USING COSMIC MICROWAVE BACKGROUND LENSING TO CONSTRAIN THE MULTIPlicative BIAS OF COSMIC SHEAR

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ABSTRACT

Weak gravitational lensing is one of the key probes of cosmology. Cosmic shear surveys aimed at measuring the distribution of matter in the universe are currently being carried out (Pan-STARRS) or planned for the coming decade (DES, LSST, EUCLID, WFIRST). Crucial to the success of these surveys is the control of systematics. In this work, a new method to constrain one such family of systematics, known as multiplicative bias, is proposed. This method exploits the cross-correlation between weak-lensing measurements from galaxy surveys and the ones obtained from high-resolution cosmic microwave background experiments. This cross-correlation is shown to have the power to break the degeneracy between the normalization of the matter power spectrum and the multiplicative bias of cosmic shear and to be able to constrain the latter to a few percent.

Key words: cosmic background radiation – cosmological parameters – cosmology: observations – large-scale structure of universe

Online-only material: color figure

1. INTRODUCTION

Cosmic shear probes the distribution of matter in the universe by measuring the distortions it induces in the ellipticities of background galaxies. In the past decade, this technique has emerged as one of the most promising probes for cosmology. Thanks to large galaxy surveys that are currently under way (Pan-STARRS, DES or in the works (LSST, EUCLID, WFIRST), weak lensing promises to tightly constrain the large-scale properties of the universe and to probe the nature of dark energy to unprecedented accuracy (Hu & Tegmark 1999).

Because systematics are one order of magnitude larger than the cosmological signal, their control is absolutely critical for the success of this experimental program. Several different algorithms exist to correct for the instrumental distortions of the point-spread function and (in the case of ground based surveys) for the atmospheric seeing (Heymans et al. 2006). As shown by the Shear Testing Programme (Heymans et al. 2006), while the application of these algorithms corrects for most systematics, it can introduce bias in the data. Such biases have been cataloged into three generic families: multiplicative and additive biases in shear measurements and errors in redshift measurements (Heymans et al. 2006; Huterer et al. 2006; Amara & Refregier 2007). The focus of the present work is multiplicative bias. The lack of precise knowledge of multiplicative bias leads to a dramatic degradation in the accuracy of the cosmological parameters thus measured (Huterer et al. 2006; Amara & Refregier 2008; Semboloni et al. 2008): for example, lack of percent-level constraints on multiplicative bias can lead to an increase in the errors in the value of Ωm and σ8 of 100% or more (Huterer et al. 2006). Quite generally, multiplicative bias is insidious because it does not show any scale dependence that can be exploited to decouple it from the weak-lensing signal. As such, it is completely degenerate with the normalization of the matter power spectrum: if the observed ellipticities were the only information available, then it would always be possible to trade a non-zero multiplicative bias for a variation in the total matter in the universe responsible for the lensing signal. A first solution to this problem was proposed in Vallinotto et al. (Vallinotto et al. 2010), where galaxies’ size and luminosity information is used to constrain this bias. Such a solution, however, relies on somehow quantifying the impact of weak lensing on the populations of galaxy sizes and luminosities.

The present work proposes an alternative way to constrain shear multiplicative bias by cross-correlating it with the weak lensing of the cosmic microwave background (CMB). Intuitively, these two probes measure essentially the same observable (albeit extending over different redshift ranges), but using two completely different techniques. Because the systematics affecting these two techniques have completely different natures, cross-correlating these two signals is an effective way to constrain their impact.

2. THE CMB LENSING FIELD

Experiments aimed at measuring small-scale temperature and polarization fluctuations of the CMB have recently reached the sensitivity required to reconstruct the effective deflection field arising from the dark matter structures present between the last scattering surface and the observer. CMB lensing was first detected by Wilkinson Microwave Anisotropy Probe (Smith et al. 2007) and recent measurements from ACT and SPT have reported the first detection of its power spectrum (Das et al. 2011; van Engelen et al. 2012). The measurement of the lensing of CMB through quadratic optimal estimators (Hu & Okamoto 2002) exploits the statistical properties of the primary CMB anisotropies. When considering lensing measurements carried out using data from CMB polarization experiments, the dominant source of noise is the sample variance of the CMB fluctuations. Further systematics include emission from unresolved radio and dusty star-forming galaxies and thermal and kinetic Sunyaev–Zeldovich effects (Millea et al. 2011). These systematics are completely uncorrelated from the ones

1 http://pan-starrs.ifa.hawaii.edu/public/
2 http://www.darkenergysurvey.org/
3 http://www.lsst.org/
4 http://www.physics.princeton.edu/act
5 http://pole.uchicago.edu/
characterizing cosmic shear measurements, which are related to the treatment of galaxy images (in particular, atmospheric seeing and the correction of anisotropies in the point-spread function) and to the measurement of their ellipticities.

The lensing convergence \( \kappa(\hat{n}, \chi_f) \) measured for a source at comoving distance \( \chi_f \) along a line of sight in the direction \( \hat{n} \) (Bartelmann & Schneider 2001) is defined as

\[
\kappa(\hat{n}, \chi_f) = C \int_0^{\chi_f} d\chi W_l(\chi, \chi_f) \frac{\delta(\hat{n}, \chi)}{a(\chi)},
\]

where \( C = 3 \Omega_m H_0^2/(2\pi^2) \) and \( W_l(\chi, \chi_f) = \chi(\chi_f - \chi)/\chi_f \) is the lensing window function. High-resolution CMB experiments allow the reconstruction of the convergence field \( \kappa_{\text{obs}}(\hat{n}, \chi_{\text{LSS}}) = \kappa_{\text{obs}}(\hat{n}) \), which extends all the way to the last scattering surface. In general, the convergence measured from cosmic shear data in a given redshift bin will be properly normalized (Hanson et al. 2011) so as not to include the low-multipole cutoff that is imposed by the geometry of the footprint is neglected. Second, it is assumed that in this (imaging) regime the noise term \( n_c, \hat{n} \) (or more precisely a set of them) and the correction of anisotropies in the point-spread function—can be introduced by the reconstruction procedure (Kesden et al. 2003). On the other hand, the optimal quadratic estimators used to reconstruct the deflection field (Bartelmann & Schneider 2001) is defined as

\[
\kappa_{\text{obs}}(\hat{n}) = \kappa_{\text{true}}(\hat{n}) + n_c(\hat{n}),
\]

where an additive bias \( n_c, \hat{n} \) (or more precisely a set of them) can be introduced by the reconstruction procedure (Kesden et al. 2003). On the other hand, the optimal quadratic estimators used to reconstruct the deflection field

\[
\kappa_{\text{obs}}(\hat{n}, \chi_{\text{LSS}}) = \kappa_{\text{obs}}(\hat{n}),
\]

which extends all the way to the last scattering surface. In general, the convergence measured from cosmic shear data in a given redshift bin will be properly normalized (Hanson et al. 2011) so as not to include the low-multipole cutoff that is imposed by the geometry of the footprint is neglected.

3. COSMIC SHEAR AND MULTIPLICATIVE BIAS

Weak-lensing surveys measure cosmic shear in redshift bins of finite thickness. The convergence they aim at measuring is

\[
\kappa_{\text{true}}(\hat{n}, z_0) = \int_0^{\infty} d\chi \eta(\chi) \kappa(\hat{n}, \chi),
\]

where \( z_0 \) denotes the center of a redshift bin of thickness \( \Delta z \) and \( \eta(\chi) \) is a selection function for the given redshift bin, normalized so that \( \int_0^{\infty} d\chi \eta(\chi) = 1 \). The convergence value measured from cosmic shear data in a given redshift bin will in general differ from the “true” value because of the (possibly redshift dependent) multiplicative bias, so that

\[
\kappa_{\text{obs}}(\hat{n}, z_0) = b(z_0) \kappa_{\text{true}}(\hat{n}, z_0).
\]

In what follows, a single redshift slice with \( z \in [0.9, 1] \) is considered and on such a redshift slice the multiplicative bias is assumed to be constant. It is straightforward to extend this analysis to other redshift ranges.

4. CORRELATION CALCULATION

Given a pair of surveys, the data set will consist of the observed \( \{\kappa_{\text{obs}}^{\text{C}}, \kappa_{\text{obs}}^{\text{P}}\} \) over the patch of sky where the two surveys overlap. From Equations (1) to (3), it is straightforward to obtain expressions for the elements appearing in the covariance matrix of the joint \( \{\kappa_{\text{obs}}^{\text{C}}, \kappa_{\text{obs}}^{\text{P}}\} \) data set. Let \( \hat{n}_i \) and \( \theta_{ij} \) denote the direction of the \( i \)th pixel and the angular separation between pixels directed along \( \hat{n}_i \) and \( \hat{n}_j \), respectively. Furthermore, let \( \alpha \) be the matter power spectrum normalization, so that \( P(k) = \alpha^2 \mathcal{P}(k) \), where \( \mathcal{P}(k) \) denotes the shape of the power spectrum. Then, defining

\[
g(\chi) \equiv \int_{\chi}^{\infty} d\chi W_L(\chi, \chi_1) \eta(\chi_1),
\]

\[
\xi(\chi, \theta, \bar{\theta}) \equiv \int_{\chi_0}^{\infty} \frac{1}{2 \pi^2} J_0(\theta) e^{-\lambda(\theta, \bar{\theta})} \alpha^2 \mathcal{P}
\]

and using Limber’s approximation, the correlations between the different data sets (denoted for brevity by \( \langle \hat{\kappa}_{\mu, \nu}^{\text{obs}}(\hat{n}_i, \kappa_{\nu, \nu}^{\text{obs}}(\hat{n}_j) \rangle \equiv \langle \hat{\kappa}_{\mu, \nu} \rangle_{ij} \) with \( \{\mu, \nu\} = \{c, s\} \)) are given by the following expressions:

\[
\langle \kappa_c, \kappa_c \rangle_{ij} = C^2 b^2 \int_0^{\infty} d\chi \frac{g(\chi) W_L(\chi, \chi_{\text{LSS}})}{\bar{a}(\chi)^2} \xi(\chi, \theta_{ij}, \bar{\theta}_{cc}),
\]

\[
\langle \kappa_c, \kappa_s \rangle_{ij} = C^2 b^2 \int_0^{\infty} d\chi \frac{W_L(\chi, \chi_{\text{LSS}})}{\bar{a}(\chi)^2} \xi(\chi, \theta_{ij}, \bar{\theta}_{cs}),
\]

\[
\langle \kappa_s, \kappa_s \rangle_{ij} = C^2 b^2 \int_0^{\infty} d\chi \frac{g^2(\chi)}{\bar{a}(\chi)^2} \xi(\chi, \theta_{ij}, \bar{\theta}_{ss}) + \delta_{ij} \sigma_{\text{rms}}^2
\]

where \( \bar{a}_{\text{rms}} \equiv \bar{a}_{\text{rms}} \bar{a}_{\text{rms}} (\bar{L}_{\text{rms}}^2 + \bar{L}_{\text{rms}}^2)^{1/2} \). Here, \( \sigma_{\text{rms}} \) is the rms of the shear measurement in absence of signal—due to the intrinsic ellipticities of galaxies and measurement errors (scaling as \( N^{-1/2} \), where \( N \) is the average number of galaxies in a \( \kappa_s \) pixel)—and \( \bar{L}_{\text{rms}} \) is fixed to be the limiting multipole corresponding to the \( \kappa_s \) pixel size. Similarly, \( \bar{L}_{\text{rms}} \) represents the cutoff on the modes contributing to \( \kappa_s \). As previously mentioned, \( \bar{L}_{\text{rms}} \) is the data covariance matrix with elements given by Equations (7)–(9). Because of the very large number of \( \kappa_{\text{obs}}^{\text{C}} \) pixels (\( 1.8 \times 10^6 \) for DES and \( 7.2 \times 10^6 \) for LSST), the evaluation of the Fisher matrix poses a numerical
Figure 1. Projected constraints on multiplicative biases and on the normalization of the matter power spectrum $\alpha$ obtained by cross-correlating cosmic shear data from DES (left panel) and LSST (right panel) with CMB lensing data. The red dashed curves show results for cross-correlating with Planck data and include a 4% prior on the value of $\alpha$. The solid black curve shows results for DES+WidePol (left) or LSST+CMBPol (right) cross-correlations. In the former case, complete overlap between the experiments’ footprints is assumed.

(A color version of this figure is available in the online journal.)

Table 1
Parameters Assumed in the Calculation for the Two Cosmic Shear Surveys

| Survey     | Pixel Size (arcmin²) | Galaxies per $\kappa_{\text{s,obs}}$ pixel | Sky Coverage (deg²) | $\sigma_{\text{s,N}}$ |
|------------|----------------------|------------------------------------------|----------------------|-----------------------|
| DES        | 10                   | 15                                       | 5000                 | 0.3                   |
| LSST       | 10                   | 100                                      | 20000                | 0.3                   |

Note. $\sigma_{\text{s,N}}$ is the rms of $\kappa_s$ in the absence of signal, due to shape noise and measurement errors for a single galaxy.

Table 2
Parameters Assumed in the Calculation for the CMB Lensing Surveys

| Survey     | $l_c$   | Sky Coverage (deg²) |
|------------|---------|----------------------|
| Planck     | 100     | All sky              |
| WidePol    | 300     | 4000                 |
| CMBPol     | 1000    | All sky              |

The left panel of Figure 1 shows the constraints on $\{\alpha, b\}$ projected for DES+Planck (red dashed contour) and for DES+WidePol (black solid contour). In general, cross-correlating cosmic shear and CMB lensing allows us to break the degeneracy between the normalization of the matter power spectrum and cosmic shear’s multiplicative bias and to constrain the latter. Because of the limited number of $\kappa_{\text{s,obs}}$ pixels that can be reconstructed using Planck data (assuming the value of $l_c = 100$), for DES+Planck and LSST+Planck a 4% prior on $\alpha$ (consistent with current constraints on $\sigma_{\alpha}$) is assumed to improve the constraints. In the DES+WidePol case, on the other hand, complete overlap between the experiments’ footprints is assumed. Under these assumptions, the Fisher matrix estimates show that DES+Planck (DES+WidePol) data should allow us to constrain shear multiplicative bias $b$ to about 4% (2%).

The right panel of Figure 1 shows projections for LSST+Planck (red dashed contour) and LSST+CMBPol (black solid contour). These data sets benefit from the increase in the cosmic shear footprint and the constraints on $b$ are, respectively, reduced to 1.7% and 0.3%.

6. DISCUSSION AND CONCLUSIONS

The interplay between CMB lensing and cosmic shear data sets allows us to lift the degeneracy between the power spectrum normalization and cosmic shear’s multiplicative bias and to constrain the latter. This is primarily due to the fact that the CMB lensing kernel is wide and even if it peaks at deeper redshifts, it is broad enough to give a significant non-zero correlation with cosmic shear measurements.6

The results reported thus far are based on the quite remarkable fact that the CMB lensing signal reconstructed from optimal quadratic estimators can be affected by additive bias but not by a scale-independent multiplicative bias. In other words, as long as optimal quadratic estimators are used, the CMB lensing signal is properly normalized (Hanson et al. 2011). It is however of practical importance to quantify the impact of relaxing such a condition. To do this, the CMB lensing multiplicative bias $c$ is introduced, so that

$$\kappa_{\text{c,obs}}(\hat{n}) \equiv c\kappa_{\text{true}}(\hat{n}) + n_c(\hat{n}).$$

The resulting enlarged set of multiplicative parameters $\{\alpha, b, c\}$ is degenerate with respect to the cosmic shear and the CMB lensing data sets. It is nonetheless possible to estimate the constraining power of the latter with respect to $b$ and $c$ by

6 These results, however, differ sensibly from the ones of (Ishak et al. 2004), where stringent priors on the bias factors are imposed.
imposing a prior on the normalization of the power spectrum. The results obtained by assuming a 4% prior on $\alpha$ (consistent with current constraints on $\sigma_8$) are reported in Table 3. Not surprisingly, the results obtained strongly depend on the prior assumed. However, they also show that under this assumption, the data allow for testing and constraining both multiplicative biases at the few percent level.

It is possible to speculate that the cross-correlation between these data sets could also be exploited to constrain the additive biases present in cosmic shear and CMB lensing measurements: while CMB lensing and cosmic shear are characterized by additive biases, their cross-correlation is free from these contaminations. Furthermore, it is also necessary to stress that the present analysis focuses on a single redshift slice, with $z \in [0.9, 1]$. One possibly relevant issue that has not been explored in this work is the interplay between a possible redshift dependence of the multiplicative bias and the redshift evolution of the growth of structure. In this case, it is possible to note that the different terms entering the Fisher matrix calculation show a different dependence on the multiplicative bias. It is therefore possible to speculate that unless multiplicative bias and structure growth are characterized by the same redshift dependence, using cross-correlation information will probably allow us to constrain the multiplicative bias.

Finally, it is possible to remark that the strong synergy between these different probes of the dark matter distribution suggests that future CMB lensing and cosmic shear surveys should greatly benefit from sharing the same footprint on the sky. This fact is not restricted to cosmic shear surveys but it applies in general to most astrophysical surveys (galaxy density surveys, dwarf galaxy surveys).

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Table 3

| Data Sets       | $b$     | $c$  |
|-----------------|---------|------|
| DES + Planck    | 4.2%    | 5.3% |
| DES + WidePol   | 4.3%    | 4.2% |
| LSST + Planck   | 4.0%    | 4.3% |
| LSST + CMBPol   | 4.0%    | 4.0% |

Notes. Because the two multiplicative biases are jointly degenerate with the normalization of the power spectrum $\alpha$, a 4% prior on $\alpha$ is assumed, consistent with current measurements of $\sigma_8$. In parenthesis is the constraint value for an analysis not including $c$. 

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