1. INTRODUCTION

The spin structure of a nucleon is one of the most interesting problems to be resolved within the framework of (nonperturbative) Quantum Chromodynamics (QCD). In particular, the singlet part \( \Sigma(x, Q^2) \) of the parton distribution functions

\[
\Sigma(x, Q^2) = \sum_{f} f_{q}(x, Q^2),
\]

where \( f \) is a number of active quarks, is intensively studied, because there is strong disagreement between the experimental data for its first Mellin moment and corresponding theoretical predictions. This disagreement is usually called a spin crisis (see, for example, reviews in [1]).

Here we consider only the non-singlet (NS) part, which the fundamental Bjorken sum rule (BSR) holds for [2]

\[
\Gamma_{1}^{p-n}(Q^2) = \int_{0}^{1} [\hat{g}_{1}^{q}(x, Q^2) - \hat{g}_{1}^{\bar{q}}(x, Q^2)] dx.
\]

It deals with the first moment \( (n = 1) \) of NS part of the structure function (SF) \( g_{1}(x, Q^2) \). For the case \( n = 1 \), the corresponding anomalous dimension of Wilson operators is zero and all the \( Q^2 \)-dependence of \( \Gamma_{1}^{p-n}(Q^2) \) is encoded in the coefficient function.

Usually, BSR is represented in the form

\[
\Gamma_{1}^{p-n}(Q^2) = \frac{G_{A}}{6} E_{NS}(Q^2) + \sum_{i=2}^{\infty} \frac{1}{2i-2} \hat{g}_{2i-2}^{p-n}(Q^2),
\]

where the first term in the r.h.s. is a twist-two part and the second one is a contribution of higher twists (HTs).

At high \( Q^2 \) values the experiment data for \( \Gamma_{1}^{p-n}(Q^2) \) and the theoretical predictions [1] are well compatible with each other. Here we will focus on low \( Q^2 \) values, at which there presently exist the very precise CLAS [3, 4] and SLAC [5] experimental data for BSR. On the other hand, there also is a great progress in theoretical calculations: recently, the terms \(-\alpha_s^4\) are evaluated in [6].

2. BASIC FORMULAE

In our analysis we will mostly follow the analyses done by the Dubna-Gomel group [7, 8]. We try, however, to resum the twist-two part with the purpose of reducing a contribution coming from the HT terms. Indeed, there is an interplay

• between HTs and higher orders of perturbative QCD corrections (see, for example, [9], where the SFs \( xF_3 \) was analyzed).

• between HTs and resummations in the twist-two part (see, for example, application of the Grunberg approach [10] in [11] to the study of SFs \( F_2 \) and \( F_L \)).

The twist-two part of BSR has the following form (see, for example, [7])

\[
E_{NS}(Q^2) = 1 - 4\Delta(Q^2),
\]

where the term \( \Delta(Q^2) \) looks like

\[
\Delta(Q^2) = \alpha_s(Q^2) \left( 1 + \sum_{k=1}^{\infty} C_k a_s^k(Q^2) \right)
\]

The first three coefficients \( C_1, C_2 \) and \( C_3 \) are already known (see [6, 12] and references therein).

We will replace the above representation (2) by the following one

\[
E_{NS}(Q^2) = \frac{1}{1 + 4\Delta(Q^2)},
\]
where

$$\tilde{\Delta}(Q^2) = \alpha_s(Q^2) \left(1 + \sum_{k=1}^{\infty} \tilde{C}_k a_k^i(Q^2)\right) \quad (4)$$

and $\tilde{C}_k$ can be obtained from the known $C_k$:

$$\tilde{C}_1 = C_1 + 4, \quad \tilde{C}_2 = C_2 + 8C_1 + 16,$$
$$\tilde{C}_2 = C_1 + 8C_2 + 4C_1^1 + 48C_1 + 64. \quad (5)$$

The reason behind this transformation is as follows: the CLAS experimental data [3, 4] demonstrate that $\Gamma_{1}^{p-n}(Q^2) \rightarrow 0$. Therefore, in the case when the HT corrections produce small contributions at $Q^2 \rightarrow 0$ we see that

$$E_{NS}(Q^2 \rightarrow 0) \rightarrow 0. \quad (6)$$

Since the strong coupling constant $\alpha_s(Q^2 \rightarrow \Lambda^2) \rightarrow \infty$, it is seen that the form (3) behaves much like the CLAS experimental data. Indeed,

$$E_{NS}(Q^2 \rightarrow \Lambda^2) = \frac{1}{1 + 4\Delta(Q^2 \rightarrow \Lambda^2)} \rightarrow 0. \quad (7)$$

As $\Lambda_{QCD}^2 \sim 0.01$ is rather small, one can conclude that the above representation (7) agrees with experiment at very low $Q^2$ values.

Note, however, that we have a very small coefficients of $\Delta(Q^2)$ and $\tilde{\Delta}(Q^2)$. Thus, for small but non-zero $Q^2$ values the above representations (1) and (3) lead to similar results (see Fig. 1, where we restricted our consideration to the next-to-next-to-leading order (NNLO) accuracy). As is seen in Fig. 1, the theoretical predictions are not too close to the shape of the experimental data.

3. GRUNBERG APPROACH

At $Q^2 \sim 0$, the value of the strong coupling constant is very large. Thus, in our approach it is better to avoid the usage of series like

$$\sum_{k=1}^{\infty} C_k a_k^i(Q^2), \quad (8)$$
as in Eqs. (2) and (4).

Instead, it is convenient to use the Grunberg method of effective charges [10], i.e. to consider the variables $\Delta(Q^2)$ and $\tilde{\Delta}(Q^2)$ as new effective “coupling constants”, which have the following properties:

$$Q^2 \rightarrow Q^2/D_k, \quad Q^2 \rightarrow Q^2/\tilde{D}_k \quad (9)$$

for the variables $\Delta(Q^2)$ and $\tilde{\Delta}(Q^2)$, respectively, with

$$D_k = e^{\frac{C_i}{\beta_0}}, \quad \tilde{D}_k = e^{\tilde{C}_i/\beta_0}, \quad (10)$$

which are in turn responsible for the vanishing of the coefficients $C_k$ and $\tilde{C}_k$ in a series similar to (8). Moreover, these shifted arguments (9) provide also a strong reduction in the magnitudes of the coefficients $C_k$ and $\tilde{C}_k$ ($k \geq 2$).

- new $\beta_i$ ($i \geq 2$) coefficients of the corresponding $\beta$-functions, which are responsible for the vanishing of the coefficients $C_k$ and $\tilde{C}_k$ ($k \geq 2$).

However, a straightforward application of the Grunberg approach to the variables $\Delta(Q^2)$ and $\tilde{\Delta}(Q^2)$ is not as convenient, because the coefficients $C_1$ and $\tilde{C}_1$ are positive and the $Q^2$ values are very small. It is in contrast with its direct applications, where the coefficients $C_1$ and $\tilde{C}_1$ are negative [14] and/or the $Q^2$ values are not so small [11, 15].

So, the new arguments $Q^2/D_k$ and $Q^2/\tilde{D}_k$ have now very small values and, as a result, we have to use the Grunberg approach associated with something else. One of the ways is to use a so-called “frozen” coupling constant.
1.5
1.5
2.0
2.0
28
1.5
1.5
2.0
2.0

It is compatible with the observation steps: small of experimental data are close enough to each other at that the shape of theoretical predictions and the form standard coupling constant (see Fig. 1), we observe well consistent with each other.

(1) and theoretical predictions obtained with
ences can be found in [17]:
following replacement should be done (a list of refer-
mally ,
altering its argument corrections is in progress.

The analysis of the Bjorken sum rule performed within the framework of perturbative QCD is presented at low \( Q^2 \). It features the following important steps:

- The new form (3) for the twist-two part was used.
  It is compatible with the observation \( E_{NS}(Q^2 \to 0) \to 0 \),

coming from the experimental data (if HTs are negli-
gible).

- The application of the Grunberg method of effective charges [10] in a combination with a “frozen” coupling constant provides good agreement with experimental data, though with a slightly larger freezing parameter \( (1.5 M^2_\rho \text{ instead of } M^2_\rho) \).

Further elaborations to be undertaken include taking into account the \( \alpha_s^2 \) and \( \alpha_s^3 \) corrections to our analysis, as well as the study of HT corrections and their correlations with a freezing parameter \( a \) (in front of \( M^2_\rho \)). We also plan to add to our analysis an analytic coupling constant [18], which has no the Landau pole and leads usually to the results, which are similar to those obtained in the case of the “frozen” coupling constant [17, 19].

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