A Note on Fuzzy Soft Ditopological Spaces

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Abstract

In this study, we define fuzzy soft ditopological spaces as the generalization of soft ditopological spaces defined by Simsekler et. al. (2016). Fuzzy soft ditopological space is a combination of fuzzy soft topological and fuzzy soft cotopological spaces which are defined by two independent structures fuzzy soft open and fuzzy soft closed sets. Also we give fuzzy soft $\tau$-continuity in fuzzy soft topology, fuzzy soft $\gamma$-continuity in fuzzy soft cotopology and by using these two types of continuity we finally define the fuzzy soft continuity in a fuzzy soft ditopology.

Keywords: Fuzzy soft sets, fuzzy soft ditopology, fuzzy soft continuity.

Bulanık Esnek Ditopolojiler Üzerine Bir Çalışma

Öz

Bu çalışmada, Simsekler Dizman ve arkadaşları tarafından (2016) tanımlanan esnek ditopolojik uzayların bir genelleştirilmesi olan bulanık esnek ditopolojik uzaylar tanımlanmıştır. Bulanık esnek ditopolojik uzaylar birbirinden bağımsız olarak tanımlandığımız bulanık esnek açık ve bulanık esnek kapalı kümler kullanılarak tanımlanmış bulanık esnek topolojiler ve bulanık esnek kotopolojilerin bir kombinasyonudur. Ayrıca bulanık esnek topolojilerde tanımlandığımız $\tau$-sürekilik ve bulanık esnek kotopolojilerde $\gamma$-sürekiliği kullanarak bulanık esnek ditopolojilerde sürekiliği verdik.

Anahtar Kelimeler: Bulanık esnek kümler, bulanık esnek ditopoloji, bulanık esnek süreklilik

1. Introduction

Since the real life problems in several areas are more objective, solving these problems by classical mathematics rules is not appropriate many times. Fuzzy set theory defined by Zadeh (1965) gives us an opportunity to define a set and its elements by a different way than the well-known approach “Black or White” and presents us a new angle to sight the connections between the set and its elements. The main opinion of the theory is defining a fuzzy set by the fuzzy membership function and hence state that which “degree” an element pertion to a fuzzy set. Chang (1968) was first investigated fuzzy set topology. The soft set was described in 1999 by Molodtsov (1999) as a new approach for uncertainty and the theory is based on defining a soft set by a parameter set of objects in the universe with a mapping. It is easily seen that fuzzy and soft sets are interdependent (Aktas and Cagman, 2007). Maji et al. (2001) first studied the hybrid
model of fuzzy and soft sets and defined fuzzy soft set (briefly fs-set) as a new type of vague sets. Ahmad and Kharal (2012) contributed to the fuzzy soft theory and described the concept of a mapping between fs-sets. Aktaş and Cagman (2007) defined the notion of soft groups and some of their properties. Feng et al. (2008) studied the soft semirings and investigated several related properties. Nazmul and Samanta (2010) contributed to the algebraic structures of soft sets. Shabir and Naz (2011) described the soft topological spaces and separation axioms of soft topological spaces. Tanay and Kandemir (2011) searched the topology of fs-sets. Throughout the paper U will denote the universe and E will denote the parameter set and \( I^U \) will denote the all fuzzy sets on U. Let A and B be nonempty sets of E.

In this section we give the main definitions of fs-set theory which can be found several papers we cited in introduction. As we state in the previous section our Fuzzy soft set definition is different from this works in the meaning of parameter set.

**Definition 1.** \( f_A \) is called the fuzzy soft set (briefly fs-set over) U if \( f_A : A \rightarrow I^U \) is a mapping defined by \( f_A(e) = \mu_A^e \) where \( \mu_A^e(u) \neq 0 \) for each \( u \in U \) otherwise i.e, \( e \notin A \), \( f_A(e) \) will not be considered.

**Definition 2.** The complement of a fs-set \( f_A \) is a fs-set denoted by \( f_A^c \) where \( f_A^c : A \rightarrow I^U \) is a mapping defined by \( \mu_A^c(u) = 1 - \mu_A^e(u) \) for all \( e \in A, u \in U \).

**Definition 3.** Let \( f_A \) and \( g_B \) be two fs-sets over U. \( f_A \) is said to be a fs-subset of \( g_B \) if \( A \subseteq B \) and \( \mu_A^e(u) \leq \mu_B^e(u) \) for each \( e \in A \) and \( u \in U \).

**Definition 4.** Let \( f_A \) and \( g_B \) be two fs-sets over U. The intersection of \( f_A \) and \( g_B \) denoted by
Now we obtain the intersection of \( f_A \cap g_B \) is a fs-set \( h_C \) where \( C = A \cap B \) and is defined by

\[
\mu_{h_C}^e(u) = \min \{ \mu_{f_A}^e(u), \mu_{g_B}^e(u) \}, \forall e \in C, \forall u \in U.
\]

**Definition 5.** Let \( f_A \) and \( g_B \) be two fs-sets over \( U \) and let \( f_A \cup g_B \) be a fs-set \( h_C \) where \( C = A \cup B \) and is defined by

\[
\mu_{h_C}^e(u) = \begin{cases} 
\mu_{f_A}^e(u), & \text{if } e \in A - B \\
\mu_{g_B}^e(u), & \text{if } e \in B - A \\
\max \{ \mu_{f_A}^e(u), \mu_{g_B}^e(u) \}, & \text{if } e \in A \cap B
\end{cases}
\]

\[ \forall u \in U. \]

**Definition 6.** The fs-set \( f_A \) over \( U \) is defined to be null fs-set and is denoted by \( \Phi \) where \( f_A(e) = 0^- \forall e \in A, \forall u \in U. \)

**Definition 7.** The fs-set \( f_A \) over \( U \) is defined to be absolute fs-set and is denoted by \( U^c \) where \( f_A(e) = 1^- \forall e \in A, \forall u \in U. \)

**Theorem 1.** Let \( f_A \) and \( g_B \) be two fs-sets over \( U \). Then the followings hold:

i) \( (f_A \cap g_B)^c \subseteq (f_A^c \cup g_B^c). \)

ii) \( f_A^c \cap g_B^c \subseteq (f_A \cup g_B)^c. \)

**Proof.**

i) Let \( (f_A \cap g_B)^c = h_C, C = A \cap B. \) Then, \( \mu_{h_C}^e(u) = \max \{ \mu_{f_A}^e(u), \mu_{g_B}^e(u) \}, \) for all \( e \in A \cap B \) and for all \( u \in U. \) On the other hand, let \( f_A^c \cup g_B^c = k_M, M = A \cup B. \)

\[ \mu_{k_M}^e(u) = \begin{cases} 
\mu_{f_A}^e(u), & \text{if } e \in A - B \\
\mu_{g_B}^e(u), & \text{if } e \in B - A \\
\max \{ \mu_{f_A}^e(u), \mu_{g_B}^e(u) \}, & \text{if } e \in A \cap B
\end{cases} \]

Hence the proof is completed.

\[ f_A^c \cup g_B^c = \{ e_1 = \{ u_1^{0.7}, u_2^{0.6}, u_3^1 \}, e_2 = \{ u_4^{0.7}, u_5^{0.6}, u_3^1 \} \}. \]

On the other side,

\[ (f_A \cap g_B)^c = \{ e_1 = \{ u_1^{0.7}, u_2^{0.6}, u_3^1 \}, e_2 = \{ u_4^{0.7}, u_5^{0.6}, u_3^1 \} \}. \]

ii) \( (f_A \cup g_B)^c \)

\[ (f_A \cup g_B)^c = \{ e_1 = \{ u_1^{0.5}, u_2^{0.3}, u_3^{0.4} \}, e_2 = \{ u_4^{0.5}, u_5^{0.3}, u_3^{0.4} \}, e_3 = \{ u_4^{0.5}, u_5^{0.3}, u_3^{0.4} \} \}. \]

Now we obtain the intersection of \( f_A^c, g_B^c. \)

\[ f_A^c \cap g_B^c = \{ e_1 = \{ u_1^{0.5}, u_2^{0.3}, u_3^{0.4} \} \}. \]

It can be easily seen that \( (f_A \cup g_B)^c \) is not a subset of \( f_A^c \cap g_B^c. \)

### 3. Fuzzy Soft Topological Spaces

Fuzzy topological spaces were defined and searched in several papers. (Aygunoglu et. al 2014, Atmaca and Zorlutuna 2014, Pazar and Aygun 2012, Roy and Samanta 2012, Roy and Samanta 2013, Simsekler and Yuksel 2013). In this work the interpretation of fs-sets is
different from the published papers and this leads different results while defining fs-topological spaces and the properties of this spaces. Also in this section we use only fs-openness as an independent structure from closedness.

**Definition 7.** Let U be the universe and E be a parameter set. The fs-topological space is a pair $(U_E, \tau_f)$ where $\tau_f$ is a family of fs-sets over U satisfying the following conditions:

i) $\Phi, U_E \in \tau_f$,

ii) If $f_A, g_B \in \tau_f$ then $f_A \cap g_B \in \tau_f$,

iii) If $f_A_i \in \tau_f$ for all $i \in I$ then $\bigcup_{i \in I} f_A_i \in \tau_f$.

$\tau_f$ is called a fuzzy soft topology (briefly fs-topology) of fs-sets over U. The pair $(U_E, \tau_f)$ is called fs-topological space. The elements of $\tau$ are defined to be fs-open sets.

**Definition 8.** (Atmaca and Zorlutuna, 2013) The fs-set $f_A$ is called a fs-point and denoted by $e^A_\lambda$ if there exists the parameter $e$ and the fuzzy point $x_\lambda$ such that $f_A^e(x) = \lambda$ where $x \in U$ and $\lambda \in (0,1]$.

**Definition 9.** (Atmaca and Zorlutuna, 2013) $e^A_\lambda$ is called an element of $f_A$ denoted by $e^A_\lambda \in \nabla f_A$ if $\lambda \leq \mu_{f_A}^e(x)$.

**Lemma 1.** If $e^A_\lambda \cap f_A = \Phi$ then $e^A_\lambda \notin \nabla f_A$.

Proof. Let $e^A_\lambda \cap f_A = \Phi$. Then $\min\{\lambda, \mu_{f_A}^e(x)\} = 0$. This shows that $\mu_{f_A}^e(x) = 0$ and hence $e^A_\lambda \notin \nabla f_A$.

**Remark 2.** The inverse inclusion of theorem does not satisfy generally.

**Example 2.** Let $U = \{x, y\}, E = \{k, l, m\}$. Let $f_A = \{l = \{x_0, y_0\}, m = \{x_0, y_0, x_2\}\}$ be a fs-set over U and $m_0^\lambda$ be a fs-point. It can be easily shown that $m_0^\lambda \notin \nabla f_A$ but $m_0^\lambda \cap f_A = \{m = \{y_0\}\}$.

**Definition 10.** Let $(U_E, \tau_f)$ be a fs-topological space, $e^A_\lambda$ be a fs-point and $f_A$ be a fs-set. $f_A$ is called fs-$\tau$-neighborhood of $e^A_\lambda$ if there exists a fs-open set $g_B$ such that $e^A_\lambda \in \nabla g_B \subseteq f_A$. The family of all fs-neighborhoods of $e^A_\lambda$ is denoted by $\nabla(e^A_\lambda)$.

**Theorem 2.** Let $(U_E, \tau_f)$ be a fs-topological space, $e^A_\lambda$ be a fs-point and $f_A, g_B$ be two fs-sets. The followings hold for $\nabla(e^A_\lambda)$:

i) If $f_A \in \nabla(e^A_\lambda)$ then $e^A_\lambda \in \nabla f_A$.

ii) If $f_A, g_B \in \nabla(e^A_\lambda)$ then $f_A \cap g_B \in \nabla(e^A_\lambda)$.

iii) If $f_A \in \nabla(e^A_\lambda)$ and $f_A \subseteq g_B$ then $g_B \in \nabla(e^A_\lambda)$.

Proof. The proof can be done similarly to the analogous statements in Simsekler and Yuksel (2013).

**Definition 11.** Let $(U_E, \tau_f)$ be a fs-topological space, $e^A_\lambda$ be a fs-point and $f_A$ be a fs-set $e^A_\lambda$ is called fs-interior point of $f_A$ if there exists a fs-open set $g_B$ such that $e^A_\lambda \in \nabla g_B \subseteq f_A$.

$\text{int} f_A = \bigcup_{i \in I} \{g_{B_i} : g_{B_i} \in \tau_f, g_{B_i} \subseteq f_A, i \in I\}$

**Theorem 3.** Let $(U_E, \tau_f)$ be a fs-topological space and $f_A$ be a fs-set. Then the following hold:

i) $\text{int} f_A \subseteq f_A$.

ii) $\text{int} f_A$ is a fs-open set.

iii) $\text{int} f_A$ is the biggest fs-open set contained in $f_A$.

iv) $f_A$ is a fs-open set if and only if $\text{int} f_A = f_A$. 


Proof. The proof can be done similarly to the analogous statements in Simsekler and Yuksel (2013).

**Definition 12.** (Atmaca and Zorlutuna, 2013) $e^v_x$ is called quasi with $f_A$ and denoted by $e^v_x q f_A$ if $\lambda + \mu^e_{f_A}(x) > 1$.

**Definition 13.** (Atmaca and Zorlutuna, 2013) $f_A$ is called quasi with $g_B$ and denoted by $f_Aq g_B$ if there exists $e \in A \cap B$, $u \in U$ such that $\mu^e_{f_A}(u) + \mu^e_{g_B}(u) > 1$.

**Definition 14.** (Atmaca and Zorlutuna, 2013) Let $(U_\xi, \tau_f)$ be a fs-topological space, $e^v_x$ be a fs-point and $v_A, w_B$ be fs-sets. $v_A$ is called a fs-Q-neighborhood of $e^v_x$ if there exists $w_B \in \tau_f$ such that $e^v_x q w_B \subseteq v_A$. The all fs-Q-neighborhoods of $e^v_x$ is denoted by $Q\mathcal{N}(e^v_x)$.

**Theorem 4.** Let $(U_\xi, \tau_f)$ be a fs-topological space, $e^v_x$ be a fs-point and $v_A, w_B$ be fs-sets. The followings hold for $Q\mathcal{N}(e^v_x)$:

i) If $v_A \in Q\mathcal{N}(e^v_x)$ then $e^v_x q v_A$.

$$\varphi_\psi(f_A)^p(v) = \begin{cases} \bigvee_{\varphi(u) = v} \psi(e)(\varphi(u)) & \text{if } u \in \varphi^{-1}(v), \\ 0^-, \text{otherwise} \end{cases}$$

or $\forall p \in (e(e))$ and $v \in V$.

Let $g_B$ be a fs-set of $FS(V,P)$. The preimage of $g_B$ under the fs-mapping $\varphi_\psi$ is a fs-set of $FS(U,E)$ and defined by:

$$\varphi_\psi^{-1}(g_B)^p(u) = (g_B)^{\psi(e)}(\varphi(u))$$

If $\varphi, \psi$ are injective then $\varphi_\psi$ is injective, if $\varphi, \psi$ are surjective then $\varphi_\psi$ is surjective.

**Definition 16.** Let $(U_\xi, \tau_f), (V_\rho, \tau_f^*)$ be two fs-topological spaces. $\varphi: U \rightarrow V, \psi: E \rightarrow P$ be mappings and $e^v_x$ be a fs-point. $\varphi_\psi = (\varphi, \psi): FS(U, E) \rightarrow FS(V, P)$ is called fs-$\tau$-continuous at $e^v_x$ if for any $\tau$- neighborhood $g_B$ of $\varphi_\psi(e^v_x)$, there exists a fs-$\tau$-neighborhood $f_A$ of $e^v_x$ such that $\varphi_\psi(f_A) \subseteq g_B$.

**Theorem 5.** Let $(U_\xi, \tau_f), (V_\rho, \tau_f^*)$ be two fs-topological spaces. $\varphi: U \rightarrow V, \psi: E \rightarrow P$ be mappings and $e^v_x$ be a fs-point. The followings are equivalent:

i) $\varphi_\psi = (\varphi, \psi): FS(U, E) \rightarrow FS(V, P)$ is fs-$\tau$-continuous at $e^v_x$.

ii) If $v_A, w_B \in Q\mathcal{N}(e^v_x)$ then $v_A \cap w_B \in Q\mathcal{N}(e^v_x)$.

iii) If $v_A \in Q\mathcal{N}(e^v_x)$ and $v_A \subseteq w_B$ then $w_B \in Q\mathcal{N}(e^v_x)$.

Proof. The proof can be done similarly to the analogous statements in Simsekler and Yuksel (2013).

**Proposition 1.** (Atmaca and Zorlutuna, 2013) Let $f_A, g_B$ be two fs-sets. If $f_A \subseteq g_B$ then $f_A$ is not quasi coincident with $g_B$.

**Definition 15.** (Pazar and Aygun, 2012) Let $U,V$ be universe sets, $E$ and $P$ the parameter sets, $FS(U,E)$ and $FS(V,P)$ be the families of fs-sets over $U$ and $V$ respectively. Let $\varphi: U \rightarrow V, \psi: E \rightarrow P$ be mappings. Then the pair $(\varphi, \psi)$ is called fs-mapping and is denoted by $\varphi_\psi = (\varphi, \psi): FS(U, E) \rightarrow FS(V, P)$.

Let $f_A$ be a fs-set of $FS(U,E)$. The image of $f_A$ under the fs-mapping $\varphi_\psi$ is a fs-set of $FS(V,P)$ and defined by:
ii) For any fs-$\tau$-neighborhood $g_B$ of $\varphi \varphi(e^3_x)$ there exists a fs-$\tau$-neighborhood $f_A$ of $e^3_x$ such that $f_A \subseteq \varphi \varphi^{-1}(g_B)$.

iii) The inverse image of every fs-$\tau$-neighborhood of $\varphi \varphi(e^3_x)$ is a fs-$\tau$-neighborhood of $e^3_x$.

Proof. The proof can be done similarly to the analogous statements in Atmaca and Zorlutuna (2013).

Example 2. Let $U = \{x, y, z\}, V = \{a, b, c\}$ be the universe sets, $E = \{e_1, e_2\}, P = \{p_1, p_2\}$ be the parameter sets, $(U^\sim_E, \tau^*_f), (V^\sim_P, \tau^*_f)$ be two fs-topological spaces where

$$\tau_f = \{\Phi, U^\sim_E, f_A = \{e_1 = \{x_{0.3}, y_{0.8}, z_{0.5}\}, e_2 = \{x_{0.7}, y_{0.8}, z_{0.4}\}\},$$

$$g_B = \{e_2 = \{x_{0.3}, y_{0.2}, z_{0.8}\}, e_3 = \{x_{0.4}, y_{0.6}, z_{0.5}\}\}, h_C = \{e_2 = \{x_{0.3}, y_{0.2}, z_{0.4}\}\},$$

$$s_E = \{e_1 = \{x_{0.3}, y_{0.8}, z_{0.5}\}, e_2 = \{x_{0.7}, y_{0.8}, z_{0.8}\}, \{e_3 = \{x_{0.4}, y_{0.6}, z_{0.5}\}\}.$$
\[ \begin{align*}
\text{i) If } r_A & \in \mathcal{RN}(e^A_x) \text{ then } e^A_x \not\in r_A. \\
\text{ii) If } r_A, s_B & \in \mathcal{RN}(e^A_x) \text{ then } r_A \cup s_B \in \mathcal{RN}(e^A_x). \\
\text{iii) If } r_A & \subseteq s_B \text{ and } s_B \in \mathcal{RN}(e^A_x) \text{ then } r_A \in \mathcal{RN}(e^A_x).
\end{align*} \]

**Proof.** 1. It is clear from the definition 18.

2. Let \( r_A, s_B \in \mathcal{RN}(e^A_x) \). Then there exist fs-closed sets \( k_{A_1}, m_{B_1} \) such that

\[ e^A_x \not\in k_{A_1} \supseteq r_A \text{ and } e^A_x \not\in m_{B_1} \supseteq s_B. \]

It is seen that \( \lambda > k_{A_1} e(x) > r_A e(x) \) and \( \lambda > m_{B_1} e(x) > s_B e(x) \) and hence \( \lambda > \max\{k_{A_1} e(x), m_{B_1} e(x)\} \). This shows that \( r_A \cup s_B \in \mathcal{RN}(e^A_x) \).

**Theorem 8.** Let \((U_E^\sim, \kappa_f)\) be a fs-cotopological space and \( f_A \) be a fs-set over \( U \). The followings hold:

\[ \text{i) } clf_A \text{ is a fs-closed set.} \]

\[ \text{ii) } f_A \subseteq clf_A. \]

\[ \text{iii) } clf_A \text{ is the smallest fs-closed set containing } f_A. \]

\[ \text{iv) } f_A \in \kappa_f \text{ iff } f_A = clf_A. \]

**Proof.** The proof can be done similarly to the analogous statements Simsekler and Yuksel (2013).

**Definition 19.** Let \((U_E^\sim, \kappa_f)\) be a fs-cotopological space, \( f_A \) be a fs-set over \( U \). The intersection of all fs-closed sets containing \( f_A \) is called the closure of \( f_A \) and is denoted by \( clf_A \).

**Definition 20.** Let \((U_E^\sim, \kappa_f)\) be a fs-cotopological space \( v_A \) be a fs-set over \( U \). \( v_A \) is called fs-Q-coneighborhood of \( e^A_x \) if there exists a fs-closed set \( k_B \) such that \( e^{1-\lambda}_x \) is not quasi coincident with \( k_B \) and \( k_B \supseteq v_A \).

**Theorem 9.** Let \((U_E^\sim, \kappa_f)\) be a fs-cotopological space and \( v_A, w_B \) be two fs-sets over \( U \). Then the followings hold:

\[ \text{i) If } v_A \text{ is a fs-Q-coneighborhood of } e^A_x \text{ then } e^{1-\lambda}_x \text{ is not quasi coincident with } v_A. \]

\[ \text{ii) If } v_A, w_B \text{ are fs-Q-coneighborhood of } e^A_x \text{ then } v_A \cup w_B \text{ is a fs-Q-coneighborhood of } e^A_x. \]

\[ \text{iii) If } v_A \text{ is a fs-Q-coneighborhood of } e^A_x \text{ and } w_B \subseteq v_A \text{ then } w_B \text{ is a fuzzy soft Q-coneighborhood of } e^A_x. \]

**Definition 21.** Let \((U_E^\sim, \kappa_f), (V_E^\sim, \kappa_f^-)\) be two fs-topological spaces, \( \varphi: U \rightarrow V, \psi: E \rightarrow P \) be mappings and \( e^A_x \) be a fs-point. \( \varphi\psi = \varphi(\psi): FS(U,E) \rightarrow FS(V,P) \) is called fs-\( \kappa \)-continuous at \( e^A_x \) if for any fs-remote neighborhood \( v_A \) of \( \varphi\psi(e^A_x) \), there exists a fs-remote neighborhood \( w_B \) of \( e^A_x \) such that \( \varphi\psi(w_B) \subseteq v_A \).

**Theorem 10.** Let \((U_E^\sim, \kappa_f), (V_E^\sim, \kappa_f^-)\) be two fs-topological spaces, \( \varphi: U \rightarrow V, \psi: E \rightarrow P \) be mappings, \( e^A_x \) be a fs-point and \( \varphi\psi = \varphi(\psi): FS(U,E) \rightarrow FS(V,P) \) is called fs-\( \kappa \)-continuous at \( e^A_x \) if for any fs-remote neighborhood \( v_A \) of \( \varphi\psi(e^A_x) \), there exists a fs-remote neighborhood \( w_B \) of \( e^A_x \) such that \( \varphi\psi(w_B) \subseteq v_A \).
(φ,ψ): FS(U, E) → FS(V, P) be a mapping. Then the followings are equivalent:

i) φψ = (φ,ψ): FS(U, E) → FS(V, P) is fs-κ-continuous mapping at e_κ^A.

ii) For any fs-remote neighborhood v_A of φψ(e^κ_2) there exists a fs-remote neighborhood w_B of e^κ_2 such that w_B ⊆ φψ^−1(v_A).

iii) The inverse image of every fs-remote neighborhood of φψ(e^κ_2) is a fs-remote neighborhood of e^κ_2.

5. Fuzzy Soft Ditopological Spaces

After the definitions and theorems mentioned in the previous sections we can give the main purpose of this paper- a fs-ditopological space, which is a synthesis of a fs-topology, connected to the property of openness in the space and a fs-cotopology, relying on the property of closedness in the space.

**Definition 22.** The triple (U^−_E, τ_f, κ_f) is said to be a fs-ditopological space if U^−_E is a fs-set, τ_f is a fs-topology and κ_f is a fs-cotopology on U^−_E. A pair δ = (τ_f, κ_f) is called a fs-ditopology on U^−_E.

**Definition 23.** Let (U^−_E, δ) be a fs-ditopological space, f_B, m_C be two fs-sets on U and e^κ_2 be a fs-point. A pair (f_B, m_C) is called a fs-neighborhood of e^κ_2 if f_B is a fs-τ-neighborhood and m_C is a fs-remote neighborhood of e^κ_2.

Fs-interior and fs-closure of a fs-set f_A in a fs-ditopological space (U^−_E, δ) are defined respectively by:

\[\text{int } f_A = \bigcup_{i \in I} \{g_B_i \subseteq U^−_E : g_B_i \in \tau_f, g_B_i \subseteq f_A, i \in I\}\]

\[\text{cl } f_A = \bigcap_{i \in I} \{v_B_i \subseteq U^−_E : v_B_i \in \kappa_f, f_A \subseteq v_B_i, i \in I\}\]

**Definition 24.** Let (U^−_E, δ_1), (V^−_P, δ_2) be two fs-ditopological spaces. A mapping φψ = (φ,ψ):(U^−_E, δ_1) → (V^−_P, δ_2) is called fs-continuous if the preimage of any fs-set from τ_2 is in τ_1 and the preimage of any fs-set from κ_2 is in κ_1.

**Theorem 11.** A mapping

φψ = (φ,ψ):(U^−_E, δ_1) → (V^−_P, δ_2) be a mapping Then the followings are equivalent:

i) φψ is fs-continuous at e^κ_2.

ii) For any fs-τ-neighborhood f_A and fs-remote neighborhood v_A of φψ(e^κ_2) there exist a fs-τ-neighborhood g_B and a fs-remote neighborhood w_B of e^κ_2 such that g_B ⊆ φψ^−1(f_A) and w_B ⊆ φψ^−1(v_A) respectively.

iii) The preimage of every fs-τ-neighborhood of φψ(e^κ_2) is a fs-τ-neighborhood of e^κ_2 and the preimage of every fs-remote neighborhood of φψ(e^κ_2) is a fs-remote neighborhood of e^κ_2.

**Proof.** The proof is obvious by Theorem 5. and Theorem 10.

6. References

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