Baryon Density in the Central Region of a Heavy-Ion Collision

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The O(4) linear sigma model of the chiral transition in QCD is similar to models of the superfluid transition in $^4$He and $^3$He. Observations of vortex formation in superfluid helium have recently improved the understanding of the dynamics of such transitions. This is exploited to estimate the baryon density in the central region of a heavy ion region and the result is consistent with the long held belief that this density is very small in comparison with the pion density.

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I. INTRODUCTION

Although QCD is believed to be the underlying theory of the quarks and gluons which constitute normal baryonic matter, very little is currently known for certain about the nature of its chiral phase transition. A purely gluonic SU(3) theory is believed to have a first order transition [1]. In a world with just two massless quarks, however, or in other words an infinitely massive strange quark, the chiral transition is plausibly second order [2]. If the strange quark were massless the transition would again be first order [3]. It is even conceivable that there is no real phase transition at all and that the change of state occurs by a smooth cross-over [4]. For the physical values of the quark masses the nature of the transition is not certain although it is frequently assumed to be second order. It is also often assumed that the chiral and deconfining transitions are one and the same and are referred to as simply the QCD transition. Here, we will adopt both these conventional assumptions and explore the consequences of the resulting similarity between the QCD transition and the superfluid phase transition.

Collisions of highly relativistic nuclei offer the possibility of producing quasi-macroscopic regions of dense nucleonic matter at a sufficiently high temperature that it might be possible to observe the QCD transition experimentally. In such ‘heavy-ion collisions’, one typically collides Pb, Au, S or O at energies between 3 and 200 GeV per nucleon. The high energies of the incident nuclei mean that, in the centre of mass frame, they are highly Lorentz contracted and resemble two pancakes approaching each other along the beam pipe at almost the speed of light. In fact, the energies of the incident nuclei are sufficiently high that the whole system is approximately Lorentz boost invariant. The practical result of this is that all physical quantities tend to be functions of proper time and hence observables are rapidity independent.

Immediately after the collision, the two nuclei recede in opposite directions down the beam-pipe, leaving a region of hot quark-gluon plasma between them. This then expands and cools through the QCD phase transition. Eventually the energy density becomes sufficiently low that the plasma hadronises to produce the pions, nucleons and kaons observed. Unsurprisingly by far the majority of the products are pions since they are so much lighter than anything else.

If one is to use such heavy-ion collisions to probe the nature of the QCD phase transition, it would clearly help to have some indication of what the experimental consequences of QCD ought to be. Due to the difficulty of applying conventional perturbative techniques to QCD or simulating the transition numerically, this is an incredibly hard problem. Nonetheless there has been much work on possible experimental signatures, involving observables such as the photon and dilepton fluxes and the $K/\pi$ ratio. There has also been interest in the effect of hydrodynamic instabilities during the cooling of the plasma. Possibly the most clear signature so far considered, however, is the deficit of neutral pions which would arise from large regions of misaligned QCD vacuum acting as pion lasers, more usually known as dis-oriented chiral condensates.

Another, very natural possibility, would be to look at the baryon density. Although the baryon number of the incident nuclei would probably almost all be contained in the receding pancakes which constitute the remnants of the original particles, it is conceivable that there could be a significant baryon number density in the central region immediately after the transition. Since the speed of the plasma is proportional to the distance from the collision point, this central region is also known as the central rapidity region. Any baryons in it would therefore have characteristically low longitudinal velocities compared with the receding nuclei.

Nonetheless, the central rapidity region is usually assumed to be baryon free, partly on the basis of string models, although there has until now been no work to predict the proton and neutron distributions directly [1]. It turns out, however, that in the context of the linear sigma model the evolution of the baryon density has many similarities with the production of topological defects [5]. Recent theoretical progress in this area [6], supported by experiments in superfluids [7,8], means that it is now possible to address the question of the
baryon density immediately after a heavy ion collision more directly. The resulting estimate of the initial baryon density is consistent with the conventional belief that the central rapidity region is almost baryon free.

II. A MODEL FOR THE QCD TRANSITION

In order to calculate the baryon density in the central rapidity region immediately after the transition, we must first choose a tractable model for its dynamics. We have already assumed the chiral and deconfining transitions to be one and the same and second order. Let us further assume that it is then reasonable to use two flavour QCD instead of the full theory. If we use superscripts $L$ and $R$ to distinguish the left and right handed sectors of QCD, then the breaking of the full chiral symmetry group:-

$$U^L(N_f) \times U^R(N_f) \equiv U_V(1) \times SU^L(N_f) \times SU^R(N_f),$$

where the vector $U_V(1)$ and axial $U_A(1)$ sub-groups correspond to multiplying the left and right quark spinors by equal and opposite phases, to the residual symmetry:-

$$SU(N_f)_{L+R}$$

would be described by non-vanishing expectation values for operators of the form:-

$$\mathcal{M}_j = \langle \sigma_L q_{Rj} \rangle.$$

Here $q_{Lj}$ and $q_{Rj}$ are left and right handed quark spinors.

The question of what model to use to describe the dynamics of such an order parameter is far from clear cut. The full QCD lagrangian might be correct in principle but makes calculation too hard since the interacting quanta are strongly interacting. Since below the critical temperature we have a good idea of what to expect phenomenologically, one often tries to deduce the more physically relevant weakly-interacting degrees of freedom and use these to construct a more tractable model. A familiar example occurs in condensed matter systems where one starts with strongly interacting atoms in a crystal lattice and transforms this into a description in terms of weakly interacting phonons. For QCD, the weakly interacting perturbative degrees of freedom are usually taken to be mesonic, or in other words the pions. Exact transformations between the original quark-gluon degrees of freedom and the effective mesonic degrees of freedom are not known, but there are two phenomenological models commonly used, the Skyrme model [11] and the $O(4)$ linear sigma model [12].

Both of these theories roughly reproduce multi-pion scattering amplitudes to order $p^2$, or in other words at tree level. This is equivalent to treating the models as classical lagrangians. If one were to treat these new phenomenological theories classically, one would expect the baryons to be solitons. The justification for a classical treatment has been much discussed in the literature [1].

In fact, the classical approximation can be very similar to a fully quantum mechanical, although still approximate treatment [12]. Physically, this is because the long wavelength modes of the scalar fields grow exponentially and their correlation length becomes larger than all other length-scales in the problem, including the inverse pion-masses and $1/T_C$. In other words they become more classical as the transition progresses. Pictorially, one can think of classical, long wavelength ocean swells coming to dominate over the short wavelength chop as the transition progresses.

Let us first consider the Skyrme model:-

$$\mathcal{L}_{\text{Skyrme}} = \frac{1}{2} \partial_\mu \phi^a \partial^\mu \phi_a + \frac{1}{4} e^2 \eta^4 \left[ (\partial_\mu \phi_\alpha \partial^\mu \phi^\alpha)^2 - (\partial_\mu \phi_\alpha)^4 \right].$$

where the vector $\phi = (\sigma, \pi)$ is an $O(4)$ multiplet of real scalar fields, the vector $\pi$ representing the three pions and the $\sigma$ a sigma particle too massive to be observed at low energies.

Due to the extra scale coming from the four derivative term, this model has stable texture-like topological defects, usually called Skyrmions. It is possible to show that these textures have baryon number one and spin one half and they are therefore identified with the protons and neutrons. Unfortunately, since exact isospin invariance was assumed there is no distinction between the two. Since experimentally it would be far easier to detect protons than neutrons this is potentially a serious problem. The minimum energy solution for such a Skyrmion is of the form:-

$$\pi = \frac{r}{r} f(r) \sin \theta(r)$$

$$\sigma = f(r) \cos \theta(r)$$

where $f$ is some function of $r$ which has to be calculated numerically by minimising the energy to give the result shown in figure [1].

The conserved topological current associated with these textures is given by:-

$$W^\mu = -\frac{1}{12\pi^2 \eta^4} \epsilon^{\mu\nu\lambda\rho} \epsilon_{abcd} \partial_\nu \phi_a \partial_\lambda \phi_b \partial_\rho \phi_c \partial_\sigma \phi_d,$$

where $\epsilon^{\mu\nu\lambda\rho}$ is the totally antisymmetric symbol in Minkowski space and $\epsilon_{abcd}$ is the equivalent in $O(4)$ field space. The zeroth component of this gives the topological charge density, or in other words what turns out to be the baryon number density:-

$$W^0 = -\frac{1}{2\pi^2} \epsilon^{ijkl} \epsilon_{abcd} \frac{\phi_a}{|\phi|} \partial_i \left( \frac{\phi_b}{|\phi|} \right) \partial_j \left( \frac{\phi_c}{|\phi|} \right) \partial_k \left( \frac{\phi_d}{|\phi|} \right).$$

Since protons and neutrons are indistinguishable in this model this corresponds to the density of nucleons minus the density of anti-nucleons.
A Skyrmion

\[ \rho, \varphi, \pi \]

\[ r, R \]

\[ p, P \]

FIG. 1. This configuration of the O(4) scalar field is stable for topological reasons. Such configurations are generically called topological defects and this particular example is called a Skyrmion. It corresponds to either a proton or neutron.

From the point of view of predicting the baryon density immediately after the QCD transition, however, this model has one fatal flaw, namely the Skyrme term itself. This term is clearly not conformally invariant since it is designed specifically to provide a scale for the protons and neutrons. Hence, a model incorporating such a term can not describe a renormalisation group fixed point such as a phase transition and in particular couldn’t describe the chiral transition. Another way of seeing the same thing is to regard the Skyrme term as a Lagrange multiplier which fixes the vacuum expectation value of the field. If this is constrained to be finite, clearly one can’t describe the symmetric phase in which, by definition, the vacuum expectation value should be zero.

The only other alternative, without going to higher orders in some form of derivative expansion and ending up with a model which is totally impractical for calculation, is the O(4) linear sigma model:

\[ \mathcal{L} = \frac{1}{2} \partial_{\mu} \phi^a \partial^{\mu} \phi_a - \frac{\lambda}{4} (\phi_a \phi^a - \nu^2)^2 + H \sigma. \]

Again \( \phi = (\sigma, \pi) \) is an O(4) multiplet of real scalar fields, \( \pi \) representing the pions and the \( \sigma \) a sigma particle too massive to be observed at low energies. A priori, this is a quantum field theory and the zero temperature values of the parameters \( \lambda, H \) and \( \nu^2 \) should be chosen to give reasonable agreement with experiment at low energies. Given the other approximations associated with our choice of model, the exact experimental data used to choose the values of these parameters is not critical. Here the following values are chosen [3,14]:

\[ v = 87.4 \text{MeV} \]
\[ H = (119 \text{MeV})^3 \]
\[ \lambda = 20 \]

which are consistent with \( m_\pi = 140 \text{MeV} \), \( m_\sigma = 600 \text{MeV} \) and the pion decay constant \( f_\pi = 92.5 \text{MeV} \). There are other equally valid possibilities, however [15]. With these parameters, the sigma model gives a reasonable description of the phenomenology at less than 1 GeV. The phase transition takes place at roughly \( T_C \approx f_\pi \) and low energy \( \pi - \pi \) scattering amplitudes come out about right. Probably the largest criticism of the model is that the \( \sigma \), which is far more massive than the quasi-goldstonian pions, has never been seen.

One advantage of the linear sigma model is that all the critical exponents of the theory can be calculated, in both the static and dynamic renormalisation group, in the limit of small \( H \). In fact, for the three dimensional theory, the linear sigma model is plausibly in the same universality class as the O(4) Heisenberg ferromagnet [3] whose indices have previously been calculated to seven loops [16]. Given the other approximations made here, this would seem to be plenty. In the conventional notation, the critical indices are as follows:

\[
\begin{align*}
\alpha &= 2 - d \nu = -0.19 \pm 0.06 \\
\beta &= \frac{\nu}{2} (d - 2 + \eta) = 0.38 \pm 0.01 \\
\gamma &= (2 - \eta) \nu = 1.44 \pm 0.04 \\
\delta &= \frac{d + 2 - \eta}{d - 2 + \eta} = 4.82 \pm 0.05 \\
\nu &= 0.73 \pm 0.02
\end{align*}
\]

where \( d \) is the spatial dimension which we take to be three, notwithstanding the apparent flatness induced by the approximate Lorentz boost invariance, since the spatial structure of the field will later turn out to be important. If we define the relative temperature \( \epsilon = 1 - T/T_C \) in the conventional way, then the correlation length of the scalar field will be:

\[ \xi = \frac{\xi_0}{\epsilon^\nu}, \]

where \( \xi_0 \approx 0.7 \text{fm} \).
Similarly, the relaxation rate of the pion field may be obtained from dynamical renormalisation group arguments:

$$\tau = \tau_0 \xi^z$$

where

$$\tau_0 = \frac{2h}{\lambda v^2 c^2} \approx 2 \times 10^{-24} s$$

which unsurprisingly is of the order of the light crossing time of a pion. In three dimensions, the critical index $z = d/2 = 3/2$.

The biggest draw-back with this model as far as we are concerned comes from Derrick’s theorem, which tells us that any renormalisable field theory involving only scalar fields can’t support stable solitons. In other words, the fact that there is no fourth order derivative term in the sigma model to provide a length scale for the baryons means that the configurations of the field which would be defects and represent the baryons are not topologically stable. Whereas the Skyrme model had stable objects corresponding to protons and neutrons but could not be a good description of the phase transition since it broke conformal invariance, the linear sigma model is conformally invariant at the transition but does not have stable protons and neutrons.

The most obvious solution to this dilemma would be to require the model to be a renormalisable field theory. Since we are looking for a phenomenological model rather than a fundamental field theory there is some justification for this. In this case, however, it would not be clear exactly what was meant by conformal invariance and it would not be easy to calculate with whatever terms were necessary to ensure stable solitons.

Another possibility would be to exploit the fact that we are only really interested in a finite sized volume of quark-gluon plasma and hence have boundary conditions so that Derrick’s theorem doesn’t necessarily apply. Certainly, if we were to use the sigma model on a 2-sphere, for example by exploiting the axial symmetry to reduce the problem to just the radial and beam-pipe co-ordinates and then imposing the condition that the field should be zero at the edge of the plasma, there would be stable solitons, even though these would lack a scale and the nucleon size would not be fixed.

Neither of these solutions is particularly appealing or clearly better than the other, however. Assuming that there really is a transition and that it is second order, then ideally one would like a renormalisable theory which is conformally invariant at the critical point and conserves baryon number written in terms of the pion fields. It is not clear, however, that this is possible since the bosonic theory does not include either glueballs or all the quark flavours. It could also be the case that in reality there is a smooth cross-over rather than a real phase transition, in which case the theory wouldn’t have to be conformally invariant at the critical point.

Here, however, we have assumed a second order transition in two flavour QCD. In order to describe the transition we therefore have to use the sigma model. In both the Skyrme and sigma model, however, the $O(4)$ scalar field represents the same physical degrees of freedom. We know from the Skyrme model what configuration of the pion and sigma fields corresponds to a baryon and hence the same configuration ought also to correspond to a baryon in the sigma model. Indeed, if we consider the sigma model in thermal field theory, at low temperatures, the field would effectively be confined to the vacuum manifold, the texture configurations would be effectively stable and the model would be equivalent to the Skyrme model. We will therefore assume that the QCD phase transition can be described by the linear sigma model and that the protons and neutrons are represented by Skyrmion-like configurations of the field notwithstanding the fact that they are not topologically stable.

In fact, Skyrmion-like configurations of the scalar field will tend to collapse at the speed of light. If we were to take this seriously this would imply violation of baryon number. This is clearly a flaw in the sigma model and presumably arises since this model doesn’t contain all of the relevant physics. Is it, however, serious for the dynamics of the phase transition? Certainly, the presence of topological defects can produce non-perturbative effects which can, for example, change the critical temperature. The time-scales for this process, however, are such that the rate of texture decay, or in other words baryon violation due to the inadequacy of the model, is always slower than the time-scale for breaking the symmetry. The time-scale for the symmetry to be broken is of order $2h/\lambda v^2 c^2 \approx \xi_0/c$. This should be compared with the minimum texture collapse time of $\xi/c$. Since a texture corresponding to a proton or a neutron will always be larger than the cold coherence length, it is safe to assume that textures will take longer to decay than the time available during the course of the transition. In fact, detailed studies of texture dynamics suggest that texture unwindings may be quite rare and even less likely than a simple time-scale argument suggests. It will therefore be assumed that the $O(4)$ field is unlikely to be much influenced by the texture unwinding events which would describe proton / neutron decay.

In conclusion, in order to calculate the baryon density immediately after a heavy ion collision, we will use the non-linear sigma model with critical exponents calculated in the $H = 0$ limit to model the dynamics and assume that baryons correspond to configurations of the $O(4)$ scalar field which look like Skyrminons.

*Although in the case of the Skyrme model there is some ambiguity in the choice of which of the components is the massive sigma.
III. INITIAL CONDITIONS AND HYDRODYNAMICS

Let us now consider the initial conditions. If the energy released in a heavy-ion collision is roughly equivalent to that produced in nucleon-nucleon collisions then one expects the energy density immediately after the collision to be of order 3 GeV / fm$^3$. Unlike many cosmological phase transitions, however, QCD is a strongly coupled system and, in the sigma model, the dimensionless coupling constant $\lambda$ must be of order 20 in order to approximate low energy pion cross-sections. At these sort of energy densities, this means that the interaction time-scale is likely to be far shorter than the cooling rate and the deconfined quark-gluon plasma is likely to come rapidly into thermal equilibrium at a temperature of a few hundred MeV.

For example, if following Bjorken [19] one assumes the initial energy density to be somewhere between 1 and 10 GeV / fm$^3$ and distributes this energy at about 400 MeV per quantum, then the mean free path works out to be roughly:-

$$\lambda_{mfp} \approx \left( \frac{10\text{mb}}{\sigma_{int}} \right) \times (0.05 - 0.5) \text{fm}$$

with a corresponding thermal equilibration time of the order of 1 fm / c. Thus, while the usual assumption of an initial thermal state is likely to be wrong in the comparatively weakly coupled cosmological models, it may be a reasonable approximation for a heavy-ion collision.

Typically people assume you reach 200 and 300 MeV at between 1 and 4 fm / c [19]. Here, as an initial condition, we will assume that the quark-gluon plasma is in thermal equilibrium at a temperature of 200-300 MeV at a proper time $\tau_I \approx 1$ fm / c after the collision.

There is in fact another reason not to try to use the sigma model at times less than 1 fm / c. If we were to treat the sigma model as a phenomenological quantum theory rather than as a classical field theory, some momentum cut-off, $\Lambda$ would be needed. Clearly $2m_\pi < m_\sigma < \Lambda$ in order to allow fluctuations of the field on the scale of the pion compton wavelength. What is less obvious is that, with $\Lambda$ of the order of ten or twenty, a cut-off much larger than 1 GeV leads to a negative effective coupling, thus constraining $600 \text{MeV} < \Lambda < 1 \text{GeV}$. This is equivalent to a length-scale cut-off of the order of 0.2fm and hence the model wouldn’t make much sense on time-scales much less than 1 fm / c. It is safe to assume that we should also not take the classical theory seriously on such length scales.

At these sorts of energies, in the centre of mass frame, both the incoming nuclei appear highly Lorentz contracted into pancake shapes. In addition, experiments see uniform particle production as a function of rapidity, at least from the collision or central rapidity region. Both of these facts imply an approximate Lorentz boost invariance and consequently that physical quantities should depend only on the proper time $\tau = \sqrt{t^2 - x^2}$, where $x$ is the co-ordinate along the beam-pipe with zero at the collision point. The consequence of this is that initially at least the expansion of the plasma will be linear along the beam pipe. This should be true for times ( or distances from the collision axis ) of the order of a nuclear radius, or $t \ll 1.2(A_1 + A_2)^{1/3} \approx 7 \text{ fm} / c$ for lead or uranium, where $A_1$ and $A_2$ are the mass numbers of the colliding nuclei. At later times, one would expect three dimensional rather than linear expansion but since this volume is at least as big as the region in which protons and neutrons are likely to be formed we ignore this.

Since we are assuming, as is conventional, that the sigma model may be treated classically in the context of the chiral transition, it follows that during the initial linear expansion of the plasma, the temperature falls off like:-

$$T = T_I \left( \frac{\tau_I}{\tau} \right)^\alpha,$$

where $\alpha$ depends on the speed of sound in the plasma and is 1/3 in the case of an ultra-relativistic plasma [10]. The fact that the temperature can only depend on $\tau$ implies that the plasma is hottest just behind the receding pancakes and coolest in the central region, and cools from the inside outwards, somewhat like a baked-alaska.

![Important Time-Scales in the Plasma](image)

FIG. 2. A pictorial representation of the evolution of the plasma in space time. The remains of the incident heavy ions recede down the beam-pipe on light-like trajectories and all physical quantities and events are either specified or take place on space-like hypersurfaces of constant proper time.

Clearly, the plasma and the sigma model can only be treated classically while there is more than one pion per pion compton wavelength:-

$$kT > kT_I \left( \frac{\tau_I}{\tau_H} \right)^\alpha = \frac{m_\pi c^2}{\lambda_\pi^3},$$

or in other words up until the proper time $\tau_H$ such that:-
For an initial temperature of about 200 MeV / fm$^3$ this works out to be about $3.4 \tau_I$ if one takes the pion Compton wavelength to be one fermi and proportionally larger if one takes the value 1.5 fm. This is worth noting, since with the smaller value it is conceivable that the plasma would hadronize sufficiently early to be of dynamical interest. The relation between these time-scales is shown in figure 2.

The question of how big the correlation domains are when defects are formed is the subject of the Zurek scenario for the formation of topological defects [7]. This scenario provides an estimate of the initial defect density immediately after the phase transition in a particular case of the Kibble mechanism, namely a rapid quench through a second order phase transition. Although this scenario arose through considering the possible formation of cosmic strings in the early universe, it has since been tested using the superfluid transitions in both $^4$He [3] and $^3$He [10], and has so far been consistent with all observations. In fact the systems in which the scenario has been tested are all strongly coupled in some sense and are hence far closer to the QCD transition in terms of their dynamics than they are to the sorts of cosmological phase transitions typically considered.

In the Zurek scenario, the prediction of the defect density formed during a symmetry breaking phase transition depends on the phenomenon of critical slowing down.

\[
\tau_H \leq \tau_I \left[ \frac{kT \lambda^3}{m_\pi c^2} \right]^{1/\alpha}
\]

IV. ZUREK SCENARIO IN A QCD PLASMA

To summarise, our picture of a heavy ion collision in the laboratory frame is of two highly lorentz contracted nuclei colliding to produce an approximately boost invariant plasma, initially in local thermal equilibrium. This then cools from the inside out, expanding initially linearly, with correlation domains which grow as the phase transition is approached. This we will describe using a classical treatment of the linear sigma model, with critical exponents calculated in the $H = 0$ limit. In order to predict the number of protons and neutrons produced in the central rapidity region, we need to know how many Skyrmion like configurations of the pion field will be produced in traversing the phase transition.

Although they will not actually be topologically stable in the sigma model, and will only become topologically stable objects when the plasma has cooled sufficiently below the transition that the field is almost always on the vacuum manifold and the Skyrme model is appropriate, the formation of these Skyrmion like field configurations will presumably be very similar to the formation of regular topological defects by the Kibble mechanism [20]. In fact, counting the number of Skyrmions produced in this symmetry-breaking phase transition is equivalent to counting the number of topological defects produced in many other phase transitions, including those in superfluid helium which have been used to experimentally test our ideas concerning defect formation. *This is the crucial point which we exploit here in order to estimate the baryon density in the central rapidity region.*

In the present context, the Kibble mechanism would work as follows. During the transition, the $O(4)$ scalar pion field begins to fall from the false ground-state into the true ground-state, choosing a point on the ground-state manifold at each point in space. We will assume that the bias induced by the sigma term is small and that the point on the vacuum manifold is chosen approximately at random. Our sigma model has a second order phase transition so this collapse to the true ground-state will occur by phase separation, the resulting field configuration being one of domains within each of which the scalar field has relaxed to a constant ground-state value.

In the conventional Kibble mechanism, one then argues that continuity and single valuedness will sometimes force the field to remain in the false ground-state between some of the domains. This requires at least one zero of the field which would have topological stability and characterise a defect. The density of defects is then closely linked to the number of domains as shown in figure 2.

In the case of the linear sigma model, however, it would be perfectly possible for the field to have Skyrmion like winding but still not be a defect since the field could unwind inside the surface over which the winding is computed. Counting protons and neutrons with the conventional topological current then might be hard. We can however still exploit heuristic evidence from simulations which indicates that there will be one real Skyrmion configuration formed for every twenty-five to a hundred domain sizes [18]. Calculating the proton-neutron density then becomes a case of finding how many correlation domains are formed within the plasma.

The question of how big the correlation domains are when defects are formed is the subject of the Zurek scenario for the formation of topological defects [7]. This scenario provides an estimate of the initial defect density immediately after the phase transition in a particular case of the Kibble mechanism, namely a rapid quench through a second order phase transition. Although this scenario arose through considering the possible formation of cosmic strings in the early universe, it has since been tested using the superfluid transitions in both $^4$He [3] and $^3$He [10] and has so far been consistent with all observations. In fact the systems in which the scenario has been tested are all strongly coupled in some sense and are hence far closer to the QCD transition in terms of their dynamics than they are to the sorts of cosmological phase transitions typically considered.

In the Zurek scenario, the prediction of the defect density formed during a symmetry breaking phase transition depends on the phenomenon of critical slowing down.
As the plasma cools and approaches the phase transition, the correlation length grows and the relaxation rate of the pion field gradually decreases like $\tau = \tau_0 \xi^2$. At some stage during the cooling process therefore the relaxation time-scale of the pion field will become longer than the time-scale on which the plasma is cooling. In other words, the pion field will no longer be in thermal equilibrium and will be unable to keep up with the cooling of the plasma. This is shown schematically in figure 

![Correlation Length in the Zurek Scenario](image)

**FIG. 4.** Pictorial representation of the evolution of the correlation length according to the Zurek scenario. Initially the plasma is sufficiently close to equilibrium that the correlation length stays quite close to the equilibrium value. Eventually, as the transition is approached, however, it must go out of equilibrium at a particular proper time, $\tau_z$. 

Hence, during the cooling of the QCD plasma, two regimes can be distinguished. Initially, sufficiently far away from the critical temperature the relaxation time-scale is much smaller than the time on which the cooling is proceeding and the pion field can maintain itself in local equilibrium with a correlation length $\xi(T)$. By contrast, when the plasma has cooled to a relative temperature in the vicinity of the phase transition, the pion field is effectively frozen compared to the time-scale on which the plasma is cooling. Thus, whatever the configuration and correlation length of the field at this relative temperature, it will be frozen in until after the phase transition when the field is again in thermal equilibrium. The time at which the plasma moves from one regime to another we will refer to as the Zurek time $\tilde{\tau}$. Quantities evaluated at this time will be denoted by a tilde.

\[ \frac{\partial T}{\partial x} = T_I \alpha T_I^2 (c^2 t^2 - x^2)^{-\frac{\alpha t}{\tau}} \]

Similarly, for any given position, the rate of change of temperature with respect to conventional time will be:

\[ \frac{\partial T}{\partial t} = -c^2 \alpha T_I t (c^2 t^2 - x^2)^{-\frac{\alpha t}{\tau}} \]

In other words, the length and time scales associated with temperature change will be:

\[ \frac{1}{\lambda_T} = \frac{1}{T} \frac{\partial T}{\partial x} = \frac{\alpha x}{\tau^2} \]

\[ \frac{1}{\tau_Q} = \frac{1}{T} \frac{\partial T}{\partial t} = \frac{c^2 \alpha t}{\tau^2} \]  

Hence, in the centre of mass frame, this front will propagate with speed:

\[ v_T = \frac{\lambda_T}{\tau_Q} = \frac{c^2 t}{x} \]

However, for the region inside the plasma, $x < ct$ except very close to the receding crepes where the exact

\[ \frac{\partial T}{\partial x} = \frac{T_I}{\alpha T_I^2} (c^2 t^2 - x^2)^{-\frac{\alpha t}{\tau}} \]

\[ \frac{\partial T}{\partial t} = -c^2 \alpha T_I t (c^2 t^2 - x^2)^{-\frac{\alpha t}{\tau}} \]

\[ \frac{1}{\lambda_T} = \frac{1}{T} \frac{\partial T}{\partial x} = \frac{\alpha x}{\tau^2} \]

\[ \frac{1}{\tau_Q} = \frac{1}{T} \frac{\partial T}{\partial t} = \frac{c^2 \alpha t}{\tau^2} \]  

\[ v_T = \frac{\lambda_T}{\tau_Q} = \frac{c^2 t}{x} \]

\[ \frac{\partial T}{\partial x} = \frac{T_I}{\alpha T_I^2} (c^2 t^2 - x^2)^{-\frac{\alpha t}{\tau}} \]

\[ \frac{\partial T}{\partial t} = -c^2 \alpha T_I t (c^2 t^2 - x^2)^{-\frac{\alpha t}{\tau}} \]

\[ \frac{1}{\lambda_T} = \frac{1}{T} \frac{\partial T}{\partial x} = \frac{\alpha x}{\tau^2} \]

\[ \frac{1}{\tau_Q} = \frac{1}{T} \frac{\partial T}{\partial t} = \frac{c^2 \alpha t}{\tau^2} \]  

\[ v_T = \frac{\lambda_T}{\tau_Q} = \frac{c^2 t}{x} \]

\[ \frac{\partial T}{\partial x} = \frac{T_I}{\alpha T_I^2} (c^2 t^2 - x^2)^{-\frac{\alpha t}{\tau}} \]

\[ \frac{\partial T}{\partial t} = -c^2 \alpha T_I t (c^2 t^2 - x^2)^{-\frac{\alpha t}{\tau}} \]

\[ \frac{1}{\lambda_T} = \frac{1}{T} \frac{\partial T}{\partial x} = \frac{\alpha x}{\tau^2} \]

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\[ v_T = \frac{\lambda_T}{\tau_Q} = \frac{c^2 t}{x} \]
boost invariance breaks down. Hence the acausal temperature front will always move at least as fast as the speed of light. In other words, the phase front propagates not only faster than the fluid which moves with a bulk velocity along the beam pipe of \( x/t \) but also faster than the sound speed which is relevant for the equilibration of the plasma. Effectively this means that there is no difference between this baked-alaska cooling and the homogeneous case as far as the formation of topological defects is concerned.

Let us therefore compute the relative temperature at which the textures will be frozen in for some point within the plasma with co-ordinates \((x,t)\) in the centre of mass frame. Clearly, the field will go out of equilibrium when the time-scale associated with the cooling, \(t_Q\), is equal to the relaxation rate of the plasma, \(t_R\). As earlier,

\[
\frac{1}{t_Q} = \frac{c^2 \alpha t}{\tau^2}
\]

and

\[
t_R = \frac{t_0}{\kappa^2}
\]

Suppose we consider a point in the plasma a fixed fraction \(\sqrt{1-f^2}\) of the distance from the point of impact to the position of the crepes at the outermost extremity of the plasma with \(x \approx ct\). At any time \(t\), this point \((t, \sqrt{1-f^2} ct)\) will have the proper time

\[
\tau = \sqrt{c^2 t^2 - x^2} = \sqrt{c^2 t^2 - c^2 t^2 (1-f^2)} = ct f
\]

and consequently the quench time-scale at this point will be \(t_Q = ft/\alpha\). Since we know how to calculate the relative temperature \(\epsilon\) as a function of proper time \(\tau\) we can relate \(ft\) to the relative temperature at the point \((t, \sqrt{1-f^2} ct)\) in the plasma as:

\[
tf = \frac{\tau f}{c} \left( 1 + \frac{T_C}{T_I} \right)^{1/\alpha}
\]

Equating the two time-scales then yields:

\[
\frac{\alpha ct_0}{\tau I} \left( \frac{T_I}{T_C} \right)^{1/\alpha} \approx \epsilon^{\nu z} \left( 1 + \frac{\tilde{\epsilon}}{\alpha} \right)
\]

This can be solved numerically for the relative temperature \(\tilde{\epsilon}\) which is frozen into the O(4) field when it goes out of equilibrium and the length-scale imprinted on the plasma will be:

\[
\tilde{\epsilon} = \frac{\xi_0}{\kappa^2}
\]

It will be seen that this relative temperature does not depend on \(f\), the position in the plasma. In other words the plasma goes out of equilibrium at the same relative temperature everywhere and a single length-scale is imprinted on the plasma.

The approximate Lorentz boost invariance of the plasma implied that physical quantities such as the initial conditions were specified on space-like hypersurfaces of constant proper time, as shown in figure 2. Although the plasma goes out of equilibrium at different times depending on how close to the receding crepes it is, the freezing in of topological defects actually occurs on a space-like hypersurface of constant proper time.

Since for \(x = 0\) the proper time is equal to \(ct\), using the fact that \(T = T_I (\tau I / \tau)^{\alpha}\) one finds that \(\tau_C = (T_I / T_C)^{1/\alpha} \tau_I\). Substituting our expression for temperature in terms of time at \(x = 0\) into our definition of relative temperature \(\epsilon = T/T_C - 1\) one finds:

\[
\tau = \tau_I \left[ \frac{T_I}{T_C} - 1 \right]^{1/\alpha} = \frac{t_0}{\kappa^2}
\]

Rather than solving for \(\tilde{\epsilon}\) numerically, however, it is possible to obtain an approximate solution by specialising to the case of \(x = 0\) or equivalently \(f = 1\) and using a slightly different criterion for when the pion field goes out of equilibrium, namely that the relaxation time is equal to the time remaining until the phase transition occurs:

\[
t_C - \tilde{\epsilon} = t_R
\]

The criterion for going out of equilibrium then becomes:

\[
t_I \left( \frac{T_I}{T_C} \right)^{1/\alpha} \left[ 1 - \left( \frac{1}{1 + \tilde{\epsilon}} \right)^{1/\alpha} \right] = \frac{t_0}{\kappa^2}
\]

Exploiting the fact that \(\epsilon\) is likely to be small near the phase transition when the field actually goes out of equilibrium and in any case will certainly be smaller than the initial value of roughly 0.25 we can series expand \((1 + \tilde{\epsilon})^{-1/\alpha}\) to give:

\[
\tilde{\epsilon}|_{x=0} = \left[ \frac{t_0}{\alpha t_I} \left( \frac{T_C}{T_I} \right)^{1/\alpha} \right]^{1/\alpha} \tilde{\epsilon}^{\nu z}
\]

Since we know that the plasma freezes out at the same relative temperature everywhere, and in particular it freezes out on a specific space-like hypersurface of constant proper time, we may as well use this approximate solution rather than solving the previous equation numerically.

Using this approximate solution for the relative temperature \(\tilde{\epsilon}\), we can calculate the length scale which is frozen into the plasma during the phase transition, the Zurek length. Since we know that volume of the QCD plasma at any particular time we can then calculate how many correlation volumes are frozen into the plasma according to the Zurek scenario and hence how many protons and neutrons we would expect to see in the central rapidity region.
Since the initial conditions are not well known the results for a variety of plausible initial temperatures and proper times are shown:

| \( \tau_f/\text{fm} \) | 1    | 1    | 2    | 4    | 4    |
|-----------------|------|------|------|------|------|
| \( T_I/\text{MeV/fm}^3 \) | 200  | 300  | 250  | 200  | 300  |
| \( \xi/c \) / fm | 2.0  | 6.6  | 7.6  | 7.8  | 26.4 |
| \( \epsilon \) | 0.25 | 0.88 | 0.56 | 0.25 | 0.88 |
| \( \xi / \text{fm} \) | 0.36 | 0.20 | 0.19 | 0.19 | 0.10 |
| \( \xi / \text{fm} \) | 1.5  | 2.3  | 2.4  | 2.4  | 3.8  |
| Plasma Volume / fm\(^3\) | 246  | 1170 | 1386 | 1417 | 6098 |
| Number of Domains | 18   | 24   | 25   | 26   | 28   |

where we have taken the QCD plasma to be a cylinder of width 14fm and length equal to 2\( \xi \). Although both the Zurek proper time, or equivalently the frozen in domain size, and the volume of the plasma at this time are quite sensitive to the initial conditions, which are not well known, the total number of correlation volumes frozen into the plasma is much less sensitive since it is the ratio \( V/\xi^3 \).

In other words there will be somewhere between 15 and 30 coherence volumes frozen into the plasma. From simulations, however, we expect to get roughly one skyrmion configuration per every 25 - 100 domains. In order to create a proton-antiproton pair then we would need to have between 50 and 200 coherence volumes and also be lucky enough that the pair didn’t immediately annihilate. Avoiding annihilation is not entirely implausible since the distance between the nucleon and anti-nucleon must be at least as great as the Zurek length which in this case is slightly greater than the proton Compton wavelength. Forming a sufficiently large number of domains that the resulting proton and neutron density would be observable on the other hand is far more difficult. Roughly speaking, however, this result indicates that the central rapidity region of the plasma is likely to remain free of protons and neutrons as is usually thought.

With the smallest possible length-scale imprinted, namely the pion compton wavelength, the situation is somewhat better and one could conceivably see a reasonable number of protons and neutrons. However, this corresponds to almost no domain growth and in this context DCCs would also be ruled out.

It is possible, however, that we might be extraordinarily lucky since the predicted values are necessarily rather approximate and do not categorically exclude the possibility of a detectable baryon number in the central rapidity region. Also, the freeze out occurs at a sufficiently large proper time that it is debatable whether we are still justified in treating the model classically. The model is already known to be flawed since it does not conserve baryon number and the results are somewhat suspect because of this. It is certainly possible to do a slightly better calculation to include the quantum mechanical aspects of the theory, but this would not cure the problem of baryon number conservation. It seems likely that the only way to make a significant improvement would be to improve the model somehow, and this would probably make further analytic work intractable and necessitate a simulation.

V. CONCLUSIONS

The conventional wisdom is that the central rapidity region of a heavy-ion collision in which a quark-gluon plasma is produced should be almost baryon free. A direct estimate of this initial baryon density in the context of the \( O(4) \) linear sigma model shows this belief to be well founded and entirely consistent with the production of dis-oriented chiral condensates.

However, a non-zero baryon density is not ruled out by many orders of magnitude and the result is somewhat sensitive to the initial conditions. It is therefore conceivable that with a slightly larger volume of QCD plasma one might occasionally produce a small but non-zero baryon density. To determine whether it is worthwhile looking for this experimentally further theoretical work is necessary.

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