Pseudo-Dirac Neutrino Scenario:
Cosmic Neutrinos at Neutrino Telescopes

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Abstract

Within the “pseudo-Dirac” scenario for massive neutrinos the existence of sterile neutrinos which are almost degenerate in mass with the active ones is hypothesized. The presence of these sterile neutrinos can affect the flavor composition of cosmic neutrinos arriving at Earth after traveling large distances from astrophysical objects. We examine the prospects of neutrino telescopes such as IceCube to probe the very tiny mass squared differences $10^{-12} \text{eV}^2 < \Delta m^2 < 10^{-19} \text{eV}^2$, by analyzing the ratio of $\mu$-track events to shower-like events. Considering various sources of uncertainties which enter this analysis, we examine the capability of neutrino telescopes to verify the validity of the pseudo-Dirac neutrino scenario and especially to discriminate it from the conventional scenario with no sterile neutrino. We also discuss the robustness of our results with respect to the uncertainties in the initial flavor ratio of neutrinos at the source.

PACS numbers: 14.60.Pq; 13.15.+g; 95.85.Ry

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1 Introduction

Analyses of the data from reactor [1], accelerator [2], atmospheric [3] and solar [4] neutrino experiments conclusively demonstrate the oscillation of neutrino flavors. The results of these experiments can be interpreted by two independent mass squared differences (between three active neutrinos). From the direct measurement of the invisible part of the decay width of $Z$ boson (i.e., $\Gamma_{Z \rightarrow \nu_\alpha \bar{\nu}_\alpha}$), the number of active neutrinos lighter than $M_Z/2$ found to be $N_\nu = 2.92 \pm 0.06$ [5,6] and from the fit of the LEP data to Standard Model prediction it found to be $N_\nu = 2.994 \pm 0.012$ [7,6]. Thus, if an extra light neutrino exists, it should be a sterile neutrino (singlet under the gauge symmetries of the Standard Model.) Historically the strongest hint for the existence of sterile neutrino came from the short baseline LSND experiment [8]. Data of the LSND experiment, with neutrino energy $E_\nu \sim 30$ MeV and baseline $\sim 30$ m, suggested a 3+1-scheme (active+sterile) with the new mass squared difference $\Delta m^2 \sim \mathcal{O}(1)$ eV$^2$. The LSND result has not been verified by the MiniBooNE experiment [9] and considerable efforts have gone into reconciling the null result of MiniBooNE with the LSND data [10]. All the other data of the neutrino experiments can be interpreted by assuming only three active massive neutrinos without any need to introduce sterile neutrinos in the data analyses. However, sterile neutrinos can still be present in the yet not probed regions of the parameter space ($\Delta m^2, \theta$). These regions correspond to sterile neutrinos almost degenerate in mass with the active ones, with very tiny mass differences $\Delta m^2 \ll \Delta m^2_{\text{sol}}$. The scenario of degenerate sterile neutrinos, the so-called “Pseudo-Dirac” scenario, has been proposed long time ago in [11] and has been studied in the literature extensively [12].

The prospect for the existence of light sterile neutrinos with masses nearly degenerate with the masses of active neutrinos is motivated in many theoretical extensions of the Standard Model [13]. From the observational point of view, probing very small $\Delta m^2$ between sterile and active neutrinos needs very long baselines. Neutrinos coming from the Sun (which is the farthest observed source of neutrinos with continuous emission) set the bound $\Delta m^2 \lesssim 10^{-12}$ eV$^2$ on the active-sterile mass splitting. Bounds from other performed or forthcoming experiments will be discussed in Sect. 2.1. In this paper we concentrate on the effects of almost degenerate sterile neutrinos on the expected flux of cosmic neutrinos coming from astrophysical sources.

The new generation of km$^3$ scale neutrino telescopes give a unique opportunity to probe the very tiny $\Delta m^2$ in pseudo-Dirac scenario. The cosmic neutrinos from sources such as GRBs [14], AGN [15] and type Ib/c supernovae [16] travel large distances over $\sim 100$ Mpc

\footnote{The reason for this nomenclature will be described in Sect. 2}
before arriving at neutrino telescopes in Earth. With such extremely long baseline, tiny mass squared differences as small as $\Delta m^2 \sim 10^{-19} \text{ eV}^2 (E_\nu/100 \text{ GeV})$ can be probed. The idea of using neutrino telescopes to discover the sterile neutrinos present in pseudo-Dirac scenario was proposed in [17, 18, 19, 20]. In order to probe small values of $\Delta m^2$, it has been suggested to look at distortions in the spectrum of $\nu_\mu$ from supernovae remnants in the average distance of $\sim 1 - 8 \text{ kpc}$ in [17] and the spectrum of $\nu_\mu$ from Galactic center in [18]. The authors of [19, 20] evaluate the effect of the pseudo-Dirac neutrinos on the flavor composition of the cosmic neutrinos; i.e., the deviation of the $F_{\nu_\mu} : F_{\bar{\nu}_\mu} : F_{\nu_\tau}$ (where $F_{\nu_\alpha}$ is the flux of $\nu_\alpha + \bar{\nu}_\alpha$ at Earth) from the expected value $1 : 1 : 1$ in the absence of sterile neutrinos.

In the measurement of flavor composition of neutrinos in neutrino telescopes, the uncertainties in the relevant parameters and experimental limitations should be taken into account. For example, there are uncertainties in the mixing parameters of neutrinos and also in the spectrum of the arriving neutrinos. Also, the current constructed or proposed neutrino telescopes, AMANDA/IceCube [21], NEMO [22], NESTOR [23], ANTARES [24] and KM3NET [25] cannot identify all three flavors of the active neutrinos. Ref. [26] considers these uncertainties and experimental limitations in the analysis of the cosmic neutrinos in order to extract mixing parameters and flavor composition of neutrinos at the source. In this paper, by considering the aforementioned uncertainties and experimental limitations, we investigate the potential of neutrino telescopes in discovering the pseudo-Dirac nature of neutrinos. The initial flavor ratio of the neutrinos at the source can also be a source of uncertainty in the calculation of event rates in neutrino telescopes. We discuss the robustness of our result to this kind of uncertainty.

The paper is organized as follows. In sect. 2, the pseudo-Dirac scenario for massive neutrinos is reviewed. In sect. 2.1 the current bounds on $\Delta m^2$ from various neutrino experiments are summarized; and in sect. 2.2 the effects of sterile neutrinos on the flavor composition of cosmic neutrinos are discussed. Sect. 3 is devoted to the production mechanism of neutrinos at the source and their detection processes in the neutrino telescopes. Various sources of uncertainties that enter the calculation of event rates in neutrino telescopes are enumerated. Sect. 4 summarizes the results of the present analysis on the capability of neutrino telescopes to discriminate between pseudo-Dirac and conventional scenarios. A summary of the results and the conclusions are given in sect. 5.
2 Pseudo-Dirac Scenario

A simple and economic way to generate mass for neutrinos in the SM is to add right-handed (sterile) neutrinos to the matter content of SM. In the presence of $N_s$ right-handed (sterile) fields $\nu_{kR}$ ($k = 1, \ldots, N_s$), we define the following column matrix $\Psi$ of $N = 3 + N_s$ left-handed fields

$$\Psi = (\nu_{eL}, \nu_{\mu L}, \nu_{\tau L}, (\nu_{1R})^C, \ldots, (\nu_{N_s R})^C)^T,$$  \hspace{1cm} (1)

where $\nu^C = C\bar{\nu}^T$ and $C$ is the charge conjugation operator. For Majorana neutrinos which we consider here $(\nu_{\alpha R})^C = \bar{\nu}_{\alpha L}$. Here we consider models with at most three sterile neutrinos ($N_s \leq 3$). In the basis $\Psi$, the generic mass term for neutrinos is

$$\mathcal{L}_m = -\frac{1}{2} \bar{\Psi}^C M \Psi + \text{H.c.},$$  \hspace{1cm} (2)

The $(3 + N_s) \times (3 + N_s)$ mass matrix $M$ is of the following form (after electroweak symmetry breaking)

$$M = \begin{pmatrix} m_L & m_D^T \\ m_D & m_R \end{pmatrix},$$  \hspace{1cm} (3)

where $m_D$ is the $N_s \times 3$ Dirac mass matrix and $m_L$ and $m_R$ are the $3 \times 3$ left-handed and $N_s \times N_s$ right-handed Majorana mass matrices, respectively. The non-vanishing elements of $m_L$ and $m_R$ violate lepton numbers while by assigning the lepton number $+1$ to sterile neutrinos, $m_D$ conserves this symmetry. The left-handed mass matrix $m_L$ is not invariant under the SM gauge group $SU(2)_L$ and should be zero unless other new particles (such as a new Higgs triplet) are present. The elements of $m_R$ can take a wide range of values, it can be as large as the GUT scale $\sim 10^{15}$ GeV which are preferred in see-saw mechanisms, or it can vanish like $m_L$ as a result of new gauge symmetries such as $SU(2)_R$ [27]. The case $m_L = m_R = 0$ and $N_s = 3$ results in pure Dirac neutrinos. In this case the six Weyl neutrinos decompose into three pairs of neutrinos with degenerate masses. The active-sterile mixing angle in each pair is maximal $\theta = \pi/4$, but the active neutrinos do not oscillate to their sterile partners because $\Delta m^2_{s_j a_j} = m^2_{s_j} - m^2_{a_j} = 0$, where $m_{s_j}$ and $m_{a_j}$ are the masses of sterile and active neutrinos in the $j$-th pair, respectively. Here we are interested in the case $m_L, m_R \ll m_D$. The non-zero but very small values of the elements of $m_L$ and $m_R$ lift the degeneracy in mass at each pair. In this “pseudo-Dirac” scenario, active-sterile mixing angle in each pair is $\theta \simeq \pi/4$ and active-sterile oscillation can in principle occur due to very small but non-zero $\Delta m^2_{sa}$. To illustrate this point, let us consider the one generation example. In this case, the mass matrices $m_L$, $m_R$ and $m_D$ in Eq. (3) are numbers (we assume that all the masses are real.) In the pseudo-Dirac limit, we obtain $\tan(2\theta) = |2m_D/(m_R - m_L)| \gg 1$.
\[ \Delta m_{sa}^2 \simeq 2m_D(m_L + m_R) \ll m_D. \]

Notice that in the pseudo-Dirac scenario, neutrinos oscillate even in one generation, in contrast to pure Dirac scenario where oscillation occurs only between generations.

In general the \( N \times N \) symmetric mass matrix \( M \) (where \( N = 3 + N_s \)) can be diagonalized by \( V_\nu^T M V_\nu = M_{\text{diag}} \), where \( V_\nu \) is a \( N \times N \) unitary matrix. We choose the elements of \( V_\nu \) such that \( M_{\text{diag}} = \text{diag}(m_{a1}, m_{a2}, m_{a3}, m_{s1}, \ldots, m_{sN_s}) \). The mixing matrix \( V \) appearing in the weak charged-current \( J_{\mu} = 2\Psi_i V_\nu^\dagger \gamma^\mu l_i \) is a \( 3 \times N \) rectangular matrix with the elements \( V_{\alpha k} = \sum_{\beta=e,\mu,\tau} (V_\nu)_{\alpha \beta} (V_\nu)_{\beta k} \), where \( V_\nu \) is the \( 3 \times 3 \) diagonalizing unitary matrix of charged leptons mass matrix. In the case \( N_s = 3 \), the \( 6 \times 6 \) matrix \( V \) can be parameterized by 12 mixing angles and 12 CP-violating phases (7 Dirac phases+5 Majorana phases.) It has been shown in [28] that in the pseudo-Dirac limit \( m_L, m_R \ll m_D \) and at first order of perturbation in the small parameters \( m_L/m_D \) and \( m_R/m_D \), the mixing matrix \( V \) has only three mixing angles (responsible for oscillation between the pairs) and three CP-violating phases (1 Dirac phases+2 Majorana phases). This fact can be seen from the explicit form of the matrix \( V_\nu \) which diagonalizes the mass matrix \( M \) (in the pseudo-Dirac limit) [28]:

\[
V_\nu = \begin{pmatrix} U_{PMNS} & 0 \\ 0 & U_R \end{pmatrix} \frac{1}{\sqrt{2}} \begin{pmatrix} I_{3\times3} & iX_{3\times N_s} \\ (X_{3\times N_s})^T & -iI_{N_s \times N_s} \end{pmatrix},
\]

(4)

where \( U_{PMNS} \) is the \( 3 \times 3 \) conventional neutrino mixing matrix of left-handed neutrinos, \( U_R \) is the \( N_s \times N_s \) unitary matrix which diagonalizes the right-handed Majorana mass matrix, \( I_{n \times n} \) is the \( n \times n \) identity matrix and the matrices \( X_{3\times N_s} \) (\( N_s \leq 3 \)) are:

\[
X_{3\times1} = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, \quad X_{3\times2} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{pmatrix}, \quad X_{3\times3} = I_{3 \times 3}.
\]

(5)

The flavor conversion probability between the active neutrinos \( P_{\alpha \beta} \equiv P_{\nu_{\alpha} \rightarrow \nu_{\beta}}(L, E_\nu) \) is

\[
P_{\alpha \beta} = \left| \left( V_\nu \exp \left\{ i \frac{M_{\text{diag}}^2 L}{2E_\nu} \right\} V_\nu^\dagger \right|_{\alpha \beta} \right|^2.
\]

(6)

Using the explicit form of the matrix \( V_\nu \) in Eq. (4), the probability \( P_{\alpha \beta} \) becomes

\[
P_{\alpha \beta} = \frac{1}{4} \left| \sum_{j=1}^{3} U_{\alpha j} \left\{ e^{i(m_j^+)^2 L/2E_\nu} + e^{i(m_j^-)^2 L/2E_\nu} \right\} U_{\beta j}^* \right|^2,
\]

(7)
where $m_j^+$ and $m_j^-$ are the mass eigenvalues in the $j$-th pair of active and sterile neutrinos; $U_{\alpha j}$ and $U_{\beta j}$ are the elements of the $3 \times 3$ mixing matrix $U_{PMNS}$. Notice that this relation reduces to the standard flavor conversion probability formula in the limit of pure Dirac neutrinos $m_j^+ = m_j^-$ ($j = 1, 2, 3$). By setting $m_j^+ = m_j^-$ for the active neutrino generations which do not have sterile partners, Eq. (7) also applies to cases with $N_s < 3$.

Using Eq. (7) in analyzing the data of oscillation experiments gives information on $(m_j^+)^2 - (m_j^-)^2$ in each pair. In subsect. 2.1 we review the current bounds on active-sterile mass square differences and the prospect of future experiments to improve these bounds. In subsect. 2.2 we discuss the implications of Eq. (7) on the flavor composition of cosmic neutrinos.

### 2.1 Current Bounds on $\Delta m^2$ and Sensitivity of Future Experiments

An oscillation experiment with baseline $L$ and neutrino energy $E_\nu$ can probe mass square difference $\Delta m^2 \sim E_\nu/(4\pi L)$. If $\Delta m^2 \ll E_\nu/(4\pi L)$, the baseline is too short for flavor oscillation to take place; on the other hand, if $\Delta m^2 \gg E_\nu/(4\pi L)$ so many oscillations take place during the propagation and the oscillatory term should be averaged out. In both of these cases it is not possible to derive the value of $\Delta m^2$ in oscillation experiments.

Solar neutrino experiments with the baseline $1 \text{ AU} \approx 1.5 \times 10^{11} \text{ m}$ and neutrino energy $E_\nu \sim 0.1 - 10 \text{ MeV}$, can probe mass squared differences $\Delta m^2 \sim 10^{-10} - 10^{-12} \text{ eV}^2$. These very small values of $\Delta m^2$ has been favored by the so-called “Vacuum Oscillation Solution” of the solar neutrino problem, but as is well-known this solution has been ruled out by KamLand [1]. However, the sterile-active oscillation with mass square differences $\Delta m^2 \lesssim 10^{-12} \text{ eV}^2$ can still be present as a subdominant effect in solar data. The recent work [29] updates the solar data and obtains $\Delta m^2 < 1.8 \times 10^{-12} \text{ eV}^2$ (at 3$\sigma$ level) for the sterile-active mass splitting. This bound is the most stringent bound on $\Delta m^2$. The flavor composition of the neutrinos from core-collapse supernovae (SNe) also can change due to an active-sterile oscillation from the SN to Earth. The mean energy of the neutrinos from a SN explosion is $E_\nu \sim 30 \text{ MeV}$. Thus, a SN explosion at a distance of $\sim 10 \text{ kpc}$ can probe $\Delta m^2 \sim 10^{-19} \text{ eV}^2$. The constraint from the data of the SN1987A data is not restrictive because of the low statistics and high uncertainties in the mechanism of SNe explosion [30]. Construction of future Mton water-Čerenkov detectors can dramatically improve the current bound or find a hint for sterile neutrinos hypothesizes in pseudo-Dirac scenario [31].
Population of the sterile neutrinos in the early universe and their effects on the Big Bang Nucleosynthesis (BBN) can change the abundance of light elements. The standard BBN, given the number of the relativistic particles $N_\nu$ and the baryon asymmetry $\eta = n_B/n_\gamma$, predicts the abundance of the light nuclei in the universe. Assuming that the sterile neutrinos are produced only in the active/sterile oscillation and the initial abundance of sterile neutrinos at temperatures $T \gg \text{MeV}$ is zero, the tightest limit comes from the $^4\text{He}$ abundance: $\Delta m^2 \lesssim 10^{-8} \text{eV}^2$ \cite{31,32}.

Two main non-oscillation neutrino experiments which probe neutrino masses kinematically are tritium beta decay and neutrinoless double beta decay ($0\nu\beta\beta$) experiments. Among them, the $0\nu\beta\beta$ decay is sensitive to the Majorana or Dirac nature of neutrinos. The rate of $0\nu\beta\beta$ decay is proportional to the effective mass of the electron neutrino which is defined as

$$\langle m_{ee} \rangle = \left| \sum_{j=1}^{6} (V_{\nu j})^2 m_j \right| = \left| \sum_{j=1}^{3} \left( \frac{U_{ej}}{\sqrt{2}} \right)^2 (m_j^+ - m_j^-) \right|$$  \hspace{1cm} (8)

As mentioned after Eq. (3), in the limit $m_L = m_R = 0$ (pure Dirac neutrino) each Dirac neutrino is the superposition of two Majorana neutrinos with degenerate masses and opposite CP eigenvalues. It is easy to see that the Majorana neutrinos in each pair interfere destructively in Eq. (8) ($m_j^+ = m_j^-$) which results in $\langle m_{ee} \rangle = 0$ for pure Dirac neutrinos. In the pseudo-Dirac scenario with non-zero Majorana masses and $m_L, m_R \ll m_D$, the cancelation is not exact and $\langle m_{ee} \rangle \neq 0$ but it is very small \cite{33}. Thus, it seems that the observation of a positive signal in the next generation $0\nu\beta\beta$ experiments, with sensitivities $\langle m_{ee} \rangle \sim 10 \text{meV}$, will rule out the small values of $m_L$ and $m_R$ in the mass matrix of neutrinos and therefore pseudo-Dirac scenario. However, two points should be considered. The first one is that the value of $\langle m_{ee} \rangle$ can still be significant if only one or two families of neutrinos have sterile partners. Contribution of each family to the value of $\langle m_{ee} \rangle$ depends on the corresponding mixing matrix element $U_{\alpha j}$. This means that, because of the small value of $U_{e3}$ ($\leq 0.041$), presence or absence of a sterile neutrino with a mass almost degenerate with $\nu_{3L}$ do not change the value of $\langle m_{ee} \rangle$ substantially; but the case with two sterile neutrinos with a masses degenerate with $\nu_{1L}$ and $\nu_{2L}$ leads to an effective mass $\langle m_{ee} \rangle$ much smaller than its value in the absence of sterile neutrinos. The second point is that the dominant contribution to $0\nu\beta\beta$ decay can come from new particles or physics beyond the SM, such as a $V + A$ interaction \cite{34}. It is shown in \cite{35} that a non-zero $0\nu\beta\beta$ decay rate generates small $m_L$ through radiative corrections, which results in pseudo-Dirac scenario for neutrino masses. Considering these points, it is not easy to draw a conclusion on pseudo-Dirac scenario from the results of the $0\nu\beta\beta$ experiments.

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2.2 Cosmic Neutrinos

Neutrinos arriving at neutrino telescopes from astrophysical sources travel distances of the order $L \sim 100$ Mpc. The flavor conversion probabilities over these large distances can be obtained by averaging out the oscillatory terms in Eq. (7). Two different scales of $\Delta m^2$ are involved in Eq. (7), one is the atmospheric, $\Delta m^2_{\text{atm}} \sim 10^{-3}$ eV$^2$, and solar, $\Delta m^2_{\text{sol}} \sim 10^{-5}$ eV$^2$, and the other is the very small $\Delta m^2$ between the mass eigenstates in each pair of active and sterile neutrinos. As it is shown in [36] the oscillatory terms depending on $\Delta m^2_{\text{atm}}$ and $\Delta m^2_{\text{sol}}$ should be completely averaged out over these large distances. The mass squared difference $\Delta m^2$ that can be probed by neutrinos with energy $E_\nu$ which propagate through distance $L$ is

$$\frac{\Delta m^2}{\text{eV}^2} = 10^{-16} \left( \frac{\text{Mpc}}{L} \right) \left( \frac{E_\nu}{\text{TeV}} \right).$$

(9)

Thus, even for $L$ as large as 10 Mpc oscillatory terms given by $\Delta m^2 \sim 10^{-17}$ eV$^2$ do not average out. After averaging out $\Delta m^2_{\text{atm}}$ and $\Delta m^2_{\text{sol}}$, Eq. (7) becomes

$$P_{\alpha\beta} = \sum_{j=1}^{3} |U_{\alpha j}|^2 |U_{\beta j}|^2 \cos^2 \left( \frac{\Delta m^2_j L}{4E_\nu} \right),$$

(10)

where $\Delta m^2_j \equiv (m^+_j)^2 - (m^-_j)^2$ is the mass squared difference in the $j$-th pair. Thus, if the initial flavor composition of neutrinos in the source is $w_e : w_\mu : w_\tau$, the flavor composition of the neutrino beam arriving at Earth will be $F_{\nu_e} : F_{\nu_\mu} : F_{\nu_\tau}$, where

$$F_{\nu_\alpha} = \sum_{\beta} w_\beta \sum_{j=1}^{3} |U_{\alpha j}|^2 |U_{\beta j}|^2 \cos^2 \left( \frac{\Delta m^2_j L}{4E_\nu} \right).$$

(11)

The average of the cosine factor for $\Delta m^2_L/4E_\nu \gg 1$ is $1/2$. Thus, if for all three pairs ($j = 1, 2, 3$) the condition $\Delta m^2_j L/4E_\nu \gg 1$ is satisfied, all three $F_{\nu_\alpha}$ in Eq. (11) are multiplied by $1/2$ such that the flavor ratios $F_{\nu_e} : F_{\nu_\mu} : F_{\nu_\tau}$ do not change with respect to the flavor ratios in the pure Dirac case $\Delta m^2_j = 0$. In this situation the only difference between the pure Dirac and pseudo-Dirac scenarios is that the number of neutrinos arriving at Earth is reduced by half in the pseudo-Dirac case. Thus, it is very hard to verify pseudo-Dirac scenario for distances or mass squared differences where $\Delta m^2_j L/4E_\nu \gg 1$ for $j = 1, 2, 3$. That is because the estimation of the overall normalization of neutrino flux needs knowledge about the details of the neutrino production mechanism in the source which is not well understood.
However, the pseudo-Dirac scenario of neutrinos can be tested in neutrino telescopes for the cases that only one or two sterile neutrinos exist ($N_s < 3$) or the distance of the source or the mass square differences are such that all the three oscillatory terms given by $\Delta m^2 L/4E_\nu$ do not average out. In these cases the average of one or two of the cosine factors in Eq. (11) are $1/2$ and the other cosine factors can be replaced by 1 (we do not consider the special situations where $\Delta m^2 L/4E_\nu \sim 1$). Thus, measuring the deviations of the flavor ratios in Eq. (11) from their standard values (in the absence of nearly degenerate sterile neutrinos) in neutrino telescopes can shed light on these cases. In the next section we discuss the details of the detection processes in the neutrino telescopes and the feasibility of identifying different neutrino flavors in these experiments. By taking into account the realistic measurable quantities in neutrino telescopes and the uncertainties in these measurements, we discuss to what extent it is possible to measure the flavor ratio of cosmic neutrinos and their deviations from the standard values.

3 Detection and Production Processes

In this section we briefly discuss neutrino flavor identification in the km$^3$ scale neutrino telescopes such as IceCube or its counterparts in the Mediterranean sea; KM3NET, NEMO, NESTOR and ANTARES. A detailed description of the flavor tagging efficiencies in a typical neutrino telescope can be found in [26, 38]. Here we summarize the main points relevant for the present analysis. Particularly, for the first time, we take into account different sources of uncertainties in the calculation of event rates and also a more realistic analysis of the detectable events, such as the contribution of $\nu_\tau$ ($\bar{\nu}_\tau$) to the $\mu$-tracks.

The flux of neutrinos arriving at Earth can come from a single luminous point source or as a diffuse flux from sum over different sources at different distances. The advantage of the point sources is that the short period of the burst and the direction of incoming neutrinos can be used to reduce the background events (especially when the source can be identified using a different method such as gamma photons for GRBs). Point sources with an intense neutrino flux detectable at km$^3$-scale neutrino telescopes can take place in the close-by galaxies located at a distance of $\lesssim 10$ Mpc and such a source of neutrinos yields about a few hundred neutrino events in IceCube [37].

Discriminating between different flavors of neutrinos is a great challenge for neutrino telescopes. Two types of events are completely distinguishable in the next generation of these experiments: $\mu$-track events and shower-like events. Charged Current (CC) and Neutral
Current (NC) interactions of different flavors can contribute to each of these events. \( \mu \)-track events, which are the Čerenkov light radiated by muons propagating through the volume of the detector, get contributions from two sources: i) \( \mu \) (\( \bar{\mu} \)) produced in the CC interaction of \( \nu_\mu \) (\( \bar{\nu}_\mu \)); ii) CC interaction of \( \nu_\tau \) (\( \bar{\nu}_\tau \)) which produce \( \tau \) (\( \bar{\tau} \)) leptons and the subsequent leptonic decay of tau leptons \( \tau \to \mu \bar{\nu}_\mu \nu_\mu \) \( \bar{\tau} \to \bar{\mu} \bar{\nu}_\mu \nu_\mu \) produce \( \mu \) (\( \bar{\mu} \)). Shower-like events have three sources: i) NC interactions of all the three flavors of neutrinos; ii) CC interactions of \( \nu_e \) and \( \bar{\nu}_e \); and iii) CC interaction of \( \nu_\tau \) (\( \bar{\nu}_\tau \)) which produce \( \tau \) (\( \bar{\tau} \)) leptons and their subsequent hadronic decays. The exact formulae for calculating the rate for each of these events can be found in Sect. 2 of [26].

The threshold energy of the detection of \( \mu \)-tracks and showers in experiments such as IceCube, respectively, is \( E_{\mu}^{th} \sim 100 \text{ GeV} \) and \( E_{\text{shower}}^{th} \sim 1 \text{ TeV} \) [21]. On the other hand, the mean free path of the neutrinos with energy \( E_\nu^{cut} \sim 100 \text{ TeV} \) becomes of the order of \( \sim 2R_\oplus \), the diameter of Earth. The exact values of the \( E_{\mu}^{th} \), \( E_{\text{shower}}^{th} \) and \( E_\nu^{cut} \) depend on the details of the experiments, such as the geometry of the photomultipliers in the volume of the detector and the direction of the incoming neutrinos; and a dedicated analysis can be done for each experiment. Here, in order to avoid considering the absorption of neutrinos in the Earth, we restrict the analysis to \( 100 \text{ GeV} < E_\nu < 100 \text{ TeV} \).

The realistic quantity that can be measured in neutrino telescopes is

$$R = \frac{\text{Number of Muon-track events}}{\text{Number of Shower-like events}}. \quad (12)$$

The value of \( R \) can be calculated if we know the initial flux and the flavor ratio of neutrinos at the source. The main mechanism of neutrino production at the astrophysical sources is the interaction of the accelerated proton by the ambient protons and photons. The decay of the secondary particles (\( \pi^\pm, K^\pm, D, \ldots \)) produced in these \( pp \) and \( p\gamma \) interactions generate neutrinos and muons. For example, the pion chain decays \( \pi^+ \to \mu^+ \nu_\mu \to (e^+ \nu_e \bar{\nu}_e) \nu_\mu \) and \( \pi^- \to \mu^- \bar{\nu}_\mu \to (e^- \bar{\nu}_e \nu_\mu) \bar{\nu}_\mu \) generate neutrinos with the flavor ratio \( (\nu_\mu + \bar{\nu}_\mu)/(\nu_e + \bar{\nu}_e) \simeq 2 \). The exact value of the flavor ratio \( w_\mu : w_\tau : w_\mu \) depends on the spectrum of the parent particles and the properties of the production medium. A class of models based on the Fermi acceleration mechanism for the particle in the source, predict a power-law spectrum for neutrinos

$$\frac{dF_\nu}{dE_\nu} = \mathcal{N}_{\nu_\beta} E_\nu^{-\alpha}, \quad (13)$$

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where $\alpha$ is the spectral index and $N_{\nu_\beta}$ is a normalization factor. Acceleration of particles through Fermi acceleration mechanism \[39\] results in $\alpha = 2$ for neutrino spectrum. However, non-linear effects change this value such that $\alpha$ can take any value in the interval $(1,3)$ \[40\]. It is shown in \[41\] that for the case of pion decay chain and assuming $\alpha = 2$, the initial flavor ratio is $w_e : w_\mu : w_\tau = 1 : 1.85 : 0$ (the difference with $1 : 2 : 0$ comes from the wrong polarization states of $\mu^\pm$ in the decay of $\pi^\pm$). Also the authors of \[42\] show that inclusion of other secondary particles (such as $K^\pm$) has a very little effect on this value. The $\mu^\pm$ generated in the decays of the secondary particles can substantially lose their energy before decay. In this case the neutrinos generated in the decay of muons do not contribute to the flux of neutrinos with $100 \text{ GeV} < E_\nu < 100 \text{ TeV}$ and the flavor ratio becomes $0 : 1 : 0$.

In the calculation of $R$ for different scenarios of neutrino production at source and propagation between source and Earth, the uncertainties in the input parameters should be considered. Uncertainties of the input parameters in the calculation of the $\mu$-track and shower-like event rates induce uncertainties in the value of $R$. Here we summarize these sources of uncertainties:

**Mixing Parameters** Flavors content of the neutrino beam arriving at Earth depends on the mixing angles $(\theta_{12}, \theta_{23}, \theta_{13})$ and Dirac CP-violating phase $\delta$ through the $|U_{\alpha j}|^2|U_{\beta j}|^2$ factors in Eq. (11). An update on the values of these parameters and uncertainties in each of them can be found in \[46\], which are also listed in Table 1.

**Spectral Index** As it is mentioned after Eq. (13), the spectral index $\alpha$ can take any value in the interval $(1,3)$. It is shown in \[38\] that IceCube can measure the spectral index $\alpha$ with 10% precision (assuming $E_\nu^2dF_\nu/dE_\nu = 0.25 \text{ GeV cm}^{-2} \text{ sr}^{-1} \text{ yr}^{-1}$ and after one year of data-taking).

$\bar{\nu}_\alpha/\nu_\alpha$ Ratio Let us define $\lambda_\beta \equiv N_{\bar{\nu}_\beta}/N_{\nu_\beta}$ for $\beta = e, \mu$. It is obvious that $\lambda_\mu = 1$ in the pion decay chain, but the value of $\lambda_e$ depends on the ratio of $\pi^+ / \pi^-$ production and can take any value in the interval $(0,1)$. To best of our knowledge, in the energy range we are interested here $(100 \text{ GeV} < E_\nu < 100 \text{ TeV})$, there is not any proposed or established method to measure $\lambda_e$.

**Neutrino-Nucleon Cross Section** The rate of the $\mu$-track and shower-like events depends on the CC and NC neutrino-nucleon cross sections $\sigma^{CC}_{\nu N}$ and $\sigma^{NC}_{\nu N}$. The current uncertainty in these cross sections is $\sim 3\%$. However, the uncertainties in $\sigma^{CC}_{\nu N}$ and $\sigma^{NC}_{\nu N}$ have a very small effect on $R$ because of the cancelation between numerator and denominator of Eq. (12).
Table 1: Relevant parameters in the calculation of $R$ with their uncertainties. The current uncertainty column represents the $3\sigma$ uncertainty interval for mixing angles [46]. The future uncertainty column shows the precisions that can be achieved in the forthcoming neutrino oscillation experiments, as described in the corresponding references. For $\theta_{13}$ it is assumed that its value is close to the current upper limit: $\sin^2 \theta_{13} = 0.03$.

| Parameter     | Best-fit | Current Allowed Range    | Future Uncertainty |
|---------------|----------|--------------------------|--------------------|
| $\sin^2 \theta_{12}$ | 0.304    | $0.25 - 0.37 \ (3\sigma \ C.L.)$ | 6 % (Ref. [43]) |
| $\sin^2 \theta_{23}$ | 0.50     | $0.36 - 0.67 \ (3\sigma \ C.L.)$ | 6 % (Ref. [44]) |
| $\sin^2 \theta_{13}$ | 0.01     | $\leq 0.056 \ (3\sigma \ C.L.)$ | 5 % (Ref. [45]) |
| $\delta$      | $-\pi, 2\pi$ | $[0,2\pi]$ | $[0,2\pi]$ |
| $\alpha$      | $-\pi, 2\pi$ | $10 \%$ | $10 \%$ |
| $\lambda_e \equiv N_{\bar{\nu}_e}/N_{\nu_e}$ | $-\pi, 2\pi$ | $[0,1]$ | $[0,1]$ |

In Table 1 we have listed the relevant parameters in the calculation of $R$ and their uncertainty intervals. The third and fourth columns respectively correspond to the current and future uncertainties in these parameters. In addition to the uncertainties in the input parameters listed above, measurement of $R$ is also done with limited precision and has an uncertainty. It is shown in [38] that by assuming that the flux of neutrinos is $E^2 dF_\nu/dE_\nu = 0.25 \text{ GeV cm}^{-2} \text{ sr}^{-1} \text{ yr}^{-1}$, the ratio $R$ can be measured with $\sim 7 \%$ precision after a couple of years of data-taking.

Now, by taking into account the uncertainties mentioned in this section, the question is to what extent it is possible to measure the deviations of the flavor composition $F_{\nu_e} : F_{\nu_\mu} : F_{\nu_\tau}$ from its value in the absence of pseudo-Dirac sterile neutrinos. In the next section we will discuss the prospect of neutrino telescopes to measure these deviations.

## 4 Results and Discussion

In this section we consider two sources with different initial flavor compositions: the pion and stopped-muon sources with flavor compositions $1 : 1.85 : 0$ and $0 : 1 : 0$, respectively. The value of $R$ in Eq. (12) in the absence of almost degenerate sterile neutrinos and assuming the best-fit values for the mixing angles, $\delta = 0$ and $\lambda_e = 1$, will be denoted by $R_\pi$ and $R_\mu$, for pion and stopped-muon sources respectively. We assume the power-law spectrum Eq. (13)
for the neutrino production, with the spectral index \( \alpha = 2 \). At the end, we investigate the robustness of the results with respect to the deviation of initial flavor ratios from the assumed values 1 : 1.85 : 0 and 0 : 1 : 0.

Fig. (1) shows \( R \) versus \( \alpha \) for pion source, where \( \bar{R}_\pi = 2.50 \). The hatched area between the two red curves corresponds to the values that \( R \) can take if the input parameters vary in the uncertainty intervals. The inputs have been varied in the current uncertainty intervals shown in the third column of Table 1. The two horizontal and vertical pairs of dashed-lines show the 7% and 10% precision intervals in the measurement of \( R \) and \( \alpha \), respectively. The rectangle created from the intersection of these dashed-lines corresponds to the region of parameter space \((R, \alpha)\) where can be limited by the measurements in neutrino telescopes. Thus, the points inside the rectangle represent the values of \( R \) for the case of no sterile neutrino and consistent with 7% (10%) precision in the measurement of \( R \) (\( \alpha \)). The green dashed-curves in Fig. (1) show the values of \( R \) in the presence of sterile neutrinos. For example, in Fig. (1-a) the hatched region between the green dashed-curves corresponds to the case when the sterile neutrino mass is almost degenerate with the mass of the active neutrino \( \nu_{1L} \) such that the oscillatory term depending on \( \Delta m^2_{13} \) in Eq. (7) can be averaged to \( 1/2 \) and the flavor conversion probabilities become

\[
P_{\alpha \beta} = \frac{1}{2} |U_{\alpha 1}|^2 |U_{\beta 1}|^2 + |U_{\alpha 2}|^2 |U_{\beta 2}|^2 + |U_{\alpha 3}|^2 |U_{\beta 3}|^2. \tag{14}
\]

As can be seen, in all six parts of the Fig. (1) the two hatched areas overlap. The overlapping of the hatched areas inside the dashed-line rectangles means that the presence of the sterile neutrinos cannot be ruled out, even if the measurement of \( R \) gives a value inside this rectangles. However, if the measurement of \( R \) gives a value much different than \( \bar{R}_\pi \), the existence of sterile neutrinos is favored. For example, the value \( R = 2.1 \) is not possible in the no sterile case and this value is favored by the scenarios depicted in Fig. (1-b,c,f).

Fig. (2) is the same as Fig. (1) with the exception that the input parameters have been varied in the future uncertainty intervals shown in the forth column of Table 1. We assumed (6%, 6%, 5%) uncertainties for \((\sin^2 \theta_{12}, \sin^2 \theta_{23}, \sin^2 \theta_{13})\), which can be achieved in the forthcoming neutrino oscillation experiments [43, 44, 45]. Also, we assumed a large value for the 13-mixing angle \( \sin^2 \theta_{13} = 0.03 \) which is near its present upper bound. The uncertainties of \( \delta, \lambda_e, \alpha \) and \( R \) are the same as in Fig. (1). As can be seen from Fig. (2), reducing the uncertainties of mixing angles results in a separation between the two hatched areas where there was an overlap in Fig. (1). For example, in Fig. (2-f), the regions corresponding to no sterile neutrino case and the case with averaged second and third pairs are completely separated. This separation means that if the measurement of \( R \) gives a value inside the
Figure 1: The dependence of $R$, ratio of $\mu$-tracks to shower-like events, on $\alpha$ for the pion source with the initial flavor ratio 1 : 1.85 : 0 and power-law spectrum with the spectral index $\alpha = 2$. Assuming the best-fit values for the mixing angles, $\delta = 0$ and $\lambda_e = 1$, the value of $R$ is $\bar{R}_\pi = 2.50$. In each figure the red curves represent the case with no sterile neutrino and the green dashed-curves correspond to the cases with average on pairs mentioned in the legends. The hatched areas show the values that $R$ can take when the input parameters vary in the current uncertainty intervals in Table I. The two vertical and horizontal dashed-lines show the 10% and 7% precisions in the measurements of $\alpha$ and $R$, respectively.
Figure 2: The same as Fig. [II] except that the input parameters have been varied in the future uncertainty intervals of Table [II]. Particularly, we assumed 5% uncertainty interval for the 13-mixing angle with the central value near the present upper bound: $\sin^2 \theta_{13} = 0.03$. 

$$
\begin{align*}
\sin^2 \theta_{13} &= 0.03.
\end{align*}
$$
rectangle in Fig. (2f), the existence of sterile neutrinos (for the case of average on second and third pairs) will be ruled out. On the other hand, a value of $R$ outside the rectangle can be interpreted as a signal for the existence of sterile neutrinos. However, recognizing which case is consistent with the measurement of $R$ should be done with care. To illuminate this point, let us assume that IceCube has measured $R = 3.0$. According to the diagrams in Fig. (2) this value of $R$ is consistent with three cases: i) average on first pair; ii) average on first and second pairs; and iii) average on first and third pairs. Notice that with the current uncertainties of the mixing angles (depicted in Fig. (1)) it is not possible to conclude that $R = 3.0$ is a signal of sterile neutrinos.

Figs. (3) and (4) show the dependence of $R$ on $\alpha$ for the stopped-muon source; where $\bar{R}_\mu = 3.07$. The input parameters have been varied in the current uncertainty intervals in Fig. (3). Comparing Fig. (3) with Fig. (1), it is obvious that the hatched areas are wider for the stopped-muon source, which means that the recognition of sterile neutrinos is harder for this kind of sources. In contrast to the pion source, reducing the uncertainty of input parameters to the future uncertainty intervals do not lead to a separation between the hatched areas in Fig. (4). However, some cases can marginally be discriminated such as the case shown in Fig. (4-d).

In drawing Figs. (1,2) and Figs. (3,4) we have assumed initial flavor ratios 1 : 1.85 : 0 and 0 : 1 : 0, respectively. However, it should be noticed that the initial flavor ratio can deviates from these values due to the interplay of different mechanisms in neutrino production or effects that have not been considered in the calculation of these values. For example, in the pion source, a part of the produced muons in the decays of $\pi^\pm$ (not all of them) can lose their energy before decay such that the initial flavor ratio of neutrinos takes a value between the two extreme cases of pion and stopped-muon sources. The question that arises here is that to what extent the results of this section (for example the separation between hatched areas in Fig. (2f)) are robust against the deviations of the initial flavor ratio $w_e : w_\mu : w_\tau$ from the assumed values. To answer this question we consider particularly the case of Fig. (2f) where the hatched areas are completely separated. Also we use the following parametrization of the initial flavor ratio:

$$w_e : w_\mu : w_\tau = n : 1 : 0.$$  \hspace{1cm} (15)

The $w_\tau = 0$ in the above parametrization comes from the fact that the number of prompt $\nu_\tau$ (from the decays $D_s \to \tau\nu_\tau, \ldots$) is very small and can be neglected. In Fig. (5-a) we

---

3Conversely, by assuming standard propagation of neutrinos between the source and detector, measurements of flavor ratios at Earth can be used to set bounds on the initial flavor ratios at the source [26, 47].

4A slightly different parametrization $w_e : w_\mu : w_\tau = 1 : n : 0$ has been used in [47].
Figure 3: The dependence of $R$, ratio of $\mu$-tracks to shower-like events, on $\alpha$ for the stopped-muon source with the initial flavor ratio $0 : 1 : 0$ and power-law spectrum with the spectral index $\alpha = 2$. Assuming the best-fit values for the mixing angles and $\delta = 0$, the value of $R$ is $\bar{R}_\mu = 3.07$. In each figure the red curves represent the case with no sterile neutrino and the green dashed-curves correspond to the cases with average on pairs mentioned in the legends. The hatched areas show the values that $R$ can take when the input parameters vary in the current uncertainty intervals in Table I. The two vertical and horizontal dashed-lines show the 10 % and 7 % precisions in the measurements of $\alpha$ and $R$, respectively.
Figure 4: The same as Fig. (3) except that the input parameters have been varied in the future uncertainty intervals of Table II. Particularly, we assumed 5% uncertainty interval for the 13-mixing angle with the central value near the present upper bound: $\sin^2 \theta_{13} = 0.03$. 
Figure 5: The hatched areas in (a) show the dependence of $R$ on $\alpha$ for the case of no sterile neutrinos (red) and the case with average on the 2nd and 3rd pairs of almost degenerate neutrinos (green). The initial flavor ratio $w_e : w_\mu : 0$ has been varied in the region depicted in (b). In drawing this figure we have assumed the best-fit values for the mixing parameters and varied $\lambda_e \in (0.8, 1)$.

have varied $n$ in Eq. (15) such that the separated regions in Fig. (2f) begin to overlap. In drawing Fig. (5a) we have varied $n \in (0.25, 0.75)$. The initial flavor ratios $w_e : w_\mu : 0$ corresponding to this interval are shown in Fig. (5b). As can be seen, for nearly large deviations of the initial flavor ratio from $1 : 1.85 : 0$, the hatched areas remain separated, which means that the lack of knowledge about the exact value of the initial flavor ratio of neutrinos at the source do not affect substantially the capability of neutrino telescopes in probing pseudo-Dirac neutrino scenario.

5 Conclusion

The new generation of km$^3$ neutrino telescopes opens a new window to study cosmos via the detection of high energy neutrinos predicted to be emitted from the astrophysical objects. Because of the extremely large distance of the sources ($\gtrsim$ Mpc) the flavor oscillation is sensitive to the very tiny mass squared differences $10^{-18}$ eV$^2 \lesssim \Delta m^2 \lesssim 10^{-12}$ eV$^2$. The existence of sterile neutrinos with masses almost degenerate with the active ones such that the mass squared differences between the active and sterile neutrinos lie in the above region is hypothesized in many models (the so-called pseudo-Dirac scenario.) We have studied the effect of these sterile neutrinos on the flux of cosmic neutrinos and discussed the capability of IceCube to verify the existence of them. In the analysis we have considered different cases
corresponding to the existence of sterile neutrinos degenerate in mass with different $\nu_{iL}$ such that the oscillatory terms driven by these mass squared differences can be averaged out.

The detection power of IceCube (and other proposed km$^3$ neutrino telescopes), in the range of neutrino energies $100 \text{ GeV} < E_\nu < 100 \text{ TeV}$, is limited to distinguishing two types of events: $\mu$-track and shower-like events. We have considered the ratio of these events, $R$ in Eq. (12), as the realistic quantity that can be measured in the IceCube. We have studied the possibility of using the measured value of $R$ as a discriminator between the pseudo-Dirac scenario and the scenario with no sterile neutrinos. We have considered various sources of uncertainties in our analysis. One part of these uncertainties comes from the imprecision of the neutrino oscillation experiments in the determination of mixing parameters. For this part we considered two sets of uncertainty intervals for the mixing parameters: i) the current uncertainty intervals from the performed experiments; and ii) the future uncertainty intervals that will be achieved in the forthcoming experiments. Both of these sets have been depicted in Table 1. Among the mixing parameters, the measurable quantity $R$ is most sensitive to the exact value of the mixing angle $\theta_{23}$. For the uncertainty of this parameter we have used the 6% precision on $\sin^2 \theta_{23}$ [44] depicted in Table 1. The other part of uncertainties comes from the not completely known mechanism of neutrino production in the sources. Many models of neutrino production predict a power-law spectrum for the cosmic neutrinos. However, the value of the spectral index in the power-law spectrum depends on the details of the neutrino production mechanism and can take values in the interval $(1, 3)$. Also, the ratio of the number of electron anti-neutrinos to the number of electron neutrinos is not known. We have taken into account all these uncertainties in our analysis.

The analysis has been done for two different initial flavor composition $w_\nu : w_\mu : w_\tau$ at the source: i) pion source $1 : 1.85 : 0$; and ii) stopped-muon source $0 : 1 : 0$. It has been shown that with a higher precision of the mixing angles achievable in the forthcoming oscillation experiments, in certain cases it is possible to demonstrate or rule out the existence of sterile neutrinos hypothesized in the pseudo-Dirac scenario. In these cases the regions corresponding to existence and absence of sterile neutrinos are well-separated in the parameter space for neutrinos coming from pion sources. For example, it is very promising to probe the case of two sterile neutrinos with masses almost degenerate with $\nu_{2L}$ and $\nu_{3L}$ (see Fig. 2-f)). For neutrinos coming from stopped-muon sources, these regions mostly overlap such that their discrimination cannot be done without ambiguity. Also, the robustness of these results has been tested against the uncertainties in the initial flavor ratio of neutrinos at the source. It has been shown that for reasonably large variations of the initial flavor ratio around the expected value for pion source, $1 : 1.85 : 0$ (see Fig. 5 for clarification), the regions
corresponding to the existence and the absence of sterile neutrinos remain separated.

Acknowledgement

The author is grateful to Y. Farzan for useful discussions and for her valuable comments on the manuscript. Also, I would like to thank H. Firouzjahi for the careful reading of the manuscript and valuable comments. I would like to thank “Bonyad-e Melli-e Nokhbegan” for partial financial support.

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