Transformations of triangle ladder diagrams

Igor Kondrashuk and Alvaro Vergara

Departamento de Ciencias Básicas, Universidad del Bío-Bío,
Campus Fernando May, Casilla 447, Chillán, Chile

Abstract

It is shown how dual space diagrammatic representation of momentum integrals corresponding to triangle ladder diagrams with an arbitrary number of rungs can be transformed to half-diamonds. In paper arXiv:0803.3420 [hep-th] the half-diamonds were related by conformal integral substitution to the diamonds which represent the dual space image of four-point ladder integrals in the four-dimensional momentum space. Acting in the way described in the present paper we do not need to use the known result for diamond (four-point) diagrams as an external input in deriving relations of arXiv:0803.3420 [hep-th], however, that result for the diamond diagram arises in the present proof as an intermediate consequence in a step-by-step diagrammatic transformation from the triangle ladder diagram to the half-diamond diagrams.

Keywords: UD functions.
1 Introduction

Triangle ladder diagrams were initially investigated in the momentum space in Refs. [1, 2, 3]. The result is UD functions. In the momentum space integration is done over internal momenta that run in loops of the graph. However, the same diagram can be considered in the position space with integrals taken over coordinates of internal vertices in the graph [4, 5]. The results of the integrations in the position space and in the momentum space are related by Fourier transform. In Refs. [4, 5] the position space representation of the ladder diagrams has been investigated and it has been found that the result is the same UD functions. This means that the Fourier transform of the UD function are the same UD functions, and the correspondence is one-to-one, namely the UD function with number \( n \) transforms to the UD function with number \( n \) [4, 5]. These relations follow ST identity [6] - [11] which was studied for \( \mathcal{N} = 4 \) supersymmetric Yang-Mills theory in Refs. [12] - [21].

In deriving relations of Ref. [5] the dual graphical representation of the four-point ladder diagram in the form of “diamond” found in Ref. [22, 23] has significantly been used as an external input in order to relate half-diamonds to the UD functions \( \Phi^{(n)}(x, y) \). In this letter we show how to perform analysis of Ref. [5] starting with the three-point ladder diagrams and transforming them via dual space images to the half-diamonds. Passing several steps in the transformation procedure from the ladder diagrams to the half-diamond diagrams, we obtain the diamond diagrams at an intermediate step. However, we do not use the known result of Refs. [22, 23] for the diamond diagram. In contrary, that result is reproduced via dual space image of the triangle ladder diagram in the momentum representation.

The paper is organized as follows. In Section 2 the iterative definition of UD functions is recalled. In Section 3 dual space is reviewed and dual space representation of momentum triangle ladder diagrams is considered as a particular example. In Section 4 conformal substitution is analysed for the most simple example. In Section 5 the conformal substitution in the ladder integrals is applied to an arbitrary number of rungs. In Conclusion we write the formula for the Fourier transform of UD functions that follows the equations derived in Ref. [5] and discuss its particular features.

2 Definition of UD functions

In Refs. [2, 3] the definition of the UD functions has been done in the momentum space. For example, the result for the three-rung diagram presented in Fig. (1) is a function \( C^{(3)}(p_1^2, p_2^2, p_3^2) \) of three independent Lorentz-invariant variables \( p_1^2, p_2^2, p_3^2 \), constructed from the external momenta of the diagram which satisfy the conservation law \( p_1 + p_2 + p_3 = 0 \). A useful parametrization for them is

\[
\begin{align*}
  p_1 &= q_3 - q_2, \\
  p_2 &= q_1 - q_3, \\
  p_3 &= q_2 - q_1.
\end{align*}
\]

Three four-dimensional vectors \( q_1, q_2, q_3 \) are independent [1, 2, 3].
A ladder diagram with an arbitrary number of rungs is represented in Fig.(2), and the result for $n$ rung diagram $C^{(n)}(p_1^2, p_2^2, p_3^2)$ is expressed in terms of UD function with number $n$. All the functions $\Phi^{(n)}\left(\frac{p_1^2}{p_3^2}, \frac{p_2^2}{p_3^2}\right)$ were calculated in Refs. [1, 2, 3] explicitly. The iterative definition can be seen in Fig.(2) and is given by the equation

$$C^{(n)}(p_1^2, p_2^2, p_3^2) = \int d^4r_n \frac{C^{(n-1)}((p_1 + r_n)^2, (p_2 - r_n)^2, p_3^2)}{(p_1 + r_n)^2(p_2 - r_n)^2r_n^2}. \quad (2)$$

3 Dual representation of the momentum integrals

In Introduction the relation between the position space representation (p.s.r.) of some diagram and its momentum space representation (m.s.r.) is explained. In addition to these representations a dual space representation (d.s.r.) [24, 22, 23] exists. The dual space is not a physical
space. However, it is useful mathematical construction that in case of ladder diagrams helps to find the relation between results of calculation in the p.s.r. and in the m.s.r. for the same ladder graph.

In the d.s.r. of the integrals in the momentum space momenta that run in loops of any Feynman diagram are treated as coordinates of internal vertices of a dual diagram in an auxiliary “position” space. The line that connects two vertices reproduces a propagator in the integrand of the momentum space integral. In the massless case the line which connects any two such vertices represents the inverse square of the interval between these two vertices. This precisely reproduces the massless propagators in the momentum space integrand (2). For example, the dual representation of the momentum integral for the three-rung ladder diagram of Fig.(1) is depicted in Fig.(3). Graphically, the dual space diagram is another schematic representation of the same momentum integral.

After re-parametrization according to Eqs.(1) we make a shift of the integration variables \( r_1, r_2, r_3 \) of the l.h.s. of Fig.(3) by the external value \( q_3 \). As the result of this shift we obtain the r.h.s. of Fig.(4). The new variables of integrations (after the shift) \( r'_1, r'_2, r'_3 \) are related to the
initial variables as \( r_1 = r'_1 - q_3 \), \( r_2 = r'_2 - q_3 \), and \( r_3 = r'_3 - q_3 \). The l.h.s. of Fig.(4) and the r.h.s. of Fig.(4) are equal, and they are equal to the l.h.s. of Fig.(5) in which \( N = 1, 2, 3 \) stands for \( q_N = q_1, q_2, q_3 \). In this notation and in according to the definition of three-rung UD function of Fig.(1) the result for the diagram of Fig.(5) should be written as

\[
\frac{1}{[12]^3} \Phi^{(3)} \left( \begin{array}{c} [23] \\ [12] \\ [12] \end{array} \right).
\]

In this paper concise notation \([Nq] = (r_N - q)^2\) of Ref.[16] for squares of intervals between two points of four-dimensional space is used.

However, the diagram of Fig.(5) in the dual space can be viewed as a diagram in the position space generated for another field theory. The letters \( q_1, q_2, q_3 \) corresponding to the external points of the d.s.r. can be replaced with the letters \( x_1, x_2, x_3 \) corresponding to any three distinct points in the position space. The important feature of the construction is that in four space-time dimensions the propagator of scalar massless field between two points in the position space is square of the inverse interval between these two points. This happens in four dimensions only [25, 26]. This correspondence allows the interpretation the dual space diagram as a position space diagram with massless scalar propagators. The concise notation of Ref.[16] for the space-time intervals is used in the rest of the paper, namely \([Nx] = (x_N - x)^2\) and \([12] = (x_1 - x_2)^2\), that is, \( N = 1, 2, 3 \) stands for \( x_N = x_1, x_2, x_3 \).

4 Conformal substitution

Now we look at the diagram of the l.h.s. of Fig.(5) as at a diagram in the position space. Our purpose is to transform this diagram to the half-diamond form of Ref.[4, 5]. An important tool to reach this purpose is a conformal substitution in the integrands. To demonstrate how to use it, we can consider the simplest example of the first UD function depicted in Fig.(6). This
\[ 3 = \frac{1}{12} \Phi^{(1)} \begin{pmatrix} [23] \\ [12] \end{pmatrix}, \begin{pmatrix} [31] \\ [12] \end{pmatrix} \]

Figure 6: First UD function in the position space

picture corresponds to the integral definition of the first UD function \(^1\) of Ref.[2],

\[ J(1, 1, 1) = \int d^4x \frac{1}{(x_1^2 - x_2^2)(x_1^2 - x_3^2)} \equiv \int d^4x \frac{1}{[1x_1][2x][3x]} = \frac{1}{12} \Phi^{(1)} \begin{pmatrix} [23] \\ [12] \end{pmatrix}, \begin{pmatrix} [31] \\ [12] \end{pmatrix} = \frac{1}{31} \Phi^{(1)} \begin{pmatrix} [12] \\ [31] \end{pmatrix}, \begin{pmatrix} [23] \\ [31] \end{pmatrix}. \]

The conformal substitution for each vector of the integrand (including the external vectors) is

\[ x_\mu = \frac{x_\mu'}{x'}^2, \quad x_{1\mu} = \frac{x_{1\mu}'}{x_1'} \Rightarrow [x] = \frac{x'[1']}{[x'][1']}; \quad [x] = x^2, \quad [1] = x_1^2 \]  

(3)

Figure 7: New leg to fourth point appears due to Jacobian of conformal substitution in the internal vertex

Making the conformal transformation on the l.h.s. and on the r.h.s. of equation in Fig.(6),

\[ \int d^4x' \frac{1}{[1'x'][2'x'][3'x']} = \frac{1}{12} \Phi^{(1)} \begin{pmatrix} [23] \\ [12] \end{pmatrix}, \begin{pmatrix} [31] \\ [12] \end{pmatrix}. \]

we have

\[ [1'][2'][3'] \int d^4x' \frac{x'^3}{[1'x'][2'x'][3'x'][x']^4} = \frac{1'}{1'2'} \Phi^{(1)} \begin{pmatrix} [2'3'] \\ [1'2'] \end{pmatrix}, \begin{pmatrix} [1'] \\ [1'2'] \end{pmatrix} \begin{pmatrix} [2'] \\ [1'2'] \end{pmatrix}, \begin{pmatrix} [3'] \\ [1'2'] \end{pmatrix}, \begin{pmatrix} [2'] \\ [1'2'] \end{pmatrix} \]  

(4)

\(^1\)Our definition for UD functions is \( \Phi^{(L)}_{New} = (\pi^2)^L \Phi^{(L)}_{Old} \), where \( \Phi^{(L)}_{New} \) is \( \Phi^{(L)} \) of this paper, and \( \Phi^{(L)}_{Old} \) is the original UD function \( \Phi^{(L)} \) of Refs. [2, 3].
and after shifting the variables
\[ x' = x'' - x''_4, \quad x'_1 = x''_1 - x''_4, \quad x'_2 = x''_2 - x''_4, \quad x'_3 = x''_3 - x''_4, \]

Eq.(4) takes the form
\[ \int d^4x'' \frac{1}{[1''x'']^2[2''x'']^3[3''x'']^4} = \frac{1}{[1''2'']^3[3''4'']} \Phi^{(1)} \left( \left[ \begin{array}{c} 2''3'' \\ 1''4'' \end{array} \right], \left[ \begin{array}{c} 3''1'' \\ 2''4'' \end{array} \right] \right). \tag{5} \]

Omitting double prime symbols we can depict Eq.(5) in Fig.(7).

The creation of a new fourth factor in the denominator of integral with help of Jacobian of the conformal substitution is in some sense an opposite trick to the uniqueness method of Refs. [27, 28, 25, 26]. The uniqueness method is based on elimination of one of the three factors in the denominator of the unique integral due to the Jacobian of the conformal substitution. That trick of elimination should be done in the Euclidean space in order to avoid possible difficulties with the imaginary part of the causal massless propagators. The Euclidean space metrics can be obtained by Wick rotation [2] from Minkowski metrics. After the integration according to the uniqueness method the Wick rotation can be done back in order to recover the signature of the Minkowski metrics. This means that a small imaginary part should be added to the square of spacetime distances in the denominator.

The same philosophy can be applied to the constructions done in the present paper. All the formulas and figures should be understood in the Euclidean space. All the transformations with creating and eliminating of a new propagator are done in the Euclidean space in analogy with the uniqueness method. The Minkowski metrics signature can be recovered with help of the Wick rotation in each of the integrations in the internal vertices of graph. A small imaginary part should be added to the square of spacetime distances which are parts of the arguments of the UD functions (the arguments are fractions of the spacetime distances) in the formulas of the present paper on the r.h.s. of the Figures.

For example, Fig.(6) and Fig.(7) are related in the Euclidean space before the Wick rotation due to the conformal substitution. After the Wick rotation a small imaginary parts appear in the propagators on the l.h.s. and in the numerators and denominators of the fractions of spacetime distances on the r.h.s. in the arguments of the UD functions. Both sides of Fig.(7) can be represented as a sum of real and imaginary parts. This imaginary part on the r.h.s. of Fig.(7) has been evaluated in Ref.[29] via Mellin-Barnes transformation.

\[ \bigotimes \left[ \begin{array}{c} 1'' \\ 2'' \\ 3'' \end{array} \right] = \frac{\left[ \begin{array}{c} 1'' \\ 2'' \end{array} \right] \Phi^{(1)} \left( \left[ \begin{array}{c} 2''3'' \\ 1''4'' \end{array} \right], \left[ \begin{array}{c} 3''1'' \\ 2''4'' \end{array} \right] \right)}{\left[ \begin{array}{c} 1''2'' \\ 3''4'' \end{array} \right]} \]

Figure 8: New leg in three-point function due to Jacobian of conformal substitution in the internal vertices
As an example, we consider transformation of the diagram depicted in Fig.(5) with three points of integration. We generalize the trick of Eq. (4) and Eq. (5) and obtain the formula of Fig.(8). After making simple algebra and removing the prime symbols the formula of Fig.(8) transforms to formula of Fig.(9).

In order to obtain Fig.(10) we make another substitution (shift of integration variables)

\[
y_i = y_i'' - x_4'' \\
x_1 = x_1'' - x_4'', \quad x_2 = x_2'' - x_4'', \quad x_3 = x_3'' - x_4''
\]

in which \(y_i\) (or \(y_i''\)) are variables of integration in the internal vertices of Fig.(9) (or Fig.(10)), \(i = 1, 2, 3\). Removing the prime symbols and putting \(x_1 = 0\) (as a particular case for which equation of Fig.(10) is valid), we obtain Fig.(11). By making conformal substitution, we remove point “0” and propagators connecting it to other internal vertices due to Jacobians of the conformal substitutions in each internal vertex of integration of Fig.(11). This results in Fig.(12). Removing the prime symbols and replacing \(x_4\) with \(x_1\) we obtain Fig.(13) from Fig.(12). The same steps from Fig.(8) to Fig.(13) can be reproduced for an arbitrary number \(n\) of points corresponding to internal vertices of integration in the line connecting points \(x_1\) and \(x_3\) (\(n\) loops in momentum space), and we obtain as a conclusion the relation depicted in Fig.(14).

The diamond diagram appears at intermediate steps from Fig.(8) till Fig.(11) from the dual image of three-point diagram in the m.s.r. However, we did not use the result of Refs. [2, 3, 23, 22] for the diamond diagram. In Fig.(9) the result for diamond diagram is obtained as a consequence of the change of variables in the arguments of UD function which is done together with the integral substitution on the l.h.s. of Fig.(8).
Figure 11: Particular case of Fig.(10)

Figure 12: Removing point "0" due to Jacobian of conformal substitution in the internal vertices

6 Conclusion

Equation in Fig.(14) combined with iterative definition (2) of the UD function produces relations between the triangle ladder diagrams and the UD functions which were found in Ref.[5]. As it was shown, the result $T_n([12], [23], [31])$ for $n$-rungs triangle ladder diagram in the position space satisfies the equations

$$\left(\partial_{(2)}^2\right)^{n-1} T_n([12], [23], [31]) = \left(-4\pi^2\right)^{n-1} \Phi^{(n+1)} \left(\frac{[12]}{[31]}, \frac{[23]}{[31]}\right),$$

$$\left(\partial_{(2)}^2\right)^n T_n([12], [23], [31]) = \left(-4\pi^2\right)^n \Phi^{(n)} \left(\frac{[12]}{[31]}, \frac{[23]}{[31]}\right),$$

$$\left(\partial_{(2)}^2\right)^n T_{n+1}([12], [23], [31]) = \left(-4\pi^2\right)^n \Phi^{(n+2)} \left(\frac{[12]}{[31]}, \frac{[23]}{[31]}, \frac{[31]}{[31]}\right).$$

These relations show that the Fourier transform of UD function with number $n$ is the same UD function with number $n$,

$$\frac{1}{[31]^2} \Phi^{(n)} \left(\frac{[12]}{[31]}, \frac{[23]}{[31]}, \frac{[31]}{[31]}\right) = \frac{1}{(2\pi)^4} \int d^4p_1 d^4p_2 d^4p_3 \delta(p_1 + p_2 + p_3) \times$$

$$\times e^{ip_2 x_2} e^{ip_1 x_1} e^{ip_3 x_3} \frac{1}{(p_2)^2} \Phi^{(n)} \left(\frac{p_1}{p_2}, \frac{p_2}{p_3}, \frac{p_3}{p_2}\right).$$

The explicit form of the function is given in Refs.[1, 2, 3],

$$\Phi^{(n)} (x, y) = \frac{1}{n!\lambda} \sum_{j=n}^{2n} \frac{(-1)^j j! \ln^{2n-j} (y/x)}{(j-n)! (2n-j)!} \left[ \Li_{j} \left( \frac{1}{\rho x} \right) - \Li_{j} \left( \frac{1}{\rho y} \right) \right],$$

in which

$$\rho = \frac{2}{1-x-y}, \quad \lambda = \sqrt{(1-x-y)^2 - 4xy}. $$
Looking at this explicit form of the UD functions $\Phi^{(n)}(x, y)$ it is difficult to imagine that they do not change their form under the Fourier transformation. However, the relations (6) were derived in Ref. [5] in diagrammatic way without any use of the explicit expression in terms of polylogarithms (8). To our knowledge, this is the unique example of the non-trivial functions (actually infinite set of functions) possessing such a property.

Nevertheless, Eq. (6) was an occasional finding of a family of functions with such a property of invariance with respect to Fourier transform. Other examples can be constructed manually. Indeed, Eqs. (7) can be cross-checked via MB transformation [19]. In the demonstration of Ref.[19] the explicit Mellin-Barnes image of the UD functions does not play any role and in principle can be replaced with any function of two complex variables with sufficiently good behaviour at complex infinity, which possesses nontrivial set of left and right residues. For example, Eq.(7) can be proved via MB transformation for the first UD function $\Phi^{(1)}(x, y)$ which does not appear in the chain of Ref. [5]. That chain of transformation starts with the second UD function $\Phi^{(2)}(x, y)$. Another example would be a combination of Appell’s hypergeometric functions of Ref.[1] which corresponds to a three-point integral of three scalar propagators with arbitrary powers in denominator.

**Acknowledgments**

This work is supported by Fondecyt (Chile) grants #1040368, #1050512 and by DIUBB grant (UBB, Chile) #082609. This paper is based on I.K.’s talks at “High energy physics in the LHC era” conference, Valparaiso (Chile), The XVIIth Oporto Meeting on Geometry, Topology and Physics “Mathematical aspects of quantum field theory”, Oporto (Portugal), and at the seminars at Departamento de Física, Universidad de Concepción (Chile), Theoretical Physics department, Karlsruhe University (Germany), Department of Mathematics, University
of Bergen (Norway), DMFA seminar, UCSC, Concepción and IMAFI seminar, Universidad de Talca (Chile). He is grateful to organizers of all these events for opportunity to present the results.

References

[1] A. I. Davydychev, “Recursive algorithm of evaluating vertex type Feynman integrals,” J. Phys. A 25, 5587 (1992).

[2] N. I. Usyukina and A. I. Davydychev, “An Approach to the evaluation of three and four point ladder diagrams,” Phys. Lett. B 298 (1993) 363.

[3] N. I. Usyukina and A. I. Davydychev, “Exact results for three and four point ladder diagrams with an arbitrary number of rungs,” Phys. Lett. B 305 (1993) 136.

[4] I. Kondrashuk and A. Kotikov, “Fourier transforms of UD integrals,” arXiv:0802.3468 [hep-th], Birkhauser book series “Trends in Mathematics”, volume “Analysis and Mathematical Physics”, B. Gustafsson and A. Vasil’ev (Eds), (2009) Birkhauser Verlag, Basel, Switzerland, 337-348

[5] I. Kondrashuk and A. Kotikov, “Triangle UD integrals in the position space,” JHEP 0808 (2008) 106 [arXiv:0803.3420 [hep-th]].

[6] A. A. Slavnov, “Ward Identities In Gauge Theories,” Theor. Math. Phys. 10 (1972) 99 [Teor. Mat. Fiz. 10 (1972) 153].

[7] J. C. Taylor, “Ward Identities And Charge Renormalization Of The Yang-Mills Field,” Nucl. Phys. B 33 (1971) 436.

[8] A. A. Slavnov, “Renormalization Of Supersymmetric Gauge Theories. 2. Nonabelian Case,” Nucl. Phys. B 97 (1975) 155.

[9] L. D. Faddeev and A. A. Slavnov, “Gauge Fields. Introduction To Quantum Theory,” Front. Phys. 50, 1 (1980) [Front. Phys. 83, 1 (1990)]; “Introduction to quantum theory of gauge fields”, Moscow, Nauka, (1988).

[10] B. W. Lee, “Transformation Properties Of Proper Vertices In Gauge Theories,” Phys. Lett. B 46 (1973) 214.

[11] J. Zinn-Justin, “Renormalization Of Gauge Theories,” SACLAY-D.PH-T-74-88, Lectures given at Int. Summer Inst. for Theoretical Physics, Jul 29 - Aug 9, 1974, Bonn, West Germany.

[12] G. Cvetiˇ c, I. Kondrashuk and I. Schmidt, “Effective action of dressed mean fields for N = 4 super-Yang-Mills theory,” Mod. Phys. Lett. A 21 (2006) 1127 [hep-th/0407251].

[13] I. Kondrashuk and I. Schmidt, “Finiteness of N = 4 super-Yang-Mills effective action in terms of dressed N = 1 superfields,” arXiv:hep-th/0411150
[14] K. Kang and I. Kondrashuk, “Semiclassical scattering amplitudes of dressed gravitons,” hep-ph/0408168.

[15] G. Cvetič, I. Kondrashuk and I. Schmidt, “On the effective action of dressed mean fields for N = 4 super-Yang-Mills theory,” in Symmetry, Integrability and Geometry: Methods and Applications, SIGMA (2006) 002 [math-ph/0601002].

[16] G. Cvetič, I. Kondrashuk, A. Kotikov and I. Schmidt, “Towards the two-loop Lcc vertex in Landau gauge,” Int. J. Mod. Phys. A 22 (2007) 1905 [arXiv:hep-th/0604112].

[17] G. Cvetič and I. Kondrashuk, “Further results for the two-loop Lcc vertex in the Landau gauge,” JHEP 0802 (2008) 023 [arXiv:hep-th/0703138].

[18] G. Cvetič and I. Kondrashuk, “Gluon self-interaction in the position space in Landau gauge,” Int. J. Mod. Phys. A 23 (2008) 4145 [arXiv:0710.5762 [hep-th]].

[19] P. Allendes, N. Guerrero, I. Kondrashuk and E. A. Notte Cuello, “New four-dimensional integrals by Mellin-Barnes transform,” arXiv:0910.4805 [hep-th].

[20] G. Cvetič and I. Kondrashuk, “QCD effective action with dressing functions: Consistency checks in perturbative regime,” Phys. Rev. D 67 (2003) 065007 [hep-ph/0210185].

[21] I. Kondrashuk, “The solution to Slavnov-Taylor identities in D4 N = 1 SYM,” JHEP 0011, 034 (2000) [hep-th/0007136].

[22] A. P. Isaev, “Multi-loop Feynman integrals and conformal quantum mechanics,” Nucl. Phys. B 662 (2003) 461 [arXiv:hep-th/0303056].

[23] J. M. Drummond, J. Henn, V. A. Smirnov and E. Sokatchev, “Magic identities for conformal four-point integrals,” JHEP 0701 (2007) 064 [arXiv:hep-th/0607160].

[24] D. I. Kazakov and A. V. Kotikov, “The method of uniqueness “Multiloop calculation in QCD” Theor. Math. Phys. 73 (1988) 1264 [Teor. Mat. Fiz. 73 (1987) 348];

[25] D. I. Kazakov, “Analytical Methods For Multiloop Calculations: Two Lectures On The Method Of Uniqueness,” JINR-E2-84-410.

[26] A.N. Vasiliev, “The field theoretic renormalization group in critical behaviour theory and stochastic dynamics”, St. Petersburg Institute of Nuclear Physics Press, 1998.

[27] M. D’Eramo, L. Peliti and G. Parisi, “Theoretical Predictions for Critical Exponents at the $\lambda$-Point of Bose Liquids,” Lett. Nuovo Cimento 2 (1971) 878.

[28] A. N. Vasiliev, Y. M. Pismak and Y. R. Khonkonen, “1/N Expansion: Calculation Of The Exponents Eta And Nu In The Order 1/N**2 For Arbitrary Number Of Dimensions,” Theor. Math. Phys. 47 (1981) 465 [Teor. Mat. Fiz. 47 (1981) 291].

[29] G. Duplančić and B. Nižić, “IR finite one-loop box scalar integral with massless internal lines,” Eur. Phys. J. C 24 (2002) 385 [arXiv:hep-ph/0201306].