Dynamics of correlations in a quasi-2D dipolar Bose gas following a quantum quench

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We study the evolution of correlations in a quasi-2D dipolar gas driven out-of-equilibrium by a sudden ramp of the interaction strength. For sufficiently strong ramps, the momentum distribution, excited fraction and density-density correlation function all display pronounced features that are directly related to the appearance of a roton minimum in the underlying spectrum. Our study suggests that the evolution of correlations following a quench can be used as a probe of roton-like excitations in a dipolar gas. We also find that the build up of density-density correlations following a quench occurs much more slowly in the dipolar gas compared to a non-dipolar gas, owing to the long-range interactions.

Introduction. Recent advances in ultra-cold atomic and molecular gases have greatly expanded the potential of these systems as tools for studying many-body physics. For example, the realization of quantum degenerate gases with large magnetic dipole moments, ongoing efforts to trap and cool polar molecules, experiments on Rydberg atoms, trapped ions, and atoms in cavities have opened up the possibility of realizing ultra-cold atomic systems with long-range interactions, such as the anisotropic dipole-dipole interaction. Concurrently, better control over experimental parameters and high resolution imaging techniques have introduced new probes for exploring many-body physics, notable among which is the ability to study the dynamics of correlations following a non-adiabatic ramp (quench) of system parameters. Here we study the evolution of correlations in a quasi-2D dipolar gas following a quench, finding qualitatively new physics arising from the long-range nature of the interactions.

A novel property of quasi-2D dipolar superfluids is that in addition to the phonon-like mode at low energies $E_k \sim ck$, the low energy excitation spectrum also features a roton-like mode ($E_k = \Delta + \hbar^2(k - k_0)^2/2m^*$) for sufficiently strong dipolar interactions. Although this mode has been spectroscopically identified in non-polar Bose condensates in optical cavities, the corresponding measurement in dipolar systems has not been performed, leaving the existence of a roton in dipolar systems still only as a theoretical prediction. Here we investigate how the roton mode is revealed in the spatio-temporal evolution of one and two-body correlations, following a sudden switching on of the dipolar interactions. Our main result, summarized in Fig. 1, shows that these correlations develop striking features that are directly related to the underlying roton minimum in the dispersion.

Quench experiments also probe how correlations spread between initially uncorrelated regions in the sample. The manner in which correlations develop is intimately linked to questions about how isolated systems approach equilibrium, and how (and if) long range order is established following a quench. Recent theoretical studies on spin models have shown that information tends to propagate slower in systems with long range versus short range interactions, a prediction which can be directly tested using Rydberg atoms or trapped ions. Here we find similar results in the continuum: the build up of density-density correlations occurs much more slowly in a dipolar gas compared to a non-dipolar gas, owing to the long range dipole interactions.

FIG. 1: (Color Online) Top: Evolution of the radial momentum distribution $n_k$ in a quasi-2D dipolar gas ($l_z$ is the width of the cloud in the direction perpendicular to the 2D plane), following a sudden ramp of the dipolar interaction. Momentum distribution develops a peak at the wave-vector (black arrow) corresponding to the roton minimum in the inset. Black curve is the the corresponding equilibrium feature for the same interaction strength. Time ($t$) is given in units of $1/(\nu_{wo} = 2\hbar^2/m l_z^2)$. Bottom: (Red, Dashed) Long time density-density correlation function following a quench to strong dipolar interactions. (Black, Solid) Equilibrium density-density correlation function in a strongly interacting dipolar gas.
Theory. We consider a quasi-2D dipolar Bose gas of mass $m$, at zero temperature, confined in a harmonic potential of the form $U(z) = \frac{1}{2} m a_z^2 z^2$, and free in the $x - y$ (transverse) directions. Furthermore, we assume that the dipoles are polarized along the $z$-direction, which yields a dipolar interaction potential of the form $V_{\text{dip}}(R) = \frac{-\epsilon}{R^3}(1 - 3 \cos^2(\theta))$, where $d$ is the dipole moment, and $\cos(\theta) = z/|R|$. Additionally, there is a short-range contribution to the interaction potential, which we model as a contact interaction $V_c(R) = g_0(R)$, where $g = 4\pi \hbar^2 a/m$ and $a$ is the s-wave scattering length. Finite temperature does not affect our conclusions provided the temperature is less than the degeneracy temperature, and the equilibrium roton energy scale.

In the limit where the zero-point energy greatly exceeds the interactions, the longitudinal and transverse degrees of freedom decouple, and the atomic density can be expressed as $n^{3D}(R = \{r, z\}) = n(r)\Psi(z) = \frac{1}{\sqrt{4\pi}} \sqrt{n(r)} e^{-z^2/2\lambda^2}$, where $r$ and $z$ are the radial and the axial co-ordinate respectively, and $\lambda = \sqrt{\hbar/m a_z}$. Integrating out the $z$-direction, the Fourier transform of the resulting quasi-2D interaction potential reads \[ V(k) = \frac{1}{\sqrt{2\pi} V(z = 0)} \left( g + g_{\text{d}} F \left( \frac{k_l}{\sqrt{2}} \right) \right), \] where $k$ is the magnitude of the radial momentum, $g_{\text{d}} = \frac{4\pi}{\sqrt{2}} d^2$, and $F(x) = 2 - 3\sqrt{\pi} \text{Erfc}(x) e^{x^2}$, where Erfc($x$) is the complimentary error function. The specific Gaussian choice of our wave-function has no effect on our conclusions.

The Hamiltonian for a uniform quasi-2D dipolar Bose gas, where the dipoles are aligned along the $z$-axis, now takes the form:

\[ \mathcal{H} = \sum_{k} (\epsilon_k - \mu) a_k^\dagger a_k + \frac{1}{2\Omega} \sum_{pqk} V(q) a_{p+q}^\dagger a_{k-q}^\dagger a_{k} a_{p} \]  

where $\epsilon_k = \hbar^2 k^2/2m$, $a_k$ is the bosonic annihilation operator at momentum $k$ and time $t$, $\Omega$ is the area, and $\mu$ is the chemical potential. We consider the evolution of the momentum distribution $n_k(t) = \langle a_k^\dagger(t) a_k(t) \rangle$, and the density-density correlation function: $g^{(2)}(\mathbf{r}, t) = \sum_{pq} e^{i \mathbf{q} \cdot \mathbf{r}} \rho_q(t) \rho_{-\mathbf{q}}(0)$, where $\rho_q(t) = \sum_{k} a_k^\dagger(t) a_k(t)$, following a sudden quench in the dimensionless interaction parameter $\tilde{g} = g_{\text{d}}/G$. In quasi-2D at zero temperature, there is a true Bose condensate \cite{35}, and we can model the dynamics using a time-dependent Bogoliubov approach. We set the density of condensate atoms $n_0 = \langle a_{k=0}^\dagger a_{k=0} \rangle^2$, and write $a_{k\neq0}(t) = u_k(t) b_k + v_k(t) b_k^\dagger$, where $b_k$ denotes the bosonic annihilation operator for the non-condensed atoms \cite{30}. The $b_k$ operators have no time dependence, and are formally treated as small. Substituting the expression for $a_{k\neq0}$ into Eq. 1 and discarding all terms cubic or higher order in $b_k$, we arrive at $u_k(t = 0) = \sqrt{\frac{1}{2} \left( 1 + \epsilon_k + V_{\text{dip}}(k) n_0 \right) E_k}$ and $v_k(t = 0) = -\text{sgn}(V_{\text{dip}}(k)) \sqrt{\frac{1}{2} \left( \epsilon_k + V_{\text{dip}}(k) n_0 \right) E_k}$, where $\text{sgn}(x)$ denotes the sign of the argument, and $E_k^f = \sqrt{\epsilon_k + 2 V_{\text{dip}}(k) n_0}$, where the index $i$, denotes the initial state. At future times, these coherence factors $u_k(t)$ and $v_k(t)$ will acquire complex values, but will always satisfy $|u_k(t)|^2 - |v_k(t)|^2 = 1$. We neglect the time dependence of $n_0$.

The equations of motion for $u_k$ and $v_k$ are obtained from the Heisenberg equations of motion for $a_k$ and read \[ i\partial_t \left( \begin{array}{c} u_k(t) \\ v_k(t) \end{array} \right) = \left( \begin{array}{cc} A_k & B_k \\ -B_k & -A_k \end{array} \right) \left( \begin{array}{c} u_k(0) \\ v_k(0) \end{array} \right) \]  

where we have introduced the functions $A_k = \epsilon_k + V_{\text{dip}}(k) n_0$, $B_k = V_{\text{dip}}(k) n_0$ and $E_k^f = \sqrt{A_k^2 - B_k^2}$ is the Bogoliubov dispersion. As we consider a sudden quench, the evolution of $u_k$ and $v_k$ depends only on the final Hamiltonian parameters, denoted by the label $f$. The matrix on the right hand side is real and time-independent, thus this time-dependent Bogoliubov approximation amounts to coherent oscillations of quasi-particles in and out of the condensate at a frequency proportional to the quasi-particle energy $E_k^f$. Excitations at different momenta evolve independently of one another.

The evolution of the momentum distribution is given by $n_k(t) = |u_k(t)|^2$, and can be probed in time-of-flight \cite{33}. We also define the excited fraction as $n_{ex}(t) = \int d^2 k n_k(t)$. Likewise, the density-density correlation function takes the form $g^{(2)}(\mathbf{r}, t) = n_0^2 + n_{ex}(t) \sum_k e^{i \mathbf{k} \cdot \mathbf{r}} \left( 2|u_k(t)|^2 + 2 \text{Re}(u_k(t) v_k(t) + u_k(t) v_k^\ast(t)) \right)$ \cite{33} and can be probed using high resolution imaging \cite{23, 24}, Bragg spectroscopy \cite{33} or noise correlations \cite{40}. The first term in this expression is the correlation between the condensate atoms, while the second term is the contribution from correlations between the condensed and non-condensed atoms. Terms which involve correlations between the non-condensed atoms (quartic in $u_{k}s$ and $v_{k}s$) are negligible on length scales $\mathbf{r} \gg \zeta$, where $\zeta$ is the healing length of the condensate \cite{30, 41, 42}, and are neglected here. We also define a dimensionless density-density correlation function $\tilde{g}^{(2)}(t) = g^{(2)}(t) E_k^f/n_0$.

The validity of the Bogoliubov approach relies on the assumption that the condensate depletion is small, $(n_{ex} \ll n_0)$ throughout the evolution. When the interactions become strong, effects beyond the time-dependent Bogoliubov theory can be captured by treating the condensate density as a dynamical variable \cite{43}. In quasi-2D, the relevant dimensionless parameter is $n_0 l_z^2$, which for typical condensate densities of $n_0 \sim 3 \times 10^{13} \text{m}^{-2}$ and $l_z \sim 0.5 \mu\text{m}$ \cite{14}, is of the order $n_0 l_z^2 \sim 5$. Below we show that even for quenches to strong dipolar interactions, $n_{ex} l_z^2 \sim 0.5$, hence we expect the corrections beyond mean-field theory to be small at experimental densities.

Although we can model arbitrary quenches, through-
the vicinity of the roton minimum reads $m_k^* \approx 2 \hbar^2/\mu l^2$ and $\tilde{g}_t = g_d/g = 0$. We then switch on the dipolar interactions ($g_d$), keeping $g$ fixed ($g_t = g_f$), to produce a non-zero $\tilde{g}_f$. In the top plot (see solid red curve), solid line is the roton dispersion curve). As a result, many modes oscillate at the frequency equal to twice the roton gap, and an envelope which decays algebraically as $1/\sqrt{t}$. The roton gap extracted from fitting the numerical data for $n_{ex}(t)$ to this long time asymptotic formula is in near perfect agreement with the gap obtained directly from the dispersion relation $E_k = \sqrt{\epsilon_k (E_k + 2V_f(k) \nu_0)}$. Hence the roton gap can be measured following the quench.

Interestingly, at intermediate values of the dipolar interactions (green, dotted curve in Fig. 2 (top)), the excited fraction approaches its asymptotic value on a timescale $\tau = 1/(g + g_d) \nu_0$. For strong dipolar interactions (blue and red curves in Fig. 2 (top)), the system preferentially populates modes near the roton minimum, all of which oscillate at a frequency roughly equal to $E_k$. This give rise to pronounced oscillations in the excited fraction. The observed “damping” of the oscillations is determined by the spread of the energies near $E_k$, which is set by the curvature of the dispersion near $k = k_t$.

At long times, the dynamics of the excited fraction takes the approximate analytic form $n_{ex}(t \to \infty) \propto \cos(2\Delta t + \phi)/\Delta^2 \sqrt{\eta t \nu_0}$, where $\eta = m/m^*$, $\Delta = \Delta/\nu_0$, and $\phi$ is an arbitrary constant phase factor. The dynamics are thus described by damped oscillations at a frequency equal to twice the roton gap, and an envelope which decays algebraically as $1/\sqrt{t}$. The roton gap extracted from fitting the numerical data for $n_{ex}(t)$ to this long time asymptotic formula is in near perfect agreement with the gap obtained directly from the dispersion relation $E_k = \sqrt{\epsilon_k (E_k + 2V_f(k) \nu_0)}$. Hence the roton gap can be measured following the quench.

We note that our results remain valid even in the presence of a trap. Modeling the trap using a local density approximation, and assuming the condensate density profile is unchanged following the quench, we have checked numerically that the oscillations in $n_{ex}$ persist for a few cycles, provided the trapping time is larger than the inverse roton gap. In typical experiments, the roton gap is of the order $\Delta \sim 0.5 \nu_0 \sim 500$Hz, while the radial trapping potential is $\omega_r \sim 10$Hz.
Density-density correlations. The density-density correlation function following the quench reads: $g^{(2)}(t) - g^{(2)}(0) = -\frac{1}{2} \int dkk J_0(kr) \sin(E_k^f t)^2 \epsilon_k B_0 \left( \frac{E_k^f}{\mu_{\text{ho}}} \right)$, where $J_0$ is the Bessel function of the first kind. The long time asymptotic value of $g^{(2)}$ can be obtained by setting $\sin(E_k^f t)^2|_{t=\infty} = (\sin(E_k^e t)^2)^{1/2}$.

In Fig. 1 we plot $g^{(2)}(r,t \rightarrow \infty)$ for a quench to $\tilde{g}_f = 2.65$. The long time non-equilibrium density-density correlation displays pronounced oscillations with a characteristic wave-length equal to $\lambda = 2\pi/k_r$. The amplitude of these oscillations is noticeably larger than the corresponding feature in the equilibrium zero temperature density-density correlation function [30], indicative of the enhanced occupation of roton modes after the quench.

In Fig. 2 we plot $g^{(2)}(r,t)$ for different values of $r$ following a quench to two different values of $\tilde{g}_f$, starting from $\tilde{g} = 0$. For weak dipolar interactions ($\tilde{g}_f \lesssim 1$), the density-density correlation function develops a dip feature, and then rapidly relaxes to its long time value 36,47, while for strong dipolar interactions, density correlations display persistent oscillations, with a frequency largely independent of $r$. The origin of these oscillations is the same as the oscillations in the excited fraction —

the excitations produced after the quench are predominantly rotons, all of which oscillate at the same frequency.

To study how the correlations spread in a dipolar gas, we plot the temporal location of the dip feature ($t_{\text{dip}}$) in the density-density correlations for different values of $r$ and $\tilde{g}_f$ (Fig. 3 (bottom-left)). The slope of the curves at large $r$, plotted on the bottom-right panel, is the characteristic velocity of spreading of correlations. Absent dipolar interactions and damping processes, quasi-particles propagate ballistically, hence correlations exhibit a light-cone like evolution at large distances 46,47. At small $\tilde{g}_f$, the velocity of spreading of correlations can be found by noting that at large distances and times, the dominant contribution to the density-density correlation function, comes from phonons. Expanding the dispersion as $E_k \approx \sqrt{c^2(1 + 2\tilde{g}_f)k}$, and the approximating the Bessel function $J_0(x) \approx \cos(x/\sqrt{2})$, we obtain that correlations propagate with a velocity $v \approx 2\lambda/\nu_{\text{ho}}$, which is what we find numerically.

Counter-intuitively, increasing dipolar interactions decreases the velocity. This is surprising, as naïvely one would expect that increasing repulsive interactions should increase the quasi-particle velocity. In a quasi-2D dipolar gas however, increasing repulsive interactions decreases the average group velocity of the quasi-particles. (The opposite is true of attractive dipolar interactions.) Consequently, correlations still spread in a light-cone manner, however the velocity of spread is slower in the dipolar gas.

For quenches to even stronger dipolar interactions, the spreading of correlations develops a non-trivial step-like feature. The width of the step grows with $\tilde{g}_f$, becoming infinitely wide at the roton instability threshold, indicating that near the roton instability, correlations take infinitely long to build up. A natural scale for the velocity with which time correlations build up in a strongly interacting dipolar gas is $v \sim 2\Delta/\hbar k_r \rightarrow 0$. As shown in Fig. 3 (bottom-right), the numerically found propagation velocity is consistent with this formula. For $\tilde{g} < 2.3$, there is no sharp roton feature in the dispersion and for $\tilde{g} > 2.6$, the data in Fig. 3 (bottom-left) can no longer be fit by a straight line at large $r$.

Conclusions. We study the dynamics of correlations in a quasi-2D dipolar Bose gas, following a sudden ramp in the dipolar interaction strength, from a non-dipolar initial state. Quenches to strong dipolar interactions enhance the population of roton-like excitations, which produces observable features in the correlation functions. This striking enhancement occurs due to the divergent density of states for excitations near the roton minimum. Our study shows that quench dynamics may complement spectroscopic and other probes of roton excitations in the dipolar gas 48,49, and could directly manifest the physics of the roton arising from the long range interactions present in the system.

A well known phenomenon in solid state is the en-
hancement of superconductivity using radiation \cite{61,62}, which has been observed in Josephson junctions \cite{63} and thin films \cite{64}. Radiation produces a non-equilibrium momentum distribution which amplifies the superconducting gap. A related direction for further study is whether the non-equilibrium enhancement of rotons could be a route to realizing (albeit metastable \cite{54,55}) supersolids or Wigner crystal phases \cite{56–62}.

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