On the mass function of star clusters

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ABSTRACT
Clusters that form in total $10^3 \lesssim N \lesssim 10^5$ stars (type II clusters) lose their gas within a dynamical time as a result of the photo-ionising flux from O stars. Sparser (type I) clusters get rid of their residual gas on a timescale longer or comparable to the nominal crossing time and thus evolve approximately adiabatically. This is also true for massive embedded clusters (type III) for which the velocity dispersion is larger than the sound speed of the ionised gas. On expelling their residual gas, type I and III clusters are therefore expected to lose a smaller fraction of their stellar component than type II clusters. We outline the effect this has on the transformation of the mass function of embedded clusters (ECMF), which is directly related to the mass function of star-cluster-forming molecular cloud cores, to the “initial” MF of bound gas-free star clusters (ICMF). The resulting ICMF has, for a featureless power-law ECMF, a turnover near $10^{4.5} M_\odot$ and a peak near $10^3 M_\odot$. The peak lies around the initial masses of the Hyades, Praesepe and Pleiades clusters. We also find that the entire Galactic population II stellar spheroid can be generated if star formation proceeded via embedded clusters distributed like a power-law MF with exponent $0.9 \lesssim \beta \lesssim 2.6$.

Key words: stellar dynamics – stars: formation – Galaxy: formation – globular clusters: general – open clusters and associations: general – early universe

1 INTRODUCTION
Star clusters form embedded in gas, and because the star formation efficiency (SFE), $\epsilon$, is low, they lose a large fraction of their stars when the gas is expelled. Open and globular clusters thus contain only a fraction of the original population of stars in the cluster.

By comparing the binding energy of the embedded cluster with the binding energy of the post-gas expulsion cluster, Hills (1980) and Mathieu (1983) showed that if more than 50 per cent of the total mass ($\epsilon < 0.5$) is removed rapidly on a time scale shorter than the crossing time of the stars through the embedded cluster ($\tau_G < t_{\text{cr}}$), then the entire cluster dissolves. This argument assumes the embedded cluster to be close to global virial equilibrium just prior to gas expulsion, a state that is considered to be the most realistic (Kroupa 2002a). The energy argument posed a serious challenge, because $\epsilon \equiv M_{\text{ecl}}/(M_{\text{ecl}} + M_G) = 0.2 – 0.4$ by observation (e.g. Lada 1999; Nürnberg et al. 2002), where $M_{\text{ecl}}$ is the mass in stars that form in the embedded cluster within a region a few times larger than the characteristic radius, $R$, of the embedded cluster, while $M_G$ is the mass in residual gas within this same region. The problem arises because bound star clusters like the Pleiades and Hyades exist. They are sufficiently populous to have contained a few O stars that photo-ionise the cloud core rapidly which, as a consequence of being thus heated to $10^4$ K, expands thermally at the sound speed, $v_s \approx 10$ km/s, which is larger than the velocities of the stars in the embedded cluster.

This rather severe problem was re-addressed in a pioneering study by Lada, Margulis \& Dearborn (1984, hereinafter LMD) using $N$–body computations of clusters containing up to $N_{\text{ecl}} = 100$ stars embedded in a gas potential that was removed over various time-scales. LMD, and subsequent work with much higher $N_{\text{ecl}}$ by Goodwin (1997, 1998) and Geyer \& Burkert (2001), confirmed Hills’ and Mathieu’s result. LMD noted, however, that a small bound core of stars formed even if gas expulsion was rapid. Their small total number of stars precluded a more detailed study of these cores. Improved $N$–body algorithms and computer power allowed this problem to be attacked anew. Kroupa, Aarseth \& Hurley (2001, hereinafter KAH) performed computations with $N_{\text{ecl}} = 10^4$ stars in 100 per cent primordial binaries and a realistic stellar IMF using a state-of-the-art high-precision $N$–body code and assuming that the mass-dominating residual gas ($\epsilon = 0.33$) is expelled...
on a thermal expansion timescale. They showed that Orion-Nebula-like embedded clusters easily evolve to Pleiades-like open clusters. These contain a fraction \( f_{ex} = 0.2 - 0.3 \) of the stars originally in the embedded cluster. The KAH models include stellar evolution and a realistic Galactic tidal field which unbinds a larger fraction of the expanding stellar population than in the previous models that neglected the tidal field contribution. The KAH result is interpreted to be a consequence of the formation of a stellar core through stellar encounters that redistribute the energy prior, during and after gas expulsion (Boily & Kroupa 2002).

Direct observational evidence for the loss of a substantial fraction of the stars from an embedded cluster is provided by Elson et al. (1987), who study a sample of 10 massive young clusters in the LMC and find that many if not all of them have up to 50 per cent of their total mass in a halo of unbound stars. Depending on the distribution of embedded cluster masses and the gas-removal time-scale, such expanding populations can add a kinematically hot component to the field population of a galaxy. This may explain the hitherto not understood age–velocity dispersion relation for solar-neighbourhood stars and possibly the origin of the Galactic thick disk (Kroupa 2002a). The formation of star clusters and the cluster mass function thus become important for shaping galaxies. The notion that star clusters form by expelling a large fraction of their stars also calls into question conclusions about the dominant mode of star formation based on studies of the number of open and globular clusters and their evaporation rate through two-body relaxation. Thus, for example, the Galactic population II spheroid is typically interpreted to be the result of a clustered mode of star formation (the globular clusters) and an isolated mode (the population II field stars).

The mass function of young star-clusters (CMF) is also fundamentally important for constraining star-cluster formation theories and for dynamical population synthesis studies that aim to account for the binary properties of a galaxy’s population (Kroupa 1995a). Furthermore, it is becoming apparent that star clusters play a crucially important role for the chemical evolution of galaxies: long-lived and thus initially massive clusters lead to a significantly enhanced production rate of type Ia supernovae through the formation of short-period double-white-dwarf and giant–white-dwarf systems due to encounters (Shara & Hurley 2002). It is thus essential to know the distribution and number of long-lived clusters.

The CMF is not well constrained though, with log-normal or power-law forms being discussed for various star-forming galaxies (e.g. Elmegreen et al. 2000; Fritze-v. Alvensleben 1999; Whitmore et al. 1999). Inferring the CMF is very difficult. Observational biases, such as obscuration by dust, flux limits, and radial truncation through the limited survey area together with the unknown stellar-dynamical state of the cluster make inferences about the mass highly uncertain. There are also significant theoretical uncertainties, such as the rate of star-cluster dissolution through internal evolution and external agents, and uncertainties in stellar models that define the mass-to-light ratio of young clusters and their ages.

The above discussion makes it apparent though that because cluster birth is associated with a large fraction of stars being lost from their embedded cluster, the physically relevant distribution function is the \( MF \) of embedded clusters (ECMF). The ECMF accounts for all the stars born in a star-cluster. The ECMF differs from the standard, or classical “initial” mass function of bound and gas free star clusters (ICMF). The ICMF is estimated either by observing ensembles of young star clusters with the above mentioned uncertainties, or by inferring initial cluster masses by correcting observed cluster masses for mass-loss through stellar and “standard” stellar-dynamical evolution.

This contribution addresses the issue of how the ICMF is related to the ECMF. We define the ICMF to be the \( MF \) of an ensemble of gas-free star clusters in which each cluster is bound and virialised within the tidal field but not older than a critical time \( t_{ICMF} \). As the critical time we take the shortest between the stellar-evolution time-scale, \( t_{st} \), and the initial half-mass relaxation time, \( t_{rel} \), of the virialised object. Restriction to ages younger than the stellar-evolution time-scale (about \( 10^{7-8} \) yr) is necessary because by this time a cluster loses more than about 10 per cent of its mass through stellar evolution (Kroupa 2002b). For clusters with \( t_{st} > t_{rel} \) the restriction to ages younger than \( t_{rel} \) is necessary in order to limit mass loss from the cluster through two-body relaxation. The definition of the ICMF is important for the construction of the ICMF from observational data. As shall be shown in this paper, structure in the ICMF allows star-cluster formation theories to be constrained.

The article is laid out as follows. In §2 the concept of inferring an initial cluster mass for an observed present-day cluster using the classical, or usually adopted, method is explained, and in §3 we consider characteristic time-scales involved in the formation of star clusters. We use these to sub-divide the ensemble of embedded clusters into three types. This sub-division is used in §4 as an ansatz for parametrising the transformation of embedded cluster mass to (initial) bound cluster mass after removal of the residual gas, allowing the transformation of the ECMF to the ICMF to be computed. The practical construction of the ICMF is outlined in §5. §6 presents a short application of the concepts raised with this contribution by addressing the origin of the population II stellar spheroid. The conclusions are presented in §7.

2 THE CLASSICAL INITIAL MASS OF A CLUSTER

The ‘standard’ method of inferring the initial mass of a star cluster in the solar vicinity amounts to estimating its stellar-dynamical evolution which is mostly driven by evaporation of stars from the cluster through two-body relaxation, and by
The present observed state of a cluster can thus be mapped to an initial state, which corresponds to the case $\epsilon = 1$ and initial dynamical equilibrium. Tidal shocking from passages of the cluster through the plane of the Milky-Way disk, which lead to additional mass loss from the cluster, can be readily incorporated in those cases where a cluster is known to have a significant out-of-plane motion.

The contrast between the actual and the classical evolution tracks is demonstrated in Fig. 1. The figure shows the evolution of an embedded cluster with $\epsilon = 0.33$ that expels its gas faster than a dynamical time after an embedded period lasting for 0.6 Myr (model A in KAH). The model cluster is on a circular orbit at the solar distance in the plane of the Milky-Way disk. The mass in bound stars decays rapidly after gas expulsion, while the core radius expands significantly. The bound cluster stabilises by about 40 Myr after core-contraction to $R_{\text{core}} \approx 1$ pc near a mass of 1500 $M_\odot$. Further evolution is driven by standard evaporation through two-body relaxation and stellar evolution which inhibits further significant core contraction.

Now assume there exists, in the solar vicinity, a star cluster on a circular orbit in the plane of the disk and with an age of 100 Myr inferred from its stellar population. It is represented by the open circles in Fig. 1. The classical evolution tracks would fit this cluster with an initial mass represented by $M_{\text{cl}} \approx 1500$ $M_\odot$ and $R_{\text{core}} \approx 1$ pc. A stellar-dynamical computation beginning from this initial state would reproduce the cluster. The initial state of a cluster is thus degenerate as far as the mass and radius are concerned, but additional information such as the distribution of binary-star periods within the observed cluster helps to lift some of the degeneracy (Kroupa 2000). Here the argument is essentially that the widest binaries constrain the maximum concentration of the cluster.

As an example, Portegies Zwart et al. (2001) infer the initial mass of the Pleiades (about 100 Myr old), Hyades (about 600 Myr) and Praesepe (400–900 Myr) to have been about the same (1600 $M_\odot$), but such classical estimates do not account for the gas-expulsion ($\epsilon < 0.5$) and associated loss of stars.

Although initial cluster parameters do not reflect a physically relevant initial configuration, they nevertheless provide a potentially powerful path for probing the physics involved in star-cluster formation.

### 3 TIMESCALES AND CLASSES OF EMBEDDED CLUSTERS

General consideration of the crossing time-scale and the time-scale involved in the assembly of star clusters relative to the gas-expulsion time-scale allows an assessment of the fraction of stars lost from an embedded cluster. This approach leads to a rough but useful sub-division of embedded clusters into three classes.

The introduction of a few generic quantities is useful for the following discussion. The nominal global crossing time of an embedded cluster can be defined as $t_{\text{cr}} = 2R/{\sigma_{3D}}$, where the nominal global three-dimensional velocity dispersion of the stars in the embedded cluster is $\sigma_{3D} = (G(M_{\text{cl}} + M_{\text{G}})/R)^{1/2}$, and $R$ is the gravitational radius of the embedded cluster ($G = 0.0045$ pc$^2$ $M_\odot^{-1}$ Myr$^{-2}$). Perusal of observational data (Lada 1999; Clarke, Bonnell & Hillenbrand 2000; Wilking 2001) shows that $R \approx 0.5$–1.5 pc despite a variation of $M_{\text{cl}}$ by many orders of magnitude. We therefore set $r = 1$ pc, and consider $M_{\text{cl}}$ to be the primary variable. A constant SFE of $\epsilon = 0.33$ is adopted for all cluster masses. This appears to be consistent with observational data for low-mass embedded clusters (Lada 1999) and massive star-burst clusters (Nürnberg et al. 2002).

The number of stars in an embedded cluster is $N_{\text{cl}} = M_{\text{cl}}/m_{\text{av}}$, where $m_{\text{av}} = 0.4$ $M_\odot$ is the average stellar mass. The star-cluster formation timescale is $t_{\text{fcl}}^{\text{cl}} = t_{\text{cr}}$ for a few Myr (Hartmann 2001). The time over which a significant fraction of the gas is removed from an embedded cluster, the gas-expulsion time scale, $\tau_{G}$, depends on the presence of O stars and on the depth of the potential well. Details are still very uncertain, but three general types of behaviour allow a rough sub-division of embedded clusters:

- **Type I**: These sparse embedded clusters contain no O stars, and thus $N_{\text{cl}} \lesssim 10^3$ assuming a standard stellar IMF (Kroupa 2002b). Sparse embedded clusters remove their gas on a time-scale comparable to the cluster-formation time-scale because the primary mechanism is the accumulation of the increasing number of outflows as the stellar population builds up (Matzner & McKee 2000). This is comparable to the nominal crossing time. We thus have

  \[
  \tau_{G} \approx t_{\text{fcl}}^{\text{cl}} \approx t_{\text{cr}}.
  \]

  Such systems may thus not be mixed and appear sub-structured. These clusters lie in the region between adiabatic and explosive evolution. Members of this class are the embedded clusters in Orion cloud L1630 (Lada & Lada 1991) and the embedded cluster ρ Oph (Bontemps et al. 2001).

  In the adiabatic limit and for an isolated cluster all initially bound stars remain bound despite significant expansion (Hills 1980; Mathieu 1983). A tidal field from the parent molecular cloud and the Milky Way unbinds a significant fraction of the expanded cluster though, so that the fraction of stars, $f_{\text{st}}$, that remain bound after adiabatic gas removal will be reduced significantly. This fraction depends on the details of the mass profile and the tidal field, and here it is assumed very roughly that the “initial” cluster contains a fraction $f_{\text{st}} \approx 0.5$ of $N_{\text{cl}}$.

  In the explosive limit and with a solar-neighbourhood tidal field and stellar evolution, $f_{\text{st}} < 0.3$ (KAH).
Figure 1. The embedded cluster has a significantly different mass and core radius than the classical initial cluster. The solid line in the upper panel shows the evolution of the stellar mass in an Orion-Nebula-like embedded cluster ($M_{\text{cl}} = 4054 M_\odot$, model A in KAH), while the solid line in the lower panel shows the evolution of the core radius. The open circle is a hypothetical open cluster datum. The dashed lines indicate the classical approach to estimate the initial configuration. The classical approach only takes account of mass loss in a tidal field driven by standard evaporation through two-body relaxation and stellar evolution, as well as the rare ejected stars. The classical approach infers an initial mass $M_{\text{icl}} \approx 1500 M_\odot$ and an initial core radius $R_{\text{core}} \approx 1$ pc.

Initial clusters with $N_{\text{icl}} = f_{\text{st}} N_{\text{necl}} \lesssim 500$ evaporate completely due to two-body relaxation within less than 1 Gyr (Terlevich 1987; Kroupa 1995b, de la Fuente Marcos 1997). Such clusters would not be evident in star-cluster catalogues because their low density precludes easy identification above the background Galactic field density.

• **Type II**: $10^3 \lesssim N_{\text{necl}} \lesssim 10^5$. These embedded clusters are rich enough to contain between one and about 150 O stars but are not so massive as to have a nominal velocity dispersion larger than the sound-speed of the ionised gas. The nominal crossing time is shorter than the cluster-formation time-scale ($t_{\text{cr}} < \tau_{\text{cl}}^{\text{cl}} \approx 1 - \text{few Myr}$). The cluster is thus approximately in virial equilibrium and mixed for times $t > t_{\text{cr}}$, since most of the proto-stars have enough time to cross the system and “virialise” before the gas is expelled.

Gas expulsion occurs on a time-scale given by the time it takes for the hot ($10^4$ K) ionised gas to expand outwards as a result of the overpressure. This expansion occurs, approximately, with the sound-speed, $v_s \approx 10$ km/s

$$\tau_{\odot} \approx \frac{R}{v_s} \lesssim t_{\text{cr}}.$$  \hfill (2)

The gas is thus removed “explosively”, implying $f_{\text{st}} < 0.3$ (KAH).

An example of such a system is the ONC (Hillenbrand & Hartmann 1998).
• Type III: \( N_{\text{cl}} \geq 10^5 \). These clusters contain more than a few hundred O stars and have a velocity dispersion \( \sigma_{3D} > v_s \).

The nominal velocity dispersion is thus larger than the sound speed of the hot ionised gas, so that the gas cannot leave through thermal expansion. Supernovae may be needed to do the job. In this case the gas-expulsion time-scale may take several Myr after the first supernova explodes, if more than one supernova is needed (Goodwin, Pearce & Thomas 2002). Thus

\[ \tau_G \gg t_{\text{cr}}. \]  

When this is true, then the embedded cluster reacts approximately adiabatically and \( f_{\text{st}} \approx 0.5 \). An example of a very young cluster of this type is R136 in the LMC which appears to have already removed most of its gas (Hunter et al. 1995; Selman et al. 1999).

Even if the details are still very uncertain, the above sub-division demonstrates that a variation of \( f_{\text{st}} \) with \( M_{\text{cl}} \) should be expected. The next section will demonstrate that such a variation leaves its imprint in the ICMF. This is why the ICMF is of interest and may be used to infer the function \( f_{\text{st}}(M_{\text{cl}}) \).

4 THE INITIAL CLUSTER MASS FUNCTION

In what follows two examples for \( f_{\text{st}}(M_{\text{cl}}) \) are considered to outline the associated transformation from an ECMF to an ICMF. These examples are intended as a demonstration of the information carried by structure in the ICMF.

The number of embedded clusters in the mass interval \( M_{\text{cl}}, M_{\text{cl}} + dM_{\text{cl}} \) is

\[ dN_{\text{cl}} = \xi_{\text{cl}}(M_{\text{cl}}) dM_{\text{cl}}, \]  

where \( \xi_{\text{cl}}(M_{\text{cl}}) \) is the ECMF. Assuming all embedded clusters form bound clusters so that \( f_{\text{st}}(M_{\text{cl}}) \) is continuous, the ICMF follows from

\[ \xi_{\text{cl}}(M_{\text{cl}}) = \xi_{\text{cl}}(M_{\text{cl}}) \frac{dM_{\text{cl}}}{dM_{\text{cl}}} \]  

To fix ideas and based on the discussion in §3 we consider the following two examples for the transformation function

\[ M_{\text{cl}} = f_{\text{st}}(M_{\text{cl}}) M_{\text{cl}}, \]  

between embedded and initial cluster mass, assuming the average stellar mass, \( m_{\text{av}} \), remains unchanged. The two examples are plotted in Fig. 3. Note that the present example functions \( f_{\text{st}}(M_{\text{cl}}) \) have no physical meaning apart from providing simple parametrisations of the type of behaviour expected (§3).

Example A:

\[ f_{\text{st}} = \begin{cases} 0.2 & : N_{\text{cl}} < 8.5 \times 10^4, \quad \text{explosive evolution for types I and II}, \\ 0.5 & : N_{\text{cl}} \geq 8.5 \times 10^4, \quad \text{adiabatic evolution for type III, assuming winds play no role}. \end{cases} \]  

Note that this example does not allow for near-adiabatic evolution of sparse clusters.

Example B:

\[ f_{\text{st}}(M_{\text{cl}}) = 0.5 - 0.4 \mathcal{G}(IM_{\text{cl}}; IM_{\text{expl}}, IM_{\text{expl}}), \]  

where \( \mathcal{G}(IM_{\text{cl}}) \) is a Gaussian function in \( IM_{\text{cl}} \equiv \log_{10}(M_{\text{cl}}/M_\odot) \),

\[ \mathcal{G}(IM_{\text{cl}}; IM_{\text{expl}}, IM_{\text{expl}}) = e^{-\left(\frac{IM_{\text{cl}} - IM_{\text{expl}}}{\sigma_{IM_{\text{cl}}}}\right)^2}, \]  

with mean \( IM_{\text{expl}} \equiv \log_{10}(M_{\text{expl}}/M_\odot) \) and variance \( \sigma_{IM_{\text{cl}}} \) and scaled such that \( \mathcal{G}(IM_{\text{cl}} = IK_{\text{expl}} = 1) \).

This form for \( f_{\text{st}} \) together with \( IM_{\text{expl}} = 4 \) and \( \sigma_{IM_{\text{cl}}} = 0.5 \), assumes that embedded clusters of type I and III evolve approximately adiabatically as the gas is removed (\( f_{\text{st}} \approx 0.5 \)), while embedded clusters with a mass near \( IM_{\text{expl}} = 10^4 M_\odot \) suffer explosive gas loss and consequently lose up to 90 per cent of their stars.

Fig. 3 shows a power-law ECMF,

\[ \xi(M_{\text{cl}}) = AM_{\text{cl}}^{-\beta}, \]  

and the resulting ICMF.

In example A the resulting ICMF has a gap in the mass range 6800 to 17000 \( M_\odot \), which arises because embedded clusters
with masses $M_{\text{ecl}} \geq 3.4 \times 10^4 M_\odot$ lose 50 per cent of their stars, while embedded clusters with $M_{\text{ecl}} < 3.4 \times 10^4 M_\odot$ lose 80 per cent of their stars.

For example B the ICMF has a depression over a similar mass range but a peak near $M_{\text{icl}} = 10^3 M_\odot$ that can be very sharp if $\sigma_{lM_{\text{ecl}}} \lesssim 0.5$. These features follow by differentiating eq. 6, but the Monte-Carlo approach adopted for the lower panel of Fig. 3 can readily be extended to include more complex effects, such as merging clusters. The turnover of the ICMF near $10^5 M_\odot$ is noteworthy considering empirical evidence for a turnover of the young-cluster MF in the Antennae galaxies near this mass (Fritze-v. Alvensleben 1999; Whitmore et al. 1999).

The fraction of clusters in the peak can be estimated by computing the number of initial clusters with masses $700 \leq \frac{M_{\text{icl}}}{M_\odot} \leq 1300$, and comparing this number to the number of all open clusters that are expected to form in the Milky Way disk in the solar vicinity and that also enter catalogues, i.e. that have evaporation time-scales longer than 1 Gyr. These span, roughly, $170 \leq \frac{M_{\text{icl}}}{M_\odot} \leq 10^4$. The fraction in the peak amounts to 9.1 per cent for $\sigma_{lM_{\text{ecl}}} = 0.8$ and to 28 per cent for $\sigma_{lM_{\text{ecl}}} = 0.5$.

5 CONSTRUCTING AN ICMF

It is interesting that the peak in the ICMF of example B lies close to the (classical) initial mass of the Pleiades, Praesepe and Hyades clusters. The computations by Portegies Zwart et al. (2001) suggest that these three clusters form approximately one evolutionary sequence (§ 3). This apparent ubiquity of such similar clusters of different age, and the absence of clusters with initial masses in the range from a few $\times 10^3$ to a few $\times 10^4 M_\odot$ within a small region of space around the Sun indicates that the shoulder seen on Fig. 3 around $1000 M_\odot$ may be a real feature of the ICMF. The previous section indicates that if $\sigma_{lM_{\text{ecl}}} \approx 0.5$ then roughly 30 per cent of all open clusters may have an initial mass near $1000 M_\odot$.

To verify the shoulder or otherwise we need an empirical estimate of the ICMF. To achieve this, the following steps will have to be undertaken:

![Figure 2. Transformation factor as a function of embedded cluster mass for the two examples considered in the text.](image-url)
On the mass function of star clusters

Figure 3. The logarithmic embedded cluster mass function (ECMF, $M_{cl} = M_{ecl}$) and the logarithmic initial cluster MF (ICMF, $M_{cl} = M_{icl}$), for the two examples considered in the text $\log_{10}(\xi_{cl}(lM_{cl})) = \log_{10}(\xi_{cl}(lM_{ecl})))$, $\xi_{cl} = (M_{cl} \ln 10) \xi^{ecl}$. **Upper panel:** the ECMF is eq. 10 with $\beta = 1.5$, and the ICMF is eq. 5. **Lower panel:** the ECMF and ICMF are obtained by picking $10^6$ cluster masses at random from the ECMF [eq. 4 with $\beta = 1.5$] and transforming each embedded cluster mass to the initial cluster mass (eq. 6), and binning both samples to create the histograms shown. The ICMF is obtained assuming $1M_{\odot}^{ecl} = 4$ for two values for $\sigma_{1M_{\odot}^{ecl}}$.

(i) Construct a representative sample of star clusters. A star-burst galaxy may be used, but here the problem is that the observer does not know which dynamical state the young clusters are in. A better approach will be to construct a volume-limited open cluster sample in the solar vicinity, say out to 500 pc. This has the disadvantage that the clusters span a very large range of ages and that only the low-mass part of the CMF is being sampled. The advantage is that the clusters can be selected to be older than, say, 60 Myr thus ensuring that the entire evolution driven by gas expulsion is finished (Fig. 1), and that the Galactic tidal field is reasonably well known.

(ii) Construct “classical” (§1) star-cluster evolution tracks (e.g. by performing $N$–body computations á la Terlevich 1987; Portegies Zwart et al. 2001).

(iii) Infer the initial cluster mass of each cluster in the representative sample by fitting to the classical tracks.

(iv) Create the ICMF from the ensemble of initial cluster masses.

Any structure within this empirical ICMF should indicate the dominant physics involved in cluster formation, if it is assumed that the ECMF is a featureless power-law. Conversely, by applying a favoured cluster-formation theory (i.e. essentially a model for $f_{st}(M_{ecl})$) to the so-obtained ICMF, the ECMF may be estimated for a comparison with the MF of molecular cloud cores. We note in passing that the MF of star-cluster-forming molecular cloud cores is related to the ECMF via

$$\xi_{core}(M_{core}) = \xi_{ecl}(M_{ecl}) \frac{dM_{ecl}}{dM_{core}}.$$  (11)
If we assume that the SFE does not depend on the mass of the cloud core, \( M_{\text{core}} = M_{\text{cl}}/\epsilon \), then
\[
\xi^{\text{core}}(M_{\text{core}}) = \epsilon \xi^{\text{cl}}(M_{\text{cl}}).
\] (12)

6 THE GALACTIC POPULATION II SPHEROID

The ancient Galactic stellar spheroid, that consists of population II field stars plus globular clusters, totals to a mass of about \( 5 \times 10^7 \leq M_{\text{ph}}(M_\odot) \leq 5 \times 10^8 \) and contains about 150 globular clusters with masses in the range \( 10^4 - 5 \times 10^6 M_\odot \) (e.g. Binney & Merrifield 1998). Genesis of this stellar halo plus globular cluster system is of cosmological interest, because it pre-dates the formation of the rest of the Galaxy. The stellar spheroid was born within a time-span of about 1-3 Gyr roughly 13 Gyr ago, corresponding to a SFR of 0.017 – 0.5 \( M_\odot/\text{yr} \).

Larsen (2001) notes a well-defined correlation between the absolute V-band magnitude of the brightest cluster, \( M_V^{\text{br}} \), and the SFR in the galaxy hosting the star-cluster system. This correlation implies, for the above SFR, that the most massive cluster formed during the assembly of the Galactic spheroid may have had \( M_V^{\text{br}} \approx -11 \) to -12 when about 20 Myr old. This corresponds to globular-cluster masses of roughly \( 10^{5-6} M_\odot \). That intensively interacting gas-rich galaxies profusely form massive clusters due to the induced high-pressures in the gas clouds is well established empirically (Elmegreen et al. 2000), and it is likely that the Galactic spheroid was likewise assembled during a brief but violent epoch of merging gas-rich sub-systems. The question we now address is if there exists an ECMF that can account for the entire Galactic spheroid, or if the population II field stars and the globular clusters stem from different SF modes.

This problem is approached by assuming the ECMF is a power-law (eq. 10), and that the SF-burst samples clusters with masses in the range \( M_{\text{low}} = 5 M_\odot \) to \( M_{\text{max}} = 1 \times 10^7 M_\odot \) from this ECMF. Note that \( M_{\text{max}} \approx M_{\text{Cen}}/f_{\text{st}} \), where \( M_{\text{Cen}} \) is the mass of the most massive globular cluster, \( \omega \), Cen, and \( f_{\text{st}} = 0.5 \) is assumed for this cluster. The total mass in the ECMF is then
\[
M_{\text{ph}} = N_{\text{GI}} \frac{1 - \beta}{2 - \beta} \left( \frac{M_{\text{max}}^{2-\beta} - M_{\text{low}}^{2-\beta}}{M_{\text{max}}^{1-\beta} - M_{\text{low}}^{1-\beta}} \right),
\]
where \( N_{\text{GI}} \) is the number of embedded clusters in the mass-range \( M_1 = 10^4 M_\odot \) to \( M_{\text{max}} \). These are the precursors of the present-day globular clusters. Vesperini (1998) and Baumgardt (1998) show that a power-law ECMF with \( \beta \approx 2 \) evolves to the observed Gaussian luminosity function of the present-day globular cluster system. According to Vesperini’s analysis, \( N_{\text{GI}} \approx 300 \).

The solution space is shown in Fig. 4. The results presented in Fig. 4 suggest that the entire Galactic spheroid could have been populated by embedded clusters being sampled from a power-law ECMF with \( 0.9 < \beta < 2.6 \) for \( N_{\text{GI}} \approx 300 \). Note that Vesperini’s result (\( \beta = 2, N_{\text{GI}} = 300 \)) is just compatible with Fig. 4. It has to be kept in mind that \( N_{\text{GI}} \) is somewhat uncertain given that the Milky-Way potential and its evolution, which determines how many clusters are tidally destroyed over a Hubble time, is ill-constrained (Dehnen & Binney 1998).

The result obtained here remains valid if the Galactic spheroid accumulated through a number of more-or less discrete star-burst events, as long as each one of them produced a similar ECMF.

7 CONCLUDING REMARKS

There are reasons to believe that a young star cluster emerges from the molecular cloud by losing more than 50 per cent of its stars. This fraction, \( 1 - f_{\text{st}} \), correlates with the rate with which the unused gas is expelled. This rate is a function of cluster mass through the presence or absence of O stars and the depth of the potential well of the cluster. Low-mass clusters that contain no O stars (type I) probably evolve adiabatically, while clusters containing between about 1000 and \( 10^5 \) stars (type II embedded clusters) probably expel their unused gas explosively, that is, more rapidly than a crossing time. More massive clusters (type III) may again evolve approximately adiabatically because the ionised gas may not be able to leave the deep potential well within a crossing time.

While the likely behaviour of the function \( f_{\text{st}}(M_{\text{cl}}) \) can be deduced from general physical arguments, important uncertainties remain. For example, we do not know yet how good the approximation \( f_{\text{st}} \approx 0.5 \) for adiabatic evolution in a tidal field is. This needs to be quantified using \( N\)-body experiments, and much more work is necessary to quantify the rate with which gas is expelled from embedded clusters of all mass.

However, the present study demonstrates that a dependence of the mass-loss rate on embedded cluster mass leads to structure in the ICMF, even if the ECMF is a featureless power law. Such structures may be a gap near \( 10^4 M_\odot \) if type I and type II embedded clusters remove their gas faster than a dynamical time. If, on the other hand, type I and type III clusters rid themselves of natal gas over time-scales comparable to or longer than a crossing time, while type II clusters suffer explosive gas loss, the MF of gas-free but bound clusters shows a broad depression between about 1000 and \( 10^{4.5} M_\odot \), while having a pronounced peak near \( 10^5 M_\odot \).
The allowed range of ECMFs that can synthesise the population II Galactic stellar spheroid. The ECMF is a power law (eq. 10) with embedded cluster masses in the mass range $5 \leq M_{\text{ecl}}/M_\odot \leq 1 \times 10^7$. The power-law index, $\beta$, is plotted as a function of the number of clusters, $lN_{\text{Gl}} \equiv \log_{10} N_{\text{Gl}}$, in the mass range $10^4 \leq M_{\text{ecl}}/M_\odot \leq 1 \times 10^7$ needed such that the total stellar spheroid mass (field stars plus globulars, eq. 13) is $5 \times 10^7 \leq M_{\text{sph}}/M_\odot \leq 5 \times 10^8$.

These examples serve to illustrate the type of features that may be present in the ICMF. The most important insight gained with this analysis is that structure in the ICMF may indicate in a statistical sense the relevant gas-removal processes. It is therefore, at least in principle, possible to constrain star-cluster formation theories by studying the ICMF.

An effect not taken into account here but that may affect the ICMF is the merging of multiple star clusters to form a single more massive cluster. Also, an ensemble of embedded clusters with equal stellar mass is likely to have a distribution of SFEs and half-mass radii, which will broaden features in the resulting ICMF. Such additional sophistication can be included with a Monte-Carlo approach once $f_{\alpha}(M_{\text{ecl}})$ is better quantified theoretically.

Finally, it was shown that the population II stellar spheroid plus its globular clusters can be assembled if all its stars formed in ensembles of embedded clusters that are sampled from a power-law ECMF. There is therefore no need to postulate two modes of star formation, where one mode made the globular clusters and the other mode the population II field stars.

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