Is the Energy Density of the Cosmic Quaternionic Field a Possible Candidate for the Black Energy?

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Abstract
We try to show that the energy density of the cosmic quaternionic field might be a possible candidate for the black energy.

1 Introduction

According to the astronomical observation, the evidence continues to mount that the expansion of the universe is accelerating rather than slowing down. New observation suggests a universe that is light-weight, is accelerating, and is flat [15] [12] [13]. To induce cosmic acceleration it is necessary to consider some components, whose equations of state are different from baryons, neutrinos, dark matter, or radiation considered in the standard cosmology.

As is well-known, in cosmology a new kind of energy is considered called quintessence ("dark energy"). Quintessence represents a dynamical form of energy with negative pressure [6]. The quintessence is supposed to obey an equation of state of the form

\[ p_Q c^{-2} = w_Q \rho_Q, \quad -1 < w_Q < 0. \]  

(1)

For the vacuum energy (static cosmological constant), it holds \( w_Q = -1 \) and \( \dot{w}_Q = 0 \).

An adequate theory of a cosmological scenario, conform with recent observation, should give answers to the following problems:

(i) The cosmological constant problem. The ‘Λ-problem’ can be expressed as discrepancies between the negligible value of Λ for the present universe and the value \( 10^{50} \) times larger.
expected by Glashow-Salam-Weinberg model [1] or by GUT [2] where it should be \(10^{107}\) times larger.

**The fine-tuning problem.** Assuming that the vacuum energy density is constant over time and the matter density decreases as the universe expands it appears that their ratio must be set to immense small value (\(\approx 10^{-120}\)) in the early universe in order for the two densities to nearly coincide today, some billions years later.

(iii) **The age problem.** This problem expresses the discrepancy connected, on the one side, with the height estimates of the Hubble parameter and with the age of globular clusters on the other side. The fact that the age of the universe is smaller than the age of globular clusters is unacceptable.

(iv) **The flatness problem.** Inflation predicts a spatially flat universe. According to Einstein’s theory, the mean energy density determines the spatial curvature of the universe. For a flat universe, it must be equal to the critical energy. The observed energy density is about one-third of critical density. The discrepancy between the value of the observed energy density and the critical energy is called the flatness problem.

(v) **Problem of the particle creation.** In variable \(\lambda\) models the creation of particles generally takes place. The question what is the mechanism for this process represents the problem of the particle creation.

It is well-known that the Einstein field equations with a non-zero \(\lambda\) can be rearranged so that their right-hand sides consist of two terms: the stress-energy tensor of the ordinary matter and an additional tensor

\[
T^{(\nu)}_{ij} = \left(\frac{c^4 \lambda}{8\pi G}\right) g_{ij} = \Lambda g_{ij}.
\]

\(\Lambda\) is identified with vacuum energy because this quantity satisfies the requirements asked from \(\Lambda\), i.e. (i) it should have the dimension of energy density, and (ii) it should be invariant under Lorentz transformation. The second property is not satisfied for arbitrary systems, e.g. material systems and radiation. Gliner [4] has shown that the energy density of vacuum represents a scalar function of the four-dimensional space-time coordinates so that it satisfies both above requirements. This is why \(\Lambda\) is identified with the vacuum energy.

From what has been said so far it follows that the following properties are required from the vacuum energy density: (i) It should be intrinsically relativistic quantity having the dimension of the energy density. (ii) It should be smoothly distributed throughout the universe. (iii) It should cause the speedup of the universe. (iv) It should balances the total mean energy density to \(\Omega = 1\).

In the next Sections we will describe a model of the universe consisting of a mixture of the ordinary matter and a so-called cosmic quaternionic field [11] whose energy density we set equal to the cosmological constant. We show that the value of the energy density of
cosmic quaternionic field is consistent with the data. Then, we describe the force exerting on the moving bodies in the cosmic quaternionic field and the possible mechanism of the particle creation in this field. Finally, we sketch the evolution of the universe with the cosmic quaternionic field.

2 The cosmic quaternionic field

In a very recent article [11], Λ has been interpreted as the field energy of a classical quaternionic field (called Φ-field, for short) [5] [10] [9] which is given by the field tensor $F_{ij} \quad i, j = 1, 2, 3, 0$ whose components are defined as $F_{ij} = 0$ for $i \neq j$ and $F_{11} = F_{22} = F_{33} = -F_{00} = Φ$. The Φ-field belongs to the family of the quaternionic fields (see [5]). The quaternionic field which we consider is given by the field tensor which, in the matrix, has the form

\[
F_{ij} = \begin{pmatrix}
Φ & 0 & 0 & 0 \\
0 & Φ & 0 & 0 \\
0 & 0 & Φ & 0 \\
0 & 0 & 0 & -Φ
\end{pmatrix}.
\]

Φ is the only field variable in it. $F_{ij}$ is a symmetric field tensor with the components $F_{ii} = Φ \quad i = 1, 2, 3, \quad F_{ii} = -Φ \quad i = 0$, and $F_{ij} = 0 \quad i \neq j$. It is easily to show that Φ is transformed as a scalar under Lorentz transformation [11]. The field equations of the Φ-field in the differential are

\[
\nabla Φ = kJ_i \quad i = 1, 2, 3 \quad \text{and} \quad -\frac{1}{c} \frac{dΦ}{dt} = k_0 J_0, \quad i = 0,
\]

(3)

where

\[
k = \frac{\sqrt{G}}{4πc} \quad \text{and} \quad k_0 = 8π\sqrt{G}.
\]

These equations are first-order differential equations whose solution can be found given the source terms. Assuming the spacial homogeneity of the Φ-field it becomes independent of spatial coordinates therefore it holds $J_1 = J_2 = J_3 = 0$. The source of the Φ-field is its own mass density associated with the field energy density, i.e. $Φ^2/8πc^2$, therefore, it holds $J_0 = Φ^2/8πc^2$. $J_0$ is dependent only on time. The energy density associated with the field is [5]

\[
E_Φ = \frac{Φ^2}{8π}.
\]

Since the current 4-vector in the everywhere local rest frame has only one non-zero component, $J_0$, Eqs.(3) become

\[
\nabla Φ = 0
\]

\[
-\frac{1}{c} \frac{dΦ}{dt} = \frac{8π\sqrt{G}Φ^2}{8πc^2} = \frac{\sqrt{G}Φ^2}{c^2}
\]
whose solution is

$$\Phi(t) = \frac{c}{\sqrt{G(t + t_0)}},$$

where $t_0$ is the integration constant given by the boundary condition. The energy density of the $\Phi$-field $E_\Phi$ is approximately equal to the observed value of the cosmological constant.

3 The black energy modeled by a time-dependent cosmological constant

Next, we set $\Lambda$ equal to the field energy density of the cosmic quaternionic field ($t_0 = c = 1$)

$$\Lambda = \frac{1}{8\pi} \left[ \frac{1}{\sqrt{Gt}} \frac{1}{\sqrt{Gt}} \right] = \frac{\Phi^2}{8\pi} \quad \Phi = \frac{1}{\sqrt{Gt}},$$

Accordingly, we have

$$\Lambda = \frac{\Phi^2}{8\pi} = \frac{1}{8\pi G t^2} \quad \text{and} \quad \lambda = \frac{1}{t^2}. \quad (5)$$

The gravitational field equations with a cosmological constant $\lambda$ and the energy conservation law are $(k=0)$

$$H^2 = \frac{8\pi G}{3} (\rho + \Lambda) \quad H = \frac{\dot{R}}{R} \quad \Lambda = \frac{\lambda}{8\pi G} \quad \dot{\Lambda} = \frac{4\pi G}{3} (\rho + 3p + 2\Lambda) \quad (6)$$

and

$$\dot{\rho} + 3 \frac{\dot{R}}{R} (p + \rho) = -\dot{\Lambda}. \quad (7)$$

Suppose we have a perfect-gas equation of state

$$p = \alpha \rho \quad (9)$$

and suppose that the deceleration parameter is constant. If the evolution of the scale factor is given in form $R \propto t^n$ then $q = -(n - 1)/n$, therefore, we set

$$q = -\frac{\ddot{R}R}{\dot{R}^2} = \frac{1}{n} - 1. \quad (10)$$

If we suppose the time dependence $\rho$ and $\Lambda$ in form

$$\rho = \frac{A}{t^2} \quad \text{and} \quad \Lambda = \frac{B}{t^2} \quad B = \text{const.} \quad \text{and} \quad A = \text{const.}, \quad (11)$$
respectively, then, inserting (11) into (6), (7) and (8), gives the following relation between $A$ and $B$ [7]

$$2B = A[(-2 + 3n)(1 + \alpha)].$$

(12)

Given $A$ or $B$ and $n$ we can uniquely determine $B$ or $A$, respectively. For $\lambda \propto 1/t^2$, there is a relation between $\Omega_M$ and the time-dependence of scaling factor $R(t)$. Assuming that $\Omega_M$ does not change during the matter-dominated era ($\alpha = 0$) [8]

$$R(t) = \left(\frac{3}{2}\right)^{\frac{2}{3\Omega_M}} (\Omega_M C_1 t)^{\frac{2}{3\Omega_M}}.$$

(13)

The quantities $q$, $R(t)$ and $\Omega_M$ are mutually related. Given one of them the remained quantities can be determined by means of Eqs. (13), (10) and (11). It seem that $\Omega_M$ is best determined by the observation, therefore, we take it for the calculation of $q$ and $R(t)$. Inserting $\Omega_M = 1/3$ into Eq.(13) we obtain $R \propto t^2$ which yields $q = -1/2 = 1/n - 1$, $n = 2$. We see from Eq.(5) that $B = 1$ which inserting into Eq.(12) gives $A = (1/2)$. The mean energy density $\rho$ and the cosmological constant is given as ($\alpha = 0$)

$$\rho = \frac{1}{16\pi t^2} \quad \text{and} \quad \lambda = \frac{1}{8\pi G t^2},$$

(14)

respectively. Their ratio

$$\frac{\rho}{\Lambda} = 1/2.$$

Supposing the flat space, we have

$$\Omega_M = \frac{1}{3} \quad \text{and} \quad \Omega_\Lambda = \frac{2}{3}.$$

There is no "fine tuning" problem in our model since the ratio of the $\lambda$-part energy density to the mass-energy density of the ordinary matter remains during the cosmic evolution constant.

4 The force exerting on the moving bodies in the cosmic quaternionic field

In analogy with the electromagnetic field, the quaternionic field acts on the moving "charged" objects with the Lorenz-like force. In [11] we supposed that the $\Phi$-field interacts with all form of energy and matter and the coupling constant $k$ is from the dimensional reason equal to $\sqrt{G}$. The "charge" of the $\Phi$-field for a point mass $m_0$ is $\sqrt{G} m_0$. Since the momentum of a moving particle is $p_i = m_0 v_i$, $i = 1, 2, 3, 0$, its current is given as $J_i = \sqrt{G} m_0 v_i = m_0 \sqrt{G} v_i$. For the Lorentz-like force acting on this particle in the $\Phi$-field we get [17]

$$F_i = c^{-1} \sqrt{G} m_0 \Phi v_i = c^{-1} \sqrt{G} \Phi p_i.$$

(14)
Now, for the sake of simplicity, we confine ourselves to the non-relativistic case, i.e. we suppose that \( m = \text{const.} \) and \( v \ll c \). Then Eq.(14) turns out to be
\[
m\dot{v} = c^{-1}\sqrt{G}\Phi mv. \tag{15}
\]
The quaternionic field affects the following kinetic quantities of the moving bodies:

(i) The velocity of the moving bodies in the presence of the \( \Phi \)-field increases. Since \( c^{-1}\sqrt{G}\Phi = 1/t \) we get the following simple equation of equation \( \dot{v} = \beta v \), where \( \beta = 1/t \), the solution of which is \( v = Ct \). A free moving object in the quaternionic field is accelerated by a constant acceleration \( C \). This acceleration is due to the immense smallness of \( \beta \approx 1/10^{18} \) in the present-day extremely small.

(ii) The kinetic energy of the moving bodies in \( \Phi \)-field increases, too. The gain of kinetic energy of a moving body per time unit in the quaternionic field if \( (F_i \parallel v_i) \) is
\[
\frac{dE}{dt} = F_i v_i = c^{-1}\sqrt{G}\beta mv^2 = 2\sqrt{G}c^{-1}\Phi E_{\text{kin}} = 2\beta E_{\text{kin}}.
\]
Again, the increase of the kinetic energy of a moving object is extremely small. However, for a rapid rotating dense body it may represent a considerable value [17].

It is noteworthy that the force exerting on cosmical body in cosmic quaternionic field is always parallel to the direction of the velocity. This means that velocity of moving bodies in cosmic \( \Phi \)-field is in all direction increasing. For example, the moving bodies stemming from a exploding cosmic body is equally speedup as those falling in the collapsing center.

5 The possible mechanism for the particle creation in the cosmic quaternionic field

According to quantum theory, the vacuum contains many virtual particle-anti-particle pairs whose lifetime \( \Delta t \) is bounded by the uncertainty relation \( \Delta E \Delta t > h \). The proposed mechanism for the particle creation in the \( \Phi \)-field is based on the force relation (14). During the lifetime of the virtual particles the Lorentz-like force (14) acts on them and so they gain energy. To estimate this energy we use simple heuristic arguments. As is well-known, any virtual particle can only exist within limited lifetime and its kinetics is bounded to the uncertainty relation \( \Delta p \Delta x > h \). Therefore, the momentum of a virtual particle \( p \) is approximately given as \( p \approx h \Delta x^{-1} \). If we insert this momentum into Eq.(14) and multiply it by \( \Delta x \), then the energy of virtual particle \( \Delta E \), gained from the ambient \( \Phi \)-field during its lifetime, is
\[
F \Delta x = \Delta E = \sqrt{G}\Phi(t)\frac{h}{c}. \tag{15}
\]
When the $\Phi$-field is sufficiently strong then it can supply enough energy to the virtual particles during their lifetime and so spontaneously create real particles from the virtual pairs. The energy necessary for a particle to be created is equal to $m_v c^2$ ($m_v$ is the rest mass of the real particle). At least, this energy must be supplied from the ambient $\Phi$-field to a virtual particle during its lifetime. Inserting $\Phi$ into Eq. (15), we have

$$\Delta E \approx \frac{\hbar}{(t + t_0)},$$

Two cases may occur: (i) If $m_v c^2 < \Delta E$, then the energy supplied from the $\Phi$-field is sufficient for creating real particles of mass $m_v$ and, eventually, gives them an additional kinetic energy. (ii) If $m_v c^2 > \Delta E$, then the supplied energy is not sufficient for creating the real particles of mass $m_v$ but only the energy excitations in vacuum.

We see that there is a plausible mechanism of particle production in the cosmic $\Phi$-field. This production is very intensive in the early universe but practically negligible in the present time.

6 The energy density of the cosmic $\Phi$-field as a possible candidate for the black energy

As has been shown in the previous Chapters, when taking the field energy density of cosmic quaternionic field as the vacuum energy density, the problems presented in the Introduction may be resolved. The problem of the cosmological constant, because the value of $\Lambda$ is consistent with data, the tuning and age problems because because the ratio of mass to vacuum energy density does not vary during the cosmical evolution and the age of the universe is large enough to evolve the globular cluster. The flatness problem is also solved because the sum of the black energy to the ordinary matter yields just the critical energy density. Moreover, in the cosmic quaternionic field, there exists a plausible mechanism of the particle creation. We conclude that the energy density of the $\Phi$-field represent a relativistic quantity satisfying Gliner’s requirements which is smoothly distributed in space. It causes the speedup of the universe, balances the total energy density to the critical one and gives a plausible mechanism for particle creation.

It is generally believed that dark energy was less important in the past and will become more important in the future. In our model the value of black energy is proportional to $1/t^2$, therefore, its value becomes very large in past and will be adequate large in the future. We remember that the force exerted by the cosmic $\Phi$-field on moving bodies acts always in the direction of the velocity. That means that the high value of the black energy in the early universe does not interfere with the structure forming, contrarily, it accelerates it.
The evolution of the universe with the cosmic quaternionic field can be briefly sketched as follows: The cosmic evolution started with purely field-dominated with the inflation, after which a massive creation of particles began together with enormous release of entropy. During the time interval $(\approx 0, 10^{-20})$ the masses of the created particles lie in the range from $10^{-5}$ to $10^{-27}$ g. Their kinetic energy was $E_{\text{kin}} = [h/(t + t_0)] - m_0 c^2$. $E_{\text{kin}}$ of the created nucleons has reached values up to $10^{-5}$ erg, which corresponds to the temperature of $10^{21}$ K. Today, energies of the virtual pairs, gained during their lifetime, are immense small, therefore, it represents only a certain local energy excitations of the vacuum. The large vacuum energy density of the cosmic quaternionic field at the early stage of the universe accelerates its structure forming. From what has been said above we conclude that the energy density of the cosmic quaternionic field might be a possible candidate for the black energy because (i) it has the value consistent with data (ii) it does not suffer from the cosmological constant, fine-tuning, age and flatness problems (iii) it yields a plausible mechanism particle production and (iv) it accelerated the structure forming in the early universe.

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