EFFECTS OF THE GEOMETRY OF THE LINE-FORMING REGION ON THE PROPERTIES OF CYCLOTRON RESONANT SCATTERING LINES

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ABSTRACT

We use a Monte Carlo radiative transfer code to examine the dependence of the properties of cyclotron resonant scattering lines on the spatial geometry and the optical depth of the line-forming region. We focus most of our attention on a line-forming region that is a plane-parallel slab threaded by a uniform magnetic field oriented at an angle Ψ to the slab normal. We also consider a cylindrical line-forming region with the magnetic field oriented along the cylinder axis. In both cases, the line-forming region contains an electron-proton plasma at the equilibrium Compton temperature, Tc, and the field strength is ~10^{12} gauss. We consider geometries in which the photon source illuminates the line-forming region from below and in which the photon source is embedded in the line-forming region. The former may correspond to a line-forming region in the magnetosphere of a neutron star, illuminated from below, the latter to a line-forming region on or near the surface of a neutron star as in an accretion column. We calculate the cyclotron line spectra produced by line-forming regions having a range of Thompson optical depths from τ_T = 8 \times 10^{-4} to τ_T = 10. Our findings have implications for accretion-powered pulsars and gamma-ray bursters. In particular, the absence of pronounced shoulders on both sides of the cyclotron first harmonic line in the spectra of accretion-powered pulsars suggests that the line-forming region is either illuminated from below or has a large optical depth (Thompson optical depth τ_T ≥ 10). However, we (like earlier workers) find that models in which the line-forming region is either a static slab or a static cylinder and has a large optical depth are unable to explain the modest equivalent widths W_e of the cyclotron lines in the observed spectra of accretion-powered pulsars. In addition, we find that approximating the injected photon spectrum as a Wien spectrum, an approximation made by almost all workers to date, is not valid because of the frequency dependence of the cyclotron scattering cross section below the cyclotron fundamental. Consequently, future work in this area should explore physical effects that have not been included so far, such as the plasma flow velocity in the accretion column and the injected photon spectrum resulting from energy deposition in the surface layers of the neutron star; more complicated geometries, such as accretion mounds and multiple component magnetic fields; and alternative models, such as cyclotron scattering in the magnetosphere. We find that slab line-forming regions in which the magnetic field is parallel to the slab are able to produce narrow lines with large equivalent widths W_e, suggesting that the lines observed in the X-ray spectra of some gamma-ray bursts might be able to be formed not only in plasma near the magnetic poles of a neutron star but also in plasma trapped at the magnetic equator of the star.

Subject headings: gamma rays: theory — line: formation — radiation mechanisms: nonthermal — radiative transfer — stars: neutron — X-rays: stars

1. INTRODUCTION

Cyclotron lines in an astrophysical X-ray spectrum are the clear signature of a superstrong magnetic field in the line-forming region. Consequently, they are an important clue to the nature of the source and the properties of the emission region, especially at energies ≥10 keV, where atomic lines are not available. Cyclotron lines have been important in identifying accreting magnetized neutron stars as the source of radiation in about a dozen X-ray pulsars (Mihara 1995; Makishima & Mihara 1992), including Her X-1 (Trümper et al. 1978), 4U 0115+63 (Wheaton et al. 1979), 4U 1538−52 (Clark et al. 1990), and A0535+26 (Grove et al. 1995). Absorption-like features have also been observed in classical gamma-ray bursts and interpreted as cyclotron lines (Mazets et al. 1980, 1981, 1996; Heuter 1984; Murakami et al. 1988; Fenimore et al. 1988; Harding & Preece 1989; Wang et al. 1989b; Alexander & Mészáros 1989; Lamb et al. 1989; Yoshida et al. 1991; Nishimura & Ebisuzaki 1992; Nishimura 1994; Briggs 1996; Briggs et al. 1998). Although the observation of spectral features in gamma-ray bursts and their interpretation continue to be controversial, they are perhaps the strongest evidence that at least some gamma-ray bursts come from strongly magnetized neutron stars associated with our own galaxy.

By building theoretical models of line-forming regions and comparing the emerging spectra with observations, theorists have been able to infer quantitatively many properties of the sources that are believed to possess cyclotron lines. For example, Mészáros & Nagel (1985) use a Fautrier calculation to show that the spectrum of Her X-1 could be produced in a region with magnetic field B_{12} = 8 \times 10^{-4} to τ_T = 10. Our findings have implications for accretion-powered pulsars and gamma-ray bursters. In particular, the absence of pronounced shoulders on both sides of the cyclotron first harmonic line in the spectra of accretion-powered pulsars suggests that the line-forming region is either illuminated from below or has a large optical depth (Thompson optical depth τ_T ≥ 10). However, we (like earlier workers) find that models in which the line-forming region is either a static slab or a static cylinder and has a large optical depth are unable to explain the modest equivalent widths W_e of the cyclotron lines in the observed spectra of accretion-powered pulsars. In addition, we find that approximating the injected photon spectrum as a Wien spectrum, an approximation made by almost all workers to date, is not valid because of the frequency dependence of the cyclotron scattering cross section below the cyclotron fundamental. Consequently, future work in this area should explore physical effects that have not been included so far, such as the plasma flow velocity in the accretion column and the injected photon spectrum resulting from energy deposition in the surface layers of the neutron star; more complicated geometries, such as accretion mounds and multiple component magnetic fields; and alternative models, such as cyclotron scattering in the magnetosphere. We find that slab line-forming regions in which the magnetic field is parallel to the slab are able to produce narrow lines with large equivalent widths W_e, suggesting that the lines observed in the X-ray spectra of some gamma-ray bursts might be able to be formed not only in plasma near the magnetic poles of a neutron star but also in plasma trapped at the magnetic equator of the star.
presence of a magnetic field approaching the critical field, $B_{\text{crit,12}} = 44.14$, from the spectrum of A0535+26. For gamma-ray bursts, Wang et al. (1989b) use a Monte Carlo radiative transfer code (Wang, Wasserman, & Salpeter 1988; Lamb et al. 1989) to show that the cyclotron lines in the spectrum of the burst GB880205 could be produced in a region with $B_{12} = 1.7, N_{e,21} = 1.2$, and with electrons at the equilibrium Compton temperature, $T_C$, where $kT_C = 5.3$ keV in this case.

Wang et al. (1989b) inject an initial photon distribution into a plane-parallel slab atmosphere from a source plane below the slab. Here we consider both this model and one in which the photons are injected at a source plane embedded inside the slab. A slab illuminated from below may correspond to a line-forming region in the magnetosphere of a neutron star (see Dermer & Sturmer 1991; Sturmer & Dermer 1994; see also Zheleznyakov & Serber 1994, 1995). A slab illuminated from within may correspond to a line forming region in a semi-infinite atmosphere at the stellar surface (see Slater, Salpeter, & Wasserman 1982; Wang, Wasserman, & Salpeter 1988, 1989a; Freeman et al. 1992).

Wang et al. (1989b) take the magnetic field to be oriented along the slab normal, $\hat{z}_S$ (i.e., $\Psi = 0$). Such fields are found near the magnetic poles of neutron stars with dipole fields. Kaminker, Pavlov & Shibanov (1983; 1982) and Burnard, Arons, & Klein (1991) consider the effects of varying the angle $\Psi$ between the magnetic field and the slab normal. However, these authors do not include the effects of scattering in the cyclotron line core (see §3); Burnard et al. (1991) calculate the spectrum in the continuum only, while the calculation of Kaminker et al. (1982, 1983) is valid for the continuum and the line wings.

In the present work, we consider scattering in line-forming regions with $\Psi \neq 0$, including core scattering. Fields at angles other than $\Psi = 0$ occur when lines are formed in structures near the magnetic pols, such as accretion columns or mounds (Burnard et al. 1991). Line-forming regions with $\Psi \neq 0$ also correspond to nonpolar regions on the surface or in the magnetospheres of neutron stars with dipole fields, as well as regions on stars with more complicated fields. For example, the surface of a neutron star may be threaded by local fields with various orientations relative to the local slab normal, instead of or in addition to a global dipole field (see, e.g., Ruderman 1991a, 1991b, 1991c; Lamb, Miller, & Taam 1996).

We will show that if the magnetic field is oriented perpendicular to the slab normal ($\Psi = \pi/2$) and the photon source is embedded in the slab, the line properties are similar to those for a cylindrical line-forming region. Thus a slab line-forming region with this geometry is a good approximation to the cylindrical line-forming region expected in the canonical model of accretion-powered pulsars.

Our Monte Carlo calculations cover a range of column depths, from $N_{e,21} = 6 \times 10^{-4} \tau_{T_0} = 4 \times 10^{-5}$ to $N_{e,21} = 15,000 \tau_{T_0} = 10$. The smaller and intermediate optical depths are appropriate for cyclotron scattering in layers of plasma suspended in the magnetospheres of neutron stars and for models of cyclotron line formation in gamma-ray bursts. The larger optical depths are appropriate for cyclotron scattering in an atmosphere on the surface and in the accretion columns of accretion-powered pulsars.

In §2 we discuss the geometry of the line-forming regions we use in our calculations. In §3 we describe the Monte Carlo radiative transfer code. We show how the equilibrium Compton temperature varies with the field orientation in §4. In §5 we present the spectra calculated by the Monte Carlo code and discuss the effects of the line-forming region geometry on the properties of the emergent lines. Finally, in §6 we discuss the implications of our calculations for accretion-powered pulsars and gamma-ray bursters.

2. SPATIAL GEOMETRY OF THE LINE-FORMING REGION

2.1. Slab Geometry

The principal spatial geometry we study in this paper, a plane parallel slab, is shown in Figure 1a. Photons are
injected at the source plane, travel through an electron-proton plasma, and emerge from one or the other of the faces of the slab. The horizontal extent of the slab is infinite. The angle \( \theta \) is the polar angle of a photon’s direction of propagation with respect to the magnetic field. The polar angle is usually specified in terms of \( \mu \equiv \cos \theta \). Similarly, \( \theta_{\parallel} \) and \( \phi_{\parallel} \) are the polar and azimuthal angles, respectively, of a photon with respect to the slab normal and \( \mu_{\parallel} \equiv \cos \theta_{\parallel} \).

As the figure shows, \( N_s \) is the column depth between the top of the slab and the source plane, while \( N_t \) is the column depth between the source plane and the bottom of the slab. In some circumstances, we will also specify the column depth in terms of the Thomson depth, \( \tau_{\text{Th}} \equiv N_e \sigma_T \), where \( \sigma_T \equiv (8\pi/3)[e^2/(m_e c^2)]^2 \) is the Thomson cross section. We stress that, in general, the optical depth encountered by a given photon will differ from the Thomson depth since the actual scattering cross section varies with photon energy, direction, and polarization.

We characterize the position of the source plane by the ratio of \( N_s \) to \( N_e \). Thus, for a slab illuminated from below, as in Wang et al. (1989b), the column depth between the source plane and the bottom of the slab is zero; we call this the 1-0 geometry. Similarly, a slab with the source plane embedded in the middle of the slab, so that \( N_s = N_e \), has a 1-1 geometry.

Slater et al. (1982) and Wang et al. (1988) determine that the mean number of scatterings between the source and the top edge that a resonant photon experiences prior to escape approaches a limiting value as the atmosphere becomes semi-infinite (i.e., as \( N_e/N_s \to \infty \)). It is generally within 10% of its limiting value in the 1-1 geometry and indistinguishable from its limiting value in the 1-4 geometry. The corresponding mean path length the photon travels between the source and the top edge displays similar behavior, as does the median frequency shift of the escaping photon. Thus the 1-4 geometry is an excellent approximation to a semi-infinite atmosphere.

Unfortunately, our Monte Carlo simulations show that the total number of scatterings per injected photon in the 1-4 geometry is approximately 3 times the number of scatterings in the 1-1. The computer time required for the simulation increases proportionately. Consequently, we use the 1-1 geometry in the present work as a compromise between the quality of the approximation to a semi-infinite atmosphere and the amount of computer time required. Our choice of the 1-1 geometry instead of the 1-4 has some effect on the calculated spectrum emerging from line-forming regions that are optically thin in the continuum (\( \tau_{\text{Th}} < 1 \)): the line shoulders (see § 5.3, below) are less prominent in the 1-1 geometry than in the 1-4. The calculated spectrum emerging from line-forming regions that are optically thick in the continuum (\( \tau_{\text{Th}} \gtrsim 1 \)) are identical in the 1-1 and 1-4 geometries, within the uncertainty of our Monte Carlo calculations.

We refer to photons that emerge from the top of the slab as “transmitted” and those that emerge from the bottom as “reflected.” The former reach the observer. In the 1-0 geometry, we assume that the latter return to the stellar surface where they are thermalized, i.e., absorbed by nonresonant inverse magnetic bremsstrahlung. In the 1-1 geometry, the reflection symmetry of the line-forming region across the source plane ensures that the transmitted and reflected spectra for isotropically injected photons are the same. We can, therefore, set the transmitted spectrum for isotropic injection equal to the sum of the transmitted and reflected spectra for semi-isotropic injection. By doing so, we can reduce the computer time required per transmitted photon by an additional factor of \( \sim 2 \) in the 1-1 geometry as compared with the 1-4 geometry.

### 2.2. Cylindrical Geometry

We also consider a cylindrical geometry, as shown in Figure 1b. The length of the cylinder is infinite. We consider only magnetic fields oriented parallel to the cylinder axis, as expected in the canonical model of the emission region of accretion-powered pulsars. Photons are injected along the cylinder axis and emerge from the surface of the cylinder. As in the slab geometry, the angle \( \theta \) is the polar angle of a photon’s direction of propagation with respect to the magnetic field. The column depth between the cylinder axis and the surface of the cylinder is \( N_c \).

### 3. MONTE CARLO RADIATIVE TRANSFER CODE

Our Monte Carlo radiative transfer code is an extended version of that used by Wang et al. (1989b). The code injects an initial photon distribution \( N(E) \) into an isothermal, fully ionized slab atmosphere with a uniform magnetic field at an angle \( \Psi \) to the slab normal. It then follows each photon through the slab and determines the emergent spectrum.

The code uses polarization-averaged cross sections and assumes that the cold plasma approximation \( (kT_e < E_b) \) applies. The cross sections are valid for \( n(E'/(m_e c^2))^{b-1} \ll 1 \), where \( n \) is the harmonic number, \( b = B/B_{\text{crit}} \), \( E \) is the photon energy, and the superscript \( \nu \) denotes quantities measured in the frame of reference where the electron is at rest \( (p_e = 0) \) prior to scattering (i.e., the prescattered electron’s rest frame; see, e.g., Daugherty & Ventura 1977). The code treats the first three harmonics, including photon spawning (production of lower harmonic photons from a higher harmonic photon) from resonant Raman scattering (Lamb et al. 1989; Wang et al. 1989b), and is valid for line forming regions that are optically thin in the line wings. For media optically thick in the line wings, we may still use the code if the photon spectrum falls off rapidly above the first harmonic so that photon spawning in the line wings is not important. Finally, we assume that the optical depth is small enough so that nonresonant inverse magnetic bremsstrahlung does not have a significant impact on the emerging spectrum. We discuss the cross sections and these approximations in greater detail in the following paragraphs.

The scattering cross sections we use in the Monte Carlo code are averaged over the initial polarization states and summed over the final states. Wang et al. (1988) argue that polarization averaged cross sections are appropriate for first harmonic scattering in optically thick media when the vacuum contribution to the dielectric tensor dominates the plasma contribution. This will be the case when

\[
\frac{w}{\delta} = 0.04 \left( \frac{n_e}{10^{20} \ cm^{-3}} \right) B_{12}^{-4} \ll 1 ,
\]

where \( w \equiv (h\omega_p/E_b)^2 \) is the plasma frequency parameter, \( \omega_p \) is the plasma frequency, \( E_b = 11.6 \text{ keV} B_{12} \) is the cyclotron energy, \( \delta \) is the magnetic vacuum polarization parameter (see, e.g., Adler 1971), and \( n_e \) is the electron number density. This condition will generally hold under the physical conditions studied in this paper.
We work in the cold plasma approximation, in which \( kT_e \ll E_B \). In this approximation, the Landau level spacing is much larger than the typical electron energy so the Landau levels are not collisionally populated. We further assume that the photon densities are sufficiently low so that the levels are not radiatively populated. Thus, in each scattering the initial and final electron state is the Landau ground state (\( n = 0 \)). We denote scattering channels by the sequence of Landau levels that the electron occupies, e.g., \( 0 \to 0, 0 \to 1 \to 0, \) and \( 0 \to 3 \to 2 \to 0 \).

In our treatment of electron-photon scattering, we adopt the approximation

\[
\left| \sum_i a_i \right|^2 \approx \left| a_{0 \to 0} + a_{0 \to 1 \to 0} \right|^2 + \sum_{i \neq 0, 0 \to 1 \to 0} |a_i|^2 ,
\]

where \( a_i \) is the matrix element for the \( i \)th scattering channel. In general, this approximation is a good one only near the line centers. Consequently, equation (2) is valid for line-forming regions that are not optically thick in the line wings of the first harmonic (see below; also Wasserman & Salpeter 1980; Lamb et al. 1989); that is

\[
\tau_1 \lesssim 1/a ,
\]

where

\[
\tau_1 = 100N_{e, 21}B_{12}^2 \left( \frac{kT_e}{1 \text{ keV}} \right)^{-1/2}
\]

is the polarization-, angle-, and frequency-averaged optical depth in the first harmonic;

\[
a \equiv \frac{\Gamma_1}{2E_j} = 1.8 \times 10^{-3} B_{12} \left( \frac{kT_e}{1 \text{ keV}} \right)^{-1/2}
\]

is the dimensionless natural line width;

\[
E_n^r = nE_B \sqrt{\frac{2kT_e}{m_e c^2}} = 0.73nB_{12} \left( \frac{kT_e}{1 \text{ keV}} \right)^{1/2} \text{ keV}
\]

is the Doppler width associated with the \( n \)th harmonic;

\[
\Gamma_n = \frac{4\pi x E_B^2}{3m_e c^2} = 2.6 \times 10^{-3} nB_{12}^2 \text{ keV}
\]

is the radiative width for the \( n \)th harmonic; and \( x \) is the fine structure constant. Even when the line-forming region is optically thick in the line wings, equation (2) is still a good approximation if the spectrum has an exponential rollover such that there are very few photons beyond the first harmonic. This is usually the case for the spectra of accretion-powered pulsars (Mihara 1995). Thus, for instance, for rollover energies of \( kT_e \approx E_B/4 \), the number of photons with energies significantly above the first harmonic is negligible. In this case only continuum and first harmonic scattering will be significant. However, if neither of these conditions hold, we cannot ignore the cross terms when squaring the matrix element, as we have in equation (2). For the most part, in the present work, we confine ourselves to cases where equation (2) is valid. However, we shall see below that when the magnetic field is parallel to the slab, photons traveling along the slab see a very large column depth and a large cross section and so can scatter multiple times with a wide range of electron velocities. Some of these photons can therefore continue to scatter far into the line wing above \( \sim E_B \), where equation (2) no longer holds. However, the range of magnetic field orientations and photon directions for which this occurs is small.

The scattering cross section we use is valid in the limit \( n(E'/(m_e c^2))^{-1} \ll 1 \). This limit applies throughout the present work. For the 0 \( \to 0 \) and 0 \( \to 1 \to 0 \) channels, these limits give the classical magnetic Compton scattering cross section. When evaluated in the prescattered electron's rest frame and averaged over the azimuthal angle \( \phi \), this cross section is given by (Canuto, Loddenkai, & Ruderman 1971; Herold 1979; Ventura 1979; Wasserman & Salpeter 1980; Harding & Daugherty 1991; Graziani 1993; first term on right-hand side of eq. [2])

\[
\frac{d\sigma'}{dQ_x} = \frac{3}{16\pi} \sigma_T \times \left[ \sin^2 \theta' \sin^2 \theta' \frac{E'}{E' + E_B} \left( 1 + \frac{\mu^2}{2} \right) \left( 1 + \mu'^2 \right) \right]
\]

is the continuum part of the cross section and

\[
\frac{d\sigma_{0 \to 1 \to 0}}{dQ_x} = \frac{9}{32} \sigma_T m_e c^2 \left[ \frac{4E_0^3}{E_B(E' + E_B)^2} \right] \times \\frac{\Gamma_1/2^2}{(E' - E_0)^2 + (\Gamma_1/2)^2} \times \left( 1 + \frac{2\mu^2 + 1 + \mu'^2}{2} \right)
\]

is the resonant part. We use the exact relativistic resonant energy for the \( n \)th harmonic \( E'_n = (2nbm_e c^2)/[1 + (1 + 2nb(1 - \mu^2)^{1/2})] \) with \( n = 1 \) in the Lorentzian factor in equation (10). The subscript \( s \) denotes parameters of the scattered photon.

For a 0 \( \to 0 \) scattering we obtain the total continuum scattering cross section \( \sigma_T \) by integrating equation (9) over the scattered angle. We define \( \sigma_n' \) as the total cross section for the resonant absorption that initiates a resonant scattering at the \( n \)th harmonic. For \( n \geq 1 \), this is given by (Daugherty & Ventura 1977; Fenimore et al. 1988)

\[
\sigma_n' = \frac{3}{8} \frac{\pi m_e c^2 \sigma_T}{\alpha} b_n^{-1} \left( \frac{n^2}{2} \right)^{n-1} \left( 1 + \mu^2 \right) \times \left( 1 - \mu^2 b^{-1} \right) \left( \frac{\Gamma_n'}{E' - E_0} \right)^{2} \left( 1 + \mu'^2 \right) \left( 1 + \mu'^2 \right)
\]

In order to achieve greater accuracy in the first harmonic line wings, we use the value of \( \sigma_1' \) obtained by integrating equation (10) over the scattered angle rather than using equation (11), which is strictly valid only near line center. The two expressions converge when \( E' \to E_1' \). For the higher harmonics, we use equation (11).

From the Lorentz invariance of \( \tau/|\mu| \) (see, e.g., Rybicki & Lightman 1979), the lab frame cross section \( \sigma_n \) is related to \( \sigma_n' \) by

\[
\sigma_n = (1 - \beta \mu) \sigma_n' .
\]

We average this cross section over \( f(p) \), the one-dimensional electron momentum distribution, and divide by \( \sigma_T \) in order
to obtain the dimensionless scattering profile for the \( n \)th harmonic,
\[
\phi_n(E, \Omega) = \frac{1}{\sigma_T} \int dp f(p) \phi_n(p).
\]

The total scattering profile is approximately the sum of the profiles for the individual Landau levels (including the continuum contribution):
\[
\phi(E, \Omega) \approx \frac{1}{\sigma_T} \int dp f(p) \int d\Omega_\|=0 \frac{d \sigma}{dE_\|=0} \approx \sum_{n=0}^{\infty} \phi_n(E, \Omega)
\]
(Wasserman & Salpeter 1980; Wang, Wasserman, & Lamb 1993).

Figures 2a, 2b, and 2c show the scattering profiles \( \phi_0(E, \Omega), \phi_1(E, \Omega), \) and \( \phi_0(E, \Omega) + \phi_1(E, \Omega) \) versus \( E \) for \( B_{12} = 3.5, kT_e = 10 \) keV, and \( \mu = 0, 0.5, \) and 1, respectively. It is clear from the figure that the line can be divided into the line core (\( |x_n/\mu| < 1 \)) and the line wings (\( |x_n/\mu| > 1 \)), where \( x_n = (E - E_n)/E_n \) is the dimensionless frequency shift (Wasserman & Salpeter 1980). In the line core, the thermal electron distribution dominates the profile so that \( \phi_n \propto \exp \left[ -\frac{E_n}{2kT_e} \right] \). In the wings, the tail of the Lorentzian distribution dominates so that \( \phi_n \propto d(x_n)^{-2} \). We refer to the wing at energies below the line center as the red wing and the wing at energies above the line center as the blue wing. Wasserman & Salpeter (1980) showed that, for the first harmonic, the core-wing boundary appears at \( |x_1/\mu| \approx 2.62 - 0.19 \ln(100\mu) \). Similarly, we define the wing-continuum boundary as the frequency shift where the profile for wing scattering along the slab normal is equal to the profile for continuum scattering. From equations (9) and (10), we see that for \( \Psi = 0 \) this boundary occurs for the first harmonic at \( |x_1| = 0.732 E_B/E_d^3 \) in the red wing and \( |x_1| = 2.73 E_B/E_d^3 \) in the blue (see Fig. 2c).

The dependence of the resonant cross section on photon energy is not strictly Lorentzian. There is a non-Lorentzian factor, \( 4E^3 E_B^{-1}(E' + E_B)^{-2} \), shown in square brackets in equation (10). In Figures 2d, 2e, and 2f, we compare \( \phi_0 + \phi_1 \), calculated with and without the non-Lorentzian factor, for \( \mu = 0, 0.5, \) and 1, respectively. The profile is unaffected at the line center where the non-Lorentzian

![Fig. 2.—Scattering profiles vs. photon energy for field strength \( B_{12} = 3.5 \) (\( E_B = 40 \) keV), \( kT_e = 10 \) keV, and incident photon direction cosine \( \mu = 0, 0.5, \) and 1. (a–c) \( \phi_0 \) (dotted lines), \( \phi_1 \) (dashed lines), and \( \phi_0 + \phi_1 \) (solid lines), including non-Lorentzian factor \( 4E^3 E_B^{-1}(E' + E_B)^{-2} \). (d–f) Comparison of \( \phi_0 + \phi_1 \) with (solid lines) and without (dot-dashed lines) non-Lorentzian factor.](image-url)
factor is equal to unity. However, this factor changes the profiles considerably in the wings. It raises the value of $\phi_l$ in the blue wing but ensures that $\phi_l \rightarrow 0$ as $E \rightarrow 0$. This has a significant effect on the total profile as $\mu \rightarrow 1$, where $\phi_0$ also goes to zero at low energies. It is clear from the figure that the non-Lorentzian factor can be ignored when the line wings are optically thin. Consequently, Wang et al. (1989b) did not include it in their simulations. But, as we shall show, the non-Lorentzian factor significantly alters the properties of radiation emerging from line-forming regions that are optically thick in the wings. We therefore include it in our calculations for large optical depths.

The code used by Wang et al. (1989b) required the magnetic field to be oriented along the slab normal ($\Psi = 0$). When this is the case, the line-forming region is azimuthally symmetric. Consequently, the original code needed only to keep track of the polar angle $\theta$ of a photon’s orientation; it ignored the azimuthal angle $\varphi$. When $\Psi \neq 0$, the symmetry is broken, as shown in Figure 1a. Thus, in the present work, we modify the code to keep track of both angles ($\theta$, $\varphi$) and bin the output accordingly. However, we continue to use cross sections that are averaged over the azimuthal angle $\varphi$. The $\varphi$-dependent part of the scattering cross section is only significant in the line wings and continuum; in the present work we consider line-forming regions that are either not optically thick in the wings or are azimuthally symmetric.

Lamb et al. (1989) showed that relativistic kinematics has a significant effect on the shape of the scattering profile, even in the limit $E, kT_c \ll m_e c^2$. We therefore use relativistic kinematics throughout this calculation, except where we indicate otherwise. For zero natural line width, relativistic kinematics prohibits scattering at the nth harmonic above a cutoff frequency $E_C = [(1 + 2n\beta)^2 - 1\beta^2/(1 - \beta^2)]^{1/2}$ (Daugherty & Ventura 1978; Harding & Daugherty 1991; see the Appendix of Wang et al. 1993 for a physical derivation). Wasserman & Salpeter (1980) show that under physical conditions where electron recoil is important, photons escape more readily in the red wing than in the blue. Since there was less of a focus on the blue wing, and for simplicity, Lamb et al. (1989) and Wang et al. (1989b, 1993) took the resonant scattering profile to be zero for $E > E_C$, ignoring the effects of the finite natural line width. For $E < E_C$, the effects of finite line width were included. The absence of resonant scattering above $E_C(\mu)$ gave rise to a spike at small $\mu$ just blueward of the first harmonic in some of the spectra generated by the original Monte Carlo code. This spike contained photons that were scattered to energies above $E_C$, either by scattering at the first harmonic or by photon spawning due to Raman scattering at the second or third harmonics, and then escaped the atmosphere without further scattering (the probability of continuum scattering is finite but very small in the thin slabs used). In the present work, we include the effect of finite natural line width for $E > E_C$, so that the resonant scattering profile is now small but finite above $E_C$. With this enhancement, the spikes are smeared by scattering and no longer appear. However, we stress that, even though the scattering profile is finite above the cutoff energy when the effects of natural line width are properly treated, the profile still falls off sharply above $E_C$, leading to a strongly asymmetric line shape at small $\mu$.

We do not include nonresonant inverse magnetic bremsstrahlung in our calculation. This is justified since we are interested in high photon energies ($\approx 1$ keV) and small column depths. Specifically, photons with energy $E$ originating from a depth $N_{e,21} > N_{e,21}^{th}$, where

$$N_{e,21}^{th} \approx 5.8 \times 10^4 \left( \frac{kT_c}{5 \text{ keV}} \right)^{1/3} \left( \frac{E}{20 \text{ keV}} \right)^{7/6}$$

(see, e.g., Nelson, Salpeter, & Wasserman 1993), will be thermalized before escape. For all cases we study in the present work, the column depth $N_{e,21} < N_{e,21}^{th}$.

4. SCATTERING ENERGETICS AND THE EQUILIBRIUM

**COMPTON TEMPERATURE**

When a photon scatters off an electron, energy is exchanged, either heating or cooling the atmosphere. The Compton equilibrium temperature $T_C$ is defined as the temperature at which the heating and cooling, summed over all scatters, balance exactly. For media optically thin in the continuum, the temperature is determined by resonant scattering. The resonant Compton temperature is reached on timescales that are short compared to most timescales of interest, such as the burst and dynamical timescales (see Lamb, Wang, & Wasserman 1990; hereafter LWW). We therefore calculate spectra for atmospheres at the resonant Compton temperature, except where we indicate otherwise. Consequently, it is important to understand how this temperature is affected by the geometry of the line-forming region.

To gain physical insight into the dependence of the resonant Compton temperature on the field angle $\Psi$, we first calculate $T_C$ analytically in the single scattering limit (valid for line-forming regions that are optically thin in the first harmonic). We present this calculation in § 4.1. In § 4.2, we present results for $T_C$ obtained from Monte Carlo simulations for line-forming regions that are optically thick in the line core, but not in the wings. We discuss line-forming regions that are also optically thick in the wings and the continuum in § 4.3.

4.1. Small Optical Depths

The single scattering analytic calculation of $T_C$ applies in the limit of column depths small enough that the medium is optically thin at the first harmonic. Our treatment is similar but not identical to the analytic treatment of LWW. In their treatment, LWW used the resonant single scattering power to calculate $T_C$. Thus, their treatment depended solely on the distribution of injected photons with respect to the magnetic field (i.e., on the direction of the nongrating component of the electrons’ velocities); it did not address the particular geometry of the scattering medium. In the present analytic treatment, we explicitly consider an optically thin slab geometry and study the dependence of $T_C$ on $\Psi$. In the evaluation of $T_C$, we assume an isothermal atmosphere that is threaded by a magnetic field whose strength is much less than the critical field $B_{crit}$ and where the electron temperature $T_e$ is much less than $m_e$ (we use $h = c = k = 1$ throughout § 4).

To calculate the Compton temperature, we require that the net power in scattered photons—that is, the scattered power minus the incident power—be equal to zero. Equivalently, the heating and cooling of the atmosphere by the plasma exactly balance at $T_C = T_e$ so that the total energy change of the photons $\Delta E$, which has been summed over scatters and averaged over the scattering photon distribu-
tion, is equal to zero. Working in the lab frame, we have
\[
\Delta E = \int \frac{d\Omega dE n(E, \Omega) \Delta E N_{\text{scat}}(\tau_{\text{f}}_E, E, \Omega)}{d\Omega dE n(E, \Omega)},
\]
where \(n(E, \Omega) dE\) is the scattering photon number density in the interval \((E, \Omega)\) to \((E + dE, \Omega + d\Omega)\) (assumed constant throughout the medium), \(N_{\text{scat}}(\tau_{\text{f}}_E, E, \Omega)\) is the total number of scatters experienced by a photon injected from position \(\tau_{\text{f}}_E\) inside the medium with energy \(E\) and direction of propagation \(\Omega\), and \(\Delta E\) is the mean energy change per electron-photon scattering.

Setting \(\Delta E = 0\) thus gives the slab Compton temperature (from the Appendix, eq. [A14]):
\[
T_{C} \frac{E}{E_B} = \frac{1}{10} \int \frac{(d\Omega/|\mu|)Q(\Omega)(2 + 7\mu^2 + 5\mu^4)}{(d\Omega/|\mu|)Q(\Omega)[1 + (s + 2)\mu^2 + (s - 3)\mu^4]}.
\]

The derivation is given in the Appendix. For the electron momentum distribution along the field, we used a nonrelativistic one-dimensional Maxwellian. To render the problem analytically tractable, we assume that the scattering photon number density is separable in energy and angle, viz. (from the Appendix, eq. [A8]),
\[
n(E, \Omega) = n(E)Q(\Omega)\]
with (from the Appendix, eq. [A12])
\[
s \equiv -E \frac{dE}{n(E) dE} \bigg|_{E = E_B}.
\]

In the single scattering limit, the photon density should remain separable and constant throughout the medium, to a good approximation. This is in contrast to the emergent spectra, which are, in general, different from the injected spectra even in this limit (see, e.g., Rybicki & Lightman 1979).

The most natural coordinate system to use to evaluate \(T_C\) is the slab coordinate system (see Fig. 1), that is, \(d\Omega = d\mu d\phi_{\delta}\), where
\[
\mu = \cos \Psi + \sqrt{1 - \mu^2} \sin \Psi \cos \phi_{\delta}.
\]

For injection along the slab normal, \(Q(\Omega) \propto \delta(\mu_{\delta} - 1)\), \(\mu = \cos \Psi\), and equation (A14) gives
\[
T_C \frac{E}{E_B} = \frac{1}{10} \left[ \begin{array}{c} 2 + 7 \cos^2 \Psi + 5 \cos^4 \Psi \\ 1 + (s + 2) \cos^2 \Psi + (s - 3) \cos^4 \Psi \end{array} \right].
\]

This reduces to the values found by LWW for injection along the field,
\[
\Psi = 0, \quad T_C \frac{E}{E_B} = \frac{7}{10s},
\]
and injection orthogonal to the field,
\[
\Psi = \pi/2, \quad T_C \frac{E}{E_B} = \frac{1}{5},
\]
as it must. If the initial photon distribution covers a range of angles, we expect the Compton temperature to fall between the two extremes given in equations (19) and (20). The \(1/|\mu|\) factor in equation (A14) originates from the slab geometry (i.e., eq. [A3]). It indicates that the dominant contribution comes from photons traveling at large angles to the slab normal, since these photons have the largest probability of scattering. However, equation (A14) is valid only when \(\tau/|\mu| \ll 1\) [see eqs. (A2), (A3), and (A5)], so the case of isotropic injection must be treated with some care. For first harmonic scattering,
\[
\tau(E = E_B, \Omega) \approx \frac{\tau_1}{\sqrt{4} 3 (1 + \mu^2)}.
\]
lower $T_C/E_B$ (for given $\Psi$) than runs without this channel. This is evident in Figure 3.

4.2. Moderate Optical Depths

At larger column depths, $\tau/|\mu_\parallel| > 1$ and equation (A3) are no longer valid. Consequently, we need to consider the effects of multiple scatterings, and we cannot use our analytic treatment to calculate $T_C$. Instead we use our Monte Carlo code to calculate the emerging photon spectrum and determine the amount of energy transferred to the photons while they pass through the slab. By varying the electron temperature, we can determine the temperature at which the net energy transferred is zero. Using this technique, LWW calculate the Compton temperature for semi-isotropic injection in a line-forming region with a 1-0 geometry and $\Psi = 0$, column depths in the range $0.12 \leq N_{e,21} \leq 12$, and magnetic fields in the range $1.50 \leq B_{12} \leq 2.10$. They find that $T_C/E_B \approx 0.27$. In the ranges considered, they find $T_C/E_B$ to be relatively insensitive to the magnetic field and to decrease slightly with increasing column depth.

In the present work, we fix the column depth and magnetic field and study the effects of the source plane position and the field orientation, as well as the effects of the higher harmonics and the natural line width. We use $B_{12} = 1.70$, $N_{e,21} = 1.2$, $\tau_{1} = 71(T_{e}/1\text{ keV})^{-1/2}$, and a semi-isotropically injected power law spectrum with $s = 1$. Figure 4 shows the Compton temperature $T_C/E_B$ as a function of the field orientation for both the 1-0 and 1-1 geometries. For $\Psi = 0$ and the 1-0 geometry, we find $kT_C/E_B \approx 0.30$, which is consistent with the results of LWW.

The temperatures in Figure 4 are generally higher than in the optically thin case. The increase is most dramatic when the field is parallel to the slab normal ($\Psi = 0$). To understand this, recall that in the optically thin case, almost all photons that scatter are moving perpendicular to the slab normal, and therefore have $\mu = \mu_\parallel \approx 0$ when $\Psi = 0$. However, in the optically thick case, there is an increase in the number of photons moving at larger values of $\mu$ before scattering, which raises the Compton temperature (see eqs. [19], [20]). The increase in high-$\mu$ photons that scatter is due to the increased optical depth for photons moving parallel to the field (along the slab normal), and the multiple scattering of photons that were injected perpendicular to the field (along the slab).

While the analytic model assumes zero natural line width and includes first harmonic scattering only, the calculations shown in Figure 4 assume finite natural line width and include scattering at the first three harmonics. As shown in Table 1, the effect of the finite line width and higher harmonics is small.

As we increase the column depth to $N_{e,21} = 12$ [$\tau_{1} = 71(T_{e}/1\text{ keV})^{-1/2}$], we find a small increase in Compton temperature, in contrast to the results of LWW. The difference is due to the effects of blue wing scattering. Because higher energy photons are able to scatter, the cooling of the electrons is less efficient, which raises the Compton temperature.

4.3. Large Optical Depths

In a closed system in thermal equilibrium, in which the photon density is low enough that stimulated scattering can be ignored, the photons have a Wien distribution with temperature $T = T_{e}$ (see, e.g., Rybicki & Lightman 1979). There is no net energy transfer between the photons and the electrons, and the photons have an isotropic angular distribution, i.e., $N(E) = 0$, where $N(E)$ is the net photon flux. We expect that, at optical depths that are large enough so that $\tau \gg 1$ and the flux to density ratio, $N(E)/[\phi(E)] \ll 1$, the photons behave approximately as in a closed system.

In Figure 4 we discussed scattering energetics in line-forming regions that are optically thick in the line core and optically thin in the wings and the continuum. In this section, we
examine the effects of larger optical depths by considering line-forming regions with $\Psi = 0$, $B_{12} = 3.5$, and $N_{e,21} = 15,000$ ($\tau_{T_0} = 10$). The line-forming region is optically thick in both the line wings ($\tau_L \gg 1$) and the continuum ($\tau_C \gg 1$). For comparison, we also consider a line-forming region with $N_{e,21} = 1,500$ ($\tau_{T_0} = 1$), which is optically thick in the wings but only marginally thick in the continuum, and a line-forming region with $N_{e,21} = 30$ ($\tau_{T_0} = 0.02$) that is marginally optically thick in the wings and optically thin in the continuum. All three line-forming regions are optically thick in the line core ($\tau_L \gg 1$). The key parameters for the runs are listed in Table 2. In addition, the mean number of scatterings experienced per escaping photon, $N_{\text{scat}}$, and the mean energy of escaping photons (averaged over the emerging photon distribution; see below) are listed.

To simulate the behavior of a system with large optical depth, we inject a Wien spectrum with $T_0 = 15,000$ and the 1-1 geometry, the mean energy of the transmitted photons is considerably smaller than the mean injected energy. For example, for $N_{e,21} = 15,000$ ($\tau_{T_0} = 10$) and the 1-1 geometry, the mean energy of the transmitted photons is 15.5 keV, compared to a mean energy of 3 keV for the injected photons. Most of the energy redistribution occurs deep in the slab, as shown in Figure 3. Here we plot $\langle E_T(\tau_1) \rangle$, the mean photon energy associated with the net (upward) flux crossing a given plane inside the slab and is calculated from

$$\langle E_T(\tau_1) \rangle = \left[ \int_0^{\infty} dE E \left[ N_{\text{up}}(E, \tau_1) - N_{\text{down}}(E, \tau_1) \right] \right].$$

At the edge of the atmosphere, $N_{\text{down}}(E, \tau_T = 0) \equiv 0$, and $\langle E_T(\tau_T = 0) \rangle$ becomes simply the mean energy of escaping photons averaged over the emergent photon distribution (see last column in Table 2).

It is evident from this figure that an optical depth of $\tau_T = 10$ is not sufficient for thermal equilibrium between the electrons and the photons. On the contrary, as the photons move upward they steadily lose energy to the electrons until they reach a depth of about $\tau_T = 7$. Above this depth, the shape of the spectrum is altered by multiple scattering, but the mean energy of the distribution remains the same until the photons get close to the surface. Above $\tau_T \approx 0.004$, the

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### TABLE 1

| Channels Open | Natural Line Width | $T_C/E_B (\Psi = 0)$ | $T_C/E_B (\Psi = \pi/2)$ |
|---------------|--------------------|---------------------|------------------------|
| First three harmonics | Finite | 0.299±0.003 | 0.425±0.007 |
| 0→1→0 only | Finite | 0.274±0.003 | 0.350±0.010 |
| 0→1→0 only | Zero | 0.262±0.005 | 0.358±0.009 |

### TABLE 2

| $N_{e,21}$ | 0→0 Channel Open | $\tau_{T_0}$ | $\tau_1$ | $\sigma_1$ | $N_{\text{scat}}$ | $\langle E_T(\tau_T = 0) \rangle$ (keV) |
|------------|------------------|---------------|-----------|-----------|------------------|-----------------------------------|
| 30 .......... | Yes              | 0.02          | 280       | 0.56      | 39                | 26.2                             |
| 1500 ....... | Yes              | 1             | 14000     | 28        | 3280              | 19.3                             |
| 15000 ...... | Yes              | 10            | 140000    | 280       | 640000            | 14.3                             |

### 1-1 Geometry

| $N_{e,21}$ | 0→0 Channel Open | $\tau_{T_0}$ | $\tau_1$ | $\sigma_1$ | $N_{\text{scat}}$ | $\langle E_T(\tau_T = 0) \rangle$ (keV) |
|------------|------------------|---------------|-----------|-----------|------------------|-----------------------------------|
| 30 .......... | Yes              | 0.02          | 280       | 0.56      | 435              | 28.8                             |
| 1500 ....... | Yes              | 1             | 14000     | 28        | 223000           | 24.6                             |
| 15000 ...... | Yes              | 10            | 140000    | 280       | 233000           | 15.5                             |
| 150000 ...... | Yes              | 10            | 140000    | 280       | 806000           | 24.7                             |
| 1500000 ...... | Yes              | 10            | 140000    | 280       | 173000           | 16.4                             |

**Note.**—For all simulations, $B_{12} = 3.5$, $\Psi = 0$, and the injected photons have a Wien distribution with temperature $kT_0 = kT_T = 10$ keV.

* Number of scatterings per transmitted photon.

* Cross section does not include non-Lorentzian factor.
line wings are optically thin and there is a slight increase in $<E_s(\tau_f)>$.

The reason that thermal equilibrium does not occur at $\tau_T = 10$ is apparent from equation (9) and Figure 2c. For $E \ll E_B$, the scattering profile for a $\mu = 1$ photon decreases with energy like $(E/E_B)^2$. Thus, at any depth $\tau_T$, there exists some energy, $E_{\text{th}}(\tau_T)$, such that the plasma appears optically thin to $\mu = 1$ photons with $E < E_{\text{th}}(\tau_T)$. If $\tau_T$ is large enough so that $E_{\text{th}} < E_B$, then, from equations (8), (9), and (10),

$$E_{\text{th}}(\tau_T) = \left( \frac{2 + \tau_T + \sqrt{8 \tau_T + \tau_T^2}}{2} \right)^{-1/2}. \quad (24)$$

Because of the strong angle and frequency dependence of the cross section in a strong magnetic field, the use of the Thomson depth to indicate optical depth in the continuum is misleading. Even at $\tau_T = 10$, a significant number of photons can escape without scattering.

The energy shift in Figure 5 occurs because photons diffuse in angle and frequency space until their angle with the magnetic field is small and $E \ll E_{\text{th}}(\tau_T)$. They are then able to escape without further scattering. The importance of this redistribution is illustrated by comparing the cumulative spectrum of the emerging photons with that of the injected photons, as shown in Figure 6. As the figure shows, even though only 14% of the photons are injected with $E < E_{\text{th}}(\tau_T) = 12.9$ keV, 45% escape below this energy. This effect could be even more pronounced if we did not use polarization-averaged cross sections; at low energies, the scattering profile for extraordinary mode photons goes like $(E/E_B)^2$, regardless of the direction they are traveling.

To test the hypothesis that the shift in the photon energy is due to the form of the scattering cross section at low $E$, we ran a 1-1, $N_{\text{e,21}} = 15,000$ ($\tau_T = 10$) simulation in which the non-Lorentzian factor was removed from the scattering cross section. As Figure 2f shows, without this factor, the scattering profile approaches $\sigma_T$ as $E \to 0$, so the line-forming region is no longer optically thin to low energy photons. As shown in Table 2, $<E_s(\tau_f)> = 24.7$ keV in this simulation, much larger than in the simulation with the non-Lorentzian factor included. In addition, without the non-Lorentzian factor, the ratio $N(E)/(cn(E))$ remains much smaller than unity through most of the slab, as expected for a thermal distribution of photons. Clearly the detailed behavior of the scattering cross section plays an important role in forming the redshifted spectrum of the escaping photons.

The transfer of energy from the photons to the electrons deep in the slab implies that the electron temperature $T_e$ is $E_B/4$ assumed in the calculation is less than the Compton temperature at large depths. In this situation, the photons will heat the electrons until $T_e$ is reached. However, as Figure 5 indicates, $T_e$ already equals $T_C$ above $\tau_T \approx 7$. The shape of the curve indicates that a self-consistent calculation requires a temperature profile, $T_e(\tau_T)$, rather than a constant value of $T_e$ throughout the slab. This result is consistent with the finding of Bulik (1993) that $T_e$ rises below $\tau_T \approx 7$ (see also Nagel 1981; Miller, Wasserman, & Salpeter 1989). Bulik (1993) calculates that the rise in temperature is small: $\lesssim 15\%$. The small size of the temperature variation, combined with the dependence of the first harmonic optical depth $\tau_1$ and Doppler width $E_p$ on the square root of the temperature, suggest that the effect on the emerging spectrum is also small.

5. MONTE CARLO SPECTRA

Our Monte Carlo simulations reveal a rich variety of line properties that vary with the viewing angle, the magnetic field orientation, the optical depth, and the slab geometry. For line-forming regions that are optically thick in the line core, but optically thin in the line wings and the continuum, we discuss the line shapes in § 5.1 and the equivalent widths in § 5.2. These simulations reveal prominent shoulders on
both sides of the first harmonic line when the radiation is viewed at $\mu_{\text{sl}} \gtrsim 0.25$, regardless of the field orientation. We discuss the line shoulders in § 5.3. In § 5.4 we consider line-forming regions that are optically thick in the line wings and the continuum, in addition to the core.

5.1. Line Shapes for Moderate Optical Depths

Figures 7, 8, and 9 show Monte Carlo scattering spectra for $\Psi = 0$, $\Psi = \pi/4$, and $\Psi = \pi/2$, respectively. In all three figures, the magnetic field is $B_{1,2} = 1.7$ and the column depth is $N_{e,21} = 1.2$, which is optically thick in the line core [$\tau_1 = 71(kT_C/1 \text{ keV})^{-1/2}$] and optically thin in the wings [$\tau_1 = 0.22(kT_C/1 \text{ keV})^{-1}$]. We include results for both the 1-0 and 1-1 geometries and for several viewing angles. To produce the $\Psi = 0$ spectra, we inject a total of 1,000,000 photons into the slab isotropically and record the photons emerging from the slab in one of eight bins in $\mu_{\text{sl}}$ ($0 < \mu_{\text{sl}} \leq 1$). To produce the $\Psi \neq 0$ spectra, we inject 8,000,000 photons and record the emerging photons in 80 angular bins (10 bins in $\phi_{\text{sl}}$ between 0 and $\pi$ for each of 8 bins in $\mu_{\text{sl}}$). In each panel the 1-1 spectrum is normalized to have unit area and the 1-0 and pure absorption spectra are normalized to match the 1-1 spectrum in the continuum.

For comparison, we also show pure absorption spectra,

$$N_{\text{abs}}(E, \Omega_{\text{sl}}) = N(E) \exp \left( - \frac{N_e \sigma_T \phi}{|\mu_{\text{sl}}|} \right),$$

where the scattering profile $\phi$ is given by equation (14). $N_{\text{abs}}(E, \Omega_{\text{sl}})$ is the spectrum in which photons that have undergone the absorption that initiates a resonant scatter are not reemitted. We can explain many of the properties of the spectra in terms of the scattering profile and the geometry of the line-forming region.

The second and third harmonics have shapes similar to absorption lines because most of the photons that scatter at these energies are Raman scattered—i.e., they are absorbed and then reemitted as 2 or 3 lower harmonic photons. The line properties of the higher harmonic features can therefore be understood in terms of equation (25) and the scattering profiles. To illustrate this, we combine equations (11), (12), (13), and (A13) and let $\Gamma_n \to 0$ and $\beta \ll 1$ to get the nonrelativistic resonant scattering profile for the $n$th harmonic with

![Fig. 7 - Spectra for $\Psi = 0$ and several viewing angles. $B_{1,2} = 1.7$ and $N_{e,21} = 1.2$ [$\tau_1 = 71(kT_C/1 \text{ keV})^{-1/2}$]. Monte Carlo spectra for the 1-1 (solid lines) and 1-0 (dotted lines) geometries are shown, as well as relativistic absorption spectra with finite natural line width (dashed lines).]
Fig. 8.—Spectra for $\Psi = \pi/4$ and several viewing angles. $B_{12} = 1.7$ and $N_e/\mu_s = 1.2$ [$\chi = 71(kT_e/1\text{ keV})^{-1/2}$]. Monte Carlo spectra for the 1-1 (solid lines) and 1-0 (dotted lines) geometries are shown, as well as relativistic absorption spectra with finite natural line width (dashed lines).

It is evident from equations (26) and (25) that while the Doppler factor $E_{\nu}^{\mu} \mu$ broadens the lines when viewed along the field, the line-of-sight column depth $N_e/\mu_s$ deepens the lines when viewed along the slab. Thus, when the magnetic field is parallel to the slab normal ($\Psi = 0$), the lines become deeper and narrower as the viewing angle moves from $\mu_s = 1$ to $\mu_s = 0$ (Fig. 7). When the field is perpendicular to the slab normal however ($\Psi = \pi/2$), these two effects combine to provide especially broad and deep lines when viewed perpendicular to the slab normal and at modest angles to the magnetic field (e.g., Fig. 9k). When the viewing angle is directly along the field (e.g., Figs. 7a and 9j), the scattering profile for the higher harmonics is dominated by the $(1 - \mu^2)^{-1}$ factor and the higher harmonics are suppressed.

The properties of the first harmonic are determined by multiple scatterings and photon spawning. Consequently, there is no simple analytic expression that describes the first harmonic scattering line. However, though the physics of line formation are fundamentally different, the full width at
half-maximum of the first harmonic scattering line tends to be similar to that of the first harmonic absorption line. Thus, equations (25) and (26) do provide some insight into the first harmonic. Up to a logarithmic factor of \((\ln \epsilon_1)^{1/2} \sim 1\), the Doppler width dominates the width of the first harmonic line. This is because \(\delta E/E \ll 1\) in core scattering, and there is negligible frequency redistribution in the line wings for the cases we study in Figures 7, 8, and 9.

5.2. Line Equivalent Widths for Moderate Optical Depths

The equivalent width of the first harmonic line is plotted as a function of viewing angle in Figures 10, 11, and 12. In calculating the first harmonic equivalent width, we need to adjust for any overlap between the first and second harmonic lines in the emergent spectra. For column depths \(N_{e,21}/|\mu_1| \gtrsim 6(kT_e/1 \text{ keV})^{1/2}\), the line wings of the first harmonic can have optical depths \(\gtrsim 1\) so frequency redistribution in the line wings can be significant. Thus, at large values of \(N_{e,21}\) or small values of \(|\mu_1|\), the first and second harmonics begin to overlap (see, e.g., Fig. 9). To correct for this, we assume that the second harmonic is approximately an absorption line and we calculate the equivalent width of the first harmonic according to

\[
W_{E_1}(\Omega_a) = \int \frac{N_g(E)g_1(E, \Omega_a) - N(E, \Omega_a)}{N_g(E)g_2(E, \Omega_a)} dE, \quad (27)
\]
where \( N(E, \Omega_{\alpha}) \) is the transmitted spectrum and \( g_{\alpha}(E, \Omega_{\alpha}) \) is as defined in equation (25) with \( \phi \to \phi_{\alpha} \). We stress that this formula assumes that the spectrum \( N(E, \Omega_{\alpha}) \) is accurate and merely corrects for the overlap in the calculation of \( W_{E1} \). It does not correct for the inaccuracy in our modeling of the transfer through a scattering medium that could be optically thick in the line wings (see eq. [2] and associated discussion).

We display \( W_{E1} \) for slabs with \( \Psi = \pi/2 \) in Figure 10 for the 1-0 geometry and in Figure 11 for the 1-1 geometry. \( B_{1,2} = 1.7 \) and \( N_{e,21} = 1.2 \) in both figures \([\epsilon_1 = 71(kT_{C}/1 \text{ keV})^{-1/2}]\). For viewing along the slab (small \( \mu_{\alpha} \)), the equivalent width decreases as the azimuth \( \varphi_{\alpha} \) moves from 0 to \( \pi/2 \). This is because the scattering profile for the first harmonic, unlike the higher harmonics, has no \( 1 - \mu^2 \) factor that causes the cross section to vanish along the field. Photons propagating along the field see a larger cross section and Doppler width, which tends to scatter them out of the line of sight, while photons propagating orthogonal to the field see a reduced scattering cross section and Doppler width, which reduces the chance of scattering. As \( \mu_{\alpha} \) increases, the equivalent width decreases because photons escape more easily in directions transverse to the slab where the column depth is lower.

We display \( W_{E1} \) for slabs with \( \Psi = 0 \) and compare the results with those for \( \Psi = \pi/2 \) in Figure 12. The figure also illustrates the dependence of equivalent width \( (W_{E1}) \) on column depth. The figure shows \( W_{E1} \) as a function of \( \mu_{\alpha} \) for \( B_{1,2} = 1.7 \) and \( N_{e,21} = 0.12, 1.2, \) and \( 12 \) \([\epsilon_1 = 71, 71, \) and \( 710 \times (kT_{C}/1 \text{ keV})^{1/2}]\), and for both the 1-0 and 1-1 geometries. The azimuthal angle is \( 0 < \varphi_{\alpha} < \pi/8 \) for \( \Psi = \pi/2 \) and \( 0 < \varphi_{\alpha} < 2\pi \) for \( \Psi = 0 \). The figure shows that the geometry can have a larger effect on the equivalent width than the column depth does. For all geometries, the equivalent widths are largest when the spectrum is viewed along the slab (\( \mu_{\alpha} \to 0 \)). In the 1-0 geometry with \( \Psi = 0 \), increasing the column depth two orders of magnitude from \( N_{e,21} = 0.12 \) to 12 increases the width at \( 0 < \mu_{\alpha} < 1/8 \) from \( W_{E1}/E_B = 0.096 \) to 0.39. However, keeping \( N_{e,21} \) at 0.12 but rotating the field so it lies along the slab increases the equivalent width in this \( \mu_{\alpha} \) bin from \( W_{E1}/E_B = 0.096 \) to 0.64.

As \( \mu_{\alpha} \) approaches unity, the equivalent width decreases and can become negative. This is especially true for the 1-1 geometry. A negative equivalent width corresponds to an emission-like feature. The presence of such a feature in an angular bin requires a surplus of photons emerging in the bin compared with the number of photons that were injected. In Figures 7, 8, and 9, such features appear as shoulders on either side of the line center. The shoulders are very prominent in the 1-1 geometry, but less so in the 1-0. For example, as we see in Figure 12d, \( W_{E1}/E_B = -0.96 \) in the 7/8 < \( \mu_{\alpha} \) < 1 bin in the 1-1 geometry, but only \( -0.066 \) in the 1-0. Physically, these shoulders are the result of angular and frequency redistribution in electron-photon resonant scattering.

5.3. Line Shoulders

Many authors discuss line shoulders. Wasserman & Salpeter (1980) predict that for physical conditions under which electron recoil is important (e.g., accretion-powered pulsar line-forming regions), there is an excess of photons escaping in the red wing compared with the blue. Alexander & Mészáros (1989), Nishimura & Ebisuzaki (1992), and Nishimura (1994) report shoulders in spectra generated by Feautrier calculations with 1-0 geometries and \( N_{e,21} \) varying from \( \sim 0.1 \) to \( \sim 100 \). The shoulders reported by Alexander & Mészáros (1989) are small, in agreement with our 1-0 results. In contrast, Nishimura (1994) finds very large shoulders with \( W_{E1}/E_B < -10 \) in some cases. Further, he reports that the shoulders are most prominent at small...
in direct contradiction to our results. Araya & Harding (1996) also report shoulders, which they observe in spectra generated by a Monte Carlo code for both 1-1 and cylindrical geometries with column depths up to \( N_{\text{e},21} \sim 5 \). In this section, we develop a better understanding of line shoulders by investigating how they are affected by viewing angle, spawning, electron temperature, and source plane position.

Chandrasekhar (1960) shows that photons that scatter isotropically in a slab atmosphere emerge disproportionately at high \( \mu_{\|} \) because of the lower column depth along the slab normal. The tendency of photons to emerge at high \( \mu_{\|} \) is the reason the shoulders in the present calculation become more prominent as \( \mu_{\|} \rightarrow 1 \). The shoulders are weak or nonexistent for low \( \mu_{\|} \) because the enhanced column depth along the slab ensures that most photons are scattered out of the line of sight, resulting in the formation of absorption-like features. Figure 13 displays Monte Carlo calculations of the emergent angular distribution of resonantly scattered photons. The photons are injected monochromatically at \( E = E_B \) so that every photon scatters. As the figure shows, the number of photons emerging generally increases with \( \mu_{\|} \), even though they were injected isotropically. This is true for both the 1-0 and 1-1 geometries and for fields both parallel and perpendicular to the slab normal. In the 1-0 geometry (Figs. 13a and 13c), there is an excess of reflected photons over transmitted photons because of the shorter path length for the former. This contrasts with the 1-1 geometry (Figs. 13b and 13d), where the line forming region is symmetric about the source plane. There is no short escape path and the transmitted and reflected spectra are (by symmetry) identical. In both geometries, few photons escape along the slab owing to the large path length. In the \( \Psi = 0 \) case, escape along the slab

![Figure 12](image-url)
normal is favored by both the short path length and by a scattering cross section that is largest for a scattered photon direction \( \mu_s = 1 \). The emergent angular distribution peaks at \( \mu_{sl} = 1 \) (see Figs. 13a, 13b). By contrast, in the \( \Psi = \pi/2 \) case, escape along the slab normal, while favored by the shorter path length, is discouraged by the scattering cross section. The emergent angular distribution is, therefore, peaked in a direction determined by a compromise between the most favored scattered angle \( (\mu_s \rightarrow 0) \) and the shortest path length \( (\mu_{sl} \rightarrow 1) \) (see Figs. 13c and 13d).

Figures 14 and 15 show spectra emerging from line-forming regions threaded by a field of strength \( B_{1,2} = 1.7 \) and \( \Psi = 0 \) and \( \pi/2 \), respectively. Both figures use column depth \( N_{\delta,21} = 1.2 \) \([\tau_1 = 71(kT_e/1 \text{ keV})^{-1/2}]\). The injected photon spectrum is \( \propto 1/E \). When the viewing angle is perpendicular to the slab normal, the scattered spectrum is similar to a pure absorption spectrum. When the viewing angle is parallel to the slab normal, shoulders can appear. It is evident from Figures 14a, 14b, 15a, and 15b that spawning enhances the shoulders by providing an additional source of first harmonic photons. However, the shoulders are prominent in the 1-1 geometry even when spawning is not included in the calculation. This is because photons injected initially into the bottom one-half of the slab in the 1-1 geometry can escape through the top one-half after multiple scatters thereby providing an effective additional source of first harmonic photons (see Fig. 17 below and associated discussion).

Figures 14 and 15 also reveal that the spacing of the line shoulders is of the same order as the width of the corresponding pure absorption lines. This simply reflects the fact that in media that are optically thin in the line wings the shoulder spacing is given approximately by the Doppler width \( 2E^1_e \mu_{esc} \) times a factor of \( \sim (\ln \tau_1)^{1/2} \sim 1 \) as a result of multiple scatters (see Osterbrock 1962; Wang et al. 1988).

The relative size of the red and blue shoulders is determined by the relationship between the electron temperature and the Compton temperature. At the Compton tem-
temperature, by definition, the photons on average lose as much energy as they gain. We therefore expect the area under the two shoulders in the photon energy spectrum to be approximately equal when $T_e = T_C$.

As we see in Figures 16c and 16d, this is the case for a 1-1 slab atmosphere with $B_{12} = 1.7$, $N_{e,21} = 1.2$ [$\tau_1 = 71(kT_C/1 \text{ keV})^{-1/2}$], $\Psi = 0$ and $\pi/2$, and a $1/E$ injected photon number spectrum. In order to illustrate the total change in energy of the photons, we use angle-integrated spectra in this figure. For $T_e < T_C$, the photons on average lose more energy than they gain and the red shoulder is larger (Figs. 16a and b). This result is consistent with Alexander & Mészáros (1989), who use a temperature (5.2 keV) that is below the Compton temperature and report a slightly greater flux in the red shoulder over the blue. For $T_e > T_C$, we find that the blue shoulder is larger than the red, as expected (Figs. 16c and 16d).

The line shoulders are more prominent in the 1-1 geometry than in the 1-0 at modest optical depths because of an excess in the flux of photons near the line center in the 1-1. We illustrate this for the case of $\Psi = 0$, $B_{12} = 1.7$, and $N_{e,21} = 1.2$ [$\tau_1 = 71(kT_C/1 \text{ keV})^{-1/2}$]. Figure 17 shows the angle-integrated flux of photons moving upward ($\mu_{\text{al}} > 0$) as a function of energy at four points in the slab: $\tau_f/\tau_{\text{to}} = 1.00$ (the source plane), 0.32, 0.02, and 0.00 (the upper surface). At the source plane, the excess of photons in the 1-1 geometry takes the form of a prominent peak (Fig. 17d). The reason for the peak is that as line photons scatter, they can cross the source plane many times. The peak contains photons that were injected toward the bottom of the slab ($\mu_{\text{inj}} < 0$) but subsequently scattered upward and crossed the source plane (an odd number of times, in general). The peak also contains photons that were injected toward the top of the slab ($\mu_{\text{inj}} > 0$) but that, in the course of multiple scatters, cross the source plane twice (or, in general, an even number of times)—once downward and once upward. In other words, the photon flux in the 1-1 geometry, like the emergent spectrum, is the sum of transmitted and reflected components (see § 2). The two components contribute about equally to the peak. The peak does not appear in the 1-0

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**Fig. 14.**—Shoulder formation for $\Psi = 0$. Spectra for absorption (dashed lines) and scattering (dotted lines) from the first harmonic only are shown. When higher harmonics (and spawning) are included (solid lines), the shoulders are enhanced.
geometry because photons that are traveling downward from the source plane have escaped the slab and are part of the reflected spectrum.

The peak is responsible for the prominence of the shoulders in the 1-1 geometry. As the photons in the peak move upward through the slab their frequencies are redistributed in multiple scatterings—they are scattered out of the line—and shoulders form. Shoulders are diminished in the 1-0 geometry, compared to their appearance in the 1-1, because there is no photon excess near the line center. Note also the formation of the higher harmonic features, which, being effectively true absorption features, is essentially independent of geometry.

5.4. Large Optical Depths

In §§ 5.1, 5.2, and 5.3 we discussed the spectra emerging from line forming regions with a moderate column depth of $N_{e,21} = 1.2 (\tau_{T_0} = 0.0008)$. However, the emission regions of accretion-powered pulsars are thought to have column depths $N_{e,21} \sim 10^3-10^4$. In this section, we discuss our Monte Carlo results for the emerging spectra for slabs with column depths in this range. Table 2, above, lists the key parameters for these simulations.

Figure 18 shows the angle-integrated flux of photons moving upward ($\mu_s > 0$) as a function of energy at four points in the slab: $\tau_d/\tau_{T_0} = 1.00$ (the source plane), 0.32, 0.02, and 0.00 (the upper surface). As expected, the line becomes broader and deeper as the column depth increases. At $N_{e,21} = 15,000$, virtually no photons escape in the line core.

The $N_{e,21} = 30$ simulation shows a prominent red wing shoulder. The blue shoulder is less evident, due in part to the exponential decline of the injected spectrum above the line energy. At $N_{e,21} = 1,500$, substantial scattering in the

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**Fig. 15.**—Shoulder formation for $\Psi = \pi/2$. Spectra for absorption (dashed lines) and scattering (dotted lines) from the first harmonic only are shown. When higher harmonics (and spawning) are included (solid lines), the shoulders are enhanced.
wings significantly shifts the centroids of the shoulders and broadens them. At $N_{e,21} = 15,000$, the slab is optically thick in the continuum. Scattering in the continuum broadens the shoulders so that they are no longer discernible.

The influence of the continuum can be seen in the mean number of scatters a photon experiences before escaping the slab. Taking a typical $\mu = 0.5$, we see from Figure 2 that continuum scattering dominates below $\sim 20$ keV. For $N_{e,21} = 30$ and 1500, the mean energy of escaping photons is about 29 and 25 keV, respectively, so that photons escape primarily in the wings after multiple core-wing excursions (though for $N_{e,21} = 30$, the wings are only marginally thick). The number of scatters, $N_{\text{scat}}$, is then dominated by core and wing scatters. Since these scatters are accumulated mostly in the region where $T_e \sim T_C$ (see Fig. 5), $N_{\text{scat}}$ follows the zero recoil scaling, viz, $N_{\text{scat}} \propto T_1$ (Adams 1972; Wasserman & Salpeter 1980). For $N_{e,21} = 15,000$, the mean energy of escaping photons is about 15 keV, so that photons escape primarily in the continuum. In this case, photons enter the continuum after multiple core-wing excursions.

Since the continuum is optically thick, they can return to the wings after many scatters in the continuum and escape in the continuum after a few such continuum-wing transitions. The addition of the continuum domain breaks the $N_{\text{scat}} \propto T_1$ scaling and increases $N_{\text{scat}}$ above that expected for pure line transfer. This increase can be seen clearly in Table 2 where $N_{\text{scat}}$ for runs both with and without the continuum are listed. For $T_1 = 10$, core and wing scatters still dominate the total number of scatters so that $N_{\text{scat}} \propto T_1$ remains good to $\lesssim 20\%$. For much larger column depths, however, photons escape in the continuum only after many
continuum-wing–core transitions, thereby resulting in much larger $N_{\text{scat}}$. Thus, at $\tau_{t_0} = 100$, for instance, we find $N_{\text{scat}} \approx 6 \times 10^6$ with the continuum turned on and about $1 \times 10^6$ with it turned off. The mean energy of escaping photons in this case is about 10 keV. The number of (line) scatters with continuum turned off ($\sim 10^6$) is less than that expected from the $N_{\text{scat}} \propto \tau_1$ scaling ($\sim 2 \times 10^6$) because this scaling ignores the non-Lorentzian factor (see eq. [10]), which substantially reduces the scattering cross section at very low energies.

5.5. Cylindrical Geometry

If the radiation from an accretion-powered pulsar emerges from the stellar surface at the polar cap, the spectrum is similar to the spectrum emerging from a 1-1 line-forming region with $\Psi = 0$. If the spectrum emerges from the sides of a cylindrical accretion column, however, it is similar to that of a 1-1 line-forming region with $\Psi = \pi/2$ viewed in the $\varphi_{\mu} \sim 0$ plane. We illustrate the latter case in Figure 19, which shows a Monte Carlo spectrum emerging from a cylindrical line-forming region. The magnetic field is oriented along the cylinder axis and has strength $B_{1.2} = 1.7$. The column depth, measured radially from the cylinder axis to the surface is $N_{e_{21}} = 1.2$, corresponding to $\tau_1 = 7/(kT/1 \text{ keV})^{-1/2}$. A power-law spectrum with $s = 1$ is injected along the cylinder axis. The figure shows that the cylindrical spectrum is qualitatively similar to a 1-1 spectrum with $\Psi = \pi/2$ viewed in the $\varphi_{\mu} \sim 0$ plane. Shoulders appear in the spectrum at small values of $\mu$ (Fig. 19a) but not at large values (Fig. 19d). The separation between the shoulder peaks is comparable to the line width for an absorption spectrum and increases as the line of sight approaches the cylinder axis because of the increase in the Doppler width. We stress that the typical radial column depth expected for an accretion column is $N_{e_{21}} \sim 10^3$–$10^4$, much larger than the value used in Figure 19. But we expect that, as the column depth of a cylindrical line-forming region increases, the properties of the emerging spectrum continue to be similar to those of a 1-1 line-forming region with $\Psi = \pi/2$, $\varphi_{\mu} \sim 0$, and similar column depth. Specifi-
cally, the spectrum emerging from a cylindrical line-forming region with high $N_e$ should possess a cyclotron line with large equivalent width at all viewing angles, and with no visible shoulders.

6. IMPLICATIONS FOR ACCRETION-POWERED PULSARS AND GAMMA-RAY BURSTERS

Our understanding of how cyclotron line properties depend on geometry can provide important insights into the sources of observed cyclotron lines. It can shed light on the optical depth and location of the line-forming region in accretion powered pulsars; for example, whether line-formation occurs in the accretion column or in a thin scattering layer located in the pulsar magnetosphere. Understanding geometrical effects can also explain how cyclotron lines in gamma-ray bursts could be formed, e.g., by sources in a galactic corona.

6.1. Accretion-Powered Pulsars

Virtually every known accretion-powered pulsar has an observed flux that is super-Eddington for polar cap
accretion (Nagase 1989). In other words,

\[ L_X (\text{ergs s}^{-1}) \gtrsim L_{\text{Edd}}^\text{cap} = F_E A_{\text{cap}} = 10^{35} \left( \frac{A_{\text{cap}}}{1 \text{ km}^2} \right) \times \left( \frac{M}{M_\odot} \right) \left( \frac{R}{10 \text{ km}} \right)^{-2} \frac{\sigma_T}{\sigma_m}, \]

where \( A_{\text{cap}} \) is the polar cap area where accretion occurs, \( F_E = 10^{25} (M/M_\odot) (R/10 \text{ km})^{-2} \text{ ergs cm}^{-2} \text{ s}^{-1} \) is the polar cap Eddington flux, and \( \sigma_m \) is the effective magnetic electron-photon scattering cross section (see, e.g., Nelson et al. 1995). Thus, a realistic, fully self-consistent calculation of the emergent spectra from these objects must couple the radiative transfer with the radiation hydrodynamics of the accretion flow (see, e.g., Arons, Klein, & Lea 1987; Klein & Arons 1989). This is a formidable multidimensional time-dependent problem. In principle, such calculations will determine both the structure of the accretion column and the associated continuum and line spectrum and be able to explain why observed spectra contain an excess of photons at low energies compared with a Wien function as well as reproduce the properties of observed cyclotron lines. Owing to the complexity of this problem, most calculations of accretion-powered pulsar spectra have focussed on more tractable (but much less realistic) static line forming regions. These studies, however, may provide qualitative descriptions of the spectral features and furnish guides to understanding the more realistic calculations. An intermediate approach is also possible wherein radiative transfer is carried out through an accretion flow with a given velocity and density profile.

Such “kinematic” studies have been carried out in the context of accretion-powered pulsars by, e.g., Burnard et al. (1991), and in the context of gamma-ray bursts by, e.g., Miller et al. (1991, 1992) and Isenberg, Lamb, & Wang (1996, 1998). In the former case, the authors calculate the emergent continuum spectra from a quasi-stationary accretion mound (representing the subsonic settling flow in the postshock region of the accretion flow near the polar cap), while in the latter case, the authors studied the physics of cyclotron scattering in an outflowing plasma (at the mag-
nnetic polar cap) that is optically thin in both the continuum and the first harmonic line wings.

If the luminosity of the pulsar is small enough so that radiation forces are not important, i.e., $L_x < L_{Edd}$, the accreting material is stopped via magnetic Coulomb collisions in the atmosphere on the stellar surface, and photons are produced down to depths $N_{e,21} \sim 15,000-150,000$ ($\tau_e \sim 10-100$) (Kirk & Galloway 1982; Miller, Salpeter, & Wasserman 1987; Miller et al. 1989). (If radiation forces are significant, the accreting material is decelerated by radiation pressure from the star [Davidson 1973; Basco & Sunyaev 1976; Burnard et al. 1991] before stopping via magnetic Coulomb collisions at smaller depths ($\tau_e \sim 1$) in the underlying atmosphere [Wang et al. 1989a].) Since the stopping depth is optically thick to (nonmagnetic) Thomson scattering, many calculations assume a Wien spectrum at these depths. However, as we discussed in § 4.3, in a strong magnetic field the photons will not generally have a Wien distribution at these depths. Consequently, the injected spectrum needs to reflect the physics of the stopping process (see, e.g., Wang et al. 1989a).

The $(E/E_0)^2$ dependence of the cross section for $E < E_0$ may play an important role in creating the photon excess at low energies. The excess in our own simulations is small compared with observations, but this is most likely because of our choice of a Wien distribution for the injected photons and our neglect of the velocity of the accreting material above the atmosphere.

Observed cyclotron features are broad, but shallow. For example, Soong et al. (1990) report a full width at half-maximum of 15.8 keV but an equivalent width of only 10.3 keV for the line in the phase-averaged spectrum of Her X-1 (centroid at ~ 35 keV). In contrast, as we saw in Figure 18, theoretical calculations for line-forming regions with column depths typical of accretion columns ($N_{e,21} \sim 10^{12}-10^{14}$; Lamb, Pethick, & Pines 1973) generate lines that are extremely deep, that is, with large equivalent widths $W_E$. Similar results were found by Bulik et al. (1992, 1995). Clearly, most existing models of accretion-powered pulsar cyclotron line formation, i.e., models with static slab and cylindrical line-forming regions with $N_{e,21} \sim 10^{12}-10^{14}$, are not able to reproduce the properties of the observed spectra.

Future work should consider the possible broadening of the line due the plasma flow velocity in the accretion column. The material accreting onto a neutron star reaches relativistic speeds before being decelerated in the stellar atmosphere. If $L_x \ll L_{Edd}$, $v \sim v_{esc} \sim 0.5c$. Wang & Frank (1981) show that even in cases where the luminosity is high enough that radiative deceleration is important, the maximum plasma velocity still reaches a significant fraction of $v_{esc}$. In a moving plasma, the cyclotron energy in the rest frame of the plasma is given by $E_c = eE_0(1 - \beta\mu)$, where $E_0$ is the energy of the photon in the star’s frame. This Doppler shifting in the accretion column will tend to broaden the line emergent from the underlying atmosphere, especially redward of line center. This is because these photons will resonantly scatter in a layer where $E_c \approx E_1$, with the layer’s location being determined by $(E_1, \mu)$ and the velocity profile of the accretion column. (Miller et al. 1991, 1992 and Isenberg et al. 1996, 1998 discuss the physics of cyclotron scattering in a moving plasma in the context of gamma-ray bursts.)

In fits to the spectra of 4U 1538-52 and Vela X-1, Bulik et al. (1992, 1995) find that they can obtain acceptable fits to the data by using a model with two field components, differing in strength by a factor $\sim 5$. They suggest that the lower field component could correspond to scattering in a dipole field, $B(r) \sim B_0(R/r)^2$ at $r \sim 1.3R$, where $B_0$ is the surface dipole field strength and $R$ is the stellar radius. This scattering could occur in an accretion mound, as in the model of Burnard et al. (1991), or in a suspended scattering atmosphere, as in the model of Dermer & Sturman (1991) and Sturman & Dermer (1994). Thus the observed spectrum could be the product of a combination of scattering near the surface and scattering in the magnetosphere.

The spectral signature of the geometry of the line-forming regions can provide insight into the contribution of each region to the spectrum. At moderate optical depths, spectra formed in the 1-1 geometry are characterized by prominent shoulders; these are smaller or absent in the 1-0. Although the prominence of the shoulders is a straightforward consequence of scattering in a semi-infinite atmosphere, shoulders have not been reported in the observed spectra of accretion-powered pulsars. If the shoulders persist after the model spectra have been folded through the detector response matrix, and they prevent an acceptable fit to an observed spectrum, their absence in the observed spectrum could be a significant clue to the geometry of the line-forming region—it indicates either a geometry analogous to a 1-0 slab with no restriction on the column depth or a geometry analogous to a 1-1 slab with a column depth $N_{e,21} \gtrsim 15,000$ ($\tau_e \gtrsim 10$). This latter possibility would be consistent with calculations of proton stopping in strongly magnetized atmospheres (Miller et al. 1987, 1989).

Using both 1-1 and cylindrical geometries, Araya & Harding (1996) consider the possible effect of line shoulders on the observed spectrum of the accretion-powered pulsar A0535+26. Kendziorra et al. (1992, 1994) report HEXE observations of 50 and 100 keV features from this pulsar at 2 and 4.5 $\sigma$, respectively. OSSE observations by Grove et al. (1995) of a feature at 110 keV confirm the high-energy feature reported by HEXE; the low-energy feature is too close to the 45 keV OSSE threshold for a conclusive observation. Araya & Harding (1996) consider the possibility that the 110 keV feature is a second harmonic line in a spectrum where the first harmonic has been filled in by the line shoulders. They fit this model to the observed spectrum, for $N_{e,21} \lesssim 5$: the fit is poor compared with a model in which the 110 keV feature is the fundamental. The latter model is plausible in light of the low significance of the 50 keV feature in the HEXE observation.

The column depths in the Araya & Harding (1996) calculation are suitable for scattering in the magnetosphere but are much smaller than the column depths usually considered typical of accretion columns. The results of § 5 in the present work show that the line shoulders disappear at large column depths. If formed in a very thick accretion column, the first harmonic feature at 50 keV should therefore have a large equivalent width, contrary to observations. Since both line formation in moderate and very thick media are not able to produce a shallow 50 keV first harmonic, these calculations suggest that the 110 keV feature observed in A0535+26 is the first harmonic. However, the spectrum of A0535+26 could possess a ~50 keV first harmonic that has been mostly filled in with photons spawned by scattering at higher harmonics in a medium that is optically thin in the wings. It is difficult to assess this conjecture until second and higher harmonic
scattering at such large optical depths are better understood. In addition, the fact that the first harmonics observed in other accretion-powered pulsars are broad but shallow, with \( W_2 \) much smaller than those given by theoretical models, gives pause.

In many accretion-powered pulsars, the pulse profiles are complex and strongly energy dependent and elude explanation by simple models. Derm & Sturner (1991) and Sturner & Derm (1994) suggest that some of these properties can be explained by photon scattering in the pulsar magnetosphere, in a layer of plasma supported by radiation pressure. Such an atmosphere has a geometry similar to the 1-0. By equating the momentum per second carried by the radiation to the gravitational force on the suspended plasma, Sturner & Dermer calculate the maximum mass that can be suspended at a given altitude. At the maximum mass, these layers are optically thick to line scattering but thin to continuum scattering. They are located at a distance of a few stellar radii from the star's center. A calculation of the actual suspended mass needs to take into account the scattering cross section, the effects of multiple scatterings, and the stability of the suspended layer. We will address these issues in a separate paper.

Sturner & Dermer calculate the effect of scattering in the suspended plasma on the radiation pulse profiles. They report that the scattering flattens the pulse profiles. The flattening corresponds to the reduction in flux at the line center that we see in our Monte Carlo spectra. In the present work, we speculate about the signature of magneto-spheric scattering on the spectrum. If a suspended layer could be formed, spectral features formed there would have small or no shoulders, in agreement with observations. At \( \Psi = 0 \), it would be difficult for a layer with \( N_e \sim 1 \) to form the broad lines required for the case of accretion-powered pulsars. However, as the present work shows, broad lines are not a problem as \( \Psi \) approaches \( \pi/2 \). The lines would be broadened even more if the line-forming region covers an extended region in which the magnetic field strength varies significantly.

However, if the lines from an accretion-powered pulsar are formed in this way, one expects that the dipole field \( B_\ell \) at the neutron star surface would be \( \sim 10 \) times larger than the dipole component of the field \( B(r) \) in the line-forming region in the magnetosphere. The field \( B(r) \) can be inferred from the cyclotron line energy. For most sources, such as Her X-1, the field inferred from the cyclotron lines is larger than the dipole field inferred from the accretion torque model of Ghosh & Lamb (1979a, 1979b; 1991), contrary to what would be expected if the lines are formed in the magnetosphere. However, it is possible that the nondipole components of the field in the line-forming region are large enough to account for the discrepancy. There are also some accretion-powered pulsars in which the field inferred from the cyclotron line energy could conceivably be much smaller than the dipole surface field inferred from the accretion torque theory (Ghosh & Lamb 1979a, 1979b, 1991; Mihara 1995).

As we have discussed above, existing models of the line-forming region are unable to reproduce even qualitatively the observed properties of the cyclotron lines in the spectra of accretion-powered pulsars. Future work in this area should therefore explore physical effects that have not been included so far, such as the plasma flow velocity in the accretion column and the injected spectrum resulting from energy deposition in the surface layers of the neutron star. In addition, nonstandard geometries such as accretion mounds and multiple component magnetic fields, or alternative models such as cyclotron scattering in the magnetosphere, may need to be considered.

6.2. Gamma-Ray Bursters

The fit to the spectrum of gamma-ray burst GB880205 made by Wang et al. (1989b) assumes a static line-forming region with a uniform magnetic field parallel to the slab normal. As we mention in § 1, this geometry is suitable, for example, for the magnetic polar cap of a neutron star with a dipole field. LWW point out that if the line-forming region is indeed at the polar cap, the static model is valid only if the bursters lie at distances less than several hundred parsecs. Otherwise, the burst luminosity exceeds the Eddington luminosity and the radiation force creates a relativistic plasma outflow along the field lines.

However, in order to explain the brightness and sky distributions of the bursts observed by BATSE, it has been suggested that, if the bursters are Galactic, the sources are in a Galactic corona at distances of 100–400 kpc (for a review, see Lamb 1995). In light of the BATSE results, it is important to explore line-formation models that are appropriate for sources at these distances.

One possibility is line formation in a relativistic outflow (Miller et al. 1991, 1992; Isenberg et al. 1996, 1998). Another possibility is line formation in a static slab at the magnetic equator. Here the line fields are parallel to the slab (\( \Psi = \pi/2 \)), and can confine the plasma magnetically (see, e.g., Zheleznyakov & Serber 1994, 1995). Using the revised Monte Carlo code developed in the present work, and varying \( \Psi \), Freeman et al. (1996) fit models to the two observed spectra corresponding to the two time intervals S1 and S2 in which lines appeared in burst GB870303. When they perform a joint fit to the two intervals, using models with a common \( B_{12} \) and \( N_{e,21} \) for both spectra but not a common viewing angle, they find that the I-1 geometry the data marginally favors the equatorial model over the polar model.

There are still many open issues concerning both models. For example, for the large magnetic fields and source distances required by the Galactic corona model, the optical depths for the processes \( \gamma \rightarrow e^+e^- \) and \( \gamma\gamma \rightarrow e^+e^- \) are both much larger than unity (Schmidt 1978; Daugherty & Harding 1983; Burns & Harding 1984; Brainerd & Lamb 1987). The production of electron-positron pairs by these processes could lead to line-forming regions that are optically thick in the continuum, which might prevent the formation of the narrow lines observed in gamma-ray bursts. In addition, pair production could truncate the spectrum at the pair production threshold, 1 MeV. Truncation is inconsistent with observations by COMPTEL (Winkler et al. 1993) and EGRET (Schneid et al. 1992; Kwok et al. 1993; Sommer et al. 1994) of photon energies up to 1 GeV with no evidence of a spectral cutoff or rollover at \( \sim 1 \) MeV.

Various solutions to these problems have been proposed (for a review, see Higdon & Lingenfelter 1990). One possibility is that the entire spectrum is produced at the magnetic polar cap in a medium with bulk motion corresponding to a Lorentz factor of \( \Gamma \sim 10 \). The radiation would then be beamed into an angle \( \sim 1/\Gamma \). For corona distances, the optical depth for the two-photon process would then be reduced enough to be consistent with the observed spectra.
above 1 MeV (Schmidt 1978; Baring & Harding 1993; Harding & Baring 1994; Harding 1994). Additional calculations are necessary to determine whether the observed cyclotron features could be formed at the polar cap under these physical conditions (see, e.g., Isenberg et al. 1998).

Alternatively, the gamma-ray burst spectrum could consist of two components produced in two distinct physical regions: a soft ($\lesssim$ 1 MeV) component produced in plasma trapped in the equatorial region of the magnetosphere (Lamb 1984; Katz 1982, 1994) and a hard component ($\gtrsim$ 1 MeV) produced by relativistic outflow at the magnetic polar cap. Because the magnetic field traps the plasma in the line-forming region, the line-forming region can be static, even for highly super-Eddington luminosities, as is the case if the bursts come from neutron stars in a Galactic corona. The hard component could be produced in a relativistic pair outflow or expanding fireball at the magnetic polar cap. If so, the cross section for the two-photon process is further reduced by the bulk motion of the pairs. A recent report by Chernenko & Mitrofanov (1995) of evidence for two components in the spectrum of GB881024 is intriguing. The soft component dominates the spectrum for $E < 250$ keV; the hard component is dominant above this energy.

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**APPENDIX**

**CALCULATION OF $T_c/E_B$ IN OPTICALLY THIN SLABS**

Equation (16),

$$
\Delta E \equiv \left[ \int d\Omega dE n(E, \Omega) \delta E N_{\text{scal}}(\tau_{\text{To}}, E, \Omega) \right] / \int d\Omega dE n(E, \Omega),
$$

gives the total energy change of the photons, summed over scatterers and averaged over the scattering photon distribution. In terms of the differential scattering cross section $\sigma/\Omega$, the mean energy change per electron-photon scattering is

$$
\delta E \equiv \frac{\int dp f(p) d\Omega_s (\sigma/\Omega) \delta E}{\int dp f(p) d\Omega_s (\sigma/\Omega)} = \int dp d\Omega_s f(p) \frac{1}{\sigma_T \phi(E, \Omega)} \frac{d\sigma}{d\Omega_s} \delta E,
$$

(A1)

where $\delta E = E_s - E$ is the energy transferred from the electron to the photon in an individual scatter and $\phi$ is as defined in equation (14). Substituting equation (A1) into equation (16) and adopting a normalized photon distribution, we obtain

$$
\Delta E = \int d\Omega dE dp d\Omega_s n(E, \Omega) f(p) \frac{1}{\sigma_T \phi(E, \Omega)} \frac{d\sigma}{d\Omega_s} \delta E N_{\text{scal}}(\tau_{\text{To}}, E, \Omega).
$$

(A2)

In the optically thin, or single-scattering, limit, the total number of scatters per photon is just the fraction that experienced scattering, viz,

$$
N_{\text{scal}}(\tau_{\text{To}}, E, \Omega) = 1 - \exp \left[-\frac{\tau(E, \Omega)}{|\mu_s|}\right] \approx \frac{\tau}{|\mu_s|},
$$

(A3)

where, for an isothermal medium,

$$
\frac{\tau}{|\mu_s|} = \frac{\tau_{\text{To}} \phi(E, \Omega)}{|\mu_s|} \ll 1,
$$

(A4)

is the depth at which photons are injected measured from the top of the slab, along the line of sight. Substituting equation (A3) into (A2) gives

$$
\Delta E = N_s \tau \int_{-\infty}^{+\infty} \frac{dp}{|\mu_s|} \int dE n(E, \Omega) \left[ \int d\Omega_s \frac{d\sigma}{d\Omega_s} E_s - E \int d\Omega_s \frac{d\sigma}{d\Omega_s} \right],
$$

(A5)

where we have used $\delta E = E_s - E$.

For first-harmonic resonant scattering with zero natural line width, the differential cross section is given by equation (10) in the limit $\Gamma \rightarrow 0$ ($h = c = k = 1$ used throughout):

$$
\frac{d\sigma'}{d\Omega'_s} = \frac{9}{32} \frac{\sigma_T m_e}{\alpha} \delta(E' - E'_s) \frac{1 + \beta'^2 + \mu'^2}{2}.
$$

(A6)

For atmospheres with $T_e \ll m_e$ and $b \ll 1$, the two naturally occurring small parameters are the electron velocity along the field ($\beta \sim (T_e/m_e)^{1/2}$) and the gyration velocity orthogonal to the field ($\mu^2 \sim b \sim E/m_e$). To evaluate $T_c$, we expand equation...
integrating over \( \Omega \). Thus, the scattered frequency is given by
\[
E_s = \gamma E(1 + \beta \mu_s),
\]
where
\[
E_s = E' \left\{ 1 - \frac{E'}{2m_e} (\mu'_s - \mu')^2 + \frac{1}{3} \left( \frac{E'}{2m_e} \right)^2 \right\}. \tag{A7}
\]

The energy dependence of the pulse profiles of accretion-powered pulsars strongly suggests that the angular distribution of radiation in the line-forming region also depends on the photon energy (see, e.g., Joss & Rappaport 1984; Nagase 1989; Mészáros 1992; Mihara 1995). Nevertheless, to permit an analytic solution for the Compton temperature in the single scattering limit, we assume that the photon density is separable; i.e.,
\[
n(E, \Omega) = n(E)Q(\Omega). \tag{A8}
\]

The effects of an energy dependent angular distribution on the Compton temperature and the emerging spectrum is beyond the scope of the present work.

Substituting equations (12), (A6), (A7), and (A8) into equation (A5), integrating over scattered angles, and integrating over \( E \) using
\[
\delta(E' - E'_s) = \frac{1}{\gamma(1 - \beta \mu)} \delta(E - E_s), \tag{A9}
\]
where \( E_s = E'/[\gamma(1 - \beta \mu)] \), gives
\[
\Delta E \approx \frac{3\tau_0 m_e}{16\pi^2} N_{e2} \int_{-\infty}^{+\infty} \int d\Omega \frac{Q(\Omega)N(\Omega)E'_s(1 + \mu^2)}{[1 - \frac{1}{\gamma^2(1 - \beta \mu)} - \frac{E'_s}{m_e}(\frac{\mu^2}{2} + \frac{1}{5})]} \tag{A10}
\]

The leading order term in brackets is \( O(\beta) \), and so we need only expand \( N(\Omega)E'_s(1 + \mu^2) \) to \( O(\beta) \) to obtain the desired result. Doing this gives
\[
\Delta E = \frac{3\tau_0 m_e}{16\pi^2} N(E_0)E_B \int_{-\infty}^{+\infty} \int d\Omega \frac{Q(\Omega)\mu_s}{[1 - \frac{1}{\gamma^2(1 - \beta \mu)} - \frac{E'_s}{m_e}(\frac{\mu^2}{2} + \frac{1}{5})]} \tag{A11}
\]

where
\[
s = -\frac{E}{n(E)} \frac{dn}{dE} \bigg|_{E=E_B}. \tag{A12}
\]

Note that, if the initial photon spectrum is a power law, \( n(E) \propto E^{-\alpha} \), then \( s \) is just the spectral index, \( \alpha \).

We assume \( f(p) \) to be a nonrelativistic one-dimensional Maxwellian; that is,
\[
f(p)dp = \frac{\exp\left[-p^2/(c m_e T_e)\right]}{\sqrt{2\pi m_e T_e}} dp. \tag{A13}
\]

Integrating over \( p \) in equation (A11) and setting \( \Delta E = 0 \) gives the slab Compton temperature
\[
\frac{T_C}{E_B} = \frac{1}{10} \left\{ \frac{1}{(d\Omega/[\mu_s])Q(\Omega)(2 + 7\mu^2 + 5\mu^4)} \right\} \tag{A14}
\]

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