DILATONIC QUANTUM-INDUCED TWO-BRANE WORLDS

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The 5d dilatonic gravity action with surface counterterms, motivated by the AdS/CFT correspondence and with contributions of brane quantum CFTs is considered with an AdS-like bulk. The role of quantum brane CFT consists in inducing complicated brane dilatonic gravity. For exponential bulk potentials, a number of AdS-like bulk spaces is found in an analytical form. The corresponding flat or curved (de Sitter or hyperbolic) dilatonic two-branes are created.

1. Introduction

The recent booming activity in brane-world studies is caused by several reasons. First, gravity on a 4d brane embedded in a higher-dimensional AdS-like Universe may be localized [2, 3]. Second, there appears a way to resolve the mass hierarchy problem [2].

An essential element of brane-world models is the presence of two free parameters in the theory (the bulk cosmological constant and the brane tension, or brane cosmological constant). The role of the brane cosmological constant is to fix the position of the brane in terms of tension (that is, why the brane cosmological constant and the brane tension are almost the same thing). Being completely consistent and mathematically reasonable, such a way of doing things may look incompletely satisfactory. Indeed, the physical origin (and prediction) of brane tension in terms of some dynamical mechanism may be required.

The ideology may be different, in the spirit of Refs. [6, 5, 4]. One considers the addition of surface counterterms to the original action on an AdS-like space. These terms are responsible for making the variational procedure well-defined (in Gibbons-Hawking’s spirit) and for eliminating the leading divergences of the action. The brane tension is no more considered as a free parameter but is fixed by the condition of finiteness of spacetime when the brane goes to infinity. Of course, leaving the theory in such a form would rule out a possible existence of consistent brane-world solutions. Fortunately, other parameters contribute to brane tension. If one assumes that there is a quantum CFT living on the brane (which is closer to the spirit of AdS/CFT correspondence [7]), then such a CFT produces a conformal anomaly (or an anomaly-induced effective action). This contributes to the brane tension. As a result, a dynamical mechanism of getting a brane world with a flat or curved (de Sitter or Anti-de Sitter) brane appears. In other words, the brane world is a consequence of the presence of matter on the brane! For example, the sign of conformal anomaly terms for usual matter is such that, in the one-brane case, the de Sitter (ever-expanding, inflationary) Universe is a preferable solution to the brane equation.

The scenario of Refs. [6, 5] may be extended to the presence of dilaton(s), as was done in Ref. [8], or to the formulation of quantum cosmology in a Wheeler-De Witt form [9]. Then the whole scenario looks even more related to the AdS/CFT correspondence, since dilatonic gravity appears naturally as the bosonic sector of 5d gauged supergravity. Moreover, there appears an extra prize in the form of dynamical determination of the 4d boundary value of the dilaton. In [8], a quantum dilatonic one-brane Universe has been presented, with a possibility of getting an inflationary, hyperbolic or flat brane with dynamical determination of the brane dilaton. An interesting question is related to generalization of such a scenario in dilatonic gravity for the multi-brane case. This will be discussed here.

In the next section we present the general action of 5d dilatonic gravity with surface counterterms and a quantum brane CFT contribution. This action is convenient for describing brane-worlds where the bulk is an AdS-like spacetime. There could be one or two (flat or curved) branes in the theory. As was already mentioned, the brane tension is not fixed in our approach, instead, the effective brane tension is induced by quantum effects. An explicit analytical solution of the bulk equation for a number of exponential bulk potentials is presented. It is interesting that quantum-created branes can be flat, de Sitter (inflationary), or hyperbolic. The 2

\footnote{A similar mechanism for anomaly driven inflation in the usual 4d world has been invented by Starobinsky [11] and generalized for the presence of a dilaton in Refs. [12].}
role of quantum brane matter corrections in getting such
branes is extremely important. Nevertheless, there are
a few particular cases where such branes appear on the
classical level, i.e., without quantum corrections.
In most cases, as usually occurs in AdS dilatonic
gravity, the solutions contain a naked singularity. How-
ever, in other cases the scalar curvature is finite, and
there is a horizon. The corresponding 4d branes may be
interpreted as wormholes.

2. Dilatonic gravity action with brane
quantum corrections
Let us present the initial action for dilatonic AdS gravity
under consideration. The (Euclidean) AdS metric has
the following form:

$$ds^2 = dz^2 + e^{2\tilde{A}(z)} \sum_{i,j=1}^{4} \tilde{g}_{ij} dx^i dx^j.$$  

(1)

Here $\tilde{g}_{ij}$ is the metric of the Einstein manifold defined
by $r_{ij} = k\tilde{g}_{ij}$, where $r_{ij}$ is the Ricci tensor constructed
from $\tilde{g}_{ij}$ and $k$ is a constant. One can consider two
copies of the regions given by $z < z_0$ and glue two re-
gions putting a brane at $z = z_0$. More generally, one
can consider two copies of regions $\tilde{z}_0 < z < z_0$ and glue
the regions putting two branes at $z = \tilde{z}_0$ and $z = z_0$.
Hereafter we call the brane at $z = z_0$ as an “inner” brane
and that at $z = z_0$ as “outer” brane.

One chooses the 4-dimensional boundary metric as

$$g(4)_{\mu\nu} = e^{2A} \tilde{g}_{\mu\nu},$$  

(2)

and a tilde specifies quantities connected with $\tilde{g}_{\mu\nu}$.
Let us consider the case where there are two branes
at $z = \tilde{z}_0$ and $z = z_0$:

$$S_{\text{two branes}} = S + \tilde{S}_{\text{GH}} + 2\tilde{S}_1 + \tilde{W},$$  

(3)

$$\tilde{S}_{\text{GH}} = \frac{1}{8\pi G} \int d^4x \sqrt{g(4)} \nabla_{\mu} \nabla^\mu,$$  

(4)

$$\tilde{S}_1 = \frac{1}{16\pi G} \int d^4x \sqrt{g(4)} \left( \frac{6}{I} + \frac{1}{4} \Phi(\phi) \right),$$  

(5)

$$\tilde{W} = \tilde{b} \int d^4x \sqrt{\tilde{g}} FA +$$

$$+ \frac{\tilde{b}'}{2} \int d^4x \sqrt{\tilde{g}} \left( \tilde{A} \left[ 2A + \tilde{R}_{\mu\nu} \tilde{\nabla}_{\mu} \tilde{\nabla}_{\nu} - \frac{4}{3} \tilde{R}^2 + \frac{2}{3} \tilde{R}_{\mu\nu} \tilde{R}^{\mu\nu} \right] + \tilde{A} \tilde{G} \right)$$

$$- \frac{1}{12} \left[ \tilde{b}'' + 3 \tilde{b}'' + \frac{2}{3} (\tilde{b}'' + \tilde{b}') \right] \int d^4x \sqrt{\tilde{g}} \tilde{R}$$

$$- 6 \tilde{G} A - 6 \tilde{G} A (\tilde{\nabla}_{\mu} A)^2 +$$

$$+ C \int d^4x \sqrt{\tilde{g}} A \phi \left[ \tilde{g}^{ij} + 2 \tilde{R}_{ij} \tilde{\nabla}_{\mu} \tilde{\nabla}_{\nu} \right]$$

$$- \frac{2}{3} \tilde{R}^2 + 3 \tilde{R}_{ij} \tilde{\nabla}_{\mu} \tilde{\nabla}_{\nu} - \frac{1}{3} \tilde{R}_{ij} \tilde{\nabla}_{\mu} \tilde{\nabla}_{\nu} \phi$$  

(6)

with $N$ scalar, $N_{1/2}$ spinor, $N_{1}$ vector fields, $N_{2} = 0$
or 1) gravitons and $N_{\text{HD}}$ higher-derivative conformal
calars; $b$, $b'$ and $b''$ are

$$b = \frac{N + 6N_{1/2} + 12N_1 + 611N_2 - 8N_{\text{HD}}}{120(4\pi)^2},$$

$$b' = -\frac{N + 11N_{1/2} + 62N_1 + 1411N_2 - 28N_{\text{HD}}}{360(4\pi)^2},$$

$$b'' = 0.$$  

(7)

For an introduction to the anomaly-induced EA see [10].
We should note that the relative sign of $S_1$ is different
from $S_1$. The parameters $\tilde{b}$, $\tilde{b}'$, $\tilde{b}''$ and $\tilde{C}$ correspond
to the matter which may be different from the outer brane
one on the inner brane as in [6]. Hence, the situation
with different CFTs on the branes may be considered.
Having the action, one can study its dynamics.
Let us start the consideration of field equations for
a two-brane model. First of all, one defines a new coor-
dinate $z$ by

$$z = \int dy \sqrt{f(y)},$$  

(8)

and solves $y$ with respect to $z$. Then the warp factor
is $e^{2A(z,k)} = y(z)$. Here one assumes the 5-di-

mensional space-time metric as follows:

$$ds^2 = f(y)dy^2 + y \frac{4}{\sqrt{f(y)}} \sum_{i,j=1}^{4} \tilde{g}_{ij}(x^k) dx^i dx^j.$$  

(9)

Using the substitution $dz = \sqrt{f} dy$ and choosing $l^2 e^{2\tilde{A}(z,k)} = y(z)$, one has metric in the form

$$ds^2 = dz^2 + e^{2A(z,k)} \tilde{g}_{\mu\nu} dx^\mu dx^\nu, \tilde{g}_{\mu\nu} dx^\mu dx^\nu = l^2 \left( ds^2 + d\Omega_3^2 \right).$$  

(10)

Here $d\Omega_3$ is the metric of a 3-dimensional unit sphere.
Then for the unit sphere $(k = 3)$

$$A(z, \sigma) = \tilde{A}(z, k = 3) - \ln \cosh \sigma,$$  

(11)

for flat Euclidean space $(k = 0)$

$$A(z, \sigma) = \tilde{A}(z, k = 0) + \sigma,$$  

(12)

and for the unit hyperboloid $(k = -3)$

$$A(z, \sigma) = \tilde{A}(z, k = -3) - \ln \sinh \sigma.$$  

(13)

We now identify $A'$ and $\tilde{g}$ in [10] with those in [6].
Then we find $\tilde{F} = \tilde{G} = 0$, $\tilde{R} = 6/l^2$, etc.
Our choice for the dilaton and bulk potential admitting
an analytical solution is

$$\phi(y) = p_1 \ln (p_2 y),$$  

(14)

$$\Phi(\phi) = c_0 + c_1 e^{p_3\phi} + c_2 e^{2p_3\phi},$$  

(15)

where $a, p_1, p_2, c_0, c_1, c_2$ are some constants. When
$p_1 = \pm 1/\sqrt{6}$, we find that $f(y)$ identically vanishes.
Therefore we should assume $p_1 \neq \pm 1/\sqrt{6}$. Then we
find the following set of exact bulk solutions (for all cases $c_0 = -12/2^2$):

**case 1:**  
$c_1 = \frac{6k p_2 p_1^2}{3 - 2p_1^2}$,  
$c_2 = 0$,  
$a = -\frac{1}{p_1}$,  
$f(y) = \frac{3 - 2p_1^2}{4ky}$,  
$p_1 \neq \pm \sqrt{6}$;  
(16)

**case 2:**  
$c_1 = -6k p_2$,  
$a = \pm \frac{1}{\sqrt{3}}$,  
$p_1 = \mp \sqrt{3}$,  
$f(y) = \frac{3 - 2p_1^2}{4ky}$;  
(17)

**case 3:**  
$c_2 = 3k p_2$,  
$a = \pm \frac{1}{\sqrt{3}}$,  
$p_1 = \mp \frac{\sqrt{3}}{2}$,  
$f(y) = \frac{21\sqrt{p_2}}{8\sqrt{q} (c_1 y + 7k p_2 y)}$.  
(18)

The equation for the case $k = 0$ is identical to that of the outer brane.

As an example, we consider case 1 in (14). Since $f(y)$ should be positive (we should also note $y > 0$), one gets

$$q^2 = \frac{4k}{3 - 2p_1^2} > 0 \quad (q > 0).$$  
(19)

In (15), we can also consider the limit of $k \to 0$ keeping $q$ finite, i.e., $p_1^2 \to \frac{3}{2}$.

When $k \neq 0$, from the equations of motion we get:

$$-8b' = F_1(y_0) = \frac{3y_0}{32\pi G} \left( y_0^{1/2} - \frac{y_0}{7} - \frac{q^2 p_1^2 l}{8} \right)$$

$$= -\frac{3}{16\pi G} y_0 \left( y_0^{1/2} - \frac{1 + \sqrt{1 - p_1^2 l^2}/q}{2} \right)$$

$$\times \left( y_0^{1/2} - \frac{1 - \sqrt{1 - p_1^2 l^2}/q}{2} \right)$$

$$= -\frac{q}{8\pi G} \frac{5}{y_0^{5/2}} + \frac{9l p_1 q^2}{8\pi G} y_0 + 6C \phi_0.$$  
(20)

The brane equations are

$$8b' = F_1(y_0),$$

$$0 = \frac{p_1 q}{8\pi G y_0} - \frac{9l p_1 q^2}{8\pi G} y_0 + 6C \phi_0.$$  
(21)

Since $p_2$ is absorbed into the definition of $q$ in (20) and (22), Eqs. (20) and (21) can be regarded as the equation which determines $p_2$, or $(\phi_0/p_1) \ln(p_2 y_0)$ and $(\phi_0/p_1) = \ln(p_2 y_0)$. We now investigate the properties of $F_1(y_0)$ as a function of $y_0$. The asymptotic behaviours are given by

$$F_1(y_0) \xrightarrow{y_0 \to 0^+} -\frac{3}{16\pi G} p_1^2 \frac{q^2}{y_0} < 0,$$

$$F_1(y_0) \xrightarrow{y_0 \to \infty} -\frac{3}{16\pi G} \frac{1}{2} y_0^2 < 0.$$  
(24)

Since

$$F_1'(y_0) = \frac{3}{16\pi G} \frac{3q}{4} y_0^{1/2} - \frac{1}{7} y_0 - \frac{p_1^2 q^2 l}{16},$$

$$F_1'(y_0)$$

has extrema when

$$0 = y_0 - \frac{3q}{4} y_0^{1/2} + \frac{p_1^2 q^2 l}{16},$$

whose solutions are given by

$$y_0^{1/2} = y_0^{1/2} = \frac{3q}{8} \left( 1 \pm \sqrt{1 - \frac{4p_1^2}{9}} \right).$$

Therefore if

$$|p_1| > 3/2,$$

Eq. (27) has no solution and $F_1(y_0)$ is a monotonically decreasing function of $y_0$. Then Eqs. (24) and (25) tell that there is no solution of the brane equation (20) for negative $b'$ in the case (24). On the other hand, when

$$|p_1| < 3/2,$$

substituting (28) into the expression for $F_1(y_0)$ in (20), one gets

$$F_1(y_0) = \frac{3}{16\pi G} 3q^4 l^3 \left( \sqrt{1 - \frac{4p_1^2}{9}} \pm 1 \right)$$

$$\times \left( \sqrt{1 - \frac{4p_1^2}{9}} \mp \frac{1}{3} \right).$$

Then we find that $y_0 = y_+$ corresponds to a maximum of $F_1(y_0)$. The maximum is positive, $F_1(y_+) > 0$, if $\sqrt{1 - 4p_1^2}/9 - 1/3 > 0$, that is,

$$p_1^2 < 2.$$  
(32)

In the case (22), if

$$F_1(y_+) \geq -8b',$$

Eq. (21) has a solution, i.e., there can be a brane. We can also consider an inner brane, which lies at $y = y_1 < y_0$. For the inner brane, the relative sign of $b'$ and $b'$ changes in the equation corresponding to (21). Then, if

$$F_1(y_-) \leq 8b'$$

there can be an inner brane. Then, if both (33) and (34) hold, we can have a two-brane dilatonic solution. For such a solution, there might be, in general, a problem in the consistency between (21) and (23). Taken together, (21) and (23), can be regarded as the equations which determine $p_1$ and $p_2$ (we should note that $p_2$ is implicitly contained in $\phi_0$ and $\phi_0$). In the classical limit, where $C = 0$, the terms containing $p_2$ (or $\phi_0$ and $\phi_0$) disappear. Then it seems to be non-trivial whether or not there is any solution which satisfies both (33) and (34).

We now consider the classical limit in case $k \neq 0$, where $b' = \tilde{b}' = \tilde{C} = 0$, and (20) and (22) become identical. Then the solutions to Eqs. (21) and (23) are given by

$$y_0^\frac{1}{2} = \left( 1 \pm \sqrt{1 - \frac{p_1^2}{2}} \right) \frac{q l}{2}.$$  
(35)
Since both solutions are positive, we can regard the smaller one (the $-$ sign in (33)) as the one corresponding to the inner brane and and larger one (+ sign) to the outer brane. On the other hand, Eqs. (21) and (23) have the following form:

$$0 = \frac{p_1 y_0 q_0^2}{2} \left( 17 + \sqrt{1 - \frac{p_1^2}{2}} \right).$$  \hspace{1cm} (36)

In (34), the upper sign (−) corresponds to the outer brane and the lower one (+) to the inner brane. We should note that there is no solution, except the trivial one, for the inner brane. This seems to tell us that we need a quantum correction from brane matter in order to obtain the two-brane dilatonic inflationary universe where the observable world can be associated with one of inflationary branes.

When $k = 0$ in case 1, as discussed before, if $q$ is finite,

$$p_1^2 \to 3/2.$$  \hspace{1cm} (37)

Then we get the following equation:

$$0 = y_0 - q_0 y_0^{1/2} + \frac{3q_0^2 l^2}{16},$$  \hspace{1cm} (38)

which has two solutions:

$$y_0^{1/2} = \frac{3q l}{4}, \quad \frac{q l}{4}. \hspace{1cm} (39)$$

These two solutions might be regarded as two-brane solutions. On the other hand, the form of the equation of motion in case $k = 0$ is identical to that of $k \neq 0$ in (21), which can be solved with respect to $\phi_0$ or $p_2$ in the one-brane solution. However, in case $k = 0$, the value of $p_1$ is fixed by (27). In the classical limit of the case $k = 0$, the equation of motion (14) can be rewritten in the form of (25), and there appear the solutions (35).

The brane equation of motion (10) has the form (36), but Eq. (37) does not satisfy (35). This demonstrates the role of quantum effects in the realization of a dilatonic inflationary two-brane-world Universe.

Solutions 2 and 3 may be considered in a similar way, for details see [1].

3. Discussion

In summary, we have presented a generalization of a quantum dilatonic brane world [3] where the brane is flat, spherical (de Sitter) or hyperbolic, and it is induced by the quantum effects of CFT living on the brane. In this generalization one may have two-brane-worlds or even multi-brane-worlds which proves a general nature of the scenario suggested in Refs. [1, 3] where, instead of an arbitrary brane tension added by hand, the effective brane tension is produced by boundary quantum fields. What is more interesting, the bulk solutions have an analytical form, at least for the specific choice of the bulk potential under consideration.

In classical dilatonic gravity, a variety of brane-world solutions have been presented in Ref. [12] where the problem of singularities was also discussed. A fine-tuned example of bulk potential where one gets a nonsingular bulk solution, has been presented.

Let us find out if our solutions contain curvature singularities. Multiplying $g^{(5)}_{\mu \nu}$ with the Einstein equation in the bulk,

$$R_{(5)\mu \nu} - \frac{1}{2} \nabla_\mu \phi \nabla_\nu \phi - \frac{1}{2} g_{(5)\mu \nu} \left( R_{(5)} - \frac{1}{2} \nabla_\rho \phi \nabla^\rho \phi + \frac{12}{l^2} + \Phi(\phi) \right),$$  \hspace{1cm} (40)

which is obtained from $S_{EH}$, one gets

$$R_{(5)} = \frac{1}{2} \nabla_\rho \phi \nabla^\rho \phi - \frac{5}{3} \left( \frac{12}{l^2} + \Phi(\phi) \right).$$  \hspace{1cm} (41)

Substituting the expressions (14) and (15) into (41), we find

$$R_{(5)} = \frac{p_1^2}{2 y^2 f} - \frac{5}{3} \left[ c_1 (p_2 y)^{p_1} + c_2 (p_2 y)^{2p_1} \right].$$  \hspace{1cm} (42)

Then, for cases $1 \div 3$, the scalar curvature $R_{(5)}$ is

- case 1: $R_{(5)} = \frac{3}{2} \frac{p_1^2 q^2}{y}$,
- case 2: $R_{(5)} = \frac{8k}{y} - \frac{2c_2}{3y^2}$,
- case 3: $R_{(5)} = -\frac{33c_1}{21} - \frac{4k}{y}$.  \hspace{1cm} (45)

In all cases the singularity appears at $y = 0$.

In case 1, when $y \sim 0$ and the coordinates besides $y$ are fixed, the infinitesimally small distance $ds$ is given by

$$ds = \sqrt{f} dy \sim \frac{dy}{q \sqrt{y}}$$  \hspace{1cm} (46)

which tells us that the distance between the brane and the singularity is finite. Then in the cases $k = 0$ and $k < 0$ the singularity is naked when we Wick-re-rotate the space-time to Lorentzian signature. When $k > 0$, the singularity is not precisely naked after the Wick re-rotation since the horizon is situated $y = 0$, i.e., the horizon coincides with the curvature singularity.

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References

[1] S. Nojiri, S.D. Odintsov and K.E. Osetrin, Phys. Rev. D 63 084016 (2001); [hep-th/0009054].
[2] L. Randall and R. Sundrum, Phys. Rev. Lett. 83, 3370 (1999), [hep-th/9905221].
[3] L. Randall and R. Sundrum, *Phys. Rev. Lett.* **83**, 4690 (1999), [hep-th/9906064].
[4] S. Nojiri and S. D. Odintsov, *JHEP* **0007**, 049 (2000), [hep-th/0006232].
[5] S.W. Hawking, T. Hertog and H.S. Reall, *Phys. Rev.* **D62**, 043501 (2000).
[6] S. Nojiri, S.D. Odintsov and S. Zerbini, [hep-th/0001192], *Phys.Rev. D62*, 064006 (2000);
S. Nojiri and S.D. Odintsov, [hep-th/0004097], *Phys. Lett. B* **484**, 119 (2000).
[7] J.M. Maldacena, *Adv. Theor. Math. Phys.* **2**, 231 (1998);
E. Witten, *Adv. Theor. Math. Phys.* **2**, 253 (1998);
S. Gubser, I. R. Klebanov and A. M. Polyakov, *Phys. Lett. B* **428** (1998) 105.
[8] S. Nojiri, O. Obregon and S.D. Odintsov, [hep-th/0005127], *Phys. Rev. D* **62**, 104003 (2000).
[9] L. Anchordoqui, C. Nunez and K. Olsen, [hep-th/0007064].
[10] I.L. Buchbinder, S.D. Odintsov and I.L. Shapiro, “Effective Action in Quantum Gravity”, IOP Publishing, Bristol and Philadelphia 1992.
[11] A. Starobinsky, *Phys. Lett. B* **91**, 99 (1980).
[12] C. Csáki, J. Erlich, C. Crojean and T.J. Hollowood, [hep-th/0004134].
[13] I. Brevik and S. D. Odintsov, *Phys. Lett. B* **455**, 104 (1999);
B. Geyer, S.D. Odintsov and S.Zerbini, *Phys. Lett. B* **460**, 58 (1999);
S. Nojiri, S.D. Odintsov and S. Zerbini, [hep-th/0006115].