Application of the autofocusing algorithm for the synthesis of X and L band radar images

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Abstract. An approach to the formation of radar images of various frequency ranges is presented. The process of forming a radar image from a radio hologram in the case with a range migration is described. An approach to autofocusing a radar image by minimizing the entropy of a radar image is considered. The results of the operation of the described algorithms on real radio holograms are discussed.

1. Introduction
The process of synthesizing the antenna aperture is a coherent accumulation of echo signals during the translational movement of the radar carrier. This process is shown schematically in figure 1.

Antenna radiation pattern illuminates a certain area on the Earth's surface, and surface elements form a group echo signal received by the antenna [1]. The repetition rate of radio pulses is such that each surface element in the capture zone is irradiated multiple times. As a result of this kind of signal accumulation, a radio hologram is formed [1-2].
For high-quality restoration of the radar image, it is necessary to know the exact phases of the signals in the direction of the track range. Unfortunately, many factors introduce an error into the phase portrait, among them: instability of the flight path, limitations of the accuracy of the micronavigation system, natural oscillations of the carrier body, inhomogeneity of the signal propagation medium, instability of master oscillators, etc.

Numerous works are investigating [3-5] the possibilities of image autofocusing in order to compensate for phase instability in the travel direction. This paper examines the application of the autofocusing algorithm based on minimizing the entropy of the image.

2. **Synthesis of radio images with compensation for range migration**

The distance to a point reflector can be described by the following expression [1]:

\[ R(t) = R_0 + \Delta R = R_0 + \frac{V^2 t^2}{2R_0}, \]  

where: \( R_0 \) – shortest synthesis distance, \( V \) – carrier speed.

With the development of synthetic aperture radars, the resolution element decreases in the directions of azimuthal and slant ranges [6]. In this case, the synthesis interval increases, which leads to an increase in the value \( \Delta R \). Thus, a situation is revealed when, in the synthesis interval \( \Delta R > r_d \), where \( r_d \) – range resolution element. In this case, the trajectory signal appears in several range channels at once. This kind of situation is shown in figure 2.

![Figure 2. Migration across range channels.](image)

The change in range is associated with the Doppler effect, in which the Doppler frequency has the form:

\[ f(t) = \frac{1}{2} \frac{d\phi(t)}{dt}, \]  

where: \( \phi(t) = \frac{4\pi}{\lambda} R(t) \) – a function that describes the phase modulation of a trajectory signal.

From expressions (1) and (2) we can obtain the following law for describing the Doppler frequency:

\[ f(t) = \frac{2V^2 t}{\lambda R_0}. \]  

This formula gives a relationship between time and Doppler frequency, however, it is worth noting that with a change in the range channel, the Doppler incursion will change.

Relation (3) makes it possible to eliminate the range migration not in the time domain, but in the frequency domain for each range channel. Before compression in azimuth, a fast Fourier transform procedure is performed. For each sample in the radar image, its own reference function is used, which indicates from which range channels the radar image should be reconstructed. Then a corrected azimuth
spectrum is formed, which is multiplied with the spectrum by a reference function in azimuth. The inverse discrete Fourier transform procedure forms the resulting image.

The advantage of this method is the ability to use arbitrary signals in the radar, other than the signals with chirp. Also, azimuthal convolution using fast Fourier transform algorithms is performed much faster than with direct compensation of distance migration.

3. Autofocusing of radio images by minimum entropy

To quantify the influence of various parameters that distort the radar image, you can use entropy, which determines the information content of the image. The path is given a radar image $N$ with readouts for the slant range and $M$ readings for the track range. To calculate the entropy of an image, we use the following definition:

$$ H = -\sum_{i} \sum_{j} p_{ij} \log(p_{ij}), $$

where: $p_{ij} = \frac{|\hat{e}_{ij}|^2}{P},$ $\hat{e}_{ij}$ — complex readout of the radar image at a point $(i, j),$ $P = \sum_{i} \sum_{j} |\hat{e}_{ij}|^2$ — the energy of the radar image records.

For images with poor focus, the measure of entropy is usually higher than for focused images [1]. In other words, the more the image is defocused, the more entropy it has.

Trajectory signal from a single target located in the $i$ range channel:

$$ s_i(t) = U_i \exp[-j(\psi_i(t) + \zeta_i(t))], $$

Where: $\psi_i(t)$ — phase change caused by a change in the geometry of the radar scene during the translational movement of the carrier; $\zeta_i(t)$ — residual phase incursion caused by deviations of the carrier from a given trajectory, inaccurate compensation of phase fluctuations from the onboard microwavigation system, inhomogeneity of the signal propagation medium.

For slow phase fluctuations, when the synthesis time $T_s$ is significantly less than the correlation interval of phase fluctuations, the distorting function $\zeta_i(t)$ is usually approximated by a power polynomial with constant coefficients [1]:

$$ \zeta_i(t) = C_0 + C_1 t + C_2 t^2 + C_3 t^3 + \ldots, $$

It is believed that the largest contribution to defocusing is made by the quadratic component of the phase error. The presence of the quadratic component is usually associated with a decrease in the maximum of a point object, an expansion of the main lobe, and an increase in the level of side lobes.

Thus, the entropy can be considered as a function of the coefficient of the quadratic component of the phase error:

$$ H(C_2), $$

To compensate for this kind of error, the complex trajectory signal $s_i(t)$ should be multiplied by the compensating phase factor $\exp\left(jC_2 t^2\right).$ By choosing the value of the coefficient $C_2,$ one can solve the problem of minimizing the image entropy. In this case, the image will be considered focused.

It is believed that the maximum allowable phase incursion at the edges of the synthesis interval is $\pi/4.$ In this case, you can add a constraint:

$$ C_2 \leq \pi/T_s^2, $$
Thus, the autofocus problem is solved by searching for the minimum of the entropy function in the range $[C_{2\text{min}}, C_{2\text{max}}]$.

4. Results
The above algorithms for the synthesis of radar images, as well as the autofocus procedure, were applied to synthesize radar images of the X and L frequency ranges.

Figure 3 shows an L-band radar image containing a fragment of the coastline. The autofocus procedure helped to form a sufficiently contrasting image with a minimum of side lobes. However, at the bottom of the image, you can see the narrow stretched response regions. It should be assumed that in this case, elements of higher order prevail in the phase error of the trajectory signal.

Figure 3 shows an X-band radar image. The radio image in figure 4, in contrast to figure 3, has no shooting artifacts.

Thus, we can assume that autofocus has successfully coped with the formation of a contrasting image.

Figure 3. Radar image in L band after autofocusing procedure with a spatial resolution of 2.5 m.

Figure 4. Radar image in the X band after autofocusing procedure with a spatial resolution of 0.5 m.
5. Conclusion
The synthesis of radar images performed in this work reflects the possibilities of applying methods for the synthesis of radar images, taking into account the range migration and autofocusing algorithms at the minimum entropy in the absence of navigation data. Anthropogenic objects in the L range represent the elements with the greatest brightness, while in the X range the contrast of the image elements is much lower.

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