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Numerical investigation of the electric field distribution and the power deposition in the resonant cavity of a microwave electrothermal thruster

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Microwave electrothermal thruster (MET), an in-space propulsion concept, uses an electromagnetic resonant cavity as a heating chamber. In a MET system, electromagnetic energy is converted to thermal energy via a free floating plasma inside a resonant cavity. To optimize the power deposition inside the cavity, the factors that affect the electric field distribution and the resonance conditions must be accounted for. For MET thrusters, the length of the cavity, the dielectric plate that separates the plasma zone from the antenna, the antenna length and the formation of a free floating plasma have direct effects on the electromagnetic wave transmission and thus the power deposition. MET systems can be tuned by adjusting the lengths of the cavity or the antenna. This study presents the results of a 2-D axis symmetric model for the investigation of the effects of cavity length, antenna length, separation plate thickness, as well as the presence of free floating plasma on the power absorption. Specifically, electric field distribution inside the resonant cavity is calculated for a prototype MET system developed at the Bogaziçi University Space Technologies Laboratory. Simulations are conducted for a cavity fed with a constant power input of 1 kW at 2.45 GHz using COMSOL Multiphysics commercial software. Calculations are performed for maximum plasma electron densities ranging from $10^{19}$ to $10^{21}$ #/m$^3$. It is determined that the optimum antenna length changes with changing plasma density. The calculations show that over 95% of the delivered power can be deposited to the plasma when the system is tuned by adjusting the cavity length. © 2017 Author(s). All article content, except where otherwise noted, is licensed under a Creative Commons Attribution (CC BY) license (http://creativecommons.org/licenses/by/4.0/).

I. INTRODUCTION

The concept of using a microwave resonant cavity as a heating chamber of a space propulsion system is put forward in the 1980s.\textsuperscript{1–3} Resistojets and arcjets have more commonly been studied as electrothermal propulsion systems for spacecraft. Thermal endurance limit of the heater and wall materials used in resistojets and the cathode erosion due to ion bombardment of arcjets are major efficiency and life limiting factors for these thrusters.\textsuperscript{4,5} Microwave Electrothermal Thruster (MET) concept employs free floating plasma at atmospheric pressure instead of a resistant heater or an electric arc to heat the propellant, and thus put forward to eliminate the inherent shortcomings of resistojet and arcjet concepts.\textsuperscript{6,7}

MET systems working in frequency ranges of 915 MHz to 17.8 GHz and the power level of a few Watts to 50 kW levels have been tested.\textsuperscript{8–11} Tests are performed using various kinds of monoatomic or molecular propellants such as $He$, $N_2$, $N_2O$, and water.\textsuperscript{12–15} Although the MET systems are still in laboratory development phase, they are promising systems among electrothermal thrusters with their achieved $I_{sp}$ levels of 800 s with water as the propellant.

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In a microwave electrothermal thruster, conversion of microwave energy to thermal energy materialize inside a resonant cavity. When electromagnetic radiation is transmitted to the resonant cavity, walls of which are made of a conductor, a standing wave is formed. Free electrons inside the propellant gas are accelerated by the electric field of the wave. Collisions between these accelerated electrons and the neutral atoms/molecules result in the transfer of energy from the electrons to these heavy particles.

If the colliding electrons have energies above the ionization energy of the gas, the neutral species could be ionized and new electrons would be produced. There is a relation like the Paschen curve between the E-field intensity and the gas pressure for microwave induced plasma.\textsuperscript{16} If the pressure of the gas and the electric field intensity are matched well for a breakdown to be initiated, the plasma discharge will begin. Once the plasma discharge begins, it will act as a resistive heater and absorb the microwave energy in accordance with “Joule Heating”. The propellant gas is heated as it swirls around the plasma. The thermal energy of the gas is turned into kinetic energy as this high-temperature gas expands through a conventional nozzle.\textsuperscript{8,17,18}

To start the breakdown and to maintain the plasma inside the cavity, adequate electromagnetic energy deposition must be achieved. The energy absorption reaches its peak value when the reflections are minimum. This condition can be ensured when the system is at resonant conditions. Since the resonant frequency of the system is directly related to its dimensions, to set the resonant conditions, cavity dimensions must be precisely calculated for a given operation frequency. Also, in order to increase the coupling efficiency, which is the ratio of absorbed power to incident power, effects of the disturbance factors must be considered. The dielectric plate that separates the microwave applicator from plasma, and the presence and the shape of the antenna provide such disturbance. Additionally, the effects of the plasma must be accounted for. The system can be tuned for given operating conditions by adjusting the antenna and cavity lengths.

In the current study, COMSOL Multiphysics electromagnetic solver is used to compute the electric field distribution and the power coupling inside the Bogazici University Space Technologies Laboratory (BUSTLab) MET resonant cavity. In the model, the plasma and the electromagnetic equations are not solved in a coupled fashion, rather the plasma properties are assumed and prescribed to evaluate the profile of the electric field and power deposition. BUSTLab MET, which uses a circular cavity, is designed to operate at 2.45 GHz microwave frequency. Simulations are carried out to obtain the optimal antenna and the cavity lengths at which the resonant conditions are achieved. Calculations are performed for a MET operating at a power level of 1000 W. To simulate plasma conditions in the cavity, based on the literature, electron number densities and the electron temperature are assumed to be $10^{19}$ to $10^{21} \text{#/m}^3$ and 1 eV, respectively.

\section*{II. BUSTLAB MET}

A prototype MET system is designed, manufactured and tested at BUSTLab.\textsuperscript{20} For the BUSTLab MET, which operates in TM\textsubscript{011} mode, the cavity radius is chosen to be 50 mm, which is above the critical radius for the operation frequency. For this mode of operation, the resulting total cavity height is evaluated to be 175 mm. The height of the cavity is kept fixed at this value and the tuning of the system is achieved using a two-stub-tuner in the power transmission line. A 10 mm thick separation plate, made of quartz, is placed inside the resonant cavity to separate the region where the antenna is located and the region where the plasma discharge occurs. A quarter wavelength ($\lambda/4$) antenna, made of copper, is used as the coupling probe. To observe the plasma conditions, an observation window of 50 mm diameter located on the wall on the plasma zone side of the cavity is used. A modular nozzle is attached to the flat wall on the plasma section of the resonant cavity.\textsuperscript{21} A technical drawing of the BUSTLab MET is seen in Figure 1.
III. THEORY

In free space, microwave propagates in TEM (Transverse Electromagnetic Mode). In this mode, wave carries electric field and magnetic field which are orthogonal to each other and as well as to the direction of propagation. Within a conducting boundary, electromagnetic waves propagate in TE (Transverse Electric: no electric field component in the direction of propagation) or TM (Transverse Magnetic: no magnetic field component in the direction of propagation) modes.\textsuperscript{22,23} The resonant cavity is a closed surface with a specific geometry (rectangular, cylindrical, etc.) where the boundaries have perfect electric conductor walls.\textsuperscript{23} When the electromagnetic wave is transmitted into this closed volume, a standing wave is produced.

In experiments to date, it is demonstrated that if $TM_{011}$ mode is applied in a circular cavity, the electric field has the highest intensity on the axis at the two ends of the cavity.

Time harmonic wave equation for electric field for source free and lossless (no attenuating) medium can be represented as:

$$\nabla^2 E = -\beta^2 E$$

(1)

where $\beta$ is the phase constant.\textsuperscript{23} If a new function, $\psi$, is defined to be used in a separable solution as

$$\psi(\rho, \phi, z) = f(\rho) g(\phi) h(z)$$

(2)

three ordinary differential equations (ODEs) for radial, angular and axial directions of a cylindrical cavity as shown in Figure 2 can be derived;

$$\rho^2 \frac{d^2 f}{d\rho^2} + \rho \frac{df}{d\rho} + \left[ (\beta_\rho \rho)^2 - m^2 \right] f = 0$$

$$\frac{d^2 g}{d\phi^2} = -m^2 g$$

$$\frac{d^2 h}{dz^2} = -\beta_z^2 h$$

(3)

(4)

(5)

Solutions for these three ODEs can be represented as:\textsuperscript{23}

$$f_1(\rho) = A_1 J_m(\beta_\rho \rho) + B_1 Y_m(\beta_\rho \rho)$$

$$f_2(\rho) = C_1 H_m^{(1)}(\beta_\rho \rho) + D_1 H_m^{(2)}(\beta_\rho \rho)$$

$$g_1(\phi) = A_2 e^{-jm\phi} + B_2 e^{jm\phi}$$

(6)

(7)

(8)
FIG. 2. (a) Schematic of a cylindrical resonant cavity, (b) Electric field distribution in $TM_{011}$ mode for a cylindrical resonant cavity.\textsuperscript{24}

\begin{align}
g_2(\phi) &= C_2 \cos(m\phi) + D_2 \sin(m\phi) \\
h_1(z) &= A_3 e^{-j\beta z} + B_3 e^{j\beta z} \\
h_2(z) &= C_3 \cos(\beta z) + D_3 \sin(\beta z)
\end{align}

where $J_m$ and $Y_m$ are the first and the second kind of Bessel function, $H_{m}^{(1)}$ and $H_{m}^{(2)}$ are the first and the second kind of Hankel function. For source free region, the electric vector potential $\mathbf{F}$ and the magnetic vector potential $\mathbf{A}$ can be defined as:

\begin{align}
\nabla \cdot \mathbf{D} &= 0 \Rightarrow \mathbf{D} = -\nabla \times \mathbf{F} \\
\nabla \cdot \mathbf{B} &= 0 \Rightarrow \mathbf{B} = \nabla \times \mathbf{A}
\end{align}

Transverse magnetic modes can be derived by letting the vector potentials $\mathbf{F}$ and $\mathbf{A}$ be equal to:

\begin{align}
\mathbf{F} &= 0 \\
\mathbf{A} &= \vec{a}_z A_z(\rho, \phi, z)
\end{align}

In resonant cavity, for a standing wave, $A_z$ can be given as:

\begin{align}
A_z(\rho, \phi, z) &= \left[ A_1 J_m(\beta_\rho \rho) + B_1 Y_m(\beta_\rho \rho) \right] \\
&\times \left[ C_2 \cos(m\phi) + D_2 \sin(m\phi) \right] \\
&\times \left[ C_3 \cos(\beta z) + D_3 \sin(\beta z) \right]
\end{align}

The boundary conditions for the cavity are

1. $E_\phi(\rho = a, \phi, z) = 0$
2. The fields must be finite everywhere
3. The fields must be periodic in $\phi$

The coefficients $A$, $B$, $C$ and $D$ can be evaluated using appropriate equations and boundary conditions, and the axial and radial electric field distribution in the cavity can be derived. The expressions for the constants $\beta$ and $m$ can be obtained. For $TM_{011}$ mode:

\begin{align}
E_z &= E_{011} J_0(\chi_{01} \rho) \cos(\frac{\pi}{h}) \\
E_\rho &= E_{011} \frac{\pi a}{\chi_{01} h} J_1(\chi_{01} \rho) \sin(\frac{\pi}{h})
\end{align}
\[ \beta_0 = \frac{\chi_{01}}{a} \]  
\[ \beta_z = \frac{\pi}{h} \]  
\[ m = 0 \]  

where \( a \) and \( h \) are the radius and the height of the cavity, respectively. \( \chi_{01} \) is the first zero of the Bessel function of the first kind of order zero. \( E_{011} \) represents the magnitude of the E-Field. Relation between the resonance frequency and the cavity dimensions for an empty cavity can be expressed as;  
\[ (f_r)_{011}^{TM} = \frac{1}{2\pi} \sqrt{\frac{\mu_0}{\varepsilon_0}} \sqrt{\left(\frac{\chi_{01}}{a}\right)^2 + \left(\frac{\pi}{h}\right)^2} \]  

(22)

\( \mu_0 \) and \( \varepsilon_0 \) are permeability and permittivity of free space, respectively. The cut-off frequency is given as;  
\[ (f_r)_{011}^{TM} = \frac{1}{2\pi} \sqrt{\mu_0\varepsilon_0} \sqrt{\left(\frac{\chi_{01}}{a}\right)^2} \]  

(23)

The power absorbed in a resonant cavity can be expressed as;  
\[ P_{abs} = \frac{\sigma}{2} \int_V |E|^2 dV \]  

(24)

where \( \sigma \) is the conductivity of the medium, \( V \) is the volume. In a MET system over 90% of the electrical power can be absorbed by the plasma if the cavity is well designed and an appropriate tuning system is utilized.

### IV. MODEL DESCRIPTION

Two different models are developed for the BUSTLab MET described in Section II. In both of these two models, only the cavity itself is included. Due to the small radius of the throat at the nozzle, the electromagnetic wave leakage is assumed to be prevented from this opening.

In the electromagnetic model, the computations are performed to determine the effect of the separation plate and the antenna on the electric field strength and the propagation mode in the empty cavity. First, electric field distribution in the cavity, dimensions of which are given in Table I, is evaluated. In the second configuration, a quartz separation plate is placed in the middle section of the cavity. In the third configuration, in addition to the separation plate, an antenna is inserted into the resonant cavity as seen in Figure 3a. A computational mesh, consisting of 1850 triangular elements as shown in Figure 3b is used in the simulations and Equation 25 is used\(^{22,25} \) to obtain the electric field distribution:

\[ \frac{1}{\mu_r} \nabla \times (\nabla \times E) + \omega^2 \varepsilon_0 \mu_0 \left( \varepsilon_r - j \frac{\sigma}{\omega \varepsilon_0} \right) E = 0 \]  

(25)

| TABLE I. Dimensions used in the model for the configuration with the separation plate and the antenna protrusion. |
|-----------------------------------------------|
| Parameter                      | Length (mm) |
|-----------------------------------------------|
| Cavity Height (\( h_c \))              | 175          |
| Cavity Radius (\( a \))                | 50           |
| Sliding Section Height (\( h_s \))     | 77.5         |
| Antenna Length (\( h_a \))             | 31           |
| Antenna Radius (\( r_a \))             | 6.35         |
| Port Radius (\( r_p \))                | 20.64        |
| Port Height (\( h_p \))                | 15           |
| Separation Plate Thickness (\( t_s \))  | 10           |
| Teflon Height (\( h_t \))              | 5            |
where $\varepsilon_r$ and $\mu_r$ are relative permittivity and relative permeability of the medium, respectively. The medium is assumed to be nonmagnetic ($\mu_r = 1$).

In a MET thruster, when the plasma discharge begins the conductivity and the dielectric constant of the medium change as in Equations 26 and 27. The conductivity and the relative permittivity of the plasma would be given by:

$$\sigma = \frac{n_e e^2}{m_e} \frac{1}{\nu_m + j\omega}$$  \hspace{1cm} (26)

$$\varepsilon_p = 1 - \frac{\omega_p^2}{\omega(\omega - j\nu_m)}$$  \hspace{1cm} (27)

where $e$ and $m_e$ are electron charge and electron mass, respectively. $\varepsilon_p$ is permittivity of the plasma, $\omega_p = \sqrt{\frac{e^2 n_e}{\pi m_e}}$ is the plasma frequency and $\nu_m$ is the electron-neutral collision frequency.

In studies conducted to date, it is reported that the plasma densities inside a MET thruster are on the order of $10^{19} - 10^{20} \text{#/m}^3$ and the electron temperature is about 1 eV.\textsuperscript{17,27,28} The free floating plasma inside the BUSTLab MET cavity is seen in Figure 4. The plasma density distribution, which is derived using plasma continuity equation for a cylindrical geometry,\textsuperscript{29} is assumed to have the profile given in Equation 28 as shown in Figure 5(a).

$$n_e(r, z) = n_0 J_0(\chi_0 r / R_p) \cos(\pi z / l)$$  \hspace{1cm} (28)

where $R_p$ and $l$ are the radius and the length of the plasma filled side of the cavity, respectively, and $n_0$ is the density of the plasma at the center of the plasma section.

The electron-neutral collision frequency, $\nu_m$, in Equations 26 and 27 is a function of the neutral density and electron temperature. $\nu_m$ is calculated using the equations given by Jonkers et al.\textsuperscript{30} In the model, the neutral density, $N$, is evaluated by:

$$N = \frac{p}{kT} - n_e(1 + \frac{T_e}{T})$$  \hspace{1cm} (29)

where $T$ is the heavy species temperature and $p$ is the pressure. In the model, the gas temperature is prescribed as 2000 K, the pressure is taken to be 1 atm and the electron temperature is assumed to be uniform at 1 eV.
V. RESULTS AND DISCUSSIONS

Numerical simulations are performed for the two different cases described in Section IV.

A. Empty cavity

The aim of the electromagnetic model is to investigate the effects of the separation plate thickness and the antenna length on the electric field distribution inside the empty cavity.

First, the electric field distribution in an empty cavity is obtained. The antenna length is set to “zero” by setting the tip of the antenna to be flush with the bottom surface of the resonant cavity. It
was observed that the maximum value of the electric field is on the order of $10^5$ V/m at the two ends of the cavity as seen in Figure 6(a).

In the second configuration, a quartz separation plate with a thickness of 10 mm is placed near the cavity center. It is seen in Figure 6(b) that the placement of this plate does not change the $TM_{011}$ mode for the cavity, however, the electric field strength drops by about ten folds in comparison the case for the empty cavity. In the third configuration, a 30.6 mm long antenna is added along with the 10 mm thick separation plate. The presence of the antenna affects the electric field strength as seen in Figure 6(c). Although the resonance mode does not change for these two configurations, it is observed that the field strength significantly decreases.

In the experiments conducted at BUSTLab it was observed that the plasma discharge inside the MET begins at a chamber pressure of about 10 torr. At this pressure level to initiate the plasma discharge with Helium gas a power level of 70 W is required. To reduce the effect of the presence of a separation plate and the protrusion of the antenna into the resonant cavity and to ensure the formation of the plasma using with minimum power, a tuning procedure must be applied. Even though in the experiments conducted, tuning is achieved by dynamically changing the power transmission line
FIG. 8. Power deposited to the propellant gas versus the antenna length for the sliding section length of 77.5 mm.

using a two-stub tuner, tuning can be achieved by changing the dimensions of the cavity and/or the antenna as analyzed and presented in this paper.

A parametric sweep is done in order investigate the optimum antenna length for a cavity with different separation plate thickness values. It is evaluated that the maximum electric field is reached when the plate thickness is 2 mm and the antenna is not protruded into the cavity.

In BUSTLab MET prototype 10 mm separation plate is used. As can be seen in Figure 7, the maximum strength of the electric field decreases to roughly one-third of this maximum value for that thickness. The electric field maximum is reached when the antenna length is about 10.5 mm.

B. Cavity with plasma discharge

When the plasma discharge begins, the electromagnetic wave propagation pattern will change because of the changes of the medium properties inside the cavity. As the plasma would act as a conductor, electromagnetic wave will be absorbed by the plasma or be scattered from the surface of the plasma. The magnitude of the plasma density is important parameters that affect the amount of power deposition. The conductivity of the medium is a function of the electron number density as expressed in Equation 26. So, the conductivity of the medium changes based on the plasma density profile given as in Equation 28. For the prescribed and assumed plasma properties, the effects of the antenna length and sliding section height on the power deposition are investigated in this section.

In the first simulation, antenna length is changed when the sliding section height and the separation plate length are fixed at 77.5 mm and 10 mm, respectively. The maximum power deposition is achieved for plasma electron density of $10^{20}$ #/m$^3$. For this plasma density, the optimum antenna length is determined to be roughly 7.5 mm, and nearly 98% of the electrical power is deposited as seen

FIG. 9. Absorbed Power vs Sliding Section Length for the antenna length of 9.5 mm.
in Figure 8. When the antenna length is set to quarter wavelength (30.6 mm for 2.45 GHz frequency), the amount of deposited power changes very little for varying plasma density values.

In order to determine the optimum antenna length for all the number density conditions, the average power deposition value (black line) which has the same distance from all the deposited power lines is drawn. The maximum power deposition is at 9.5 mm antenna length. This value is taken as the optimum antenna length.

The second simulation is done to evaluate the effect of cavity length on power deposition. The sliding section height is varied from 10 mm to 100 mm. For this simulations, the antenna length is fixed at 9.5 mm and the separation plate thickness is kept constant at 10 mm. The peak of the power deposition line sweeps to the left as seen in Figure 9. The maximum power deposition is reached at different cavity lengths for various plasma densities.

VI. CONCLUSION

An electromagnetic model is developed to investigate the effect of the antenna length and the thickness of the separation plate on the electric field distribution inside the MET cavity. Optimum antenna and cavity length values at which the power deposition is maximized inside the cavity are evaluated. COMSOL Multiphysics electromagnetic solver is used for the computations. Simulations are done for electron densities between $10^{19} – 10^{21} \text{#m}^{-3}$.

It is shown that the presence of a separation plate and protrusion of an antenna inside the resonant cavity affect the electric field intensity for an empty cavity. Based on the analyses for the BUSTLab MET, the maximum power deposition is achieved when the electron density is $10^{20} \text{#m}^{-3}$, and the cavity length and separation plate thickness values are 175 mm and 10 mm, respectively. An average optimum antenna length of 9.5 mm is calculated for all plasma densities considered. Also the calculations show that over 92% of delivered power is deposited to the plasma by varying the cavity length for a fixed antenna length.

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