Stable ‘antiferromagnetic’ vortex lattice imprinted into a type-II superconductor

V N Gladilin$^{1,2}$, J Tempere$^{2,3}$, J T Devreese$^2$ and V V Moshchalkov$^1$

$^1$ INPAC—Institute for Nanoscale Physics and Chemistry, Katholieke Universiteit Leuven, Celestijnenlaan 200D, B-3001 Leuven, Belgium

$^2$ TQC—Theory of Quantum and Complex Systems, Universiteit Antwerpen, Universiteitsplein 1, B-2610 Antwerpen, Belgium

E-mail: Jacques.Tempere@ua.ac.be

New Journal of Physics 14 (2012) 103021 (9pp)
Received 4 July 2012
Published 15 October 2012
Online at http://www.njp.org/
doi:10.1088/1367-2630/14/10/103021

Abstract. In type-II superconductors, where vortices and antivortices tend to annihilate, only a ‘ferromagnetic’ vortex lattice, with the same orientation of vortex magnetic moments, is usually formed in a homogeneous external magnetic field. Using the time-dependent Ginzburg–Landau formalism, we demonstrate that a checkerboard vortex–antivortex lattice (‘antiferromagnetic vortex lattice’), imprinted onto a superconducting film by a periodic array of underlying clockwise and counterclockwise microcoils generating spatially periodic positive and negative magnetic field pulses and then trapped by an array of artificial pinning centers, remains stable even after the imprinting magnetic field pulse is switched off.

Online supplementary data available from stacks.iop.org/NJP/14/103021/mmedia

$^3$ Author to whom any correspondence should be addressed.

Content from this work may be used under the terms of the Creative Commons Attribution-NonCommercial-ShareAlike 3.0 licence. Any further distribution of this work must maintain attribution to the author(s) and the title of the work, journal citation and DOI.

New Journal of Physics 14 (2012) 103021
1367-2630/12/103021+09$33.00 © IOP Publishing Ltd and Deutsche Physikalische Gesellschaft
Magnetic moments in solids can order ferro- or antiferromagnetically [1–3]. In some analogy with that, in type-II superconductors the ordered vortex state can be built up from entities of the same or opposite polarity—vortices and antivortices. Such vortex lattices can be referred to as ‘ferromagnetic’ and ‘antiferromagnetic’ vortex states. In this terminology, the Abrikosov vortex lattice [4] is an example of a ‘ferromagnetic’ state. During the last decade, vortex patterns consisting of superconducting vortices and antivortices have attracted significant theoretical and experimental efforts. A number of stable vortex–antivortex (VAV) configurations have been predicted and experimentally detected in symmetric mesoscopic superconducting samples subjected to a homogeneous magnetic field [5–7]. VAV pairs can be naturally created and stabilized by spatially inhomogeneous fields of micromagnets [8–11], leading to various commensurability effects and novel stable VAV configurations in superconductor–ferromagnet hybrids with regular arrays of magnetic dipoles [9, 12, 13].

Whereas VAV lattices were induced using an array of micromagnets, the creation of a VAV state that remains stable without permanent presence of an external spatially alternating magnetic field is still an open problem. An appealing idea relying on superconducting relaxation dynamics was formulated in [14, 15]. This proposal considers vortices and antivortices that are spontaneously generated during the recovery of superconductivity after a sample was heated locally with a laser pulse. In the presence of strong pinning centers some of those vortices and antivortices can be frozen and persist practically indefinitely. In this context, it was suggested [16] that an analysis of the superconducting phase recovery, accompanied by the creation of vortices and antivortices, provides a convenient tool to test the Kibble–Zurek cosmological scenario [17, 18] for the nucleation of topological defects during a symmetry-breaking phase transition.

In this paper, we consider the use of a specially designed pulse of spatially periodic magnetic field for imprinting a regular VAV lattice, which, as we predict, remains stable after the removal of the external magnetic field used for VAV imprinting. In particular, we will demonstrate that due to mutual cancellation of forces, which act on a vortex (antivortex) in the equilibrium state of such a lattice, the antiferromagnetic vortex state can be stabilized even in the case when pinning centers in the superconductor are relatively weak.

To describe the vortex dynamics, we apply the time-dependent Ginzburg–Landau (TDGL) formalism as presented in [19, 20]. For the sake of convenience, we use dimensionless variables by expressing lengths in units of \( \sqrt{2} \xi \) and currents in units of \( \Phi_0 \xi/(\sqrt{2} \pi \mu_0 \lambda^2) \), where \( \Phi_0 = \pi h/e \) is the magnetic flux quantum, \( \mu_0 \) is the vacuum permeability, \( \xi \) is the coherence length and \( \lambda \) is the penetration depth. The unit of magnetic field, \( \Phi_0/(4\pi \xi^2) \), is half the second critical field. Our unit of time is \( 2u\tau_{GL} \), where \( \tau_{GL} \) is the Ginzburg–Landau relaxation time and \( u = \pi^4/[14\zeta(3)] \) with \( \zeta(x) \) being the Riemann zeta function.

We consider a thin infinite superconductor film subjected to a pulse of spatially periodic negative and positive magnetic fields (figure 1(a)). In the model, this field is generated by an underlying square array of circular quasi-one-dimensional current loops with diameter \( D \), located at a distance \( d_1 \) below the middle of the superconductor layer. The current direction changes from one loop to another in the checkerboard order as shown in figure 1(a), thus generating a periodic array of local magnetic fields of the opposite polarities. It is assumed that due to an appropriate arrangement of the leads connecting the loops to each other, the magnetic field induced by these leads is negligible. In the present calculations, the spatial period of the
Figure 1. (a) Unit cell of a periodic structure, which contains a superconducting film and a square array of $\Omega$-shaped current loops of diameter $D$ at a distance $d_1$ from the film. When applying a current $I(t)$ as shown with green arrows, the magnetic field induced by the loops alternates in checkerboard order. Spatial modulations of the superconductor thickness provide an array of vortex-pinning centers, which are shifted along the $x$-axis by a distance $s_x$ with respect to the centers of the nearest current loops. (b) At the end of the pulse plateau, a lattice of vortices and antivortices, seen as local suppressions in the square modulus of the order parameter $|\psi|^2$, is created by the magnetic field $B_z$ of the loops. The red (blue) arrows indicate the direction of the magnetic moment of vortices (antivortices). (c) When the applied current decreases and eventually vanishes, the vortices and antivortices drift toward the nearest pinning centers. This results in a stable VAV lattice characterized by an ‘antiferromagnetic’ distribution of the induced magnetic field $B_z$. 
structure is \( L = 15 \), the superconductor film thickness is \( d = 0.3 \) and the Ginzburg–Landau parameter of the superconductor is \( \kappa \equiv \lambda / \xi = 1 \). The current pulse in the loops has a trapezoidal shape: the current \( I(t) \) linearly increases from 0 to \( I_{\text{max}} \) in the time interval from \( t = 0 \) to \( t = t_1 \), remains constant \( I(t) = I_{\text{max}} \) in the time interval \( (t_1, t_2) \) and then linearly decreases down to 0 within the interval \( (t_2, t_3) \). A periodic lattice of up and down out-of-plane magnetic fields, induced by such a current pulse, creates in the superconducting layer a square lattice of vortices and antivortices located just above the loops with counterclockwise and clockwise current directions, respectively. Vortices and antivortices, arranged to form a periodic checkerboard-like square lattice, remain in equilibrium even after completely switching off the applied magnetic field. This equilibrium is obviously unstable in an ideally uniform superconducting film: an arbitrarily weak perturbation would destroy the lattice due to the VAV attraction and subsequent recombination of the VAV pairs. However, in reality superconducting films often contain numerous naturally occurring pinning centers, which may prevent such a collapse of the VAV lattice. Alternatively, in a more controlled way, a periodic array of artificial pinning centers can be intentionally induced, e.g., in the form of a periodic array of ‘blind’ or complete antidots [21, 22].

Here, we consider the square array of artificial pinning centers formed by indentations of depth \( \delta \ll d \) made in the superconducting film of thickness \( d \) (blind antidots), as shown in figure 1(a). The number and periodicity of the pinning sites correspond to the number and periodicity of the current loops, but the arrays of loops and blind antidots are shifted with respect to each other. We have analyzed shifts along the side of a unit cell, over a distance denoted by \( s_x \), and shifts along the diagonal of the unit cell, over a distance \( s_d \).

Figures 1(b) and (c) illustrate the formation of a VAV lattice in the case of \( \delta = 0.06 \), \( d_1 = 0.5 \), \( D = 1 \), \( s_x = 2 \), \( I_{\text{max}} = 4 \), \( t_1 = 10 \), \( t_2 = 210 \) and \( t_3 = 250 \). Ramping up the periodical array of up and down magnetic fields during the time interval \( (0, t_1) \) strongly suppresses the order parameter \( |\psi| \) just above the centers of the current loops. Due to the radial gradient of the magnetic field, induced by a current loop, a VAV pair is generated at each loop. This process is quite similar to the VAV pair generation by a magnetic dipole considered in [23]. When the current pulse reaches its plateau, some recombinations take place: the vortex induced by the field of a current loop with counterclockwise current remains pinned to this loop, whereas the corresponding antivortex is driven away from the loop and recombinates with a vortex pushed away from one of the neighboring loops with clockwise current. After that, single vortices and antivortices, which correspond to one flux quantum \( \Phi_0 \) each, are left at the sites of the counterclockwise and clockwise current loops, respectively. Due to the aforesaided processes, a VAV lattice, pinned to the current-loop array, is imprinted into the superconductor (see figure 1(b)). This lattice pertains until the end \( (t = t_2) \) of the plateau in the applied field pulse. During the time interval \( (t_2, t_3) \), when the applied magnetic field decreases and eventually vanishes, the vortices and antivortices gradually drift toward the nearest pinning sites, resulting in the formation of an ‘antiferromagnetic’ vortex state, which is shown in figure 1(c) and which remains stable for an arbitrarily long time after the applied field has been switched off.

Figure 2 summarizes the results of the TDGL simulations for the structure shown in figure 1(a) at \( \delta = 0.06 \), \( d_1 = 0.5 \), \( D = 1 \), \( t_1 = 10 \), \( t_2 = 210 \) and \( t_3 = 250 \). Depending on the magnitude of the current in the loops, \( I_{\text{max}} \), and the horizontal \( (s_x) \) or diagonal \( (s_d) \) shift between the centers of the current loops and the pinning centers, different vortex configurations, shown in figure 2 with different symbols, appear to be stable after switching off the magnetic field pulse. In the case of zero shifts \( s_x \) and \( s_d \), no VAV lattice is formed in the film for current magnitudes...
lower than $I_{\text{max}} \approx 3.6$, even though at $I_{\text{max}} < 3.6$ the applied magnetic field pulse can lead to strong local suppressions of the order parameter. Interestingly, a moderate nonzero shift $s_x$ facilitates nucleation and spatial separation of vortices and antivortices from these suppressions. As follows from figure 2, this can result in the formation of a stable VAV lattice even for current magnitudes $I_{\text{max}}$ slightly below 3.6. A similar but somewhat less pronounced effect is seen to be caused by a diagonal shift $s_d$.

At $3.6 < I_{\text{max}} < 4.6$, an antiferromagnetic lattice is stabilized both at zero shifts $s_x$ or $s_d$ and at moderate nonzero values of these shifts. Our calculations show that for this range of $I_{\text{max}}$ the vortex pattern created at the end of the pulse plateau ($t = t_2$) has exactly one single flux quantum vortex or antivortex near the center of each current loop, thus forming a VAV checkerboard lattice. Taking as a unit cell the square with side $L$ (see figure 1(a)), this leads to two VAV pairs per unit cell. At moderate shifts $s_x$ or $s_d$, when switching off the applied magnetic field, this lattice simply shifts to the pinning-site array. However, when the distance between the centers of the loops and the pinning sites exceeds a critical value, a complete VAV annihilation, rather than the antiferromagnetic-lattice stabilization, occurs after switching off the applied magnetic field.
At larger $I_{\text{max}}$, i.e. for higher magnetic fields, each current loop generates more than one VAV pair due to the application of the magnetic field pulse. As a result, a giant vortex (antivortex), which corresponds to two or more flux quanta [24, 25], or a dense cluster of single flux quantum vortices (antivortices) is accommodated by most current loops at the end of the pulse plateau. Ramping down the applied field during the time interval $(t_2, t_3)$ leads to dissociation of these giant vortices (antivortices) or vortex (antivortex) clusters into single-$\Phi_0$ vortices (antivortices). The further evolution of the system is determined by an intricate interplay of the vortex–(anti)vortex interactions as well as the interaction of vortices and antivortices with the pinning potentials and the gradually decreasing applied inhomogeneous magnetic field. This leads to a rather complicated recombination dynamics of the vortices and antivortices. After that, generally one of two outcomes is realized: either we again obtain a stable VAV checkerboard pattern with two VAV pairs per unit cell as before, or full recombination has taken place. Remarkably, a moderate diagonal shift $s_d$ of the current loops with respect to the pinning sites can promote the antiferromagnetic-vortex-lattice stabilization for this range of $I_{\text{max}}$. No such effect is found for a shift $s_x$ along the side of the unit cell. The following qualitative difference between diagonal and horizontal shifts seems to be important: in the former case, the pinning sites lie on the lines that connect loops with the same direction of current, while in the latter case the pinning sites are shifted from a current loop toward a loop with the opposite current direction. As further seen from figure 2, besides the two ‘limiting cases’ (stabilization of a lattice with two VAV pairs per unit cell or full annihilation of pairs), sometimes an ‘intermediate’ configuration is observed for parameter values near the boundary between the regions corresponding to the aforementioned two ‘limits’. This configuration has one VAV pair per unit cell, and an example of this pattern will be given below.

Large shifts between the pinning lattice and the current-loop lattice impede the stabilization of the VAV lattice. In order to avoid this, the density of artificial pinning sites can be increased so that a pinning site is always nearby for the vortices generated by a current loop. An example of such a structure is shown in figure 3(a). As in the cases analyzed above, stable antiferromagnetic vortex lattices in this structure may contain either two VAV pairs per unit cell or one VAV pair per unit cell. The latter is illustrated by figure 3(b), showing the evolution of the magnetic field patterns, corresponding to vortices and antivortices, for the particular set of parameters, $\delta = 0.09, d_1 = 1, D = 2, I_{\text{max}} = 5.5, t_1 = 10, t_2 = 160$ and $t_3 = 260$. By the end of the magnetic-pulse plateau ($t = 160$), giant vortices (antivortices), carrying two flux quanta, are formed at each current loop. When decreasing the applied magnetic field, these giant vortices and antivortices still persist up to $t \approx 244$ (see figure 3(b)). However, with further reduction of the applied field (at $t \approx 252$), they dissociate into single-$\Phi_0$ vortices and antivortices, which then partially recombine (see the snapshots for $t = 254$ and 255 in figure 3(b)). Since the pinning potentials in the structures under consideration are relatively weak, they do not prevent the corresponding motion of vortices and antivortices. This means that these pinning centers can provide fine-tuning in stabilizing a state with coexisting vortices and antivortices. The crucial prerequisite for such a stabilization is mutual cancellation of vortex–vortex and VAV interactions in a regular antiferromagnetic VAV lattice. In the case under consideration, the order parameter finally settles into a stable state with one VAV pair per unit cell, corresponding to periodic VAV chains, separated from each other by a distance $L$ (see the corresponding magnetic field distribution at $t = 350$ in figure 3(b)).

Each current loop, independently of the sense of the current in it, can generate both vortices and antivortices in the superconductor film. This suggests that a stable VAV lattice can also
be formed in a periodic structure where the current direction is the same for all the current loops. The results of our calculations for those structures (see supplementary information, available at stacks.iop.org/NJP/14/103021/mmedia) confirm such a possibility. Moreover, these results demonstrate that (at least for certain sets of the relevant parameters), stabilization of a VAV lattice is possible even when the number of vortices and antivortices, induced by each current loop in the course of the magnetic field pulse, is much larger than 1. At the same time,
we find that the most promising and reliable regime for the experimental realization of the imprinted antiferromagnetic VAV lattices that remain stable after the external magnetic field is switched off corresponds to the generation of just one VAV pair by each current loop.

Since the unit of length used in our calculations is proportional to the coherence length $\xi$, the examples given above can correspond to different geometric parameters, depending on a particular superconducting material and temperature. Thus, for the coherence length $\xi \approx 250\,\text{nm}$ found in [20] for an Al film at $T = 0.63T_c$, the structure shown in figure 3 corresponds to the unit-cell size of about $5.3\,\mu\text{m}$, the superconducting-layer thickness of $100\,\text{nm}$, the diameter of current loops of $700\,\text{nm}$ and the spacing between the loops and the superconducting layer of about $350\,\text{nm}$. For the depth and lateral size of the indentations, which serve as pinning sites, we then obtain the values of about $30$ and $600\,\text{nm}$, respectively. Of course, all the indicated values would appear larger (smaller) for higher (lower) temperatures.

Experimentally, stable imprinted VAV lattices, theoretically predicted in this paper, can be directly visualized in nanopatterned systems (similar to the one shown in figure 1) by using scanning Superconducting Quantum Interference Device [26–29] or Hall probe [30] microscopies.

To conclude, we have proposed the application of a pulse of spatially periodic up and down magnetic fields to imprint a VAV lattice on a superconducting film. Stabilization of the imprinted VAV pattern can be provided by an artificial array of pinning centers, so that the antiferromagnetic vortex pattern remains stable in zero magnetic field, after the magnetic field pulse is turned off. The presence of pinning centers (especially of their periodic arrays) in the superconducting layer is a necessary but—in general—not sufficient condition for the antiferromagnetic-vortex-lattice stabilization: it is also sensitive to the magnitude of the applied magnetic field pulse as well as to its spatial and temporal shape. Nevertheless, our results clearly demonstrate that the relevant parameters, which ensure the creation of stable antiferromagnetic vortex lattices with a relatively high density of vortices and antivortices, can vary in a rather broad range. Moreover, the stabilization of the antiferromagnetic vortex lattices is even easier in the presence of a denser periodic array of artificial pinning centers.

Acknowledgments

This work was supported by Methusalem funding by the Flemish Government, the Flemish Science Foundation (FWO-Vl) through FWO projects numbers G.0356.05, G.0115.06, G.0370.09N and G.0115.12N, the Scientific Research Community through project number WO.033.09N, the Belgian Science Policy and the European Science Foundation NES network.

References

[1] Weiss P 1907 J. Phys. 6 661
[2] Heisenberg W 1928 Z. Phys. 49 619
[3] Néel L 1948 Ann. Phys. 3 137
[4] Abrikosov A A 1957 Sov. Phys.—JETP 5 1174
[5] Chibotaru L F, Ceulemans A, Bruyndonck V and Moschalkov V V 2000 Nature 408 833
[6] Misko V R, Fomin V M, Devreese J T and Moschalkov V V 2003 Phys. Rev. Lett. 90 147003
[7] Geurts R, Milošević M and Peeters F M 2006 Phys. Rev. Lett. 97 137002
[8] Lyuksyutov I F and Pokrovsky V I 1998 Phys. Rev. Lett. 81 2344

New Journal of Physics 14 (2012) 103021 (http://www.njp.org/)
[9] Lange M, Van Bael M J, Bruynseraede Y and Moshchalkov V V 2003 Phys. Rev. Lett. 90 197006
[10] Kayali M A 2004 Phys. Rev. B 69 012505
[11] Erdin S 2005 Phys. Rev. B 72 014522
[12] Priour D J Jr and Fertig H A 2004 Phys. Rev. Lett. 93 057003
[13] Milošević M V and Peeters F M 2006 Physica C 434–438 208
[14] Ghinovker M, Shapiro B Ya and Shapiro I 1998 Europhys. Lett. 44 354
[15] Ghinovker M, Shapiro I and Shapiro B Ya 1999 J. Low Temp. Phys. 116 9
[16] Ghinovker M, Shapiro B Ya and Shapiro I 2001 Europhys. Lett. 53 240
[17] Kibble T W B 1976 J. Phys. A: Math Gen. 9 1387
[18] Zurek W H 1996 Phys. Rep. 276 177
[19] Chapman S J, Du Q and Gunzburger M D 1996 Z. Angew. Math. Phys. 47 410
[20] Silhanek A V, Gladilin V N, Van de Vondel J, Raes B, Ataklti G W, Gillijns W, Tempere J, Devreese J T and Moshchalkov V V 2011 Supercond. Sci. Technol. 24 024007
[21] Baert M, Metlushko V V, Jonckheere R, Moshchalkov V V and Bruynseraede Y 1995 Phys. Rev. Lett. 74 3269
[22] Harada K, Kamimura O, Kasai H, Matsuda T, Tonomura A and Moshchalkov V V 1996 Science 274 1167
[23] Gladilin V N, Tempere J, Devreese J T, Gillijns W and Moshchalkov V V 2009 Phys. Rev. B 80 054503
[24] Moshchalkov V V, Baert M, Metlushko V V, Rosseel E, Van Bael M J, Temst K, Jonckheere R and Bruynseraede Y 1996 Phys. Rev. B 54 7385
[25] Moshchalkov V V, Baert M, Metlushko V V, Rosseel E, Van Bael M J, Temst K, Bruynseraede Y and Jonckheere R 1998 Phys. Rev. B 57 3615
[26] Kirtley J R, Tsuei C C and Tafuri F 2003 Phys. Rev. Lett. 90 257001
[27] Tafuri F, Kirtley J R, Medaglia P G, Orgiani P and Balestrino G 2004 Phys. Rev. Lett. 92 157006
[28] Carillo F, Papari G, Stornaiuolo D, Born D, Montemurro D, Pingue P, Beltram F and Tafuri F 2004 Phys. Rev. B 81 054505
[29] Kirtley J R, Tsuei C C, Tafuri F, Medaglia P G, Orgiani P and Balestrino G 2004 Supercond. Sci. Technol. 17 S217
[30] Gutierrez J, Raes B, Silhanek A V, Li L J, Zhigadlo N D, Karpinski J, Tempere J and Moshchalkov V V 2012 Phys. Rev. B 85 094511