Giant Fano factor and bistability in a Corbino disk in the quantum Hall effect breakdown regime

Tokuro Hata¹, Tomonori Arakawa¹, Kensaku Chida¹,², Sadashige Matsuo¹,² and Kensuke Kobayashi¹

¹ Graduate School of Science, Osaka University, Toyonaka, Osaka 560-0043, Japan
E-mail: tokuro@meso.phys.sci.osaka-u.ac.jp

Received 6 July 2015, revised 5 November 2015
Accepted for publication 19 November 2015
Published 13 January 2016

Abstract
We performed noise measurements for a Corbino disk in the quantum Hall effect breakdown regime. We investigated two Corbino-disk-type devices with different sizes and observed that the Fano factor increases when the length between the contacts doubles. This observation is consistent with the avalanche picture suggested by the bootstrap electron heating model. The temperature dependence of the Fano factor indicates that the avalanche effect becomes more prominent as temperature decreases. Moreover, in the highly nonlinear regime, negative differential resistance and temporal oscillation due to bistability are found. A possible interpretation of this result is that Zener tunneling of electrons between Landau levels occurs.

Keywords: quantum Hall effect, current noise, nonequilibrium, nonlinearity

I. Introduction
Quantum Hall (QH) state emerges when a perpendicular magnetic field is applied to a two-dimensional electron gas (2DEG) at low temperature [1]. The QH state is robust against small perturbations because of the protection provided by gauge symmetry [2], resulting in a quantized Hall resistance in units of $\frac{h}{e^2}$, where $h$ is the Planck constant and $e$ is the charge quantum. However, when an applied electric field exceeds a certain threshold, the state is broken to enter the nonequilibrium regime and the Hall resistance deviates from the quantized value [3, 4]. This phenomenon, which is called QHEBD [5], has been one of the most important problems for researchers in the field of metrology [6].

Two models have typically been used to discuss how the QH state is broken in a high electric field. First, the quasi-elastic-inter-Landau-level scattering (QUILLS) model was proposed by two groups [8, 9]. According to this model, a high electric field induces Zener tunneling of electrons between Landau levels, which breaks the QH state. Second, the bootstrap electron heating (BSEH) model was proposed by Komiyama et al [10–13]. In this model, QHEBD is not caused by the high electric field; instead, it is driven by thermal excitation of electrons with accompanying electron avalanches. Thus far, a number of experiments have been performed to test the validity of these two models. For example, the dependence of the threshold electric field on the constriction width has been addressed to confirm that both the QUILLS model [14] and the BSEH model [15] can account for several experimental results. Nevertheless, comprehensive understanding of QHEBD mechanisms has not yet been obtained [16].
Most experiments on QHEBD have been carried out using conventional conductance or resistance measurements in which the time-averaged data are investigated [4, 9–18]. On the other hand, recently, considerable interest has focused on current noise measurements, which allow electron conduction in the QHEBD state to be investigated in more detail [7, 19–21]. Current noise is the fluctuation of the current around its average value over a finite time, which is quantified using the power spectral density, $S_I$. The noise in a conductor can be broadly classified into three types. The first type is thermal noise represented by $S_{\text{thermal}} = 4k_BT/G$, where $k_B$, $T$, and $G$ are the Boltzmann constant, electron temperature, and conductance of the conductor, respectively. This kind of noise is induced by thermal fluctuation of electrons and can be observed even in the equilibrium situation. The second type is shot noise, as represented by $S_{\text{shot}} = 2eIF$ (at zero temperature), where $I$ is current and $F$ is called the Fano factor [22]. This kind of noise is caused by the partition process of electrons accompanied by transmission or reflection at the potential barrier. The Fano factor tells us how the bunching or antibunching of electrons takes place. For example, $F = 1$ if electrons pass through the barrier independently, namely, via the Poisson process. When electron bunching occurs, such as in an avalanche diode, $F \gg 1$ is realized [23]. The third type is $1/f$ noise. This noise is proportional to the inverse of the frequency $f$, that is, $S_{1/f} \propto 1/f$, and is dominant at low frequencies. Thus, the low frequency dynamics such as slow resistance fluctuation can be revealed by probing the $1/f$ noise [24].

Typically, Hall bars and Corbino disks have been used for QHEBD research [4, 7, 9–17, 19–21]. The essential difference between the two is that the edge current is absent in a Corbino disk in the QH state, whereas it is present in a Hall bar. Thus, the Corbino disk would be more appropriate to purely investigate the contribution of current caused by QHEBD than the Hall bar. In our previous noise study with a Corbino disk, the Fano factor was greatly enhanced, exceeding a Poisson value of unity [21]. This result indicates the presence of electron bunching in the current, which is consistent with the avalanche picture of QHEBD [10–13].

In this paper, we report an experimental work of the noise measurement in a Corbino disk, which is an extended work of our previous noise study in a Corbino disk [21] to include a very high electric field range in nonlinear regime. We report three main findings. The first one is the difference in the Fano factor between the two lengths of electron conduction. The experimental observation that the Fano factor increases when the length between the contacts doubles suggests that electron bunching is enhanced while electrons travel. This result is consistent with the avalanche picture of the QHEBD state. The second one is the temperature dependence of the Fano factor, which suggests that the QH state at lower temperature leads to a more prominent avalanche effect. The last one is the observation of the bistability of the states at an electric field well above the threshold field of QHEBD. Negative differential resistance and temporal oscillation caused by bistability are found to occur in this highly nonlinear regime.

Our noise measurements reveal details regarding electron transport in the QHEBD state and show that this transport is consistent with the avalanche model. The present work proves the advantage of the noise measurement as it is as sensitive or more sensitive to QHEBD compared to conventional conductance measurement.

This paper is organized as follows. In sections II.A and II.B, we describe our device, experimental setup, and noise analysis method. In section III.A, the difference in the Fano factors between two different Corbino devices is discussed. In section III.B, we present the temperature dependence of the Fano factor. In sections III.C and III.D, we examine the negative differential resistance with temporal oscillation of the QHEBD state and the possible Zener tunneling. In section IV, we summarize our work.

II. Experiment

II.A. Device and setup

We prepared two Corbino-disk devices fabricated on a 2DEG in a GaAs/AlGaAs heterostructure with an electron density $n_e = 1.85 \times 10^{15} \text{cm}^{-2}$ and an electron mobility $\mu = 11.9 \text{m}^2\text{V}^{-1}\text{s}^{-1}$. Their geometry is shown in figure 1(a). One of the devices has a length between contacts $W = 80 \mu\text{m}$ (Sample 1), and the other has a length between contacts $W = 40 \mu\text{m}$ (Sample 2). These samples are used to investigate how QHEBD depends on $W$.

The schematic diagram of the measurement circuit is shown in figure 1(b). The measurements are performed using a variable temperature insert (VTI) (Oxford Inc.) between 1.7 K and 3.8 K with a magnetic field applied perpendicularly to the 2DEG. Two resistors, $R_1 = 100 \text{k}\Omega$ and $R_2 = 100 \text{k}\Omega$, are inserted in series at room temperature and at the low temperature, respectively, so as to reduce the external inessential noise. The voltage ($V_{\text{out}}$) is applied across the sample through $R_1$ and $R_2$. The voltage ($V_{\text{ad}}$) across the device is measured by a voltmeter (Keithley 2000). The current ($I_{\text{ad}}$) is determined by $I_{\text{ad}} = \frac{V_{\text{ad}} - V_{\text{out}}}{R_1 + R_2}$. The average electric field between the contacts is nominally defined by $E = \frac{V_{\text{ad}}}{W}$.

The noise power spectral density (PSD) is obtained by two low-noise amplifiers (NF Corporation, LI-75A). The external noise is minimized using the cross-correlation technique [25, 26], where the voltage signal is collected by an onboard digitizer (National Instruments, PCI-5922), where the fast Fourier transform is applied to the data to obtain the voltage-noise PSDs. The resistance of a Corbino disk in the QH state is, ideally, infinite and is, in practice, too high to be measured. In such a case, the damping frequency for the noise measurement, $1/2\pi ZC$, where $Z$ is the device impedance and $C$ is the capacitance of the measurement circuit, is much smaller than roughly 1 kHz, limiting the measurement bandwidth to this frequency. At such low frequencies, the $1/f$ noise due to either the device or the amplifiers would dominate the observed noise, which disturbs quantitative analysis. For this reason, a capacitor $C_p = 1 \mu\text{F}$ and a resistor $R_p = 1 \text{k}\Omega$ were put in parallel with the sample in our setup, as shown in figure 1(b) [21, 27]. In this setup, the damping frequency estimated for $C \sim 400 \mu\text{F}$ and $Z \sim 1 \text{k}\Omega$ is...
roughly 400 kHz, which effectively increases the measurement bandwidth.

Figure 1(c) shows the magnetic field dependence of $I_d$ measured at $V_{out} = 1$ mV for Sample 1 at $T = 1.7$ K. At $B = 3.9$ T, $I_d$ is significantly suppressed (less than 100 pA), indicating that the quantum Hall state is well formed. The filling factor estimated from $n_e$ is 1.96. In this paper, we focus on a discussion of the result of the noise measurement of QHEBD at $\nu = 1.96$, although the result reported here is qualitatively the same near $\nu \sim 2.0$.

II.B. Analysis of the noise

Figure 1(d) shows typical voltage-noise PSDs for 1 at different electric fields. The amplitude at $E = 0$ kV m$^{-1}$ is due to the thermal noise of the resistor $R_p = 1$ kΩ (marked as (1) in figure 1(d)). When $E$ reaches 3.5 kV m$^{-1}$, QHEBD occurs and the voltage noise increases (marked as (2) in figure 1(d)). The spectrum shows no frequency dependence above $\sim 100$ kHz. As $E$ is further increased to $E = 3.7$ kV m$^{-1}$, the voltage noise is rapidly enhanced by $\sim 10^5$ (marked as (3) in figure 1(d)). Although the spectrum has a slight frequency dependence (possibly because of a 1/f noise contribution), it does not affect our discussion at all, because we focus on the voltage noise change on the order of $10^5$. The voltage noise, $S_V$, is estimated by averaging the observed voltage-noise PSD between 140 kHz and 150 kHz, which is then converted into the current noise, $S_I$, using $S_I = S_V/Z^2$.

III. Results and discussion

III.A. Fano factor of two types of QHEBD

Figures 2(a) and (b) shows the differential conductance $dI/dV$ and $S_I$ values as a function of $E$ for Samples 1 and 2, respectively. For 1, the differential conductance is less than 1 $\mu$S below $E = 3.5$ kV m$^{-1}$, indicating that a quantum Hall state is robustly formed. Between $E = 0$ and 3.5 kV m$^{-1}$, as shown in the lower panel of figure 2(a), the current noise remains $1.2 \times 10^{-25}$ A$^2$ Hz$^{-1}$, which corresponds to the thermal noise at $E = 0$ kV m$^{-1}$. Above $E = 3.5$ kV m$^{-1}$, the conductance and the current noise suddenly increase, indicated by ‘A’ in figure 2(a). The noise increases from $1.2 \times 10^{-25}$ A$^2$ Hz$^{-1}$ to $9.6 \times 10^{-21}$ A$^2$ Hz$^{-1}$ (that is, by about $10^5$) while the differential conductance changes by $10^2$. This observation indicates that the current noise is as sensitive to QHEBD as the conductance and provides a useful indicator of QHEBD. At $E = 3.7$ kV m$^{-1}$, the conductance and the noise take their maximum values at ‘B’, marked in figure 2(a). As $E$ is further increased, the noise decreases slowly above point ‘B’ in the direction of the arrow. The Fano factor estimated using $F = \frac{S_V-S_0}{S_0}$ at ‘B’ is $\sim 2 \times 10^2$, where $S_0$ is the voltage noise at $E = 0$ kV m$^{-1}$. It would be unreasonable to interpret the observed $F$ value being larger than the Poisson value ($F = 1$) simply as the effect of an increase in the effective temperature. As our previous work demonstrated, this very large Fano factor indicates QHEBD accompanied by electron
bunching, which is consistent with the avalanche picture of QHEBD [10–13, 21].

Next, we focus on Sample 2, which has a length $W$ that is half that of 1. As shown in figure 2(b), the conductance and the current noise rapidly increase, indicating QHEBD at $E = 0.2 \text{ kV m}^{-1}$, indicated by 'A'. The current noise increases to the maximum value $4.5 \times 10^{-23} \text{ A}^2 \text{ Hz}^{-1}$ at $E = 1.1 \text{ kV m}^{-1}$, indicated by ‘B’ in figure 2(b). The Fano factor estimated at this point is roughly $10^{-5}$, which is (again) much larger than the Poisson value. Although the behavior of 1 and 2 is qualitatively similar, the Fano factor of 1 is more than one hundred times larger than that of 2. The results further support the avalanche picture, because longer electron conduction distances are expected to result in larger electron multiplication.

### III.B. Temperature dependence of Fano factor

Figure 3(a) shows the temperature dependence of $dI/dV$ and $S_I$ for Sample 1. The quantum Hall state is broken at ‘A’. ‘B’ is the point at which the noise takes its maximum value. (b) Corresponding electric field dependence of $dI/dV$ and $S_I$ for Sample 2.

Fano factor increases by $10^2$ times while the system temperature is lowered by only 2 K. This phenomenon indicates that the robustness of the QH state significantly affects electron bunching in the avalanche regime.

Figure 3(b) shows the temperature dependence of $F$ for 2, too. The Fano factors are 400, 132, 114, and 90 at 1.7 K, 3.4 K, 3.6 K, 4.1 K, and 4.4 K, respectively. The Fano factor increases by four times as the temperature decreases by roughly 2 K, which agrees with the results for 1. However, the temperature dependence of the Fano factor is less significant in 2 than in 1. This phenomenon may reflect the fact that the length between contacts is shorter in 2, and thus, the QH state is less robust than in 1.

### III.C. Bistability

We now discuss the region in which the applied electric field is much higher than previously considered. Figure 4(a) shows...
the electric field dependence of the conductance and the voltage noise for Sample ♯2. This figure is an extended view of figure 2(b) to show the electric field above 2.5 kV m$^{-1}$, up to 5.7 kV m$^{-1}$. The voltage noise is plotted instead of the current noise because negative differential resistance appears, as we will explain later. Points labeled ‘A’ and ‘B’ correspond to those in figure 2(b). The conductance and voltage noise increase abruptly at $E = 5$ kV m$^{-1}$, indicated by ‘C’ in figure 4(a). Figure 4(b) is an expanded view of the electric field dependence of the current near this abrupt change. The current has negative slopes as a function of $E$, showing that negative differential resistance appears (at locations marked (2) and (4) in figure 4(b)). Note that this phenomenon occurs only in the limited electric field region and is not seen in the plot in the upper panel of figure 4(a).

Negative differential resistance (NDR) is generally classified as N-shape and S-shape based on the shape of the current as a function of the electric field [28]. For instance, NDR which is seen in tunnel diodes, is classified as a N-type. An S-shaped NDR is known to occur in n-GaAs in a high electric field where the system becomes unstable because of the breakdown accompanied by the impact ionization [28, 29].

The present NDR is composed of two S-type NDRs. Komiyama et al claim that impact ionization and avalanche occur at QHEBD based on the bistability of electron temperature [10–13]. Our result suggests that there is bistability because of QHEBD accompanied by impact ionization.

In practice, we observe a more direct indicator of the bistability. Figure 4(c) shows the spectra obtained at five different points in figure 4(b). An oscillation whose frequency period is 9.625 kHz is observed in the spectrum (5). Note that the two peaks which can be found in all spectra at 143 kHz and 183 kHz are caused by external noise.

The observed oscillation can be explained by a periodic fluctuation between two states as is usually caused by bistability of the state. We perform a simple simulation with the assumption that the periodic fluctuation has a voltage of 1 V, a period of 103.9 μs ($= 9.625$ kHz), and a time for the voltage to be finite of 2 μs, as shown in the inset of figure 4(d). The spectrum computed by Fourier transformation is qualitatively similar to the experimental spectrum (figure 4(d)). Although the real-time observation of bistability was difficult because of the small signal, the evaluation of the voltage noise using the FFT technique supports the claim that there is periodic bistability.

Very few studies have reported on time-dependent fluctuation. For example, Ahler et al [30] observed the fluctuation in a Hall bar and discussed bistability near QHEBD. On the other hand, the present time-dependence fluctuation is observed at higher electric fields in the highly nonlinear regime.

III.D. Possible Zener tunneling

Finally, we discuss the observed peak at very high electric fields, indicated by ‘C’ in Fig 4(a), in terms of Zener tunneling. The lower panel of figure 5(a) shows a 2D plot of $I_d$ as a function $B$ and $E$ for Sample ♯2. The upper panel shows three $dI/dV$ curves for $B = 3.9, 1.96, 1.3$, and 0.96 T. The circles
in the 2D plot indicate the \((E, B)\) condition at which \(dI/dV\) markedly increases. It is found that the threshold \(E\) value is proportional to \(B^{3/2}\), indicated by \(E \propto B^{3/2}\) in figure 5(b). This observation agrees with the theoretical relation based on Zener tunneling between Landau levels, \(eE l_B \propto \hbar \omega_z\), where \(l_B(\approx 1/B^{1/2})\) is the quantum magnetic length and \(\omega_z(\approx B)\) is the cyclotron frequency [9, 14]. \(S_V\) in this regime (at ‘C’ in figure 4(a)) is much larger than near QHEBD, indicating that Zener tunneling leads to a larger fluctuation than in QHEBD that occurs at lower electric fields.

We note that Sample ♯1 shows similar peak behavior (at ‘C’) as ♯2. The peaks for Sample ♯1 appear at 4.7 kV m\(^{-1}\), 1.5 kV m\(^{-1}\), 0.9 kV m\(^{-1}\), and 0.6 kV m\(^{-1}\) at \(B = 3.9, 1.96, 1.3,\) and 0.96 T, respectively. The values are almost the same as those for Sample ♯2, which is consistent with the interpretation that Zener tunneling occurs at the threshold field.

It is interesting to note what the relation of \(E \propto B^{3/2}\) indicates. If we use \(R_S(\propto 1/B)\), namely the cyclotron radius, as a characteristics length of electrons in the Zener tunneling condition [31–33], \(E\) would be proportional to \(B^2\). In this case, the electrons behave classically. Instead of this, we found that the points in the lower panel of figure 5(a) can be fitted with \(E \propto B^{3/2}\) (full line) more nicely than with \(E \propto B^2\) (dashed line). This may indicate that the quantum characteristics is not lost even in such a highly nonequilibrium state after the QH state is broken.

IV. Summary

We presented the three experimental results for the QHEBD state of Corbino disks. First, the Fano factor increases when the length between the contacts doubles, which is consistent with the avalanche picture of QHEBD state. Second, the temperature dependence of the Fano factor suggests that the QH state at lower temperature leads to a more prominent avalanche. Finally, the bistability of the state at higher electric fields is observed. Negative differential resistance and temporal oscillation caused by bistability are found to occur in this regime. Our experiments show that noise measurement is an effective method for determining the nonlinearity of QHEBD states; thus, we propose its use to investigate the nonlinearity in detail, such as how the nonequilibrium state changes temporally.

Acknowledgments

We are grateful to S Nakamura for fruitful discussions. This work was partially supported by a Grant-in-Aid for Scientific Research (S) (No. 26220711) from the Japan Society for the Promotion of Science, a Grant-in-Aid for Scientific Research on Innovative Areas, ‘Fluctuation and Structure’ (No. 25103003), from the Ministry of Education, Culture, Sports, Science, and Technology of Japan and the Yazaki Memorial Foundation for Science and Technology.

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