Repeatable measurements and the collapse postulate

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Abstract

J. v. Neumann justified the collapse postulate by the empirical fact of the repeatability of a measurement at a single quantum system. However, in his quantum mechanical treatment of the measurement process repeatability emerges without collapse. The entangled state of the measurement device and the measured system after their interaction ensures it already. Furthermore, this state gives the same predictions for the measured system alone as the description demanded by the collapse postulate.

Keywords: measurement process, repeatable measurement, v. Neumann measurement, collapse, reduction, projection

1 Introduction and overview

None of the fundamental postulates of quantum mechanics is as controversial as the collapse (reduction, projection) postulate. It demands for a measurement an instantaneous, non-deterministic state transition, which is seemingly in conflict with the continuous, deterministic state evolution governed by the Schrödinger equation. In pilot wave (Bohm 1952) or ensemble (Ballentine 1998) interpretations this postulate is omitted, but it is part of the orthodox Copenhagen interpretation (Heisenberg 1958) and of the modern quantum computing (Nielsen and Chuang 2000).

v. Neumann (1932) founded his reasoning for this postulate on the Compton-Simon experiment, where the measurement results for the momentum of a scattered photon and the associated recoil electron determine each other (Compton 1927). He explained this “sharp (causal) correlation” by the state reduction caused by the first measurement.

However, in v. Neumann’s quantum mechanical treatment of the measurement process the repeatability of the measurement emerges without collapse, just by application of Born’s probabilistic interpretation of the wave function. The entangled state of the measurement device and the measured system after their interaction ensures already, that an immediate repetition of the process with a second measurement device of the same type will give with probability 1 the same result as the first. Furthermore, this entangled state of the compound system gives the same probabilities as the description demanded by the collapse postulate for all following measurements at the measured system.
2 Basic notions and notations

Let $S$ be a quantum mechanical system, described with Hilbert space $H$, and $A = \sum_k a_k |\alpha_k\rangle\langle\alpha_k| \in \mathcal{L}(H)$ a discrete, non-degenerate observable. For a measurement of the observable $A$ at the system $S$ in the state $|\psi\rangle \in H$

- the **probability postulate** (Born’s rule) demands, that the probability, to get the measurement result $a_j$, is $|\langle \alpha_j | \psi \rangle|^2$,

- the **collapse postulate** demands, that, if the measurement result is $a_j$, the state immediately after the measurement will be $|\alpha_j\rangle$.

As a consequence of both postulates the system $S$ has to be described after the measurement, if the result is unknown, by a mixture of states represented by the statistical operator

$$W = \sum_j |\langle \alpha_j | \psi \rangle|^2 |\alpha_j\rangle\langle\alpha_j| \tag{2.1}$$

v. Neumann’s quantum mechanical treatment of the measurement process describes a measurement as an interaction between the system $S$ and a measurement device $M$, which is itself a quantum system with Hilbert space $H_M$. Pairwise orthogonal pointer states $|\phi_k(M)\rangle \in H_M$ indicate the measurement results. The interaction of the measured system and the measurement device is described by an unitary transformation $U^{(SM)}$ in the tensor product Hilbert space $H \otimes H_M$ of the compound system $SM$. The assumption, that an ideal measurement of an eigenstate $|\alpha_k\rangle$ of the measured observable $A$ should give exactly the corresponding pointer state $|\phi_k(M)\rangle$ as result, without disturbing the system $S$, can be expressed by

$$U^{(SM)} |\alpha_k\rangle |\phi_0(M)\rangle = |\alpha_k\rangle |\phi_k(M)\rangle \tag{2.2}$$

Therefore, with the initial state $|\psi\rangle$ of the system $S$ the unitary transformation $U^{(SM)}$ will give the entangled final state

$$|\Phi\rangle = U^{(SM)} |\psi\rangle |\phi_0(M)\rangle = \sum_k |\langle \alpha_k | \psi \rangle| |\phi_k(M)\rangle$$

The reading of the pointer can be considered as a secondary measurement. The probability of the result $A_j(M) = 1 \otimes |\phi_j(M)\rangle\langle\phi_j(M)|$, that the pointer state $|\phi_j(M)\rangle$ is observed, is according the probability postulate for the final state $|\Phi\rangle$ given by

$$p(A_j(M)) = |\langle \phi_j(M) | \langle \phi_j(M) | \Phi \rangle|^2 = |\langle \alpha_j | \psi \rangle|^2 \tag{2.3}$$

As long as the compound system $SM$ is in the state $|\Phi\rangle$, the system $S$ alone has to be described by the statistical operator given by the partial trace

$$\text{tr}_{H_M}(|\Phi\rangle\langle\Phi|) = \sum_k |\langle \alpha_k | \psi \rangle| |\alpha_k\rangle\langle\alpha_k| = W \tag{2.4}$$

which is identical with (2.1). But this description gives no statement about the correlation with the measurement result.
3 Repeatability of the measurement

The wave function of the compound system can be used to compute the conditional probabilities of the results of further measurements. When a second measurement device $M'$ of the same type interacts with the measured system $S$ as part of the compound system $SM$ in state $|\Phi\rangle$ in the enlarged Hilbert space $\mathcal{H} \otimes \mathcal{H}_M \otimes \mathcal{H}_{M'}$, this gives the state

$$|\Phi'\rangle = U^{(SM')}|\Phi\rangle|\Phi_0^{(M')}\rangle = \sum_k \langle \alpha_k | \psi \rangle |\alpha_k\rangle |\phi_k^{(M')}\rangle |\phi_k^{(M')}\rangle$$

The pointer results of both measurement devices define a Boolean event algebra and a common probability space, because all projections onto the pointer states $A_j^{(M)} = 1 \otimes |\phi_j^{(M)}\rangle \langle \phi_j^{(M)}| \otimes 1^{(M')}$ and $A_k^{(M')} = 1 \otimes 1^{(M)} \otimes |\phi_k^{(M')}\rangle \langle \phi_k^{(M')}|$ commute pairwise. Therefore, the conditional probability to get the pointer result $A_k^{(M')}$ in the second measurement, given the pointer result $A_j^{(M)}$ in the first, is well-defined

$$p(A_k^{(M')}|A_j^{(M)}) = \frac{p(A_j^{(M')}&A_k^{(M')})}{p(A_j^{(M)})} = \frac{\langle \Phi'|\phi_j^{(M')}\rangle \langle \phi_j^{(M')}| \otimes \phi_k^{(M')}\rangle \langle \phi_k^{(M')}| \Phi'\rangle}{\langle \Phi'|\phi_j^{(M')}\rangle \langle \phi_j^{(M')}| \Phi'\rangle} = \delta_{j,k}$$

(3.1)

This means: The probability is 1, that an immediate repetition of the measurement gives the same result as the first. The same statement is valid for further repetitions\(^1\) and, when the initial state of the system is a mixture or when the measured observable is degenerate\(^2\).

This repeatability of the measurement is a consequence of condition (2.2). The weaker condition

$$U^{(SM')}|\alpha_k\rangle |\phi_0^{(M')}\rangle = |\psi_k\rangle |\phi_k^{(M')}\rangle$$

with some not further specified $|\psi_k\rangle \in \mathcal{H}$, gives the same probability distribution for the pointer results \(^2\)(3.2), but in general without repeatability.

4 Description of the measured system

With v. Neumann’s description of a repeatable measurement all further measurements without collapse the same conditional probability distributions as demanded by the collapse postulate. To see that, let instead of $M'$ another device $M''$ for the measurement of an arbitrary non-degenerate observable $B = \sum_k |\beta_k\rangle \langle \beta_k| \in \mathcal{L}(\mathcal{H'})$ interact with the measured system $S$ as part of the compound system $SM$ in state $|\Phi\rangle$. With the resulting state

$$|\Phi''\rangle = \sum_{k,j} \langle \alpha_k | \psi \rangle \langle \beta_j | \alpha_k \rangle |\beta_j\rangle |\phi_k^{(M')}\rangle |\phi_j^{(M'')}\rangle$$

\(^1\)This gives a simple explanation of avalanche effects, which are part of some measurement devices, where many molecules interact as measurement devices with a measured particle and the resulting state is macroscopic visible, because all molecules are observed in the same pointer state.

\(^2\)But the spectrum of the observable has to be discrete (Busch et al. 1991).
the conditional probability, to get in the second measurement the pointer result $B_k^{(M')} = 1 \otimes 1^{(M)} \otimes |\psi_j^{(M')}\rangle \langle \varphi_j^{(M')}|$, if the first measurement result is $A_j^{(M)}$, is also well-defined

$$p(B_k^{(M')}|A_j^{(M)}) = \frac{\langle \Phi' | \varphi_j^{(M)} \rangle \langle \varphi_j^{(M')} | \Phi' \rangle \langle \varphi_j^{(M')} | \Phi' \rangle}{\langle \Phi' | \varphi_j^{(M)} \rangle \langle \varphi_j^{(M)} | \Phi' \rangle} = |\langle \beta_k | \alpha_j \rangle|^2$$

This exactly the same value as with the collapse postulate; (3.1) is just a special case with $B = A$.

The total probability of the result $B_k^{(M')}$

$$p(B_k^{(M')}) = \sum_j p(B_k^{(M')}|A_j^{(M)}) p(A_j^{(M)}) = \sum_j |\langle \alpha_j | \psi \rangle|^2 \langle \beta_k | \alpha_j \rangle \langle \alpha_j | \beta_k \rangle = \text{tr}(W | \beta_k \rangle \langle \beta_k |)$$

is the same as the probability of the result $b_k$ for the mixture $W$ in (2.1) and (2.4).

So far we have considered only the situation immediately after the measurement $M$. With the collapse postulate it is possible to describe the system $S$ by a pure state $|\alpha_j \rangle \in \mathcal{H}$, whose evolution is governed by a group of unitary transformations $U_t \in \mathcal{L}(\mathcal{H})$, as long as $S$ is isolated. If the measurement result was $a_j$, the state at time $t$ after the measurement will be

$$|\alpha_j(t) \rangle = U_t |\alpha_j \rangle$$

If $S$ is isolated and the pointer state $|\varphi_k^{(M)} \rangle$ is an eigenstate of the evolution of $M$, the state of the compound system $SM$ at the time $t$ after the measurement will be

$$|\Phi_t \rangle = \sum_k \langle \alpha_k | \psi \rangle |\alpha_k(t) \rangle |\varphi_k^{(M)} \rangle$$

With this state the conditional probability, to get in a second measurement $M''$ at time $t$ the result $B_k^{(M'')}$, if the first measurement result at time $t = 0$ was $A_j^{(M)}$, is

$$p(B_k^{(M'')}|A_j^{(M)}) = \frac{\langle \Phi_t | \varphi_j^{(M)} \rangle \langle \varphi_j^{(M')} | \Phi_t \rangle \langle \varphi_j^{(M')} | \Phi_t \rangle}{\langle \Phi_t | \varphi_j^{(M)} \rangle \langle \varphi_j^{(M)} | \Phi_t \rangle} = |\langle \beta_k | \alpha_j(t) \rangle|^2 \ (4.1)$$

This is the same value as with the collapse postulate.

Interactions of other systems with the measurement device $M$ can destroy the measurement result and change the state of the system $S$ remotely. It is an open question how a measurement result becomes irreversible. However, pointer readings by a repeatable measurement at $M$ with another device $\hat{M}$ will give a state

$$|\tilde{\Phi}_t \rangle = \sum_k \langle \alpha_k | \psi \rangle |\alpha_k(t) \rangle |\varphi_k^{(M)} \rangle$$

which does not change the conditional probabilities for the results of a further measurement $M''$ (4.1). A similar mechanism is assumed by decoherence theory (Schlosshauer 2004).
5 Conclusions

Applying the probability postulate we have shown that v. Neumann’s treatment of the measurement process describes repeatable measurements without collapse of the wave function. However, this description has to take into account not only the measured system but also the measurement devices. A simpler description of the measured system alone is possible, which gives the same conditional and total probabilities for all further measurement results: It is the description demanded by the collapse postulate.

One can regard this as a derivation of the collapse postulate, without contradicting impossibility proofs like Bassi and Ghirardi [2000]. From this point of view the wave function is merely a computational tool. And the collapse is no physical process; it is only the change to an equivalent but simpler probabilistic description with a cut between the observed system and the rest of the world. The dynamical conditions during and after the measurement interaction determine, if this cut is possible.

Of course, our analysis does not explain how definite measurement results arise. That is already presupposed by the probability postulate. Nevertheless, it explains why the collapse of the wave function gives a correct probabilistic description of the measured system, whenever a definite measurement result was obtained by a repeatable measurement. And it explains how it is possible to omit the collapse postulate at all, without losing the capability to describe sequential measurements.

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