Application of improved GM (1, m) model for transformer faults prediction

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Abstract. The traditional GM (1, m) prediction model is improved, the original data sequence is transformed, and its data generation method is changed, so that the transformed data sequence has a more approximate exponential change property, which meets the gray model's smoothness Requirements, to be able to predict fluctuation series. At the same time, in order to improve the prediction accuracy of the model, the model background value is optimized, so that the prediction accuracy of the model is greatly improved. The improved GM (1,7) prediction model is used to predict the volume fraction of various gas characteristics of the transformer. Compared with the traditional GM (1,1) and GM (1,7) prediction results, it has a good approximation to the original data sequence the effect shows the effectiveness of the model.

1. Introduction
The operating state of the transformer directly affects the level of safe operation of the power system. If the transformer fails, the direct and indirect economic losses are huge. Therefore, it is very important to ensure the safe operation of power transformers in the safety and economic operation of power systems. It is of great significance to detect latent faults of transformers and their development trends, effectively diagnose transformer faults, and accurately determine the operating status of transformers. The majority of transformer fault predictions at home and abroad are based on dissolved gas analysis technology (DGA) in transformer oil. DGA is widely used as a non-destructive, inexpensive, and effective diagnostic technique for transformer insulation conditions.

At present, DGA fault prediction methods mainly include regression analysis prediction, gray prediction, neural network prediction and other models. Among them, the regression analysis prediction model is simple and easy to implement, but the prediction accuracy is relatively low; the neural network prediction model requires a large number of samples for training, which is more difficult to achieve.

Reference [1] proposed to introduce the GM(1,1) prediction model into the prediction of gas concentration in the oil of power transformers, but this model is only suitable for data prediction with strong exponential change law. Reference [2] has improved the traditional gray prediction method and improved the prediction accuracy. Reference [3] established the corresponding MGM (1,7) model, and performed gray correlation analysis on the seven characteristic gases. Compared with the traditional GM
(1,1) model and its improved form, the prediction effect is better. In this paper, the traditional GM (1,7) model is improved, and the seven characteristic gas volume fractions of the transformer are predicted. It is verified that the prediction accuracy of this model is significantly higher than the traditional GM (1,7) model and (1,1) Model.

2. GM (1, m) gray multivariate prediction model [4]
There is the original data matrix \( X^{(0)} = \{X_1^{(0)}, X_2^{(0)}, \ldots, X_m^{(0)}\}^T \). Where \( X_j^{(0)} \) represents the observation sequence of the JTH variable at time 1, 2, ..., \( n \). \( X_1^{(0)}, X_2^{(0)}, \ldots, X_m^{(0)} \) are accumulated once, and the matrix, and the matrix \( X^{(1)} \) is called the first-order accumulation generator matrix, which is recorded as \( X^{(1)} = \{X_1^{(1)}, X_2^{(1)}, \ldots, X_m^{(1)}\}^T \). then

\[
X_j^{(1)} = x_j^{(1)}(1), x_j^{(1)}(2), \ldots, x_j^{(1)}(n)
\]

\[
x_j^{(1)}(k) = \sum_{l=1}^{m} a_{jl} z_l^{(1)}(k) \quad j=1,2,\ldots,m, k=2,3,\ldots,n
\]

The matrix form of the multivariable GM (1, m) model is

\[
dX^{(1)}(t)/dt = AX^{(1)}(t) + B
\]

Where

\[
X_j^{(1)}(t) = (x_j^{(1)}(t), x_j^{(2)}(t), \ldots, x_j^{(n)}(t))^T
\]

\[
A = (a_{ij})_{m \times m}
\]

\[
B = (b_1, b_2, \ldots, b_m)^T
\]

The response of formula (2) is

\[
X^{(1)}(t) = e^{A(t-1)}(X^{(1)}(1) + A^{-1}B) - A^{-1}B
\]

Discrete formula (2)

\[
x_j^{(0)}(k) = \sum_{l=1}^{m} a_{jl} z_l^{(1)}(k) + b_j \quad j=1,2,\ldots,m, k=2,3,\ldots,n
\]

The response of GM (1, m) is

\[
x_j^{(0)}(k) = 0.5(x_j^{(1)}(k-1) + x_j^{(1)}(k))
\]

From the least squares method

\[
\hat{a}_j = (\hat{a}_{j1}, \hat{a}_{j2}, \ldots, \hat{a}_{jm}, \hat{b}_j)^T = (\bar{p}^TP)^{-1}\bar{p}^T\bar{y}_j
\]

Where

\[
P = \begin{bmatrix}
z_1^{(1)}(2) & z_2^{(1)}(2) & \cdots & z_m^{(1)}(2) & 1 \\
z_1^{(1)}(3) & z_2^{(1)}(3) & \cdots & z_m^{(1)}(3) & 1 \\
\vdots & \vdots & \ddots & \vdots & \vdots \\
z_1^{(1)}(n) & z_2^{(1)}(n) & \cdots & z_m^{(1)}(n) & 1
\end{bmatrix}
\]

\[
\bar{y}_j = (x_j^{(0)}(2), x_j^{(0)}(3), \ldots, x_j^{(0)}(n))^T
\]

It can be obtained that the identification values of the parameter matrix A and the parameter vector B are

\[
\hat{A} = (\hat{a}_{ij})_{m \times m}, \hat{B} = (\hat{b}_1, \hat{b}_2, \ldots, \hat{b}_m)
\]

The response of GM (1, m) is

\[
\hat{X}^{(1)}(k) = e^{\hat{A}(k-1)}(X^{(1)}(1) + \hat{A}^{-1}\hat{B}) - \hat{A}^{-1}\hat{B}
\]
The model data is restored as follows

\[ \tilde{X}^{(0)}(k) = \tilde{X}^{(1)}(k) - \tilde{X}^{(1)}(k - 1) \]  

(7)

Where \( k = 2, 3, \ldots, n \).

3. Accuracy test of prediction model

The prediction error always exists when the state parameters of the power transformer are estimated by the prediction model, because the prediction model and the objective reality described by it will always be different. In order to measure whether a prediction model is reasonable and whether the prediction result is credible, there must be an evaluation standard, which is the accuracy test. The common precision test method in gray theory is the residual test. Let the original data sequence be \( X^{(0)} = (x^{(0)}(1), x^{(0)}(2), \ldots, x^{(0)}(n)) \), the corresponding model simulation sequence is \( \tilde{X}^{(0)} = (\tilde{x}^{(0)}(1), \tilde{x}^{(0)}(2), \ldots, \tilde{x}^{(0)}(n)) \), then

\[ \varepsilon^{(0)}(k) = \frac{x^{(0)}(k) - \tilde{x}^{(0)}(k)}{x^{(0)}(k)} \times 100\% \]  

(8)

is the residual of the gray model. The residual sequence is

\[ \varepsilon^{(0)} = (\varepsilon^{(0)}(1), \varepsilon^{(0)}(2), \ldots, \varepsilon^{(0)}(n)) \]

The average residual is

\[ \varepsilon(avg) = \frac{1}{n-1} \sum_{k=2}^{n} |\varepsilon(k)| \]  

(9)

4. Improved GM (1, m) prediction model

4.1. Data generation

In actual engineering applications, due to the influence of various factors, the collected raw data sequence may have certain volatility, but the traditional GM (1, m) model has certain limitations to deal with the volatility sequence. Sequences cannot meet the traditional requirements for smoothness. In order to better predict the original data sequence with volatility, the original data sequence is processed to make the processed sequence meet the smoothness requirements of the model.

There is a raw data matrix, \( X^{(0)} = \{ X_i^{(0)}, X_2^{(0)}, \ldots, X_m^{(0)} \}^T, X_j^{(0)} = (x_j^{(0)}(1), x_j^{(0)}(2), \ldots, x_j^{(0)}(n)), j=1,2,\ldots, m. \)

In order to make the original data sequence change exponentially, the sequence \( X_j(0) \) is transformed as follows:

\[ y_j^{(1)}(i) = \begin{cases} \sum_{k=i-1}^{i} (y_j^{(0)}(k))^2 & i \geq 3 \\ x_j^{(0)}(1) & i = 1 \end{cases} \]  

(10)

Where \( i = 1, 2, \ldots, n. \)

\( X_j(0) \) transform to get the sequence

\[ Y_j^{(0)} = (y_j^{(0)}(1), y_j^{(0)}(2), \ldots, y_j^{(0)}(n)) \]

The obtained sequence can be used as the original data sequence of the GM (1, m) model. After obtaining the predicted value, the corresponding inverse transformation can be performed.

4.2. Grey relational degree of GM (1, m) model variables

GM (1, m) model for fault prediction, it is necessary to clarify the relationship between various variables. For the processing of m original data sequences, the relationship between these sequences can be judged by the geometric similarity of the sequence curves. If the curves are closer, the correlation between the corresponding sequences is greater, and vice versa. This is the grey correlation analysis. Take \( X_1^{(0)}, X_2^{(0)}, \ldots, X_m^{(0)} \) as the parent sequence \( x_0(k) \) of gray correlation analysis in turn, and the remaining m-
Since the optimized gray differential equation is

\[ \text{4.4. Optimized background value modeling} \]

Let \( x_0'(k) = x_0(k)/x_0(1) \), then

\[ \xi_i(k) = \frac{\min_{t} m[k|x_0'(k) - x_i'(k)| + \rho \max_{t} \max_{k} x_0'(k) - x_i'(k)]}{|x_0'(k) - x_i'(k)| + \rho \max_{k} \max_{k} x_0'(k) - x_i'(k)} \]  

(11)

Is the gray correlation coefficient of the sequence \( x_0 \) and \( x_i \) at \( k \) points, \( \rho = 0.5 \), and the gray correlation degree between the sequence and the reference sequence is

\[ r_i = \frac{1}{m} \sum_{k=1}^{m} \xi_i(k) \]  

(12)

If \( r_i \geq 0.5 \), it can be considered that there is a correlation between the mother and child sequences.

### 4.3. Optimization of background value of GM (1, m) prediction model

Since the construction method of improving the background value of the model can improve the prediction accuracy of the model, optimizing the background value of the model is one of the important means to improve the prediction model. Let the original data matrix

\[ X^0 = \{ X_1^0(0), X_2^0(0), \ldots, X_m^0(0) \}^T \]

where \( X_j^0 = (x_j^0(1), x_j^0(2), \ldots, x_j^0(n)) \), \( j=1,2,\ldots, m \). Therefore, the optimized background value formula is[5]

\[ \bar{z}_j(k) = \int_{k-1}^{k} [b_j e^{a_j(k-1)} + c_j] \, dt = x_j^0(k)/a_j + c_j \]  

(13)

Finally get

\[ \bar{z}_j(k) = \frac{x_j^0(k)}{\ln x_j^0(k) - \ln x_j^0(k-1)} + x_j^0(1) + \frac{(x_j^0(k-1))^{k-1}}{(x_j^0(k))^k - (x_j^0(k-1)-x_j^0(k))} \]  

(14)

### 4.4. Optimized background value modeling

Since the optimized gray differential equation is[5]

\[ X_j^0(k) = \sum_{l=1}^{m} a_{jl} \bar{z}_j(k) + b_j \]  

(15)

Where

\[ j=1,2,\ldots, m, k=2,3,\ldots, n, \quad l=1,2,\ldots, m \]

Parameter column is

\[ \bar{a}_j = (\bar{a}_{j1}, \bar{a}_{j2}, \ldots, \bar{a}_{jm}, b_j)^T = (\bar{p}^T \bar{p})^{-1} \bar{p}^T \bar{y}_j \]  

(16)

Where

\[ \bar{p} = \begin{bmatrix} \bar{z}_1^1(2) & \bar{z}_2^1(2) & \cdots & \bar{z}_m^1(2) & 1 \\ \bar{z}_1^1(3) & \bar{z}_2^1(3) & \cdots & \bar{z}_m^1(3) & 1 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ \bar{z}_1^1(n) & \bar{z}_2^1(n) & \cdots & \bar{z}_m^1(n) & 1 \end{bmatrix} \]

\[ \bar{y}_j = (x_j^0(2), x_j^0(3), \ldots, x_j^0(n))^T \]

\[ \bar{A} = (\bar{a}_{ij})_{m \times m} \]

\[ \bar{B} = (\bar{b}_1, \bar{b}_2, \ldots, \bar{b}_m) \]

The time response of formula (15) is

\[ \bar{X}^1(k) = e^{\bar{A}(k-1)}(\bar{X}^1(1) + \bar{A}^{-1}\bar{B}) - \bar{A}^{-1}\bar{B} \]  

(17)

Restore formula is

\[ \tilde{X}^0(k) = \bar{X}^1(k) - \tilde{X}^1(k-1) \]  

(18)
5. Examples of improved GM (1, m) model fault prediction

5.1. Faults prediction

Using the improved GM (1,7) prediction model algorithm to analyze the oil chromatography data of multiple power transformers, it is verified that the improved GM (1,7) algorithm is superior to GM (1,1) and traditional in certain indicators GM (1,7). Examples of prediction are given below. Before the prediction, the original data sequence is first normalized to reduce the difference between the various gas levels and affect the prediction results.

Example 1 The oil chromatogram data monitored by the oil chromatograph online monitoring device of a 220kV substation No. 1 of Taiyuan Power Supply Company from October 17 to October 23, 2009 are shown in Table 1. The oil chromatographic data are monitoring data around 20 o'clock at night. In order to verify the prediction effectiveness of the improved multivariable GM (1,7) model, the first 5 data of the original data series are used for modeling, that is, m = 5, and the remaining data are used to verify the effectiveness of the prediction algorithm.

Table 1. Data of the dissolved gases in power transformer oil. μL/L

| Date    | CO  | CO₂ | H₂  | CH₄ | C₂H₆ | C₂H₄ | C₂H₂ |
|---------|-----|-----|-----|-----|------|------|------|
| 2009-10-18 | 649 | 3487 | 127 | 545 | 204  | 712  | 0.68 |
| 2009-10-19 | 674 | 3545 | 121 | 578 | 226  | 747  | 0.66 |
| 2009-10-20 | 653 | 3470 | 148 | 602 | 219  | 781  | 0.70 |
| 2009-10-21 | 654 | 3495 | 146 | 620 | 236  | 819  | 0.83 |
| 2009-10-22 | 650 | 3417 | 154 | 626 | 245  | 854  | 0.92 |
| 2009-10-23 | 641 | 3487 | 151 | 660 | 239  | 906  | 0.90 |

First, the gray correlation degree is used to analyze the closeness of the correlation between the seven gases. If the gray correlation coefficient rᵢ ≥ 0.5, it can be considered that there is a correlation between the mother and child sequences. Calculate the gray correlation of 7 gases, as shown in Table 2. It can be seen that the gray correlation coefficient between them is greater than 0.5, indicating that there is a coupling relationship between them.

Table 2. Grey relational degree of different gases.

| Gases | CO  | CO₂ | H₂  | CH₄ | C₂H₆ | C₂H₄ | C₂H₂ |
|-------|-----|-----|-----|-----|------|------|------|
| CO    | 1   | 0.9375 | 0.6117 | 0.6815 | 0.6562 | 0.6677 | 0.6222 |
| CO₂   | 0.9408 | 1   | 0.6280 | 0.6688 | 0.6501 | 0.6536 | 0.6344 |
| H₂    | 0.5283 | 0.5345 | 1   | 0.7614 | 0.7846 | 0.7752 | 0.6443 |
| CH₄   | 0.5989 | 0.5698 | 0.7550 | 1   | 0.7929 | 0.8455 | 0.5961 |
| C₂H₆  | 0.5655 | 0.5458 | 0.7782 | 0.7913 | 1   | 0.8219 | 0.6062 |
| C₂H₄  | 0.6321 | 0.6040 | 0.8021 | 0.8714 | 0.8495 | 1   | 0.7016 |
| C₂H₂  | 0.6321 | 0.6344 | 0.7278 | 0.6967 | 0.7053 | 0.7496 | 1   |

The first 5 sets of oil chromatographic data from October 18 to 22 were used for the improved GM (1,7) model modeling. The results are shown in Table 3. The first 5 data are simulated values, and the last one is the predicted value on October 23.
Table 3. Prediction of the dissolved gases in power transformer oil by improved GM (1, 7). μL/L

| Date       | CO  | CO₂ | H₂  | CH₄ | C₂H₆ | C₂H₄ | C₂H₂  |
|------------|-----|-----|-----|-----|------|------|-------|
| 2009-10-18 | 649 | 3487| 127 | 545 | 204  | 712  | 0.68  |
| 2009-10-19 | 673.5| 3536| 128 | 562.7| 219.9| 746.8| 0.70  |
| 2009-10-20 | 674.6| 3528| 138.9| 590.5| 227  | 780.5| 0.72  |
| 2009-10-21 | 670.5| 3524| 148.6| 618.2| 233.9| 814.4| 0.77  |
| 2009-10-22 | 664.6| 3524| 156.4| 645.9| 241.5| 848  | 0.82  |
| 2009-10-23 | 658.7| 3523| 162.4| 673.2| 250.1| 881  | 0.88  |

Table 4. Prediction of the dissolved gases in power transformer oil by GM (1, 7). μL/L

| Date       | CO  | CO₂ | H₂  | CH₄ | C₂H₆ | C₂H₄ | C₂H₂  |
|------------|-----|-----|-----|-----|------|------|-------|
| 2009-10-18 | 649 | 3487| 127 | 545 | 204  | 712  | 0.68  |
| 2009-10-19 | 671.4| 3524.7| 125.4| 577.5| 222.5| 746.5| 0.65  |
| 2009-10-20 | 659.2| 3495.3| 142.6| 600.7| 223.9| 782.9| 0.72  |
| 2009-10-21 | 662.4| 3454.9| 149.3| 608.3| 235.6| 822.1| 0.82  |
| 2009-10-22 | 661.4| 3385.4| 155 | 613.1| 246.9| 858  | 0.92  |
| 2009-10-23 | 655.7| 3285.5| 159.8| 615.0| 257.3| 890.0| 1.01  |

Table 5. Prediction of the dissolved gases in power transformer oil by GM (1, 1). μL/L

| Date       | CO  | CO₂ | H₂  | CH₄ | C₂H₆ | C₂H₄ | C₂H₂  |
|------------|-----|-----|-----|-----|------|------|-------|
| 2009-10-18 | 649 | 3487| 127 | 545 | 204  | 712  | 0.68  |
| 2009-10-19 | 661.27| 3499.5| 137.2| 598.2/1 | 227.6| 781.38| 0.73  |
| 2009-10-20 | 654.14| 3463.6| 146.61| 614.3| 235.1| 817.22| 0.82  |
| 2009-10-21 | 647.08| 3428.1| 156.67| 630.8³ | 242.84| 854.71| 0.92  |
| 2009-10-22 | 640.09| 3392.9| 167.42| 647.8| 250.84| 893.92| 1.03  |
| 2009-10-23 | 633.19| 3358.1| 178.91| 665.2/3 | 259.1| 934.93| 1.16  |

5.2. Precision inspection

The average residual error ε and the posterior relative error δ of the GM (1,1) and improved GM (1,7) models are calculated, as shown in Table 6. It can be seen from the comparison that the improved GM (1,7) model has an average residual error ε. Except that the prediction accuracy of H₂ is lower than the prediction accuracy of GM (1,7), the prediction accuracy of other gases is better than the other two predictions. model.

It can be seen from Figure 1 that in the calculation of the simulated values of the first 5 data, the accuracy of the GM (1,7) and improved GM (1,1) models is not much different, but in the prediction of the numerical value, the improved GM (1,7) model is significantly higher than the GM (1, 1) and GM (1, 7) models.
Table 6. Compared prediction accuracy of several prediction models

| Gases  | Improved GM(1,7) | GM(1,7) | GM(1,1) |
|--------|-----------------|---------|---------|
|        | ε%   | δ%  | ε%   | δ%  | ε%   | δ%  |
| CO     | 2.04 | 2.75 | 1.10 | 2.30 | 1.16 | 1.22 |
| CO₂    | 1.47 | 1.04 | 0.84 | 5.78 | 1.02 | 3.70 |
| H₂     | 3.80 | 7.58 | 2.55 | 5.81 | 7.59 | 18.48 |
| CH₄    | 2.00 | 2.00 | 1.07 | 6.82 | 2.69 | 0.79 |
| C₂H₆   | 2.17 | 4.65 | 1.18 | 7.65 | 3.34 | 8.41 |
| C₂H₄   | 0.34 | 2.75 | 0.29 | 1.76 | 4.57 | 3.19 |
| C₂H₂   | 6.87 | 2.02 | 1.45 | 12.47 | 12.40 | 29.3 |
| Average| 2.67 | 3.26 | 1.21 | 6.08 | 4.68 | 9.30 |

6. Conclusion
An improved GM (1,7) prediction model is proposed, which is used to predict the volume fraction of 7 dissolved gases in transformers.

(1) Transform the original data sequence of the traditional GM (1, 7) prediction model, so that the original data sequence after the transformation has better exponential properties, to meet the model's requirements for sequence smoothness, and can predict the volatility sequence. In order to improve the prediction accuracy of the model, the construction method of background value optimization is introduced to greatly improve the prediction accuracy of the model.

(2) Through example verification, the average residual ε% of the improved GM (1,7) model is smaller than that of other gray prediction models. Compared with the prediction results of traditional GM (1,1) and GM (1,7) The data sequence has a good approximation effect, indicating the effectiveness of the model. The superiority of the model is proved.

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