Kinematic and dynamic analysis of a planar tensegrity-based mechanism

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Abstract. Tensegrity-based mechanisms are proposed to be used in the field of robotics due to their advantages such as deployable, easily tunable, redundant, and easily modelled. In this work, the kinematic and dynamic analysis of a planar tensegrity-based mechanism was researched. At first, the forward and inverse kinematic analysis of the mechanism was studied by using the energy method. Then, stiffness of the mechanism was investigated. Afterwards, the dynamic model was developed and the numerical simulation was conducted. The results indicate that the stiffness of the mechanism is always a maximum when the mechanism is in equilibrium and it decreases with an increase in the external load. The dynamic model lays the foundation for the control of the mechanism.

1. Introduction

As the complication of the applications of mechanisms arise, new demands for lighter and quicker mechanisms are increasing. In order to meet these demands, many mechanisms composed of rigid rods and tensile strings are considered as alternative solutions over conventional mechanisms. For this reason, tensegrity mechanisms can be viewed as one such alternative. The goal of this paper is to analyze the kinematics and statics of a planar tensegrity mechanism.

The word tensegrity is an abbreviation of words tension and integrity, coined by Buckminster Fuller [1]. In fact, the first tensegrity structure was created by artist Kenneth Snelson [2]. Moreover, a detailed history of tensegrity structures was given by Motro [3]. Buckminster Fuller [1] gave a quite intuitive description of tensegrity as follows.

“Islands of compression inside an ocean of tension.”

Tensegrity structures have many attractive characters such as efficient, deployable, easily tunable, and precisely controllable. Due to these natures, tensegrity structures have been widely applied in several fields such as civil engineering [4-5], architecture [6], geometry [7], art [8] and even biology [9]. In such fields, tensegrities have been mainly used for static applications.

Furthermore, an interesting character of a tensegrity structure is that the structure experiences a state of self-stress by loading the strings and rods appropriately. The geometry of a tensegrity structure varies with the lengths of strings and rods inside the structure. Therefore, tensegrity structures can be possibly used as mechanisms when some of the components of the structure are actuated. It is not
since very recent years that some relevant works have been found towards this goal. Paul et al. [11] proposed a mobile robot based on a triangular tensegrity prism which was actuated by the strings. A genetic algorithm was employed to study the forward locomotion of the proposed robot. A new planar 3-DOF tensegrity robot manipulator was developed by Vasquez and Correa [12]. The kinematics and dynamics of the manipulator were researched. Recently, some new tensegrity mechanisms have been proposed in [13-18]. Tensegrity mechanisms do have many advantages over conventional mechanisms. As mentioned, the components of tensegrity mechanisms are axially loaded which leads to modeling such mechanisms easier. Since strings are introduced to construct tensegrity mechanisms, the weight of such mechanisms is reduced. The mass and inertia of moving parts are reduced too. For this reason, it is promising for tensegrity mechanisms to be applied for pick and place operations, as mentioned by Mohr and Arsenault [19]. A planar tensegrity mechanism was introduced by Bayat and Crane III [16]. Moreover, the force equilibrium of the planar tensegrity has been analyzed. However, when the mechanism is put to use, the kinematics and dynamics can not be neglected. The kinematic and dynamic analysis of the mechanism was thus researched in this paper.

This paper is organized as follows. In section 2, the tensegrity mechanism is described. Afterwards, the forward and inverse kinematic analysis and stiffness for the tensegrity mechanism have been studied based on an energy formulation in section 3. In section 4, the dynamic model has been developed using Newton-Euler formulation. Finally, conclusions are reported in section 5.

2. Mechanism description
A diagram of the planar tensegrity parallel mechanism researched in this paper is shown in figure 1. It can be observed that the mechanism consists of two platforms. The mobile platform is composed of three rigid rods joining node pairs DE, EF and DF while the fixed platform is composed of three rigid rods joining node pairs AB, BC, and AC. Moreover, the mobile platform is connected to the fixed platform by two strings and one prismatic actuator. The strings are joining node pairs BE and CF. Moreover, the prismatic actuator is used to vary the distance between nodes A and D, whose length is \( L_3 \).

![Figure 1 Planar Tensegrity Parallel Mechanism](image)

As illustrated in figure 1, the angle between the \( X \) axis and the rigid rod joining node pairs AD is defined as \( \gamma_1 \) while the angle between the \( X \) axis and the strut joining node pairs DE is defined as \( \gamma_2 \). In addition, it is assumed that the rods joining node pairs AB, AC, DE, EF, and DF are of the same length \( L \) and the springs are linear with the same string constant \( K \) and zero free lengths. Moreover, the length of the rod BC is chosen to be \( \sqrt{3} L \). The hypothesis that the springs have zero free lengths is not problematic, as explained by Gosselin [14], since virtual zero-free-length spring can be created by extending the actual spring beyond their attachment point. Moreover, the rigid rods in the mechanism cannot vary their lengths and the stiffness of the prismatic actuators is considered to be infinite relative to those of springs.
For such mechanism, it should be noted that the components of the mechanism are connected to each other at each node by revolute joints and the whole mechanism lies in a horizontal plane. Moreover, the fixed platform ABC is fixed to the ground. The position of the mobile platform DEF can be controlled by the prismatic actuator AD. In this work, the coordinates of the node E are chosen as the output variables of the mechanism while the angle $\gamma_1$ and the length $L_3$ are chosen as the input variables of the mechanism. In addition, the ranges imposed on angles $\gamma_1$ and $\gamma_2$ are chosen as follows.

$$\pi \leq \gamma_1 \leq 2\pi, \quad \pi \leq \gamma_2 \leq 2\pi$$

The planar tensegrity mechanism shown in figure 1 was introduced by Bayat and Crane III [16] and the force equilibrium of such mechanism was performed. This mechanism has some attractive characters such as light weight, high ratio of strength to weight, low inertia of moving parts. When such mechanism is put to use, the kinematics and dynamics should not be ignored. In the following sections, the kinematics, stiffness, and dynamics of such mechanism have been studied.

3. Kinematic analysis

For the tensegrity mechanism, shown in Figure 1, the static configurations depend not only on the geometry of the mechanism but also on the internal forces in the strings. Therefore, the kinematics and statics need to be considered simultaneously. Moreover, it is always assumed that the mechanism studied in this section is in equilibrium.

3.1. Forward kinematic analysis

The forward kinematic analysis consists in computing the output variables for the given input variables of the mechanism. From figure 1, it can be observed that when $\gamma_1$ and $L_3$ are specified, the position of node D is determined. Then, the movement of node E is thus constrained to a circle of radius $L$ centred on node D. The coordinates of node E can be found using an energy based method.

From figure 1, it can be seen that the coordinates of node C denoted by $x_c$ and $y_c$ are $x_c = -L/2$ and $y_c = \sqrt{3}L/2$. The coordinates of node D, E and F can be easily obtained as follows.

$$\mathbf{P}_D = \begin{bmatrix} x_D \\ y_D \end{bmatrix} = \begin{bmatrix} L_3 \cos \gamma_1 \\ L_3 \sin \gamma_1 \end{bmatrix}$$

$$\mathbf{P}_E = \begin{bmatrix} x_E \\ y_E \end{bmatrix} = \begin{bmatrix} L_3 \cos \gamma_1 + L \cos \gamma_2 \\ L_3 \sin \gamma_1 + L \sin \gamma_2 \end{bmatrix}$$

$$\mathbf{P}_F = \begin{bmatrix} x_F \\ y_F \end{bmatrix} = \begin{bmatrix} L_3 \cos \gamma_1 + L \cos(\gamma_2 - \frac{\pi}{3}) \\ L_3 \sin \gamma_1 + L \sin(\gamma_2 - \frac{\pi}{3}) \end{bmatrix}$$

With the coordinates of nodes D, E and F now known, the lengths of strings BE and FC can be expressed as a function of $\gamma_1$, $\gamma_2$ and $L_3$. Therefore the potential energy of the mechanism can be obtained.

$$U = \frac{K}{2} \sum_{i=1}^{2} L_i^2 = K \left[ L_3^2 + 2L_2^2 + L_3 L \left( \cos(\gamma_1 - \gamma_2) + \cos(\gamma_1 - \gamma_2 + \frac{\pi}{3}) - \cos(\gamma_1 - \frac{\pi}{3}) \right) \right]$$

For the mechanism considered here, the potential energy will reach its minimum when the mechanism is in equilibrium. Therefore, differentiating $U$ with respect to $\gamma_2$ and equating the result to zero, we obtain

$$KL_3 \left[ \sin(\gamma_1 - \gamma_2) + \sin(\gamma_1 - \gamma_2 + \frac{\pi}{3}) \right] = 0$$

Considering the range imposed to $\gamma_1$ and $\gamma_2$, the following expressions can be derived.
\[ \gamma_2 = \gamma_1 + \frac{\pi}{6} \quad (7) \]

Substituting equation (7) into equation (3), a unique solution to the forward kinematic analysis is found.

### 3.2. Inverse kinematic analysis

The inverse kinematic analysis corresponds to the computation of the input variables for the given output variables of the system. For the tensegrity-based mechanism studied here, it consists in computing the length \( L_3 \) and angle \( \gamma_1 \) for the given coordinates of node E. Substituting equation (7) into equation (3), the following equations can be derived.

\[
L_3 = \frac{x_E - L \cos \left( \gamma_1 + \frac{\pi}{6} \right)}{\cos \gamma_1} \quad (8)
\]

\[
y_E = L_3 \sin \gamma_1 + L \sin \left( \gamma_1 + \frac{\pi}{6} \right) \quad (9)
\]

By combining equation (8) with equation (9), we obtain

\[
x_E \sin \gamma_1 + y_E \cos \gamma_1 + \frac{L}{2} = 0 \quad (10)
\]

The tan-half angle of \( \gamma_1 \) can be defined as

\[ t = \tan \frac{\gamma_1}{2} \quad (11) \]

Then, the trigonometric identities are introduced.

\[ \sin \gamma_1 = \frac{2t}{1 + t^2}, \quad \cos \gamma_1 = \frac{1 - t^2}{1 + t^2} \quad (12) \]

By substituting equation (12) into equation (10), the following equation is generated.

\[
\left( \frac{L}{2} - y_E \right) t^2 + 2x_E t + \left( \frac{L}{2} + y_E \right) = 0 \quad (13)
\]

Solving the above equation for \( t \) yields

\[
t = \left( \frac{\frac{L}{2} - y_E}{L^2 - y_E} \right)^{-1} \left\{ \delta \left[ x_E - \left( \frac{L}{2} - y_E \right) \left( \frac{L}{2} + y_E \right) \right]^\frac{1}{2} - x_E \right\} \quad (14)
\]

where \( \delta = \pm 1 \). Substituting equation (14) into equation (11) and computing the arctangent of equation (11) generate two solutions for \( \gamma_1 \). By substituting these results into equation (8), two solutions to the inverse kinematic analysis are found. Considering the range imposed to \( \gamma_1 \), we can obtain the requirement that \( t \geq 0 \). Therefore, negative solutions for \( t \) should be eliminated.

### 3.3. Stiffness

The stiffness of mechanism corresponds to the ability to resist to the deformation caused by external loads. In this section, the relation between the external loads and the corresponding deformation are developed. Then, the stiffness of the mechanism is analysed.

In figure 1, \( L_3 \) and \( \gamma_1 \) are controlled by two actuators. The position of node E is considered as the output of the mechanism. In addition, node E can only generate a rotation centred on node D with actuators locked. Therefore, the external torque applied to node E can be obtained by differentiating \( U \) with respect to the angle \( \gamma_2 \).
\[ \tau = \frac{\partial U}{\partial \gamma_2} = -K \left\{ L_3 L \left[ \sin(\gamma_1 - \gamma_2) + \sin(\gamma_1 - \gamma_2 + \frac{\pi}{3}) \right] \right\} \]  
(15)

where \( \tau \) is defined as being negative when it tends to increase \( \gamma_2 \). Moreover, the stiffness of the mechanism can be obtained as the slope of the torque profile.

\[ K_{\gamma_2} = \frac{\partial \tau}{\partial \gamma_2} = K L_3 L \left[ \cos(\gamma_1 - \gamma_2) + \cos(\gamma_1 - \gamma_2 + \frac{\pi}{3}) \right] \]  
(16)

From equations (15) and (16), it can be seen that the torque \( \tau \) and stiffness \( K_{\gamma_2} \) can be considered as functions of \( \gamma_2 \). Such plots are shown in Figures 2 and 3.

![Figure 2. Torque \( \tau \) as a function of \( \gamma_2 \) for the mechanism with \( \gamma_1 = 7\pi/6, L_3 = 10 \text{ m}, L = 8 \text{ m} \) and \( K = 10 \text{ N/m} \).](image1.png)

![Figure 3. Stiffness \( K_{\gamma_2} \) as a function of \( \gamma_2 \) for the mechanism with \( \gamma_1 = 7\pi/6, L_3 = 10 \text{ m}, L = 8 \text{ m} \) and \( K = 10 \text{ N/m} \).](image2.png)

According to equation (7), we can obtain \( \gamma_2 \) for the given \( \gamma_1 \), when the mechanism is in equilibrium (\( \tau = 0 \)). Therefore, the value of \( \gamma_2 \) is \( 4\pi/3 \) with the condition that \( \gamma_1 = 7\pi/6 \), which can also be seen from figure 2. By combining figure 2 with figure 3, we can obtain that when the mechanism is in equilibrium, the stiffness \( K_{\gamma_2} \) reaches its maximum. An increase in the torque led to a decrease in the stiffness of the mechanism. When the torque \( \tau \) reaches its maximum, the stiffness \( K_{\gamma_2} \) will be zero. In this case \( \gamma_2 \) is \( 11\pi/6 \). Moreover, should \( \gamma_2 \) be increased slightly past \( \gamma_2 = 11\pi/6 \) and kept constant, the mechanism would collapse upon itself. Therefore, we can conclude that the stiffness of the mechanism is always a maximum at equilibrium and it decreases with an increase in the external load. Obviously, this character is not an attractive nature.

4. Dynamic analysis

When the mechanism is put to use, it is not always in equilibrium. Therefore, it is necessary to study the dynamics of the tensegrity mechanism. In this section, the dynamic model of the mechanism is developed.

4.1. Hypotheses

In order to develop the dynamic model and analyse the dynamic characters of the mechanism, the following hypotheses are made.

- There is no friction in the mechanism’s revolute joints.
- The springs are massless.
- The whole mechanism lies in a horizontal plane.
- Each of the rods which consist the mobile platform is modelled as a rigid rod of mass \( m \) and of moment of inertia \( I = mL^2/12 \) (with respect to an axis perpendicular to the plane and passing through the rod’s centroid).

4.2. Dynamic equation
The purpose of this section is to obtain the Newton-Euler equations of motion for such mechanism using plücker coordinates. Using the Newton-Euler formulation, the following equations are used to define the dynamics of the mobile platform.

\[ F = m_o \begin{bmatrix} \ddot{x}_o \\ \ddot{y}_o \end{bmatrix} \]  

(17)

\[ M = I \ddot{\phi} \]  

(18)

where \( m_o \) is the mass of the mobile platform while \( x_o \) and \( y_o \) are the coordinates of the centre of mass of the mobile platform. Moreover, \( F \) is the sum of forces acting on the platform DEF and \( M \) is the sum of moments acting about the centre of mass of the platform DEF. \( I \) is the inertia of mobile platform about its centre. \( \ddot{\phi} \) is the angular acceleration of the platform DEF.

Suppose the point \( O \) is the center of mass of the platform DEF. According to equations (2), (3) and (4), the coordinates of the point \( O \) can be derived as follows.

\[ P_o = \begin{bmatrix} x_o \\ y_o \end{bmatrix} = \begin{bmatrix} L_3 \cos \gamma_1 + \frac{L}{3} \cos \gamma_2 + \frac{L}{3} \cos(\gamma_2 - \frac{\pi}{3}) \\ L_3 \sin \gamma_1 + \frac{L}{3} \sin \gamma_2 + \frac{L}{3} \sin(\gamma_2 - \frac{\pi}{3}) \end{bmatrix} \]  

(19)

The sum of forces and moments acting on the platform DEF can be expressed as

\[ F = f_1 \overrightarrow{EB} + f_2 \overrightarrow{FC} + f_3 \overrightarrow{AD} \]  

(20)

\[ M = \overrightarrow{OE} \times \overrightarrow{EB} \cdot f_1 + \overrightarrow{OD} \times \overrightarrow{AD} \cdot f_3 + \overrightarrow{OF} \times \overrightarrow{FC} \cdot f_2 \]  

(21)

where \( f_1, f_2 \) and \( f_3 \) are the values of forces acting on the mobile platform by the sting BE, string FC and the prismatic actuator AD respectively. \( \overrightarrow{EB} \) is the unit vector BE while \( \overrightarrow{FC} \) is the unit vector FC. In addition, \( \overrightarrow{OE}, \overrightarrow{OD} \) and \( \overrightarrow{OF} \) are the vectors OE, OD, and OF respectively.

With the coordinates of nodes \( D, E \) and \( F \) now given by equations (2), (3) and (4), the unit vectors \( \overrightarrow{EB}, \overrightarrow{FC} \) and \( \overrightarrow{AD} \) can be obtained as follows.

\[ \overrightarrow{BE} = \frac{1}{L_1} \left\{ \left[ L_3 \cos \gamma_1 + L \cos \gamma_2 - L \right] i + \left[ L_3 \sin \gamma_1 + L \sin \gamma_2 \right] j \right\} \]  

(22)

\[ \overrightarrow{CF} = \frac{1}{L_2} \left\{ \left[ L_3 \cos \gamma_1 + L \cos(\gamma_2 - \frac{\pi}{3}) + L \right] i + \left[ L_3 \sin \gamma_1 + L \sin(\gamma_2 - \frac{\pi}{3}) - \frac{\sqrt{3}L}{2} \right] j \right\} \]  

(23)

\[ \overrightarrow{AD} = \left\{ \cos \gamma_1 i + \sin \gamma_1 j \right\} \]  

(24)

where \( i \) is the unit coordinate vector paral- lelling to the \( X \) axis and \( j \) is the unit coordinate vector paral- lelling to the \( Y \) axis. \( L_1 \) and \( L_2 \) are the lengths of the strings BE and CF respectively. Moreover, with the coordinates of node \( O \) given by equation (19), the vectors \( \overrightarrow{OE}, \overrightarrow{OD}, \overrightarrow{OF} \) can be expressed as

\[ \overrightarrow{OE} = \frac{L}{3} \left\{ 2 \cos \gamma_2 - \cos \left( \gamma_2 - \frac{\pi}{3} \right) \right\} i + \frac{L}{3} \left\{ 2 \sin \gamma_2 - \sin \left( \gamma_2 - \frac{\pi}{3} \right) \right\} j \]  

(25)

\[ \overrightarrow{OD} = -\frac{L}{3} \left\{ \cos \gamma_2 + \cos \left( \gamma_2 - \frac{\pi}{3} \right) \right\} i - \frac{L}{3} \left\{ \sin \gamma_2 + \sin \left( \gamma_2 - \frac{\pi}{3} \right) \right\} j \]  

(26)

\[ \overrightarrow{OF} = \frac{L}{3} \left\{ 2 \cos \left( \gamma_2 - \frac{\pi}{3} \right) - \cos \gamma_2 \right\} i + \frac{L}{3} \left\{ 2 \sin \left( \gamma_2 - \frac{\pi}{3} \right) - \sin \gamma_2 \right\} j \]  

(27)
With Parallel Axis Theorem, the inertia of mobile platform about its centre can be written in the following form.

\[ I = 3 \left[ \frac{m}{12} L^2 + m \left( \frac{\sqrt{3}}{6} - L \right)^2 \right] = \frac{mL^2}{2} \]  

Suppose the angle of the vector OD is denoted by \( \theta \), according to equation (26), we have

\[ \tan \theta = \frac{\sin \gamma_2 + \sin \left( \gamma_2 - \frac{\pi}{3} \right)}{\cos \gamma_2 + \cos \left( \gamma_2 - \frac{\pi}{3} \right)} \]  

When the mobile platform generates a rotation centred on its mass centre, the vector OD rotates centered on node O. In this case, the vector OD and the mobile platform have the same angular acceleration. Thus we obtain

\[ \ddot{\phi} = \dot{\theta} \]  

By substituting equations (29) into (30), the following equation is derived.

\[ \ddot{\phi} = \ddot{\gamma}_2 \]  

Substituting equations (22), (23) and (24) into equation (20), we can obtain an expression for the sum forces \( F \) acting on the platform DEF. In addition, substituting equations (25), (26) and (27) into equation (21), we obtain an expression for the sum moments \( M \) acting on the platform DEF.

Finally, by substituting expressions for \( F \) and \( M \) into equations (17) and (18), the dynamic equations of the planar tensegrity mechanism are obtained as follows.

\[ M \ddot{\mathbf{q}} + F \mathbf{q} + G \mathbf{q} + H \mathbf{f} + \mathbf{w} = 0 \]  

where \( \mathbf{q} = [\gamma_1, \gamma_2, L_3]^T \), \( \mathbf{q} = [\dot{\gamma}_1, \dot{\gamma}_2, \dot{L}_3]^T \), \( \mathbf{q} = [\ddot{\gamma}_1, \ddot{\gamma}_2, \ddot{L}_3]^T \) and \( \mathbf{f} = [0, 0, f_3]^T \), \( f_3 \) is the actuator force by the actuator AD. Furthermore, \( M_0 \), \( F \), \( G \) and \( H \) are all matrices whose elements are defined as follows.

\[ M_{11} = 3mL_3 \sin \gamma_1, \quad M_{12} = mL \left[ \sin \left( \gamma_2 - \frac{\pi}{3} \right) + \sin \gamma_2 \right], \quad M_{13} = -3m \cos \gamma_1 \]

\[ M_{21} = 3mL_3 \cos \gamma_1, \quad M_{22} = mL \left[ \cos \left( \gamma_2 - \frac{\pi}{3} \right) + \cos \gamma_2 \right], \quad M_{23} = 3m \sin \gamma_1 \]

\[ M_{31} = M_{32} = 0, \quad M_{33} = -\frac{1}{2} mL^2 \]

\[ F_{11} = 3mL_3 \cos \gamma_1, \quad F_{12} = mL \left[ \cos \gamma_2 - \frac{L}{3} \cos \left( \gamma_2 - \frac{\pi}{3} \right) \right] \]

\[ F_{21} = -3mL_3 \sin \gamma_1, \quad F_{22} = -mL \left[ \sin \gamma_2 + \sin \left( \gamma_2 - \frac{\pi}{3} \right) \right] \]

\[ F_{13} = F_{31} = F_{32} = F_{33} = 0 \]

\[ G_{11} = G_{13} = G_{21} = G_{23} = G_{31} = G_{32} = G_{33} = 0 \]

\[ G_{12} = 6m \sin \gamma_1, \quad G_{22} = 6m \cos \gamma_1 \]

\[ H_{11} = H_{12} = H_{21} = H_{22} = H_{31} = H_{32} = H_{33} = 0 \]

\[ H_{13} = \cos \gamma_1, \quad H_{32} = -\sin \gamma_1 \]

In equation (32), \( w \) is a 1×3 matrix whose elements are defined as follows.
\[ w_{11} = -K \left[ 2L_3 \cos \gamma_1 + L \cos \gamma_2 + L \cos \left( \gamma_2 - \frac{\pi}{3} \right) - \frac{L}{2} \right] \]
\[ w_{12} = K \left[ 2L_3 \sin \gamma_1 + L \sin \gamma_2 + L \sin \left( \gamma_2 - \frac{\pi}{3} \right) - \frac{\sqrt{3}L}{2} \right] \]
\[ w_{13} = \left( KLL_3 + \frac{L}{3} \right) \sin \left( \gamma_2 - \gamma_1 - \frac{\pi}{3} \right) + \left( KLL_3 + \frac{2L}{3} \right) \sin (\gamma_1 - \gamma_2) - \frac{\sqrt{3}}{3} KL^2 - \frac{\sqrt{3}}{3} KL \sin \gamma_2 \]  

(37)

4.3. Numerical simulation

Equation (32) is a second-order differential equation. The numerical solutions to equation (32) can be found for a set of given initial conditions. The parameters of the tensegrity-based mechanism are chosen to be \( K = 10 \, \text{N/m}, \, L = 8 \, \text{m} \) and \( m = 5 \, \text{kg} \). When the mobile platform begins to move, the initial length of the actuator AD is chosen to be \( L_3 = 8 \, \text{m} \) and the angle \( \gamma_1 \) is chosen to be \( \gamma_1 = 7\pi/6 \). The following input laws are chosen:

\[ \gamma_1 = \frac{7\pi}{6} + t, \quad L_3 = 10 + t \]  

(38)

where \( t \) is the time. The range imposed to \( t \) is chosen to be \([0 \, 2]\). When \( t \) is equal to zero, the mobile platform does not generate any movement. In this case, the whole mechanism is in equilibrium. From section 3.1, we can obtain the value for \( \gamma_2 \) when \( t = 0 \). Therefore, the initial conditions for equation (32) can be obtained as follows.

\[ \gamma_2 = \frac{4}{3} \pi, \quad \dot{\gamma}_2 = 0 \]  

(39)

Considering the initial conditions given by equation (39), the numerical solutions to the dynamic equation (equation (32)) can be found using the Runge-Kutta method, which is shown in figure 4.

From figure 4, it can be seen that the value of \( \gamma_2 \) is identified for a given time. According to equation (38) and figure 4, we can easily obtain the position and rotation of the mobile platform for a given time.

5. Conclusions

In this study, the kinematics, statics and dynamics of a planar tensegrity-based mechanism was investigated. At first, the solutions to the forward and inverse kinematic analysis were found using an
energy formulation. Then, the stiffness of the mechanism has been investigated. It has been found that the stiffness of the mechanism is always a maximum at equilibrium and it decreases with an increase in the external load. Afterwards, the dynamic model has been developed using Newton-Euler formulation. The numerical simulation has been conducted using the Runge-Kutta method. The dynamic model can be used to analyse the dynamics of the tensegrity-based mechanism. Moreover, the dynamic model can also be used to study the control of such mechanism.

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