A novel and economical explanation for SM fermion masses and mixings

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Abstract I propose the first multiscalar singlet extension of
the Standard Model (SM), that generates tree level top quark
and exotic fermion masses as well as one and three loop
level masses for charged fermions lighter than the top quark
and for light active neutrinos, respectively, without invoking
electrically charged scalar fields. That model, which is based
on the $S_3 \times Z_8$ discrete symmetry, successfully explains
the observed SM fermion mass and mixing pattern. The charged
exotic fermions induce one loop level masses for charged
fermions lighter than the top quark. The $Z_8$ charged scalar
singlet $\chi$ generates the observed charged fermion mass and
quark mixing pattern.

1 Introduction

Despite its great consistency with the experimental data, the
Standard Model (SM) is unable to explain several issues
such as, for example, the number of fermion generations,
the observed pattern of fermion masses and mixings, etc. In
this letter I propose the first multiscalar singlet extension
of the SM, that generates tree level top quark and exotic
fermion masses as well as one and three loop level masses
for charged fermions lighter than the top quark and for light
active neutrinos, respectively, without invoking electrically
charged scalar fields. That multiscalar singlet extension is
consistent with the SM fermion mass and mixing pattern.

2 The Model

The model has the SM gauge symmetry, which is supple-
mented by the $S_3 \times Z_8$ discrete group. It is noteworthy that
among the discrete symmetries, I introduced the symmetry
group $S_3$ since it is the smallest non-Abelian group that has
been considerably studied in the literature. The $S_3$ symme-
try is assumed to be preserved whereas the $Z_8$ discrete group
is broken at the scale $v$. The breaking of the $Z_8$ symme-
try gives rise to the observed charged fermion mass and
quark mixing pattern. The scalar sector of the SM is ex-
tended by introducing three EW scalar singlets, i.e., $\eta_1$, $\eta_2$
and $\chi$, assumed to be charged under the $Z_8$ symmetry. Out
of these three SM scalar singlets, two scalar fields ($\eta_1$, $\eta_2$) are
grouped in a $S_3$ doublet, namely $\eta$, whereas the remaining
one ($\chi$) is assigned to be a trivial $S_3$ singlet. The SM Higgs
doublet $\phi$ is assigned to be a trivial $S_3$ singlet, neutral un-
der the $Z_8$ discrete symmetry. Since the $S_3$ symmetry is pre-
served, the $S_3$ scalar doublet $\eta = (\eta_1, \eta_2)$ does not acquire
a vacuum expectation value. The remaining scalar fields, i.e,
$\phi$ and $\chi$, which are assigned as trivial $S_3$ singlets, acquire
nonvanishing vacuum expectation values, as it should be.
Regarding the SM fermion sector, I assign the left handed
fermionic fields, the right handed top quark field as trivial
$S_3$ singlets and the remaining right handed SM fermionic
fields as $S_3$ nontrivial singlets, implying that the top quark
is the only SM charged fermion that acquires a tree level
mass. The remaining SM charged fermions get their masses
from a one loop radiative seesaw mechanism and the hier-
archy among their masses will arise from the different $Z_8$
charge assignments of the fermionic fields. As it will be
shown in the following, light active neutrinos masses will
arise from a three loop radiative seesaw mechanism. In or-
der that all fermions lighter than the top quark acquire non
vanishing masses, the fermion sector of the Standard Model
is extended by including: two heavy right handed Majorana
neutrinos $\nu_{1R}$, $\nu_{2R}$, six SM gauge singlet charged leptons
$E_{\kappa L}$ and $E_{\kappa R}$ ($\kappa = 1, \cdots , 6$), ten heavy $SU(2)_L$
singlet exotic quarks $B_{kL}$, $B_{kR}$ ($\kappa = 1, \cdots , 6$), $T_{3L}$, $T_{3R}$ ($\lambda = 1, \cdots , 4$).
The heavy exotic $T_{3}$ ($\lambda = 1, \cdots , 4$) and $B_{k}$ ($\kappa = 1, \cdots , 6$)
quarks should have electric charges equal to $\frac{2}{3}$ and $-\frac{1}{3}$, re-

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respectively, in order to implement a one loop radiative seesaw mechanism that generates masses for quarks, lighter than the top quark. To build the Yukawa terms invariant under the $\mathbb{S}_3$ discrete group, I assign the two heavy right handed Majorana neutrinos as non trivial $\mathbb{S}_3$ singlets, whereas the non SM charged fermions are grouped into the $\mathbb{S}_3$ doublets $T^{(r)}_{L,R}$ ($r = 1, 2, 3$), $B^{(r)}_{L,R}$ and $E^{(k)}_{L,R}$ ($k = 1, 2, 3$). I do not unify the two heavy right handed Majorana neutrinos in a $\mathbb{S}_3$ doublet since that assignment will result in two massless active neutrinos, which is in clear contradiction with the neutrino oscillation experimental data. The non SM fermionic fields described above, together with the $\mathbb{S}_3$ doublet $\eta = (\eta_1, \eta_2)$ will induce one and three loop radiative seesaw mechanisms to generate the masses for fermions lighter than the top quark and for the light active neutrinos, respectively. The aforementioned non SM fermion content is the minimal required to generate the masses for fermions lighter than the top quark and for the light active neutrinos. I further assume that the fermionic fields transform under the $\mathbb{Z}_8$ symmetry, as follows:

\[
q_{jL} \rightarrow e^{-\frac{2\pi j}{8}} q_{jL}, \quad u_{jR} \rightarrow e^{\frac{2\pi (j+1)}{8}} u_{jR}, \quad d_{jR} \rightarrow e^{\frac{2\pi j}{8}} d_{jR},
\]

\[
l_{jL} \rightarrow e^{-\frac{2\pi j}{8}} l_{jL}, \quad l_{jR} \rightarrow e^{\frac{2\pi (j+1)}{8}} l_{jR}, \quad j = 1, 2, 3,
\]

\[
T^{(r)}_L \rightarrow e^{\frac{2\pi r}{8}} T^{(r)}_L, \quad T^{(r)}_R \rightarrow e^{\frac{2\pi r}{8}} T^{(r)}_R, \quad r = 1, 2,
\]

\[
B^{(r)}_L \rightarrow e^{-\frac{2\pi r}{8}} B^{(r)}_L, \quad B^{(r)}_R \rightarrow e^{\frac{2\pi r}{8}} B^{(r)}_R, \quad k = 1, 2, 3,
\]

\[
E^{(k)}_L \rightarrow e^{-\frac{2\pi}{8} E^{(k)}}_L, \quad E^{(k)}_R \rightarrow e^{\frac{2\pi}{8} E^{(k)}}_R, \quad k = 1, 2, 3,
\]

\[
\nu_{SR} \rightarrow e^{-\frac{2\pi}{8} \nu_{SR}}, \quad s = 1, 2.
\]

(1)

The EW scalar singlets $\eta = (\eta_1, \eta_2)$ and $\chi$ are charged under the $\mathbb{Z}_8$ symmetry as $\eta \rightarrow e^{-\frac{2\pi}{8}} \eta, \chi \rightarrow e^{\frac{2\pi}{8}} \chi$. With the above particle content, the following charged quark, charged lepton and neutrino Yukawa terms invariant under the symmetries of the model arise:

\[
- \mathcal{L}^{(u)}_Y = \sum_{j=1}^{3} \sum_{l=1}^{3} y_{lj}^{(u)} q_{jL} \overline{\nu}_l \eta \begin{pmatrix} T^{(r)}_L \end{pmatrix} \eta \Lambda^{3-j} + \begin{pmatrix} \Lambda^{3-k} \end{pmatrix} + \sum_{j=1}^{3} \sum_{l=1}^{3} y_{lj}^{(u)} q_{jL} \overline{\nu}_l \eta \begin{pmatrix} T^{(r)}_R \end{pmatrix} \eta \Lambda^{3-j} + \begin{pmatrix} \Lambda^{3-k} \end{pmatrix} + 2 \begin{pmatrix} \nu \end{pmatrix} \begin{pmatrix} T^{(r)}_L \end{pmatrix} \begin{pmatrix} T^{(r)}_R \end{pmatrix} \eta + h.c.
\]

(2)

\[
- \mathcal{L}^{(d)}_Y = \sum_{j=1}^{3} \sum_{l=1}^{3} y_{jl}^{(d)} q_{jL} \overline{\nu}_l \eta \begin{pmatrix} B^{(k)}_L \end{pmatrix} \eta \Lambda^{3-j} + \begin{pmatrix} \Lambda^{3-k} \end{pmatrix} + \sum_{j=1}^{3} \sum_{l=1}^{3} y_{jl}^{(d)} q_{jL} \overline{\nu}_l \eta \begin{pmatrix} B^{(k)}_R \end{pmatrix} \eta \Lambda^{3-j} + \begin{pmatrix} \Lambda^{3-k} \end{pmatrix} + \begin{pmatrix} y \end{pmatrix} \begin{pmatrix} B^{(k)}_L \end{pmatrix} \begin{pmatrix} B^{(k)}_R \end{pmatrix} \eta + h.c.
\]

(3)

Where, for the sake of simplicity, I have neglected the mixing terms between the different $\mathbb{S}_3$ fermionic doublets as well as the mixings between the right handed Majorana neutrinos. I have assumed that the non SM fermions are physical fields. After the spontaneous breaking of the electroweak and the $\mathbb{Z}_8$ discrete symmetry and considering that the $\mathbb{S}_3$ symmetry is preserved, the Yukawa interactions given above will generate tree level masses for the top quark and for the non SM fermions and one loop level masses for the remaining SM charged fermions. To generate the one loop level masses for the charged fermions lighter than the top quark, the $\mathbb{S}_3$ symmetry has to be softly broken by adding a $\mu_{12}^2 \eta_1 \eta_2$ term in the scalar potential for the $\mathbb{S}_3$ scalar doublet $\eta = (\eta_1, \eta_2)$. Since the $\mathbb{S}_3$ scalar doublet $\eta = (\eta_1, \eta_2)$ has a vanishing vacuum expectation value, light active neutrinos do not acquire tree level masses, they get masses via a three loop radiative seesaw mechanism (as follows from the $\mathbb{S}_3$ invariance of the neutrino Yukawa interactions) that involves the two heavy Majorana neutrinos $\nu_{1R}, \nu_{2R}$ as well as the real and imaginary parts of the $\eta_1, \eta_2$ scalar fields running in the loops. It is remarkable that this three loop level radiative seesaw mechanism for light active neutrino masses does not require charged scalar fields as in the other three loop level mechanisms discussed in the literature [1].

Since the hierarchy of charged fermion masses and quark mixing angles arises from the breaking of the $\mathbb{Z}_8$ discrete group, and in order to relate the quark masses with the quark mixing parameters, the vacuum expectation value (VEV) of the SM scalar singlet $\chi$ is set as follows: $v_\chi = \lambda \Lambda$, where $\lambda = 0.225$ is one of the Wolfenstein parameters and $\Lambda$ corresponds to the model cutoff.

3 Fermion masses and mixings

From the Yukawa terms, it follows that the quark, charged lepton and light active neutrino mass matrices have the form:

\[
M_{u_l} = \left( \begin{array}{ccc}
\lambda_1 & \lambda_2 & \lambda_3 \\
\lambda_2 & \lambda_3 & \lambda_4 \\
\lambda_3 & \lambda_4 & \lambda_5 \\
\end{array} \right) \begin{pmatrix} v \end{pmatrix} \left( \begin{array}{ccc}
\lambda_1 & \lambda_2 & \lambda_3 \\
\lambda_2 & \lambda_3 & \lambda_4 \\
\lambda_3 & \lambda_4 & \lambda_5 \\
\end{array} \right)^{-1}.
\]

(6)
Model and experimental values of the quark masses and CKM parameters.

| Observable     | Model value | Experimental value |
|----------------|-------------|--------------------|
| $m_u$(MeV)     | 1.44        | 1.45±0.56          |
| $m_d$(MeV)     | 656         | 635±86             |
| $m_s$(GeV)     | 177.1       | 172.1±0.6±0.9      |
| $m_c$(MeV)     | 2.9         | 2.9±0.4            |
| $m_t$(MeV)     | 57.7        | 57.7±16.8±15.7     |
| $m_b$(GeV)     | 2.82        | 2.82±0.09±0.04     |
| $\sin\theta_{12}$ | 0.225      | 0.225              |
| $\sin\theta_{13}$ | 0.0412     | 0.0412             |
| $\sin\theta_{13}$ | 0.00351    | 0.00351            |
| $\delta$       | 64°         | 68°                |

In what follows I will explain the reason for choosing the $Z_3$ discrete symmetry. It is noteworthy that the $Z_3$ discrete group is the smallest cyclic symmetry that allows to get the $\lambda^2$ suppression in the 13 entry of the up type quark mass matrix from a $\frac{1}{\sqrt{3}}$ insertion on the $\theta_{13}$, $\theta_{13}$ operator. However, since the masses of the non SM fermions arise from their renormalizable Yukawa interactions with the SM scalar singlet $\chi$, charged under the discrete cyclic group, the invariance of these Yukawa interactions under the cyclic symmetry requires to consider the $Z_3$ instead of the $Z_4$ discrete symmetry.

In the concerning to the charged lepton sector, I adopt a benchmark where I set $a_{ijk}^{(l)} = a_{ijk}^{(l)} \delta_{jk}$. This benchmark scenario that leads to a diagonal charged lepton mass matrix is justified by the requirement of forbidding unobserved lepton flavor violating processes such as $\mu \rightarrow e\gamma$. This assumption is also made in the framework of an extended inert two Higgs doublet model addressed to explain the charged lepton mass hierarchy [4]. Within the benchmark previously described, the charged lepton masses take the form:

$$m_e^{(l)} = a_1^{(l)} \lambda \frac{v}{\sqrt{2}}, \quad m_{\mu}^{(l)} = a_2^{(l)} \lambda \frac{v}{\sqrt{2}}, \quad m_{\tau}^{(l)} = a_3^{(l)} \lambda \frac{v}{\sqrt{2}}.$$
neutrinos $\nu_R$ or the lightest of the scalar fields $Re(\eta_1)$ and $Im(\eta_1)$ ($s = 1, 2$). In order to show that the light active neutrino texture $M_\nu$ can fit the experimental data, I set $\varphi = 2\rho$ only for the case of normal hierarchy (NH). Varying the lepton sector model parameters $a_{1}^{(l)}, a_{2}^{(l)}, a_{3}^{(l)}, \rho, W_1, W_2$ and $W_3$ (as well as $\varphi$ for IH only), I fitted the charged lepton masses, the neutrino mass squared splittings $\Delta m_{21}^2, \Delta m_{31}^2$ and the leptonic mixing parameters $\sin^2 \theta_{12}, \sin^2 \theta_{13}$ and $\sin^2 \theta_{23}$ to their experimental values for NH and IH. The results shown in Table 2 correspond to the following best-fit values:

$$\rho \approx 38.73^\circ, \quad W_1 \approx -0.063 eV^2, \quad W_2 \approx 0.18 eV^2,$$
$$W_3 \approx 0.15 eV^2, \quad \text{for NH}$$
$$a_{1}^{(l)} \approx 0.1, \quad a_{2}^{(l)} \approx 1.02, \quad a_{3}^{(l)} \approx 0.88, \quad (11)$$
$$\rho \approx 162.26^\circ, \quad \varphi \approx 79.44^\circ, \quad W_1 \approx 0.22 eV^2,$$
$$W_2 \approx 0.15 eV^2, \quad W_3 \approx 0.17 eV^2, \quad \text{for IH}$$
$$a_{1}^{(l)} \approx 0.1, \quad a_{2}^{(l)} \approx 1.02, \quad a_{3}^{(l)} \approx 0.88, \quad (12)$$

Using the best-fit values given above, I get for NH and IH, respectively, the following neutrino masses:

$$m_1 = 0, \quad m_2 \approx 9\text{meV}, \quad m_3 \approx 50\text{meV}, \quad \text{for NH}$$
$$m_1 \approx 49\text{meV}, \quad m_2 \approx 50\text{meV}, \quad m_3 = 0, \quad \text{for IH} \quad (13)$$

The obtained and experimental values of the observables in the lepton sector are shown in Table 2. The experimental values of the charged lepton masses, which are given at the $M_Z$ scale, have been taken from Ref. [2], whereas the experimental values of the neutrino mass squared splittings and leptonic mixing angles for both normal (NH) and inverted (IH) mass hierarchies, are taken from Ref. [5]. The obtained charged lepton masses, neutrino mass squared splittings and lepton mixing angles are in excellent agreement with the experimental data for both normal and inverted neutrino mass hierarchies. For the sake of simplicity, I assumed all leptonic parameters to be real, but a non-vanishing CP violating phase in the PMNS mixing matrix can be generated by making one of the entries of the neutrino mass matrix $M_\nu$ to be complex. It is noteworthy to mention that the consistency of the Higgs couplings to SM fermions and gauge bosons with the SM expectation requires that the mixing between the 126 GeV Higgs and the SM scalar singlet $\chi$ to be suppressed. In addition, the model cutoff has to be of the order of few TeVs to prevent the one loop level masses and Yukawa couplings for charged fermions lighter than the top quark to be arbitrary small. An ultraviolet completion of this model will consist in replacing the EW scalar singlets $\eta = (\eta_1, \eta_2)$ by two $SU(2)$ inert scalar doublets unified in a $S_3$ triplet. This will make the model cutoff very high thus avoiding large effects in the low energy flavour physics (FCNC, etc).

### Table 2

| Observable | Model value | Experimental value |
|------------|-------------|--------------------|
| $m_e (MeV)$ | 0.487 | 0.487 |
| $m_\mu (MeV)$ | 102.8 | 102.8 ± 0.0003 |
| $m_\tau (GeV)$ | 1.75 | 1.75 ± 0.0003 |
| $\Delta m_{21}^2 (10^{-3} eV^2) \ (\text{NH})$ | 7.22 | 7.60^{+0.19}_{-0.18} |
| $\Delta m_{31}^2 (10^{-5} eV^2) \ (\text{NH})$ | 2.50 | 2.48^{+0.05}_{-0.07} |
| $\sin^2 \theta_{12} \ (\text{NH})$ | 0.334 | 0.323 ± 0.016 |
| $\sin^2 \theta_{13} \ (\text{NH})$ | 0.567 | 0.567^{+0.032}_{-0.128} |
| $\sin^2 \theta_{13} \ (\text{IH})$ | 0.0228 | 0.0234 ± 0.0020 |
| $\Delta m_{21}^2 (10^{-3} eV^2) \ (\text{IH})$ | 7.60 | 7.60^{+0.19}_{-0.18} |
| $\Delta m_{31}^2 (10^{-3} eV^2) \ (\text{IH})$ | 2.48 | 2.48^{+0.05}_{-0.06} |
| $\sin^2 \theta_{12} \ (\text{IH})$ | 0.323 | 0.323 ± 0.016 |
| $\sin^2 \theta_{23} \ (\text{IH})$ | 0.573 | 0.573^{+0.025}_{-0.043} |
| $\sin^2 \theta_{23} \ (\text{IH})$ | 0.0240 | 0.0240 ± 0.0019 |

4 Conclusions

I have proposed the first multiscalar singlet extension of the SM that generates one and three loop level masses for charged fermions lighter than the top quark and for the light active neutrinos, respectively, via radiative seesaw mechanisms, without invoking electrically charged scalar fields. These one and three loop radiative seesaw mechanisms are mediated by the $\eta_1$ and $\eta_2$ scalar fields as well as by exotic charged fermions and right handed Majorana neutrinos, respectively. The model has the $S_3 \times Z_8$ discrete symmetry, where $S_3$ is preserved and $Z_8$ is spontaneously broken. The breaking of the $Z_8$ discrete group generates the non SM fermion masses as well as the observed SM charged fermion mass and quark mixing pattern. The unbroken $S_3$ symmetry of the model allows for natural dark matter candidates, which could be either the right handed Majorana neutrinos $\nu_R$ ($s = 1, 2$) or the lightest of the $S_3$ charged scalar fields $Re(\eta_1), Re(\eta_2), Im(\eta_1)$ and $Im(\eta_2)$. This possibility is left beyond the scope of this Letter.

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