Assessment of numerical procedures for determining shallow foundation failure envelopes

STEPHEN K. SURYASENATA*, HELEN P. DUNNE†, CHRISTOPHER M. MARTIN†, HARVEY J. BURD†, BYRON W. BYRNE† and AVI SHONBERG‡

The failure envelope approach is commonly used to assess the capacity of shallow foundations under combined loading, but there is limited published work that compares the performance of various numerical procedures for determining failure envelopes. This paper addresses this issue by carrying out a detailed numerical study to evaluate the accuracy, computational efficiency and resolution of these numerical procedures. The procedures evaluated are the displacement probe test, the load probe test, the swipe test (referred to in this paper as the single swipe test) and a less widely used procedure called the sequential swipe test. Each procedure is used to determine failure envelopes for a circular surface foundation and a circular suction caisson foundation under planar vertical, horizontal and moment (VHM) loading for a linear elastic, perfectly plastic von Mises soil. The calculations use conventional, incremental-iterative finite-element analysis (FEA) except for the load probe tests, which are performed using finite-element limit analysis (FELA). The results demonstrate that the procedures are similarly accurate, except for the single swipe test, which gives a load path that under-predicts the failure envelope in many of the examples considered. For determining a complete VHM failure envelope, the FEA-based sequential swipe test is shown to be more efficient and to provide better resolution than the displacement probe test, while the FELA-based load probe test is found to offer a good balance of efficiency and accuracy.

KEYWORDS: bearing capacity; finite-element modelling; footings/foundations; limit state design/analysis; numerical modelling; offshore engineering; soil/structure interaction

INTRODUCTION
In recent decades, there has been significant interest in the failure envelope approach for assessing the ultimate capacity of foundations under combined loading. The failure envelope is a hypersurface that defines the n-dimensional combination of loads (n ≥ 1) that results in the ultimate limit state (or plastic failure) of a foundation. The advantages of this approach over classical bearing capacity methods (Terzaghi, 1943; Meyerhof, 1951; Vesić, 1973) are manifold and have been widely discussed (Schotman, 1989; Tun, 1990; Nova & Montrasio, 1991; Gottardi & Butterfield, 1993; Bransby & Randolph, 1998; Martin & Houlsby, 2000; Houlsby & Byrne, 2001; Gourvenec, 2007).

The failure envelope approach was first introduced by Roscoe & Schofield (1957) to analyse the interaction between a steel frame and its foundations using envelopes of normalised forces. Since then, it has been widely adopted to represent the results of numerical studies of foundation bearing capacity, for a broad range of foundation types. For example, failure envelopes have been determined for surface foundations (Bell, 1991; Taiebat & Carter, 2000, 2010; Gourvenec, 2007; Vulpe et al., 2014; Shen et al., 2016, 2017), skirted or caisson foundations (Bransby & Randolph, 1998; Bransby & Yun, 2009; Gourvenec & Barnett, 2011; Hung & Kim, 2014; Karapiperis & Gerolymos, 2014; Gerolymos et al., 2015; Vulpe, 2015; Mehravar et al., 2016), spudcan foundations (Zhang et al., 2011) and mudmat foundations (Feng et al., 2014; Fu et al., 2014; Nouri et al., 2014; Dunne & Martin, 2017). However, there is limited published work that quantifies the performance of the numerical procedures used to determine these failure envelopes. Given the increasing need for site- and foundation-specific failure envelopes, either for macro-element modelling (e.g. Martin & Houlsby, 2001; Cassidy et al., 2004) or for the assessment of ultimate limit states using the failure envelope approach, the performance of these numerical procedures is an important consideration.

The contributions of this paper are two-fold. First, it addresses the uncertainty around the performance of various numerical procedures by carrying out a systematic comparison of the failure envelopes determined by each procedure and by making an assessment of relative computational efficiency, albeit for a limited range of foundation types and loading conditions. The aim is to provide guidance for researchers to identify which procedure they should adopt for their studies, based on the criteria of accuracy, efficiency and resolution. This paper does not make assumptions on which particular parts of the failure envelope are more, or less, significant for design and thus there is no attempt to quantify or include the practical significance of errors (on the basis of where they occur in load space) in the criteria of the comparative study. Second, this study provides insights into the implementation of one of the less widely used numerical procedures called the sequential swipe test. As will be shown...
NUMERICAL PROCEDURES FOR DETERMINING FAILURE ENVELOPES

The numerical procedures investigated in this paper can be categorised into two main groups: displacement-controlled and load-controlled. The displacement-controlled analyses (i.e. displacement probe test, single swipe test and sequential swipe test) are performed using the three-dimensional (3D) finite-element analysis (FEA) software, Abaqus version 6.13 (Dassault Systèmes, 2014). The load-controlled analyses (i.e. load probe test) are performed using the 3D finite-element limit analysis (FELA) software, OxLim (Makrodimopoulos & Martin, 2006, 2007; Martin, 2011), which has been used to analyse various bearing capacity problems in plane strain (Martin & White, 2012; Mana et al., 2013; Dunne et al., 2015) and more recently in three dimensions (Dunne & Martin, 2017).

All of the analyses reported in this paper are total stress analyses carried out for undrained clay, modelled using the von Mises yield criterion. The von Mises criterion was chosen over the Tresca criterion primarily for convenience, as it is more efficient to solve von Mises problems than Tresca problems with the 3D FELA software, OxLim. However, it has been shown by Gourvenec et al. (2006) that vertical bearing capacity calculations using the von Mises criterion (with the strength in simple shear set equal to the undrained shear strength, \( s_d \)) are reasonably close to those using the Tresca criterion. Furthermore, once the failure envelope has been normalised by the uniaxial capacities, the resulting shape of the non-dimensional failure envelope is qualitatively similar for both von Mises and Tresca soil – for example, compare the \( V_HM \) failure envelopes for a circular surface foundation in this paper (shown later as Fig. 11(a)) to Fig. 10(a) in the paper by Gourvenec (2007). This paper is concerned more with the numerical approaches, rather than a particular soil model, and thus the adoption of a single soil model for the comparative study is accepted as a limitation of the scope of the paper.

In this paper, \( V, H \) and \( M \) refer to the vertical, horizontal and moment loads, respectively, and \( w, u \) and \( \theta \) refer to the corresponding vertical, horizontal and rotational displacements. The loading reference points (LRP) are located at the centres of the surface foundation base and the suction caisson lid base (refer to Fig. 1 for the adopted sign conventions). Note that this is different from some previous research where the LRP is located at the level of the caisson skirt base. Furthermore, failure envelopes are presented in terms of normalised loads \( (V = V/V_0, H = H/H_0, M = M/M_0) \), which refer to loads normalised by their respective uniaxial capacities \( (V_0, H_0, M_0) \) as determined using the same numerical procedure.

Displacement probe test

In the displacement probe test, a displacement increment in a prescribed direction is applied to the foundation from a zero load state, with the final (steady) load state determining a single point on the failure envelope. To find the full failure envelope, a series of these probe tests with varying displacement directions must be completed. The displacement probe test has robust convergence properties, and provided that the prescribed displacement magnitude is sufficiently large, a well-defined failure load (or combination of loads) can be obtained.

However, this approach is relatively inefficient as each calculation only determines a single point on the failure envelope. Furthermore, it does not allow a straightforward investigation of the failure envelope as the load path followed during a displacement probe test is typically non-linear and difficult to predict. For example, the schematic diagram in Fig. 2(a) shows a representative, non-linear load path followed during a displacement probe. The initial load path is determined by the elastic stiffness of the soil-foundation system and the prescribed displacement direction. However, as soil yielding occurs, the stiffness reduces by differing amounts in each of the loading directions and the load path changes direction before arriving at (and possibly tracking along) the failure envelope, eventually maintaining a steady load state as the displacements continue to increase.

Load probe test

In the load probe test, combined loading components in a prescribed ratio are applied to the foundation until failure occurs. It can be difficult to determine accurate failure loads with load control in FEA, as convergence generally cannot be obtained if the final prescribed load exceeds the foundation capacity. A series of trial-and-error load cases, or a careful approach to the failure envelope, is therefore required to determine the maximum load that can converge. However, the FELA technique does not suffer from such issues and hence, FELA was adopted for the load probe tests in this study. Furthermore, the use of both lower-bound and upper-bound FELA provides a rigorous bracket on the theoretical failure load. A key advantage of the load probe test is that the predefined direction is followed throughout the analysis, which enables a more straightforward approach to determining the entire failure envelope. The schematic diagram in Fig. 2(a) shows the process of determining a \( VH \) failure envelope by probing in load space. Once a loading ratio is defined, each of the load paths travels from the origin.

![Fig. 1. Sign conventions for loads \((V, H, M)\) and displacements \((w, u, \theta)\): (a) surface foundation; (b) caisson foundation. LRP denotes loading reference point](image-url)
Sequential swipe test

Although the sequential swipe test is a less widely used procedure for determining failure envelopes, it can resolve the potential under-prediction behaviour of the single swipe test referred to above. A sequential swipe test is a multi-swipe test, which applies a more gradual change in direction (in displacement space) by way of a series of discrete swipe stages, compared with the abrupt directional change that occurs in the single swipe test. This type of test first appeared in physical experiments (Martin, 1994; Byrne, 2000; Martin & Houlsby, 2000) under the term ‘loop test’, as a closed loop path applied in displacement space. More recently, Taiebat & Carter (2010) suggested that this would maintain a greater plastic displacement than the elastic displacement in the first DoF while the plastic displacements were developing in the other DoF, which would maintain normality over the whole load path and, thus, the load path would stay on the failure envelope.

Regardless of the different names adopted (loop test, modified swipe test, sequential swipe test), the key principle behind these tests is the same, which is that changes in displacement direction should be applied gradually. Fig. 2(b) shows the different load paths taken by representative displacement probe tests. The sequential swipe test can be considered as a ‘discrete’ version of the loop or modified swipe test, in which the user can control how gradually the displacement direction changes through the number of discrete swipe stages (denoted below as \( m \)). This will be made clearer in the following exposition.

Suppose that the directional change in the displacement space is controlled by \( \psi \), the angle between the current and previous increments in displacement space. In this paper, the sequential swipe test is implemented by keeping \( \psi \) constant between all stages of the swipe sequence. For example, a two-swipe sequential swipe test in the first quadrant of\(-w-u\) displacement space (assuming the initial pre-swipe displacement is in the \( w \) direction) applies \( \psi = \pi/4 \) for all swipes, resulting in \( \delta u_0w = \tan(\pi/4) \) followed by \( \delta u_1w = \tan(\pi/2) \), where \( \delta u \) and \( \delta w \) are the horizontal and vertical displacement increments respectively. Correspondingly, an \( m \)-swipe sequential swipe test in the same displacement space applies \( \psi = \pi/2m \) for all swipes, where the direction of the displacement increment in the \( m \)th swipe is given in equation (1). Here \( q_1 \) and \( q_2 \) denote generic normalised displacements.
corresponding to the first and second DoF, respectively, while $\psi_t$ is the total directional change in the displacement space during the swipe phase (e.g. $q_1 = w/D$, $q_2 = u/D$ and $\psi_t = \pi/2$ for the above swipe).

$$
\left( \frac{\partial q_2}{\partial q_1} \right)_i = \tan \left( \frac{i \psi_t}{m} \right) \quad \text{for} \quad 1 \leq i \leq m
$$

The larger $m$ is, the more gradually the displacement direction changes. A single swipe test can be obtained as a special case of the sequential swipe test by letting $m = 1$. A preliminary investigation to illustrate the effect of $m$, different $m$-valued sequential swipe tests were carried out for a surface strip foundation on von Mises soil.

Figure 3 shows the two-dimensional (2D) FEA mesh for the surface strip foundation, which consists of 7200 second-order, fully integrated, hybrid quadrilateral elements (Abaqus code CPE8H). The von Mises yield strength in pure shear, $k$, was equated with the undrained shear strength of the clay, $s_u$, and was modelled as homogeneous throughout the soil domain. The Poisson’s ratio of the soil, $\nu$, was set as 0.49, while its Young’s modulus, $E$, was set as $1000\sqrt{3}s_u$. The soil was modelled as a weightless material, as soil weight does not affect the capacity for this type of problem (i.e. horizontal ground surface; pressure-insensitive von Mises yield criterion for the soil; no contact breaking between foundation and soil).

The surface strip foundation was modelled indirectly by applying a rigid body constraint to the soil nodes underneath the foundation.

Figure 4 compares the $VH$ failure envelopes obtained from different $m$-valued sequential swipe tests with the analytical solution (Green, 1954). Two types of swipe analysis were carried out, with one reaching $V_0$ before swiping to $H_0$ and the other taking the opposite route. For each analysis, three sequential swipe tests were carried out, with $m$ ranging from 2 to 16. Key observations from Fig. 4 are listed below.

(a) All the tests swiping to $H_0$ end at point A, where the analytical solution indicates no further change in failure envelope gradient, as shown in Fig. 4(a).
The single swipe test marginally under-predicts the failure envelope in Fig. 4(a) but significantly under-predicts it in Fig. 4(b). In contrast, the sequential swipe tests show accurate tracking of the failure envelope, regardless of the starting point of the swipe phase.

It can be observed that the load paths of the sequential swipe tests are essentially indistinguishable from the analytical failure envelope when \( m \geq 8 \). This suggests that if \( m \) is above some critical value, the load path will track the failure envelope with negligible deviation.

When completing the analyses for Fig. 4, it was observed that the rate of increase in the total computational time decreased as the number of discrete swipe stages increased (this is because the FEA requires fewer incrementation cutbacks and equilibrium iterations for smaller \( \varphi \) than for larger \( \varphi \)). For example, the total additional computational times (relative to the single swipe test) taken by the two-swipe, eight-swipe and 16-swipe tests were approximately 19\%, 24\% and 28\%, respectively. This indicates a marginal penalty in choosing a higher number of stages for the sequential swipe test. Hence, it is more practical to select a high number of stages at the outset (e.g. \( m = 8 \)) than to waste computational resources attempting to find the optimal \( m \), which in any case is likely to vary with the problem type and the current load state.

For more systematic mapping of high-dimensional \( (n \geq 3) \) failure envelopes, it is advisable that the sequential swipe test is restricted to two dimensions, while constant load envelopes, it is advisable that the sequential swipe tests show accurate tracking of the failure envelope, regardless of the starting point of the swipe phase.

APPLICATION OF NUMERICAL PROCEDURES

To further evaluate the numerical procedures described above, each procedure was used to find the failure envelopes for planar VHM loading of two types of shallow foundation (circular surface and suction caisson foundations) bearing on undrained clay.

Foundation and soil properties

Both the surface and caisson foundations were modelled as fully rigid, with a diameter \( D \). The caisson foundation was modelled as having an embedded length \( L = D \) and a skirt of thickness \( t_r = 0.005D \). The undrained clay was modelled in FEA as a homogeneous, linear elastic (\( v = 0.49; E = 1000\sqrt{3}s_u \)), perfectly plastic material and in FELA as a homogeneous, rigid, perfectly plastic material. For both sets of analyses, the von Mises yield criterion (with a yield strength in pure shear of \( s_u \)) and an associated flow rule were adopted. The soil and foundations were modelled as weightless materials, as soil weight does not affect the capacity for the problems considered here (for the same reasons as described above).

The 3D FEA model

First-order, fully integrated, hybrid brick elements (Abaqus code C3D8H) were used for the soil as these are generally recommended for modelling near-incompressible materials (Dassault Systèmes, 2014). Brick elements were also used for the foundation, but the foundation was made fully rigid by the application of a rigid body constraint. Sliding and contact breaking between the foundation and soil were not allowed.

Figure 5 shows the 3D FEA meshes for the surface and caisson foundations, with symmetry exploited. Displacement boundary conditions were set to prevent radial displacements on the circumferential faces and out-of-plane displacements on the plane of symmetry. In addition, the base of the mesh was fixed in all directions. The meshes were sufficiently large that boundary effects on the failure response of the foundation were verified to be negligible. The meshes for the surface and caisson foundations comprised approximately 40 000 and 44 000 elements, respectively.

The 3D FELA model

The FELA software OxLim first discretises the soil domain into a mesh of tetrahedral elements using TetGen (Si, 2015) and applies the boundary conditions. It then sets up two constrained optimisation problems that together bound the load multiplier (i.e. the factor by which the specified live loads must be increased to cause failure). For this study, the lower-bound (LB) analyses used a piecewise linear stress field, and the upper-bound (UB) analyses used a piecewise linear velocity field. The average of the bounds, \((LB + UB)/2,\) was taken as the best estimate solution for the load multiplier. The use of the von Mises criterion meant that both the LB and UB analyses could be cast as standard second-order cone programming problems and solved with high efficiency using specialised numerical optimisation software (Mosek, 2014). OxLim uses adaptive mesh refinement to improve the bracketing of the exact collapse load multiplier, where the
adapтивity is based on the spatial variation of the deviatoric strain rate in the UB velocity field. For the surface foundation, the initial mesh was adaptively refined twice to increase the number of elements from approximately 6500 to 25 000, as shown in Fig. 6(a). For the caisson foundation, the initial mesh was adaptively refined once to increase the number of elements from approximately 14 000 to 30 000, as shown in Fig. 6(b). To keep the number of elements comparable with the FEA mesh, a second refinement was not undertaken for the caisson foundation. It should be noted, however, that the average of the LB and UB solutions (which is the main measure of comparison with the FEA results) typically does not vary significantly as the bounds converge. The mesh domain was sufficiently large to render boundary effects negligible. Fixed boundary conditions were applied to the base and sides of the domain (excluding the symmetric plane).

Fig. 6. FELA meshes for load probe tests. For surface foundation of diameter \( D \), mesh domain is 3·5D deep, 7D wide and 3·5D thick. For caisson foundation of diameter \( D \) and skirt length \( L = D \), mesh domain is 4·5D deep, 9D wide and 4·5D thick. (a) Surface foundation, refined mesh under moment loading; (b) caisson foundation, refined mesh under moment loading

### Loading methodology

For this study, the failure envelopes were explored in increasing dimensionality of load components. First, the uniaxial capacities were identified for pure \( V \), \( H \) and \( M \) loading. Thereafter, failure envelopes for combined \( VH \), \( VM \) and \( HM \) loading were found. Owing to the symmetry in the \( VH \) and \( VM \) load spaces, only one quadrant of the failure envelope needs to be determined. Similarly, symmetry in the \( HM \) load space dictates that only two adjoining quadrants are needed to define the full failure envelope.

For the displacement probe tests, nine equally spaced displacement probe directions were used in each quadrant. For comparison purposes, eight discrete swipe stages (using the same set of probe directions) were adopted for the sequential swipe tests. The displacement probe directions can be identified from equation (1) by letting \( \delta q_1 \) and \( \delta q_2 \) be the

![Table 1. Uniaxial capacities of surface and caisson foundations](image)

|               | \( V_0 \) | \( H_0 \) | \( M_0 \) |
|---------------|-----------|-----------|-----------|
| Surface       |           |           |           |
| Displacement probe | 5·63      | 1·02      | 0·714     |
| Load probe (LB) | 5·45      | 1·00      | 0·667     |
| Load probe (UB) | 5·77      | 1·00      | 0·715     |
| Load probe (average) | 5·61      | 1·00      | 0·691     |
| Caisson       |           |           |           |
| Displacement probe | 13·12     | 5·86      | 3·64      |
| Load probe (LB) | 12·52     | 5·52      | 3·36      |
| Load probe (UB) | 13·68     | 6·28      | 3·96      |
| Load probe (average) | 13·10     | 5·90      | 3·66      |

\( A = \pi D^2/4 \) refers to the foundation plan area. Note that the procedure and results for the displacement probe, single swipe and sequential swipe tests are identical for uniaxial loading.

Finally, the full \( VH \) failure envelope was determined. Mixed load and displacement controls were used for the FEA-based tests. Load control was used for \( V \), while displacement control was used in the \( HM \) load space – that is, the \( VH \) failure envelope was explored by determining \( HM \) contours of the failure envelope at fixed levels of \( V \). Five vertical load levels were considered: \( F = 0 \), 0·25, 0·5, 0·625, 0·75 and 0·875. A similar procedure was followed for the load probe tests performed using FELA, with the \( HM \) contours being determined by probing in \( HM \) load space under the same set of fixed \( V \) loads.

### RESULTS

#### Pure \( V \), \( H \) and \( M \) loading

To validate whether the FEA- and FELA-based procedures would provide similar answers for the same problems, Table 1 compares the results obtained by the various procedures for the uniaxial foundation capacities \( (V_0, H_0 \) and \( M_0 ) \), which shows that the results from the displacement probes using FEA are within the bounds obtained using the 3D FELA procedure, except for \( H_0 \) for the surface foundation. Furthermore, the FELA load probe (average) results generally agree very well with the FEA-based results.

#### Combined \( VH \), \( VM \) and \( HM \) loading

Figures 7–9 show the \( VH \), \( VM \) and \( HM \) failure envelopes for both foundations. Because of symmetry, only one or two quadrants are shown in these figures, as appropriate. The small black markers in Figs 7–9 for the sequential swipe test results represent intermediate equilibrium load states during each discrete stage of the sequential swipe, which are determined by ABAQUS’s automatic step size incrementation.
Fig. 7. Dimensionless $VH$ failure envelopes: (a) surface foundation; (b) caisson foundation

Fig. 8. Dimensionless $VM$ failure envelopes: (a) surface foundation; (b) caisson foundation

Fig. 9. Dimensionless $HM$ failure envelopes: (a) surface foundation; (b) caisson foundation
scheme. The density of the black markers (i.e. the resolution of the failure envelope) can be controlled by changing the step size incrementation scheme. The final load states from the displacement probe tests, and the load paths from the sequential swipe tests, were all within the bounds obtained from the load probe tests. In fact, there are no significant differences between these two sets of FEA-generated results and the FELA load probe (average) results. In contrast, there is noticeable under-prediction of the failure envelopes by the single swipe test. The under-prediction of the failure envelopes for the surface foundation is minor for most cases, except for the $HM$ failure envelope when $H \geq 0.7$. However, under-prediction of the failure envelopes for the caisson foundation is apparent for all the load spaces explored.

**Combined VHM loading**

Figure 10 shows the $HM$ failure envelopes obtained for both foundations under three selected levels of normalised vertical load $\tilde{V}$. Again, the results of the displacement probe and sequential swipe tests are all within the LB and UB envelopes obtained using FELA. Furthermore, Fig. 11 shows the $HM$ failure envelopes obtained from both the single swipe and sequential swipe tests, for all of the vertical load levels considered. It is evident that the single swipe test under-predicts the $HM$ failure envelopes for all vertical load levels.

**DISCUSSION**

To assess the performance of the various numerical procedures in determining the above failure envelopes, the following performance criteria were adopted: accuracy, computational efficiency and resolution.

To allow for a quantitative (albeit approximate) comparison of the accuracy of the various numerical procedures, an accuracy measure $\eta$ (relative to the displacement probe test) is introduced as follows

$$\eta = \frac{A_i}{A_{ref}}$$

where $A_i$ refers to the area enclosed within a failure envelope that was determined by any numerical procedure, and $A_{ref}$ refers to the area enclosed within a reference failure envelope that was determined by the displacement probe method (which is the most widely used among the FEA-based procedures). The area calculations were performed by...
taking the set of failure points as the vertices of a polygon (the failure points are taken to be the average of the bounds for the FELA analyses). Fig. 12 shows an illustration of a typical computation of the accuracy measure $\eta$. Note that values of $\eta$ above 1 do not necessarily imply inaccuracy, as some of the procedures have either a higher number of failure points (e.g. single swipe and sequential swipe tests) or more evenly spaced failure points (e.g. load probe test) to better approximate the computation of the failure envelope area.

Table 2 shows the comparison of the accuracy measure $\eta$ (as per equation (2)) for the surface and caisson foundations, for each failure envelope shown in Figs 7–10.

| Failure envelope | $\eta$ (single swipe) | $\eta$ (sequential swipe) | $\eta$ (load probe) |
|------------------|------------------------|---------------------------|----------------------|
| Surface          |                        |                           |                      |
| Figure 7(a)      | 0·99                   | 1·01                      | 1·01                 |
| Figure 8(a)      | 1·01                   | 1·03                      | 1·04                 |
| Figure 9(a)      | 0·89                   | 1·01                      | 1·03                 |
| Figure 10(a)     | 0·87                   | 1·01                      | 1·04                 |
| Figure 10(b)     | 0·83                   | 1·01                      | 1·05                 |
| Figure 10(c)     | 0·81                   | 1·01                      | 1·07                 |
| Caisson          |                        |                           |                      |
| Figure 7(b)      | 0·92                   | 1·01                      | 1·01                 |
| Figure 8(b)      | 0·94                   | 1·03                      | 1·02                 |
| Figure 9(b)      | 0·88                   | 1·04                      | 1·00                 |
| Figure 10(d)     | 0·90                   | 1·04                      | 1·01                 |
| Figure 10(e)     | 0·87                   | 1·04                      | 1·02                 |
| Figure 10(f)     | 0·78                   | 1·04                      | 1·04                 |
| Average          | 0·89                   | 1·02                      | 1·03                 |

Table 3 shows the computational time taken by each numerical procedure to find all failure envelopes ($V_H, V_M, H_M, V_H M$) of the surface and caisson foundations.

|                      | Number of probes | Total time: h | Average time per probe: h |
|----------------------|------------------|---------------|--------------------------|
| Surface              |                  |               |                          |
| Displacement probe   | 120              | 68·6          | 0·571                    |
| Single swipe         | 22               | 21·4          | 0·971                    |
| Sequential swipe     | 120              | 25·5          | 0·212                    |
| Load probe           | 120              | 31·2          | 0·260                    |
| Caisson              |                  |               |                          |
| Displacement probe   | 120              | 152·8         | 1·27                     |
| Single swipe         | 22               | 23·0          | 1·05                     |
| Sequential swipe     | 120              | 59·5          | 0·496                    |
| Load probe           | 120              | 22·3          | 0·186                    |

Fig. 11. Dimensionless $H M$ failure envelopes at selected $\tilde{V}$ levels ($\tilde{V} = 0, 0·25, 0·5, 0·625, 0·75, 0·85$): (a) surface foundation; (b) caisson foundation

Fig. 12. Computation of the accuracy measure $\eta$ for a typical single swipe test in $V H$ space, where $A_i$ is the area enclosed by the failure envelope from the single swipe test (i.e. the shaded area) and $A_{ref}$ is the area enclosed by the failure envelope from the displacement probe tests.

Table 2. Comparison of the accuracy measure $\eta$ (as per equation (2)) for the surface and caisson foundations, for each failure envelope shown in Figs 7–10

Table 3. Computational time taken by each numerical procedure to find all failure envelopes ($V H, V M, H M, V H M$) of the surface and caisson foundations

swipe stages. In contrast, only 22 displacement increments (corresponding to the first and last probe directions in each quadrant of the displacement space) were performed for the single swipe tests. Using FELA, 120 load probe tests were performed to identify 120 failure loads. All the analyses were set up using scripts and the difference in set-up time is thus negligible. The computer used to run the analyses had an Intel Xeon 3·60 GHz processor (eight central processing units) with 16 GB RAM (random access memory).

Table 3 is revealing in several ways. The single swipe test was found to be the most efficient procedure if the total time
taken is adopted as the efficiency measure. However, different procedures provide different numbers of reliable failure points; only the final failure points which have reached steady state at the end of each probe are dependably accurate for all cases. Thus, an alternative efficiency measure, the time taken per reliable failure point (defined as the average time to analyse one probe), was compared. Based on this efficiency measure, the load probe test was found to be the most efficient procedure. For the analyses of the surface and caisson foundations, the sequential swipe test was, respectively, 2.7 and 2.6 times faster than the displacement probe test. This is an interesting result as it shows the existence of a numerical procedure capable of providing failure envelope predictions that are as accurate as the displacement probe test, but with greater efficiency. The single swipe test, on the other hand, has lower efficiency than the sequential swipe test when evaluated on a per probe basis.

In terms of resolution, the single swipe and sequential swipe tests provide more failure points than the other procedures. However, Figs 7–9 have shown that the load path followed during a single swipe test may be far from the FELA load probe (average) results (and outside the bounds). Thus, the intermediate points during the single swipe test may not be accurate failure points. In contrast, the same figures show that the intermediate points obtained during each stage of a sequential swipe test are sufficiently close to the FELA load probe (average) results (and within the bounds) to be considered as reasonably accurate failure points. Thus, the sequential swipe test provides higher failure envelope resolution than the other procedures.

Overall, the sequential swipe test appears to provide the best balance of accuracy, efficiency and resolution among the FEA-based procedures, while the FELA-based load probe test provides a good balance of accuracy and efficiency (if resolution is not an important criterion). However, if the accuracy of intermediate failure points is not an important criterion, the load path from a single swipe test can provide a quick and conservative estimation of the failure envelope; although users should be aware that the shape of a failure envelope determined from the intermediate points can sometimes be significantly different from the reference failure envelope (e.g. see Figs 4(b) and 11(a)).

There are some limitations of this comparative study. First, the conclusions of this study have only been obtained for von Mises soil. It is unknown whether the same conclusions apply for other soil models such as the Mohr–Coulomb model, especially if a non-associated flow rule is adopted. Second, the influence of features such as non-homogeneous soil strength profiles and the allowance for contact breaking between foundation and soil have not been investigated. Further studies are required to address these issues.

CONCLUSIONS

The primary goal of this paper was to evaluate the performance of various numerical procedures for determining undrained VHM failure envelopes of shallow foundations: the displacement probe test, the single swipe test and the sequential swipe test (all performed using FEA) as well as the load probe test (performed using FELA). Two circular foundation types with significantly different failure envelope shapes were considered.

In general, there is little to differentiate between the procedures in terms of accuracy, except for the single swipe test, where the load path was sometimes found to under-predict (i.e. deviate inside) the reference failure envelope. For the examples considered in this paper, the sequential swipe test appears to offer the best balance of accuracy, efficiency and resolution. The FELA-based load probe test has higher efficiency but lower resolution. The findings suggest that the sequential swipe test offers an attractive alternative to the widely used displacement probe test, since it is just as accurate, but is faster and has the additional benefit of higher failure envelope resolution.

Finally, this study investigated the influence of the number of discrete swipe stages used in a sequential swipe test. It was found that there is a critical number above which the load path appears to track the failure envelope with negligible deviation. Based on the findings of this paper, a minimum of eight discrete swipe stages in each quadrant of the displacement space is recommended to ensure that the load path stays close to the failure envelope throughout the analysis. As the number of discrete swipe stages decreases, the accuracy of the sequential swipe test decreases and the load path becomes more sensitive to the starting point of the swipe phase, as shown by the single swipe test results in Fig. 4.

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NOTATION

\( A \) \narea enclosed by failure envelope from any numerical procedure

\( A_{ref} \) \narea enclosed by reference failure envelope from displacement probe tests

\( B \) \nwidth of surface strip foundation

\( H \) \nhorizontal load

\( H_0 \) \nnormalised horizontal load

\( M \) \nmoment load

\( M_0 \) \nmoment uniaxial capacity

\( n \) \nnumber of discrete sequential swipe stages

\( q_1 \) \nnormalised generalised first degree of freedom

\( q_2 \) \nnormalised generalised second degree of freedom

\( u \) \nhorizontal displacement

\( V \) \nv vertical load

\( \tilde{V} \) \nnormalised vertical load

\( V_0 \) \nv vertical uniaxial capacity

\( w \) \nvertical displacement

\( \eta \) \nrelative accuracy measure for failure envelopes

\( \theta \) \nrotational displacement

REFERENCES

Bell, R. W. (1991). *The analysis of offshore foundations subjected to combined loading*. MSc thesis, University of Oxford, Oxford, UK.

Bransby, M. F. & Randolph, M. F. (1998). Combined loading of skirted foundations. *Géotechnique* **48**, No. 5, 637–655, https://doi.org/10.1680/geot.1998.48.5.637.

Bransby, M. F. & Yun, G. J. (2009). The undrained capacity of skirted strip foundations under combined loading. *Géotechnique* **59**, No. 2, 115–125, https://doi.org/10.1680/geot.2007.00098.

Byrne, B. W. (2000). *Investigations of suction caissons in dense sand*. DPhil thesis. University of Oxford, Oxford, UK.

Cassidy, M. J., Byrne, B. W. & Randolph, M. F. (2004). A comparison of the combined load behaviour of spudcan and caisson foundations on soft normally consolidated clay. *Géotechnique* **54**, No. 2, 91–106, https://doi.org/10.1680/geot.2004.54.2.91.

Dassault Systèmes (2014). *Abaqus user manual*, version 6.13. Providence, RI, USA: Simulia Corp.

Dunne, H. P. & Martin, C. M. (2017). Capacity of rectangular mudmat foundations on clay under combined loading. *Géotechnique* **67**, No. 2, 168–180, https://doi.org/10.1680/jgeot.16.P079.
