Equilibrium configuration of self-gravitating charged dust clouds: Particle approach

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A three dimensional Molecular Dynamics (MD) simulation is carried out to explore the equilibrium configurations of charged dust cloud subjected to self-gravity. The equilibrium of such system is characterized by three parameters: charge to mass ratio of dust grain, number of particle in the cloud and average kinetic energy of the grain. Below a critical charge to mass ratio and temperature, dust particles coagulate to form a spherical cloud. The interior of such clouds is probed by the radial density profile and its variation with different parameters is investigated. The problem of equilibrium is also formulated in the mean field limit where dust pressure which is the sum of kinetic and electro-static pressure, balances the self-gravity. The mean field solutions are compared with the results of MD simulations.

Keywords: Dusty plasma, self-gravity, Jeans instability, MD simulation

I. INTRODUCTION

Dust is one of the main constituents of universe. A variety of astrophysical systems such as interstellar clouds, solar system, planetary rings, cometary tails, Earth’s environment etc. contain micron to sub-micron sized dust particles in different amounts. However, in presence of ambient plasma and radiative environments, these particles are acted upon by the electron-ion currents, photons and energetic particles. As a result, the macroscopic particles become electrically charged and start interacting with each other, as well as with ions and electrons via long-range electric fields giving rise to collective behavior. This leads to the formation of special medium called the “dusty plasma”.

The macroscopic particles in dusty plasmas interact not only via electric field but also via gravitation field which is also a long ranged force. Which of the forces dominate is determined by the charge to mass ratio which in turn depends on the dust size. For dust particle of mass $m_d$ and charge $Q_d$, the gravitational attraction may becomes important if the ratio $Gm_d^2/Q_d^2 \approx O(1)$. For spherical dust grain, mass is given by $m_d = \frac{4}{3}\pi r_d^3 \rho/3$ where $r_d$ is dust radius and $\rho$ is mass density. The two forces are roughly equal if $r_d \approx r_{cr}$ and $r_{cr} = \left[\frac{9Q_d^2/(16\pi^2\rho^3G)}{1/6}\right]$. For minimum possible charge i.e. $Q_d \approx 1000 e$ critical grain radius comes out to be $r_{cr} \approx 10^{-2} \text{cm}$. For realistic dust charge i.e. $Q_d \approx 1 e$ and $\rho = 1 \text{gm/cc}$ critical grain radius comes out to be $r_{cr} \approx 10^{-1} \text{cm}$. Thus the submicron and micron-size grains interact mainly via electric forces whereas gravitational force may become important for particles of a few hundred microns. There are many astrophysical objects where self-gravity plays an important role along with the electric field e.g. Saturn’s rings which contain both submicron grains and fragments of a few meters in size. Other examples where such particle may exist are galactic disk, including dark clouds, protoplanetary nebulae, planetary rings, and cometary atmospheres.

The gravitational collapse is an important mechanism for formation of charged dust cloud with self-attraction. In the quasi-neutral state. In such a situation, what usually done is to artificially construct an infinite homogeneous equilibrium by neglecting the zero order gravitational force and then study stability of linear perturbations.

Avinash and Shukla proposed a new way of balancing self-gravity of charged dust cloud by the electrostatic (ES) pressure arising from the inhomogeneous distribution of dust in the background of plasma. In their paper, the problem of equilibrium is posed in terms of an equation for the force balance together with an equa-
tion of state of ES pressure. At low dust density the ES pressure scales quadratically with number density and the adiabatic index is two. At very high dust densities ES pressure becomes independent of number density due to charge reduction caused by mutual screening of the grains\textsuperscript{18,19} and as a consequence, the adiabatic index becomes zero. This change of adiabatic index from two to zero provides an upper threshold on the total mass ($M_{\text{AS}}$) supported by ES pressure against gravity. The physics of this mass limit is very similar to Chandrasekhars mass limit for white dwarfs where adiabatic index becomes zero. This change of adiabatic index for hydrostatic pressure changes from 5 to 4/3. Chandrasekhar’s mass limit for white dwarfs where adiabatic index for hydrostatic pressure changes from 5/3 to 4/3 due to relativistic effects. If the mass of cloud exceeds this upper mass ($M > M_{\text{AS}}$) the ES pressure is not strong enough to balance the gravity and the system will undergo symmetric collapse under self-gravity\textsuperscript{17,20,21}. We, in this communication, revisit the equilibrium problem using molecular dynamics (MD) simulations. The particles are subject to Yukawa repulsion as well as gravitation attraction force. The equilibrium of these two forces above a critical charge to mass ratio results as gravitation attraction force. The equilibrium of these charge clouds is formulated and it is established that collapse responsible for $M > M_{\text{AS}}$ is due to stability of radial eigen modes\textsuperscript{22–24}.

Paper is organized in the following manner. In Sec. II the details of MD simulation is given. In Sec. III we present a theoretical mean field solution based on hydrodynamic force balance. Simulation results are presented in section IV where we also compare our results with theory. In Sec. V we summarize our results and discuss the future scope of work.

II. SIMULATION DETAILS

The electrostatic interaction among grains in the background of electron-ion plasma is given by an isotropically screened Coulomb potential or Yukawa potential $\phi(r)$ while gravitational interaction among dust is given by $\psi(r)$

$$\phi(r) = \frac{Q_d \exp(-r/\lambda_D)}{4\pi\epsilon_0 r}, \quad \psi(r) = -G\frac{m_d}{r},$$

where $Q_d$ is dust charge, $r$ is the distance between two dust particles and $\lambda_D$ is the screening length and $G$ is universal gravitational constant. Screening length $\lambda_D$ depends on density and temperature of the background plasma as $1/\lambda_D^2 = \left(\frac{e^2 n_e}{\epsilon_0 k_B T_e}\right) + \left(\frac{e^2 n_i}{\epsilon_0 k_B T_i}\right)$ where $n_e(n_i)$ is electron(ion) density, $e(q)$ is electron(ion) charge and $T_e(T_i)$ is electron(ion) temperature. It should be noted that while simulating dust grains, the background electrons are assumed to be unaffected by the gravity. This assumption is justified because for the typical astrophysical dust $m_d \sim 10^{15} - 10^{20}$ times $m_i$, therefore Jeans frequency for dust is much much greater than that of ions and electrons, i.e., $\omega_J > \omega_{Ji}, \omega_{Je}$. It means that dust collapses in a much shorter time scale than ions and electrons do and on the the time scale of dust collapse, electrons and ions can be treated as unaffected by the gravity.

All the simulation are performed using a large scale OpenMP parallel Three Dimensional Molecular Dynamics (3DMD) code developed by authors. The further details regarding the units, equations and methodology are given in the following Secs. II A II B.

A. Equations and Units

The Hamiltonian of the system is given by:

$$H = \sum_{i=1}^{N_d} \frac{p_i^2}{2m_d} + \frac{1}{2} \sum_{i,j}^{N_d} \left( \frac{Q_d^2}{4\pi\epsilon_0} \frac{e^{-r_{ij}/\lambda_D}}{r_{ij}} - \frac{Gm_d^2}{r_{ij}} \right),$$

where $r_{ij} = |\vec{r}_i - \vec{r}_j|$ and $p_i = |\vec{p}_i|$. We have assumed that all grains have equal charge $Q_d$ and mass $m_d$. The equation of motion are given by

$$\frac{d\vec{r}_i}{dt} = \frac{\vec{p}_i}{m_d}, \quad \frac{d\vec{p}_i}{dt} = -\vec{v}_\ast \left( \sum_{j \neq i}^{N_d} \left( \frac{Q_d^2}{4\pi\epsilon_0} \frac{e^{-r_{ij}/\lambda_D}}{r_{ij}} - \frac{Gm_d^2}{r_{ij}} \right) \right),$$

where $\vec{v}_\ast = \left( \sum_{j \neq i}^{N_d} \left( 1 + \frac{\vec{r}_{ij}}{r_{ij}} \right) \frac{e^{-r_{ij}/\lambda_D}}{r_{ij}} - \frac{4\pi\epsilon_0 Gm_d^2/Q_d^2}{r_{ij}} \right) \vec{r}_{ij}$.

The dimensionless kinetic and potential energy per particles can be written as

$$\hat{E}_K = \frac{1}{2N_d} \sum_{i=1}^{N_d} \hat{v}_i^2, \quad \hat{E}_{pot} = \frac{1}{2N_d} \sum_{i,j}^{N_d} \sum_{j \neq i}^{N_d} \left( e^{-r_{ij}/\lambda_D} - \Gamma_d \right),$$

where $\Gamma_d$ is the distance between two dust particles and $\lambda_D$ is the screening length and $G$ is universal gravitational constant.
FIG. 1. Different states of self-gravitating Yukawa particles is shown for \( N_d = 2000 \). At the initial state particles are uniformly distributed in a cubical box with number density \( \tilde{n}_d = 0.001 \) as shown in Fig.1(a) and corresponding radial density distribution is shown in Fig.1(b). Fig.1(c) refers to the equilibrium structure of the system at dust temperature \( \tilde{T}_d = 1.50 \) and the corresponding density profile is shown in Fig.1(d). At low temperature \( \tilde{T}_d = 0.5 \) equilibrium structure has the shape of sphere with a very dense core as shown in Fig.1(e). The averaged radial density of such dust cloud is shown in Fig.1(d).
where \( \Gamma_g \) is the ratio of gravitational interaction energy to electrostatic interaction energy between of two grains i.e.,

\[
\Gamma_g = \frac{Gm^2}{(Q_d/4\pi\epsilon_0)^2}.
\]

It should be noted here that \( \Gamma_g \) is also related with the charge to mass ratio and the relation among the two is given by:

\[
Q_d/m_d = \left( \frac{4\pi\epsilon_0 G}{\Gamma_g} \right)^{1/2}.
\]

The value of \( \sqrt[4]{A\pi\epsilon_0 G} \) is 8.614 \times 10^{-11} C/kg in S.I. units and 2.582 \times 10^{-4} statC/g in cgs units. The effect of charge to mass ratio on equilibrium structure is studied via \( \Gamma_g \) in this paper. Temperature is given by mean kinetic energy per particle which, in dimensionless units in 3D, turns out to be

\[
\tilde{T}_d = \frac{1}{3N_d} \sum_{i=1}^{N_d} \tilde{v}_i^2.
\]

Normalized number density is given as,

\[
\tilde{n}_d = n_d \lambda_D^3.
\]

In dusty plasma community it is customary to define equilibrium in terms of two dimensionless parameters: \( \kappa = a/\lambda_D \) ; the ratio of the mean inter-particle distance \( a = (3/4\pi n_d)^{1/3} \) to the screening length \( \lambda_D \) and \( \Gamma = Q_d^2/4\pi\epsilon_0 T_d \); the inverse of dust temperature \( T_d \) measured in units of \( Q_d^2/4\pi\epsilon_0 a \). The coupling parameter \( \Gamma^* = \Gamma \exp(-\kappa) \) which is the ratio of the mean inter-particle potential energy to the mean kinetic energy, is used as a measure of coupling strength in dusty plasmas. For \( \Gamma^* \ll 1 \), Yukawa system behaves like an ideal gas, \( \Gamma^* \sim 1 \) corresponds to an interacting fluid whereas \( \Gamma^* \gg 1 \) refers to a condensed solid state. The dimensional parameters \( \kappa \) and \( \Gamma \) can be related to normalized dust density and temperature as:

\[
\kappa = \left( \frac{3}{4\pi \tilde{n}_d} \right)^{1/3}, \quad \Gamma = \frac{1}{\kappa \tilde{T}_d}.
\]

Instead of choosing \( (\Gamma, \kappa) \) space to work in, we prefer to work in \( (\tilde{n}_d, \tilde{T}_d) \) space, however, one can switch from one space to other space using the relations given in Eq.(7).

**B. Methodology**

Simulations begin with an initial condition where \( N_d \) particles are distributed randomly over a large volume \( n_d = 10^{-3} \). The equation of motion given in Eq.(4) is integrated taking time step of size 0.01 \( \omega_0^{-1} \). This step size is appropriate for conservation of total energy (i.e. sum of kinetic and potential energy). The system is kept in contact with a heat bath using Berendsen thermostat for fist \( 2 \times 10^5 \) steps to bring the system at fixed temperature and then system is isolated and remains there for \( 2 \times 10^5 \) steps. For runs where \( \tilde{T}_d \geq 1.0 \) step size is halved and the total number of steps is doubled to achieve numerical accuracy. All observations are taken during the micro-canonical run when system is isolated. As the system evolves, depending upon \( \Gamma_g, N_d \) and \( \tilde{T}_d \), different equilibrium structures are obtained [Fig.(1)]. To get the information about interior structure of the cloud we plot spherically symmetric radial density distribution \( \tilde{n}_d(r) \). While plotting \( \tilde{n}_d \) as a function of \( r \), the radial distance is measured from the center of mass of the cloud. The final radial distribution is ensemble average of hundreds of individual copies of \( \tilde{n}_d(r) \) taken at different time intervals.

Before showing the MD simulation results, let us first formulate the the problem of equilibrium in mean-field limit using fluid approximation. In the later section, we would compare the results obtained from the mean field solution with MD simulation results and would examine the validity of the two approaches.

**III. MEAN FIELD SOLUTIONS**

In the mean field (continuum) limit, the system of particles is replaced by a self-gravitating charged fluid. The electric force acting on the grains behave like an effective ES pressure force and the gravitation potential \( \psi \), defined in Eq.(11), is no more a pair potential but rather defined by the Poisson’s equation. The equation of motion for such self-gravitating fluid is given by

\[
\rho_d \frac{du_d}{dt} = -\nabla P_d - \rho_d \nabla \psi, \tag{8}
\]

where \( \rho_d = m_d n_d \) is dust mass density, \( P_d \) is total dust pressure (sum of kinetic pressure and ES pressure) and \( \psi \) is gravitational potential. In static equilibrium \( u_d = 0 \), therefore

\[
\nabla P_d = -\rho_d \nabla \psi. \tag{9}
\]

Eq.(9) is closed by equation of states for \( P_d \) and \( \psi \). Equation of state for \( \psi \) comes from Poisson’s equation:

\[
\nabla^2 \psi = 4\pi G \rho_d. \tag{10}
\]

Taking the divergence of Eq.(10) (after dividing by \( \rho_d \)) and using Poisson’ Eq.(11), the force balance equation in radial coordinates can be written as

\[
\frac{1}{r^2} \frac{d}{dr} \left( \frac{r^2 dP_d}{dr} \right) = -4\pi G m_d n_d \tilde{T}_d, \tag{11}
\]

As we want to compare the results of MD simulation with the mean field solution, let us normalize Eq.(11) as we
did in Sec. [11] i.e. using, \( r \to \tilde{r} \lambda_D \), \( P_d \to \tilde{P}_d(\frac{Q_d^2}{4\pi\gamma_0\lambda_D^3}) \) and \( n_d \to \tilde{n}_d/\lambda_D^3 \).

\[
\frac{1}{r^2} \frac{d}{dr} \left( \frac{r^2 d\tilde{P}_d}{\tilde{n}_d} \right) = -4\pi G m_d^2 \tilde{P}_d/4\pi \varepsilon_0 \tilde{n}_d
\]

\[
= -4\pi \Gamma_g \tilde{n}_d
\]

Equation (12)

where we still need equation of state for dust pressure to solve Eq. (12). Recently, Shukla et al. [22] have obtained the equation of state for Yukawa fluid by using rigorous MD simulations. Their expression for total dust pressure in normalized units is given by

\[
\tilde{P}_d = \tilde{n}_d \tilde{\rho}_d + \beta \tilde{n}_d^2,
\]

where \( \tilde{\rho}_d \) is dust temperature and \( \beta \) is a number of the order of \( \pi \). In Eq. (13), the first term which scales linearly with number density, is usual kinetic pressure term whereas second term which is proportional to the square of number density, corresponds to ES pressure. Substituting Eq. (13) in Eq. (12), we get the following differential equation

\[
\tilde{n}_d'' + \frac{2}{r} \tilde{n}_d' + \frac{\tilde{\rho}_d}{\tilde{n}_d} \tilde{n}_d' - \frac{2\pi\Gamma_g}{\beta} \tilde{n}_d = 0
\]

Equation (14) along with Eq. (15) may be solved numerically to get \( \tilde{n}_d(\tilde{r}) \) for a given temperature \( \tilde{\rho}_d \), central density \( \tilde{n}_d(0) \) and \( \Gamma_g \). The radius of the cloud (\( R \)) is estimated using the relation \( \tilde{n}_d(R) = 0 \), whereas the mass of the cloud is obtained using the relation

\[
M_d = m_d N_d = 4\pi m_d \int_0^R \tilde{n}_d'(r) d\tilde{r}.
\]

IV. RESULTS

As discussed in the previous section, the size of the cloud critically depends on three parameters, first, the number of particles in the cloud, i.e. \( N_d \), second, the relative strength of gravity, i.e. \( \Gamma_g \) (which also gives the measure of mass to charge ratio) and third, on the kinetic pressure which is characterized by \( \tilde{\rho}_d \). Following these observations, the simulation results may broadly be divided into three parts accordingly. One, comprising the effect of \( N_d \) on \( \tilde{n}_d(\tilde{r}) \) keeping \( \Gamma_g \) and \( \tilde{\rho}_d \) fixed, second, the effect of \( \Gamma_g \) on \( \tilde{n}_d \) for fixed \( N_d \) and \( \tilde{\rho}_d \) and third, effect of \( \tilde{\rho}_d \) on \( \tilde{n}_d(\tilde{r}) \) keeping \( \Gamma_g \) and \( N_d \) fixed.

Special Case: Solution for \( \tilde{\rho}_d = 0 \)

There is an interesting case with \( \tilde{\rho}_d = 0 \) when the Eq. (14) can be solved analytically. As stated earlier, it is not just the kinetic the pressure which contributes to the dust pressure but it is the sum of kinetic and ES pressure which together give the dust pressure [see Eq. (13)]. In this special case, the gravitation force is balanced completely by ES pressure force and kinetic pressure has no role to play. In this limit Eq. (14) reduces to the form given as;

\[
\tilde{n}_d'' + \frac{2}{r} \tilde{n}_d' = -\frac{2\pi\Gamma_g}{\beta} \tilde{n}_d.
\]

The solution of above differential equation along with BCs defined in Eq. (15), is given by

\[
\tilde{n}_d(\tilde{r}) = \tilde{n}_d(0) \sin \left( \frac{\sqrt{2\pi\Gamma_g}}{\beta} \tilde{r} \right).
\]

As mentioned earlier the radius of the cloud is obtained by setting \( \tilde{n}_d(R) = 0 \), which gives \( \sqrt{\frac{2\pi\Gamma_g}{\beta}} R = \pi \). Hence the radius of cloud is given by

\[
R = \sqrt{\frac{\pi\beta}{2\Gamma_g}}.
\]

The relation between central density \( \tilde{n}_d \) and \( N_d \) can be obtained by integrating number density as follows

\[
N_d = \int_0^R \tilde{n}_d(\tilde{r}) 4\pi \tilde{r}^2 d\tilde{r}.
\]

Substituting the value of \( \tilde{n}_d \) and \( R \) from Eq. (15) and (19) respectively in above equation gives,

\[
\tilde{n}_d(0) = \frac{1}{\sqrt{2\pi}} \left( \frac{\Gamma_g}{\beta} \right)^{3/2} N_d
\]

It should be noted here that in the absence of kinetic pressure, radius of cloud \( R \) is independent of number of particles \( N_d \) and is function of \( \Gamma_g \) only [Eq. (19)] whereas \( \tilde{n}_d(0) \) is directly proportional to \( N_d \) and it also depends on \( \Gamma_g^{3/2} \) [Eq. (21)].
FIG. 2. Effect of Number of particles on the radial density is shown for fixed value of $\Gamma_g = 0.08$ and $T_d = 0.10$. Symbols represent the data points from MD simulation whereas curves represent the best fit of the corresponding data using trial function of the form given in right hand side of Eq. (18). It can be seen from the plot that the radius of dust cloud is almost independent of $N_d$ and any increases in number of particles only increases the density of the core.

A. Effect of $N_d$ on density profile

To examine the effect of $N_d$ of $\tilde{n}_d(\tilde{r})$, we fix $\Gamma_g = 0.08$ and $T_d = 0.10$. As the temperature is close to zero, solution for $T_d = 0.10$ case will suffice for $\tilde{n}_d(\tilde{r})$. To compare the results of MD simulation with mean field solution, the simulation data of $\tilde{n}_d(\tilde{r})$ vs $\tilde{r}$ are fitted in the trial function of the form;

$$f(x) = a \frac{\sin(kx)}{kx}, \quad (22)$$

where parameters $a$ and $k$ are obtained by fit. Here $a$ serves as central density for MD simulation results and $k$ represents the factor $\sqrt{2\pi\Gamma_g/\beta}$ i.e. $a = \tilde{n}_{d0}$ and $k = \sqrt{2\pi\Gamma_g/\beta}$. In Fig. (2), the variation of number density with number of particles is shown where symbols are data points obtained from simulation and curves represent the fitted function. The numerical values of central density $\tilde{n}_{d0}$ and $\sqrt{2\pi\Gamma_g/\beta}$ obtained from MD data are tabulated in Table I.

| $N_d$ | $\tilde{n}_{d0}$ | $\sqrt{2\pi\Gamma_g/\beta}$ |
|-------|----------------|-----------------------------|
| 500   | 0.380          | 0.321                       |
| 1000  | 0.736          | 0.316                       |
| 1500  | 1.106          | 0.317                       |
| 2000  | 1.450          | 0.315                       |
| 3000  | 2.149          | 0.313                       |
| 4000  | 2.839          | 0.313                       |

It can be seen from Fig. (2) that the radii of dust clouds is almost independent of number of particles. Numerically $R = \pi/\sqrt{2\pi\Gamma_g/\beta}$ and it is evident from the Table I that value of $\sqrt{2\pi\Gamma_g/\beta}$ is almost independent of $N_d$ and so does $R$. This result is consistent with the theoretical predictions given by Eq. (19). Also the constancy of $\sqrt{2\pi\Gamma_g/\beta} \approx 0.3158$ fixes $\beta \approx 1.604\pi$. Fig. (2) also suggests that the increase in $N_d$ results in increases in $\tilde{n}_{d0}$. The Eq. (21) states that central density $\tilde{n}_{d0}$ should be a linear function of $N_d$ and the predicted slope is $\frac{1}{\sqrt{2\pi}} \left(\frac{\Gamma_g}{\beta}\right)^{3/2}$ which comes out to be $7.98 \times 10^{-4}$ for $\Gamma_g = 0.08$ and $\beta = 1.604\pi$. To verify Eq. (21), we fit central density with number of particles in Fig. (3) using the data from Table I. The slope of $\tilde{n}_{d0}$ vs. $N_d$ line comes out to be $7.165 \times 10^{-4}$ against the predicted value of $7.98 \times 10^{-4}$ thereby validating the results with a deviation of about 10%.

B. Effect of $\Gamma_g$ on density profile

The effect of $\Gamma_g$ on radial number density $\tilde{n}_d(\tilde{r})$ for fixed number of particles $N_d = 4000$ and constant temperature $T_d = 0.10$ is shown in Fig. (3) where symbols represent the simulation data points and curves correspond to the mean field solution obtained from Eq. (14). The initial value of central density in Eq. (14) is provided from the simulation data. It can be seen that the agreement of the two approaches is excellent. It is also clear from the Fig. (3(a), 3(c)) that the radial density, $\tilde{n}_d(\tilde{r})$, radius of the cloud, $R$ and central density, $\tilde{n}_{d0}$, is directly related to $\Gamma_g$. In the zero temperature limit, the simulation data can be fitted in the form given in Eq. (22) and fitted parameters are tabulated in Table I. For the fixed parameters $\beta = 1.604\pi$ and $N_d = 4000$, the relation between $R$ and $\Gamma_g$, using Eq. (20), reduces to $R = 2.813 \Gamma_g^{-1/2}$. A direct plot between $\Gamma_g$ and $R$ using Table I gives the relation $R = 2.861 \Gamma_g^{-1/2}$. Similarly the relation between $\tilde{n}_{d0}$ and
FIG. 4. Effect of $\Gamma_g$ on radial density profile: Radial density $\tilde{n}_d$ is plotted against radial distance $\tilde{r}$ for different values of $\Gamma_g$ for $N_d = 4000$ and $T_d = 0.10$ in Fig.5(a)-5(e). symbols show the density profile obtained from MD simulation where as solid lines correspond to mean field solution of $\tilde{n}_d(\tilde{r})$ obtained from Eq.(14). It is clear from the results that the radius of the cloud $R$ decreases whereas core density $\tilde{n}_{d0}$ increases with $\Gamma_g$. Fig. 5(f) shows the variation of $R$ and $\tilde{n}_{d0}$ with $\Gamma_g$.

$C$. Effect of dust temperature on density profile

Effect of dust temperature on the radial density profile for 2000 particles and $\Gamma_g = 0.15$ is shown in Fig.5 for $T_d = 0.5, 1.0, 1.5$ and 2.0. Mean field solution is plotted along with the simulation data in plots. While plotting mean field solution, the central density is taken from simulation results and Eq.(14) is solved numerically. At low temperatures, equilibrium structure has a very dense core of central density is about $10^3$ to $10^4$ order of magnitudes denser than the initial density. Here the system is phase separated and all the particles are concentrated in a very small volume inside the simulation box [Fig.5(c)]. As the temperature increases, the central density of the cloud slowly decreases while radius gradually increases [Fig.5(a) and 5(b)]. This trend keeps on going until a critical temperature is met beyond which the central core density falls abruptly and becomes of the order of the initial density [Fig.5(c) and 5(d)]. Now, the system is no more phase separated and particles are all over the volume inside the simulation box touching the walls of the box. Any further increase in temperature further homogenizes the system. It is remarkable to see here that equation of state given in Eq.(14) along with equation of hydrostatic balance Eq.(13) predicts the density profile excellently all the way from low temperature, high

| $\Gamma_g$ | $\tilde{n}_{d0}$ | $\sqrt{2\pi \Gamma_g}/\beta$ | $R$ |
|-----------|----------------|-----------------|------|
| 0.04      | 0.965          | 0.216           | 14.544 |
| 0.06      | 1.785          | 0.265           | 11.855 |
| 0.08      | 2.839          | 0.313           | 10.037 |
| 0.10      | 4.102          | 0.355           | 8.849  |
| 0.12      | 5.583          | 0.395           | 7.953  |
FIG. 5. Effect of dust temperature on radial number density is shown for $\Gamma_g = 0.15$ and $N_d = 2000$. Fig 5(a) and 5(b) are plotted for $\tilde{T}_d = 0.1$ and $\tilde{T}_d = 1.0$ respectively where the system leaves the boundary of the box and occupies a very small volume inside the box. On the other hand at $\tilde{T}_d = 1.5$ and $\tilde{T}_d = 2.0$ shown in Fig.5(c) and 5(d) respectively, system is occupying the full volume but density is not uniform.

V. SUMMARY AND CONCLUSION

To summarize, we have examined the problem of equilibrium of self-gravitating dusty plasmas using particle level MD simulations. Dust grains interact with each other via repulsive Yukawa potential and attractive gravitational potential. The equilibrium of the system is characterized by three parameters, $\Gamma_g (= 4\pi\epsilon_0 Gm_d^2/Q_d^2)$, number of particles $N_d$ and mean kinetic energy or temperature $T_d$ and depending upon these three parameters, different equilibrium structures are formed. The interior of these equilibrium structures is studied using radial density function where, center of mass of the cloud is taken as the $r = 0$ point. We have rigorously investigated the effect of $T_d$, $N_d$ and $\Gamma_d$ on radial density. We have also formulated the problem of equilibrium in the mean field limit where dust pressure, which is the sum of kinetic pressure and electrostatic pressure, balances the self gravity. The results of mean field limit are compared with simulation results and the two approaches are found to be consistent with each other.

It should be mentioned here that while formulating the problem of equilibrium, dust charge is taken to be constant and independent of number density while there are enough evidences to conform that charge of dust decreases with number density. As mentioned in Sec. we reduction of dust charge at high density can limit the total mass supported by the ES pressure. Therefore our study is useful to the scenarios where dust density is low and dust charge is constant. A number of other effects like magnetic field, dust rotation etc. are also not taken into account and will be addressed in future communication.

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