Vanishing $\text{Str } M^2$ in the presence of anomalous $U_A(1)$

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Abstract

We show that the presence of an anomalous $U_A(1)$ factor in the gauge group of string-derived models may have the new and important phenomenological consequence of allowing the vanishing of $\text{Str } M^2$ in the “shifted” vacuum, that results in the process of cancelling the anomalous $U_A(1)$. The feasibility of this effect seems to be enhanced by a vanishing vacuum energy, and by a “small” value of $\text{Str } M^2$ in the original vacuum. In the class of free-fermionic models with vanishing vacuum energy that we focus on, a necessary condition for this mechanism to be effective is that $\text{Str } M^2 > 0$ in the original vacuum. A vanishing $\text{Str } M^2$ ameliorates the cosmological constant problem and is a necessary element in the stability of the no-scale mechanism.
Many realistic string models built to date, especially all those constructed within the free-fermionic formulation, contain U(1) factors in their gauge groups with non-vanishing traces. These so-called anomalous U(1)’s may be understood as the result of truncating the string spectrum to the massless sector (over which the traces are taken), and do not imply an anomaly in the full string model. Nonetheless, the low-energy effective theory takes notice of this effect in the form of a Fayet-Iliopoulos contribution to the D-term of the anomalous U_A(1). This contribution can be calculated by examining the conditions under which the anomaly will cancel in the full theory, and is given by a one-loop string calculation:

\[ D_A \rightarrow D_A + \epsilon; \quad D_A = \sum_i q^i_A |\phi^i|, \quad \epsilon = \frac{q^2 M^2}{192 \pi^2} \text{Tr} U_A , \]  

where \( q^i_A \) is the charge of the \( \phi^i \) field under \( U_A(1) \), and \( M \approx 10^{18} \text{GeV} \) is the relevant mass scale (or reduced Planck mass). The non-zero shift in \( D_A \), if not compensated, has the dire consequence of breaking supersymmetry at the Planck scale. Interestingly enough, in all instances where anomalous U(1)’s have been reported in consistent models, it has been always possible to give vacuum expectation values (vevs) to some scalar fields charged under \( U_A(1) \), such that \( D_A + \epsilon \) vanishes at the nearby vacuum determined by these vevs. This shifted vacuum is “nearby” because typically \( \langle \phi \rangle \sim \sqrt{\epsilon} = \mathcal{O}(\frac{1}{10} M) \).

The vacuum shift needed to cancel the anomalous U(1) is not without consequence. Indeed, one now has to consider carefully other pieces of the scalar potential that in the absence of \( U_A(1) \) would have vanished automatically, with all vevs equal to zero. These are the usual F- and D-flatness conditions:

\[ \langle \partial W / \partial \phi^i \rangle = 0, \quad \langle D_a \rangle = \sum_i q^i_a |\langle \phi^i \rangle|^2 = 0 , \]  

where the \( a \) label runs over all non-anomalous U(1) factors in the gauge group. In practice, several of the U(1) factors may have non-zero traces, and one needs to find the linear combination which is truly anomalous, leaving the remaining orthogonal linear combinations traceless. This anomalous combination is given by:

\[ U_A = \frac{1}{\text{Tr} U_A} \sum_i [\text{Tr} U_i] U_i , \]  

with \( \text{Tr} U_A = \sum_i [\text{Tr} U_i]^2 \). The result of this exercise is the restoration of supersymmetry in the shifted vacuum, which is not fully specified, as typically there are many more scalar fields capable of obtaining vevs than constraint equations restricting the values of these vevs. Moreover, this mechanism has an unsuspected phenomenological benefit, as the ratio:

\[ \frac{\langle \phi \rangle}{M} \sim \frac{\sqrt{\epsilon}}{M} \sim \frac{1}{10} , \]  

can be used to generate hierarchies in the fermion mass matrices, when the corresponding Yukawa couplings appear at the non-renormalizable level (see e.g., Ref. [1]).
In this note we would like to point out a new and important consequence of the vacuum shifting required to cancel the anomalous $U_A(1)$. This pertains to the calculation of the quantity $\text{Str} \mathcal{M}^2$ in spontaneously broken supergravity models, as those obtained in the string models mentioned above. This quantity is particularly important because it parametrizes a one-loop quadratic divergence in the scalar potential

$$\frac{1}{32\pi^2} \text{Str} \mathcal{M}^2 M_{P1}^2, \quad \text{Str} \mathcal{M}^2 = 2Q m_{3/2}^2,$$

where the second expression defines the quantity $Q$ and makes explicit the dependence on the supersymmetry-breaking order-parameter $m_{3/2}$. After supersymmetry breaking, if $Q \neq 0$ one would generate a cosmological constant at high scales, which is unlikely to be cancelled by lower energy effects. This should be motivation enough to seek models with vanishing values of $Q$. Yet, there are further reasons to desire a suppressed value of $Q$. The gravitino mass may remain as an undetermined parameter down to low energies, as advocated in the context of no-scale supergravity \cite{6, 7, 8}, where the tree-level cosmological constant is naturally zero ($V_0 = 0$). If the condition $\text{Str} \mathcal{M}^2 = 0$ is satisfied \cite{7}, then the scalar potential does not depend on large mass scales, and lower energy dynamical effects may lead to the determination of the gravitino mass via the no-scale mechanism \cite{4}. The no-scale supergravity program has been recently extended to string models \cite{5, 10}, where the most pressing question has become the search for models or mechanisms by which $Q$ may be sufficiently suppressed.

Ignoring the effect of the anomalous $U_A(1)$, calculations of $Q$ in a few string models exist. Except for the “toy” models considered in Ref. \cite{5}, no realistic model has yet been found where $Q$ vanishes. Whatever effects the cancellation of $U_A(1)$ may have, these will probably be “small”, simply because the vacuum is shifted to a nearby one. In fact, the non-trivial dimensionless numbers one can form with the vevs are proportional to the small ratio in Eq. (4). As we will see, this expectation is borne out in specific model-dependent calculations. Therefore, if $Q$ itself may be shifted towards zero as a consequence of the vacuum shift, $Q_0$ (the initial value of $Q$) better be “close” to zero to begin with. The known model with $Q_0$ closest to zero is that derived in Ref. \cite{11}, which gives $Q_0 = 4$ (and $V_0 = 0$) \cite{10}. Other known calculations of $|Q_0|$ yield much larger values, perhaps not unrelated to the fact that $V_0 \neq 0$ in those models.

The calculation of $Q$ can be performed in two complementary ways: (i) one can employ an explicit formula that depends on the Kähler function ($G = K + \ln|W|^2$) and the gauge kinetic function, or (ii) one can compute the supersymmetry-breaking spectrum explicitly and then calculate $\text{Str} \mathcal{M}^2$ directly. Here we follow the second approach, as we find it physically more intuitive. For concreteness, we will restrict our explicit calculations to free-fermionic string models, although we expect that similar effects should occur in other string constructions. Furthermore, we will consider the class of (level-one) free-fermionic models with vanishing vacuum energy. In such class
of models the Kähler potential is generically given by [10]

\[ K = -\ln(S + \bar{S}) - \ln \left( (\tau + \bar{\tau})^2 - \sum_{i}^{n_{U}} [\alpha_i^{(1)} + \bar{\alpha}_i^{(1)}]^2 \right) + \sum_{i}^{n_{U}} \alpha_i^{(2)} \bar{\alpha}_i^{(2)} + \sum_{i}^{n_{U}} \alpha_i^{(3)} \bar{\alpha}_i^{(3)} + \sum_{i}^{n_{\beta_i(1)}} \beta_i^{(1)} \bar{\beta}_i^{(1)} \]

where \( S \) and \( \tau \) represent the dilaton and modulus fields, \( \alpha_i^{(1,2,3)}, \beta_i^{(1,2,3)} \) represent untwisted and twisted matter fields in each of the three “sets” into which the spectrum divides itself in this class of models. The number of untwisted and twisted fields in each of these sets is represented by \( n_{U1,2,3}, n_{T1,2,3} \), and vary from model to model. Equation (6) indicates that the modulus field \( \tau \) corresponds to a field in the first untwisted set. If we ignore the possible presence of the anomalous U(1) level) vacuum energy can be readily shown to vanish (\( V_0 = 0 \)) for the Kähler potential in Eq. (6) [10]. As indicated above, we calculate \( \text{Str} \mathcal{M}^2 = \sum_{j} (-1)^{2j}(2j + 1) \mathcal{M}_{j}^2 \) directly. With the explicit form of \( K \) above, we can calculate each of the mass matrices separately [10]. The masses of the complex scalars \( (j = 0) \) are given by the following multiples of \( m_{3/2} \)

\[
\begin{align*}
U^{(1)}: \quad & \alpha_i^{(1)} = 0 \\
U^{(2)}: \quad & \alpha_i^{(2)} = 1 \\
U^{(3)}: \quad & \alpha_i^{(3)} = 1 \\
T^{(1)}: \quad & \beta_i^{(1)} = 1 \\
T^{(2)}: \quad & \beta_i^{(2)} = 0 \\
T^{(3)}: \quad & \beta_i^{(3)} = 0
\end{align*}
\]

and contribute to the supertrace (in units of \( m_{3/2}^2 \)) in the amount of \( 2(n_{U2} + n_{U3} + n_{T1}) \). The masses of the Majorana fermions \( (j = 1/2) \) are given by

\[
m_{\alpha_i^{(2)}} = m_{\alpha_i^{(3)}} = 0 , \quad m_{\beta_i^{(1)}} = m_{\beta_i^{(2)}} = m_{\beta_i^{(3)}} = 0 ,
\]

and by the eigenvalues of the following mass matrix

\[
(M_f)_{IJ} = m_{3/2} S \begin{pmatrix}
S & \tau & \alpha_i^{(1)} \\
\tau & -\sqrt{2}/3 & 0 \\
\alpha_i^{(1)} & 0 & \delta_{ij}
\end{pmatrix}
\]

That is

\[
m_{\eta_\perp} = m_{3/2} , \quad m_{\alpha_i^{(1)}} = m_{3/2}
\]

where \( \eta_\perp \) is the state orthogonal to the massless goldstino (given by \( \eta \propto S + \sqrt{2} \tau \)). Thus, the Majorana fermions contribute \(-2(1 + n_{U1})\) to \( \text{Str} \mathcal{M}^2 \). The Majorana gaugino masses are all equal to \( m_{3/2} \) and contribute \(-2d_f\), where \( d_f \) is the dimension of the gauge group. Finally the gravitino contributes \(-4\). Putting it all together gives

\[
Q_0 = n_{U2} + n_{U3} + n_{T1} - n_{U1} - d_f - 3 .
\]
This expression for $Q_0$ may be positive or negative, and will be “small” if there is a definite correlation among the numbers of fields in the different sets and the dimension of the gauge group. This form of the expression (i.e., the relative signs of the various terms) depends on the number of moduli, which in turn determine the vacuum energy $(V_0)$. In specific examples (not considered here) one can see that if $V_0 \neq 0$, then the relative signs in $Q_0$ do not favor a large cancellation among the various contributions, resulting in large values of $|Q_0|$. If $V_0 = 0$, the signs are well balanced and a small value of $Q_0$ is possible. In the only known model of this class where $V_0 = 0$ one has $n_{U_1} = 13$, $n_{U_2} = 14$, $n_{U_3} = 16$; $n_{T_1} = 80$, $n_{T_2} = 80$, $n_{T_3} = 68$; and $d_f = 90$, and therefore

$$Q_0 = 14 + 16 + 80 - 13 - 90 - 3 = 110 - 106 = 4.$$  \hspace{1cm} (12)

We say $Q_0 = 4$ is “small” in the sense that it is only 2% of the total obtained if the terms had been added in absolute value.

When one takes into account the presence of the anomalous $U_A$, Eqs. \((1,2)\) need to be satisfied in non-trivial ways. The possible vacuum expectation values of the scalar fields are further constrained by the desire to maintain $V_0 = 0$. This can be assured by allowing non-zero vevs only for the scalar fields that do not acquire supersymmetry-breaking masses, i.e., those in the $U^{(1)}$, $T^{(2)}$, and $T^{(3)}$ sets (see Eq. \((7)\)):

$$\langle \alpha_{i}^{(2)} \rangle = \langle \alpha_{i}^{(3)} \rangle = \langle \beta_{i}^{(1)} \rangle = 0.$$  \hspace{1cm} (13)

To determine $Q$ in the presence of non-zero vevs, we need to revisit the contributions to $\text{Str} \mathcal{M}^2$. The scalar masses are not affected by shifts in vevs. The gaugino masses are also unaffected, as (at tree-level) they only depend on the dilaton contribution to the gauge kinetic function. The gravitino contribution is also unaffected since $m_{3/2}$ is an overall factor. All we need to consider are the Majorana fermion masses, which are given by \[3\]

$$(M_f)_{IJ} = m_{3/2} \left( G_{IJ} - G_{IJK}G^{JK} + \frac{1}{3} G_{I}G_{J} \right).$$  \hspace{1cm} (14)

Examining the Kähler potential in Eq. \[(9)\], we see that we still have

$$m_{\alpha_{i}^{(2)}} = m_{\alpha_{i}^{(3)}} = m_{\beta_{i}^{(1)}} = 0,$$  \hspace{1cm} (15)

whereas the matrix in Eq. \[(9)\] is extended to also include the $\beta_{i}^{(2)}$, $\beta_{i}^{(3)}$ fields. In what follows we will make the simplifying assumption that the vev shifts are small compared to the vev of the modulus field: $\langle \alpha, \beta \rangle / \langle \tau \rangle \ll 1$. This assumption is motivated by the fact that one would expect $\langle \tau \rangle \sim M$, whereas the anomalous $U_A(1)$ cancellation

\footnote{It is amusing to note that the model of Ref. \[11\] contains extra matter representations $(10, 10)$ sufficient to postpone the gauge coupling unification scale naturally up to the string scale $M$.}

\footnote{This is true at tree-level. The one-loop string effect that gives the Fayet-Iliopoulos contribution to $D_\Lambda$, induces mass shifts $\sim \sqrt{\epsilon}$ for all scalars charged under $U_A(1)$. These mass shifts do not contribute to $\text{Str} \mathcal{M}^2$ since, when $D_\Lambda$ is cancelled, compensating one-loop fermionic mass shifts are generated, such that supersymmetry is restored in the shifted vacuum \[12\].}
generically implies \( \langle \alpha, \beta \rangle \sim \frac{1}{10^2} M \). (Note that the modulus field is a gauge singlet and does not participate in the \( U_A(1) \) cancellation mechanism.) Should this expectation not be realized, the following calculations would need to be performed numerically. The resulting symmetric matrix of properly normalized fields is given by

\[
(M_f)_{IJ} = m_{3/2} \begin{pmatrix}
S & \tau & \alpha_1^{(1)} & \alpha_2^{(1)} & \beta_1^{(2,3)} & \beta_2^{(2,3)} \\
\tau & \frac{2}{3} - \sqrt{2} (1 + X + \frac{1}{2} Y) & \frac{1}{3} (1 + 5X + 4Y) & \frac{1}{3} \sqrt{X_1} & -\frac{1}{3} \sqrt{Y_1} & \frac{2}{3} \sqrt{Y_2} \\
\alpha_1^{(1)} & \frac{1}{3} \sqrt{X_1} & -\frac{4}{3} \sqrt{X_2} & \frac{1}{3} \sqrt{Y_1} & -\frac{1}{3} \sqrt{Y_2} & \frac{2}{3} \sqrt{Y_2} \\
\alpha_2^{(1)} & -\frac{1}{3} \sqrt{X_1} & \frac{1}{3} \sqrt{X_2} & \frac{2}{3} \sqrt{Y_1} & -\frac{1}{3} \sqrt{Y_2} & \frac{2}{3} \sqrt{Y_2} \\
\beta_1^{(2,3)} & \frac{1}{3} \sqrt{Y_1} & -\frac{1}{3} \sqrt{Y_2} & \frac{2}{3} \sqrt{Y_2} & -\frac{1}{3} \sqrt{Y_2} & \frac{2}{3} \sqrt{Y_2} \\
\beta_2^{(2,3)} & \frac{1}{3} \sqrt{Y_1} & -\frac{1}{3} \sqrt{Y_2} & \frac{2}{3} \sqrt{Y_2} & -\frac{1}{3} \sqrt{Y_2} & \frac{2}{3} \sqrt{Y_2}
\end{pmatrix},
\]

Here we have restricted the number of generic fields to the minimal that show the emergent pattern. In this matrix we have defined the ratios

\[
X_i = \frac{(\alpha_i^{(1)} + \bar{\alpha}_i^{(1)})^2}{(\tau + \bar{\tau})^2}, \quad Y_i = \frac{\beta_i^{(2,3)} \bar{\beta}_i^{(2,3)}}{\tau + \bar{\tau}},
\]

and the sums of ratios

\[
X = \sum_i X_i, \quad Y = \sum_i Y_i,
\]

which run over all the fields in the first untwisted set \( \alpha_i^{(1)} \) and the second and third twisted sets \( \beta_i^{(2)}, \beta_i^{(3)} \). Also, the coefficients of the higher-order off-diagonal terms \( \sqrt{X_i Y_j} \) in Eq. (16) have been omitted. Note that the \( X_i \) and \( Y_i \) corresponding to fields charged under the Standard Model quantum numbers vanish on phenomenological grounds. For our present purposes, it is enough to compute the trace of the square of the Majorana mass matrix. This amounts to summing over the squares of all of the elements of the \( M_{IJ} \) mass matrix. We obtain

\[
\text{Tr} M^2_{IJ} = m_{3/2}^2 \left[ 1 + n_{U_1} + \frac{14}{3} X + \frac{5}{3} Y \right],
\]

keeping terms to only first order in \( X \) and \( Y \), since the twisted sector Kähler potential is only known to first order. The expression for \( Q \) then becomes

\[
Q = Q_0 - \Delta Q = Q_0 - \frac{1}{3} (14X + 5Y),
\]

with \( Q_0 \) as given in Eq. (11). Since \( X \) and \( Y \) are both positive, we conclude that a necessary condition for a possibly vanishing \( Q \) in the shifted vacuum, is that \( Q_0 > 0 \) in the original vacuum. If \( Q_0 \) is positive and “small”, then \( Q \) may vanish in a suitably chosen shifted vacuum.

The choice of shifted vacuum is a rather model-dependent exercise, as the possible choices of vacuum expectation values are restricted by the D- and F-flatness.
constraints in Eqs. (1,2), plus the $V_0 = 0$ constraint in Eq. (13). The idea is to scan the parameter space of simultaneous solutions to these set of equations, looking for values of the vevs which bring $Q$ closest to zero. Is this a feasible possibility? At this point we can say that we have carried out this exercise in the model of Ref. [11], which has $Q_0 = 4$, and have found sets of values for the vevs that indeed shift $Q_0$ down to zero. Details of this particular application are given elsewhere [13]. We can however show that this specific case is rather typical. Let us assume that all vevs have the same magnitude: $\langle \phi \rangle \sim \sqrt{\epsilon}$, and consider the expressions for $X$ and $Y$:

$$X \sim (n_{U_1}) \gamma, \quad Y \sim (n_{T_2} + n_{T_3}) \sqrt{\gamma} \frac{\sqrt{\epsilon}}{M},$$

(21)

where we have defined

$$\gamma = \left( \frac{\sqrt{\epsilon}}{\tau + \bar{\tau}} \right)^2.$$

(22)

Taking typical values of the number of untwisted and twisted fields in a set ($n_{U_1} \sim 10$, $n_T \sim 80$) we see that

$$\frac{1}{3}(14X + 5Y) \sim \frac{140}{3} \gamma + \frac{80}{3} \sqrt{\gamma},$$

(23)

where we have set $\sqrt{\epsilon}/M \sim \frac{1}{10}$. Plugging in numbers one finds that $\frac{1}{3}(14X + 5Y) \sim \text{few}$ for $\gamma \sim 0.01$. Since we expect $\langle \tau \rangle \sim M$, and thus $\gamma \sim (\sqrt{\epsilon}/M)^2 \sim 0.01$, we see that this mechanism is quite feasible, if $Q_0 \sim \text{few}$. The explicit model-dependent calculations in Ref. [13] yield similar results.

Let us point out that restricting the shifted vacua to those where $Q$ nearly vanishes is a phenomenological procedure. In fact, adding to the scalar potential the one-loop contribution in Eq. (2) does not shift the vevs in any noticeable way, since this contribution ($\sim m_{3/2}^2 M^2$) is so much smaller than that coming from $D_A^2$ ($\sim \epsilon^2 \phi^2 \sim M^4$). One may speculate that in the full string model new (loop) contributions to the scalar potential arise that make the chosen vacuum (where $Q$ vanishes) energetically preferred. Our phenomenological procedure would then be the “effective” way of realizing such scenario. These corrections will further shift the value of $Q$, as the Kähler potential and the gauge kinetic function receive string loop corrections. Thus, it is not clear that at this level of approximation one should be fixated on an exactly vanishing value of $Q$. It has been suggested [3], that in analogy with the string loop corrections to the gauge kinetic function, which vanish at two and higher loops, perhaps $Q$ would behave similarly. In this case, the higher-loop quadratic divergences pointed out recently in Ref. [14], would effectively vanish in the full string theory.

In sum, we have shown that the presence of an anomalous $U_A(1)$ factor in the gauge group of string models may have yet another welcomed phenomenological consequence, since its cancellation may allow the vanishing of $\text{Str} \mathcal{M}^2$ in the shifted vacuum. This result seems to be the more likely if the vacuum energy vanishes, and if $\text{Str} \mathcal{M}^2$ is “small” in the original vacuum. In the class of free-fermionic models that we have focused on, $\text{Str} \mathcal{M}^2$ must also be positive in the original vacuum. Evidently, having $\text{Str} \mathcal{M}^2 = 0$ has important consequences for the cosmological constant problem, and for the stability of the no-scale mechanism.
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