RELATIVISTIC GENERALIZATION OF THE INCENTIVE TRAP OF INTERSTELLAR TRAVEL WITH APPLICATION TO BREAKTHROUGH STARSHOT

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ABSTRACT
As new concepts of sending interstellar spacecraft to the nearest stars are now being investigated by various research teams, crucial questions about the timing of such a vast financial and labor investment arise. If humanity could build high-speed interstellar lightsails and reach α Centauri 20 yr after launch, would it be better to wait a few years, then take advantage of further technology improvements and arrive earlier despite waiting? The risk of being overtaken by a future, faster probe has been described earlier as the incentive trap. Based on 211 yr of historical data, we find that the speed growth of artificial vehicles, from steam-driven locomotives to Voyager 1, is much faster than previously believed, about 4.72% annually or a doubling every 15 yr. We derive the mathematical framework to calculate the minimum of the wait time to launch \( t \) plus travel time \( \tau(t) \) and extend it into the relativistic regime. We show that the \( t + \tau(t) \) minimum disappears for nearby targets. There is no use of waiting once we can reach an object within about 20 yr of travel, irrespective of the actual speed. In terms of speed, the \( t + \tau(t) \) minimum occurs at 19.6% the speed of light (\( c \)), in agreement with the 20% \( c \) proposed by the Breakthrough Starshot Initiative. If interstellar travel at 20% \( c \) could be achieved within 45 yr from today and the kinetic energy be increased at a rate consistent with the historical record, then humans can reach the ten most nearby stars within 100 yr from today.

Keywords: methods: analytical — relativistic process — solar neighborhood — space vehicles — stars: individual: α Centauri — time

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1. INTRODUCTION

The exponential growth of the maximum speed of man-made vehicles, from wind-driven ships, to steam- driven ships and trains, to cars, planes, and space rockets suggests that mankind will reach reasonable speeds for interstellar travel either in this century or the next. The possibility of pushing lightsails to interstellar velocities using Earth-based lasers or the solar photon pressure has in fact been studied as long as half a century ago (Marx 1966; Redding 1967; Forward 1984; Macchi et al. 2010; Macdonald 2016). But it has not been until very recently that gram-sized spacecraft have actually been re-considered for interstellar journeys using ultra-light photon sails (Lubin 2016; Manchester & Loeb 2017; Heller & Hippke 2017; Christian & Loeb 2017; Hoang et al. 2017) with top speeds of up to 20 % the speed of light (c). Further into the future, fly-bys around nearby stars could use gravity assists (Forgan et al. 2013) and the stellar photonic pressures (Heller et al. 2017) to go beyond the solar neighborhood with extremely low demands for on-board propellant.

1.1. The incentive trap

The time to reach interstellar targets is potentially larger than a human lifetime, and so the question arises of whether it is currently reasonable to develop the required technology and to launch the probe. Alternatively, one could effectively save time and wait for technological improvements that enable gains in the interstellar travel speed, which could ultimately result in a later launch with an earlier arrival.

Intuitively, one might be inclined to expect that the continuing growth of speed should make it more reasonable for mankind to wait before we set out to the stars, because future spacecraft would be fast enough to overtake any probe that we could send out soon. This conflict has been described as the incentive trap, i.e., the risk of an interstellar space probe to be overtaken by a future probe that has been launched with a velocity high enough to intercept the first probe owing to the ongoing technological progress.\(^1\) Kennedy (2006) showed that the total time from now, that is to say, the waiting time to launch \(\tau\) plus the travel time \(\tau(t)\), to reach an arbitrary stellar target has a minimum if we assume an exponential growth of the interstellar travel speed \(v(t)\). Given the fastest speed of travel at the time (referring to NASA’s New Horizon mission), and assuming a 1.4 % average growth rate in speed, Kennedy (2006) showed that the minimum of \(t + \tau(t)\) to reach Barnard’s star, at a distance of about 6 ly, is 712 yr from 2006. Kennedy (2006) also stated that the minimum of the total time will be reached long before relativistic speeds will be achieved.

Here we address the incentive trap under the notion that \(v = 20\% c\) can be reached within a few decades from now, as proposed by the Breakthrough Starshot Initiative\(^2\), or Starshot for short (Popkin 2017). This would fundamentally change both the assumptions and the implications of the incentive trap because the speed doubling and the compounded annual speed growth laws would collapse as \(v\) approaches \(c\).

1.2. Historical speed growth

Let us consider humanity’s speed improvements in modern history. The first steam locomotive, Richard Trevithick’s “Penydarren”, made about 4 km h\(^{-1}\) in the year 1804. The world’s first production car, the Benz Velocipede of 1894, obtained top speeds of up to 19 km h\(^{-1}\), which was overruled by a 204 km h\(^{-1}\) world record by a Stanley Steamer race car in 1903. At the same time, the top speeds of planes increased from about 11 km h\(^{-1}\) of the Wright Flyer in 1903 to about 1000 km h\(^{-1}\) by the German Messerschmitt rocket-powered planes in the 1940s.

Later on, rockets allowed interplanetary cruises on a timescale of years in the 1960s. In 2015, the Voyager 1 mission has been observed to leave the solar system at a speed of about 17 km s\(^{-1}\) or 5.7 \(\times 10^{-5}\) \(c\) relative to the Sun. And next year, the Solar Probe Plus mission

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\(^1\) An early illustration of this scenario was given in the 1944 science fiction short story “Far Centauri” by van Vogt (1944). It pictures a manned mission that takes 500 yr to reach a Centauri only to find that the system has already been colonized by humans who actually launched later.

\(^2\) http://breakthroughinitiatives.org/Initiative/3
will perform a close solar fly-by with a top heliocentric speed of $195 \text{ km s}^{-1}$ or $6.5 \times 10^{-4} c$ (Fox et al. 2016). The nominal launch of interstellar lightsails with Starshot is in about the year 2040 with a speed of $2 \times 10^{-1} c$.

All these values are symbolized with black-rimmed circles in Figure 1, with additional top speed measurements of trains, cars, planes, and rockets shown with different symbols (see legend). The dashed black line illustrates an exponential growth law connecting the 1 m s\(^{-1}\) speed of the “Penydarren” steam locomotive in 1804 with the $5.7 \times 10^{-5} c$ solar system escape speed of Voyager 1 in 2015. Although this exponential growth captures the development of historic top speeds, we do not claim in this report that it will continue as such. Instead, we investigate the implications for interstellar travel if it does continue. Moreover, note the substantial offset of the yellow symbol referring to Starshot. In Section 4 we demonstrate that this jump in velocity in the year 2040 would save about 150 yr of speed growth according to the historic record.

2. ANALYTICAL MODELS OF WAITING TIMES AND TRAVEL TIMES

2.1. Doubling laws

2.1.1. The speed doubling law

The historical and future speed developments can be addressed with mathematical frameworks, e.g. with an exponential growth of the maximum speed of a vehicle or probe. In its most simple version, the exponential growth law can be written as

\[ v(t) = v_0 2^{t/h} , \]

where $v_0$ is the current maximum speed, $h$ is the time between doublings, and $t$ is time (Kennedy 2006). Then $\tau_0 = s/v_0$ is the current travel time and $\tau(t) = s/v(t)$ is the future travel time to an object at a distance $s$ from us. With

\[
\frac{\tau_0}{\tau(t)} = \frac{s/v_0}{s/v(t)} = 2^{t/h} \\
\Leftrightarrow \quad \tau(t) = \frac{\tau_0}{2^{t/h}} ,
\]

the total time from now to reach the target becomes

\[ t + \tau(t) = t + \frac{\tau_0}{2^{t/h}} . \quad (3) \]

In this framework, $t + \tau(t)$ depends neither on the distance of the target nor on the current maximum speed. Moreover, relativistic effects are neglected.

The above set of equations has been worked out by Kennedy (2006) to identify the incentive trap. As an extension to that, we determine the location of the minimum of Equation (3), at a time $t_{\text{min}}$ from now, which can be evaluated analytically by means of the derivative. We use the fact that $(u')' = u''(v' \ln(u) + vu'/u)$ for $u > 0$, where $u$ and $v$ are functions of $x$ and $u'$ and $v'$ denote their derivatives. Applied to Equation (3), this yields

\[
\frac{d}{dt} (t + \tau(t)) = 1 + \tau_0 \frac{d}{dt} 2^{-t/h} = 1 - \frac{\tau_0 \ln(2)}{h} 2^{-t/h} , \quad (4)
\]

where $\ln(x)$ symbolizes the natural logarithm of $x$. We then solve

\[
\frac{d}{dt} (t + \tau(t)) = 0, \quad t = t_{\text{min}}
\]

\[
\Leftrightarrow \quad t_{\text{min}} = \frac{-\ln \left(\frac{h}{\tau_0 \ln(2)}\right)}{\ln(2)} . \quad (5)
\]

The historical record (see Section 1.2) traces a speed growth of about four orders of magnitude over the 211 yr from the 1804 “Penydarren” (about 1 m s\(^{-1}\)) to Voyager 1 (about 17,000 m s\(^{-1}\)) in 2015 or, alternatively, 14 speed doublings with a speed doubling time of

\[
h = \frac{t}{\log_2(v/v_0)} = \frac{(2015 - 1804) \text{yr}}{(\log_{10}(17,000)/\log_{10}(2))}
\]

\[
= 15.0 \text{ yr} \quad (6)
\]

(see Section 4 for a discussion of the reference speeds chosen). Plugging this value of $h$ into Equation (1) yields the black dashed line in Figure 1. Most important, this value, which is based on the historical record of top speeds is, much smaller than the nominal 100 yr assumed by Kennedy (2006). As a consequence, we can expect to derive much smaller waiting times to the launch of interstellar probes in our model compared to Kennedy (2006).

If this growth can be maintained for another $7\frac{1}{2}$ doublings, i.e. for another about 112 yr, then humanity would achieve $1\% c$. That said, the historical record of top speed achievements exhibits jumps whenever new technologies have been introduced. The interstellar speeds aimed at by Starshot could initiate such a speed jump within the next few decades and enable interstellar speeds much earlier.

2.1.2. The energy growth law
The speed doubling law is naturally restricted to non-relativistic speeds. Beyond 0.5c, the speed cannot possibly double anymore. We now consider a model, in which the growth of the kinetic energy \( E_{\text{kin}} \) is considered instead. If technological progress permits, then \( E_{\text{kin}} \) could allow relativistic speeds of gram-to-kilogram sized probes within decades from now (Lubin 2016). Figure 2 shows the gain in speed for an increase of the kinetic energy pumped into a 1 kg sail (dashed line) and a 0.01 kg sail (solid line), the latter of which corresponds roughly to the nominal weight of a Starshot sail. Note that the most powerful power plant on Earth today, the Three Gorges Dam in China, can reach power outputs of up to 22.5 GW. A 1 kg (or 0.01 kg) probe gaining kinetic energy at the same rate for the duration of 100 s would reach terminal speeds of 0.7 %c (or 7.1 %c).

In the non-relativistic regime, Equation (1) is equivalent to

\[
v(t) = \sqrt{\frac{2E_{\text{kin},0}}{M_s}} t^{2t/h}, \tag{7}
\]

where \( M_s \) is the rest mass of the space probe and \( E_{\text{kin},0} \) is the maximum kinetic energy that can possibly be transferred to the probe today. Hence, the development of the kinetic energy obeys the following law:

\[
E_{\text{kin}}(t) = E_{\text{kin},0} \cdot 2^{2t/h}. \tag{8}
\]

The total energy of the probe then is \( E = \gamma M_s c^2 \), with \( \gamma(v) = (1 - (v/c)^2)^{-1/2} \), so that

\[
v(t) = c \sqrt{1 - \left( \frac{M_s c^2}{E(t)} \right)^2}. \tag{9}
\]

With \( E(t) = M_s c^2 + E_{\text{kin}}(t) \) we have

\[
v(t) = c \sqrt{1 - \left( \frac{M_s c^2 + E_{\text{kin},0} 2^{2t/h}}{M_s c^2 + E_{\text{kin},0} 2^{2t/h}} \right)^2}. \tag{10}
\]

In analogy to Equations (2) and (3), we can calculate the future travel time to an object as

\[
t + \tau(t) = t + \frac{\tau_0 v_0}{c} \sqrt{1 - \left( \frac{M_s c^2 + E_{\text{kin},0} 2^{2t/h}}{M_s c^2 + E_{\text{kin},0} 2^{2t/h}} \right)^2}, \tag{11}
\]

which has a derivative

\[
\frac{d}{dt} (t + \tau(t)) = 1 - \frac{\tau_0 v_0}{hc} \frac{(M_s c^2)^2 \ln(4) 2^{-t/h}}{\sqrt{E_{\text{kin},0} (2M_s c^2 + E_{\text{kin},0} 4^{t/h})^3}}. \tag{12}
\]

The minimum of Equation (12) will be determined numerically below. Note that the total wait plus travel time in Equation (11) depends on \( v_0 \), which is different from the speed doubling law described in Equation (3).

2.2. Compounded growth of speed or energy

2.2.1. Compounded growth of speed

As an alternative to the speed doubling law, Kennedy (2006) proposed that a compound annual growth law, as it is often used in industry and investment business, could offer a more realistic description of future speed developments:

\[
v(t) = v_0 (1 + r)^t/yr, \tag{13}
\]

where \( r \) is the annual percenetal growth rate. In fact, Equation (13) is equivalent to Equation (1) for

\[
r = 2^{r/yr - 1}, \quad h = \frac{yr}{\log_2 (1 + r)}, \tag{14}
\]

which can be derived by equating Equations (1) and (13) and then solving for either \( r \) or \( h \).

The total time to arrival is then given as

\[
t + \tau(t) = \frac{\tau_0}{(1 + r)^t/yr} + t \tag{15}
\]

and we determine the derivative as

\[
\frac{d}{dt} (t + \tau(t)) = 1 - \frac{\tau_0 \ln (1 + r) yr}{(1 + r)^t/yr}. \tag{16}
\]

\[3\] Note that the exponent \( t \) in Equation (3) of Kennedy (2006) must be divided by 1 yr.
Figure 3. Waiting times $t$ and travel times $\tau(t)$ assuming a speed doubling (black lines) or relativistic energy (red lines) growth law. (a) Total time to the target (black solid line) as in the Kennedy (2006) non-relativistic model, assuming speed doubling after a time $h$ (Equation 3). The dashed and dotted lines show the contributions of waiting time and travel time. The red line shows the relativistic correction assuming continuous energy growth (Equation 11). (b) Minimum of the total waiting plus travel time as a function of the current travel time $\tau_0$. The doubling time is taken to be $h = 15 \pm 5\,\text{yr}$, with the models using the nominal value shown with solid lines. The black line and the grey envelope illustrate the non-relativistic speed doubling (Equation 5). Red lines visualizes the relativistic energy growth law based on numerical calculations of the minimum of Equation (12).

The minimum of which is located at

$$d\frac{dt}{dt}(t + \tau(t)) = 0, \quad t = t_{\min}$$

$$t_{\min} = -\frac{\ln\left(\frac{\tau_0}{\text{yr}} \ln(1 + r)\right)}{\ln(1 + r)} \text{yr}. \quad (16)$$

The historical speed record outlined in Section 1.2 tracks an annual speed growth of

$$r = \left(\frac{v}{v_0}\right)^{\text{yr/yr}} - 1 = (17,000)^{1/211} - 1 = 4.72\% \quad (17)$$

(see black dashed line in Figure 1), suggesting that humanity will achieve $1\% c$ within $112\,\text{yr}$. This value for the annual growth rate is substantially larger than the fiducial $1.4\%$ assumed by Kennedy (2006).

2.2.2. Compounded growth of energy

Just like the speed doubling model, the compounded growth of speed becomes impossible near the speed of light, which is why we now address the compounded growth of energy as per

$$E_{\text{kin}}(t) = E_{\text{kin,0}}(1 + r)^{2t/\text{yr}}, \quad (18)$$

which means that the speed grows as

$$v(t) = c\sqrt{1 - \left(\frac{M_0c^2}{M_0c^2 + E_{\text{kin,0}}(1 + r)^{2t/\text{yr}}}\right)^2}, \quad (19)$$

so that the total wait plus travel time becomes

$$t + \tau(t) = t + \frac{\tau_0v_0}{\text{yr}} \frac{1}{c} \sqrt{1 - \left(\frac{M_0c^2 + E_{\text{kin,0}}(1 + r)^{2t/\text{yr}}}{M_0c^2 + E_{\text{kin,0}}(1 + r)^{2t/\text{yr}}}\right)^2}. \quad (20)$$

The minimum of the derivative

$$d\frac{dt}{dt}(t + \tau(t)) = 1 - 2\frac{v_0}{c} \frac{\tau_0}{\text{yr}} \times \frac{(M_0c^2)^2 \ln(1 + r)(1 + r)^{-t/\text{yr}}}{\sqrt{E_{\text{kin,0}}(2M_0c^2 + E_{\text{kin,0}}(1 + r)^{2t/\text{yr}})^3}}. \quad (21)$$

is calculated numerically in the next section.

3. RESULTS

3.1. Speed doublings and relativistic correction

3.1.1. The speed doubling law

The black lines in Figure 3(a) show the different contributions of the travel time (dashed) and the waiting time (dotted) in the non-relativistic approach, assuming a doubling time $h = 15\,\text{yr}$ and $\tau_0 = 100\,\text{yr}$ for the travel time, e.g. to a hypothetical nearby star. In this example, the minimum occurs within about $t_{\min} = 33\,\text{yr}$, when the sum of $t$ and $\tau(t_{\min})$ is about $55\,\text{yr}$ (see the black circle).

More precisely, Equation (5) yields $t_{\min} = 33.1\,\text{yr}$. The gain in numerical precision compared to a graphical
interpretation in Figure 3(a) is irrelevant given the substantial uncertainties in the growth law, but the formula allows us to conveniently study the expected optimum time for the launch of interstellar probes over a range of possible doubling times and contemporary travel times \( \tau_0 \).

Figure 3(b) shows the location of the minimum of \( t + \tau(t) \) for a doubling time of 15 yr (black solid line), with a grey strip visualizing a tolerance of \( \pm 5 \) yr in \( h \), as a function of the current travel time \( \tau_0 \). The black circle on the solid line (for \( h = 15 \) yr) at \( \tau_0 = 100 \) yr refers to its counterpart in Figure 3(a) at the minimum of \( t + \tau(t) \).

An interesting result from this plot is that, taking \( h = 15 \) yr as a reference, there is no future minimum in \( t + \tau(t) \) if the current travel time to the target is smaller than about 22 yr. For larger doubling times, the minimum disappears at larger values of \( \tau_0 \) and vice versa. In words, if the travel time to an object is short in the first place, then there is no point in waiting for speed improvements. Any further growth in the maximum possible speed would be negligible due to the short travel time, whereas the waiting time would dominate.

We refer to this minimum value of \( \tau_0 \) for which the expected speed improvements make it worth waiting as the incentive travel time,

\[
\tau_{inc} = \min \left( \tau_0(t_{min}) \right) = \tau_0(t_{min} = 0) = \tau_{0}(t_{min} = 0) = \tau_{0}(t_{min} = 0). \tag{22}
\]

In the non-relativistic regime of the speed doubling law, we can determine \( \tau_{inc} \) by setting \( t_{min} = 0 \) in Equation (5) and solving for \( \tau_0 \), thus

\[
\tau_{inc} = \frac{h}{\ln(2)}. \tag{23}
\]

For the above-mentioned example (solid line in Figure 3b), we find \( \tau_{inc} = 21.6 \) yr as the minimum current travel time to the target for which it is worth waiting for speed improvements. In other words, any target that we can reach today within 21.6 yr of travel cannot be reached earlier based on a doubling of speed every 15 yr.

3.1.2. Relativistic correction of the speed doubling law

The red line in Figure 3(a) shows \( t + \tau(t) \) in the relativistic regime as per Equation (11). Different from the non-relativistic model in Equation (3), Equation (11) requires the input of a specific \( v_0 \) and \( M_s \) as the doubling of the speed of travel breaks down near the speed of light.\(^4\)

For the purpose of illustration, let us consider the travel of a nominal \( M_s = 0.01 \) kg Starshot probe to the \( \alpha \) Centauri (\( \alpha \) Cen) system, which has recently become attractive for interstellar exploration after the detection of an Earth-mass planet in the stellar habitable zone of \( \alpha \) Cen C (Anglada-Escudé et al. 2016). We adopt a distance of 4.3 ly to \( \alpha \) Cen and assume \( v_0 = 4.3 \% c \) to construct an initial travel time \( \tau_0 = 100 \) yr. The to-

\( E_{kin,0} \) in Equation (11) is also determined by \( v_0 \) and \( M_s \), that is to say, as per \( E_{kin,0}(v_0, M_s) = M_s c^2(\sqrt{1 + (v_0/c)^2}) \).
tal time to the target agrees in both the non-relativistic model (solid black line) and the relativistic model (solid red line) within 1% up to about 50 yr from now. Note that “now” implies that we could achieve \( v_0 = 4.3\% c \) with the given technology today, which has not actually been demonstrated.

The divergence between the non-relativistic speed doubling and the relativistic model in Figure 3(a) is a manifestation of the fact that the travel time in the relativistic model (not shown) cannot possibly converge to zero but is always restricted to a value \( > s/c, \) or \( > 4.3 \) yr for \( \alpha \) Cen. In fact, this value corresponds to the offset between the black solid and the red solid lines.

In particular, while Equation (5) yields an optimal wait time \( t_{\text{min}} = 33.1 \) yr in the non-relativistic model, our numerical evaluation of Equation (12) for the derivative of the total wait plus travel time in the relativistic regime yields \( t_{\text{min}} = 32.8 \) yr. The difference in the two models is small since \( v_0 \) is relatively small compared to \( c \) and the distance to the target is also relatively small. Special-relativistic effects kick in for more far away objects (see Section 3.3). It is critical to note that the minimum in the relativistic model occurs earlier than in the non-relativistic case. That is because the speed doubling breaks down in the relativistic model.

Knowing \( t_{\text{min}} \), we can now derive the critical interstellar speed that serves as a benchmark value above which gains in the kinetic energy to be transferred into the probe would not result in smaller wait plus travel times to \( \alpha \) Cen. Assuming \( \tau_0 = 100 \) yr and \( v_0 = 4.3\% c \), a 0.01 kg probe would have a kinetic energy of \( E_{\text{kin},0} = M c^2(\gamma(v_0) - 1) = 0.86 \) TJ. We plug \( E_{\text{kin},0} = 0.86 \) TJ and \( t_{\text{min}} = 32.8 \) yr into Equation (10) and obtain \( v(t_{\text{min}}) = 19.6\% c \).

If we consider larger values of \( \tau_0 \), then \( t_{\text{min}} \) becomes larger than 32.8 yr. On the other hand, for a given distance, \( v_0 = s/\tau_0 \) would decrease and so would \( E_{\text{kin},0} \). We performed numerical simulations for arbitrary values of \( \tau_0 \), which show that \( v(t_{\text{min}}) \) is independent of the assumption of \( \tau_0 \). In other words, whenever humanity will achieve the capability of reaching \( v(t_{\text{min}}) = 19.6\% c \), there is no need to wait for speed improvements according to the relativistic correction of the speed doubling law for going to \( \alpha \) Cen. This value is in agreement with the 20\% \( c \) proposed by Starshot.

3.2. Compounded speed growth and relativistic correction

3.2.1. Compounded growth of speed

Figure 4(a) shows \( t + \tau(t) \) for a compounded speed growth, again assuming \( \tau_0 = 100 \) yr as in Figure 3(a) but now with an annual speed gain of \( r = 4.72\% \), illustrated by the black line. The grey envelope refers to a variation of \( r \) by 1%. Given this parameterization of the model, Equation (16) yields \( t_{\text{min}} = 33.1 \) yr with a minimum wait plus travel time of 54.8 yr.

As in the case of speed doubling, we observe that the minimum of \( t + \tau(t) \) disappears for initial travel times \( \tau_0 < \tau_{\text{inc}} \), the limit of which we determine by setting Equation (16) equal to zero and by solving for \( \tau_0 \), thus

\[
\tau_{\text{inc}} = \frac{1 \text{ yr}}{\ln(1 + r)}. \tag{24}
\]

For our example of \( \tau_0 = 100 \) yr and \( r = 4.72\% \) this yields \( \tau_{\text{inc}} = 21.7 \) yr. Any target that can be reached earlier than that is not worth waiting for further speed improvements in this model.

3.2.2. Relativistic correction of the law of compounded speed growth

The red solid line in Figure 4(a) denotes the relativistic compounded growth of energy assuming that we could reach \( \alpha \) Cen at \( \tau_0 = 100 \) yr. The minimum of the total wait plus travel time occurs at \( t_{\text{min}} = 32.8 \) yr where \( t + \tau(t) = 55.1 \) yr.

Plugging \( E_{\text{kin},0} = 0.86 \) TJ and \( t_{\text{min}} = 32.8 \) yr into Equation (19) yields \( v(t_{\text{min}}) = 19.6\% c \), in agreement with the 20\% \( c \) proposed by Starshot. Again, numerical simulations show that this value is independent of the assumption of \( \tau_0 \).

The red lines in Figure 4(b) refer to the relativistic compounded energy growth law. The red solid line shows \( t_{\text{min}} \) as a function of \( \tau_0 \), and the red dotted lines illustrate a variation of \( r \) by 1%. Just like in the speed doubling law model or in its relativistic energy growth law, the red lines lie under their non-relativistic counterparts, which is to say that \( t_{\text{min}} \) occurs earlier if special-relativistic effects are taken into account.

3.3. Application to nearby stars with Starshot

We now apply the relativistic correction of the compounded growth law to our most nearby stars (see Table 1), assuming a 0.01 kg sail and an available travel speed of \( v_0 = 20\% c \) envisioned by Starshot. Although \( v_0 = 20\% c \) is not accessible for a space probe today, the results of this section inform us about the optimal strategy of the exploration of nearby stars once Starshot goes on line. The relativistic correction of the speed doubling law is not studied in detail, but we confirmed that its predictions do not differ significantly from the ones derived with the relativistic compounded growth law.

In Figure 5 we plot \( t + \tau(t) \) for the ten targets as per Equation (20), where we substitute \( \tau_0 v_0 \) with the stellar distance \( s \). Black lines refer to \( r = 4.72\% \) and the colored shades illustrate a 1% tolerance. The origin of
Figure 5. Total wait plus travel times to ten of the most nearby stellar systems as a function of time after Starshot starts operating at 20 % c. A compounded growth law for the available energy is assumed, with black lines referring to an annual increase in the available energy of $r = 4.72\%$ based on the last roughly 200 yr of human speed improvements. The colored strips illustrate 1% of tolerance in $r$. The names of the systems are indicated with labels. The inset shows the speed available to humanity at the minimum of the projected wait plus travel time ($v(t_{\text{min}})$) as a function of distance to the star.

The coordinate system is chosen at a time $t_{\text{BS}} = 0$ when Starshot goes into service and allows $v_0 = 20\% c$ at the time. Table 1 lists $t_{\text{BS}}$ at the minima of $t + \tau(t)$ as well as the corresponding speeds $v(t_{\text{BS}})$ for each star. These are the launch speeds beyond which the compounded growth law does not suggest any further reduction in the wait plus travel time.

Figure 5 shows that as soon as interstellar speeds of 20 % c become available, there is no use in waiting for further speed growth to go to α Cen. Furthermore, the minima of the waiting plus travel times for the next nine systems in our list all occur within 25 yr or less after the time Starshot will have gone into service, with total times to the target $\lesssim 55$ yr. In other words, if Starshot would become available in as late as 45 yr from today and if the kinetic energy pumped into 0.01 kg probes can be increased at a rate consistent with the historical record of the last two centuries, then humans can reach the ten most nearby bright stars within 100 yr from today.

The inset in the lower right of Figure 5 shows $v(t_{\text{min}})$, the speed (in units of c) at the time of the minimum wait plus travel time, as a function of distance out to 1000 ly. The solid curve assumes an annual speed growth rate of $r = 4.72\%$. The green strip illustrates a 1% tolerance in $r$. For the closest targets, this optimal speed converges to small values, e.g. to $v(t_{\text{min}}) = 0.5\% c$ for a target at 0.1 ly = 6320 AU. The function then rises to $v(t_{\text{min}}) = 50\% c$ at 13.5 ly and eventually converges to c for larger distances.

For objects closer than 2 ly, the non-relativistic model (dashed black line) and the relativist model differ by less than 0.1% in $v(t_{\text{min}})$. For targets beyond about 10 yr, however, the non-relativistic model predicts $v(t_{\text{min}}) = 46\% c$ as an optimal departure speed, whereas the relativistic model suggests $v(t_{\text{min}}) = 40\% c$, the latter of which takes into account that future gains in the energy transferred to the probe will not result in as much an increase in speed as in the non-relativistic scenario (but rather in increased mass). We performed
Table 1. Solar distances, optimal launch times after Starshot starts operations, and optimal velocities for minimization of \( t + \tau(t) \) to reach ten of the most nearby bright star systems.

| #   | Name               | Distance (ly) | \( t_{BS} \) (yr) | \( v(t_{BS}) \) \(^b\) % c |
|-----|--------------------|---------------|-------------------|----------------------------|
| 1.  | \( \alpha \) Cen A/B/C | 4.3           | 0                 | 20                         |
| 2.  | Sirius A/B         | 8.6           | 13                | 36                         |
| 3.  | \( \epsilon \) Eri | 10.5          | 17                | 42                         |
| 4.  | 61 Cyg A/B         | 11.4          | 19                | 45                         |
| 5.  | Procyon A/B        | 11.5          | 19                | 45                         |
| 6.  | \( \epsilon \) Ind A/Ba/Bc | 11.8    | 19                | 46                         |
| 7.  | \( \tau \) Cet    | 11.9          | 20                | 46                         |
| 8.  | GJ 380             | 15.9          | 25                | 55                         |
| 9.  | 40 Eri A/B/C       | 16.5          | 25                | 57                         |
| 10. | 70 Oph A/B         | 16.6          | 25                | 57                         |

\(^a\) Minimum of the wait plus travel time after Starshot starts operating at 20\% c, see the abscissa in Figure 5. The values have been determined numerically from the curves in Figure 5.

\(^b\) Optimal speed for the reduction of \( t + \tau(t) \) if improvements of the Starshot technology would permit an annual speed gain by \( r = 4.72\% \).

numerical simulations to validate that all these values of \( v(t_{min}) \) are independent of the sail’s rest mass and only depend on \( r \) in the compounded growth law.

4. DISCUSSION

Free parameters in exponential growth laws are notoriously hard to determine or predict. Small variations in the free parameters, e.g. of \( h \) in Equation (6) or of \( r \) in Equation (17), have dramatic effects on the predictions of the respective model. Hence, the numerical values that we derived from historical data shall only be accepted with reservation.

That said, our long data baseline of speed records spans 211 yr. Variations of the initial 1 m s\(^{-1}\) or the final 17,000 m s\(^{-1}\) by a factor of two over this baseline would result in variations of \( h \) by about 1 yr or of \( r \) by about 0.34\%. The colored strips of 5 yr tolerance in Figure 3 and of 1\% tolerance in Figure 4 thus encapsulate the somewhat arbitrary choice of the reference speeds.

Coming back to Figure 1, we find that the symbol referring to Starshot sits about three orders of magnitude above the exponential speed doubling law in the year 2040. An equivalent transformation of Equation (6), solving it for \( t \), suggests that \( v(t) = 0.2c \) would be achieved in the year 2191 if the top speeds would follow the historical records of the past about 200 yr. This confirms the truly transformative effect that Starshot would have on humanity as a migrating and exploring civilization as a whole.

New velocity regimes impose new physical or engineering challenges. In our understanding, this is all absorbed in the growth law as illustrated in Figure 1. As an example, in the early 19th century people were afraid of losing consciousness if travelling faster than running speed (\( \approx 20 \) km h\(^{-1}\)). Later on, vibrations that occur when planes approach the sonic barrier were thought to cause fatal damage (\( \approx 1200 \) km h\(^{-1}\)) (Portway 1940). And the heat generated during atmospheric re-entry (\( \approx 28,000 \) km h\(^{-1}\)) was considered a main challenge for any manned spaceship returning to Earth (Heppenheimer 2014). Certainly other challenges will be identified towards relativistic speeds, e.g. the structural integrity and stability during acceleration (Manchester & Loeb 2017), the interaction of the space probe with the interstellar medium (Hoang et al. 2017), or the aiming accuracy towards the target (Heller et al. 2017), to name just a few. That said, these might not turn out to be ultimate limits on the maximum possible speed to reach.

5. CONCLUSIONS

We studied the top speeds obtained by human-made vehicles over the past 211 yr, which can well be described
by exponential growth laws, and projected the historical data into the future to investigate the possibility of interstellar travel within the next century. According to our estimates, the historical speed growth is much faster than previously believed, about 4.72% annually or a doubling every 15 yr, from steam-driven locomotives to Voyager 1.

Surprisingly, we found that the minimum of the wait plus travel time \( t + \tau(t) \), previously described by Kennedy (2006), disappears for targets that can be reached earlier than a critical travel time, which we refer to as the incentive travel time \( \tau_{\text{inc}} \). In the non-relativistic domain, \( \tau_{\text{inc}} \) depends only on the doubling time (see Equation 23) or, alternatively, on the annual rate of speed growth (see Equation 24). As an example, for \( h = 15 \) yr as derived from historical data, \( \tau_{\text{inc}} \) is about 22 yr in both the relativistic and the non-relativistic model, i.e., targets that we can reach within about 22 yr of travel are not worth waiting for further speed improvements if speed doubles every 15 yr. The identification of an incentive travel time is irrespective of the parameterization of the underlying law of the speed growth, though its actual value depends, of course.

In terms of the optimal interstellar velocity for launch, the most nearby interstellar target \( \alpha \) Cen will be worthy of sending a space probe as soon as about 20% \( c \) can be achieved because future technological developments will not reduce the travel time by as much as the waiting time increases. This value is in agreement with the 20% \( c \) proposed by Starshot for a journey to \( \alpha \) Cen. We also investigated the speeds beyond which further speed improvements according the historical data would not result in reduced wait plus travel times to ten of the most nearby bright star systems (see Table 1). It turns out that these speeds, from 20% \( c \) for \( \alpha \) Cen (at 4.3 ly) to 57% \( c \) for 70 Oph (at 16.6 ly), would all become available within 25 yr after Starshot will have started operations. These values were derived under the assumption that Starshot or alternative technology would continuously be upgraded according the historical speed record and they are independent of the sail’s rest mass.

If Starshot would go on line within the next 45 yr and if the kinetic energy transferred into the probes can be increased at a rate consistent with the historical speed record of the last 211 yr, then humans can reach the ten most nearby stars within 100 yr from today.

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