The Fundamental Need for a SM Higgs
and the Weak Gravity Conjecture

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Abstract

Compactifying the SM down to 3D or 2D one may obtain AdS vacua depending on the neutrino mass spectrum. It has been recently shown that, by insisting in the absence of these vacua, as suggested by Weak Gravity Conjecture (WGC) arguments, intriguing constraints on the value of the lightest neutrino mass and the 4D cosmological constant are obtained. For fixed Yukawa coupling one also obtains an upper bound on the EW scale $\langle H \rangle \lesssim \Lambda_4^{1/4}/Y_{\nu_i}$, where $\Lambda_4$ is the 4D cosmological constant and $Y_{\nu_i}$ the Yukawa coupling of the lightest (Dirac) neutrino. This bound may lead to a reassessment of the gauge hierarchy problem. In this letter, following the same line of arguments, we point out that the SM without a Higgs field would be inconsistent with a quantum gravity embedding, giving a fundamental basis for the very existence of the SM Higgs. Furthermore one can derive a lower bound on the Higgs vev $\langle H \rangle \gtrsim \Lambda_{QCD}$ which is required by the absence of AdS vacua in lower dimensions. This would explain the relative proximity of the EW and hadronic scales in the SM. The lowest number of quark/lepton generations in which this need for a Higgs applies is three, giving a justification for family replication. We also reassess the connection between the EW scale, neutrino masses and the c.c. in this approach. The EW fine-tuning is here related to the proximity between the c.c. scale $\Lambda_4^{1/4}$ and the lightest neutrino mass $m_{\nu_i}$ by the expression $rac{\Delta H}{H} \lesssim \frac{(a\Lambda_4^{1/4} - m_{\nu_i})}{m_{\nu_i}}$. 
1 Introduction

It is a frustrating fact how poor our present understanding of the origin of the different fundamental mass scales in Particle Physics is. Simplifying a bit, there are essentially three regions of scales in fundamental physics. There is a deep-infrared region in which there are only two fundamental massless particles, photon and graviton with the three neutrinos with masses in the region $m_{\nu_i} \lesssim 10^{-3} - 10^{-1}$ eV, where one of the neutrinos could even be massless. Interestingly, this is also very close to the scale of the observed cosmological constant $\Lambda_4 = (2.25 \times 10^{-3} \text{eV})^4$. The second region is that of the masses of most elementary particles which are around $10^{-3} - 10^2$ GeV. These masses are dictated both by the value of the QCD condensate $\Lambda_{\text{QCD}} \approx 10^{-1}$ GeV and the Higgs vev $\langle H^0 \rangle = 246$ GeV. Finally there is the Planck scale and presumably a unification/string scale somewhat below. We would like, of course, to understand why the scales are what they are and what is the information that this distribution of scales is giving us concerning the fundamental theory. In particular it is difficult to understand why $\Lambda_4$ and the EW scale are so small compared to the fundamental scales of gravity and unification. Also, the proximity of neutrino masses to $\Lambda_4^{1/4}$ as well as the (relative) proximity of $\Lambda_{\text{QCD}}$ to the EW scale could be just coincidences or could be telling us something about the underlying theory.

A natural question is whether all these scales are independent or whether they are related or constrained within a more fundamental theory including quantum gravity coupled to the SM physics. Recently it has been pointed out that quantum gravity constraints could have an impact on Particle Physics [1][3]. The origin of these constraints is based on the Weak Gravity Conjecture (WGC) [4][5], see [6] for a review and [7][9] for some recent references. A sharpened variation of the WGC was proposed by Ooguri and Vafa in [1] which states that a non-SUSY Anti-de Sitter stable vacuum cannot be embedded into a consistent theory of quantum gravity (see also [10]). This general statement, together with an assumption of background independence, may be applied to the Standard Model (SM) itself [1] implying that no compactification of the SM to lower dimensions should lead to a stable AdS vacuum, if indeed the SM is to be consistently coupled to quantum gravity.

Interestingly, the exercise of compactifying the SM down to 3D and 2D was already done by Arkani-Hamed et al. [11][12] a long time ago, with a totally different motivation. They found that there may be 3D and 2D SM AdS vacua depending on the values of neutrino masses, via a radion potential induced by the Casimir effect. By assuming those results OV claimed that their sharpened WGC would imply the inconsistency of Majorana neutrino masses. In [2] a thorough analysis of this question was presented. It was further found that the 4D cosmological constant is bounded below by the value of

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1We are assuming here that the origin of dark energy is a 4D cosmological constant.
the lightest neutrino mass, providing an explanation for the apparent proximity of both quantities. Furthermore, it was shown that the same bound, for fixed $\Lambda_4$ and Yukawa couplings, induces an upper bound on the Higgs vev, giving an explanation for the stability of the Higgs potential of the SM [2,3]. This bound implies that the gauge hierarchy stability may be a consequence of quantum gravity constraints for fixed values of $\Lambda_4$ and the neutrino mass. We reassess this issue at the end of this note.

For the WGC constraints to apply, the AdS SM minima must be stable. There are potential sources of instability from the massless scalars associated to the SM Wilson lines, which may give rise to runaway scalar directions [13]. Nevertheless, it turns out that those scalars are projected out from the spectrum in certain vacua, like toroidal $Z_N$ 2D Standard Model vacua [14], so that again one can recover the same bounds of the circle or toroidal compactifications. However, one also finds within this class of vacua examples in which the minimal SM necessarily develops AdS stable vacua irrespective of the value of neutrino masses nor any other SM parameter [14]. Thus, if these WGC arguments are correct, the SM by itself would be in the swampland, it would be inconsistent with quantum gravity. On the other hand a SUSY completion of the SM like the MSSM does not have such stable AdS minima and preserves all the predictions on neutrino masses and $\Lambda_4$ described above. This is true provided the $U(1)_{B-L}$ symmetry (or a discrete subgroup of it) is gauged at some scale [14].

In the present note we obtain new constraints on the Higgs vev by imposing the absence of stable lower dimensional SM AdS vacua. These new bounds are independent from the neutrino bounds. We find that in order to avoid AdS vacua:

- A Higgs with non-vanishing vev and Yukawa couplings must exist.

- There is a lower bound on the Higgs vevs for fixed Yukawa couplings $\langle H \rangle \gtrsim \Lambda_{\text{QCD}}$.

- The minimum number of generations for which the existence of a Higgs is mandatory is three.

In deriving these conclusions we are setting fixed the values of the dimensionless couplings (Yukawa and gauge couplings) as well as the measured value of the cosmological constant. We discuss the combination of the above lower bound with the upper bound coming from the absence of neutrino generated AdS vacua. We also rephrase the upper bound on neutrino masses as a constraint relating the EW fine-tuning with the proximity between the c.c. and the neutrino mass scale.

2 A world with no Higgs is in the swampland

Let us consider first the fermion and gauge boson content of the SM (plus the graviton) with $n_g$ quark/lepton generations. In the absence of the Higgs (or in the presence of the Higgs with a zero vev), the theory has an (approximate) $U(2n_g)_L \times U(2n_g)_R$ accidental global symmetry in the quark sector. This symmetry is spontaneously broken by the QCD condensate of the quarks down to $U(2n_g)_{L+R}$, generating a total of $4n_g^2$ massless Goldstone bosons. Three of them become massive by combining with the $W^\pm$ and $Z$ bosons, which acquire masses given by:

$$M_W = \sqrt{n_g} \frac{gf_\pi}{2}$$  \hspace{1cm} (2.1)

$$M_Z = m_W / \cos \theta_W.$$  \hspace{1cm} (2.2)
where \( n_g \) is the number of generations and \( f_\pi \) is the Goldstone boson decay constant. In the physical QCD case with \( n_g = 3 \) the latter is given for the pion by \( f_\pi = 93 \text{ MeV} \). More generally one has for the Goldstone boson decay constants \( f_G \approx \Lambda_{\text{QCD}} \). One more Goldstone boson is expected to become massive due to the QCD anomaly, so that below the \( \Lambda_{\text{QCD}} \) scale we are left with a total of \( 4(n_g^2 - 1) \) massless Goldstone bosons. In addition to those there are 4 more bosonic degrees of freedom from the massless photon and graviton, so that the number of massless bosonic degrees of freedom below the QCD scale is \( N_B = 4n_g^2 \). The total fermionic minus bosonic degrees of freedom below \( \Lambda_{\text{QCD}} \) is then

\[
(N_F - N_B)^{\Lambda_{\text{QCD}}} = 8n_g - 4n_g^2 = 4n_g(2 - n_g) \ , \tag{2.3}
\]

where the fermionic degrees of freedom correspond to charged leptons and (Dirac) neutrinos (we are taking Dirac rather than Majorana neutrinos because the latter lead necessarily to AdS vacua, as shown in [2]). Note that above \( \Lambda_{\text{QCD}} \) one has leptons and unconfined quarks and then one rather has

\[
(N_F - N_B)^{>\Lambda_{\text{QCD}}} = 32n_g - 24 - 2 \ , \tag{2.4}
\]

where the 24 comes from the SM gauge bosons and the 2 from the graviton. The value of \( (N_F - N_B) \) is crucial since the Casimir potential of the radion upon compactification of the SM down to 3D or 2D depends linearly on it. Since above the QCD transition it is always positive, an AdS minimum will develop if it is negative below \( \Lambda_{\text{QCD}} \). From Eq. (2.3) that will happen only for three or more generations. We will go through the details below, but before describing the computation of the radion potential let us mention that in fact the electro-magnetic interactions explicitly (but weakly) break the global symmetry to \( U(6) \to U(3) \times U(3) \), due to the different charges for up and down types of quarks. So in fact only the \( 2n_g^2 \) neutral Goldstones remain massless whereas the remaining \( 2n_g^2 \) are pseudo-Goldstones with charge \( \pm 1 \). This is like the \( \pi^\pm \) pions in the Standard Model which get an excess of mass over the \( \pi^0 \) due to electro-magnetic interactions (in addition to the quark mass contributions, which are absent in our case). Inspired by what happens in the Standard Model in the zero quark-mass limit [15] we will estimate the masses for the charged pseudo-Goldstones as \( m_{ij}^2 \approx (\alpha_{\text{em}}/4\pi)\Lambda_{\text{QCD}}^2 \).

Let us discuss for simplicity the case of a compactification of the SM in a circle, it may be seen that analogous numerical results are obtained for 2D compactifications on a torus or \( Z_N \) 2D orbifolds [14]. One obtains a scalar potential for the radion scalar \( R \):

\[
V(R) = \frac{2\pi\Lambda_4}{R^2} + \sum_p (-1)^{2s_p + 1}n_p V_C[R, m_p] \ , \tag{2.5}
\]

where

\[
V_C[R, m_p] = \frac{m_p^2}{8\pi^4 R^4} \sum_{n=1}^{\infty} \frac{K_2(2\pi n R m_p)}{n^2} \tag{2.6}
\]

The first term comes from the dimensional reduction of the cosmological constant piece of the 4D action. The second piece comes from the one-loop Casimir energy contribution of any particle \( p \) with multiplicity \( n_p \) and mass \( m_p \) in the spectrum. Here \( s_p = 1/2, 0 \) for fermions and scalars respectively, and \( K_2 \) is a modified Bessel function of the second kind. The fermions we consider will all be assigned periodic boundary conditions. For massless particles the Casimir term simplifies to

\[
V_C[R, 0] = \frac{1}{720\pi R^6} \ . \tag{2.7}
\]
Figure 1: Effective radion potential for different numbers of quark/lepton generations $n_g$ in the absence of a Higgs. For $n_g \geq 3$ an AdS vacuum develops slightly below the corresponding QCD scale.

Note that for massless particles the coefficient of the Casimir piece is proportional to $(N_F - N_B)$. It is important also to remark that in computing the potential around a mass value $m$, the effect of heavier states with mass $M$ exponentially decouples like $e^{-M/m}$. Also, higher loop corrections to this expression are sub-leading at weak coupling, so the potential is reliable except for a finite region around the QCD hadronic threshold in which non-perturbative techniques would be required for the actual computation.

Since there are no current quark masses, we can compute the running of the strong coupling constant from the top-quark mass $m_t$ down to $\Lambda_{\text{QCD}}$ using the one-loop RGE expression

$$\frac{1}{\alpha_s(\mu, n_f)} = \frac{1}{\alpha_s(m_t)} + b(n_f) \log \frac{m_t}{\mu}. \quad (2.8)$$

Here $n_f$ is the number of quark flavours and $b(n_f) = -\frac{1}{2\pi} (11 - \frac{2}{3} n_f)$, so that in our case with vanishing current quark masses $b(6) = -\frac{7}{2\pi}$. If the coupling constants start running at some UV scale like e.g. a unification/string scale $M_X$, the value of $\alpha_s(m_t, n_f)$ depends on the number of flavors. To calculate these different values we start with the known 3-generation case with $\alpha_s(m_t) = 0.117$ and run up in energies to an UV value $\alpha_s(M_X)$. From there one can then run $\alpha_s(n_f)$ for the different flavor numbers down to $m_t$. In this way one obtains different values for $\Lambda_{\text{QCD}}$ depending on the number of generations, as shown in Fig. 1 in which we have taken for definiteness $M_X = 10^{15}$ GeV. It is important to remark, though, that the conclusions below are independent of the precise initial conditions taken for each value of $n_f$. Note also that we do not include errors in the input data which would have little effect in the conclusions. The values of $\Lambda_{\text{QCD}}$ so computed mark the separation between a low-energy region in which the Goldstone and pseudo-Goldstone bosons description is appropriate and the high energy region with unconfined quarks.

Using Eqs. (2.5) and (2.6) and the above discussed spectrum in the different regions above and below the corresponding $\Lambda_{\text{QCD}}$ values, one can readily compute the radion potential. The results shown have been computed for the case of a 3D compactification on a circle with the Wilson lines set to zero. As explained in [14] the results obtained for 2D orbifold vacua in which Wilson lines are projected out would be practically identical. We plot in Fig. 1 the effective potential of the radion $R$ multiplied by
Figure 2: Effective radion potential for different values of the Higgs vev $\langle H \rangle$ in units of the SM value $v = 246$ GeV, with $n_g = 3$. The Yukawa couplings are fixed at their SM values. For Higgs vevs larger than $10^{-3}v$ the AdS vacua ceases to develop.

$R^6$ and a numerical factor so that the vertical axis gives the number of effective degrees of freedom for the Casimir contribution, $(N_F - N_B)$, in each region. At large $R$ the contribution of the cosmological constant dominates as seen on the left-hand side of the plot. The vertical bands represent the QCD scale in which a perturbative calculation cannot be trusted. We see that, as advanced, for $n_g = 1$ and $n_g = 2$ the radion potential does not become negative before the QCD scale. However for $n_g = 3$, due to the presence of many Goldstones $(N_F - N_B = -12)$, the potential does become negative and an AdS vacuum develops. This is interesting because it shows that for $n_g \geq 3$ a situation with no Higgs (or no vev) would be inconsistent with quantum gravity. A 3-generation Standard Model requires the existence of a Higgs field coupled through Yukawas to quarks and leptons in order to avoid the appearance of an AdS vacuum, as we now discuss.

3 A lower bound on the Higgs vev

Let us concentrate now in the physical case with 3 generations. A Higgs with a vev and coupling to quarks and leptons may avoid the presence of an AdS vacuum. Let us assume that the Yukawa and gauge couplings are those observed experimentally and let us switch on slowly a vev $\langle H \rangle$ for the Higgs. When the heaviest fermion, the top, reaches a mass $m_t > \Lambda_{QCD}$ (which also means $\langle H \rangle \gtrsim \Lambda_{QCD}$), the approximate chiral symmetry is only $U(5)_L \times U(5)_R$ and there are only 25 Goldstone bosons. Three of them get masses of order $\Lambda_{QCD}$, combining with the EW bosons, and a fourth one also becomes massive through the QCD anomaly. Added to the photon and graviton there is a total of 25 bosonic degrees of freedom below $\Lambda_{QCD}$. This is only slightly larger than the 24 leptonic degrees of freedom, so that indeed one expects that with vevs slightly above $\Lambda_{QCD}$, the AdS vacua will disappear.

Indeed this is what one obtains doing a more detailed computation, the results of which are shown in Fig. 2. To find the precise value of $\langle H \rangle$ for which the AdS vacua disappears one has to take into account a number of small effects. To start with, the value of $\Lambda_{QCD}$ in these configurations with a Higgs vev different from the experimental one $\langle H \rangle = v = 246$ GeV is different from what is observed in
the SM, since, for example, the running of the strong coupling has a different sequence since the quark thresholds are much lighter than the experimental ones. We will content ourself with the running at one-loop given by

$$\frac{1}{\alpha(\mu, n_f)} = \frac{1}{\alpha(m_t)} + b(n_f) \log \frac{m_t}{\mu},$$  \hspace{1cm} (3.1)

where $n_f$ is the number of flavours and now $b(n_f)$ varies as thresholds are crossed. We start the running from the measured value at the top mass and run down in energies with a step variation of the one-loop beta-function as we reach each threshold. It turns out however that the values of $\Lambda_{\text{QCD}}$ obtained for different values of the Higgs vev is always close to 100 MeV, as in the SM. Another effect is that the neutral and charged Goldstones get an additional mass from the presence of Yukawa couplings. Mimicking what happens in the chiral $SU(2)_L \times SU(2)_R$ pion theory we parametrize the (quark mass induced) pseudo-Goldstone masses as $m^2_{Gb} = Y_q \langle H \rangle \Lambda_{\text{QCD}}$, where $(Y_q \langle H \rangle)$ is the mass of the heaviest quark in the Goldstone for the given Higgs vev. For scales smaller than these masses the corresponding scalars do not contribute to the Casimir potential. We introduce the SM values of the Yukawa couplings and take into account how they run according to their QCD anomalous dimensions $\gamma = 2$, i.e.

$$Y_q(\mu) = Y_q(m_t) \times \left( \frac{\alpha_s(\mu)}{\alpha_s(m_t)} \right)^{-\frac{\gamma}{\pi b(n_f)}},$$  \hspace{1cm} (3.2)

where we are neglecting here the contribution from the EW gauge interactions. In fact all these details do not practically modify at all the results displayed in Fig. 2 which is the reason why we did not include the 2-loop running. After all we are only computing the Casimir energy at one-loop. In this figure we again plot the effective potential of the radion $R$ multiplied by $R^6$ and a numerical factor so that the vertical axis gives the number of effective degrees of freedom for the Casimir contribution. As the value of $\langle H \rangle$ increases, the region in which the potential is negative is reduced and for $\langle H \rangle \gtrsim 10^{-3} v \simeq 200$ MeV the potential is always positive and AdS vacua disappear. This sets a lower bound around $\langle H \rangle \gtrsim \Lambda_{\text{QCD}}$ as already advanced. This computation has been performed here for a 3D circle compactification of the SM but practically identical results are obtained for the 2D compactifications considered in [14]. Note finally that in this computation and the one in the previous section we take for $\Lambda_4$ its experimental value. The bounds remain true as long as the cosmological constant is $\Lambda_4 \lesssim \Lambda_{\text{QCD}}^4$.

## 4 Combined constraints on the Higgs vev

The above lower bound on $\langle H \rangle$ is totally independent from the bounds in [2] in which the existence or not of AdS vacua depended on the mass of the lightest neutrino. In [2] it was shown that there is an upper bound on the value of the lightest neutrino mass $m_{\nu_i} \lesssim a \Lambda_4^{1/4}$ in order to avoid the appearance of 3D or 2D AdS SM vacua. Neutrinos must be Dirac or pseudo-Dirac. Here $a \simeq 3.2(0.9)$ and $i = 1(3)$ depending on whether the neutrino hierarchy is normal (NH) or inverted (IH) [2,14]. For fixed neutrino Yukawa and fixed value of $\Lambda_4$, this implies a bound on the Higgs vev [2]

$$\langle H \rangle \lesssim a \frac{\Lambda_4^{1/4}}{Y_{\nu_i}}.$$  \hspace{1cm} (4.1)

We can now combine this upper bound with the lower bound just found above. The results are depicted in Fig. 3. We plot the value of the Higgs vev as a function of the cosmological constant scale.
Figure 3: Constraints on the Higgs vev as a function of the c.c. scale $\Lambda^{1/4}$ for fixed neutrino Yukawa couplings. The vertical (horizontal) dashed line gives the experimental value of the c.c. (Higgs vev) respectively. We also show with a vertical band bounds on the cosmological constant from anthropic constraints [16]. We have divided the plot in AdS safe (in blue) and unsafe (in red) zones. For fixed values of the c.c. an upper bound on the Higgs vev is obtained.

$\Lambda^{1/4}$. The horizontal blue line around $\langle H \rangle \approx \Lambda_{\text{QCD}}$ sets the lower bound discussed above whereas the leaning blue line is the upper bound coming from Eq. (4.1) for a value $Y_{\nu_1} = 10^{-14}$ (also, in dashed line, the $Y_{\nu_1} = 10^{-15}$ case is shown). A vertical dashed line marks the observed value of the c.c. and a horizontal dashed line shows the physical Higgs vev. Both lines cross in between both limits, within the blue region in which no AdS vacua develops. For a fixed value of $\Lambda_4$ and the Yukawa coupling, the Higgs vev is bounded above. As discussed in [2,3] this puts the hierarchy problem into a new perspective: the Higgs vev is stable against quantum corrections because larger values of the Higgs vev would give rise to AdS vacua which do not admit a consistent quantum gravity embedding. In this sense no fine-tuning is required since the possible values of the Higgs vev are both bounded above and below. The upper constraint in Eq. (4.1) may be re-expressed in a slightly more elegant way as follows. Let us write the Higgs vev as $\langle H \rangle = \langle H^0 \rangle + \langle \Delta H^0 \rangle$, where $\langle H^0 \rangle = 246$ GeV. Then the same expression may be written as

$$\frac{\langle \Delta H^0 \rangle}{\langle H^0 \rangle} \leq \frac{a\Lambda_4^{1/4} - m_{\nu_i}}{m_{\nu_i}},$$

(4.2)

where $i = 1(3)$, $a = 3.2(0.9)$ for NH(IH) [14]. This expressions tell us that possible corrections on the physical Higgs value $\langle \Delta H^0 \rangle$ are bounded above by the difference between the cosmological constant scale $\Lambda_4^{1/4}$ (weighted by $a$) and the lightest neutrino mass, measured in neutrino mass units. Thus the stability of the Higgs on its measured value is related to the proximity of the neutrino and cosmological constant scales. Note that the upper bound on the lightest neutrino mass is explicit in the formula, as the left-hand side is positive. This expression also shows that the upper bound disappears as $m_{\nu_i} \to 0$, so that this solution to the Higgs stability problem requires that $m_{\nu_i} \simeq a\Lambda_4^{1/4}$. If one insists in having $\langle \Delta H^0 \rangle = 0$, so that the experimental value $\langle H^0 \rangle$ saturates the upper bound, one obtains predictions...
for the lightest (Dirac) neutrino mass as \[2\]

\[
m_{\nu_1}^{\text{NH}} = 3.2 \times \Lambda_4^{1/4} = 7.2 \times 10^{-3} \text{ eV} \\
m_{\nu_3}^{\text{IH}} = 0.9 \times \Lambda_4^{1/4} = 2.0 \times 10^{-3} \text{ eV}.
\]

(4.3)

Here we are using the values of \(a\) from the orbifold case in \[14\] which are very close to those found in \[2\] for the circle.

A comment about the value of \(\Lambda_4\) is in order. The bounds here discussed take the cosmological constant as a fixed fundamental parameter of the 4D action. So when we compare here the theory for different values of the Higgs vev we only consider theories in which \(\Lambda_4\) remains around its observed experimental value. This would be natural if \(\Lambda_4\) was a fundamental constant at a deeper level. Maintaining the cosmological constant around its observed value is also suggested by anthropic considerations that fix \(\Lambda_4^{1/4}\) in a narrow region around \((1 - 10) \times 10^{-3}\) eV, see e.g. \[16\]. We also show this range of values in Fig. \[3\] with a vertical band. We see that for the range of values allowed by the anthropic constraints, the Higgs vev is always bounded above (for finite \(m_{\nu_i}\)) and below. Let us however emphasise that the argumentation in this paper has nothing to do with anthropic considerations. Theories or ranges of parameters are excluded here because they are inconsistent with quantum gravity, not because otherwise we would not exist.

It is remarkable how a very abstruse condition like imposing the absence of AdS vacua in compactifications of the SM seems to lead to a number of constraints on SM facts and parameters and no obvious contradiction with experiment. We have shown in this note that a world with no Higgs would be inconsistent with quantum gravity, motivating the very existence of a Higgs particle. This need for a Higgs is only true for 3 or more generations, suggesting family replication as coming along with the existence of the Higgs. One also finds that, for the Yukawa couplings of the SM, the absence of AdS vacua implies a lower bound on the Higgs vev, \(\langle H^0 \rangle \gtrsim \Lambda_{\text{QCD}}\), in agreement with the (relative) proximity of that scale with the EW scale. In the works \[1,2\] it was also shown that absence of AdS vacua imply that neutrinos must be Dirac and that the c.c. is bounded below in terms of the lightest neutrino mass. The same bound leads to an upper bound on the Higgs vev, summarized in Eq. (4.2). This would provide for a new avenue to understand the stability of the Higgs field, i.e., the EW hierarchy problem. In \[14\] it was shown however that the minimal SM is itself problematic since it was found there are 2D SM compactifications leading to AdS vacua. However, it was also shown that this conclusion does not apply e.g. in a SUSY extension of the SM like the MSSM, although the scale of the SUSY particles needs not be low.

Many issues remain to be clarified. First of all it would be important to find additional evidence for the sharpened Weak Gravity Conjecture of \[1\]. Furthermore, the question of the stability of the AdS vacua found for the SM compactifications is very important. If the compactifications had some hidden source of instability, the obtained constraints and predictions would be gone. In this respect it would be interesting to further explore all the different possible vacua that one can obtain starting from the SM or extensions like the MSSM. Also an exploration of the constraints/predictions in the presence of different sources of new physics like axions, sterile neutrinos, dilatons or hidden sector particles in general would be interesting. We believe that the results and conditions obtained so far are very intriguing and deserve serious study. They may provide us with a unique opportunity to obtain quantum gravity predictions for Particle Physics.
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