Anomalous diffusion in non-equilibrium relativistic heavy-ion rapidity spectra

A. Lavagno

Dipartimento di Fisica, Politecnico di Torino and INFN, Sezione di Torino, Torino, Italy

Abstract

There is experimental and theoretical evidence that the broad rapidity distribution of net proton yield ($p - \bar{p}$) in central heavy-ion collisions at SPS energies could be a signal of non-equilibrium properties of the system. We show that the broad rapidity shape can be well reproduced in the framework of a non-linear relativistic Fokker-Planck dynamics which incorporates non-extensive statistics and anomalous diffusion.

PACS: 25.75.-q; 05.20.-y; 05.90.+m

Keywords: High energy collisions, non-extensive statistics, rapidity spectra.

1 Introduction

In relativistic heavy-ion collisions nuclear matter is believed to be compressed to high baryon density and nuclei undergoing central collisions reduce strongly their original longitudinal momentum. This loss of rapidity, usually referred as baryon stopping, is an important characteristic to understand the reaction mechanism at high energy density.

Isotropic thermal models fail to reproduce the broad and flat rapidity distribution observed in the central Pb-Pb collisions at 158 A GeV/c [1]. Collective and anisotropic flows must be considered (with the introduction of new parameters) to take into account the experimental spectra [2,3]. Complicate evolution codes take only partially into account of such distributions and the physical evolution of the system is not completely understood [4].

In this paper we study the evolution of the rapidity distribution from a macroscopic point of view by using a non-linear relativistic Fokker-Planck equation and we show that the observed broad rapidity shape could be a signal of non-equilibrium properties of the system. Let us note that a similar approach, within a linear Fokker-Planck equation, has been previously studied in Ref.[5] by using a linear drift in the space of the rapidity and a free parameter diffusion coefficient. With this choice, the author found a strongly violation of the
fluctuation-dissipation theorem. We will see that generalizing the Brownian motion to the relativistic kinetic variables, the standard Einstein relation is satisfied and Tsallis non-extensive statistics emerges in a natural way from the non-linearity of the Fokker-Planck equation.

2 Non-linear Fokker-Planck equation in relativistic dynamics

A class of anomalous diffusions are currently described through the non-linear Fokker-Planck equation (NLFPE)

\[
\frac{\partial}{\partial t}[f(y, t)]^\mu = \frac{\partial}{\partial y} \left[ J(y)[f(y, t)]^\mu + D \frac{\partial}{\partial y}[f(t, y)]^\nu \right],
\]

where \(D\) and \(J\) are the diffusion and drift coefficients, respectively. Tsallis and Bukman [6] have shown that, for linear drift, the time dependence solution of the above equation is a Tsallis-like distribution with \(q = 1 + \mu - \nu\). The norm of the distribution is conserved for all times only if \(\mu = 1\), therefore we will limit the discussion to the case \(\nu = 2 - q\).

We are going to study the time evolution of the particle distribution by means the above NLFPE in the rapidity space. Basic assumption of our analysis is that the rapidity distribution \(y\) is not appreciably influenced by transverse dynamics which is considered in thermal equilibrium. Such hypothesis is well confirmed by the experimental data and adopted in many theoretical works [1–3]. A crucial rôle in the solution of the above NLFPE plays the choice of the diffusion and the drift coefficients. Such a choice influences the time evolution of the system and its equilibrium distribution.

Since the temperature at freeze-out exceeds 100 MeV, Boltzmann approximation is usually adopted and the single particle equilibrium distribution is written as

\[
E \frac{d^3 N}{d^3 p} \propto E \exp(-E/T) \equiv m_\perp \cosh(y) \exp(-m_\perp \cosh(y)/T),
\]

where \(y\) is the rapidity, \(m_\perp = \sqrt{m^2 + p_\perp^2}\) is the transverse mass, \(T\) is the temperature. Imposing the validity of the Einstein relation for Brownian particle, we can generalize to the relativistic case the standard expressions of diffusion and drift coefficients as follows

\[
D = \gamma T, \quad J(y) = \gamma m_\perp \sinh(y) \equiv \gamma p_\|,
\]

where \(p_\|\) is the longitudinal momentum and \(\tau\) is a common constant. It is easy to see that the above coefficients give us the Boltzmann stationary distribution
(2) in the linear Fokker-Planck equation \((q = \nu = 1)\), while the stationary solution of the NLFPE (1) with \(\nu = 2 - q\) is a Tsallis-like distribution with the relativistic energy \(E = m_{\perp} \cosh(y)\). In the light of the above results, we generalize the results of Tsallis-Bukman by searching as a solution of the NLFPE (1) the following time dependent Tsallis-like distribution

\[
f_q(y, m_{\perp}, t) = \left\{1 - (1 - q) \beta(t) m_{\perp} \cosh[y - y_m(t)]\right\}^{1/(1-q)}.
\]

The unknown functions \(\beta(t) = 1/T(t)\) and \(y_m(t)\) have been derived by means numerical integration of Eq.(1) with initial \(\delta\)-function condition depending on the value of the experimental projectile rapidities. The rapidity distribution at fixed time is then obtained by numerical integration over the transverse mass \(m_{\perp}\) (or transverse momentum) as follows

\[
\frac{dN}{dy}(y, t) = c \int_{m}^{\infty} m_{\perp}^2 \cosh(y) f_q(y, m_{\perp}, t) \, dm_{\perp},
\]

where \(c\) is the normalization constant fixed by the total number of the particles. Rapidity spectra calculated from (5) will ultimately depend on two parameters: the “interaction” time \(\tau = \gamma t\) and the non-extensive parameter \(q\). Therefore, no more free parameters are used in this analysis respect to previous theoretical studies \([2–4]\).

### 3 Results and Conclusions

We have studied the time dependent rapidity distribution (5) in the case of central Pb+Pb collisions at 158 GeV/c and our results are compared with the net proton experimental NA49 data \([1]\). The obtained spectra are normalized to 164 protons and the beam rapidity is fixed to \(y_{cm} = 2.9\) (in the c.m. frame) \([1]\). In Fig.1 we show the calculated rapidity spectra compared with the experimental data. The full line corresponds to the NLFPE solution (5) at \(\tau = 0.82\) and \(q = 1.25\); the dashed line corresponds to the solution of the linear case \((q = 1)\) at \(\tau = 1.2\). Only in the non-linear case \((q \neq 1)\) exists a (finite) time for which the obtained rapidity spectra well reproduces the broad experimental shape. A value of \(q \neq 1\) implies anomalous superdiffusion in the rapidity space, i.e., \([y(t) - y_M(t)]^2\) scale like \(t^\alpha\) with \(\alpha > 1\) \([6]\).

We have studied the rapidity spectra from a macroscopic point of view. A good agreement with the experimental data is reached assuming a deviation from the equilibrium and a non-linear evolution which implies non-extensive Tsallis-like distribution. Non-extensive features in relativistic collisions are

\[1\] This result cannot be obtained if one assumes a linear drift coefficient as in Ref.\([5]\).
Fig. 1. Rapidity spectra for net proton production ($p-\bar{p}$) in central Pb+Pb collisions at 158A GeV/c (grey circles are data reflected about $y_{cm} = 0$) [1]. Full line corresponds to our results by using a non-linear evolution equation ($q = 1.25$), dashed line corresponds to the linear case ($q = 1$).

outlined in several works [7–11], even if a microscopic justification of these effects is still lacking. In Ref.[4], the authors show that diquark breaking during inelastic collisions in dual parton model could be responsible for the observed baryon stopping. The possible connection between the nature of these inelastic collisions and the above observed non-extensive statistical effects is under investigations.

Acknowledgments. It is a pleasure to thank Prof. P. Quarati and Prof. G. Wilk for fruitful discussions and comments.

References

[1] H. Appelshäuser et al. (NA49 Collaboration), Phys. Rev. Lett. 82 (1999) 2471.
[2] I.G. Bearden et al. (NA44 Collaboration), Phys. Rev. Lett. 78 (1997) 2080.
[3] P. Braun-Munzinger et al., Phys. Lett. B 344 (1995) 43; B 356 (1996) 1.
[4] A. Capella, C.A. Salgado, Phys. Rev. C 60 (1999) 054906.
[5] G. Wolschin, Eur. Phys. J. A 5 (1999) 85.
[6] C. Tsallis, D.J. Bukman, Phys. Rev. E 54 (1996) R2197.
[7] W.M. Alberico, A. Lavagno, P. Quarati, Eur. Phys. J. (2000) 499.
[8] G. Wilk, Z. Wlodarczyk, Phys. Rev. Lett. 84 (2000) 2770.
[9] D.B. Walton, J. Rafelski, Phys. Rev. Lett. 84 (2000) 31.
[10] I. Bediaga, E.M.F. Curado, J.M. de Miranda, Physica A 286 (2000) 156.
[11] C. Beck, Physica A 286 (2000) 164.