A simple calibration-free method of complex permittivity extraction

S. Kurdjumov¹, D. Filonov¹,², E. Kretov¹, V. Ivanov¹ and P. Ginzburg¹,²

1. Department of Nanophotonics and Metamaterials, ITMO University, St-Petersburg, Russia, 197101
2. School of Electrical Engineering, Tel-Aviv University, Tel-Aviv, Israel, 69978
E-mail: s.kurdjumov@metalab.ifmo.ru

Abstract. Here we present a new calibration-free method for relative complex permittivity extraction, which allows using accessible and cheap materials and simplifies the measurement process. The method combines the advantages of resonant and non-resonant techniques and provides results for liquids and solids in 0.5-5GHz frequency band in dependence on the material type. The essence of the proposed method is based on measuring of the magnetic dipole resonance of a spherical sample. The size of such a sphere can be dynamically changed making it possible to evaluate its permittivity at different frequencies. Mie theory serves as the basis for connecting the resonant frequency of the sample with the real part of the permittivity, while imaginary part can be found from Q-factor. In this work we demonstrate the efficiency of the method for water. In addition, we evaluate the applicability of the method for lossy materials.

1. Introduction

Extraction of material parameters, i.e. permittivity and permeability, is a very important task in applications which involve interaction between electromagnetic waves and matter [1]. The existing techniques for material parameters extraction can be divided into 2 groups: resonant and non-resonant techniques. Methods of the resonant group (e.g. cavity perturbation technique [2]) are the most accurate (the mean percentage error does not overcome 1-2%), however they can operate only at the single frequency, whereas non-resonant methods, such as open-ended coaxial probe technique [3] or waveguide/transmission line techniques [4, 5], are broadband but their accuracy is lower, up to 10%. All these methods have significant drawbacks related to their complicity: to implement them one needs to perform calibration which consists of several steps, or the extraction algorithm is pretty difficult. For example, to implement the open-ended coaxial probe technique one needs to calibrate the probe using 3-steps calibration which must be performed very precisely and, in several cases, not once, otherwise the measurement results are not reliable. Also, the algorithms of material parameters extraction using waveguide/transmission line techniques contains slightly difficult equations. The proposed method is free of these disadvantages. Moreover, it combines the advantages of both resonant and non-resonant approaches simultaneously. Mie theory serves as a theoretical basis to our approach. It provides the condition of the magnetic dipole resonance of a dielectric sphere: [6]

\[ \lambda_{res} = \sqrt{\varepsilon \cdot D} \]  

The equation (1) is applicable only for dielectrics with relative permeability is equal to 1. [7] The magnetic dipole resonance frequency is obtained by measuring S₁₁ of the dielectric sphere filled with a liquid under the test. With the increase of the frequency of the magnetic dipole resonance the imaginary part of permittivity grows, and it results in enhancement of the error of the method in higher frequencies where losses are significant. Using the eq. (1) one can extract only real part of permittivity. Imaginary
part can be found using the half-power method of Q-factor calculation. Another way to determine the imaginary part of permittivity can be based on Kramers-Kroning relations, though this approach has significant limitations when available experimental data has a finite bandwidth and a relatively small number of experimental data points. [8]

The half-power method is applicable in general case when the following proposals are made:

- material losses are greater than radiation leakage
- the resonances are well-defined in a spectrum

Then the imaginary part of permittivity can be calculated from the Q-factor:

$$\varepsilon'' = \frac{\varepsilon'}{Q}$$

The novel proposed method for extraction of material parameters is based on equations (1) and (2). The method is calibration-free and does not require expensive additional equipment (for the exception of network analyzer, phase information is also not mandatory). Furthermore, the data post-processing is transparent and straightforward, which makes the method cheap and easy for handling.

2. Methods

To measure permittivity value within a frequency range, the diameter of the sphere $D$ was varied with the help of a flexible shell filled by measured liquid. The surface tension of the shell with proper area allows to automatically reach the shape of the sample close to spherical for extraction of the permittivity at several frequencies. Though the gravity force distorts the spherical shape, the elasticity forces spread evenly over the shell area are much greater than gravity in case of the considered sizes of the sphere. In the case of solid substances, a set of prepared samples with different diameters can be used instead. Schematic representation of the experimental setup for liquids measurement and the photo of the experimental setup are depicted in Figure 1 (a, b) respectively. The sphere formed by the stretching latex shell is connected by a flexible tube to a syringe with a liquid. Non-matched magnetic loop antenna is connected to the port of the VNA and provides excitation of a magnetic dipole resonance.

![Figure 1](image_url)

Figure 1. a) Schematic representation of the experimental setup for extraction of complex permittivity. Arrows illustrate the electric (blue) and magnetic (red) fields corresponding to the magnetic dipole resonance. b) A photo of the experimental setup for extraction of material parameters of liquid solutions

As a preliminary stage for the evaluation of the proposed method, a numerical simulation was performed. A sphere made of distilled water was simulated in CST Microwave Studio 2017 with Frequency Domain method. Dispersion of the real and imaginary parts of the material permittivity was interpolated with 20 points in the range 0.5-3.5 GHz previously obtained with commercially available dielectric assessment kit (DAK). [9] The diameter of the sphere in the simulations varied from 10 mm to 60 mm with 20 fixed points using a logarithmic step. Such step variation is necessary for uniform filling of the spectrum with calculated data points. For each sphere with different size, the first magnetic dipole resonance frequency was found, and the real part of permittivity was calculated in accordance
with expression (1). After that, the half-power bandwidth method described above was used for
calculation of the Q-factor for each case and imaginary part was determined using the expression (2).
The obtained values are plotted as red and blue circles in Fig. 2(a).

**Figure 2.** (a) Real and imaginary parts of permittivity specified in CST MWS for the sphere with
distilled water material while modeling (red and blue lines) and extracted values (red and blue circles)
obtained by using the considered method. (b) DAK data for complex permittivity (red and blue lines)
and values calculated by the considered method (red and blue circles) with dynamic changing of the
sphere’s size.

To approve the method in practice, an experimental setup shown at Figure 1 (b) was assembled. The
sphere formed by a shell made of widely available medical latex is fastened with threads at the end of a
tube. The other end of the tube is connected to a 200 ml syringe filled with measured liquid. By
controlling the volume of liquid supplied from the syringe, it is possible to form a sphere of a required
diameter to perform resonance frequency measurements. The loop antenna was placed around the
supplying tube. This place is optimal for its location in terms of saving a constant distance between the
antenna and the surface of the sphere during changes of its size. $S_{11}$ parameter of the loop antenna was
measured by Agilent E8362C VNA during 17 iterative steps of changing of the sphere diameter. Achieved data were processed using the same procedures as described above for numerical modeling.
Extracted values of real and imaginary parts of permittivity are shown in Figure 2 (b) (red and blue
circles) together with reference DAK measurements data (solid red and blue lines).

3. Results and discussion

The mean percentage deviation for real permittivity value extracted by the proposed method in case
of numerical modeling with water was estimated as 2.61% with the highest value of 3.3%. Values of
mean percentage deviation estimated as 2.38% with the highest value equal to 5.46% were obtained in
the experiment. From Figure 2(c) we can see that at higher frequencies (greater than 2.5 GHz) the
deviation in both real and imaginary parts of permittivity grows. At the same time, for the case of
numerical simulation (Figure 2(a)), this effect is not observed. It can be explained by the fact that it is
more difficult in practice to create a perfectly shaped sphere with a relatively small diameter, so the
sphere becomes distorted in different proportions and the resonance frequency shifts leading to the error
increase. The errors of extraction were caused by 3 factors. Firstly, the formula (1) is approximate and
it takes into account only the first magnetic dipole resonance. Secondly, the measured $S_{11}$ was quite
noisy. Averaging can improve measurements. The last cause of errors was the sample’s shape
imperfection. To obtain more accurate results one can use spherical templates which allow to hold a
better shape of sphere.

Our method has specific bounds of applicability based on the dielectric losses of the measured
material. To specify these limitations, a sphere from material with real permittivity of 80 and radius of
10 mm was simulated in CST Microwave Studio. The loss tangent of this sphere varied from $10^{-4}$ to 1.
The dependence of the resonance magnitude from loss tangent and frequency is demonstrated in Figure 3. From this color plot we can see that when the loss tangent exceeds 0.5, the magnetic dipole resonance of the sphere becomes indistinguishable, so it is hard to extract real and imaginary parts of permittivity using the proposed technique when loss tangent value is more than 0.5.

![Figure 3](image)

**Figure 3.** Color plot, showing the dependency of the resonance depth vs. loss tangent and frequency. The magnetic dipole resonance is the lower blue area.

4. **Conclusion**

In conclusion, we have experimentally demonstrated a new method for extraction of material parameters, which is simple in implementation, cheap and requires only accessible materials. Furthermore, this method allows to avoid difficult additional calibration procedures of measuring equipment. The accuracy of this technique does not overcome 5% so one can apply this method of permittivity extraction within all limitations described in the paper.

5. **Acknowledgements**

The research was supported in part by PAZY Foundation, Ministry of Science and Technology (project “Integrated 2D & 3D Functional Printing of Batteries with Metamaterials and Antennas”), and 3PEMS Ltd. The experimental part of research was supported by the Ministry of Education and Science of the Russian Federation (Project 3.1500.2017/4.6) and by Russian Foundation for Basic Research (project no. 18-79-10167).

**References**

[1] S. Costanzo, *MICROWAVE MATERIALS Edited by Sandra Costanzo* (2012), ISBN 978-953-51-0848-1.

[2] N. Krismer, D. Silbernagl, M. Malferttheiner, and G. Specht 2016 *CEUR Workshop Proc. 1594* 74

[3] J.P. Grant, R.N. Clarke, G.T. Symm, and N.M. Spyrou 1989 *J. Phys. E*. 22 757

[4] A.M. Nicolson and G.F. Ross 1970 *IEEE Trans. Instrum. Meas. 19* 377

[5] L.F. Chen, C.K. Ong, C.P. Neo, V. V. Varadan, and V.K. Varadan, *Microwave Electronics* (2004).

[6] G. Mie 1908 *Annalen der Physik (in German). 25* (IV): 377–445

[7] A.I. Kuznetsov, A.E. Miroshnichenko, M.L. Brongersma, Y.S. Kivshar, and B. Luk’yanchuk, 2016 *Science (80-. ). 354*, aag2472

[8] K.R. Waters, J. Mobley, and J.G. Miller 2005 *IEEE Trans. Ultrason. Ferroelectr. Freq. Control 52*, 822

[9] URL: [https://speag.swiss/products/dak/vna/](https://speag.swiss/products/dak/vna/)