Centre Vortex Effects on the Overlap Quark Propagator

Daniel Trewartha
Derek Leinweber and Waseem Kamleh

CSSM, School of Chemistry and Physics
University of Adelaide

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Overview

- Identifying centre vortices on the lattice via MCG fixing
- Overlap quark propagator on vortex-free and vortex-only backgrounds
  
  Qualitatively different results to previous ASQTAD results
- Effects of cooling on vortex-only backgrounds
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Identifying Centre Vortices on the Lattice

Transform to Maximal Centre Gauge, where links are brought close to centre elements

\[ Z_\mu(x) = z \mathbf{I}, \quad z^3 = 1 \]

\[ = \exp \left[ \frac{2\pi i}{3} m_\mu(x) \right] \mathbf{I}, \quad m_\mu(x) \in \{-1, 0, 1\} \] (1)

Require transformation \( \Omega(x) \) maximising overlap between gauge links and centre elements

\[ \sum_{x, \mu} \text{Re} \text{Tr} \left[ U_\mu^\Omega(x) Z_\mu^\dagger(x) \right] \rightarrow \text{Max} \] (2)
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- Implemented through ’mesonic’ centre gauge fixing condition

\[ R_{mes} = \sum_{x,\mu} |\text{Tr} \ U_\mu^\Omega(x)|^2 \rightarrow \text{Max} \]  

(3)

- Then we project onto \( Z_3 \)

\[ \frac{1}{3} \text{Tr}U_\mu^\Omega(x) = r_\mu(x) \exp(i\phi_\mu(x)) \]  

(4)

Choose \( m_\mu(x) \in \{-1, 0, 1\} \) with \( \frac{2\pi m_\mu(x)}{3} \) closest to \( \phi_\mu(x) \)
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We use the overlap operator, which has a lattice-deformed version of chiral symmetry, leading to greater sensitivity to topological effects.

Results calculated on 50 $20^3 \times 40$ gauge-field configurations using Lüscher-Weisz $O(a^2)$ mean-field improved action with a lattice spacing of 0.125 fm.
Simulation Details

- We use the overlap operator, which has a lattice-deformed version of chiral symmetry, leading to greater sensitivity to topological effects.
- Results calculated on 50 $20^3 \times 40$ gauge-field configurations using Lušcher-Weisz $\mathcal{O}(a^2)$ mean-field improved action with a lattice spacing of 0.125 fm.
MCG-fixed phases
Identifying Centre Vortices on the Lattice

3 sets of configurations:

- Untouched configurations

\[ U_\mu(x) \]  

(5)

- Vortex-only configurations

\[ Z_\mu(x) = \exp \left[ \frac{2\pi i}{3} m_\mu(x) \right] I \]  

(6)

- Vortex removed configurations

\[ R_\mu(x) = Z^\dagger_\mu(x) U^\Omega_\mu(x) \]  

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Centre Vortices and Confinement

From Bowman et al, Phys. Rev. D 84, 034501 (2011)
Previous Results Using an ASQTAD action

Performed with $m_0a = 0.048$, $a = 0.122$ on a $16^3 \times 32$ lattice

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Write momentum-space propagator as

\[ S(p) = \frac{Z(p)}{i\vec{q} + M(p)}, \tag{8} \]

with \( q_\mu \) the tree-level improved kinematic lattice momentum[1]

Fixed to Landau gauge using a Fourier transform accelerated algorithm [2] to the \( \mathcal{O}(a^2) \) improved gauge-fixing functional [3].

[1] F.D.R. Bonnet et al, Phys. Rev. D 65, 2002
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Mass function on Untouched Configurations

\[ m_q = 70 \text{ MeV} \]

- Untouched
- Bare Mass

\[
\begin{align*}
M(p) \text{ MeV} & \quad \text{versus} \quad p \text{ GeV} \\
0 & \quad 1 \quad 2 \quad 3 \quad 4 \quad 5 \quad 6 \\
\end{align*}
\]
Mass function with Vortex Removed Configurations

\[ m_q = 70 \text{ MeV} \]

- Untouched
- Vortex Removed
- Bare Mass

\[ \text{MeV} \]

\[ \text{GeV} \]
Renormalization function on UT Configurations

\[ Z(p) \]

\[ m_q = 70 \text{ MeV} \]

untouched
Renormalization function with VR Configurations

\[ m_q = 70 \text{ MeV} \]

- Untouched
- VR
Quark Propagator on Vortex Removed Configurations

- ASQTAD propagator unable to show loss of dynamical mass generation with vortex removal
- Overlap propagator shows loss of dynamical mass generation coincident with vortex removal
- Loss of confinement on vortex removed backgrounds using overlap
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Mass function on Vortex Only Configurations

\[ m_q = 70 \text{ MeV} \]

- x Untouched
- o Vortex Only
- -- Bare Mass
Renormalization function on VO Configurations

\[ m_q = 70 \text{ MeV} \]

- Untouched
- VO
The story so far...

- Vortex-only backgrounds cannot reproduce dynamical mass generation
- Vortex-only backgrounds not trivial; evidence of confinement
- The question: what information about the original configurations do vortex-only configurations retain?
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Cooling

- Vortex-only configurations consist only of center elements ⇒ high action
- We will perform cooling on vortex-only configurations
- Cooling is performed using an $\mathcal{O}(a^4)$-three-loop improved action, and the topological charge density is calculated using an $\mathcal{O}(a^4)$-five-loop improved definition of the field-strength tensor.
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Untouched Configurations with Cooling
Untouched Configurations with Cooling
Vortex Only Configurations with Cooling
Vortex Only Configurations with Cooling
40 sweep comparison
Mass function with cooling

- Under a UV filter, the overlap mass function retains its form qualitatively, with some loss of dynamical mass generation[1]

[1] D. T, W. Kamleh, D. Leinweber and D. S. Roberts, Phys. Rev. D 88, 034501 (2013) [arXiv:1306.3283 [hep-lat]].
Renormalization function with cooling

\[ Z(p) \]

- \( m_q = 70 \text{ MeV} \)
- Untouched
- 40 sweeps cooling

\( p \text{ GeV} \)
Mass function with cooling

\[ m_q = 70 \text{ MeV} \]

- Vortex Only
- --- Bare Mass

\[ M(p) \text{ MeV} \]

\[ p \text{ GeV} \]
Mass function with cooling

\[ m_\eta = 70 \text{ MeV} \]

- \( \times \) Vortex Only
- \( \Box \) Vortex Only Cooled
- \( - - - - \) Bare Mass

![Graph showing mass function with cooling](image-url)
Renormalization function with cooling

\[ Z(p) \]

\[ m_q = 70 \text{ MeV} \]

\[ \text{Vortex Only} \]
Renormalization function with cooling

\[ Z(p) \]

- \[ m_q = 70 \text{ MeV} \]
- \( \square \) Vortex Only
- \( \times \) Vortex Only Cooled
Mass function with cooling

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Renormalization function with cooling

\[ Z(p) \]

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- Untouched
- VO
Mass function with cooling

\[ m_q = 70 \text{ MeV} \]

- Untouched
- Vortex Removed
- Vortex Only
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Renormalization function with cooling

\[ m_q = 70 \text{ MeV} \]
- Untouched
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Conclusion

- Shown for the first time removal of centre vortices coincident with loss of dynamical mass generation
- A centre vortex background alone does not support dynamical mass generation, but shows evidence of confinement
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Additional Slides
Preconditioning Landau-gauge fixing

\[ m_q = 70 \text{ MeV} \]

- Untouched
- VO Method A
- VO Method B
- VO Method C

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Bare Mass

\[ M(p) \text{ MeV} \]

\[ p \text{ GeV} \]
MCG fixing

- Wish to maximise the local quantity

\[ R_x = \sum_{\mu} |\text{Tr}\{G(x)U_{\mu}(x)\}|^2 + \sum_{\mu} |\text{Tr}\{U_{\mu}(x - \mu)G^\dagger(x)\}|^2 \quad (9) \]

- Use an \( SU(2) \) matrix \( g = g_4 I - ig_i \sigma_i \) embedded in one of the 3 \( SU(2) \) subgroups of \( SU(3) \)

- Can be re-written as

\[ R_x = g_i A_{ij} g_j + g_i b_i + c, \quad (10) \]

with \( A \) real, symmetric \( 4 \times 4 \) matrix, \( b \) a real 4-vector, \( c \) a real constant.

Method of A. Montero, Phys. Lett. B 467, 106 (1999)
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Lower Bare Masses

\[ m_q = 12 \text{ MeV} \]

- Untouched
- VO
- VR

\[ \text{Bare Mass} \]
Lower Bare Masses

\[ m_q = 70 \text{ MeV} \]
- Untouched
- VR
- VO
$m_q = 70$ MeV

- Untouched
- Vortex Only
- Vortex Removed

--- Bare Mass
$z(p)$

$m_q = 70$ MeV

- Untouched
- VR
- VO