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Boston University
SIMBA: Scalable Inversion in Optical Tomography using Deep Denoising Priors

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Abstract—Two features desired in a three-dimensional (3D) optical tomographic image reconstruction algorithm are the ability to reduce imaging artifacts and to do fast processing of large data volumes. Traditional iterative inversion algorithms are impractical in this context due to their heavy computational and memory requirements. We propose and experimentally validate a novel scalable iterative mini-batch algorithm (SIMBA) for fast and high-quality optical tomographic imaging. SIMBA enables high-quality imaging by combining two complementary information sources: the physics of the imaging system characterized by its forward model and the imaging prior characterized by a denoising deep neural net. SIMBA easily scales to very large 3D tomographic datasets by processing only a small subset of measurements at each iteration. We establish the theoretical fixed-point convergence of SIMBA under nonexpansive denoisers for convex data-fidelity terms. We validate SIMBA on both simulated and experimentally collected intensity diffraction tomography (IDT) datasets. Our results show that SIMBA can significantly reduce the computational burden of 3D image formation without sacrificing the imaging quality.

Index Terms—Optical tomography, regularization by denoising, plug-and-play priors, stochastic optimization.

I. INTRODUCTION

Optical tomographic imaging seeks to recover the three-dimensional (3D) distribution of the refractive index of an object from its light measurements. In a standard setup (see Figure 1 for an example), the sample is illuminated multiple times from different angles and the scattered light-field is recorded with a camera. In the interferometry-based microscopy, one measures both the amplitude and the phase of the scattered field [1]–[3], while in the intensity-only setups one measures only the amplitude of the light-field [4]–[6]. A tomographic reconstruction algorithm is then used to computationally reconstruct the 3D distribution of the sample’s refractive index. The quantitative characterization of the refractive index is important in biomedical imaging since it allows to visualize the internal structure of a tissue, as well as characterize physical changes within biological samples.

The reconstruction of the refractive index is often formulated for optical tomographic imaging. The brightfield measurements of the scattered light-field are collected with a standard computational microscope platform. An online reconstruction algorithm, SIMBA, facilitated by a convolutional neural network (CNN) denoiser is then used to form a 3D phase image. SIMBA can significantly reduce both computational and memory requirements, compared to traditional batch algorithms, when processing large tomographic datasets.

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Fig. 1. The conceptual illustration of the proposed inversion algorithm for optical tomographic imaging. The brightfield measurements of the scattered light-field are collected with a standard computational microscope platform. An online reconstruction algorithm, SIMBA, facilitated by a convolutional neural network (CNN) denoiser is then used to form a 3D phase image. SIMBA can significantly reduce both computational and memory requirements, compared to traditional batch algorithms, when processing large tomographic datasets.

Optical tomography, regularization by denoising, plug-and-play priors, stochastic optimization.

Online Processing

Denoiser

Online Reconstruction

Less Noisy

Online Denoising

Denoised Output

Noisy Input

Online Processing

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iterative inversion. By leveraging advanced image denoisers, such as BM3D [10] and DnCNN [11], PuP methods have achieved the state-of-the-art performance in various imaging applications [12]–[20]. An alternative framework for using image denoisers is the regularization by denoising (RED) [21], where the denoiser is used to formulate an explicit regularizer that has a simple gradient. The work [22] has clarified the existence of RED regularizers for certain class of denoisers, and the excellent performance of the framework has been demonstrated in phase retrieval [23] and image super-resolution [24] using DnCNN and the deep image prior, respectively. In short, using advanced denoisers has proven to be effective for improving the reconstruction quality in various imaging contexts.

In this paper, we present a new scalable iterative mini-batch algorithm (SIMBA) for the regularized inversion in optical tomography. SIMBA is an online extension of the traditional RED framework. It can thus leverage powerful convolutional neural network (CNN) denoisers as imaging priors, while also taking advantage of the physical information available through the forward model. However, unlike traditional RED algorithms, SIMBA is scalable to datasets that are too large for batch processing since it only uses a subset of measurements at a time. We prove that SIMBA converges in expectation to the same set of fixed points as its batch counterparts under a set of transparent assumptions. Thus, SIMBA benefits from the excellent imaging quality offered by RED, but does so in a computationally tractable way for optical tomographic imaging.

We validate SIMBA in the context of intensity-only microscopy called intensity diffraction tomography (IDT). IDT microscopes are relatively cheap and easy to implement since they do not collect the phase of the light. We adopt the IDT forward model in [25] that establishes a linear relationship between the desired object and the intensity measurements by neglecting the terms corresponding to higher order light scattering. We show that SIMBA can efficiently reconstruct a high-resolution (1024 × 1024 × 25 pixels) IDT image while also offering improvements in the 3D sectioning capability. The preliminary version of this work was presented in [26]. The current paper significantly extends [26] by including the IDT model, providing additional simulations, and validating the method on an experimentally collected 3D IDT dataset.

This paper is organized as follows. In Section II, we introduce the IDT forward model and the RED framework. In Section III, we present the algorithmic details of SIMBA. In Section IV, we analyze the fixed-point convergence under a set of assumptions. In Section V, we provide simulations and experiments that illustrates the efficiency and effectiveness of SIMBA. Section VI concludes the paper.

II. BACKGROUND

In this section, we provide the background on IDT and image-denoising priors. We start by describing the IDT forward model, then formulate the corresponding inverse problem, and finally introduce the RED framework as a strategy to leverage image denoisers as priors.

A. Linearized IDT

Consider a 3D object with the permittivity distribution $\epsilon(r)$ in a bounded sample domain $\Omega \subset \mathbb{R}^3$, immersed into the background medium of permittivity $\epsilon_b$. We use $\Delta \epsilon = \Delta \epsilon_{\text{Re}} + i\Delta \epsilon_{\text{Im}} = \epsilon - \epsilon_b$ to denote the permittivity contrast between the object and the background medium. The real part $\Delta \epsilon_{\text{Re}}$ corresponds to the phase effect, and the imaginary part $\Delta \epsilon_{\text{Im}}$ accounts for the absorption. The object is illuminated by an angled incident light field $u_{\text{in}}(r)$. The incident field $u_{\text{in}}$ is assumed to be known inside $\Omega$ as well as at the camera domain $\Gamma \subset \mathbb{R}^2$. The total light-field $u(r)$ is measured only through its intensity at the camera. Here, $r = (x, y, z)$ denotes the 3D spatial coordinates. Under the first Born approximation [27], the light-sample interaction is described by the following equation

$$u(r) = u_{\text{in}}(r) + \int_{\Omega} g(r - r') \, v(r') \, u_{\text{in}}(r') \, dr', \quad r \in \Omega \quad (1)$$

where $u(r) = u_{\text{in}}(r) + u_{\text{sc}}(r)$ is the total light field, $v(r) = \frac{1}{2} k^2 \Delta \epsilon$ is the scattering potential, $k = 2\pi/\lambda$ is wave number in free space, and $\lambda$ is the wavelength of the illumination. In the 3D space, the Green’s function at the camera plane $\Gamma$ is given by

$$g(r) = \frac{e^{i k_b |r| \|z\|}}{\|r\|_2},$$

where $k_b = \sqrt{\epsilon_b} k$ is the wavenumber of the background medium, and $\|\cdot\|_2$ denotes the $\ell_2$-norm. For a single illumination, the intensity of the light field after propagating through the sample is given by

$$I = |u(r) \ast p|^2, \quad (2)$$

where $p$ is the point spread function of the microscope, and the operator $\ast$ denotes the 2D convolution. Eq. (2) can be expanded into the summation of four components

$$I = I^{si} + I^{ss} + I^{\ast i} + I^{\ast s}, \quad (3)$$

where $I^{si}$ is the constant background intensity, $I^{ss}$ is the squared modulus of the scattered field, and $I^{\ast i} = (I^{\ast s})^*$ are the cross terms that relate the unscattered and scattered field. Here, $(\cdot)^*$ denotes the complex conjugate. Due to the first Born approximation, $I^{ss}$ can be assumed to be small and thus neglected. By modeling the 3D object as a series of slices along the axial dimension $z$, one can represent the spectrum of the total scattered field as the summation of the sub-scattered fields produced by each slice [25]

$$\tilde{I} = \tilde{I}^{si} + \int \left[ H_{\text{Re}}(z) \Delta \epsilon_{\text{Re}}(z) + H_{\text{Im}}(z) \Delta \epsilon_{\text{Im}}(z) \right] \, dz \quad (4)$$

where $\tilde{\cdot}$ denotes 2D Fourier transform, and $\tilde{I}^{si}$ is the background intensity spectrum measured at $\Gamma$. In (4), $H_{\text{Re}}$ and $H_{\text{Im}}$ are the angle-dependent phase and absorption transfer functions (TF) for each sample slice at depth $z$, respectively. These TFs linearly map the Fourier transform of the permittivity contrast to the intensity spectrum of the scattered field. We refer the reader to [25] for the full details of the TF for IDT.
By discretizing (4) and explicitly including the Fourier transform into the equation, we obtain the following linear model in the spatial domain for the $i^{th}$ illumination

$$I_i = I_i^0 + \sum_{j=0}^{J} A_{ij} x_j, \text{ with } A_{ij} = F^H y_{ij} F, \quad (5)$$

where $j = 0, \ldots, J$ discretely indexes the axial direction $z$, $x_j \in \mathbb{C}^M$ is the discretized complex permittivity contrast of the $j^{th}$ slice, $I_i$ is the measured intensity of the total field, $I_i^0$ is the discretized intensity of the background, and $H_{ij}$ is the discretized TF accounting for both phase and absorption at $z_j$. We use $F$ and $F^H$ to denote the 2D discrete Fourier transform and its inverse, respectively. By re-arranging the terms, we can obtain the following linear forward model

$$y_i = A_i x + e, \quad \text{with } A_i^H = \begin{bmatrix} A_{i0}^H & \cdots & A_{iJ}^H \end{bmatrix}, \quad x = \begin{bmatrix} x_0 \\ \vdots \\ x_J \end{bmatrix} \quad (6)$$

where the operator $(\cdot)^H$ denotes the conjugate transpose, $y_i := I_i - I_i^0 \in \mathbb{R}^M$ is the measured intensity with the removal of the background intensity for the $i^{th}$ illumination, and the noise $e \in \mathbb{R}^M$ is assumed to be additive white Gaussian noise (AWGN). Note that, as was discussed in [25], the IDT forward model does not contain any information on the DC component of the phase.

### B. Inverse Problem

Since image reconstruction in optical tomography is often ill-posed, it is typically formulated as the regularized inversion problem

$$\hat{x} = \arg \min_{x \in \mathbb{C}^N} \{ g(x) + h(x) \}, \quad (7)$$

where $g$ is the data-fidelity term that ensures the consistency with the measured data, and $h$ is the regularization term that imposes the prior knowledge on the desired image. For example, the Tikhonov regularization [28] assumes a Gaussian prior on the unknown image. It has been previously used in IDT for deriving a closed form solution [25]. More recent regularizers, such as the sparsity-promoting $\ell_1$-norm penalty [29] and the edge-preserving total variation (TV) [30], are nonsmooth and do not have closed-form solutions, thus requiring iterative algorithms for image formation. In particular, the family of proximal algorithms—such as proximal gradient method (PGM) [31]–[34] and alternating direction method of multipliers (ADMM) [35]–[38]—avoid the need to differentiate the regularizer by using the proximal map [39].

Recently, deep learning has gained popularity in imaging inverse problems [40]–[48]. Traditional strategy trains the convolutional neural network (CNN) to learn the direct mapping from the measurements to some ground-truth image. Despite their excellent performance in some image reconstruction problems, this strategy does not leverage the known physics of the imaging system and does not insure consistency with the measured data. In this paper, we propose SIMBA to reconcile the model-based and learning-based approaches by infusing deep denoising priors into online iterative algorithms.

### C. Regularization by Denoising

RED [21] is a recently introduced framework to leverage powerful image denoisers. It has been successfully applied in many regularized imaging tasks, including image deblurring [21], super-resolution [24], and phase retrieval [23]. The framework aims to find a fixed point $x^*$ that satisfies

$$G(x^*) = \nabla g(x^*) + \tau (x^* - D_\sigma(x^*)) = 0, \quad (8)$$

where $\nabla g$ denotes the gradient of $g$, $D_\sigma$ is the image denoiser, and $\tau > 0$ adjusts the tradeoff between the data-fidelity and the prior. RED algorithms seek a vector $x^*$ that lies in the zero set of $G : \mathbb{R}^n \rightarrow \mathbb{R}^n$

$$x^* \in \text{zer}(G) := \{ x \in \mathbb{R}^n : G(x) = 0 \}. \quad (9)$$

For example, the gradient-method variant of RED (denoted as GM-RED) can be implemented as

$$x_k = x_{k-1} - \gamma (\nabla g(x_{k-1}) + H(x_{k-1}))$$

where $H(x) := \tau (x - D_\sigma(x)). \quad (10)$

Here, the parameter $\gamma > 0$ is the step-size. When the denoiser $D_\sigma$ is locally homogeneous and has a symmetric Jacobian [21], [22], the operator $H$ corresponds to the gradient of the following regularizer

$$h(x) = \frac{\tau}{2} x^T (x - D_\sigma(x)). \quad (11)$$

By having a closed-form objective function, one can use the classical optimization theory to analyze the convergence of RED algorithms [21]. On the other hand, fixed-point convergence has also been established without having an explicit objective function [19], [22]. Reehorst et al. [22] have shown that RED proximal gradient methods (RED-PG) converges to a fixed point by utilizing the monotone operator theory. Sun et al. [49] have established the explicit convergence rates for the block coordinate variant of RED (BC-RED) under a nonexpansive $D_\sigma$. In this paper, we extend these prior analyses to the randomized processing of the measurements instead of image blocks, which opens up applications to tomographic imaging with a large number of projections.

### III. Proposed Method

We now introduce SIMBA that combines the iterative usage of the forward model with a deep denoising prior. At each iteration, SIMBA updates $x$ by combining a stochastic gradient for increasing data-consistency with a CNN denoiser for artifact reduction. SIMBA is ideal for data-intensive biomedical imaging applications where the object features are difficult to characterize using traditional regularizers.

#### A. Iterative Online Procedure

In IDT, the data-fidelity term can be written as an average over a set of distinct components functions

$$g(x) = \mathbb{E}[g_i(x)] = \frac{1}{T} \sum_{i=1}^{T} g_i(x), \quad (12)$$

where $g_i(x)$ is the data-fidelity term that ensures the consistency with the measured data, and $T$ is the number of projections.
Algorithm 1 SIMBA

1: input: $x^0 \in \mathbb{R}^n$, $\tau > 0$, $\sigma > 0$, and $B \geq 1$
2: for $k = 1, 2, \ldots$ do
3: \quad $\nabla g(x^{k-1}) \leftarrow$ minibatchGradient$(x^{k-1}, B)$
4: \quad $\hat{G}(x^{k-1}) \leftarrow \nabla g(x^{k-1}) + \tau (x^{k-1} - D_0(x^{k-1}))$
5: \quad $x^k \leftarrow x^{k-1} - \gamma \hat{G}(x^{k-1})$
6: end for

where each component function $g_i$ is evaluated only on the subset $y_i$ of the full measurements $y$

$$g(x) = \frac{1}{2} \|A_i x - y_i\|^2_2.$$ (13)

The computation of the gradient of $g$

$$\nabla g(f) = \mathbb{E}[\nabla g_i(f)] = \frac{1}{f} \sum_{i=1}^{f} \nabla g_i(f),$$

where $\nabla g_i(x) = A_i^\top (A_i x - y_i)$, (14)

is proportional to the total number of illuminations $I$. Note that the expectations in (12) and (14) are taken over a uniformly distributed random variables $i \in \{1, \ldots, f\}$. A large $f$ effectively precludes the usage of batch RED algorithms because of large memory requirements or impractical computation times. The key idea of SIMBA is to approximate the gradient at every iteration by averaging $B$ component gradients

$$\hat{\nabla} g(x) = \frac{1}{B} \sum_{b=1}^{B} \nabla g_{i_b}(x),$$ (15)

where $i_1, \ldots, i_B$ are independent random indices that are distributed uniformly over $\{1, \ldots, f\}$. The minibatch size parameter $1 \leq B \ll f$ controls the number of gradient components used at every iteration.

Algorithm 1 summarizes the algorithmic details of SIMBA, where the operation minibatchGradient computes the averaged gradients with respect to the selected minibatch components. Note that at each iteration, the minibatch is randomly sampled from the entire set of measurements.

B. CNN-based Denoiser

In recent years, CNNs have been shown to achieve the state-of-the-art performance on image denoising [11], [50]. We propose a simple denoising network DnCNN* as the deep learning module in SIMBA. The architecture of the neural network, illustrated in Figure 1, is adapted from the popular DnCNN. In general, DnCNN* consists of two parts. The first part contains $N_e - 1$ sequential composite convolutional layers, each of which has one convolutional layer followed by a rectified linear unit (ReLU) layer. The second part is a single convolutional layer that outputs the final denoised image, resulting the total number of layers in DnCNN* to be $N_e$. All the convolution filters are implemented with size $3 \times 3$, and every feature map has 64 channels. In SIMBA, we apply this 2D image denoising network to the 3D sample by performing the layer-by-layer denoising along the axial direction $z$.

We generated the training dataset by adding AWGN to the natural images from BSD400 and applying standard data augmentation strategies including flipping, rotating, and rescaling. Note that our training dataset does not include any biomedical image. We employed the residual learning technique [51] in DnCNN* so that the network is forced to learn the noise residual in the noisy input. DnCNN* was trained to minimize the following loss

$$L_\theta = \frac{1}{n} \sum_{i=1}^{n} \left\{ \|f_\theta(x_i) - y_i\|^2_2 + \rho \|f_\theta(x_i) - y_i\|_1 \right\},$$ (16)

where $x_i$ is the noisy input, $y_i$ is the noise, and $f_\theta(x)$ represents the noise predicted by the neural network. Eq. (16) penalizes both the mean squared error (MSE) and the mean absolute error (MAE) between the estimated noise and the ground truth. A loss parameter $\rho > 0$ is thus introduced to adjust the tradeoff between the two errors for the best training performance. Our results show that our simple DnCNN* is competitive with traditional denoisers in terms of the imaging quality.

IV. CONVERGENCE ANALYSIS

Our analysis relies on the fixed-point convergence of averaged operators, which is well known as the Krasnosel’ski-Mann theorem [52]. Here, we extend the result to the iterative online algorithms under the RED formulation and show the worst-case convergence rates. Note that our analysis does not assume that the denoiser corresponds to any explicit RED regularizer. We first introduce the assumptions necessary for our analysis and then present the main results.

Assumption 1. We make the following assumptions on the data-fidelity term $g$:

(a) The component functions $g_i$ are all convex and differentiable with the same Lipschitz constant $L > 0$. 

(b) At every iteration, the gradient estimate is unbiased and has a bounded variance:

$$\mathbb{E}[\nabla g(x)] = \nabla g(x), \quad \mathbb{E}[\|\nabla g(x) - \hat{\nabla} g(x)\|^2_2] \leq \nu^2/B,$$

for some constant $\nu > 0$.

Assumption 1 (a) implies that the overall data-fidelity $g$ is also convex and has Lipschitz continuous gradient with constant $L$. 

![Fig. 2. Eight test images used in the experiments. Top row from left to right: Aircraft, Boat, Cameraman, Foreman. Bottom row from left to right: House, Monarch, Parrot, Pirate.](image-url)
Assumption 1 (b) assumes that the minibatch gradient is an unbiased estimate of the full gradient. The bounded variance assumption is a standard assumption used in the analysis of online and stochastic algorithms [53]–[55].

Assumption 2. Let operator $\mathbf{G}$ have a nonempty zero set $\text{zer}(\mathbf{G}) \neq \emptyset$. The distance between the the farthest point in $\text{zer}(\mathbf{G})$ and the sequence $\{x^k\}_{k=0,1,...}$ generated by SIMBA is bounded by a constant $R_0$

$$\max_{x^* \in \text{zer}(\mathbf{G})} ||x^k - x^*||_2 \leq R_0, \quad k \geq 0$$

This assumption indicates that the iterates of SIMBA lie within a Euclidean ball of a bounded radius from $\text{zer}(\mathbf{G})$.

Assumption 3. Given $\sigma > 0$, the denoiser $D_\sigma$ is a nonexpansive operator such that

$$||D_\sigma(x) - D_\sigma(y)||_2 \leq ||x - y||_2 \quad x, y \in \mathbb{R}^n,$$

One can train a nonexpansive CNN denoisers by using the spectral normalization techniques [49], [56]. Under the above assumptions, we can establish the following for SIMBA.

**Theorem 1.** Run SIMBA for $t \geq 1$ iterations under Assumptions 1-3 using a fixed step-size $\gamma \in (0, 1/(L + 2\tau))$ and a fixed minibatch size $B \geq 1$. Then, we have

$$\mathbb{E} \left[\min_{k \in \{1, \ldots, t\}} ||\mathbf{G}(x^{k-1})||_2^2\right] \leq \mathbb{E} \left[\frac{1}{t} \sum_{k=1}^{t} ||\mathbf{G}(x^{k-1})||_2^2\right] \leq \frac{(L + 2\tau) \gamma}{\gamma} \left[\frac{\nu^2 \gamma^2}{B} + \frac{2 \gamma \nu}{B} R_0 + \frac{R_0^2}{t}\right].$$

**Proof.** See Appendix A.

When $t$ goes to infinity, this theorem shows that the accuracy of the expected convergence of SIMBA to an element of $\text{zer}(\mathbf{G})$ improves with larger $B$. For example, we can have the convergence rate of $O(1/\sqrt{t})$ by setting $\gamma = 1/(L + 2\tau)$ and $B = t$

$$\mathbb{E} \left[\frac{1}{t} \sum_{k=1}^{t} ||\mathbf{G}(x^{k-1})||_2^2\right] \leq \frac{C}{\sqrt{t}},$$

where $C > 0$ is a constant and we use the bound $\frac{1}{t} \leq \frac{1}{\sqrt{t}}$ that is valid for $t \geq 1$.

**V. EXPERIMENTAL VALIDATION**

In this section, we validate SIMBA on both simulated and experimental data. We first numerically demonstrate the efficiency and practical convergence of SIMBA in simulations. Next, we apply SIMBA to reconstruct a 3D model from a set of real intensity-only measurements. Our results highlight the applicability and effectiveness of SIMBA for the iterative inversion in optical tomography.

**A. Setup**

In simulations, we reconstruct eight grayscale natural images, displayed in Figure 2. They are assumed to be on the focal plane $z = 0 \mu m$ with LEDs located at $z_{LED} = -70 \mu m$. We generate $I = 60$ simulated intensity measurements with $40 \times$ microscope objectives (MO) and 0.65 numerical aperture (NA).
All simulated measurements are corrupted by AWGN corresponding to 20 dB of input signal-to-noise ratio (SNR). As a quantitative metric for measuring the quality of reconstructions, we use the SNR defined as follows

$$\text{SNR}(\hat{y}, y) = \max_{a, b \in \mathbb{R}} \left\{ 20 \log_{10} \left( \frac{\|y\|_2^2}{\|y - ay + b\|_2} \right) \right\}$$

where $\hat{y}$ represents the noisy vector and $y$ denotes the ground truth. In experiments, we recover a 3D algae sample from real IDT measurements. The 3D sample is located over the range ($-20, 100$) µm and $s_{\text{LED}} = -79$ mm. We set the slice spacing as 5 µm, so each slice represents the average over the sample thickness. We take $I = 89$ measurements with $10 \times$ MO and 0.25 NA for reconstruction. We refer to Table I for the detailed summary of the experimental parameters. All experiments in this paper were performed on a machine equipped with an Intel Xeon E5-2620 v4 Processor that has 4 cores of 2.1 GHz and 256 GBs of DDR memory. We trained all neural nets using NVIDIA RTX 2080 GPUs.

The algorithmic hyperparameters are summarized in Table II. All algorithms start from $x^0 = 0 \in \mathbb{R}^N$. We trained DnCNN* for the removal of AWGN at four noise levels corresponding to $\sigma \in \{5, 10, 15, 20\}$. The same set of $\sigma$ is used for BM3D. All algorithmic parameters are optimized for the best performance.

### B. Simulated Data

In this section, we numerically illustrate the advantages of SIMBA in tomographic imaging over the batch GM-RED. The advantages are: (1) better SNR under a limited memory budget; (2) better time efficiency when all the measurements are used.

Figure 3 (top) plots the average SNR over test images against the iteration number for SIMBA and GM-RED (20), both using DnCNN* as the denoiser. GM-RED (20) uses a fixed set of 20 (out of 60) measurements, while SIMBA selects a random subset of 20 at every iteration. Under the same computational complexity, SIMBA achieves a significant SNR boost of 3.2 dB over GM-RED (20) because the former has access to all the measurements. Visual examples in Figure 4 further illustrate the improvement in terms of visual quality. Specifically, the images reconstructed by SIMBA succeed in recovering the mountains and masts in the region denoted with (a) of Aircraft and Boat, respectively. The same areas remain blurry when recovered by GM-RED (20). SIMBA also clearly reconstructs the words in the region denoted by (b) in both example images, while the words recovered by GM-RED (20) are far from readable. As a reference, we also plot the SNR for GM-RED using all 60 measurements, denoted as GM-RED (full).

Figure 3 (bottom) highlights the faster time convergence of SIMBA compared to GM-RED (full) to the same level of SNR. Figure 4 highlights that the SNR values and the visual quality obtained by SIMBA and GM-RED (full) are nearly identical. SIMBA, however, significantly reduces the reconstruction time by processing one third of all measurements at each iteration. Specifically, the average per-iteration times of GM-RED (20), SIMBA, and GM-RED (full) are 0.30 second, 0.31 second, and 0.52 second, respectively. We also note that by processing only a subset of measurements, SIMBA has lower memory requirements compared to GM-RED (full), which increases its scalability. This makes SIMBA favorable for processing datasets containing a large number of tomographic measurements.

Table III shows final SNRs of all reconstructions we performed. We run all simulations using the accelerated versions of these algorithms, which are analogous to the accelerated gradient method by Nesterov [57]. Empirically, they converge to the same solution as the non-accelerated counterparts. For reference, we show the evolution of SNR for non-accelerated versions by the dotted lines in Figure 3. Table III shows that our DnCNN denoiser has higher average SNR than BM3D. The compatibility of SIMBA with DnCNN*, which is a low-complexity denoiser, increases the potential of applying SIMBA to large scale image reconstructions.

### C. Experimental IDT Dataset

In this section, we use SIMBA to reconstruct a 3D algae sample of $1024 \times 1024 \times 25$ pixels from 89 high-resolution measurements. The large sample volume dramatically increase the memory usage and computational cost, and prohibits the applicability of the full batch algorithms. Experimental results show that SIMBA successfully overcomes these difficulties by processing a small subset of all measurements ($B = 10$) at every iteration and leads to significant performance improvements compared to the method reported in [25].

Figure 5 provides a 3D visualization of SIMBA reconstruction with different algae labeled by circled numbers (there are 6 of them). Figure 6 compares three slices of our SIMBA results and the Tikhonov (full) results obtained by algorithm in [25], which uses all 89 measurements. As discussed in [25], the DC component of the phase is lost in the IDT forward model, we
thus set the mean of all the results to the one of the Tikhonov reconstruction for a more uniform comparison. We evaluate the quality of different reconstructions by comparing their axial sectioning effect and the ability to eliminates artifacts. In the 3D tomographic model with strong sectioning effect, a pattern emerges only in the slice it belongs to and fades away as we go axially to different depths. Sectioning enables us to better predict the axial location of the patterns within a 3D object and thus better understand its internal structure, which is crucial for biomedical imaging applications. Tikhonov regularization is attractive from computational perspective, however, it corresponds to a Gaussian prior on the image, which might lead to excessive smoothing. This complicates the understanding of the axial structure of the sample. On the other hand, by leveraging the DnCNN* prior, SIMBA improves the performance, while also mitigating the computational complexity with online processing. Our results show that SIMBA with DnCNN* enables better sectioning of the object compared to the Tikhonov prior. For example, maintaining the clarity and sharpness of algae 2 in slice $z = 25 \mu m$, SIMBA successfully reduces the artifacts generated by the content of adjacent slices, which exist in the region (a) of Tikhonov. In the other two slices, algae 2 fades away and does not generate strong shadowy artifacts as indicated by arrows (c) and (f). By horizontally comparing the two rows, the algae cluster in region (a) is visually better resolved by SIMBA than Tikhonov. Moreover, in SIMBA reconstructions, the top half of algae 5 in region (b) looks sharp in slice $z = 25 \mu m$ and the bottom half appears clear in slice $z = 35 \mu m$. This inter-slice information implies that algae 5 penetrate through $z = 25 \mu m$ and $z = 35 \mu m$. However, the whole structure of algae 5 is present in both slices of Tikhonov reconstructions, which fails in illustrating the axial position. Note that SIMBA also better eliminates artifacts pointed out by arrows (d) and (e). To further analyze the performance of the priors, we bring BM4D, the 3D version of the well-known denoiser BM3D, into comparison. In zoom-in region (a) of Figure 6, Tikhonov reconstruction contains grid-shape artifacts. BM4D generates small blocks due to its block-matching mechanism. DnCNN* provides a more real and sharper result than the other two. In region (b), Tikhonov reconstruction is of satisfactory visual quality but the shadow of algae 5 and 6 in the background interferes with the actual content in this slice. BM4D erases the shadow in the background but it again generates blocky artifacts which
Fig. 5. Visualization of the 3D algae reconstruction. Algae are labeled by circled numbers. We select three slices of the sample to illustrate the improvement of performance by our proposed method SIMBA with $B = 10$ over Tikhonov (full), which uses all 89 measurements. Regions (a) and (b) demonstrate the better axial sectioning effect of SIMBA and arrows (c) to (f) point out the areas where SIMBA suppresses the artifacts present in the Tikhonov reconstruction.

Fig. 6. Comparison of SIMBA under BM4D and DnCNN* against Tikhonov. Regions (a) and (b) are zoomed in to highlight visual differences. Tikhonov reconstructed image contains grid-shape artifacts and interfering contents from other slices, while BM4D generates blocks and nonsmoothness. SIMBA under DnCNN* produces the most real recovery with the clearest shape of the algae.

makes its reconstruction not as real as DnCNN* result.

Finally, we present one slice of the full $1024 \times 1024 \times 25$ reconstruction by SIMBA under DnCNN* in Figure 7. For comparison, we run GM-RED (full) under DnCNN* but only for the dotted region because of the high computational cost of the full batch reconstruction. The result is juxtaposed with our SIMBA result. These two algorithms are run with the same $\tau$ value until convergence. Visually, they look almost identical and we present the absolute value of the residual between the two for reference. The residual is negligible compared to the numerical scale of the two results. Quantitatively, if we assume the result of the full batch algorithm to be the “ground truth”, the SNR of SIMBA is 47.03 dB. This substantiates that SIMBA sufficiently matches the full batch algorithms in terms of the final reconstruction quality. Specifically, the average per-iteration running time of SIMBA for reconstructing the dotted region is 22 seconds, while that of GM-RED is 192 seconds, which corresponds to a $9 \times$ speed-up. SIMBA also requires less memory at every iteration by processing only about one ninths of full measurements. The reduced running time and memory usage in processing such an intensive amount of data highlighted the efficiency improvement of SIMBA compared to the traditional batch GM-RED.

VI. CONCLUSION

To conclude, we proposed SIMBA as an extension of the RED framework for solving the imaging inverse problems in optical tomography. Our method is scalable to large measurements and uses a deep denoising prior to improve the final estimate. We proved the fixed-point convergence of SIMBA without assuming an explicit objective function, which complements the current theoretical analysis of RED for large-scale image reconstruction. We validated SIMBA by both IDT simulations and experiments. Especially, the 3D
reconstruction of a large algae sample fully elucidates the benefits of our method in data-intensive imaging problems. Future work includes the application of SIMBA in other advanced IDT modalities with coded illumination patterns \[58\] and accelerated data acquisition \[59\].

**APPENDIX**

We consider the following two operators

\[ P := I - \gamma G \quad \text{and} \quad \hat{P} := I - \gamma \hat{G} \]

where \( \hat{P} \) is the online variant of \( P \). The iterates of SIMBA can be expressed as

\[ x^k = \hat{P}(x^{k-1}) = x^{k-1} - \gamma \hat{G}(x^{k-1}), \quad \text{with} \quad \hat{G} = \hat{\nabla}g + H. \]

Note also the following equivalence

\[ x^* \in \text{zer}(G) \iff x^* \in \text{fix}(P) \]

**Proposition 1.** Consider an operator \( P \) and its online variant \( \hat{P} \). If the data-fidelity \( g(\cdot) \) satisfies Assumption 1, then we have

\[ \mathbb{E}[\hat{P}(x)] = P(x), \quad \mathbb{E}[\|P(x) - \hat{P}(x)\|_2^2] \leq \frac{\gamma^2 \nu^2}{B}. \]

**Proof.** First, we can show

\[ \mathbb{E}[\hat{G}(x)] = \mathbb{E}[\hat{\nabla}g(x)] + H(x) = G(x) \]

and

\[ \mathbb{E}[\|G(x) - \hat{G}(x)\|_2^2] = \mathbb{E}[\|\nabla g(x) - \hat{\nabla}g(x)\|_2^2] \leq \frac{\nu^2}{B}. \]

Then, we can prove the desired result

\[ \mathbb{E}[\hat{P}(x)] = I - \gamma \mathbb{E}[\hat{G}(x)] = P(x) \]

and

\[ \mathbb{E}[\|P(x) - \hat{P}(x)\|_2^2] = \gamma^2 \mathbb{E}[\|G(x) - \hat{G}(x)\|_2^2] \leq \frac{\gamma^2 \nu^2}{B}. \]

\[ \square \]

**Proposition 2.** Let the denoiser \( D_\sigma \) be such that it satisfies Assumption 3 and \( \nabla g \) is L-Lipschitz continuous. For any \( \gamma \in (0, 1/(L + 2\tau)) \), the operator \( P \) is nonexpansive

\[ \|P(x) - P(y)\|_2 \leq \|x - y\|_2 \quad \forall x, y \in \mathbb{R}^n \]

**Proof.** The proposition is a direct result of the part (c) of the proof of Theorem 1 (Section A) in the Supplementary Material.
of [49] by setting $U = U^T = I$ and $G_t = G$, which corresponds to the full-gradient RED algorithm of (10).

Now we prove Theorem 1 in the paper. Consider a single iteration $x^k = \hat{P}(x^{k-1})$, then we can write for any $x^* \in \text{zer}(G)$ that

$$ \|x^k - x^*\|^2 = \|\hat{P}(x^{k-1}) - P(x^*)\|^2 $$
$$ = \|\hat{P}(x^{k-1}) - P(x^{k-1}) + P(x^{k-1}) - P(x^*)\|^2 $$
$$ = \|P(x^{k-1}) - P(x^*)\|^2 + \|\hat{P}(x^{k-1}) - P(x^{k-1})\|^2 $$
$$ + 2(\hat{P}(x^{k-1}) - P(x^{k-1}))^T(P(x^{k-1}) - P(x^*)) $$
$$ \leq \|x^k - x^*\|^2 - \left(\frac{\gamma}{L + 2\tau}\right) \|G(x^{k-1})\|^2 $$
$$ + \|\hat{P}(x^{k-1}) - P(x^{k-1})\|^2 $$
$$ + 2\|\hat{P}(x^{k-1}) - P(x^{k-1})\|_2 \cdot \|P(x^{k-1}) - P(x^*)\|_2, $$

where we use the Cauchy-Schwarz inequality and adapt the bound (14) in the part (d) of the proof of Theorem 1 (Section A) in the Supplementary Material of [49] by setting $U = U^T = I$ and $G_t = G$. According to Assumption 2 and Proposition 4, we have

$$ \|P(x^{k-1}) - P(x^*)\|_2 \leq \|x^{k-1} - x^*\|_2 \leq R_0. \quad (18) $$

Additionally, by using Jensen’s inequality, we can have for all $x \in \mathbb{R}^n$ that

$$ \mathbb{E}\left[\|P(x) - \tilde{P}(x)\|_2^2\right] = \mathbb{E}\left[\sqrt{\|P(x) - \tilde{P}(x)\|^2}\right] $$
$$ \leq \sqrt{\mathbb{E}\left[\|P(x) - \tilde{P}(x)\|^2\right]} \leq \frac{\gamma \nu}{\sqrt{B}}. \quad (19) $$

By rearranging and taking a conditional expectation of (20) and using these bounds, we can obtain

$$ \mathbb{E}\left[\|x^k - x^*\|^2 - \|x^{k-1} - x^*\|^2 \mid x^{k-1}\right] $$
$$ \leq 2\gamma \nu \|x^k - x^*\|^2 + \frac{\gamma^2 \nu^2}{B} - \left(\frac{\gamma}{L + 2\tau}\right) \|G(x^{k-1})\|^2 $$

which can be reorganized as

$$ \|G(x^{k-1})\|^2 \leq \left(\frac{L + 2\tau}{\gamma}\right) \left[\frac{2\gamma \nu^2}{B} + \frac{\gamma^2 \nu}{\sqrt{B} R_0}\right] $$
$$ + \mathbb{E}\left[\|x^{k-1} - x^*\|^2 - \|x^k - x^*\|^2 \mid x^{k-1}\right]. $$

By averaging the inequality over $t \geq 1$ iterations, taking the total expectation, and dropping the last term, we obtain

$$ \mathbb{E}\left[\frac{1}{t} \sum_{k=1}^{t} \|G(x^{k-1})\|^2\right] $$
$$ \leq \frac{L + 2\tau}{\gamma} \left[\frac{2\gamma \nu^2}{B} + \frac{\gamma^2 \nu}{\sqrt{B} R_0} + \frac{R_0^2}{7}\right] $$

where we apply the law of total expectation and Assumption 2. This establishes the Theorem 1.

### APPENDIX

We consider the following two operators

$$ \tilde{P} := 1 - G \quad \text{and} \quad \hat{P} := 1 - \hat{G} $$

where $\hat{P}$ is the online variant of $P$. The iterates of SIMBA can be expressed as

$$ x^k = \hat{P}(x^{k-1}) = x^{k-1} - \gamma \hat{G}(x^{k-1}), \quad \text{with} \quad \hat{G} = \nabla g + H. $$

Note also the following equivalence

$$ x^* \in \text{zer}(G) \iff x^* \in \text{fix}(P). $$

**Proposition 3.** Consider an operator $P$ and its online variant $\hat{P}$. If the data-fidelity $g(\cdot)$ satisfies Assumption 1, then we have

$$ \mathbb{E}[\hat{P}(x)] = P(x), \quad \mathbb{E}[\|P(x) - \tilde{P}(x)\|^2] \leq \frac{\gamma^2 \nu^2}{B}. $$

**Proof.** First, we can show

$$ \mathbb{E}[\hat{G}(x)] = \mathbb{E}[\nabla g(x)] + H(x) = G(x) $$

and

$$ \mathbb{E}[\|G(x) - \hat{G}(x)\|^2] = \mathbb{E}[\|\nabla g(x) - \hat{g}(x)\|^2] \leq \frac{\nu^2}{B}. $$

Then, we can prove the desired result

$$ \mathbb{E}[\hat{P}(x)] = 1 - \gamma \mathbb{E}[\hat{G}(x)] = P(x) $$

and

$$ \mathbb{E}[\|P(x) - \tilde{P}(x)\|^2] = \gamma^2 \mathbb{E}[\|G(x) - \hat{G}(x)\|^2] \leq \frac{\gamma^2 \nu^2}{B}. $$

**Proposition 4.** Let the denoiser $D_x$ be such that it satisfies Assumption 3 and $\nabla g$ is L-Lipschitz continuous. For any $\gamma \in (0, 1/(L + 2\tau))$, the operator $P$ is nonexpansive

$$ \|P(x) - P(y)\|_2 \leq \|x - y\|_2 \quad \forall x, y \in \mathbb{R}^n $$

**Proof.** The proposition is a direct result of the part (c) of the proof of Theorem 1 (Section A) in the Supplementary Material of [49] by setting $U = U^T = I$ and $G_t = G$, which corresponds to the full-gradient RED algorithm of (10).

Now we prove Theorem 1 in the paper. Consider a single iteration $x^k = \hat{P}(x^{k-1})$, then we can write for any $x^* \in \text{zer}(G)$ that

$$ \|x^k - x^*\|^2 = \|\hat{P}(x^{k-1}) - P(x^*)\|^2 $$
$$ = \|\hat{P}(x^{k-1}) - P(x^{k-1}) + P(x^{k-1}) - P(x^*)\|^2 $$
$$ = \|P(x^{k-1}) - P(x^*)\|^2 + \|\hat{P}(x^{k-1}) - P(x^{k-1})\|^2 $$
$$ + 2(\hat{P}(x^{k-1}) - P(x^{k-1}))^T(P(x^{k-1}) - P(x^*)) $$
$$ \leq \|x^k - x^*\|^2 - \left(\frac{\gamma}{L + 2\tau}\right) \|G(x^{k-1})\|^2 $$
$$ + \|\hat{P}(x^{k-1}) - P(x^{k-1})\|^2 $$
$$ + 2\|\hat{P}(x^{k-1}) - P(x^{k-1})\|_2 \cdot \|P(x^{k-1}) - P(x^*)\|_2, $$

where we use the Cauchy-Schwarz inequality and adapt the bound (14) in the part (d) of the proof of Theorem 1 (Section A) in the Supplementary Material of [49] by setting $U = U^T = I$. 


and \( G = G \). According to Assumption 2 and Proposition 4, we have
\[
\| P(x^{k-1}) - P(x^*) \|_2 \leq \| x^{k-1} - x^* \|_2 \leq R_0. \tag{21}
\]
Additionally, by using Jensen’s inequality, we can have for all \( x \in \mathbb{R}^n \) that
\[
E \left[ \| P(x) - \tilde{P}(x) \|_2 \right] = E \left[ \left( \sqrt{\| P(x) - \tilde{P}(x) \|_2} \right)^2 \right] \leq \sqrt{E \left[ \| P(x) - \tilde{P}(x) \|_2^2 \right]} \leq \frac{\gamma \mu}{\sqrt{B}}. \tag{22}
\]
By rearranging and taking a conditional expectation of (20) and using these bounds, we can obtain
\[
E \left[ \| x^k - x^* \|_2^2 - \| x^{k-1} - x^* \|_2^2 \mid x^{k-1} \right] \leq \frac{2\gamma \mu}{\sqrt{B}} R_0 + \frac{\gamma^2 \nu^2}{B} - \left( \frac{\gamma}{L + 2\tau} \right) \| G(x^{k-1}) \|_2^2,
\]
which can be reorganized as
\[
\| G(x^{k-1}) \|_2^2 \leq \left( \frac{L + 2\tau}{\gamma} \right) \left( \frac{\gamma^2 \nu^2}{B} + \frac{2\gamma \mu}{\sqrt{B}} R_0 \right)
+ E \left[ \| x^{k-1} - x^* \|_2^2 - \| x^k - x^* \|_2^2 \mid x^{k-1} \right].
\]
By averaging the inequality over \( t \geq 1 \) iterations, taking the total expectation, and dropping the last term, we obtain
\[
E \left[ \frac{1}{T} \sum_{k=1}^{t} \| G(x^{k-1}) \|_2^2 \right] \leq \frac{L + 2\tau}{2} \left( \frac{\gamma^2 \nu^2}{B} + \frac{2\gamma \mu}{\sqrt{B}} R_0 + \frac{R^2_0}{t} \right)
\]
where we apply the law of total expectation and Assumption 2. This establishes the Theorem 1.

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