Deterministic Identification Over Poisson Channels

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Outline

1. Motivation
2. Main Contributions
3. Definitions
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5. Conclusions
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1. Motivation
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Transmission vs. Identification

- **Shannon’s setting:** Bob recovers the message.

  ![Diagram of Shannon's setting](image)

- **Identification setting:** Bob asks if a message was sent or not?

  ![Diagram of Identification setting](image)
Transmission vs. Identification

- **Shannon’s setting**: Bob recovers the message.

- **Identification setting**: Bob asks if a message was sent or not?

- **V2X and P2MP communications**
- **Cancer treatment and smart drug delivery**
- **Any event-triggered scenario**
Randomized Identification (RI) \(^1\)

- Originally introduced by Ahlswede and Dueck (1989)
- Capacity was established with randomness at encoder
- Encoder employs distribution to select codewords

**Remarkable Property**

- Reliable identification is possible with code size growth \(\sim 2^{2nR}\)
- Sharp difference to transmission with code size growth \(\sim 2^{nR}\)

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\(^1\) R. Ahlswede, and G. Dueck, "Identification via channels", 1989
Deterministic Identification (DI) \(^2\) \(^3\)

- Encoder uses deterministic mapping for coding

**Why deterministic?**
- Simpler implementation (random resource not required)
- Suitable for Jamming scenarios
- Suitable for molecular communication

\(^2\) R. Ahlswede and N. Cai. “Identification without randomization”, 1999

\(^3\) M. J. Salariseddigh, U. Pereg, H. Boche, and C. Deppe, “Deterministic identification over channels with power constraints,” IEEE Int'l Conf. Commun. (ICC), 2021 [arXiv:2010.04239, 2021]
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Main Contributions

- We develop lower and upper bounds on the DI capacity for the memoryless discrete time Poisson channels (DTPC) subject to both average and peak power constraints.

- We use the bounds to determine the **correct scale**.

- We show that the optimal code size scales as $\sim 2^{(n \log n)R}$. 
DI Codes

**Definition**

An \((L(n, R), n, \lambda_1, \lambda_2)\)-DI code for DTPC \(W\) is a system

\[
\{(u_i, D_i)\}_{i \in [1:L(n, R)]}
\]

subject to

1. **Code size**: \(L(n, R) = 2^{(n \log n)R}\)
2. **Code-word**: \(u_i \in \mathcal{X}^n\), decoding sets: \(D_i \subset \mathcal{Y}^n\)
3. **Input constraints**:
   - \(0 < u_{i,t} \leq P_{\text{max}}\)
   - \(n^{-1} \sum_{t=1}^{n} u_{i,t} \leq P_{\text{avg}}\)
4. **Error requirement type I**: \(W^n(D_i | u_i) > 1 - \lambda_1\)
5. **Error requirement type II**: \(W^n(D_i | u_j) < \lambda_2\) for \(i \neq j\)
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DI for Poisson Channel

$Y(t) \sim \text{Pois}(\lambda + u_i(t))$
DI for Poisson Channel

\[ Y(t) \sim \text{Pois} (\lambda + u_i(t)) \]

**Definitions**

- Dark current $\rightarrow \lambda \in (0, \infty)$
- Realization of channel output $\rightarrow y \in \mathbb{N}_0^n$
- Power const. $0 < u_{i,t} \leq P_{\text{max}}$ and $\frac{1}{n} \sum_{t=1}^n u_{i,t} \leq P_{\text{avg}}$
- Channel law $\rightarrow W^n(y|u_i) = \prod_{t=1}^n \frac{e^{-(\lambda+u_{i,t})} (\lambda+u_{i,t})^{y_t}}{y_t!}$
**DI for Poisson Channel**

**Theorem**

4 Let $\mathcal{W}$ be a DTPC with dark current $\lambda \in (0, \infty)$. Then the DI capacity subject to power constraints $n^{-1} \sum_{t=1}^{n} u_{i,t} \leq P_{\text{avg}}$ and $0 < u_{i,t} \leq P_{\text{max}}$ for $L(n, R) = 2^{(n \log n)R}$ is bounded by

$$\frac{1}{4} \leq C_{\text{DI}}(\mathcal{W}, L) \leq \frac{3}{2}$$
**DI for Poisson Channel**

**Theorem**

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$$\frac{1}{4} \leq C_{DI}(\mathcal{W}, L) \leq \frac{3}{2}$$

**Corollary (Traditional Scales)**

**DI capacity in traditional scales is given by**

$$C_{DI}(\mathcal{W}, L) = \begin{cases} \infty & \text{for } L(n, R) = 2^{nR} \\ 0 & \text{for } L(n, R) = 2^{2^{nR}} \end{cases}$$

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4 M. J. Salariseddigh, U. Pereg, H. Boche, and C. Deppe, and R. Schober, "Deterministic identification over Poisson channels," Submitted to the IEEE Glob. Commun. Conf. (GLOBECOM), 2021 [arXiv:2107.06061]
DI for Poisson Channel

Theorem

Let $\mathcal{W}$ be a DTPC with dark current $\lambda \in (0, \infty)$. Then the DI capacity subject to power constraints $n^{-1} \sum_{t=1}^{n} u_{i,t} \leq P_{\text{avg}}$ and $0 < u_{i,t} \leq P_{\text{max}}$ for $L(n, R) = 2^{(n\log n)^R}$ is bounded by

$$\frac{1}{4} \leq \mathbb{C}_{\text{DI}}(\mathcal{W}, L) \leq \frac{3}{2}$$

Corollary (Traditional Scales)

DI capacity in traditional scales is given by

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Achiev. proof: sphere pkg. of rad. $n^{\frac{1}{4}} \Rightarrow 2^{\frac{1}{4}(n\log n)}$ codewords

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Proof Sketch. (Achievability)

- Dense sphere packing arrangement with radius $\sqrt{n\epsilon_n}$
- **Minkowski-Hlawka Theorem** guarantees a density $\Delta \geq 2^{-n}$
- $2^n \log(n)R \geq \Delta \cdot \frac{\text{Vol}(Q_0[n,A])}{\text{Vol}(S_{u_1}(n,\sqrt{n\epsilon_n}))} \geq 2^{-n} \cdot \frac{A^n}{\text{Vol}(S_{u_1}(n,\sqrt{n\epsilon_n}))}$
- $R \geq \frac{1}{n\log n} \left[ o(n \log n) + \frac{n}{2} \log n - \frac{1}{4} (1 + b) \cdot n \log n \right] \xrightarrow{n \to \infty} \frac{1}{4}$
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**Chebyshev's inequality** leads to the following error bounds:

1. $P_{e,1}(i) \leq \frac{c_1^2}{n^2 \epsilon_n^2}$
2. $P_{e,2}(i,j) \leq \frac{c_2^2}{n^2 \epsilon_n^2}$
Proof Sketch. (Achievability)

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$$R \geq \frac{1}{n\log n} \left[ o(n\log n) + \frac{n}{2} \log n - \frac{1}{4} (1 + b) \cdot n \log n \right] \xrightarrow{n \to \infty} \frac{1}{4}$$

**Chebyshev’s inequality** leads to the following error bounds:

1. $P_{e,1}(i) \leq \frac{c_1}{n\epsilon_n^2}$
2. $P_{e,2}(i,j) \leq \frac{c_2}{n\epsilon_n^2}$

- Cond. 1 & 2 $\rightarrow \epsilon_n = \frac{A}{n^{\frac{1}{2}(1-b)}}$ for $b > 0$ being arbitrarily small
Proof Sketch. (Converse)

We show that if two distinct code-words $u_i$ and $u_j$ satisfy

$$\left| 1 - \frac{v_{i_2,t}}{v_{i_1,t}} \right| \leq \epsilon'_n , \text{ for all } t \in [1 : n],$$

where $v_{i,t} = \lambda + u_{i,t}$ is the letter for shifted codeword, then using the continuity of the Poisson PDF, we obtain

$$P_{e,1}(i) + P_{e,2}(i,j) \geq 1 - \kappa_n$$
Proof Sketch. (Converse)

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  where \( v_{i,t} = \lambda + u_{i,t} \) is the letter for shifted codeword, then using the continuity of the Poisson PDF, we obtain

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P_{e,1}(i) + P_{e,2}(i,j) \geq 1 - \kappa_n
  \]

- We have

  \[
  |u_{i_1,t} - u_{i_2,t}| = |v_{i_1,t} - v_{i_2,t}| \geq \epsilon'_n v_{i_1,t} > \lambda \epsilon'_n
  \]

- Hence

  \[
  \|u_{i_1} - u_{i_2}\| > \lambda \epsilon'_n
  \]
Proof Sketch Cont. (Converse)

- **Tight** upper-bound requires:
  1. $\epsilon'_n$ large as possible
  2. $\kappa_n$ tends to zero

- By conditions 1 & 2 we obtain

$$
\epsilon'_n = \frac{P_{\text{max}}}{n^{1+b}}
$$

for $b > 0$ being an arbitrarily small rate

$$
\text{rate} \uparrow \iff \epsilon'_n \downarrow
$$
Coding Scale

$L(n, R)$

$2^{2nR}$ • randomized id

$2^{(n \log n)R}$ • deterministic identification

$2^{nR}$ • transmission

$2^{\sqrt{n}R}$ • covert commun.

S., Pereg, Boche & Deppe, ITW 2020

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M. J. Salariseddigh, U. Pereg, H. Boche, and C. Deppe, "Deterministic identification over fading channels," IEEE Inf. Theory Workshop (ITW), 2020 [arXiv:2010.10010, 2021]
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Conclusions

- We have determined DI capacity for
  - discrete time Poisson channel \( 2^{(n \log n)C} = n^nC \) behavior
  As opposed to \( 2^{2^{nR}} \) for randomized identification

- We observed that DI coding scale is the same for both DTPC and fading channels

- Future directions
  - Address other molecular communication channel models
  - Try Multi-user scenarios
Thank You!