Optimal control of plant disease model with roguing, replanting, curative, and preventive treatment

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Abstract. In this paper, we determine the optimal control of plant disease model with roguing, replanting, curative, and preventive treatment using the Maximum Pontryagin principle. Numerical simulation results show that the procedure can reduce the population of infected plants. Therefore, controlling by roguing, replanting, curative, and preventive treatment is highly recommended to increase the number of susceptible, removed, and protected plants.

1. Introduction
Crop cultivation is a promising business opportunity, but it is not uncommon for farmers to experience losses. This loss is caused by attacking pathogens, such as fungi, oomycetes (water fungi), bacteria, viruses, viroid (plant pathogens), and nematodes that cause plants to become infected (growth and development of these plants are not optimal) [1]. Researchers have carried out many studies about plant diseases, such as the analysis of epidemic models [2] and the spread of disease [3].

There are several ways commonly used to overcome the problem of disease in plants, one of which is roguing and replanting. Development of mathematical models about roguing and replanting has been carried out such as a structured population model for dynamics of SIR type epidemics by considering the total number of infections and the number of post-infectious infections in a tree [4], sensitivity analysis of two explicit spatial simulation models [5], and models of plant viral diseases with periodic and bloody rogue environments [6]. Several studies have explored mathematical models of roguing and replanting, such as creating analytic models of plant virus disease dynamics [7], determining the dynamics of infectious disease control strategies in plants in gardens [4], discrete models for plant diseases [8], plant disease models using the fractional method [9] and uses a system of ordinary differential equations to simulate the effects of roguing in controlling plant diseases [10].

In addition to roguing and replanting, controlling and repairing can also be done, by giving insecticides. The dynamical models of roguing plant diseases and replanting with curative, preventive, and curative and preventive treatment have been carried out by previous researchers [11-14].

Although providing preventive or curative insecticides can reduce protected plants, increase the number of plants removed and vulnerable plants. However, given excessive insecticide will increase costs. Therefore, it is necessary to find the optimal control of insecticides to use to reduce the plants promoted. Some researchers have researched optimal control, such as modelling to control fungicide applications [15], controlling plant diseases with cost-effectiveness [16], using pesticides to use biological control, and using technology [17], and optimizing plant disease control [17].

Also, mathematical models can provide useful recommendations for managing the spread of Huanglongbing [18], showing optimal control of preventive maintenance [19], as well as controlling the...
use of botanical fungicides to reduce infected host populations [20, 21]. The use of an optimal control model can be used to determine the application of *Verticillium lecanii* (*V. lecanii*) to reduce the intensity of the spread of the yellow virus [22]. Besides, optimal control can also help the success of eradicating the disease [23] and investigating the effects of using roguing and insecticides to maximize healthy plants that are harvested [24].

From previous research ideas [11-14], this study will discuss optimal control of the application of insecticides by considering costs. We assume that providing curative treatment in the form of insecticides on infected plants, preventive treatment in the form of insecticides on latent and infected plants, or curative and preventive treatment in the form of insecticides on susceptible plants, can increase susceptible and removed plants and reduce plants that are infected.

### 2. Optimal Control Model

Optimal control carried out in this study includes roguing and replanting plant disease models, by providing curative treatment (Model 1), preventive treatment (Model 2) [14], as well as curative and preventive treatment (Model 3) [12] using the principle of Maximum Pontryagin [25]. Where the Susceptible plants denote $S(t)$, $E(t)$ is the Latent compartment, the Infected compartment denotes $I(t)$, the Post Infectious/Removed compartment denote $R(t)$ and $P(t)$ is the Protected Plants compartment.

Dynamic model of plant diseases by roguing and replanting with the help of supervision is curative treatment by giving insecticides to infected plants to minimize the population of infected plants by considering the costs. Following are the objective functions of the optimal control model 1:

\[
J(u) = \int_{t_0}^{t_1} \left( A I(t) + C_c u_c^2(t) \right) dt
\]

where $A$ is the weight of the plant in the infected compartment, $C_c$ is the cost weight for $u_c$, while $u_c$ is the curative treatment control. Where is the status variable $x(t) = [S(t) \quad E(t) \quad I(t) \quad R(t)]^T$ and the constraints are:

\[
\frac{dS}{dt} = r(K - N) - \mu S - k_s \frac{I}{K}
\]

\[
\frac{dE}{dt} = k_s \frac{I}{K} - (\mu + k_2 + \alpha_1)E
\]

\[
\frac{dI}{dt} = k_2 E - (\mu + k_3 + \alpha_2 + \eta)I - pu_c I
\]

\[
\frac{dR}{dt} = k_3 I - (\mu + \alpha_3)R + pu_c I
\]

with boundary conditions

$t_0 < t < t_1$, $0 \leq u_c(t) \leq 1$, $S(0) = S_0 \geq 0, E(0) = E_0 \geq 0, I(0) = I_0 \geq 0, R(0) = R_0 \geq 0$, by using Pontryagin’s Maximum Principle we get $u_c = \frac{(\lambda_3 - \lambda_1)pI}{2c_c}$, so $u_c = \min \left\{ 0, \max \left( \frac{(\lambda_3 - \lambda_0)pI}{2c_c}, 1 \right) \right\}$.

Dynamic models of plant diseases by sweeping and replanting with preventive maintenance controls by applying insecticides on susceptible plants to minimize the population of infected plants by considering the costs. The following objective functions are optimal control of model 2:

\[
J(u) = \int_{t_0}^{t_1} \left( A I(t) + C_p u_p^2(t) \right) dt
\]

where $A$ is the weight of the plant in the infected compartment, $C_p$ is the cost weight for $u_p$, while $u_p$ is the preventive treatment control.

Where is the status variable $x(t) = [S(t) \quad P(t) \quad E(t) \quad I(t) \quad R(t)]^T$ and the constraints are:

\[
\frac{dS}{dt} = r(K - N) - \mu S - k_s \frac{I}{K} - \beta u_p S + \delta P
\]

\[
\frac{dP}{dt} = \beta u_p S - \delta P - \mu P
\]
\[
\begin{align*}
\frac{dE}{dt} &= k_1 S \frac{l}{K} - (\mu + k_2 + \alpha_1) E \\
\frac{dl}{dt} &= k_2 E - (\mu + k_3 + \alpha_2 + \eta) l \\
\frac{dR}{dt} &= k_3 l - (\mu + \alpha_3) R \\
\end{align*}
\]

with boundary conditions
\[t_0 < t < t_1, 0 \leq u_p(t) \leq 1, S(0) = S_0 \geq 0, P(0) = P_0 \geq 0, E(0) = E_0 \geq 0, I(0) = I_0 \geq 0, R(0) = R_0 \geq 0,\]

by using Pontryagin’s Principle we get \[u_p = \frac{(\lambda_1 - \lambda_2) \beta S}{2c_p} \] so \[u_p = \min \left\{ 0, \max \left( \frac{(\lambda_3 - \lambda_4) p l}{2c_p}, 1 \right) \right\}.\]

Dynamic models of plant diseases by roguing and replanting with curative and prevention treatments in the form of insecticides on susceptible and infected plants that aim to minimize the number of infected plant populations at the lowest cost. The following objective functions are optimal control of model 3:

\[ J(u) = \int_{t_0}^{t_1} \left( A I(t) + C_c u_c^2(t) + C_p u_p^2(t) \right) dt \]

where \( A \) is the weight of the number of plants in the infected compartment, \( C_c \) and \( C_p \), respectively the cost weights for \( u_c \) and \( u_p \). Whereas \( u_c \) is curative treatment control and \( u_p \) is preventive treatment control. Where is the status variable \( x(t) = \begin{bmatrix} S(t) \\ P(t) \\ E(t) \\ I(t) \\ R(t) \end{bmatrix} \) and the constraints are:

\[
\begin{align*}
\frac{dS}{dt} &= r(K - N) - \mu S - k_2 S \frac{l}{K} - \beta u_p S + \delta P \\
\frac{dP}{dt} &= \beta u_p S - \delta P - \mu P \\
\frac{dE}{dt} &= k_1 S \frac{l}{K} - (\mu + k_2 + \alpha_1) E \\
\frac{dl}{dt} &= k_2 E - (\mu + k_3 + \alpha_2 + \eta) l - pu_c l \\
\frac{dR}{dt} &= k_3 l - (\mu + \alpha_3) R + pu_c l \\
\end{align*}
\]

with boundary conditions
\[t_0 < t < t_1, 0 \leq u_c(t) \leq 1, 0 \leq u_p(t) \leq 1, S(0) = S_0 \geq 0, P(0) = P_0 \geq 0, E(0) = E_0 \geq 0, I(0) = I_0 \geq 0, R(0) = R_0 \geq 0, \]

by using Pontryagin’s Principle we get: \[u_c = \frac{(\lambda_1 - \lambda_2) \beta S}{2c_c} \] and \[u_p = \min \left\{ 0, \max \left( \frac{(\lambda_3 - \lambda_4) p l}{2c_p}, 1 \right) \right\}).\]

3. Numerical Simulation

To illustrate the dynamics of infected plant populations, we provide numerical examples with control and without control with parameters and initial values as in Table 1 [14]. Figure 1 shows that with the control that is curative treatment control, with weight values \( A = 1 \) and \( C_c = 1 \), the number of infected plants decreases more than those without control \( (u_c = 0) \). Changes in weight values will affect the optimal control of the model.
Figure 1. Differences with and without control of infected plants in model 1

Figure 2 shows that with controls that are preventive maintenance controls, with weight values $A = 1$ and $C_p = 1$ and the number of plants infected with or without control ($u_p = 0$) does not change. Thus, the control effect does not exist.

Figure 2. Difference with and without control of infected plants in model 2

Figure 3 shows that with preventive maintenance control and preventive maintenance control, with weight values $A = 1$, $C_c = 1$, and $C_p = 1$, the number of infected plants decreased more than those without control (values $u_c = 0$ and $u_p = 0$). However, the comparison is not very significant.
Figure 3. Difference with and without control of infected plants in model 3

Previous research [12, 14] discuss the dynamical system model and analyze the effect of the treatment on plant disease transmission dynamics. From [12, 14] we developed optimal control model. The result shows that roguing and replanting and controlling efforts, namely providing curative and preventive treatment in the form of insecticides on susceptible plants and infected plants, is better done to control the spread of plant diseases (Figure 1-3).

4. Conclusion
In this paper, we develop optimal control models of plant diseases through roguing and replanting by providing curative treatment (model 1), preventive treatment (model 2), and preventive and curative treatment (model 3) using Pontryagin’s Maximum Principle. The purpose of control is to minimize the number of infected plant populations by making effective treatments. Optimal control simulation results show that the number of plants infected with control is lower than those without control. As a result, the number of susceptible plants removed and protected increases if given control in the form of insecticides. For model 2, the graph shows that the number of infected plants, both with and without control, shows the same results. It means that infected plants do not change for preventive treatment. So that effective treatment is to provide curative treatment or both treatments are carried out.

5. Acknowledgments
The work was supported by Kementrian Riset dan Teknologi/Badan Riset dan Inovasi Nasional 2020, with contract number 1827/UN6.3.1/LT/2020 through Penelitian Dasar Unggulan Perguruan Tinggi.

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