The procedure for determination of the dependence of the cost of insulation materials on their thermophysical characteristics

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Abstract. Optimal insulation for the walls of the house is selected based on the material of construction and must meet several requirements. This article describes the method of justifying the dependence of the cost of heaters on their technical characteristics, based on a statistical analysis of price and thermophysical characteristics, used later in the development of optimization models for the selection and use of thermal insulation materials (heaters). To assess the quality of the resulting economic and mathematical model, the coefficient of determination was used. In order for the number of regressors (factors) not to affect the statistics of this coefficient, an adjusted coefficient of determination was also calculated, which gives a “penalty” for the factors included in the model.

Introduction

A good home is a warm home. That is why today apply a variety of insulation in the construction and renovation of buildings and structures.

The issues of energy saving and, in particular, the insulation of the external walls of buildings and structures have always attracted the attention of Russian and foreign researchers [1-20].

Livchak V.I. and Gorshkov A.S. made a description of the evolution and development of regulatory requirements for enclosing structures over an almost 100-year period. They showed how the calculated formulas, terms, dimensions of physical quantities changed and transformed [1]. Livchak V.I. gives specific suggestions for improving the methodology for calculating the heat balance of residential and public buildings, makes recommendations on how to certify buildings when conducting energy audits in accordance with international standards [2].

Gorshkov A.S. and Nemova D.V. paid much attention to the issue of improving the energy efficiency of buildings, as well as the economic efficiency of energy-saving measures [3, 4].

Yelchishcheva T.F. and Slivin R.V. presented a feasibility study of wall insulation options on the example of a 9-storey residential building [5].

The thermophysical parameters of enclosing structures and the justification of measures to improve the energy efficiency of external walls were carried out by such Russian and foreign researchers as: Petrichenko M.R., Kotov E.V., Nemova D.V., Vatin N.I., Korniyenko S.V., Tarasova D.S., Sergeev V.V., Zaborova D.D., Kukolev M.I., Mussorina M.T., Borodinec A. and etc. [6-17]. Some of these researchers considered the energy efficiency of individual series of residential buildings [18-20].
The requirements of standards for thermal insulation are formed on the basis of economic feasibility, while the required values of heat transfer resistance of enclosing structures should be achieved without a special increase in the cost of construction. But it turns out that in conditions of a temperate climate, and even more so cold, single-layer walls with thickness calculated from the bearing capacity are simply cold. This will require that the thickness of the wall (layer) must be increased in order to achieve the requirements of the standard for heat saving.

An alternative is the use of insulation. On the modern construction market there are many heat-insulating materials, differing in cost, thickness and other technical characteristics. In this case, many questions arise: what insulation to choose? What are the characteristics? But how will the characteristics and cost of insulation affect the payback period of energy saving measures?

1. Materials and Methods

Within this section, a description will be given of the procedure for determination of the dependence of the cost of insulation materials on their technical characteristics.

The main provisions of the procedure:

- The object of the study is the dependence of the specific cost of the heater (per 1 m²) on its technical characteristics (thickness, thermal conductivity, density, etc.).
- To substantiate this dependence, various samples of insulation materials from mineral (basalt) wool, polystyrene foam and extruded polystyrene foam are considered.
- It is assumed that the specific cost of a heater depends on its technical characteristics, which determines the expediency of using a linear multifactor regression model to form a dependence. This model has the form:

\[ \hat{y} = a_0 + \sum_j a_j \cdot x_j, \]

where \( \hat{y} \) – the dependent characteristic, \( a_0 \) – free member of the regression model; \( a_j \) – regression coefficients; \( x_j \) – factor \( j \) which influences the dependent characteristic.

- Justification of the dependence of the cost of insulation on its technical characteristics assumes the justification of the model parameters - coefficients \( \{a_j\} \) - according to the criterion of minimizing the sum of squares of deviations of the actual values of the cost of insulation samples from the predicted values determined with known technical characteristics \( \{x_j\} \) of samples.
- To assess the adequacy of the regression model, it is supposed to use the coefficient of determination \( R^2 \), the corresponding interval of values \((\infty;1]\) and its derived indicators.

Baseline data of the developed procedure are presented in Table 1.

| No | The name of the characteristics | Designation | Unit rev. |
|----|---------------------------------|-------------|-----------|
| 1  | The number of samples of insulation | \( n \) | units |
| 2  | Number of insulation parameters | \( m \) | units |
| 3  | Insulation sample index | \( i=1,...,n \) | - |
| 4  | Insulation parameter index | \( j=1,...,m \) | - |
| 5  | The value of parameter \( j \) for sample insulation \( i \) | \( x_{ji} \) | - |
| 6  | The actual values of the cost of insulation \( i \) | \( y_i \) | rub. |
The structure of the developed procedure is described by a sequence of stages, each of which involves the calculation of a specific quantitative characteristic. The list of characteristics being presented in the chronological order of their calculation is given in Table 2.

| No | The name of the characteristics | Unit rev. | Formula |
|----|---------------------------------|-----------|---------|
| 1  | The element of the main matrix of the system of equations of the regression model corresponding to the row \( l \) \((l = 1, 2, ..., m)\) and column \( k \) \((k = 1, 2, ..., m)\) | - | \[ A_{lk} = \begin{cases} \sum_{i=1}^{n} x_{il}, & \text{if } l = 1, k = 1; \\ \sum_{i=1}^{n} x_{il}, & \text{if } l > 1, k = 1; \\ \sum_{i=1}^{n} x_{il} \cdot x_{ik}, & \text{in other case.} \end{cases} \] |
| 2  | The matrix element of the system of equations for the parameter \( j \) of the predictive model \((j = 0, 1, ..., m)\), corresponding to the row \( l \) \((l = 1, 2, ..., m)\) and column \( k \) \((k = 1, 2, ..., m)\) | - | \[ A_{jk} = \begin{cases} \sum_{i=1}^{m} y_{i}, & \text{if } l = 1, k = j'; \\ \sum_{i=1}^{m} y_{i} \cdot x_{ij}, & \text{if } l > 1, k = j'; \\ A_{kj}, & \text{in other case.} \end{cases} \] |
| 3  | The value of the parameter \( j \) \((j' = 0, 1, ..., m)\) | - | \[ a_{j'} = \frac{A_{jk}}{A_{kj}} \] |
| 4  | Forecast (calculated) values of the cost of insulation \( i \) | rub. | \[ \hat{y}_i = a_0 + \sum_{j=1}^{m} a_j \cdot x_{ij} \] |
| 5  | The average value of the actual cost of insulation in the amount of \( n \) | rub. | \[ \bar{y} = \frac{1}{n} \sum_{i=1}^{n} y_i \] |
| 6  | Coefficient of determination | - | \[ R^2 = 1 - \frac{\sum_{i=1}^{n} (y_i - \hat{y}_i)^2}{\sum_{i=1}^{n} (y_i - \bar{y})^2} \] |
| 7  | Number of parameters, including free member | units | \( k = j + 1 \) |
| 8  | Adjusted coefficient of determination | - | \[ R_{adj}^2 = 1 - \left(1 - R^2\right) \cdot \frac{n-1}{n-k} \] |

* The value \( j' = 0 \) corresponds to the value constant; otherwise, the value determines the proportionality coefficient of the specific cost of the insulation to the value of the parameter \( j = j' \).

It is important to note that the main characteristic that determines the adequacy of the developed predictive model for the dependence of the cost of insulation on its technical characteristics (determined by the expression of item 4 of Table 2) is the coefficient of determination \( R^2 \) reaching its highest value.
– 1 – in the case when the predicted values of the cost are equal to actual ones for all considered samples of insulation. In general case, the coefficient $R^2$ has the value that is less than 1, but the condition of acceptability of the developed predictive model for further calculations is determined by the expression

$$R^2 \geq 0.6$$

(2)

At the same time, factors such as the number of thermal insulation characteristics taken into account, as well as the number of thermal insulation samples considered, have a great importance on the adequacy of the predictive model.

For taking into account of the first of these factors, on the basis if the statement that the value of the coefficient of determination $R^2$ does not decrease when additional factors are added to the model, it was also proposed to calculate the corrected coefficient of determination $R^2_{adj}$ (item 8 in Table 2), which gives a “penalty” for the additional factors included and, accordingly, more reliably assesses the model.

The second factor is connected to the law of large numbers, which determines the direct relationship between the number of observations and the representativeness of the sample set, i.e. the degree of compliance of the characteristics of the sample composition with the phenomenon or conformity under investigation). According to the overwhelming number of literary sources in the field of statistics, the sample set is representative if the number of elements included in its composition is at least 30 units.

2. Results and Discussion

The developed tools were implemented on a practical example (determination of insulation parameters for external walls). All calculations were made in the program Microsoft Excel. The initial data for testing the analytical method – model names, values of technical characteristics and cost for 30 insulation samples sold in the North-West Federal District of the Russian Federation – are presented in Table 3. As these characteristics were considered the following:

- Thickness.
- Coefficient of thermal conductivity.
- Average density.
- Water absorption in 24 hours, % by volume.
- Flammability group.

It is important to note that the “Flammability group” parameter is discrete and for its quantitative description it was decided to use a set of positive integer values.

The elements of the main matrix of the system of equations of the regression model (formula 3) were calculated in accordance with item 1 of Table 2.

In a similar way, the particular determinants of the system of equations for the parameters $a_0, \ldots, a_5$ were calculated. The remaining parameters of the computational model (Table 4) are calculated by the formulas items 3 – 8 of Table 2.

As a result, the model will take the form:

$$\hat{y} = -887.86 + 4273.02 \cdot x_1 - 30137.84 \cdot x_2 + 2.66 \cdot x_3 + 1295.8 \cdot x_4 + 392.76 \cdot x_5$$

(3)

As can be seen from expression (3), the cost constant, as well as the specific cost for the parameter of the thermal conductivity coefficient, turned out negative. The negative value of the value constant is determined by the decrease in the actual value of the insulation cost in comparison with objective value, determined only by the values of technical characteristics, which, in turn, is possible during the process of goods’ selling in conditions of market with significant competition either within the country or at the interregional level.

The inverse dependence of the cost of insulation on the coefficient of thermal conductivity is obvious: the worse the insulation conducts heat, the greater its operational efficiency.

It is important to note that the standard and adjusted coefficients of determination for the developed predictive model are equal to 0.938 and 0.918, respectively, which indicates the high adequacy of the specified model, and, as a consequence, its high practical significance. As for the absolute deviations of
the predicted values of the cost for the different models (samples) of insulation being considered from the actual ones, the largest deviation was 93.39 rubles / m², which is about 28.6% of the average cost of 1 m² of insulation within the considered sample set. The specified deviation is acceptable according to the standard scope of work on the warming of structural elements of buildings and structures and the amount of funding allocated for these purposes.

| No | Heater | Thickness | Coefficient of thermal conductivity | Average density | Water absorption in 24 hours, % by volume | Flammability group | Actual cost |
|----|--------|-----------|-------------------------------------|-----------------|------------------------------------------|-------------------|------------|
|    |        | m         | W/(m·°C)                            | kg/m³           | %                                       |                   | rub./m²    |
| 1  | PENOLEX Wall, 1185x585x50 mm | 0.05 | 0.32 | 20 | 0.5 | 3 | 280 |
| 2  | PENOLEX Comfort, 1200x600x20 mm | 0.02 | 0.03 | 30 | 0.4 | 3 | 110 |
| 3  | PENOLEX Comfort, 1200x600x30 mm | 0.03 | 0.03 | 30 | 0.4 | 3 | 167.5 |
| 4  | PENOLEX Comfort, 1200x600x40 mm | 0.04  | 0.03 | 30 | 0.4 | 3 | 217 |
| 5  | PENOLEX Comfort, 1200x600x50 mm | 0.05  | 0.03 | 30 | 0.4 | 3 | 263 |
| 6  | PENOLEX Comfort, 1200x600x60 mm | 0.06  | 0.03 | 30 | 0.4 | 3 | 277 |
| 7  | PENOLEX Comfort, 1200x600x80 mm | 0.08  | 0.03 | 30 | 0.4 | 3 | 368 |
| 8  | PENOLEX Basis, 1200x600x20 mm | 0.02  | 0.03 | 30 | 0.4 | 3 | 92 |
| 9  | PENOLEX Basis, 1200x600x30 mm | 0.03  | 0.03 | 30 | 0.4 | 3 | 138 |
| 10 | PENOLEX Basis, 1200x600x40 mm | 0.04  | 0.03 | 30 | 0.4 | 3 | 184 |
| 11 | PENOLEX Basis, 1200x600x50 mm | 0.05  | 0.03 | 30 | 0.4 | 3 | 232 |
| 12 | PENOLEX Basis, 1200x600x60 mm | 0.06  | 0.03 | 30 | 0.4 | 3 | 276 |
| 13 | PENOLEX Basis, 1200x600x80 mm | 0.08  | 0.03 | 30 | 0.4 | 3 | 368 |
| 14 | PENOLEX Basis, 1200x600x100 mm | 0.1   | 0.03 | 30 | 0.4 | 3 | 440 |
| 15 | BASWOOL Standard 50, 1200x600x100 mm | 0.1 | 0.038 | 50 | 1 | 1 | 207 |
| 16 | BASWOOL Standard 60, 1200x600x50 mm | 0.05  | 0.038 | 60 | 1 | 1 | 116 |
| 17 | BASWOOL Standard 60, 1200x600x100 mm | 0.1   | 0.038 | 60 | 1 | 1 | 231 |
| 18 | BASWOOL Standard 70, 1200x600x50 mm | 0.05  | 0.038 | 70 | 1 | 1 | 129 |
| 19 | BASWOOL Standard 70, 1200x600x100 mm | 0.1  | 0.038 | 70 | 1 | 1 | 257 |
| 20 | BASWOOL Façade 110, 1200x600x50 mm | 0.05 | 0.038 | 110 | 1 | 1 | 436 |
| 21 | BASWOOL Façade 110, 1200x600x100 mm | 0.1 | 0.038 | 110 | 1 | 1 | 653 |
| 22 | BASWOOL Façade 110, 1200x600x150 mm | 0.15 | 0.038 | 110 | 1 | 1 | 236 |
| 23 | BASWOOL Façade 120, 1200x600x50 mm | 0.05 | 0.038 | 120 | 1 | 1 | 473 |
| 24 | BASWOOL Façade 120, 1200x600x100 mm | 0.1 | 0.038 | 120 | 1 | 1 | 709 |
| 25 | BASWOOL Façade 120, 1200x600x150 mm | 0.15 | 0.038 | 120 | 1 | 1 | 246 |
| 26 | BASWOOL Façade 140, 1200x600x50 mm | 0.05 | 0.038 | 140 | 1 | 1 | 491 |
| 27 | BASWOOL Façade 140, 1200x600x100 mm | 0.1 | 0.038 | 140 | 1 | 1 | 737 |
| 28 | BASWOOL Façade 140, 1200x600x150 mm | 0.15 | 0.038 | 140 | 1 | 1 | 490 |
| 29 | BASWOOL Façade 160, 1200x600x100 mm | 0.1 | 0.038 | 160 | 1 | 1 | 735 |
| 30 | BASWOOL Façade 160, 1200x600x150 mm | 0.15 | 0.038 | 160 | 1 | 1 | 325.88 |

Table 3. Initial data and characteristics.
The main determinant of the system of equations:

\[
\Delta = \sum_{i=1}^{n} x_{1i} - \sum_{i=1}^{n} x_{2i} - \sum_{i=1}^{n} x_{3i} - \sum_{i=1}^{n} x_{4i} - \sum_{i=1}^{n} x_{5i}
\]

Table 4. Model parameters.

| No | Parameter name | Unit rev. | Identified / Formula | Value  |
|----|----------------|-----------|----------------------|--------|
| 1  | Cost constant  | rub./m²   | \( a_0 = \Delta a_0 / \Delta \) | -887.861 |
| 2  | The coefficient of proportionality of the unit cost of insulation value of the parameter \( \delta \) | rub./m³ | \( a_1 = \Delta a_1 / \Delta \) | 4273.02 |
| 3  | The coefficient of proportionality of the unit cost of insulation value of the parameter \( \lambda \) | rub./°C m·W | \( a_2 = \Delta a_2 / \Delta \) | -30137.83 |
| 4  | The coefficient of proportionality of the unit cost of insulation value of the parameter \( \rho \) | rub./n kg | \( a_3 = \Delta a_3 / \Delta \) | 2.66 |
| 5  | The coefficient of proportionality of the unit cost of insulation value of the parameter \( \mu \) | rub./m²·% | \( a_4 = \Delta a_4 / \Delta \) | 1295.8 |
| 6  | The coefficient of proportionality of the unit cost of insulation value of the parameter \( \varphi \) | rub./m² | \( a_5 = \Delta a_5 / \Delta \) | 392.758 |
| 7  | Coefficient of determination | - | \( R^2 \) | 0.932 |
| 8  | Number of parameters, including free member | units | \( k \) | 6 |
| 9  | Adjusted coefficient of determination | - | \( R^2_{adj} \) | 0.918 |
3. Conclusions

Thus, the implementation of the developed analytical methodology on a practical example confirmed the adequacy of the obtained predictive model, which led to the conclusion of the high practical importance of the developed tool. At the same time, the structure of the formed predictive model, which determines the analytical dependence of the cost of insulation on its technical characteristics, can be effectively used to solve problems connected to determination of the optimal characteristics for insulation for a given surface area of the insulated element according to the criterion for minimization of the insulation cost with taking into account of the constraints for calculated values of thermal resistance. Relevant procedures are planned for the next stages of the study.

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