Variation of Stress Intensity Factor and Strain Energy Release Rate in Human Cortical Bone using Finite Element Analysis

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Abstract. Finite element modelling and analysis is an alternative way to study human fracture behaviour in biomechanics field compared to experimental test in laboratory. The aim of this study is to analyse the stress intensity factor (K) and strain energy release rate (J-integral) when three-point bending test and tensile test are applied on a two-dimensional (2D) cortical bone model developed by using finite element software with different applied loads and crack-to-width ratios. Two methods are used to to evaluate the values of K and J-integral which are Displacement Extrapolation Method (DEM) and CINT method. The values then been compared with other theoretical expression done by other researchers for validation purposes. It has been proved that by using CINT method, the error is very small compared to DEM method. As a result, stress intensity factor values increased as crack-to-width ratio increased and strain energy release rate also increased as crack-to-width ratio and loads increased. It can be concluded that finite element analysis can be used to study the fracture behaviour of human cortical bone. Therefore, researchers can easily study the behaviour of cortical bone using simulation instead of mechanical experiment.

1. Introduction
The bone is a complex hierarchical microstructure that made up of water, collagen and mineral. The bone consists of two layer which are cortical and trabecular bone. Cortical bone is the compact bone and outer layer bone, protecting the inner cavity which is trabecular bone. Human bones are exposed to many possible ways of failure such as bending and tension. It is because, the complex microstructure of human cortical bone has a significant effect on its mechanical and fracture properties [1].

This research was done as finite element modelling of cortical bone is tested with two testing method in a closed fracture. Finite element is a method which is used to solve complex problems using simpler solution [2]. Usually, problems are expressed in partial differential equation solved using analytical method. But, as some of the problems cannot be settled using analytical method, numerical technique is used instead. Finite element modelling, an effective computed method is utilised in solving biomedical engineering problems such as bone mechanics [3], observing how a product behaves under a particular condition. Using this method, 2D of human cortical bone can be formed and tested using finite element software which is Mechanical ANSYS Parametric Design Language (APDL) 14.0. Ansys is one of the computational method software used to solve the finite element
analysis (FEA) in working out mechanical problems such as static or dynamic structural analysis [4]. The DEM is a proven numerical method used for modelling particle systems as this numerical technique is able to describe the mechanical behaviour of a material, makes it widely used in biomechanics to study the fracture toughness of bone [5]. Meanwhile, CINT method is the most preferable method to find the SERR for fracture analysis using FE method. This technique is based on the assumption that the energy required to form a crack is equal to the energy needed to close the crack.

Then, Linear Elastic Fracture Mechanics (LEFM) is the basic theory of fracture developed using SIF determined by stress analysis [6]. In fracture mechanics, SIF is one of the most fundamental and a practical parameter developed by George R Irwin (1957) [7]. In a material with a crack, the stress at the crack tip is represented by stress intensity factor (SIF) which is correlated to the crack propagation and the failure of the material. Moreover, strain energy releaser rate (SERR) is introduced by Griffith which also represents fracture toughness, which known as energy dissipated during fracture per unit [8]. When a material is loaded, the crack propagates and produced new fracture surface area by the help of the strain energy. Other than observing the fracture behaviour, two parameters were also have been observed which are stress intensity factor (SIF) and strain energy release rate (SERR).

As the world keeps evolving, the demands for advancement and development in medical field specifically in orthopaedic field is growing. So, many studies are made to feed the needs of the people proving the determination of previous researchers. This scenario somehow seeds the motivation for this research to be done. However, solving this problem using 2D model on simulation software is not yet done by many. Previous studies showed that experiments are done on the fresh bone either animal or human bone as specimen to observe the crack propagation and crack behaviour using various testing method. To obtain the fresh bone specimens are not that easy under certain circumstances including ethics issue that might rise from it. Furthermore, most of them did analyse on stress intensity factor and strain energy release rate in different testing methods. So, this is the suitable platform to observe the behaviours of cortical bone with respect to different testing methods. Thus, this will be a new challenge where closed fracture on human bone with different crack lengths and applied loads were tested by using simulation software. This will be a point where other researchers feel encourage to shift their focus on problem relate to finite element modeling and analysis.

2. Methods
The mechanical properties possessed by cortical bone was defined and been specified where the Young’s Modulus and Poisson’s ratio is 14 GPa and 0.3 respectively. The single layer model was set in solid plane strain, structural, linear, isotropic quadrilateral elements (8 node PLANE 183) with singularity elements and around the crack tip having 8 nodes throughout the analysis as shown in figure 1.

![Figure 1. The quad element type 8 node (PLANE 183)](image)

After model was created in Ansys, postprocessing is been done where two mechanical properties will be analysed from the models which are SIF and SERR from both testing methods.
2.1. Stress Intensity Factor, K
SIF values were calculated using two different FEA methods which are DEM and CINT method. In this section, K-values for two different testing methods are analyzed separately. A load of 1000 N was applied in different crack-to-width ratio (α/W).

For three-point bending test, K values obtained is compared between DEM and CINT method with the theoretical expression proposed by Bower (2009) [9] and Tada (2000) [10]. The theoretical expression of Bower is expressed in equation 1 and equation 2 and Tada’s in equation 3, where P is load, B represents specimen thickness and S is span length of the specimen.

\[
K = \frac{P}{B\sqrt{W}} f\left(\frac{a}{W}\right) \quad (1)
\]

\[
f\left(\frac{a}{W}\right) = \frac{3\pi a}{2(1+2\frac{a}{W})} \left\{ 1.99 - \frac{a}{W}(1-\frac{a}{W})[2.15 - 3.93\left(\frac{a}{W}\right) + 2.7\left(\frac{a}{W}\right)^2]\right\} \quad (2)
\]

\[
K_I = 4\frac{P}{B\sqrt{W}} \left[1.6\left(\frac{a}{W}\right)^2 - 2.6\left(\frac{a}{W}\right)^3 + 12.3\left(\frac{a}{W}\right)^5 - 21.2\left(\frac{a}{W}\right)^7 + 21.8\left(\frac{a}{W}\right)^{9/2}\right] \quad (3)
\]

Meanwhile, K values obtained from DEM were compared with CINT method for tensile test.

2.2. Strain Energy Release Rate, J-integral
J-integral values were obtained through FE software by using CINT method. Five different loads and eight different crack-to-width ratios are tested onto the cortical bone model by using two different testing methods separately.

3. Results & Discussion
3.1. Analysis of Stress Intensity Factor (SIF), K
The values gained from FE software were compared with theoretical expression and the average percent of error for DEM compared to theoretical expressions is 1.415% while CINT method has average percent of error of 1.173%. Therefore, CINT method is used to compare with DEM in tensile testing as it is proved reliable with percent of error less than 10%. Comparisons of data are tabulated in table 1.

| α/W | K_{Bower} | K_{Tada} | K_{DEM} | Error % (Bower) | Error % (Tada) | K_{CINT} | Error % (Bower) | Error % (Tada) |
|-----|-----------|----------|---------|----------------|----------------|----------|----------------|----------------|
| 0.126 | 30.981 | 32.299 | 31.301 | 1.033 | 3.090 | 31.582 | 1.941 | 2.219 |
| 0.1926 | 38.879 | 39.639 | 38.879 | 1.242 | 1.917 | 39.163 | 1.980 | 1.202 |
| 0.2520 | 45.741 | 46.317 | 45.741 | 1.305 | 1.244 | 45.996 | 1.980 | 0.693 |
| 0.3190 | 54.467 | 54.923 | 54.467 | 1.447 | 0.830 | 54.646 | 1.780 | 0.505 |
| 0.3780 | 63.794 | 64.206 | 63.794 | 1.597 | 0.642 | 63.846 | 1.680 | 0.561 |
| 0.5040 | 92.623 | 92.748 | 92.623 | 1.599 | 0.135 | 91.942 | 0.852 | 0.870 |
| 0.5700 | 116.860 | 116.266 | 116.86 | 1.761 | 0.511 | 115.083 | 0.214 | 1.017 |
| 0.622 | 143.800 | 141.762 | 143.8 | 2.847 | 1.438 | 140.400 | 0.415 | 0.961 |

For both testing method as indicated in figure 2, the SIF increased as crack length increased. This indicated that the fracture toughness of the cortical bone keeps increasing in order to resist fracture which brings to failure of the bone. Therefore, this property of bone protected the cortical bone when it is exposed to force.
Figure 2. Comparison of $K$ values in tensile test with different crack-to-width ratio, $\alpha/W$

3.2. Analysis of Strain Energy Release Rate, $J$-Integral

For both testing method, it can be seen that as the crack-to-width ratio is raised, the $J$-integral will be escalated too. Besides, as the loads were increased, the $J$-integral values increased too. This drastic change is affected by the fracture toughness of the cortical bone since $J$-integral is related with fracture toughness of a material. The energy release rate is affected by the loads and cracks. That is why, as the crack length increase together with the crack length, SERR increased.

3.2.1. Three-Point Bending Test.

Referring to table 2, a graph in figure 3 is constructed to show the correlation between $J$-integral and crack-to-width ratio. It clearly showed that $J$-integral increase along with crack-to-width ratio and load. No matter what value of load exerted, the lines are displaying nonlinear increment of the relationship.

Table 2. $J$-integral values when different loads applied at different crack-to-width ratio for three-point bending test

| Crack-to-Width Ratio, $\alpha/W$ | 1000N | 2000N | 3000N | 4000N | 5000N |
|-------------------------------|-------|-------|-------|-------|-------|
| 0.126                         | 0.0617| 0.2470| 0.5557| 0.9879| 1.5437|
| 0.193                         | 0.0949| 0.3798| 0.8545| 1.5192| 2.3737|
| 0.252                         | 0.0976| 0.5257| 1.1788| 2.0956| 3.2744|
| 0.319                         | 0.1849| 0.7395| 1.6638| 2.9579| 4.6218|
| 0.378                         | 0.2524| 1.0095| 2.2713| 4.0378| 6.3091|
| 0.504                         | 0.5235| 2.0941| 4.7118| 8.3765| 13.0883|
| 0.570                         | 0.8224| 3.2940| 7.4014| 13.1583| 20.5597|
| 0.622                         | 1.2221| 4.9140|10.9980| 19.5527| 30.5510|
3.2.2. Tensile Test.

Referring to Table 3, a graph (figure 4) was constructed to show the correlation between $J$-integral and crack-to-width ratio for tensile test. It can be seen that as the crack-to-width ratio is raised, the $J$-integral will be escalated too. Besides, as the loads were increased, the $J$-integral values increased too. This drastic change is affected by the fracture toughness of the cortical bone since $J$-integral is related with fracture toughness of a material. The energy release rate is affected by the loads and cracks. That is why, as the crack length increase together with the crack length, SERR increased. However, $J$-integral values obtained were smaller compared to three-point bending test.

Table 3. $J$-integral values when different loads applied at different crack-to-width ratio for tensile test

| Crack-to-Width Ratio, $a/W$ | Strain Energy Release Rate, $J$-Integral |
|-----------------------------|----------------------------------------|
|                             | 1000N        | 2000N        | 3000N        | 4000N        | 5000N        |
| 0.126                       | 0.0027       | 0.0109       | 0.0246       | 0.0437       | 0.0683       |
| 0.193                       | 0.0051       | 0.0203       | 0.0456       | 0.0811       | 0.1268       |
| 0.252                       | 0.0082       | 0.0328       | 0.0739       | 0.1313       | 0.2052       |
| 0.319                       | 0.0137       | 0.0549       | 0.1236       | 0.2198       | 0.3434       |
| 0.378                       | 0.0216       | 0.0865       | 0.1946       | 0.3460       | 0.5407       |
| 0.504                       | 0.0592       | 0.2367       | 0.5327       | 0.9470       | 1.4796       |
| 0.570                       | 0.1060       | 0.3408       | 0.7667       | 1.3630       | 2.0080       |
| 0.622                       | 0.1736       | 0.5020       | 1.1296       | 2.0080       | 3.1377       |
Figure 4. Graph of comparison between $J$-integral of tensile test when different loads are applied on different crack-to-width ratio, $\alpha/W$

4. Conclusion
It is proved that fracture toughness of cortical bone which are SIF and SERR increased as crack-to-width ratio or load applied increased. This indicated that the fracture toughness of the cortical bone keeps increasing in order to resist fracture which brings to failure of the bone since SIF increased with crack-to-width ratio. Besides, FE method were proved as accurate as theory when DEM and CINT method were compared with theoretical expression. Moreover, it can be deduced that both forces exerted and crack-to-width, $\alpha/W$ ratio affect the $J$-integral during both tests since as the crack length increase together with the loads, SERR increased. However, SIF and SERR values for tensile test is lower than three-point bending test because of the force distributed by the load, as the force for bending test is directly above the crack while for the tensile test, the force is far from the crack.

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