Event-triggered Adaptive Dynamic Surface Output Feedback Inverse Control for a Class of Hysteresis Nonlinear Systems

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Abstract. Aiming at the difficult problem of the nonlinear system with hysteresis, the Prandtl-Ishlinskii model is used to describe the system. A neural network-based output feedback adaptive dynamic surface inverse control (DSIC) scheme is proposed. Firstly, in order to overcome the hysteresis compensation error, an auxiliary design parameter is introduced into the observer to construct an improved high-gain k-order filter state observer to accurately estimate the unmeasurable state. Then, the inverse of the asymmetric displacement matrix of the Prandtl-Ishlinskii model is used as feedforward compensation. The unknown density function with different thresholds is not estimated, the analysis compensation error is obtained, the over-parameterization problem is effectively avoided. At the same time, the event trigger mechanism is introduced in the controller design process to discretize the continuous control signal under the condition of event triggering to improve the computational efficiency of the controller. Finally, the simulation is verified, the results shows the control scheme has higher control precision and faster control speed.

1. Introduction
In the past few decades, piezoelectric materials have been widely used in ultra-high-precision positioning systems such as micro-nano systems and metal cutting systems [1], such as scanning probe microscopes, optical calibration, diamond lathes, active vibration control, Biologically operated equipment, etc. More attention has been given to their ability to meet ultra-high precision requirements and their versatility in a variety of applications. However, the inevitable shortcoming is the hysteresis nonlinearity present in piezoelectric material actuators, because the hysteresis is non-differentiable and multi-valued, the control system lacks compensation for hysteresis, it may exhibit undesirable characteristic. A common method of dealing with hysteresis is to establish an inverse model of hysteresis and cascade it as a compensation for the actuator before the control system; another method to use a robust adaptive scheme [2-4]. For the first method, since the hysteresis is usually unknown, it is difficult to establish an analytical error expression between the hysteresis model and its corresponding inverse model. In the second method, the unknown hysteresis is divided into linear and nonlinear parts and is generally regarded as interference processing, but the system only outputs measurable, this method is not feasible because of the nonlinear part of the hysteresis [5-6]. It may be unbounded, means there may be no upper limit to the observation error. So far, in the output feedback control system, the use of the second method is still limited.
Considering the resource utilization and network channel limitation in the signal transmission process, an event trigger control strategy is used [6]. When the system is less interfered or the system has reached an ideal state, the controller is not correct. Control; the controller controls the system only the system deviates from the expected state. It can ensure the system has the ideal control performance, reduces network load and resource waste and improves the utilization of network channels.

2. Problem statement

Consider the following nonlinear systems with hysteresis inputs:

\[ \dot{x}_i = x_{i+1} + f_i(\bar{x}_i) + d_i(t), \]

\[ x_n = b_n w(u) + f_n(\bar{x}_n) + d_n(t), \]

\[ y = x_i, i = 1, \ldots, n-1, \]

Where \( \bar{x}_i := [x_1, x_2, \ldots, x_i]^T \in \mathbb{R}^i \) are State variable, \( f_i(\bar{x}_i), i = 0, 1, \ldots, n \) is unknown smooth function, \( d_i(t) \) is external interference, \( b_n \) is unknown constant, \( y \in \mathbb{R} \) is the output of the plant, \( w \in \mathbb{R} \) represents an unknown hysteresis and can be expressed in the following form,

\[ w(u) = P[u](t) \]

Where \( u \) is the input signal, \( P \) is the hysteresis operator.

For the above nonlinear systems, we made the following assumptions

A1: Disturbance \( d_i(t), i = 1, \ldots, n \) satisfy \( |d_i(t)| \leq \bar{d}_i, \) wherer \( \bar{d}_i \) are unkonw positive parameter.

A2: Expected trajectory \( y_e \) is smooth and \( y_e(0) \) can be set according to the designer, \([y_e, \dot{y}_e, \ddot{y}_e]^T\) belongs to a known compact set for all \( t \geq 0. \)

A3: \( b_n \) is known parameter, without loss of generality, considered to be greater than zero.

Remark 1: A1 shows that the interference is bounded and its widely used in robust adaptive control systems [5]. A2 is commonly used in dynamic surface control, to facilitate stability analysis and to keep all signals consistent and bounded [6]. A3 is the basic need in adaptive control.

By equation (1) we have

\[ \dot{x}_i = x_{i+1} + f_i(\bar{x}_i) + d_i(t), \]

\[ \dot{x}_n = b_n w(u) + f_n(\bar{x}_n) + d_n(t), \]

\[ y = x_i, i = 1, \ldots, n-1, \]

Where \( b_n \) is a positive parameter and satisfy \( b_n = b_n \phi(A) \).

The RBFNN shows in (4) is used to approximate the unknown smooth function in (3),

\[ f_i(\bar{x}_i) = \psi_i^T(\bar{x}_i) \Theta_i^T + \varepsilon_i \]

Where \( \varepsilon_i, i = 1, \ldots, n \) is positive constant \( \varepsilon_i = (\varepsilon_{i1}, \ldots, \varepsilon_{i1}) \).

Now substitute the (4) into (3), we can get

\[ \dot{x}_i = x_{i+1} + \psi_i^T(\bar{x}_i) \Theta_i^T + \varepsilon_i + d_i(t), \]

\[ \dot{x}_n = b_n w(u) + \psi_n^T(\bar{x}_n) \Theta_n^T + \varepsilon_n + b_n d_n(t) + d_n(t), \]

\[ y = x_i, i = 1, \ldots, n-1, \]

Converting the above form to a state space form, we have
\[
\dot{x} = Ax + \Psi^T(\xi) \vartheta^* + bu_d + B, \\
y = e_1^T x
\]

Where \( B = D_n + \varepsilon + d \), from (5) and (6) we can obtain

\[
A = \begin{bmatrix}
0 & 1 & 0 \\
0 & \ddots & \\
0 & \cdots & 1
\end{bmatrix},
\quad d = \begin{bmatrix}
d_1(t) \\
\vdots \\
d_{n-1}(t) \\
d_n(t)
\end{bmatrix},
\quad \varepsilon = \begin{bmatrix}
\varepsilon_1 \\
\vdots \\
\varepsilon_{n-1} \\
\varepsilon_n
\end{bmatrix},
\]

\[
b = \begin{bmatrix}
0 \\
\vdots \\
0 \\
b_n
\end{bmatrix},
\quad e_i = \begin{bmatrix}
x_1 \\
x_2 \\
\vdots \\
x_n
\end{bmatrix},
\quad x = \begin{bmatrix}
x_1 \\
x_2 \\
\vdots \\
x_n
\end{bmatrix},
\quad D_n = \begin{bmatrix}
0 \\
\vdots \\
0
\end{bmatrix},
\]

\[
\vartheta^* = \begin{bmatrix}
\vartheta_1^* \\
\vdots \\
\vartheta_{n-1}^* \\
\vartheta_n^*
\end{bmatrix} \in \mathbb{R}^{n},
\Psi^T(\xi) = \begin{bmatrix}
\psi_1 \\
\vdots \\
\psi_n
\end{bmatrix}
\]

Where \( \psi_i = [\psi_{i,1}(\xi_1), \ldots, \psi_{i,N}(\xi_n)] \), \( \psi_n = [\psi_{n,1}(\xi_1), \ldots, \psi_{n,Nn}(\xi_n)] \) \( N_i, i = 1, \ldots, n \) are the dimension of the weight vector in the RBF neural network.

**Remark 2:** The control objective is establish an adaptive neural network control system based on output feedback dynamic surface inversion for a class of nonlinear hysteresis systems; the output \( y \) has a good tracking effect and all signals in the control system are semi-global and ultimately uniformly bounded.

Let \( A_0 = A - qe_1^T \), we rewritten (6) as

\[
\dot{x} = A_0 x + qy + \Psi^T(\xi) \vartheta^* + bu_d + B \\
y = e_1^T x
\]

Where

\[
A_0 = A - qe_1^T = \begin{bmatrix}
-q_1 & 1 \\
-q_2 & \ddots \\
\vdots & \ddots & \ddots \\
-q_n & 0 & \cdots & 0
\end{bmatrix},
\quad q = \begin{bmatrix}
q_1 \\
q_2 \\
\vdots \\
q_n
\end{bmatrix}
\]

Where \( A_0 \) is Hurwitz matrix, \( q \) is a positive parameter.

From the literature [6], the following high-gain K-order filter observers are available to estimate the unmeasurable state \( x \) in equation (8)

\[
\dot{v}_0 = kA_0v_0 + \Phi^{-1}e_n u_d \\
\dot{\xi}_0 = kA_0\xi_0 + kqy \\
\dot{\Xi} = kA_0\Xi + \Phi^{-1}\Psi^T
\]

\[
(10) \\
(11) \\
(12)
\]
Where \( K \geq 1 \) is a positive design parameter. \( e \) is the \( n \)th coordinate vector. Let 
\[ \Phi = \text{diag} \{1, k, \ldots, k^{n-1}\} \]  
and according to the equation (10) to (12) we can obtain an estimate of the following state vectors

\[ \hat{x} = \Phi \xi_0 + \Phi \hat{b} \dot{v}_0 + \Phi \Xi \hat{\theta} \]  
(13)

Define the following estimation error \( \epsilon = x - \hat{x} \) we have 
\[ \dot{\epsilon} = A \epsilon - k \Phi q \epsilon_i + B \]  
(14)

Where \( \epsilon_i \) is the first value of \( \epsilon \) and already defined in equation (6).

**Lemma 1:** Considering the above high gain \( k \)-order filter state observer, the following quadratic functions are now defined

\[ V_\epsilon := \epsilon^T P \epsilon \]  
(15)

Where \( P = (\Psi^{-1})^T \bar{P} \Psi^{-1}, \bar{P} = \bar{P}^T > 0 \) are positive definite matrix and satisfies

\[ A_1^T \bar{P} + \bar{P} A_0 = -2I \]  
(16)

Let

\[ \zeta_\epsilon := \frac{k}{\lambda_{\text{max}}(\bar{P})} \]  
\[ \delta_\epsilon := k \left( \frac{\| \bar{B} \|_{\text{max}}}{k^n} \right)^2 \]  
(17)

\( \| \bar{B} \|_{\text{max}} \) is the maximum value of \( \| \bar{B} \| \), without any \( k > 1 \) we have.

\[ \dot{V}_\epsilon \leq -\zeta_\epsilon V_\epsilon + \delta_\epsilon \]  
(18)

According to equation (13) we can get the true state estimated to

\[ \hat{x} = \Phi \xi_0 + \Phi \hat{b} \dot{v}_0 + \Phi \Xi \hat{\theta} \]  
(19)

Where \( \hat{b}_x \) is the estimated value of \( b_x \), \( \hat{\theta} \) is the estimated value of \( \theta \).

**Remark 2:** The established high-gain \( K \)-order filter observer is used to deal with bounded terms \( B \). on the one hand, the observer error can be arbitrarily small. on the other hand by properly selecting design parameters and matrices.

3. **Adaptive DSC Design**

In this section the controller using an adaptive dynamic surface inverse control scheme is prososed. The control structure is shown in Figure 1.

**Step 1:** Define the first error surface \( S_1 \),

\[ S_1 = y - y_r \]  
(20)
Where \( y_r \) is expected trajectory, according (5) we can get the derivative of \( S_1 \).

\[
\dot{S}_1 = \dot{y} - \dot{y}_r
= x_2 + \psi^T_i \left( \xi \right) \theta^* + \varepsilon_1 + d_i \left( t \right) - \dot{y}_r
\]

According to (14), we can get,

\[
x_2 = \hat{x}_2 + \varepsilon_2
= k \xi_{0,2} + k b_v \nu_{0,2} + k \Xi_{(2)} \theta^* + \varepsilon_2
\]

Where \( \Xi \) is the second vector of \( \Xi_{(2)} \), substitute the (22) into (21), we can get,

\[
\dot{S}_1 = k \xi_{0,2} + b_v k \nu_{0,2} + k \Xi_{(2)} \theta^* + \theta^* \psi_i \left( \xi \right) + \varepsilon_1 + d_i \left( t \right) - \dot{y}_r + \varepsilon_2
\]

We rewritten the equation (23) can get,

\[
\dot{S}_1 = k \xi_{0,2} + b_v k \left( \nu_{0,2} - \overline{v}_{0,2} \right) + b_v k \overline{v}_{0,2}
+ \theta^* \left( k \Xi_{(2)} + \Psi_{(1)} \right)^T + \varepsilon_1 + d_i \left( t \right) - \dot{y}_r + \varepsilon_2
\]

Let \( \overline{v}_{0,2} \) is a virtual control signal,

\[
\overline{v}_{0,2} = \hat{\zeta} \overline{v}_{0,2}
\]

Where \( \zeta \) is estimated of \( \zeta = 1 / b_v \) and we can get,

\[
\overline{v}_{i,2} = \left( -l_i S_1 - \hat{\theta}^* \left( k \Xi_{(2)} + \Psi_{(1)} \right) - k \zeta_{0,2} + \dot{y}_r \right) / k
\]

Where \( l_i \) is a positive design parameter, \( \hat{\theta} \) is the estimated of \( \theta^* \). We choose adaptive law \( \hat{\zeta}, \hat{\theta} \) as follow,

\[
\dot{\zeta} = -\gamma_\zeta \left( k \overline{v}_{0,2} \theta^* + \sigma_\zeta \hat{\zeta} \right)
\]

\[
\dot{\theta} = \gamma_\theta \left( \left( k \Xi_{(2)} + \Psi_{(1)} \right)^T \theta - \sigma_\theta \theta \right)
\]

Let \( \overline{v}_{0,2} \) through the first-order low pass filter,

\[
z_2 + \nu_2 \dot{z}_2 = \overline{v}_{0,2}, \quad \overline{v}_{0,2} \left( 0 \right) = \overline{v}_{0,2} \left( 0 \right)
\]

Where \( \nu_2 \) is a positive parameter, \( z_2 \) is new state variable.

**Step 2:** Define the first error surface \( S_2 \)

\[
S_2 = y - y_r
\]

According to (29) we can get the derivative of \( S_2 \).

\[
\dot{S}_2 = -k q_2 v_{0,1} + k v_{0,3} - \dot{z}_2
= k \left( v_{0,3} - \overline{v}_{0,3} \right) + k \overline{v}_{0,3} - k q_2 v_{0,1} - \dot{z}_2
\]

Let \( \overline{v}_{0,3} \) is a virtual control signal

\[
\overline{v}_{0,3} = \left( -l_2 S_2 + k q_2 v_{0,1} + \dot{z}_2 - \hat{b}_v k S_1 \right) / k
\]

Where \( l_2 \) is a positive design parameter, we choose adaptive law \( \hat{b}_v \) as follow,

\[
\dot{\hat{b}}_v = \gamma_\theta \left( k S_1 S_2 - \sigma \hat{b}_v \right)
\]
Let $\bar{v}_{0.3}$ throught the first-order low pass filter
\begin{equation}
    z_2 + \nu_2 \dot{z}_2 = \bar{v}_{0.2}, \quad z_2 (0) = \bar{v}_{0.2} (0)
\end{equation}
Where $\nu_1$ is a positive parameter, $z_3$ is new state variable

**Step 3:** $i \quad (3 \leq i \leq n-1)$ Define the $i$ th error surface
\begin{equation}
    S_i = v_{0,i} - z_i
\end{equation}
According to (34) the derivative of $S_i$.
\begin{equation}
    \dot{S}_i = -k q_{i} v_{0,i} + k v_{0,s+1} - \dot{z}_i
\end{equation}
Let $\bar{v}_{0.2}$ is a virtual control signal
\begin{equation}
    \bar{v}_{0,i+1} = (-l_i S_i + k q_{i} v_{0,1} + \dot{z}_i) / k
\end{equation}
Where $l_i$ is a positive design parameter, we choose adaptive law $\bar{v}_{0,i+1}$ as follow
\begin{equation}
    \dot{\hat{b}}_\lambda = \gamma_0 (k S_i S_i - \sigma_0 \dot{\bar{b}}_\lambda)
\end{equation}
Let $\bar{v}_{0,i+1}$ throught the first-order low pass filter
\begin{equation}
    z_2 + \nu_2 \dot{z}_2 = \bar{v}_{0.2}, \quad z_2 (0) = \bar{v}_{0.2} (0)
\end{equation}
Where $\nu_{i+1}$ is a positive parameter, $z_{i+1}$ is new state variable

**Step n:** Define the $n$ th error surface
\begin{equation}
    S_n = v_{0,n} - z_n
\end{equation}
According to (34) we can get the derivative of $S_n$.
\begin{equation}
    \dot{S}_n = -k q_{n} v_{0,1} + k^{1-n} v_{0,n} - \dot{z}_n
\end{equation}
Let $\bar{v}_{0,n}$ is a virtual control signal
\begin{equation}
    \bar{v}_{0,n} = k^{n-1} (k q_{n} v_{0,1} - l_n S_n + \dot{z}_n)
\end{equation}
Where $l_n$ is a positive design parameter

Next, an event triggering mechanism will be introduced on the basis of the above control structure. First, the following adaptive controller is given:
\begin{equation}
    \ddot{u}_d (t) = \bar{v}_{0,n} - \bar{g} \tanh \left( \frac{k l^N S_n \bar{g}}{\rho} \right)
\end{equation}
The trigger event:introduce as follow
\begin{equation}
    \ddot{u}_d (t) u_d (t) = \ddot{u}_d (t_j), \forall t \in [t_j, t_{j+1})
    \quad t_{j+1} = \inf \left\{ t > t_j \mid e (t) \geq g \right\}, t_1 = 0
\end{equation}
Where $e(t) = \ddot{u}_d (t) - u_d (t)$ is measurement error, $\rho, g$ and $\bar{g} > g$ are positive design parameters, $t_j, j \in z^+$ is the controller update time.

**Remark 3:** At the moment $t_{j+1}$, the control signal $u_d (t)$ will be updated to $\ddot{u}_d (t_{j+1})$ , on the other hand ,at the moment $t \in [t_j, t_{j+1})$, the control signal remains unchanged, i.e $u_d (t) = \ddot{u}_d (t_j)$. 
4. Stability analysis

In this section, the stability analysis for the proposed DSC scheme will be presented, we define

\[
\dot{y}_2 = \dot{z}_2 - \dot{v}_2 = -\frac{y_2}{u_2} - \dot{\zeta} \left(-k_2 s_{0,2} - l_2 S_1 + \dot{y}_r - \hat{\Theta}^T \left(\Psi (1) + k \Xi (2)\right)^T \right) / k
\]

\[
-\dot{\zeta} \left(-l_2 \hat{S}_1 - k_2 \dot{s}_{0,2} - \left[\hat{\Theta}^T \left(\Psi (1) + k \Xi (2)\right)^T\right] + \dot{y}_r\right) / k
\]  \hspace{1cm} (44)

\[
\dot{y}_{i+1} = -\frac{y_{i+1}}{u_{i+1}} + \frac{1}{k} l_{i+1} \hat{S}_1 - q_i v_{i+1} - \frac{1}{k} \xi_i, i = 3, \ldots, n-1.
\]  \hspace{1cm} (45)

Where \( v_{0,2} \) and \( v_{0,n} \) are given by (26) and (32), according to (31) and (38), we can get

\[
\dot{z}_2 = \frac{v_{0,2} - z_2}{u_2} = \frac{v_{0,2} - y_2 - v_{0,2}}{u_2} = -\frac{y_2}{u_2}
\]  \hspace{1cm} (46)

Therefore, the derivative of (45) can be written as

\[
\dot{y}_2 = \dot{z}_2 - \dot{v}_2 = -\frac{y_2}{u_2} - \dot{\zeta} \left(-k_2 s_{0,2} - l_2 S_1 + \dot{y}_r - \hat{\Theta}^T \left(\Psi (1) + k \Xi (2)\right)^T \right) / k
\]

\[
-\dot{\zeta} \left(-l_2 \hat{S}_1 - k_2 \dot{s}_{0,2} - \left[\hat{\Theta}^T \left(\Psi (1) + k \Xi (2)\right)^T\right] + \dot{y}_r\right) / k
\]  \hspace{1cm} (47)

Now define the following Lyapunov function:

\[
V = \frac{1}{2} \sum_{i=1}^{\infty} s_i^2 + \frac{1}{2} \sum_{i=1}^{\infty} y_{i+1}^2 + \frac{b_{\lambda}}{2} \zeta^2 + \frac{1}{2} \gamma_\phi \bar{\theta}^2 + \frac{1}{2} \gamma_\theta \bar{\theta}^2 + V_i
\]  \hspace{1cm} (48)

We can get the derivative of \( V \) as follow

\[
\dot{V} \leq -\left(l_2 - \frac{2 + b_{\lambda} k}{2} - \frac{k_2^2}{2}\right) S^2_2 - (l_2 - k) S^2_2 + \sum_{i=3}^{\infty} \left[l_2 - \frac{3k}{2}\right] S^2_i
\]

\[
-\left(l_2 - \frac{k}{2}\right) S^2_2 - \frac{\sigma_\phi}{2} \bar{\theta}^2 \bar{\theta} + \frac{\sigma_\phi}{2} \bar{\theta}^2 \bar{\theta} + \frac{b_{\lambda} \sigma_\zeta}{2} \zeta^2 + \frac{b_{\lambda} \sigma_\zeta}{2} \zeta^2
\]

\[
- \frac{\sigma_\phi}{2} \bar{\theta}^2 + \sigma_\theta \bar{\theta} - \left(\frac{1}{\nu_2} - \frac{M_{2,2}^2}{2k} - \frac{b_{\lambda} k^2}{4}\right) y_2^2
\]

\[
- \sum_{i=2}^{\infty} \left[\frac{1}{\nu_{i+1}} - \frac{M_{i+1}^2}{2k} - \frac{k}{4}\right] y_{i+1}^2 - \left(\frac{1}{\lambda_{\text{max}}(\bar{P})} - \frac{1}{2\lambda_{\text{min}}(\bar{P})}\right) V_i
\]  \hspace{1cm} (49)

Choose the following design parameters below
\[ k \geq \lambda_{\text{max}}(\bar{P})C_1 + \frac{\lambda_{\text{max}}(\bar{P})}{2\lambda_{\text{min}}(\bar{P})}, \]

\[ l_i \geq 2 + b_i k + \frac{k^2}{2} + C_1, l_2 \geq k + C_1, \]

\[ l_i \geq \frac{3k}{2} + C_1, i = 2, \ldots, n - 1, \]

\[ l_n \geq \frac{k}{2} + C_1, \frac{1}{\nu_2} \geq \frac{b^2 k^2}{4} + \frac{M^2}{2\kappa} + C_1, \]

\[ \frac{1}{\nu_{l+1}} \geq \frac{k}{2} + \frac{M^2_{l+1}}{2\kappa} + C_1, i = 2, \ldots, n - 1, \]

Substituting equations (48) and (50) into (49), we have

\[ V \leq -2CV + C_2 \]

Where \( C_2 = (n - 1)\kappa/2 + \sigma_d\theta^t \theta^t/2 + b_i \sigma_x \xi^2 /2 + \sigma_d d_i^2/2 + (e_i^2 + d_i^2) /2 + 0.2785\rho \) is a positive design parameter, and satisfy \( C_i \geq C_2 / 2\rho \)

Let \( S_i(0) = 0, S_i(0) = 0, i = 3, \ldots, n, y_i(0) = 0, i = 2, \ldots, n \), we can get the upper boundary \( V(t) \)

\[ V(t) \leq \frac{C_2}{2C_1} + \frac{1}{k^2} \lambda_{\text{max}}(\bar{P})\|f(0)\|^2 \]

It can be further seen from the above equation, the \( L_\infty \) norm tracking error is shown in (52).

\[ \|S_i\|_\infty = \sup_{t \geq 0} |S_i| = \|x_i - y_i\|_\infty \leq \sqrt{2V}\]

According to (53), the appropriate selection of design parameters \( \|S_i\|_\infty \) can be arbitrarily small. It is now proved the event triggering mechanism, exist \( t^* > 0 \) to make time interval \( \{t_{j+1} - t_j\} > t^* \), \( \forall j \in z^+ \) have a lower bound, and according to (42) we can get

\[ \hat{\dot{u}}_d = \hat{\dot{v}}_{0,n} = \frac{\sqrt{g^2 S_n}}{\rho \cosh^2 \left( \frac{k^{1-n} S_n \sqrt{g}}{\rho} \right)} \]

From (54) we know \( \hat{\dot{u}}_d \) is a continuous function, so \( |\hat{\dot{u}}_d| \) is bounded.

\[ \hat{\dot{u}}_d = \hat{\dot{v}}_{0,n} = \frac{\sqrt{g^2 S_n}}{\rho \cosh^2 \left( \frac{k^{1-n} S_n \sqrt{g}}{\rho} \right)} \]

5. Simulation Results

This section will apply a second-order hysteresis nonlinear system to verify the effectiveness of the above control method.

Consider the following nonlinear system with an unknown hysteresis input
\[
\begin{align*}
\dot{x}_1 &= x_2 + 0.4\sin(x_1) x_1^2 + 0.3 x_1^2 + 0.1\cos(t) \\
\dot{x}_2 &= w + 0.8\cos(x_1) x_2 + x_1 \sin(x_1) + 0.3\sin(t)
\end{align*}
\]

(56)

Where \( w \) represents hysteresis nonlinearity. The corresponding density function is selected as

\[ p(r) = e^{-0.067r^2} \]

\( r \in [0,5] \). The goal of this simulation is to make the system output a better tracking reference signal \( y_r = 0.5 \sin(0.5t) + 0.5 \cos(t) \).

The design parameters of the state observer are selected as follows:

\[
q = \begin{bmatrix} q_1 \\ q_2 \end{bmatrix} = \begin{bmatrix} 5 \\ 3 \end{bmatrix},
\]

\[
A_0 = \begin{bmatrix} -q_1 & 1 \\ -q_2 & 0 \end{bmatrix}, \Phi^{-1} = \begin{bmatrix} 1 & 0 \\ 0 & 1/k \end{bmatrix}
\]

(57)

In the simulation, the initial value of the design parameter value of the state and the adaptive rate is selected as \( x_1(0) = x_2(0) = 0, \hat{\xi}(0) = 0, \hat{\theta}(0) = 0, \hat{\beta}_\lambda(0) = 0.1 \). Radial basis function neural network \( \psi_1(\hat{\xi}_1), \psi_2(\hat{\xi}_2) \) each of the 7 and 11 nodes is selected, and the centers of the basis functions are respectively \(-1, -0.8, -0.4, 0, 0.4, 0.8, 1\); The corresponding width is \( \eta_i = 1, i = 1, \cdots, 7 \); \( \hat{\xi}_1 = (\hat{x}_1) \) and \( \hat{\xi}_2 = (\hat{x}_1, \hat{x}_2) \). In addition,

\[
\Psi^T(\hat{\xi}) = \begin{bmatrix} \psi_1 \\ \psi_2 \end{bmatrix}
\]

\[
= \begin{bmatrix} \psi_{1,1}(\hat{\xi}_1), \cdots, \psi_{1,7}(\hat{\xi}_7) \\ \psi_{2,1}(\hat{\xi}_1), \cdots, \psi_{2,7}(\hat{\xi}_2) \end{bmatrix}
\]

(58)

The simulation results shown in figure 2-4.

Figure 2. Control signal
Figure 3. Tracking performance

Figure 4. Tracking error

6. Conclusion
The dynamic surface method can avoid the "differential explosion" problem caused by the backstepping method and reduce the computational burden. The stability analysis proves that all signals of the closed-loop system are semi-global and ultimately uniformly bounded.

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