In the dual superconductivity description of quark confinement the core of the flux tube connecting a quark pair belongs to a deconfined, hot phase. This can be checked in numerical experiments on 3D \( \mathbb{Z}_2 \) gauge model. The Svetitsky-Yaffe conjecture provides analytic expressions for the distribution of the flux density around quark sources at critical temperature.

1 Introduction

The internal structure of the colour flux tube (CFT) joining a quark pair in the confining phase of any gauge model provides an important test of the dual superconductivity (DS) conjecture\(^1\), because it should show, as the dual of an Abrikosov vortex, a core of normal, hot vacuum as contrasted with the surrounding medium, which is in the dual superconducting phase. A general way to study the internal structure of the flux tube is to test it with suitable gauge invariant probes. More specifically, the vacuum state of a lattice gauge model is modified by the insertion in the action of a quark source (for instance a Wilson loop). In this modified vacuum (called W-vacuum) one can evaluate the expectation value of various probes as a function of their position with respect the quark sources. Some general results of such an analysis has been already reported in Ref.\(^2\). Here I will describe some new results which are specific of the 3D \( \mathbb{Z}_2 \) gauge model.

2 The Disorder Parameter around Quark Sources

The location of the core of the CFT is given in DS conjecture by the vanishing of the disorder parameter \( \langle \Phi_M(x) \rangle \), where \( \Phi_M \) is some effective magnetic Higgs field. In a pure gauge theory, the formulation of this property poses some problems, because in general no local, gauge invariant, disorder field \( \Phi_M(x) \) is known. In the special case of 3D \( \mathbb{Z}_2 \) gauge model there is an exact duality, namely the Kramers-Wannier transformation, which maps the gauge theory in the Ising model. The spontaneous magnetization \( \mu = \langle \sigma \rangle \) is precisely the wanted disorder parameter: it vanishes in the deconfined phase, while it is different from zero in the confining phase. As an example, in Fig.\(^1\) the spontaneous magnetization in a W-vacuum generated by a pair of parallel Polyakov
loops is reported. One can clearly see the formation of a flux tube with a core where the disorder parameter vanishes, as required by the DS conjecture.

![Figure 1. Spontaneous magnetization around a quark pair.](image1)

![Figure 2. Total magnetization as a function of the loop area. The black dots are square Wilson loops, the open symbols are pairs of Polyakov loops.](image2)

The total thickness of the flux tube is the sum of two different contributions: one is due to quantum fluctuations of string-like modes of the CFT, which produce an effective squared width growing logarithmically with the in-
terquark distance \( d \), the other is the intrinsic thickness of the flux tube, which according to the DS conjecture is non-vanishing.

The total magnetization of the W-vacuum provides us with a method to evaluate such an intrinsic thickness: describing the CFT approximately as a cylinder of vanishing magnetization immersed in a mean of magnetization \( \mu \neq 0 \) we get that the total magnetization of the W-vacuum in a finite volume decreases linearly with the volume \( V \) spanned by the CFT as shown in Fig.2, with \( V = AL_c \), where \( L_c \) is the intrinsic thickness of the tube and \( A \) is the area of the minimal surface bounded by the Wilson loop (black dots) or by a Polyakov pair (open squares). The slope of such a linear behaviour yields an intrinsic thickness \( L_c \sqrt{\sigma} = 0.98(2) \) (\( \sigma \) is the string tension) in reasonable agreement with the theoretical value of \( \sqrt{\pi/3} \) suggested by a conformal field theory argument.

### 3 The Colour Flux Tube at Criticality

According to the widely tested Svetitsky-Yaffe conjecture\[^6\] any gauge theory in \( d + 1 \) dimensions with a continuous deconfining transition belongs to the same universality class of a \( d \)-dimensional \( C(G) \)-symmetric spin model, where \( C(G) \) is the center of the gauge group. It follows that at the critical point all the critical indices describing the two transitions and all the adimensional ratios of correlation functions of corresponding observables in the two theories should coincide.

In particular, since the order parameter the gauge theory is mapped into the corresponding one of the spin model, the correlation functions among Polyakov loops should be proportional to the corresponding correlators of spin operators:

\[
(P_1 \ldots P_n)_{T = T_c} \propto \langle \sigma_1 \ldots \sigma_n \rangle \ .
\]

The crucial point is that for \( d = 2 \) the form of these universal functions is exactly known. Then one can use these analytic results to get useful informations on the internal structure\[^1\] of the colour flux tube at \( T = T_c \). For instance, the correlator

\[
(P_1 \ldots P_{n+2}) = \langle P(x_1, y_1) \ldots P(x, y) P(x + \epsilon, y) \rangle \ ,
\]

thought as a function of the spatial coordinates \( x, y \) of the last two Polyakov loops (used as probes), describes, when \( \epsilon \) is chosen small with respect to the other distances entering into the game, the distribution of the flux around \( n \) Polyakov loops with spatial coordinates \( x_i, y_i \) (\( i = 1, \ldots n \)). In Fig.3 the contour lines of the flux distribution \( \rho(x, y) = \langle P_1 \ldots P_6 \rangle / \langle P_1 \ldots P_4 \rangle - \langle P_4 P_5 \rangle \)
in a critical gauge system with $C(G) = \mathbb{Z}_2$ are reported. The Polyakov lines are located at the corners of a rectangle $d \times r$ with $d > r$. Denoting by $r_i$ the distance of the probe ($P_5P_6$) from the source $P_i$ one has simply

$$\rho(x, y) \propto \epsilon^{\frac{3}{4}} \left\{ \frac{rd}{\sqrt{r^2 + d^2}} \sum_i \frac{r_i^2}{\prod_i r_i} + O(\epsilon) \right\} .$$

One clearly sees the formation of two flux tubes connecting the two pairs of nearest sources. Comparison with the distribution obtained by the sum of the fluxes generated by two non-interacting (i.e. $d = \infty$) flux tubes (dotted contours) indicates an attractive interaction between them, as expected.

Figure 3. Contours of flux density around two pairs of parallel Polyakov loops at criticality. The dotted lines correspond to the contours in the non-interacting case.

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