Balance Functions, Correlations, Charge Fluctuations and Interferometry

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Connections between charge balance functions, charge fluctuations and correlations are presented. It is shown that charge fluctuations can be directly expressed in terms of a balance functions under certain assumptions. The distortion of charge balance functions due to experimental acceptance is discussed and the effects of identical boson interference is illustrated with a simple model.

I. INTRODUCTION

Charge balance functions \[] and charge fluctuations \[, , ,] have been proposed as a means for gaining insight into the dynamics of hadronization in relativistic heavy ion collisions. Both observables are sensitive to the separation, in momentum space, of balancing charges. Such a pair is composed of a positive and negative particle whose charge derives from the same point in spacetime. As a quark-gluon plasma scenario entails a large production of new charges late in the reaction, a tight correlation between the balancing charge/anti-charge pairs would provide evidence of the creation of a novel state of matter.

Both balance functions and charge fluctuations can be expressed in terms of one-particle and two-particle observables. In the following section we present expressions for both balance functions and charge fluctuations in terms of spectra and correlations, and show how the charge fluctuations can be simply expressed in terms of balance functions for neutral systems.

Unfortunately, balance functions and charge fluctuations can both be rather sensitive to detector acceptance. In Sec. \[\] we present a variant of balance functions which reduces acceptance effects for detectors with sharp cutoffs in rapidity. Idential pion correlations also affects both observables in a non-trivial manner. In a simple model utilizing parameters consistent with observed spectra and correlations from RHIC, we illustrate the distortion of the balance functions due to Bose-Einstein correlations in Sec. \[\]. The insights gained from these studies are summarized in Sec. \[\].

II. RELATING BALANCE FUNCTIONS, FLUCTUATIONS AND CORRELATIONS

As mentioned in the introduction, both balance-function and charge-fluctuation observables are generated from one-body and two-body observables which necessitates that they may be expressed in terms of spectra and two-particle correlation functions. In order to express the balance functions in terms of the elementary correlation functions, first define

\begin{equation}
\langle N(a, \Delta_1) \rangle = \int_{\Delta_1} d^3 p \frac{dn_a}{d^3 p} \tag{1}
\end{equation}

and

\begin{equation}
\langle N(b, \Delta_2; a, \Delta_1) \rangle = \int_{\Delta_2} d^3 p_a \int_{\Delta_1} d^3 p_b \frac{dn_{ab}}{d^3 p_a d^3 p_b} \tag{2}
\end{equation}

\begin{equation}
\int_{\Delta_1} d^3 p_a \int_{\Delta_2} d^3 p_b \frac{dn_{a}{\cdot}dn_b}{d^3 p_a d^3 p_b} C(b, \Delta_2; a, \Delta_1) \tag{3}
\end{equation}

where \(\Delta_{1,2}\) are phase space criteria such as rapidity intervals.

In terms of these quantities, the balance function is expressed as

\begin{equation}
B(\Delta_2|\Delta_1) = \frac{1}{2} \{D(-, \Delta_2|+, \Delta_1) - D(+, \Delta_2|+, \Delta_1) \}
+ D(+, \Delta_2|-, \Delta_1) - D(-, \Delta_2|-, \Delta_1) \}, \tag{5}
\end{equation}

where

\begin{equation}
D(b, \Delta_2|a, \Delta_1) = \frac{\langle N(b, \Delta_2; a, \Delta_1) \rangle}{\langle N(a, \Delta_1) \rangle}, \tag{6}
\end{equation}

which can be considered as a conditional probability.

Thus correlation functions and spectra are sufficient to determine balance functions, although the required integration can be somewhat convoluted, depending on the binning, \(\Delta_1\) and \(\Delta_2\). The criteria \(\Delta_1\) is based solely on the momenta of the first particle, while the criteria \(\Delta_2\) might be any function of the momenta of both particles,
e.g., it might be determined by the relative rapidity of the two particles.

To establish the correspondence between charge fluctuations and balance functions, consider a balance function binned as a function of the rapidity difference where both particles are required to reside within a fixed rapidity window of size $Y$. For this case $\Delta_1$ constrains the first particle to be within the rapidity window, and $\Delta_2$ constrains the second particle to have a relative rapidity $|y_b - y_a| = \Delta y$ while also existing inside the rapidity window. This binning was applied in preliminary results from STAR reported in \[6\]. Referring to this balance function as $B(\Delta y|Y)$, one can find the charge fluctuation within the rapidity window $0 < y < Y$ by integrating $B(\Delta y|Y)$ in the interval $0 < \Delta y < Y$. In this case,

$$B(Y|Y) = \int_0^Y d\Delta y \, B(\Delta y|Y)$$

$$= \frac{1}{2} \left\{ \frac{\langle N_+ N_- \rangle \Delta}{\langle N_+ \rangle \Delta} + \frac{\langle N_+ N_- \rangle \Delta}{\langle N_- \rangle \Delta} - \frac{\langle N_+ (N_+ - 1) \rangle \Delta}{\langle N_+ \rangle \Delta} - \frac{\langle N_- (N_- - 1) \rangle \Delta}{\langle N_- \rangle \Delta} \right\}$$

where $\langle \cdots \rangle_{\Delta}$ denotes averages in the phase space region $\Delta$. Writing $N_{\pm} = \langle N_{\pm} \rangle + \delta N_{\pm}$, it is not hard to show

$$\frac{\langle (Q - \langle Q \rangle)^2 \rangle}{\langle (\delta N_{ch}) \rangle} = 1 - \int_0^Y d\Delta y \, B(\Delta y|Y) + O \left( \frac{\langle Q \rangle}{\langle (\delta N_{ch}) \rangle} \right)$$

where $Q = N_+ - N_-$ and $N_{ch} = N_+ + N_-$. For electric charge, the size of the correction is usually less than 5% in relativistic heavy ion collisions where the number of produced charges is much greater than the net charge. However, for baryon number the additional term is not negligible even at RHIC.

In a boost-invariant system (independent of rapidity) the balance function $B(\Delta y|Y)$ can be related to the balance function for an infinite interval.

$$B(\Delta y|Y) = B(\Delta y|Y = \infty)(1 - \Delta y/Y).$$

The factor $(1 - \Delta y/Y)$ accounts for the probability that a particle’s partner will fall within the rapidity window given that they are separated by $\Delta y$. Also, assuming boost invariance allows one to express the balance functions simply in terms of correlation functions as described in Eq. (\[\ref{eq:balance} \]).

$$B(\Delta y|Y = \infty) = \frac{1}{2} \left\{ \frac{dn_+}{dy} C_{++}(\Delta y) + \frac{dn_-}{dy} C_{--}(\Delta y) - \left( \frac{dn_+}{dy} + \frac{dn_-}{dy} \right) C_{+-}(\Delta y) \right\}.$$  

From the above discussion it is clear that the charge fluctuation is the global measure of the charge correlation and the balance function is a differential measure of the charge correlation and therefore carries more information. The advantage of charge fluctuations is that they carry a clear physical meaning in terms of a grand canonical ensemble \[\text{GCE}\], and can therefore be easily connected to more ideal theoretical models, e.g. Lattice QCD calculations. However, since there are no external sources of charge in heavy ion collisions to warrant a grand canonical treatment, both observables are effectively driven by the dynamics of how balancing charges are formed and separate.

We emphasize here that charge fluctuations were not intended to provide a derivative measure. As can be seen from Eq. (\[\ref{eq:balance} \] ) the charge fluctuation summarizes the balance functions in one number. It gives somewhat different information than the width of the balance function since it is also affected by the height. We do not recommend analyzing charge fluctuations as a function of the size of the rapidity window. If the different sized windows included the same pairs, the values would no longer be statistically independent when plotted against the window size. If the windows are used only once, the information from pairs which occupy adjacent windows is thrown away.

A similar set of issues surfaced in making the connection between fluctuations and correlations in the study of multiplicity distributions analyzed as a function of rapidity \[\ref{eq:balance} \]. A more general connection between fluctuation and inclusive observables can be found in \[\ref{eq:balance} \]. However, it should be noted that factorial moments and scaled factorial moments, which are measures of fluctuation \[\ref{eq:balance} \], offer the opportunity to study $n$-body correlations for $n > 2$ in a manner which, unlike correlations, can be easily collapsed into a single variable.

## III. MINIMIZING ACCEPTANCE EFFECTS IN BALANCE FUNCTIONS

Balance functions analyzed by the STAR collaboration \[\ref{eq:balance} \] were constructed according to the prescription that $p_1$ would refer to any pion that is measured within a specified rapidity window while $p_2$ referred to the relative rapidity, again with the requirement that the second particle was within the rapidity window. In that case,

$$B(\Delta y|Y) = \frac{1}{2} \left\{ \frac{\langle N_{++}(\Delta y) \rangle - \langle N_{++}(\Delta y) \rangle}{\langle N_+ \rangle} \right\}$$

$$+ \frac{\langle N_{--}(\Delta y) \rangle - \langle N_{--}(\Delta y) \rangle}{\langle N_- \rangle} \right\}.$$  

Here $N_{++}(\Delta y)$ counts pairs with opposite charge that satisfy the criteria that their relative rapidity equals $\Delta y$, whereas $N_+$ is the number of positive particles in the same interval. Here the angular bracket represents averaging over the events and $Y$ is the size of the detector rapidity window. From this example, one can readily understand how balance functions identify balancing charges. For any positive charge, there exists only one negative particle whose negative charge derived from the point at which the positive charge was created. By subtracting from the numerator the same object created with
positive-positive pairs, one is effectively subtracting the uncorrelated negatives from the distribution and identifying the balancing charge on a statistical basis.

From the construction of Eq. (11), one can understand the sensitivity of $B(\Delta y/Y)$ to the acceptance size $Y$ by considering a detector which covers a finite range in rapidity.

\[ y_{\text{min}} < y < y_{\text{max}}. \]  

(12)

with $y_{\text{max}} - y_{\text{min}} = Y$. For this example, the balance function must go to zero as $\Delta y$ approaches $Y$. This occurs because the particle satisfying the condition $p_1$ must lie at the extreme boundary of the acceptance in order for the second particle to have a relative rapidity $\Delta y \sim Y$ while remaining in the acceptance. The balance function is thus forced to zero at the limits of the acceptance for trivial reasons.

Of course, the balance function corresponding to a perfect detector, $B(\Delta y/\infty)$, is independent of any particular detector size $Y$. As described in Eq. (4), one can easily correct for the detector acceptance in the boost-invariant case by dividing the balance function by a factor $(1 - \Delta y/Y)$.

These balance functions would not have more information than those created without the correction factor, but the information would more directly address the physics of charge separation rather than reflecting the experimental acceptance. We note that the statistical uncertainties of the corrected balance function will however be quite large as $\Delta y \sim Y$.

More generally, one can correct the balance functions for the acceptance by dividing the numerators in Eq. (8), $N(Q_2, p_2|Q_1, p_1)$, by acceptance factors, $A(Q_1, p_2|Q_1, p_1)$.

\[ B(p_2|p_1) = \frac{1}{2} \left\{ \frac{N(-, p_2|+, p_1)}{A(-, p_2|+, p_1)N(+, p_1)} - \frac{N(+, p_2|+, p_1)}{A(+, p_2|+, p_1)N(+, p_1)} + \frac{N(+, p_2|-, p_1)}{A(+, p_2|-, p_1)N(-, p_1)} - \frac{N(-, p_2|-, p_1)}{A(-, p_2|-, p_1)N(-, p_1)} \right\}. \]  

(13)

The acceptance factor represents the probability that, given a particle $i$ satisfies the criteria $p_1$, a second particle that satisfies $p_2$ would be detected. Since the criteria $(Q_2, p_2)$ may depend on the individual particle that satisfied $(Q_1, p_1)$, it may be simpler to calculate $A$ in terms of $a_i(Q_2, p_2)$ which represents the acceptance into $(Q_2, p_2)$ given the particular particle $i$.

\[ A(Q_2, p_2|Q_1, p_1) = \frac{\sum_{i\in Q_2, p_2} a_i(Q_2, p_2)}{N(Q_1, p_1)}. \]  

(14)

The acceptance probability $a_i(Q_2, p_2)$ would be between zero and unity. We note that the acceptance is effectively accounted for by performing a substitution for the denominators in Eq. (8): 

\[ N(Q_1, p_1) \rightarrow \sum_{i\in Q_1, p_1} a_i(Q_2, p_2) \]  

(15)

For the boost-invariant case above where $p_2$ referred to the relative rapidity, and where the acceptance is represented by simple step functions in rapidity, the probabilities would become

\[ a_i(p_2) = \begin{cases} 
0, & y_{\text{max}} - y_i < \Delta y \text{ and } y_i - y_{\text{min}} < \Delta y \\
1/2, & y_{\text{max}} - y_i < \Delta y \text{ and } y_i - y_{\text{min}} > \Delta y \\
1, & y_{\text{max}} - y_i > \Delta y \text{ and } y_i - y_{\text{min}} < \Delta y \\
1/2, & y_{\text{max}} - y_i > \Delta y \text{ and } y_i - y_{\text{min}} > \Delta y 
\end{cases} \]  

(16)

Given that the bins $p_2$ would be of finite extent, the values might differ from 1/2, 0 or unity if the bin straddled the acceptance. For a boost invariant system, averaging over $y_i$ results in the simple correction factor $(1 - \Delta y/Y)$ mentioned previously.

In general, if the acceptance depends on where in the $(Q_2, p_2)$ bin the second particle lies, one can not calculate the acceptance correction exactly without knowledge of the charge correlation which is unavailable except by measuring the balance function in sufficiently small $p_2$ bins such that the acceptance is effectively uniform within the small bins. This may not be feasible due to statistics. It is our recommendation that such factors $a_i(Q_2, p_2)$ should be kept simple. One can always correct theoretical results for the detector response by applying whatever factor is applied to the experimental analysis. Although comparisons with models could have been made without any corrections, acceptance-corrected balance functions can allow for a more physical interpretation while not compromising the integrity of the analysis.

IV. BOSE-EINSTEIN CORRELATIONS AND BALANCE FUNCTIONS

Although Bose-Einstein correlations only affect identical particles at small relative momentum, they manifest themselves in balance functions despite the fact that the binning in balance functions typically covers a large volume in momentum space. In a related topic, Bose-Einstein correlations (also known as the Hanbury-Brown Twiss effect, HBT) have been observed in rapidity correlations where all charged particles, both positive and negative, were used in the analysis. The manifestations of HBT in balance function derives from the fact that it induces a correlation between a given charge and all other charges, not just those that were created to balance the given charge.

In balance functions the HBT effect should enhance the probability that same-charge particles have small relative momentum, thus providing a dip in the balance function at small relative rapidity. In order to model this effect, we consider pairs of pions with momenta $p_a$ and $p_b$ and opposite charge that are created according to a boost-invariant thermal distribution with a temperature of 190 MeV, thus roughly reproducing the pion spectra measured in Au + Au collisions at RHIC. In addition to the usual contribution to the balance function between $p_a$
and $p_b$, a second component derives from the interaction with other pions from other pairs which in this case have momenta $p_c$ and $p_d$. The thermal distribution describing the first two particles was centered at zero rapidity, while the thermal distribution responsible for emission of the second pair was randomly chosen within ±4 units.

The particles $p_a$ and $p_c$ were assumed to have the same sign, as were the particles with momenta $p_b$ and $p_d$. A contribution to the balance function was constructed using these particles, but with a weight,

$$w = C_{++}(p_a, p_c)C_{--}(p_b, p_d)C_{++}(p_a, p_d)C_{--}(p_b, p_c).$$

(17)

This accounts for the weight due to two-particle interactions. The correlation functions were simple functions of $Q_{ab}(p_a, p_b) \equiv \sqrt{(p_a - p_b)^2}$, which were generated by calculating correlation functions for a spherically symmetric Gaussian source of radius, $R_{\text{unc}} = 7$ fm, again crudely in line with measurements at RHIC[4]. The weights were calculated by averaging the squared relative wave function for two particles, including the Coulomb interaction between the pions. The weight was multiplied by the number of such pairs which came from assuming that there were 200 pion pairs per unit rapidity. Only a fraction, $\lambda = 0.7$, of the pairs were assumed to interact due to the fact that some pions would be created in long-lived decays. The acceptance of the STAR detector, and the fact that only a fraction of the pions would truly be balanced by other pions (rather than by charged kaons or other particles) was roughly accounted for by accepting only 60% of the particles with transverse momenta between 100 MeV/c and 700 MeV/c.

The resulting balance functions are displayed in Fig. 4. When the interactions between particles is neglected, the resulting balance function falls monotonically, and has a width consistent with the temperature. The inclusion of the HBT effect results in a large dip near $\Delta y = 0$, and an enhancement at somewhat larger $\Delta y$. The dip derives from the enhancement of same-sign pairs which results in a negative contribution to the balance function. Since the weight is assigned to the emission of the $cd$ pair, the positive HBT weight contributes to opposite sign pairs with equal strength, but is spread out over a wider range of $\Delta y$. Hence, the balance function is slightly enhanced for $\Delta y \sim 1/2$ from HBT effects.

Also shown in Fig. 4 are calculations where the Coulomb interaction is included. Since Coulomb interactions result in attractions for opposite-sign pairs, and repulsions for same sign pairs, the dip due to HBT is mitigated.

Although the shape of the balance function is visibly altered by the inclusion of two-particle interactions, the mean width changed by only a few percent. The strength of the distortion was proportional to the multiplicity, but the effect is not necessarily weaker for peripheral events. This follows because the HBT correction contributes with a strength proportional to $R^{-3}$. Since the product of $dn/dy$ and $R^{-3}$ stays roughly constant over a wide range of centralities in heavy ion collisions, the distortion due to interactions should not appreciably affect the centrality dependence of the balance function’s width.

V. SUMMARY

This paper covered several technical issues related to balance functions. The conclusion of Sec. II is that charge fluctuations can be related to balance functions in a straightforward manner unless the average net charge is large. In fact, the charge fluctuation can be thought of as a measure of the integrated balance function $B(\Delta y | Y)$ from zero to $Y$. In that sense, it represents a one-component measure of the balance function, just like the mean width.

Section II provided an illustration of how balance functions can be created in such a way as to minimize sensitivity to experimental acceptance. Although the example addressed only problems with finite acceptance in rapidity where the balance function was binned according to relative rapidity, the principles could be applied to balance functions in any variables.

The last section considered the inclusion of two-particle interactions into Balance functions. The effects were shown to be quite visible at small relative rapidity. However, the width of the balance function was not significantly affected by the two-particle interactions. This is encouraging, as it justifies interpreting balance functions as objects that statistically identify balancing partners, while subtracting out contributions from other pairs.

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FIG. 1: The balance function from the simple thermal Bjorken model (line) has been parameterized and filtered to roughly provide rough consistency with measurements of STAR. The inclusion of HBT effects (triangles) gives a dip at small $\Delta y$, while the extra addition of Coulomb (circles) modifies the dip.

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