Effects of Typical Emission Waveform on Laser Reflective Tomography Imaging

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Abstract. Laser reflective tomography imaging is a new imaging method of long distance and high resolution. The echo could be expressed as the convolution of incident waveform and the reflectance projection distribution, so the deconvolution effect of the echo obtained by different emission waveforms is also different. In order to explore the reflectance projection distribution obtained by deconvolution of different emission pulses and from the comparison of the RMS of Euclidean distance between deconvolution results and reality results, we found that Gaussian pulse could be selected as the most suitable emission waveform when the SNR was less than 40, and when the SNR was larger, Sawtooth pulse was the best choice.

1. Introduction
Remote dark detection has gotten much interests following the development of deep space explorations. Traditional detection methods lack the ability to distinguish space debris targets with the magnitude of cm-level. Laser reflective tomography imaging (LRTI) is a new imaging method of long distance and high resolution. The space-based lidar is not restricted by geographical location or meteorological conditions, and the coverage of space domain and time domain is greatly increased. Also, it can realize long-range and high-precision detection throughout the day, and is an ideal means for space debris detection.

 Developed from Computed Tomography (CT), LRTI was first proposed in 1988 by Parker, Kniningt et al. from the Lincoln laboratory in the United States[1]. In 1989, the experimental system was modified to replace the photomultiplier with an EG&G streak camera by Knight F K et al[2]. Matson et al. have further studied the theory and application of reflection tomography imaging[3,4,5]. In addition to, they obtained reconstruction images of satellite from a ground-based lidar[6].

In image reconstruction, the modulation of the pulse waveform by the target is a typical convolution process, which can result in the reduction of the resolution of the reconstructed target image and the blurred image details. Generally, the method of deconvolution is used to obtain the reflectance projection distribution, and the effect of deconvolution of different emission waveforms is different. Moreover, the results of reflectance projection distribution obtained by deconvolution of different emission waveforms under the same Signal to Noise Ratio (SNR) are different. Therefore, in this research, we analysed the emission and reception regularly by establishing analytic model, and
compared the results of reflectance projection distribution obtained by deconvolution of several different emission waveforms. Finally, we found the most suitable emission waveform for LRTI, and proposed the requirements for waveform selection and SNR.

The rest of this paper is organized as follows: in Section 2, we briefly review the principle of laser reflective tomography. The model of laser reflection tomography is established in Section 3. The results of reflectance projection distribution obtained by deconvolution of different emission waveforms under the same SNR are displayed and analyzed in Section 4. Finally, the concluding remarks are made in Section 5.

2. Principle of laser reflective tomography imaging

The basic principles of LRTI are to illuminating the object of laser at multiple angles, collecting the echo of the target at multiple angles, and reconstruct the 2D cross-sectional image of the target according to the reflectance projection distribution. As shown in Fig. 1(a), the parallel laser beams irradiate a 2-D target, when the irradiation angle is \( \phi \). The reflectance projection distribution of the target at angle \( \phi \) is defined as

\[
p(r, \phi) = \int_{L_{r, \phi}} f(x, y) ds.
\]

(1)

Where \( L_{r, \phi} \) is a straight line perpendicular to the direction of light, with a function of \( r = x \cos \phi + y \sin \phi \), \( f(x, y) \) is the reflectance distribution of the target. An angle projection image is shown in Fig. 1(b), the parallel lines with arrows unfold in the laser beams irradiate region, and the distance of the projection depends on the depth of the region irradiate by the laser. It should be noted that the reflectance projection distribution is actually nonzero only on the surface, as laser beams can’t penetrate the target. That is

\[
f(x, y) = 0, (x, y) \notin D.
\]

(2)

where D is the set of points of the target surface. Therefore, the actual projection can be represented as

\[
p(r, \phi) = \int_{L_{r, \phi} \cap D} f(x, y) ds.
\]

(3)

Fig. 1 Schematic of the principle of LRTI. (a) Target projection; (b) Data backprojection

The main idea of the direct back-projection method is smearing the scanned path("original path") of the measured projection data in angle to the pixel passed by the path. That is, the values of all the points passing through this coordinate point are accumulated or averaged after accumulated, which are taken as the values of the pixels in the reconstructed 2D image. However, this method can generate artifacts and
decrease the resolution of image, so the general method of LRTI is Filtered Back projection (FBP) algorithm.

3. Model of LRTI

3.1 Echo convolution model. In laser reflective tomography imaging, the depth information is contained in the echo wave, from which the reflectance projection distribution can be extracted. Theoretically, the echo can be expressed as the convolution of the emission pulse and the reflectance projection distribution

\[ w(r, \phi) = p(r, \phi) \otimes g + \xi(r, \phi) = \int_{-\infty}^{\infty} p(s, \phi) g(r-s) + \xi(r, \phi). \]  

Where \( w(r, \phi) \) is the echo at angle \( \phi \), \( p(r, \phi) \) is the reflectance projection distribution at angle \( \phi \). \( g \) is the expression of the emission pulse, and \( \xi(r, \phi) \) is the additive noise. The additive white gaussian noise (AWGN) is one of the most basic models of noise and interference[8]. In the process of simulation, the random normal distribution function is firstly used to generate white noise, and then the average energy of signal and noise is calculated. Then given SNR, the variance value set by noise is calculated. Finally, white noise is formed according to the average energy of noise, and the actual echo is the superposition of convolution results and noise. In order to get the expression of reflectance projection distribution, some deconvolution methods should be applied in the solving process such as multi-frame iterative blind deconvolution method[9] and maximum likelihood method[10].

3.2 Multi-angle echo model. The geometry of LRTI is shown in Fig. 2(a), and the target is situated in the origin of the coordinate system. The laser pulse source and avalanche photodiode (APD) detector are put together as a LRTI system to rotate around the target along the circle, or equivalently the system stays static and the target rotate around. the \( \overrightarrow{ox} \) axis points to the system at the beginning when the angle between the \( \overrightarrow{ox} \) axis and the sight of view of the system is \( \phi = 0^\circ \).

Fig.2 Schematic diagram of echo model.
(a) Diagram of the geometry of LRTI; (b) Echo from different angles

When the target rotates slowly along the \( \overrightarrow{ox} \) axis of rotation, it is equivalent to the laser beams moving slowly on the target surface with a certain fixed track. If the target rotates at a constant angular
speed, the echo received by the detector can be arranged at fixed projection angle intervals. The projection angle starts from $\phi = 0^\circ$ with the $\text{ox}$ axis and rotates clockwise to $\phi = 90^\circ$, $\phi = 180^\circ$, etc. The laser beams irradiates the target from three directions of 1-3 in the Fig. 2(a), and the echoes of the receiving system at these three specific angles are shown in Fig. 2(b).

4. Simulation results and analysis

In order to explore the reflectance projection distribution obtained by deconvolution of different emission waveforms under the same SNR. We simulated four kinds of emission pulses: Gaussian pulse, Triangular pulse, Square pulse, and Sawtooth pulse. Through the analysis of the reflectance projection distribution, we found the most suitable emission waveform for LRTI, and proposed the requirements for waveform selection and SNR. The schematic diagrams of different emission waveform are shown in Fig. 3.

![Fig. 3 Schematic diagrams of different emission waveform](image)

Fig. 3 indicates that the full width at half maximum(FWHM) of Gaussian pulse is set as $FWHM = 1\text{ns}$, and the pulse amplitude of the Gaussian pulse is set as $A = 1W$. In order to control the invariant, we set the other pulses to have the same amplitude. Furthermore, we set the signal pulse energy of each pulse to be the same, correspond to the pulse width of the Square pulse is $width = 1\text{ns}$, and the FWHM of the Triangular pulse and the Sawtooth pulse is $FWHM = 1\text{ns}$. 

According to the distance resolution equation of LRTI

\[ R = \frac{ct}{2} \]  \hspace{1cm} (5) \]

Where \( c \) is the speed of light, \( \tau \) is the FWHM of the Gaussian pulse. We set a target with the reflection projection distribution, with the depth range of 0.1 to 5m. The laser beams irradiate the target horizontally from the left, and the reflection projection distribution is obtained by deconvolution of the echo of the target irradiated by the Gaussian pulse. As shown in Fig. 4(b), The red curve shows the reality reflectance projection distribution and the blue curve shows the deconvolved reflectance projection distribution. The noncausal wiener filter (NCWF) is a regularization inverse filter which has the advantage of simple calculation and efficiency \[9\], and this method is widely applied in LRTI. Therefore, in the process of deconvolution, we used NCWF to estimate the reflection projection distribution, and the SNR was set to 30. We believed that the deconvolution result at 150mm was significantly different from the reality result due to the Gaussian pulse had the certain FWHM, which could not be responded well by the change of reflectance with a large slope.

Euclidean distance is the most easily understood distance calculation method, which derives from the distance formula between two points in Euclidean space \[11\]. In order to denote the quality of reflectance projection distribution obtained by deconvolution more intuitively, it is vital to consider that the points of the curve are discrete. The evaluation standard is defined as the root mean square (RMS) of Euclidean distance in discrete form

\[ R_E = \sqrt{\frac{\sum_{i=1}^{N} (p_d(i) - p(i))^2}{N}} ] \hspace{1cm} (6) \]  

Where \( N \) is the number of points on the curve, \( p_d(i) \) is the deconvolution result and \( p(i) \) is the reality. Due to the noise was random, we repeated each experiment 100 times to avoid accidental errors. The RMS of Euclidean distance of the reflectivity of the four pulses under a certain SNR are shown in Table 1.
As shown in Table 1, the deconvolution effect of Gaussian pulse is obviously superior to others in the case of low SNR, and Square pulse has the worst deconvolution effect. This is different from the estimated results, as it is believed that the NCWF loses part of the high frequency signal in order to suppress the noise, and the echo bandwidth of Square pulse after modulation is relatively wide. In order to verify these analysis, the frequency domain analysis of echo without the noise was tested, and the result are shown in Fig. 5. Through the frequency domain analysis of the echo without noise, it is found that the convolution results of Square pulse and reflection projection distribution has more high frequency components, so NCWF deconvolution has the worst effect under the condition of low SNR, while Gaussian pulse has the least high frequency components, so the deconvolution effect is the best. That is, Gaussian pulse has strong anti-noise ability. In addition, it can be informed from Table 1 that the deconvolution effects of both of Sawtooth pulse and Square pulse are better than Gaussian pulse under the condition of high SNR, this might be because the pulse front of Sawtooth pulse and Square pulse are narrow, which can be well responded by the change of reflectance with a large slope. In general, in order to suppress noise, part of high frequency signals will be lost in the deconvolution process. Therefore, Gaussian pulse can be selected as the most suitable emission waveform when the SNR is less than 40, while, when the SNR is larger, Sawtooth pulse is the best choice.

![Fig. 5 Frequency domain analysis results of echo without the noise](image-url)
5. Conclusion

We studied the basic principle of LRTI, discussed the deconvolution problem in echo processing, and used NCWF to get the reflection projection distribution from the deconvolution in echo. Moreover, in order to explore the reflectance projection distribution obtained by deconvolution of different emission waveforms under the same SNR, we simulated four kinds of typical emission pulses. Additionally, from the comparison of the RMS of Euclidean distance between deconvolution results and reality results, we found that Gaussian pulse could be selected as the most suitable emission waveform for LRTI when the SNR was less than 40, while, Sawtooth pulse was the best choice when the SNR was larger. However, in this research, only NCWF deconvolution was used, in the future research, subsequent simulations can be carried out based on the commonly used deconvolution methods of LRTI, through which the conclusion might be more universal and convincing.

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