Abstract

A careful investigation of $1/m_b$ power suppressed effects is crucial for enhancing predictive power of the QCD factorization approach to charmless B decays. It is instructive to systematically investigate the $1/m_b$ effects from soft and hard gluon exchanges, in addition to annihilation topology in the charmless B decays. In this work we try to give a systematic discussion on impact of such soft and hard exchanges on the penguin-dominated $B \rightarrow K\pi$ decays within the framework of the light-cone QCD sum rules (LCSR). For the weak phase $\gamma(= \text{Im}V_{ub}^*)$ ranging from 40$^\circ$ to 80$^\circ$, we find that because of the annihilation and soft and hard exchanges a numerical increase of $(20 - 30)\%$ is expected in the branching ratios up to $O(1/m_b^2)$ corrections; the resultant soft and hard corrections are less important than the annihilation contributions and amount only to a level of 10%. Possible sources of uncertainties are discussed in some details.

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I. INTRODUCTION

A considerable progress has been achieved in theoretical study of $B$ decays since the naive factorization ansatz [1] was proposed for the non-leptonic decays of heavy-mesons. With the QCD background one makes all effort to approach the physics of $B$ decays, by developing various theoretical frameworks such as large energy effective theory (LEET)[2], three-scale perturbative QCD factorization theorem [3], generalized factorization [4], QCD factorization [5] and soft-collinear effective theory (SCET) [6, 7, 8]. Especially, more attentions are paid to the QCD factorization and SCET. Both of them are formulated from the first principle of QCD and provide a rigorous theoretical basis to the factorization for a large class of non-leptonic $B$ decays. In comparison, the QCD factorization approach turns out to be much simpler in structure and more intuitive in physical picture, while SCET possesses stronger predictive powers and provides a more complete theoretical framework, in which, for instance, an elegant proof of the factorization for $B$ decays into two light mesons and into $D\pi$ is given in Ref.[7] and some relations can be established among different decay modes [8].

The QCD factorization formula has been proved to be a substantial improvement of the naive factorization assumption. It allows us to compute radiative corrections to factorizable amplitudes to all orders in the heavy quark limit $m_b \to \infty$, meanwhile leaving the $1/m_b$ power-suppressed corrections to be estimated with help of concrete models. The reasoning behind this theory is that as one works in the heavy quark limit, the soft gluons with momentum of order $\Lambda_{QCD}$ decouple and so the interaction kernel responsible for the transitions can be calculated with the perturbative QCD (PQCD) in the case of the $B$ decays into two light mesons. To be specific, the hadronic matrix elements for, say, $B \to \pi\pi$ can be expressed as,

$$\langle \pi\pi| O_i |B \rangle = \langle \pi| j_1 |B \rangle \langle \pi| j_2 |0 \rangle \left[ 1 + \sum r_n \alpha_s^n + O\left(\frac{\Lambda_{QCD}}{m_b}\right) \right],$$

where $O_i$ are the concerned local four-quark operators in the weak effective Hamiltonian, $j_{1,2}$ are the bilinear quark currents and the other two terms correspond to the perturbative corrections and non-perturbative contributions respectively. Nevertheless, an existing problem in the approach is that while the power corrections in $\alpha_s$ can be calculated in a systematical way, the $1/m_b$ power-suppressed effects cannot. Thus an accurate theoretical prediction on nonleptonic $B$ decays still is a challenge.

In the QCD factorization, there are a variety of sources of power-suppressed effects. Among them the annihilation topology, soft and hard exchanges and final state interaction ($FSI$) effects etc. are the main ones. The recent work of Mantry, Pirjol and Stewart [8] indicates that there exists an additional source of power suppressed contributions. It is obvious that including all the power-suppressed effects is so far almost impossible in any practical computation. But it is plausible to make an order of magnitude estimate of the overall power-suppressed effects
by calculating the effects due to annihilation topology and soft and hard-gluon exchanges. To this end, a better understanding or at least a reliable estimation of the order of magnitude for both effects is crucial to enhance the predictive power of the QCD factorization. Based on PQCD [9], contributions of annihilation to $B \to \pi \pi, K \pi$ have been estimated, showing a subleading behavior in $1/m_b$. Soft effects can be understood as processes where a background field gluon is exchanged between, for instance, an emitted meson and the other meson which picks up the spectator quark in the case of emission topology. Therefore, they can be viewed as a higher-twist effect. Power suppressed hard effects originate from hard gluon absorption by a spectator quark, which is only relevant to the penguin contractions of effective operators and the chromomagnetic dipole operator. There have already been some earlier attempts [10] to understand such soft effects, and recently a systematic discussion is given by Khodjamiriani [11]. In Ref.[11] the author suggests using the QCD light-cone sum rule(LCSR), which is originally developed in Ref.[12], to evaluate the non-factorizable corrections to B decays into two light mesons. Using the generalized LCSR technique [11, 13, 14] non-factorizable soft corrections to $B \to \pi \pi$ have been discussed in details. It is found that despite its numerical smallness, the $\mathcal{O}(1/m_b)$ soft effect is at the same level as the corresponding $\mathcal{O}(\alpha_s)$ corrections to the factorizable amplitudes. This implies that such soft effects are indispensable for an accurate evaluation of nonfactorizable contributions to charmless B decays.

Recently, the CLEO-II and-III, Belle and BaBar Collaborations reported their data on the branching ratios of the $B \to K \pi$ decays [15]. The experimental averages are [16]

$$B(B^- \to \bar{K}^0 \pi^-) = (21.8 \pm 1.4) \times 10^{-6},$$
$$B(\bar{B}^0 \to K^- \pi^+) = (18.2 \pm 0.8) \times 10^{-6},$$
$$B(B^- \to K^- \pi^0) = (12.5 \pm 1.0) \times 10^{-6},$$
$$B(\bar{B}^0 \to \bar{K}^0 \pi^0) = (11.7 \pm 1.4) \times 10^{-6}.$$  

Contrast to $B \to \pi \pi$ decays, these decay modes are penguin-dominated, for the tree contributions are CKM suppressed. As well known, all new physics effects can only manifest themselves via loops, so that these processes deserve a detailed investigation in search for new physics. In this work we will make an evaluation of the power-suppressed effects in the $B \to K \pi$ decays by investigating all the possible soft-gluon effects, as well as the power-suppressed hard effects associated with the penguin topology and chromomagnetic dipole operator in the framework of the LCSR.

Our paper is organized as follows. In the following section, we present a systematic LCSR analysis of the soft contributions to $B \to K \pi$. The calculation is performed to $\mathcal{O}(1/m_b)$. In particular, the soft effects stemming from emission topology are investigated at some length. Also, the special role of the penguin topology and chromomagnetic dipole operator is discussed.
They contribute, in addition to a soft effect, a $1/m_b$-suppressed hard effect which is absent in the QCD factorization. We estimate both of them with help of the LCSR results for $B \to \pi\pi$ [14]. In section III, we give the formula for calculating the branching ratios at subleading orders in $\alpha_s$ and $1/m_b$, including all the estimated power-suppressed corrections. In Sec.IV, we present our numerical discussions along with all the necessary input parameters, and a somewhat detailed analysis on the main sources of uncertainties, which influence our numerical results more or less. The last section is devoted to a brief summary.

II. LCSR FOR SOFT-GLUON EFFECTS IN $B \to K\pi$

We begin with the weak effective Hamiltonian $H_{\text{eff}}$ for the $\Delta B = 1$ transition as [17]

$$H_{\text{eff}} = \frac{G_F}{\sqrt{2}} \sum_{p = u, c} \lambda_p (C_1(\mu)O_1(\mu)^p + C_2(\mu)O_2(\mu)^p + \sum_{i=3}^{10} C_i(\mu)O_i(\mu) + C_{8g}(\mu)O_{8g}(\mu)) + h.c., \quad (3)$$

where $\lambda_p = V_{pb}V_{pq}^*(q = d, s)$, $C_i(\mu)$ and $C_{8g}(\mu)$ denote the Wilson coefficients, $O_{1,2}$, $O_i(i = 3 - 6)$ ($O_i(i = 7 - 10)$) and $O_{8g}$ are the tree, QCD (electroweak) penguin and chromomagnetic dipole operators, respectively. For a completeness, we list the relevant effective operators below

$$O_1^p = (\bar{p}b)_{V-A}(\bar{s}p)_{V-A},$$

$$O_2^p = (\bar{p}_\alpha b_\beta)_{V-A}(\bar{s}_\beta p_\alpha)_{V-A},$$

$$O_3 = (\bar{s}b)_{V-A} \sum_q (\bar{q}q)_{V-A},$$

$$O_4 = (\bar{s}_\alpha b_\beta)_{V-A} \sum_q (\bar{q}_\beta q_\alpha)_{V-A},$$

$$O_5 = (\bar{s}b)_{V-A} \sum_q (\bar{q}q)_{V+A},$$

$$O_6 = (\bar{s}_\alpha b_\beta)_{V-A} \sum_q (\bar{q}_\beta q_\alpha)_{V+A},$$

$$O_7 = (\bar{s}b)_{V-A} \sum_q \frac{3}{2} e_q (\bar{q}q)_{V+A},$$

$$O_8 = (\bar{s}_\alpha b_\beta)_{V-A} \sum_q \frac{3}{2} e_q (\bar{q}_\beta q_\alpha)_{V+A},$$

$$O_9 = (\bar{s}b)_{V-A} \sum_q \frac{3}{2} e_q (\bar{q}q)_{V-A},$$

$$O_{10} = (\bar{s}_\alpha b_\beta)_{V-A} \sum_q \frac{3}{2} e_q (\bar{q}_\beta q_\alpha)_{V-A},$$

$$O_{8g} = \frac{-g_s}{8\pi^2} m_b \bar{s}a^{\mu\nu}(1 + \gamma_5)G_{\mu\nu}b, \quad (4)$$

where $\alpha$ and $\beta$ are the color indices, $q$ runs over $u, d, s, c$ and $b$, $G_{\mu\nu} \equiv G^a_{\mu\nu} \frac{\lambda^a}{2}$ is the gluon field strength.
To estimate the soft-exchange effects from emission topology in the $B \to K\pi$ decays, one would decompose the relevant 4-quark effective operators into a color singlet part and a color octet one. Then the soft corrections to the matrix elements of the color octet operators can be estimated by studying the interactions between an emitted quark-antiquark pair and a background field gluon. In the first place, we concentrate ourselves on the case of $\bar{B}^0 \to K^-\pi^+$ and calculate such effects in terms of the LCSR method. The operators, which may induce soft-exchange effects, contain the tree operator $O_2$, QCD penguin operators $O_3, 5$, and EW penguin operators $O_{7,9}$. The color-octet operators which we encounter in this work, are of two types of structure: $(V - A)(V - A)$ and $(S + P)(S - P)$. The former case has been taken into account for $B \to \pi\pi$ in Ref. [11, 13]. Here we would like to provide a detailed derivation of the soft contributions arising from this type of operators in the $\bar{B}^0 \to K^-\pi^+$ case. To be definite, we are going to calculate the soft correction to the matrix element of the operator

$$\mathcal{O} = (\bar{u}^a \lambda^a V^- A)(s^a \lambda^a u) V^- A.$$  

(5)

Given that the $K^-$ meson is produced as an emitted hadron, the vacuum-pion correlation function, as the beginning point of the LCSR calculation, is written as

$$F_\alpha(p, q, k) = -\int d^4x e^{-i(p-q)\cdot x} \int d^4y e^{i(p-k)\cdot y} \langle 0 | T\{ j^{(K)}_a(y) \mathcal{O}(0) j^{B}_5(x) \} | \pi(q) \rangle,$$  

(6)

where $j^{(K)}_a = \bar{u} \gamma_\alpha \gamma_5 s$ and $j^{(B)}_5 = m_b \bar{b} \gamma_5 d$ are the interpolating fields for $K^-$ and $\bar{B}^0$ mesons respectively. Then we decompose the correlation function (7) with respect to the independent momenta into four invariant pieces:

$$F_\alpha(p, q, k) = (p - k)_\alpha F + q_\alpha F_1 + k_\alpha F_2 + \epsilon_{\alpha\beta\lambda\rho} q^\beta p^\lambda k^\rho F_3.$$  

(7)

Here an unphysical 4-momentum $k \neq 0$ is introduced as an auxiliary external momentum in the weak operator vertex. Thus the total momentum of the final state becomes $P = p - k - q$, which is independent of the momentum $p - q$ in the B channel. The advantage of introducing $k$ is to help avoiding artificial ambiguities in the dispersion relation for the B-meson channel. Of course, the unphysical $k$ has to vanish automatically in the physical matrix elements. This can be guaranteed, as will be seen, by choosing kinematical regions in such a way that we let $k^2 = 0$.

Saturating the correlator (6) with a complete set of intermediate states of the $K$ quantum numbers and utilizing the definition of the $K$ meson decay constant $\langle 0 | \bar{u} \gamma_\alpha \gamma_5 s | K^- (p-k) \rangle = if_K (p-k)_\alpha$, it follows that only the invariant function $F$ is relevant to our concern. Explicitly, the resultant hadronic expression for $F$ reads,

$$F_H((p - k)^2, (p - q)^2, P^2) = \frac{if_K \Pi_{\pi K}((p - q)^2, P^2)}{m_K^2 - (p - k)^2} + \int_{s_0}^\infty ds \frac{\rho^K(s, (p - q)^2, P^2)}{s - (p - k)^2},$$  

(8)
where $s_0^K$ is the threshold parameter and the spectral function $\rho_h^{(K)}$ stands for the higher state contribution in the K channel and $\Pi_{\pi K}((p - q)^2, P^2)$ is a correlator,

$$
\Pi_{\pi K}((p - q)^2, P^2) = i \int d^4xe^{i(p-q)x}\langle K^- (p - k)|T\{\overline{\mathcal{O}}(0)|j_5^{(B)}(x)\}|\pi^-(q)\rangle.
$$

On the other hand, applying the Operator Product Expansion (OPE) to Eq.(6) $F$ can be calculated in large space-like regions of both $p - k$ and $p - q$. In terms of the quark-hadron duality and then making the Borel transformation $(p - k)^2 \rightarrow M^2$, we have

$$
\Pi_{\pi K}((p - q)^2, P^2) = -\frac{i}{\pi f_K} \int_0^{s_0^K} ds \text{Im} F_{\text{QCD}}(s, (p - q)^2, P^2)e^{-\frac{m_B^2 - s}{M^2}}.
$$

The correlator $\Pi_{\pi K}((p - q)^2, P^2)$ is applicable for only large space-like $P^2$, therefore one needs to make an analytic continuation of Eq.(10) from a large space-like region $P^2 \ll 0$ to a large time-like region $p^2 = m_B^2$ for the realistic $\bar{B}^0 \rightarrow K^-\pi^+$ decay. We then have

$$
\Pi_{\pi K}((p - q)^2, M_B^2) = \frac{i}{\pi f_K} \int_0^{s_0^K} ds \text{Im} F_{\text{QCD}}(s, (p - q)^2, M_B^2)e^{-\frac{m_B^2 - s}{M^2}}.
$$

Inserting further the intermediate states of $\bar{B}^0$ quantum numbers in the correlator of Eq (11), the hadronic matrix element $\langle K^-\pi^+|\overline{\mathcal{O}}|B \rangle$ can be extracted in the light of the standard procedure for the QCD sum rule calculation. Not giving any technical details, we end up with the following LCSR expression for the matrix element in question,

$$
\langle K^- (p)\pi^+ (q)|\overline{\mathcal{O}}|B(p - q)\rangle = \frac{-i}{f_K f_B m_B^2} \int_0^{s_0^K} ds \text{Im} e^{-\frac{m_B^2 - s}{M^2}} \int_{m_B^2}^{R(s,m_B^2,m_B^2,s_0^B)} ds' \rho_{\text{QCD}}(s, s', m_B^2) \times e^{-\frac{m_B^2 - s'}{M^2}},
$$

where $f_B$ is the B decay constant, $s_0^B$ and $M^2$ are the threshold and Borel parameters in the B-channel and the QCD double spectral density $\rho_{\text{QCD}}(s, s', m_B^2) = 1/\pi^2 \text{Im}_s \text{Im}_r F_{\text{QCD}}(s, s', m_B^2)$. We will use the notation $A_1$ to denote such matrix element from now on.

Now we expand the correlation function (6) near the light-cone $x^2 \sim y^2 \sim (x - y)^2 \sim 0$ in order to get $\rho_{\text{QCD}}(s, s', m_B^2)$. The kinematical regions we choose are summarized as follows:

$$
q^2 = 0, p^2 = m_K^2, k^2 = 0,
$$

and

$$
(p - k)^2 \sim (p - q)^2 \sim P^2 \rightarrow -\infty,
$$

where the LCSR calculation is efficient and self-consistent. Moreover, we set all the light quark masses to be zero and neglect all the terms proportional to order of $1/m_b^2$ which emerge in the calculations. As has been mentioned, the soft effect is due to the interactions between emitted
quark-antiquark pairs and a background field gluon, thus the underlying quark propagator would receive a correction term [18]:

\[ \overline{S}(x, 0) = \frac{g_s \Gamma(n/2 - 1)}{16\pi^2(-x^2)^{n/2-1}} \int_0^1 dv \{ (1-v)f_\sigma G^{\mu\nu}(vx) + v\sigma G^{\mu\nu}(vx)f_\sigma \} \]

where \( n \) is the space-time dimension.

Using Eq.(15) and parameterizing the nonperturbative QCD effects with the three particle distribution functions of the pion, a straightforward calculation leads to the following light-cone QCD result

\[ F_{QCD} = F_{tw3} + F_{tw4} \]

with the twist-3 contribution

\[ F_{tw3} = \frac{m_b f_\pi}{4\pi^2} \int_0^1 dv \int d\alpha_i \varphi_3(\alpha_i) \frac{1}{(m_b^2 - (p-q)^2(1 - \alpha_i))(-P^2v\alpha_3 - (p-k)^2(1 - \alpha_3))} \]

\[ \times [(2-v)(q \cdot k) + 2(1-v)q \cdot (p-k)]q \cdot (p-k), \]

(17)

and the twist-4 contribution

\[ F_{tw4} = -\frac{m_b^2 f_\pi}{\pi^2} \left( \int_0^1 dv \int d\alpha_1 d\alpha_2 \Psi_1(\alpha_1, \alpha_2) \frac{1}{2[m_b^2 - (p - (1 - \alpha_1)q)^2]} \frac{(2v-3)(p-k) \cdot q}{(p-k - v\alpha_3 q)^2} \right. \]

\[ - \int_0^1 dv \int d\alpha_1 d\alpha_3 \Psi_1(\alpha_1, \alpha_3) \frac{1}{m_b^2 - (p - (1 - \alpha_1)q)^2} \frac{(p \cdot q - vq \cdot k)(p-k) \cdot q}{(p-k - v\alpha_3 q)^2} \]

\[ + \int_0^1 dv \int d\alpha_1 d\alpha_2 \Psi_2(\alpha_2) \frac{1}{m_b^2 - (p - (1 - \alpha_1)q)^2} \frac{(p \cdot q - vq \cdot k)(p-k) \cdot q}{(p-k - v\alpha_3 q)^2} \]

\[ - \int_0^1 dv \int d\alpha_1 d\alpha_3 \Psi_2(\alpha_3) \frac{1}{p \cdot q[m_b^2 - (p - \alpha_3 q)^2]} \frac{[(p-k) \cdot q]^3}{(p-k - v\alpha_3 q)^4} \]

\[ + \int_0^1 dv (v-1)v \int d\alpha_1 d\alpha_3 \Psi_2(\alpha_3) \frac{1}{m_b^2 - (p - \alpha_3 q)^2} \frac{[(p-k) \cdot q]^3}{(p-k - v\alpha_3 q)^4} \].

(18)

In Eqs.(17) and (18), \( D\alpha_i \equiv d\alpha_1 d\alpha_2 d\alpha_3 (1 - \alpha_1 - \alpha_2 - \alpha_3); \) \( f_\pi \) is the decay constant of the pion, \( f_\pi \) indicates a nonperturbative parameter defined by the matrix element \( \langle 0|d\sigma_{\mu\nu}\gamma_5 g_s G_{\alpha\beta} u|\pi^+ \rangle \); \( \varphi_3(\alpha_i) \) is a pionic twist-3 distribution function, whereas \( \varphi_\perp(\alpha_i) \) is of twist-4 and it, together with another twist-4 distribution amplitude \( \varphi_\parallel(\alpha_i) \), defines the functions \( \Psi_{1,2} \) as

\[ \Psi_1(u, v) = \int_0^u d\eta \left[ \varphi_\perp(\eta, 1 - \eta - v, v) + \varphi_\parallel(\eta, 1 - \eta - v, v) \right], \]

\[ \Psi_2(u) = \int_0^u d\eta \int_0^{1-\eta} d\xi \left[ \varphi_\perp(\xi, 1 - \xi - \eta, \eta) + \varphi_\parallel(\xi, 1 - \xi - \eta, \eta) \right]. \]

(19)

Readers are advised to refer to Ref.[19] for the definitions of various wavefunctions involved here and hereafter.

A simple manipulation can make Eq.(16) expressed in a preferred form of dispersion integral, from which extraction of \( \rho^{QCD}(s, s', m_b^2) \) is straightforward. Substituting the yielded
\[ \rho^{\text{QCD}}(s', s, m_B^2) \] into Eq.(12), we derive the final LCSR results for \( A_1 \) as

\[
A_1 = \left( \frac{m^2_B}{4\pi^2 f_K} \right) \left( \frac{m^2_B}{2f_B m_B^2} \right) \int_0^{s_0} dse^{-\frac{m^2_B}{M^2}} \int_0^1 \frac{du}{u} e^{\frac{m^2_B}{M^2} - \frac{m^2_B}{uM^2}} \times \left[ m_b f_{3\pi} \int_0^u \frac{dv}{v} \varphi_{3\pi}(1 - u, u - v, v) + f_{K} \int_0^u \frac{dv}{v} [3 \varphi_{A}(1 - u, u - v, v) - \left( \frac{m^2_B}{uM^2} - 1 \right) \psi_1(1 - u, v) + f_{K} \left( \frac{m^2_B}{uM^2} - 2 \right) \psi_2(u)] \right),
\]

where \( u_0^B = m_B^2/s_B^2 \). It is noted that for the twist-3 part we obtain the same result as that in \( B \rightarrow \pi \pi \) case [11], whereas the obtained twist-4 parts are not quite the same (in contrast with the corresponding term \( (\frac{m^2_B}{uM^2} - \frac{s_B^2}{uM^2}) \) in Ref. [11], our result is \( (\frac{m^2_B}{uM^2} - 2) \psi_2(u) \)). Numerically, however, the two forms result in close numbers.

Applying the same procedure to the \((S + P)(S - P)\) operators, one can notice that they do not result in any soft contributions to the amplitudes at all.

Now let us turn to a discussion on the other three decay modes. The case of \( B^- \rightarrow K^0 \pi^- \) is simple, where only the \((V - A)(V - A)\) operators \( O_{3,9} \) are concerned so that the LCSR result (20) may apply directly. The situations for the \( B^- \rightarrow K^- \pi^0 \) and \( B^0 \rightarrow K^0 \pi^0 \) decays are a bit complicated, in which both \( K \) mesons may either be an emitted hadron or include a spectator quark (antiquark). It is obvious that the LCSR result (20) holds for the emission case where only the \((V - A)(V - A)\) operators contribute to the decay amplitudes (note that there is an additional factor of \( 1/\sqrt{2} \) \((-1/\sqrt{2}\) for the operator \((\bar{u} \frac{\lambda}{2} b)_{V - A}(\bar{s} \frac{\lambda}{2} d)_{V - A}(\bar{d} \frac{\lambda}{2} d)_{V - A}\)). For the latter case, however, we have to modify the correlation function (6) with a necessary replacement, and besides the \((V - A)(V - A)\) and \((S + P)(S - P)\) operators we have to deal with the operators of \((V - A)(V + A)\) structure. Omitting the concrete derivations to save space, we only present a simple summary of our results. The operator \( (\bar{s} \frac{\lambda}{2} d b)_{V - A}(\bar{u} \frac{\lambda}{2} d)_{V - A}(\bar{d} \frac{\lambda}{2} d)_{V - A}\) provides the relevant matrix elements associated with the soft contribution \(1/\sqrt{2}A_2\) \((-1/\sqrt{2}A_2\),

\[
A_2 = \left( \frac{m^2_B}{4\pi^2 f_K} \right) \left( \frac{m^2_B}{2f_B m_B^2} \right) \int_0^{s_0} dse^{-\frac{m^2_B}{M^2}} \int_0^1 \frac{du}{u} e^{\frac{m^2_B}{M^2} - \frac{m^2_B}{uM^2}} \times \left[ m_b f_{3\pi} \int_0^u \frac{dv}{v} \varphi_{3\pi}(1 - u, u - v, v) + f_{K} \int_0^u \frac{dv}{v} [3 \varphi_{A}(1 - u, u - v, v) - \left( \frac{m^2_B}{uM^2} - 1 \right) \psi_1(1 - u, v) + f_{K} \left( \frac{m^2_B}{uM^2} - 2 \right) \psi_2(u)] \right). \]

The \((S + P)(S - P)\) operators make a vanishing contribution, as in the \(K\) emission case. It is interesting to notice that the \((V - A)(V + A)\) operators have an equal matrix element to those of the corresponding \((V - A)(V - A)\) operators.

Expanding the LCSR results for \(A_1\) and \(A_2\) in \(1/m_b\) and then comparing them with the corresponding factorizable amplitudes, we observe that the soft effects of the emission topology are typically of order \(1/m_b\).
Finally, the soft contributions of the emission topology to the $B \to K\pi$ decay amplitudes can be parametrized in terms of the resultant $A_1$ and $A_2$ as the following,

$$M_s^{(O)}(B^- \to \bar{K}^0\pi^-) = -\frac{G_F}{\sqrt{2}} V_{tb} V_{ts}^*(2C_3 - C_9)A_1,$$

$$M_s^{(O)}(\bar{B}^0 \to K^-\pi^+) = \sqrt{2} G_F (V_{ub} V_{us}^* C_2 A_1 - V_{tb} V_{ts}^* (C_3 + C_9))A_1,$$

$$M_s^{(O)}(B^- \to K^-\pi^0) = G_F \{V_{ub} V_{us}^* (C_1 A_2 + C_2 A_1) - V_{tb} V_{ts}^* [(C_3 + C_9)A_1 - \frac{3}{2} (C_8 A_2 + C_{10})A_2]\}, \quad (22)$$

$$M_s^{(O)}(\bar{B}^0 \to \bar{K}^0\pi^0) = G_F \{V_{ub} V_{us}^* C_1 A_2 + V_{tb} V_{ts}^* [(C_3 - \frac{1}{2} C_9)A_1 - \frac{3}{2} (C_8 + C_{10})A_2]\}. \quad (23)$$

which evidently respect the isospin symmetry.

Besides the soft contributions due to the emission topology, the two-body B decays, generally speaking, receive the power-suppressed corrections from the chromomagnetic dipole operator $O_{8g}$ and penguin topology [14]. The relevant contributions contain a soft and a hard parts. The former is owing to a soft gluon which is emitted off either from the $O_{8g}$ vertex or from a quark loop of the penguin contraction and finally immerses into the meson which absorbs a spectator quark. The latter is of two different origins. One is the hard gluon exchange between the $O_{8g}$ or penguin vertices and a spectator quark(antiquark). Another is related to the factorizable quark condensate contributions with QCD radiative correction being involved. The LCSR approach also is suitable for a quantitative study on these corrections. In fact, a detailed discussion has been made on their influences on $B \to \pi\pi$ in the LCSR framework [14]. The behaviors of such contributions in the heavy quark limit $m_b \to \infty$ comply with the following power counting: (1) The penguin diagrams make no contributions up to $1/m_b^2$ order. (2) The $O_{8g}$ operator supplies a hard effect of $\mathcal{O}(1/m_b)$ and a soft effect of $\mathcal{O}(1/m_b^2)$. Despite being formally suppressed by $\mathcal{O}(1/m_b^2)$ compared with the leading-order factorizable amplitudes, the soft correction from $O_{8g}$ is free of $\alpha_s$ suppression and numerically comparable with the $\mathcal{O}(\alpha_s)$ hard part. (3) The part with quark condensate has a chiral enhancement factor $r_{\pi}^\chi$ via the PCAC relation and it suffers formally from $\mathcal{O}(\alpha_s)$ and $\mathcal{O}(1/m_b)$ double suppression but numerically turns out to be a large effect. However, it has a counterpart in the QCD factorization and thus is not included in our calculation to avoid double counting.

It is reasonable to assume that the same power counting holds for $B \to K\pi$. Furthermore, since no strange quark appears as a spectator in the concerned case, the emitted soft gluons can only combine with a quark pair to form a three-particle Fock state of the pion, we may
directly use the corresponding LCSR results [14] by simply replacing the relevant parameters to achieve an estimate of the soft and hard effects due to the $O_{8g}$ operator and penguin topology in the $B \to K\pi$ case. As a consequence, the resultant corrections of $O_{8g}$ to the decay amplitudes can be expressed, at the subleading order in $1/m_b$ as follows,

$$
M^{(O_{8g})}_{h+s}(B^- \to \bar{K}^0\pi^-) = M^{(O_{8g})}_{h+s}(\bar{B}^0 \to \bar{K}^-\pi^+)
$$

$$= \sqrt{2} M^{(O_{8g})}_{h+s}(B^- \to K^-\pi^0)
$$

$$= -\sqrt{2} M^{(O_{8g})}_{h+s}(\bar{B}^0 \to \bar{K}^0\pi^0)
$$

$$= \frac{G_F}{\sqrt{2}} V_{tb} V^*_{ts} C_{8g}(A^{(O_{8g})}_h + A^{(O_{8g})}_s). \quad (24)
$$

Here $A^{(O_{8g})}_h$ and $A^{(O_{8g})}_s$ express the hard and soft parts of the matrix element $<\bar{K}^0\pi^-|O_{8g}|B^->$, and have the following LCSR results:

$$
A^{(O_{8g})}_h = \frac{i \alpha_s C_F}{2\pi} m_b^2 \left( \frac{1}{4\pi^2 f_K} \int_{s_0}^{s_K} ds \left( \frac{m_b^2}{M^2 - m_b^2/2uM^2} \right) \int_{u_0}^{1} du \frac{1}{u} e^{m_b^2/2M^2 - m_b^2/2uM^2} \right)
$$

$$\times \left[ \varphi_P(1) \bar{u} \left( 1 + \frac{3m_b^2}{um_B^2} \right) - \frac{\varphi'(1)}{6} \left( \frac{5m_b^2}{um_B^2} - 1 \right) \right], \quad (25)
$$

$$
A^{(O_{8g})}_s = i m_b^2 \left( \frac{1}{4\pi^2 f_K} \int_{s_0}^{s_K} ds \left( \frac{m_b^2}{M^2 - m_b^2/2uM^2} \right) \int_{u_0}^{1} du \frac{1}{u} e^{m_b^2/2M^2 - m_b^2/2uM^2} \right)
$$

$$\times \left( 1 + \frac{m_b^2}{uM_B^2} \right) \left[ \varphi_\perp(1 - u, 0, u) + \tilde{\varphi}_\perp(1 - u, 0, u) \right], \quad (26)
$$

where both distribution amplitudes $\varphi_P(u)$ and $\varphi_\sigma(u)$ are of twist-3 and are used to describe the pionic valence Fock state, while $\varphi_\perp(u)$ is a twist-4 three-particle wavefunction in analogy to $\bar{\varphi}_\perp(\alpha_i)$ and $\bar{\varphi}_\parallel(\alpha_i)$.

**III. DECAY AMPLITUDES AND BRANCHING RATIOS WITH SUBLEADING CORRECTIONS**

The LCSR results for the $O(1/m_b)$ soft and hard corrections to the $B \to K\pi$ decay amplitudes may serve as an order of magnitude estimate of the overall power-suppressed effects. We add them, together with the annihilation contributions $M_a$, to the QCD factorization results $M_f$, to get a decay amplitude with the subleading power corrections in both $\alpha_s$ and $1/m_b$. In Ref.[9], the $B \to K\pi$ decay amplitudes have been computed by including the $O(\alpha_s)$ corrections, and the annihilation effects have also been estimated in PQCD. Their results, which will be used for our upcoming numerical discussion, are:

$$
M_f(B^- \to \bar{K}^0\pi^-) = \sum_{p=u,c} V_{pb} V^*_{ps} \left[ (a_p^0 - \frac{1}{2}a_{10}^p) + r^K_{\psi}(a_0^p - \frac{1}{2}a_8^p) \right] A_{\pi K},
$$
\[
M_f(B^- \to K^-\pi^0) = \frac{1}{\sqrt{2}} ([V_{ub}V_{us}^*a_1 + \sum_{p=u,c} V_{pb}V_{ps}^*(a^p_4 + a^p_{10}) + \sum_{p=u,c} V_{pb}V_{ps}^r \chi (a^p_6 + a^p_8)]A_{\pi K} + \sum_{p=u,c} V_{pb}V_{ps}^3 (-a_7 + a_9)]A_{K\pi}),
\]

\[
M_f(\bar{B}^0 \to K^-\pi^0) = [V_{ub}V_{us}^*a_2 + \sum_{p=u,c} V_{pb}V_{ps}^*(a^p_4 + a^p_{10}) + \sum_{p=u,c} V_{pb}V_{ps}^r \chi (a^p_6 + a^p_8)]A_{\pi K},
\]

\[
M_f(B^- \to K^-\pi^0) = M_f(\bar{B}^0 \to K^-\pi^0) - \frac{1}{\sqrt{2}} M_f(B^- \to K^0\pi^-) - \frac{1}{\sqrt{2}} M_f(\bar{B}^0 \to K^-\pi^0),
\]

(27)

and

\[
M_a(B^- \to K^-\pi^-) = [V_{ub}V_{us}^*b_2 + (V_{ub}V_{us}^* + V_{cb}V_{cs}^*)(b_3 + b_{3EW}^E)]B_{\pi K},
\]

\[
M_a(B^- \to K^-\pi^0) = \frac{1}{\sqrt{2}} M_a(B^- \to K^0\pi^-),
\]

(28)

In Eq.(26), \(a_i\) and \(a^p_i\) are the QCD modified effective coefficients, and the low energy effects for the heavy to light transitions are included in the parameters \(A_{\pi K}(A_{K\pi})\) with an obvious dependence on the \(B \to \pi(B \to K)\) form factor \(F_{B \to \pi}(m^2_{\pi})(F_{B \to K}(m^2_{\pi}))\), the parameter \(r^K_\chi\) is a so called chiral enhancement factor which is related to the running quark masses. As the parameters existing in Eq.(27) are concerned, \(b_2\) and \(b_3(b_{3EW}^E)\) are the parameters related to the tree operator \(O_2^p\) and QCD (electroweak) penguin operators and embody the QCD dynamics in the annihilation processes, while \(B_{\pi K}\) is closely related to the decay constants. The explicit definitions of all these quantities are given in Ref. [9] and we do not repeat them here.

At present, we can write down the decay amplitudes of \(B \to K\pi\) with the subleading corrections in both \(\alpha_s\) and \(1/m_b\),

\[
M(B \to K\pi) = M_f(B \to K\pi) + M_a(B \to K\pi) + M_s^{(O_1)}(B \to K\pi) + M_s^{(O_{8u})}(B^- \to K\pi),
\]

(29)
Using the resultant decay amplitudes, it is straightforward to calculate the branching ratios of the $B \rightarrow K\pi$ decays, which are given by

$$\mathcal{B}(B \rightarrow K\pi) = \frac{\tau_B}{8\pi} |M(B \rightarrow K\pi)|^2 \frac{|P|}{m_B^2},$$

where $P$ is the c.m momentum of the outgoing mesons in the center of mass frame of B meson,

$$|P| = \frac{[(m_B^2 - (m_K + m_\pi)^2)(m_B^2 - (m_K - m_\pi)^2)]^{1/4}}{2m_B},$$

and $\tau_B$ is the $B$ lifetimes.

IV. NUMERICAL RESULTS AND DISCUSSIONS

Now we are in a position to make a numerical analysis and then to see how the subleading effects in $1/m_b$ impact the results of the QCD factorization. The set of important parameters of the present concern contains the QCD modified effective coefficients, quark masses, form factors, decay constants and distribution amplitudes. For the QCD modified effective coefficients and current quark masses, we adopt the numerical values presented in Ref.[9]. There have been a number of model-dependent estimates for the form factors $F^{B \rightarrow \pi,K}(q^2)$ and decay constants $f_B$ in the literature. Consistently we pick up, as inputs, the sum rule evaluations [20]: $F^{B \rightarrow \pi}(0) = 0.28 \pm 0.06$, $F^{B \rightarrow K}(0) = 0.32 \pm 0.05$ and $f_B = 180 \pm 30$ MeV with $m_b = 4.7 \pm 0.1$ GeV. The decay constants of $\pi$ and $K$ mesons are taken as $f_\pi = 132$ MeV and $f_K = 160$ MeV, respectively. As for the distribution amplitudes for the pion and $K$ meson, we don’t take into account SU(3) breaking effect for consistency, and adopt the following forms [19]:

$$\varphi_{\pi,K}(\mu) = 6u(1 - u)[1 + a_2(\mu)C_2^{3/2}(2u - 1)],$$

$$\varphi_p(u, \mu) = 1 + 30 \frac{f_3\pi}{\mu_\pi f_\pi} C_2^{1/2}(2u - 1) - 3 \frac{f_3\pi}{\mu_\pi f_\pi} C_4^{1/2},$$

$$\varphi_{\sigma}(u, \mu) = 6u(1 - u) \left[1 + 5 \frac{f_3\pi}{\mu_\pi f_\pi} \left(1 - \frac{\omega_{3\pi}}{10}\right) C_2^{3/2}(2u - 1)\right],$$

$$\varphi_{3\pi}(\alpha_i) = 360\alpha_1\alpha_2\alpha_3 \left[1 + \frac{\omega_{3\pi}}{2}(7\alpha_3 - 3)\right],$$

$$\varphi_{\perp}(\alpha_i) = 30\delta_2^2(\alpha_1 - \alpha_2)\alpha_3^2 \left[\frac{1}{3} + 2\epsilon_\pi(1 - 2\alpha_3)\right],$$

$$\bar{\varphi}_{\parallel}(\alpha_i) = -120\delta_2^2\alpha_1\alpha_2\alpha_3 \left[\frac{1}{3} + \epsilon_\pi(1 - 3\alpha_3)\right],$$

$$\bar{\varphi}_{\perp}(\alpha_i) = 30\delta_2^2(1 - \alpha_3)\alpha_3^2 \left[\frac{1}{3} + 2\epsilon_\pi(1 - 2\alpha_3)\right],$$

where the the leading twist-2 distribution amplitudes $\varphi_{\pi,K}(\mu)$ enter in calculation of the QCD factorization and annihilation contributions. The various coefficients in above equation have been determined [19] at $\mu_b = \sqrt{m_B^2 - m_b^2} \simeq 2.4$ GeV: $f_3\pi = 0.0026$ GeV$^2$, $\omega_{3\pi} = -2.18$, $\epsilon_\pi = 0.14$. The leading twist amplitudes $\varphi_{\pi,K}(\mu)$, $\varphi_p(u, \mu)$, $\varphi_{\sigma}(u, \mu)$, $\varphi_{3\pi}(\alpha_i)$, $\varphi_{\perp}(\alpha_i)$, $\bar{\varphi}_{\parallel}(\alpha_i)$, $\bar{\varphi}_{\perp}(\alpha_i)$ in this paper have been evaluated using Ref.[12, 19].
\[ \mu_\pi = \frac{m_\pi^2}{(m_u + m_d)} = 2.02 \text{GeV}, \ \delta^2 = 0.17 \text{GeV}^2, \ \epsilon = 0.36. \]  

The scale-dependence of them is achievable by use of the renormalization group equations [14, 19]. The parameters inherent in the sum rules can be fixed via the two-point sum rules at: 

\[ s_0^\pi = 0.7 \text{GeV}^2 \text{ and } M^2 = 0.5 - 1.2 \text{ GeV}^2 \]  
\[ s_0^K = 1.2 \text{ GeV}^2 \text{ and } M^2 = 1.5 - 3 \text{ GeV}^2 \]  
\[ s_0^{B_0} = 35 \pm 2 \text{ GeV}^2 \text{ and } M'^2 = 8 - 12 \text{ GeV}^2 \]  

[20], corresponding to the pion, K and B channels respectively. The B-lifetimes are measured experimentally as [22]: 

\[ \tau_{\bar{B}^0} = 1.542 \times 10^{-12} \text{ps} \text{ and } \tau_{B^-} = 1.674 \times 10^{-12} \text{ps}. \]  

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\[ \tau_{\bar{B}^0} = 1.542 \times 10^{-12} \text{ps} \text{ and } \tau_{B^-} = 1.674 \times 10^{-12} \text{ps}. \]  

The reasons are the following: (1) The penguin contributions dominate in these decays, since the tree ones are less important by the CKM suppression. (2) It is generally desirable that the branching ratios for \( B^- \rightarrow K^0\pi^- \) and \( B^0 \rightarrow K^-\pi^+ \) must be very close due to the smallness of the electroweak penguin effect, the slight difference between them arises from a destructive interference between the tree and QCD penguin contributions to \( B^0 \rightarrow K^-\pi^+ \). (3) The \( \pi^0 \) wavefunction makes the first two modes in Eq.(33) larger than the other two, and \( \Gamma(B^- \rightarrow K^0\pi^-) > \Gamma(B^- \rightarrow K^-\pi^0) > \Gamma(B^0 \rightarrow K^0\pi^0). \)  

Although the numerical calculation can be done using the inputs given above, we should note that the uncertainties resulting from such calculations are difficult to be quantitatively evaluated because of our poor knowledge about the inputs, especially the higher-twist distribu-
tion amplitudes. However, the yielded results are adequate to serve as an order of magnitude estimate of the effects in question.

We present our predictions based on the QCD factorization formula $\mathcal{B}^{(f)}(B \to K\pi)$ for each mode, respectively in Fig.1-4, together with those with the annihilation effects included $\mathcal{B}^{(f+a)}(B \to K\pi)$ and the results all the estimated subleading corrections in $\alpha_s$ and $1/m_b$ $\mathcal{B}^{(nl)}(B \to K\pi)$. The yielded $\mathcal{B}^{(f)}(B \to K\pi)$ and $\mathcal{B}^{(f+a)}(B \to K\pi)$ are plotted for the four different modes, respectively in Fig.5 and Fig.6. It should be understood that only the central values, which are obtained for $\mu = \mu_b$ and $\gamma = 60^\circ \pm 20^\circ$, are shown there. Working within the QCD factorization framework, one observes from Fig.5: (1) The yielded branching ratios answer to the pattern

$$\mathcal{B}^{(f)}(\bar{K}^0\pi^-) > \mathcal{B}^{(f)}(K^-\pi^+) > \mathcal{B}^{(f)}(K^-\pi^0) > \mathcal{B}^{(f)}(\bar{K}^0\pi^0).$$

(35) (2) The branching ratios of both $\bar{K}^0\pi^-$ and $B \to \bar{K}^0\pi^0$ are less sensitive to the change of $\gamma$ than the other two, as expected.

As the $1/m_b$ power-suppressed effects are involved from the annihilations and soft and hard gluons, the branching ratios get an evident modification as shown in Fig.6. Letting $\mu$ vary between $\mu_b/2$ and $2\mu_b$, we find that the $1/m_b$ power-suppressed effects from the annihilations and soft and hard gluons can enhance the branching ratios by $(20 - 30)\%$, depending on the decay modes and $\gamma$.

Now, for taking a closer look at the individual roles of the soft and hard exchange and annihilation effects in the subleading contributions in $1/m_b$, we estimate the two ratios $\mathcal{B}^{(nl)}/\mathcal{B}^{(f+a)}$ and $\mathcal{B}^{(f+a)}/\mathcal{B}^{(f)}$ which can make sense about the involved physics. It is found that for $\mu$ ranging from $\mu_b/2$ to $2\mu_b$ the effects of annihilation topology can make the branching ratios $\mathcal{B}^{(f)}(B \to K\pi)$ increase by $(20 - 30)\%$, whereas the soft and hard effects modify $\mathcal{B}^{(f+a)}(B \to K\pi)$ at a level of about $10\%$.

As emphasized, most of the inputs suffer from theoretical uncertainties, which can affect the numerical results and should be carefully examined. Obviously, these uncertainties are related to the non-perturbative QCD, about which a solid knowledge is absent at present. Thus we are not so ambitious to make a complete quantitative estimate of them, instead, we just list the main sources of uncertainties which we can conjecture, and discuss how they influence the present results. By our observation, the most important sources of uncertainties are the following: (1) Distribution amplitudes. The models, which are here adopted for the distribution amplitudes of light mesons, are based on an expansion in conformal spin. For the leading twist-2 distribution amplitudes, the asymptotic forms are known precisely and the non-asymptotic corrections, in spite of their model-dependence, are also believed to be under control. In contrast, little is known about the uncertainties in higher-twist wavefunctions of light-mesons and distribution amplitudes of $B$ mesons. Therefore, this could have a significant
impact on the reliability of theoretical prediction. For example, the higher-twist wavefunctions can greatly affect the accuracy of the $1/m_b$ correction parts and the hard spectator scattering contributions, which are proportional to $1/\Lambda_{QCD}$. When we carry out the calculation by using the extensively adopted form of the $B$ meson wavefunctions, a considerable uncertainty may exist. (2) “Chirally enhanced” terms. A term proportional to the parameter $r^K_X$ appears in the calculations of the matrix elements of the $(S - P)(S + P)$ operators. The part is called the “chiral enhancement” term, being formally $1/m_b$-suppressed, but numerically large. Because the factor is sensitive to the current quark masses whose precise values are not known, their variations may cause a considerable uncertainty in the numerical results. (3) Form factors. One may believe that long-distance effects dominate the hadronic matrix elements of the heavy-to-light transitions from a naive power counting. If it is true, the LCSR results for the form factors, which are used in our calculations, should be relatively reliable. Nevertheless, there are other viewpoints contrary to it [3], that is, the short-distance contributions are predominant over long-distance ones so that PQCD is applicable in this case. A better understanding of the uncertainties due to the form factors asks for a clarification of the transition mechanism. (4) Decay constants. In QCD sum rule calculations the decay constants for the $B$ mesons are sensitive to the $b$ quark mass, as we know. In contrast, the decay constants of the light pseudoscalar mesons have been experimentally measured to high accuracy, the uncertainties from this part is the least. (5) Final state interactions (FSI). Besides those effects which have already been estimated, the FSI effects may play a non-negligible role in the evaluation of decay amplitudes. These effects are usually regarded as being power suppressed in $1/m_b$ owing to the large energy released in the $B$ decays. Unfortunately, so far, we lack a convincing quantitative evaluation on them. The $B \to K\pi$ modes can receive the final state rescattering contributions from a number of intermediate states. The FSI contributions from the $B \to D_s \bar{D}$ followed by $D_s \bar{D} \to K\pi$ are anticipated to be especially important. The reason is obvious that the $B \to D_s \bar{D}$ modes are $CKM$ favored and thus can provide a large branching ratio; on the other hand, they are typically dominated by long-distance dynamics and violate the QCD factorization. (6) Other sources of power suppression. In SCET a new source of power suppressed contributions has been identified in study on the color suppressed process $\bar{B}^0 \to D^0 \pi^0$. It is necessary to explore the corresponding impacts on $B \to K\pi$. In addition, there might be other new sources of power suppressed contributions awaiting to be dug out and their effects should be estimated. (7) Theoretical approach. The numerical results presented here are based on a combination of the QCD factorization formula, PQCD approach and LCSR method. A possible inconsistency might manifest as the calculations are done by using all the three different approaches. Of course, to avoid this ambiguity is to do calculation within the same theoretical framework. The LCSR approach provides such a
possibility. Practical manipulation, however, would be considerably difficult in view of the existing complications in technique.

Once all these uncertain factors are taken into account and clarified, either theoretically or phenomenologically, with our new knowledge, the numerical results presented here can and should eventually be updated.

V. SUMMARY

We obtain the LCSR results for the $1/m_b$ suppressed soft and hard corrections to the $B \rightarrow K\pi$ decays. Then combining them with the previously obtained results of QCD factorization and estimates of annihilation topology, the branching ratios are calculated and the numerical impacts of such effects, as an order of magnitude estimate of the overall power-suppressed effects, are investigated to subleading order in $1/m_b$. Also, the important sources of uncertainties are discussed in some details.

Subject to a quantitative estimate of the existing uncertainties, for $\gamma = 60^\circ \pm 20^\circ$ we have observed the following. (1) The $1/m_b$ power suppressed effects resulting from the annihilations and soft and hard exchanges can make the branching ratios getting an increase of $(20 - 30)\%$ with respect to the results of the QCD factorization, depending on concrete decay modes. (2) The annihilation effect is predominate over the soft and hard ones, which modify the branching ratios by only $10\%$ of the results which include the $O(\alpha_s)$ radiative corrections and annihilation effects.

Our present work helps to make sense about the contributions of the soft and hard-gluon exchange along with other power suppressed terms to the $B \rightarrow K\pi$ decays. No doubt, it still is too early to draw a decisive conclusion at present whether or not the theoretical estimates can accommodate the experimental data and the power-suppressed soft and hard effects are negligibly small in the $B \rightarrow K\pi$ decays. We have to await the improvement in experiment and progress in theoretical or phenomenological study on, amongst other things, the higher-twist wavefunctions as well as behaviors of FSI effects in the heavy quark limit. Certainly, a better understanding of the other sources of uncertainties is indispensable to arrive at a reliable conclusion. Anyhow a further investigation, whether theoretical or phenomenological, on the power-suppressed effects in charmless B decays may be needed.

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Figure 1: $\mathcal{B}(B^- \to \bar{K}^0\pi^-)$ versus the weak phase $\gamma$. The solid, dotted and dashed lines correspond to $\mathcal{B}^{(f)}(B^- \to \bar{K}^0\pi^-)$, $\mathcal{B}^{(f+a)}(B^- \to \bar{K}^0\pi^-)$ and $\mathcal{B}^{(nl)}(B^- \to \bar{K}^0\pi^-)$, respectively.

Figure 2: $\mathcal{B}(\bar{B}^0 \to K^-\pi^+)$ versus the weak phase $\gamma$. The definitions of the lines are the same as in Figure 1.
Figure 3: $B(B^- \to K^-\pi^0)$ versus the weak phase $\gamma$. The definitions of the lines are the same as in Figure 1.

Figure 4: $B(\bar{B}^0 \to \bar{K}^0\pi^0)$ versus the weak phase $\gamma$. The definitions of the lines are the same as in Figure 1.
Figure 5: $\mathcal{B}^{(f)}(B \rightarrow K\pi)$ versus the weak phase $\gamma$. The solid, dotted, dashed and dash-dotted lines correspond to $B^-(\rightarrow K^0\pi^-)$, $\bar{B}^0 \rightarrow K^-\pi^+$, $B^- \rightarrow K^-\pi^0$ and $\bar{B}^0 \rightarrow \bar{K}^0\pi^0$, respectively.

Figure 6: $\mathcal{B}^{(nl)}(B \rightarrow K\pi)$ versus the weak phase $\gamma$. The definitions of the lines are the same as in Figure 5.