Radial dependence of self-organized criticality behavior in TCABR tokamak

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Abstract. In this work we present evidence of the self-organized criticality behavior of the plasma edge electrostatic turbulence in the tokamak TCABR. Analyzing fluctuation data measured by Langmuir probes, we verify the radial dependence of self-organized criticality behavior at the plasma edge and scrape-off layer. We identify evidence of this radial criticality in statistical properties of the laminar period distribution function, power spectral density, autocorrelation, and Hurst parameter for the analyzed fluctuations.

1. Introduction

Electrostatic turbulence observed at the plasma edge tokamaks is a severe limitation to plasma confinement [1, 2]. There is considerable theoretical and experimental progress on the understanding of this turbulence, however a complete description of the observations does not have yet been achieved [3]. Thus, basic knowledge of the statistical fluctuation or driven transport fluctuations are still necessary to improve the understanding of plasma edge turbulence [4].

Quantitative investigations of the electrostatic fluctuations using spectral analysis show that drift waves are destabilized in the confining magnetic field generating a turbulent spectrum [5]. Moreover, dynamical diagnostics used to describe fluid turbulence have been applied to analyze the plasma edge turbulence, as the return-time statistics [4, 6], recurrence analyses [7] and multifractality [8]. Furthermore, the structure of intermittent plasma signals can be studied by analyzing fat tails and long-range correlations. Some of the methods to characterize the long-range correlations from experimental time series evolve autocorrelation functions, power spectral densities, and probability distribution functions (PDFs).

Recent results show analysis of electrostatic fluctuations from confined magnetically plasmas that have been described by their space and time self-organized similarity behavior (SOC) [9, 10].
Self-organized criticality (SOC) is a general property of dynamical systems with an attracting critical state, displaying scale invariance in both spatial and temporal degrees of freedom [11]. The time evolution of systems displaying SOC is characterized by the presence of self-organized structures, or avalanches, which obey a power-law statistical distribution. The paradigmatic example of a system displaying SOC is a sandpile having a constant injection of sand [8-10]. The excess mass in the surface of the sandpile is released through avalanches which were found to obey such power-law distribution. SOC has been viewed as a mechanism by which there is spontaneous emergence of complexity from simple local interactions. This encourages the search for SOC in complex systems as magnetically confined plasmas [12, 13]. In the last years, several SOC characteristics were identified in the plasma edge turbulence, as the existence of critical average gradients and profile resilience to the power-law power spectra of plasma parameters fluctuations (density, temperature, magnetic field) [14]. Many numerical models with SOC properties reproduce experimental turbulence phenomenology [3-7] and features such as transport barriers [3, 5]. SOC may be used to explain the existence of noise $1/f$ ($f$ is the frequency) in several experimental observations [14], and the self-similar character of electrostatic fluctuations at the plasma edge [13]. Moreover, SOC gives us connection between the scale invariance in space and time.

Here, we present evidence of the self-organized criticality behavior in the electrostatic turbulence data obtained with Langmuir probes in TCABR tokamak [15]. Our analysis also reveals the radial dependence of the algorithms that characterize the observed criticality.

The paper is organized as follows: Section 2 we present the experimental setting. Section 3 treats of the analysis of the experimental results relate to self-organized criticality behavior. The last section is devote to our conclusions.

2. Tokamak data
The analyzed experiments were performed in a hydrogen circular plasma in the Brazilian tokamak TCABR [15] (major radius $R=61$ cm and minor radius $a=18$ cm). The maximum value of the plasma current is 110 kA, with duration 100 ms, the hydrogen filling pressure is $3 \times 10^{-4}$ Pa, and the toroidal magnetic field is $B_T = 1.1$ T. The floating potential was measured by two Langmuir probes, poloidally separated by 0.4 cm. The probes are mounted on a movable shaft that can be displaced radially from $r = 15.0$ to 23.0 cm, with respect to the center of the plasma column. As a matter of fact, we focus on the range from 16.5 to 21.0 cm ($r/a$ from 0.92 to 1.16) to cover the plasma edge and also the so-called scrape-off layer (SOL), i. e., the low density plasma layer comprised between the plasma column and the vessel wall.

The probe displacement occurs only for separate discharges, such that it does not disturb the plasma due to the movement of the probe. The measurements were performed at a sampling frequency of 1 MHz, and the measuring circuit has a 300 kHz bandwidth to avoid aliasing [5]. Fig. 1 shows the typical time evolution of a considered plasma discharge in TCABR. The plasma current Fig. 1(a) grows slowly in the first 85 ms. The electron density evolution indicated by Fig. 1(b) exhibits the region analysed that is a plateau with value $n_e = 1.1 \times 10^{19}$ m$^{-3}$. We are interested in the floating electrostatic potential signals in the radial region between inside (Fig. 1c) and outside the plasma column, which shows irregular fluctuations. We analyze data from time intervals, as the one in the window indicated by vertical lines in Fig. 1, chosen before MHD activity start increasing and changing the plasma turbulence [16]. Moreover, to study the self-organized criticality we consider the fluctuations around the mean electrostatic potential as the time series of Fig 1(d).

3. Radial dependence of self-organized criticality behavior in experimental data
In order to investigate the radial dependence of self-organized criticality on the TCABR plasma turbulence we analyze some typical SOC evidences we found in the probability density function
Figure 1. Time evolution of plasma discharge in TCABR tokamak. (a) Plasma current, (b) central chord plasma mean density, (c) floating electrostatic potential for a typical discharge inside the limiter \(r=17\) cm and (d) its fluctuation during the analyzed time interval, from \(t=60\) ms to 70 ms (indicated by dashed line).

(PDF) of laminar periods, auto-power frequency spectrum, autocorrelation function, and Hurst parameter on TCABR float potential fluctuations [9].

Figure 2 shows the PDFs of the laminar period duration for different radial positions. The laminar period is the time the amplitude of the fluctuations remains below a threshold (Fig. 2 presents the PDFs for thresholds defined as \(\delta = 2\sigma\), where \(\sigma\) is the standard deviation). Regarding Figs. 2(b) and 2(c), we can observe a linear fit that suggests a power-law decay, while in Fig. 2(a) and (d) a power-law decay is not possible to be fitted. In doing so, the PDF shows a power-law dependence around the plasma column radius.

In SOC systems, the Fourier spectra are expected to be similar to those applied to \(1/f\) noise signal for a given range of frequencies [17]. This behavior can be inferred from the existence of avalanches or transport. The shape of the power frequency spectrum is characterized by a \(1/f^\alpha\) power-law spectrum, where \(\alpha\) is from 0 to 2 or higher separated by three regions quite different according to the theory. It has been discussed theoretically that transport events and electrostatic fluctuations at the plasma edge in magnetically confined plasmas have some characteristics of self-organized critical systems [9]. For the TCABR tokamak data measured at \(r/a = 1\) (Fig. 3b) and \(r/a = 1.05\) (Fig. 3c), the frequency spectrum shows two distinct regions in the frequency range with decay indexes of -1 and -4, respectively. The range involving the \(1/f^4\) dependence occurs for the higher frequency part of the spectrum \((\geq 200\) kHz\). It means that small scale events are related to low power events in the obtained spectrum due to plasma background or instrumental noise. Furthermore, in Fig. 3 (b) and (c) we can observe a range with a \(1/f\) dependence, this behavior has been related to the transport events linked with
self-organized similarity behavior. Moreover, in this figure the separation between frequency ranges are smooth. As a matter of fact, these results are in agreement with other experimental observations [14, 13]. On the other hand, Fig. 3(a) and (d) do not present regions with power-law $1/f$.

Figure 3. Power spectrum of the floating potential fluctuation at radial position (a) $r/a = 0.92$, (b) $r/a = 1$, (c) $r/a = 1.05$ and (d) $r/a = 1.16$.

The autocorrelation function (ACF) was obtained from the same shots considered in Fig. 3
is showed in Fig. 4. The width of the peak (the e-folding time of the ACF) is taken to be the decorrelation time of the local turbulence (see black dashed line). In Fig. 4(b) the e-folding time is nearby 0.05 ms [18] (at \( r/a = 1 \)). Fig. 4(a-d) shows that the ACF in the the e-folding time depends on the radial coordinate [9]. Moreover, the autocorrelation function close to the plasma border shows an extended tail at large delay time. The existence of long-time correlation, at plasma edge, enforces the evidence of SOC behavior on TCABR plasma edge.

![Autocorrelation function (ACF) of the floating potential fluctuations measured at radial position](image)

**Figure 4.** Autocorrelation function (ACF) of the floating potential fluctuations measured at radial position (a) \( r/a = 0.92 \), (b) \( r/a = 1 \), (c) \( r/a = 1.05 \) and (d) \( r/a = 1.16 \).

Other evidence of self-organized criticality on TCABR fluctuations is obtained through the Hurst parameter (with R/S method) [13, 19]. The parameter \( H \) is the self-similarity parameter. The value of the Hurst parameter varies between 0 and 1. For \( H = 0.5 \), the process is not strongly correlated. Moreover, for \( 0 \leq H \leq 0.5 \) the time series are characterized like anti-persistent or anti-correlated, and for \( 0.5 < H \leq 1 \) the signal is persistent or auto-correlated. Fig. 5 shows the radial dependence of the Hurst parameter of the TCABR plasma turbulence (red triangles), where we can see that the Hurst parameter decreases from the radial position \( r/a = 1 \) towards inside the plasma column (plasma edge). The same situation is observed for Hurst parameters that decreases in the scrape-off layer plasma according to the radial position moving away from \( r/a = 1 \). The variation of the TCABR Hurst values varies from 0.55 (on the scrape-off layer) to a maximum equal to 0.92 on the limiter (\( r/a = 1 \)), indicating that the plasma turbulence, as well as plasma transport, present high self-organized behavior.

4. Conclusions

We analyzed the plasma edge turbulence in TCABR tokamak to verify the radial dependence of self-organized criticality behavior of the experimental fluctuating floating electrostatic potential, measured by Langmuir probes. The dependence of the laminar period probability density function at \( r/a \approx 1 \) is best recognized in a log-log plot, where the linear fit suggests a power-law decay. The frequency spectrum analysis presents scaling behavior. Moreover, we can observe that the autocorrelation function shows an extended tail at large delay times and we also obtain a higher Hurst parameter, indicating the existence of self-organized criticality in TCABR fluctuations.
Figure 5. Radial dependence of the Hurst parameter (red triangles). The vertical black line marks the position of the plasma column ($r/a = 1$).

We have observed that our results are in agreement with SOC behavior around the radial position $r/a = 1$, as well as plasma transport mechanisms based on avalanches.

In tokamaks, the SOC radial dependence at the plasma edge reported in this article may be associated with coherent or recurrence structures propagating at this region. A theoretical description of plasma edge turbulence should predict these statistical properties and explain their influence on the anomalous particle transport in tokamaks.

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