Modelling and simulating of a multiple input and multiple output system to control the liquid level and temperature by using model predictive control

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Abstract. In recent years, Model Predictive Control has been regarded as one of the most reliable advanced control methods widely used in industrial and nuclear processes. Model predictive control uses a system model to forecast its future response, and also treats a real-time optimization algorithm to choose the best control action that leads the predicted output to the reference. The overall design goal of a predictive control model is to compute the trajectory of a future manipulated variable input signal to optimize the future behavior of the plant output. Optimization is performed within a limited time window by providing information at the beginning of the time window. This paper proposes a predictive controller MPC model for multiple input-output systems. The design of the controller includes the development of a model of the state space system for the tank system, then the design and the simulation of the MPC controller for the developed model system. The variables to be controlled are the temperature and the liquid level of the tank by changing the MPC’s parameters.

1. Introduction
Model Predictive Control is an advanced control method. That has been in use in the process industries such as nuclear and chemical plants and Oil Refineries. Model predictive controllers are based on real-time models of the process, the linear models are obtained by system identification. Model predictive control refers to computer control algorithms that use the process model to forecast the future response of a plant. At each control step, the MPC attempts to optimize future plant response by computing the variables' future adjustments sequence. The first input sequence is sent into the plant, and the whole calculations are repeated at subsequent control intervals. MPC was originally developed to meet the specific control needs of power plants and petroleum refineries, MPC technology is now used in a wide variety of application areas including nuclear chemicals and aerospace applications. The MPC design has several tuning parameters, such as prediction horizon, control horizon, and weight matrices in the cost function. Therefore, the tuning parameters are very important for the successful implementation of multivariable MPC, it is crucial to study MPC tuning methodology that allows finding the right formulas form for the tuning parameters selection.

2. MPC principle of work:
Control A block diagram of a model predictive control system is shown in figure 1.
A process model is used to predict the current values of the output variables. The residuals, the differences between the actual and predicted outputs, serve as the feedback signal to a Prediction block. The predictions are used in two types of MPC calculations that are performed at each sampling instant: set-point calculations and control calculations. Inequality constraints on the input and output variables, such as upper and lower limits, can be included in either type of calculation.

However, the coordination of the control and set-point calculations is a unique feature of MPC. Furthermore, MPC has had a much greater impact on industrial practice, because it is more suitable for MIMO control problems.

Figure 1: Model predictive control block diagram.

In the MPC approach, the current control action is computed on-line rather than using a pre-computed, off-line, control law.

A model predictive controller uses, at each sampling instant, the plant’s current input and output measurements, the plant’s current state, and the plant’s model to:
- calculate, over a finite horizon, a future control sequence that optimizes a given performance index and satisfies constraints on the control action;
- use the first control in the sequence as the plant’s input.

The MPC strategy is illustrated in Figure 2, where Np is the prediction horizon, \( u(t + k|t) \) is the predicted control action at \( t + k \) given \( u(t) \). Similarly, \( y(t + k|t) \) is the predicted output at \( t + k \) given \( y(t) \).

3. The space model of the model predictive controller

Single-input and Single-output System was chosen for simplicity, we begin our study by assuming that the underlying plant is a single-input and single-output system, described by:

\[
\begin{align*}
x_m(k + 1) &= A_m x_m(k) + B_m u(k) \\
y_m(k + 1) &= C_m x_m(k)
\end{align*}
\]

where \( u \) is the manipulated variable or input variable; \( y \) is the process output;
and \( x_m \) is the state variable vector with assumed dimension \( n \times 1 \). Note that this plant model has as its input. Thus, we need to change the model to suit our design purpose in which an integrator is embedded. Note that a general formulation of a state-space model has a direct term from the input signal \( u(k) \) to the output \( y(k) \) as:

\[
y(k) = C_m x_m(k) + D_m u(k)
\]
However, due to the principle of receding horizon control, where current information of the plant is required for prediction and control, we have implicitly assumed that the input $u(k)$ cannot affect the output $y(k)$ at the same time. Thus, $D_m = 0$ in the plant model.

Taking a difference operation on both sides of (1.1), we obtain that

$$x_m(k + 1) - x_m(k) = A_m(x_m(k) - x_m(k - 1)) + B_m(u(k) u(k - 1)).$$

Let us denote the difference of the state variable by

$$\Delta x_m(k + 1) = x_m(k + 1) - x_m(k);$$
$$\Delta x_m(k) = x_m(k) - x_m(k - 1),$$

and the difference of the control variable by

$$\Delta u(k) = u(k) - u(k - 1)$$

These are the increments of the variables $x_m(k)$ and $u(k)$. With this transformation, the difference of the state-space equation is:

$$\Delta x_m(k + 1) = A_m \Delta x_m(k) + B_m \Delta u(k).$$

Note that the input to the state-space model is $\Delta u(k)$. The next step is to connect $\Delta x_m(k)$ to the output $y(k)$. To do so, a new state variable vector is chosen to be

$$x(k) = [\Delta x_m(k)^T y(k)]^T$$

where superscript indicates matrix transpose. Note that

$$y(k + 1) - y(k) = C_m(x_m(k + 1) - x_m(k)) = C_m \Delta x_m(k + 1) = C_m A_m \Delta x_m(k) + C_m B_m \Delta u(k).$$

Putting together (1.3) with (1.4) leads to the following state-space model:

$$\begin{bmatrix} x_m(k + 1) \\ y(k + 1) \end{bmatrix} = \begin{bmatrix} A_m \\ C_m A_m \end{bmatrix} \begin{bmatrix} x(k) \\ y(k) \end{bmatrix} + \begin{bmatrix} B_m(k) \\ C_m B_m(k) \end{bmatrix} \Delta u(k).$$

4. Multi-inputs/Multi-output system modeling under Model Predictive Control

Most nuclear and industrial control systems have many input and output variables. In our case, we consider the mixing process shown in Figure as a multiple-input/multiple-output (MIMO) systems.
The level \( h \) in the tank and the temperature \( T \) are to be controlled by adjusting the two main variables in the system are the liquid’s level. These two parameters are related to the flow rate of the hot and cold liquid denoted by \( w_H \) and \( w_C \), respectively. In this case the disturbance variables the inlet temperature \( T_H \) and \( T_C \) are considered to be disturbance variables. The outlet flow rate \( w \) and properties of the liquid are assumed to be constant. The mass and energy balances in this system can vary with time.

\[
\rho C \frac{d}{dt} \left[ V (T - T_{\text{ref}}) \right] = w_H C (T_H - T_{\text{ref}}) + w_C C (T_C - T_{\text{ref}}) - C (T_C - T_{\text{ref}}) \tag{4.1}
\]

\[
\rho C \frac{dv}{dt} = w_H + w_C - w \tag{4.2}
\]

\[
\frac{d}{dt} \left[ V (T - T_{\text{ref}}) \right] = \left( T_C - T_{\text{ref}} \right) \frac{dv}{dt} + V \frac{dT}{dt} \tag{4.4}
\]

\[
V = Ah. \tag{4.5}
\]

\[
\frac{dT}{dt} = \frac{1}{\rho_{ah}} [w_H T_H + w_C T_C - (w_H + w_C) T] \tag{4.6}
\]

\[
\frac{dh}{dt} = \frac{1}{\rho A} (w_H + w_C - w) \tag{4.7}
\]

After linearizing (2.4) and (2.5), putting them in the derivative from, and taking Laplace transforms, we obtain the transfer function matrix:

\[
[T(s)] = \begin{bmatrix}
\frac{T_{h-T}}{s} & \frac{T_{c-T}}{s} & \frac{w_H}{s} & \frac{w_C}{s} \\
\frac{1}{\rho A} & \frac{1}{\rho A} & 0 & 0 \\
\end{bmatrix} \tag{4.8}
\]
Accordingly, the disturbance variables, $T_H$ and $T_C$, and manipulated variables, $w_H$ and $w_C$, can be separated by using two transfer function

$$
\begin{bmatrix}
T(s) \\
H(s)
\end{bmatrix} = \begin{bmatrix}
\frac{T_H - T}{w \tau s + 1} & \frac{T_C - T}{w \tau s + 1} \\
\frac{1}{\rho A} & \frac{1}{\rho A}
\end{bmatrix}
\begin{bmatrix}
w_H(s) \\
w_C(s)
\end{bmatrix} + \begin{bmatrix}
w_H \frac{1}{\tau s + 1} & w_C \frac{1}{\tau s + 1} \\
0 & 0
\end{bmatrix}
\begin{bmatrix}
T_H(s) \\
T_C(s)
\end{bmatrix}
$$

(4.9)

The corresponding block diagram of the system is shown in the figure below:

![Block Diagram](image)

**Figure 3**: block diagram of the system.

### 5. MATLAB Simulation and Results

Firstly the program code was written in Matlab editor, then with the help of MPCtool, the model was exported to Simulink to generate the simulation of the system studied as shown in Figure 4.
After running the simulation we have obtained the following results:

Figure 4: MPCtool in Matlab: graph of the inputs and the outputs of the system.

Figure 5: Graph of the temperature response to the changes in the control (a) and predictive horizon (b).

From the graph, it’s noticed that:
- For the temperature case 1: changes of the control horizon: the corresponding graphs are shown in Figure 5 (a) and (b).
- The rise time is almost the same and is almost not affected by the MPC’s parameter changes (control horizon).
- Peak time is proportionally rising with the control horizon.
- Settle time is almost the same regardless control horizon.
- For the temperature case 2: changes of the predictive horizon: the corresponding graphs are shown in Figure 5 (a) and (b).
- the rising and peak time are slightly affected by the predictive horizon.
- For the level case 1: changes of the control horizon the corresponding graphs are shown in Figure 6 (a) and (b).

Figure 6: Graph of the temperature response to the changes in the control(a) and predictive horizon (b).

From the graph, it’s noticed that:
- The rise time is almost the same and is almost not affected by the MPC’s parameter changes (predictive and control horizon).
- Peak time is proportionally rising with the control horizon.
- Settle time is almost the same regardless control horizon.

- For the temperature case 2: changes of the predictive horizon the corresponding graphs are shown in Figure 6 (a) and (b):
- the rising and peak time are slightly affected by the control horizon.

From the observation and the analysis, the response of the system to the control and predictive horizon it was noticed that the best response occurs when: Control horizon \( N_c \leq \) Prediction horizon \( N_p \). This means with a short predictive horizon the MPC controller works more like a traditional feedback controller. And with a long control horizon results in a more aggressive change in the control action.

6. Conclusion
In this paper, a model predictive controller is designed for monitoring the liquid temperature and level comprising many tools of control. From the simulation results, it is noticed that MPC control is very suitable for handling nonlinear processes. the system identification of the system and the mathematical model was obtained.

The model predictive controller is designed for monitoring the liquid temperature and level comprising many tools of control. The real-time graph implies that the controller would able to track the
target continuously as well as rejects the disturbances and resettle over the reference point. The performance investigation of the systems including comparisons of the proposed controllers for the outputs (Temperature and level) analysis is done using Matlab/Simulink. From the simulation results, it is clear that the MPC control is very suitable for nonlinear processes with the MIMO system that are most often encountered in industry and nuclear processes.

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