Connecting Low Energy Leptonic CP-violation to Leptogenesis

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It was commonly thought that the observation of low energy leptonic CP-violating phases would not automatically imply the existence of a baryon asymmetry in the leptogenesis scenario. This conclusion does not generically hold when the issue of flavour is relevant and properly taken into account in leptogenesis. We illustrate this point with various examples studying the correlation between the baryon asymmetry and the CP-violating asymmetry in neutrino oscillations and the effective Majorana mass in neutrinoless double beta decay.

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Leptogenesis [1] is a simple mechanism to explain the baryon number asymmetry (per entropy density) of the Universe $Y_B = (0.87 \pm 0.02) \times 10^{-10}$ [2]. A lepton asymmetry is dynamically generated and then converted into a baryon asymmetry due to $(B+L)$-violating sphaleron interactions [3,4] which exist in the Standard Model (SM). A simple model in which this mechanism can be implemented is the “seesaw” (type I) [5], consisting of the SM plus three right-handed (RH) Majorana neutrinos. In thermal leptogenesis [6] the heavy RH neutrinos are produced by thermal scatterings after inflation and subsequently decay out-of-equilibrium in a lepton number and CP-violating way, thus satisfying Sakharov’s constraints [4]. At the same time the smallness of neutrino masses suggested by oscillation experiments [7] can be ascribed to the seesaw mechanism where integrating out heavy RH Majorana neutrinos generates mass terms for the left-handed flavour neutrinos which are inversely proportional to the mass of the RH ones.

Establishing a connection between the CP-violation in low energy neutrino physics and the CP-violation at high energy necessary for leptogenesis has received much attention in recent years [8] and is the subject of the present paper. In the case of three neutrino mixing, CP-violating way, thus satisfying Sakharov’s constraints [4]. At the same time the smallness of neutrino masses suggested by oscillation experiments [7] can be ascribed to the seesaw mechanism where integrating out heavy RH Majorana neutrinos generates mass terms for the left-handed flavour neutrinos which are inversely proportional to the mass of the RH ones.

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diated by the charged lepton Yukawa couplings are out of equilibrium and there is no notion of flavour. One is allowed to perform a rotation in flavour space to store all the lepton asymmetry in one flavour, the total lepton number. However, at $T \sim M_1 \sim 10^{12}$ GeV, the interactions mediated by the charged tau Yukawa coupling come into equilibrium followed by those mediated by the charged muon Yukawa coupling at $T \sim M_1 \sim 10^{9}$ GeV and the notion of flavour becomes physical. Including the issue of flavour can significantly affect the result for the final baryon asymmetry [15–17]. Thermal leptogenesis is a dynamical process, involving the production and destruction of RH neutrinos and of the lepton asymmetry that is distributed among distinguishable flavours. The processes which wash out lepton number are flavour dependent, e.g. the inverse decays from electrons can destroy the lepton asymmetry carried by, and only by, the electrons. The asymmetries in each flavour are therefore washed out differently, and will appear with different weights in the final formula for the baryon asymmetry. This is physically inequivalent to the treatment of washout in the one-flavour approximation, where the flavours are taken indistinguishable, thus obtaining the unphysical result that inverse decays from all flavours are taken to wash out asymmetries in any flavour (that is, e.g., an asymmetry stored in the first family may be washed out by inverse decays involving the second or the third family).

When flavour is accounted for, the final value of the baryon asymmetry is the sum of three contributions. Each term is given by the CP asymmetry in a given flavour $\alpha$ properly weighted by a washing out factor induced by the lepton $\alpha$ violating processes. Taking into account the flavour dependence one may show that observing low energy CP-violating phases automatically implies, barring accidental cancellations, generation of the baryon asymmetry. Before going into details though, let us summarize why this conclusion is not possible in the ‘one-flavour’ approximation. The starting point is the Lagrangian of the SM with the addition of three right-handed neutrinos $N_i$ ($i = 1, 2, 3$) with heavy Majorana masses $M_i$ and Yukawa couplings $\lambda_{i\alpha}$. Working in the basis in which the Yukawa couplings for the charged leptons are diagonal, the Lagrangian reads

$$\mathcal{L} = \mathcal{L}_{\text{SM}} + \frac{M_i}{2} N_i^2 + \lambda_{i\alpha} N_i \ell_\alpha H + \text{h.c.} .$$

Here $\ell_\alpha$ indicates the lepton doublet with flavour ($\alpha = e, \mu, \tau$) and $H$ is the Higgs doublet whose vacuum expectation value is $v$. For the time being, we assume that right-handed neutrinos are hierarchical, $M_{2,3} \gg M_1$ so that restricting to the dynamics of $N_1$ suffices.

The total lepton asymmetry per entropy density generated by the $N_1$ decays is given by $Y_L \simeq (\epsilon_1/g_*) \eta(\bar{m}_1)$, where $\eta(\bar{m}_1)$ accounts for the washing out of the total lepton asymmetry due to $\Delta L = 1$ inverse decays, $\bar{m}_1 = (\lambda \lambda^\dagger)_{11} v^2/M_1$, $g_*$ counts the relativistic degrees of freedom and the CP asymmetry generated by $N_1$ decays reads

$$\epsilon_1 \equiv \frac{\sum_\alpha \Gamma(N_1 \rightarrow H\ell_\alpha) - \Gamma(N_1 \rightarrow H\ell_\alpha^\dagger)}{\sum_\alpha \Gamma(N_1 \rightarrow H\ell_\alpha) + \Gamma(N_1 \rightarrow H\ell_\alpha^\dagger)} = \frac{3M_1}{16\pi} \sum_{j \neq 1} \text{Im} \left[ (\lambda \lambda^\dagger)_{11} \right] \frac{1}{M_j^2} .$$

(2)

Notice, in particular, that the CP asymmetry in the ‘one flavour approximation’ depends upon the trace of the CP asymmetries over flavours. In the basis where the charged lepton Yukawa coupling and the RH mass matrix are diagonal, the neutrino Yukawa matrix can be written as $\lambda = V_R^\dagger \text{Diag}(\lambda_1, \lambda_2, \lambda_3) V_L$ and the low energy leptonic phases may arise from the phases in the left-handed (LH) sector, in RH sector, or from both. The CP-asymmetry can be expressed in terms of the diagonal matrix of the light neutrino mass eigenvalues $m = \text{Diag}(m_1, m_2, m_3)$, the diagonal matrix of the right-handed neutrino masses $M = \text{Diag}(M_1, M_2, M_3)$ and an orthogonal complex matrix $R = v M^{-1/2} \lambda U m^{-1/2}$ [20], which ensures that the correct low energy parameters are obtained. CP-violation in the RH sector is encoded in the phases of $V_R$ and, from $\lambda_{i\alpha} = V_R^\dagger \text{Diag}(\lambda_{1\alpha}, \lambda_{2\alpha}, \lambda_{3\alpha}) V_R = M^{1/2} R m^{1/2} / v^2$, one sees that the phases of $R$ are related to those of $V_R$. Now, summing over all phases, one finds

$$\epsilon_1 = -\frac{3M_1}{16\pi v^2} \text{Im} \left( \sum_\rho m_\rho^2 R^2_\rho \right) \sum_\beta m_\beta |R_{1\beta}|^2 .$$

(3)

In the ‘one-flavour’ approximation a future observation of CP-violating phases in the neutrino sector does not imply the existence of a baryon asymmetry. Indeed, low energy CP phases might stem entirely from the LH sector and hence be irrelevant for leptogenesis which would be driven by the phases in $R$, i.e. of the RH sector.

The ‘one-flavour’ approximation rigorously holds, however, only when the interactions mediated by the charged lepton Yukawas are out of equilibrium, that is at $T \sim M_1 \gtrsim 10^{12}$ GeV. In this regime, flavours are indistinguishable and there is effectively only one flavour, the total lepton number. At smaller temperatures, though, flavours are distinguishable: the $\tau$ ($\mu$) lepton doublet is a distinguishable mass eigenstate for $T \sim M_1 \lesssim 10^{12}$ ($10^9$) GeV. The asymmetry in each flavour is given by

$$\epsilon_\alpha = -\frac{3M_1}{16\pi v^2} \text{Im} \left( \sum_\rho m_\rho^{3/2} m_\rho^{1/2} U^*_{\alpha\rho} U_{\alpha\rho} R_{1\beta} R_{1\rho} \right) \sum_\beta m_\beta |R_{1\beta}|^2 .$$

(4)

The trace over the flavours of $\epsilon_\alpha$ coincides of course with $\epsilon_1$. Similarly, one may define a parameter for each flavour $\alpha$, $\bar{m}_\alpha = |\lambda_{1\alpha}|^2 v^2 / M_1$ parametrizing the decay rate of $N_1$ to the $\alpha$-th flavour and the trace $\sum_\alpha \bar{m}_\alpha$
coincides with the $\tilde{m}_1$ parameter defined for the one-single flavour case. Solving the Boltzmann equations for each flavour one finds $Y_\alpha \simeq (\epsilon_\alpha / g_*) \eta (\tilde{m}_\alpha)$ [15–17]. The way the total baryon asymmetry depends upon the individual lepton asymmetries is a function of temperature. For instance, for $(10^9 \lesssim T \lesssim M_1 \lesssim 10^{12}$) GeV, only the interactions mediated by the $\tau$ Yukawa coupling are in equilibrium and the final baryon asymmetry is

$$Y_B = -\frac{(12/37g_*) (\epsilon_\tau \eta (0.7 \tilde{m}_\tau) + \epsilon_\nu \eta (0.67 \tilde{m}_\nu))}{\eta \tilde{m}_\tau}$$

where $\epsilon_\tau = \epsilon_{\nu} + \epsilon_{\nu R}$, $m_\tau = \tilde{m}_\tau + m_\nu$. $Y_\nu = Y_{\nu+\mu}$ [17]. As the CP asymmetry in each flavour is weighted by the corresponding wash out parameter, $Y_\nu$ is generically not proportional to $\epsilon_1$, but depends on each $\epsilon_\alpha$. The dependence on the PMNS matrix elements in (4) is such that non-vanishing low energy leptonic CP-violating phases imply, in the context of leptogenesis and barring accidental cancellations, a nonvanishing baryon asymmetry [16,17].

We can go even further. CP invariance would correspond to a real matrix $R$ provided that the CP-parities of the heavy and light Majorana neutrinos are equal to $+i$ [19]. In this case the low energy Majorana phases vanish (modulo $2\pi$) and $\delta = 0$ (modulo $\pi$). $R$ real [16,17] corresponds to the class of models where CP is an exact symmetry in the RH neutrino sector [17]. In this case, the flavour CP asymmetries and the baryon asymmetry depend exclusively on the low energy phases in the PMNS matrix. Consequently, leptogenesis is maximally connected to the low energy leptonic CP-violation. This conclusion is clear from the expression of the flavour CP asymmetries in terms of a real $R$ matrix,

$$\epsilon_\alpha \propto \sum_{\beta,\rho>\beta} \sqrt{\tilde{m}_\beta \tilde{m}_\rho} (m_\rho - m_\beta) R_{\beta \beta} R_{\rho \rho} \text{Im} \left( U_{\alpha \beta}^* U_{\alpha \rho} \right).$$

Notice that $\epsilon_1 = 0$ if $R$ is real and $\epsilon_\alpha = 0$ if $R$ is real and diagonal. Once flavour effects are taken into account, a baryon asymmetry is generically generated from nonzero phases in the PMNS matrix.

To illustrate better this point, we provide two examples where the baryon asymmetry is generated uniquely by the CP phases in the PMNS matrix. We will consider the range of values $(10^9 \lesssim M_1 \lesssim 10^{12})$ GeV, for which it is sufficient to consider $\epsilon_\tau$, being $\epsilon_\nu = -\epsilon_\tau$. In the first example, we consider the NH spectrum. In the limit $M_1 \ll M_2 \ll M_3$, we obtain

$$\epsilon_\tau \simeq \frac{3M_1}{16\pi v^2} \left( \frac{\Delta m^2_{12}}{\Delta m^2_{13}} \right)^{1/4} R_{12} R_{13} c_{13}$$

$$\times \left( \frac{1}{2} c_{23} \sin 2\theta_{23} \sin \frac{\alpha_{23}}{2} - s_{23}^2 c_{33} \sin (\delta - \frac{\alpha_{32}}{2}) \right), \quad (5)$$

where $c_{ij} = \cos \theta_{ij}$ and $s_{ij} = \sin \theta_{ij}$. Only the Majorana phase $\alpha_{32} = \alpha_{31} - \alpha_{21}$ plays a role being the contribution of $m_1$ negligible. With these expressions, it is straightforward to compute the final baryon asymmetry solving the flavoured Boltzmann equations of Ref. [17]. In the IH case, a similar expression holds for $\epsilon_\tau$, but is suppressed for real $R$ with respect to the one in the NH case by a factor $\sim (\Delta m^2_{12}/\Delta m^2_{13})^{3/4}$, leading generically to an asymmetry which is small. A sufficiently large asymmetry can be recovered in the case of purely imaginary product $R_{11} R_{12}$ or in the supersymmetric version of leptogenesis [19]. In the expression (5) the dominant contribution comes from the Majorana CP-violating phase, while the effects due to $\delta$ are suppressed by $\sin \theta_{13}$. The Majorana phase $\alpha_{32}$ appears in the expression for the effective Majorana mass ($m_\nu$). The baryon asymmetry depends also on the combination sin $\theta_{13}$ sin $\delta$, which enters in the CP-asymmetry measurable in future long baseline oscillation experiments.

![FIG. 1. The invariant $J_{CP}$ versus the baryon asymmetry varying (in blue) $\delta = [0, 2\pi]$ in the case of hierarchical RH neutrinos and NH light neutrino mass spectrum for $\theta_{13} = 0.2$, $\alpha_{32} = 0$, $R_{12} = 0.86$, $R_{13} = 0.5$ and $M_1 = 5 \times 10^{11}$ GeV. The red region denotes the 2$\sigma$ range for the baryon asymmetry.](image1)

![FIG. 2. The baryon asymmetry $|Y_B|$ versus the effective Majorana mass in neutrinoless double beta decay, $\langle m_\nu \rangle$, in the case of Majorana CP-violation, hierarchical RH neutrinos and IH light neutrino mass spectrum, for $\delta = 0$, $\theta_{13} = 0$, purely imaginary $R_{11} R_{12}$, $|R_{11}| = 1.05$ and $M_1 = 2 \times 10^{11}$ GeV. The Majorana phase $\alpha_{21}$ is varied in the interval $[-\pi/2, \pi/2]$.](image2)
RH neutrino masses as well, $M_1 \sim M_2 \sim M_3$; all RH neutrinos contribute to the baryon asymmetry. The washing out of a given flavour is parametrized by $\bar{m}_\alpha = \sum_\beta |\lambda_{\alpha\beta}|^2 v / M_1$. For $R$ real, it is approximately the same for all flavours, $\bar{m}_\alpha \sim m$. Again, for $(10^9 \lesssim M_1 \lesssim 10^{12})$ GeV and $R$ real, $\epsilon_2 = -\epsilon_\tau$. If we consider the case in which $M_1 \approx M_2 \approx M_3$, the total CP asymmetry in the third flavour $\epsilon_\tau$ is resonantly enhanced when the decay rate $\Gamma_{N_2} \sim (M_2 - M_1)$ and [19]

$$\epsilon_\tau \simeq \frac{1}{2m^2} \left( \Delta m^2_{\odot} R_{11} R_{21} - \Delta m^2_{\odot} R_{13} R_{23} \right) \times \sum_{\rho \neq \beta} (R_{1\rho} R_{2\beta} - R_{1\beta} R_{2\rho}) \text{Im} \left( U_{3\beta} U^*_{3\rho} \right).$$

We may write the matrix $R$ under the form $R = e^A$, where $A$ is a real matrix satisfying $A^T = -A$. In Fig. 3, we show the correlation of the baryon asymmetry with the effective Majorana mass in neutrinoless double beta decay. A number of projects aim to reach a sensitivity to $|\langle m_\nu \rangle| \sim (0.01 - 0.05)$ eV [12] and can certainly probe the region of values of $|\langle m_\nu \rangle|$ for successful baryon asymmetry from the PMNS phases only.

In particular, a direct information on the Majorana phase $\alpha_{21}$ may come from the measurement of $\langle m_\nu \rangle$, $m$, and $\sin^2(\alpha_{21}) \simeq \left( 1 - \left( |\langle m_\nu \rangle|^2 / m^2 \right) \right) (1/ \sin^2 2\theta_{12})$ and might tell us if enough baryon asymmetry may be generated uniquely from the PMNS phases.

Our examples show that the observation of effects of the CP-violating phases of $U$ in neutrino oscillations and/or in the neutrinoless double beta decay would generically ensure a nonvanishing baryon asymmetry through leptogenesis. We will present a more detailed analysis, including the supersymmetric generalization, in a forthcoming publication [19].

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