LIGHT-FRONT CQM CALCULATIONS OF BARYON ELECTROMAGNETIC FORM FACTORS

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Abstract

The parameter-free predictions for the $N - P_{11}(1440)$ and $N - P_{33}(1232)$ electromagnetic transition form factors, obtained within our light-front constituent quark model using eigenfunctions of a baryon mass operator which includes a large amount of configuration mixing, are reported. The effects due to small components in the baryon wave functions, such as S’- and D-wave, are also investigated.

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1 INTRODUCTION

The electromagnetic (e.m.) excitation of Nucleon resonances, in the space-like region, represents a very interesting tool for gathering information on the internal structure of baryons and it is one of the major issues of the TJNAF research programme [1]. For investigating this topic we have developed a phenomenological approach [2], based on a constituent quark (CQ) model which features: i) a proper treatment of relativistic effects, achieved by formulating the model on the light-front (LF), see e.g. [3]; ii) baryon eigenfunctions of a mass operator that describes fairly well the mass spectrum [4], at variance with the widely adopted gaussian-like ansatz, see e.g. Ref. [5]; iii) a one-body approximation for the e.m. current able to reproduce the experimental data on the Nucleon and pion form factors.

2 GENERAL FORMALISM

In the LF hamiltonian dynamics [3] a baryon state, in the $u - d$ sector, $|\Psi_{J\lambda_n}^{TT}, \vec{P}\rangle$, is an eigenstate of: i) isospin, $T$ and $T_3$; ii) parity, $\pi$; iii) kinematical (non-interacting) LF angular momentum operators $j^2$ and $j_n$, where the vector $\hat{n} = (0, 0, 1)$ defines the spin quantization axis; iv) total LF baryon momentum $\vec{P} \equiv (P^+, \vec{P}_\perp) = \vec{p}_1 + \vec{p}_2 + \vec{p}_3$, where $P^+ = P^0 + \hat{n} \cdot \vec{P}$ and $\vec{P}_\perp \cdot \hat{n} = 0$. The state $|\Psi_{J\lambda_n}^{TT}, \vec{P}\rangle$ factorizes into $|\Psi_{J\lambda_n}^{TT}, \vec{P}\rangle |\vec{P}\rangle$, and the intrinsic part $|\Psi_{J\lambda_n}^{TT}, \vec{P}\rangle$ can be constructed from the eigenstate $|\psi_{J\lambda_n}^{TT}, \vec{P}\rangle$ of the canonical angular momentum, i.e. $|\Psi_{J\lambda_n}^{TT}, \vec{P}\rangle = \mathcal{R}^\dagger |\psi_{J\lambda_n}^{TT}, \vec{P}\rangle$, where the unitary operator $\mathcal{R}^\dagger = \prod_{j=1}^3 R_{Med}(\vec{k}_j, m_j)$, with $R_{Med}(\vec{k}_j, m_j)$ being the generalized Melosh rotation [3]. Then $|\psi_{J\lambda_n}^{TT}, \vec{P}\rangle$ satisfies the following mass equation

\begin{equation}
(M_0 + V) |\psi_{J\lambda_n}^{TT}, \vec{P}\rangle = M |\psi_{J\lambda_n}^{TT}, \vec{P}\rangle
\end{equation}

where $M_0 = \sum_{i=1}^3 \sqrt{m_i^2 + \vec{k}_i^2}$ is the free mass operator, $m_i$ the CQ mass ($m_u = m_d = 0.220 \text{ GeV}$ accordingly to [1]), $M$ the baryon mass, and $J(J + 1)$. $J_n$ are the eigenvalues of the operators $j^2$, $j_n$, respectively. The interaction $V$ has to be independent of the total momentum $P$ and invariant upon spatial rotations and translations. We can identify Eq. (1) with the baryon mass equation proposed by Capstick and Isgur ($CI$) [1]. Their CQ interaction is composed by a linear confining term, dominant at large separations, and a one-gluon-exchange ($OGE$) term, dominant at short separations, given by a central Coulomb-like potential and a spin-dependent part, responsible for the hyperfine splitting of baryon masses. The mass equation [1] has been accurately solved by expanding the eigenstates onto a large harmonic oscillator ($HO$) basis (up to 20 $HO$ quanta) and then by applying the Rayleigh-Ritz variational principle.

The CQ momentum distribution calculated from the baryon eigenfunctions of Eq. (1), with the $CI$ interaction, has a striking feature [3]: for a CQ three-momentum larger than 1 $\text{GeV}/c$, it is order of magnitude larger than momentum distributions evaluated from model functions, such as gaussian or power-law functions (cf. [3]). This fact is due to the smoothly singular $OGE$ part and has immediate consequences on the interpretation of the
resonances, for instance, the Roper resonance is not a simple (first) radial excitation of the Nucleon and it has a large mixed-symmetry S’ component ($P_{S_{Roper}} = 9.3\%$).

In the LF formalism the space-like e.m. form factors are related to the matrix elements of the plus component of the current, $I^+ = T^0 + \hat{n} \cdot \vec{I}$, with the standard choice $q^+ = q^0 + \hat{n} \cdot \vec{q} = P_f^+ - P_i^+ = 0$, that allows to suppress the contribution of the pair creation from the vacuum $|\bar{\psi}_{vf}\psi_f\rangle$.

For a $\frac{1}{2}^+$ baryon in the final state, e.g. the Nucleon ($f = N$) or the Roper resonance ($f = R$), the Dirac and Pauli form factors, $F^{I\tau}_{1(2)}(Q^2)$ ($\tau = p$ or $n$, are given by

$$F^{I\tau}_{1}(Q^2) = \frac{1}{2} \text{Tr}[I^+(\tau)] \quad , \quad F^{I\tau}_{2}(Q^2) = i \frac{M_f + M_N}{2Q} \text{Tr}[\sigma_{2} I^+(\tau)]$$

with $I^+_{\nu,\nu}(\tau) = \bar{u}_L(\hat{P}_f, \nu_f) \left\{ F^{I\tau}_{1}(Q^2) \gamma^\nu + F^{I\tau}_{2}(Q^2) i\sigma^{\nu\rho}q_\rho/(M_f + M_N) \right\} u_L(\hat{P}_i, \nu)$, where $Q^2 = -q \cdot q$ is the squared four-momentum transfer, $\sigma^{\nu\rho} = \frac{i}{2}[\gamma^\nu, \gamma^\rho]$, $u_L(\hat{P}_i, \nu)$ [$u_L(\hat{P}_f, \nu_f)$] the Nucleon [final baryon] LF-spinor, $\sigma_2$ a Pauli matrix. For the excitation to a $\Delta$ resonance, or in general to a $\frac{3}{2}^+$ baryon, the kinematic-singularity free form factors $G_{1,2,3}$ [7] are related to the LF matrix elements of $I^+$ as follows

$$I^+_{\frac{1}{2}} = \frac{Q}{\sqrt{2}} \left[ G_1(Q^2) + M_\Delta - \frac{M_N}{2} G_2(Q^2) \right]$$
$$I^+_{\frac{1}{2}} = -\frac{Q^2}{\sqrt{6}} \left[ G_1(Q^2) + M_\Delta \frac{G_2(Q^2)}{2} - \frac{M_\Delta - M_N}{M_\Delta} G_3(Q^2) \right]$$
$$I^+_{\frac{1}{2}} = \frac{Q}{\sqrt{6}} \left[ G_1(Q^2) \frac{M_N}{M_\Delta} - \frac{M_\Delta - M_N}{2M_\Delta} G_2(Q^2) - \frac{Q^2}{M_\Delta} G_3(Q^2) \right]$$
$$I^+_{\frac{1}{2}} = -\frac{Q^2}{2\sqrt{2}} G_2(Q^2)$$

with $I^+_{\nu,\nu}(\tau) = \langle \frac{1}{2}\tau, 10|TT_3| \bar{u}_L(\hat{P}_f, \nu_f) \Gamma^\nu + u_L(\hat{P}_i, \nu) \rangle$, with $\Gamma^\nu = G_1(Q^2) K_{1\nu}^+ + G_2(Q^2) K_{2\nu}^+ + G_3(Q^2) K_{3\nu}^+$ (tensors $K_{im}^\nu$ are defined in [4]). Differently from the case of a $\frac{1}{2}^+$ baryon, the number of form factors for the excitation of a $\frac{3}{2}^+$ baryon is not equal to the one of the LF matrix elements of $I^+$, cf. Eq. (3). For the exact current the inversion of Eq. (3) is unique. Since we adopt a one-body approximation for $I^+$ (see below) different choices of LF matrix elements can lead to different predictions for the $G_i$ form factors. In the actual calculation for the N-Â transition we consider two different prescriptions for extracting the $G_i$ form factors from Eq. (3): i) $G_1$ and $G_3$ are obtained from the first three equations in (3), while $G_2$ is directly taken from the fourth one (prescription I); ii) all the $G_i$ form factors are extracted from the first three equations (prescription II).

3 RESULTS

Elastic and transition form factors have been evaluated using eigenvectors of Eq. (11) and approximating the $I^+$ component of the e.m. current by the sum of one-body $CQ$ currents.
the N-Roper helicity amplitudes $A^{(n)}_{5/2}$ can provide an overall description of relative-ly small quantities, such as the ratios $E_1/M_1$ and $S_1/M_1$. After fixing the $CQ$ form factors, we have calculated, without free parameters, the N-Roper helicity amplitudes $A^{(n)}_{1/2}(Q^2)$ and $S^{(n)}_{1/2}(Q^2)$, shown in Fig. 1 (see [2](c)).

Our results both for $A^{(n)}_{1/2}(Q^2)$ and $S^{(n)}_{1/2}(Q^2)$ exhibit a remarkable reduction (bringing our predictions closer to the experimental analyses [2](a,b,c)) with respect to non-relativistic [3] as well as relativistic [4] predictions, based on simple gaussian-like wave functions. It is worth noting that the helicity amplitudes $S^{(n)}_{1/2}(Q^2)$ and $S^{(n)}_{3/2}(Q^2)$ are sizably sensitive to the presence of the mixed-symmetry $S'$ component in the $CI$ wave function.

The magnetic form factor $G^{N-\Delta}_{M_{1/2}}(Q^2)$ and the ratios $E_1/M_1 = -G^{N-\Delta}_{E}(Q^2) / G^{N-\Delta}_{M_{1/2}}(Q^2)$ and $S_1/M_1 = (\sqrt{K^+K^-}/4M^2) G^{N-\Delta}_{C}(Q^2) / G^{N-\Delta}_{M_{1/2}}(Q^2)$ (see Ref. [3] for the relation with the $G_i$ form factors), calculated within our model without free-parameters (see also [2](b)) are shown in Fig. 2 (a,b,c), respectively. The differences between the prescription I and II are not so relevant for $G^{N-\Delta}_{M_{1/2}}$ as they are for $E_1/M_1$ and $S_1/M_1$; however the effect due to the D-wave component is always small for both prescriptions (indeed $P^{1/2} = 1.1\%$). Although two-body $CQ$ currents are lacking in the approximation [2] it is encouraging that our effective current can provide an overall description of relatively small quantities, such as the ratios $E_1/M_1$ (≈ 5 %) and $S_1/M_1$ (≈ 10 %) for $Q^2$ up to few GeV/c$^2$. Finally in Fig. 2 (d) the ratio of $G^{N-\Delta}_{M_{1/2}}(Q^2)$ (obtained in prescription II) and the isovector part of the Nucleon magnetic form factor, $G^{0}_{M}(Q^2) - G^{0}_{M}(Q^2)$, is shown. It can clearly be seen that this ratio is largely insensitive to the presence of $CQ$ form factors, whereas it is sharply affected by the spin-dependent part of the $CI$ potential, which is generated by the chromomagnetic interaction.

In conclusion, we have reported the calculations of the e.m. form factors for the transition $N-P_{11}(1440)$ and $N-P_{33}(1232)$, obtained within our approach based on a relativistic $CQ$ model, baryon eigenfunctions of a mass operator and an effective one-body $CQ$ current. The results allow an overall description of the data, but they also indicate the necessity of the introduction of at least a two-body term in the $CQ$ current, in order to give accurate predictions for ”small” form factors (like, e.g. $E_1$ or $S_1$ for the $N-\Delta$ transition). Finally, we have shown that the determination of $G^{N-\Delta}_{M_{1/2}}(Q^2) / (G^{0}_{M}(Q^2) - G^{0}_{M}(Q^2))$ could provide relevant information on SU(6) breaking effects in N and $\Delta$ wave functions, without substantial model dependence.
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Figure 1. The $N-P_{11}(1440)$ helicity amplitudes $A_{1/2}^{p(n)}(Q^2)$ and $-S_{1/2}^{p(n)}(Q^2)$ vs. $Q^2$. Solid line: LF calculation obtained by using baryon wave functions corresponding to the $CI$ interaction and the e.m. current with the $CQ$ form factors determined in (a); dashed line: the same as the solid one, but without the $S'$ component in the baryon eigenfunctions; dot-dashed line: LF calculation obtained by using the gaussian functions of Ref. without CQ form factors. The long-dashed and triple-dot-dashed lines correspond to the non-relativistic $q^3G$ and $q^3$ models of Ref. . Full dots: PDG values; full squares and open dots: phenomenological analyses of Ref. (a) and (b), respectively. Within the hybrid $q^3G$ model $S_{1/2}^{p(n)}(Q^2) = 0$, whereas within the non-relativistic $q^3$ model only $S_{1/2}^{n}(Q^2) = 0$. (After (c))
Figure 2. (a) $G_{M}^{N-\Delta}(Q^2)/3G_{D}(Q^2)$, with $G_{D}(Q^2) = 1/(1 + Q^2/0.70)^2$, vs $Q^2$. The thick and thin lines correspond to the LF calculations with and without the D-wave in the $\Delta$ eigenstate. The e.m. current (4) with the $CQ$ form factors determined in [2](a) has been adopted. Solid lines: prescription I (see text). Dashed lines: prescription II (see text). Triangles: [11](a); full squares: [11](b); open dots: [11](c); full dots: [11](d). - (b) The same as in (a), but for $E_1/M_1$. Full dots: PDG [9]; diamonds: [12](a); open squares: [12](b); triangles: [12](c). - (c) The same as in (b) but for $S_1/M_1$. - (d) The ratio $G_{M}^{N-\Delta}(Q^2)/(G_{M}^{p}(Q^2) - G_{M}^{n}(Q^2))$ vs $Q^2$. Solid line: our calculation (prescription II) with the $CI$ baryon eigenfunctions and $CQ$ form factors; dashed line: the same as the solid line, but without $CQ$ form factors; short-dashed line: the same as the dashed line, but with the baryon eigenfunctions corresponding to the spin-independent part of the $CI$ interaction [4]; dotted line: the same as the dashed line, but retaining only the confining part of the $CI$ potential. (After [3](b)).