Global solvability of the initial-boundary value problem for Navier–Stokes–Fourier type equations describing flows of viscous compressible heat-conducting multifluids

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Abstract. We consider the initial-boundary value problem governing unsteady motions of viscous compressible heat-conducting multifluids in a bounded three-dimensional domain. The operator of the material derivative is assumed to be common for all components and defined by the average velocity of the multifluid, but in the remaining terms, the individual velocities are kept. Pressure is considered common and dependent on total density and temperature. The existence of weak solutions of the initial-boundary value problem is proved without simplifying assumptions about the structure of viscosity matrices, except the standard physical requirements of positive definiteness.

1. Formulation of the equations of the multifluids dynamics of general form

Motion in a bounded domain $\Omega \subset \mathbb{R}^3$ of a multifluid, which consists of $N \geq 2$ viscous compressible heat-conducting fluids, over time $t \in [0,T]$, $T = \text{const} > 0$, is described by the following system of differential equations [1], [2], [3]:

\begin{equation}
\frac{\partial \rho_i}{\partial t} + \text{div}(\rho_i \mathbf{u}_i) = 0, \quad i = 1, \ldots, N,
\end{equation}

\begin{equation}
\frac{\partial (\rho_i \mathbf{u}_i)}{\partial t} + \text{div}(\rho_i \mathbf{u}_i \otimes \mathbf{u}_i) + \nabla p_i = \text{div} \mathbf{S}_i + \mathbf{J}_i + \rho_i \mathbf{f}_i, \quad i = 1, \ldots, N,
\end{equation}

\begin{equation}
\frac{\partial \mathcal{E}}{\partial t} + \text{div} \left( \sum_{i=1}^N \mathcal{E}_i \mathbf{u}_i \right) + \text{div} \left( \mathbf{q} - \sum_{i=1}^N \mathbf{S}_i \mathbf{u}_i + \sum_{i=1}^N \rho_i \mathbf{f}_i \right) = \sum_{i=1}^N \rho_i \mathbf{f}_i \cdot \mathbf{u}_i + \rho g.
\end{equation}

These equations are, respectively, the mathematical formulations of the laws of mass conservation for each component, the laws of conservation of momentum for each component and the law of conservation of total energy of the multifluid and are a generalization of the well-known system of Navier—Stokes—Fourier equations for single-component viscous compressible heat-conducting media. Here $\rho_i \geq 0$ is the density of the $i$-th component; $\rho = \sum_{i=1}^N \rho_i$ is the total density of the
multifluid; $u_i$ is the velocity of the $i$-th component; $E_i = \frac{\rho_i |u_i|^2}{2} + \rho_i e_i$ is the total energy of the $i$-th component, where $e_i$ is the specific internal energy of the $i$-th component; $E = \sum_{i=1}^{N} E_i$ is the total energy of the multifluid; $p_i$ is the pressure of the $i$-th component;

$$S_i = \sum_{j=1}^{N} ((\lambda_{ij}\text{div} u_j)I + 2\mu_{ij} \mathbb{D}(u_j)) = \sum_{j=1}^{N} \left( (\eta_{ij}\text{div} u_j)I + 2\mu_{ij} \left( \mathbb{D}(u_j) - \frac{1}{3}(\text{div} u_j)I \right) \right)$$

is the viscous part of the stress tensor in the $i$-th component, where $I$ is the unit tensor, $\mathbb{D}(v) = \frac{1}{2}((\nabla \otimes v) + (\nabla \otimes v)^*)$ is the rate of deformation tensor of the vector field $v$ (the superscript $*$ stands for the transposition), and the viscosity coefficients $\lambda_{ij}$, $\mu_{ij}$ and $\eta_{ij}$ compose the matrices

$$M = \{\mu_{ij}\}_{i,j=1}^{N} > 0, \quad \Lambda = \{\lambda_{ij}\}_{i,j=1}^{N}, \quad \text{and} \quad H = \{\eta_{ij}\}_{i,j=1}^{N} = \Lambda + 2\frac{2}{3}M \geq 0,$$

which, in particular, gives that

$$N = \{\nu_{ij}\}_{i,j=1}^{N} = \Lambda + 2M > 0; \quad (5)$$

moreover,

$$J_i = \sum_{j=1}^{N} a_{ij}(u_j - u_i), \quad a_{ij} = a_{ji}, \quad i, j = 1, \ldots, N$$

is the momentum influx into the $i$-th component from the others;

$$q = -k(\theta)\nabla \theta$$

(here $\theta > 0$ is the temperature of the multifluid) is the total heat flux, $k$ is the thermal conductivity; $f_i$ is the density of the mass forces acting from the external environment on the $i$-th component; finally, $g$ is the density of the heat sources of the external environment.

As can be seen, the main feature of the equations (1)–(3) of the multifluids apart from their nonlinearity is the presence (in the conservation laws of momenta and energy) of first and second order derivatives of the velocities of all components due to the compound form of viscous stress tensors. In contrast to the one-component case, in which the viscosities are scalars, in the multicomponent case they form matrices, entries of which are responsible for viscous friction. Diagonal entries are responsible for viscous friction inside each component, and non-diagonal entries correspond to the friction between the components. This leads to the fact that the results known for one-component models are not automatically transferred to the multifluid model considered in the paper.

2. Formulation of equations with average velocity and total pressure

Let us denote the average velocity of the multifluid as $v = \frac{1}{N} \sum_{i=1}^{N} u_i$. Observe that

$$\frac{\partial \rho_i}{\partial t} + \text{div}(\rho_i v) = \text{div}(\rho_i(v - u_i)), \quad i = 1, \ldots, N,$$

$$\frac{\partial (\rho_i u_i)}{\partial t} + \text{div}(\rho_i v \otimes u_i) + \text{div}(\rho_i(u_i - v) \otimes u_i) - J_i + \nabla p_i = \text{div}S_i + \rho_i f_i, \quad i = 1, \ldots, N$$

(8)
\[
\frac{\partial E}{\partial t} + \text{div} \left( \mathcal{E} \mathbf{v} \right) + \text{div} \left( \sum_{i=1}^{N} \mathcal{E}_i \left( \mathbf{u}_i - \mathbf{v} \right) \right) + \text{div} \left( \mathbf{q} - \sum_{i=1}^{N} \mathbf{S}_i \mathbf{u}_i + \sum_{i=1}^{N} \rho_i \mathbf{u}_i \right) = \sum_{i=1}^{N} \rho_i \mathbf{f}_i \cdot \mathbf{u}_i + \rho g.
\] (9)

Suppose that the underlined terms in the momenta (8) and energy (9) equations, and also the right-hand sides in the continuity equations (7) are small (which is valid e. g. if the velocities \( \mathbf{u}_i \) are close to each other). Suppose also that in all components (partial) pressures are the same: \( p_1 = \ldots = p_N = \frac{p}{N} \) (where \( p \) is the total pressure of the multifluid). In the isothermal case, these assumptions are used in the articles [4]–[14]. Let us also note the results obtained for related models of mixtures [15]–[23]. Thus, we obtain the equations

\[
\frac{\partial \rho_i}{\partial t} + \text{div} \left( \rho_i \mathbf{v} \right) = 0, \quad i = 1, \ldots, N,
\] (10)

\[
\frac{\partial (\rho_i \mathbf{u}_i)}{\partial t} + \text{div} \left( \rho_i \mathbf{v} \otimes \mathbf{u}_i \right) + \frac{1}{N} \mathbf{v} \cdot \nabla p = \text{div} \mathbf{S}_i + \rho_i \mathbf{f}_i, \quad i = 1, \ldots, N,
\] (11)

\[
\frac{\partial E}{\partial t} + \text{div} \left( \mathcal{E} \mathbf{v} \right) + \text{div} \left( \mathbf{q} - \sum_{i=1}^{N} \mathbf{S}_i \mathbf{u}_i + \rho g \right) + \text{div} \left( \mathbf{q} - \sum_{i=1}^{N} \mathbf{S}_i \mathbf{u}_i + \rho g \right) = \sum_{i=1}^{N} \rho_i \mathbf{f}_i \cdot \mathbf{u}_i + \rho g.
\] (12)

for \( N + 1 \) scalar-valued \( (\rho_i, i = 1, \ldots, N, \text{ and } \theta) \) and \( N \) vector-valued \( (\mathbf{u}_i, i = 1, \ldots, N) \), i. e. totally \( 4N + 1 \) scalar-valued unknown functions, provided that \( p \) and \( e_i, i = 1, \ldots, N \), are somehow defined as functions of \( \rho_i, i = 1, \ldots, N, \) and \( \theta \).

**Remark 1.** The equations (10)–(12) allow the equivalent form

\[
\rho_i \frac{\partial \mathbf{u}_i}{\partial t} + \mathbf{v} \cdot \nabla \rho_i + \rho_i \text{div} \mathbf{v} = 0, \quad i = 1, \ldots, N,
\]

\[
\rho_i \frac{\partial \mathbf{u}_i}{\partial t} + \rho_i (\nabla \otimes \mathbf{u}_i)^* \mathbf{v} + \frac{1}{N} \mathbf{v} \cdot \nabla p = \text{div} \mathbf{S}_i + \rho_i \mathbf{f}_i, \quad i = 1, \ldots, N,
\]

\[
\sum_{i=1}^{N} \left( \rho_i \frac{\partial \left( \frac{1}{2} |\mathbf{u}_i|^2 + e_i \right)}{\partial t} + \rho_i \mathbf{v} \cdot \nabla \left( \frac{1}{2} |\mathbf{u}_i|^2 + e_i \right) \right) + \text{div} \left( \mathbf{q} - \sum_{i=1}^{N} \mathbf{S}_i \mathbf{u}_i + \rho g \right) = \sum_{i=1}^{N} \rho_i \mathbf{f}_i \cdot \mathbf{u}_i + \rho g,
\]

moreover, \( (\nabla \otimes \mathbf{u}_i)^* \mathbf{v} = (\mathbf{v} \cdot \nabla) \mathbf{u}_i \). Such non-divergent notation is unsuitable for the study of weak solutions, but it allows us to observe common material derivative operator \( \frac{d}{dt} = \frac{\partial}{\partial t} + \mathbf{v} \cdot \nabla \).

3. Constitutive equations

We will assume that \( p = p(\rho, \theta), e_i = e_i(\rho, \theta), i = 1, \ldots, N \).

The constitutive equations relating the thermodynamic parameters to each other must satisfy certain constraints, in particular, the Gibbs relations [24]

\[
\theta ds_i = de_i + pd \left( \frac{1}{\rho} \right), \quad i = 1, \ldots, N,
\] (13)

where \( s_i = s_i(\rho, \theta) \) is the specific entropy of the \( i \)-th component, that is equivalent to the Maxwell relations

\[
\rho^2 \frac{\partial e_i}{\partial \rho} = p - \theta \frac{\partial p}{\partial \theta}, \quad i = 1, \ldots, N,
\] (14)

as well as the conditions of thermodynamic stability

\[
\frac{\partial p}{\partial \rho} > 0, \quad \frac{\partial e_i}{\partial \theta} > 0, \quad i = 1, \ldots, N.
\] (15)
Remark 2. The equation (12) may be written in one of the following equivalent forms:

\[
\frac{\partial}{\partial t} \sum_{i=1}^{N} \rho_i e_i + \text{div} \left( \sum_{i=1}^{N} \rho_i e_i \mathbf{v} \right) + p \text{div} + \text{div} \mathbf{q} = \sum_{i=1}^{N} S_i : \mathbf{D}(u_i) + \rho g,
\]

(16)

\[
\frac{\partial}{\partial t} \sum_{i=1}^{N} \rho_i s_i + \text{div} \left( \sum_{i=1}^{N} \rho_i s_i \mathbf{v} \right) + \text{div} \left( \frac{\mathbf{q}}{\theta} \right) = \frac{1}{\theta} \sum_{i=1}^{N} S_i : \mathbf{D}(u_i) - \mathbf{q} \cdot \nabla \theta + \frac{\rho g}{\theta},
\]

(17)

\[
\sum_{i=1}^{N} \frac{\partial e_i}{\partial \theta} \left( \frac{\partial (\rho_i \theta)}{\partial t} + \text{div}(\rho_i \theta \mathbf{v}) \right) + \text{div} \mathbf{q} = \sum_{i=1}^{N} S_i : \mathbf{D}(u_i) - \theta \frac{\partial p}{\partial \theta} \text{div} \mathbf{v} + \rho g.
\]

(18)

Following the approach suggested in [25], we will assume that

\[
p(\rho, \theta) = p_e(\rho) + \theta p_\theta(\rho)
\]

(19)

with some functions \(p_e\) and \(p_\theta\). Then (14) provides that

\[
e_i(\rho, \theta) = \int_{1}^{\rho} \frac{p_e(z)}{z^2} \, dz + Q_i(\theta), \quad i = 1, \ldots, N,
\]

(20)

and up to non-essential additive constants, the representation

\[
Q_i(\theta) = \int_{0}^{\theta} c_{\theta i}(z) \, dz, \quad i = 1, \ldots, N
\]

(21)

holds with some functions \(c_{\theta i}\). The conditions (15) are satisfied provided that \(p'_e\) and \(p'_\theta\) are non-negative, and \(p'_e\) or \(p'_\theta\) is positive (it is implied that \(\theta > 0\), and \(c_{\theta i}(z) \geq c_1 = \text{const} > 0 \quad \forall \, z \geq 0\).

Now we deduce from (13) that

\[
s_i(\rho_i, \theta) = \int_{1}^{\theta} \frac{c_{\theta i}(z)}{z} \, dz - \int_{1}^{\rho} \frac{p_\theta(z)}{z^2} \, dz + s_{0i}, \quad i = 1, \ldots, N,
\]

(22)

where \(s_{0i}\) are arbitrary constants. The equation (18) in this case takes the form

\[
\frac{\partial}{\partial t} \sum_{i=1}^{N} \rho_i Q_i(\theta) + \text{div} \left( \sum_{i=1}^{N} \rho_i Q_i(\theta) \mathbf{v} \right) + \text{div} \mathbf{q} + \theta p_\theta(\rho) \text{div} \mathbf{v} = \sum_{i=1}^{N} S_i : \mathbf{D}(u_i) + \rho g.
\]

(23)

4. Statement of the problem

The main goal of the paper is to solve the following problem.

Problem A. It is required to find scalar fields \(\rho_i \geq 0, i = 1, \ldots, N, \theta > 0\) and vector fields \(\mathbf{u}_i, i = 1, \ldots, N\), which are defined in the closure \(\Omega_T\) of the domain \(\Omega_T = (0, T) \times \Omega\), where \(\Omega \subset \mathbb{R}^3\) is the flow domain, and \(T > 0\) is an arbitrary number, satisfy the system of equations (10), (11), (23) and the following initial and boundary conditions

\[
\rho_i|_{t=0} = \rho_{0i}, \quad \mathbf{u}_i|_{t=0} = \mathbf{u}_{0i}, \quad i = 1, \ldots, N, \quad \theta|_{t=0} = \theta_0,
\]

(24)
\[ u_i = 0, \quad i = 1, \ldots, N, \quad q \cdot n = 0 \quad \text{at} \quad (0, T) \times \partial \Omega. \] (25)

Here \( \rho_{0i} \) (the initial densities), \( u_{0i} \) (the initial velocities) and \( \theta_0 \) (the initial temperature) are given functions; \( \partial \Omega \) is the boundary of the flow domain \( \Omega \); and \( n \) is the outer unit normal of \( \partial \Omega \).

**Remark 3.** Strictly speaking, the initial conditions must be given in terms of \( \rho_i(0, \cdot), \) \( (\rho_\gamma u_i)(0, \cdot) \) and \( \rho_i Q_i(\theta)(0, \cdot), \) however, it is more convenient to work mathematically with the initial conditions written in the form (24).

**Remark 4.** Both physically and mathematically it is necessary to ensure the non-negativity of the entropy production. The total entropy \( S = \sum_{i=1}^{N} \int_{\Omega} \rho_i s_i \, dx \) of the system should not decrease with time when the system is thermodynamically closed, i. e. when \( g = 0. \) In a general (thermodynamically open) case we obtain from (6), (17) and (25) that

\[
\frac{dS}{dt} = \int_{\Omega} \frac{1}{\theta} \sum_{i=1}^{N} S_i : \mathbb{D}(u_i) \, dx + \int_{\Omega} k(\theta)|\nabla \theta|^2 \, dx + \int_{\Omega} \frac{\rho g}{\theta} \, dx.
\]

Thus, it suffices to require the condition on the coefficient

\[ k \geq 0, \] (26)

and the following condition for viscous stress tensors:

\[ \sum_{i=1}^{N} S_i : \mathbb{D}(u_i) \geq 0. \]

However, under the conditions on the viscosity matrices listed in (4), the fulfillment of this condition is obvious, in view of the equality

\[
\sum_{i=1}^{N} S_i : \mathbb{D}(u_i) = \sum_{i,j=1}^{N} \left( \eta_{ij}(\text{div} u_i)(\text{div} u_j) + 2\mu_{ij} \left( \mathbb{D}(u_i) - \frac{1}{3} (\text{div} u_i)I \right) : \left( \mathbb{D}(u_j) - \frac{1}{3} (\text{div} u_j)I \right) \right).
\]

Moreover, from the conditions (4) (see (5)), due to (25), follows the inequality

\[ \sum_{i=1}^{N} \int_{\Omega} S_i : (\nabla \otimes u_i) \, dx \geq B_0 \sum_{i=1}^{N} \int_{\Omega} |\nabla \otimes u_i|^2 \, dx, \]

which is very important mathematically, with some positive constant \( B_0 = B_0(\Lambda, M). \)

### 5. Conditions for the constitutive functions

For the functions \( p_e \) and \( p_\theta, \) the following conditions are supposed

\[ p_e, p_\theta \in C^1[0, \infty), \quad p_e(0) = p_\theta(0) = 0, \quad \frac{1}{c_2} z^{\gamma-1} \leq p'_e(z) \leq c_2 z^{\gamma-1} + c_3 \quad \forall z \geq 0, \]

\[ p_\theta(z) \leq c_4(1 + z^2) \quad \forall z \geq 0, \quad p'_\theta(z) > 0 \quad \forall z \geq 0, \]

where \( c_2 = \text{const} \geq 1, \) \( c_3, c_4 = \text{const} > 0, \) \( \gamma = \text{const} > 3. \) For the function \( k, \) we suppose the following:

\[ k \in C^2[0, \infty), \quad \frac{1}{c_5} (1 + z^m) \leq k(z) \leq c_5 (1 + z^m) \quad \forall z \geq 0, \] (28)
where \( c_5 = \text{const} \geq 1, m = \text{const} \geq 2 \). For the functions \( c_{\theta i}, i = 1, \ldots, N \) (see (21)), the following hypotheses are accepted:
\[
\frac{1}{c_6} \left( 1 + \frac{\theta_{\text{max}}}{\theta_i} \right) \leq c_{\theta i}(\theta) \leq \frac{1}{c_6} \left( 1 + \frac{\theta_{\text{max}}}{\theta_i} \right) \quad \forall \theta \geq 0, \quad i = 1, \ldots, N,
\]
where \( c_6 = \text{const} \geq 1 \).

**Example 5.** Let us give the simplest example of a situation when the assumptions on pressure and energy are fulfilled:
\[
p_c(\rho) = \rho^\gamma, \quad p_\theta(\rho) = \rho, \quad c_{\theta i}(\theta) = 1 + \theta^{\frac{m}{m-1}}, \quad i = 1, \ldots, N.
\]
Then it follows from (19) that
\[
p(\rho, \theta) = \rho^\gamma + \rho \theta, \quad e_i(\rho, \theta) = \frac{\rho^{\gamma-1}}{\gamma-1} + \theta + \frac{2}{m} \theta^\frac{m}{m-1} - \frac{1}{\gamma-1}, \quad i = 1, \ldots, N,
\]
\[
s_i(\rho, \theta) = \ln \left( \frac{\theta}{\rho} \right) + \frac{2}{m-2} \theta^\frac{m}{m-1} + s_{\theta i} - \frac{2}{m-2}, \quad m > 2, \quad i = 1, \ldots, N,
\]
\[
s_i(\rho, \theta) = \ln \left( \frac{\theta^2}{\rho} \right) + s_{\theta i}, \quad m = 2, \quad i = 1, \ldots, N.
\]
The conditions (27), (29) are obviously fulfilled, and the equation (18)=(23) in this case takes the form
\[
\left( 1 + \theta^{\frac{m}{m-1}} \right) \left( \frac{\partial(\rho \theta)}{\partial t} + \text{div} \left( \rho \theta v \right) \right) + \text{div}q + \theta \rho \text{div}v = \sum_{i=1}^N S_i : D(u_i) + pg.
\]

6. Conditions for the input data
The initial conditions in the Problem \( \mathcal{A} \) are of the class
\[
\rho_{0i} \in L_\gamma(\Omega), \quad \rho_{0i} \geq 0, \quad \theta_{0i} \in L_\infty(\Omega), \quad \text{ess inf}_{\Omega} \theta_{0i} > 0, \quad u_{0i} \in L_\infty(\Omega), \quad i = 1, \ldots, N,
\]
and external forces and sources satisfy the requirements
\[
f_i \in L_\infty(Q_T), \quad i = 1, \ldots, N, \quad g \in L_\infty(Q_T), \quad g \geq 0.
\]

7. Definition of solution
**Definition 6.** Let the conditions (4), (19), (21), (26), (27), (28), (29), (30) and (31) be satisfied. By a weak solution to the Problem \( \mathcal{A} \) we mean the collection of functions \( \rho_i \geq 0, u_i, \quad i = 1, \ldots, N, \) and \( \theta > 0 \) of the following class:
\[
\rho_i \in L_\infty(0, T; L_\gamma(\Omega)), \quad u_i \in L_2(0, T; W_2^1(\Omega)), \quad i = 1, \ldots, N,
\]
\[
\rho_i u_i \in L_\infty(0, T; L_{\sigma_1}(\Omega)) \quad \text{with} \quad \sigma_1 > \frac{6}{5}, \quad i = 1, \ldots, N,
\]
\[
\rho_i Q_i(\theta) \in L_\infty(0, T; L_1(\Omega)) \bigcap L_2(0, T; L_{\sigma_2}(\Omega)) \quad \text{with} \quad \sigma_2 > \frac{6}{5}, \quad i = 1, \ldots, N,
\]
\[
\ln \theta, \theta p_\theta(\rho) \in L_2(Q_T), \quad \int_0^\theta k(z)dz \in L_1(Q_T),
\]
which satisfy the equations (10), (11), (23) and the boundary conditions (24), (25) in the weak sense (as it is accepted in the Navier—Stokes—Fourier theory for single-component heat-conducting viscous compressible fluids [25]).
8. Main result
The main result of the paper is contained in the following theorem.

**Theorem 7.** Let $\Omega \subset \mathbb{R}^3$ be a bounded domain of the class $C^{2+\sigma_3}$, where $\sigma_3 \in (0, 1)$, and $T > 0$ be an arbitrary finite number. Then, for any input data of the class described in Definition 6, and under the conditions specified in it on the parameters of the equations, there exists at least one weak solution to the Problem $A$.

9. Sketch of the proof
Proof of Theorem 7 is carried out according to the following scheme:

(i) formulation of an approximate problem in which the parameters $q \in \mathbb{N}$, $0 < \varepsilon \leq 1$ and $0 < \delta \leq 1$ appear, the equations (11) are satisfied in the Galerkin sense (i.e. via the projection on $q$-D space of basis functions), regularizing terms are inserted into all equations: $\varepsilon \Delta \rho_i$ in the right-hand side of (10), $\varepsilon (\nabla \otimes u_i)^* \nabla \rho_i$ in the left-hand side of (11), $\delta \theta^{m+1}$ in the left-hand side of (23) etc., the input data are regularized, the boundary conditions are corrected;

(ii) proof of solvability of the approximate problem with fixed $q$, $\varepsilon$ and $\delta$ using the Schauder fixed point theorem;

(iii) passage to the limit as $q \to +\infty$ (proof of solvability of the approximate problem with the momentum equations satisfied in the usual sense);

(iv) passage to the limit with respect to the small parameters: first $\varepsilon \to 0$, and second $\delta \to 0$;

(v) at each stage, obtaining estimates of solutions that do not depend on the parameter via which the passage to the limit is carried out.

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