Features of renormalization induced by interaction in 1D transport.

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(April 24, 1977)

One-dimensional interacting electrons in a quantum wire connected to reservoirs are studied theoretically. The difference in the Tomonaga-Luttinger interaction constants between the wire (g) and reservoirs (g\textsubscript{∞}) produces the cross-correlation between the right- and left-going chiral components of the charge density wave field. The low energy asymptotics of this field correlator, which is determined by (g) and (g\textsubscript{∞}), specifies renormalization of physical quantities. We have found that charge of the carriers in the shot noise is determined by g\textsubscript{∞} (no renormalization for the Fermi liquid reservoirs) at any energy, meanwhile the factor g renormalizing the charge and spin susceptibilities emerges in the threshold structures at some rational fillings.

71.10.Pm,72.15.Nj, 73.23.Ps

I. INTRODUCTION

Recent development in the nano-fabrication technique makes 1D interacting electron systems an experimental reality. This allows comparison of the transport experiments with the predictions of the 1D quantum field theory, which has been developing since the early 1930’s. A basic property of this theory suggests that even a weak electron-electron interaction changes the nature of elementary quasi-particles in 1D. Instead of free-electron behavior they acquire a fractional charge and statistics. To deal with the metallic phase of these systems, a simple model called the Tomonaga-Luttinger liquid (TLL) has been developed. This model is specified by one constant g of the interaction. Its deviation from the free-electron value g = 1 renormalizes the density of states of the sound wave excitations, the compressibility, and the charge of TLL quasiparticles. Therefore, it had been expected that the same factor of the fractional charge has to appear in the conductance \[\frac{e^2}{h}\] and in the shot noise \[e^2/2h\]. It had been assumed that this renormalization would show up in 1D transport through a quantum wire and through the edge of a Fractional Quantum Hall Liquid (FQHL). First transport measurements have shown that, in contrast to the above expectation, there is no renormalization of the 1D conductance of the clean quantum wire \[\frac{e^2}{h}\], although there is the renormalization of the conductance and shot noise in transport through the edge of the FQHL \[\frac{e^2}{h}\]. The first result has been explained by taking into account the Fermi liquid source and drain reservoirs with the inhomogeneous TLL model (ITLL) of 1D transport \[\frac{e^2}{h}\]. Its solution clarified that the zero-frequency conductance is unchanged in the TLL wire at any temperature/voltage and has a simple physical explanation: The interaction in the TLL wire reduces to forward scattering, and cannot change the outgoing flows of electrons from the reservoirs whose chemical potentials are shifted by the applied voltage. It has been suggested later \[\frac{e^2}{h}\] that the same model describes transport through the FQHL connected with the reservoirs via two point-like contacts. However, in typical experiments these contacts are wide. Then the equilibration of the edge chemical potential with the reservoir is expected \[\frac{e^2}{h}\] to account for the fractional value of conductance equal to the Hall conductivity. This suggests \[\frac{e^2}{h}\] that both problems might be treated in a unified fashion in terms of an effective voltage which, up to a factor $e$ of electron charge, coincides with the difference between the chiral electrochemical potentials. The latter is equal to the real voltage for the FQHL transport according to the above hypothesis of the equilibration and has to be rescaled by a factor $g^{-1}$ in the case of the TLL wire \[\frac{e^2}{h}\]. Therefore, it was claimed \[\frac{e^2}{h}\] that an exact solution to the problem of a point impurity in uniform TLL describes both the FQHL edge and the TLL wire transport with the above choice of the effective voltage.

In this paper we discuss transport through the 1D quantum wire in terms of the inhomogeneous TLL (ITLL) model \[\frac{e^2}{h}\] to gain a further insight into the role of the reservoirs. Our final objective is specification of an easy observable parameter in 1D transport through the wire which may confirm existence of the TLL phase in the latter when its conductance shows an approximately power suppression \[\frac{e^2}{h}\] with lowering temperature $T$. In particular, we examine shot noise whose relation to the current doesn’t contain the applied voltage $V$ and therefore could along the lines of the above argument, reveal a fractional charge of the carriers as it appears in the uniform wire solution \[\frac{e^2}{h}\] addressed to the FQHL transport \[\frac{e^2}{h}\].

This paper is organized as follows. We describe our model in Section II. For electrons with spin the model assumes that the reservoirs are Fermi Liquid, and may be mapped onto the free TLL with the constant $g_{\infty} = 1$, whereas the wire contains repulsive TLL with $g < 1$. Inside the wire a weak backscattering is produced by random impurities, and by Umklapp interaction due to a periodic potential near some rational fillings. In the spinless case $g_{\infty} < 1$ could also occur when the reservoirs are in a principal FQH state with a single edge state along
the boundary.

In Section III we derive an expression for the backscattering current which is an operator proportional to the rate of particle exchange between the right and left chiralities. Its average shows a deviation of the current average below its unsuppressed value $2e^2g_{\infty}V/h$. In our derivation this operator is determined by a low frequency asymptotics of the retarded correlator of the charge density wave field. The operator specifies an energy and charge transferred by the one-particle exchange as $e^2g_{\infty}V$ and $g_{\infty}e$, respectively. Although this energy neither fixes the difference between fluctuations of the direct current, i.e., shot noise, and the average of the backscattered current, it allows us to derive a linear relation between fluctuations of the direct current, i.e., shot noise, and the average of the backscattered current. For the metallic wire this relation always has $g_{\infty}$ as the coefficient. It reveals that the zero-frequency current fluctuations are determined by the backscattering of carriers of the charge $g_{\infty}e$. It is not only valid for energies less than that $(T_L)$ of the wire length, where the average current behavior corresponds to the reservoir type spectrum (the Fermi liquid behavior at $g_{\infty} = 1$ [13], but also for energies above $T_L$. At these energies the current is ruled by the TLL spectrum of the wire. This means, in particular, that the above idea [13] of accounting for the reservoir effect by a proper choice of the effective voltage oversimplifies the problem and the exact solution by Lesage et al cannot be addressed to the wire transport in the way they suggested [10].

Our further search for an observable parameter, whose renormalization by the interaction endures connecting the wire to the reservoirs, starts from observation that the factor $g$ appears in the low frequency asymptotics of the retarded correlator. Then it eventually multiplies the chemical potential of the wire, i.e., the renormalization of TLL compressibility is not affected by the reservoirs. In Section V we suggest how this factor can be observed in the threshold structures [13] produced by the Umklapp scattering on the periodic potential of the wire in current vs voltage. When a filling factor $\nu$ of electrons in the wire related to the period of the potential is close to its rational value (1, 1/2, 1/3, etc), this structure arises as the current suppression by the Umklapp backscattering is strengthened when voltage exceeding an energy $E_{\text{thr}}$ proportional to the quasi-momentum is transferred. The resonant Umklapp backscattering at an even denominator $\nu$ induces charge transfer only, and the coefficient of the proportionality is $v_{\nu}$, the velocity of the charge excitation. Meanwhile at odd denominator $\nu$ there are two threshold voltages proportional to velocities of the charge and spin excitations $v_{\nu}$ and $v_{\nu'}$, respectively. This structure has been observed [13] in transport through a 1D wire with the periodic potential induced artificially. However, the interpretation of the experiment lacks an understanding of the interaction effect. Recently Tarucha et al. [14] succeeded in introducing potential of a shorter period into a more narrow 1D wire. The electron density $\rho_c$ can be continuously controlled by the gate voltage, and one can satisfy the half-filling condition within an accessible value of $\rho_c$. Then observation of a variation of $E_{\text{thr}}$ with changing the average chemical potential would probe the TLL inside the wire. However, in this section we will see that the method may be used only under a severe restriction to the capacitance density between a close screening gate and the wire. Otherwise the classical electrostatics essentially modifies the TLL compressibility. If the density is large enough, $E_{\text{thr}}$ related to the charge transfer shifts as $\Delta E_{\text{thr}} = geV$ under asymmetrically applied voltage when only one chemical potential of the reservoirs is biased. The threshold voltage related to the spin transfer shifts as $\Delta E_{\text{thr}} = geVv_{\nu}/v_c$ $(c = h = 1$ below). Similarly, under application of a magnetic field $H$, $E_{\text{thr}}$ produced by the Umklapp processes involving the spin transfer splits into two thresholds divided by a gap equal to $\mu_c H$, with the electron magnetic momentum $\mu_c$ renormalized by the interaction. Finally, we summarize our findings and sketch an attempt at their generalization in Section VI.

II. MODEL

Our model can be derived following [1] from a 1 channel electron Hamiltonian

$$\mathcal{H} = \int dx \left\{ \sum_\sigma \psi_\sigma^+(x)(-\frac{\partial^2}{2m^*} - E_F)\psi_\sigma(x) + \varphi(x)\rho^2(x) + [V_{\text{imp}}(x) + V_{\text{period}}(x)]\rho(x) \right\}, \quad (1)$$

with the periodic potential $V_{\text{period}}(x)$ (period $a$) producing Umklapp backscatterings and the random impurity potential $V_{\text{imp}}(x)\rho(x)$. The Fermi momentum $k_F$ and the Fermi energy $E_F$ is determined by the filling factor $\nu$ as $\nu = k_F a/\pi$ and $E_F \approx v_F k_F$. In Eq. (1) the function $\varphi(x) = \theta(x)(L - x)$ switches on the electron-electron interaction inside the wire confined in $0 < x < L$. Following Haldane’s generalized bosonization procedure [7] to account for the non-linear dispersion one has to write the fermionic fields as $\psi_\sigma(x) = \sqrt{k_F/(2\pi)} \sum \exp(i(n + 1)(k_F x + \varphi_\sigma(x)/2) + \theta_\sigma(x)/2)$ and the electron density fluctuations as $\rho(x) = \sum \rho_\sigma(x)$, $\rho_\sigma(x) = \langle \partial_x \phi_\sigma(x)/(2\pi) \sum \exp(i(n + 1)k_F x + \varphi_\sigma(x)/2) \rangle$ where summation runs over even $n$ and $\phi_\sigma, \theta_\sigma$ are mutually conjugated bosonic fields $[\phi_\sigma(x), \theta_\sigma(y)] = i2\pi \text{sgn}(x - y)$.

After substitution of these expressions into (1) and introduction of the charge and spin bosonic fields as
\[ \phi_{c,s} = (\phi_1 \pm \phi_2)/\sqrt{2} \] we come to the bosonic form \( \mathcal{H}_B \) of the Hamiltonian \([\text{I}]\). We skip writing it down (see \([\text{II}] \)), as below we will use its associate Lagrangian with respect to the \( \phi \)-fields. It will appear in our calculation of the averages of operators which are functions of the \( \phi_{c,s} \) fields. Considering \( 1/(4\pi) \partial_t \phi_{c,s}(x) \) as the conjugated momentum to the field \( \phi_{c,s}(x) \), respectively, one can find the density of the Lagrangian as
\[ \mathcal{L} = \sum_{b=c,s} \mathcal{L}_b(x, \phi_b, \partial_t \phi_b) + \mathcal{L}_{bs}(x, \phi_c, \phi_s). \tag{2} \]

The first part of the Lagrangian describes a free-electron movement modified by the forward scattering interaction. Its density is a quadratic form \([\text{III}]\):
\[ \mathcal{L}_b = -\frac{E_F^2 \varphi(x)}{v_F} \left[ \sum_{\text{even} \ m > 0} U_m \cos(2k_{mF}mx + \frac{m\phi_c(x)}{\sqrt{2}}) + \sum_{\text{odd} \ m \geq 1} (\delta_{m,1} \frac{V_{\text{imp}}(x)}{2\pi E_F} + U_m \cos(\frac{\phi_s(x)}{\sqrt{2}}) \cos(2k_{mF}mx + \frac{m\phi_c(x)}{\sqrt{2}})) \right]. \tag{4} \]

Here \( \varphi(x) = \theta(x)\theta(L - x) \) switches on the backscattering interaction inside the wire confined in \( 0 < x < L \). The Fermi momentum \( k_F \) relates to the Fermi energy \( E_F \) as \( E_F \approx v_F k_F \), to the filling factor \( \nu = k_F a/\pi \), and to the transferred momenta \( 2k_{mF} \) in the absence of applied voltage: \( k_{mF} = k_F - \pi l/(ma) > 0 \), where \( l \) is an integer chosen to minimize \( k_{mF} \). Depending on the value of \( g \) a few terms of the Umklapp backscattering could open gaps \( M \) in the spectrum of the infinite wire at small \( k_{mF} \). The most singular is the first term of the second sum \( (m = 1) \) responsible for opening of the band gap in the infinite wire at \( \nu = 1 \). The first term of the first sum \( (m = 2) \) produces a Hubbard gap at half filling. Away from these fillings all backscattering is produced by the random impurity potential \( V_{\text{imp}} \) and the wire is in a metallic phase (TLL) or an Anderson insulator. General consideration of the backscattering current and the current fluctuations in Sections III and IV does not assume any small parameters. To probe shot noise, however, we treat the backscattering in the lowest perturbative order. In Section V we neglect the random potential and assume the perturbative regime of the Umklapp interaction \( M/T_L \ll 1 \) which has been observed experimentally \([\text{I}]\).

Application of smooth electric \((-\partial_x V(x))\) and magnetic \((H(x))\) fields may be accounted for by addition of the following Lagrangian density,
\[ \mathcal{L}_V + \mathcal{L}_H = \frac{1}{\sqrt{2\pi}} \{ \phi_c \partial_x V(x) + \frac{1}{2} \phi_s H(x) \}, \tag{5} \]

to the one in \([\text{III}]\). The function \( V(x) \) is equal to the electrochemical potential of the left (right) reservoir \( V_L(V_R) \) outside the wire \( x < 0(x > L) \). Inside the wire it will be determined self-consistently in the case of a uniformly screened wire by a close gate in the absence of impurities in Section V. To describe the effect of the uniform magnetic field we put \( H(x) = H \) inside the wire, \( 0 < x < L \), and switch the field off at the infinity \( H(\pm\infty) = 0 \).

With a finite voltage applied between the reservoirs \( V = V_L - V_R \), the system is in non-equilibrium. Then calculation of a physical quantity average at some moment of time involves consideration of the time evolution of the system along the Keldysh contour \( \Gamma \) which runs from \(-\infty \) to \(+\infty \) above the real time axis and returns back to \(-\infty \) below it. In particular, the current flowing through the point \( x \) at the time \( t_0 \) is given by the average of its operator
\[ I(V) = \langle \hat{I} \rangle = -\langle \partial_t \phi_c(x, t_0) \rangle / (\sqrt{2\pi}) \]

It does not depend on \( x \) and \( t_0 \) for the stationary voltage. Describing the evolution in functional integral technique allows us to write it as
\[ I = -\int \left( \prod_b D\phi_b \right) \frac{\partial_x \phi_c(x, t_0 - i\theta)}{\sqrt{2\pi}} e^{i(S + S_V + S_H)} \tag{6} \]
\[ S = \sum_{b=c,s} S_\phi(\phi_b, \partial_t \phi_b) + S_{bs}(\phi_c, \phi_s) = \int dy \int dt \sum_b \mathcal{L}_b(y, \{ \phi_b, \partial_t \phi_b \}) + \mathcal{L}_{bs}(y, \{ \phi_{c,s} \}) \tag{7} \]
\[ S_{V,H} = \int dy \int dt \mathcal{L}_{V,H}(y, \{ \phi_{c,s} \}) \tag{8} \]

Here the differential operator acting on the Keldysh time contour in the first two Gaussian actions \( S_{c,s} \) in \([\text{III}]\) needs further specification. It could be gathered from comparison with the equilibrium model in the absence of \( S_{bs}, S_{V,H} \). We may write these operators as a \( 2 \times 2 \)
matrix operator working on the ordinary time contour going from $-\infty$ to $\infty$ if vectors $\phi_b(x,t)$, $(b = c,s)$ are introduced with components $\phi_b = (\phi_b(x,t_+), \phi_b(x,t_-))$ $(t_\pm = t \pm i0)$. Then the quadratic Lagrangian density of $S_b$ takes its matrix form $L_b = \frac{i}{2} \hat{T}_b^{-1} \hat{T}_b \phi_b$, where the matrix operator $\hat{T}_b$ may be composed from the equilibrium finite temperature correlators $T_{b,\pm,\pm}(x,y,t) =$ $< \hat{T}_b \hat{T}_b^\dagger \phi_b \phi_b \phi_b^\dagger \phi_b >$ in the absence of $L_b$ and $L_V \phi_b$. These correlators are proper solutions to the differential equation related to $K_b$. They are connected via a specific relationship with the retarded correlator $S_b \phi_b \phi_b \phi_b^\dagger \phi_b$, calculated in $[11]$. In general, only Fourier transform of these correlators with respect to $t$ ($\omega$-form) is well determined because of $1/\omega$ singularity. We will use the $t$-form symbolically to make notation shorter.

The above consideration could be easily modified to cover the spinless case, where $\phi_s$ is absent ($\phi_s = 0$) and the charge density is $\rho_c(x,t) = (\partial_x \phi_c(x,t))/\sqrt{2\pi}$. The $1/\sqrt{2}$ factor at the $\phi_c$ field drops out from the backscattering Lagrangian $[10]$. In the spinless case $g_\infty = 1/(2n+1) = \nu$ could occur in experiment if the reservoirs are in a proper FQHL phase.

III. DUALITY TRANSFORM AND BACKSCATTERING CURRENT

Part of the action $S_V + S_H$ induced by the external fields tends to shift an average value of $\phi_{c,s}$. This effect can be accounted for by changing the variables of the functional integration in $[10]$, $\phi_b(x,t_\pm) = \eta_b(x,t_\pm) + \Delta \phi_b(x,t_\pm)$, $b = c, s$, $\Delta \phi_c(x,t) = \sqrt{2} \left[ g_\infty (V(\infty) - V(-\infty)) t + \int_x dy \frac{g_c(y)}{v_c(y)} (V(+\infty) + V(-\infty) - 2V(y)) \right]$.

The time dependence of $\Delta \phi_c$ relates to the current shift under a finite voltage. It feels the entire drop of the potential $V = V(-\infty) - V(\infty)$ only. The spatial dependence reflects the electron density redistribution following the new potential profile. The shift of the spin field may be written by analogy with $[13]$ as $\Delta \phi_s(x) = -\sqrt{2x} g_{s} H/v_{s}$ for $x \in [0, L]$. It does not contain a time dependence, since with the choice of $H(x)$ we have assumed the effect of $L_V$ is equilibrium.

Applying the above change of variables $[10],[13]$ under the integral in $[10]$, we come to the relation between the so that the charge field satisfies:

$$S_c(\phi_c, \partial_t \phi_c) + S_V(\phi_c) = S_c(\eta_c, \partial_t \eta_c) + \text{const.}$$

and the spin field does the same. Let us first consider a non-equilibrium effect of the Lagrangian $[3]$ produced by the voltage. It results in such a shift of the charge field, $\Delta \phi_c(x,t) = \frac{i}{\sqrt{2\pi}} \int dt' \int dy T_{c,R}(t - t', x, y) \partial_y V(y)$,

where the integration over $y$ is, in fact, only inside the wire $y \in [0, L]$, as $V(y)$ is constant equal to the electrochemical potential of the left $V_L$ or right $V_R$ reservoir outside it. The low energy asymptotics of the retarded correlator for $x, y \in [0, L]$ extracted from $[11]$ as

$$T_{c,R}(x, y, \omega \approx 0) = 2\pi g_{\infty} \left( \frac{1}{\omega} + ic(x,y) \right),$$

$$c(x,y) = \frac{t_L}{2} \left( \frac{g_\infty}{g} - \frac{g}{g_\infty} \right) + \frac{g|y-x|}{v_c g_\infty},$$

are specified with two constants $g_{\infty}, g$ in addition to the dimensional and, hence, non-universal parameters: charge wave velocity in the wire $v_{c}$ and traversal time $t_L = L/v_c$. These constants determine the coefficients at low $\omega$ and small $x - y$ singularities respectively. The difference between these two constants $g$ and $g_{\infty}$ indicates the cross correlation between the left and right chiral components of $\phi_c$ in the wire. Remarkably, the same constant $g$ in $[12]$ also determines the $1/\omega$ behavior for high energies $\omega t_L \gg 1$, and therefore the density of states for the charge wave excitations of the TLL inside the wire.

Substituting these asymptotics $[11],[12]$ one can come to

average direct and backscattering currents:

$$I = \sigma_0 V - \int (\prod_b \delta \eta_b) \frac{i}{\sqrt{2\pi}} e^{i \sum_b S_b(\eta_b, \partial_t \eta_b) + S_{\text{ext}}(\eta + \Delta \phi)}.$$

Here $\sigma_0$ is the universal conductance $g_{\infty}^2/\pi$, and the second term which we denote $\Delta I$ ensues from the backscattering Lagrangian $[3]$. Indeed, this term may be found by expanding the exponent under the integral in $S_b$, and noticing that the gaussian $\eta$-field correlators coincide with the $T$-correlators introduced in the previous Section.

$$\Delta I = \sum_{b = c,s} \frac{-i}{\sqrt{2\pi}} \int_{-\infty}^{\infty} dt \int_0^L dy \partial_y T_{c,-,\pm}(t_0, t, x, y) \int (\prod_b \delta \eta_b) \frac{\delta S_{\text{ext}}}{\delta \eta_c(t, x, y)} e^{i \sum_b S_b(\eta_b, \partial_t \eta_b) + S_{\text{ext}}(\eta + \Delta \phi)}.$$
Since \( \pm < \delta S_{bs}(\{\eta_b + \Delta \phi_b\}_{b=c,s})/\delta \eta_c(t_{\pm},y) > \) are equal for both signs and do not depend on \( t \), the \( T \)-correlators may be collected into the retarded correlator. Substituting its zero frequency asymptotics (11), we gather that \( \Delta I(t) \) is the average of an operator \( \Delta I(t) = < \Delta \hat{I}(t) > \) of the backscattering current,

\[
\Delta \hat{I}(t) \equiv -g_\infty \sqrt{2} \int dx j(x,t),
\]

(16)

whose density is

\[
j(x,t_{\pm}) \equiv \mp \frac{\delta S_{bs}(\{\eta_b + \Delta \phi_b\}_{b=c,s})}{\delta \eta_c(t_{\pm},x)}.
\]

(17)

Let us relate operator (14) with the one which counts transitions of particles between the right and left chiralities per a unit of time. In Hamiltonian formalism, the latter may be found as

\[
\partial_t \hat{N} = \frac{-i}{(2\sqrt{2}\pi)} \int dx [\partial_x \theta_c(x), \mathcal{H}_B] = \sqrt{2} \int dx \frac{\partial \mathcal{H}_{bs}(\phi_{c,s})}{\partial \phi_c(x)},
\]

(18)

where \( \mathcal{H}_{bs}(\phi_{c,s}) = \int dy \mathcal{L}_{bs}(y, \phi_{c,s}) \), and we use notation \( \mathcal{H}_B \) for the bosonic form of Hamiltonian (1). Comparison of (14) with (18) proves \( \Delta \hat{I}(t) = g_\infty \partial_t \hat{N} \), and that \( g_\infty \) in the coefficient in (14) is the charge of the particles transferred between the chiralities. It will be corroborated with the shot noise analysis in Section IV. This charge is \( g_c(x) \) time less than \( \epsilon \) as a cloud of opposite charge surrounds each particle. However, the charge is taken not at the point of the transition \( x \) but at the point of the final destination \( x = \pm \infty \) for each particle, where \( g_c(x) = g_\infty (= 1) \). Substitution of the expression for \( \Delta \phi_c \) in (14) reveals that the TLL parameter \( g \) enters into the backscattering current (10), (11) in two more ways: (ii) The charge in front of the voltage drop \( V(-\infty) - V(\infty) \) in (13), (14) turns out to be \( g_\infty \) too, since both charges and also the one in \( \sigma_0 \) originate from the same parameter of the asymptotics of the retarded correlator in (11); (iii) There appear local values of \( g \) unchanged by the reservoirs in the spacious dependence of \( \Delta \phi_c \) in (3). We will argue in Section V that they are related to the charge compressibility whose values inside the wire are not affected by the reservoirs.

IV. SHOT NOISE

Next, we consider spectrum of the current fluctuations: \( \delta I^2(V,\omega) = \sum_{\pm} P(\pm \omega)/2 \), where \( P(\omega) \) is Fourier transform of the current-current correlator:

\[
P(\omega) = \int dt e^{i\omega t} < \hat{I}(t)\hat{I}(0) > - < \hat{I}(t) >^2 = P^*(\omega).
\]

(19)

Assuming that the current is measured in the right lead, we find

\[
P(t) = \frac{-\partial_\tau \eta_c(L,t_{\pm}) \partial_\eta_c(L,0_{\pm})}{2\pi^2} \sim < \Delta I >^2.
\]

(20)

Making a similar calculations to those we did for the average current one can find the spectrum at non-zero \( \omega \):

\[
P(\omega) = \frac{i\omega}{2\pi^2} \partial_\tau T_{c,-,+}(\omega,L,L) - \frac{i}{2\pi^2} \int dx \frac{\delta^2 S_{bs}}{\delta \eta^2_0(0,x)} \sum_{\alpha,\pm} \partial_\tau T_{c,-,\alpha}(L,x,\omega) \partial_\eta T_{c,+,-}(L,x,\omega) - \frac{1}{(2\pi)^2} \sum_{\alpha,\beta=\pm} \int dx \int dx' e^{i\omega t} \int (\prod_b D\eta_b) \frac{\delta S_{bs}}{\delta \eta_c(t,\alpha,x)} \frac{\delta S_{bs}}{\delta \eta_c(t,\beta,x')} e^{i\int_{-\infty}^{\infty} S_0 + S_{bs}(\eta + \Delta \phi) / \omega) dt}.
\]

(21)

The first term describes the current-current correlator in the absence of the backscattering. It is always of equilibrium and, hence, relates (14) to the frequency dependent conductance of the TLL wire without backscattering through the fluctuation-dissipation theorem (FDT). In the limit \( \omega \rightarrow 0 \) the right-hand side of (21) drastically simplifies after applying the low energy asymptotics for the \( \partial_\tau T_{c,\alpha,\beta} \). They may be found from the spectral representations

\[
T_{c,\alpha,\beta}(x,y,\omega) = \frac{2\pi p_c(x,y,\omega)}{1 - e^{-\omega/T}} T_{c,<}(x,y,\omega)
\]

(22)

(\( \langle , > \) stand for \(+, -\) and \(-, +\), respectively), with the spectral density:

\[
\rho_c(x,y,\omega) = \frac{1}{\pi} \text{Re}[T_{c,R}(x,y,\omega)]
\]

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\[
\rho_c(x,y,\omega) = \frac{1}{\pi} \text{Re}[T_{c,R}(x,y,\omega)]
\]

(23)

Availing ourselves of the low frequency asymptotics of representations (22), the zero frequency limit for the second term (II) in (21) reduces to

\[
(II) = \frac{i}{2\pi^2} \int dx \frac{\delta j(x,0)}{\delta \eta_c(0,x)} \partial_\eta T_{c,\alpha}(L,x,\omega) \sum_{\pm} \partial_\eta T_{c,R}(L,x,\pm\omega)
\]

\[
\rightarrow_{\omega \rightarrow 0} -8T g_\infty^2 \int dx c(x) \frac{\delta j(x,0)}{\delta \eta_c(0,x)},
\]

(24)

where \( c(x) = c(L,x) \). Calculation of the third term on the right-hand side of Eq. (21) involves asymptotics of two others \( \phi_c \)-correlators:
\[ T_{e,+}(x, y, \omega) = T_{e,-}(x, y, -\omega) = 
= iM[T_{e,R}(x, y, \omega)] + \coth \left( \frac{\omega}{2T} \right) \text{Re}[T_{e,R}(x, y, \omega)] 
\approx_{\omega \to 0} 2\pi g\infty c(x, y) + \frac{2\pi g\infty}{\omega} \coth \left( \frac{\omega}{2T} \right) (1 + O(\omega^2)). \]  

(25)

\[
(III) = 4T \partial_\omega \bar{P}(\omega) + 2\bar{P}(\omega) + (X) \text{ where } \omega \to 0
\]

\[
(X) = -4iT \delta_0^2 \int dt \, dx \, dy \left[ c(y) \left( \langle j(x, t+)j(y, 0+) \rangle - \langle j(x, t-)j(y, 0+) \rangle \right) + c(x) \left( \langle j(x, t+)j(y, 0+) \rangle - \langle j(x, t-)j(y, 0+) \rangle \right) \right].
\]  

(27)

Here \( \bar{P}(\omega) \) is Fourier transform of the backscattering current correlator:

\[
\bar{P}(\omega) = \int dt e^{i\omega t} < \Delta \bar{I}(t) \Delta \bar{I}(0) > .
\]  

(28)

The piece \((X)\) plus the second term \((II)\) turn out to be zero. It is just a Ward identity for the average density \( < j(x, t) > \) of the backscattering current following from the time translational invariance of the action in \((III)\). Indeed, making a change of the variable \( \eta_c \to \eta_c + \text{cst} \) of the functional integration in the expression for \( < j(x, t) > \) one can find that:

\[ i \left( \frac{\delta j(x,t)}{\delta \eta_c(t,x)} \right) = \int_{-\infty}^{\infty} dt' \sum_{\pm} < T_V \{ j(x,t)j(x',t') \} > .\]  

(29)

The derivative in \((26)\) is equal to deviation of the differential conductance from the maximum value \( \sigma_0 \) of the linear bias conductance:

\[ \partial_\omega \bar{P}(\omega) \big|_{\omega \to 0} = -\partial_V < \Delta I(V) > .\]  

(30)

Then the relation between the noise of the direct current \( \delta I^2(V, 0) \) and the backscattering one \( \delta I^2(V, 0) = \bar{P}(\omega) \big|_{\omega \to 0} \) at zero frequency reduces to

\[ \delta I^2(V, 0) = 2T \sigma_0 + 4T \frac{\partial < \Delta \bar{I} >}{\partial V} + \delta (\Delta I)^2(V, 0). \]  

(31)

This relation has been earlier derived \([13]\) for the backscattering in the uniform TLL. It meets the fluctuation-dissipation theorem in the equilibrium limit \( V \ll T \). Indeed making use of \((30)\), we can write the third term on the right-hand side of \((31)\) as

\[ \coth \left( \frac{\omega}{2T} \right) [\bar{P}(\omega) - \bar{P}(\omega)] / 2 \big|_{\omega \to 0} = -2T \partial_V < \Delta I(V) > .\]  

Hence, it cancels a half of the second term and the rest is equal to \( 2TI/V, V \to 0 \). On the shot noise side \( V > T \to 0 \) the two first terms on the right-hand side of \((31)\) are going to zero and zero-frequency fluctuations of the current become equal to the fluctuations of the backscattering one.

These asymptotics contain the \( x \)-independent parts and the \( x \)-dependent ones proportional to \( c(x, y) \). Their substitution into the third term \((III)\) produces, respectively, the piece where integration over \( x \) may be accumulated into the whole backscattering current and the piece \((X)\) where it may not:

\[
\delta (\Delta I)^2 = -2g\infty \coth(g\infty \omega/(2T)) < \Delta \bar{I} > .
\]  

(33)

Its substitution into \((31)\) proves that \( \delta I^2 = g\infty < \Delta \bar{I} > \) for \( V \gg T \), i.e., the charge of the carriers showing up in the current fluctuations Eq. \((31)\) determined by the impurity scattering is the one of the reservoir quasi-particle \( g\infty \). This result holds on at any energy while \( T \ll V \). At low energy (less than \( T_L \)) the average current \( < \Delta \bar{I} > \) follows a power law of the voltage determined by the reservoirs \([14]\). In particular, for the Fermi liquid reservoirs: \( < \Delta \bar{I} > /V \approx -(V_L/E_F)^{(g-1)/2}/(\tau_{se} \epsilon_c) \) if the random impurity potential \( V_{imp}(x)V_{imp}(y) \approx \frac{V}{\tau_{se} E_F} \delta(x-y) \) is weak. Meanwhile at the high energy the leading \( V^2 \) power comes from the wire TLL spectrum. Therefore, the above renormalization of the shot noise charge cannot deny that the transport is carried by the TLL quasi-particles of the wire. It just shows that the transformation of the wire quasi-particles into the reservoir ones is extremely robust at the low frequency.

Approaching an insulating phase of the wire some \( m \)-Umklapp processes could be resonantly strengthened if \( k_{mF}L < 1 \) \([15]\). In general we represent \( \Delta \bar{I} = \Delta \bar{I}_{\text{imp}} + \sum_{m \geq 1} \Delta \bar{I}_m \) in obvious correspondence to \([14]\). Substitution of this into \((28)\) and \((32)\) shows that each part of \( \Delta \bar{I} \)
brings additive contribution to both the average current noise, which are related by
\[
\delta(\Delta I_{\text{imp}})^2 = -g_{\infty} \coth(g_{\infty} V/(2T)) < \Delta \tilde{I}_{\text{imp}},
\]
\[
\delta(\Delta I_m)^2 = -mg_{\infty} \coth(mg_{\infty} V/(2T)) < \Delta \tilde{I}_m.
\]

Then the zero-temperature relation \(\delta I^2 / < \Delta \tilde{I} > | \) between the current fluctuations and the backscattering average is larger than \(g_{\infty} \) for \( V > T_L \). Eventually when \( V \ll T_L \) the one-electron backscattering dominates and this relation decreases to \(g_{\infty} \) as \( < \Delta I_m > \) is proportional to \(-V^{m^2-1}\) for the even \(m\)'s and \(-V^{m^2}\) for the odd \(m\)'s.

V. RENORMALIZED CHARGE/SPIN COMPRESSIBILITY

In this section we consider variation of the charge density in the wire under a change of the reservoir electrochemical potentials neglecting any backscattering. The local density is determined by the operator \(\partial_x \phi_c(x, t_0)/(\sqrt{2\pi})\). Calculation of a variation of its average results in
\[
\Delta \rho_c(x) = \left(\prod_b D \phi_b \right) \frac{\partial_x \phi_c(x, t - i0)}{\sqrt{2\pi}} e^{i\left(\sum_{a<R,L} S_a + S_{V+S_H}\right)}
\]

after making change of the variables of the integration \(H\) in the first line of (34). The notation for the actions was written down in (4). Keeping in mind such a setup where the long wire is uniformly screened by a close gate we suggest that \(V(x)\) equal to \(V_g(V_L)\) for \(x > L(x < 0)\), respectively, is constant \(V(x) = V_{\text{bott}}\) inside the wire [4]. Then the variation of the electron density \(\Omega\) is also constant in the wire:
\[
\Delta \rho_c = \frac{1}{\sqrt{2\pi}} \partial_x \phi_c = g \frac{\left(\sum_{a=R,L} V_a - 2V_{\text{bott}}\right)}{\pi v_c}.
\]

From Eq. (36) one can gather that variation of the average chemical potential \(\mu\) inside the wire under the applied voltage is
\[
\mu = (V_R + V_L)/2 - V_{\text{bott}}.
\]

Indeed, the energy density accumulated inside the wire under a constant shift of the charge density \(\Delta \rho \) is \(E_{\text{wire}}/L = (\Delta \rho)^2 \pi v_c / (2\mu)\) according to (3), and
\[
\frac{\partial \mu}{\partial \rho_c} = \frac{1}{L} \frac{\partial^2 E_{\text{wire}}}{\partial \rho_c^2} = \frac{\pi v_c}{2g}.
\]

The above expression for the average chemical potential contrasts with the one for the difference between the chiral chemical potentials. The latter emerges if the current \(I = -\frac{1}{2\pi e} \frac{\partial \phi_c}{\partial \mu}\) related to \(\Delta \phi_c\) in (13) is represented as a difference between the chiral density variations, i.e., \(I = v_c (\rho_R - \rho_L)\). Then the difference between the chiral chemical potentials can be found using (35) as follows:
\[
\Delta \mu = \frac{g_{\infty}}{g} (V_L - V_R)
\]

This quantity however lacks direct physical sense in the situation where chiralities do not conserve. In particular, its matching with the energy transferred by the backscattering current \(I(0)\) per one particle implies that the particle carries charge \(g\) but not \(g_{\infty}\).

The average chemical potential \(\mu\) includes an obscure electrostatic potential \(V_{\text{bott}}\). In experiment, instead of \(V_{\text{bott}}\), one could measure potential \(V_g\) of the screening gate having a capacitance \(C_g\) with respect to the wire proportional to the wire length: \(C_g = c_g L\). To determine \(V_{\text{bott}}\) and the charge density in the wire as a function of \(V_g, V_R\), and \(V_L\), we will consider electrostatics of the whole setup self-consistently. If the voltage is applied symmetrically, \(V_R = -V_L = -V/2\), there is no re-distribution of the charge and \(V_{\text{bott}} = 0\). A non-zero variation of \(V_R + V_L\), on the other hand, changes the average chemical potential and, hence, the density in the wire. The additional charge of the wire \(Q\) has to minimize the entire energy
\[
E = E_{\text{wire}}(Q) + Q^2/(2C_g) + QV_g - (V_R + V_L)Q/2,
\]

consisting of the electrostatic one and an internal energy of the wire \(E_{\text{wire}}(Q)\). To evaluate the \(L\)-proportional part of the latter, we can use its equilibrium form because of the translational symmetry. Therefore, the charge density variation meets
\[
\Delta \rho_c = c_{\text{eff}} \Delta \left(\frac{V_R + V_L}{2} - V_g\right),
\]

with the density \(c_{\text{eff}} = C_{\text{eff}}/L\) of the effective capacitance:
\[
C_{\text{eff}}^{-1} = C_g^{-1} + (\partial Q/\partial \mu)^{-1}.
\]

Here \(Q(\mu)/L\) is dependence of the charge density on the chemical potential for the uniformly interacting 1D electrons: \(\mu(Q/L) = L^{-1} \partial_q E_{\text{wire}}\). A similar expression for the effective capacitance has been derived by Büttiker et al. [21] in the free electron case when the last term accounts for the final density of the electron states inside the conductor [2]. In the TLL model (3) the above derivative \(\partial \mu / \partial \rho_c\) has been found in (38). Its substitution brings out the density of the effective capacitance as follows:
\[
c_{\text{eff}} = \frac{\partial \rho_c}{\partial \mu} = \frac{2g}{\pi v_c}.
\]

This expression shows that in the experimentally relevant dependence of \(\Delta \rho_c\) on \((V_R + V_L)/2\) at fixed \(V_g\) the
TLL compressibility of the wire is renormalized by a factor \((1 + 2g/(e \pi N))^{-1}\) due to electrostatic Coulomb screening. In the limit: \(2e^2L/(\pi C_g v_F) \ll 1\) (without 2 for the spinless electrons), this factor becomes unimportant and \(V_{\text{att}}\) remains independent of \(V_R\) and \(V_L\). On the other hand, a finite density of the gate capacitance veils the short range interaction effect on the charge density. Let us estimate the gate capacitance contribution for a GaAs wire. Evaluating \(e_g^{-1} \approx d/(\varepsilon R)\) if the radius of the wire is larger than the distance \(d\) from the wire to the gate, the dielectric constant \(\varepsilon = 12\), \(v_F = 2 \times 10^5 \text{m/s}\) and \(e^2/\hbar = 3.3 \times 10^3 \text{m/s}\), we come to \(2e^2L/(\pi C_g v_F) \approx 0.2d/R\). This shows that the effect of the finite gate capacitance may be suppressed in experiment. Then a direct measurement of the charge density variation, which is equal to minus the variation of the density in the screening gate, would allow us to find a ratio of the TLL parameters: \(g/v_s\).

To find the constant \(g\), however, the Umklapp backscattering process in (II) dependent on the transferred momentum, turns out to be useful. Its manifestation in current vs voltage was described in [18] in the lowest order in the strength of the backscattering for \(V_L = -V_R\) as the appearance of a threshold structure. In a general case the charge density variation specified by (13) will show up in this effect causing a shift of this structure. Indeed, the average Umklapp backscattering current \(\Delta I\) is decomposed into the sum of the different backscattering mechanism contributions \(<\Delta I_m>\) in the lowest order. Substituting (13) into (14), one can find that the even \(m\) terms involving only \(\phi_c\) field are equal to

\[
<\Delta I_m> = -\frac{m}{4} \left( \frac{U_m E_F^2}{v_F} \right)^2 \int_0^\infty dt \int_0^L dx_1 dx_2 <\text{e}^{im\phi_c(x_1,t)}/\sqrt{2} e^{-im\phi_c(x_2,0)}/\sqrt{2}> [\text{e}^{im(2m_F + \Delta k)F(x_1 - x_2) + g_\infty Vt} - h.c.],
\]

where \(\Delta k_F = \partial_\varepsilon \phi_c/(2\sqrt{2})\) is identified with a variation of the Fermi momentum \(k_F\) in agreement with the Luttinger theorem in the TLL. \(\Delta \rho_F = 2\Delta k_F/\pi\). The odd \(m\) terms additionally include a spin field correlator \(<\text{e}^{i\phi_s(x,t)}/\sqrt{2} e^{-i\phi_s(x,0)}/\sqrt{2}>\) under the integrals in (13). This correlator is chiral: It is a product of two functions, which are negative powers of their arguments: \((x_1 - x_2)/v_s \pm t\), respectively, if the latters are less than \(1/T\). Similarly, the charge correlator in (14) approaches its chiral high temperature asymptotics depending on \((x_1 - x_2)/v_s \pm t\) at \(T > T_L\), whereas the interference stemming from the boundary scattering makes it more complex at the low temperature. Therefore, the \(L\)-proportional part of \(<\Delta I_m>\) in (14) shows a singular dependence on \(L\) on its arguments, \(E_{\text{thr,c}} - g_\infty V\) and \(E_{\text{thr,s}} - g_\infty V\), smeared over the max\{\(T, T_L\)\} range near their zeros if \(m\) is odd. Here the charge and spin threshold energies are \(E_{\text{thr,c}} = v_s(2m_F + 2\Delta k_F)\) and \(E_{\text{thr,s}} = v_s(2m_F + 2\Delta k_F) < E_{\text{thr,c}}\), respectively. In particular, suppression of the differential conductance produced by \(\Delta I_1\) has peaks at \(g_\infty V = E_{\text{thr,s}}\) and \(g_\infty V = E_{\text{thr,c}}\), which are precursors of the band gap at integer filling, whereas the suppressive contribution to the conductance vanishes as \((k_F + \Delta k_F) L^{-2(1+rg)}\) if \([k_F + \Delta k_F] L \gg 1\) at low voltage [15]. Similarly, \(\Delta I_2\) brings out a peak in the differential conductance versus voltage at \(g_\infty V = E_{\text{thr,c}}\) near half filling, which is a precursor of the Mott-Hubbard insulator. We see that the positions of these structures vary with the chemical potential inside the wire as

\[
\Delta E_{\text{thr,c}} = \pi v_c \sigma_{\text{eff}} \Delta \left( \frac{V_R + V_L}{2} - V_s \right) = \frac{v_c}{v_s} \Delta E_{\text{thr,s}},
\]

with \(\sigma_{\text{eff}}\) written in (11). Measurement of this variation allows us to find \(g/g_\infty\) and \(v_c/v_s\) if the density of capacitance \(c_g\) between the wire and the gate located closely is large enough.

\[
\text{F.

Finally, we discuss effect of the magnetic field on the threshold structure. The linear in \(x\) shift \(\Delta \phi_s\) produced by this field changes momentum transferred by the odd \(m\) terms of \(L_{\text{bs}}\) in (11). It results in a symmetrical split of both of the threshold energies \(E_{\text{thr,b}} = \Delta H E_{\text{thr,b}}/2\) associated with the \(m\)-th threshold structure. There is no additional redistribution of the charge and \(\Delta H E_{\text{thr,c}} = v_s g_s H/v_s\) and \(\Delta H E_{\text{thr,s}} = g_s H\). The even \(m\) thresholds are not affected by the magnetic field. In the case of the spin \(SU(2)\) symmetry, the charge threshold splitting shows renormalization of the electron magnetic moment in \(v_s/v_s\approx g^{-1}\) times even though there is no renormalization of the magnetic susceptibility \(g_s = 1\). Albeit, there could be a deviation of \(g_s \propto (\text{log}[E/F]/\max\{T_L, T\})^{-1}\) from its renormalization group limit [22].

VI. CONCLUSION

In the model we have analyzed, the low energy form of the retarded correlator is basically determined by the two Tomohaga-Luttinger interaction constants \(g\) in the wire and \(g_\infty\) in the reservoirs: \(g_\infty\) is a coefficient at the low energy \(1/\omega\) singularity, and \(g\) at the high momentum \(1/k^2\) singularity. The latter also determines the density wave spectrum at energies higher than \(1/4T_L\) and, hence, the scaling powers at these energies. Then we saw that the same constant \(g_\infty\) determines the normalization of the conductance, the low energy charge in the shot noise, and the charge at the voltage drop. All of these are equal, therefore, in this model. In the TLL phase of the wire the same charge \(g_\infty\) appears in the shot noise at high energy, whereas the scaling of the backscattering...
current unambiguously shows that the transport is carried by the intrinsic excitation of the TLL wire. This is because the mechanism of transformation of the quasi-particles of the reservoirs into those of the wire reduces to an interference of the charge density waves scattered on the inhomogeneities of the interaction. It is classical by its nature and therefore robust, as opposed to the case of the wire in the correlated insulator phase.

Let us assume that the above form of the low energy asymptotics holds for the wider variety of models describing transport between the reservoirs through a 1D TLL, even though this could be difficult to prove in the absence of exact solutions. Then we could conclude that in these models charges in the coefficients at the conductance, the Hall conductivity for a FQHL with \( \nu = g_L = 1/3 \). On the other hand, measurement of the tunneling conductance \( \frac{1}{g} \) have found a power law scaling with the exponent related to a different \( g : 1/g = 2.6 \). This conclusion seems to comply with known results of experiments on transport through a principal FQHL. Indeed, normalization of the conductance and the charge in the shot noise are determined by a different constant. This conclusion is drawn from the Hall conductivity for a FQHL with \( \nu = g_L = 1/3 \). On the other hand, measurement of the tunneling conductance \( \frac{1}{g} \) have found a power law scaling with the exponent related to a different \( g : 1/g = 2.6 \).

VII. ACKNOWLEDGMENTS

The authors acknowledge H. Fukuyama and M. Ogata for useful discussions. One of us (V.P.) endured a criticism of M. Buttiker we benefitted from during the 1996 Curacao workshop. This work was supported by the Center of Excellence at the Japanese Society for Promotion of Science and partly by the Swiss National Science Foundation.

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\[
\frac{1}{L} \frac{\partial L}{\partial P} = \frac{1}{L} \left( \frac{\partial^2 E_{\text{wire}}}{\partial L^2} \right)^{-1} = \left( \frac{\partial \mu}{\partial \rho_c \rho_c^0} \right)^{-1} = \frac{\partial \rho_c}{\partial \mu} \left( \frac{\pi}{2k_F} \right)^2,
\]

where \( P \) is the pressure in a 1D system of length \( L \).
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