Comprehensive investigation of the symmetric space-star configuration in the nucleon-deuteron breakup

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Abstract

We examine a description of available cross section data for symmetric space star (SST) configurations in the neutron-deuteron (nd) and proton-deuteron (pd) breakup reaction using numerically exact solutions of the three-nucleon (3N) Faddeev equation based on two- and three-nucleon (semi)phenomenological and chiral forces. The predicted SST cross sections are very stable with respect to the underlying dynamics for incoming nucleon laboratory energies below $\approx 25$ MeV. We discuss possible origins of the surprising discrepancies between theory and data found in low-energy nd and pd SST breakup measurements.

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I. INTRODUCTION

Since the advent of numerically exact 3N continuum Faddeev calculations the elastic nucleon-deuteron (Nd) scattering and deuteron breakup reactions have become a powerful tool to test modern models of nuclear forces [1–3]. With the appearance of high precision (semi)phenomenological nucleon-nucleon (NN) potentials and first models of the 3N force (3NF) the question about the importance of the 3NF has become the main topic of studies in the 3N system. That issue was given a new impetus from chiral perturbation theory (ChPT), which opened the possibility to employ consistent two- and many-body nuclear forces derived within this framework in 3N continuum calculations.

First applications of (semi)phenomenological NN and 3N forces to elastic Nd scattering and to the nucleon-induced deuteron breakup reactions revealed interesting cases of discrepancies between theoretical predictions based solely on two-nucleon (2N) potentials and data, indicating a possibility of large 3NF effects [4, 5]. The exclusive breakup reaction offers a rich spectrum of kinematically complete geometries and the SST configuration from the very beginning attracted attention as a possible candidate to reveal significant 3NF effects. In this kinematically complete geometry of Nd breakup the momenta of the three outgoing nucleons have the same magnitudes and they form a three-pointed ”Mercedes-Benz” star perpendicular to the beam direction in the 3N center-of-mass (c.m.) frame. Measurements

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of the pd and nd breakup performed at low incoming nucleon energies in different laboratories indeed revealed large discrepancies between predicted theoretical cross sections and data for that geometry. The SST measurements for nd breakup have been performed at the following energies: $E = 10.25$ MeV Bochum [6], $E = 10.5$ MeV Erlangen [7] and TUNL [8], $E = 13.0$ MeV Erlangen [9, 10] and TUNL [8, 11, 12], $E = 16.0$ MeV TUNL [13, 14], $E = 19.0$ MeV TUNL [14], and $E = 25.0$ MeV CIAE [15]. For the corresponding pd breakup reaction, data were taken at $E = 10.5$ MeV Köln [16], $E = 13.0$ MeV Köln [17] and Fukuoka [18, 19], $E = 19.0$ MeV Köln [20], and $E = 65.0$ MeV PSI [21]. The data provided by different groups at the same energy are consistent with each other both for the pd and nd systems, with nd SST cross sections clearly different from and larger than the pd ones.

First analyses of the data performed in the framework of the 3N Faddeev formalism with the Bonn and Paris potentials [22, 23] showed that theory underpredicted the low-energy SST nd cross sections while simultaneously overpredicting the pd ones. The development of NN potentials resulted in the construction of high precision (semi)phenomenological interactions, such as Av18 [24], CDBonn [25], Nijm1, and Nijm2 [26], and their application in 3N continuum calculations confirmed the SST discrepancies between theory and data found with the older potentials. Further progress in numerical techniques permitted to include the 3NF in 3N Faddeev calculations [27] and first reported results with standard 3NF models such as Urbana IX [28] or $2\pi$-exchange Tucson-Melbourne (TM99) [29] exhibited only moderate 3NF effects [4, 5] at low energies.

All analyses of the pd breakup performed by the Cracow-Bochum group have a permanent drawback: they neglected the proton-proton (pp) long-range Coulomb interaction present in the pd system and were based on nd calculations. The successful implementation of the long-range proton-proton (pp) Coulomb force in the Faddeev formalism achieved in Ref. [30] permitted, for the first time, to perform exact calculations of pd breakup. It turned out that the pp Coulomb interaction effects are practically negligible for the SST cross sections [30], which validated the results of analyses of that geometry in pd breakup based on nd calculations.

Recent progress in constructing nuclear forces within chiral effective field theory [31–33] resulted in several high precision NN potentials. Also, for the first time, a possibility appeared of applying 2N and 3N forces derived consistently within the same formalism. Understanding of nuclear spectra and reactions based on these consistent chiral two- and
many-body forces has grown into a main topic of present day few-nucleon and many-nucleon studies.

In view of these new developments it is timely to check if it is possible to get new insights about the origin of the discrepancies between theory and data for the SST configuration by using the newly developed chiral two- and three-body interactions.

The paper is organized as follows: in Sec. II we present the available SST data together with their description by (semi)phenomenological forces and additionally discuss sensitivity of the SST cross sections to particular NN force components. Results with chiral NN potentials alone are presented in Sec. III, while in Sec. IV the importance of 3NF is discussed. In Sec. V we surmise on possible origins for the low-energy discrepancies between theoretical predictions and SST cross section data. We summarize and conclude in Sec. VI.

II. RESULTS WITH (SEMI)PHENOMENOLOGICAL FORCES

Theoretical predictions which are shown in the present paper are obtained within the 3N Faddeev formalism using various 2N and 3N forces. The formalism itself and information regarding our numerical performance were presented in numerous publications so for details we refer the reader to [1, 27, 34, 35].

In Fig. 1 we show the available Nd SST cross section data and compare them to theory based on the CD Bonn NN potential [25]. It is seen that at low laboratory energies $E$ of the incoming nucleon, theoretical predictions clearly underestimate nd data by $\approx 15 - 30\%$ and simultaneously overestimate pd data by $\approx 10\%$. The difference between theory and data decreases with the growing energy, and at $E = 65$ MeV theory describes the pd data.

The predicted low-energy SST cross sections do not change when instead of the CD Bonn potential another (semi)phenomenological interaction is used. The very narrow (red) dark shaded band at 13 MeV in Fig. 2, which comprises AV18 [24], CD Bonn [25], Nijm1 and Nijm2 [26] predictions, reflects the astonishing stability of the low-energy SST cross section to the underlying dynamics. This stability is lost at 65 MeV as evidenced by broadening of the band showing the predictions with NN interactions only.

The predicted low-energy SST cross sections are not only stable with respect to the underlying NN (semi)phenomenological potentials. They are also insensitive to the standard $2\pi$-exchange 3NF’s. In Fig. 2 we show also (cyan) light shaded bands containing predictions
based on the AV18, CD Bonn, Nijm1 and Nijm2 interactions combined with the $2\pi$-exchange Tucson-Melbourne (TM99) 3NF. The cut-off parameter $\Lambda$ of that 3NF is adjusted for each particular NN potential and TM99 3NF combination to reproduce the experimental triton binding energy. In Fig. 2 the band of NN+3NF predictions contains also the cross section for the combination of the AV18 potential and the Urbana IX 3NF. At 13 MeV, effects of these 3NF’s are practically negligible, and the resulting band of predictions is very narrow and overlaps with the band representing NN-only predictions. The astonishing stability with respect to the underlying dynamics present at low energies is lost at 65 MeV. Here, both bands broaden significantly and slightly move apart, indicating small effects of the 3NF. It is a region of energy where also in elastic Nd scattering 3NF effects start coming into play.

The small effects of the 3NF and insensitivity to the underlying dynamics raise the question about the dominant NN force components, contributing to the SST cross section. It turns out that at low energies practically the whole input stems from the $^1S_0$ and $^3S_1$ NN force components only, with a dominating $^3S_1$ contribution (see Fig. 3). These force components provide nearly the whole cross section at low energies as evidenced by nearly overlapping (red) solid (CD Bonn prediction) and (blue) dashed (CD Bonn restricted to $^1S_0 +^3S_1 -^3D_1$ NN partial waves only) lines at 13 and 19 MeV in Fig. 3. With increasing energy the contributions from the remaining partial waves become visible and at 65 MeV they start to outweigh the $^1S_0$ part.

The low-energy dominance of the $^3S_1$ and $^1S_0$ contributions brings up the question to what extent uncertainties of these NN force components could be responsible for the observed discrepancies between theory and data. To answer this question we investigated changes of the SST cross section caused by varying the strengths of the $^3S_1$ and $^1S_0$ NN force components. To this end we multiplied the corresponding potential matrix elements by a factor $\lambda$. It was shown in that changes of the $^1S_0$ interaction induced by the $\lambda$ values in a vicinity of $\lambda = 1$ do not affect exclusive Nd elastic scattering observables and the total cross sections significantly.

In the first step, we investigated what changes of the $^1S_0$ nn or pp interaction are required to get a proper description of the low-energy SST cross section data. It turned out that it was not possible to modify the $^1S_0$ nn or pp potential in a way which would shift the cross section predictions to the nd SST data. However, increasing the strength of $^1S_0$ nn interaction by
≈ 20 − 30% brings theoretical predictions close to the pd SST cross sections at low energies (see Fig. 1). Such a large increase of the strength (λ = 1.21 or λ = 1.3) allows two neutrons to form a 1S0 bound state with the binding energy $E_{b}^{\lambda=1.21} = -0.144$ MeV or $E_{b}^{\lambda=1.3} = -0.441$ MeV. Existence of such a di-neutron state does not spoil the description of nd elastic scattering data [37] but it would have severe consequences for the 3H binding energy, increasing it from the CD Bonn value $E_{3H}^{CD \, Bonn} = -7.923$ MeV to $E_{3H}^{\lambda=1.21} = -9.717$ MeV or $E_{3H}^{\lambda=1.3} = -10.560$ MeV. Admittedly, one could argue that an action of repulsive 3NF’s could provide again proper binding of that system. However, such a strong 1S0 force would also spoil the description of nuclear structure. As can be seen in Fig. 4 such modifications of the 1S0 nn force component would also lead to a significant overestimation of the pd SST cross section data at $E = 65$ MeV but this could change upon including the 3NF tuned to the 3H binding energy.

In the case of the 3S1 component, a proper description of nd SST data would require a reduction of its strength by ≈ 5% (λ = 0.95) (see Fig 5), what leads to a complete deterioration of the np data description and to a deuteron binding energy $E_{d}^{\lambda=0.95} = -1.412$ MeV, drastically different from the experimental value $E_{d}^{exp} = -2.224575(9)$ MeV [38]. Furthermore, 3H would be bound by only $E_{3H}^{\lambda=0.95} = -6.338$ MeV and a large effect of an attractive 3NF would be required to regain the CD Bonn 3H binding. On the other hand, the SST pd data require only a 2% increase of 3S1 − 3D1 strength (λ = 1.02) (see Fig. 5), what could be still tolerated by NN data. However, even such a small change of the strength would increase the deuteron binding to $E_{d}^{\lambda=1.02} = -2.592$ MeV, in contradiction to the very precise experimental value. Also 3H would be bound stronger, with $E_{3H}^{\lambda=1.02} = -8.598$ MeV.

Summarizing, despite the fact that the SST cross sections are strongly dominated by the S-wave NN force components, modifications of their strengths cannot serve to explain differences between theory and low-energy data for that configuration. Namely those NN force components are very much restricted by available 2N and 3N data and their variations have to be considered with great caution.

III. RESULTS WITH CHIRAL NN POTENTIALS

From the available chiral NN interactions we choose four of the most advanced potentials which provide a satisfactory description of NN data in a large energy range. One is an
older set of Bochum forces \cite{31,32} developed up to fourth order (N^3LO) of chiral expansion. It reproduces experimental NN phase-shifts in a wide energy range with an almost comparable accuracy as the high precision (semi)phenomenological NN potentials. We employ five versions of that N^3LO chiral NN potential corresponding to different sets of cut-off parameters used to regularize the Lippmann-Schwinger equation and in spectral function regularization, namely (450, 500) MeV, (450, 700) MeV, (550, 600) MeV, (600, 500) MeV, and (600, 700) MeV, denoted in the following by 201, 202, 203, 204, and 205, respectively.

Two other choices are the new-generation chiral NN potentials introduced and developed up to N^4LO by the Bochum-Bonn \cite{39,40} and Idaho-Salamanca \cite{41} groups. While in the Idaho-Salamanca force, the nonlocal momentum-space regularization was applied with a cutoff parameter \( \Lambda \), in the Bochum-Bonn potential the one-pion and two-pion exchange contributions are regularized in coordinate space using the cutoff parameter \( R \), and for the contact interactions a simple Gaussian nonlocal momentum-space regulator with the cutoff \( \Lambda = 2R^{-1} \) was used. The Idaho-Salamanca N^4LO force is available for three values of the cutoff parameter \( \Lambda = 450, 500, \) and 550 MeV, while the semilocal coordinate-space regularized (SCS) chiral Bochum-Bonn potential has been developed for five cutoff parameters \( R = 0.8, 0.9, 1.0, 1.1, \) and 1.2 fm. Both versions provide a very good description of the NN data set (Idaho-Salamanca) or the phase shifts and mixing angles of the Nijmegen partial wave analysis \cite{42} (Bochum-Bonn), used to fix the low-energy constants (LECs) accompanying the NN contact interactions.

The last chiral potential considered here is the so-called semilocal momentum-space regularized (SMS) chiral potential of the Bochum group \cite{43}, developed up to fifth order (N^4LO) of chiral expansion and augmented by an additional (the so-called N^4LO^+ ) version including some sixth-order terms (also the N^4LO Idaho-Salamanca potential is augmented by the same Q^6 contact terms). In this 2N chiral force, a new momentum-space regularization scheme for the long-range contributions is employed, and a nonlocal Gaussian regulator for the minimal set of independent contact interactions is introduced. These new features have also been applied to the corresponding 3N forces at the N^3LO level \cite{44}. This new family of semilocal chiral 2N potentials provides an outstanding description of the NN data and is available up to N^4LO^+ for five values of the cutoff \( \Lambda = 350, 400, 450, 500, \) and 550 MeV.

In Fig. 6 we show predictions of these NN potentials at \( E = 13 \) and 65 MeV as bands comprising the available range of cutoffs or regulator parameters for each of the four models.
At 13 MeV the bands are very narrow and practically overlap with each other. Similarly to the (semi)phenomenological NN potentials, the chiral interactions also provide predictions for the SST cross sections at low energies that are very stable with respect to the type of the underlying interaction and its parameters as far as they provide a satisfactory description of the NN data. The predictions of the chiral potentials agree with those of (semi)phenomenological forces, leading to the same disagreement with the low-energy SST cross section data.

As can be seen in Fig. 6 these bands are broadened at 65 MeV, especially for the older Bochum-Bonn potential (the (red) dark shaded band in Fig. 6), reflecting the increased dependence of the predictions on potential parameters at that energy as well as the worse description of NN data by the older Bochum-Bonn potential. The bands of SCS and SMS interactions are significantly constricted, in line with a good representation of the NN phase-shifts by these potentials.

The applied chiral potentials differ not only in their regularization scheme. The older Bochum-Bonn potential leads to the deuteron wave function, which is quite different from the ones obtained with other chiral potentials [45]. Despite these differences the calculated low-energy SST cross sections are practically the same.

IV. RESULTS WITH CHIRAL 3N-FORCES

First nonvanishing 3NF contributions appear at N²LO [46, 47] and contain, in addition to the $2\pi$-exchange term, two short-range contributions with the strength parameters $c_D$ and $c_E$ [48]. The latter two can be determined from the $^3$H binding energy and the Nd differential cross section minimum at about $E_{lab} = 70$ MeV, which is the energy at which effects of 3NF start to appear in the Nd elastic scattering cross section [4, 36, 44, 49]. Specifically, first the so-called $(c_D, c_E)$ correlation line is established, which for a particular chiral NN potential combined with a N²LO 3NF yields values of $(c_D, c_E)$ reproducing the $^3$H binding energy. Then, a fit to the experimental data for the elastic Nd cross section is performed and the values of both strengths, $c_D$ and $c_E$, are uniquely determined.

In Fig. 7a, b, and c we show predictions for the SST cross section at $E = 13$ MeV for the SCS chiral potential with the regularization parameter $R = 0.9$ fm at N²LO, N³LO, and N⁴LO, respectively, combined with the N²LO 3NF for four sets of the strength param-
eters taken from the corresponding correlation lines. Also, predictions of particular chiral potentials are shown by (red) dashed line. All lines practically overlap showing that effects of \( N^3 \text{LO} \) 3NF on 13 MeV SST cross section are negligible. The predicted cross section is insensitive to the order of the chiral NN potential used.

Effects of the \( N^2 \text{LO} \) 3NF start to appear at 65 MeV (see Fig. 7d). This is the energy region, where effects of 3NF’s start to come into play also in elastic Nd scattering \cite{4,36}. The overlapping predictions for four sets of strengths combinations from the correlation line \((c_D, c_E)\) shown in Fig. 7d indicates, that the magnitude of 3NF effects in this energy range does not depend on strength values as far as they are taken from the correlation line.

To investigate further how effects of the \( N^2 \text{LO} \) 3NF depend on the strengths of the contact terms we took the most precise chiral SMS potential at \( N^4 \text{LO}^+ \) with the regulator \( \Lambda = 450 \) MeV and combined it with the \( N^2 \text{LO} \) 3NF \cite{48}. In Fig. 8 we show predictions for SST cross sections at \( E = 13 \) and 65 MeV for eight combinations of strengths taken from the correlation line \((c_D, c_E)\). Again, in spite of a very wide range of \( c_D \) values, taken between \( c_D = -20 \) and \( c_D = +20 \), the predicted 13 MeV cross sections are found to lie within a relatively narrow band. Contrary to that, at 65 MeV a very broad range of predictions is seen confirming the observation that in this energy region 3NF effects become important.

One may now raise the question of the role of 3NF components from higher chiral orders and their impact on the SST cross section. The necessary work to derive the chiral 3NFs at \( N^3 \text{LO} \) has been done in \cite{50,51} using dimensional regularization. At that order, five different topologies contribute to the 3NF. Three of them are of long-range character \cite{50} and are given by two-pion (2\( \pi \)) exchange graphs, by two-pion-one-pion (2\( \pi - 1 \pi \)) exchange graphs, and by the so-called ring diagrams. They are supplemented by the short-range one-pion-exchange-contact (1\( \pi \)-contact) and two-pion-exchange-contact (2\( \pi \)-contact) terms \cite{51}. The 3NF at \( N^3 \text{LO} \) order does not involve any new unknown low-energy constants (LECs), see, however, a related discussion in \cite{52}, and depends only on two parameters, \( c_D \) and \( c_E \), that parameterize the leading one-pion-contact term and the 3N contact term appearing already at \( N^2 \text{LO} \). Their values need to be fixed at a given order from a fit to few-nucleon data, as in the \( N^2 \text{LO} \) case.

In the first preliminary investigation of \( N^3 \text{LO} \) 3NF effects \cite{45} we considered the action of the 3NF only in 3N states with the total 3N angular momenta \( J = 1/2 \) and 3/2 and included all long-range contributions with the exception of 1/\( m \) corrections. Additionally,
the $2\pi$-exchange-contact term was omitted in the short-range part of 3NF. The strengths parameters $c_E$ and $c_D$ were determined at that time from the correlation line and the nd doublet scattering length $^2a_{nd}$ was used in addition to the $^3$H binding energy to uniquely determine both values.

In Fig. 9a we show the SST cross sections at 13 MeV in form of a red band comprising five predictions of the N$^3$LO Bochum-Bonn potentials (versions 201-205). Combining these potentials with the N$^3$LO chiral 3NF gives the blue band. For the sake of comparison also the CD Bonn prediction is shown by the (orange) solid line. In b) the corresponding predictions at N$^2$LO are also presented. It again turns out that the cross section for the SST configuration of the nd breakup is very stable with respect to the underlying dynamics. Not only (semi)phenomenological potentials, alone or combined with standard 3N forces, provide practically the same SST cross sections. Also the chiral 2N forces supplemented by the N$^3$LO 3NF without relativistic $1/m$ corrections and short-range $2\pi$-contact term yield similar predictions and cannot explain the discrepancy between the theory and the data found for the low-energy SST configurations. Notice further that a consistent regularization of the 3NF beyond N$^2$LO has not yet been achieved, see [53].

Due to the restriction to the low total 3N angular momenta this result has to be confirmed. A systematic investigation of effects of the 3NF beyond N$^2$LO is the main aim of the LENPIC collaboration [49].

At fifth order (N$^4$LO) the chiral 3NF comprises thirteen purely short-range operators [54] in addition to the long- and intermediate-range interactions generated by pion-exchange diagrams [55, 56]. In an exploratory study of [57], effects of these subleading short-range terms were investigated in pd scattering below $E_{lab} = 3$ MeV within a hybrid approach based on phenomenological two- and three-nucleon forces.

To get insight into the expected 3NF effects on SST cross sections from these N$^4$LO short-range 3NF contributions we choose two out of the ten terms, namely the isoscalar central and spin-orbit interactions coming with strengths $c_{E_1}$ and $c_{E_7}$, respectively [48, 54] and add them to the N$^2$LO 3NF. In Fig. 10 we show predictions for the SST cross section at $E = 13$ and 65 MeV for the chiral SMS N$^4$LO$^+$ potential with regularization parameter $\Lambda = 450$ MeV combined with that 3NF for a set of eight combinations of strengths from the correlation lines $(c_D, c_E, c_{E_1}, c_{E_7})$ for fixed values of $c_{E_1}$ and $c_{E_7}$. For five of these combinations also the lines of predictions are drawn. It is seen that the inclusion of the $c_{E_1}$- and $c_{E_7}$-terms
has a negligible effect on the 13 MeV SST cross section and the predicted cross sections essentially coincide with each other and with the results of SCS N^4LO and SMS N^4LO^+ potentials alone. Again, at 65 MeV, a wide spread of predictions indicates significant effects of the 3NF at this energy.

The observed discrepancy between theoretical predictions and the nd and pd low-energy SST cross section data thus indeed appears to be puzzling. Due to the observed strong stability of the low-energy space-star cross sections to the underlying dynamics it seems very unlikely that this puzzle can be resolved by the inclusion of omitted N^3LO terms or remaining N^4LO and higher order contributions to the 3NF. We checked for a combination of SMS N^4LO^+ NN and N^2LO 3NF that even removing the requirement to reproduce \(^3\)H binding energy does not help to come into the vicinity of the 13 MeV nd SST cross section data. Namely taking strengths \(c_D = 2.0\) and \(c_E = 0.287\) from the correlation line and increasing \(c_E\) to \(c_E = 1.433\) lowers the predicted cross sections and brings them close to pd SST data (see (indigo) long dashed line in Fig. 10). For the combination of strengths \(c_D = 2.0, c_E = 1.433\), \(^3\)H is strongly bound with \(E_{3H} = -10.783\) MeV. However, decreasing the \(c_E\) value and even changing its sign does not have any significant effect on the predicted cross section, which remains close to the stable region of predictions for strength values from the correlation line (see Fig. 10 and the (red) dash-double-dotted line, which is prediction for \(c_D = 2.0\) and \(c_E = -1.433\)). For the combination of strengths \(c_D = 2.0, c_E = -1.433\), \(^3\)H is bound with \(E_{3H} = -6.792\) MeV.

V. DISCUSSION OF THE LOW-ENERGY DISCREPANCY

The presented results support the conjecture that 3NF effects are not responsible for the discrepancies between data and theory in the low-energy SST cross sections. One could argue that, perhaps, modifications of the \(^1S_0\) and/or \(^3S_1 - ^3D_1\) NN force components, more refined than a simple change of their strengths, would provide an explanation for at least pd SST low-energy cross section data. However, this seems to be difficult since there is no room for modifications of np and pp forces compatible with NN data [42, 58, 59]. In spite of the fact that the pd SST discrepancy is relatively small (\(\approx 10\%\)) in comparison to the nd SST one, since the theoretical predictions lie well outside statistical error bars and the systematic errors are claimed to be small [19], it presents a significant discrepancy. A dedicated pd SST
measurement aimed to determine precise normalization of the SST cross sections would help to put some light on this discrepancy. Even larger is the discrepancy to nd SST data and between pd and nd data themselves. However, due to the strong insensitivity of low-energy SST cross sections to the underlying dynamics it seems rather unlikely, that any conceivable charge symmetry breaking mechanism in the NN and/or 3N force would be able to explain the difference between pd and nd SST data and allow to describe nd data. This situation poses an interesting puzzle for theory and its solution has to be probably sought in some exotic mechanism contributing to the nd breakup and irrelevant for the pd one. When looking for such a mechanism one could consider the contributions of hypothetical bound state of two neutrons in the state $^1S_0$ to a region of SST breakup phase-space. Such contributions could create in nd SST measurements additional background originating from accidental coincidences between breakup neutrons and di-neutrons produced in nd scattering, increasing thus the measured cross section. That such a scenario is conceivable follows from the fact that detection of neutrons in nd SST measurements was performed using liquid scintillators with pulse-shape discrimination and their energy was determined by the time-of-flight technique [12]. Such a detection system does not distinguish between di-neutrons and breakup neutrons. As a result, accidental coincidences between breakup neutrons and di-neutrons could appear in the region of SST phase-space which cannot be distinguished from true events by the applied measurement technique. The kinetic energy assigned to di-neutron when using such an experimental arrangement, which is determined by time of flight measurement of its velocity, will be in consequence twice as small as its real kinetic energy. To be specific, assuming that di-neutron is bound by $E_{b}^{d_{i} - n} = -0.144$ MeV (what corresponds to the factor $\lambda_{1S_0} = 1.21$) would lead at incoming neutron lab. energy $E = 13$ MeV to the energy of outgoing di-neutrons from $d(n, d_{i} - n) p$ reaction at laboratory angle of the SST configuration $\theta = 50.5^\circ$ $E_{lab}^{d_{i} - n} = 3.12$ MeV and its energy detected by the TOF system will be 1.56 MeV. In Fig. 11 positions of breakup events (S-curve) in the plane of kinetic energies $E_1 - E_2$ of two outgoing neutrons detected in coincidence are shown together with positions of di-neutron energies determined by the detection system for SST configurations at $E = 13, 19,$ and 65 MeV. Also the range of S-curve covered by data is indicated by two circles. Since at 13 MeV di-neutrons would come nearest to the SST region, the data at that energy would be influenced most by the assumptive background of accidental coincidences. The intensity of accidental coincidences depends on
the number of neutrons or di-neutrons arriving at detectors, which is determined by the
energy spectra of outgoing neutrons in incomplete $d(n,n)np$ breakup and by laboratory
angular distribution of di-neutrons from the $d(n,di-neutron)p$ reaction. The predictions
for these quantities based on solutions of the 3N Faddeev equation with a modified (in the
$^1S_0$ partial wave) CD Bonn potential \[37\] at the above considered three energies are shown
in Fig. 12. Since the cross section for di-neutrons production is comparable to the cross
section for production of neutrons in incomplete nd breakup, it indeed seems plausible that
accidental coincidences could impact the measured low-energy SST cross sections. Rapid
diminishing of di-neutrons production with energy (see Fig. 12) would also explain why the
discrepancy between SST data and theory decreases with energy. New measurements of the
low-energy nd SST cross sections as well as a measurement of nd SST at $E = 65$ MeV, using
a detection system able to distinguish between neutrons and hypothetical di-neutrons, like
the one proposed in \[60\], would be very welcome.

VI. SUMMARY AND CONCLUSIONS

In this investigation we performed a comprehensive analysis of the available SST Nd
breakup cross section data using high precision (semi)phenomenological NN potentials alone
or combined with the standard 3N forces as well as selected chiral forces. Four different chiral
NN potentials including the most precise SMS $N^4LO^+ \text{ of Ref. } 43 \text{ have been applied alone
or in combination with chiral 3NF's at different orders of chiral expansion. The main results}
are summarized as follows.

- The available nd SST data cover the range of incoming neutron lab. energies between
  $E = 10 – 25$ MeV while the pd data were measured for proton energies in a region
  $E = 10 – 65$ MeV. The experiments were performed by different groups using different
  experimental arrangements or techniques. When at a particular energy several pd or
  nd data sets are available, the data from different measurements are consistent with
  each other.

- Using (semi)phenomenological NN potentials alone or accompanied by the TM99 or
  Urbana IX 3NF one is not able to explain the low-energy SST pd and nd SST data.
  All theoretical predictions practically overlap in nd and pd systems with pd data over-
estimated by $\approx 10\%$ and nd data underestimated by $\approx 20 - 30\%$. The discrepancies between theory and data diminish with increasing energy of the incoming nucleon. At $E = 65$ MeV NN force predictions agree with the pd SST cross sections, while inclusion of 3NF provides a slight overestimation of the data.

- Predicted low-energy SST cross sections based on different chiral NN potentials are independent from the type of regularization used or from the regularization parameters, and are practically identical to predictions of (semi)phenomenological interactions. At $E = 65$ MeV that independence starts to be lost.

- Adding the considered chiral 3NF at different orders of chiral expansion has no significant influence on the SST cross sections at low energy. Even a broader range of strengths from the correlation line $(c_D, c_E)$ for N$^2$LO 3NF contact terms yields practically the same low-energy SST cross sections. Again, at $E = 65$ MeV, this stability with respect to changes of strengths vanishes and 3NF effects come into play.

- The low-energy SST cross sections originate practically from $^1S_0$ and $^3S_1 - ^3D_1$ NN force components. Changes introduced by a simple multiplication of the corresponding matrix elements by a factor $\lambda$ could explain the SST pd data but not nd ones. However, the required changes of the $^1S_0$ and/or $^3S_1 - ^3D_1$ are excluded by the NN data and/or by the $^3$H binding energy.

- In view of the astonishing stability of the low-energy SST cross sections to the underlying dynamics it seems very unlikely that charge symmetry breaking mechanism of any conceivable kind in 2N or 3N forces could be able to explain the low-energy nd SST cross sections. The explanation of the nd data should be thus sought in some exotic phenomena such as e.g. the hypothetical bound state of two neutrons. Supposable existence of the di-neutron would provide additional background in the region of the SST breakup phase-space, which could not be discerned in measurements performed so far.

Further investigations and theoretical as well as experimental efforts are required to solve that low-energy SST puzzle. From the experimental side, measurements of nd SST cross sections at low-energies with experimental arrangement able to discern supposable di-neutron background would be needed. Also dedicated pd SST measurement directed
to determine precise normalization of SST cross sections would be welcome. From the theoretical side, efforts to fully include in 3N continuum calculations consistently regularized \( N^3 \)LO and \( N^4 \)LO 3NF components are required. This is the aim of the LENPIC project.

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FIG. 1. (Color online) The nd breakup five-fold differential cross section in the SST complete geometry at the incoming neutron laboratory energies $E = 10.5, 13, 16, 19, 25, \text{ and } 65 \text{ MeV}$, shown as a function of arc-length of the S-curve. The solid (red) lines are predictions of the CD Bonn NN potential. At $E = 10.5 \text{ MeV}$ the (red) squares, the (maroon) diamonds, and the (green) squares are 10.3 MeV TUNL ([8]), 10.25 MeV Bochum ([6]), and 10.5 MeV Erlangen ([7]) nd data. The (marron) circles are 10.5 MeV Köln ([16]) data. At $E = 13 \text{ MeV}$ the (red) squares, the (maroon) circles, and the (green) squares are 13 MeV TUNL ([12]), 13 MeV TUNL ([8]), and 13 MeV Erlangen ([18]) nd data. The (maroon) circles are 13 MeV Köln ([17]) and (violet) diamonds 13 MeV Fukuoka ([18]) pd data. At $E = 16$ and 25 MeV the (red) squares are TUNL ([13]) and CIAE ([15]) nd data. The (maroon) circles at $E = 19$ and 65 MeV are Köln ([20]) and PSI ([21]) pd data.
FIG. 2. (Color online) The same as in Fig. 1 for the incoming neutron laboratory energies \( E = 13 \) and \( 65 \) MeV. The (red) dark shaded and (cyan) light shaded bands comprise predictions of the AV18, CD Bonn, Nijm1 and Nijm2 NN potentials alone or combined with TM99 and for AV18 also with Urbana IX 3NF, respectively. At \( E = 13 \) MeV the (red) squares are TUNL ([12]) nd data and the (maroon) circles are Köln ([20]) pd data. The (maroon) circles at \( E = 65 \) MeV are PSI ([21]) pd data.
FIG. 3. (Color online) Contributions to the SST cross section at $E = 13, 19$ and 65 MeV from different partial waves. The (red) solid line is prediction of the CD Bonn NN potential. The (black) dotted and (green) dash-dotted lines show cross sections obtained with 3N partial waves restricted to those which contain only $^1S_0$ and $^3S_1$-$^3D_1$ NN partial waves, respectively. The result obtained with only $^1S_0 + ^3S_1$-$^3D_1$ is given by (blue) dashed lines and prediction with all partial waves excluding $^1S_0 + ^3S_1$-$^3D_1$ is shown by (brown) dash-double-dotted lines. The (maroon) circles at $E = 19$ MeV are pd Köln data [20]. For description of the data points at 13 and 65 MeV see Fig. 2.
FIG. 4. (Color online) Sensitivity of the SST cross section at $E = 10.5, 13, 19,$ and 65 MeV to changes of the $^1S_0$ nn force component. The (red) solid line is prediction of the CD Bonn potential. The (black) dotted curve is the corresponding cross section when the strength of the nn $^1S_0$ force component is reduced by multiplying its matrix elements with a factor $\lambda = 0.9$. The (orange) dashed and (magenta) dash-dotted lines shows the cross section when that strength is increased by multiplying with factor $\lambda = 1.21$ and $\lambda = 1.3$, respectively. For description of the data points see Fig. 1.
FIG. 5. (Color online) Sensitivity of the SST cross section at $E = 13, 19,$ and $65$ MeV to changes of the $^3S_1 - ^3D_1$ np force component. The (red) solid line is prediction of the CD Bonn potential. The (magenta) dashed curve is the corresponding cross section when the strength of that force component is reduced by 5 \% by multiplying its matrix elements with a factor $\lambda = 0.95$. The (black) dotted line shows the cross section when that strength is increased by 2 \% by multiplying matrix elements with factor $\lambda = 1.02$. For description of the data points see Fig. 4.
FIG. 6. (Color online) The predicted SST cross sections by chiral NN potentials at incoming neutron laboratory energies $E = 13$ and $65$ MeV. The (red) shaded band comprises five predictions of $N^3LO$ chiral potentials (201, 202, 203, 204, 205) of Ref. [32] and the (magenta) band three predictions of $N^4LO$ potentials of Ref. [41] with regulator parameters $\Lambda = 450, 500, 550$ MeV. The (cyan) band covers five predictions of the SCS $N^4LO$ potential of Ref. [39] with regulator parameters $R = 0.8, 0.9, 1.0, 1.1, 1.2$ fm and the (yellow) band four predictions of the SMS $N^4LO^+$ potential of Ref. [43] with regulators $\Lambda = 400, 450, 500, 550$ MeV. For description of the data points see Fig. [2].
FIG. 7. (Color online) Effects of N^2LO 3NF on SST cross section at \( E = 13 \) and 65 MeV when combined with chiral SCS NN potential at different orders of chiral expansion. The dashed (red) lines are predictions of the SCS NN potential with the regulator \( R = 0.9 \) fm at N^2LO (a), N^3LO (b), and N^4LO (c) and (d). Combining that potential with the N^2LO 3NF with four strengths of the contact terms from the correlation lines \((c_D, c_E)\) leads to results shown by different curves: solid (blue) (a: \((-4.0, 0.344)\), b: \((4.0, 0.103)\), c,d: \((4.0, -0.270)\)), dotted (red) (a: \((-2.0, 0.131)\), b: \((6.0, -0.960)\), c,d: \((6.0, -1.094)\)), double-dotted-dashed (blue) (a: \((0.0, -0.097)\), b: \((8.0, -1.937)\), c,d: \((8.0, -2.032)\)), and dashed (maroon) (a: \((2.0, 0.345)\), b: \((10.0, -3.063)\), c,d: \((10.0, -3.108)\)). For description of the data points see Fig. 2.
FIG. 8. (Color online) SST cross sections at $E = 13$ and 65 MeV predicted by SMS chiral N$^4$LO+ NN potential (the regulator parameter $\Lambda = 450$ MeV) alone (shown by dashed (red) lines) or combined with consistently regularized N$^2$LO 3NF with different strengths of the contact terms taken from the correlation line ($c_D, c_E$) and shown by lines: dotted (blue) (2.0, 0.287), short dashed (brown) (4.0, 0.499), long dashed (cyan) (15.0, 1.362), dash-dotted (orange) (−15.0, −2.010), dash-double-dotted (maroon) (20.0, 1.533), double-dash-dotted (black) (−20.0, 1.362), and short dashed (turquoise) (−10.0, −1.257). For description of the data points see Fig. 2.
FIG. 9. (Color online) Effects of N$^3$LO 3NF, restricted to 3N total angular momenta $J = 1/2$ and $3/2$, with all long-range contributions with exception of $1/m$ corrections and omitted $2\pi$-exchange-contact term in the short-range part on SST cross section at $E = 13$ MeV (a). The red band in (a) comprises predictions of five chiral N$^3$LO Bochum-Bonn potentials (201-205). Combining them with N$^3$LO 3NF with strengths of the contact terms determined by the correlation line $(c_D, c_E)$ and requirement to reproduce $^2a_{nd}$ leads to the blue band. The (orange) solid line is prediction of the CD Bonn potential. In (b) the same is shown for chiral Bochum-Bonn potentials and 3NF at N$^2$LO. In this case the N$^2$LO 3NF acted up to $J = 7/2$. For description of the data points see Fig. 2.
FIG. 10. (Color online) SST cross sections at $E = 13$ and 65 MeV predicted by the chiral SMS N$^4$L0$^+$/NN potential with the regulator parameter $\Lambda = 450$ MeV alone (the long dashed (yellow) lines) or combined with 3NF comprising N$^2$L0 and two out of ten N$^4$L0 contact terms, with strengths of the contact terms from the correlation line ($c_D, c_E, c_{E_1}, c_{E7}$). The results with 3NF are shown by the following lines: dotted (orange) (2.741, 0.367, 0.0, 0.0), dash-double-dotted (turquoise) (10.818, −0.295, −4.0, 0.0), long dashed (maroon) (−11.977, −0.969, 4.0, 0.0), double-dash-dotted (indigo) (10.471, 1.079, 0.0, −4.0), and dash-double-dotted (green) (−8.205, −1.002, 0.0, 2.0). The (cyan) shaded band covers region of predictions given by five above and three additional sets of strengths: (6.971, 0.207, −2.0, 0.0), (8.533, 0.925, 0.0, −2.0), (3.133, 0.041, 2.0, 0.0). For comparison also the short dashed (red) line is drawn which is the prediction of the SCS N$^4$L0 potential with the regulator parameter $R = 0.9$ fm. In a), two additional lines are drawn which correspond to the combinations of strengths ($c_D, c_E$) outside the correlation line, namely (indigo) long dashed: (2.0, 1.433) and (red) dash-double-dotted: (2.0, −1.433). For description of the data points see Fig. 2.
FIG. 11. (Color online) Position of $d(n,nn)p$ breakup events (S curve) in the laboratory kinetic energy plane $E_1 - E_2$ of two detected in coincidence neutrons for SST configurations at $E = 13, 25, \text{and } 65 \text{ MeV}$. The dashed lines show half of the energy of the outgoing di-neutron from $d(n, di-neutron)p$ reaction at lab. angle of the corresponding SST geometry. The position on the S-curve where the SST condition is fulfilled is shown by square and the range of S-curve covered by nd (pd) SST data is indicated by dots.
FIG. 12. (Color online) The angular distributions of di-neutrons and the energy spectra of outgoing neutrons from incomplete breakup reaction \(d(n, n)np\) for lab. angle of the detected neutron equal to the SST configuration angle, predicted by the CD Bonn potential with a \(^1S_0\) nn force component modified by increasing its strength by a factor \(\lambda\) to get two neutrons bound. The (blue) dashed line corresponds to \(\lambda = 1.21\) with di-neutron binding energy \(E^{d_{n-n}}_b = -0.144\) MeV and the dashed-dotted line to \(\lambda = 1.3\) with \(E^{d_{n-n}}_b = -0.441\) MeV. The a), b) and c) are predictions for incoming neutron lab. energies \(E = 13, 25,\) and \(65\) MeV, respectively.