Internet has drastically changed the mode of communication in the last few decades. E-mail has made communication efficient, fast and easy across the globe and helps business, scientific and social interactions. The structure of this communication network is dynamic, where each user is a node and the packet of information transmitted (an e-mail) constitutes a link between a pair of nodes (users, as sender and recipient). The structure of such a network has been found to be scale-free and is somewhat similar to network structure of the internet, E-mail has been looked upon as a social network, whose degree distribution and mode of information flow has also been studied. Given the network topology, how the individual nodes respond temporally in terms of their activity has not yet been studied. A very important and immediate example in this respect is a study of lifetimes of different logins in an e-mail server. The fundamental question stands as: How much time does an user spend during a particular login in his/her e-mail account? It can range from very small time-scales (bounded by the time required to log out immediately after logging in) to practically infinity. But in practice, it ranges from the order of seconds to many days, depending on the purpose and necessity of the login.

Analyzing data from stock-market, Internet (number of links to a page, number of pages within a site), as well as natural phenomena like earthquake, rainfall etc. yielded interesting observations, and our study on the distribution of login times is not an exception. Login time data from different countries are distributed in time according to an universal power law: there are many logins of short durations, whereas, very few extend for very large times. This universal distribution can be explained by a simple mean-field theory.

We collected login time data from e-mail servers of all major research centres in India and a few places in different parts of the world. The computers register login times to the extent of detailed particulars as user, date, login entry time, login exit time and net login time. The registered login times are in units of minutes, as it is the standard time resolution set to measure login times in the machines. Hence, all data collected were in terms of integral minutes. The login time distribution (see Fig. 1) turned out to be scale-free, i.e.,

$$P(t) \sim t^{-\alpha}$$

with the decay exponent $\alpha$ varying between 1.2 to 2.2 from server to server. Table I shows the decay exponents for different e-mail servers.

The data suggest that there are more logins of shorter durations than of longer durations, which is quite natural. Speculations could only predict the decaying nature of the distribution, but not why it is scale-free. The data shows that there is no typical time-scale of a login.

The origin of different login times are due to human behavior to check e-mails frequently between work, while those of larger time scales are due to logins which take place once or twice in a day to check e-mails coming in the previous few hours. The even larger time scales arise due to file-transfer processes, downloads and even due to reasons like ‘carelessness’!

We explain the scale-free nature of the login times by a simple mean field theory. We define $p$ as the probability of any user to log out at any time. We, however, consider no interaction between users for simplicity, and

FIG. 1: The probability distribution $P(t)$ of logins at different login times $t$ in minutes for some e-mail servers. The data has been binned logarithmically to reduce noise. The decay exponent varies between 1.2 to 2.2 (see Table I). The dotted line is the mean-field solution (Eqn. 3).
effects of heterogeneity and frustration are only realized by assigning a distribution of $p$. Hence, we treat this as a single-particle problem, and the cumulative effect in the many-particle situation is realized by assuming a distribution of $p$.

As per our assumption, the probability of an user to have not logged out at time $t$, $P(t)$, is $(1-p)^{t-1}$ and logs out at time $t$ is $\rho(p,t) = p(1-p)^{t-1}$. For a multi-user system, where $p$ follows a distribution $f(p)$, the probability of logging out at time $t$ is

$$P(t) = \int_0^1 \rho(p,t)f(p)dp. \quad (2)$$

We assume that the distribution of $p$ among the users is fairly uniform, i.e., $f(p) = 1$, which gives,

$$P(t) = \frac{1}{t(t+1)} \quad (3)$$

which in asymptotic limit gives $P(t) \sim t^{-2}$. Fig.1 shows a comparison of the real data and Eqn.(3).

We have studied the empirical distribution of login times in e-mail servers. This observed behavior is unique for e-mail servers and differ from general purpose computers and computational servers, which show a peaked distribution with an exponential (or stretched exponential) decay. The distribution of login times in e-mail servers, however, also has a peak which is not observed due to the fact that the minimum time resolution there is greater than the typical time-scale at which this peak occurs. The origin of the observed distribution is but a result of human behavior. Human behavior giving rise to frustration and fluctuations have been approximated by a simple mean-field theory. Our mean-field result is compatible with the real data and can reasonably explain the origin of scale-free nature of login time distribution.

Apart from the e-mail systems studied, this interesting behavior of a large number of components may also lead to similar observations in systems with similar dynamical topology as in other modes of social communication like telephone network and verbal conversation.

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**TABLE I:** The details of the empirical observations: Name of E-mail servers and their approximate decay exponents.

| E-mail server             | Decay exponent |
|--------------------------|----------------|
| MPIPKS Dresden           | 1.8            |
| IISc Mrc, Bangalore      | 1.9            |
| IISc Physics, Bangalore  | 2.2            |
| SNBNCBS Kolkata          | 1.5            |
| IMSc Chennai             | 1.3            |
| SINP Theory              | 1.2            |
| SINP Cmp                 | 1.6            |
| SINP Surf                | 1.7            |
| SINP Lotus               | 1.5            |
| SINP Petal               | 1.5            |

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[2] Albert, R., Jeong, H. & Barabási, A.-L. Nature 401 130 (1999).
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[5] Mandelbrot, B. B. J. Business 36 394 (1963).
[6] Huberman, B. A. & Adamic, L. A. Nature 401 131 (1999).
[7] Gutenberg, B. & Richter, C. F. Seismicity of the Earth and Associated Phenomena (Princeton Univ. Press, Princeton, N.J, 1954).
[8] Peters, O., Hertlein, C. & Christensen, K. Phys. Rev. Lett. 88 018701 (2001).
[9] Data for login times have been collected from e-mail servers of:
   (i) Saha Institute of Nuclear Physics, Kolkata, India;
   (ii) IISc-MRC, Bangalore, India; homepage: http://mrc.iisc.ernet.in/.
   (iii) The Institute of Mathematical Sciences, Chennai, India, homepage: http://www.imsc.ernet.in/.
   (iv) Materials Research Centre (MRC), Indian Institute of Science, Bangalore, India; homepage: http://mrc.iisc.ernet.in/.
   (v) Deptt. of Physics, Indian Institute of Science, Bangalore, India; homepage: http://physics.iisc.ernet.in/.
   (vi) Indian Institute of Technology, Kanpur, India; homepage: http://www.iitk.ac.in/.
   (vii) IISc-Physics, Bangalore, India; homepage: http://physics.iisc.ernet.in/.

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E-mail servers analyzed are: cmp, surf, theory, lotus, petal.

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