Evolution of a Schwarzschild Black Hole in Phantom-like Chaplygin gas Cosmologies

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Abstract

In the classical relativistic regime, the accretion of phantom energy onto a black hole reduces the mass of the black hole. In this context, we have investigated the evolution of a Schwarzschild black hole in the standard model of cosmology using the phantom-like modified variable Chaplygin gas and the viscous generalized Chaplygin gas. The corresponding expressions for accretion time scale and evolution of mass have been derived. Our results indicate that mass of the black hole will decrease if the accreting phantom Chaplygin gas violates the dominant energy condition and will increase in the opposite case. Thus our results are in agreement with the results of Babichev et al [21] who first proposed this scenario.

Keywords: Accretion; dark energy; Chaplygin gas; black hole; bulk viscosity; relativistic energy conditions; phantom energy.
1 Introduction

The evidence of accelerated expansion in the observable universe is quite compelling and has been confirmed by various astrophysical investigations including observations of supernovae of type Ia \cite{1,2}, anisotropies of the cosmic microwave background radiation \cite{3,4}, large scale structure and galaxy distribution surveys \cite{5}. This expansion of the universe is supposedly driven by an exotic energy commonly called ‘dark energy’ possessing negative pressure $p < 0$ and positive energy density $\rho > 0$, related by equation of state (EoS) $p = \omega \rho$. It should be noted that $p = \omega \rho$ is not a true EoS for dark energy rather a phenomenological description valid for a certain configuration \cite{6}. Astrophysical data suggests that about two third of the critical energy density is stored in the dark energy component. The corresponding parameter $\omega$ is then constrained in the range $-1.38 < \omega < -0.82$ \cite{7}. It shows that the EoS of cosmic fluids is not exactly determined. The genesis of this exotic energy is still unknown. The simplest and the earliest explanation of this phenomenon was provided by the general theory of relativity through the cosmological constant $\Lambda$. The observational value of its energy density is 56 to 120 orders of magnitude smaller than that derived from the standard theory \cite{8}. The satisfactory explanation of this phenomenon requires extreme fine tuning of the cosmological parameters. Other problem associated with $\Lambda$ is the coincidence problem (i.e. Why did the cosmic accelerated expansion start in the presence of intelligent beings? alternatively why the energy densities of matter and dark energy are of the same order at current time?) which is as yet explained either through anthropic principle \cite{9}, variable cosmological constant scenario \cite{10} or by invoking a dark matter-dark energy interaction \cite{11,12,13}. In this context, several other models have been proposed among them are models based on homogeneous and time dependent scalar fields termed as quintessence \cite{14}, quintom \cite{15,16} and k-essence \cite{17}, to name a few.

The interest in phantom energy arose when Caldwell et al \cite{18} explored the cosmological consequences of the EoS, $\omega < -1$. The dark energy can achieve this EoS if it is assumed to be variable quantity i.e. $\omega(z)$, where $z$ is the redshift parameter. Thus $\omega$ evolves as: for matter dominated universe $\omega = 0$, in quintessence phase $-1 < \omega \leq -1/3$, for cosmological constant dominated arena $\omega = -1$, while in phantom regime $\omega < -1$. This scenario appears to be consistent with the observations \cite{19}. In phantom cosmology, the energy density of phantom energy will become infinite in a finite time leading the ‘big rip’, a kind of future singularity. Moreover, due to strong negative pressure of phantom energy, all stable gravitationally bound objects will be dissociated near the big rip. These findings were later confirmed in \cite{20} by doing numerical analysis for the solar system and the Milky Way galaxy. In this context, the accretion of phantom dark energy onto a black hole was first modeled by Babichev et al \cite{21} who proved that black hole mass will gradually decrease due to strong negative pressure of phantom energy and tend to zero near the big rip where it will finally disappear. Note that $\omega > -1$ leads to the opposite scenario where black hole mass increases by accreting dark energy until its event horizon swells up to swallow the whole universe \cite{22}. Later studies \cite{23} showed that quantum effects dominate near the big rip singularity and consequently the mass of black hole although decreases.
but stops decreasing at a finite value. In another investigation [24], it was demonstrated that the physical black hole mass will increase due to accretion of phantom energy consequently the black hole horizon and the cosmological horizon will coincide leading the black hole singularity to become naked, all in a finite time. This analysis has been extended for the Riessner-Nördstrom, Kerr-Neumann and Schwarzschild-de Sitter black holes as well [25, 26, 27, 28, 29, 30]. This result apparently refutes the cosmic censorship conjecture (or hypothesis) which forbids the occurrence of any naked singularity. However, the formation of naked singularities is not completely ruled out. Numerical simulations of gravitational collapse of spheroids show that if the collapsing spheroid is sufficiently compact, the singularities are hidden inside black hole while they become naked (devoid of apparent horizon) if the spheroid is sufficiently large [31]. The future singularity of big rip is alternatively avoided by the ‘big trip’ where the accretion of phantom energy onto a wormhole will increase the size of its throat so much to engulf the whole universe [32, 33]. Another interesting scenario appears in cyclic cosmology where the masses of black holes first decrease and then increase through the phantom energy accretion and never vanish [34]. The implications of generalized second law of thermodynamics to the phantom energy accretion onto a black hole imply that accretion will be significant only near the big rip. If this law is violated then the black hole mass will decrease [35]. The thermodynamical investigations of phantom energy imply its positive definite entropy which tends to become constant if the phantom energy largely dominates the universe [36]. This results in the late universe to be hotter compared to the present.

We here discuss the accretion of phantom like modified variable Chaplygin gas and the viscous Chaplygin gas separately onto a black hole. This accretion of the phantom fluid reduces the mass of the black hole. This works serves as the generalization of the earlier work by Babichev et al [21, 37] who initiated the concept of accretion of exotic matter on the black hole. We have built our model on the same pattern by choosing more general EoS for the dark energy.

The outline of the paper is as follows: In the next section, we discuss the relativistic model of accretion onto a black hole. In third section, we investigate the evolution of the mass of black hole by the accretion of modified variable Chaplygin Gas (MVG) while in the fourth section, we discuss the similar scenario with the viscous generalized Chaplygin gas (VCG). Finally, we present conclusion of our paper. The formalism adopted here is from [21].

2 Accretion onto black hole

We consider a Schwarzschild black hole of mass $M$ which is gravitationally isolated and is specified by the line element (in geometrical units $c = 1 = G$):

$$ds^2 = \left(1 - \frac{2M}{r}\right)dt^2 - \left(1 - \frac{2M}{r}\right)^{-1}dr^2 - r^2(d\theta^2 + \sin^2 \theta d\phi^2).$$

(1)
The black hole is accreting Chaplygin gas which is assumed to be a perfect fluid specified by the stress energy tensor

\[ T^{\mu\nu} = (\rho + p)u^\mu u^\nu - pg^{\mu\nu}. \]  

(2)

Here \( p \) and \( \rho \) are the pressure and energy density of the Chaplygin gas respectively. Due to static and spherically symmetric nature of the black hole we assume the velocity four vector \( u^\mu = (u^t, u^r, 0, 0) \) which satisfies the normalization condition \( u^\mu u_\mu = -1 \). Thus we are considering only radial in-fall of the Chaplygin gas on the event horizon. Using the energy-momentum conservation for \( T^{\mu\nu} \), we get

\[ u x^2 (\rho + p) \sqrt{1 - \frac{2}{x} + u^2} = C_1, \]  

(3)

where \( x = r/M \) and \( u = u^r = dr/ds \) is the radial component of the velocity four vector \( u^\mu \) and \( C_1 \) is a constant of integration. The second constant of motion is obtained from \( u_\mu T^{\mu\nu}_{,\nu} = 0 \), which gives

\[ u x^2 \exp \left[ \int_{\rho_\infty}^{\rho_h} \frac{d\rho'}{\rho' + p'(\rho')} \right] = -A, \]  

(4)

where \( A \) is a constant of integration which is determined below for two models of Chaplygin gas. The quantities \( \rho_\infty \) and \( \rho_h \) are the densities of Chaplygin gas at infinity and at the black hole horizon respectively. Further using Eqs. (3) and (4), we obtain

\[ (\rho + p) \sqrt{1 - \frac{2}{x} + u^2} \exp \left[ - \int_{\rho_\infty}^{\rho_h} \frac{d\rho'}{\rho' + p'(\rho')} \right] = C_2, \]  

(5)

where \( C_2 = -C_1/A = \rho_\infty + p(\rho_\infty) \). In order to calculate \( \dot{M} \), the rate of change of mass of black hole we integrate the Chaplygin gas flux over the entire horizon as, \( \dot{M} = \oint T^r_t dS \) where \( T^r_t \) denotes the momentum density of Chaplygin gas in the radial direction and \( dS = \sqrt{-g} d\theta d\phi \) is the surface element of black hole horizon. Using Eqs. (2 - 5), we get this rate of change as

\[ \frac{dM}{dt} = 4\pi AM^2(\rho + p). \]  

(6)

Integration of Eq. (6) yields

\[ M = M_i \left( 1 - \frac{t}{\tau} \right)^{-1}, \]  

(7)

which determines the evolution of mass of black hole of initial mass \( M_i \) and \( \tau \) is the characteristic accretion time scale given by

\[ \tau^{-1} = 4\pi AM_i(\rho + p). \]  

(8)

The number density and energy density of Chaplygin gas are related as

\[ \frac{n(\rho_h)}{n(\rho_\infty)} = \exp \left[ \int_{\rho_\infty}^{\rho_h} \frac{d\rho'}{\rho' + p'(\rho')} \right], \]  

(9)
here \( n(\rho_h) \) and \( n(\rho_\infty) \) are the number densities of the Chaplygin gas at the horizon and at infinity respectively. Further the constant \( A \) appearing in Eq. (8) is determined as

\[
\frac{n(\rho_h)}{n(\rho_\infty)}ux^2 = -A,
\]

which is an alternative form of energy momentum conservation Eq. (4). Moreover, the critical points of accretion (i.e. the points where the speed of flow achieves the speed of sound \( V^2 = c_s^2 = \partial p/\partial \rho \)) are determined as follows

\[
u^2 = \frac{1}{2x}, \quad V^2 = \frac{u^2}{1 - 3u^2}, \tag{11}\]

where \( V^2 \equiv \frac{n}{\rho + p} \frac{d(\rho + p)}{dn} - 1 \). Finally, the above Eqs. (9 - 11) are combined in a single expression as

\[
\frac{\rho + p(\rho)}{n(\rho)} = [1 + 3c_s^2(\rho)]^{1/2} \frac{\rho_\infty + p(\rho_\infty)}{n(\rho_\infty)}. \tag{12}\]

### 3 Accretion of modified variable Chaplygin gas

The Chaplygin gas had been proposed to explain the accelerated expansion of the universe \[38\]. It is represented by a simple EoS, \( p = -A'/\rho \) where \( A' \) is positive constant. The corresponding expression for the evolution of energy density is

\[
\rho = \sqrt{A' + \frac{B}{a^6}}, \tag{13}\]

where \( B \) is a constant of integration. From Eq. (13) we find the following asymptotic behavior for the density \[39\]:

\[
\rho \sim \sqrt{Ba^3}, \quad a \ll (B/A')^{1/6}, \tag{14}
\]

\[
\rho \sim p \sim \sqrt{A}, \quad a \gg (B/A')^{1/6}. \tag{15}\]

Thus for small \( a \), it gives matter dominated era at earlier times while for large \( a \) we get dark energy dominated era at late times. Thus Chaplygin gas has the property of giving a unified picture of the evolution of the universe. The observational evidence in favor of cosmological models based on Chaplygin gas is quite encouraging \[40, 41, 42, 43\]. The Chaplygin gas model favors a spatially flat universe which agrees with the observational data of Sloan Digital Sky Survey (SDSS) and Supernova Legacy Survey (SNLS) with 95.4 % confidence level \[44\]. Consequently, various generalizations of Chaplygin gas have been proposed in the literature to incorporate any other dark component in the universe (see e.g. \[45, 46, 47, 48\] and references therein).

We here consider an equation of state which combines various EoS of Chaplygin gas given by \[49\]

\[
p = A'\rho - \frac{B(a)}{\rho^m}, \quad B(a) = B_0a^{-m}. \tag{16}\]
Here $A'$, $B_o$ and $m$ are constant parameters with $0 \leq \alpha \leq 1$. For $A' = 0$, Eq. (16) gives generalized Chaplygin gas. Further if $B = B_o$ and $\alpha = 1$, it yields the usual Chaplygin gas. Also Eq. (16) reduces to modified Chaplygin gas if only $B = B_o$. Moreover, if only $A' = 0$, the same equation represents variable Chaplygin gas.

We consider the background spacetime to be spatially flat ($k = 0$), homogeneous and isotropic represented by Friedmann-Robertson-Walker (FRW) metric. The spacetime is assumed to contain only one component fluid i.e. the phantom energy represented by the Chaplygin gas EoS. The corresponding field equations are

\begin{equation}
H^2 \equiv \left(\frac{\dot{a}}{a}\right)^2 = \kappa^2 \rho. \tag{17}
\end{equation}

\begin{equation}
\dot{H} + H^2 = \frac{\dot{a}}{a} = -\frac{\kappa^2}{2}(\rho + 3p), \tag{18}
\end{equation}

where $\kappa^2 = 8\pi/3$. The conservation of energy is

\begin{equation}
\dot{\rho} + 3H(\rho + p) = 0. \tag{19}
\end{equation}

Using Eqs. (16) and (19), the density evolution is given by

\begin{equation}
\rho = \left[\frac{3B_o(1 + \alpha)}{\{3(1 + \alpha)(1 + A') - m\} a^m + \frac{\Psi}{a^{3(1+\alpha)(1+A')}}}\right]^{\frac{1}{1+\alpha}}. \tag{20}
\end{equation}

Here $\Psi$ is a constant of integration. Note that to obtain the increasing energy density of phantom energy with respect to scale factor $a(t)$, we require the coefficients of $a(t)$ in Eq. (20) to be positive i.e. $\Psi \geq 0$, $B_o(1 + \alpha) > 0$ and $3(1 + \alpha)(1 + A') - m > 0$. Moreover, the exponents of $a(t)$ must be negative i.e. $m < 0$ and $3(1 + \alpha)(1 + A') < 0$ to obtain increasing $\rho$. These constraints together imply that $m > 3(1 + \alpha)(1 + A')$. Another way of getting positive $\rho$ is by setting $m > 0$, $1 + A' > 0$ and $m < 3(1 + \alpha)(1 + A')$. Further, using Eq. (9) the ratio of the number density of Chaplygin gas near horizon and at infinity is calculated to be

\begin{equation}
\frac{n(\rho_h)}{n(\rho_\infty)} = \left[\frac{\rho_h^{1+\alpha}(1 + A') - B(a)}{\rho_\infty^{1+\alpha}(1 + A') - B(a)}\right]^{\frac{1}{(1+\alpha)(1+A')}} \equiv \Delta_1. \tag{21}
\end{equation}

Notice that the function $B(a)$ can be expressed in terms of $\rho$ implicitly and is determined from Eq. (20). Making use of Eq. (11), the critical points of accretion are given by

\begin{equation}
a_*^2 = \frac{\Delta_2}{1 + 3\Delta_2}, \quad x_* = \frac{1 + 3\Delta_2}{2\Delta_2}, \tag{22}
\end{equation}

where

\begin{equation}
V_*^2 = A' + \frac{\alpha B(a)}{\rho_*^{\alpha+1}} \equiv \Delta_2 \tag{23}
\end{equation}

Thus for the critical points to be finite and positive, we require either $\Delta_2 > 0$ or $\Delta_2 < 0$ and $\Delta_2 < -1/3$. For the accretion to be critical, the quantity $V^2$ must become supersonic from the initial
subsonic somewhere near the black hole horizon. For the MVG, we have \( \omega = A' - B/\rho^{1+\alpha} < 0 \), since \( A' < -1 \). One can observe that fluids having EoS \( \omega < 0 \) are hydrodynamically unstable i.e. the speed of sound in that medium cannot be defined since \( c_s^2 < 0 \). In order to overcome this problem Babichev et al \[50\] proposed a redefinition of \( \omega \) with the help of a generalized linear EoS given by \( p = \beta(\rho - \rho_o) \), where \( \beta \) and \( \rho_o \) are constant parameters. Here \( \beta > 0 \) refers to a hydrodynamically stable while \( \beta < 0 \) corresponds to hydrodynamically unstable fluid. We will not be interested in the later case here. Note that now two parameters \( \omega \) and \( \beta \) are related by \( \omega = \beta(\rho - \rho_o)/\rho \). Further \( \omega < 0 \) now corresponds to \( \beta > 0 \) and \( \rho_o > \rho \) thereby making the previously unstable fluid, now stable. We also have \( c_s^2 \equiv \partial p/\partial \rho = \beta \). Since for stability, we require \( \beta > 0 \) and \( 0 < c_s^2 < 1 \), it leads to \( 0 < \frac{1}{\rho - \rho_o}(A'\rho - B/\rho^\alpha) < 1 \) and \( 0 < \beta < 1 \). Hence the EoS parameter is now well-defined with \( A' < -1 \) and \( \rho_o > \rho \). Thus the stability of the phantom like MVG is guaranteed with the use of generalized linear EoS.

The constant \( A \) is determined from Eq. (10) to give

\[
-A = \frac{\Delta_1}{4} \left( \frac{1 + 3\Delta_2}{\Delta_2} \right)^{3/2}.
\]

Using Eq. (8) the characteristic evolution time scale becomes

\[
\tau^{-1} = \pi M_i(\rho + p) \frac{\Delta_1}{4} \left( \frac{1 + 3\Delta_2}{\Delta_2} \right)^{3/2}.
\]

Using Eq. (25) in (7), the black hole mass is given by

\[
M(t) = M_i \left[ 1 - \pi M_i t(\rho + p) \frac{\Delta_1}{4} \left( \frac{1 + 3\Delta_2}{\Delta_2} \right)^{3/2} \right]^{-1},
\]

which determines the evolution of mass of black hole accreting phantom MVG. It can be seen that if the phantom MVG violates the dominant energy condition \( \rho + p > 0 \) than mass \( M \) of the black hole will decrease. Contrary if this condition is satisfied than \( M \) will increase. Thus in the classical relativistic regime, this result is in conformity with the result of Babichev et al \[21\]. We also stress here that although our metric (1) is static, we get a dynamical mass \( M(t) \) in Eq. (26). Astrophysically the mass of a black hole is a dynamical quantity: the mass will increase if the black hole accretes classical matter (which satisfies \( \rho + p > 0 \)) however it will decrease for the exotic phantom energy accretion. The mass can also decrease if the Hawking evaporation process is invoked. Hence the static black holes may not necessarily correspond to the astrophysical black holes. We also stress that \( \omega > 0 \) (\( \omega < 0 \)) corresponds to non-phantom (phantom) MVG fluid; although the accretion through the critical point is possible in both the cases, only phantom MVG violating the dominant energy condition will reduce the mass of black hole.
4 Accretion of viscous generalized Chaplygin gas

In viscous cosmology, the presence of viscosity corresponds to space isotropy and hence is important in the background of FRW spacetime [51, 52, 53]. The presence of viscous fluid can explain the observed high entropy per baryon ratio in the universe [54]. It can cause exponential expansion of the universe and can rule out the initial singularity which mares the standard big bang picture. The matter power spectrum in bulk viscous cosmology is also well behaved as there are no instabilities or oscillations on small perturbation scale [55]. Any cosmic fluid having non-zero bulk viscosities has the EoS $p_{\text{eff}} = p + \Pi$, where $p$ is the usual isotropic pressure and $\Pi$ is the bulk viscous stress given by $\Pi \equiv \xi(\rho) u^\mu_\mu$ [56]. The scaling of viscosity coefficient is $\xi = \xi_o \rho^n$ where $n$ is a constant parameter and $\xi(t_o) = \xi_o$. Note that for $0 \leq n \leq 1/2$, we have de Sitter solution and for $n > 1/2$ we get deflationary solutions. The viscosity coefficient is generally taken to be positive for positive entropy production in conformity with the second law of thermodynamics [57]. Moreover, the entropy corresponding to viscous cosmology is always positive and increasing which is consistent with the thermodynamic arrow of time. In fact the cosmological model with viscosity is consistent with the observational SN Ia data at lower redshifts while it mimics the $\Lambda$CDM model in the later cosmic evolution [58]. It is proved in [59, 60] that FRW spacetime filled with perfect fluid and the bulk viscous stresses will violate the dominant energy condition.

Thus the effective pressure is given by

$$p_{\text{eff}} \equiv p + \Pi,$$

where $\Pi = -3H\xi$ and $p = \chi/\rho^\alpha$ with $\chi$ is a constant. Thus in the VCG case, the standard FRW equation becomes [61]

$$\frac{\dot{a}}{a} = -\frac{\kappa^2}{2}(p + 3p_{\text{eff}}).$$

Further the energy conservation principle gives

$$\dot{\rho} + 3H(\rho + p_{\text{eff}}) = 0,$$

which shows that the viscosity term serves as the source term. Using Eqs. (17) and (27) in (29), we get

$$a \frac{d \rho}{3 da} + \rho + \frac{\chi}{\rho^\alpha} - 3\kappa \xi(\rho) \sqrt{\rho} = 0.$$

Thus solving Eq. (30) we get

$$a(t) = a_o \exp \left[ - \int_{\rho_o}^{\rho} \frac{\rho^\alpha d\rho'}{\rho'^{\alpha+1} - 3\kappa \xi(\rho') \rho'^{\alpha+\frac{1}{2}} + \chi} \right].$$

For our further analysis we shall assume $\xi$ to be constant.
The ratio of the number density of VCG near black hole horizon and at infinity is given by
\[
\frac{n(\rho_h)}{n(\rho_\infty)} = \exp \left[ \int_{\rho_\infty}^{\rho_h} \frac{\rho'^\alpha d\rho'}{\rho'^{\alpha+1} - 3\kappa \xi \rho'^{\frac{\alpha+1}{2}} + \chi} \right] \equiv \Delta_3.
\] (32)

The corresponding critical points of accretion are
\[
a_*^2 = \frac{\Delta_4}{3\Delta_4 - 1}, \quad x_* = \frac{3\Delta_4 - 1}{2\Delta_4},
\] (33)
where
\[
V_*^2 = -\left( \frac{\alpha \chi}{\rho_*^{\alpha+1}} + \frac{3}{2\sqrt{\rho_*}} \kappa \xi \right) \equiv \Delta_4.
\] (34)

Notice that for the critical points to be finite and positive valued we require either \(\Delta_4 < 0\) or \(\Delta_4 > 1/3\). Using Eq. (11) we see that the speed of flow at the critical point is \(V^2 = -\Delta_4\). Further, the EoS parameter is \(\omega = \chi/\rho^{1+\alpha} - 3\xi H/\rho = \chi/\rho^{1+\alpha} - \sqrt{3\kappa \xi}/\sqrt{\rho}\). Note that if \(\chi < 0\) then \(\omega < 0\) and stability of VCG is lost. However, if we here invoke the argument presented in the last section, we can consider accretion with \(\omega < 0\). Using the generalized linear EoS \(p = \beta (\rho - \rho_o)\) for the phantom energy, we obtain \(\beta > 0\) and \(\rho_o > \rho\) for \(\omega < 0\). Using the definition \(c_s^2 \equiv \partial p/\partial \rho = \beta\) and stability requirements \(\beta > 0\) and \(0 < c_s^2 < 1\) lead to \(0 < \frac{1}{\rho - \rho_o} (\chi/\rho^\alpha - \sqrt{3\rho \kappa \xi}) < 1\) and \(0 < \beta < 1\). The EoS parameter \(\beta\) is now well-defined with \(\chi < 0\) and \(\rho_o > \rho\). Therefore the stability of the phantom like VCG is assured with the use of generalized linear EoS.

Using Eq. (10) the constant \(A\) is now determined to be
\[
-A = \Delta_3 \left( \frac{3\Delta_4 - 1}{2\Delta_4} \right)^{3/2}.
\] (35)

The characteristic evolution time scale is
\[
\tau^{-1} = 4\pi M_i (\rho + p) \Delta_3 \left( \frac{3\Delta_4 - 1}{2\Delta_4} \right)^{3/2}.
\] (36)

Using Eqs. (35) and (36) in (7), we get black hole mass evolution as
\[
M(t) = M_i \left[ 1 - 4\pi M_i t (\rho + p) \Delta_3 \left( \frac{3\Delta_4 - 1}{2\Delta_4} \right)^{3/2} \right]^{-1}.
\] (37)

It can be seen that black hole mass will decrease when \(\rho + p < 0\) and increase in the opposite case. It is emphasized that this result is valid till the contribution of viscous stress is negligible compared to isotropic stress. For the sake of clarity, we emphasize that fluid violating the standard energy conditions is termed ‘exotic’ and hydrodynamically unstable i.e. its existence is not fully guaranteed. But this conclusion is drawn due to the ‘bad’ choice of the EoS \((p = \omega \rho)\) in the analysis. The result is reversed and remedied when we introduce the generalized linear EoS in our model which makes the accretion of exotic fluid much more practical.
5 Conclusion

We have investigated the accretion of two different forms of phantom-like Chaplygin gas onto a Schwarzschild black hole. The time scale of accretion and the evolution of mass of black hole are derived in the context of two widely studied Chaplygin gas models namely the modified variable Chaplygin gas and the viscous generalized Chaplygin gas. Although the phantom energy is an unstable fluid as it corresponds to a medium with indeterminate speed of sound and superluminal speeds. These pathologies arise due to bad choices of the equations of state for the phantom energy and hence can be removed by choosing some suitable transformation from one EoS to another or a totally new EoS for this purpose. This work serves as the generalization of the earlier work by Babichev et al [21]. It should be noted that we have ignored matter component in the accretion model. Thus it will be more insightful to incorporate the contributions of matter along with the Chaplygin gas during accretion onto black hole. Moreover our analysis can be extended to the case of rotating black holes as well.

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