Design and simulation of an adjustable amplitude vibration system for mechanical linear friction welding equipment

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Abstract
This paper reports the design of an adjustable amplitude vibration system for mechanical linear friction welding (LFW) equipment. The system consists of two crank slider mechanisms that are connected by a phase modulator. The amplitude of the vibration system is determined by the average displacement of the two sliders, which can be changed by adjusting the phase difference between the two cranks. When the vibration amplitude is zero, the vibration system stays at a fixed position called positioning point. This study analyses the cause of the small amplitude vibrations of the system about the positioning point and improve the length of the connecting rod to minimize these small vibrations and ensure positioning accuracy. A kinematic and kinetic model under linear friction welding working conditions is developed based on Simulink. The simulation results indicate that after improvement of the mechanism, the system can satisfy the large loads of mechanical LFW equipment (40 kN frictional force) as well as the rapid amplitude adjustment capability and positioning accuracy requirements.

Keywords: Phase modulator, Adjustable amplitude vibration system, Linear friction welding, Kinematic and kinetic model, Simulation

1. Introduction

Linear friction welding is a solid state welding method. Welding of homogeneous or heterogeneous materials and of components with complex welding surfaces can be performed by driving two tightly fit parts with cyclic vibrations, which is especially suited for welding aero-engine and turbine blades. The key component of LFW equipment is the vibration system, which may be mechanical, electro-magnetic, or hydraulic. In addition to driving the workpieces at a certain amplitude and frequency during the welding process, the vibration system must also provide accurate positioning when the welding is completed. The best way to attain accurate positioning is to adjust the amplitude to zero while keeping the vibration frequency. The positional point is the vibration origin. This control method is simple, but the amplitude of the vibration system must be adjusted during the operation process. Due to its advantages of high load capacity and adjustable amplitude, hydraulic vibration systems are commonly used in LFW equipment, but their shortcomings of high costs and control difficulty cause the extreme high prices of LFW equipment. While electromagnetic vibration systems can adjust the amplitude, their low load bearing capacity cannot satisfy the requirements of LFW. The main component of mechanical vibration systems is the crank-slider mechanism. Due to the difficulty in adjusting the amplitude or the complexity of the amplitude adjustment mechanism, mechanical systems are usually not suitable for use in LFW equipment. However, if a breakthrough in amplitude adjustment can be achieved, their advantages of low cost and control simplicity could greatly reduce the cost of LFW equipment.

The basic principle of LFW is shown in Fig. 1. A normal pressure is applied to fit the two welding parts together, and one of the parts is driven to move reciprocally along the fitting surface. Due to the high pressure and high friction, the two parts are welded together. In practice, part 1 is usually installed on a vibration platform that can only move reciprocally in the vibration direction, and part 2 is connected to a hydraulic cylinder that supplies the pressure. Thus,
in addition to the positive pressure, the vibrating platform also experiences a frictional force, which is proportional to the pressure and is in the direction opposite to the movement of the vibrating platform. Because this frictional force is usually large, it should be considered the main load of the vibration system.

This paper reports the design of an adjustable amplitude vibration system for LFW, simulates the load that is exerted on the vibration system during the operation of the LFW equipment, and analyses the kinematics and dynamics of the vibration system to verify the system’s feasibility.

Fig. 1 Basic principle of LFW

2. Related work

There are three major types of vibration systems: mechanical, electromagnetic, and hydraulic (Guo, 2003; Wayne, 2011; Wayne, 2012). Mechanical vibration systems usually employ a crank-slider mechanism (Biechl, et al., 1998), in which the amplitude adjustment is the most difficult issue. Xu designed an adjustable amplitude vibration system for mechanical LFW equipment, as shown in Fig. 2, that is able to adjust the amplitude of a single crank-slider mechanism by a complicated hydraulic device (Xu, 2007). Electromagnetic vibration systems are usually used for lower load and high frequency conditions. Fan and Pan used an electromagnetic exciter to eliminate the whip of a rotor-bearing system (Fan, et al., 2011), and Kim, Jeong and Han designed an electromagnetic vibration exciter for electrostatic precipitation and verify its feasibility (Kim, 2012). Due to their high load bearing capacity and adjustable amplitude, hydraulic vibration systems can satisfy almost all of the requirements of a vibration system (Silva, et al., 2002; Jia, et al., 2008; Carden, 2012; Zhang, et al., 2012) and thus have been used widely. Carle introduced a hydraulic vibration exciter and developed a method to cool the exciter (Carle, 1980). The control of an electro-hydraulic exciter is very complicated (Olszewski, et al., 2004; Johnson, 2010; Konieczny, et al., 2013). Plummer et al. provided an effective electro-hydraulic exciter control system (Plummer, 2007; Zhu, et al., 2010). To address the problem of the low frequency of electro-hydraulic exciters, Ruan et al. developed a 2-D valve solution to increase the frequency of an electro-hydraulic exciter and study its bias control strategy (Ruan, et al., 2009; Yan, et al., 2010a). Furthermore, Yan, Jian and Ji analysed the excitation waveform of a 2-D valve electro-hydraulic exciter (Yan, et al., 2010b).

Fig. 2 The LFW equipment with a single crank-slider mechanism (Xu, 2007)
(1-Pressure cylinder; 2-Piston rod; 3-Rear bearing; 4-Sliding table; 5-Guide shaft; 6-Front bearing; 7-Weldment; 8-Fixture; 9-Weldment; 10-Vibration fixture; 11- Linear guide; 12- Rear bracket; 13-Connecting rod; 14-Roller; 15- Roller frame; 16-Piston rod; 17-Amplitude adjusting cylinder; 18-Guide; 19-Bracket; 20- Eccentric shaft bearing; 21- Connecting rod; 22-Eccentric shaft; 23- Motor)
3. Vibration system design

Although a single crank slider can provide the required vibration for LFW with a mechanical vibration system, it is usually difficult to achieve workpiece positioning at the completion of welding. For example:

1) If the motor is directly shut off, the workpiece may stop at any position within the range of vibration, and the positioning cannot be achieved.
2) Due to the high load and high motor rotation speed, using a clutch to cut off the power source combined with a position limiting method for the positioning will introduce a large shock on the relevant mechanism.
3) Due to the high load, using a motor rotation angle closed loop control method to achieve positioning places a high demand on the motor performance.

To address these problems, this paper employs a double crank phase adjustment method that resolves the workpiece positioning problem by adjusting the amplitude of the vibration system. While compared with the hybrid LFW equipment shown in Fig. 2, the proposed method is simpler in structure and control and easier to implement.

3.1 Design requirements

The main demands of the LFW process on the vibration system’s performance are as follows:

1) Load bearing capacity: a frictional force of 40 kN.
2) Amplitude modulation duration: adjust the amplitude from 3 mm to zero within 1 s.
3) Positioning accuracy: 10 μm.

3.2 Overall design

Figure 3 shows the overall design of the vibration system. Two identical crank slider mechanisms are driven by an AC motor (1), and the crank-slider mechanism is composed of a crank (8), connecting rod (2), and slider (3). The two sliders (3) are located at both ends of the beam (7) and slide along with the beam (7), and the vibration platform (5) can slide up and down along the linear track (6). The vibration platform (5) is hinged at the midpoint of the beam (7), and the displacement is the average displacement of the two sliders (3). The linear track (4) under the sliders (3) are fixed symmetrically on both sides of the linear track (6). The two cranks (8) are coaxially connected through the phase modulator (9); they have the same rotation speed, and the phase differences can be changed by the phase modulator (9). When the phase difference is 0, the two sliders (3) have the same displacement, and the vibration platform (5) has the largest amplitude. As the phase difference changes from 0 to 180°, the amplitude of the vibration platform (5) decreases gradually. When the phase difference is 180°, the displacement of the two sliders (3) are the same but in the opposite direction, and the amplitude of the vibration platform (5) is the smallest. At this time, the LFW process is completed, and the workpiece is positioned.

Fig. 3 Overall design of the vibration system

(1-AC motor; 2-Connecting rod; 3-Slider; 4-Linear track; 5-Vibration platform; 6-Linear track; 7-Beam; 8-Crank; 9-Phase modulator)
3.3 Phase modulator design

The phase modulator is the key mechanism of the proposed vibration system; it can change the phase difference between the two cranks during operation of the vibration system and ensures that the two cranks have the same rotation speed before and after the phase modulation. Figure 4 shows the motion of the phase modulator. The planetary gear mechanism is composed of the input shaft (a), the planetary gear (b), the carrier (c), and the output shaft (d), and each bevel gear has the same number of teeth. The worm wheel (e) is fixed to the carrier (c), and the worm shaft (d) is fixed to the phase modulator frame. The worm shaft (d) is driven by a stepping motor and rotates the worm wheel (e) and the carrier (c). The lead angle of the worm shaft (d) is less than 4° to achieve self-locking of the worm gear. For the planetary gear mechanism that is an epicyclic gear train,

\[ \frac{\omega_c}{\omega_i} = \frac{z_i}{z_c} = -1 \]  

where \( i \) is the transmission ratio of the gear, \( z \) is the number of teeth of the gear, and superscript ‘c’ means that the carrier (c) is treated as the reference frame of the epicyclic gear train.

The output shaft rotation speed can be obtained as

\[ \omega_f = -(\omega_c - 2\omega_i) \]  

When the planet carrier is stationary (i.e., \( \omega_c = 0 \) and \( \omega_f = -\omega_a \)), the input and output shafts have the same speed but rotate in opposite directions. When the worm shaft is driving the worm wheel and the carrier (i.e., \( \omega_c \neq 0 \)), the rotation angle of the output shaft at any time \( t \) is

\[ \theta_t = \int_0^t \omega_t dt = \int_0^t -(\omega_c - 2\omega_i) dt = -\left( \int_0^t \omega_c dt - 2\int_0^t \omega_i dt \right) = -\left( \theta_c - 2\theta_i \right) \]  

Thus, the leading (or lagging) angle of \( \theta_i \) relative to \( \theta_c \) is \( 2\theta_i \); i.e., for every 1° of rotation of the worm wheel that is driven by the shaft, the phase difference between the two crank slider mechanisms is 2°.

![Kinematic diagram of phase modulator](image-url)

As shown in Fig. 5, the carrier that is indirectly driven by the stepping motor is affected by the force of the internal gear during the phase modulating process; thus, the driving requirement of the phase modulator needs to be determined to select the suitable stepping motor. The rotation speed of the designed input shaft is \( n_a = 3000 \text{ rpm} \), the number of threads of worm shaft is \( z_d = 1 \), the number of teeth of the worm gear is \( z_e = 50 \), and the worm gear’s transmission efficiency is \( \eta = 0.7 \). Assuming that the torque of the input shaft (a) \( T_a \) is half of the AC motor’s load torque \( T_L \), then

\[ T_a = 0.5T_L \]  

On the carrier,

\[ T_c = \frac{d}{2}(2F_a + 2F_i) \]
where
\[ T_a = \frac{d}{2} \cdot 2F_a \]  \hspace{1cm} (6)
\[ T_l = \frac{d}{2} \cdot 2F_a \]  \hspace{1cm} (7)

Ignoring the friction force between the gears and rotating pairs,
\[ T_a = T_l \]  \hspace{1cm} (8)

From Eq. (4) to Eq. (8),
\[ T_c = T_L \]  \hspace{1cm} (9)

Thus, the load torque of the worm shaft (i.e., stepping motor) is
\[ T_d = \frac{T_c}{\varepsilon \eta} = \frac{T_L}{\frac{50}{1} \times 0.7} = \frac{1}{35} T_L \]  \hspace{1cm} (10)

Because the amplitude of the vibration system must be modulated from 3 mm to 0 mm within 1 s (i.e., the phase modulator needs to make a phase difference of 180° within 1 s), the worm wheel rotates 90° within 1 s, and the stepping motor should complete a rotation of 25π (i.e., 50×π/2) radians in 1 s. An S shape output curve shown in Fig. 6 is applied to the angle control of the stepping motor so that a good dynamic performance is obtained. The expression of the S shape curve is
\[ \theta_d = 25\pi \cdot \frac{1 + e^{-5}}{e^5 - e^{-5}} \left( \frac{1 + e^{i \omega t}}{1 + e^{i \omega t - 0.3t}} - 1 \right) \]  \hspace{1cm} (11)

where \( \theta_d \) is the rotation angle of the stepping motor, \( t \) is the time. So
\[ \alpha_d = \frac{d\omega_d}{dt} = \frac{d^2 \theta_d}{dt^2} \]  \hspace{1cm} (12)

where \( \omega_d \) and \( \alpha_d \) are the angular velocity and angular acceleration of the stepping motor. According to Eq.(11) and (12), the maximum angular velocity and maximum angular acceleration of the stepping motor are \( \omega_{d\text{max}} = 199 \text{ rad/s} \) and \( \alpha_{d\text{max}} = 766 \text{ rad/s}^2 \).

The relationship between the output torque of the stepping motor \( T_s \), \( T_d \) and \( \alpha_d \) is
\[ T_s - T_d = J_w \alpha_d \]  \hspace{1cm} (13)

where \( J_w \) is the equivalent moment of inertia of each moving element driven by the stepping motor relative to the stepping motor shaft.
\[ J_{sc} = J_d + J_{bce} \left( \frac{\omega_a}{\omega_a} \right)^2 + J_f \left( \frac{\omega_b}{\omega_a} \right)^2 + 2J_h \left( \frac{\omega_h}{\omega_a} \right)^2 + J_{csv} \left( \frac{\omega_h}{\omega_a} \right)^2 \] (14)

The meanings and values of the variables in Eq.(14) are obtained from the 3D models and shown in Table (1). Considering that \( \omega_a \), the rotation speed of (a) in Fig.4, keeps unchanged during the phase modulation and the stepping motor only contributes to the increments of rotation speeds and driving torques of b, c, d, e and f, which are independent of \( \omega_a \), \( \omega_a \) is assumed to be 0 to simplify the calculation of the output torque of the stepping motor. In this case, \( \omega_e/\omega_a = 1/50 \), being equal to the transmission ratio between the worm shaft and worm wheel. As \( \omega_c = \omega_a \) and \( \omega_f = 2\omega_a \) from Eq.(2), \( \omega_f/\omega_a = 1/25 \). As shown in Fig. 4, when \( \omega_a \) is 0, the velocity of the center of (b) \( \omega_{bo} \) equals \( \omega_b r \), and also equals \( \omega_c r \), where \( r \) is the pitch radius of the bevel gears (a) and (b). Thus, \( \omega_b = \omega_c \), that is \( \omega_f/\omega_a = 1/50 \).

The maximum rotational speed of the stepping motor is:

\[ n_{max} = 60 \frac{\omega_{d_{max}}}{2\pi} \text{ rpm} = 60 \times \frac{199}{2\pi} \text{ rpm} = 1900 \text{ rpm} \] (15)

![Graph](image)

**Fig. 6** The S shape output curve of the stepping motor

| Parameter name                                      | Symbol | Value | Unit     |
|----------------------------------------------------|--------|-------|----------|
| Moment of inertia of worm shaft                     | \( J_d \) | 1.95\times10^{-4} | kg·m²    |
| Total moment of inertia of planetary gear revolution, carrier and worm wheel | \( J_{bce} \) | 2.36\times10^{-2} | kg·m²    |
| Moment of inertia of output shaft                  | \( J_f \) | 2.16\times10^{-2} | kg·m²    |
| Moment of inertia of planetary gear rotation       | \( J_h \) | 1.66\times10^{-4} | kg·m²    |
| Equivalent moment of inertia of connecting rod, slider, beam and vibration platform relative to the crank | \( J_{csv} \) | Refer to Eq.(32) | kg·m²    |

4. Kinematic analysis of the vibration system

The vibration system is mainly composed of two crank-slider mechanisms, a phase modulator, a beam, and a vibration platform. Because the phase modulator is used to change the phase difference between the two crank-slider mechanisms, it can be excluded from the kinematic and dynamic analysis. The kinematic diagram of the vibration system is shown in Fig. 7.
According to the design conditions, \( \theta_4 = \theta_1 + \varphi, \omega_4 = \omega_1, \) and \( l_5 = l_2 \) and for the crank slider mechanism 1-2-3,

\[
\sin \theta_2 = \frac{L}{l_2} \sin \theta_1 = \lambda \sin \theta_1
\]  
(16)

\[
\omega_2 = \frac{d\theta_2}{dt} = \lambda \omega_1 \cos \theta_1 \cos \theta_2
\]  
(17)

\[
s_3 = l_1(1 - \cos \theta_1) - l_2(1 - \cos \theta_2)
\]  
(18)

\[
v_3 = \frac{ds_3}{dt} = \frac{\omega_1 l_1 \sin(\theta_1 - \theta_2)}{\cos \theta_2}
\]  
(19)

Similarly, for the slider-crank mechanism 4-5-6,

\[
\sin \theta_5 = \lambda \sin \theta_4
\]  
(20)

\[
\omega_5 = \lambda \omega_4 \cos \theta_4 \cos \theta_5
\]  
(21)

\[
s_6 = l_1(1 - \cos \theta_1) - l_2(1 - \cos \theta_2)
\]  
(22)

\[
v_6 = \frac{ds_6}{dt} = \frac{\omega_4 l_1 \sin(\theta_4 - \theta_5)}{\cos \theta_5}
\]  
(23)

For the center of the beam,

\[
s_7 = \frac{1}{2}(s_3 + s_6) = \frac{1}{2}[l_1(2 - \cos \theta_1 - \cos \theta_4) - l_2(2 - \cos \theta_2 - \cos \theta_5)]
\]  
(24)

\[
v_7 = \frac{1}{2}(v_3 + v_6)
\]  
(25)
5. Kinetic analysis of the vibration system

5.1 Power balance equation

Because the two crank-slider mechanisms mainly work against the friction exerted by the LFW, based on the power balance principle, the power of the AC motor is equal to the power that is consumed to overcome the frictional force:

\[ T_L \omega_1 = |fv| \]  \hspace{1cm} (26)

5.2 Equivalent moment of inertia

Because the AC motor drives the double crank-slider mechanisms to operate at a high rotational speed, the moment of inertia of each rotating component has a significant influence on the motor shaft and the motor operation state, and it is necessary to analyse the moment of inertia of the moving parts. The method of the equivalent moment of inertia is used for convenience. The moment of inertia of each moving element is equivalent to that of the motor shaft; the expression is:

\[ J_e = \sum_{i=1}^{m} \left( \frac{m_i}{\omega_1} \right)^2 + \sum_{j=1}^{n} \left( \frac{v_{j}}{\omega_1} \right)^2 \]  \hspace{1cm} (27)

That is,

\[ J_e = J_1 + J_2 \left( \frac{\omega_2}{\omega_1} \right)^2 + J_3 \left( \frac{\omega_3}{\omega_1} \right)^2 + m_2 \left( \frac{v_2}{\omega_1} \right)^2 + m_3 \left( \frac{v_3}{\omega_1} \right)^2 + m_4 \left( \frac{v_4}{\omega_1} \right)^2 + m_5 \left( \frac{v_5}{\omega_1} \right)^2 + m_6 \left( \frac{v_6}{\omega_1} \right)^2 \]  \hspace{1cm} (28)

Considering the connection rod 2, the velocity of its centroid is equal to the vector sum of the velocity of its big end, which is \( l_i \omega_1 \), and the relative velocity between the centroid and the big end, which is \( l_i \omega_2 \) where \( l_i \) is the distance between the centroid and the big end. Calculated by the 3D model of the rod, \( l_i \) is 2/5 of the rod length \( l_2 \), so

\[ v_2 = \left( l_i \omega_1 \cos \theta - \frac{2}{5} l_i \omega_2 \cos \theta_2 \right)^2 + \left(l_i \omega_1 \sin \theta - \frac{2}{5} l_i \omega_2 \sin \theta_2 \right)^2 \]

\[ = l_2^2 \omega_1^2 + \frac{4}{25} l_2^2 \omega_2^2 - \frac{4}{5} l_1 l_2 \omega_1 \omega_2 \cos (\theta_1 - \theta_2) \]  \hspace{1cm} (29)

Similarly

\[ v_2 = l_1^2 \omega_1^2 + \frac{4}{25} l_1^2 \omega_2^2 - \frac{4}{5} l_1 l_2 \omega_1 \omega_2 \cos (\theta_1 - \theta_2) \]  \hspace{1cm} (30)

The relationship between \( T_L, J_e \) and the electromagnetic torque of the AC motor \( T_e \) can be written as

\[ T_e - T_L = J_e \frac{d\omega_1}{dt} \]  \hspace{1cm} (31)

With Eq. (20), (21), (23), (25) and (30), \( J_{csv} \) in Eq.(14) can be obtained by \( J_{csv} = J_3 \left( \frac{\omega_3}{\omega_1} \right)^2 + m_2 \left( \frac{v_2}{\omega_1} \right)^2 + m_3 \left( \frac{v_3}{\omega_1} \right)^2 + m_4 \left( \frac{v_4}{\omega_1} \right)^2 + m_5 \left( \frac{v_5}{\omega_1} \right)^2 \). Its maximum value is used for the design in the paper

\[ J_{csvmax} = J_1 \left( \frac{l_1}{l_2} \right)^2 + \frac{64}{25} m_1 l_1^2 + m_2 l_1^2 + \frac{1}{2} m_5 l_1^2 \]  \hspace{1cm} (32)
6. Simulink-based modeling and simulation

6.1 Simulink-based modeling

The dynamic simulation model of the vibration system is developed in Simulink as shown in Fig. 8 to Fig. 10, where the expressions of all the Function Blocks $f(u)$ can be obtain from Eq. (16) to Eq. (31). The system is mainly composed of a general three-phase asynchronous AC motor model with $U_a$, $U_b$, $U_c$ being the three-phase input voltages, a kinematic model, and a kinetic model. The parameters of the vibration system obtained from its 3D model and the parameters of the AC motor (Huang, et al., 2013) are listed in Tables (2) and (3), respectively.

![Fig. 8 Dynamic simulation model of the vibration system](image)

![Fig. 9 Kinematic model](image)
Fig. 10 Kinetic model

### Table 2  Vibration system parameters

| Parameter name                                              | Symbol | Value | Unit   |
|-------------------------------------------------------------|--------|-------|--------|
| Total inertia of the crank and motor shaft                  | $J_1$  | 0.227 | kg·m$^2$ |
| Inertia of connecting rod                                   | $J_2, J_5$ | 0.068 | kg·m$^2$ |
| Mass of connecting rod                                       | $m_2, m_5$ | 10    | kg     |
| Slider mass                                                 | $m_3, m_6$ | 10    | kg     |
| Total mass of the beam and vibration platform               | $m_7$  | 110   | kg     |
| Crank eccentricity                                          | $l_1, l_4$ | 0.003 | m      |
| Length of connecting rod                                    | $l_2, l_5$ | 0.25  | m      |
| Friction force                                              | $f$    | 40    | kN     |

### Table 3  Three-phase asynchronous AC motor parameters

| Parameter name                                              | Symbol | Value   | Unit   |
|-------------------------------------------------------------|--------|---------|--------|
| Rated power                                                 | $P_N$  | 37      | kW     |
| Rated voltage                                               | $U_N$  | 380     | V      |
| Rated frequency                                             | $f_N$  | 50      | Hz     |
| Stator winding resistance                                   | $R_s$  | 0.08233 | Ω      |
| Rotor winding resistance                                    | $R_r$  | 0.0503  | Ω      |
| Stator winding self-inductance                              | $L_s$  | 27.834  | mH     |
| Rotor winding self-inductance                               | $L_r$  | 27.834  | mH     |
| Stator-rotor mutual inductance                              | $L_m$  | 27.11   | mH     |
| Pole pairs                                                  | $n_p$  | 1       | -      |

### 6.2 Analysis of Simulation Results

The two most important parameters of the proposed vibration system are the amplitude and frequency. The amplitude $A$ can be obtained from the vibration platform’s displacement $s_7$:

$$A = \frac{1}{2}(s_{7_{\text{max}}} - s_{7_{\text{min}}})$$  \hspace{1cm} (33)

The frequency $f$ can be obtained by the motor speed $\omega$:

$$f = \frac{\omega}{2\pi}$$  \hspace{1cm} (34)
The initial phase difference is set to be $\varphi = 0^\circ$. When $t_1 = 1.5$ s, the phase difference is set to be $\varphi = 180^\circ$ in 1 s. Figure 11(a) and Fig. 11(b) show the simulation results, where $t_s$ is the starting time of the AC motor.

1) When the phase difference $\varphi = 0^\circ$, $\omega = 312.25$ rad/s, $s_{7\text{max}} = 6$ mm, and $s_{7\text{min}} = 0$. From Eq. (33) and Eq. (34), $f = 49.69$ Hz, and $A = 3$ mm.

2) When the phase difference $\varphi = 180^\circ$, $\omega = 314.11$ rad/s. As shown in Fig. 11(c), $s_{7\text{max}} = 3$ mm, and $s_{7\text{min}} = 2.982$ mm. From Eq. (33) and Eq. (34), $f = 49.99$ Hz, and $A = 0.009$ mm.
6.3 Error analysis and improvement

When the phase difference $\varphi = 180^\circ$, the vibration platform should be stationary at the origin, which corresponds to the position at the completion of the LFW process. However, the simulation results show that the vibration platform is still vibrating with a fixed amplitude of $A = 0.009$ mm at this time. This amplitude has a significant impact on the positioning accuracy at the completion of the LFW process and thus needs to be improved.

6.3.1 Impact of the fitting gap on the vibration system

As shown in Fig. 3, the vibration platform (5) is connected to the midpoint of the beam (7) through a hinge connection; the fitting method is a shaft-hole clearance fit (H8 / g7). Considering the impact of this gap on the vibration platform, $s_7$ is actually the displacement of the beam’s centre. As shown in Fig. 12, when the phase difference $\varphi = 180^\circ$, the maximum vibration displacement of the beam’s centre $\Delta h$ only needs to be less than the minimum gap $X_{\text{min}}$; thus, this vibration can be cancelled by the fitting gap and does not affect the positioning accuracy of the vibration system.

The basic dimension of the gap fitting is 25 mm. Using a look-up table, we obtain $EI = 0$ and $es = -7 \mu m$; that is

$$X_{\min} = EI - es = 7 \mu m$$

(35)

![Fig. 12 The relationship between $\Delta h$ and $X_{\text{min}}$ in the shaft-hole clearance fit](image)

6.3.2 Error analysis and optimization considering the fitting gap

When the phase difference $\varphi = 180^\circ$, the movement of the vibration system can be simplified as shown in Fig. 13. When the crank rotation angle is $\alpha$ degrees (i.e., $\theta_1 = 180^\circ + \alpha$, $\theta_4 = \alpha$), Eq. (16) and Eq. (20) show that

$$\theta_5 = -\theta_2 = \theta$$

(36)

Let $l_2 = l_1 = r = 3$ mm, $l_5 = l_2 = l$; then

$$h_\alpha = l \cos \theta + r \cos \alpha$$

(37)

$$h_\beta = l \cos \theta - r \cos \alpha$$

(38)

The height of the beam centre at this time is

$$h = \frac{1}{2} (h_\alpha + h_\beta) = l \cos \theta$$

(39)

Because

$$\theta_{\min} = 0$$

$$\theta_{\min} = \arccos \frac{l}{\sqrt{l^2 + r^2}}$$

(40)
the maximum displacement of the midpoint of the beam’s centre is

$$\Delta h = h(\theta_{\text{min}}) - h(\theta_{\text{max}}) = l(1 - \cos \theta_{\text{max}}) = l - \frac{l^2}{\sqrt{l^2 + r^2}}$$  \hspace{1cm} (41)

The variation of $\Delta h$ with respect to $l$ can be obtained from Eq. (41) and is shown in Fig. 14. When $l > 2.4$ mm, $\Delta h$ decreases with increasing $l$; As the length of connecting rod is larger than 2.4mm(also larger than 3mm, the eccentricity of the cranks), increasing $l$ can reduce the maximum displacement of the beam’s centre. When $\Delta h = X_{\text{min}}$, $l = 643$ mm; that is, when $l > 643$ mm, the vibration of the beam’s centre will be cancelled by the fitting gap, and the positioning accuracy of the vibration system will not be affected by the vibration. To increase the reliability, the design uses $l = 1000$ mm; at this length, $\Delta h = 4.5 \mu$m, and the length of connecting rod $l$ is within the acceptable range. The improved parameters are shown in Table (4).

![Fig. 13 Vibration system analysis diagram when $\varphi = 180^\circ$](image-url)

![Fig. 14 Variation of the maximum vibration platform displacement with the length of connecting rod when $\varphi = 180^\circ$](image-url)

| Table 4  | Adjusted parameters after improvement |
|----------|--------------------------------------|
| Parameter name | Symbol | Value | Unit |
| Length of connecting rod | $l_2, l_5$ | 1 | m |
| Mass of connecting rod | $m_2, m_5$ | 50 | kg |
| Inertia of connecting rod | $J_2, J_5$ | 2.221 | kg·m$^2$ |
6.3.3 Analysis of simulation results after improvement

The simulation conditions are kept the same as before, and the results are shown in Fig. 15(a) and Fig. 15(b). The results show that:

1) When the phase difference $\varphi = 0^\circ$, $\omega = 312.25$ rad/s, $s_{7\text{max}} = 6$ mm, and $s_{7\text{min}} = 0$. From Eq. (33) and Eq. (34), $f = 49.69$ Hz, and $A = 3$ mm.

2) When the phase difference $\varphi = 180^\circ$, $\omega = 314.13$ rad/s. Figure 15(c) shows that $s_{7\text{max}} = 3$ mm, and $s_{7\text{min}} = 2.9955$ mm. Eq. (33) and Eq. (34), $f = 49.995$ Hz, and $A = 2.25$ μm.

3) As shown in Fig. 15(d), when the phase difference $\varphi = 0^\circ$, the AC motor load torque $T_L$ reaches its maximum of 120 N·m. Eq. (10), (13), (14) and (32) and Table (1), (2) and (4) show that the required output torque of the stepping motor $T_s = 3.61$ N·m.

These simulation results show that the amplitude of the vibration platform changes with the variation in the phase difference; it reaches a maximum when $\varphi=0^\circ$ and has a value of 0 when $\varphi = 180^\circ$. The workpiece is well positioned (the vibration amplitude of the beam centre at this time is 2.25 μm, but it is cancelled by the fitting gap and thus does not affect the positioning accuracy). Moreover, the AC motor can drive the vibration system to vibrate at a frequency of approximately 50 Hz, which satisfies the requirement of the LFW process and thus is considered to be stable.

To enable the phase modulation, an output torque of 3.61 N·m and a rotation speed of 1900 rpm are required for the stepping motor, which is satisfied by 86J1895EC, the stepping motor we use.
7. Conclusions

This study designed an adjustable amplitude vibration system for LFW equipment. Amplitude adjustment of the vibration system was achieved by utilizing a phase modulator to alter the phase difference between two crank-slider mechanisms. This study analysed the working principle of the phase modulator, developed kinematic and kinetic models for the vibration system, and utilized Simulink to simulate the vibration system. The following conclusions were obtained:

1) The phase difference between the two crank-slider mechanisms can be modulated using a stepping motor driving worm and worm wheel mechanism, and the worm wheel mechanism can self-lock.

2) Under a frictional force of 40 kN, the AC motor can drive the vibration system to vibrate at a stable frequency of approximately 50 Hz, which satisfies the demands of common LFW processes.

3) The amplitude of the vibration platform changes with the phase variation. The amplitude reaches a maximum of 3 mm when the phase difference is $\phi = 0^\circ$. When the phase difference is $\phi = 180^\circ$, the vibration platform has vibrates at a small amplitude, but the amplitude can be reduced by increasing the length of the connecting rod in the crank-slider mechanism. When the connecting rod is longer than 643 mm, this small vibration amplitude has no effect on the positioning accuracy of the vibration system.

4) The working conditions of the system are satisfied by this system.

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