REWARD MACHINES FOR COOPERATIVE MULTI-AGENT REINFORCEMENT LEARNING

Cyrus Neary, Zhe Xu, Bo Wu, and Ufuk Topcu
The University of Texas at Austin
Task decomposition in multi-agent reinforcement learning

Cooperative multi-agent RL: A team of agents learn interact in a shared environment to achieve a common objective.
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**Cooperative multi-agent RL:** A team of agents learn interact in a shared environment to achieve a common objective.

Source: Blizzard Entertainment

Source: OpenAI

Source: Starship Technologies
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Observation: Often, agents only interact in several crucial moments of the task.
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Objectives:

1) Present a framework for specifying structured representations of team tasks.
2) Use this specification to decompose problems into necessary individual behaviors.
3) Present a reinforcement learning algorithm that uses decomposition to simplify multi-agent learning.

Observation: Often, agents only interact in several crucial moments of the task.
An Illustrative Running Example

Team objective:
Have $A_1$ safely reach the goal location $Goal$. 

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Pressing the red button requires two agents.
An Illustrative Running Example

Team objective:
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Set of team environment states
$S = S_1 \times S_2 \times S_3$

Environment actions
$A = A_1 \times A_2 \times A_3$

Actions available to agent 3
An Illustrative Running Example

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Representing Temporally Extended Team Tasks – Reward Machines
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\[ U = \{ u_1, u_1, u_2, \ldots, u_7 \} \] – Set of states
Representing Temporally Extended Team Tasks – Reward Machines

$U = \{u_1, u_2, ..., u_7\}$ – Set of states

$\Sigma = \{Y_B, G_B, R_B, A_2^{R_B}, A_2^{\neg R_B}, A_3^{R_B}, A_3^{\neg R_B}, \text{Goal}\}$ – Set of events
Representing Temporally Extended Team Tasks – Reward Machines

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\[ \delta : U \times \Sigma \rightarrow U \] – Transition function
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\[ \delta : U \times \Sigma \rightarrow U \] — Transition function
\[ \sigma : U \times U \rightarrow \mathbb{R} \] — Output function
\[ F \] — Set of final states
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Connecting Environment States to a Reward Machine

The labeling function: Relate the environment state to collections of high-level events.

- Reward machine states $U$
- High-level Events $\Sigma$
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Environment Dynamics
Model: Transition distribution $p(\cdot | s, a)$

- Environment states $S = S_1 \times S_2 \times S_3$
- Environment actions $A = A_1 \times A_2 \times A_3$
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Using Reward Machines for Reinforcement Learning

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Perform q-learning to learn a separate policy for each state of the reward machine.

- $q_1(s,a)$
- $q_2(s,a)$
- $q_3(s,a)$
- $q_4(s,a)$
- $q_5(s,a)$
- $q_6(s,a)$
- $q_7(s,a)$

Start

Selected action

Environment Transition

Next state

Labeling function $L$

High level events

Previous reward machine state

Reward machine $R$
Using Reward Machines for Reinforcement Learning

Perform q-learning to learn a separate policy for each state of the reward machine.
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Perform q-learning to learn a separate policy for each state of the reward machine.
Reward Machine Projection

How does the reward machine change if one only has access to events from $\Sigma_1 \subseteq \Sigma$?

Example: Event subset for $A_1$ is $\Sigma_1 = \{Y_B, R_B, Goal\}$
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How does the reward machine change if one only has access to events from $\Sigma_i \subseteq \Sigma$?

Example: Event subset for $A_1$ is $\Sigma_1 = \{Y_B, R_B, Goal\}$

- Transitions triggered by missing events are not observed.
- Merge states that cannot be differentiated by events from $\Sigma_1$. 

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Projected Reward Machine $R_1$

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- Merge states that cannot be differentiated by events from $\Sigma_1$. 
Reward Machine Projection

How does the reward machine change if one only has access to events from $\Sigma_1 \subseteq \Sigma$?

Projected Reward Machine $R_1$

Projected Reward Machine $R_2$

Projected Reward Machine $R_3$

Note: reward machine projections may be computed automatically.
Reward Machine Projection

How does the reward machine change if one only has access to events from $\Sigma_i \subseteq \Sigma$?

Projected reward machines encode the sub-tasks of individual agents who only observe events in $\Sigma_i$.

Note: reward machine projections may be computed automatically.
Problem Equivalence

When is the task described by the team reward machine equivalent to the composition of its projections?
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Bisimulation: Behavioral equivalence of reward machines

Parallel composition: Concurrent combination of reward machines
Local Labeling Functions

Connecting environment dynamics with projected reward machines?

Environment Dynamics \( \rightarrow \) Team Reward Machine \( R \) \( \xrightarrow{\text{Labeling function } L} \) Projected Reward Machine \( R_1 \) \( \parallel \) Projected Reward Machine \( R_2 \) \( \parallel \) \ldots \( \parallel \) Projected Reward Machine \( R_N \)
Local Labeling Functions

Connecting environment dynamics with projected reward machines?

Environment Dynamics

Team environment states

Labeling function $L$

Projected Reward Machine $R$

Local labeling function $L_1$

Projected Reward Machine $R_1$

Local labeling function $L_2$

Projected Reward Machine $R_2$

Local labeling function $L_N$

Projected Reward Machine $R_N$

Local environment states
Local Labeling Functions

Connecting environment dynamics with projected reward machines?

Team Reward Machine $R$

Labeling function $L$

Team environment states

Environment Dynamics

Projected Reward Machine $R_1$

Local labeling function $L_1$

Local environment states

Synchronization on shared events

Projected Reward Machine $R_2$

Local labeling function $L_2$

Projected Reward Machine $R_N$

Local labeling function $L_N$
Problem Equivalence

Task complete $\iff$ Subtask 1 complete AND Subtask 2 complete AND ... AND Subtask $N$ complete

Team Reward Machine $R$ $\overset{L}{\Rightarrow}$ Projected Reward Machine $R_1$ $\overset{L_1}{\Rightarrow}$ Projected Reward Machine $R_2$ $\overset{L_2}{\Rightarrow}$ ... $\overset{L_N}{\Rightarrow}$ Projected Reward Machine $R_N$

Labeling function $L$ $\iff$ Local labeling function $L_1$ $\iff$ Local labeling function $L_2$ $\iff$ ... $\iff$ Local labeling function $L_N$

Team environment states $\overset{\text{Team Reward Machine } R}{\Rightarrow}$ Local environment states $\overset{\text{Projected Reward Machine } R_i}{\Rightarrow}$ Environment Dynamics

Subtask complete $\iff$ Subtask complete AND Subtask complete AND ... AND Subtask complete

Environment Dynamics

Environment Dynamics

Team environment states
**Problem Equivalence**

Task complete $\iff$ Subtask 1 complete $\land$ Subtask 2 complete $\land$ ... $\land$ Subtask $N$ complete

Team Reward Machine $R$ $\iff$ Projected Reward Machine $R_1$ $\parallel$ Projected Reward Machine $R_2$ $\parallel$ ... $\parallel$ Projected Reward Machine $R_N$

Labeling function $L$ $\iff$ function $L_1$ $\parallel$ function $L_2$ $\parallel$ ... $\parallel$ function $L_N$

Team environment states $\rightarrow$ Local labeling function $L_N$

Environment Dynamics

**Observation:** Agent $i$ may use $R_i$ to learn its subtask, without observing the states of its teammates.
Decentralized Q-Learning with Projected Reward Machines (DQPRM)

Agents learn to accomplish their subtasks in the absence of teammates.
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**Decentralized Q-Learning with Projected Reward Machines (DQPRM)**

![Team reward machine $R$](image)
Decentralized Q-Learning with Projected Reward Machines (DQPRM)

While ensuring the resulting composite behavior **accomplishes the team task.**
**Team Task:** Each agent must *simultaneously* occupy the rendezvous location $R$, before proceeding to their respective goals $G_i$. 
DQPRM Scaling with the Number of Agents

Rendezvous Experiment

Team Task: Each agent must *simultaneously* occupy the rendezvous location \( R \), before proceeding to their respective goals \( G_i \).

- Two agent rendezvous.
- Ten agent rendezvous.
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DQPRM scales well with the number of agents.

Each agent learns **in the absence of its teammates**.
DQPRM scales well with the number of agents.

Each agent learns in the absence of its teammates.

Composite behavior accomplishes the team task.
DQPRM scales well with the number of agents.

Two agent rendezvous

Ten agent rendezvous
Reward machines for MARL task decomposition

Reward machines to **specify** and **decompose** cooperative RL tasks.

Cyrus Neary
cneary@utexas.edu
Reward machines for MARL task decomposition

Reward machines to specify and decompose cooperative RL tasks.

Conditions guaranteeing equivalence between original and decomposed tasks.

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cneary@utexas.edu
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