Huyghens Principle, Planck Law: Peculiarities in the Behavior of Planar Photons

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Abstract

Huyghens principle and Planck law are studied in Maxwell and Maxwell-Chern-Simons frameworks in (2+1) dimensions. Contrary to (3+1) dimensions, massless photons are shown to violate Huyghens principle in planar world. In addition, we obtain that Planck law is no longer proportional to $\nu^3$, but to the squared frequency, $\nu^2$, of the planar photons. We also briefly discuss possible physical consequences of these results.

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1 Introduction

Field theories lying in three space-time dimensions have attracted a great deal of efforts since nearly two decades [1]. Such studies have given us a number of new theoretical insights which have been useful for a better theoretical understanding of several aspects, not only related to these particular theories, but also to a wider class of models, including some ones defined in other dimensions. On the other hand, planar physics has also been shown to provide good explanations of important phenomena observed in the Condensed Matter domain, among them, those concerned to the Quantum Hall Effect [2] and to the High-Tc Superconductivity [3, 4].

A quite important underlying aspect of these theoretical applicabilities is undoubtedly a number of peculiar characteristics exhibited by such models, among them we may quote charge and spin fractionization [4, 5]. Actually, these interesting characteristics are intimately related to topological and dimensional peculiarities carried out by 3d space-time, like as the possibility of defining the so-called Chern-Simons term, which provides remarkable novelties whenever suitably added to a given model. For instance, the massless degree of freedom (d.f.) which comes about from the pure Maxwell action acquires a mass gap, without breaking the gauge symmetry, when such an action is supplemented by the Chern-Simons term, leading us to the so-called Maxwell-Chern-Simons (MCS) model, whose Lagrangian reads:

$$\mathcal{L}_{MCS} = -\frac{1}{4} F^{\mu\nu} F_{\mu\nu} + m\epsilon^{\mu\nu\kappa} A_\mu \partial_\nu A_\kappa.$$  (1)

As it is well-known, this mass generation mechanism holds in Abelian and non-Abelian frameworks, and a similar one also takes place in planar gravity (see Ref. [6] for further details).

Another interesting characteristic that takes place in electrodynamical-like models in (2+1) dimensions is the violation of the so-called Huyghens principle by the classical (retarded and advanced) Green functions associated to the free massless vector field [4] (for details, see Refs. [7, 8, 9, 10]). This, in turn, was shown to lead us to some remarkable phenomena, such as the classical reverberation of electromagnetic signals and to a still missing Larmor-like formula relating the acceleration of the sources to the radiated power. Indeed, these aspects were verified in both Maxwell and Maxwell-Chern-Simons electrodynamics (see Ref. [7]; see also Ref. [11]).

In this work, we wish to go further into such a subject and deal with these points at the quantum level. More precisely, we shall show that a similar violation takes also place here. This is explicitly demonstrated by showing that the Pauli-Jordan-like commutators between two components of the field-strength has non-vanishing support for light-like and time-like intervals as well, i.e., $(x-y)^2 \geq 0$. However, although Huyghens principle no longer holds, microcausality is kept, since these commutators are shown to vanish for every space-like interval. Namely, such a result states us that the measure of the interference (correlation, more precisely) between the electric and magnetic field will be non-vanishing whenever these fields be separated by non-space-like vectors.

Here, it is worthy noticing that such a result is in deep contrast with (3+1)-dimensional facts, where such a correlation vanishes unless the interval is light-like. The latter, as it is well-known, is an equivalent statement that (3+1)-dimensional quantized radiation (genuine photons) travels at speed $c$, what implies that they are massless (for details, see Refs. [12, 13, 14, 15]). Actually, it is

\(^1\)Here, for the violation of Huyghens principle by such functions, we are meaning that their mathematical support no longer lie only on the surface of the light-cone, where $(x-y)^2 = 0$, but also spread throughout other space-time intervals.
widely-known that, other physical phenomena are deeply connected to these characteristics of the radiation, like as those processes concerning absorption, emission, and scattering of light.

Therefore, we may wonder whether some well-established (3+1)-dimensional results are still maintained unaltered in the planar case. As we have already said, the “matching” between the validity of the Huyghens principle and the massless character of the radiation is a first result that must be abandoned, at least when (2+1)-dimensional facts are concerned. Indeed, as we shall see later, there are some emission and absorption processes that have to be also substantially modified whenever dealing with planar electrodynamic-like models. For instance, Planck law appears to be proportional to the square frequency of the radiation, $\nu^2$, instead of $\nu^3$ (the case in (3+1) dimensions).

2 The Pauli-Jordan commutators and the violation of the Huyghens principle

Although the Hamiltonian and equal-time commutation relations constitute in the whole machinery necessary to canonically quantize the electromagnetic radiation and extract its physical content, sometimes it is very important to calculate and explore the consequences of non-equal-time commutation relations between fields and/or conjugated momenta.

For instance, we know that (3+1)-dimensional quantized radiation (photons) has vanishing CR’s between two components of its potential unless the space-time interval is light-like, say:

$$[A_\mu(x), A_\nu(y)]_{x^\rho \neq y^\rho} = -i \eta_{\mu\nu} D^{3+1}_{P,J}(x - y),$$

where the massless Pauli-Jordan function is given by ($\Theta(z)$ is the step-function):

$$D^{3+1}_{P,J}(z) = \frac{1}{4\pi} (\Theta(z^0) - \Theta(-z^0)) \delta^{3+1}(z^2).$$

Here, since the support of the function above lies only on the surface of the (3+1)-dimensional light-cone, we say that $D^{3+1}_{P,J}(x - y)$ satisfies Huyghens’ principle. In addition, it is not difficult to show that relation (2) implies, for example, that ($\vec{E} = -\nabla A^0 - \partial_1 \vec{A}$ and $\vec{B} = \nabla \wedge \vec{A}$, as usual):

$$[\vec{B}_1(x), \vec{E}_2(y)]_{x^\rho \neq y^\rho} = i \frac{\partial^2}{\partial x^0 \partial y^3} D^{3+1}_{P,J}(x - y),$$

which is clearly non-vanishing if and only if $(x - y)^2 = 0$ (see Refs. [12, 13, 14, 15]). Therefore, we conclude that there is interference between measurements of field-strength components if and only if they are connected by light-like intervals. This statement, in turn, is a direct consequence from the massless character of the radiation, which implies that the physical excitations will always travel with velocity $c$, or in other words, their information may only be connected by light-like vectors. Therefore, the statement of the massless feature of the Maxwell radiation is equivalent to state the validity of the Huyghens principle, by Pauli-Jordan function, in (3+1) dimensions.

\footnote{We use the conventions: $\mu, \nu, \text{etc} = 0, 1, 2, 3$ with \text{diag}(\eta_{\mu\nu}) = (+, -, -, -).$ We also use $\mu, \nu, \text{etc} = 0, 1, 2, \text{while}$ $i, j, \text{etc} = 1, 2$. In addition, \text{diag}(\eta_{\mu\nu}) = (+, -, -)$ and $\epsilon^{012} = \epsilon_{012} = \epsilon^{12} = \epsilon_{12} = 1$. We also denote by $\Box$ the (2+1)-dimensional D’Alembertian differential operator: $\Box = \partial_\mu \partial^\mu = \partial_1^2 - \partial_i^2$.}
2.1 The massless case

As we shall see below, such a “matching” above will no longer hold in (2+1) dimensions. More precisely, even though free Maxwell Lagrangian (as usual, $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$),

$$\mathcal{L}_M = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu},$$  \hspace{1cm} (5)

describes a massless degree of freedom, $\Box A_\mu = 0$, we shall show that a similar commutator between two components of the gauge field (or of its associated field-strength) will be non-vanishing not only if they were separated by light-like vectors, but also by time-like ones, say, $(x - y)^2 \geq 0$.

In order to see this, let us consider the (2+1)-dimensional analogue of relation (5), say:

$$[A_\mu(x), A_\nu(y)]_{x^0 \neq y^0} = -i \eta_{\mu\nu} D_{P,J}^{2+1}(x - y).$$  \hspace{1cm} (6)

Now, if we take the general form of the massless Pauli-Jordan function,

$$D_{P,J}^{(d+1)}(z) = \frac{i}{(2\pi)^{d+1}} \int d^{d+1}k e^{i(k^0)\delta^{d+1}(k^2)} \frac{e^{-ik_\mu z^\mu}}{k^2},$$  \hspace{1cm} (7)

with $\epsilon(k^0) = \Theta(k^0) - \Theta(-k^0)$ and $\delta^{d+1}(k^2) = \delta^{d+1}(k_0^2 - \vec{k}^2)$, and particularize it to the planar case, we obtain:

$$D_{P,J}^{(2+1)}(z) = \frac{1}{2\pi} [\Theta(z^0) - \Theta(-z^0)] \frac{\Theta(z^2)}{\sqrt{z^2 - \vec{z}^2}}.$$  \hspace{1cm} (8)

Here, it is worthy noticing a remarkable feature of the function above: as previously announced, it yields a non-vanishing result whenever the space-time interval is light-like or time-like, $z^2 = (x - y)^2 \geq 0$. This is in deep contrast with $D_{P,J}^{3+1}$, compare with eq. (3), whose support lies only over light-like vectors. Thus, we say that $D_{P,J}^{2+1}$ does not satisfy Huyghens principle. Notice, however, that micro-causality is not lost: $D_{P,J}^{2+1}$ vanishes whenever $z^2 = (x - y)^2 < 0$. However, we loose the “matching” between the statements of the massless character of a field and the validity of Huyghens principle as presented by its associated Pauli-Jordan function in (2+1) dimensions.

Now, let us return to the (2+1)-dimensional Pauli-Jordan commutator. From expression (6) we may obtain similar relations between any two components of the field-strength. After some algebra we can show the following non-equal-time equality ($\tilde{F}^\mu = \epsilon^{\mu\nu\kappa} \partial_\nu A_\kappa = (-B, -\epsilon^{ij} E^j)$, as usual):

$$[\tilde{F}^\mu(x), \tilde{F}^\nu(y)] = -2i (\Box \eta_{\mu\nu} - \partial^\mu \partial^\nu) D_{P,J}^{(2+1)}(x - y),$$  \hspace{1cm} (9)

which gives us, among other, that:

$$[E^j(x), B^j(y)]_{x^0 \neq y^0} = +2i\epsilon^{ij} \frac{\partial^2}{\partial y_j \partial x_0} D_{P,J}^{(2+1)}(x - y) = +3i \frac{(x^0 - y^0)\epsilon^{ij}(x^j - y^j)}{(\sqrt{(x - y)^2})^4} D_{P,J}^{(2+1)}(x - y).$$  \hspace{1cm} (10)

Clearly, the result above states us that the interference between the measurements of $\tilde{E}$ and $B$ are non-vanishing whenever such quantities are separated by non-space-like intervals, $(x - y)^2 \geq 0$. 

2.2 The topologically massive case

In a standard massive theory, the non-equal-time CR’s between two components of the gauge potential would be read as:

$$[\phi_I(x), \phi_J(y)]_{x^0 \neq y^0} = -i g_{IJ} D^{d+1}(x - y; \mu),$$  \hspace{1cm} (11)

where $D^{d+1}$ is the $(d+1)$-dimensional massive Pauli-Jordan function,

$$D^{d+1}(z) = \frac{i}{(2\pi)^{d+1}} \int d^{d+1}k \, \epsilon(k^0) \delta^{d+1}(k^2 - \mu^2) \frac{e^{-ik^\mu z_\mu}}{(k^2 - \mu^2)},$$  \hspace{1cm} (12)

which smoothly recover its massless counterpart as $\mu \to 0$.

Nevertheless, whenever topologival-like mass generation mechanisms are involved, then relation (11) may acquire new terms. This is the case for MCS model, eq. (1). Actually, due to the Chern-Simons term, a similar expression to (11), for $A^\mu$-potential, gets the following form (see, for example, Ref.[18] for further details):

$$[A^\mu(x), A^\nu(y)]_{x^0 \neq y^0} = -i \left( \eta^{\mu\nu} + \frac{\partial_x^\mu \partial_y^\nu}{m^2} - \frac{\epsilon^{\mu\nu\kappa}}{m} \partial_\kappa \right) D^{2+1}(x - y; m) + i m^2 \left( \partial_x^\mu \partial_y^\nu - m \epsilon^{\mu\nu\kappa} \partial_\kappa \right) D_{P,J}^{2+1}(x - y),$$  \hspace{1cm} (13)

whose first term is the contribution due to the massive physical excitation, while the second one comes from the (non-dynamical) massless pole. The latter answers for the magnetic vortex-like flux attached to the electric charge. Notice also that as $m \to 0$ relation (13) reduces to its massless counterpart, eq.(6).

Now, taking eq. (12) for the (2+1)-dimensional case, we obtain that:

$$D^{2+1}(z) = \frac{1}{2\pi} \left[ \Theta(z^0) - \Theta(-z^0) \right] \frac{\cos(m\sqrt{z^2}) \Theta(z^2)}{\sqrt{z^2}},$$  \hspace{1cm} (14)

which is clearly non-vanishing for all $z^2 \geq 0$. Hence, we readily see that the function above does not satisfy Huyghens principle (but respects micro-causality, since it identically vanishes for all $z^2 < 0$). However, such a ‘violation’ is not so drastic here. It was indeed expected to happen, since as we know very well, massive excitations are hadrons, thus propagating slower than light. Indeed, similar ‘violation’ may also take place in (3+1) dimensions whenever dealing with massive particles (see, for example, Ref.[19]).

Although both massless and massive poles contribute to expression (13), when we calculate a similar relation for the field-strength components, we are led to (notice that, as $m \to 0$ the commutator below recovers its massless counterpart, (1))

$$[F^\mu(x), F^\nu(y)]_{x^0 \neq y^0} = i \left( \square \eta^{\mu\nu} - \partial^\mu \partial^\nu + m \epsilon^{\mu\nu\kappa} \partial_\kappa \right) D^{2+1}(x - y; m),$$  \hspace{1cm} (15)

which highlights that only the massive pole gives us a non-vanishing value when the electric and magnetic fields are measured. In other words, only the massive degree of freedom can be properly detected as a physical excitation (see also Ref.[18]). It is also clear that, even though in a quite particular way, the interference between measurements of the field-strength will give a non-trivial
result whenever the space-time interval that separates them is light-like or time-like, say, \((x-y)^2 \geq 0\). For example:

\[
[E^i(x), B(y)]_{x^0 \neq y^0} = i \left[ \frac{(x_0 - y_0) \epsilon^{ij} (x^j - y^i)}{\sqrt{(x-y)^2}} \left( m^2 + 3 \frac{\tan(m \sqrt{(x-y)^2})}{\sqrt{(x-y)^2}} \right) + \frac{3}{\sqrt{(x-y)^2}} \right] D^{i}(x-y, m)
\]

which, despite its rather complicated form is clearly non-vanishing whenever \((x-y)^2 \geq 0\), what confirms the fact that massive radiation is detected as propagating slower than light. In addition, it is worthy noticing that the last term in expression above, proportional to \(m \epsilon^{ij} (x_j - y_j)\), comes from the Chern-Simons term and thus is odd under spin flipping, say, \(m \rightarrow -m\). Notice also that as \(m \rightarrow 0\), expression above recover its massless counterpart, eq. (10).

3 Emission and absorption of planar photons

Now, we shall study some physical processes concerning emission and absorption of planar radiation by non-relativistic atomic electrons, namely, how Einstein-like coefficients and Planck read in the planar world. Notice, hereafter, the explicit presence of the constants \(\hbar\) and \(c\).

First, let us consider an atom in a initial state \(A\), which may interact with a \(n\)-photon state, \(n\vec{k}, \vec{\xi}\), with momentum \(\vec{k}\) and polarization vector \(\vec{\xi}\). Now, the transition matrix element for absorption of an unique photon (first-order process) reads:

\[
< B; n\vec{k}, \vec{\xi} - 1 | H^{(1)}_{\text{int}} | A; n\vec{k}, \vec{\xi} > = -\frac{e}{m} \sqrt{\frac{n\vec{k}, \vec{\xi}}{2 \omega(\vec{k})}} < B | \sum_i e^{i\vec{k}\cdot \vec{x}_i} \vec{\xi}(\vec{k}) \cdot \vec{p}_i | A > ,
\]

which similarly to its (3+1)-dimensional analogue, is proportional to the squared root of the total number of photons. In expression above, \(H^{(1)}_{\text{int}}\) is the (1st order) interaction Hamiltonian between photons and electrons, given by:

\[
H^{(1)}_{\text{int}} = -\frac{e}{m} \sum_i \vec{A}(\vec{x}_i, t) \cdot \vec{p}_i,
\]

with \(\vec{p}_i\) being the momentum of the i-th electron located at \(\vec{x}_i\), where \(\vec{A}(\vec{x}_i, t)\) acts at time \(t\). The field \(\vec{A}\), in turn, is decomposed in plane-waves:

\[
\vec{A}^\mu(\vec{x}, t) = \frac{1}{(2\pi)^2} \int \frac{d^2 \vec{k}}{\sqrt{2 \omega(\vec{k})}} \xi^\mu(\vec{k}) [a(\vec{k}) e^{-i k^\nu x^\nu} + a^\dagger(\vec{k}) e^{+i k^\nu x^\nu}],
\]

with \(\xi^\mu(\vec{k})\) being the polarization vector, whose explicit form reads like below:

\[
\xi^\mu(\vec{k}) = (0; 1, 0),
\]

whenever the radiation is massless, or else (at rest frame -Landau gauge)

\[
\xi^\mu_L(\vec{k} = 0) = \left( 0; \frac{1}{\sqrt{2}}, -\frac{i m}{\sqrt{2} |m|} \right),
\]
for photons appearing in the MCS framework. In the latter case, such a structure of the polarization vector has provided a natural (say, Lorentz and gauge covariant) explanation why such excitations display spin ±1 (see Refs. [16, 17], for further details).

For first-order emission processes, we also have:

\[
< B; n\vec{k},\vec{\xi} + 1 | H_{\text{int}}^{(1)} | A; n\vec{k},\vec{\xi} > = -\frac{e}{m} \sqrt{\frac{n_{\vec{k},\vec{\xi}} + 1}{2\omega(k)}} < B | \sum_i e^{i\vec{p}_i \cdot \vec{r}(\vec{k})} A > .
\]  

(20)

It is worth noticing that result above coincide, in form, with usual (3+1)-dimensional ones, for both cases, stimulated \((n \neq 0)\) and spontaneous \((n = 0)\) emissions. Namely, for \(n = 0\) we realize that the spontaneous emission of planar photons takes place, like in the 3-spacial case: the transition matrix element is non-vanishing although no radiation is present.

Now, we may readily show that, while matrix elements differ whenever the photon carries mass or not, since they strongly depend on the structure of the polarization vector, eqs. (18) and (19), the probability transition, basically \(| < B; n \pm 1 | H_{\text{int}}^{(1)} | A | n > |^2\), may be shown to give the same results, independing on the massive character. At this point we should then stress that such a quantity is not modified by the (topological) mass gap of the excitations, at least in the planar case. This is because such a gap does not increase the number of degrees of freedom of the planar radiation.

Going further, and searching for how Planck law reads in the planar case, we find that:

\[
U^{2+1}(\nu) = 2\pi \frac{\hbar\nu^2}{c^2} \frac{1}{(e^{\hbar\nu/KT} - 1)},
\]

(21)

which contrasts with its (3+1)-dimensional counterpart,

\[
U^{3+1}(\nu) = 8\pi \frac{\hbar\nu^3}{c^3} \frac{1}{(e^{\hbar\nu/KT} - 1)},
\]

(22)

by a multiplicative factor of \(4\nu/c\). Indeed, it is not difficult show that such a factor arises by virtue of dimensional matters, like as the “volume element” and the number of physical degrees of freedom carried by massless radiation in both space-time. As it is well-known, \(U(\nu)\) represents the energy density\(^3\) (per unity frequency, \(d\nu\)) of radiation in thermal equilibrium, at temperature \(T\), distributed over a given “volume”, with frequency ranging from \(\nu\) to \(\nu + d\nu\) (see Refs. [12, 13, 14] for further details). Moreover, comparing eqs. (21) and (22), we easily realize that ultra-violet divergences concerning electromagnetic radiation is better handled in planar world than in 3D-spatial case.

Furthermore, working on the topologically massive version of the planar Planck law, we must recall that the excitations now present rest energy, say, \(E^2 = |\vec{p}|^2 c^2 + m^2 c^4\), what implies that the frequency of the massive photons, \(\nu' = \omega'/2\pi = E/h\) is no longer determined only by its wave-vector, \(\vec{k} = \vec{p}/h\), but also by its mass. Actually, following the same steps like in the previous case, we find

\[
U^{2+1}(\nu', m) = 2\pi \frac{\hbar\nu'^2}{c^2} \frac{1}{(e^{\hbar\nu'/KT} - 1)},
\]

(23)

\(^3\)Essentially, \(U(\nu)\) is the total energy enclosed in a given “volume” times the density of allowed physical states (per unity frequency), \(\rho_{\nu,d\nu} = N \cdot \prod dk_i\), \(\vec{k}\) is the wave-vector and \(N\) the number of degrees of freedom carried by the excitations.
with \( \nu' = c\sqrt{(k/2\pi)^2 + (mc/\hbar)^2} = c\sqrt{\nu^2 + \nu_0^2} \), where \( \nu \) and \( \nu_0 \) are the kinetic and “rest” frequencies of the massive radiation. Then, we now understand \( U^{2+1}(\nu, m) \) as the distributed energy density (per unity frequency, \( d\nu' \)) of photons, in thermal equilibrium, with frequency between \( \nu' \) and \( \nu' + d\nu' \), and (constant) physical mass \( m \).

In addition, we may easily show that in the small mass limit, \( |m| << 1 \) or equivalently, \( \nu >> \nu_0 \), eq. (23) gives us

\[
U^{2+1}(\nu, m)|_{m<<1} = 2\pi \frac{h\nu}{c^2} \left( e^{h\nu/kT} - 1 \right) + O(\nu_0^2),
\]

which recovers the massless result, eq. (21), as \( m \to 0 \).

4 Conclusions and Prospects

We have shown that Huyghens principle is not satisfied by Pauli-Jordan-like functions in (2+1) dimensions (massless or massive cases). In the massless scenario, this has led us to the lost of the matching which connects the massless character of photons and its propagation at light-speed, \( c \).

Indeed, by applying to the quantum framework, a similar interpretation to that we have done in the classical case (see Ref.[7]), we are led to some surprisingly questions, for instance, the following ones.

First, by facing an electromagnetic signal rather as a wave, reverberation affects its propagation and we can no longer speak of sharp pulses. On the other hand, whenever quantizing such a wave in order to give it the status of a particle (a “planar photon”), we may wonder whether the concept of a photon as a localized energy packet should not be reassessed in the planar case. Second, since \( A_{\mu} \) describes an elementary particle (an irreducible representation of the SO(2, 1) Lorentz-like group), how could we conciliate such a structureless character with the division of the planar massless photons into minor parts?

We have also seen that probability transition elements for emission and absorption of planar photons by non-relativistic atomic electrons do not generally change, neither by lowering the dimension of space-time, (3+1)D to (2+1)D, nor by virtue of the (topological) mass gap. On the other hand, we have seen that Planck law has to be substantially modified in the planar world. For instance, in the massless case a frequency-dependent multiplicative factor was found to contrast it from its 4-dimensional counterpart. Moreover, the topologically massive case was worked out and shown to recover massless result as the mass parameter vanishes.

At this point, we should also discuss about possible conditions that could provide the dynamics of genuine photons as they were planar ones. Indeed, under certain circumstances (low temperature, high magnetic field, and so forth) electrons are shown to perform a (quasi) planar dynamics, which is the case when quantum Hall effect and high-Tc superconductivity come about. Therefore, we may wonder whether some physical conditions, unknown to us at the present, could lead us to a similar scenario for photons. For instance, inside a superconductor vector bosons acquire mass by virtue of the Meissner effect. Now, if we intend to describe the electrodynamics inside such a sample by means of Maxwell-Chern-Simons model (the anyonic superconductivity proposal, for example), then the observed vector gauge excitations should be expected to behave, at some extent, as much as they were “planar photons”. Perhaps, some experiments towards the measurement of the interference between electric and magnetic fields associated to such excitations could yield to results well-fit by massive Pauli-Jordan commutators; or still they could determine the energy distribution of a “bath” of photons and confirm the present results concerning this issue.
In forthcoming communications, we intend to go further into the abovementioned analysis and investigate possible modifications on other physical results whenever dealing with (2+1)d electrodynamic-like models, namely those subjects concerning emission, absorption, and scattering of radiation.

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