Mathematical Modeling of the Transfer of Energy Forces from the Engine through Hydro Transmission and Hydro Differential to Executive Bodies

Farkhod Matmurodov1*, Bozorboy Sobirov2, Isomiddin Tulanov3, Jakhongir Mirzaabdullayev4, Jamshid Khakimov4, Oybek Daminov4

1Turinsky the Polytechnical University in Tashkent, Tashkent, Uzbekistan
2Urgench Branch of the Tashkent University of Information Technologies, Tashkent, Uzbekistan
3Research Institute of Mechanization of Agriculture, Settlement of Gulbakh, Uzbekistan
4Tashkent State Technical University, Tashkent, Uzbekistan

Email: *matmurodov@yahoo.com

Abstract
Mathematically simulated energy transfers from the energy source to the chassis through hydro transmissions and hydro differential. The developed unified mathematical model of a dynamic system allows, at the design stage, of many branched drive mechanisms, including transmission hydraulic and hydraulic differential actuators, to explore dynamic processes and select rational parameters.

Keywords
Energy Forces, Dynamic System, Many-Branched Mechanism, Engine, Hydro Transmission, Hydro Differential

1. Introduction
The results of experimental studies of experimental structures indicate a high dynamic load during transient processes of moving off, shifting gears and locking hydraulic differentiation, as well as in steady-state motion modes of mobile machines, which limits the durability of the elements of power mechanisms [1]. This determines the need for in-depth research aimed at reducing dynamic loading.

At present, the available theoretical and experimental data for previously de-
signed machines do not allow taking into account the potential properties of the
designed machines, features of new design solutions, operating conditions, etc.
Analytical methods for predicting durability and reliability based on the works
of scientists and specialists were created for machines with low power at steady
motion, for which the probability of moving at high speeds and high-loaded
modes is not high. The mode of operation of many branched mechanisms of
promising machines has not been scientifically studied.

Known mathematical models do not allow sufficiently taking into account the
real design features, conditions and modes of motion control of machines, their
interaction of parallel-sequentially installed driving mechanisms.

2. Problem Definition

One of the promising areas for improving the designs of wheeled vehicles and
creating high-performance equipment is the use of volumetric hydraulic drives
as a drive for the drive wheels of the undercarriage system. At present, industrial
and agricultural mobile energy equipment (wheeled and tracked tractors)
has been improved and has reached the level of mobile power facilities (MPF).
MPF universal and unified tractor. It can simultaneously hang and fasten sev-
deral different many operating machines in front, side and rear. In this case,
there will be a lot of branching of the transmission of power from the engine
to the actuators. In particular, energy is transmitted to the undercarriage,
working actuators—to the drive shaft, a number of active working bodies. To
describe these phenomena by mathematical expression is considered an im-
portant task.

Taking into account the influencing factors in the design of many branched
mechanisms, including the flexibility of the working fluid and hydraulic drive
elements, allows the design stage to provide a high technical level, reduce the
amount of testing by increasing the reliability of calculations.

The creation of many branched mechanisms of a high technical level is ham-
pered by the fact that the magnitudes of the influence of the flexibility of the
working fluid, the elements of the hydraulic drive and the links of the mechan-
isms on its dynamic loading are insufficiently investigated. The influence of the
flexibility of the working fluid and hydraulic drive elements on the optimal val-
ues of the parameters of the boom lifting mechanisms and the handle drive has
not been taken into account.

The development of more accurate mathematical models of working processes
of hydraulic mechanisms taking into account leaks in the hydraulic system, op-
eration of safety systems, pliability of the working fluid and hydraulic drive ele-
ments makes it possible to fully and objectively determine the loads overcome by
a many-branched mechanism during operation, and therefore evaluate the
pressure state of the links, including number and in transient conditions.

The laboratory field tests of experimental tractors of the Research and Devel-
opment Institute of Automobile and Tractor of Russia and the Institute of Agri-
cultural Machinery (England) showed increased slipping of tractor wheels with hydro-transmission in comparison with mechanical transmission. At the same time, the Institute of Agricultural Machinery in Leipzig (Germany) and MTZ specialists note a reduction in slipping, which confirms the opinion of the influence of the type and parameters of the transmission, hydraulic machines and characteristics of working fluids on the traction properties of self-propelled machines.

From the side of Panasenko S.M. it was determined that the hydraulic drive of propulsion with high-torque hydraulic motors with kinematic perturbation or uneven rotation in the entire load range exceeds the skidding of the tractor with a mechanical transmission; Installing low-torque hydro motors helps reduce slippage when exposed to forced vibrations. The most rational volume hydraulic actuators of the tractor chassis are characterized by the use of high-speed hydraulic motors followed by a manual gearbox and torque reduction to the propulsion units. Such a scheme has higher dissipation properties, which exclude the sources of oscillations that tell the effect on the driving movement of the thrusters.

Usually, traditionally, after the transmission gearbox, the differential, semi-axle and front-wheel reducer is installed. All these mechanisms serve to reduce the power during transmission from the hydraulic motor to the wheel.

In the thesis [2], the hypothesis about the destruction of metal-ceramic disks of friction elements, the control system of hydromechanical transmission due to the occurrence of resonant modes, caused by high-frequency disturbances generated by the torque converter, is put forward and substantiated. Based on the results of the study, an improved method of friction elements is presented.

Energy transfer previously performed on all mathematical models is described in parts and in a vague form. The transmissions of generated power or power from the engine by a mathematical expression are not fully recorded in any scientific work. The creation of a multi-operational and multi-functional mobile power and super-power engine makes a single transfer model of the power from the engine through a lot of branched transmission mechanisms to the executive bodies written. Unity is a unified system of equations, which are written by the expression the generated energy of an engine in parallel-sequential order transmitting to the executive bodies. Basically, MPF uses mechanical, hydraulic and electric drive transmissions. In this paper, we model the mechanical and hydraulic drive mechanisms.

To write a single mathematical model, we accept the following assumptions: oscillation phenomena in all mechanisms and nodes are not taken into account; sustainability is not considered; not studied external perturbing all sorts of phenomena.

3. Results and Discussion

3.1. Draw an Equivalent Design Scheme (Figure 1)
3.2. Describe the Transfer of Energy from the Engine to Branched Mechanisms Involving Hydraulic Transmission

Equations describing the rotational motion of mechanisms from the engine

\[(J_d + J_{21}) \ddot{\varphi}_d + k_{d2} (\dot{\varphi}_d - \dot{\varphi}_2) + c_{d2} (\varphi_d + \varphi_2) = M_d - \frac{M_2}{i_2}, \varphi_d = \varphi_2\]

\[(J_{21} + J_{22}) \ddot{\varphi}_2 + c_2 (\varphi_d + \varphi_2) = -M_2 \text{sign} (\ddot{\varphi}_2) i_2 ,\]

\[(J_{\text{pto}1} + J_{\text{pto}2}) \ddot{\varphi}_{\text{pto}} + c_{\text{pto}} (\varphi_d + \varphi_{\text{pto}}) = -M_{\text{pto}} \text{sign} (\ddot{\varphi}_{\text{pto}}) x_i_{\text{pto}} ,\]

PTO n-clutch

\[J_{\text{pto}} \ddot{\varphi}_{\text{pto}} + k_{\text{pto}} (\dot{\varphi}_d - \dot{\varphi}_{\text{pto}}) + c_{\text{pto}} (\varphi_d + \varphi_{\text{pto}}) = -\frac{M_{\text{pto}} \text{sign} (\ddot{\varphi}_{\text{pto}})}{i_{\text{pto}}} ,\]

n-PTO

Describes the transfer of energy through an extensive mechanism involving hydraulic transmission.
\[ J_1^i \phi_1^i - k_{23}^i (\phi_2 - \phi_3^i)^2 - e_{23}^i (\phi_2 - \phi_3^i) = -p_1 Rf \sin \gamma \sum_{i=0}^{n-1} \sin (\phi_i^1 - k \beta) / \eta_{HTM}, \]
\[ J_1^i \phi_1^i - k_{23}^i (\phi_2 - \phi_3^i)^2 - e_{23}^i (\phi_2 - \phi_3^i) = -p_2 Rf \sin \gamma \sum_{i=0}^{n-1} \sin (\phi_i^1 - k \beta) / \eta_{HTM}, \]
\[ J_5^i \phi_5^i R_{genc} \sin \gamma \sum_{i=0}^{n-1} \sin (\phi_i^1 + k \beta) = \phi_5^1 F \sum_{i=0}^{a} v_\varphi + p_r + \bar{p} V_1 / E, \]
\[ J_1^i \phi_3^i + k_{45} (\phi_3^i - \phi_4^i) + e_{45} (\phi_3^i - \phi_4^i) = p_1 F \eta_{HTM} \sum_{i=0}^{a} v_\varphi, \]
\[ J_1^i \phi_3^i + k_{45} (\phi_3^i - \phi_4^i) + e_{45} (\phi_3^i - \phi_4^i) = p_2 F \eta_{HTM} \sum_{i=0}^{a} v_\varphi, \]
\[ J_1^i \phi_3^i - k_{45} (\phi_3^i - \phi_4^i) + k_{56} (\phi_3^i - \phi_6) - e_{45} (\phi_4^i - \phi_6) + c_{56} (\phi_6 - \phi_6) = -0.5 M_\varphi, \]
\[ J_1^i \phi_3^i - k_{45} (\phi_3^i - \phi_4^i) + k_{56} (\phi_3^i - \phi_6) - e_{45} (\phi_4^i - \phi_6) + c_{56} (\phi_6 - \phi_6) = -0.5 M_\varphi, \]

And consider hydraulic transmissions from hydraulic leaks and leakages.

\[ k_a \phi_3^1 \gamma - c_s (p_1 - p_2) - c_r p_1 - q_{gw} \phi_4 - 2 e_{23} \dot{p}_1 = \begin{cases} 0, & \text{при } p_1 > p_{sdk} \\ r_{sdk} P_1 - Q_m & \text{при } p_1 \leq p_{sdk} \end{cases}; \]
\[ k_a \phi_3^1 \gamma - c_s (p_1 - p_2) - c_r p_2 - q_{gw} \phi_4 - 2 e_{34} \dot{p}_2 = \begin{cases} 0, & \text{при } p_2 > p_{sdk} \\ r_{sdk} P_2 - Q_m & \text{при } p_2 \leq p_{sdk} \end{cases}; \]
\[ M_{\text{gw}} = q_{gw} (p_1 - p_2) - f_{\text{gw}} \varphi_4, \]
\[ M_n = k_n \gamma (p_1 - p_2) - f_{\text{snk}} \varphi_4, \]

where, \( J_1^i, J_4^i, J_1^i, J_4^i, J_5^i, J_7^i \)—are the reduced moments of inertia of the concentrated masses of the right and left pumps, right and left hydrometers, and the leading right and left wheels; \( \phi_2^i, \phi_6^i \)—angular movement of the clutch and an active executive body; \( \phi_2^i, \phi_6^i \)—angular speed of the clutch and an active executive body; \( k_{23}^i, k_{23}^i \)—damping factor of the pump shaft; \( e_{23}, e_{34} \)—hydraulic compliance pressure pump parts; \( R \)—created efforts RJ; \( \gamma \)—angle of rotation of the pump control device; \( r \)—leakage ratio of RJ; \( V_1, V_2 \)—volume in pressure and drain cavities; \( E \)—volume modulus of elasticity RJ; \( k_{45} \)—damping coefficient of hydraulic motor shaft; \( e_{45} \)—hydraulic compliance of the working part of the motor; \( e_{23}, e_{34} \)—hydraulic compliance of pressure and drain lines between the pump and the hydraulic unit; \( k_{56} \)—shaft damping ratio of the active executive body; \( c_{56} \)—circumferential rigidity of the shaft of the active executive body; \( \phi_3^i, \phi_4^i \)—angular displacement of the shaft in the right pump 3 and the right hydraulic motor 4; \( \phi_3^{i1}, \phi_4^{i1} \)—the angular displacement of the shaft in the left pump 3 and in the left hydraulic motor 4; \( \phi_5^i, \phi_6^i \)—are the angular displacements of the shaft of the link 5 from the side of the respective hydraulic motors; \( R \)—cross-sectional area; \( \eta_{HTM}, \eta_{HMM} \)—efficiency of the pump and hydraulic motor; \( v_\varphi \)—reduced speed of a high-speed hydraulic motor; \( c_s, c_r \)—hydraulic leakage and leakage rates; \( p_1, p_2 \)—pressure in the pressure and discharge lines; \( \dot{p}_1, \dot{p}_2 \)—time derivatives of pressure in the pressure and discharge lines; \( p_{sdk} \)—pressure setting of the make-up valve; \( r_{sdk} \)—specific flow through the return pick valve; \( k_n \)—coefficient specific feed pump; \( q_{gw} \)—specific consumption of
the hydraulic motor; $f_{g\text{gm}}, f_{aw}$ — are the coefficients of the generalized equivalent damping of the pump and the hydraulic motor; $M_{gm}, M_n$ — the moment created by the hydraulic motor and the pump; $M_o$ — the moment of engagement of thrusters with the ground.

The resulting components of the differential equations for the hydraulic pump NAR-53 in the form $\sum_{k-0}^{n-1} \sin (\varphi + k/\beta) = 0.5 \sin \varphi + 2.83 \cos \varphi, -20^\circ \leq \varphi \leq 20^\circ$; $\varphi', \varphi''_k$ — for angular displacement is written accordingly; hydromotor MG-265T

$$\sum_{k=0}^{v_\varphi} a + n_\varphi = 18.55 + 3.044 \varphi.$$ 

Moment of adhesion of propulsion with the ground is determined taking into account vibrodynamic effects of disturbing loads and variable wheel speed according to the formula [3]

$$M_\varphi = \left[ m q_\varphi p + (1-m)(c + qg_\varphi) \right] F_0 r_i \sqrt{\frac{\delta}{\delta_{\max}}} \sum_{i=1}^{n} \sqrt{i} \exp \left[ -\alpha_i (|\dot{\phi}_c - \phi_0| r_i) \right]$$

here, $m$ — saturation coefficient tire tread; $q$ — normal tire pressure on the ground; $\varphi_p$ — the walking angle; $c$ — connectedness of the soil; $\varphi$ — the angle of internal friction of the soil; $F_0$ — contact area; $r_i$ — the radius of the wheel; $\alpha$ — an indicator depending on the type of soil; $\delta, \delta_{\max}$ — shear characteristics; $\alpha_i$ — a constant coefficient characterizing the physical and mechanical properties of soil; $\dot{\phi}_c$ — acceleration of the load on the soil; $i$ — quantitatively lugs that are engaged.

3.3. Describes Energy Transfer in Hydro Differential

According to the calculation scheme (4H, 4gm links) (Figure 1) and accepted assumptions, the mathematical model of the hydro differential action will be the system of equations [4]

$$\begin{align*}
J_{4H} \dot{\phi}_{4H} &= M_{4H} - M_p - \beta_{4H} \dot{\phi}_{4H} \\
J_{4gm} \dot{\phi}_{4gm} &= M_p + M_{4gm} \\
(V_{4H} / 2\pi) \dot{\phi}_{4H} - (V_{4gm} / 2\pi) \dot{\phi}_{4gm} &= ep + k_o p + f_0 \dot{x}_0 \\
p - p_0 &= 0.5 k^{-1} \rho \dot{x}_0 \dot{x}_0 \\
f_0 \dot{x}_0 \dot{p}_0 &= V_2 p_1 p_0
\end{align*}
$$

$$M_p = (V_{4H} / 2\pi) p \quad \text{moment of pressure created RJ, } M_{4gm} = M_s - M_i, \text{ Initial conditions } \dot{\phi}_{4H} = 150c^{-1}, \dot{\phi}_{4gm} = 150c^{-1}, p = 0, p_0 = p_1, \dot{x}_0 = 0.$$

Here, $J_{4H}$ — moment of inertia reduced to the pump axis; $J_{4gm}$ — moment of inertia of the hydraulic motor brought to the axis; $M_{4H}$ —torque axis of the pump; $M_{4gm}$ —torque of the motor axis; $M_s$ —moment of resistance to the hydraulic motor axes; $M_i$ —moment of the $i$-th link located in turn; $V_{4H}, V_{4gm}$ —volumes of the pump and hydraulic motor; $V_2, p_1, p_0$ — volume, pre-charge pressure and pressure in the accumulator; $p$ — pressure in the pressure line; $\beta_{4H}$ —damping coefficient of the pump; $\varphi_{4H}, \varphi_{4gm}$ —angular displacements of the axis of the pump and the hydraulic motor; $k_o, c$ —coefficient of volumetric losses in the hydraulic drive and the flexibility of the pressure line; $f_0, \dot{x}_0, \dot{p}_0$ — cross-section and velocity of the RJ at the inlet of the hydro accumulator; $k, \rho$
— flow coefficient and the density of the RJ.

In the system of Equation (1), the first two equations reflect the rotation of the pump and the hydraulic motor, the third one—the flow rate of the fluid, the fourth one—the fluid flow in the throttle of the accumulator, the fifth—the change in pressure in the hydro accumulator.

In the absence of a hydro accumulator, the system of Equation (1) is simplified to three equations

\[
\begin{align*}
J_{4H} \dot{\phi}_{4H} &= M_{4H} - M_p \\
J_{4gm} \dot{\phi}_{4gm} &= M_p + M_{4gm} \\
\left( V_{4H} / 2\pi \right) \dot{\phi}_{4H} - \left( V_{4gm} / 2\pi \right) \dot{\phi}_{4gm} &= e \dot{p} + k_0 p
\end{align*}
\]

(2)

Initial conditions \( \dot{\phi}_{4H} = 150c^{-1}, \phi_{4gm} = 0, p = 0 \).

In relative coordinate \( \theta = \phi_{4H} - 5\phi_{4gm} \), two equations of the system are distinguished (2)

\[
\begin{align*}
\dot{\theta} + \left( V_0 / 2\pi \right) p &= e_0 \\
-\left( V_0 / 2\pi \right) \dot{\theta} + e \dot{p} + k_0 p &= 0
\end{align*}
\]

(3)

System of Equation (2) with \( \theta = 0 \), \( p_1 = -p \) take the form

\[
\begin{align*}
J_{4gm} \dot{\phi}_{4gm} - \left( V_{4gm} / 2\pi \right) p &= M_{4gm} \\
e \dot{p} + k_0 p + \left( V_{4gm} / 2\pi \right) \dot{\phi}_{4gm} &= 0
\end{align*}
\]

As a result of operational calculus, the angle, time and ways of braking are determined.

\[
\varphi_T = \varphi_0 / \epsilon_T ; \quad \tau_T = \varphi_0^2 / \epsilon_T ; \quad \epsilon_T = V_{4gm} p_0 .
\]

Load maxima on the motor axis

\[
p_{4gm} \approx p_0 + J_{4gm} \dot{\phi}_{4gm} (2 p_0 \epsilon)^{-1} ; M_{4gm} \approx \left( V_{4gm} / 2\pi \right) p_{4gm} .
\]

For the functioning of the hydro differential with the connection with the drive wheels, we supplement the system of equations (1) and (2) corresponding to the transfer of power

\[
\begin{align*}
J_{4H} \ddot{\phi}_{4H} &= M_{4H} - M_p - \beta_{4H} \dot{\phi}_{4H} \\
J_{4gm} \ddot{\phi}_{4gm} &= M_p + M_{4gm} \\
\left( V_{4H} / 2\pi \right) \dot{\phi}_{4H} - \left( V_{4gm} / 2\pi \right) \dot{\phi}_{4gm} &= e \dot{p} + k_0 p + f_0 \dot{x}_0 \\
p - p_0 &= 0.5 k^{-2} \rho \dot{x}_0 \left| \dot{x}_0 \right| \\
f_0 \dot{x}_0 p_0^n &= V_s p_s p_0 \\
J_{4gm} \ddot{\phi}_{4gm} + k_{4gm} \left( \phi_{4gm} - \phi_s \right) + e_{4gm} \left( \phi_{4gm} + \phi_4 \right) &= p_{2s} F_2 \eta_{4gm} \sum_0^{\delta} v_{\phi 4} \\
J_s \dot{\phi}_s + k_{45} \left( \phi_s - \phi_s \right) + c_{45} \left( \phi_s + \phi_s \right) &= -M_s \text{sign} \left( \dot{\phi}_s \right) i_s \\
J_{4H} \ddot{\phi}_{4H} &= M_{4H} - M_p \\
J_{4gm} \ddot{\phi}_{4gm} &= M_p + M_{4gm} \\
\left( V_{4H} / 2\pi \right) \dot{\phi}_{4H} - \left( V_{4gm} / 2\pi \right) \dot{\phi}_{4gm} &= e \dot{p} + k_0 p \\
J_{4gm} \ddot{\phi}_{4gm} + k_{4gm} \left( \phi_{4gm} - \phi_4 \right) + e_{4gm} \left( \phi_{4gm} + \phi_4 \right) &= p_{2s} F_2 \eta_{4gm} \sum_0^{\delta} v_{\phi 4} \\
J_s \dot{\phi}_s + k_{45} \left( \phi_s - \phi_s \right) + c_{45} \left( \phi_s + \phi_s \right) &= -M_s \text{sign} \left( \dot{\phi}_s \right) i_s
\end{align*}
\]

(6)
here, \( k_{gm}, \varphi_{gm} \) —coefficient of damping and angular displacement and shafting between the hydraulic motor and link 4; \( \varepsilon_{gm} \) —hydro hydraulics compliance; \( p_{24} \) —RJ pressure in the side of the link 4; \( F_4 \) —sectional area; \( \eta_{gm} \) —hydro motor efficiency; \( v_{\phi 4} \) —reduced speed high-speed hydraulic motors of hydro differential.

The systems of Equations (5) and (6) are subject to the mechanisms under the condition \( M_{\psi 5} = 0, \ M_{\phi 5} \neq 0 \), and vice versa with \( M_{\phi 5} = 0, \ M_{\psi 5} \neq 0 \).

The system of Equation (5) reflects the hydromechanical network with a hydro accumulator and (6) without it.

4. Conclusions

A single mathematical model of the transfer of energy from the engine through a multi-branched transmission mechanism to the executive bodies is described by the expression of the transfer of energy from the joint work of mechanical and hydraulic actuators. The developed mathematical model of a dynamic system allows, at the design stage, of many branched drive mechanisms, including transmission hydraulic and hydraulic differential actuators, to explore dynamic processes and choose rational parameters of hydraulic drive compliance, time and damping coefficient, which are variable by selecting kinematic dynamic parameters, hydraulic motors working volume, the volume of the injection line and the reduced mass of inertia.

A unified mathematical model of the transfer of energy from the engine through many branched transmission mechanisms to the executive bodies take into account the hydraulic transmission and hydraulic differential.

Conflicts of Interest

The authors declare no conflicts of interest regarding the publication of this paper.

References

[1] Taratorkin, I.A. (2009) Development of Computational and Experimental Methods for Reducing Dynamic Loading and Increasing the Durability of Hydromechanical Transmissions of Transport Vehicles. Dissertation, Bauman Moscow State Technical University (MSTU), Russian Federation (RF), Kurgan, 1.

[2] Homichev, A.S. (2010) Improvement of the Design Calculation Methodology for Friction Elements of Hydromechanical Transmissions of Transport Vehicles. Dissertation, Izhevsk State Technical University (ISTU), Russian Federation (RF), Kurgan, 12.

[3] Panasenko, S.M. (1992) Influence of Kinematic and Dynamic Qualities of the Volumetric Hydraulic Drive of the Tractor Chassis on the Traction and Operational Performance. Dissertation, Kharkiv Automobile and Road Institute (KARI), Ukraine, Kharkov, 8-10.

[4] Spiridonov, S.V. (1993) Improving the Efficiency of Pruning with a Vertical Broaching Tree by Reducing the Dynamic Load. Dissertation, St. Petersburg Forest Technical Academy, Russian Federation (RF), St. Petersburg, 7-9.