar{q}_L^j) - \frac{1}{2} - 2 x_W r_{12k}(e_R^k).
$$

Using the SM value of the weak charge $Q_{W}^{SM} = -73.17 \pm 0.13$ [10] and the new experimental number $Q_{V}^{exp} = -72.11 \pm 0.93$ [14], the upper limits on the $\lambda^i_{12}$ couplings can be significantly improved as $\leq 0.035$ for $\bar{m} = 100$ GeV [14, 13]. Note that these couplings are relevant for the R squark explanation of the recent large-$Q^2$ HERA anomaly [18].

2.9. $\nu_e$ deep-inelastic scattering

The left- and the right-handed couplings of the $d$-quark in neutrino interactions are modified by the $R$ couplings as

$$
g_L = (-\frac{1}{2} + \frac{1}{3} x_W)(1 - r_{12k}(e_R^k)) - r_{21k}(d_R^k),
$$
$$
g_R = \frac{1}{3} x_W + r_{2j1}(\bar{d}_L^j) - \frac{1}{3} x_W r_{12k}(e_R^k).
$$
The bounds, for $\tilde{m} = 100$ GeV, are $\lambda'_{21k} \leq 0.11$ (1\(\sigma\)) and $\lambda'_{2j1} \leq 0.22$ (2\(\sigma\)) [13].

2.10. Quark mixing: $K^+ \to \pi^+ \bar{\nu} \nu$ or $D^0 - \bar{D}^0$ mixing

Consideration of only one non-zero $R$ coupling with indices related to the weak basis of fermions generates more than one non-zero coupling with different flavour structure in the mass basis. However, all what we know about quark mixing is the relative rotation between the left-handed up and down sectors given by the CKM matrix ($V_{\text{CKM}} = U_u^L U_d^L$). The absolute mixing in either sector is not known. If we assume $V_{\text{CKM}} = U_u^L$, the strongest bounds come from $K^+ \to \pi^+ \bar{\nu} \nu$ which, in the presence of $R$-interactions, proceeds at tree level. The bounds are $\lambda'_{ijk} \leq 0.012$ (90\% CL), for $m_{\tilde{\phi}_R} = 100$ GeV and for $j = 1$ and 2 [19]. If we assume the other extreme, i.e. $V_{\text{CKM}} = U_d^L$, the bounds from $K^+$ decay become invalid. The best bounds in this case arise from $D^0 - \bar{D}^0$ mixing and the upper limits are considerably relaxed becoming 0.20 [14, 19]. Although the latter is a much more conservative estimate than the former, all of them are nevertheless basis-dependent bounds.

2.11. $\tau$-decays

The decay $\tau^- \to \bar{u}d\nu_\tau$ proceeds in the SM through a tree-level $W$-exchanged graph. The scalar-exchanged graph induced by $\lambda'_{31k}$ can be written in the same $(V - A) \otimes (V - A)$ form by a Fierz rearrangement. Using the experimental input [20]:

$$Br(\tau^- \to \pi^- \nu_\tau) = 0.113 \pm 0.0015,$$

(11)

with $f_{\pi^-} = (130.7 \pm 0.1 \pm 0.36)$ MeV, one obtains $\lambda'_{31k} \leq 0.10$ (1\(\sigma\)) for $m_{\tilde{\phi}_R} = 100$ GeV [21] (I have updated this bound).

2.12. $D$-decays

The tree-level process $c \to se^+ \nu_e$ is mediated by a $W$ exchange in the SM and by a scalar boson exchange in $\lambda'$-induced interaction. By a Fierz transformation it is possible to express the latter in the same $(V - A) \otimes (V - A)$ form as the former. Using the experimental input [20]:

$$\frac{Br(D^+ \to \bar{K}^{0*} e^+ \nu_e)}{Br(D^+ \to K^{0*} u^+ \nu_u)} = 0.94 \pm 0.16,$$

(12)

one obtains (at 1\(\sigma\)) $\lambda'_{12k} \leq 0.29$ and $\lambda'_{22k} \leq 0.18$, for $\tilde{m} = 100$ GeV [21]. The form factors related to the hadronic matrix elements cancel in the ratios, thus making the prediction free from the large theoretical uncertainties associated with those matrix elements.
2.13. LEP precision measurements

Heavy virtual chiral fermions induce sizable loop corrections to \( \Gamma(Z \rightarrow f \bar{f}) \) (\( f \) is a light fermion) \textit{via} fermion-sfermion mediated triangle graphs. Since vertices involving \( \lambda'_{3ik} \) \cite{22} or \( \lambda''_{3jk} \) \cite{23} could allow top quark in internal lines of a triangle diagram, the bounds on them are most interesting. For \( \tilde{m} = 100 \text{ GeV} \) and at 1\( \sigma \), the following bounds emerge (\( R_l = \Gamma_{\text{had}}/\Gamma_l \); \( R_{l}^{\text{SM}} = 20.756 \) with \( m_H \) treated as a free parameter):

\[
\begin{align*}
\lambda'_{13k} & \leq 0.34 \leftarrow R_{e}^{\text{exp}} = 20.757 \pm 0.056, \\
\lambda'_{23k} & \leq 0.36 \leftarrow R_{\mu}^{\text{exp}} = 20.783 \pm 0.037, \\
\lambda'_{33k} & \leq 0.48 \leftarrow R_{\tau}^{\text{exp}} = 20.823 \pm 0.050, \\
\lambda''_{3jk} & \leq 0.50 \leftarrow R_{l}^{\text{exp}} = 20.775 \pm 0.027.
\end{align*}
\]

I have updated these limits\(^2\) using the most recent experimental numbers for the LEP observables presented at the EPS meeting at Jerusalem \cite{24}.

3. Summary

To summarise, I have presented, in Table 1 and Table 2, the best indirect upper bounds to date on the 45 \( R \) Yukawa couplings and the processes from which they are constrained. I also present, in Table 3, some important product couplings – their upper limits and the processes that constrain them \cite{24, 26}. The limits on \( \lambda''_{123}, \lambda''_{212}, \lambda''_{213} \) and \( \lambda''_{223} \), presented in Table 1, correspond to the requirement that these couplings remain perturbative up to the GUT scale \cite{27}. In this short review, I have basically followed, with some modifications, the style of my earlier review on \( R \)-parity-violating couplings \cite{28} but updated quite a few limits.

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\(^2\) While extracting limits on \( \lambda'' \), leptonic universality in \( R_l \) is assumed since \( \lambda'' \)-Yukawa couplings do not directly couple to any leptonic flavour.
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Table 1. Upper limits (1σ) on $\lambda$- and $\lambda''$-couplings for $\tilde{m} = 100$ GeV. The numbers with (*) correspond to 2σ limits and those with (†) are not phenomenological limits.

| $ijk$ | $\lambda_{ijk}$ Sources | $ijk$ | $\lambda'_{ijk}$ Sources |
|-------|--------------------------|-------|--------------------------|
| 121   | 0.05(*) CC univ.         | 112   | $10^{-6}$ Double nucleon decay |
| 122   | 0.05(*) CC univ.         | 113   | $10^{-4}$ $\mu$-$\mu$ osc. |
| 123   | 0.05(*) CC univ.         | 123   | 1.25(†) Pert. unitarity   |
| 131   | 0.06 $\Gamma(\tau \to e\nu\bar{\nu})/\Gamma(\tau \to \mu\nu\bar{\nu})$ | 212   | 1.25(†) Pert. unitarity   |
| 132   | 0.06 $\Gamma(\tau \to e\nu\bar{\nu})/\Gamma(\tau \to \mu\nu\bar{\nu})$ | 213   | 1.25(†) Pert. unitarity   |
| 133   | 0.03 $\nu_e$-mass       | 223   | 1.25(†) Pert. unitarity   |
| 231   | 0.06 $\Gamma(\tau \to e\nu\bar{\nu})/\Gamma(\tau \to \mu\nu\bar{\nu})$ | 312   | 0.50 $R_l$ (LEP1)         |
| 232   | 0.06 $\Gamma(\tau \to e\nu\bar{\nu})/\Gamma(\tau \to \mu\nu\bar{\nu})$ | 313   | 0.50 $R_l$ (LEP1)         |
| 233   | 0.06 $\Gamma(\tau \to e\nu\bar{\nu})/\Gamma(\tau \to \mu\nu\bar{\nu})$ | 321   | 0.50 $R_l$ (LEP1)         |
| 212   | 0.02(*) APV              | 221   | 0.18 $D$-decay            |
| 222   | 0.02 $\nu_e$-mass       | 222   | 0.18 $D$-decay            |
| 223   | 0.20(†) $D^0$-$\bar{D}^0$ mix. | 223   | 0.20(†) $D^0$-$\bar{D}^0$ mix. |
| 131   | 0.035(*) APV             | 231   | 0.22(*) $\nu_\mu$ d.i scatter. |
| 132   | 0.34 $R_\mu$ (LEP)       | 232   | 0.36 $R_\mu$ (LEP)        |
| 133   | 0.0007 $\nu_e$-mass     | 233   | 0.36 $R_\mu$ (LEP)        |

Table 2. Upper limits (1σ) on $\lambda'$-couplings for $\tilde{m} = 100$ GeV. The numbers with (*) correspond to 2σ limits and those with (‡) are basis-dependent limits.

| $ijk$ | $\lambda'_{ijk}$ Sources | $ijk$ | $\lambda'_{ijk}$ Sources | $ijk$ | $\lambda'_{ijk}$ Sources |
|-------|--------------------------|-------|--------------------------|-------|--------------------------|
| 111   | 0.00035 $(\beta\beta)_{bb}$ | 211   | 0.09 $Re$ ($\pi$-decay) 311   | 0.10 $\tau^- \to \pi^- \nu_\tau$ |
| 112   | 0.02(*) CC univ.         | 212   | 0.09 $Re$ ($\pi$-decay) 312   | 0.10 $\tau^- \to \pi^- \nu_\tau$ |
| 113   | 0.02(*) CC univ.         | 213   | 0.09 $Re$ ($\pi$-decay) 313   | 0.10 $\tau^- \to \pi^- \nu_\tau$ |
| 121   | 0.035(*) APV             | 221   | 0.18 $D$-decay            | 321   | 0.20(‡) $D^0$-$\bar{D}^0$ mix. |
| 122   | 0.02 $\nu_e$-mass       | 222   | 0.18 $D$-decay            | 322   | 0.20(‡) $D^0$-$\bar{D}^0$ mix. |
| 123   | 0.20(‡) $D^0$-$\bar{D}^0$ mix. | 223   | 0.18 $D$-decay            | 323   | 0.20(‡) $D^0$-$\bar{D}^0$ mix. |
| 131   | 0.035(*) APV             | 231   | 0.22(*) $\nu_\mu$ d.i scatter. |
| 132   | 0.34 $R_\mu$ (LEP)       | 232   | 0.36 $R_\mu$ (LEP)        | 333   | 0.48 $R_\tau$ (LEP)       |
| 133   | 0.0007 $\nu_e$-mass     | 233   | 0.36 $R_\mu$ (LEP)        | 333   | 0.48 $R_\tau$ (LEP)       |

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Table 3. Upper limits on some important product couplings for $\tilde{m} = 100$ GeV.

| Combinations | Limits  | Sources  | Combinations | Limits  | Sources  |
|--------------|---------|----------|--------------|---------|----------|
| $\lambda'_{11k}\lambda_{11k}$ | $10^{-22}$ | Proton decay | $\lambda'_{ijk}\lambda'_{mn}$ | $10^{-10}$ | Proton decay |
| $\lambda_{1j1}\lambda_{1j2}$ | $7.10^{-7}$ | $\mu \rightarrow 3\epsilon$ | $\lambda_{231}\lambda_{131}$ | $7.10^{-7}$ | $\mu \rightarrow 3\epsilon$ |
| $\text{Im} \lambda'_{12}\lambda'_{121}$ | $8.10^{-12}$ | $\epsilon_K$ | $\lambda'_{112}\lambda'_{121}$ | $1.10^{-9}$ | $\Delta m_K$ |
| $\lambda'_{1j1}\lambda'_{131}$ | $8.10^{-8}$ | $\Delta m_B$ | $\lambda'_{1k1}\lambda'_{2k2}$ | $8.10^{-7}$ | $K_L \rightarrow \mu \epsilon$ |
| $\lambda_{1k1}\lambda_{2k1}$ | $5.10^{-8}$ | $\mu\text{Ti} \rightarrow e\text{Ti}$ | $\lambda'_{11j}\lambda'_{21j}$ | $5.10^{-8}$ | $\mu\text{Ti} \rightarrow e\text{Ti}$ |