The importance of correct specification of tribological parameters in dynamical systems modelling

S Alaci¹, F C Ciornei¹, I C Romanu¹ and M C Ciornei²

¹“Stefan cel Mare” University of Suceava, Mechanics and Technologies Department, Universitatii str., no. 13, 720229 Suceava, Romania
²“Carol Davila” University of Medicine and Pharmacy, Department 2 Physiology I, Bd. Eroilor Sanitari, no. 8, 050474 Bucuresti, Romania

E-mail: alaci@fim.usv.ro

Abstract. When modelling the behaviour of dynamical systems, the friction phenomenon cannot be neglected. Dry and fluid friction may occur, but dry friction has more severe effects upon the behaviour of the systems, based on the fact that the introduced discontinuities are more important. In the modelling of dynamical systems, dry friction is the main cause of occurrence of the bifurcation phenomenon. These aspects become more complex if, in the case of dry friction, static and dynamic frictions are put forward. The behaviour of a simple dynamical system is studied, consisting in a prismatic body linked to the ground by a spring, placed on a conveyor belt. The theoretical model is described by a nonlinear differential equation which after numerical integration leads to the conclusion that the steady motion of the prism is an un-damped oscillatory motion. The system was qualitatively modelled using specialised software for dynamical analysis. It was impractical to obtain a steady uniform translational motion of a rigid, therefore the conveyor belt was replaced by a metallic disc in uniform rotation motion. The attempts to compare the CAD model to the theoretical model were unsuccessful because the efforts of selecting the tribological parameters directed to the conclusion that the motion of the prism is a damped oscillation. To decide which of the methods depicts reality, a test-rig was assembled and it indicated a sustained oscillation. The conclusion is that the model employed by the dynamical analysis software cannot describe the actual model and a more complex model is required in the description of the friction phenomenon.

1. Introduction

The friction phenomenon is found whenever two solid bodies come into contact [1], [2]. There are numerous effects of friction occurring in mechanical systems and describing them is not an easy task.

In various applications the presence of friction is neglected but a correct analysis requires the consideration of friction. One of the most important criteria in the classification of friction is the presence or absence of a lubricant between the boundary surfaces of the contacting solid bodies, therefore fluid friction and dry friction is found, respectively. Both types of friction have the effects of wear and energy dissipation as heat, consequences of relative velocity on the contact area, but they are completely different from a qualitative point of view. Thus, in the case of fluid friction, the magnitude of friction force depends on the size of relative velocity between the two surfaces and according to Newton’s law [3-5], for a given relative velocity only one value for friction force corresponds.

However, for the case of dry friction, the magnitude of the friction force is described using inequalities, and for the same kinematical configuration of the system, there is the possibility that
several values of the friction force may exist. The behavior of mechanical systems where dry friction occurs is described by means of nonlinear differential equations. The integration of these equations, typically by numerical procedures, leads to the conclusion that in the presence of dry friction in the dynamic system may tend to different equilibrium states, arriving at bifurcation and chaos notions [6-7]. When the behaviour of dynamic systems is modelled via specialized software, the friction from kinematical pairs of the system must be accurately characterized. Regardless of the complexity of the friction model accepted by the software for dynamical modelling, this model considers simplifying hypotheses concerning friction. There are situations when the behaviour predicted by the modelling software is analogous to the behaviour of the actual model, but for other cases, there are major discrepancies between model and tangible system, the conclusion being that the simplifying hypotheses lead to an overly simplified model, and therefore a new, more complex model of friction is required. To illustrate this, a simple dynamic system whit dry friction is presented, modelled both directly and via software, and different evolutions are obtained. An experimental set-up is used and it is concluded that, although a series of simplifying hypothesis concerning friction was employed, the behaviour of the system is correctly described by the model.

2. A theoretical model with dry friction

The dynamic system form Figure 1 is considered. A prismatic body is set on an horizontal conveyor belt that has constant peripheral velocity, \(v_0\). The body is attached to a spring with linear characteristic, having the elastic constant \(k\). There is no lubricant between the body and the belt.

![Figure 1](image.png)

**Figure 1.** Scheme of the proposed model

The theorem of centre of mass is applied in order to characterize the motion of the body:

\[
\begin{align*}
\dot{m}\ddot{x} &= -T - F_e \\
N - G &= 0
\end{align*}
\]  

(1)

where \(F_e\) is the elastic force:

\[
F_e = k_e x
\]  

(2)

and the friction force is given by the relation, [8-11]:

\[
T = -\mu N \, \text{sgn} v_{rel}
\]  

(3)

where \(\mu\) is the coefficient of dry friction, defined as a function of the relative sliding velocity \(v\) between the belt and the body. A careful definition of the static friction force must consider the fact that, if no relative velocity exists between the contacting surfaces, the friction force may have any
value between \(-\mu_s N\) and \(\mu_d N\), while when relative velocity exists, the friction force is according to Amonton-Coulomb law, proportional to the normal force \(N\) and oriented as to oppose to the relative displacement between the two surfaces. The dependency of the coefficient of friction on velocity can be defined by the following function:

\[
\mu(v) = \begin{cases} 
\mu_d, & v < 0 \\
\mu_s \leq \mu \leq \mu_s, & v = 0 \\
\mu_d, & v > 0 
\end{cases}
\]

and is represented in Figure 2,

![Figure 2. Theoretical dependence of dry friction coefficient versus relative velocity](image)

where \(\mu_s\) and \(\mu_d\) are the limit coefficients for static friction and dynamic friction, respectively. The fact that for \(v = 0\) the coefficient of friction takes values in the domain \([-\mu_s, \mu_s]\) assumes utilizing a complicated mathematic apparatus. The same aspect occurs in the instants immediately after the exit from the equilibrium state, when the coefficient of friction suddenly jumps from \(\mu_s\) to \(\mu_d\) and from \(-\mu_s\) to \(-\mu_d\) imposes the utilization of functions with hard discontinuities whose description require the employment of distribution (generalized functions). To avoid these aspects, the hypothesis is formulated that transition from repose to motion is made in a narrow velocity domain, \([-v_{tr}, v_{tr}]\) where \(v_{tr}\) is the stiction transition velocity used in characterisation of immobile body. The second assumption is based on accepting that the conversion from the maximum values of static friction force \(\pm \mu_s N\) to the value of dynamic friction force \(\pm \mu_d N\) is made obeying an exponential law. The dependence of the coefficient of friction on velocity under the new hypotheses is described by the function:

\[
\mu(v) = \begin{cases} 
\mu_s v / v_{tr}, & |v| < v_{tr} \\
(\mu_s - \mu_d) \exp[\alpha(v - v_{tr})] + \mu_d, & v > v_{tr} \\
-(\mu_d - \mu_s) \exp[\alpha(-v - v_{tr})] + \mu_d, & v < -v_{tr} 
\end{cases}
\]

The plot of the dry coefficient of friction versus relative velocity is shown in Figure 3. With the stipulated relation between coefficient of friction and velocity, the equation of motion of the body is written:

\[
m \ddot{x} + \mu(\dot{x} - v_0)mg + k_v \dot{x} = 0
\]

In relation 6, \(v_0\) is the constant velocity of the belt and \(\dot{x} - v_0\) is the relative velocity of the body with respect to the belt. The equation 6 is a nonlinear homogenous equation and the Runge-Kutta 4 methodology is applied in solving it [12]. Following the integration of the differential equation of motion, the elongation, velocity and acceleration graphs versus time were traced, as well as the phase diagram.
Figure 3. Approximation of dependence of the coefficient of dry friction versus relative velocity

Figure 4. Elongation, velocity, acceleration and phase diagram for $v_0 < v_{0\text{cr}}$

The integration of the differential equation for different values of belt velocity, $v_0$, revealed the existence of a critical value denoted $v_{0\text{cr}}$. The four dependencies mentioned above are presented in
Figures 4 and 5 for two values of the conveyor belt velocity. Thus, for a belt velocity smaller than the critical value, Figure 4, the body starts to move from the equilibrium position and gains an oscillatory motion, without reaching the initial position. In another situation, adopting for the belt a velocity greater than the critical one, $v_0 > v_{0cr}$, as presented in Figure 5, the body takes a periodical motion and attains the initial position. As it can be remarked from Figures 4 and 5, in both cases, the motion is undamped since the characteristic point-in the phase diagram, moves on a circular trajectory towards the end of the analysis; for the damped case, the motion is characterized by asymptotic displacement towards a fixed point.

![Figure 4. Elongation](image1)

![Figure 5. Elongation, velocity, acceleration and phase diagram for $v_0 > v_{0cr}$](image2)

3. **Modelling the dynamic system using specialised software**

For the dynamical system considered above, the dynamic behaviour was simulated using the dedicated software MSC.ADAMS [13]. A first issue which arises consists in the fact that it is impossible to model a solid body in uniform rectilinear motion, with constant velocity $v_0$ during a time interval of any dimension – motion ensured in the initial model by the conveyor belt. The fact that a point of the
belt periodically comes into contact with the surface of the mobile body leads to the idea of replacing the belt by a disc, 1 as shown in Figure 6, which rotates about the vertical axis of the rotation pair \( \overline{R} \) between the disc and the ground. A small prismatic body is placed on the frontal surface of the disc, having small dimensions compared to the radius of the disc, and situated at a distance large enough with respect to the axis of the disc, these aspects supporting the hypothesis that on the contact surface between the body and the disc the relative velocity is the same. An ideal planar parallel pair (frictionless) \( P \) was introduced in the structure of the mechanism, normal to the radius vector of the body, in order to ensure rectilinear translational motion to the prismatic body. The mobile body is also subjected to other constraints, for example \( C \), the contact with friction between the body and the disc and \( 3 \) the helical spring ensuring the link between the ground and the prismatic body.

In order to describe the friction in the system, the tribological parameters characteristic to the contact \( C \) and to the helical spring \( 3 \) are required. The specification of the tribological parameters is not a simple task [14], [15] and to prove this, the tables required to be completed for the contact \( C \) and spring \( 3 \) are presented in Figures 7 and 8 respectively.

For the present analysis, the effect of friction from contact \( C \) is more important and therefore the adopted damping coefficient of the spring was very small, \( 3 \cdot 10^{-9} \text{Ns/m} \), that is a value corresponding to negligible damping effect. The dynamic behaviour of the system was simulated with the tribological parameters shown in Figures 7 and 8, and the same dependencies were sought as in Figures 4 and 5.

These variations are presented in Figure 9 and the *damped motion of the mobile body* may be remarked. Simulations were also made for different values of the rotation velocity of the disc and of the static and dynamic friction coefficients. The results did not differ qualitatively, since the mobile body presented a damped motion. Additionally, the manner in which the amplitudes of the oscillations decrease is an exponential function, characteristic to fluid friction. It can be easily shown that the decrease of amplitudes of oscillations for dry friction conditions is a linear function.

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**Figure 6.** Dynamical simulation using specialized software
Figure 7. Tribological parameters for contact characterization

Figure 8. Characteristic parameters of helical spring

Figure 9. Elongation, velocity, acceleration and phase diagram plotted by the MSC.ADAMS software
4. Experimental validation of the correct solution

The fact that the two models, the mathematical one and the model obtained via a dynamic analysis software lead to completely different behaviours from a qualitative point of view – the mathematical model prognoses an oscillatory motion, while the dynamic simulation provides a damped motion, imposes the experimental validation of the correct model. To this purpose, an experimental device was designed and executed, consisting in a disc run into rotation via a d.c. electric motor with worm gear speed reducer. The prismatic body was replaced by a bronze cylinder placed on the horizontal surface of the disc. The need of employing a soft spring was observed after the experimental adjustments. Therefore, elastic belts of different lengths were used, Figure 10. It was observed that for a suitable selection of the length of the elastic belt and for a specified rotation velocity of the disc, the body presents periodical undamped motion, and this confirms the correctness of the mathematical model. Because the mobile body and the disc have comparable radii, the assumption of constant relative velocity on the contact surface is not valid. A consequence resides in the occurrence of the moment of friction forces and of an oscillatory rotation motion of the cylindrical body, in addition to the translation motion.

![Figure 10. Experimental test-rig: a) general view; b) top view](image)

5. Conclusions

The main purpose of the paper consists in highlighting the importance of correct stipulation of the parameters characterizing the friction forces that occur in the pairs of a mechanical system. To this end, a simple system is considered, consisting in a horizontal conveyor belt on top of which a body - attached to the ground via a helical spring, can be in motion. Dry friction contact is assumed between the mobile body and the belt.

In the first part of the work, a law for the dependency of the coefficient of friction versus relative velocity between the belt and the body is proposed. The actual form of the function was chosen to ensure a well established value for the coefficient of friction for any velocity, but this is in opposition to the actual case when, for zero relative velocity, the friction force can take any value from a certain domain. For the proposed model of friction, the nonlinear differential equation that describes the motion of the system is obtained. After numerical integration, the elongation, velocity, and acceleration versus time are traced together to the phase diagram. The conclusion emerging is that the motion of the body is a forced oscillatory motion with constant amplitude.

The same system was then modeled utilizing specialized software, with the mention that instead of the conveyor belt, a disc in uniform rotation motion about its vertical axis was considered. This simulation, regardless of the values of the friction forces, leads to the conclusion that the body presents a damped oscillatory motion, contrary to the conclusion of the mathematical model. Moreover, the damping type of the amplitude of the motion is characteristic to fluid friction, not to dry friction.
An experimental set-up with a structure similar to the model used by the software was constructed, to determine which of the two models provides the correct solution. Experimental tests were carried out, and the motion of the mobile body proved to be an un-damped oscillatory motion, as the mathematical model predicts.

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