Mass melting and pair production in strong fields

Stefan Evans and Johann Rafelski

Department of Physics, The University of Arizona, Tucson, AZ 85721, USA

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The electron’s electromagnetic (EM) and Higgs mass contents respond differently to strong external fields. The EM mass melts entirely in the presence of a critical external electric field, as we show explicitly using a model. For strong quasi-constant electromagnetic fields we apply this effect to compute an enhancement of pair production rates allowing direct measurement of mass melting. Virtual electron mass melting using the perturbative loop QED insert narrows the muon anomalous magnetic moment discrepancy.

We explore the mass of an electron. Multiple mechanisms likely contribute: an electromagnetic (EM) part, a Higgs (minimal coupling) component, and eventually, a beyond standard model (BSM) part possibly connected to neutrino mass. Here we focus on the EM mass component of the electron which can be explored in strong external fields. We do not believe that the BSM component responds to an external field, and the Higgs component requires field strengths many orders beyond the EM response.

To determine the amount of EM mass in the electron, we study how in nonlinear electromagnetism mass is modified by an external field. We find that mass can be melted entirely by an external electric field near the critical field strength $E_{\text{BI}} = 1.187 \times 10^{18}$ V/m arising in the Born-Infeld (BI) model \[1\][2]. We expect that a similar mass melting effect arises in QED and behaves as a scalar potential, which we label $\phi$ that enters the Dirac equation as

$$[\gamma \cdot (i\partial - eA) - m_e (1 - \phi)] \psi = 0. \quad (1)$$

The mass quantity $\phi$ is recognized in the limit of quasi-constant fields within perturbative QED as the electron self-energy $\Sigma(p \to 0) \to \phi$. Perturbative QED treatment of $\phi$ in the presence of an external field produces dynamical field-dependent terms \[3\][4]. Pending further investigation, we explore a model for $\phi$ in context of observable effects in experiment.

Considering the QED vacuum in presence of external fields on the order of the Schwinger critical field $E_s = m_e^2/e = 1.323 \times 10^{18}$ V/m we explore how the electron’s EM mass component affects the nonlinear QED vacuum response to an electric field. We demonstrate how pair production is enhanced by this effect via computation of the imaginary part of Euler-Heisenberg-Schwinger (EHS) effective action \[5\][7].

The electron mass melting enters high-precision perturbative QED experiments: The melting of electron mass in the vacuum polarization contribution to the muon vertex diagram produces an increase in the theoretical value of the muon AMM, bringing it closer to the experimental value. We here recall the mass melting model’s behavior as a scalar potential reminiscent of the proposed resolution to the muon AMM discrepancy between theory and experiment, in which a scalar particle was introduced, for a recent discussion see \[8\] and references therein.

**Nonlinear action:** We write two nonlinear EM actions: the QED EHS action relevant to an experimental probe of mass melting, and a simpler model: the BI action providing an exactly solvable example for how EM mass is affected by external fields. EHS action is described by Lagrangian \[5\][7]

$$\mathcal{L}_{\text{EHS}} = -\frac{1}{8\pi^2} \int_0^\infty ds s^2 e^{-m_e^2 s} \times \left( \frac{e^2 a b s^2}{\tan|eas| \tanh|ebs|} - 1 + \frac{e^2 s^2 (a^2 - b^2)}{3} \right), \quad (2)$$

where invariants $a$ and $b$ defined in terms of $S$ and $P$

$$a^2 - b^2 = 2S, \quad a^2b^2 = P^2$$

$$S = \frac{1}{2} (E^2 - B^2), \quad P \equiv E \cdot B, \quad (3)$$

and the pre-factor $1/8\pi^2$ follows units used by Schwinger in which $\alpha = e^2/4\pi$, see \[9\]. The function has been subtracted twice to remove the zero-point energy and the logarithmically divergent contribution, the latter absorbed by charge renormalization. The BI Lagrangian is given by \[11\][2]

$$\mathcal{L}_{\text{BI}}(\vec{E}, \vec{B}) = -\mathcal{L}_{\text{BI}}^e \left( \sqrt{1 - \frac{a^2 - b^2}{2e^2 B_{\text{BI}}} - \frac{a^2 b^2}{e^2 B_{\text{BI}}^2}} - 1 \right). \quad (4)$$

In the BI model all electron mass is in the EM field.

We consider an electron’s electric field $E_p$ in the presence of a constant and homogeneous external electromagnetic field described by $E_{\text{ex}}, B_{\text{ex}}$. Due to nonlinearity of Eqs. (2) and (4), $E$ cannot be written as a linear superposition of $E_p$ and $E_{\text{ex}}$. Instead the displacement fields generated by free charge may be superposed, regardless of the form of the electromagnetic Lagrangian. We use displacement fields to distinguish the external and particle field:

$$\vec{D} = \vec{D}_p + \vec{D}_{\text{ex}} = \hat{\rho} \frac{e}{4\pi |\vec{r}|^2} + \vec{D}_{\text{ex}}. \quad (5)$$

We compute self-energy via application of the appropriate Legendre transform detailed in \[2\]. An analytical
form for energy density in terms of $D$, $B$ is a feature of BI theory that is otherwise not easily achieved in most nonlinear effective actions:

$$U(\vec{D}, \vec{B}) = \mathcal{E} \cdot D - \mathcal{L} = \mathcal{E}_{\text{BI}} \mathcal{L}_{\text{BI}}$$

$$= \mathcal{E}_{\text{BI}} \left( \mathcal{L}_{\text{BI}} + \frac{D^2 + B^2}{\mathcal{E}_{\text{BI}}^2} + \frac{(D \times B)^2}{\mathcal{E}_{\text{BI}}^4} - 1 \right) ,$$

where the displacement field

$$\vec{D} = \frac{\partial \mathcal{L}_{\text{BI}}}{\partial \mathcal{E}} = \mathcal{E} + \mathcal{B} (\mathcal{E} \cdot B) \sqrt{1 - (\mathcal{E}^2 - B^2)/\mathcal{E}_{\text{BI}}^2 - (\mathcal{E} \cdot B)^2/\mathcal{E}_{\text{BI}}^4}. \quad (7)$$

A exact expression for energy density using EHS action is more challenging.

The connection between electromagnetic mass behavior in QED EHS action and that in the BI model lies in the weak field expansions of the actions, which to sixth order in EM fields read (subscript ‘EM’ denotes usual EM-Maxwell action)

$$\mathcal{L}_{\text{EHS}} = \frac{2a^2}{45m^2} \left\{ 4S^2 + 7P^2 + \frac{8\pi\alpha}{7m} (16S^3 + 26SP^2) \right\} ,$$

$$\mathcal{L}_{\text{BI}} - \mathcal{L}_{\text{EM}} = \frac{1}{8\mathcal{E}_{\text{BI}}} \left\{ S^2 + 4P^2 + \frac{1}{2\mathcal{E}_{\text{BI}}} (S^3 + 4SP^2) \right\} . \quad (8)$$

In the example of a pure electric field, the two series have the same sign and thus we expect electromagnetic mass melting to occur in both cases. We recall that a complete comparison between the BI model and QED action requires inhomogeneous corrections to the EHS result, which is valid for fields quasi-constant over the electron Compton wavelength.

**Example of electromagnetic mass melting:** We compute the BI mass, starting with the well-known result in the absence of external fields:

$$m_{\text{BI}}|_{D_{\text{ex}}=B_{\text{ex}}=0} = \int d^3r U(\vec{D}_p)$$

$$= \mathcal{E}_{\text{BI}} \int d^3r \left( \sqrt{1 + \mathcal{D}_p^2/\mathcal{E}_{\text{BI}}^2} - 1 \right)$$

$$= m_e , \quad (9)$$

in the last line set equal to the electron mass $m_e$. To include an external field into the mass, we subtract from the energy density the external field contribution:

$$\delta U(\vec{D}, \vec{B}) = U(\vec{D}_p + \vec{D}_{\text{ex}}, \vec{B}_{\text{ex}}) - U(\vec{D}_p, \vec{B}_{\text{ex}}) . \quad (10)$$

Eq. (10) is integrated to obtain the external field-dependent mass:

$$m_{\text{BI}} = \int d^3r \delta U(\vec{D}) . \quad (11)$$

The modification of the electromagnetic mass is entirely a consequence of the nonlinearity of the theory. If the Maxwell energy density were applied instead, we would have the square of linearly superposed fields, in which the mixing (interaction) term integrated over position space produces zero.

We compute Eq. (11) numerically (solid line in figure 1) as a function of the external field $\mathcal{E}_{\text{ex}}$, where for $\mathcal{B}_{\text{ex}} = 0$

$$\mathcal{E}_{\text{ex}} = \lim_{|\vec{x}| \to \infty} \mathcal{E} = \frac{\mathcal{D}_{\text{ex}}}{\sqrt{1 + \mathcal{D}_{\text{ex}}^2/\mathcal{E}_{\text{BI}}^2}} , \quad (12)$$

defined as the electric field far from the electron. While linear superposition of the two electric fields is violated, evaluation of $m_{\text{BI}}$ (Eq. (11)) for $\mathcal{E}_{\text{ex}}$ within $0 < \mathcal{E}_{\text{ex}}/\mathcal{E}_{\text{BI}} < 1$ is equivalent to computation using $\mathcal{D}_{\text{ex}}$ within the domain $0 < \mathcal{D}_{\text{ex}}/\mathcal{E}_{\text{BI}} < \infty$. We find that as the electric field approaches the critical field, mass disappears entirely and series expansion (dashed line) no longer applies.

**Perturbative series:** To verify the computation of mass melting above and to show its nonperturbative character near to critical field, we expand the integrand of Eq. (11) in powers of weak external fields $\mathcal{E}_{\text{ex}}/\mathcal{E}_{\text{BI}}, \mathcal{B}_{\text{ex}}/\mathcal{E}_{\text{BI}} \ll 1$ and integrate for an analytical result. We present three cases expanded to sixth order in EM fields: a pure electric field, a pure magnetic field, and crossed fields.

$$\frac{m_{\text{BI}}}{m_e} = \begin{cases} 
1 - \frac{3}{8} \mathcal{E}_{\text{ex}}^2 & \mathcal{B}_{\text{ex}} = 0 \\
1 + \frac{3}{40} \mathcal{B}_{\text{ex}}^4 - \frac{19}{280} \mathcal{B}_{\text{ex}}^6, & \mathcal{E}_{\text{ex}} = 0 \\
1 - \frac{3}{8} \mathcal{E}_{\text{ex}}^2 + \frac{39}{128} \mathcal{E}_{\text{ex}}^4 - \frac{1861}{7168} \mathcal{E}_{\text{ex}}^6, & \mathcal{S}_{\text{ex}} = P_{\text{ex}} = 0 . \end{cases} \quad (13)$$

The dashed line in figure 1 shows the first case, the pure electric field. In a pure magnetic field, mass grows ($< 5\%$ increase at the critical field). For crossed fields mass also melts, as magnetic field ($|\mathcal{B}_{\text{ex}}|$ = $|\mathcal{E}_{\text{ex}}|$) contributions do not appear until the fourth order in the external field.

**Pair production in strong fields:** We explore how to measure electron mass melting in QED by considering pair production in the EHS action. We have shown a
model example demonstrating mass melting in which the entire mass is electromagnetic, and is solved exactly via computation of self-energy. Such a melting effect also occurs in QED, yet by an amount that awaits the resolution of the electron EM mass portion.

Motivated by the leading electric-field dependent term in the mass melting result in Eq. (13), we introduce as parametric model for the mass melting scalar potential φ:

\[ m = m_e \left( 1 - \phi \right) = m_e \left( 1 - k_e \frac{a^2 - b^2}{E^2} \right), \quad (14) \]

Parameter \( k_e \) denotes the fraction of mass that melts at field \( E \), and may indicate what fraction of the electron mass is electromagnetic in origin. We apply this melting effect to the imaginary part of EHS action, given by

\[ \text{Im}[\mathcal{L}_{EHS}] = \frac{\alpha^2 a b}{8 \pi^3} \sum_{n=1}^{\infty} \frac{\coth[n \pi b/a]e^{-n \pi m_e^2 / e a}}{n}, \quad (15) \]

from which the rate of pair production and vacuum decay time is obtained [10]. In figure 2 the enhancement of pair production according to Eq. (15) with mass modified by Eq. (14) is shown. For the example where \( k_e = 1/5 \), at \( E_{\text{ex}} = E_e \) the imaginary part of action is nearly tripled.

![FIG. 2. Enhancement of pair production \text{Im}[\mathcal{L}_{EHS}] due to mass melting according to Eq. (14) normalized to the original result as a function of external \( E_{\text{ex}} \), with \( B_{\text{ex}} = 0 \).](image)

The real part of the EHS effective action is also modified by mass melting. In the weak field limit using first line in Eq. (8) with mass as seen in Eq. (14),

\[ \mathcal{L}_{EHS} = \frac{2 \alpha^2}{45 m_c^4} \left\{ \left(4S^2 + 7P^2\right) \left(1 + k_e S \frac{32 \pi \alpha}{m_e^4} \right) \right. \]
\[ + \frac{8 \pi \alpha}{im_e^4} \left(16 S^3 + 26 S P^2\right) \left. \right\} + \mathcal{O}(\mathcal{E}_e^6). \quad (16) \]

That is, the mass melting effect does not modify the perturbative form of EHS action until order \( \mathcal{E}_e^6 \).

Should the mass of the electron melt entirely, the non-perturbative behavior of \( \text{Re}[\mathcal{L}_{EHS}] \) requires further attention: the real part of EHS action would diverge and radiative corrections become important. We will return to discuss the incorporation of radiative corrections to EHS action with total mass melting in an upcoming paper, for both internal photon line [11, 12] and reducible photon line [13, 14] corrections to the EHS action. Here we have presented examples where no more than half the electron mass melts at \( E_e \), in which case radiative corrections to the effective action are small.

**Implications in perturbative QED:** The mass melting influences the electron mass entering vacuum polarization contributions to muon AMM and muonic Lamb shift. For the muon AMM, see figure 3. This diagram doesn’t comprise the entire mass melting effect, but it is illustrative of how the QED contribution to mass melting arises.

![FIG. 3. Mass melting electron vacuum polarization contribution to muon AMM. ‘x’ denotes the differing external fields seen by the muon and virtual electron.](image)

The muon sees \( B_{\text{ex}} \), the magnetic field applied in measuring the AMM, while the electron sees \( \mathcal{E}_p, B_p, B_{\text{ex}} \), the combination of the applied magnetic field and the electromagnetic fields of the muon. The electron mass is melted by the field of the muon, which is far stronger than \( B_{\text{ex}} \). Our intuition tells us that the electron mass is more impacted by external fields than the muon’s, since in addition to being subject to a weaker external field, we expect that the electron proportionally has more EM mass than the muon.

As an example we consider the electron mass melting due to \( B_p \). We apply field strength \( \langle \mathcal{E}_p \rangle = 12 \alpha \mathcal{E}_e \), the absolute value of the muon Coulomb field averaged over volume \((4/3)\pi \lambda_e^3\), where \( \lambda_e \) is the electron Compton wavelength. Choosing parameter \( k_e = 0.2 \) for the leading perturbative scalar field expression (Eq. (14)), we obtain a mass melting of \( m = 0.99839m_e \), which we apply to the leading electron vacuum polarization correction to the muon AMM:

\[ \Delta a_\mu(m_e) \sim \frac{\alpha^2}{\pi^2} \left( \frac{1}{3} \ln \left[ \frac{m_e}{m_\mu} \right] - \frac{25}{36} \right), \quad (17) \]

given by Eq. (77) in [15]. The AMM increases by

\[ \Delta a_\mu(0.99839m_e) - \Delta a_\mu(m_e) = 2.90 \cdot 10^{-9}, \quad (18) \]

doing the discrepancy between experimental and theoretical values for \( a_\mu [15] \).

Precision experiments of muonic atoms are also important to consider. For a long time there has been a
Lamb shift discrepancy in measurement of muonic hydrogen [11], which in part motivated this work. However this may have been resolved recently [17]. We note in passing that since a muonic atom’s Bohr radius is close in value to the electron Compton wavelength, the virtual electrons experience much stronger fields and mass is more likely to melt than in ordinary atoms. However the muon and proton fields interfere, making computation of virtual electron mass melting effects much more involved than in the muon $g-2$ study. Since the mass melting fields change with $Z$ of the (light) nuclei, there is possibility for study of these effects, which we return to in the future.

Heavy atoms also probe mass melting of the electron. We comment on the real electrons, considering the well-studied hydrogen-like uranium 1s state. The leading QED contributions to mass melting are the self-energy and Wichmann-Kroll corrections to the 1s binding energy, see table 17 of [15] and references therein. The ratio of these corrections to the 1s binding in the Coulomb field of point-like nucleus provides an estimate of the electron mass melting: $m = 0.997m_e$. This amounts in our model to parameter $k_e = 0.0295$. For an electric field approximated to the Coulomb field at 574 fm: $E \approx 0.304E_e$. The value of $k_e$ here is smaller than that considered for the muon AMM moment above. However, one value doesn’t necessarily restrict the other since the external field configurations are different between the two cases. For higher $n$-shells in high $Z$-atoms the electrons experience weaker fields: the range of allowed $k_e$ increases, e.g. for $n = 2$ we find $k_e \approx 0.8$. A resolution to $k_e$ requires extensive study of QED action in strong inhomogeneous external fields.

**Conclusion:** We have shown that modification of electromagnetic mass in external fields is caused by violation of linear superposition of EM fields. Melting of electron mass impacts pair production in strong fields, which is at the center of much theoretical attention [9, 13, 20]. Vacuum instability is motivating a renaissance of strong field physics and the development of novel ultra-intense-pulsed-laser technologies [21-25]. As laser fields approach strengths suitable for probing vacuum instability, it will become possible to measure mass melting via enhancement of pair production rates.

Electron mass melting also impacts the results of perturbative QED, especially as it enters the muon AMM, narrowing the discrepancy between theory and experiment. It is important to recall that the mass melting model we apply is an estimate even when fields are quasi-constant. The value of $k_e \approx 0.2$ entering the scalar potential for inhomogeneous fields in perturbative QED differs from that in quasi-constant fields probing EHS vacuum instability. For further study, nonperturbative effective action for inhomogeneous fields [26, 29] must be considered.

Mass melting effects offer an opportunity for exploring the origin of lepton mass, which is currently unexplained in particle physics. We expect the relation between the EM and Higgs portions of electron mass to be further refined by pair production and perturbative QED experiment. In near future we refine our study to allow pair production computation in presence of a full melting of electron mass, and application of this method in the study of the nonperturbative QED mass melting.

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