Three generation vacuum oscillations and the solar neutrino problem

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Abstract

We investigate the solar neutrino problem in the scenario of three generation neutrino oscillation hypothesis, taking into account other phenomenological constraints to the neutrino mixing and mass parameters.
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Although direct measurements of the neutrino masses are all consistent with zero [1], it is well known that some issues, as the solar [2] and atmospheric [3] neutrino problems, may be an indication of nonvanishing neutrino masses once that an economical and successful way to understand both results relies on the neutrino oscillation hypothesis [4]. Even neutrino oscillations in vacuum continue to be quoted [3] as a possible solution to the solar neutrino problem [2]. Most of these analysis deal with two neutrino oscillations, based on the simplified assumption that only the mixing of the electron neutrino with another active or sterile neutrino, as well as their squared mass differences are nonvanishing parameters generating neutrino oscillations.

Nevertheless, there is no reason, in principle, for not considering three generation neutrino oscillations in the interpretation of solar neutrino data [3]. Moreover, some experimental evidence has been accumulated for the existence of three light neutrinos [7].

The main difficulty concerning three generation neutrino oscillations is connected with the appearance of too many free oscillating parameters, namely, three mixing angles, one phase and two neutrino squared mass differences which can not be phenomenologically fixed taking into account only oscillation effects. In fact, when three neutrino generation oscillation phenomenon is considered in literature such problem is usually overcome fixing arbitrarily some of the free parameters [3].

Nevertheless, in Ref. [8], assuming the minimal extension of the standard electroweak model [9], when only three right-handed neutrino singlets are introduced to generate Dirac neutrino masses, two mixing angles and one neutrino mass were constrained by experimental data from accelerators, reactors and underground facilities. It is interesting to emphasize that these angles and mass not include the values of these parameters which would lead to the limit where three generation case behaves as a usual two generation oscillating system. From this result we can conclude that the three generation oscillations are a phenomenological necessity.

In this paper we analyse the three generation neutrino oscillations as a solution to the solar neutrino problem taking into account the phenomenological constraints from Ref. [8]
for some of the mixing angles and neutrino masses entering the three generation neutrino oscillation phenomenon.

Considering the very little restrictive hierarchy among neutrino masses $m_1 \lesssim m_2 \ll m_3$, using the Maiani parametrization \cite{7,10} of the mixing matrix

$$V = \begin{pmatrix}
  c_\theta c_\beta & s_\theta c_\beta & s_\beta \\
  -s_\theta c_\gamma - c_\theta s_\gamma s_\beta & c_\theta c_\gamma - s_\theta s_\gamma s_\beta & s_\gamma c_\beta \\
  s_\theta s_\gamma - c_\theta c_\gamma s_\beta & -c_\theta s_\gamma - s_\theta c_\gamma s_\beta & c_\gamma c_\beta
\end{pmatrix} \quad (1)$$

and investigating $\tau$ leptonic decay, pion decay, $Z^0$ invisible width, $\tau$ decay end–point into five pions and assuming world average data for the ratio $G_\tau/G_\mu$, the lower masses $m_1, m_2$ and one mixing angle $\theta$ remain undetermined, but $m_3 \sim 165$ MeV, $11.54^\circ < \beta < 12.82^\circ$ and $\gamma < 4.05^\circ$. Thus, we have one mixing angle $\theta$ and the two lightest neutrino squared mass difference $\delta m^2 = m_2^2 - m_1^2$ to be determined in neutrino oscillation processes such as the solar neutrino issue. Note that the phase which would appear in Eq. (1) is irrelevant for the case of solar neutrino so we have ignored it.

The solar neutrino problem has been confirmed by many experiences. In the following we will consider experimental data from Homestake, Kamiokande and $^{71}Ga$ experiences \cite{2}. Each of them are sensitive to different types of neutrinos. In the Homestake experience 78% of the neutrinos detected are the so-called $^8B$ neutrinos, 14% are $^7Be$ ones and about 4% of them are $^{15}O$ neutrinos \cite{11}. Other sources of solar neutrinos contribute significantly less to the total theoretical capture rate in the $Cl$-detector than the total uncertainties involved in the calculations and will be neglected. This approximation is justified since for the case of $Cl$ detector the theoretical uncertainties are about 33%. Kamiokande is sensitive only to the $^8B$ neutrinos. And, finally, neutrinos detected by $^{71}Ga$ experiences are composed by 26%, 11%, 5% and 54% of $^7Be$, $^8B$, $^{15}O$ and $pp$ neutrinos, respectively. Again we are neglecting sources of neutrinos which contribute to the total $Ga$ detector rate significantly less than the total theoretical uncertainties (15%).

Neutrinos produced in different reactions have different energies. While $^7Be$ neutrinos
are almost monochromatic \cite{12}, neutrinos produced in other source-reactions have different energy spectra \cite{11} which have to be considered since, as we will see in the following, the survival probability of the solar neutrinos is sensitive to their energy $E$ or their momentum $p$.

We can compare the theoretical neutrino flux ($\phi_{th}$) calculated from the solar standard model \cite{11} with the observed flux ($\phi_{exp}$) measured by each experiment \cite{2}. The ratio $R = \phi_{exp}/\phi_{th}$ is given by

\begin{align*}
R_{\text{Homestake}} &= 0.28 \pm 0.04, \\
R_{\text{Kamiokande}} &= 0.49 \pm 0.12, \text{ and for the two experiences based on } ^{71}\text{Ga detectors:} \\
R_{\text{Gallex}} &= 0.66 \pm 0.12 \text{ and } R_{\text{Sage}} = 0.58 \pm 0.11 \quad \text{(Note that these two last numbers are compatible and we will consider in the present analysis only the Gallex result and do not use the corresponding weighted average result.)}
\end{align*}

Here we introduced the survival transition probability for the electron neutrino observed at a point $x$ if neutrinos were produced deep in the Sun at a point $x_0$ in reaction $X$:

\begin{equation}
P^J(X) = \sum_{E_i > E^J_{\text{thre}}} f^X(E_i) P_{\nu_e \rightarrow \nu_e}(E_i, \delta m^2, \theta, R). \tag{2}
\end{equation}

$J = H, K$ and $G$ index indicates Homestake, Kamiokande and Gallex. The threshold energy for each one of these experiences and the energy spectrum of neutrinos produced in reaction $X$ are denoted by $E^J_{\text{thre}}$ and $f^X(E_i)$, respectively. The spectral functions $f^X(E_i)$ are given in Ref. \cite{14}.

The probability of finding a neutrino $\nu_e$ after a length $x - x_0$ if at the origin it was a $\nu_e$ is $P_{\nu_e \rightarrow \nu_e}(E, \delta m^2, \theta, x) = |\langle \nu_e(x)|\nu_e(x_0) \rangle|^2$, or explicitly

\begin{equation}
P_{\nu_e \rightarrow \nu_e}(E, \delta m^2, \theta, r) = 1 - 2 s^2_\beta s^2_\alpha \left(1 - \cos \frac{2\pi r}{L_0}\right) - 2 c^2_\beta s^2_\alpha c_\beta \left(1 - \cos \frac{2\pi r}{L_{12}}\right), \tag{3}
\end{equation}

where $r \equiv x - x_0 \approx L_\odot$, being $L_\odot$ the Earth-Sun distance. For the mass range we are considering here, neutrino oscillations occur with essentially two wavelengths. We have defined in Eq. (3) $L_{ij} = 2\pi/(E_i - E_j)$ that is, $L_{12} = 4\pi p/\delta m^2 = 2.5(p/\text{MeV})/(\delta m^2/\text{eV}^2)$ meter, and $L_{13} = L_{23} = L = 2\pi/m_3 = 1.24/(m_3/\text{MeV}) \times 10^{-12}$ meter. The shorter wavelength is of order of $10^{-15}$ meter, for $m_3 \sim 165$ MeV. With respect to the solar neutrinos $r \sim 10^{11}$ meter, with this condition we can average out the cosine term involving $L$ and Eq. (3) becomes
\[ P_{\nu_e \rightarrow \nu_e}(E, \delta m^2, \theta, r) = 1 - 2c_{\beta}^2 s_{\beta}^2 - 2s_{\beta}^2 s_{\beta} c_{\beta}^4 \left( 1 - \cos \frac{2\pi r}{L_{12}} \right). \] (4)

We can also write down the transition probability for each experience. For Homestake (H)

\[ R(\text{Homestake}) = 0.78 P^H(8B) + 0.14 P^H(7Be) + 0.04 P^H(15O). \] (5a)

The neutrino flux measured by Kamiokande facilities is not merely the electron neutrino one since detector electrons will interact with other neutrino flavors via neutral currents. For energies involved in the solar neutrino experiences, the \( \nu_e \)-electron scattering cross section is about seven times larger than other neutrino flavor (\( \nu_\mu \)-electron and \( \nu_\tau \)-electron) cross sections. Hence, for Kamiokande (K) we have

\[ R(\text{Kamiokande}) = P^K(8B) + \frac{1}{7} [1 - P^K(8B)]. \] (5b)

Finally, for \( ^{71}\text{Ga} \) detectors (G)

\[ R(\text{G}) = 0.26 P^G(8B) + 0.11 P^G(7Be) + 0.05 P^G(15O) + 0.54 P^G(pp). \] (5c)

We can introduce Eq. (4) for the electron neutrino survival probability after vacuum oscillations into Eqs. (5) using Eq. (2). And finally compare the results with the experimental ratios \( R(J) \) for each of the relevant experiments \( (J = H, K, G) \). From this procedure we can find the parameter region where oscillation effects make the theoretical values of the survival solar neutrino probability compatible with the smaller than expected solar neutrino experimental flux.

In Fig.1.a we show the average probability as a function of \( \delta m^2 \) when \( \sin^2 \theta \) runs from 0.25 (highest curve) to 0.75 (lowest curve). In Fig.1.b we show the same probability as a function of \( \sin^2 \theta \) with \( \delta m^2 \) running from \( 8 \times 10^{-11} \) (highest curve) to \( 3 \times 10^{-11} \) (lowest curve), using Eqs. (3). The allowed region in the \( \delta m^2 - \sin^2 \theta \) plane (at 95% c.l.) is displayed in Fig.1.c for each of the three experiences: Homestake (upper row), Kamiokande (middle row) and Gallex (lower row).

In Fig. 2 we show the compatibility region for the three experiences at 90% (Fig.2a) and 95% (Fig.2b) of confidence level.
We have analysed the three generation neutrino oscillations in vacuum as a possible solution to the solar neutrino problem. Fixing some of the oscillating parameters (two mixing angles and one neutrino mass) through the procedure described in Ref. [8], we come to the following conclusions. The mixing angle $\theta$ as well as the squared mass difference $\delta m^2$ remain free parameters to be used to fit the solar neutrino data and the theoretical neutrino flux. The result obtained in Fig. 2 show that the values of these parameters are $0.3 \lesssim \sin^2 \theta \lesssim 0.7$ and $3 \times 10^{-11} \text{eV}^2 \lesssim \delta m^2 \lesssim 8 \times 10^{-11} \text{eV}^2$. Interesting enough, such values are of the same magnitude of those ones found in two generation analysis [15]. This can be understood remembering that the large value of $m_3 \sim 165 \text{MeV}$ implies very short wavelength $L$ (see Eq.(3)) and consequently the effective oscillation occurs among the two lightest generations. Nevertheless, the nonvanishing value of $\beta$ in Eq.(4) guarantees that the three generation oscillation effects we are analysing differ from the usual pure two generation oscillation phenomenon. When we put $\beta = \gamma = 0$ we obtain the usual two generation results [13].

We have not addressed here the atmospheric neutrino problem because it is not a well established experimental problem. Evenmore it is not clear that is related to neutrino oscillations.

Finally, we would like to stress that although the numerical results we have obtained in this work depend on the values of the angles $\beta$ and $\gamma$ and of the mass $m_3$ we have used, our general approach remains valid even if future experimental data imply in different values for these parameters.
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FIG. 1. Using Eqs. (3) we have plotted the probability transition as a function of $\delta m^2$ (a), as a function of $\sin^2 \theta$ (b) and the contour plot at 95\% C.L. The Homestake data appears in the first row, the Kamiokande ones in the middle row and the Gallex data in the bottom row.
FIG. 2. Compatibility common region for the 3 experiences at 90% C.L. (a) and at 95% C.L. (b).
This figure "fig1.gif" is available in "gif" format from:

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