Chapter 3

Antenna-cavity hybrids: matching polar opposites for Purcell enhancements at any linewidth

In this chapter, we demonstrate that a hybrid antenna-cavity system can achieve stronger Purcell enhancements than the cavity or antenna alone. We show that these systems can in fact break the fundamental limit governing a single antenna. Additionally, hybrid systems can be used as a versatile platform to tune the bandwidth of operation to any desired value between that of the cavity and the antenna, while simultaneously boosting Purcell enhancement. The self-consistent analytical model, which we derived in Chapter 2, allows to identify the underlying mechanisms of boosted Purcell enhancement in hybrid systems, including radiation damping and constructive interference between multiple-scattering paths. Finally, we demonstrate that hybrid systems can simultaneously boost Purcell enhancement and maintain a near-unity out-coupling efficiency into a single cavity decay channel, such as a waveguide.
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3.1 Introduction

For many nanophotonic applications, such as single photon sources operated at high frequency [23, 31, 145], nano-scale lasers [25], quantum logical gates for photons [29, 30] and highly sensitive, low detection volume sensing devices [26, 146, 147], strong interactions between a single quantum emitter and light are vital. This interaction can be enhanced by coupling emitters to nanophotonic structures that enhance their emission rates by increasing the local density of states (LDOS) available to the emitter [12], also known as the Purcell effect [14]. Traditionally, this is done by placing emitters in dielectric microcavities. The relative LDOS enhancement of an emitter at resonance with a cavity mode, i.e., the Purcell factor ($F_P$), then relates to the quality factor ($Q$) and the mode volume ($V$) as

$$F_P = \left(\frac{3}{4\pi^2}\right) \left(\frac{\lambda}{n}\right)^3 \left(\frac{Q}{V}\right),$$

(3.1)

with $n$ the index of the medium around the emitter. Microcavity modes typically reach large Purcell factors because of their long photon lifetimes and consequently high quality factors [7]. Additionally, most light is then emitted into a single cavity mode, facilitating efficient collection through e.g., a waveguide, which is a major advantage for applications such as single photon sources [23, 148]. Plasmonic nano-antennas are a popular alternative solution [117, 149]. Rather than storing photons for a very long time, antennas are able to concentrate their energy in volumes far below the diffraction limit [10, 150], thus achieving unparalleled LDOS enhancement over large bandwidths [68].

Both microcavities and antennas also suffer from important drawbacks. Microcavities are limited in their mode volume by the diffraction limit, hence requiring high quality factors to compensate. Unfortunately, high-$Q$ cavities are often extremely sensitive to minor fabrication errors and changes in temperature or environment, making it difficult to scale to multiple connected devices in e.g., a quantum photonic network [29, 30]. Moreover, such narrow resonances typically do not match with the broad emission spectra of room temperature single-photon emitters. Antennas, on the other hand, suffer from strong radiative and dissipative losses, which limit $Q$ to $\sim$10-50. This limits their application in quantum information processing, which requires single emitter-antenna strong coupling, i.e., coupling rates higher than the antenna loss rate [34, 151]. Although strong coupling has been demonstrated very recently for optical antennas supporting highly confined gap modes [70, 77], quantum logical operations remain difficult due to the extremely short coherence times. Also, their non-directional emission patterns tend to make efficient collection of the emission difficult. Ideally, one would be free to choose any desired $Q$, independent of the Purcell factor. An attractive candidate for such tunability is a hybrid antenna-cavity system. Recently such systems were proposed for a selection of applications including emission enhancement [104, 110], molecule or nano-particle detection [85–87, 91–93] and nano-scale
lasers [97, 98]. Recent theoretical work has suggested that an emitter coupled to a high-\(Q\) cavity could gain in LDOS through the inclusion of a small nanoparticle [105]. A similar effect was found in a very recent work for larger nanocore antennas [109]. Another study, however, found a strong suppression of the Purcell effect for a larger, strongly scattering antenna coupled to a cavity [118].

Here we propose hybrid systems as a versatile platform for LDOS enhancements that are not only significantly larger than those of cavities and antennas, but can also be tuned to work over any desired intermediate bandwidth. Using the simple but self-consistent coupled harmonic oscillator model discussed in Chapter 2, we show that enhancements in these systems result from a trade-off between additional losses and confinement, and we elucidate under what conditions one can profit maximally from these effects. We demonstrate that hybrid systems allow to tune the bandwidth of emission — often up to several orders of magnitude increase — while maintaining Purcell factors comparable to or even higher than the bare cavity. Since our model is applicable to any cavity or antenna geometry, this provides a general guideline for designing devices that can match any desired emitter spectrum. Moreover, we propose a realistic design for a hybrid system that can be fabricated lithographically, and find excellent agreement between LDOS spectra from our model and from finite-element simulations on this design. Finally, we demonstrate that hybrid systems can boost LDOS while retaining a high power out-coupling efficiency into a single cavity decay channel (such as a waveguide), making them excellent candidates for single photon sources.

### 3.2 LDOS in hybrids and bare components

We begin by comparing hybrid LDOS enhancements with those in the bare cavity and antenna. For concreteness we focus on a particular example cavity and antenna, for which Fig. 3.1a shows LDOS spectra. In Chapter 2, the relative LDOS in a hybrid system was found as (Eq. (2.46))

\[
\text{LDOS}_{\text{tot}} = 1 + \frac{6\pi\epsilon_0 c^3}{\omega_0^3 n} \text{Im} \left\{ \alpha_H G_{bg}^2 + 2G_{bg}\alpha_H\chi_{\text{hom}} + \chi_H \right\}, \tag{3.2}
\]

with \(\alpha_H\) and \(\chi_H\) the hybridized antenna polarizability and cavity response function, respectively, \(\chi_{\text{hom}}\) the bare cavity response and \(G_{bg}\) the (projected) Greens function of the surrounding environment describing the direct antenna-emitter coupling. The expressions for the bare antenna and cavity LDOS are given in Eqs. (2.47) and (2.48). We assume an antenna in vacuum with a resonance frequency of \(\omega_0/(2\pi) = 460\) THz, an oscillator strength \(\beta = 3V_{\text{ant}}\epsilon_0\omega_0^2\) with \(V_{\text{ant}}\) the volume of a sphere of 50 nm radius, and the ohmic damping rate \(\gamma/(2\pi) = 19.9\) THz of gold [139]. We place the source at 60 nm distance from the antenna center, chosen such that we can safely neglect quenching by dark multipoles [152]. Its dipole moment points away...
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Figure 3.1: LDOS in hybrids and their bare components. (a) LDOS for a dipole coupled to a bare antenna (blue line) or to a set of bare cavity modes (other colors). Cavity resonances are spaced half an antenna linewidth (i.e. 27.1 THz) from each other. Each cavity peak represents a different calculation, indicated by a different color. The antenna limit $r_{\text{ant}}^{\text{lim}}$ discussed in Section 3.3 is shown by the dashed dark grey line. Note that throughout this chapter, LDOS is always taken relative to that of the surrounding medium (vacuum). (b) LDOS for the hybrid system (colored lines) composed of the same elements as shown in (a), compared to $r_{\text{ant}}^{\text{lim}}$ (dashed dark grey line). The peak LDOS derived from a super-emitter approximation (LDOS$_{\text{SE}}$, light grey dashed line) shows good agreement with the narrow peaks away from the antenna resonance. The inset contains a zoom-in on the peak with highest LDOS, showing antenna (blue), cavity (red) and hybrid (green) LDOS. (c) Broadening (yellow) and confinement (purple) of the hybrid system, approximated as a super-emitter, relative to the bare cavity. The cyan line shows the ratio of the confinement and the broadening, which equals LDOS$_{\text{SE}}$ relative to the bare cavity Purcell factor $F_P$.

from the antenna. This yields an LDOS of $\sim$200 at resonance. For the cavity we assume $Q \equiv \omega_c / \kappa = 10^4$ and $V_{\text{eff}}$ to be 10 cubic wavelengths ($\lambda$), leading to a cavity Purcell factor of 76, and typical of modest-confinement cavities, like microdisks. We present results for several different cavity resonance frequencies $\omega_c$.

Fig. 3.1b shows LDOS spectra for hybrid systems at various detunings. Each spectrum has 2 features, corresponding to the two eigenmodes of the system: a broad and a narrow resonance due to modes similar to the bare an-
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tenna and the bare cavity resonance, respectively. A single spectrum example was shown in Fig. 2.3. In the remainder of this chapter, we will focus only on the narrow resonance. Because the source excites both hybrid eigenmodes, the narrow resonance presents a distinct Fano-type lineshape. Importantly, these Fano-resonances show a peak LDOS that can far exceed the LDOS in the bare components. The hybrid system outperforms the antenna at resonance by more than a factor 3, and the cavity by more than a factor 8. At the same detuned frequency, the antenna can be outperformed by up to a factor 25 for the lowest frequency peaks shown. Similar behaviour was also predicted in earlier work for much smaller, quasistatic antennas [105], and very recently for larger nano-cone antennas coupled to a Fabry-Perot microcavity [109]. Contrary to intuition, however, the strongest LDOS is not found for a cavity and an antenna tuned to resonance, but rather for cavities significantly red-detuned from the antenna. On resonance the cavity and antenna modes destructively interfere to yield a strongly suppressed LDOS, consistent with the findings of Frimmer et al. for hybrid system with a strongly radiatively damped antenna [118].

To understand the strong LDOS increase, we can employ a ‘super-emitter’ point of view. This concept was originally proposed by Farahani et al., who claimed that an emitter coupled to an antenna could be considered as one large effective dipole when interacting with its environment [153]. In this view, for a super-emitter coupled to a cavity the emitted power should be given by

\[
P_{\text{dr,SE}} = \frac{\omega}{2} |p_{\text{SE}}|^2 \text{Im} \left\{ \chi \right\},
\]

where \(p_{\text{SE}} = p_{\text{dr}} + p = p_{\text{dr}} (1 + G_{bg} \alpha)\) is the effective dipole moment of the super-emitter, \(\chi\) is the cavity response and \(\alpha\) is the antenna polarizability. First intuition suggests to use both the bare antenna polarizability \(\alpha_{\text{hom}}\) and the bare cavity response \(\chi_{\text{hom}}\) (Eqs. (2.26) and (2.35)). However, Frimmer et al. demonstrated that this procedure fails to describe the dispersive Fano lineshapes and the strongly suppressed LDOS at the antenna resonance [118], which indicates that either antenna, or cavity, or both, are spoiled when tuned on resonance. Better results are obtained if the hybridized polarizability \(\alpha_{\text{H}}\) (Eq. (2.36)) paired with \(\chi_{\text{hom}}\) is used instead. A third, alternative approach would be to use \(\alpha_{\text{hom}}\) and the hybridized cavity response \(\chi_{\text{H}}\) (Eq. (2.37)). Note that, compared to the full, self-consistent expression Eq. (3.2) for LDOS, all three super-emitter descriptions are oversimplified. The merit of using \(\alpha_{\text{hom}}\) and \(\chi_{\text{H}}\) is that it accurately predicts the envelope function (grey dashed curve in Fig. 3.1b) encompassing the Fano features. In this approach, at a hybrid resonance the LDOS experienced by a drive dipole in a super-emitter reads

\[
\text{LDOS}_{\text{SE}} = 3/(4\pi^2) Q'/V'_{\text{eff}},
\]

with \(V'_{\text{eff}} = V_{\text{eff}}/|1 + G_{bg} \alpha_{\text{hom}}|^2\) a perturbed cavity mode volume (in cubic wavelengths) and \(Q' \approx \omega_c/\kappa'\), where \(\kappa' = \kappa + (\omega_c/\epsilon_0 V_{\text{eff}}) \text{Im} \left\{ \alpha_{\text{hom}}(\omega_c) \right\}\). In the second term of \(\kappa'\), one recognizes the familiar result from perturbation theory, which states that a cavity resonance
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is broadened by the scatterer [133]. This super-emitter description thus allows us to describe the LDOS increase as a balance between enhanced broadening and improved confinement.

Fig. 3.1c shows the extra confinement $V_{\text{eff}}/V'_{\text{eff}}$ and broadening $Q/Q'$ of the super-emitter relative to the bare cavity. We see broadening is dominant on the blue side of the resonance, because of increased radiation damping of the antenna for higher frequencies [12]. Confinement, instead, favours detunings to the red of the antenna resonance. This is due firstly to the lower radiation damping, and secondly to the positive sign of $\text{Re} \{\alpha_{\text{hom}}\}$, which leads to constructive interference between source and antenna when radiating into the cavity. On the blue side the effect is opposite.$^*$ Combined, these effects cause the LDOS relative to the bare cavity (cyan line in Fig. 3.1c) to be largest on the red side of the antenna resonance. Based on the expressions for $Q'$ and $V'_{\text{eff}}$, we speculate that confinement can be further boosted without increasing broadening using an antenna with stronger coupling to emitters. For instance, bow-tie antennas [76] and nano-cone antennas [80, 109] have similar dipole moments yet larger field enhancements (captured in $G_{\text{bg}}$). In fact, simulations on a hybrid system composed of a nano-beam cavity and a bow-tie antenna showed a reduction of the cavity mode volume, due to inclusion of the antenna, of more than a factor 1000, with only a minor effect on $Q$ [93]. These results show that hybrid systems can achieve the best of two worlds: a high Q-factor typical for dielectric cavities, combined with a strongly decreased mode volume due to the high field confinement by the antenna. As an example, the inset in Fig. 3.1b shows a hybrid mode with $Q=6.9 \cdot 10^3$ very similar to the bare cavity ($10^4$), but mode volume decreased by an order of magnitude (from $10\lambda^3$ to $0.82\lambda^3$).

3.3 Breaking the antenna limit with hybrid systems

Hybrid systems can improve not only the bare cavity LDOS, but also that of the antenna. In fact, we find that these systems can break the fundamental limit governing antenna LDOS. This limit follows from the well-known upper bound of $3\lambda^2/(2\pi n^2)$ set by energy conservation on the extinction cross section of a single dipolar scatterer, also known as the unitary limit [118, 154, 155]. Consequently its polarizablity is limited to $|\alpha_{\text{lim}}| = \text{Im} \{\alpha_{\text{lim}}\} = (3\epsilon_0/(4\pi^2 n))\lambda^3$. An antenna with an albedo $A = \gamma_r/(\gamma_i + \gamma_r)$ of 1 reaches this limit at its resonance frequency. For an antenna with a finite albedo at resonance, $\text{Im} \{\alpha\} = \text{Im} \{\alpha_{\text{lim}}\} A$. Following Eq. (2.47), this limit on $\alpha$ leads to

$^*$Note that at this small antenna-source distance, $G_{\text{bg}}$ is almost entirely real over the spectrum shown in Fig. 3.1.
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Figure 3.2: LDOS for the hybrid system, split by multiple-scattering path. The LDOS is decomposed into 3 contributions corresponding to the terms in brackets in Eq. (3.2) — the ‘antenna’ term (a), the ‘cross-terms’ (b) and the ‘cavity’ term (c). Each contribution corresponds to a multiple-scattering path, shown in the insets. The grey dotted lines in (a) and (c) show LDOS$_{\text{ant}}^{\text{lim}}$ and the bare cavity Purcell factor $F_P$, respectively.

A limit on antenna LDOS given by

$$\text{LDOS}_{\text{ant}}^{\text{lim}} = 1 + 6\pi\epsilon_0 c^2 / (\omega^3 n) \text{Im} \{ \alpha_{\text{lim}}^2 G_{\text{bg}}^2 \} A(\omega)$$  \hspace{1cm} (3.4)

for an antenna with albedo $A(\omega)$. How a hybrid system can break this limit is best understood by analyzing Eq. (3.2), which indicates that three different multiple-scattering pathways contribute to the LDOS. We will refer to the first, second and last term in brackets in Eq. (3.2) as the ‘antenna’ term, ‘cross-term’ and ‘cavity’ term, respectively. Fig. 3.2 shows the hybrid LDOS from Fig. 3.1b decomposed into these three terms. Fig. 3.2a evidences that the antenna term, corresponding to scattering paths that start and end with an antenna-source interaction, is dominant over most of the spectrum. However, we also recognize that this term alone cannot break the bare antenna limit, shown as the grey dotted line. In other words, not only a bare antenna but also the antenna term in Fig. 3.2a obeys the antenna limit.

In principle there is no reason for a hybrid system, which involves a cavity mode that is not assumed to be dipolar, to be bound by the limit governing a
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single dipolar antenna. Yet it is tempting to think that, since the antenna has a far greater dipole moment than the source and consequently couples more strongly to the cavity, energy transfer between the source and the cavity is completely dominated by the path that passes through the antenna first. In that case, only the antenna term in Fig. 3.2a would contribute, and the limit would be obeyed. This is because the antenna is still a dipolar scatterer bound by energy conservation, and so long as all energy passes through the antenna, LDOS is therefore also bound to the same limit. However, we see in Fig. 3.2b and c that the cavity term and particularly the cross-terms, all of which require direct interaction between cavity and source, contribute significantly to the LDOS. The cavity term in Fig. 3.2c, which represents all scattering paths starting and ending with a direct source-cavity interaction, remains below the cavity Purcell factor \( F_p \), since the perturbed cavity response \( \chi_H \) is always weaker than that of the unperturbed cavity \( \chi_{\text{hom}} \). This stands to reason, given that the antenna spoils the cavity Q. The cross-terms in Fig. 3.2b, on the other hand, contribute strongly to the hybrid LDOS. These terms describe scattering paths starting at the antenna and ending at the cavity, and vice versa. Their contribution is largest on the red side of the antenna resonance \( \omega_0 \) (up to nearly half the total LDOS for the lowest frequency peaks), and switches in sign at \( \omega_0 \). The sign of the cross-term indicates constructive or destructive (negative contribution) interference. In this hybrid system, the interference is between source and antenna radiation into the cavity. From Fig. 3.1b we conclude that the sum of all three LDOS terms breaks the antenna limit, indicated by the dark dashed grey curve, for frequencies where this constructive interference takes place. Thus, through a subtle interference phenomenon, hybrids can attain larger LDOS than the antenna alone could ever achieve.

3.4 The range of effective hybrid Q and V

Hybrid systems do not only offer increased LDOS, they also open up an entirely new range of quality factors and mode volumes. Fig. 3.3 shows a ‘phase diagram’ of Q and V. Plasmonic antennas are found in the bottom left of this diagram, at low Q and V. Conversely, cavities are in the top right, with high Q and V. However, for most applications, neither of these extrema is optimal. For example, if one desires a high Purcell factor, yet wants to avoid strong coupling — demands that are critical to a good, low-jitter single photon source [23] — the high quality factors of cavities are unpractical. A device with an intermediate Q would be ideal, provided that the Purcell factor remains high. Such an intermediate Q would also better match the emission spectrum of an emitter, which is often broader than that of a high-Q cavity yet narrower than that of an antenna [156]. Moreover, to obtain a an optimal trade-off between stability and tunability, one should be able to reach this regime of intermediate Q: high Q renders cavities easily detuned by undesired perturbations,
3.4 The range of effective hybrid $Q$ and $V$

Figure 3.3: Phase diagram of quality factors $Q$ and dimensionless mode volumes $V/\lambda^3$. Shown are the values for the bare antenna (dark circle) and a set of bare cavities (■), as well as the values of the corresponding hybrid modes. The colored lines show corresponding hybrid results for these components, for all cavity-antenna detunings used (see text). For decreasing $\omega_c$ (that is, further red-detuning of the cavity) hybrid $Q$ and $V$ lie closer to those of the bare cavity. The light blue area indicates the location of the hybrid values attained for cavities with $500 < Q < 10^6$ and $0.5^3 < V_{\text{eff}}/\lambda^3 < 20$ (bare cavity parameters indicated by the light red area). Dashed grey lines are lines of constant Purcell factor $F_P$ — that is, constant relative LDOS.

whereas the very low $Q$ of antennas makes them difficult to tune. Here we will show that hybrid systems allow precisely this — choosing the $Q$-factor to a desired, intermediate value, while retaining or even improving on the bare cavity Purcell factor.

In Fig. 3.3, we compare $Q$ and $V_{\text{eff}}$ of modes in hybrid systems with those in the bare cavities and antenna. We assume the same antenna as in Figs. 3.1 and 3.2. Cavities were used with $500 < Q < 10^6$ and $0.5^3 < V_{\text{eff}}/\lambda^3 < 20$, and for each combination of $Q$ and $V_{\text{eff}}/\lambda^3$ we take several cavity resonance frequencies $100 \text{ THz} < \omega_c/2\pi < 433 \text{ THz}$, corresponding to cavity-antenna detunings ranging from 0.5 to 6.6 antenna linewidths. Cavities were always red-detuned from the antenna. To position hybrid structures in this diagram, we calculate LDOS for frequencies around the cavity resonance. We retrieve $Q$ from the linewidth of the Fano-resonance, which is the linewidth of the perturbed cavity mode $\kappa' = \kappa + \delta \kappa$, with $\delta \kappa$ given by Eq. (2.29). While mode volume is only well defined for a single (non-leaky) mode [61–64], here we employ an operational definition through Purcell’s formula.
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(Eq. (3.1)) and the peak value of the LDOS \( \text{LDOS}_{\text{peak}} \). This leads to \( V_{\text{hyb}}^{\text{eff}} = \left( \frac{3}{4\pi^2} \right) Q/\text{LDOS}_{\text{tot}}^{\text{peak}} \), with \( V_{\text{hyb}}^{\text{eff}} \) in units of the cubic resonance wavelength. We use the same definition for the antenna mode volume. Note that, because we keep cavity \( Q \) and \( V_{\text{eff}}/\lambda^3 \) constant when varying \( \omega_c \), cavities with different \( \omega_c \) appear at the same point in Fig. 3.3. Hybrid \( Q \) and \( V \), however, depend strongly on cavity-antenna detuning, as we have seen in Fig. 3.1. Therefore the hybrid systems composed of cavities with different \( \omega_c \) appear as lines in Fig. 3.3.

From Fig. 3.3 we see that hybrid systems provide exactly the tunability discussed earlier: through variation of the cavity-antenna detuning, any practical \( Q \) between that of the cavity and the antenna can be chosen. The subset displayed by the colored lines shows that this extreme tunability typically does not come at the price of LDOS enhancement. If the bare cavity provides an LDOS far below that of the antenna (blue and green), hybrid systems can gain strongly in LDOS compared to the cavity, yet the \( Q \)-factor remains close to that of the bare cavity. For cavities with LDOS similar to the bare antenna (red, purple and yellow), one can gain with respect to both bare components, and \( Q \) can be tuned over a large range while maintaining very high LDOS. As can be expected, the LDOS of the cavities with highest \( Q \) (light blue) is reduced by inclusion of the antenna, as cavities with such narrow resonances are easily spoiled by the losses introduced by an antenna. Yet it is remarkable that an LDOS of order \( 10^3 \) can be maintained over a large range of strongly reduced \( Q \)-factors in such systems. To illustrate the full attainable range of hybrid \( Q \) and \( V \), the light grey area shows where all the hybrid systems are located, for the full range of cavities examined here. From this we see that any \( Q \) between that of the cavity and the antenna can be obtained, at high Purcell factor. In summary, hybrid systems can bridge the gap in \( Q \) and \( V_{\text{eff}} \) between cavities and plasmonic antennas, reaching any desired, practical \( Q \) with similar or better LDOS.

3.5 Finite-element simulations on a realistic hybrid system

Let us now discuss a possible physical implementation of the proposed hybrid systems. We perform finite-element simulations on a realistic antenna-cavity design using COMSOL Multiphysics 5.1, which also serve to verify the validity of our analytical oscillator model. As a cavity, we take a silicon nitride \( (n=1.997) \) disk in vacuum with a radius of 2032 nm and a thickness of 200 nm. To tune the cavity \( Q \) and to help trace how much power flows into the cavity mode we include a small amount of absorption as imaginary component \( (4 \times 10^{-6}) \) in the permittivity of the silicon nitride. The disk supports a radially polarized \( m=22 \) whispering-gallery mode (WGM) at 382.584 THz (\( \sim 784 \) nm).
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with $Q=7.28 \cdot 10^4$ (see Fig. 3.4a and c). The antenna we use is a gold prolate ellipsoid with a long (short) axis radius of 70 (20) nm. Optical constants are described by a modified Drude model [139]. Fig. 3.4b shows the antenna field profile. The hybrid systems is obtained by placing the antenna 50 nm above the disk, just next to the source. In an experiment, one could use an antenna that is fabricated (e.g. by e-beam lithography) directly on top of the disk, as demonstrated earlier for qualitatively similar geometries [97, 157]. In Chapter 5, we will further discuss the experimental implementation of such a system.

To verify the predictions of the oscillator model, we first retrieve LDOS spectra for the bare components from the simulations, and through a fit retrieve all the input parameters for our oscillator model. We then compare the oscillator model prediction for the LDOS spectrum of the hybrid to that obtained from finite-element simulation of the hybrid system.

Figure 3.4: Cross-cuts of the cavity, antenna and hybrid mode profiles. All fields are normalized to their maximum values. Cross-cuts are taken at symmetry planes of the structures. White lines indicate the edges of the structures. (a) Top view and side view of the bare cavity eigenmode. Only the dominant (radial) field component is shown. (b) Field profile of the bare antenna in vacuum, illuminated by an x-polarized plane wave at its resonance frequency. The x-component of the scattered field is shown. The small white circle above the antenna tip indicates where we will place the source dipole. (c) Zoom-in of the bare cavity eigenmode profile. The position of the antenna in the hybrid system is indicated with the dashed line. Note that no antenna was used in this simulation. The position of the drive dipole is indicated beside the antenna tip. (d) Zoom-in of the hybrid eigenmode profile. Hot-spots are visible near the antenna tips.
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From the fit to the bare cavity emission and absorption spectra (see Section 3.A), we find the cavity parameters $\omega_c$, $\kappa_r/2\pi = 5$ GHz, $\kappa_{\text{abs}}/2\pi = 0.3$ GHz and $V_{\text{eff}} = 22.8\lambda^3$. This leads to a peak LDOS (Purcell factor) of 242. The bare antenna spectra yield the antenna parameters $\omega_0/2\pi = 436$ THz, $\gamma_i/2\pi = 18.1$ THz, $\beta = 0.073$ C$^2$/kg and an effective source-antenna distance of 55.2 nm (smaller than the physical source-to-center distance of 70+12 nm owing to the lightning rod effect). These values lead to a bare radiative (absorptive) antenna LDOS of 186 (174) at maximum. Fig. 3.5 shows the comparison between the oscillator model prediction based on these values and the full simulations on the hybrid system where the antenna was placed beside the source, just above the disk, as shown in Fig. 3.4d. We find a peak LDOS of $\sim 914$ in the hybrid system, which is a large increase with respect to the bare cavity (242) and antenna (360 at resonance and $\sim 65$ near cavity resonance). The bandwidth over which this LDOS occurs is increased by a factor 9.4 (to 49 GHz) with respect to the cavity. There is excellent agreement between the model and the simulation for all components of the LDOS. Remaining differences can be largely attributed to errors in the antenna fit (see Section 3.A). These results demonstrate that the oscillator model correctly predicts LDOS in a coupled antenna-cavity system, based on the response of the bare components. Moreover, it shows that a realistic antenna-cavity system can combine the best features of both cavity and antenna, achieving much stronger LDOS than the bare components.

![Figure 3.5: LDOS in a hybrid system from the oscillator model (dashed) and from simulations (solid). We LDOS contributions from scattering into free space (blue) and antenna absorption (red), as well as total LDOS (green). LDOS from to cavity absorption (purple) in the hybrid system is visible in the inset. LDOS from the bare cavity (yellow) is shown for comparison.](image)
3.6 Efficiency of radiation into the cavity

In the previous sections, we demonstrated that hybrid systems allow strongly boosted LDOS at any desired quality factor $Q$. Here we will show that one can also control by hybridization into what channels energy is emitted. Depending on the application, one may e.g. wish to design a system that emits all power into free-space, or rather into a single-mode waveguide. The latter is often the case for an on-chip single-photon source, for example. In Section 2.4, we derived expressions for the power dissipated in the antenna, radiated by antenna and source into free space, and the power flowing into the cavity decay channel. Here we use this to study the fraction of power going into the cavity decay channel, as this is usually most efficiently extracted in e.g. a waveguide. This fraction, i.e. the efficiency of extraction into single mode output channel, is also known as the $\beta$-factor in the context of single-photon sources [23]. Note that, as we generally have not specified the origin of the cavity loss $\kappa$, one could assume it to be dominated by coupling to a waveguide. In experiments this is commonly achieved by evanescent coupling of a cavity to a nearby integrated waveguide or fibre taper [128, 129]. Over-coupling then ensures that the waveguide or taper is the dominant loss channel.

Fig. 3.6a and b show the relative cavity outflux and the peak value of the total hybrid LDOS ($\text{LDOS}^{\text{peak}}_{\text{tot}}$) as function of cavity resonance $\omega_c$ and

![Figure 3.6: Radiation efficiency and LDOS in hybrids. (a) Fraction of power into the cavity decay channel $\kappa$, as function of cavity resonance $\omega_c$ and bare cavity Purcell factor $F_P$. This fraction was evaluated at the peak of the total hybrid LDOS ($\text{LDOS}^{\text{peak}}_{\text{tot}}$, Eq. (3.2)). We use the same antenna as in Figs. 3.1 and 3.3. (b) Peak value of total LDOS in the hybrid system ($\text{LDOS}^{\text{peak}}_{\text{tot}}$), relative to $F_P$. The same cavities and antenna were used as in (a).]
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bare cavity Purcell factor $F_P$. The same cavities and antenna were used as in Fig. 3.3, and detuning now ranged between 0 and 6.6 antenna linewidths. Note that relative cavity outflux and LDOS$_{\text{tot}}^{\text{peak}}$ are fully determined by antenna properties, detuning and $F_P$ (i.e. $Q/V_{\text{eff}}$), not by $Q$ and $V_{\text{eff}}$ separately. There is a large region in which hybrid LDOS can be increased with respect to the cavity, while maintaining a very high fraction of power flux into the cavity channel. This implies that the plasmonic antenna helps to boost LDOS though its field confinement while adding almost no additional losses, consistent with the results in Fig. 3.1c. Fig. 3.6 shows that this works particularly well for cavities with $F_P$ between 10 and $\sim 10^3$. Close to the antenna resonance (460 THz), cavity outflux drops, as power outflux is dominated by the antenna. For very good cavities with $F_P$ around $10^4$, power outflux is also dominated by the antenna, even for far red-detunings. This reflects the fact that either intrinsic cavity losses are very low (high $Q$), or coupling to the antenna is very strong (low $V$). Both cases lead to the antenna decay channels being dominant. Importantly, dominant outcoupling through the antenna does not mean that all the power is dissipated: it is distributed between dipolar radiation and dissipation according to the bare antenna albedo. For applications where radiative efficiency rather than coupling to a waveguide is important, these antenna-dominated regimes can be highly interesting.

In conclusion, one can generally engineer the system in such a way that the power flows in any of the desired channels. Specifically, we have shown that it can be designed for a high extraction efficiency into a single cavity loss channel, such as a waveguide. This is of particular interest for applications such as an on-chip single-photon source with a high $\beta$-factor.

3.7 Conclusions and outlook

We have shown that hybrid antenna-cavity systems can support larger local density of states (LDOS) than either the antenna or the cavity alone. These systems can benefit simultaneously from the high cavity quality factor and the low mode volume of the antenna. This benefit occurs only when the cavity is red-detuned from the antenna. We have demonstrated that this is partly due to the reduced radiation damping of the antenna, and partly due to constructive interference between source and antenna radiation. The latter also allows the LDOS in hybrid systems to break the fundamental limit governing antenna LDOS. Moreover, we have shown that hybrid structures allow to design any desired quality factor while maintaining similar or higher LDOS than the bare cavity and antenna. A study of the cavity power outflux as a fraction of total emitted power demonstrated that one can furthermore engineer the system to emit efficiently into a desired output channel, such as a waveguide. Finally, a physical implementation using a WGM cavity and a gold antenna was proposed and tested using finite-element simulations,
showing strongly increased LDOS and excellent agreement with the oscillator model.

These results highlight hybrid systems as a highly versatile and promising platform for enhancement of light-matter interactions. Such systems can leverage the existing expertise on high-Q cavities and plasmonic antennas for devices that combine the best of both worlds, while avoiding the disadvantages such as losses in the metal. While one has to pay the price of a multi-step fabrication process to integrate cavity, antenna and emitter, the advantage is that it could open up arbitrary bandwidth cavity QED to fit a wide variety of emitters, including single molecules, quantum dot nano-crystals and nano-diamond color centers. This paves the way to further studies, e.g. an experimental demonstration of Purcell enhancements in the proposed design or studies of hybrid systems as efficient interfaces between free-space radiation and on-chip waveguides.
Appendices

3.A Finite-element simulations

In this section we describe the finite-element simulations of the bare cavity and antenna, and of the hybrid system, which were discussed in this chapter. We describe how we retrieved radiative and dissipative LDOS from the simulations, and how we fitted bare component LDOS spectra to obtain the cavity and antenna parameters.

The antenna and cavity geometries are described above in Section 3.2, and shown in Fig. 3.4 there. Cavity spectra are obtained by sweeping the oscillation frequency of a point source placed 50 nm above the disk surface, 300 nm inward from the disk edge (see Fig. 3.4c) and oriented in the radial direction (in a cylindrical coordinate system with the center of the disk as origin). We integrate the Poynting flux over a surface enclosing the cavity and source, and calculate the absorption in the disk, which are then both normalized to Larmor’s formula (Eq. (2.45)) to obtain respectively the radiative and absorptive LDOS.

A similar procedure was used for the antenna LDOS spectra, with the source now placed 12 nm from the tip of the antenna (see Fig. 3.4b) and oriented along the antenna long axis. Radiative and absorptive LDOS was calculated as described above. In the simulation of the hybrid system, the antenna is placed 50 nm above the surface of the disk. The presence of high-index silicon nitride in the antenna near-field red-shifts the antenna resonance frequency slightly (by ~5 THz), which is not captured by the coupled-oscillator model. We account for it by simulating the bare antenna 50 nm above an infinite substrate of silicon nitride.

In Fig. 3.7 we show the LDOS spectra obtained from COMSOL simulations for the bare antenna and bare cavity. Also shown are the fits to these spectra. We use Eq. (2.53) and Eq. (2.56) to fit antenna radiation and dissipation, respectively, with \( \omega_0, \gamma, \beta \) and the antenna-source center-to-center distance \( \Delta r \) as fitting parameters. Fitting the cavity radiation and absorption is done using Eq. (2.59). To account for the two separate cavity decay mechanisms, i.e. radiation and absorption, we replace the prefactor \( \kappa_1 \) in Eq. (2.59) by radiative loss rate \( \kappa_r \) and absorptive loss rate \( \kappa_{\text{abs}} \), respectively, while setting the total cavity loss rate \( \kappa = \kappa_{\text{abs}} + \kappa_r \). Fit parameters are \( \omega_c, \kappa_r, \kappa_{\text{abs}} \) and \( V_{\text{eff}} \). Importantly, for both the cavity and the antenna, the fit routine fits absorption and radiation simultaneously. That is, it calculates the sum of the squared errors between the radiation data and fit, and between the absorption data and fit, and minimizes the sum of the two, using a nonlinear minimization routine. This allows an unambiguous retrieval of the cavity and antenna parameters.

During the simulation of the hybrid system, the antenna was centered 50 nm above the disk and 218 nm from the edge of the disk. This ensured that
3.A Finite-element simulations

Figure 3.7: Fitting the bare cavity and antenna. (a) Fits to the bare antenna LDOS spectra. The grey dashed line indicates the cavity resonance frequency $\omega_c$. The bare antenna was placed 50 nm above an infinite silicon nitride substrate. It can be seen that the fit to the antenna radiation slightly underestimates the radiative LDOS at $\omega_c$, which explains why the prediction of the oscillator model also underestimates radiative LDOS for the hybrid system, as shown in Fig. 3.5. Deviations of the antenna spectra from the lorentzian fits are likely because only a spherical or elipsoidal antenna in vacuum, with metal parameters described by the unmodified Drude model, has a strictly lorentzian lineshape. Here we use a modified Drude model [139]. (b) and (c): Fits to the bare antenna radiative (b) and absorptive (c) LDOS spectra.

The source position with respect to neither disk nor antenna was changed with respect to the simulations of the bare components. The antenna and source positions above the disk were chosen such that cavity mode intensity was approximately equal (within 12%) at the source and the antenna positions, as was assumed in the oscillator model (see Chapter 2).

All finite-element simulations were performed with the frequency domain module (radio frequency) of COMSOL Multiphysics, version 5.1. The simulation domain for the cavity and the hybrid structure was a sphere of 2932 nm radius (i.e. extending 900 nm beyond the disk edge), surrounded by a perfectly matched layer (PML) of 390 nm thickness. For the simulations of the antenna above an infinite substrate, a spherical domain of 819 nm radius with a 390 nm thick PML was used. Because of the mirror symmetry of the geometries, all the simulations used only half of the simulation domain, with a perfect magnetic mirror placed on the symmetry plane. For the antenna and a small (~50 nm) region around the source dipole, a tetrahedral mesh with an element size of 9.8 nm and 12 nm, respectively, was used. The disk cavity was meshed by creating a triangular mesh on the top surface with element sizes of 40 nm and 78 nm at the disk edge and center, respectively,
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and subsequently sweeping this mesh down to the bottom of the disk in 8 partitions. A tetrahedral mesh with maximal element size of 195 nm was used in the vacuum surrounding the structures. The PML used a swept mesh with 6 partitions. For the simulations with a dipolar source on the cavity and hybrid, the iterative solver GMRES was used. Those on the antenna used the direct solver MUMPS. The eigenfrequency calculations done to make Fig. 3.4a,c and d used MUMPS as well. Quadratic element discretization of the electric field was used in all simulations.