Gravitational waves from $p$-form inflation

Tsutomu Kobayashi$^1$ and Shuichiro Yokoyama$^2$

1 Department of Physics, Waseda University, Okubo 3-4-1, Shinjuku, Tokyo 169-8555, Japan
2 Department of Physics and Astrophysics, Nagoya University, Aichi 464-8602, Japan

Recently it was shown that an inflationary background can be realized by any $p$-form field non-minimally coupled to gravity. In this paper, we study gravitational waves generated during $p$-form inflation. Even though the background evolution is identical to that in conventional scalar field inflation, the behavior of gravitational waves is different in $p$-form inflation. In particular, we find that the propagation speed of gravitational waves differs from unity in 2- and 3-form inflationary models. We point out that the squared speed becomes negative in the large field models. The small field models are free from pathologies and the correction to the spectrum of gravitational waves turns out to be very small.

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I. INTRODUCTION

Our universe is well described by a homogeneous and isotropic Friedmann-Lemaître-Robertson-Walker background with small fluctuations on it. This picture is supported by the observations of smoothly distributed large scale structure in the universe such as the cosmic microwave background (CMB) radiation. A quasi-de Sitter expansion at early times, i.e., inflation, is a very attractive paradigm to account for the universe which is homogeneous and isotropic to a high degree, and hence the inflationary paradigm is widely accepted. One or more scalar fields (inflatons) are commonly believed to be responsible for inflation, but it is still unclear what inflatons really are. Therefore, it is important to discuss alternative inflationary models that rely not on scalar fields but on other fields. One can even radically argue that since no scalar fields have been discovered in nature, it might be natural to consider inflation driven by other fields! Indeed, an inflationary model driven by vector fields was proposed recently [1] (see also earlier works [2, 3] and other related papers [4, 5, 6, 7, 8, 9, 10, 11]). The essential ingredients of vector inflation are a large number of randomly oriented vector fields and their non-minimal coupling to gravity. A large number of fields are used to make the model compatible with isotropy of the background. The non-minimal coupling to gravity is required in order to realize slow-roll inflation. Later, it was shown that inflation can be driven by any $p$-form field, with scalar and vector inflation being the special cases with $p = 0$ and $p = 1$, respectively [12].

In this paper, we study the behavior of gravitational waves generated during $p$-form inflation. Gravitational waves are in general produced quantum mechanically in the quasi-de Sitter stage of the early universe [13]. They can be observed indirectly via imprints on the CMB and directly by future detectors such as LISA [14] and DECIGO/BBO [15]. Therefore, the inflationary gravitational wave is a powerful probe into the early universe. Gravitational waves from 1-form inflation were already investigated in [16].

This paper is organized as follows. In the next section we give a brief review of $p$-form inflation. Then, in Sec. III we consider tensor perturbations (gravitational waves) and derive the actions governing their behavior on the $p$-form inflationary background. We quantize the derived actions in Sec. IV. Finally we draw our conclusions in Sec. V. Calculation details are presented in Appendix.

II. $p$-FORM INFLATION

We begin with a brief review of a $p$-form inflationary background. The metric is given by

$$ds^2 = a^2(\eta) \left( -d\eta^2 + \delta_{ij}dx^i dx^j \right).$$

(1)

The first example is vector (i.e., 1-form) inflation proposed in [1]. Apparently, vector fields are incompatible with isotropy of background cosmology because they induce off-diagonal spatial components of the energy-momentum...
tensor. However, one can evade this problem by invoking three mutually orthogonal vector fields or a large number $N$ of randomly oriented fields. The former case was first analyzed in the context of dark energy [3]. In the latter case, anisotropy is statistically suppressed [1]. The action for vector inflation driven by a large number of fields is given by

$$S = \int d^4x \sqrt{-g} \left\{ \frac{R}{2\kappa^2} + \sum_{a=1}^{N} \left[ -\frac{1}{4} F_{\mu\nu}^{(a)2} - V(I^{(a)}) + \frac{1}{12} R I^{(a)} \right] \right\},$$

(2)

where $I^{(a)} := A^{(a)2}_\mu$ and $F_{\mu\nu}^{(a)} := \nabla_\mu A^{(a)}_\nu - \nabla_\nu A^{(a)}_\mu$. Note here that the vector fields are non-minimally coupled to gravity. Without this coupling the vector fields would have an effective mass term of order the Hubble rate, $H$, which makes it difficult to realize slow-roll inflation. The non-minimal coupling in Eq. (2) cancels this contribution, and the equation of motion for each $B_i^{(a)} := A^{(a)1}_i/a$ reduces to the equation for a (minimally coupled) scalar field [1]. (The equation of motion also implies $A^{(a)}_0 = 0$.) Let us assume that $N$ fields all have the magnitude of order $B$ initially (i.e., $B_i^2 = B^2$). Then, the energy-momentum tensor is given by $T_{00}^i \simeq -N[(B')^2/2a^2 + V]$ and $T_{ij} \simeq N[(B')^2/2a^2 - V]\delta_i^j$, where a prime stands for the derivative with respect to $\eta$. In deriving the expression for the energy-momentum tensor we used the formula

$$\sum_{a=1}^{N} B_i^{(a)} B_j^{(a)} \simeq \frac{N}{3} B^2 \delta_{ij} + \mathcal{O}(1) \sqrt{N} B_i B_j,$$

(3)

and omitted terms corresponding to the subleading contributions. Thus, for a wide class of potentials we get an inflationary background which is very similar to usual scalar field inflation, or, more precisely, what is called Nflation [17].

Recently, vector inflation was generalized to the cases with $p$-form fields in [12]. 2-form inflation is driven by a large number of 2-form fields and can be described by the action

$$S = \int d^4x \sqrt{-g} \left\{ \frac{R}{2\kappa^2} + \sum_{a=1}^{N} \left[ -\frac{1}{12} F_{\mu\nu\rho}^{(a)2} - V(I^{(a)}) + \frac{R}{6} I^{(a)} + \frac{1}{2} A^{(a)}_\mu R^{\rho\sigma} A^{\rho\sigma}_\mu \right] \right\},$$

(4)

where $I^{(a)} := A^{(a)2}_\mu$ and

$$F_{\mu\nu\rho}^{(a)} := \nabla_\mu A^{(a)}_{\nu\rho} + \nabla_\nu A^{(a)}_{\rho\mu} + \nabla_\rho A^{(a)}_{\mu\nu}.$$

(5)

The field equations for each $A_{\mu\nu}$ obtained from the action (4) imply $A_{0i} = 0$ and

$$B''_i + 2\mathcal{H} B'_i + 4a^2 V_I B_i = 0,$$

(6)

where $\mathcal{H} := a'/a$, and, instead of $A_{ij}$, we used the field $B_i$ defined by

$$A_{ij} = a^2 \varepsilon_{ijk} B_k(\eta)$$

(7)

with $\varepsilon_{ijk}$ being the totally antisymmetric symbol. Noting that $I = 2B^2$ for the background, one sees that Eq. (6) coincides with the equation of motion for a inflaton field. We are considering a large number $N$ of 2-form fields so that we may use the formula (3) also in this case. Assuming again that $N$ fields all have the magnitude of order $B$ initially (i.e., $B_i^2 = B^2$), we have the estimate $T_{00}^i \simeq -N[(B')^2/2a^2 + V], T_{ij} \simeq 0$, and $T_{ij}^j \simeq N[(B')^2/2a^2 - V]\delta_i^j$, where subleading terms are statistically suppressed. We thus have the background Einstein equations

$$3\mathcal{H}^2 = \kappa^2 N \left[ \frac{(B')^2}{2} + a^2 V(I) \right],$$

(8)

$$2\mathcal{H}' + \mathcal{H}^2 = -\kappa^2 N \left[ \frac{(B')^2}{2} - a^2 V(I) \right].$$

(9)

In the case of chaotic inflation, the potential is given by $V(I) = m^2 I/4$.

Let us move on to the case of the 3-form field. The action for 3-form inflation is

$$S = \int d^4x \sqrt{-g} \left\{ \frac{R}{2\kappa^2} - \frac{1}{48} F_{\mu\nu\rho\sigma}^{2} - V(I) + \frac{1}{8} RI - \frac{1}{2} A_{\mu\nu\rho} R^{\rho\sigma} A^{\mu\nu}_\sigma \right\},$$

(10)
where $I = A^2_{\mu\nu\rho}$ and

$$F_{\mu\nu\rho\sigma} := \nabla_{\mu} A_{\nu\rho\sigma} - \nabla_{\nu} A_{\mu\rho\sigma} + \nabla_{\rho} A_{\mu\nu\sigma} - \nabla_{\sigma} A_{\mu\nu\rho}.$$  

(11)

Note that a 3-form field is compatible with spatial isotropy and so in this case we do not need to use a large number of fields. We write the ansatz as follows:

$$A_{0ij} = \alpha_{ij}(\eta), \quad A_{ijk} = a^3 \phi(\eta) \varepsilon_{ijk}.$$  

(12)

Substituting this into the field equations for the 3-form field derived from the action (10), one arrives at $\alpha_{ij} = 0$ and

$$\phi'' + 2H\phi' + 12a^2 V_2 \phi = 0,$$  

(13)

where $I$ for the background is given by $I = 6a^2$. Thus, the field $\phi$ evolves according to the same equation of motion as in conventional scalar field inflation. The energy-momentum tensor is found to be $T^0_0 = -(^{\phi'})^2/2a^2 - V$ and $T^{ij}_2 = [(^\phi')^2/2a^2 - V] \delta^i_j$, implying that the 3-form field is indeed compatible with isotropy. We would like to stress that the non-minimal coupling of the 3-form to gravity is essential to obtain the desired inflationary background similar to the standard one.

As is almost clear from the above derivation, 2- and 3-form inflation models have their dual description in terms of 1- and 0-forms. However, they are not equivalent to vector and standard scalar field inflation models. In the dual description, they correspond to vector and scalar field theories with non-minimal kinetic terms [12]. In this sense, p-forms provide novel inflationary models.

### III. ACTION FOR GRAVITATIONAL WAVES

Let us consider tensor perturbations $h_{ij} = h_{ij}(\eta, x)$ on a p-form inflationary background:

$$ds^2 = a^2(\eta) \left[ -d\eta^2 + (\delta_{ij} + h_{ij}) dx^i dx^j \right],$$  

(14)

where $h_{ij}$ is transverse and traceless, $h^i_i = \partial_j h_{ij} = 0$. In the standard inflationary scenarios driven by one or more scalar fields, the behavior of gravitational waves is completely determined by the background geometry, and hence identical expansion histories give the identical evolution of gravitational waves. In the case of p-form inflation, however, the situation is more involved, and p-form inflation models predict different evolution of gravitational waves than that in the corresponding scalar field inflation. The 1- and 2-form cases are particularly difficult to analyze in general, because in contrast to the standard linear perturbation theory, scalar, vector, and tensor modes are coupled due to the presence of the background form fields. This point is however circumvented by considering a large number of randomly oriented fields that suppress the couplings statistically [10]. For example, terms like $B_i \delta B_j$ do not contribute to the equation of motion at leading order. Therefore, we may separate the evolution of gravitational waves from the contributions of vector and scalar perturbations. The behavior of gravitational waves from vector inflation was studied in [10]. In the present paper, we generalize the analysis of [10] to 2- and 3-form inflation.

In order to obtain the action for gravitational waves, we must expand Eqs. (1) and (10) to second order in $h_{ij}$. Since lengthy calculations need to be done for this, the detailed derivation is presented in Appendix A. Here we only provide the final result.

#### A. 2-form inflation

The action for the gravitational waves from 2-form inflation is given by

$$S_2 \approx \int \frac{a^2}{8\kappa^2} \left[ (h'_{ij})^2 - c_s^2 (\partial_k h_{ij})^2 - m_g^2 h_{ij}^2 \right] d\eta d^3x,$$  

(15)

where

$$c_s^2 := 1 - \frac{2}{3} \kappa^2 N B^2$$  

(16)

is the propagation speed of the gravitational waves and

$$m_g^2 := \frac{4\kappa^2 N}{3} \left[ 4V_f a^2 B^2 + \frac{8}{5} V_1 a^2 B^4 - \frac{a''}{a} B^2 - (B' + HB)'^2 \right]$$  

(17)
is the graviton mass. The equation of motion can be derived from the action (15), or, directly from the linearized Einstein equations:

\[ h''_{ij} + 2\mathcal{H}h'_{ij} - c_s^2\nabla^2 h_{ij} = -m_g^2 h_{ij}. \]  

(18)

Let us, for example, consider a simple chaotic potential \( V = m^2 H/4 \). In this case, we may use the approximation

\[ 3\mathcal{H}^2 \simeq \kappa^2 N a^2, \quad a'' / a^3 = \mathcal{H}' + \mathcal{H}^2 \simeq 2\mathcal{H}^2 - \kappa^2 N (B')^2 / 2, \quad \text{and} \quad (B' + \mathcal{H}B)^2 \simeq a^2 H^2 B^2 (1 - 2m^2 / 3\mathcal{H}^2). \]

Then, Eq. (17) is simplified to

\[ m_g^2 \simeq 4H^2 \left( 4 - \kappa^2 NB^2 \right). \]  

(19)

The similar graviton mass term also arises in vector inflation [16]. If \( \kappa^2 N B^2 > 4 \), we have the large tachyonic mass, \( m_g^2 < 0 \), with \( m_g^2 \sim \mathcal{O}(H^2) \), implying the unstable evolution of tensor perturbations. In vector inflation, \( \kappa^2 N B^2 \gtrsim 1 \) is required in order for chaotic inflation to take place [1]. This is also the case for 2-form inflation. Therefore, it is difficult to realize the large field inflationary models within the context of 2-form inflation.

The most striking nature of 2-form inflation appears in the propagation speed of the gravitational waves. In vector inflation, one finds the usual propagation speed, i.e., \( c_s^2 = 1 \) [16]. In 2-form inflation, however, \( c_s^2 \) depends on the background field value and hence differs from unity in general. In particular, if \( \kappa^2 N B^2 > 3/2 \), the propagation speed squared becomes negative, which is pathological. This shows that, in addition to the above mentioned tachyonic mass, 2-form inflation suffers from the negative sound speed squared in the case of the large field models. The varying speed of gravitational wave propagation also arises in [18].

B. 3-form inflation

The action for the gravitational waves from 3-form inflation is given by

\[ S_3 = \int \frac{a^2}{8\kappa^2} \Omega^2 \left[ (h'_{ij})^2 - c_s^2 (\partial_k h_{ij})^2 \right] d\eta d^3 x, \]  

(20)

where

\[ \Omega^2 := 1 + \frac{3}{2} \kappa^2 \phi^2 \]  

(21)

and

\[ c_s^2 := \frac{2 - \kappa^2 \phi^2}{2 + 3\kappa^2 \phi^2}. \]  

(22)

The equation of motion is

\[ h''_{ij} + 2 \left( \mathcal{H} + \frac{\Omega'}{\Omega} \right) h'_{ij} - c_s^2 \nabla^2 h_{ij} = 0. \]  

(23)

Contrary to 1- and 2-form inflation, 3-form inflation does not give rise to the graviton mass term, and so in this case one does not need to worry about the tachyonic mass. However, 3-form inflation has the varying speed of propagation of gravitational waves, as in 2-form inflation. As is clear from Eq. (22), one obtains \( c_s^2 < 0 \) for \( \kappa \phi > \sqrt{2} \), which basically rules out the large field models.

IV. GENERATION OF GRAVITATIONAL WAVES FROM P-FORM INFLATION

In this section, we quantize the actions for the \( p \)-form inflationary gravitational waves to discuss the power spectrum, following Refs. [16] and [19].

The tensor perturbation is expanded into Fourier modes as

\[ h_{ij}(\eta, x) = \int \frac{d^3 k}{(2\pi)^{3/2}} h_k(\eta) e_{ij}(k) e^{ikx}, \]  

(24)
TABLE I: The graviton mass $m_g^2$, the propagation speed of gravitational waves $c_s^2$, and the prefactor $\Omega^2$ in $p$-form inflation for different $p$. For completeness we include the cases with $p = 0$ (standard scalar field inflation) and $p = 1$ (vector inflation).

| $p$ | $m_g^2$ | $c_s^2$ | $\Omega^2$ |
|-----|---------|---------|-----------|
| 0   | 0       | 1       | 1         |
| 1   | Eq. (6) of Ref. [16] | 1 | $1 + \frac{1}{6} \kappa^2 NB^2$ |
| 2   | Eq. (17) | $1 - \frac{2}{3} \kappa^2 NB^2$ | 1 |
| 3   | 0       | $\frac{2 - \kappa^2 \phi^2}{2 + 3\kappa^2 \phi^2}$ | $1 + \frac{3}{2} \kappa^2 \phi^2$ |

where $e_{ij}(k)$ is the polarization tensor. The spectrum of gravitational waves is commonly defined by

$$\langle h_k h_{k'} \rangle \equiv \frac{2\pi^2}{k^3} P_T(k) \delta^{(3)}(k + k').$$

(25)

In terms of a new variable and a new time coordinate defined by

$$v_k := zh_k, \quad z := \frac{a\sqrt{c_s\Omega}}{2},$$
$$dy := c_s d\eta,$$

(26)
(27)
the action can be rewritten as

$$S_{GW} = \frac{1}{2\kappa^2} \int d^3k dy \left\{ |v_{k,y}|^2 - \left[ k^2 - \left( \frac{z_{yy}}{z} - \frac{m_g^2}{c_s^2} \right) \right] |v_k|^2 \right\},$$

(28)

where $m_g^2$, $c_s^2$, and $\Omega^2$ can be found in Table I. Then, the equation of motion for $v_k$ reduces to

$$v_{,yy} + \left[ k^2 - (1 + \alpha) \frac{a_{yy}}{a} \right] v = 0,$$

(29)

where we have abbreviated a suffix $k$ and introduced a parameter $\alpha$ to describe the deviation from the well-known formula for the gravitational waves from the standard scalar field inflation model.\(^1\) The appropriate initial condition is given by

$$v \rightarrow \frac{1}{\sqrt{2k}} e^{-iky} \quad \text{for} \quad ky \rightarrow -\infty \quad \text{(the Bunch-Davies vacuum)}.$$

(30)

It is useful to express $\alpha$ in terms of slow-roll parameters. We define the slow-roll parameters as

$$\epsilon := 1 - \frac{H'}{H^2}, \quad s := \frac{c_s^2}{Hc_s}, \quad \omega := \frac{\Omega'}{H\Omega},$$

(31)

with the slow-roll condition $\epsilon \ll 1$. Since $c_s$ and $\Omega$ are functions of the field, they may be thought of as slowly varying functions of time. We assume that $s, \omega \lesssim O(\epsilon^{1/2})$, and the estimate will be verified later. Using these slow-roll parameters, the deviation parameter $\alpha$ can be written as

$$\alpha = \alpha_1 + \alpha_2$$

(32)

\(^1\) In the limiting case with $H$, $c_s$, $\Omega$, $\alpha = \text{const.}$, we have an exact solution

$$v = \frac{\sqrt{\pi}}{2} e^{i(2\nu+1)\pi/4} a Hc_s^{1/2}(-\eta)^{3/2} H^{(1)}_{\nu}(c_s k\eta), \quad \nu := \frac{3}{2} \left( 1 + \frac{8}{9} \alpha \right)^{1/2},$$

where $H^{(1)}_{\nu}$ is the Hankel function of the first kind of order $\nu$.  

with
\[
\alpha_1 = \frac{3}{4}s + \frac{3}{2}H + \frac{1}{4} s' + \frac{1}{2} \omega' + \frac{1}{4} s^2 + \frac{1}{2} \omega^2 + \frac{3}{4} s \omega + \mathcal{O}(s^{3/2}), \\
\alpha_2 = -\frac{m_g^2}{2H^2} \left( 1 - \frac{1}{2} \epsilon - \frac{1}{2} s \right)^{-1},
\]
where \(\alpha_1\) represents the correction from the time variation of \(c_s\) and \(\alpha_2\) the correction from the graviton mass.

In the limit where the slow-roll conditions for \(\epsilon, s, \omega\) are satisfied and the graviton mass is small, we have \(\alpha \approx 0\), leading approximately to the scale invariant spectrum of gravitational waves:
\[
P_T^{1/2} \sim \frac{\kappa H}{2\pi c_s^{3/2}} \left|_{\alpha H = c_s} \right..
\]

If the graviton mass squared is large and positive, i.e., \(m_g^2 \sim \mathcal{O}(H^2) > 0\), the growth of the tensor perturbations is suppressed. If, on the other hand, the mass squared is large and negative, then we have the unstable evolution of tensor perturbations. As mentioned in the previous section, the mass squared clearly becomes large and negative in chaotic inflation. In addition to this problem, we have to be careful about the negative sound speed squared in the large field models of 2- and 3-form inflation, which is pathological. For this reason, in what follows we focus on the cases with \(c_s^2 > 0\) and evaluate in more detail the correction term \(\alpha\) which gives the scale dependence of the spectrum.

### A. 2-form inflation

In the case of 2-form inflation, we explicitly have
\[
\epsilon = \frac{1}{2\kappa^2 N} \left( \frac{4V_1 B}{V} \right)^2, \quad s = \sigma(2\epsilon)^{1/2} \frac{2\beta}{3 - 2\beta^2}, \quad \omega = 0,
\]
where \(\beta := \kappa \sqrt{N} B\) and \(\sigma = 1\) (respectively \(\sigma = -1\)) for \(V_1 B/V > 0\) (respectively \(V_1 B/V < 0\)). Now it is easy to see \(s \lesssim \mathcal{O}(\epsilon^{1/2})\). The graviton mass reduces to
\[
m_g^2 = -\beta^2 \left( 1 - \frac{\epsilon}{3} \right) - \frac{2\epsilon}{3} + \frac{5}{3} \sigma(2\epsilon)^{1/2} \beta + \frac{8}{5} \frac{V_1 I B^4}{V}.
\]

In deriving the above equations we have used the approximation \(3H^2 \simeq \kappa^2 N a^2 V\) and \(3H_B' \simeq -4aV_1 B\). It is straightforward to check that for the chaotic potential \(V = m^2 I/4\) Eq. (37) reproduces Eq. (19). One sees that the leading correction is given by
\[
\alpha \sim \max \left\{ \epsilon, \beta^2, \epsilon^{1/2}, \frac{V_1 I}{V} B^4 \right\}.
\]

In principle we can take \(\beta \sim \mathcal{O}(1)\) while keeping \(c_s^2 > 0\), so that the correction is large. However, this case seems irrelevant because we have \(m_g^2 \approx -4H^2 < 0\) unless the last term in Eq. (37) is fine-tuned to cancel this negative contribution.

### B. 3-form inflation

Let us next consider 3-form inflation. Using \(3H^2 \simeq \kappa^2 a^2 V\) and \(3H\phi' \simeq -12a^2 V_1 \phi\), one finds
\[
\epsilon = \frac{1}{2\kappa^2} \left( \frac{12V_1 \phi}{V} \right)^2, \quad s = \sigma(2\epsilon)^{1/2} \frac{8\beta}{(2 - \beta^2)(2 + 3\beta^2)}, \quad \omega = -\sigma(2\epsilon)^{1/2} \frac{3\beta}{2 + 3\beta^2},
\]
where \(\beta := \kappa \phi\) and \(\sigma = 1\) (respectively \(\sigma = -1\)) for \(V_1 \phi/V > 0\) (respectively \(V_1 \phi/V < 0\)). One can verify \(s, \omega \lesssim \mathcal{O}(\epsilon^{1/2})\). In the small field models (\(\kappa \phi \ll 1\)), the leading correction is given by
\[
\alpha \simeq -\frac{3}{4} \sigma(2\epsilon)^{1/2} \kappa \phi.
\]

With a relatively large field value \(\kappa \phi \sim 1\) and positive \(c_s^2\), the correction could be as large as \(\mathcal{O}(\epsilon^{1/2})\).
V. CONCLUSIONS

We have studied gravitational waves generated during $p$-form inflation. In the case of 2-form inflation, we considered a large number of 2-form fields so that the model is compatible with the background isotropy [1,12]. The main feature of gravitational waves from 2-form inflation can be found in the mass term, $m_2^2$, and the propagation speed, $c_2^s$. We obtained the mass term very similar to that in vector inflation. The mass squared becomes negative in large field models, as in vector inflation, implying the unstable evolution of gravitational waves. In addition to this, 2-form inflation predicts the varying speed of gravitational wave propagation, and the large field models are found to give $c_2^s < 0$. Thus, the large field models of 2-form inflation are unlikely to be viable. In contrast to 1- and 2-forms, a 3-form is compatible with isotropy. With some particular coupling to gravity, the background evolution of 3-form inflation is very similar to that driven by a scalar field. However, the behavior of gravitational waves is different again. Although 3-form inflation does not give rise to the mass term of gravitons, we showed that the propagation speed of gravitational waves differs from unity also in 3-form inflation. As the squared speed becomes negative when the field value is large, it is difficult to construct working large field models in the context of 3-form inflation. We also showed that the correction to the spectrum of gravitational waves is very small in the small field models of 2- and 3-form inflation.

Finally, we would like to remark that in a spatially curved universe the background evolution of $p$-form inflation will be different from the corresponding scalar field case. The non-zero spatial curvature indeed affects the onset of 1-form inflation [20]. It would be interesting to study the dynamics of general $p$-form inflation in the case of a spatially curved universe.

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Note added

A day after this paper appeared on arXiv, a related paper [21] has also appeared, in which the authors study cosmological perturbations from vector inflation.

APPENDIX A: CALCULATION DETAILS

1. The metric and Ricci tensor

The perturbed metric we consider is

$$ds^2 = a^2(\eta) \left[ -d\eta^2 + (\delta_{ij} + h_{ij})dx^i dx^j \right],$$

(A1)

where $\delta^{ij}h_{ij} = \partial^i h_{ij} = 0$. The inverse metric is then given by

$$g^{ij} = a^{-2} \left( \delta^{ij} - h^{ij} + h^{ik}h_{kj} \right).$$

(A2)
For this metric we have
\[ \sqrt{-g} = a^4 \left( 1 - \frac{1}{4} h_{ij}^2 \right), \]  
(A3)
\[ R_{00} = -3 \mathcal{H}' + \frac{1}{2} h_{ij} h_{,ij} + \frac{1}{4} h_{ij}^2 h_{ij}' + \frac{1}{2} \mathcal{H} h_{ij} h_{ij}', \]  
(A4)
\[ R_{ij} = (\mathcal{H}' + 2 \mathcal{H}^2) (\delta_{ij} + h_{ij}) + \frac{1}{2} (h_{ij}' + 2 \mathcal{H} h_{ij}' - \nabla^2 h_{ij}) - \frac{1}{2} h_{ik}' h_{jk}' - \frac{1}{2} \mathcal{H} h_{ij} h_{kl} h_{ij}' h_{kl}' \] 
\[ + \frac{1}{4} (\partial_i h_{kl} \partial_j h_{kl} + \frac{1}{2} h_{kl} \partial_i \partial_j h_{kl} - \frac{1}{2} h_{kl} \partial_k h_{il} + \partial_j h_{kl} + \partial_k h_{ij}) - \frac{1}{2} \partial_k h_{il} \partial_l h_{ij} + \frac{1}{2} \partial_k h_{il} \partial_k h_{jl} , \]  
(A5)
\[ R = \frac{1}{a^2} \left[ 6 a'' - h_{ij} h_{,ij}' - \frac{3}{4} h_{ij}^2 h_{ij}' + 3 \mathcal{H} h_{ij} h_{ij}' + h_{ij} \nabla^2 h_{ij} + \frac{3}{4} (\partial_k h_{ij})^2 - \frac{1}{2} \partial_k (h_{ij} \partial_k h_{ij}) \right], \]  
(A6)
leading to
\[ \sqrt{-g} R = \frac{a^2}{4} \left[ (h_{ij}')^2 - (\partial_k h_{ij})^2 \right] - \frac{a^2}{2} \left( \mathcal{H}^2 + 2 \mathcal{H}' \right) h_{ij}^2, \]  
(A7)
where summation over repeated indices is understood. Note that a total derivative term is omitted in Eq. (A7).

2. The 2-form

We start with computing \( I^{(a)} := A_{\mu \nu}^{(a)} \). Each \( I \) is explicitly given by
\[ I = \frac{b^2}{4} (h_{ij}' - 2 h_{ij} b_{ik} b_{jk} + h_{ij} h_{kl} b_{ik} b_{jl} + 2 h_{ik} h_{jk} b_{il} b_{jl} \] 
\[ = 2 B^2 + 2 h_{ij} B_i B_j + h_{ij} h_{kl} \varepsilon_{ijk} \varepsilon_{ln} B_m B_n + 2 h_{ij} B_k^2 - 2 h_{ik} h_{jk} B_i B_j, \]  
(A8)
so that
\[ \sum_{a=1}^{N} I^{(a)} \approx 2 NB^2 + NB^2 h_{ij}^2, \]  
(A9)
where we used Eq. (3). The field strength is \( F_{0ij} = a^2 \varepsilon_{ij} (B_i' + 2 \mathcal{H} B_i) \), and the kinetic term of the 2-form field is simply given by
\[ \sqrt{-g} \sum_{a=1}^{N} 12 F_{\mu \nu}^{(a)} \approx \frac{Na^2}{4} (B_i' + 2 \mathcal{H} B_i)^2 \left( 1 + \frac{1}{4} h_{ij}' \right), \]  
(A10)
while the potential term reduces to
\[ \sqrt{-g} \sum_{a=1}^{N} V(I^{(a)}) \approx a^4 \left[ NV + \left( -\frac{N}{4} V + NV_1 B^2 + \frac{4N}{10} V_1 B^4 \right) h_{ij}^2 \right], \]  
(A11)
where we used the formula
\[ \sum_{a=1}^{N} B_i^{(a)} B_j^{(a)} B_k^{(a)} h_{ij} h_{kl} \approx \frac{2}{15} NB^4 h_{ij}^2, \]  
(A12)
Finally, the coupling terms are
\[ \sum_{a=1}^{N} \sqrt{-g} \frac{1}{6} R I^{(a)} \approx 2 Na' a'' B^2 + \frac{Na^2 B^2}{12} \left( h_{ij}' \right)^2 - (\partial_k h_{ij})^2 \] 
\[ + \frac{Na^2}{3} \left[ \frac{a''}{a} (B')^2 + \frac{1}{2} \mathcal{H}^2 B^2 + \mathcal{H} B' + 4a^2 V_1 B^2 \right] h_{ij}^2, \]  
(A13)
and
\[
\sum_{a=1}^{N} \frac{1}{2} \sqrt{-g} A_{\mu}^{(a)} R_{\nu}^{\rho} A_{\alpha}^{\mu(a)} \approx -N a^2 B^2 (H' + 2H^2) - \frac{3}{4} N a^2 B^2 (H' + 2H^2) h_{ij}^{(1)}
\]
\[+ \frac{1}{2} N a^2 B^2 h_{ij} \delta R_{ij}^{(1)} - \frac{1}{3} N a^2 B^2 \delta_{ij} \delta R_{ij}^{(2)}, \tag{A14}
\]
with
\[
Na^2 B^2 h_{ij} \delta R_{ij}^{(1)} = -\frac{1}{2} Na^2 B^2 \left[ (h'_{ij})^2 - (\partial_k h_{ij})^2 \right] + Na^2 \left[ \frac{1}{2} (B')^2 - 2a^2 B^2 V_4 + \left( \frac{a''}{a} + H^2 \right) B^2 \right] h_{ij}^{(2)}, \tag{A15}
\]
and
\[
Na^2 B^2 \delta_{ij} \delta R_{ij}^{(2)} = Na^2 B^2 \left[ -\frac{1}{2} (h'_{ij})^2 + \frac{1}{2} (\partial_k h_{ij})^2 \right] + \frac{3}{4} Na^2 \left[ 2HBB' + \left( \frac{a''}{a} + H^2 \right) B^2 \right] h_{ij}^{(3)}, \tag{A16}
\]
where we used the background equation (13) and integration by parts, and removed total derivative terms.

3. The 3-form

\[I := A_{\mu\nu\rho} \text{ is given by}
\]
\[I = 6\phi^2 \left( 1 + \frac{1}{2} h_{ij}^2 \right). \tag{A17}
\]
The field strength is \( F_{\mu\nu} = a^3 (\phi' + 3H\phi) \varepsilon_{\mu\nu} \), and the kinetic term of the 3-form field is
\[
\sqrt{-g} \frac{1}{48} F_{\mu\nu\rho}^2 = -\frac{1}{2} a^2 (\phi' + 3H\phi)^2 \left( 1 + \frac{1}{4} h_{ij}^2 \right). \tag{A18}
\]
The potential term is
\[
\sqrt{-g} V(I) = a^4 \left[ V + \left( -\frac{1}{4} V + V_4 \right) \phi^2 \right] h_{ij}^2. \tag{A19}
\]
The first one of the coupling terms is
\[
\frac{1}{8} \sqrt{-g} RI = \frac{9}{2} a^2 \phi^2 + \frac{3a^2 \phi^2}{16} [(h'_{ij})^2 - \partial k h_{ij}^2] + \frac{3a^2}{4} \left( \frac{2a''}{a} \phi^2 - (\phi')^2 + \frac{1}{2} \phi'^2 + H\phi' + 2a^2 V_4 \phi^2 \right) h_{ij}^2. \tag{A20}
\]
Noting that
\[
A_{\mu
u} i A^{i\mu\nu} = 2\phi^2 g_{ij} + a^{-2} \phi^2 \left[ 2 \left( h_{ij} h_{kl} - h_{ik} h_{jk} \right) + \varepsilon_{ikm} \varepsilon_{jln} h_{kl} h_{mn} \right], \tag{A21}
\]
we get
\[
\sqrt{-g} A_{\mu \nu \rho} R_{\rho}^{\sigma} A_{\sigma}^{\mu \nu} = 6a^2 \phi^2 \left( \frac{a''}{a} + H^2 \right) - \frac{1}{2} a^2 \phi^2 (\partial_k h_{ij})^2
\]
\[+ a^2 \left( 3\phi^2 \left( \frac{a''}{a} + H^2 \right) + 3\phi' H - (\phi')^2 + 12a^2 \phi^2 V_4 \right) h_{ij}^2. \tag{A22}
\]
In computing the coupling terms we used the background equation (13) and integration by parts, and removed total derivative terms.

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