A NEW SUPERSYMMETRIC CP VIOLATING CONTRIBUTION TO NEUTRAL MESON MIXING

Mihir P. Worah

Stanford Linear Accelerator Center
Stanford University, Stanford, CA 94309

Abstract

We study the contribution to flavor changing neutral current processes from box diagrams with light higgsinos and squarks. Starting with just the Cabbibo Kobayashi Maskawa (CKM) phase, we find contributions to the $K^0$ and $B^0$ meson mass matrices that are out of phase with the Standard Model contributions in the case of substantial mixing between the up-type squarks. This difference in phase could be large enough to be detected at the proposed $B$ factories, with interesting implications for the unitarity triangle of CKM matrix elements.

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1 Introduction

Flavor changing neutral currents (FCNC) are predicted by, and constrain most extensions of the Standard Model. They will be well studied at the proposed $B$ factories both in the hope of uncovering new physics, as well as pinning down the parameters of the Standard Model more accurately. In this regard it is important to see what the predictions of various extensions of the Standard Model are. The minimal supersymmetric standard model (MSSM) is a well motivated and popular extension of the Standard Model \cite{1}. Its predictions for FCNC have been extensively studied both in the constrained version \cite{2, 3, 4, 5}, and for the more general case \cite{6, 7} (the first of Refs. \cite{6} also considers the consequences of embedding the MSSM in various GUTs).

We partially redo the analysis of the previous papers, concentrating, however, on a region of supersymmetric parameter space that has not been explored in this context: namely the case of low $\tan \beta$ ($\tan \beta$ denotes the ratio of vacuum expectation values of the two Higgs bosons in the MSSM), where the lightest charginos are mostly higgsinos, and where the right-handed up-type squarks are light and substantially mixed with each other.

Although this scenario cannot occur in the constrained MSSM, our motivation for studying it are twofold. Firstly, it has recently been proposed that light higgsinos and right-handed stops in the small $\tan \beta$ limit could partially explain the anomalously large $Z \rightarrow b \bar{b}$ partial width \cite{9, 10}. As we will explain in the next section, this particular configuration of the theory leads to robust predictions for supersymmetric contributions to neutral meson mixing. Secondly, even if one starts with just the Cabbibo Kobayashi Maskawa (CKM) phase, mixing between the right-handed up-type squarks can give rise to large contributions to the neutral meson mixing matrices that are out of phase with the usual Standard Model contributions. Thus, for example, the angle $\beta'_{KM}$ measured by the CP asymmetry in the decay $B_d \rightarrow \Psi K_S$ is not the angle $\beta_{KM}$ of the CKM matrix and the unitarity triangle constructed using the angle $\beta'_{KM}$ will fail to close. It is the feasibility of uncovering this interesting possibility that we wish to explore.

In section 2 we show how these large FCNC can arise for light higgsinos and right handed squarks, and derive formulas for the new contributions to the $K - \bar{K}$, $B_d - \bar{B}_d$ and $B_s - \bar{B}_s$ mixing matrices. We study the implications of these new contributions to the angles and lengths of the unitarity triangle, and the experi-

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1This particular scenario, in the absence of squark mixing, has been recently studied in detail in \cite{8}. The absence of squark mixing, however, precludes the existence of the interesting CP violating effects we discuss here.

2Mixing between the squarks reduces the effect on the $Z \rightarrow b \bar{b}$ partial width, but alleviates the problems the scenario of \cite{10} has with the decays of the top quark.
mental tests of this scenario in Section 3. Section 4 contains our conclusions.

2 Light Higgsinos and FCNC

The importance of light higgsinos and squarks to FCNC processes is simply that
the presence of the quark masses at the quark-squark-higgsino vertices removes
the super GIM cancellation present in the usual supersymmetric box graphs.
Thus higgsino mediated box graphs give non-zero contributions to neutral meson
mixing independent of details of the squark mass matrices. Given the factor of
$m_t/\sin \beta$ that appears in the $d_L - \tilde{u}_R - \tilde{h}$ vertex, we are guaranteed to get large
supersymmetric contributions to $K - \bar{K}$, $B_d - \bar{B}_d$ and $B_s - \bar{B}_s$ mixing for light
right-handed up-type squarks, higgsinos, and small tan $\beta$.

Consider the basic quark-squark-higgsino vertex:

$$\mathcal{L}_i = \frac{g}{\sqrt{2} M_W \sin \beta} d_L [V^\dagger_{KM} \hat{M}_U \tilde{V}_U] \tilde{u}_R \tilde{h}$$

where $V_{KM}$ is the CKM matrix, $\hat{M}_U$ is the diagonal matrix of up-type quark
masses, and $\tilde{V}_U = V^\dagger_{UR} \tilde{V}_{UR}$ is a product of the unitary matrices that diagonalize
the right-handed up-type quark and squark mass matrices respectively. Starting
with Eq. (1) we can derive the following very simple formula for the supersym-
metric contribution to the off diagonal terms in the mass matrix of the neutral
meson consisting of the quarks ($\bar{a}b$) with $a, b = 1, 2, 3$:

$$(M_{ab})_{12} = K_{\bar{a}b} [V^\dagger_{KM} \hat{M}_U \tilde{V}_U \frac{M^{-1}}{2\sqrt{3} \sin^2 \beta} \tilde{V}^\dagger_{UR} \hat{M}_U V_{KM} M_{ab}]$$

where

$$K_{\bar{a}b} = \frac{G_F^2}{12\pi^2} (B_{\bar{a}b} f_{\bar{a}b}^2 m_{\bar{a}b} \eta)$$

with $B_{\bar{a}b}$, $f_{\bar{a}b}$, $m_{\bar{a}b}$ being the bag factor, decay constant and mass of the meson,
and $\eta$ a QCD correction factor which we always set equal to the corresponding
QCD correction for the Standard Model box diagram with top quarks in the
loop. $\hat{M}$ is the diagonal matrix of right-handed up-type squark masses. We have
ignored any difference between the charged higgsino mass and the squark masses
in deriving Eq. (2). This approximation does not significantly affect the accuracy

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3A similar argument could be made for charged Higgs boson mediated boxes. These, however, are generally smaller because of the heavy top quark in the loops. We have checked that over the parameter range we study here, the charged Higgs bosons can be made heavy enough to satisfy constraints from other processes like $b \rightarrow s\gamma$, and not affect our analysis.
of our results for the range of masses we consider (this was noted in [8]), while allowing us to derive the simple expression of Eq. (2).

Let us now assume the following form for the mixing matrix \( \tilde{V} \):

\[
\tilde{V} = \begin{pmatrix}
1 & 0 & 0 \\
0 & 1/\sqrt{2} & 1/\sqrt{2} \\
0 & -1/\sqrt{2} & 1/\sqrt{2}
\end{pmatrix}
\] (4)

i.e. the right-handed scalar charm and top are maximally mixed. This form of the mixing matrix could be motivated in some of the models for fermion masses based on Abelian horizontal symmetries [7, 11]. We will denote the common mass of the lightest squark and charged higgsino by \( \tilde{m} \), and assume no special degeneracy between the physical squark masses. In this case, we can use Eq. (2) to obtain the following expressions for the meson mixing to first order in \( m_c/m_t \):

\[
M^{B_d}_{12} = \frac{G_F^2}{12\pi^2} B_{B_d} f_{B_d}^2 m_{B_d} \eta_{B_d} m_t^2 [a V_{td}^* V_{tb}^2 + b V_{td}^* V_{tb}^2 + c V_{cd}^* V_{td} V_{tb}]
\] (5)

\[
M^{B_s}_{12} = \frac{G_F^2}{12\pi^2} B_{B_s} f_{B_s}^2 m_{B_s} \eta_{B_s} m_t^2 [a V_{ts}^* V_{tb}^2 + b V_{ts}^* V_{tb}^2 + c V_{cs}^* V_{ts} V_{tb}]
\] (6)

\[
M^K_{12} = \frac{G_F^2}{12\pi^2} B_K f_K^2 m_K \eta_K m_t^2 [a V_{td}^* V_{ts}^2 + b V_{td}^* V_{ts}^2 + c V_{cd}^* V_{td}^* V_{ts} + c V_{td}^* V_{cs} V_{ts}]
\]

\[+ \frac{m_{\tilde{m}}^2}{m_t^2} f_2(y_c) V_{cd}^* V_{cs}^2 + \frac{m_{\tilde{m}}^2}{m_t^2} f_3(y_c, y_t) V_{cd}^* V_{cs} V_{ts} V_{td}]
\] (7)

where

\[
a = f_2(y_t), \quad b = \frac{1}{48 \sin^4 \beta} \frac{m_t^2}{m^2}, \quad c = \frac{1}{24 \sin^4 \beta} \frac{m_{\tilde{m}} m_t}{m^2},
\] (8)

\[y_i = m_i^2/M_W^2,\text{ and the functions } f_2(x), f_3(x, y) \text{ are defined in [12, 13]:}
\]

\[
f_2(x) = \frac{1}{4} + \frac{9}{4(1-x)} - \frac{3}{2(1-x)^2} - \frac{3x^2 \ln x}{2(1-x)^3},
\]

\[
f_3(x, y) = \ln \left( \frac{y}{x} \right) - \frac{3y}{4(1-y)}(1 + \frac{y \ln y}{1-y})
\] (9)

The terms proportional to \( b \) and \( c \) in Eqs. (5-7) are the supersymmetric contributions, and have important consequences for the determination of the CKM matrix elements as we show in the next section. The dominant supersymmetric contribution proportional to \( b \) is present also in the usual analyses based on the constrained MSSM, and is always in phase with the Standard Model contribution proportional to \( m_t^2 \). Although we started with only the CKM phase, in this
model, both the $B_d - \bar{B}_d$ and the $K - \bar{K}$ mass matrices have a second out of phase contribution given by the term proportional to $c$. This contribution is a result of the mixing between the right-handed scalar top and charm, and should be observable at the $B$ factory $CP$ violating experiments. An estimate of the importance of this term compared to the “in phase” supersymmetric contribution is given by

$$\frac{c|V_{cd}|}{b|V_{td}|} \approx \frac{2m_c}{A\lambda^2 m_t} \approx 50 m_c m_t.$$  

(10)

where $A$ and $\lambda$ parametrize elements of the CKM matrix as shown below, and we have used $(A\lambda^2)^{-1} = |V_{cb}|^{-1} \approx 25$.

3 The Unitarity Triangle

In the Wolfenstein parametrization [14], the CKM matrix is given by

$$V_{CKM} = \begin{pmatrix}
1 - \frac{1}{2}\lambda^2 & \lambda & A\lambda^3(\rho - i\eta) \\
-\lambda & 1 - \frac{1}{2}\lambda^2 & A\lambda^2 \\
A\lambda^3(1 - \rho - i\eta) & -A\lambda^2 & 1
\end{pmatrix}$$

(11)

We can best visualize the effects of the new contributions on the CKM parameters by plotting the allowed regions in the $\rho - \eta$ plane. We will plot the usual three constraints coming from the experimentally measured quantities $|V_{ub}|/|V_{cb}|$, $\Delta m_{B_d}$, and $|\epsilon|$, as well as the constraint from $Arg(M_{12}^{B_d})$ which will be cleanly measured at the $B$ factories by the $CP$ asymmetry in the decay $B_d \to \Psi K_S$. Although the model satisfies the constraint from $\Delta m_K$, we do not include it in the subsequent analysis because of the large uncertainty in the standard model prediction for this quantity due to long distance effects.

The curves to be plotted are determined by the following equations

1) $$\frac{|V_{ub}|}{|V_{cb}|} = \lambda\sqrt{\rho^2 + \eta^2}$$  

(12)

this determines a circle centered at the origin of the $\rho - \eta$ plane. Since this quantity is determined by tree-level decays, it is not affected by the presence of new physics.

2) $$\Delta m_{B_d} = 2|M_{12}^{B_d}|$$  

(13)

\[\text{The parameter } \lambda \text{ corresponds to the Cabbibo angle, and is extremely well measured in tree-level standard model decays. Although the parameter } A \text{ occurs in all of the expressions for neutral meson mixing, and we allow it to vary in our subsequent fits, its best fit value is always close to that determined from the CKM element } V_{cb} \text{ whose determination is again dominated by tree-level standard model physics. Thus the effects of new physics are dominantly felt by the parameters } \rho \text{ and } \eta.\]
which gives
\[(1 - \frac{c}{2A\lambda^2(a+b)} - \rho)^2 + \eta^2 = \frac{\Delta m_{B_d}}{2K_\eta m_t^2(a+b)}\] (14)
where $K_\eta$ and $a, b, c$ have been defined in Eqs. (3,8). This once again determines a circle on the $\rho - \eta$ plane. The presence of new physics has two effects here. The in phase supersymmetric contribution given by $b$ in the denominator on the right-hand-side of Eq. (14) reduces the radius of the circle, and the out of phase contribution proportional to $c$ displaces the center from $\rho = 1$.

3) \[|\epsilon| = \frac{\text{Im} M_{12}^K}{\sqrt{2}\Delta m_K}\] (15)
which gives
\[\eta[(a+b)A^2\lambda^4(1-\rho) - cA\lambda^2(1-\rho) - \frac{c}{2}A\lambda^2 + P'_0] = \frac{\Delta m_K|\epsilon|}{\sqrt{2K_\eta m_t^2A^2\lambda^6}}\] (16)
with
\[P'_0 = \frac{m^2_c}{m_t^2}[\frac{\eta_{et}}{\eta_K}f_\beta(y_t, y_h) - \frac{\eta_{ec}}{\eta_K}]\] (17)
This curve determines a hyperbola in the $\rho - \eta$ plane, with the new physics once again having two effects. The term proportional to $b$ reduces the distance of the directrix from the origin, and the term proportional to $c$ shifts the coordinates to which the hyperbola is referred. Finally,

4) \[V_{td} = |V_{td}|e^{-i\beta_{KM}}\] (18)
which leads to the straight line
\[\eta = (1 - \rho)\tan \beta_{KM}\] (19)
where $\beta_{KM}$ is determined from the expression
\[\text{Arg}(M_{12}^{B_d}) = \tan^{-1}\left[\frac{(a+b)\sin 2\beta_{KM} - c'\sin \beta_{KM}}{(a+b)\cos 2\beta_{KM} - c'\cos \beta_{KM}}\right],\] (20)
with
\[c' = \frac{c}{A\lambda^2\sqrt{(1-\rho)^2 + \eta^2}}.\] (21)
Here the shift from the standard model expectation is entirely due to the “out-of-phase” contribution proportional to $c$, which tends to increase the phase of $B_d - \bar{B}_d$ mixing as compared to the standard model (in Eqs. (14, 16, 20), the
| Parameter                  | Value                                           |
|---------------------------|-------------------------------------------------|
| $|V_{ub}|/|V_{cb}|$         | 0.08 ± 0.02                                      |
| $\Delta m_{B_d}$          | $(0.306 ± 0.0158) \times 10^{-12}$ GeV           |
| $|\epsilon|$               | $(2.26 ± 0.02) \times 10^{-3}$                  |
| $|V_{cb}|$                 | 0.039 ± 0.002                                    |
| $\lambda$                | 0.2205                                          |
| $m_t$                     | 170 ± 10 GeV                                    |
| $m_c$                     | 1.3 GeV                                          |
| $\sqrt{B_B f_B}$          | $180 \pm 30$ MeV                                 |
| $B_K$                     | 0.8 ± 0.2                                        |
| $\eta_B$                 | 0.55                                            |
| $\eta_K$, $\eta_{ct}$, $\eta_{cc}$ | 0.57, 0.47, 1.32                      |
| $\tilde{m}$, $\tan \beta$ | 85 GeV, 1                                       |

Table 1: Input parameters for the $\rho - \eta$ analysis presented in Fig. 1 and Table 2.

The standard model limit can be recovered by setting $b = c = 0$. This corresponds to the limit $\tilde{m} \to \infty$.

We plot the constraints from these curves using the inputs from Table 1 in Figs. 1. Our inputs are the same as those in [15] except for $|V_{cb}|$ where we use the value in [8], and that we have been slightly less conservative in our estimates of the uncertainties in $\sqrt{B_B f_B}$ and $B_K$. Fig. 1(a) corresponds to the standard model case i.e. no new physics. Fig. 1(b) and Fig. 1(c) include the supersymmetric contribution with $\tilde{m} = 85$ GeV, and $\tan \beta = 1$. However Fig. 1(c) contains the wrong analysis where we incorrectly assume that the supersymmetric contribution is always in phase with the standard model one. In all of these figures we include the error from $m_t$ only in its effect on the leading coefficients of Eqs. (5, 7), ignoring its effect on the terms in the square brackets. We also ignore the effects of the error in $|V_{cb}|$. The straight lines in the figures corresponding to $Arg(M_{12}^{B_d})$ are obtained in the following way: we determine the point in the overlap region of the other three curves that give us the largest values for the phase $\beta_{KM}$. This is then plugged into Eq. (20), and we include an error of ±0.059 in the determination of $sin(Arg(M_{12}^{B_d}))$ as quoted in [16].

Comparing Fig. 1(a) with Figs. 1(b) and 1(c), we notice that although there is a large overlap between the allowed regions for the Standard Model and for the supersymmetric case, it could be possible that the supersymmetric contributions, as discussed above shift, $\rho$ and $\eta$ into a region excluded by the Standard Model. This possibility which has been noticed in Refs. [5, 8] is not very interesting from
Figure 1: Constraints on $\rho$ and $\eta$ based on the parameters of Table 1. The small circles (dash) are from $|V_{ub}|/|V_{cb}|$, the large circles (dot dash) are from $\Delta m_{B_d}$, the hyperbolae (dot dot dash) from $\epsilon$ and the straight lines (solid) from $Arg(M_{12}^{B_d})$. 
(a) The standard model. (b) The correct supersymmetric analysis. (c) The incorrect supersymmetric analysis.
the point of view of the $B$-factory CP violation experiments. This is because supersymmetric particles in the mass range we are considering would already be detected at the high energy colliders like LEP2 before the $B$-factories turn on, thus we would know already that the Standard Model analysis is incorrect. More interesting is the fact that the line denoting $\text{Arg}(M_{B_d}^{12})$ lies outside the allowed region for some values of the phase $\beta_{KM}$, a possibility that can never occur in the constrained version of the MSSM. Thus, if we incorrectly interpreted this phase as measured by the CP asymmetry in $B_d \to \Psi K_S$ as the CKM phase $\beta_{KM}$ (as in Fig. 1(c)), we would come to the false conclusion that the unitarity triangle does not close, and that there are additional sources of CP violation in the theory besides the complex Yukawa couplings between the quark and Higgs fields.

The obvious way to check for this supposed deviation from unitarity is by the clean measurement of phases that the CP violation experiments at the $B$-factories allow. The CP violating rate asymmetries in the decays $B_d \to \pi \pi$, $B_d \to \Psi K_S$, $B_s \to \rho K_S$ measure the quantities $\sin 2\alpha'$, $\sin 2\beta'$, and $\sin 2\gamma'$ where

$$2\alpha' = \text{Arg}(M_{B_d}^{12}) + 2\gamma, \quad 2\beta' = \text{Arg}(M_{B_s}^{12}), \quad 2\gamma' = \text{Arg}(M_{B_s}^{12}) + 2\gamma.$$  \hspace{1cm} (22)

and $\gamma$ is the phase of $V_{ub}$. In the standard model, $\text{Arg}(M_{B_d}^{12}) = 2\beta_{KM}$ and $\text{Arg}(M_{B_s}^{12}) = 0$ so the measured quantities reduce to $-\sin 2\alpha$, $\sin 2\beta_{KM}$, and $\sin 2\gamma$ (after making the replacement $\beta_{KM} + \gamma = \pi - \alpha$) where $\alpha$, $\beta_{KM}$ and $\gamma$ are the angles of the “unitarity” triangle. Thus if the phases of $B_d - \bar{B}_d$ or $B_s - \bar{B}_s$ mixing are affected by new physics, the three measured angles $\alpha'$, $\beta'$ and $\gamma'$ will not correspond to the angles of the unitarity triangle, and in general will not add up to $180^\circ$. This test, however, does not work in this case, because although the phase of $B_d - \bar{B}_d$ mixing is affected by the supersymmetric contribution, that of $B_s - \bar{B}_s$ mixing isn’t [Eq. (3)]. Thus, if we repeat the above analysis making the replacement $\beta' + \gamma = \pi - \alpha'$, we will still obtain a triangle that closes, with angles $\alpha - \delta$, $\beta_{KM} + \delta$ and $\gamma$, where $\delta$ is the amount by which the phase of $B_d - \bar{B}_d$ mixing is shifted from the standard model value. This possibility that the angles measured by the above CP violating experiments would still add up to $180^\circ$ if the phase of $B_d - \bar{B}_d$ mixing is changed, but that of $B_s - \bar{B}_s$ mixing isn’t was pointed out in [17]. An alternative method to measure $\gamma$ is in the CP violating decay $B_d \to D_{CP}^0 K^*$ [18]. However, since this measurement is a result of interfering tree-level amplitudes, it is not affected by the new physics, and we would still measure the true angle $\gamma$. Thus, as in the case above, we would mistakenly interpret the three angles obtained as summing to $180^\circ$.

Another interesting manifestation of this new phase in the $B_d - \bar{B}_d$ mixing matrix could be in the existence of CP violating asymmetries in decays where the standard model predicts none. A simple example of this is the penguin mediated decay $B_d \to K_S K_S$ where the phase of the top mediated penguin exactly cancels
the phase $-2\beta_{KM}$ of $B_d - \bar{B}_d$ mixing in the standard model. In the model we are considering, this cancellation would not be exact because of the new phase in $B_d - \bar{B}_d$ mixing matrix, and there could be observable $CP$ asymmetries in the decay. It has recently been observed, however, that sub-dominant penguins mediated by up and charm quarks could contribute to $CP$ violation in this channel [19]. We have checked that this contribution is not only comparable in magnitude to the one due to the new mixing phase, but is also uncertain in sign. Thus the observation (or non-observation) of $CP$ violation in this decay could not distinguish this model from either the standard model or the constrained MSSM.

The considerations of the previous paragraphs show us that only phase information is not enough to tell us that we are wrong in assuming that supersymmetric contributions do not modify the phase of neutral $B$ meson mixing. In order to detect this, we need to combine the phase information from the $CP$ violating experiments with independent information on magnitudes (and phases) of the CKM matrix elements available in the quantities $|V_{ub}|/|V_{cb}|$, $\Delta M_{B_d}$ and $|\epsilon|$ discussed earlier. To this end we do a $\chi^2$ analysis for the central values of $\rho$ and $\eta$ using the quantities listed in Table 1. Our experimental inputs are $|V_{ub}|/|V_{cb}|$, $\Delta M_{B_d}$, $|\epsilon|$, $|V_{cb}|$, $m_t$, and “projected values” for $\sin 2\beta'$ and $\sin 2\alpha'$, while allowing $\rho$, $\eta$, $A$ and $m_t$ to vary. We display our results in Table 2, where analyses I and II correspond to two different choices for the inputs $\sin 2\beta'$ and $\sin 2\alpha'$.

In both analyses we first do the $\chi^2$ minimization without any input for $\sin 2\beta'$ and $\sin 2\alpha'$, to obtain central values and errors on $\rho$ and $\eta$ (these would correspond to the allowed regions of Figs. 1 without including the constraints from the straight lines representing $Arg(M_{12}^{B_d})$). In analysis I, we then include as inputs, $\sin 2\beta'$ and $\sin 2\alpha'$ calculated using these central values, and repeat the $\chi^2$ minimization to obtain a new minimum $\chi^2$ and central values for $\rho$ and $\eta$. These are the values displayed in Table 2. Analysis II follows the same procedure, except that the inputs $\sin 2\beta'$ and $\sin 2\alpha'$ are calculated using values of $\rho$ and $\eta$ that are one standard deviation above the central values obtained in the first part of the procedure (the central values and errors obtained here would correspond to the allowed regions of Figs. 1 where we have included all the constraints including those from $Arg(M_{12}^{B_d})$). In both the analyses we include an experimental error on $\sin 2\beta'$ of $\pm 0.059$ and on $\sin 2\alpha'$ of $\pm 0.085$ which are the errors quoted by the BABAR collaboration.

The three cases in Table 2 correspond to those of Fig. 1 i.e. case (a) corresponds to the standard model where there is no new physics, in case (b) we

\footnote{Although the determination of $\sin 2\beta'$ is not affected by “penguin pollution” in this model, the determination of $\sin 2\alpha'$ could be affected by out-of-phase supersymmetric penguins in addition to the standard model ones. We assume that these effects could be accounted for by an isospin analysis [20].}
correctly include the supersymmetric contribution, whereas in case (c) we include the supersymmetric contributions but neglect the out-of-phase part. We have checked that the results of our $\chi^2$ analysis for the standard model agree with those of [15] for similar choices of inputs.

We see that the results presented in Table 2 corroborate the visual information of Fig. 1. Firstly the central values for $\rho$ and $\eta$ for the supersymmetric case are indeed different from those for the standard model. In particular more positive values for $\rho$ are preferred by the supersymmetric case. Secondly, it is only if the actual values for $\rho$ and $\eta$ were to lie near their current $1\sigma$ upper bounds, as in analysis II, that the $B$ factory experiments would be sensitive to the new phase in $M_{B_\tau}^{12}$. This is signalled by the large value of the minimum $\chi^2$ for the incorrect analysis in II where we assume that there are no new phases in $M_{B_\tau}^{12}$ (since we have seven experimental inputs and four variables, we consider $\chi^2 < 3$ indicative of a good fit). This is as in Fig. 1(c) where we can see that for $\rho$ and $\eta$ near their central values, the area predicted by the incorrect analysis would lie within the allowed region, whereas with $\rho$ and $\eta$ close to their $1\sigma$ upper bounds, the area predicted by the incorrect analysis clearly lies outside the allowed region. Thus, it seems that even with the precise phase information provided by the $B$ factory experiments, we would still have to be lucky in order to be able to notice any deviation from the usual expectations of no new phases in neutral meson mixing. However, the insensitivity of this analysis to the new phase is mostly due to the large errors in the experimental and theoretical inputs into the analysis. Since we expect most of these to decrease before the $B$ factory data analyses begin, we redo the analysis of Table 2 using the same central values for the inputs, but with the improved errors expected in the future.

We display our new inputs in Table 3, and the results of our $\chi^2$ analysis in Table 4. We base our estimates for the improved errors on the inputs on [21]. Figs. 2 display the same constraints as Figs. 1, but are plotted using the reduced

| Analysis         | $(\rho, \eta)$               | $\chi^2_{\text{min}}$ |
|------------------|-------------------------------|------------------------|
| I a) Standard Model | $(-0.04 \pm 0.03, 0.35 \pm 0.04)$ | 0.013 |
| b) Correct Susy  | $(0.12 \pm 0.03, 0.34 \pm 0.03)$ | 0.0051 |
| c) Incorrect Susy | $(0.13 \pm 0.03, 0.36 \pm 0.03)$ | 0.88 |
| II a) Standard Model | $(0.17 \pm 0.04, 0.42 \pm 0.03)$ | 2.0 |
| b) Correct Susy  | $(0.23 \pm 0.05, 0.39 \pm 0.03)$ | 1.9 |
| c) Incorrect Susy | $(0.25 \pm 0.05, 0.42 \pm 0.03)$ | 3.8 |

Table 2: Results of the $\chi^2$ analysis for $\rho$ and $\eta$ based on the inputs of Table 1.
| Parameter                  | Value                                      |
|---------------------------|--------------------------------------------|
| $|V_{ub}|/|V_{cb}|$       | 0.08 ± 0.01                                |
| $\Delta m_{B_d}$          | $(0.306 \pm 0.0158) \times 10^{-12}$ GeV   |
| $|\epsilon|$              | $(2.26 \pm 0.02) \times 10^{-3}$           |
| $|V_{cb}|$                 | 0.039 ± 0.001                              |
| $\lambda$                 | 0.2205                                     |
| $m_t$                     | 170 ± 5 GeV                                |
| $m_c$                     | 1.3 GeV                                    |
| $\sqrt{B_B f_B}$          | $180 \pm 10 MeV$                           |
| $B_K$                     | 0.8 ± 0.05                                 |
| $\eta_B$                  | 0.55                                       |
| $\eta_K, \eta_{ct}, \eta_{cc}$ | 0.57, 0.47, 1.32                         |
| $\bar{m}$, $\tan \beta$ | 85 GeV, 1                                  |

Table 3: Inputs for the $\rho - \eta$ analysis presented in Fig. 2 and Table 4. The central values are the same as those of Table 1, however the errors reflect our expectations for experimental and theoretical improvements in estimating these quantities.

errors of Table 3.

Table 4 (as well as Fig. 2) contains what we believe to be an accurate representation of the physics results obtained at the $B$ factories if the scenario outlined in this paper were to hold, i.e., the existence of low energy supersymmetry with small $\tan \beta$, light right-handed up-type squarks and no new CP violating phases. Once again we notice that the central value for $\rho$ is more positive and clearly different from what would be the standard model value. Here however, in contrast with the results of Table 2, we see that incorrectly assuming that $Arg(M_{12}^{B_d})$ is not affected by the new physics yields a poor fit over most of the allowed region for $\rho$ and $\eta$ (cases I(c) and II(c)). Interestingly though, this deviation from the standard model is not due to the existence of new CP violating phases in the theory as one would naively infer, but simply due to the fact that the mixing patterns of the squarks could be different from those of the quarks, resulting in the CKM phase showing up in physical quantities in combinations different from those in the standard model. It is exciting to know that the experiments at the proposed $B$ factories are sensitive to this possibility. Although we have based our analysis on one particular choice for $\tan \beta$ and the mass of the lightest squark, the explicit formulas presented make generalizations to other values trivial. In particular, the effects we discuss become larger for smaller squark mass and $\tan \beta$,
Figure 2: Constraints on $\rho$ and $\eta$ based on the reduced errors on the input parameters we expect in the future (Table 3). The small circles (dash) are from $|V_{ub}|/|V_{cb}|$, the large circles (dot dash) are from $\Delta m_{B_d}$, the hyperbolae (dot dot dash) from $\epsilon$ and the straight lines (solid) from $\text{Arg}(M_{12}^{B_d})$. (a) The standard model. (b) The correct supersymmetric analysis. (c) The incorrect supersymmetric analysis.
Table 4: Results of the $\chi^2$ analysis for $\rho$ and $\eta$ based on the inputs of Table 3.

and are reduced in the opposite limit (as long as $\tan \beta \lesssim 30$, after which this analysis no longer holds).

4 Conclusions

We have presented a scenario based on the MSSM where although the CKM paradigm for $CP$ violation still holds, substantial mixing between the right-handed top and charm squarks introduces new $CP$ violating phases into the neutral meson mixing matrices. We have analyzed a specific case where the lightest right-handed squark weighs 85 GeV and $\tan \beta = 1$, and show that in this case, the experiments at the proposed $B$ factories would be sensitive to these new phases. We stress that the presence of these new phases in the meson mixing matrices should not be interpreted as proving the existence of new fundamental $CP$ violating phases, but rather as a novel manifestation of the usual CKM phase due to different patterns of mixing for the quarks and the squarks.

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References

[1] By Minimal Supersymmetric Standard Model we mean the supersymmetric extension of the standard model with minimal particle content. Its constrained version includes the assumptions of coupling constant unification, and universal scalar and gaugino masses at a high “unification” scale.
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