Investigation of half-quantum vortex in superfluid $^3$He-$A$ phase

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Abstract. In superfluid $^3$He-$A$, it was theoretically predicted that the half-quantum vortex (HQV) is stable where the order parameters $^d$ and $^\ell$ are perpendicular to each other in the order parameter configuration. However, the existence of the HQV has not been reported experimentally. Now we are trying to detect the HQV in superfluid $^3$He-$A$ confined in the parallel-plates sample cell with 12 $\mu$m gap, where $^d$ and $^\ell$ are perpendicular to each other, by using a NMR technique under a rotation at ISSP. In order to detect the HQV, we improved the magnet system for NMR so far, but we have not observed the HQV yet. We report here the experimental details and some results for investigating the HQV.

1. Introduction
The half-quantum vortex (HQV), which has a winding number 1/2, in the superfluid $^3$He-$A$ phase was first proposed by Volovik and Mineev in 1976 [1] and many theoretical [2–6] and experimental [7, 8] studies have been done on this subject. However the existence of the HQV has not been confirmed so far in experiments. The superfluid $^3$He is well known as the spin-triplet $p$-wave condensate, particularly the superfluid $^3$He-$A$ phase is the equal-spin pairing state, with the anisotropic energy gap. The order parameter for the $A$ phase, $A_{\mu j}$, is described as $A_{\mu j} = A_{d\mu}(\hat{m}_j + i\hat{n}_j)$ by using two vector fields $^d$, which is the spin part of the order parameter, and $^\ell = \hat{m} \times \hat{n}$, which is the orbital part. $^\ell$ must be perpendicular to the sample container wall due to the pair breaking effect. In the presence of a magnetic field $H$, $^d$ is perpendicular to $H$ due to the anisotropy of the magnetic energy. In the bulk liquid, $^\ell$ is parallel to $^d$ due to the dipole interaction. On the other hand, we can obtain the $^\ell \perp ^d$ configuration in the parallel-plate geometry as follows. A strong magnetic field whose strength is larger than that of the dipole field, $H \gtrsim 3$ mT, is applied directed to perpendicular to the surface of the parallel-plates which has a gap as thin as the dipole coherence length $\xi_D \sim 10$ $\mu$m [7, 8]. In the $^\ell \perp ^d$ configuration, it is shown that the HQV can be energetically stable than the singular vortex under certain conditions, such as a magnetic field on the order of 1 T, a rotation speed larger than several rad/sec, and a temperature near $T_c$ [6, 9]. In the $A$ phase, the information about an angle $\alpha$ between two vectors $^\ell$ and $^d$ is provided by cw-NMR measurements. The NMR resonance frequency $f$ is written as $f = f_0 + \frac{f_L^2}{2f_0} \cos 2\alpha$.
under the strong magnetic field [10], where \( f_L \) is the Larmor frequency and \( f_L^A \) is the pressure- and temperature-dependent longitudinal NMR frequency of the bulk A phase. The frequency shift from the Larmor frequency, \( \Delta f = f - f_L = f_L^A/2f_L \cos 2\alpha \), is positive in the bulk liquid, whereas it is negative in the parallel-plate geometry with \( \alpha = 90^\circ \) [7,8].

2. Experiments

We are trying to detect the HQV in the superfluid A phase confined in the parallel-plate sample cell, with cw-NMR method and the rotating cryostat at ISSP [11]. The sample cell is the same as that was used in ref. [8]. It consists of 220 alternately stacked polyimide films of a 12.5 µm thickness and a 25 µm thickness. In each 12.5 µm film, there are two holes of a 3.0 mm diameter and a channel connecting between them, 0.3 mm wide and 9.5 mm long. There is also one hole of a 3.0 mm diameter in 25 µm film. By vertically sandwiching the 12.5 µm film between the 25 µm films, a space of 12.5 µm in thickness and 3.0 mm in diameter and a through hole in stacked films were obtained. The sample cell is set to the rotating cryostat at ISSP; the surface of the parallel-plate sample is perpendicular to the rotation axis, \( \hat{z} \) axis.

At the ISSP rotating cryostat we could apply only a longitudinal magnetic field, \( H_L \), whose direction was parallel to \( \hat{z} \) and whose maximum strength was 100 mT, by one solenoidal coil. Recently we improved the current source power and the current lead ampacity. In order to improve the homogeneity of the magnetic field, we also added a new magnet composed of two pairs of saddle-shaped coils perpendicular to each other, named the inner coil and the outer coil (see Figure 1). So we can apply a transverse magnetic field, \( H_t \), to any direction perpendicular to \( \hat{z} \), and we can tilt the magnetic field direction. As a result of replacements of the current source and the cable ampacity, we are now able to apply the maximum current about 10 A corresponding to the maximum magnetic field of 1 T. In cw-NMR measurements, we set the operating frequency at 875 kHz corresponding to the NMR magnetic field \( H_0 = 27.0 \) mT in the normal phase. The absorption signal is observed by sweeping \( H_L \) under a liquid pressure at 3.01 MPa corresponding to \( T_c = 2.4 \) mK. Additionally, in order to prevent solid \(^3\)He from being formed on the surface of parallel-plates, we coated all the sample cell surface with 2.5 layers \(^4\)He before introducing \(^3\)He.

3. Results and Discussions

Figure 2(a) shows the cw-NMR absorption spectra in the normal phase with respect to a change of a current passing through the outer coil, \( I_s \), as a function of the longitudinal magnetic field \( H_L \). Applying the transverse magnetic field \( H_t = \beta I_s \) by the saddle-shaped coils, where \( \beta \) is the magnetic current ratio, it was found that the longitudinal resonance field of the NMR spectrum was shifted (Figure 2(a)) and the peak height of the NMR spectrum was changed (Figure 2(b)).
Maximum peak height in Figure 2(b) shows that the homogeneity of the magnetic field became better. A current which optimized the homogeneity of a magnetic field for the inner coil and the outer coil was around 2.0 A and 2.5 A, respectively.

The changes of the resonating longitudinal field are explained as follows. Since $H_l$, $H_t$, and $H_0$ satisfy the relation $H_0 = \sqrt{H_t^2 + H_l^2}$ at the resonant condition (Figure 2(d)), it is found that $H_l^2 = -\beta^2 I_s^2 + H_0^2$. Figure 2(c) shows the square of $H_l$ is well fitted by a quadratic function of the outer coil current $I_s$. We evaluated the value of $\beta$ from the fitting parameter, and we found that the value of $\beta$ for the inner coil and the outer coil was 0.454 mT/A and 0.377 mT/A, respectively. The ratio of $\beta$, 0.454/0.377, is equal to the ratio of the number of turns in the saddle-shaped coils, 180/150. Using the value of $\beta$, we calculated the tilt angle $\varphi$, which is written as $\varphi = \tan^{-1}\{H_t/H_l\}$ (Figure 2(d)). The tilt angle given by the outer coil current $I_s = 2.5$ A optimizing the magnetic field was $2^\circ$.

Figure 3 shows the cw-NMR absorption spectra with respect to a change of temperature and $H_l$ as a function of $\Delta f$. In the A phase, two peaks were observed; one small peak near the Larmor frequency and another large peak which has $\Delta f \approx -1.0$ kHz at 2.3 mK or $\Delta f \approx -3.2$ kHz at
2.0 mK. It is thought that negatively shifted large peaks are attributed to the NMR response of the A phase in the slab geometry as described above. By tilting the magnetic field of 2°, the amount of $\Delta f$ at the large peak increased as indicated by two arrows for each temperature. This means that applying $I_s = 2.5$ A to the saddle-shaped coil produces not only the better homogeneity of the magnetic field but also the correction for $\alpha$ closed to 90°. Tilting the magnetic field of $\phi$ corresponds to the changes of the angle $\alpha$ to $\alpha + \phi$. Then $\Delta f$ is rewritten as $\Delta f = f_L^2 / 2 f_L \cos 2(\alpha_0 + \phi)$, where $\alpha_0$ is an original angle between $\hat{l}$ and $\hat{d}$ without $H_t$.

By using $f_L = 875$ kHz, we evaluated that $f_L^2 = 1.9 \times 10^9$ Hz² and $\alpha_0 = 81^\circ$ at 2.3 mK, and $f_L^2 = 5.7 \times 10^9$ Hz² and $\alpha_0 = 85^\circ$ at 2.0 mK. According to ref. [12], $f_L^2$ is $2 \times 10^9$ Hz² at 2.3 mK and $6 \times 10^9$ Hz² at 2.0 mK. Experimental values of $f_L^2$ almost agree with literature values. The angle $\alpha_0$ should not depend on the temperature, however, the obtained angle is different from each other. The reason of this discrepancy is not clear. That the negatively shifted spectrum has an asymmetry shape having the tail toward to the Larmor frequency seems to show the existence of the non-uniform texture of $\hat{l}$ between parallel-plates. Both theoretical arguments on the NMR spectrum and more precise measurements are needed to explain the observed spectrum quantitatively.

Our recent theoretical calculation suggests that if the HQVs exist, the higher a rotation speed is, the smaller the amount of $|\Delta f|$ becomes. Unfortunately the experimental result does not show such a feature of the rotation dependent resonance frequency. Non-uniform texture may prevent the HQV from being formed. We have not tried to perform the cw NMR measurement a combined condition of a high magnetic field and a large rotation speed, which is suggested as the stabilized condition of the HQV theoretically. We will try to stabilize the HQV under a rotation applying a high $H_l$ and an optimized $H_t$, and detect the HQV by performing the cw-NMR measurements under rotation.

4. Summary
We are trying to detect the HQV in the superfluid A phase. In order to detect the HQV, we just improved the apparatus, and fabricated the new superconducting magnet. By performing cw-NMR measurements applying $H_t$ in the normal phase, we evaluated the performance of the new magnet and we found that the homogeneity of the NMR field was improved. We have not detected the HQV so far but found that $\hat{l}$ seemed to be not uniform.
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References
[1] Volovik G and Mineev V 1976 *JETP Lett.* 24 561
[2] Cross M C and Brinkman W F 1977 *J. Low Temp. Phys.* 27 683
[3] Salomaa M M and Volovik G E 1985 *Phys. Rev. Lett.* 55 1184
[4] Vakaryuk V and Leggett A J 2009 *Phys. Rev. Lett.* 103 057003
[5] Kawakami T, Tsutsumi Y and Machida K 2010 *J. Phys. Soc. Jpn.* 79 044607
[6] Nakahara M and Ohmi T 2014 *Phys. Rev. B* 89 104515
[7] Hakonen P J, Nummila K K, Simola J T, Skrbek L and Mamniashvili G 1987 *Phys. Rev. Lett.* 58 678
[8] Yamashita M, Ezumina K, Matsubara A, Sasaki Y, Ishikawa O, Takagi T, Kubota M and Mizusaki T 2010 *J. Low Temp. Phys.* 158 353
[9] Kondo K, Ohmi T, Nakahara M, Kawakami T, Tsutsumi Y and Machida K 2012 *J. Phys. Soc. Jpn.* 81 104603
[10] Vollhardt D and Wöllfle P 1990 *The Superfluid Phases of Helium 3* 1st ed (Taylor & Francis)
[11] Kubota M, Obata T, Ishiguro R, Yamashita M, Igarashi T, Hayata E, Ishikawa O, Sasaki Y, Mikhin N, Fukuda M, Kovacik V and Mizusaki T 2003 *Physica B: Condensed Matter* 329-333 1577
[12] Abragam A and Goldman M 1982 *Nuclear magnetism: order and disorder* (Clarendon Press)