Abstract

In this paper, as a preparation of developing data analysis procedures for using 3-dimensional information offered by directional Dark Matter (DM) detection experiments, we study the patterns of the angular distribution of the Monte Carlo–generated 3-D velocity of halo Weakly Interacting Massive Particles (WIMPs) as well as apply the Bayesian fitting technique to reconstruct the radial distribution of the 3-D WIMP velocity. Besides the diurnal modulation of the angular WIMP velocity distribution, the so-called “directionality” of DM signals proposed in the literature, we will demonstrate a possible “annual” modulation of both of the angular and the radial distributions of the 3-D WIMP velocity. Our Bayesian reconstruction results of the (annual modulation of the) radial WIMP velocity distribution will also be discussed in detail. For readers’ reference, the angular distribution patterns of the 3-D WIMP velocity in the “laboratory (location)–dependent” reference frames for several underground laboratories are also given in the Appendix.
Contents

1 Introduction 5

2 Toolbox 7

2.1 Our definitions of different celestial coordinate systems 7

2.1.1 Definition of the Galactic coordinate system 7

2.1.2 Definitions of the Ecliptic and the Equatorial coordinate systems 8

2.1.3 Definition of the Earth coordinate system 9

2.1.4 Definition of the horizontal coordinate system 10

2.1.5 Definition of the laboratory coordinate system 10

2.2 Event generation in the Galactic coordinate system 11

2.2.1 Radial component of the 3-D WIMP velocity distribution 12

2.2.2 Angular component of the 3-D WIMP velocity distribution 13

2.2.3 Measuring time and observation periods/shifts of WIMP events 14

2.3 Bayesian reconstruction of the radial distribution of the 3-D WIMP velocity 18

2.3.1 Bayesian analysis 18

2.3.2 Bayesian reconstruction of \( f(v) \) 18

2.3.3 Fitting velocity distribution \( f_{v,th}(v) \) 20

3 Angular distributions of the 3-D WIMP velocity in different coordinate systems 22

3.1 Angular WIMP velocity distribution in the Ecliptic frame 22

3.2 Angular WIMP velocity distribution in the Equatorial frame 23

3.2.1 Annual modulation of the angular velocity distribution in the Equatorial frame 24

3.3 Angular WIMP velocity distribution in the Earth frame 25

3.3.1 Annual modulation of the angular velocity distribution in the Earth frame 26

3.4 Angular WIMP velocity distribution in the horizontal frame 27

3.4.1 Annual modulation of the angular velocity distribution in the horizontal frame 28

3.5 Angular WIMP velocity distribution in the laboratory frame 32

3.5.1 Diurnal modulation of the angular velocity distribution in the laboratory frame 32

4 Bayesian reconstruction of the radial distribution of the 3-D WIMP velocity 34

4.1 Radial WIMP velocity distribution in the Equatorial frame 34

4.2 Fitted radial WIMP velocity distribution in the Equatorial frame 36

4.2.1 With the one–parameter velocity distribution \( f_{1,v,th}(v; v_0) \) 36

4.2.2 With the \( v_0 \)-fixed velocity distribution \( f_{1,v,th}(v; v_0) \) 38

4.2.3 With the simplified velocity distribution \( f_{1,v,th}(v; v_0, v_e) \) 39

4.2.4 With the modified velocity distribution \( f_{1,v,th}(v; v_0, \Delta v) \) 41

4.3 Annual modulation of the radial distribution of the 3-D WIMP velocity 42

4.3.1 With the one–parameter velocity distribution \( f_{1,v,th}(v; v_0) \) 43

4.3.2 With the \( v_0 \)-fixed velocity distribution \( f_{1,v,th}(v; v_e) \) 45

4.3.3 With the simplified velocity distribution \( f_{1,v,th}(v; v_0, v_e) \) 47

4.3.4 With the modified velocity distribution \( f_{1,v,th}(v; v_0, \Delta v) \) 49

4.4 Fitted radial WIMP velocity distribution in the Galactic frame 52
5 With a raised total event number
5.1 Angular distributions of the 3-D WIMP velocity in the laboratory-independent frames
5.1.1 Angular WIMP velocity distribution in the Ecliptic frame
5.1.2 Angular WIMP velocity distribution in the Equatorial frame
5.1.3 Angular WIMP velocity distribution in the Earth frame
5.2 Bayesian reconstruction of the radial distribution of the 3-D WIMP velocity
5.2.1 Fitted radial WIMP velocity distribution in the Equatorial frame
5.2.2 Annual modulation of the reconstructed WIMP velocity distribution
5.2.3 Fitted radial WIMP velocity distribution in the Galactic frame

6 Summary and conclusions

A Definitions of and transformation between the celestial coordinate systems
A.1 Definitions of the horizontal and the laboratory coordinate systems
A.1.1 Conventional definition
A.1.2 Our definitions
A.2 Definitions of the Ecliptic and the Equatorial coordinate systems
A.2.1 Conventional definitions
A.2.2 Our definitions
A.2.3 Velocity of the Earth in the Ecliptic coordinate system
A.3 Definition of the Galactic coordinate system
A.3.1 Conventional definition
A.3.2 Our definition
A.3.3 Direction of the movement of the Solar system around the Galactic center
A.3.4 Annual modulation of the Earth’s velocity in the Ecliptic coordinate system
A.3.5 Annual modulation of the Earth’s velocity in the Equatorial coordinate system
A.3.6 Dates considered for demonstrating the diurnal modulation of the angular WIMP velocity distribution

B Angular distributions of the 3-D WIMP velocity observed at underground laboratories
B.1 Agua Negra Deep Experiment Site (ANDES)
B.2 Boulby Laboratory
B.3 Callio Laboratory
B.4 China Jinping Underground Laboratory (CJPL)
B.5 Deep Underground Science and Engineering Laboratory (DUSEL)
B.6 Kamioka Observatory
B.7 Laboratori Nazionali del Gran Sasso (LNGS)
B.8 Laboratoire Souterrain Canfranc (LSC)
B.9 Laboratoire Souterrain de Modane (LSM)
B.10 Sudbury Neutrino Observatory (SNOLAB)
B.11 Stawell Underground Physics Laboratory (SUPL)

C Annual modulation of the radial WIMP velocity distribution in four normal seasons
C.1 With the one–parameter velocity distribution $f_{1,sh,v_0}(v; v_0)$
C.2 With the $v_0$–fixed velocity distribution $f_{1,sh,v_e}(v; v_e)$
C.3 With the simplified velocity distribution $f_{1,sh}(v; v_0, v_e)$ ........................................ 174
C.4 With the modified velocity distribution $f_{1,sh,\Delta v}(v; v_0, \Delta v)$ ........................................ 179

D Analytic forms of the shift Maxwellian velocity distribution .................................................. 182

References ........................................................................................................................................ 182
1 Introduction

So far Weakly Interacting Massive Particles (WIMPs) $\chi$ arising in several extensions of the Standard Model of particle physics are still one of the most favorite candidates for cosmological Dark Matter (DM). In the last (more than) three decades, a large number of experiments has been built and is being planned to search for different WIMP candidates by direct detection of the scattering recoil energy of ambient WIMPs off target nuclei in low–background underground laboratory detectors (see Refs. [1, 2, 3, 4, 5, 6] for reviews).

Among these direct detection experiments measuring the recoil energy deposited in detectors, the “directional” detection of Galactic DM particles has been proposed more than one decade to be a promising experimental strategy for using 3-dimensional information (the recoil track and/or the head–tail sense) of WIMP–nucleus scattering events to discriminate these WIMP signals from backgrounds (see Refs. [7, 8, 9, 10, 11, 12, 13]). Several experimental collaborations are investigating different detector materials and techniques and have achieved recently great progress.

For instance, as the first directional detection experiment, the DRIFT Collab. demonstrated the reconstruction ability of the range component signature by using their DRIFT–Ild detector with a gas mixture of $\text{CS}_2 + \text{CF}_4 + \text{O}_2$. Meanwhile, the MIMAC experiment started with $^3\text{He}$ as detector material for searching for light WIMPs [22, 23, 24], but turned later to develop micromegas $\mu$TPC detector with a gas mixture of $\text{CF}_4 + \text{CHF}_3 + \text{C}_4\text{H}_{10}$ and also demonstrated the ability of determining 3-D positions of primary ionization electrons for reconstructing in turn the track of the recoiled nucleus [25, 26, 27, 28, 29].

In contrast to the use of gas mixtures, pure $\text{CF}_4$ gas has also been studied by the NEWAGE Collab. with also the $\mu$TPC technique [30, 31, 32] as well as by the DMTPC Collab. using an optical (CCD pixel image) readout combined with a transient charge readout [33, 34, 35, 36, 37, 38, 39]. Recently, while the D3 Collab. considers $\text{SF}_6$ gas with sulfur as target nucleus [40, 41], the NEWSdm Collab. investigates the nuclear emulsion technique with $\text{AgBr(I)}$ as detector material [42, 43, 44]. Furthermore, a larger CYGNUS Collab. has been built for combining members, efforts and achievements from different experimental collaborations [45].

The basic concept of directional Dark Matter detection is based on the rotation of the Earth. As shown in Figs. 1, there are two kinds of possible diurnal modulation of WIMP signals to observe: the diurnal modulation of the (main) direction of WIMP events, the so–called “directionality” of the WIMP wind, as well as that of the number of WIMP events caused by Earth’s shielding of the WIMP flux. Directional DM detection experiments aim hence, as the first step, to identify positive WIMP signals and discriminate them from theoretically (approximately) isotropic background events.

Once positive WIMP signals could be observed and more and more events could be accumulated, reconstructions of particle and/or astronomical properties of halo WIMPs would be considered as the next stage. For this purpose, different methods have been developed. In the literature, a large number of earlier studies is focused on the (use of the) distribution patterns of the nuclear recoil energy and WIMP flux, e.g., works done by N. Bozorgnia, G. B. Gelmini and P. Gondolo [46, 47, 48, 49] as well as by B. Morgan, A. M. Green, C. A. J. O’Hare and B. J. Kavanagh [50, 51, 52, 53, 54, 55, 56]. Meanwhile, J. Billard, F. Mayet and D. Santos developed their Bayesian analysis, which considers not only the 3-D velocity dispersion of the local WIMP velocity distribution and the main direction of the recoiling nuclei, but also the WIMP mass and the WIMP–nucleon cross section as fitting parameters simultaneously [57, 58, 59]. Moreover, some authors focused on theoretically analyzing of the (local) Dark Matter velocity distribution, e.g., works done by S. K. Lee and A. H. G. Peter [60, 61].
Figure 1: A sketch of the basic concept of directional Dark Matter detection experiments: (a) the diurnal modulation of the main direction of incident WIMPs, the so-called “directionality” of the WIMP wind, for a laboratory located in the Northern Hemisphere (in Summer); (b) except of the directionality of the WIMP wind, the event number of WIMP signals observed at a laboratory located in the Southern Hemisphere (in Winter) could also have the diurnal modulation caused by the Earth’s shielding of the WIMP flux. The darkened/lightened (left/right-hand) sphere indicates that the laboratory of interest is in the night/day. See Figs. [10] and [11] as well as the text there for more details.

Besides these works, in this paper, as a preparation for our future study on the development of data analysis procedures for using and/or combining 3-D information offered by directional detection experiments, to, e.g., reconstruct the 3-dimensional WIMP velocity distribution, we study the patterns of the angular distribution of the Monte Carlo–generated 3-dimensional WIMP velocity as well as apply the Bayesian fitting technique to reconstruct the radial distribution (magnitude) of the 3-D WIMP velocity. Besides the diurnal modulation of the angular WIMP velocity distribution mentioned above, we will also demonstrate a possible “annual” modulation of both of the angular and the radial distributions of the 3-D WIMP velocity.

The remainder of this paper is organized as follows. In Sec. 2, we describe all needed tools for our simulations and analysis procedures presented in this paper: our definitions of different celestial coordinate systems, the event generation process based on the Monte Carlo method, as well as the Bayesian fitting procedure used for reconstructing the radial component of the 3-dimensional WIMP velocity distribution. Then we present at first the (annual/diurnal modulated) patterns of the angular WIMP velocity distribution in different celestial coordinate systems one by one in Sec. 3 and the reconstructions of the (annual modulation of the) radial WIMP velocity distribution in both of the Equatorial and the Galactic coordinate systems in Sec. 4. In Sec. 5, we raise the total number of generated WIMP events and demonstrate the simulation results with an improved analysis resolution. We conclude in Sec. 6. Some technical details for our analyses and a summary of the laboratory–dependent simulation results will be given in Appendix.
In this subsection, we give our definitions of four “laboratory–independent” (Galactic, Ecliptic, Equatorial and Earth) and two “laboratory–dependent” (horizontal and laboratory) coordinate systems\footnote{Note that some of our definitions are different from their astronomical conventions, but would be more convenient for our data analyses, in particular, for comparisons of results obtained by using data from different underground laboratories.}. The analytic and/or the numerical forms of all transformation matrices between these coordinate systems will be derived in detail in Appendix A.

2.1.1 Definition of the Galactic coordinate system

As shown in Fig. 2, the Galactic coordinate system in our simulations presented in this paper is defined as follows: the origin is at the Galactic Center (GC), the primary direction (the $X_G$–axis) points from the Solar center to the approximate center of our Galaxy, and the $Z_G$–axis points to the Galactic North Pole (GNP). As usual, the right–handed convention is used for defining the $Y_G$–axis and the fundamental ($X_G – Y_G$) plane is the approximate Galactic plane \cite{62}.
Figure 3: (a) A sketch of the definitions of the (red) Ecliptic and the (blue) Equatorial coordinate systems: their origins are located at the center of the Sun and that of the Earth, respectively, the common primary direction (the $X_S/X_{Eq}$-axis) is the direction pointing from the Solar center to that of the Earth at 12 midnight (the *end* of the date of the vernal equinox), the $Z_S$- and the $Z_{Eq}$-axes are perpendicular to the (yellow) Ecliptic or the (blue) Equatorial plane, respectively, and their $Y_S$- and $Y_{Eq}$-axes are then defined as usual by the right–handed convention. The blue circular band indicates the Earth’s orbit around the Sun. (The purple arrow points to the celestial Equinox, which is the conventional (common) primary direction of the Ecliptic and the Equatorial coordinate systems.) (b) A sketch of the relative orientations between the (black) Galactic, the (red) Ecliptic and the (blue) Equatorial coordinate systems (on the date of the vernal equinox).

Note that, as discussed in detail in Appendix A.3.3 and sketched in Fig. A5, the direction of the Solar movement towards the CYGNUS constellation is *not parallel to*, but only *approximately along* the $Y_G$-axis of the Galactic coordinate system.

### 2.1.2 Definitions of the Ecliptic and the Equatorial coordinate systems

As shown in Fig. 3(a), the Ecliptic and the Equatorial coordinate systems are defined as follows: their origins are located at the center of the Sun and that of the Earth, respectively, the common primary direction (the $X_S/X_{Eq}$-axis) is the direction pointing from the Solar center to that of the Earth at 12 midnight (the *end* of the date of the vernal equinox) the $Z_S$- and the $Z_{Eq}$-axes are perpendicular to the Ecliptic or the Equatorial plane, respectively, and their $Y_S$- and $Y_{Eq}$-axes are then defined as usual by the right–handed convention.

Additionally, in Fig. 3(b), we sketch the relative orientations between the Galactic, the Ecliptic and the Equatorial coordinate systems (on the date of the vernal equinox). Note that, the Ecliptic and the Equatorial coordinate systems only move, but do not rotate with the Sun nor with the Earth.
2.1.3 Definition of the Earth coordinate system

As shown in Fig. 4, we define the Earth coordinate system as follows: while the origin is also located at the Earth’s center and the $Z_E$–axis is still the Earth’s north polar axis, the primary direction (the $X_E$–axis) points now from the Earth’s center to the (yellow) Prime Meridian (the longitude $0^\circ$) at 12 midnight (i.e., the moment when the Prime Meridian passes the purple arrow pointing from the Solar center to that of the Earth, $r_{yr}$) of “each single” day. Finally, the fundamental ($X_E - Y_E$) plane is also the (blue) Earth’s Equatorial plane and the right–hand convention is used to define the $Y_E$–axis (see also Figs. 5 and 6). The (red) Ecliptic and the (blue) Equatorial coordinate systems as well as the (blue) Earth’s orbit around the Sun are also given here.

Note that, the Earth coordinate system is fixed but rotates with the Earth, during the Earth’s orbital motion around the Sun. Hence, in our simulations presented in this paper, we consider the position of the Earth coordinate system at 12 midnight (the beginning) of (the Coordinated Universal Time (UTC) of) each single “Solar” day. This means then that our Earth coordinate system changes daily and discretely (see Table 1 for the summary of the styles of the movement and the rotation of different coordinate systems).
2.1.4 Definition of the horizontal coordinate system

Moreover, for demonstrating the (annual/diurnal modulation of the) angular distribution of the 3-D WIMP velocity observed in a specified underground laboratory, we define first the horizontal coordinate system with the origin at the geographic location of the laboratory of interest at 12 midnight (the beginning) of (the UTC time of) each single day, the primary direction (the \( X_H \)-axis) and the \( Z_H \)-axis pointing towards north and the zenith, respectively, and the right-handed convention for defining the \( Y_H \)-axis. Our (light-green) Earth coordinate system is also given here and \( \phi \) and \( \theta \) indicate the longitude and the latitude of the location of the laboratory, respectively.

2.1.5 Definition of the laboratory coordinate system

Finally, considering the long running time of (directional) direct Dark Matter detection experiments as well as for identifying the diurnal modulation of the angular WIMP velocity distribution, we define, besides the discretely variated horizontal coordinate system, the laboratory coordinate system by taking into account the instantaneous measuring time of each recorded WIMP scattering event. This means that our laboratory coordinate system is defined basically the same as the horizontal coordinate system, but rotates with the laboratory of interest around the Earth’s north polar (\( Z_{Eq}/Z_E \)-axis) instantaneously by an angle of \( \omega t_{PM} \) (see Fig. 6), where

\[ \omega = \frac{2 \pi}{T_{Earth}} \]

Note that, in our simulations presented in this paper, this date has been fixed as the 79th day (May 20th) of a 365-day year.

Originally, the Earth coordinate system was defined for connecting the horizontal and the laboratory coordinate systems (defined in Secs. 2.1.4 and 2.1.5) with the Equatorial and the Ecliptic coordinate systems [63].
we define
\[ \omega \equiv \frac{2\pi}{1 \text{ day}}, \]  
and \( t_{PM} \) indicates the measuring UTC time of each recorded WIMP event in unit of day. Note that, our laboratory coordinate system changes (rotates around the Earth’s north polar (\( Z_{\text{Eq}}/Z_E \)-axis) event by event.

In Table 1 we summarize the styles of the movement and the rotation of all six celestial coordinate systems defined in our simulations presented in this paper. Remind that, except of the “linear + orbital \( \rightarrow \) spiral” motion of the Equatorial coordinate system, the Earth, the horizontal and the laboratory coordinates systems are fixed on the Earth and only rotate around the Earth’s north polar (i.e., \( Z_{\text{Eq}}/Z_E \)-axis). Hence, the radial component (magnitude) of the 3-D WIMP velocity distribution in these three coordinate systems are equal to that in the Equatorial coordinate system.

2.2 Event generation in the Galactic coordinate system

In this subsection, we describe the generation procedure of the 3-dimensional information on the WIMP velocity (the magnitude and the direction as well as the measuring time) in the Galactic coordinate system based on the Monte Carlo method. These generated 3-D WIMP velocities will be transformed into different celestial coordinate systems for data analyses and distribution reconstructions presented in Secs. 3, 4, and 5 as well as in Appendices B and C.
| Coordinate system | Movement | Rotation | Style |
|-------------------|----------|----------|-------|
| Galactic          | ×        | ×†       | —     |
| Ecliptic          | √        | ×        | Orbital → approximately linear |
| Equatorial        | √        | ×        | Linear + orbital → spiral |
| Earth             | ×‡       | √        | Daily and discrete |
| Horizontal        | ×‡       | √        | Daily and discrete |
| Laboratory        | ×‡       | √        | Instantaneous and continuous |

Table 1: The summary of the styles of the movement and the rotation of all six celestial coordinates systems defined in our simulations presented in this paper.
†: The tiny angle swept by the connection between the Solar and the Galactic centers during to the orbital motion of the Solar system in the Galaxy is ignored here.
‡: Fixed on the Earth and combined additionally with the “linear + orbital → spiral” movement of the Equatorial coordinate system.

### 2.2.1 Radial component of the 3-D WIMP velocity distribution

For generating the radial component (magnitude) of the 3-D WIMP velocity in the Galactic coordinate system, we consider the simple Maxwellian velocity distribution function truncated at the Galactic escape velocity [1]:

\[
f_{1,\text{Gau}}(v) = \begin{cases} 
N_{\text{Gau}} \left( \frac{v^2}{v_0^2} \right) e^{-v^2/v_0^2}, & \text{for } v < v_{\text{esc}}, \\
0, & \text{for } v > v_{\text{esc}}, 
\end{cases}
\]

with the normalization constant

\[
N_{\text{Gau}} = \left[ \left( \frac{\sqrt{\pi}}{4} \right) \text{erf} \left( \frac{v_{\text{esc}}}{2v_0} \right) - \left( \frac{v_{\text{esc}}}{2v_0} \right) e^{-v_{\text{esc}}^2/v_0^2} \right]^{-1},
\]

where \( v_0 \approx 220 \text{ km/s} \) is the Solar orbital speed around the Galactic center and \( 498 \text{ km/s} < v_{\text{esc}} < 608 \text{ km/s} \) is the escape velocity from our Galaxy at the position of the Solar system [64].

In Fig. 7 we show the radial distribution of the 3-dimensional WIMP velocity in the Galactic coordinate system generated by Eqs. (2) to (6). One entire year (0 to 365 day) for the measuring time of WIMP events has been considered. 50 total events on average in one experiment have been generated and binned into 8 bins. 5,000 experiments have been simulated. The solid red curve is the generating simple Maxwellian velocity distribution \( f_{1,\text{Gau}}(v) \) given in Eq. (2) with the Solar Galactic velocity \( v_0 = 220 \text{ km/s} \), while the dashed black histogram shows the generated WIMP velocities and the thin vertical dashed black lines indicate the 1σ Poisson statistical uncertainties on the recorded event numbers in the \( v- \)bins. The Galactic escape velocity has been set as \( v_{\text{esc}} = 550 \text{ km/s} \).

Not surprisingly, the recorded event numbers (averaged by all simulated experiments) in all \( v- \)bins match our generating simple Maxwellian velocity distribution perfectly.
2.2.2 Angular component of the 3-D WIMP velocity distribution

Since the simplest model of the Galactic Dark Matter halo is assumed to be isothermal, spherical and isotropic, the angular distribution of the 3-D WIMP velocity in the Galactic coordinate system has been considered to be isotropic and thus the velocity directions (i.e., the $\phi-$ and $\theta-$angles in the longitude and the latitude directions, respectively) are generated with a constant probability in our simulations:

$$f_{\phi, G}(\phi) = 1, \quad \phi \in (-\pi, \pi],$$

and

$$f_{\theta, G}(\theta) = 1, \quad \theta \in [-\pi/2, \pi/2].$$

In Fig. 8 we show the angular distribution of the 3-dimensional WIMP velocity in the Galactic coordinate system generated by Eqs. (2) to (6). 50 total events on average in one experiment have been binned into $6 \times 6$ bins in the longitude and the latitude directions, respectively. The horizontal color bar on the top of the plot indicates the mean value of the
Figure 8: The angular distribution of the 3-dimensional WIMP velocity in the Galactic coordinate system generated by Eqs. (2) to (6). 50 total events on average in one experiment (in one entire year) have been generated and binned into $6 \times 6$ bins in the longitude and the latitude directions, respectively. The horizontal color bar on the top of the plot indicates the mean value of the actual event number in each angular bin (averaged over all simulated experiments) in unit of the all–sky average value ($50 \text{ events} / 36 \text{ bins} \approx 1.39 \text{ events/bin}$ here). See the text for further details.

As expected, the event numbers in all angular bins are between 70% and 1.4 times of the all–sky average value (0.97 events/bin to 1.94 events/bin). More exactly, with $O(50)$ total WIMP events, the maximal perturbation of the angular distribution of the simulated isotropic WIMP halo would be less than $\pm 2.5\%$ of the all–sky average.

2.2.3 Measuring time and observation periods/shifts of WIMP events

Although due to the orbital motion of the Earth around the Sun and the Earth’s rotation around its axis, the theoretically predicted radial and angular WIMP velocity distributions as well as the event rate for WIMP–nucleus scattering should be time–dependent (the so–called “annual modulation”) [65], in the Galactic point of view, WIMP–nucleus scattering events should be observed randomly and constantly. Hence, in our simulations we considered a constant probability for generating the measuring time of the recorded WIMP scattering events:

$$f_t, G(t) = 1, \quad t \in [t_{\text{start}}, t_{\text{end}}].$$

For generating the WIMP events shown in Figs. 7 and 8, the observation period has been set as
Figure 9: Sketches of two options for the observation periods (lightened areas) considered in our simulations presented in this paper (see also Table 2). (a) Four normal seasons with a common 60-day period on the central dates of 79.0 day, 170.25 day, 261.50 day, and 352.75 day, respectively. (b) Four “advanced” seasons with a 60-day period on the central dates of 49.49 day, 140.74 day, 231.99 day, and 323.24 day, respectively. The golden arrow indicates the moving direction of the Solar system towards the CYGNUS constellation with the velocity of $v_0 \simeq 220$ km/s, and the four short dark–blue arrows (in front of the Earths) indicate the (average) orbital velocity of the Earth, $|v_{E,S}| \simeq 30$ km/s, on the central dates of four normal or four advanced seasons. While the small purple point at the bottom of each sketch indicates the location, where the Earth’s orbital speed is maximal on the date around June 2nd ($t_p = 152.5$ day) [65], two Earths without the velocity arrow indicate the locations, where the theoretical main direction of the WIMP wind (the opposite direction of the Solar movement) points straightly to the Prime Meridian in the night (207.66 day) or the day (25.16 day), corresponding (approximately) to the locations drawn in Figs. 1, 10, and 11. Other notations are the same as in Fig. 3(a). See the text as well as Appendices A.3.4 and A.3.6 for further details.

$[t_{\text{start}}, t_{\text{end}}] = [0, 365 \text{ day}]$.

**Annual modulation of the 3-D WIMP velocity**

As shown in Fig. 9(a), on the central dates of four normal seasons, the orbital velocity of the Earth’s rotation around the Sun in the Ecliptic coordinate system is along the $X_S$– or the $Y_S$–axis. This would thus be the most natural choice for demonstrating the annual modulation of the Earth’s velocity relative to the Dark Matter halo. However, as shown in Fig. 9(b) and discussed in detail in Appendix A.3.4, the relative velocity of the Earth to the DM halo should be the maximum (minimum), when its orbital velocity is (anti–)parallel to the projection of the direction of the Solar movement on the Ecliptic plane, namely, on the date around the 21st of May (140.74 day) (the 19th of November, 323.24 day). Hence, for demonstrating the annual modulation of the radial and angular distributions of the 3-D WIMP velocity, besides the 60-day ($\pm 30$ days) observation periods of four normal seasons on the central dates of 79 day, 170.25 day, 261.5 day, and 352.75 day, respectively, we considered also the periods of four “advanced” ($\sim 30$ days earlier) seasons on the central dates of 49.49 day, 140.74 day, 231.99 day, and 323.24 day, respectively. In Figs. 9 we sketch these two simulation options as the lightened areas.
Table 2: The list of four options for the observation periods in a 365-day year considered in our simulations presented in this paper. Note that, first, in each of the normal and advanced season, a period of 60 days (±30 days) has been set. This means that each pair of the corresponding season has around 30 days overlap (see the lightened and darkened areas in two sketches of Figs. 9). Second, the last option is considered only for demonstrating the diurnal modulation of the 3-D WIMP velocity (see Appendix A.3.6 for the detailed calculation).

Moreover, in Table 2 we list, including the normal and advanced seasons, four different options for the observation periods considered in our simulations presented in this paper. Note that, since in each of the normal and advanced season, a period of 60 days (±30 days) has been set, each pair of the corresponding season has around 30 days overlap (compare two sketches in

Figure 10: As in Figs. 1, except that the laboratory in the Northern Hemisphere (a) is now in Winter, while that in the Southern Hemisphere (b) is in Summer.
Figure 11: A sketch of two options for the observation periods (lightened areas) considered for demonstrating the diurnal modulation of the angular distribution of the 3-D WIMP velocity presented in this paper (see also Table 2): two 60-day (±30 days) observation periods on the central date of 207.66 day and 25.16 (= 390.16) day, respectively, at which the (light–green) WIMP wind points straightly to the (yellow) Prime Meridian in the night (207.66 day) or the day (25.16 day), corresponding (approximately) to the locations drawn in Figs. 1 and 10, respectively. Other notations are the same as in Figs. 9. Note that the angle of view in this sketch turns ∼63° counterclockwise from the sketches in Figs. 9. See the text and Appendix A.3.6 for further details.

Figs. 9).

Diurnal modulation of the 3-D WIMP velocity

Due to the very small unexcluded WIMP–nucleus cross section, nowadays it is required to run a direct Dark Matter detection experiment for a (very) long period, especially for directional low–mass gas detectors. Then, by comparing Figs. 1 with Figs. 10 and as shown in detail in Figs. 9 and 11, one can find that the original motivation of directional DM detection experiments — identifying the directionality of incident WIMPs — could be (strongly) reduced or even cancelled. For demonstrating however the original motivation of directional detection experiments, we considered four observation intervals of 4 hours (±2 hours) at the central times of 0 o’clock, 6 o’clock, 12 o’clock, and 18 o’clock, respectively (see Table 3), in the 60-day periods centered at the 207.66 day and 390.16 (= 25.16) day, respectively (see Table 2 and Fig. 11), at which the theoretical main direction of the WIMP wind points straightly to the Prime Meridian in the night (207.66 day) or the day (25.16 day)4, corresponding (approximately) to the locations drawn in Figs. 1 and 10, respectively (detailed calculation see Appendix A.3.6).

In Figs. 11 we sketch these two specified observation periods as the lightened areas (see also Table 2). And in Table 3 we list, two options for the observation intervals in a (Solar) day

4 Note that these pure theoretically estimated dates (under some simplified assumptions) have in fact an ∼8-hour difference from the midnight or the noon, respectively, which could nevertheless be neglected in the 60-day observation periods. See Appendices A.3.4 and A.3.6 for further details.
| Option              | Central time (hour) | Interval (hour) |
|---------------------|---------------------|-----------------|
| A whole day         | —                   | 0 – 24          |
| Four daily shifts   | 0                   | 0 – 2, 22 – 24  |
|                     | 6                   | 4 – 8           |
|                     | 12                  | 10 – 14         |
|                     | 18                  | 16 – 20         |

Table 3: The list of two options for the observation intervals in a (Solar) day considered for demonstrating the diurnal modulation of the angular distribution of the 3-D WIMP velocity presented in this paper. Note that, in each shift, an interval of 4 hours (±2 hours) has been set.

considered in our simulations presented in this paper.

2.3 Bayesian reconstruction of the radial distribution of the 3-D WIMP velocity

In this subsection, we describe briefly the Bayesian fitting procedure applied for reconstructing the radial component (magnitude) of the 3-D WIMP velocity distribution presented in Secs. 4 and 5.2 as well as in Appendix C.

2.3.1 Bayesian analysis

We start with the basic formula for Bayesian analysis \[66\]:

\[
p(\Theta|\text{data}) = \frac{p(\text{data}|\Theta)}{p(\text{data})} \cdot p(\Theta).
\]

Here \( \Theta = \{a_1, a_2, \cdots, a_{N_{\text{Bayesian}}} \} \) denotes a specified (combination of the) value(s) of the fitting parameter(s); \( p(\Theta) \), called the “prior probability”, represents our degree of belief about \( \Theta \) being the true value(s) of fitting parameter(s), which is often given in form of the (multiplication of the) probability distribution(s) of the fitting parameter(s). \( p(\text{data}) \), called the “evidence”, is the total probability of obtaining the particular set of data, which is in practice irrespective of the actual value(s) of the parameter(s) and can be treated as a normalization constant; it will not be of interest in our analysis presented in this paper. \( p(\text{data}|\Theta) \) denotes the probability of the observed result, once the specified (combination of the) value(s) of the fitting parameter(s) happens, which can usually be described by the likelihood function of \( \Theta \), \( L(\Theta) \). Finally, \( p(\Theta|\text{data}) \), called the “posterior probability density function” for \( \Theta \), represents the probability of that the specified (combination of the) value(s) of the fitting parameter(s) happens, given the observed result.

2.3.2 Bayesian reconstruction of \( f_r(v) \)

Below we describe the procedure of our Bayesian reconstruction of the radial distribution of the 3-D WIMP velocity in detail.
First, as shown in Fig. 7 and 25, the magnitudes of the (transformed) 3-D velocity of the recorded WIMP scattering events are binned into $B$ bins as

$$v_{n,\text{min}} \equiv v_n - \frac{b_v}{2} \leq v_{n,i} \leq v_{n,\text{max}} \equiv v_n + \frac{b_v}{2}, \quad i = 1, 2, \cdots, N_n, \quad n = 1, 2, \cdots, B. \quad (8)$$

Here the entire velocity range below the maximal cut–off ($v_{\text{esc}}$ in the Galactic coordinate system and $v_{\text{max}}$ in the Equatorial coordinate system) has been divided into $B$ bins with central points $v_n$ and a common width $b_v$. In the $n$th bin, $N_n$ events are recorded and

$$N_{\text{tot}} = \sum_{n=1}^{B} N_n \quad (9)$$

is the number of total WIMP events in the dataset to be analyzed. This means that in the $n$th $v$–bin $[v_{n,\text{min}}, v_{n,\text{max}}]$, the normalized event number is

$$f_{r,\text{expt}}(v_n) = \frac{1}{N_{\text{tot}}} \left( \frac{N_n}{b_v} \right). \quad (10)$$

Choosing a theoretical prediction of the one–dimensional WIMP velocity distribution:

$$f_{r,\text{th}}(v_n; a_1, a_2, \cdots, a_{N_{\text{Bayesian}}}),$$

where $(a_1, a_2, \cdots, a_{N_{\text{Bayesian}}})$ are the $N_{\text{Bayesian}}$ fitting parameters, and since the recorded event number in each $v$–bin should be Poisson–distributed around the theoretical predictions $f_{r,\text{th}}(v_n; a_1, a_2, \cdots, a_{N_{\text{Bayesian}}})$, the likelihood function for $p(\text{data}|\Theta)$ can be defined by

$$L\left(f_{r,\text{expt}}(v_n), \quad n = 1, 2, \cdots, B; \quad a_i, \quad i = 1, 2, \cdots, N_{\text{Bayesian}}\right) \equiv \prod_{n=1}^{B} \text{Poi}\left(v_n; f_{r,\text{expt}}(v_n); a_1, a_2, \cdots, a_{N_{\text{Bayesian}}} \right), \quad (11)$$

where

$$\text{Poi}\left(v_n, f_{r,\text{expt}}(v_n); a_1, a_2, \cdots, a_{N_{\text{Bayesian}}} \right) = \frac{f_{r,\text{exp}}(v_n; a_1, a_2, \cdots, a_{N_{\text{Bayesian}}})}{\Gamma \left( f_{r,\text{expt}}(v_n) + 1 \right)} e^{-f_{r,\text{th}}(v_n; a_1, a_2, \cdots, a_{N_{\text{Bayesian}}})}, \quad (12)$$

and

$$\Gamma(x) = \int_{0}^{\infty} t^{x-1} e^{-t} dt \quad (13)$$

is the gamma function. Then the posterior probability density on the left–hand side of Eq. (7) can be given as

$$p\left(a_i, \quad i = 1, 2, \cdots, N_{\text{Bayesian}} \quad | \quad f_{r,\text{expt}}(v_n), \quad n = 1, 2, \cdots, B\right) \propto L\left(f_{r,\text{expt}}(v_n), \quad n = 1, 2, \cdots, B; \quad a_i, \quad i = 1, 2, \cdots, N_{\text{Bayesian}}\right) \prod_{i=1}^{N_{\text{Bayesian}}} p_i(a_i). \quad (14)$$

Regarding our degree of belief about each fitting parameter $a_i$, i.e. $p_i(a_i)$ in Eq. (14), two probability distribution functions have been considered. The simplest one is the flat–distribution:

$$p_i(a_i) = 1, \quad \text{for} \quad a_{i,\text{min}} \leq a_i \leq a_{i,\text{max}}, \quad (15)$$
where \( a_{i,(\text{min, max})} \) denote the minimal and maximal bounds of the scanning interval of the fitting parameter \( a_i \). On the other hand, for the case that we have already prior knowledge about one fitting parameter, a Gaussian-distribution:

\[
p_i(a_i; \mu_{a,i}, \sigma_{a,i}) = \frac{1}{\sqrt{2\pi} \sigma_{a,i}} e^{-(a_i - \mu_{a,i})^2 / 2\sigma_{a,i}^2},
\]

with the expectation value \( \mu_{a,i} \) of and the 1\( \sigma \) uncertainty \( \sigma_{a,i} \) on the fitting parameter \( a_i \) is used. Note that, in one simulated experiment, we scan the parameter space \((a_1, a_2, \ldots, a_{N_{\text{Bayesian}}})\) in the volume \( a_i \in [a_{i,\text{min}}, a_{i,\text{max}}], i = 1, 2, \ldots, N_{\text{Bayesian}}\), to find a particular point \((a_1^*, a_2^*, \ldots, a_{N_{\text{Bayesian}}}^*)\), which maximizes the (numerator of the) posterior probability density:

\[
p(a_i, i = 1, 2, \ldots, N_{\text{Bayesian}} | f_{r,\text{exp}}(v_n), n = 1, 2, \ldots, B)\].

And after that all simulations have been done, we determine the mean value of the event number in each \( v \)-bin from all simulated experiments, denoted as \( N_{i,\text{mean}} \), and define then

\[
f_{r,\text{mean}}(v_n) = \frac{1}{N_{\text{tot,ave}}} \left( \frac{N_{n,\text{mean}}}{b_v} \right),
\]

where \( N_{\text{tot,ave}} \) is the expectation value of the event number in one simulated experiment\(^5\). Hence, we can define further

\[
P_{\text{mean}}(a_i, i = 1, 2, \ldots, N_{\text{Bayesian}}) = \equiv p\left(a_i, i = 1, 2, \ldots, N_{\text{Bayesian}} | f_{r,\text{mean}}(v_n), n = 1, 2, \ldots, B\right),
\]

and scan the points \((a_{1,i}, a_{2,i}, \ldots, a_{N_{\text{Bayesian}},i})\) obtained from all simulated experiments one-by-one to find the "best-fit" point \((a_{1,\text{Pmax}}, a_{2,\text{Pmax}}, \ldots, a_{N_{\text{Bayesian}},\text{Pmax}})\), which maximizes \( P_{\text{mean}}(a_i, i = 1, 2, \ldots, N_{\text{Bayesian}})\).

### 2.3.3 Fitting velocity distribution \( f_{r,\text{th}}(v) \)

By taking into account the orbital motion of the Solar system around our Galaxy as well as that of the Earth around the Sun, the shifted Maxwellian velocity distribution has been derived in detail by Lewin and Smith [67]:

\[
f_{i,\text{sh,vesc}}(v) = \begin{cases} 
N_{i,\text{sh,vesc}} \left( \frac{v}{v_0 v_{\text{e}}} \right) \left[ e^{-(v-v_{\text{e}})^2/2v_0^2} - e^{-(v+v_{\text{e}})^2/2v_0^2} \right], & \text{for } v \leq v_{\text{esc}} - v_e, \\
N_{i,\text{sh,vesc}} \left( \frac{v}{v_0 v_{\text{e}}} \right) \left[ e^{-(v-v_{\text{e}})^2/2v_0^2} - e^{-v_{\text{esc}}^2/v_0^2} \right], & \text{for } v_{\text{esc}} - v_e \leq v \leq v_{\text{esc}} + v_e, \\
0, & \text{for } v \geq v_{\text{esc}} + v_e \equiv v_{\text{max}},
\end{cases}
\]

with the normalization constant

\[
N_{i,\text{sh,vesc}} = \left[ \sqrt{\pi} \, \text{erf} \left( \frac{v_{\text{esc}}}{v_0} \right) - \left( \frac{2v_{\text{esc}}}{v_0} \right) e^{-v_{\text{esc}}^2/v_0^2} \right]^{-1}.
\]

Here \( v_e \) is the time-dependent Earth’s speed in the Galactic frame, which has been given in Ref. [65] by

\[
v_e(t) = v_0 \left[ 1.05 + 0.07 \cos \left( \frac{2\pi(t - t_p)}{1 \, \text{yr}} \right) \right],
\]

\(^5\) Note that in our numerical simulations presented in this paper, the total number of the generated WIMP events in each experiment, \( N_{\text{tot}} \), is Poisson-distributed around the expectation value \( N_{\text{tot,ave}} \).
with \( t_p \simeq \) June 2nd (the purple points sketched in Figs. 9), and, since the Galactic escape velocity has been set as \( v_{\text{esc}} = 550 \) km/s in our simulations, the maximal cut–off on the radial distribution of the 3-D WIMP velocity in the Equatorial coordinate system is given as \( v_{\text{max}} = 781 \) km/s. Meanwhile, a simplified expression for the shifted Maxwellian velocity distribution \( f_{1, \text{sh}, v_{\text{esc}}}(v) \) given in Eq. (19) is often adopted in the literature [1]:

\[
f_{1, \text{sh}}(v) = \begin{cases} 
N_{\text{sh}} \left( \frac{v}{v_0 v_e} \right) \left[ e^{-(v-v_e)/\nu_0^2} - e^{-(v+v_e)/\nu_0^2} \right], & \text{for } v \leq v_{\text{max}}, \\
0, & \text{for } v > v_{\text{max}},
\end{cases}
\]

with the normalization constant

\[
N_{\text{sh}} = \left\{ \left( \frac{\sqrt{\pi}}{2} \right) \left[ \text{erf}\left( \frac{v_{\text{max}} + v_e}{v_0} \right) + \text{erf}\left( \frac{v_{\text{max}} - v_e}{v_0} \right) \right] \right.^{-1}
+ \left( \frac{v_0}{2v_e} \right) \left[ e^{-\left((v_{\text{max}} + v_e)/\nu_0^2\right)} - e^{-\left((v_{\text{max}} - v_e)/\nu_0^2\right)} \right] \right.^{-1}.
\]

In Appendix D, we will show that, with the Galactic escape velocity of \( v_{\text{esc}} = 500 \) km/s, the shape difference between the exact analytic expression \( f_{1, \text{sh}, v_{\text{esc}}}(v) \) given in Eq. (19) and the simplified \( f_{1, \text{sh}}(v) \) in Eq. (22) is pretty tiny and negligible compared to the annual variation of these expressions as well as the much larger statistical uncertainties on the recorded WIMP events (shown in e.g., Fig. 25); this tiny shape difference could even vanish once the Galactic escape velocity would be as large as \( v_{\text{esc}} = 600 \) km/s. Hence, for our Bayesian reconstruction of the radial distribution of the 3-D WIMP velocity presented in this paper, we took the simplified expression (22) and considered the following four fitting distribution functions.

The first one is the “one–parameter” shifted Maxwellian velocity distribution with the Solar Galactic velocity \( v_0 \) as the unique fitting parameter [65]:

\[
f_{1, \text{sh}, v_0}(v; v_0) = N_{\text{sh}}(v_0) \left( \frac{v}{v_0 v_e} \right) \left[ e^{-(v-v_e)/\nu_0^2} - e^{-(v+v_e)/\nu_0^2} \right],
\]

and the constraint on \( v_e \):

\[
v_e = 1.05 \, v_0.
\]

Considering the (annual) variation of the Earth’s Galactic velocity in different observation periods, we introduce a “\( v_0 \)–fixed” shifted Maxwellian velocity distribution with now the Earth’s Galactic velocity \( v_e \) as the fitting parameter:

\[
f_{1, \text{sh}, v_e}(v; v_e) = N_{\text{sh}}(v_e) \left( \frac{v}{v_0 v_e} \right) \left[ e^{-(v-v_e)/\nu_0^2} - e^{-(v+v_e)/\nu_0^2} \right],
\]

and the constraint of \( v_0 = 220 \) km/s. Moreover, in order to obtain reconstruction results which can match the recorded radial WIMP velocity distribution as well as possible, the simplified shifted Maxwellian velocity distribution with \( v_0 \) and \( v_e \) as two independent fitting parameters should certainly be considered:

\[
f_{1, \text{sh}}(v; v_0, v_e) = N_{\text{sh}}(v_0, v_e) \left( \frac{v}{v_0 v_e} \right) \left[ e^{-(v-v_e)/\nu_0^2} - e^{-(v+v_e)/\nu_0^2} \right].
\]

\[\text{Although in Appendix A.3.4 we will show that the first–order term of the time–dependence between } v_0 \text{ and } v_e \text{ would only be 1.0078, we take still the commonly used value of 1.05 for our reconstructions presented in this paper.}\]
And, finally, as the auxiliary fitting function for confirming and/or improving our reconstruction results, the “modified” shifted Maxwellian velocity distribution with $v_0$ and $\Delta v \equiv v_e - v_0$ as two independent fitting parameters introduced in our earlier work \[68\] has also be used:

$$f_{1,sh,\Delta v}(v; v_0, \Delta v) = N_{sh}(v_0, \Delta v) \left[ \frac{v}{v_0(v_0 + \Delta v)} \right] \left\{ e^{-\frac{(v-(v_0+\Delta v))^2}{v_0^2}} - e^{-\frac{(v+(v_0+\Delta v))^2}{v_0^2}} \right\} . \quad (28)$$

### 3 Angular distributions of the 3-D WIMP velocity in different coordinate systems

In this section, we present the angular distributions of the (transformed) 3-D WIMP velocity in three “laboratory–independent” (Ecliptic, Equatorial, and Earth) coordinate systems and then in two “laboratory–dependent” (horizontal and laboratory) coordinate systems one by one. As readers’ reference, a summary of the angular distributions observed in the horizontal and laboratory coordinate systems for different underground laboratories will be given in Appendix B.

#### 3.1 Angular WIMP velocity distribution in the Ecliptic frame

In this subsection, we present at first the angular distribution of the 3-D WIMP velocity in the Ecliptic coordinate system. Remind that the Ecliptic coordinate system only moves (approximately) linearly with the Solar Galactic orbital velocity $v_0 \simeq 220 \text{ km/s}$ and its tiny rotation is imperceptible.

In Fig. 12, we show the angular distribution of the 3-D WIMP velocity transformed from events shown in Fig. 8 to the Ecliptic coordinate system. One entire year (0 to 365 day) and 50 total events on average in one experiment have been considered. It can be found that, while in the most part of the northern hemisphere and a small part of the southern hemisphere, the numbers of WIMP events would be less than 35% (0.49 events/bin) of the all–sky average value (1.39 events/bin), the event numbers in the bins from the center (i.e., the direction of the $X_S$–axis) to southwest would be larger than 1.4 times (1.94 events/bin) of the all–sky average value; in particular, in the dark–red and dark–purple bins in the southern hemisphere, the numbers of WIMP events are at least 4 times (5.56 events/bin) or even larger than 5.6 times (7.78 events/bin) of the all–sky average value, respectively. This means that the average event numbers from the center to the southwest part could be at least 4 times or even 16 times larger than the rest part of the sky and would hence be a clear confirmation of the anisotropy of the main direction of the WIMP wind.

However, comparing with the dark–green star on the map, which indicates the theoretical main direction of incident WIMPs (the opposite direction of the Solar Galactic movement) in the Ecliptic coordinate system: 57.40°S, 29.10°W\[7\] the bin with the most WIMP events would not match the theoretically predicted direction of the WIMP wind and have a 30° to 40° northward deviation. In Sec. 5.1.1 we will see that, with $O(500)$ total WIMP events and a higher analysis resolution, this systematic bias could be strongly reduced, but a small amount of deviation would still exist.

\[7\] See Appendix A.3.3 for detailed calculations.
3.2 Angular WIMP velocity distribution in the Equatorial frame

In this subsection, we present the angular distribution of the 3-D WIMP velocity in the Equatorial coordinate system. Remind that the Equatorial coordinate system moves orbitally around (and also linearly with) the Sun, but doesn’t rotate. Thus its axes are fixed.

In Fig. 13, we show the angular distribution of the 3-D WIMP velocity transformed from events shown in Figs. 8 and 12 to the Equatorial coordinate system. One entire year (0 to 365 day) and 50 total events on average in one experiment have been considered. As in Fig. 12, it can also be seen clearly here that, while in the most part of the northern and the eastern hemispheres, the numbers of WIMP events would be less than 35% (0.49 events/bin) of the all-sky average value (1.39 events/bin), the event numbers in the southwest part of the sky would be at least 2 times (2.78 events/bin) or even more than 5.6 times (7.78 events/bin) of the all-sky average value, respectively. This means that the average event numbers from the center (i.e., the direction of the $X_{\text{Eq}}$-axis) to the southwest part could be at least 5.7 times or even 16 times larger than the rest part of the sky and would hence also be a clear confirmation of the anisotropy of the main direction of the WIMP wind.

In Fig. 13, we also put a dark–green star to indicate the theoretical main direction of incident WIMPs in the Equatorial coordinate system \cite{63}: 42.00°S, 50.70°W. Now the deviation between the bin with the most WIMP events and the theoretically predicted direction of the WIMP wind would only be $\sim 10°$ northwestward, but they can still not match each other. In Sec. 5.1.2
Figure 13: The angular distribution of the 3-D WIMP velocity transformed from events shown in Fig. 12 to the Equatorial coordinate system. The dark–green star indicates now the theoretical main direction of incident WIMPs in the Equatorial coordinate system [63]: 42.00°S, 50.70°W. All simulation setup and other notations are the same as in Fig. 12.

we will show that, with \( \mathcal{O}(500) \) total WIMP events and a higher analysis resolution, a small deviation might still exist.

### 3.2.1 Annual modulation of the angular velocity distribution in the Equatorial frame

Moreover, in order to demonstrate the annual modulation of the angular distribution pattern of the 3-D WIMP velocity, we show in Figs. 14 and 15 the angular distribution in the Equatorial coordinate system in four observation periods of the normal and the advanced seasons listed in Table 2 and sketched in Figs. 9, respectively. Note that 50 total events on average in each 60-day observation period have been simulated. This means although that \( \sim 300 \) total events in one year would be required, considering the laboratory–independence of the Equatorial coordinate system, we could in fact collect the WIMP events observed in several different underground laboratories for such an analysis.

In each plot of Figs. 14 and 15 besides the dark–green star indicating the theoretical main direction of incident WIMPs, we also put a blue–yellow point to indicate the opposite direction of the Earth’s relative velocity to the Dark Matter halo on the central date of the observation period.\(^8\) By comparing the plots in four normal and four advanced seasons carefully, one could find that the event numbers indeed variate (become more and then fewer) slightly and this

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\(^8\) See Appendix A.3.4 for detailed calculations.
Angular distribution of the WIMP velocity in the Equatorial frame, 49.0 – 109.0 day, 50 events

(a) 49.0 – 109.0 day

Angular distribution of the WIMP velocity in the Equatorial frame, 140.25 – 200.25 day, 50 events

(b) 140.25 – 200.25 day

Angular distribution of the WIMP velocity in the Equatorial frame, 231.5 – 291.5 day, 50 events

(c) 231.50 – 291.50 day

Angular distribution of the WIMP velocity in the Equatorial frame, 322.75 – 382.75 day, 50 events

(d) 322.75 – 382.75 day

Figure 14: The angular distribution of the 3-D WIMP velocity in the Equatorial coordinate system. Four observation periods of the normal seasons listed in Table 2 and sketched in Fig. 9(a) as well as 50 total events on average in each 60-day observation period have been considered. Besides the dark–green star indicating the theoretical main direction of incident WIMPs, the blue–yellow point in each plot indicates the opposite direction of the Earth’s relative velocity to the Dark Matter halo on the central date of the observation period (listed in Table A2). All other simulation setup and notations are the same as in Fig. 13.

3.3 Angular WIMP velocity distribution in the Earth frame

In this subsection, we present the angular distribution of the 3-D WIMP velocity in the Earth coordinate system. Remind that the Earth coordinate system not only moves orbitally around (and also linearly with) the Sun, but also rotates daily and discretely.

In Fig. 16 we show the angular distribution of the 3-D WIMP velocity transformed from events shown in Figs. 8, 12, and 13 to the Earth coordinate system. One entire year (0 to 365
Figure 15: As in Figs. 14, except that four observation periods of the advanced seasons listed in Table 2 and sketched in Fig. 9(b) have been considered.

day) with 50 total events on average have been considered. It could be seen that, first, due to the Earth’s orbital motion around the Sun and thus the Earth coordinate system rotates daily, the (anisotropy of the) angular distribution of the 3-D WIMP velocity spreads out latitudinally. However, one could still find that the event numbers are at fewest (less than 35% of the all–sky average value, < 0.49 events/bin among O(50) total events) in the bins more northern than 60°N, and at least 2 times of the all–sky average (> 2.78 events/bin) in the southwest part of the sky.

3.3.1 Annual modulation of the angular velocity distribution in the Earth frame

In Figs. 17 and 18, we show the angular distribution of the 3-D WIMP velocity in the Earth coordinate system in four observation periods of the normal and the advanced seasons, respectively. Remind that, as in Figs. 14 and 15, 50 total events on average in each 60-day observation period have been simulated.

By comparing with Figs. 14 and 15, one can see clearly the similarity of the angular distribution patterns of the 3-D WIMP velocity observed in each 60-day observation period. The reason is understandable: since by definition the Earth coordinate system rotates daily with a
one–year period, the directions of the $X_E$–axis on the central dates of the observation period in four plots shown in Figs. 17 and 18 rotates 90° eastwards on the celestial Equator. Hence, four distribution patterns in the normal and the advanced seasons rotates 90° westwards around the $Z_E$–axis. Due to the same reason, the distribution patterns in each pair of the normal and the advanced seasons would also rotate ~ 30° westwards. In general, the angular distribution patterns in the Earth coordinate system should approximately be those in the Equatorial coordinate system combined with a westward rotation of a one–year period. This periodic variation of the anisotropic angular distribution of the 3-D WIMP velocity could be observed more clearly in our simulations with $O(500)$ total WIMP events and a higher analysis resolution given in Sec. 5.1.3.

3.4 Angular WIMP velocity distribution in the horizontal frame

In this subsection, we present the angular distribution of the 3-D WIMP velocity in the laboratory–dependent horizontal coordinate system. Remind that, as the Earth coordinate system, the horizontal coordinate system not only moves orbitally around (and also linearly with) the Sun, but also rotates daily and discretely.
Figure 17: The angular distribution of the 3-D WIMP velocity in the Earth coordinate system. Four observation periods of the normal seasons as well as 50 total events on average in each 60-day observation period have been considered. All other simulation setup and notations are the same as in Fig. 16.

### 3.4.1 Annual modulation of the angular velocity distribution in the horizontal frame

In Figs. [19](#), we show the angular distribution of the 3-D WIMP velocity transformed from events shown in Figs. [16](#) and [18](#) to the horizontal coordinate system at the location of the LNGS laboratory (42.45°N, 13.58°E) as a demonstration for an underground laboratory located in the Northern Hemisphere. 50 total events on average in one experiment in one entire year (a) and in each of four observation periods of the advanced seasons (b – e) have been considered.

From the sketch of an underground laboratory located in the Northern Hemisphere in Winter in Fig. [10](#)(a) combined with the sketches of the Earth’s positions in Figs. [9](#) and [11](#) one can find that, first, during the observation period of the advanced Spring (Fig. [19](#)(b)), the main direction of incident WIMPs should be approximately towards the $-X_{H, LNGS}$-axis. Thus the highest–WIMP–flux bins is between 30°N and 30°S, from 150°E to 180°. Note here that there is a difference of $\sim 24$ days between the central date of the advanced Spring (49.49 day) and the date sketched in Fig. [10](#)(a) (25.16 day). This corresponds to a shift of the angular distribution
Figure 18: As in Figs. 17, except that four observation periods of the advanced seasons have been considered.

pattern of $\sim 24^\circ$ westwards. Additionally, by definition of the horizontal coordinate system, the geographical difference between the LNGS laboratory and the Prime Meridian enlarge the shift of the angular distribution pattern by $13.58^\circ$. Totally, the overall shift of the angular distribution of the LNGS laboratory in the advanced Spring is $\sim 38^\circ$ westwards.

Similar reason can explain the angular distribution of the advanced Winter (Fig. 19(e)). A difference of $\sim 67$ days between the central date of the advanced Winter (323.24 day) and the date sketched in Fig. 10(a) (390.16 day) corresponds to a shift of the angular distribution pattern of $\sim 66^\circ$ eastwards now. In addition, in this observation period, more events should come from the direction above the horizon and result in the angular distribution with slightly higher event numbers in the southern hemisphere.

On the other hand, from the sketch of a laboratory in the Northern Hemisphere in Summer (Fig. 1(a)) combined with the sketches in Figs. 9(b) and 11, one can also conclude that, from the (advanced) Summer to the (advanced) Autumn, most WIMPs should come from the zenith and the most part in the northern hemisphere in Figs. 19(c) and (d) would hence have the lowest event numbers.

Finally, the angular distribution shown in Fig. 19(a) in one entire year is approximately a
Figure 19: The angular distribution of the 3-D WIMP velocity in the horizontal coordinate system at the location of the LNGS laboratory (42.45°N, 13.58°E). 50 total events on average in one entire year (a) and in each of four advanced seasons (b–e) have been considered. All simulation setup and notations are the same as in Figs. 16 or 18, respectively.
Figure 20: As in Figs. 19, except that the SUPL laboratory (37.07°S, 142.81°E), as the so far unique functionable underground laboratory in the Southern Hemisphere, has been considered.
combination of the plots of four advanced seasons. Hence, as the angular distribution shown in Fig. 16 due to the Earth’s orbital motion around the Sun, the anisotropy of the main direction of the WIMP wind would be averaged out, except of the lowest–WIMP–flux bins from the center (i.e., the north at the laboratory location) to the north pole (the zenith of the laboratory). From the sketches in Figs. 1(a) and 10(a) combined with the sketches in Figs. 9(b) and 11 one can find that these bins correspond exactly to the sky around the Earth’s North Pole (see Fig. 16).

Moreover, in order to demonstrate the location–dependence of the (annual modulation of the) angular distribution pattern, in Figs. A67 we show also the angular distribution of the 3-D WIMP velocity in the horizontal coordinate system at the location of the SUPL laboratory (37.07°S, 142.81°E), which is so far the unique functionable underground laboratory in the Southern Hemisphere.

From Figs. 1(b) and 10(b) and the detailed discussions above, one would expect that, from the (advanced) Spring to the (advanced) Summer (Figs. A67(b) and (c)), the main direction of incident WIMPs should be towards the \(-X_{\text{H,SUPL}}\), SUPL axis. This implies that the high–WIMP–flux bins should be close to the 180° longitude. Moreover, from the (advanced) Autumn to the (advanced) Winter, most WIMPs should come from the ground and thus the most part in the southern hemisphere in Figs. A67(d) and (e) have the lowest event numbers. On the other hand, in contrast to Fig. 19(a), the angular distribution in one entire year in Fig. A67(a) shows the pattern with the lowest–WIMP–flux bins from the center to the south pole.

### 3.5 Angular WIMP velocity distribution in the laboratory frame

In this subsection, we present the angular distribution of the 3-D WIMP velocity in the laboratory coordinate system. Remind that the laboratory coordinate system not only moves orbitally around (and also linearly with) the Sun, but also rotates continuously. Thus the angular distribution in the laboratory coordinate system would also be averaged out, when a long observation period for accumulating enough WIMP events is required. Hence, in this subsection we only discuss the diurnal modulation of the angular distribution patterns. The angular distributions in one entire year and in four advanced seasons at the locations of several underground laboratories will be summarized in Appendix B as readers’ reference.

#### 3.5.1 Diurnal modulation of the angular velocity distribution in the laboratory frame

In Figs. 21 and 22, we show the angular distribution of the 3-D WIMP velocity observed at the location of the LNGS laboratory (42.45°N, 13.58°E) in four daily shifts listed in Table 3 in the observation period of 177.66 – 237.66 day and 360.16 – 420.16 (= 55.16) day, respectively. As usual, 50 total events on average in one experiment in each 4-hour daily shift in the 60-day observation period have been considered.

Not surprisingly, by comparing these two figures with each other, we can find first that, in two observation periods with a half–year time difference, the angular distribution patterns show indeed a 12-hour shift. Moreover, the diurnal modulations (from the morning and noon shifts to the evening and midnight shifts) in both of two periods look also similar to the annual modulation shown in Figs. 19.

As in Sec. 3.4, in order to demonstrate the location–dependence of the diurnal modulation of the angular distribution of the 3-D WIMP velocity, in Figs. 23 and 24 we show also the angular distribution patterns observed at the location of the SUPL laboratory (37.07°S, 142.81°E) in four daily shifts in two observation periods. As expected, by comparing these two figures with each
Figure 21: The angular distribution of the 3-D WIMP velocity observed at the location of the LNGS laboratory (42.45°N, 13.58°E) in four daily shifts listed in Table 3 in the observation period of 177.66 – 237.66 day. 50 total events on average in each 4-hour daily shift in the 60-day observation period have been considered. All other simulation setup and notations are the same as in Figs. 19.

In Appendix B, one can find more simulation results and discussions for other underground laboratories. Note only that, although the diurnal modulation in the laboratory coordinate system (or, in fact, in any laboratory’s own reference frame) can be pretty clearly observed, a few tens of WIMP events accumulated in each a–few–hour daily shift in one observation period of a few tens of days would be required. This means in turn that a couple of thousands of WIMP events would be needed to observe in a few years. A very hard challenge for (directional) direct Dark Matter detection experiments in the near future.
Figure 22: As in Figs. 21, except that the observation period is now 360.16 – 420.16 (= 55.16) day.

4 Bayesian reconstruction of the radial distribution of the 3-D WIMP velocity

In this section, we turn to analyze the radial distribution (magnitude) of the (transformed) 3-D WIMP velocity in the Equatorial coordinate system. We discuss at first the radial distribution of the 3-D WIMP velocity transformed from events shown in Fig. 7 to the Equatorial coordinate system. Then we present the reconstruction results of the (annual modulation of the) radial WIMP velocity distribution by using the Bayesian fitting procedure described in Sec. 2.3.

4.1 Radial WIMP velocity distribution in the Equatorial frame

In Fig. 25 we show the radial distribution of the 3-D WIMP velocity transformed from events shown in Fig. 7 to the Equatorial coordinate system. The solid red curve is the shifted Maxwellian
Angular distribution of the WIMP velocity in the laboratory frame, 177.66 - 237.66 day, 50 events, at SUPL AMIDAS-2D

Figure 23: As in Figs. 21, except that the SUPL laboratory (37.07°S, 142.81°E), as the so far unique functionable underground laboratory in the Southern Hemisphere, has been considered.

velocity distribution $f_{1,sh,vesc}(v)$ given in Eq. (19) with an input value of $v_0 = 220$ km/s, while the dashed black histogram shows the binned radial component of the transformed 3-D WIMP velocities and the thin vertical dashed black lines indicate the $1\sigma$ Poisson statistical uncertainties on the recorded event numbers in the $v$–bins. One entire year (0 to 365 day) and 50 total events on average in one experiment have been considered. Remind that the maximal cut–off velocity here is given by $v_{\max} = 781$ km/s.

It can be found that, firstly, although the theoretical (solid red) curve falls inside the $1\sigma$ statistical uncertainty range of the radial distribution of the 3-D WIMP velocity, the theoretical distribution seems to be slightly higher than the histogram in the velocity ranges of $100$ km/s $< v < \sim 200$ km/s and $400$ km/s $< v < \sim 700$ km/s, while in the velocity range around the peak (close to the average value) of the theoretical distribution ($200$ km/s $< v < \sim 400$ km/s), the radial distribution of the simulated WIMP velocity are obviously higher than the theoretical prediction. This indicates that there would be more WIMPs with middle–value velocities, but fewer WIMPs with relatively lower or higher velocities than theoretically predicted. In Sec. 5.2 we will see that, with $O(500)$ total WIMP events and a higher analysis resolution, the difference between the simulated radial WIMP velocity distribution and the theoretical prediction would
become more obviously.

4.2 Fitted radial WIMP velocity distribution in the Equatorial frame

Since we found unexpectedly that the simulated radial WIMP velocity distribution in the Equatorial coordinate system could not match the theoretical prediction well, we considered then to apply the Bayesian fitting technique to reconstruct the radial component of the transformed 3-D WIMP velocity shown in Fig. 25, in order to understand the best–fit radial WIMP velocity distribution as well as to obtain the best–fit values of the Solar and Earth’s Galactic velocities, \( v_0 \) and \( v_e \). In this and the next subsections, we discuss our reconstruction results by using the Bayesian fitting procedure with four fitting velocity distributions given in Sec. 2.3.

4.2.1 With the one–parameter velocity distribution \( f_{1,\text{sh},v_0}(v; v_0) \)

We consider at first the one–parameter shifted Maxwellian velocity distribution \( f_{1,\text{sh},v_0}(v; v_0) \) given by Eq. (24) with the constraint that \( v_e = 1.05 v_0 \). The fitting parameter \( v_0 \) has been scanned in the range of \( 160 \text{ km/s} \leq v_0 \leq 270 \text{ km/s} \).
Figure 25: The radial distribution of the 3-D WIMP velocity transformed from events shown in Fig. 7 to the Equatorial coordinate system. The solid red curve is the shifted Maxwellian velocity distribution $f_{1, \text{sh}, \text{vesc}}(v)$ given in Eq. (19) with an input value of $v_0 = 220 \text{ km/s}$, while the dashed black histogram shows the binned radial component of the transformed 3-D WIMP velocities and the thin vertical dashed black lines indicate the 1$\sigma$ Poisson statistical uncertainties on the recorded event numbers in the $v$–bins. One entire year (0 to 365 day) and 50 total events on average have been considered. Remind that the maximal cut–off velocity here is given by $v_{\text{max}} = 781 \text{ km/s}$.

In Fig. 26(a), we show the reconstructed radial distributions of the 3-D WIMP velocity and the 1(2)$\sigma$ statistical uncertainty bands by using $f_{1, \text{sh}, v_0}(v; v_0)$ to fit data shown in Fig. 25. The solid red curve is the shifted Maxwellian velocity distribution $f_{1, \text{sh}, \text{vesc}}(v)$ given by Eq. (19) with an input value of $v_0 = 220 \text{ km/s}$. While the dashed green curve labeled with the subscript “median” indicates the reconstructed velocity distribution with the fitting parameter $v_0$ given by the median value of all simulated experiments, the dash–dotted blue curve labeled with the subscript “$P_{\text{max}}$” indicates the reconstructed velocity distribution with $v_0$ maximizing $P_{\text{mean}}(a_i, i = 1, 2, \cdots, N_{\text{Bayesian}})$.

Additionally, the light–green (light–blue) area shown here indicate the 1(2)$\sigma$ statistical uncertainty bands of the Bayesian reconstructed velocity distribution, which has been determined as follows. After scanning the reconstructed fitting parameter $v_0$ obtained from all simulated experiments and ordering according to their $P_{\text{mean}}$ values defined in Eq. (18) descendingly, we can not only determine the point which maximizes $P_{\text{mean}}$ (labeled with the subscript “$P_{\text{max}}$” in our plots hereafter), but also the smallest and largest values of the first 68.27% (95.45%) of all reconstructed $v_0$’s. We then use the smallest (largest) value of the first 68.27% (95.45%) reconstructed $v_0$’s to give the 1(2)$\sigma$ lower (upper) boundaries of the Bayesian reconstructed velocity.
Figure 26: (a) The reconstructed radial distributions of the 3-D WIMP velocity and the 1(2)σ statistical uncertainty bands by using the one–parameter shifted Maxwellian velocity distribution $f_{1, \text{sh}, v_0}(v; v_0)$ given by Eq. (24) to fit data shown in Fig. 25. The solid red curve is the shifted Maxwellian velocity distribution $f_{1, \text{sh}, \text{vesc}}(v)$ given by Eq. (19) with an input value of $v_0 = 220$ km/s. While the dashed green curve labeled with the subscript “median” indicates the reconstructed velocity distribution with the fitting parameter $v_0$ given by the median value of all simulated experiments, the dash–dotted blue curve labeled with the subscript “Pmax” indicates the reconstructed velocity distribution with $v_0$ maximizing $P_{\text{mean}}(a_i, i = 1, 2, \ldots, N_{\text{Bayesian}})$ defined in Eq. (18). (b) The distribution of the fitting parameter $v_0$ in all simulated experiments. The red vertical line labeled with the subscript “input” indicates the input value of $v_0$, whereas the green vertical line labeled with the subscript “median” and the blue one labeled with the subscript “Pmax” indicate the median value of the simulated results and the value which maximizes $P_{\text{mean}}$, respectively. In addition, the horizontal thick (thin) green bars show the 1(2)σ ranges of the reconstructed results. See the text for further details.

distribution. This means that all of the velocity distributions with $v_0$’s which give the largest 68.27% (95.45%) $P_{\text{mean}}$ values should be in the 1(2)σ light–green (light–blue) areas.

Meanwhile, Fig. 26(b) shows the distribution of the fitting parameter $v_0$ in all simulated experiments. The red vertical line labeled with the subscript “input” indicates the input value of $v_0$, whereas the green vertical line labeled with the subscript “median” and the blue one labeled with the subscript “Pmax” indicate the median value of the simulated results and the value which maximizes $P_{\text{mean}}$, respectively. In addition, the horizontal thick (thin) green bars show the 1(2)σ ranges of the reconstructed results.

It can be seen that the best–fit value of $v_0 = 215.0$ km/s is only a little bit (0.45σ, see Table 4) smaller than the input value of 220 km/s and thus the best–fit distribution curve would be very close to the theoretically derived shifted Maxwellian velocity distribution $f_{1, \text{sh}, v_\text{esc}}(v)$. However, as found in Sec. 4.2.1, the best–fit velocity distribution shown here differs clearly from the simulated radial distribution of the 3-D WIMP velocity.

4.2.2 With the $v_0$–fixed velocity distribution $f_{1, \text{sh}, v_e}(v; v_e)$

As shown in Sec. 4.2.1, by using $v_0$ as the unique fitting parameter with the fixed relation between $v_0$ and $v_e$, the reconstructed velocity distribution can not fit the simulated radial distribution of the 3-D WIMP velocity as well as we hoped. Hence, we consider here the $v_0$–fixed shifted
Maxwellian velocity distribution $f_{1,sh,v_e}(v; v_0, v_e)$ given by Eq. (26) with the input condition that $v_0 = 220$ km/s (although the best–fit value of $v_0$ obtained in Sec. 4.2.1 is 215.0 km/s) and scan the Earth’s Galactic velocity $v_e$ in the range of 90 km/s $\leq v_e \leq 330$ km/s.

In Fig. 27(a), we show the reconstructed radial distributions of the 3-D WIMP velocity and the 1(2)$\sigma$ statistical uncertainty bands by using $f_{1,sh,v_e}(v; v_0, v_e)$ to fit data shown in Fig. 25. Also, in Fig. 27(b), we give the distribution of the fitting parameter $v_e$ in all simulated experiments. The red vertical line labeled with the subscript “input” indicates the theoretical value of $v_e = 1.05 v_0 = 231$ km/s. Unfortunately and unexpectedly, we have obtained a similar result as in Sec. 4.2.1: the best–fit value of $v_e = 224.4$ km/s is only a little bit ($0.25\sigma$, see Table 4) smaller than the theoretical value of 231 km/s, and thus the best–fit distribution curve would be very close to the theoretical derivation $f_{1,sh,v_{esc}}(v)$, but not improved very much (as we hoped).

### 4.2.3 With the simplified velocity distribution $f_{1,sh}(v; v_0, v_e)$

Our fitting results shown in Secs. 4.2.1 and 4.2.2 indicate that, although the best–fit values of $v_0 = 215.0$ km/s and $v_e = 224.4$ km/s are only a little bit smaller than the theoretical values and thus the best–fit distribution curves would be very close to the theoretically predicted shifted Maxwellian velocity distribution $f_{1,sh,v_{esc}}(v)$, there would obviously be some systematic bias between the theoretical derivation and our (Monte Carlo–simulated) radial velocity distribution. Therefore, we release now the constraints on both of the fitting parameters $v_0$ and $v_e$, use the simplified shifted Maxwellian velocity distribution $f_{1,sh}(v; v_0, v_e)$ given by Eq. (27) and scan the parameter plane of 80 km/s $\leq v_0 \leq 340$ km/s and 0 $\leq v_e \leq 380$ km/s in order to obtain a better–fitted WIMP velocity distribution.

In Fig. 28(a), we show the reconstructed radial distributions of the 3-D WIMP velocity and

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10 Note that the scanning ranges of both of $v_0$ and $v_e$ are extended from those used for the one–parameter and $v_0$–fixed velocity distributions in Secs. 4.2.1 and 4.2.2.
Figure 28: (a) The reconstructed radial distributions of the 3-D WIMP velocity and the 1(2)σ statistical uncertainty bands by using the simplified shifted Maxwellian velocity distribution $f_{1,sh}(v; v_0, v_e)$ given by Eq. (27) to fit data shown in Fig. 25. Notations are the same as in Fig. 26(a). (b) The distribution of the fitting parameters $v_0$ and $v_e$ in all simulated experiments on the $v_0 - v_e$ plane. The light-green (light-blue, golden) square points indicate the 1(2)($>$ 2)σ areas of the reconstructed combination of $v_0$ and $v_e$. While the red upward–triangle labeled with the subscript “input” indicates the theoretical values of $v_0 = 220$ km/s and $v_e = 231$ km/s, the green disk labeled with the subscript “median” and the blue downward–triangle labeled with the subscript “Pmax” indicate the median values of the simulated results and the point which maximizes $P_{\text{mean}}$, respectively. The meaning of the horizontal and vertical thick (thin) green bars are the same as in Figs. 26(b) and 27(b). (c) As in Figs. 26(b). (d) As in Figs. 27(b).

the 1(2)σ statistical uncertainty bands by using $f_{1,sh}(v; v_0, v_e)$ to fit data shown in Fig. 25. It can be seen that, while the red theoretically derived velocity distribution [19] could still be covered by the 1σ statistical uncertainty band, except of in the ranges around two turning points: $v \sim 270$ km/s and $v \sim 550$ km/s, the best–fit velocity distributions would now clearly differ from the theoretical distribution, but indeed fit to our simulated radial distribution of the 3-D WIMP velocity much better.

Meanwhile, Fig. 28(b) shows the distribution of the fitting parameters $v_0$ and $v_e$ in all simu-
lated experiments on the $v_0 - v_e$ plane. The light–green (light–blue, golden) square points indicate the $1(2)(> 2)\sigma$ areas of the reconstructed combination of $v_0$ and $v_e$. While the red upward–triangle labeled with the subscript “input” indicates the theoretical values of $v_0 = 220$ km/s and $v_e = 231$ km/s, the green disk labeled with the subscript “median” and the blue downward–triangle labeled with the subscript “Pmax” indicate the median values of the simulated results and the point which maximizes $P_{\text{mean}}$, respectively. Additionally, Figs. 28(c) and (d) show the distributions of the fitting parameters $v_0$ and $v_e$ in all simulated experiments separately.

Corresponding to the reconstructed radial distributions, from these three plots one can find that, while the best–fit values of $v_0 \simeq 190$ km/s are $\sim 14\%$ smaller than the input value, those of $v_e = 258.4$ km/s are now $\sim 12\%$ larger than its theoretical value (see also Table 4), although the theoretical values of our fitting parameters could still fall inside their $1\sigma$ statistical uncertainty ranges. Consequently, the ratio between the best–fit values of $v_0$ and $v_e$ is $\sim 1.4$, much larger than the commonly adopted theoretical value of 1.05 [65] and our estimate of 1.0078 [11]. Note also that, due to the pretty small ($\mathcal{O}(50)$) simulated total event number and thus a large statistical fluctuation, there is a long distribution tail at the lower scanning boundary of $v_e$ in Fig. 28(b). In Sec. 5.2 we will see that, once the total event number is raised to $\mathcal{O}(500)$ and thus the statistics is improved, this tail would disappear.

4.2.4 With the modified velocity distribution $f_{1,\text{sh},\Delta v}(v; v_0, \Delta v)$

In Ref. [68], we introduced the modified shifted Maxwellian velocity distribution $f_{1,\text{sh},\Delta v}(v; v_0, \Delta v)$ given by Eq. (28) for reducing the systematic bias of the best–fit results obtained by using the simplified velocity distribution $f_{1,\text{sh}}(v; v_0, v_e)$. For checking and potentially also improving our reconstruction results shown in Sec. 4.2.3 we apply now the modified velocity distribution $f_{1,\text{sh},\Delta v}(v; v_0, \Delta v)$ and scan the parameter plane of $80 \text{ km/s} \leq v_0 \leq 340 \text{ km/s}$ and $-190 \text{ km/s} \leq \Delta v \leq 230 \text{ km/s}$.

In Fig. 29(a) we show the reconstructed radial distributions of the 3-D WIMP velocity and the 1(2)$\sigma$ statistical uncertainty bands by using $f_{1,\text{sh},\Delta v}(v; v_0, \Delta v)$ to fit data shown in Fig. 25. Not surprisingly, the reconstructed velocity distributions as well as the 1(2)$\sigma$ statistical uncertainty bands have similar shapes as those shown in Fig. 28(a).

Meanwhile, Fig. 29(b) shows the distribution of the fitting parameters $v_0$ and $\Delta v$ in all simulated experiments on the $v_0 - \Delta v$ plane. The light–green (light–blue, golden) square points indicate the $1(2)(> 2)\sigma$ areas of the reconstructed combination of $v_0$ and $\Delta v$. While the red upward–triangle labeled with the subscript “input” indicates the theoretical values of $v_0 = 220$ km/s and $\Delta v = 11$ km/s, the green disk labeled with the subscript “median” and the blue downward–triangle labeled with the subscript “Pmax” indicate the median values of the simulated results and the point which maximizes $P_{\text{mean}}$, respectively. As shown in 28(b), one can also see here a long distribution tail at the lower scanning boundary of $\Delta v$. Additionally, Figs. 29(c) and (d) show the distribution of the fitting parameters $v_0$ and $\Delta v$ in all simulated experiments separately.

From these three plots one can confirm that, first, the best–fit values of $v_0 \simeq 190$ km/s are the same as the results obtained by using the simplified velocity distribution (see also Table 4). Second, the best–fit values of $\Delta v \simeq 70$ km/s are also equal to the difference between $v_0$ and $v_e$ obtained previously. Note however that, the 1(2)$\sigma$ statistical uncertainties on $\Delta v$ would be at least 1.5 times larger than those on $v_e$ (see Table 4).

\(^{11}\) See Appendix A.3.4 for detailed discussions.
4.3 Annual modulation of the radial distribution of the 3-D WIMP velocity

In this subsection, we discuss the annual modulation of the radial distribution of the 3-D WIMP velocity and present the reconstructed radial WIMP velocity distributions in the observation.
Table 4: The summary of the reconstructed results of the fitting parameters and their 1(2)σ statistical uncertainty ranges of the median values by using four considered fitting velocity distributions with 50 total events on average in one entire year shown in Figs. [26, 27, 28] and [29].

4.3.1 With the one–parameter velocity distribution $f_{1,sh,v_0}(v; v_0)$

As in Sec. 4.2, we consider at first the one–parameter velocity distribution $f_{1,sh,v_0}(v; v_0)$ with $v_0$ as the fitting parameter and the constraint that $v_e = 1.05v_0$. Remind however that, although the fitting parameter $v_0$ is in fact the Solar Galactic velocity and should be fixed in the whole year, it has been used here as a normal fitting parameter.\(^{13}\)

In Figs. [30], we show the reconstructed radial WIMP velocity distributions and the 1(2)σ statistical uncertainty bands by using $f_{1,sh,v_0}(v; v_0)$ as well as the distributions of the fitting parameter $v_0$ in all simulated experiments for four advanced seasons. Firstly, from the plots in the left column of Figs. [30], one can see the variation of the radial distributions of the 3-D WIMP velocity in different seasons: the event numbers in high–velocity bins $(v > 290 \text{ km/s})$ vary (from top to bottom) at first more and then fewer, in contrast, the event numbers in low–velocity bins $(v < 290 \text{ km/s})$ at first fewer and then more. This can be observed in particularly clearly from the variation of the difference between the third and the forth $v$–bins around the distribution peak. This implies that the (average) velocity of the simulated WIMP events indeed increases at first to the maximum (in the advanced Summer) and then reduces to the minimum (in the advanced Winter). Secondly and more importantly, the reconstructed velocity distributions and their 1(2)σ statistical uncertainty bands follow the variation of the histogram in different seasons as tightly as possible. Note that the red theoretically derived curve shown in the left column of Figs. [30] is time–independent.

Moreover, from the plots in the right column of Figs. [30], the symmetric periodic variation

\(^{12}\) For the sake of completeness and readers’ reference, the reconstructed radial WIMP velocity distributions for four normal seasons will be given in Appendix [C].

\(^{13}\) In Secs. 4.3.3 and 4.3.4, we will see that, once our constraints on $v_0$ and $v_e$ are released and two fitting parameters, $v_0$ and $v_e$ (or $\Delta v$), are used, the best–fit values of $v_0$ would indeed be a constant.
Radial component of the 3-D WIMP velocity distribution in the Equatorial frame, 110.74 - 170.74 day, 50 events

Radial component of the 3-D WIMP velocity distribution in the Equatorial frame, 201.99 - 261.99 day, 50 events

Radial component of the 3-D WIMP velocity distribution in the Equatorial frame, 293.24 - 353.24 day, 50 events

Figure 30: As in Figs. 26, reconstructed with the one–parameter velocity distribution $f_{1,\text{sh},0}(v; v_0)$, except that four 60-day observation periods of the advanced seasons have been considered.
### Fitting distribution: one–parameter velocity distribution \( f_{1,sh,v_0}(v; v_0) \)

| Central date  | Parameter | Max. P\(_{\text{median}}\) | Median \( \pm \sigma \) | 1\(\sigma \) range | 2\(\sigma \) range |
|---------------|-----------|----------------------------|-----------------|-----------------|-----------------|
| 49.49 (19.49 – 79.49) | \( v_0 \) [km/s] | 215.0 | 215.0 ± 11.0 \((^{+22.0}_{-20.9})\) | [204.0, 226.0] | [194.1, 237.0] |
| 140.74 (110.74 – 170.74) | \( v_0 \) [km/s] | 220.5 | 220.5 ± 11.0 \((^{+23.1}_{-20.9})\) | [209.5, 231.5] | [199.6, 243.6] |
| 231.99 (201.99 – 261.99) | \( v_0 \) [km/s] | 215.0 | 215.0 ± 11.0 \((^{+22.0}_{-20.9})\) | [204.0, 226.0] | [194.1, 237.0] |
| 323.24 (293.24 – 353.24) | \( v_0 \) [km/s] | 209.5 | 208.4 ± 11.0 \((^{+22.0}_{-19.8})\) | [198.5, 219.4] | [188.6, 230.4] |

50 total events on average in one observation period of 60 days

Table 5: The summary of the reconstructed results of the fitting parameter \( v_0 \) and their 1(2)\(\sigma \) statistical uncertainty ranges of the median values by using the one–parameter velocity distribution \( f_{1,sh,v_0}(v; v_0) \) with 50 total events on average in each 60–day observation period of the advanced seasons.

Of the fitting parameter \( v_0 \) of 215.0 \((± \sim 5.5)\) km/s can also be seen clearly. It would be worth to notice here that the \( \sim ±5.5 \) km/s annual variation of the best–fit value of \( v_0 \) is \( \sim 0.5\sigma \) of the statistical uncertainties on \( v_0 \) (see the summary given in Table 5) and approximately equal to the difference between the annual average value of \( v_0 = 215.0 \) km/s (see Table 4) and its theoretical value. In Sec. 5.2.2, we will show that, with \( O(500) \) total WIMP events and thus \( \sim 1/3 \) reduced statistical uncertainties on \( v_0 \), the annual variation of the fitting parameter \( v_0 \) and in turn that of the radial WIMP velocity distribution could be identified with a confidence level of 2\(\sigma \) to 3\(\sigma \).

In Table 5, we summarize the reconstructed results of the fitting parameter \( v_0 \) and their 1(2)\(\sigma \) statistical uncertainty ranges of the median values by using the one–parameter velocity distribution \( f_{1,sh,v_0}(v; v_0) \) with 50 total events on average in each 60–day observation period of the advanced seasons.

### 4.3.2 With the \( v_0 \)–fixed velocity distribution \( f_{1,sh,v_e}(v; v_e) \)

In order to check the identification possibility of the variation of the Earth’s Galactic velocity \( v_e \), we consider then the \( v_0 \)–fixed velocity distribution \( f_{1,sh,v_e}(v; v_e) \) with \( v_e \) as the fitting parameter and the input condition that \( v_0 = 220 \) km/s.

In Figs. 31, we show the reconstructed radial WIMP velocity distributions and the 1(2)\(\sigma \) statistical uncertainty bands by using \( f_{1,sh,v_e}(v; v_e) \) as well as the distributions of the fitting parameter \( v_e \) in all simulated experiments for four advanced seasons. Firstly, from the plots in the left column of Figs. 31, one can find that the reconstructed velocity distributions and their 1(2)\(\sigma \) statistical uncertainty bands also follow the variation of the simulated radial velocity distribution in different seasons. Meanwhile, from the plots in the right column of Figs. 31, one
Figure 31: As in Figs. 27, reconstructed with the $v_0$-fixed velocity distribution $f_{1,sh,v_0}(v; v_0)$, except that four 60-day observation periods of the advanced seasons have been considered.
Table 6: The summary of the reconstructed results of the fitting parameter $v_e$ and their 1(2)$\sigma$ statistical uncertainty ranges of the median values by using the $v_0$–fixed velocity distribution $f_{1,sh,v_e}(v; v_e)$ with 50 total events on average in each 60–day observation period of the advanced seasons.

| Central date (observation period) (day) | Parameter | Max. P | Median $v_e$ [km/s] | 1$\sigma$ range | 2$\sigma$ range |
|----------------------------------------|-----------|--------|---------------------|----------------|--------------|
| 49.49 (19.49 – 79.49) | $v_e$ [km/s] | 224.4 | $224.4^{+26.4}_{-28.8}$ ($^{+50.4}_{-58.2}$) | [195.6, 250.8] | [166.2, 274.8] |
| 140.74 (110.74 – 170.74) | $v_e$ [km/s] | 238.8 | $238.8 \pm 26.4$ ($^{+50.4}_{-52.8}$) | [212.4, 265.2] | [186.0, 289.2] |
| 231.99 (201.99 – 261.99) | $v_e$ [km/s] | 224.4 | $224.4 \pm 26.4$ ($^{+50.4}_{-57.0}$) | [198.0, 250.8] | [166.8, 274.8] |
| 323.24 (293.24 – 353.24) | $v_e$ [km/s] | 207.6 | $207.6 \pm 28.8$ ($^{+52.8}_{-60.0}$) | [178.8, 236.4] | [147.6, 260.4] |

Table 6: The summary of the reconstructed results of the fitting parameter $v_e$ and their 1(2)$\sigma$ statistical uncertainty ranges of the median values by using the $v_0$–fixed velocity distribution $f_{1,sh,v_e}(v; v_e)$ with 50 total events on average in each 60–day observation period of the advanced seasons.

can also see the (approximately) symmetric periodic variation of the fitting parameter $v_e$ of 224.4 ($^{+14.4}_{16.8}$) km/s clearly; this $^{+14.4}_{16.8}$ km/s annual variation of the best–fit value of $v_e$ is also $\sim 0.5\sigma$ of the statistical uncertainties on $v_e$ (see the summary given in Table 5). In Sec. 5.2.2, we will show that, with $\mathcal{O}(500)$ total WIMP events and thus $\sim 1/3$ reduced statistical uncertainties on $v_e$, its annual variation could be identified with a confidence level of $> 2\sigma$.

In Table 6, we summarize the reconstructed results of the fitting parameter $v_e$ and their 1(2)$\sigma$ statistical uncertainty ranges of the median values by using the $v_0$–fixed velocity distribution $f_{1,sh,v_e}(v; v_e)$ with 50 total events on average in each 60-day observation period.

### 4.3.3 With the simplified velocity distribution $f_{1,sh}(v; v_0, v_e)$

Now we release the constraints on the fitting parameters $v_0$ and $v_e$ and consider the simplified velocity distribution $f_{1,sh}(v; v_0, v_e)$, in order to identify the variation of the Earth’s Galactic velocity $v_e$ as well as to check the invariability of the Solar Galactic velocity $v_0$ with the better–fitted radial WIMP velocity distributions in the advanced seasons.

In Figs. 32, we show the reconstructed radial WIMP velocity distributions and the 1(2)$\sigma$ statistical uncertainty bands by using $f_{1,sh}(v; v_0, v_e)$ as well as the distributions of the fitting parameters $v_0$ and $v_e$ in all simulated experiments on the $v_0 – v_e$ plane for four advanced seasons. Firstly, as found in Sec. 4.2.3 from the plots in the left column of Figs. 32, one can observe clearly that the reconstructed velocity distributions and their 1(2)$\sigma$ statistical uncertainty bands indeed follow the variation of the simulated radial velocity distribution in different seasons almost perfectly. Meanwhile, from the plots in the right column of Figs. 32 as well as our summary given in Table 7, we can find that, although the first fitting parameter $v_0$ is unconstraint, its
Radial component of the 3-D WIMP velocity distribution in the Equatorial frame, 19.49 - 79.49 day, 50 events

Radial component of the 3-D WIMP velocity distribution in the Equatorial frame, 201.99 - 261.99 day, 50 events

Radial component of the 3-D WIMP velocity distribution in the Equatorial frame, 293.24 - 353.24 day, 50 events

Radial component of the 3-D WIMP velocity distribution in the Equatorial frame, 110.74 - 170.74 day, 50 events

Figure 32: As in Figs. 28, reconstructed with the simplified velocity distribution $f_{1,sh}(v; v_0, v_e)$, except that four 60-day observation periods of the advanced seasons have been considered.
Finally, as a confirmation of the invariability of the Solar Galactic velocity $v_0$ and the variation of the Earth’s Galactic velocity $v_e$ shown in Sec. 4.3.3, we consider here the modified velocity distribution $f_{1,sh}(v; v_0, \Delta v)$ with $v_0$ and $\Delta v$ as two free fitting parameters.

In Figs. [33] we show the reconstructed radial WIMP velocity distributions and the 1(2)$\sigma$
Figure 33: As in Figs. 29, reconstructed with the modified velocity distribution $f_{1,sh,\Delta v}(v; v_0, \Delta v)$, except that four 60-day observation periods of the advanced seasons have been considered.
Table 8: The summary of the reconstructed results of the fitting parameters \(v_0\) and \(\Delta v\) as well as their \(1(2)\sigma\) statistical uncertainty ranges of the median values by using the modified velocity distribution \(f_{1,sh,\Delta v}(v; v_0, \Delta v)\) with 50 total events on average in each 60–day observation period of the advanced seasons.

| Central date (observation period) (day) | Parameter | Max. \(P_{\text{median}}\) | Median | 1\(\sigma\) range | 2\(\sigma\) range |
|----------------------------------------|-----------|---------------------------|--------|-------------------|-------------------|
| 49.49 (19.49 – 79.49)                  | \(v_0\) [km/s] | 189.2 | 186.6\(^{+36.4}_{-26.0}\) \(^{+88.4}_{-46.8}\) | [160.6, 223.0] | [139.8, 275.0] |
|                                        | \(\Delta v\) [km/s] | 70.4 | 74.6\(^{+46.2}_{-75.6}\) \(^{+88.2}_{-264.6}\) | [−1.0, 120.8] | [−190.0, 162.8] |
| 140.74 (110.74 – 170.74)              | \(v_0\) [km/s] | 191.8 | 186.6\(^{+33.8}_{-23.4}\) \(^{+93.6}_{-46.8}\) | [163.2, 220.4] | [139.8, 280.2] |
|                                        | \(\Delta v\) [km/s] | 78.8 | 87.2\(^{+49.2}_{-71.4}\) \(^{+84.0}_{-273.0}\) | [15.8, 133.4] | [−185.8, 171.2] |
| 231.99 (201.99 – 261.99)              | \(v_0\) [km/s] | 189.2 | 186.6\(^{+33.8}_{-23.4}\) \(^{+91.0}_{-46.0}\) | [163.2, 220.4] | [139.8, 277.6] |
|                                        | \(\Delta v\) [km/s] | 70.4 | 74.6\(^{+50.4}_{-75.6}\) \(^{+88.2}_{-264.6}\) | [−1.0, 125.0] | [−190.0, 162.8] |
| 323.24 (293.24 – 353.24)              | \(v_0\) [km/s] | 189.2 | 186.6\(^{+36.4}_{-26.0}\) \(^{+88.4}_{-46.8}\) | [160.6, 223.0] | [137.2, 275.0] |
|                                        | \(\Delta v\) [km/s] | 57.8 | 62.0\(^{+50.4}_{-88.2}\) \(^{+88.2}_{-252.0}\) | [−26.2, 112.4] | [−190.0, 150.2] |

50 total events on average in one observation period of 60 days

Statistical uncertainty bands by using \(f_{1,sh,\Delta v}(v; v_0, \Delta v)\) as well as the distributions of the fitting parameters \(v_0\) and \(\Delta v\) in all simulated experiments on the \(v_0 - \Delta v\) plane for four advanced seasons. Firstly, by comparing the plots in the left column of Figs. 33 with the plots in the left column of Figs. 32, one could see that, except of the \(2\sigma\) statistical uncertainty bands\(^\text{13}\), the velocity distributions and their \(1\sigma\) statistical uncertainty bands reconstructed with the simplified and the modified velocity distributions would be identical.

Additionally, from the plots in the right column of Figs. 33 as well as our summary given in Table 8, one could confirm that the best–fit values of the first fitting parameter \(v_0\) in four advanced seasons are fixed around the annual average value of \(v_0 \simeq 190 \text{ km/s}\) with the similar \(1(2)\sigma\) statistical uncertainties. Meanwhile, the best–fit values of the second fitting parameter \(v_e\) show also clearly an (approximately) symmetric periodic variation of \(70.4 \text{ (±8.4)}\) km/s in four advanced seasons, which is approximately equal to both of the annual average value of \(\Delta v \simeq 70 \text{ km/s}\) and the difference between the best–fit values of \(v_0\) and \(v_e\) obtained in Sec. 4.3.3. It would be worth to notice that, while the \(±8.4\) km/s annual variation of the best–fit value of \(\Delta v\) is smaller than the \(±11.4\) km/s annual variation of \(v_e\), the \(1\sigma\) statistical uncertainties on \(\Delta v\)

\(^{13}\)Note that, due to the wide–extended distribution tails at the lower scanning boundary, the lower bounds of the \(2\sigma\) statistical uncertainty on \(v_e\) and in turn the shapes of the \(2\sigma\) uncertainty bands are actually limited by our simulation setup for the scanning range. In Sec. 5.2.2, we will see that, with \(O(500)\) total WIMP events the \(1(2)\sigma\) uncertainty contours become closed and the lower bounds of the \(2\sigma\) uncertainty on \(v_e\) can be determined accurately. Then the shape differences between the \(2\sigma\) uncertainty bands of the WIMP velocity distributions reconstructed with the simplified and the modified velocity distributions becomes tiny.
4.4 Fitted radial WIMP velocity distribution in the Galactic frame

As preparation for analyzing (future) real experimental data to understand the 3-dimensional velocity distribution of halo WIMPs, we have also tried to analyze the saved 3-D velocity information of the simulated WIMP events (including measuring times) in the laboratory coordinate system to reconstruct the radial WIMP velocity distribution in the Galactic coordinate system.

In Figs. 34 we show the reconstructed radial distributions of the 3-D WIMP velocity and the 1(2)σ statistical uncertainty bands by using the simple Maxwellian velocity distribution $f_1, \text{Gau}(v)$ given by Eq. (2) to fit data shown in Fig. 7 as well as the distribution of the fitting parameter $v_0$ in all simulated experiments. Not surprisingly, our best–fit reconstruction distributions could be in contrast much larger than that on $v_e$.

In Table 9 we summarize the reconstructed results of the fitting parameters $v_0$ and $\Delta v$ as well as their 1(2)σ statistical uncertainty ranges of the median values by using the modified velocity distribution $f_{1,\text{sh},\Delta v}(v; v_0, \Delta v)$ with 50 total events on average in each 60-day observation period.

| Fitting dist. | Parameter | Max. $P_{\text{median}}$ | Median | 1σ range | 2σ range |
|---------------|-----------|--------------------------|--------|----------|----------|
| $f_{1, \text{Gau}}(v)$ | $v_0$ [km/s] | 220.2 | $220.2 \pm 14$ ($^{+29.4}_{-26.6}$) | [206.2, 234.2] | [193.6, 249.6] |

Table 9: The summary of the reconstructed results of the fitting parameter $v_0$ and their 1(2)σ statistical uncertainty ranges of the median values by using the simple Maxwellian velocity distribution with 50 total events on average in one experiment in one entire year shown in Figs. 34.
match the input simple Maxwellian velocity distribution perfectly and the fitting parameter $v_0$ could also be pinned down very precisely\textsuperscript{15} (see the summary given in Table 9).

5 With a raised total event number

In this section, we raise the total event number in one observation period (365 days/year, 60 days/season, or 4 hours/shift $\times$ 60 days) to 500 events on average in one experiment. We present at first the angular distribution patterns of the 3-dimensional WIMP velocity and then discuss the Bayesian reconstructions of its radial distributions.

5.1 Angular distributions of the 3-D WIMP velocity in the laboratory–independent frames

In this subsection, we present only the angular distributions of the (transformed) 3-D WIMP velocity in three laboratory–independent (Ecliptic, Equatorial, and Earth) coordinate systems. The angular distributions in two laboratory–dependent (horizontal and laboratory) coordinate systems observed in different underground laboratories can be found in Appendix B.

5.1.1 Angular WIMP velocity distribution in the Ecliptic frame

In Fig. 35, we show the angular distribution of the 3-D WIMP velocity transformed to the Ecliptic coordinate system. 500 total events on average in one entire year have been generated and binned into $12 \times 12$ bins for the longitude and latitude directions, respectively.

It can be found that, first, with a higher analysis resolution, the bins with higher event number spreading from the center to the southwest part can be seen more obviously, including the bins with the highest event number ($> 5.6$ times of the all–sky average value of 500 events / 144 bins = 3.47 events/bin, i.e., $> 19.44$ events/bin) between $45^\circ$S and $0^\circ$ latitude, $30^\circ$W and $0^\circ$ longitude. Second, the deviation between the center of the band of the high–WIMP–flux bins and the theoretical main direction of incident WIMPs in the Ecliptic coordinate system ($57.40^\circ$S, $29.10^\circ$W) (the dark–green star) would now be $\lesssim 15^\circ$ and $\lesssim 30^\circ$ in the latitude and the longitude direction, respectively.

5.1.2 Angular WIMP velocity distribution in the Equatorial frame

In Fig. 36, we show at first the angular distribution of the 3-D WIMP velocity transformed to the Equatorial coordinate system with 500 total events on average in one entire year. With a higher analysis resolution, one can also find that, first, as in the Ecliptic coordinate system, the bins with higher event number spreading from the center to the southwest part can be seen more obviously and there seems still to be a small deviation between the center of the band of the high–WIMP–flux bins and the theoretical main direction of incident WIMPs in the Equatorial coordinate system ($42.00^\circ$S, $50.70^\circ$W) (the dark–green star). Second, not only the bins around the north pole (more northern than $45^\circ$N) and in the most part of the eastern hemisphere, a few bins around the south pole (more southern than $75^\circ$S) would also be a “WIMP hole” with event numbers less than 35% of the all–sky average (i.e., $< 1.22$ events/bin among $O(500)$ total events).

\textsuperscript{15} The tiny difference of 0.2 km/s between the best–fit and the input values of $v_0$ is only due to the scanning resolution.
Figure 35: As in Fig. 12, the angular distribution of the 3-D WIMP velocity transformed to the Ecliptic coordinate system, except that 500 total events on average in one entire year have been generated and binned into $12 \times 12$ bins for the longitude and latitude directions, respectively.

Moreover, in order to demonstrate the annual modulation of the angular distribution pattern of the 3-D WIMP velocity, in Figs. 37 and 38 we show the angular distribution in the Equatorial coordinate system in four normal and four advanced seasons, respectively. Remind that 500 total events on average in each 60-day observation period have been simulated. Now, with a higher analysis resolution, the clockwise circular variation of the angular distribution patterns following the blue–yellow point around the theoretical main direction of incident WIMPs could be observed more clearly. In addition, the difference of the angular distribution patterns between the corresponding observation periods of four normal and four advanced seasons can already be observed. In fact, one could even combine all eight plots of the normal and the advanced seasons to build a more–detailed rotated pattern of the angular WIMP velocity distribution.

5.1.3 Angular WIMP velocity distribution in the Earth frame

In Fig. 39, we show the angular distribution of the 3-D WIMP velocity transformed to the Earth coordinate system with 500 total events on average in one entire year. With a higher analysis resolution, as in the Equatorial coordinate system, not only the bins around the north pole (more northern than 30°N), a few bins around the south pole (more southern than 75°S) would also be the “WIMP hole” with event numbers less than 35% of the all–sky average (i.e., $< 1.22$ events/bin among $O(500)$ total events).

Moreover, in order to demonstrate the annual modulation of the angular distribution pattern of the 3-D WIMP velocity, in Figs. 40 and 41 we show the angular distribution in the Earth coordinate system in four normal and four advanced seasons, respectively. Remind that 500
Figure 36: As in Fig. 13, the angular distribution of the 3-D WIMP velocity transformed to the Equatorial coordinate system, except that 500 total events on average in one entire year have been simulated.

total events on average in each 60-day observation period have been simulated. Now, with a higher analysis resolution, the rotation of the angular distribution patterns around the $Z_E$-axis can be clearly observed, not only in four normal and four advanced seasons, but also between each pair of the corresponding seasons.

5.2 Bayesian reconstruction of the radial distribution of the 3-D WIMP velocity

In this subsection, we present our Bayesian reconstructions of the radial distribution of the (transformed) 3-D WIMP velocity in the Equatorial coordinate system. As in Sec. 4, we discuss at first the one–year simulations and then the results of the annual modulation in the observation periods of four advanced seasons\textsuperscript{16}. All four fitting velocity distributions given in Sec. 2.3.3 will be considered. At the end of this subsection, we will also show the reconstruction result of the WIMP velocity distribution in the Galactic coordinate system.

5.2.1 Fitted radial WIMP velocity distribution in the Equatorial frame

We discuss at first our reconstruction results with 500 total events on average in one experiment recorded in one entire year.

\textsuperscript{16} The reconstructed radial WIMP velocity distributions in the observation periods of four normal seasons will be given in Appendix C for readers’ reference.
Figure 37: As in Figs. 14, the angular distribution of the 3-D WIMP velocity in the Equatorial coordinate system, except that 500 total events on average in each 60-day observation period of four normal seasons have been considered.

With the one–parameter and the $v_0$–fixed velocity distributions $f_{1,sh,v_0}(v; v_0)$ and $f_{1,sh,v_e}(v; v_e)$

As in Secs. 4.2.1 and 4.2.2 we consider at first the one–parameter and the $v_0$–fixed velocity distribution functions, $f_{1,sh,v_0}(v; v_0)$ and $f_{1,sh,v_e}(v; v_e)$, with the constraints that $v_e = 1.05 v_0$ and $v_0 = 220$ km/s, respectively. Note that the scanning ranges of the fitting parameters have been shrunk to $190$ km/s $\leq v_0 \leq 240$ km/s or $180$ km/s $\leq v_e \leq 270$ km/s, respectively.

In Figs. 42 and 43, we show the reconstructed radial distributions of the 3-D WIMP velocity and the 1(2)$\sigma$ statistical uncertainty bands by using the one–parameter and the $v_0$–fixed velocity distributions. One can find that, although two 1$\sigma$ statistical uncertainty bands could still cover the theoretical (solid red) velocity distribution, the difference between the best–fit and the input values of $v_0$ is already $\sim 1.6 \sigma$, while the difference between these two values of $v_e$ is also $\sim 0.7 \sigma$ (see the summary given in Table 10). More importantly, both of the theoretical velocity distribution and those reconstructed with two fitting distribution functions are now clearly $\sim 1\sigma$ deviated from the radial distribution of the simulated 3-D WIMP velocity in most of the $v$–bins.
Figure 38: As in Figs. 15, the angular distribution of the 3-D WIMP velocity in the Equatorial coordinate system, except that 500 total events on average in each 60-day observation period of four advanced seasons have been considered.

These would imply that the radial distribution of the 3-D WIMP velocity in the Equatorial as well as in the laboratory coordinate systems would differ from the theoretically derived analytic form $f_{1,sh,vesc}(v)$ given in Eq. 19 [67].

With the simplified and the modified velocity distributions $f_{1,sh}(v; v_0, v_e)$ and $f_{1,sh,\Delta v}(v; v_0, \Delta v)$

As in Secs. 4.2.3 and 4.2.4, we release now the constraints that $v_e = 1.05 v_0$ and $v_0 = 220$ km/s and consider the simplified and the modified velocity distribution functions, $f_{1,sh}(v; v_0, v_e)$ and $f_{1,sh,\Delta v}(v; v_0, \Delta v)$. Note that the scanning ranges of the fitting parameters have been shrunk to $140$ km/s $\leq v_0 \leq 240$ km/s, $200$ km/s $\leq v_e \leq 310$ km/s, and $-20$ km/s $\leq \Delta v \leq 150$ km/s.

In Figs. 44 and 45, we show the reconstructed radial distributions of the 3-D WIMP velocity and the $1(2)\sigma$ statistical uncertainty bands by using the simplified and the modified velocity distributions. One can find firstly that, although the long distribution tails at the lower scanning boundary on the $v_0-v_e$ and the $v_0-\Delta v$ planes disappear now and the $1(2)\sigma$ statistical uncertainty
Figure 39: As in Fig. 16, the angular distribution of the 3-D WIMP velocity transformed to the Earth coordinate system, except that 500 total events on average in one entire year have been simulated.

Contours become closed, two 1(2)σ uncertainty bands could not cover the theoretical (solid red) velocity distribution any more. Additionally, two reconstructed radial distributions as well as their 1(2)σ uncertainty bands seem not to fit the radial distribution of the simulated 3-D WIMP velocity in the velocity range of 200 km/s $\lesssim v \lesssim 300$ km/s very well. Moreover, in Figs. 44(b) and 45(b), one can see that the theoretical values of the fitting parameters (the red upward–triangle) are now clearly out of the (light–blue) 2σ statistical uncertainty contours: the difference between the best–fit and the input values of $v_0$ becomes now $\sim 3.4\sigma$, and the difference between these two values of $v_e$ is also enlarged to $\sim 2.8\sigma$ (see the summary given in Table 10). These would imply not only that the radial distribution of the 3-D WIMP velocity in the Equatorial as well as in the laboratory coordinate systems would differ from the theoretically derived analytic form (19) of $f_{1,sh,vesc}(v)$, but also a requirement of the modification of the analytic form of the fitting velocity distribution function.

In Table 10, we summarize the reconstructed results of the fitting parameters and their 1(2)σ statistical uncertainty ranges of the median values with 500 total events on average in one entire year for all four considered fitting velocity distributions shown in Figs. 26, 27, 28, and 29.

### 5.2.2 Annual modulation of the reconstructed WIMP velocity distribution

Now we come to present the reconstruction results of the annual modulation of the radial WIMP velocity distributions in the observation periods of four advanced seasons.
Figure 40: As in Figs. 17, the angular distribution of the 3-D WIMP velocity in the Earth coordinate system, except that 500 total events on average in each 60-day observation period of four normal seasons have been considered.

With the one–parameter and the $v_0$–fixed velocity distributions $f_{i,sh,v_0}(v;v_0)$ and $f_{i,sh,v_e}(v;v_e)$

As in Secs. 4.3.1 and 4.3.2 we consider at first the one–parameter and the $v_0$–fixed velocity distribution functions, $f_{i,sh,v_0}(v;v_0)$ and $f_{i,sh,v_e}(v;v_e)$, with the constraints that $v_e = 1.05 v_0$ and $v_0 = 220$ km/s, respectively.

In Figs. 46 and Figs. 47, we show the reconstructed radial distributions of the 3-D WIMP velocity and the $1(2)\sigma$ statistical uncertainty bands as well as the distributions of the fitting parameters $v_0$ and $v_e$ in all simulated experiments by using the one–parameter and the $v_0$–fixed velocity distributions in four observation periods of the advanced seasons, respectively. It can be find that the (approximately) symmetric periodic variations of the fitting parameters $v_0$ and $v_e$ observed in Secs. 4.3.1 and 4.3.2 can now be seen more obviously and pinned down more precisely as $214.5 \pm 6.0$ km/s and $225.0 \left(\mp15.3\right)$ km/s, respectively; the $\pm6.0$ km/s and the $\pm15.3$ km/s annual variations of the best–fit values of $v_0$ and $v_e$ are already $\sim 2\sigma$ of their statistical uncertainties. Additionally, it would be worth to also notice that, in (the advanced) Winter the
Figure 41: As in Figs. 18, the angular distribution of the 3-D WIMP velocity in the Earth coordinate system, except that 500 total events on average in each 60-day observation period of four advanced seasons have been considered.

deviations of the best–fit values of \( v_0 \) and \( v_e \) from their theoretical (annual–average) values of 220 km/s and 231 km/s could be as large as \( \sim 3.3\sigma \) and \( \sim 2.6\sigma \) of their statistical uncertainties, respectively.

In Tables 11 and 12 we summarize the reconstructed results of the fitting parameters \( v_0 \) and \( v_e \) as well as their 1(2)\( \sigma \) statistical uncertainty ranges of the median values by using the one–parameter and the \( v_0 \)–fixed velocity distribution with 500 total events on average in each 60-day observation period of the advanced seasons, respectively.

With the simplified and the modified velocity distributions \( f_{1,sh}(v; v_0, v_e) \) and \( f_{1,sh,\Delta v}(v; v_0, \Delta v) \)

As in Secs. 4.3.3 and 4.3.4 we release now the constraints that \( v_e = 1.05v_0 \) and \( v_0 = 220 \) km/s and consider the simplified and the modified velocity distribution functions, \( f_{1,sh}(v; v_0, v_e) \) and \( f_{1,sh,\Delta v}(v; v_0, \Delta v) \), respectively, in order to check the identification possibility for the invariability of the Solar Galactic velocity \( v_0 \) as well as for the variation of the Earth’s Galactic velocity \( v_e \).
Figure 42: As in Figs. 26, reconstructed with the one–parameter velocity distribution $f_{1,sh,v_0}(v;v_0)$, except that 500 total events on average in one entire year have been simulated. Note that the scanning range of the fitting parameter $v_0$ is shrunk to between 190 km/s and 240 km/s.

observed in Secs. 4.3.3 and 4.3.4.

In Figs. 48 and Figs. 49, we show the reconstructed radial distributions of the 3-D WIMP velocity and the 1(2)$\sigma$ statistical uncertainty bands as well as the distributions of the fitting parameters $v_0$, $v_e$ and $\Delta v$ in all simulated experiments on the $v_0 - v_e$ and $v_0 - \Delta v$ planes by using the simplified and the modified velocity distributions in four observation periods of the advanced seasons, respectively. As observed in Secs. 4.3.3 and 4.3.4 although the first fitting parameter $v_0$ is unconstraint, its best–fit values in four advanced seasons would be fixed as its annual average value of $\simeq 185$ km/s, with the similar 1(2)$\sigma$ statistical uncertainties of $+10.0 (-16.0)$ km/s. In contrast, in four advanced seasons the best–fit values of the second fitting parameter $v_e$ and

Figure 43: As in Figs. 27, reconstructed with the $v_0$–fixed velocity distribution $f_{1,sh,v_0}(v;v_e)$, except that 500 total events on average in one entire year have been simulated. Note that the scanning range of the fitting parameter $v_e$ is shrunk to between 180 km/s and 270 km/s.
Figure 44: As in Figs. 28 reconstructed with the simplified velocity distribution \( f_{1,sh}(v; v_0, v_e) \), except that 500 total events on average in one entire year have been simulated. Note that the scanning ranges of the fitting parameters \( v_0 \) and \( v_e \) are shrunk to between 140 km/s and 240 km/s and between 200 km/s and 310 km/s, respectively.

\( \Delta v \) show clearly the (approximately) symmetric periodic variations of 262.7 (± ∼ 13) km/s and 78.6 (± ∼ 12) km/s, respectively. Additionally, these annual variations would now be comparable with (or even larger than) their strongly reduced 1σ statistical uncertainties. Remind that, as shown in Figs. 44(b) and 45(b), the theoretical values of the fitting parameters (the red upward–triangle) are now clearly out of the (light–blue) 2σ statistical uncertainty contours.

In Tables 13 and 14 we summarize the reconstructed results of the fitting parameters \( v_0 \), \( v_e \) and \( \Delta v \) as well as their 1(2)σ statistical uncertainty ranges of the median values by using the simplified and the modified velocity distribution with 500 total events on average in each 60-day observation period of the advanced seasons, respectively.

### 5.2.3 Fitted radial WIMP velocity distribution in the Galactic frame

Finally, we present here also our Bayesian reconstruction result of the radial distribution of the 3-D WIMP velocity in the Galactic coordinate system with 500 total events on average recorded in one entire year. Note that the scanning range of the fitting parameter has been shrunk to
Figure 45: As in Figs. 29, reconstructed with the modified velocity distribution $f_{1,sh,\Delta v}(v; v_0, \Delta v)$, except that 500 total events on average in one entire year have been simulated. Note that the scanning ranges of the fitting parameters $v_0$ and $\Delta v$ are shrunk to between 140 km/s and 240 km/s and between $-20$ km/s and 150 km/s, respectively.

$190 \text{ km/s} \leq v_0 \leq 250 \text{ km/s}$.

In Figs. 50, we show the reconstructed radial distributions of the 3-D WIMP velocity and the $1(2)\sigma$ statistical uncertainty bands by using the simple Maxwellian velocity distribution $f_{1,Gau}(v)$ given by Eq. (2). As in Sec. 4.4, our best–fit reconstruction distributions could match the input simple Maxwellian velocity distribution perfectly and the fitting parameter $v_0$ could also be pinned down very precisely with the $\sim 1/3$ reduced statistical uncertainties.

In Table 15, we summarize the reconstructed results of the fitting parameter $v_0$ and their $1(2)\sigma$ statistical uncertainty ranges of the median values by using the simple Maxwellian velocity distribution with 500 total events on average in one entire year shown in Figs. 50.

6 Summary and conclusions

In this paper, as a preparation for our future study aiming to develop data analysis procedures using and/or combining 3-dimensional information offered by directional Dark Matter detection.
Radial component of the 3-D WIMP velocity distribution in the Equatorial frame, 19.49 – 79.49 day, 500 events

Radial component of the 3-D WIMP velocity distribution in the Equatorial frame, 293.24 - 353.24 day, 500 events

Radial component of the 3-D WIMP velocity distribution in the Equatorial frame, 19.49 - 79.49 day, 500 events

Radial component of the 3-D WIMP velocity distribution in the Equatorial frame, 201.99 - 261.99 day, 500 events

Radial component of the 3-D WIMP velocity distribution in the Equatorial frame, 110.74 - 170.74 day, 500 events

Radial component of the 3-D WIMP velocity distribution in the Equatorial frame, 201.99 - 261.99 day, 500 events

Radial component of the 3-D WIMP velocity distribution in the Equatorial frame, 293.24 – 353.24 day, 500 events

Radial component of the 3-D WIMP velocity distribution in the Equatorial frame, 293.24 – 353.24 day, 500 events

(a) 19.49 – 79.49 day

(b) 110.74 – 170.74 day

(c) 201.99 – 261.99 day

(d) 293.24 – 353.24 day

Figure 46: As in Figs. 30, reconstructed with the one-parameter velocity distribution \( f_{1,sh,m}(v; v_0) \), except that 500 total events on average in each 60-day observation period of the advanced seasons have been simulated.

...
Figure 47: As in Figs. 31, reconstructed with the \(v_0\)-fixed velocity distribution \(f_{1,sh,v_0}(v; v_e)\), except that 500 total events on average in each 60-day observation period of the advanced seasons have been simulated.
Radial component of the 3-D WIMP velocity distribution in the Equatorial frame,

(a) 19.49 – 79.49 day

(b) 110.74 – 170.74 day

(c) 201.99 – 261.99 day

(d) 293.24 – 353.24 day

Figure 48: As in Figs. 32 reconstructed with the simplified velocity distribution $f_{1,sh}(v; v_0, v_e)$, except that 500 total events on average in each 60-day observation period of the advanced seasons have been simulated.
Radial component of the 3-D WIMP velocity distribution in the Equatorial frame, 201.99 – 261.99 day, 500 events

Radial component of the 3-D WIMP velocity distribution in the Equatorial frame, 110.74 - 170.74 day, 500 events

Radial component of the 3-D WIMP velocity distribution in the Equatorial frame, 293.24 - 353.24 day, 500 events

Radial component of the 3-D WIMP velocity distribution in the Equatorial frame, 19.49 - 79.49 day, 500 events

Figure 49: As in Figs. 33, reconstructed with the modified velocity distribution $f_{1,sh,\Delta v}(v; v_0, \Delta v)$, except that 500 total events on average in each 60-day observation period of the advanced seasons have been simulated.
Table 10: The summary of the reconstructed results of the fitting parameters and their $1(2)\sigma$ statistical uncertainty ranges of the median values by using four considered fitting velocity distributions with 500 total events on average in one entire year shown in Figs. 42, 43, 44, and 45.

| Fitting dist. | Parameter | Max. $P_{\text{median}}$ | Median | $1\sigma$ range | $2\sigma$ range |
|---------------|-----------|---------------------------|--------|----------------|-----------------|
| $f_{1,sh,v_0}(v)$ | $v_0$ [km/s] | 214.5 | 214.5 $\pm$ 3.5 (± 16.2) | [211.0, 218.0] | [208.0, 221.0] |
| $f_{1,sh,v_1}(v)$ | $v_0$ [km/s] | 225.0 | 225.0 $\pm$ 8.1 ($\pm 16.2$) | [216.0, 233.1] | [207.9, 241.2] |
| $f_{1,sh}(v)$ | $v_0$ [km/s] | 186.0 | 185.0 $\pm$ 10.0 ($\pm 20.0$) | [177.0, 195.0] | [169.0, 205.0] |
| $f_{1,sh,\Delta v}(v)$ | $v_0$ [km/s] | 186.0 | 185.0 $\pm$ 10.0 ($\pm 20.0$) | [177.0, 195.0] | [169.0, 205.0] |
| $\Delta v$ [km/s] | 75.2 | 76.9 $\pm$ 17.0 ($\pm 32.3$) | [58.2, 93.9] | [36.1, 109.2] |

500 total events on average in the observation period of 0 – 365 day

Table 11: The summary of the reconstructed results of the fitting parameter $v_0$ and their $1(2)\sigma$ statistical uncertainty ranges of the median values by using the one–parameter velocity distribution $f_{1,sh,v_0}(v; v_0)$ with 500 total events on average in each 60–day observation period of the advanced seasons.

| Central date (observation period) (day) | Parameter | Max. $P_{\text{median}}$ | Median | $1\sigma$ range | $2\sigma$ range |
|---------------------------------------|-----------|---------------------------|--------|----------------|-----------------|
| 49.49 (19.49 – 79.49) | $v_0$ [km/s] | 214.5 | 214.5 $\pm$ 3.0 ($\pm 16.5$) | [211.0, 217.5] | [208.0, 221.0] |
| 140.74 (110.74 – 170.74) | $v_0$ [km/s] | 220.5 | 220.5 $\pm$ 3.5 ($\pm 16.5$) | [217.0, 224.0] | [214.0, 227.0] |
| 231.99 (201.99 – 261.99) | $v_0$ [km/s] | 214.5 | 214.5 $\pm$ 3.5 ($\pm 16.5$) | [211.5, 218.0] | [208.0, 221.0] |
| 323.24 (293.24 – 353.24) | $v_0$ [km/s] | 208.5 | 208.5 $\pm$ 3.5 ($\pm 16.5$) | [205.5, 212.0] | [202.0, 215.0] |

500 total events on average in one observation period of 60 days
### Fitting distribution: $v_0$–fixed velocity distribution $f_{1,sh,v_e}(v; v_e)$

| Central date (observation period) (day) | Parameter $v_e$ [km/s] | Max. $P_{\text{median}}$ | Median $v_e$ [km/s] | $1\sigma$ range | $2\sigma$ range |
|----------------------------------------|------------------------|--------------------------|---------------------|-----------------|-----------------|
| 49.49 (19.49 – 79.49)                  | 225.0                  | 225.0 $^{+7.2}_{-9.0}$ ($^{+15.3}_{-17.1}$) | [216.0, 232.2]     | [207.9, 240.3]  |
| 140.74 (110.74 – 170.74)              | 240.3                  | 240.3 $^{+7.2}_{-8.1}$ ($^{+15.3}_{-17.1}$) | [232.2, 247.5]     | [223.2, 255.6]  |
| 231.99 (201.99 – 261.99)              | 225.0                  | 225.0 $\pm$ 8.1 ($^{+16.2}_{-17.1}$) | [216.9, 233.1]     | [207.9, 241.2]  |
| 323.24 (293.24 – 353.24)              | 207.9                  | 207.9 $^{+9.0}_{-8.1}$ ($^{\pm17.1}$) | [199.8, 216.9]     | [190.8, 225.0]  |

500 total events on average in one observation period of 60 days

Table 12: The summary of the reconstructed results of the fitting parameter $v_e$ and their $1(2)\sigma$ statistical uncertainty ranges of the median values by using the $v_0$–fixed velocity distribution $f_{1,sh,v_e}(v; v_e)$ with 500 total events on average in each 60–day observation period of the advanced seasons.

Experiments, to, e.g., reconstruct the 3-dimensional WIMP velocity distribution, we simulated 3-D velocity information (magnitude, direction and measuring time) of WIMP events in the Galactic coordinate system, transformed them to different celestial coordinate systems, and

![Radial component of the 3-D WIMP velocity distribution in the Galactic frame, 0 – 365 day, 500 events](a)

![Distribution of the fitting parameter $v_0$, scan resolution: 100 points](b)

Figure 50: As in Figs. 34, reconstructed with the simple Maxwellian velocity distribution $f_{1,\text{Gau}}(v)$, except that 500 total events on average in one entire year have been simulated. Note that the scanning range of the fitting parameter $v_0$ is shrunk to between 190 km/s and 250 km/s.
then investigated the angular distribution patterns of the 3-dimensional WIMP velocity as well as reconstructed its radial distribution (magnitude) by applying the Bayesian fitting technique.

Our simulations indicate that, first, with $O(50)$ total WIMP events recorded in one entire year, the angular distribution patterns of the 3-D WIMP velocity transformed to the Ecliptic and Equatorial coordinate systems would already show clear anisotropy. And, more precisely, the most frequent directions of the 3-D WIMP velocity would be close to, but somehow deviate from the theoretically predicted direction of the WIMP wind, i.e., the opposite direction of the Solar movement in our Galaxy. Once $O(500)$ total events could be accumulated and thus a higher analysis resolution would be used, the systematic bias could be reduced, but a small deviation might still exist.

Moreover, for demonstrating the “annual” modulation of the main direction of the WIMP wind due to the Earth’s orbital motion around the Sun, we considered two sets of observation periods: four normal seasons and four advanced seasons. $O(50)$ total recorded events in 60 observation days ($\pm 30$ days from the central date) have been considered for each season. It has been found that, with only $O(50)$ total WIMP events recorded in one season, the pattern of the angular distribution of the 3-D WIMP velocity in the Equatorial coordinate system would already vary slightly but clearly with a clockwise rotation around the theoretical main direction of incident WIMPs. This would be, besides the pure “directionality” of the WIMP wind, a second (important) characteristic for identifying directional WIMP signals and discriminating from any (unexpected) backgrounds from some specified incoming directions. Meanwhile, the

Table 13: The summary of the reconstructed results of the fitting parameters $v_0$ and $v_e$ as well as their $1(2)$σ statistical uncertainty ranges of the median values by using the simplified velocity distribution $f_{1,sh}(v; v_0, v_e)$ with 500 total events on average in each 60–day observation period of the advanced seasons.

| Central date (observation period) (day) | Parameter | Max. P$_{\text{median}}$ | Median | 1σ range | 2σ range |
|----------------------------------------|-----------|--------------------------|--------|----------|----------|
| 49.49 (19.49 – 79.49)                  | $v_0$ [km/s] | 185.0                    | 185.0 ± 9.0 ($^{+20.0}_{-16.0}$) | [176.0, 194.0] | [169.0, 205.0] |
|                                       | $v_e$ [km/s] | 262.7                    | 262.7 $^{+9.9}_{-11.0}$ ($^{+18.7}_{-24.2}$) | [251.7, 272.6] | [238.5, 281.4] |
| 140.74 (110.74 – 170.74)              | $v_0$ [km/s] | 186.0                    | 185.0 $^{+9.9}_{-8.0}$ ($^{+19.0}_{-16.0}$) | [177.0, 194.0] | [169.0, 204.0] |
|                                       | $v_e$ [km/s] | 274.8                    | 274.8 $^{+8.8}_{-9.9}$ ($^{+17.6}_{-22.0}$) | [264.9, 283.6] | [252.8, 292.4] |
| 231.99 (201.99 – 261.99)              | $v_0$ [km/s] | 185.0                    | 185.0 $^{+9.9}_{-8.0}$ ($^{+19.0}_{-16.0}$) | [176.0, 194.0] | [169.0, 204.0] |
|                                       | $v_e$ [km/s] | 262.7 $^{+9.9}_{-8.0}$   | 262.7 $^{+9.9}_{-8.0}$ ($^{+18.7}_{-24.2}$) | [251.7, 272.6] | [238.5, 281.4] |
| 323.24 (293.24 – 353.24)              | $v_0$ [km/s] | 186.0                    | 185.0 $^{+10.0}_{-9.0}$ ($^{+21.0}_{-17.0}$) | [176.0, 195.0] | [168.0, 206.0] |
|                                       | $v_e$ [km/s] | 248.4 $^{+9.9}_{-11.0}$ ($^{+18.7}_{-26.4}$) | 249.5 $^{+9.9}_{-11.0}$ ($^{+18.7}_{-26.4}$) | [238.5, 259.4] | [223.1, 268.2] |

500 total events on average in one observation period of 60 days
Table 14: The summary of the reconstructed results of the fitting parameters $v_0$ and $\Delta v$ as well as their $1(2)\sigma$ statistical uncertainty ranges of the median values by using the modified velocity distribution $f_{1,sh,\Delta v}(v; v_0, \Delta v)$ with 500 total events on average in each 60–day observation period of the advanced seasons.

Table 15: The summary of the reconstructed results of the fitting parameter $v_0$ and their $1(2)\sigma$ statistical uncertainty ranges of the median values by using the simple Maxwellian velocity distribution with total events on average in one entire year shown in Figs. 50.

71
strated also the laboratory–dependent diurnal modulation of the angular WIMP velocity distribution pattern in our laboratory coordinate system with $O(50)$ total WIMP events recorded in each 4-hour daily shift (in the 60-day observation period). Not surprisingly, such a laboratory–dependent diurnal modulation shows a similar variation pattern to the annual modulation in our horizontal coordinate system, requires however much more recorded events.

On the other hand, we found unexpectedly that, with only $O(50)$ total WIMP events recorded in one entire year, the radial component of the transformed 3-D WIMP velocity would already show a distribution shape, which is a little bit more concentrated than the theoretically derived shifted Maxwellian velocity distribution in the range around the average WIMP velocity, but a little bit reduced in the lower and higher velocity ranges. By using the Bayesian fitting technique, we obtained also slightly smaller best–fit values of the fitting parameter $v_0$ (under the constraint of $v_e = 1.05 v_0$) and $v_e$ (under the constraint of $v_0 = 220 \text{ km/s}$), although the reconstructed velocity distributions could match the theoretical velocity distribution very well.

However, once we release the constraints on $v_0$ and $v_e$ and treat them as two independent fitting parameters, the reconstructed velocity distributions would have a clearly different shape from the theoretical prediction, although, with only $O(50)$ total events recorded in one entire year and thus a relatively large statistical uncertainties, the $1\sigma$ statistical uncertainty bands of the reconstructed velocity distributions could still cover the theoretical distribution in most part of the velocity range; the best–fit values of the fitting parameters $v_0$ and $v_e$ would also be $\sim 16\%$ smaller and $\sim 14\%$ larger than their theoretical values, respectively.

Once we raise the event number to $O(500)$ total WIMP events, the best–fit velocity distributions would differ pretty strongly from the theoretical distribution and their narrower $1\sigma$ statistical uncertainty bands could not cover the expected curve in most part of the velocity ranges; the fitting parameters $v_0$ and $v_e$ would also be $3\sigma – 4\sigma$ smaller or larger than their theoretical values, respectively. Additionally, our reconstructed radial distributions as well as their $1(2)\sigma$ statistical uncertainty bands seems not to fit the radial distribution of the simulated 3-D WIMP velocity in the middle velocity range very well. These would imply not only that the radial distribution of the 3-D WIMP velocity in the Equatorial as well as the in laboratory coordinate systems would differ from the theoretically derived analytic form, but also a requirement of the modification of the analytic form of the fitting velocity distribution function.

Nevertheless, with $O(50)$ total WIMP events recorded in each (60-day) season, the annual modulation of the radial distribution of the 3-D WIMP velocity could already be confirmed very clearly by observing the $\sim 5\%$ annual variation of the best–fit values of the second fitting parameter $v_e$, in contrast and interestingly, the best–fit values of the first fitting parameter $v_0$ in all observation periods stay fixed, although it is also unconstraint.

In our simulations presented in this paper, 50 and 500 total WIMP events on average in (one daily shift of) one observation period (365 days/year, 60 days/season, or 4 hours/shift $\times$ 60 days) have been considered. In this and probably the next decades, even only $O(50)$ total WIMP signals (per year) to observe would be a strong challenge for all (non)directional direct Dark Matter detection experiments. Fortunately, the recorded 3-D velocity information of WIMP signals offered by different underground laboratories could (in principle) be transformed to all laboratory–independent (Galactic, Ecliptic, Equatorial, and Earth) coordinate systems. Hence, data offered by more than one directional DM detection experiment could be combined for:

- the identification of the anisotropy of the angular velocity distribution of incident WIMPs;
- the demonstration of the annual modulation of the angular velocity distribution pattern of incident WIMPs;


• the Bayesian reconstruction of the radial distribution of the 3-D WIMP velocity;
• the demonstration of the annual modulation of the (Bayesian reconstructed) radial distribution of the 3-D WIMP velocity.

These analyses could be applied for reconstructing the 3-dimensional WIMP velocity distribution in both of the Equatorial and Galactic coordinate systems, for checking our theoretical models of Galactic Dark Matter halo.

In the long term, with enough WIMP events observed in one underground laboratory, the angular distribution pattern of the 3-D WIMP velocity in the laboratory–dependent (horizontal and laboratory) coordinate systems could be used to demonstrate:

• the diurnal modulation of the angular velocity distribution pattern of incident WIMPs;
• the latitude–dependence of the angular velocity distribution pattern of incident WIMPs;

Regarding the observation periods considered in our simulations presented in this paper, we used several approximations about the Earth’s orbital motion in the Solar system. First, the Earth’s orbit around the Sun is perfectly circular on the Ecliptic plane and the orbital speed is thus a constant. Second, the date of the vernal equinox is exactly fixed at the end of the 79th day (May 20th) of a 365-day year and the few extra hours in an actual Solar year has been neglected. Nevertheless, considering the very low WIMP scattering event rate and thus only a few tens of total (combined) WIMP events required in at least a few tens of days (an overall event rate of ~ 1 event/day) for the first–phase analyses, these approximations should be acceptable.

For our simulations presented in this paper, we also assumed implicitly that 3-D information on the velocity of incident WIMPs (magnitudes and directions/angles) can be measured by using data from directional detection experiments. However, so far we could only measure, besides recoil energies, recoil tracks and in turn recoil angles of the recoiled nuclei. A way to use the experimental measurable data (recoil energies and recoil tracks/angles of the recoiled nuclei) to reconstruct 3-D WIMP velocities requires more investigations.

Furthermore, for generating the 3-D WIMP velocity we considered in this paper only the simplest isotropic Maxwellian velocity distribution. Effects of other theoretically predicted structures of Galactic Dark Matter halo, including some possible streams and/or the bulk rotation discussed in the literature, and the possibility of discriminating these models would also be interesting to investigate.

In summary, we have begun to extend our earlier works and to study how to use information provided by directional Dark Matter detection experiments to understand the 3-dimensional (velocity) distribution of Galactic DM particles. Hopefully this and more works achieved in the future could offer useful information on the structure of our Galaxy as well as for data analyses in indirect Dark Matter detection experiments.

Acknowledgments

The author appreciates N. Bozorgnia and P. Gondolo for useful discussions about the transformations between the celestial coordinate systems. The author would also like to thank the friendly hospitality of the Gran Sasso Science Institute, the School of Physics at the University of New South Wales, the School of Physics and Astronomy at the Monash University, and the Department of Nuclear Physics at the Australian National University, during the completion and finalization of this work.
Figure A1: A sketch of the conventional definition of the (dark-green) horizontal/laboratory coordinate system: it is basically the same as our definition of the horizontal coordinate system described in Sec. 2.1.4 and sketched in Fig. 5, except that the primary direction (the $X_{H/Lab}$–axis) points here towards east. Our (light-green) Earth coordinate system is also given here.

A Definitions of and transformation between the celestial coordinate systems

In this section, we describe the definitions of the celestial coordinate systems used in our simulations and data analyses presented in this paper as well as the transformation matrices between these coordinate systems in detail. In order to avoid possible confusion, the conventional astronomical definitions of these coordinate systems will also be mentioned.

A.1 Definitions of the horizontal and the laboratory coordinate systems

Firstly, in order to connect the celestial (laboratory–independent) (Galactic, Ecliptic, and Equatorial) coordinate systems with the geographic (laboratory–dependent) (horizontal and laboratory) coordinate systems, we define particularly the Earth coordinate system as described in Sec. 2.1.3 and sketched in Fig. 4.

Then we discuss at first two coordinate systems depending on the geographic location of the underground laboratory of interest as well as the measuring time of WIMP scattering events.

A.1.1 Conventional definition

Conventionally, as shown in Fig. A1, the horizontal/laboratory coordinate system is defined with the origin at the geographic location of the laboratory of interest at, e.g., 12 midnight (the beginning) of (the UTC time of) each single Solar day, the primary direction (the $X_{H/Lab}$–axis) and the $Z_{H/Lab}$–axis pointing towards east and the zenith, respectively, and the right–handed convention for defining the $Y_{H/Lab}$–axis.
In Fig. A1, we sketch the conventionally defined horizontal/laboratory coordinate system with our Earth coordinate system together. It can easily find out that, by rotating the conventional horizontal/laboratory coordinate system at first \(-\frac{\pi}{2} - \theta\) around the \(X_{H/Lab}\)–axis and then \(-\frac{\pi}{2} + \phi\) around the \(Z_{H/Lab}\)–axis, one can obtain our Earth coordinate system. Hence, the transformation matrix from the conventional horizontal/laboratory coordinate system to our Earth coordinate system can be expressed by

\[
M_{H \rightarrow E, \text{conv}} = \begin{bmatrix}
\cos \left[-\left(\frac{\pi}{2} + \phi\right)\right] & \sin \left[-\left(\frac{\pi}{2} + \phi\right)\right] & 0 \\
-\sin \left[-\left(\frac{\pi}{2} + \phi\right)\right] & \cos \left[-\left(\frac{\pi}{2} + \phi\right)\right] & 0 \\
0 & 0 & 1
\end{bmatrix}
\times
\begin{bmatrix}
1 & 0 & 0 \\
0 & \cos \left[-\left(\frac{\pi}{2} - \theta\right)\right] & \sin \left[-\left(\frac{\pi}{2} - \theta\right)\right] \\
0 & -\sin \left[-\left(\frac{\pi}{2} - \theta\right)\right] & \cos \left[-\left(\frac{\pi}{2} - \theta\right)\right]
\end{bmatrix}
\]

\[
= \begin{bmatrix}
-\sin \phi & -\cos \phi \sin \theta & \cos \phi \cos \theta \\
\cos \phi & -\sin \phi \sin \theta & \sin \phi \cos \theta \\
0 & \cos \theta & \sin \theta
\end{bmatrix}, \quad (A1a)
\]

where \(\phi\) and \(\theta\) are the longitude and the latitude of the location of the laboratory, respectively. Conversely, the transformation matrix from our Earth coordinate system to the conventional horizontal/laboratory coordinate system can be given directly as

\[
M_{E \rightarrow H, \text{conv}} = M_{H \rightarrow E, \text{conv}}^T = \begin{bmatrix}
-\sin \phi & \cos \phi & 0 \\
-\cos \phi \sin \theta & -\sin \phi \sin \theta & \cos \theta \\
\cos \phi \cos \theta & \sin \phi \cos \theta & \sin \theta
\end{bmatrix}. \quad (A1b)
\]

A.1.2 Our definitions

In order to understand and compare the angular distribution patterns of the 3-D WIMP velocity offered by different underground laboratories more easily, as shown in Fig. 5 for our simulations presented in this paper we define the horizontal coordinate system by rotating the conventional coordinates \(90^\circ\) around the \(Z_{H/Lab}\)–axis, so that the \(X_H\)–axis points now towards north. Remind that, as our Earth coordinate system, for each single Solar day, our horizontal coordinate system are fixed at 12 midnight (the beginning) of (the UTC time of) the day.

By this definition, the transformation from our horizontal coordinate system to our Earth coordinate system can be done by rotating at first \(\pi/2 - \theta\) around the \(Y_H\)–axis and then \(\pi - \phi\) around the \(Z_H\)–axis. Thus the transformation matrix can be given by

\[
M_{H \rightarrow E} = \begin{bmatrix}
\cos (\pi - \phi) & \sin (\pi - \phi) & 0 \\
-\sin (\pi - \phi) & \cos (\pi - \phi) & 0 \\
0 & 0 & 1
\end{bmatrix}
\]
\[
\begin{bmatrix}
\cos (\pi/2 - \theta) & 0 & -\sin (\pi/2 - \theta) \\
0 & 1 & 0 \\
\sin (\pi/2 - \theta) & 0 & \cos (\pi/2 - \theta)
\end{bmatrix}
\times
\begin{bmatrix}
-\cos \phi \sin \theta & \sin \phi & \cos \phi \cos \theta \\
-\sin \phi \sin \theta & -\cos \phi & \sin \phi \cos \theta \\
\cos \theta & 0 & \sin \theta
\end{bmatrix}
=\begin{bmatrix}
-\cos \phi \sin \theta & \sin \phi \sin \theta & \cos \theta \\
\sin \phi & -\cos \phi & 0 \\
\cos \phi \cos \theta & \sin \phi \cos \theta & \sin \theta
\end{bmatrix},
\]  
(A2a)

and then, conversely, we have

\[
\mathbf{M}_{E\rightarrow H} = \mathbf{M}_{H\rightarrow E}^T =
\begin{bmatrix}
-\cos \phi \sin \theta & \sin \phi \sin \theta & \cos \theta \\
\sin \phi & -\cos \phi & 0 \\
\cos \phi \cos \theta & \sin \phi \cos \theta & \sin \theta
\end{bmatrix}
\]  
(A2b)

On the other hand, considering the long running time of (directional) direct Dark Matter detection experiments as well as for identifying the diurnal modulation of the angular WIMP velocity distribution, we also define the laboratory coordinate system by taking into account the instantaneous measuring time of each recorded WIMP scattering event. As sketched in Fig. 6 it is defined by rotating our horizontal coordinate system of the laboratory of interest around the Earth’s north polar \((\mathbf{Z}_{\mathbf{E}q}/\mathbf{Z}_{\mathbf{E}q})\) axis instantaneously by the angle of \(\omega t_{\text{PM}}\), where we define

\[
\omega \equiv \frac{2\pi}{1 \text{ day}},
\]  
(1)

and \(t_{\text{PM}}\) indicates the measuring UTC time of each recorded WIMP event in unit of day. Hence, by adding the longitude of the laboratory of interest in the transformation matrices (A2a) and (A2b) this extra time–dependent term:

\[
\phi \rightarrow \phi + \omega t_{\text{PM}},
\]  
(A3)

we can obtain the transformation matrices between the laboratory and the Earth coordinate systems straightforwardly as

\[
\mathbf{M}_{\text{Lab}\rightarrow E} = 
\begin{bmatrix}
-\cos (\phi + \omega t_{\text{PM}}) \sin \theta & \sin (\phi + \omega t_{\text{PM}}) & \cos (\phi + \omega t_{\text{PM}}) \cos \theta \\
-\sin (\phi + \omega t_{\text{PM}}) \sin \theta & -\cos (\phi + \omega t_{\text{PM}}) & \sin (\phi + \omega t_{\text{PM}}) \cos \theta \\
\cos \theta & 0 & \sin \theta
\end{bmatrix},
\]  
(A4a)

and

\[
\mathbf{M}_{E\rightarrow \text{Lab}} = 
\begin{bmatrix}
-\cos (\phi + \omega t_{\text{PM}}) \sin \theta & -\sin (\phi + \omega t_{\text{PM}}) \sin \theta & \cos \theta \\
\sin (\phi + \omega t_{\text{PM}}) & -\cos (\phi + \omega t_{\text{PM}}) & 0 \\
\cos (\phi + \omega t_{\text{PM}}) \cos \theta & \sin (\phi + \omega t_{\text{PM}}) \cos \theta & \sin \theta
\end{bmatrix}.
\]  
(A4b)
A.2 Definitions of the Ecliptic and the Equatorial coordinate systems

Now we turn to define the most important coordinate systems in our data analysis procedures: the Ecliptic and the Equatorial coordinate systems. The transformation matrices between the Earth and these two coordinate systems will be derived in detail here.

A.2.1 Conventional definitions

As shown in Fig. [A2], in astronomy the Ecliptic and the Equatorial coordinate systems are conventionally defined as follows: their origins are located at the center of the Sun and that of the Earth.
the Earth, respectively, the common primary direction (the $X_S/X_{Eq}$-axis) is now the direction pointing from the Earth’s center to that of the Sun at 12 midnight (the end) of the date of the vernal equinox (i.e., pointing towards the celestial Equinox), the fundamental ($X_S-Y_S$ and $X_{Eq}-Y_{Eq}$) planes are the Ecliptic plane and the Equatorial plane, respectively, the $Z_S$- and $Z_{Eq}$-axes are perpendicular to the Ecliptic or the Equatorial plane, respectively, and their $Y_S$- and $Y_{Eq}$-axes are then defined as usual by the right–handed convention.

A.2.2 Our definitions

Our definitions of the Ecliptic and the Equatorial coordinate systems are described in Sec. 2.1.2 and sketched in Fig. 3(a). Note only that, first, the Equatorial coordinate system is fixed and moves with the Earth, during the Earth’s orbital motion around the Sun combined with the motion of the Solar system in the Galaxy. Second, since in our definitions the common primary direction (the $X_S/X_{Eq}$-axis) point towards the opposite direction of the conventional definition, in our calculations given in Appendix A.3 there is always a $180^\circ$ (or $-90^\circ$) difference from the conventional data (and those used in Ref. [63]).

In Fig. A3, we sketch the relations between the Earth, the Equatorial, and the Ecliptic coordinate systems (cf. Fig. 4). Here $\psi_{yr}$ indicates the angle swept by the connection between the Solar and the Earth’s centers, $r_{yr}$, from the day of the vernal equinox, $\psi_\oplus = 23.4^\circ$ is the Earth’s obliquity, and $\psi_{PM}$ is the angle between the $X_{Eq}$- and the $X_E$-axes. It can be found that, firstly, the Equatorial coordinate system can be obtained by simply rotating the Ecliptic coordinate system by the Earth’s obliquity $\psi_\oplus = 23.4^\circ$ around the common $X_S/X_{Eq}$-axis. Hence, the transformation matrix from the Ecliptic to the Equatorial coordinate system can be
given directly as

\[ M_{S \rightarrow Eq} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos \psi_{\oplus} & \sin \psi_{\oplus} \\ 0 & -\sin \psi_{\oplus} & \cos \psi_{\oplus} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0.91775 & 0.39715 \\ 0 & -0.39715 & 0.91775 \end{bmatrix} . \]

(A6a)

and, conversely,

\[ M_{Eq \rightarrow S} = M_{S \rightarrow Eq}^T = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos \psi_{\oplus} & -\sin \psi_{\oplus} \\ 0 & \sin \psi_{\oplus} & \cos \psi_{\oplus} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0.91775 & -0.39715 \\ 0 & 0.39715 & 0.91775 \end{bmatrix} . \]

(A6b)

On the other hand, following the calculations done by A. Bandyopadhyay and D. Majumdar in Ref. \[63\], since \( \mathbf{r}_{y \gamma} \) is the intersection vector of the \( \mathbf{X}_E - \mathbf{Z}_E \) plane and the Ecliptic (\( \mathbf{X}_S - \mathbf{Y}_S \)) plane, one has

\[ \mathbf{r}_{y \gamma} = \cos \psi_{y \gamma} \mathbf{X}_S + \sin \psi_{y \gamma} \mathbf{Y}_S \\
= \cos \psi_{y \gamma} \mathbf{X}_{Eq} + \sin \psi_{y \gamma} \cos \psi_{\oplus} \mathbf{Y}_{Eq} - \sin \psi_{y \gamma} \sin \psi_{\oplus} \mathbf{Z}_{Eq}, \]

(A7)

and the vector perpendicular to the \( \mathbf{Z}_E - \mathbf{X}_E - \mathbf{r}_{y \gamma} \) plane (i.e., parallel to the \( \mathbf{Y}_E - \mathbf{axis} \)) can be obtained by

\[ \mathbf{Z}_E \times \mathbf{r}_{y \gamma} = \mathbf{Z}_{Eq} \times \left( \cos \psi_{y \gamma} \mathbf{X}_{Eq} + \sin \psi_{y \gamma} \cos \psi_{\oplus} \mathbf{Y}_{Eq} - \sin \psi_{y \gamma} \sin \psi_{\oplus} \mathbf{Z}_{Eq} \right) \\
= \cos \psi_{y \gamma} \mathbf{Y}_{Eq} - \sin \psi_{y \gamma} \cos \psi_{\oplus} \mathbf{X}_{Eq}. \]

(A8)

Here the angle \( \psi_{y \gamma} \) can be estimated by the measuring time \( t \) of one WIMP event as

\[ \psi_{y \gamma}(t) \equiv \frac{2\pi}{365} \left[ (t - t_{PM}) - 79.0 \right]. \]

(A9)

Then the vector perpendicular to the \( \mathbf{Y}_E - \mathbf{Z}_E \) plane (i.e., parallel to the \( \mathbf{X}_E - \mathbf{axis} \)) can be given by

\[ \left( \mathbf{Z}_E \times \mathbf{r}_{y \gamma} \right) \times \mathbf{Z}_E = \left( \cos \psi_{y \gamma} \mathbf{Y}_{Eq} - \sin \psi_{y \gamma} \cos \psi_{\oplus} \mathbf{X}_{Eq} \right) \times \mathbf{Z}_{Eq} \\
= \cos \psi_{y \gamma} \mathbf{X}_{Eq} + \sin \psi_{y \gamma} \cos \psi_{\oplus} \mathbf{Y}_{Eq}, \]

(A10)

and its length can thus be obtained as

\[ \left| \left( \mathbf{Z}_E \times \mathbf{r}_{y \gamma} \right) \times \mathbf{Z}_E \right| = \sqrt{\cos^2 \psi_{y \gamma} + \sin^2 \psi_{y \gamma} \cos^2 \psi_{\oplus}} . \]

(A11)

Hence, the unit vector \( \mathbf{X}_E \) of the Earth coordinate system can be expressed as

\[ \mathbf{X}_E = \frac{(\mathbf{Z}_E \times \mathbf{r}_{y \gamma}) \times \mathbf{Z}_E}{\left| (\mathbf{Z}_E \times \mathbf{r}_{y \gamma}) \times \mathbf{Z}_E \right|} \\
= \frac{\cos \psi_{y \gamma} \mathbf{X}_{Eq} + \sin \psi_{y \gamma} \cos \psi_{\oplus} \mathbf{Y}_{Eq}}{\sqrt{\cos^2 \psi_{y \gamma} + \sin^2 \psi_{y \gamma} \cos^2 \psi_{\oplus}}} \\
= \gamma \cos \psi_{y \gamma} \mathbf{X}_{Eq} + \gamma \sin \psi_{y \gamma} \cos \psi_{\oplus} \mathbf{Y}_{Eq} , \]

(A12a)
and thus
\[ Y_E = Z_E \times X_E = Z_{Eq} \times \left( \gamma \cos \psi_{yr} X_{Eq} + \gamma \sin \psi_{yr} \cos \psi_\oplus Y_{Eq} \right) = \gamma \cos \psi_{yr} Y_{Eq} - \gamma \sin \psi_{yr} \cos \psi_\oplus X_{Eq}, \] (A12b)

where we define
\[ \gamma \equiv \frac{1}{\sqrt{\cos^2 \psi_{yr} + \sin^2 \psi_{yr} \cos^2 \psi_\oplus}}. \] (A13)

Therefore, the transformation matrix from the Equatorial to the Earth coordinate system can be given by
\[
M_{\text{Eq}\rightarrow\text{E}} = \begin{bmatrix}
\gamma \cos \psi_{yr} & \gamma \sin \psi_{yr} \cos \psi_\oplus & 0 \\
-\gamma \sin \psi_{yr} \cos \psi_\oplus & \gamma \cos \psi_{yr} & 0 \\
0 & 0 & 1
\end{bmatrix},
\] (A14a)

and, conversely,
\[
M_{\text{E}\rightarrow\text{Eq}} = M_{\text{Eq}\rightarrow\text{E}}^T = \begin{bmatrix}
\gamma \cos \psi_{yr} & -\gamma \sin \psi_{yr} \cos \psi_\oplus & 0 \\
\gamma \sin \psi_{yr} \cos \psi_\oplus & \gamma \cos \psi_{yr} & 0 \\
0 & -\sin \psi_\oplus & \cos \psi_\oplus
\end{bmatrix}.
\] (A14b)

Finally, by combining the matrices given in Eqs. (A6a) and (A14a), the transformation matrix from the Ecliptic to the Earth coordinate system can be obtained by
\[
M_{\text{S}\rightarrow\text{E}} = M_{\text{Eq}\rightarrow\text{E}} M_{\text{S}\rightarrow\text{Eq}} = \begin{bmatrix}
\gamma \cos \psi_{yr} & \gamma \sin \psi_{yr} \cos^2 \psi_\oplus & \gamma \sin \psi_{yr} \sin \psi_\oplus \cos \psi_\oplus \\
-\gamma \sin \psi_{yr} \cos \psi_\oplus & \gamma \cos \psi_{yr} \cos \psi_\oplus & \gamma \cos \psi_{yr} \sin \psi_\oplus \\
0 & -\sin \psi_\oplus & \cos \psi_\oplus
\end{bmatrix},
\] (A15a)

and, conversely,
\[
M_{\text{E}\rightarrow\text{S}} = M_{\text{S}\rightarrow\text{E}}^T = \begin{bmatrix}
\gamma \cos \psi_{yr} & -\gamma \sin \psi_{yr} \cos \psi_\oplus & 0 \\
\gamma \sin \psi_{yr} \cos^2 \psi_\oplus & \gamma \cos \psi_{yr} \cos \psi_\oplus & -\sin \psi_\oplus \\
\gamma \sin \psi_{yr} \sin \psi_\oplus \cos \psi_\oplus & \gamma \cos \psi_{yr} \sin \psi_\oplus & \cos \psi_\oplus
\end{bmatrix}.
\] (A15b)

**A.2.3 Velocity of the Earth in the Ecliptic coordinate system**

The orbital velocity of the Earth’s rotation around the Sun in the Ecliptic coordinate system can be expressed as
\[
v_{\oplus,S}(t) = v_{\oplus,S} \left( -\sin \psi_{yr}(t) X_S + \cos \psi_{yr}(t) Y_S \right) = v_{\oplus,S} \begin{bmatrix}
-\sin \psi_{yr}(t) \\
\cos \psi_{yr}(t) \\
0
\end{bmatrix}_S,
\] (A16)
where the Earth’s orbital speed can be estimated by \[64\]
\[
 v_{\oplus, S} = \frac{2\pi \times 1.495978707 \times 10^8 \text{ km}}{3.15569252 \times 10^7 \text{ s}} \simeq 29.79 \text{ km/s}. \tag{A17} 
\]

Note that the Earth’s orbit around the Sun has been assumed here to be perfectly circular on the Ecliptic plane and the orbital speed is thus a constant.

## A.3 Definition of the Galactic coordinate system

Finally, we come to define the Galactic coordinate system. The Earth’s velocity relative to the Dark Matter halo in the Ecliptic and the Equatorial coordinate systems and its annual modulation in four normal and four advanced seasons given in Table 2 will also be discussed in detail.

### A.3.1 Conventional definition

In Fig. A4, we sketch the astronomical definitions of the Galactic coordinate system in both of the Cartesian and the spherical coordinates with the origin located at the center of the Sun (not the Galactic Center (GC)). The primary direction (the $X_G$–axis) points from the Solar center to the approximate center of the our Galaxy, the $Z_G$–axis points to the Galactic North Pole (GNP), and the $Y_G$–axis is then defined as usual by the right–handed convention \[62\]. On the other hand, the Galactic longitude ($l$) is measured from the $X_G$–axis on the Galactic plane, while the Galactic latitude ($b$) measures the angle of the object above or below the approximate Galactic plane \[62\].
A.3.2 Our definition

Our definition of the Galactic coordinate system is described in Sec. 2.1.1 and sketched in Fig. 2. Then, according to Ref. [62], the direction of the Galactic Center in the Equatorial coordinate system can be expressed by

\[
X_{G, \text{Eq}} = \begin{bmatrix}
\cos \theta_{GC, \text{Eq}} \cos \phi_{GC, \text{Eq}} & \cos \theta_{GC, \text{Eq}} \sin \phi_{GC, \text{Eq}} & \sin \theta_{GC, \text{Eq}}
\end{bmatrix}_{\text{Eq}}
\]

where we have adopted the values provided by Ref. [62] as

\[
\phi_{GC, \text{Eq}} = 5^{h}45.6^{m} = 86.40^\circ,
\]

and

\[
\theta_{GC, \text{Eq}} = -28.94^\circ
\]

as the right ascension and the declination of the Galactic Center in the Equatorial coordinate system (from the \(X_{\text{Eq}}\)–axis (= \(X_{S}\)–axis) and the Earth’s/celestial Equator (\(X_{\text{Eq}} - Y_{\text{Eq}}\) plane), respectively. Meanwhile, the direction of the Galactic North Pole in our Equatorial coordinate system can also be given as

\[
Z_{G, \text{Eq}} = \begin{bmatrix}
\cos \theta_{GNP, \text{Eq}} \cos \phi_{GNP, \text{Eq}} & \cos \theta_{GNP, \text{Eq}} \sin \phi_{GNP, \text{Eq}} & \sin \theta_{GNP, \text{Eq}}
\end{bmatrix}_{\text{Eq}}
\]

where we have

\[
\phi_{GNP, \text{Eq}} = 51.4^m = 12.85^\circ,
\]

and

\[
\theta_{GNP, \text{Eq}} = 27.13^\circ
\]

as the right ascension and the declination of the Galactic North Pole in the Equatorial coordinate system, respectively. By combining Eqs. (A18) and (A20), the \(Y_{G}\)–axis of the Galactic coordinate system in the Equatorial coordinate system can be calculated by

\[
Y_{G, \text{Eq}} = Z_{G, \text{Eq}} \times X_{G, \text{Eq}}
\]

\[
= \begin{bmatrix}
\cos \theta_{GNP, \text{Eq}} \sin \phi_{GNP, \text{Eq}} & \sin \theta_{GNP, \text{Eq}} \cos \phi_{GNP, \text{Eq}} & -\cos \theta_{GNP, \text{Eq}} \sin \phi_{GNP, \text{Eq}} - \sin \theta_{GNP, \text{Eq}} \cos \phi_{GNP, \text{Eq}}
\end{bmatrix}_{\text{Eq}}^T
\]

\[
= \begin{bmatrix}
0.49406 & 0.44492 & 0.74696
\end{bmatrix}_{\text{Eq}}
\]

\[17\] Remind that, in the Ecliptic and the Equatorial coordinate systems defined in Sec. 2.1.2, the common \(X_{S}/X_{\text{Eq}}\)–axis points from the center of the Sun to that of the Earth, which is opposite to the \(X\) direction defined conventionally. Hence, the right ascensions of the Galactic Center and the Galactic North Pole in the Equatorial and the Ecliptic coordinate systems adopted in this section as well as those calculated later would differ from the conventional values by \(12^h = 180^\circ\).
Then the transformation matrix from the Equatorial to the Galactic coordinate system can be given by

\[
\mathbf{M}_{\text{Eq} \rightarrow \text{G}} = \begin{bmatrix}
\mathbf{X}_{\text{G}, \text{Eq}} \\
\mathbf{Y}_{\text{G}, \text{Eq}} \\
\mathbf{Z}_{\text{G}, \text{Eq}}
\end{bmatrix} = \begin{bmatrix}
0.05495 & 0.87340 & -0.48389 \\
-0.49406 & 0.44492 & 0.74696 \\
0.86769 & 0.19793 & 0.45601
\end{bmatrix}, \tag{A23a}
\]

and, conversely, we have

\[
\mathbf{M}_{\text{G} \rightarrow \text{Eq}} = \mathbf{M}_{\text{Eq} \rightarrow \text{G}}^T = \begin{bmatrix}
0.05495 & -0.49406 & 0.86769 \\
0.99374 & 0.11168 & 0.00055 \\
-0.09723 & 0.86223 & 0.49711
\end{bmatrix}. \tag{A23b}
\]

Furthermore, by using the transformation matrices between the Equatorial and the Ecliptic coordinate systems, the transformation matrices between the Ecliptic and the Galactic coordinate systems can be given by

\[
\mathbf{M}_{\text{G} \rightarrow \text{S}} = \mathbf{M}_{\text{Eq} \rightarrow \text{S}} \mathbf{M}_{\text{G} \rightarrow \text{Eq}} = \begin{bmatrix}
0.05495 & -0.49406 & 0.86769 \\
0.99374 & 0.11168 & 0.00055 \\
-0.09723 & 0.86223 & 0.49711
\end{bmatrix}, \tag{A24a}
\]

and

\[
\mathbf{M}_{\text{S} \rightarrow \text{G}} = \mathbf{M}_{\text{G} \rightarrow \text{S}}^T = \begin{bmatrix}
0.05495 & 0.99374 & -0.09723 \\
-0.49406 & 0.11168 & 0.86223 \\
0.86769 & 0.00055 & 0.49711
\end{bmatrix}. \tag{A24b}
\]

Finally, by using the matrix \(\mathbf{M}_{\text{G} \rightarrow \text{S}}\) given in Eq. (A24a), one can obtain the \(\mathbf{X}_G\)– and \(\mathbf{Z}_G\)–axes in the Ecliptic coordinate system as

\[
\mathbf{X}_{\text{G}, \text{S}} = \mathbf{M}_{\text{G} \rightarrow \text{S}} = \begin{bmatrix}
1 \\
0 \\
0
\end{bmatrix}_S = \begin{bmatrix}
0.05495 \\
0.99374 \\
-0.09723
\end{bmatrix}_S, \tag{A25}
\]

and

\[
\mathbf{Z}_{\text{G}, \text{S}} = \mathbf{M}_{\text{G} \rightarrow \text{S}} = \begin{bmatrix}
0 \\
0 \\
1
\end{bmatrix}_S = \begin{bmatrix}
0.86769 \\
0.00055 \\
0.49711
\end{bmatrix}_S. \tag{A26}
\]

These give in turn the right ascensions and the declinations of the Galactic Center and the Galactic North Pole in the Ecliptic coordinate system as

\[
\phi_{\text{GC}, \text{S}} = 86.84^\circ = 5.79^h, \tag{A27a}
\]

83
Figure A5: The same as in Fig. 3(b), except that the additional (golden) arrow here indicates the direction of the movement of the Solar system around the Galactic center. Note that the moving direction of the Solar system is not parallel to, but only approximately along the $Y_G$–axis (with an included angle of 8.87° = 35.48 m), nor on the (approximate) $X_G - Y_G$ (Galactic) plane (but 0.60° above).

$$\theta_{GC,S} = -5.58^\circ,$$  \hspace{1cm} (A27b)

and

$$\phi_{GNP,S} = 0.036^\circ = 2.17^m,$$ \hspace{1cm} (A28a)

$$\theta_{GNP,S} = 29.81^\circ,$$ \hspace{1cm} (A28b)

respectively.

### A.3.3 Direction of the movement of the Solar system around the Galactic center

In Fig. A5 we add the (golden) arrow to indicate the direction of the movement of the Solar system around the Galactic center (cf. Fig. 3(b)). The Galactic movement of the Solar system pointing currently towards the CYGNUS constellation can be given in the Equatorial coordinate system as \[63\]

$$v_{\odot,Eq} = v_{\odot,G} \begin{bmatrix} \cos \theta_{Cyg,Eq} \cos \phi_{Cyg,Eq} \\ \cos \theta_{Cyg,Eq} \sin \phi_{Cyg,Eq} \\ \sin \theta_{Cyg,Eq} \end{bmatrix}_{Eq} \approx v_{\odot,G} \begin{bmatrix} -0.47069 \\ 0.57508 \\ 0.66913 \end{bmatrix}_{Eq} \sim \begin{bmatrix} -103.55 \text{ km/s} \\ 126.52 \text{ km/s} \\ 147.21 \text{ km/s} \end{bmatrix}_{Eq}. \hspace{1cm} (A29)$$

Here we have used

$$v_{\odot,G} \simeq 220 \text{ km/s}, \hspace{1cm} (A30)$$
and the right ascension and the declination of the direction of the CYGNUS constellation in the Equatorial coordinate system are

$$\phi_{\text{Cyg,Eq}} = 8.62^h = 129.30^\circ,$$  \hspace{1cm} (A31a)

and

$$\theta_{\text{Cyg,Eq}} = 42^\circ.$$  \hspace{1cm} (A31b)

Moreover, by using the transformation matrices $M_{\text{Eq}\rightarrow S}$ and $M_{\text{Eq}\rightarrow G}$ given in Eqs. (A6b) and (A23a), the moving direction of the Solar system in the Ecliptic and the Galactic coordinate systems can be obtained as

$$v_{\odot, S} = M_{\text{Eq}\rightarrow S} v_{\odot, \text{Eq}}$$

$$= v_{\odot, G} \begin{bmatrix} \cos \theta_{\text{Cyg,Eq}} \cos \phi_{\text{Cyg,Eq}} \\ \cos \psi_{\odot} \cos \theta_{\text{Cyg,Eq}} \sin \phi_{\text{Cyg,Eq}} - \sin \psi_{\odot} \sin \theta_{\text{Cyg,Eq}} \\ \sin \psi_{\odot} \cos \theta_{\text{Cyg,Eq}} \sin \phi_{\text{Cyg,Eq}} + \cos \psi_{\odot} \sin \theta_{\text{Cyg,Eq}} \end{bmatrix}_S$$

$$= v_{\odot, G} \begin{bmatrix} -0.47069 \\ 0.26203 \\ 0.84249 \end{bmatrix} S$$

and

$$v_{\odot, G} = M_{\text{Eq}\rightarrow G} v_{\odot, \text{Eq}} = v_{\odot, G} \begin{bmatrix} 0.15262 \\ 0.98823 \\ 0.01054 \end{bmatrix}_G \approx \begin{bmatrix} 33.58 \text{ km/s} \\ 217.41 \text{ km/s} \\ 2.32 \text{ km/s} \end{bmatrix}_G.$$  \hspace{1cm} (A32)

These give in turn the right ascension and the declination of the moving direction of the Solar system in the Ecliptic coordinate system as

$$\phi_{\text{Cyg,S}} = 150.90^\circ = 10.06^h,$$  \hspace{1cm} (A34a)

$$\theta_{\text{Cyg,S}} = 57.40^\circ;$$  \hspace{1cm} (A34b)

and, in the Galactic coordinate system, one has

$$\phi_{\text{Cyg,G}} = 81.22^\circ = 5.41^h,$$  \hspace{1cm} (A35a)

$$\theta_{\text{Cyg,G}} = 0.60^\circ.$$  \hspace{1cm} (A35b)
A.3.4 Annual modulation of the Earth’s velocity in the Ecliptic coordinate system

In this and the next subsections, we discuss the annual modulation of the Earth’s velocity relative to the Dark Matter halo in the Ecliptic and the Equatorial coordinate systems separately. Remind that our calculations are based on two assumptions. First, the Earth’s orbit around the Sun is perfectly circular on the Ecliptic plane and the orbital speed is thus a constant. Second, the date of the vernal equinox is exactly fixed at the end of the 79th day (May 20th) of a 365-day year and the few extra hours in an actual Solar year has been neglected.

We begin with four normal seasons, in which the orbital velocity of the Earth’s rotation around the Sun in the Ecliptic coordinate system on the central date of each season is along the $X_S$- or the $Y_S$-axis, corresponding to the Earth’s positions shown in Fig. 9(a). By setting $\psi_{\text{yr}} = 0$, $\pi/2$, $\pi$, and $3\pi/2$, or, equivalently, $t = 79.0$ (the beginning of the 21st of March), 170.25 (June 20th), 261.50 (September 19th), and 352.75 day (December 19th), respectively, into the expression for the Earth’s orbital velocity around the Sun given in Eq. (A16), we have the Earth’s velocity (vectors) in four normal seasons in the Ecliptic coordinate system as

\[
v_{\oplus, S}, 79.00 = v_{\oplus, S} Y_S , \tag{A36a}
\]

\[
v_{\oplus, S}, 170.25 = -v_{\oplus, S} X_S , \tag{A36b}
\]

\[
v_{\oplus, S}, 261.50 = -v_{\oplus, S} Y_S , \tag{A36c}
\]

and

\[
v_{\oplus, S}, 352.75 = v_{\oplus, S} X_S , \tag{A36d}
\]

respectively. By combining with the velocity of the Solar system given in Eq. (A32) and taking the Earth’s orbital speed given in Eq. (A17), one can obtain the Earth’s velocity relative to the Dark Matter halo in four normal seasons in the Ecliptic coordinate system as

\[
v_{\oplus, \chi, S}, 79.00 = v_{\oplus, S} + v_{\oplus, S}, 79.00 = \begin{bmatrix} -103.55 \text{ km/s} \\ 87.43 \text{ km/s} \\ 185.35 \text{ km/s} \end{bmatrix}_S , \tag{A37a}
\]

\[
v_{\oplus, \chi, S}, 170.25 = v_{\oplus, S} + v_{\oplus, S}, 170.25 = \begin{bmatrix} -133.34 \text{ km/s} \\ 57.65 \text{ km/s} \\ 185.35 \text{ km/s} \end{bmatrix}_S , \tag{A37b}
\]

\[
v_{\oplus, \chi, S}, 261.50 = v_{\oplus, S} + v_{\oplus, S}, 261.50 = \begin{bmatrix} -103.55 \text{ km/s} \\ 27.86 \text{ km/s} \\ 185.35 \text{ km/s} \end{bmatrix}_S , \tag{A37c}
\]

and

\[
v_{\oplus, \chi, S}, 352.75 = v_{\oplus, S} + v_{\oplus, S}, 352.75 = \begin{bmatrix} -73.77 \text{ km/s} \\ 57.65 \text{ km/s} \\ 185.35 \text{ km/s} \end{bmatrix}_S . \tag{A37d}
\]
respectively.

On the other hand, as shown in Fig. 9(b), the Earth’s velocity relative to the Dark Matter halo should be maximal (minimal), once the Earth’s velocity is (anti–)parallel to the projection of the moving direction of the Solar system on the Ecliptic plane. This requires that, from Eq. (A34a),

\[ \psi_{yr} = 60.90^\circ = 0.3383 \pi = 1.0628, \quad \text{(A38)} \]

or, equivalently,

\[ t_{\max} = 140.74 \ \text{day}, \quad \text{(A39)} \]

i.e., around the 21st of May. Then, by setting \( \psi_{yr} = 1.0628, 4.2044, -0.5080, \) and 2.6336 (corresponding to the Earth’s positions in four advanced seasons shown in Fig. 9(b)), or, equivalently, \( t = 140.74 \) (May 21st), 323.24 (November 20th), 49.49 (February 19th), and 231.99 day (August 20th), respectively, into the expression (A16), we have the Earth’s velocity as

\[
\begin{pmatrix}
-0.874 \\ 0.486 \\ 0
\end{pmatrix}_S = \begin{pmatrix}
-26.02 \ \text{km/s} \\ 14.49 \ \text{km/s} \\ 0
\end{pmatrix}_S, \quad \text{(A40a)}
\]

\[
\begin{pmatrix}
0.874 \\ -0.486 \\ 0
\end{pmatrix}_S = \begin{pmatrix}
26.02 \ \text{km/s} \\ -14.49 \ \text{km/s} \\ 0
\end{pmatrix}_S, \quad \text{(A40b)}
\]

\[
\begin{pmatrix}
0.486 \\ 0.874 \\ 0
\end{pmatrix}_S = \begin{pmatrix}
14.49 \ \text{km/s} \\ 26.02 \ \text{km/s} \\ 0
\end{pmatrix}_S, \quad \text{(A40c)}
\]

and

\[
\begin{pmatrix}
-0.486 \\ -0.874 \\ 0
\end{pmatrix}_S = \begin{pmatrix}
-14.49 \ \text{km/s} \\ -26.02 \ \text{km/s} \\ 0
\end{pmatrix}_S, \quad \text{(A40d)}
\]

respectively. Then the maximal, the minimal and the middle values of the Earth’s velocity relative to the Dark Matter halo in four advanced seasons can be obtained as

\[
\begin{pmatrix}
-129.58 \ \text{km/s} \\ 72.14 \ \text{km/s} \\ 185.35 \ \text{km/s}
\end{pmatrix}_S, \quad \text{(A41a)}
\]

\[
\begin{pmatrix}
-77.53 \ \text{km/s} \\ 43.16 \ \text{km/s} \\ 185.35 \ \text{km/s}
\end{pmatrix}_S, \quad \text{(A41b)}
\]
| Date (day) | Magnitude $v_{\oplus, S}$ (km/s) | Right ascension $\phi_{X, S}$ | Declination $\theta_{X, S}$ |
|-----------|-------------------------------|-----------------------------|------------------------|
| 79.0      | 229.61                        | 139.82° = 9.32°h            | 53.83°                 |
| 170.25    | 235.49                        | 156.62° = 10.44°h           | 51.91°                 |
| 261.50    | 214.13                        | 164.94° = 11.00°h           | 59.95°                 |
| 352.75    | 207.65                        | 141.99° = 9.47°h            | 63.20°                 |
| 49.49     | 222.01                        | 136.79° = 9.12°h            | 56.60°                 |
| 140.74    | 237.38                        | 150.90° = 10.06°h           | 51.34°                 |
| 231.99    | 222.01                        | 165.00° = 11.00°h           | 56.60°                 |
| 323.24    | 205.49                        | 150.90° = 10.06°h           | 64.42°                 |

Table A1: The list of the magnitudes, the right ascensions and the declinations of the Earth’s velocity relative to the Dark Matter halo in four normal and four advanced seasons in the Ecliptic coordinate system.

Furthermore, one can in general have that

$$\mathbf{v}_{\oplus, S, 49.49} = \mathbf{v}_{\odot, S} + \mathbf{v}_{\oplus, S, 49.49} = \begin{bmatrix} -89.06 \text{ km/s} \\ 83.67 \text{ km/s} \\ 185.35 \text{ km/s} \end{bmatrix}_S$$

and

$$\mathbf{v}_{\oplus, S, 231.99} = \mathbf{v}_{\odot, S} + \mathbf{v}_{\oplus, S, 231.99} = \begin{bmatrix} -118.04 \text{ km/s} \\ 31.62 \text{ km/s} \\ 185.35 \text{ km/s} \end{bmatrix}_S$$

respectively. One can thus estimate the magnitudes and the directions (the right ascensions and the declinations) of the Earth’s velocity relative to the Dark Matter halo in four normal and four advanced seasons straightforwardly, which we summarize in Table A1 for readers’ reference.

Furthermore, one can in general have that

$$\mathbf{v}_{\oplus, S, 49.49} = \mathbf{v}_{\odot, S} + \mathbf{v}_{\oplus, S, 49.49} = \begin{bmatrix} -89.06 \text{ km/s} \\ 83.67 \text{ km/s} \\ 185.35 \text{ km/s} \end{bmatrix}_S$$

and

$$\mathbf{v}_{\oplus, S, 231.99} = \mathbf{v}_{\odot, S} + \mathbf{v}_{\oplus, S, 231.99} = \begin{bmatrix} -118.04 \text{ km/s} \\ 31.62 \text{ km/s} \\ 185.35 \text{ km/s} \end{bmatrix}_S$$

respectively. One can thus estimate the magnitudes and the directions (the right ascensions and the declinations) of the Earth’s velocity relative to the Dark Matter halo in four normal and four advanced seasons straightforwardly, which we summarize in Table A1 for readers’ reference.
Hence, we can get that
\[
v_{\odot}^{\chi}(t) = v_{\odot,G} \left[ 1 + \left( \frac{v_{\odot,S}}{v_{\odot,G}} \right)^2 + 2 \left( \frac{v_{\odot,S}}{v_{\odot,G}} \right) \left( 0.47069 \sin \psi_{\text{yr}} + 0.26203 \cos \psi_{\text{yr}} \right) \right]^{1/2}
\]
\[
= v_{\odot,G} \left[ \alpha_{v,0} + \alpha_{v,1} \cos \left( \frac{2\pi(t - t_{\text{max}})}{1 \text{ yr}} \right) + \alpha_{v,2} \cos \left( \frac{2 \cdot 2\pi(t - t_{\text{max}})}{1 \text{ yr}} \right) + \cdots \right]. \quad (A43)
\]

Here, in the last line, we expand \(v_{\odot}^{\chi}(t)\) by the Fourier cosine series with the date of the maximal Earth’s relative velocity \(t_{\text{max}} = 140.74\) day. By some numerical calculations, one can find easily that
\[
\alpha_{v,0} = 1.0078, \quad (A44a)
\]
\[
\alpha_{v,1} = 0.0724. \quad (A44b)
\]
and
\[
\alpha_{v,2} = -0.0013. \quad (A44c)
\]

Hence, the time–dependent Earth’s speed relative to the Dark Matter halo can be expressed as
\[
v_{\odot}^{\chi}(t) = v_{\odot,G} \left[ 1.0078 + 0.0724 \cos \left( \frac{2\pi(t - t_{\text{max}})}{1 \text{ yr}} \right) - 0.0013 \cos \left( \frac{2 \cdot 2\pi(t - t_{\text{max}})}{1 \text{ yr}} \right) \right]. \quad (A45)
\]

In Figs. A6, we show the fitted \(v_{\odot}^{\chi}(t)\) with the order of 1 (upper) and 2 (below), respectively.

A.3.5 Annual modulation of the Earth’s velocity in the Equatorial coordinate system

First, by using the transformation matrix from the Ecliptic to the Equatorial coordinate system \(M_{S \rightarrow Eq}\) given in Eq. (A6a), the Earth’s velocity (vectors) relative to the Dark Matter halo in four normal seasons in the Equatorial coordinate system can be calculated directly as

\[
v_{S, \odot, \chi, \text{Eq}, 79.00} = M_{S \rightarrow Eq} v_{S, \odot, \chi, 79.00} = \begin{bmatrix} -103.55 \text{ km/s} \\ 153.85 \text{ km/s} \\ 135.38 \text{ km/s} \end{bmatrix}_{\text{Eq}}. \quad (A46a)
\]

\[
v_{S, \odot, \chi, \text{Eq}, 170.25} = M_{S \rightarrow Eq} v_{S, \odot, \chi, 170.25} = \begin{bmatrix} -133.34 \text{ km/s} \\ 126.52 \text{ km/s} \\ 147.21 \text{ km/s} \end{bmatrix}_{\text{Eq}}. \quad (A46b)
\]

\[
v_{S, \odot, \chi, \text{Eq}, 261.50} = M_{S \rightarrow Eq} v_{S, \odot, \chi, 261.50} = \begin{bmatrix} -103.55 \text{ km/s} \\ 99.18 \text{ km/s} \\ 159.04 \text{ km/s} \end{bmatrix}_{\text{Eq}}. \quad (A46c)
\]
Figure A6: Fourier cosine fitting of the time–dependent Earth’s speed relative to the Dark Matter halo $v_{\oplus \chi}(t)$ with the order of 1 (upper) and 2 (below), respectively. The date of the maximal Earth’s relative velocity is $t_{\text{max}} = 140.74$ day.
Table A2: The list of the magnitudes, the right ascensions and the declinations of the Earth’s velocity relative to the Dark Matter halo in four normal and four advanced seasons in the Equatorial coordinate system.

| Date (day) | Magnitude \( v_{\oplus, \chi, Eq} \) (km/s) | Right ascension \( \phi_{\chi, Eq} \) | Declination \( \theta_{\chi, Eq} \) |
|------------|--------------------------------|---------------------------------|-----------------|
| 79.0       | 229.61                          | \( 123.94^\circ = 8.26^h \)    | 36.13°           |
| 170.25     | 235.49                          | \( 136.50^\circ = 9.10^h \)    | 38.69°           |
| 261.50     | 214.13                          | \( 136.24^\circ = 9.08^h \)    | 47.96°           |
| 352.75     | 207.65                          | \( 120.24^\circ = 8.01^h \)    | 45.15°           |
| 49.49      | 222.01                          | \( 120.63^\circ = 8.04^h \)    | 38.06°           |
| 140.74     | 237.38                          | \( 132.82^\circ = 8.85^h \)    | 36.58°           |
| 231.99     | 222.01                          | \( 138.99^\circ = 9.27^h \)    | 45.21°           |
| 323.24     | 205.49                          | \( 124.40^\circ = 8.29^h \)    | 48.11°           |

and

\[
v_{\oplus, \chi, Eq, 352.75} = M_{S \rightarrow Eq} v_{\oplus, S, 352.75} = \begin{bmatrix} -73.77 \text{ km/s} \\ 126.52 \text{ km/s} \\ 147.21 \text{ km/s} \end{bmatrix}_{Eq}, \tag{A46d}
\]

respectively. Similarly, for four advanced seasons, one has

\[
v_{\oplus, \chi, Eq, 140.74} = M_{S \rightarrow Eq} v_{\oplus, S, 140.74} = \begin{bmatrix} -129.58 \text{ km/s} \\ 139.81 \text{ km/s} \\ 141.45 \text{ km/s} \end{bmatrix}_{Eq}, \tag{A47a}
\]

\[
v_{\oplus, \chi, Eq, 323.24} = M_{S \rightarrow Eq} v_{\oplus, S, 323.24} = \begin{bmatrix} -77.53 \text{ km/s} \\ 113.22 \text{ km/s} \\ 152.96 \text{ km/s} \end{bmatrix}_{Eq}, \tag{A47b}
\]

\[
v_{\oplus, \chi, Eq, 49.49} = M_{S \rightarrow Eq} v_{\oplus, S, 49.49} = \begin{bmatrix} -89.06 \text{ km/s} \\ 150.40 \text{ km/s} \\ 136.87 \text{ km/s} \end{bmatrix}_{Eq}, \tag{A47c}
\]

and

\[
v_{\oplus, \chi, Eq, 231.99} = M_{S \rightarrow Eq} v_{\oplus, S, 231.99} = \begin{bmatrix} -118.04 \text{ km/s} \\ 102.63 \text{ km/s} \\ 157.54 \text{ km/s} \end{bmatrix}_{Eq}, \tag{A47d}
\]
respectively. Then one can have the directions (the right ascensions and the declinations) of the Earth’s velocity relative to the Dark Matter halo in four normal and four advanced seasons in the Equatorial coordinate system (i.e., the blue—yellow points shown in Figs. 14 and 37 as well as in Figs. 15 and 38) summarized in Table A2 for readers’ reference.

### A.3.6 Dates considered for demonstrating the diurnal modulation of the angular WIMP velocity distribution

At the end of this section, we calculate the dates considered for demonstrating the diurnal modulation of the angular WIMP velocity distribution shown in Sec. 3.5.1 and Appendix B.

From Fig. A3 and Eq. (A12a), one can have

\[
X_E = \cos \psi_{PM} X_{Eq} + \sin \psi_{PM} Y_{Eq} = \gamma \cos \psi_{yr} X_{Eq} + \gamma \sin \psi_{yr} \cos \psi_{\oplus} Y_{Eq} .
\]

Then, by taking \(\psi_{PM} = \phi_{Cyg, Eq} = 129.30^\circ\) given in Eq. (A31a), the angle \(\psi_{yr}\) can be solved directly as

\[
\psi_{yr} = \tan^{-1} \left( \frac{\tan \psi_{PM}}{\cos \psi_{\oplus}} \right) = 126.9^\circ .
\]

This means in turn that the date on which the WIMP wind points straightly to the Prime Meridian in the night is\(^{18}\)

\[
126.9^\circ \left( \frac{365 \text{ day}}{360^\circ} \right) + 79 \text{ day} = 207.66 \text{ day} .
\]

And the date on which the WIMP wind points straightly to the Prime Meridian in the day is 25.16 day.

### B Angular distributions of the 3-D WIMP velocity observed at underground laboratories

For readers’ reference, we summarize in this section the angular distributions of the 3-D WIMP velocity in two laboratory–dependent (horizontal and laboratory) coordinate systems at the locations of several underground laboratories. Due to their geographical advantages, two under-constructed laboratories have also been considered: the Agua Negra Deep Experiment Site (ANDES) on the border between Argentina and Chile as the second underground laboratory at the Southern Hemisphere and the Callio Laboratory located at the Pyhäsalmi Mine in Finland, which will be the northernmost (close to the Arctic Circle) underground laboratory in the future (the Boulby Laboratory is so far the northernmost one).

For each laboratory, we provide the following figures:

- The angular distribution of the 3-D WIMP velocity in the horizontal coordinate system in one entire year (a) and in the 60-day observation periods of four advanced seasons on the central date of 49.49 day, 140.74 day, 231.99 day, and 323.24 day, respectively.

\(^{18}\) Remind that these dates are only pure theoretical estimations under some simplified assumptions. It is obviously that the Prime Meridian should be in the day (night) on the 207.66 (25.16) day, respectively.
• The angular distribution of the 3-D WIMP velocity in the laboratory coordinate system in one entire year (a) and in the 60-day observation periods of four advanced seasons on the central date of 49.49 day, 140.74 day, 231.99 day, and 323.24 day, respectively.

• The angular distribution of the 3-D WIMP velocity in the laboratory coordinate system in four daily shifts in two 60-day observation periods with the central dates of 207.66 day and 25.16 (= 390.16) day.

50 and 500 total WIMP events on average in one observation period (365 days/year, 60 days/season, or 4 hours/shift × 60 days) have been simulated. By comparing the angular distribution patterns observed at different laboratories, it can be found that, with $O(500)$ total WIMP events and a higher analysis resolution, we might even be able to demonstrate a more detailed latitude–dependent distribution pattern.

For the identification of the annual modulation of the angular distribution pattern with real experimental data in the future, as the first confirmation of the directionality of the WIMP wind, we also summarize the directions of the simulated 3-D WIMP velocity with the highest event numbers (> 4 times of the all-sky average value) in the horizontal coordinate system in one entire year and in four advanced seasons in one table for each laboratory. Note that in these tables we mark the ranges with a double underline to indicate that, with $O(500)$ total WIMP events and a higher analysis resolution, some extra bins would exceed the high–WIMP–flux areas identified with only $O(50)$ total events and a lower resolution.
### B.1 Agua Negra Deep Experiment Site (ANDES)

| Central date (observation period) (day) | Most–event directions | 50 events | 500 events |
|----------------------------------------|------------------------|-----------|------------|
| 0 – 365                                | —                      | —         | —          |
| 49.49 (19.49 – 79.49)                  | 0° – 60°N              | 120°W – 60°W | 0° – 60°N, 120°W – 60°W, 60°N – 75°N, 120°E – 180° |
| 140.74 (110.74 – 170.74)              | 0° – 60°N              | 60°E – 180° | 0° – 60°N, 60°E – 150°E |
| 231.99 (201.99 – 261.99)              | 30°S – 30°N            | 60°E – 180° | 30°S – 30°N, 90°E – 150°W |
| 323.24 (293.24 – 353.24)              | 30°S – 60°N            | 180° – 120°W | 45°S – 60°N, 180° – 120°W |

Table A3: The summary of the directions of the simulated 3-D WIMP velocity with the highest event numbers (> 4 times of the all-sky average value) in the horizontal coordinate system in one entire year and four advanced seasons at the location of the ANDES laboratory (30.19°S, 69.82°W). 50 and 500 total events on average in each observation period of 60 days.

Remark: in Fig. A8(b) one can find that, with O(500) total WIMP events and a higher analysis resolution, a second “hot point” from 60°N to 75°N and 120°E to 180° in the angular distribution pattern in the horizontal coordinate system would be observed at the ANDES laboratory.
Figure A7: The angular distribution of the 3-D WIMP velocity in the horizontal coordinate system at the location of the ANDES laboratory (30.19°S, 69.82°W), 50 total events on average in one entire year (a) and in each of four advanced seasons on the central date of 49.49 day (b), 140.74 day (c), 231.99 day (d), and 323.24 day (e), respectively, have been simulated.
Figure A8: As in Figs. A7, except that 500 total events on average in one entire year (a) and in each of four advanced seasons (b – e) have been simulated.
Figure A9: The angular distribution of the 3-D WIMP velocity transformed from events shown in Fig. A7 to the laboratory coordinate system at the location of the ANDES laboratory (30.19°S, 69.82°W). All simulation setup and notations are the same as in Fig. A7.
Figure A10: As in Figs. A9 except that 500 total events on average in one entire year (a) and in each of four advanced seasons (b – e) have been simulated.
Figure A11: The angular distribution of the 3-D WIMP velocity observed at the location of the ANDES laboratory (30.19°S, 69.82°W) in four daily shifts in the observation periods of 177.66 – 237.66 day (a – d) and 360.16 – 420.16 (= 55.16) day (e – h), respectively. 50 total events on average in each 4-hour daily shift in the 60-day observation period have been simulated.
Figure A12: As in Figs. A11 except that 500 total events on average in each 4-hour daily shift in the 60-day observation period have been simulated.
### B.2 Boulby Laboratory

| Central date (observation period) (day) | Most–event directions |
|----------------------------------------|------------------------|
|                                        | 50 events              |
|                                        | 500 events             |
| 0 – 365                                | —                      |
|                                        | —                      |
| 49.49 (19.49 – 79.49)                  | 30°S – 30°N            |
|                                        | 120°E – 180°           |
|                                        | 45°S – 30°N            |
|                                        | 90°E – 150°W           |
| 140.74 (110.74 – 170.74)              | 60°S – 0°              |
|                                        | 60°E – 120°E           |
|                                        | 60°S – 15°N            |
|                                        | 60°E – 150°E           |
| 231.99 (201.99 – 261.99)              | 60°S – 30°S            |
|                                        | 180° – 120°W + 0° – 60°E |
|                                        | 150°W – 90°W + 30°W – 60°E |
| 323.24 (293.24 – 353.24)              | 30°S – 0°              |
|                                        | 180° – 60°W            |
|                                        | 45°S – 0°              |
|                                        | 150°E – 60°W           |

Table A4: The summary of the directions of the simulated 3-D WIMP velocity with the highest event numbers (> 4 times of the all-sky average value) in the horizontal coordinate system in one entire year and four advanced seasons at the location of the Boulby laboratory (54.55°N, 0.82°W). 50 and 500 total events on average in each observation period of 60 days.

Remark: Fig. A13(d) shows that, with only $O(50)$ total WIMP events one could already observe two high-WIMP-flux bins in the angular distribution pattern in the horizontal coordinate system at the Boulby laboratory; with $O(500)$ total events and a higher analysis resolution, these two hot points could be identified more clearly (see Fig. A14(d)).
Figure A13: The angular distribution of the 3-D WIMP velocity in the horizontal coordinate system at the location of the Boulby laboratory (54.55°N, 0.82°W). 50 total events on average in one entire year (a) and in each of four advanced seasons on the central date of 49.49 day (b), 140.74 day (c), 231.99 day (d), and 323.24 day (e), respectively, have been simulated.
Figure A14: As in Figs. A13, except that 500 total events on average in one entire year (a) and in each of four advanced seasons (b – e) have been simulated.
Figure A15: The angular distribution of the 3-D WIMP velocity transformed from events shown in Fig. A13 to the laboratory coordinate system at the location of the Boulby laboratory (54.55°N, 0.82°W). All simulation setup and notations are the same as in Fig. A13.
Figure A16: As in Figs. A15, except that 500 total events on average in one entire year (a) and in each of four advanced seasons (b – e) have been simulated.
Figure A17: The angular distribution of the 3-D WIMP velocity observed at the location of the Boulby laboratory (54.55°N, 0.82°W) in four daily shifts in the observation periods of 177.66 – 237.66 day (a – d) and 360.16 – 420.16 (= 55.16) day (e – h), respectively. 50 total events on average in each 4-hour daily shift in the 60-day observation period have been simulated.
Figure A18: As in Figs. A17, except that 500 total events on average in each 4-hour daily shift in the 60-day observation period have been simulated.
Table A5: The summary of the directions of the simulated 3-D WIMP velocity with the highest event numbers (> 4 times of the all-sky average value) in the horizontal coordinate system in one entire year and four advanced seasons at the location of the Callio laboratory (63.66°N, 26.04°E). 50 and 500 total events on average in each observation period of 60 days.

Remark: Comparing Fig. A20(d) with Fig. A19(d), one can find that, with \( O(500) \) total WIMP events and a higher analysis resolution, the high–WIMP–flux bins in the angular distribution pattern in the horizontal coordinate system at the Callio laboratory could separate into two areas.
Figure A19: The angular distribution of the 3-D WIMP velocity in the horizontal coordinate system at the location of the Callio laboratory (63.66°N, 26.04°E), 50 total events on average in one entire year (a) and in each of four advanced seasons on the central date of 49.49 day (b), 140.74 day (c), 231.99 day (d), and 323.24 day (e), respectively, have been simulated.
Figure A20: As in Figs. A19, except that 500 total events on average in one entire year (a) and in each of four advanced seasons (b – e) have been simulated.
Figure A21: The angular distribution of the 3-D WIMP velocity transformed from events shown in Fig. A19 to the laboratory coordinate system at the location of the Callio laboratory (63.66°N, 26.04°E). All simulation setup and notations are the same as in Fig. A19.
Figure A22: As in Figs. A21, except that 500 total events on average in one entire year (a) and in each of four advanced seasons (b – e) have been simulated.
Figure A23: The angular distribution of the 3-D WIMP velocity observed at the location of the Callio laboratory (63.66°N, 26.04°E) in four daily shifts in the observation periods of 177.66 – 237.66 day (a – d) and 360.16 – 420.16 (= 55.16) day (e – h), respectively. 50 total events on average in each 4-hour daily shift in the 60-day observation period have been simulated.
Figure A24: As in Figs. A23, except that 500 total events on average in each 4-hour daily shift in the 60-day observation period have been simulated.
### B.4 China Jinping Underground Laboratory (CJPL)

| Central date (observation period) (day) | Most–event directions |
|----------------------------------------|------------------------|
|                                        | 50 events              | 500 events            |
| 0 – 365                                | —                      | —                     |
| 49.49 (19.49 – 79.49)                  | 60°S – 0°              | 75°S – 15°N           |
|                                        | 60°E – 120°E           | 60°E – 150°W          |
| 140.74 (110.74 – 170.74)              | 60°S – 0°              | 60°S – 0°             |
|                                        | 180° – 120°W           | 150°W – 90°W          |
| 231.99 (201.99 – 261.99)              | 30°S – 30°N            | 30°S – 30°N           |
|                                        | 180° – 60°W            | 150°E – 90°W          |
| 323.24 (293.24 – 353.24)              | 30°S – 30°N            | 45°S – 45°N           |
|                                        | 120°E – 180°           | 120°E – 150°W         |

Table A6: The summary of the directions of the simulated 3-D WIMP velocity with the highest event numbers (> 4 times of the all-sky average value) in the horizontal coordinate system in one entire year and four advanced seasons at the location of the CJPL laboratory (28.15°N, 101.71°E). 50 and 500 total events on average in each observation period of 60 days.

Remark: Fig. A26(b) shows that, with $O(500)$ total WIMP events and a higher analysis resolution, the high–WIMP–flux bins in the angular distribution pattern in the horizontal coordinate system at the CJPL laboratory could spread towards the southeast pretty widely.
Figure A25: The angular distribution of the 3-D WIMP velocity in the horizontal coordinate system at the location of the CJPL laboratory (28.15° N, 101.71° E), 50 total events on average in one entire year (a) and in each of four advanced seasons on the central date of 49.49 day (b), 140.74 day (c), 231.99 day (d), and 323.24 day (e), respectively, have been simulated.
Figure A26: As in Figs. A25, except that 500 total events on average in one entire year (a) and in each of four advanced seasons (b – e) have been simulated.
Figure A27: The angular distribution of the 3-D WIMP velocity transformed from events shown in Fig. A25 to the laboratory coordinate system at the location of the CJPL laboratory (28.15°N, 101.71°E). All simulation setup and notations are the same as in Fig. A25.
Figure A28: As in Figs. A27, except that 500 total events on average in one entire year (a) and in each of four advanced seasons (b – e) have been simulated.
Figure A29: The angular distribution of the 3-D WIMP velocity observed at the location of the CJPL laboratory (28.15°N, 101.71°E) in four daily shifts in the observation periods of 177.66 – 237.66 day (a – d) and 360.16 – 420.16 (= 55.16) day (e – h), respectively. 50 total events on average in each 4-hour daily shift in the 60-day observation period have been simulated.
Figure A30: As in Figs. A29, except that 500 total events on average in each 4-hour daily shift in the 60-day observation period have been simulated.
### B.5 Deep Underground Science and Engineering Laboratory (DUSEL)

| Central date (observation period) (day) | Most–event directions          |
|----------------------------------------|--------------------------------|
|                                        | 50 events                      | 500 events                     |
| 0 – 365                                | —                              | 30°S – 0°                      |
|                                        |                                 | 150°E – 150°W                  |
| 49.49 (19.49 – 79.49)                  | 30°S – 30°N                    | 45°S – 15°N                    |
|                                        | 180° – 60°W                    | 180° – 60°W                    |
| 140.74 (110.74 – 170.74)               | 30°S – 30°N                    | 30°S – 45°N                    |
|                                        | 120°E – 120°W                  | 120°E – 120°W                  |
| 231.99 (201.99 – 261.99)               | 60°S – 0°                      | 75°S – 30°N                    |
|                                        | 60°E – 180°                    | 90°E – 150°E                   |
| 323.24 (293.24 – 353.24)               | 60°S – 30°S                    | 60°S – 15°S                    |
|                                        | 180° – 120°W                   | 150°W – 120°W                  |

Table A7: The summary of the directions of the simulated 3-D WIMP velocity with the highest event numbers (> 4 times of the all-sky average value) in the horizontal coordinate system in one entire year and four advanced seasons at the location of the DUSEL laboratory (44.35°N, 103.75°W). 50 and 500 total events on average in each observation period of 60 days.

Remark: although due to the Earth’s orbital motion around the Sun and thus the horizontal coordinate system rotates daily, by using data recorded in one entire year, the angular distribution of the 3-D WIMP velocity spreads out latitudinally and the anisotropy of the main direction of the WIMP wind would be in principle averaged out, in Fig. A32(a) one can unexpectedly find that, with O(500) total WIMP events and a higher analysis resolution, some clear high–WIMP–flux bins from 15°S to 0° and 150°E to 150°W could be identified in the angular distribution pattern in the horizontal coordinate system at the DUSEL laboratory.
Figure A31: The angular distribution of the 3-D WIMP velocity in the horizontal coordinate system at the location of the DUSEL laboratory ($44.35^\circ$N, $103.75^\circ$W), 50 total events on average in one entire year (a) and in each of four advanced seasons on the central date of 49.49 day (b), 140.74 day (c), 231.99 day (d), and 323.24 day (e), respectively, have been simulated.
Figure A32: As in Figs. A31, except that 500 total events on average in one entire year (a) and in each of four advanced seasons (b – e) have been simulated.
Figure A33: The angular distribution of the 3-D WIMP velocity transformed from events shown in Fig. A31 to the laboratory coordinate system at the location of the DUSEL laboratory (44.35°N, 103.75°W). All simulation setup and notations are the same as in Fig. A31.
Figure A34: As in Figs. A33 except that 500 total events on average in one entire year (a) and in each of four advanced seasons (b – e) have been simulated.
Angular distribution of the WIMP velocity in the laboratory frame, 177.66 – 237.66 day, 50 events, at DUSEL

Angular distribution of the WIMP velocity in the laboratory frame, 360.16 – 420.16 day, 50 events, at DUSEL

Figure A35: The angular distribution of the 3-D WIMP velocity observed at the location of the DUSEL laboratory (44.35°N, 103.75°W) in four daily shifts in the observation periods of 177.66 – 237.66 day (a – d) and 360.16 – 420.16 (= 55.16) day (e – h), respectively. 50 total events on average in each 4-hour daily shift in the 60-day observation period have been simulated.
Figure A36: As in Figs. A35, except that 500 total events on average in each 4-hour daily shift in the 60-day observation period have been simulated.
### B.6 Kamioka Observatory

| Central date (observation period) (day) | Most–event directions |
|----------------------------------------|------------------------|
|                                        | 50 events              | 500 events              |
| 0 – 365                                | —                      | —                      |
| 49.49
  (19.49 – 79.49)                      | 60°S – 30°S
  60°E – 120°E                          | 75°S – 30°S, 150°W – 90°W
  60°S – 15°S, 30°E – 90°E              |
| 140.74
  (110.74 – 170.74)                     | 60°S – 0°
  180° – 60°W                           | 60°S – 0°
  180° – 30°W                           |
| 231.99
  (201.99 – 261.99)                     | 30°S – 30°N
  120°E – 120°W                         | 30°S – 30°N
  150°E – 90°W                          |
| 323.24
  (293.24 – 353.24)                     | 60°S – 30°N
  120°E – 180°                          | 60°S – 45°N
  120°E – 180°                          |

Table A8: The summary of the directions of the simulated 3-D WIMP velocity with the highest event numbers (> 4 times of the all-sky average value) in the horizontal coordinate system in one entire year and four advanced seasons at the location of the Kamioka laboratory (36.43°N, 137.31°E). 50 and 500 total events on average in each observation period of 60 days.

Remark: as at the ANDES laboratory, in Fig. A38(d) one can find clearly that, with $O(500)$ total WIMP events and a higher analysis resolution, a second “hot point” from 75°S to 30°S, and 150°W to 90°W in the angular distribution pattern in the horizontal coordinate system would also be observed at the Kamioka laboratory.
Figure A37: The angular distribution of the 3-D WIMP velocity in the horizontal coordinate system at the location of the Kamioka laboratory (36.43°N, 137.31°E), 50 total events on average in one entire year (a) and in each of four advanced seasons on the central date of 49.49 day (b), 140.74 day (c), 231.99 day (d), and 323.24 day (e), respectively, have been simulated.
Figure A38: As in Figs. A37, except that 500 total events on average in one entire year (a) and in each of four advanced seasons (b – e) have been simulated.
Figure A39: The angular distribution of the 3-D WIMP velocity transformed from events shown in Fig. A37 to the laboratory coordinate system at the location of the Kamioka laboratory (36.43°N, 137.31°E). All simulation setup and notations are the same as in Fig. A37.
Figure A40: As in Figs. A39 except that 500 total events on average in one entire year (a) and in each of four advanced seasons (b – e) have been simulated.
Angular distribution of the WIMP velocity in the laboratory frame, 177.66 – 237.66 day, 50 events, at Kamioka

Angular distribution of the WIMP velocity in the laboratory frame, 360.16 – 420.16 day, 50 events, at Kamioka

Figure A41: The angular distribution of the 3-D WIMP velocity observed at the location of the Kamioka laboratory (36.43°N, 137.31°E) in four daily shifts in the observation periods of 177.66 – 237.66 day (a – d) and 360.16 – 420.16 (= 55.16) day (e – h), respectively. 50 total events on average in each 4-hour daily shift in the 60-day observation period have been simulated.
Figure A42: As in Figs. A41, except that 500 total events on average in each 4-hour daily shift in the 60-day observation period have been simulated.
### B.7 Laboratori Nazionali del Gran Sasso (LNGS)

| Central date (observation period) (day) | Most–event directions |
|----------------------------------------|------------------------|
|                                        | 50 events              |
|                                        | 500 events             |
| 0 – 365                                | —                      |
|                                        | —                      |
| 49.49 (19.49 – 79.49)                  | 30°S – 30°N            |
|                                        | 120°E – 180°           |
|                                        | 45°S – 45°N            |
|                                        | 90°E – 180°            |
| 140.74 (110.74 – 170.74)               | 60°S – 0°              |
|                                        | 60°E – 120°E           |
|                                        | 60°S – 15°N            |
|                                        | 60°E – 120°E           |
| 231.99 (201.99 – 261.99)               | 60°S – 30°S            |
|                                        | 180° – 120°W           |
|                                        | 60°S – 15°S, 150°W – 120°W |
|                                        | 60°S – 45°S, 30°W – 30°E |
| 323.24 (293.24 – 353.24)               | 30°S – 30°N            |
|                                        | 120°E – 60°W           |
|                                        | 30°S – 15°N            |
|                                        | 150°E – 90°W           |

Table A9: The summary of the directions of the simulated 3-D WIMP velocity with the highest event numbers (> 4 times of the all–sky average value) in the horizontal coordinate system in one entire year and four advanced seasons at the location of the LNGS laboratory (42.45°N, 13.58°E). 50 and 500 total events on average in each observation period of 60 days.

Remark: as at the ANDES and the Kamioka laboratories, in Fig. A44(d) one can also find clearly a second “hot point” from 60°S to 45°S, and 30°W to 30°E in the angular distribution pattern in the horizontal coordinate system would also be observed at the LNGS laboratory.
Figure A43: The angular distribution of the 3-D WIMP velocity in the horizontal coordinate system at the location of the LNGS laboratory (42.45°N, 13.58°E), 50 total events on average in one entire year (a) and in each of four advanced seasons on the central date of 49.49 day (b), 140.74 day (c), 231.99 day (d), and 323.24 day (e), respectively, have been simulated.
Figure A44: As in Figs. A43 except that 500 total events on average in one entire year (a) and in each of four advanced seasons (b – e) have been simulated.
Figure A45: The angular distribution of the 3-D WIMP velocity transformed from events shown in Fig. A43 to the laboratory coordinate system at the location of the LNGS laboratory (42.45°N, 13.58°E). All simulation setup and notations are the same as in Fig. A43.
Figure A46: As in Figs. A45, except that 500 total events on average in one entire year (a) and in each of four advanced seasons (b – e) have been simulated.
Figure A47: The angular distribution of the 3-D WIMP velocity observed at the location of the LNGS laboratory (42.45°N, 13.58°E) in four daily shifts in the observation periods of 177.66 – 237.66 day (a – d) and 360.16 – 420.16 (= 55.16) day (e – h), respectively. 50 total events on average in each 4-hour daily shift in the 60-day observation period have been simulated.
Angular distribution of the WIMP velocity in the laboratory frame, 360.16 – 420.16 day, 500 events, at LNGS.

Figure A48: As in Figs. A47, except that 500 total events on average in each 4-hour daily shift in the 60-day observation period have been simulated.
### B.8 Laboratoire Souterrain Canfranc (LSC)

| Central date (observation period) (day) | Most–event directions |
|----------------------------------------|------------------------|
|                                        | 50 events              | 500 events |
| 0 – 365                                | _                       | _          |
| 49.49 (19.49 – 79.49)                  | 30°S – 30°N            | 45°S – 60°N |
|                                        | 120°E – 180°            | 120°E – 150°W |
| 140.74 (110.74 – 170.74)              | 60°S – 0°               | 60°S – 60°N |
|                                        | 60°E – 120°E           | 90°E – 150°E |
| 231.99 (201.99 – 261.99)              | 60°S – 30°S            | 60°S – 15°S, 150°W – 120°W |
|                                        | 180° – 120°W           | 60°S – 45°S, 0° – 60°E |
| 323.24 (293.24 – 353.24)              | 30°S – 0°               | 45°S – 15°N |
|                                        | 180° – 60°W            | 150°E – 60°W |

Table A10: The summary of the directions of the simulated 3-D WIMP velocity with the highest event numbers (> 4 times of the all–sky average value) in the horizontal coordinate system in one entire year and four advanced seasons at the location of the LSC laboratory (42.81°N, 0.56°W). 50 and 500 total events on average in each observation period of 60 days.

Remark: as at the ANDES, the Kamioka, and the LNGS laboratories, in Fig. A50(d) one can also find clearly a second “hot point” from 60°S to 45°S, and 0° to 60°E in the angular distribution pattern in the horizontal coordinate system would also be observed at the LSC laboratory.
Figure A49: The angular distribution of the 3-D WIMP velocity in the horizontal coordinate system at the location of the LSC laboratory (42.81°N, 0.56°W), 50 total events on average in one entire year (a) and in each of four advanced seasons on the central date of 49.49 day (b), 140.74 day (c), 231.99 day (d), and 323.24 day (e), respectively, have been simulated.
Figure A50: As in Figs. A49, except that 500 total events on average in one entire year (a) and in each of four advanced seasons (b – e) have been simulated.
Figure A51: The angular distribution of the 3-D WIMP velocity transformed from events shown in Fig. A49 to the laboratory coordinate system at the location of the LSC laboratory (42.81°N, 0.56°W). All simulation setup and notations are the same as in Fig. A49.
Figure A52: As in Figs. A51 except that 500 total events on average in one entire year (a) and in each of four advanced seasons (b – e) have been simulated.
Figure A53: The angular distribution of the 3-D WIMP velocity observed at the location of the LSC laboratory (42.81°N, 0.56°W) in four daily shifts in the observation periods of 177.66 – 237.66 day (a – d) and 360.16 – 420.16 (= 55.16) day (e – h), respectively. 50 total events on average in each 4-hour daily shift in the 60-day observation period have been simulated.
Figure A54: As in Figs. A53, except that 500 total events on average in each 4-hour daily shift in the 60-day observation period have been simulated.
### B.9 Laboratoire Souterrain de Modane (LSM)

| Central date (observation period) (day) | Most–event directions | 50 events | 500 events |
|----------------------------------------|------------------------|-----------|------------|
| 0 – 365 | — | — | — |
| 49.49  (19.49 – 79.49) | 30°S – 30°N 120°E – 180° | 45°S – 45°N 90°E – 150°W |
| 140.74 (110.74 – 170.74) | 60°S – 0° 60°E – 120°E | 75°S – 15°N 60°E – 120°E |
| 231.99 (201.99 – 261.99) | 60°S – 30°S 180° – 120°W | 60°S – 15°S, 150°W – 120°W 60°S – 45°S, 30°W – 30°E |
| 323.24 (293.24 – 353.24) | 30°S – 0° 120°E – 60°W | 30°S – 15°N 150°E – 60°W |

Table A11: The summary of the directions of the simulated 3-D WIMP velocity with the highest event numbers (> 4 times of the all–sky average value) in the horizontal coordinate system in one entire year and four advanced seasons at the location of the LSM laboratory (45.14°N, 6.70°E). 50 and 500 total events on average in each observation period of 60 days.

Remark: as at the ANDES, the Kamioka, the LNGS, and the LSC laboratories, in Fig. [A56](d) one can also find clearly a second “hot point” from 60°S to 45°S, and 30°W to 30°E in the angular distribution pattern in the horizontal coordinate system would also be observed at the LSM laboratory.
Figure A55: The angular distribution of the 3-D WIMP velocity in the horizontal coordinate system at the location of the LSM laboratory (45.14°N, 6.70°E), 50 total events on average in one entire year (a) and in each of four advanced seasons on the central date of 49.49 day (b), 140.74 day (c), 231.99 day (d), and 323.24 day (e), respectively, have been simulated.
Figure A56: As in Figs. A55, except that 500 total events on average in one entire year (a) and in each of four advanced seasons (b – e) have been simulated.
Figure A57: The angular distribution of the 3-D WIMP velocity transformed from events shown in Fig. A55 to the laboratory coordinate system at the location of the LSM laboratory (45.14°N, 6.70°E). All simulation setup and notations are the same as in Fig. A55.
Figure A58: As in Figs. A57, except that 500 total events on average in one entire year (a) and in each of four advanced seasons (b – e) have been simulated.
Angular distribution of the WIMP velocity in the laboratory frame, 177.66 – 237.66 day, 50 events, at LSM

Angular distribution of the WIMP velocity in the laboratory frame, 360.16 – 420.16 day, 50 events, at LSM

**Figure A59:** The angular distribution of the 3-D WIMP velocity observed at the location of the LSM laboratory (45.14°N, 6.70°E) in four daily shifts in the observation periods of 177.66 – 237.66 day (a – d) and 360.16 – 420.16 day (= 55.16) day (e – h), respectively. 50 total events on average in each 4-hour daily shift in the 60-day observation period have been simulated.

155
Figure A60: As in Figs. A59 except that 500 total events on average in each 4-hour daily shift in the 60-day observation period have been simulated.
### B.10 Sudbury Neutrino Observatory (SNOLAB)

| Central date (observation period) (day) | Most–event directions |
|----------------------------------------|-----------------------|
|                                        | 50 events             | 500 events          |
| 0 – 365                                | —                     | 15°S – 0°           |
|                                        |                       | 150°E – 150°W       |
| 49.49 (19.49 – 79.49)                  | 30°S – 30°N           | 30°S – 15°N         |
|                                        | 180° – 60°W           | 150°E – 90°W        |
| 140.74 (110.74 – 170.74)               | 30°S – 30°N           | 45°S – 45°N         |
|                                        | 120°E – 180°          | 120°E – 150°W       |
| 231.99 (201.99 – 261.99)               | 30°S – 0°             | 60°S – 15°N         |
|                                        | 60°E – 120°E          | 90°E – 150°E        |
| 323.24 (293.24 – 353.24)               | 60°S – 0°             | 60°S – 15°S         |
|                                        | 180° – 120°W          | 150°W – 120°W       |

Table A12: The summary of the directions of the simulated 3-D WIMP velocity with the highest event numbers (> 4 times of the all-sky average value) in the horizontal coordinate system in one entire year and four advanced seasons at the location of the SNOLAB laboratory (46.47°N, 81.19°W). 50 and 500 total events on average in each observation period of 60 days.

Remark: as at the DUSEL laboratory, with $O(500)$ total WIMP events recorded in one entire year and a higher analysis resolution, in the angular distribution pattern in the horizontal coordinate system at the SNOLAB laboratory shown in Fig. A62(a), some clear high–WIMP–flux bins from 15°S to 0° and 150°E to 150°W could be identified.
Figure A61: The angular distribution of the 3-D WIMP velocity in the horizontal coordinate system at the location of the SNOLAB laboratory (46.47°N, 81.19°W), 50 total events on average in one entire year (a) and in each of four advanced seasons on the central date of 49.49 day (b), 140.74 day (c), 231.99 day (d), and 323.24 day (e), respectively, have been simulated.
Angular distribution of the WIMP velocity in the horizontal frame, 0 - 365 day, 500 events, at SNOLAB

Figure A62: As in Figs. A61, except that 500 total events on average in one entire year (a) and in each of four advanced seasons (b – e) have been simulated.
Figure A63: The angular distribution of the 3-D WIMP velocity transformed from events shown in Fig. A61 to the laboratory coordinate system at the location of the SNOLAB laboratory (46.47°N, 81.19°W). All simulation setup and notations are the same as in Fig. A61.
Figure A64: As in Figs. A63, except that 500 total events on average in one entire year (a) and in each of four advanced seasons (b – e) have been simulated.
Figure A65: The angular distribution of the 3-D WIMP velocity observed at the location of the SNOLAB laboratory (46.47°N, 81.19°W) in four daily shifts in the observation periods of 177.66 – 237.66 day (a – d) and 360.16 – 420.16 (= 55.16) day (e – h), respectively. 50 total events on average in each 4-hour daily shift in the 60-day observation period have been simulated.
Angular distribution of the WIMP velocity in the laboratory frame, 360.16 – 420.16 day, 500 events, at SNOLAB

Angular distribution of the WIMP velocity in the laboratory frame, 177.66 – 237.66 day, 500 events, at SNOLAB

Angular distribution of the WIMP velocity in the laboratory frame, 177.66 – 237.66 day, 500 events, at SNOLAB

Angular distribution of the WIMP velocity in the laboratory frame, 177.66 – 237.66 day, 500 events, at SNOLAB

Angular distribution of the WIMP velocity in the laboratory frame, 360.16 – 420.16 day, 500 events, at SNOLAB

Angular distribution of the WIMP velocity in the laboratory frame, 360.16 – 420.16 day, 500 events, at SNOLAB

Angular distribution of the WIMP velocity in the laboratory frame, 360.16 – 420.16 day, 500 events, at SNOLAB

Angular distribution of the WIMP velocity in the laboratory frame, 360.16 – 420.16 day, 500 events, at SNOLAB

Figure A66: As in Figs. A65, except that 500 total events on average in each 4-hour daily shift in the 60-day observation period have been simulated.
## B.11 Stawell Underground Physics Laboratory (SUPL)

| Central date (observation period) (day) | Most–event directions | 50 events | 500 events |
|---------------------------------------|------------------------|-----------|-----------|
| 0 – 365                               | —                      | —         | —         |
| 49.49 (19.49 – 79.49)                 | 30°S – 30°N            | 120°E – 120°W | 45°S – 30°N |
|                                       | 120°E – 120°W          |            | 120°E – 120°W |
| 140.74 (110.74 – 170.74)             | 30°S – 60°N            | 180° – 120°W | 45°S – 60°N |
|                                       | 150°W – 90°W           |            |           |
| 231.99 (201.99 – 261.99)             | 30°N – 60°N            | 120°W – 60°W + 120°E – 180° | 15°N – 60°N, 120°W – 60°W |
|                                       | 30°N – 60°N, 120°E – 150°E |            |           |
| 323.24 (293.24 – 353.24)             | 0° – 60°N              | 60°E – 180° | 15°S – 45°N |
|                                       | 60°E – 180°             |            |           |

Table A13: The summary of the directions of the simulated 3-D WIMP velocity with the highest event numbers (> 4 times of the all–sky average value) in the horizontal coordinate system in one entire year and four advanced seasons at the location of the SUPL laboratory (37.07°S, 142.81°E). 50 and 500 total events on average in each observation period of 60 days.

Remark: as at the Boulby laboratory, with only $O(50)$ total WIMP events one could already observe two high–WIMP–flux bins in the angular distribution pattern in the horizontal coordinate system at the SUPL laboratory shown in Fig. A67(d); with $O(500)$ total events and a higher analysis resolution, these two hot points could be identified more clearly (see Fig. A68(d)).
Figure A67: The angular distribution of the 3-D WIMP velocity in the horizontal coordinate system at the location of the SUPL laboratory (37.07°S, 142.81°E). 50 total events on average in one entire year (a) and in each of four advanced seasons on the central date of 49.49 day (b), 140.74 day (c), 231.99 day (d), and 323.24 day (e), respectively, have been simulated.
Figure A68: As in Figs. A67, except that 500 total events on average in one entire year (a) and in each of four advanced seasons (b – e) have been simulated.
Figure A69: The angular distribution of the 3-D WIMP velocity transformed from events shown in Fig. A67 to the laboratory coordinate system at the location of the SUPL laboratory (37.07°S, 142.81°E). All simulation setup and notations are the same as in Fig. A67.
Figure A70: As in Figs. A69, except that 500 total events on average in one entire year (a) and in each of four advanced seasons (b – e) have been simulated.
Figure A71: The angular distribution of the 3-D WIMP velocity observed at the location of the SUPL laboratory (37.07°S, 142.81°E) in four daily shifts in the observation periods of 177.66 – 237.66 day (a – d) and 360.16 – 420.16 (= 55.16) day (e – h), respectively. 50 total events on average in each 4-hour daily shift in the 60-day observation period have been simulated.
Angular distribution of the WIMP velocity in the laboratory frame, 177.66 – 237.66 day, 500 events, at SUPL

Figure A72: As in Figs. A71, except that 500 total events on average in each 4-hour daily shift in the 60-day observation period have been simulated.
C Annual modulation of the radial WIMP velocity distribution in four normal seasons

For the sake of completeness and readers’ reference, in this section, we provide the reconstruction results of the annual modulation of the radial distribution of the 3-D WIMP velocity in the Equatorial coordinate system in the observation periods of four normal seasons. 50 and 500 total events on average in each 60-day observation period have been simulated.

C.1 With the one–parameter velocity distribution $f_{1,sh,v_0}(v; v_0)$

As in Sec. 4.3.1, we start our reconstruction by using the one–parameter velocity distribution $f_{1,sh,v_0}(v; v_0)$ given by Eq. (24) with $v_0$ as the fitting parameter scanned in the ranges of $160 \text{ km/s} \leq v_0 \leq 270 \text{ km/s}$ (50 events) and $190 \text{ km/s} \leq v_0 \leq 240 \text{ km/s}$ (500 events) as well as the constraint that $v_e = 1.05 v_0$.

In Figs. A73 and A74, we show the reconstructed radial distributions of the 3-D WIMP velocity and the 1(2)$\sigma$ statistical uncertainty bands by using $f_{1,sh,v_0}(v; v_0)$ as well as the distributions of the fitting parameter $v_0$ in all simulated experiments with 50 and 500 total events on average in each 60-day observation period of four normal seasons, respectively. Comparing with our results for the advanced seasons shown in Figs. 30 and 46, one can see here, as expected, an asymmetric periodic variation of the best–fit values of the fitting parameter $v_0$ (see the summary of the reconstructed results and the 1(2)$\sigma$ statistical uncertainty ranges of the median values in Tables A14 and A15).

| Central date (observation period) | Parameter | Max. P_median | Median | 1$\sigma$ range | 2$\sigma$ range |
|----------------------------------|-----------|---------------|--------|-----------------|----------------|
| 79.0 (49.0 – 109.0)              | $v_0$ [km/s] | 218.3         | 217.2$^{+11.0}_{-9.9}$ ($^{+23.1}_{-20.9}$) | [207.3, 228.2] | [196.3, 240.3] |
| 170.25 (140.25 – 200.25)        | $v_0$ [km/s] | 220.5         | 219.4$^{+11.0}_{-9.9}$ ($^{+23.1}_{-20.9}$) | [209.5, 230.4] | [198.5, 242.5] |
| 261.5 (231.50 – 291.50)         | $v_0$ [km/s] | 212.8         | 211.7$^{+11.0}_{-9.9}$ ($^{+22.0}_{-20.9}$) | [201.8, 222.7] | [190.8, 233.7] |
| 352.75 (322.75 – 382.75)        | $v_0$ [km/s] | 209.5         | 209.5$^{±11.0}$ ($^{+22.0}_{-20.9}$) | [198.5, 220.5] | [188.6, 231.5] |

50 total events on average in one observation period of 60 days

Table A14: The summary of the reconstructed results of the fitting parameter $v_0$ and their 1(2)$\sigma$ statistical uncertainty ranges of the median values by using the one–parameter velocity distribution $f_{1,sh,v_0}(v; v_0)$ with 50 total events on average in each 60–day observation period of the normal seasons.
Figure A73: As in Figs. 30, reconstructed with the one–parameter velocity distribution $f_{1,sh,v0}(v; v_0)$, except that four 60-day observation periods of the normal seasons have been considered.
Figure A74: As in Figs. A73, except that 500 total events on average in one observation period of 60 days have been simulated. Remind that the scanning range of the fitting parameter $v_0$ is shrunk to between 190 km/s and 240 km/s.
Fitting distribution: one–parameter velocity distribution $f_{1,sh,v_0}(v; v_0)$

| Central date (observation period) (day) | Parameter | Max. P$_{\text{median}}$ | Median $v_0$ [km/s] | 1$\sigma$ range | 2$\sigma$ range |
|----------------------------------------|-----------|--------------------------|---------------------|-----------------|----------------|
| 79.0 (49.0 – 109.0)                    | $v_0$ [km/s] | 217.5                   | 217.5$^{+3.0}_{-3.5}$ ($^{+6.5}_{-7.0}$) | [214.0, 220.5] | [210.5, 224.0] |
| 170.25 (140.25 – 200.25)              | $v_0$ [km/s] | 220.0                   | 220.0$^{+3.0}_{-3.5}$ ($^{+6.5}_{-7.0}$) | [216.5, 223.0] | [213.0, 226.5] |
| 261.5 (231.50 – 291.50)               | $v_0$ [km/s] | 211.5                   | 211.5$^{+3.5}_{-3.0}$ ($^{+7.0}_{-6.5}$) | [208.5, 215.0] | [205.0, 218.5] |
| 352.75 (322.75 – 382.75)              | $v_0$ [km/s] | 209.5                   | 209.5$^{+3.0}_{-3.5}$ ($^{+6.5}_{-7.0}$) | [206.0, 212.5] | [203.0, 216.0] |

500 total events on average in one observation period of 60 days

Table A15: The summary of the reconstructed results of the fitting parameter $v_0$ and their 1(2)$\sigma$ statistical uncertainty ranges of the median values by using the one–parameter velocity distribution $f_{1,sh,v_0}(v; v_0)$ with 500 total events on average in each 60–day observation period of the normal seasons.

C.2 With the $v_0$–fixed velocity distribution $f_{1,sh,v_e}(v; v_e)$

As in Sec. 4.3.2, we now consider the $v_0$–fixed velocity distribution $f_{1,sh,v_e}(v; v_e)$ given by Eq. (26) with $v_e$ as the fitting parameter scanned in the ranges of 90 km/s $\leq v_e \leq 330$ km/s (50 events) and 180 km/s $\leq v_e \leq 270$ km/s (500 events) as well as the input condition that $v_0 = 220$ km/s. In Figs. A75 and A76, we show the reconstructed radial distributions of the 3-D WIMP velocity and the 1(2)$\sigma$ statistical uncertainty bands by using $f_{1,sh,v_e}(v; v_e)$ as well as the distributions of the fitting parameter $v_e$ in all simulated experiments with 50 and 500 total events on average in each 60-day observation period of four normal seasons, respectively. As in Sec. C.1, by comparing with our results for the advanced seasons shown in Figs. 31 and 47, we can also find here an asymmetric periodic variation of the best–fit values of the fitting parameter $v_e$ (see the summary of the reconstructed results and the 1(2)$\sigma$ statistical uncertainty ranges of the median values in Tables A16 and A17).

C.3 With the simplified velocity distribution $f_{1,sh}(v; v_0, v_e)$

As in Sec. 4.3.3, we release now the constraints on $v_0$ and $v_e$ and consider the simplified velocity distribution $f_{1,sh}(v; v_0, v_e)$ given by Eq. (27) with $v_0$ and $v_e$ as two free fitting parameters scanned in the ranges of 80 km/s $\leq v_0 \leq 340$ km/s (50 events) and 140 km/s $\leq v_0 \leq 240$ km/s (500 events) as well as 0 $\leq v_e \leq 380$ km/s (50 events) and 200 km/s $\leq v_e \leq 310$ km/s (500 events), respectively. In Figs. A77 and A78, we show the reconstructed radial distributions of the 3-D WIMP velocity and the 1(2)$\sigma$ statistical uncertainty bands by using $f_{1,sh}(v; v_0, v_e)$ as well as the distributions of the fitting parameters $v_0$ and $v_e$ in all simulated experiments on the $v_0 – v_e$ plane with
Figure A75: As in Figs. 31, reconstructed with the $v_0$-fixed velocity distribution $f_{1,sh,v_0}(v; v_e)$, except that four 60-day observation periods of the normal seasons have been considered.
Figure A76: As in Figs. A75 except that 500 total events on average in one observation period of 60 days have been simulated. Remind that the scanning range of the fitting parameter $v_e$ is shrunk to between 180 km/s and 270 km/s.
Figure A77: As in Figs. 32, reconstructed with the simplified velocity distribution $f_{1,sh}(v; v_0, v_e)$, except that four 60-day observation periods of the normal seasons have been considered.
Figure A78: As in Figs. A77, except that 500 total events on average in one observation period of 60 days have been simulated. Remind that the scanning ranges of the fitting parameters $v_0$ and $v_e$ are shrunk to between 140 km/s and 240 km/s and between 200 km/s and 310 km/s, respectively.
Fitting distribution: \( v_0 \)-fixed velocity distribution \( f_{1,sh,v_e}(v; v_e) \)

| Central date (observation period) (day) | Parameter | Max. \( P_{\text{median}} \) | Median | 1\(\sigma \) range | 2\(\sigma \) range |
|----------------------------------------|-----------|-----------------------------|--------|------------------|------------------|
| 79.0 (49.0 – 109.0)                    | \( v_e \) [km/s] | 231.6 | 231.6 \( \pm 26.4 \) \((^{+50.4}_{-55.2})\) | [205.2, 258.0] | [176.4, 282.0] |
| 170.25 (140.25 – 200.25)              | \( v_e \) [km/s] | 238.8 | 238.8 \( ^{+24.0}_{-28.8} \) \((^{+50.4}_{-55.2})\) | [210.0, 262.8] | [183.6, 289.2] |
| 261.5 (231.50 – 291.50)               | \( v_e \) [km/s] | 217.2 | 217.2 \( ^{+26.4}_{-28.8} \) \((^{+50.4}_{-60.0})\) | [188.4, 243.6] | [157.2, 267.6] |
| 352.75 (322.75 – 382.75)             | \( v_e \) [km/s] | 210.0 | 210.0 \( ^{+26.4}_{-28.8} \) \((^{+52.8}_{-62.4})\) | [181.2, 236.4] | [147.6, 262.8] |

50 total events on average in one observation period of 60 days

Table A16: The summary of the reconstructed results of the fitting parameter \( v_e \) and their 1(2)\(\sigma \) statistical uncertainty ranges of the median values by using the \( v_0 \)-fixed velocity distribution \( f_{1,sh,v_e}(v; v_e) \) with 50 total events on average in each 60-day observation period of the normal seasons.

50 and 500 total events on average in each 60-day observation period of four normal seasons, respectively. Besides that the asymmetric periodic variation of the best–fit values of the fitting parameter \( v_e \) can be seen clearly here, the invariability of the best–fit values of \( v_0 \) as its annual average value can also be confirmed (see the summary of the reconstructed results and the 1(2)\(\sigma \) statistical uncertainty ranges of the median values in Tables A18 and A19).

**C.4 With the modified velocity distribution \( f_{1,sh,\Delta v}(v; v_0, \Delta v) \)**

Finally, as in Sec. 4.3.4, we consider the modified velocity distribution \( f_{1,sh,\Delta v}(v; v_0, \Delta v) \) given by Eq. (28) with \( v_0 \) and \( \Delta v \) as two free fitting parameters scanned in the ranges of 80 km/s \( \leq v_0 \leq 340 \) km/s (50 events) and 140 km/s \( \leq v_0 \leq 240 \) km/s (500 events) as well as \(-190 \) km/s \( \leq \Delta v \leq 230 \) km/s (50 events) and \(-20 \) km/s \( \leq \Delta v \leq 150 \) km/s (500 events), respectively.

In Figs. A79 and A80, we show the reconstructed radial distributions of the 3-D WIMP velocity and the 1(2)\(\sigma \) statistical uncertainty bands by using \( f_{1,sh,\Delta v}(v; v_0, \Delta v) \) as well as the distributions of the fitting parameters \( v_0 \) and \( \Delta v \) in all simulated experiments on the \( v_0 - \Delta v \) plane with 50 and 500 total events on average in each 60-day observation period of four normal seasons, respectively. Not surprisingly, one can find the asymmetric periodic variation of the best–fit values of the fitting parameter \( \Delta v \) as well as the invariability of the best–fit values of \( v_0 \) as its annual average value (see the summary of the reconstructed results and the 1(2)\(\sigma \) statistical uncertainty ranges of the median values given in Tables A20 and A21).
Figure A79: As in Figs. 33, reconstructed with the modified velocity distribution $f_{1,sh,\Delta v}(v; v_0, \Delta v)$, except that four 60-day observation periods of the normal seasons have been considered.
Figure A80: As in Figs. A79, except that 500 total events on average in one observation period of 60 days have been simulated. Remind that the scanning ranges of the fitting parameters $v_0$ and $v_e$ are shrunk to between 140 km/s and 240 km/s and between $-20$ km/s and 150 km/s, respectively.
Fitting distribution: \( v_0 \)-fixed velocity distribution \( f_{1,sh,v_e}(v; v_e) \)

| Central date (observation period) (day) | Parameter \( v_e \) [km/s] | Max. P\(_{\text{median}}\) | Median \( v_e \) | 1\( \sigma \) range | 2\( \sigma \) range |
|----------------------------------------|---------------------------|-------------------------|-------------|----------------|----------------|
| 79.0 (49.0 – 109.0)                    | 232.2                     | 232.2 ± 8.1 (+16.2 \(-17.1\)) | [224.1, 240.3] | [215.1, 248.4] |
| 170.25 (140.25 – 200.25)              | 238.5                     | 238.5 \(+7.2 \(-9.0\)) (+15.3 \(-17.1\)) | [229.5, 245.7] | [221.4, 253.8] |
| 261.5 (231.50 – 291.50)               | 216.9                     | 216.9 ± 8.1 (±17.1) | [208.8, 225.0] | [199.8, 234.0] |
| 352.75 (322.75 – 382.75)              | 210.6                     | 210.6 \(+8.1 \(-9.0\)) (+16.2 \(-18.0\)) | [201.6, 218.7] | [192.6, 226.8] |

Table A17: The summary of the reconstructed results of the fitting parameter \( v_e \) and their 1(2)\( \sigma \) statistical uncertainty ranges of the median values by using the \( v_0 \)-fixed velocity distribution \( f_{1,sh,v_e}(v; v_e) \) with 500 total events on average in each 60-day observation period of the normal seasons.

D Analytic forms of the shift Maxwellian velocity distribution

In this section, we show the plots of the exact analytic expression of the shift Maxwellian velocity distribution \( f_{1,sh,v_e}(v) \) given in Eq. (19) and the simplified form \( f_{1,sh}(v) \) given in Eq. (22). The Galactic escape velocity has been set as \( v_{\text{esc}} = 500 \) km/s in Figs. A81 and raised to \( v_{\text{esc}} = 600 \) km/s in Figs. A82. Additionally, as a comparison of the shape difference between two expressions, their boundaries due to the annual modulation of the Earth’s Galactic velocity \( v_e(t) \) given by Eq. (21) have also been given.

It can be seen clearly that, with the Galactic escape velocity of \( v_{\text{esc}} = 500 \) km/s, the shape difference between the exact analytic expression of the shift Maxwellian velocity distribution \( f_{1,sh,v_e}(v) \) and the simplified form \( f_{1,sh}(v) \) is much smaller than the annual variation of these expressions. This tiny difference would also be negligible compared to the much larger statistical uncertainties on the recorded WIMP events (shown in e.g., Fig. 25). Moreover, once the Galactic escape velocity would be as large as \( v_{\text{esc}} = 600 \) km/s, the tiny shape difference between two expressions could even vanish.

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Table A18: The summary of the reconstructed results of the fitting parameters $v_0$ and $v_e$ as well as their 1(2)$\sigma$ statistical uncertainty ranges of the median values by using the simplified velocity distribution $f_{1,sh}(v; v_0, v_e)$ with 50 total events on average in each 60-day observation period of the normal seasons.

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### Fitting distribution: simplified velocity distribution $f_{1,sh}(v; v_0, v_e)$

| Central date (observation period) (day) | Parameter | Max. $P_{\text{median}}$ | Median $v_0$ [km/s] | $\sigma$ range | $2\sigma$ range |
|----------------------------------------|-----------|--------------------------|----------------------|-----------------|-----------------|
| 79.0 (49.0 – 109.0)                    | $v_0$ [km/s] | 185.0                    | $185.0^{+9.0}_{-8.0} (^{+19.3}_{-16.0})$ | [177.0, 194.0]  | [169.0, 204.3]  |
|                                        | $v_e$ [km/s] | 269.3                    | $269.3^{+8.8}_{-11.0} (^{+17.6}_{-23.1})$ | [258.3, 278.1]  | [246.2, 286.9]  |
| 170.25 (140.25 – 200.25)              | $v_0$ [km/s] | 186.0                    | $186.0^{+8.0}_{-9.0} (^{+18.0}_{-16.0})$ | [177.0, 194.0]  | [170.0, 204.0]  |
|                                        | $v_e$ [km/s] | 272.6                    | $273.7^{+8.8}_{-9.9} (^{+16.8}_{-22.0})$ | [263.8, 282.5]  | [251.7, 290.5]  |
| 261.5 (231.50 – 291.50)               | $v_0$ [km/s] | 185.0                    | $185.0 \pm 9.0 (^{+19.0}_{-17.0})$       | [176.0, 194.0]  | [168.0, 204.0]  |
|                                        | $v_e$ [km/s] | 257.2                    | $257.2^{+9.9}_{-11.0} (^{+18.7}_{-24.2})$ | [246.2, 267.1]  | [233.0, 275.9]  |
| 352.75 (322.75 – 382.75)              | $v_0$ [km/s] | 186.0                    | $186.0 \pm 9.0 (^{+21.0}_{-17.0})$       | [177.0, 195.0]  | [169.0, 207.0]  |
|                                        | $v_e$ [km/s] | 250.6                    | $250.6^{+9.9}_{-11.0} (^{+19.8}_{-25.3})$ | [239.6, 260.5]  | [225.3, 270.4]  |

500 total events on average in one observation period of 60 days

Table A19: The summary of the reconstructed results of the fitting parameters $v_0$ and $v_e$ as well as their $1(2)\sigma$ statistical uncertainty ranges of the median values by using the simplified velocity distribution $f_{1,sh}(v; v_0, v_e)$ with 500 total events on average in each 60–day observation period of the normal seasons.

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Table A20: The summary of the reconstructed results of the fitting parameters \(v_0\) and \(\Delta v\) as well as their 1(2)\(\sigma\) statistical uncertainty ranges of the median values by using the modified velocity distribution \(f_{1,sh,\Delta v}(v; v_0, \Delta v)\) with 50 total events on average in each 60–day observation period of the normal seasons.

| Central date (observation period) (day) | Parameter | Max. \(P_{\text{median}}\) | Median \(v_0\) [km/s] | \(\Delta v\) [km/s] | \(1\sigma\) range | \(2\sigma\) range |
|----------------------------------------|-----------|--------------------------|-----------------------|-----------------------|-----------------|-----------------|
| 79.0 (49.0 – 109.0)                    | \(v_0\) [km/s] | 191.8                    | 186.6\( ^{+36.4}_{-26.0} \) (\( ^{+88.4}_{-46.8} \)) | 70.4                    | [163.2, 220.4] | [139.8, 278.3] |
|                                        | \(\Delta v\) [km/s] | 83.0\( ^{+40.2}_{-75.6} \) (\( ^{+84.0}_{-273.0} \)) | [7.4, 129.2] | [−190.0, 167.0] |
| 170.25 (140.25 – 200.25)              | \(v_0\) [km/s] | 191.8                    | 186.6\( ^{+33.8}_{-23.4} \) (\( ^{+93.6}_{-46.8} \)) | 74.6                    | [163.2, 220.4] | [139.8, 280.2] |
|                                        | \(\Delta v\) [km/s] | 87.2\( ^{+42.0}_{-71.4} \) (\( ^{+84.0}_{-274.1} \)) | [15.8, 129.2] | [−186.9, 171.2] |
| 261.5 (231.50 – 291.50)               | \(v_0\) [km/s] | 191.8                    | 186.6\( ^{+33.8}_{-26.0} \) (\( ^{+89.1}_{-49.4} \)) | 57.8                    | [160.6, 220.4] | [137.2, 275.7] |
|                                        | \(\Delta v\) [km/s] | 70.4\( ^{+46.2}_{-79.8} \) (\( ^{+88.2}_{-260.4} \)) | [−9.4, 116.6] | [−190.0, 158.6] |
| 352.75 (322.75 – 382.75)              | \(v_0\) [km/s] | 191.8                    | 186.6\( ^{+39.0}_{-26.0} \) (\( ^{+88.4}_{-49.4} \)) | 53.6                    | [160.6, 225.6] | [137.2, 275.0] |
|                                        | \(\Delta v\) [km/s] | 66.2\( ^{+46.2}_{-88.2} \) (\( ^{+84.0}_{-256.2} \)) | [−22.0, 112.4] | [−190.0, 150.2] |

50 total events on average in one observation period of 60 days
Table A21: The summary of the reconstructed results of the fitting parameters $v_0$ and $\Delta v$ as well as their $1\sigma$ and $2\sigma$ statistical uncertainty ranges of the median values by using the modified velocity distribution $f_{1,sh,\Delta v}(v; v_0, \Delta v)$ with 500 total events on average in each 60-day observation period of the normal seasons.

186
Figure A81: The exact analytic form of the shift Maxwellian velocity distribution $f_{1,sh,vesc}(v)$ given in Eq. (19) (dash-dotted blue) and the simplified form $f_{1,sh}(v)$ given in Eq. (22) (solid red) with the Galactic escape velocity of $v_{esc} = 500$ km/s. In the lower frame, the boundaries of two velocity distributions due to the annual modulation of the Earth’s Galactic velocity $v_e(t)$ given by Eq. (21) have been additionally given as the dashed blue and the dotted red curves for the exact and the simplified expressions, respectively.
Figure A82: As in Figs. A81 except that the Galactic escape velocity has been raised to $v_{\text{esc}} = 600 \text{ km/s}$. 

$v_0 = 220 \text{ km/s}, v_{\text{esc}} = 600 \text{ km/s}, t = 243.75 \text{ d}, 152.5 \text{ d}, 335 \text{ d}$
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