The Generic Supersymmetric Standard Model
as the Complete Theory of Supersymmetry without R-parity

Otto C.W. Kong
Institute of Physics, Academia Sinica, Nankang, Taipei, TAIWAN 11529

Abstract

The generic supersymmetric standard model is a model built from a supersymmetrized standard model field spectrum the gauge symmetries only. The popular minimal supersymmetric standard model differs from the generic version in having R-parity imposed by hand. We review an efficient formulation of the model in which all the admissible R-parity violating terms are incorporated without bias. The model gives many new interesting R-parity violating phenomenological features only started to be studied recently. Some of our recent results will be discussed, including newly identified 1-loop contributions to neutrino masses and electric dipole moments of neutron and electron. This is related to the largely overlooked R-parity violating contributions to squark and slepton mixings, which we also present in detail.

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The generic supersymmetric standard model is a model built from a supersymmetrized standard model field spectrum the gauge symmetries only. The popular minimal supersymmetric standard model differs from the generic version in having R-parity imposed by hand. We review an efficient formulation of the model in which all the admissible R-parity violating terms are incorporated without bias. The model gives many new interesting R-parity violating phenomenological features only started to be studied recently. Some of our recent results will be discussed, including newly identified 1-loop contributions to neutrino masses and electric dipole moments of neutron and electron. This is related to the largely overlooked R-parity violating contributions to squark and slepton mixings, which we also present in detail.

1. INTRODUCTION

From the early history of supersymmetry (SUSY), there had been thinking about its usage in the obviously non-supersymmetric low-energy phenomenology. One of the first ideas was the identification of the neutrino as a goldstino, i.e. the Goldstone mode from (global) SUSY breaking. Nowadays, the question: “Is the masslessness of the neutrino a result of SUSY (breaking)?” is obvious an uninteresting one. Nevertheless, neutrinos and SUSY just may have everything to do with one another; after all, nonzero masses of neutrinos may be a result of SUSY. The latter is related to the notion of R-parity violation — the topic discussed here.

The notion of R-parity came about also early in the history of SUSY. In those days, baryon and lepton number symmetries might look even better than the standard model (SM) itself. R-parity, being basically baryon and lepton number symmetries of a supersymmetric SM, seemed quite natural. However, global symmetries are understood to be less than sacred. The basic theoretical building block of the SM is the field spectrum and the gauge symmetries, while baryon and lepton number symmetries come out only as an accident. In fact, there are now strong evidence of nonzero neutrino masses. Moreover, such evidence is our only definite experimental indication of existence particle physics beyond the SM. On the contrary, SUSY itself still awaits discovery — that is, if it has anything to do with nature at all. Most, if not all, neutrino mass models actually violate lepton number symmetry. Of course if one simply adds (softly broken) SUSY to the basic building blocks of the SM, the (generic) supersymmetric SM thus obtained admits violations of baryon and lepton number symmetries and nonzero neutrino masses. There is the acute problem of superparticle mediated proton decay. But even in the consideration of the issue, R-parity certainly overkills. It is not the only candidate for the job, nor is it the most effective. It is the most restrictive though, in terms of what terms are admitted in the renormalizable Lagrangian and otherwise. May be the only advantage of R-parity is to provide a much simpler model for phenomenological analyses — the minimal supersymmetric SM (MSSM). However, the generic supersymmetric SM is, at least conceptually, the simplest model with SUSY and neutrino masses. Here, the latter can in fact be considered as a consequence of supersymmetrizing the SM. Hence, we take a simple phenomenological perspective here, taking the generic supersymmetric SM and study the experimental constraints on the various couplings with a priori bias. From the theoretical point of view, some sort of baryon number, in relation to proton decay, is expected
2. THE GENERIC SUPERSYMMETRIC STANDARD MODEL

The most general renormalizable superpotential for the generic supersymmetric SM (without R-parity) can be written as

\[
W = \varepsilon_{ab} \left[ \mu_a \hat{H}_u^a \hat{L}_a^b + h_v \hat{Q}_v^a \hat{H}_u^{ab} \hat{U}_k^c + X_{aij} \hat{L}_a^i \hat{Q}_j^b \hat{D}_k^c \right]
\]

\[
+ \frac{1}{2} \lambda_{abk} \hat{L}_a^i \hat{L}_b^j \hat{E}_k^c \left( + \frac{1}{2} \lambda'_{abk} \hat{U}_a^i \hat{U}_b^j \hat{D}_k^c \right),
\]

where \((a, b)\) are \(SU(2)\) indices, \((i, j, k)\) are the usual family (flavor) indices, and \((\alpha, \beta)\) are extended flavor index going from 0 to 3. In the limit where \(\lambda_{ij,k}, \lambda'_{ij,k}, X_{ijk}\) and \(\mu_i\) all vanish, one recovers the expression for the R-parity preserving case, with \(\hat{L}_0\) identified as \(\hat{H}_d\). Without R-parity imposed, the latter is not \emph{a priori} distinguishable from the \(\hat{L}_i\)'s. Note that \(\lambda\) is antisymmetric in the first two indices, as required by the \(SU(2)\) product rules, as shown explicitly here with \(\varepsilon_{12} = -\varepsilon_{21} = 1\). Similarly, \(\lambda'\) is antisymmetric in the last two indices, from \(SU(3)_C\).

R-parity is exactly an \emph{ad hoc} symmetry put in to make \(\hat{L}_0\) stand out from the other \(\hat{L}_i\)'s as the candidate for \(\hat{H}_d\). It is defined in terms of baryon number, lepton number, and spin as, explicitly, \(R = (-1)^{3B+L+2S}\). The consequence is that the accidental symmetries of baryon number and lepton number in the SM are preserved, at the expense of making particles and superparticles having a categorically different quantum number, R-parity. As mentioned above, R-parity hence kills the dangerous proton decay but also forbids neutrino masses within the model.

There are certainly no lack of studies on various “R-parity violating models” in the literature. However, such models typically involve strong assumptions on the form of R-parity violation. In most cases, no clear statement on what motivates the assumptions taken is explicitly given. In fact, there are quite some confusing, or even plainly wrong, statements on the issues concerned. It is important to distinguish among the different RPV “theories”, and, especially, between such a theory and the unique generic supersymmetric SM\(^3\). The latter is the \textit{complete} theory of SUSY without R-parity, one which admits all the RPV terms without \emph{a priori} bias.

R-parity violating (RPV) parameters come in various forms. These include the more popular trilinear \((\lambda_{ijk}, \lambda'_{ijk}, X_{ijk})\) and bilinear \((\mu_i)\) couplings in the superpotential, as well as soft SUSY breaking parameters of the trilinear, bilinear, and soft mass (mixing) types. From a phenomenological point of view, there is the related notion of (RPV) “neutrino VEV’s”. In order not to miss any plausible RPV phenomenological features, it is important that all of the RPV parameters be taken into consideration. For example, they all have a role to play in neutrino mass generations\(^4\). The soft SUSY breaking part of the Lagrangian is more interesting, if only for the fact that many of its interesting details have been overlooked in the literature. However, we will postpone the discussion for the moment, to the latter part of the article.

2.1. Supersymmetrizing the Standard Model

Let us review here the supersymmetrization of the SM. In the matter field sector, all fermions and scalars have to be promoted to chiral superfields containing both parts. It is straightforward for the quark doublets and singlets, and also for the leptonic singlet. The leptonic doublets, however, have the same quantum number as the Higgs doublet that couples to the down-sector quarks. Nevertheless, one cannot simply get the Higgs, \(H_d\), from the scalar partners of the leptonic doublets, \(L_i\). Holomorphicity of the superpotential requires a separate superfield to contribute the Higgs coupling to the up-sector quarks. This \(\hat{H}_u\) superfield then contributes a fermionic doublet, the Higgsino, with non-trivial gauge anomaly. To cancel the latter, an extra fermionic doublet with the quantum number of \(H_d\) or \(L\) is needed. So, the result is that we need four superfields with that quantum number. As they are \emph{a priori} indistinguishable, we label them by \(\hat{L}_\alpha\) \((\alpha = 0\) to \(3)\). With the superfield content and the SM gauge symmetries, we have the superpotential given above.
2.2. The Single-VEV Parametrization

After the supersymmetrization, however, some of the superfields lose the exact identities they have in relation to the physical particles. The latter has to be mass eigenstates, which have to be worked out from the Lagrangian of the model. Assuming electroweak symmetry breaking, we have now five (color-singlet) charged fermions, for example. There are also 1+4 VEV’s admitted, together with a SUSY breaking gaugino mass. If one writes down naively the (tree-level) mass matrices, the result is extremely complicated (see 6 for an explicit illustration), with all the \( \mu_i \) and \( \lambda_{\alpha\beta\epsilon} \) couplings involved, from which the only definite experimental data are the three physical lepton masses as the light eigenvalues, and the overall magnitude of the electroweak symmetry breaking VEV’s. The task of analyzing the model seems to be formidable.

Doing phenomenological studies without specifying a choice of flavor bases is, however, ambiguous. It is like doing SM quark physics with 18 complex Yukawa couplings, instead of the 10 real physical parameters. As far as the SM itself is concerned, the extra 26 real parameters are simply redundant, and attempts to relate the full 36 parameters to experimental data will be futile. In the case at hand, the choice of an optimal parametrization mainly concerns the 4 \( L_\alpha \) flavors. We use here the single-VEV parametrization (SVP), in which flavor bases are chosen such that: 1/ among the \( L_\alpha \)’s, only \( \hat{L}_0 \) bears a VEV, i.e. \( \langle \hat{L}_i \rangle \equiv 0 \); 2/ \( h^i_{jk} \equiv \lambda_{ijk} = \sqrt{2} \text{diag} \{ m_1, m_2, m_3 \} \); 3/ \( u^i_{jk} \equiv \lambda_{ijk} = \sqrt{2} \text{diag} \{ m_4, m_5, m_6 \} \); 4/ \( v^i_{jk} \equiv \sqrt{2} v^i_{G\text{KM}} \text{diag} \{ m_u, m_c, m_t \} \), where \( v_0 \equiv \sqrt{2} \langle \hat{L}_0 \rangle \) and \( v_u \equiv \sqrt{2} \langle \hat{H}_u \rangle \). Thus, the parametrization singles out the \( L_0 \) superfield as the one containing the Higgs. As a result, it gives the complete RPV effects on the tree-level mass matrices of all the states (scalars and fermions) the simplest structure. The latter is a strong technical advantage.

3. The (Color-singlet) Fermions

The SVP gives quark mass matrices exactly in the SM form. For the masses of the color-singlet fermions, all the RPV effects are paramatrized by the \( \mu_i \)’s only. For example, the five charged fermions \( \{ \text{gaugino} + \text{higgsino} + 3 \text{ charged leptons} \} \), we have

\[
\mathcal{M}_{\xi} = \begin{pmatrix} M_2 & \frac{g_2 v}{\sqrt{2}} & 0 & 0 & 0 \\ \frac{g_2 v}{\sqrt{2}} & \mu_0 & \mu_1 & \mu_2 & \mu_3 \\ 0 & \mu_1 & m_1 & 0 & 0 \\ 0 & 0 & m_2 & 0 & 0 \\ 0 & 0 & 0 & m_3 & 0 \end{pmatrix}.
\]

Moreover each \( \mu_i \) parameter here characterizes directly the RPV effect on the corresponding charged lepton \( (\ell_i = e, \mu, \tau) \). Hence, in the limit of small \( \mu_i \)’s (relative to \( M_2 \) and \( \mu_0 \)), the superfields \( \hat{L}_i \)’s and \( \hat{E}^{\mu}_{\tau} \)’s have small deviations from the \( \ell_i \) superfields, with \( m_1 \)’s being roughly the physical masses. In general, for any set of other parameter inputs, the \( m_i \)’s can still be determined, through a numerical procedure, to guarantee that the correct mass eigenvalues of \( m_e, m_\mu, \text{ and } m_\tau \) are obtained — an issue first addressed and solved in Ref. 8.

Under the SVP, neutral fermion (neutralino-neutrino) mass matrix has also RPV contributions from the three \( \mu_i \)’s only. The mass matrix can be written in the \( 3 + 4 \) block form

\[
\mathcal{M}_\nu = \begin{pmatrix} M & \xi^T \\ \xi & m_\nu^0 \end{pmatrix},
\]

where, at the tree-level,

\[
\mathcal{M} = \begin{pmatrix} M_1 & 0 & -g_1 v \mu_0 & -g_2 v \mu_0 \\ 0 & M_2 & -g_1 v \mu_0 & -g_2 v \mu_0 \\ -g_1 v \mu_0 & -g_2 v \mu_0 & -\mu_0 & 0 \\ 0 & 0 & -\mu_0 & 0 \end{pmatrix},
\]

\[
\xi = \begin{pmatrix} 0 & 0 & -\mu_1 & 0 \\ 0 & 0 & -\mu_2 & 0 \\ 0 & 0 & -\mu_3 & 0 \end{pmatrix},
\]

\[
m_\nu^0 = 0_{3 \times 3}.
\]

For small \( \mu_i \)’s, it has a “seesaw” type structure, with the effective neutrino mass matrix given by

\[
m_\nu = -\xi \mathcal{M}^{-1} \xi^T.
\]

with one non-zero mass eigenvalue given as

\[
\frac{1}{2 \mu_0} \left[ g_1 M_1 + g_2 M_2 \right] \mu_3^2 - \frac{v^2 \cos^2 \beta \left( g_1^2 M_1 + g_2^2 M_2 \right) \mu_3^2}{2 \mu_0 \left[ 2 M_1 M_2 \mu_0 - \left( g_1^2 M_1 + g_2^2 M_2 \right) v^2 \sin \beta \cos \beta \right]},
\]

where \( \mu_3^2 = \mu_1^2 + \mu_2^2 + \mu_3^2 \) and the corresponding eigenstate is an admixture of the three basis...
neutrino states of $m_\nu$ here exactly in proportion $\frac{\Delta m}{\mu}$. Note that at the limit of small $\mu_i$'s, the three neutrino states correspond to $\nu_e$, $\nu_\mu$, and $\nu_\tau$.

4. Some Phenomenological Implications

Taking the fermion mass matrices above and analyzing the resulted $Z^0$- and $W^\pm$- couplings of the physical states, an interesting list of tree-level RPV phenomenology from the gauge interactions can be exploited to get some constraints on the $\mu_i$ parameters. The topic is studied in detail in Ref.\[3\], which we summarized here in Table 1.

As for neutrino masses, apart from the tree-level contribution given above, there are the well-studied 1-loop contributions from the $X$- or $\lambda$-couplings, with interesting implications on the flavor structure of the classes of parameters\[9\]. There are also contributions involving a bilinear together with a trilinear parameter, as first pointed out in the study\[10\] of a SUSY version of the Zee model\[1\]. Such kind of contributions are closely related to RPV contributions to scalar mixings. The topic is much overlooked till our recent analysis\[12\].

The most interesting RPV contributions to scalar masses involve both the bilinear and trilinear parameters, coming into the $LR$-mixings part of the mass matrices. It is then very easy to see that they give rise to RPV contributions to electric dipole moments (EDM’s)\[13\], as well the important flavor changing processes such as $b \rightarrow s \gamma$\[14\] and $\mu \rightarrow e \gamma$\[17\]. In the discussion below, we will illustrate some of these interesting recent results.

5. SOFT TERMS AND SQUARKS

5.1. The Soft SUSY Breaking Terms

The soft SUSY breaking part of the Lagrangian can be written as

$$V_{\text{soft}} = \tilde{Q}^\dagger m_\tilde{Q} \tilde{Q} + \tilde{U}^\dagger m_\tilde{U} \tilde{U} + \tilde{D}^\dagger m_\tilde{D} \tilde{D} + \tilde{L}^\dagger m_\tilde{L} \tilde{L}$$

$$+ \tilde{L}^\dagger \mu \tilde{H}_d + \tilde{U} \tilde{D} - \frac{\mu_B}{2} \tilde{B} \tilde{B} + \frac{\mu_W}{2} \tilde{W}^\dagger \tilde{W}$$

$$+ \frac{\mu_0}{2} \tilde{g} \tilde{g} + \kappa_D \left( \mathcal{H}_1 \tilde{H}_d^{\dagger} \tilde{H}_d + \mathcal{H}_2 \tilde{H}_d \tilde{H}_d \right)$$

$$+ A_{ij} \tilde{Q}^i \tilde{H}_d^j \tilde{U}^j + A_{ij} \tilde{D}^i \tilde{H}_d^j \tilde{D}^j + A_{ijk} \tilde{L}^i \tilde{Q}^j \tilde{D}^k$$

$$+ \frac{1}{2} \lambda_{ijk} \tilde{L}^i \tilde{L}^j \tilde{E}^k + \frac{1}{2} A_{ijk} \tilde{U}^i \tilde{D}^j \tilde{D}^k + \text{h.c.} \right]$$

(6)

where we have separated the R-parity conserving $A$-terms from the RPV ones (recall $\tilde{H}_d = \tilde{L}$). Note that $\tilde{L}^1 \tilde{Q}^2 \tilde{D}$, unlike the other soft mass terms, is given by a $4 \times 4$ matrix. Explicitly, $\tilde{m}_{\tilde{L}}^2$ corresponds to $\tilde{m}_{\tilde{b}}^2$ of the MSSM case while $\tilde{m}_{\tilde{e}}^2$'s give RPV mass mixings. Going from here, it is straightforward to obtain the squark and slepton masses.

5.2. Down Squark Mixings

The SVP also simplifies much the otherwise extremely complicated expressions for the mass-squared matrices of the scalar sectors. Firstly, we will look at the squarks sectors. The masses of up-squarks obviously have no RPV contribution. The down-squark sector, however, has interesting result. We have the mass-squared matrix as follows:

$$M^2_{\tilde{u}} = \begin{pmatrix} M^2_{\tilde{u}_{LL}} & M^2_{\tilde{u}_{LR}} \\ M^2_{\tilde{u}_{RL}} & M^2_{\tilde{u}_{RR}} \end{pmatrix}$$

(7)

where

$$M^2_{\tilde{u}_{LL}} = \tilde{m}^2_{\tilde{Q}} + m_0 \tilde{m}_D + M^2_{\tilde{u}} \cos 2\beta \left[ -\frac{1}{2} + \frac{1}{3} \sin^2 \theta_W \right]$$

$$M^2_{\tilde{u}_{RR}} = \tilde{m}^2_{\tilde{Q}} + m_0 \tilde{m}_D + M^2_{\tilde{u}} \cos 2\beta \left[ -\frac{1}{3} \sin^2 \theta_W \right]$$

and

$$(M^2_{\tilde{u}_{LL}})^T = A^D \frac{\mathbf{\tilde{e}}}{\sqrt{2}} - (\mu^*_i X_{ijk}) \frac{\mathbf{\tilde{e}}}{\sqrt{2}}$$

$$= [A_d - \mu^*_i \tan \beta] m_0 + \frac{\sqrt{2} M_W \cos \beta}{g_2} \delta A^D$$

$$- \frac{\sqrt{2} M_W \sin \beta}{g_2} (\mu^*_i X_{ijk})$$

(9)

Here, $m_0$ is the down-quark mass matrix, which is diagonal under the parametrization adopted; $A_d$ is a constant (mass) parameter representing the “proportional” part of the $A$-term and the matrix $\delta A^D$ is the “proportionality” violating part; $\mu^*_i X_{ijk}$, and similarly $\mu^*_i X_{ijk}$, denotes the $3 \times 3$ matrix ($\mu^*_i X_{ijk}$) with elements listed. The $(\mu^*_i X_{ijk})$ term is the full $F$-term contribution, while the $(\mu^*_i X_{ijk})$ part separated out in the last expression gives the RPV contributions. It is important to note that the term contains flavor-changing ($j \neq k$) parts which, unlike the $A$-terms ones, cannot be suppressed through a flavor-blind
SUSY breaking spectrum; even for the diagonal part, the phase cannot be suppressed as that of the $A$-terms from gauge mediation. The novel issue here is that the RPV contributions come from supersymmetric, rather than SUSY breaking parameters.

6. CONTRIBUTIONS TO NEUTRON EDM

6.1. The Illustrative Gaugino Loop Contributions

It is familiar in SUSY phenomenology that diagonal $LR$-scalar mixings giving rise to EDM’s. For the $d$ quark EDM, we have then a direct contribution coming from a gaugino loop. The diagram looks the same as the MSSM gluino and neutralino diagram with two gauge coupling vertices. The new RPV contributions here is a simple result of the RPV $LR$ squark mixings [cf. Eq.(3)]. In Ref.[13], we focused on the illustrative gluino loop contribution; while the complete SUSY loop contributions to neutron EDM are analyzed in detail in Ref.[14], with a comprehensive exact numerical study. Notice that though both $u$ and $d$ quarks get EDM from gaugino loops in MSSM, only the $d$ quark has the RPV contribution. The $u$-squark sector simply has no RPV $LR$ mixings.

Neglecting inter-family mixings among the squarks, we have the gluino diagram contribution in the often quoted expression

$$\left( \frac{d_e}{e} \right) = -\frac{2\alpha_\chi}{3\pi} \frac{M_\chi}{M_2} Q_\chi \text{Im}(\delta^\alpha_a) F\left( \frac{M^2}{M_2^2} \right), \quad (10)$$

where $\delta^\alpha_a$ is $M^2_{\alpha h}/M^2_a$ (with $M^2_{\alpha h}$ restricted to the $\tilde{d}$ family) and

$$F(x) = \frac{1}{(1-x)^3} \left[ \frac{1+5x}{2} + \frac{(2+x)x \ln x}{(1-x)} \right]. \quad (11)$$

The expression above is, in fact, the same as that of the MSSM case, except that $\delta^\alpha_a$, or equivalently $M^2_{\alpha h}$, has now an extra RPV part. From the general result given in Eq.(14), we have for the $\tilde{d}$ squark,

$$\delta^\alpha_a M^2_d = [A_d - \mu^*_d \tan\beta] m_d + \frac{\sqrt{2} M_\nu \cos\beta}{g_2} \delta A^\alpha_{11},$$

where

$$-\frac{\sqrt{2} M_\nu \sin\beta}{g_2} (\mu^*_d X^i_{11}) . \quad (12)$$

Note that the $\mu^*_d X^i_{11}$ term does contain nontrivial CP violating phases and gives RPV contribution to $d$ quark EDM. Including inter-family mixings would complicate the mass eigenstate analysis but not modify the EDM result in any substantial way. We want to point out, without going into the details, that the analog of the type contribution to electron EDM, through a neutral (color-singlet) gaugino with a $\mu^*_d \lambda^{\prime}_i$ slepton mixing, is obvious.

If one naively imposes the constraint for this RPV contribution itself not to exceed the experimental bound on neutron EDM, one gets roughly $\text{Im}(\mu^*_d X^i_{11}) \lesssim 10^{-6}$ GeV, a constraint that is interesting even in comparison to the bounds on the corresponding parameters obtainable from asking no neutrino masses to exceed the super-Kamiokande (super-K) atmospheric oscillation scale[9]. In fact, the most stringently interpreted bounds on the individual parameters involved are given by

$$X^i_{11} \lesssim 0.05 \sim 0.1$$

and

$$\mu^*_d \cos\beta \lesssim 10^{-4} \text{ GeV},$$

while the $\mu^*_d$ bounds admitting a heavier neutrinos are much weaker, as shown in Table 1.

6.2. Fermion Mixings and EDM

Once we see the above discussed EDM contribution through $LR$-scalar mixing, it is quite natural to expect the same kind of RPV parameter combinations could contribute in other diagrams. In fact, there are other 1-loop contributions. In the case of MSSM, the chargino contribution is known to be competitive or even dominates over the gluino one in some regions of the parameter space[8]. The major part of the chargino contribution comes from a diagram with a gauge and a Yukawa coupling for the loop vertices, with pure $L$-squark running in the loop. Here we give the corresponding formula generalized to the case of SUSY without R-parity. This is given by[15]

$$\left( \frac{d_e}{e} \right)_{\chi^+} = -\frac{\alpha_m}{4\pi \sin^2\theta_W} \sum_{f' n = 1}^5 \text{Im}(\xi^a_{11}) \frac{M_{\chi^+}}{M_{\chi^+}}$$
for \( f \) being \( u \) (\( d \)) quark and \( f' \) being \( d \) (\( u \)), where

\[
C_{un\tau} = \frac{y_e}{g_2} V_{2u} D_{d1\tau} \cdot \left( U_{1n} D_{d1\tau}^* - \frac{\lambda_{ii}}{g_2} U_{(i+2)n} D_{d2\tau}^* \right),
\]

\[
C_{dn\tau} = \left( \frac{y_e}{g_2} U_{2n} + \frac{\lambda_{ii}}{g_2} U_{(i+2)n} \right) D_{u1\tau} \cdot \left( V_{1n} D_{u1\tau}^* - \frac{y_e}{g_2} V_{2n} D_{u2\tau}^* \right),
\]

(only repeated index \( i \) is to be summed)

(14)

and the loop integral function \( B(x) \) and \( A(x) \) given by

\[
B(x) = \frac{1}{2(x-1)^2} \left[ 1 + x + \frac{2x \ln x}{1-x} \right],
\]

\[
A(x) = \frac{1}{2(1-x)^2} \left[ 3 - x + \frac{2 \ln x}{1-x} \right].
\]

(15)

7. SLEPTON MASSES

AND PHENOMENOLOGY

7.1. The Charged Scalars

Things in the slepton sector are more complicated. We have eight charged scalar states, including an unphysical Goldstone mode, The \( 8 \times 8 \) mass-squared matrix of the following \( 1 + 4 + 3 \) form:

\[
\mathcal{M}_{\tilde{E}}^2 = \begin{pmatrix}
\tilde{m}_{\tilde{e}}^2 + M_{\tilde{e}}^2 \cos2\beta \left[ \frac{1}{2} - \sin^2\theta_W \right] & + M_{\tilde{e}}^2 \sin^2\beta \left[ 1 - \sin^2\theta_W \right] & , \\
+ M_{\tilde{e}}^2 \cos2\beta \left[ -\frac{1}{2} + \sin^2\theta_W \right] & + M_{\tilde{e}}^2 \cos2\beta \left[ 1 - \sin^2\theta_W \right] & , \\
+ M_{\tilde{e}}^2 \cos2\beta \left[ -\frac{1}{2} + \sin^2\theta_W \right] & + M_{\tilde{e}}^2 \cos2\beta \left[ 1 - \sin^2\theta_W \right] & , \\
& & \\
\end{pmatrix};
\]

(16)

where

\[
\tilde{m}_{\tilde{e}} = \tilde{m}_{\tilde{e}}^0 + \mu_{\tilde{e}} \mu_3 + M_{\tilde{e}}^2 \cos2\beta \left[ \frac{1}{2} - \sin^2\theta_W \right] + M_{\tilde{e}}^2 \sin^2\beta \left[ 1 - \sin^2\theta_W \right],
\]

\[
\tilde{M}_{\tilde{e}_{LL}} = \tilde{m}_{\tilde{e}} + m_{\tilde{e}} \mu_3 + (\mu_{\tilde{e}}^* \mu_3),
\]

\[
\tilde{M}_{\tilde{e}_{RR}} = \tilde{m}_{\tilde{e}} + m_{\tilde{e}} m_{\tilde{e}}^\dagger + M_{\tilde{e}}^2 \cos2\beta \left[ -\sin^2\theta_W \right],
\]

\[
\tilde{M}_{\tilde{e}_{HL}} = (B_{\tilde{e}}^*) + \left( \frac{1}{2} M_{\tilde{e}}^2 \sin2\beta \left[ 1 - \sin^2\theta_W \right] \right),
\]

\[
\tilde{M}_{\tilde{e}_{HR}} = - (\mu_{\tilde{e}}^* \lambda_{\tilde{e}k}) \frac{v_0}{\sqrt{2}} (\text{no sum over } k),
\]

\[
\tilde{M}_{\tilde{e}_{H}}^2 = \begin{pmatrix}
0 & (0 \lambda_{\tilde{e}k}^*) \frac{v_0}{\sqrt{2}} \\
(0 \lambda_{\tilde{e}k}^*) \frac{v_0}{\sqrt{2}} & (0 \lambda_{\tilde{e}k}^*) \frac{v_0}{\sqrt{2}} \\
\end{pmatrix};
\]

(17)

Notations and results here are similar to the squark case above, with some difference. We have \( A_\chi \) and \( \Delta A^\psi \), or the extended matrices \( (\tilde{e}^*) \) incorporating them, denote the splitting of the \( A \)-term, with proportionality defined with respect to \( m_{\tilde{e}} \); \( m_\chi = \text{diag}\{0, m_{\tilde{e}}\} = \text{diag}\{0, m_1, m_2, m_3\} \). Recall that the \( m_i \)'s are approximately the charged lepton masses.
A $4 \times 3$ matrix $(\mu_i^* \lambda_{ijk})$ gives the RPV contributions to $(\mathcal{M}_{iLL}^2)^T$ which is the LR-mixing part. In the above expression, we separate explicitly the first row of the former, which corresponds to mass-squared terms of the type $\hat{l}^i h_j^+$ type ($h_j^+ \equiv \tilde{l}_j^+$). This is the piece that gives rise to the Zee neutrino mass diagram within the present SUSY framework, in which the $R$-handed sleptons play the role of the Zee scalar [1]. The remaining $3 \times 3$ part given by $(\mu_i^* \lambda_{ij2})$ is the exact analog of the squark mixings discussed above. We have already pointed out the contributions to electron EDM from the RPV parameter combination $\mu_i^* \lambda_{i22}$ in analog to that of the $d$ quark.

Unlike the squarks, however, we have also the $\mu_i^* \mu_j$ flavor changing LL-mixing in $\mathcal{M}_{iLL}^2$. Among other things, this also contributes to $\mu \to e \gamma$ [17]. The nonzero $\mathcal{M}_{i22}^2$ and the $B_i^*$'s in $\mathcal{M}_{i2}^2$ are also RPV contributions. The former is a $\tilde{l}^i (h_j^+)^\dagger$ type, while the latter is a $\tilde{l}^i h_j^+$ term. Note that the parts with the $[1 - \sin^2 \theta_W]$ factor are singled out as they are extra contributions to the masses of the charged-Higgses (i.e. $\tilde{l}_i^\pm \equiv h_i^\pm$ and $h_i^0$). The latter is the result of the quartic terms in the scalar potential and the fact that the Higgs doublets bear VEV's. Such scalar mixings also play a role in contributing to EDM's. For example, that is a top loop contribution to $d$ quark EDM of the form depicted in Fig. 1 [4].

### 7.2. The Neutral Scalars

The neutral scalar mass terms, in terms of the $(1 + 4)$ complex scalar fields, $\phi_n$'s, can be written in two parts — a simple $(\mathcal{M}_{\phi\phi}^2)_{mn} \phi_m^\dagger \phi_n$ part, and a Majorana-like part in the form $\frac{1}{2} (\mathcal{M}_{\phi \phi}^2)_{mn} \phi_m \phi_n + \text{h.c.}$. As the neutral scalars are originated from chiral doublet superfields, the existence of the Majorana-like part is a direct consequence of the electroweak symmetry breaking VEV's, hence restricted to the scalars playing the Higgs role only. They come from the quartic terms of the Higgs fields in the scalar potential. We have explicitly

$$\mathcal{M}_{\phi\phi}^2 = \frac{1}{2} M^2_{\phi} \begin{pmatrix} \sin^2 \beta & - \cos \beta \sin \beta & 0_{1 \times 3} \\ - \cos \beta \sin \beta & \cos^2 \beta & 0_{1 \times 3} \\ 0_{1 \times 3} & 0_{1 \times 3} & 0_{1 \times 3} \end{pmatrix} ;$$

(18)

and

$$\mathcal{M}_{\phi \phi}^2 = (\bar{m}_\phi^2 + \mu^* \mu_n - \frac{1}{2} z) - (B_n) \tilde{m}_\phi^2 + (\mu^* \mu_3) + \frac{1}{2} z ^2,$$

(19)

with

$$z = M_2 \cos 2 \beta.$$

Note that $\mathcal{M}_{\phi \phi}^2$ here is real (see the appendix), while $\mathcal{M}_{\phi \phi}^2$ does have complex entries. The full $10 \times 10$ (real and symmetric) mass-squared matrix for the real scalars is then given by

$$\mathcal{M}_{\phi}^2 = \begin{pmatrix} \mathcal{M}_{\phi \phi}^2 & \mathcal{M}_{\phi \rho}^2 \\ (\mathcal{M}_{\phi \rho}^2)^T & \mathcal{M}_{\rho \rho}^2 \end{pmatrix},$$

(21)

where the scalar, pseudo-scalar, and mixing parts are

$$\mathcal{M}_{\phi \phi}^2 = \text{Re}(\mathcal{M}_{\phi \phi}^2),$$

$$\mathcal{M}_{\phi \rho}^2 = \text{Re}(\mathcal{M}_{\phi \rho}^2),$$

$$\mathcal{M}_{\rho \rho}^2 = -\text{Im}(\mathcal{M}_{\phi \rho}^2),$$

(22)

respectively. If $\text{Im}(\mathcal{M}_{\phi \phi}^2)$ vanishes, the scalars and pseudo-scalars decouple from one another and the unphysical Goldstone mode would be found among the latter.

The most interesting part of the neutral scalar masses involves the RPV parameters $B_i$'s and the corresponding mixing part in $\tilde{m}_\phi^2$. These parameters are not all independent, as discussed below in the next subsection. The $B_i$'s, for example, lead
to seesaw type Majorana-like mass terms for the “sneutrinos” [19], as depicted in Fig. 2. One of the interesting consequence here is a gauge loop contribution to neutrino masses [20], depicted in Fig. 3.

7.3. A Look at the Scalar Potential

We would like to emphasize that the above scalar mass results are complete — all RPV contributions, SUSY breaking or otherwise, are included. The simplicity of the result is a consequence of the SVP. Explicitly, there are no RPV A-term contributions due to the vanishing of VEV’s $v_i \equiv \sqrt{2} \langle \phi_i \rangle$. The Higgs-slepton results given as in Eqs. (16) and (21) contain redundancy of parameters and hide the unphysical Goldstone states. The latter can be easily identified, after implementation of the tadpole equations. The equations also identify important relations among dependent parameters.

In terms of the five, plausibly electroweak symmetry breaking, neutral scalars fields $\phi_n$, the generic (tree-level) scalar potential, as constrained by SUSY, can be written as:

$$V_s = Y_n |\phi_n|^4 + X_{mn} |\phi_m|^2 |\phi_n|^2 + \tilde{m}_n^2 |\phi_n|^2 - (\tilde{m}_\nu^2 e^{i\theta_\nu} \phi^\dagger_n \phi_n + \text{h.c.}) \quad (m < n).$$

Here, we count the $\phi_n$’s from $-1$ to $3$ and identify a $\phi_0$ (recall $\alpha = 0$ to $3$) as $\tilde{\phi}_0$ and $\phi_1$ as $\tilde{h}_1$. Parameters in the above expression for $V_s$ (all real) are then given by

$$\tilde{m}_\nu^2 = \tilde{m}_{\nu_{\alpha\alpha}}^2 + |\mu_\alpha|^2,$$

$$\tilde{m}_n^2 = \tilde{m}_{n_{\alpha\alpha}}^2 + \mu_\alpha^* \mu_\alpha,$$

$$\tilde{m}_{\nu_{\alpha\beta}} e^{i\theta_{\nu\beta}} = -\tilde{m}_{\nu_{\alpha\beta}}^2 - \mu_\alpha^* \mu_\beta \quad \text{(no sum)},$$

$$\tilde{m}_{n_{\alpha\beta}} e^{i\theta_{n\beta}} = B_\alpha \quad \text{(no sum)},$$

$$Y_n = \frac{1}{8} (g_1^2 + g_2^2),$$

$$X_{\nu_\alpha} = -\frac{1}{4} (g_1^2 + g_2^2) = -X_{\nu_{\alpha}}. \quad (24)$$

Under the SVP, we write the VEV’s as follows:

$$v_\alpha (\equiv \sqrt{2} \langle \phi_\alpha \rangle) = v_\alpha,$$

$$v_0 (\equiv \sqrt{2} \langle \phi_0 \rangle) = v_0 e^{i\theta_0},$$

$$v_i (\equiv \sqrt{2} \langle \phi_i \rangle) = 0, \quad (25)$$

where we have put a complex phase in the VEV $v_0$, for generality.

The equations from the vanishing derivatives of $V_s$ along $\phi_0$ and $\phi_0$ give

$$\left[ \frac{1}{8} (g_1^2 + g_2^2)(v_\alpha^2 - v_\beta^2) + \tilde{m}_{n_{\alpha\beta}}^2 \right] v_\alpha = B_0 \ v_0 \ e^{i\theta_0},$$

$$\left[ \frac{1}{8} (g_1^2 + g_2^2)(v_\alpha^2 - v_\beta^2) + \tilde{m}_{n_{\alpha\beta}}^2 \right] v_\beta = B_0 \ v_0 \ e^{i\theta_0}. \quad (26)$$

Hence, $B_0 e^{i\theta_0}$ is real. In fact, the part of $V_s$ that is relevant to obtaining the tadpole equations is no different from that of MSSM apart from the fact that $\tilde{m}_{n_{\alpha\beta}}^2$ and $\tilde{m}_{n_{\alpha\beta}}^2$ of the latter are replaced by $\tilde{m}_{n_{\alpha\beta}}^2$ and $\tilde{m}_{n_{\alpha\beta}}^2$ respectively. As in MSSM, the $B_0$ parameter can be taken as real. The conclusion here is therefore that $\theta_0$ vanishes, or all VEV’s are real, despite the existence of complex parameters in the scalar potential. Results from the other tadpole equations, in a $\phi_i$ direction, are quite simple. They can be written as complex equations of the form

$$\tilde{m}_{\nu_{\alpha i}} e^{i\theta_{\nu_{\alpha i}}} \tan \beta = -e^{i\theta_{\nu_{\alpha i}}} \tilde{m}_{\nu_{\alpha i}} e^{i\theta_{\nu_{\alpha i}}}, \quad (27)$$

which is equivalent to

$$B_i \ \tan \beta = -\tilde{m}_{\nu_{\alpha i}}^2 + \mu_{\alpha}^* \mu_i, \quad (28)$$
where we have used $v_u = v \sin \beta$ and $v_d = v \cos \beta$. Note that our $\tan \beta$ has the same physical meaning as that in the R-parity conserving case. For instance, $\tan \beta$, together with the corresponding Yukawa coupling ratio, gives the mass ratio between the top and the bottom quark.

The three complex equations for the $B_i$’s reflect the redundance of parameters in a generic $L_\alpha$ flavor basis. The equations also suggest that the $B_i$’s are expected to be suppressed, with respect to the R-parity conserving $B_0$, as the $\mu_i$’s are, with respect to $\mu_0$. They give consistence relationships among the involved RPV parameters (under the SVP) that should not be overlooked.

8. CONCLUDING REMARKS

The complete theory of SUSY without R-parity is just the generic supersymmetric SM. Giving up R-parity is particularly well-motivated by the expectation of Majorana neutrino masses. The model gives many interesting results, that would otherwise be missed in limited version of RPV models. It is important to have a consistent and efficient framework to deal with the various kinds of RPV parameters; a specific and optimal parametrization of the model is needed to match parameters unambiguously with experimental data. Our formulation (SVP) reviewed here provides such a parametrization. It simplifies all (tree-level) mass matrices very substantially, hence giving a strong technical advantage to phenomenological studies of the model. We have summarized some of our interesting new result here. Many other features of the model awaits careful analysis.

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Table 1
Summary of Phenomenological Constraints from Leptonic Z^0 and W^\pm Couplings.

| Quantity | \(\mu_i\), combo. constrained | Experimental bounds |
|----------|-------------------------------|---------------------|
| **Z^0-coupling:**             |                               |                     |
| \(Br(\mu^- \rightarrow e^- e^+ e^-)\) | \(|\mu_1\mu_2|\)             | < 1.0 \times 10^{-12} |
| \(Br(\tau^- \rightarrow e^- e^+ e^-)\) | \(|\mu_1\mu_3|\)             | < 2.9 \times 10^{-6}  |
| \(Br(\mu^- \rightarrow \mu^- e^+ e^-)\) | \(|\mu_2\mu_3|\)             | < 1.7 \times 10^{-6}  |
| \(Br(\tau^- \rightarrow \mu^- e^+ e^-)\) | \(|\mu_2^2\mu_3\mu_5|\)       | < 1.5 \times 10^{-6}  |
| \(Br(\mu^- \rightarrow e^- \mu^+ \mu^-)\) | \(|\mu_1\mu_3|\)             | < 1.8 \times 10^{-6}  |
| \(Br(\mu^- \rightarrow e^+ \mu^- \mu^-)\) | \(|\mu_1\mu_2\mu_3|\)       | < 1.5 \times 10^{-6}  |
| \(Br(\tau^- \rightarrow \mu^- \mu^+ \mu^-)\) | \(|\mu_2\mu_3|\)             | < 1.9 \times 10^{-6}  |
| \(Br(Z^0 \rightarrow e^+ e^-\mu^+ \mu^-)\) | \(|\mu_1\mu_2|\)             | < 1.7 \times 10^{-6}  |
| \(Br(Z^0 \rightarrow e^+ \tau^-\tau^+)\) | \(|\mu_1\mu_3|\)             | < 9.8 \times 10^{-6}  |
| \(Br(Z^0 \rightarrow \mu^+ \tau^-\tau^+)\) | \(|\mu_2\mu_3|\)             | < 1.2 \times 10^{-5}  |
| \(Br(Z^0 \rightarrow \chi^\pm \ell^\mp)\) | complicated               | < 1.0 \times 10^{-5}  |
| \(Br(Z^0 \rightarrow \chi^0 \chi_1^0, \chi_2^0 \nu)\); \(j \neq 1\) | \(\mu_5\)                  | < 1.0 \times 10^{-5}  |
| **W^\pm-coupling:**           |                               |                     |
| \(\Gamma^\mu_{e\nu}\) (e-\mu universality) | \(\mu_1^2 - \mu_2^2\)     | (0.596 \pm 4.37) \times 10^{-3} |
| \(\Gamma^\tau_{e\nu}\) (e-\tau universality) | \(\mu_1^2 - \mu_3^2\)     | (0.955 \pm 4.98) \times 10^{-3} |
| \(\Delta A_{\mu e}\) (e-\mu L-R asymmetry) | \((\mu_1^2 - \mu_2^2) +\text{Rt. contribution}\) | (0.346 \pm 2.54) \times 10^{-3} |
| \(\Delta A_{\tau e}\) (e-\tau L-R asymmetry) | \((\mu_1^2 - \mu_3^2) +\text{Rt. contribution}\) | 0.0043 \pm 0.104 |
| \(\Delta A_{\mu\tau}\) (e-\mu L-R asymmetry) | \((\mu_2^2 - \mu_3^2) +\text{Rt. contribution}\) | 0.082 \pm 0.25 |
| \(\Gamma_\ell\) (total Z^0-width) | \(\mu_5\)                  | 2.4948 \pm 0.075 GeV |
| \(\Gamma_{\tau e}\) (*) | \(\mu_5\)                  | 500.1 \pm 5.4 MeV |
| \(Br(Z^0 \rightarrow \chi^0_1, \chi^0_2, \chi^0_3 \nu); j \neq 1\) | \(\mu_5\)                  | < 1.0 \times 10^{-5}  |
| **mass constraints:**         |                               |                     |
| \(\nu_3\) mass                | \(\mu_3\)                  | < 18.2 MeV if \(\nu_3 = \nu_\tau\) |
| \(\mu_5\)                    |                              | < 149 MeV if \(\nu_3 \neq \nu_\tau\) |