On Hydrodynamical Description of Thermal Photons

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The WA98 collaboration in the CERN SPS has reported an excess of photons over those originating from the decays of final hadrons in the lead-lead collisions. These photons can originate either from primary interactions of partons from colliding nuclei or from secondary interactions among produced particles. Photons produced in the secondary interactions, often called thermal photons, can be calculated by using thermal production rates and equilibrium hydrodynamics for the evolution of the expanding matter. I will review the main features of hydrodynamical studies for the WA98 data. The data can be reproduced both with or without a phase transition to the QGP, but high initial temperature, over the values predicted for the phase transition temperature, is required by the data. I will also show a prediction for the photon excess for central gold-gold collisions at the Brookhaven RHIC collider. In this prediction, the initial state for the hydrodynamical expansion is obtained from a perturbative QCD calculation.

1. Introduction and Theoretical Framework

Photons are suggested to be an ideal probe for the hot initial state of the relativistic heavy ion collisions, since, due to the large mean free path in a hadronic matter, they escape from the collision zone right after being emitted without any further interactions. Unfortunately, photons emitted at early stages of the collision are difficult to obtain experimentally, since the decays of the final state hadrons, especially neutral pions and etas, dominate the production of photons in the collision. So far only the WA98 collaboration at the CERN SPS has reported an excess of photons over those originating from decays of the final state hadrons [1, 2]. Photons are measured also by the PHENIX collaboration at Brookhaven RHIC collider, but more statistics is needed to reach conclusions [3].

In a hydrodynamical description of relativistic heavy ion collision the secondary interactions are assumed to lead to thermalization of the fireball. With this assumption, the evolution of the system can be described by equilibrium hydrodynamics and transverse momentum spectrum of the thermal photons emitted during the expansion can be calculated if thermal emission rate in the matter is known. Experimentally measured direct photon spectrum has a contribution from primary interactions as well [4], referred as pQCD (or prompt) photons in following.

Several groups [5, 6, 7, 8, 9] have performed hydrodynamical studies to explain WA98 data. The authors of [5] use a (2+1)-dimensional hydrodynamical code that takes into account also the finite longitudinal size of the initial collision zone, whereas the studies in [6, 7] have performed full 3D calculations.
are based on the boost-invariant approximation. In the boost-invariant scenario very hot initial state [5,6], considerable initial radial velocity [7,8,9] or modification of hadron masses [9] is needed to explain the WA98 data. I have made a choice to omit discussion on modifications of hadron masses [9] and also chosen references [6,7] to represent boost-invariant studies in this note.

Important improvements have been obtained in a perturbative calculation of the thermal emission rate of photons from the thermalized QGP [10,11,12]. In hadron gas mesonic processes $\pi\pi \rightarrow \rho\gamma$ and $\pi\rho \rightarrow \rho\gamma$, described with a pseudo-vector Lagrangian, are included in the photon emission rate [13]. The $\pi\rho$ scattering channel gets a large contribution from the interaction with the $a_1$ axial meson: $\pi\rho \rightarrow a_1 \rightarrow \rho\gamma$ [14]. Parametrizations of the emission rates are provided by the authors of [14,15]. The effects of baryons are not included in the rates.

2. Details of Hydrodynamics and Results

In section 2.1 results of three different groups [6,7,9] are compared to the WA98 data [1,2]. Groups [6,7] use 2-loop photon emission rates in QGP [16,17] while in [5] resummed rate [12], complete in order $\alpha_s$, is used. In works [5,6,8] contribution of the pQCD photons is taken from [18]. The authors of [5,6] present their own estimate for the pQCD photons.

In section 2.2 a prediction for direct photon spectrum in Au+Au collisions with $\sqrt{s} = 200$ AGeV is given. Results for hadronic observables in this study, where the initial state is based on perturbative QCD + saturation model, can be found in [19,20]. For this energy, hard pQCD photons from the primary interactions are studied in [21].

The following discussion will concentrate on the role of the initial state in hydrodynamical models, since especially high transverse momentum $k_t$ part of the thermal photon spectrum is sensitive to the initial conditions. In particular, temperature $T$ in the initial state plays a big role due to the Boltzmann suppression $e^{-k_t/T}$ in the emission rates.

2.1. Comparison with the WA98 Data

In boost-invariant hydrodynamical studies one must give the initial energy density $\epsilon(\mathbf{r},\tau_0)$ and velocity $v(\mathbf{r},\tau_0)$ distributions in the transverse plane at fixed formation (thermalization) time $\tau_0$, that are the initial conditions for the hydrodynamical evolution. Let us start with a case $v(\mathbf{r},\tau_0) \equiv 0$ for all $\mathbf{r}$. For the shape of the initial energy density $\epsilon(\mathbf{r},\tau_0)$ various choices can be found in the literature. For example, it can be proportional to wounded nucleon distribution [8], Woods-Saxon distribution [8] or nuclear overlap function [14]. Within these choices, the formation time $\tau_0$ remains as the main uncertainty in the boost-invariant picture. At SPS energies, $\tau_0$ is expected to be $\sim 1$ fm/c based on the geometrical argument, that it takes a time $\sim 2R_A/\gamma$, where $R_A$ is the nuclear radius and $\gamma$ the Lorentz gamma-factor, for the colliding nuclei to pass through each other. Fixing $\tau_0 = 1$ fm/c leaves only the normalization of the initial energy density as a free parameter, that can be fixed from experimentally measured hadron spectra.

Studies by the authors of [4] reveal a problem in this simple approach. If initial time is chosen to be of the order 1 fm/c, the initial state is way too cool to reproduce high transverse momentum, $k_t$, part of the spectrum as measured by the WA98 collaboration.
Figure 1. Direct photon production in ref. [6] with the WA98 data. QM refers to emission in QGP and QGP part of mixed phase, HM likewise for hadronic matter. $T_c$ is a critical temperature in EoS, $\tau_0$ thermalization time and $T_0$ average initial temperature.

Since decreasing of initial time considerably raises the temperature in the initial state[4] and final state hadron spectra are fairly insensitive to small changes of $\tau_0$, the data seems to suggest very rapid thermalization and hot initial state within the boost-invariant scenario.

Figure 1 shows the main results in [6]. The data is well reproduced with initial time $\tau_0 = 0.2$ fm/c which corresponds to an average initial temperature $\langle T \rangle_{\text{ini}} = 335$ MeV in their choice of the initial state. This temperature is considerably higher than the expected critical temperature $T_c = 150 - 180$ MeV from recent lattice calculations [22] and hence the authors of [6] conclude that this confirms the formation of the QGP. However, in my opinion, one should consider the initial time $\tau_0 = 0.2$ fm/c carefully, although the WA98 data seem to favor it. The geometrical argument above, on the transit time of the nuclei, suggests that at such a short initial time primary collisions are still taking place, and only part of the matter has been produced [23].

Let us turn to discuss effects from non-zero initial radial velocity. The authors of [7] have studied the effects of radially increasing initial velocity $v(r, \tau_0) = (r/R_A) \Theta(R - r)V_{\text{max}}$, where $V_{\text{max}}$ is treated as a free parameter. They have performed a simultaneous $\chi^2$ analysis of $\pi^0$ and $\gamma$ spectra measured by the WA98 collaboration [1] to find the best combination of the thermalization time $\tau_0$ and the velocity parameter $V_{\text{max}}$. Their results [7] with a phase transition to QGP are presented in figure 2. Curves in the figure correspond to the choices of the initial parameters presented in the table [1]. The results show clear interplay between the initial temperature and the strength of the initial radial velocity: thermal photons with large $k_t$ are emitted at first moments of the evolution when the temperature is high. Lowering of initial temperature, or equivalently increasing the thermalization time $\tau_0$, requires an introduction of initial radial velocity, which boosts the

\[ \text{If radial expansion is omitted, } T(r, \tau) = T(r, \tau_0) \times (\tau/\tau_0)^{-1/3} \text{ for massless ideal gas.} \]
emitted photons to larger $k_t$. With this compensation, initial temperature can be lowered below 200 MeV and the thermalization time rises to values $\tau_0 > 1$ fm/c. These values of the temperature are quite close to the critical temperature and hence the authors of [7] conclude, that, considering uncertainties of the model, formation of QGP or its absence cannot be distinguished.

The origin and the magnitude of the initial radial velocity is a somewhat uncertain. The authors of [7] explain its origin “as a result of significant radial energy gradient coming from the shapes of the colliding nuclei”. They do not present a (quantitative) model that could explain how this leads to a collective motion into the radial direction at initial time $\tau_0$. In this sense, I would consider $V_{\max}$ to be an extra fitting parameter in the model. In any case, initial radial velocity can not be large. The authors of [5] have made a boost-invariant calculation with initial radial velocity profile taken from [7]. They found a good agreement with the photon data by choosing initial parameters $\tau_0 = 1.0$ fm/c and $V_{\max} = 0.30$, but the resulting $\pi^0$ spectrum clearly overshoots the experimental one. It should be noted that the initial energy density profile — normalization is fixed from multiplicity in both cases — and the equation of state are different in these two studies, and in [5] chemical and kinetic freeze-out temperatures are the same. These differences may explain the discrepancy on the $\pi^0$ spectrum.

The authors of [5] used a (2+1)-dimensional hydrodynamical code, where also the longitudinal extend of the fireball is finite and the longitudinal velocity evolves dynamically from a given initial state. Details of this model can be found in references [24, 25, 26]. My first remark is, that there is no uniquely defined initial time, when the assumption of boost-invariant flow is relaxed. In the boost-invariant case the source is pointlike in the $zt$-plane and one can trace all the particles back to the collision point. This cannot be done when the longitudinal flow profile of the system is not that of the scaling flow, $v_z = z/t$. To estimate the thermalization time one can relate a timescale to the longitudinal extent by considering the collision geometry. In the model [25] the initial length of the system was chosen to be 3.4 fm. Since $\gamma \sim 10$ at SPS energies, the system is roughly 1.3 fm thick when the colliding nuclei overlap and it would take $\sim 1$ fm/c for the system to reach longitudinal extent of 3.4 fm [27]. The rapidity distribution of initial energy density is assumed to be Gaussian. The energy per unit transverse area, $e(r) = \int dz T^{00}(r, z)$, is assumed to equal the energy per unit transverse area of the incoming nucleons. To convert this distribution to spatial energy density, one must define initial velocity profiles. Initial conditions, chosen to reproduce the observed hadron spectra, depend also on the equation of state (EoS). In [5] EoS H describes a hadron resonance gas without phase
transition and EoS A has a first order phase transition to QGP. Initial radial velocity is set to zero. For the longitudinal velocity profile two different choices were studied [26]; in the case called IS 1 the longitudinal flow rapidity is assumed to increase linearly with the distance $z$. This gives a large peak, by a factor of two over the Bjorken estimate, to the initial energy density. In the other choice, IS 2, the $z$ dependence is nonlinear and the resulting energy density profile is almost flat.

Combinations IS 1 + EoS A and IS 2 + EoS H reproduce simultaneously the NA49 data for the rapidity distributions of hadrons [23], and the WA98 data for neutral pions and direct photons [2]. Figure 3 shows the results for measured rapidity density of negative charged hadrons, comparison to $\pi^0$ spectrum can be found in [5] and figure 4 shows the results for direct photons. Maximum and average temperatures in $z = 0$ in the initial state are given in the table 2.

|               | IS 1 + EoS A | IS 2 + EoS H |
|---------------|--------------|--------------|
| $T_{\text{max}}$ [MeV] | 325          | 245          |
| $\langle T \rangle_{\text{ini}}$ in $z = 0$ [MeV] | 255          | 213          |

At the given temperature, the thermal emission rate of photons is larger in hadronic matter, which makes IS 2 with lower initial temperature more suitable for purely hadronic scenario. Choosing IS 1 + EoS A leads to a high initial temperature, but gives a good
agreement also with the high $k_t$ part of the measured photon spectrum, as shown in the figure 4. Despite the $\sim 50$ MeV difference in the initial temperature, later evolution is so similar in these two choices for the initial state, that hadronic spectra are equally well reproduced. These observations reflect the ambiguity in the choice of the initial state.

After pQCD photon spectrum from the ref. [18] is added to the thermal one (upper set of curves, scaled by a factor of 100, in the figure 4), the authors of [5] find that the WA98 data can be reproduced both with or without a phase transition to QGP. However, the maximum (average) temperature 245 (213) MeV in the initial state with hadron gas is fairly high, and clearly over the expected phase transition temperature 150–180 MeV [22]. Also the shape of the solid curve in the figure 4, which corresponds to the results from IS 1 + EoS A with the phase transition, seems to follow the data better.

### 2.2. Thermal Photons at RHIC Collider

References [19, 20] show results for hadronic observables from boost-invariant hydrodynamics, when the initial state is obtained from a pQCD + saturation model. With this model for the initial state, only free parameter in the hydrodynamical description is the decoupling temperature. The initial time and energy density, including normalization, can be calculated from the model when the center of mass energy of the collision, $\sqrt{s}$, mass number of the colliding nuclei, $A$, and the experimental centrality cut are given. Construction of the initial state is explained in [19], where also results for global observables (e.g. multiplicity) are shown. In [20] the effect of the decoupling temperature is discussed and results are compared to hadron spectra measured by the PHENIX collaboration [23, 30]. Following results for thermal photons are calculated with initial parameters $\sqrt{s} = 200$ AGeV and $A = 197$, which lead to $\tau_0 = 0.17$ fm/c, $\langle E_T \rangle = 86.33$ mbGeV and $A_{eff} = 178$ for a 6 % most central collisions. For details see [19].

The initial time is determined from the saturation momentum scale, $p_{sat}$, of the perturbative minijet calculation [31]. This leads to a very short initial time $\tau_0 = 1/p_{sat} = 0.17$ fm/c compared to values $\gtrsim 0.6$ fm/c, that are used in many recent hydrodynamical studies at RHIC energies, for a review see e.g. [32]. The short initial time $\tau_0 = 0.17$ fm/c is consistent with the geometrical argument presented in section 2.1. The produced minijet system also looks thermal from the point of view of the number of the partons and the energy per particle at the initial time [31]. Starting hydrodynamics at $\tau_0 = 0.17$ fm/c can be considered to give an upper limit for the initial temperature and the thermal photon production.

The solid line in the figure 8 shows the transverse momentum spectrum of thermal photons. The dashed line shows contribution from quark-gluon plasma and QGP part of the mixed phase, and the dotted line likewise from hadron gas. The plasma contribution dominates the thermal emission for $k_t > 3$ GeV. Comparing results from SPS and RHIC energies one can see roughly a factor of 100 increase in thermal emission at $k_t = 4$ GeV, but the difference is less than order of magnitude at $k_t = 1$ GeV, which reflects a clear increase of the maximum temperature.

To analyze the effect from initial temperature (thermalization time) I have scaled the initial state from $\tau_0 = 0.17$ to 1.0 fm/c with longitudinal Bjorken expansion so that entropy in unit rapidity does not change. Neglecting transverse expansion before $\tau = 1.0$ fm/c is not physical, but in this way one can define a cooler initial state in such a way
that the results for hadronic observables do not change much. Table 3 shows the changes in the maximum and average values of the initial temperature and energy density. Effect on the thermal photon spectrum is presented in the figure 6.

Increasing initial time from $\tau_0 = 0.17$ to 1.0 fm/c reduces the thermal emission by a factor of 20 at $k_T = 4$ GeV. Unfortunately, the amount of thermal photons in this region may be hard to resolve experimentally, because pQCD photons from primary interactions may dominate the direct photon spectrum there. The analysis of the pQCD photons can be found in [21]. In [21] the centrality cut is 10% instead of 6, so the pQCD results presented in the figure 6 are multiplied with a factor of 1.1 [33]. Results for two different values of the intrinsic transverse momentum, $\langle p_T^2 \rangle = 0$ and 2.4 GeV$^2$, are given.

These theoretical results on excess photons at RHIC can be summarized as an upper limit given by the sum of the solid and dashed lines and a lower limit given by the sum of the dotted and dash-dotted lines. Uncertainties are fairly large, but in any case one should see a change in the slope in the region $k_T \sim 2.0 - 3.5$ GeV, above which the pQCD photons start to dominate the spectrum. In principle a very clean data around $k_T \sim 2$ GeV could fix the value of the initial time, but one should keep in mind the possible effects from initial transverse velocity and longitudinal geometry discussed in the section.

Figure 5. Thermal photon spectrum in Au+Au collision with $\sqrt{s} = 200$ AGeV. Plasma refers to the contribution from QGP and QGP part of the mixed phase. Also hadron gas includes emission from hadronic part of the mixed phase.

Figure 6. Effect of initial time $\tau_0$ to the thermal photon emission. Spectra of pQCD photons, with two different values for intrinsic transverse momentum $\langle p_T^2 \rangle$, are from work by A. Dumitru et al. [4, 21]. Initial temperatures are in the table 3.
Table 3
Maximum and average initial temperatures and energy densities in Au+Au collisions with √s = 200 AGeV.

|                  | T_{max} [MeV] | ⟨T⟩_{ini} [MeV] | ɛ_{max} [GeV/fm^3] | ⟨ɛ⟩_{ini} [GeV/fm^3] |
|------------------|----------------|------------------|---------------------|----------------------|
| τ_0 = 0.17 fm/c  | 580            | 340              | 208                 | 60                   |
| τ_0 = 1.0 fm/c   | 320            | 220              | 20                  | 6.8                  |

These effects may be smaller at RHIC energies, because rapid thermalization means shorter time interval to build up the initial transverse velocity. Also the rapidity range is broader at RHIC, and hence scaling hydrodynamics should work better.

3. Conclusions

I have reviewed hydrodynamical studies to explain the WA98 excess photon spectrum. The role of the initial state and the uncertainty in the values of initial temperature were emphasized in the discussion. The data can be explained with or without QGP phase transition, but high initial temperatures, well above the predicted critical temperature, are favored. In the case without boost-invariance the shape of the spectrum is closer to the data, when a phase transition is assumed. Still, both theoretical and experimental uncertainties are somewhat too large to rule out either alternative.

Using boost-invariant hydrodynamics with an initial state calculated from a pQCD + saturation model, theoretical estimates of the upper and lower limits for the thermal photons were given and compared with the results for the pQCD photons [3, 21].

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