Semiample invertible sheaves with semipositive continuous hermitian metrics

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Let $(L, h)$ be a pair of a semiample invertible sheaf and a semipositive continuous hermitian metric on a proper algebraic variety over $\mathbb{C}$. In this paper, we prove that $(L, h)$ is semiample metrized, answering a generalization of a question of S. Zhang.

Introduction

Let $X$ be a proper algebraic variety over $\mathbb{C}$. Let $L$ be an invertible sheaf on $X$, and let $h$ be a continuous hermitian metric of $L$. We say that $(L, h)$ is semiample metrized if, for any $\epsilon > 0$, there is $n > 0$ such that, for any $x \in X(\mathbb{C})$, we can find $l \in H^0(X, L^{\otimes n}) \setminus \{0\}$ with

$$\sup \{ h^{\otimes n}(l, l)(w) \mid w \in X(\mathbb{C}) \} \leq e^{\epsilon n} h^{\otimes n}(l, l)(x).$$

Shouwu Zhang proposed the following question:

**Question 0.1 [Zhang 1995, Question 3.6]**. If $L$ is ample and $h$ is smooth and semipositive, does it follow that $(L, h)$ is semiample metrized?

Theorem 3.5 of the same reference gives an affirmative answer in the case where $X$ is smooth over $\mathbb{C}$. The purpose of this paper is to give an answer for a generalization of the above question. First of all, we fix some notation: We say that $L$ is semiample if there is a positive integer $n_0$ such that $L^{\otimes n_0}$ is generated by global sections. Moreover, $h$ is said to be semipositive (or we say that $(L, h)$ is semipositive) if, for any point $x \in X(\mathbb{C})$ and a local basis $s$ of $L$ on a neighborhood of $x$, $-\log h(s, s)$ is plurisubharmonic around $x$ (for the definition of plurisubharmonicity on a singular variety, see Section 1). Note that $h$ is not necessarily smooth. By using the recent work of Coman, Guedj and Zeriahi [Coman et al. 2013], we have the following answer:

**Theorem 0.2.** If $L$ is semiample and $h$ is continuous and semipositive, then $(L, h)$ is semiample metrized.

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1. Plurisubharmonic functions on singular complex analytic spaces

Let $T$ be a reduced complex analytic space. An upper-semicontinuous function
\[ \varphi : T \to \mathbb{R} \cup \{-\infty\} \]
is said to be plurisubharmonic if $\varphi \neq -\infty$ and, for each $x \in T$, there is an analytic closed embedding $\iota_x : U_x \hookrightarrow W_x$ of an open neighborhood $U_x$ of $x$ into an open set $W_x$ of $\mathbb{C}^{n_x}$ together with a plurisubharmonic function $\Phi_x$ on $W_x$ such that $\varphi|_{U_x} = \iota_x^*(\Phi_x)$. For an analytic map $f : T' \to T$ of reduced complex analytic spaces and a plurisubharmonic function $\varphi$ on $T$, it is easy to see that $\varphi \circ f$ is either identically $-\infty$ or plurisubharmonic on $T'$. By [Fornæss and Narasimhan 1980, Theorem 5.3.1], an upper-semicontinuous function $\varphi : T \to \mathbb{R} \cup \{-\infty\}$ is plurisubharmonic if and only if, for any analytic map $\varphi : \mathbb{D} \to T$, $\varphi \circ \varphi$ is either identically $-\infty$ or subharmonic on $\mathbb{D}$, where $\mathbb{D} := \{ z \in \mathbb{C} \mid |z| < 1 \}$. Moreover, if $T$ is compact and $\varphi$ is plurisubharmonic on $T$, then $\varphi$ is locally constant.

Let $\omega$ be a smooth $(1, 1)$-form on $T$, that is, in the same way as in the definition of plurisubharmonic functions, $\omega$ is a smooth $(1, 1)$-form on the regular part of $T$ and, for each $x \in T$, there is an analytic closed embedding $\iota_x : U_x \hookrightarrow W_x$ of an open neighborhood $U_x$ of $x$ into an open set $W_x$ of $\mathbb{C}^{n_x}$ together with a smooth $(1, 1)$-form $\Omega_x$ on $W_x$ such that $\omega|_{U_x} = \iota_x^*(\Omega_x)$. We assume that $\omega$ is locally given by $dd^c(u)$ for some smooth function $u$ on a neighborhood of $x$. Let $\phi$ be a quasiplersubharmonic function on $T$; that is, for each $x \in T$, $\phi$ can be locally written as the sum of a smooth function and a plurisubharmonic function around $x$. We say that $\phi$ is $\omega$-plurisubharmonic if there is an open covering $T = \bigcup_{\lambda} U_{\lambda}$, together with a smooth function $u_{\lambda}$ on $U_{\lambda}$ for each $\lambda$, such that $\omega|_{U_{\lambda}} = dd^c(u_{\lambda})$ and $\phi|_{U_{\lambda}} + u_{\lambda}$ is plurisubharmonic on $U_{\lambda}$. The condition for $\omega$-plurisubharmonicity is often denoted by $dd^c(\phi) + \omega \geq 0$.

Here we consider the following lemma:

**Lemma 1.1.** Let $f : X \to Y$ be a surjective and proper morphism of algebraic varieties over $\mathbb{C}$. Let $\varphi$ be a real-valued function on $Y(\mathbb{C})$.

1. $\varphi$ is continuous if and only if $\varphi \circ f$ is continuous.

2. Assume that $\varphi$ is continuous. Then $\varphi$ is plurisubharmonic if and only if $\varphi \circ f$ is plurisubharmonic.

**Proof.** (1) It is sufficient to see that if $\varphi \circ f$ is continuous, then $\varphi$ is continuous. Otherwise, there are $y \in Y(\mathbb{C})$, $\epsilon_0 > 0$ and a sequence $\{y_n\}$ on $Y(\mathbb{C})$ such that $\lim_{n \to \infty} y_n = y$ and $|\varphi(y_n) - \varphi(y)| \geq \epsilon_0$ for all $n$. We choose $x_n \in X(\mathbb{C})$ such that $f(x_n) = y_n$. As $f : X \to Y$ is proper, we can find a subsequence $\{x_{n_i}\}$ of $\{x_n\}$ such that $x := \lim_{i \to \infty} x_{n_i}$ exists in $X(\mathbb{C})$. Note that
\[ f(x) = \lim_{i \to \infty} f(x_{n_i}) = \lim_{i \to \infty} y_{n_i} = y, \]
so that, as $\varphi \circ f$ is continuous,
\[
\varphi(y) = (\varphi \circ f)(x) = \lim_{i \to \infty} (\varphi \circ f)(x_{n_i}) = \lim_{i \to \infty} \varphi(f(x_{n_i})) = \lim_{i \to \infty} \varphi(y_{n_i}),
\]
which is a contradiction, so that $\varphi$ is continuous.

(2) We need to check that if $\varphi \circ f$ is plurisubharmonic, then $\varphi$ is plurisubharmonic. By using Chow’s lemma, we may assume that $f : X \to Y$ is projective. Moreover, since the assertion is local with respect to $Y$, we may further assume that there is a closed embedding $\iota : X \hookrightarrow Y \times \mathbb{P}^N$ such that $p \circ \iota = f$, where $p : Y \times \mathbb{P}^N \to Y$ is the projection to the first factor. The remaining proof is same as the last part of the proof of [Demailly 1985, Theorem 1.7]. Let $g : (\mathbb{D}, 0) \to (Y, y)$ be a germ of an analytic map. By the theorem of Fornæss and Narasimhan, it is sufficient to show that $\varphi \circ g$ is subharmonic. Clearly we may assume that $g$ is given by the normalization of a 1-dimensional irreducible germ $(C, y)$ in $(Y, y)$. Using hyperplanes in $\mathbb{P}^N$, we can find $x \in X$ and a 1-dimensional irreducible germ $(C', x)$ in $(X, x)$ such that $(C', x)$ lies over $(C, y)$. Let $g' : (\mathbb{D}, 0) \to (X, x)$ be the germ of an analytic map given by the normalization of $(C', x)$. Then we have an analytic map $\sigma : (\mathbb{D}, 0) \to (\mathbb{D}, 0)$ with $g \circ \sigma = f \circ g'$:
\[
(\mathbb{D}, 0) \xrightarrow{g'} (X, x) \xrightarrow{f} (Y, y)
\]
Changing a variable of $(\mathbb{D}, 0)$, we may assume that $\sigma$ is given by $\sigma(z) = z^m$ for some positive integer $m$. Then $\varphi \circ g \circ \sigma$ is subharmonic because $\varphi \circ f$ is plurisubharmonic. Therefore, as $\sigma$ is étale over the outside of $0$, $\varphi \circ g$ is subharmonic on the outside of $0$, and hence $\varphi \circ g$ is subharmonic on $(\mathbb{D}, 0)$ by the removability of singularities of subharmonic functions. □

2. Descent of a semipositive continuous hermitian metric

Here, we consider a descent problem of a semipositive continuous hermitian metric.

**Theorem 2.1.** Let $f : X \to Y$ be a surjective and proper morphism of algebraic varieties over $\mathbb{C}$ with $f_*\mathcal{O}_X = \mathcal{O}_Y$. Let $L$ be an invertible sheaf on $Y$. If $h'$ is a semipositive continuous hermitian metric of $f^*(L)$, then there is a semipositive continuous hermitian metric $h$ of $L$ such that $h' = f^*(h)$.

**Proof.** Let $h_0$ be a continuous hermitian metric of $L$ on $Y$. There is a continuous function $\phi$ on $X(\mathbb{C})$ such that $h' = \exp(\phi) f^*(h_0)$. Let $F$ be a subvariety of $X$ such that $F$ is an irreducible component of a fiber of $f : X \to Y$. Then, as
\[
(f^*(L), h')|_F \simeq (\mathcal{O}_F, \exp(\phi|_F)),
\]
we can see that $-\phi|_F$ is plurisubharmonic, so that $\phi|_F$ is constant. Therefore, for any point $y \in Y(\mathbb{C})$, $\phi|_{\mu^{-1}(y)}$ is constant because $\mu^{-1}(y)$ is connected, and hence there is a function $\psi$ on $Y(\mathbb{C})$ such that $\psi \circ f = \phi$. By Lemma 1.1(1), $\psi$ is continuous, so that, if we set $h := \exp(\psi)h_0$, then $h$ is continuous on $Y(\mathbb{C})$ and $h' = f^*(h)$.

Finally, let us see that $h$ is semipositive. As this is a local question on $Y$, we may assume that there is a local basis $s$ of $L$ over $Y$. If we set $\varphi = -\log h(s, s)$, then $\varphi \circ f$ is plurisubharmonic because $h'$ is semipositive. Therefore, by Lemma 1.1(2), $\varphi$ is plurisubharmonic, as required.

\[\square\]

3. The proof of Theorem 0.2

In the case where $X$ is smooth over $\mathbb{C}$, $L$ is ample and $h$ is smooth, this theorem was proved by Zhang [1995, Theorem 3.5]. First we assume that $L$ is ample. Then there are a positive integer $n_0$ and a closed embedding $X \hookrightarrow \mathbb{P}^N$ such that $\mathcal{O}_{\mathbb{P}^N}(1)|_X \cong L^\otimes n_0$. Let $h_{FS}$ be the Fubini–Study metric of $\mathcal{O}_{\mathbb{P}^N}(1)$. Let $\phi$ be the continuous function on $X(\mathbb{C})$ given by $h^\otimes n_0 = \exp(-\phi)h_{FS}|_X$. We set $\omega = c_1(\mathcal{O}_{\mathbb{P}^N}(1), h_{FS})$. Then $\phi$ is $(\omega|_X)$-plurisubharmonic. Therefore, by [Coman et al. 2013, Corollary C], there is a sequence $\{\varphi_i\}$ of smooth functions on $\mathbb{P}^N(\mathbb{C})$ with the following properties:

1. $\varphi_i$ is $\omega$-plurisubharmonic for all $i$.
2. $\varphi_i \geq \varphi_{i+1}$ for all $i$.
3. For $x \in X(\mathbb{C})$, $\lim_{i \to \infty} \varphi_i(x) = \phi(x)$.

Since $X$ is compact and $\phi$ is continuous, (3) implies that the sequence $\{\varphi_i\}$ converges to $\phi$ uniformly on $X(\mathbb{C})$. We choose $i$ such that $|\phi(x) - \varphi_i(x)| \leq \epsilon n_0/2$ for all $x \in X$. We set $h_i = \exp(-\varphi_i)h_{FS}$. Then $h_i$ is a semipositive smooth hermitian metric of $\mathcal{O}_{\mathbb{P}^N}(1)$. Therefore, there is a positive integer $n_1$ such that, for $x \in \mathbb{P}^N(\mathbb{C})$, we can find $l \in H^0(\mathbb{P}^N, \mathcal{O}_{\mathbb{P}^N}(n_1)) \setminus \{0\}$ with

$$\sup\{h_i^\otimes n_1(l, l)(w) \mid w \in \mathbb{P}^N(\mathbb{C})\} \leq c_{n_1}(\epsilon n_0/2)h_i^\otimes n_1(l, l)(x).$$

In particular, if $x \in X(\mathbb{C})$, then $l(x) \neq 0$ (so that $l|_X \neq 0$) and

$$\sup\{h_i^\otimes n_1(l, l)(w) \mid w \in X(\mathbb{C})\} \leq e^{\epsilon n_0/2}h_i^\otimes n_1(l, l)(x).$$

Note that

$$h_i^\otimes n_0 e^{-\epsilon n_0/2} \leq h_i \leq h_i^\otimes n_0$$

(3-1)

on $X(\mathbb{C})$, because $h_i = h_i^\otimes n_0 \exp(\phi - \varphi_i)$ and $-\epsilon n_0/2 \leq \phi - \varphi_i \leq 0$ on $X(\mathbb{C})$. Therefore,

$$\sup\{h_i^\otimes n_0 n_1(l, l)(w) \mid w \in X(\mathbb{C})\} e^{-n_0 n_1 \epsilon/2} \leq \sup\{h_i^\otimes n_1(l, l)(w) \mid w \in X(\mathbb{C})\}$$
and
\[ h_1^{\otimes n_1}(l, l)(x) \leq h^{\otimes n_0 n_1}(l, l)(x), \]
and hence
\[ \sup\{h^{\otimes n_0 n_1}(l, l)(w) \mid w \in X(\mathbb{C})\} \leq e^{n_1 n_0 \epsilon} h^{\otimes n_0 n_1}(l, l)(x), \]
as required.

In general, as \( L \) is semiample, there are a positive integer \( n_2 \), a projective algebraic variety \( Y \) over \( \mathbb{C} \), a morphism \( f : X \to Y \) and an ample invertible sheaf \( A \) on \( Y \) such that \( f_* \mathcal{O}_X = \mathcal{O}_Y \) and \( f^*(A) \simeq L^{\otimes n_2} \). Thus, by Theorem 2.1, there is a semipositive continuous hermitian metric \( k \) of \( A \) such that \((f^*(A), f^*(k)) \simeq (L^{\otimes n_2}, h^{\otimes n_2})\). Therefore, the assertion of the theorem follows from the previous observation.

4. A variant of Theorem 0.2

The following theorem is a consequence of Theorem 0.2 together with the arguments in [Zhang 1995, Theorem 3.3]. However, we can give a direct proof using ideas in the proof of Theorem 0.2.

**Theorem 4.1.** Let \( X \) be a projective algebraic variety over \( \mathbb{C} \). Let \( L \) be an ample invertible sheaf on \( X \) and let \( h \) be a semipositive continuous hermitian metric of \( L \). Let us fix a reduced subscheme \( Y \) of \( X \), \( l \in H^0(Y, L|_Y) \) and a positive number \( \epsilon \). Then, for the given \( X, L, h, Y, l \) and \( \epsilon \), there is a positive integer \( n_1 \) such that, for all \( n \geq n_1 \), we can find \( l' \in H^0(X, L^{\otimes n}) \) with \( l'|_Y = l^{\otimes n} \) and
\[ \sup\{h^{\otimes n}(l', l')(w) \mid w \in X(\mathbb{C})\} \leq e^{n \epsilon} \sup\{h(l, l)(w) \mid w \in Y(\mathbb{C})\} \]

**Proof.** In the case where \( X \) is smooth over \( \mathbb{C} \) and \( h \) is smooth and positive, the assertion of the theorem follows from [Zhang 1995, Theorem 2.2], in which \( Y \) is actually assumed to be a subvariety of \( X \). However, the proof works well under the assumption that \( Y \) is a reduced subscheme. First of all, let us see the theorem in the case where \( X \) is smooth over \( \mathbb{C} \) and \( h \) is smooth and semipositive. As \( L \) is ample, there is a positive smooth hermitian metric \( t \) of \( L \) with \( t \leq h \). Let us choose a positive integer \( m \) such that \( e^{-\epsilon/2} \leq (t/h)^{1/m} \leq 1 \) on \( X(\mathbb{C}) \). If we set \( t_m = h^{1-1/m} t^{1/m} \), then \( t_m \) is smooth and positive, so that, for a sufficiently large integer \( n \), there is \( l' \in H^0(X, L^{\otimes n}) \) such that \( l'|_Y = l^{\otimes n} \) and
\[ \sup\{t_m^{\otimes n}(l', l')(w) \mid w \in X(\mathbb{C})\} \leq e^{n \epsilon/2} \sup\{t_m(l, l)(w) \mid w \in Y(\mathbb{C})\} \]
and hence the assertion follows because \( e^{-\epsilon/2} h \leq t_m \leq h \) on \( X(\mathbb{C}) \).

For a general case, we use the same symbols \( n_0 \), \( X \hookrightarrow \mathbb{P}^N \), \( h_{FS} \), \( \phi \), \( \omega \) and \( \{\varphi_i\} \) as in the proof of Theorem 0.2. Clearly we may assume that \( l \neq 0 \). Since \( L \) is ample, if \( a_0 \) is a sufficiently large integer, then, for each \( j = 0, \ldots, n_0 - 1 \), there is
$l_j \in H^0(X, L^\otimes n_{\alpha_0} + j)$ with $l_j|_Y = l^\otimes n_{\alpha_0} + j$. Let us fix a positive number $A$ such that

$$\sup\{h^\otimes n_{\alpha_0} + j(l_j, l_j)(w) | w \in X(\mathbb{C})\} \leq e^A \sup\{h(l, l)(w) | w \in Y(\mathbb{C})\}^{n_{\alpha_0} + j} \quad (4-1)$$

for $j = 0, \ldots, n_0 - 1$. We choose $i$ with $|\phi(x) - \varphi_i(x)| \leq \epsilon n_0 / 2$ for all $x \in X$, and we set $h_i = \exp(-\varphi_i)h_{FS}$. As $h_i$ is smooth and semipositive, for the given $p^n, \mathcal{O}_{p^n}(1)$, $h_i$, $Y, l^\otimes n_0$ (as an element of $H^0(Y, \mathcal{O}_{p^n}(1)|_Y)$) and $n_0 \epsilon / 4$, there is a positive integer $a_1$ such that the assertion of the theorem holds for all $a \geq a_1$. We put

$$n_1 := n_0 \max\left\{a_1 + a_0 + 1, \frac{4A}{n_0 \epsilon} - 3a_0 + 1\right\}.$$ 

Let $n$ be an integer with $n \geq n_1$. If we set $n = n_0(a + a_0) + j$ ($0 \leq j \leq n_0 - 1$), then

$$a \geq a_1 \quad \text{and} \quad a \geq \frac{4A}{n_0 \epsilon} - 4a_0,$$

so that we can find $l'' \in H^0(p^n, \mathcal{O}_{p^n}(a))$ with $l''|_Y = l^\otimes n_{\alpha_0}$ and

$$\sup\{h_i^\otimes a(l'', l'')(w) | w \in p^n(\mathbb{C})\} \leq e^{a(n_{\alpha_0} / 4)} \sup\{h_i(l^\otimes n_0, l^\otimes n_0)(w) | w \in Y(\mathbb{C})\}^a,$$ 

which implies that

$$\sup\{h^\otimes n_{\alpha_0}(l'', l'')(w) | w \in X(\mathbb{C})\} \leq e^{(3/4)n_{\alpha_0} \epsilon} \sup\{h(l, l)(w) | w \in Y(\mathbb{C})\}^{n_{\alpha_0}}, \quad (4-2)$$

because of (3-1). Here we set $l' = l'' \otimes l_j$. Then, $l'|_Y = l^\otimes n$ and, using (4-1) and (4-2), we have

$$\sup\{h^\otimes n(l', l')(w) | w \in X(\mathbb{C})\} \leq \sup\{h^\otimes n_{\alpha_0}(l'', l'')(w) | w \in X(\mathbb{C})\} \sup\{h^\otimes n_{\alpha_0} + j(l_j, l_j)(w) | w \in X(\mathbb{C})\}$$

$$\leq e^{(3/4)n_{\alpha_0} \epsilon + A} \sup\{h(l, l)(w) | w \in Y(\mathbb{C})\}^{n},$$

which implies the assertion because $(3/4)n_{\alpha_0} \epsilon + A \leq \epsilon n$. \hfill $\square$

## 5. Arithmetic application

As an application of Theorem 0.2, we have the following generalization of the arithmetic Nakai–Moishezon criterion (see [Zhang 1995, Corollary 4.8]).

**Corollary 5.1.** Let $\mathcal{X}$ be a projective and flat integral scheme over $\mathbb{Z}$. Let $\mathcal{L}$ be an invertible sheaf on $\mathcal{X}$ such that $\mathcal{L}$ is nef on every fiber of $\mathcal{X} \to \mathbb{Z}$. Let $h$ be an $F_{\infty}$-invariant semipositive continuous hermitian metric of $\mathcal{L}$, where $F_{\infty}$ is the complex conjugation map $\mathcal{X}(\mathbb{C}) \to \mathcal{X}(\mathbb{C})$. If $\widetilde{\deg}(\hat{c}_1((\mathcal{L}, h)|_{\mathcal{Y}})^{\dim(\mathcal{Y})}) > 0$ for all horizontal integral subschemes $\mathcal{Y}$ of $\mathcal{X}$, then, for an $F_{\infty}$-invariant continuous hermitian invertible sheaf $(\mathcal{M}, k)$ on $\mathcal{X}$, $H^0(\mathcal{X}, \mathcal{L}^\otimes n \otimes \mathcal{M})$ has a basis consisting of strictly small sections for a sufficiently large integer $n$. 

Proof. Let $X$ be the generic fiber of $\mathcal{X} \rightarrow \text{Spec}(\mathbb{Z})$ and let $Y$ be a subvariety of $X$. Let $\mathcal{Y}$ be the Zariski closure of $Y$ in $\mathcal{X}$. As

$$\hat{\deg}(\hat{c}_1(\mathcal{L}, h)|_{\mathcal{Y}})^{\text{dim}(\mathcal{Y})} > 0,$$

$(\mathcal{L}, h)|_{\mathcal{Y}}$ is big by [Moriwaki 2012, Theorem 6.6.1], so that $H^0(\mathcal{Y}, \mathcal{L}^{\otimes n_0}|_{\mathcal{Y}}) \setminus \{0\}$ has a strictly small section for a sufficiently large integer $n_0$. Moreover, if we set $L = \mathcal{L}|_{X}$, then $L|_{Y}$ is big, and hence $\deg(L^{\text{dim} Y} \cdot Y) > 0$ because $L$ is nef. Therefore, $L$ is ample by the Nakai–Moishezon criterion for ampleness. In particular, by Theorem 0.2, $h$ is semiample metrized. Thus the assertion follows from the arguments in [Zhang 1995, Theorem 4.2].

\[\square\]

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