Angle-resolved spectroscopy of electron-electron scattering in a 2D system

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Abstract. – Electron-beam propagation experiments have been used to determine the energy and angle dependence of electron-electron (ee) scattering in a two-dimensional electron gas (2DEG) in a very direct manner. The experimental results provide direct evidence for novel ee-scattering effects in 2D degenerate conductors. Most striking is the increased importance of small-angle scattering in a 2D system with decreasing excitation energy. In particular, in a 2DEG ee-scattering can, at sufficiently low energies, be purely dephasing in character, i.e. changing the phase but not the direction of electron motion.

Research on electron-electron (ee) scattering phenomena has recently gained a new impetus due to the interest in dephasing of coherent wave functions. Most work in this field has been done in nanostructured semiconductor heterostructures [1]. Therefore, there exists a vital mutual interest to understand these scattering phenomena in detail.

The scattering characteristics of electrons in systems with reduced dimensionality exhibit decisive differences with respect to the situation in bulk. Theoretically, ee-scattering in two-dimensional (2D) systems was first considered in [2,3]. It was shown that the energy-relaxation time in a 2DEG is shorter by a factor of order \(\ln \varepsilon_F/\varepsilon\) compared to the three-dimensional (3D) case (\(\varepsilon\) is the electron’s excess energy counted from the Fermi energy \(\varepsilon_F\)). A reduction of the dimensionality (from three to two) induces much more drastic changes in momentum transfer processes [4–9]. This is due to the fact that energy and momentum conservation lead to substantial restrictions on the final states of colliding particles in a 2D system. The angular dependence of the scattering probability is determined by the parameter \(\varepsilon/\varepsilon_F\) (see fig. 1). For small excess energies the typical scattering angles are of the order of \((\varepsilon/\varepsilon_F)^{1/2} \ll 1\) while the energy transfer is of the order of \(\varepsilon\). Thus ee-scattering leads to a dephasing of the electron wave function but leaves its trajectory practically unaffected. This process can be viewed as an ideal “dephaser”.

In general, two types of ee-collisions with nearly the same probability characterize scattering in 2D degenerate systems [4]: collisions of a non-equilibrium electron with momentum...
Fig. 1 – ee-scattering angular distribution function \( g(\alpha) \) in a 2D system, 2DEG temperature \( T = 0 \), (1) \( \varepsilon = 0.12\varepsilon_F \), (2) \( \varepsilon = 0.4\varepsilon_F \), dashed line: 3D case (Callaway’s Ansatz). Here angle \( \alpha \) is measured with respect to the momentum of scattering electron, \( p \). By definition, \( |g(\alpha)|\,d\alpha \) characterizes the probability that a non-equilibrium electron, \( g(\alpha) > 0 \) (or hole for \( g(\alpha) < 0 \)), emerges in an interval \( d\alpha \) after scattering. The function \( g(\alpha) \) is normalized to the unity (this corresponds to the scattering of one electron).

\( p \) with excess energy \( \varepsilon \) i) with equilibrium electrons \( p_1 \), which usually result in scattering of both electron by a small angle \( \alpha \sim \varepsilon/\varepsilon_F \) into states \( p_2 \) and \( p_3 \) leaving a hole (an empty state below the Fermi surface) in state \( p_1 \), with \( p + p_1 = p_2 + p_3 \), and ii) with electrons of nearly opposite momentum, \( p \approx -p_1 \). In the latter case, the electrons at \( p_2 \) and \( p_3 \approx -p_2 \) are scattered by a much larger angle, on average \( \alpha \approx \sqrt{\varepsilon/\varepsilon_F} \).

Let us consider fig. 1, the angular distribution function of scattered electrons, \( g(\alpha) \), in more detail. For comparison we have also plotted the most commonly used approximation for \( g(\alpha) \) for 3D systems (dashed line), sometimes referred to as the Callaway Ansatz \( [g(\alpha) \propto 1 + 2\cos(\alpha)] \), and independent of \( \varepsilon \). For 3D systems \( g(\alpha) \) is very smooth, exhibiting a broad distribution of electrons moving in forward direction and holes moving backwards. In the 2D case \( g(\alpha) \) shows several distinct features. The most striking one is a very narrow distribution of electrons moving in approximately forward direction (\( |\alpha| \leq 0.05 \) rad for \( \varepsilon = 0.12\varepsilon_F \)). The height of this peak is determined by the small-angular processes of type i); its width is determined by type-ii) processes and increases with energy according to \( \delta\alpha \sim \sqrt{\varepsilon/\varepsilon_F} \). The type-ii) scattering events also cause a secondary peak at \( \alpha \approx \pi - 2\sqrt{\varepsilon/\varepsilon_F} \) [8] and a narrow hole dip of width \( \sqrt{\varepsilon/\varepsilon_F} \) at \( \alpha = \pi \) [4,6]. Another intriguing feature of \( g(\alpha) \) is a dip in forward direction for very small angles, with a width \( \propto (\varepsilon/\varepsilon_F)^{3/2} \), which can be seen more clearly in the inset of fig. 1. This dip is caused by the conservation laws: An electron may give away its surplus energy to equilibrium partners only upon scattering by a finite angle. This effect exists also in 3D systems, however, in this case it is much weaker [5,10] and has never actually been observed. The peak due to the small angular scattering processes has a complicated lineshape and a height which is proportional to \( (\varepsilon/\varepsilon_F)^{-3/2} \). For further discussion we restrict ourselves to scattering angles \( \alpha < 1 \) (rad), which are accessible in experiments and where the specific 2D phenomena due to small angular scattering processes dominate the scattering characteristics.

To study experimentally the effects mentioned above we use a technique which allows for a spectroscopy of the electron-electron interaction in a magnetic field. This method is based on the separation of groups of particles, which are scattered at different angles, by magnetic field. In the experiment, an electron beam injected into the 2DEG via an electrostatically defined
quantum point contact (QPC) “i” is detected by a second QPC “d” in a certain distance (cf. fig. 2) [11, 12]. When a magnetic field is applied perpendicular to the 2DEG plane, the injected beam is deflected and only scattered electrons can reach the detector QPC. At low electron excess energies ($\varepsilon \ll \varepsilon_F$) one can neglect the energy dependence of the cyclotron radius $r_c$. When the opening angle $\Phi$ of the QPCs “i” and “d” is sufficiently small, i.e., $\Phi \ll 1$ [11], we have a situation that for a given magnetic field $B$ the detector signal is determined only by one trajectory, i.e., the signal results solely from electrons that were scattered across an angle $\alpha = 2 \arcsin(L/2r_c)$ at point $O$ (indicated in fig. 2). Thus, by changing the magnetic field we can directly measure the angular distribution function of scattered electrons $g(\alpha)$ in a wide range of angles $\alpha$.

As discussed above, $g(\alpha)$ will essentially depend on the excess energy $\varepsilon$ of the injected electrons. This is why we apply a differential measurement technique, which is equivalent to the use of mono-energetic electron beams. $\varepsilon$ is controlled by adjusting the bias voltage $V_i$ applied between contacts 1 and 2. The voltage drop $V_{d}$, measured between contacts 3 and

![Fig. 2](image)

**Fig. 2** – Schematic view of the sample structure showing the Schottky gates (black areas) defining the injector (i) and detector (d) point contacts. Also indicated is the only possible trajectory of an injected electron in a perpendicular magnetic field $B$, where the electron scattered in point $O$ over an angle $\alpha$ reaches the detector. Hatched areas and crosses represent the depleted 2DEG regions and Ohmic contacts, respectively.

![Fig. 3](image)

**Fig. 3** – Behaviour of the electron beam signal at different injector voltages, from top to bottom: (1) $V_i = 0.8 \text{ mV}$, (2) $V_i = 1.2 \text{ mV}$, (3) $V_i = 1.6 \text{ mV}$, (4) $V_i = 2.6 \text{ mV}$, (5) $V_i = 3.5 \text{ mV}$ and (6) $V_i = 4.5 \text{ mV}$ as a function of magnetic field $B$. Inset: curve for $V_i = 0.1 \text{ mV}$.
4, results from electrons which have reached the QPC detector \( d \). A small ac-modulation \( \delta V_i \ll V_i \) is added to the dc-bias. Although an electron beam injected via a QPC consists of electrons of all energies from \( \varepsilon_F \) up to \( \varepsilon_F + \varepsilon \), only the contribution \( \delta V_d \) of the high-energy part of the beam to the signal can be detected by measuring the signal with a lock-in at the same frequency as \( \delta V_i \).

For the experiments, conventional Si-modulation–doped GaAs-(Ga,Al)As heterostructures were used, with a carrier concentration of \( n_s \approx 2.8 \times 10^{11} \text{ cm}^{-2} \) and an electron mobility of \( \mu \approx 100 \text{ m}^2(\text{V s})^{-1} \), which implies an impurity mean free path of \( l_{\text{imp}} \geq 10 \mu \text{m} \). A pair of QPCs, about \( L \approx 4 \mu \text{m} \) apart, were fabricated using split-gate technology. By applying a negative voltage to the gate contacts, the conductance of the QPCs (\( G_{\text{QPC}} = N^2 e^2/h \)) could be adjusted from several conducting modes \( N \) into the tunneling regime \( (N < 1) \). Throughout all experiments injector and detector QPCs were adjusted to \( N = 1 \) to ensure narrow opening angles [11]. The injection dc-voltage, \( V_i = V_{12} \), was varied between 0 and 5 mV. The ac-modulation voltage was kept constant at 30 \( \mu \text{V} \), so that \( \delta V_i \leq k_B T_0/e \ll V_i \). The sample was kept at a lattice temperature \( T_0 \approx 200 \text{ mK} \) in a dilution refrigerator.

Figure 3 displays some examples of the measured detector signal for various injection voltages as a function of magnetic field. The inset of fig. 3 shows the measured signal for the lowest injection energy \( eV_i = 0.1 \text{ meV} \), when the ee-scattering mean free path \( l_{\text{ee}} \) is much larger than \( L \) and the electrons reach the detector QPC ballistically. From this we determine the characteristic opening angle [11] of injector and detector, \( \Phi \approx 12^\circ \). The detector signal is maximum at zero magnetic field. With increasing injection energy \( V_i \) ee-scattering becomes more important, leading to i) a decrease of the signal of non-scattered electrons near \( B = 0 \), ii) a broadening of the signal with \( B \) and iii) the appearance of a dip in the signal around \( B = 0 \) for energies \( V_i \geq 3.5 \text{ mV} \).

For further consideration we have to investigate how this experimental behaviour relates quantitatively to the expected 2D-scattering characteristics. Therefore, we describe the problem by a linearized Boltzmann equation in a magnetic field,

\[
\omega_c \partial f / \partial \varphi + v_x \partial f / \partial x + v_y \partial f / \partial y = \hat{J} f ,
\]

where \( \omega_c \) is the cyclotron frequency and \( \hat{J} \) is the linearized operator for the ee-collisions, which can be written as

\[
\hat{J} f(p) = -\nu f(p) + \int dp' \nu_{pp'} f(p') ,
\]

with \( \nu = \int dp' \nu_{pp'} \). Integration of the collision integral kernel \( \nu_{pp'} \) over energy yields the angular distribution function of the scattered electrons:

\[
g(\alpha) = m \nu^{-1} \int d\varepsilon' \nu_{p'p} ,
\]

where \( \alpha \) is the angle between \( p \) and \( p' \), and \( p' \) refers to the electrons (holes) at \( p_1, p_2 \) and \( p_3 \) mentioned above. If the probability is small for an electron to be scattered over a distance \( L \) (i.e., \( l_{\text{ee}}(\varepsilon) \approx v \cdot \nu^{-1} \geq L \)), eq. (1) can be solved using perturbation theory on the collision integral. If we write the electron distribution function at the exit of the injector as \( f_0 = \delta V_i \delta(\varepsilon - eV_i) \lambda_p \delta(y) \rho_i(\varphi) \) and consider only the first iteration of the collision integral, we can obtain an expression for the current through the detector QPC. For low injection energies \( eV_i = \varepsilon \ll \varepsilon_F \), the detector signal can be written as
Here \( C = 2mL\lambda_F\delta V_i/h, \) \( \lambda_F \) is the Fermi wavelength and \( \rho_i(\varphi) \) is the angular emittance (acceptance) function of the injector (detector) QPC \([11]\). From this equation it is clear that \( g(\alpha) \) can be obtained from the magnetic-field dependence of \( \delta V^s_d \).

When \( g(\alpha) \) varies only slightly on the scale of the opening angle \( \Phi \), a local approximation of eq. (5) is valid for large scattering angles, \( \pi - \alpha < 2\sqrt{\Phi} \).

However, it is possible to extend the range of validity for this one-collision approximation. This is because in all experiments we have that \( eV_i \gg k_B T \), implying that the probability for secondary ee-collisions is approximately an order of magnitude lower than that of the first one \([6,8]\). It turns out that a one-collision approximation is valid as long as

\[
\frac{\delta V}{ee}(\alpha, V_i) \approx 10 l_{ee}(eV_i/3) \approx 10 l_{ee}(eV_i) \text{ i.e. for a much wider range of parameters than the perturbation theory.}
\]

Partial summation of the corresponding iteration series of eq. (1) results in the following expression for \( g(\alpha, V_i) \):

\[
\tilde{g}(\alpha, V_i) = \exp \left[ -\frac{\Lambda}{l_{ee}} \right] g(\alpha, V_i),
\]

which replaces \( g(\alpha, V_i) \) in eqs. (4) and (5). The exponential factor on the r.h.s. gives the probability for an electron to travel ballistically to a point of scattering, after which it reaches the detector without further collisions. In the local approximation of eq. (5), \( \Lambda = L\alpha/4\sin(\alpha/2) \) can be interpreted as the length of the trajectory from the injector to point \( O \) (cf. fig. 2).

In order to compare the experimental data with theory, it is necessary to extract the contribution of scattered electrons, \( \delta V^s_d \), from the observed signal \( \delta V_d \). We have

\[
\delta V^s_d = \delta V_d - \exp \left[ -\frac{2r_c}{l_{ee}(V_i)} \arcsin \frac{L}{2r_c} \right] \delta V^0_d.
\]

Here, \( \delta V^0_d(B) \) is the signal which would be observed in the absence of scattering, so that the second term on the r.h.s. of eq. (7) is the contribution of electrons that reach the detector ballistically. Experimentally, \( \delta V^0_d(B) \) can be obtained from the experiment at lowest injection energy, \( eV_i = 0.1 \text{ meV} \). In this case \( l_{ee}/L \sim 10^2 \) and thus collisions can be neglected. For \( l_{ee} \) we use the expression for the energy relaxation length in a 2DEG obtained by Giuliani and Quinn (cf. ref. [3], eq. (13)).

Figure 4(a) shows the angular distribution functions \( g(\alpha) \) for various injector energies, obtained from the experimental data in fig. 3 using eqs. (5), (6), and (7). The various \( g(\alpha) \) clearly display the expected small-angle scattering behaviour. For small \( V_i \) (curves (1) and
Fig. 4 – (a) ee-scattering distribution function $g(\alpha)$ retrieved from the experiment in fig. 3. Inset: width $\delta \alpha$ of $g(\alpha)$ (squares) as a function of injector bias voltage $V_i$. $\delta \alpha$ is defined as the angle below which 2/3 of the electrons are scattered. Solid and dashed lines represent theoretical prediction for a 2D ($\delta \alpha \propto (eV_i/E_F)^{1/2}$) and a 3D system, respectively. (b) Comparison of the theoretical (symbols, eq. (8)) and experimental (lines, inferred from the experiment using eq. (7)) $\delta V_s^d$, the detector signal due only to scattered electrons. Curves are displayed with an offset for clearness.

(2), $V_i = 0.8$ and 1.2 mV, respectively) the observed peak width $\delta \alpha$ is only slightly larger than the point contact opening angle $\Phi$. As discussed above, in this limit the experimentally recovered $g(\alpha)$ is smoothed, and we cannot expect to observe the dip at very small angles.

The peak in $g(\alpha)$ broadens when the energy of the injected electrons is increased (curves 3-4). In the inset of fig. 4(a) the width of $g(\alpha)$ is displayed as a function of injection energy. It shows a clear square-root behaviour $\delta \alpha \propto \sqrt{eV_i/E_F}$ in striking contrast to the 3D situation where $g$ is essentially energy independent. The increase of $\delta \alpha$ with $V_i$ also directly implies that the observed small-angle scattering cannot be attributed to the weak screening in 2D systems. In this case the scattering angle should actually decrease with excess energy.

When, for higher $V_i$, $\delta \alpha$ becomes larger than the QPC opening angle $\Phi$, one can clearly observe the expected dip in forward direction (curves 5-6, fig. 4(a)). The amplitude of the dip is much larger than would be the case for a 3D electron system.

As discussed above, the local approximation of eq. (5) is not valid at small scattering angles $\alpha < \Phi$. For these angles, $g$ is more precisely given by the integral equation

$$
\delta V_s^d \simeq 2C \nu \int d\varphi \rho_i(\varphi) \int d\varphi' \rho_d(\varphi') \times \tilde{g}(\varphi' - \varphi + L/r_c, V_i) \kappa(\varphi' - \varphi + L/r_c),
$$

where $\kappa(x) = 1/|x|$ for $x > \Phi$ and $\kappa(x) = 1/\Phi$ for $x < \Phi$. Here again we use $\tilde{g}(\alpha)$ as defined in eq. (6); $\Lambda = L(2\varphi' + L/r_c)/2(\varphi' - \varphi + L/r_c)$ is the distance between the injector and the crossing point (O) of electron trajectories injected at angle $\varphi$ and detected at angle $\varphi'$; the integration in (8) has to be evaluated for all $\Lambda$ such that $0 < \Lambda < L$, while $l_{ee} \equiv l_{ee}(V_i)$.

For comparison, the results of eq. (8) for various values of $V_i$ are presented together with experimental data for $\delta V_s^d$ in fig. 4(b). As is evident from the figure, we find a gratifying agreement between theory (symbols) and experiment (solid lines), justifying the assumptions made in extracting $g(\alpha)$ from the experimental data.

The results presented in this paper demonstrate that a suitably performed electron-beam experiment can provide a wealth of details on fundamental electron scattering processes, not only on the energy dissipation, but certainly also on momentum scattering.

Note that by
changing the detector gate voltage in a controlled manner it is also possible to analyze the energy dependence of the scattered electrons [12]. Combining this and the $B$-field–dependent measurements presented here should enable a direct determination of the collision integral kernel $\nu_{pp'}$ as a function of detection energy ($\varepsilon'$), injection energy ($\varepsilon = eV_i$), and scattering angle $\alpha$. This method can also be used to study other electron-scattering processes (e.g., electron-phonon or electron-impurity scattering) in various generalized systems (e.g., 3D conductors and non-degenerated systems) with small modifications. Furthermore, the method allows for a direct study of the influence of spin-orbit interaction on ee-scattering processes and the spin dependence of angular scattering characteristic of spin-polarized targets by using spin-polarized electron beams. Additionally, the small angular character of ee-scattering processes in a 2DEG implies that hot-electron injection through a QPC can be used as a dephasing tool for electron interference experiments in the quasi-ballistic transport regime.

In conclusion, we have demonstrated direct experimental evidence for essential changes in momentum transfer processes in degenerated electron gas in reduced dimensions by realizing a spectroscopy method which gives the possibility to locate scattering events and to detect the energy-dependent angular distribution of scattered particles.

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