CURRENT ISSUES FOR INFLATION

D. H. LYTH

Physics Department, Lancaster University, Lancaster LA1 4YB, U.K.

Brief review of some current topics, including gravitino creation and large extra dimensions.

Most inflation models create a lot of gravitinos. I will focus on papers that appeared in 1999, building on a fairly comprehensive review of earlier work, and starting with gravitino creation. Gravitinos are created at reheating by thermal collisions. If the gravitino mass $m_{3/2}$ is of order 100 GeV, as in gravity-mediated models of SUSY breaking, these gravitinos upset nucleosynthesis unless $\gamma T_R \lesssim 10^9$ GeV, where $T_R$ is the reheat temperature, and $\gamma^{-1}$ is the increase in entropy per comoving volume (if any) between reheating and nucleosynthesis. If instead $m_{3/2} \sim 100$ keV, as in typical gauge-mediated models of SUSY breaking, the gravitino is stable and will overclose the Universe unless $\gamma T_R \lesssim 10^4$ GeV. Only if $m_{3/2} \gtrsim 60$ TeV, as might be the case in anomaly-mediated models of SUSY breaking, are the gravitinos from thermal collisions completely harmless.

Gravitinos will also be created after inflation, by the amplification of the vacuum fluctuation. The evolution equations for the helicity $1/2$ and $3/2$ mode functions, required to calculate this second effect, have been presented only this year. A suitably chosen helicity $3/2$ mode function satisfies the Dirac equation in curved spacetime, with mass $m_{3/2}(t)$ (the gravitino mass in the background of the time-dependent scalar field(s) which dominate the Universe after inflation). This implies that helicity $3/2$ gravitinos created from the vacuum are cosmologically insignificant, compared with those created from particle collisions.

The situation for helicity $1/2$ is more complicated, because this state mixes with the fermions involved in SUSY breaking (the super-Higgs effect). So far, the evolution equation for the mode function has been presented only for the simplest possible case, that the only relevant superfield is a single chiral superfield. Using this idealized equation, its authors estimated (see also) that gravitinos created just after inflation have, at nucleosynthesis, the abundance

$$\frac{n}{s} \simeq 10^{-2} \frac{\gamma T_R M^3}{V}. \quad (1)$$

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$^a$Updated version of a talk given at COSMO99 International Workshop on Particle Physics and the Early Universe, 27 September–2 October 1999, Trieste, Italy.
The abundance is specified by the ratio of $n$, the gravitino number density, and $s$, the entropy density. It is determined by $V$, the potential at the end of inflation, and $M$, the mass of the oscillating field which is responsible for the energy density just after inflation.

Entropy increase can come from a late-decaying particle, with or without thermal inflation \cite{9,10,11,12}. If there is no thermal inflation, the requirement that final reheating occurs before nucleosynthesis gives

$$\gamma T_R \gtrsim 10 \text{ MeV}. \quad (2)$$

One bout of thermal inflation typically multiplies $\gamma$ by a factor of order $e^{-10} \sim 10^{-15}$.

Eq. (1) is not the end of the story. Rather, close examination \cite{13} of the idealized mode function equation reveals that gravitino creation continues until $H$ falls below the true gravitino mass $m_{3/2}$. This increases the abundance to

$$\frac{n}{s} \sim 10^{-2} \frac{\gamma T_R M^3}{M^4 S}, \quad (3)$$

where $M_S = \sqrt{M_\text{P} m_{3/2}}$ is the intermediate scale. (The energy density is of order $M^4 S$ when $H \sim m_{3/2}$.)

The idealized mode function equation, used to obtain the above results, assumes that the superfield responsible for SUSY breaking in the vacuum is the same as the superfield(s) describing inflation. This will presumably not be the case in reality. On the other hand, the non-adiabaticity responsible for gravitino creation, present in the idealized case that these two superfields are identical, is unlikely to disappear just because they are different. Therefore, Eq. (3) should provide a reasonable estimate of the gravitino abundance if reheating takes place after the epoch $H \sim m_{3/2}$.

If, in contrast, reheating

\footnote{This late-time creation occurs only when SUSY is broken in the vacuum, leading to a nonzero value for $m_{3/2}$. It occurs because global supersymmetry then ceases to be a good approximation, every time the potential dips through zero. The models considered in \cite{5,6,7,8} have unbroken SUSY in the vacuum, so that global SUSY is a good approximation at all times, and helicity 1/2 gravitino production becomes the same as Goldstino production. As is the case for any spin 1/2 particle, the production of the Goldstino ceases soon after inflation ends.}

\footnote{Just after inflation ends, a significant fraction of the energy of the oscillating field may be drained off by preheating, into marginally relativistic bosons and/or fermions. If this occurs, the idealized model will certainly be invalidated for a while, but because the new energy redshifts, and is anyhow never completely dominant, the idealized model is likely to become reasonable again after a few Hubble times. If so, it will survive until reheating, defined as the epoch when practically all of the oscillating energy is converted into thermalized radiation.}
occurs earlier, gravitino creation will certainly stop then because there is no coherently oscillating field, and the abundance will be
\[
\frac{n}{s} \sim 10^{-2} \gamma \left( \frac{M}{T_R} \right)^3 \quad (T_R > M_S).
\] (4)

Combining Eqs. (3) and (4), we see that the maximal abundance occurs if \( T_R \approx M_S \), with smaller abundance if we either decrease or increase \( T_R \).

In typical models of inflation and reheating, these gravitino abundances are huge compared with the abundance from thermal collisions, and lead to far stronger constraints on the \( T_R \) and \( \gamma \). Consider first the case of gravity-mediated supersymmetry breaking, corresponding to \( m_{3/2} \approx 100 \) GeV and \( M_S \approx 10^{10} \) GeV. Then, nucleosynthesis requires \( n/s \lesssim 10^{-13} \), and
\[
\gamma \lesssim 10^{-11} \left( \frac{10^{10} \text{ GeV}}{T_R} \right) \left( \frac{10^{10} \text{ GeV}}{M} \right)^3 \quad (T_R \lesssim 10^{10} \text{ GeV}) \] (5)
\[
\gamma \lesssim 10^{-11} \left( \frac{T_R}{M} \right)^3 \quad (10^{10} \text{ GeV} \lesssim T_R). \] (6)

Alternatively, consider the case of gauge-mediated SUSY breaking, with the favoured values \( m_{3/2} \approx 100 \) keV and \( M_S \approx 10^7 \) GeV. Then the gravitino is stable, and the requirement that it should not overclose the Universe gives \( n/s \lesssim 10^{-5} \), and
\[
\gamma \lesssim 10^{-3} \left( \frac{10^7 \text{ GeV}}{T_R} \right) \left( \frac{10^7 \text{ GeV}}{M} \right)^3 \quad (T_R \lesssim 10^7 \text{ GeV}) \] (7)
\[
\gamma \lesssim 10^{-3} \left( \frac{T_R}{M} \right)^3 \quad (10^7 \text{ GeV} \lesssim T_R). \] (8)

These constraints are very strong in most models of inflation. For instance, the popular D-term inflation model (and other models) requires \( V^{1/4} \approx M \approx 10^{15} \) GeV. Then, Eqs. (3), (5) and (6) require at least one bout of thermal inflation if SUSY-breaking is gravity-mediated. If instead it is gauge-mediated, Eqs. (3), (7) and (8) require \( T_R > 10^{11} \) GeV, and again entropy production (though not necessarily thermal inflation). The only popular models where the constraints are completely ineffective are those with soft supersymmetry breaking during inflation, leading to \( M \) perhaps of order \( m_{3/2} \). Such models include modular inflation and hybrid inflation with soft supersymmetry breaking (using a tree-level or loop-corrected potential).
**What sort of field is the inflaton?**  The rest of this review deals with various issues in inflation model-building. At the most primitive level, a model of inflation is simply a specification of the form of the potential, but one normally requires also that the form of the potential looks reasonable in the context of particle physics. In particular, one might be concerned if the field values are big compared with the ultra-violet cutoff $\Lambda_{UV} < M_f < M_P$. However, string theory gives us different kinds of scalar field. There are, indeed, the ordinary fields (matter fields) whose values should be small compared with $\Lambda_{UV}$, if the form of the potential is to be under control. Most models of inflation have been built with such fields in mind, though all too often one notices at the end of the calculation that the magnitude of the inflaton field is at the Planck scale or bigger.

On the other hand, there are also moduli, which determine things like the gauge couplings and the size of extra dimensions. String theory can give guidance about the form of their potential at field values of order $M_P$, even if $M_f$ is much less than $M_P$ owing to the presence of large extra dimensions. It is marginally flat enough to support inflation, a detailed investigation being necessary to see whether viable inflation occurs in a given model. Yet more exotic fields might be contemplated. For instance, it has been suggested that the inflaton corresponds to the distance between $D$-branes, which are coincident now but were separated at early times. The canonically normalized inflaton field is $\phi \simeq M_f^2 r$, where $r$ is the distance between the branes and $M_f$ is the fundamental quantum gravity scale. The regime $r \gtrsim M_f^{-1}$ presumably required by quantum gravity now corresponds to $\phi$ bigger than $\phi \gg M_f$. (From this viewpoint it is not clear how to justify also the regime $\phi \ll M_f$, invoked in.) At present, we do not know which type of model Nature has chosen. On the other hand, future measurements of the spectral index will confirm or rule out most of the forms of the inflationary potential, that are natural in the context of matter fields.

**Hybrid inflation needs fairly large field values**  During hybrid inflation, the slowly-rolling inflaton field $\phi$ couples to a second field $\chi$, holding the latter at the origin during inflation. Ignoring loop corrections, both $\phi$ during inflation, and the vev $\langle \chi \rangle$ can be taken to be very small on the Planck scale. Somewhat remarkably, it has been shown recently that this is no longer the case with the loop correction included. For instance, it is found that in the usual case that the hybrid inflation is supposed to give the primordial curvature

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\[d\] The fundamental quantum gravity scale $M_f$ is less than the 4-dimensional Planck scale $M_P$ if there are large extra dimensions. We are, of course, talking about the values of the canonically-normalized fields, with the origin at a fixed point of the symmetries.
perturbation, \( \phi \) during inflation and/or \( \langle \chi \rangle \) must be at least \( 10^9 \) GeV. While far below the Planck scale, this number is far above the electroweak scale. This means that hybrid inflation, with matter fields, cannot work in the context of TeV-scale quantum gravity. Also, if \( \chi \) is identified with an electroweak Higgs field, \( \phi \) has to be bigger than \( M_P \), even if the curvature perturbation comes from an earlier era of inflation. This second result calls into question the viability of an otherwise attractive model of electroweak baryogenesis.

**Extra dimensions** The growth industry this year has been the possibility that we live on a three-dimensional brane, with \( n \geq 1 \) large extra dimensions. I will confine my remarks to the case \( n > 1 \), because the situation for the case \( n = 1 \) is changing too rapidly to say anything useful.

It is assumed that Einstein gravity holds in the \( 4+n \) dimensions with some Planck scale \( M_t \). To avoid obvious conflict with collider experiments one needs at least \( M_t \sim \) TeV, and this extreme case is the one that has received the most attention. With \( n > 1 \), and the extra dimensions stabilized, Einstein gravity holds in our 4-dimensional spacetime on scales bigger than the radius \( R \) of the extra dimensions. The 4-dimensional Planck scale \( M_P \) is given by \( M_P^2 \sim R^n M_t^{2-n} \). The thickness of our brane is presumably of order \( M_t^{-1} \). Then, in the regime where the \( 4+n \) dimensional energy density is much less than \( M_t^{4+n} \) (i.e., well below the quantum gravity scale) the energy density on our brane is much less than \( M_t^4 \). Assuming that the extra dimensions are stabilized, the Hubble parameter in this regime is given by \( 3 H^2 = \rho/M_P^2 \ll R^{-2} \). We learn that, well below the quantum gravity regime, Einstein gravity will correctly describe the evolution of the Robertson-Walker Universe, through the usual Friedmann equation.

While cosmological scales are leaving the horizon during inflation, the extra dimensions must indeed be stabilized, since significant variation would spoil the observed scale independence of the spectrum of the primordial curvature perturbation. The simplest hypothesis is that they remain stabilized thereafter, so that they have their present value while cosmological scales leave the horizon. In that case, the mass of the inflaton during inflation (not necessarily in the vacuum) must be tiny, \( m_{\phi} \lesssim M_t^2/M_P \). This mass presumably requires protection from supersymmetry but sufficient protection is problematic because the inflaton has to communicate with the visible sector so as to reheat, while in that sector the chiral supermultiplets have TeV mass splitting. Leaving aside that problem, new as opposed to hybrid inflation may be quite viable. Another proposal is to use the field corresponding to the distance between D-branes, though this does not seem to give a viable curvature perturbation.
An alternative is to assume that while the curvature perturbation is generated, the extra dimensions are stabilized, while cosmological scales are leaving the horizon, with sizes much smaller than at present. One still needs a second, short period of inflation to get rid of the dangerous cosmological relics (moduli) associated with the oscillation of the extra dimension about its present value. (Indeed, it has been shown that when entropy production finally ends, the moduli must have their present size, with an accuracy \(10^{-14}(T_R/10\text{ MeV})^{3/2}\).) This late inflation might be thermal or slow-roll, thermal having the advantage that it allows a bigger inflaton mass (though one that will still require protection from supersymmetry).

Some other recent work  Many other papers on inflation have appeared in 1999. Some of them address the problem of keeping the inflationary potential flat, in the face of supergravity corrections. For instance, presents a no-scale type model, while several works pursue the paradigm of assisted inflation. There has been further consideration of hybrid inflation with a running mass. Finally, a completely new paradigm of inflation has been proposed, in which the coefficient of the kinetic term of the inflaton passes through zero.

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