Research on Calibration Model of Bridge Health Monitoring Based on Vibrating Wire Strain Sensor

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Abstract. Bridge structure quality inspection and health monitoring are important research contents of the transportation industry. This study uses the national standard 6061 aluminum alloy cylindrical bar, and vibrating the vibrating wire strain sensor used in the mainstream of the two markets in a symmetrical position. This study used a superimposed force standard machine to perform a 0-100KN compression test on a bar. We performed data regression analysis on the data of the two sensors collected and obtained a new model. By comparing the output values with the traced force standard machine, we conclude that the trend of the model is consistent with the trend of the force standard machine. This study provides a theoretical basis for the rapid calibration detection of bridges before installation by strain monitoring sensors.

Keywords: traffic metering, vibrating wire strain gage, bridge health monitoring, data model.

1. Introduction
Deformation is the inevitable result of bridge facilities. From the beginning of construction, the bridge facilities will undergo various deformations due to natural, geological conditions and human factors. This deformation exceeds the allowable value and will evolve into a disaster, causing loss of life and property. If the deformation exceeds the allowable value, it will evolve into a disaster, which will result in the loss of life and property [1]. It is to make the micro-deformation (quantity change) process tangible that the monitoring technology for the healthy operation of the bridge structure. We can make the micro-deformation trend through long-term monitoring, thus achieving predictable [2, 3]. How to accurately assess deformation? It is only meaning of assessing deformation that regular, irregular or continuous high-precision online measurement [4]. It can prevent the deformation of bridge structure from quantitative change to qualitative change to prevent disasters, thus ensuring life safety and avoiding major economic losses that correct and reliable large-scale data, scientific analysis and system management of trend models. Only by the combination of high-precision measuring equipment and multi-measurement methods, we can accurately evaluate the relationship between deformation and locality [5], accumulate a lot of local and overall data, monitor the micro-variables, judge the trend of micro-deformation and prevent micro-duration [6].

After consulting the data, we understand that digital close-range measurement technology needs to mark targets [7]. The position of such a test bed is not easy to be allowed in the laboratory, and at this stage, the accuracy of our holding equipment cannot be used to characterize the micro-strain [8].
According to the test principle, the strain monitoring sensor can be divided into resistive, vibrating wire, fiber grating, etc. Because of its shortcomings, ordinary resistive sensors cannot be used for long-term monitoring. The application technology of vibrating wire sensor is relatively mature, and its long-term stability is good. Its most prominent advantage is that its measured value is not affected by the length of the connecting line. Other sensors, fiber grating type test accuracy, but it is expensive and also plagued by the laying process and temperature compensation scheme [9, 10]. After a brief introduction to the testing principle of the vibrating wire strain gage, we use a laboratory-tracked superimposed force standard machine with an accuracy of 0.003%. We used a method of comparative analysis and a numerical theoretical analysis method to perform traceability of the two sensors in the laboratory. We enable them to meet the test requirements for traffic metering [11].

2. Methods

2.1. Theoretical Model of Vibrating Wire Sensor

We use the variation of the vibration frequency of the steel string to characterize the force of the vibrating wire sensor. The actual output signal of this sensor is frequency, so there are not many problems, such as on-site calibration of strain gauges, signal drift, poor long-distance transmission, and poor durability for long-term use. And because of the good performance of aluminum rods, we have solved the shortcomings of unstable strain gauges for long-term use. Therefore, vibrating wire sensors are used on a large scale in current bridge monitoring.

The measurement characteristics of the vibrating wire sensor are very good. Its non-linear characteristic is less than 0.1%; and its sensitivity is 0.05%; and its temperature error is less than 0.1%/10°C. The initial tensile force of the steel string of the strain gauge after the factory is made $T_0$, so its initial frequency is $F_0$. After installing the strain gauge after the layout, the tension of the vibrating wire changes with the deformation. We can measure the strain by using the tension change of the vibrating wire. We assume that the tension of the vibrating wire is $T$, the natural frequency is $f$, and the relationship between tension and frequency can be expressed as:

$$ T = Kf^2 $$

In the formula, it is related to the length of the string and the mass per unit length. Obviously, there is

$$ \Delta T = T - T_0 = K(f^2 - f_0^2) $$

We assume that the strain increment of the strain gauge is $\varepsilon_h$, and the strain increment of the vibrating string is set to $\varepsilon_K$, then:

$$ \varepsilon_h = \varepsilon_K = \frac{\Delta k}{EA} $$

The axial stiffness of the steel string is $EA$, so:

$$ \varepsilon_h = \frac{K}{EA}(f^2 - f_0^2) = k_h(f^2 - f_0^2) $$

The mathematical model of the vibrating wire sensor is:

$$ F = k(f^2 - f_0^2) $$

$$ F = A(f^2 - f_0^2) + B(f_f_0) $$
Figure 1. Working principal diagram of vibrating wire strain gauge

As shown in figure 1, when the length is L, the mass is m, and the tension of the string is F, the natural frequency it produces is f:

\[
f = \frac{1}{2l} \sqrt{\frac{E \Delta l}{\rho l}} = \frac{1}{2l} \sqrt{\frac{E \Delta l}{\rho l}}
\]

\[
f = \varphi(F)
\]

Sensitivity:

\[
f^2 = \frac{E \Delta l}{4l^2 \rho l} = K \epsilon
\]

After differentiation:

\[
2 f df = K d \epsilon
\]

\[
k = \frac{df}{d \epsilon} = \frac{K}{2f}
\]

Material coefficient:

\[
k = \frac{1}{4l^2 \rho} = \frac{E S}{4l^2 \rho}
\]

\[
\epsilon = \frac{\Delta l}{l}
\]

From the above formula we can see that the sensitivity k is proportional to the material coefficient K and inversely proportional to the vibration frequency of the string.

Nonlinear error:

When the measured tension is \(F_0\), then the initial frequency is \(f_0\), and when the measured tension is \(F_1 = f_0 + f\), the vibration frequency is \(f_1\):

\[
f_1 = \frac{1}{2l} \sqrt{\frac{F_0 + \Delta F}{ml}} = \frac{1}{2l} \sqrt{\frac{F_0}{ml}} \sqrt{1 + \frac{\Delta F}{F_0}} = f_0 \sqrt{1 + \frac{\Delta F}{F_0}} = f_0 \sqrt{1 + \epsilon_F} = f_0 (1 + \epsilon_F)^{\frac{1}{2}}
\]

\[
f_1 = f_0 (1 + \frac{1}{2} \epsilon_F - \frac{1}{8} \epsilon_F^2 - \frac{1}{16} \epsilon_F^3 - \cdots)
\]

When

\[
F_2 = F_0 - \Delta F
\]

\[
f_2 = f_0 \left(1 - \frac{1}{2} \epsilon_F - \frac{1}{8} \epsilon_F^2 - \frac{1}{16} \epsilon_F^3 - \cdots\right)
\]

Its quadratic nonlinearity error is:

\[
\frac{1}{2} f_0 \epsilon_F^2 = \frac{1}{4} \epsilon_F
\]
It can be seen from the above formula that the larger $\varepsilon_F$ is, the larger $\delta_m$ will be.

Frequency stability:
The main influencing factor of frequency stability is the change in temperature of the environment. The bulk density $\rho_v$ and the $\Delta l$ caused by $F$ are not dependent on the ambient temperature:

$$\gamma_f = \frac{df}{f} = \frac{dE}{E} = \frac{3}{2} \frac{dl}{l}$$

2.2. Experimental

Usually in the process of use, the acquisition frequency of vibrating string strain gauge is very low. In the practical application of engineering, the strain value will be affected by the vibration of the structure itself due to environmental factors and other phenomena. Therefore, in a stable environment, we have used a high precision superposition force standard machine to compress the specimens. In this way, we can reduce the force transfer error, and can collect the output value of two symmetrical sensors, and then the error data can be neglected.

In this test, we used a superimposed force standard machine calibrated by China National Metrology Institute, and selected Vibrating wire strain sensor from A and B manufacturers that are used in the domestic inspection market.

In this study, as shown figure 2, in order to make the measured data points evenly distributed in the measurement range, we designed the following test scheme by using different gradients. We used a nut in the 6061 T6 aluminum bar and fixed the vibrating wire strain sensor at both ends of the aluminum rod. In this test, we divided the experiments into three groups according to the test pressure, namely 50KN group, 75KN group and 100KN group. According to formula $F_{in} = 5*(n-1)$, we determined the pressure values of the 50KN test. (In the formula, 1 represents the 50KN test group; $n = 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11$ represents the test level of the group; $F$ represents the test level pressure, unit KN)

In each set of pressure tests, we applied pressure to the A and B sensors for 120 s and repeated the test 10 times and collected test data.

![Figure 2. Experimental device diagram](image)

**Table 1.** The index of 6061 aluminum

| tensile strength $\sigma_b$ | Yield Strength $\sigma_{0.2}$ | Elongation $\delta_5$ (%) | Elastic coefficient | Bearing Yield Strength | Fatigue strength |
|-----------------------------|-------------------------------|--------------------------|---------------------|-----------------------|-----------------|
| $\geq 180$MPa               | $\geq 110$MPa                 | $\geq 14$                | 68.9GPa             | 103MPa                | 62.1MPa         |

Note: Longitudinal mechanical properties of bar materials at room temperature

3. Results and discussion

In the study, we used SPSS20.0 to analyze the three sets of test data, and used Origin8.5 to draw the data analysis chart. We use 3, 4, and 5 to respectively show the test results of the 50KN group, the 75KN group, and the 100KN group.

We performed regression analysis on the experimental data of the two sensors to obtain a new model $Y$. 
Where

\[ Y = 2.207 + 0.01 \cdot X_1 - 0.003 \cdot X_2 \]

Y: new model
X1: Frequency measured by the A sensor, Hz
X2: measured frequency of B sensor, με

In this paper, we mainly study the correlation between two kinds of sensor measurements and force standard machine. In the new model, the units of parameters X_1 and X_2 are different.

\[
\begin{align*}
Y &= 2.207 + 0.01 \cdot X_1 - 0.003 \cdot X_2 \\
Y &= 2.207 + 0.01 \cdot X_1 - 0.003 \cdot X_2
\end{align*}
\]

In figure 3-6, we can see that the trend of the new model is consistent with the trend of the force standard machine, and both the A sensor and the B sensor are significantly different from the force standard machine trend.

It is the peak inflection point at 15KN-25KN. But the maximum peak value of A sensor is 20 KN. The salient point of B sensor appears at 15KN. We can see that the linearity instability of the force standard machine appears at 5KN-15KN. Therefore, we can verify the output linearity of the standard machine in subsequent experiments.
In figure 7-10, we can see that the trend of the A sensor is a parabolic trend. And this change is completely different from the linear change of the force standard machine. The trend of the B sensor after 22.5KN is very slow. This result is different from the trend of the force standard machine. As the value of force increases, the trend of the new model increases, and its trend is similar to that of the standard machine.

At the same time, we have solved the problem of output linearity of force standard machine in this group of experiments. However, we find that there is no strain at the position of 15KN-25KN for A sensor to characterize the material stress. And B sensor also produces unstable output at 15KN-25KN. We find that the characterization of deformation by B sensor is better than that by A sensor.
In figure 11-14, we can see that the trend of the A sensor after 90KN is an approximate straight line. It does not characterize the effects of external force changes on aluminum rods. The trend of the new model is consistent with the trend of the force standard machine.

In figure 12, we can see that the trend of the A sensor after 20KN is an approximate straight line. It does not characterize the effects of external force changes on aluminum rods. The trend of the new model is consistent with the trend of the force standard machine.

In this group of experiments, we found that A and B sensors simultaneously characterize the feature change nodes, and the output of the two sensors is stable. However, the linearity of our new model is not optimized enough between 10KN and 40KN. The results provide an effective basis for data screening in the later stage of the new model.

4. Conclusion
We studied the data changes of A sensor and B sensor under different force values, and carried out regression analysis on the data to obtain a new model. The trend of the model is consistent with the trend of the force standard machine, which provides a theoretical basis for the development of real-time detection technology for bridges in the future.

At the same time, we verify the instability of the repeatability of the vibrating string strain gauge in the acquisition process. It can also be explained that the vibrating string strain gauge needs a certain time to stabilize the impact of the vibration of the vibrating string sensor string itself. During the experiment, we can find that the change of temperature has a certain effect on the stability of the sensor.
After the sensor is subjected to long-term stress, the fatigue property of the bar used in the experiment is found to be a better material to characterize the deformation after the stress. Vibrating string strain gauge can measure micro-deformation more steadily after long working time. In conclusion, the research shows that we can trace the origin of vibration string strain gauge by in-situ comparison of standard force output. This method of in-situ comparison through standard force output can trace the value of vibration string strain gauge.

In the field engineering process, due to environmental impact, temperature compensation and other factors, we need to calculate the uncertainty. This experiment is under stable laboratory environment, so we only consider the sensor's own acquisition error. In the follow-up research process, we will conduct a large number of experiments after in-depth analysis.

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