Wavelet based fractal analysis of El Niño/La Niña episodes

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Abstract:

The fractal dimension $H$, which is frequency-dependent within Quasi-Biennial (QB) period, was explored to measure the noise characteristics of Niño 3.4 Sea Surface Temperature (SST), where “noise” specifies the cycles within QB such as the Tropospheric Biennial Oscillation (TBO). The results show that the oscillation pattern of $H$ corresponds mostly to development of El Niño, particularly during two strong Tropical Pacific Decadal Oscillation (TDO) periods of 1894 to 1923 and 1978 to 2000. This represents a stochastic resonance mechanism when a positive-phase noise overlaps a stronger positive-phase of TDO. In this case, SST would exceed a critical value to trigger an El Niño. The mechanism provides a favorable condition by which the onset of El Niño becomes more sensitive to noise. Self-organized criticality (SOC) explains that a small disturbance on an uncertain system will result in an avalanche, including scale-invariance (scaling) and criticality (threshold) features. The results show that strong and medium El Niño events regularly show scaling within QB period especially after the 1970s. Therefore, scaling is a critical state for onset of a strong El Niño and noise modulation by SOC within QB period plays a significant role in the El Niño developments.

KEYWORDS Fractal; Self-organized criticality; Scaling; Wavelets; ENSO

INTRODUCTION

El Niño-Southern Oscillation (ENSO) has been widely studied for sixty years and has been regularly observed for more than a couple of decades (TAO). The ENSO variability affects global precipitation and hydrological cycle affecting the water resources distribution. The frequency and strength of El Niño have been reported to have increased since the late 1970s together with global warming (Wang, 1995). These changes usually result in variation in fractal characteristics, and changes in fractal characteristics may also indicate changes in the background of El Niño (Tsonis et al., 1998).

The fractal dimension, also known as the Hurst coefficient $H$ (first proposed by Hurst, 1951), is important for climate study (Koutsoyiannis, 2006). It has been applied to fields including the study of precipitation and river discharge (Hurst, 1951), ice and clouds (see Ausloos, 2004 for a review). $H$ is dependent on time-space frequency. Both data-based and model-based studies (for example, Fraedrich et al., 2004) have documented the temporal and spatial fractal dimensions of sea surface temperature (SST). The relationships between the time-frequency dependent $H$ for SST and the development of an El Niño event have not been documented so far. The importance and necessity of it is based on the following experiences.

Noise in the ENSO signal refers to high-frequency forcing and small spatial scale processes such as a westerly wind burst or Madden-Julian Oscillation (MJO, 30–60 days cycle). The importance of noise for forecasting El Niño is controversial with some researchers supporting its use (Dijkstra et al., 2002, amongst others) and others not (e.g. Chen et al., 2004).

Additionally, the Tropospheric Biennial Oscillation (TBO; Meehl et al., 1997) greatly affects ENSO. The Indian Ocean Dipole (IOD, Saji et al., 1999) is mainly due to Quasi-Biennial (QB) cycle and as an aspect of TBO (Rao et al., 2002), interacting with ENSO (Behera et al., 2006) and influencing ENSO through the Darwin pressure (Behera et al., 2003). The biennial variability of ENSO is a part of TBO (Li et al., 2006). TBO and IOD are important factors for ENSO variability.

An essential element of any TBO mechanism is the memory of the ocean, and certain short-term oscillations such as Kelvin waves also contribute to the TBO mechanism (Meehl et al., 2003). This TBO mechanism provides us with possibilities to explore an index $H$ of a short-term memory, which only depends on frequencies within QB period, to investigate the noise characteristics of SST accompanying the development of El Niño. ENSO has a cycle of two to seven years. In this study, cycles under two years are considered “noise” and cycles longer than seven years are the “background state”. Therefore, $H$, which has restricted frequencies within QB period, is used to index the noise characteristics (mean power of noise) of SST. This study aims to find the relationship of $H$ with the development of an El Niño/La Niña event and the way noise and background interact leading up to the onset of an El Niño event.

Another profile of fractal, Self-organized criticality (SOC) (Bak et al., 1987) is one possible explanation for fractal behavior (Fraedrich et al., 2004). SOC explains that a little disturbance on an uncertain system will result in an avalanche. SOC has also been applied to study soil moisture, ice surges, and so on (see the review of Turcotte, 1999). Scale-invariance (scaling) and critical state (threshold) are the two main independent components of SOC; SOC should include both.

Andrade et al. (1995) suggested that an El Niño event was a phenomenon of SOC. Physically, El Niño can be viewed as a series of events when the ocean releases energy into the atmosphere. This study also aims to determine whether an avalanche of energy spectrum in time-frequency space on a specific time-scale, for example QB period, will result in an El Niño event, and also define the role of noise based on SOC.

DATA SOURCES

Sea surface temperature data was sourced from the
Niño 3.4 region covering 5°N-5°S, 120°W-170°W, comprising monthly data from the Hadley Centre Sea Ice and SST Data Set (HadISST), over the period from 1870 to 2005. The data was provided by the Hadley Centre, Met Office, UK (Rayner et al., 2003).

The ENSO event-index is a value representing the intensity of an El Niño/La Niña event for a given year (Severov et al., 2004), which is one of the following (shown in Table I): -1.5 (strong La Niña), -1 (medium strength La Niña), -0.5 (weak La Niña), 0 (neutral year), 0.5 (weak El Niño), 1 (medium strength El Niño), and 1.5 (strong El Niño), respectively.

The intensity of the IOD that is named Dipole Mode Index (DMI, Saji et al., 1999) is obtained from the JAMSTEC Frontier Research Center for Global Change (FRCGC) website (http://www.jamstec.go.jp/frcgc/research/d1/iod/).

**METHODOLOGY**

We selected (Torrence and Compo, 1998) the second order of the Daubechies orthogonal wavelet function (shown in Figure 1) for estimating $H$ (see Huang et al., 2006) and references therein for details of the definition of $H$.

The plot of the logarithm of wavelet power spectrum against the logarithms of timescale is called a “log-log plot”, and is presented in the results below (e.g. Figure 4). The following procedure was used to analyze the Niño 3.4 SST in this study:

1. A time window of the SST time series was selected with a period of 64 months, equivalent to the average ENSO cycle, starting from January. The years used for definition of $H$ are supposed to be the start years.
2. The Daubechies wavelet based spectral analysis was applied to the selected SST time window.
3. The Hurst coefficient $H$ was calculated using $H = (K - 1)/2$, where $K$ is the slope rate of the linear fitted line of a log-log plot, shown in Figure 4.
4. The time window was then shifted forward one year and steps 1 to 3 were repeated.

In the log-log plot (Figure 4) the slope shows the average energy of wavelet spectra subjected to noise in that frequency inside QB periodicity and $H$ relates to the slope of the log-log plot. Therefore, $H$ is an index of the mean power of noises.

The Morlet wavelet was also applied to estimate $H$, but the results were a little different. We also applied the Morlet wavelet to investigate the power spectra of a Niño3.4 SST.

**RESULTS**

**Fractal characteristics of SST**

Sensitivity of the noise characteristic index $H$ to El Niño/La Niña

Figure 2 shows the relationship between the oscillations of $H$ and that of the ENSO event-index. The shaded areas indicate the in-phase oscillations. The oscillation patterns of $H$ and the ENSO event-index were well-matched in phase oscillation, meaning that noise corresponds well to the development of an El Niño event. Two periods longer than twenty years with a co-oscillation rate of at least 85 percent between the two time series were selected for further analysis: 1894 to 1923 and 1978 to 2000.

An AR(1) process model (the univariate lag-1 autoregressive process) (Torrence and Compo, 1998) was used to simulate both the ENSO-event index and $H$ time series, and the Monte Carlo method employed to perform statistical significance tests. The tests showed that these two periods occurred significantly at the five percent level. A Chi-square test was then used to find the goodness of phase-fit between the two time series during these two periods. The test also showed a 95 percent confidence level. However, the Monte Carlo test for investigating the amplitude-fitness of the two time series shows an 85 percent confidence level. A little lower confidence here reflects that the ENSO event-index is influenced by TDO and inter-annual cycles but $H$ is not.

For example, there are seven strong and medium El Niño years after 1968 such as 1972, 1982, 1986, 1987, 1991, 1994 and 1997. We compared $H$ and ENSO event-index of these years with the intensity of IOD respectively. It is worth to note that all of them are positive-phase IOD years except 1986. We found the same in phase oscillation patterns happened in these years between them except 1986.

The difference in phase oscillation between the $H$ and ENSO event-index in Figure 2, such as 1978, 1984 and 1988 is due to negative-phase of IOD accompanying a La Niña. This shows that the noise modulated by IOD interacts with ENSO (Rao et al., 2002; Behera et al., 2006).

Stochastic resonance as a possible modulation mechanism

Figure 3 shows the Morlet wavelet spectra for Niño 3.4 SST. During 1894 to 1923 and 1978 to 2000, respectively, there were very energetic oscillations of the decadal/inter-decadal cycle, in other words, strong tropical Pacific Decadal Oscillation (TDO) signals (Prof. Michael McPhaden of NOAA, personal communication). Similarly, Torrence and Compo (1998) also found that during 1880 to 1920 and 1960 to 1990 respectively, significantly higher powers existed in the Southern

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**Table I. Relationship between the ENSO event-indices and the intensity of El Niño/La Niña events.**

| Type          | El Niño events | Neutral | La Niña events |
|---------------|---------------|---------|----------------|
| Intensity     | Strong | Medium | Weak          | Neutral | Weak | Medium | Strong |
| ENSO event-indices | 1.5    | 1.0   | 0.5           | 0       | -0.5 | -1.0   | -1.5   |
Oscillation (SOI) and the Niño3 SST. This implies that noise contributes more to the onset of an El Niño event in the presence of a stronger positive-phase TDO background. This is a stochastic resonance mechanism (Nicolis, 1993) when a positive-phase noise overlaps with a stronger positive-phase TDO, the Niño 3.4 SST would exceed easily a critical state to launch an El Niño. This mechanism gives a condition when the onset of El Niño is more sensitive to noise.

A strong TDO was evident after 1978 though the TDO signal was not so clear from 1894 to 1923. In fact, the significance test of wavelet spectrum was sensitive to the type of red-noise background selected (Torrence and Compo, 1998). A 95 percent confidence level was achieved when a propriety rate of noise to signal was selected.

The contribution of SOC to the onset of El Niño events

Scaling features of SOC for El Niño events

Scaling or scale-invariance means that an invariant property exists in the distribution of energy to frequency. Log-log plots of energy versus frequency will show an almost-straight line in such a case. Figure 4 shows the log-log plot for 1997, and the almost-straight line indicates the presence of scaling features in 1997. The plots for other years are not shown here. In this study, the almost-straight line assumes that the last
Table II. The ratio of the number of scaling to the number of El Niño events.

| El Niño type/year | 1870-1899 | 1900-1929 | 1930-1959 | 1960-1989 | 1990-2000 | Total |
|-------------------|-----------|-----------|-----------|-----------|-----------|-------|
| Strong El Niño    | 5/5       | 2/3       | 1/3       | 3/3       | 1/1       | 12/15 |
| Medium El Niño    | 2/3       | 3/8       | 2/3       | 2/2       | 2/2       | 11/18 |
| Weak El Niño      | 1/1       | 0/1       | 0/4       | 2/3       | 0/3       | 3/12  |
| Total of Strong & medium event | 7/8 | 5/11 | 3/6 | 5/5 | 3/3 | 23/33 |

Table III. The ratio of the number of thresholds to the number of El Niño events.

| El Niño type/year | 1870-1899 | 1900-1929 | 1930-1959 | 1960-1989 | 1990-2000 | Total |
|-------------------|-----------|-----------|-----------|-----------|-----------|-------|
| Strong El Niño    | 2/5       | 1/3       | 1/3       | 2/3       | 1/1       | 7/15  |
| Medium El Niño    | 1/3       | 4/8       | 2/3       | 0/2       | 2/2       | 9/18  |
| Weak El Niño      | 1/1       | 0/1       | 1/4       | 1/3       | 0/3       | 3/12  |
| Total of Strong & medium event | 3/8 | 5/11 | 3/6 | 2/5 | 3/3 | 16/33 |

three points should fall within a range of a 5° angle from the unit line (linear fitted line) on the log-log plots. These three points on the scale of 3, 4 and 5 refer to the semiannual cycle, annual cycle and QB, respectively. Table II describes the ratios of the number of scaling to the number of El Niño events during each 30-year and 11-year period.

During the periods 1870 to 1899 and 1960 to 2000, the number of scaling to the number of strong El Niño events had ratios of 5:5 and 4:4 respectively. Medium El Niño events had ratios of 2:3 and 4:4, respectively. Thus, strong and medium El Niño events mainly showed scaling during these two periods, for example after 1968 all seven events witness scaling. Scaling represents a strong signal of the short-term memory of the Pacific Ocean that is an essential element of the IOD/TBO mechanism and interacts with IOD (Meehl _et al._, 2003). Incidentally, except 1986, all of these years are strong or medium positive IOD years.

The ratios of the total number of scaling to the number of events decreased from strong (12:15) to medium (11:18) to weak (3:12) El Niño events. This shows that stronger El Niño events had a higher occurrence of scaling. Scaling is a critical state for strong El Niño.

Threshold features of SOC for El Niño events subject to QB

The tail of a log-log plot with a timescale of 32 months is the top-right point (for example, Figure 4). The height of the tail represents the average energy of the QB signal over 64 months.

The threshold refers to the existence of a critical state in the process of energy spectrum oscillations at the QB frequency, where the energy spectrum is raised from a low level before an El Niño event, and then reaches the straight line of a log-log plot during El Niño evolution, and finally drops after the event. The straight line of the log-log plot (scaling) is the critical state of SOC.

For example, in Figure 4, the “tail” goes from 1996 (normal year) up to that of 1997 (strong El Niño year) and drops in 1998 (La Niña year). Therefore, 1997 is the threshold state.

Table III summarizes the ratios of the number of thresholds to the number of El Niño events in each period. The ratio of the total number of thresholds to the total number of strong and medium events is 16:33, showing that the threshold features of SOC exist in an El Niño episode. This is particularly the case after 1990, with a ratio of 3:3. A 95 percent confidence level is observed for values in both Tables II and III through the Monte Carlo method based on the simulation of Niño3.4 SST using the AR(1) process model (Torrence and Compo, 1998).

Threshold feature represents that the system tends to flip-flop back and forth from year to year similar to the TBO mechanism (Meehl _et al._, 2003). As an example, we have discussed the interactions between _H_ and the ENSO event-index and the intensity of IOD by the phase-fitness in section 4.1.1.

SOC for El Niño events subjected to QB

Tables II and III imply that some strong and medium El Niño events are the result of SOC subjected to QB. In particular, most strong and medium El Niño events indicate SOC after the 1970s. Strong and medium El Niño events have higher ratios of scaling (23:33) than weak El Niño events (3:12). Strong and medium El Niño events have higher ratios of the threshold (16:33) subjected to QB than a weak El Niño (3:12) (Table III). Therefore, a strong QB does not necessarily imply that an El Niño event will occur, but a strong El Niño event regularly accompanies a strong QB. This result supports previous studies, for example Meehl _et al._ (2003). Whether QB will cause a strong El Niño depends on whether the system reaches a critical state and is ready for an avalanche.

**DISCUSSION**

The roles of annual and intra-annual cycles for El Niño based on SOC were discussed as follows. From the view of SOC, a little disturbance will result in a large-scale avalanche for a sandpile only situation when the system is near the critical point (Bak _et al._, 1987). El Niño can be viewed as a result of a chain reaction of energy releases on a large scale, without being sensitive to the “avalanche” at the high frequencies. High frequencies may play a role in the build-up of energy, starting the first chain reaction of energy releases on a large scale, leading to the onset of an El Niño event.

This view supports previous findings. Intra-seasonal cycles, i.e. high frequency events such as winds related to MJO, tropical cyclones and Yanai waves, alter the SST anomaly, which may help trigger the development and demise of an El Niño event (Bergman _et al._, 2001). However, they do not directly cause El Niño as they do not produce sufficient conditions for El Niño to occur. El Niño occurs when energy in time-frequency space reaches a critical state, and when background oceanic...
and atmospheric conditions are conducive to the rapid growth of random disturbances (Moore and Kleeman, 1999).

CONCLUSION

This study has discussed the ENSO variability, which has a significant role in surface water budget through changes in mean precipitations. Roles of noise with frequency within QB oscillations and background state including the TDO are investigated for El Niño evolutions based on fractal analysis and SOC. The main results are summarized below:

The oscillation of the noise characteristics, represented by $H$, which was frequency-dependent within QB, mostly corresponds with the development of El Niño, particularly for periods 1894 to 1923 and 1978 to 2000. This was due to the contribution of strong signals of the TDO background. A stronger TDO background provides the conditions for high frequencies to be more sensitive to the onset of an El Niño. This represented the stochastic resonance mechanism and gave a condition when the onset of El Niño was more sensitive to noise.

Strong and medium El Niño events regularly show scaling within QB period and on QB threshold, especially after the 1970s. Scaling is a critical state for the onset of a strong El Niño. From the view of SOC, TDO provides the background for the onset of an El Niño. Interannual cycles directly affect El Niño and noise may be the first in the chain-reaction leading to the onset of an El Niño event. SOC gives a condition in time-frequency space for noise to trigger an El Niño, confirming QB is significant for ENSO.

Many areas remain to be researched. Scaling and the critical state of SST oscillation exist during El Niño episodes, whereas the exact critical points of spectrum variation for El Niño/La Niña are not yet clear. Moreover, the way that El Niño and La Niña affect the short-term memory of the ocean interacting with TBO or IOD requires further investigation.

ACKNOWLEDGMENTS

We are thankful to Dr. Michael McPhaden of the Pacific Marine Environmental Laboratory, National Ocean & Atmospheric Administration (NOAA) for privately discussing the signal of TDO. We are indebted to the editor in chief and two anonymous reviewers for many useful comments that helped us improve this manuscript.

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