3D-xy critical properties of YBa$_2$Cu$_4$O$_8$ and magnetic-field-induced 3D to 1D crossover

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Abstract

We present reversible magnetization data of a YBa$_2$Cu$_4$O$_8$ single crystal and analyze the evidence for 3D-xy critical behavior and a magnetic-field-induced 3D to 1D crossover. Remarkable consistency with these phenomena is observed in agreement with a magnetic-field-induced finite size effect, whereupon the correlation length transverse to the applied magnetic field cannot grow beyond the limiting magnetic length scale $L_{Hi} = (\Phi_0/(aH_i))^{1/2}$. By applying the appropriate scaling form we obtain the zero-field critical temperature, the 3D to 1D crossover, the vortex melting line and the universal ratios of the related scaling variables. Accordingly there is no continuous phase transition in the $(H, T)$ plane along the $H_{c2}$ lines as predicted by the mean-field treatment.

(Some figures in this article are in colour only in the electronic version)

1. Introduction

Fluctuation effects are known to be strongly enhanced in high temperature cuprate superconductors due to their anisotropic behavior and their high zero-field transition temperature $T_c$ [1, 2]. For YBa$_2$Cu$_4$O$_8$ and related compounds several studies point to the importance of critical fluctuations [3–6]. To circumvent the smearing of the phase transition due to inhomogeneities YBa$_2$Cu$_4$O$_8$ is an exquisite candidate due to its nearly stoichiometric structure and the availability of excellent single crystals [7, 8]. As YBa$_2$Cu$_4$O$_8$ is intrinsically doped there is no anomalous precursor diamagnetism expected, as reported for example in aluminum-doped MgB$_2$ [6].

In this study we present reversible magnetization data of a YBa$_2$Cu$_4$O$_8$ single crystal and analyze the evidence for 3D-xy critical behavior and a magnetic-field-induced 3D to 1D crossover. We observe remarkable consistency with these phenomena. Since near-$T_c$ thermal fluctuations are expected to dominate [1, 9–12], Gaussian fluctuations point to a magnetic-field-induced 3D crossover [13]. Whereby the effect of fluctuations is enhanced, it appears inevitable to take thermal fluctuations into account. Indeed, invoking the scaling theory of critical phenomena we show that the data are inconsistent with the traditional mean-field interpretation. In contrast, we observe agreement with a magnetic-field-induced finite size effect, whereupon the correlation length transverse to the magnetic field $H_i$, applied along the $i$ axis, cannot grow beyond the limiting magnetic length

$$L_{Hi} = (\Phi_0/(aH_i))^{1/2},$$ (1)

with $a \simeq 3.12$ [14]. $L_{Hi}$ is related to the average distance between vortex lines. Indeed, as the magnetic field increases, the density of vortex lines becomes greater, but this cannot continue indefinitely. The limit is roughly set on the proximity of vortex lines by the overlapping of their cores. This finite size effect implies that, in type II superconductors, superconductivity in a magnetic field is confined to cylinders with diameter $L_{Hi}$ [12, 15]. Accordingly, below $T_c$ there is the 3D to 1D crossover line

$$H_{pr}(T) = (\Phi_0/(a\xi_0^j\xi_0^k))(1-T/T_c)^{4/3},$$ (2)

with $i \neq j \neq k$. $\xi_0^{ij,ijkl}$ denotes the critical amplitudes of the correlation lengths below $T_c$ along the respective axes.
It circumvents the occurrence of the continuous phase transition in the \((H_c, T)\) plane along the \(H_{c2}\) lines predicted by the mean-field treatment \([16]\). Indeed, the relevance of thermal fluctuations emerges already from the reversible magnetization data shown in figure 1. As a matter of fact, the typical mean-fluctuations emerges already from the reversible magnetization allowing us to achieve a resolution of \(10^{-8}\) emu. For this experiment the applied magnetic field was oriented along the \(c\) axis of the crystal. For different magnitudes of the field, temperature-dependent magnetization curves were measured. The zero-field-cooled (ZFC) and field-cooled (FC) data have been compared in order to probe the reversible magnetization only. The superconducting susceptibility was finally obtained by correcting the measured data for the normal state and sample holder contributions. In figure 1 we depicted some of the measured magnetization curves \(m\) versus \(T\) for magnetic fields \(H_c\) applied along the \(c\) axis. At a first glance the data fall on rather smooth curves, revealing that the extraction of critical and crossover behavior requires a rather detailed analysis. When thermal fluctuations dominate and the coupling to the charge is negligible the magnetization per unit volume, \(m = M/V\), adopts the scaling form \([1,9–11]\)

\[
\frac{m}{TH^{1/2}} = \frac{Q^\pm k_B \xi_{ab}}{\Phi_0^{1/2} \xi_c} F^\pm(z),
\]

\[
F^\pm(z) = z^{-1/2} \frac{dG^\pm}{dz},
\]

\[
z = x^{-1/2\nu} \left( \frac{\ell_{ab0}}{\ell_{c0}} \right)^2 |t|^{-2\nu} H_c.
\]

\(Q^\pm\) is a universal constant and \(G^\pm(z)\) is a universal scaling function of its argument, with \(G^\pm(z = 0) = 1\). \(\gamma = \xi_{ab}/\xi_c\) denotes the anisotropy, \(\xi_{ab}\) the zero-field in-plane correlation length and \(H_c\) the magnetic field applied along the \(c\) axis. In terms of the variable \(x\) the scaling form (3) is similar to Prange’s \([17]\) result for Gaussian fluctuations. Approaching \(T_c\) the in-plane correlation length diverges as

\[
\xi_{ab} = \xi_{ab0} |t|^{-\nu}, \quad t = T/T_c - 1, \quad \pm = \text{sgn}(t). \quad (4)
\]

Supposing that 3D-xy fluctuations dominate, the critical exponents are given by \([18]\)

\[
v \simeq 0.671 \simeq 2/3, \quad \alpha = 2\nu - 3 \simeq -0.013, \quad (5)
\]

and there are the universal critical amplitude relations \([1,9–12,18]\)

\[
\frac{\xi_{ab0}}{\xi_{c0}} = \frac{\xi_{c0}}{\xi_{ab0}} \simeq 2.21, \quad \frac{Q^-}{Q^+} \simeq 11.5, \quad A^+ = A^- = 1.07, \quad (6)
\]

\[Q^\pm c_0^\pm \simeq 0.7, \quad Q^+ c_0^+ \simeq 0.9, \quad Q^\pm c_\infty^\pm \simeq 0.5, \quad (11)
\]

\[c_1 \simeq 1.76. \quad (12)
\]

We are now prepared to analyze the magnetization data. To estimate \(T_c\) we note that, according to equations (3), (10)

\[\lambda_{c0} = \frac{\Phi_0^2}{16\pi^2 \lambda_{ab0}^2}, \quad \frac{\ell_{c0}}{\ell_{ab0}} = \frac{\Phi_0^2}{16\pi^2 \gamma \lambda_{ab0}^2}. \quad (9)
\]

Furthermore, the existence of the magnetization at \(T_c\) of the penetration depth below \(T_c\) and of the magnetic susceptibility above \(T_c\) imply the following asymptotic forms of the scaling function \([1,9–12]\):

\[
Q^\pm \left. \frac{dG^\pm}{dz} \right|_{z \to \infty} = Q^\pm c_\infty^\pm, \quad (10)
\]

\[
Q^\pm c_0^\pm = \left. \frac{dG^\pm}{dz} \right|_{z \to 0} = Q^\pm c_0^+ \ln(z + c_1), \quad (10)
\]

\[
Q^\pm c_0^\pm = \left. \frac{dG^\pm}{dz} \right|_{z \to 0} = Q^\pm c_0^+, \quad (10)
\]

with the universal coefficients

\[
Q^\pm c_0^\pm \simeq -0.7, \quad Q^+ c_0^+ \simeq 0.9, \quad Q^\pm c_\infty^\pm \simeq 0.5, (11)
\]

\[c_1 \simeq 1.76. \quad (12)
\]
and (12), the plot \( m / T H_z^{1/2} \) versus \( T \) should exhibit a

crossing point at \( T \sim 79.6 \) K. The full arrow marks the zero-field critical
temperature \( T_c \), the dashed arrow the 3D to 1D crossover and the
dotted arrow the vortex melting line.

From the straight lines we obtain \( m / T H_z^{1/2} \) tends to the value
\( m / T_c H_z^{1/2} = -0.5 k_B / \Phi_0^{3/2} \). The inset in figure 1 reveals
that there is a crossing point near \( T_c \sim 79.6 \) K. Given
this estimate, consistency with 3D-xy critical behavior then requires according to the scaling form (3) that the data plotted
as \( m / T H_z^{1/2} \) versus \( t H_z^{-1/2} \) should collapse near
\( t H_z^{-1/4} \to 0 \) on a single curve. Evidence for this collapse emerges from figure 2 with \( T_c \sim 79.6 \) K. Considering the limit
\( z \to 0 \) below \( T_c \) the appropriate scaling form is

\[
m/T = -Q c^2 B \phi / \phi_0 \xi_0 \left( \ln \left( \frac{H_c \xi_{ab}^2}{\xi_0} \right) + c_1 \right),
\]

according to equations (3), (10) and (12). Thus, given the magnetization data of a homogeneous system, attaining the
limit \( \xi_{ab} = H_c \xi_{ab}^2 / (\Phi_0 / T) \ll 1 \), the growth of \( \xi_{ab} \) and \( \xi_0 \) is
unlimited and estimates for \( \xi_0 \) and \( \xi_{ab} \) can be deduced from

\[
|t|^{-2/3} m / T = -Q c^2 B \phi / \phi_0 \xi_0 \left( \ln \left| t \right|^{-4/3} / \phi_0 \right) + c_1 \right). \tag{14}
\]

In figure 3 we depicted \( |t|^{-2/3} m / T \) versus \( \ln|t|^{-4/3} \).

From the straight lines we obtain

\[
- Q c^2 B \phi / \phi_0 \xi_0 \approx 0.025, \tag{15}
\]

and with that

\[
\xi_0 \approx 1.87 \text{ Å}. \tag{16}
\]

Furthermore, from \( \ln(H_c \xi_{ab}^2 / \phi_0) \) versus \( \ln(H_c) \) we deduce

\[
\xi_{ab} \approx 15.6 \text{ Å}. \tag{17}
\]

For the anisotropy we obtain then the estimate

\[
\gamma = \frac{\xi_{ab}}{\xi_0} \approx 8.34, \tag{18}
\]

compared to \( \gamma \approx 13.4 \), \( \gamma \approx 14.7 \) [19] and \( \gamma \approx 12.3 \) [20].

To explore the magnetic-field-induced 3D to 1D crossover further and to probe the vortex melting line directly we invoke Maxwell’s relation

\[
C \frac{\partial (C/T)}{\partial H_c} = \frac{\partial^2 M / \partial T^2}{H_c}, \tag{19}
\]

uncovering the vortex melting transition in terms of a
singularity, while the magnetic-field-induced finite size effect
leads to a dip. These features seem to differ drastically from the
nearby smooth behavior of the magnetization. Together with
the scaling form of the specific heat (equation (8)), extended to the presence of a magnetic field:

\[
c = \frac{A}{\alpha} \left| t \right|^{-\alpha} f(x), \quad x = \frac{t}{H_c^{-1/2}}, \tag{20}
\]

we obtain the scaling form

\[
T H_c^{1+\alpha/2} \frac{\partial (c / T)}{\partial H_c} \frac{\partial^2 M}{\partial T^2} = - \frac{k_B A}{\alpha} \frac{1}{\alpha - 1} \frac{\partial f}{\partial x} = T H_c^{1+\alpha/2} \frac{\partial^2 M}{\partial T^2}. \tag{21}
\]

In figure 4 we depicted \( T H_c d^2 m / d T^2 \) versus \( x \) for various
magnetic fields \( H_c \). Apparently, the data collapses reasonably
well on a single curve. There is a peak and a dip marked by
an arrow and a vertical line, respectively. Their occurrence
differs clearly from the traditional mean-field behavior where
\( \partial^2 m / \partial T^2 = 0 \). The finite depth of the dip is controlled by the
magnetic-field-induced finite size effect. It replaces the reputed
singularity at the upper critical field obtained in the Gaussian
approximation [17]. Note that both the peak and the dip are
hardly visible in the magnetization shown in figure 2. There
we marked the location of the peak, the dip and \( T_c \) in terms of
dashed and solid arrows. The location of the dip determines the line

\[
x_p = t_p H_c^{-3/4} \approx -2.85 \times 10^{-5} \text{ Oe}^{-3/4}, \tag{22}
\]

in the \( (H_c, T) \) plane where the 3D to 1D crossover occurs. Along this line, rewritten in the form \( H_p(T) = \Phi_0 / (a (\xi_{ab}^2)^2 (1 - T / T_c)^{1/3}) \), the in-plane correlation length is
limited by \( L_{Hi} \) (equation (1)). In addition there is a peak at

\[
x_m = t_m H_c^{-3/4} \simeq -8.35 \times 10^{-5} \text{Oe}^{-3/4},
\]

(23)
corresponding to the vortex melting transition. Rewritten, the vortex melting line follows in the form \( H_{mc} \simeq 2.7 \times 10^8 \text{Oe} \cdot (1 - T_m/T_c)^{1/3} \) which agrees very well with the previous estimate \( H_{mc} \simeq 1.8 \times 10^9 \text{Oe} \cdot (1 - T_m/T_c)^{1/3} \) obtained by Katayama et al.\[21\] as far as the temperature dependence is concerned. Accordingly, we obtain the universal ratios of the scaling variables of the reduced temperatures for the vortex melting line and the 3D to 1D crossover line as

\[
\frac{z_m}{z_p} = \left( \frac{t_p(H_c)}{t_m(H_c)} \right)^{2\nu} \simeq 0.24,
\]

(24)
\[
t_p(H_c)/t_m(H_c) \simeq 0.34.
\]

These values agree well with the estimates \( t_p(H_c)/t_m(H_c) \simeq 0.3 \) for an NdBa\(_2\)Cu\(_4\)O\(_{4.4}\) single crystal\[22\] and \( t_p(H_c)/t_m(H_c) \simeq 0.35 \) for a YBa\(_2\)Cu\(_4\)O\(_{6.97}\) single crystal\[23\] derived from the respective references.

Finally, invoking the universal relation (9) we obtain with \( T_c = 79.6 \text{K} \) and \( \xi_{0,0} \simeq 1.87 \text{Å} \) (equation (16)) for the critical amplitude of the in-plane magnetic field penetration depth the value \( \lambda_{a0} \simeq 1.37 \times 10^{-5} \text{cm} \), in reasonable agreement with the estimate \( \lambda_{a0} \simeq 1.7 \times 10^{-5} \text{cm} \) obtained from magnetization data of polycrystalline YBa\(_2\)Cu\(_4\)O\(_8\) samples\[24\].

3. Summary

We have shown that the analysis of reversible magnetization data of a YBa\(_2\)Cu\(_4\)O\(_8\) single crystal provides considerable insight into the effect of thermal fluctuations and the magnetic-field-induced 3D to 1D crossover. In particular we demonstrated that the fluctuation-dominated regime is experimentally accessible and uncovers remarkable consistency with 3D-x-y critical behavior. Furthermore there is, however, the magnetic-field-induced finite size effect. It implies that the correlation length transverse to the magnetic field \( H_c \), applied along the \( i \) axis, cannot grow beyond the limiting magnetic length \( L_{Hi} = (\Phi_0/(a H_c))^{1/2} \), related to the average distance between vortex lines. Invoking the scaling theory of critical phenomena, clear evidence for this finite size effect has been provided.

In type II superconductors it comprises the 3D to 1D crossover line \( H_{pi}(T) = (\Phi_0/(a_0^2 \gamma_{0,0}))(1 - T/T_c)^{1/3} \) with \( i \neq j \neq k \) and \( \gamma_{0,0} \), \( \gamma_{0,40} \) denoting the critical amplitude of the correlation length below \( T_c \). As a result, below \( T_c \) and above \( H_{pi}(T) \) superconductivity is confined to cylinders with diameter \( L_{Hi} \) (1D). Accordingly, there is no continuous phase transition in the \((H_c, T)\) plane along the \( H_{Hi} \) lines as predicted by the mean-field treatment. In addition, we confirmed the universal relationship between the 3D to 1D crossover and vortex melting line. The universal relation (9) and Maxwell’s relation (19) also imply that the effects of isotope exchange and pressure on \( T_c \), in-plane magnetic field penetration depth, correlation lengths, specific heat and magnetization are not independent.

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