COLLECTIVE EXCITATIONS OF MASSIVE DIRAC PARTICLES IN HOT AND DENSE MEDIUM

O.K.Kalashnikov
High-Energy Physics
ICTP, Trieste

Abstract

The one-loop dispersion equation which defines the collective excitations of the massive Dirac particles in hot and dense quark-gluon medium is obtained in the high temperature limit for the case $m<<T$ and solved explicitly for all $|q|$ when $\mu = 0$. Four well-separated spectrum branches (quasi-particle and quasi-hole excitations) are found and their behaviors for the small and large $|q|$ are investigated. All calculations are performed using the temperature Green function technique and fixing the Feynman gauge. The gauge dependency of the spectra found are briefly discussed.

\[1\]Permanent address: Department of Theoretical Physics, P.N.Lebedev Physical Institute, Russian Academy of Sciences, 117924 Moscow, Russia. E-mail address: kalash@td.lpi.ac.ru
1 Introduction

The studying of the collective excitations in hot and dense medium is a very actual problem for the current physics and for the chromodynamics in the first rate. In the medium all particles (fermions the same as bosons) lose their individual properties and the collective excitations arise which (unlike the ordinary vacuum physics at \( T, \mu = 0 \)) have many new peculiarities. Namely these collective excitations determine the bulk of kinetical and thermodynamical properties of hot and dense medium and are very important for many processes taken place, for example, inside the quark-gluon medium. Moreover, inside the quark-gluon medium (when \( \mu \) or \( T \) are nonzero) the new collective (hole) excitations of fermions arise \([1,2]\) which are different from the quasi-particle ones and their peculiarities (e.g. the minimum of the quasi-hole branches at the finite momentum and the "wrong" relation between chirality and helicity) can be exploited for searching the new physical consequences. All these collective modes due to the medium have the effective masses (the same as the plasmon masses for gluons \([3]\)) which independently from the bare masses are generated dynamically and these dynamical masses are not small for the large \( T, \mu \)-parameters. In particular, for the initially massive Dirac particles there is the set of four effective masses \([4,5,6]\) which, in a general case, are well-separated and are always nonzero in the medium.

The goal of this paper is to present the one-loop dispersion equation which defines the collective excitations of the massive Dirac particles in hot and dense quark-gluon plasma in the high temperature limit for the case \( m << T \) and to solve it explicitly for all \( |q| \) when \( \mu = 0 \). We use the standard temperature Green function technique and fix the Feynman gauge for explicit calculations. The case of a zero damping is only considered and many additional problems connected with calculating the damping rate \([7]\) are not discussed. Four well-separated spectrum branches are established and their behaviors for the small and large \( |q| \) are investigated. The gauge dependency of the spectra found are briefly discussed. To start we choose hot and dense QCD although many results are model independently.

2 QCD Lagrangian and quark self-energy

The QCD Lagrangian in covariant gauges has the form

\[
\mathcal{L} = - \frac{1}{4} G_{\mu\nu}^a \,^2 + N_f \bar{\psi} [\gamma_\mu (\partial_\mu - \frac{1}{2} i g \lambda^a V^a_\mu) + m] \psi \\
- \mu N_f \bar{\psi} \gamma_4 \psi + \frac{1}{2 \alpha} (\partial_\mu V^a_\mu)^2 + \bar{C}^a (\partial_\mu \delta^{ab} + g f^{abc} V^c_\mu) \partial_\mu C^b
\]

where \( G_{\mu\nu}^a = \partial_\mu V^a_\nu - \partial_\nu V^a_\mu + g f^{abc} V^b_\mu V^c_\nu \) is the Yang-Mills field strength; \( V^a_\mu \) is a non-Abelian gauge field; \( \psi \) and \( \bar{\psi} \) are the quark fields in the SU(\( N \))-fundamental representation (\( \frac{1}{2} \lambda^a \) are its generators and \( f^{abc} \) are the SU(\( N \))-structure constants) and \( C^a \) and \( \bar{C}^a \) are the ghost Fermi fields. In Eq.(1) \( \mu \) and \( m \) are the quark chemical potential and the bare quark mass, respectively, \( N_f \) is the number of quark flavours and \( \alpha \) is the gauge fixing parameter (\( \alpha = 1 \) for the Feynman gauge). The metric is chosen to be Euclidean and \( \gamma_4^2 = 1 \). We use the exact Schwinger-Dyson equation for the temperature quark Green
function

\[ G^{-1}(q) = G_0^{-1}(q) + \Sigma(q) \]  

(2)

where the quark self-energy in any gauge has the simple representation [8]

\[ \Sigma(q) = \frac{N^2 - 1}{2N} \frac{g^2}{\beta} \sum_{p \neq q} \int \frac{d^3p}{(2\pi)^3} \frac{i\gamma_{\mu}\hat{\gamma}_\mu + 2m}{(p^2 + m^2)(p - q)^2} \]  

(3)

The representation (3) is exact but we calculate \( \Sigma(q) \) only in the one-loop approximation using the bare Green functions in Eq.(3) and fixing the Feynman gauge (i.e. taken the appropriate \( D \)-function). All ultraviolet divergencies are renormalized as usual but the infrared ones (which also arise in the high temperature expansion when \( m << T \)) will be eliminated phenomenologically.

At first the summation over the spinor indices is performed in Eq.(3) using the standard \( \gamma \)-matrix algebra

\[ \Sigma(q) = \frac{N^2 - 1}{2N} \frac{g^2}{\beta} \sum_{p \neq q} \int \frac{d^3p}{(2\pi)^3} \frac{i\gamma_{\mu}\hat{\gamma}_\mu + 2m}{(p^2 + m^2)(p - q)^2} \]  

(4)

and then the summation is performed over the Fermi frequencies \( p_4 = 2\pi T(n + 1/2) \) using the well-known prescription [8]. Here \( \hat{p} = \{(p_4 + i\mu), \mathbf{p}\} \) is the convenient abbreviation for vectors with \( \mu \). All terms found are collected in the convenient form using the simple algebraic trasformations and the final result is given by

\[ \Sigma(q) = -\frac{g^2(N^2 - 1)}{N} \int \frac{d^3p}{(2\pi)^3} \left\{ \frac{1}{\epsilon_p} \left[ \frac{n^+_p [\gamma_4 \epsilon_p + (i\gamma \mathbf{p} + 2m)]}{|q_4 + i(\mu + \epsilon_p) + (q - \mathbf{p})|^2} \right] + \frac{n^B_p}{|\mathbf{p}|} \left[ \frac{m - i q_4 \gamma_4 - [i\gamma(q - \mathbf{p}) + 2m]}{|q_4 + i(\mu + |\mathbf{p}|)} + \epsilon_{p-q}^2 \right] \right\} - \text{h.c.}(m, \mu) \rightarrow -(m, \mu) \]  

(5)

where \( \epsilon_p = \sqrt{\mathbf{p}^2 + m^2} \) is the bare quark energy; \( n^B_p = \{\exp(\beta|\mathbf{p}| - 1)^{-1} \) and \( n^+_p = \{\exp(\beta(\epsilon_p - \mu) + 1)^{-1} \) are the Bose and Fermi occupation numbers, respectively.

For further calculations it is convenient to introduce two new functions and to rewrite Eq.(5) as follows

\[ \Sigma(q) = i\gamma_\mu K_\mu(q) + m Z(q) \]  

(6)

where \( K_\mu(q) = q_\mu a(q) + iu_\mu b(q) \) and \( u_\mu = \{1, 0\} \) is the unit medium vector. All functions separately depend on \( q_4 \) an \( |\mathbf{q}| \) as usual in the medium case. Eq.(6) presents the one-loop decomposition of \( \Sigma(q) \) which, however, is not complete here (see [9] for detail) since a number of functions are generated only in the multi-loop calculations. Using the decomposition (6) we transform Eq.(2) into the form

\[ G(q) = \frac{-i\gamma_\mu (\hat{q}_\mu + K_\mu) + m (1 + Z)}{(\hat{q}_\mu + K_\mu)^2 + m^2 (1 + Z)^2} \]  

(7)

which gives the correct nonperturbative structure for this function. Setting up the determinant of Eq.(7) to be zero, we find the dispersion equation

\[ (\hat{q}_\mu + K_\mu)^2 + m^2 (1 + Z)^2 = 0 \]  

(8)

which defines the collective excitation spectra after the standard analytic continuation.
3 Collective excitations in the high temperature limit

Here we use Eq.(8) to find the dispersion equation for the collective excitations of the massive Dirac particles in hot and dense quark-gluon plasma when $m << T$. The different limits of this equation are discussed and it is solved exactly for the massive fermion case with $\mu = 0$. The spectrum branches are found for all $|q|$ and their limits for the small and large momenta are presented explicitly. The case of a zero damping is only considered and due to this fact our analytical continuation is trivial.

Our starting point is the dispersion equation (8)

$$\left[(iq - \mu) - \vec{K}\right]^2 = q^2 (1 + K)^2 + m^2 (1 + Z)^2$$

(9)

with $m \neq 0$ and we use Eq.(5) to find its high temperature expansion when $m << T$. Here $K = i\vec{K}$ and we take into account only the leading $T^2$-terms with the $\mu/T$-corrections within Eq.(9). In this case all functions which define Eq.(9) can be simplified as follows

$$K(q, \mathbf{q}) = \frac{I_K}{q^2} \left( 1 + \frac{\xi}{2} \ln \frac{\xi - 1}{\xi + 1} \right) + I_B \left( \frac{\xi - 1}{2} (1 - \xi^2) \ln \frac{\xi - 1}{\xi + 1} \right)$$

(10)

$$-\vec{K}(q, \mathbf{q}) = \frac{I_K}{2|q|} \ln \frac{\xi - 1}{\xi + 1} + I_B, \quad -Z(q, \mathbf{q}) = 2I_Z + \frac{2I_B}{|q|} \ln \frac{\xi - 1}{\xi + 1}$$

(11)

that gives a possibility to solve Eq.(9) explicitly. Here $\xi = \omega/|q|$ is a convenient variable and the integrals are defined to be

$$I_K = g^2(N^2 - 1) \int_0^\infty \frac{d|p|}{4\pi^2} |p| \left[ \frac{n^+_p n^-_p}{2} + n^B_p \right]$$

(12)

$$I_B = -g^2(N^2 - 1) \int_0^\infty \frac{d|p|}{8\pi^2} \frac{n^+_p - n^-_p}{2}$$

(13)

$$I_Z = g^2(N^2 - 1) \int_0^\infty \frac{d|p|}{8\pi^2} \frac{n^+_p + n^-_p}{2\epsilon_p}.$$  

(14)

The integral $I_Z$, however, is redefined to avoid the infrared divergencies which arise after the high temperature expansion has been performed for $Z(q, |\mathbf{q}|)$

$$Z(q) = -\frac{g^2(N^2 - 1)}{N} \int \frac{d^3p}{(2\pi)^3} \left\{ \frac{1}{\epsilon_p} \left[ \frac{n^+_p}{[q_4 + i(\mu + \epsilon_p)]^2 + (q - \mathbf{p})^2} \right] + \left[ h.c. (\mu \rightarrow -\mu) \right] \right\}.$$  

(15)

The last expression is extracted from Eq.(5).

Now one should plug the expressions found above into Eq.(9) and perform a number of the algebraic transformations to find $\omega = \xi|\mathbf{q}|$. Here $\omega = (iq_4 - \mu)$ and the variable $\xi$ is more convenient than $|\mathbf{q}|$. The result is the equation of the fourth power with respect
to $\omega(\xi)$

$$
\omega^4[\xi^2 - (1 + b(\xi)I_B)^2] + 2\omega^3 \xi^2 I_B + \omega^2 \xi^2 \left[ I_B^2 - m_R^2 + 2d(\xi)I_K \right]
- 2(1 + b(\xi)I_B)(1 + d(\xi))I_K + 2\omega \xi^2 d(\xi) I_B \left[ I_K + 4m_R^2 \right]
+ I_K^2 \xi^2 \left[ d(\xi)^2 - \xi^2 (1 + d(\xi))^2 \right] - 16m^2 \xi^2 d(\xi)^2 I_B^2 = 0
$$

(16)

where $m_R = m(1 - 2I_Z)$ is the renormalized fermionic mass and functions $d(\xi)$ and $b(\xi)$ are given by

$$
d(\xi) = \frac{\xi}{2} \ln \frac{\xi - 1}{\xi + 1}
$$

$$
b(\xi) = \xi - \frac{1}{2}(1 - \xi^2) \ln \frac{\xi - 1}{\xi + 1}.
$$

(17)

The obtained dispersion equation being very complicated is not solved exactly. However in the long wavelength limit (when $\xi \to \infty$) it can be simplified as follows

$$
[\omega^2 + \omega(I - \eta m_R) - (I_K + 4\eta m_I B)] \cdot [\omega^2 + \omega(I_B + \eta m_R) - (I_K - 4\eta m_I B)] = 0
$$

(18)

and one finds a rather simple solution [6]

$$
\omega(0) = \frac{1}{2} \eta m_R - I_B \pm \sqrt{\frac{\eta m_R - I_B^2}{4} + (I_K + 4\eta m_I B)}
$$

(19)

which demonstrates four well-separated effective masses: two of them are related to the quasi-particle excitations and other two present the quasi-hole ones. Here $\eta = \pm 1$ and the parameters $m$ and $\mu$ are nonzero.

The solutions for all $|q|$ are possible to find within Eq.(16) if either $m$ or $\mu$-parameters are equal to zero.

The case $m = 0$ with $\mu \neq 0$ has been recently considered in detail and the result has the form [6]

$$
E(\xi) = \mu - \frac{\xi I_B}{2(\xi - \eta)} \pm \sqrt{\frac{\xi^2 I_B^2}{4(\xi - \eta)^2} + I_K \xi^2 \left( \frac{\eta}{\xi - \eta} + \frac{\eta}{2} \ln \frac{\xi - 1}{\xi + 1} \right)}
$$

(20)

which extends the well-known result found in [1,2] to the case $\mu \neq 0$. Here we restore the physical variable $E = ip_4$. The variable $\xi$ runs in $1 < \xi < \infty$ and the long wavelength limit corresponds to $\xi \to \infty$. For this limit one finds the very simple result

$$
E(0) = \mu - \frac{I_B}{2} \pm \sqrt{\frac{I_B^2}{4} + I_K}
$$

(21)

which can be compared with the interpolation formula in [10].

Another possibility is the case $m \neq 0$ but with $\mu = 0$ when Eq.(16) can be solved exactly for all $|q|$ as well. Namely this possibility is the subject of this paper and will be discussed below when $m << T$. Now $I_B = 0$ and keeping the proper accuracy of calculations the solution of Eq.(16) is found to be

$$
\omega(\xi)^2 = \frac{\xi^2 (2I_K + m_R^2)}{2(\xi^2 - 1)} \pm \sqrt{\frac{\xi^4}{(\xi^2 - 1)^2} \left[ (b(\xi)I_K)^2 + m_R^2 (I_K + m_R^2/4) \right]}
$$

(22)
These spectra are our main result. They present the collective excitations of massive Dirac particles in hot medium for all $|q|$ when $m \ll T$. Two spectrum branches (when the sign is plus) correspond to the quasi-particle excitations and other two (when the sign is minus) are the quasi-hole ones. These spectrum branches have a rather different asymptotical behaviors and many other different properties.

The long wavelength behaviour of these spectra (when $\xi \to \infty$) has the form

$$\omega_{\pm}(|q|)^2 = M_{\pm}^2 + \left( M_{\pm}^2 \pm \frac{4}{9} \frac{I_K^2}{m_R^2(m_R^2 + 4I_K)} \right) \frac{|q|^2}{M_{\pm}^2} + O(|q|^4)$$

where the effective masses squared are given by

$$M_{\pm}^2 = \frac{m_R^2}{2} + I_K \pm \sqrt{m_R^2\left(\frac{m_R^2}{4} + I_K\right)}.$$  

(24)

These masses are different for four spectrum branches $M_{\pm} = \frac{1}{2}(\eta m_R \pm \sqrt{m_R^2 + 4I_K})$ and are in agreement with the results [4,5]. Here $\eta = \pm 1$.

However, this is not the case when the second term in Eq.(23) is taken into account. This term is not in agreement with one obtained in [4,5]. Although it qualitatively coincides with the result presented in [5] but there is the essential difference with [4] where the linear term was found. It is also important that the quasi-hole spectra $\omega_{-}(|q|)^2$ are very sensitive to the choice of $m,T$-parameters. In many cases these spectra are the monotonical functions for the small $|q|^2$ and the well-known minimum [1] disappears. This minimum always exists only for the massless particles but when $m \neq 0$ the special conditions are necessary to generate it.

In the high momentum region the asymptotical behaviors found for the quasi-particles and the quasi-hole excitations are unlike completely. The quasi-particle spectrum branches are approximated as follows

$$\omega_{+}(|q|)^2 = |q|^2 + (2I_K + m_R^2) - \frac{I_K^2}{|q|^2} \ln \frac{4|q|^2}{2I_K + m_R^2}$$

(25)

where the nonanalytical term is unessential. Another situation takes place for the quasi-hole excitations which do not exist in the vacuum (when $T$ and $\mu$ are equal to zero). They very fast disappear and their asymptotical behaviour is found to be

$$\omega_{-}(|q|)^2 = |q|^2 + 4|q|^2 \exp\left(-\frac{|q|^2(2I_K + m_R^2)}{I_K^2}\right).$$

(26)

In the high momentum region these spectrum branches approach to the line $\omega^2 = |q|^2$ more quickly than (25).

4 Conclusion

To summarize we have obtained and solved the one-loop dispersion equation for the massive fermions at finite temperature. Our solution gives the collective Fermi excitations for all $|q|$ and we establish that they have four well-separated branches: two of them
present the quasi-particle excitations and two other correspond to the quasi-hole ones. The splitting calculated demonstrates that the effective masses for all branches are different when \( m \neq 0 \) and these masses are always nonzero in the medium. The asymptotical behavior found for the small \(|q|\) shows that the difference between the initially massive and massless fermions is kept although the dynamical mass is always generated and all their collective excitations are massive. For the massless fermions one finds that the spectrum minimum always exist and the leading asymptotical term for the small \(|q|\) is linear. However this is not the case for the initially massive fermions. When \( m \neq 0 \) the spectrum minimum, as rule, disappears as well as the linear term and the term \(|q|^2\) gives the leading asymptotic behavior for the small \(|q|\). The gauge invariance of the results found, unfortunately, is not proved and there are not any guaranties that this is, indeed, so. Here the situation is completely unclear and there is only the fact that the dynamical mass for the case \( m, \mu = 0 \) is the gauge invariant object. All other quantities are gauge dependent, in any case, within the one-loop calculations. Of course it is not excluded that the Braaten-Pisarski resummation is necessary to improve this situation but this question is not so evident as for the usual damping rate calculations.

**Acknowledgments**

I would like to thank S. Randjbar-Daemi for the invitation me to the International Centre for Theoretical Physics in Trieste and to all the colleagues of this center for the kind hospitality.

**References**

1.) V. V. Klimov, Yad.Fiz. **33** (1981) 1734 (Sov. J. Nucl. Phys. **33** (1981) 934); Zh. Eksp. Teor. Fiz. **82** (1982) 336 (Sov. Phys. JETP **55** (1982) 199).

2.) H. A. Weldon, Phys. Rev. **D26** (1982) 2789.

3.) O. K. Kalashnikov and V. V. Klimov, Yad. Fiz. **31** (1980) 1357 (Soviet J. Nucl. Phys. **31** (1980) 699).

4.) R. D. Pisarski, Nucl. Phys. **A498** (1989) 423c.

5.) C. Quimbay and S. Vargas-Castrillon, Nucl. Phys. **B451** (1995) 265.

6.) O. K. Kalashnikov, Mod. Phys. Lett. **A12** (1997) 347.

7.) Jean-Paul Blaizot and E. Iancu, Phys. Rev. **D55** (1997) 973.

8.) E. S. Fradkin, Proc. (Trudy ) P.N.Lebedev Physics Inst. **29** (1967) 1.

9.) O. K. Kalashnikov, Pis’ma Zh. Eksp. Teor. Fiz. **41** (1985) 477; (JETP Lett. **41** (1985) 582).

10.) K. Kajantie and P. V. Ruuskaven, Phys.Lett. **B 121** (1983) 352.