Cosmic acceleration in LRS bianchi type I space-time with bulk viscosity in general relativity

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Abstract: In present paper our intention to construct a locally rotational symmetric bianchi type I space time under framework of massless scalar field have flat potential. In existence of bulk viscosity the developed model provides inflationary solutions. The System of nonlinear Einstein field equations are solved by using suitable condition $V(\phi) = K$ (constant) and the bulk viscosity coefficient is inversely proportional to scalar of expansion i.e. $\xi \theta = \alpha$ yields the negative deceleration parameter, which favorable to cosmic accelerating universe in current scenario. The model isotropize at particular cases. The Proper volume for model increase with time represents eternal inflation of present universe. The physical and structural aspects of model are discussed in significantly manner.

Keywords: Bianchi Type I space time, Bulk viscosity, Eternal inflation, Flat Potential

1. Introduction

In recent years, Einstein theory [1] has become subject of interest for its attainment in explaining the mystery of early universe. Bianchi models of cosmology provide a mathematical framework to study the realistic picture of present universe. Bianchi Type I space time becomes remarkable to understand the origin of early evolution of universe more general than standard model. It also provides the essential description of large scale behavior of physical universe in a simple manner in modern cosmology. In general relativity it is curious to study the cosmological problem to developed mathematical model that exactly predict the result of astronomical facts. Many cosmological problems are needed to be investigated by cosmologist to understand the cosmic structure formation after big-bang.

Inflation means extremely quick expansion of earliest universe by a factor of $10^{78}$ with volume driven by negative pressure with vacuum energy density. Guth [2] first introduced the basic idea of inflation while faced the problem why we monopole do not exist today. He investigated the false vacuum with positive energy leads an expansion of space exponentially. Inflationary scenario of universe provides a potential solution to cosmological problem like Horizon problem, Isotropy, Homogeneity and Magnetic Monopole. Inflationary phenomenon for homogenous and isotropy space time (FRW model) is investigated by many authors [3-5]. Rothman and Ellis [6] have resulted out solution of isotropic problem if we deal with anisotropic space time metric that isotropize in special case. Some Bianchi Type I inflationary model for flat potential in general relativity is constructed by Bali and Jain[7] and Bali[8]. Bali and Singh [9] provide LRS Poonia Bianchi Type I space time metric for stiff fluid with variable bulk viscosity. Bali and [10] studied Bianchi Type VI0 cosmological model to
study inflationary scenario of universe and observe late time acceleration. Naidu et.al [11] constructed inflationary model in Kantowski–Sachs space with constant deceleration parameter. Many authors [12-15] discussed the cosmological phenomenon under consideration of bulk viscosity and study the effect on evolution. Khalatnikov and Belinskii [16] studied FRW model by taking bulk viscosity as function of energy density. Reddy [17] have developed spatially, anisotropic and homogenous Bianchi Type V space time to study inflation and structure of early universe.

In current paper, we have presented a LRS Bianchi Type I inflationary space time with bulk viscosity in presence of flat potential to discuss inflationary scenario of universe and cosmic acceleration. The spatial volume increased with time which represent accelerating phase of universe. Model isotropize at special case. This study will provide some sufficient fact to study the astrophysical phenomenon. The negative deceleration parameter yield de-sitter universe. Geometrical and physical behavior of the model are discussed.

2. The metric and fields equations

We consider LRS Bianchi Type I metric can be written in the form

$$ds^2 = -dt^2 + A^2 dx^2 + B^2 (dy^2 + dz^2)$$  \hspace{1cm} (1)

where A and B are metric coefficients and function of parameter t only.

The gravitational field coupled minimally with scalar field \(V(\varphi)\) is given by

$$L = \int \left( R - \frac{1}{2} g^{ij} \varphi_{,i} \varphi_{,j} - V(\varphi) \right) \sqrt{-g} \, dx^4$$  \hspace{1cm} (2)

The Einstein’s field equation \((8\pi G = c = 1)\) for massless less scalar field \(\varphi\) is given by

$$R_{ij} - \frac{1}{2} R g_{ij} = -T_{ij}$$  \hspace{1cm} (3)

The energy momentum tensor \(T_{ij}\) in presence of viscosity for scalar field is given by

$$T_{ij} = \varphi_{,i} \varphi_{,j} - \left( \frac{1}{2} \varphi_{,i} \varphi^{,i} + V(\varphi) \right) g_{ij} - \xi \theta (g_{ij} + u_{i} u_{j})$$  \hspace{1cm} (4)

with

$$\frac{1}{\sqrt{-g}} \partial_{i} \left( \sqrt{-g} \varphi_{,i} \right) = - \frac{dV(\varphi)}{d\varphi}$$  \hspace{1cm} (5)

where \(V\) is the scalar potential, \(\varphi\) is Higgs field, \(\xi\) is coefficient of viscosity and \(\theta\) is the expansion scalar.

The commoving derivative is considered as \(u^{i} = (0,0,0,1)\)

The system of field equations (3) for metric (1) become

$$\frac{B_{44}}{B^2} + 2 \frac{B_{44}}{B} = \frac{1}{2} \varphi_{,4}^2 - K - \alpha$$  \hspace{1cm} (6)

$$\frac{A_{44}}{A} + \frac{B_{44}}{B} + \frac{A_{4}B_{4}}{AB} = \frac{1}{2} \varphi_{,4}^2 - K - \alpha$$  \hspace{1cm} (7)
for inflationary solution, we have assumed \( V(\varphi) = K, \xi \theta = \alpha \)

We consider \( \xi \theta = \alpha \) (constant) from Brevik et al.[18] because it have significant role to connect with existence of LR cosmology using FRW metric.

From equation (5)

\[
\varphi_{44} + \left( \frac{A_4}{A} + 2 \frac{B_4}{B} \right) \varphi_4 = 0
\]

on solving provide

\[
\varphi_4 = \frac{\mu}{AB^2}
\]

where \( \mu \) is constant of integration.

From equation (6) and (7) we have

\[
\frac{A_4 B_4}{AB} + \frac{A_4}{A} - \frac{B_4^2}{B^2} - \frac{B_4}{B} = 0
\]

which gives

\[
\left( \frac{A_4}{A} - \frac{B_4}{B} \right)_4 + \left( \frac{A_4}{A} - \frac{B_4}{B} \right) \left( \frac{A_4}{A} + 2 \frac{B_4}{B} \right) = 0
\]

on integration we get

\[
\left( \frac{A_4}{A} - \frac{B_4}{B} \right) = \frac{\mu}{AB^2}
\]

now from equation (6), (7) and (8) we have

\[
\left( \frac{A_4}{A} + 2 \frac{B_4}{B} \right) = \frac{\lambda^2}{A^2 B^4} - K - \frac{1}{2} \alpha
\]

Let assume \( AB^2 = \eta \) in equation (13) and provide eq. (17)

\[
\left( \frac{A_4}{A} - \frac{B_4}{B} \right) = \frac{\mu}{\eta}
\]

Equation (14) become

\[
\left( \frac{A_4}{A} + 2 \frac{B_4}{B^2} \right) = \frac{\lambda^2}{\eta^2} - K - \frac{1}{2} \alpha
\]

\[
\left( \frac{A_4}{A} + 2 \frac{B_4}{B} \right) = \frac{\eta_4}{\eta}
\]
Equations (15) and (17) gives relation

\[ 3 \frac{A_4}{A} = \frac{n_4}{\eta} + 2 \frac{\mu}{\eta} \]  

(18)

and

\[ 3 \frac{B_4}{B} = \frac{n_4}{\eta} - \frac{\mu}{\eta} \]  

(19)

using equations (18), (19) in (16) we obtained

\[ \left( \frac{n_4}{\eta} \right)^4 + \frac{1}{9} \left( \frac{n_4}{\eta} + 2 \frac{\mu}{\eta} \right) + \frac{2}{9} \left( \frac{n_4}{\eta} - \frac{\mu}{\eta} \right)^2 = \frac{\lambda^2}{\eta^2} - K - \frac{1}{2} \alpha \]  

(20)

on solving gives

\[ \eta_4^4 - \frac{2}{3} \eta_4^2 + K \eta^2 + \frac{1}{2} a \eta^2 + \frac{2}{3} \mu^2 - \lambda^2 = 0 \]  

(21)

Let use the transformation

\[ \eta_4 = f(\eta) \]

so that

\[ \eta_4 = \int \frac{df}{d\eta} \]

Equation (21) become

\[ 2f \frac{df}{d\eta} - \frac{4 f^2}{3 \eta} = (2K + \alpha)\eta + \frac{2\lambda^2 - 4\mu^2}{\eta} \]

(22)

Provide

\[ \eta_4^2 = \frac{3}{2} \eta^2(2K + \alpha) - \frac{3}{4} \left(2\lambda^2 - \frac{4}{3} \mu^2 \right) + D\eta^4 \]

(23)

where D is the constant of integration.

Since equation (6),(7),(8) and (9) have four unknown A,B, φ and K = V(φ)

For find solution take \( D = 0 \), which gives

\[ \frac{d\eta}{dt} = \sqrt{\frac{3}{2} \eta^2(2K + \alpha) - \frac{3}{4} \left(2\lambda^2 - \frac{4}{3} \mu^2 \right)} \]

(24)

leads to

\[ \frac{d\eta}{\sqrt{\frac{3}{2}(2K + \alpha)(\eta^2 + \beta^2)}} = dt \]

(25)

where

\[ \beta^2 = \frac{\mu^2 - \frac{2}{3} \lambda^2}{\frac{3}{2}(2K + \alpha)} \]

(26)

gives

\[ \eta = \beta sinh \sqrt{\frac{3}{2}(2K + \alpha)(t + t_0)} \]

where \( t_0 \) is constant of integration

(27)
and

$$\eta_A = \beta \sqrt{\frac{3}{2}} (2K + \alpha) \cosh \sqrt{\frac{3}{2}} (2K + \alpha) (t + t_0)$$  \hspace{1cm} (28)$$

From equation (18), (27) and (28) we get

$$3 \frac{A_A}{A} = \frac{\eta_A}{\eta} + 2 \frac{\mu}{\eta} = \frac{\beta \sqrt{\frac{3}{2}} (2K + \alpha) \cosh \sqrt{\frac{3}{2}} (2K + \alpha) (t + t_0) + 2 \mu}{\beta \sinh \sqrt{\frac{3}{2}} (2K + \alpha) (t + t_0)}$$  \hspace{1cm} (29)$$

$$3 \frac{B_A}{B} = \frac{\eta_A}{\eta} - \frac{\mu}{\eta} = \frac{\beta \sqrt{\frac{3}{2}} (2K + \alpha) \cosh \sqrt{\frac{3}{2}} (2K + \alpha) (t + t_0) - \mu}{\beta \sinh \sqrt{\frac{3}{2}} (2K + \alpha) (t + t_0)}$$  \hspace{1cm} (30)$$

on integrating we get

$$A = \frac{1}{\tau} \sinh \frac{1}{\tau} \left[ \frac{3}{2} (2K + \alpha) (t + t_0) \right] \tanh \frac{3\beta}{2} \sqrt{\frac{3}{2}} (2K + \alpha) \left[ \frac{1}{2} \sqrt{\frac{3}{2}} (2K + \alpha) (t + t_0) \right]$$  \hspace{1cm} (31)$$

From equation (19), (27) and (28) we get

$$B = \frac{1}{\gamma} \sinh \frac{1}{\gamma} \left[ \frac{3}{2} (2K + \alpha) (t + t_0) \right] \tanh \frac{3\beta}{2} \sqrt{\frac{3}{2}} (2K + \alpha) \left[ \frac{1}{2} \sqrt{\frac{3}{2}} (2K + \alpha) (t + t_0) \right]$$  \hspace{1cm} (32)$$

where \( \tau \) and \( \gamma \) are the constant of integration

From equation (10)

$$\varphi_4 = \frac{\lambda}{AB^2} = \frac{\lambda}{\beta \sinh \sqrt{\frac{3}{2}} (2K + \alpha) (t + t_0)}$$  \hspace{1cm} (33)$$

Using the transformation \( t + t_0 = T, \frac{1}{\tau} x = X, \frac{1}{\gamma} y = Y \) and \( \frac{1}{\gamma} z = Z \) we get the metric in the form

$$ds^2 = -dT^2 + \sinh^3 \left[ \frac{3}{2} (2K + \alpha) T \right] \tanh \frac{3\beta}{2} \sqrt{\frac{3}{2}} (2K + \alpha) \left[ \frac{1}{2} \sqrt{\frac{3}{2}} (2K + \alpha) (t + t_0) \right] dX^2$$

$$+ \sinh^3 \left[ \frac{3}{2} (2K + \alpha) T \right] \tanh \frac{3\beta}{2} \sqrt{\frac{3}{2}} (2K + \alpha) \left[ \frac{1}{2} \sqrt{\frac{3}{2}} (2K + \alpha) (t + t_0) \right] (dY^2 + dZ^2)$$

\hspace{1cm} (34)$$
3. Physical and geometrical features of model

Scalar of the expansion for model is given by

$$\theta = \frac{A_a}{A} + 2 \frac{b_x}{b} = \frac{\eta}{\eta} = \sqrt{\frac{3}{2}} (2K + \alpha) \coth \left[ \sqrt{\frac{3}{2}} (2K + \alpha) T \right]$$  \hspace{1cm} (35)

Shear scalar for model is given as

$$\sigma = \frac{\mu}{\sqrt{3} \beta \sinh \left[ \frac{1}{2} (2K + \alpha) T \right]}$$  \hspace{1cm} (36)

The Hubble parameters in X, Y and Z direction is given as

$$H_1 = \frac{\beta \sqrt{\frac{3}{2}} (2K + \alpha) \cosh \left[ \frac{3}{2} (2K + \alpha) T + \mu \right]}{3 \beta \sinh \frac{3}{2} (2K + \alpha) T}$$  \hspace{1cm} (37)

and

$$H_2 = H_3 = \frac{\beta \sqrt{\frac{3}{2}} (2K + \alpha) \cosh \left[ \frac{3}{2} (2K + \alpha) T - \mu \right]}{3 \beta \sinh \frac{3}{2} (2K + \alpha) T}$$  \hspace{1cm} (38)

Also

$$\frac{\sigma}{\theta} = \frac{\mu}{\sqrt{3} \beta \sinh \left[ \frac{3}{2} (2K + \alpha) T \right]}$$  \hspace{1cm} (39)

Higgs field for model is given by

$$\varphi = \int \frac{\lambda}{\beta \sinh \left[ \frac{3}{2} (2K + \alpha) T \right]} + N$$  \hspace{1cm} (40)

The Proper volume V is obtained as

$$V = \beta \sinh \left[ \frac{3}{2} (2K + \alpha) T \right]$$  \hspace{1cm} (41)

Deceleration Parameter is given as

$$q = - \left( 1 + 3 \text{sech}^2 \left( \sqrt{\frac{3}{2}} (2K + \alpha) T \right) \right)$$  \hspace{1cm} (42)

4. Conclusion and discussion

The proper volume for developed model increases with time. When \(T \to \infty\) then the proper volume \(R^3\) tends to infinity, it represent inflationary scenario of universe in Bianchi Type I model contain massless scalar field with flat potential. The expansion (\(\theta\)) tends to infinite when \(T\) tends to zero and \(\theta \to \text{finite term}\) when \(T\) tends to infinity. The physical quantity \(\frac{\mu}{\beta}\) measures the anisotropy in the
model. The Shear ($\sigma$) scalar tends to zero when $T$ tends to infinity. The rate of Higgs field decreases with time. The model (34) start with a big-bang at $T=0$. The deceleration parameter ($q$) is negative represents the accelerating phase of universe. Since $\frac{\sigma}{\sigma} \to 0$ for large value of $T$ it shows that the model approaches isotropy. The Hubble parameter decreases with time.

References:

[1] Einstein, A. 1916 The Foundation of generalized theory of relativity *Annalen der physik* 49 pp.769-822.
[2] Guth, A.H.1980 Inflationary Universe: A Possible solution of the horizon and flatness problem *Phys. Lett. B* 91 pp. 99-102
[3] Linde, A.D. 1982 A New inflationary scenario: A Possible solution of horizon, flatness, homogeneity, isotropy and primordial monopole problems *Phys. Lett. B*108 pp. 389-393
[4] La, D. and Steinhardt, P.J. 1989 Extended Inflationary Cosmology *Physical Review Letters* 62, pp. 376-378.
[5] Burd, A.B. and Barrow, J.D. 1988 Inflationary Models with Exponential Potentials *Nucl. Physics* 308, pp.929-945.
[6] Rothman, T. and Ellis, G.F.R.1986 Can inflation occur in anisotropic cosmologies *Phys. Lett. B*180 pp. 19-24
[7] Bali, R., Jain, V.C. 2002 Bianchi Type I Inflationary Universe in General Relativity *Pramana* 59 pp.1-7.
[8] Bali, R. 2011 Inflationary Scenario in Bianchi Type I Space-time *International Journal of Theoretical Physics* 50 pp. 3043-3048.
[9] Bali, R., Singh, S. 2014 LRS Bianchi Type I Stiff Fluid Inflationary Universe with Variable Bulk Viscosity *Canadian Journal of Physics* 92 pp. 365-369.
[10] Bali, R. and Poonia, L.2013 Bianchi Type VI Inflationary Cosmological Model in General Relativity *International Journal of Modern Physics* 22 pp. 593-602.
[11] Reddy, D.R.K. and Naidu, R.L. 2008 A Higher Dimensional Inflationary Universes in General Relativity *International Journal of Theoretical Physics* 47 pp. 2339-2343.
[12] Zimdahl, W. 1996 Bulk Viscous Cosmology *Physical Review D* 53 pp-5483-5493.
[13] Saha, B. 2005 Bianchi Type I Universe with Viscous Fluid *Modern Physics Letters* 20 pp.2127-2143.
[14] Peebles, P.J.E. and Ratra, B.2003 The Cosmological Constant and Dark Energy *Review of Modern Physics* 75 pp.559-606.
[15] Sahni, V. and Starobinsky, A. 2000 The Case for a Positive Cosmological Lambda term *International Journal of Modern Physics D* 9 pp. 373-443.
[16] Belinskii, V.A. and Khalatnikov, I.M. 1975 Influence of Viscosity on the character of cosmological evolution *Journal of Experimental and Theoretical Physics* 42 pp.205-210.
[17] Reddy, D.R.K. 2009 Bianchi Type V Inflationary Universe in General Relativity *International Journal of Theoretical Physics* 48 pp. 2036-2040.