Supplemental Material

“Saturation of the inverse cascade in surface gravity wave turbulence”

E. Falcon, G. Michel, G. Prabhudesai, A. Cazaubiel, M. Berhanu, N. Mordant, S. Aumaître, and F. Bonnefoy

1 Université de Paris, Univ Paris Diderot, MSC, UMR 7057 CNRS, F-75 013 Paris, France
2 Sorbonne Université, IJLRA, UMR 7190 CNRS, F-75 005 Paris, France
3 Ecole Normale Supérieure, LPS, UMR 8550 CNRS, F-75 205 Paris, France
4 Université Grenoble Alpes, LEGI, UMR 5519 CNRS, F-38 000 Grenoble, France
5 CEA-Saclay, Sphynx, DSM, URA 2464 CNRS, F-91 191 Gif-sur-Yvette, France
6 Ecole Centrale de Nantes, LHEEA, UMR 6598 CNRS, F-44 321 Nantes, France

In this Supplemental Material, we present movies (§1) and additional data analyses related to the observation of the inverse cascade in gravity wave turbulence on the surface of a fluid: Wave homogeneity (§2), wave height distribution probability (§3), and timescale separation (§4). Notations as in the aforementioned paper.

1. MOVIES

- BasinMEDcutLow.mpeg (35s - 24Mo): Large-scale wave basin seen from the shore showing the wave makers, the wall at the opposite end, the probe array, and the control room. Weak random forcing conditions.
- WeakForcingIMG_7799.mp4 (15s - 17Mo): Wave field seen from the shore. Weak random forcing conditions: Wave steepness $\epsilon = 0.063$ ($\sigma_{sat}^{\eta} = 0.59$ cm) and narrow spectral bandwidth ($1.8 \pm 0.2$ Hz).
- HigherForcingIMG_7800.mp4 (15s - 16Mo): Wave field seen from the shore. Higher random forcing conditions: Wave steepness $\epsilon = 0.98$ ($\sigma_{sat}^{\eta} = 1.84$ cm) and narrow spectral bandwidth ($1.8 \pm 0.2$ Hz).

2. WAVE HOMOGENEITY

![Graph showing time-averaged standard deviations of wave height $\sigma_{\eta}(x)$ recorded at different distances $x$ from the wave makers for different forcing amplitudes during 60 min. Good homogeneity for weak enough forcing. Same symbols as in the main paper.](image)

FIG. 1: Time-averaged standard deviations of wave height $\sigma_{\eta}(x)$ recorded at different distances $x$ from the wave makers for different forcing amplitudes during 60 min. Good homogeneity for weak enough forcing. Same symbols as in the main paper.
3. WAVE PROBABILITY DISTRIBUTION

FIG. 2: (Left) Probability distribution functions (PDF) of normalized wave height $\eta(t)/\sigma_\eta$ recorded in the middle of the basin ($x = 20$ m) during 60 min and averaged on different probes ($n^5$ (blue), 10 (red), 11 (black), and 12 (magenta)). $\sigma_\eta \equiv \sqrt{\langle \eta(t)^2 \rangle_t} = 1.14$ cm. Black dash-dotted line displays a Gaussian of zero mean and unit standard deviation. Red dashed line shows a Tayfun distribution for a wave steepness of 0.08. A weak asymmetry $S = \langle \eta^3 \rangle / \langle \eta^2 \rangle^{3/2} = 0.23$ and a weak Kurtosis $P = \langle \eta^4 \rangle / \langle \eta^2 \rangle^2 = 3.3$ are observed as expected for a weakly nonlinear wave field (Normal distribution would lead to $S = 0$ and $P = 3$). Inset displays a part of the corresponding signal $\eta(t)$ for probe n°12. (Right) PDF of $\eta(t)/\sigma_\eta$ computed before ($t \leq 16$ min - blue) and after ($35 < t \leq 51$ min - red) the saturation of the inverse cascade, and parts of the corresponding temporal signals $\eta(t)$ (insets). $x = 20$ m (probe n°12). $\sigma_\eta = 1.14$ cm.

4. WAVE TURBULENCE TIMESCALE SEPARATION

FIG. 3: Wave turbulence timescale separation. (Black solid line) Linear propagation timescale $\tau_{lin} = 1/\omega$. (Blue solid lines) Nonlinear interaction timescales: $\tau_{nl}^i = c^i g^{3/8} Q^{-3/8} k(\omega)^{-3/8}$ (inverse cascade) and $\tau_{nl}^s = c^s g^{3/2} P^{-1/2} k(\omega)^{-3/2}$ (direct cascade) [4], using the values of $Q(\epsilon)$ and $P(\epsilon)$ inferred experimentally [fixed wave steepness $\epsilon = 0.1$ ($\sigma_\eta^2 = 1.92$ cm)], the dimensionless constant value $c^i = 0.03$ found experimentally [28], and assuming $c^s = c^i$. (Green solid line) Discreteness time $\tau_{disc}$ computed as the number of eigenmodes found in a frequency band divided by this bandwidth, taking into account both transverse and lateral eigenmodes. No discreteness effect for $\tau_{nl} < 2\tau_{disc}$ (i.e. nonlinear spectral widening > half frequency separation between adjacent eigenmodes). Linear viscous dissipation timescales [36]: (red dashed line) viscous surface $\tau_{diss} = 1/[2\nu k(\omega)^2]$, (red solid line) surface boundary layer with an inextensible film $\tau_{diss}^f = 2\sqrt{2/[k(\omega)]}$ negligible for $f \lesssim 2$ Hz [31], and (red dotted-dash line) lateral boundary layer $\tau_{diss}^l = 2\sqrt{2L_x L_y/[3\sqrt{\nu}(L_x + L_y)]}$, $L_x = 40$ m, $L_y = 30$ m, whereas bottom boundary layer is negligible. Water kinematic viscosity $\nu = 10^{-6}$ m$^2$/s. $f_0$ is the central forcing frequency, and $f_{max}$ the frequency of the spectrum maximum at the saturation time $t_{sat}$. $\omega(k) = \sqrt{gk}$. 