Impact of fracture length distribution on seepage characteristics of fractured rock masses

J Zhang¹, R Liu¹, L Yu¹,², * and D Liu¹

¹ State key laboratory for Geomechanics and Deep Underground Engineering, China University of Mining and Technology, Xuzhou, Jiangsu 221116, China
² State Key Laboratory of Disaster Prevention and Mitigation of Explosion and Impact, Army University of Engineering, Nanjing, Jiangsu 210007, China

*Corresponding author:
L Yu
Email address: yuliyuan@cumt.edu.cn

Abstract. This study presents a numerical investigation on the effect of fracture length distribution on the equivalent permeability of fractured rock masses. Three-dimension discrete fracture network (DFN) models with different fracture length distributions were generated using Monte Carlo method. The equivalent permeability was calculated by simplifying the 3D DFN models to equivalent pipe network (EPN) models. The results show that the equivalent permeability becomes less scattered with increasing side length of sub-models (Lₐ), showing a clear tendency of reaching a representative element volume (REV) size. The root mean square (RMS) = 0.2 is utilized as the threshold to determine the REV size. The REV size of the DFN model is approximately 4.17×10⁴ m³, in which the power exponent (α) of fracture length distribution is 4.5. The fracture density (P₃₂) decreases exponentially with the increment of α. This is because the number of short fractures in the DFN model with a larger α is larger. The P₃₂ of the DFN model is positively correlated to the equivalent permeability.

1. Introduction

The techniques of modelling hydraulic behaviors of fractured rock masses have been widely applied to geothermal exploitation, reservoir stimulation by hydraulic stimulation, and high-level nuclear waste repository (Ayden, 2000; Rutqvist and Stephansson, 2003; Tsang et al., 2005; Taron et al., 2009; Gale et al., 2014; Tsang et al., 2015; Sun et al., 2017; Zhang et al., 2019). Owing to multi-stochastic fracture patterns incorporated in the model and computational limitations, hydraulic simulations of fractured rock masses are often simplified to two-dimension problems, which underestimate the conductivity/permeability of fracture networks (Nick et al., 2011; Rutqvist et al., 2013; Huang et al., 2016). It is thus important to characterize and model fluid flow through fracture networks using three-dimensional models.

Borgos et al. (2000) demonstrated that the deviations from ideal spatial process of fracture and size frequency distribution without power law scaling may arise the discrepancies of extrapolating
one-dimensional fracture size distribution to higher dimensional samples. They presented new approaches to the difficulties of extrapolation in the theory. Perfect et al. (2006) presented a theoretical approach for generating two-dimensional multifractal saturated hydraulic conductivity fields, which seems to be relatively straightforward extended to three-dimension. Lang et al. (2014) described a method to determine the full permeability tensor of three-dimension discrete fracture model, which is based on the element-wise averaging of pressure and flux. Huang et al. (2016) developed a numerical method to simulate fluid flow through three-dimension fracture networks by solving the Reynolds equation using Galerkin method. Iglauer et al. (2016) imaged wet sandstones with X-ray microcomputed tomography (micro-CT) at high resolution in three-dimension. They analyzed the impact of wettability on residual gas trapping in pore-scale. Moein et al. (2019) presented a methodology to generate fracture networks with fractal characteristics in one, two, and three dimensions. Zhu (2020) developed a new approach to calculate the flow rate through a fractal fracture network. Three dimensionless parameters were defined to estimate the relative effects of power-law fluid in fractal fracture network. However, the previous studies commonly did not focus on the scale dependence of the seepage parameter, meaning especially that the equivalent permeability of three-dimension discrete fracture network (DFN) models can dramatically fluctuate with model size.

In this study, the representative element volume (REV) of the 3D DFN model is calculated to determine the suitable size of DFN model used for investigating the influence of fracture length distribution on equivalent permeability. The fracture density \( P_{32} \) of DFN model is obtained to reveal the trend of equivalent permeability with increasing power law exponent \( (a) \) of fracture length distribution.

2. Flow modelling

The REV size of the three-dimension DFN model with \( a = 4.5 \) was calculated by transforming the three-dimension DFN model to equivalent pipe network (EPN) models, in which the fracture length follows a power law distribution. A power law function can be correctly used to model the length distribution of fractures as follows (De Dreuzy et al., 2000):

\[
n(l) = \alpha l^a
\]

where \( l \) is the fracture length, \( n(l)dl \) is the number of fractures which distribute in the range \([l, l+dl]\), \( \alpha \) is the proportionality coefficient, and \( a \) is the power law exponent. The increment of \( a \) would lead to the increment of the number of shorter fractures. The \( l \) is ranged from \( l_{min} \) to \( l_{max} \), which are the lower and upper limits of the fracture lengths, respectively.

2.1. Generation of 3D DFN model

The fractures were characterized by ellipse discs distributed in impermeable matrix, and were located within the model volume as a network by translation and rotation. The statistical distribution of fractures after translation and rotation refers to the distributions of centre point positions, dips, and dip angles of fracture discs. The statistical geometry distribution of fracture length needs to be defined before the DFN model generation.

As shown in figure 1, the side length of the original DFN model \((L)\) is assumed to 100 m, in which fracture length follows a power law distribution between 16.7 m and 100 m with \( a = 4.5 \). We kept the ratio of fracture length \((L_f)\) to \( L \) ranging from 0.167 to 1 which is same as the ratio in the reference paper (Huang et al., 2016) to indicate the effectiveness of the EPN model. The number of fractures in the model that is consisted of fractures with power law exponent of 4.5 is determined by the density \( P_{32} \) \((0.4 \text{ m}^2/\text{m}^3)\). The density is also same with that in the reference paper (Huang et al., 2016). The aperture of all fractures is set to be a constant value (1 mm) to study the effect of fracture length distribution on the equivalent permeability. The fracture dip direction angles and fracture dips are assumed to be uniformly and randomly distributed in the range of \([0, 2\pi]\). Three original DFN models that have the same statistical geometry characteristics and different randomness were generated. The cube sub-models were extracted from the original DFN models with side lengths \((L_a)\) ranging from 5 to 95 m with a separation of 5 m as shown in figure 1.
2.2. Boundary conditions

Figure 2 shows the cube sub-models and corresponding pressure head distributions with different boundary conditions, in which the side length is 50 m. The cube sub-model shown in figure 2a was extracted from the original DFN model shown in figure 1. And it was equivalent to EPN model for calculating the flow rate through the sub-model based on open source software ADFNE (Alghalandis, 2017). The flow behaviour of EPN models was characterized by the finite difference method (FDM). Three hydraulic boundaries are applied to each cube sub-model: flow from the left side to the right side; flow from the top side to the bottom side; flow from the front side to the back side. The hydraulic head difference is fixed as 1 m and the other boundaries are assumed to be impermeable. The three fluid flow directions are presented in figures 2b, c, and d, respectively. Therefore, there are 9 directional equivalent permeability for each $L_a$.

![Figure 1. Schematic view of the generation of sub-models.](image)

![Figure 2. Hydraulic boundary conditions of cube sub-models.](image)

2.3. Estimation of REV size

Figure 3 shows variations in equivalent permeability and RMS versus $L_a$. When $L_a$ is small, the equivalent permeability fluctuates significantly due to the influence of randomness utilized to generate fracture location and orientation. When $L_a$ exceeds a critical value, equivalent permeability holds almost unchanged despite of flow direction. The critical scale can be regarded as the REV size. RMS = 0.2 is utilized as the criterion of reaching REV size, and the corresponding volume of DFN ($V_{REV}$) can be calculated. The side length of DFN model is 34.68 m when RMS = 0.2. So the REV size is reached at an approximated $V_{REV}$ of 4.17E+04 m$^3$. 
3. Results and analysis

The statistical geometry characteristics of fractures have considerable influences on fracture network structure and consequently affect its hydraulic properties. Field investigations have demonstrated that the fracture length follows power law distributions, and the power exponent varies from 2.0 to 4.5 (De Dreuzy et al., 2000). To investigate the impact of fracture length distribution on seepage properties, six 3D DFN models, which are composed of fractures with power exponents covering 2.0 ~ 4.5 with an interval of 0.5, were generated as shown in figures 4a, c, e, g, i and k, respectively. The fracture length distribution range of each model is the same. The ratio of $L_f$ to $L$ is consistent with that in Section 2.1. The density of the model, in which fractures follow power law distribution and the exponent is 4.5, is also set to be 0.4 m$^2$/m$^3$. The numbers of fractures in the models, which power law exponent range from 2.0 to 4.0, are consistent with that in the model with larger $a$ (4.5) to investigate the effect of $a$ on the $K$. The apertures of all fractures are also set to be a constant value (1 mm). The locations of centre points, dips and dip angles follow the same distribution in Section 2.1. The $L$ of each DFN model is assumed to be 50 m, in which the volume (1.25E+05 m$^3$) is much larger than the REV size (4.17E+04 m$^3$) calculated in Section 2.3. The numbers of fractures distributed in the six DFN models keep the same.

![Figure 3](image1.png)

(a) Relationship between $K$ and $L_n$
(b) Relationship between $L_n$ and RMS

**Figure 3.** The impact of $L_n$ on equivalent permeability coefficient ($K$).

![Figure 4](image2.png)

(a) $a = 2.0$
(b) $a = 2.0$
(c) $a = 2.5$
(d) $a = 2.5$
(e) $a = 3.0$
(f) $a = 3.0$
(g) $a = 3.5$
(h) $a = 3.5$
(i) $a = 4.0$
(j) $a = 4.0$
(k) $a = 4.5$
(l) $a = 4.5$

**Figure 4.** Three dimension fracture networks and corresponding pipe networks with increasing $a$.

The six DFN models (figures 4a, c, e, g, i and k) were equivalent to EPN models to compute the corresponding equivalent permeability coefficients. The pressure head distributions of the six EPN
models are shown in figures 4b, d, f, h, j and l, respectively. The results show that the frequency of short fractures in the DFN model with a larger $a$ is larger than that with a smaller $a$. The short fractures can account for the decrease of $P_{32}$ with the increment of $a$ in a negative exponential function as shown in figure 5b. The $P_{32}$ of DFN model decreases from 0.76 m$^2$/m$^3$ to 0.41 m$^2$/m$^3$ when $a$ varies from 2.0 to 4.5. This evidences that the volume of DFN model (1.25E+05 m$^3$), which is larger than the $V_{REV}$ when $a = 4.5$, is large enough to avoid scale dependence of the seepage parameter when $a$ is smaller than 4.5. As shown in figure 5c, the equivalent permeability increases with the increment of $P_{32}$, indicating that the denser fractures lead to a stronger connectivity/permeability. The relationship between them follows an exponential relationship instead of a linear relationship. This suggests that the equivalent permeability of the DFN model is affected not only by $P_{32}$ but also other geometry factors. The simulation results agree well with the results of Huang et al. (2016), which can indicate that the EPN model is effective. The equivalent permeability decreases exponentially with the increment of $a$ as shown in figure 5d. The equivalent permeability of the model with $a = 2.0$ is nearly five times larger than that with $a = 4.5$.

![Figure 5](image)

**Figure 5.** The effect of $a$ on geometry characteristics and $K$ of DFN model.

4. Conclusions

An original DFN model was generated to evaluate the appropriate scale of estimating the influence of fracture length distribution on equivalent permeability of fracture networks. A series of DFN models that are consisted of fractures with different power exponents were generated with suitable scales. The DFN models were equivalent to EPN models to calculate their equivalent permeability.

The results show that the $P_{32}$ decreases with the increment of $a$ in a negative exponential function. The frequency of short fractures in DFN model with a larger $a$ is larger. More short fractures are generated in DFN models with a large $a$, and consequently lead to a smaller $P_{32}$. The equivalent permeability increases exponentially with the increment of $P_{32}$ instead of a linear relationship. This suggests that the equivalent permeability of DFN model is affected by not only $P_{32}$ but also other geometry factors. The larger $P_{32}$ corresponds to a better connectivity of DFN. The equivalent permeability of DFN model approximately decreases to one fifth as $a$ varies from 2.0 to 4.5.

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