Improving the mathematical stability in the numerical simulation of the vehicle movement with an electronic movement control system

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Abstract. The aim of the study is to determine the influence of the calculated radius type on the calculated parameters of the vehicle movement, equipped with an electronic movement control system. A numerical simulation of the vehicle movement equipped with an electronic movement control system was carried out. Under calculated conditions, there are forces that disrupt the stable and controlled vehicle movement. The studies carried out have shown that in the numerical simulation of the parameters of the vehicle movement, the use of a dynamic radius instead of a rolling radius never affects the calculated values of the vehicle’s longitudinal shifts. In this case, the values of the lateral shifts and the turning angle of the vehicle on a dry hard surface change insignificantly, but there is a significant mathematical instability of the solution. On a wet hard surface, the influence of the calculated radii types on the characteristics of the simulated vehicle movement is preserved, but this influence is less pronounced.

1. Introduction
Currently, there is a variety of electronic systems that control a vehicle movement, and this direction of improving the vehicle is rapidly developing:

1. ABS – Anti-lock Braking System;
2. AEBS – Advanced Emergency Braking Systems;
3. ASR – Anti-Slip Regulation;
4. ESP – Electronic Stability Program;
5. LDWS – Lane Departure Warning System;
6. VIDC – Vehicle Identification Control is a multifunctional system including Anti-Lock Braking System, Anti-Slip Regulation and Lane Departure Warning System.

Stability, controllability and braking dynamics of the created vehicle are first evaluated at the stages of its design modeling. In the modeling of the vehicle movement, particularly, in modeling the performance of test maneuvers, it is necessary to mathematically describe the wheels with tires having many geometric and elastic characteristics. The most important geometric characteristics are wheel radii. Different authors in different problems use different types of wheel radii [1-4], [5-8], [9-13], [14-17].
2. **Aim of the work**
Determine the effect of the wheel radius type used on the calculated movement trajectory parameters of the vehicle, equipped with an electronic movement control system.

3. **Problem statement**
Determine the effect of the wheel radius type used on the calculated movement trajectory parameters of the vehicle, equipped with an electronic movement control system. The wheel has the following radii: free radius $R_0$, static radius $R_s$, dynamic radius $R_d$, rolling radius $R_i$ [1-3, 13]. The difference between them can be up to 20%, depending on the radial deformation of the tire [1, 7, 8]. In accordance with the state standard 17697-72 «Cars. Wheel rolling. Terms and Definitions»:
1. The free radius of the wheel is half the outer wheel diameter.
2. The outer diameter of the wheel is the diameter of the largest circumferential of the wheel running track in the absence of contact between the wheel and the supporting surface.
3. The static wheel radius is the distance from the center of the wheel loaded only by the static normal load to the reference plane.
4. The dynamic radius is the distance from the center of the wheel to the reference plane when the wheel is moving.
5. The rolling radius of a wheel is the ratio of the longitudinal component of the wheel translational speed to its angular speed during rolling without sliding. The rolling radius, which is determined by taking into account the possible wheel sliding, is also called the kinematic radius.

The types of wheel radii are shown in figure 1.

![Figure 1. Wheel scheme.](image)

The figure uses the following designations: $R_0$ – free radius; $R_d$ – dynamic radius; $P_n$ – normal load; $l_c$ – contact patch length; $Z$ – radial deformation; $\omega$ – wheel angular speed; $V_x$ – wheel axle forward speed.
In fact, the rolling radius is the average radius of a wheel rolling without sliding. It can be determined by the approximate dependence obtained earlier by the authors:

\[
R_r = R_0 \left[ \left( 1 - \frac{\arcsin \left( \sqrt{n(2-n)} \right)}{\pi} \right) + \frac{\sqrt{n(2-n)}}{\pi} \right],
\]

(1)

where \( n \) – relative radial tire deformation \( (n = \frac{Z}{R_0}) \).

V.A. Petrushov also established the corrective dependence of the rolling radius on the tangential elasticity of the diagonal tire and the applied torque [6]:

\[
R_r = R_{r0} - C_{t\beta} \cdot M_k,
\]

(2)

where \( C_{t\beta} \) – tire torsional stiffness coefficient; \( M_k \) – torque applied to the wheel; \( R_{r0} \) – rolling radius in free mode.

The radii of the elastic wheel (tire) have different values. At the same time, always: \( R_{st} < R_r \); \( R_{st} \leq R_{d} \).

Depending on the radial deformation of the tire: \( R_{r} < R_{st} \land R_{r} = R_{st} \). Rolling radius: \( R_{r} \leq R_{r0} \) \( R_{r} > R_{st} \land R_{r} \geq R_{d} \).

Figure 2 shows the relative radii calculated by the authors: dynamic and rolling at different relative tire deformations. It can be seen from the figure that at operating deformations of the tire, the rolling radius decreases by no more than 2%.

The rolling radius is also determined experimentally, through the path traversed by a real wheel in 10 turns, as is customary in the scientific school of professor A.I. Fedotov [8]. The experimental results show that with an increase in the radial tire deformation, the rolling radius of the wheel decreases to 3% [8].

**Figure 2.** Calculated relative radii: dynamic and rolling at different tire relative deformations \( n \).
However, in the problems of the first design prediction of the vehicle active safety properties, it is not possible to experimentally determine the rolling radius, therefore, in such cases it is permissible to calculate it using the proposed theoretical dependence (1).

Figures 3 and 4 show the rolling radii calculated by the authors and the dynamic radii of all wheels in the process of performing the «braking in a turn» maneuver by automobile passenger car without ABS and with ABS.

Despite the large number of studies on the theory of elastic wheel rolling, experts have not yet developed a common opinion on which radius could be used in which tasks. In this case, the difference between $R_d$ and $R_r$, for example, can be up to 20%, that naturally, is accompanied by a difference in the calculations in which these radii are used.

Much has been done by S.P. Pozhidaev, who showed that to describe the kinematic, power and energy parameters of the wheel, it would be correct to use the rolling wheel radius [7]. An application of the dynamic radius in the rolling theory of an elastic wheel contradicts the basic laws of mechanics.
Figure 3. Changes in the radii of the vehicle wheels during the «braking in a turn» maneuver on dry asphalt concrete without ABS with an initial speed of 10 m/s: a – front left wheel; b – rear left wheel; c – front right wheel; d – rear right wheel; 1 – rolling radius; 2 – dynamic radius.
Figure 4. Changes in the radii of the vehicle wheels during the «braking in a turn» maneuver on dry asphalt concrete with ABS with an initial speed of 10 m/s: a – front left wheel; b – rear left wheel; c – front right wheel; d – rear right wheel; 1 – rolling radius; 2 – dynamic radius.

Figure 5 shows the scheme proposed by the authors for the application of the wheel radius types in various problems related to the movement modeling of the vehicle equipped with an electronic movement control system.
Figure 5. The proposed scheme for the application of the wheel radius types in various problems related to the movement modeling of the vehicle.

It lists the active safety properties of the vehicle and the main phenomena that determine the parameters of these operational properties. There is slip of an elastic wheel, large and small oscillations of the operated wheels, longitudinal and lateral redistribution of loads on the wheels, longitudinal and lateral sliding in the contact patch. The intensity of these phenomena is associated with a variety of parameters of the first and second levels, but Figure 5 shows not all, but only those that somehow depend on one of the wheel radii.

There is a need to know the contact patch length in calculating the slip parameters. It is determined simultaneously by the free radius (the large wheel has a long contact patch) and by the dynamic radius associated with the current radial tire deformation.

The disturbing moment on the operated wheel from the lateral reaction of the support surface depends on two factors that determine the arm of this reaction: the longitudinal incline of the wheel rotation axis and the longitudinal displacement of the static friction area, which implements the lateral
reaction and to the center of which it is applied. Therefore, to calculate the specified moment, the free radius and radii are required, which appear in calculating the contact patch length.

The objective of this study is to calculate the trajectory parameters of the vehicle equipped with an electronic movement control system when performing standard test maneuvers using different types of wheel radius.

The solution of this problem will be the basis for selection the type of wheel radius in the movement modeling of the vehicle equipped with an electronic movement control system.

4. Results
A calculated analysis of the influence of the used radius type on the of longitudinal wheel sliding coefficient $s_s$ and on the shape of the obtained $\phi - s_s$-diagrams was carried out. Based on the analysis, it was concluded that it is advisable to use the rolling radius in the calculations of longitudinal wheel sliding coefficient. It has been established that in the calculations of the $\phi - s_s$-diagrams, instead of the rolling radius, it is possible to use the free radius (but not the dynamic one!). Of course, from a mechanical point of view, it would be better to use the rolling radius. But, since it differs from the free radius, even with the maximum permissible tire deformations by no more than 2%, and this discrepancy does not have a visible effect on the calculated $\phi - s_s$-diagrams at different values of the lateral force that appeared before or after the start of wheel braking, then it is not necessary to complicate the task by calculating the rolling radius at each moment of movement, since this increases the counting time, and using the free radius gives the same result. And it is possible to approximately calculate the rolling radius with an accuracy sufficient for practical calculations using the above dependence (1).

Modeling of the movement of the vehicle equipped with an electronic control system was carried out, with the following standard test maneuvers: «entering a turn of a radius of 35 m», «linear braking on a surface with a high friction coefficient», «linear braking on a surface with a reduced friction coefficient», «linear braking on the surface with different friction coefficients», «braking in a turn». During these maneuvers, forces are present that disrupt the stable and controlled vehicle movement.

The studies carried out by the authors have shown that the use of the dynamic radius instead of the rolling radius in calculating the trajectory parameters of the vehicle does not affect the obtained values of the braking distance on any road surface. In this case, the values of linear deviations and the turning angle of the vehicle on a dry asphalt concrete surface change insignificantly, but the use of the dynamic radius in modeling leads to solution instability. This does not always provide an adequate assessment of active safety parameters in mathematical modeling of the movement of a projected vehicle with an electronic movement control system. On wet asphalt concrete, the physical picture of the influence of the radius types on the characteristics of the vehicle simulated trajectory is preserved, but this influence is less pronounced.

5. Conclusion
There are five types of vehicle wheel radii, and the difference in their values reaches 20%. At the same time, the wheel radius has a complex, ambiguous effect on the properties of a vehicle equipped with an electronic movement control system: stability, controllability, braking dynamics.

The influence analysis of the wheel radius types used on the results of movement modeling of the vehicle equipped with an electronic movement control system, when performing standard test maneuvers, was carried out.

It has been confirmed that it will be correct to use the rolling radius in calculating the longitudinal wheel sliding. It has been established that, in calculating the $\phi - s_s$-diagrams, instead of the rolling radius, it is permissible to use the free radius.

It has been determined that the use of the dynamic radius instead of the rolling radius in design modeling of standard test maneuvers of vehicles does not affect the obtained values of braking distance on any road surface, regardless of the presence of an electronic movement control system.
It has been established that the use of the dynamic radius instead of the rolling radius in the design modeling of standard test maneuvers of vehicles affects the solution stability in calculating linear deviations and turning angles of the vehicle, especially on dry asphalt concrete pavement.

It is recommended to use the rolling radius in modeling of the vehicle the trajectory equipped with an electronic movement control system.

The results of the work can be used in the design of vehicles equipped with an electronic movement control system, at the stage of solving the problems of design prediction of their stability, controllability and braking dynamics.

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