Mechanical model of a boundary layer for the parallel tracks of kilohertz quasi-periodic oscillations in accreting neutron stars

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ABSTRACT

Kilohertz-scale quasi-periodic oscillations (kHz QPOs) are a distinct feature of the variability of neutron star low-mass X-ray binaries. Among all the variability modes, they are especially interesting as a probe for the innermost parts of the accretion flow, including the accretion boundary layer (BL) on the surface of the neutron star. All the existing models of kHz QPOs explain only part of their rich phenomenology. Here, we show that some of their properties can be explained by a very simple model of the BL that is spun up by accreting rapidly rotating matter from the disk and spun down by the interaction with the neutron star. In particular, if the characteristic time scales for the mass and the angular momentum transfer from the BL to the star are of the same order of magnitude, our model naturally reproduces the so-called parallel tracks effect, where the QPO frequency is correlated with luminosity at time scales of hours but becomes uncorrelated at time scales of days. The closeness of the two time scales responsible for mass and angular momentum exchange between the BL and the star is an expected outcome of the radial structure of the BL.

Key words. accretion, accretion disks – stars: neutron – stars: oscillations – X-rays: binaries

1. Introduction

Stitching an accretion disk rotating at about the Keplerian rate with the central object rotating much more slowly leads to the concept of the accretion boundary layer (BL). The reason for talking about the BL as an entity separate from the accretion disk is the inevitable breakdown of the basic assumptions of the standard disk theory in a very narrow region just above the surface of the accretor (Lynden-Bell & Pringle 1974; Papaloizou & Stanley 1986).

In neutron star (NS) low-mass X-ray binaries (LMXBs), the BL is thought to be an important source of radiation when the magnetic field of an accreting NS is too weak to support a magnetosphere. Shining at a luminosity comparable to that of the accretion disk (Lynden-Bell & Pringle 1974; Sibgatullin & Sunyaev 2000), but being much more compact, the BL is expected to have a harder spectrum and shorter variability time scales. Such a component has indeed been identified in LMXBs, spectrally (Suleimanov & Poutanen 2006; Revnivtsev et al. 2013) and via its timing properties, in particular as a source of kilohertz quasi-periodic oscillations (kHz QPOs; Gilfanov et al. 2003). The position of the BL at the surface of the NS makes it a valuable probe for the fundamental properties of the star: its size, radius, and the physical conditions on its surface.

The kHz QPOs have been observed in many NS LMXBs (van der Klis 2000). Their frequencies span the range between about 200 Hz and the Keplerian frequency near the surface (about 1.3 kHz; see Méndez et al. 1999; Belloni et al. 2005). Either one or two peaks, with a frequency difference of about 300 Hz (Méndez & Belloni 2007), are observed. In individual sources, kHz QPO frequencies may vary by a factor of 1.5–2. The frequencies are correlated with flux on time scales of hours (Méndez et al. 1999, 2001), while the correlation disappears on time scales of days. This phenomenon, known as QPO parallel tracks, was explained in a purely phenomenological way by van der Klis (2001). He suggested that the instantaneous X-ray luminosity of the source is a linear combination of the mass accretion rate $\dot{M}$ and its running average $\langle \dot{M} \rangle$, while the oscillation frequency is a function of $M/\langle \dot{M} \rangle$ only. In this paper, we propose a mathematically similar but more physically motivated solution to the parallel tracks problem.

We have developed a simple mechanical model of the BL, which is treated as a thin massive belt supplied by the mass and angular momentum from the accretion disk and which is losing mass and angular momentum to the NS. We consider the rotation frequency of the BL to be the characteristic frequency responsible for kHz QPOs, though the real situation is probably much more complicated (e.g., Abolmasov et al. 2020). In Sect. 2, we introduce the main equations based on conservation laws. In Sect. 3, we consider the properties of the model by solving the equations numerically. We discuss the results in Sect. 4.

2. Model setup

We consider the BL as an infinitely thin equatorial belt on the surface of a NS of radius $R$ and mass $M_{\text{NS}}$ rotating at an angular frequency $\Omega_{\text{NS}}$. The rotation of the layer is aligned with both the rotation of the star and the disk. The dynamics of the layer can be reduced to two equations, one for the mass and the other for the angular momentum conservation. The conservation law for
the BL mass $M$ can be written as

$$\frac{dM}{dt} = M - M_{\text{depl}}, \quad (1)$$

where $M$ is the mass supply rate from the disk. The second term describes mass precipitation from the BL onto the NS surface with the depletion time scale $t_{\text{depl}}$ that exceeds the characteristic dynamical (Keplerian) time scale $t_{\text{dyn}} = 1/\Omega_K = \sqrt{R^3/GM_{\text{NS}}}$, where $\Omega_K$ is the Keplerian frequency.

Conservation of the angular momentum also involves sources and sinks related to the interaction with the surface of the star. Hydrodynamic numerical simulations (e.g., Belyaev et al. 2013) suggest that the interaction between the BL and the surface of the star mediated by Reynolds stress is relatively weak. The relevant tangential stress $W_{\text{re}} = 10^{-4} P$, where $P$ is the pressure at the bottom of the layer. The impact of magnetic fields on the internal dynamics of the layer is probably important (Armitage 2002), but it is unclear if they can provide an efficient angular momentum transfer between the BL and the star. We will assume that the stress at the bottom of the BL is proportional to the pressure with a small proportionality coefficient $\alpha \ll 1$,

$$W_{\text{re}} = \alpha P = \alpha g_{\text{eff}} \Sigma, \quad (2)$$

where $\Sigma$ is the BL surface density and

$$g_{\text{eff}} = \frac{GM_{\text{NS}}}{R^2} - \Omega^2 R \quad (3)$$

is the effective surface gravity, where $\Omega$ is the rotation frequency of the layer. This allows us to express the braking torque acting on the layer as

$$T^- = ARW_{\text{re}} = \alpha g_{\text{eff}} MR, \quad (4)$$

where $A$ is the surface area of the BL (projected onto the surface of the star) and the BL mass is $M = \Delta \Sigma$.

The angular momentum conservation law that includes mass depletion and friction takes the form

$$\frac{dJ}{dt} = M j_\text{d} - \frac{J}{t_{\text{depl}}} - \alpha g_{\text{eff}} MR, \quad (5)$$

where $J = \Omega MR^2$ is the total angular momentum of the layer and $j_\text{d} = \sqrt{GM_{\text{NS}}R}$ is the specific angular momentum of the matter entering from the disk. We ignore viscous interaction between the disk and the BL. This corresponds to the “accretion gap” scenario (Kluzniak & Wagoner 1985), where the last stable orbit is located above the surface of the NS and thus the disk is causally disconnected from the BL. Recent constraints for the NS radius ($N\tilde{a}t\tilde{l}\tilde{a}$ et al. 2017; Miller et al. 2019; Riley et al. 2019; Capano et al. 2020) suggest that this should be the case, at least below the Eddington limit.

Two equations, Eqs. (1) and (5), are sufficient to describe the evolution of the physical parameters of the BL with time, given $M(t)$ and initial conditions. In our framework, the energy released during accretion and dissipation does not affect the dynamics of the layer. However, luminosity is an important observable. Some of the kinetic energy of the flow contributes to the spin-up of the star; the rest is converted to heat and contributes to the luminosity. The dissipated luminosity can be found as the change in the kinetic energy (see, e.g., Appendix B of Popham & Narayan 1995). Our model splits this spin-down of the gas being accreted into two episodes: Some dissipation occurs when the matter from the disk enters the BL at the rate $M$, and some dissipation occurs during the matter depletion from the BL (at the rate of $M/t_{\text{depl}}$). In addition to these two components, there is viscous dissipation unrelated to mass exchange, equal to one-half of the stress $W_{\text{re}}$ times the strain $Rd\Omega/dR$ (see Landau & Lifshitz 1987, section 16). Together, the luminosity associated with the BL can be written as the sum of three terms:

$$L = \frac{1}{2} MR^2 \left(\Omega_K^2 - \Omega^2\right) + \frac{1}{2} \alpha g_{\text{eff}} MR (\Omega - \Omega_{\text{NS}})$$

$$+ \frac{1}{2} M \frac{t_{\text{depl}}}{R^2} \left(\Omega^2 - \Omega_{\text{NS}}^2\right). \quad (6)$$

The first term on the right-hand side is the kinetic energy lost by the matter that enters the BL from the disk with the angular frequency $\Omega_K = j_d/R^2$. The second term is the viscous dissipation associated with the Reynolds stress (Eq. (2)). The last term corresponds to the kinetic energy of the BL material that precipitates onto the NS and acquires its rotation velocity.

Below, we assume that the BL is fed by a variable source of mass. We assume stochastic variability of the mass accretion rate, modeled as a white noise source convolved with a kernel corresponding to a power-law power-density spectrum (PDS) with a random Fourier image phase (which corresponds to a random moment in time and unsynchronized variability at different frequencies). Integrating white noise leads to a normally distributed quantity as it involves the summation of a large number of independent random numbers. To reproduce the log-normal flux distribution reported in many observational works (Uttley et al. 2005), we then exponentiated the result of the convolution and renormalized it to match the mean value of $M$.

3. Results

3.1. Approach to the equilibrium solution

For a fixed BL mass and mass accretion rate, the rotation of the BL can be described as evolution toward an equilibrium state. Using Eqs. (1) and (5), we can derive an evolutionary equation for $\Omega$:

$$\frac{d\Omega}{dt} = \frac{d}{dt} \left(\frac{J}{MR^2}\right) = \frac{J}{MR^2} \left(\frac{M j_\text{d}}{J} + \frac{\alpha g_{\text{eff}} MR}{J}\right)$$

$$= \frac{M}{\Omega} \left(\Omega - \Omega_{\text{d}} - \alpha \left(\Omega_K^2 - \Omega^2\right)\right). \quad (7)$$

The right-hand side of this equation is quadratic in $\Omega$, which allows us to rewrite it in the form

$$\frac{d\Omega}{dt} = \alpha (\Omega - \Omega_{\text{d}}) (\Omega_{\text{e}} - \Omega_{\text{d}}), \quad (8)$$

where

$$\Omega_{\text{e}} = \frac{M}{2\alpha M} \pm \sqrt{\left(\frac{M}{2\alpha M} - \Omega_K\right)^2 + \frac{M}{\alpha M} \left(\Omega_K^2 - \Omega_{\text{d}}^2\right)}. \quad (9)$$

For $\Omega_{\text{d}} = \Omega_K$, one of the frequencies becomes $\Omega_{\text{e}} = \Omega_K$ and the other $\Omega_{\text{e}} = M/(\alpha M) - \Omega_{\text{d}}$. The lower of the two roots, which is always $\Omega_{\text{e}}$, for the parameter values we consider (see Sect. 3.3 for more details), is stable.

Our approximation is valid only if $\Omega < \Omega_K$; otherwise, effective gravity becomes negative and the flow is unbound. Unless $\Omega_{\text{d}}$ becomes smaller than $\Omega_{\text{NS}}$, the BL will evolve toward this equilibrium state. Otherwise, the layer stalls at $\Omega = \Omega_{\text{NS}}$, and $W_{\text{re}}$ works as static friction.
At a fixed mass accretion rate, the system reaches both equi-
librium mass and equilibrium rotation frequency. Mass equi-
librium is given by
\[ M = M_{\text{eq}} = M_{\text{depl}}. \] (10)
The equilibrium rotation frequency depends on the ratio \( \Omega_d/\Omega_K \),
which we fix to 1, and on only one additional parameter \( \alpha M_{\text{eq}}/M \). It is easy to check that this quantity, multiplied by the
Keplerian frequency, is equal to the ratio of the characteristic
depletion and friction time scales,
\[ q = \frac{t_{\text{depl}}}{t_{\text{fric}}} = \alpha \Omega_K t_{\text{depl}}. \] (11)
For \( \Omega_d = \Omega_K \), the equilibrium rotation frequency is
\[ \Omega_{\text{eq}} = \left( \frac{1}{q} - 1 \right) \Omega_K. \] (12)
When the friction becomes more efficient than depletion, the
layer brakes down to \( \Omega = \Omega_{\text{NS}} \), which leads to trivial rota-
tional evolution. Hence, in the simulations with the variable mass
accretion rate, we keep \( \alpha \leq 1/(\Omega_K t_{\text{depl}}) \).

3.2. Variable mass accretion rate

If the mass accretion inflow to the layer is variable, the BL works
as a filter for the variability of \( M \). The system of equations we
consider is practically linear, though there is nonlinearity intro-
duced by \( q_{\text{depl}} \) in the friction term in Eq. (5). The characteristic
depletion and friction time scales are presumably much longer than
the dynamical time, and they probably also exceed the vis-
cous time scales in the inner disk. The outer disk, however,
evolves even more slowly. In the relevant frequency range, the
shapes of the PDSs of LMXBs are generally close to a power-
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by $\dot{M}/\alpha M$. Hence, in this limit, the Fourier image of the rotation frequency is

$$\tilde{\Omega} \approx \frac{1}{2\pi} \dot{M} \left( \Omega - \Omega_0 \right).$$

As $L$ is mainly affected by the first term, the cross-spectrum becomes

$$C(L, \Omega) \approx \frac{1}{2} \Omega^2 \left( \Omega_0^2 - \Omega^2 \right) \dot{\Omega}^2$$

$$\approx \frac{i}{4\pi} \Omega^2 \left( \Omega_0^2 - \Omega^2 \right) \left( \Omega - \Omega_0 \right) \left| \dot{M} \right|^2.$$ \hspace{1cm} (15)

The argument of this expression is $\pi/2$.

### 3.3. Rotation frequency variations

The behavior of the BL, including its rotation frequency, depends strongly on the balance between mass and angular momentum loss, which can be described by the dimensionless quantity $q$ (see Eq. (11)). In Fig. 2, we show the mean rotation frequency and its variations for different values of $q$. Apparently, the mean value is well predicted by the $\Omega_0$ given by Eq. (9). When the depletion time scale is much shorter ($q \leq 0.5$), the BL corotates with the disk. In the opposite limit, friction spins the BL down to $\Omega_{NS}$. Strong variations in $\Omega$ are present only when the two time scales (friction and depletion) are comparable ($q = 0.5-0.9$).

In general, the relation between the observed luminosity and rotation frequency of the layer is nonunique, and the parallel tracks picture is qualitatively reproduced (see the lower panels of Fig. 2). On the shortest time scales, much smaller than $t_{\text{depl.fric}}$, the variability of the luminosity is dominated by the first term in Eq. (6), which is uncorrelated with $\Omega$. However, if the luminosity is averaged in time bins that are several times smaller than the time scales of the BL, it becomes correlated with $\Omega$. On these time scales, the variations of $\Omega_0$ in Eq. (8) dominate over the variations of $\Omega$ (see Fig. 3), hence the rotation frequency derivative

$$\frac{d\Omega}{dt} \approx (\Omega_0 - \Omega) \frac{M}{\dot{M}}.$$ \hspace{1cm} (16)

Neglecting mass depletion, this yields

$$M \propto \frac{1}{\Omega_0 - \Omega},$$ \hspace{1cm} (17)

where the proportionality coefficient is a slowly variable function of time. This scaling is well reproduced in the evolution...
of the parallel tracks e

ing between the short-term tracks increases from about 30% to nearly two orders of magnitude.

4. Discussion

4.1. Friction and deplletion times

As mentioned in the introduction, the observed kHz QPO frequencies vary by a factor of 1.5–2 in individual sources. While our model reproduces the parallel tracks effect in a broad range of parameters, strong variations in the rotation frequency of the BL appear only when the characteristic friction and mass deplletion time scales are comparable. If friction is more efficient \((q \gtrsim 0.8)\), the BL corotates with the star. If deplletion is faster \((q \lesssim 0.5)\), the BL corotates with the disk and loses angular momentum only with mass. Effectively, the second independent parameter necessary to reproduce the parallel tracks behavior exists only in a narrow range of \(q\), meaning that there should be a physical reason for the deplletion and friction time scales to be close to each other.

Such a similarity in the time scales may be explained if the BL is resolved in the radial direction. The radial flux of angular momentum consists of two parts, viscous \(w_{\nu R} R\) and advective \(\omega R^2 \nu v\), where \(v\) is vertical velocity, \(h \ll R\) is the height above the NS surface, \(w_{\nu R} = w_{\nu R}(h)\) is the viscous stress component, and \(\omega = \Omega(h)\) is the rotation frequency, decreasing from \(\Omega\) somewhere inside the BL to \(\Omega_{\text{NS}}\) at the NS surface. Because the viscous angular momentum transfer is directed outward in the disk and inward at the bottom of the BL, at some altitude it should be zero. Let us assume that \(w_{\nu R} = 0\) at the same altitude where \(\omega = \Omega\), and describe the angular momentum transfer along the radial coordinate by the following equation

\[
\frac{\partial}{\partial t} (\omega R^2) + \nu \frac{\partial}{\partial h} (\omega R^2) = -\frac{1}{R \rho} \frac{\partial}{\partial h} (w_{\nu R} R^2).
\]  

(18)

In a steady-state case, \(\nu v = \text{const}\) and the time derivative in Eq. (18) is zero. Integration yields

\[
\rho \nu v R^2 + w_{\nu R} R = \rho \nu \Omega R^2.
\]  

(19)

At the surface of the NS, \(\omega(h) = \Omega_{\text{NS}}\) and \(w_{\nu R} = W_{\nu R}\), which implies

\[
\rho v (\Omega - \Omega_{\text{NS}}) R^2 = RW_{\nu R}.
\]  

(20)

Multiplying this equation by \(A\) and taking Eq. (4) into account yields

\[
M_{\text{lept}} (\Omega - \Omega_{\text{NS}}) R^2 = \alpha g_{\text{eff}} M R.
\]  

(21)

We note that the mass flux \(\rho v\) is related to the mass motion from the BL onto the surface of the star, and hence we replaced \(\rho A\) with \(M/J_{\text{lept}}\). Substituting \(g_{\text{eff}}\) from Eq. (3), we can express the \(q\) parameter using Eq. (11) as

\[
q = \frac{\Omega_{\text{K}} (\Omega - \Omega_{\text{NS}})}{\Omega_{\text{K}} - \Omega^2}.
\]  

(22)

These estimates suggest that, instead of being an independent parameter, \(q\) should depend on the rotation frequency of the BL. It is unclear if \(q\) should change with the variations of \(\Omega\). If \(q\) depends on the mean or instantaneous value of \(\Omega\), Eq. (12) predicts an attractor for \(\Omega/\Omega_{\text{K}}\) and \(q\). Combining Eqs. (12) and (22), we get
The depletion time depends on rotation frequency as

\( t = \frac{2}{\Omega - \Omega_{\text{eq}}/2} \).

and for the equilibrium rotation frequency we obtain

\( \Omega_{\text{eq}} = \Omega_K + \Omega_{\text{NS}}/2 \).

In Fig. 7, we show how our dynamical model behaves if the depletion time depends on rotation frequency as \( t_{\text{depl}} = \frac{2}{\Omega - \Omega_{\text{eq}}/2} \). Here, we consider the rotation frequency of the BL as a characteristic QPO frequency. Though this is probably not the case,
the real dynamical processes behind kHz QPOs are still likely sensitive to $\Omega$. If the real oscillation frequencies are functions of $\Omega$ and $L$ or $M$, the parallel tracks effect is equally well reproduced, though the parameters of the correlation with radiation flux change.

In particular, for the Rossby-wave model considered in Abolmasov et al. (2020), the characteristic oscillation frequencies are the epicyclic frequency

$$\Omega_\ast = 2\Omega \cos \theta,$$  

(25)

where $\theta$ is the colatitude of the region where the oscillations are excited as well as its aliases with rotation frequency $\Omega_\ast + n\Omega$, where $n$ is a whole number. The oscillations are likely excited in the region of strongest latitudinal velocity shear, which is unstable to supersonic shear instability. This naturally explains the multiplicity of kHz QPO frequencies and the difference between the frequencies that tends to be close to $\Omega_{\text{NS}}$ (though not necessarily; see Méndez et al. 2001). Such a model also explains the characteristic values of the QPO frequencies and their correlation with the flux ($\cos \theta$ is likely a growing function of $L$; see Inogamov & Sunyaev 1999; Suleimanov & Poutanen 2006), as well as the different quality factors of the two QPO peaks (quality factors of the axisymmetric mode $n = 0$ and all others should differ since visibility effects enhance the periodic component in a non-axisymmetric case). It is unclear, however, how to explain the existence of only two QPO peaks (probably $n = 0$ and $-1$). Higher harmonics may be below the sensitivity level, or their excitation conditions may be different. If, instead of rotation frequency, we plot $\Omega_\ast(L)$, the qualitative picture remains the same: a tight correlation on time scales similar to those of the BL that worsens on longer scales. The crucial point is the existence of the second variable, BL mass, which is slowly changing with time.

In beat-frequency models of kHz QPO (Miller et al. 2001), the higher peak corresponds to a rotation frequency somewhere in the disk, and the lower peak corresponds to the beat between the higher frequency and the stellar rotation. Both frequencies in such models change with a single variable parameter: the radius in the disk where the oscillations are excited. This radius should apparently change on the viscous time scale of the inner disk, and, on longer time scales, the flux from the disk and the characteristic frequency should tightly correlate. A way to reproduce a parallel track picture in the framework of such a model is to add a contribution on the disk flux rather than the total flux, and the dependence $\Omega(L)$ retains its slope but not the constant. Apparently, this is not the case as the slope of the short-time relation between flux and frequency also changes considerably (Méndez et al. 1999), suggesting that the frequency itself is sensitive to the parameters of the BL rather than the disk.

5. Conclusions

We show that a very simple, zero-dimensional model of a BL accumulating mass and angular momentum from the disk allows us to explain some of the properties of kHz QPOs. In particular, the model naturally reproduces the parallel tracks effect: The rotation frequency of the BL correlates with its luminosity at small time scales but becomes uncorrelated at longer time scales.

Such an “integrator” BL should have a distinct phase-lag signature: At high frequencies, its mass and rotation frequency should lag the variations of the mass accretion rate by $\Delta \omega \approx \pi/2$. We expect the variations in kHz QPO frequencies in LMXBs to lag the variations of bolometric flux, with a phase lag related to the contribution of the BL. Studying the cross-correlation properties of the kHz QPOs and the flux variations in LMXBs will be an important test for the model and for our understanding of LMXBs in general.

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