Weak hyperon decays in heavy baryon chiral perturbation theory: Renormalization and applications

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Abstract

The complete renormalization of the weak Lagrangian to chiral order $q^2$ in heavy baryon chiral perturbation theory is performed using heat kernel techniques. The results are compared with divergences appearing in the calculation of Feynman graphs for the nonleptonic hyperon decay $\Lambda \to p\pi^-$ and an estimate for the size of the counterterm contributions to the s-wave amplitudes in nonleptonic hyperon decays is given.
1 Introduction

Weak decays of hyperons have been examined using effective field theory methods for more than three decades \cite{1}, but still a number of mysteries exist. The matrix elements of nonleptonic hyperon decays, e.g., can be described in terms of two amplitudes — the parity-violating s-wave and the parity-conserving p-wave. Chiral perturbation theory provides a framework whereby these amplitudes can be expanded in terms of small four-momenta and the current masses $m_q$ of the light quarks, $q = u, d, s$. At lowest order in this expansion the amplitudes are expressed in terms of two unknown coupling constants, so-called low-energy constants (LECs). However, there is no consensus for the determination of these two weak parameters. If one employs values which provide a good fit for the s-waves, one obtains a poor description of the p-waves. On the other hand, a good p-wave representation yields a poor s-wave fit. In order to overcome this problem, one must go beyond leading order. In Refs. \cite{2, 3}, a first attempt was made in calculating the leading chiral corrections to such decays. But the inability to fit s- and p-waves simultaneously remains even after including the lowest nonanalytic contributions. In a recent paper \cite{4} a calculation was performed which included all terms at one-loop order. An exact fit to the data was possible but not unique, and other model-dependent assumptions had to be made in order to estimate the LECs. Another intriguing possibility was examined by Le Yaouanc et al., who asserted that a reasonable fit for both s- and p-waves can be provided by appending pole contributions from $SU(6) (70, 1^-)$ states to the s-waves \cite{5}. Their calculations were performed in a simple quark model and appear to be able to provide a resolution of the s- and p-wave dilemma. The validity of this approach has been confirmed within the framework of chiral perturbation theory, but only after contributions from the lowest lying $1/2^+$ baryon octet resonant states were also taken into account \cite{6}.

Another topic of interest are the radiative hyperon decays: $\Sigma^+ \rightarrow p\gamma$, $\Lambda \rightarrow n\gamma$, etc. Here, the primary problem has been and remains to understand the size of the asymmetry parameter in polarized $\Sigma^+ \rightarrow p\gamma$ decay: $\alpha_\gamma = -0.76 \pm 0.08 \cite{7}$. The difficulty here is associated with the restrictions posed by Hara’s theorem, which requires the vanishing of this asymmetry in the U-spin symmetric limit \cite{8}. Of course, in the real world, U-spin is broken and one should not be surprised to find a nonzero value for the asymmetry — what is difficult to understand is its size. Recent work involving the calculation of chiral loops has also not lead to a resolution, although slightly larger asymmetries can be accommodated \cite{9}. Within that work it was claimed, that in order to obtain a better understanding for the decays one should account for all terms at one-loop order, i.e., including all counterterms, not just the leading log corrections. For a different approach including explicitly baryon resonances which leads to improved agreement with experimental data from radiative hyperon decays see Refs. \cite{10, 11}. This might indicate that weak hyperon decays cannot be described appropriately without the inclusion of baryon resonances. We will not elaborate on this possibility in the present investigation. Rather, one of the main purposes of our paper is to estimate the size of higher order counterterms which might shed some light in the understanding of the origin of the discrepancy between theory and experiment when baryon resonances are not taken into account.

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At one-loop order, divergences appear and are absorbed by infinite LECs from counterterms of the same chiral order. The renormalized low-energy constants are scale-dependent and measurable, i.e., they can in principle be determined from a fit to some observables. They satisfy renormalization group equations under scale changes and, therefore, the choice of another scale leads to modified values of the renormalized LECs. The divergences of the generating functional determine the renormalization group equations and the behavior of the renormalized LECs under scale changes. The sum of the irreducible one-loop functional and the counterterm functional is, of course, finite and scale-independent. Some of these divergences were treated in Ref. [4], which, to our knowledge, is the only work in the weak baryonic sector that performed renormalization explicitly. Obviously, only a subset of the leading divergences were treated in that work. It is our aim to work out all leading divergences in the generating functional of the weak baryon-meson Lagrangian, thus extending the work of Müller and Meißner [12] in the strong sector, using two different techniques and to discuss a few applications. The complete divergence structure might be used as a check in future calculations.

In the next section we present the weak Lagrangian at lowest order. The generating functional to one-loop order is worked out in Sec. 3. The divergent parts of the irreducible tadpole, self-energy and the so-called eye-graph are isolated by using heat kernel techniques. Sec. 4 contains a sample calculation of the divergent pieces of the diagrams for the nonleptonic hyperon decay $\Lambda \to p\pi^-$. The divergent pieces of these diagrams are compared with the results from the heat kernel technique. In Sec. 5 we give an estimate for some counterterm contributions to the s-wave amplitudes in nonleptonic hyperon decays and we summarize our findings in Sec. 6. In the Appendix we extend the recently proposed super heat kernel technique to the weak effective Lagrangian.

2 Effective Lagrangian

We perform our calculations using the lowest order effective Lagrangian within the heavy baryon formalism. To this end, one writes down the most general relativistic Lagrangian which is invariant under chiral and $CP$ transformations. Imposing invariance of the Lagrangian under the transformation $S$, which interchanges down and strange quarks in the Lagrangian, one can further reduce the number of counterterms. We will work in the $CP$ limit so that all LECs are real. This Lagrangian is then reduced to the heavy fermion limit by the use of path integral methods, which deliver the relativistic corrections as $1/\hat{m}$ terms in higher orders. The baryons are described by a four-velocity $v_\mu$ and a consistent chiral counting scheme emerges, i.e., a one-to-one correspondence between the Goldstone boson loops and the expansion in small momenta and quark masses. However, we will not present the relativistic Lagrangian explicitly here, but rather quote only the form of the heavy baryon limit.

The pseudoscalar Goldstone fields ($\phi = \pi, K, \eta$) are collected in the $3 \times 3$ unimodular, unitary matrix $U(x)$,

$$U(\phi) = u^2(\phi) = \exp\{2i\phi/F_0\} \quad (1)$$
with $F_0$ being the pseudoscalar decay constant (in the chiral limit), and

$$\phi = \frac{1}{\sqrt{2}} \begin{pmatrix} \frac{1}{\sqrt{2}} \pi^0 + \frac{1}{\sqrt{6}} \eta & \pi^+ & K^+ \\ \pi^- & -\frac{1}{\sqrt{2}} \pi^0 + \frac{1}{\sqrt{6}} \eta & K^0 \\ K^- & \bar{K}^0 & -\frac{2}{\sqrt{6}} \eta \end{pmatrix}.$$ (2)

Under $SU(3)_L \times SU(3)_R$, $U(x)$ transforms as $U \rightarrow U' = LUR^\dagger$, with $L, R \in SU(3)_{L,R}$. The matrix $B$ denotes the baryon octet,

$$B = \begin{pmatrix} \frac{1}{\sqrt{2}} \Sigma^0 + \frac{1}{\sqrt{6}} \Lambda & \Sigma^+ & p \\ \Sigma^- & -\frac{1}{\sqrt{2}} \Sigma^0 + \frac{1}{\sqrt{6}} \Lambda & n \\ \Xi^- & \Xi^0 & -\frac{2}{\sqrt{6}} \Lambda \end{pmatrix},$$ (3)

which under $SU(3)_L \times SU(3)_R$ transforms as any matter field,

$$B \rightarrow B' = KBK^\dagger,$$ (4)

with $K(U, L, R)$ the compensator field representing an element of the conserved subgroup SU(3)$_V$. At the order we are working, the effective Lagrangian has the form

$$\mathcal{L}_{\text{eff}} = \mathcal{L}_{\phi B} + \mathcal{L}^W_{\phi B} + \mathcal{L}_\phi,$$ (5)

where $\mathcal{L}_\phi$ is the usual (strong and electromagnetic) mesonic Lagrangian, see, e.g., Ref. \[13\].

For the strong meson-baryon Lagrangian $\mathcal{L}_{\phi B}$ one writes

$$\mathcal{L}_{\phi B} = \mathcal{L}^{(1)}_{\phi B} = i < \bar{B}[v \cdot \nabla, B] > + D < \bar{B}S_\mu \{u^\mu, B\} > + F < \bar{B}S_\mu[u^\mu, B] >$$ (6)

with $2S_\mu = i\gamma_5 \sigma_\mu \sigma^\nu$ denoting the Pauli-Lubanski spin vector, $< \ldots >$ is defined as the trace in flavor space and the superscript denotes the chiral order. At lowest order the meson-baryon Lagrangian contains two axial-vector couplings, denoted by $D$ and $F$. The covariant derivative $\nabla_\mu$ of the baryons is decomposed as

$$[\nabla_\mu, B] = \partial_\mu B + [\Gamma_\mu, B]$$ (7)

with $\Gamma_\mu$ being the so-called chiral connection

$$\Gamma_\mu = \frac{1}{2} [u^\dagger(\partial_\mu - i r_\mu)u + u(\partial_\mu - i l_\mu)u^\dagger].$$ (8)

The external fields $v_\mu, a_\mu$ appear in the combinations $r_\mu = v_\mu + a_\mu$ and $l_\mu = v_\mu - a_\mu$. The meson fields are summarized in the quantity

$$u_\mu = \frac{1}{2} [u^\dagger(\partial_\mu - i r_\mu)u - u(\partial_\mu - i l_\mu)u^\dagger].$$ (9)
We will also need the expression \( \chi_+ = 4B_0 \mathcal{M} + \mathcal{O}(\phi^2) \) with \( \mathcal{M} = \text{diag}(m_u, m_d, m_s) \) being the quark mass matrix and \( B_0 = -(0|\bar{q}q|0)/F_0^2 \) the order parameter of the spontaneous symmetry violation.

Having dealt with its strong counterpart, the weak meson-baryon Lagrangian \( \mathcal{L}_{\phi B}^W \) at lowest order is
\[
\mathcal{L}_{\phi B}^{W(0)} = d < \bar{B}\{h_+, B\} > + f < \bar{B}[h_+, B] > .
\] (10)

Here, we have defined
\[
h_+ = u^\dagger h u + u^\dagger h^\dagger u,
\] (11)
with \( h^a_b = \delta^a_2 \delta^b_3 \) being the weak transition matrix. Note that \( h_+ \) transforms as a matter field.

### 3 Renormalization of the weak one-loop generating functional using standard heat kernel techniques

In this section, we turn to the calculation of the complete one-loop generating functional in SU(3) heavy baryon chiral perturbation theory, i.e., to order \( q^3 \) for the strong interaction and to order \( q^2 \) for the weak sector in the small momentum expansion. This extends the renormalization of the strong interacting functional [12] to the weak interaction. The method used in this section was first proposed by Ecker [14] in the two flavor case. We will focus our calculation on applications that are of physical interest, so we neglect weak hyperon decays into two or more pions. Radiative and nonleptonic hyperon decays are of specific interest.

For the calculation it is useful to write the fields in terms of the physical basis, e.g., \( B = B^a \lambda^a \) with \( <\lambda^a\dagger\lambda^b> = \delta^a_2 \delta^b_3 \). We can then write the meson-baryon interaction in the form
\[
S_{\phi B} = \int d^4x B^a \left( A_{\text{str}}^{ab} + A_{\text{W}}^{ab}\right) B^b
\] (12)
with the strong and weak interaction pieces \( A_{\text{str}}^{ab} \) and \( A_{\text{W}}^{ab} \), respectively. Following Refs. [12] and [14] one has to expand
\[
\mathcal{L}_\phi^{(2)} + \mathcal{L}_\phi^{(4)} - \bar{R}^a \left[ A_{(1),\text{str}}^{ab} + A_{(0),\text{W}}^{ab}\right]^{-1} R^b
\] (13)
in the functional integral around the classical solution, \( u_{\text{cl}} = u_{\text{cl}}[j] \), which is obtained by the variation \( \delta \int d^4x \mathcal{L}_\phi^{(2)}/\delta U \) at lowest order. Here, \( j \) collectively denotes the external fields \( v_\mu, a_\mu \) and the quark mass matrix \( \mathcal{M} \). The baryon source fields have also been decomposed into a light component, denoted by \( R \), and a heavy component where the heavy components are not needed for a consistent renormalization and will therefore be omitted. One thus arrives at a set of irreducible and reducible diagrams. From Eq. (13) we immediately derive
\[
\mathcal{L}_\phi^{(2)} + \mathcal{L}_\phi^{(4)} - \bar{R}^a \left[ A_{(1),\text{str}}^{ab}\right]^{-1} R^b + \bar{R}^a \left[ A_{(1),\text{str}}^{ab}\right]^{-1} \left[ A_{(0),\text{W}}^{ab} \right] \left[ A_{(1),\text{str}}^{ab}\right]^{-1} R^b ,
\] (14)
where the first baryon term leads to the well known tadpole, self-energy contributions and the last baryon term is the new eye-graph part with a weak insertion on the intermediate baryon.
line. This problem was solved in Refs. [13] and [16] for the strong interaction with one insertion from the second order Lagrangian. As in the mesonic sector, we choose the fluctuation variables \(\xi\) in a symmetric form [13],

\[
\xi_R = u_{cl} \exp \{i \xi/2\}, \quad \xi_L = u_{cl}^\dagger \exp \{-i \xi/2\},
\]

with \(\xi^\dagger = \xi\) traceless 3x3 matrices. Consequently, we also have

\[
U = u_{cl} \exp \{i \xi\} u_{cl}^\dagger.
\]

At second order in \(\xi\), the covariant derivative \(\nabla_\mu\), the chiral connection \(\Gamma_\mu\), the axial-vector \(u_\mu\) and \(h_+\) take the form

\[
\Gamma_\mu = \Gamma_{cl}^{ab} + \frac{1}{4} [u_{cl}^{ab}, \xi] + \frac{1}{8} \xi \nabla_\mu \xi + O(\xi^3),
\]

\[
[h_+^{ab}, \xi] = \partial_\mu \xi + [\Gamma_{cl}^{ab}, \xi], \quad \xi \nabla_\mu \xi = \xi [\nabla_\mu, \xi] - [\nabla_\mu, \xi],
\]

\[
u_\mu = u_{cl}^{ab} - [\nabla_\mu, \xi] + \frac{1}{8} [\xi, [u_{cl}^{ab}, \xi]] + O(\xi^3),
\]

\[
h_+ = h_+^{ab} + \frac{i}{2} [h_+^{ab}, \xi] + \frac{1}{8} [\xi, [h_+^{ab}, \xi]] + O(\xi^3).
\]

Inserting this into the expression for \(A^{ab}\) and retaining only the terms up to and including order \(\xi^2\) gives

\[
A^{ab}_{(1),\text{str}} = A_{(1),\text{str}}^{ab,cl} + \frac{d}{8} \lambda^{ab} \left[ [v \cdot u_{cl}^{ab}, \xi], \lambda^b \right] > -D/F < \lambda^{ab} \left( [S \cdot u_{cl}^{ab}, \xi], \lambda^b \right)_\pm >
\]

\[
+ \frac{i}{8} \lambda^{ab} \left[ [\xi v \cdot u_{cl}^{ab}, \xi], \lambda^b \right] + \frac{1}{8} D/F < \lambda^{ab} \left( [\xi, [S \cdot u_{cl}^{ab}, \xi]], \lambda^b \right)_\pm > +O(\xi^3),
\]

\[
A^{ab}_{(0),W} = A_{(0),W}^{ab,cl} + \frac{i}{2} \lambda^{ab} \left[ h_+^{ab}, \xi \right], \lambda^b > \frac{1}{8} d/f < \lambda^{ab} \left( [\xi, [h_+^{ab}, \xi]], \lambda^b \right)_\pm > +O(\xi^3).
\]

where we have introduced the compact notation

\[
D/F(\lambda^a,\lambda^b) = D\{\lambda^a,\lambda^b\} + F[\lambda^a,\lambda^b].
\]

The corresponding generating functional reads

\[
Z_{\text{str}}[j, R^a, R^e] = \int d^4x d^4x' d^4y d^4y' R^a(x) S_{(1),\text{str}}^{ac,cl}(x,y) \times [\Sigma_{\text{tad}}(y, y') \delta(y - y') + \Sigma_{\text{self}}(y, y') + \Sigma_{\text{eye}}(y, y')] S_{(1),\text{str}}^{ed,cl}(y', x') R^e(x')
\]

in terms of the tadpole, self-energy and eye-graph functionals. Here, \(S_{(1),\text{str}}^{cd}\) is the full classical fermion propagator with the weak interactions turned off. The functionals are given by

\[
\Sigma_{\text{self}}^{ab} = -\frac{2}{F_0^2} V_{(2)}^{ac} G_{ij} [A_{(1),\text{str}}^{cd,cl}]^{-1} V_{(1)}^{db} = -\frac{2}{F_0^2} V_{(1)}^{ac} G_{ij} S_{(1),\text{str}}^{cd,cl} V_{(1)}^{db}
\]
\[ \Sigma_{\text{eye}}^{ab} = \frac{2}{F_0^2} V_i^{ac} G_{ij} \left[ A^{ce,cl}_{(1),\text{str}} \right] \left[ A^{ef,cl}_{(0),W} \right] \left[ A^{fd,cl}_{(1),\text{str}} \right]^{-1} V_j^{db} \]

\[ \Sigma_{\text{tad}}^{ab} = \frac{1}{8 F_0^2} \left\{ D/F < \lambda_i^\dagger \left( [\lambda_i^G, [S \cdot u^{cl}, \lambda_i^G]] \right), \lambda^b \right\}_+ \geq G_{ij} + d/f < \lambda_i^\dagger \left( [\lambda_i^G, [h^+, \lambda_i^G]] \right), \lambda^b \right\}_+ \geq G_{ij} + i < \lambda_i^\dagger \left( \lambda_i^G (G_{ij} v \cdot \hat{d}_{jk} - v \cdot d_{ij} G_{jk}) \lambda_i^G \right) \}

\[ \Sigma^{ab} = \Sigma_{\text{eye}}^{ab} + \Sigma_{\text{tad}}^{ab} \]

with the following definitions of the vertices

\[ V_i^{ab} = V_{i,\text{str}}^{ab} + V_{i,W}^{ab} \]

\[ V_{i,\text{str}}^{ab(1)} = \frac{i}{4\sqrt{2}} < \lambda_i^\dagger \left( [v \cdot u^{cl}, \lambda_i^G] \right), \lambda^b >, \]

\[ V_{i,\text{str}}^{ab(2)} = -\frac{D/F}{\sqrt{2}} < \lambda_i^\dagger \left( \lambda_i^G S \cdot d_{ji}, \lambda^b \right)_+ > \]

\[ V_{i,W}^{ab(1)} = \frac{d/f}{2\sqrt{2}} < \lambda_i^\dagger \left( [h_i^{cl}, \lambda_i^G] \right), \lambda^b > \]

\[ \Sigma_{\text{weak}}^{ab}(y, y) = \frac{1}{(4\pi F_0)^2} \frac{1}{\epsilon} \left[ \Sigma_{\text{tad}}^{ab}(y, y) + \Sigma_{\text{self}}^{ab}(y, y) + \Sigma_{\text{eye}}^{ab}(y, y) \right], \]

where \( i, j, k, a, b, c = 1, \ldots, 8 \) and \( \lambda_i^G \) denote Gell-Mann’s SU(3) matrices, which are related to the ones in the physical basis by \( \lambda_p = (\lambda^4_i + i\lambda^5_i)/2, \lambda_n = (\lambda^6_i + i\lambda^7_i)/2, \) etc. The quantity \( G_{ij} \) is the full meson propagator \[13\]

\[ G_{ij} = (d_{ij} d^\mu \delta^{ij} + \sigma^{ij})^{-1} \]

with

\[ \left[ \nabla_{cl}^\mu, \xi \right] = \frac{1}{\sqrt{2}} \lambda_{\alpha_i}^G d_{\mu j}^\alpha \xi_k, \quad \xi = \frac{1}{\sqrt{2}} \lambda_{\alpha_i}^G \xi_i, \]

\[ d_{\mu j}^\alpha = \delta_{ij} \partial^\mu + \gamma_{\mu}^\alpha, \quad \gamma_{\mu}^\alpha = -\frac{1}{2} \left( \Gamma_{\alpha}^{\mu \nu} \left[ \lambda_{\alpha_i}^G, \lambda_{\alpha_j}^G \right] \right), \]

\[ \sigma^{ij} = \frac{1}{8} \left[ u_{\mu}^{cl}, \lambda_{\alpha_i}^G \right] \left[ \lambda_{\alpha_j}^G, u_{\mu}^{cl} \right] + \chi_+ \left\{ \lambda_{\alpha_i}^G, \lambda_{\alpha_j}^G \right\} >. \]

Note that the differential operator \( d_{ij} \) is related to the covariant derivative \( \nabla_{cl}^\mu \) and it acts on the meson propagator \( G_{ij} \). The connection \( \gamma_{\mu} \) defines a field strength tensor,

\[ \gamma_{\mu} = \partial_\nu \gamma_{\mu} - \partial_\mu \gamma_{\nu} + [\gamma_{\mu}, \gamma_{\nu}], \]

where we have omitted the flavor indices. We are now in a position to derive the divergences of the one-loop generating functional for weak and strong interaction after contracting trace indices in flavor space. In the heat kernel representation, the divergences appear as simple poles in \( \epsilon = 4 - d \). The beta functions of the strong interacting functional are given in Ref. \[12\] and the new divergent contribution of the weak interaction can be cast in the form (rotated back to Minkowski space)
where \( \hat{\Sigma}^{ab}(y, y) \) are finite monomials in the fields of chiral dimension two. Let us start with the tadpole contribution which is given by

\[
\hat{\Sigma}^{ab}_{\text{tad}}(y, y) = -\frac{1}{4} \frac{d/f}{f} \left\{ -\frac{3}{2} \lambda^a \left\{ \{ h_+, \chi_+ \}, \lambda^b \right\} < \right. \\
- \left. \lambda^a (h_+, \lambda^b) < \chi_+ \right\} - \frac{1}{2} d < \lambda^a \lambda^b < h_+ \chi_+ \right. .
\] (27)

Notice that we have neglected terms of order \( u^2 \) which contain at least two external pions, since they do not contribute to nonleptonic and radiative hyperon decays. The self-energy contribution is given by

\[
\hat{\Sigma}^{ab}_{\text{self}}(y, y) = 6(Df + Fd) < \lambda^a \left\{ \{ h_+, \Sigma^a \Gamma_{\mu \nu} v^\nu \}, \lambda^b \right\} > \\
+ \left( \frac{10}{3} Dd + 6Ff \right) < \lambda^a \left\{ [[ h_+, \Sigma^a \Gamma_{\mu \nu} v^\nu ]], \lambda^b \right\} > \\
+ \frac{3}{2} f < \lambda^a \{ v \cdot u, \{ h_+, [i v \cdot \nabla, \lambda^b] \} > + \text{h.c.} \\
+ \frac{3}{2} f < \lambda^a [v \cdot u, [h_+, [iv \cdot \nabla, \lambda^b]]] > + \text{h.c.} \\
+ \frac{3}{2} d < \lambda^a \{ v \cdot u, [h_+, [iv \cdot \nabla, \lambda^b]] \} > + \text{h.c.} \\
+ \frac{3}{2} d < \lambda^a [v \cdot u, \{ h_+, [iv \cdot \nabla, \lambda^b] \}] > + \text{h.c.} \\
+ 3d < \lambda^a \{ [h_+, [iv \cdot \nabla, v \cdot u]], \lambda^b \} > + 3d < \lambda^a \{ [[iv \cdot \nabla, h_+], v \cdot u], \lambda^b \} > \\
+ 3f < \lambda^a [[h_+, [iv \cdot \nabla, v \cdot u]], \lambda^b] > + 3f < \lambda^a [[iv \cdot \nabla, h_+], v \cdot u], \lambda^b \} > \\
+ 4f < \lambda^a [iv \cdot \nabla, \lambda^b] > v \cdot u h_+ > + \text{h.c.} \\
+ 4f < \lambda^a h_+ > v \cdot u [iv \cdot \nabla, \lambda^b] > + \text{h.c.} .
\] (28)

Here, \( \Gamma_{\mu \nu} \) denotes the field strength tensor of the fields \( \Gamma_\mu \). The last contribution stems from the eye-graph and takes the form

\[
\hat{\Sigma}^{ab}_{\text{eye}}(y, y) = [S^a, S^b] \left\{ (-4(D^2 + F^2) f - 8DFd) < \lambda^a_\dagger \lambda^b > h_+ \Gamma_{\mu \nu} > \\
+ 4f(D^2 - F^2) < \lambda^a_\dagger h_+ > < \Gamma_{\mu \nu} \lambda^b > + \text{h.c.} \\
+ (3DFd - \frac{3}{2} f(D^2 + F^2)) < \lambda^a_\dagger \{ \Gamma_{\mu \nu} \{ h_+, \lambda^b \} \} > + \text{h.c.} \\
+ (-\frac{1}{3} Dd + \frac{3}{2} f(D^2 + F^2)) < \lambda^a_\dagger [\Gamma_{\mu \nu} [h_+, \lambda^b]] > + \text{h.c.} \\
+ (3DF - \frac{3}{2} d(D^2 + F^2)) < \lambda^a_\dagger \{ \Gamma_{\mu \nu} [h_+, \lambda^b] \} > + \text{h.c.} \\
+ (-\frac{17}{3} DFd + \frac{3}{2} d(D^2 + F^2)) < \lambda^a_\dagger [\Gamma_{\mu \nu} \{ h_+, \lambda^b \}] > + \text{h.c.} \right\} \\
+ (6D^2 d - 18 F^2 d + 36 DF f) < [\lambda^a_\dagger, v \cdot \nabla] \{ h_+, [v \cdot \nabla, \lambda^b] \} >
\]
\[
+ (-10D^2f - 18F^2f + 20DFd) < [\chi^a, v \cdot \nabla] [h_+ , [v \cdot \nabla, \lambda^b]] > \\
+ (-2D^2d + 6F^2d - 12DFf) < \lambda^a \{ [v \cdot \nabla, [v \cdot \nabla, h_+]], \lambda^b \} > \\
+ (\frac{10}{3}D^2f + 6F^2f - \frac{20}{3}DFd) < \lambda^a [[v \cdot \nabla, [v \cdot \nabla, h_+]], \lambda^b] > \\
\left\{ \frac{3}{16} \left( \frac{68}{9} D^2d + 4F^2d - 8DFf \right) < \lambda^a \{ h_+, \lambda^b \} > \chi_+ > \\
+ (\frac{68}{9} D^2f + 4F^2f - 8DFd) < \lambda^a [ h_+, \lambda^b ] > \chi_+ > \\
+ (8(D^2 + F^2)d + 8DFf) < \lambda^a \lambda^b > h_+ \chi_+ > \\
+ (\frac{136}{9} D^2d - 8F^2d) < \lambda^a h_+ > \chi_+ \lambda^b > + \text{h.c.} \\
+ (\frac{-23}{3} D^2d - 6DFf + 3F^2d) < \lambda^a \{ \chi_+ \{ h_+, \lambda^b \} \} > + \text{h.c.} \\
+ (-3D^2d + \frac{2}{3} DFf - 3F^2d) < \lambda^a [ \chi_+ [ h_+, \lambda^b ] ] > + \text{h.c.} \\
+ (-\frac{7}{3} D^2f - \frac{2}{3} DFd + 3F^2f) < \lambda^a \{ \chi_+ [ h_+, \lambda^b ] \} > + \text{h.c.} \\
+ (-3D^2f + \frac{2}{3} DFd - 3F^2f) < \lambda^a [ \chi_+ [ h_+, \lambda^b ] ] > + \text{h.c.} \right\} \\
+ (-\frac{68}{9} D^3d - 12D^2Ff - 12DF^2d - 4F^3f) < \lambda^a [ iv \cdot \nabla, \lambda^b ] > S \cdot u h_+ > + \text{h.c.} \\
+ (-4D^3d + 4D^2Ff + 4DF^2d - 4F^3f) < \lambda^a h_+ > S \cdot u [ iv \cdot \nabla, \lambda^b ] > + \text{h.c.} \\
+ (3D^3d + 3D^2Ff + 3DF^2d - 3F^3f) < \lambda^a \{ h_+, \{ S \cdot u [ iv \cdot \nabla, \lambda^b ] \} \} > + \text{h.c.} \\
+ (\frac{1}{3} D^3d + 5D^2Ff + \frac{13}{3} DF^2d - 3F^3f) < \lambda^a [ h_+, [ S \cdot u [ iv \cdot \nabla, \lambda^b ] ] ] > + \text{h.c.} \\
+ (-\frac{7}{3} D^3f + D^2Fd + 3DF^2f - 3F^3d) < \lambda^a \{ h_+, [ S \cdot u [ iv \cdot \nabla, \lambda^b ] ] \} > + \text{h.c.} \\
+ (-D^3f - \frac{1}{3} D^2Fd + 3DF^2f - 3F^3d) < \lambda^a \{ h_+, \{ S \cdot u [ iv \cdot \nabla, \lambda^b ] \} \} > + \text{h.c.} \\
+ (\frac{2}{9} D^3f - \frac{26}{9} D^2Fd - 2DF^2f + 2F^3d) < \lambda^a \{ [[iv \cdot \nabla, h_+], S \cdot u], \lambda^b \} > \\
+ (\frac{2}{3} D^3d - 2D^2Ff + \frac{14}{9} DF^2d + 2F^3f) < \lambda^a \{ [[iv \cdot \nabla, h_+], S \cdot u], \lambda^b \} > \\
+ (-\frac{2}{9} D^3f + \frac{26}{9} D^2Fd + 2DF^2f - 2F^3d) < \lambda^a \{ [ h_+, [ iv \cdot \nabla, S \cdot u ] ], \lambda^b \} > \\
+ (-\frac{2}{3} D^3d + 2D^2Ff + \frac{22}{9} DF^2d - 2F^3f) < \lambda^a \{ [ h_+, [ iv \cdot \nabla, S \cdot u ] ], \lambda^b \} > . \\
\text{(29)}
\]

It is now straightforward to pin down the full weak counterterm Lagrangian at order \(q^2\). We also use the curvature relation for \(\Gamma_{\mu\nu}\)

\[
\Gamma_{\mu\nu} = \frac{1}{4} [u_{\mu}, u_{\nu}] - \frac{i}{2} F_{\mu\nu}^+ \\
\text{(30)}
\]
where $F_{\mu\nu}^+ = uF_{\mu\nu}^{L}u^i + u^i F_{\mu\nu}^{R}u$ with $F_{\mu\nu}^{L/R}$ being the field strength tensors related to the external fields $l_\mu$ and $r_\mu$, respectively. The generating functional can be renormalized by introducing the counterterm Lagrangian for the strong interaction \[12\]

$$L_{\phi B}^{(3)}(x) = \frac{1}{(4\pi F_0)^2} \sum_i d_i B^a(x) \tilde{O}_{i,\text{str}}^{ab}(x) B^b(x)$$

where the $d_i$ are dimensionless coupling constants and the field monomials $\tilde{O}_{i,\text{str}}^{ab}(x)$ are of order $q^3$ and by introducing for the weak interaction the counterterm Lagrangian

$$L_{\phi B}^{(2)}W(x) = \frac{1}{(4\pi F_0)^2} \sum_i h_i B^a(x) \tilde{O}_{i,W}^{ab}(x) B^b(x)$$

where the $h_i$ are dimensionless coupling constants and the field monomials $\tilde{O}_{i,W}^{ab}(x)$ are of order $q^2$. The low-energy constants are decomposed into

$$d_i = d_i^R(\mu) + (4\pi)^2 \beta_i L(\mu)$$
$$h_i = h_i^R(\mu) + (4\pi)^2 \beta_i L(\mu)$$

with $\mu$ being the scale introduced in dimensional regularization and

$$L(\mu) = \mu^{d-4} \left( \frac{1}{(4\pi)^2} - \frac{1}{2} \left[ \log(4\pi) + 1 - \gamma \right] \right)$$

where $\gamma = 0.5772215...$ is the Euler-Mascheroni constant. The $\beta_i$ are dimensionless functions of $F$, $D$ and $f$, $d$ constructed such that they cancel the divergences of the one-loop functional. The renormalized LECs $d_i^R(\mu)$ ($h_i^R(\mu)$) are measurable quantities. They satisfy the renormalization group equations and therefore, the choice of another scale leads to modified values of the renormalized LECs. We remark that the scale-dependence in the counterterm Lagrangian is, of course, balanced by the scale-dependence of the renormalized finite one-loop functional for observable quantities.

The next two sections are devoted to the application of the formulae derived in the present investigation. First, we will show how the divergences can be used to check results obtained from the ordinary computation of Feynman diagrams. Second, the scale-dependence of the renormalized LECs as given in Eq. (33) can be employed to make an estimate of the size of such higher order counterterms.

### 4 A sample calculation

In this section we will discuss the renormalization of the nonleptonic hyperon decay $\Lambda \to p\pi^-$ by calculating explicitly the Feynman graphs and comparing them with the results obtained in the
previous section. In the rest frame of the heavy baryon, \( v_\mu = (1, 0, 0, 0) \), the decay amplitude reduces to the non-relativistic form

\[
A(B_i \to B_j \pi) = \bar{u}_{B_j} \left( A_{ij}^{(s)} + S \cdot k A_{ij}^{(p)} \right) u_{B_i}, \tag{35}
\]

where \( k \) is the outgoing momentum of the pion. Here, \( A_{ij}^{(s)} \) is the parity-violating s-wave amplitude and \( A_{ij}^{(p)} \) is the corresponding parity-conserving p-wave term. In this frame, the energy of the outgoing pion is

\[
v \cdot k = \frac{1}{2m_i} \left( m^2_i - m^2_j + M^2_\pi \right) \tag{36}\]

and the energy of the decaying hyperon in the heavy baryon formulation can be written as

\[
v \cdot p = m_i - \hat{m}. \tag{37}\]

Here, \( m_{i,j} \) are the physical masses of the baryons and \( \hat{m} \) is the mass of the baryon octet in the chiral limit. Since the baryon masses are analytic to linear order in the quark masses we see that \( v \cdot p \) and \( v \cdot k \) count effectively as order \( O(q^2) \). We will therefore restrict ourselves to the computation of the one-loop divergences to this decay which are proportional to the quark mass matrix and do not contain the momenta of the external particles. In our example we present only the renormalization of the weak vertices. The divergent structure of the purely strong baryonic Lagrangian which also contributes to this decay is already given in Ref. [12].

We start by renormalizing the p-wave amplitude. There are four Feynman graphs contributing to the p-wave and containing mass-dependent divergences (see Fig. 1). For the renormalization procedure it is sufficient to consider only the irreducible tadpole from Figures 1a and 1b and the irreducible eye-graph from Figs. 1c and 1d by neglecting the parts from the internal baryon propagator and the strong vertex. The contributions \( P^a \) and \( P^b \) of the irreducible tadpole in Figures 1a and 1b to the decay then read

\[
P^a = -\frac{i}{4F_0^2} (d - f) L(\mu) \left( 3M^2_\pi + 6M^2_K + 3M^2_\eta \right),
\]

\[
P^b = \frac{i}{4F_0^2 \sqrt{6}} (d + 3f) L(\mu) \left( 3M^2_\pi + 6M^2_K + 3M^2_\eta \right). \tag{38}\]

The divergent pieces can be recovered by using the results from the previous section. Employing the counterterms from Eq. (27) and using the Gell-Mann-Okubo relation for the pseudoscalar mesons, \( 4M^2_K = 3M^2_\eta + M^2_\pi \), which is consistent to the order we are working, cancels the divergences from the calculation of the one-loop graphs. Note that the last term in Eq. (27) does not contribute here, since \( \langle h_+ \chi_+ \rangle = 0 \).

The mass-dependent divergences from diagrams 1c and 1d read after neglecting the internal baryon propagator and the strong vertex

\[
P^c = \frac{i}{F_0} L(\mu) \left( M^2_\pi \left[ -\frac{2}{3} D^2 d - 8DF f + 6F^2 d - \frac{10}{3} D^2 f + 4DF d - 6F^2 f \right] \right)\]
\[
P^d = \frac{i\sqrt{6}}{F_0^2} L(\mu) \left( M_\pi^2 \left[ \frac{14}{9} D^2 d - 2 D F f - \frac{4}{3} D^2 f + \frac{4}{3} D F d \right] + M_K^2 \left[ -\frac{19}{18} D^2 d + 5 D F f - \frac{3}{2} F^2 d - \frac{7}{6} D^2 f + \frac{11}{3} D F d - \frac{9}{2} F^2 f \right] \right).
\]

The divergent pieces are in agreement with the mass-dependent divergences in Eq. (29).

The renormalization of the s-wave is somewhat more subtle. In this case it turns out that in addition to the simple irreducible tadpole diagram in Fig. 2a contributing to the s-wave there are three more diagrams (2b,c,d) which lead to the same divergent structure. The pertinent divergent contributions which are proportional to the quark mass matrix for Fig. 2a read

\[
S^a = \frac{1}{24\sqrt{3}F_0^3} (d + 3f) L(\mu) \left( 5M_\pi^2 + 16M_K^2 + 9M_\eta^2 \right)
\]

and for Figures 2c and 2d

\[
S^c + S^d = -\frac{1}{4\sqrt{3}F_0^3} (d + 3f) L(\mu) \left( 2M_\pi^2 + M_K^2 \right).
\]

The sum of graphs 2a, 2c and 2d leads to a divergence structure different than that obtained from the tadpole occurring in the p-wave. Agreement between s- and p-waves is only achieved if one includes the Z-factor of the pion which is divergent and, therefore, contributes to the divergences for nonleptonic hyperon decay. This was not treated correctly in Ref. [4]. From an explicit calculation of the pion Z-factor including the mesonic Lagrangian of fourth chiral order \( L^{(4)} \) one obtains

\[
\sqrt{Z_\pi} = -\frac{1}{F_0^2} L(\mu) \frac{2}{3} \left( 2M_\pi^2 + M_K^2 \right) + \text{finite term}
\]

which is multiplied by the pertinent tree level contribution to the decays

\[
-\frac{1}{2\sqrt{3}F_0}(d + 3f).
\]

Expanding \( h_+ \) in Eq. (27) in terms of the meson fields

\[
h_+ = h - \frac{i}{F_0^2} [\phi, h] + \mathcal{O}(\phi^2, h^1)
\]

the term which is linear in the meson fields reproduces the divergences of the one-loop contributions to the s-wave discussed above. Note that the pseudoscalar decay constant in the chiral limit \( F_0 \) is finite and does not lead to additional divergences.
Finally, Fig. 2e leads to the divergences

\[ S^e = \frac{\sqrt{3}}{F_0} L(\mu) \left( M_\pi^2 \left[ \frac{14}{9} D^2 d - \frac{4}{3} D^2 f + \frac{4}{3} D F d \right] + M_K^2 \left[ -\frac{19}{18} D^2 d + \frac{3}{2} F^2 d - \frac{7}{6} D^2 f + \frac{11}{3} D F d - \frac{9}{2} F^2 f \right] \right) . \] (45)

This result agrees with the mass dependent divergences in Eq. (29).

5 Importance of counterterms

Finally, we would like to address the issue of the importance of higher order counterterms for nonleptonic hyperon decays. There exist seven such transitions: \( \Sigma^+ \to n \pi^+ \), \( \Sigma^+ \to p \pi^0 \), \( \Sigma^- \to n \pi^- \), \( \Lambda \to p \pi^- \), \( \Lambda \to n \pi^0 \), \( \Xi^- \to \Lambda \pi^- \), and \( \Xi^0 \to \Lambda \pi^0 \). Isospin symmetry of the strong interactions implies the relations

\[ A(\Lambda \to p \pi^-) + \sqrt{2} A(\Lambda \to n \pi^0) = 0 \]

\[ A(\Xi^- \to \Lambda \pi^-) + \sqrt{2} A(\Xi^0 \to \Lambda \pi^0) = 0 \]

\[ \sqrt{2} A(\Sigma^+ \to p \pi^0) + A(\Sigma^- \to n \pi^-) - A(\Sigma^+ \to n \pi^-) = 0 \] (46)

which hold both for s- and p-waves. We choose \( \Sigma^+ \to n \pi^+ \), \( \Sigma^- \to n \pi^- \), \( \Lambda \to p \pi^- \), and \( \Xi^- \to \Lambda \pi^- \) to be the four independent decay amplitudes which are not related by isospin. As mentioned in the introduction, both a good s- and p-wave fit cannot be achieved just by taking the lowest order couplings \( d, f \) and chiral corrections into account. This suggests that one has to consider higher order counterterms, but there are so many of them that they cannot be determined uniquely \([4]\). However, as we will show in this section the divergent structure of the one-loop functional can be used to give an estimate of the size of these counterterm contributions. In order to keep the presentation more compact, we restrict ourselves to the case of the s-waves. Neglecting divergences which are proportional to \( v \cdot k \) and \( v \cdot q \) (see last section) the possible counterterms are linear in \( \chi_+ \) and read

\[ L^{W(2,br)}_{\phi B} = h_1 \left\{ \text{tr} \left( \bar{B} \{ h_+, \{ \chi_+, B \} \} \right) \right\} + h_2 \left\{ \text{tr} \left( \bar{B} \{ h_+, [ \chi_+, B ] \} \right) \right\} + h_3 \left\{ \text{tr} \left( \bar{B} \{ h_+, \{ \chi_+, B \} \} \right) + \text{tr} \left( \bar{B} \{ \chi_+, [ h_+, B ] \} \right) \right\} + h_4 \left\{ \text{tr} \left( \bar{B} \{ h_+, [ \chi_+, B ] \} \right) + \text{tr} \left( \bar{B} \{ \chi_+, \{ h_+, B \} \} \right) \right\} + h_5 \left\{ \text{tr} \left( B h_+ \right) \text{tr} ( \chi_+ B ) + \text{tr} \left( B \chi_+ \right) \text{tr} ( h_+ B ) \right\} + h_6 \text{tr} \left( \bar{B} \{ h_+, B \} \right) \text{tr} ( \chi_+ ) + h_7 \text{tr} \left( \bar{B} \{ h_+, B \} \right) \text{tr} ( \chi_+ ) . \] (47)
Note that we did not make any use of Cayley-Hamilton identities in order to have the same set of counterterms as in Eqs. (27) and (29). After renormalization of the mass-dependent divergences the contributions of these counterterms read

\[ A_{\Sigma^-n}^{(s)} = 0 \]

\[ A_{\Sigma^-n}^{(s)} = \frac{2\sqrt{2}}{F_\pi} \left( M_\pi^2 [h_1^r - h_2^r - h_3^r + h_4^r - \frac{1}{2} h_6^r + \frac{1}{2} h_7^r] \right. \]

\[ + M_\pi^2 [h_1^r + h_2^r - h_3^r - h_4^r - h_6^r + h_7^r] \right) \]

\[ A_{\Lambda^p}^{(s)} = \frac{2}{\sqrt{3} F_\pi} \left( M_\pi^2 [3 h_1^r - 3 h_2^r + h_3^r - h_4^r + 2 h_5^r - \frac{3}{2} h_6^r - \frac{1}{2} h_7^r] \right. \]

\[ + M_\pi^2 [-5 h_1^r + 3 h_2^r - 7 h_3^r + h_4^r - 2 h_5^r - 3 h_6^r - h_7^r] \right) \]

\[ A_{\Xi^-\Lambda}^{(s)} = \frac{2}{\sqrt{3} F_\pi} \left( M_\pi^2 [3 h_1^r - 3 h_2^r - h_3^r + h_4^r + 2 h_5^r + \frac{3}{2} h_6^r - \frac{1}{2} h_7^r] \right. \]

\[ + M_\pi^2 [-5 h_1^r + 3 h_2^r + 7 h_3^r - h_4^r - 2 h_5^r + 3 h_6^r - h_7^r] \right) \]

where we have not shown explicitly the scale-dependence of the \( h_i^r \). At the order we are working, we replace \( F_0 \) by the pion decay constant \( F_\pi = 92.4 \text{ MeV} \) since the differences between the two show up at higher orders only. The dependence on a chosen scale \( \mu_1 \) of these counterterm contributions is, of course, compensated by the scale-dependence of the renormalized finite one-loop functional for observable quantities. The choice of another scale \( \mu_2 \) leads to modified values of the renormalized LECs according to the equation

\[ h_i^r(\mu_2) = h_i^r(\mu_1) + \beta_i \log \frac{\mu_1}{\mu_2}. \]

Varying the scale from the \( \rho \)-meson mass \( M_\rho = 770 \text{ MeV} \) to the \( \Delta \) mass \( M_\Delta = 1232 \text{ MeV} \) one obtains the following differences in the counterterm contributions for the s-waves, in units of \( 10^{-7} \),

\[ A_{\Sigma^-n}(\mu_\rho) - A_{\Sigma^-n}(\mu_\Delta) = -0.77 \]  \hspace{1cm} (4.27)

\[ A_{\Lambda^p}(\mu_\rho) - A_{\Lambda^p}(\mu_\Delta) = 0.60 \]  \hspace{1cm} (3.25)

\[ A_{\Xi^-\Lambda}(\mu_\rho) - A_{\Xi^-\Lambda}(\mu_\Delta) = 1.94 \]  \hspace{1cm} (4.51)

which is about 19% for \( \Sigma^- \to n \pi^- \), \( \Lambda \to p \pi^- \) and even 43% for \( \Xi^- \to \Lambda \pi^- \); the experimental values for the decays being given in brackets. There are no contributions at one-loop order for the decay \( \Sigma^+ \to n \pi^+ \)\( [4] \), neither from the counterterms nor from the chiral loops; we have therefore omitted its presentation here. Assuming that the differences of these counterterm contributions for different realistic choices of the scale give an estimate of their absolute value \( \delta A^{(s)} \), we obtain, in units of \( 10^{-7} \),

\[ |\delta A_{\Sigma^-n}^{(s)}| = 0.77 \]

\[ |\delta A_{\Lambda^p}^{(s)}| = 0.60 \]

\[ |\delta A_{\Xi^-\Lambda}^{(s)}| = 1.94. \]  \hspace{1cm} (51)
While the counterterm contributions to the s-waves seem to be well behaved for $\Sigma^- \to n\pi^-$ and $\Lambda \to p\pi^-$, our calculation indicates that they might be significant for $\Xi^- \to \Lambda\pi^-$. 

6 Summary

In this paper, we have performed the chiral invariant renormalization of the weak effective baryon-meson Lagrangian up to one-loop order within heavy baryon chiral perturbation theory. The complete set of counterterms at leading one-loop order $q^2$ with $q$ being an external momentum or meson mass has been constructed. This extends work by Müller and Meißner [12], who considered the strong $SU(3)$ baryon-meson Lagrangian. The present calculation has been performed both using the standard heat kernel formalism and the recently proposed super heat kernel method which has the advantage of simplifying the calculation in intermediate steps. We also compared our results with a direct calculation of the Feynman graphs for the nonleptonic hyperon decay $\Lambda \to p\pi^-$. It turns out that the divergences contained in the $Z$-factor of the pion are essential for achieving agreement between the renormalization of s- and p-wave amplitudes. Since, to our knowledge, there exists only one calculation in the weak baryon-meson sector where some of the divergences at order $O(q^2)$ have been evaluated [4], our work might serve as a reference to check further calculations in the future.

The low-energy constants of higher order counterterms contain, in general, divergent pieces which cancel the divergences from one-loop graphs. The finite remainder of the counterterms is scale-dependent and compensates the scale-dependence of the one-loop functional to give scale independent expressions for physical quantities. For radiative and nonleptonic hyperon decays there exist more counterterms than there are experimental data [4, 4], so that the counterterms cannot be determined uniquely from experiment. Nevertheless, the scale-dependence of the finite remainder of the LECs after renormalization can be used to give an estimate of the size of the counterterm contributions. Since the divergent structure of the one-loop functional determines the scale-dependence of the renormalized LECs, we are able to give such estimates and as an example we have chosen the contributions of the counterterms linear in the quark mass matrix $M$ to the s-waves of the independent nonleptonic hyperon decays that are not related by isospin. We find that for the s-waves the contributions from these counterterms relatively to the experimental values range from 19% for $\Sigma^- \to n\pi^-$ and $\Lambda \to p\pi^-$ up to 43% for the decay $\Xi^- \to \Lambda\pi^-$. 

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Appendix: Renormalization using the super heat kernel formalism

We provide an alternative renormalization prescription by using the super heat kernel formalism as proposed by Ecker, Gasser, and Neufeld [17]. This formalism has already been applied to the effective two- and three-flavor Lagrangian in Refs. [18] and [19], respectively, where it served as a check for previous work [12, 14]. We use this approach to calculate the divergences of the weak effective Lagrangian. This method allows us to verify our previous calculation presented in Sec.3.

The fluctuation action generated by the lowest order meson-baryon Lagrangian has the general form

\[
S^{(2)} = -\frac{1}{2} \xi^T (\partial_\mu D^\mu + Y) \xi + \bar{\eta} (\alpha + \beta_\mu D^\mu) \eta + \xi^T (\delta - \beta_\mu D^\mu) \eta + \bar{\eta} \eta D_\mu \eta , \tag{52}
\]

where \(\psi(\eta)\) are the bosonic (fermionic) fluctuations and

\[
D_\mu = \partial_\mu + X_\mu , \quad D_\mu = \partial_\mu + f_\mu ,
\]

\[
\delta = \alpha - \beta_\mu \partial_\mu , \quad v^2 = 1 , \quad v \cdot \beta = 0 .
\] (53)

\(X_\mu, Y, \) and \(f_\mu\) are bosonic (matrix) fields, whereas \(\alpha\) and \(\beta_\mu\) are fermionic objects. The form of \(\delta\) in Eq. (53) is required by the reality of Eq. (52). Apart from the condition \(v \cdot \beta = 0\) no further assumption about the terms entering in Eq. (52) is made. To be specific, in HBCHPT we find:

\[
X_\mu = \gamma_\mu + g_\mu , \quad (g_\mu)^{ij} = -i \frac{\eta_\mu}{8 F_0^2} < \bar{B} [\lambda_G^i, \lambda_G^j, B] > , \quad i, j = 1, ..., 8 ,
\]

\[
Y = \sigma + s ,
\]

\[
s^{ij} = -\frac{D/F}{4 F_0^2} < B (\lambda_G^i [S \cdot u, \lambda_G^j], B)_+ > - \frac{d/f}{4 F_0^2} < B (\lambda_G^i [S \cdot u, \lambda_G^j], B)_- > ,
\]

\[
f_\mu = f^{str}_\mu + f^W_\mu ,
\]

\[
f^{ab, str}_\mu = < \lambda^{a \dagger} [\Gamma_\mu, \lambda^b] > - i v_\mu D/F < \lambda^{a \dagger} (S \cdot u, \lambda^b)_\pm > ,
\]

\[
f^{ab, W}_\mu = -i v_\mu d/f < \lambda^{a \dagger} (h_+, \lambda^b)_\pm > , \quad a = 1, ..., 8 ,
\]

\[
\alpha^{ai} = \frac{i}{4 F_0} < \lambda^{a \dagger} [v \cdot u, \lambda_G^i], B ] > + \frac{i}{2 F_0} < \lambda^{a \dagger} [h_+, \lambda_G^i], B ] > ,
\]

\[
(\beta_\mu)^{ai} = - \frac{D/F}{F_0} S_\mu < \lambda^{a \dagger} (\lambda_G^i, B)_\pm >
\] (54)

with the definitions of Section 3.

Details of super heat kernel calculation in SU(3) can be found in Ref. [19]. We only use the final result to calculate the structure of the divergences. Using these definitions we easily derive from the following formula the pertinent beta functions and the counterterms

\[
W^{\text{div}}_{L=1 | F_{-\Gamma}} = \frac{i}{48 \pi^2 (d-4)} \int d^4 x \ tr \left\{ -12 \bar{\eta} \eta \nabla_\alpha + 6 \left[ \bar{\eta} \beta_\mu X^{\mu \nu} v_\nu + \beta_\mu \alpha X^{\mu \nu} v_\nu \right] \right\}
\]

16
\[-3 \left[ \bar{\beta} \cdot \beta v \cdot \hat{\nabla}Y + 2 \bar{\beta}_\mu (v \cdot \hat{\nabla} \beta^\mu)Y \right] - 4 \bar{\beta}_\mu (v \cdot \hat{\nabla})^3 \beta^\mu + \bar{\beta} \cdot \beta \hat{\nabla}_\mu X^{\mu \nu} v_\nu \\
+ 6 \bar{\beta}_\mu (v \cdot \hat{\nabla} \beta_\nu) X^{\mu \nu} + 4 \bar{\beta}_\mu \beta_\nu v \cdot \hat{\nabla} X^{\mu \nu} + 2 \bar{\beta}_\mu \beta_\nu \hat{\nabla}_\mu X^{\nu \rho} v_\rho \right] \] 

(55)

where $X^{\mu \nu}$ is the field strength tensor of the fields $X^\mu$. This result can be used since the weak interaction has a relatively simple structure and does not contain any divergences which act on the bosonic or fermionic fluctuation variables. The new eye-graph contribution can be evaluated by redefining the covariant derivative in the most economic way, i.e., $f_\mu = f_\mu^{\text{str}} + f_\mu^{W}$. This shows that the eye-graph without any derivative is immediately obtained in the super heat kernel formalism as it is the case in the standard heat kernel method. But the situation changes dramatically in the presence of derivatives. So far the result is given in a very compact form, and both the beta functions and the corresponding monomials must be evaluated by contracting the various trace indices. In the SU(3) case this is the most tedious part of the calculation and therefore, we skip the presentation of these technicalities. After a lot of cumbersome algebra one ends up with the result presented in the last section.

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Figure captions

Fig.1 Given are the diagrams which contribute to the mass-dependent divergences of the p-wave in the nonleptonic hyperon decay $\Lambda \to p\pi^-$. Solid and dashed lines denote baryons and pseudoscalar mesons, respectively. The solid square represents a weak vertex and the solid circle denotes a strong vertex.

Fig.2 Given are the diagrams which contribute to the mass-dependent divergences of the s-wave in the nonleptonic hyperon decay $\Lambda \to p\pi^-$. Solid and dashed lines denote baryons and pseudoscalar mesons, respectively. The solid square represents a weak vertex and the solid circle denotes a strong vertex.
Figure 1
Figure 2