Second-harmonic generation in vortex-induced waveguides

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We study the second-harmonic generation and localization of light in a reconfigurable waveguide induced by an optical vortex soliton in a defocusing Kerr medium. We show that the vortex-induced waveguide greatly improves conversion efficiency from the fundamental to the second harmonic field.

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Spatial optical solitons have become a topic of active research promising many realistic applications and opening new directions in nonlinear physics [1]. In its simplest form, a spatial soliton is a single self-guided beam of a specific polarization and frequency. Two (or more) mutually trapped components with different polarizations or frequencies can forma vector soliton.

One important application of spatial optical solitons is to induce stable nondiffractive steerable waveguides that can guide and direct another beam, thus creating a reconfigurable all-optical circuit. The soliton-induced optical waveguides have been studied theoretically and demonstrated experimentally in many settings [2, 3, 4, 5]. It was also shown that soliton waveguides can be used for a number of important applications, including the second-harmonic generation [6], directional couplers and beam splitters [7], and optical parametric oscillators [8].

Second-harmonic generation [6], directional couplers and optical parametric oscillators [8] are more attractive for soliton waveguiding applications because of their greater stability and steerability [1].

First, we analyze the stationary solutions of the model (1) for three-component vector solitons [9, 10] created by a vortex beam. An optical vortex can guide both weak and strong probe beams, and that in the latter case the vortex creates a stable vector soliton with its guided component [1].

In this Letter, we study the second-harmonic generation in a reconfigurable vortex-induced waveguide and determine conditions for significant enhancement of the conversion efficiency. We also describe novel types of three-component vector solitons created by a vortex beam together with both fundamental and second-harmonic parametrically coupled localized modes guided by the vortex-induced waveguide.

We consider two incoherently coupled beams with frequencies ω0 and ω1 propagating in a bulk nonlinear Kerr medium. The ω0-beam propagates in a self-defocusing regime and carries a phase dislocation. We assume that the phase-matching conditions of the second-harmonic generation (SHG) are fulfilled for the fundamental wave of frequency ω1 guided by the vortex waveguide, so that it generates a second-harmonic (SH) wave with the frequency 2ω1. The SH wave is parametrically coupled to the fundamental one and is also guided by the vortex waveguide. Evolution of the slowly varying beam envelopes of the vortex beam, the fundamental guided wave, and the SH wave can be described by the following system of three coupled dimensionless equations

\[ i \frac{\partial u}{\partial z} + \Delta_\perp u - (|u|^2 + |v|^2 + |w|^2) u = 0, \]

\[ i \frac{\partial w}{\partial z} + \Delta_\perp w + w^* v - \sigma |w|^2 v = 0, \]

\[ 2i \frac{\partial v}{\partial z} + \Delta_\perp v - \beta v + \frac{1}{2} |w|^2 + \rho |v|^2 v = 0. \]

where \( u, w, \) and \( v \) are the normalized slowly varying complex envelopes of the vortex beam, the fundamental field, and the SH field, respectively. Other notations are: the Laplacian \( \Delta_\perp \) refers to the transverse coordinate \( r = (x, y) \), the beam propagation coordinate measured in units of \( z_0 = 2k_1 r_0^2 \). The parameter \( \beta = 2z_0 \Delta k \) is proportional to the wavevector mismatch \( \Delta k = 2k_1 - k_2 \), whereas the nonlinear coupling coefficients \( \sigma \) and \( \rho \) are proportional to the corresponding third-order tensor components [11], and the self-action effects for the fundamental and SH fields are neglected. Equations (1) are valid when spatial walk-off is negligible and the fundamental frequency \( \omega_1 \) and its second harmonic are far from resonance.

We emphasize that the model (1) is the simplest of its kind, which is most suitable for our feasibility study of SHG in vortex-induced waveguides. It is clear that modelling of particular experimental setups for realization of this concept would require modifications of Eqs. (1), according to the geometry of an experiment and properties of nonlinear materials. For example, in photorefractive crystals [3] one should take into account the nonlinearity saturation effect.

First, we analyze the stationary solutions of the model (1) in the form of the \((2+1)\)-dimensional radially symmetric nonlinear modes. We look for spatially localized solutions in the polar coordinates \((r, \phi)\) of the form

\[ u = u(r)e^{-i\omega_1 z} e^{i\phi}, \]

\[ w = w(r) e^{i\lambda z}, \] and

\[ v = v(r) e^{i2\lambda z}, \]

with the following asymptotic: \( u(r) \to 1 \), and \( v(r), w(r) \to 0 \) for \( r = \sqrt{x^2 + y^2} \to \infty \). Then, the mode amplitudes sat-
induced waveguide. Examples of such solutions are pre-
be regarded as two guided modes of the effective vortex-
are localized only in the presence of the vortex, and can

tions for selected values of the parameters
(divided into
twocategories
(dashed) found from a simple analysis of Eqs. (2).
 numerically and the asymptotic lines
 λ
λ ≤ 0, and (ii) quadratic solitons regime, λ > 0 . For
 λ th < λ < 0, the parametrically coupled modes w and v
are localized only in the presence of the vortex, and can
be regarded as two guided modes of the effective vortex-
induced waveguide. Examples of such solutions are pre-

mum phase-matching parameter, β.

In order to study the SHG process in the vortex-
induced waveguide, we employ the stationary solutions
obtained above and analyze numerically the evolution of
the beams in the case when the SH component is ab-
sent at the input. We perform all our calculations for
the case of a finite-extent input vortex beam, obtained
by superimposing the stationary vortex profile u(r) onto
a broad super-Gaussian beam, u_{SG} = u(r) \exp[-(r^6/d)],
where d = 10^6. This form of initial conditions makes our
predictions more suitable for experimental verifications.

The numerical results indicate that the generation of
the SH field from such an input differs dramatically for
the vortex-waveguiding and quadratic soliton regimes.
Indeed, for λ < 0 we observe a good correspondence with
the SHG theory. For large β, the generated SH field is
weak and the process corresponds to the so-called nonde-
plicated pump approximation in the SHG theory. Almost
perfect SHG is observed for β close to zero, and in all
such cases the distortion of the vortex waveguide is weak.

Figure 3(upper row) and Fig. 4 show an example of the
SHG process with u(r) corresponding to the point D in
Fig. 2. A good confinement of both fundamental and SH
guided modes can be seen with a very good conversion
efficiency and weak distortion of the vortex beam.

However, in the quadratic soliton regime, when λ > 0,
the strong parametric interaction between the guided
components does not allow good energy conversion be-
tween the harmonics. Instead, even for a high-intensity
fundamental input, both the fundamental and SH fields
approach a stationary state with nonzero but low-

FIG. 1: Spatial profiles of the three-wave vector soliton com-
ponents for the points A to F marked in Fig. 2. Shown are:
the vortex amplitude u(r) (thin solid), the fundamental field
w(r) (thick solid), and the SH field v(r) (dashed) at the indi-
cated values of β and λ.

satisfy the system of z-independent equations

\[ \Delta_r u - \frac{1}{r^2} u + u - (w^2 + 2w^2 + 8v^2) u = 0, \]

\[ \Delta_r w - \lambda w + wv - 2u^2 w = 0, \]  \( \text{(2)} \)

\[ \Delta_r v - (4\lambda + \beta) v + \frac{1}{2} w^2 - 8u^2 v = 0. \]

where \( \Delta_r = (1/r)d/dr(rd/dr) \) is the radial part of the
Laplacian, and for definiteness we have specified the
parameters of the cross-phase modulation interaction,
\( \sigma = 2 \) and \( \rho = 8 \). In Eqs. (2), the real propagation con-
stant \( \lambda \) must be above cutoff, \( \lambda > \lambda_r = \max(0, -\beta/4), \)
for w and v to be exponentially localized.

Using the standard relaxation numerical technique, we
find the families of localized solutions of the system (2)
for allowed values of \( \beta \) and \( \lambda \). In Fig. 1 we show several
examples of the profiles of the three-wave localized solutions
for selected values of the parameters \( \beta \) and \( \lambda \).

The numerical results are summarized in Fig. 2 which shows the
existence domain as a shaded region of the plane
(\( \lambda, \beta \)) with the boundary \( \lambda = \lambda_{th} \) (solid curve) found nu-
umerically and the asymptotic lines \( \lambda = 0 \) and \( \lambda = -\beta/4 \)
(dashed) found from a simple analysis of Eqs. (2).

All three-wave solutions of Eqs. (2) can formally be divided into two categories according to the dominant
regime of their formation: (i) vortex-waveguiding regime,
\( \lambda < 0 \), and (ii) quadratic solitons regime, \( \lambda > 0 \).

FIG. 2: Region of existence (shaded) of the three-component
vector solitons of the model (1) in the plane (\( \lambda, \beta \)). Marked
points correspond to the localized modes shown in Fig. 1.
FIG. 3: Examples of SHG in the vortex-induced waveguide with no SH field at the input and the parameters corresponding to the point D (upper row) and point E (lower row) in Fig. 1 and 2. Notice the scale differences between the top and bottom rows.

FIG. 4: Grey-scaled images of the vortex waveguide and the guided modes for the SHG process. Initial conditions correspond to a vortex carried by a Gaussian beam and the fundamental wave, both corresponding to the point D in Fig. 2.

amplitude components. The SHG process becomes even worse for the negative phase-matching. Figure 3 (lower row) shows an example of a very strong mode coupling and vortex distortion corresponding to the parameter region $\beta < 0$ and $\lambda > 0$.

If the vortex is removed at the input in the vortex-waveguiding regime ($\lambda < 0$), the SHG conversion efficiency drops by at least one order of magnitude or more, and both the strong fundamental and weak SH fields diffract rapidly. In the quadratic soliton regime ($\lambda > 0$), the effective self-focusing nonlinearity of the second-order parametric interaction between the fields $u$ and $v$ allows the formation of two-wave parametric solitons even without the vortex component. However, in this case the input power does not transfer into the SH field, it undergoes a redistribution between the harmonics in such a way that both fields either approach a stationary state corresponding to a $(2+1)$-dimensional quadratic soliton (above the existence threshold), or just diffract (below the threshold). To summarize, our study of SHG in vortex-induced waveguides in different regimes suggests that the enhanced conversion efficiency can be achieved only in the vortex-waveguiding regime.

Possible experimental realizations of the concept of the SHG in vortex-induced waveguides can be achieved in a crystal of Fe:LiNbO$_3$ where phase-matching can be satisfied through the birefringence effect at the angle $\theta = 81^\circ$ with respect to the $z$-axis, provided the four-wave mixing effect is suppressed. The other possibility is to employ photorefractive crystals and the temperature tuning technique, similar to that reported earlier.

In conclusion, we have analyzed the simultaneous guidance of both the fundamental and second-harmonic waves by an optical vortex soliton. We have described novel classes of three-wave parametric solitons with a vortex-soliton component, and have studied the second-harmonic generation in the vortex-induced waveguides. For the first time to our knowledge, we demonstrated that larger conversion efficiency of the SHG process can be achieved in the vortex-waveguiding regime.

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