A direction of arrival estimation method based on deep learning

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Abstract. Direction of arrival (DOA) estimation is a research subject of many experts and scholars in information, control and communication, and it is also a key technology in smart antenna and sonar array system. In this paper, the direction of arrival of the signal is estimated by establishing a classification network model of deep learning. After training, the network model can effectively identify the direction of arrival of the unknown signal. Under the same signal-to-noise ratio (SNR) environment, it is verified by software simulation that the accuracy is improved compared with the classical music algorithm.

1. Introduction
With the development of sonar array system, direction of arrival (DOA) estimation is an important parameter in sonar array system. The accuracy of DOA estimation will directly affect the performance of the whole sonar system. After decades of development, DOA estimation has formed a large number of algorithms[1]. There are conventional beamforming (CBF) algorithm and adaptive minimum variance distortion less response (MVDR) algorithm based on beamforming technology, and multiple signal classification (Music) algorithm based on subspace decomposition[2].

This paper introduces a method of DOA estimation based on deep learning, which mainly identifies the direction of the unknown signal by establishing a classification neural network model of deep learning. Deep learning is a new direction and hot spot in the field of machine learning. Deep learning studies the internal laws, representation levels and corresponding features of data. The information obtained through learning is very meaningful for data interpretation. At present, deep learning has achieved very good results in speech and image recognition[3].

2. model of array reception
This paper studies that the effect of the algorithm does not require the shape of the array, so this paper uses a plane linear array as an example, the number of elements is M, and the spacing of elements is d. This paper mainly studies the direction of arrival estimation in the far field, so let N narrow-band signals enter the ray array. Figure 1 shows the incident model of the signal.
Figure 1. Model of signal incidence.

Since the far-field signal is received, the wave front at the receiving array can be approximately a plane, as shown in Figure 1. There is a wave path difference $D_m$ when the signal arrives at different array elements, which can be obtained from the trigonometric function:

$$D_m = (m - 1)d\sin\theta_n$$  \hspace{1cm} (1)

The time difference can be derived from formula (1):

$$\tau_m = \frac{D_m}{v}$$  \hspace{1cm} (2)

In formula (2), $v$ is the speed of signal propagation in water. Then the phase difference of the received signal of the array element is:

$$\beta = e^{-j2\pi f\tau_m}$$  \hspace{1cm} (3)

Substituting formula (1-2) into formula (3),

$$\beta = e^{-j2\pi \frac{(m-1)d\sin\theta_n}{\lambda}}$$  \hspace{1cm} (4)

Since $f = \frac{v}{\lambda}$, formula (4) can be reduced to:

$$\beta = e^{-j2\pi \frac{(m-1)d\sin\theta_n}{\lambda}}$$  \hspace{1cm} (5)

The actual signal is a snapshot signal, so it is a discrete signal. Then the signal $x_m(k)$ collected by the uniform linear array:

$$x_m(k) = \sum_{n=1}^{N} s_n(k)\beta + n_m(k)$$  \hspace{1cm} (6)

In formula (6), $n_m(k)$ is the interference noise signal for the $Mth$ receiving array element. Formula (6) is written in matrix form as follows:

$$X = AS + N$$  \hspace{1cm} (7)

In formula (7),

$X = [x_1(k), x_2(k), \ldots, x_M(k)]^T$ is the received signal matrix of the array

$S = [s_1(k), s_2(k), \ldots, s_N(k)]^T$ is the source signal matrix

$N = [n_1(k), n_2(k), \ldots, n_M(k)]^T$ is the additive Gaussian white noise matrix received by the array

$$A = \begin{bmatrix}
1 & e^{-j2\pi \frac{d\sin\theta_1}{\lambda}} & \cdots & e^{-j2\pi \frac{d\sin\theta_N}{\lambda}} \\
e^{-j2\pi \frac{(M-1)d\sin\theta_1}{\lambda}} & e^{-j2\pi \frac{(M-1)d\sin\theta_2}{\lambda}} & \cdots & e^{-j2\pi \frac{(M-1)d\sin\theta_N}{\lambda}} \\
\vdots & \vdots & \ddots & \vdots \\
e^{-j2\pi \frac{(M-1)d\sin\theta_1}{\lambda}} & e^{-j2\pi \frac{(M-1)d\sin\theta_2}{\lambda}} & \cdots & e^{-j2\pi \frac{(M-1)d\sin\theta_N}{\lambda}}
\end{bmatrix}$$ is the flow pattern matrix of the array

Because $A$ is a Vandermonde matrix, when $\theta_1$, $\theta_2$, $\ldots$, $\theta_N$ are different from each other, $A$ matrix is full rank. The autocorrelation matrix $R_{xx}$ of the signal $X(k)$ can be obtained as follows:

$$R_{xx} = E\{XX^H\}$$  \hspace{1cm} (8)
3. The basic principles of MUSIC algorithm

MUSIC algorithm decomposes the eigenvalues of the autocorrelation matrix of array signals to obtain the signal subspace corresponding to the signal component and the noise subspace orthogonal to the signal component, and then estimates the incident direction of the signal using the orthogonality of the two subspaces.

The autocorrelation matrix $R_{xx}$ of formula (8) is transformed as follows:

$$R_{xx} = E[(AS + N)(AS + N)^H]$$

$$R_{xx} = AE[SS^H]A^H + E[NN^H]$$

In equation (10), $E[SS^H]$ is the autocorrelation matrix of the source signal, which is recorded as $R_{SS}$. $E[NN^H]$ is the autocorrelation matrix of the noise signal, which is recorded as $R_{NN}$, which can be rewritten as follows:

$$R_{NN} = \sigma^2 I$$

In formula (11), $\sigma^2$ is the noise power and $I$ is the unit matrix of order $M$. Therefore, formula (10) can be written as follows:

$$R_{xx} = AR_{SS}A^H + \sigma^2 I$$

The white Gaussian noise is not related to the signal. Since $\sigma^2 > 0$ and $R_{xx}$ has full rank, the eigenvalues and eigenvectors of $R_{xx}$ can be obtained:

$$\lambda = [\lambda_1, \lambda_2, \cdots, \lambda_M]$$

$$\nu = [\nu_1, \nu_2, \cdots, \nu_M]$$

In formula (13-14), $N$ eigenvalues and eigenvectors are related to signals, and the rest $M - N$ eigenvalues and eigenvectors are related to noise. Therefore, the eigenvalues are arranged in descending order. The first $N$ are the eigenvalues of the signal, and the corresponding $N$ eigenvectors of these eigenvalues are the eigenvectors of the signal, and the rest are the eigenvalues and eigenvectors of the noise. The eigenvectors of $N$ signals form the signal subspace:

$$E_s = [\nu_1, \nu_2, \cdots, \nu_N]$$

The eigenvectors of $M - N$ noises are used to form the noise subspace:

$$E_n = [\nu_{N+1}, \nu_{N+2}, \cdots, \nu_M]$$

Because signal subspace and noise subspace are orthogonal to each other. When $\theta = \theta_i, i \in [1, N]$

$$a^H(\theta)E_n = 0$$

According to the characteristics of formula (17), a function similar to power spectrum is defined:

$$P(\theta) = \frac{1}{a^H(\theta)E_nE_n^H a(\theta)}$$

In formula (18), the peak value of $P(\theta)$ corresponds to $\theta$, which is the wave direction of the signal [4].

4. Deep learning classification model

In order to solve the classification problem, deep neural network needs to firstly use the training data to learn the weight parameters. When reasoning, using the weight just learned to can classify the output data [5].

Since the correlation matrix $R_{xx}$ in formula (12) is Hermitian matrix, the elements on the main diagonal are all real numbers instead of containing the orientation information. So in this paper, only the elements of the upper triangle region are selected to form the reference vector $L$ [6]:

$$L = [R_{12}, R_{13}, \cdots, R_{1M}, R_{23}, \cdots, R_{2M}, \cdots, R_{(M-1)M}]$$

In this paper, the reference vector $L$ is used as the input of the whole classification model, and [-90°, +90°] is divided into 181 categories, and the probability vector $Z$ of 181 categories is used as the output of the model. In this paper, a neural network with three layers and 128 neurons in each layer is used to predict the direction of arrival of unknown signals. The known training is set as $\{(L_1, Z_1), (L_2, Z_2), \cdots, (L_i, Z_i)\}$, $L_i$ is the input data, $Z_i$ is the output data. The network is trained through the known training data. In order to train samples better, the network must know the gap between the predicted value and the real value of the model, so the importance of loss function is self-evident. The
The classification model established in this paper uses sparse category cross entropy as the loss function, whose function is almost the same as that of category cross entropy, but sparse category cross entropy does not need to encode the target vector uniquely [7]. Because of the properties of sigmoid function, the derivative of the commonly used mean square error loss function is very small in most of the time, which leads to the slow updating of w and b. However, the classification cross entropy can overcome this deficiency. The classification cross entropy formula is as follows:

$$E = - \sum_{n} \sum_{k} y_{nk} \log \hat{y}_{nk}$$  \hspace{1cm} (20)

In formula (20), $y_{nk}$ represents the expected value of the kth element of the nth data, and $\hat{y}_{nk}$ is the actual output value of the neural network. The role of cross entropy is to measure the difference between the current output probability distribution and the expected probability distribution of the network. The larger the cross entropy value, the farther the current output situation is from the expected, the worse the effect.

The activation function of the output layer of this paper uses softmax. Softmax is the most commonly used function to deal with multi-classification problems. It is a generalization of logistic regression model on multi-classification problems [8]. The definition of softmax function is expressed as follows:

$$y_k = \frac{e^{a_k}}{\sum_{i=1}^{n} e^{a_i}}$$  \hspace{1cm} (21)

In formula (21), $y_k$ is the output of the kth neuron, and N represents the number of neurons in the output layer of the network. The formula indicates that the numerator of the softmax function is the exponential function of the input signal $a_k$, and the denominator is the sum of the exponential functions of all input signals. Because not only the output of softmax function is a real number between 0 and 1, but also the sum of the output value of softmax function is 1, the output of softmax function can be interpreted as "probability". Generally, neural network takes the corresponding category of the neuron with the largest output value as the recognition result, which is the basis of the final judgment of the direction angle of the target signal in this paper. Since [-90°, +90°] are divided into 181 categories in this paper, the output layer neurons of this neural network are set to 181.

5. Computer simulation comparison

5.1. Simulation based on deep learning algorithm

In this paper, the array receiving model is built in MATLAB, using a single frequency target sound source in ocean far field, whose frequency is 1kHz. The number of elements in linear array is 10, and the array spacing $d = \frac{l}{2}$. The incidence angle traverses [-90°, +90°], each angle is tested and sampled 1000 times, and the correlation matrix is obtained to form the reference vector $L$ required by the test set and data set. Therefore, the corresponding relationship between 181000 groups of reference vectors $L$ and the vector $Z$ related to the probability of angle $\theta$ is obtained.

The neural network model is established under the tensorflow learning framework based on python, 181000 groups of data are randomly divided into 101812 training sets, 33938 verification sets and 45250 test sets by train_test_split function.

In this paper, simulation is carried out under different water environment conditions with signal-to-noise ratio of 20dB, 10dB, 0dB and -10dB respectively. The results are shown in Figure (2-5).

It can be seen from Figure (2-4) that in the environment of positive signal-to-noise ratio, the training effect of deep learning neural network model is very good, the accuracy is very high, and there is almost no fitting phenomenon. At the 10th epoch, the accuracy is more than 90%, and the loss function has reached a very low value, so the training speed is very fast.

It can be seen from Figure 5 that in the environment of poor signal-to-noise ratio, the training effect of deep learning neural network model is not as good as before, the accuracy is about 75%, and the over fitting phenomenon is worse than that in the case of positive signal-to-noise ratio. This is due to the fact
that the values of reference vectors in low SNR environment are very similar, the degree of discrimination is not high, and the difficulty of feature extraction is increased.

Figure 2. The evaluation model in snr = 20dB.

Figure 3. The evaluation model in snr = 10dB.

Figure 4. The evaluation model in snr = 0dB.

Figure 5. The evaluation model in snr = -10dB.

5.2. Comparison of simulation results with music algorithm

In the MATLAB environment, the experimental conditions and the depth learning algorithm are the same, respectively take 20dB, 10dB, 0dB and -10dB SNR, incidence angle traversal [-90°, +90°], and the same angle test 1000 times. The MUSIC algorithm is used to search the spectrum peak, and the predicted angle value is output to compare with the incident angle. The accuracy under four different SNR environments is calculated respectively, and compared with the accuracy of the depth learning algorithm, as shown in Table 1.

| SNR/dB | Deep learning algorithm | MUSIC algorithm |
|--------|-------------------------|-----------------|
| 20     | 98.35%                  | 98.65%          |
| 10     | 97.85%                  | 96.58%          |
| 0      | 94.22%                  | 89.83%          |
| -10    | 74.87%                  | 53.63%          |

It can be seen from the table that in the environment of high SNR, the accuracy of deep learning algorithm is equivalent to music algorithm, which can identify the far-field sound source direction well. However, when the SNR is reduced, the deep learning algorithm has better anti-jamming ability and higher accuracy than music algorithm. When the SNR is -10dB, the accuracy of deep learning is 50%
higher than music algorithm, which proves that deep learning has more advantages in low SNR environment.

6. Conclusion
This paper discusses a method of DOA estimation, which is based on the depth neural network. It can learn the autocorrelation matrix information of array signals at different incident angles, and build the classification model. Through learning a large number of data, this method can accurately predict the incident angle in the environment of high SNR and adaptively learn and adjust the model during the reduction of low SNR, which is better than MUSIC algorithm. The method is proved to be feasible by computer simulation.

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