Optimization of steel frames with the choice of materials’ grades with restrictions on general and local stability, strength and stiffness

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Abstract. A computational scheme for the optimal design of steel flat frames made of thin-walled rods with closed cross sections has been developed. The structural elements’ total cost is minimized by searching for the materials’ grades and the rods’ cross sections sizes on the variable parameters’ discrete sets. The rods are separately grouped according to the condition of using one steel grade and the criterion for the cross sections’ identity. Active restrictions on the overall structural system’s stability, local stability of the rods’ walls, strength and stiffness are taken into account. The optimization process using a genetic algorithm using a mixed approach to the mutation procedure, a selection option that provides the inoperative design options, and a single-point crossing-over procedure for the simple exclusions from consideration is implemented. The deformable object stress-strain state analysis is carried out on the rod finite element model’s basis. The assessment of ensuring overall stability is carried out by checking the positive definiteness of the tangent stiffness matrix of a finite element system. Local stability using an analytical relationship for rectangular plates, generally subjected to compression-tension and bending in their plane is confirmed. The results of optimization of a three-span frame made of square pipes are presented. Steel grades and profiles were selected for the groups of the structural system’s rods.

Introduction
Optimal design is an important prerequisite for the cost-effective building structures’ development. One of the most effective approaches to optimizing the load-bearing systems is metaheuristic algorithms. For these tasks, metaheuristic schemes such as genetic algorithms [1], harmony search [2], particle swarm approach [3], firefly algorithm [4], dolphin echolocation [5], simulated annealing [6], colliding bodies optimization [7], etc. have been tested. In the construction industry, steel core systems are widely used. In [8–11] the optimization of steel frames using metaheuristic schemes was considered. Moreover, the conditions of local stability of the shelves and walls of the rods are not considered as active restrictions. It is envisaged to ensure this requirement either by selecting the acceptable cross-sectional profile options, or by installing the additional reinforcing elements. Such simplifications can lead to the loss of the most effective design solutions. An optimization algorithm for the steel frames made of I-beams was developed in [12]. The volume of material is minimized. The rods’ cross sections sizes vary. The requirements to fulfill the conditions of strength, stiffness, general and local stability have been identified. To solve the optimization problem, random and direct search methods are used. The
optimization process is divided into two stages. At the first stage, the restrictions on rigidity and general stability are taken into account, at the second - on strength and local stability. It should be noted that the decomposition of this type also narrows the possibilities for searching the options for the design parameters’ combinations. The issue of choosing steel grades for rods is not considered in [8-12]. At the same time, it seems advisable to set more durable steels for the rods in which tensile stresses predominate. For rods, the bearing capacity of which is determined, first of all, by ensuring the stability conditions, less durable steels with a lower unit cost can be used. In this paper, we present a procedure for the flat steel frames’ optimal design, in which, based on the genetic algorithm, the total cost of the rods is minimized by varying the grades of steel and the rods’ cross sections. At the same time, the complex takes into account the active restrictions on the conditions of general and local stability, stresses and displacements.

Statement and algorithm for solving the problem
We believe that a flat steel frame is made of rectilinear thin-walled rods with constant closed cross-sections along the length. The frame is loaded in its plane. Taking into account the need to check the stability of the structure, we take into account for the rods’ compression-tension deformation, bending in two main planes and pure torsion. The stress-strain state of the object is calculated using the finite element method as a part of the displacement method based on the core model. In this case, each of the rods should be divided into several finite elements. We accept that the construction work materials in the conditions of linear elasticity. We carry out the stability tests according to the Euler scheme. We take into account the reliability coefficients and the working conditions coefficients in the framework of the requirements of BC 16.13330.2017 “SNiP II-23-81*. Steel structures”.

We set the task of minimizing the total cost of the rods:

$$C(M_1, M_2, ..., M_I, H_1, H_2, ..., H_N) = \sum_{i=1}^{I} C_i l_k \Rightarrow \min$$

where $M_i$ is the set of acceptable grades of steel for the rods of the group $i$ ($i = 1, 2, ..., I$), $I$ - this is the total number of the rod groups combined by the material, $H_n$ - this is the permissible cross sections’ set for the rods of the group $n$ ($n = 1, 2, ..., N$), $N$ - this is the total number of groups united by the cross sections, $k_c$ - this is the total number of the rods $C_i$, this is the cost per linear meter of the rod $k$, $l_k$ - this is the length of the rod $k$ ($k = 1, 2, ..., k_c$).

The search is done on the sets $M_i$, $H_n$. We consider the following restrictions:

1) The overall stability of the core system, including the separate rods’ stability.

We check the general stability on the basis of the tangential stiffness matrix positive definiteness confirmation $[K]_e$ [13] of the finite element model:

$$[K]_e = [K] + [K_0(N_k)]$$

where $[K]$, is the usual rigidity matrix of a finite element system, $[K_0(N_k)]$ - a global geometric matrix that is expressed through the normal forces $N_k$ in the rods ($k = 1, 2, ..., k_c$), obtained when calculating the stress-strain state of the frame.

2) Local stability of thin-walled elements of the rods.

When checking the local stability of the plates for the case of a polygonal profile of the rods’ cross section, we consider the plate section as a rectangular plate articulated along the contour with the sides $a$ and $b$ (Figure 1). We believe that this section is generally subjected to the action of compression-tension and bending in its plane, identical in cross sections, with the occurrence of compressive stresses for the entire section or its part. Let normal stresses $\sigma$ at $y = 0$ be negative. We accept the linear law of the stress distribution along the axis $y$ [14] as:
\[
\sigma = -\sigma_o \left(1 - \beta \frac{y}{b}\right),
\]

where \(\sigma_o\) is the absolute value of \(\sigma\) at \(y = 0\), \(\beta\) is the coefficient.

**Figure 1.** Plate loading in its plane.

Euler critical values \(\sigma_o\) in this case can be determined using the equation [14]

\[
\sigma_\text{ek} = K \frac{\pi^2 D}{b^2 h},
\]

where \(K\) is a coefficient dependent on \(\beta\) and the relationships \(a/b\); \(D\) is the cylindrical stiffness of the plate:

\[
D = \frac{E h^3}{12(1-\mu^2)}.
\]

Here \(h\) is the thickness of the plate, \(E, \mu\) is the elastic modulus and the Poisson’s ratio of the material.

We will take into account the minimum value \(K\) for each \(\beta\): \n
\[
K_{\text{min}}(\beta) = \min_{a/b} K(a/b, \beta)
\]

Based on the approximation of the data presented in [14], the following approximate dependence was obtained:

\[
K_{\text{min}}(\beta) = 4 + 3.4083 \beta^2 + 0.3917 \beta^4
\]

which was used in the optimal search.

3) **Strength Condition:**

\[
\max_{i=1...k} \frac{\sigma_{mk}}{R_k} \leq 1,
\]

where \(\sigma_{mk}\) is the greatest tension von Mises in the rod \(k\), \(R_k\) is the calculated steel resistance for the bar \(k\), assigned yield strength (BC 16.13330.2017).

4) **Stiffness requirements.** For each \(j\) node of the discrete design in the direction of the axis \(OX_k\) cartesian coordinate system \(OX_1X_2\), associated with the plane of the frame, the condition

\[
\max_{i=1...k} \left(\left|\delta_{ij}\right| / \left|\delta_{j}\right|\right) \leq 1
\]

where \(\delta_{ij}\) this is the projection of the node’s displacement vector \(j\) on the axis \(OX_i\), \(\left|\delta_{j}\right|\) is a modulo value of a given quantity, \(j_n\) is the total number of nodes, should be satisfied.
All the considered limitations are related to the active ones, directly taking them into account in the optimization process. We form a genetic algorithm using a modification of the work scheme [15] based on the complete rejection of any penalty functions. Such an approach is related to taking into account the limitations on overall stability in this problem without directly assessing the critical loads. We construct each discrete set of permissible parameter values from smaller to larger: for sets of steel grades - according to their cost criterion, for sets of bar profiles - according to the cross-sectional area. In the evolutionary scheme, we take into account the main population \( \Omega_1 \) from \( N \) projects and the additional populations \( \Omega_2 \) from the elite projects, the number of which depends on the results of the evolutionary algorithm, but does not exceed \( N \). Initially, we form the population projects \( \Omega_1 \) from the same design options with the highest permissible parameter values. Next, we carry out an iterative process (movement through generations), each iteration of which includes the following basic actions:

1) Verification of restrictions for individuals \( \Omega_1 \). The calculations of the stress-strain state and the stability, strength and stiffness assessment of the considered design options are performed. If at least one of the set limits is not satisfied for any of the individuals, then it is replaced by an individual from the population that is not used in the main population \( \Omega_2 \) or a newly formed version of the carrier system.

2) Population change \( \Omega_2 \). Each of the individuals in the population \( \Omega_1 \) is checked according to two criteria: does such an individual exist in the population \( \Omega_2 \), and does the value \( C \) exceed the individual under consideration has the highest value of the objective function in this population. If both answers are negative, then the individual is included in the population \( \Omega_2 \). If at the same time the number of individuals in this population exceeds \( N \), then the individual with the highest value \( C \) is excluded from it.

3) Mutation. Several parameters can be changed randomly for a part of the population \( \Omega_1 \). A mixed mutation procedure [15], which provides the choice of the value of each parameter from the entire set of permissible values and from the 1-2 nearest values from this set for random alternation, is used.

4) Repeating compliance checks for individuals in a population \( \Omega_1 \) and editing the population \( \Omega_2 \) (see 1, 2).

5) Breeding and Crossover. A single-point crossing-over procedure is implemented for the individuals \( \Omega_1 \). Moreover, in accordance with the scheme described in [15], the selection is performed according to the objective function value criterion.

6) The criterion satisfaction verification for the iterations’ finishing. The calculations show that with an optimal synthesis of structures for the type in question using the presented evolutionary scheme, there are no changes in the population \( \Omega_2 \) during 300 iterations indicates the feasibility of the optimization’s finishing. The continuation of the iterative process usually does not lead to any significant change in the parameters for the most rational projects.

Results
Let us illustrate the possibilities of the proposed methodology by the example of the 18 m long steel frame optimization (Figure 2). It was believed that the frame is made of square shaped pipes in accordance with GOST 32931-2015 “Steel profile pipes for metal structures. Technical conditions”. The properties of the materials were set in accordance with BC 16.13330.2017. The axes of all the rods are located in the same plane. Rigid support anchors H and ties T were introduced from the plane of the frame. Two combinations of loads were taken into account. In the first combination, a system of vertical concentrated forces \( P_i \) was considered from payloads and snow loads distributed by wind loads \( q_i \) with the wind direction to the left and gravity from the rods, adjusted during the optimization process depending on the choice of cross sections. Moreover, for the first group of the limiting states (BC 16.13330.2017) the following parameters were accepted: \( P_1=44.4 \) kN, \( P_2=34.56 \) kN, \( P_3=37.8 \) kN, \( \ldots \).
\[ P_1 = 30 \text{ kN}, \ P_2 = 60 \text{ kN}, \ P_3 = 54 \text{ kN}, \ P_4 = 68 \text{ kN}, \ P_5 = 15 \text{ kN}, \ P_6 = 7.5 \text{ kN}, \ q_1 = 1200 \text{ kN/m}, \ q_2 = 840 \text{ N/m}, \ q_3 = 720 \text{ N/m}, \ q_4 = 660 \text{ N/m}. \]

The second combination of loads differs from the first combination only by considering the direction of the wind to the right. It was possible to use the steel types S235, S245, S255, S285 and the bar profiles in accordance with Table 1, where \( N \) – is the profile number in the considered numbering scheme, \( d \) – is the side of the square, \( t \) – defines the thickness of the profile wall (Figure 3). The rods were combined to vary in 5 groups according to the choice of steel grade and 12 groups according to the cross-sectional sizes in accordance with Tables 2 and 3. At the same time, the symmetry condition of the structural system was ensured.

30 independent algorithm starts for optimal search were performed. In each start, 300 iterations were performed. The objective function values were obtained in the range from 19.80 to 20.25 thousand rubles. The final results were recorded for 206-2040 iterations. The best value was achieved in more than 10% of starts. Tables 2, 3 show the obtained values of the varied parameters for the groups of rods for this solution. Figure 4 shows the graphs of convergence in the objective function for the solution with the lowest \( C \) value obtained at the fastest and slowest convergence.

**Discussion**

The algorithm developed in this article made it possible, in comparison with the existing approaches [8-12], to complement the possibilities of optimal design for the steel frames in terms of selecting the steel grades for bar groups and expanding the initial sets of acceptable options for cross sections, taking into account the active restrictions on the local stability for the bar plates. In contrast to the methodology [10], the assessment of ensuring overall stability is realized only by checking the positive definiteness of the tangent matrix of the finite element model without performing an iterative process for calculating the system in a deformed state. Moreover, as in [10, 11], it is not required to solve a rather laborious task of determining the eigenvalues of this matrix. The optimal design of a real frame structure led to a number of solutions with fairly close values of the objective function. The best-found version of the frame supporting system included S235 and S245 steels with relatively low strength indices, which can be explained by the fact that stability is essential for all thin-walled rods of the considered construction. Moreover, for the rods working mainly on bending, the fulfillment of the local stability condition may be determining. It is advisable to use the proposed algorithm to improve the optimal design schemes for steel frame structures in the construction industry.

**Figure 2.** The scheme of the frame with a breakdown into finite elements and an indication of forces \( P_i \), \( q_j \) from the first combination of loads: 1-23 - rod numbers, \( U \) – nodes of the finite element model. **Figure 3.** The rods’ cross section of

**Table 1.** Acceptable options for the rods’ cross sections.
Table 2. Grouping rods by materials and the result of the steel grades’ selection.

| Group number | Rod Numbers   | Steel grade |
|--------------|--------------|-------------|
| 1            | 1, 2, 22, 23 | S245        |
| 2            | 3, 11, 15, 21 | S245        |
| 3            | 4, 10, 14, 20 | S235        |
| 4            | 5, 9, 13, 19 | S235        |
| 5            | 6, 7, 8, 12, 16, 17, 18 | S245        |

Table 3. Cross-sectional grouping of rods and the result of selecting the size combinations.

| Group number | Rod Numbers | b (cm) | t (cm) |
|--------------|-------------|--------|--------|
| 1            | 1, 23       | 12     | 0.3    |
| 2            | 2, 22       | 8      | 0.3    |
| 3            | 5, 19       | 5      | 0.2    |
| 4            | 4, 20       | 5      | 0.2    |
| 5            | 3, 21       | 8      | 0.4    |
| 6            | 6, 16       | 12     | 0.3    |
| 7            | 7, 17       | 12     | 0.3    |
| 8            | 8, 18       | 8      | 0.4    |
| 9            | 9, 13       | 5      | 0.2    |
| 10           | 10, 14      | 5      | 0.3    |
| 11           | 11, 15      | 12     | 0.3    |
| 12           | 12          | 5      | 0.3    |

Figure 4. Convergence graphs for starts with the best result for the objective function.
Summary

1. A methodology for optimizing the flat steel frames made of thin-walled rods with a closed cross-sectional profile has been developed. A search for steel grades and bar profiles for groups of structural elements is provided. Active restrictions are taken into account on the general stability of the frames, the local stability of the plates, rods, strength and stiffness.

2. A genetic scheme for the optimal search implementation without use of penalty functions without paying attention to the inoperative design options, has been proposed, which makes it possible to simplify the limitations’ consideration on overall stability without determining the critical load level.

3. The results of the optimal search for a flat three-span frame led to a number of the design solutions that differ from each other in the value of the objective function by no more than 2.3 %.

4. The proposed computational scheme for optimizing the steel frames can be recommended for use in the software packages for the finite element analysis.

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