Hyperon suppression in hadron-quark mixed phase

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Abstract. We investigate the property of the hadron-quark mixed phase using the Brueckner-Hartree-Fock model for hadron (hyperon) phase and the MIT bag model for quark phase. To satisfy the Gibbs conditions, charge density as well as baryon number density becomes non-uniform in the mixed phase, accompanying phase separation. We clarify the roles of the surface tension and the charge screening effect. We show that the screened Coulomb interaction tends to make the geometrical structure of the mixed phase less stable, and the resultant EOS becomes similar to the one given by the Maxwell construction. The composition of the mixed phase, however, is very different from that of the Maxwell construction; in particular, hyperons are completely suppressed in the mixed phase, because hadron phase is positively charged. This is a novel mechanism of hyperon suppression in compact stars.

(Some figures in this article are in color only in the electric version)

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1. Introduction

It is well known that at several times the normal nuclear density $\rho_0$, hyperons emerge in matter and lead to a strong softening of the equation of state (EOS). Consequently the maximum neutron star mass is reduced to the one much lower than currently observed values of $\sim 1.4M_\odot$. For example the microscopic Brueckner-Hartree-Fock approach gives much lower masses.

On the other hand, the hadron-quark deconfinement transition is believed to occur in hot and/or dense matter. Then one may expect the maximum mass increases to the Chandrasekhar limit once the deconfinement transition occurs in hyperon matter [1]. The deconfinement transition from the hadron to quark phase may be of first order. It brings about a thermodynamic instability of uniform matter to have phase separation. In
other words, matter should have the nonuniform mixed phase (MP) around the critical density. Since hadron and/or quark matter consists of many kinds of particles, the Gibbs conditions must be properly taken into account. The usual Maxwell construction (MC) can be no more applied in this case. Due to the interplay of the Coulomb interaction and the surface tension between two phases, the MP can have exotic shapes called “pasta” structures [2] (as a review, see Ref. [3]). With increase of density, the stable shape of pasta structures may change from droplet to rod, slab, tube, and to bubble. The name “pasta” comes from rod and slab structures figuratively spoken as “spaghetti” and “lasagna”.

Generally, the appearance of the MP in matter results in a softening of the EOS. The bulk Gibbs calculation (BG) of the MP, without the effects of the Coulomb interaction and surface tension, leads to an appearance of the MP in a broad density region [4]. If one takes into account the geometrical structures of the MP, however, the EOS deviates from that of the BG. It approaches to the one given by the MC [3].

In this report we explore the EOS and the structure of the MP during the hyperon-quark transition, properly taking account of the Gibbs conditions together with the pasta structures. Then we demonstrate a novel phenomenon, suppression of hyperons in the MP.

2. Numerical Calculation

The numerical procedure to determine the EOS and the geometrical structure of the MP is explained in detail in Ref. [5]. We employ the Wigner-Seitz cell approximation in which the whole space is divided into equivalent cells with a given geometrical symmetry, sphere for three dimensional (3D) case, cylinder for 2D, and slab for 1D. In each cell the quark and hadron phases are spatially separated by a sharp boundary. The energy density of the MP is then written as

\[ \epsilon = \frac{1}{V_W} \left[ \int_{V_H} d\mathbf{r} \, \epsilon_H(\mathbf{r}) + \int_{V_Q} d\mathbf{r} \, \epsilon_Q(\mathbf{r}) + \int_{V_W} d\mathbf{r} \left( \epsilon_e(\mathbf{r}) + \frac{(\nabla \epsilon_e(\mathbf{r}))^2}{8\pi e^2} \right) + \sigma S \right], \]

where the volume of the Wigner-Seitz cell \( V_W \) is the sum of those of hadron and quark phases \( V_H \) and \( V_Q \), and \( S \) the quark-hadron interface area. The surface energy is taken into account with a surface-tension parameter \( \sigma \). The quantities \( \epsilon_H, \epsilon_Q \) and \( \epsilon_e \) are energy densities of hadrons, quarks and electrons, respectively, which are functions of local densities of \( n, p, \Lambda, \Sigma^- \), \( u, d, s, e \). For a given density \( \rho_B \), the optimum configuration of the cell (uniform hadron, quark droplet, rod, slab, tube, bubble, or uniform quark), the cell size \( R_W \), the lump size \( R \), and the density profile of each component are searched for to give the minimum energy density.

To calculate \( \epsilon_H \) we use the Thomas-Fermi approximation for the kinetic energy density. The interaction-energy density is calculated by the nonrelativistic BHF approach [1] based on the microscopic NN and NY potentials. With these potentials, the various \( G \) matrices are evaluated by solving numerically the Bethe-Goldstone equation,
which can be written in operator form as
\[ G_{ab}[W] = V_{ab} + \sum_c \sum_{p,p'} V_{ac}[pp'] \frac{Q_c}{W - E_c + i\epsilon} \langle pp' | G_{cb}[W] \rangle. \]  

(2)

where the indices \(a, b, c\) indicate pairs of baryons and the Pauli operator \(Q\) and energy \(E\) determine the propagation of intermediate baryon pairs. The pair energy in a given channel \(c = (ij)\); \(i, j = n, p, \Lambda, \Sigma\) is
\[ E_{(ij)} = T_i(k_i) + T_j(k_j) + U_i(k_i) + U_j(k_j) \]  

(3)

with \(T_i(k) = m_i + k^2/2m_i\). The various single-particle potentials are given self-consistently from the \(G\) matrices as,
\[ U_i(k) = \sum_{j=n,p,\Lambda,\Sigma} \sum_{k<k^{(j)}} \text{Re}\langle kk'|G_{(ij)(ij)}[E_{(ij)}(k,k')]|kk'\rangle. \]  

(4)

Once the different single-particle potentials are known, the total nonrelativistic hadronic energy density, \(\epsilon_H\), can be evaluated:
\[ \epsilon_H = \sum_{i=n,p,\Lambda,\Sigma} \sum_{k<k_i^{(i)}} \left[ T_i(k) + \frac{1}{2} U_i(k) \right], \]  

(5)

and \(\epsilon_H\) is thus represented as a function of particle number densities \(\rho_i(i = n, p, \Lambda, \Sigma)\). The parameter set used in this calculation reproduces the scattering phase shifts and nuclear saturation property.

For the quark phase, we employ the MIT bag model with massless \(u\) and \(d\) quarks and massive \(s\) quark with \(m_s = 150\) MeV. The energy density \(\epsilon_Q\) consists of the kinetic term by the Thomas-Fermi approximation, the leading-order one-gluon-exchange term and the bag constant \(B\) as
\[ \epsilon_Q = B + \sum_f \epsilon_f, \]  

(6)

\[ \epsilon_f(\rho_f) = \frac{3m_f^4}{8\pi^2} \left[ x_f \left( 2x_f^2 + 1 \right) \sqrt{1 + x_f^2} - \text{asinh} x_f \right] - \alpha_s \frac{m_f^4}{\pi^3} \left[ x_f^4 - \frac{3}{2} \left( x_f \sqrt{1 + x_f^2} - \text{asinh} x_f \right)^2 \right], \]  

(7)

where \(m_f\) is the \(f\) current quark mass, \(x_f = k_F^{(f)}/m_f\), and the number density of \(f\) quarks \(\rho_f = k_F^{(f)^3}/\pi^2\).

Demanding that the quark EOS crosses the hadronic EOS at a reasonable density, we choose \(B = 100\) MeV/fm\(^3\) and \(\alpha_s = 0\).

The surface tension of the hadron-quark interface is poorly known, but some theoretical estimates based on the MIT bag model for strangelets and lattice gauge simulations at finite temperature suggest a range of \(\sigma \approx 10–100\) MeV/fm\(^2\). We employ \(\sigma = 40\) MeV/fm\(^2\) in the present study.
3. Hadron-Quark Mixed Phase

Figure 4(a) illustrates an example of the density profile in a 3D cell for $\rho_B = 0.4 \, \text{fm}^{-3}$. One can see the non-uniform density distribution of each particle species together with the finite Coulomb potential. Charged particle distributions are rearranged by the Coulomb potential. For example, the quark phase is negatively charged, so that $d$ and $s$ quarks are repelled to the boundary of the negatively charged quark phase, while $u$ quarks gather at the center. The protons in the hadron phase are attracted by the negatively charged quark droplet, while the electrons mostly exist in the hadron phase. This density rearrangement of the charged particles causes the screening of the Coulomb interaction between two phases.

In panels (b) and (c), depicted are the cases of MC and BG for comparison. MC assumes the local charge neutrality, while the BG does not. One can see that the local charge neutrality in the full calculation lies between two cases.

4. Effects of the Coulomb Screening and the Surface Tension

Here let us discuss the effects of the Coulomb screening and the surface tension. The volume fraction $V_i/V_W$ ($i = Q, H$) is determined by a bulk calculation without any surface tension or the Coulomb interaction. Then the size of the structure is determined by the balance of the Coulomb repulsion and the surface tension, as schematically explained in Fig. 2(a). Taking three dimensional case for example, the Coulomb energy per particle $E_C/A$ is roughly proportional to the second power of the structure size $R$, while the surface energy per particle $E_S/A$ is to the inverse of $R$. Then the sum of these two energies has a minimum at a certain value of $R$ with $E_S = 2E_C$.

As the $E_C/A$ curve gets lower, the minimum point moves to the right hand side, i.e., the structure size becomes large. In the same way, if the surface tension is stronger (weaker), the structure size becomes larger (smaller).

When the screening of the Coulomb interaction is incorporated, the situation should be changed: the $R^2$ dependence breaks and $E_C/A$ decreases to zero as $R \to \infty$. Therefore the minimum point of $(E_C+E_S)/A$ disappears, if both the screening effect and

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{fig1.png}
\caption{(a) Density profiles and Coulomb potential $V_C$ within a 3D (quark droplet) Wigner-Seitz cell of the MP at $\rho_B = 0.4 \, \text{fm}^{-3}$. (b) Same as (a) for MC case. The radius $r$ is in arbitrary unit. (c) The case of BG calculation.}
\end{figure}
the surface tension become strong enough. In Fig. 2 (b) we demonstrate an example, where the energy per particle is depicted by changing the strength of the surface tension. One can see that there is no minimum for $\sigma > 70$ MeV.

5. Maxwell Construction and the Bulk Gibbs Calculation

Figure 3 (a) compares the resulting EOS with that of the pure hadron and quark phases. The thick black curve indicates the case of the MC, while the colored lines indicate the MP with various geometries starting at $\rho_B = 0.326$ fm$^{-3}$ with a quark droplet structure and terminating at $\rho_B = 0.666$ fm$^{-3}$ with a quark bubble structure. Note that the charge screening effect always tends to make matter locally charge-neutral and to reduce the Coulomb energy. Combined with the surface tension, it makes the non-uniform structures mechanically less stable and limits the density region of the MP [3, 5]. Figure 3 (b) shows the particle fraction by the full calculation. One can see that there appears no hyperon in matter. We shall discuss this point later.
Next let us consider the dependence on the surface tension. If the surface tension is strong, the structure size becomes large. In the larger scale, the Coulomb screening effect becomes more prominent and the local charge neutrality will be approximately achieved. Consequently the energy of the MP is close to that of the MC. On the other hand, if the surface tension is weak, the structure size becomes small. Then the Coulomb interaction becomes ineffective. Therefore the aspects of the MP becomes close to that of the BG.

If we use $\sigma \approx 60 \text{ MeV/fm}^2$, the EOS of the MP will coincide with the MC (thick black curve). If we use small value of $\sigma$, it approaches to the BG (thick gray curve). Our surface tension parameter $\sigma = 40 \text{ MeV/fm}^2$ is strong enough for the MP to be close to the MC case.

6. Structure of Hybrid Stars

Knowing the EOS comprising hadronic, mixed, and quark phase in the form $P(\epsilon)$, the equilibrium configurations of static NS are obtained in the standard way by solving the Tolman-Oppenheimer-Volkoff (TOV) equation for the pressure $P(r)$ and the enclosed mass $m(r)$,

$$\frac{dP}{dr} = - \frac{Gm\epsilon (1 + P/\epsilon)(1 + 4\pi r^3 P/m)}{r^2 (1 - 2Gm/r)} , \quad \text{(8)}$$

$$\frac{dm}{dr} = 4\pi r^2 \epsilon , \quad \text{(9)}$$

with $G$ being the gravitational constant. Starting with a central mass density $\epsilon(r = 0) \equiv \epsilon_c$, one integrates out until the surface density equals the one of iron. This gives the stellar radius $R$ and its gravitational mass $M = m(R)$.

![Figure 4](image-url) Neutron star mass-radius relations for different EOS. For the hybrid stars, the dashed lines indicate the MC (upper curve) or BG calculation (lower curve) and the solid line the MP of the full calculation.
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Figure 5. (a) Particle fractions of neutral matter with electrons. (b) The same quantity for charged matter without electrons, the low-density part of which corresponds to symmetric nuclear matter.

Figure 6. Charge number density per baryon number density in each phase $\rho_{\text{ch}}^{(i)}/\rho_B^{(i)}$ ($i = Q, H$) as a function of baryon number density. Neutral hadron matter, for instance, has $\rho_{\text{ch}}^{(H)}/\rho_B^{(H)} = 0$, and symmetric nuclear matter 0.5.

Our EOS gives results similar to those given by the MC. The maximum mass of a hybrid star is around 1.5 $M_\odot$, larger than that of the purely hadronic (hyperonic) star, $\approx 1.3 M_\odot$. Hence we may conclude that a hybrid star is still consistent with the canonical NS mass of 1.4 $M_\odot$, while the masses of purely hyperonic stars lie below it.

7. Suppression of Hyperons

The structure and the composition of the MP, however, are very different from those of the MC. Though a relevant hyperon ($\Sigma^-$) fraction is finite in the MC case, it is completely suppressed up to very high density in the full calculation (see Fig. 5 (b)). The suppression of hyperon mixture in the MP is due to the fact that the hadron phase is positively charged. As shown in Fig. 5, hyperons ($\Sigma^-$) appear in charge-neutral hadronic matter at low density ($0.34 \text{ fm}^{-3}$) to reduce the Fermi energies of electron and neutron. In the absence of the charge-neutrality condition, on the other hand, symmetric nuclear matter will be realized at lower density and hyperons will be mixed at higher density ($> 1.15 \text{ fm}^{-3}$) due to the large hyperon masses. Although the Coulomb screening effect diminishes the local charge density, the MP has positively charged hadron phase.
and negatively charged quark phase. Thus, the mixture of hyperons is suppressed in the MP where the hadron phase is positively charged.

In Fig. 6 we plot charge number density per baryon number density in hadron and quark phases. It is clear that the hadron/quark phase is positively/negatively charged. At $\rho_B \approx 0.3 \text{ fm}^{-3}$ where quark phase has a very small volume fraction, negative charge of the quark phase is large. This is because it should compensate the positive charge of the hadron phase with a large volume fraction. On the other hand, at high densities where hadron phase has small volume fraction, the hadron phase is positively charged and the value approaches to that of symmetric nuclear matter. This high charge of hadron phase at high density leads to the suppression of hyperons.

8. Summary

In this article we have studied the properties of the MP in the quark deconfinement transition in hyperonic matter, and their influence on compact stars.

The hadron-quark MP was consistently treated with the basic thermodynamical requirement due to the Gibbs conditions. We have seen that the resultant EOS is close to the one given by the MC. This is because the finite-size effects, i.e. the strong surface tension and the Coulomb screening, enlarge the structure size and promote the local charge neutrality. They also tend to diminish the density region of the MP through the mechanical instability. The masses and radii of compact stars given by our EOS are similar to those given by the MC. The maximum mass of a hybrid star is around $1.5 M_\odot$, larger than that of the purely hadronic star, $\approx 1.3 M_\odot$.

Although the EOS of matter and the resultant bulk properties of compact stars are close to those in the case of the MC, the ingredients of the MP are found to be different. Hyperons are completely suppressed in the MP. This is a novel feature of the hadron-quark MP and should have important consequences for the elementary processes inside compact stars. For example, coherent scattering of neutrinos off lumps in the MP may enhance the neutrino opacity [6]. Also, the absence of hyperons prevents a fast cooling mechanism by way of the hyperon Urca processes [7,8,9]. These results directly modify the thermal evolution of compact stars.

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