Nonlinear dynamics of soft fermion excitations in hot QCD plasma I: soft-quark–soft-gluon scattering

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Abstract

Within the framework of the hard thermal loop effective theory we derive a system of Boltzmann-like kinetic equations taking into account the simplest processes of nonlinear interaction of soft fermionic and bosonic QCD plasma excitations: elastic scattering of soft-(anti)quark excitations off soft-gluon and soft-quark excitations, pair production of soft quark-antiquark excitations, annihilation into two soft-gluon excitations. The matrix elements of these processes to leading order in the coupling constant $g$ are obtained. The iterative method of calculation of the matrix elements for the higher processes of soft-mode interactions is proposed. The most general expression for the emitted radiant power induced by the effective currents and effective sources in a quark-gluon plasma (QGP) taking into account an existence of fermion sector of plasma excitations is defined. The explicit form of the linearized Boltzmann equation accounting for scattering of color(less) plasminos off color(less) plasmons is written out.

PACS: 12.38.Mh, 24.85.+p, 11.15.Kc

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1 Introduction

In our previous papers [1, 2] on the basis of a purely gauge sector of the Blaizot-Iancu equations [3] complemented by the Wong equation [4] we have studied in great detail the nonlinear dynamics of soft boson excitations in hot QCD plasma. In this and two accompanying papers we would like to enlarge the analysis carried out in Refs. [1, 2] on the fermion sector of soft plasma excitations. For solving this problem it is necessary to use the whole system of the Blaizot-Iancu equations. It is well known that hot QCD plasma including massless quarks (and antiquarks) possesses doubled dispersion relation for soft fermion plasma excitations [5]. The first branch describes normal particle excitations (in the subsequent discussion designated by a symbol “+”) with relation between chirality and helicity at zero temperature. The second branch is purely collective excitation (designated further by symbol “−”), where usual relation between chirality and helicity is flipped [6]. It has been called a plasmino [7] by analogy with the plasmon (longitudinal) mode of gluons having also a purely collective character.

Here, in the first part we have restricted ourselves to study of the processes connected with interaction of the soft fermion and boson excitations among themselves without exchange of energy with hard thermal (or external) partons. We consider the processes of nonlinear interaction of soft excitations within the framework of the kinetic approach by the determination of corresponding set of coupled Boltzmann-like kinetic equations describing some non-linear aspects of the dynamics of the soft collective excitations in high-temperature QGP. The construction of such equations is generally straightforward at a conceptual level, but it is tedious and intricate in practice. By virtue of complexity and awkwardness of the kinetic equations for soft plasma modes taking into account simultaneously the fermion and boson degrees of freedom, we will consider in details only the simplest processes of the nonlinear interaction: soft-(anti)quark–soft-gluon and soft-(anti)quark–soft-quark elastic scattering, production of soft quark-antiquark pair by fusion of two soft gluons, and annihilation of soft quark-antiquark pair into two soft-gluon excitations. At least for weakly excited system corresponding to the level of thermal fluctuations at the soft momentum scale these processes are the basic those of the nonlinear interaction of soft excitations in the medium. In deriving the kinetic equations for soft fermion and boson modes we use oversimplified approach. Proceeding from the semiclassical character of the problem, we initially assume that the structure of the collision terms is determined by Fermi’s golden rules and thus we focus our efforts exclusively on calculating the scattering probabilities. We calculate these probabilities in a direct fashion by simple extracting all relevant contributions in scattering processes \(2 \rightarrow 2\) enumerated above. We don’t give an explicit proof of a gauge invariance of the scattering probabilities obtained. This can be performed in spirit of our previous paper [8] by using the effective
Ward identities for hot gauge theories \[9, 10\].

It should be particularly noted that isolation and study of the nonlinear dynamics of soft-mode interactions only in QGP presented in this work have sufficiently artificial character and bear rather limited sense. Really, by the efforts of a number of authors (see, for example \[11, 12\]) it was shown that the dominant interactions in QGP are generally those between the soft modes of interest and the hard particles in the thermal bath. Moreover, the general transport or equilibration properties of the plasma are in fact controlled by the hard particles (those with typical momenta of order \(T\), the temperature of the system). In our following papers the interaction processes of soft and hard modes of different statistics among themselves and also radiation processes of soft modes induced by collisions of hard thermal particles will be considered in details. The construction of the effective theory for such processes has shown that it is more convenient to consider separately a theory for simpler (in the conceptual plan) interaction processes of the soft modes. Matrix elements of interaction processes of soft and hard excitations include, as constitutive blocks, the expressions defining matrix elements of self-interaction of soft excitations provided that external momenta are put on relevant mass-shells. This significantly facilitates general analysis and classification of individual terms in very complicated and tangled expressions for matrix elements of the interaction processes between the soft excitations and the hard particles including simultaneously the soft and hard modes of Fermi and Bose statistics.

Incidentally there are circumstances at which the kinetic equations for the soft quasi-particles may be relevant independently: for instance, when we are interested in the thermalization of such soft modes or in their damping rates. Besides at studying the processes with soft-gluon excitations only we have shown \[1, 2\] that there exists a certain level of intensity of plasma excitations above which the nonlinear interaction processes of soft boson excitations among themselves start to dominate over the interaction processes of soft modes with hard thermal particles, thus defining in particular qualitatively different mechanism for the damping of a soft gluons. In other words, for large values of soft-gluon occupation number (i.e. when a subsystem of soft boson excitations is far from equilibrium) dynamics of hot QCD plasma is defined by scattering processes of soft quasiparticles off each other. In the second part \[13\] we will show that and for the case of fermion sector of plasma excitations in QGP there exists a critical threshold above which the interaction processes of soft fermion and boson excitations among themselves start to play dominating role in general dynamics of the system.

In the light of discussed above a study of soft-excitations dynamics only can represent independent interest, however this is beyond the scope of our work. Our ultimate goal is the study of new (not considered earlier in the literature) possible mechanisms of energy losses of high-energy particles propagating through hot QCD medium associated with an existence in medium soft-(anti)quark excitations on a level with soft-gluon excitations.
This is a main subject of the following papers. Research of the energy losses of energetic partons in QGP at present is of great interest with respect to jet quenching phenomenon \cite{14,15}. The basic purpose of present work is development of the general technique for calculation of relevant probabilities and in particular, deriving the semiclassical formula for the emitted radiant power in QGP taking into account fermion sector of plasma excitations (section 8). This formula (with the further updating) is basic formula for defining energy losses of energetic parton (quark or gluon) travelling through quark-gluon plasma.

The derivation of the Boltzmann equation describing evolution of the number density of fermion quasiparticles in QGP was also considered by Niégawa \cite{16}. Within the framework of the Keldysh-Schwinger formalism the kinetic equations for normal and abnormal modes here, are defined from the requirement of the absence of the large contributions due to pinch singularities of the perturbative scheme proposed in Ref. \cite{16}. The formalism suggested by Niégawa is quite rigorous and possesses great generality at least for quasi-uniform systems near equilibrium. Unfortunately in this paper particular expressions for collision terms wasn’t given. In this sense, our work having more phenomenological character supplements the paper \cite{16} with examples of concrete calculation of the collision terms within the HTL effective theory.

The paper is organized as follows. In section 2, preliminary comments with regard to deriving the system of the Boltzmann equations describing the processes of nonlinear interaction of soft fermion and boson excitations in QGP are given. In section 3, the system of nonlinear integral equations for gauge potential $A^a_\mu$ and quark wave function $\psi_\alpha^i$ is written out. On the basis of perturbative solutions of these equations the notions of the effective currents and sources playing a key position in our research are given. In section 4, an algorithm of the successive calculation of the effective amplitudes defining matrix elements of the processes we are interested is presented. In sections 5, 6 and 7, the details of calculations of the probabilities for the simplest processes of interaction: soft-(anti)quark–soft-gluon and soft-(anti)quark–soft-quark elastic scattering, production of soft quark-antiquark pair by fusion of two soft gluons, and annihilation of soft quark-antiquark pair into two soft-gluon excitations are presented. Section 8 is devoted to deriving the most general formula for the emitted radiant power in QGP. It was shown how this formula within the framework of Tsytovich correspondence principle allows by a direct way to calculate the probabilities of higher order processes of nonlinear interaction of soft plasma modes. In section 9, the explicit form of linearized Boltzmann equation for colorless plasminos is written out and some speculations concerning the extension of the formalism developed to colorcharged plasminos are given. In Conclusion some features of dynamics of soft excitations is briefly discussed and connection with more traditional approaches based on imaginary- and real-time field theories is traced. Finally,
2 Preliminaries

In this paper we consider a construction of effective kinetic theory for colorless soft-quark and soft-gluon plasma excitations propagating in a hot quark-gluon plasma. We assume that localized number densities of soft-quark \(n_\pm(q,x) \equiv (n_{q}^{\pm})\), soft-antiquark \(\bar{n}_\pm(q,x) \equiv (\bar{n}_{q}^{\pm})\), and soft-gluon \(\mathcal{N}_{t,l}^{\pm}(k,x) \equiv (N_{k}^{t,l})\) excitations are diagonal in a color space

\[ n_{q}^{\pm} = \delta^{ij}n_{k}^{\pm}, \quad \bar{n}_{q}^{\pm} = \delta^{ij}\bar{n}_{k}^{\pm}, \quad \mathcal{N}_{k}^{t,l} = \delta^{ab}\mathcal{N}_{k}^{t,l}, \]

where \(i,j = 1,2,\ldots,N_c\) and \(a,b = 1,\ldots,N_c^2 - 1\) for \(SU(N_c)\) gauge group. We consider a change of the number densities of the colorless soft-(anti)quark excitations as a result of their interactions among themselves and with the colorless soft-gluon excitations.

The dispersion relations for soft-quark modes \(\omega^{\pm}(q) \equiv \omega_{q}^{\pm}\) and soft-gluon modes \(\omega^{t,l}(k) \equiv \omega_{k}^{t,l}\) are defined by equations

\[
\text{Re} \ast \Delta^{-1}(q^{0}, q) = 0, \quad \text{Re} \ast \Delta^{-1,t,l}(k^{0}, k) = 0, \quad (2.1)
\]

where

\[
\ast \Delta^{-1}(q^{0}, q) = q^{0} + |q| + \frac{\omega_{q}^{2}}{|q|} \left[ 1 - \left( 1 + \frac{|q|}{q^{0}} \right) F\left( \frac{q^{0}}{|q|} \right) \right],
\]

and

\[
\ast \Delta^{-1,t,l}(k^{0}, k) = k^{2} - \frac{3}{2} \omega_{pl}^{2} \left[ \frac{(k^{0})^{2}}{k^{2}} - \frac{k^{2}}{k^{2}} F\left( \frac{k^{0}}{|k|} \right) \right],
\]

\[
\ast \Delta^{-1}(k^{0}, k) = k^{2} \left[ 1 + \frac{3 \omega_{pl}^{2}}{k^{2}} \left[ 1 - F\left( \frac{k^{0}}{|k|} \right) \right] \right]
\]

\[ F(x) \equiv \frac{x}{2} \left[ \ln \left( \frac{1 + x}{1 - x} \right) - i\pi\theta(1 - |x|) \right] \]

are inverse quark and gluon scalar propagators. \(\omega_{q}^{2} = g^{2}C_{F}T^{2}/8\) and \(\omega_{pl}^{2} = g^{2}(2N_c + N_{f})T^{2}/18\) are plasma frequencies squared of the quark and gluon sectors of plasma excitations. Hereafter, we denote momenta of the soft-quark fields by \(q, q', q_{1}, \ldots\) and momenta of the soft-gauge fields by \(k, k', k_{1}, \ldots\).

We expect a time-space evolution of the scalar functions \(n_{q}^{(f)}\), \(f = \pm\) and \(N_{k}^{(b)}\), \(b = t, l\) to be described by the self-consistent system of Boltzmann-like equations

\[
\frac{\partial n_{q}^{(f)}}{\partial t} + v_{q}^{(f)} \cdot \frac{\partial n_{q}^{(f)}}{\partial x} = -n_{q}^{(f)} \Gamma_{d}^{(f)}[n_{q}^{\pm}, N_{k}^{t,l}] + (1 - n_{q}^{(f)}) \Gamma_{i}^{(f)}[n_{q}^{\pm}, N_{k}^{t,l}], \quad (2.2)
\]

\[
\frac{\partial N_{k}^{(b)}}{\partial t} + v_{k}^{(b)} \cdot \frac{\partial N_{k}^{(b)}}{\partial x} = -N_{k}^{(b)} \Gamma_{d}^{(b)}[n_{q}^{\pm}, N_{k}^{t,l}] + (1 + N_{k}^{(b)}) \Gamma_{i}^{(b)}[n_{q}^{\pm}, N_{k}^{t,l}], \quad (2.3)
\]
where \( \mathbf{v}_q^{(f)} = \partial \omega_q^{(f)} / \partial \mathbf{q} \) and \( \mathbf{v}_k^{(b)} = \partial \omega_k^{(b)} / \partial \mathbf{k} \) are the group velocities of soft fermionic and bosonic excitations, respectively. The generalized decay rates \( \Gamma^{(f,b)}_d \) and inverse decay rates \( \Gamma^{(f,b)}_i \) are (non-linear) functionals of the soft-(anti)quark and soft-gluon number densities. Here, for the sake of brevity we drop dependence on soft-antiquark occupation numbers \( \bar{n}_q^\pm \) on the right-hand side of Eqs. (2.2) and (2.3). The equation for \( \bar{n}_q^{(f)} \) is obtained from (2.2) by replacement \( n_q^{(f)} \rightarrow \bar{n}_q^{(f)} \).

The decay and regenerating rates can be formally represented in the form of functional expansion in powers of the soft-(anti)quark and soft-gluon number densities

\[
\Gamma^{(f,b)}_d[n_q^\pm, N_k^{t,l}] = \sum_{n=1}^{\infty} \Gamma_d^{(f,b)(2n+1)}[n_q^\pm, N_k^{t,l}], \quad \Gamma^{(f,b)}_i[n_q^\pm, N_k^{t,l}] = \sum_{n=1}^{\infty} \Gamma_i^{(f,b)(2n+1)}[n_q^\pm, N_k^{t,l}],
\]

where \( \Gamma_d^{(f,b)(2n+1)}[n_q^\pm, N_k^{t,l}] \) collect the contributions of the total \( n \)th power in \( n_q^\pm, \bar{n}_q^\pm \) and \( N_k^{t,l} \). Unfortunately, the general structure of the expressions for arbitrary \( n \) is very cumbersome and therefore we restrict our consideration to the simplest case for \( n = 1 \).

The fermion decay and regenerating rates in the lowest order of the nonlinear interaction \( (n = 1 \text{ in } \text{Eq. (2.2)}) \) can be formally represented in the following form:

\[
\Gamma^{(f)}_d[n_q^\pm, N_k^{t,l}] = \sum_{f_1 = \pm} \sum_{b_1, b_2 = t, l} \left\{ 2 \int d\mathcal{T}_{qq'\rightarrow qq''} w_{qq'\rightarrow qq''}^{(f)}(\mathbf{q}, \mathbf{k}_1; \mathbf{q}_1, \mathbf{q}_2) \left( 1 - n_{q_1}^{(f)} \right) N_{k_1}^{(b_1)} \left( 1 + N_{k_2}^{(b_2)} \right) \right.
\]

\[
+ \int d\mathcal{T}_{qq'\rightarrow gg} w_{qq'\rightarrow gg}^{(f)}(\mathbf{q}, \mathbf{q}_1; \mathbf{k}_1, \mathbf{k}_2) n_{q_1}^{(f_1)} \left( 1 + N_{k_1}^{(b_1)} \right) \left( 1 + N_{k_2}^{(b_2)} \right) \right\}
\]

\[
+ \sum_{f_1, f_2, f_3 = \pm} \left\{ \int d\mathcal{T}_{qq'\rightarrow gg} w_{qq'\rightarrow gg}^{(f)}(\mathbf{q}, \mathbf{q}_1; \mathbf{k}_1, \mathbf{k}_2) n_{q_1}^{(f_1)} \left( 1 - n_{q_2}^{(f_2)} \right) \left( 1 - n_{q_3}^{(f_3)} \right) \right.
\]

\[
+ 2 \int d\mathcal{T}_{qq'\rightarrow q\bar{q}} w_{qq'\rightarrow q\bar{q}}^{(f)}(\mathbf{q}, \mathbf{q}_1; \mathbf{q}_2, \mathbf{q}_3) \left( 1 - \bar{n}_{q_1}^{(f_1)} \right) n_{q_2}^{(f_2)} n_{q_3}^{(f_3)} \right\},
\]

and in turn,

\[
\Gamma^{(f)}_i[n_q^\pm, N_k^{t,l}] = \sum_{f_1 = \pm} \sum_{b_1, b_2 = t, l} \left\{ 2 \int d\mathcal{T}_{qq'\rightarrow gg} w_{qq'\rightarrow gg}^{(f)}(\mathbf{q}, \mathbf{k}_1; \mathbf{q}_1, \mathbf{q}_2) n_{q_1}^{(f_1)} \left( 1 + N_{k_1}^{(b_1)} \right) N_{k_2}^{(b_2)} \right.
\]

\[
+ \int d\mathcal{T}_{qq'\rightarrow gg} w_{qq'\rightarrow gg}^{(f)}(\mathbf{q}, \mathbf{k}_1; \mathbf{k}_1, \mathbf{k}_2) \left( 1 - n_{q_1}^{(f_1)} \right) N_{k_1}^{(b_1)} N_{k_2}^{(b_2)} \right\}
\]

\[
+ \sum_{f_1, f_2, f_3 = \pm} \left\{ \int d\mathcal{T}_{qq'\rightarrow gg} w_{qq'\rightarrow gg}^{(f)}(\mathbf{q}, \mathbf{q}_1; \mathbf{k}_1, \mathbf{k}_2) \left( 1 - n_{q_1}^{(f_1)} \right) N_{q_2}^{(f_2)} n_{q_3}^{(f_3)} \right.
\]

\[
+ 2 \int d\mathcal{T}_{qq'\rightarrow q\bar{q}} w_{qq'\rightarrow q\bar{q}}^{(f)}(\mathbf{q}, \mathbf{q}_1; \mathbf{q}_2, \mathbf{q}_3) \left( 1 - \bar{n}_{q_1}^{(f_1)} \right) n_{q_2}^{(f_2)} n_{q_3}^{(f_3)} \right\},
\]
The functions \( w^{(f_1; b_1 b_2)}_{qg \to qg}, w^{(f_1; b_1 b_2)}_{qg \to gq} \) are probabilities for ‘elastic’ scattering of soft-quark excitations off soft-gluon excitations and annihilation of soft quark-antiquark pair into two soft-gluon excitations, and \( w^{(f_1; f_2 f_3)}_{qg \to qg}, w^{(f_1; f_2 f_3)}_{gq \to qg} \) are probabilities for ‘elastic’ scattering of soft-quark excitations off soft-quark and soft-antiquark excitations, respectively. The phase-space measures for these processes are

\[
\int d\mathcal{T}^{(f_1; b_1 b_2)}_{gq \to qg} = \int \frac{d\mathbf{q}_{1}}{(2\pi)^3} \int \frac{d\mathbf{k}_{1}}{(2\pi)^3} \int \frac{d\mathbf{k}_{2}}{(2\pi)^3} (2\pi)^4 \delta(\mathbf{q} + \mathbf{k}_1 - \mathbf{q}_1 - \mathbf{k}_2) \delta(\omega^{(f)}_{q} + \omega^{(b)}_{k_1} - \omega^{(b)}_{q_1} - \omega^{(b)}_{k_2}),
\]

\[
\int d\mathcal{T}^{(f_1; f_2 f_3)}_{gq \to qg} = \int \frac{d\mathbf{q}_{1}}{(2\pi)^3} \int \frac{d\mathbf{k}_{1}}{(2\pi)^3} \int \frac{d\mathbf{k}_{2}}{(2\pi)^3} (2\pi)^4 \delta(\mathbf{q} + \mathbf{k}_1 - \mathbf{q}_1 - \mathbf{k}_2) \delta(\omega^{(f)}_{q} + \omega^{(f)}_{q_1} - \omega^{(b)}_{k_1} - \omega^{(b)}_{k_2}),
\]

\[
\int d\mathcal{T}^{(f_1; f_2 f_3)}_{gq \to qg} = \int \frac{d\mathbf{q}_{1}}{(2\pi)^3} \int \frac{d\mathbf{k}_{1}}{(2\pi)^3} \int \frac{d\mathbf{k}_{2}}{(2\pi)^3} (2\pi)^4 \delta(\mathbf{q} + \mathbf{k}_1 - \mathbf{q}_1 - \mathbf{k}_2) \delta(\omega^{(f)}_{q} + \omega^{(f)}_{q_1} - \omega^{(f)}_{k_1} - \omega^{(f)}_{k_2}),
\]

For boson sector of the plasma excitations the generalized rates \( \Gamma_{d}^{(b)} \) and \( \Gamma_{i}^{(b)} \) to the lowest order in the nonlinear interactions of soft modes have a similar structure

\[
\Gamma_{d}^{(b)}[n_{q_{1}}^{\pm}, N_{k_{1}}^{l_{1}}] = \sum_{b_{1}=t, l, f_1, f_2=\pm} \left\{ \int d\mathcal{T}^{(b_{1}; b_{2})}_{gq \to gq} w^{(b_{1}; b_{2})}_{gq \to gq} (\mathbf{k}, \mathbf{q}_{1}, \mathbf{k}_{1}, \mathbf{q}_{2}) \left( 1 + N_{k_{1}}^{(b_{1})} \right) \tilde{n}_{q_{1}} \left( 1 - \tilde{n}_{q_{2}} \right) + \int d\mathcal{T}^{(b_{1}; f_{2})}_{gq \to gq} \left( \mathbf{k}, \mathbf{q}_{1}, \mathbf{k}_{1}, \mathbf{q}_{2} \right) \left( 1 + N_{k_{1}}^{(b_{1})} \right) \tilde{n}_{q_{1}} \left( 1 - \tilde{n}_{q_{2}} \right) + \int d\mathcal{T}^{(b_{1}; f_{1})}_{gq \to gq} \left( \mathbf{k}, \mathbf{q}_{1}, \mathbf{k}_{1}, \mathbf{q}_{2} \right) \left( 1 + N_{k_{1}}^{(b_{1})} \right) \tilde{n}_{q_{1}} \left( 1 - \tilde{n}_{q_{2}} \right) \right\},
\]

\[
\Gamma_{i}^{(b)}[n_{q_{1}}^{\pm}, N_{k_{1}}^{l_{1}}] = \sum_{b_{1}=t, l, f_1, f_2=\pm} \left\{ \int d\mathcal{T}^{(b_{1}; b_{2})}_{gq \to gq} \left( \mathbf{k}, \mathbf{q}_{1}, \mathbf{k}_{1}, \mathbf{q}_{2} \right) N_{k_{1}}^{(b_{1})} \left( 1 - n_{q_{1}} \right) n_{q_{2}} + \int d\mathcal{T}^{(b_{1}; f_{2})}_{gq \to gq} \left( \mathbf{k}, \mathbf{q}_{1}, \mathbf{k}_{1}, \mathbf{q}_{2} \right) N_{k_{1}}^{(b_{1})} \left( 1 - \tilde{n}_{q_{1}} \right) \tilde{n}_{q_{2}} + \int d\mathcal{T}^{(b_{1}; f_{1})}_{gq \to gq} \left( \mathbf{k}, \mathbf{q}_{1}, \mathbf{k}_{1}, \mathbf{q}_{2} \right) \left( 1 + N_{k_{1}}^{(b_{1})} \right) \tilde{n}_{q_{1}} \left( 1 - \tilde{n}_{q_{2}} \right) \right\},
\]

where, \( w^{(b_{1}; b_{2})}_{gq \to gq}, w^{(b_{1}; f_{2})}_{gq \to gq} \) are probabilities for ‘elastic’ scattering of soft-gluon excitations off soft-quark and soft-antiquark excitations, respectively, and \( w^{(b_{1}; f_{1})}_{gq \to gq} \) is the probability for soft quark-antiquark pair creation by two soft-gluon fusion. The phase-space measures for these processes are

\[
\int d\mathcal{T}^{(b_{1}; b_{2})}_{gq \to gq} = \int d\mathcal{T}^{(b_{1}; b_{2})}_{gq \to gq} \equiv \int \frac{d\mathbf{k}_{1}}{(2\pi)^3} \int \frac{d\mathbf{q}_{1}}{(2\pi)^3} \int \frac{d\mathbf{k}_{2}}{(2\pi)^3} \int \frac{d\mathbf{q}_{2}}{(2\pi)^3} (2\pi)^4 \delta(\mathbf{k} + \mathbf{q}_1 - \mathbf{k}_1 - \mathbf{q}_2).
\]
\[
\int dT_{g_{gg}}^{(b_1; f_1 f_2)} \equiv \int \frac{dk_1}{(2\pi)^3} \int \frac{dq_1}{(2\pi)^3} \int \frac{dq_2}{(2\pi)^3} (2\pi)^4 \delta(k + k_1 - q_1 - q_2) \delta(\omega^{(b)}(k_1) - \omega^{(b)}(k_1) - \omega^{(f)}(q_1) - \omega^{(f)}(q_2)).
\]

We have written out above the most general expressions for the fermion and boson decay and regenerating rates to the lowest order in the nonlinear interaction taking into account all possible channels of transitions from initial two-quasiparticle states to different types of the final two-quasiparticle states. However it is clear that not all these processes of scattering, annihilations and fusions are kinematically permissible\(^1\). In this work we do not set as a purpose to determine all permissible channels of reactions that require as a first step a study of a system of equations determining the conservation laws of energy and momentum of two-quasiparticles interacting system. Unfortunately at present there not exist general analytic methods\(^2\) permitting in a direct way to define whether or not \(\delta\)-functions have supports different from zero in integration measures \((2.7)\) and \((2.9)\) and thus for solving this problem it should be used numerical methods. In the remaining part of the paper we restrict our consideration to calculation of the probabilities of only processes, which certainly exist (for example, the elastic scattering) leaving the general case for a separate research.

### 3 Effective currents and effective amplitudes

We use the metric \(g^{\mu\nu} = \text{diag}(1, -1, -1, -1)\), choose units such that \(c = k_B = 1\) and note \(x = (t, \mathbf{x})\), \(k = (k_0, \mathbf{k})\), \(q = (q_0, \mathbf{q})\) etc. As was mentioned above, we consider \(\text{SU}(N_c)\) gauge theory with \(N_f\) flavors of massless quarks. The color indices for the adjoint representation \(a, b, \ldots\) run from 1 to \(N_c^2 - 1\), while those for the fundamental representation \(i, j, \ldots\) run from 1 to \(N_c\). The Greek indices \(\alpha, \beta, \ldots\) for the spinor representation run from 1 to 4. In the following discussion we will use notation adopted in our previous paper \[8\].

The input equations for the effective theory under consideration are self-consistent system of the field equations: the Yang-Mills equation for gauge potential \(A^a_\mu(k)\) (Eq. (4.9))

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\(^1\)Here, as a kinematic permissibility we mean generally speaking not only the existence of a solution for the system of conservation laws of energy and momentum corresponding to concrete reaction of the nonlinear interaction of two quasiparticles, but also selection over helicity conservation of a quasiparticle system.

\(^2\)In the work of Kadomtsev and Kontorovich \[17\] the geometrical method of solving the system of conservation laws for two-to-two scattering processes of quasiparticles with arbitrary dispersion laws was suggested. Unfortunately, this approach having sufficiently general character is restricted only to the case of two-dimensional momenta of quasiparticles.
in Ref. [3])

\[ *D_{\mu\nu}^{-1}(k)A^{\mu\nu}(k) = - j_{\mu}^{A(2)}(A, A)(k) - j_{\mu}^{A(3)}(A, A)(k) - j_{\mu}^{\Psi(0,2)}(\bar{\psi}, \psi)(k) \]

\[ - j_{\mu}^{\Psi(1,2)}(A, \bar{\psi}, \psi)(k), \]

(3.1)

and the Dirac equation for soft-quark field (Eq. (3.3) in [8]) supplemented by its Dirac conjugate equation for completeness of the description

\[ *S_{(3,1)}^{-1}(q)\psi_{(3,1)}^j(q) = - \eta^{(1,1)i}_{\alpha}(A, \psi)(q) - \eta^{(2,1)i}_{\alpha}(A, \psi)(q), \]

\[ \bar{\psi}_{(3,1)}^j(-q) *S_{(3,1)}^{-1}(-q) = \bar{\eta}^{(1,1)i}_{\alpha}(A^*, \bar{\psi})(-q) + \bar{\eta}^{(2,1)i}_{\alpha}(A^*, \bar{\psi})(-q). \]

Here, in the last line we take into account the following relations: \( \gamma_0^* S^1(-q) \gamma_0 = - *S(q) \) and \( \bar{\eta}(-q) = \eta^0(q) \gamma_0 \). On the right-hand side of Eqs. (3.1), (3.2) in the expansion of induced currents and sources we keep the terms up to the third order in interacting fields containing a relevant information on the two-to-two scattering processes. The expansion terms of induced currents\(^3\) \( j^A \) and \( j^\psi \) on the right-hand side of Eq. (3.1) in the hard thermal loop (HTL) approximation have the following structure:

\[ j_{\mu\nu}^{A(2)}(A, A)(k) = \frac{1}{2!} g(T^a)^{bc} \int \Gamma_{\mu\nu\lambda}(k, -k_1, -k_2) A^{\mu\nu}(k_1) A^{\alpha\lambda}(k_2) \delta(k - k_1 - k_2) dk_1 dk_2, \]

\[ j_{\mu\alpha}^{\Psi(0,2)}(A, \bar{\psi}, \psi)(k) = g^2 (T^a)^{ij} \int \Gamma_{\mu\nu\alpha\beta}(k; q_1, -q_2) \bar{\psi}_\alpha^j(q_1) \psi_\beta^j(q_2) \delta(k + q_1 - q_2) dq_1 dq_2, \]

\[ j_{\mu\alpha}^{\Psi(1,2)}(A, \bar{\psi}, \psi)(k) = g^2 \int \delta \Gamma_{\mu\nu, \alpha\beta}^{(G)ab, ij}(k, -k_1; q_1, -q_2) A^{\mu\nu}(k_1) \bar{\psi}_\alpha^j(q_1) \psi_\beta^j(q_2) \]

\[ \times \delta(k + q_1 - k_1 - q_2) dk_1 dq_1 dq_2, \]

where \( (T^a)^{bc} \equiv -if^{abc} \). \( \Gamma_{\mu\nu\lambda} \) is HTL-resummed three-gluon vertex and \( \Gamma_{\mu\nu, \alpha\beta}^{(G)} \) is HTL-resummed vertex between quark pair and gluon. They represent a sum of bare vertex and corresponding HTL-correction. \( \delta \Gamma_{\mu\nu, \alpha\beta}^{(G)ab, ij} \) is an effective (HTL-induced) vertex between quark pair and two gluons. The current \( \Gamma^{A(3)} \) defines a self-action of soft bosonic field considered in Ref. [11, 18]. The explicit form of HTL-amplitudes in integrands in Eq. (3.3) can be found in Ref. [3]. The superscript \( (G) \) points to the fact that in the coordinate representation the time argument of external gluon leg incoming in the vertex functions \( \Gamma_{\mu\nu, \alpha\beta}^{(G)} \) and \( \delta \Gamma_{\mu\nu, \alpha\beta}^{(G)ab, ij} \) is largest.

Furthermore, the induced sources on the right-hand side of Dirac equation (3.2) have the following structure:

\[ \eta^{(1,1)i}_{\alpha}(A, \psi)(q) = g (T^a)^{ij} \int \Gamma_{\mu\nu, \alpha\beta}^{(G)ab, ij}(k_1; q_1, -q_2) A^{\mu\nu}(k_1) \bar{\psi}_\alpha^j(q_1) \delta(q - q_1 - k_1) dk_1 dq_1, \]

(3.4)

\(^3\)Here, we somewhat redefine HTL-induced currents \( j^A(2), j^A(3), j^{\Psi(0,2)} \) and source \( \eta^{(1,1)} \) including bare vertices in their definition (see, below).
\[ \eta^{(2,1)i}_\alpha(A, A, \psi)(q) = \frac{1}{2^i} g^2 \int \delta \Gamma^{(i)(A)ab,ij}_{\mu,\alpha\beta}(k_1, k_2; q_1, -q) A^{\mu}(k_1) A^{\nu}(k_2) \bar{\psi}^j(k_1) \]
\[ \times \delta(q - q_1 - k_1 - k_2) dq_1 dk_1 dk_2. \]

Here, \( \Gamma^{(i)(A)ab,ij}_{\mu,\alpha\beta} \) and \( \delta \Gamma^{(i)(A)ab,ij}_{\mu,\alpha\beta} \) are HTL-resummed vertices between quark pair and gluon, and quark pair and two gluons, respectively. The superscript (\( Q \)) denotes that these vertices in the coordinate representation have the largest time argument for external quark incoming leg.

For conjugate induced sources \( \eta^{(1,1)a}_\alpha \) and \( \bar{\eta}^{(2,1)a}_\alpha \) by virtue of properties (A.3) and (A.4), we have from (3.4)
\[ \eta^{(1,1)i}_\alpha(A^*, \bar{\psi})(-q) = g(t^a)^i j \int \Gamma^{(i)(A)\mu,\alpha\beta}(q_1, -q_1, q) A^{\mu}(k_1) \bar{\psi}^j(k_1) \delta(q - q_1 - k_1) dq_1 dk_1 dq_2, \]
\[ \bar{\eta}^{(2,1)i}_\alpha(A^*, A^*, \bar{\psi})(-q) = -\frac{1}{2!} g^2 \int \delta \Gamma^{(i)(A)ba,ij}_{\mu,\alpha\beta}(q_1, q_2, k_1, k_2) A^{\mu}(k_1) A^{\nu}(k_2) \bar{\psi}^j(k_2) \delta(q - q_1 - k_1 - k_2) dq_1 dk_1 dk_2. \]

The \( \delta \Gamma^{(i)(A)ab,ij}_{\mu,\alpha\beta} \) and \( \delta \Gamma^{(i)(A)ab,ij}_{\mu,\alpha\beta} \) vertices don’t exist at tree level and to leading order they arise entirely from the HTL [9, 10, 3]. In the subsequent discussion we are needed their color structure, therefore an explicit form of the vertex functions is given in Appendix A.

Lastly \( \ast D_{\mu\nu}(k) \) and \( \ast S(q) \) are medium modified gluon (in the covariant gauge) and quark propagators
\[ \ast D_{\mu\nu}(k) = -P_{\mu\nu}(k) \ast \Delta^0(k) - Q_{\mu\nu}(k) \ast \Delta^i(k) + \xi D_{\mu\nu}(k) \Delta^0(k), \]
\[ \ast S(q) = h_+(\hat{q}) \ast \Delta_+(q) + h_- (\hat{q}) \ast \Delta_-(q). \]

Here in the first line, the Lorentz matrices are defined by
\[ P_{\mu\nu}(k) = g_{\mu\nu} - D_{\mu\nu}(k) - Q_{\mu\nu}(k), \quad Q_{\mu\nu}(k) = \frac{\bar{u}_\mu(k) \bar{u}_\nu(k)}{\bar{u}(k)}, \quad D_{\mu\nu}(k) = \frac{k_\mu k_\nu}{k^2}, \]
\[ \Delta^0(k) = \frac{1}{k^2}, \quad \bar{u}_\mu(k) = k^2 u_\mu - k_\mu (k \cdot u). \]

The scalar transverse and longitudinal gluon propagators are \( \ast \Delta^0-I(k) = 1/(k^2 - \delta \Pi^0-I(k)) \), where \( \delta \Pi^0-I(k) \) are transverse and longitudinal soft-gluon self-energies, \( \xi \) is a gauge fixing parameter. Let us assume that we are in a rest frame of a heat bath, so that \( u_\mu = (1, 0, 0, 0) \). In the soft-quark propagator the matrix functions \( h_\pm(\hat{q}) = (\gamma^0 \mp \hat{q} \cdot \gamma)/2 \) with \( \hat{q} \equiv q/|q| \) are the spinor projectors onto eigenstates of helicity and
\[ \ast \Delta_\pm(q) = -\frac{1}{q^0 \mp [q + \delta \Sigma_\pm(q)]}, \]
where \( \delta \Sigma_\pm(q) \) are scalar quark self-energies for normal (+) and plasmino (−) modes.
The system of nonlinear integral equations (3.1), (3.2) can be perturbatively solved (at least in the weak-field limit) by the approximation scheme method. Discarding the nonlinear terms in $A$, $\psi$ and $\bar{\psi}$ on the right-hand side of Eqs. (3.1), (3.2), we obtain in zero approximation

$$^*D^{-1}_{\mu\nu}(k)A^{\alpha\nu}(k) = 0, \quad S^{-1}_{\alpha\beta}(q)\psi^{i}_{\beta}(q) = 0, \quad \bar{\psi}^{i}_{\alpha}(-q) S^{-1}_{\beta\alpha}(-q) = 0.$$  

The solutions of these equations denoted by $A^{(0)a}_{\mu}(k)$, $\psi^{(0)i}_{\alpha}(q)$ and $\bar{\psi}^{(0)i}_{\alpha}(-q)$ are the solutions for free fields. By iterating Eqs. (3.1), (3.2) in general case we obtain the series

$$A^{a}_{\mu}(k) = \sum_{s=1}^{\infty} A^{(s-1)a}_{\mu}(k), \quad \psi^{i}_{\alpha}(q) = \sum_{s=1}^{\infty} \psi^{(s-1)i}_{\alpha}(q), \quad \bar{\psi}^{i}_{\alpha}(-q) = \sum_{s=1}^{\infty} \bar{\psi}^{(s-1)i}_{\alpha}(-q), \quad (3.7)$$

where $A^{(s-1)a}_{\mu}(k)$, $\psi^{(s-1)i}_{\alpha}(q)$ and $\bar{\psi}^{(s-1)i}(-q)$ are contributions of $g^{s-1}$ order to the soft-gluon and soft-quark interacting fields. Substituting expansions (3.7) into system (3.1), (3.2), we obtain iterative solutions of higher order in $g$:

to the first order

$$A^{(1)a}_{\mu}(k) = -^*D_{\mu\nu}(k)[\tilde{j}^{A(2)\nu}(A^{(0)}, A^{(0)}) + \tilde{j}^{\Psi(0,2)\nu}(\bar{\psi}^{(0)}, \psi^{(0)})], \quad (3.8)$$

$$\psi^{(1)i}_{\alpha}(q) = -S_{\alpha\beta}(q) \tilde{\eta}^{(1,1)i}_{\beta}(A^{(0)}, \psi^{(0)}), \quad \bar{\psi}^{(1)i}_{\alpha}(-q) = \tilde{\eta}^{(1,1)i}_{\beta}(A^{*0}, \bar{\psi}^{(0)}) S_{\beta\alpha}(-q),$$

to the second order

$$A^{(2)a}_{\mu}(k) = -^*D_{\mu\nu}(k)[\tilde{j}^{A(3)\nu}(A^{(0)}, A^{(0)}, A^{(0)}) + \tilde{j}^{\Psi(1,2)\nu}(A^{(0)}, \bar{\psi}^{(0)}, \psi^{(0)})], \quad (3.9)$$

$$\psi^{(2)i}_{\alpha}(q) = -S_{\alpha\beta}(q) [\tilde{\eta}^{(2,1)i}_{\beta}(A^{(0)}, \psi^{(0)}) + \tilde{\eta}^{(0,3)i}_{\beta}(\bar{\psi}^{(0)}, \psi^{(0)}), \psi^{(0)}], \quad \bar{\psi}^{(2)i}_{\alpha}(-q) = [\tilde{\eta}^{(2,1)i}_{\beta} (A^{*0}, A^{*0}, \bar{\psi}^{(0)}) + \tilde{\eta}^{(0,3)i}_{\beta} (\bar{\psi}^{(0)}, \bar{\psi}^{(0)}, \psi^{(0)})] S_{\beta\alpha}(-q),$$

and so on. Here, on the right-hand side new effective currents and sources being functionals of free fields appear. The effective currents and sources to first order approximation (3.8) are associated with initial currents (3.3) and sources (5.4), (5.5) by a simple way

$$\tilde{j}^{A(2)\nu}(A^{(0)}, A^{(0)}) \equiv j^{A(2)\nu}(A^{(0)}, A^{(0)}), \quad \tilde{j}^{\Psi(0,2)\nu}(\bar{\psi}^{(0)}, \psi^{(0)}) \equiv j^{\Psi(0,2)\nu}(\bar{\psi}^{(0)}, \psi^{(0)}),$$

$$\tilde{\eta}^{(1,1)i}_{\beta} (A^{(0)}, \psi^{(0)}) \equiv \eta^{(1,1)i}_{\beta} (A^{(0)}, \psi^{(0)}), \quad \tilde{\eta}^{(1,1)i}_{\beta} (A^{*0}, \bar{\psi}^{(0)}) \equiv \eta^{(1,1)i}_{\beta} (A^{*0}, \bar{\psi}^{(0)}).$$

Note that by virtue of choice of the right-hand sides of Eqs. (3.1) and (3.2), expansions (3.7) accurate up to $s = 3$. For higher $s$ it is necessary to add the subsequent terms of the expansions of induced currents $j^{A}$, $j^{\Psi}$ and sources $\eta$, $\tilde{\eta}$ to the right-hand sides of Eqs. (3.1), (3.2).

Hereafter all effective quantities such as currents, sources, vertices etc. will be denoted by the same letters as initial quantities with tilde above only.
The explicit form of the effective current \( \tilde{j}^{A(3)\mu} \) to second approximation order \([3.10]\) was obtained in Ref. \([13]\). The remaining effective currents and sources have the following structures:

\[
\tilde{j}_\mu^{(1,2)a}(A^{(0)}, \bar{\psi}(0), \psi(0)) = g^2 \int \tilde{\Gamma}^{(G)a\alpha_1;i\beta_1}(k, -k_1; q_1, -q_2) A^{(0)a_1\mu_1}(k_1) \bar{\psi}^{(0)i}_\alpha(-q_1) \psi^{(0)}_\beta(q_2)
\]

\[
\times \delta(k + q_1 - k_2 - q_2) dk_1 dq_1 dq_2,
\]

\[
(3.10)
\]

\[
\tilde{\eta}_\alpha^{(2,1)i}(A^{(0)}, A^{(0)}, \psi(0)) = \frac{g^2}{2!} \int \tilde{\Gamma}^{(Q)a_1a_2;i\beta_1}(k_1, k_2; q_1, q_2) A^{(0)a_1\mu_1}(k_1) A^{(0)a_2\mu_2}(k_2) \psi^{(0)i}_\beta(q_1)
\]

\[
\times \delta(q - q_1 - k_1 - k_2) dq_1 dq_2 dq_3,
\]

\[
(3.11)
\]

\[
\tilde{\eta}_\alpha^{(0,3)i}(\bar{\psi}(0), \psi(0), \psi(0)) = \frac{g^2}{2!} \int \tilde{\Gamma}^{\epsilon_{\alpha_1\alpha_2\alpha_3}}(q_1, q_2, q_3) \bar{\psi}^{(0)i}_\alpha(-q_1) \psi^{(0)i}_\beta(q_2) \psi^{(0)i}_\gamma(q_3)
\]

\[
\times \delta(q + q_1 - q_2 - q_3) dq_1 dq_2 dq_3,
\]

\[
(3.12)
\]

and correspondingly, (Dirac) conjugate effective sources are

\[
\tilde{\eta}_\alpha^{(2,1)i}(A^{*}(0), A^{*}(0), \bar{\psi}(0)) = \frac{g^2}{2!} \int \tilde{\Gamma}^{(Q)a_1a_2;i\beta_1}(k_1, k_2; q_1, q_2) A^{*}(0)a_1\mu_1(k_1) A^{*}(0)a_2\mu_2(k_2) \bar{\psi}^{(0)i}_\beta(-q_1)
\]

\[
\times \delta(q - q_1 - k_1 - k_2) dq_1 dq_2 dq_3,
\]

\[
(3.13)
\]

\[
\tilde{\eta}_\beta^{(0,3)i}(\bar{\psi}(0), \psi(0), \psi(0)) = \frac{g^2}{2!} \int \tilde{\Gamma}^{\epsilon_{\alpha_1\alpha_2\alpha_3}}(q_1, q_2, q_3) \bar{\psi}^{(0)i}_\alpha(-q_1) \psi^{(0)i}_\beta(q_2) \psi^{(0)i}_\gamma(q_3)
\]

\[
\times \delta(q + q_1 - q_2 - q_3) dq_1 dq_2 dq_3.
\]

\[
(3.14)
\]

The effective amplitudes entering into the integrand of the second order \([3.10] - [3.14]\) (and also entering into higher iterations \(s > 3\)) represent highly nontrivial combinations of the HTL-amplitudes and equilibrium soft-gluon and soft-quark propagators. These effective amplitudes can be defined in a direct way starting from the connection between the effective and initial currents. Thus for example the effective amplitude in \([3.10]\) is defined from the relation

\[
\tilde{j}_\mu^{(1,2)a}(A^{(0)}, \bar{\psi}(0), \psi(0))(k) = j_\mu^{(1,2)a}(A^{(0)}, \bar{\psi}(0), \psi(0))(k)
\]

\[
+ \{j^{A^{(2)a\mu,-}D_j}{\tilde{\Psi}}^{(0,2)}(\bar{\psi}(0), \psi(0)), A^{(0)}) + j^{A^{(2)a\mu,-}D_j}{\tilde{\Psi}}^{(0,2)}(\bar{\psi}(0), \psi(0))\}
\]

\[
+ \{j^{\Psi^{(0,2)a\mu}}(\tilde{\eta}^{(1,1)}(A^{(0)}, \bar{\psi}(0))^S, \psi(0)) + j^{\Psi^{(0,2)a\mu}}(\tilde{\eta}^{(1,1)}(A^{(0)}, \psi(0))^S)\}.
\]

However, such a direct approach for determination of the explicit form of the effective amplitudes is very complicated even in the second order approximation and as a consequence, ineffective. In the forthcoming section we will consider somewhat different approach to the calculation of the effective amplitudes, which allows us to avoid many intermediate operations and to a certain extent to automate the calculation procedure. This approach is extension of the calculating method for purely gluonic effective amplitudes suggested in Ref. \([1]\) to the case of an existence of soft-quark degree of freedom in the system. New features here, is appearing effective sources \([3.12], [3.14]\), which don’t have counterparts in the expansion of the induced currents and sources in initial system \([3.1], [3.2]\).
4 Calculation of effective amplitudes

The calculating algorithm of the effective amplitudes is based on the following idea. The total induced current \( j^A_\mu[A, \bar{\psi}, \psi] \), and induced source \( \eta^i_\alpha[A, \psi] \) have two representations: by means of interacting and free fields that must be equal each other

\[
\begin{align*}
j^A_\mu[A] + j^\Psi_\mu[A, \bar{\psi}, \psi] &= \sum_{s=2}^{\infty} j^A(s)_\mu(A, \ldots, A) + \sum_{n=0}^{\infty} j^{\Psi(n,2)}_\mu(A, \ldots, A, \bar{\psi}, \psi) = \quad (4.1) \\
\sum_{s=2}^{\infty} j^A(s)_\mu(A^{(0)}, \ldots, A^{(0)}) + \sum_{n=0}^{\infty} \sum_{l=1}^{\infty} j^{\Psi(n,2l)}_\mu(A^{(0)}, \ldots, A^{(0)}, \bar{\psi}^{(0)}, \psi^{(0)}, \ldots, \bar{\psi}^{(0)}, \psi^{(0)}) &= \quad (4.2)
\end{align*}
\]

Equation (4.2) is to be supplemented by a similar equation for conjugate source \( \bar{\eta}^i_\alpha[A, \psi] \). The explicit form of the functions \( j^A(s)_\mu(A, \ldots, A) \) and \( j^{\Psi(n,2)}_\mu(A^{(0)}, \ldots, A^{(0)}) \) was defined in Ref. [1] (Eqs. (3.9) and (7.8)). The original currents \( j^{\Psi(n,2)}_\mu(A, \ldots, A, \bar{\psi}, \psi) \) and sources \( \bar{\eta}^{(n,1)}_\alpha(A, \ldots, A, \psi) \) are expressed as

\[
\begin{align*}
j^{\Psi(n,2)}_\mu(A, \ldots, A, \bar{\psi}, \psi)(k) &= \frac{1}{n!} g^{n+1} \int \Gamma^{(G)}{\mu_{\alpha_1} \ldots \alpha_n, ij}(k, -k_1, \ldots, -k_n; q_1, -q_2) \\
&\quad \times A^{a_1 \mu_1}(k_1) \ldots A^{a_n \mu_n}(k_n) \bar{\psi}^i_\alpha(-q_1) \psi^j_\beta(q_2) \\
&\quad \times \delta\left(k - \sum_{i=1}^{n} k_i + q_1 - q_2\right) dq_1 dq_2 \prod_{i=1}^{n} dk_i, \quad n = 0, 1, \ldots , \quad (4.3)
\end{align*}
\]

\[
\begin{align*}
\bar{\eta}^{(s,1)}_\alpha(A, \ldots, A, \psi)(q) &= \frac{1}{s!} g^s \int \Gamma^{(Q)}{\mu_{ij} \ldots \alpha_{s}, \alpha_{s}, ij}(k_1, \ldots, k_s; q_1, -q) \\
&\quad \times A^{a_1 \mu_1}(k_1) \ldots A^{a_s \mu_s}(k_s) \psi^j_\beta(q_1) \delta\left(q - \sum_{i=1}^{s} k_i\right) dq_1 \prod_{i=1}^{s} dk_i, \quad s = 1, 2, \ldots . \quad (4.4)
\end{align*}
\]

Here, the HTL-amplitudes \( \Gamma^{(G)}_{\mu_{\alpha_1} \ldots \alpha_n, ij} \) for \( n = 0 \) and \( \Gamma^{(Q)}_{\mu_{ij} \ldots \alpha_{s}, \alpha_{s}, ij} \) for \( s = 1 \) represent the sum of bare vertices and corresponding HTL-correction. For \( n > 0, s > 1 \) we rename \( \Gamma^{(G)} = \delta \Gamma^{(G)}, \Gamma^{(Q)} = \delta \Gamma^{(Q)} \).

The interacting fields on the left-hand side of equations (4.1) and (4.2) are defined by the expansion

\[
A^{a_\mu}(k) = A^{(0)}{a_\mu}(k) - \hat{D}^{a_\mu}(k) \sum_{s=2}^{\infty} \bar{\eta}^{(s,1)}_\alpha(A, \ldots, A) \]

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\[
+ \sum_{n=0}^{\infty} \sum_{l=1}^{\infty} \tilde{j}_\mu^{\Psi(n, 2l)\alpha}(A^{(0)}, \ldots, A^{(0)}, \bar{\psi}^{(0)}, \psi^{(0)}, \ldots, \bar{\psi}^{(0)}, \psi^{(0)})[n, 2l],
\]

(4.4)

\[
\psi^{\dagger}_\alpha(q) = \psi^{(0)i}_\alpha(q) - \ast S_{\alpha\beta}(q) \left[ \sum_{s=1}^{\infty} \eta^{(s,1)i}_\beta(A^{(0)}, \ldots, A^{(0)}, \bar{\psi}^{(0)})
\right.
\]

\[
+ \sum_{n=0}^{\infty} \sum_{l=1}^{\infty} \tilde{\eta}^{(s,2l+1)\alpha}_\alpha(A^{(0)}, \ldots, A^{(0)}, \bar{\psi}^{(0)}, \psi^{(0)}, \ldots, \bar{\psi}^{(0)}, \psi^{(0)})[n, 2l+1],
\]

Similar expansion holds for conjugate function \(\bar{\psi}^{\dagger}_\alpha(-q)\). The calculation of purely gluonic effective currents \(\tilde{j}_\mu^{A(s)\alpha}(A^{(0)}, \ldots, A^{(0)})\) was considered in Ref. [1]. The remaining effective currents and sources have the following structure:

\[
\tilde{j}_\mu^{\Psi(n, 2l)\alpha}(A^{(0)}, \ldots, A^{(0)}, \bar{\psi}^{(0)}, \psi^{(0)}, \ldots, \bar{\psi}^{(0)}, \psi^{(0)})(k) = \frac{1}{n!} \frac{1}{(l!)^2} g^{n+l} \frac{g_{\mu_1 \mu_2} g_{\nu_1 \nu_2}}{G^{\mu_1 \mu_2} G^{\nu_1 \nu_2}} \left[ \sum_{i=1}^{n} k_i - \sum_{j=1}^{l} q_j \right]
\]

(4.5)

\[
* G^{\mu_1 \mu_2} G^{\nu_1 \nu_2} \delta \left( k - \sum_{i=1}^{n} k_i + \sum_{j=1}^{l} q_j \right) \prod_{i=1}^{n} dk_i \prod_{j=1}^{l} dq_j,
\]

\[
\tilde{\eta}^{(s,1)i}_\alpha(A^{(0)}, \ldots, A^{(0)}, \psi^{(0)})(q) = \frac{1}{s!} g^s \int * G^{(s)\mu_1 \ldots \mu_s, \alpha \beta}(k_1, \ldots, k_s, q_1, -q) \times A^{(0)\alpha_1 \mu_1}(k_1) \ldots A^{(0)\alpha_n \mu_n}(k_n) \bar{\psi}^{(0)i}_\alpha(-q_1) \ldots \bar{\psi}^{(0)i}_\alpha(-q) \psi^{(0)i}_\beta(q_1) \ldots \psi^{(0)i}_\beta(q_l) \delta(k - q - \sum_{i=1}^{s} k_i) dq_1 \prod_{i=1}^{s} dk_i,
\]

eq \text{etc. The substitution of Eqs. (4.4), (4.5) into (4.1) and (4.2) turns the last equations into identities. Now we functionally differentiate the right- and left-hand sides of equalities (4.1) and (4.2) with respect to free fields } A^{(0)\mu} = \psi^{(0)i} = \bar{\psi}^{(0)i} = 0 \text{ after all calculations. The desired effective amplitudes will appear both on the left-hand side and on the right-hand side. However principal effect here, is that the effective amplitudes on the left-hand side will have at least one less external soft leg than on the right-hand side. This enables us to calculate them in a recurrent way. Below we will give some examples.}

The second functional derivative of the induced current \(j^{\Psi\alpha}_\mu\) with respect to \(\bar{\psi}^{(0)}\) and \(\psi^{(0)}\) yields

\[
\frac{\delta^2 j^{\Psi\alpha}_\mu[A, \bar{\psi}, \psi](k)}{\delta \bar{\psi}^{(0)i}_\beta(q_2) \delta \psi^{(0)i}_\alpha(-q_1)} \bigg|_{A^{(0)} = \psi^{(0)} = \bar{\psi}^{(0)} = 0} = g * \Gamma^{(G)\alpha_1 \ldots \alpha_s \nu_1 \ldots \nu_s}_\mu^{\mu_1 \ldots \mu_s, \alpha \beta}(k; q_1, -q_2) \delta(k + q_1 - q_2)
\]

(4.6)

\[
= g (l^a)^{ij} * \Gamma^{(G)}_{\mu, \alpha \beta}(k; q_1, -q_2) \delta(k + q_1 - q_2).
\]

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and the second functional derivative of the induced source $\eta^{a}_{\alpha\beta}$ with respect to $A^{(0)}$ and $\psi^{(0)}$ gives

$$\frac{\delta^2 \eta^{a}_{\alpha\beta}[A, \psi](q)}{\delta A^{(0)a_1\mu_1}(q_1) \delta \psi^{(0)i_1}_{\alpha_1}(q_1)} \bigg|_{A^{(0)}=\psi^{(0)}=\bar{\psi}^{(0)}=0} = g^{i_1i_2} \Gamma^{(Q)}_{\mu_2, \alpha_2} \delta(q - q_1 - k) \quad (4.7)$$

$$= g \left( t^{a_1} \right)^{i_1} \Gamma^{(Q)}_{\mu_1, \alpha_1} (k; q_1, -q) \delta(q - q_1 - k).$$

Eqs. (4.6) and (4.7) show that the effective amplitudes of decay of soft-gluon excitation into pair of soft quark-antiquark excitations (or inverse process of pair annihilation into soft gluon) coincide with usual HTL-resummed vertex between quark pair and gluon. This decay process is kinematically forbidden and hence the $\delta$-functions don’t support on the mass-shell of plasma excitations.

The first nontrivial examples arise in calculation of the next derivative. It defines the effective amplitudes for the processes with four soft plasma excitations: the process of nonlinear interaction of two soft bosonic and two fermionic excitations and the process of four soft fermionic interaction. For the first process two effective amplitudes are possible. The first amplitude arises from the variation

$$\frac{\delta^2 \left( j^{Aa}_{\mu}(A)(k) + j^{qa}_{\mu}(A, \bar{\psi}, \psi)(k) \right)}{\delta A^{(0)a_1\mu_1}(k_1) \delta \psi^{(0)i_1}_{\alpha_1}(q_1)} \bigg|_{A^{(0)}=\psi^{(0)}=\bar{\psi}^{(0)}=0} =$$

$$= \int \left\{ \frac{\delta^2 j^{A(2)a}_{\mu}(k)}{\delta A^{a_1\mu_1}(k_1) \delta \psi^{(0)i_1}_{\alpha_1}(q_1)} \delta A^{a_2\mu_2}(k'_2) \delta \psi^{(0)i_2}_{\alpha_2}(q'_2) \delta \psi^{(0)i_1}_{\alpha_1}(q_1) \right. \right.$$

$$+ \frac{\delta^2 \bar{\psi}^{(2)a}_{\beta_1}(k'_1) \delta \psi^{(0)i_1}_{\alpha_1}(q_1)}{\delta A^{(0)a_1\mu_1}(k_1) \delta \psi^{(0)i_2}_{\alpha_2}(q_2) \delta \psi^{(0)i_1}_{\alpha_1}(q_1)} \right.$$

$$+ \left. \frac{\delta^2 j^{\Psi(2)a}_{\mu}(k)}{\delta A^{a_1\mu_1}(k_1) \delta \psi^{(0)i_2}_{\alpha_2}(q_2) \delta \psi^{(0)i_1}_{\alpha_1}(q_1)} \delta \psi^{(0)i_1}_{\alpha_1}(q_1) \right. \right.$$

$$+ \left. \frac{\delta^2 j^{\Psi(2)a}_{\mu}(k)}{\delta \psi^{(0)i_2}_{\alpha_2}(q_2) \delta \psi^{(0)i_1}_{\alpha_1}(q_1)} \right. \right.$$

Here, on the right-hand side we keep the terms different from zero only. Taking into account Eqs. (3.3), (4.4), (4.6) and (4.7), from the last expression we find the effective amplitude determining effective current (3.10)

$$\Gamma^{(G)\alpha_1\beta_1}_{\mu_1, \alpha_2}(k, -k_1; q_1, -q_2) = \delta \Gamma^{(G)\alpha_1\beta_1}_{\mu_1, \alpha_2}(k, -k_1; q_1, -q_2) \quad (4.8)$$

$$- \left[ t^{a_1} \right]_{ij} \delta \Gamma^{(G)}_{\mu_1, \alpha_1}(k, -k_1; k_1) \Gamma^{(G)}_{\nu_1, \alpha_1}(q_1 + q_2; q_1, -q_2)$$

$$- [t^a, t^{a_1}]_{ij} \delta \Gamma^{(G)}_{\mu_1, \alpha_1}(k, -k + k_1; -k_1) \Gamma^{(G)}_{\nu_1, \alpha_1}(q_1 + q_2; q_1, -q_2)$$

[15]
\[ -(t^a t^a)^{ij} \star \Gamma_{\mu_1,\alpha_\gamma}^{(G)}(k; q_1, -k - q_1) \star S_{\gamma_\gamma'}(k + q_1) \star \Gamma_{\mu_1,\gamma_\beta}^{(Q)}(k_1; q_2, -k_1 - q_2) \]
\[ + (t^a t^a)^{ij} \star \Gamma_{\mu_1,\alpha_\gamma}^{(G)}(k_1; q_1, -k_1 - q_1) \star S_{\gamma_\gamma'}(k_1 - q_1) \star \Gamma_{\mu_1,\gamma_\beta}^{(Q)}(k; -k + q_2, -q_2), \]
where \([,]\) denotes a commutator. The second similar effective amplitude results from variation
\[
\frac{\delta^3 \eta^i_{\alpha} [A, \psi](q)}{\delta A(0) a_1 \mu_1(k_1) \delta A(0) a_2 \mu_2(k_2) \delta \psi^{(0) j}_{\beta}(q_1)} \bigg|_{A(0) = \psi(0) = \bar{\psi}(0) = 0}.
\]
The calculations analogous to previous ones lead to the effective amplitude determining effective source \([8.11]\)
\[
\star \Gamma_{\mu_1,\alpha_\gamma}^{(Q) a_1 a_2; ij}(k_1, k_2; q_1, -q) = \delta \Gamma_{\mu_1,\mu_2,\alpha_\beta}^{(Q) a_1 a_2; ij}(k_1, k_2; q_1, -q) \quad (4.9)
\]
\[ + [t^a t^a]^{|ij} \star \Gamma_{\nu,\alpha_\gamma}^{(Q)}(-q + q_1; q_1, -q_1) \star D_{\nu\mu'}(k_1 + k_2) \star \Gamma_{\mu_\nu,\mu_\gamma}^{(Q)}(k_1 + k_2, -k_1, -k_2) \]
\[ - (t^a t^a)^{|ij} \star \Gamma_{\mu_1,\alpha_\gamma}^{(Q)}(-k_1; q_1, -q_1 + k_1) \star S_{\gamma_\gamma'}(q - k_1) \star \Gamma_{\mu_1,\gamma_\beta}^{(Q)}(-k_2; q_1 + k_2, -q_1) \]
\[ - (t^a t^a)^{|ij} \star \Gamma_{\mu_2,\alpha_\gamma}^{(Q)}(-k_2; q_1, -q_1 + k_2) \star S_{\gamma_\gamma'}(q - k_2) \star \Gamma_{\mu_1,\gamma_\beta}^{(Q)}(-k_1; q_1 + k_1, -q_1). \]
From the definition of this effective amplitude it follows symmetry property with respect to permutation of soft-gluon legs useful in the subsequent discussion
\[
\star \tilde{\Gamma}_{\mu_1,\mu_2,\alpha_\beta}^{(Q) a_1 a_2; ij}(k_1, k_2; q_1, -q) = \star \tilde{\Gamma}_{\mu_2,\mu_1,\alpha_\beta}^{(Q) a_2 a_1; ij}(k_2, k_1; q_1, -q). \quad (4.10)
\]
The effective amplitude of four soft-quark interaction follows from the variation
\[
\frac{\delta^3 \eta^i_{\alpha} [A, \psi](q)}{\delta \psi^{(0) j}_{\alpha_\gamma}(q_1) \delta \psi^{(0) j}_{\alpha_\beta}(q_2) \delta \psi^{(0) j}_{\alpha_\beta}(q_1)} \bigg|_{A(0) = \psi(0) = \bar{\psi}(0) = 0}
\]
\[ = \int \left\{ - \frac{\delta^2 \eta^{(1,1)}_{\alpha}(q)}{\delta A^{(0)\mu_1\mu_2}(k_1) \delta \psi^{(0) j}_{\alpha_\gamma}(q_1)} \frac{\delta^2 A^{(0)\mu_1}(k_1)}{\delta \psi^{(0) j}_{\alpha_\gamma}(q_1) \delta \psi^{(0) j}_{\alpha_\beta}(q_2)} \frac{\delta^2 \eta^{(1,1)}_{\alpha}(q_1)}{\delta A^{(0)\mu_1}(k_1) \delta \psi^{(0) j}_{\alpha_\gamma}(q_1) \delta \psi^{(0) j}_{\alpha_\beta}(q_1)} \right\}
\]
\[ + \frac{\delta^2 \eta^{(1,1)}_{\alpha}(q)}{\delta A^{(0)\mu_1}(k_1) \delta \psi^{(0) j}_{\alpha_\gamma}(q_1)} \frac{\delta^2 A^{(0)\mu_1}(k_1)}{\delta \psi^{(0) j}_{\alpha_\gamma}(q_1) \delta \psi^{(0) j}_{\alpha_\beta}(q_2)} \frac{\delta^2 \eta^{(1,1)}_{\alpha}(q_1)}{\delta A^{(0)\mu_1}(k_1) \delta \psi^{(0) j}_{\alpha_\gamma}(q_1) \delta \psi^{(0) j}_{\alpha_\beta}(q_1)} \bigg|_{A(0) = \psi(0) = \bar{\psi}(0) = 0}.
\]
We kept again the terms different from zero only on the right-hand side. Taking into account Eqs. \([3.1], [4.5]\) and \([4.7]\), we easily derive the explicit form of the effective amplitude in integrand of effective source \([3.12]\)
\[
\star \tilde{\Gamma}_{\alpha_1,\alpha_2,\alpha_3}^{(Q) a_1 a_2 a_3}(q, q_1, -q_2, -q_3) = \quad (4.11)
\]
\[ = (t^a)^{ij} (t^a)^{ij} \star \Gamma_{\alpha_1,\alpha_2}^{(Q) \mu}(q - q_2; q_2, -q) \star D_{\mu\mu'}(q - q_2) \star \Gamma_{\alpha_1,\alpha_3}^{(Q) \mu}(q - q_1 + q_3; q_1, -q_3) \]
\[ + (t^a)^{ij} (t^a)^{ij} \star \Gamma_{\alpha_1,\alpha_3}^{(Q) \mu}(q - q_3; q_3, -q) \star D_{\mu\mu'}(q - q_3) \star \Gamma_{\alpha_1,\alpha_2}^{(Q) \mu}(q - q_1 + q_2; q_1, -q_2). \]
Here, the following property of antisymmetry with respect to permutation of two last soft-quark legs holds

\[ i \tilde{\Gamma}_{\alpha_1 \alpha_2 \alpha_3}^{ij} (q, q_1, -q_2, -q_3) = -i \tilde{\Gamma}_{\alpha_1 \alpha_2 \alpha_3}^{ij} (q, q_1, -q_3, -q_2). \]

There is no such a property in general case for permutation of two first soft-quark legs by virtue of inequality of the HTL-amplitudes with different time ordering of external legs: \( i \Gamma_{\alpha \beta}^{(Q)\mu} (q - q_1; q_1, -q) \neq i \Gamma_{\alpha \beta}^{(Q)\mu} (q - q_1; q_1, -q). \)

For convenience of subsequent references in Appendix B we give the explicit form of the effective amplitudes for conjugate effective current \( j_{\mu}^{(1,2)\alpha} (A^{*\alpha \beta}, \bar{\psi}(0), \psi^{(0)}) \) and Dirac conjugate effective sources \( \tilde{\eta}_{\alpha}^{(2,1)i} (A^{*\alpha \beta}, \bar{\psi}(0), \psi^{(0)}) \) and \( \tilde{\eta}_{\beta}^{(0,3)i} (\bar{\psi}(0), \bar{\psi}(0), \psi^{(0)}). \)

## 5 Boltzmann equation for elastic scattering of soft-quark excitations off soft-gluon excitations

The initial equation for deriving the Boltzmann equation describing a change of the number densities of the soft fermion excitations owing to their scattering off the soft boson excitations is equation (Eq. (4.4) in Ref. \[3\])

\[ \text{Sp} \left( \frac{\partial}{\partial q^\mu} \left[ \hat{\eta} + \delta \Sigma^\alpha (q) \right] \right) = \]

\[ = i g \int dq' dq_1 dk_1 \left\{ \left\langle \bar{\psi}_q^i (-q) A^{\mu} (k_1) \left[ * \Gamma_{\mu}^{(Q)\alpha} (k_1; q_1, -q') \psi(q_1) \right] \right\rangle \delta (q' - q_1 - k_1) \right. \]

\[ - \left. \left\langle \left[ \bar{\psi}_q^i (k_1) \right] * \Gamma_{\mu}^{(Q)\alpha} (-k_1; q_1, q) \right\rangle \bar{\psi}_q^j (q') \delta (q - q_1 - k_1) \right\} \]

\[ + i g^2 \int dq' dq_1 dk_1 dk_2 \left\{ \left\langle A^{\mu} (k_1) A^{\nu} (k_2) \bar{\psi}_q^i (-q) \left[ \delta \Gamma_{\mu \nu}^{(Q)ab} (-k_1, -k_2; q', -q_1) \psi(q_1) \right] \right\rangle \right. \]

\[ \times \left. \delta (q' - q_1 - k_1 - k_2) \right. \]

\[ - \left. \left\langle A^{\mu} (k_1) A^{\nu} (k_2) \bar{\psi}_q^i (-q_1) \delta \Gamma_{\mu \nu}^{(Q)ab} (k_2, k_1; -q, q_1) \right\rangle \bar{\psi}_q^j (q') \delta (q - q_1 - k_1 - k_2) \right\} \}

Here, \( \hat{\eta} = \gamma^\mu q_\mu \), the Dirac trace is denoted by Sp, and \( \delta \Sigma^\alpha (q) \) is “Hermitian” part of the quark self-energy. The angle brackets denote an average with respect to any density matrix.

Using above-obtained expressions \[3.3\] and \[4.4\] for gauge potential and quark wave functions by a simple search we extract all sixth-order correlators of free fields responsible for the scattering processes of soft fermion excitations off the soft boson excitations to the lowest order in the coupling constant. As a guide rule for choice of the relevant terms a simple fact will be used. When we make decoupling of the sixth-order correlators in the
terms of the pairs (in order to define the product \( n_{q_1}^{(f_1)} N_{k_1}^{(b_1)} N_{k_2}^{(b_2)} \)) we keep the terms, which correctly reproduce relevant momentum-energy conservation laws. These conservation laws encoded in \( \delta \)-functions of the first two measures of integration in Eq. (2.7). Besides the number of appearing terms can be considerably cut if we note that it is necessary to keep only such terms in intermediate expressions, which contain the quark propagators \( \delta S(q') \) and \( *S(q') \). These propagators give later on the terms proportional to \( Z_{\pm q}(q) \delta(q^0 - \omega_q^\pm) \), i.e., the factors take into account the existence of soft-quark excitations with wave vector \( q \) and energies \( \omega_q^\pm \) in spite of the fact that the number densities of soft-quark excitations \( n_q^\pm \) are explicitly absent on the right-hand side.

Let us consider the first term on the right-hand side of Eq. (5.1) containing the third-order correlation function of interacting fields

\[
ig \int dq' dq_1 dk_1 \langle \bar{\psi}_\alpha(q) A^{\mu\nu}(k_1) * \Gamma^{(Q)a_{ij}j'}(k_1; q_1, -q') \psi_\beta(q_1) \rangle \delta(q' - q_1 - k_1). \tag{5.2}
\]

For this term there exist two substitutions resulting in the sixth-order correlation function of free fields with conservation laws (2.7). The first substitution is

\[
\bar{\psi}^j_\alpha(q) \rightarrow \tilde{\eta}^{(2.1)i}_\gamma (A^*(0), A^*(0), \bar{\psi}^{(0)})(-q) * S_{\gamma \alpha}(-q), \quad \psi_\beta^{(0j')}(q_1) \rightarrow \psi_\beta^{(0j')}(q_1),
\]

\[
A^{\mu\nu}(k_1) \rightarrow - * D^{\mu\nu}(k_1) \tilde{\eta}^{(2.1)i}_\gamma (A^*(0), A^*(0))(k_1)
\]

and the second one is

\[
\bar{\psi}^j_\alpha(q) \rightarrow \tilde{\eta}^{(2.1)i}_\gamma (A^*(0), A^*(0), \bar{\psi}^{(0)})(-q) * S_{\gamma \alpha}(-q), \quad A^{\mu\nu}(k_1) \rightarrow A^{(0)\mu\nu}(k_1),
\]

\[
\psi_\beta^{(0j')}(q_1) \rightarrow - * S_{\beta \rho}(q_1) \tilde{\eta}^{(1.1)i}_\rho (A^*(0), \bar{\psi}^{(0)})(q_1).
\]

Here, the product of the correlation functions of free gauge and quark fields appears in the following form:

\[
\langle A^{*(0)a_1 \mu_1}(k_1) A^{*(0)a_2 \mu_2}(k_2) A^{(0)a_3 \mu_3}(k_3) A^{(0)a_4 \mu_4}(k_4) \rangle \langle \bar{\psi}^{(0)i}_\alpha(-q) \psi^{(0j)}_\beta(q_1) \rangle
\]

\[
\simeq \left\{ \delta^{a_1 a_4} I^{\mu_1 \mu_4}(k_1) \delta(k_1 - k_4) \delta^{a_2 a_3} I^{\mu_2 \mu_3}(k_2) \delta(k_2 - k_3) \right\} \delta^{j j'} \Gamma_{\beta \alpha}(q_1) \delta(q_1 - q), \tag{5.3}
\]

where on the right-hand side we use definitions of the correlation functions \( I^{\mu\nu}(k) \) and \( \Gamma_{\alpha \beta}(q) \) in the equilibrium \( \textbf{8} \). Simple algebraic transformations enables us to lead term (5.2) to the following form:

\[
\frac{ig^4}{2!} \int dq_1 dq_2 dk_1 dk_2 * \Gamma^{(Q)a_{ij}a_{ij}'}(k_1, k_2; q_1, -q) * S_{\alpha \alpha'}(-q)
\]

\[
\times \left\{ \eta^{a_1, a_2}_i * \Gamma^{(Q)}_{\mu_1 \mu_2, \alpha \beta'}(q - q_1; q_1, -q) * D^{\nu \nu'}(k_1 + k_2) * \Gamma_{\nu' \mu'_2 \mu'_2}(k_1 + k_2, -k_1, -k_2) \right\}. \tag{5.4}
\]
In deriving Eq. (5.4) we have used property (B.4) for the effective amplitude \( \tilde{\Gamma}_{\mu_1 \mu_2, \alpha \beta} \).

Now we turn to the third term on the right-hand side of Eq. (5.1) containing the fourth-order correlation function of interacting fields

\[
ig^2 \int dq' dq_1 dk_1 dk_2 \langle A^{\mu}(k_1) A^{\nu}(k_2) \psi_{\alpha}^i(q) \delta \Gamma^{(Q)_{a_1 a_2, i l}}(q_1) \rangle (q, q', q_1, q_2) = \delta(q' - q_1 - k_1 - k_2). \tag{5.5}
\]

Here, there exists a unique replacement, which leads to relevant conservation laws (2.7)

\[
\tilde{\psi}_{\alpha}^i(-q) \rightarrow \tilde{\psi}_{\alpha}^i(A^{(0)}, A^{(0)}, \tilde{\psi}^{(0)} \ast S_{\gamma \alpha}(-q), \ast S_{\gamma \alpha}(-q) \rightarrow \psi^{(0)j} \psi^{(0)j}(q_1),
\]

\[
A^{\mu}(k_1) \rightarrow A^{(0)\mu}(k_1), \quad A^{\nu}(k_2) \rightarrow A^{(0)\nu}(k_2).
\]

This replacement with decomposition (5.3) results equation (5.3) in the following form:

\[
ig^4 \int dq_1 dk_1 dk_2 \ast \tilde{\Gamma}_{\mu_1 \mu_2, \alpha \beta} (k_1, k_2; q_1, q) \ast S_{\alpha \alpha'}(-q) \ast S_{\gamma \alpha}(-q) \ast \tilde{\Gamma}_{\mu_1 \mu_2, \alpha' \beta} (k_1, k_2; q_1, q)
\]

\[
\times \Upsilon_{\beta \beta}(q_1) I^{\mu_1 \mu_1'}(k_1) I^{\mu_2 \mu_2'}(k_2) \delta(q - q_1 - k_1 - k_2). \tag{5.6}
\]

If we now add the last expression to Eq. (5.4) and take into account the definition of effective amplitude (4.19), then we get

\[
ig^4 \int dq_1 dk_1 dk_2 \ast \tilde{\Gamma}_{\mu_1 \mu_2, \alpha \beta} (k_1, k_2; q_1, q) \ast S_{\alpha \alpha'}(-q) \ast \tilde{\Gamma}_{\mu_1 \mu_2, \alpha' \beta} (k_1, k_2; q_1, q)
\]

\[
\times \Upsilon_{\beta \beta}(q_1) I^{\mu_1 \mu_1'}(k_1) I^{\mu_2 \mu_2'}(k_2) \delta(q - q_1 - k_1 - k_2). \tag{5.6}
\]

The remaining second and fourth terms on the right-hand side of Eq. (5.1) after similar reasoning give in a sum expression complex conjugate to that of (5.6). To consider the simplest interaction process of plasminos with plasmons, it is necessary to fulfill the following replacements:

\[
I^{\mu_1 \mu_1'}(k_1) \rightarrow Q^{\mu_1 \mu_1'}(k_1) I^{1}(k_1), \quad I^{\mu_2 \mu_2'}(k_2) \rightarrow Q^{\mu_2 \mu_2'}(k_2) I^{1}(k_2), \tag{5.7}
\]

\[
\Upsilon_{\beta \beta}(q_1) \rightarrow (h_{-}(q_1))_{\beta \beta} T^{-}(q_1) \delta(q_1^0 - \omega_q) + (h_{+}(q_1))_{\beta \beta} T^{-}(q_1) \delta(q_1^0 + \omega_q),
\]

\[
\ast S_{\alpha \alpha'}(-q) \rightarrow - (h_{-}(q))_{\alpha \alpha'}(\ast \Delta_{-}(q))^{*}.
\]

We have determined the function \( \Upsilon_{\beta \beta}(q_1) \) in the quasiparticle approximation. The spectral density \( \Upsilon^{-}(q_1) \) describes the plasmino branch of the soft quark excitations and \( \Upsilon^{-}(q_1) \)
describes antiplasmino branch. To take into account weakly inhomogeneous and non-
stationary in the medium it is sufficient within an accepted accuracy to replace equilibrium
spectral densities $I_i^l(k_i)$, $\Upsilon^-(q_1)$ and $\Upsilon^-(q_1)$ by off-equilibrium ones in the Wigner form
(Eq. (3.6) in Ref. [8]):

$$I_i^l(k_i) \rightarrow I_i^l(k_i, x), \quad i = 1, 2,$$

$$\Upsilon^-(q_1) \rightarrow \Upsilon^-(q_1, x), \quad \Upsilon^-(q_1) \rightarrow \Upsilon^-(q_1, x)$$

slowly depending on $x = (t, x)$.

After replacements (5.7) we perform a trivial integration in $dq_1^0$. For antiplasmino part
of the function $\Upsilon_{\beta\beta}(q_1)$ we make a replacement of variable in integrand (5.6): $q_1 \rightarrow -q_1$
($\omega_{q_1} \rightarrow \omega_{q_1}$). Finally instead of (5.6) for this special case of nonlinear interaction of soft
modes in QGP we obtain

$$\frac{g^4}{2!} (i * \Delta_-(q)) \int dq_1 dk_1 dk_2 \frac{1}{u^2(k_1)u^2(k_2)}$$

$$\times \left[ \tilde{u}^{\mu_1}(k_1)\tilde{u}^{\mu_2}(k_2) \Gamma_{\mu_1\mu_2, \alpha\beta}^{(Q)a_1a_2, \mu l}(k_1, k_2; q_1, -q)(h_-(\hat{q}))_{\alpha\beta} \\
\times \tilde{u}^{\mu_1}(k_1)\tilde{u}^{\mu_2}(k_2) \Gamma_{\mu_1\mu_2, \alpha\beta}^{(Q)a_1a_2, \mu l}(k_1, k_2; q_1, -q)(h_-(\hat{q}))_{\beta\beta} \\
\times \Upsilon_{q_1} I_{k_1}^l I_{k_2}^l \delta(q - q_1 - k_1 - k_2) \right] (5.8)$$

The expression obtained enables us to define the probability of the plasmino-plasmon
scattering $w^{(-l; -l)}_{qq \rightarrow gg}(q, q_1; k_1, k_2)$ and the probability of pair annihilation into two plasmons
$w^{(-l; -l)}_{qq \rightarrow gg}(q, q_1; k_1, k_2)$. At first we express projectors $h_-(\hat{q})$ and $h_-(\hat{q}_1)$ in terms of simulta-
neous eigenspinors of chirality and helicity

$$(h_-(\hat{q}))_{\alpha\beta} = \sum_{\lambda = \pm} v_\alpha(\hat{q}, \lambda)\bar{v}_\alpha(\hat{q}, \lambda), \quad (h_-(\hat{q}_1))_{\beta\beta} = \sum_{\lambda_1 = \pm} v_\beta(\hat{q}_1, \lambda_1)\bar{v}_\beta(\hat{q}_1, \lambda_1). \quad (5.9)$$

Furthermore, we take also the boson spectral functions $I^l_{k_1}$ and $I^l_{k_2}$ in the form of the quasiparticle approximation

$$I^l_{k_i} = I^l_{k_i} \delta(k_0^0 - \omega_{k_i}^0) + I_{-k_i}^l \delta(k_0^0 + \omega_{k_i}^0), \quad I^l_{k_i} \equiv I^l(k_i, x), \quad i = 1, 2. \quad (5.10)$$

For the first term in integrand of (5.8) in product $I^l_{k_1} I^l_{k_2}$ it is necessary to keep only
‘crossed’ terms

$$I^l_{k_1} \delta(k_0^0 - \omega_{k_1}^0)I^l_{-k_2} \delta(k_0^0 + \omega_{k_2}^0) + I^l_{-k_1} \delta(k_0^0 + \omega_{k_1}^0)I^l_{k_2} \delta(k_0^0 - \omega_{k_2}^0).$$
Employing properties (4.10) and (B.4) it can be shown that the second term here with replacement \( k_1 \rightleftharpoons k_2 \) gives a contribution, which is equal to the first term.

For the second term in integrand of (5.8) in product \( I_{k_1}^l, I_{k_2}^l \) we are needed only to keep one term of a ‘direct’ product

\[
I_{k_1}^l \delta(k_1^0 - \omega_{k_1}^l) I_{k_2}^l \delta(k_2^0 - \omega_{k_2}^l).
\]

The \( \delta \)-functions give us a possibility to perform integration in \( dk_1^0 dk_2^0 \). Now we turn to (anti)plasmino and plasmon number densities accordingly setting

\[
n_q^- = (2\pi)^3 2Z_q^{-1}(q)T_q^-, \quad 1 - n_q^- = (2\pi)^3 2Z_q^{-1}(q)\bar{T}_q^-, \quad N_k^l = - (2\pi)^3 2\omega_k^l Z_k^{-1}(k)I_k^l,
\]

where \( Z_-(q) \) and \( Z_i(p) \) are residues of the HTL-resummed quark and gluon propagators at plasmino and plasmon poles, respectively.

As was stated above, a presence of the second and fourth terms gives us an expression complex conjugate to that of (5.8). It is not difficult to see that in practice this comes to a simple replacement of the factor \( (i^*\Delta_-(q))^* \) in Eq. (5.8) by

\[
(i^*\Delta_-(q))^* + i^*\Delta_-(q) \simeq -2\pi Z_-(q)\delta(q^0 - \omega_q^-).
\]

Taking into account the above-mentioned reasoning, we introduce the following matrix element of plasmino-plasmon elastic scattering

\[
T_{\lambda_1}^{q_1 a_1, \mu_1}(-k_1, k_2; q_1, -q) \equiv g^2 \left( \frac{Z_-(q_1)}{2} \right)^{1/2} \left( \frac{Z_-(q_1)}{2} \right)^{1/2} \left( \frac{Z_i(k_1)}{2\omega_k^l} \right)^{1/2} \left( \frac{Z_i(k_2)}{2\omega_k^l} \right)^{1/2} \left( \frac{Z_i(k_2)}{2\omega_k^l} \right)^{1/2} (5.12)
\]

\[
\times \left[ \tilde{u}_\beta(q_1, \lambda_1) T_{\mu_1\mu_2, \alpha_\beta}^{(Q) a_1 a_2, \mu_1} (-k_1, k_2, q_1, -q) v_\alpha(q, \lambda) \right]_{\text{on-shell}}.
\]

The probability of plasmino-plasmon scattering is defined as

\[
\delta^{ji} w_{qq}^{l(k_1, k_2)}(q, k_1; q_1, k_2) = \sum_{\lambda_1 \lambda_2 = \pm} T_{\lambda_1}^{q_1 a_1, \mu_1}(-k_1, k_2; q_1, -q) (T_{\lambda_2}^{q_2 a_2, \mu_2}(-k_1, k_2; q_1, -q))^*.
\]

(5.13)

Now we consider the problem of deriving the probability for elastic scattering of normal quark excitations off soft transverse gluon excitations. In this case instead of replacements (5.7) it should be used the following those in Eq. (5.6)

\[
I_i^{\mu_1 \mu_1'}(k_1) \rightarrow P^{\mu_1 \mu_1'}(k_1) I^l(k_1), \quad I_i^{\mu_2 \mu_2'}(k_2) \rightarrow P^{\mu_2 \mu_2'}(k_2) I^l(k_2),
\]

\[
\Upsilon_{\beta \gamma}(q_1) \rightarrow (h_+(q_1))_{\beta \gamma} \Upsilon^+(q_1) \delta(q_1^0 - \omega_{q_1}^+) + (h_-(q_1))_{\beta \gamma} \Upsilon^+(q_1) \delta(q_1^0 + \omega_{q_1}^+),
\]

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where $e^\delta_1 Z_i$ at soft normal quark and soft transverse gluon poles, respectively.

Furthermore the transverse projectors $P^{\mu_1 \nu_1'}(k_i)$, $i = 1, 2$ will be also presented in the form of expansion in eigenvectors of transverse polarization (Kalashnikov, Klimov [19])

$$P^{\mu_1 \nu_1'}(k_i) = \sum_{\xi_i = 1, 2} \left( \frac{\epsilon^{\mu_1}(k_i, \xi_i)}{\sqrt{\epsilon^2(k_i, \xi_i)}} \right) \left( \frac{\epsilon^{\nu_1'}(k_i, \xi_i)}{\sqrt{\epsilon^2(k_i, \xi_i)}} \right),$$  

where $\epsilon^{\mu_1}(k_i, \xi_i)$ are some four-vectors such that $\epsilon^{\mu_1}(k_i, \xi_i) T^{\nu_1 \nu_1'}(\xi_i) = 0$, $i = 1, 2$, where $\epsilon^{\mu_1}(k_i, \xi_i)$ are four-vectors such that $\epsilon^{\mu_1}(k_i, \xi_i)$ are linearly independent.

The reasoning similar to that used in deriving the probability of plasmino-plasmon scattering (5.13) results in the following expression for desired probability

$$\delta^{j_2} w^{(t; +t)}_{qg \rightarrow qg}(q, k_1; q_1, k_2) = \sum_{\xi_1, \xi_2 = 1, 2} \sum_{\lambda_1 = \pm} T^{a_1 a_2, j_2}_{\xi_1 \xi_2, \lambda_1}(-q_1, k_2; q_1, -q)(T^{a_1 a_2, j_2}_{\xi_1 \xi_2, \lambda_1}(-k_1, k_2; q_1, -q))^*,$$

where the matrix element for elastic scattering of soft normal quark by soft transverse gluon is defined now by the following expression:

$$T^{a_1 a_2, j_2}_{\xi_1 \xi_2, \lambda_1}(-k_1, k_2; q_1, -q) = g^2 \left( \frac{Z_+(q)}{2} \right)^{1/2} \left( \frac{Z_+(q_1)}{2} \right)^{1/2} \left( \frac{Z_-(k_1)}{2\omega_{k_1}} \right)^{1/2} \left( \frac{Z_-(k_2)}{2\omega_{k_2}} \right)^{1/2} \left( \frac{\epsilon^{\mu_1}(k_1, \xi_1)}{\sqrt{\epsilon^2(k_1, \xi_1)}} \right) \left( \frac{\epsilon^{\mu_2}(k_2, \xi_2)}{\sqrt{\epsilon^2(k_2, \xi_2)}} \right) \left[ \bar{u}_\beta(q_1, \lambda_1)^* \Gamma^{\mu_1 \mu_2, a_\alpha}(-k_1, k_2; q_1, -q) u_\alpha(q, \lambda) \right]_{\text{on-shell}}.$$

Here, $Z_+(q)$ and $Z_-(p)$ are residues of the HTL-resummed quark and gluon propagators at soft normal quark and soft transverse gluon poles, respectively.

The calculation of probabilities for other types of elastic scattering processes $w^{(-t; +t)}_{qg \rightarrow qg}$ and $w^{(+t; +t)}_{qg \rightarrow qg}$ is quite obvious. In both cases it is reduced to corresponding replacements of the gluon wave functions in Eqs. (5.12), (5.13) and (5.16), (5.17)

$$\left( \frac{Z_+(k_i)}{2\omega_{k_i}} \right)^{1/2} \left( \frac{\bar{u}_\mu(k_i)}{\sqrt{\bar{u}^2(k_i)}} \right) \leftrightarrow \left( \frac{Z_+(k_i)}{2\omega_{k_i}} \right)^{1/2} \left( \frac{\epsilon^{\mu_1}(k_i, \xi_i)}{\sqrt{\epsilon^2(k_i, \xi_i)}} \right),$$

$i = 1, 2$ and proper choice of mass-shell conditions on the right-hand side of Eqs. (5.12) and (5.17). The factors $Z_i^{1/2}(k_i)$ provide renormalization of transverse and longitudinal gluon wave functions by thermal effects.
The probabilities for more complicated scattering processes, which result in change of a type of one or both quasiparticles in a final state (for example, the probabilities \( w_{qg \rightarrow qg}^{(l-t)} \), \( w_{qg \rightarrow qg}^{(l+t)} \) and so on) can be obtained by similar replacements of gluon and quark wave functions. However, according to remark at the end of section 2 in this case it should be performed further research on the existence of solutions for the system of energy-momentum conservation laws.

Matrix elements (5.12) and (5.17) owing to the structure of effective amplitude (B.3) have a simple diagrammatic representation drawn in Fig. 1. The first graph in Fig. 1 represents a direct interaction of soft-quark modes with soft-gluon modes induced by HTL-amplitude \( \delta \Gamma_{4}^{(Q)} \) (Eq. (A.2)). The remaining terms are connected with soft-quark–soft-gluon interaction induced by two-quark-gluon HTL-resummed vertex and three-gluon HTL-resummed vertex with intermediate virtual oscillations representing \( s \)- and \( t \)-channel contributions. Matrix elements (5.12), (5.17) also contain \( u \)-channel contribution, which it is not drawn in Fig. 1.

Let us consider in more detail a structure of matrix element (5.12). For this purpose in the initial definition of effective amplitude (B.3) we proceed from the product of color matrices \( t^{a_1} t^{a_2} \) and \( t^{a_2} t^{a_1} \) to their symmetric (\( S \)) and anti-symmetric (\( A \)) combinations

\[
 t^{a_1} t^{a_2} \rightarrow \frac{1}{2} \{ t^{a_1}, t^{a_2} \} + \frac{1}{2} [ t^{a_1}, t^{a_2}], \quad t^{a_2} t^{a_1} \rightarrow \frac{1}{2} \{ t^{a_1}, t^{a_2} \} - \frac{1}{2} [ t^{a_1}, t^{a_2}], \tag{5.18}
\]

where \( \{ , \} \) denotes anticommutator in a color space.

The first term only in the effective amplitude (B.3) is needed some comment. Making use of an explicit definition of HTL-induced vertex function (A.2), we rewrite the vertex in the following form (we suppress spinor indices and color indices in the fundamental

![Figure 1: The matrix element for soft-quark–soft-gluon scattering. The straight and wave lines denote soft-quark and soft-gluon excitations, correspondingly. The blob stands for HTL resummation.](image-url)
The functions \( \delta \Gamma^{(Q;S,A)}_{\mu_1 \mu_2} (k_1, k_2; q_1, -q) \) possess evident properties

\[
\delta \Gamma^{(Q;S,A)}_{\mu_1 \mu_2} (k_1, k_2; q_1, -q) = \pm \delta \Gamma^{(Q;S,A)}_{\mu_1 \mu_2} (k_2, k_1; q_1, -q),
\]

\[
\delta \Gamma^{(Q;S,A)}_{\mu_1 \mu_2} (-k_1, -k_2; -q_1, q) = \gamma^0 \left( \delta \Gamma^{(Q;S,A)}_{\mu_1 \mu_2} (k_1, k_2; q_1, -q) \right)^\dagger \gamma^0.
\]

Taking into account color decomposition (5.19) and change (5.18), we can present the matrix element \( T_{\lambda_\lambda_1}^{\alpha_\beta_1, \iota_\iota_1} \) as follows:

\[
T_{\lambda_\lambda_1}^{\alpha_\beta_1, \iota_\iota_1} (-k_1, k_2; q_1, -q) = \frac{1}{2} \left\{ \tau^{\alpha_2, \alpha_1} \right\}^{\iota_\iota_1} \Gamma^{S^{-1}}_{A} T^{(S)}_{\lambda_\lambda_1} (-k_1, k_2; q_1, -q)
\]

\[
+ \frac{1}{2} \left\{ \tau^{\alpha_2, \alpha_1} \right\}^{\iota_\iota_1} \Gamma^{A^{-1}}_{A} T^{(A)}_{\lambda_\lambda_1} (-k_1, k_2; q_1, -q),
\]

where

\[
\Gamma^{(S)}_{\mu_1 \mu_2} (-k_1, k_2; q_1, -q) \equiv g^2 C_{S,A} \left( \frac{Z_-(q_1)}{2} \right)^{1/2} \left( \frac{Z_-(-q_1)}{2} \right)^{1/2} \left( \frac{Z_+(k_1)}{2 \alpha_{k_1}} \right)^{1/2} \left( \frac{Z_+(k_2)}{2 \alpha_{k_2}} \right)^{1/2}
\]

\[
T^{(A)}_{\mu_1 \mu_2} (-k_1, k_2; q_1, -q) \equiv \delta \Gamma^{(Q;A)}_{\mu_1 \mu_2} (k_1, k_2; -q_1, q)
\]

\[
- \Gamma^{(Q)}_{\mu_2} (k_2; -q_1 - k_2, q_1) S(-q_1 - k_2) \Gamma^{(Q)}_{\mu_1} (-k_1; q_1, q + k_1)
\]

\[
- \Gamma^{(Q)}_{\mu_1} (-k_1; -q_1 + k_1, q_1) S(-q_1 + k_1) \Gamma^{(Q)}_{\mu_2} (k_2; -q_1 - k_2, q_1)
\]

and

\[
\Gamma^{(Q)}_{\mu_2} (k_2; -q_1 - k_2, q_1) S(-q_1 - k_2) \Gamma^{(Q)}_{\mu_1} (-k_1; q_1, q + k_1)
\]

\[
+ 2 \Gamma^{(Q)}_{\mu_2} (q - q_1; -q_1, q_1) \mathcal{D}^{(\mu_2)} (k_1 - k_2) \Gamma^{(Q)}_{\nu_1 \mu_1 \mu_2} (k_1 - k_2, -k_1, k_2)
\]

\[
- \Gamma^{(Q)}_{\mu_2} (k_2; -q_1 - k_2, q_1) S(-q_1 - k_2) \Gamma^{(Q)}_{\mu_1} (-k_1; q_1, q + k_1)
\]
The functions $T_{\lambda \lambda_1}^{(S, A)}$ possess the properties following from their definitions

$$T_{\lambda \lambda_1}^{(S, A)}(-k_1, k_2; q_1, -q) = \pm T_{\lambda \lambda_1}^{(S, A)}(k_2, -k_1; q_1, -q).$$

(5.24)

The advantage of the choice of new color basis [5, 18] is defined by the following properties:

$$\{\{t^{a_1}, t^{a_2}\}\{t^{a_2}, t^{a_1}\}\} = 2C_F\left(C_F - \frac{1}{2N_c}\right)\delta^{ij},$$

$$\{\{t^{a_1}, t^{a_2}\}[t^{a_2}, t^{a_1}]\} = 2C_F\left(C_F + \frac{1}{2N_c}\right)\delta^{ij},$$

(5.25)

i.e., crossed terms vanish. Making use of the color algebra [5, 20], we obtain instead of Eq. (5.13)

$$w_{qq \rightarrow gg}^{(-t;-t)}(q, k_1; q_1, k_2) = w_{qq \rightarrow gg}^{(S)}(q, k_1; q_1, k_2) + w_{qq \rightarrow gg}^{(A)}(q, k_1; q_1, k_2),$$

(5.26)

where

$$w_{qq \rightarrow gg}^{(S, A)}(q, k_1; q_1, k_2) = \sum_{\lambda \lambda_1 = \pm} \left| T_{\lambda \lambda_1}^{(S, A)}(-k_1, k_2; q_1, -q)\right|^2_{\text{on-shell}}.$$

Completely similar decomposition is true and for probability $w_{qq \rightarrow gg}^{(+t;+t)}$ (Eq. (5.16)) and also for probabilities $w_{qq \rightarrow gg}^{(+t;-t)}$ and $w_{qq \rightarrow gg}^{(-t;+t)}$.

The second term in integrand of Eq. (5.8) defines the probability for plasmino-antiplasmino annihilation into two plasmons $w_{qq \rightarrow gg}^{(-t;-t)}$. The probability is obtained from (5.26) by replacement $T_{\lambda \lambda_1}^{(S, A)}(-k_1, k_2; q_1, -q) \rightarrow T_{\lambda \lambda_1}^{(S, A)}(k_1, k_2; -q_1, -q)$ (without change of a sign of vector $q_1$ in spinor $\bar{\psi}(q_1, \lambda_1)$). Accordingly the probability for annihilation of soft normal quark-antiquark pair into two soft transverse gluons $w_{qq \rightarrow gg}^{(+t;+t)}$ is obtained by replacement $T_{\xi \xi \lambda_1}^{(S, A)}(-k_1, k_2; q_1, -q) \rightarrow T_{\xi \xi \lambda_1}^{(S, A)}(k_1, k_2; -q_1, -q)$ also without change of a sign of a vector $q_1$ in spinor $\bar{u}(q_1, \lambda_1)$.

From Eq. (5.26) we see that the processes of elastic scattering and annihilation proceed through two physical independent channels determined by a parity of state of two gluon system with respect to permutation of external soft-gluon legs (Eq. (5.24)). The symbols $(S, A)$ belong to states of two gluons being in even and odd states correspondingly (in the c.m.s. of gluons).

We now return to initial equation (5.1). Let us transform the left-hand side of this equation similar to Ref. [8]. We result in

$$-\delta^{ij}q^0 - \omega^{(f)}_q \left( \frac{\partial n^{(f)}_q}{\partial t} + v^{(f)}_q \cdot \frac{\partial n^{(f)}_q}{\partial x} \right).$$

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Taking into account above-mentioned, finally we write out the expression for kinetic equation defining the change in the colorless soft-quark number densities caused by ‘spontaneous’ processes of soft-quark–soft-gluon elastic scattering and annihilation of soft-quark-antiquark pair

\[
\left( \frac{\partial n_{q}^{(f)}}{\partial t} + v_{q} \cdot \frac{\partial n_{q}^{(f)}}{\partial x} \right) = \sum_{f_{1}=\pm} \sum_{b_{1}, b_{2}=t, l} \left\{ 2 \int d^{T}_{qg \rightarrow gg} \, w_{qg \rightarrow gg}^{(f_{1} b_{1}: f_{2} b_{2})}(q, k_{1}, k_{2}) \, \rho_{q_{1}}^{(f_{1})} N_{k_{1}}^{(b_{1})} N_{k_{2}}^{(b_{2})} \right. \\
\left. + \int d^{T}_{qq \rightarrow gg} \, w_{qq \rightarrow gg}^{(f_{1} b_{1}: f_{2} b_{2})}(q, q_{1}, k_{1}, k_{2}) \left( 1 - \tilde{n}_{q_{1}}^{(f_{1})} \right) N_{k_{1}}^{(b_{1})} N_{k_{2}}^{(b_{2})} \right\}.
\]

This equation follows from (2.2), (2.6) in the limit of a small intensity of soft-quark mode \(n_{q}^{(f)}\) → 0 and making use of the fact that the boson occupation numbers \(N_{b_{i}}\) are much more than one, \(1 + N_{b_{i}} \simeq N_{b_{i}^{(i)}}, \ i = 1, 2\).

### 6 Probabilities of soft-quark–soft-(anti)quark elastic scattering

Now we consider the problem of determination of the probabilities for the scattering processes of soft-quark excitations off soft-quark and soft-antiquark excitations. For this purpose we return to term (5.2) and examine the following replacement:

\[
\bar{\psi}_{\alpha}^{(3)}(-q) \rightarrow \bar{n}_{\gamma}^{(0,3)}(\bar{\psi}^{(0)}, \psi^{(i)}; \psi^{(0)})(-q)^{*} S_{\gamma\alpha}(-q), \quad \psi_{\beta}^{(0)}(q_{1}) \rightarrow \psi_{\beta}^{(0)j'}(q_{1}),
\]

\[
A_{\mu_{1} \mu_{2}}^{(3)}(k_{1}) \rightarrow ^{*} D_{\mu_{1} \mu_{2}}^{(3)}(k_{1}) \gamma_{\mu_{1}}^{(0,2)\alpha_{1}}(\bar{\psi}^{(0)}, \psi^{(0)})(k_{1}),
\]

where the effective source \(\bar{n}_{\gamma}^{(0,3)}\) is defined by Eq. (3.14). This substitution results in sixth-order correlator with respect to free quark fields \(\psi^{(0)}\) and \(\bar{\psi}^{(0)}\). Furthermore, we decouple the sixth-order correlation function in terms of pair correlators \(\langle \bar{\psi}^{(0)} \psi^{(0)} \rangle\). One can show that just two terms of this decoupling reproduce finally necessary factor in integrand: \(\delta(q + q_{1} - q_{2} - q_{3})\). Taking into account these terms and the definition of the effective vertex \(^{*} \Xi_{\alpha_{1} \alpha_{2} \alpha_{3}}\), Eq. (4.11), we get instead of (5.2)

\[
- \frac{ig^{4}}{2!} S_{\alpha_{1} \alpha_{2}}^{*}(-q) \int dq_{1} dq_{2} dq_{3} \delta(q + q_{1} - q_{2} - q_{3}) \, ^{*} \Xi_{\alpha_{1} \alpha_{2} \alpha_{3}}^{*} \, \Xi_{\alpha_{1} \alpha_{2} \alpha_{3}}(q, q_{1}, -q_{2}, -q_{3})
\]

\[
\times ^{*} \Xi_{\alpha_{1} \alpha_{2} \alpha_{3}}^{*}(q, q_{1}, -q_{2}, -q_{3}) \gamma_{\alpha_{1} \alpha'_{1}}(q_{1}) \gamma_{\alpha_{2} \alpha'_{2}}(q_{2}) \gamma_{\alpha_{3} \alpha'_{3}}(q_{3}).
\]

At first we will obtain an expression for probability of elastic scattering of plasmino off plasmino \(w_{qq \rightarrow gg}^{(-q, q_{1}, q_{2}, q_{3})}(q, q_{1}, q_{2}, q_{3})\). For this purpose we set

\[
S_{\alpha_{1} \alpha_{2}}^{*}(-q) \rightarrow (h_{-}(q))_{\alpha_{1} \alpha_{2}}^{*} \Delta_{-}(q)\).
The probability of plasmino-plasmino scattering is defined as

$$\delta_{ji} w_{qq\rightarrow qq} (q, q_1; q_2, q_3) = \sum_{\lambda_1, \ldots, = \pm} \Gamma_{\lambda_1, \lambda_2, \lambda_3}^{ii, jk\ell} (q, q_1; -q_2, -q_3) \Gamma_{\lambda_1, \lambda_2, \lambda_3}^{ii, jk\ell} (q, q_1; -q_2, -q_3)^*.$$
The diagrammatic interpretation of the different terms of matrix element (6.4) is presented in Fig. 2.

Figure 2: Møller-like elastic soft-quark by soft-quark scattering.

The probability of elastic scattering of plasmino off soft normal quark excitation \( u^\text{q}_{qq\rightarrow qq}^{(-+;-+)} \) is obtained from Eqs. (6.5) and (6.4) by simple replacement of quark wave functions

\[
\left( \frac{Z_-(q_1)}{2} \right)^{1/2} v_{\alpha_1} (q_1, \lambda_1) \rightarrow \left( \frac{Z_+(q_1)}{2} \right)^{1/2} u_{\alpha_1} (q_1, \lambda_1), \quad \left( \frac{Z_-(q_3)}{2} \right)^{1/2} \bar{v}_{\alpha_3} (q_3, \lambda_3) \rightarrow \left( \frac{Z_+(q_3)}{2} \right)^{1/2} \bar{u}_{\alpha_3} (q_3, \lambda_3),
\]

and corresponding choice of mass-shell conditions. For deriving the probability of mutually elastic scattering of soft normal quark excitations \( u^\text{q}_{qq\rightarrow qq}^{(++;++)} \) it should be also added the following replacements to the above-stated those:

\[
\left( \frac{Z_-(q)}{2} \right)^{1/2} v_{\alpha} (q, \lambda) \rightarrow \left( \frac{Z_+(q)}{2} \right)^{1/2} u_{\alpha} (q, \lambda), \quad \left( \frac{Z_-(q_2)}{2} \right)^{1/2} \bar{v}_{\alpha_2} (q_2, \lambda_2) \rightarrow \left( \frac{Z_+(q_2)}{2} \right)^{1/2} \bar{u}_{\alpha_2} (q_2, \lambda_2).
\]

In the remaining part of this section we shall restrict our attention to study of the scattering processes including only plasmino and antiplasmino. Results for the scattering processes with soft normal quark modes follows from previous by above-mentioned replacements of quark wave functions.

For more detailed analysis of structure of probability (6.5) we regroup the terms in the effective amplitudes \( \Gamma_{\alpha_{i_1} \alpha_{i_2} \alpha_{i_3}}^{ii_1 i_2 i_3} (q, q_1, -q_2, -q_3) \) and \( \Gamma_{\alpha'_{i_1} \alpha'_{i_2} \alpha'_{i_3}}^{ii_1 i_2 i_3} (q, q_1, -q_2, -q_3) \). For this purpose we replace initial color factors by ‘orthogonal’ ones

\[
(t^{a}_{i i_1} (t^{a}_{i i_2} t^{a}_{i i_3}) + (t^{a}_{i i_2} t^{a}_{i i_3}) t^{a}_{i i_1}) + \frac{1}{2} \left( (t^{a}_{i i_1} t^{a}_{i i_2} t^{a}_{i i_3}) - (t^{a}_{i i_3} t^{a}_{i i_1} t^{a}_{i i_2}) \right),
\]

\[
(t^{a}_{i i_1} (t^{a}_{i i_2}) + (t^{a}_{i i_2}) t^{a}_{i i_1}) + \frac{1}{2} \left( (t^{a}_{i i_1} t^{a}_{i i_2}) t^{a}_{i i_3} - (t^{a}_{i i_3}) t^{a}_{i i_1} t^{a}_{i i_2} \right) - \frac{1}{2} \left( (t^{a}_{i i_1} t^{a}_{i i_2}) t^{a}_{i i_3} - (t^{a}_{i i_3}) t^{a}_{i i_1} t^{a}_{i i_2} \right).
\]

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Thus we can represent matrix element \((6.4)\) in the following form:

\[
T^{(S,A)}_{\lambda_1 \lambda_2 \lambda_3}(q_1, q_2, q_3) =
\]

\[
= \frac{1}{2} \left\{ (t^a)^{ij} (t^a)^{ij} + (t^b)^{ij} (t^b)^{ij} \right\} \tilde{c}_{S,A}^{-1} T^{(A)}_{\lambda_1 \lambda_2 \lambda_3}(q_1, q_2, -q_3)
\]

\[
+ \frac{1}{2} \left\{ (t^a)^{ij} (t^a)^{ij} - (t^b)^{ij} (t^b)^{ij} \right\} \tilde{c}_{S,A}^{-1} T^{(S)}_{\lambda_1 \lambda_2 \lambda_3}(q_1, q_2, -q_3),
\]

where

\[
\tilde{c}_{S,A} \equiv \left\{ \frac{1}{4} C_F \left( 1 \pm \frac{1}{N_c} \right) \right\}^{1/2},
\]

\[
T^{(S,A)}_{\lambda_1 \lambda_2 \lambda_3}(q_1, q_2, -q_3) \equiv g^2 \tilde{c}_{S,A} \left( \frac{Z_w(q_1)}{2} \right)^{1/2} \prod_{i=1}^{3} \left( \frac{Z_w(q_i)}{2} \right)^{1/2}
\]

\[
\times \left[ \Gamma^Q_{\alpha\beta}(-q + q_2; -q_2, q) \right] \Gamma^{G'\rho\lambda}(-q + q_3; -q_3, q) \Gamma^{G'\rho\lambda}(-q + q_3; -q_1, q_3) \]

\[
\times \left[ \Gamma^{G'\rho\lambda}(-q + q_3; -q_3, q_3) \right] \Gamma^{G'\rho\lambda}(-q + q_3; -q_1, q_3) \right\}_{on-shell}.
\]

The functions \(T^{(S,A)}_{\lambda_1 \lambda_2 \lambda_3}\) possess properties

\[
T^{(S,A)}_{\lambda_1 \lambda_2 \lambda_3}(q_1, q_2, -q_3) = \pm T^{(S,A)}_{\lambda_1 \lambda_2 \lambda_3}(q_1, -q_2, q_3).
\]

Taking into account a color algebra

\[
\left\{ (t^a)^{ij} (t^a)^{ij} \pm (t^b)^{ij} (t^b)^{ij} \right\} \left\{ (t^b)^{ij} (t^b)^{ij} \pm (t^b)^{ij} (t^b)^{ij} \right\} = 2 \left\{ (t^a t^b)^{ij} (t^a t^b)^{ij} \pm (t^a t^b)^{ij} (t^a t^b)^{ij} \right\} = \delta^{ij} C_F \left( 1 \pm \frac{1}{N_c} \right),
\]

we can cast probability \((6.5)\) into a sum of two independent part

\[
w^{(S,A)}_{qq \rightarrow qq}(q_1, q_2, q_3) = w^{(S)}_{qq \rightarrow qq}(q_1, q_2, q_3) + w^{(A)}_{qq \rightarrow qq}(q_1, q_2, q_3),
\]

where

\[
w^{(S,A)}_{qq \rightarrow qq}(q_1, q_2, q_3) = \sum_{\lambda_1 \lambda_2 \lambda_3} \left| T^{(S,A)}_{\lambda_1 \lambda_2 \lambda_3}(q_1, q_2, q_3) \right|_{on-shell}^2.
\]

Thus, as in the case of elastic scattering of soft-quark excitations off soft-gluon excitations (see previous section), the elastic scattering process of soft-quark excitations off each other proceed through two physical independent channels determined by parity of final state of soft-quark quasiparticles system. We pay attention to the fact that in the color decomposition of matrix element \((6.6)\) function \(T^{(A)}_{\lambda_1 \lambda_2 \lambda_3}\) stands with symmetric combination of color matrices, and \(T^{(S)}_{\lambda_1 \lambda_2 \lambda_3}\) stands with anti-symmetric combination.
The second term in Eq. (6.2) defines the process of elastic scattering of plasmino off antiplasmino. Reasoning similar to previous one results in expression for the probability $w^{(\alpha_1'\alpha_2')}_{\bar{q}_2q_1}(q, q_1; q_2, q_3)$ analogous to (6.8), (6.9) with replacement of the helical amplitudes $\hat{T}_{\alpha_1'\alpha_2'}(q, q_1; -q_2, -q_3)$ by

$$\hat{T}_{\lambda_1\lambda_2\lambda_3}(q, q_1; -q_2, -q_3) \equiv g^2 \hat{\mathcal{C}}(\frac{Z_1(q)}{2}) \prod_{i=1}^{3} \left( \frac{Z_{-i}(q_i)}{2} \right)^{1/2}$$

$$\times v_\alpha(q, \lambda) v_{\alpha_2}(q_2, \lambda_2) \bar{v}_{\alpha_1}(q_1, \lambda_1) \bar{v}_{\alpha_3}(q_3, \lambda_3)$$

$$\times \left[ \star \Gamma^{(Q)\nu'}_{\alpha_1\alpha}(-q - q_1; q_1, q) \star \mathcal{D}_{\nu\nu'}(-q - q_1) \star \Gamma^{(G)\nu'}_{\alpha_3\alpha_2}(-q_2 - q_3; q_2, q_3) \right]_{\text{on-shell}}.$$

The diagrammatic interpretation of the different terms of matrix element in this case is presented in Fig. 3.

![Diagram](image)

Figure 3: Bhabha-like elastic scattering of soft-quark off soft-antiquark excitations.

Finally, the third term in Eq. (6.2) also defines the process of elastic scattering of plasmino off antiplasmino. The matrix element of this process $\hat{T}_{\lambda_1\lambda_2\lambda_3}(q, q_1; -q_2, -q_3)$ is associated with the preceding matrix element by relation

$$\hat{T}_{\lambda_1\lambda_2\lambda_3}(q, q_1; -q_2, -q_3) = \pm \hat{T}_{\lambda_1\lambda_3\lambda_2}(q, q_1; -q_3, -q_2).$$

The whole contribution associated with the third term in (6.2) can be resulted in previous one by a simple replacement $q_2 \leftrightarrow q_3$.

---

6 Besides, in the kinetic equation it is necessary to replace $(1 - n_{q_1})n_{q_2}n_{q_3}$ by $(1 - \bar{n}_{q_1})\bar{n}_{q_2}\bar{n}_{q_3}$. 
Taking into account the aforesaid we write out the final expression for ‘spontaneous’ part of collision terms caused by the processes of soft-quark – soft-(anti)quark elastic scattering, which should be added to the right-hand side of Eq. (5.27)

\[
\sum_{f_1, f_2, f_3 = \pm} \left\{ \int dT_{qq \rightarrow qq}^{(f_1; f_2 f_3)} w_{qq \rightarrow qq}^{(f_1; f_2 f_3)}(q, q_1; q_2, q_3) \left( 1 - \eta_{q_1}(f_1) \right) \eta_{q_2}(f_2) \eta_{q_3}(f_3) \right\} (6.10)
\]

\[
+ 2 \int dT_{qq \rightarrow qq}^{(f_1; f_2 f_3)} w_{qq \rightarrow qq}^{(f_1; f_2 f_3)}(q, q_1; q_2, q_3) \left( 1 - \eta_{q_1}(f_1) \right) \eta_{q_2}(f_2) \eta_{q_3}(f_3). \]

7 Elastic scattering of soft-gluon excitations off soft-quark excitations

Here, we consider derivation of the kinetic equation describing a change of soft gluon number densities \( N_k^{l} \) caused by their interaction with soft-quark excitations. In this case the initial equation is

\[
\frac{\partial}{\partial k_\lambda} \left[ k^2 g^{\mu \nu} - (1 + \xi^{-1}) k^\mu k^\nu + \delta \Pi^{\mu \nu}(k) \right] \frac{\partial \Pi^{ab}(k, x)}{\partial x^\lambda} = \frac{\partial}{\partial k_\lambda} \left[ k^2 g^{\mu \nu} - (1 + \xi^{-1}) k^\mu k^\nu + \delta \Pi^{\mu \nu}(k) \right] \frac{\partial \Pi^{ab}(k, x)}{\partial x^\lambda} = (7.1)
\]

\[
- \frac{i g}{2} \int dk' dk_1 dk_2 \left\{ (T^{b})^{cd} \Gamma^{\mu \nu \lambda}(k, -k_1, -k_2) \langle A_{\mu}^{a}(k) A_{\nu}^{c}(k_1) A_{\lambda}^{d}(k_2) \rangle \delta(k - k_1 - k_2) \right. \\
+ (T^{a})^{cd} \langle A_{\mu}^{b}(k) A_{\nu}^{c}(k_1) A_{\lambda}^{d}(k_2) \rangle \delta(k' - k_1 - k_2) \right\} \\
+ ig \int dk' dq_1 dq_2 \left\{ (t^{b})^{ij} \Gamma^{(G)\mu}(k; q_1, -q_2) \langle A_{\mu}^{a}(k') \psi_{\alpha}^{i}(q_1) \psi_{\beta}^{j}(q_2) \rangle \delta(k + q_1 - q_2) \right. \\
- (t^{a})^{ij} \langle \gamma^{0} \Gamma^{(G)\mu}(k; q_1, -q_2) \Gamma^{0}_{\alpha \beta} \langle A_{\mu}^{b}(k) \psi_{\alpha}^{i}(q_1) \psi_{\beta}^{j}(q_2) \rangle \delta(k' + q_1 - q_2) \right\} \\
+ ig^2 \int dk' dk_1 dk_1 dq_1 dq_2 \left\{ \delta \Gamma^{(G)c, ij}(k, -k_1, q_1, -q_2) \langle A_{\mu}^{a}(k') A_{\nu}^{c}(k_1) \psi_{\alpha}^{i}(q_1) \psi_{\beta}^{j}(q_2) \rangle \right. \\
\times \delta(k + q_1 - k_1 - q_2) \\
- (\gamma^{0} \delta \Gamma^{(G)ac}(k', -k_1, q_1, -q_2) \Gamma^{0}_{\alpha \beta} \langle A_{\mu}^{b}(k) A_{\nu}^{c}(k_1) \psi_{\alpha}^{i}(q_1) \psi_{\beta}^{j}(q_2) \rangle \delta(k' + q_1 - k_1 - q_2) \right\}.
\]

In order to obtain the desired equation we must take into account only the substitutions that lead to the correlation function depending on two pairs of free-quark fields \( \psi^{(0)} \bar{\psi}^{(0)} \), \( \bar{\psi}^{(0)} \psi^{(0)} \) and two free-gluon fields \( A^{(0)} \), \( A^{* (0)} \). We consider the first term on the right-hand side of Eq. (7.1) containing the third-order correlator of the interacting fields

\[
- \frac{i g}{2} (T^{b})^{cd} \int dk' dk_1 dk_2 \langle A_{\mu}^{a}(k', k_1, -k_2) \rangle \langle A_{\mu}^{c}(k_1) A_{\lambda}^{d}(k_2) \rangle \delta(k - k_1 - k_2). \]
The replacement resulting in relevant correlation function has the following form:

\[ A^{a \mu}(k') \rightarrow - (\ast \mathcal{D}^{\mu \nu}(k')) \ast \Sigma_{\mu}^{\psi(1,2) \alpha}(A^{a(0)}, \bar{\psi}^{(0)}, \psi^{(0)})(k'), \]

\[ A^{cv}(k_1) \rightarrow - \ast \mathcal{D}^{\nu \sigma}(k_1) \Sigma^{\psi(0,2) \alpha}(\bar{\psi}^{(0)}, \psi^{(0)})(k_1), \quad A^{d \lambda}(k_2) \rightarrow A^{(0) d \lambda}(k_2). \]

Here and henceforth, we consider only the replacements that contain gluon propagators \( \ast \mathcal{D}^{\mu \nu}(k) \) and \( \ast \mathcal{D}^{\nu \sigma}(k') \). These propagators give later on the factors proportional to \( \delta(\omega - \omega_{k,l}^{a \mu}) \) and \( \delta(\omega + \omega_{k,l}^{a \mu}) \).

The factors take into account an existence of soft transverse gluon modes and plasmons with wave vector \( k \) and energy \( \omega_{k,l}^{a \mu} \) (see remark after Eq. (5.1)).

For this substitution the following product of correlation functions of free fields arises:

\[ \langle \bar{\psi}_{\alpha_1}^{(0)i_1}(-q_1)\psi_{\alpha_2}^{(0)i_2}(q_2)\bar{\psi}_{\alpha_3}^{(0)i_3}(-q_3)\psi_{\alpha_4}^{(0)i_4}(q_4)\rangle \langle A^{a(0)\alpha}(k_1)A^{(0)\beta \nu}(k_2) \rangle \]

\[ \simeq \left\{ \delta^{i_1 i_2} \Upsilon^{\alpha_2 \alpha_1}(q_2)\delta(q_2 - q_1)\delta^{i_3 i_4} T^{\alpha_4 \alpha_3}(q_4)\delta(q_4 - q_3) \right. \]

\[ \left. - \delta^{i_1 i_4} T^{\alpha_4 \alpha_1}(q_4)\delta(q_4 - q_1)\delta^{i_2 i_3} \Upsilon^{\alpha_2 \alpha_3}(q_2)\delta(q_2 - q_3) \right\} \delta^{ab} f^{\mu \nu}(k_1)\delta(k_1 - k_2). \]

On the right-hand side of the last expression only the second term in braces results in collision terms with required conservation laws of energy and momentum, Eq. (2.9).

Performing the calculations similar to those of sections 5 and 6, we obtain instead of (7.2)

\[ - \frac{ig^4}{2} [t^b, t^c]^{ij} \ast \mathcal{D}^{\mu \nu}(-k) \int dk_1 dq_1 dq_2 \ast \Gamma_{\mu \nu \lambda}(k, -k + k_1, -k_1) \ast \mathcal{D}^{\rho \sigma}(k - k_1) \]

\[ \times \Gamma^{(G)}_{\rho', \alpha', \beta'}(k - k_1; q_1, -q_2) \ast \bar{\Sigma}^{(G) ac, ij}_{\mu', \alpha', \beta'}(k, -k_1; q_1, -q_2) \]

\[ \times f^{\rho \sigma}(k_1) \Upsilon_{\alpha \alpha'}(q_1) \Upsilon_{\beta' \beta}(q_2) \delta(k + q_1 - k_1 - q_2). \]

For term (7.2) there exists another relevant substitution

\[ A^{a \mu}(k') \rightarrow - (\ast \mathcal{D}^{\mu \nu}(k')) \ast \Sigma_{\mu}^{\psi(1,2) \alpha}(A^{a(0)}, \bar{\psi}^{(0)}, \psi^{(0)})(k'), \quad A^{cv}(k_1) \rightarrow A^{(0) cv}(k_1), \]

\[ A^{d \lambda}(k_2) \rightarrow - \ast \mathcal{D}^{\lambda \nu}(k_2) \Sigma^{\psi(0,2) \alpha}(\bar{\psi}^{(0)}, \psi^{(0)})(k_2). \]

The expression obtained for this replacement by a simple change of integration variable can be resulted in (7.3).

Furthermore, we consider the third term on the right-hand side of basic equation (7.1) also containing the third order correlation function of interacting fields

\[ ig (t^b)^{ij} \int dk' dq_1 dq_2 \ast \Gamma^{(G) \mu}_{\alpha \beta}(k; q_1, -q_2) \langle A^{a \mu}(k') \bar{\psi}_{\alpha}^{i}(q_1) \psi_{\beta}^{j}(q_2) \rangle \delta(k + q_1 - q_2). \]
For this term there exist two substitutions leading to sixth-order correlation function of free fields with conservation laws (2.9):

\[ A^{*a\mu}(k') \rightarrow - (\star \mathcal{D}^{\mu\nu}(k'))^{*} \mathcal{J}_\mu^{(1,2)\alpha} (A^{*0}(0), \bar{\psi}^{(0)}, \psi^{(0)})(k'), \bar{\psi}_\alpha^{i}(-q_1) \rightarrow \bar{\psi}_\alpha^{(0)i}(-q_1), \]

\[ \bar{\psi}_\beta^{j}(q_2) \rightarrow - * S_{\beta\beta'}(q_2) \tilde{\mathcal{J}}_{\beta'}^{(1,1)j} (A^{00}, \psi^{0})(q_2), \]

and

\[ A^{*a\mu}(k') \rightarrow - (\star \mathcal{D}^{\mu\nu}(k'))^{*} \mathcal{J}_\mu^{(1,2)\alpha} (A^{*0}(0), \bar{\psi}^{(0)}, \psi^{(0)})(k'), \bar{\psi}_\alpha^{i}(-q_1) \rightarrow \tilde{\mathcal{J}}_{\alpha}^{(1,1)i} (A^{00}, \bar{\psi}^{0})(-q_1), \bar{\psi}_\beta^{j}(q_2) \rightarrow \bar{\psi}_\beta^{(0)j}(q_2). \]

These replacements after simple algebraic transformations lead term (7.4) to the following expression:

\[ -ig^{4} (\star \mathcal{D}^{\mu\nu}(-k)) \int dk_{1}dq_{1}dq_{2} \]

\[ \times \left\{ (t^{a}t^{b})^{ij} \mathcal{I}_{\mu,\alpha,\gamma}(k) * \mathcal{I}_{\mu,\gamma,\beta}(k+q_1) * \mathcal{I}_{\lambda,\gamma,\beta}(k_1; q_2, -k_1 - q_2) \right. \]

\[ + (t^{a}t^{b})^{ij} \mathcal{I}_{\mu,\alpha,\gamma}(k; -q_1, q_1 - k_1) * \mathcal{I}_{\mu,\gamma,\beta}(k - q_2) * \mathcal{I}_{\lambda,\gamma,\beta}(k; -k + q_2, -q_2) \right\} \]

\[ \times * \mathcal{I}_{\mu',\lambda',\alpha\beta}(k, -k_1; q_1, -q_2) I_{a\alpha'}^{(1)}(k_1) Y_{\alpha\alpha'}(q_1) Y_{\beta\beta'}(q_2) \delta(k + q_1 - k_1 - q_2). \]

Finally, we will consider the terms containing fourth-order correlation functions on the right-hand side of Eq. (7.1). Here, there exists a unique relevant substitution

\[ A^{*a\mu}(k') \rightarrow - (\star \mathcal{D}^{\mu\nu}(k'))^{*} \mathcal{J}_\mu^{(1,2)\alpha} (A^{*0}(0), \bar{\psi}^{(0)}, \psi^{(0)})(k'), \]

\[ A^{\nu}(k_1) \rightarrow A^{(0)\nu}(k_1), \]

\[ \bar{\psi}_\alpha^{i}(-q_1) \rightarrow \bar{\psi}_\alpha^{(0)i}(-q_1), \quad \bar{\psi}_\beta^{j}(q_2) \rightarrow \bar{\psi}_\beta^{(0)j}(q_2). \]

This replacement results the first term containing the fourth-order correlation function of interacting fields in the following form:

\[ -ig^{4} (\star \mathcal{D}^{\mu\nu}(k'))^{*} \int dk_{1}dq_{1}dq_{2} \delta \mathcal{J}_{\mu,\lambda,\alpha\beta}(k, -k_1; q_1, -q_2) * \mathcal{I}_{\mu',\lambda',\alpha\beta}(k, -k_1; q_1, -q_2) \]

\[ I_{a\alpha'}^{(1)}(k_1) Y_{\alpha\alpha'}(q_1) Y_{\beta\beta'}(q_2) \delta(k + q_1 - k_1 - q_2). \]

Now we add together expressions obtained (7.3), (7.5) and (7.6). Taking into account the definition of effective vertex (4.10), we find that the sum of the first, third and fifth terms on the right-hand side of Eq. (7.1) can be presented in the form similar to Eq. (5.6)

\[ -ig^{4} \mathcal{D}^{\mu\nu}(-k) \int dk_{1}dq_{1}dq_{2} \mathcal{J}_{\mu,\lambda,\alpha\beta}(k, -k_1; q_1, -q_2) \mathcal{I}_{\mu',\lambda',\alpha\beta}(k, -k_1; q_1, -q_2) \]

\[ I_{a\alpha'}^{(1)}(k_1) Y_{\alpha\alpha'}(q_1) Y_{\beta\beta'}(q_2) \delta(k + q_1 - k_1 - q_2). \]
Further, we transform expression (7.7) within the scheme suggested in section 5. Let us obtain at first the probability of plasmon-plasmino elastic scattering. For this purpose at the beginning we extract a purely plasmon part from gluon spectral density $I^{\lambda\lambda}(k_1)$ setting

$$I^{\lambda\lambda}(k_1) \rightarrow \frac{\bar{u}^{\lambda}(k_1)u^{\lambda}(k_1)}{\bar{u}^2(k_1)} \left\{ I_{k_1}^I \delta(k_1^0 - \omega_{k_1}) + I_{-k_1}^I \delta(k_1^0 + \omega_{k_1}) \right\},$$

$$(*D^{\mu\nu}(k))^* \rightarrow -\frac{\bar{u}^{\mu}(k)\bar{u}^{\nu}(k)}{\bar{u}^2(k)} (*\Delta^I(k))^*.$$

By virtue of $\delta$-functions we can perform integration with respect to $dk_1^0$. In term containing $I^I_{-k_1}$ we make a replacement of integration variable: $k_1 \rightarrow -k_1$ ($\omega^\mu_{k_1} \rightarrow \omega^\mu_{-k_1}$). Further, we extract the plasmino part of the functions $\Upsilon_{\alpha\alpha'}(q_1)$ and $\Upsilon_{\beta\beta'}(q_2)$

$$\Upsilon_{\alpha\alpha'}(q_1) \rightarrow (h_-(\hat{q}_1))_{\alpha\alpha'} \hat{\Upsilon}_{-q_1} \delta(q_1^0 - \omega_{q_1}) + (h_+(\hat{q}_1))_{\alpha\alpha'} \hat{\Upsilon}_{q_1} \delta(q_1^0 + \omega_{q_1}),$$

$$\Upsilon_{\beta\beta'}(q_2) \rightarrow (h_-(\hat{q}_2))_{\beta\beta'} \hat{\Upsilon}_{-q_2} \delta(q_2^0 - \omega_{q_2}) + (h_+(\hat{q}_2))_{\beta\beta'} \hat{\Upsilon}_{q_2} \delta(q_2^0 + \omega_{q_2}).$$

In the product $\Upsilon_{\alpha\alpha'}(q_1)\Upsilon_{\beta\beta'}(q_2)$ for collision term containing conservation law $\delta(k + q_1 - k_1 - q_2)$ (Eq. (7.8) it is necessary to keep only two terms of a ‘direct’ production, i.e.,

$$\Upsilon_{\alpha\alpha'}(q_1)\Upsilon_{\beta\beta'}(q_2) \rightarrow (h_-(\hat{q}_1))_{\alpha\alpha'} \hat{\Upsilon}_{-q_1} \delta(q_1^0 - \omega_{q_1})(h_-(\hat{q}_2))_{\beta\beta'} \hat{\Upsilon}_{-q_2} \delta(q_2^0 - \omega_{q_2})$$

$$+ (h_+(\hat{q}_1))_{\alpha\alpha'} \hat{\Upsilon}_{q_1} \delta(q_1^0 + \omega_{q_1})(h_+(\hat{q}_2))_{\beta\beta'} \hat{\Upsilon}_{q_2} \delta(q_2^0 + \omega_{q_2})$$

and in the term containing conservation law $\delta(k + q_1 + k_1 - q_2)$ it is necessary to keep only ‘crossed’ term

$$\Upsilon_{\alpha\alpha'}(q_1)\Upsilon_{\beta\beta'}(q_2) \rightarrow (h_+(\hat{q}_1))_{\alpha\alpha'} \hat{\Upsilon}_{q_1} \delta(q_1^0 + \omega_{q_1})(h_-(\hat{q}_2))_{\beta\beta'} \hat{\Upsilon}_{-q_2} \delta(q_2^0 - \omega_{q_2}).$$

The first term on the right-hand side of Eq. (7.8) is associated with the scattering process of plasmon off plasmino and the second one is associated with the scattering of plasmon off antiplasmino. Finally, contribution (7.9) defines the process of creation of plasmino-antiplasmino pair by two plasmon fusion.

Let us consider for the sake of definiteness the first term on the right-hand side of Eq. (7.8). We substitute this term into Eq. (7.7) and perform integration in $dq_1^0dq_2^0$. For spinor projectors $h_-(\hat{q}_1)$ and $h_-(\hat{q}_2)$ we use representation (5.9). The spectral densities $I^I_{k_1}$, $\hat{\Upsilon}_{q_1}$, and $\hat{\Upsilon}_{-q_2}$ are replaced by plasmon and (anti)plasmino number densities according to rules (5.11) and (5.3). The remaining conjugate terms on the right-hand side of Eq. (7.7) lead to the expression similar to (7.7) with a unique replacement $(*D^{\mu\nu}(k))^* \rightarrow (*\Delta^I(k))^*$. Thus in the case of plasmon-plasmino interaction for a weak-absorption medium when $\text{Im} *\Delta^{-1}(k) \rightarrow 0$, this results in simple replacement $(*\Delta^I(k))^*$ by

$$(*\Delta^I(k))^* - *\Delta^I(k) = -2i \text{Im} *\Delta^I(k) \simeq 2\pi i \text{sign}(k^0) \left( \frac{Z^I(k)}{2\omega_k} \right) \delta(k^0 - \omega_k).$$

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Here, on the right-hand side we keep only positively-frequency part.

Taking into account above-mentioned, we introduce the following matrix element of plasmon-plasmino scattering:

\[ M^{a a_1, i_1 i_2}_{\lambda_1 \lambda_1}(k, -k_1; q_1, -q_2) \equiv g^2 \left( \frac{Z_t(k)}{2\omega_k} \right)^{1/2} \left( \frac{Z_t(k_1)}{2\omega_{k_1}} \right)^{1/2} \left( \frac{Z_+ (q_1)}{2} \right)^{1/2} \left( \frac{Z_+ (q_2)}{2} \right)^{1/2} \]  

(7.10)

\[
\times \left( \frac{\bar{u}^\mu(k)}{\sqrt{\bar{u}^2(k)}} \right) \left( \frac{\bar{u}^\mu_1(k_1)}{\sqrt{\bar{u}^1(k_1)}} \right) \left[ \bar{v}_\beta(q_2, \lambda_2) \ast \tilde{\Gamma}^{(G) a a_1, i_1 i_2}_{\mu_1 \lambda_1, \alpha_\beta} (k, -k_1; q_1, -q_2) v_\alpha(q_1, \lambda_1) \right]_{\text{on-shell}}. 
\]

The probability of Compton-like plasmon-plasmino scattering in this case is defined as

\[ \delta^{ab} w^{(t-;t-)}_{qg \rightarrow qg}(k, q_1; k_1, q_2) = \sum_{\lambda_1, \lambda_2 = \pm} M^{a a_1, i_1 i_2}_{\lambda_1 \lambda_2}(k, -k_1; q_1, -q_2) \ast (M^{a a_1, i_1 i_2}_{\lambda_1 \lambda_2}(k, -k_1; q_1, -q_2))^*, \]  

(7.11)

The different terms of matrix element (7.10) have a similar diagrammatic interpretation as is depicted in Fig.4 with relevant replacements of momenta and HTL-resummed vertices only. The probability for the elastic scattering of soft transverse gluon excitation off soft normal quark excitation is defined by the following expression:

\[ \delta^{ab} w^{(t+;t+)}_{qg \rightarrow qg}(k, q_1; k_1, q_2) = \sum_{\xi, \xi_1 = 1,2 \lambda_1, \lambda_2 = \pm} \sum_{\lambda_3 = \pm} M^{a a_1, i_1 i_2}_{\xi_1 \lambda_1 \lambda_2}(k, -k_1; q_1, -q_2) \ast (M^{a a_1, i_1 i_2}_{\xi \lambda_1 \lambda_2}(k, -k_1; q_1, -q_2))^*, \]  

(7.12)

where now the matrix element is

\[ M^{a a_1, i_1 i_2}_{\xi \lambda_1 \lambda_2}(k, -k_1; q_1, -q_2) \equiv g^2 \left( \frac{Z_t(k)}{2\omega_k} \right)^{1/2} \left( \frac{Z_t(k_1)}{2\omega_{k_1}} \right)^{1/2} \left( \frac{Z_+ (q_1)}{2} \right)^{1/2} \left( \frac{Z_+ (q_2)}{2} \right)^{1/2} \]  

(7.13)

\[
\times \left( \frac{\epsilon_\mu(k, \xi)}{\sqrt{\epsilon^2(k, \xi)}} \right) \left( \frac{\epsilon_\mu_1(k_1, \xi_1)}{\sqrt{\epsilon^2(k_1, \xi_1)}} \right) \left[ \bar{u}_\beta(q_2, \lambda_2) \ast \tilde{\Gamma}^{(G) a a_1, i_1 i_2}_{\mu_1 \lambda_1, \alpha_\beta} (k, -k_1; q_1, -q_2) u_\alpha(q_1, \lambda_1) \right]_{\text{on-shell}}. 
\]

The probabilities for other types of the elastic scattering \( w^{(t-;t-)}_{qg \rightarrow qg}, w^{(t+;t+)}_{qg \rightarrow qg} \) are obtained from previous by the appropriate replacements of gluon and quark wave functions and choice of mass-shell conditions on the right-hand sides of Eqs. (7.10) and (7.13).

As in section 5 we regroup the terms in the effective amplitudes \( \tilde{\Gamma}^{(G) a a_1, i_1 i_2}_{\mu_1 \lambda_1, \alpha_\beta} \) and \( \Gamma^{(G) a a_1}_{\mu_1 \lambda_1, \alpha_\beta} \) according to transition (5.18). We present the HTL-induced vertex \( \delta \Gamma^{(G) a a_1}_{\mu_1 \lambda_1} \) in the form similar to (5.19), where now by virtue of initial definition (A.1) instead of \( \delta \Gamma^{(Q,S,A)}_{\mu_1 \lambda_1} \) we imply

\[ \delta \Gamma^{(G,S,A)}_{\mu_1 \lambda_1}(k, -k_1; q_1, -q_2) = \omega_0^2 \int \frac{d\Omega}{4\pi} \frac{v_\mu v_{\mu_1}}{v \cdot (q_1 - i\epsilon)(v \cdot q_2 + i\epsilon)} \times \left( \frac{1}{v \cdot (q_1 + k) + i\epsilon} \pm \frac{1}{v \cdot (q_1 - k_1) - i\epsilon} \right). \]
Taking into account this expression, we present matrix element \((7.10)\) in the following form:

\[
M_{\lambda_1\lambda_2}^{a_1,i_1i_2}(k, -k_1; q_1, -q_2) = \frac{1}{2} \left\{ t^{a_1}, t^{a} \right\}^{i_1i} \tilde{C}^{-1}_{S, \lambda} M_{\lambda_1\lambda_2}^{(S)}(k, -k_1; q_1, -q_2) + \frac{1}{2} \left\{ t^{a_1}, t^{a} \right\}^{i_1i} \tilde{C}^{-1}_{A} M_{\lambda_1\lambda_2}^{(A)}(k, -k_1; q_1, -q_2),
\]

where

\[
\tilde{C}_{S, A} \equiv \left\{ \frac{1}{2} T_F \left( C_F + \frac{1}{2N_c} \right) \right\}^{1/2},
\]

\[
M_{\lambda_1\lambda_2}^{(S, A)}(k, -k_1; q_1, -q_2) \equiv g^2 \tilde{C}_{S, A} \left( \frac{Z_0(k)}{2\omega_k} \right)^{1/2} \left( \frac{Z_0(k_{1})}{2\omega_{k_1}} \right)^{1/2} \left( \frac{Z_-(q_1)}{2} \right)^{1/2} \left( \frac{Z_-(q_2)}{2} \right)^{1/2}
\]

\[
\times \left[ \frac{\bar{\nu}(\hat{q}_2, \lambda_2)}{\sqrt{\bar{\nu}^2(k)}} \left( \frac{\bar{\nu}^{\mu_1}(k_{1})}{\sqrt{\bar{\nu}^2(\hat{k})}} \right) \left[ \bar{\nu}(\hat{q}_1, \lambda_1) \right]_{\text{on-shell}} \right].
\]

The functions \(M_{\mu_1}^{(S, A)}\) are defined by the following expressions:

\[
M_{\mu_1}^{(S)}(k, -k_1; q_1, -q_2) \equiv \delta_{\mu_1}^{(G; S)}(-k, k_1; -q_1, q_2)
\]

\[-*\Gamma^{(Q)}_{\mu_1}(-k_1; -q_2, k_1 + q_2) *S(-k - q_1) *\Gamma^{(G)}_{\mu}(-k; k + q_1, -q_1)
\]

\[+*\Gamma^{(G)}_{\mu}(-k; k - q_2, q_2) *S(-k_1 + q_1) *\Gamma^{(Q)}_{\mu_1}(-k_1; q_1, -q_1 + k_1)
\]

and

\[
M_{\mu_1}^{(A)}(k, -k_1; q_1, -q_2) \equiv \delta_{\mu_1}^{(G; A)}(-k, k_1; -q_1, q_2)
\]

\[+2*\Gamma^{(G)}_{\mu_1}(q_1 - q_2; -q_1, q_2) *D_{\mu_1}^{\nu \nu'}(-k + k_1) *\Gamma_{\mu_1 \nu'}(-k; k_1, k - k_1)
\]

\[-*\Gamma^{(Q)}_{\mu_1}(-k_1; -q_2, k_1 + q_2) *S(-k_1 - q_1) *\Gamma^{(G)}_{\mu}(-k; k + q_1, -q_1)
\]

\[-*\Gamma^{(G)}_{\mu}(-k; q_1 - q_2, q_2) *S(-k_1 + q_1) *\Gamma^{(Q)}_{\mu_1}(-k_1; q_1, -q_1 + k_1).
\]

Here, \(T_F\) is index of the fundamental representation.

By virtue of representation \((7.14)\) probability \((7.11)\) can be introduced in the form of a sum of two independent parts similar to \((5.20)\)

\[
u^{(S; -)}_{gg \to gq}(k, q_1; k_1, q_2) = \nu^{(S)}_{gg \to gq}(k, q_1; k_1, q_2) + \nu^{(A)}_{gg \to gq}(k, q_1; k_1, q_2),
\]

where

\[
\nu^{(S, A)}_{gg \to gq}(k, q_1; k_1, q_2) = \sum_{\lambda_1, \lambda_2 = \pm} \left| M_{\lambda_1\lambda_2}^{(S, A)}(k, -k_1; q_1, -q_2) \right|^2_{\text{on-shell}}.
\]

In deriving this expression a color algebra was used

\[
\text{tr}\left\{ t^{a_1} t^{a} \right\} \left\{ t^{b}, t^{a_1} \right\} = 2\delta^{ab} T_F \left( C_F - \frac{1}{2N_c} \right),
\]

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The decomposition (7.16) is true also for probability (7.12) and for other probabilities of elastic scattering: \( w_{gq \rightarrow gq}^{(l-; l-)} \) and \( w_{gq \rightarrow gq}^{(l+; l+)} \).

Now we consider the second term on the right-hand side of Eq. (7.8). As was mentioned above this term defines the process of plasmon scattering off antiplasmino. The reasoning similar to previous one results in scattering probability \( w_{gq \rightarrow gq}^{(l-; l-)}(k, k_1; q_1, q_2) \), as was defined by Eq. (7.16), where now

\[
\begin{align*}
M^{(S,A)}_{\lambda_1, \lambda_2}(k, -k_1; q_1, -q_2) &= \sum_{\lambda_3, \lambda_4} \left| M^{(S,A)}_{\lambda_1, \lambda_2}(k, -k_1; q_1, -q_2) \right|^2_{\text{on-shell}},
\end{align*}
\]

and

\[
\begin{align*}
M^{(S,A)}_{\lambda_1, \lambda_2}(k, -k_1; q_1, -q_2) &= g^2 \tilde{C}_{S,A} \left( \frac{Z_{l}(k)}{2\omega_k} \right)^{1/2} \left( \frac{Z_{l}(k_1)}{2\omega_{k_1}} \right)^{1/2},
\end{align*}
\]

\[
\begin{align*}
\times \left( \frac{\bar{u}^\mu(k)}{\sqrt{\bar{u}^2(k)}} \right) \left( \frac{\bar{u}^{\mu_1}(k_1)}{\sqrt{\bar{u}^2(k_1)}} \right) \left[ \bar{v}(q_1, \lambda_1)M^{(S,A)}_{\mu\mu_1}(k, -k_1; q_1, -q_2) v(q_2, \lambda_2) \right]_{\text{on-shell}}.
\end{align*}
\]

In the kinetic equation it is necessary to replace \( n^{-}_{q_1}, n^{-}_{q_2} \) by \( \bar{n}^{-}_{q_1}, \bar{n}^{-}_{q_2} \). The integration measure here remains invariable, as it was defined on the first line of Eq. (2.9). The diagrammatic interpretation of the different terms entering into matrix element (7.7) is presented in Fig. 4.

![Diagram](image_url)

Figure 4: Compton scattering of soft-gluon off soft-antiquark excitations.

Finally we will consider the contribution of term (7.9) determining a pair production \( gg \rightarrow q\bar{q} \). The probability of this process \( w_{gq \rightarrow gq}^{(l_1; -l_1)}(k, k_1; q_1, q_2) \) is defined by an expression, which is similar to Eqs. (7.16), (7.15) with a unique difference\(^7\) that functions

\(^7\)In the kinetic equation we replace \( 1 - n^{-}_{q_1} \) by \( \bar{n}^{-}_{q_1} \) and \( 1 + N^l_{k_1} \) by \( N^l_{k_1} \). The integration measure is defined by the second equation in (2.9)
\( \mathcal{M}_{\mu_1}^{(S, A)}(\mathbf{k}, -\mathbf{k}_1; \mathbf{q}_1, -\mathbf{q}_2) \) in (7.13) should be replaced by \( \mathcal{M}_{\mu_1}^{(S, A)}(\mathbf{k}, \mathbf{k}_1; -\mathbf{q}_1, -\mathbf{q}_2) \). The diagrammatic interpretation of the different terms entering into the matrix element of pair production is presented in Fig. 4.

![Diagram](image)

**Figure 5:** Production of soft quark-antiquark pair.

Now we return to initial equation (7.1). We transform the left-hand side similar to (18). As result we have

\[
\begin{align*}
- \delta^{ab} \delta(k^0 - \omega_k^{(b)}) \left( \frac{\partial N_{k_{k}}^{(b)}}{\partial t} + v_{k}^{(b)} \cdot \frac{\partial N_{k_{k}}^{(b)}}{\partial x} \right).
\end{align*}
\]

Taking into account above-mentioned, we write out the final expression for the kinetic equation defining the change of the colorless soft-gluon number densities caused by ‘spontaneous’ processes of soft-gluon–soft-quark elastic scattering and pair production by fusion of two soft-gluon excitations

\[
\begin{align*}
\left( \frac{\partial N_{k_{k}}^{(b)}}{\partial t} + v_{k}^{(b)} \cdot \frac{\partial N_{k_{k}}^{(b)}}{\partial x} \right)^{sp} = & \sum_{b_1=t, l \ f_1, f_2=\pm} \left\{ \int dT_{g_{f_1}g_{f_2}}^{(b_1; b_1 f_2)} u_{g_{f_1}g_{f_2}}^{(b_1; b_1 f_2)} (k, \mathbf{q}_1, \mathbf{k}_1, \mathbf{q}_2) N_{k_{k_1}}^{(b_1)} (1-n_{\mathbf{q}_1}^{(f_1)}) n_{\mathbf{q}_2}^{(f_1)} \\
+ & \int dT_{g_{f_1}g_{f_2}}^{(b_1; b_1 f_2)} u_{g_{f_1}g_{f_2}}^{(b_1; b_1 f_2)} (k_1, \mathbf{q}_1, \mathbf{k}_1, \mathbf{q}_2) N_{k_{k_1}}^{(b_1)} (1-n_{\mathbf{q}_1}^{(f_1)}) n_{\mathbf{q}_2}^{(f_1)} \\
+ & \int dT_{g_{f_1}g_{f_2}}^{(b_1; b_1 f_2)} u_{g_{f_1}g_{f_2}}^{(b_1; b_1 f_2)} (k, \mathbf{k}_1, \mathbf{q}_1, \mathbf{q}_2) N_{k_{k_1}}^{(b_1)} n_{\mathbf{q}_1}^{(f_1)} n_{\mathbf{q}_2}^{(f_1)} \right\}.
\end{align*}
\]

This equation follows from (2.3), (2.8) in the limit of a small intensity of soft-gluon excitations \( N_{k_{k_1}}^{(b_1)} \to 0 \) and making use of the condition \( 1 + N_{k_{k_1}}^{(b_1)} \sim N_{k_{k_1}}^{(b_1)} \).
8 Generalized formula of emitted radiant power in QGP

In previous sections we have explicitly shown how one can define the probabilities of the simplest processes of nonlinear interaction between the soft collective modes Fermi and Bose statistics in QGP. This has been done by immediate extracting all possible contributions to matrix elements of these processes. However the use of such a direct approach in the general case of the processes including an arbitrary number of soft fermion and boson excitations becomes ineffective because of awkwardness and complexity of computations. In this case the method developed by Tsytovich [20, 21] in the theory of usual plasma known as the correspondence principle is more convenient for deriving an explicit form of the scattering probabilities. We have already used this approach for deriving the general expression for the probability of $2n + 2$-colorless soft-gluon decay processes, $n = 1, 2, ...$ [1]. Here, we don’t set ourselves a purpose to obtain similar general expressions for the probabilities of the processes including an arbitrary (even) number of soft boson and fermion excitations. We concentrate our attention on deriving the most general expression for the emitted radiant power, which would take into account an existence of fermionic sector of plasma excitations in QGP. Having at hand this expression and following standard scheme stated in section 4 of the paper [1] with employing general expressions for effective currents and sources (4.5), it is not difficult to obtain the general expressions for the probabilities in this more complicated case. Thus the problem stated is reduced to calculation of explicit form of the effective amplitudes in integrand of effective currents and sources (4.5). Calculating algorithm of the effective amplitudes was proposed in section 4 of the present paper.

For derivation of the probabilities of soft-gluon decay processes in [1] we have used the following formula for the emitted radiant power:

$$I = \lim_{\tau, V \to \infty} \frac{1}{\tau V} \frac{\tau/2}{\tau/2 V} \int dx dt \langle E^a(x, t) \cdot j^a(x, t) \rangle,$$

where $V$ is a spatial volume of integration. Averaging over the time is made for elimination of oscillating terms in $I$. Thus the emitted radiant power is defined as a work per time done by radiation field with color current (in this case $j^a(x, t) = j^{Aa}(x, t)$) creating this field. The chromoelectric field $E^a(x, t)$ is associated with $j^a(x, t)$ by the field equation.

We propose that generalization of the above-written formula for emitted radiant power taking into account an existence of soft fermion excitations in the system is

$$I = I_B + I_F,$$
where

\[ I_\mathcal{E} = \lim_{\tau, V \to \infty} \frac{1}{\tau V} \int_{-\tau/2}^{\tau/2} dx dt \left\{ \mathbf{E}^a(x, t) \cdot \mathbf{j}^{\alpha a}(x, t) \right\} + \lim_{\tau, V \to \infty} \frac{1}{\tau V} \int_{-\tau/2}^{\tau/2} dx dt \left\{ \mathbf{E}^a(x, t) \cdot \mathbf{j}^{\psi a}(x, t) \right\} \]

\[ = \lim_{\tau, V \to \infty} \frac{(2\pi)^4}{\tau V} \int d\mathbf{k} d\mathbf{q} \langle \mathbf{E}^a(\mathbf{k}, k^0) \cdot \mathbf{j}^{\alpha a}(\mathbf{k}, k^0) \rangle + \lim_{\tau, V \to \infty} \frac{(2\pi)^4}{\tau V} \int d\mathbf{k} d\mathbf{q} \langle \mathbf{E}^a(\mathbf{k}, k^0) \cdot \mathbf{j}^{\psi a}(\mathbf{k}, k^0) \rangle \]

(8.1)

and

\[ I_\mathcal{F} = -\frac{1}{2} \lim_{\tau, V \to \infty} \frac{1}{\tau V} \int_{-\tau/2}^{\tau/2} dx dt \left\{ \langle \bar{\psi}_\alpha(x, t) \bar{D}^\dagger_0 i \eta_\alpha^i(x, t) \rangle + \langle \bar{\eta}_\alpha^i(x, t) \bar{D}_0 \psi_\alpha(x, t) \rangle \right\} \]

(8.2)

\[ = -\frac{i}{2} \lim_{\tau, V \to \infty} \frac{(2\pi)^4}{\tau V} \int d\mathbf{q} d\mathbf{q}^0 \langle \bar{\psi}_\alpha^i(-q) \eta_\alpha^i(q) \rangle - \langle \bar{\eta}_\alpha^i(-q) \psi_\alpha^i(q) \rangle \}

\[ + \frac{ig}{2} \lim_{\tau, V \to \infty} \frac{(2\pi)^4}{\tau V} \int d\mathbf{q} d\mathbf{q} d\mathbf{k} \left\{ (t^a)^{ij} \langle \bar{\psi}_\alpha^j(-q) A_0^a(k) \eta_\alpha^i(q') \rangle \delta(q - q' - k) \right. \]

\[ - (t^a)^{ij} \langle \bar{\eta}_\alpha^i(-q') A_0^a(k) \psi_\alpha^j(q) \rangle \delta(q - q' + k) \}. \]

Here, on the first line\(^8\) of Eq. (8.2) we make use of covariant derivative \( D_0 \equiv \partial/\partial t + ig A_0^a(x, t) t^a \). For Fourier transformation we have used a relation

\[ \lim_{\tau \to \infty} \int_{-\tau/2}^{\tau/2} e^{i\omega t} dt = 2\pi \delta(\omega). \]

The appearance of the second term on the right-hand side of Eq. (8.1) containing the induced current \( \mathbf{j}^{\psi a}(x, t) \) is rather evident. The appearance of contribution (8.2) is a principal new feature, and therefore our further attention will be concentrated on a proof of the fact that expression (8.2) correctly reproduces the scattering probabilities obtained in sections 5 and 6 within the framework of the generalized correspondence principle.

According to the correspondence principle for determination of the scattering probabilities of the processes of a type

\[ qg \to qg, \quad qq \to qq, \quad q\bar{q} \to q\bar{q}, \quad \text{etc} \]

\(^8\)The authors are grateful to reviewer pointed to the necessity of entering covariant derivative \( D_0 \) instead of usual time derivative \( \partial/\partial t \) in Eq. (8.2) that is necessary for maintenance of gauge symmetry. The terms in the emitted radiant power \( I_\mathcal{F} \) including \( A_0 \) should be taken into account in the consideration of more complicated scattering processes than the processes presented in this work. For this reason in the subsequent discussion these terms will be omitted (or one can assume that we work in \( A_0\)-gauge). For recovery of Lorentz symmetry of Eq. (8.2) instead of \( D_0 \) it should be used \( (u \cdot D) \), where \( u \) is four-velocity of plasma.
the expression (8.2) should be compared with the expression determining a change of energy of soft fermionic plasma excitations caused by the spontaneous processes of soft-quark and soft-gluon waves emission only

$$\left( \frac{dE}{dt} \right)_{sp}^{(f)} = \sum_{f_1=\pm} \sum_{b_1=b_2=t_1,1} \left\{ 2 \int \frac{d\mathbf{q}}{(2\pi)^3} \int d\mathbf{T}(f_{b_1} f_{b_2}) \omega_{\mathbf{q}}^{(f)} w_{\mathbf{q}}^{(f_{b_1} f_{b_2})}(\mathbf{q}, \mathbf{q}_1; \mathbf{k}_1, \mathbf{k}_2) n_{\mathbf{q}_1}^{(f_1)} N_{\mathbf{k}_1}^{(b_1)} N_{\mathbf{k}_2}^{(b_2)} \right.$$  

$$+ \int \frac{d\mathbf{q}}{(2\pi)^3} \int d\mathbf{T}(f_{b_1} f_{b_2}) \omega_{\mathbf{q}}^{(f)} w_{\mathbf{q}}^{(f_{b_1} f_{b_2})}(\mathbf{q}, \mathbf{q}_1; \mathbf{k}_1, \mathbf{k}_2) \left(1 - \bar{n}_{\mathbf{q}_1}^{(f_1)} \right) N_{\mathbf{k}_1}^{(b_1)} N_{\mathbf{k}_2}^{(b_2)} \right.$$  

$$+ \sum_{f_1=\pm} \left\{ \int \frac{d\mathbf{q}}{(2\pi)^3} \int d\mathbf{T}(f_{b_1} f_{b_2}) \omega_{\mathbf{q}}^{(f)} w_{\mathbf{q}}^{(f_{b_1} f_{b_2})}(\mathbf{q}, \mathbf{q}_1; \mathbf{q}_2, \mathbf{q}_3) \left(1 - n_{\mathbf{q}_1}^{(f_1)} \right) n_{\mathbf{q}_2}^{(f_2)} \bar{n}_{\mathbf{q}_3}^{(f_3)} \right.$$  

$$+ 2 \int \frac{d\mathbf{q}}{(2\pi)^3} \int d\mathbf{T}(f_{b_1} f_{b_2}) \omega_{\mathbf{q}}^{(f)} w_{\mathbf{q}}^{(f_{b_1} f_{b_2})}(\mathbf{q}, \mathbf{q}_1; \mathbf{q}_2, \mathbf{q}_3) \left(1 - \bar{n}_{\mathbf{q}_1}^{(f_1)} \right) n_{\mathbf{q}_2}^{(f_2)} \bar{n}_{\mathbf{q}_3}^{(f_3)} \right.$$  

$$+ \left(n_{\mathbf{q}_1}^{(f_1)} \equiv \bar{n}_{\mathbf{q}_1}^{(f_1)}, \ i = 1, 2, 3 \right).$$

In deriving the right-hand side of (8.3) we have used equations (5.27) and (6.10). To proof (8.2) it is sufficient restored some scattering probability on the right-hand side of Eq. (8.3). For definiteness we consider the probability of elastic scattering of plasmino off plasmon entering into the first term.

At first step we transform Eq. (8.2) to more suitable form. The Fourier components of quark fields $\psi_\alpha^i(q)$, $\bar{\psi}_\alpha(-q)$ are associated with induced sources $\eta_\alpha^i(q)$ and $\bar{n}_\alpha^i(-q)$ by the Dirac equation

$$\psi_\alpha^i(q) = -S_{\alpha\beta}(q) \eta_\beta^i(q), \quad \bar{\psi}_\alpha^i(-q) = \bar{n}_\alpha^i(-q) S_{\alpha\beta}(-q).$$

Substituting the last expressions into Eq. (8.2), we obtain instead of (8.2)

$$I_F = -\frac{i}{2} \lim_{V \to \infty} \int \frac{d\mathbf{q} \ d\mathbf{q}^0 \ d\bar{q}}{(2\pi)^4} \eta_\alpha^i(-q) \eta_\beta^i(q) \left\{ S_{\alpha\beta}(-q) + S_{\alpha\beta}(q) \right\}. \quad (8.4)$$

By virtue of the properties of the quark scalar propagators $\Delta_\pm(-q) = (\Delta_\mp(q))^*$ the sum in braces in integrand of (8.4) can be written in the form

$$2i \operatorname{Im}(\Delta_+(q))(h_+(\hat{\mathbf{q}}))_{\alpha\beta} + 2i \operatorname{Im}(\Delta_-(q))(h_-(\hat{\mathbf{q}}))_{\alpha\beta}.$$

Furthermore, we express the projection operators $h_\pm(\hat{\mathbf{q}})$ in terms of simultaneous eigen-spinors of chirality and helicity

$$(h_+(\hat{\mathbf{q}}))_{\alpha\beta} = \sum_{\lambda=\pm} u_\alpha(\hat{\mathbf{q}}, \lambda) \bar{u}_\beta(\hat{\mathbf{q}}, \lambda), \quad (h_-(\hat{\mathbf{q}}))_{\alpha\beta} = \sum_{\lambda=\pm} v_\alpha(\hat{\mathbf{q}}, \lambda) \bar{v}_\beta(\hat{\mathbf{q}}, \lambda),$$

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and approximate imaginary parts of the scalar propagators for a weak-absorption medium by the following expressions:

\[ \text{Im} \Delta_\pm(q) \simeq \mp \pi Z_\pm(q) \delta(q^0 - \omega_\pm^*) \pm \pi Z_\mp(q) \delta(q^0 + \omega_\mp^*). \]

It is not difficult to see that the term containing function \( \delta(q^0 + \omega_\mp^*) \) after substitution into (8.4), performing integration in \( dq_0 \) and replacement of variable \( \mathbf{q} \to -\mathbf{q} \) results in contribution to emitted radiant power \( I_\mathcal{F} \) exactly equal the contribution of term with \( \delta(q^0 - \omega_\mp^*) \). Taking into account above-mentioned, we define the final expression for the emitted radiant power \( I_\mathcal{F} \), which is more convenient for concrete applications

\[ I_\mathcal{F} = 2\pi \lim_{\tau, V \to \infty} \frac{(2\pi)^4}{\tau V} \sum_{\lambda = \pm} \int dq \omega_\mp(q) \left\{ \omega_\mp^* Z_+(q) \langle |\tilde{\eta}_\alpha^i(-\mathbf{q}) u_\alpha(q, \lambda)|^2 \rangle_{q^0 = \omega_\mp^*} + \omega_\mp Z_-(-\mathbf{q}) \langle |\tilde{\eta}_\alpha^i(-\mathbf{q}) v_\alpha(q, \lambda)|^2 \rangle_{q^0 = \omega_\mp^*} \right\}. \tag{8.5} \]

For derivation of the probability for the elastic scattering of plasmino off plasmon in the last term on the right-hand side of Eq. (8.5) we perform the following replacement:

\[ \tilde{\eta}_\alpha^i(-\mathbf{q}) \to \tilde{\eta}_\alpha^{(2,1)i}(A^{*(0)}, A^{*(0)}, \bar{\psi}^{(0)})(-\mathbf{q}), \]

where the effective source \( \tilde{\eta}_\alpha^{(2,1)i}(-\mathbf{q}) \) is defined by equation (3.13). As a result of such a replacement for ‘plasmino’ part of expression (8.5) we have

\[ (I_\mathcal{F})_{\text{plasmino}} = \pi g^4 \lim_{\tau, V \to \infty} \frac{(2\pi)^4}{\tau V} \sum_{\lambda = \pm} \int dq \omega_\mp(q) \left( \tilde{\Gamma}_{\mu_1 \mu_2, \alpha \gamma}^{Q}(-\mathbf{q}) \langle A^{*(0)}(k_1, k_2; q_1, -\mathbf{q}) v_\alpha(q, \lambda) \rangle \right) \]

\[ \times \left( \tilde{\Gamma}_{\mu_1' \mu_2', \beta' \gamma'}^{Q}(-\mathbf{q}) \langle A^{*(0)}(k_1', k_2'; q_1', -\mathbf{q}) \bar{\psi}_{\beta'}(q, \lambda) \rangle \right) \Delta(q - q_1 - k_1 - k_2) \delta(q - q_1' - k_1' - k_2') dq_1 dk_1 dq_2 dk_2 dq_1' dk_1' dk_2'. \]

Here, the product of correlation functions is transformed by formula (5.3). Performing integration in \( dq_1' dk_1' dk_2' \), we obtain instead of the last expression

\[ (I_\mathcal{F})_{\text{plasmino}} = \pi g^4 \lim_{\tau, V \to \infty} \frac{(2\pi)^4}{\tau V} \sum_{\lambda = \pm} \int dq \omega_\mp(q) \left( I^{\mu_1 \mu_2'}(k_1) I^{\mu_2 \mu_1'}(k_2) \Upsilon_{\gamma \gamma'}(q_1) \right) \]

\[ \times \left( \tilde{\Gamma}_{\mu_1 \mu_2, \alpha \gamma}^{Q}(k_1, k_2; q_1, -\mathbf{q}) v_\alpha(q, \lambda) \right) \left( \tilde{\Gamma}_{\mu_1' \mu_2', \beta' \gamma'}^{Q}(k_1, k_2; q_1', -\mathbf{q}) \bar{\psi}_{\beta'}(q, \lambda) \right) \]

\[ \times \left[ \delta(q - q_1 - k_1 - k_2) \right]^2 dq_1 dk_1 dk_2. \]

By \( \delta \)-function squared we mean

\[ \left[ \delta(q - q_1 - k_1 - k_2) \right]^2 = \frac{1}{(2\pi)^4} \tau V \delta(q - q_1 - k_1 - k_2). \]
In functions $I^{\mu_1 \nu_1}(k_1)$, $I^{\mu_2 \nu_2}(k_2)$ and $\Upsilon_{\gamma \gamma}(q_1)$ we extract plasmon and plasmino part\(^9\) according to Eq. \((5.7)\). Further, going to the plasmino and plasmon number densities (Eq. \((5.11)\)), we result equation \((8.3)\) in the final form

$$(\mathcal{I}_p)_{\text{plasmino}} = 2 \int \frac{d\mathbf{q}}{(2\pi)^3} \frac{d\mathbf{q}_1}{(2\pi)^3} \frac{d\mathbf{k}_1}{(2\pi)^3} \frac{d\mathbf{k}_2}{(2\pi)^3} (2\pi)^4 \delta(\mathbf{q} + \mathbf{k}_1 - \mathbf{q}_1 - \mathbf{k}_2) \delta(\omega_\mathbf{q} + \omega_{\mathbf{k}_1} - \omega_{\mathbf{q}_1} - \omega_{\mathbf{k}_2})$$

$$\times \omega_{\mathbf{q}} \left( \sum_{\lambda, \lambda_1 = \pm} T^{\alpha_1 \alpha_2, \beta_1 \beta_2}_{\lambda \lambda_1} (-\mathbf{k}_1, \mathbf{k}_2; \mathbf{q}_1, -\mathbf{q}) (T^{\alpha_1 \alpha_2, \beta_1 \beta_2}_{\lambda \lambda_1} (-\mathbf{k}_1, \mathbf{k}_2; \mathbf{q}_1, -\mathbf{q}))^* \right) n_{\mathbf{q}_1} N_{\mathbf{k}_1} N_{\mathbf{k}_2},$$

where the function $T^{\alpha_1 \alpha_2, \beta_1 \beta_2}_{\lambda \lambda_1}$ is defined by Eq. \((5.12)\). The expression obtained exactly coincides with corresponding term on the right-hand side of Eq. \((8.3)\).

Thus suggested expression \((8.2)\) correctly reproduces the scattering probabilities obtained by a different way. Formulae for emitted radiant power \((8.1)\) and \((8.2)\) open a way for more direct and economic deriving the scattering probabilities of the other higher processes of nonlinear interaction of soft fermion and boson excitations in QGP.

### 9 Linearized version of Boltzmann equation

In this section we briefly consider the linearization procedure of self-consistent system of kinetic equations \((2.2)\) and \((2.3)\). We examine plasmino kinetic equation \((2.2)\) with generalized rates \((2.5)\) and \((2.6)\), where we will consider only the contribution with $f_1 = -$ and $b_1 = b_2 = l$. We assume that the off-equilibrium fluctuations are perturbative small and present the number densities of colorless (anti)plasminos and plasmons in the form

$$n_{\mathbf{q}}^- = n_{eq}^- (\mathbf{q}) + \delta n_{\mathbf{q}}^-, \bar{n}_{\mathbf{q}}^- = \bar{n}_{eq}^- (\mathbf{q}) + \delta \bar{n}_{\mathbf{q}}^-, \quad N_{\mathbf{k}}^l = N_{eq}^l (\mathbf{k}) + \delta N_{\mathbf{k}}^l,$$

where $n_{eq}^- (\mathbf{q}) = \bar{n}_{eq}^- (\mathbf{q}) = (\frac{\omega_\mathbf{q}}{T} + 1)^{-1}$ and $N_{eq}^l (\mathbf{k}) = (\frac{\omega_{\mathbf{k}}}{T} - 1)^{-1}$. Parametrizing off-equilibrium fluctuations of the occupation numbers $\delta n_{\mathbf{q}}^-$, $\delta \bar{n}_{\mathbf{q}}^-$ and $\delta N_{\mathbf{k}}^l$ as follows

$$\delta n_{\mathbf{q}}^- \equiv - \frac{dn_{eq}^- (\mathbf{q})}{d\omega_{\mathbf{q}}^-} w_{\mathbf{q}}^- = \frac{1}{T} n_{eq}^- (\mathbf{q}) (1 - n_{eq}^- (\mathbf{q})) w_{\mathbf{q}}^-,$$

$$\delta \bar{n}_{\mathbf{q}}^- \equiv - \frac{dn_{eq}^- (\mathbf{q})}{d\omega_{\mathbf{q}}^-} \bar{w}_{\mathbf{q}}^- = \frac{1}{T} n_{eq}^- (\mathbf{q}) (1 - n_{eq}^- (\mathbf{q})) \bar{w}_{\mathbf{q}}^-,$$

$$\delta N_{\mathbf{k}}^l \equiv - \frac{dN_{eq}^l (\mathbf{k})}{d\omega_{\mathbf{k}}^l} W_{\mathbf{k}}^l = \frac{1}{T} N_{eq}^l (\mathbf{k}) (1 + N_{eq}^l (\mathbf{k})) W_{\mathbf{k}}^l,$$

\(^9\)In the product of the spectral densities $I_{\mathbf{k}_1}^l I_{\mathbf{k}_2}^l$ considering quasiparticle approximation \((5.10)\) we keep only the terms corresponding to the scattering of plasmino off plasmon.
we derive from Eqs. (2.2), (2.5) and (2.6), after simple algebraic transformations the linearized Boltzmann equation for the function $w_q^-$

$$\frac{\partial w_q^-}{\partial t} + v_q^- \cdot \nabla w_q^- =$$

$$-2 \int dT_{qq \rightarrow gg} \frac{n_{eq}^-(q_1)N_{eq}^{lt}(k_2)(1 + N_{eq}^{lt}(k_1))}{n_{eq}^-(q)} w_{qq \rightarrow gg}(q, k_1; q_1, k_2) \{w_q^- + \mathcal{W}_{k_1}^l - w_{q_1}^- - \mathcal{W}_{k_2}^l\}$$

$$- \int dT_{qq \rightarrow gg} \frac{(1 - n_{eq}^-(q_1))N_{eq}^{lt}(k_1)N_{eq}^{lt}(k_2)}{n_{eq}^-(q)} w_{qq \rightarrow gg}(q, q_1, k_1, k_2) \{w_q^- + \bar{w}_{q_1}^- - \mathcal{W}_{k_1}^l - \mathcal{W}_{k_2}^l\}$$

$$- \int dT_{qq \rightarrow gg} \frac{(1 - n_{eq}^-(q_1))n_{eq}^-(q_2)n_{eq}^-(q_3)}{n_{eq}^-(q)} w_{qq \rightarrow gg}(q, q_1, q_2, k_1, k_2) \{w_q^- + \bar{w}_{q_1}^- + \bar{w}_{q_2}^- - w_{q_3}^-\}$$

In deriving Eq. (9.1) we have used the identities

$$(1 - n_{eq}^-(q))n_{eq}^-(q_1)(1 + N_{eq}^{lt}(k_1))N_{eq}^{lt}(k_2) = n_{eq}^-(q)(1 - n_{eq}^-(q_1))N_{eq}^{lt}(k_1)(1 + N_{eq}^{lt}(k_2)),$$

$$(1 - n_{eq}^-(q))(1 - n_{eq}^-(q_1))N_{eq}^{lt}(k_1)N_{eq}^{lt}(k_2) = n_{eq}^-(q)n_{eq}^-(q_1)(1 + N_{eq}^{lt}(k_1))(1 + N_{eq}^{lt}(k_2)),$$

$$(1 - n_{eq}^-(q))(1 - n_{eq}^-(q_1))n_{eq}^-(q_2)n_{eq}^-(q_3) = n_{eq}^-(q)n_{eq}^-(q_1)(1 - n_{eq}^-(q_2))(1 - n_{eq}^-(q_3)),$$

which hold by virtue of corresponding conservation laws of energy.

In the remaining part of this section we make a little digression from the general line of our present work and consider the problem of the minimal generalization of Boltzmann equation (2.2) to the case of the dynamics of color soft-quark and soft-gluon excitations. For this purpose we assume that a time-space dependent external perturbation (e.g. external color current $j_{\mu}^{ext}(x)$) starts acting on the system. In the presence of the external color perturbation the soft gauge field develops an expectation value $\langle A_{\mu}^a(x) \rangle \equiv A_{\mu}^a(x) \neq 0$ (but nevertheless $\langle j_{\mu}^{ext}(x) \rangle = 0$) and the soft-quark and soft-gluon number densities acquire a non-diagonal color structure. In this case, we expect the time-space evolution of $n_q^- = (n_{q,i}^-)$ to be described by the (matrix) plasmino Vlasov-Boltzmann equation instead of (2.2)

$$\left(\mathcal{D}_t + v_q^- \cdot \mathcal{D}_x\right) n_q^- - \frac{1}{2} g \left\{ \left( \mathcal{E}(x) + (v_q^- \times \mathcal{B}(x)) \right)_i \nabla_p n_q^- \right\} = -C [n_q^-, N_k^l], \quad (9.2)$$

where $\mathcal{D}_\mu$ is the covariant derivative acting as

$$\mathcal{D}_\mu n_q^- = \partial_\mu n_q^- + ig[A_\mu(x), n_q^-], \quad A_\mu(x) = t^a A_{\mu}^a(x).$$

\footnote{We can formally suggest that in the system there exists also an external color source $n_3^{ext}(x)$ produced mean quark field $\langle \bar{q}^a_3(x) \rangle \neq 0$ (see, for example 22). Here, we don’t consider such a case for simplification of the problem.}
The functions $E^i(x)$ and $B^i(x)$ are mean chromoelectric and chromomagnetic fields. The collision term $C[n_q^-, N_k^l]$ has the following structure:

$$C[n_q^-, N_k^l] = \frac{1}{2} \{ n_q^-, \Gamma_d^{(-)}[n_q^-, N_k^l] \} - \frac{1}{2} \{ (1 - n_q^-), \Gamma_i^{(-)}[n_q^-, N_k^l] \} - \ldots,$$  \hfill (9.3)

where $\Gamma_d^{(-)}[n_q^-, N_k^l] = (\Gamma_d^{ii}[n_q^-, N_k^l])$ and $\Gamma_i^{(-)}[n_q^-, N_k^l] = (\Gamma_i^{ii}[n_q^-, N_k^l])$ represent the generalized decay and regenerating rates of color plasminos, respectively. The dots on the right-hand side of Eq. (9.3) is refereed to possible contributions containing commutators of $[\text{Re} \Sigma_R, n_q^-]$ type, where $\Sigma_R$ is retarded soft-quark self-energy.

As above, we restrict our attention to the process of the elastic scattering of color plasmino off color plasmon. In this particular case the decay and regenerating rates have the following form:

$$\Gamma_d^{ii'}[n_q^-, N_k^l] = \sum_{\lambda, \lambda_1 = \pm} \int dT_{qq \rightarrow qg}^{(-; l; l)} T_{\lambda \lambda_1}^{a l_a l_1, i i'} (-k_1, k_2; q_1, -q) \left( T_{\lambda \lambda_1}^{a l_a l_1, i i'} (-k_1, k_2; q_1, -q) \right)^*$$

$$\times (1 - n_q^-)^i j (N_{l_1}^l)^a a_j^l (1 + N_{k_2}^l)^a a_2^l,$$  \hfill (9.4)

$$\Gamma_i^{ii'}[n_q^-, N_k^l] = \sum_{\lambda, \lambda_1 = \pm} \int dT_{qq \rightarrow qg}^{(-; l; l)} T_{\lambda \lambda_1}^{a l_a l_1, i i'} (-k_1, k_2; q_1, -q) \left( T_{\lambda \lambda_1}^{a l_a l_1, i i'} (-k_1, k_2; q_1, -q) \right)^*$$

$$\times (n_q^-)^i j (1 + N_{l_1}^l)^a a_j^l (N_{k_2}^l)^a a_2^l,$$

where matrix element $T_{\lambda \lambda_1}^{a l_a l_1, i i'}$ is defined by Eq. (5.12). We recall that this matrix element has a physical sensible color decomposition:

$$T_{\lambda \lambda_1}^{a l_a l_1, i i'} (-k_1, k_2; q_1, -q) = \frac{1}{2} \{ t^{a_2}, t^{a_1} \}^i j \left( C_S^{-1} T_{\lambda \lambda_1}^{(S)} (-k_1, k_2; q_1, -q) \right)$$

$$+ \frac{1}{2} \{ t^{a_2}, t^{a_1} \}^j i \left( C_A^{-1} T_{\lambda \lambda_1}^{(A)} (-k_1, k_2; q_1, -q) \right).$$  \hfill (9.5)

Here, helical amplitudes $T_{\lambda \lambda_1}^{(S, A)}$ are given by Eq. (5.22) and $C_{S, A} = \{ C_F (C_F = 1/(2N_c)) / 2 \}^{1/2}$.

Furthermore, we consider the linearized version of Vlasov-Boltzmann equation (9.2). We write the number densities of color plasminos and plasmons as follows:

$$(n_q^-)^i j = n_q^- (k) \delta^{ij} - \frac{d n_q^- (q)}{d \omega_q^-} w_q^- (t^a)^i j,$$

$$(N_q^l)^{ab} = N_q^l (k) \delta^{ab} - \frac{d N_q^l (k)}{d \omega_k^l} \mathcal{W}_k^l c (T^c)^{ab}.$$  \hfill (9.6)

After some cumbersome algebraic transformations we derive the linearized kinetic equation for color plasminos from Eqs. (9.2) – (9.5)

$$\left( D_t + \mathbf{v}^- \cdot \mathbf{D}_x \right) w_q^- = - g \left( \mathbf{v}_q^- \cdot \mathbf{E} (x) \right) - \delta C \left[ w_q^-, \mathcal{W}_k^l \right],$$

$$\hfill (9.6)$$
where plasmino-plasmon linearized collision term $\delta C$ has the form:

$$
\delta C [w^{-}_q, \mathcal{W}^{l}_k] = \sum_{\lambda, \lambda_1 = \pm} \int dT^{(-)}_{qg ightarrow qg} n_{eq}(q_1) n_{eq}(q_2) (1 + N_{eq}^{l}(k_1)) n_{eq}(q) \left[ \left\{ |T^{(S)}_{\lambda\lambda_1}|^2 + |T^{(A)}_{\lambda\lambda_1}|^2 \right\} w^{-}_q 
+ \left\{ \frac{1}{4} \left[ \left( C_F - \frac{1}{N_c} \right) \frac{N_c}{2} - \frac{1}{4} \right] C^{-2}_s |T^{(S)}_{\lambda\lambda_1}|^2 + \frac{1}{4} \left( C_F + \frac{1}{N_c} \right) \frac{N_c}{2} - \frac{1}{4} \right] C^{-2}_A |T^{(A)}_{\lambda\lambda_1}|^2 
+ \frac{1}{2} \left( C_F + \frac{1}{2N_c} \right) \frac{N_c}{2} (C_s C_A)^{-1} \mathrm{Re} \left( T^{(S)}_{\lambda\lambda_1} T^{(A)}_{\lambda\lambda_1} \right) \right\} \mathcal{W}^{l}_k 
- \left\{ \frac{1}{4} \left[ \left( C_F - \frac{1}{N_c} \right) \frac{N_c}{2} - \frac{1}{4} \right] C^{-2}_s |T^{(S)}_{\lambda\lambda_1}|^2 + \frac{1}{4} \left( C_F + \frac{1}{N_c} \right) \frac{N_c}{2} - \frac{1}{4} \right] C^{-2}_A |T^{(A)}_{\lambda\lambda_1}|^2 
- \frac{1}{2} \left( C_F + \frac{1}{2N_c} \right) \frac{N_c}{2} (C_s C_A)^{-1} \mathrm{Re} \left( T^{(S)}_{\lambda\lambda_1} T^{(A)}_{\lambda\lambda_1} \right) \right\} \mathcal{W}^{l}_k 
- \frac{1}{2} \left[ \left( 1 + \frac{N_c^2}{4N_c^2} + \frac{1}{4} \right) \right] C^{-2}_s |T^{(S)}_{\lambda\lambda_1}|^2 + \frac{1}{2} \left[ \frac{1}{4N_c^2} + \frac{1}{4} \right] C^{-2}_A |T^{(A)}_{\lambda\lambda_1}|^2 \right\} w^{-}_q \right]. 
$$

(9.7)

Here, $w^{-}_q \equiv w^{-}_q \cdot t^a$, $\mathcal{W}^{l}_k \equiv \mathcal{W}^{l}_k \cdot t^a$. In deriving (9.7) we use the following identities:

$$
\{ \{ t^{a_1}, t^{a_2} \} t^{a_1}, t^{a_2} \} \cdot t^{a_1} = \left[ \left( C_F - \frac{1}{N_c} \right) \frac{N_c}{2} - \frac{1}{4} \right] \{ \{ t^{a_1}, t^{a_2} \} t^{a_1} \},
$$

$$
\{ \{ t^{a_1}, t^{a_2} \} t^{a_1} \} \cdot t^{a_1} = \frac{1}{2} \left( C_F + \frac{1}{2N_c} \right) N_c \{ \{ t^{a_1}, t^{a_2} \} t^{a_1} \},
$$

$$
\{ \{ t^{a_1}, t^{a_2} \} t^{a_1} \} \cdot t^{a_1} = \frac{1}{2} \left[ \frac{1}{4N_c^2} + \frac{1}{4} \right] \{ \{ t^{a_1}, t^{a_2} \} t^{a_1} \},
$$

etc. Since the equilibrium plasmino number density is proportional to the identity, the commutator terms on the right-hand side of Eq. (9.3) vanish. Therefore if the system is in conditions, when the off-equilibrium function $n^{-}_q$ is perturbatively small, then linearized Vlasov-Boltzmann equation (9.6), (9.7) in a certain sense is exact. Finally by analogy with color current induced by color plasmons (Eq. (9.12) in [3]), we can define the color current resulting from the color-plasmino number density $j^{-}_\mu(x) = (j^{-}_0(x), j^{-}_1(x))$, where now

$$
j^{-}_0(x) = -gt \int \frac{dq}{(2\pi)^3} \left( \frac{\partial n_{eq}(q)}{\partial \omega^{-}_q} \right) (w^{-}_q - \tilde{w}^{-}_q),
j^{-}_1(x) = -gt \int \frac{dq}{(2\pi)^3} \left( \frac{\partial n_{eq}(q)}{\partial \omega^{-}_q} \right) (w^{-}_q - \tilde{w}^{-}_q).
$$

For closing kinetic equation (9.6) and similar equations for the functions $\tilde{w}^{-}_q$ and $\mathcal{W}^{l}_k$, it remains only to write out the mean field equation defining a change of mean gauge field in a system in a self-consistent manner

$$
\mathcal{D}^{\nu}(x) \mathcal{F}_{\mu\nu}(x) = j^{-}_\mu(x) + j^{(0)}_\mu(x) + j^{\text{ext}}_\mu(x).
$$

46
Here, the color-plasmon induced current \( j^{(l)}_{\mu}(x) \) is defined by Eq. (9.12) in [1] and \( j^{\text{ext}}_{\mu} \) is the external current that plays a part of initial color perturbation. It should be noted that an existence of a time-space dependence of background gauge field in the system leads, generally speaking, to the replacement of dispersion equations (2.1) by

\[
\text{Re} \Delta^{-1}(q^0, q; t, x) = 0, \quad \text{Re} \Delta^{-1}(k^0, k; t, x) = 0,
\]
slowly depending on \( x = (t, x) \) that in turn results in modification of the dispersion relations for plasminos and plasmons: \( \omega^- = \omega^-(q, t, x) \), \( \omega^l = \omega^l(k, t, x) \). Besides a color background gauge field removes an degeneracy of spectrum of plasma excitations, when the same dispersion relation corresponds to various oscillations with different color indices. The consequence of the last property is the fact that the dispersion relations for color plasminos and plasmons acquire a matrix character: \( \omega^- = (\omega^{ij}) \), \( \omega^l = (\omega^{l_{ab}}) \). In this case the left-hand side of Eq. (9.2) should be supplemented by term of the following type:

\[
\frac{1}{2} \left\{ \frac{\partial \omega^- (q, x)}{\partial x}, \frac{\partial n_q^-}{\partial q} \right\},
\]
which can be considered as additional ‘force’ term acting on the color quasiparticles. We can assume nevertheless that in the weak-field limit background gauge field weakly influences on dispersion properties in the system and for the first approximation one can successfully neglected by it.

10 Conclusion

In this paper within the framework of the hard thermal loops effective theory we have presented the formal scheme for deriving the system of the semiclassical Boltzmann equations, which describes the space-time evolution of the soft-quark and soft-gluon number densities due to their interactions among themselves in hot non-Abelian plasma. This type of the nonlinear interactions forms in itself an important element in the general dynamics of the processes occurring in quark-gluon plasma at least in a weak coupling regime. For highly excited plasma states, when typical time of relaxation of the hard thermal particle distributions is commensurable with the typical time of relaxation of soft oscillations or even significantly exceeds it, along with the kinetic equations for hard thermal particles it should be used the kinetic equations for soft modes of type (2.2), (2.3). However, for completeness of the description the right-hand sides of kinetic equations (2.2), (2.3) should be added by collision terms taking into account the scattering processes of soft plasma excitations on hard thermal particles. For moderate level of plasma excitations the processes of this type can even play more important role in the dynamics of the system in comparison with the above-mentioned processes of self-interaction of soft
modes. In our forthcoming papers a formalism of regular calculation of such collision
terms will be presented to the full extent.

Furthermore, we would like to discuss briefly how the approach presented here and
in our early papers, is agreed with the more traditional calculations from the Feynman
graphs (like the original calculations of the damping rates for soft quasiparticles within
the framework of the imaginary-time formalism).

One of the strongest arguments of plausibility of this approach is the fact that damping
rate of a plasmino at rest calculated on its basis, exactly coincides with the damping
rate of the standing plasmino derived by Kobes, Kunstater, and Mak in the framework
of the Braaten-Pisarski effective theory. The same can be said about the damping rate
of a plasmon computed within the framework of our approach. Thus remaining in
the context of the Blaizot-Iancu equations and deriving from them the effective kinetic
equations for the soft plasma modes, we are able to reproduce the gauge-invariant damping
rates of the soft quasiparticles (to the leading order in the coupling), which corresponds
to the damping rates from the resummed perturbation theory.

Notice that in the work, Braaten and Thoma have pointed to an existence of cor-
respondence of such a kind in calculation of the soft contribution to energy loss of a heavy
fermion in hot QED plasma. They have shown that the result of calculation using the rig-
orous imaginary-time formalism and the result employing the alternative field-theoretical
calculation (that consists in a simple replacement of photon propagator by effective pho-
ton propagator) coincide among themselves. At the same time they have noted that “…a
rigorous justification must await the development of resummation methods for the real-
time formalism that are as powerful as the resummation methods for the imaginary-time
formalism”. This in full measure can be related to our approach of deriving the effective
kinetic equations for the soft modes.

In recent years significant efforts have been undertaken to construction of the hard
thermal loop resummation technique in the non-equilibrium field theory allowing one
effectively to work not only with the single-particle propagators, but
also with three- and four-point functions. The methods and approaches suggested in these
works enable one at least in principle within the framework of standard methods of real-
time finite temperature field theory, to construct a calculation scheme of kinetic equations
for soft quasiparticles in QGP. As a guide idea here, it can be used elegant, thoroughly
considered approach suggested by Blaizot and Iancu in deriving the Boltzmann equation
for hard transverse gluons in high-temperature plasma. Their derivation relies on a
gauge-covariant gradient expansion of the Kadanoff-Baym equations for the gluon two-
point function. The Boltzmann equation has emerged as the quantum transport equation
to leading order in $g$ for the gauge-covariant fluctuation $\delta G$ of a hard gluon propagator.
Besides in the above-mentioned paper, the Kadanoff-Baym equations for the off-equilibrium propagator of the soft gluon $D_{\mu\nu}(X,Y)$, which are formally identical to those for a hard gluon propagator $G_{\mu\nu}(X,Y)$, are written out. These equations for $D_{\mu\nu}(X,Y)$ are used in [28] only to deduce the relation between the off-equilibrium gauge-covariant fluctuation $D^<(k,x)$ and the gauge-covariant fluctuation of the leading-order soft gluon polarization tensor $\delta\Pi^<(k,x)$. The problem of self-interactions of the soft fields is not considered here. However, in principal, there is nothing to forbid the use of these equations for research of the soft-field dynamics and construction of the relevant transport equations within the framework of the scheme suggested by Blaizot and Iancu. Here, by $\delta\Pi^<(k,x)$ we mean the fluctuation of the next-to-leading order of the soft-gluon self-energy involving three- and four-gluon off-equilibrium vertices with soft external lines. The relevant effects of self-interaction of the soft fields are encoded in these vertex functions. It would allow us to derive the transport equations for the soft gluons directly from the underlying quantum field theory and compare them with the equation obtained in this paper in the context of the semiclassical approximation. For deriving the kinetic equations for soft fermion modes it should be used the Kadanoff-Baym equations for the quark two-point function. The development of this approach is also needed to specify the limits of validity of the semiclassical kinetic approach suggested in present work to the research of the processes of nonlinear interaction of the soft fields in hot QCD plasma.

Unfortunately on this way considerable technical complications resulting from the doubling of degrees of freedom arise. Although it is suggested a number of receptions [25], which to a certain extent simplify intermediate computations in deriving convolutions of propagators and vertices, nevertheless a practical application of these methods to solving specific problem on deriving soft-quasiparticle kinetic equations is at present very complicated and requires using the symbolic manipulation program. In this sense our less strict approach gives more economic method for calculation of desired kinetic equations, although it also is very complex and lengthy. The “memory” of initial rigorous theory here, is a presence of different types of vertices $\Gamma^{(G)}$ and $\Gamma^{(Q)}$ connected with different chronological orders of incoming lines. However their amount is small that gives a possibility to perform calculation in visible form without resorting to a program for evaluating real time Feynman amplitudes. It can be assumed that in rigorous approach the final kinetic equations (at least their linearized versions) will be practically the same, as the kinetic equations obtained within the framework of our more phenomenological approach.
Acknowledgments

This work was supported by the Russian Foundation for Basic Research (project no 03-02-16797).
Appendix A

The explicit form of the HTL-induced vertices between quark pair and two gluons is defined by the following expressions:

\[
\begin{align*}
\delta \Gamma_{\mu
u}^{(G)}(k_1, k_2; q_1, q_2) &= -\omega_0^2 \int \frac{d\Omega}{4\pi} \frac{v_{\mu} v_{\nu} \not{q}}{(v \cdot q_1 - i\epsilon)(v \cdot q_2 - i\epsilon)} \left( \frac{t^a t^b}{v \cdot (q_1 + k_1) + i\epsilon} + \frac{t^b t^a}{v \cdot (q_1 + k_2) + i\epsilon} \right), \\
\delta \Gamma_{\mu
u}^{(Q)}(k_1, k_2; q_1, q_2) &= -\omega_0^2 \int \frac{d\Omega}{4\pi} \frac{v_{\mu} v_{\nu} \not{q}}{(v \cdot q_1 + i\epsilon)(v \cdot q_2 - i\epsilon)} \left( \frac{t^a t^b}{v \cdot (q_1 + k_1) + i\epsilon} + \frac{t^b t^a}{v \cdot (q_1 + k_2) + i\epsilon} \right). 
\end{align*}
\]

Below we list the properties of HTL-resummed three-gluon, two-quark – one-gluon and two-quark – two-gluon vertex functions, which used in the text

\[
\begin{align*}
(*\Gamma_{\mu_1 \mu_2}(-k_1 - k_2, k_1, k_2))^* &= -(*\Gamma_{\mu_1 \mu_2}(k_1 + k_2, -k_1, -k_2) = \Gamma_{\mu_1 \mu_2}(k_1 + k_2, -k_2, -k_1), \\
\gamma^0(*\Gamma^{(Q)}(k; q_1, q_2))^\dagger \gamma^0 &= \Gamma^{(Q)}(k; q_2, q_1) = \Gamma^{(Q)}(-k; -q_1, -q_2), \\
\gamma^0(*\Gamma^{(G)}(k; q_1, q_2))^\dagger \gamma^0 &= \Gamma^{(G)}(-k; -q_1, -q_2) = \Gamma^{(G)}(-k; -q_2, -q_1), \\
\delta \Gamma^{(Q)ab}(k_1, k_2; q_1, q_2) &= \delta \Gamma^{(Q)ab}(-k_2, -k_1; -q_2, -q_1) = \delta \Gamma^{(Q)ba}(k_2, k_1; q_1, q_2), \\
\gamma^0(\delta \Gamma^{(Q)ab}(k_1, k_2; q_1, q_2))^\dagger \gamma^0 &= -\delta \Gamma^{(Q)ab}(k_1, k_2; q_1, q_2) = -\delta \Gamma^{(Q)ba}(-k_1, -k_2; q_1, q_2), \\
\delta \Gamma^{(G)ab}(k_1, k_2; q_1, q_2) &= -\delta \Gamma^{(G)ab}(k_1, k_2; q_2, q_1), \\
\gamma^0(\delta \Gamma^{(G)ab}(k_1, k_2; q_1, q_2))^\dagger \gamma^0 &= -\delta \Gamma^{(G)ab}(-k_1, -k_2; q_1, q_2). 
\end{align*}
\]

Appendix B

In this Appendix we give the explicit expressions for the coefficient functions entering into integrands of the conjugate effective currents and sources. For the conjugate effective current

\[
\tilde{J}_\mu^{a*Q(1,2)a}(A^{(0)}, \tilde{\psi}, \psi) = -g^2 \int \Gamma^{(G)a_{a_1,a_2}ij}_{\mu_1,\alpha\beta}(k, -k_1; -q_1, -q_2) A^{(0)a_1\mu_1}(k_1) \tilde{\psi}_{\beta}^{(0)ij}(q_2) \psi^{(0)ij}(q_1) \\
\times \delta(k - k_1 - q_1 - q_2) dk_1 dq_1 dq_2
\]

(B.1)
we have
\[ \tilde{\Gamma}^{(G)\alpha a_{1}, ij}_{\mu_{1}, \alpha \beta} (k, -k_{1}; -q_{1}, -q_{2}) = \delta \Gamma^{(G)\alpha a_{1}, ji}_{\mu_{1}, \alpha \beta} (-k, k_{1}; q_{1}, q_{2}) \]  \hspace{1cm} (B.2)

\[ - [t^{a_{1}}, t^{a_{2}}]^{ji} \ast \Gamma_{\mu_{1}, \rho_{1} \alpha} (-k, -k_{1}, k_{1}) \ast \mathcal{D}^{\mu_{\rho}} (-k + k_{1}) \ast \Gamma^{(G)\mu_{\rho}, \alpha}_{\rho_{1}, \beta} (-q_{1} - q_{2}; q_{1}, q_{2}) \]

\[ - (t^{a_{1}}t^{a_{2}})^{ji} \ast \Gamma^{(Q)\mu_{1}, \rho_{1} \gamma}_{\rho_{1}, \gamma \beta} (-k_{1} - q_{2}, k_{1} + q_{2}) \ast S_{\gamma \gamma'} (-k + q_{1}) \ast \Gamma^{(G)\mu_{1}, \beta \gamma}_{\rho_{1}, \gamma \alpha} (-k; k - q_{1}, q_{1}) \]

\[ + (t^{a_{1}}t^{a_{2}})^{ji} \ast \Gamma^{(G)\mu_{1}, \rho_{1} \gamma}_{\rho_{1}, \gamma \beta} (-k; k - q_{2}, q_{2}) \ast S_{\gamma \gamma'} (-k_{1} - q_{1}) \ast \Gamma^{(Q)\mu_{1}, \beta \gamma}_{\rho_{1}, \gamma \alpha} (-k_{1}; -q_{1}, q_{1} + k_{1}). \]

Note that this expression cannot be obtained by replacements \( a \Leftrightarrow a_{1}, \alpha \Leftrightarrow \beta, k \Leftrightarrow -k, k_{1} \Leftrightarrow -k_{1}, \) and \( q_{1, 2} \Leftrightarrow -q_{1, 2} \) from expression \( (4.8) \) by virtue of nontrivial (spinor) structure of two last terms. However, here we can point to the following formula connecting these effective amplitudes

\[ \ast \tilde{\Gamma}^{(G)\alpha a_{1}, ij}_{\mu_{1}, \alpha \beta} (k, -k_{1}; -q_{1}, -q_{2}) = - \ast \tilde{\Gamma}^{(G)\alpha a_{1}, ji}_{\mu_{1}, \beta \alpha} (-k, k_{1}; q_{1}, q_{2}). \]

For Dirac conjugate effective source \( \tilde{\eta}^{(2.1)}_{\alpha_{1}} \) \( (5.13) \) we have expression for conjugate effective amplitude in integrand

\[ \ast \tilde{\Gamma}^{(Q)\alpha a_{2}, ii_{1}}_{\mu_{1}, \alpha \beta} (k_{1}, k_{2}; q_{1}, -q) = - \left\{ \delta \Gamma^{(Q)\alpha a_{2}, ii_{1}}_{\mu_{1}, \alpha \beta} (-k_{1}; -k_{2}; -q_{1}, q) \right\} \]  \hspace{1cm} (B.3)

\[ + [t^{a_{2}}, t^{a_{1}}]^{ii_{1}} \ast \Gamma^{(Q)\nu_{1}, \alpha \beta}_{\nu_{1}, \beta \alpha} (q - q_{1}; -q, q_{1}) \ast \mathcal{D}^{\mu_{\rho}} (-k_{1} - k_{2}) \ast \Gamma^{(Q)\mu_{1}, \mu_{2}} (-k_{1} - k_{2}, k_{1}, k_{2}) \]

\[ - (t^{a_{2}}t^{a_{1}})^{ii_{1}} \ast \Gamma^{(Q)\mu_{1}, \rho_{1} \gamma}_{\rho_{1}, \gamma \beta} (k_{1}; -q_{1} - k_{2}, q_{1}) \ast S_{\gamma \gamma'} (-q_{1} - k_{2}) \ast \Gamma^{(Q)\mu_{1}, \beta \gamma}_{\rho_{1}, \gamma \alpha} (k_{1}; -q, q - k_{1}) \]

\[ - (t^{a_{1}}t^{a_{2}})^{ii_{1}} \ast \Gamma^{(Q)\rho_{1}, \rho_{1} \gamma}_{\rho_{1}, \gamma \beta} (k_{1}; -q_{1} - k_{1}, q_{1}) \ast S_{\gamma \gamma'} (-q_{1} - k_{1}) \ast \Gamma^{(Q)\rho_{1}, \beta \gamma}_{\rho_{1}, \gamma \alpha} (k_{2}; -q, q - k_{2}) \}. \]

As in the case of effective amplitude \( (B.2) \) this effective one cannot be obtained from expression \( (B.9) \) by a simple replacement of indices and momenta of the following type: \( a_{1} \Leftrightarrow a_{2}, i \Leftrightarrow i_{1}, k_{1} \Leftrightarrow -k_{1} \) etc. Effective amplitude \( (B.3) \) possesses a property

\[ \ast \tilde{\Gamma}^{(Q)\alpha a_{2}, ii_{1}}_{\mu_{1}, \alpha \beta} (k_{1}, k_{2}; q_{1}, -q) = \ast \tilde{\Gamma}^{(Q)\alpha a_{2}, ii_{1}}_{\mu_{2}, \alpha \beta} (k_{2}, k_{1}; q_{1}, -q). \]  \hspace{1cm} (B.4)

Finally we consider the conjugate effective source \( \tilde{\eta}^{(0.3)}_{\beta_{1}} \), \( (3.14) \). By direct calculation it is not difficult to show that the conjugate effective amplitude \( \tilde{\Gamma}^{ii_{1}ii_{2}i_{3}}_{\alpha a_{1}a_{2}a_{3}} \) is associated with \( \tilde{\Gamma}^{ii_{1}ii_{2}i_{3}}_{\alpha a_{1}a_{2}a_{3}} \) by the following expression:

\[ \ast \tilde{\Gamma}^{ii_{1}ii_{2}i_{3}}_{\alpha a_{1}a_{2}a_{3}} (q, q_{1}, -q_{2}, -q_{3}) = - \ast \tilde{\Gamma}^{ii_{2}ii_{3}i_{1}}_{\alpha a_{2}a_{3}a_{1}} (-q, -q_{1}, q_{2}, q_{3}). \]  \hspace{1cm} (B.5)
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