Fully-strange tetraquark states with the exotic quantum number $J^{PC} = 4^{+-}$

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We study the fully-strange tetraquark states with the exotic quantum number $J^{PC} = 4^{+-}$. We systematically construct all the diquark-antidiquark interpolating currents, and apply the method of QCD sum rules to calculate both the diagonal and off-diagonal correlation functions. The obtained results are used to construct three mixing currents that are nearly non-correlated. We use one mixing current to extract the mass of the lowest-lying state to be $2.85^{+0.19}_{-0.22}$ GeV. We apply the Fierz rearrangement to transform this mixing current to be the combination of three meson-meson currents. The obtained Fierz identity suggests that this lowest-lying state dominantly decays into the $P$-wave $\phi(1020)f_{2}^{\pm}(1525)$ channel.

Keywords: exotic hadron, tetraquark state, QCD sum rules

I. INTRODUCTION

In the past twenty years many candidates of exotic hadrons were observed in particle experiments [1], which can not be well explained in the traditional quark model [2-22]. Especially, some of them have the exotic quantum numbers that the traditional $q\bar{q}$ mesons and $qqq$ baryons can not reach, such as the spin-parity quantum numbers $J^{PC} = 0^{-}/0^{+-}/1^{+-}/2^{+-}/3^{+-}/4^{+-}$, etc. These hadrons are of particular interests, and their possible interpretations are compact multiquark states [23-31], hadronic molecular states [32-35], glueballs [36-45], and hybrid states [46-64], etc.

Among these exotic quantum numbers, the states of $J^{PC} = 1^{-+}$ have been extensively studied in the literature [23, 24, 32-34, 36, 46-63, 65, 66] together with some experimental observations [67-71]. Besides, the states of $J^{PC} = 0^{-}/0^{+-}/2^{+-}/3^{+-}/4^{+-}$ have also been studied to some extent [25-31, 35, 37-45, 64]. However, there has not been any investigation on the exotic quantum number $J^{PC} = 4^{+-}$ yet. In this paper we shall study for the first time the fully-strange tetraquark states with such an exotic quantum number through the QCD sum rule method.

In this paper we shall work within the diquark-antidiquark picture and systematically construct all the fully-strange tetraquark currents of $J^{PC} = 4^{+-}$. We shall calculate both the diagonal and off-diagonal correlation functions. The obtained results are used to construct three mixing currents that are nearly non-correlated. We shall use one mixing current to extract the mass of the lowest-lying state to be $2.85^{+0.19}_{-0.22}$ GeV. With a large amount of $J/\psi$ sample, the BESIII collaboration are intensively studying the physics happening around here. Such experiments can also be performed by Belle-II, COMPASS, and GlueX, etc. Hence, this state is an exotic hadron to be potentially observed in future particle experiments.

In this paper we shall also systematically construct the fully-strange meson-meson currents, and relate them to the diquark-antidiquark currents through the Fierz rearrangement. The obtained Fierz identity suggests that the lowest-lying state dominantly decays into the $P$-wave $\phi(1020)f_{2}^{\pm}(1525)$ channel. Accordingly, we propose to search for it in the $X \to \phi(1020)f_{2}^{\pm}(1525) \to \phi K \bar{K}$ decay process in future particle experiments.

This paper is organized as follows. In Sec. II we systematically construct the fully-strange tetraquark currents with the exotic quantum number $J^{PC} = 4^{+-}$. Their diagonal and off-diagonal correlation functions are calculated in Sec. III. We separately perform the single-channel analysis in Sec. IV and the multi-channel analysis in Sec. V. The obtained results are summarized in Sec. VI, where we also discuss the decay behaviors.

II. INTERPOLATING CURRENTS

In this section we construct the fully-strange tetraquark currents with the exotic quantum number $J^{PC} = 4^{+-}$. This quantum number can not be reached by simply using one quark and one antiquark, and moreover, we need two quarks and two antiquarks together with at least two derivatives to reach such a quantum number.

There are two possible configurations, the diquark-antidiquark configuration and the meson-meson configuration. When investigating the former configuration, the two covalent derivatives $D_{a}(\equiv \partial_{a} + ig_{a}A_{a})$ and $D_{\bar{b}}$ can be either inside the diquark/antidiquark field or between them:

$$\eta = [s_{a}^{T}C_{T}^{-1}D_{a}D_{\bar{b}}s_{\bar{b}}](\bar{s}_{c}\Gamma_{2}C\bar{s}_{d}^{T}) \pm h.c.$$, \ \ \ (1)

$$\eta' = [s_{a}^{T}C_{T}D_{a}s_{\bar{b}}]D_{\bar{b}}(\bar{s}_{c}\Gamma_{4}C\bar{s}_{d}^{T}) \pm h.c.$$, \ \ \ (2)

$$\eta'' = [s_{a}^{T}C_{T}s_{\bar{b}}]D_{a}D_{\bar{b}}(\bar{s}_{c}\Gamma_{6}C\bar{s}_{d}^{T})$$, \ \ \ (3)

where $[X D_{a} Y] \equiv X[D_{a} Y] - [D_{a} X] Y$, $a \cdots d$ are color indices, and $\Gamma_{1\cdots 6}$ are Dirac matrices. We find that only the $\eta$ currents can reach $J^{PC} = 4^{+-}$, while the $\eta'$ and $\eta''$ currents can not.
Altogether we can construct three independent diquark-antidiquark currents of $J^{PC} = 4^{-+}$:

$$\eta_{1,2,3}^i_{\alpha_1\alpha_2\alpha_3\alpha_4} = \epsilon^{abc}e^{cde} \times \left( \begin{array}{c} s^6 \\ 247m_s^2s^5 \\ -2(\bar{s}s)^2 \\ 8(\bar{s}s)^2 \\ 4\bar{s}s \\ 4s^2 \end{array} \right) \times \left( \begin{array}{c} \delta_{\rho}^{\sigma} \\ -\delta_{\sigma}^{\rho} \\ \delta_{\rho}^{\sigma} \\ -\delta_{\sigma}^{\rho} \\ \delta_{\rho}^{\sigma} \\ -\delta_{\sigma}^{\rho} \end{array} \right) \left( \begin{array}{c} \eta_1^i \\ \eta_2^i \\ \eta_3^i \end{array} \right).$$ (10)

This Fierz identity will be used to study the decay behaviors in Sec. VI.

### III. QCD SUM RULE ANALYSIS

In this section we apply the QCD sum rule method [72, 73] to study the fully-strange tetraquark currents $\rho_{ij}^{1,2,3} = \rho_{ij}^{1,2,3}(ss\bar{q}q)$ with the exotic quantum number $J^{PC} = 4^{-+}$. This non-perturbative method has been successfully applied to study various conventional and exotic hadrons in the past fifty years [74, 75]. We have also applied it to study various fully-strange tetraquark states in Refs. [76–80].

We generally assume that the current $\eta_{ij}^i_{\alpha_1\alpha_2\alpha_3\alpha_4}$ (i = 1 · · · 3) couples to the fully-strange tetraquark states $X_n$ (n = 1 · · · N) through

$$\langle 0|\eta_{ij}^i_{\alpha_1\alpha_2\alpha_3\alpha_4}|X_n\rangle = f_{in}\epsilon_{\alpha_1\alpha_2\alpha_3\alpha_4},$$ (11)

where $f_{in}$ is the $3 \times N$ matrix for the decay constants, and $\epsilon_{\alpha_1\alpha_2\alpha_3\alpha_4}$ is the traceless and symmetric polarization tensor satisfying

$$\epsilon_{\alpha_1\alpha_2\alpha_3\alpha_4}\epsilon_{\beta_1\beta_2\beta_3\beta_4}^\dagger = S'[\bar{g}_{\alpha_1\beta_1}\bar{g}_{\alpha_2\beta_2}\bar{g}_{\alpha_3\beta_3}\bar{g}_{\alpha_4\beta_4}],$$ (12)

with $\bar{g}_{\mu\nu} = g_{\mu\nu} - q_{\mu}q_{\nu}/q^2$. The symbol $S'$ denotes symmetrization and subtracting the trace terms in the sets $\{\alpha_1\alpha_2\alpha_3\alpha_4\}$ and $\{\beta_1\beta_2\beta_3\beta_4\}$.

Based on Eq. (11), we can investigate both the diagonal and off-diagonal correlation functions:

$$\Pi_{ij}^{\rho_{ij}^i}(q^2) = \rho_{ij}^{phen}(s) = \int_{s_0}^{\infty} ds\rho_{ij}^{\rho_{ij}^i}(s),$$ (13)

where $s_0 = 16m_s^2$ the physical threshold. We parameterize the spectral density $\rho_{ij}^{phen}(s)$ for the states $X_n$ together with a continuum contribution as

$$\rho_{ij}^{phen}(s) = \sum_n \delta(s - M_n^2)\rho_{ij}^{\rho_{ij}^i}(0) + \cdots,$$ (15)

$$= \sum_n f_{in}\bar{f}_{jn}\delta(s - M_n^2) + \cdots,$$

with $M_n$ the mass of $X_n$.

At the quark-gluon level we calculate $\Pi_{ij}(q^2)$ using the dispersion relation as

$$\Pi_{ij}(q^2) = \int_{s_0}^{\infty} ds\rho_{ij}^{\rho_{ij}^i}(s),$$ (14)

with $s_0 = 16m_s^2$ the physical threshold. We parameterize the spectral density $\rho_{ij}^{phen}(s)$ for the states $X_n$ together with a continuum contribution as

$$\rho_{ij}^{phen}(s) = \rho_{ij}^{\rho_{ij}^i}(s).$$ (15)

$$= \sum_n f_{in}\bar{f}_{jn}\delta(s - M_n^2) + \cdots,$$
The calculations are performed up to the twelfth dimension. In the calculations we have considered the perturbative term, the quark condensate $\langle ss \rangle$, the quark-gluon mixed condensate $\langle g_s s \sigma G_s \rangle$, the gluon condensate $\langle g^2 G G \rangle$, and their combinations.

Then we perform the Borel transformation at both the hadron and quark-gluon levels. After approximating the continuum using $\rho_{ij}(s)$ above the threshold value $s_0$, we obtain the sum rule equations:

\[
\Pi_{ij}(s_0, M_H^2) = \sum_n f_n f_{\bar{n}} e^{-M_n^2/M_H^2} + \int_{s<s_0} e^{-s/M_H^2} \rho_{ij}(s) ds .
\]

We shall investigate them through two steps, the single-channel analysis and the multi-channel analysis, as follows.

\[
\rho_{22}(s) = \frac{s^6}{316800\pi^6} - \frac{31m_s\langle g_s s \sigma G_s \rangle}{5184\pi^4} s^3 - \frac{19\langle g^2 G G \rangle m_s^2(s)}{3240\pi^4} s^2 + \frac{2m_s^2(s)}{27\pi^2},
\]

\[
\rho_{34}(s) = \frac{s^6}{316800\pi^6} - \frac{13\langle g^2 G G \rangle m_s(s)}{60480\pi^4} s^4 + \frac{11m_s^2(s) s}{216\pi^2} + \frac{m_s^2(s)}{2880\pi^6},
\]

\[
\rho_{12}(s) = \frac{\langle g^2 G G \rangle s^4}{464486\pi^4} + \frac{\langle g^2 G G \rangle m_s(s)}{48384\pi^6} + \frac{\langle g^2 G G \rangle m_s(s)}{2016\pi^4} s^2.
\]

\[
\rho_{13}(s) = -\frac{m_s^2 s^5}{6912\pi^5} + \frac{m_s(s)}{1728\pi^4} s^4 + \frac{5\langle g^2 G G \rangle m_s(s)}{14512\pi^6} s^3 - \frac{43\langle g^2 G G \rangle m_s(s)}{69120\pi^4} s^2 - \frac{23m_s^2(s)}{15\pi^2} s^2 + \frac{269m_s\langle g_s s \sigma G_s \rangle}{181440\pi^4} s^3 - \frac{263\langle g^2 G G \rangle m_s(s)}{4320\pi^2} s.
\]

\[
\rho_{23}(s) = \frac{41m_s\langle g_s s \sigma G_s \rangle}{60480\pi^4} - \frac{5\langle g^2 G G \rangle m_s^2(s)}{48384\pi^6} + \frac{43\langle g^2 G G \rangle m_s(s)}{23040\pi^4} s^2 - \frac{23040\pi^4}{5760\pi^4} - \frac{19\langle g^2 G G \rangle m_s(s)}{1440\pi^2} s + \frac{17\langle g^2 G G \rangle m_s(s)}{1728\pi^2} s.
\]

IV. SINGLY-CHANNEL ANALYSIS

As the first step, we neglect the off-diagonal correlation functions by setting $\rho_{ij}(s)|_{i \neq j} = 0$, and perform the single-channel analysis. This condition means that the three currents $\eta_{i,j}^{1,2,3}$ are “non-correlated”, and any two of them can not mainly couple to the same state $X$, otherwise,

\[
\rho_{ij}(s) \equiv \sum_n \delta(s - M_n^2)\langle 0|\eta_i|n\rangle \langle n|\eta_j^\dagger|0 \rangle + \cdots
\]

\[
\approx \delta(s - M_X^2)\langle 0|\eta_i|X \rangle \langle X|\eta_j^\dagger|0 \rangle + \cdots
\]

\[
\not= 0.
\]

Accordingly, we assume that there are three states $X_{1,2,3}$ corresponding to the three currents $\eta_{i,j}^{1,2,3}$ through

\[
\langle 0|\eta_{i,j}^\dagger|X_{i,j}\rangle = f_{ii}t_{i}c_{i,j}c_{i,j} .
\]

After parameterizing the spectral density $\rho_{ij}(s)$ as one pole dominance for the state $X_i$ together with a continuum, 
um contribution, Eq. (22) is simplified to be
\[
\Pi_i(s_0, M_B^2) = \int_{s<s_0} e^{-M_i^2/M_B^2} \rho_i(s) ds,
\]
which can be used to calculate \( M_i \) through
\[
M_i^2(s_0, M_B) = \frac{\int_{s<s_0} e^{-s/M_B^2} \rho_i(s) ds}{\int_{s<s_0} e^{-s/M_B^2} \rho_i(s) ds}.
\]  \( \text{(26)} \)

We use the spectral density \( \rho_1(s) \) extracted from the current \( \eta_{\alpha_1,\alpha_2,\alpha_3,\alpha_4} \) as an example to perform the numerical analysis. We take the following values for various QCD parameters [1, 81–87]:
\[
m_s(2 \text{ GeV}) = 93^{+11}_{-10} \text{ MeV},
\]
\[
\langle \bar{s}s \rangle = -(0.8 \pm 0.1) \times (0.240 \text{ GeV})^3,
\]
\[
\langle g_s s G_s \rangle = -M_0^2 \times (\langle \bar{s}s \rangle),
\]
\[
M_0^2 = (0.8 \pm 0.2) \text{ GeV}^2.
\]  \( \text{(27)} \)

As shown in Eq. (26), the mass \( M_1 \) of the state \( X_1 \) depends on two free parameters, the Borel mass \( M_B \) and the threshold value \( s_0 \). We investigate three aspects to find their proper working regions: a) the convergence of OPE, b) the sufficient amount of the pole contribution, and c) the mass dependence on these two parameters.

![FIG. 1: CVG_{12} (short-dashed curve, defined in Eq. (28)), CVG_{10} (middle-dashed curve, defined in Eq. (29)), CVG_{8} (long-dashed curve, defined in Eq. (30), and PC (solid curve, defined in Eq. (31)) with respect to the Borel mass \( M_B \). These curves are obtained using the current \( \eta_{\alpha_1,\alpha_2,\alpha_3,\alpha_4} \) when setting \( s_0 = 16.0 \text{ GeV}^2 \).

Firstly, we investigate the convergence of OPE, which is the cornerstone for a reliable QCD sum rule analysis. We require the \( D = 12 \) terms (CVG_{12}) to be less than 5\%, the \( D = 10 \) terms (CVG_{10}) to be less than 10\%, and the \( D = 8 \) terms (CVG_{8}) to be less than 20\%:
\[
\text{CVG}_{12} = \frac{\Pi_{11}^{D=12}(\infty, M_B^2)}{\Pi_{11}(\infty, M_B^2)} < 5\%,
\]  \( \text{(28)} \)

\[\text{CVG}_{10} = \frac{\Pi_{11}^{D=10}(\infty, M_B^2)}{\Pi_{11}(\infty, M_B^2)} < 10\%, \]
\[\text{CVG}_{8} = \frac{\Pi_{11}^{D=8}(\infty, M_B^2)}{\Pi_{11}(\infty, M_B^2)} < 20\%. \]  \( \text{(29)} \)
\( \text{(30)} \)

As shown in Fig. 1, the lower bound of the Borel mass is determined to be \( M_B^2 > 2.40 \text{ GeV}^2 \).

Secondly, we investigate the one-pole-dominance assumption by requiring the pole contribution (PC) to be larger than 40\%:
\[\text{PC} = \frac{\Pi_{11}(s_0, M_B^2)}{\Pi_{11}(\infty, M_B^2)} > 40\%. \]  \( \text{(31)} \)

As shown in Fig. 1, the upper bound of the Borel mass is determined to be \( M_B^2 < 2.65 \text{ GeV}^2 \) when setting \( s_0 = 16.0 \text{ GeV}^2 \). Altogether the Borel window is determined to be \( 2.40 \text{ GeV}^2 < M_B^2 < 2.65 \text{ GeV}^2 \) for \( s_0 = 16.0 \text{ GeV}^2 \). Redoing the same procedures, we find that there are non-vanishing Borel windows for \( s_0 > s_0^{\text{min}} = 14.6 \text{ GeV}^2 \). Accordingly, we choose \( s_0 \) to be slightly larger, and determine its working region to be \( 13.0 \text{ GeV}^2 < s_0 < 19.0 \text{ GeV}^2 \).

Thirdly, we show the mass \( M_1 \) in Fig. 2, and investigate its dependence on \( M_B \) and \( s_0 \). It is stable against \( M_B \) inside the Borel window 2.40 \text{ GeV}^2 < M_B^2 < 2.65 \text{ GeV}^2, \) and its dependence on \( s_0 \) is moderate inside the working region 13.0 \text{ GeV}^2 < s_0 < 19.0 \text{ GeV}^2, \) where the mass is calculated to be
\[M_1 = 3.50^{+0.21}_{-0.25} \text{ GeV}. \]  \( \text{(32)} \)

Its uncertainty is due to \( s_0 \) and \( M_B \) as well as various QCD parameters given in Eqs. (27).

We repeat the same procedures to study the other two currents \( \eta_{\alpha_1,\alpha_2,\alpha_3,\alpha_4} \) and \( \eta_{\alpha_1,\alpha_2,\alpha_3,\alpha_4} \). The obtained results are summarized in Table I.

V. MULTI-CHANNEL ANALYSIS

In this section we take into account the off-diagonal correlation functions, and perform the multi-channel analysis. As listed in Eqs. (16–21), the off-diagonal spectral densities are actually non-zero, i.e., \( \rho_{ij}(s)|_{i \neq j} \neq 0 \). It is interesting to see how large they are, so we choose \( s_0 = 11.0 \text{ GeV}^2 \) and \( M_B^2 = 1.85 \text{ GeV}^2 \) to obtain:
\[\Pi_{ij}(s_0, M_B^2) = \begin{pmatrix} 2.77 \, 0.04 & -3.83 \\ -0.04 \, 0.98 & 0.46 \\ -3.83 \, 0.46 & 2.38 \end{pmatrix} \times 10^{-6} \text{ GeV}^{14}. \]  \( \text{(33)} \)

Hence, the off-diagonal terms of \( \rho_{ij}(s) \) are non-negligible.

To diagonalize the \( 3 \times 3 \) matrix \( \rho_{ij}(s) \), we construct three mixing currents \( J_{ij}^{1,2,3} \):
\[
\begin{pmatrix} J_x^1 \\ J_x^2 \\ J_x^3 \end{pmatrix} = T_{3 \times 3} \begin{pmatrix} \eta_1^1 \\ \eta_1^2 \\ \eta_1^3 \end{pmatrix}, \]  \( \text{(34)} \)
with $T_{3\times 3}$ the transition matrix.

We apply the method of operator product expansion to extract the spectral densities $\rho_{ij}'(s)$ from the mixing currents $J_{\alpha_1\alpha_2\alpha_3\alpha_4}^{1,2,3}$. After choosing

$$T_{3\times 3} = \begin{pmatrix} 0.72 & -0.06 & -0.69 \\ 0.68 & -0.13 & 0.72 \\ 0.14 & 0.99 & 0.05 \end{pmatrix},$$

we obtain:

$$\Pi_{ij}'(s_0,M_B^2) = \begin{pmatrix} 6.43 & 0 & 0 \\ 0 & -1.30 & 0 \\ 0 & 0 & 1.00 \end{pmatrix} \times 10^{-6} \text{GeV}^{14},$$

at $s_0 = 11.0 \text{ GeV}^2$ and $M_B^2 = 1.85 \text{ GeV}^2$. Hence, the off-diagonal terms of $\rho_{ij}'(s)$ are negligible around here, suggesting that the three mixing currents $J_{\alpha_1\alpha_2\alpha_3\alpha_4}^{1,2,3}$ are nearly non-correlated around here.

Consequently, we use the procedures previously applied on the single currents to study these mixing currents, and the obtained results are also summarized in Table I. Especially, the mass extracted from the current $J_{\alpha_1\alpha_2\alpha_3\alpha_4}^1$ is significantly reduced to be

$$M_1' = 2.85^{+0.19}_{-0.22} \text{ GeV}.$$  \hfill (37)

We show it in Fig. 3 with respect to $M_B$ and $s_0$.

**VI. SUMMARY AND DISCUSSIONS**

In this paper we apply the method of QCD sum rules to study the fully-strange tetraquark states with the exotic quantum number $J^{PC} = 4^+-$.

We work within the diquark-antidiquark picture and systematically construct their interpolating currents. We calculate both the diagonal and off-diagonal correlation functions. The obtained results are used to construct three mixing currents that are nearly non-correlated. We use the mixing current $J_{\alpha_1\alpha_2\alpha_3\alpha_4}^1$ to extract the mass of the lowest-lying state to be $2.85^{+0.19}_{-0.22} \text{ GeV}$.

In this paper we also systematically construct the fully-strange meson-meson currents, and relate them to the diquark-antidiquark currents through the Fierz rearrangement. Especially, we apply Eq. (34) and Eq. (10) to transform the mixing current $J_{\alpha_1\alpha_2\alpha_3\alpha_4}^1$ to be

$$J_{1..}^1 = 4.3 \xi_1 + 1.2 \xi_2 - 1.3 \xi_3. \hfill (38)$$

This Fierz identity suggests that the lowest-lying state dominantly decays into the $P$-wave $\phi(1020) f_2^0(1525)$ channel through the meson-meson current $\xi_{\alpha_1\alpha_2\alpha_3\alpha_4}$. Accordingly, we propose to search for it in the $X \rightarrow \phi(1020) f_2^0(1525) \rightarrow \phi K K$ decay process in the future Belle-II, BESIII, COMPASS, and GlueX experiments.

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TABLE I: QCD sum rule results of the fully-strange tetraquark states with the exotic quantum number $J^{PC} = 4^{+-}$, extracted from the diquark-antidiquark interpolating currents $\eta^{1,2,3}_{a_1a_2a_3a_4}$ and their mixing currents $J^{1,2,3}_{a_1a_2a_3a_4}$.

| Currents | $s_0^{\text{min}}$ [GeV$^2$] | Working Regions | Pole [%] | Mass [GeV] |
|----------|-----------------|-----------------|--------|-----------|
| $\eta_{a_1a_2a_3a_4}$ | 14.6 | 2.40-2.65 | 16 ± 3.0 | 40-50 | 3.50±0.21-0.25 |
| $\eta_{a_1a_2a_3a_4}$ | 19.2 | 2.80-3.13 | 21 ± 4.0 | 40-51 | 4.08±0.26-0.31 |
| $\eta_{a_1a_2a_3a_4}$ | 11.0 | 1.25-1.65 | 12 ± 2.0 | 40-58 | 3.34±0.39-0.18 |
| $J^1_{a_1a_2a_3a_4}$ | 10.1 | 1.78-1.92 | 11 ± 2.0 | 40-48 | 2.85±0.19-0.22 |
| $J^2_{a_1a_2a_3a_4}$ | - | - | - | - | - |
| $J^3_{a_1a_2a_3a_4}$ | 19.1 | 2.79-3.14 | 21 ± 4.0 | 40-51 | 4.08±0.26-0.31 |

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