Reliability optimization of crankshaft for high-speed punch based on genetic algorithm method

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Abstract. The crankshaft is one of the most important components of high-speed punch and the design and optimization of crankshaft has fateful consequence to the performance and reliability. The high reliability value of crankshaft contributes to the increasing of the reliability of the whole punch system in the manufacturing and using process. This study builds a reliability optimization model of the crankshaft for punching process under constraints of the strength, stiffness and geometry. Genetic algorithm method is employed to make reliability optimization for crankshaft based on ANSYS and MATLAB. The strength and stiffness reliability are both set as 0.997, namely, the minimum reliability of crankshaft is 0.997. The optimum design parameters based on reliability are obtained under the above condition. The obtained results demonstrate that genetic algorithm method can be effectively employed to investigate the reliability optimization of crankshaft for high-speed punch.

1. Introduction

High-speed punch is an efficient, high-precision and high-automation punching machine. It has been widely used in aeronautics and astronautics, automobile, traffic and transportation, metallurgical chemical industries and so on [1-2]. The crankshaft is a key part of main transmission system for high-speed punch and its reliability has great impact on the reliability of high-speed punch. Design and optimization of the crankshaft is obviously not new, as the crankshaft is a very popular mechanism used in many different types of machinery. Sun [3] made multi-objective optimization design of crankshaft-bearing based on the crankshaft-bearing system, which reduced the frictional power loss of crankshaft-bearings and increased the crankshaft safety factor. Renzi [4] built an integrated design environment based on genetic algorithm, which is used for the preliminary design and optimization of aeronautical piston engine components, and the test results show that it could reduce 20% weight of the crankshaft. Liu [5] programmed a multi-island genetic algorithm combined with multidisciplinary cooptimization method, which is used to carry out the multi-objective optimization of crankshaft structure. Sun [6] carried out the optimum design of crankshaft bearings considering crankshaft strength. Song [7] carried out a multi-objective optimization considering the mutually exclusive changes of local machining allowances of the final forged product. Kim [8] proposed a hybrid method based on micro-genetic algorithm and regression-based sequential approximate optimizer, the result shows the radial stiffness and axial stiffness of the main shaft have been greatly improved. Additionally, there are more references for the design optimization of crankshaft [9-10].

As stated previously, there is few study on reliability optimization of crankshaft for high-speed punch. This work introduces the reliability optimization model. Then, the reliability optimization model of
crankshaft is built in ANSYS and MATLAB and the reliability optimization based on genetic algorithm method is carried out.

2. Reliability optimization model

2.1. Fundamental of reliability optimization

Reliability based design optimization is defined as:

\[
\min \; f(d, \mu_d, \mu_p) \\
\text{s.t. } P\{g_i(d, X, P) \leq 0 \} \geq R^* \quad i=1,2,\ldots,n_g
\]

Where \( d \) is the certain design variable, \( \mu_d \) is the mean value of the uncertain design variable, \( \mu_p \) is the mean value of the uncertain state parameters, \( n_g \) is the number of reliability constraints, \( g_i \) is the limit state function, \( R^* \) is the reliability value needed to be reached.

The reliability analysis is to calculate the probability when the limit state function is less than zero by the effect of input random variable, as shown in Eq. (2).

\[
R_i = P\{g_i(x) \leq 0 \} = \int_{g_i(x)=0} f_x(x) \, dx
\]

Where \( X = \{x_1, x_2, \ldots, x_n\} \) is the random design variable, \( f_x(X) \) is the joint probability density function, \( g_i(X) \leq 0 \) is the integral area.

2.2. Reliability optimization model in narrow sense

For illustrative purposes, take the relationship between the reliability and cost of component as an example.

2.2.1. The cost of component is given (or volume, weight, performance). Maximize the reliability value under the constraint of cost, the objective function is the reliability of the component, the model is shown as:

\[
\max \; R(X) \\
\text{s.t. } f(X) \leq C_0 \quad \forall u = 1,2,\ldots, m
\]

Where \( R(X) \) is the required reliability, \( X \) is the basis random variable, \( C_0 \) is the pre-concerted cost of component, \( f(X) \) and \( g_u(X) \) are the function of design variable (cost, performance or parameter).

2.2.2. The reliability of component is given. Minimize the cost (or volume, weight) of component or maximize the performance and reliability is the constraint, the objective function is the cost of component, the model is shown as:

\[
\max \; f(X) \\
\text{s.t. } R(X) \leq R_0 \quad \forall u = 1,2,\ldots, m
\]

Where \( R(X) \) is the actual reliability of the component, \( R_0 \) is the pre-concerted reliability of the component.

2.3. Reliability optimization model in broad sense

If considering the reliability or probability of all the constraints, the probabilistic reliability optimization model should be built, which is called the reliability optimization model in broad sense for mechanical component. There are several models for different purposes.

The dynamic reliability model is shown as:

\[
\min \; E\left\{\sum_{i=1}^{m} f_i(X)\right\} , (i = 1, 2, \ldots, m) \\
\text{s.t. } \prod_{i=1}^{m} R_i \geq R^*(j = 1, 2, \ldots, n) \quad a_u \leq X_u \leq b_u, (u = 1, 2, \ldots, p) \quad g_i(X) = 0, (l = 1, 2, \ldots, q)
\]
Where \( f_i(X) \) is the \( i \)-th objective function, \( R_j \) is the reliability of the \( j \)-th system, \( R_0^k \) is the limit reliability value, \( X_u \) is the \( u \)-th design variable, \( a_u \) and \( b_u \) are the upper and lower boundary values of the \( u \)-th design variable \( X \), \( g(X) \) is the constraint of the \( l \)-th design variable \( X \).

3. Reliability optimization model of crankshaft

3.1. The objective function of crankshaft

The reliability optimization model of crankshaft is shown as:

\[
\min F(x) \\
\text{s.t. } R_0^i - P(g_i(x,X) > 0) \leq 0, (i=1,2,\ldots,m) \quad x^l_k \leq x_k \leq x^u_k, (k=1,2,\ldots,n) \quad x = \mu(X)
\]  

Where \( F \) is the objective function, \( x \) is the vector of the design variable, \( \mu \) is the mean value of the random variable \( X \), \( R_0^i \) is the objective reliability of the \( i \)-th probability constraint, \( P(g_i(x,X) > 0) \) is the probability when \( g_i(x,X) > 0 \), \( m \) is the number of probability constraints, \( x^l_k \) and \( x^u_k \) is the upper and lower boundary values of the \( k \)-th design variable, \( n \) is the number of the design variable.

Set \( F \) as the volume of the crankshaft, it is shown as:

\[
F(x) = \frac{\pi}{4} (0.324D_1^2 + 0.47D_2^2 + 0.3D_3^2 + 0.001312D_4 + 0.17D_5^2 + 0.0056874)
\]  

Where \( D_1 \) is the diameter of constrained neck journal, \( D_2 \) is the diameter of neck journal of main connecting rod, \( D_3 \) is the diameter of constrained neck journal, \( D_4 \) is the diameter of neck journal of balancing connecting rod.

3.2. Design variables of crankshaft

There are a lot of parameters which can have an effect on the performance and reliability of the crankshaft. According to the result of the reference [11-12], the design variables \( D_1, D_2, D_3, D_4, L_1 \) (Length of constrained neck journal), \( L_2 \) (Length of neck journal of main connecting rod), \( L_3 \) (Length of constrained neck journal) and \( L_4 \) (Length of neck journal of balancing connecting rod) have an impact on the reliability of the crankshaft. Therefore, these design variables are selected to the optimization variables, it is shown as:

\[
x = (x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8)^T = (D_1, D_2, D_3, D_4, L_1, L_2, L_3, L_4)^T
\]  

3.3. Constraints of crankshaft

3.3.1. Strength constraint

According to the stress-strength interference theory, the equation of state is described in the ultimate stress state:

\[
\text{SUBS} = \text{YIES} - \text{maxstr} > 0
\]  

Where \( \text{YIES} \) is the yield strength, \( \text{maxstr} \) is the maximum stress of crankshaft.

The reliability constraint of the strength is shown as:

\[
g_1(X) = P(\text{SUBS} > 0) - R_0^1 \geq 0
\]  

Where \( P(\text{SUBS} > 0) \) is the reliability of strength for crankshaft, \( R_0^1 \) is the minimum allowed value of the reliability of strength for crankshaft.

3.3.2. Stiffness constraint

To ensure the crankshaft to work properly, the stiffness should be considered in the design process. Take the allowed value of the stiffness: \( U_{\text{max}} = 0.05 \text{mm} \). The equation of state for the stiffness is shown as:

\[
\text{SUBU} = U_{\text{max}} - \text{maxu} > 0
\]  

Where \( \text{SUBU} \) is the stiffness reliability of the crankshaft, \( \text{maxu} \) is the maximum allowable displacement of crankshaft in the vertical direction.

The reliability constraint of the stiffness is shown as:

\[
g_2(X) = P(\text{SUBU} > 0) - R_0^2 \geq 0
\]  

Where \( P(\text{SUBU} > 0) \) is the reliability of stiffness for crankshaft, \( R_0^2 \) is the minimum allowed value of the reliability of stiffness for crankshaft.
3.3.3. Geometrical constraint. To ensure the crankshaft to be suited for complex assembly requirements, the equation of state for the geometry is shown as:

\[
\begin{align*}
g_3(X) &= 0.13 - x_1 = 0.13 - D_1 \geq 0 \\
g_4(X) &= x_2 - 0.11 = D_1 - 0.11 \geq 0 \\
g_5(X) &= 0.16 - x_2 = 0.16 - D_2 \geq 0 \\
g_6(X) &= x_3 - 0.14 = D_2 - 0.14 \geq 0 \\
g_7(X) &= 0.19 - x_3 = 0.19 - D_3 \geq 0 \\
g_8(X) &= x_4 - 0.17 = D_3 - 0.17 \geq 0 \\
g_9(X) &= 0.15 - x_4 = 0.15 - D_4 \geq 0 \\
g_{10}(X) &= x_5 - 0.13 = D_4 - 0.13 \geq 0 \\
g_{11}(X) &= 0.065 - x_5 = 0.065 - L_4 \geq 0 \\
g_{12}(X) &= x_6 - 0.045 = L_4 - 0.045 \geq 0 \\
g_{13}(X) &= 0.136 - x_6 = 0.136 - L_3 \geq 0 \\
g_{14}(X) &= x_7 - 0.116 = L_3 - 0.116 \geq 0 \\
g_{15}(X) &= 0.084 - x_7 = 0.084 - L_3 \geq 0 \\
g_{16}(X) &= x_8 - 0.064 = L_3 - 0.064 \geq 0 \\
g_{17}(X) &= 0.11 - x_8 = 0.11 - L_4 \geq 0 \\
g_{18}(X) &= x_9 - 0.09 = L_4 - 0.09 \geq 0
\end{align*}
\]

Considering that there are two failure modes: strength failure and stiffness failure, the reliability interval estimation of the crankshaft is shown as:

\[
\prod_{i=1}^{m} R_i \leq R_S \leq R_{\text{min}} \leq R_i
\]

Where \(m\) is the number of the failure, set \(m=2\), \(R_i\) is the reliability of each failure mode, \(R_{\text{min}}\) is the minimum value of \(R_i\), \(R_S\) is the system reliability.

3.4. Optimization algorithm

The genetic algorithm (GA) is employed to make reliability optimization for crankshaft. GA is a metaheuristic inspired by the process of natural selection and it is commonly used to generate high-quality solutions to optimization and search problems by relying on bio-inspired operators such as mutation, crossover and selection. The genetic algorithm which is based on reliability is written in MATLAB and ANSYS is employed to make strength and stiffness reliability analysis for crankshaft. The basic flowchart is shown as Figure 1.

Planning approaches and procedures of the genetic algorithm are shown as follows:

- According to the reliability optimization model of the crankshaft, determine the population size of genetic algorithm.
- According to the initial design value, determine the range of the design variable.
- Initialize the population in binary random way, decode the design variable, obtain the initial design variable value.
- Write the initial design variable value to a file.
- Invoke the ANSYS, read the initial design variable value, import them in reliability calculation file, calculate the strength and stiffness reliability of crankshaft.
Establish objective function
Beginning
Establish the population and evolution generation
Encode the chromosome
Generate initial population randomly
Decode to get the variable value
Assign the variable value and write it to a file
Invoke the ANSYS
Read the variable value
Calculate the reliability of strength and stiffness
Normalize the strength and stiffness reliability
Calculate the sum of reliability constraints
Add to the objective function as penalty term
Calculate the value of total objective function
Choose the best value of this generation
Satisfy stop standard?
Y
Output the optimum value
Decode the design variable
N
Check calculation in ANSYS
End
Generate a new colony
Mutation operator
Cross operator
Select operator

Figure 1. The basic flowchart of genetic algorithm
- The reliability analysis result is normalized with a base of the minimum allowed reliability value.
- Calculate the summation of reliability constraints for all individuals of each generation.
- Add the reliability constraint to the objective function as penalty term.
- Calculate the value of the total objective function, select the individual with the minimum value as the optimization value of this generation.
- Make a judgement of whether the evolution generations are satisfied.
- If the evolution generations are satisfied, then output the optimization value, decode the design variable, invoke the ANSYS to check the reliability result, finally exit the program.
- If the evolution generations are not satisfied, select, cross and mutate operators, produce a new generation, return to the third step.

4. Results and discussion
The convergence process of genetic algorithm for reliability optimization of crankshaft is shown as Figure 2. As seen in the chart, after the 4th generation, the change of the value of the total objective function presents a fairly stable status with two small fluctuations. The reliability optimization result is shown as Table 1.

| Design variable | D1  | D2  | D3  | D4  | L1  | L2  | L3  | L4  |
|-----------------|-----|-----|-----|-----|-----|-----|-----|-----|
| Initial design value/mm | 0.12| 0.15| 0.18| 0.14| 0.055| 0.126| 0.074| 0.1 |
| Optimization value/mm  | 0.11| 0.155| 0.195| 0.14| 0.055| 0.125| 0.095| 0.105|

The result shows that the design variable D4 and L1 do not change, the design variable D1 gets smaller, the rest of the design variable gets bigger. The strength and stiffness reliability value of the initial design for crankshaft are 0.94863 and 0.99953, the reliability optimization analysis both set the strength and stiffness reliability values as 0.997. For this reason, most design variable value become bigger and the total reliability of the optimized crankshaft is higher than that of the initial crankshaft.
5. Conclusion
This paper studies the reliability optimization of crankshaft for high-speed punch. The reliability optimization model (including the model in narrow sense and in broad sense) is introduced. The reliability optimization model of crankshaft is built, which considers the strength, stiffness, and geometry constraints. Genetic algorithm method is employed to make reliability optimization for crankshaft based on ANSYS and MATLAB. The reliability optimization results present that the most design variable value become bigger than the initial value under the condition of the strength and stiffness reliability are both set as 0.997. And the total reliability of the optimized crankshaft is higher than that of the initial crankshaft. The obtained results demonstrate that the genetic algorithm method can be effectively employed to investigate the reliability optimization of crankshaft for high-speed punch.

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