Scalable fault-tolerant quantum computation in DFS blocks

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We investigate how to concatenate different decoherence-free subspaces (DFSs) to realize scalable universal fault-tolerant quantum computation. Based on tunable XXZ interactions, we present an architecture for scalable quantum computers which can fault-tolerantly perform universal quantum computation by manipulating only single type of parameter. By using the concept of interaction-free subspaces we eliminate the need to tune the couplings between logical qubits, which further reduces the technical difficulties for implementing quantum computation.

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The fragility of quantum superposition is the major stumbling block to achieving the physical implementation of quantum computers. As a potential approach to prevent decoherence, DFS encoding of qubits shields quantum coherence from environmental noise. The essence of this noise immune proposal is the assumption that the quantum systems suffer identical environmental noise. At present, under the assumption of collective decoherence the theory of universal fault-tolerant quantum computation (QC) in the DFSs we must be able to implement local universal fault-tolerant operations in every DFS subblock and realize fault-tolerant entangling operations between adjacent subblocks. Recently, several architectures for scalable quantum computing have been presented in ion-trap and one-dimensional array of solid-state systems.

To overcome this difficulty, a natural strategy is to partition the whole qubit system into several DFS subblocks. Coherent quantum information can be stored in them. To realize scalable fault-tolerant quantum computation (SFTQC) in DFSs we must be able to implement local universal fault-tolerant operations in every DFS subblock and realize fault-tolerant entangling operations between adjacent subblocks. Recently, several architectures for scalable quantum computing have been presented in ion-trap and one-dimensional array of solid-state systems.

In this letter, we show how to carry out SFTQC in DFS subblocks for qubit systems with tunable XXZ exchange interaction. By using appropriate encoding, one can make quantum information in every subblock interaction-free from the other subblocks even though the physical couplings to connect different DFS subblocks is always on. This is exactly the central idea of interaction-free encoding. To implement nonlocal operations between different subblocks one can transform local encoded states to “switch on” the interaction between adjacent subblocks. All the operations ensure the evolution of the system in decoherence-free subspaces. Our scheme leads to three additional prominent advantages. First, since interaction-free encodings spontaneously screen the subblocks from the couplings to other subblocks it is not necessary to use recoupling pulses to eliminate the effect of fixed couplings between subblocks. Second, physical switches are not necessary for interaction-free subspaces, and this reduces the technical difficulties for the design of quantum computers. Finally, in our scheme, the universal encoded QC can be realized by manipulating only one type of interaction parameter. This further reduces the difficulty of the manipulation.

DFS and IFS: Let us consider a one dimensional array of subblocks. Each subblock, which contains a certain number of qubits, is surrounded by an independent environment. We assume that the subblocks are only coupled to their nearest subblocks (see Fig.1).

For the $L$th subblock, there are three types of interactions to drive the evolution of the qubits in it.

$$H_{total} = H_{LL} + H_{LB} + \sum_{L'=L\pm 1} H_{LL'}.$$  (1)

Here, $H_{LL}$, $H_{LB}$ and $H_{LL'}$ refer to interactions between qubits in the $L$th subblock, qubits in the $L$th subblock and its environment, and interactions between adjacent subblocks. (For simplicity, we omit the free Hamiltonian of single qubit in this letter by using two degenerate states or working in the rotating frame.) In general, these interaction terms have the following forms:

$$H_{LL} = \sum_{i,j} J_{ij}^{\alpha\beta} \sigma_i^\alpha \sigma_j^\beta,$$

$$H_{LL'} = \sum_{i,j} g_{ij}^{\alpha\beta} \sigma_i^\alpha \sigma_j^\beta,$$

$$H_{LB} = \sum_{i,j} \gamma_i^{\alpha} \sigma_i^\alpha b_{Lj}.$$  Here, $\sigma_i^{(\alpha)}$ refers to the

![FIG. 1: A one dimensional array of subblocks. Each subblock surrounded by dotted-dashed lines suffers independent environment noise. Solid lines represent the couplings between the qubits in one subblock. Dotted lines represent the couplings between qubits in different subblocks.](image-url)
Pauli operator of the $i$th qubit in the $L(')th$ subblock, $\alpha(\beta) = x, y, z$. $b^\alpha_i$ is the environmental operator which couples to the $i$th qubit in the $Lth$ subblock. $J_{ij}^\alpha, \gamma_{i,\alpha}^\beta$ and $\gamma_{i,\alpha}^\alpha$ are coupling coefficients. According to our assumption, noise from different subblocks are independent: $b^\alpha_i b^\alpha_i = 0$ for $\forall \alpha, \beta$ and $L \neq L'$. Moreover, all the qubits in each subblock encounter collective decoherence $\gamma_{i,\alpha}^\alpha = \gamma_{i,\alpha}^\beta = b^\alpha_i$. Thus, $H_{LB} = \sum_i S^\alpha b^\alpha_i$, where $S^\alpha = \sum_i \sigma_i^\alpha B^\alpha_i = \gamma_{i,\alpha}^\alpha b^\alpha_i$. For the qubits in the $Lth$ subblock, due to interaction with the environment and other subblocks the evolution is usually nonunitary. To preserve the coherence of the qubits in the $Lth$ subblock, we wish all the couplings ($H_{LB}$ and $H_{LL'}$) be switched off in the idle mode. However, couplings between the system and the environment are usually unavoidable. This obstacle can be overcome by DFS encoding. The main idea on DFS is that by taking advantage of the symmetry of the interaction between the system and the environment, one may find a special subspace $H^*_D FS$ in the $Lth$ subblock Hilbert space $H^L$ such that

$$\forall \alpha : S^\alpha |\psi^D_i\rangle = e^\alpha |\psi^D_i\rangle, e^\alpha \in R \quad (2)$$

if $|\psi^D_i\rangle \in H^*_D FS$. Thus, for any initially unentangled system-environment state $|\psi^D_i\rangle \otimes |\psi^B_i\rangle, H_{LB} |\psi^D_i\rangle \otimes |\psi^B_i\rangle = |\psi^D_i\rangle \otimes \sum \alpha \gamma_{i,\alpha}^\alpha |\psi^B_i\rangle$, where $|\psi^B_i\rangle$ is the state of the environment. The space $H^*_D FS$ is the so called decoherence-free subspace (DFS) in which $H_{LB}$ equivalently reduces to an environmental operator so that the effect from the environment can be eliminated. In general, DFS can be defined by stabilizers [1]. In our model, since collective operators $S^\alpha$ are Hermitian we may introduce a type of single parameter stabilizer $D^\Gamma = \prod \exp[-\Gamma (S^\alpha - c^\alpha I)]$, which is an identity operator on the DFS state in $H^L$:

$$D^\Gamma |\psi^D_i\rangle = |\psi^D_i\rangle, \text{ iff } |\psi^D_i\rangle \in H^*_D FS \quad (3)$$

where $\Gamma$ is a positive real number. The stabilizer $D^\Gamma$ reduces to the projector $P^L_{D FS}$ of DFS when $\Gamma \to \infty$:

$$P^L_{D FS} = \lim_{\Gamma \to \infty} D^\Gamma. \quad (4)$$

Since switching the couplings between adjacent subblocks complicates the operation and may add additional noise to the system, we would prefer to avoid such physical switching. Recently, there have been QC proposals with always-on couplings [8, 10]. If the couplings between subblocks are fixed and always on, the persistent interactions between adjacent subblocks will distort their states in idle mode. To eliminate this effect the idea of IFS is introduced. Assume that the couplings between adjacent subblocks have highly symmetrical forms: $H_{LL'} = \sum g_{LL'} A^\alpha_{LL'} A^\alpha_{LL'}$, where $A^\alpha_{LL'}$ is the collective operator of the $L(')th$ subblock. We may define the interaction free subspace (IFS) $H^L_{IFS}$ in Hilbert space $H^L$ which satisfies $A^\alpha_{LL'} |\psi^L_i\rangle = \alpha^\alpha |\psi^L_i\rangle$ for any $|\psi^L_i\rangle \in H^L_{IFS}$. Similar to the case of DFS, the interaction Hamiltonian $H^L_{IFS}$ reduces to an operator $\sum g_{LL'} A^\alpha_{LL'} A^\alpha_{LL'}$ acting on the $L(')th$ Hilbert space $H^L$ when the state of the $Lth$ subblock is in IFS. Thus, the stabilizer and projector of IFS of the $Lth$ subblock have the following form: $I^L = \prod_{\alpha} \exp[-\Gamma (A^\alpha_{L'} - \alpha^\alpha I)]$ and $P^L_{IFS} = \lim_{\Gamma \to \infty} I^L$, respectively.

Since DFS and IFS separately keep the system from interacting with the environment and other subblocks, the intersection space of DFS and IFS naturally screen all couplings from them. For the $Lth$ subblock, we define the intersection space of DFS and IFS by $H^L_{IFS}$, whose projector is $P^L_{IFS} = P^L_{IFS} P^L_{IFS}$. The dimension $d_{IFS}$ of the intersection space $H^L_{IFS}$ can be obtained by tracing its projector: $d_{IFS} = Tr I^L$. Usually the space $H^L_{IFS}$ is trivial, $d_{IFS} = 0$. Only for suitably designed systems the intersection space is nontrivial. Clearly, $d_{IFS} \geq 2$ is necessary for encoding quantum information. We find that the coupling Hamiltonian $H_{IFS}$ can have different effects when the states in the $L$th and $L'\th$ subblocks are encoded in different ways. By combining the ideas of DFS and IFS, we provide an architecture of scalable fault-tolerant quantum computer with fixed couplings between subblocks. For a carefully designed QC system we may have $P^L_{IFS} \neq P^L_{IFS}$ and $d_{IFS} \geq 2$. Thus, in idle mode we encode quantum information in the subspace $H^L_{IFS}$. The local fault-tolerant QC in the $Lth$ subblock can be realized by dynamical processes in the subspace $H^L_{IFS} \otimes H^L_{IFS} \otimes H^L_{IFS} \otimes H^L_{IFS} \otimes ... \otimes H^L_{IFS} \otimes H^L_{IFS} \otimes H^L_{IFS} \otimes H^L_{IFS} \otimes ...$, in which the fixed couplings $H_{IFS}$ reduce to constants or local operators. Nonlocal fault-tolerant QC between the $Lth$ and $(L + 1)th$ subblocks can be realized by the dynamical processes in the space $... \otimes H^L_{IFS} \otimes H^L_{IFS} \otimes H^L_{IFS} \otimes H^L_{IFS} \otimes H^L_{IFS} \otimes ...$. To preserve the state within a subspace the controllable Hamiltonian should satisfy the following condition, which follows from theorem 3 in [3].

The necessary and sufficient condition for Hamiltonian $H$ to keep the state at all times entirely within subspace $H_s$ of Hilbert space $H_s$ is $[P, H] = 0$, where $P$ is the projector of the subspace $H_s$.

In the following we will concretely present our architecture based on tunable $XXZ$ exchange interaction.

**SFTQC based on tunable $XXZ$ interaction:** The tunable $XXZ$ type interaction has the following form:

$$H^X_{XXZ} = K_{ij} (\sigma_i^x \sigma_j^x + \sigma_i^y \sigma_j^y) + J_{ij} \sigma_i^z \sigma_j^z. \quad (5)$$

Here, $i$ and $j$ refer to two neighboring qubits. We assume that the parameter $J_{ij}$ is fixed but $K_{ij}$ can be tuned. This model describes the interactions in a few physical systems, such as electrons floating on helium [11] and charge qubits based on superconducting charge boxes [12].
Initially, in idle mode, we adjust the coupling coefficients \( K_{ij} = 0 \), and \( H^{XXZ} \) reduces to the Ising type.

We consider the case of collective phase decoherence: 
\[
H_{L,B} = \sum_{i=1}^{m} \sigma_i^z B_i^2, \\
\text{where } m \text{ is the number of qubits in one subblock. DFS of the } L\text{th subblock is the eigenspace of the operator } S_L = \sum_{i=1}^{m} \sigma_i^z \text{ which has eigenvalues } l = -m, -m + 2, ..., m. \text{ The dimension of DFS with eigenvalue } l \text{ is } d_l = c^{(m-l)/2}. 
\]

We consider subblocks composed of four particles, which are symmetrically arranged on the vertices of a rectangle (see Fig.2). We choose a special DFS with zero eigenvalue of \( S_L \). This is a 6-dimensional space. For two neighboring subblocks \( L \) and \( L + 1 \), the fixed interaction Hamiltonian has the form: 
\[
H_{L,L+1} = J(\sigma_i^z + \sigma_j^z) \otimes (\sigma_i^x + \sigma_j^x). 
\]
Thus, the projectors of different IFSs can be obtained according to the formulation in the above. Furthermore, we may hunt out the maximal intersection space of IFS and DFS, whose projector is 
\[
\sum_{i,j=0}^1 \langle i \langle_j \| \langle i \langle_j \rangle \rangle \rangle, \\
\text{where } \langle \rangle \text{ refers to } 1 - i(j), \langle 0 \rangle \text{ and } \langle 1 \rangle \text{ are the eigenstates of } \sigma_z. \text{ (For simplicity, we will arrange qubits in-order and omit their subscripts in the following context.) This is a 4-dimensional space. we choose its 2-dimensional subspace } \{|001\rangle, |010\rangle \rangle \text{ as the space of logical qubit } \{|0L\rangle, |1L\rangle \}. 
\]

In our scheme for QC, we simply manipulate the interaction in local subblocks by switching on the \( K_{ij} \) coupling between some two qubits \( i \) and \( j \) but fix the couplings between adjacent subblocks. We may prove that 
\[
[H_{L,L+1}, P^{L}_{\text{DFS}}] = 0 \text{ and } [H_{L,L+1}, P^{L}_{\text{DFS}}] = 0. 
\]
This ensures that QC is performed in a fault-tolerant fashion.

To realize universal QC, for the \( L\text{th subblock, we choose a 3-dimensional subspace of DFS with zero eigenvalue of operator } S_L \text{ as the computational space } H_L, \text{ which has the projector } P_L = \sum_{i=0}^2 \langle i \| L \rangle \langle L \| i \rangle \rangle. \text{ Here, } |2L\rangle = |0011\rangle \rangle \text{ is an auxiliary state. When logical qubits are encoded in the intersection space } H_{L-D} \text{ spanned by } \{|0L\rangle, |1L\rangle \} \text{ the effect of nonlocal Hamiltonian coupling with the neighboring subblocks will be eliminated. However, since the state } |2L\rangle \text{ is not in } H_{L-D}, \text{ nonlocal effects can be induced once the quantum states in two neighbor-}
\]

![FIG. 2: A Strategy for concatenating different DDFSs. In the idle mode, the coupling (marked by solid lines) between qubits \( i \) and \( j \) reduces to Ising type interactions (\( K_{ij} = 0 \)). The couplings represented by horizontal and diagonal lines have the same strength \( J \). The couplings represented by vertical lines have the strength \( J' \). Each subblock subject to local collective dephasing environment is surrounded by dotted-dashed lines.](image)

boring subblocks are driven into \(|2L\rangle\rangle. \text{ For simplicity, in the following we represent the effective Hamiltonian in the space } H_L^\perp \text{ by } h_{L}, \text{ and that in the space } H_{L}^\perp \otimes H_{L+1}^\perp \text{ by } h_{L,L+1}. \text{ The operators } X_k^L, Y_k^L, Z_k^L \text{ denote Pauli operators in the subspace spanned by } \{|L_L\rangle, |K_L\rangle \} \rangle (l, k = 0, 1, 2; l \neq k). \]

In idle mode, \([H_{L,L}, P_L] = 0 \text{ therefore the computational space } H_L^\perp \text{ will be driven by the following effective Hamiltonian } h_{L,L}^{id}. \]

\[
h_{L,L}^{id} = \begin{bmatrix} -2J' & 0 & 0 & 0 \\ 0 & -2J' & 0 & 0 \\ 0 & 0 & 2J' - 4J & 0 \\ 0 & 0 & 0 & 2J' - 4J \end{bmatrix}. \quad (6)
\]

When we only adjust \( K_{12} = \mu_L \neq 0, (K_{23} = \nu_L \neq 0) \) the corresponding Hamiltonian \( H_{L}^a \text{ (} H_{L,L}^{a,b} \text{) satisfies } \)

\[
[H_{L,L}^{a,b}, P_L] = 0, \text{ which keeps the space } H_L^\perp \text{ unchanged.} 
\text{Thus, the effective Hamiltonian } h_{L}^{a} \text{ and } h_{L}^{b} \text{ have the following forms:} 
\]

\[
h_{L}^{a} = \begin{bmatrix} -2J' & 2\mu_L & 0 & 0 \\ 2\mu_L & -2J' & 0 & 0 \\ 0 & 0 & 2J' - 4J & 0 \\ 0 & 0 & 0 & 2J' - 4J \end{bmatrix}, \quad (7) 
\]

Here, we assume that \( \mu_L \) and \( \nu_L \) are pre-chosen parameters and the transition from \( h_{L}^{a} \) to \( h_{L}^{b} \) or \( h_{L}^{a} \) can be instantaneously achieved. Therefore, when we adjust the coefficient \( K_{12} \) it is equivalent to performing \( X_1^{(a)} \) operation on the \( L \text{th logical qubit. Logical } Z_2^{(a)} \text{ operation can be implemented by switching on the } K_{23} \text{ coupling. After some time } t = 2\pi/\sqrt{2 + (2\mu_L)^2}, \text{ where } \zeta = 2(J - J'), \text{ the system will undergo an evolution given by: } U_L(\theta) = \exp(-i\theta Z_2^{(a)}). \text{ Here, } \theta = \pi\zeta/\sqrt{2 + (2\mu_L)^2}. \text{ One can properly choose the parameter } \nu_L \text{ to make } \zeta/\sqrt{2 + (2\nu_L)^2} \text{ an irrational number. Then, positive integer powers of } U_L(\theta) \text{ can approach } U_L(\lambda) = \exp(-i\lambda Z_2^{(a)}) \text{ to arbitrary precision, for any real } \lambda. \text{ Since any single qubit gate can be decomposed into rotations around the } z \text{ and } x \text{ axis any fault-tolerant single logical qubit gate can be realized.} 
\text{In the following we will show how to implement nonlocal operations between the } L\text{th and (}L + 1\text{)th subblocks. The representation of the fixed Hamiltonian } H_{L,L+1} \text{ in the space } H_L^\perp \otimes H_{L+1}^\perp \text{ has the effective form:} 
\]

\[
h_{L,L+1} = -4J \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}. \quad (8) 
\]

We find that controlled phase gates can be realized by switching on and off the \( K_{23} \) and \( K_{22'} \) couplings in adjacent subblocks \( L \) and \( L + 1 \). In this process, the whole
effective Hamiltonian $h_{tot}$ is:

$$h_{tot} = h_L + h_{L+1} + h_{LL+1}.$$  \hfill (9)

In the following we show the implementation of controlled phase gate in two cases.

i) $|J|, |J'| \ll |v_L| (= |v_{L+1}|)$. In this case, simultaneously switch on $K_{23}$ and $K_{2'3'}$ couplings, after a suitable time interval $t = \frac{\pi}{4v_L}$ switch them off. The system is driven by strong pulses and the state $|1_L\rangle\langle 1_{L+1}|$ can be flipped to $|2_L\rangle\langle 2_{L+1}|$. In the space $\{|1_L\rangle, |2_L\rangle\} \otimes \{|1_{L+1}\rangle, |2_{L+1}\rangle\}$ the whole idle Hamiltonian is equivalent to $h_{tot}^{id} = h_L^{id} + h_{L+1}^{id} + h_{LL+1} = -5J + (3J - 2J') (Z_L^{12} + Z_{L+1}^{12}) - JZ_L^{12} \otimes Z_{L+1}^{12}$. Let the system evolve for a suitable time, drive it back into IFS by using strong pulses again. A controlled phase gate can be realized by the above 2-qubit interaction $h_{tot}^{id}$ and local operations $\mathbb{I}$. Unfortunately, the condition i) is not realistic in certain physical systems $\mathbb{I}$.\mathbb{I}.

ii) We assume that $v_L$ and $v_{L+1}$ can be slowly changed with time so that Hamiltonian $h_{tot}$ can be adiabatically implemented. To implement the nonlocal operation, we first transform logical state $|1_L\rangle$ to $|2_L\rangle$. Since $h_L^{id}$ and $h_{L+1}^{id}$ separately reduce to noncommuting operators $-2J + (2J - 2J')Z_L^{12}$ and $-2J + (2J - 2J')Z_{L+1}^{12} + 2vL_{L}^{12}$ in the space spanned by $\{|1_L\rangle, |2_L\rangle\}$ this transformation can always be achieved by Hamiltonian $h_L^{id}$ and $h_{L+1}^{id}$. After this, we switch on the $K_{2'3'}$ coupling in the $(L+1)th$ subblock. At that time, Hamiltonian $h_{LL+1}$ starts to work. If we further assume $|J - J'|, |2J - J'| \gg |v_{L+1}(t)|$, the adiabatic condition can be reduced to $|J - J'|^2, |2J - J'|^2 \gg \frac{1}{\delta}[v_{L+1}(t)]$. These instantaneous eigenstates of Hamiltonian $h_{tot}(t) = h_L^{id} + h_{L+1}^{id} + h_{LL+1}$ will slowly change along with the varying $h_{tot}(t)$. Finally, the strength of the $K_{2'3'}$ coupling goes back to its starting value $v_{L+1}(t_f) = v_{L+1}(0) = 0$, and the instantaneous eigenstates return to their initial forms $\mathbb{I}$.\mathbb{I}.

During the adiabatic evolution every logical state will obtain corresponding phases. Since it is easy to prove that the Berry phase is zero during this evolution we need only calculate the dynamical phase $\varphi^d$:

$$\varphi^d = -\int_0^{t_f} \langle \psi(t)| h_{tot}(t)| \psi(t) \rangle \, dt.$$  \hfill (10)

Thus, we can obtain the corresponding phases for all combinations of the states of the two logical qubits:

$$\varphi^d_{00} = 4J_f t_f; \varphi^d_{01} = 4J_f t_f + \eta$$
$$\varphi^d_{20} = 4J_f t_f; \varphi^d_{21} = 4J_f t_f + \kappa$$  \hfill (11)

where $\eta = \int_0^{t_f} (\alpha_{L+1}(t))^2 \, dt, \kappa = \int_0^{t_f} (\alpha_{L+1}(t))^2 \, dt$. This evolution can be represented by a unitary operator $U_{eq}$ in the space $\{|0_L\rangle, |1_L\rangle\} \otimes \{|0_{L+1}\rangle, |1_{L+1}\rangle\}$:

$$U_{eq} = \exp(i \theta Z_L^{12} \otimes Z_{L+1}^{12}),$$  \hfill (12)

Here, $a, b, c$ and $d$ are determined by dynamical phases $\varphi_{mn}(m = 0, 2; n = 0, 1)$ and the coefficient $d = (\kappa - \eta)/4$ is vital for the nonlocal operation. When we appropriately adjust the function $v_{L+1}(t)$ and the time of the adiabatic evolution $t_f$ we can always make $d$ a suitable value. Thus, the adiabatic evolution produces an equivalent nonlocal unitary transformation in the logical 2-qubit space: $U_{eq} = \exp(i \theta Z_L^{12} \otimes Z_{L+1}^{12})$.

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[1] P. Zanardi and M. Rasetti, Phys. Rev. Lett. 79, 3306 (1997); L.-M. Duan and G.-C. Guo, ibid. 79, 1953 (1997).
[2] E. Knill et al., Phys. Rev. Lett. 84, 2525 (2000).
[3] D.A. Lidar et al., Phys. Rev. Lett. 81, 2594 (1998).
[4] D. Bacon et al., Phys. Rev. Lett. 85, 1758 (2000).
[5] J. Kempe et al., Phys. Rev. A 63, 042307 (2001).
[6] D. Kielpinski et al., Nature 417, 709 (2002).
[7] D.A. Lidar and L.-A. Wu, Phys. Rev. Lett. 88, 017905 (2002); L.-A. Wu and D.A. Lidar, ibid. 88, 207902 (2002).
[8] X. Zhou et al., Phys. Rev. Lett. 89, 197903 (2002).
[9] J.S. Waugh et al., Phys. Rev. Lett. 20, 180 (1968).
[10] S.C. Benjamin and S. Bose, Phys. Rev. Lett. 90, 247901 (2003).
[11] P.M. Platzman and M.I. Dykman, Science 284, 1967 (1999).
[12] X. Zhou et al., Phys. Rev. A 69, 030301(R) (2004).
[13] J. Preskill, Lecture Notes for Physics 229: Quantum Information and Computation, Caltech, 1998.
[14] M.A. Nielsen and I.L. Chuang, Quantum Computation and Quantum Information, Cambridge University Press, 2000.
[15] W. Dittrich and M. Reuter, Classical and Quantum Dynamics, Springer-Verlag, 1994.
[16] Y. Makhlin et al., Rev. Mod. Phys. 73, 357 (2001).