We study the consequences which the presence of an elementary electric charge in the \( \mu^\pm, \pi^\pm \) and \( K^\pm \) mesons has for the rest mass of \( \mu^\pm, \pi^\pm \) and \( K^\pm \). The addition of the electric charges \( e^\pm \) to the massive neutral bodies of these particles does not increase the energy in the rest mass of the muon and pion, but rather decreases their energy by the binding energy of the electric charge to the neutral bodies of the muon and pion. The addition of a charge to the neutral neutrino lattices of the muon or pion changes the simple cubic lattices of the neutral particles to face-centered cubic lattices of the charged particles, which is essential for the stability of the particles.

Key words: electron, muon, pion, particle masses, lattice theory.

Introduction

The elementary electric charge \( e^\pm \) is bound to the neutral bodies which make up most of the muon, pion and \( K^\pm \) masses. The mass of the electron or positron is but a very small part of the mass of \( \mu^\pm, \pi^\pm \) or \( K^\pm \), \( m(\mu^\pm) \cong 207 \, m(e^\pm) \), \( m(\pi^\pm) \cong 273 \, m(e^\pm) \) and \( m(K^\pm) \cong 966 \, m(e^\pm) \). It has to be explained how the electric charges \( e^\pm \) affect the mass of the muons, pions and K mesons. We have described in [1] an explanation of the neutral bodies of the muons and pions as consisting of simple cubic lattices made up by muon neutrinos, anti-muon neutrinos, electron neutrinos and anti-electron neutrinos and their oscillations. In Section 11 in [1] we have also explained the rest masses of the electron or positron as consisting of an electron neutrino.
or anti-electron neutrino lattice and of electric oscillations. The accurately known ratios \( m(\mu^\pm)/m(e^\pm) \) and \( m(\pi^\pm)/m(e^\pm) \) seem to be the most sensitive gauge for the influence of the charge on the mass of the particles. Any valid model of the elementary particles must, in the end, be able to determine the ratios of the particle masses to the electron mass. In particular the one hundred years old problem of why is the mass of the proton 1836 times larger than the mass of the electron has to be solved.

The ratios \( m(\mu^\pm)/m(e^\pm) \) and \( m(\pi^\pm)/m(e^\pm) \) were first discussed by Nambu [2] who noted that \( m(\mu^\pm) \) is approximately \( 3/2\alpha_f \cdot m(e^\pm) \) and \( m(\pi^\pm) \) is approximately \( 2/\alpha_f \cdot m(e^\pm) \), where \( \alpha_f \) is the fine structure constant. A more accurate relation between \( m(\mu^\pm) \) and \( m(e^\pm) \) was later suggested by Barut [3]. We have shown in Section 12 of [1] that accurate values of the ratios \( m(\mu^\pm)/m(e^\pm) \) and \( m(\pi^\pm)/m(e^\pm) \) can be obtained from our explanation of the masses of the muons, pions and electron. Even when the consequences of the charge of \( \mu^\pm \) and \( \pi^\pm \) were neglected we arrived at values for the mass ratios which differed from the measured \( m(\mu^\pm)/m(e^\pm) \) by only 0.38\% and from the measured \( m(\pi^\pm)/m(e^\pm) \) by 1.08\%. We will now improve our calculations of the ratios \( m(\mu^\pm)/m(e^\pm) \) and \( m(\pi^\pm)/m(e^\pm) \) by taking also into account the consequences of the charge on the mass of the particles. The consequences of the addition of an electric charge \( e^\pm \) to the neutrino body of the K mesons can be expected to be much smaller than in the case of \( \mu^\pm \) and \( \pi^\pm \) because of the much larger mass of the K mesons.

1 The charge in the \( \mu^\pm \) mesons

In [1] we have shown that the masses of the muons and pions without charge can be explained, within 1\% accuracy, by the mass in the oscillation energies and the sum of the rest masses of electron neutrinos and muon neutrinos and their antiparticles in simple cubic lattices. Similarly the electron mass can be explained by the sum of the rest masses of electron neutrinos in a cubic lattice plus the mass in the sum of the energy of electric oscillations. Familiarity with the essence of our model of the particles will be helpful in the following. The oscillation energy of the neutrinos in the neutral cubic
The subscipt $n$ refers to neutral. Although the oscillations of a neutrino lattice are neutral, we must retain the $\pm$ superscript of $\mu$ and $\pi$ in Eq.(1) because the neutrino lattices of e.g. the $\mu^+$ and $\mu^-$ mesons are not the same, but they have the same oscillation energy. The number of all neutrinos in the neutrino lattice of the $\pi^\pm$ mesons without charge, i.e. of $\pi_n^\pm$, is

$$N = 2.854 \cdot 10^9,$$

Eq.(17) in [1]. In Eq.(1) $h$ is Planck’s constant, the frequency $\nu_0$ is given by $\nu_0 = c/2\pi a$, where $c$ is the velocity of light and $a$ is the lattice constant, and $\phi = 2\pi a/\lambda$. One half of the energy in $\pi_n^\pm$ is in the sum of the oscillation energies of the $N$ neutrinos of the lattice, the neutrino masses make up the other half of the energy in the rest mass of $\pi_n^\pm$.

The neutral neutrino lattices of the $\mu^\pm$, $\pi^\pm$ and $K^\pm$ mesons seem to exist only in conjunction with an electric charge $e^\pm$. As noted on p.16 of [1], a simple cubic lattice held together by a central force between the particles of the lattice is not stable according to Born [4]. A cubic lattice consisting only of neutrinos falls into this category. The charge added to the neutrino lattices of $\mu^\pm$, $\pi^\pm$, $K^\pm$ makes their neutrino lattices “stable”, because the neutrino lattices are then no longer simple cubic, but face-centered cubic, as we will see soon. *Face-centered cubic* lattices are stable according to Born [4]. The face-centered cubic neutrino lattices of $\mu^\pm$ and $\pi^\pm$ have a lifetime on the order of $10^{-6}$ respectively $10^{-8}$ seconds, which means that there is, e.g. in the $\mu^\pm$ mesons, a sequence of $O(10^{20})$ oscillations before they decay. As we observed already in [1], there are startling differences in the lifetimes of the charged particles as compared to the lifetimes of the corresponding neutral particles. For example, the lifetimes of the charged $\pi$ mesons is $O(10^8)$ times longer than the lifetime of the neutral $\pi^0$ meson. And the lifetime of the neutron is 885.7 sec, whereas the lifetime of its charged counterpart, the proton, is infinite.

In the $\mu^\pm$ mesons an electric charge $e^\pm$ is bound to the massive neutral body of the muons, which is about 205.768 m($e^\pm$). The body of the muons contains the remains of the body of the pions from which the $\mu^\pm$ mesons
emerge in the decay $\pi^\pm \to \mu^\pm + \nu_\mu(\bar{\nu}_\mu)$. In a good approximation the body of the muons is $3/4$ of the body of the pions, $m(\mu^\pm) = 1.00937 \cdot 3/4 \cdot m(\pi^\pm)$. The neutral body of the muons consists of $3/4 \cdot N$ neutrinos, where $N = 2.854 \cdot 10^9$ is the number of all neutrinos in the neutral lattice of the $\pi^\pm$ mesons, Eq.(2). The composition of the $\mu^\pm$ mesons is given by Eq.(66) in [1]. This equation does not consider the consequences of the charge on the mass of $\mu^\pm$, it reads

$$m(\mu_n^\pm) = E_\nu(\mu_n^\pm)/c^2 + N/4 \cdot m(\nu_\mu) + N/4 \cdot m(\nu_e) + N/4 \cdot m(\bar{\nu}_e), \quad (3)$$

or correspondingly with the antiparticles. $m(\mu_n^\pm)$ is the mass of the muons without considering the charge. $m(\nu_\mu) = 49.91$ milli-eV/c$^2$ is the mass of the muon neutrino, and $m(\nu_e) = 0.365$ milli-eV/c$^2$ is the mass of the electron neutrino, Eqs.(67,70) in [1], both can be exchanged by their antiparticles. Actually, conservation of neutrino numbers requires that $N/8$ muon neutrinos and $N/8$ anti-muon neutrinos are in the $\mu_n^\pm$ lattice after the decay of $\pi^\pm$. But since $m(\nu_\mu) = m(\bar{\nu}_\mu)$ we use, for the sake of simplicity, the term $N/4 \cdot m(\nu_\mu)$ or $N/4 \cdot m(\bar{\nu}_\mu)$ for the sum $N/8 \cdot [m(\nu_\mu) + m(\bar{\nu}_\mu)]$. Eq.(3) means that the rest mass of the muon $m(\mu_n^\pm) = 105.658$ MeV/c$^2$ is equal to the oscillation energy $E_\nu(\mu_n^\pm)/c^2$ of the neutrinos in the lattice of the muons plus the sum of the rest masses of the neutrinos in this lattice.

The oscillation energy in the neutral muon lattice is the same as the oscillation energy in the neutral pion lattice, as follows from the measured masses of the muons and pions. According to Eq.(63) in [1] we have

$$E_\nu(\pi_n^\pm) = E_\nu(\mu_n^\pm). \quad (4)$$

The formula for the oscillation energy of a neutral neutrino lattice, Eq.(1), applies to the neutrino lattice of the muons only if $N$ neutrinos are in $\mu_n^\pm$, not the $3/4 \cdot N$ neutrinos one expects after either $N/4$ muon neutrinos or $N/4$ anti-muon neutrinos have been removed from the lattice of the pions in the $\pi^\pm$ decay. This is expressed by Eq.(64) in [1] which, applied to the neutral lattices, reads

$$m(\pi_n^\pm) - m(\mu_n^\pm) = N/4 \cdot m(\nu_\mu) = N/4 \cdot m(\bar{\nu}_\mu). \quad (5)$$

According to our explanation of the electron in Section 11 of [1] the electric charge $e^\pm$ carries $N/4$ electron neutrinos respectively anti-electron neutrinos. Conservation of neutrino numbers requires that these $N/4$ neutrinos must be added to the $3/4 \cdot N$ neutrinos in the neutrino lattice of the neutral muons,
when a charge $e^\pm$ is added to the lattice of $\mu_n^\pm$. There are then in total $N$ neutrinos in the lattice of the charged muon. It is likely that the additional electron neutrinos or anti-electron neutrinos brought by $e^\pm$ into the cell-sides of $\mu_n^\pm$ occupy the places in the lattice vacated by the muon neutrinos or anti-muon neutrinos in the decay of $\pi^\pm$. The neutrinos in the cells of $\mu^\pm$ would then have a cubic configuration. Additionally $N/4$ positive or negative charge elements, or the electric oscillations introduced by $e^\pm$, must be in the centers of the sides of the cubic neutrino cells. The charge elements in the centers of the cell-sides of the $\pi^\pm$ lattice do apparently not change their position during the conversion from the $\pi^\pm$ lattice to the $\mu^\pm$ lattice in

$$\pi^\pm \rightarrow \mu^\pm + \nu_\mu(\bar{\nu}_\mu).$$

The cells in the $\mu^\pm$ lattice are therefore face-centered cubic.

The energy in the sum of the $N$ neutrino rest masses in the charged neutrino lattice of the muons is

$$\Sigma m(\text{neutrinos})(\mu^\pm)c^2 = N/4 \cdot m(\nu_\mu)c^2 + 3/4 \cdot N m(\nu_e)c^2,$$

with

$$m(\bar{\nu}_\mu) = m(\nu_\mu) \quad \text{and} \quad m(\bar{\nu}_e) = m(\nu_e),$$

from Eqs.(68,71) in [1]. The oscillation energy of the $N$ neutrinos in the lattice of the neutral pions was found in Eq.(34) of [1] and is given by Eq.(1), which means that the oscillation energy $E_\nu(\pi^\pm_n)$ is equal to the energy in the sum of the neutrino rest masses in $\pi^\pm_n$, i.e. equal to $N/2 \cdot [m(\nu_\mu) + m(\nu_e)]c^2$. The oscillation energy of the neutrino lattice of the neutral muons is, according to Eq.(1), the same, to which we now add the oscillation energy in the electron or positron, which is

$$E_\nu(e^\pm) = 1/2 \cdot m(e^\pm)c^2 = N/4 \cdot m(\nu_e)c^2 = N/4 \cdot m(\bar{\nu}_e)c^2,$$

according to Eqs.(69,84) in [1]. The oscillation energy in the electron or positron must be preserved when they are added to the neutral neutrino lattice, because the oscillation energy in the electron or positron represents the electric charge. Consequently the oscillation energy in the charged muons is

$$E_\nu(\mu^\pm) = N/2 \cdot [m(\nu_\mu) + m(\nu_e)]c^2 + N/4 \cdot m(\nu_e)c^2,$$

or we have

$$E_\nu(\mu^\pm) = N/2 \cdot m(\nu_\mu)c^2 + 3/4 \cdot N m(\nu_e)c^2.$$

Eq.(10) expresses the oscillation energy in the charged muons through the energy in neutrino rest masses. The oscillation energy in the charged muons
is not equal to the energy in the sum of the rest masses of its neutrinos. In a first approximation we can neglect the energy in the rest masses of the $\nu_e$ or $\bar{\nu}_e$ neutrinos, which is justified by the relation

$$m(\nu_e) = \alpha_f \cdot m(\nu_\mu),$$

(11)

from Eq.(72) in [1]. We then arrive at a value of \(\approx 2\) for the ratio of the oscillation energy in $\mu^\pm$ to the energy in the sum of the neutrino masses of the charged muons.

Adding to the oscillation energy of the charged muons, Eq.(10), the energy in the sum of the rest masses of the neutrinos in the charged muon lattice, Eq.(6), we arrive, not considering a binding energy, at the energy in the charged muon

$$m(\mu^\pm)c^2(\text{theor}) = [3/4 \cdot N m(\nu_\mu) + 3/2 \cdot N m(\nu_e)]c^2 = 108.395\,\text{MeV},$$

(12)

whereas

$$m(\mu^\pm)c^2(\text{exp}) = 105.6584\,\text{MeV}.\,$$

The energy in the rest mass of $e^\pm$,

$$m(e^\pm)c^2 = E_\nu(e^\pm) + \Sigma m(\text{neutrinos})(e^\pm)c^2 = 0.5109989\,\text{MeV},$$

is, according to Eq.(85) in [1],

$$m(e^\pm)c^2 = N/2 \cdot m(\nu_e)c^2 = N/2 \cdot m(\bar{\nu}_e)c^2.$$  

(13)

Dividing Eq.(12) by Eq.(13) gives the ratio of the mass of the charged muon to the mass of the electron or positron

$$\frac{m(\mu^\pm)}{m(e^\pm)}(\text{theor}) = \frac{3}{2} \cdot \frac{m(\nu_\mu)}{m(\nu_e)} + 3 = \frac{3}{2\alpha_f} + 3 = 208.554 = 1.0086 \frac{m(\mu^\pm)}{m(e^\pm)}(\text{exp}),$$

(14)

which differs by + 2 from Barut’s empirical formula

$$\frac{m(\mu^\pm)}{m(e^\pm)}(\text{emp}) = \frac{3}{2\alpha_f} + 1 = 206.554 = 0.99896 \frac{m(\mu^\pm)}{m(e^\pm)}(\text{exp}),$$

(15)

with $m(\mu^\pm)/m(e^\pm)(\text{exp}) = 206.768$. We attribute the difference + 2 between Eq.(14) and Eq.(15) to the binding energy of the electron or positron to the neutrino lattice of the neutral muon, that means that

$$\Delta E_b(\mu^\pm_n, e^\pm) = 2m(e^\pm)c^2.$$  

(16)

The ratio of the binding energy in $\mu^\pm$ to the oscillation energy in the muon is, in a first approximation, equal to $2\alpha_f$. 

6
Only one muon neutrino or anti-muon neutrino and one electron neutrino and one anti-electron neutrino, i.e. three neutrinos, hold the $\nu_e$ or $\bar{\nu}_e$ neutrinos introduced by $e^\pm$ into the sides of the muon cells in place. The $\nu_e$ or $\bar{\nu}_e$ neutrinos brought by $e^\pm$ into the neutrino lattice of $\mu^\pm$ do not sit in the center of a cell-side of the $\mu^\pm$ lattice, but rather in the corner of the cell-sides vacated originally by the emission of a $\nu_\mu$ or $\bar{\nu}_\mu$ neutrino in the decay of the $\pi^\pm$ mesons. Since the mass of the $\nu_\mu$ or $\bar{\nu}_\mu$ neutrinos is 137 times larger than the mass of the $\nu_e$ or $\bar{\nu}_e$ neutrinos we can neglect, in a first approximation, the interaction of the other two light $\nu_e, \bar{\nu}_e$ neutrinos in the corners of the cell-sides of the $\mu^\pm$ lattice with the single $\nu_e$ or $\bar{\nu}_e$ neutrino brought by $e^\pm$ into the corner of the cell-side opposite to the corner in which the $\nu_\mu$ or $\bar{\nu}_\mu$ neutrino is.

The binding energy of the $N/4$ electron neutrinos or anti-electron neutrinos and of the charge elements brought into the neutral muon lattice by an electron or positron is, according to Eq.(16), equal to $2 m(e^\pm)c^2$. One half of the binding energy originates from the binding of the charge elements, the other half of the binding energy comes from the $\nu_e$ or $\bar{\nu}_e$ neutrinos brought by $e^\pm$ into the $\mu^\pm$ lattice and is therefore equal to $m(e^\pm)c^2$. In $\mu^\pm$ are $N/4$ light neutrinos, either $\nu_e$ or $\bar{\nu}_e$ coming from $e^\pm$, which bind with $N/4$ heavy neutrinos, either $\nu_\mu$ or $\bar{\nu}_\mu$, which are in one corner of the cell-sides of the muon. The binding energy of a single light neutrino brought by $e^\pm$ into the corner of a cell-side of the $\mu^\pm$ lattice which is opposite to the corner which carries the heavy muon neutrino is, using Eq.(13),

$$E_b(\nu_\mu, \nu_e) = \frac{m(e^\pm)c^2}{N/4} = 2m(\nu_e)c^2. \quad (17)$$

The other half of the binding energy $2 m(e^\pm)c^2$ of the muon is caused by the charge elements.

Adding to the energy in the sum of the neutrino masses of the charged muons, Eq.(6), the energy in the lattice oscillations, Eq.(10), and subtracting the binding energy $\Delta E_b(\mu^\pm, e^\pm) = 2 m(e^\pm)c^2 = N m(\nu_e)c^2$ we arrive at the total energy in the charged muons. Simplifying with Eq.(7) we have

$$m(\mu^\pm)c^2(\text{theor}) = \left[3/4 \cdot N m(\nu_\mu) + 1/2 \cdot N m(\nu_e)\right]c^2 = 107.35 \text{ MeV} = 1.0160 m(\mu^\pm)c^2(\text{exp}), \quad (18)$$

with $m(\mu^\pm) = 105.658 \text{ MeV/c}^2$. From Eq.(18) follows for the ratio of the
mass of the charged muons to the mass of $e^\pm$ in Eq.(13) that

$$\frac{m(\mu^\pm)}{m(e^\pm)}(\text{theor}) = \frac{3}{2} \cdot \frac{m(\nu_\mu)}{m(\nu_e)} + 1,$$

(19)

or we find with $m(\nu_e) = \alpha_f m(\nu_\mu)$, (Eq.11), that

$$\frac{m(\mu^\pm)}{m(e^\pm)}(\text{theor}) = \frac{3}{2\alpha_f} + 1 = 206.554 = 0.99896 \frac{m(\mu^\pm)}{m(e^\pm)}(\text{exp}).$$

(20)

The experimental ratio $m(\mu^\pm)/m(e^\pm)(\text{exp}) = 206.768$ is 1.00104 times larger than our theoretical value 206.554. The dependence of $m(\mu^\pm)/m(e^\pm)$ on $3/2\alpha_f$ is the same as was once anticipated by Nambu [2] and Barut [3].

We have thus shown that the binding energy of the electric charge $e^\pm$ brought into the neutrino lattice of the neutral muons is $2 m(e^\pm)c^2$ and that then the ratio $m(\mu^\pm)/m(e^\pm)$ can be explained with an accuracy of $1 \cdot 10^{-3}$.

2 The charge in the $\pi^\pm$ mesons

When a $\pi^\pm$ meson is formed N/4 additional electron neutrinos or anti-electron neutrinos should come with the electron or positron into the neutral $\pi_n^\pm$ lattice, because $1/2 \cdot m(e^\pm) = N/4 \cdot m(\nu_e) = N/4 \cdot m(\bar{\nu}_e)$, as the explanation of the electron we have put forward in [1] demands. Conservation of neutrino numbers requires that these neutrinos are also in the charged pions. But the additional neutrinos have apparently no place in the cubic lattice geometry of the pions. An additional electron neutrino which originates from an electron added to a neutral cubic neutrino lattice can, however, sit in the center of any of the six sides of each cubic cell of the neutral pion lattice. The cells would then be face-centered cubic, Fig. 1. It is fundamental for the stability of the $\pi^\pm$ mesons that their lattice is face-centered. The lifetime of the $\pi^\pm$ mesons is $2.6 \cdot 10^{-8}$ sec, whereas the lifetime of the $\pi^0$ meson, which has a simple cubic lattice, is $8.4 \cdot 10^{-17}$ sec.

The N/4 electric oscillations brought into the neutral pion lattice by the charge $e^\pm$ must remain intact because the sum of the charges in the electric oscillations is equal to the elementary electric charge. Where then do the charge elements of $e^\pm$ sit in the lattice of the $\pi^\pm$ mesons? According to our explanation of the electron in [1] the charge elements in $e^-$, or the electric
oscillations, and the electron neutrinos in $e^-$ form a simple cubic lattice, Fig. 8 in [1]. If this structure is maintained when a charge $e^\pm$ attaches to the neutral cubic lattice of, say, the $\pi^\pm$ mesons, and if a $\nu_e$ or $\bar{\nu}_e$ neutrino coming from $e^\pm$ sits in the center of a side of an originally simple cubic neutrino cell, then a charge element should sit in the center of another side of the same cell, the other side being perpendicular to the side which carries the $\nu_e$ or $\bar{\nu}_e$ neutrino coming from $e^\pm$. Thus a charged cubic cell of the $\pi^\pm$ lattice would be face-centered cubic with two $\nu_e$ or $\bar{\nu}_e$ neutrinos in the centers of two opposite sides of the cell, and two charge elements in the centers of two opposite sides of the cell which are perpendicular to the sides which carry the electron neutrinos or anti-electron neutrinos introduced by $e^\pm$.

![Face-Centered Cubic Cell](image)

Fig. 1: A face-centered cubic cell. The cells of a face-centered lattice can have, but do not have to have, a particle in the center of each side of the cells. The thin lines connecting the points at the centers of the cell-sides are only a visual aid.

Four $\nu_\mu$, $\bar{\nu}_\mu$, $\nu_e$, $\bar{\nu}_e$ neutrinos are on each side, e.g. on the front or rear side, of the simple cubic cells of the $\pi^\pm_n$ neutrino lattice. The charge $e^\pm$ adds $N/4$ additional electron neutrinos respectively anti-electron neutrinos to the neutral $\pi^\pm_n$ lattice, or one additional neutrino to e.g. the center of the front as well as to the rear side of the cells. There are three possible locations for the neutrino brought by the charge into the sides of a simple cubic cell. It can sit either in the center of the $x$-$y$ surface, or of the $z$-$x$ surface, or of the $z$-$y$ surface of the cells and on the opposite sides.
The additional neutrinos originating from the charge of $\pi^\pm$ in the centers of the sides of a face-centered cubic neutrino cell of the $\pi^\pm$ lattice cannot oscillate with the frequency of the neutrinos in the cell corners, because the oscillation energy of the $\pi^\pm$ lattice would then be proportional to $5/4 \cdot N$, according to Eq.(1), which is not possible because the oscillation energy of the $\pi^\pm$ mesons would then exceed $m(\pi^\pm)c^2/2$ by 25%. If, however, the neutrinos in the centers of the cell-sides oscillate with the same frequencies they have in $e^\pm$, namely with a frequency proportional to $\alpha_f\nu_0$, as required by the oscillation frequencies in the electron or positron, as in Eq.(79) in [1]

$$E_\nu(e^\pm) = \frac{N h \nu_0 \cdot \alpha_f}{2\pi(e^{hc/kT} - 1)} \int_{-\pi}^{\pi} \phi d\phi = \frac{N}{2} \cdot \frac{e^2}{a} \cdot \frac{1}{f(T)} \int_{-\pi}^{\pi} \phi d\phi,$$

(21)

then this would add, because of the fine structure constant $\alpha_f \cong 1/137$, so little to the oscillation energy $E_\nu(\pi^\pm)$ (Eq.1) in $\pi^\pm$ in order to be negligible in the first approximation. The addition of a fifth electron neutrino or anti-electron neutrino to the cell-sides of the neutrino lattice $\pi^\pm_n$ does likewise add so little (0.36%) to the sum of the neutrino masses that it is also negligible in the first approximation.

As we have shown in [1] the ratio $m(\pi^\pm)/m(e^\pm)$ is, not considering the consequences of the charge on the mass of $\pi^\pm$, given by Eq.(102) therein

$$\frac{m(\pi^\pm_n)}{m(e^\pm)}(theor) = 2 \left[ \frac{m(\nu_\mu)}{m(\nu_e)} + 1 \right] \cong \frac{2}{\alpha_f} + 2 = 276.072 = 1.0107 \frac{m(\pi^\pm)}{m(e^\pm)}(exp),$$

(22)

using the relation $m(\nu_e) = \alpha_f m(\nu_\mu)$, (Eq.11). The last term in Eq.(22) contains the experimental ratio

$$m(\pi^\pm)/m(e^\pm)(exp) = 273.132,$$

(23)

with $m(\pi^\pm)c^2 = 139.570$ MeV and $m(e^\pm)c^2 = 0.5109989$ MeV. The improved empirical formula of Nambu [2] for the ratio $m(\pi^\pm)/m(e^\pm)$ is

$$\frac{m(\pi^\pm)}{m(e^\pm)}(emp) = 2/\alpha_f - 1 = 273.072 = 0.99978 \frac{m(\pi^\pm)}{m(e^\pm)}(exp).$$

(24)

The neutrinos and the oscillation energy brought by $e^\pm$ into the neutral neutrino lattice of the pions must add to the oscillation energy and to the energy in the sum of the masses of the neutrinos in the neutral pion lattice.
On the other hand, there will be a binding energy if an electron or positron is brought into the neutrino lattice of the neutral pions. Adding to Eq.(34) of [1] for the oscillation energy of the neutrino lattice of the $\pi^\pm_n$ mesons

$$E_\nu(\pi^\pm_n) = \frac{N}{2} \cdot [m(\nu_\mu) + m(\nu_e)]c^2$$

(25)

the oscillation energy in the electron or positron

$$E_\nu(e^\pm) = \frac{1}{2} \cdot m(e^\pm)c^2 = \frac{N}{4} \cdot m(\nu_e)c^2,$$

(26)

according to Eq.(8), and using

$$m(\bar{\nu}_\mu) = m(\nu_\mu) \quad \text{and} \quad m(\bar{\nu}_e) = m(\nu_e),$$

(27)

from Eq.(7), we arrive at the oscillation energy in the charged pions

$$E_\nu(\pi^\pm) = \frac{N}{2} \cdot [m(\nu_\mu) + m(\nu_e)]c^2 + \frac{N}{4} \cdot m(\nu_e)c^2,$$

(28)

because the oscillation energy in the neutral pion lattice expressed in neutrino masses is given by Eq.(25).

We now add to the sum of the rest masses of the neutrinos in the lattice of the neutral pions $\Sigma m(\text{neutrinos})(\pi^\pm_n) = \frac{N}{2} \cdot [m(\nu_\mu) + m(\nu_e)]$ the $N/4$ electron neutrinos or anti-electron neutrinos which come with the charge $e^\pm$ into the pions and find with $m(\nu_e) = m(\bar{\nu}_e)$ that

$$\Sigma m(\text{neutrinos})(\pi^\pm) = \frac{N}{2} \cdot [m(\nu_\mu) + m(\nu_e)] + \frac{N}{4} \cdot m(\nu_e).$$

(29)

The energy in the sum of the rest masses of the neutrinos in the lattice of the charged pion is equal to its oscillation energy $E_\nu(\pi^\pm)/c^2$, Eq.(28); as the energy in the sum of the rest masses of the neutrinos in the electron is equal to the oscillation energy in the electron, Eq.(8). From Eq.(29) follows that the energy in the sum of the neutrino rest masses in the charged $\pi^\pm$ mesons is

$$\Sigma m(\text{neutrinos})(\pi^\pm)c^2 = \left[\frac{N}{2} \cdot m(\nu_\mu) + \frac{3}{4} \cdot N m(\nu_e)\right]c^2.$$  

(30)

And the energy in the mass of $e^\pm$ is, according to Eq.(13)

$$m(e^\pm)c^2 = \frac{N}{2} \cdot m(\nu_e)c^2 = \frac{N}{2} \cdot m(\bar{\nu}_e)c^2.$$  

(31)

If we now add to the oscillation energy in the neutral pion lattice $E_\nu(\pi^\pm_n)$ (Eq.25), the energy in the sum of the rest masses of the neutrinos in the
neutral pion lattice, $N/2 \cdot [m(\nu_\mu) + m(\nu_e)]c^2$, and the energy in the mass of $e^\pm$ in Eq.(31) we arrive at the total energy $m(\pi^\pm)c^2$ of the charged pions

$$E_\nu(\pi^+_n) + \Sigma m(\text{neutrinos})(\pi^+_n)c^2 + m(e^\pm)c^2$$

$$= E_\nu(\pi^+_n) + N/2 \cdot [m(\nu_\mu) + m(\nu_e)]c^2 + N/2 \cdot m(\nu_e)c^2$$

$$= Nm(\nu_\mu)c^2 + 3N/2 \cdot m(\nu_e)c^2 = m(\pi^\pm)c^2,$$  \hspace{1cm} (32)

making use of $E_\nu(\pi^+_n) = \Sigma m(\text{neutrinos})(\pi^+_n)c^2$ in the neutrino lattice of the neutral pion.

After division by $m(e^\pm)c^2$ follows from Eq.(32) that the ratio of the mass of the charged pions to the mass of the electron or positron should be

$$\frac{m(\pi^\pm)}{m(e^\pm)}(\text{theor}) = 2 \frac{m(\nu_\mu)}{m(\nu_e)} + 3 = 2 + 3 = 277.072 = 1.0144 \frac{m(\pi^\pm)}{m(e^\pm)}(\text{exp}). \hspace{1cm} (33)$$

Nambu's empirical ratio $m(\pi^\pm)/m(e^\pm)$ is $2/\alpha_f - 1$, Eq.(24); the experimental ratio is given by Eq.(23). From the difference between the experimental mass of the pions and the theoretical mass of the charged pions follows the binding energy of the electric charge

$$\Delta E_b = mc^2(\text{exp}) - mc^2(\text{charged, theor}). \hspace{1cm} (34)$$

We can always approximate $mc^2(\text{exp})$ by a suitable empirical formula for the mass of the particle. The binding energy of the electron or positron to the neutrino lattice of the pions is equal to the difference between Eq.(33) and Eq.(24) and must be approximately

$$\Delta E_b(\pi^+_n, e^\pm) \approx 4 m(e^\pm)c^2. \hspace{1cm} (35)$$

The ratio of the binding energy in Eq.(35) to the oscillation energy in the neutrino lattice of the neutral pion, Eq.(25), is, in a first approximation, equal to $4\alpha_f$.

The binding energy can come only from the oscillation energy of the neutrinos in the corners of the cell-sides in $\pi^+_n$, because of conservation of neutrino numbers, and because the oscillation energy of the charge elements brought by $e^\pm$ into $\pi^+_n$ must be conserved since they represent the charge of the electron. The electron neutrinos or anti-electron neutrinos brought by $e^\pm$ into the centers of the cell-sides of the originally simple cubic lattice of $\pi^+_n$ cause what is practically a node of the oscillations. The oscillations
in the centers of the cell-sides are near a standstill, because the oscillations frequencies there are $\alpha_f$ times the oscillation frequencies at the corners of the cell-sides.

From Eq.(35) we learn about the binding energy between the constituents of the $\pi^\pm$ lattice. We must keep in mind that a charge $e^\pm$ added to the neutral neutrino lattice $\pi_n^\pm$ does not only introduce neutrinos but also the charge elements of $e^\pm$, i.e. the electric oscillations. According to Section 11 of [1] there are $N/4$ electron neutrinos or anti-electron neutrinos as well as $N/4$ positive or negative charge elements in $e^\pm$. The energy in the sum of the charge elements is the same as the energy in the sum of the neutrino rest masses. We can therefore treat the charge elements and neutrino masses the same way and use the neutrinos as the representative sample. The $N/4$ electron neutrinos or anti-electron neutrinos brought by $e^\pm$ into the neutral neutrino lattice $\pi_n^\pm$ sit in the centers of the sides of the cubic cells of the neutrino lattice and interact with the massive muon neutrinos or anti-muon neutrinos which are in opposite corners of the cell-sides of the $\pi_n^\pm$ lattice. Since $m(\nu_e) = \alpha_f m(\nu_\mu)$ we will neglect, in a first approximation, the interaction of the light neutrinos brought by $e^\pm$ into the centers of the cell-sides with the two light $\nu_e$ and $\bar{\nu}_e$ neutrinos which are in the other two corners of the cell-sides, see Fig. 2 in [1]. For the binding energy of a single electron neutrino or anti-electron neutrino in the center of a side of a face-centered cubic cell to the pair of $\nu_\mu$ and $\bar{\nu}_\mu$ neutrinos in the corners of the same side of the cell we find from Eq.(35) that

$$E_b(2\nu_\mu, \nu_e) = 2m(e^\pm)c^2/(N/4) = 4m(\nu_e)c^2,$$

(36)

with $2m(e^\pm)c^2 = N m(\nu_e)c^2$. The other half of the $4m(e^\pm)c^2$ of the binding energy in Eq.(35) is caused by the interaction of the charge elements with the $\nu_\mu$ or $\bar{\nu}_\mu$ neutrinos in the corners of their cell-sides. We must keep in mind that the energy in the charge elements, or the electric oscillations, is only on the average equal to the fixed energy in the rest mass of the $\nu_e$ or $\bar{\nu}_e$ neutrinos, because the energy of the electric oscillations varies with their frequencies, which differ according to Eq.(1). However, the energy in the sum of the electric oscillations is equal to the energy in the sum of the rest masses of the $\nu_e$ or $\bar{\nu}_e$ neutrinos in $e^\pm$.

The binding energy Eq.(36) between a single $\nu_e$ or $\bar{\nu}_e$ neutrino brought by $e^\pm$ into the center of the cell-sides of the $\pi^\pm$ lattice and a pair of $\nu_\mu$, $\bar{\nu}_\mu$ neutrinos in the corners of a cell-side of the $\pi^\pm$ lattice is twice the binding
energy Eq.(16) of the single $\nu_e$ or $\bar{\nu}_e$ neutrino, introduced by $e^\pm$ into $\mu^\pm$, to a single $\nu_\mu$ or $\bar{\nu}_\mu$ neutrino in a corner of a cell-side of the $\mu^\pm$ lattice. And since there are just as many $\nu_\mu$, $\bar{\nu}_\mu$ pairs in $\pi^\pm$ as single $\nu_\mu$ or $\bar{\nu}_\mu$ neutrinos are in $\mu^\pm$, the binding energy of an electron or positron in the $\pi^\pm$ lattice is twice the binding energy of $e^\pm$ in the $\mu^\pm$ lattice.

Subtracting the binding energy in Eq.(35) divided by the energy in the rest mass of the electron, i.e. $\Delta E_b(\pi^\pm, e^\pm)/m(e^\pm)c^2 = 4$, from Eq.(33) we arrive at the mass ratio $2/\alpha_f - 1$ given by Nambu’s formula (Eq.24) and find

$$m(\pi^\pm)c^2(\text{theor}) = 139.5395 \text{ MeV} = 0.99978 m(\pi^\pm)c^2(\text{exp}), \quad (37)$$

or that we have explained the mass of the charged pions with an accuracy of $2.2 \cdot 10^{-4}$.

To summarize: We have found that the electron neutrinos or anti-electron neutrinos brought by $e^\pm$ into the neutral neutrino lattice of the pions must sit in the center of the sides, e.g. the front and rear sides, of the simple cubic cells of the neutral neutrino lattice of the pions, and that the charge elements sit in the centers of cell-sides which are perpendicular to the sides with the $\nu_e$ or $\bar{\nu}_e$ neutrinos coming from $e^\pm$. That means that the cells in the $\pi^\pm$ lattice are face-centered cubic. And we have found that bringing $e^\pm$ into the neutral neutrino lattice of the pions releases a binding energy which is $\approx 4m(e^\pm)c^2$ and that then the mass $m(\pi^\pm)$ has been explained with an accuracy of $2.2 \cdot 10^{-4}$.

In simple terms: The ratio of the mass in the neutrino lattice of the neutral muons to the mass of the electron or positron has been derived in Eq.(98) of [1]. It is

$$m(\mu^\pm)/m(e^\pm)(\text{theor}) = 3/2\alpha_f + 2. \quad (38)$$

Adding to the right hand side of this equation + 1 for the mass of $e^\pm$ added to the neutrino lattice of the neutral muon and deducting 2 for the binding energy of $e^\pm$ to the neutrino lattice of the neutral muon gives Barut’s formula $3/2\alpha_f + 1$ for the ratio of the mass of the charged muon to the mass of the electron or positron, Eq.(15) above. Similarly, Eq.(102) in [1] gives the ratio of the mass of the neutrino lattice of the neutral pion to the mass of $e^\pm$

$$m(\pi^\pm)/m(e^\pm)(\text{theor}) = 2/\alpha_f + 2. \quad (39)$$

Adding to the right hand side of this equation + 1 for the mass of the electron or positron added to the neutrino lattice of the neutral pions and deducting
4 for the binding energy of $e^\pm$ to the neutrino lattice of the neutral pions gives Nambu’s empirical formula $2/\alpha_f - 1$ for the ratio of the mass of the charged pions to the mass of the electron or positron, Eq.(24), as it must be.

3 The charge in the K mesons.

It follows from the measured mass of the $K^\pm$ mesons that the ratio of the mass of the $K^\pm$ mesons to the mass of the electron or positron is

\[
m(K^\pm)/m(e^\pm)(exp) = 966.102 = 0.99984 \cdot (7/\alpha_f + 7).
\]

As we found in Section 6 of [1] the $K^\pm$ mesons consist of the second mode of the neutrino lattice oscillations of the $\pi^\pm$ mesons plus a $\pi^0$ meson, or are the state $(2.)\pi^\pm + \pi^0$. The energy in the second mode of the neutral neutrino lattice oscillations is given by Eq.(35) in [1],

\[
E((2.)\pi^\pm_n) = 4E_\nu(\pi^\pm_n) + N/2 \cdot [m(\nu_\mu) + m(\nu_e)]c^2,
\]

that means by the oscillation energy of the second mode of the neutrino oscillations of $\pi^\pm_n$, plus the sum of the energies of the rest masses of the neutrinos of the lattice, $N/2 \cdot [m(\nu_\mu) + m(\nu_e)]c^2$. That means with Eq.(25) that

\[
E((2.)\pi^\pm_n) = 5/2 \cdot N [m(\nu_\mu) + m(\nu_e)]c^2.
\]

Approximating the electromagnetic energy of the $\pi^0$ meson in the state $(2.)\pi^\pm + \pi^0$ by the energy in the charged $\pi^\pm$ mesons, or with

\[
m(\pi^\pm)c^2 = Nm(\nu_\mu)c^2 + 3N/2 \cdot m(\nu_e)c^2,
\]

from Eq.(32) we find for the energy of the charged $K^\pm$ mesons

\[
m(K^\pm)c^2(\text{theor}) \cong E((2.)\pi^\pm_n) + m(\pi^\pm)c^2 \cong [7N/2 \cdot m(\nu_\mu) + 4N m(\nu_e)]c^2.
\]

Dividing that by the energy in $m(e^\pm)c^2$ from Eq.(31) we have

\[
\frac{m(K^\pm)}{m(e^\pm)}(\text{theor}) = \frac{7m(\nu_\mu)}{m(\nu_e)} + 8 = 7/\alpha_f + 8 = 967.252 = 1.0012 \frac{m(K^\pm)}{m(e^\pm)}(\text{exp}).
\]
We have thus determined the theoretical ratio \( m(K^\pm)/m(e^\pm) \) from the sum of the energy in the second mode of the lattice oscillations and the energy in the rest mass of \( \pi^\pm \) with an accuracy of \( 1.2 \cdot 10^{-3} \). The binding energy of the charge \( e^\pm \) to the neutral neutrino lattices of the \( K^\pm \) mesons is given by the difference between Eq.(40) and (45) and by the binding energy of \( e^\pm \) in \( \pi^\pm \), and is therefore equal to \( 5 m(e^\pm)c^2 \).

It appears from the form of Eq.(44), and the accuracy with which we then can calculate \( m(K^\pm)/m(e^\pm) \), that the correct mode of the \( K^\pm \) mesons is not \( (2.)\pi^\pm + \pi^0 \), as we suggested in [1], but rather is \( (2.)\pi^+_n + \pi^\pm \). If a \( \pi^\pm \) meson is in \( K^\pm \), not a \( \pi^0 \) meson, then the neutral body of the \( K^\pm \) mesons would consist of neutrinos only, which is a much more simple composition than with the electromagnetic component of which the \( \pi^0 \) meson is made.

The ratio of the measured mass of the \( K^0,\bar{K}^0 \) mesons to the mass of the electron or positron is given by

\[
m(K^0,\bar{K}^0)/m(e^\pm)(exp) = 973.873 = 0.99961 (7/\alpha_f + 15).
\]  

The measured mass of the \( K^0,\bar{K}^0 \) mesons can be written as

\[
m(K^0,\bar{K}^0)(exp) = 497.648 \text{ MeV} = 0.99976 [m(K^\pm) + 8m(e^\mp)].
\]  

With the introduction of \( e^\pm \) into \( K^\pm \) the structure of the particle has changed. The \( K^\pm \) mesons consist of a face-centered cubic lattice with \( N/4 \) electron neutrinos or anti-electron neutrinos in the center of the sides of the cells of the lattice, and of \( N/4 \) positive or negative charge elements in the centers of the cell-sides perpendicular to the sides with the neutrinos introduced by \( e^\pm \). If we add to this lattice another elementary electric charge of opposite sign we add, according to our explanation of the structure of the electron in Section 11 of [1], another \( N/4 \) anti-electron neutrinos or \( N/4 \) electron neutrinos to the \( K^\pm \) lattice. We also add \( N/4 \) positive or negative charge elements to the \( K^\pm \) lattice. That means that \( N/4 \nu_e, \bar{\nu}_e \) neutrino dipoles are in the centers of the sides of the cells of the \( K^0,\bar{K}^0 \) lattice, as well as \( N/4 \) electric dipoles made of positive and negative charge elements. There are now twice as many bindings between the neutrino dipoles and charge dipoles in the centers of the cell-sides to the \( \nu_\mu \) and \( \bar{\nu}_\mu \) neutrinos in the corners of the cell-sides of the \( K^0,\bar{K}^0 \) mesons as were before in the case of \( K^\pm \). Although the signs of the two opposite electric charges introduced into
the $K^\pm$ lattice cancel, the charge elements of opposite sign remain as dipoles as conservation of charge requires. After division by $m(e^\pm)c^2$ follows that

$$\frac{m(K^0,\overline{K}^0)}{m(e^\pm)}(\text{theor}) = \frac{7m(\nu_\mu)}{m(\nu_e)} + \frac{8Nm(\nu_e)}{N/2 \cdot m(\nu_e)} = \frac{7}{\alpha_f} + 16 = 975.252$$

$$= 1.0014 \frac{m(K^0,\overline{K}^0)}{m(e^\pm)}(\text{exp}) . \quad (48)$$

The binding energy of the pair of electric charges $e^\pm e^\mp$ to the neutrino lattice of the $K$ mesons is then twice the binding energy of a charge $e^\pm$ in the $\pi^\pm$ mesons plus the difference of one $m(e^\pm)c^2$ from Eqs.(46,48) and is equal to

$$\Delta E_b(K^\pm_n, e^\pm e^\mp) = 9m(e^\pm)c^2 . \quad (49)$$

We have now explained the ratio $m(K^\pm)/m(e^\pm)$ with an accuracy of about $1.2 \cdot 10^{-3}$ and the ratio $m(K^0,\overline{K}^0)/m(e^\pm)$ with an accuracy of $1.4 \cdot 10^{-3}$.

**Conclusions**

We have been concerned with the consequences of the presence of the electric charge $e^\pm$ for the mass of the $\mu^\pm, \pi^\pm$ and $K$ mesons and with the explanation of the ratios of the masses of the charged $\mu^\pm, \pi^\pm$ and $K$ mesons to the mass of the electron or positron,

$$m(\mu^\pm)/m(e^\pm) = 206.7683 = 1.00104 \cdot (3/2\alpha_f + 1),$$
$$m(\pi^\pm)/m(e^\pm) = 273.1320 = 1.00022 \cdot (2/\alpha_f - 1),$$
$$m(K^\pm)/m(e^\pm) = 966.102 = 0.99984 \cdot (7/\alpha_f + 7),$$
$$m(K^0)/m(e^\pm) = 973.873 = 0.99961 \cdot (7/\alpha_f + 15).$$

In each case the term on the right hand side with half integer or integer multiples of $1/\alpha_f$ agrees exactly with what one expects from our model of the particles consisting of lattices of equal numbers of muon neutrinos, anti-muon neutrinos, electron neutrinos and anti-electron neutrinos, whose masses are related by $m(\nu_e) = \alpha_f \cdot m(\nu_\mu)$.

From the ratios of the measured particle masses to the mass of the electron or positron follows furthermore that integer multiples of $1/\alpha_f$ are in

$$m(n)/m(e^\pm) = 1838.684 = 1.000098 \cdot (14/\alpha_f - 80),$$
$$m(p)/m(e^\pm) = 1836.153 = 1.000081 \cdot (14/\alpha_f - 82.5),$$
\[ \frac{m(D^\pm)}{m(e^\pm)} = 3658.13 = 0.997 \cdot (28/\alpha_f - 168), \]
as it should be if \( m(n) \cong 2 m(K^0) \), and as it should be if \( m(D^\pm) \cong m(p + \bar{n}) \).

On the other hand, the integer numbers added to the \( 1/\alpha_f \) terms in the mass ratios do not agree, in each of the mass ratios listed above, with what one expects from our model of the neutral neutrino bodies of the particles. These differences are caused by the introduction of the electric charges \( e^\pm \) into the neutrino bodies of the particles. The introduction of \( e^\pm \) into the neutral body of a particle changes the number of the electron neutrinos or anti-electron neutrinos in the particles, not the number of the muon neutrinos or anti-muon neutrinos.

When an electric charge \( e^\pm \) is introduced into the large neutrino bodies in the \( \pi^\pm \) and \( \mu^\pm \) mesons an exothermic binding energy is released which amounts in the case of \( \pi^\pm \) to \( 4 m(e^\pm)c^2 \), and in the case of \( \mu^\pm \) to \( 2 m(e^\pm)c^2 \). The difference of both binding energies appears to be a consequence of the presence of only one muon neutrino or anti-muon neutrino in a side of each cell of the muon lattice, whereas one muon neutrino and one anti-muon neutrino are in the sides of each cell of the pion lattice. We have learned that the ratio of the mass of the charged muons to the mass of the electron or positron can be explained with an accuracy of about \( 1 \cdot 10^{-3} \) and that the ratio of the mass of the charged pions to the mass of the electron or positron can be explained with an accuracy of \( 2.2 \cdot 10^{-4} \), if the binding energy of the charge to the neutrino lattices of the muons and pions is taken into account.

We have also learned that the electron neutrinos or anti-electron neutrinos as well as the charge elements brought by the charge into the simple cubic neutrino lattice of the pions must sit in the center of the sides of the cubic cells of the neutrino lattice of the pions, making them face-centered cubic cells, which is essential for the stability of the \( \pi^\pm \) mesons. It seems likely that the electron or anti-electron neutrinos brought by \( e^\pm \) into the muon lattice occupy the places vacated by the emission of the muon neutrinos or anti-muon neutrinos in the \( \pi^\pm \) decay and that therefore the neutrinos in the cells of the \( \mu^\pm \) lattice have a cubic configuration. However, the charge elements brought also with \( e^\pm \) into the neutral lattice of the muons make the cells in the charged muons face-centered cubic. The consequences of the addition of \( e^\pm \) to the neutrino lattice of the K mesons have also been studied.
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