Split Hopkinson pressure bar test and cell model of the damage threshold of siltstone

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Abstract

Study the damage threshold of siltstone, we conducted the large-diameter series of split Hopkinson pressure bar cyclic impact tests on siltstone samples. When the damage value of siltstone was 0.16-0.4, increased exponentially with an increase in stress wave amplitude. Damage evolution in siltstone obeys the power-law distribution. This feature is consistent with the self-similarity theory in fractal mechanics. Thus, self-organized criticality theory can be used to determine the rock damage threshold. Combined with the microscopic tectonic features of siltstone, a 2 × 3 cell model was established by based on the renormalization group method, and the failure probability of seven 2 × 3 cell models was established by combining the force transfer coefficients among cell units. The seven damage models, showed that the damage accumulation threshold of siltstone without damage is 0.119, and the damage threshold during failure is 0.4, which is consistent with the SHPB test results. Thus, the cellular model established based on the renormalization group method can be used to study the damage evolution of siltstone.

Keywords: Siltstone; Damage evolution; Renormalization group method; Damage threshold

1. Introduction

Currently, many damage models, such as the LoLand model(Loland KE, 1980), component model, KUS model(Mazars J, 1986), and segmentation Curve Model(Feng Xiqiao, 1995) have been established for hard brittle rock materials. These models all assume that rock damages after a peak. That is, the upper limit of the rock damage threshold is 1. However, there are several pores and cracks in the initial state. These models are inconsistent with rocks in practice, because rocks are compacted under load, and negative damage occurs, indicating that the model has a certain applicability. Furthermore, granite and limestone do not begin to rupture when the damage value is 1(Zhang Qingcheng, 2014), and related theoretical studies have shown some threshold values for rock damage, such as 0.2 < D < 0.8, 0.4 < D < 0.8(Yu Shouwen and Feng Xiqiao, 1997). It indicates that the damage value is less than 1 when the rock is destroyed. After introducing the statistical damage theory into the mechanical properties and failure mechanism of rock failure, many scholars have established various rock statistical damage constitutive models, which assume that rock damage obeys the normal distribution and other forms. Until now, more attention has been paid to the damage threshold of rock. Currently, only a few test methods can be used to determine the damage threshold of rock. Herein, we investigated isotropic siltstone both theoretically and experimentally, to clarify the damage threshold of such rock.

2. Damage evolution of siltstone in SHPB

In geotechnical engineering, changes in rock damage are mostly caused by external disturbances, such as blasting operations. Therefore, in this study, the SHPB device was used to study the cyclic impact test of siltstone at a constant strain rate. The test piece has a diameter of 50 mm and a length is 40 mm. The experimental scheme of “indirect characterization of dynamic mechanical properties by static mechanical properties” was adopted. Because rock joints, fractures, and pores are very sensitive to the response of wave velocity, the average
change in wave velocity in siltstone after multiple shocks was used to characterize the damage. Therefore, the wave velocity of the siltstone after multiple impacts is used to characterize the damage\cite{8}. Thus, the rock damage was measured by the change in the longitudinal wave velocity, as expressed in Eq (1):

\[
D = 1 - \left( \frac{C_{p2}}{C_{p1}} \right)^2 \tag{1}
\]

where \( D \) is the amount of damage, and \( C_{p1} \) and \( C_{p2} \) are the longitudinal wave velocities of the rock material before and after impact.

The damage of the siltstone specimens under bullet speed of different lengths and impact velocities was obtained, and the data are listed in Table 1.

| No. | Cycle index | Bullet speed (m/s) | Amplitude (MPa) | PV before impact (m/s) | PV after impact (m/s) | damage \( D \) | Cumulative damage \( D \) | States |
|-----|-------------|-------------------|----------------|-----------------------|-----------------------|----------------|---------------------------|--------|
| 1   | 1           | 1.46              | 28.81          | 2200                  | 2196                  | 0.0036        | 0.0036                    | intact |
| 2   | 1           | 1.57              | 30.98          | 2196                  | 2190                  | 0.0055        | 0.0091                    | intact |
|     | 3           | 1.52              | 29.99          | 2190                  | 2200                  | invalid       | invalid                   | intact |
| 1   | 1           | 2.06              | 40.65          | 2200                  | 2133                  | 0.06          | 0.06                      | intact |
| 2   | 1           | 1.76              | 34.73          | 2133                  | 2075                  | 0.05          | 0.11                      | intact |
|     | 3           | 2.14              | 42.23          | 2075                  | 1968                  | 0.10          | 0.2                       | lateral crack |
| 1   | 1           | 2.37              | 46.77          | 2133                  | 2079                  | 0.11          | 0.10                      | intact |
| 2   | 1           | 2.14              | 2190           | 2079                  | 1933                  | 0.14          | 0.22                      | lateral crack |
| 1   | 2           | 2.54              | 50.12          | 1933                  | 1722                  | 0.10          | 0.29                      | face off |
| 2   | 3           | 3.37              | 66.50          | 1849                  | 1732                  | 0.16          | 0.16                      | lateral crack |
| 3   | 2           | 3.45              | 68.08          | 1732                  | 1649                  | 0.24          | 0.32                      | face off |
| 4   | 3           | 3.2               | 62.60          | 1649                  | 1532                  | 1.00          | 1.00                      | crush  |
3. Cellular damage threshold model of siltstone

3.1 Self-organization of rock failure

Studies on acoustic emissions during rock damage have shown that the signal strength of various frequency scales is proportional to the frequency (Per Bak, 1988 and Cao Wengui, 2014), which is manifested in the time domain as a typical 1/f noise feature, that is, the signal attenuation follows the power-law distribution. Fractal rock mechanics focus on the self-similarity and symmetry of rocks [8]. The self-similarity is not only on the boundary, but also in the internal structure of the rock. The rock damage evolution on the spatial structure is scale invariant of self-similarity, and self-similarity obeys power-law distribution [9]. Several experiments have shown that the strength of the rock evolution characteristics also follows the normal and Weibull Distributions (Liu Dongqiao, 2014), and the essence of these distributions is special power-law distribution. Combined with the above SHPB test results, the evolution law of rock damage also follows the power function distribution, which is mutually confirmed with the self-similarity theory in fractal mechanics. The self-organized criticality theory can be employed in theoretical studies of rock damage thresholds. In the meantime, the self-organized criticality theory is a statistical theory, which does not depend on any detailed changes, or force path and loading conditions. Therefore, there is no need for a detailed description of the graphic changes.

In several SHPB impact tests, we found that when siltstone damage is small, the rock is only locally defective. The partial defects are random. Also, the first failure occurs in the part with the lowest strength. The failure results in the redistribution of the original stress of the
rock, and then, failure occurs in the part with lower strength under the action of external forces, which again causes the redistribution of the stress of the rock. This chain reaction, after a limited number of cycles, eventually leads to rock damage. The rock damage process shows clustering behavior, and renormalization group method can well describe the cluster behavior well. The degradation behavior between units is studied on a small scale, and the results are reflected on a larger scale. This method is also applicable to the rock self-organized criticality, that is, the damage threshold of rock.

3.2 Two-dimensional cellular model of the renormalization group method for siltstone

Rock damage evolution is a self-organization phenomenon; hence, the damage itself also satisfies the self-similar property. The cellular model of siltstone has the following requirements. First, each unit in the model is subjected to the same force $F$ action. When the value is greater than the uniaxial compressive strength, the unit goes through the failure stage. Also, when each failure unit transmits force to adjacent units, only the cellular level of the unit will carry on, and the change in force in other units will not be affected. Second, the failure probability of each unit is subject to the Weibull distribution. Third, each unit and the other five units can be considered as a cell, six first-order cells can be seen as a second-order cell, until the $n$-order cell can also be seen as an $n+1$ cell. Fourth, rock damage is a failure probability $p(F)$.

Siltstone is a sedimentary rock with a stratified structure in microscopic state. We established a $2 \times 3$ two-dimensional(2D) renormalized group cell model. Six units form a primary cell, and six primary cells form a secondary cell. That is, a relatively simple system is studied on the smallest scale, and then, the problem is renormalized (re-scaled) to use the same system on a larger scale. The whole process is repeated continuously on a much larger scale. The rock can be represented by a grid consisting of $2n \times 3n$ elements. The black squares in Fig. 3 represents the failure of the element.

![Fig 3 Unit restructuring sketch](image)

The probability calculation method of first-order cell destruction comprising six units is shown in Fig. 4.
The failure probability of a unit is $p$, and the probability of the whole unit is $1-p$. Assuming that two adjacent unit fails, the next level of cell damage can occur. In a case where stress transferred to adjacent elements after failure of one element, the $2 \times 3$ cell model is converted into a linear superposition of two $2 \times 2$ cell models in the failure probability calculation. Then, the probability of failure model is expressed as follows:

1) 6 units are undamaged, then $p$ is 0;

2) If a unit fails, the probability of destruction of the cell in the b configuration is given by

$$8p(1-p)^5[2p_{3,1}(1-p_{3,4})+p_{3,1}p_{3,4}] - 2p(1-p)^5[2p_{3,1}(1-p_{3,4})+p_{3,1}p_{3,4}]^2$$

3) In configuration c, two of the seven types are adjacent. Then, the probability of cell failure is $7p^2(1-p)^2$. When the two diagonal elements fail, c3 is, for example, first calculate in IV1, units 1 and 4 fail simultaneously, the force $F$ on it transfers $2\Delta F$ to unit 2 or unit 3, or $2\Delta F$ at the same time to unit 2 or unit 3, in which case the cell may the probability of failure is $4p^2(1-p)^5[2p_{1,2,3}(1-p_{1,4,3})+p_{1,2,3}p_{1,4,3}]$: in IV2, this model is similar to (2) in the relevant analysis. There are $p^2(1-p)^5[2p_{3,1}(1-p_{3,4})+p_{3,1}p_{3,4}]$, cell failure under this type. The probability is given by

$$4p^2(1-p)^5[2p_{1,2,3}(1-p_{1,4,3})+p_{1,2,3}p_{1,4,3})+p_{4,3}p_{6,4}] - 16p^4(1-p)^8[2p_{1,2,3}(1-p_{1,4,3})+p_{1,2,3}p_{1,4,3}] [2p_{4,3}(1-p_{4,6})+p_{4,3}p_{6,4}]$$

(3)

For the c3, c6, c12, and c13 types, models IV1 and IV2 are the same. Then,

$$4 \times 2p^2(1-p)^5[2p_{1,2,3}(1-p_{1,4,3})+p_{1,2,3}p_{1,4,3})-16p^4(1-p)^8[2p_{1,2,3}(1-p_{1,4,3})+p_{1,2,3}p_{1,4,3}]^2$$

(4)

The probability of destruction of the cells in the c configuration is the sum of the above three types.

4) In configuration d, two of the 18 types are adjacent. Then the probability of cell failure of this type is $18p^3(1-p)^3$. d8 and d13 are D2 types. Dividing this into two quaternion cells is similar to the analysis in (3). Therefore, in the IV1 and IV2 types, $p^3(1-p)^3[2p_{1,4,2}(1-p_{1,4,3})+p_{1,4,2}p_{1,4,3}]$. The failure probability of the D2 cell type is

$$2p^3(1-p)^3[4p_{1,4,2}(1-p_{1,4,3})+2p_{1,4,2}p_{1,4,3}] - 2[(p^3(1-p)^3[2p_{1,4,2}(1-p_{1,4,3})+p_{1,4,2}p_{1,4,3})]^2$$

(5)
The probability of destruction of cells in configuration d is the sum of the probabilities for the two above types.

5) In configuration e, there is only one failure mode. The probability of cell failure is $15p^4(1-p)^2$.

6) In configuration f, there is only one failure mode, and the probability of cell failure is $6p^5(1-p)^1$.

7) In configuration g, there is only one failure mode, and the probability of cell failure is $p^6$.

Then, the failure probability of the first-order cell model is the sum of the failure probability of the above seven configurations. Among them, $p_{3,1}$ and $p_{14,2}$ and other conditional probabilities, assuming the combined force of each unit after force transmission is $F'$, then $p_{3,1}$ and $p_{14,2}$ etc. are represented $p$.

$$p = \frac{p_{14}(F) - p_{14}(F)}{1 - p_{14}(F)}$$  \hspace{1cm} (6)$$

Therefore, it is necessary to quantify the $\Delta F$ to solve for $F'$.

### 3.3 Damage threshold of siltstone

The parameter $\alpha$ is used to describe the interaction between the self-organized critical OFC model elements. When $\alpha$ is 0, there is no interaction between adjacent units, and when $\alpha$ is 0.25, the interaction is maximized. $\alpha$ is actually a weight, that is, a stress condition is assigned to a certain direction. Since rock damage involves the formation of new cracks and the development of existing cracks, $\alpha$ is closely related to the rock properties and the trend of crack development. There are relatively few quantification theories for crack development (Dong Chunliang, 2015; YUAN Xiao-ping, 2012). Most of them use the concept of averaging. Herein, we study the stress transfer and employ the principle of averaging transfer to investigate the effects of stress transfer on the unit failure process. Assuming an infinitesimal cell model, it was established. The coefficient absorbed by the adjacent unit is $\alpha$, then the additional force received by the adjacent unbroken cell increases by $\Delta F$ or $2\Delta F$. Then $F'$ is

$$F' = F + \Delta F = F + aF = (1 + \alpha)F;$$

$$F' = (1 + 2\alpha)F$$ \hspace{1cm} (7)$$

The rock follows the following Weibull distribution:

$$p(F) = 1 - \exp \left(-\left(\frac{F}{F_0}\right)^m\right)$$ \hspace{1cm} (8)$$

Where $m$ and $F_0$ are Weibull distribution parameters, which reflect the mechanical properties of rock materials. Combining the above formula and the Weibull distribution law obeyed by rock damage, the above conditional probabilities are expressed as follows:

$$p_{3,1} = 1 - [1 - p(F)]^{2(2\alpha+\alpha^2)}; \quad p_{14,2} = 1 - [1 - p(F)]^{4(4\alpha+4\alpha^2)}$$ \hspace{1cm} (9)$$

The self-similarity and self-organization of rock show that the failure expression of the first-order cell is the same as that of the n-level cell. When $n$ is large enough, the principle of renormalization has $p_n = p_{n+1}$. Then, $p_n = f(p_n)$, i.e., $f(p_n) - p_n = 0$, which is a typical fixed point theorem. To replace $p_n$ with $p$,

$$8p(1-p)^5[2(1 - (1 - p)^{2\alpha+\alpha^2})(1 - p)^2(1 - p)^{2\alpha+\alpha^2}] + 7p^2(1-p)^4 + 4 \times 2p^2(1-p)^3 [2(1-p)^{4\alpha+4\alpha^2}] + 18p^2(1-p)^3 + 2p^2(1-p)^3 [4(1 - (1 - p)^{4\alpha+4\alpha^2})] (1 - p)^{4\alpha+4\alpha^2} + 15p^2(1-p)^2 + 6p^3(1-p)^2 + p^6 - p^9 = 0 $$ \hspace{1cm} (10)$$

When $\alpha$ is 0, $p$ is 0, 1, 0.4; when $\alpha$ is 0.25, $p$ is 0, 1, 0.119.

For other values of $\alpha$, the failure probability has two same values of 0 and 1, indicating that 0 and 1 are stable fixed points. The two values represent the integrity and failure of rock. It also shows that the failure probability equation that expresses the damage degree of rock is
accurate. When α is 0, it indicates that the cell unit cannot transmit stress to the periphery, which also indicates that the rock has been damaged, and the bearing strength is almost zero. This is consistent with the result of the SHPB test, where the rock damage is obtained to be 0.4. When α is 0.25, the damage is 0.119 which differs from the initial crack of the siltstone, where the damage was 0.16 in the SHPB test. The similarity and difference in the results may be attributed to the accuracy of the SHPB test. Meanwhile, when there is a crack inside siltstone, α is less than 0.25. By changing the cellular automaton model of α between the units to study the damage evolution of the rock, rock strength weakening and failure of the two thresholds can be observed, showing that the renormalization group method to is feasible for studying rock damage evolution. Also, siltstone SHPB impact test results validate each other series, showing that the method is reliable.

4. CONCLUSION

We conducted SHPB cyclic impact test on siltstone to study the damage mechanism. When the damage value of siltstone was greater than 0.16, the rock damage showed a cumulative effect. When the damage value was between 0.16 and 0.4, the rock damage varied exponential with the stress amplitude. When the damage value was 0.4, the siltstone immediately changed to the failure state.

A 2×3 two-dimensional renormalization group cell model was developed, which exhibits the power-law distribution feature and microscopic layered knot of siltstone damage. With different force transfer coefficients for the model, the damage threshold of the rock failure was calculated to be 0.4, and the threshold of non-damage accumulation of the rock was 0.119. Theoretical and experimental analyses proved that the upper limit of siltstone damage is less than 1.

According to the self-organization phenomenon of rock damage, the renormalization group method was employed to develop a cell model suitable for calculating the damage probability of siltstone structures. This method serves as a reference significant for establishing universal rock damage thresholds or critical value theory and the predicting of rock engineering damage.

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