\( \omega(\rightarrow \pi^+\pi^-\pi^0) \) meson photoproduction on proton

Swapan Das

Nuclear Physics Division, Bhabha Atomic Research Centre
Mumbai-400085, India

Abstract

The cross section is estimated for the \( \pi^+\pi^-\pi^0 \) invariant mass distribution in the \( \gamma p \) reaction in the GeV region. This reaction is assumed to proceed through the formation of the \( \omega \) meson in the intermediate state, since the production cross section for this meson in the \( \gamma p \) reaction in GeV region is significant and it has large branching ratio (88.8\%) in the \( \pi^+\pi^-\pi^0 \) channel. The cross sections for this reaction have been calculated using the energy dependent reaction amplitude, i.e., \( f_{\gamma p \rightarrow \omega p}(0) \), extracted from the latest \( \omega \) meson photoproduction data. We use established procedure to calculate other factors, like width and propagator of the \( \omega \) meson, so that our calculation can provide reliable cross section. The calculated results reproduce the measured \( \pi^+\pi^-\pi^0 \) invariant mass distribution spectra in the \( \gamma p \) reaction.

PACS number(s): 25.20.Lj

1 Introduction

The vector meson production in the nuclear and particle reactions shows growing interest over the years, since it revealed many rewarding physics in various topics of the hadronic physics. The importance of the vector meson in context to the pion production in the GeV region was realized long back [1, 2, 3]. The dilepton production in this energy region is undoubtedly explained due to the production of the vector mesons in the intermediate state [4, 5]. The vector dominance model (VDM) gives special status to the vector meson in describing the electromagnetic interaction between the lepton and hadron [6]. The vector meson can probe the low-lying nucleonic resonances [7] since it couples to these resonances through the tail of its mass distribution. To be added, the vector meson production process can be used to search the missing resonances [8]. For the later purpose, the \( \omega p \) system is a preferable choice because of the narrow width (i.e., 8.43 MeV) of the \( \omega \) meson. Of course, this system has a restriction to identify the nucleonic resonance of isospin \( I = \frac{1}{2} \).

The vector meson has a significant role in understanding the quark-gluon picture of the hadron, since the Quantum Chromodynamics (QCD) subtles it as a spin-triplet bound state of the specific valence quark (\( q \)) and anti-quark (\( \bar{q} \)) pair in the sea.
of $q\bar{q}$ pairs of all flavors including gluons. Indeed, the static quark model (a simplified picture elucidating the vector meson as a spin-triplet bound state of specific $q\bar{q}$ pair only) [9] is deceptively successful to account some properties for the vector meson, such as spin-isopin for this meson, decay of vector meson, the potential energy between the $q\bar{q}$ pair, .... etc. In the energy region of hard scattering, the fluctuation of the quark-gluon configuration inside a hadron can be studied through the vector meson production process. This fluctuation makes better transparency (called color transparency) for the vector meson propagation through the nucleus [10].

Recent past, CBELSA/TAPS collaboration at ELSA did experiment for the $\omega$ meson photoproduction on proton and nuclear targets [11]. In this measurement, the $\omega$ meson was probed by the $\pi^0\gamma$ invariant mass distribution spectrum. We have studied the mechanism for this reaction and calculated the cross section for it [12]. To be mentioned, the $\omega$ meson has only 8.5% branching ratio in the $\pi^0\gamma$ channel whereas it dominantly (88.8%) decays into the $\pi^+\pi^-\pi^0$ channel. Therefore, we calculate the cross section for the $\pi^+\pi^-\pi^0$ invariant mass distribution (since it gives better signal for the $\omega$ meson production) in the $\gamma p$ reaction. We assume that this reaction proceeds as $\gamma p \rightarrow \omega p; \omega \rightarrow \pi^+\pi^-\pi^0$. To describe the $\omega$ meson photoproduction in the intermediate state, we extract the energy dependent $\gamma p \rightarrow \omega p$ reaction amplitudes from the latest measurement of the four momentum transfer distribution (elaborated later). We follow the widely used procedure to evaluate all other factors appearing in the cross section (e.g., the propagator and width of the $\omega$ meson) to get the reliable cross section for the $\pi^+\pi^-\pi^0$ invariant mass distribution in the above mentioned reaction.

The formalism for this reaction is described in sec. 2. The calculated results (along with the data) have been described in sec. 3. The conclusion of this study is presented in sec. 4.

## 2 Formalism

The differential cross section for the $\pi^+\pi^-\pi^0$ invariant mass distribution in the reaction: $\gamma p \rightarrow \omega p; \omega \rightarrow \pi^+\pi^-\pi^0$ can be expressed as

$$\frac{d\sigma(m, E_\gamma)}{dm} = \int d\Omega_\omega[KF]|\Gamma_{\omega\rightarrow 3\pi}(m)|G_\omega(m)|^2|F(\gamma p \rightarrow \omega p')|^2,$$

where $[KF]$ represents the kinematical factor for the above reaction. It is given by

$$[KF] = \frac{1}{(2\pi)^3 k_\gamma|k_\omega(E_\gamma + m_p - k_\gamma \cdot k_\omega E_\omega)|^2 m^2 |k_\omega m|},$$

$$k_\gamma = k_\omega E_\gamma = \frac{1}{2\pi}. $$
All symbols carry their usual meanings.

\( \Gamma_{\omega \to 3\pi}(m) \) in Eq. (1) denotes the width for \( \omega \) (at rest) \( \to \pi^+\pi^-\pi^0 \), i.e., \( \Gamma_{\omega \to 3\pi}(m) \equiv \Gamma_{\omega \to \pi^+\pi^-\pi^0} \), governed by the Lagrangian density \( \mathcal{L}_{\omega 3\pi} = \frac{G_{\omega 3\pi}}{m_\omega^2} \epsilon_{\mu\nu\lambda\sigma} \omega^\mu k^\nu_\pi k^\lambda_\pi k^\sigma_\pi \) [13].

The expression for \( \Gamma_{\omega \to \pi^+\pi^-\pi^0} \) is given in Eq. (5). The \( \omega \) meson propagator \( G^{\mu\nu}_{\omega}(m) \) is given by \( G^{\mu\nu}_{\omega}(m) = (-g^{\mu\nu} + \frac{1}{m^2} k^{\mu}_\omega k^{\nu}_\omega) \Gamma_{\omega}(m) \). \( g^{\mu\nu} \) couples the \( \pi^+\pi^-\pi^0 \) field (in the final state) to the vector field. The second part of the propagator does not contribute because of the antisymmetric coupling of \( \omega \) meson to three pion (see in \( \mathcal{L}_{\omega 3\pi} \), written above). The factor \( \Gamma_{\omega}(m) \), which also appears in Eq. (1), describes the scalar part of the \( \omega \) meson propagator in the free space. The expression for it is

\[
G_{\omega}(m) = \frac{1}{m^2 - m_{\omega}^2 + im_{\omega} \Gamma_{\omega}(m)},
\]

with \( m_{\omega} \approx 782 \text{ MeV} \). \( \Gamma_{\omega}(m) \) denotes the total free space decay width for the \( \omega \) meson. It is composed of hadronic, semi-hadronic and leptonic decay channels [14]:

\[
\Gamma_{\omega}(m) \approx \Gamma_{\omega \to \pi^+\pi^-\pi^0}(88.8\%) + \Gamma_{\omega \to \pi^0\gamma}(8.5\%) + \Gamma_{\omega \to \pi^+\pi^-}(2.21\%) + \Gamma_{\omega \to \gamma\gamma}(\sim 10^{-4}\%).
\]

(4)

Since the leptonic decay channels (i.e., \( \Gamma_{\omega \to \gamma\gamma} \)) for the \( \omega \) meson are insignificant in compare to other decay channels, they are ignored in this calculation.

The form for \( \Gamma_{\omega \to \pi^+\pi^-\pi^0}(m) \) in Eq. (4), as shown by Sakurai [13], is given by

\[
\Gamma_{\omega \to \pi^+\pi^-\pi^0}(m) = \frac{m}{m_{\omega}} \frac{(m - 3m_\pi)^4}{(m_{\omega} - 3m_\pi)^4} U(m),
\]

with \( \Gamma_{\omega \to \pi^+\pi^-\pi^0}(m_{\omega}) \approx 782 \text{ MeV} \) \( \approx 7.49 \text{ MeV} \) [14]. The function \( U(m) \) is described in Ref. [13] as \( U(m) \to 1 \) for \( m \to 3m_\pi \) and \( U(m) \to 1.6 \) for \( m \to 787 \text{ MeV} \). It is also taken equal to 1.6 for \( m > 787 \text{ MeV} \).

The width \( \Gamma_{\omega \to \pi^+\pi^-\pi^0}(m) \) appearing in Eq. (4) is evaluated using the Lagrangian density: \( \mathcal{L}_{\pi^+\pi^-\pi^0} = \frac{G_{\pi^+\pi^-\pi^0}}{m_\pi^2} \epsilon_{\mu\nu\lambda\sigma} \partial^\mu A^\nu_\pi \partial^\lambda \omega^\sigma \) [15]. The expression for it is

\[
\Gamma_{\omega \to \pi^+\pi^-\pi^0}(m) = \Gamma_{\omega \to \pi^+\pi^-\pi^0} \left[ \frac{k(m)}{k(m_{\omega})} \right]^3,
\]

with \( \Gamma_{\omega \to \pi^+\pi^-\pi^0}(m_{\omega}) \approx 0.72 \text{ MeV} \) at \( m_{\omega} \approx 782 \text{ MeV} \) [14]. \( k(m) \) denotes the momentum of pion originating due to the \( \omega \) meson of mass \( m \) decaying at rest.

In Eq. (4), \( \Gamma_{\omega \to \pi^+\pi^-}(m) \) denotes the width for the \( \omega \) meson decaying to \( \pi^+\pi^- \) channel. In fact, this channel arises due to small isovector component present in the physical \( \omega \) meson, i.e., \( \omega = \omega_1(0,0) + \epsilon \rho_1(1,0) \) [5]. Here, \( \omega_1(0,0) \) and \( \rho_1(1,0) \) denote the isoscaler and isovector fields respectively. \( \epsilon \) is the small mixing parameter. Therefore, the isovector \( \pi^+\pi^- \) current can strongly couple to \( \rho_1(1,0) \) in the
Lagrangian density \( \mathcal{L}_{\omega \pi \pi} = f_{\omega \pi \pi}(\vec{p} \times \partial_{\mu} \vec{p}) \cdot \omega_{\mu} \), and \( \omega_T(0,0) \) can be ignored for this purpose. We do not use isovector sign on the \( \omega \) meson appearing in the Lagrangian since the \( \rho_T(1,0) \) content in the \( \omega \) meson is very small. To be mentioned, it has been shown clearly in the Ref. [5] that even if \( \omega_T(0,0) \to \pi^+ \pi^- \) is allowed due to charge symmetry violation (CSV), the \( \pi^+ \pi^- \) emission from the \( \omega \) meson is possible only due to \( \rho_T(1,0) \to \pi^+ \pi^- \) for \( m_\rho \sim m_\omega \) and \( \Gamma_\rho \gg \Gamma_\omega \). The width for the \( \omega \to \pi^+ \pi^- \) channel is worked out as

\[
\Gamma_{\omega \to \pi^+ \pi^-}(m) = \Gamma_{\omega \to \pi^+ \pi^-}(m_\omega) \frac{m_\omega}{m} \left[ \frac{k(m)}{k(m_\omega)} \right]^3.
\]  

(7)

The value for \( \Gamma_{\omega \to \pi^+ \pi^-}(m_\omega) \approx 782 \text{ MeV} \), according to Ref. [14], is approximately equal to 0.19 MeV. \( k(m) \) represents the pion momentum in the \( \pi^+ \pi^- \) cm of system.

The \( \omega \) meson dominantly decays to \( \pi^+ \pi^- \pi^0 \) channel (see \( \Gamma_\omega(m) \) in Eq. (4)). Therefore, the mass distribution of the \( \omega \) meson is significantly governed by the decay width of this channel, i.e., \( \Gamma_{\omega \to \pi^+ \pi^- \pi^0}(m) \), expressed in Eq. (5). The \( \omega \) meson possesses narrow width (about 8.43 MeV) in the free space. To justify it, we plot the mass \( m \) dependence of \( \Gamma_\omega(m) \) in Eq. (4) as well as \( \Gamma_{\omega \to \pi^+ \pi^- \pi^0}(m) \) in Eq. (5) in Fig. 1. This figure shows that expressions used for these widths duly reproduce the respective measured values at \( m = m_\omega \), quoted in Ref. [14].

The generalised potential for the \( \omega \) meson photoproduction in the nuclear reaction can be expressed as \( F(\gamma p \to \omega p') \phi(r) \) [16], where \( \phi(r) \) represents the density distribution of the target nucleus. For the point particle (i.e., proton target), the density distribution for it is \( \phi(r) = \delta(r) \). The form for \( F(\gamma p \to \omega p') \), which appears in Eq. (1), in the center of mass system is given by

\[
F(\gamma p \to \omega p') = -4\pi \left[ 1 + \frac{E_\omega}{E_{p'}} \right] f_{\gamma p \to \omega p'}(0),
\]  

(8)

where \( f_{\gamma p \to \omega p'}(0) \) is the forward amplitude for the \( \gamma p \to \omega p' \) reaction. In the cross section in Eq. (1), \( f_{\gamma p \to \omega p'}(0) \) appears in the form of \( |f_{\gamma p \to \omega p'}(0)|^2 \) which is related to the four momentum \( q^2 \) transfer distribution \( d\sigma(\gamma p \to \omega p')/dq^2 \) [17] as

\[
\frac{d\sigma}{dq^2}(\gamma p \to \omega p'; q^2 = 0) = \frac{\pi}{k_\gamma^2} |f_{\gamma p \to \omega p'}(0)|^2.
\]  

(9)

The forward \( d\sigma(\gamma p \to \omega p')/dq^2 \) is used to obtain from the extrapolation of the measured \( d\sigma(\gamma p \to \omega p')/dq^2 \). In fact, the energy dependent values for it are reported in Refs. [17, 18] for \( E_\gamma \geq 1.6 \text{ GeV} \). In the present study, we deal with the \( \omega \) meson photoproduction for the beam energy range: \( E_\gamma(\text{GeV}) = 1.1 - 9.3 \). In the lower energy region, i.e., \( E_\gamma \leq 2.6 \text{ GeV} \), the data for the \( d\sigma(\gamma p \to \omega p')/dq^2 \) distribution...
are taken from the measurement (done in recent past) with the SAPHIR detector at electron stretcher ring (ELSA), Bonn [19]. In this measurement, they have reported the measured $d\sigma(\gamma p \rightarrow \omega p')/dq^2$ vs $|q^2 - q_{\text{min}}^2|$ ($q_{\text{min}}^2$ is defined in Ref. [17, 20]). Therefore, we extract $|f_{\gamma p \rightarrow \omega p'}(0)|^2$ from the SAPHIR data for $E_\gamma \leq 2.6$ GeV. For $E_\gamma \geq 2.6$ GeV, the energy dependent $|f_{\gamma p \rightarrow \omega p'}(0)|^2$ is evaluated from the forward $d\sigma(\gamma p \rightarrow \omega p')/dq^2$ given in Refs. [17, 18].

The Eq. (1) can be used to calculate the differential cross section for the $\pi^+\pi^-\pi^0$ invariant mass distribution $d\sigma/dm$ due to $\omega \rightarrow \pi^+\pi^-\pi^0$ for a fixed beam energy $E_\gamma$. To describe this reaction for the $\gamma$ beam of certain energy range, as it happens for the tagged photon, we modulate the cross section given in Eq. (1) with the beam profile function $W(E_\gamma)$ [21], i.e.,

$$\frac{d\sigma(m)}{dm} = \int_{E_{\gamma_{\text{min}}}^m}^{E_{\gamma_{\text{max}}}} dE_\gamma W(E_\gamma) \frac{d\sigma(m, E_\gamma)}{dm}.$$  

(10)

The profile function $W(E_\gamma)$ for the $\gamma$ beam, originating due to the bremsstrahlung radiation of the electron, varies as $W(E_\gamma) \propto \frac{1}{E_\gamma}$ [21].

### 3 Results and Discussion

We calculate the cross sections $d\sigma/dm$ for the $\omega(\rightarrow \pi^+\pi^-\pi^0)$ meson mass distribution spectra using the Eq. (1) for beam energies ($E_\gamma$ in GeV) taken equal to 2.8, 4.7 and 9.3. The calculated results (solid curves) are compared in Fig. 2 with the $\pi^+\pi^-\pi^0$ invariant mass distribution spectra (presented by histograms) measured by Ballam et al., [2]. In this figure, the calculated cross section is normalized to the measured spectrum at the peak. The sharp peak appearing in the calculated spectrum at $\sim 782$ MeV is the characteristic feature for the $\omega$ meson. The calculated spectra, as shown in this figure, reproduce well the respective peak positions of all measured distributions. These agreements elucidate the production of the $\omega$ meson in the intermediate state. The mismatch in widths between the calculated and measured spectra (even in other figures also) can be presumed due to resolution width associated with the detector. The calculated cross section at the peak is increased to $\sim 10.51$ mb/GeV at $E_\gamma = 9.3$ GeV from 7.25 mb/GeV at $E_\gamma(\text{GeV}) = 2.8$ GeV.

We present in Fig. 3 the calculated cross section, i.e., $d\sigma/dm$ due to Eq. (10), along with the data for $\pi^+\pi^-\pi^0$ invariant mass distribution in the energy bins: $E_\gamma$ (in GeV) = 1.1 – 1.5; 1.5 – 1.8; 1.8 – 2.5 and 2.5 – 6.0. In this figure, the histograms represent the $\pi^+\pi^-\pi^0$ invariant mass distribution spectra measured by the BUBBLE chamber group [3]. The dashed curves describe the phase-space distributions. The solid
curve in each energy bin is obtained multiplying the calculated cross section by a
normalizing factor and adding it with the respective phase space. The normalizing
factors are found equal to 14.5, 2.9, 3.4 and 3.2 for the beam energy ($E_\gamma$) bin:
$1.1 - 1.5$ GeV, $1.5 - 1.8$ GeV, $1.8 - 2.5$ GeV and $2.5 - 6$ GeV respectively. This
figure shows that the calculated result reproduces the peak position in each energy
bin. The actual magnitudes of the calculated cross sections are presented in Fig. 4.
This figure shows that the peak cross section is enhanced to $\sim 7.73$ mb/GeV at
$E_\gamma$(GeV) = 2.5 − 6.0 from $\sim 2.31$ mb/GeV at $E_\gamma$(GeV) = 1.1 − 1.5.

Recent past, the SAPHIR collaboration measured the $\pi^+\pi^-\pi^0$ invariant mass
distribution spectrum in the energy bin: $1.2 < E_\gamma$(GeV) < 1.25 with an additional
constraint: $0.3 < q^2_{min} - q^2$(GeV$^2$) < 0.4 [19]. Since the variation in the beam
energy $E_\gamma$ is negligibly small (less than 50 MeV) in compare to the energy in the
bin, we calculate $\frac{d\sigma}{dm}$ using Eq. (1) at $E_\gamma = 1.225$ GeV for this spectrum. We
compare the calculated result with the measured spectrum (stated above) in Fig. 5.
In this figure, the measured distribution is shown by the histogram whereas the solid
curve represent the calculated spectrum (normalised to the peak of the measured
distribution). The magnitude of the calculated cross section at the peak is about 1.7
(mb/GeV). In this case also, the calculated result is well accord with the measured
distribution.

4 Conclusions

We have calculated the differential cross section for the $\pi^+\pi^-\pi^0$ invariant mass
distribution in the $\gamma p$ reaction in the GeV region. Since the $\omega$ meson couples strongly
to $\pi^+\pi^-\pi^0$ in this energy region, we consider that this event in the final state arises
due to the decay of the $\omega$ meson produced in the intermediate state. The agreement
between the calculated and measured peak positions corroborates this considera-
tion. The reaction amplitude $f_{\gamma p \rightarrow \omega p}$, which is extracted from the latest $\omega$
meson photoproduction data, is used to estimate the magnitude of the cross section. Other
factors in this calculation are evaluated using the well known procedure for them.
Therefore, our calculation gives reliable cross section for the $\pi^+\pi^-\pi^0$ invariant mass
distribution in the $\gamma p$ reaction.

5 Acknowledgements

I gratefully acknowledge A. K. Mohanty, R. K. Choudhury and S. Kailas.
References

[1] Aachen -Berlin -Bonn -Hamburg -Heidelberg -München Collaboration, Phys. Rev. 175, 1669 (1968); M. Davier et al., Phys. Rev. D 01, 790 (1970); Y. Eisenberg et al., Phys. Lett. B 34, 439 (1971).

[2] J. Ballam, et al., Phys. Rev. D 07, 3150 (1973).

[3] Brown -Harvard -MIT -Padova -Weizmann Institute Bubble Chamber Group, Phys. Rev. 155, 1468 (1967).

[4] T. H. Bauer, R. D. Spital, D. R. Yennie and F. M. Pipkin, Rev. Mod. Phys. 50, 261 (1978); D. R. Yennie, ibid. 47, 311 (1975).

[5] H. B. O’Connell, B. C. Pearce, A. W. Thomas and A. G. Williams, Prog. Part. Nucl. Phys. 39, 201 (1997).

[6] J. J. Sakurai, Currents and Mesons (The University of Chicago Press, Chicago and London, 1969) p. 48; R. K. Bhadury, Models of the Nucleon (Addison-Wesley Publishing Company, Inc., New York and Singapore, 1988) p. 261.

[7] G. M. Huber et al., Phys. Rev. C 68, 065202 (2003); G. M. Huber, G. J. Lolos, and Z. Papandreou, Phys. Rev. Lett. 80, 5285 (1998); G. J. Lolos et al., ibid. 80, 241 (1998).

[8] R. Koniuk, Nucl. Phys. B 195, 452 (1982); S. Capstick CEBAF-TH-93-18; S. Capstick and W. Roberts, nucl-th/0008028; S. Capstick Phys. Rev. D 46, 2864 (1992); S. Capstick and W. Roberts, Phys. Rev. D 49, 4570 (1994).

[9] Donald H. Perkins, Introduction to High Energy Physics, second edition (Addison-Wesley Publishing Company, London and Tokyo, 1982) p. 180.

[10] P. Jain, B. Pire and J. P. Ralston, Phys. Rep. 271, 67 (1996).

[11] D. Trnka et al. (CBELSA/TAPS Collaboration), Phys. Rev. Lett. 94, 192303 (2005).

[12] Swapan Das, Phys. Rev. C 78, 045210 (2008).

[13] J. J. Sakurai, Phys. Rev. Lett. 8, 300 (1962).

[14] Particle Data Group, Phys. Rev. D 54, 334 (1996).
[15] J. A. Gómez Tejedor and E. Oset, Nucl. Phys. A 600, 413 (1996); G. I. Lykasov, W. Cassing, A. Sibirtsev and M. V. Rzhanin, Eur. Phys. J. A 6, 71 (1999); M. Gourdin, in Meson Resonance and Related Phenomena, edited by R. H. Dalitz and A. Zichichi (Editrice Compositori Publishers, Bologna, Italy, 1972) p. 219.

[16] A. Pautz and G. Shaw, Phys. Rev. C 57, 2648 (1998).

[17] A. Sibirtsev, H.-W. Hammer, U.-G. Meißner and A. W. Thomas, Eur. Phys. J. A 29, 209 (2006).

[18] A. Sibirtsev, K. Tsushima and S. Krewald, nucl-th/0202083; Phys. Rev. C 67, 055201 (2003).

[19] J. Barth et al., Eur. Phys. J. A 18, 117 (2003).

[20] W. J. Schwille, F. J. Klein, F. Klein, and J. Barth (private communication); Particle Data Group, Phys. Lett. B 667, 340 (2008).

[21] M. Kaskulov, E. Hernandez and E. Oset, nucl-th/0610067; Eur. Phys. J. A 31, 245 (2007).
Figure Captions

1. The dependence of $\Gamma_\omega(m)$ in Eq. (4) and $\Gamma_{\omega\rightarrow\pi^+\pi^-\pi^0}(m)$ in Eq. (5) on the $\omega$ meson mass.

2. The calculated $\omega(\rightarrow \pi^+\pi^-\pi^0)$ meson mass distribution spectra for various beam (gamma) energies are presented. $m$ denotes the mass of the $\omega$ meson, i.e., the $\pi^+\pi^-\pi^0$ invariant mass in the measurement. The histograms (a) represent the $\pi^+\pi^-\pi^0$ invariant mass distribution spectra (along with the background < 10%) measured by Ballam et. al., [2]. They are given in Events/0.02 GeV. The solid curves (b) show the calculated results, i.e., $\frac{d\sigma}{dm}$ due to the Eq. (1). The calculated results are normalized to the measured spectra at the respective peaks.

3. The calculated $\omega(\rightarrow \pi^+\pi^-\pi^0)$ meson mass distribution spectra are compared with the data due to BUBBLE chamber group [3]. In each energy bin, the histogram (a) represents the number of counts for the measured $\pi^+\pi^-\pi^0$ invariant mass distribution spectrum and the dashed curve describes the phase space. The solid curves (b) are related to the calculated cross section $\frac{d\sigma}{dm}$ in Eq. (10), explained in the text.

4. The calculated $\omega(\rightarrow \pi^+\pi^-\pi^0)$ meson mass distribution spectra for various beam (gamma) energy bins have been presented. The curves appearing in this figure are the calculated results due to Eq. (10). This figure distinctly shows the enhancement in the cross section with the beam energy.

5. The calculated $\omega(\rightarrow \pi^+\pi^-\pi^0)$ meson mass distribution spectrum is compared with the data due to SAPHIR collaboration [19]. The histogram (a) represents the measured number of events for the $\pi^+\pi^-\pi^0$ invariant mass distribution spectrum [19]. The solid curve (b) corresponds to the calculated cross section ($\frac{d\sigma}{dm}$ in Eq. (1)), normalized to the peak of the measured distribution.
The width of the $\omega$ meson ($\Gamma_\omega(m)$) is given in Eq. (4) as a function of $m$ (GeV). The width in Eq. (5) for $\omega \rightarrow \pi^+ \pi^- \pi^0(m)$ is also shown in the graph.
\[ p(\gamma,\omega \rightarrow \pi^+\pi^-\pi^0)p \]
\[ E_\gamma = 2.8 \text{ GeV} \]
\[ \text{(a) Experiment} \]
\[ \text{(b) Theory} \]

\[ p(\gamma,\omega \rightarrow \pi^+\pi^-\pi^0)p \]
\[ E_\gamma = 4.7 \text{ GeV} \]
\[ \text{(a) Experiment} \]
\[ \text{(b) Theory} \]

\[ p(\gamma,\omega \rightarrow \pi^+\pi^-\pi^0)p \]
\[ E_\gamma = 9.3 \text{ GeV} \]
\[ \text{(a) Experiment} \]
\[ \text{(b) Theory} \]
p(γ,ω→π⁺π⁻π⁰)p
Eγ(GeV)=1.1-1.5

(a) Experiment
(b) Theory

p(γ,ω→π⁺π⁻π⁰)p
Eγ(GeV)=1.5-1.8

(a) Experiment
(b) Theory

p(γ,ω→π⁺π⁻π⁰)p
Eγ(GeV)=1.8-2.5

(a) Experiment
(b) Theory

p(γ,ω→π⁺π⁻π⁰)p
Eγ(GeV)=2.5-6.0

(a) Experiment
(b) Theory

[dσ/df]_{exp} [normalized to counts]

[dσ/df]_{th} [normalized to counts]
$d\sigma/dm$ (mb/GeV) vs. $m$ (GeV) for $p(\gamma,\omega \rightarrow \pi^+\pi^-\pi^0)p$

$E_\gamma$(GeV):
- (a) 1.1-1.5
- (b) 1.5-1.8
- (c) 1.8-2.5
- (d) 2.5-6.0
$p(\gamma, \omega \rightarrow \pi^+ \pi^- \pi^0)p$

$E_p$(GeV) = 1.225 GeV

$0.3 < q^2_{\text{min}} - q^2(\text{GeV}^2) < 0.4$

(a) Experiment
(b) Theory

\[ \frac{d\sigma}{dm} \times 355 \]