ONE-LOOP EFFECTIVE ACTION IN OPEN-STRING THEORY

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This is a summary of the analysis of the one-loop effective action in $Z_2$-orbifold compactifications of type-I theory presented in references [1,2]. We show how, for non-abelian group factors, the threshold effects are ultraviolet finite though given entirely by a six-dimensional field theory expression. We also discuss implications for the equivalence between Type-I and Heterotic four dimensional $N = 2$ superstring theories.

1 Motivations

1.1 Gauge Couplings Unification in open-string theory

Assuming the Minimal Supersymmetric Standard Model spectrum at low energy, and using the renormalization group equations, the extrapolation of low-energy values of gauge coupling constants predicts unification at the scale $M_U \simeq 2 \times 10^{16}$ GeV. In string theory, unification with gravity implies an additional tree-level relation at the string scale $M_{\text{string}}$, between $\alpha_U$ - the fine structure constant at unification - and $G_N$, the Newton constant. Since we know the values of $\alpha_U$ and $G_N$ this relation can be turned into a prediction for $M_{\text{string}}$.

In heterotic string theory, a single closed-string diagram - the sphere - is responsible for $\alpha_U$ and $G_N$ at tree-level. Then, in the previous unification relation one can eliminate the dependence both on the dilaton and on the compactification volume, and one obtain a model independent prediction for $M_{\text{string}} \simeq 5 \times 10^{17}$ GeV which is bigger than $M_U$ by a factor 20.

In open-string theory $G_N$ is given at tree-level by a closed-string diagram while $\alpha_U$ is given by an open-string one - the disk - so that the string unification scale depends on the compactification volume. Though it is less predictive than the heterotic theory, it seems easier to realize tree-level unification in open-string theory by adjusting the compactification volume. The next step is to compute the string threshold corrections.

1.2 Heterotic/Type-I duality

There has been some very convincing evidence accumulated so far for the equivalence of theories which were believed in the past to describe truly different types of superstrings. Type I, Type II and Heterotic theories seem merely to provide complementary descriptions of a more complicated theory of fundamental interactions, and the larger framework of superstring dualities now includes also M-theory and F-theory descriptions.

Among the four-dimensional models, the most familiar examples of dual pairs are based on Type II and Heterotic constructions. Type I theory remained a wild card in duality conjectures until quite recently Polchinski and Witten presented several arguments for the equivalence of Type I and Heterotic theories in ten dimensions. Although in $D = 10$ this is a strong-weak coupling duality, it turns out that upon appropriate compactification on $K_3 \times T^2$, one obtains four dimensional $N = 2$ supersymmetric dual pairs with the Heterotic gauge coupling mapped to a Type I gauge coupling in a way that some weakly coupled regions overlap on both sides. One can check their equivalence in the quantitative comparisons of their low-energy effective actions.

2 One-loop threshold computation

2.1 The open-string model

In reference [1] we have studied the one-loop Lagrangian for slowly-varying gauge-field strengths in some $Z_2$-orbifold models first constructed by Bianchi and Sagnotti and analyzed in detail recently by Gimon and Polchinski. These models have N=1 supersymmetry in six dimensions, and a maximal gauge group $G = U(16) \times \tilde{U}(16)$, which can be broken by both discrete and continuous (antisymmetric) moduli. Upon toroidal compactification to four dimensions, one finds N=2 supersymmetries and extra (adjoint) Wilson-line moduli. The Type-I theory on $R^6 \times T^4/Z^2$ contains untwisted and twisted closed strings, as well as
open strings of three different types: those with freely moving endpoints (NN), those whose endpoints are stuck on some 5-branes transverse to the orbifold (DD), and those with one endpoint stuck and one moving freely (DN). Consistency fixes both the number of 9-branes and the number of 5-branes to be 32. It also fixes the action of the orientation reversal ($\Omega$) and orbifold-twist ($R$) on the open-string end-point states.

The massless spectrum of this theory in six dimensions has:

(i) the N=1 supergravity, one tensor and four gauge-singlet hypermultiplets from the untwisted closed-string sector,

(ii) sixteen gauge-singlet hypermultiplets, one from each fixed-point of the orbifold, in the twisted closed-string sector

(iii) $U(16)$ vector multiplets and two hypermultiplets in the antisymmetric 120 representation from the NN sector,

(iv) identical content, i.e. an extra $\tilde{U}(16)$ gauge group and two antisymmetric hypermultiplets, from the DD sector, and

(v) one hypermultiplet transforming in the representation $(16,16)$ of the full gauge group and coming from the DN sector.

Notice that each twisted-sector hypermultiplet contains a RR 4-form field $C^{(4)}$ localized at the $i$th fixed point of the orbifold, which plays a special role in what follows. We will furthermore choose to work on $R^4 \times T^2 \times T^4/Z_2$, so that we may break the gauge group by Wilson-line moduli in four dimensions.

2.2 Method and results

In calculating the effective gauge-field action we have followed reference, where a similar calculation was carried out for toroidal compactifications of the type-I SO(32) superstring. In order to determine the threshold effects at one-loop compute the free energy in the presence of a magnetic background $B$ and then extract the coefficient of $B^2$ in the weak-field limit. This model has a T-duality, which interchanges NN and DD sectors and hence also the two U(16) gauge groups. Without losing generality, we may thus restrict ourselves to background fields arising from the NN sector.

The one-loop free energy is the sum of contributions from the various sectors:

$$F_{\text{orbifold}} = F_{\text{closed}} + F_{\text{NN}} + F_{\text{ND}} + F_{\text{DD}}$$

Since only Neumann endpoints couple to the background field, we can ignore $F_{\text{closed}}$ and $F_{\text{DD}}$ which vanish by space-time supersymmetry. In the remaining terms one find contributions of the annulus, and the Möbius strip. Using the Jacobi identity, one finds that all these amplitudes expanded to quadratic order in the magnetic field collapse to contributions of six-dimensional massless states. This is to be expected: indeed, only BPS states can contribute to threshold effects, and the only BPS states of the open string are the massless (before Higgsing) modes of the six-dimensional model. A similar result is well known in the heterotic string, where, however, the spectrum of BPS states includes infinite string excitations with no simple field-theoretic description.

The final result for the full free energy, including classical and one-loop contributions is a sum over annulus and Möbius contributions:

$$F(B)/V^{(4)} = \frac{B^2}{2g_{(4)}^2} + \frac{B^2}{8\pi^2} \int_0^\infty \frac{dt}{t} \times$$

$$\left[ -\sum_{ij} s_{ij}(q_i + q_j)^2 \sum_{a_++a_-+\Gamma_2} \frac{1}{4} e^{-\pi tp^2/2} + \sum_i q_i^2 \left( \sum_{a_++\Gamma_2} 4e^{-\pi tp^2/2} - \sum_{2a_++(\Gamma_2)} e^{-\pi tp^2/2} \right) \right] + o(B^4)$$

where the SU(16) generators are normalized to $\tilde{g}_{16}Q^2 = \frac{4}{3}$, $a_i$ the Wilson-line moduli, and we recall that the Chan-Paton charges $q_i$ run over both the 16 and the 16 representations separately. The orbifold-twist operator acts as a simple sign, $s_{ij} = -1$ or $+1$ ,

according to whether the end-point states $|i>$ and $|j>$ belong to the same or to conjugates representations.

As a check let us extract the leading infrared divergence of the coupling-constant renormalization in the limit of vanishing Wilson lines. Cutting off $t < 1/\mu^2$ one finds after some straightforward algebra

$$\left. \frac{4\pi^2}{g_{(4)}^2} \right|_{1\text{-loop}} = \left. \frac{4\pi^2}{g_{(4)}^2} \right|_{\text{tree}} - 6 \log \mu + \text{IR finite}$$

in agreement with the correct $\beta$-function coefficient of the N=2 theory in four dimensions,

$$C_{\text{adj}} - 2C_{120} - 16C_{\text{fund}} = -6.$$
2.3 What about ultraviolet behaviour?

Considered as an expression of field theory, the threshold effects would be quadratically divergent in the ultraviolet limit. How then is the string result finite?

In heterotic string theory ultraviolet finiteness at one loop follows from the restriction of the integration over all world-sheet tori to a single fundamental domain - thanks to modular invariance. This presupposes conformal invariance, or equivalently the absence of classical tadpoles. In open-string theory we do not have modular invariance any more but we learn from references [12] that the ultraviolet divergences come from tadpoles of the massless closed string states sandwiched with an on-shell propagator.

Compared with the toroidal case, the orbifold model has only one additional piece of potential ultraviolet divergence. It comes from the twisted RR 4-forms, which couple to the background field through the generalized Green-Schwarz interaction

\[
(2\pi)^{-5/2} \sum_i \int d^6x \frac{1}{4!} \mu^{\rho\sigma\kappa\tau} C^{(4)}_{\mu\nu\rho\sigma} tr F_{\kappa\tau}
\]

for canonically-normalized 4-forms. The coupling gives mass to the $U(1)$ (abelian) gauge field, rendering a background inconsistent. For non-abelian background fields, on the other hand, the structure of ultraviolet divergences is identical to that of the toroidal model. Furthermore since in the toroidal theory the gauge coupling is not renormalized, we may conclude that in the orbifold the renormalization is ultraviolet finite.

The first physical divergence appears at order $o(B^4)$ and comes from tadpoles of order $o(B^2)$ of the graviton and the dilaton. It can be used to derive the relation between gauge and gravitational couplings: taking into account the halving of the volume, we may see that, with $SO(32)$ normalizations for the generators of $U(16)$, the relation between gauge and gravitational couplings remains the same in the orbifold model as in the toroidal one.

One may be puzzled by the fact that our final result for the free energy is formally identical to that of Kaluza-Klein theory compactified from six to four dimensions, and yet is ultraviolet finite. If we were to impose a uniform ultraviolet momentum cutoff, $t > 1/\Lambda^2$, the result would indeed be quadratically divergent. The cutoff dictated by string theory is, however, much smarter! It is uniform in transverse time $l$. The relation between $l$ and the proper time in the direct channel is different for each surface

\[
l = \begin{cases}
1/t & \text{annulus} \\
1/4t & \text{M"{o}bius} \\
1/4t & \text{Klein bottle}
\end{cases}
\]

so that if we cutoff the annulus at $t = 1/\Lambda^2$, we must cutoff the M"{o}bius strip at $t = 1/4\Lambda^2$. The ultraviolet divergence vanishes as we sum over the annulus and the M"{o}bius contributions in the free energy.

Our calculation illustrates in a very simple context the way in which open-string theory produces finite answers: in the case at hand it is simply field theory but with a very smart cutoff on the momenta.

3 Heterotic - Type I duality

These results have non-trivial implications, both in the context of heterotic-type-I duality [2] and for the study of moduli spaces of D-branes [24]. In this latter context in particular they imply that the metric in the moduli space of $N=2$ configurations of D-branes is given entirely by simple and massless closed-string exchange.

We now give one specific model which has simultaneous Type I, Heterotic and type II descriptions. On the type I side it originates from a six-dimensional model with one tensor multiplet and a completely broken gauge group. This model can be obtained from the class of orientifold constructions that is presented in section (2.1). Upon toroidal compactification to $D = 4$ one finds the 3 universal vector multiplets $S$, $S'$ and $U$. A Heterotic-Type II dual pair with the same massless spectrum has been considered before in refs.[9] - the so called $S - T - U$ model.

Thanks to $N = 2$ supersymmetry, the low-energy effective actions are entirely given by the respective prepotentials, which are well known for the Heterotic-Type II dual pair. For the Type I construction the tree-level prepotential is

\[
F^{(0)} = SS'U - \frac{1}{4} \sum_i (SA_i^2 + S'A_i'^2)
\]

where $\tilde{A}$ and $\tilde{A}'$ refer to the open-string gauge multiplets from NN and DD sectors respectively, $S$
and \( S' \) are the couplings to this two kind of gauge field and \( U \) is the usual complex modulus which determines the complex structure of the torus. The complex scalars \( S, S' \) and \( U \) belong to vector multiplets, and parametrize a \([SU(1,1)]^3\) manifold in the absence of open-string vector multiplets. We can deduce from Type I-Heterotic duality in \( D = 10 \) the corresponding duality in four dimensions:

\[
S_I = S_H, \quad S'_I = T_H, \quad U_I = U_H
\]

Here, \( S_H \) is the heterotic dilaton and \( T_H \) is the usual Kähler structure modulus of the torus.

The type-I theory exhibits two continuous Peccei-Quinn symmetries associated to \( S \) and \( S' \) which dictate the following form of Type I prepotential:

\[
F(S, S', U, A) = F(0) + f_I(U, A) + \text{non-perturbative corrections}
\]

where \( f_I(U, A) \) is the one-loop correction. Type-I non-perturbative terms are suppressed in the region we consider.

In order to determine perturbative corrections to the prepotential, one could in principle follow the method applied on the Heterotic side, by extracting the one-loop Kähler potential \( K^{(1)} \) from the universal part of threshold corrections to gauge couplings \( \Delta \). But unlike the Heterotic case, in Type I theory there are also one-loop corrections to the Planck mass \( \delta \) so that the one-loop Kähler potential \( K^{(1)} \) is given by:

\[
\partial_U \partial_{\bar{U}} \Delta + \frac{1}{2S_0^2} \sqrt{G} \partial_U \partial_{\bar{U}} \delta = -\frac{b}{(U - \bar{U})^2} + K^{(1)}_{U \bar{U}}
\]

where \( b \) is the one-loop beta function coefficient. Threshold computation has been presented in section (2). The one-loop correction to Planck mass can be calculated in a similar way, and is related to an index of the Ramond open-string sector.

As expected from the superstring triality conjecture, we have shown \( \) that all three prepotentials agree in the appropriate limits:

\[
\text{Im}(S) > \text{Im}(S') \to \infty \quad \text{in the open-string case}
\]

and

\[
\text{Im}(S) > \text{Im}(T) \to \infty \quad \text{in the Heterotic one}.
\]

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