An Approach to Construct a Control Chart for Standard Deviation Based on Six Sigma

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Abstract In present scenario companies started applying six sigma concept in their manufacturing process, which results in lesser number of defects compared with existing Shewhart control chart. In this article an attempt is made to construct a control chart based on standard deviation with six sigma especially planned for companies applying this technique in their business and constructed table also presented for the experts to take quick decisions.

Keywords Control Chart; Process Control; Six Sigma; Six Sigma Quality Level

1. Introduction

The control charts suggested by W.A. Shewhart (1931) was based on 3 sigma control limits. Radhakrishnan (2009a, 2009b, 200c, 2010a, 2010b, 2010c, 2011 and 2012) described as if the same charts are used for the products of the companies which adopt Six sigma concept in their process, and then no point will fall outside the control limits because of the improvement in the quality. So a separate control chart is necessary to check the results of the companies, which adopt Six sigma. The practice was aimed at taking action to improve the overall performance and the companies, which are practicing Six sigma, are expected to produce 3.4 or less number of defects per million opportunities. In this research an effort is made to construct a control chart based on the concept Six sigma for Standard deviation and the Table 1 is also provided for the experts to build rapid judgments on the floor itself.

2. Concepts and Terminologies

Upper Specification Limit (USL)

It is the greatest amount specified by the producer for a process or product to have the acceptable performance.
Lower Specification Limit (LSL)

It is the smallest amount specified by the producer for a process or product to have the acceptable performance.

Tolerance Level (TL)

It is the difference between USL and LSL, TL = USL - LSL

Process Capability (Cp)

This is the ratio of tolerance level to six times standard deviation of the process.

$$c_p = \frac{TL}{6\sigma} = \frac{USL - LSL}{6\sigma}$$

Standard Deviation (σ)

For many purposes standard deviation is the most useful measure of dispersion of a set of numbers. It is the root mean square value.

Quality Control Constant¹ (O₆σ)

The constant O₆σ introduced in this paper to determine the control limits based on six sigma initiatives for standard deviation.

Quality Control Constant² (C₄)

The constant C₄ introduced in this paper to determine the control limits based on 3sigma for standard deviation.

3. Construction of Control Chart Based on Six Sigma Initiatives for Standard Deviation

Fix the tolerance level (TL) and process capability (Cₚ) to determine the process standard deviation (σ₆σ). Apply the value of σ₆σ in the control limits \( \bar{S} \pm \left( O_{6\sigma} \times \sqrt{1-c_z^2} \right) \times \sigma_{6\sigma} \), to get the control limits based on six sigma initiatives for Standard deviation. The value of O₆σ is obtained using \( p(z \leq z_{6\sigma}) = 1 - \alpha_i, \alpha_i = 3.4 \times 10^{-6} \) and z is a standard normal variate. For a specified TL and Cₚ of the process, the value of σ (termed as σ₆σ) is calculated from \( c_p = \frac{TL}{6\sigma} \) using a C program and presented in Table 1.2 for various combinations of TL and Cₚ. The control limits based on six sigma initiatives for Standard deviation chart are

\[
UCL_{6\sigma} = \bar{S} + \left( O_{6\sigma} \times \sqrt{1-c_z^2} \right) \times \sigma_{6\sigma}
\]

Center line CL₆σ = \( \bar{S} \)

\[
LCL_{6\sigma} = \bar{S} - \left( O_{6\sigma} \times \sqrt{1-c_z^2} \right) \times \sigma_{6\sigma}
\]
Table 1: Values for a specified \( C_p \) and \( TL \)

| \( C_p \) | 0.0131 | 0.0132 | 0.0133 | 0.0134 | 0.0135 |
|---|---|---|---|---|---|
| 1.0 | 0.0022 | 0.0022 | 0.0022 | 0.0022 | 0.0023 |
| 1.1 | 0.0020 | 0.0020 | 0.0020 | 0.0020 | 0.0020 |
| 1.2 | 0.0018 | 0.0018 | 0.0018 | 0.0019 | 0.0019 |
| 1.3 | 0.0017 | 0.0017 | 0.0017 | 0.0017 | 0.0017 |
| 1.4 | 0.0016 | 0.0016 | 0.0016 | 0.0016 | 0.0016 |
| 1.5 | 0.0015 | 0.0015 | 0.0015 | 0.0015 | 0.0015 |
| 1.6 | 0.0014 | 0.0014 | 0.0014 | 0.0014 | 0.0014 |
| 1.7 | 0.0013 | 0.0013 | 0.0013 | 0.0013 | 0.0013 |
| 1.8 | 0.0012 | 0.0012 | 0.0012 | 0.0012 | 0.0012 |
| 1.9 | 0.0011 | 0.0011 | 0.0011 | 0.0011 | 0.0011 |
| 2.0 | 0.0010 | 0.0010 | 0.0010 | 0.0010 | 0.0010 |
| 2.1 | 0.0010 | 0.0010 | 0.0010 | 0.0010 | 0.0010 |
| 2.2 | 0.0010 | 0.0010 | 0.0010 | 0.0010 | 0.0010 |
| 2.3 | 0.0009 | 0.0009 | 0.0009 | 0.0009 | 0.0009 |
| 2.4 | 0.0009 | 0.0009 | 0.0009 | 0.0009 | 0.0009 |
| 2.5 | 0.0009 | 0.0009 | 0.0009 | 0.0009 | 0.0009 |

4. Conditions for Application

- Human involvement should be less in the manufacturing process
- The company adopts Six sigma quality initiatives in its processes

Example

The following results of inside Diameter Measurement (mm) for Automobile Engine Piston Rings.

Table 2: Inside Diameter Measurement (mm) for Automobile Engine Piston Rings and Standard deviation \( (S_i) \)

| Sample Number | Observation | \( S_i \) |
|---|---|---|
| 1 | 74.030 74.002 74.019 73.992 | 74.008 0.0148 |
| 2 | 73.995 73.992 74.001 74.011 | 74.004 0.0075 |
| 3 | 73.986 74.024 74.021 74.005 | 74.002 0.0147 |
| 4 | 74.002 73.996 73.993 74.015 | 74.009 0.0091 |
| 5 | 73.992 74.007 74.015 73.989 | 74.014 0.0122 |
| 6 | 74.009 73.994 73.997 73.985 | 73.993 0.0087 |
| 7 | 73.995 74.006 73.994 74.000 | 74.005 0.0055 |
| 8 | 73.985 74.003 73.993 74.015 | 73.988 0.0123 |
| 9 | 74.008 73.995 74.009 74.005 | 74.004 0.0055 |
| 10 | 73.998 74.000 73.990 74.007 | 73.995 0.0063 |
| 11 | 73.994 73.998 73.994 73.995 | 73.990 0.0029 |
| 12 | 74.004 74.000 74.007 74.000 | 73.996 0.0042 |
| 13 | 73.983 74.002 73.998 73.997 | 74.012 0.0105 |
| 14 | 74.006 73.967 73.994 74.000 | 73.984 0.0153 |
| 15 | 74.012 74.014 73.998 73.999 | 74.007 0.0073 |
| 16 | 74.000 73.984 74.005 73.998 | 73.996 0.0078 |
| 17 | 73.994 74.012 73.986 74.005 | 74.007 0.0106 |
| 18 | 74.006 74.010 74.018 74.003 | 74.000 0.0070 |
| 19 | 73.984 74.002 74.003 74.005 | 73.997 0.0085 |
n = 5 and $\bar{S} = 0.0094$

4.1a. Three Sigma Control Limits for Standard Deviation Chart

The $3\sigma$ control limits suggested by Shewhart (1931) are

$$\bar{S} \pm \left(3 \sqrt{1-c_i^2} \left(\frac{S}{c_i}\right)\right)$$

For $n = 4$, $c_4 = 0.9400$ (Quality control factor, W.A. Shewhart)

$$\begin{align*}
\text{UCL}_{3\sigma} &= \bar{S} + \left(3 \sqrt{1-c_i^2} \left(\frac{S}{c_i}\right)\right) = 0.0094 + \left(3 \sqrt{1-0.9400^2} \left(0.0094 / 0.9400\right)\right) \\
&= 0.0094 + 0.0102 = 0.0196 \\
\text{Central Line CL}_{1\sigma} &= \bar{S} = 0.0094 \\
\text{LCL}_{3\sigma} &= \bar{S} - \left(3 \sqrt{1-c_i^2} \left(\frac{S}{c_i}\right)\right) = 0.0094 - \left(3 \sqrt{1-0.9400^2} \left(0.0094 / 0.9400\right)\right) \\
&= 0.0094 - 0.0102 = 0
\end{align*}$$

From the resulting Figure 1 that the process is in control, since all the samples lie inside the control limits.

4.1b. Control Limits Based on Six Sigma Initiatives for Standard Deviation Chart

For a given TL = 0.0133 (USL-LSL = 0.0162-0.0029) & $C_p = 1.5$, it is found from the Table 1 that the value of $\sigma_{6\sigma}$ is 0.0015. The control limits based on six sigma initiatives for Standard deviation chart for a specified TL and $O_{6\sigma}$ are

$$\bar{S} \pm \left(4.831 \times \sqrt{1-c_i^2} \right) \times \sigma_{6\sigma}$$

with

$$\begin{align*}
\text{UCL}_{6\sigma} &= \bar{S} + \left(O_{6\sigma} \times \sqrt{1-c_i^2}\right) \times \sigma_{6\sigma} = 0.0094 + \left(4.831 \times \sqrt{1-0.9400^2} \times 0.0015\right) = 0.0119 \\
\text{Center line CL}_{6\sigma} &= \bar{S} = 0.0094 \\
\text{LCL}_{6\sigma} &= \bar{S} - \left(O_{6\sigma} \times \sqrt{1-c_i^2}\right) \times \sigma_{6\sigma} = 0.0094 - \left(4.831 \times \sqrt{1-0.9400^2} \times 0.0015\right) = 0.0069
\end{align*}$$

From the resulting Figure 1 that the sample numbers 1, 3, 5, 8, 14, 21 and 25 goes above the upper control limit and the sample numbers 7, 9, 10, 11 and 12 goes below the lower control limit. Therefore the process does not exhibit statistical control.
5. Conclusion

In this paper, a procedure is given to construct a control chart based on six sigma initiatives for Standard deviation with an example. It is found that the process was not in control even when Six Sigma initiatives are adopted. It is very clear from the comparison that when the process is centered with reduced variation many points fall outside the control limits than the 3 sigma control limits, which indicate that the process is not in the level it was expected. So a correction in the process is very much required to reduce the variations. In future, all the companies are adopting this technique instead of the existing Shewhart chart in their organization.

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