Flat Higgs Potential from Planck Scale Supersymmetry Breaking

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The observed Higgs boson mass poses a new puzzle in addition to the longstanding problem of the origin of the electroweak scale; the shallowness of the Higgs potential. The Higgs quartic coupling even seems to vanish at around the Planck scale within the uncertainties of the top quark mass and the strong gauge coupling. We show that the shallowness of the Higgs potential might be an outcome of supersymmetry breaking at around the Planck scale. There, the electroweak fine-tuning in the Higgs quadratic terms leads to an almost vanishing quartic coupling at around the Planck scale.

With the discovery of the Higgs boson at the LHC experiments [1,2], the investigation of the detailed structure of the Higgs sector has just started. Among other things, the measured Higgs boson mass, $m_h = 125.9 \pm 0.4$ GeV [3], seems to pose a new puzzle in addition to the longstanding problem of the origin of the electroweak scale; why the Higgs potential is so shallow. In fact, the extrapolated Higgs quartic coupling seems to vanish at around the Planck scale within the uncertainties of the top quark mass and the strong gauge coupling if we assume that there are no new physics below the Planck scale [4–7].

So far, a lot of attempts to provide such a boundary condition of the flat Higgs potential at around the Planck scale have been discussed based on, such as the asymptotic safety [8], or the multiple point criticality principle [9] (for recent works, see e.g. Ref. [10–13]). In this letter, we propose a new possibility where the almost vanishing quartic Higgs coupling at the Planck scale is an outcome of supersymmetry breaking at around the Planck scale. We will show, the electroweak fine-tuning in the Higgs mass parameters automatically leads to an almost vanishing quartic coupling either when the supersymmetry breaking sector is weakly coupled to the Higgs sector, or when the soft squared masses of the two Higgs doublets are close with each other.

Fine-tuning in the Higgs quadratic terms

To explain how the quartic coupling constant is determined at around the Planck scale, $M_{PL}$, let us take the simplest Higgs sector as an example, where the Kähler and the superpotential are given by

$$K = Z^\dagger Z + H_u^\dagger H_u + H_d H_d^\dagger + (c H_u H_d + h.c.) ,$$

$$W = \Lambda_{\text{SUSY}}^2 Z + m_{3/2}^2 M_{PL}^2 .$$

Here, $c$ denotes a dimensionless constant of $O(1)$, and $\Lambda_{\text{SUSY}}$ and $m_{3/2}$ are the supersymmetry breaking scale and the gravitino mass, respectively. The supersymmetry breaking field $Z$ obtains an $F$-term vacuum expectation value, $F_z = -\Lambda_{\text{SUSY}}^2$, and the flat universe condition gives $\Lambda_{\text{SUSY}}^2 \simeq 3 m_{3/2}^2 M_{PL}^2$. We assume that the supersymmetry breaking scale is at around the Planck scale and higher dimensional operators which couple supersymmetry breaking field and the Higgs doublets are somehow suppressed.

With these potentials, the Higgs mass terms are given by

$$V_2 = \bar{m}_{H_u}^2 |H_u|^2 + \bar{m}_{H_d}^2 |H_d|^2 + (b H_u H_d + h.c.)$$

$$\simeq (|\mu_H|^2 + m_{3/2}^2 |H_u|^2 + (|\mu_H|^2 + m_{3/2}^2) |H_d|^2$$

$$+ (b H_u H_d + h.c.) ,$$

where $\mu_H$ and $b$ are given by,

$$\mu_H = cm_{3/2} , \quad b = 2cm_{3/2}^2 .$$

Hereafter, we take $b$ to be real and positive by redefining the phases of $H_u$ and $H_d$ appropriately.

The higher dimensional operators which couple the Higgs doublets to the supersymmetry breaking field such as,

$$K = \frac{c_{u,d}}{M_{PL}^2} |Z|^2 |H_{u,d}|^2 ,$$

lead to additional contributions to the Higgs mass parameters. In the followings, we assume that the coefficients are rather suppressed, i.e. $c_{u,d} < O(0.1)$ or the coefficients are almost universal, i.e. $(c_u - c_d)/(c_u + c_d) < O(0.1)$, so that the soft squared masses of the two Higgs doublets are close with each other, i.e. $\bar{m}_{H_u}^2 \simeq \bar{m}_{H_d}^2$.

For successful electroweak symmetry breaking, we need fine-tuning so that one of the linear combinations of the two Higgs doublets, $h = \sin \beta H_u - \cos \beta H_d$, remains very

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1 For example, the almost universality can also be realized by an approximate symmetry which interchanges $H_u$ and $H_d$. 

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light with a mass much smaller than the Planck scale. In terms of the Higgs mass parameters, this requires
\[ m_{Hu}^2 m_{Hd}^2 - b^2 \ll O(M_{PL}^4), \]
which leads to \( b \simeq m_{Hu}^2 \simeq m_{Hd}^2 \). Therefore, by remembering that the Higgs mixing angle is determined by
\[ \tan 2\beta \simeq \frac{2b}{m_{Hu}^2 - m_{Hd}^2}, \]
we find that the electroweak fine-tuning predicts
\[ |\tan 2\beta| \gg 1, \]
and hence,
\[ \tan \beta \simeq 1, \]
for almost universal Higgs doublet masses, \( m_{Hu}^2 \simeq m_{Hd}^2 \).

At higher loop levels, the radiative corrections change the mass parameters in Eq. (3). Thus, the fine-tuning condition and the Higgs mixing angle are accordingly changed to
\[ (m_{Hu}^2 + \Delta_{Hu})(m_{Hd}^2 + \Delta_{Hd}) - (b^2 + \Delta_b) \ll O(M_{PL}^4), \]
\[ \tan 2\beta \simeq \frac{2(b + \Delta_b)}{(m_{Hu}^2 + \Delta_{Hu}) - (m_{Hd}^2 + \Delta_{Hd})}. \]
Here, \( \Delta_{Hu,Hd} \) denote the radiative corrections to the mass parameters. The fine-tuning condition for the light Higgs boson is imposed only after the radiative corrections to the mass parameters of all orders are included. Those corrections are, however, expected to be at most about a 10% compared to the tree-level mass parameters since the Standard Model interactions are rather suppressed at around the Planck scale (e.g., the top Yukawa coupling is \( y_t \simeq 0.4 \) at around the Planck scale). Therefore, the prediction of \( \tan \beta \simeq 1 \) at the tree level is not significantly affected by the radiative corrections.

In Fig. 1 we show the predicted value of \( \tan \beta \) as a function of finely tuned light Higgs boson mass parameter \( \tilde{m}_h \). We varied \( \tilde{m}_{Hu} \) from \( \tilde{m}_{Hd} \) by 10% (blue band) and 20% (light blue band).

Here, we redefined the mass parameters in the right hand side so that they include the radiative corrections. In the figure, we varied \( \tilde{m}_{Hu} \) from \( \tilde{m}_{Hd} \) by 10% (blue band) and 20% (light blue band) to explore how the non-universality as well as the radiative corrections change the prediction. The figure shows that for \( \tilde{m}_h \ll M_{PL} \), the predicted value of \( \tan \beta \) immediately converges to \( \tan \beta \simeq 1 \). The figure also shows that the prediction is not significantly affected even when \( \tilde{m}_{Hu} \) deviates from \( \tilde{m}_{Hd} \) by 20%.

It should be noted that unlike the low energy supersymmetry, the renormalization group effects to the Higgs mass parameters are negligible. For example, the up-type Higgs squared mass receives a correction from the renormalization group effects,
\[ \Delta m_{Hu}^2 \simeq \frac{6\mu^2}{16\pi^2} m_t^2 \log \frac{m_{3/2}^2}{M_{PL}^2}, \]
where \( m_t \) denotes the typical mass of the top squarks. These corrections are, however, not significant and lead to only a few percent changes to the Higgs mass parameters, \( \tilde{m}_{Hu}^2 \), as long as \( m_{3/2} \ll M_{PL} \) and \( m_t \simeq m_{3/2} \).

Let us emphasize that the prediction of \( \tan \beta \simeq 1 \) is not altered as long as \( \tilde{m}_{Hu}^2 \simeq \tilde{m}_{Hd}^2 \), and hence, the prediction does not rely on the particular model defined in Eqs. (1) and (2). Therefore, in a class of models with \( \tilde{m}_{Hu}^2 \simeq \tilde{m}_{Hd}^2 \), the electroweak fine-tuning predicts \( \tan \beta \simeq 1 \) when supersymmetry is broken at around the Planck scale.

**Quartic coupling at the Planck scale**

Below the supersymmetry breaking scale at around the Planck scale, the Higgs sector consists of the light Higgs

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2 In terms of the Kähler potential, the radiative corrections to the Higgs parameters leads to
\[ K \simeq (1 + \delta_{u,d})(Z^2/M_{PL}^2)|H_{u,d}|^2 + ((c + \delta_c)H_uH_d + h.c.) \]
where \( \delta \)'s are expected to be small.

3 In a model with \( \mu_H \ll m_{3/2} \) while \( b = O(m_{3/2}^2) \), the Higgsino can be a viable dark matter candidate when \( \mu_H \gtrsim 10^8 \) GeV [14].
boson $h$ and its scalar potential is given by,

$$V(h) = \frac{\lambda}{2} (h^\dagger h - v^2)^2,$$  \hspace{1cm} (15)

where $v \approx 174.1$ GeV is achieved as a result of the fine-tuning of the quadratic terms as discussed above. As a notable feature of the supersymmetric standard model, the Higgs quartic coupling $\lambda$ is given by the $SU(2)_L \times U(1)_Y$ gauge coupling constants,

$$\lambda \simeq \frac{1}{4} \left( \frac{3}{5} g_1^2 + g_2^2 \right) \cos^2 2\beta,$$  \hspace{1cm} (16)

at the tree-level. Thus, the prediction of $\tan \beta \simeq 1$ from the electroweak fine-tuning results in the almost vanishing quartic coupling.

At the higher loop-level, the Higgs quartic coupling receives threshold corrections from the top squarks,

$$\Delta \lambda \simeq \frac{6y_t^4}{16\pi^2} \left( \frac{X_t^2}{m_t^2} - \frac{1}{12} \frac{X_t^4}{m_t^4} \right),$$  \hspace{1cm} (17)

where $X_t = A_t - \mu_H \cot \beta$ [15,17]. This contribution is, however, suppressed due to a small top Yukawa coupling at the Planck scale, $y_t \simeq 0.4$.

The Higgs quartic coupling also gets contributions from higher dimensional operators which connect the supersymmetry breaking fields and the Higgs sector. For example, higher dimensional operators

$$\Delta K = \frac{c}{M_{PL}} |Z|^2 |H_u,d|^4,$$  \hspace{1cm} (18)

lead to additional contributions to the quartic coupling,

$$\Delta \lambda = O \left( \frac{\Lambda_{\text{SUSY}}^2}{M_{PL}^2} \right).$$  \hspace{1cm} (19)

These contributions are suppressed either when the supersymmetry breaking sector is slightly separated from the Higgs sector, i.e. $c \ll 1$, or when the supersymmetry breaking scale is somewhat smaller than the Planck scale. However, it should be noted that the later possibility, e.g. $\Lambda_{\text{SUSY}} \simeq 10^{17}$ GeV, does not affect the prediction of $\tan \beta \simeq 1$.5

In Fig. 2 we show the predicted quartic coupling at the Planck scale. In our analysis, we assumed that $X_t^2/m_t^2$ ranges between 0 to 10, in which $\Delta \lambda$ becomes maximal for $X_t^2/m_t^2 \simeq 6$ for a given $\tilde{m}_h$.6 In the figure, the (light-)red shaded regions show the predicted quartic coupling for $X_t^2/m_t^2 = 0$ while allowing $m_{H_u}^2$ varying from $m_{H_d}^2$ by 10% (20%). The (light-)blue shaded regions show the ones for $X_t^2/m_t^2 = 6$. The values of the quartic coupling with error bars show the Higgs quartic coupling extrapolated from the electroweak scale for a given physical Higgs boson mass.

FIG. 2: The predicted Higgs quartic coupling at the Planck scale. We allowed $X_t^2/m_t^2$ from 0 to 10. The (light-)red shaded regions show the predicted quartic coupling for $X_t^2/m_t^2 = 0$ while allowing $m_{H_u}^2$ varying from $m_{H_d}^2$ by 10% (20%). The (light-)blue shaded regions show the ones for $X_t^2/m_t^2 = 6$. The values of the quartic coupling with error bars show the Higgs quartic coupling extrapolated from the electroweak scale assuming that there is no new physics below the Planck scale [6]. The figure shows that the predicted quartic coupling is vanishingly small once the electroweak fine-tuning is required. Therefore, we find that the electroweak fine-tuning leads to a shallow Higgs potential in a class of models with $m_{H_u}^2 \simeq m_{H_d}^2$ when supersymmetry is broken at around the Planck scale.7

Discussions

We have shown that the almost vanishing Higgs quartic coupling is predicted for $m_{H_u}^2 \simeq m_{H_d}^2$ with the Planck scale supersymmetry breaking. It is an intriguing feature of this mechanism that the shallowness of the Higgs potential is caused by the electroweak fine-tuning in the Higgs mass parameters.

Since the predicted quartic coupling is almost vanishing but is positive valued in most parameter space, the future precise measurements of the Higgs mass parameters as well as the top Yukawa coupling and the strong coupling constants provide an important test of this mechanism. At the ILC, for example, the Higgs mass can be measured with high precision.

For a related discussion, see also Ref. [15,18].

5 The predicted value of $\tan \beta$ is significantly deviated from 1 for a much lower supersymmetry breaking scale, $\Lambda_{\text{SUSY}} \ll 10^{17-18}$ GeV, where the renormalization group effects spoil the universality of the soft masses of the two Higgs doublets even if the universal soft masses, $m_{H_u}^2 = m_{H_d}^2$, are realized at the mediation scale around the Planck scale.

6 If we allow much larger value of $X_t^2/m_t^2$, e.g. $X_t^2/m_t^2 \gtrsim 15$, the predicted $\lambda$ takes a negative value.

7 It is also possible to provide $\lambda(M_{PL}) \simeq 0$ if the $SU(2)_L \times U(1)_Y$ gaugino masses are dominated by the Dirac mass, which results in the vanishing D-term contributions to the Higgs potential [19,20].
determined with a precision of 30 MeV for the integrated luminosity $L = 250 \text{fb}^{-1}$\textsuperscript{[21, 22]}. The uncertainties of the top Yukawa coupling will be also reduced by about one order of magnitude at the ILC\textsuperscript{[23]}. Improvements in lattice calculations could reduce the error of the strong coupling constant $\alpha_s$ down to 0.1%\textsuperscript{[24]}. With these improvements, it is possible to refute this mechanism if the central value of the extrapolated Higgs quartic coupling at around the Planck scale is close to the current central values, unless there is a small, but non-negligible contributions from Eq. (18).

Finally, let us comment on a more ambitious interpretation of this mechanism. In the simplest model we discussed in Eqs. (1) and (2), the electroweak fine-tuning condition is nothing but the requirement of $c \simeq 1$. In this case, the Kähler potential can be rewritten by $K = |H_u + H_d^1|^2$, and hence, the model has a shift symmetry, $H_{u,d} \rightarrow H_{u,d} + i \alpha$ with $\alpha$ being a real parameter. This suggests that the prediction of $\tan \beta \simeq 1$ can be related to the existence of the shift symmetry.\textsuperscript{8} In fact, the prediction of $\tan \beta \simeq 1$ is not altered even if we take a more generic Kähler potential as long as the shift symmetry is preserved, i.e.

$$K = K(H_u + H_d^1, Z). \quad (20)$$

Here, we do not need to assume that the couplings between the supersymmetry breaking sector and the Higgs sector are suppressed, since the above Kähler potential does not contribute to the scalar potential of the light Higgs boson, $h \simeq H_u - H_d^1$. It is notable that the shallowness of the Higgs potential can be interrelated to the shift symmetry of the Higgs sector despite the fact that the shift symmetry is explicitly broken by the gauge interactions which provide the leading contribution to the quartic coupling in the supersymmetric standard model (Eq. (16)).

It is also possible to extend this mechanism to more generic models in which the Higgs doublets emerge as Goldstone modes of approximate symmetries\textsuperscript{[27, 30]} such as models with $SU(3)/SU(2) \times U(1)$\textsuperscript{[31, 32]}. There, again, the prediction of $\tan \beta \simeq 1$ is guaranteed by the non-linearly realized symmetry by the Higgs doublets which non-trivially leads to the vanishing quartic coupling.

\textbf{Note Added}

After this paper was posted to arXiv.org, it came to the author’s attention that Refs.\textsuperscript{[34, 35]} have observed that the electroweak fine-tuning with the boundary condition with $m_{H_u}^2 \simeq m_{H_d}^2$ at the intermediate to the scale of the unification leads to the appropriate Higgs boson mass, i.e. $m_H \simeq 126 \pm 3$ GeV. These observations partially overlap with our arguments that the electroweak fine-tuning from the Planck scale supersymmetry breaking leads to the flat Higgs potential at the Planck scale. Their boundary conditions at the intermediate scale, however, may not be easily realized in a simple framework of supergravity due to non-negligible radiative corrections to the higgs boson masses.\textsuperscript{9}

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\textsuperscript{8} See also Refs.\textsuperscript{[25, 26]} which discussed the connection between the prediction of $\tan \beta \simeq 1$ and the shift symmetries as well as their realization in string theory.

\textsuperscript{9} We stress that the our setup does not require any specific mediation mechanisms of the supersymmetry breaking effects other than supergravity mediation.
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