Coherent control of a multi-qubit dark state in waveguide quantum electrodynamics

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Superconducting qubits in a waveguide have long-range interactions mediated by photons that cause the emergence of collective states. Destructive interference between the qubits decouples the collective dark states from the waveguide environment. Their inability to emit photons into the waveguide render dark states a valuable resource for preparing long-lived quantum many-body states and realizing quantum information protocols in open quantum systems. However, they also decouple from fields that drive the waveguide, making manipulation a challenge. Here we show the coherent control of a collective dark state that is realized by controlling the interactions between four superconducting transmon qubits and local drives. The dark state’s protection against decoherence results in decay times that exceed those of the waveguide-limited single qubits by more than two orders of magnitude. Moreover, we perform a phase-sensitive spectroscopy of the two-excitation manifold and reveal bosonic many-body statistics in the transmon array. Our dark-state qubit provides a starting point for implementing quantum information protocols with collective states.

Waveguide quantum electrodynamics (QED) has become a very active research field to study localized quantum emitters that are coupled to one-dimensional photonic channels. The platform offers novel opportunities to study interacting quantum systems and promises various applications in quantum information processing based on the fundamental physics of light–matter interaction. Typical realizations include natural and artificial atoms, such as superconducting qubits and quantum dots, coupled to optical or microwave waveguides. Well-developed control techniques and the possibility to engineer high coupling efficiencies make superconducting qubits an excellent platform to study waveguide-mediated interactions between multiple emitters. The engineering capabilities of superconducting qubits led to the observation of a broad range of quantum optical phenomena such as the Mollow triplet, ultrastrong coupling, generation of non-classical photonic states, qubit–photon bound states, topological physics and collective effects.

Collective states appear in waveguide QED as a result of waveguide-mediated interactions and interference effects in an ensemble of emitters. The relative phase between individual emitters determines whether the collective state obtains a sub- or super-radiant decay rate, that is, whether it becomes a dark or bright state. Collective bright states have been measured in various waveguide QED systems, whereas dark states have only been spectroscopically observed in superconducting waveguide QED. More recently, a multi-qubit dark state has been used to build a microwave cavity; however, full coherent control of the dark state has not been achieved yet. Multi-qubit dark and bright states provide a possibility to investigate the dynamics of interacting quantum systems, study many-body localization in disordered arrays or even build a quantum computer and simulation platform within an open quantum system. A key element to realize these concepts is coherent control of the system. The difficulty of controlling a dark state arises from its main property—it decouples from the electromagnetic environment of the waveguide. In addition, coherent control requires accurate knowledge of the energy and decay characteristics beyond the one-excitation manifold. However, higher-excited states of collective systems have been barely explored so far.

In this work, we realize a collective dark state by exploiting near-field and waveguide-mediated interactions between four superconducting transmon qubits. The collective dark state is coherently controlled via two physically separate drive ports, which allows the adjustment of their relative phase and thus solve the problem of driving a state that is decoupled from the waveguide. By tuning the qubits to the decoherence-free frequency, we demonstrate long coherence times along with full coherent control in an open quantum system. We utilize the state-dependent scattering of the collective bright transition to read out the ground- and dark-state populations.

Further, we perform a pulsed spectroscopy to characterize the collective two-excitation states and demonstrate the use of superradiant transitions to reset the dark–state qubit. Remarkably, when a transmon array has two or more excitations, their bosonic statistics become crucial for the energy spectrum. Our theoretical and experimental analysis show that this leads to collective high-excitation states that are fundamentally different from those of two-level arrays.

Superconducting qubits can be coupled to a microwave transmission line, much like a natural atom to a nanophotonic fibre or photonic-crystal waveguide (Fig. 1a). The qubit decoherence rate $\Gamma = (\gamma + \gamma_{nr})/2 + \gamma_{\text{ph}}$ is a sum of radiative decay $\gamma$ into the waveguide modes, non-radiative energy loss $\gamma_{nr}$ and pure dephasing $\gamma_{\text{ph}}$ (refs 27–29). When the qubit is coupled to a waveguide, the linewidth can be extracted in scattering experiments by measuring the waveguide transmission or reflection, to obtain the coupling strength to the waveguide and the non-radiative decoherence rate $\gamma_{\text{nr}} = \gamma_{\text{in}}/2 + \gamma_{\text{ph}}$. 

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The device sketched in Fig. 1d comprises four frequency-tunable transmon qubits acting as artificial atoms. Transmons Q₁ and Q₂ are located on the left, closer to the input side of the waveguide. Transmons Q₁ and Q₄ are located on the right, closer to the output side of the waveguide, such that the physical separation between the pairs is \( d_{\text{nl}} = 46.0 \pm 0.5 \) mm. Within the pairs, the transmons are separated by \( d_{\parallel} = 1.0 \pm 0.2 \) mm, which sets the capacitive coupling strengths \( J_{\parallel} \) and \( J_{\perp} \). The fundamental waveguide mode TE₁₀ has a cutoff frequency of \( \omega_{c}/2\pi = 6.55 \) GHz (ref. 13) and its electrical field is polarized parallel to the dipole moment of the transmons such that they efficiently couple to the waveguide. Four superconducting coils can individually control the resonance frequency of all the transmons. (Fig. 1b). The signal propagating through the waveguide is antisymmetric (\( \phi = \pi \)). Thus, we can only excite the symmetric bright state \( |\text{loc} \rangle = (|\text{ge} \rangle + |\text{ge} \rangle)/\sqrt{2} \) and the antisymmetric bright state \( |\text{loc} \rangle = (|\text{ge} \rangle - |\text{ge} \rangle)/\sqrt{2} \). For a distance of \( \lambda/2 \), the phase relation of the electromagnetic field in the waveguide is asymmetric (\( \phi = \pi \)); thus, we can only excite the antisymmetric bright state. The dark-state symmetry is opposite to the field symmetry of the waveguide, eliminating the coupling to the drive field and decay into the waveguide.

Two nearby transmons are directly coupled through the capacitance between the metallic pads of their antennae. Unlike interactions mediated by the waveguide, the capacitive coupling for transmons in this configuration has an effective \( 1/r^3 \) dependence, leading to short-range coupling. On resonance, an excitation can coherently swap between the local transmons, resulting in new eigenstates, particularly a symmetric state \( |\text{loc} \rangle = (|\text{ge} \rangle + |\text{ge} \rangle)/\sqrt{2} \) and a symmetric bright state \( |\text{loc} \rangle = (|\text{ge} \rangle - |\text{ge} \rangle)/\sqrt{2} \). For a distance of \( \lambda/2 \), the phase relation of the electromagnetic field in the waveguide is asymmetric (\( \phi = \pi \)); thus, we can only excite the antisymmetric bright state. The dark-state symmetry is opposite to the field symmetry of the waveguide, eliminating the coupling to the drive field and decay into the waveguide.
Before tuning all the qubits into resonance, we characterize individual qubit and pairwise couplings. The individual qubit and collective bright-state radiative decay rates are extracted from transmission measurements, using a circle-fit routine on the complex-valued scattering parameters. In Fig. 2a, we show the magnitude of the normalized transmission for a single transmon, as well as for two and four transmons tuned to frequency \( \omega_c \). For two and four qubits, we observe the typical broadening of the linewidth, caused by superradiant decay. The capacitively coupled transmon pairs have direct coupling strengths \( J_2/2\pi = 43 \text{ MHz} \) and \( J_4/2\pi = 47 \text{ MHz} \), which can be extracted from an avoided crossing (shown for \( Q_1 \) and \( Q_2 \); Fig. 2b). The difference in coupling strengths is a result of imperfections in the alignment and machining precision, which lifts the degeneracy between the local dark states \( |D_1\rangle \) and \( |D_3\rangle \). The coherent exchange interaction lifts the degeneracy of \( |B_{40}\rangle \) and \( |D_{10}\rangle \) and allows us to observe the decoupling of the dark state when we tune the qubits into resonance.

We calibrate the decoherence-free frequency \( \omega_a \) using the dark state formed by two distant transmons. To control the dark states, we introduce two sideports that are weakly coupled to the local transmon pairs (Fig. 1d). For over a decade, individual drive ports have been applied in on-chip circuit QED experiments on the complex-valued scattering parameters. In our experiment, we capitalize on the symmetry between the four qubits and sideport drives. The sideports provide an amplitude gradient over the local pairs to access the dark states \( |D_{10}\rangle \), but also the possibility to independently adjust the relative phase between the local pairs, which allows us to apply a symmetric drive and access the non-local dark state \( |D_{0}\rangle \). The field of the drive port does not coincide with the polarization of the TE\(_{0}\) waveguide mode and decays exponentially along the propagation direction of the waveguide; thus, the drive is effectively local (Supplementary Section 6).

We measure the ground-state population by employing a state-dependent scattering scheme, adapted from quantum non-demolition state detection in trapped-ion quantum computing. If the collective system is in the ground state \( |G\rangle \), we can coherently scatter photons between the ground state \( |G\rangle \) and super-radiant state \( |B_{0}\rangle \), which reduces the transmission through the waveguide (Fig. 2a). If the bright state \( |D_{0}\rangle \) is populated, the microwave signal is not scattered, resulting in unit transmission. By selectively exciting the dark state using microwave signals applied through the sideports with \( \phi = 0 \), we can experimentally search for the longest dark-state relaxation time around the analytical decoherence-free frequency and indeed find it within the uncertainty at \( \omega_a/2\pi = 7.321 \text{ GHz} \) (Fig. 2c).

From now on, we consider the full system; therefore, we tune all the four transmons into resonance such that the bright transitions of the capacitively coupled pairs match the decoherence-free frequency \( \omega_a \). Both local two-qubit bright states interact via the waveguide and create the collective four-qubit states \( |B_{0}\rangle \) and \( |D_{0}\rangle \), whereas the local two-qubit dark states \( |D_{0}\rangle \) and \( |D_{0}\rangle \) cannot interact via the waveguide. These four states span the first excitation manifold (Fig. 1c). In Fig. 2a, we extract the linewidth \( \Gamma_{a}\) = 60.9 MHz resulting from the constructive interference of all the transmons, namely, \( \Gamma_{a} = \sum \Gamma_{n} \).

To characterize the dark state, we study the time-resolved dynamics. When driving the transmon array through the sideports, the transition amplitudes from the ground state to non-local dark and bright states depend on the driving phase \( \phi \) as

\[
|G\rangle \rightarrow |D_{1}\rangle : \quad \frac{h\Omega}{2} (1 + e^{i\phi}).
\]

\[
|G\rangle \rightarrow |B_{1}\rangle : \quad \frac{h\Omega}{2} (1 - e^{i\phi}).
\]

Rabi oscillations between \( |G\rangle \) and \( |D_{1}\rangle \) are shown in Fig. 3a when the amplitude of the drive field \( \Omega \) is increased and the phase difference between the sideports matches \( \phi = 2n\pi \) (\( n \in \mathbb{Z} \)). For an anti-symmetric drive with odd integer multiple \( \phi = (2n - 1)\pi \), we only drive the bright state \( |B_{1}\rangle \) that decays very rapidly to the ground state.
Fig. 3 | Coherent control of the dark state. a. We apply a Gaussian-shaped pulse of total length $t = 240\,\text{ns}$ and standard deviation $\sigma = 40\,\text{ns}$ to observe Rabi oscillation between the ground state $|G\rangle$ and non-local four-qubit dark state $|D\rangle$ as a function of Rabi frequency $\Omega$ and sideport phase difference $\phi$. By applying the pulse through the sideports, we can independently set the phase $\phi$. The ground-state population is read out by sending a $5\,\mu\text{s}$-long rectangular pulse through the waveguide, resonant with the transition between states $|G\rangle$ and $|B_i\rangle$. The right panel shows a vertical linecut on the white dashed lines of the colour map for phase difference $\phi = 0$ and $\phi = \pi$. The bottom panel shows a horizontal linecut for a Rabi frequency of $\Omega/2\pi = 1\,\text{MHz}$. For the theory curve, we simulate the Hamiltonian (equation (3)) and master equation in Supplementary Information with the parameters specified in Supplementary Table 1. b. A symmetric excitation pulse with Rabi frequency $\Omega/2\pi = 1\,\text{MHz}$ and relative phase difference $\phi = 0$ between both sideports is used to populate the collective dark state $|D\rangle$. After a variable delay time, the ground-state population is read out to find an average relaxation time $T_\alpha = (1.7\pm 0.06)\,\mu\text{s}$. In a Ramsey experiment, we find an average coherence time of $T_\gamma = (0.58\pm 0.06)\,\mu\text{s}$. On resonance ($\Delta/2\pi = 0$), we observe an exponential decay; for a detuned pulse ($\Delta/2\pi = 9\,\text{MHz}$), we induce oscillations with frequencies corresponding to detuning between the drive frequency and transition frequency between states $|G\rangle$ and $|D\rangle$.

state with rate $\Gamma_{k\alpha}$. Again, we employ the state-dependent scattering readout scheme as for the two-qubit case, now using transition $|G\rangle$ to $|B_i\rangle$ to scatter the waveguide photons. To determine the ground-state population, we conduct a reference measurement of the transmitted readout pulse for the case where all the transmons are tuned below the waveguide cutoff frequency. With a calibrated $\pi$ and $\pi/2$ pulse, we can investigate the coherence properties of the dark state. For the collective dark state, we measure an average relaxation time $T_\alpha = (1.71\pm 0.06)\,\mu\text{s}$ and coherence time $T_\gamma = (0.58\pm 0.06)\,\mu\text{s}$ (Fig. 3b). In this system, dephasing and frequency fluctuations of the individual qubits cause imperfections in the dark-state symmetry. This results in a finite decay rate into the waveguide; thus, the dark-state relaxation time $T_\alpha$ depends on the pure transmon dephasing rate $\gamma_{\phi}$ (Supplementary Section 4).

To simulate the collective dynamics shown by the black lines in Fig. 3a, we model the transmons and their direct couplings with the Hamiltonian\cite{34,36,37}

$$\hat{H}_T/\hbar = \sum_{j=1}^{3} \left[ \omega_j \hat{n}_j - \frac{U_i}{2} \hat{n}_j(\hat{n}_j - 1) \right] + J_{12} \left[ \hat{a}_i^\dagger \hat{a}_2 + \text{h.c.} \right] + J_{34} \left[ \hat{a}_3^\dagger \hat{a}_4 + \text{h.c.} \right].$$  (3)

where $\hbar$ is the reduced Planck constant, $\omega_j$ is the fundamental resonance frequency and $U_i$ is the anharmonicity of the individual transmon. Operators $\hat{a}_i$ and $\hat{a}_i^\dagger$ are the bosonic annihilation and creation operators of the $j$th transmon and $\hat{n}_j = \hat{a}_j^\dagger \hat{a}_j$ is the corresponding number operator. In the presence of the waveguide radiation field, the dynamics are governed by a master equation\cite{31,11,15} taking into account the coherent exchange interaction $J_{\alpha\beta}$ and correlated decay $\gamma_{\alpha\beta}$ between the transmons at sites $j$ and $k$. The properties of the system are then described by the effective non-Hermitian Hamiltonian

$$\hat{H}_{\text{eff}}/\hbar = \hat{H}_T/\hbar + \sum_{jk} \left( J_{jk} - \frac{\gamma_{\alpha\beta} e^{-i\phi}}{2} \right) \hat{a}_j^\dagger \hat{a}_k + \frac{1}{2} \sum_{j} \gamma_{\alpha\beta} \hat{n}_j \hat{a}_j^\dagger \hat{a}_j;$$  (4)

where parameter $\gamma_{\phi}$ describes the non-radiative dissipation of individual transmons, whereas we neglect pure dephasing for simplicity.

As shown in Fig. 3a, the Rabi oscillations decay for large drive amplitudes. This is caused by leakage into fast decaying states of the higher-excitation manifold. To study this in more detail, we explore the two-excitation manifold of the collective four-transmon system (Fig. 4) by concatenating a spectroscopy pulse after populating the dark state $|D\rangle$. For the spectroscopy, we change the frequency and relative phase to unveil the symmetry and energy of the states in the two-excitation manifold. When the spectroscopy pulse is resonant with a transition, for example, $|W_{\alpha}\rangle$, $|W_{\beta}\rangle$, $|B_i\rangle$, or $|B_j\rangle$, the system is reset to the ground state due to the rapid decay of these states dominantly via the bright state $|B_i\rangle$. We denote the collective states by $|D\rangle$, $|B_i\rangle$, and $|W\rangle$, where the letter refers to their waveguide radiation characteristics, namely, dark, bright or weakly radiating, respectively. The subscript is an ascending enumeration based on their energy value. In Fig. 4, the collectiveness of these states is apparent in the phase dependence of the measured ground-state population (left), which is consistent with the simulation (right) of the model Hamiltonian (equation (4)).

It is essential to note that a transmon is a bosonic multilevel system with anharmonicity $U$. The many-body excited-state
In conclusion, the experiment demonstrates that collective dark states constitute a resource for coherent quantum information and can be controlled by local drives with an adjustable phase relation. The collective four-transmon system comprises a one-excitation state manifold with long-lived dark states, as well as one rapidly decaying bright state. In particular, we achieve an effective protection from the waveguide, leading to a decrease in the relaxation rate by a factor of 160 compared with the single-qubit coupling rate or a factor of 650 compared with the collective bright state. The degenerate bright state can be used to read out the system. Although in conventional resonator-based architectures, detuning between the readout cavity and qubit plays an important role for its coherence time, the experiment shows that the protection can be engineered by taking into account the symmetry properties of the system as both transitions are resonant. Unlike in previous experiments, the observation of the weakly radiant states $|W_4\rangle$ and $|W_5\rangle$ is a direct manifestation of the transmons’ bosonic nature and demonstrates the necessity to go beyond the two-level approximation when trying to engineer many-body physics with artificial atoms.

Looking forward, coherent control of multi-qubit dark states opens up the possibility to investigate the dynamics of interacting quantum many-body systems, to study many-body localization in disordered arrays or even realize a quantum computation and simulation platform within an open quantum system. On one hand, the adiabatic elimination of higher-excited states promises the possibility to further optimize the coherent control of the dark state; on the other hand, the two-excitation manifold can be used to reset the dark-state qubit and transfer quantum information into itinerant photons. This mechanism is a source of cluster-state creation, whereas the cascaded decay can be utilized to study...
entanglement between photons of different frequencies. Finally, the interplay between long-lived subradiant states and weakly radiative states can give new insights into incoherent scattering properties and photon–phonon correlations.\textsuperscript{41–46}

Online content
Any methods, additional references, Nature Research reporting summaries, source data, extended data, supplementary information, acknowledgements, peer review information; details of author contributions and competing interests; and statements of data and code availability are available at https://doi.org/10.1038/s41567-022-01527-w.

Received: 9 June 2021; Accepted: 25 January 2022; Published online: 14 March 2022

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Data availability
The data that support the findings of this study are available on Zenodo (https://zenodo.org/record/5772190).

Code availability
The code used for data analysis and simulated results is available from the corresponding author upon reasonable request.

Acknowledgements
We thank A. Strasser for fabricating the waveguide sample. We would like to thank E. I. Rosenthal for valuable comments on the manuscript. M.Z. and S.O. acknowledge funding by the European Research Council (ERC) under the European Union’s Horizon 2020 research and innovation programme (714235). M.Z. and C.M.F.S. acknowledge support by the Austrian Science Fund FWF within the DK-ALM (W1259-N27). R.A. acknowledges support from the Austrian Science Fund FWF within the SFB-BeyondC (F7106-N38). T.O. and M.S. acknowledge funding by the Emil Aaltonen Foundation and by the Academy of Finland (316619 and 320086). M.L.J. acknowledges funding from the Canada First Research Excellence Fund.

Author contributions
M.Z. and G.K. conceived and designed the experiment. M.Z. simulated and fabricated the devices and also conducted the measurements. M.Z. and C.M.F.S. analysed the data. T.O. and M.S. developed the theoretical model and performed the simulations. M.Z. and G.K. wrote the manuscript. All the authors discussed the results and contributed to the writing of the manuscript.

Competing interests
The authors declare no competing interests.

Additional information
Supplementary information The online version contains supplementary material available at https://doi.org/10.1038/s41567-022-01527-w.

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Peer review information Nature Physics thanks the anonymous reviewers for their contribution to the peer review of this work.

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