Summary of a Numeric Method for Calculating Heat Transfer in the Rotary Air Preheater

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Abstract. Due to the introduction and implementation of new standards and new specifications, the structure of the heat transfer elements inside the air preheater has undergone relatively large changes. The thermal calculation method of the rotary air preheater is based on the Soviet 1973"Standard Method for Thermal Calculation of Boiler Units" It has been impossible to directly establish the heat transfer formula. According to the actual operating conditions of the rotary air preheater, the boundary conditions of the calculated heat transfer model can be obtained, including: the inlet temperature and inlet flow rate of the flue gas and air of the preheater; the heat transfer element of the regenerator is repeated Rotation, there are continuous conditions of the temperature of the heat storage element heat transfer element between the different air flow sub-bins. And a method for solving discrete equations is proposed.

1. Introduction
Heat transfer characteristic is important for the design and transformation of three-compartment rotary air preheater, which is a direct embodiment of economic benefit and energy benefit of an air preheater. The distribution of temperature field, flow and heat transfer parameter value of flue gas and air after the transformation need heat transfer calculation to determine.

In the thermodynamic calculation theory of rotary air preheater, the thermodynamic calculation algorithm in steady state of rotary air preheater is usually carried out according to the Boiler Unit Thermodynamic Calculation Standard Method of 1973 of the former Soviet Union. This method introduces the thermodynamic calculation process of two-compartment rotary air preheater and calculates the average outlet temperature of flue gas and air by using standard. The method is suitable for calculating the steady-state outlet average temperature of rotary air preheater in two silos.

Due to the introduction and implementation of new standards and new specifications, the air preheater of internal heat transfer element structure has made big changes, such as the improvement of heat transfer element, and the sealing system optimization, some of the current theoretical models for rotary air preheater and had not well used in air preheater of design calculation process, especially after the denitrification air preheater, flue gas composition and temperature and the situation of the big changes have taken place in three points storehouse rotary air preheater, the heat transfer process is the runtime binary storehouse air preheater is more complex, heat transfer formula is established by using thermal resistance method cannot directly, so it can't be calculated directly by the standard method of boiler thermal calculation, and there is no calculation method of heat transfer of three-point warehouse standard rotary air preheater in the boiler thermal calculation standard, so three warehouse heat transfer calculation method of the rotary air preheater is very diverse, after reforming for denitrification of
rotary air preheater, especially for enamel heat transfer components modified three points warehouse performance calculation methods of rotary air preheater is also imperfect.

2. Performance calculation of air preheater

According to the working principle of rotary air preheater, the rotor space of rotary air preheater is discretized by finite difference method, and the space of heat accumulator is meshed to obtain several small units. Euler method was adopted in the model establishment, that is, focusing on the control volume of heat accumulator and fluid, which cross through each cell for heat transfer, to obtain the limited control volume as shown in Figure 1. The \( r \), \( z \), and \( \theta \) in the figure represent the radial, axial, and tangential directions of the rotary air preheater rotor, respectively. It can be seen that there are five net energy flows into the control volume. Based on the principle of energy conservation, the heat balance equation of the control body can be obtained.

\[
-mc \frac{\partial t}{\partial z} - MC \frac{1}{r} \frac{\partial T}{\partial \theta} + \left[ \lambda \psi_z \frac{\partial^2 T}{\partial z^2} + \lambda \psi_{\theta} \frac{1}{r} \frac{\partial^2 T}{\partial \theta \partial r} + \lambda \psi_r \left( \frac{1}{r} \frac{\partial T}{\partial r} + \frac{\partial^2 T}{\partial r^2} \right) \right] = 0
\]

Where, \( t \) represents the temperature of the gas; \( m \) is the mass flow rate of the gas; \( c \) is the specific heat capacity of the gas; \( T \) represents the temperature of the heat transfer element of the regenerator; \( M \) represents the mass flow rate of the heat accumulator into the control element as the rotor rotates, which is related to the rotor rotation rate. \( C \) represents the specific heat capacity of the regenerator metal; \( \psi \) represents the heat transfer element of the rotor space. The three items on the left side of the equation represent the energy carried by flue gas or air, heat accumulator and heat accumulator into the control volume.

In the control volume, there is mutual heat transfer between the fluid and the heat accumulator, and the heat transfer equation can be expressed as:

\[
mc \frac{\partial t}{\partial z} = h\sigma (T - t)
\]

The left side of the equation represents the increment of fluid energy, and the right side represents the convective heat transfer from the heat reservoir to the fluid.

According to the characteristics of the actual operation of the rotary air preheater, the following assumptions were made for the convenience of mathematical solution:

- The temperature and composition of flue gas and air at the rotor inlet are evenly distributed.
- The physical parameters of flue gas and heat transfer element metal are only related to temperature.
- Ignore the heat conduction of flue gas and air, as well as the radiative heat exchange with heat transfer elements.
- Ignore the impact of carrying air leakage on the heat transfer of the preheater.
- The physical parameters of flue gas and heat transfer element metal are only related to temperature.
- According to the loading method of the heat transfer element in the rotor, the heat conduction of the heat storage body in the tangential direction is considered to be zero, the heat conduction of the heat storage body in the radial direction is ignored, and only the heat conduction of the heat storage body in the axial direction is considered.

\[
-mc \frac{\partial t}{\partial z} - MC \frac{1}{R} \frac{\partial T}{\partial \theta} + \lambda \psi_z \frac{\partial^2 T}{\partial z^2} = 0
\]
Figure 1. Schematic diagram of rotor finite control volume
(1- flue gas carries heat, 2, 3, 5- heat transfer element conducts heat, 4- heat transfer element carries heat)

The three-dimensional heat transfer model of the rotary air preheater can be simplified into a two-dimensional heat transfer model, as shown in Figure 2. Figure \( \Theta \) and \( H \), respectively in the rotor in a gas tangential Angle and axial height of the warehouse; \( f_i \) and \( f_j \) respectively represent the share of each microelement in the axial and tangential heat accumulator; The subscripts \( i \) and \( j \) are used to represent the positions of discrete microelements in the two-dimensional model, where, \( i = 2,3,\cdots,n \) \( j = 1,2,3,\cdots,m \).

Figure 2. Two-dimensional heat transfer model of rotary air preheater

By discretizing the energy conservation equation with the finite difference method, it can be obtained that:

\[
-mc \frac{t_{j+1/2,j} - t_{j-1/2,j}}{f_jH} - MC \frac{T_{i,j+1/2} - T_{i,j-1/2}}{f_j \Theta} + \lambda \varphi \frac{1}{f_jH} \left( \frac{\partial T}{\partial z} \right)_{i+1/2,j} - \left( \frac{\partial T}{\partial z} \right)_{i-1/2,j} = 0
\]

Among them
\[
\frac{\partial T}{\partial z} \bigg|_{r=1/2,j} = \frac{\left( T_{i+1,j-1/2} + T_{i+1,j+1/2} - T_{i,j-1/2} - T_{i,j+1/2} \right)}{2} \left( f_i + f_{i+1} \right) H \frac{1}{2} \tag{5}
\]

\[
\frac{\partial T}{\partial z} \bigg|_{r=1/2,j} = \frac{\left( T_{i-1,j-1/2} + T_{i-1,j+1/2} - T_{i,j-1/2} + T_{i,j+1/2} \right)}{2} \left( f_{i-1} + f_i \right) H \frac{1}{2} \tag{6}
\]

Therefore, the equation (4) can be further discretized into:

\[
-mA_i f_j c \left( t_{i,j-1/2} - t_{i-1,j-1/2} \right) - M H f_j C \left( T_{i,j-1/2} - T_{i,j-3/2} \right) + 2\lambda f_j R \Theta \phi \left( \frac{T_{i+1,j-1/2} + T_{i+1,j+1/2}}{2} - \frac{T_{i,j-1/2} + T_{i,j+1/2}}{2} \right) \left( f_i + f_{i+1} \right) H \frac{1}{2} = 0 \tag{7}
\]

Among them, \( i = 2, 3, \ldots, n-1; j = 1, 2, 3, \ldots,m \); The mass flow rate of the regenerator \( \dot{M} \) can be expressed as:

\[
\dot{M} = \rho \cdot A \cdot H \cdot \psi \cdot \Omega / 60 \tag{8}
\]

Type of \( \Omega \) says the speed of the rotor.

Similarly, discretization of the heat transfer equation can be obtained as follows:

\[
m f_j c_{p,j} \left( t_{i+1/2,j} - t_{i,j} \right) = h_{i,j} S f_j \left( \frac{T_{i+1,j-1/2} + T_{i+1,j+1/2}}{2} - \frac{t_{i,j-1/2} + t_{i,j+1/2}}{2} \right) \tag{9}
\]

Among them, \( i = 2, 3, \ldots,n-1; j = 1, 2, 3, \ldots,m \); \( h \) represents the convective heat transfer coefficient of the heat transfer element. The heat transfer resistance characteristic parameters of the heat transfer element in the rotary air preheater are obtained based on the wind tunnel experiment of the heat transfer element. \( S \) represents the heat transfer area of heat transfer elements in the same reservoir, which can be expressed as:

\[
S = \sigma A_i H \tag{10}
\]

Where, \( A_i \) represents the flow area of a gas compartment on the rotor; \( \sigma \) represents the heat transfer area density of the heat storage body.

Equations (7) and equations (8) can be further simplified and arranged into algebraic equations:

\[
A_{1i} t_{i+1/2,j} + A_{2i} t_{i+1/2,j} + A_{3i} t_{i+1/2,j} + B T_{i,j+1/2} = 0 \tag{11}
\]

Where, \( i = 1, 2, 3, \ldots,m \).

\[
A_{1i} t_{i+1/2,j} + A_{2i} t_{i+1/2,j} + A_{3i} t_{i+1/2,j} + B T_{i,j+1/2} = 0 \tag{12}
\]

Among them, \( i = 2, 3, \ldots,n-1; j = 1, 2, 3, \ldots,m \).

\[
A_{1i} t_{i-1/2,j} + A_{2i} t_{i-1/2,j} + A_{3i} t_{i-1/2,j} + B T_{i,j-1/2} = 0 \tag{13}
\]

Where, \( i = n; j = 1, 2, 3, \ldots,m \).

\[
T_{i,j+1/2} + T_{i-1/2,j} = C t_{i,j+1/2,j} + C' t_{i+1/2,j} \tag{14}
\]

Among them, \( i = 1, 2, 3, \ldots,n, j = 1, 2, 3, \ldots,m \).

Where \( A, B, \) and \( C \) represent the coefficients in the simplified equation. According to the actual operating conditions of the rotary air preheater, the boundary conditions of the heat transfer model can be obtained, including the inlet temperature and inlet flow of the flue gas and air in the preheater; Because the heat transfer element of the regenerator rotates repeatedly, there are continuous conditions of temperature of the heat transfer element of the regenerator between different flow compartments. Combined with the linear algebraic equations and the boundary conditions of the heat transfer model, the temperature of the rotary air preheater can be solved.
3. Numerical calculation method of linear algebraic equations

After analysis, we find that the algebraic equation of each fluid temperature node can be written in the following form:

$$a_{i}^{t} = a_{i+1}^{t} + a_{i-1}^{t} + a_{i-2}^{t} + b$$  \hspace{1cm} (15)

Where, subscript $i$ represents the location of local node, and $i+1$, $i-1$ and $i-2$ respectively represent one node downstream of local node and two nodes upstream. The algebraic equation of each node contains at most the unknown value of four nodes, so we can combine the algebraic equation of each node of a discrete column into a system of equations, whose coefficient matrix is a quad diagonal matrix. A quad diagonal matrix is characterized by that only the diagonal elements and the two elements above and below adjacent elements are not zero, and all other elements are zero.

If elimination and iteration methods are used for this kind of equation, a lot of unnecessary operations must be performed. According to the characteristics of pent diagonal matrix, an amazingly effective method, Thomas algorithm, is proposed. It is an accurate algorithm based on matrix decomposition theory, and the Thomas algorithm based on Gaussian elimination is adopted in this chapter.

The process of Thomas algorithm includes two steps: elimination and back generation. In the elimination process, the non-zero elements in each row are eliminated one by one starting from the second row of the coefficient matrix, and the quaternion equation is reduced to a binary equation. When the elimination goes to the last line, the discrete equation of the last node is reduced to a unitary equation, thus obtaining the value of the unknown quantity on the node. Then, one by one, the temperature values of other nodes are obtained by solving each binary equation. Here we need to figure out the general formula for the elimination and the back-substitute process, and here is the derivation.

For convenience, the formula (15) can be rewritten as:

$$A_{i}^{t} = B_{i}^{t} + C_{i}^{t} + D_{i}^{t} + E_{i}$$  \hspace{1cm} (16)

Suppose there are $N$ nodes, namely, $i=1, ..., N$. When $i=1$, $C_{1}=0$ and $D_{1}=0$, that is, the first equation contains only 2 unknowns. When $i=2$, $D_{2}=0$, that is, the first equation contains only 2 unknowns. When $i$ is equal to $N$, $B_{N}$ is equal to 0 and the last equation contains only three unknowns.

The purpose of elimination is to reduce the equation (16) to the following form:

$$t_{i} = P_{i}t_{i+1} + Q_{i}$$  \hspace{1cm} (17)

In order to find the relationship between the coefficients $P_{i}$, $Q_{i}$ and $A_{i}$, $B_{i}$, $C_{i}$, $D_{i}$ and $E_{i}$, when $i=3...N$, we assume that:

$$\begin{align*}
    t_{i} & = P_{i}t_{i+1} + Q_{i} \\
    t_{i+1} & = P_{i+1}t_{i+2} + Q_{i+1} \\
    t_{i+2} & = P_{i+2}t_{i+3} + Q_{i+2}
\end{align*}$$  \hspace{1cm} (18)

Substitute the equation (18) into the equation (16) and eliminate $t_{i+1}$ and $t_{i+2}$ to get:

$$A_{i}^{t} = B_{i}^{t} + C_{i}P_{i}t_{i} + C_{i}Q_{i-1} + D_{i}P_{i-2}t_{i} + D_{i}P_{i-2}Q_{i-2} + D_{i}Q_{i-2} + E_{i}$$  \hspace{1cm} (19)

And then:

$$P_{i} = \frac{B_{i}}{A_{i} - C_{i}P_{i-1} - D_{i}P_{i-2}}$$  \hspace{1cm} (20)

$$Q_{i} = \frac{C_{i}Q_{i-1} + D_{i}P_{i-2}Q_{i-2} + D_{i}Q_{i-2} + E_{i}}{A_{i} - C_{i}P_{i-1} - D_{i}P_{i-2}}$$  \hspace{1cm} (21)

When $i$ is equal to 1, the equation (16) can be written as:

$$A_{1}^{t} = B_{1}t_{2} + C_{0}t_{0} + E_{1}$$  \hspace{1cm} (22)
So,

\[ P_i = \frac{B_i}{A_i} \]  

(23)

\[ Q_i = \frac{C_i t_0 + E_i}{A_i} \]  

(24)

When \( i \) is equal to 2, the equation (16) can be written as

\[ A_2 t_2 = B_2 t_3 + C_2 t_1 + D_2 t_0 + E_2 \]  

(25)

So,

\[ P_2 = \frac{B_2}{A_2 - C_2 P_1} \]  

(26)

\[ Q_2 = \frac{C_2 Q_1 + D_2 t_0 + E_2}{A_2 - C_2 P_1} \]  

(27)

When the elimination goes to the equation of the last node, that is, when \( i = N \), there is:

\[ t_N = P_{N,N+1} t_{N+1} + Q_N, \text{ and } P_{N,N+1} = 0, \text{ so } t_N = Q_N \]  

(28)

Then, from (28) the beginning, using (17),(20),(21),(23),(24),(26),(27) one by one back to solve \( t_i \) (\( i = N-1, \ldots, 1 \)).

4. Summary

Both the analytical and numerical methods can accurately calculate the thermal performance of the preheater, including the temperature distribution of the fluid and the rotor. However, the numerical method can solve the multi-layer arrangement of the heat transfer element in the preheater rotor, showing stronger adaptability than the analytical method.

It can be concluded that the above method can solve discrete equations. In the actual calculation of the spatial temperature distribution of the air preheater rotor, it is necessary to solve multiple nodes and write the code to solve the discrete equations at the innermost part of the loop. Thomas algorithm does not need to be iterated repeatedly, so compared with other algorithms, it has a faster calculation speed, which can greatly improve the efficiency of the overall calculation.

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