Reflected wavefront manipulation by acoustic metasurfaces with anisotropic local resonant units

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Abstract – In this work, we develop a new gradient acoustic metasurface to manipulate reflective wavefronts arbitrarily. Each unit of the gradient metasurface is constructed of a locally anisotropic resonant structure, comprising a steel cylinder with an elliptical rubber coating embedded in epoxy. Phase shifts of the reflected wave over the full $2\pi$ range provided by different units can be realized by selecting the radius of the steel cylinder. With an appropriate design of the phase profiles along the acoustic metasurface, we can achieve anomalous reflection, a planar acoustic lens, and an acoustic cloak. The locally anisotropic resonant units have significant potential for engineering and manipulating acoustic wavefronts.

Metamaterials form a class of subwavelength artificial composite structures with exotic properties that cannot be found in natural materials. A new type of metamaterials —metasurfaces— has emerged. Metasurfaces can produce phase shifts and amplitude modulation, and thus, they form an active research topic in the physics and engineering communities. Inspired by optical wave manipulation and because acoustic waves also follow the generalized Snell’s law [1], metasurfaces may be useful for manipulating acoustic wavefronts. Some acoustic metasurfaces have a compact subwavelength thickness, which highlights their potential for miniaturization and integration. Acoustic metasurfaces can freely tailor the wave field across a single layer and have many fascinating wave-based features.

Significant progress has been made in theoretical analyses and experimental demonstrations of non-resonant or resonant acoustic metasurfaces. Based on the generalized Snell’s law, many acoustic metasurfaces have been produced that engineer reflected or transmitted acoustic phase modulation. Non-resonant acoustic metasurfaces designed with space-coiling structures [2–8] are able to increase the momentum of an incident wave, leading to acoustic waves being reflected or refracted at an abnormal angle. These structures can generate an abrupt phase shift (up to $2\pi$), causing a large phase delay within a small space. Xie et al. [8] applied tapered labyrinthine units with high impedance matching to realize a conversion from a propagating wave to a surface mode, extraordinary beam steering, and apparent negative refraction through higher-order diffraction. A special class of acoustic metasurfaces has shown the phase adjustment mechanism at a resonant frequency, including a mass-weighted membrane [9–13] or a Helmholtz resonator [14–21]. It has been proven that metasurfaces based on a mass-weighted membrane can be particularly efficient in absorbing sound at low frequencies. Acoustic metasurfaces based on Helmholtz resonators have a tunable phase velocity. These metasurfaces are easily fabricated and assembled. Li et al. [16] utilized a metascreen composed of elements with four Helmholtz resonators in series and a straight pipe to realize anomalous refraction, even if the incident wave is propagating obliquely with a large angle. Besides the successful work on phase engineering, acoustic control with simultaneous phase and amplitude modulation can improve the accuracy of acoustic holography [22,23]. Acoustic metasurfaces with different inner structures can almost propagate acoustic waves in a fluid matrix.

Acoustic metasurfaces with a subwavelength thickness have many applications in engineering and physics. An acoustic metasurface can be used to make an acoustic ground-plane cloak [11,24,25]. These can shield an object from an incoming acoustic wave without generating a disturbance. Compared with a perfect cloak based on a coordination transformation, invisible cloaks using acoustic metasurfaces are so small that they have potential for miniaturization and integration. Zhai et al. [11] designed

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2D acoustic ultrathin skin cloaks constructed from a cavity coupled with membranes, which compensate entirely for the wavefront discrepancy generated by the scattering of the hidden object.

Like the resonant mechanism of Helmholtz resonators, three-component composites have also an outstanding resonant performance. These composites have a matrix of silicone-coated metallic spheres embedded in epoxy and were first proposed by Liu et al. [26,27]. They can generate a negative dynamic mass density at resonant frequencies. These resonant structures have been widely studied in physics and engineering [28–30]. The three-component resonators might be hard to make acoustic metasurfaces because of its complexity. For example, the dipolar resonances contain two degeneracy modes. We can borrow the ideal of anisotropic metamaterials to remove one of the two degenerate modes in the dipolar resonance. An elastic metamaterial built from lead cylinders with an elliptical rubber coating embedded in epoxy matrix had an anisotropic effective mass density, based on a multi-displacement microstructure continuum model [31]. Following theoretical studies, microstructural designs with an anisotropic mass density have been experimentally validated [32].

In this paper, we propose a new resonant-based acoustic metasurface. Each of its anisotropic resonant units contains a steel cylinder with an elliptical rubber coating embedded in epoxy. The rubber ellipses induce two non-degeneracy modes, which are difficult to implement in the circular inclusions. Only one mode will be excited at a certain work frequency. The dipolar resonant mechanism can be used to create adjustable phase shifts. These acoustic metasurfaces have an extraordinary reflected phase modulation and can be used to arbitrarily manipulate low-frequency acoustic waves in a water matrix.

Our locally anisotropic resonant units, which contain three available materials, manifest an excellent resonant effect and are able to produce a remarkable phase adjustment. Anomalous reflection, acoustic focusing, and acoustic cloaking can be realized by selecting suitable phase delay profiles of the resonant-based acoustic metasurfaces. The commercial software COMSOL Multiphysics (based on the Finite Element Method) was employed for numerical simulations. These acoustic metasurfaces have additional degrees of freedom and pave the way for thin planar surfaces that can manipulate acoustic wavefronts.

Figure 1(a) shows that when an acoustic wave is normally incident, the locally anisotropic resonant unit reflects it completely. The dark-purple and orange arrows indicate the propagation directions of the incident wave and reflected wave, respectively. Figure 1(b) is a schematic of a locally anisotropic resonant unit. Each unit is composed of a steel cylinder with an elliptical rubber coating embedded in epoxy. The parameters of the materials used are: \( \rho = 1000 \text{ kg/m}^3 \), \( c = 1490 \text{ m/s} \) for water; \( \rho_e = 1180 \text{ kg/m}^3 \), \( \lambda_e = 4.4 \times 10^8 \text{ N/m}^2 \) and \( \mu_e = 1.6 \times 10^9 \text{ N/m}^2 \) for epoxy; \( \rho_r = 980 \text{ kg/m}^3 \), \( \lambda_r = 1.96 \times 10^9 \text{ N/m}^2 \), and \( \mu_r = 5.5 \times 10^5 \text{ N/m}^2 \) for rubber; \( \rho_s = 7900 \text{ kg/m}^3 \), \( \lambda_s = 1 \times 10^{11} \text{ N/m}^2 \), and \( \mu_s = 8.1 \times 10^{10} \text{ N/m}^2 \) for steel, where \( \rho \) is the mass density, \( c \) is the speed of sound, and \( \lambda \) and \( \mu \) are the Lamé constants.

We designed and optimized 16 types of anisotropic resonant units with trihedral rigidity, to produce phase delay profiles with a relatively high resolution. The units have a square cross-section of length \( w \). The three boundaries of the anisotropic resonant unit have been defined as the prescribed velocity to make each unit independent of the others. (c) Phase of the reflected wave as a function of the radius of the steel cylinder for incident wavelength \( \lambda = 8.8w \). Pressure strips of the reflected wave are shown for 16 anisotropic resonant units.
The wavelength of the incident acoustic wave is \(8.8\) alized Snell's law, which agrees well with the simulated result. The reflected angle is calculated as \(33^\circ\) utilizing the generalized Snell's law, which agrees well with the simulated result. The wavelength of the incident acoustic wave is \(8.8w\).

The reflected angle is calculated as \(33^\circ\). Note that the pressure fields show that the reflected acoustic wave is oblique. The reflected angle is calculated as \(33.4^\circ\) utilizing the generalized Snell's law, which agrees well with the simulated result. The wavelength of the incident acoustic wave is \(8.8w\).

To confirm the validity of the anomalous reflection from the anisotropic resonant units, we numerically simulated the acoustic metasurface. Note that the phase gradient of the acoustic metasurface designed is set as \(d\varphi(x)/dx = \pi/8w\). In fig. 2, when an acoustic wave from the bottom is normally incident at the designed metasurface, the pressure fields show that the reflected acoustic wave is oblique. The angle of the reflected wave is calculated as \(33.4^\circ\) utilizing the generalized Snell's law with a normal incident wave. The simulated result agrees well with the theoretical result for the reflected pressure distribution.

Next, we design a flat lens using a well-designed acoustic metasurface to demonstrate further the remarkable potential for wavefront modulation. By delaying the acoustic reflected phase, the phase profile becomes circular. Thus, these anisotropic resonant units can realize acoustic focusing. For a given focal length \(f\), the phase shift \(\varphi(x)\) everywhere must satisfy the following equation:

\[
\varphi(x) = k_0 \left( \sqrt{x^2 + f^2} - f \right).
\]

From this condition, when the focal length is set as \(3\lambda\), the structure of the anisotropic resonant units at any \(x\) position is known. Figure 3(a) illustrates the continuous phase shift along the \(x\)-axis of the reflected wave. The reflected acoustic pressure intensity distribution is shown in fig. 3(b).

In addition to their extraordinary ability to manipulate acoustic waves, our acoustic metasurfaces built from anisotropic resonant units can also shield an object from an incident wave. Compared with the bulky cloaks based on coordination transformation, this type of acoustic cloak is smaller. It is possible for carpet cloaks to control wave amplitude and wave phase simultaneously. Our anisotropic resonant units provide precise phase modulation and hence, retain the invariability of the specific scattered sound field.
For cloaking, we chose an isosceles triangle region of width \(4a\) and height \(0.5\lambda\), shown as the grey-colored regions at the top of figs. 4(a) and (b). We covered both inclined planes with the anisotropic resonant units. The anisotropic resonant units on the surface of the area were carefully selected for each position. To verify the performance of the cloak, we ran numerical simulations for an acoustic wave impinging normally from the bottom toward i) a region covered by the acoustic metasurface and ii) an uncloaked region. It is clear in fig. 4(a) that the reflected acoustic pressure field exhibits invisibility, no matter what the object is in the triangular area. The pressure field of the reflected acoustic wave shows the good performances of carpet cloaking. This excellent performance of the cloak is due to the fine spatial resolution, the complete reflection, and the fully controlled phase shift. In contrast, the uncloaked object induces a strong perturbation in the scattered acoustic pressure field (fig. 4(b)). We have achieved nearly perfect cloaking.

In this work, we present a class of acoustic metasurfaces based on anisotropic resonant units, which have sufficient degrees of freedom for wavefront modulation. By selecting an appropriate gradient phase profile, the acoustic metasurfaces can redirect the incident acoustic wave as described by the generalized Snell’s law. We also demonstrate that, when we control the reflected phase profile to a circle, acoustic focusing can be realized. Finally, due to the fine spatial resolution, complete reflection, and the fully controlled phase shift, this kind of acoustic metasurface has significant potential for cloaking by making the isosceles triangle region invisible. The acoustic metasurfaces built from our anisotropic resonant units are a new approach for engineering and manipulating wavefronts. The anisotropic resonant units have significant potential for manipulating elastic waves in a solid matrix.

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