Effect of anisotropic impurity scattering on a density of states of a d-wave superconductor

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We discuss the effect of an anisotropic impurity potential on the critical temperature, local density of states in the vicinity of a single impurity, and the quasiparticle density of states for a finite impurity concentration in a d-wave superconductor. Different scattering regimes are concerned.

I. INTRODUCTION

Impurities make a useful tool in probing the ground state of high-temperature superconductors. The most direct impurity probe of the superconducting state is provided by the scanning tunneling microscopy (STM) measurement of the local quasiparticle density of states (LDOS) around a single impurity. The STM images of Bi2Sr2CaCu2O8+δ reveal a tetragonal symmetry representative for the d-wave state. In addition to a possible identification of the superconducting state, this experiment may shed light on the nature of the quasiparticle scattering process - issue, which is important to the interpretation of the impurity pair-breaking effect in the d-wave superconductor. It has been shown that the weak impurity-induced suppression of the critical temperature, which is unexpected for d-wave superconductivity, can be understood assuming a momentum-dependent (anisotropic) impurity potential. In the present paper we discuss the effect of anisotropic impurity scattering on the critical temperature, local density of states in the vicinity of a single impurity, and the quasiparticle density of states for a finite impurity concentration in a d-wave superconductor. We establish how the internal structure of a defect, which reproduces the observed out of phase potential defined by $\psi = 1$, and the amplitude of $\Delta = \sqrt{2} \Delta_0$.

II. CRITICAL TEMPERATURE AND ORDER PARAMETER SUPPRESSION

The role of the anisotropy of the impurity potential is particularly clear for the potential $\psi$ with an oscillating potential factorizes $\langle e \rangle^2 + \langle ef \rangle^2 - 1 = \left[ \psi \left( \frac{1}{2} + \pi N_0 \left( v_i^2 + v_a^2 \right) \right) \right]^2$ and for unitary scattering is given by $\ln \frac{T_c}{T_{c_0}} = \left( \langle e \rangle^2 + \langle ef \rangle^2 - 1 \right) \left[ \psi \left( \frac{1}{2} + \frac{\pi N_0 \left( v_i^2 + v_a^2 \right)}{2 \pi T_e} \right) - \psi \left( \frac{1}{2} \right) \right]$.

where $v_i, v_a$ are isotropic and anisotropic scattering amplitudes, respectively, and $f(k)$ is the anisotropy function that vanishes after integration over the Fermi surface. We study the effect of anisotropic scattering by referring to an impurity pair-breaking is minimized for the in phase scattering, that is for $\langle ef \rangle = 1$ or close to 1, which is achieved for $f(k) = \sqrt{2} (k_x^2 - k_y^2)$ or $f(k) = sgn (k_x^2 - k_y^2)$. It has been shown that when the scattering is close to being in phase it reproduces the observed $T_c$ suppression and the $H_{c_2}$ critical field initial slope in the cuprates. On the other hand, when the scattering is out of phase with the order parameter $f(k) = 2\sqrt{2}k_xk_y$, $\frac{T_c}{T_{c_0}} = \left( \langle e \rangle^2 + \langle ef \rangle^2 - 1 \right) \left[ \psi \left( \frac{1}{2} + \frac{\Gamma}{2 \pi T_e} \right) - \psi \left( \frac{1}{2} \right) \right]$.
The Green’s function is obtained from Dyson’s equation

\[ \alpha \]

FIG. 1: Order parameter vs. partition parameter \( \alpha \) for the in phase scattering (solid line) and out of phase scattering (dashed line). Scattering strength \( c = 0.1/\pi \), impurity concentration \( n = 0.5 \Delta_0 N_0 \)

or \( f(\mathbf{k}) = \text{sgn}(k_x k_y) \) it gives \( \langle ef \rangle = 0 \) and strong suppression of the critical temperature. The differing effect of these two groups of impurity potential is also seen in the solution to the gap equation. In Fig. 1 we show the gap at a temperature of \( 0.1T_c \) as a function of the anisotropy degree \( \alpha \) for in phase (solid line) and out of phase scattering (dashed line) in the unitary limit. We note that the anisotropy leading to the maximal \( T_c \) for the in phase impurity potential, that is \( \alpha \sim 0.5 \), gives the minimal \( T_c \) for out of phase scattering.

III. LOCAL DENSITY OF STATES

The position-dependent change of the quasiparticle density of states around a single impurity is determined by the real space transform of the retarded Green’s function \( \hat{G} \) and reads \( \delta N(\mathbf{r}, \omega) = -\frac{1}{\pi} \text{Im} \{ \delta G_{11}(\mathbf{r}, \omega) \} \).

The Green’s function is obtained from Dyson’s equation \( \hat{G}^{-1}(\mathbf{r}, \omega) = \hat{G}_0^{-1}(\mathbf{r}, \omega) - \hat{\Sigma} \), where \( \hat{G}_0 \) is the quasiparticle propagator for a pure system and the impurity-induced self-energy \( \hat{\Sigma} \) is obtained within a single impurity approximation, i.e., the t-matrix equation is solved with the Green’s function of a pure system. We have evaluated the LDOS at the impurity site for different partitions of the impurity potential starting from isotropic scattering (\( \alpha = 1 \)) and ending on purely anisotropic scattering (\( \alpha = 0 \)). For in phase scattering (Fig. 2) we observe a shift of the spectral density from the hole-like resonant state for isotropic scattering to the electron-like resonant state for scattering close to being purely anisotropic. This effect is similar to the changes in the LDOS induced by deviations from particle-hole symmetry, or by changes in sign of the scattering potential \( [14] [15] \). The influence of the out of phase anisotropy of the scattering potential is entirely different. It turns out that the increasing degree of anisotropy broadens and finally destroys the impurity resonant state (Fig. 3).

IV. DENSITY OF STATES

The quasiparticle density of states for a uniform impurity distribution is obtained self-consistently within the t-matrix approximation, i.e., the matrix \( \hat{T} \) is obtained with the use of a dressed single-particle propagator \( \hat{G}^{-1} = \hat{G}_0^{-1} - \hat{\Sigma} \), and the self-energy is determined by \( \hat{\Sigma} = n \hat{T} \), where \( n \) is the impurity concentration \( [10] \). We have evaluated the density of states, given by the imaginary part of the retarded Green’s function \( N(\omega) = -\frac{1}{\pi} \text{Im} \sum_{\mathbf{k}} G_{11}(\mathbf{k}, \omega) \), in the limit of Born scattering \( (c = 10/\pi) \), intermediate scattering \( (c = 1/\pi) \) and unitary scattering \( (c = 0.001/\pi) \). For each considered scattering strength we have compared the effect of the isotropic impurity potential, the anisotropic potential in phase with the order parameter and the out of phase anisotropic potential (Fig. 4). The degree of anisotropy was set to \( \alpha = 0.8 \). Although the discussed potentials

FIG. 2: Local density of states at the impurity site (\( r=0 \)) induced by the in phase impurity potential, \( f(\mathbf{k}) = \sqrt{2}k_x k_y \), for different partitions of the impurity potential: \( \alpha = 1 \) (isotropic scattering), \( 0.5, 0.1, 0 \) (purely anisotropic scattering). Scattering strength \( c = 0.1 \).

FIG. 3: Local density of states at the impurity site (\( r=0 \)) induced by the out of phase impurity potential, \( f(\mathbf{k}) = \sqrt{2}k_x k_y \), for different partitions of the impurity potential: \( \alpha = 1 \) (isotropic scattering), \( 0.5, 0.1, 0 \) (purely anisotropic scattering). Scattering strength \( c = 0.1 \).
FIG. 4: The density of states for Born scattering $c = 10/\pi$ and impurity concentration $n=1 \Delta_0 N_0$: a) isotropic potential, b) anisotropic in phase, c) anisotropic out of phase; intermediate scattering $c = 1/\pi$ and impurity concentration $n=1 \Delta_0 N_0$: d) isotropic potential, e) anisotropic in phase, f) anisotropic out of phase; resonant scattering $c = 0.001/\pi$ and impurity concentration $n=0.01 \Delta_0 N_0$: g) isotropic potential, h) anisotropic in phase, i) anisotropic out of phase. The partition of the impurity potential is set to $\alpha = 0.8$.

lead to similar densities of states in the Born limit, observable differences between anisotropic in phase and out of phase potentials are seen for intermediate scattering and they become significant for the resonant scattering when the in phase potential gives $N(\omega = 0) = 0$, and the out of phase potential enhances the non-zero value of $N(\omega = 0)$.

V. CONCLUSIONS

We have shown that the anisotropy of the impurity potential can broaden or destroy the impurity resonant state. The energy level of the impurity resonant state relative to Fermi energy can be shifted by the anisotropy of the impurity potential, in particular the hole-like resonant state is transformed into the electron-like resonant state for highly anisotropic in phase scattering. Significant is also effect of anisotropy on the quasiparticle density of states which is best seen for unitary impurities. The nonzero value of the low-energy density of states for isotropic scattering is enhanced by the anisotropic impurity potential which is out of phase with the order parameter. On the other hand, the in phase impurity potential leads to a vanishing density of states at the Fermi level. These results show that the quasiparticle properties of a disordered d-wave superconductor depend, apart from the way of modeling the disorder \[\text{[16, 17]}\], on the internal structure of a scattering center.

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[1] S. H. Pan, E. W. Hudson, K. M. Lang, H. Eisaki, S. Uchida, and J. C. Davis, Nature 403, 746 (2000).
[2] E. W. Hudson, K. M. Lang, V. Madhavan, S. H. Pan, H. Eisaki, S. Uchida, and J. C. Davis, Nature 411, 920 (2001).
[3] J. M. Byers, M. E. Flatté, and D. J. Scalapino, Phys. Rev. Lett. 71, 3363 (1993).
[4] M. I. Salkola, A. V. Balatsky, and D. J. Scalapino, Phys. Rev. Lett. 77, 1841 (1996).
[5] J. Giapintzakis, D. M. Ginsberg, M. A. Kirk, and S. Ockers, Phys. Rev. B 50, 15967 (1994).
[6] S. Tolpygo, J. -Y. Lin, M. Gurvitch, S. Y. Hou, and J. M. Phillips, Phys. Rev. B 53, 12454 (1996).
[7] S. Tolpygo, J. -Y. Lin, M. Gurvitch, S. Y. Hou, and J. M. Phillips, Phys. Rev. B 53, 12462 (1996).
[8] J. -Y. Lin, S. J. Chen, S. Y. Chen, C. F. Chang, H. D. Yang, S. K. Tolpygo, M. Gurvitch, Y. Y. Hsu, and H. C. Ku, Phys. Rev. B 59, 6047 (1999).
[9] G. Harani and A. D. S. Nagi, Phys. Rev. B 54, 15463 (1996).
[10] G. Harani and A. D. S. Nagi, Phys. Rev. B 58, 12441 (1998).
[11] G. Harani and A. D. S. Nagi, Phys. Rev. B 63, 012503 (2001).
[12] G. Harani and A. D. S. Nagi, Acta Phys. Pol. B 32, 3459 (2001).
[13] H. Won and K. Maki, Physica C 282-287, 1837 (1997); Physica B 244, 66 (1998).
[14] A. Polkovnikov, S. Sachdev, and M. Vojta, Phys. Rev. Lett. 86, 296 (2000).
[15] I. Martin, A. V. Balatsky, and J. Zaanen, Phys. Rev. Lett. 88, 097003 (2002).
[16] W. A. Atkinson, P. J. Hirschfeld, A. H. MacDonald, and K. Ziegler, Phys. Rev. Lett. 85, 3926 (2000).
[17] A. M. Martin, G. Litak, B. L. Györfy, J. F. Annett, and K. I. Wysokiński, Phys. Rev. B 60, 7523 (1999).