Unification of Gravity with Electromagnetism

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Principles of Maxwell, Lorentz, Milne, Dirac and Feynman are combined to unify gravity with electromagnetism. Special-relativistic settled reality (SR) evolves, as universe age increases, via cosmological Feynman paths. Although SR is ‘classical’, its evolution is ‘quantum mechanical’. A unitary Hilbert-space Lorentz-group representation—a lightlike-fiber-bundle—allows definition of divergenceless Lorentz-tensor self-adjoint retarded-potential operators. Feynman-path action (real but not SR) is invariant under a 7-parameter group that augments 6-parameter SL(2,c) by a 1-parameter compact (Kaluza-Klein) group generated by discrete electric charge. Sublightlike charged-particulate matter (a component of ‘objective reality’) reflects ‘zitterbewegung’—fluctuation of a lightlike-velocity direction specified by 2 angles of a 3-sphere fiber. The third fiber angle is Dirac conjugate to a discrete-spectrum electric-charge operator, whose commutation with all potential operators renders discrete the spatially-localized temporally-stable charged-particle component of SR.
Introduction

Physics at molecular or smaller scales has relied on the special-relativistic ‘particle’ concept, but the absence of finite-dimensional unitary Lorentz-group representations precludes a Dirac-theoretic \(^{(1)}\) Fock space that houses finite-spin ‘Lorentz particles’. [The \(S\) matrix is not a ‘Dirac theory’.] The present paper employs Gelfand-Naimark (GN) infinite-dimensional Hilbert-space unitary representation of the Lorentz-group \(^{(2)}\) to define classical retarded gravitational-electromagnetic fields. A special-relativistic settled reality (\(5R\)) reconciles Dirac principles with ‘detectable particles’. Particle properties such as electron mass are determined by an invariant Feynman-path action \(^{(3)}\) that combines gravity with electromagnetism.

Deferring path action to another paper, we here locally represent (classical) gravity and electromagnetism by expectations of self-adjoint retarded Lorentz-tensor potential operators that ‘Dirac-extend’ the classical Lienard-Wiechert 4-vector retarded electromagnetic potential. Maxwell’s equations are satisfied by \(5R\).

Dalembertians of potential-operator expectations are current densities of conserved electric charge, energy and momentum. A ‘detectable particle’ is a discretely-charged temporally-stable ‘concentration’ in classical Lorentz-tensor densities. Our theory admits no Dirac meaning for ‘particle wave function’. Although the Lemaître-Hubble redshift displayed by our proposal \(^{(4)}\) might seem incompatible with special relativity, all our reality is Lorentz-group based.

The Gelfand-Naimark Hilbert space, that unitarily represents the 6-parameter semisimple ‘Lorentz’ Lie group \(SL(2,c)\), comprises normed twice-differentiable functions of location in a 6-dimensional super manifold—a fiber bundle of compact, \(SU(2)\), 3-sphere fiber over an invariantly metricized (noncompact) 3-hyperboloid base space. Astonishingly, this Hilbert space also represents a 12-parameter Lie group isomorphic to transformations of a unimodular \(2\times2\) complex matrix through right or left multiplication by other such matrices.

The 6 generators of \(SL(2,c)_R\) commute with the 6 generators of \(SL(2,c)_L\). Referring to the product group as \(12L\), we here show how electric-charge discreteness, accompanying action symmetry under a 7-parameter \(12L\) subgroup, underpins the experimental-physics (special-relativistic) meaning of ‘individual particle’. Our (‘cosmologically-relativistic’) unified quantum theory of gravity and electromagnetism defines ‘particle’ to be a discretely-charged temporally-stable component of ‘settled’ (classical) reality.

Within each of 6 \(12L\) commuting-generator ‘matched’ pairs, one pair member generates displacements along a (noncompact) real line while the other generates displacements around a (compact) circle. Eigenvalues of the former are continuous while, of the latter, discrete. The action that determines universe Feynman-path (quantum) dynamics is invariant under both \(SL(2,c)_R\) and a 1-parameter compact left subgroup of \(12L\). The septet of symmetry generators—\(GN\)-represented by 7 self-adjoint (Dirac) operators—are ‘universe constants of motion’.

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Central to this paper is a pair of left generators we call ‘quc energy’ (continuous) and ‘quc electric charge’ (discrete). The abbreviation ‘quc’ stands not for reality but for a lightlike ‘quantum-universe constituent’ that, in collaboration with other qucs, Dirac-defines reality. An individual quc, despite lacking the temporal stability essential to objective reality, carries conserved momentum, angular momentum and electric charge in a Dirac wave-function sense. ‘Dirac particle reality’ is achieved through expectations of self-adjoint operators acting on a finite (although currently-huge) tensor product of GN single-quc Hilbert spaces.

Any quc momentum component—a right Lorentz-group generator in a single-quc Hilbert space—commutes with the corresponding (same direction) component of the quc’s angular momentum. Two different-direction quc momentum components do not commute with each other. Although a corresponding property of angular-momentum operators is familiar in previous quantum theory, quc-momentum eigenvalues are continuous. Quc-angular-momentum eigenvalues (unsurprisingly) are discrete. Momentum operators generate displacements in the (non-compact) 3-dimensional metricized hyperbolic quc-fiber-bundle base space.

For pedagogical reasons we avoid the term ‘Lorentz boost’ that associates to a change of velocity. The velocity of a quc is unchangeably lightlike (always c in magnitude). On the other hand, quc momentum, despite being a ‘boost generator’, is a component of a 3-vector set of quc spatial-displacement generators within a generator 6-vector. Our classical SR meaning for particle momentum is more familiar; status as 3-vector component of a positive 4-vector there prevails.

A fixed-direction component of quc momentum generates an infinitesimal displacement in the curved base space. But, because of the curvature, a non-infinitesimal displacement in fixed direction does not follow a geodesic. In contrast, a lightlike displacement generated by the quc’s energy operator follows a geodesic that parallels quc velocity. All displacements of GN fiber-bundle quc location are at fixed ‘age’ within a ‘big-bang’ spacetime—the interior of a forward lightcone where redshift-specifying age is Minkowski distance from lightcone vertex. (4)

Particle momentum accompanies (settled) positive-energy objective-reality. A particle’s temporal stability requires its energy to be positive. Probabilistic (Copenhagen) interpretation of the S matrix reflects ‘hidden’—inaccessible to ‘observation’—negative-energy reality. (4) Another paper, dealing with Feynman-path action, will define a classical quc-path momentum that, although not a positive-energy particle momentum, is a component of a right-Lorentz 4-vector.

Quc-path momentum differs from the 6-vector-component self-adjoint operator we call ‘quc momentum’, while both these concepts differ from our reality’s 4-vector ‘particle momentum’—conserved by vanishing divergence of gravitational potential rather than by action invariance. Quc-path 4-vector momentum (in contrast) is conserved by the definition of ‘cosmological Feynman path’ (CFP). Neither in ray nor in path is ‘quc’ to be understood as ‘particle’. (‘Hidden reality’, manifesting negative-energy qucs, is addressed in Reference (4).)
The universe spacetime representing the group $12L$ is a forward-lightcone interior. ‘Age’ enjoys a purely-classical ‘arrowed’ and scale-setting (redshift-specifying) status. Neither a group Casimir nor other nontrivial self-adjoint operator, age is invariant under the full $12L$ group. Age, ‘arrowed’ with unambiguous origin, plays a role similar in many ways to that of the single (classical-Newtonian, Euclidean-group-invariant) time of nonrelativistic (Heisenberg-Schrödinger-Dirac) quantum theory, despite the arbitrariness of that (non-arrowed) time’s origin.

**Symmetry**

The group $12L$ is isomorphic to transformations of a unimodular $2 \times 2$ complex matrix through left or right multiplication by some (other) such matrix. Any left $12L$ element commutes with any right element ($6$-parameter left and right semisimple subgroups). $12L$ invariance of a Haar measure accords this group a unitary $GN$ Hilbert-space representation. (4)

Although all $12L$ generators are represented by self-adjoint (Dirac) operators, not all $12L$ elements correspond to symmetries. Path action is invariant under a 7-parameter symmetry subgroup, whose generator septet comprises the universe’s ‘constants of motion’. A left-Lorentz symmetry generator-- electric charge--joins the six right-Lorentz generators of momentum and angular momentum (a Lorentz 6-vector). Zero total-universe angular momentum corresponds to Mach’s principle.

$GN$ unitary $12L$ Hilbert-space representation is by functions of location in a 6-dimensional fiber-bundle at fixed age. Each bundle attaches a 3-sphere fiber to every location in a $12L$-invariantly metricized 3-dimensional hyperbolic base space that (independently of fiber) provides 4-vector representation of left and right Lorentz groups. Location in base space is ‘quc spatial location’. Two fiber angles specify ‘quc-velocity direction’; the third angle is Dirac (Kaluza-Klein) conjugate to quc electric charge. (No quc attributes, either in Hilbert space or in Feynman path, correspond to SR.)

**Definition of ‘Reality’ by Expectation**

Exceptional ages, spaced by a Planck-scale age interval $\delta$, each carry a universe ray—a vector in a quc quasi-Fock space. Meaning for ‘settled reality’, within (quc-less although quc-CFP-traversed) slices of continuous ‘classical spacetime’ (where age is not exceptional), is through classical retarded right-Lorentz-tensor fields. These fields are prescribed by expectations, with respect to that earlier quasi-Fock-space ray whose age is closest to the field-point age, of self-adjoint multi-quc right-tensor retarded-field operators. The finite number of qucs doubles with each successive step of exceptional universe age. The minimum number-- that at universe beginning--was 1. Prominent among the classical fields prescribing settled local reality is a divergenceless second-rank symmetric (right) Lorentz-tensor--conserved energy-momentum current density. This tensor field is the Dalembertian of our retarded gravitational potential, divided by Newton’s gravitational constant.

We caution readers against attribution of ‘ordinary-language’ significance to the Hilbert-space term ‘expectation’-- a real number prescribed by pairing some ray with a self-adjoint operator. Absent from our meaning for ‘expectation’ is any probabilistic or consciousness aspect. There is no ‘Schrödinger cat’—only a ‘cat’.
Electromagnetic ‘expectation reality’ has two distinct components, matching the unambiguous decomposition of the Lienard-Wiechert retarded vector potential into a zero-Dalembertian ‘radiation’ component proportional to electric-charge ‘accelerations’ and a ‘Coulombic’ component proportional to charge ‘velocities’. (The Dalembertian of the latter is the conserved electric-charge current density.) ‘Photon reality’ (as recognized by experimental physicists) resides partly in the former vector-potential component and partly in the energy-momentum tensor.

Unitary Hilbert-Space 12L Representation

We now introduce the GN Hilbert-space unitary Lorentz-group representation. A later section will define the self-adjoint operators whose expectations prescribe reality. We invoke Pauli’s 2×2 matrices, a tool familiar in particle theory although unexploited in Reference (2). A lightlike single-quc rigged Hilbert space provides ‘regular’ 12L representation. Single-quc Hilbert-space vectors are twice-differentiable normed complex functions of the coordinates of a 6-dimensional (‘super’) manifold. Three (noncompact) dimensions spatially locate a quc, two (compact) specify its velocity direction and one (compact) underpins its charge. The latter dimension also distinguishes lightlike ‘bosonic’ qucs from ‘fermionic’. (Analytic 5-matrix CPT symmetry entangles ‘internal’ particle quantum numbers with the complex Lorentz group which preserves energy-momentum complex 4-vector inner products.)

Gelfand and Naimark defined a Hilbert space of functions of a unimodular 2×2 complex matrix \( a \) through three complex variables \( s, y, z \) (six real variables) according to the following product of three unimodular 2×2 matrices, each of which coordinates the manifold of a 2-parameter abelian 12L subgroup:

\[
a(s, y, z) = \exp(-\sigma_3 s) \times \exp(\sigma_+ y) \times \exp(\sigma_- z).
\]  

In Formula (1) the symbols \( \sigma_1, \sigma_2, \sigma_3 \) denote the standard (handed) set of Pauli hermitian traceless self-inverse 2×2 matrices (determinant \(-1\)), with \( \sigma_{\pm} \equiv \frac{1}{2}(\sigma_1 \pm i\sigma_2) \). The matrix \( \sigma_3 \) is real diagonal while the real hermitian-conjugate matrix pair, \( \sigma_+ \) and \( \sigma_- \), each has a unit off-diagonal element. Three two-parameter subgroups are represented, respectively, by ‘s’, ‘y’ and ‘z’ submanifolds of the 6-manifold \( a \). The z manifold represents a 2-parameter ‘velocity-direction’ subgroup of right SL(2,c) while the s manifold represents the 2-parameter left subgroup of diagonal matrices that is central to this paper. The 2-dimensional y manifold, although associating neither to a right nor to a left subgroup, enjoys geometrical significance: A directed geodesic of Milne’s hyperbolic 3-space \(^5\) is specified by the pair \( y, z \) of complex variables.

The complex coordinate \( z \) specifies geodesic direction while y spatially locates the geodesic in a 2-dimensional surface transverse to this direction. Finally, a point along the \( z, y \) geodesic is longitudinally coordinated by \( \text{Re} s \). Noteworthy is absence of any geometrical role for \( \text{Im} s \); this absence is crucial to what follows.

Alternative to the unimodular-matrix factorization (1) is the factorization \( a(s, y, z) = u(a) \times h(a_s) \), where \( u(a) \) is unitary unimodular while \( h(a_s) \) is positive-hermitian unimodular with \( a_s = \exp(-\sigma_1 \text{Re} s) \times \exp(\sigma_+ y) \times \exp(\sigma_- z) \). The matrix functions \( u(a) \) and \( h(a_s) \) we do not display here but are straightforwardly computable. The unitary \( u(a) \) maps \( a \) onto a compact 3-dimensional fiber space (a 3-sphere) that covers a noncompact 12L-invariantly-metricized 3-
dimensional base space onto which $a_5$ is mapped by $h(a_5)$. Base space will below alternatively be coordinated by a positive 4-vector of fixed invariant magnitude.

We employ Dirac’s shorthand (1) of denoting, by a single symbol, both a (real classical) quc coordinate and a self adjoint quc operator whose spectrum comprises the possible values of this coordinate. An example is the symbol $Re s^\sigma$—linearly related to what might either be called ‘the local time’ of Quc $\sigma$ or its ‘longitudinal coordinate’. The symbol $E^\sigma$ will denote Quc-$\sigma$ energy in (below-defined) ‘$\sigma$ local frame’. The ‘canonically-conjugate’ (when appropriately normalized) Quc-$\sigma$ time and energy operators, $Re s^\sigma$ and $E^\sigma$, are below seen not to commute.

The 6-dimensional Haar measure,

$$da^\sigma = ds^\sigma \, dy^\sigma \, dz^\sigma,$$  \hspace{1cm} (2)

is invariant under $a^\sigma \rightarrow a'^{\sigma} = a^\sigma \Gamma^{-1}$, with $\Gamma$ a 2×2 unimodular matrix representing a right $\text{SL}(2,c)$ transformation of the coordinate $a^\sigma$. The measure (2) is also invariant under analogous left transformation. The ‘volume-element’ symbol $d\zeta$ in (2), with $\zeta$ complex, means $d \text{Re } \zeta \, d \text{Im } \zeta$.

Any Hilbert-space individual-quc vector is a complex differentiable function $\psi(a^\sigma)$ with invariant (finite) norm,

$$\int |\psi(a^\sigma)|^2 \, da^\sigma.$$  \hspace{1cm} (3)

The integration in (3) spans the full $y^\sigma$ and $z^\sigma$ complex planes and the full $Re s^\sigma$ line but only a $2\pi$ interval of $Im s^\sigma$. Expectations of self-adjoint operators such as the Quc-$\sigma$ discrete electric-charge,

$$Q^\sigma \equiv i\hbar g \, \partial/\partial \text{Im } s^\sigma,$$  \hspace{1cm} (4)

with $g$ an elsewhere-addressed universal dimensionless constant, are specified by the norm (3).

The Quc-$\sigma$ continuous-spectrum energy-operator companion to (4) is

$$E^\sigma_\tau \equiv i(\hbar/2\tau) \, \partial/\partial Re \, s^\sigma,$$  \hspace{1cm} (5)

the symbol $\tau$ here standing for some discrete and exceptional value of universe age that labels a quc Hilbert space. (Our velocity unit is such that $c = 1$.) The six generators of (right) $\text{SL}(2,c)$ are also self-adjoint linear homogeneous superpositions of partial first derivatives (in the $s,y,z$ basis)—derivative superpositions with coefficients dependent on $a^\tau$, but that all commute with (4) and (5) while each commutes with exactly one other member of the right-generator sextet.

Two positive-trace self-adjoint hermitian 2×2 unimodular-matrix functions, bilinear in $a^\sigma$, $a^{\sigma\dagger}$ and linear in $\tau$ that, together, are equivalent to the coordinate quintet $a_5^\sigma = Re s^\sigma, y^\sigma, z^\sigma$—plus the age $\tau$ are

$$x^\sigma(a_5^\sigma, \tau) \equiv \tau a^{\sigma\dagger} a^\sigma,$$  \hspace{1cm} (6)

$$v^\sigma(a_5^\sigma) \equiv a^{\sigma\dagger}(\sigma_0 - \sigma_3) a^\sigma.$$  \hspace{1cm} (7)

The hermitian matrix $x^\sigma(a_5^\sigma, \tau)$ is a positive-timelike 4-vector with ‘Lorentz magnitude’ $x^\sigma \cdot x^\sigma = \tau^2$, while the hermitian matrix $v^\sigma(a_5^\sigma)$ is a positive-lightlike 4-vector ($v^\sigma \cdot v^\sigma = 0$) such that
\[ \mathbf{x}^\alpha \cdot v^\alpha = \tau. \] The dimensionful 4-vector \( \mathbf{x}^\alpha \) locates \( \text{Quc} \sigma \) in spacetime (with respect to lightcone vertex) while dimensionless \( v^\alpha \) specifies this quc’s lightlike velocity 4-vector--whose timelike component equals 1 in the \( \text{Quc-}\sigma \) local frame where \( \mathbf{x}^\alpha = (\tau, 0, 0, 0) \).

Our definition of ‘reality’ supposes present-universe age \( \tau \) to be huge compared to an (‘inflation’-interpretable) ‘starting-age’ \( \tau_0 \) that in turn was huge compared to the Planck-scale exceptional-age spacing \( \delta \). The universe starting age \( \tau_0 \) establishes the ‘macro’ scale of laboratory physics (~ 1 km)—in effect determining Avogadro’s number. Physics (as pursued by humanity) recognizes a unique right-Lorentz local frame--the frame in which cosmic background radiation is isotropic, reflecting initial-universe spherical symmetry.

Failure of the commutator,

\[ [E^\alpha(\tau), v^\alpha(a_5^\sigma)] = i\hbar/\tau v^\alpha(a_5^\sigma), \] to vanish at finite \( \tau \) reflects redshift.\(^{(4)}\)

One might suppose ‘quc lightlikeness’ to preclude massive particles, but we shall define reality via expectations of self-adjoint lightlike gravitational and electromagnetic potential operators that are functions of the (right) 4-vector Quc-\( \sigma \) operators \( \mathbf{x}^\alpha(a_5^\sigma, \tau), v^\alpha(a_5^\sigma) \) and the invariant operators \( E^\alpha(\tau) \) and \( Q^\alpha \). Fluctuation of quc velocity-direction, at fixed momentum, helicity, energy and electric charge, is expected in the universe wave function. Conserved (‘classical’) current densities of energy-momentum and electric-charge are then generally not ‘lightlike’. Although Dirac’s lightlike electron-velocity operator, which led Schrödinger to coin the term, ‘zitterbewegung’, differs importantly from the quc-velocity operator \( (7) \), Schrödinger’s language is appropriate for describing ‘cosmological origin of rest mass’.

**Classical Retarded Settled Reality**

We now blend the classical Lienard-Wiechert (LW) retarded-field notion with fixed-age \( \tau = N\delta \) Hilbert-space self-adjoint quc operators. The reader is warned against confusing the self-adjoint field operators defined here with the quantum radiation fields employed by the Standard Model-- the latter operators not being Dirac-associable to SR.

Let the positive-timelike 4-vector symbol \( \mathbf{x} \), with \((N\delta)^2 < \mathbf{x} \cdot \mathbf{x} < [(N+1)\delta]^2\), denote a spacetime location between the ages \( N\delta \) and \((N+1)\delta \). We shall define a continuous set of \( \mathbf{x} \)-labeled single-quc self-adjoint operators on the \( N \) (exceptional-age) Hilbert space housed at Age \( N\delta \). Summing over all qucs, settled reality is then prescribed by interpreting a corresponding continuum of Ray \( N \) expectations as the divergenceless (retarded, classical) LW electromagnetic vector potential. First derivatives of this vector potential yield Maxwell’s electric and magnetic fields; the potential’s Dopplerian is the conserved electric-charge current density. Maxwell’s (classical) equations, for electric and magnetic fields in terms of current density, apply throughout the interior of the spacetime slice.

In what follows the single superscript \( \sigma \) is to be understood as identifying \( \text{Quc} \sigma \) at age \( N\delta \). For the retarded electromagnetic vector-potential operator \( A^\mu_\sigma(\mathbf{x}) \), associated to \( \text{Quc} \sigma \) and the spacetime-location \( \mathbf{x} \) (the ‘field-point’), we postulate

\[ A^\mu_\sigma(\mathbf{x}) \equiv \Theta_{\text{rel}}(\mathbf{x}, a_5^\sigma) \, Q^\sigma \, v^\mu / \mathbf{v} \cdot \mathbf{x} \]

\[ (9) \]
the retardation step function \( \theta_{ret}(x, a^\sigma) \)—defined two paragraphs below—*not* depending on \( \text{Im} \ s^\tau \). All operators in (9) commute. The (right) 4-vector operators \( x^\sigma \) and \( v^\sigma \) have been defined, respectively, by Formulas (6) and (7). Because the guc-velocity 4-vector \( v^\sigma \) is lightlike, the Lorentz-divergence of \( A^\mu_\mu(x) \) vanishes—a consideration that will render electric-charge conservation an aspect of ‘classical reality’.

The \( x \) dependence of the potential is seen to reside in the step function \( \theta_{ret}(x, a^\sigma) \) and in the invariant LW-denominator operator, \( v^\sigma \cdot (x - x^\sigma) \). Because \( v^\sigma \cdot v^\sigma = 0 \), this denominator has the same value at all guc spacetime locations (not only that of age \( N_0 \)) along the lightlike trajectory with guc-\( \sigma \) velocity that passes through \( x^\sigma \). If the \( a^\sigma \) trajectory intersects the \( x \) backward lightcone, classical LW language refers to that intersection’s location as the spacetime location of the ‘retarded source’ for the electromagnetic potential \( A^\mu_\mu(x) \). Age \( N_0 \) gucs whose spatial locations are far from that of the slice-interior point \( x \), thereby admit LW description as being located in the ‘distant past’ of \( x \).

The symbol \( \theta_{ret}(x, a^\sigma) \) in (9) denotes a function equal to 1 iff the \( a^\sigma \) trajectory (passing with velocity \( v^\sigma \) through the guc-\( \sigma \) spacetime location \( x^\sigma \)) intersects the \( x \) backward lightcone. Otherwise \( \theta_{ret}(x, a^\sigma) \) vanishes. (Any lightlike trajectory not located on the \( x \) lightcone intersects the \( x \) forward-lightward lightcone exactly once.) Summed over all sources, the Ray-\( N \) expectation of (9) prescribes the classical electromagnetic vector potential \( A^\mu_\mu(x) \) within the Ray-\( N \) immediate future.

Although the symbol \( A_\mu(x) \), without superscript \( \mathcal{N} \) may be employed to designate the retarded vector potential *almost everywhere* in spacetime, exclusion must be remembered of the exceptional ray ages where \( \tau = \mathcal{N}_0 \). Classical reality is not defined on the hyperboloids that house rays. Second-order differential equations connecting (classical) potentials to current densities, that are meaningful inside any ‘spacetime slice’, only *approximately* extrapolate these fields from one slice to the next. Universe reality evolution is quantum-mechanically determined by cosmological Feynman paths (CFPs) --via action-specified phases of complex unimodular numbers. [CFP prescription is *not* addressed here.]

We now attend to a settled *gravitational* reality founded on the self-adjoint *guc* energy operator (5) rather than the *guc* electric-charge operator (4). The corresponding pair of commuting left-Lorentz-group generators, invariant under the right Lorentz group and representable by individual *gues*, are Dirac conjugate to real and imaginary parts of the complex *guc* coordinate \( s^\sigma \).

Paralleling the electromagnetic divergenceless symmetric-tensor potential \( \Phi_\mu_\nu(x) \) is a gravitational divergenceless symmetric-tensor potential \( \Phi_\mu_\nu(x) \). When divided by \( G \), the Dalembertian of \( \Phi_\mu_\nu(x) \) prescribes (without Heisenberg uncertainty) the 4-vector current density of conserved energy-momentum. We anticipate qualitative difference between electromagnetic and gravitational objective reality, symptomized by positivity of *Maxwell-field* energy density. We expect to categorize *all* electromagnetic reality as ‘objective’ whereas gravitational reality comprises not only objective but also ‘hidden’ negative-energy components that have necessitated a probabilistic interpretation for Copenhagen quantum theory.

The (Newton-LW-Dirac) gravitational-potential operator

\[
\Phi_\mu_\nu(x) = G \left[ E^\sigma V^\nu_\mu(x) + V^\nu_\mu(x) E^\sigma \right], \tag{10}
\]

where the divergence-less symmetric-tensor retarded-field self-adjoint operator \( V^\nu_\mu(x) \), defined by

\[
V^\nu_\mu(x) \equiv \theta_{ret}(x, a^\sigma) v^\nu_\mu v^\sigma_\nu / v^\sigma_\nu (x - x^\sigma), \tag{11}
\]

*right* transforms as a symmetric second-rank Lorentz tensor of zero invariant trace. Paralleling the electromagnetic vector potential, the tensor classical gravitational potential \( \Phi_\mu_\nu(x) \) is the Ray \( \mathcal{N} \) expectation of (10), summed over all *gues* (whose *total* number is finite). The Dalembertian of \( \Phi_\mu_\nu(x) \), when divided by \( G \), is the energy-momentum tensor *--presumed to manifest both massless and massive particles (e.g., photons, electrons, atoms) and electromagnetic-field energy-momentum, all positive-
energy components of objective reality. ‘Hidden reality’, associated to negative energy but also manifested by $\Phi_\mu(x)$, is elsewhere addressed. (4)

Conclusion

We have unified gravity and electromagnetism through a succession of $quc$ quasi-Fock-space rays that unitarily represent a 12-parameter ‘right-left doubling’ of SL(2,c). Self-adjoint $quc$ energy (source of gravity) and $quc$ electric charge (source of electromagnetism) are, respectively, Dirac conjugate to real and imaginary parts of a complex Gelfand-Naimark coordinate for a 6-dimensional (super ) fiber-bundle manifold. Sustained has been ‘Dirac reality’-- via self-adjoint Hilbert-space operators--as well as principles of Maxwell and Feynman.

A 7-parameter symmetry group (with a 6-parameter Lorentz subgroup) is generated by 7 conserved self-adjoint operators—momentum, angular momentum and electric charge—whose expectations separately aggregate to zero for the universe as a whole. Elsewhere described is a single-$quc$ spherically-symmetric initial condition that specifies ‘zero total-universe energy’. Classical ‘settled’ reality resides in electromagnetic and gravitational fields within invariant spacetime slices of Planck-scale age width. Each universe ray is separated from its successor by such a slice.

Any ray (after the first at Age $\tau_0$) is determined from its predecessor by the actions of cosmological Feynman paths that traverse the intervening slice. Gravitational and electromagnetic line-integral path action, proportional to the potentials here defined, is specified in a separate paper. Although no spacetime-slice-inhabiting particle (an SR component for some age interval where $t>>\tau_0$) lives forever, a $quc$ never dies. The number of $qucs$ at age $N\delta$ -- equalling $2^{N\delta - N_0}$, where $N_0 = \tau_0/\delta$, increases monotonically with universe age.

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