Integral Adaptive Sliding Mode Control for Quadcopter UAV Under Variable Payload and Disturbance

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This work was supported in part by the Universiti Teknologi Malaysia High Impact Research under Grant Q.J130000.2451.08G80 and in part by the King Fahd University of Petroleum and Minerals.

ABSTRACT The quadcopter unmanned aerial vehicle (UAV) system is considered a good platform for control scheme design as it is highly nonlinear with coupled dynamics and an under-actuated system. Considering these challenges, this manuscript aimed to propose a control algorithm to control the quadcopter system in the presence of uncertainty and disturbance influences. The integral adaptive sliding mode control scheme has been proposed to control the system. The proposed control scheme is composed of the outer loop controller to control the position of the quadcopter, while the inner loop controls the attitude of the quadcopter. The proposed control law has three major terms, firstly the equivalent control which is developed based on the Lyapunov approach to handle most of the uncertainty and disturbance, secondly the adaptive switching gain, which is achieving fast adaptation against uncertainty, finally the switching function which has been approximated by a tangent hyperbolic function to reduce the unwanted chattering phenomena. The proposed control scheme and its performance have been investigated via a MATLAB/Simulink. The results prove that the implemented control scheme is robust even in the presence of uncertainties and disturbance and the quadcopter tracks the predefined trajectories with limited chattering influence.

INDEX TERMS Quadcopter UAV, integral SMC control, adaptive control, chattering reduction.

I. INTRODUCTION The quadcopter is a type of unmanned aerial vehicle (UAV). It has some advantages against the other types of UAVs, such as fast maneuverability, easy take-off and landing, and payload capacity. Considering these advantages, the quadcopter UAV can be used to achieve many precise and critical tasks, for instance, military applications, medical applications, agriculture applications, and some other dangerous applications to avoid the risk to the human being.

Prior mentioned applications attract researchers to design robust control algorithms to achieve the assigned tasks precisely. Therefore, this research follows the model-based control design approach where the model dynamic is used in the proposed control design phase [1], [2].

The quadcopter is a complex system due to high nonlinearity in the dynamics, unbounded dynamics and uncertainties [3]. Furthermore, the quadcopter often operates in a harsh environment where external disturbance plays a significant role. Therefore, all these challenges must be taken into consideration in the control design stage.

Several advanced control techniques have been developed to control the quadcopter systems in order to handle these challenges. For example, sliding mode control [4], [5], [6], backstepping control [7], [8], feedback linearization [9], [10], adaptive SMC control [11], [12], [13], and high-order sliding mode control [14].

The SMC is classified among the robust nonlinear control algorithm, which provides better performance against
uncertain dynamics and disturbances [11], [15], [16]. Essentially, the SMC composes the equivalent and discontinuous (switching) control terms. The equivalent control deals with the nonlinear system dynamic, whereas the switching control term handles any influences that may occur due to uncertainty and unmodeled dynamics.

The chattering phenomenon is considered a major disadvantage associated with the traditional SMC [17], [18], where the unwanted chattering increases dramatically by increasing the switching gain. Meanwhile, for robust stability against uncertainty, disturbance and unmodeled dynamics, the value of the switching gain must be big enough to guarantee the stability of the quadcopter system.

The unwanted chattering leads to serious issues, for instance, vibration in the mechanical parts of the quadcopter and heat in onboard electronics elements, which results in rapid power loss. This study aims to utilize the SMC control advantages and reduce the impact of the chattering to a satisfactory bound. Some works are reported in the literature and focus on chattering reduction, such as the adaptive SMC control [17], higher-order SMC control [18], and fuzzy gain scheduling SMC [19], [20], [21], [22].

An adaptive SMC has been designed where the switching gain is tuned adaptively following the changes on parameters uncertainties [23], [24]. However, the designed controller was not investigating the chattering influences. An adaptive SMC control law is developed in [25] to achieve fast adaptation and reduce unwanted chattering.

This study proposes a control scheme based on the adaptive SMC control technique. The quadcopter dynamic is divided into two subsystems: position and attitude. Subsequently, an outer loop controller is developed based on the adaptive SMC control technique to control the quadcopter position. An inner loop controller is developed based on the adaptive SMC control technique to control the quadcopter attitude. The overall proposed control scheme provided robust tracking and contributed to chattering reduction.

The developed control law of the proposed adaptive SMC control scheme consists of three major terms, the equivalent term, the switching function and finally, the switching gain. The combination of these three terms contributes to a robust tracking and chattering reduction. Therefore, the contributions of this study are listed as follows:

- The equivalent control term is developed based on Lyapunov approach to handle the bulk of disturbance and uncertainty impact and relieve the burden on the switching control term.
- The switching function (sign(s)) is approximated by hyperbolic function tangent function (tanh(s)) where it contributes to the chattering reduction.
- The switching gain is calculated based on an adaptive formula to achieve fast adaptation against disturbance and uncertainty.

This paper is written in the following pattern. First, section II briefly introduces quadcopter modeling. Then, Section III discusses the proposed control scheme. Section IV evaluates the proposed control scheme via the MATLAB/Simulink platform, where the simulation results are presented and discussed. Finally, section V concludes the article.

II. THE QUADCOPTER MODELLING AND DESCRIPTION
In this section, a brief description of the quadcopter has been introduced, then the quadcopter kinematics and the dynamics equations have been presented.

A. QUADCOPTER SYSTEM DESCRIPTION
The quadcopter system composes of four rotors which attached to the mainframe in symmetric cross shape [26], [27], as illustrated in Figure 1.

B. QUADCOPTER KINEMATICS EQUATIONS
To obtain the kinematics equations of the quadcopter, the coordinates space can be given as follows.

\[ q = [\xi, \eta]^T \]  

where,

\[ \xi = [x, y, z]^T \]  

and,

\[ \eta = [\phi, \theta, \psi]^T \]  

Therefore, the translational motion equations of the quadcopter can be written as follows.

\[ \dot{\xi} = RV \]  

where \( \xi \) represents the linear velocity in the E-frame where \( E=(x_e, y_e, z_e) \), and \( V \) denotes the linear velocity in the B-frame where \( B=(x_b, y_b, z_b) \), while and \( R \) is the transformational matrix (5), as shown at the bottom of the next page.
Similarly, the quadcopter rotational motion equations can be obtained as follows.

\[
\dot{\eta} = T \omega
\]  

(6)

where \( \dot{\eta} \) denotes the angular velocity in the E-frame, and \( \omega \) denotes the angular velocity in the B-frame. \( T \) is the transfer matrix [29], and it’s given as follows.

\[
T = \begin{bmatrix}
1 \sin \phi \tan \theta & \cos \phi \tan \theta \\
0 & \cos \phi \\
0 & \sin \phi / \cos \theta
\end{bmatrix}
\]  

(7)

C. QUADCOPTER DYNAMIC EQUATIONS

The quadcopter dynamic equations in 6-DOF (space) are presented as follows.

\[
\begin{align*}
\dot{x} &= (\cos \phi \sin \theta \cos \psi + \sin \phi \sin \psi) \frac{u_1}{m} \\
\dot{y} &= (\cos \phi \sin \theta \sin \psi - \sin \phi \cos \psi) \frac{u_1}{m} \\
\dot{z} &= -g + (\cos \phi \cos \theta) \frac{u_1}{m} \\
\dot{\phi} &= a_1 \dot{\theta} \dot{\psi} + a_2 \dot{\theta} \Omega_d + \frac{1}{I_x} u_2 \\
\dot{\theta} &= a_3 \dot{\phi} \dot{\psi} + a_4 \dot{\phi} \Omega_d + \frac{1}{I_y} u_3 \\
\dot{\psi} &= a_5 \dot{\phi} \dot{\theta} + \frac{1}{I_z} u_4
\end{align*}
\]  

(8)

where

\[
a_1 = \frac{I_y - I_z}{I_x}, \quad a_2 = \frac{J_r}{I_x}, \quad a_3 = \frac{I_x - I_z}{I_y}, \quad a_4 = \frac{J_r}{I_y}, \quad a_5 = \frac{I_x - I_y}{I_z}
\]

and, \((u_1, u_2, u_3, u_4)\) are the control inputs, where

\[
\begin{align*}
u_1 &= b(\Omega_1^2 + \Omega_2^2 + \Omega_3^2 + \Omega_4^2) \\
u_2 &= b(\Omega_1^2 - \Omega_2^2) \\
u_3 &= b(\Omega_2^2 - \Omega_3^2) \\
u_4 &= d(\Omega_3^2 + \Omega_2^2 - \Omega_3^2 - \Omega_1^2)
\end{align*}
\]  

(9)

and, \(\Omega_d\) is written as follows.

\[
\Omega_d = -\Omega_1 + \Omega_2 - \Omega_3 + \Omega_4
\]  

(10)

where the quadcopter model parameters are listed and defined as in Table 1.

III. CONTROL DESIGN

This section discussed the proposed control algorithm design based on the integral adaptive SMC to control the quadcopter attitude and position subsystems, via the inner loop and the outer loop controllers, respectively.

| TABLE 1. Model parameters definitions. |
|--------------------------------------|
| Parameter | Definition |
| \(m\) | Mass |
| \(I_x\) | Inertia on x-axis |
| \(I_y\) | Inertia on y-axis |
| \(I_z\) | Inertia on z-axis |
| \(b\) | Thrust coefficient |
| \(d\) | Drag coefficient |
| \(J_r\) | Rotor inertia |
| \(l\) | Arm length |
| \(\Omega\) | The rotor angular speed |

A. THE INNER LOOP CONTROLLER DESIGN

The inner loop controller has been proposed based on the integral adaptive SMC to control the quadcopter attitude dynamics, and it is designed based on the following approach.

\[
\begin{align*}
\dot{\phi} &= a_1 \phi \dot{\psi} + a_2 \dot{\theta} \Omega_d + \frac{1}{I_x} u_2 + \mu_\phi \\
\dot{\theta} &= a_3 \phi \dot{\psi} + a_4 \dot{\phi} \Omega_d + \frac{1}{I_y} u_3 + \mu_\theta \\
\dot{\psi} &= a_5 \phi \dot{\theta} + \frac{1}{I_z} u_4 + \mu_\psi
\end{align*}
\]  

(11)

where,

\[
\begin{align*}
a_1 &= \frac{I_y - I_z}{I_x}, \quad a_2 = \frac{J_r}{I_x}, \quad a_3 = \frac{I_x - I_z}{I_y}, \quad a_4 = \frac{J_r}{I_y}, \quad a_5 = \frac{I_x - I_y}{I_z}
\end{align*}
\]

and \(\mu_\phi, \mu_\theta, \mu_\psi\) denote the external disturbances for \(\phi, \theta, \psi\) dynamics, respectively.

The control objective is to develop an integral adaptive SMC control law to stabilize the error dynamics of the quadcopter attitude. The error attitude dynamics are given as follows:

\[
\begin{align*}
e_{\phi} &= \phi - \phi_d \\
e_{\theta} &= \theta - \theta_d \\
e_{\psi} &= \psi - \psi_d
\end{align*}
\]  

(12)
where \((\phi, \theta, \psi)\) is the measured attitude, and \((\dot{\phi}, \dot{\theta}, \dot{\psi})\) is the reference attitude. The proposed integral adaptive SMC control laws for the attitude have been developed following the below methodology:

**Step 1:** is to write the dynamics of the errors as in (12).

**Step 2:** is to select the integral sliding surface as in (13) [5].

\[
s = \left(\frac{d}{dt} + k_1\right)^{n-1} + k_2 \int_{0}^{t} e d\tau \tag{13}
\]

Consequently, the integral SMC surface for attitude angles individually is chosen as follows.

\[
s_\phi = \dot{\phi} + k_1 \phi \dot{\phi} + k_2 \int_{0}^{t} \phi d\tau
\]

\[
s_\theta = \dot{\theta} + k_1 \phi \dot{\theta} + k_2 \int_{0}^{t} \theta d\tau
\]

\[
s_\psi = \dot{\psi} + k_1 \phi \dot{\psi} + k_2 \int_{0}^{t} \psi d\tau \tag{14}
\]

where, \(s_\phi, s_\theta\) and \(s_\psi\) represent the integral SMC surfaces. While \(k_1, k_1, k_1 > 0\) and \(k_2, k_2, k_2 > 0\) are the inner loop integral SMC control gains.

**Step 3:** is to implement the sliding mode condition as in the below (15).

\[
\dot{s} = -C_1 sgn(s) - C_2 s
\]

where \(C_1\) and \(C_2\) are positive constants.

Substitute (12) into (14), then derivative both sides result to.

\[
\dot{s}_\phi = (\ddot{\phi} - \ddot{\phi}_d) + k_1 \phi \dot{\phi} + k_2 \phi \dot{\phi}
\]

\[
\dot{s}_\theta = (\ddot{\theta} - \ddot{\theta}_d) + k_1 \phi \dot{\theta} + k_2 \phi \dot{\theta}
\]

\[
\dot{s}_\psi = (\ddot{\psi} - \ddot{\psi}_d) + k_1 \phi \dot{\psi} + k_2 \phi \dot{\psi} \tag{16}
\]

Substitute (11) into (16), yields to.

\[
\dot{s}_\phi = a_1 \phi \dot{\psi} + a_2 \phi \dot{\Omega} + \frac{1}{I_x} u_2 + \mu_\phi - \ddot{\phi}_d + k_1 \phi \dot{\phi} + k_2 \phi \dot{\phi}
\]

\[
\dot{s}_\theta = a_3 \phi \dot{\psi} + a_4 \phi \dot{\Omega} + \frac{1}{I_y} u_3 + \mu_\theta - \ddot{\theta}_d + k_1 \phi \dot{\theta} + k_2 \phi \dot{\theta}
\]

\[
\dot{s}_\psi = a_5 \phi \dot{\psi} + \frac{1}{I_z} u_4 + \mu_\psi - \ddot{\psi}_d + k_1 \phi \dot{\psi} + k_2 \phi \dot{\psi} \tag{17}
\]

where \(k_1, k_1, k_1 > 0\) and \(k_2, k_2, k_2 > 0\) are the design parameters.

**Step 4:** is to select the control input \(u_2, u_3, u_4\) as follows.

\[
u_2 = I_x (\ddot{\phi}_d - a_1 \dot{\phi} \dot{\psi} - a_2 \dot{\phi} \dot{\Omega} - k_1 \phi \dot{\phi} - k_2 \phi \dot{\phi} + \mu_\phi + U_1)
\]

\[
u_3 = I_y (\ddot{\theta}_d - a_3 \dot{\phi} \dot{\psi} - a_4 \dot{\phi} \dot{\Omega} - k_1 \phi \dot{\theta} - k_2 \phi \dot{\theta} + \mu_\theta + U_2)
\]

\[
u_4 = I_z (\ddot{\psi}_d - a_5 \dot{\phi} \dot{\psi} - k_1 \phi \dot{\psi} - k_2 \phi \dot{\psi} + \mu_\psi + U_3) \tag{18}
\]

Substitute (18) into (17), yields to.

\[
\dot{s}_\phi = U_1 + \Gamma_\phi
\]

\[
\dot{s}_\theta = U_2 + \Gamma_\theta
\]

\[
\dot{s}_\psi = U_3 + \Gamma_\psi \tag{19}
\]

where,

\[
\Gamma_\phi = I_x \mu_\phi
\]

\[
\Gamma_\theta = I_y \mu_\theta
\]

\[
\Gamma_\psi = I_z \mu_\psi \tag{20}
\]

**Step 5:** is to calculate the estimated \(\Gamma_\phi, \Gamma_\theta, \Gamma_\psi\) based on selected Lyapunov functions as follows.

\[
\dot{V}_\phi = \frac{1}{2} \dot{\phi}^2 + \frac{1}{2} \ddot{\phi}_d \dot{\phi} \dot{\phi}_d
\]

\[
\dot{V}_\theta = \frac{1}{2} \dot{\theta}^2 + \frac{1}{2} \ddot{\theta}_d \dot{\theta} \dot{\theta}_d
\]

\[
\dot{V}_\psi = \frac{1}{2} \dot{\psi}^2 + \frac{1}{2} \ddot{\psi}_d \dot{\psi} \dot{\psi}_d \tag{21}
\]

where, \(\dot{\Gamma}_\phi, \dot{\Gamma}_\theta, \dot{\Gamma}_\psi\) are errors between the actual and estimated uncertainties which can be written as, \(\dot{\Gamma}_\phi = \Gamma_\phi - \dot{\Gamma}_\phi, \dot{\Gamma}_\theta = \Gamma_\theta - \dot{\Gamma}_\theta,\) and \(\dot{\Gamma}_\psi = \Gamma_\psi - \dot{\Gamma}_\psi,\) respectively, while \(\gamma_\phi, \gamma_\theta, \gamma_\psi\) are positive constants.

Differentiating the equation (21) leads to.

\[
\dot{V}_\phi = s_\phi \dot{s}_\phi + \dot{\Gamma}_\phi \gamma_\phi \dot{\Gamma}_\phi
\]

\[
\dot{V}_\theta = s_\theta \dot{s}_\theta + \dot{\Gamma}_\theta \gamma_\theta \dot{\Gamma}_\theta
\]

\[
\dot{V}_\psi = s_\psi \dot{s}_\psi + \dot{\Gamma}_\psi \gamma_\psi \dot{\Gamma}_\psi \tag{22}
\]

As per (22) the adaption laws are calculated as follows.

\[
\dot{\dot{\Gamma}}_\phi = \frac{1}{\gamma_\phi} s_\phi
\]

\[
\dot{\dot{\Gamma}}_\theta = \frac{1}{\gamma_\theta} s_\theta
\]

\[
\dot{\dot{\Gamma}}_\psi = \frac{1}{\gamma_\psi} s_\psi \tag{23}
\]

Therefore, considering (15) and (19), the control laws are obtained as follows.

\[
U_1 = -\dot{\Gamma}_\phi - C_1 \dot{\phi} sgn(s_\phi) - C_2 s_\phi
\]

\[
U_2 = -\dot{\Gamma}_\theta - C_1 \dot{\theta} sgn(s_\theta) - C_2 s_\theta
\]

\[
U_3 = -\dot{\Gamma}_\psi - C_1 \dot{\psi} sgn(s_\psi) - C_2 s_\psi \tag{24}
\]

**Step 6:** to reduce the chattering, the sign(s) is approximated by the hyperbolic function tangent function (tanh(s)). Therefore, the switching function in (24) will be replaced by (tanh(s)), where its formula is given as follows.

\[
tanh(s) = \frac{e^{s} - e^{-s}}{e^{s} + e^{-s}} \tag{25}
\]

and has properties:

\[
tanh(s) = \begin{cases}
1 & \text{if } s \text{ is a positive big} \\
0 & \text{if } s = 0, \\
-1 & \text{if } s \text{ is a negative big}
\end{cases}
\]

**Step 7:** the switching gains are designed as in (26) to sense and adapt the changes in the uncertainties quickly. There are three parameters \(\mu_i, \alpha_i\) and \(\delta_i(t)\) can be used and selected to
achieve fast adaptation, which contributes to the chattering reduction.

\[
\dot{\hat{C}}_i = \begin{cases} 
\varphi_i \left( \alpha_i^{-1} \cdot |s_i(t)| \right)^{-\delta_i(t)} \cdot \delta_i(t) & \text{if } \hat{C}_i > 0 \\
\varphi_i \cdot \alpha_i^{-1} \cdot |s_i(t)| & \text{if } \hat{C}_i = 0 
\end{cases} 
\]  

(26)

The parameter \( \varphi_i \) can be tuned as follows:

\[
\delta_i(t) = \tanh\left(\|s_i(t)\|_{\infty} - \epsilon_i\right) 
\]

(27)

\[
\varphi_i = \begin{cases} 
\varphi_{i_{\text{up}}} & \text{if } \delta_i(t) > 0 \\
\varphi_{i_{\text{down}}} & \text{if } \delta_i(t) \leq 0 
\end{cases} 
\]

(28)

where, \( \varphi_{i_{\text{up}}} \) is controlling the increment rate and \( \varphi_{i_{\text{down}}} \) is controlling the decrement rate.

**B. THE OUTER LOOP CONTROLLER DESIGN**

As it can be observed, the overall dynamics of the quadcopter (attitude and position) as given in (8) are underactuated where there are 6-DOFs \([x, y, z, \phi, \theta, \psi]^T\) controlled via \([u_1, u_2, u_3, u_4]^T\) only [30], [31]. However, as shown in (8), the underactuated part is coming from the quadcopter position dynamics subsystem. Therefore, the Cartesian positions \((u_1, \phi, \theta)\) are utilized to generate three virtual control inputs to control each output individually. Considering the quadcopter position dynamics, the virtual controls can be written as follows.

\[
\begin{bmatrix} u_x \\ u_y \\ u_z \end{bmatrix} = \begin{bmatrix} \cos \phi \sin \theta \cos \psi + \sin \phi \sin \psi \\ \cos \phi \sin \theta \sin \psi - \sin \phi \cos \psi \\ \cos \phi \cos \theta \end{bmatrix} \begin{bmatrix} u_1 \\ -0 \\ g \end{bmatrix} + \begin{bmatrix} \mu_x \\ \mu_y \\ \mu_z \end{bmatrix} 
\]

(29)

where, \( \mu_x, \mu_y, \text{and } \mu_z \) are the position disturbances.

From (29), the Cartesian positions are calculated as follows.

\[
u_1 = \sqrt{u_x^2 + u_y^2 + (u_z + g)^2} 
\]

(30)

and,

\[
\phi_d = \sin^{-1} \left( \frac{u_x \sin \psi_d - u_x \cos \psi_d}{u_1} \right) 
\]

(31)

\[
\theta_d = \tan^{-1} \left( \frac{u_x \sin \psi_d + u_x \cos \psi_d}{u_z + g} \right) 
\]

(32)

where \( u_x, u_y \) and \( u_z \) denote the virtual controls generated to control the quadcopter position through \((u_1, \phi_d, \text{and } \theta_d)\). Particularly, as shown in Figure 2, the outer loop controller takes the errors signals which is the difference between the desired \((x_d, y_d, z_d)\) and the measured \((x, y, z)\) position and produces the virtual controls \((u_x, u_y, u_z)\). Then, the converter generates the lift force \(u_1, \phi_d, \text{and } \theta_d\) angles.

The aim of this section is to design an outer loop controller based on the integral adaptive SMC control to stabilize the quadcopter position.

Step 1: Subsequently, the error dynamics of the quadcopter position subsystem are written as follows.

\[
x_e = x - x_d \\
y_e = y - y_d \\
z_e = z - z_d 
\]

(33)

Step 2: Similarly, as in the inner loop controller design section, the integral sliding surface for the outer loop controller is written as follows.

\[
x_s = \dot{x}_s + k_1 e_x + k_2 \int_0^t e_x d\tau \\
y_s = \dot{y}_s + k_1 e_y + k_2 \int_0^t e_y d\tau \\
z_s = \dot{z}_s + k_1 e_z + k_2 \int_0^t e_z d\tau 
\]

(34)

where, \( x_s, y_s \) and \( z_s \) represent the sliding surfaces for \( x, y, \) and \( z \) dynamics, respectively. While \( k_1, k_1, k_2 > 0 \) and \( k_2, k_2, k_2 > 0 \) are the outer loop integral adaptive SMC control gains.

Step 3: is to apply the sliding mode condition as in (15).

\[
\dot{s}_x = (\ddot{x} - \ddot{x}_d) + k_1 \dot{e}_x + k_2 e_x \\
\dot{s}_y = (\ddot{y} - \ddot{y}_d) + k_1 \dot{e}_y + k_2 e_y \\
\dot{s}_z = (\ddot{z} - \ddot{z}_d) + k_1 \dot{e}_z + k_2 e_z 
\]

(35)

Then, from (29) and (35), we obtain.

\[
\dot{s}_x = u_x + \mu_x - \ddot{x}_d + k_1 \dot{e}_x + k_2 e_x \\
\dot{s}_y = u_y + \mu_y - \ddot{y}_d + k_1 \dot{e}_y + k_2 e_y \\
\dot{s}_z = u_z + \mu_z - \ddot{z}_d + k_1 \dot{e}_z + k_2 e_z 
\]

(36)

Step 4: is to select the control input \( u_x, u_y, u_z \) as follows:

\[
u_x = (\ddot{x}_d - k_1 \dot{e}_x - k_2 e_x) + U_x \\
u_y = (\ddot{y}_d - k_1 \dot{e}_y - k_2 e_y) + U_y \\
u_z = (\ddot{z}_d - k_1 \dot{e}_z - k_2 e_z) + U_z 
\]

(37)

Therefore, from (36) and (37), we get

\[
\dot{s}_x = \mu_x + U_x \\
\dot{s}_y = \mu_y + U_y \\
\dot{s}_z = \mu_z + U_z 
\]

(38)

Thus,

\[
\dot{s}_x = U_x + \Gamma_x \\
\dot{s}_y = U_y + \Gamma_y \\
\dot{s}_z = U_z + \Gamma_z 
\]

(39)

where,

\[
\Gamma_x = \mu_x \\
\Gamma_y = \mu_y \\
\Gamma_z = \mu_z 
\]

(40)
TABLE 2. The quadcopter model Parameters [32].

| Name             | Parameter | Value     | Unit  |
|------------------|-----------|-----------|-------|
| Mass             | \( m \)   | \( 65 \times 10^{-2} \) | kg    |
| Inertia on x-axis| \( I_x \)  | \( 75 \times 10^{-4} \) | kgm²  |
| Inertia on y-axis| \( I_y \)  | \( 75 \times 10^{-4} \) | kgm²  |
| Inertia on z-axis| \( I_z \)  | \( 13 \times 10^{-3} \) | kgm²  |
| Thrust coefficient| \( b \)   | \( 313 \times 10^{-6} \) | Ns²   |
| Drag coefficient | \( d \)    | \( 7.5 \times 10^{-7} \) | Nms²  |
| Rotor inertia    | \( J_r \)  | \( 6 \times 10^{-5} \) | kgm²  |
| Arm length       | \( l \)    | \( 23 \times 10^{-2} \) | m     |

Step 5: is to calculate the estimated \( \Gamma_x, \Gamma_y, \Gamma_z \) based on selected Lyapunov functions as follows.

\[
\begin{align*}
V_x &= \frac{1}{2} s_x^2 + \frac{1}{2} \Gamma_x \gamma_x \Gamma_x \\
V_y &= \frac{1}{2} s_y^2 + \frac{1}{2} \Gamma_y \gamma_y \Gamma_y \\
V_z &= \frac{1}{2} s_z^2 + \frac{1}{2} \Gamma_z \gamma_z \Gamma_z \\
&= \frac{1}{2} s_x^2 + \frac{1}{2} \Gamma_x \gamma_x \Gamma_x \\
&= \frac{1}{2} s_y^2 + \frac{1}{2} \Gamma_y \gamma_y \Gamma_y \\
&= \frac{1}{2} s_z^2 + \frac{1}{2} \Gamma_z \gamma_z \Gamma_z
\end{align*}
\]  

(41)

where, \( \Gamma_x, \Gamma_y, \Gamma_z \) are the uncertainty errors, and calculated as \( \Gamma_x = \Gamma_x - \hat{\Gamma}_x, \Gamma_y = \Gamma_y - \hat{\Gamma}_y, \) and \( \Gamma_z = \Gamma_z - \hat{\Gamma}_z \), while, \( \gamma_x, \gamma_y, \) and \( \gamma_z \) are positive constants.

Therefore, the first derivative for both sides of the selected Lyapunov function (41) yields to.

\[
\begin{align*}
\dot{V}_x &= s_x \dot{s}_x + \hat{\Gamma}_x \gamma_x \hat{\Gamma}_x \\
\dot{V}_y &= s_y \dot{s}_y + \hat{\Gamma}_y \gamma_y \hat{\Gamma}_y \\
\dot{V}_z &= s_z \dot{s}_z + \hat{\Gamma}_z \gamma_z \hat{\Gamma}_z \\
&= s_x \dot{s}_x + \hat{\Gamma}_x \gamma_x \hat{\Gamma}_x \\
&= s_y \dot{s}_y + \hat{\Gamma}_y \gamma_y \hat{\Gamma}_y \\
&= s_z \dot{s}_z + \hat{\Gamma}_z \gamma_z \hat{\Gamma}_z
\end{align*}
\]  

(42)

The adaption laws are calculated as follows.

\[
\begin{align*}
\dot{\hat{\Gamma}}_x &= \frac{1}{\gamma_x} \dot{s}_x \\
\dot{\hat{\Gamma}}_y &= \frac{1}{\gamma_y} \dot{s}_y \\
\dot{\hat{\Gamma}}_z &= \frac{1}{\gamma_z} \dot{s}_z
\end{align*}
\]  

(43)

Thus, the control laws will be as follows.

\[
\begin{align*}
U_x &= -\hat{\Gamma}_x - C_{1_x} \text{sgn}(s_x) - C_{2_x} s_x \\
U_y &= -\hat{\Gamma}_y - C_{1_y} \text{sgn}(s_y) - C_{2_y} s_y \\
U_z &= -\hat{\Gamma}_z - C_{1_z} \text{sgn}(s_z) - C_{2_z} s_z
\end{align*}
\]  

(44)

To reduce the chattering, steps 6 and 7 are implemented as in the previous section.

IV. SIMULATION MODEL

The proposed control scheme has been simulated and evaluated via the MATLAB/Simulink platform. The designed control parameters are listed in Table 3 and Table 4 for the inner and the outer loop controllers, respectively.

V. SIMULATION RESULTS

To evaluate the performance of the proposed control scheme, three different scenarios have been investigated. In the first case, the quadcopter has been commanded to reach the desired position. In the second case, the quadcopter has been commanded to track a predefined reference trajectory. Finally, the proposed control scheme has been evaluated, considering both the external disturbance and the quadcopter mass uncertainty, where the quadcopter is following the predefined track.

A. CONTROL SYSTEM PERFORMANCE: ACHIEVING THE DESIRED POSITION

In this scenario, the desired setpoint in space is defined as \((x_d, y_d, z_d)\), and the quadcopter UAV has been commanded to reach that point. The proposed control scheme is designed to achieve the assigned task precisely and fastly as shown in Figure 3 and Figure 4 for the position...
and attitude, respectively. Figure 5 and Figure 6 presented the associated errors for the position and attitude, respectively. Figure 7 presents the control efforts to accomplish the assigned tasks.

B. CONTROL SYSTEM PERFORMANCE: FOLLOWING REFERENCE TRAJECTORY TRACKING

In this case, the quadcopter is simulated and commanded to track a predefined trajectory in space 6-DOFs (for the position as in Figure 8 and Figure 9, and for the attitude as in Figure 10), considering the ideal environment where there is no influence neither from the uncertainties nor disturbances. This scenario is almost similar and applicable for indoor quadcopter applications where the operation area is limited and closed.

The simulation results in this scenario prove that the proposed controller is robust against the reference tracking modification. In the previous scenario, the performance was evaluated for the quadcopter to reach the reference setpoint, whereas, in this scenario, the proposed controller is evaluated
to follow a full predefined trajectory tracking in space. Figure 11 and Figure 12 visualize the error signals for the position and attitude, respectively. The control efforts that enabled the quadcopter to achieve this task successfully are shown in Figure 13.

**C. THE CONTROL SYSTEM PERFORMANCE: FOLLOWING REFERENCE TRAJECTORY TRACKING AGAINST THE VARIABLE PAYLOAD AND DISTURBANCE**

In this scenario, the quadcopter is simulated to mimic the real and harsh environment applicable in some outdoor applications such as spraying agricultural pesticides where the disturbances and uncertainties are challenging. In this case the uncertainty has been represented by a variable payload where it is assumed as an added mass (such as liquid pesticides) 0.18 kg to the quadcopter nominal mass 0.65 kg; the added mass is about 30%. As shown in Figure 14 the added mass discharges linearly until the liquid pesticides finish and the quadcopter mass returns to the nominal value. Meanwhile, the disturbances have been considered as well as depicted in Figure 15.

Despite these challenges, the proposed controller still enables the quadcopter to track a predefined trajectory in

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**FIGURE 10.** Quadcopter attitude-following the desired angles in space-nominal parameters.

**FIGURE 11.** Quadcopter position errors signals (nominal parameters).

**FIGURE 12.** Quadcopter attitude errors signals-nominal parameters.

**FIGURE 13.** Quadcopter control signals-nominal parameters.

**FIGURE 14.** Quadcopter variable payloads (mass uncertainty).

**FIGURE 15.** External disturbances acting on x, y and z-axes.
FIGURE 16. 3D view of desired trajectory tracking in the presence of disturbance and mass uncertainty.

FIGURE 17. Quadcopter position-following the desired position in space-in the presence of disturbance and mass uncertainty.

FIGURE 18. Quadcopter attitude-following the desired angles in space-in the presence of disturbance and mass uncertainty.

FIGURE 19. Quadcopter position errors signals-in the presence of disturbance and mass uncertainty.

FIGURE 20. Quadcopter attitude errors signals-in the presence of disturbance and mass uncertainty.

FIGURE 21. Quadcopter control signals-in the presence of disturbance and mass uncertainty.

space 6-DOFs for (the position as in Figure 16 for 3D view and Figure 17 and the attitude as in Figure 18). The assigned task has been achieved successfully, and the controller performance can be observed as in Figure 19, and

Figure 20, which presented the errors in quadcopter position and attitude, respectively. The control efforts that enabled the quadcopter to overcome these challenges with satisfactory chattering as shown in Figure 21. The performance of
the proposed control scheme has been compared with the benchmark integral adaptive SMC controller in which the gain is considered as constant and the switching function is considered as sign(s) without an approximation to study and investigate the performance of the proposed integral adaptive SMC control scheme in terms of the chattering attenuation and robustness. The results have been shown in Figure 21 and Figure 22 which represent the performance of the proposed controller and benchmark controller, respectively, where the performance of the proposed controller outperformed the benchmark controller. While Figure 23 visualizes the behavior of the adaptive gains against the uncertainties and the disturbance.

VI. CONCLUSION

In this paper, the proposed control scheme has been developed based on the adaptive SMC control algorithm. The quadcopter dynamics is divided into two parts, the position and the attitude subsystems. Subsequently, an outer loop controller has been designed based on the adaptive SMC control method to control the quadcopter position. Also, an inner loop controller has been developed based on the adaptive SMC control technique to control the quadcopter attitude. The overall proposed control scheme provided robust tracking and contributed to chattering reduction. Based on the proposed integral SMC, it is suggested to proceed further to identify the unknown dynamics, which is also important for UAV control. Currently, intelligent algorithms are widely used in online estimation, e.g., neuroadaptive control for complicated underactuated systems with simultaneous output and velocity constraints; adaptive fuzzy control for a class of MIMO underactuated systems with plant uncertainties and actuator dead-zones. These topics are the subject of future research.

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