Social dynamics with peer support on heterogeneous networks

The “mafia model”

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Received 6 May 2010 / Received in final form 7 August 2011
Published online 17 October 2011 – © EDP Sciences, Società Italiana di Fisica, Springer-Verlag 2011

Abstract. Human behavior often exhibits a scheme in which individuals adopt indifferent, neutral, or radical positions on a given topic. The mechanisms leading to community formation are strongly related with social pressure and the topology of the contact network. Here, we discuss an approach to model social behavior which accounts for the protection by alike peers proportional to their relative abundance in the closest neighborhood. We explore the ensuing non-linear dynamics emphasizing the role of the specific structure of the social network, modeled by scale-free graphs. We find that both coexistence of opinions and consensus on the default position are possible stationary states of the model. In particular, we show how these states critically depend on the heterogeneity of the social network and the specific distribution of external control elements.

1 Introduction

Opinion and group formation in human societies have been investigated in the last years as complex phenomena, well described with the methods of non-linear dynamics and statistical physics (for a review see [1]). It has become clear that the evolutionary dynamics of societies, based on social pressure and imitation, must be understood together with the intricate network of contacts established among single agents. Individuals are represented by the nodes of a graph and interact with their neighbors under a given set of rules to form their own opinion. Interactions may take place in several ways, and many models have dealt with various mechanisms in which individuals might meet as well as with different rules to update their opinions.

In the simplest voter model [2–4] individuals on the nodes of a regular lattice may hold two opinions encoded by a binary spin variable $\sigma = \pm 1$. Two randomly selected individuals interact with each other (one-to-one interaction) in which the second simply adopts the opinion of the first. In one dimension, the voter model is identical to the kinetic Ising model with zero-temperature Glauber dynamics [5–7]. The latter is exactly solvable for regular lattices in any dimension [8,9]. This simple dynamics leads to consensus in finite systems, which is the only stable solution: all spins are aligned, while both states are equally likely to be the consensus opinion – provided they are equally abundant in the initial state – i.e. ensemble magnetization is conserved [5].

On heterogeneous networks, special updating mechanisms like the link-update [10–12] approach are needed to ensure conservation of the ensemble magnetization. For the reverse-voter [7,11], where a node is randomly selected and its opinion copied to a randomly selected neighbor, it was found that the time to consensus differs from the direct voter model. A common feature of the voter dynamics on various complex networks, such as in small-world [13], scale-free [10,12,14], and random [15] graphs, is the absence of consensus in the thermodynamic limit, which has also been supported by analytical investigations on uncorrelated graphs [15]. However, consensus is systematically reached on finite graphs [12,13], even though metastable states of coexisting opinions are observed for long periods before the system evolves towards consensus for a time which scales with the system’s size.

In the voter model, the inclusion of more than two opinions [16,17], non-confident vacillating voters [18], zealots with a fanatic position [19,20], or a threshold number of successful encounters [21] allows the coexistence of opinions. Encounters among many agents have been considered in the majority model [22–24], in which a group of individuals is randomly selected and all of them adopt the opinion of the majority (all-to-all interactions), or in the rumor spreading model [25,26], in which more than one group-encounters take place simultaneously. In both of these models consensus on one or the other opinion depending on the initial conditions is achieved for various structures. Nevertheless, if noise is introduced in this group dynamics, as in the majority-minority [27] or the majority-vote [5] models, the system evolves either towards a consensus or towards a state in which both
opinions are equally represented (zero magnetization state in the Ising model analogy). The same behavior is observed in the Sznajd model [28,29] where two individuals sharing the same opinion impose it on their neighbors.

Another approach for describing social interactions that accounts of the influence of the whole neighborhood on a single individual (one-to-all interactions) has been discussed in the social impact theory [30–32]. An agent changes her opinion if the pressure in favor of the opinion change overcomes the support to keep the current position. This model yields stable coexistence of opinions, unless the presence of external fields is taken into account, for which metastable states are observed in which domains with the minority opinion successively shrink [32].

The Abrams-Strogatz (AS) model describes the competition between two languages in a population, with non-linear transition rates proportional to $x^a$, where $x$ is the population fraction of one particular language, and $a$ the volatility characterizing the tendency to change state [33]. For high volatilities, $a < 1$, coexistence of both languages has been found, whereas for $a > 1$ one of the two languages becomes dominant [33]. Studying the AS model on networks, it has been shown that a decrease in network connectivity leads to a reduced parameter regime with language coexistence [34]. In the same study, the AS model was also compared with a variation including a bilingual state, which has been observed to hinder coexistence.

Here, we propose a model which aims to capture the mechanisms of community formation and how these depend on the specific structure of the contact network. It describes the dynamics of a society in which individuals may either be neutral (hold a default opinion) or belong to a minority with radical positions on a current topic. Examples of human behavior with such a dynamics are the membership and involvement in political parties or organizations, belonging to religious communities, the interest in leisure activities as online games, the behavior of consumers with new technologies, or the typical cycle of addiction to chemicals. Neutral individuals may at some point in time make acquaintance with a new product or idea and subsequently become customers or adopt it as their default position. Hereafter, they can either like it and consolidate as their default state or not and return to neutral behavior. Additionally, if agents in a default state are in contact with fanatics (radical agents) they can also turn into radicals. Radical agents, in turn, may get bored or disappointed after a while reducing their level of commitment and turn back to a default position or even completely leave the community and become neutral agents again.

There are two characteristic features of our model, defined mathematically in the next chapter, which are essential for its dynamics. First, we account for the fact that the population density may vary over the contact network, i.e. in addition to individuals with different states (opinions) we also consider empty sites. Second, the probability of a given individual to change its state is taken to be proportional to both the number of like and alike peers in its neighborhood. This product form of the transition rates is the main difference to previously studied non-linear voter-like models [35,36] and the AS model [33]. This form implies, that transition rates may change from a linear to a non-linear form upon changing the composition of the population.

The outlined scheme bears similarities with the formation of mafias, in which normal citizens are the analog to default opinions, while mafias represent the radical minority. Neutral individuals are those who do not take active part in the society and that are represented by empty sites. Allegorically these sites may enter the community as citizens by a birth event and join the mafia later due to the social pressure of their vicinity. In this paper, for illustration we will use the metaphoric language of mafias.

We will show that the mafia model yields a rich phase behavior which contains both coexistence of opinions and consensus on the default position in well-mixed societies. In scale-free networks, coexistence appears in a larger regime of the parameter space due to the effect of local interactions: the stability of the absorbing state (extinction) is lost. We will discuss the role played by the nature of interactions – one-to-all, i.e. the whole neighborhood determines the state of single agents, as compared to one-to-one interactions like in the voter model – for the loss of stability of the absorbing state. Furthermore, we will show that the inclusion of external fields in the model, representing the presence of elements controlling the interactions (which we refer to as police individuals), gives rise to micro-phase separation where regions with different dynamics are observed to coexist in the system.

This paper is organized as follows: we first describe the dynamics of the mafia model and then we analytically discuss the phase diagram expected for well-mixed populations. Then, we analyze the behavior on structured societies showing different degrees of heterogeneity, which we investigate through extensive stochastic simulations. We finally address the role of external control elements in the system and explore how specific distributions of those into highly or sparsely connected nodes modifies the system’s behavior. An extended mean-field theory accounting for the local structure of networks is given in the Appendix.

2 The mafia model
We model human interactions in social networks by graphs whose nodes allocate interacting agents and whose edges describe the relations between pairs of individuals (Fig. 1). The nodes of a network can either be empty ($\Phi$) or occupied by an individual. In the simplest case agents belong to one of two existing groups identified with what we refer to as two different strategies, namely they either belong to the mafia ($M$) or to the group of lawful citizens ($C$). The network's size is assumed to be fixed and, therefore, the sum of the number of citizens, mafiosi, and empty places (or their corresponding fractions) remains constant: $C + M + \Phi = N$.

We employ an agent-based model in which the society evolves according to the following stochastic dynamics: (i) at empty nodes citizens are born with a site-independent