On the gravitational stability of a galactic disc as a two-fluid system

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ABSTRACT

The gravitational stability of a disc with gaseous and stellar components is studied in the linear regime when the gaseous component is turbulent. A phenomenological approach is adopted to describe the turbulence, in which both the effective surface density and the velocity dispersion of the gaseous component are scale-dependent as power-law functions of the wavenumber of the perturbations. In addition, the stellar component, which interacts gravitationally with the gas, is considered as a fluid. We calculate growth rates of the perturbations, and find that in most cases the stability of the disc depends strongly on the existence of the stars and on the exponents of the functions for describing the turbulence. Our analysis suggests that the conventional gas and star threshold is not adequate for analysing the stability of two-component discs when turbulence is considered.

Key words: instabilities – turbulence – ISM: structure – galaxies: ISM – galaxies: structure.

1 INTRODUCTION

Various instability mechanisms can operate in an accretion disc, depending on the physical properties of the disc itself. An imbalance of the heating and cooling rates in a thin accretion disc may lead to a disc that is thermally unstable (e.g. Lightman & Eardley 1974; Pringle 1976; Piran 1978). Under some conditions, an accretion disc is hot enough to be fully or partially ionized, which implies that significant dynamical effects of the magnetic field can be expected. In fact, an instability related to the rotational profile and the magnetic field of an accretion disc is known to be the driving mechanism of turbulence inside these discs (Balbus & Hawley 1991). In most accreting systems, the mass of the disc is negligible in comparison to the mass of the central object, and so the self-gravity of the disc can be safely neglected. However, gaseous or stellar discs are subject to the gravitational instability if the Toomre parameter drops below unity (Toomre 1964) and the cooling time-scale is less than a few times the dynamical time-scale (Gammie 2001). The formation of structures such as planets in protoplanetary discs (e.g. Boss 1998; Rafikov 2005, 2007) or stars at the Galactic Centre (e.g. Goodman & Tan 2004; Nayakshin & Cuadra 2005; Nayakshin 2006; Levin 2007) are described based on the gravitational instability.

Analysis of the gravitational stability of a disc started with the pioneering works of Safronov (1960) and Toomre (1964). The onset of the axisymmetric gravitational instability is determined by the following condition (Toomre 1964):

\[ Q = \frac{c_s \kappa}{\pi G \Sigma} < Q_0, \]

where \( Q_0 = 1 \) and \( c_s, \kappa \) and \( \Sigma \) are the sound speed, epicyclic frequency and surface density of the disc, respectively. The non-dimensional Toomre parameter \( Q \) has a vital role in the gravitational stability of an accretion disc. The non-axisymmetric perturbations grow for higher values of \( Q_0 \).

The Toomre condition (1), however, is not the only criterion for the fragmentation of a disc. In fact, the thermodynamics of the disc and its efficiency of cooling dictate another condition for the fragmentation of a disc, as was suggested in a seminal work by Gammie (2001) and investigated through extensive numerical simulations (e.g. Johnson & Gammie 2003; Rice, Lodato & Armitage 2005; Clarke, Harper-Clark & Lodato 2007; Cossins, Lodato & Clarke 2009).

Although the validity of the Toomre condition for the stability of self-gravitating discs has been confirmed by direct numerical simulations, in some astrophysical systems such as galaxies the one-component description is inadequate for studying their stability (e.g. Yim et al. 2011; Bournaud et al. 2010; Elmegreen 2011). A typical galaxy consists of gas and stars. Over the last two decades, many authors have studied the gravitational stability of such two-components discs (e.g. Jog & Solomon 1984; Romeo 1992; Wang & Silk 1994; Jog 1996). The main outcome is that the stellar component has a destabilizing role, and either a fluid approximation or a collisionless description is used to model this component. Unfortunately, it is not possible to represent the stability condition of a two-component disc in a closed analytical form as for the one-component case, although some authors introduce approximate analytical formulae for the onset of instability in a disc (e.g. Wang & Silk 1994; Romeo & Wiegert 2011).

Recently, Romeo, Burkert & Agertz (2010) (hereafter RBA) studied the possible effects of turbulence on the gravitational stability of accretion discs using a simple but very illustrative model for the turbulence. They used scale-dependent relations for the effective surface density and the velocity dispersion of the gaseous...
component. This enabled them to obtain the conditions under which the self-gravitating disc becomes unstable depending on the properties of the turbulence. For a range of input parameters, they showed that the conventional Toomre parameter is modified to correctly describe the stability of the disc when the turbulence of the gas is considered. Such an approach for studying the gravitational stability of discs has been applied by a few authors, but for specific cases (e.g. Elmegreen 1996). In another relevant study, Elmegreen (2011) investigated gravitational instabilities in two-component disc galaxies when the gas dissipates in the local crossing time. However, he performed the linear analysis by considering the energy equation and prescribing a scale-dependent dissipation rate instead of following a phenomenological approach to describe the turbulence. The main finding was an increase in the conventional gas and star threshold by a factor of two or three because of the existence of the dissipation, which destabilizes the turbulent pressure.

In this paper, we study the gravitational stability of a two-fluid disc in which the turbulence of the gaseous component is considered in a similar way to in RBA. However, we will show that different regimes of the (in)stability according to RBA are significantly modified in the presence of stars. Our results can also be compared with those of Jog & Solomon (1984) to see how the stability of a two-fluid disc changes when the turbulence of the gaseous component is considered. In the next section, our basic assumptions and the main dispersion relation are presented. Then, we analyse the stability of the system for a wide range of input parameters and compare the results with those of RBA.

2 LINEAR PERTURBATIONS

The basic equations of our model are identical to those introduced in Jog & Solomon (1984) (JS), in particular regarding the two-fluid description of the system. The gaseous and stellar components are considered as two separate fluids that can interact only gravitationally. Although we assume that the stellar component is collisional (i.e. a fluid description), it is also possible to consider the stellar component to be collisionless (Rafikov 2001). Both the gaseous and the stellar component are assumed to be isothermal, characterized by the sound speed $c_s$ and the velocity dispersion $\sigma_v$, respectively. The unperturbed surface densities for the gas and the stellar fluid are denoted by $\Sigma_{g0}$ and $\Sigma_{s0}$. Furthermore, the scale-heights of the gaseous component and the stellar mass distribution are given by $2h_g$ and $2h_s$. However, we include the turbulence of the gaseous component in the way that was introduced by RBA. In this approach, the initial density and the velocity dispersion of the gaseous component are both scale-dependent because of the existence of the turbulence:

$$\Sigma_{\text{eff}} = \Sigma_{g0} \left(\frac{k}{k_0}\right)^{-a},$$

$$\sigma_\delta = \sigma_{\text{g0}} \left(\frac{k}{k_0}\right)^{-b},$$

where $a$ and $b$ are input parameters that describe the nature of the turbulence. The density fluctuation has a power-law spectrum, $\delta g(k) \propto k^{-\beta}$, which implies that $\rho \propto k^{-(r-1)/2}$ (see e.g. Elmegreen & Scalo 2004; Scalo & Elmegreen 2004). Assuming $\Sigma_{\text{eff}} \sim \rho h_g$, it can easily be shown that $a = (1/2)(r - 1)$. Moreover, the spectrum of the velocity fluctuations is a power law; that is, $\delta v(k) \propto k^{-\alpha}$, and then $\sigma_v \propto k^{-(r-1)/2}$. The above power-law relations are appropriate for the cold interstellar medium, and the length $1/k_0$ is

the fiducial scale at which the Toomre parameter and other stability quantities are measured (RBA). Moreover, the mass-size scaling relation implies that $-2 \leq a \leq 1$ (RBA). However, observational and theoretical studies imply more constraints on the acceptable ranges for $a$ and $b$. HI observations show that a Kolmogorov spectrum can describe the density and the velocity fluctuations in these regions; that is, $a \sim 1/3$ for scales less than 10 kpc, and $b \sim 1/3$ for scales less than 1 kpc (e.g. Lazarian & Pogosyan 2000; Dutta et al. 2009). In giant molecular clouds, however, the density and the velocity exponents are $a \sim 0$ and $b \sim 1/2$ (e.g. Larson 1981; Heyer et al. 2009). On the other hand, high- and low-resolution simulations of supersonic turbulence imply typical values of $(a, b)$ to be $(1/2, 1/2)$ and $(2/3, 1/2)$, respectively (RBA).

If we assume a functional form of $\exp[i(kr + o t)]$ for the perturbed variables in the linearized hydrodynamical equations, the dispersion relation is obtained as

$$\omega^2 - \omega^2(\alpha + \beta) + (\alpha \alpha - \beta \beta) = 0,$$

where

$$\alpha = k^2 + h^2 \sigma_v^2 - 2\pi Gk \Sigma_{s0} \left[1 - \exp(-kh_s)\right],$$

$$\beta = 2\pi Gk \Sigma_{s0} \left[1 - \exp(-kh_s)\right],$$

We identify $\omega$ as the growth rate of the gas and star gravitational instability. Because only the two-dimensional forcing is considered, a correction factor of $[1 - \exp(-kh_s)]/(kh_s)$ appears in the above equations (Toomre 1964). However, Vandervoort (1970) introduced a correction factor of $(1 + kh_s)^{-1}$ from a more detailed analysis. Note that the appropriate correction factor appears for the gas as well. Thus, if we set $a = b = 0$, it means that the correction factors are considered for the stellar fluid and the non-turbulent gas component and the equations are reduced to JS. Equation (4) is solved analytically as

$$\omega^2(k) = \frac{1}{2} \left[(\alpha + \beta) \pm \sqrt{(\alpha + \beta)^2 - 4(\alpha \alpha - \beta \beta)}\right],$$

where only the following root leads to the instability (JS):

$$\omega^2(k) = \frac{1}{2} \left[(\alpha + \beta) - \sqrt{(\alpha + \beta)^2 - 4(\alpha \alpha - \beta \beta)}\right].$$

We note that if the stellar or the gaseous contributions to the (in)stability of the system are neglected, then the growth rate of the unstable perturbations is determined based on the sign of $\alpha_0$ or $\beta_0$. In other words, the stellar or the gaseous components are unstable when $\alpha_0 < 0$ or $\beta_0 > 0$, respectively.

For a purely gaseous disc, RBA studied the instability regimes corresponding to $\alpha_0 < 0$ for various values of $a$ and $b$. We now illustrate how this parameter study is modified when the stellar contribution is considered. Thus, we must determine under what circumstances equation (10) gives a negative value, namely when $\omega^2 < 0$. However, it is very unlikely that we will be able to present the stability condition based on the growth rate equation (10) in a closed analytical form. Instead, we adopt the parameters corresponding to the Galaxy as an illustrative example, but for different values of $a$ and $b$. For a gaseous disc, RBA showed that
the stability of the disc is classified into seven categories depending on the values of $a$ and $b$. We now explore how these different regimes of instability are modified in the presence of a stellar fluid.

3 ANALYSIS

JS adopted their input parameters for the Galaxy based on observational data. We also consider a similar set of input parameters, because it helps us to compare our results with previous findings.

In all plots, our input parameters correspond approximately to conditions at the distance $R = 6$ kpc from the centre of the Galaxy. Although the stars in the disc have a range of velocity dispersions, we characterize the stellar fluid by a single velocity dispersion $\sigma_0 = 34.7$ km s$^{-1}$. The epicyclic frequency is $\kappa = 65$ km s$^{-1}$ kpc$^{-1}$.

The sound-speed in the gas and the ratio of the gas density to the stellar density are $5$ km s$^{-1}$ and 0.1, respectively. Furthermore, we assume $\Sigma_0 = 19 M_\odot$ pc$^{-2}$ and $\Sigma_0 = 190 M_\odot$ pc$^{-2}$. We also assume that the disc is in vertical equilibrium, and the scale-heights are assumed to be $2h_z \approx 150$ pc and $2h_z \approx 180$ pc (JS).

Having the above input parameters, we can plot growth rate versus the wavenumber of the perturbations using equation (10). Fig. 1 shows $\omega^2$ versus the inverse wavelength of the perturbations, $\lambda^{-1}$, when $a = 1$ and $b \neq 1$. In this case, RBA showed that a gaseous disc is stable for all wavenumbers as long as $k_0 \leq k_T = \kappa^2/(2\pi G \Sigma_0)$. Here, $k_T$ is the Toomre wavenumber. We found, however, that this condition is not valid when the stellar fluid is considered. In this case, we note that for the gaseous component the self-gravity term is independent of the wavenumber, but for the stellar fluid it is not. The Kolmogorov turbulence corresponds to $a = 1$ and $b = 1/3$, which lies in the explored regime of Fig. 1. HI observations also suggest a Kolmogorov scaling for the velocity and density fluctuations (e.g. Lazarian & Pogosyan 2000; Dutta et al. 2009). The growth rate curves in Fig. 1 are drawn for three values of $k_0$, namely $k_0 = 0.5k_T$ (top panel), $k_0 = k_T$ (middle) and $k_0 = 1.5k_T$ (bottom). In all plots, dashed curves correspond to a case without turbulence, namely $a = b = 0$. The top panel of Fig. 1 clearly shows that the system is not stable for all wavenumbers and there is always a range of $k$ for which the disc is unstable, irrespective of the value of $b$. When the critical wavenumber $k_0$ increases, however, more interesting cases emerge according to the middle and bottom plots of Fig. 1. For example, RBA predicts that for $k_0 = 1.5k_T$ a gaseous disc is always unstable, but the bottom panel of Fig. 1 shows stable behaviour when $b = 2$. Moreover, the range of $k$ for which the disc is unstable becomes larger as $k_0$ increases.

In Fig. 2, we explore another case: $a \neq 1$ and $b = 0.5(1 + a)$. So, both the pressure and the self-gravity terms of the gaseous component have the same $k$-dependence. Each curve is labelled with the corresponding values of $a$ and $b$, and again the dashed curves show the growth rate of the perturbations when the turbulence of the gas is neglected. Three values of $k_0$ are adopted. For this case, RBA showed that a gaseous disc is stable for all $k$ as long as $k_0 \geq k_T = (2\pi G \Sigma_0)/\sigma_0^2$, where $k_T$ is the conventional Jeans wavenumber. However, we see that this condition is modified, at least for some of the input parameters, for example for $a = 0$ and $b = 0.5$.

Figs 3 and 4 show the growth rate of the perturbations for the case $a < 1$ and $b > 0.5(1 + a)$. RBA did not explore this case in detail because the zero(s) of $\omega^2(k)$ should be determined numerically, case-by-case. They noted, however, that a one-component gaseous disc is unstable at small scales, irrespective of the existence of the

Figure 1. Plot of $\omega^2$ versus $\lambda^{-1}$ when $a = 1$ and $b \neq 1$ and $k_0 = 0.5k_T$ (top), $k_0 = k_T$ (middle) and $k_0 = 1.5k_T$ (bottom). The other input parameters are $\kappa = 65$ km s$^{-1}$ kpc$^{-1}$, $\sigma_0 = 34.7$ km s$^{-1}$, $\sigma_0 = 5$ km s$^{-1}$, $\Sigma_0 = 19 M_\odot$ pc$^{-2}$ and $\Sigma_0 = 190 M_\odot$ pc$^{-2}$. The dashed curve corresponds to a case without turbulence.
turbulence. Here, we also study this case for a limited range of input parameters. For a particular set of parameters $a$ and $b$ from this category, Elmegreen (1996) studied the stability of a turbulent gaseous disc corresponding to the Larson-type scaling relations, namely $a = -1$ and $b = 1/2$. He found that the disc is always stable at large scales and unstable at small scales. In both Figs 3 and 4, different values of $k_0$ are considered. We can see that the disc becomes more stable with increasing $k_0$, so that when $k_0 = 1.5k_J$ the system is stable for all values of $a$ and $b$, but when $k_0 = 0.2k_J$ and $a = 0$ the system is unstable for two intervals of the wavenumber. These intervals of the instability diminish with increasing $k_0$. Comparing Figs 3 and 4, it can be seen that the system becomes more stable as the value of $a$ decreases. So, not only the
results of RBA but also Elmegreen’s results are modified in the presence of a stellar fluid.

Fig. 5 shows the growth rate of perturbations for another case considered by RBA, namely $a > 1$ and $b < 0.5(1 + a)$. For this case, RBA showed that a turbulent gaseous disc is unstable at large scales. Our Fig. 5 confirms this behaviour for a turbulent gaseous disc in the presence of stellar fluid.

The case $a < 1$ and $b < 0.5(1 + a)$ is a Toomre-like situation, as discussed by RBA. They found a stability threshold $Q$ in terms of the input parameters, and the turbulent gaseous disc is stable at all wavenumbers if $Q > \bar{Q}$. Fig. 6 explores this case. Each curve is labelled with the relation between the corresponding parameters $Q$ and $\bar{Q}$ based on RBA. For example, curves corresponding to $k_0 = 1.5k_T$ and $k_0 = 0.75k_T$ in the middle panel show stability although $Q$ is less than $\bar{Q}$. In other panels, the system becomes unstable for limited ranges of the wavenumber of the perturbations. In all panels, we also note that for fixed $\Sigma_0$ and $\Sigma_{g0}$, varying $k_0$ has the effect of varying the effective surface density. Thus, the system has a higher effective surface density and becomes more unstable at lower $k_0$ when $a < 0$. For positive values of $a$, however, we have a less effective surface density and more stability at lower $k_0$.

4 CONCLUSIONS

We have studied the gravitational stability of a disc consisting of two components (i.e. gaseous and stellar fluids) using a perturbation analysis. We also considered the turbulent nature of the gaseous fluid using a phenomenological description that is supported by observations and numerical simulations. Although the stability of a two-fluid disc without turbulence of the gas (e.g. Jog & Solomon 1984; Jog 1996; Wang & Silk 1994; Rafikov 2001) and of a one-component turbulent gaseous disc (Romeo et al. 2010) have already been studied analytically, our analysis is a first step towards considering not only the two-fluid nature of such systems but also the turbulence of the gas. We found that the turbulence of the gas strongly affects the stability of two-component stellar and fluid discs. According to our analysis, not only is there not a closed analytical relation for the onset of instability in a two-component disc, but the conventional gas and star threshold does not correctly address the stability of such systems in the presence of turbulence of the gas. We also explored situations without the thickness correction for the gas. In fact, this correction factor has a stabilizing effect, in particular for high wavenumbers $k$. For perturbations much larger
than the thickness of the disc, the thickness correction becomes unity. However, as the wavenumber increases to scales larger than the thickness of the disc, the correction factor introduces an additional inverse $k$ dependence to the effective surface density. This weakens the self-gravitational effect significantly. In the presence of turbulence of the gas, we also found this stabilizing effect.

In dwarf galaxies or the far-outer regions of the galaxy discs, the conventional stability criterion suggests a high level of stability (e.g. van Zee et al. 1997; Hunter et al. 2011). Although Elmegreen (2011) showed that such regions are more unstable when the turbulence of the gas is considered, our analysis shows that the stability of such systems depends strongly on the nature of the turbulence (i.e. values of $a$ and $b$) in the presence of stars. However, detailed numerical simulations of galactic discs (including the turbulence of the gas component) are needed to confirm our linear stability analysis.

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Figure 6. As Fig. 1, but for the case $a < 1$ and $b < 0.5(1 + a)$. Various values of $a$ and $b$ are considered.