Cobimaximal Neutrino Mixing from $S_3 \times Z_2$

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Abstract

It has recently been shown that the phenomenologically successful pattern of cobimaximal neutrino mixing ($\theta_{13} \neq 0$, $\theta_{23} = \pi/4$, and $\delta_{CP} = \pm \pi/2$) may be achieved in the context of the non-Abelian discrete symmetry $A_4$. In this paper, the same goal is achieved with $S_3 \times Z_2$. The residual lepton $Z_3$ triality in the case of $A_4$ is replaced here by $Z_2 \times Z_2$. The associated phenomenology of the scalar sector is discussed.
Introduction:

Present neutrino data \[1, 2\] are indicative of $\theta_{13} \neq 0$, $\theta_{23} = \pi/4$ and $\delta_{CP} = -\pi/2$. Calling this **cobimaximal** mixing \[3\], it has been shown that it may be derived in two equivalent ways. (I) The Majorana neutrino mass matrix, in the basis where charged-lepton masses are diagonal, is of the form \[4, 5, 6\]

$$
\mathcal{M}^{(e, \mu, \tau)}_\nu = \begin{pmatrix} A & C & C^* \\ C & D^* & B \\ C^* & B & D \end{pmatrix},
$$

where $A, B$ are real. (II) The neutrino mixing matrix is of the form \[7, 8, 9, 10\]

$$
U_{\nu} = U_\omega \mathcal{O},
$$

where \[11, 12\]

$$
U_\omega = \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & 1 & 1 \\ 1 & \omega & \omega^2 \\ 1 & \omega^2 & \omega \end{pmatrix},
$$

with $\omega = \exp(2\pi i/3) = -1/2 + i\sqrt{3}/2$, and $\mathcal{O}$ is any arbitrary real orthogonal matrix. This yields $|U_{\mu i}| = |U_{\tau i}|$ which leads to cobimaximal mixing. Using the fact that $U_\omega$ is derivable from $A_4$ \[13\], and the scotogenic generation of neutrino mass from a set of real scalars \[9, 14, 15, 16\], Eq. (2) is naturally achieved. This conceptual shift from tribimaximal \[17, 18\] to cobimaximal mixing may also be understood as the result of a residual generalized $CP$ symmetry \[6, 19, 20, 21, 22, 23, 24\]. Here we show how cobimaximal mixing may be obtained from the soft breaking of $S_3 \times Z_2$ to $Z_2 \times Z_2$ instead of the soft breaking of $A_4$ to $Z_3$.

Soft breaking of $S_3$ to $Z_2$ with two Higgs doublets:

Let $(\Phi_1, \Phi_2) \sim 2$ under $S_3$, then the most general $S_3$ invariant scalar potential for $\Phi_{1,2}$ is given by \[25\]

$$
V_0 = \mu_0^2(\Phi_1^\dagger \Phi_1 + \Phi_2^\dagger \Phi_2) + \frac{1}{2} \lambda_1(\Phi_1^\dagger \Phi_1 + \Phi_2^\dagger \Phi_2)^2 \\
+ \frac{1}{2} \lambda_2(\Phi_1^\dagger \Phi_1 - \Phi_2^\dagger \Phi_2)^2 + \lambda_3(\Phi_1^\dagger \Phi_2)(\Phi_2^\dagger \Phi_1).
$$
The soft breaking of $V_0$ to a residual $Z_2$ symmetry may be accomplished in three ways, according to how the $Z_2$ is chosen. (1) $\Phi_1 \to \Phi_1$, $\Phi_2 \to -\Phi_2$. The soft term required is then $\Phi_1^\dagger \Phi_1 - \Phi_2^\dagger \Phi_2$. (2) $\Phi_1 \to \Phi_2$, $\Phi_2 \to \Phi_1$. The soft term required is then $\Phi_1^\dagger \Phi_2 + \Phi_2^\dagger \Phi_1$. (3) $\Phi_1 \to i\Phi_2$, $\Phi_2 \to -i\Phi_1$. The soft term required is then $i(\Phi_1^\dagger \Phi_2 - \Phi_2^\dagger \Phi_1)$. In (1), the residual $Z_2$ symmetry corresponds to $\langle \phi_0^1 \rangle \neq 0$, $\langle \phi_0^2 \rangle = 0$. In (2), it is $\langle \phi_0^1 + \phi_0^2 \rangle \neq 0$, $\langle \phi_0^1 - \phi_0^2 \rangle = 0$. In (3), it is $\langle \phi_0^1 + i\phi_0^2 \rangle \neq 0$, $\langle \phi_0^1 - i\phi_0^2 \rangle = 0$.

Two lepton families under $S_3$:

Let $(\nu_1, l_1)_L, (\nu_2, l_2)_L \sim 2$ under $S_3$, and $l_{1R}, l_{2R} \sim \mathbf{1'}, \mathbf{1}$ under $S_3$. The $S_3$ invariant Yukawa terms for charged-lepton masses are then

$$-\mathcal{L}_Y = f_{\mu}(\bar{l}_{1L}\phi_0^1 - \bar{l}_{2L}\phi_0^2)l_{1R} + f_{\tau}(\bar{l}_{1L}\phi_0^1 + \bar{l}_{2L}\phi_0^2)l_{2R} + H.c.$$  

(5)

Choosing the third option for $Z_2$ with $i\mu_{12}^2(\Phi_1^\dagger \Phi_2 - \Phi_2^\dagger \Phi_1)$ with $\mu_{12}^2 < 0$, so that $v/\sqrt{2} = \langle \phi_0^1 \rangle = i\langle \phi_0^2 \rangle$, the $2 \times 2$ mass matrix linking $(\bar{l}_{1L}, \bar{l}_{2L})$ to $(l_{1R}, l_{2R})$ is then given by

$$\mathcal{M}_l = \begin{pmatrix} f_\mu & f_\tau \\ i f_\mu & -i f_\tau \end{pmatrix} \frac{v}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ i & -i \end{pmatrix} \begin{pmatrix} f_\mu v \\ 0 \end{pmatrix} = \begin{pmatrix} f_\mu v \\ 0 \end{pmatrix}$$  

(6)

Three lepton families under $S_3 \times Z_2$:

A third lepton family may be added which transforms as $(\mathbf{1}, -)$ under $S_3 \times Z_2$, so that it couples to a third Higgs doublet which transforms as $(\mathbf{1}, +)$. The $3 \times 3$ unitary matrix linking the diagonal charged-lepton mass matrix to the neutrino mass matrix is then

$$U_2 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1/\sqrt{2} & -i/\sqrt{2} \\ 0 & i/\sqrt{2} & i/\sqrt{2} \end{pmatrix}.$$  

(7)

This serves the same purpose as $U_\omega$ of Eq. (3), because

$$U_{\nu l} = U_2 O$$  

(8)

also yields $|U_{\mu l}| = |U_{\tau l}|$ which leads to cobimaximal mixing. In fact, since $U_{ei} = O_{1i}$, the $\theta_{12}$ and $\theta_{13}$ angles are the same in both $U_{\nu l}$ and $O$.  

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More about $\mathcal{O}$:

The $3 \times 3$ Majorana neutrino mass matrix $\mathcal{M}_\nu$ is symmetric but also complex in general with three physical phases. It is thus not diagonalized by an orthogonal matrix. However, if the origin of this mass matrix is radiative and comes from a set of three real scalars, and there are no extraneous phases from the additional interactions, then it is possible [9, 14, 15, 16] to achieve this result.

In general $\mathcal{M}_\nu$ is diagonalized by a unitary matrix with 3 angles and 3 phases:

\[
U = \begin{pmatrix}
1 & 0 & 0 \\
0 & c_{23} & s_{23} \\
0 & -s_{23} & c_{23}
\end{pmatrix}
\begin{pmatrix}
1 & s_{13}e^{-i\delta} & 0 \\
-s_{12} & c_{12} & 0 \\
0 & 0 & 1
\end{pmatrix}
\begin{pmatrix}
1 & 0 & 0 \\
0 & 1/e^{i\alpha_{21}/2} & 0 \\
0 & 0 & e^{i\alpha_{31}/2}
\end{pmatrix},
\]

(9)

where $c_{ij} = \cos \theta_{ij}$ and $s_{ij} = \sin \theta_{ij}$. If $\delta = 0$, then it is equal to an orthogonal matrix times a diagonal matrix involving only Majorana phases. Upon multiplication on the left by $U_2$ of Eq. (7), it will still lead to cobimaximal neutrino mixing. Now

\[
U_2 \begin{pmatrix}
1 & 0 & 0 \\
0 & c_{23} & s_{23} \\
0 & -s_{23} & c_{23}
\end{pmatrix} = \begin{pmatrix}
1 & 0 & 0 \\
0 & e^{i\theta_{23}} & 0 \\
0 & 0 & e^{-i\theta_{23}}
\end{pmatrix}
\begin{pmatrix}
1 & 0 & 0 \\
0 & 1/\sqrt{2} & -i/\sqrt{2} \\
0 & -1/\sqrt{2} & -i/\sqrt{2}
\end{pmatrix}.
\]

(10)

The diagonal matrix of phases on the left may be absorbed into the charged leptons, and the remaining part of $U_2 U$ becomes

\[
\begin{pmatrix}
1 & 0 & 0 \\
0 & 1/\sqrt{2} & 1/\sqrt{2} \\
0 & -1/\sqrt{2} & 1/\sqrt{2}
\end{pmatrix}
\begin{pmatrix}
1 & s_{13}e^{-i\delta} & 0 \\
-s_{12} & c_{12} & 0 \\
is_{13}e^{i\delta} & 0 & c_{13}
\end{pmatrix}
\begin{pmatrix}
1 & 0 & 0 \\
0 & 1/e^{i\alpha_{21}/2} & 0 \\
0 & 0 & -i/e^{i\alpha_{31}/2}
\end{pmatrix}.
\]

(11)

This means that if $\delta = 0$, cobimaximal mixing is achieved with $e^{-i\delta_{CP}} = e^{i\pi/2} = i$ as expected. However, even if $\delta \neq 0$, so that $\delta_{CP}$ deviates from $-\pi/2$, $\theta_{23}$ remains at $\pi/4$. This is a remarkable result and it is only true because of $U_2$ of Eq. (7), and does not hold for $U_\omega$ of Eq. (3). The deviation from cobimaximal mixing is model-dependent and has been calculated [26] in a specific model, showing the correlation of $\delta_{CP}$ with $\theta_{23}$. Here only $\delta_{CP}$ deviates.
Soft breaking of $S_3$ to $Z_2$ with three Higgs doublets:

Adding $\Phi_3 \sim 1$ under $S_3$, the scalar potential of our model becomes

$$V = \mu_0^2(\Phi_1^\dagger \Phi_1 + \Phi_2^\dagger \Phi_2) + i\mu_{12}(\Phi_1^\dagger \Phi_2 - \Phi_2^\dagger \Phi_1) + \mu_3^2\Phi_3^\dagger \Phi_3 + \left[ \frac{1}{2}\mu_{30}^2\Phi_3^\dagger (\Phi_1 + i\Phi_2) + H.c. \right]$$

$$+ \frac{1}{2}\lambda_1(\Phi_1^\dagger \Phi_1 + \Phi_2^\dagger \Phi_2)^2 + \frac{1}{2}\lambda_2(\Phi_1^\dagger \Phi_1 - \Phi_2^\dagger \Phi_2)^2 + \lambda_3(\Phi_1^\dagger \Phi_2)(\Phi_2^\dagger \Phi_1) + \frac{1}{2}\lambda_4(\Phi_3^\dagger \Phi_3)^2$$

$$+ \lambda_5(\Phi_3^\dagger \Phi_3)(\Phi_1^\dagger \Phi_1 + \Phi_2^\dagger \Phi_2) + \lambda_6(\Phi_1^\dagger \Phi_1 + \Phi_2^\dagger \Phi_2)\Phi_3 + [\lambda_7(\Phi_3^\dagger \Phi_1)(\Phi_3^\dagger \Phi_2) + H.c.] \quad (12)$$

The $\mu_{12}$ and $\mu_{30}^2$ terms break $S_3$ softly to $Z_2$, under which $\Phi_3$ and $\Phi_+ = (\Phi_1 + i\Phi_2)/\sqrt{2}$ are even and $\Phi_- = (\Phi_1 - i\Phi_2)/\sqrt{2}$ is odd. The $S_3$ allowed quartic term $(\Phi_3^\dagger \Phi_1)(\Phi_2^\dagger \Phi_1) + (\Phi_3^\dagger \Phi_2)(\Phi_1^\dagger \Phi_2)$ is forbidden by making $\Phi_3$ odd under an extra $Z_2$ symmetry, which is then broken softly by the $\mu_{30}^2$ term. With this modification, $(\nu_e, e)_L$ is even and $e_R$ is odd under this softly broken extra $Z_2$ to allow the Yukawa coupling $f_e\bar{e}_L e_R \phi^0_3$, with $m_e = f_e v_3$. Assuming a small $\mu_{30}^2$ term, $v_3$ is naturally much smaller than $v$. Hence $m_e << m_\mu, m_\tau$ and the charged leptons are distinguished from each other according to

$$e \sim (+, -), \quad \mu \sim (-, +), \quad \tau \sim (+, +). \quad (13)$$

The $Z_3$ triality $[27, 28]$ coming from $A_4$ has now been replaced by the above under $Z_2 \times Z_2$. This serves to forbid $\mu \rightarrow e\gamma$, etc. as in the case of $Z_3$ lepton triality. The odd Higgs doublet $\Phi_-$ transforms as $(-, +)$ and couples to $\bar{\mu}_L \tau_R$ and $\bar{\tau}_L \mu_R$ as in Ref. $[29]$. 

Phenomenology of scalar interactions:

The leptonic Yukawa interactions are given by

$$-\mathcal{L}_Y = f_\tau(\bar{\tau}_L \phi^0_+ + \bar{\mu}_L \phi^0_-) \tau_R + f_\tau(\bar{\nu}_\tau \phi^+_+ + \bar{\nu}_\mu \phi^+_-) \tau_R$$

$$+ f_\mu(\bar{\mu}_L \phi^0_+ + \bar{\tau}_L \phi^0_-) \mu_R + f_\mu(\bar{\nu}_\mu \phi^+_+ + \bar{\nu}_\tau \phi^+_-) \mu_R$$

$$+ f_e \bar{e}_L \phi^0_3 e_R + f_e \bar{\nu}_e \phi^+_3 \nu_R + H.c. \quad (14)$$

The scalar interactions are given by

$$V = (\mu_0^2 + \mu_{12}^2)\Phi_+ \Phi_+ + (\mu_0^2 - \mu_{12}^2)\Phi_- \Phi_- + \mu_3^2\Phi_3^\dagger \Phi_3 + \left[ \frac{1}{\sqrt{2}}\mu_{30}^2\Phi_3^\dagger \Phi_+ + H.c. \right]$$
\[ v^2 = \frac{-3\mu_0^2}{\lambda_1 + (\lambda_3/2)}, \]
\[ v_3 \simeq \frac{-\mu_{30}^2 v}{\sqrt{2} \mu_3^2 + (\lambda_5 + \lambda_6 + Im(\lambda_7))v^2}. \]

The states \( \sqrt{2}[v Im(\phi_0^0) + v_3 Im(\phi_3^0)]/\sqrt{v^2 + v_3^2} \) and \( [v \phi_3^\pm + v_3 \phi_3^\mp]/\sqrt{v^2 + v_3^2} \) are the would-be massless Goldstone modes for the Z and \( W^\pm \) bosons. The states \( A = \sqrt{2}[v Im(\phi_3^0) - v_3 Im(\phi_3^0)]/\sqrt{v^2 + v_3^2} \) and \( H^\pm = [v \phi_3^\pm - v_3 \phi_3^\mp]/\sqrt{v^2 + v_3^2} \) have masses given by

\[ m_A^2 = -Im(\lambda_7)(v^2 + v_3^2) - \frac{\mu_{30}^2 (v^2 + v_3^2)}{2 vv_3} \simeq \mu_3^2 + (\lambda_5 + \lambda_6 - Im(\lambda_7))v^2, \]
\[ m_{H^\pm}^2 = -(\lambda_6 + Im(\lambda_7))(v^2 + v_3^2) - \frac{\mu_{30}^2 (v^2 + v_3^2)}{2 vv_3} \simeq \mu_3^2 + \lambda_5 v^2. \]

The states \( h = \sqrt{2} Re(\phi_3^0) \) and \( H = \sqrt{2} Re(\phi_3^0) \) are approximate mass eigenstates with

\[ m_h^2 \simeq (2\lambda_1 + \lambda_3)v^2, \quad m_H^2 \simeq \mu_3^2 + (\lambda_5 + \lambda_6 + Im(\lambda_7))v^2, \]

and \( h - H \) mixing given by

\[ \epsilon \simeq \frac{-v_3}{v} \left[ \frac{\mu_3^2 - (\lambda_5 + \lambda_6 + Im(\lambda_7))v^2}{\mu_3^2 + (\lambda_5 + \lambda_6 + Im(\lambda_7))v^2} \right]. \]
The $\Phi^\pm_-$ doublet has odd $Z_2$ and does not mix with $\Phi^+_+$ or $\Phi^3_3$. The masses of its components are given by

$$m^2(\phi^\pm_-) \simeq \mu^2_0 - \mu^2_{12} + \left(\lambda_1 - \frac{1}{2}\lambda_3\right)v^2,$$

$$m^2(\sqrt{2}\text{Re}(\phi^0_-)) \simeq \mu^2_0 - \mu^2_{12} + \left(\lambda_1 - \frac{1}{2}\lambda_3 + 2\lambda_2\right)v^2,$$

$$m^2(\sqrt{2}\text{Im}(\phi^0_-)) \simeq \mu^2_0 - \mu^2_{12} + \left(\lambda_1 + \frac{1}{2}\lambda_3\right)v^2.$$  

Phenomenology of lepton interactions:

From Eq. (14), the lepton interactions of this model are given by

$$-\mathcal{L}_Y = \frac{m_e}{\sqrt{2}}h\tau\tau + \frac{m_\mu}{\sqrt{2}}h\mu\mu + \frac{m_\tau}{\sqrt{2}}[(H + iA)\bar{e}_Le_R + H^+\bar{\nu}_e e_R + H.c.]$$

$$+ \left[\frac{m_\tau}{v}\left[\phi^0_\mu L\tau_R + \phi^+\nu_\mu \tau_R\right] + \frac{m_\mu}{v}\left[\phi^0_\tau L\mu_R + \phi^+\nu_\tau \mu_R\right]\right] + H.c. \quad (27)$$

to a very good approximation. Since $v_3 << v$ is assumed, the heavy $H$ and $A$ couple predominantly to $e^- e^+$. If they are produced, through a virtual $Z$ for example, at the Large Hadron Collider (LHC), the $e^- e^+ e^- e^+$ final state is very distinctive and potentially measurable. In the same way, $\sqrt{2}\text{Re}(\phi^0_-) + \sqrt{2}\text{Im}(\phi^0_-)$ may be produced. They decay to $\mu^- \tau^+$ and $\mu^+ \tau^-$ which are again rather distinctive if $\tau^\pm$ can be reconstructed experimentally. On the other hand, the decay of $\phi^\pm_-$ is predominantly to $\tau^+ \nu_\mu$, $\tau^- \bar{\nu}_\mu$.

Conclusion:

The notion of cobimaximal neutrino mixing, i.e. $\theta_{13} \neq 0$, $\theta_{23} = \pi/4$, and $\delta_{CP} = -\pi/2$, is shown to be a consequence of the residual $Z_2 \times Z_2$ symmetry of an $S_3 \times Z_2$ model of lepton masses. This is an alternative theoretical understanding from the usual $A_4$ realization. It has verifiable decay signatures in its three Higgs doublets, as well as the prediction that even if $\delta_{CP}$ deviates from $-\pi/2$, $\theta_{23}$ will remain at $\pi/4$, in contrast to other models.

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