Multiparton interactions: Theory and experimental findings

Markus Diehl
Deutsches Elektronen-Synchroton (DESY)

DIS 2013, Marseille
22 April 2013
Hadron-hadron collisions

- standard description based on factorization formulae

\[ \text{cross sect} = \text{parton distributions} \times \text{parton-level cross sect} \]

example: \( Z \) production

\[ pp \rightarrow Z + X \rightarrow \ell^+\ell^- + X \]

- factorization formulae are for inclusive cross sections \( pp \rightarrow Y + X \)
  where \( Y \) = produced in parton-level scattering, specified in detail
  \( X \) = summed over, no details
Hadron-hadron collisions

- standard description based on factorization formulae
  
  \[ \text{cross sect} = \text{parton distributions} \times \text{parton-level cross sect} \]

  example: \( Z \) production

  \[ pp \rightarrow Z + X \rightarrow \ell^+\ell^- + X \]

- factorization formulae are for inclusive cross sections \( pp \rightarrow Y + X \)
  where \( Y \) = produced in parton-level scattering, specified in detail
  \( X \) = summed over, no details

- have also interactions between “spectator” partons
  their effects cancel in inclusive cross sections thanks to unitarity
  but they affect the final state (namely \( X \))
Multiparton interactions (MPI)

- secondary (and tertiary etc.) interactions generically take place in hadron-hadron collisions
- predominantly low-$p_T$ scattering
  - underlying event (UE)
- at high collision energy (Tevatron, LHC) can be hard
  - multiple hard scattering

"MPI" used either for mult. hard scatt. or for hard+soft

- many studies:
  - theory: phenomenological studies, theoretical foundations (recent activity)
  - experiment: ISR, SPS, HERA (photoproduction), Tevatron, LHC
  - Monte Carlo generators: Pythia, Herwig++, Sherpa
- expected to be important for many processes at LHC

see e.g. workshops: http://mpi11.desy.de; MPI@LHC 2012, CERN
Relevance for LHC

example: $pp \rightarrow H + Z \rightarrow b\bar{b} + Z$  

- multiple interactions contribute to signal and background

analogous for $pp \rightarrow H + W \rightarrow b\bar{b} + W$ study for Tevatron: Bandurin et al,
Double vs. single hard scattering

- double hard scattering:
  net $p_T$ of produced system ($Z$ or $b\bar{b}$ pair) $\ll$ hard scale $Q$ (e.g. $M_Z$)
- single hard scattering:
  $p_T$ distribution up to values $\sim Q$
- no generic suppression for transv. mom. $\ll Q$:

$$\frac{d\sigma_{\text{single}}}{d^2p_{T,Z}d^2p_{T,b\bar{b}}} \sim \frac{d\sigma_{\text{double}}}{d^2p_{T,Z}d^2p_{T,b\bar{b}}} \sim \frac{\Lambda^2}{Q^2}$$

but since single scattering populates larger phase space:

$$\sigma_{\text{single}} \sim \frac{1}{Q^2} \gg \sigma_{\text{double}} \sim \frac{\Lambda^2}{Q^4}$$

MD, Schäfer 2011; Blok, Dokshitzer, Frankfurt, Strikman 2011
Double vs. single hard scattering

- double hard scattering:
  net $p_T$ of produced system ($Z$ or $b\bar{b}$ pair) $\ll$ hard scale $Q$ (e.g. $M_Z$)

- single hard scattering:
  $p_T$ distribution up to values $\sim Q$

- no generic suppression for transv. mom. $\ll Q$:

$$\frac{d\sigma_{\text{single}}}{d^2p_{T,Z}} \sim \frac{d\sigma_{\text{double}}}{d^2p_{T,b\bar{b}}} \sim \frac{\Lambda^2}{Q^2}$$

but since single scattering populates larger phase space:

$$\sigma_{\text{single}} \sim \frac{1}{Q^2} \gg \sigma_{\text{double}} \sim \frac{\Lambda^2}{Q^4}$$

at small $x$ double scattering enhanced due to growth of parton densities

MD, Schäfer 2011; Blok, Dokshitzer, Frankfurt, Strikman 2011
Double parton scattering: cross section formula

\[ \frac{d\sigma_{\text{double}}}{dx_1 \, dx_1 \, dx_2 \, d\bar{x}_2} = \frac{1}{C} \, \hat{\sigma}_1 \, \hat{\sigma}_2 \int d^2b \, F(x_1, x_2, b) \, F(\bar{x}_1, \bar{x}_2, b) \]

- \( C \) = combinatorial factor
- \( \hat{\sigma}_i \) = parton-level cross section
- \( F(x_1, x_2, b) \) = double parton distribution (DPD)
- \( b \) = transv. distance between partons

- follows from Feynman graphs and hard-scattering approximation
  no semi-classical approximation required
- can make \( \hat{\sigma}_i \) differential in further variables (e.g. for jet pairs)
- can extend \( \hat{\sigma}_i \) to higher orders in \( \alpha_s \)
  get usual convolution integrals over \( x_i \) in \( \hat{\sigma}_i \) and \( F \)
Double parton scattering: cross section formula

Paver, Treleani 1982, 1984; Mekhfi 1985, . . ., MD, Ostermeier, Schäfer 2012

\[
\frac{d\sigma_{\text{double}}}{dx_1 dx_1' dx_2 dx_2'} = \frac{1}{C} \hat{\sigma}_1 \hat{\sigma}_2 \int d^2 b \ F(x_1, x_2, b) F(\bar{x}_1, \bar{x}_2, b)
\]

- \( C = \) combinatorial factor
- \( \hat{\sigma}_i = \) parton-level cross section
- \( F(x_1, x_2, b) = \) double parton distribution (DPD)
- \( b = \) transv. distance between partons

- follows from Feynman graphs and hard-scattering approximation
  no semi-classical approximation required
- can make \( \hat{\sigma}_i \) differential in further variables (e.g. for jet pairs)
- can extend \( \hat{\sigma}_i \) to higher orders in \( \alpha_s \)
  get usual convolution integrals over \( x_i \) in \( \hat{\sigma}_i \) and \( F \)
- Fourier transform: \( F(x_1, x_2, b) \rightarrow F(x_1, x_2, r) \)
  \( r = \) mismatch between parton momenta in scatt. amplitude
  and its conjugate

sometimes called “generalized parton distribution (GPD)”
Double parton scattering: pocket formula

- if two-parton density factorizes as
  \[ F(x_1, x_2, b) = f(x_1) f(x_2) G(b) \]
  where \( f(x_i) = \) usual PDF

- if assume same \( G(b) \) for all parton types
  then cross sect. formula turns into

\[
\frac{d\sigma_{\text{double}}}{dx_1 \, d\bar{x}_1 \, dx_2 \, d\bar{x}_2} = \frac{1}{C} \frac{d\sigma_1}{dx_1 \, d\bar{x}_1} \, \frac{d\sigma_2}{x_2 \, \bar{x}_2} \, \frac{1}{\sigma_{\text{eff}}}
\]

with \( \sigma_{\text{eff}} = 1/\int d^2b \, G^2(b) \)

\( \sim \) scatters are completely independent

- also works for \( \sigma_i \) at higher orders in \( \alpha_s \)

- requires independent event selection criteria for particles produced in scatters 1 and 2
Double parton scattering: pocket formula

- if two-parton density factorizes as
  \[ F(x_1, x_2, b) = f(x_1) f(x_2) G(b) \]
  where \( f(x_i) \) = usual PDF

- if assume same \( G(b) \) for all parton types
  then cross sect. formula turns into

  \[
  \frac{d\sigma_{\text{double}}}{dx_1 \, dx_1 \, dx_2 \, dx_2} = \frac{1}{C} \frac{d\sigma_1}{dx_1 \, dx_1} \frac{d\sigma_2}{x_2 \, x_2} \frac{1}{\sigma_{\text{eff}}}
  \]

  with \( \sigma_{\text{eff}} = 1/\int d^2b \, G^2(b) \)
  \( \sim \) scatters are completely independent

- ansatz can be extended from 2 to \( n \) hard scatters

- underlies bulk of phenomenol. estimates

- underlies implementation of MPI in \textsc{Pythia}, \textsc{Herwig++}
  and \textsc{Sherpa} (\textsc{AMISIC++})
  together with model for non-perturbative region below some \( p_T^{\text{min}} \)
  and for subsequent soft interactions \( \sim \) “color reconnection”
Double parton scattering: pocket formula

- If two-parton density factorizes as
  \[ F(x_1, x_2, b) = f(x_1) f(x_2) G(b) \]
  where \( f(x_i) = \) usual PDF

- If assume same \( G(b) \) for all parton types
  then cross sect. formula turns into
  \[
  \frac{d\sigma_{\text{double}}}{dx_1 \, dx_1 \, dx_2 \, dx_2} = \frac{1}{C} \frac{d\sigma_1}{dx_1 \, dx_1} \frac{d\sigma_2}{x_2 \, x_2} \frac{1}{\sigma_{\text{eff}}}
  \]
  with \( \sigma_{\text{eff}} = \frac{1}{\int d^2 b \, G^2(b)} \)
  \[ \sim \] scatters are completely independent

- Relies on strong simplifications
  must expect that is only approximate
  \[ \sim \] suitable as guideline, but not when precision is needed
Underlying event studies at LHC

choose observables sensitive to soft particle production
Underlying event studies at LHC

used for tuning of Monte Carlo parameters

Plots: S. Wahrmund for the ATLAS Collaboration, MPI@LHC 2012

see parallel talks: D Kar (WG4, Tue 15:00), K Mazumdar (WG2, Tue 15:20), O Kepka (WG2, Thu 8:30)
apologies if here or in the following I have missed references to parallel sessions!
Determinations of $\sigma_{\text{eff}}$ from double hard scattering

- all determinations in same ballpark for $\sigma_{\text{eff}}$
- no clear variation with kinematics reported so far
Double parton scattering in $pp \rightarrow W + 2 \text{ jets} + X$

$W + \text{exactly 2 jets with } p_T > 20 \text{ GeV}$

- **ATLAS fits distribution to two templates**
  - A (single hard scatt.) and B (double hard scatt.)
  - extract double scattering fraction $f_{\text{DP}} = 0.08 \pm 0.01 \pm 0.02$

see parallel talks: P Bartalini (CMS) and M Myska (ATLAS), WG2, Tue 17:10 and 17:30

M. Diehl  Multiparton interactions  Experiment  16
Double parton scattering in \( pp \to W + 2 \text{ jets} + X \)

\[
\Delta S = \angle (p_T W, p_{T_{\text{jet}1}} + p_{T_{\text{jet}2}})
\]

for single hard scattering peaked at \( \pi \)

for double hard scattering flat if two scatters are completely independent

but need not be flat if have correlations between two partons in proton (see later)

special thanks to S Bansal, P Bartalini and H Jung for discussions
LHCb: double charm production ($\bar{c}ccc\bar{c}$)

Plots: LHCb, arXiv:1205.0975

J/$\Psi$ + D channels: $\sigma$ much larger than computed for single hard scatt.

size of cross sect. ratio in ballpark of $\sigma_{\text{eff}}$ from other processes

double J/$\Psi$ production: similar size estimated for single and double scattering

LHCb, arXiv:1109.0963 and several theory papers
LHCb: double charm production (c\bar{c}c\bar{c})

Plot: LHCb, arXiv:1205.0975

see parallel talks (theory): N Zotov and R Maciula, WG4/5, Wed 15:00 and 15:20
Parton correlations

pocket formula $\sigma_{\text{double}} = (\sigma_1 \sigma_2)/(C \sigma_{\text{eff}})$ is invalid if there are correlations between

- $x_1$ and $x_2$ of partons
  - most obvious: energy conservation $\Rightarrow x_1 + x_2 \leq 1$
  - often used: $F(x_1, x_2, b) = f(x_1) f(x_2) (1 - x_1 - x_2)^n G(b)$
  - significant $x_1 - x_2$ correlations found in constituent quark model

- $x_i$ and $b$
  - even for single partons see correlations between $x$ and $b$ distribution
    - HERA results on $\gamma p \rightarrow J/\Psi p$ give
      $$\langle b^2 \rangle \propto \text{const} + 4\alpha' \log(1/x) \quad \text{with} \quad \alpha' \approx (0.08 \text{ fm})^2$$
      for gluons with $x \approx 10^{-3}$
    - lattice simulations $\rightarrow$ strong decrease of $\langle b^2 \rangle$ with $x$ above $\sim 0.1$
    - precise mapping of single-parton distributions $f(x, b)$ over wide $x$ range in future lepton-proton experiments
      JLab 12, COMPASS, EIC, LHeC
    - $\rightarrow$ parallel talks in WG6 and WG7

plausible to expect similar correlations in two-parton distributions
Consequence for multiple interactions

- indications for decrease of $\langle b^2 \rangle$ with $x$
- if interaction 1 produces high-mass system
  $\rightarrow$ have large $x_1, \bar{x}_1$
  $\rightarrow$ smaller $b$, more central collision
  $\rightarrow$ secondary interactions enhanced

Frankfurt, Strikman, Weiss 2003
study in Pythia: Corke, Sjöstrand 2011

$\sigma_{tot}$ revisited

- exercise: assume absence of parton correlations
  and Gaussian $b$ distribution of single parton with average $\langle b^2 \rangle$

$$
\Rightarrow \quad \sigma_{eff} = 4\pi \langle b^2 \rangle = 41 \text{ mb} \quad \frac{\langle b^2 \rangle}{(0.57 \text{ fm})^2}
$$

determinations of $\langle b^2 \rangle$ range from $\sim (0.57 \text{ fm} - 0.67 \text{ fm})^2$
if $b$ distrib. is Fourier trf. of dipole then get extra factor $7/8$ in $\sigma_{eff}$
is $\gg \sigma_{eff} \sim 10$ to $20 \text{ mb}$ from experimental extractions
Parton spin correlations

- possible even in unpolarized proton
- detailed study for double Drell-Yan process \textit{(two gauge bosons)}
  \text{Kasemets, MD 2012}
  \begin{itemize}
    \item longitudinal pol. correlations $\rightarrow$ change $\sigma_{\text{double}}$
    \item correlations between transverse quark
      $\rightarrow$ azimuthal correlations between lepton decay planes of two bosons
    \item expect analogous effects with dijets instead of gauge bosons
  \end{itemize}
- strong spin correlations found in MIT bag model
  \text{Chang, Manohar, Waalewijn 2012}
- unknown: how important are spin correlations at small $x$?

Color correlations

- correlations between color of two partons suppressed by Sudakov logarithms
  \text{Mekhfi 1988; Manohar, Waalewijn 2012}

\[\text{Sudakov suppr. factor for quarks}\]

\begin{itemize}
  \item $\bar{U}_\mu U_\nu$
  \item $\bar{U}_\mu$ only
\end{itemize}

\text{Manohar, Waalewijn arXiv:1202.3794}
Behavior at small interparton distance

- for $b \ll 1/\Lambda$ in perturbative region $F(x_1, x_2, b)$ dominated by graphs with splitting of single parton

- find **strong** spin and color correlations between two partons  
  e.g. 100% correlation for longitudinal pol. of $q$ and $\bar{q}$

- can compute short-distance behavior:

\[ F(x_1, x_2, b) \sim \frac{1}{b^2} \text{ splitting fct } \otimes \text{ usual PDF} \]
Scale evolution

consider only distributions for partons without color correlation

- if define two-parton distributions as operator matrix elements
  in analogy with usual PDFs

\[ F(x_1, x_2, b; \mu) \sim \langle p|O_1(0; \mu)O_2(b; \mu)|p \rangle \quad f(x; \mu) \sim \langle p|O(0; \mu)|p \rangle \]

where \( O(b; \mu) = \) twist-two operator renormalized at scale \( \mu \)

- \( F(x_i, b) \) for \( b \neq 0 \):
  separate DGLAP evolution for partons 1 and 2

\[ \frac{d}{d \log \mu} F(x_i, b) = P \otimes x_1 F + P \otimes x_2 F \]

two independent parton cascades

- \( \int d^2b F(x_i, b) \):
  extra term from 2 \( \rightarrow \) 4 parton transition
  since \( F(x_i, b) \sim 1/b^2 \)

Kirschner 1979; Shelest, Snigirev, Zinovev 1982
Gaunt, Stirling 2009; Ceccopieri 2011

- which evolution eq. is relevant for double hard scattering?
Deeper problems with the splitting graphs

- contribution from splitting graphs in cross section gives divergent integrals
  \[ \int d^2b \, F(x_1, x_2, b) F(\bar{x}_1, \bar{x}_2, b) \sim \int db^2 / b^4 \]
- double counting problem between double scattering with splitting and single scattering at loop level

MD, Ostermeier, Schäfer 2011; Gaunt, Stirling 2011; Gaunt 2012
Blok, Dokshitzer, Frankfurt, Strikman 2011; Ryskin, Snigirev 2011, 2012
same problem for jets: Cacciari, Salam, Sapeta 2009

- possible solution:
  subtract splitting contribution from two-parton dist's when \( b \) is small
  will also modify their scale evolution; remains to be worked out

What is double parton scattering?
Deeper problems with the splitting graphs

- contribution from splitting graphs in cross section gives divergent integrals \( \int d^2b \, F(x_1, x_2, b) \, F(x_1, x_2, b) \sim \int db^2 / b^4 \)
- double counting problem between double scattering with splitting

also have graphs with single PDF for one proton and double PDFs for other

Blok, Dokshitzer, Frankfurt, Strikman 2011

What is double parton scattering?
Towards a factorization proof for double scattering?

- simplest case: double Drell-Yan
  processes with colored final states much more difficult

- open problem in ultraviolet region: parton splitting

- in infrared region: soft gluon exchange between scatters 1 and 2
  - partially under control
  - relevant to Sudakov factors
  - → parton showers

- open problem: soft gluon exchange in Glauber region

MD, Ostermeier, Schäfer 2011
Topics not discussed in this talk

- phenomenological studies for many pp processes
  double dijets: Domdey, Pirner, Wiedemann 2009; Berger, Jackson, Shaughnessy 2009
  W/Z + jets: Maina 2009, 2011
  like sign W pairs: Kulesza, Stirling 2009; Gaunt et al 2011; Berger et al 2011
  double Drell-Yan: Kom, Kulesza, Stirling 2011
  double charmonium: Kom, Kulesza, Stirling 2011; Baranov et al. 2011, 2012; Novoselov 2011
  double charm: Berezhnoy et al 2012; Łuszczak et al 2011; Maciula, Szczurek 2012, 2013

- pA collisions
  extra layer of complexity: two partons from same or from different nuclei in nucleus
  Calucci, Treleani 2009, 2012; Strikman, Vogelsang 2009; Blok, Strikman, Wiedemann 2011;
  d’Enterria, Snigirev 2012, 2013

- small x approach
  Flensburg et al 2011; Bartels, Ryskin 2011

- ‘ridge effect’ in pp and pA collisions
  CMS; ATLAS; ALICE; many theory papers

only list references after 2008
Summary

• multiparton interactions are ubiquitous in hadron-hadron collisions

• populate characteristic part of phase space there they can be substantial part of rate

• important theory progress for hard double scattering

• but many open questions:
  ★ size of correlations between partons
  ★ parton splitting contributions ↔ evolution of DPDs

• promising experimental developments:
  ★ different processes
  ★ kinematic distributions

• use $\sigma_{\text{eff}}$ as a handy tool, not as a precision instrument
  (if I have one free wish)