Latest Observational Constraints on Cardassian Models

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Abstract

Constraints on the original Cardassian model and the modified polytropic Cardassian model are examined from the latest derived 397 Type Ia supernova (SNe Ia) data, the size of baryonic acoustic oscillation peak from the Sloan Digital Sky Survey (SDSS), the position of first acoustic peak of the Cosmic Microwave Background radiation (CMB) from the five years Wilkinson Microwave Anisotropy Probe (WMAP), the x-ray gas mass fractions in clusters of galaxies, and the observational H(z) data. In the original Cardassian model with these combined data set, we find $\Omega_m = 0.271^{+0.014}_{-0.014}, n = 0.035^{+0.049}_{-0.049}$ at 1$\sigma$ confidence level. And in the modified polytropic Cardassian model, we find that $\Omega_m = 0.271^{+0.014}_{-0.015}, n = -0.091^{+0.331}_{-1.908}$ and $\beta = 0.824^{+0.750}_{-0.622}$ within 1$\sigma$ confidence level. According to these observations, the acceleration of the universe begins at $z_T = 0.55^{+0.05}_{-0.05}(1\sigma)$ for the original Cardassian model, and at $z_T = 0.58^{+0.12}_{-0.12}(1\sigma)$ for the modified polytropic Cardassian model. Evolution of the effective equation of state $w_{eff}$ for the modified polytropic Cardassian model is also examined here and results show that an evolutionary quintessence dark energy model is favored.

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I. INTRODUCTION

The astrophysical observations of recent years, including Type Ia supernovae (SNe Ia; [1, 2, 3, 4, 5, 6, 7]), the large scale structure [8], and the cosmic microwave background radiation (CMB; [9, 10, 11, 12, 13, 14]) et al, show that the present expansion of our universe is accelerating. In order to explain this observed accelerating expansion, a large number of cosmological models have been proposed by cosmologists. There are two main categories of proposals. The first ones (dark energy models) are proposed by assuming the existence of an energy component with negative pressure in the universe, this dark energy dominates the total energy density of the universe and drives its acceleration of expansion at late times. Currently there are many candidates for dark energy, such as the cosmological constant with equation of state $\omega_{DE} = p_{DE}/\rho_{DE} = -1$ where $p_{DE}$ and $\rho_{DE}$ are pressure and density of the dark energy, respectively [15], the quiessence whose equation of state $\omega_Q$ is a constant between $-1$ and $-1/3$ [16], and the quintessence which is described in terms of a scalar field $\phi$ [17, 18]. The other proposals suggest that general relativity fails in the present cosmic scale, and the extra geometric effect is responsible for the acceleration, such as the braneworld models which explain the acceleration through the fact that the general relativity is formulated in 5 dimensions instead of the usual 4 [19], and the Cardassian models which investigate the acceleration of the universe by a modification to the Friedmann-Robertson-Walker (FRW) equation [20].

In this work we focus on the Cardassian models, including the original Cardassian model and the modified polytropic Cardassian model. The original Cardassian model is based on the modified Friedmann equation and has two parameters $\Omega_{m0}$ and $n$. The modified polytropic Cardassian model can be obtained by introducing an additional parameter $\beta$ into the original Cardassian model which reduces to the original model if $\beta = 1$.

As we know, many observational constraints have been placed on Cardassian models, including those from the angular size of high-z compact radio sources [21], the SNe Ia [22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32], the shift parameter of the CMB [23, 28, 32, 33], the baryon acoustic peak from the SDSS [23, 33], the gravitational lensing [34], the x-ray gas mass fraction of clusters [29, 35], the large scale structure [32, 36, 37], and the Hubble parameter versus redshift data [33, 38].

The main purpose of this work is to give out constraints on Cardassian models with
the latest observational data, including the recently compiled 397 SNe Ia data set \cite{39}, the size of baryonic acoustic oscillation peak from the Sloan Digital Sky Survey (SDSS) \cite{8}, the position of first acoustic peak of the Cosmic Microwave Background radiation (CMB) from the five years Wilkinson Microwave Anisotropy Probe (WMAP) \cite{14}, the x-ray gas mass fraction of clusters \cite{40}, and the Hubble parameter versus red shift data \cite{41}. As a result, we find that the stronger constraints can be given out with this combined data set than the former results.

This paper is organized as follows: In section 2, we give out the basic equations of Cardassian models. In section 3, we describe the analysis method for the observational data. In section 4, we present the results with different data sets and some discussions for results.

II. THE BASIC EQUATIONS OF CARDASSIAN MODELS

In 2002, Freese and Lewis \cite{20} proposed Cardassian model as a possible explanation for the acceleration by modifying the FRW equation without introducing the dark energy. The basic FRW equation can be written as

\[ H^2 = \frac{8\pi G}{3} \rho, \]

where \( G \) is the Newton gravitation constant and \( \rho \) is the density of summation of both matter and vacuum energy. For the Cardassian model, which is modified by adding a term on the right side of Eq.(1), the FRW equation is shown as below

\[ H^2 = \frac{8\pi G}{3} \rho_m + B \rho_m^n. \]

The latter term is so called Cardassian term may show that our observable universe as a 3 + 1 dimensional brane is embedded in extra dimensions. Here \( n \) is assumed to satisfy \( n < 2/3 \), and \( \rho_m \) only represents the matter term without considering the radiation for simplification. The first term in Eq.(2) dominates initially, so the equation becomes to the usual Friedmann equation in the early history of the universe. At a red shift \( z \sim O(1) \) \cite{20}, the two terms on the right side of the equation become equal, and thereafter the second term begins dominate, and drives the universe to accelerate. If \( B = 0 \), it becomes the usual FRW equation, but with only the density of matter. If \( n = 0 \), it is the same as the cosmological constant model. By using
\[ \rho_m = \rho_{m0}(1 + z)^3 = \Omega_{m0}\rho_c(1 + z)^3, \]

we obtain

\[ E^2 = \frac{H^2}{H_0^2} = \Omega_{m0}(1 + z)^3 + (1 - \Omega_{m0})(1 + z)^{3n}, \]

where \( z \) is the red shift, \( \rho_{m0} \) is the present value of \( \rho_m \) and \( \rho_c = 3H_0^2/8\pi G \) represents the present critical density of the universe. Obviously, this model predicts the same distance-red shift relation as the quiescence with \( \omega_Q = n - 1 \), but with totally different intrinsic nature.

The luminosity distance of this model is

\[ d_L = cH_0^{-1}(1 + z) \int_0^z dz\left[\Omega_{m0}(1 + z)^3 + (1 - \Omega_{m0})(1 + z)^{3n}\right]^{-1/2}, \]

where \( c \) is the velocity of light.

The modified polytropic Cardassian universe is obtained by introducing an additional parameter \( \beta \) into the original Cardassian model, which reduces to the original model if \( \beta = 1 \),

\[ H^2 = H_0^2[\Omega_{m0}(1 + z)^3 + (1 - \Omega_{m0})f_X(z)], \]

where

\[ f_X(z) = \frac{\Omega_{m0}}{1 - \Omega_{m0}}(1 + z)^3(1 + \frac{\Omega_{m0}^\beta - 1}{(1 + z)^{(1-n)\beta}})^{1/\beta} - 1. \]

Here if the \( f_X(z) \) is equal to 1 at the same time, this model just corresponds to \( \Lambda \)CDM. The corresponding luminosity distance of Eq. (6) is

\[ d_L = cH_0^{-1}(1 + z) \int_0^z dz\left[\Omega_{m0}(1 + z)^3[1 + \frac{\Omega_{m0}^\beta - 1}{(1 + z)^{(1-n)\beta}}]^{1/\beta}\right]^{-1/2}. \]

III. DATA ANALYSIS

For the SNe Ia data, we use the recently combined 397 data points \[39\], which consist with the 307 Union data set \[7\] and 90 CFA data set. The Union set includes the Supernova Legacy Survey \[42\] and the ESSENCE Survey \[23, 43, 44\], the former observed SNe Ia
data, and the extended dataset of distant SNe Ia observed with the Hubble space telescope. Constraints from these SNe Ia data can be obtained by fitting the distance modulus $\mu(z)$

$$\mu(z) = 5 \log_{10} d_L + M. \quad (9)$$

Here $M$ being the absolute magnitude of the object.

In 2005, Eisenstein et al. [8] successfully found the size of baryonic acoustic oscillation peak by using a large spectroscopic sample of luminous red galaxy from the SDSS and obtained a parameter $A$, which is independent of dark energy models and for a flat universe can be expressed as

$$A = \frac{\sqrt{\Omega_{m0}}}{E(z_1)^{1/3}} \frac{1}{z_1} \int_0^{z_1} \frac{dz}{E(z)} \frac{dz}{E(z)}^{2/3}, \quad (10)$$

where $z_1 = 0.35$ and the corresponding $A$ is measured to be $A = 0.469(0.96/0.98)^{0.35} \pm 0.017$. Using parameter $A$ we can obtain the constraint on Cardassian models from the SDSS.

The shift parameter $R$ of the CMB data can be used to constrain the Cardassian models and it can be expressed as

$$R = \sqrt{\Omega_{m0}} \int_0^{z_r} \frac{dz}{E(z)}. \quad (11)$$

Here $z_r = 1089$ for a flat universe. From the five years WMAP result [13], the shift parameter is constrained to be $R = 1.715 \pm 0.021$ [14].

On the assumption that the baryon gas mass fraction in clusters is constant which is independent of the red shift, and is related to the $\Omega_b/\Omega_{m0}$, the baryon gas mass fraction can be used to constrain cosmological parameters. Here we adopt the usually used 26 cluster data [40] to constrain the Cardassian models. The baryon gas mass fraction can be present as

$$f_{SCDM}^{gas}(z) = \frac{b\Omega_b}{(1 + 0.19\sqrt{h})\Omega_m} \left[ \frac{d_{SCDM}(z)}{d_{mod}(z)} \right]^{1.5}, \quad (12)$$

where $b$ is a bias factor motivated by gas dynamical simulations.

Simon, Verde & Jimenez has obtained the Hubble parameter $H(z)$ at nine different red shifts from the differential ages of passively evolving galaxies [41]. The form of $H(z)$ is

$$H(z) = -\frac{1}{1 + z} \frac{dz}{dt}. \quad (13)$$

We can determine the value of $H(z)$, if $dz/dt$ is known. Recently, the authors in [46] obtained $H(z = 0.24) = 83.2 \pm 2.1$ and $H(z = 0.43) = 90.3 \pm 2.5$. We also add the prior
$H_0 = 72 \pm 8 \text{ km/s/Mpc}$ given by Freedman et al. [47]. So now we have 11 Hubble parameter to constrain the Cardassian models.

In order to place limits on model parameters with the observation data, we make use of the maximum likelihood method, that is, the best fit values for these parameters can be determined by minimizing

$$
\chi^2 = \sum_{i=1}^{397} \frac{[\mu(z_i) - \mu_{\text{obs}}(z_i)]^2}{\sigma_i^2} + \frac{(A - 0.0469)^2}{0.017^2} + \frac{(R - 1.715)^2}{0.021^2} + \sum_{j=1}^{26} \frac{[f_{\text{SCDM}}(z_j) - f_{\text{gas},j}]^2}{\sigma_{f_{\text{gas}},j}^2} + \sum_{k=1}^{11} \frac{[H(z_k) - H_{\text{obs}}(z_k))^2}{\sigma_{H_i}^2},
$$

where the $\mu_{\text{obs}}, \sigma_i$ represent the corresponding observational values for the SNe Ia, the $f_{\text{gas}}, \sigma_{f_{\text{gas}}}$ represent the corresponding observational values for the gas mass fraction, and $H_{\text{obs}}(z), \sigma_H$ represent the corresponding observational values for the Hubble parameter.

IV. RESULTS AND DISCUSSIONS

The latest observational data set, which is 397 SNe Ia + CMB + BAO + 26 gas mass fraction + 11 Hubble parameter, is used here to constrain parameters of the original Cardassian model. By minimizing the corresponding total $\chi^2$ in Eq. (12), we find at $1\sigma$ confidence level $\Omega_{m0} = 0.271^{+0.014}_{-0.010}$ and $n = 0.035^{+0.049}_{-0.040}$, which is shown in Fig. 1 and is consistent with the $\Lambda$CDM cosmology ($40$, $\Omega_{m0} = 0.25^{+0.04}_{-0.01}$). We find that combining these observational data can tighten the constraints significantly comparing to the results from former academic papers [22, 33, 48, 49]. Our result gives out an even much stronger constraint than other observational results, such as the results from Cao 2003 with 37 SNe Ia data [25], Sen & Sen 2003 with WMAP data set [50], Frith 2004 with about 200 SNe Ia data set [28], Godlowski, Szydlowski & Krawiec 2004 with several different data groups [27], Szydlowski and Czaja 2004 with SNe Ia data [26], Davis et al 2007 with 200 SNe Ia + BAO + CMB data set [23], Zhu, Fujimoto & He 2004 [29] with the dimensionless coordinate distance data of SNe Ia + FRIIb radio galaxies + the X-ray mass fraction data of clusters, Bento et al 2005 [30] with SNe Ia golden sample, Bento et al 2006 [31] with 157 SNe Ia + BAO + CMB data set.

For the modified polytropic Cardassian model, we find at $1\sigma$ confidence level $\Omega_{m0} = 0.271^{+0.014}_{-0.015}$, $n = -0.091^{+0.331}_{-1.908}$ and $\beta = 0.824^{+0.750}_{-0.620}$. Details for constraints are shown in Figs. 2-4, which is tighter than that obtained in [38]. The modified polytropic Cardassian model
reduces to the flat ΛCDM when $\beta = 1, n = 0$. So the flat ΛCDM cosmology is consistent with observations.

With these data using in this paper, we can determine when the universe acceleration began in Cardassian models by investigating the deceleration parameter $q(z)$. As shown in Fig. 5, we give out the evolution of $q(z)$ in the original Cardassian expansion model, and find the transition from deceleration to acceleration occurs at red shift $z_T = 0.55 \pm 0.05$ in $1\sigma$ confidence level, which is later than the result ($z_T = 0.70 \pm 0.05$) obtained in [38], but is consistent with the result by using the gold sample data in [2] ($z_T = 0.46 \pm 0.13$). Fig. 6 shows the evolution of $q(z)$ in the modified polytropic Cardassian model, and we obtain the phase transition red shift is $z_T = 0.58^{+0.12}_{-0.12}$ at $1\sigma$ confidence level, which is consistent with $z_T = 0.58^{+0.17}_{-0.18}$ in [38]. But our result gives out a stronger constraint. With this latest data set, we obtain very tight $1\sigma$ error regions of the phase transition red shift for both of the Cardarsssian models, and both of the Cardassian models’ results support that the universe began to accelerate at red shift $\sim 0.5 - 0.6$.

In addition we give the evolution of the effective equation of state for the modified polytropic Cardassian model. The results are shown in Fig. 7 and from the best fit line we find the observations favor an evolutionary quintessence dark energy model without an crossing of $-1$ line. We also obtain that the $ΛCDM$ model is consistent with the observations and the phantom model can not be ruled out at $1\sigma$ confidence level.

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FIG. 1: Constraints on $\Omega_{m0}$ and $n$ from 1$\sigma$ to 3$\sigma$ are obtained from 397 SNe Ia + CMB + BAO + 26 gas mass fraction + 11 Hubble parameter data set for original Cardassian model.

FIG. 2: Constraints on parameters $n$ and $\beta$ of the modified polytropic Cardassian model by setting the best fit value over $\Omega_{m0}$ from 1$\sigma$ to 3$\sigma$ are obtained from 397 SNe Ia + CMB + BAO + 26 gas mass fraction + 11 Hubble parameter data set.
FIG. 3: Constraints on parameters $\Omega_{m0}$ and $\beta$ of the modified polytropic Cardassian model by setting the best fit value over $n$ from 1$\sigma$ to 3$\sigma$ are obtained from 397 SNe Ia + CMB + BAO + 26 gas mass fraction + 11 Hubble parameter data set.

FIG. 4: Constraints on parameters $\Omega_{m0}$ and $n$ of the modified polytropic Cardassian model by setting the best fit value over $\beta$ from 1$\sigma$ to 3$\sigma$ are obtained from 397 SNe Ia + CMB + BAO + 26 gas mass fraction + 11 Hubble parameter data set.
FIG. 5: The evolution of the deceleration parameter $q(z)$ for the original Cardassian expansion model. The thick solid line is drawn with the best fit parameters. The shaded region shows the 1$\sigma$ errors.

FIG. 6: The evolution of the deceleration parameter $q(z)$ for the modified polytropic Cardassian expansion model. The thick solid line is drawn with the best fit parameters. The shaded region shows the 1$\sigma$ errors.
FIG. 7: The evolution of the effective equation of state for the modified polytropic Cardassian expansion model. The thick solid line is drawn with the best fit parameters. The shaded region shows the 1σ errors.

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