Factorization in processes of graviton
scattering off electron for Z and W productions

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Abstract

The study of factorization in linearized gravity is extended to the graviton scattering processes with an electron for the massive vector boson productions such as $ge \rightarrow Ze$ and $ge \rightarrow W\nu e$. It is shown that every transition amplitude is completely factorized due to gravitational gauge invariance and Lorentz invariance. Also the explicit values of vector boson polarizations are obtained.

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In the standard gauge theory every four-body Born amplitude with a massless gauge boson as an external particle has been well known to be factorizable into one factor which depends only on charge or other internal-symmetry indices and the other factor which depends on spin or polarization indices.
In our previous works [5,6], we have shown that gravitational gauge invariance and graviton transversality force the transition amplitudes of four-body graviton interactions to be factorized. In particular, we have investigated in a more extensive way the four-body graviton interactions like $g(k_1) + X(p_1) \rightarrow \gamma(k_2) + X(p_2)$ and $g(k_1) + X(p_1) \rightarrow g(k_2) + X(p_2)$ in the context of linearized gravity where $X$ is any kind of particles with spin less than 2 or graviton itself. We have found that every amplitude can be completely factorized into a simple form composed of a kinematic factor, QED-like Compton scattering form, and another gauge invariant terms. The factorization property can be used as a powerful tool to investigate the gravitational interactions and the polarization effects.

So far, the processes of a graviton conversion into a photon and gravitational Compton scattering are considered. Also we have considered the case that the incoming matter is the same as the outgoing one in each process. But in this paper, we extend the study of factorization in the process of high energy graviton conversion into a massive vector boson such as $ge \rightarrow Ze, ge \rightarrow W\nu_e$. Also it is interesting to note that factorization is held even if matter particles in the initial state and final state have different masses as in the process $ge \rightarrow W\nu_e$.

The Lagrangian $\mathcal{L}_I$ of the interaction of an electron and massive vector boson ($Z$ or $W$) with gravitational field is given by

$$\mathcal{L}_I = \mathcal{L}_{ge}(A) + \mathcal{L}_{gW}(A) + \mathcal{L}_{gZ} + \mathcal{L}_{geW} + \mathcal{L}_{geZ},$$  \hspace{1cm} (1)

$$\mathcal{L}_{ge}(A) = \sqrt{-g} \left[ \frac{i}{2} \left( \bar{\psi} \gamma^\mu (\nabla_\mu - ieA_\mu) \psi - \bar{\psi} (\nabla_\mu + ieA_\mu) \gamma^\mu \psi \right) - m_e \bar{\psi} \psi \right],$$  \hspace{1cm} (2)

$$\mathcal{L}_{gW}(A) = -\frac{1}{2} \sqrt{-g} g^{\mu\nu} g^{\alpha\beta} (D_\mu W_\alpha - D_\alpha W_\mu)(D_\nu W_\beta - D_\beta W_\nu)$$
$$+ \sqrt{-g} g^{\mu\nu} m_W^2 W^*_\mu W_\nu - ie \sqrt{-g} g^{\mu\nu} g^{\alpha\beta} W^*_\mu W_\alpha F_{\nu\beta},$$  \hspace{1cm} (3)

$$\mathcal{L}_{gZ} = -\frac{1}{4} \sqrt{-g} g^{\mu\nu} g^{\alpha\beta} (\partial_\mu Z_\alpha - \partial_\alpha Z_\mu)(\partial_\nu Z_\beta - \partial_\beta Z_\nu)$$
$$+ \frac{1}{2} \sqrt{-g} g^{\mu\nu} m_Z^2 Z^*_\mu Z_\nu, \hspace{1cm} (4)$$

$$\mathcal{L}_{geW} = g_W \sqrt{-g} g^{\mu\nu} \left[ \bar{\psi}_e W^*_\mu \gamma_\nu (1 - \gamma_5) \psi_e + \bar{\psi}_\nu W_\mu \gamma_\nu (1 - \gamma_5) \psi_e \right],$$  \hspace{1cm} (5)

$$\mathcal{L}_{geZ} = g_Z \sqrt{-g} g^{\mu\nu} \bar{\psi} \gamma_\mu [\epsilon_L (1 - \gamma_5) + \epsilon_R (1 + \gamma_5)] \psi Z_\nu, \hspace{1cm} (6)$$
where \( g, g_W, g_Z, \epsilon_L, \) and \( \epsilon_R \) are defined as
\[
g = \det g_{\mu\nu},
\]
\[
g_W = \left(2 - \frac{1}{2}G_F m_W^2\right)^{\frac{1}{2}}, \quad g_Z = \left(2\frac{1}{2}G_F m_Z^2\right)^{\frac{1}{2}},
\]
\[
\epsilon_L = -\frac{1}{2} + \sin^2 \theta_W, \quad \epsilon_R = \sin^2 \theta_W,
\]

where \( \theta_W \) is the Weinberg angle.

In the procedure one introduces a symmetric tensor field \( h_{\mu\nu} \) denoting the deviation of the metric tensor \( g_{\mu\nu} \) from the flat space Minkowski metric tensor \( \eta_{\mu\nu} \):
\[
g_{\mu\nu} = \eta_{\mu\nu} + \kappa h_{\mu\nu},
\]

where \( \kappa = \sqrt{32\pi G_N} \). After the expansion any curved space geometrical object is expressed as an infinite series in terms of \( h_{\mu\nu} \). The process of graviton scattering off electron for the \( Z \) boson production \( ge \rightarrow Ze \) is made up of four Feynman diagrams (See Fig. 1) and the explicit form of its tree-level amplitude \( M_{ge\rightarrow Ze} \) is determined from the Lagrangian \( \mathcal{L}_{ge} \), \( \mathcal{L}_Z \), and \( \mathcal{L}_{geZ} \) as
\[
M_{ge\rightarrow Ze} = M_a^{Ze} + M_b^{Ze} + M_c^{Ze} + M_d^{Ze},
\]

\[
M_a^{Ze} = \frac{i \kappa g_Z}{4} \epsilon_1 \cdot p_1 \bar{u}(p_2, s_2) \phi_2 [\epsilon_L(1 - \gamma_5) + \epsilon_R(1 + \gamma_5)] (k_1 \cdot p_1 + 2\epsilon_1 \cdot p_1) u(p_1, s_1),
\]
\[
M_b^{Ze} = -\frac{i \kappa g_Z}{4} \epsilon_1 \cdot p_2 \bar{u}(p_2, s_2) (2\epsilon_1 \cdot p_2 - \epsilon_1 \cdot k_1) \phi_2 [\epsilon_L(1 - \gamma_5) + \epsilon_R(1 + \gamma_5)] u(p_1, s_1),
\]
\[
M_c^{Ze} = \frac{i \kappa g_Z}{4} \frac{1}{(k_1 \cdot k_2)} \bar{u}(p_2, s_2) [2(\epsilon_1 \cdot k_2) \{\epsilon_1 \cdot \epsilon_2 \} \cdot \cdot \cdot - 2(k_1 \cdot k_2)(\epsilon_1 \cdot \epsilon_2) \cdot \cdot \cdot 2(\epsilon_1 \cdot k_2) \cdot \cdot \cdot + 2(\epsilon_1 \cdot k_2)(\epsilon_1 \cdot \epsilon_2) \cdot \cdot \cdot] \epsilon_L(1 - \gamma_5) + \epsilon_R(1 + \gamma_5)] u(p_1, s_1),
\]
\[
M_d^{Ze} = -\frac{i \kappa g_Z}{2} \epsilon_1 \cdot p_2 \bar{u}(p_2, s_2) \phi_1 [\epsilon_L(1 - \gamma_5) + \epsilon_R(1 + \gamma_5)] u(p_1, s_1),
\]

where \( \epsilon_1^\mu \epsilon_1^\nu \) and \( k_1^\mu \) are the polarization and momentum of the graviton, \( \epsilon_2^\mu \) and \( k_2^\mu \) are the polarization and momentum of the \( Z \) boson, and \( p_1^\mu \) and \( p_2^\mu \) are the initial and final momenta of the massive electron. From the above results, we can obtain the completely factorized transition amplitude \( M_{ge\rightarrow Ze} \) as
\[
\mathcal{M}_{ge \rightarrow Ze} = -\frac{\kappa}{2e} \frac{p_1 \cdot k_1 p_2 \cdot k_1}{k_1 \cdot k_2} \left[ \frac{p_1 \cdot \epsilon_1}{p_1 \cdot k_1} - \frac{p_2 \cdot \epsilon_1}{p_2 \cdot k_1} \right] \mathcal{M}_{\gamma e \rightarrow Ze} \\
= -\frac{\kappa}{2e^3} \frac{p_1 \cdot k_1 p_2 \cdot k_1}{k_1 \cdot k_2} \mathcal{M}_{\gamma s \rightarrow Ye} \mathcal{M}_{\gamma e \rightarrow Ze}, \tag{14}
\]

where \(\mathcal{M}_{\gamma e \rightarrow Ze}\) is the transition amplitude of an electroweak process,

\[
\mathcal{M}_{\gamma e \rightarrow Ze} = \frac{i e g_Z}{2} \bar{u}(p_2, s_2) \left[ \left. \phi_2 \left( \frac{\phi_1}{k_1 \cdot p_1} + 2 \frac{p_1 \cdot \epsilon_1}{k_1 \cdot p_1} \right) - \frac{2 \epsilon_2 \cdot (1 - \epsilon_1 k_1)}{k_1 \cdot p_2} \right] \right] \times \left[ \epsilon_L (1 - \gamma_5) + \epsilon_R (1 + \gamma_5) \right] u(p_1, s_1). \tag{15}
\]

Next, the transition amplitude \(\mathcal{M}_{ge \rightarrow W_{\nu e}}\) is obtained from the Lagrangian \(\mathcal{L}_{ge}, \mathcal{L}_{gW}\), and \(\mathcal{L}_{geW}\) as

\[
\mathcal{M}_{ge \rightarrow W_{\nu e}} = \mathcal{M}_{a W_{\nu e}} + \mathcal{M}_{b W_{\nu e}} + \mathcal{M}_{c W_{\nu e}} + \mathcal{M}_{d W_{\nu e}} \tag{16}
\]

If \(\epsilon_L, \epsilon_R, \text{ and } g_Z\) in Eqs. \((10) - (13)\) are replaced by 1, 0, and \(g_W\), respectively, the transition amplitudes \(\mathcal{M}_{i W_{\nu e}} (i = a, b, c, d)\) can be obtained in the same form as \(\mathcal{M}_{i Ze}\) of the process \(ge \rightarrow Ze\). Then the transition amplitude of the process \(ge \rightarrow W_{\nu e}\) is completely factorized as

\[
\mathcal{M}_{ge \rightarrow W_{\nu e}} = -\frac{\kappa}{2e} \frac{p_1 \cdot k_1 p_2 \cdot k_1}{p_2 \cdot k_1} \left[ \frac{p_1 \cdot \epsilon_1}{p_1 \cdot k_1} - \frac{p_2 \cdot \epsilon_1}{p_2 \cdot k_1} \right] \mathcal{M}_{\gamma e \rightarrow W_{\nu e}} \\
= \frac{\kappa}{2e} (p_1 \cdot k_1) \left[ \frac{p_1 \cdot \epsilon_1}{p_1 \cdot k_1} - \frac{p_2 \cdot \epsilon_1}{p_2 \cdot k_1} \right] \mathcal{M}_{\gamma e \rightarrow W_{\nu e}}, \tag{17}
\]

where \(\mathcal{M}_{\gamma e \rightarrow W_{\nu e}}\) is the transition amplitude of electroweak process defined as

\[
\mathcal{M}_{\gamma e \rightarrow W_{\nu e}} = \frac{i e g_W}{2} \bar{u}(p_2, s_2) \left\{ \phi_2^* \left( \frac{\phi_1}{k_1 \cdot p_1} + 2 \frac{p_1 \cdot \epsilon_1}{k_1 \cdot p_1} \right) \phi_1 \phi_2 \\
+ \frac{2}{k_1 \cdot k_2} \left[ (\epsilon_1 \cdot \epsilon_2^*) \phi_1 - (\epsilon_2^* \cdot k_1) \phi_1 - (\epsilon_1 \cdot k_2) \phi_2^* \right] \right\} (1 - \gamma_5) u(p_1, s_1). \tag{18}
\]

Using the simply factorized transition amplitudes we can obtain the polarization effects of massive vector bosons. Factorization allows us to use the well-known polarization effects in the ordinary QED for the investigation of polarization effects in the graviton processes. The processes \(ge \rightarrow Ze\) and \(ge \rightarrow W_{\nu e}\) have the same polarization property as \(\gamma e \rightarrow Ze\) and \(\gamma e \rightarrow W_{\nu e}\), respectively \([7]\), because we have freedom to choose \(\epsilon^\mu\) which makes the first
braket in Eq. (14) is independent of the graviton helicity [3,8]. The polarization of $Z^0$ and $W$ can be defined through the density matrix [3]

$$I^\mu_\nu = \frac{1}{3} I^{\mu \nu} - \frac{i}{2m} \epsilon^{\mu \nu \lambda \tau} k_{2 \lambda} P_\tau - \frac{1}{2} Q^{\mu \nu},$$

(19)

$$P_\tau = \frac{1}{2} I^{\mu \nu} + \frac{k_2^\mu k_2^\nu}{m^2},$$

(20)

where

and $P_\tau$ and $Q^{\mu \nu}$ are called the polarization vector and polarization tensor, respectively. The explicit forms of the differential cross section and the polarization vector and tensor of $Z^0$ in the $ge \rightarrow Ze$ in the massless limit of the electron are as follow;

$$\left[ \frac{d\sigma}{dt} \right]_{ge \rightarrow Ze} = \frac{\kappa^2 g_2^2 t}{2\pi u(s+u)^2 s^3} A_0,$$

(22)

$$P_\mu = \frac{2}{m_Z} \{ i(\epsilon_R)^2 + (\epsilon_L)^2 R_1^\mu + (\epsilon_R)^2 - (\epsilon_L)^2 R_2^\mu \} / A_0,$$

$$Q^{\mu \nu} = -\frac{1}{3} I^{\mu \nu} - 2 \{ (\epsilon_R)^2 + (\epsilon_L)^2 R_1^{\mu \nu} + i(\epsilon_R)^2 - (\epsilon_L)^2 R_2^{\mu \nu} \} / A_0,$$

(23)

where $s$, $t$, and $u$ are the usual Mandelstam variables and $A_0$, $R_1^\mu$, $R_2^\mu$, $R_1^{\mu \nu}$, and $R_2^{\mu \nu}$ are defined [3] as

$$A_0 = su \left\{ -(\epsilon_R)^2 + (\epsilon_L)^2 \right\} \{ 2tm_Z^2(1 + \xi_3) + s^2 + u^2 \} + \xi_2(\epsilon_R)^2 - (\epsilon_L)^2(s - u)(t + m_Z^2) \},$$

$$R_1^\mu = \frac{is}{2 su \xi_3} \{ 2m_Z^2(tk_1^\mu - sp_1^\mu + up_2^\mu) + (s^2 + u^2)k_2^\mu \},$$

$$R_2^\mu = s u m_Z^2 \{ (s - u)K^\mu - (1 + \xi_3) [A^\mu + t(p_1 + p_2)^\mu] + 2\xi_1 < \mu k_1 p_1 p_2 > \},$$

$$R_1^{\mu \nu} = \frac{s u}{t} \left\{ -t(su I^{\mu \nu} - 2m_Z^2 K^\mu K^\nu)

- (1 - \xi_3) \left[ t^2 k_1^\mu k_1^\nu + s t \eta^{\mu \nu} - tk_1^\mu (up_1^\nu - sp_2^\nu) - tk_1^\nu (up_2^\mu - sp_1^\mu) + A^\mu A^\nu \right]

+ (1 + \xi_3) [A^\mu + t(p_1 + p_2)^\mu] [A^\nu + t(p_1 + p_2)^\nu]

- 2\xi_1 [ < \mu k_1 p_1 p_2 > (A^\nu + t(p_1 + p_2)^\nu) + (\mu \leftrightarrow \nu) \}],$$

$$R_2^{\mu \nu} = \frac{i}{2 s u \xi_2} \left\{ (u - s)(t + m_Z^2) \frac{k_2^\mu k_2^\nu}{m_Z^2} + 2 \left[ m_Z^2 k_1^\mu (p_1 + p_2)^\nu - k_2^\mu A^\nu + (\mu \leftrightarrow \nu) \right] \right\}.$$

(24)
In Eqs. (24) the $\xi_i$ ($i = 1, 2, 3$) are Stokes parameters of the graviton beam \[^{[3]}\] and $A^\mu, K^\mu$, and $< \mu k_1 p_1 p_2 >$ are defined as

\[
A^\mu = u p_1^\mu + s p_2^\mu,
\]
\[
K^\mu = k_1^\mu - \frac{k_1 \cdot k_2 k_2^\mu}{m_Z^2} = k_1^\mu - \frac{(s + u)}{2m_Z^2} k_2^\mu,
\]
\[
< \mu k_1 p_1 p_2 > = \epsilon^{\mu \nu \alpha \beta} k_\nu p_{1\alpha} p_{2\beta}.
\]

The corresponding values for the $ge \rightarrow W \nu_e$ process can be obtained from Eqs. (22)-(24) by replacing $\epsilon_L, \epsilon_R, m_Z$, and $g_Z$ by 1, 0, $m_W$, and $g_W$. The effect of polarization of gravity can be obtained through its effect to $P^\mu$ and $Q^{\mu \nu}$. It is noted that the graviton can couple to any particle and the two processes $ge \rightarrow Ze$ and $ge \rightarrow W \nu_e$ have the four Feynman diagrams of the same form as given in Fig. 1. Among them we have chosen two independent diagrams in order to compare them with the result of the electroweak theory as in Eqs. (15) and (18). However, in the standard model of the electroweak theory two independent diagrams are different, but they are related through a mathematical identity as shown in Ref. \[^{[7]}\]. This is a special case of universality held in the SM of electroweak theory. The factorization is independent of the chirality of $Z^0, W$ couplings to the fermion line and mass does not affect the factorization at all.

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**FIGURE CAPTION**

**Fig. 1** Feynman diagrams for the process $ge \rightarrow Vf$. The curly line is for a graviton. $V$, denoted by a wiggly line, can be $Z$ or $W$. $f$, denoted by a solid line, can be an electron or a neutrino.
Fig. 1