Unified Theory of Surface-Plasmonic Enhancement and Extinction of Light Transmission through Metallic Nanoslit Arrays

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Metallic nanostructures are of immense scientific interest owing to unexpectedly strong interaction with light in deep subwavelength scales. Resonant excitations of surface and cavity plasmonic modes mediate strong light localization in nanoscale objects. Nevertheless, the role of surface plasmon-polaritons (SPP) in light transmission through a simple one-dimensional system with metallic nanoslits has been the subject of longstanding debates. Here, we propose a unified theory that consistently explains the controversial effects of SPPs in metallic nanoslit arrays. We show that the SPPs excited on the entrance and exit interfaces induce near-total internal reflection and abrupt phase change of the slit-guided mode. These fundamental effects quantitatively describe positive and negative effects of SPP excitation in a self-consistent manner. Importantly, the theory shows excellent agreement with rigorous numerical calculations while providing profound physical insight into the properties of nanoplasmonic systems.

Renewed by the discovery of extraordinary optical transmission (EOT)1–2, extensive study has been devoted to explaining light transmission through metal films with subwavelength aperture arrays. Initially, EOT through hole arrays was understood by local field enhancement with interfacial excitations evanescently coupled through subwavelength holes. Surface plasmon-polaritons (SPPs) in the optical domain3–5 and geometrical surface resonances in the THz or microwave spectral ranges6 induce such coupled interfacial excitations.

However, subwavelength slit arrays have shown many distinguished behaviors from those of hole arrays. A widely accepted enhancement mechanism for slit arrays is based on Fabry-Pérot resonances of slit-guided modes7–18, i.e., cavity modes (CMs). Many recent papers conclude that CMs provide the enhancement mechanism while SPPs play only a negative role13–16. In this view that deters SPPs, spectral location of the transmission minimum corresponds to the SPP resonance condition and the associated field pattern shows typical SPP character with its null at the aperture opening. Strong surface-plasmonic absorption15,16, excitation of a nonresonant SPP13, and the surface-plasmonic bandgap effect17,18 have been suggested explanations for these negative effects of SPPs. The negative effect of an SPP has also been reported for hole arrays, and different viewpoints based on the nonresonant SPP excitation19 and destructive interference between transmission pathways via surface and hole-guided modes20 have been published as suggested explanations.

Nevertheless, another bundle of recent papers have reported enhanced transmission with clear SPP characters in surface field patterns and frequency-dispersive properties7–12. In these analyses, SPPs enhance transmission with coexisting CMs10–12. Associated with SPP-CM hybrids, transmission peaks become much narrower than the pure CM resonances8–10,12,17,18 — the CM resonance condition shifts abruptly9,12 and an asymmetric Fano profile appears21–23. Therefore the role of SPPs in metallic nano aperture arrays is still under debate, and the essential physics remain unclear due to these diverging interpretations.

In this paper, we show that controversial SPP-related effects can be consistently described by a single unified model that treats a metallic nanoslit array as an optical cavity with SPP-resonant boundaries. We theoretically prove that all aforementioned SPP-related effects such as the antiresonant extinction, null field at the aperture opening, bandwidth narrowing, and abrupt shift of the CM resonance condition are rooted in a single resonance interaction: a surface-plasmonic Fano resonance that occurs when the external light and CM couple at the entrance and exit interfaces. Contributed by the SPP excitation, metal-film interfaces act as a Fano-resonant...
gate that closes or opens nanoslit cavities and causes associated phase changes in the internal reflection of the CM. This interfacial interaction successfully describes various metamorphic SPP-related effects in a physically intuitive manner.

**Analytic theory**

Consider transmission of transverse-magnetic (TM) polarized light through a metal film perforated by an array of slits with period \( \Lambda \), thickness \( d \), and slit width \( w \) as shown in Fig. 1a. For deep subwavelength slits (\( w \ll \lambda \)) in an optically thick metal film (\( d > \) skin depth \( \lambda/2\pi|\varepsilon_M|^{1/2} \)), the light transmission can be described by a Fabry-Pérot formula for the fundamental CM.\(^{2,12,22,23}\) as

\[
T = \frac{\varepsilon^{2} e^{i 2q d}}{1 - \rho_{in}^{2} e^{i 2q d}},
\]

where \( \beta = \beta' + i\beta'' \) is the complex propagation constant of the CM, \( \tau \) is the coupling coefficient between the CM and external plane wave, and \( \rho_{in} \) is the internal reflection coefficient of the CM as illustrated in Fig. 1b. Equation (1) is generally applicable to cases with arbitrary angles of incidence as long as only zero-order waves propagate in the surrounding media. Enhanced transmission peaks appear when the multiple scattering denominator in Eq. (1) becomes minimal at the phase-matching condition \( \beta' d_q = (q + 1)\pi - \arg(\rho_{in}) \), where \( q \) is an integer. The role of an SPP is implicitly held in \( \tau \) and \( \rho_{in} \) as a mechanism causing a Fano resonance to occur at the top and bottom interfaces.

Here, we further examine the coupling processes at each film interface. We treat each interface as a Fano-resonant boundary where an SPP acts as a discrete state that interferes with the nonresonant continuum. In Fig. 1c, an SPP-resonant pathway interferes with a nonresonant pathway in the scattering processes of the incident CM. An SPP originally excited by the incident CM emits the reflected CM with probability \( \eta_{in} \) and the transmitted external radiation with probability \( \eta_{ex} \). This process provides the resonant components of the single-interface reflection \( \rho_{SP} \) and transmission \( \tau_{SP} \) coefficients. The incident CM at each interface also couples to the non-plasmonic reflection \( \rho_{D} \) and transmission \( \tau_{D} \). Using the optical Fano resonance theory developed by \(^{22}\), the single-interface transmission \( \tau \) and reflection \( \rho_{in} \) coefficients are written as

\[
\tau(\delta) = \tau_{D} + \frac{2\eta_{in}}{1 - i\delta} \eta_{ex} e^{i\phi},
\]

\[
\rho_{in}(\delta) = \rho_{D} + \frac{2\eta_{in}}{1 - i\delta} \eta_{ex} e^{i\phi},
\]

where \( \delta = (\omega - \omega_{SP}/2)\gamma_{tot} \) is the normalized frequency, \( \omega_{SP} \) is the resonance frequency of the SPP, \( \gamma_{tot} \) is the total decay rate of the SPP, \( \eta_{in} \) and \( \eta_{ex} \) represent radiation probabilities of the SPP to the CM and the external plane wave, respectively, \( \phi \) is the plasmonic transmission phase at \( \delta = 0 \), and \( \phi = \arg(\rho_{SP}) \) is the plasmonic reflection phase at \( \delta = 0 \). In this view to treat the interface responses, the time reversibility requirement dictates the phase differences between the plasmonic and non-plasmonic contributions, i.e., \( \xi = \arg(\tau_{SP}) - \arg(\tau_{opt}) \) and \( \zeta = \arg(\rho_{D}) - \arg(\rho_{SP}) \) at \( \delta = 0 \), to be determined by \(^{22}\)

\[
\epsilon_{\xi} = \frac{-\eta_{rad}|\tau_{opt}|}{2\sqrt{\eta_{in}^{2} + \eta_{ex}^{2}}} + i\frac{1 - \eta_{rad}^{2}|\tau_{opt}|^{2}}{4\eta_{in} \eta_{ex}},
\]

\[
\epsilon_{\zeta} = \frac{1}{|\rho_{D}|} + \frac{\tau_{D}}{\rho_{D}} \frac{\eta_{ex}}{\eta_{in}},
\]

where \( \eta_{rad} = \eta_{in} + \eta_{ex} \) is the total radiation probability of the SPP. Once establishing the real-valued parameters such as radiation probabilities (\( \eta_{in} \) and \( \eta_{ex} \)) and magnitudes of the nonresonant pathways (\( |\rho_{D}| \) and \( |\tau_{opt}| \)), we can describe the interference between the coupling pathways and the spectral properties of \( \tau \) and \( \rho_{in} \) in a fully deterministic manner with these phase relations.

**Lossless and nondispersive system**

We check the theory with numerical calculation based on the rigorous coupled-wave analysis (RCWA) and finite element method (FEM). The metal is modeled by a complex dielectric constant \( \varepsilon_{M} = \varepsilon_{M}' + i\varepsilon_{M}'' \), where \( \varepsilon_{M}' < 0 \) and \( \varepsilon_{M}'' \ll |\varepsilon_{M}'| \). Figures 2a and 2b show the absolute value of the scattering matrix \( S \) for a metal nanoslit array. The transmission spectrum at the interface exhibits resonant peaks that match the resonance frequencies of the SPP, \( \gamma_{SP} \), and the non-plasmonic contributions, i.e., \( \gamma_{SP} = \gamma_{SP} \) and \( \gamma_{SP} > \gamma_{SP} \). The resonant peaks are shown to be highly dependent on the slit width \( w \), thickness \( d \), and incidence angle \( \theta \). The experimental results show excellent quantitative agreement with the theoretical predictions, confirming the essential physics of the Fano resonance in metamorphic SPP arrays.
symmetric Lorentzian profiles with a common resonance center $\lambda_{SP} = 1.0624 \, \lambda$ and full-width at half-maximum (FWHM) $\Delta \lambda_{SP} = 0.03211 \, \lambda$. We attribute this surface excitation to a pure SPP on a patterned surface. Using a pure-SPP model developed by Liu and Lalanne, we predict the SPP resonance wavelength and bandwidth (FWHM) on a metallic slit array to be

$$\lambda_{SP} = \frac{\lambda_{SP}}{1 - \arg(t_s + r_s)/2\pi},$$

$$\Delta \lambda_{SP} = \lambda_{SP} \left[ \frac{\lambda_{SP}}{\pi \lambda_{SP}} \ln \left( |t_s + r_s|^{-1} \right) + \frac{2n_{SP}^\prime n_{SP}^\prime}{n_{SP}^\prime} \right],$$

where $t_s$ and $r_s$ are in-plane SPP transmission and reflection coefficients at a single isolated slit, respectively, $\lambda_{SP} = n_{SP} \lambda$ is the SPP resonance wavelength on a flat metal surface, and $n_{SP}^\prime + n_{SP}^\prime = [r_{CM}/(1 + r_{CM})]^{1/2}$ is the complex effective index of the SPP on a flat metal surface. FEM calculation of SPP scattering by a single isolated slit yields $\arg(t_s + r_s) = -0.1047\pi$ and $|t_s + r_s|^{-1} = 1.1097$. The SPP resonance wavelength and bandwidth due to Eqs. (6) and (7) are $\lambda_{SP} = 1.0624 \, \lambda$ and $\Delta \lambda_{SP} = 0.03346 \, \lambda$. These values quantitatively agree with those obtained from the surface excitation spectrum in Fig. 2c. See Supplementary Section I for the derivation of Eqs. (6) and (7) and Supplementary Section II for the FEM calculation of the in-plane SPP transmission $t_s$ and reflection $r_s$ coefficients.

The pure SPP excitation and associated Fano resonance at the interface successfully describe the drastic modification of CM resonance properties. The single-interface transmittance $|t|^2$ due to the analytic theory (solid curve $\cdots$) in Fig. 2d shows excellent agreement with the numerical calculation result (square symbols $\square$) due to FEM. A typical Fano resonance profile appears with its resonant enhancement peak at $\lambda_{AR} = 1.058 \, \lambda$ ($\lambda < \lambda_{SP}$) and antiresonant extinction at $\lambda_{AR} > 1.30 \, \lambda$. First, the destructive interference between the plasmonic ($t_{SP}$) and non-plasmonic ($t_{D}$) coupling coefficients explains the antiresonant extinction of CM resonance peaks at $\lambda_{AR}$. Figure 2e shows the spectral behavior of the phase difference between $t_{SP}$ and $t_{D}$ ($\zeta = \arg(t_{D}) - \arg(t_{SP})$) due to Eq. (4). Note at $\lambda = \lambda_{AR}$, the phase difference $\zeta = \pi$ where $|t_{SP}| = |t_{D}|$ as shown in Fig. 2d, leading to complete destructive interference followed by $\tau = 0$. The null field at the slit opening for the excitation by external planewave in Fig. 3a is a natural consequence of the null excitation of the CM due to the complete destructive interference at $\lambda_{AR}$. This interference description, the null field at the slit opening does not require any special electromagnetic excitation such as nonresonant

SPP$^{15,16}$. Note that in Fig. 3b the field at the slit opening is non-zero for the interface excitation by a CM at $\lambda = \lambda_{AR}$. The effect of the antiresonant extinction of $\tau$ on the film-transmission $T$ is obvious: the internal reflectance of the CM becomes total ($|t_{in}|^2 = 1 - |t|^2 \rightarrow 1$) as $\tau$ approaches 0. Therefore, the slits at $\lambda = \lambda_{AR}$ behave as closed cavities that generally support vanishingly narrow and extremely high-quality cavity resonance peaks.

On the other hand, at the SPP enhancement condition for $\lambda = \lambda_{SP}^\prime$, $|t|^2$ is maximal and consequently the slits behave as open cavities with partial transmission and reflection at the interface as shown in Figs. 3c and 3d. We note strong SPP excitation at the interface that leads to resonant phase change in the internal reflection of the CM. In Fig. 2f, the internal reflection phase $\arg(\rho_{in})$ shows a sharp 2$\pi$ change centered at $\lambda = \lambda_{SP}$. This phase behavior clearly describes the abrupt shift of the CM resonance peaks observed in Figs. 2a and 2b. Recalling the CM resonance condition $\beta d_q = (q + 1)\pi - \arg(\rho_{in})$, 2$\pi$ phase-change in $\rho_{in}$ results in a transition of the resonant film thickness $d_q \rightarrow d_q + 2$. This transition in the CM resonance condition is widely found in the literature$^{41,42}$ but has not been explained in terms of internal reflection phase change associated with the surface-plasmonic Fano resonance.

**Lossy system**

Now we consider the effect of material dissipation. Figures 4a and 4b show transmission spectra for several different film thicknesses ($d/\lambda = 1.03, 1.06, 1.1$) in lossless ($\epsilon_{CM} = -5$) and lossy ($\epsilon_{CM} = 5 + 0.01i$) cases, respectively. All solid curves calculated by the analytic theory in Eqs. (1) -- (5) are again in excellent agreement with the RCWA simulation results (square symbols $\square$). For the lossy case in Fig. 4b, the material dissipation is included in the analytic theory by including the non-radiative decay rate of the SPP $\gamma_{AR}$ ($2\pi c/\lambda_{SP}^\prime (n_{SP}^\prime n_{SP}^\prime)$) (see Supplementary Section I for details) and the complex propagation constant of the CM $\beta = \beta^* + i\beta^*$ that is given by the equation$^{25}$ $\epsilon_{CM} \tanh(\beta^* - k_0^2)/(2\epsilon_{CM}^2 (\beta^* - k_0^2)^2) = -(\beta^* - k_0^2)/(\beta^* - k_0^2)^2)^{1/2}$. The transmission peaks for $q = 4$ in Figs. 4a and 4b clearly reveal the effect of material absorption on the CM resonance peaks near the antiresonant extinction condition $\lambda_{AR} = 1.13 \, \lambda$. For the lossless case in Fig. 4a, the linewidth of the transmission peak tends to vanish as the peak approaches $\lambda_{AR}$. Diverging CM localization lifetime in the closed cavity regime leads to an extremely narrow linewidth and a high local field enhancement. This is pointed out as the origin of the diverging inter-slit coupling matrix at the surface-mode resonance condition in the modal expansion method$^{39}$. A transmission peak with an extremely narrow linewidth quickly disappears with material
Figure 3 | Magnetic field ($H_y$) distributions for single interface excitation for (a), planewave incidence from air at the antiresonance condition ($\lambda = \lambda_{AR}$), and (b), guided-mode (CM) incidence from slit at $\lambda = \lambda_{AR}$, (c), planewave incidence from air at the SPP resonance center ($\lambda = \lambda_{SP}$), and (d), guided-mode (CM) incidence from slit at $\lambda = \lambda_{SP}$.

dissipation while those with a relatively wide linewidth near $\lambda = \lambda_{SP} = 1.062\Lambda$ or $\lambda_R = 1.058\Lambda$ are loss-insensitive as shown in Figs. 4a and 4b.

The relation between the peak transmittance ($T_{max}$) and the resonance quality factor $Q$, i.e., the number of effective oscillations in the cavity, quantitatively explains a narrow peak's high sensitivity to material dissipation. From Eq. (1) and the surface-plasmonic absorption $A_{SP} = 1 - |\tau|^2 - |\rho|^2$, we obtain $4\pi T_{max} \approx Q^2 |\tau|^2$ and $Q \approx 2\pi A_{SP}^{-1}\exp(-\beta d_y)$ in a high $Q$-factor regime ($Q \gg 1$). Whereas $T_{max} = 1$ in the lossless case, it decreases with $Q^2$ as ohmic damping of free electrons causes surface-plasmonic absorption ($A_{SP}$) at the interface and propagation loss of the CM ($A_{CM}$) inside the cavity. In Fig. 4c, the decrease in $Q$ for the lossy case ($\varepsilon_M'' = 0.01$) is negligible near $\lambda = \lambda_{SP}$ or $\lambda_R$ where $Q$ is relatively small. In the extremely high $Q$ band near $\lambda_{AR}$, however, $Q$ is remarkably suppressed for the lossy case (red curve), resulting in strong suppression of $T_{max}$ indicated by the gray curve enveloping the peaks in Fig. 4b. It is also worth noting that the surface-plasmonic absorption is not a dominant absorption channel responsible for the CM resonance extinction at $\lambda_{AR}$. The ratio of $A_{SP}$ to $A_{CM}$ in Fig. 4d is 0.1 at $\lambda_{AR}$ and thereby $A_{CM}$ is nearly 10 times stronger than $A_{SP}$. In contrast, at $\lambda = \lambda_{SP} = 1.062\Lambda$ where surface-plasmonic absorption is maximized ($A_{SP}/A_{CM} = 7.44$), the effect of loss on $Q$ and $T_{max}$ is not remarkable as shown in Figs. 4b and 4c. Therefore, the propagation loss of a CM is dominantly responsible for the resonance extinction in this model system with $\varepsilon_M = -5 + 0.01i$.

**Lossy and dispersive system**

In our description, $\lambda_R$, $\lambda_{SP}$, and $\lambda_{AR}$ are crucial parameters strongly dependent on the metal dielectric constant $\varepsilon_M$. We use FEM to calculate $\lambda_R$, $\lambda_{SP}$, and $\lambda_{AR}$ as a function of the real part $\varepsilon_M'$ of $\varepsilon_M$, and the result is shown in Fig. 5. As $\varepsilon_M'$ decreases to far negative values, $\lambda_R$

Figure 4 | Effect of material dissipation on the film transmittance ($T$) and resonance quality factor. The film transmittance ($T$) through (a), lossless ($\varepsilon_M'' = 0$) and (b), lossy ($\varepsilon_M'' = 0.01$) metal films for $d/\Lambda = 1.03$ (blue), 1.06 (black), and 1.1 (red). Square symbols (□) are obtained by RCWA simulation, and curves (−) represent the analytic theory based on Eqs. (1) − (5). (c), Resonance $Q$-factor spectra for $\varepsilon_M'' = 0$ (blue) and 0.01 (red). (d), Ratio of the SPP-induced absorption ($A_{SP}$) to the CM-induced absorption ($A_{CM}$) in a lossy metal film ($\varepsilon_M'' = 0.01$). $\lambda_R = 1.058\Lambda$ and $\lambda_{AR} = 1.130\Lambda$.

Figure 5 | Dependence of characteristic wavelengths of the surface-plasmonic Fano resonance on metal dielectric constant ($\varepsilon_M'$): the antiresonant extinction ($\lambda_{AR}$), SPP resonance center ($\lambda_{SP}$), resonant enhancement ($\lambda_{R}$), and SPP resonance center on a flat, unpatterned surface ($\lambda_{SP}$). We assume slit width $w = 0.05\Lambda$. 

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and \( \lambda_{AR} \) approach the canonical Rayleigh anomaly \( \lambda_{Rayleigh} = \Lambda \) and the SPP resonance wavelength on a flat, unpatterned metal surface \( \lambda_{SPP} \), respectively. Dependence of these characteristic locations on the metal dielectric constant \( \varepsilon_R' \) suggests that previous confusion in the role of SPPs may originate from the spectral proximity of \( \lambda_R \) and \( \lambda_{AR} \) to \( \lambda_{Rayleigh} \) and \( \lambda_{SPP} \). For example, at \( \varepsilon_R' = -15 \) where many previous analyses have been performed, \( |\lambda_R - \lambda_{SPP}| \approx 4 \times 10^{-3} \) and \( |\lambda_R - \lambda_{Rayleigh}| \approx 1.5 \times 10^{-5} \). In this situation, it is likely to form a hasty conclusion that the SPP plays only a negative role and that the actual SPP-associated effects, such as the abrupt shift of the CM resonance condition, are confused with the effect of Rayleigh anomaly\(^{28}\). In \([15,16]\) for example, the authors analyzed Au slit arrays in the near- and mid-infrared spectral domains where \( \varepsilon_R' < -30 \). Concluded by the coincidence of the antisoresonant CM resonance extinction with the SPP resonance wavelength \( \lambda_{SPP} \) on a flat, unpatterned metal surface, they attribute the CM resonance extinction solely to the surface-plasmonic absorption. However, our model shows that an SPP induces high-Q CM resonances at the antiresonance condition, and the CM resonances in this case are highly sensitive to losses in any kind including both surface-plasmonic and cavity-modal absorption. Moreover, it is evident in Fig. 5 that the true SPP resonance wavelength \( \lambda_{SPP} \) on a perforated metal surface differs from both \( \lambda_{SPP} \) and \( \lambda_{Rayleigh} \) even down to \( \varepsilon_M \approx -30 \), which corresponds to Ag for wavelength \( \sim 800 \) nm.

Finally, we show that all aspects of the simple Fano resonance model presented above are also present in realistic systems with lossy dispersive metals. In Fig. 6, we show the film-transmittance \( T(\lambda, d) \), average field intensity \( |E_\perp|^2 \), and single-interface transmittance \( |\tau(\lambda)|^2 \) for silver nanoslits arrays with several different periods and slit widths. We use RCWA for \( T(\lambda, d) \) and FEM for \( |E_\perp|^2 \) and \( |\tau(\lambda)|^2 \) with the realistic \( \varepsilon_M \) of silver experimentally obtained by Johnson and Christy\(^{29}\). For all three different cases, we confirm the same characteristic features as for the simplified case in Fig. 2; they include CM resonance modifications at \( \lambda = \lambda_{SP} \) and \( \lambda_{AR} \) in \( T(\lambda, d) \) associated with a single Lorentzian surface excitation in the \( |E_\perp|^2 \) spectrum and a typical Fano profile in \( |\tau(\lambda)|^2 \). In Figs. 6a \( \sim \) 6c for \( \Lambda = 400 \) nm and \( w = 20 \) nm, \( \lambda_{AR} \) is fairly separated from \( \lambda_{SP} \) and resonance shift \( (\delta_q - \delta_{q+2}) \) due to a \( 2\pi \) reflection-phase change that fully appears around \( \lambda_{SP} \). In Figs. 6d \( \sim \) 6f for \( \Lambda = 500 \) nm and \( w = 25 \) nm, no additional feature appears. Affected by larger \( |\delta_q'| \) for the longer wavelength, \( \lambda_{AR} \) almost coincides with \( \lambda_{SP} \) (\( \lambda_{SP} - \lambda_{SP}| \sim 3 \times 10^{-4} \)) and the resonance shift near \( \lambda_R \) is seemingly associated with the Rayleigh anomaly at \( \lambda = \Lambda \) as previously discussed in Fig. 5. In Figs. 6g \( \sim \) 6i for \( \Lambda = 400 \) nm and a wider slit width \( w = 40 \) nm, the wider slit results in a wide bandwidth in the SPP excitation. The Rayleigh anomaly, defined as sharp intensity variations occurring when an evanescent higher-order wave turns into a propagating wave, causes a corresponding decrease in the zero-order intensity. Our model is unable to describe effects associated with the Rayleigh anomaly because it is limited to a subwavelength-period regime where only the zero-order waves are allowed in the radiation continuum. Nevertheless, the rigorous calculation results for different cases in Figs. 1 and 6 consistently show no necessary effect of the Rayleigh anomaly on the resonance properties while the characteristic features of our interfacial Fano resonance model persistently appear. Therefore, the results in Figs. 1 and 6 suggest that the interfacial Fano resonance is the fundamental origin of the cavity resonance modification. In the previous literature, Sarrazin et al.\(^{12}\) reported a comprehensive spectral and surface field analysis also showing that the Rayleigh anomaly is unnecessary and they suggested the significance of surface-plasmonic Fano resonances with a phenomenological argument based on complex poles and zeros of scattering amplitudes\(^{25}\).

We have shown that various aspects of SPPs in EOT through metallic nanoslit arrays can be consistently understood by the surface-plasmonic Fano resonance in the coupling of external radiation to the slit cavity mode. The Fano resonance interpretation was first suggested by Genet et al.\(^{21}\) in order to explain the asymmetric profile and red shift of the enhanced transmission feature. They assumed a single discrete state without any other localized states such as a slit- or hole-guided mode. With detailed coupling processes unclear, the original Fano resonance interpretation has been used for phenomenological analyses of experimental and numerical data\(^{12}\). Our theory clearly describes where the Fano-type interference occurs, how it modifies the optical response of a metal surface with periodic nanoslits, and how the SPP-resonant metal surface finally contributes to metamorphic cavity-resonance properties.

**Consistency with previous theories**

In addition, our theory provides deeper physical insight into the microscopic theory of EOT developed by Liu and Lalanne\(^{2}\). For two-dimensional hole arrays, they found the single-interface transmission coefficient

\[
\tau(\omega) = \tau_D + \frac{2ab}{\exp(-in_{SP}(\omega/\omega_0)) - (\tau_3 + \tau_4)}
\]

for normal incidence, where \( a \) and \( b \) denote coupling coefficients of an SPP with the hole-guided mode and external radiation, respectively. The Fano-type interference is an inevitable consequence of this expression as its first and second terms on the right-hand side...
represent the nonresonant and resonant contributions, respectively. Equations (2) ~ (5) for Fano resonance in the single-interface coupling processes are applicable to two-dimensional hole arrays in principle. Therefore Eq. (8) should be consistent with our Fano resonance interpretation. Indeed, Eq. (8) reduces to Eq. (2) with radiation probabilities

$$\eta_{\text{ex}} = \frac{c}{|\kappa_{\text{SP}}(\Lambda_{\text{SP}}|s + r_s)|^2} \quad \text{and} \quad \eta_{\text{in}} = \frac{c}{|\kappa_{\text{SP}}(\Lambda_{\text{SP}}|s + r_s)|^2}, \quad (9)$$

and resonant transmission phase

$$\phi = \text{arg}(a) + \text{arg}(b) - \text{arg}(s + r_s) - \text{arg}(\kappa_{\text{SP}}). \quad (10)$$

In Eq. (9), the total decay rate $\gamma_{\text{rad}} = (\pi/c)\kappa_{\text{SP}}^2 \Lambda_{\text{SP}}$ with $\Lambda_{\text{SP}}$ and $\kappa_{\text{SP}}$ in Eqs. (6) and (7), respectively. See Supplementary Section III for detailed derivation. These relations describe how elementary scattering processes of electromagnetic fields at the metal-film interfaces are associated with the more fundamental wave kinematic effect of Fano resonances. The formal consistency of our model with the microscopic theory of EOT suggests further importance of the Fano resonance interaction in longer wavelength ranges beyond the visible domain. A series of theoretical\(^{3,8,55}\) and experimental\(^{17,23}\) analyses recently showed that an additional contribution from the quasi-cylindrical wave can also be described by the same elementary scattering coefficients $a$, $b$, $r_S$, and $r_D$ of an SPP. In the near-infrared and longer wavelength domains, the quasi-cylindrical wave is known to significantly contribute to the resonances in periodic arrays of metallic nanoapertures\(^{36,37}\).

**Conclusions**

In conclusion, we propose a surface-plasmonic Fano resonance theory of the light transmission through metallic nanoaperture slits. Importantly, seemingly paradoxical, metamorphic SPP-related effects are clearly explained by the pure surface-plasmonic Fano resonance effects at the film interfaces, which cause drastic modification of the cavity-resonances inside nanoapertures. We also show that the interfacial Fano resonance interpretation is formally consistent with the microscopic theory of EOT through two-dimensional hole arrays. Therefore, for a two-dimensional array of large holes that allow propagating guided modes, a surface mode must lead to fundamentally the same effects on the cavity-mode resonance properties as those in one-dimensional slit arrays. For example, Catrysse and Fan\(^{20}\) reported the antiresonant extinction of transmission peaks associated with hole-guided modes when the surface mode is resonantly excited in an SiC film with cylindrical holes. We believe that our theory unifies different interpretations and illuminates the origin of previous confusion regarding the role of SPPs. For example, the antiresonant extinction of CM resonance peaks is not simply a negative SPP effect but is rooted in the SPP-induced total internal reflection of the CM (SPP-induced cavity closing). Note that, in this case, the SPP actually contributes in a positive way as it leads to very high-Q CM resonances. Therefore, appropriate loss compensation methods\(^{38,39}\) are of great interest at the antiresonant extinction condition as extremely high-Q nanocavity resonances are expected. We summarize how our theory unifies previous partial interpretations in Supplementary Table 1. Our model is limited to the zero-order regime and deep subwavelength slits that allow only the fundamental guided mode. Further development of our approach to more general cases of interfacial Fano resonance coupling with higher-order propagating waves and multiple localized modes may yield deeper physical insight into various nanophotonic and surface-plasmonic systems where interplay of coexisting modes induces versatile spectral properties and novel optical effects.

**Methods**

To estimate basic resonance parameters $\omega_{\text{SP}}$, $\Lambda_{\text{SP}}$, $\eta_{\text{in}}$, and $\eta_{\text{ex}}$ for the analytic theory, we use surface excitation spectra in Fig. 2c. Two excitation spectra (red squares) for planewave incidence from air (blue squares) and for CM incidence from slit are determined by $\omega_{\text{SP}}$ and $\eta_{\text{SP}}$, respectively.

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Author contributions

This research was planned by J.W.Y., S.H.S. and R.M. J.W.Y. developed the analytic theory. Numerical simulation was performed by J.H.L. under supervision by S.H.S. and J.W.Y. The authors J.W.Y., J.H.L., S.H.S. and R.M. discussed the results. J.W.Y., S.H.S. and R.M. wrote the manuscript.

Additional information

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