Brane waves

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Abstract

In braneworld cosmology gravitational waves can propagate in the higher dimensions (i.e., in the ‘bulk’). In some appropriate regimes, the bulk gravitational waves may be approximated by plane waves. We systematically study five-dimensional gravitational waves that are algebraically special and of type N. In the most physically relevant case, the projected non-local stress tensor on the brane is formally equivalent to the energy–momentum tensor of a null fluid. Some exact solutions are studied to illustrate the features of these branes; in particular, we show explicitly that any plane wave brane can be embedded into a five-dimensional Siklos spacetime. More importantly, it is possible that in some appropriate regime the bulk can be approximated by gravitational plane waves which may act as initial conditions for the gravitational field in the bulk (thereby enabling the field equations to be integrated on the brane).

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1. Introduction

In braneworld cosmology, matter fields and gauge interactions are confined to a four-dimensional brane moving in a higher-dimensional ‘bulk’ spacetime. This paradigm is motivated by string and M-theory; in particular, generalized Randall–Sundrum-type models [1] are relatively simple phenomenological five-dimensional (5D) models which capture some of the essential features of the dimensional reduction of Hořava–Witten theory [2]. In a recent analysis of the asymptotic dynamical evolution of perfect fluid braneworld cosmological models close to the initial singularity, it was found that for an appropriate range of the equation of state parameter an isotropic singularity is a past attractor [3]. It was subsequently argued that the initial cosmological singularity is isotropic in braneworld cosmological models.

The 5D field equations are

\[ (5)G_{ab} = -\Lambda_5 (5)g_{ab} + \kappa_5^2 (5)T_{ab}, \quad \Lambda_5 = -\frac{6}{\ell^2}. \]

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Here, $a, b = 0, \ldots, 4$, and $\Lambda_5$ and $g_{ab}$ are the 5D cosmological constant and metric, respectively. The projected field equations on the brane are [4]

$$G_{\mu\nu} = -\Lambda g_{\mu\nu} + \kappa^2 T_{\mu\nu} + \frac{6}{\Lambda} S_{\mu\nu} = \mathcal{E}_{\mu\nu},$$  

(2)

where $\mu, \nu$ are brane indices (i.e. $\mu, \nu = 0, \ldots, 3$) $\Lambda = \Lambda_5/2 + \lambda^2 \kappa_5^4/12$ and $\kappa^2 = \lambda \kappa_5^4/6$. The term $S_{\mu\nu}$ is quadratic in $T_{\mu\nu}$ and dominates at high energies ($T_{00} = \rho > \lambda$), and the five-dimensional Weyl tensor is felt on the brane via its projection, $\mathcal{E}_{ab} = C_{abcd} n^d$, where $n^a$ is the unit normal vector to the brane. There may also be terms that arise from 5D sources in the bulk other than the vacuum energy $\Lambda_5$, such as a bulk dilaton field. In general, in the four-dimensional picture, the conservation equations do not determine all of the independent components of $\mathcal{E}_{\mu\nu}$ on the brane (and a complete higher-dimensional analysis, including the dynamics in the bulk, is necessary) [5].

In these models, the gravitational field can also propagate in the extra dimensions (i.e., in the ‘bulk’). For example, there might occur thermal radiation of bulk gravitons [6]. In particular, at sufficiently high energies particle interactions can produce 5D gravitons which are emitted into the bulk. Conversely, in models with a bulk black hole, there may be gravitational waves hitting the brane. At sufficiently large distances from the black hole, these gravitational waves may be approximated as of type N. Alternatively, if the brane has low energy initially, energy can be transferred onto the brane by bulk particles such as gravitons; an equilibrium is expected to set in once the brane energy density reaches a limiting value. (For an alternative approach see, e.g., [7].) In this paper, we shall study the consequences of assuming that in some appropriate regimes the bulk gravitational waves can be approximated by plane waves.

2. An example of a type N bulk with a plane wave brane

First, let us consider an example of an exact 5D solution of type N with a negative cosmological constant. This particular example will give us some hints what we can expect from the braneworld analysis. In particular, the example makes it possible to interpret some of the exact solutions on the brane discussed later.

Consider the Siklos metric [8] (where we have dropped an overall scaling)

$$ds^2 = \frac{1}{z^2}(2 du \, dv + H(u, x, y, z) \, du^2 + dx^2 + dy^2 + dz^2), \quad z > 0. \tag{3}$$

This metric is an Einstein space (i.e., $R_{ab} = \Lambda g_{ab}$) if the function $H$ solves the equation:

$$\left( \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} - \frac{3}{z} \frac{\partial}{\partial z} \right) H(u, x, y, z) = 0. \tag{4}$$

These solutions describe gravitational waves propagating in a negatively curved Einstein space. Hence, these metrics generalize the AdS$_5$ spaces to solutions of the Einstein equation with a negative cosmological constant containing gravitational waves. These are exactly the types of models we want to investigate for the bulk.

To see how these can generate anisotropic stresses in the bulk, we can calculate the Weyl tensor of the above solutions. In general, the Weyl tensor has the following non-vanishing components (in a coordinate basis):

$$C_{uiuj} = -\frac{1}{2z^2} \frac{\partial^2}{\partial x^i \partial x^j} H,$$

$$C_{uiui} = -\frac{1}{6z^2} \left( 2 \frac{\partial^2}{\partial x^i \partial x^j} - \frac{\partial}{\partial x^j} - \frac{\partial^2}{\partial x^k \partial x^k} \right) H. \tag{5}$$
where \((i, j, k)\) is a permutation of \((x, y, z)\). Since \(E_{ab} = C_{a\alpha b\gamma n^\alpha n^\gamma}\), the Weyl tensor can induce anisotropic stresses on the brane. For example, the Weyl tensor corresponding to the gravitational mode \(H = x^2 - y^2\) yields the bulk stresses on the brane given by \(C_{uux} = -C_{uyuy} = -\frac{1}{2}\). We also have oscillatory modes by choosing, for example,

\[
H(u, x, y, z) = (A + Cz^4)(x^2 - y^2) \cos(u + \varphi_1) + (D + Bz^4) \cos(u + \varphi_2),
\]

where \(A, B, C, D, \varphi_1\) and \(\varphi_2\) are arbitrary constants. We will later see that similar modes naturally enter the analysis of the brane.

Let us, for the sake of illustration, consider a simple exact five-dimensional solution of a brane of this type. We consider a case in which a brane is embedded in the five-dimensional solution, equation (3). The brane is located at \(z = z_0\) with normal unit vector \(\mathbf{n} = z\partial/\partial z\).

We note that any plane wave brane can be embedded (locally) in a Siklos spacetime. Given \(H(u, x, y)\), a solution can be found by choosing \(H(u, x, y, z) = D(z, \nabla^2_z)\mathcal{H}(u, x, y)\),

\[
\nabla^2_z \equiv \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2},
\]

where \(D(z, \nabla^2_z)\) is the operator

\[
D(z, \nabla^2_z) = \sum_{i=0}^{\infty} F_i(z)(\nabla^2_z)^i,
\]

and \(F_i(z)\) are defined iteratively:

\[
F_0(z) = 1, \quad F_{i+1}(z) = -\int_{z_0}^{z} \left( \frac{1}{z^3} F_i(z) \right) dz.
\]

The brane is located at \(z_0\) and \(H\) is chosen such that equation (7) is fulfilled. In the special case where \(\mathcal{H}(u, x, y)\) is a polynomial in \(x\) and \(y\), the above sum will terminate and \(H\) will only contain a finite number of terms. Furthermore, by choosing the function \(\mathcal{H}(u, x, y)\) appropriately, we note that the above example can include branes of Bianchi types III, IV, V, VI \(_h\) and VII \(_h\). The relationship between brane and bulk can be further studied in a full five-dimensional setting.

1 Also note that a propagating electromagnetic wave on the brane can support a non-zero \(\frac{\partial H}{\partial z}\) near \(z_0\).
3. A general type N bulk

Let us now be more systematic and assume that the 5D bulk is algebraically special and of type N. This puts a constraint on the 5D Weyl tensor which makes it possible to deduce the form of the non-local stresses from a brane point of view. For a 5D type N spacetime there exists a frame \( \ell_a, \tilde{n}_a, m^i_a \) such that:

\[
C_{abcd} = 4C_{i1j} \ell_a m^i_b \ell_c m^j_d.
\]

The frame \( \ell_a, \tilde{n}_a, m^i_a \) (the frame vector \( \tilde{n}_a \) is not to be confused with the brane normal vector \( n^a \)) is defined via the only non-zero contractions:

\[
\ell_a \tilde{n}_a = \ell_a \tilde{n}_a = 1, \quad m^i_a m^j_a = \delta^{ij}.
\]

The ‘electric part’ of the Weyl tensor, \( E_{ab} = C_{acbd} n^c n^d \), where \( n^a \) is the normal vector on the brane, can for the type N bulk be written as

\[
E_{ab} = C_{i1j} \left[ \ell_a (m^i_e n^e) - m^i_a (\ell_e n^e) \right] \left[ \ell_b (m^j_e n^e) - m^j_b (\ell_e n^e) \right].
\]

Furthermore, it can easily be checked that \( E_{ab} n^b = E_{a a} = 0 \). Note that for a type N bulk we also have

\[
E_{ab} \ell^b = 0.
\]

This can be rewritten using the projection operator on the brane, \( \tilde{g}^{ab} = g^{ab} - n^a n^b \),

\[
E_{ac} \tilde{g}^c \ell^b = E_{ab} \ell^b = 0, \quad \ell^b \equiv \tilde{g}^b c \ell^c.
\]

Hence, the vector \( \ell_b \) is the projection of the null vector \( \ell_b \) onto the brane. By contracting this vector with itself, we get

\[
\ell^b \ell_b = -(\ell^a n_a)^2.
\]

The following analysis splits into two cases, according to whether \( \ell^a n_a \) equals zero or not:

- (1) \( \ell^a n_a = 0 \): \( \ell_a = \ell_a \) and null.
- (2) \( \ell^a n_a \neq 0 \): \( \ell_a \) timelike.

These cases have to be treated separately and have different interpretations on the brane. From a 5D point of view they are of the same type, but since the 5D spacetime is anisotropic the orientation of the brane with respect to \( \ell_a \) is of significance. More precisely, considering plane-wave spacetimes, the case \( \ell^a n_a = 0 \) corresponds to when the wave propagates parallel to the brane, and in the case \( \ell^a n_a \neq 0 \) the wave hits the brane.

3.1. The case \( \ell^a n_a = 0 \)

Let us first investigate the consequence of \( \ell_\mu \) being a null vector. We note that \( E_{\mu\nu} \), the four-dimensional projected Weyl tensor on the brane, can be written in this case as

\[
E_{\mu\nu} = -\left( \frac{6}{\lambda \kappa^2} \right) \epsilon \hat{\ell}_\mu \hat{\ell}_\nu,
\]

where \( \epsilon \) is some appropriate function. Hence, this is formally equivalent to the energy–momentum tensor of a null fluid. Equivalently we can consider it as the energy–momentum tensor of an extreme tilted perfect fluid. Using a covariant decomposition of \( E_{\mu\nu} \) with respect to a preferred timelike vector \( u_\mu \) being orthogonal to some 3-surface with metric \( h_{\mu\nu} \), we have

\[
E_{\mu\nu} = -\left( \frac{6}{\lambda \kappa^2} \right) \left[ U \left( u_\mu u_\nu + \frac{1}{3} h_{\mu\nu} \right) + P_{\mu\nu} + 2 Q_{(\mu} u_{\nu)} \right].
\]
The non-local energy terms are thus given by [5]
\[ U = \epsilon (\hat{\ell}, u^\nu)^2, \quad Q_\mu = \epsilon (\hat{\ell}, u_\mu) \hat{\ell}_\nu, \quad P_{\mu\nu} = \epsilon \hat{\ell}_\mu \hat{\ell}_\nu. \] (20)
Here, angle brackets \( \langle \cdots \rangle \) denote the projected, symmetric and tracefree part with respect to the metric \( h_{\mu\nu} \) of the spatial 3-surfaces. The equations on the brane now close and the dynamical behaviour can be analysed (see later, and [11]). Note that
\[ U P_{\mu\nu} = Q_\mu Q_\nu - \frac{1}{3} g_{\mu\nu} Q_\lambda Q^\lambda. \] (21)
so that in this case \( E_{\mu\nu} \) is determined completely by \( U \) and \( Q_\mu \).

3.2. The case \( \ell^a n_a \neq 0 \)

In this case the vector \( \ell^a \) has a component orthogonal to the brane. This implies that the vector \( \hat{\ell}^a \) is timelike. This vector lives on the brane, and hence we can set \( u^\mu \parallel \hat{\ell}_\mu \). In this frame, the requirement \( \hat{\ell}_\mu E_{\mu\nu} = 0 \) implies that we have \( U = 0 = Q_\mu \) and hence, we can write
\[ E_{\mu\nu} = - \left( \frac{6}{\lambda \epsilon^2} \right) P_{\mu\nu}. \] (22)
This is formally equivalent to a fluid which possesses anisotropic stresses with no energy density or energy flux. From a brane point of view it appears as if these stresses are superluminal; however, as can be seen from a 5D point of view, this is just an artefact of living on a brane in a higher-dimensional spacetime. The stresses do have a gravitational origin, namely from gravitational waves in the 5D bulk.

3.3. Exact solutions describing radiating branes

Next, we will find some exact solutions in the case \( \ell^a n_a \neq 0 \), which are very illustrative and describe features of branes that seem to have gone unnoticed in the literature. We assume that the brane contains an isotropic fluid, i.e. the energy–momentum tensor takes the form
\[ T_{\mu\nu} = (p + \rho) u_\mu u_\nu + p g_{\mu\nu}, \] (23)
and that \( p = (\gamma - 1) \rho, \ Q_\mu = U = 0 \). We also assume that the brane has isotropic curvature, \( R_{(\mu\nu)} = 0 \) (e.g., is of Bianchi type I or V). From the propagation equation for the non-local dark energy \( U \), the simple constraint
\[ \sigma_{\mu\nu} P^{\mu\nu} = 0, \] (24)
where \( \sigma_{\mu\nu} = \nabla_{\mu} u_\nu \) is the shear, is obtained (see [12]). Since both \( \sigma_{\mu\nu} \) and \( P^{\mu\nu} \) are symmetric, spacelike, trace-free and of maximal rank 3, and defining the natural inner product, the constraint \( \sigma_{\mu\nu} P^{\mu\nu} = 0 \) implies that \( \sigma_{\mu\nu} \) and \( P^{\mu\nu} \) are orthogonal. We have the evolution equation
\[ \partial_t \sigma_{\mu\nu} + \Omega^\mu_{\alpha\nu} \sigma_{\alpha\nu} - \omega_{\mu\nu} \Omega^\alpha_{\nu} + \theta \sigma_{\mu\nu} = \frac{6}{\lambda \epsilon^2} P^{\mu\nu}, \] (25)
where \( \theta = \nabla^\mu u_\mu \) and \( \Omega^\nu_{\nu} \) is antisymmetric and is the rotation tensor with respect to a set of Fermi-propagated axes (recall that an overdot is defined by \( \dot{\equiv} u^\mu \nabla_\mu \)). Writing \( \theta = 3 \dot{a} / a \), the shear equations then give
\[ \partial_t (\sigma_{\mu\nu} \sigma^{\mu\nu}) + 6 \frac{\dot{a}}{a} \sigma_{\mu\nu} \sigma^{\mu\nu} = 0. \] (26)
This can easily be integrated to give \( \sigma_{\mu\nu} \sigma^{\mu\nu} = A^2 a^{-6} \). Thus, the shear scalar decays with the usual \( a^{-6} \) behaviour and is unaffected by the non-zero \( P_{\mu\nu} \). This is simply due to the fact that
\( \mathcal{P}_{\mu \nu} \) is orthogonal to \( \sigma_{\mu \nu} \) and thus it cannot affect the shear scalar. The generalized Friedmann equation now becomes [12]
\[
\frac{\dot{a}^2}{a^2} = \frac{\Lambda}{3} + \frac{A^2}{3} \frac{1}{a^6} - \frac{k}{6} \frac{1}{a^2} + \frac{k^2}{3} \frac{\rho_0}{a^{3y}} \left( 1 + \frac{1}{2\lambda} \frac{\rho_0}{a^{3y}} \right),
\]
where, for example, \( k = 0 \) for Bianchi type I and \( k = -1 \) for Bianchi type V. This equation can now easily be solved in quadrature. Due to the quadratic terms in the energy densities arising from brane effects, the shear mode is subdominant in the past as long as the isotropic fluid is stiffer than dust.

It is convenient to define two new variables
\[
X_{\mu \nu} = a^3 \sigma_{\mu \nu}, \quad Y_{\mu \nu} = a^3 \mathcal{P}_{\mu \nu}.
\]
In tensor form, the shear evolution equations are
\[
\partial_t X + [\Omega, X] = \omega Y,
\]
where \( \omega = 6/(\kappa^2 \lambda) \). This equation can be integrated in quadrature once we know \( Y \) (recall that from a brane point of view, there are no evolution equations for \( Y \)). We solve the above equations by choosing the particular gauge in which \( \Omega = 0 \) (\( X \) is not diagonal in general), so that the axes are Fermi propagated. A solution can now be written as
\[
X = X_0 \cos \omega t + Y_0 \sin \omega t, \quad Y = Y_0 \cos \omega t - X_0 \sin \omega t.
\]
In this solution it is easy to show that if \( X_0, Y_0 \) are orthogonal initially, then \( X \) and \( Y \) will be orthogonal at all times. From the assumption of a periodic \( Y \), it follows that the function \( H \) satisfies a wave equation as desired.

The above solutions describe braneworlds where the shear oscillates with a frequency
\[
f = \frac{2\pi}{\omega} = \frac{\pi \kappa^2 \lambda}{3}.
\]
This oscillation is driven by a similar oscillation in the non-local anisotropic stresses \( \mathcal{P}_{\mu \nu} \). The frequency of the oscillation is given by the brane tension and the gravitational constant on the brane and hence the physics on the brane. If these gravitational waves came from a source in the bulk, then that source must somehow know of the physics on the far-away brane. Unless there is a very special configuration in the bulk, this situation does not seem very likely. A more likely scenario is that it is the \textit{brane itself} which is the source of the gravitational radiation. The brane oscillates with a given frequency which is dictated by the brane tension and the gravitational constant on the brane. This oscillation causes the brane to emit gravitational radiation into the bulk. The frequency of the gravitational radiation will thus carry a distinct signature given by its frequency. If the bulk consists solely of an incoming or an outgoing wave then the type N approximation is valid. Suppose there is a source in the bulk emitting radiation with a different characteristic frequency from the brane. Then close to the membrane there will be a mixture of incoming and outgoing waves, unless some sort of equilibrium is established. This implies that the simple assumption that there exists a preferred wave vector just outside the brane (and the bulk just outside the brane is of type N) breaks down.

### 4. The effect of gravitational waves on the dynamical behaviour of the brane

Finally, we will investigate what kind of effect this type N bulk may have on the cosmological evolution of the brane. It is believed that an isotropic singularity is the most likely initial
starting point for a classical braneworld [3]. For this to be correct, the initial singularity has to be stable into the past. Let us assume, therefore, that we are in the regime of an isotropic past and the cosmological evolution is dominated by an isotropic perfect fluid with equation of state \( p = (\gamma - 1)\rho \). In this approximation, and assuming the case \( \ell^{\alpha n_\alpha} = 0 \), the equations for \( U \) and \( Q_\mu \) are

\[
\dot{U} + \frac{4}{3} \theta U = 0, \quad \dot{Q}_\mu + \frac{4}{3} \theta Q_\mu = 0. \tag{31}
\]

For the isotropic singularity, the expansion factor is given by \( \theta = 1/(\gamma t) \). Hence, defining the expansion-normalized non-local density, \( U \equiv U/\theta^2 \), and non-local energy flux, \( Q_\mu \equiv Q_\mu/\theta^2 \), we get

\[
\dot{U} = \frac{2}{3\gamma} (3\gamma - 2) U, \quad \dot{Q}_\mu = \frac{2}{3\gamma} (3\gamma - 2) Q_\mu. \tag{32}
\]

Hence, the isotropic singularity is stable to the past with regard to these stresses if \( \gamma > 2/3 \).

Similarly, for the case \( \ell^{\alpha n_\alpha} = 0 \) we have a flat FRW universe to the future with \( \theta = 2/(\gamma t) \), and thus

\[
\dot{U} = \frac{2}{3\gamma} (3\gamma - 4) U, \quad \dot{Q}_\mu = \frac{2}{3\gamma} (3\gamma - 4) Q_\mu. \tag{33}
\]

This implies that if the isotropic fluid on the brane is stiffer than radiation (\( \gamma = 4/3 \)), then the flat FRW universe is unstable to the future with respect to the non-local stresses. However, since dust has \( \gamma = 1 \), our physical universe is believed to be stable with respect to these non-local stresses.

5. Discussion

These phenomenological results are further supported by a more detailed analysis of the asymptotic behaviour of two tilting \( \gamma \)-law fluids in a class of Bianchi type VI\(_0\) models [11]. In particular, the physically relevant case of interest here, namely that the second fluid is a null fluid or fluid with extreme tilt, is investigated. All equilibrium points are found and their stability determined, so that the local attractors can be established. It is found that the dynamical effects of the projected Weyl tensor are not significant asymptotically at early and late times.

In brane cosmology the only degrees of freedom in the bulk are the higher-dimensional gravitational waves, which propagate in the bulk [4, 6]. The problem of initial conditions for these bulk gravitational waves is of great importance [13]. Indeed, we need initial conditions for the gravitational field in the bulk to be able to integrate the field equations and determine the dynamics on the brane. We have argued that in some situations, the bulk gravitational waves can be approximated as type N plane-wave solutions. Consequently, perhaps an appropriate set of initial conditions is to assume plane wave behaviour in a suitable regime of the bulk. This can be achieved by demanding that the five-dimensional bulk is algebraically special and of type N as above. The effects this type N bulk may have on the cosmological evolution of the brane are studied in [11].

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