The time evolution of the quark gluon plasma in the early Universe

S. M. Sanches Jr., D. A. Fogaça and F. S. Navarra

Instituto de Física, Universidade de São Paulo, Rua do Matão Travessa R, 187, 05550-090, São Paulo, SP, Brazil
E-mail: samelsanches@usp.br

Abstract.
Our knowledge of the equation of state of the quark gluon plasma has been continuously growing due to the experimental results from heavy ion collisions and also due to the advances in lattice QCD calculations. The new findings about this state may have consequences on the time evolution of the early Universe, which can estimated by solving the Friedmann equations. The solutions of these equations give the time evolution of the energy density and also of the temperature in the beginning of the Universe. In this work we compute the time evolution of the QGP in the early Universe, comparing several equations of state, some of them based on the MIT bag model (and on its variants) and some of them based on lattice QCD calculations. Among other things, we investigate the effects of a finite baryon chemical potential in the evolution of the early Universe.

1. Introduction
In the last ten years relativistic heavy-ion collision experiments have provided us with information about the properties of matter in the early Universe (at the time when its age was about 10 microseconds and its temperature was higher than 150 MeV). It is believed that, during this period, the Universe was formed by a hot phase of deconfined quarks and gluons, i.e., a quark gluon plasma (QGP). In parallel with these experimental developments there has been a significant progress on the theoretical side, coming from the numerical simulation of finite temperature QCD on a lattice. The new findings about the nature of the QGP motivate us to investigate their consequences in the primordial Universe. This can be done by solving the Friedmann equations, which allow us to determine the precise time evolution of the thermodynamic quantities in the early Universe.

Previous works along this line and with the same motivation already exist in the literature. For a review see, e.g., [1] and for recent papers on the subject see [2, 3] and references therein. Most of these works focused on the nature of the phase transition from the QGP to the hadron gas. There are exotic phenomena associated with the order of the phase transition. In [3] a realistic EOS was used in cosmological calculations. In this EOS the transition was actually a crossover and not a first order transition as commonly believed until recent years. The results showed a very smooth time dependence of various thermodynamic quantities and suggested indirectly that there are small chances for the observation of various exotic phenomena such as quark nuggets, strangelets, cold dark matter clumps, etc. Such phenomena are associated typically with first order phase transitions. Apart from these exotic phenomena, changing the
equation of state we change the space-time evolution of the early Universe and this (specially when there is a phase transition) will change the emission of gravitational waves, as pointed out in [4] and also in [5] and [6].

In the early universe the baryon chemical potential was small (and usually neglected in cosmological calculations) but we do not know exactly how small. Moreover there may have been fluctuations in the chemical potential in the early Universe associated with the anisotropy of positively and negatively charged particles in the QGP phase, as pointed out in [7]. It is therefore interesting to estimate the effects of a non-vanishing chemical potential on the solution of the Friedmann equations.

In this work we solve numerically the Friedmann equations using some recently proposed equations of state and investigate how the results depend on details of these equations of state. We compute the time evolution of some thermodynamical quantities of the Universe, such as the energy density, pressure, temperature and sound speed. We also study the time evolution of these quantities at finite chemical potential.

2. The QGP equation of state

In the standard model of cosmology the space-time is be parametrized by the Friedmann-Lemaître-Robertson-Walker (FLRW) metric which, used in the Einstein equations, leads to the Friedmann equations, which, after some manipulation [2, 3, 8] yield the following time evolution equation:

$$-\frac{d\varepsilon}{3\sqrt{\varepsilon(\varepsilon+p)}} = \sqrt{\frac{8\pi G}{3}} dt$$ (1)

which allows us to find the temporal evolution of the energy density $\varepsilon$ for any given equation of state $p \equiv p(\varepsilon)$. In this section we present some equations of state which have been either phenomenologically extracted from the study of heavy ion collisions (and also of compact stars) or derived from first principles. They can be grouped into two categories, the first one being the "MIT family" and the second one being the "Lattice QCD family". We start presenting the MIT bag model [9] EOS and its modern variants. In what follows we use natural units $\hbar = c = k_B = 1$.

2.1. The MIT bag model

The energy density and pressure are given by [9, 10]:

$$\varepsilon = \frac{37\pi^2}{30} T^4 + B$$
$$p = \frac{37\pi^2}{90} T^4 - B$$ (2)

where $B$ is the bag constant and $T$ is the temperature. Considering these thermodynamical quantities as functions of time, we have from (2) the following relation:

$$p[\varepsilon(t)] = \frac{1}{3} [\varepsilon(t) - 4B]$$ (3)

A particular case, with only free gluons is described by [9, 10]:

$$\varepsilon_g = \frac{8\pi^2}{15} T^4 + B$$
$$p_g = \frac{8\pi^2}{45} T^4 - B$$ (4)

and also satisfies (3).
2.2. Model 1

This model (and also the next one) was presented in [11]. The pressure and energy density as function of temperature are given by:

\[ p_1 = \frac{\sigma_1}{3} T^4 - AT - B_1 \quad \text{and} \quad \varepsilon_1 = \sigma_1 T^4 + B_1 \]  

(5)

from which we can write the following relation:

\[ p_1[\varepsilon_1(t)] = \frac{1}{3} [\varepsilon_1(t) - 4B_1] - A \left[ \frac{\varepsilon_1(t) - B_1}{\sigma_1} \right]^{1/4} \]  

(6)

where the parameters were fixed in [11]: \( \sigma_1 = 4.73 \), \( A = 3.94 T_c^3 \) and \( B_1 = -2.37 T_c^4 \). Here \( T_c \) is the critical temperature of the QGP.

2.3. Model 2

This model was also introduced in [11]. The pressure and energy density as function of temperature are given by:

\[ p_2 = \frac{\sigma_2}{3} T^4 - CT^2 - B_2 \quad \text{and} \quad \varepsilon_2 = \sigma_2 T^4 - CT^2 + B_2 \]  

(7)

which can be combined to give:

\[ p_2[\varepsilon_2(t)] = \frac{1}{3\sigma_2} \left\{ [\varepsilon_2(t) - 4B_2] - C \left[ C + \sqrt{C^2 + 4\sigma_2 [\varepsilon_2(t) - B_2]} \right] \right\} \]  

(8)

and again from [11] we have: \( \sigma_2 = 13.01 \), \( C = 6.06 T_c^2 \) and \( B_2 = -2.34 T_c^4 \).

2.4. Model 3

This model was proposed in [12] to study the cold and dense quark gluon plasma in the inner core of neutron stars [13]. It is derived from the QCD Lagrangian treated in the mean field approximation. Here we use its extension to the finite temperature case. We repeat all the calculations described in [12] combining the mean field approximation with the finite temperature formalism developed in [14]. After some algebra (the details will be given elsewhere) we obtain the pressure:

\[ p_3 = \frac{3g^2}{16m_F^2} \rho^2 - B_{\text{QCD}} + \sum_f \frac{\gamma_f}{6\pi^2} \int_0^\infty dk \frac{k^4}{m_f^2 + k^2} \left( d_f + \bar{d}_f \right) + \frac{\gamma_q}{6\pi^2} \int_0^\infty dk k^2 (e^{k/T} - 1)^{-1} \]  

(9)

and the energy density:

\[ \varepsilon_3 = \frac{3g^2}{16m_F^2} \rho^2 + B_{\text{QCD}} + \sum_f \frac{\gamma_f}{2\pi^2} \int_0^\infty dk k^2 \sqrt{m_f^2 + k^2} \left( d_f + \bar{d}_f \right) + \frac{\gamma_q}{2\pi^2} \int_0^\infty dk k^2 (e^{k/T} - 1)^{-1} \]  

(10)

where the Fermi distribution functions are given by:

\[ d_f \equiv \frac{1}{1 + e^{(\varepsilon_f - \nu_f)/T}} \quad \text{and} \quad \bar{d}_f \equiv \frac{1}{1 + e^{(\varepsilon_f + \nu_f)/T}} \]  

(11)
The energy of the quark of flavor $f$ is given by $E_f = \sqrt{m_f^2 + k^2}$ and $\nu_f$ is its chemical potential. The quark density is given by:

$$\rho = \sum_f \frac{\gamma_f}{(2\pi)^3} \int d^3k \left( d_f - \bar{d}_f \right)$$  \hspace{1cm} (12)

For simplicity, we consider only two quark flavors, up and down, with equal masses and we work in the high temperature regime, where $T \gg \nu_f, T \gg m_f$ and $E_f/T > \nu_f/T$. The statistical factors are given by $\gamma_g = 2$(polarizations) $\times$ 8(colors) = 16 for gluons and $\gamma_f = 2$(spins) $\times$ 3(colors) = 6 for each quark. In the high temperature limit we can solve the integrals in (9), (10) and (12) analytically [15] obtaining:

$$p_3 = \frac{3g^2}{16m_g^2} T^4 \mu^2 + \frac{37\pi^2}{90} T^4 - B_{QCD} + \frac{1}{2} T^2 \mu^2$$  \hspace{1cm} (13)

and

$$\varepsilon_3 = \frac{3g^2}{16m_g^2} T^4 \mu^2 + \frac{37\pi^2}{30} T^4 + B_{QCD} + \frac{3}{2} T^2 \mu^2$$  \hspace{1cm} (14)

where $\mu \equiv \nu_u = \nu_d$ is the chemical potential. Expressions (13) and (14) are the asymptotic EOS for high temperature and finite chemical potential. Setting $\mu = 0$ we recover the MIT bag model results [9, 10].

2.5. Model 4

The equation of state below was derived in [16], where deconfined matter in $SU(3)$ pure gauge theory was treated as an ideal gas with quasi-particle modes, which have a temperature-dependent mass given by $m(T)$. This EOS reproduces lattice results, such as the trace anomaly $(\varepsilon - 3p)/T^4$, as it was shown in [16]. The pressure and energy density as functions of the temperature are given by:

$$p_4 = \frac{8 T^2 m^2(T)}{\pi^2} K_2 \left[ \frac{m(T)}{T} \right]$$  \hspace{1cm} (15)

and

$$\varepsilon_4 = \frac{8 T^2 m^2(T)}{\pi^2} \left\{ 3K_2 \left[ \frac{m(T)}{T} \right] + \left[ \frac{m(T)}{T} - \left( \frac{dm}{dT} \right) \right] K_1 \left[ \frac{m(T)}{T} \right] \right\}$$  \hspace{1cm} (16)

where $K_i[x]$ is the modified Bessel function of the second kind. The quasi-particle mass is given by:

$$m(T) = \frac{a}{(\frac{T}{T_0} - 1)^{\epsilon}} + b \frac{T}{T_0}$$  \hspace{1cm} (17)

with the constants $a = 0.47 GeV, b = 0.125 GeV$ and $c = 0.385$ fixed in [16] to reproduce the trace anomaly. The critical temperature is $T_0 \simeq 280 MeV$.

2.6. Model 5

Here we present the equation of state obtained in [17, 18] from a lattice simulation of $SU(3)$ QCD at finite temperature and chemical potential with three quark flavors ($u$, $d$ and $s$) with equal masses and gluons. At finite chemical potential the trace anomaly reads:

$$\frac{\varepsilon_5(T, \mu) - 3p_5(T, \mu)}{T^4} = T \frac{\partial}{\partial T} \left[ \frac{p_5(T, \mu)}{T^4} \right] + \frac{\mu^2}{T^2} \chi_2$$
with (1) we use the relation:

\[ \frac{\varepsilon_5(T,0) - 3p_5(T,0)}{T^4} \quad \text{implies that the evolution starts at different initial temperatures for different models.} \]

In working primordial QGP. Also, since the equations of state are different, fixing the initial energy density is interested in determining how changes in the equation of state affect the time evolution of the initial conditions but this is not important for the present study, since we are primarily discussed above. We use the following initial condition:

\[ 3. \text{ Numerical results and discussion} \]

In this section we present the numerical solution of (1) for the different equations of state discussed above. We use the following initial condition:

\[ \varepsilon_i(t_i) = 10^7 \text{MeV}/\text{fm}^3 \quad \text{at} \quad t_i = 10^{-9} \text{ s} \]

We run the time evolution from \( t_i = 10^{-9} \text{ s} \) to \( t_f = 10^{-4} \text{ s} \). There is some uncertainty on these initial conditions but this is not important for the present study, since we are primarily interested in determining how changes in the equation of state affect the time evolution of the primordial QGP. Also, since the equations of state are different, fixing the initial energy density implies that the evolution starts at different initial temperatures for different models. In working with (1) we use the relation:

\[ \frac{dT}{dt} = \left( \frac{d\varepsilon}{dT} \right) \times \frac{d\varepsilon}{dt} \]

and solve it for \( T(t) \).
3.1. The MIT bag model and its variants

We start our numerical analysis by plotting in Fig. 1 the equation of state of the MIT bag model (3) and comparing it with its closest variants, models 1 and 2 at zero chemical potential, given by Eqs. (2) to (8). As a straightforward exercise we plot in Fig. 2 the sound speed given by:

\[ c_s^2 = \frac{\partial p}{\partial \varepsilon} \]  

(24)

For the MIT bag model we use \( B = 150 \text{ MeV/fm}^3 \). Inserting the pressure as a function of energy density in equation (1) and using the initial condition (23) we can compute the time evolution of the energy density (Fig. 3) and of the temperature (Fig. 4). Fig. 3 shows the ratio of energy density (energy density of the model considered divided by the energy density of the MIT model) as a function of time. Comparing the models, we observe that model 1 and model 2 have a similar behavior and that, during the chosen time interval, they predict a much stronger energy dilution and a much weaker cooling than the MIT bag model. This is probably related to the critical temperature in these models, which is \( T_c = 150 \text{ MeV} \). We now repeat the study performed above using the equation of state of model 3, Eqs. (13) and (14), which
allow us to consider systems with finite chemical potential. Model 3 has two parameters, the dynamical gluon mass, \(m_g\), and the QCD coupling between quarks and hard gluons, \(g\). In Fig. 5 we show the pressure as a function of the energy density and then we calculate the sound speed, shown in Fig. 6. In most of the cases, model 3 is harder than the MIT one. Looking at the expressions (13) and (14) it is easy to see qualitatively what are the effects of changing the gluon mass, the coupling constant and the chemical potential. From these expressions we can conclude that model 3 will give always a harder equation of state, except when \(\mu = 0\), in which case it reproduces the MIT results. The numerical evaluation of (13) and (14), presented in Figs. 5 and 6, shows that taking a very small value for \(m_g\), a relatively large value for \(g\) and a sizeable chemical potential we may obtain a speed of sound, which is only 50% larger than the MIT value. The time evolution of the energy density and temperature of matter described by model 3 are shown in Fig. 7 and Fig. 8 respectively. As it could have been anticipated from the previous figures, there is only a small difference between model 3 and the MIT bag model and this difference tends to decrease with time in the case of the energy density and to remain approximately constant in the case of the temperature.

![Figure 5. Pressure versus energy density.](image1)

![Figure 6. Sound speed.](image2)

![Figure 7. Time evolution of the energy density. Comparison between model 3 and the MIT bag model.](image3)

![Figure 8. Time evolution of the temperature. Comparison between model 3 and the MIT bag model.](image4)
3.2. Lattice models

The two lattice models (models 4 and 5) differ mainly because in the former there are only gluons whereas in the latter quarks are included. We can hence isolate the effect of the quarks in the equation of state and in the time evolution as well. We first compare the EOS of models 4 and 5, given by (15), (16), (21) and (22), with the EOS of a gas of gluons (4) and with the MIT EOS (2). The time evolution of the energy density is shown in Fig. 9. As it can be seen there is only a small difference when we add quarks. In fact all four models have nearly the same energy density during most of the evolution and a sizeable difference appears only at very late times. In Fig. 10 we show the time evolution of the temperature calculated with the same models. It should be noted that model 4 (solid line) has a discontinuity at its critical temperature $T = 280\, \text{MeV}$. In model 5 the quark-hadron transition occurs at $T = 200\, \text{MeV}$. In the figure we can see that for similar values of the energy density the models with more degrees of freedom have a smaller temperature, as we would expect from simple thermodynamical considerations. Interestingly, models with stronger interactions have a higher temperature. In Figs. 11 and 12 we show the time evolution of the energy density and temperature computed with model 5. In the figures $\varepsilon(t)$ and $T(t)$ are scaled by the corresponding values of the energy density and temperature at zero chemical potential. From the figures we can conclude that, in the range considered, the chemical potential does not affect the time evolution. In Fig. 13 and Fig. 14 we compare the results of model 3 and model 5, the two models with which we can obtain results at finite chemical potential. Again there are only moderate differences and only at late times.

4. Conclusion

As it was mentioned in the introduction, in view of the future experimental facilities for measurements relevant to cosmology, it is very interesting and very timely to identify observable effects of the QGP phase in the primordial Universe. Most of these effects have so far been associated with quark-hadron phase transition. However we think that it is also worth studying the time evolution of the QGP phase and check whether changes in the QGP equation of state induce changes in the dilution and cooling of the early Universe. These changes might, in turn, change the emission of gravitational waves or the generation of baryon number fluctuations. Taking the equation of state of the MIT bag model as a baseline, we have considered other EOS
Figure 11. Time evolution of the energy density ratio calculated with model 5.

Figure 12. Time evolution of the temperature ratio calculated with model 5.

Figure 13. Time evolution of the energy density at finite chemical potential.

Figure 14. Time evolution of the temperature at finite chemical potential.

with different dynamical ingredients and estimated their effects on the solutions of Friedmann equations. The main conclusion is that, with the exception of results shown in Fig. 3, there are no dramatic changes. The time evolution of the energy density is only weakly sensitive to changes in the chemical potential, to changes in the degrees of freedom (addition of quarks) and to changes in the dynamical information encoded in the effective gluon mass. The temperature evolution is somewhat more sensitive to these changes, specially to the inclusion of quarks in a pure gauge theory, as it can be seen in Fig. 10. Although our results are already very suggestive, our preliminary conclusions need to be confirmed by a more complete calculation, with the inclusion of the phase transition and the evolution of the hadronic phase. Finally, we emphasize that there are still some other ingredients of the QGP phase to be considered, such as, for example the number of leptons and a strong magnetic field. Calculations along this line are in progress.

References
[1] T. Boeckel, S. Schettler and J. Schaffner-Bielich 2011 *Prog. Part. Nucl. Phys.* 66 266; T. Boeckel and J. Schaffner-Bielich 2012 *Phys. Rev. D* 103506 and references therein.
[2] G. L. Guardo, V. Greco, M. Ruggieri arXiv:1401.7613 [hep-ph]
[3] W. Florkowski 2011 *Nucl. Phys. A* 853 173
[4] V. R. C. Mouro Roque and G. Lugones, *Phys. Rev. D* 87, no. 8, 083516 (2013).
[5] D. J. Schwarz and M. Stuke 2009 JCAP 0911, 025; [Erratum-ibid. 1010, E01 (2010)]
[6] D. Boyanovsky, H. J. de Vega and D. J. Schwarz 2006 Ann. Rev. Nucl. Part. Sci. 56, 441; C. Schmid, D. J. Schwarz and P. Widerin, Phys. Rev. D 59, 043517 (1999); D. J. Schwarz, Nucl. Phys. A 642, 336 (1998).
[7] Avijeet Ray and Soma Sanyal arXiv:1309.1236 [hep-ph]
[8] U. Ornik and R. M. Weiner 1987 Phys. Rev. D 36 1263
[9] A. Chodos, R. L. Jaffe, K. Johnson, C. B. Thorn and V. F. Weisskopf 1974 Phys. Rev. D 9 3471
[10] D. A. Fogaça, L. G. Ferreira Filho and F. S. Navarra 2010 Phys. Rev. C 81 055211
[11] V. V. Begun, M. I. Gorenstein and O. A. Mogilevsky 2012 Phys. Atom. Nucl. 75 873
[12] D. A. Fogaça and F. S. Navarra 2011 Phys. Lett. B 700 236
[13] B. Franzon, D. A. Fogaça, F. S. Navarra and J. E. Horvath 2012 Phys. Rev. D 86 065031
[14] R. J. Furnstahl and Brian D. Serot 1990 Phys. Rev. C 41 262
[15] Robert F. Tooper 1969 The Astrophysical Journal 156 1075
[16] P. Castorina, D. E. Miller and H. Satz 2011 Eur. Phys. J. C 71 1673
[17] S. Borsanyi, G. Endrodi, Z. Fodor, S. D. Katz, S. Krieg, C. Ratti and K. K. Szabo 2012 JHEP 1208 053
[18] S. Borsanyi, Z. Fodor, C. Hoelbling, S. D. Katz, S. Krieg and K. K. Szabo 2014 Phys. Lett. B 730 99