Probing the micromechanics of a multi-contact interface at the onset of frictional sliding

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Abstract. Digital Image Correlation is used to study the micromechanics of a multi-contact interface formed between a rough elastomer and a smooth glass surface. The in-plane elastomer deformation is monitored during the incipient sliding regime, i.e. the transition between static and sliding contact. As the shear load is increased, an annular slip region, in coexistence with a central stick region, is found to progressively invade the contact. From the interfacial displacement field, the tangential stress field can be further computed using a numerical inversion procedure. These local mechanical measurements are found to be correctly captured by Cattaneo and Mindlin (CM)'s model. However, close comparison reveals significant discrepancies in both the displacement and stress fields that reflect the oversimplifying hypothesis underlying CM's scenario. In particular, our optical measurements allow us to exhibit an elastoplastic-like friction constitutive equation that differs from the rigid-plastic behavior assumed in CM's model. This local constitutive law, which involves a roughness-related length scale, is consistent with the model of Bureau \textit{et al.} (Proc. R. Soc. London, Ser. A \textbf{459}, 2787 (2003)) derived for homogeneously loaded macroscopic multi-contact interfaces, thus extending its validity to mesoscopic scales.

1 Introduction

The transition from static to sliding friction is a crucial process in various fields, ranging from contact mechanics \cite{1}, earthquakes dynamics \cite{2} to human/humanoid object grasping \cite{3}. In the classical Amontons-Coulomb’s framework, when two solids are brought in contact under a normal load $P$ and subjected to a shear force $Q$, no relative motion occurs until $Q$ exceeds some threshold value $Q_s = \mu_s P$, where $\mu_s$ is called the static-friction coefficient. However, in most real situations, the transition from static to dynamic friction does not follow this ideal simple scenario. As soon as $Q > 0$, partial slippage generally sets in owing to the large stress heterogeneity within the contact zone, which depends on the geometry of the objects in contact as well as on the loading conditions. Understanding this incipient sliding regime thus requires to gain access to the interfacial micromechanics within the contact zone.

In the past ten years, several experimental groups developed new optical methods to obtain spatially resolved mechanical measurements \cite{4-8}, which triggered intense subsequent theoretical and numerical investigations \cite{9-15}. Fineberg and collaborators studied the onset of sliding of a multi-contact interface, in a plane-plane contact configuration, submitted to an adiabatic tangential loading \cite{6,7}. Using fast imaging of the interface illuminated with an evanescent laser sheet, they were able to measure local changes in the real area of contact. This simple optical measurement allowed them to reveal that, prior to macroscopic sliding, a series of dynamical rupture fronts travelled along the interface. In their experiments however, the contact was one-dimensional. Chateauminois and collaborators have considered more realistic, fully two-dimensional, contacts \cite{16,17}. By patterning a smooth elastomer’s surface with a regular grid of micro-markers, they were able to monitor, using Particle Image Velocimetry techniques, the entire $2D$ displacement field at the interface. They applied this procedure to a smooth sphere loading against a smooth flat elastomer block in a torsional configuration \cite{17}. The authors showed that the transition from static to kinetic friction involves an annular microslip front propagating from the outer edge of the circular contact towards the center. Using an inversion procedure, they computed the interfacial shear stress field from the measured displacements field. We emphasize that in their experiment, macroscopic adhesion was important due to the smoothness of the surfaces in contact. In most practical situations however, interfaces are rough at microscopic length scales, giving rise to a multi-contact frictional interface at which macroscopic adhesive effects are strongly reduced.

In this paper, we expand on this latter two-dimensional approach. We report on micromechanical measurements...
at the onset of sliding between a smooth sphere and a flat elastomer block whose surface is microscopically rough. There are several important differences with respect to [17]. First, the interface is of the multi-contact type. Second, the loading is linear instead of torsional. Third, the elastomer block whose surface is microscopically rough.

2.1 Experimental setup

Figure 1 shows a sketch of the experimental setup. It consists of a planoconvex glass lens (optical grade, Thorlabs LA1301, BK7, radius of curvature $R = 128.8$ mm) glued onto a lens holder and rigidly attached to an optical table. The lens surface is in frictional contact against a thick elastomer block maintained by van der Waals adhesion to a supporting glass plate. The latter is connected via a set of two orthogonal cantilevers to a translation stage that can be driven at constant velocity $V$ using a DC actuator (LTA-HL, Newport Inc.). Two capacitive position sensors (respectively, MCC-20 and MCC-10, Fogale Nanotech), each facing the mobile part of one cantilever, allow one to measure both $P$ and $Q$, respectively the normal and tangential (shear) force, with a 1 mN resolution in the range [0–2 N]. Both force signals are digitized and recorded at a sampling rate of 1 kHz using a NI-PCI6251 DAQ board. Imaging of the contact is done by illuminating the interface through the transparent elastomer block with a white LED diffusive array and a long-working distance Navitar objective. Images of the interface are recorded with a CCD camera (Redlake ES2020M, 1600 × 1200 pixels$^2$, 8 bits, 24 frames/s at maximum). Synchronization between the camera and the DAQ acquisition device is ensured by having the DAQ board trigger the camera.

2.2 Sample preparation and characterization

The elastomer block (50 × 50 mm, thickness 15 mm) is made of crosslinked PolyDiMethylSiloxane (PDMS Sylgard 184, Dow Corning). It is obtained by mixing in a 10:1 stoichiometric ratio a PDMS melt and a cross-linker agent in a rectangular mold. The mixture is cured for 48 hours in an oven at 70 °C. The free surface of the elastomer block is rendered rough by mechanically abrading the lid’s upper surface with a silicon carbide powder solution of typical grain size 17 μm. After careful demoulding, the surface roughness of the PDMS sample is characterized with an optical profilometer (M3D, Fogale Nanotech). Its height power spectrum [18] is found to decay as a power law $C(q)$ with $q$ the norm of the wave vector, as defined in [18], for the PDMS block used in these experiments. The thin red horizontal line is a guide for the eye, while the oblique one is a power law fit of exponent $\sim −2.67$. Such PDMS elastomers have been reported to have a bulk elastic Young’s modulus $E$ in the range [2–4 MPa] [16,19–21] (depending on the preparation protocol) and a Poisson’s ratio $\nu$ close to 0.5 [22]. For the sample used in these experiments, a JKR test is performed between its smooth back side and the bare glass lens, yielding a Young’s modulus $E = 3.43 \pm 0.05$ MPa. For the friction experiments, the glass lens surface is passivated in a PerfluoroDecylTriCloroSilane saturated vapor phase in order to reduce the macroscopic PDMS/glass adhesion and to minimize heterogeneities in the interfacial properties. Prior to each experiment, both glass and PDMS surfaces are cleaned with ethanol and dried with filtered air.

2.3 Contact imaging and displacement fields measurements

2.3.1 Contact imaging

Figure 2a shows a typical image of the elastomer/glass interface under normal load. This image is obtained in
transmission geometry by illuminating the sample with a diffuse white light. The interface appears spatially heterogeneous as a result of the diffusive nature of the rough layer. In the contact region, additional bright spots are present corresponding to the micro-junctions that favorably transmit light at the glass-PDMS interface. At the chosen magnification, images have a field of view of about 11.2 × 8.4 mm (with a pixel size of ~7.04 μm). The contact region is difficult to visualize from the raw image (fig. 2a) but can be clearly identified in fig. 2b by subtracting a reference background image recorded prior to the contact region is difficult to visualize from the raw image (fig. 2a) but can be clearly identified in fig. 2b by subtracting a reference background image recorded prior to the contact. This operation allows one to reveal hundreds of contact-induced micro-junctions contained within a circular (apparent) contact region (fig. 2b for P = 0.1 N). These characteristic features of the frictional joint remain true for the explored range of normal load P used in these experiments.

Taking advantage of the axial symmetry of the contact, the apparent contact radius is computed in the following way. Centers of the apparent contact radius r are first determined using a standard center of symmetry search algorithm. For each load P, the image intensity I(x, y) is then averaged azimuthally over the angle θ (fig. 3a)

\[ r = \frac{P}{\pi E R} (1 - \nu)^{1/2} \]

The apparent radius of contact a is then estimated by taking the minimum radius such that I(r) = 0. Figure 3b shows the resulting contact radius a and its load dependence along with Hertz contact radius a_H derived for E = 3.43 MPa. As clearly seen, the estimated radius a_H is systematically larger than a_H. This deviation can be accounted for by taking into account the multi-contact nature of the interface. As established by Greenwood and Tripp [23,24], the surface roughness extends the apparent contact region by a quantity of the order of \( \sqrt{R} \) with respect to the smooth configuration. As shown in fig. 3b, such a correction to Hertz’s contact radii does allow one to recover the measured apparent contact radii a.

2.3.2 Displacement field measurements

Interfacial displacement fields were computed from snapshots acquired at 8 frames/s (Δt = 0.125 s), and 4 frames/s for the highest P (Δt = 0.25 s), using a Digital Image Correlation technique (DIC) (see e.g. [25] and references therein). This method consists in finding, for a given sub-image centered at position (x, y) in a reference frame, the displacement \((u_x, u_y)\) that provides the maximum intensity correlation with a subsequent (deformed) image (figs. 4a and b). A 2D correlation function (fig. 4c) was computed using a direct calculation. Sub-pixel resolution was achieved by fitting the correlation function with a 2D Gaussian surface using the pixel of maximum correlation and its 8 nearest neighbors. The error on the displacements was evaluated using a series of images of the surface of the elastomer block, not in contact and uniformly displaced at constant velocity along the x-direction. DIC was performed between an image at \( t = 0 \) and images at increasing times \( t \), allowing one to extract the displacements \( u_x \) between ~0.14 pixels and ~14 pixels. The error was then taken as the standard deviation of the \( u_x - V t \) displacements distribution. This error was found to decrease with the box size \( \lambda \), in the range [10–100] pixels (fig. 4d).

Optimal \( \lambda \) was chosen based on the best compromise between spatial resolution and displacement measurement accuracy. For the current experiments, the smallest apparent contact radius is ~217 pixels long. In order to extract at least 10 independent displacement measurements along the contact, the displacement fields were computed...
C tomer block was driven at a prescribed velocity. We performed a series of 6 experiments in which the elas-

3 Force and displacement fields

We performed a series of 6 experiments in which the elastomer block was driven at a prescribed velocity $V = 5 \mu m/s$, under constant normal force $P$ in the range $[0.1–1.4] N$. For all runs, $P$ was found to vary by less than 1% over the duration of the experiment and the apparent contact zone remains circular. The velocity $V$ was chosen low enough for visco-elastic interfacial dissipation to be negligible [26]. Since the normal loading of the contact produces a significant shear force due to the small finite compliance effect, we independently measured the force threshold $Q_s$, at which macroscopic sliding sets on from the sole global force measurements. $Q_s$ was thus determined using the measured displacement fields at the center of the contact as detailed further down.

Displacement fields $u_x$ and $u_y$ at time $t$ were determined by correlating images at time $t$ with a reference image at $t = 0$ prior to any tangential loading using the algorithm described earlier. Since the lens holder is not infinitely rigid, any applied shear force is expected to induce minute rigid-body displacement of the lens that needs to be subtracted from the measured displacement $u_x$. To quantify this compliance effect, we independently measured the displacement of the lens holder while tangentially loading the elastomer block under controlled shear force using the same correlation technique. The solid lens displacement was found to vary linearly with $Q$, yielding a shear stiffness $\approx 0.68 \mu m N^{-1}$. Actual interfacial displacements along the $x$-direction were then corrected for this finite compliance effect.

Figure 6 shows the typical $u_x$ and $u_y$ displacement fields at $P = 0.5 N$ for all 6 positions in fig. 5. As soon as $Q$ increases, the displacement in the outer region of the apparent contact increases. In contrast, the measured displacement remains essentially null within a central circular region. As $Q$ increases, the radius $c$ of this stick region decreases and eventually vanishes, marking the onset of the macroscopic sliding phase. Figure 7 shows the time evolution of the displacement $u_x(r)$, averaged over the azimuthal angle $\theta$, for different radii $r = \{0, 0.5a, 0.75a, a\}$.

In practice, the actual velocity was slightly smaller than the prescribed velocity, decreasing monotonically from $4.9 \mu m/s$ at $P = 0.1 N$ to $4.76 \mu m/s$ at $P = 1.4 N$. 

load $P = \{0.1, 0.3, 0.5, 0.7, 1.0, 1.4\} N$. Each curve exhibits two distinct phases: an initial quasi-linear loading associated with the incipient sliding regime, followed by a plateau associated with a macroscopic steady sliding regime. The transition between these two regimes involves a monotonous decrease of the slope. In particular, we do not observe any static-friction peak in the loading curve, which hampers a direct determination of the force threshold $Q_s$, at which macroscopic sliding sets on from the sole global force measurements. $Q_s$ was thus determined using the measured displacement fields at the center of the contact as detailed further down.
Fig. 6. (Color online) Snapshots of the 2D displacement fields $u_x$ and $u_y$ at $P = 0.5$ N for the loading experiment of fig. 5 taken at instants labeled 1 to 6. On all displacement fields, the red dashed circle delimits the initial apparent contact area of radius $a$ as determined in sect. 2.3.1, without any shear applied. Checks were done that upon shearing, $a$ does not change. The yellow curves (respectively black curve) on snapshots indexed 1 to 5 (respectively 6) are cuts of the $u_x$ 2D displacements fields at $y = 0$ and are meant as visual guides. (a) $u_x$ displacement fields. (b) $u_y$ displacement fields.

Fig. 7. (Color online) Angularly averaged radial displacement $u_x(r)$, with $r = 0$ being the center of the contact, for values of $r = 0, 0.5a, 0.75a, a$ at a normal load $P = 0.5$ N. The ratio $\frac{\Delta u_x}{\Delta t}$ is $\sim 4.81 \mu$m/s. The horizontal black dotted line is the $y = 0$ axis. The dashed dotted line is a linear fit of $u_x(r = 0)$ at long times. The intersection of both lines, shown with the vertical black arrow, was arbitrarily chosen as the definition for the onset time of macroscopic sliding.

4 Tangential stress fields

Tangential stress fields at the contacting interface were derived using a Green’s tensor inversion procedure as described in [16]. For a semi-infinite elastic medium, the Green’s tensor characterizes the displacements at the interface induced by a point force applied at the free surface [30]. In the limit of a semi-infinite and incompressible elastic body, two assumptions which are well suited to our experiments, the lateral and vertical displacements are decoupled, allowing one to express the lateral displacements as a function of the lateral shear stresses only [20]. For a point loading $(Q_x, Q_y)$, the surface displacements $u_x$ and $u_y$...
and $\mu_d$ (upper black disks) versus $P$. $\mu_s$ is defined as the ratio $Q_s/P$, where $Q_s$ (shown in the inset) has been determined as described earlier and schematically shown in fig. 7. $\mu_d$ is taken as the mean of $Q/P$ for all $Q \geq Q_s$. Minimum and maximum values of $\mu_s$ over this range are shown with the black vertical segments surrounding each point.

$u_x$ is thus given, respectively, by

$$u_x = G_{xx}Q_x + G_{xy}Q_y,$$

$$u_y = G_{yx}Q_x + G_{yy}Q_y,$$

with the components of the Green’s tensor $G$ given by

$$G_{xx} = \frac{3}{4\pi E} \left( \frac{1}{r} + \frac{x^2}{r^3} \right),$$

$$G_{xy} = G_{yx} = \frac{3}{4\pi E} \left( \frac{xy}{r^3} \right),$$

$$G_{yy} = \frac{3}{4\pi E} \left( \frac{1}{r} + \frac{y^2}{r^3} \right).$$

For an extended contact, $u_x$ and $u_y$ are obtained by convolving $G$ with the shear stress at the interface $\sigma$ and can be formally written as

$$u_i = G_{ij} * \sigma_{jz},$$

where subscripts $i,j$ stand for $x$ or $y$. The stress fields $\sigma_{xz}$ and $\sigma_{yz}$ can then be obtained by deconvolution. This was done as in [16] using a classic iterative Van-Cittert algorithm. The stress at step $n + 1$ is obtained by adding to the one at time $n$ a corrective term proportional to the difference between the experimental displacement and the convolved one obtained using eq. (3). Convergence was considered to be attained when the rms difference between the calculated and the measured displacements was less than the displacement resolution, i.e., 0.033 pixels.

$\mu_s$ in practice, an initial guess at step $n = 0$ is taken as proportional to the experimental displacements. Such an iterative deconvolution procedure can very rapidly lead to numerical divergence if not slowed down. Empirically, only 5% of the difference in displacements is thus added to the stress at step $n$, enabling convergence in typically hundreds of iterations.

The inversion procedure provides shear stress fields in units of Young’s modulus $E$ (see eqs. (2) and (3)). The value of $E$ can then be directly fitted from the comparison between the shear force signal $Q(t)$, obtained through spatial integration of the calculated shear stress over the contact, and the actual measured force signal. As shown in fig. 9 for $P = 0.5$ N and $P = 0.7$ N, the match between both signals is very satisfactory, which validates the inversion method. The extracted Young’s modulus shows a weak though systematic dependence with the load (see inset) which is presumably due to the effect of finite thickness of the PDMS block as discussed in [31]. However, except for the two extreme load values, it remains within the error bars of $E$ as independently measured with the JKR test. Note that in the determination of the stress fields, we have used the fitted values for $E$ rather than the JKR value.

Figure 10 displays the shear stress fields $\sigma_{xz}$ computed at successive moments (indicated by the red dashed circle) during the incipient sliding phase. As expected, in all 6 configurations, the shear stress vanishes outside the apparent contact zone whose border is indicated by a red dashed circle. Within the contact, $\sigma_{xz}$ is radially symmetric with respect to the center of the contact. Once macroscopic sliding has initiated (position 6), $\sigma_{xz}$ is maximum at the center of the contact and decreases continuously towards the edge of the contact. During the transient loading, however, the shear stress exhibits a local minimum at the center of the contact.

5 Comparison with models predictions

These precise local mechanical measurements are directly amenable to comparison with existing theoretical models of incipient sliding. The first model, for a non-adhesive elastic sphere-on-plane contact, was derived independently by Cattaneo and Mindlin (CM) [32,33]. Since
then, this classic model has been refined and extended in various ways (see e.g. [1,34,35] and references therein), for instance by introducing macroscopic adhesion [36,37] or elasto-plasticity of the materials [38]. Since i) adhesive forces were found to be negligible in our experiments (no measurable pull-off force upon retraction of the contact) and ii) PDMS was used well within its linear elastic limit\(^6\), our measurements were compared to CM’s model.

CM’s calculations assume that 1) both surfaces are smooth, 2) the pressure distribution \(\sigma_{zz}\) within the contact is unchanged upon shearing and given by Hertz contact theory, and 3) Amontons-Coulomb’s law of friction is valid locally at any position within the contact, i.e. slip occurs wherever the shear stress \(\sigma_{xz}\) reaches \(\mu \sigma_{zz}\), \(\mu\) being the macroscopic friction coefficient\(^7\). CM’s model predicts the coexistence of an inner adhesive circular region of radius \(c\), which decreases with \(Q\) according to

\[
c = a_H \left(1 - \frac{Q}{\mu P}\right)^{1/3},
\]

\(^6\) Strains obtained by deriving the displacement fields do not exceed 3\%. This is well within the linearly elastic regime of the material [39].

\(^7\) CM’s model postulates the existence of a single, stress-independent friction coefficient, i.e. static- and dynamic-friction coefficients are equal, and thus predicts no static overshoot in the \(Q\) curve.

Figure 10. (Color online) Snapshots of the 2D stress fields \(\sigma_{xz}(x,y)\) at \(P = 0.5\) N for the loading experiment of fig. 5 taken at instants labeled 1 to 6. The yellow curves are cuts along \(y = 0\) and are intended to ease the visualization of a lower stress region around the center. On all stress fields, the red dashed circle delimits the initial apparent contact area \(a\) as determined in sect. 2.3.1.

Fig. 11. (Color online) \(u_x/(\mu a_H)^2\) in \(m^{-1}\), in steady sliding versus \(r/a_H\) at all \(P\) (the color code is the same as in figs. 3 and 5). The dashed black line represents Johnson’s prediction with \(E = 3.43\) MPa. The value of \(\mu\) is obtained by averaging over all images in steady sliding, i.e. for all \(t \geq t_s\).

Figures 11 and 12 show a direct comparison (i.e. without any adjustable parameter) between the measured and predicted displacement fields, in steady sliding and during the loading phase, respectively (see appendix A for the predicted \(u_x\) field expressions). In steady sliding (fig. 11), all \(u_x(r)\) curves at all loads have been rescaled by \(\mu a_H^2\), with \(\mu = \langle \mu a \rangle = (Q(t \geq t_s))/P\), where \(\langle \rangle\) stands for time averaging, and \(a_H\) is the Hertz contact radius computed using the value of Young’s modulus \(E\) deduced from the JKR test, i.e. \(E = 3.43\) MPa. The agreement with CM is found to be good at all normal loads \(P\). During the transient loading (fig. 12), a similar overall agreement is achieved, for both \(u_x\) averaged over the azimuthal angle \(\theta\) (fig. 12a) and for its angular dependence (fig. 12b). However, a closer look at fig. 12a reveals that significant deviations occur as \(Q\) increases, becoming more pronounced as \(Q\) reaches \(Q_s\). To quantify these deviations, fig. 12c displays, for the example of \(P = 0.3\) N, the difference \(\Delta u_x(r)\) between the measured displacements and CM’s predictions at 4 different positions, 3 within the contact \((r = \{0, 0.5a_H, a_H\})\) and 1 outside of it at \(r = 1.5a_H\), as a function of \(Q\). As clearly shown, \(\Delta u_x(r)\) increases with \(Q\) reaching a maximal value of a few \(\mu m\) when \(Q \sim Q_s\) at all points \(r\), with \(Q_s = 0.3\) N. Such deviations will be extensively discussed further down.

Similarly, the tangential stress fields \(\sigma_{xz}(x,y)\) were angularly averaged to obtain \(\sigma_{xz}(r)\) at all shear loads \(Q\) surrounded by an outer slip annulus, which is in full qualitative agreement with our experimental results. Using a superposition principle, CM’s calculations provide complete analytic expressions for \(\sigma_{xz}, \sigma_{zz}, u_x\) and \(u_y\) within the contact, in both stick and slip regions [1,34]. Further derivations by Johnson [1] also give complete analytic expressions for the interface. Note that CM’s model has previously been supported by global force and displacement measurements as well as by wear trace inspection [40,41]. However, no comparison had yet been performed on the displacement and stress distributions at mesoscopic scales.
Fig. 12. (Color online) (a) $u_x(r)/a_H$ versus $r/a_H$ in the loading phase for $P = 0.3\,N$ ($a_H \approx 1.85\,\text{mm}$), shown for 1 out of 8 images, i.e. every 1 second. Points are the measured displacements. Solid lines represent CM’s predictions with $E = 3.43\,\text{MPa}$. All curves have been arbitrarily shifted vertically to ease visualization. (b) $(u_x(a,\theta) - u_x(a,0))/a_H$ versus $\theta/\pi$ shown for 1 out of 16 images, i.e. every 2 seconds, with the same color code and marker convention as in (a). (c) Deviation $\Delta u_x$ between $u_x(r)$ measured and CM’s predictions versus $Q$ for $r \sim 0$ (•), $r = 0.5a_H$ (□), $r = a_H$ (x), and $r = 1.5a_H$ (+). The color code is the same as in (a) and (b). The threshold force $Q_s$ is 0.3 N as indicated in the figure.

Fig. 13. (Color online) Angularly averaged stress fields $\sigma_{xz}(r)$ at $P = 0.3\,N$ (a) and $P = 0.7\,N$ (b). On both plots, the black solid lines represent $\mu_s\sigma_{zz}(r)$ where $\sigma_{zz}(r)$ is here the Hertz pressure profile taking for $E$ the optimum values obtained from the inversion procedure and given in the inset of fig. 9. Small-$Q$ curves are shown in blue, while high-$Q$ curves are in red. Profiles are shown every 1.25 second. Solid lines are the measured stresses, while dashed lines correspond to CM’s predictions. (c), (d) $\sigma_{xz}(r = 0)$ and $\sigma_{xz}(r = 0.5a_H)$ versus $Q$ for the data shown in (a) and (b), respectively. Symbols correspond to the measured stresses, while solid lines are CM’s predictions.

When $Q = Q_s$, one assumes that Amontons-Coulomb’s friction law remains valid at a local length scale, one expects the shear and normal stresses to be related with $\sigma_{xz} = \mu_s\sigma_{zz}$. Taking Hertz’s pressure profile for $\sigma_{zz}$, we have computed $\mu_s\sigma_{zz}$. Figures 13a and b clearly show a rather good agreement between Hertz (black solid line) and our experimental measurements, except for a small tail at the edge of the contact. The latter presumably results from the multi-contact characteristics of the interface, and is to be related to the tail in the intensity profiles as discussed earlier in sect. 2.3.1. When $Q < Q_s$, the stress profiles have qualitatively the same radial dependence as predicted by CM’s model and indicated with the dashed lines (figs. 13a and b) (see appendix A for the predicted $\sigma_{xz}(r)$ field expressions). Quantitative analysis however reveals that deviations are clearly present as shown for $\sigma_{xz}(r = \{0, 0.5a_H\})$ in figs. 13c and d. Yet, these stress profiles provide a mean to extract the diameter of the stick region $c$. In CM’s model, $\sigma_{xz}$ is maximum at $r = c$. Assuming that it is still true in our case, it is thus possible to give an estimate of $c$ and compare it to CM’s predictions given by eq. (4) (fig. 14). Clearly, the agreement for such a macroscopic quantity is good, even though the amplitudes of $\sigma_{xz}$ are not exactly captured by CM’s stress predictions.

6 Local constitutive law of friction

At this point, we have shown that our measurements agree with CM’s model, not only qualitatively with the existence of an inner stick region surrounded by a growing annulus...
of slip, but also quantitatively since both displacement and stress distributions are found to follow reasonably well the predicted shape and amplitude. However, close comparison reveals discrepancies, the most striking being that the displacement in the vicinity of the center of the contact is not strictly null during the transient phase, but slowly increases with $Q$ as evidenced in fig. 12c. This observation is at odds with the rigid-plastic-like constitutive law assumed in CM’s model, which implies that the displacement $u_z$ at the center of the contact should remain null as long as $\sigma_{zz} \leq \mu \sigma_{zz}$. A possible explanation can be found when considering the principle of the DIC measurement itself, which is based on correlation boxes containing pixels corresponding to both micro-contacts and out-of-contact regions. Contrary to experiments with smooth marked PDMS such as in [16,17], the measured displacements are thus averaged over the thickness $h$ of the rough layer and combine two intricate contributions: true slip at the micro-contacts and elastic shear deformation of the rough layer.

In order to characterize the local shear behavior of the interface, we examine the relationship between the tangential stress $\sigma_{zz}$ and displacement $u_z$ measured at various positions $r$ within the contact. Assuming that the pressure profile $\sigma_{zz}$ is given by Hertz’s contact theory, we compute the ratio $\frac{\sigma_{zz}(r)}{\sigma_{zz}(r)}$ as a function of the displacement at positions $r$, $u_z(r)$. Figure 15a shows, on the example of $P = 0.7$ N, the typical measured behavior obtained for all normal loads $P$. Two distinct regimes can be identified. At small $u_z$, $\sigma_{zz}/\sigma_{zz}$ increases quasi-linearly with $u_z$. At large $u_z$, however, $\sigma_{zz}/\sigma_{zz}$ asymptotically reaches a constant value. This observation can be understood as a direct consequence of the shear response of the multi-contact interface. This problem has been theoretically analyzed by Bureau and coworkers in the context of a plane-on-plane frictional contact configuration [42]. The surface topology is described using Greenwood-Williamson’s model [43] by an assembly of spherical asperities of equal radius, and whose heights are randomly distributed with an exponential distribution. In addition, the response of each asperity upon tangential loading is described by CM’s model. The macroscopic normal and shear forces $P$ and $Q$ are distributed uniformly over the multi-contact interface. The model predicts the evolution of the ratio $Q/P$ as a function of the relative displacement $\delta$ of the centers of mass of both solids in contact, in the form

$$\frac{Q}{P} = \mu \left(1 - e^{-\frac{\delta}{\mu L}}\right), \tag{5}$$

where $\mu$ is a microscopic friction coefficient and where $L = \frac{2(1 - \nu)}{\pi(1 - 2\nu)} h$ is an elastic length whose value is controlled by the rms roughness of the interface $h$, and which depends on the material properties only through the Poisson’s ratio $\nu$. This prediction allowed the authors to interpret global friction force versus displacement measurements at the interface between two Plexiglas surfaces submitted to minute shear oscillations.

In our sphere-on-plane configuration, the interfacial stress field is not uniform. However, one may expect that at intermediate length scales, smaller than the macroscopic contact size but larger than the inter-asperity distance, the local ratio between both components $\sigma_{zz}$ and $\sigma_{zz}$ also obeys eq. (5). The validity of this result is demonstrated in fig. 15, in which the local stress ratio is plotted as a function of the measured displacement, for various radii $r$ and for $P = 0.7$ N (fig. 15a). The different curves exhibit very similar behaviors, with however a slight pressure dependence visible at large displacements. Each individual curve was fitted using eq. (5) with $\mu$ and $L$ as two independent fitting parameters, yielding a master curve shown in fig. 15b. Considering all 6 experiments at different normal loads $P$, the fitting parameters are found to have very little dependence on $\sigma_{zz}$ with $\mu = 0.93 \pm 0.08$
and $L = 0.80 \pm 0.23 \mu m$. The local friction coefficient $\mu$ is found consistent with the macroscopic static-friction coefficient $\mu_s$ (see fig. 8). The value of the elastic length is to be compared with $L = 0.89 \mu m$ predicted within Bureau et al.’s model when considering the Poisson’s ratio $\nu = 0.5$ of PDMS and the rms roughness $h = 0.595 \mu m$ of our surface (see sect. 2.2).

This quantitative agreement suggests that Bureau et al.’s macroscopic model for the shear response of multi-contact interfaces can be extended to mesoscopic scales. It also shows that the observed deviations from CM’s model are fully compatible with an elasto-plastic-like behavior of the rough interface, not taken into account in Amontons-Coulomb friction law. In this respect, the type of measurements performed here can be used to estimate the shear stiffness $k$ of a multi-contact interface, a quantity which has recently received significant attention (see e.g. [44–47]). By extending Bureau et al.’s model locally, one can define $k$ as the initial slope of the curves in fig. 15, i.e.

$$k = \left( \frac{\partial u}{\partial s} \right)_{u_s = 0} = \frac{\mu_L}{\mu_m}. \quad \text{Taking } 25 \text{kPa as a typical pressure value in our experiments, one finds } k \sim 30 \text{kPa}/\mu m \text{ to be the corresponding typical shear stiffness.}
$$

A few additional experiments were performed for a different roughness ($h \sim 1.3 \mu m$) and different driving velocities up to $20 \mu m/s$ in order to test the robustness of these measurements. In the transient regime ($Q < Q_s$), the results appear to be consistent with the observations reported in this work (figs. 11–14 and fig. 15), provided the data is rescaled with respect to the velocity and maximum shear force. In contrast, the long-time behavior ($Q > Q_s$) appears to depend on both the driving velocity and the elastomer’s curing protocol [48]. The characteristics of these long transients remain unexplained to date and are beyond the scope of the present work.

7 Conclusion

Building on Chateauminois et al.’s earlier work [16,17], we have developed a novel imaging technique to probe locally the spatial distribution of tangential displacement and associated shear stress averaged over the micrometric thickness of a heterogeneous multi-contact interface. The method, based on Digital Image Correlation, uses both micro-contacts and micro-asperities as displacement tracers, yielding a sub-micrometer resolution in the measured displacement with a typical $140 \mu m$ spatial resolution. We emphasize that this technique is only suited for rough elastic surfaces against smooth rigid bodies.

This method was used to study the transition from static friction to macroscopic sliding of a smooth glass sphere tangentially loaded against a microscopically rough elastomer plane. This model geometry allowed us to directly compare the measured micro-mechanical fields to a classical model by Cattaneo and Mindlin. We showed that their model does capture reasonably well both the shape and amplitude of the measured displacement and stress fields. However, close comparison reveals that significant deviations occur, which have been shown to involve a characteristic length scale of the order of the micrometric surface roughness. In this respect, we characterized the elastic shear response of our multi-contact interface prior to slippage, which is ignored in CM’s model. The latter was shown to be well captured by the model of Bureau et al.’s developed in [42].

This surprisingly good agreement suggests that a description of the roughness in terms of a collection of identical spherical asperities provides a reasonably good approximation of the PDMS surface topography. This may reflect the existence of a dominant length scale ($l_c \sim 30 \mu m$) introduced by the abrading method, which is set by the size of the abrading grains. The surface is essentially smooth at length scales larger than $l_c$, consistent with the observation that the micro-contacts appear homogenously distributed within the contact zone when imaged at length scales greater than $l_c$. This length scale also shows up in the height power spectrum as a cut-off $l_c^{-1}$ (fig. 1, inset) beyond which this function rapidly decays.

Consequences of multi-scale surface roughness, ignored in Bureau et al.’s model, still need to be examined. Within the context of contact asperity models, like that of Bureau et al. or [49], the shape of loading curves like the ones shown in fig. 15 is controlled by the distribution of micro-contact areas. A first step towards a more faithful description of the surface topography is to introduce distributions of radii of curvature for the asperities. For normal contacts, such refinements (see e.g. [50] and [51]) were found to change only marginally the micro-contact area distribution, so we expect Bureau et al.’s results to be rather robust. Alternative and more recent models based on a spectral description of the roughness [18,52] may allow to revisit this topic. However, to our best knowledge, predictions about the tangential behaviour of such multi-scale contacts are not yet available in the literature, and are thus well beyond the scope of the present study.

Overall, the present study suggests the need to replace the rigid-plastic-like Amontons-Coulomb friction law with an elasto-plastic constitutive friction law in CM-like derivations of the displacement/stress fields, and more generally in any micromechanical analysis of contact mechanics problems (as done in e.g. [53]). The effective modulus of the elastic part of this constitutive law is i) proportional to the local applied pressure and ii) inversely proportional to the thickness of the rough interfacing layer. The type of measurements developed and validated in this work opens the way for more focused studies in any other contact geometry or loading configurations, for which no explicit model might be available. The time resolution of the measurements being entirely controlled by the frame rate of the imaging system, we anticipate that the very same method could also be used in the fast transient regimes involved in frictional instabilities.

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Appendix A.

This appendix summarizes Cattaneo-Mindlin (CM) as well as Johnson’s results [1,34] for both $u_x$ and $\sigma_{xz}$, respectively, the tangential displacement and stress fields in the direction of loading, for a sphere of radius $R$ pressed under a load $P$ against an elastic half-space of Poisson ratio $\nu$ and reduced Young’s modulus $E^* = E/(1 - \nu^2)$.

For a perfectly adhering contact of radius $a$ submitted to a shear stress field given by

$$\sigma_{xz}(r) = \mu\sigma_0(1 - r^2/\alpha^2)^{1/2},$$

the displacement field $u_{x,\alpha}(r, \theta)$ is given by

$$u_{x,\alpha}(r, \theta) = A\frac{\pi}{4}[4(2 - \nu)a^2 - (4 - 3\nu)r^2\cos^2 \theta - (4 - \nu)r^2\sin^2 \theta],$$

where $A = \frac{\mu\alpha^2}{2G\sigma_0}$, with $G = \frac{E}{2(1+\nu)}$ the shear modulus, $\mu$ the friction coefficient and $\sigma_0 = (\frac{4P}{4\pi R})^{1/2}$ using Hertz contact model. The associated angularly averaged displacements thus read

$$r \leq \alpha, \quad u_{x,a}(r, \theta) = A\frac{\pi}{2}(2 - \nu)(2\alpha^2 - r^2),$$

$$r > \alpha, \quad u_{x,a}(r, \theta) = A(2 - \nu)(2\alpha^2 - r^2)\sin^2(\alpha/r) + \alpha r(1 - (\alpha/r)^2)^{1/2}.$$}

Using a superposition principle, CM predict that for a sphere on plane contact with partial slip, with a radius of contact $a_H$ and a radius of the stick field $c$

$$r \leq c, \quad u_x(r, \theta) = u_{x,a_H} - c\frac{u_{x,c}}{a_H},$$

$$c < r < a_H, \quad u_x(r, \theta) = u_{x,a_H} - c\frac{u_{x,c}}{a_H},$$

$$r \geq a_H, \quad u_x(r, \theta) = u_{x,a_H} - c\frac{u_{x,c}}{a_H}. $$

Note that these expressions correspond to displacement fields relative to points distant from the contact. In the present experiment, the displacements are measured relative to the rigid sphere. One thus has to subtract $u_x(r = 0, \theta)$ from the previous expressions in order to compare with the measurements.

In steady sliding, there is no stick zone ($c = 0$) and using the previous expressions, it is straightforward to show that, for $\nu = 1/2$, the angularly averaged displacement fields satisfy the following relations given by

$$r \leq a_H, \quad \frac{u_x(r)}{\mu a_H^2} = \frac{3}{4R}(2 - (r/a_H)^2),$$

$$r > a_H, \quad \frac{u_x(r)}{\mu a_H^2} = \frac{3}{2\pi R}\{(2 - (r/a_H)^2)\sin^{-1}(a_H/r) + (r/a_H)(1 - (a_H/r)^2)^{1/2}\}. $$

Finally, the corresponding stress fields $\sigma_{xz}$ have no angular dependence and are given by

$$0 \leq r \leq c, \quad \sigma_{xz}(r) = \frac{3\mu P}{2\pi a_H\gamma}(a_H^2 - r^2)^{1/2} - (c^2 - r^2)^{1/2},$$

$$c < r < a_H, \quad \sigma_{xz}(r) = \frac{3\mu P}{2\pi a_H\gamma}(a_H^2 - r^2)^{1/2},$$

$$r > a_H, \quad \sigma_{xz}(r) = 0.$$
22. Mark J.E. (Editor), *Polymer Data Handbook* (Oxford University Press, 1999).
23. J.A. Greenwood, J.H. Tripp, Trans. ASME, Ser. E, J. Appl. Mech. 34, 153 (1967).
24. J. Scheibert, *Mécanique du Contact aux Échelles Mésoscopiques* (Edilivre, Collection Universitaire, 2008).
25. F. Hild, S. Roux, Strain 42, 69 (2006).
26. O. Ronson, K.L. Coeyrehourcq, Proc. R. Soc. London, Ser. A 457, 1277 (2001).
27. K.R. Shull, Matter Sci. Eng. R. 36, 1 (2002).
28. K. N.G. Fuller, D. Tabor, Proc. R. Soc. London, Ser. A 345, 327 (1975).
29. E. Wandersman, R. Candelier, G. Debrégeas, A. Prevost, Phys. Rev. Lett. 107, 164301 (2011).
30. L. Landau, E. Lifshitz, *Theory of Elasticity* (Butterworth Heinemann, 1986).
31. E.K. Dimitriadis, F. Horkay, J. Maresca, B. Kachar, R.S. Chadwick, Biophys. J. 82, 2798 (2002).
32. C. Cattaneo, Rend. Accad. Naz. Lincei, 27, 214 (1938).
33. R.D. Mindlin, Trans. ASME, Ser. E, J. Appl. Mech. 16, 259 (1949).
34. D. Hills, D. Nowell, *Mechanics of Fretting Fatigue* (Kluwer Academic Publishers, 1994).
35. M. Ciavarella, Int. J. Solids Struct. 35, 2349 (1998).
36. A. Savkoor, *Dry adhesive contact of elastomers*, Mechanical engineering dissertation, Technical University of Delft (1987).
37. A. Savkoor, G. Briggs, Proc. R. Soc. London, Ser. A 356, 103 (1977).
38. I. Etsion, J. Tribol., Trans. ASME 132, 020801 (2010).
39. D.T. Nguyen, P. Paolino, M.C. Audry, A. Chateauminois, C. Frétigny, Y. Le Chenadec, M. Portigliatti, E. Barthel, J. Adhes. 87, 235 (2011).
40. K.L. Johnson, Proc. R. Soc. London, Ser. A 230, 531 (1955).
41. K.L. Johnson, J. Mech. Eng. Sci. 3, 362 (1961).
42. L. Bureau, C. Caroli, T. Baumberger, Proc. R. Soc. London, Ser. A 459, 2787 (2003).
43. J.A. Greenwood, J.B.P. Williamson, Proc. R. Soc. London, Ser. A 295, 300 (1966).
44. M. Gonzalez-Valadez, A. Baltazar, R.S. Dwyer-Joyce, Wear 268, 373 (2010).
45. S. Akarapu, T. Sharp, M.O. Robbins, Phys. Rev. Lett. 106, 204301 (2011).
46. C. Campaná, B.N.J. Persson, M.H. Müser, J. Phys.: Condens. Matter 23, 085001 (2011).
47. M.E. Kartal, D.M. Mulvihill, D. Nowell, D.A. Hills, Tribol. Int. 44, 1188 (2011).
48. A. Kurian, S. Prasad, A. Dhinojwala, Macromolecules 43, 2438 (2010).
49. G.N. Boitnott, R.L. Biegel, C.H. Scholz, N. Yoshioka, W. Wang, J. Geophys. Res. 97, 8965 (1992).
50. G. Carbone, F. Bottiglione, J. Mech. Phys. Solids 56, 2555 (2008).
51. S.R. Brown, C.H. Scholz, J. Geophys. Res., Solid Earth Planets 90, 5531 (1985).
52. M.H. Müser, Phys. Rev. Lett. 100, 055504 (2008).
53. A. Brzoza, V. Pauk, Arch. Appl. Mech. 78, 531 (2008).