Gravity-Mediated Supersymmetry Breaking in the Brane World

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Abstract

We study the transmission of supersymmetry breaking via gravitational interactions in a five-dimensional brane-world compactified on $S^1/Z_2$. We assume that chiral matter and gauge fields are confined at the orbifold fixed points, where supersymmetry is spontaneously broken by effective brane superpotentials. Using an off-shell supergravity multiplet we integrate out the auxiliary fields and examine the couplings between the bulk supergravity fields and boundary matter fields. The corresponding tree-level shift in the bulk gravitino mass spectrum induces one-loop radiative masses for the boundary fields. We calculate the boundary gaugino and scalar masses for arbitrary values of the brane superpotentials, and show that the mass spectrum reduces to the Scherk-Schwarz limit for arbitrarily large values of the brane superpotentials.
1 Introduction

An important question in the supersymmetric standard model is how supersymmetry is spontaneously broken in the low-energy world. This question has been mainly addressed in the context of four-dimensional effective theories with limited success, but recently the idea of extra dimensions has allowed for new possibilities \[1\]–\[9\]. The extra dimensional framework is particularly interesting because it provides a new geometrical perspective in understanding some of the problems of conventional theories. In particular, the nonlocal nature of communicating supersymmetry breaking across the compact bulk from one boundary to another can soften the divergences of the soft mass spectrum \[3, 7\].

The Horava-Witten scenario \[2\] provides the prototype model in which to study the transmission of supersymmetry breaking via an extra dimension. In this model the transmission of supersymmetry breaking can become quite involved, and to calculate boundary soft masses one requires the bulk/boundary couplings. However, the essential features can be captured by studying a simpler five-dimensional super-Yang-Mills theory coupled to chiral matter on the boundary \[4\]. In this toy model the couplings between five-dimensional supermultiplets and four-dimensional boundary fields are obtained by working with off-shell supermultiplets and including the auxiliary fields. As noticed in Ref. \[5\] the dimensional reduction of bulk fields leads to new couplings between bulk and boundary fields which are required for consistency. In particular this allows for the construction of realistic low-energy models with bulk gauge fields in flat \[7\] and warped space \[9\].

In this work we study brane-world supersymmetry breaking in the case where only gravity propagates in the bulk while the chiral matter and gauge fields are confined to the four-dimensional boundaries. We assume that due to brane dynamics supersymmetry is spontaneously broken by effective brane superpotentials. This causes the tree-level gravitino mass spectrum to shift by a constant amount depending on the values of the brane superpotential \[10\]. At tree-level boundary chiral matter and gauge fields are massless but due to their gravitational interactions with the bulk gravitinos they will receive a supersymmetry breaking mass at one loop. Just like the bulk gauge field case, one can use an off-shell formulation to study the bulk gravitational case as well.

Supersymmetric brane-world scenarios from off-shell supergravity have been formulated in Refs \[11, 12\]. We will predominantly use the results in Ref. \[11\] to study an off-shell formulation of supergravity in the context of supersymmetry breaking with brane superpotentials. In particular, we will show that after integrating out the auxiliary fields of the off-shell supergravity multiplet there are new couplings between bulk and boundary fields. Just like the bulk gauge field case these couplings are required in order to obtain a consistent supersymmetric limit.

The one-loop mass spectrum will continuously depend on the brane superpotential parameter, and due to the nonlocal nature of the supersymmetry breaking the masses will be finite. In fact we will see that one particular limit of our mass spectrum is the familiar Scherk-Schwarz limit \[13, 3\]. Actually depending on the size of the extra dimension our
one-loop results can be of order the anomaly-mediated contributions \cite{6,14} which arise from the one-loop rescaling anomalies. Thus, although we do not discuss this issue in detail, our results could be relevant in solving the tachyonic slepton mass problem \cite{6}.

The plan of this paper is as follows: In Section 2, after briefly reviewing the bulk vector multiplet case we consider the off-shell supergravity multiplet coupled to boundary fields. In particular we show that after integrating out auxiliary fields there are new couplings between boundary gauge fields and bulk supergravity fields. Supersymmetry breaking is considered in Section 3, where we derive the unitary matrix responsible for diagonalising the Kaluza-Klein gravitino mass spectrum. This is important for determining the couplings between the boundary and bulk fields. As a further check, the same results will also be derived more directly using an explicit five-dimensional calculation. In section 4 we calculate the one-loop gaugino and scalar masses for arbitrary values of the brane superpotential. We comment on the cancellations that are required for consistency and are satisfied by the new couplings. Again, for completeness we will present the calculation of the soft mass spectrum using both the Kaluza-Klein sum in four dimensions, and the direct five-dimensional calculation. Finally, our conclusion and comments will be presented in Section 5.

\section{Off-shell Bulk Supergravity on $S^1/Z_2$}

We start from a pure $N = 2$ five-dimensional Poincaré supergravity \cite{15}, compactified on an orbifold $S^1/Z_2$. Our model will assume that only gravity propagates in the bulk whereas chiral matter and gauge fields will be confined to the 4D boundaries. Thus all supersymmetry breaking effects will be transmitted by gravity and in particular the gravitino mass spectrum will shift. In order to study the transmission of supersymmetry breaking effects between the 4D boundaries it is necessary to work with an off-shell formulation of supergravity \cite{11,12}. In this way all bulk-boundary couplings can be derived.

A similar procedure for gauge fields and hypermultiplets in the bulk was considered in Ref. \cite{5}. Before launching ourselves into the more involved case of supergravity coupled to boundary chiral and vector multiplets, we briefly summarize the procedure and results of Ref. \cite{5} for the case of a $U(1)$ bulk vector multiplet which is coupled to a chiral matter multiplet on the boundary. This will allow us to emphasize some key features which will also be present in the supergravity case.

\subsection{5D Yang-Mills multiplet coupled to boundary chiral matter}

The five-dimensional $U(1)$ multiplet with coupling constant $g_5$ contains a vector field $A^M$, a real scalar field $\Phi$, and a gaugino $\lambda^i$. The five-dimensional Yang-Mills multiplet is then extended to an off-shell multiplet by adding an $SU(2)$ triplet $X^a$ of real-valued auxiliary fields. Here capitalized indices $M,N$ run over 0,1,2,3,5, lower-case indices $m$ run over 0,1,2,3, and $i,a$ are internal $SU(2)$ spinor and vector indices, with $i = 1, 2$, $a = 1, 2, 3$. 

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We now compactify the theory on $S^1/Z_2$ and assign even $Z_2$–parity to the fields
\[ \eta_1^L, A^m, \lambda_1^L, X^3, \] (1)
and odd $Z_2$–parity to the fields
\[ \eta_2^L, A^5, \Phi, \lambda_2^L, X^1, X^2, \] (2)
where $\eta_i^L$ is the supersymmetry parameter of the $N = 1$ supersymmetry transformations on the boundary at $x_5 = 0$. A simple inspection of the supersymmetry transformations reveals that the fields $A^m, \lambda_1^L,$ and $(X^3 - \partial_5 \Phi)$ transform as the vector, gaugino, and the auxiliary $D$-field of a 4D $N = 1$ vector multiplet [5]. It is then obvious how to couple the five-dimensional gauge multiplet to a 4D dimensional chiral multiplet $(\phi, \psi_L, F)$ on the boundary. One writes the Lagrangian as
\[ S = \int d^5x \left\{ \mathcal{L}_5 + \sum_i \delta(x_5 - x_i^*) \mathcal{L}_{4i} \right\}, \] (3)
where the sum includes the walls at $x_i^* = 0, \pi R$. The bulk Lagrangian $\mathcal{L}_5$ is the standard one for a 5D super-Yang-Mills multiplet, and the boundary Lagrangian has the standard form of a four-dimensional chiral model built from the chiral multiplet and with the gauge fields $(A_m, \lambda_L, D)$ replaced by the boundary values of the bulk fields $(A_m, \lambda_1^L, X^3 - \partial_5 \Phi)$.

To determine the couplings of the boundary chiral matter to the bulk vector multiplet one has to integrate out the auxiliary fields. Integrating out the auxiliary field $X^3$ gives a boundary Lagrangian of the form
\[ \int d^4x \left[ -\phi^\dagger (\partial_5 \Phi)\phi - \frac{1}{2} g_5^2 (\phi^\dagger \phi)^2 \delta(0) \right], \] (4)
and one finds new interaction terms at the boundaries (apart from the usual ones in $N = 1$ 4D Yang-Mills theory coupled to chiral multiplet) involving the scalar components of the chiral multiplet and the odd field $\Phi$ [5].

By including the kinetic term of the field $\Phi$, the singular terms can be rearranged into a perfect square
\[ -\int d^5x \left[ \frac{1}{2g_5^2} (\partial_5 \Phi)^2 + \phi^\dagger (\partial_5 \Phi)\phi \delta(x_5) + \frac{1}{2} g_5^2 (\phi^\dagger \phi)^2 \delta^2(x_5) \right] = \frac{-1}{2g_5^2} \int d^5x \left[ \partial_5 \Phi + g_5^2 \phi^\dagger \phi \delta(x_5) \right]^2. \] (5)
Varying this action with respect to $\Phi$, one finds that the background expectation value of $\Phi$ is given by
\[ \partial_5 \langle \Phi \rangle = -g_5^2 \phi^\dagger \phi \left( \delta(x_5) - \frac{1}{2\pi R} \right). \] (6)
Substituting this solution into the Lagrangian [5] one finds that the various singular terms $\delta(0)$ cancel and one is left with the usual $D$-term interaction
\[ S = -\frac{1}{2} \int d^5x \frac{g_5^2}{4\pi^2 R^2} (\phi^\dagger \phi)^2 = -\frac{1}{4} \int d^4x g_4^2 (\phi^\dagger \phi)^2, \] (7)
where \( g_4^2 = \left( g_5^2 / \pi R \right) \). To summarize, we have learned that starting from an off-shell formulation of five-dimensional Yang-Mills theory compactified on \( S^1 / Z_2 \) which is coupled to chiral matter on the boundaries, new singular interaction terms appear after integrating out the auxiliary fields. The origin of these terms is clear: they are due to the presence of the physical propagating odd field \( \Phi \) in the effective auxiliary \( D \)-term on the boundary. At the level of the effective 4D theory, the singular terms disappear after we substitute in the Lagrangian the solution of the classical equation of motion for the odd field \( \Phi \). It is worth emphasizing though that the singular terms play a crucial role at the quantum level since they provide counterterms which are necessary in explicit computations to maintain supersymmetry [5]. In particular, the role of the interaction term proportional to \( \delta(0) \) is to cancel the singular behaviour induced in diagrams where the \( \Phi \)-field is exchanged.

### 2.2 The supergravity case

We now want to extend the analysis of the previous subsection to the case of supergravity. The on-shell supergravity multiplet contains the fünfbein \( e_M^A \), the symplectic Majorana gravitino \( \Psi_M \), and the graviphoton \( A_M \). The five-dimensional bulk Lagrangian reads [13]

\[
\tilde{\mathcal{L}}_{\text{bulk}} = -\frac{1}{2} M_5^2 \varepsilon_5 R_5 - \frac{1}{4} M_5 \varepsilon_5 F_{MN} F^{MN} - \frac{1}{6\sqrt{6}} \epsilon^{MNPQ} F_{MN} F_{OP} A_Q \\
+ i M_5 \varepsilon^{MNPQ} \bar{\Psi}_{OP} \gamma_D \Psi_N - i \sqrt{\frac{3}{22}} \varepsilon_5 F_{MN} \bar{\Psi}^M \Psi^N \\
+ i \sqrt{\frac{31}{24}} \epsilon^{MNPQ} F_{MN} \bar{\Psi}_{OP} \gamma_P \Psi_Q + 4\text{--fermion terms},
\]

where the five–dimensional coordinates are \( x^M = (x^m, x^5) \); \( M_5 \) is the five–dimensional Planck mass (we will set it equal to one from now on unless otherwise stated); \( \varepsilon_5 = \det e_M^A \); \( R_5 \) is the five–dimensional scalar curvature; \( \varepsilon_4 = \det e_m^a \), where the latter are the components of the fünfbein with four-dimensional indices; finally, \( \epsilon^{MNPQ} = \varepsilon_5 \cdot e_M^A e_N^B e_P^C e_Q^D \epsilon^{ABCDE} \), \( \epsilon^{mnop} = \varepsilon_4 \cdot e_m^a e_n^b e_p^c e_o^d \epsilon^{abcd} \), \( \epsilon^{01235} = \epsilon^{0123} = +1 \).

The smallest five-dimensional off-shell supermultiplet contains 48 bosonic and 48 fermionic degrees of freedom. This decomposes into a minimal multiplet with \((40 + 40)\) components, containing the fünfbein, gravitino, graviphoton and several auxiliary fields which include an isotriplet scalar \( \vec{t} \), an antisymmetric tensor \( v_{AB} \), a gauge field \( \vec{V}_M \), a spinor \( \lambda \), and a scalar \( C \). In addition one has to introduce a compensator multiplet with \((8 + 8)\) degrees of freedom [11]. The compensator multiplet allows for the breaking of the gauged \( SU(2)_R \) symmetry. There exist several possibilities for the compensator multiplet and we will assume that it is given by the tensor multiplet containing an isotriplet scalar \( \vec{Y} \), a spinor \( \rho \), a scalar \( N \) and a vector \( W_A \) (which can be expressed as a supercovariant field strength of a three-form \( B_{MNP} \)). The \( SU(2)_R \) symmetry is gauged by the auxiliary field \( \vec{V}_M \), and the gauge is fixed by requiring

\[
\vec{Y} = e^u(0, 1, 0)^T,
\]
where $u$ is a scalar field. Thus after gauge fixing the field content of the five-dimensional theory is given by

$$(e^A_M, \Psi_M, A_M, \bar{t}, v_{AB}, \bar{V}_M, \lambda, C, u, \rho, N, B_{MNP}) .$$

Note that in flat space the choice of the tensor multiplet for the remaining $(8+8)$ components is not unique. However, it is advantageous to use the tensor multiplet because it allows a straightforward generalisation to warped spaces. The off-shell bulk Lagrangian is given by

$$\mathcal{L}_{\text{bulk}} = e^u \left[ -\frac{1}{4} R(\bar{\omega})_{AB}^{AB} + 4C - \frac{1}{6} \bar{F}_{AB} \bar{F}^{AB} + v_{AB} v^{AB} + 20 \bar{t} \bar{t} - 36(t^2)^2 ight.$$ \\
$$- \frac{1}{4} \partial^A u \partial_A u - \frac{1}{4} V_A V^{A1} - \frac{1}{4} V^3 V^{A3} + 8\sqrt{3} \Lambda_5 t^2 - \frac{i}{2} \bar{\Psi}_p \gamma^{PMN} D_M \Psi_N$$ \\
$$- 2i \bar{\Psi}_A \gamma^A \lambda - \frac{\sqrt{3} \Lambda_5}{4} \bar{\Psi}_A \gamma^A \gamma^2 \gamma^B \Psi_B - \frac{i}{2} \bar{\Psi}_A \gamma^A \Psi_B v^{AB} \right] - 12N t^2 + \sqrt{3} \Lambda_5 N$$ \\
$$- \frac{1}{\sqrt{3}} F_{AB} v^{AB} - \frac{1}{6\sqrt{3}} \varepsilon^{ABCDE} A_A F_{BC} F_{DE} - 4 \sqrt{3} \tau^2 \rho - 2i \bar{\lambda} \gamma^A \Psi_A$$ \\
$$+ \frac{1}{2} \bar{\rho} \tau^A \Psi_A \partial^A u - \frac{1}{2} \bar{\rho} \tau^A \bar{\Psi}^M V_M + \frac{1}{2} \bar{\rho} \tau^3 \bar{\Psi}^M V_M + \frac{1}{2} \bar{\Psi}_A \tau^2 \gamma^A \gamma^B \bar{D}_B \rho$$ \\
$$+ 2i \bar{\rho} \gamma^A \Psi_A \tau^2 - 2 \bar{\Psi}_A \tau^1 \gamma^A \rho t^3 + 2 \bar{\Psi}_A \tau^3 \gamma^A \rho t^4 - \frac{1}{2} \bar{\Psi}_A \tau^2 \gamma^A \rho \partial_B u$$ \\
$$- \frac{1}{12} \varepsilon^{MNQR} (V_M^2 - 2 \Lambda_5 A_M) \partial_N B_{PQR} - 32 \bar{t} \bar{t} - \frac{\sqrt{3} i \Lambda_5}{2} \bar{\Psi}_A \gamma^A \rho$$ \\
$$- \frac{1}{2} \bar{\rho} \tau^A \gamma^A \gamma^N \bar{D}_M \Psi_N + \bar{\rho} \tau^2 \gamma^B \Psi_A v^{AB} - \frac{1}{2} \sqrt{3} \bar{\rho} \tau^2 \gamma^A \gamma^B \bar{\Psi}_A \bar{F}_{BC}$$ \\
$$- \frac{i}{4\sqrt{3}} \bar{\Psi}_A \gamma^{ABCD} \Psi_B \left( e^u \bar{F}_{CD} + \frac{1}{2} F_{CD} \right) + (1 - e^u) \bar{\Psi}_A \gamma^A \Psi_B \bar{F}_{CD}$$ \\
$$+ e^{-u} \left[ -\frac{i}{4} \bar{\rho} \gamma^A \rho (v_{AB} + \frac{1}{\sqrt{3}} \bar{F}_{AB}) - 3 \bar{\rho} \tau^2 \rho t^2 + W_A W^A - N^2 ight.$$ \\
$$- \frac{i}{2} \bar{\rho} \gamma^A \bar{D}_A \rho + i \bar{\rho} \Psi_A W^A \right] - 4C - \frac{1}{e^{-2u}} \left( \bar{\rho} \tau^2 \rho N - \bar{\rho} \tau^2 \gamma^A \rho W_A \right) + \mathcal{L}_{\text{AF}},$$

where $\bar{F}_{AB} = F_{AB} + i(\sqrt{3}/2) \bar{\Psi}_A \bar{\Psi}_B$ and $\mathcal{L}_{\text{AF}}$ contains four-fermion interaction terms. We have also suppressed all the $SU(2)_R$ indices. All the definitions of the covariant derivatives can be found in Ref. [1], and we have allowed – for generality – the presence of a bulk cosmological constant $\Lambda_5$ (which will be set to zero later). Note that it will turn out that on-shell we have $u = 0$, as can be seen from the variation of $(11)$ with respect to $C$.

The fifth dimension is compactified on the orbifold $S^1/Z_2$, obtained by the identification $x_5 \leftrightarrow -x_5$. Under the orbifold symmetry fields can be classified as either even ($P = +1$) or odd ($P = -1$). Distinguishing between four-dimensional and extra-dimensional indices, and decomposing the generic five-dimensional spinor $\Psi$ and its conjugate $\bar{\Psi}$ into four-dimensional ones, with the convention that $\Psi \equiv (\psi^1_\alpha, \psi^2_\alpha)^T$ and $\bar{\Psi} \equiv (\psi^2_\alpha, \bar{\psi}^1_\bar{\alpha})$, we
assign even $Z_2$–parity to
\begin{equation}
\eta^1, \ e_m^a, \ e_{55}, \ A_5, \ \psi^1_m, \ \psi^2_5, \ v_{a5}, \ \lambda, \ C, \ V^3_m, \ V^1_5, \ V^2_5, \ t^1, \ t^2, \tag{12}
\end{equation}
and odd $Z_2$–parity to
\begin{equation}
\eta^2, \ e_5^a, \ e_{m5}, \ A_m, \ \psi^2_m, \ \psi^1_5, \ V^1_m, \ V^2_m, \ V^3_5, \ v_{ab}, \ t^3, \tag{13}
\end{equation}
where $\eta$ is the supersymmetry parameter (and from now on we will set $e_{55}$ to unity unless otherwise stated).

At the $Z_2$-fixed points half of the degrees of freedom are eliminated, reducing the number of supercharges by one half. Thus, we can locate three-branes with $N = 1$ supersymmetric chiral matter content at the orbifold fixed points. Note that the orbifold also breaks the $SU(2)_R$ symmetry at the fixed points to a residual $U(1)_R$. Since in the following we will be interested in the supersymmetric couplings between the gravitational sector and the boundary matter fields, we notice that only the fields which are even under the $Z_2$-symmetry will possess such couplings. For instance, only the Kaluza-Klein tower of $\psi^1_m$ couples to boundary chiral matter.

2.3 The intermediate multiplet

The crucial point in understanding how to couple chiral and vector multiplets to gravity at the boundaries is to construct a four-dimensional gravitational multiplet involving the even fields. One can show that all the even fields of the $N = 2$ minimal multiplet form a non-minimal $N = 1$ supergravity multiplet in four dimensions with $(16 + 16)$ components [11]. If we conveniently define the four-component 4D Majorana spinors as
\begin{equation}
\psi_m = \begin{pmatrix}
\psi^1_m \\
\overline{\psi}^1_m
\end{pmatrix}
\text{ and } \psi_5 = \begin{pmatrix}
\psi^2_5 \\
\overline{\psi}^2_5
\end{pmatrix}, \tag{14}
\end{equation}
then the $N = 1$ supergravity multiplet in four dimensions with $(16 + 16)$ components is given by
\begin{equation}
\left(e^a_m, \psi_m, b_a, a_m, \lambda, S, t^1, t^2\right). \tag{15}
\end{equation}
This multiplet [13] is known as the intermediate multiplet and was first studied in Ref. [16]. The auxiliary fields are identified as [11]
\begin{align}
b_a &= v_{a5}, \tag{16} \\
a_m &= -\frac{1}{2} \left(V^3_m - \frac{2}{\sqrt{3}} F_{m5} e_5^a + 4 e_m^a v_{a5}\right), \tag{17} \\
S &= C - \frac{1}{2} e_5^5 \left( \partial_s t^3 - \lambda t^3 \psi_5 + V^1_5 t^2 - V^2_5 t^1 \right), \tag{18}
\end{align}
where in particular the auxiliary field $a_m$ is a combination of $V^3_m, v_{a5}$ and the (even) field strength $F_{m5} = F_{m5} + i\sqrt{3}/2 \overline{\psi}_m \psi_5$ of the propagating graviphoton field, $A_M$. 

6
The situation is then completely analogous to what happens in the case of an off-shell bulk vector multiplet in 5D analyzed previously. There, the presence of the propagating odd field $\Phi$ in the effective $D$-term $D = (X^3 - \partial_5 \Phi)$ on the boundary induced new interactions between the the chiral matter fields living at the boundary and the $\Phi$ field. This suggests that after integrating out the supergravity auxiliary fields, one should expect new interaction terms at the boundaries (compared to the usual ones in $N = 1$ 4D supergravity coupled to chiral or vector multiplets) involving the components of the chiral and vector multiplets and the field strength $F_{m5}$. A similar argument can be made for the even part of the gravitino field $\psi_5$, even though we anticipate that $\psi_5$ will be set to zero in the unitary gauge after supersymmetry breaking.

2.4 Coupling to boundary matter fields

It is now quite clear how to couple boundary matter fields to the five-dimensional $N = 2$ gravity multiplet. We write

$$S = \int d^5x \left[ \mathcal{L}_{\text{bulk}} + \sum_i \delta(x_5 - x_i^*) \mathcal{L}_{4i} \right],$$

where $\mathcal{L}_{\text{bulk}}$ is given by (11) and the orbifold fixed points are located at $x^* = 0$ and $x^* = \pi R$. To obtain the explicit form of the boundary Lagrangian, $\mathcal{L}_{4}$ one simply uses the expressions given in Ref. [16] and replaces the auxiliary fields of the intermediate multiplet with the boundary values given by the expressions in Eq. (17).

In particular for an $N = 1$ vector multiplet $(u_m, \chi, D)$ on the boundary one finds that

$$\mathcal{L}_4 = \text{Tr} \left[ \frac{1}{4} \hat{D} \hat{D} - \frac{1}{4} \hat{u}_{ab} \hat{u}^{ab} + i \bar{\chi} \gamma^a \hat{D}_a \chi - 3i \bar{\chi} \gamma^a \gamma^5 \chi b_a - \frac{i}{2} \bar{\psi}_m \gamma^m \gamma^5 \chi D 
- \frac{i}{4} \bar{\psi}_m \gamma^m \gamma^a \gamma^b \chi \hat{u}_{ab} + \frac{1}{4} \bar{\psi}_m \tau^2 \gamma^a \bar{\psi}_a \chi \tau^2 \chi + \frac{1}{4} \bar{\psi}_m \tau^2 \gamma^m \gamma^5 \psi_n \chi \tau^2 \gamma^5 \chi \right].$$

where

$$\hat{u}_{ab} = u_{ab} - i \bar{\psi}_a \gamma_b \chi + i \bar{\psi}_b \gamma_a \chi,$$

$$\hat{D}_m \chi = D_m \chi - \gamma^5 \chi a_m,$$

and $D_m \chi$ is the usual covariant derivative for a gaugino field coupled to gravity.

Following the procedure of Ref. [1] and setting

$$v_{AB}' = v_{AB} - \frac{1}{2 \sqrt{3}} \hat{F}_{AB},$$

to canonically normalize the kinetic term of the graviphoton field strength, we can integrate out the auxiliary fields for the case of a boundary vector multiplet. The Lagrangian for the auxiliary fields $V_m^3$ and $v_{a5}' \equiv b_a'$ is given by

$$\mathcal{L}_{\text{aux}} = -2(b_m')^2 - \frac{1}{4} (V_m^3)^2 + \frac{1}{2} F_{m5}^2 - i \bar{\chi} \gamma^m \gamma^5 \chi \left( b_m' + \frac{\sqrt{3}}{2} F_{m5} - \frac{1}{2} V_m^3 \right) \delta(x_5).$$
which leads to the following vacuum expectation values

\[ V^3_m = i \bar{\chi} \gamma_m \gamma^5 \delta(x_5), \]
\[ b'_m = -\frac{i}{4} \bar{\chi} \gamma_m \gamma^5 \delta(x_5). \]  

(24)

When inserted back into the Lagrangian (20), the conditions (24) give the usual four-dimensional interaction terms of a vector multiplet coupled to gravity plus – as argued above – new interaction terms involving the graviphoton field strength, namely

\[ \int d^5x \frac{1}{2} \left( F_{a5} + i \frac{\sqrt{3}}{2} \bar{\psi}_a \psi_5 - i \frac{\sqrt{3}}{2} \delta(x_5) \bar{\psi} \gamma_5 \gamma^5 \right)^2. \]  

(25)

After performing an \( x_5 \) integration, we see that singular terms proportional to \( \delta(0) \) appear in the Lagrangian\(^\text{1}\), and as expected they contain the physical fields \( F_{m5} \) and \( \psi_5 \) which appear in the boundary auxiliary fields (17). This situation is therefore analogous to that described in Ref. \([5]\) where a 5D vector multiplet is coupled to chiral matter on the boundary, or to that occurring in \( E_8 \times E_8 \) strongly coupled heterotic string theory \([2]\).

Since the singular terms can be written as a perfect square, at least formally they can be eliminated by a field redefinition \( F_{a5} = \bar{F}_{a5} - i \frac{\sqrt{3}}{2} \delta(x_5) \bar{\psi} \gamma_5 \gamma^5 \). At the level of the effective 4D theory, the singular terms disappear after we substitute in the Lagrangian the solution of the classical equation of motion for the odd field \( A_a \). However, the singular terms at the quantum level play a crucial role since they provide counterterms which are necessary in explicit computations to maintain supersymmetry.

A similar, but much more involved procedure can also be followed to obtain the four-dimensional Lagrangian for a boundary chiral multiplet \( (\varphi, \psi_\varphi, F) \).

2.5 The tensor multiplet at the boundaries

We have derived the couplings between the gravity fields of the intermediate multiplet and the matter fields. There will also be couplings between the gravity fields of the intermediate multiplet and the fields of the tensor multiplet, since in addition to the minimal multiplet, parities can also be assigned to the tensor multiplet \([11]\). We assign even \( Z_2 \)-parity to

\[ Y^1, \quad Y^2, \quad \rho, \quad N, \quad B_{mnp}, \]  

(26)

and odd \( Z_2 \)-parity to

\[ Y^3, \quad B_{mn5}. \]  

(27)

On the boundary the even fields of the tensor multiplet form a chiral multiplet

\[ (A, B, \psi, F, G) \]  

(28)

\(^1\)The same result can be recovered by a full on-shell procedure \([17]\).
with chiral weight \( w = 2 \) at the fixed points. The precise correspondence on the boundary is

\[
(A, B, \psi, F, G) = (Y^2, Y^1, \rho, -2N + \tilde{D}_5 Y^3, + 2W^5 + 12(t^1 Y^2 - Y^1 t^2)) .
\]

(29)

Using the result for the \( F \)-term density of a chiral multiplet \([16]\) this leads to the complete off-shell action

\[
S = \int d^5x \left\{ \mathcal{L}_{\text{bulk}} + \frac{1}{M_5^3} [W_0 \delta(x_5) + W_\pi \delta(x_5 - \pi R)] \mathcal{L}_{4T} \right\} ,
\]

(30)

where \( W_0 \) and \( W_\pi \) are complex constants with dimension of \((\text{mass})^3\), and the boundary Lagrangian is given by

\[
\mathcal{L}_{4T} = -2N + e^u V^1_5 - \bar{\rho} \tau^3 \psi_5 + i \bar{\psi}_m \gamma^m \rho + \frac{1}{2} e^u \bar{\psi}_a \tau^2 \gamma^{ab} \psi_b - 12 e^u t^2 .
\]

(31)

Eliminating the auxiliary fields in (30) we finally obtain the on-shell action

\[
S = \int d^5x \left\{ \tilde{\mathcal{L}}_{\text{bulk}} + \frac{1}{2M_5^3} [W_0 \delta(x_5) + W_\pi \delta(x_5 - \pi R)] \bar{\psi}_a \tau^2 \gamma^{ab} \psi_b \right\} ,
\]

(32)

where we have set \( \Lambda_5 = 0 \) and \( \tilde{\mathcal{L}}_{\text{bulk}} \) is the on-shell Lagrangian of bulk supergravity \([8]\). The Killing spinor equation \( \delta \psi_M = 0 \) reduces to

\[
\partial_5 \eta = -i [W_0 \delta(x_5) + W_\pi \delta(x_5 - \pi R)] \gamma^5 \gamma^2 \eta .
\]

(33)

Decomposing the 5D symplectic spinor \( \eta \) into two component objects \( \eta_i^T = (\eta_i^+, \eta_i^-) \) \((i = 1, 2)\), Eq. (33) has a non trivial solution only if \( W_0 + W_\pi = 0 \). The solution is: \( \eta_1^+ = \epsilon \) and \( \eta_2^- = -i \theta(x^5) \epsilon \), where \( \epsilon \) is a four-dimensional Weyl spinor which generates the supersymmetry of the ground state. Therefore, the flat space solution is supersymmetric provided that \( W_0 + W_\pi = 0 \) \([11]\).

### 3 Supersymmetry breaking

We now consider the case in which supersymmetry is broken. If \( W_0 + W_\pi \neq 0 \) then we will see that the flat space solution spontaneously breaks supersymmetry. The supersymmetry breaking will be transmitted to matter on branes located at the orbifold fixed points via gravity. The brane action is assumed to be

\[
S_{\text{brane}} = \frac{1}{2} \int d^4x \int_{-\pi R}^{+\pi R} dx_5 e_4 \left\{ \delta(x_5) \mathcal{L}^{(0)}_4 + \delta(x_5 - \pi R) \mathcal{L}^{(\pi)}_4 \right\}
\]

\[
- \frac{1}{2M_5^3} \left[ \delta(x_5) W_0 + \delta(x_5 - \pi R) W_\pi \right] \psi_m^\dagger \sigma^m \psi_n^\dagger + \text{h.c.} \right\} ,
\]

(34)
where \( \mathcal{L}^{(i)} \) are the boundary Lagrangians describing the interaction of matter with the bulk. Expanding the fermions in Fourier modes consistently with their boundary conditions and \( Z_2 \)-parity assignments leads to

\[
\begin{align*}
\psi^+(x_5) &= \frac{1}{\sqrt{\pi R}} \left[ \psi^+_0 + \sqrt{2} \sum_{\rho=1}^{\infty} \psi^+_\rho \cos \frac{\rho x_5}{R} \right], \\
\psi^-(x_5) &= \frac{1}{\sqrt{\pi R}} \left[ \sqrt{2} \sum_{\rho=1}^{\infty} \psi^-_\rho \sin \frac{\rho x_5}{R} \right],
\end{align*}
\]

where \( \psi^+ \) stands for \((\psi^1_m, \psi^2_5)\) and \( \psi^- \) for \((\psi^2_m, \psi^1_5)\). The presence of the brane superpotential induces a mixing between the different Kaluza-Klein levels. The fields \( \psi^1_{5,\rho}, \psi^2_{5,0} \) and \( \psi^2_{5,\rho} (\rho > 0) \) are Goldstinos, eaten up by the gravitinos via the super–Higgs effect \([18, 10]\).

As we saw earlier when \( W_0 + W_\pi = 0 \) it is still possible to define a Killing spinor and the \( N = 1 \) supersymmetry is not spontaneously broken by the presence of the brane superpotentials. This means that the amount of supersymmetry breaking is fixed by the order parameter \( F = (W_0 + W_\pi)/M_5 \).

The infinite-dimensional gravitino mass matrix can be easily diagonalized \([10, 19]\). Defining

\[
\begin{align*}
\psi^{\pm}_{m,\rho} &= \frac{1}{\sqrt{2}} (\psi^1_{m,\rho} \pm \psi^2_{m,\rho}), \\
P^{\pm} &= \frac{1}{2\pi M_5^2} (W_0 \pm W_\pi),
\end{align*}
\]

one finds that the modes of \( \psi^1_m \) and \( \psi^2_m \) combine to form nearly degenerate pairs of Majorana states \([10]\) with masses

\[
M^{(\rho)}_{3/2} = \frac{1}{R} (\rho + \Delta) , \quad (\rho = 0, \pm 1, \pm 2, \ldots),
\]

where

\[
\Delta = \frac{1}{\pi} \arctan \left[ \frac{4\pi P_+}{\pi^2 (P_+^2 - P_-^2) + 4} \right].
\]

Note that the mass eigenvalues \([37]\) are both positive and negative. Of course, the absolute values give the physical masses, \( \Delta R, \frac{1+\Delta}{R}, \frac{2+\Delta}{R}, \ldots \), where the physical range of the supersymmetry breaking parameter \( \Delta \) is \( 0 \leq \Delta \leq 1/2 \). Notice that in the supersymmetric limit, \( W_0 = -W_\pi \) (\( P_+ = 0 \)) the gravitino mass spectrum remains unshifted as expected.

After the super–Higgs mechanism, from the four-dimensional point of view, the physical spectrum contains one massless \( N = 1 \) gravitational multiplet with spins \((2, 3/2)\) built up with the zero modes of \( \epsilon^a_m \) and \( \psi^1_m \); one radion multiplet composed of the zero modes \( \epsilon_{55}, A_5 \) and \( \psi^2_5 \) and an infinite series of massive multiplets of \( N = 2 \) supergravity with spins \((2, 3/2, 3/2, 1)\).

Let us now consider two interesting physical limits in the instance where \( W_0 \) vanishes identically and \( W_\pi \) is nonzero. This means that \( P_+ = -P_- = \frac{W_\pi}{2\pi M_5^2} \). In this case the only source of supersymmetry breaking appears as a constant superpotential on the “hidden” brane.
i) If the absolute value of the superpotential $|W_\pi|$ is much smaller than $M_5^3$, $|W_\pi| \ll M_5^3$, the function $\Delta$ is well approximated by $\mathcal{P}_+ = \frac{W_\pi}{2\pi M_5}$. This means that the gravitino zero-mode mass is given by
$$\mathcal{M}^{(0)}_{3/2} = \frac{W_\pi}{M_4^2},$$
(39)
where we have invoked the relation $M_4^2 \simeq M_5^2 \pi R$. This is the familiar four-dimensional expression for $N = 1$ supergravity. The other massive modes are well separated from the lowest mode by a multiple of $R^{-1}$. This means that the low-energy effective four-dimensional theory (describing the physics below the scale $R^{-1}$ and consisting only of zero modes) is simply the $N = 1$ supergravity theory with supersymmetry spontaneously broken by the nonvanishing superpotential $W_\pi$.

ii) If the absolute value of the superpotential $|W_\pi|$ is much larger than $M_5^3$, $|W_\pi| \gg M_5^3$), then the function $\Delta$ is well approximated by $1/2$. This means that the gravitino zero-mode mass is given by
$$\mathcal{M}^{(0)}_{3/2} = \frac{1}{2R},$$
(40)
Notice that the zero-mode gravitino mass no longer depends on the superpotential parameter. All the massive modes are again separated from the zero mode by a multiple of $R^{-1}$. Therefore, the gravitino mass spectrum is identical to that obtained from the Scherk-Schwarz supersymmetry breaking mechanism which makes use of non-trivial (anti-periodic) boundary conditions for the five-dimensional gravitino field upon compactification of the fifth dimension. This observation will turn out to be useful in the following when we will show how supersymmetry breaking is communicated to the visible brane at one loop through the gravitational sector living in the bulk. Indeed, mass splittings for the observable fields have been computed in the context of M-theory where supersymmetry breaking by gaugino condensation in the strongly coupled heterotic string can be described by an analogue of Scherk-Schwarz compactification on the eleventh dimension. At the lowest order, supersymmetry is broken only in the gravitational and moduli sector at a scale $m_{3/2} \sim R^{-1}$, where $R$ is the radius of the eleventh dimension, and it is transmitted to the observable world by gravitational interactions. We will therefore be able to reproduce the results of Ref. in the limit $|W_\pi| \gg M_5^3$.

### 3.1 From the interaction to the mass gravitino eigenstates

The infinite unitary matrix $U$ which diagonalizes the infinite gravitino mass matrix, $\mathcal{M}_{3/2}$ is defined by
$$U \mathcal{M}_{3/2} U^\dagger = \mathcal{M}_D,$$
(41)
where $\mathcal{M}_D$ is the diagonal mass matrix whose eigenvalues are given in (37). Knowledge of the unitary matrix $U$ is necessary in order to perform the one-loop computation of the soft supersymmetry breaking masses of the fields living on the boundaries because the
interactions are not in the mass eigenstate basis. Correspondingly, the gravitino mass eigenstates $\tilde{\psi}$ are obtained from the relation

$$\tilde{\psi} = U \psi, \quad (42)$$

where $\psi = (\psi_0^1, \psi_1^+, \psi_0^-, \psi_1^-, \psi_2^+, \psi_2^-, \cdots)$ represents the infinite gravitino eigenvector for the mass matrix $M_{3/2}$. For arbitrary values of the brane superpotentials, $W_0$ and $W_π$, the gravitino mass eigenvector $\tilde{\psi}_ρ$ with mass eigenvalue $M_{3/2}$ is given by

$$\tilde{\psi}_ρ = N_{λ(ρ)} \begin{pmatrix} 1, \frac{λ(ρ)ξ}{λ(ρ) - 1}, \frac{λ(ρ)ξ}{λ(ρ) + 1}, \frac{λ(ρ)ξ}{λ(ρ) - 2}, \frac{λ(ρ)ξ}{λ(ρ) + 2}, \frac{λ(ρ)ξ}{λ(ρ) - 3}, \frac{λ(ρ)ξ}{λ(ρ) + 3}, \cdots \end{pmatrix}, \quad (43)$$

where $λ(ρ) = ρ + Δ$, $N_{λ(ρ)}$ is a normalisation constant and

$$ξ = \frac{2P_+P_-}{P_+^2 + P_-^2 + (P_+^2 - P_-^2)(1 + πP_+ tan \frac{Δπ}{2})}. \quad (44)$$

Requiring that the vector $\tilde{\psi}_ρ$ has norm equal to unity gives for the normalisation constant

$$N_{λ(ρ)} = \frac{1}{λ(ρ)π} \sqrt{2 sin \lambda(ρ)π \sqrt{1 + ξ^2 + (1 - ξ^2) cos λ(ρ)π}}. \quad (45)$$

The matrix $U$ can therefore be written as

$$U = \begin{bmatrix} N_{λ(0)} & \frac{λ(0)N_{λ(0)}}{λ(0) - 1} & \frac{λ(0)N_{λ(0)}}{λ(0) + 1} & \frac{λ(0)N_{λ(0)}}{λ(0) - 2} & \frac{λ(0)N_{λ(0)}}{λ(0) + 2} & \frac{λ(0)N_{λ(0)}}{λ(0) - 3} & \frac{λ(0)N_{λ(0)}}{λ(0) + 3} & \cdots \\ N_{λ(1)} & \frac{λ(1)N_{λ(1)}}{λ(1) - 1} & \frac{λ(1)N_{λ(1)}}{λ(1) + 1} & \frac{λ(1)N_{λ(1)}}{λ(1) - 2} & \frac{λ(1)N_{λ(1)}}{λ(1) + 2} & \frac{λ(1)N_{λ(1)}}{λ(1) - 3} & \frac{λ(1)N_{λ(1)}}{λ(1) + 3} & \cdots \\ N_{λ(2)} & \frac{λ(2)N_{λ(2)}}{λ(2) - 1} & \frac{λ(2)N_{λ(2)}}{λ(2) + 1} & \frac{λ(2)N_{λ(2)}}{λ(2) - 2} & \frac{λ(2)N_{λ(2)}}{λ(2) + 2} & \frac{λ(2)N_{λ(2)}}{λ(2) - 3} & \frac{λ(2)N_{λ(2)}}{λ(2) + 3} & \cdots \\ N_{λ(3)} & \frac{λ(3)N_{λ(3)}}{λ(3) - 1} & \frac{λ(3)N_{λ(3)}}{λ(3) + 1} & \frac{λ(3)N_{λ(3)}}{λ(3) - 2} & \frac{λ(3)N_{λ(3)}}{λ(3) + 2} & \frac{λ(3)N_{λ(3)}}{λ(3) - 3} & \frac{λ(3)N_{λ(3)}}{λ(3) + 3} & \cdots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots \end{bmatrix}. \quad (46)$$

It is not difficult to check that this matrix is unitary. In addition an infinite sum over the unitary matrix elements can be performed and leads to the result

$$\sum_{n=-∞}^{∞} (±1)^nU_{kn} = \frac{1 ± ξ + (1 ± ξ) cos λ(k)π}{\sqrt{2} \sqrt{1 + ξ^2 + (1 - ξ^2) cos λ(k)π}}, \quad (47)$$

and similarly for $\sum_{n} (±1)^nU_{kn}^*$. In particular for $W_0 = 0$ and $W_π = 0$ (i.e. $ξ = -1$) we obtain

$$\sum_{n=-∞}^{∞} U_{kn} = 1 \quad \text{and} \quad \sum_{n=-∞}^{∞} (−1)^nU_{kn} = cos λ(k)π, \quad (48)$$

while for $W_0 ≠ 0$ and $W_π = 0$ (i.e. $ξ = 1$) we have

$$\sum_{n=-∞}^{∞} U_{kn} = cos λ(k)π \quad \text{and} \quad \sum_{n=-∞}^{∞} (−1)^nU_{kn} = 1. \quad (49)$$

These relations will be useful later when we consider the gravitational interaction of gravitinos with boundary matter.
3.2 5D interpretation

Before closing this section, we wish to give an alternative derivation of the mass eigenvalue formula (37), and a more transparent explanation of the relations (48) and (49). Therefore, let us consider the equations of motion of the five-dimensional gravitino fields after we have set \( W_0 = 0 \) and chosen the unitary gauge (the opposite case in which \( W_\pi = 0 \) can be analyzed in a similar fashion). The gravitino equations of motion read

\[
e^{mpq} \sigma_n \partial_p \psi^1_q + 2 \sigma^{mn} \partial_5 \psi^2_n + 2 \frac{W_\pi}{M_5^3} \delta(x_5 - \pi R) \sigma^{mn} \psi^1_n = 0 ,
\]

(50)

\[
e^{mpq} \sigma_n \partial_p \psi^2_q - 2 \sigma^{mn} \partial_5 \psi^1_n = 0 .
\]

(51)

If we now write \( \psi^i_n(x_a, x_5) = \eta^i_n(x_a) f_i(x_5) \) \((i = 1, 2)\), and integrate Eq. (50) in the interval \((\pi R - \varepsilon, \pi R + \varepsilon)\) we obtain

\[
\eta^2_n(x_a) = \frac{W_\pi}{2M_5^3} \frac{f_1(\pi R)}{f_2(\pi R)} \eta^1_n(x_a) .
\]

(52)

The \( \eta^i_n \) component of the gravitino satisfies the Rarita-Schwinger equation

\[
e^{mpq} \sigma_n \partial_p \eta^i_q = 2 \frac{\lambda}{R} \sigma^{mn} \eta^i_n ,
\]

(53)

and using the relation (52) one gets from Eqs. (50) and (51) that \( f_1(x_5) \propto \cos \left( \frac{\lambda}{R} x_5 \right) \) and \( f_2(x_5) \propto \sin \left( \frac{\lambda}{R} x_5 \right) \) together with the consistency relation

\[
\tan(\lambda \pi) = \frac{W_\pi}{2M_5^3} .
\]

(54)

The solution of this equation reproduces the Kaluza-Klein mass spectrum \( \lambda^{(\rho)} = \rho + \Delta \) given in Eq. (37) for \( P_+ = -P_- = \frac{W_\pi}{2\pi M_5^3} \). Note also that (52) reduces to \( \eta^2_n(x_a) = \eta^1_n(x_a) \) when we use the relation (54).

From this 5D picture we can also extract a transparent interpretation of the relations (48). Since only the even gravitino \( \psi^1_m \) couples to the boundaries, the interaction between the gravitino mass eigenstates \( \eta^1_m(x_a) \) and the boundary chiral and vector multiplets are accompanied by the five-dimensional wave-function \( f_1(x_5) \propto \cos \left( \frac{\lambda}{R} x_5 \right) \). Such a wavefunction is equal to unity if matter and vector multiplets live on the \( x_5 = 0 \) boundary, or alternatively equal to \( \cos(\lambda^{(\rho)} \pi) \) if they live on the \( x_5 = \pi R \) boundary. This is precisely equivalent to the relation (48). We can also alternatively describe the limit in which there is a large supersymmetry breaking on the brane at \( x_5 = \pi R \), \( W_\pi \gg M_5^3 \). In this case the eigenvalue \( \lambda^{(\rho)} \) tends to the value \( \left( \rho + \frac{1}{2} \right) \) and the wavefunction of the gravitino is suppressed at the brane where supersymmetry breaking occurs. In other words, for simple energetic reasons, the gravitino prefers to live in the bulk as far away as possible from the boundary where it acquires a large mass.
4 Communication of supersymmetry breaking via the bulk

As we have seen in the previous section, the introduction of a constant superpotential $W_\pi$ on the brane located at $x_5 = \pi R$ induces a breaking of supersymmetry in the five-dimensional gravitational sector, while it remains unbroken in the visible sector living on the brane located at $x_5 = 0$. The communication of supersymmetry breaking to the visible sector is then expected to arise radiatively via gravitational interactions. This is the issue which we will study below.

4.1 Boundary vector multiplet

Let us consider a vector supermultiplet $(u_m, \chi, D)$ on the boundary at $x_5 = 0$. Our goal is to study how supersymmetry breaking on the brane at $x_5 = \pi R$ is transmitted by gravity to the boundary vector supermultiplet.

The coupling between the intermediate multiplet representing bulk gravity and boundary vector multiplets is given in (20). At tree-level the gauginos are all massless, but the interaction with gravitinos will induce a one-loop radiative mass. From the parity assignments in (35) only the even gravitino modes, $\psi^1_{m,\rho}$ will couple to the boundary gauginos. Inspection of the off-shell Lagrangian does not reveal any new couplings between gaugino fields and the gravitino. Instead the new couplings that do appear, such as the $\delta(0)$ terms, are required in order to obtain the usual result consistent with the $N = 1$ supersymmetric limit.

The interaction between the gauginos and the gravitons do not give any contribution to the gaugino mass (as can be easily understood from chirality arguments). Therefore, there is no supersymmetric cancellation between graviton and gravitino loops; the gravitino contributions have to sum up and give a finite result [20].

![Figure 1: The one-loop diagrams containing the gravitino $\psi_m$ which give contributions to the gaugino mass.](image)

Figure 1: The one-loop diagrams containing the gravitino $\psi_m$ which give contributions to the gaugino mass.

We work in the harmonic gauge $\gamma^m \psi_m = 0$ (integrating out the auxiliary field $\lambda$ in the
bulk Lagrangian (11) gives $\gamma^A \psi_A = 0$ which reduces to $\gamma^m \psi_m = 0$ for the gauge choice $\psi_5 = 0$. Note that there is no $\lambda$ dependence in $\mathcal{L}_{4F}$ (11). In $N = 4 + \epsilon$ dimensions, every Kaluza-Klein state of the gravitino field gives for the gravitino and gauge loops of Fig. 1, the respective contributions

$$i \delta \Sigma_{1a} = \frac{(3 + \epsilon)}{12} \int \frac{d^N k}{(2\pi)^N} \left[ m_n (4 + \epsilon)(3 + \epsilon) - 2 \frac{k^2}{m_n (2 + \epsilon)} \right] \frac{1}{k^2 + m_n^2}, \quad (55)$$

$$i \delta \Sigma_{1b} = \frac{(3 + \epsilon)}{12} \int \frac{d^N k}{(2\pi)^N} \left[ -m_n (12 + 3 \epsilon - \epsilon^2) + 2 \frac{k^2}{m_n (2 + \epsilon)} \right] \frac{1}{k^2 + m_n^2}. \quad (56)$$

We see that the leading divergent parts going like $k^2$ cancel exactly. This means that every Kaluza-Klein gravitino state gives a contribution to the gaugino mass $m_\chi$ of the form

$$\epsilon \int \frac{d^N k}{(2\pi)^N} \frac{m_n}{k^2 + m_n^2}. \quad (57)$$

The next step is to isolate the divergent pieces in the (sum of the) integrals (57). To do that, one can simplify the sum over the Kaluza-Klein states using the contour trick from finite temperature field theory which allows one to identify the different possible divergences [21]. We can write the sum as

$$\sum_{n=-\infty}^{\infty} \frac{m_n}{k^2 + m_n^2} = \frac{1}{2\pi i} \int_{\mathcal{C}} dk_5 \frac{k_5}{k^2 + k_5^2} \mathcal{P}(k_5, \Delta), \quad (58)$$

where the contour $\mathcal{C}$ is the line from the left to the right below the real axis and another line from the right to the left above this axis and enclosing the poles at $k_5 = (n + \Delta)/R$ of the function [21]

$$\mathcal{P}(k_5, \Delta) = \frac{1}{2} \frac{\pi R}{\tan \left[ \pi \frac{R}{(k_5 - \frac{\Delta}{R})} \right]}. \quad (59)$$

Notice that in the limit of exact supersymmetry ($\Delta = 0$) the sum (58) vanishes identically as it should.

The expression (58) can be rewritten as

$$\frac{1}{2\pi i} \int_{-\infty}^{+\infty} dk_5 \frac{k_5}{k^2 + k_5^2} \left[ \mathcal{P}(k_5, \Delta) - \mathcal{P}(k_5, -\Delta) \right]. \quad (60)$$

The dimensionally regulated integrals usually split into two pieces: a pure 5D divergent piece and a finite piece. The constant parts $\pm \frac{1}{2} \pi R$ of the pole functions $\mathcal{P}(k_5, \pm \Delta)$ induce the pure 5D divergent pieces, but – since they do not depend upon $\Delta$ – they sum up to zero. The remainders $\rho_{\pm \Delta}$ of the functions $\mathcal{P}(k_5, \pm \Delta)$ are highly convergent functions and each gives a finite contribution to the momentum integration and – therefore – no contribution to the gaugino masses after we set $\epsilon = 0$. One can understand this result using the analogy with what happens in 4D [21]. There gaugino masses are nonvanishing at one-loop if there is no physical cut-off in the theory. However, if a physical cut-off $\Lambda_c$ is
present (such as the scale of gaugino condensation) one has to cut off the integrals \((55)\) and \((56)\) at \(k^2 = \Lambda^2\) and work in exactly four dimensions \((\epsilon = 0)\). The two contributions \((55)\) and \((56)\) then exactly cancel and we are left with no contribution to gaugino masses from massive gravitinos. In 5D the five-dimensional divergent pieces cancel and each finite loop contribution to gaugino masses can be seen as potentially 4D divergent integrals made finite by setting a cutoff at \(p \sim R^{-1}\). Therefore we find that the gaugino mass do not get any contribution at the one-loop order if gravity is the mediator of supersymmetry breaking through the bulk. This result will be confirmed in subsection 4.3 by a full 5D computation.

Note that if the gauginos were living on the 3-brane at \(x_5 = \pi R\) then the diagrams in Fig. 1 would be proportional to the sum \(\sum_{n,m}(-1)^{n+m} \sum_k U_{nk}^* U_{km} \bar{\psi}_k \bar{\psi}_k\). In this case the sum would reduce to \(\sum_k \cos^2(\lambda(k) \pi) \bar{\psi}_k \bar{\psi}_k\). As we learned at the end of section 3, the \(\cos^2(\lambda(k) \pi)\) factor represents the wave-function suppression of the gravitino at the boundary where supersymmetry is broken. In such a case, the gravitino sum is multiplied by the coefficient \(\beta_n = \cos^2[(n + \Delta) \pi]\). Repeating the procedure adopted above, we again find that gaugino masses vanish at one-loop.

### 4.2 Boundary chiral multiplet

We now add a generic matter chiral supermultiplet \((\varphi, \psi_\varphi, F_\varphi)\) on the boundary at \(x_5 = 0\). The tree-level scalar masses are zero and will again be induced at the one-loop level. We assume that the supersymmetry breaking is on the brane at \(x_5 = \pi R\) and is transmitted by gravity to the boundary matter supermultiplet.

The interaction terms between bulk gravity and the chiral multiplet on the boundary can be found by coupling the supergravity intermediate multiplet \((15)\) to the chiral multiplet. The resulting Lagrangian is quite involved \([16]\) and we do not report it here. Integrating out all the auxiliary fields turns out to be a complicated task. Apart from the usual interaction terms present in the \(N = 1\) 4D supergravity Lagrangian coupled to chiral matter, new singular interaction terms appear. However, as we already explained in section 2, these new terms may involve only the field strength \(\tilde{F}_{m5}\) and the gravitino component \(\psi_5\). In particular, the field strength and the current \(J^m = i(\varphi^\dagger \partial^m \varphi - \varphi \partial^m \varphi^\dagger) + \psi_\varphi \sigma^m \bar{\psi}_\varphi\) combine to form again a perfect square

\[
\int d^5x \frac{1}{2} \left( F_{a5} + i \sqrt{3} \psi_\varphi \bar{\psi}_5 - i \sqrt{3} J_a \delta(x_5) \right)^2.
\]  

(61)

These interaction terms induce a one-loop correction to the scalar masses. In particular, the diagram where the odd graviphoton field \(A_m\) is exchanged leads to singular behaviour which is however cancelled by the singular \(J_a J^a \delta(0)\) term. This can be seen by writing \(\delta(0)\) as

\[
\delta(0) = \frac{1}{2\pi R} \sum_{n=-\infty}^{\infty} \frac{k^2 - (n/R)^2}{k^2 - (n/R)^2}.
\]  

(62)
We find
\[ \delta m^2_\varphi \propto \frac{1}{M^3_5} \sum_{n=-\infty}^{\infty} \int \frac{d^4k}{(2\pi)^4} \left( \frac{1}{2\pi R k^2 - (n/R)^2} + \delta(0) \right), \]

\[ = \frac{1}{2M^2_4} \sum_{n=-\infty}^{\infty} \int \frac{d^4k}{(2\pi)^4} \left( \frac{k^2}{k^2 - (n/R)^2} \right), \]

(63)

where we have used the relation \( M^2_4 \simeq M^3_5 \pi R \). This contribution to the scalar masses, when summed up to the contribution of the whole tower of massive gravitino states belonging to the \( N = 2 \) multiplet in 4D, gives a finite result. Similarly, the diagram where the even field \( A_5 \) propagates is cancelled, in the supersymmetric limit, by the contribution from the \( \psi_5^2 \) gravitino (they live in the same radion multiplet). In the unitary gauge the gravitino component \( \psi_5 \) is eaten up, and cannot induce such a cancellation. Its role is then played by the lightest mode of the \( \psi_1^1 \) gravitino.

After these general comments we now proceed to the explicit computation of the one-loop scalar masses induced by gravity after supersymmetry breaking. It is important to notice that the coupling between the supergravity intermediate multiplet (15) and the boundary chiral multiplet contains a generic Kähler potential \( \Omega(\varphi, \varphi^\dagger) \) of the scalar fields \( \varphi \). Thus, the procedure described in Ref. [16] gives rise to noncanonical kinetic terms of the form
\[ S_{\text{kin}} = \frac{1}{2M^5_3} \int d^4x \int_{-\pi R}^{\pi R} dx_5 e_4 \delta(x_5) \left\{ \frac{1}{6} \Omega \left[ R_4 - \frac{1}{2} \epsilon^{klmn} \left( \bar{\psi}_k^1 \sigma_l \psi_m^1 - \psi_k^1 \sigma_l \bar{\psi}_m^1 \right) \right] \right. \]
\[ - \left. \frac{1}{\sqrt{2}} \left( \frac{\Omega_{\varphi}}{\Omega} \psi_\varphi \sigma^{mn} \psi^1_m + \text{h.c.} \right) \right\}, \]

(64)

where
\[ \psi^1_{mn} = \partial_m \psi^1_n - \partial_n \psi^1_m, \]
\[ \Omega_{\varphi} = \frac{\partial \Omega}{\partial \varphi}, \]

(65)

(66)

and \( R_4 \) is the Ricci scalar computed using the vierbein \( e^a_m(x_5 = 0) \) of the intermediate multiplet.

At this stage one is free to perform a Weyl rescaling that renders the gravity and gravitino kinetic terms canonical. However, we prefer to compute the one-loop scalar masses in the unrescaled Weyl basis using the Lagrangian (64). This choice is dictated by the fact that in the unrescaled Weyl basis both gravitons and gravitinos give rise to one-loop scalar masses and the supersymmetric cancellations are more transparent. Furthermore, in this basis there are no direct couplings between the radion supermultiplet containing the even fields \( e_{55}, A_5 \) and \( \psi_5^2 \) (the radion, for instance, arises from fluctuations of \( g_{55} \); by general covariance it can only couple to the 55-component of the matter energy-momentum tensor which vanishes for chiral matter on the brane [22]). On the contrary,
Figure 2: The one-loop diagrams which give contributions to the scalar mass.

In the Weyl rescaled basis gravitons do not give a contribution to the scalar masses if we start from vanishing tree-level scalar masses and supersymmetric cancellations are hidden.

At the one-loop level, the diagrams that contribute to the scalar masses are those shown in Fig. 2 where the graviton and gravitino vertices come from the kinetic terms (64). As we already noticed, the fields from the boundary in the $N = 1$ chiral multiplet $(\phi, \psi, F_{\psi})$, always appear in pairs, as dictated by the $Z_2$ invariance. Just like the gaugino case, only the parity even gravitinos couple to the boundary chiral multiplet.

Every diagram is proportional to the following integral

$$
\frac{1}{M_4^2} \sum_n \int \frac{d^4k}{(2\pi)^4} \frac{k^2}{k^2 + m_n^2} = \frac{1}{M_4^2} \sum_n \int \frac{d^4k}{(2\pi)^4} k^2 \int_0^\infty ds e^{-s(k^2 + m_n^2)} = \frac{1}{8\pi^2 M_4^2} \sum_n \int_0^\infty \frac{ds}{s^3} e^{-s m_n^2}.
$$

(67)
As we saw for the gaugino case each diagram containing a gravitino in Fig. 2 gives rise to the sum $\sum_k \beta_k \tilde{\psi}_k \tilde{\psi}_k$ where the coefficient $\beta_k$ depends on the location of the boundary chiral multiplet. Thus, after subtracting the fermionic contributions from the bosonic contributions the scalar mass-squared will be proportional to

$$I_B - I_F = \frac{1}{8\pi^2 M_4^2} \int_0^\infty \frac{ds}{s^3} \sum_n \left[ e^{-sn^2/R^2} - \beta_n e^{-s(n+\Delta)^2/R^2} \right] ,$$

where we have made use of the expression (37). The infinite sum can be performed using standard techniques as explained in the Appendix. For chiral matter on the brane $x_5 = 0$, the coefficient $\beta_k = 1$, and the sum of the diagrams in Fig. 2 gives a contribution to the soft-breaking mass squared

$$m_\varphi^2 = -K^{-1}_{\varphi^\dagger \varphi} K_{\varphi^\dagger \varphi} \left\{ \zeta(5) - \frac{1}{2} \left[ \text{Li}_5 \left( e^{-2\pi i \Delta} \right) + \text{Li}_5 \left( e^{2\pi i \Delta} \right) \right] \right\} ,$$

where $K(\varphi, \varphi^\dagger) = -3 \ln \left[ \frac{-\Omega(\varphi, \varphi^\dagger)}{3} \right]$ is the Kähler potential and the term $K^{-1}_{\varphi^\dagger \varphi} K_{\varphi^\dagger \varphi}$ comes from the wave-function renormalization. The supersymmetry-breaking contributions only come from the gravitino diagrams, Fig. 2(a) and (b). The fact that the sign of $m_\varphi^2$ is negative can be easily understood by working in the rescaled Weyl basis. In that basis loops involving gravitons do not give any contribution to the scalar masses which can only be induced by gravitino loops. The latter carry a negative sign because of their fermionic nature.

The one-loop scalar mass-squared induced by the gravitational transmission of supersymmetry breaking is negative and diagonal in flavour space provided that the matter metric is diagonal. However, introducing other moduli fields $z$ in the bulk with gravitational strength coupling to the boundary fields, one can get similar contributions to (69) and make the total scalar mass-squared positive \[^3\]. In such a case the Kähler potential will be a function of all scalar fields, $K = K(\varphi, \varphi^\dagger, z, z^\dagger)$, and different moduli dependence for the various scalar fields may create potentially dangerous flavour-changing neutral currents.

Let us now take two different and physically interesting limits of the expression (69). Consider first the case where the chiral multiplet is on the brane at $x_5 = 0$. If the absolute value of the superpotential $|W_\pi|$ is much smaller than $M_5^3$; ($|W_\pi| \ll M_5^3$), then we have seen that the function $\Delta$ is well approximated by $\frac{W_\pi}{2\pi M_5^2}$. Using the fact that for $\Delta \to 0$ we have

$$\frac{1}{2} \left[ \text{Li}_5 \left( e^{-2\pi i \Delta} \right) + \text{Li}_5 \left( e^{2\pi i \Delta} \right) \right] \simeq \zeta(5) - 2\pi^2 \zeta(3) \Delta^2 + O(\Delta^4) ,$$

we find that the leading contribution from the diagrams in Fig. 3 to the soft breaking scalar mass is proportional to

$$m_\varphi^2 \propto \left( \frac{W_\pi}{R^2 M_4 M_5^3} \right)^2 \sim \left( \frac{1}{RM_4} M_3^{(0)} \right)^2 ,$$

(71)
where we have used \((39)\). In this limit we see that the scalar mass acquires a suppression factor \((RM_4)^{-1}\) relative to the gravitino mass. When \(RM_4 \sim 10\) then the scalar mass will be comparable to the anomaly-mediated contribution \([3]\). The result \((71)\) can be easily understood by noting that in the effective 4D supergravity there are no quadratic divergences. The (divergent) one-loop scalar mass squared result in four-dimensions would be

\[
m^2_\phi \sim \frac{m_{3/2}^2}{M_4^2} \int^\Lambda \frac{d^4p}{p^2 + m_{3/2}^2} \sim \frac{m_{3/2}^2}{M_4^2} \Lambda^2,
\]

where \(\Lambda\) is the ultraviolet cutoff. However, in the brane world scenario the gravitino contribution to the scalar masses is the sum of a pure 5D-divergent piece (cancelled by the graviton contribution) and a finite piece

\[
m^2_\phi \sim \frac{m_{3/2}^2}{M_4^2} \int^{1/R} \frac{d^4p}{p^2 + m_{3/2}^2} \sim \frac{m_{3/2}^2}{M_4^2} \frac{1}{R^2},
\]

where – since the gravitino interacting with the chiral matter at \(x_5 = 0\) has to travel a distance at least as large as the radius of compactification to probe supersymmetry breaking at \(x_5 = \pi R\) – the loop integral is over gravitino momenta satisfying \(p < R^{-1}\) (gravitino wavelengths larger than \(R\)). Loop momenta \(p > R^{-1}\) are not sensitive to the supersymmetry breaking effects on the brane at \(x_5 = \pi R\). Thus, the effective ultraviolet cutoff is provided by the interbrane distance and substituting \(\Lambda = 1/R\) reproduces the result \((71)\). This situation resembles what happens in 4D theories at finite temperature where the ultraviolet cutoff is represented by the temperature \(T\). In the imaginary time formalism 4D loop integrals become integrals over the three spatial momenta and a sum over the so-called Matsubara frequencies and the one-loop contributions to the mass squared of an interacting scalar field can be split into the usual zero temperature 4D divergent piece plus a finite temperature dependent piece. The finiteness is due to the fact that particles in the plasma with wavelengths smaller than \(T^{-1}\) (or momenta larger than \(T\)) have exponentially suppressed abundances.

From Eq. \((71)\) we find that for \(M_{3/2} R^{-1} \sim (10^{11} \text{ GeV})^2\), the soft scalar masses are of order of the TeV scale. For instance, if \(W_\pi/M_5^3 \sim 10^{-2}\), we get \(M_5 \sim 5 \times 10^{16} \text{ GeV}\) and \(R^{-1} \sim 10^{12} \text{ GeV}\).

If the absolute value of the superpotential \(|W_\pi|\) is much larger than \(M_5^3\), \(|W_\pi| \gg M_5^3\), then we have seen that the function \(\Delta\) is well approximated by \(1/2\), and the polylogarithm functions can be expanded as

\[
\frac{1}{2} \left[ \text{Li}_5 \left( e^{-2\pi i \Delta} \right) + \text{Li}_5 \left( e^{2\pi i \Delta} \right) \right] \simeq -\frac{15}{16} \zeta (5) + O \left( (\Delta - \frac{1}{2})^2 \right).
\]

In such a case we find that for large values of \(|W_\pi|\) the integral \(\mathcal{I} = \frac{a_5}{2\pi i} \zeta (5)\), as defined in the Appendix. This reproduces the value found in Ref. \([3]\) where supersymmetry was broken by boundary conditions via the Scherk-Schwarz mechanism. This does not come as a surprise, though. As we have already pointed out, the gravitino spectrum in the
limit $|W_\pi| \gg M_5^2$, is exactly the same as obtained in the Scherk-Schwarz supersymmetry breaking mechanism. The soft breaking scalar mass is therefore proportional to

$$m_\varphi \propto \frac{1}{R^2 M_4} \approx \frac{\left(\mathcal{M}^{(0)}_{3/2}\right)^2}{M_4},$$

(75)

where we have used (10) and there is an $RM_4$ suppression. Again this result can be understood by noting that the effective cutoff for the brane-world scenario is $\Lambda = 1/R$. If $R^{-1} \sim 10^{11}$ GeV, we find $m_\varphi$ of the order of TeV.

It is also instructive to discuss the alternate possibility when the chiral matter is located on the same brane where supersymmetry breaking occurs ($\beta_n = \cos^2(n + \Delta)\pi$). In this case since loop momenta are on the same brane that has a nonzero superpotential the interbrane distance $R$ no longer plays any role. As happens in the four-dimensional case, scalar masses should be ultraviolet sensitive to the cutoff $\Lambda$. This is because gravitinos are now repelled from the brane and the cancellation of the five-dimensional divergences no longer takes place. This can be explicitly seen by performing the integral (78) and the details can be found in the Appendix. If the supersymmetry breaking parameter $|W_\pi|$ is much smaller than $M_5^2$ one finds that $m_\varphi \sim \mathcal{M}_{3/2}(\Lambda/M_4)(\Lambda R)$ which is the usual 4D dimensional divergences increased by the number of Kaluza-Klein states ($\Lambda R$) excited up to the cutoff scale. If $|W_\pi|$ is much larger than $M_5^2$, one finds $m_\varphi \sim \sqrt{\Lambda R}(\Lambda^2/M_4)$ which reflects the absence of the cancellation of the pure five-dimensional divergences.

4.3 The 5D calculation

The communication of supersymmetry breaking to the boundary matter fields can also be obtained using the five-dimensional propagator for the gravitino field. In order to calculate the 5D propagator it is simplest to Fourier transform the four-dimensional spatial coordinates, while leaving the fifth spatial coordinate $x_5$ explicit [23]. The propagator of the massless gravitino in 5D can be written in the form

$$G_{\mu\nu}(k, x_5) = G(k, x_5) \left( g_{\mu\nu} - \frac{1}{3} \gamma_\mu \gamma_\nu \right) \equiv G(k, x_5) P_{\mu\nu},$$

(76)

where we have omitted the longitudinal parts because they do not give any contribution to the gaugino and scalar masses. After the addition of the boundary mass term this reproduces the propagator of the gravitino transverse degrees of freedom. To evaluate $G(k, x_5)$ in the case of supersymmetry breaking we have to solve, starting from Eqs. (50) and (51) and their conjugates, the equations for the Green’s function $G_{1,2}$. For simplicity we assume that there is a boundary mass term, $m \equiv W_\pi/M_5^2$, at $x_5 = \pi R$. Following the procedure developed in Refs. [24, 25] we first solve for the gravitino propagator in infinite space. It reads

$$\tilde{G}_1(k_E, x_5) = \frac{-i \sigma \cdot k_E}{2k_E} \left( e^{-k_E|x_5|} - \frac{m^2}{4 + m^2} e^{-k_E\pi R - k_E|x_5 - \pi R|} \right) + \frac{i}{2k_E} \partial_5 e^{-k_E|x_5|} - \frac{m}{4 + m^2} e^{-k_E\pi R - k_E|x_5 - \pi R|},$$

(77)
where we have analytically continued the propagator to be a function of the Euclidean four-momentum $k_E$. Let us consider an arbitrary point $x_5$ in the interval $[0, \pi R]$. The amplitude to propagate from $x_5$ to $x_5$ is simply $\tilde{G}_1(k_E, x_5)$. But since our space is compact and not infinite, the point $x_5 + 2\pi n R$ where $n$ is an integer, is equivalent to $x_5$ and we need to sum over all the contributions, $\tilde{G}_1(k_E, x_5 + 2\pi n R)$. This sum can be easily performed and leads to the result for a compact space with $0 \leq x_5 \leq \pi R$

$$G^{(\Delta)}_1(k_E, x_5) = \frac{-i\sigma \cdot k_E}{2k_E \sinh(\pi k_E R)} \left[ \cosh[k_E(\pi R - x_5)] - \frac{m^2 \cosh(k_E x_5) \cosh(\pi k_E R)}{4 \sinh^2(\pi k_E R) + m^2 \cosh^2(\pi k_E R)} \right]$$

$$= \frac{-i\sigma \cdot k_E}{2k_E} \left[ \frac{\sinh[k_E(2\pi R - x_5)] + \cos(2\pi \Delta) \sinh(k_E x_5)}{\cosh(2\pi k_E R) - \cos(2\pi \Delta)} \right] - \frac{m \sinh(k_E x_5) \sin(2\pi \Delta)}{2[\cosh(2\pi k_E R) - \cos(2\pi \Delta)]}, \quad (78)$$

where $m = 2 \tan \Delta \pi$, using the relation (38) for $W_0 = 0$. Similarly, one can follow the same procedure to obtain for $0 \leq x_5 < \pi R$

$$G^{(\Delta)}_2(k_E, x_5) = \frac{-i\bar{\sigma} \cdot k_E}{2k_E \sinh(\pi k_E R)} \left[ \cosh[k_E(\pi R - x_5)] - \frac{2m \sinh(k_E x_5)}{4 \sinh^2(\pi k_E R) + m^2 \cosh^2(\pi k_E R)} \right]$$

$$= \frac{-i\bar{\sigma} \cdot k_E}{2k_E \sinh(\pi k_E R)} \left[ \frac{2m \sinh(k_E x_5) \cosh(\pi k_E R)}{2[4 \sinh^2(\pi k_E R) + m^2 \cosh^2(\pi k_E R)]} \right]. \quad (79)$$

Notice that we have not written the $\gamma_5$ terms since they give no contribution to the masses.

Let us consider the case where matter is on the brane at $x_5 = 0$. We find that the $G_2$ propagator is particularly simple,

$$G^{(\Delta)}_2(k_E, 0) = \frac{-i\bar{\sigma} \cdot k_E}{2k_E} \coth(\pi k_E R), \quad (80)$$

and does not depend on the supersymmetry breaking parameter, $\Delta$. However, the $G_1$ propagator becomes

$$G^{(\Delta)}_1(k_E, 0) = \frac{-i(\sigma \cdot k_E)/k_E}{2[\cosh(2\pi k_E R) - \cos(2\pi \Delta)]} \sinh(2\pi k_E R) - \sin(2\pi \Delta). \quad (81)$$

In the supersymmetric limit, $\Delta \to 0$ we recover the usual 5D propagator

$$G^{(0)}_1(k_E, 0) = \frac{-i\sigma \cdot k_E}{2k_E} \coth(\pi k_E R), \quad (82)$$

while in the maximally supersymmetry-breaking limit, $\Delta \to 1/2$ (or Scherk-Schwarz limit), we obtain

$$G^{(1/2)}_1(k_E, 0) = \frac{-i\sigma \cdot k_E}{2k_E} \tanh(\pi k_E R). \quad (83)$$
The propagator (81) thus continuously interpolates between these two limits. Notice also that after analytically continuing the momentum back to four-dimensional Minkowski space the poles of the propagator (81) occur at the values

\[ k_4 \equiv k_4 = (n + \Delta)/R, \]

in complete agreement with (37). It is also interesting to consider the four-dimensional limit \( k ER \ll 1 \). In this limit the propagator (81) becomes

\[ G_1^{(\Delta)}(k_E, 0) = \frac{1}{2\pi R} \frac{-i\sigma \cdot k_E - \Delta/R}{k_E^2 + \Delta^2/R^2}, \]  

where we have also taken the limit \( \Delta \to 0 \).

When matter is located on the brane at \( x_5 = \pi R \) we obtain

\[ G_1^{(\Delta)}(k_E, \pi R) = -i\sigma \cdot k_E \frac{M_3}{k_E} \left[ \frac{\cos^2(\pi \Delta) \sinh(\pi k_E R)}{\cosh(2\pi k_E R) - \cos(2\pi \Delta)} \right] - \frac{\cosh(\pi k_E R) \sin(2\pi \Delta)}{2[\cosh(2\pi k_E R) - \cos(2\pi \Delta)]}, \]  

and we see that there is an extra cosine factor which can be thought of as being due to the wavefunction of the gravitino at \( x_5 = \pi R \).

It is now straightforward to obtain the one-loop contributions to the boundary matter. Consider first the case of the gaugino mass on the boundary \( x_5 = 0 \). One finds that the contribution to the gaugino mass from the Feynman diagrams in Fig. 1 (a) is simply

\[ -\frac{i}{16\pi^2} \int_0^\infty dx^3 \frac{\sin(2\pi \Delta)}{\cosh(2\pi x) - \cos(2\pi \Delta)} \left[ \text{Li}_4(e^{-2\pi i \Delta}) - \text{Li}_4(e^{2\pi i \Delta}) \right], \]

and similarly for Fig. 1 (b)

\[ -\frac{1}{64\pi^5} \int_0^\infty dx^3 \frac{\sin(2\pi \Delta)}{\cosh(2\pi x) - \cos(2\pi \Delta)} \left[ \text{Li}_4(e^{-2\pi i \Delta}) - \text{Li}_4(e^{2\pi i \Delta}) \right]. \]

Thus, as expected we see that the sum of the one-loop contributions to the gaugino mass is zero. This also agrees with the result in Ref. [20], which found that there are no gravitational radiative corrections when the theory has a cutoff. In the 5D theory the effective cutoff is \( \sim 1/R \), and we obtain a similar result. Notice also that each separate contribution to the gaugino mass vanishes identically for \( \Delta = 0, 1/2 \). In the Kaluza-Klein picture this can be easily seen since for \( \Delta = 0, 1/2 \) the Kaluza-Klein mass spectrum is symmetric about zero mass and leads to a vanishing sum (58). Similarly, one obtains the same results for matter on the brane at \( x_5 = \pi R \), since the only relevant difference is the \( \cosh \) factor in the propagator (85).

In the case of the boundary scalar fields located at \( x_5 = 0 \), and using the Lagrangian (64) the contribution to the scalar mass-squared from the gravitino loops is given by

\[ -\frac{1}{6M_5^3} \int \frac{d^4k}{(2\pi)^4} \epsilon^{\mu\nu\rho\sigma} \text{Tr} \left[ \gamma_5 \gamma_\mu k_\nu G_\Delta(k, 0) P_{\rho\sigma} \right], \]  

23
where $G_\Delta(k, 0)$ contains $G^{(\Delta)}_1$ and $\bar{G}^{(\Delta)}_1$. There are also contributions from the graviton and graviphoton, which in the limit of zero supersymmetry breaking must cancel the gravitino contribution (88). Thus, the total contribution to the scalar mass-squared is given by

\[
\frac{1}{6M^3_6} \int \frac{d^4k}{(2\pi)^4} (-8)k^2 E_k \left[ \coth(\pi k E) - \frac{\sinh(2\pi k E R)}{\cosh(2\pi k E R) - \cos(2\pi \Delta)} \right]
\]

\[
= \frac{-1}{12\pi^2(\pi R)^5M^3_6} \int_0^\infty dx x^4 \left[ \coth(x) - \frac{\sinh(2x)}{\cosh(2x) - \cos(2\pi \Delta)} \right]
\]

\[
= \frac{-1}{8\pi^6R^4M^4_4} \left\{ \zeta(5) - \frac{1}{2} \left[ \text{Li}_5(e^{-2\pi i \Delta}) + \text{Li}_5(e^{2\pi i \Delta}) \right] \right\} .
\] (89)

This agrees with the result obtained from the Kaluza-Klein sum (89). Notice that we obtain a finite result because the leading divergences cancel in the integral (89), and the remaining part is exponentially suppressed.

When matter is located on the brane at $x_5 = \pi R$ this cancellation no longer happens because the gravitino propagator (85) contains a cosine factor which depends on $\Delta$. Only in the supersymmetric limit ($\Delta = 0$) does the cancellation in (89) occur. This is just the 5D interpretation of the result (102) that we obtained earlier from the Kaluza-Klein summation.

## 5 Conclusions and discussions

In this paper we have considered a supersymmetric five-dimensional brane-world scenario where the fifth dimension is compactified on $S^1/Z_2$. In our set-up chiral matter and gauge fields are restricted to live on the boundaries while gravity propagates in the bulk. We have assumed that supersymmetry is broken at the orbifold fixed points and that supersymmetry breaking is parametrized by a constant boundary superpotential. The bulk gravitino mass spectrum is consequently shifted relative to the bosonic bulk supergravity fields. Integrating out the supergravity auxiliary fields allows one to derive the couplings between the boundary chiral matter or gauge fields and the bulk supergravity fields. If chiral matter or gauge fields live on a brane different from the one where supersymmetry breaking occurs, the latter is communicated to our observable world by gravitational interactions.

We have computed the contribution to the soft supersymmetry breaking scalar and gaugino masses for generic values of the brane superpotential and the radius of compactification. The one-loop computation of the mass splittings generically provides a hierarchy of soft masses at the TeV scale with nonvanishing scalar masses and zero gaugino masses.

The nonvanishing superpotential on the boundaries and the relative shift in the bulk gravitino mass spectrum parametrize various possible sources of supersymmetry breaking at the fixed-points coming from Fayet-Iliopoulos $D$- and $F$-terms inducing nonvanishing brane superpotentials to keep the branes tensionless. In this paper we have assumed that
these terms are coupled to the bulk only through supergravity. In addition for certain values of the radius $R$ the anomaly-mediated contributions of scalar masses will be of roughly the same order. It may be that a combination of brane supersymmetry breaking and the anomaly-mediated contribution can solve the well-known slepton mass problem. On the other hand since the one-loop gaugino masses vanish, the anomaly-mediated gaugino mass contributions will dominate.

In deriving the results of this paper we have worked with the components of the supermultiplets. In principle the same results can also be more simply derived using the $N = 1$ superfield calculus, such as that considered in Refs. [26] for the case of bulk vector fields. This would require writing the $N = 2$ supergravity Lagrangian in terms of $N = 1$ superfields, and then repeating the procedure already done for the bulk gauge fields.

One should note that the supersymmetry breaking mechanism respects the tensionlessness of the branes even though we have added matter on the branes. This is because the Goldstino is a bulk field. Of course, a brane tension may eventually be generated due to gauge symmetry breaking (or even other forms of brane supersymmetry breaking). This may require considering more general warped bulk backgrounds. Since we have used the tensor multiplet it is straightforward to generalize the procedure used here for warped geometries. In particular one could obtain the gravitational analogue of the warped soft mass spectrum calculated in Ref. [9].

Finally, even if the branes remain tensionless, the vacuum energy is nonzero [10] and the radius is not stabilized. Thus, one is likely to require the addition of further fields in the bulk in order to stabilize the radius [22, 27]. These fields will arise when one embeds the brane-world setup in a more fundamental theory, such as string theory, and deserves further study.

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Appendix

We provide the details on evaluating the infinite Kaluza-Klein sum (68). The key is to introduce the Θ-functions defined as

\[ \Theta_\beta^\alpha(\tau) = \sum_{n=-\infty}^{\infty} e^{2\pi in\beta} e^{\pi i \tau (n+\alpha)^2}, \]  

which obey the Poisson resummation formula

\[ \Theta_\beta^\alpha(-1/\tau) = \sqrt{-i\tau} e^{-2\pi i \alpha \beta} \Theta_\alpha^{-\beta}(\tau). \]  

Consider the expression (68) with \( \beta_k = 1 \) which can be rewritten in the form

\[ I_B - I_F = \frac{1}{8\pi^2 M_4^2} \int_0^{\infty} ds \left( \Theta_0^0 \left( \frac{is}{\pi R^2} \right) - \Theta_0^\Delta \left( \frac{is}{\pi R^2} \right) \right), \]

\[ = \frac{1}{8\pi^2 M_4^2} \int_0^{\infty} ds \left( \frac{\pi R^2}{s} \right)^{1/2} \left[ \Theta_0^0 \left( \frac{i\pi R^2}{s} \right) - \Theta_0^\Delta \left( \frac{i\pi R^2}{s} \right) \right], \]  

where we have used (91). After redefining the integration variable to be \( y = R^2/s \), we obtain

\[ I_B - I_F = \frac{1}{8\pi \sqrt{\pi R^4 M_4^2}} \int_0^{\infty} dy y^{3/2} \left[ \Theta_0^0 (i\pi y) - \Theta_0^\Delta (i\pi y) \right], \]

\[ = \frac{1}{8\pi \sqrt{\pi R^4 M_4^2}} \int_0^{\infty} dy y^{3/2} \sum_{n=-\infty}^{\infty} \left( 1 - e^{2\pi in\Delta} \right) e^{-\pi^2 n^2 y}, \]

\[ \equiv \frac{1}{8\pi R^4 M_4^2} \mathcal{I}, \]  

where we have defined

\[ \mathcal{I} = \frac{1}{\sqrt{\pi}} \int_0^{\infty} dy y^{3/2} \sum_{n=-\infty}^{\infty} \left( 1 - e^{2\pi in\Delta} \right) e^{-\pi^2 n^2 y}. \]  

When \( n \) is nonzero the exponential factor guarantees that each term in the sum has a finite integral. However, notice that the potentially dangerous \( n = 0 \) term vanishes. Thus, simplifying the infinite sum gives

\[ \sum_{n=-\infty}^{\infty} \left( 1 - e^{2\pi in\Delta} \right) e^{-\pi^2 n^2 y} = 4 \sum_{n=1}^{\infty} \sin^2(\pi n \Delta) e^{-\pi^2 n^2 y}. \]  

so that performing the \( y \) integration and summing up the finite integral pieces gives the finite answer

\[ \mathcal{I} = \frac{3}{\pi^5} \sum_{n=1}^{\infty} \sin^2(\pi n \Delta) \frac{1}{n^5}, \]

\[ = \frac{3}{2\pi^5} \left\{ \zeta(5) - \frac{1}{2} \left[ \text{Li}_5 \left( e^{-2\pi i \Delta} \right) + \text{Li}_5 \left( e^{2\pi i \Delta} \right) \right] \right\}, \]  

(96)
where \( \text{Li}_k(x) = \sum_{n=1}^{\infty} \frac{x^n}{n^k} \) are the polylogarithm functions. Thus, substituting the value of \( I \) back into the expression (93) gives the final result

\[
I_B - I_F = \frac{3}{2\pi^2} \frac{1}{8\pi R^4 M_4^2} \left\{ \zeta(5) - \frac{1}{2} \left[ \text{Li}_5 \left( e^{-2\pi i \Delta} \right) + \text{Li}_5 \left( e^{2\pi i \Delta} \right) \right] \right\},
\]

which leads to the result (93).

In the case where the chiral matter lives on the same brane as the supersymmetry breaking, the evaluation of the infinite sum is only slightly more involved because of the presence of the factor \( \beta_n = \cos^2 \left( \frac{n+\Delta}{\pi} \right) \). In this case (93) becomes

\[
I_B - I_F = \frac{1}{8\pi^2 M_4^2} \int_0^\infty \frac{ds}{s^3} \sum_{n=-\infty}^{\infty} \left[ e^{-sn^2/R^2} - \cos^2 \left[ (n + \Delta) \frac{\pi}{s} \right] e^{-s(n+\Delta)^2/R^2} \right],
\]

and the only complication involves the evaluation of the infinite sum with the cosine factor. Using the expansion \( \cos \left[ (n + \Delta) \frac{\pi}{s} \right] = \left[ e^{i\pi(n+\Delta)}/2 + e^{-i\pi(n+\Delta)}/2 \right] \), one can easily show that

\[
\sum_{n=-\infty}^{\infty} \cos^2 \left[ (n + \Delta) \pi \right] e^{-s(n+\Delta)^2/R^2} = \cos^2 \left( \Delta \pi \right) \sqrt{\frac{\pi R^2}{s}} \Theta_0^0(i\pi R^2/s). \]

Thus, after changing variables to \( y = R^2/s \) the infinite sum (98) can be rewritten in the form

\[
I_B - I_F = \frac{1}{8\pi \sqrt{\pi} R^4 M_4^2} \int_0^\infty dy y^{3/2} \left[ \Theta_0^0(i\pi y) - \cos^2(\Delta \pi) \Theta_0^0(i\pi y) \right],
\]

\[
\equiv \frac{1}{8\pi R^4 M_4^2} I_\pi,
\]

where we have defined

\[
I_\pi = \frac{1}{\sqrt{\pi}} \int_0^\infty dy y^{3/2} \sum_{n=-\infty}^{\infty} \left( 1 - \cos^2(\Delta \pi) e^{2\pi i n \Delta} \right) e^{-\pi^2 n^2 y}.
\]

Notice now that the \( n = 0 \) term in the sum no longer vanishes, and there is a divergent piece in \( I_\pi \). Thus, simplifying the infinite sum gives

\[
\sum_{n=-\infty}^{\infty} \left[ 1 - \cos^2(\Delta \pi) e^{2\pi i n \Delta} \right] e^{-\pi^2 n^2 y} = \sin^2 \Delta \pi + 2 \sum_{n=1}^{\infty} \left[ 1 - \cos^2(\pi n \Delta) \cos(2\pi n \Delta) \right] e^{-\pi^2 n^2 y^2}.
\]

Only in the limit \( \Delta = 0 \) does the divergent piece from the \( n = 0 \) term vanish. This makes sense since there is now no superpotential breaking term.
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