Finite Element Analysis of Second-order Effect of Plane High Strength Steel Frame Structures

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Abstract. High-strength structural steels have been gradually used in building structures and bridge structures because of their good mechanical properties and machinability. The mechanical behavior of high strength steel members is significantly different from that of ordinary steel members. Based on the approximate second-order calculation formula in the “Code for Design of Steel Structures” GB50017-2003, the non-linear increase coefficient of the material are introduced. The finite element analysis method is used to check the second-order nonlinear effect of typical high-strength steel frame. The results of the approximation second-order analysis and the second-order elastic-plastic analysis are compared. The second-order effect and the applicability of approximation method to high strength steel frame is analysed. The result of the calculation and analysis is valuable to engineering application.

1. Introduction
Steels with nominal yield strength not less than 460 MPa are now commonly referred to as high-strength steels. The problem of ordinary steel use oversize construction member in the high-rise building can be solved and reduce the amount of steel by the application of high-strength steel. Making the structure program becomes more economical and reliable. With the continuous improvement of steel production technology, the strength and the related performance of steel products have been improved. However, high strength steel after heat treatment and reached ultimate strength, the plastic development of the structure is insufficient which hinders the redistribution of internal forces. On the other hand, there could have large differences in mechanical properties when high-strength steel in the heat treatment process has tiny windage. The approximate second-order analysis method of the steel frame proposed in “Design Code for Steel Structures” GB50017-2003 is mainly directed at the simple frame form of the steel strength not more than 420 MPa, whether it is applicable to the high-strength steel frame still needs to be analyzed.

The first-order analysis and the second-order elastic-plastic analysis were conducted by the finite element software ABAQUS in several typical high-strength steel frame structures. A comparative analysis is performed with the result of the approximate second-order analytical method. To discuss the GB50017 formula whether applies to high strength steel frame structure, then the results of the calculation and analysis is valuable to engineering application.

2. Analysis method of Steel frame structure
The analysis of the structure in the undeformed state is usually called the first-order analysis, and the analysis of the structure in the deformed state is called the second-order analysis. In the second-order
analysis is only consider the geometric nonlinearity called the second-order elastic analysis; considered the geometric nonlinearity and the material nonlinearity is called the second-order elastic-plastic analysis. GB50017 [1] states: "The first-order elastic analysis of the frame structure can be used." At the same time, the second-order analysis should be adopted for the steel frame satisfying the formula (1).

\[
\frac{\sum N \cdot \Delta u}{\sum H \cdot h} > 0.1
\]

Where, \( \Sigma N \) — the sum of the design values of the axial pressure on each column of the calculated floor;
\( \Delta u \) — the inter-story lateral displacement of the calculated floor obtained by first-order elastic analysis;
\( \Sigma H \) — the sum of the horizontal forces on the calculated floor and the above layers for the generation of the lateral displacement \( \Delta u \);
\( h \) — The height of the calculated floor.

Precise second order analysis of the framework can be done by numerical methods, such as beam-column method, finite element method, virtual load method, etc. However, accurate nonlinear analysis methods are time consuming and general engineering designs are not suitable for use. It is easier to consider second-order effects using an approximate second-order analysis method. Therefore, GB50017 [1] proposes an approximate second-order analysis method for the internal force analysis, which stipulates that when performing the second-order elastic-plastic analysis. Regardless of whether it is exact or approximate calculations, whether or not there are supporting structures, it is necessary to consider the imaginary horizontal force \( H_{ni} \) at the top of each column to take into account the influence of various defects of the structure and components (such as the initial tilt of the column, initial eccentricity, and residual stress) to match the accuracy of the calculation on the frame, and use Equation (2) for calculation.

\[
H_{ni} = \frac{\alpha_y Q_i}{250} \sqrt{0.2 + \frac{1}{n_y}}
\]

Where, \( Q_i \) — The total gravity load design value of the i-th floor;
\( n_y \) — The total number of layers of the frame, and \( \sqrt{0.2 + 1/n_y} > 1 \) take this root number value as 1.0;
\( \alpha_y \) — The influence factor of steel strength is 1.0 for Q235 steel, 1.1 for Q345 steel, 1.2 for Q390 steel, and 1.25 for Q420 steel.

In order to avoid the tedious calculation of an accurate second-order elastic analysis, GB50017 [1] recommends an approximate solution for the calculation of an unsupported pure frame structure.

For unsupported frame structures, when using the second-order elastic analysis can use the equations (3) and (4) to calculate the bending moments at the rod end of each element.

\[
M_{ii} = M_{i1} + \alpha_i M_{i2}
\]

\[
\alpha_i = \frac{1}{1 + \frac{\sum N \cdot \Delta u}{\sum H \cdot h}}
\]

Where, \( M_{i0} \) — bending moment of each rod end obtained by first-order elastic analysis assuming that the frame is not laterally displaced;
\( M_{i2} \) — bending moment of each rod end obtained by first-order elastic analysis when each node of the frame moves sideways;
\( \alpha_i \) — the increcent coefficient of the lateral bending moment of considering the second-order effect i-th layer member.
3. *Finite Element Analysis Method*

Article 5 mentioned, a large number of tests on the tensile strength of high-strength steels were conducted, and proposed a nonlinear constitutive model which based on the Ramberg-Osgood model for high-strength steels without apparent yielding platforms. Through establish mathematical expressions to describe the stress-strain relationship of high-strength steel which under monotonic loading. Therefore, the non-linear constitutive model proposed in article 5 is adopted in this paper. The nominal stress-nominal strain relationship between high-strength steel and ordinary steel is shown in Figure 1.

![Figure 1. The nominal stress-nominal strain relationship between high-strength steel and ordinary steel](image)

The frame is calculated by using the linear beam element B21[^2] in ABAQUS. Based on the Timoshenko beam theory, the influence of transverse shear deformation is considered in addition to the deformation such as axial, bending and torsion. It is suitable for structural analysis of large strain and large deformation. For the geometric nonlinearity, the large deformation switch is required to be opened in the analysis step. At the same time, the initial roll of the aluminum alloy frame is $H/1000$ ($H$ is the total height of the frame) as the initial geometrical defect of the whole structure[^5]. The analysis did not take into account the semi-rigid, the column feet and nodes are set to rigid connection.

According to the provisions of the regulations, it used a precise second-order elasto-plastic analysis method and an approximate second-order elastic analysis method to make comparative calculations. The example (Figure 2) uses single-span single frame, single-span two-story frame and two-span two-story frame.

The material properties used in this paper are elastic modulus $E = 2.06 \times 10^5$ MPa, Poisson's ratio 0.3, high-strength steel strength 550 MPa, and ultimate tensile strength 670 MPa[^5]. The section properties of each study are shown in table 1.

| Example | Section type | Cross section area/cm² | Moment of inertia/cm⁴ | Modulus of section/cm³ |
|---------|--------------|-------------------------|-----------------------|------------------------|
| Example 1 | Column HW300×300×8×12 | 94.08 | 16340.19 | 1089.34 |
|         | Beam HN400×150×10×12 | 73.59 | 17983.06 | 899.15 |
| Example 2 | Column HW400×400×10×15 | 157 | 48711.08 | 800.15 |
|         | Beam HN400×150×10×12 | 73.59 | 17983.06 | 899.15 |
| Example 3 | Column HW300×300×8×12 | 94.08 | 16340.19 | 1089.34 |
|         | Beam HN400×150×10×12 | 73.59 | 17983.06 | 899.15 |

In addition, the calculation examples all meet the requirements of the deformation tolerance of the structures or members specified in GB50017, i.e., the beam disturbance is not greater than $L/400$ of the span $L$, the relative displacement between the frames is not greater than $L/400$ of the layer height $H$, and the column top displacement not more than $L/500$ of $H$ of the total height of the frame structure.
During constructing the example, it was found that the smaller the uniform load (ie, gravity load) on the beam, the greater the horizontal concentrated force and the smaller the frame's resistance to lateral displacement, and the second-order effect is more significant.

![Figure 2. Calculation sketch of high-strength steel frame](image)

4. **Comparison between the exact analysis result and the approximate analysis result**

The relevant parameters of each example calculation are shown in table 2, and the values in the table 2 are calculated by Equation (4) and Equation (2). Since GB50017 for yield strength higher than Q420 does not have corresponding influence coefficient of steel strength $\alpha_y$. Therefore, using the influence coefficient of steel strength of Q420 which is closer to the strength 550 MPa, takes the influence coefficient as 1.25.

The exact analysis results and the approximate analysis results of each example are shown in table 3, table 4, table 5 and table 6. The numbering rules for the nodes at both ends of the element in the table are A for the bottom end of the column numbered by the identified element member and B is the top of the column numbered by the element member.

| Parameter values | Example1       | Example2       | Example3       |
|------------------|----------------|----------------|----------------|
| $\sum N \cdot \Delta u / (\sum H \cdot h)$ | 0.138           | 0.103          | 0.131          | 0.169           | 0.04            |
| $\alpha_{2i}$    | 1.16           | 1.11           | 1.15           | 1.20            | 1.04            |
| $H_w$            | 5.00           | 7.91           | 3.30           | 12.91           | 4.62            |

From table 2, it can be seen that when the horizontal concentration force is not large, the effect of the imaginary horizontal force $H_{ni}$ added to the top of each layer on the lateral displacement of the frame will increase. The greater the number of frame layers, the smaller the impact of $H_{ni}$; the increase in the number of spans (that is, the increase in the gravity load of each layer) will increase the impact of $H_{ni}$. For a single-story single-span frame, if it is meet the requirements of the specification, a smaller anti-stiffness or a larger layer height is required, that is, when the column section size is small or when the layer height is large, that is, the second-order effect of the frame is even more pronounced. For the two-layer high-strength steel frame, the second-order effect of the bottom layer is significantly larger than that of the second layer. This is mainly because the bottom column is subjected to a greater gravity load, and its second-order effect is more pronounced. The lateral shift moment increase coefficient $\alpha_{2i}$ also has a similar law, but with the increase of the anti-lateral stiffness (ie, the increase of the column cross-section size), for the two-story frame, the horizontal force of the second floor is
very small, and when the uniformly distributed load on the beam is larger, the second-order effect is more pronounced.

### Table 3. Comparison of horizontal displacement

| Parameter values                                      | Example1 | Example2 | Example3 |
|-------------------------------------------------------|----------|----------|----------|
|                                                      | First floor | First floor | Second floor | First floor | Second floor |
| First order elastic $U_i$/mm.                         | 13.05     | 14.00    | 11.22    | 13.99    | 2.41    |
| Second order elastic-plastic $U''_i$/mm.              | 16.74     | 20.39    | 19.24    | 16.08    | 4.01    |
| $\frac{(U''_i-U_i)}{U''_i} \times 100\%$            | 22.04     | 31.34    | 41.68    | 13.00    | 39.90    |

### Table 4. Rod End Moment of Example 1

| Rod end number | First order elastic $M_i$/kN·m | Approximate second order elastic $M''_i$/kN·m | Second order elastic-plastic $M''_i$/kN·m | $\frac{(M''_i-M_i)}{M_i} \times 100\%$ | $\frac{(M''_i-M_i)}{M_i} \times 100\%$ |
|----------------|---------------------------------|-----------------------------------------------|------------------------------------------|----------------------------------------|----------------------------------------|
| 1-A            | -172.29                         | -170.37                                      | -209.92                                  | 17.93                                  | 18.84                                  |
| 1-B            | 386.20                          | 426.94                                       | 464.00                                   | 16.77                                  | 7.99                                   |
| 2-A            | -217.72                         | -243.10                                      | -273.60                                  | 20.42                                  | 11.15                                  |
| 2-B            | 420.32                          | 467.24                                       | 508.83                                   | 17.39                                  | 8.17                                   |

### Table 5. Rod End Moment of Example 2

| Rod end number | First order elastic $M_i$/kN·m | Approximate second order elastic $M''_i$/kN·m | Second order elastic-plastic $M''_i$/kN·m | $\frac{(M''_i-M_i)}{M_i} \times 100\%$ | $\frac{(M''_i-M_i)}{M_i} \times 100\%$ |
|----------------|---------------------------------|-----------------------------------------------|------------------------------------------|----------------------------------------|----------------------------------------|
| 1-A            | -54.32                          | -43.90                                        | -25.81                                   | -110.46                                | -70.09                                 |
| 1-B            | 252.56                          | 247.03                                       | 256.90                                   | 1.69                                   | 3.84                                   |
| 2-A            | -214.23                         | -223.87                                      | -248.65                                  | 13.84                                  | 9.97                                   |
| 2-B            | 324.29                          | 328.54                                       | 328.83                                   | 1.38                                   | 0.09                                   |
| 3-A            | -418.53                         | -416.84                                      | -445.36                                  | 6.02                                   | 6.40                                   |
| 3-B            | 432.82                          | 428.56                                       | 446.25                                   | 3.01                                   | 3.96                                   |
| 4-A            | -429.36                         | -428.73                                      | -448.59                                  | 4.29                                   | 4.43                                   |
| 4-B            | 483.30                          | 486.26                                       | 528.51                                   | 8.55                                   | 7.99                                   |

### Table 6. Rod End Moment of Example 3

| Rod end number | First order elastic $M_i$/kN·m | Approximate second order elastic $M''_i$/kN·m | Second order elastic-plastic $M''_i$/kN·m | $\frac{(M''_i-M_i)}{M_i} \times 100\%$ | $\frac{(M''_i-M_i)}{M_i} \times 100\%$ |
|----------------|---------------------------------|-----------------------------------------------|------------------------------------------|----------------------------------------|----------------------------------------|
| 1-A            | -56.16                          | -48.32                                        | -52.36                                   | -7.26                                  | 7.72                                   |
| 1-B            | 161.35                          | 154.92                                       | 157.32                                   | -2.36                                  | 1.53                                   |
| 2-A            | -47.87                          | -55.88                                        | -55.70                                   | 14.06                                  | 0.32                                   |
| 2-B            | 60.08                           | 67.06                                        | 71.54                                    | 16.02                                  | 6.26                                   |
| 3-A            | -112.30                         | -119.39                                       | -128.24                                  | 12.43                                  | 6.90                                   |
| 3-B            | 197.05                          | 197.42                                       | 207.50                                   | 5.04                                   | 4.86                                   |
| 4-A            | -231.6                          | -231.43                                       | -240.67                                  | 3.77                                   | 3.84                                   |
| 4-B            | 193.70                          | 193.22                                       | 197.72                                   | 2.03                                   | 2.28                                   |
| 5-A            | -40.16                          | -39.93                                        | -39.16                                   | -2.55                                  | 1.97                                   |
From table 3, table 4, table 5 and table 6, it can be found that the bottom frame column where the horizontal concentration force acts (i.e., the bar number is 1) is most affected by the second-order effect. Since the absolute values of the rod-end bending moments on these frame columns are relatively small, the first-order analysis results are more pronounced than the second-order analysis results compared to other rods with the same horizontal side. The second-order effect for the $\Sigma N \cdot \Delta u / (\Sigma H \cdot h) < 0.1$ framework is smaller. The comparison between the calculated results of the approximate second-order analysis and the second-order elastic-plastic analysis shows that the error ratio between the first-order analysis and the second-order elastic-plastic analysis result is small. It is entirely possible to use the first-order analysis results for component design on the $\Sigma N \cdot \Delta u / (\Sigma H \cdot h) < 0.1$ framework.

In addition, it can also be found that the approximate second-order bending moment $M_{II}$ calculated by equation (3) is smaller than the precise analysis bending moment $M'_{II}$, showing that high-strength steels continue to use the influence coefficient of steel strength of Q420 is unsafe.

5. Conclusions
Through the calculation and analysis of the paper, the following conclusions can be drawn:

(1) It can be found that the lateral displacement and bending moment of the second-order elastic-plastic analysis of the high-strength steel frame are larger than those of the first-order analysis. There should be noticed that the value is obviously larger than the increase amplitude of the internal force. It also shows that the second-order elastic-plastic analysis is necessary for the high-strength steel frame.

(2) The results of the second-order approximation analysis and the second-order accurate analysis have significant deviations at the bottom frame column where the horizontal concentration force is applied. And the second-order effect of the bottom frame column in the entire frame is the most obvious.

(3) The approximate second-order analysis method of the ordinary steel frame proposed in GB50017 is applied for the high-strength steel frame. It can be found that the result of the internal force is less than the second-order elastic-plastic analysis method. It shows that high-strength steel frame apply for GB50017 to design is unsafe. So, it is very necessary to propose an approximate second order analysis method for high strength steel frame design.

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