An Online Algorithm for Generating Fractal Hash Chains
Applied to Digital Chains of Custody*

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February 1, 2008

Abstract

This paper gives an online algorithm for generating Jakobsson’s fractal hash chains [14]. Our new algorithm compliments Jakobsson’s fractal hash chain algorithm for preimage traversal since his algorithm assumes the entire hash chain is precomputed and a particular list of \( \lceil \log n \rceil \) hash elements or pebbles are saved. Our online algorithm for hash chain traversal incrementally generates a hash chain of \( n \) hash elements without knowledge of \( n \) before it starts. For any \( n \), our algorithm stores only the \( \lceil \log n \rceil \) pebbles which are precisely the inputs for Jakobsson’s amortized hash chain preimage traversal algorithm. This compact representation is useful to generate, traverse, and store a number of large digital hash chains on a small and constrained device. We also give an application using both Jakobsson’s and our new algorithm applied to digital chains of custody for validating dynamically changing forensics data.

Keywords: Fractal hash chains, hash chain preimage traversal, hash chain traversal, digital chains of custody, digital forensics, time-stamping.

1 Introduction

This paper proposes a digital forensics system for proactively capturing time-sensitive digital forensics evidence. Intuitively, we are trying to build fully digital chains of custody for digital evidence that provide forensics investigators with opportunities to enhance their data’s integrity. Perhaps this system is best applied to monitor suspects that have already been identified. In some cases, it is important to capture and validate dynamic evidentiary data. This may range from tracking a very fast virus infection to tracking more slow changes directly made by human touch. Even in monitoring slow changes to a file, capturing and validating an inappropriate email—as it is generated—is far more convincing than just getting a snapshot of it retrospectively.

Generally, there is a web-of-trust for validating classical forensic evidence. This web may include both witness and expert testimony as well as logical deduction given basic facts about a situation. The evidence in this web is held together by a chain of custody. A chain of custody is careful documentation of the evidence including details of all transfers of its possession for examination. A chain of custody is used to authenticate evidentiary exhibits as well as to verify these exhibits have not been modified.

*An extended abstract of this paper is to appear in the *Intelligence and Security Informatics 2007 (ISI 2007)* Conference.
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Our proposed system also initially depends on a web-of-trust. In particular, we must trust the system administrators or even law enforcement for initiating our system under proper circumstances. This may be fleshed out in the usual ways using witnesses, experts, and logical deduction given facts of the case. We maintain a full hash chain of the dynamic evidence for the following reasons: (1) to document and synchronize the dynamic and time-specific nature of change in a system, (2) to allow repeated verifications of the data under scrutiny, and (3) in the cases where the hash elements include diverse swaths of compressed data may give extensive opportunities for validation using webs-of-trust.

Our system is based on carefully timed hash chains on small constrained tamper-resistant devices. These timed hash chains are constructed with compressed snapshots of evidence while using (suspected) one-way hash functions. Generally, the amount of data or the length of time for data capture is not known in advance for proactively monitoring forensics data. Thus, the constrained nature of the hardware, variable granularity of the data, and unknown: time and number of hash chains makes it likely that these devices cannot hold sufficient complete hash chains. Likewise, such devices cannot hold a large set of time-stamps with compressed digital finger-prints such as hashes.

It is standard forensics practice to use at least three well-known hash functions to verify data integrity in case one or more of these functions is ever compromised [5, 17]. Such integrity checks may be used for verifying a disk’s data. In our case, they may also be used to validate dynamic changes in a critical file. Alternatively, hash chains use deferred disclosure to establish time-synchronized authenticity. A hash chain can chronologically document evidence by generating a timed hash chain forward while including diverse data of interest in the hash inputs. Such evidence may be verified by traversing the hash chain backwards while supplying the correct inputs at the proper times. This may include data that may be logically deduced about data captured in the broader system itself. The backwards hash chain traversal allows repeated verification by only going back in the hash chain as far as appropriate—leaving the other concealed hash elements in the chain for subsequent deferred verification. Data may be verified by both sides in a trial, in addition to the forensic investigator. To preserve data integrity, each time data is verified, each constrained device will release another hash element generated prior to all already verified hash elements. This deferred disclosure validates the known elements in the hash chain and it may tie in to different webs-of-trust.

Another approach is to have each device hold the first and last hash element of a hash chain. Generally, a forensics verifier would start at the beginning of the hash chain and present the appropriate data to verify the entire hash chain or a large subset of the hash chain. However, such large subsets of the hash chain should include either the first or last element of the hash chain for validity and verification.

Assuming the investigator or system administrator that initiated the hash chain-based data capture is trustworthy. The data then is in the domain of the evidence clerk maintaining the chain of custody. Subsequently, the hash chain evidence may be called in to question: is this the correct data? The first challenge to this approach is if the start (or an early element) of the hash chain is discovered by an attacker, then another fictitious constrained device may be created that ‘verifies’ incorrect or modified data. In evidence storage, replacing a gun with another with different serial number or bullet grooves is hard not to miss. That is, given the first element of a trusted hash chain gives an attacker an opportunity to generate and fake the rest of the hash chain, except for the last element. The last hash element may be at the center of the contention. If there is a single end element of the hash chain that disagrees, whose do we trust? Thus, we have three hash chains that only reveal their elements on-demand by deferred validation: the trusted investigator or system administrator can validate the first or early hash
elements. In fact, ideally there would be enough hash elements that they would not have to reveal the last hash element. This situation is reminiscent of the motivation for Schneier and Kelsey [24]'s audit logs.

Another possibility is to determine the first time step that verifies critical data. However, authenticating the first critical data item requires the prior hash element. If it is useful to validate this 'prior hash element,' we must traverse the hash chain back two hash elements from the critical data, etc. This is the second challenge to storing the first and last hash elements of a hash chain on a constrained device: repeatedly validating critical data each time the data is examined through deferred disclosure requires us to traverse the hash chain backwards.

Computer forensics strives to understand the relationships between suspects and events so they can be verified by third parties such as jurors in a court of law. In such situations it is critical to establish strong credence of the validity of digital forensics evidence. Digital evidence is abstract, ephemeral, time-sensitive, compact, complex, and often encoded. In some cases, biological evidence has measurable decay characteristics that allow chronologic analysis [20]. Digital evidence does not have such measurable decay characteristics. Also, digital evidence is easily copied and copies may be readily manipulated to challenge valid evidence and diminish its credibility.

The digital forensics system proposed here is applied to maintaining timed digital evidence. Figure 1 illustrates a central application addressed in this paper. This challenge is the interval time-stamping problem [1, 11, 18, 28]. Consider the left-to-right time line containing the time interval $[t_0, t_1]$. A central challenge is to demonstrate that the evidence $E$ was in a particular user’s possession between times $t_0$ and $t_1$. The value $t_1 - t_0$ may be very small. It is well known that when using public key systems, along with prominent and well distributed data, it is easy to show $E$ existed after time $t_1$. However, how does one show $E$ existed at or before time $t_0$? One approach is to divide the interval $[t_0 - \frac{t_1-t_0}{k}, t_1]$ into $k+1$ smaller intervals each of size $\frac{t_1-t_0}{k}$. In each smaller interval $[t_{0,i}, t_{1,i}]$ it may be shown that $E$ existed after time $t_{1,i}$, for each $i : k > i \geq 1$ where $t_{1,0} = t_1$. Thus, we can show $E$ existed before time $t_{1,k} = t_0$.

Throughout this paper, all logs are base 2.

1.1 Our Contributions

This paper proposes constrained devices for securing and validating time-sensitive (and dynamic) forensic data. It assumes tamper-resistant hardware, which may be viewed as a depen-
dency on a trusted third party. A special online algorithm is given here to prevent a modify-
and-copy attack. This online algorithm allows computer forensics specialists to maintain the
verifiability of timed digital evidence.

Take an easy to compute hash function $h$. Assume $h$ is one-way \[10\]: So, on average it is
intractable to invert $h$. That is, given $y$ where $y \leftarrow h(x)$ it is on average intractable to find $x$
given $y$. Given a hash function $h$ where $x \leftarrow h(y)$, then $y$ is the preimage of $x$.

Next is a $n + 1$ element hash chain:

$$v_0 \xleftarrow{} h(v_1) \xleftarrow{} v_1 \xleftarrow{} h(v_2) \xleftarrow{} v_2 \xleftarrow{} h(v_3) \xleftarrow{} v_3 \cdots \xleftarrow{} h(v_{n-1}) \xleftarrow{} v_{n-1} \xleftarrow{} h(v_n) \xleftarrow{} v_n.$$ (1)

Since the hash function is one-way, this hash chain must be initially generated from right
to left.

**Definition 1** Consider a hash chain as described by Equation [1]. The value $v_n$ is the seed of
the hash chain.

*Generating the hash values forward in this order $v_n, v_{n-1}, \cdots, v_0$ is hash chain traversal.*

*Computing hash elements backward in this order $v_0, v_1, \cdots, v_n$ is preimage traversal.*

For efficient hash chain traversal see [13]. We always assume the hash function is well known.
Using deferred disclosure of hash elements backwards validates prior knowledge of elements in
the hash chain. In time step 0, given $v_0$, then waiting to time step 1 one can verify that
$v_0 = h(v_1)$ indicating with high probability $v_0$ and $v_1$ are from the same source.

At time step $i$, our scheme inputs a chunk of digital evidence $E_i$. Let $c(E_i)$ be the compressed
version of $E_i$. Say $c(E_i)$ contains a modest number of fixed bits, for instance 160 bits [26 27].
For example, for the function $c$ we can use a technique such as Merkle-Damgård construction
of a collision-resistant compression function. Then $v_i \oplus c(E_i)$ is the input to the hash function
$h$ giving:

$$v_{i-1} \leftarrow h(v_i \oplus c(E_i))$$

and this process is continued in a carefully timed fashion to give an entire hash chain. The
function ‘$\oplus$’ may be either xor or concatenation.

If a hash chain is completely exposed an adversary has access to all elements of

$$C_1 = v_n, v_{n-1}, v_{n-2}, \cdots, v_0$$

then this hash chain’s relative and carefully timed deferred disclosure based authenticity may
be easily challenged. For instance, an attacker may take $v_{n-k}$ and falsify the input $E_{n-k}$ by
changing it to $E'_{n-k}$, then generate the incorrect hash chain:

$$C_2 = v_n, \cdots, v_{n-k}, v'_{n-k-1}, \cdots, v'_0$$

where $v'_{n-k-i} \neq v_{n-k-i}$, for all $i : n - k \geq i \geq 0$.

Now, without proofs of identity or authenticity, which chain $C_1$ or $C_2$ represents the
authenticated data is not clear. Of course, proofs of identity or authenticity are only as good as
the systems checking them.
Given a hash chain with hash seed $v_n$ and the last hash element $v_0$. Suppose we only save $v_n$ and $v_0$ on a small constrained device to validate the data from $n$ different times

$$E_{n-1}, \ldots, E_0.$$  

If $E_i$ is the first piece of critical data in hash element $v_i$, then to validate $v_i$, we must know $v_{i+1}$. But, the next time the same hash chain is validated, we take $E_{i+1}$ and validate it with $v_{i+2}$. As these validation steps are repeated each time investigators weigh the evidence, we will be performing a (slow and deliberate) preimage traversal of the hash chain.

The hash chain forward traversal algorithm given in this paper extends Jakobsson’s work by augmenting his hash chain preimage backward traversal algorithm [14]. See also, [7]. Our algorithm uses hash chains to validate forensics evidence by generating the hash chain elements forward

$$v_n, v_{n-1}, \ldots, v_0$$

online but only storing $\lceil \log_2 n \rceil$ hash elements for a hash chain of $n$ total hash elements. The preimages of such a hash chain may then be output using Jakobsson’s algorithm and validated by an evidence clerk. To best ensure the evidence’s credence, it is best for the clerk to only compute the backward preimage traversal as far as necessary. Then, additional deferred disclosures may be performed as needed to re-authenticate the validity of the exposed hash elements.

A difficulty is in validating a hash chain by backward preimage traversal in memory times computational complexity of less than $O(n)$ where $n$ is the number of hashes required, see for example [14, 7, 8, 25, 19].

This paper gives an online algorithm for generating a forward hash chain traversal while always storing $\lceil \log_2 n \rceil$ hash elements or pebbles, where $n$ hash elements have been generated so far. This is important on memory constrained devices. For any $n$, the $\lceil \log n \rceil$ pebbles can be directly plugged into Jakobsson’s algorithm to start backward preimage traversal for verification.

If $n$ is the number of hash elements stored, other recent methods applicable to emitting hash elements, double the size to $2n$ to store a single additional element—the $(n+1)$-st element. This is not acceptable here for several reasons: (1) our approach depends on precisely timed hash element generation and generating $n$ more hash elements may cause a simple and constrained device to miss data collection; and (2) the constrained devices may not have the storage to hold $n$ additional hash elements.

1.2 Previous Work

Secure audit logs applied to digital forensics were developed by Schneier and Kelsey [24]. This work assumes three machines: a trusted secure server $T$, an small machine who secure state is untrusted $U$ for keeping audit logs and a sometimes trusted verifier $V$. The audit log machine $U$ is a small constrained machine that is only occasionally securely connected to $T$. A machine $U$ is trusted until it is compromised. If it is compromised, then it cannot change its audit log or read audit elements before the compromise occurred. This system uses a hash chain to secure the audit logs on the untrusted machines. But, each untrusted machine deletes all but the last element in the hash chain as the audit log grows. Only $T$ has the seed of hash chain for verification. Our online algorithm along with Jakobsson’s can be applied to variations of Schneier and Kelsey’s audit log system. This would save space and potentially allow numerous
hash chains to serve as audit logs. They close their audit log with a ‘normalCloseMessage’ to prevent an adversary from extending it illicitly.

To our knowledge, all hash chain preimage traversal work to date assumes either (1) the hash chain is computed in advance, or (2) the length of the hash chain is known or a bound on the length is known in advance, or (3) given $2^t$ hash elements, the hash chain may be doubled in size to $2^{t+1}$ hash elements when the $(2^t + 1)$-st element is needed, for $t$ a positive integer.

The first case does not seem directly applicable to our digital forensics scheme. In the last two of these cases, there may be a good deal of excess unused memory.

1.2.1 Time-Stamping

Time-stamping is a significant area of research. Haber and Stornetta [11] gave time-stamping schemes using both hash chains and digital signatures. This is similar to the hash chain scheme used in this paper. The first method they give is a linking scheme using hash chains to maintain temporal integrity. All of their schemes depend on a trusted third party. The trusted third party provides a time-stamping service which applies a hash function and digitally signs it. Other related schemes and improvements were given in this paper as well as Bayer, Haber, Stornetta [2] and Haber and Stornetta [12]. Buldas, Laud, Lipmaa and Villemson [4] focus on ‘relative temporal authentication’ and give both time-stamping requirements and new algorithms suited to these requirements. Their methods require a trusted third party to provide time-stamping services. Ansper, Buldas, Saarepera, and Villemson [1] discuss linkage based protocols (i.e., hash chain based protocols) compared to hash-and-sign time-stamping protocols. Building on work of Willemson [28], Lipmaa [18] gives efficient algorithms to traverse skewed trees. Our paper uses basic time-stamping by hash chains. Though the focus is on constrained devices and digital forensics.

1.2.2 Hash Chain Traversal

Our forward hash chain traversal is based on Jakobsson’s backward hash chain preimage traversal work, see also Coppersmith and Jakobsson [7]. Jakbosson gives an asymptotically optimal algorithm to compute consecutive preimages of hash chains. His algorithm requires $\log n$ storage and $\log n$ hash evaluations per hash element output. This is assuming $O(n)$ preprocessing was used to build the hash chain of $n$ elements.

Coppersmith and Jakobsson [7] give an algorithm with amortized time-space product cost of about $\frac{1}{2}\log^2 n$ per hash chain element. This is also assuming $O(n)$ preprocessing was used to build the hash chain of $n$ elements. They also give the following lower bound: Computing any element of the hash chain in the worst-case requires a time-space trade-off $T S \geq \frac{\log^2 n}{T}$, where $T$ is the number of invocations of the hash function and $S$ is the number of stored hash elements.

Sella [25] gives a general solution that applies $k$ hash function evaluations to generate any element in a hash chain, while storing $(k-1)n^{1/(k-1)}$ total hash elements. His algorithm initially stores hash elements that are at constant intervals of distance $(k-1)n^{1/(k-1)}$. Kim [15, 16] gives algorithms that improve Sella’s in saving up to half of the space Sella’s algorithms use while keeping the same parametric space and hash evaluation costs. This means Kim’s algorithm uses at most the same space as Jakobsson’s algorithm.

Ben-Amram and Petersen [3] give an algorithm for backing up in a singly linked list of length $n$ in $O(n^\epsilon)$ time, for any $\epsilon > 0$. This requires $O(n)$ pre-processing of the linked list.

Matias and Porat [19] give list and graph traversal data structures that allow efficient back-
ward list traversal and very efficient forward traversal. Their ‘skeleton’ data structures allow complete back traversals in \(O(kn^{1/k})\) amortized time given \(k\) elements in storage. Thus, storing \(k = O(\log n)\) elements, their algorithm requires \(O(\log i)\) amortized element evaluations for the \(i\)-th backward traversal, where \(i \leq n\). They bring out several interesting applications beyond hash chains. If their data structures are built for lists of \(n\) elements, then to accommodate \(n + 1\) elements they build a new data structure (list synopsis) for \(2n + 1\) total elements, see subsection 4.3 of the full version of [19].

1.3 Structure of this Paper

In the remainder of the paper we develop the ideas behind our proposed approach. Section 2 gives our model. Section 3 reviews Jakobsson’s algorithm in detail. We give proofs of correctness for this algorithm in the appendix. In order to prove the correctness of our algorithm we found it necessary to supply detailed proofs of Jakobsson’s algorithm (which were not given directly in Jakobsson’s paper). Section 4 introduces the specifics of our online algorithm, discusses how it interfaces with Jakobsson’s amortized algorithm, and gives a proof of correctness. In Section 5 we give some conclusions.

2 Chains of Custody: Physical and Digital

Next is a formal definition of a chain of custody.

Definition 2 A chain of custody is a detailed account documenting the handling and access to evidence.

We quote Colquitt [6, Page 484] on the purpose of a chain of custody:

“The purpose, then, of establishing a chain of custody is to satisfy the court that it is reasonably probable that the exhibit is authentic and that no one has altered or tampered with the proffered physical exhibit.”

A chain of custody, as described in Definition 2, may sometimes be referred to as a classic chain of custody. Maintaining a chain of custody is a standard practice investigators use to inextricably link the evidence that ties a crime to the suspects.

Digital evidence is often stored using a classical chain of custody. For example, documenting when a particular individual first picked up a disk drive with critical evidence, the state of the disk drive, to whom and when they transferred the drive, etc.

Digital evidence is extraordinarily easy to copy. Using standard techniques, each copy of digital evidence is easily authenticated. Physical evidence is somewhat different [11] [20]. Consider a crime committed by using a gun. Manufacturing a new .357 Magnum Ruger Blackhawk Flattop revolver with an identical ‘look,’ serial number, and bullet grooves, to one used in a crime is extraordinary work. Just finding experts to replicate such physical evidence would generally leave a substantial paper trail. Moreover, biological evidence generally also provides time frames. Thus, classic chains of custody often focus on the basic identification, authentication, uniqueness, and time. Digital data lacks such uniqueness and timing characteristics. That is, many copies may be made of digital evidence both for legitimate and illicit reasons. Furthermore, timestamps alone may not be sufficiently convincing.
Definition 3 A digital chain of custody is the information preserved about the data and its changes that shows specific data was in a particular state at a given date and time.

A DCoC (Digital Chain of Custody) is a small constrained device for holding, authenticating, and verifying a digital chain of custody.

Take a one-way hash function $h$. Suppose $h$ has deterministic and tightly bounded time complexity. There are some hash functions with bounded time complexity that are already in use: the RSA SecurID [23] as well as systems like TESLA [21, 22] depend on timed hash functions. This paper goes one step further advocating extremely precisely timed hash functions.

This paper is focused on several aspects of evidence maintenance that are related to time stamping, see for example [8]. One side of the challenge is to show a piece of evidence existed after a particular point in time. Using the Merkle-Damgård technique, the evidence may be combined with non-predictable, widely distributed, and time sensitive documents such as a newspaper or financial market data.

Similar discussions to the next definition can be found in several places, for example [11, 27].

Definition 4 Data that is non-predictable, widely distributed, verifiably stored, and time sensitive is socially bound data.

A critical issue is that socially bound and highly granular data are not common. In certain situations, socially bound data must be highly granular, for example in milliseconds. Thus, carefully timed hash chains may be used as a proxy for socially bound data. See also, timestamp linkages in [11].

2.0.1 The Adversary

The adversary this paper assumes is either (1) an untrustworthy verifier; or (2) a general attack against known hash functions on a DCoC.

Suppose our system computes an $n$ element hash chain $v_n, \ldots, v_0$, and say $n > t$. An untrustworthy verifier may get an element of a hash chain $v_t$ along with the files of digital evidence $E = E_t, \ldots, E_0$. The adversary may illicitly modify the evidence to $E' = E'_t, \ldots, E'_0$ and compute a ‘competing’ hash chain using $v_t$ and $E'$. Thus, adding doubt to the validity of the real evidence. Provided $t < n$, and the one-way hash function $h$ is not broken, we can validate the hash chain based on $E$ and not $E'$ by producing $v_{t+1}$ and showing that $h(v_{t+1} \oplus c(E_{t+1})) = v_t$.

The issue of a general breach of a hash function is primarily dealt with by following the forensics policy of having at least three different known hash functions for the data. This procedure is to ensure trust in the hash functions in case one of them is no longer trustworthy [5].

2.1 Overview of Our Approach

This paper assumes a model consisting of at least three constrained devices (DCoCs or cards) each having a different hash function. These constrained devices may be interfaced using a USB 2.0, for instance. In any case, these devices have a small processor as well as limited memory. They are tamper resistant.

*As financial markets become more automated and distributed, it may be feasible for the trading volume to be granulated down to minute fractions of a second. For example, in 2004 the average NYSE trading day volume per second was about $1,500,000,000/(60 \cdot 60 \cdot 6.5)$ or more than 64,000 shares per second. See www.nyse.com. The volume is known down to the individual share. This highly granular data may be widely distributed.
2.1.1 Data Capture

The DCoCs may each interface through separate USB 2.0 ports simultaneously to gather common snapshots of critical system states and memory. Each DCoC may be physically secured and transported independently by a different member of law enforcement. We assume these devices will be mass produced so that each DCoC has unique authentication software. (They should also have unique physical identification.) Different members of law enforcement should handle each card making it less likely that all cards may be manipulated by a single member of law enforcement. Furthermore, from physical identification, the DCoCs may become part of the classical chains of custody.

Finally, each of the DCoCs may periodically authenticate each other while capturing data. Records of these authentications can be included in the hashed values. This authentication should be zero-knowledge interactive proofs of identity such as the Feige-Fiat-Shamir protocol \cite{Feige1987}. Such a proof of identity is important in this application domain since these proofs of identity are not transitive. Thus, a fake DCoC cannot impersonate a real one by just mimicking its proof of identity.

2.1.2 Forensic Data Verification

The plaintext evidence and diversifying data has been stored as the files $M$ in plain sight on storage such as an optical disk. Different states or snapshots of the evidence will be periodically concatenated into timed inputs of the hash chain. Suppose the hash computations run at a fixed and known speed on tamper resistant hardware. Thus, computing a hash chain on this special hardware can be used to certify the initial data states at specified times, see also Haber and Stornetta \cite{Haber1991}.

The verification procedure is done by an evidence clerk and consists of the following steps:

1. Connect all three DCoCs to a secured verification machine. Initially, each DCoC is authenticated to all other DCoCs using a zero-knowledge interactive proof of identity.

2. The DCoCs only output hash elements backwards on an as-needed basis to verify the evidence under consideration.

3. As hash chain elements are output, the verification must continue with each hash element step. If a DCoC cannot authenticate another device, then it will alert the evidence clerk or even shut down.

4. The hash elements from a DCoC are sent out from left to right:

$$h^n(v_n \oplus c(E_n)), \ h^{n-1}(v_{n-1} \oplus c(E_{n-1})), \ \cdots, \ h^t(v_t \oplus c(E_t)),$$

for $n-1 \geq t$. Moreover, $h^n(v_n \oplus c(E_n))$ is verified by computing

$$h(h^{n-1}(v_{n-1} \oplus c(E_{n-1})) \oplus c(E_n)),$$

and so on.
2.1.3 Putting It All Together

Given the tamper resistant hardware, just storing data with associated time stamps may quickly overflow a constrained device—especially if the time granularity is very fine. Thus, compressed representation of hash chains can be used to verify time stamps along with the authenticity of other constrained devices.

It is possible to publicly post data from a system under forensic investigation by way of a proxy server. This data may be signed by a small device and posted to a public (or private and trusted) location. Small devices may have trouble signing large amounts of data due to their constraints. Furthermore, for very fine time granularity, fast and consistent network bandwidth may not be available. In some contexts, if data is captured before a formal investigation is initiated, then it may be important to keep the suspects unaware that data is being captured as potential evidence.

A DCoC may run its hash chain until a forensic examiner or law enforcement officer carefully checks it into the physical chain of custody while noting the time in detail. This allows a forensic examiner to determine the time of the evidence by counting the exact number of hashes. A key motivator of our online algorithm is in proactive forensics the value of \( n \) is not known in advance. Where \( n \) is the number of time-steps recorded by the DCoCs. In the case of a digital chain of custody, it can not be known precisely how long it will take to get potential evidence to an evidence clerk. Or how long an evidence clerk will take to validate the data, etc. This precise time as well as the interval between hashes, computed by our algorithm, will have to be known in advance to fake a hash chain.

The preprocessing phase of Jakobsson’s amortized algorithm must know the value of \( n \) in advance of when it is run \([14]\). Our new algorithm prepares the appropriate \( \lceil \log n \rceil \) pebbles for Jakobsson’s algorithm, for any \( n \). During hash element generation, for any \( n \) the online algorithm never stores more than \( \lceil \log n \rceil \) pebbles. Once the online algorithm has no more requests to generate new hash elements, then the amortized algorithm \([14]\) may be immediately started.

### 3 Jakobsson’s Algorithm

This section reviews Jakobsson’s Algorithm \([14]\). The proofs of correctness are in the Appendix. Some of these proofs are used in the results of Section 4.

#### 3.1 A Review of Jakobsson’s Algorithm

Given a hash chain with 16 elements, Jakobsson’s algorithm is initialized as indicated in Table 1. In this Table, hash element 16 is the seed of the hash chain. Where the preimages are exposed sequentially in the following order: 1, 2, · · · , 16. In general, the position of a hash element in a hash chain ranges from 1, · · · , \( n \) where hash element 1 is the first element to be exposed in a pre-image traversal. Thus, hash element \( n \) is the seed of the hash chain.

| Element Position | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 |
|------------------|---|---|---|---|---|---|---|---|---|----|----|----|----|----|----|----|
| Pebble Placement | × | × | × |   |   | × |   |   |   |    |    |    |    |    |    |    |

Table 1: Initial pebble locations for Jakobsson’s algorithm

Suppose \( n = |H| \) is the number of hash values in the entire hash chain. In this case,
Jakobsson’s amortized algorithm only stores $\lceil \log n \rceil$ hash pebbles.

Suppose $n = 2^k$ for some integer $k$. Now, Jakobsson’s algorithm [14] is given:

3.1.1 Jakobsson’s Setup.

Compute the entire hash chain: $v_0, v_1, \ldots, v_n$.

For pebble $p[j]$, where $j : k = \log_2 n \geq j \geq 1$:

\[
\begin{align*}
p[j].\text{Start} & \rightarrow 3 \times 2^i \\
p[j].\text{Dest} & \rightarrow 2^{i+1} \\
p[j].\text{Position} & \rightarrow 2^i \\
p[j].\text{Destination} & \rightarrow 2^i \\
p[j].\text{Value} & \rightarrow v_{2^j}
\end{align*}
\]

Furthermore,

\[
\begin{align*}
\text{Current.} & \text{Position} \rightarrow 0 \\
\text{Current.} & \text{Value} \rightarrow v_0
\end{align*}
\]

3.1.2 Jakobsson’s Main.

This algorithm in Figure 2 updates the pebbles and values.

Using Jakobsson’s amortized technique [14], a hash chain of size $n$ requires a total of $k$ pebbles where $k = \lceil \log n \rceil$. This amortized algorithm performs $O(\log n)$ hash applications per hash element output. Initially, in the setup phase, the pebbles store hash elements from hash-chain positions $2^1, 2^2, 2^3, \ldots, 2^k$, respectively.

4 Online Output of Jakobsson’s Pebbles for any $n$

Jakobsson’s amortized algorithm works to conserve both stored pebbles and hash evaluations. This allows his algorithm to verify hash values on small sensors. This algorithm assumes preprocessing where all hash elements are pre-computed, perhaps by a more powerful processor [14].

Our aim with the online algorithm is to have a small constrained device that generates all requested hash elements. The online algorithm broadens the applicability of Jakobsson’s amortized algorithm. In particular, the online algorithm generates all hash elements, but only stores $\lceil \log n \rceil$ pebbles at any one time. Where $n$ is the total number of hash element generated so far. Every time a new hash element is generated, no additional hash evaluations must be performed.

These $\lceil \log n \rceil$ hash pebbles are positioned so that at any point the online algorithm is no longer invoked, then Jakobsson’s amortized algorithm can start to be run directly on the pebbles. Thus, we believe the amortized and online algorithms are complimentary and fit well together.

In Jakobsson’s notation each hash element keeps its index throughout the computation,

$v_0, \ldots, v_n$. 
Given an entire precomputed computed hash chain: 
\( v_1, v_2, \ldots, v_n \), then compute the pebble positions & auxiliary information.

1. if Current.Position = n, then
   
   Stop

   else
   
   Current.Position ← 1 + Current.Position

endif

2. for \( j \leftarrow 1 \) to \( k \) do
   
   if \( p[j].Position \neq p[j].Destination \) then
   
   2.1 \( p[j].Position \leftarrow p[j].Position - 2 \)
   
   2.2 \( p[j].Value \leftarrow h(h(p[j].Value)) \)
   
   endif

endfor

3. if Current.Position is odd then
   
   output \( h(p[1].Value) \)

   else
   
   output \( p[1].Value \)

   3.1 \( p[1].Position \leftarrow p[1].Position + p[1].Start_Increment \)

   3.2 \( p[1].Destination \leftarrow p[1].Destination + p[1].Dest_Increment \)

   if \( p[1].Destination > n \) then
   
   \( p[1].Position \leftarrow +\infty \)

   \( p[1].Destination \leftarrow +\infty \)

   else
   
   \( p[1].Value \leftarrow \text{FindValue} \)

   endif

   Sort Pebbles by Position

endif

Figure 2: Jakobsson’s Hash-Chain Pebble Position update

This is because \( n \) is fixed, so all hash element numbers remain the same. The pebbles in Jakobsson’s algorithm are re-numbered, according to the initial fixed hash element numbers, and sorted by their positions.

The notation for the online algorithm uses hash element index notation that changes the index values over time.

For instance, in Jakobsson’s preimage traversal algorithm, suppose the initial \( n \) hash elements are:

\[ u_1, \ldots, u_n. \]

Then, after the first element is verified, it is discarded and the following hash elements remain:

\[ u_2, \ldots, u_n. \]

Alternatively, since \( n \) is not fixed in our online algorithm, suppose the following list of elements has already been generated (with only the appropriate pebbles saved),

\[ w_1, \ldots, w_n. \]
Figure 3: Jakobsson’s Algorithm: Each row from top to bottom represents the state of a hash chain traversed by Jakobsson’s algorithm. Each row represents a single hash element exposure. For each row, bold and underlined positions represent the location of pebbles where Position = Destination. Bold positions represent a temporary location for a pebble in transit only. When a pebble is reached and moved it is moved to the back of a grey area and moves to the front of the grey area or sequence of grey areas until it reaches its destination (and is underlined and in the next row).

Then, generating another hash value $w_0 \leftarrow h(w_1)$. This gives $n + 1$ total hash elements, thus for simplicity all hash indices are increased by 1. This gives the following renumbering,

$$w_1, \ldots, w_{n+1}.$$

Take $v_i$, then it corresponds to $w_j$, where

$$j = i - (2^\sigma - \text{totalHashElements}),$$

where $\sigma$ is the total number of pebbles and totalHashElements is the number of hash elements constructed thus far, also totalHashElements $\leq 2^\sigma$. Note, that when totalHashElements $= 2^\sigma$, then we verify $i = j$ by Observation [1] in the appendix.

**Fact 1** Jakobsson’s notation and our notation corresponds exactly when $n = 2^k$ for some integer $k \geq 0$.

The online algorithm has two distinct, non-interweavable phases: The growth phase, and the exposure phase.

### 4.1 The Growth Phase

Next is the growth phase.
4.1.1 The Online Algorithm’s Setup

\[
\begin{aligned}
\text{seed} & \leftarrow \text{the initial hash value} \\
\text{totalHashElements} & \leftarrow 1 \\
\text{grow.value} & \leftarrow \text{seed} \\
\text{grow.pebble} & \leftarrow 1 \\
\text{exponent} & \leftarrow 1
\end{aligned}
\]

4.1.2 The Online Algorithm’s Init Pebble

\[
\begin{aligned}
\text{InitializePebble}(p, j, \text{grow.value}) \\
p[j].\text{Move.Increment} & \leftarrow 2^{j+1} \\
p[j].\text{Start.Increment} & \leftarrow 3 \times 2^j \\
p[j].\text{Dest.Increment} & \leftarrow 2^{j+1} \\
p[j].\text{Position} & \leftarrow 2^j \\
p[j].\text{Destination} & \leftarrow 2^j \\
p[j].\text{Value} & \leftarrow \text{grow.value}
\end{aligned}
\]

Figure 5: Initialize Pebble

4.1.3 The Online Algorithm’s Main

Figure 6 updates the pebbles and values.

4.2 The Online Algorithm’s Exposure Phase

In this phase, only the setup is different from Jakobsson’s original algorithm, because we are no longer assuming that exactly \(2^j\) pebbles must be present.
while not done growing hash chain do
  grow.value ← h(grow.value)
  totalHashElements ← totalHashElements + 1
  if totalHashElements − 1 = 2^k for some integer k ≥ 1, then
    exponent ← exponent + 1
    for all pebbles l do
      p[l].Position ← p[l].Position + 2^{exponent−1}
    endfor
    j ← grow.pebble
    Create pebble p[j]
    InitializePebble(p, j, grow.value)
    p[j].distancefromSeed ← totalHashElements
    grow.pebble ← grow.pebble + 1
  else
    for all pebbles l do
      t ← p[l].Move_Increment + p[l].distancefromSeed + 1
    endfor
    if totalHashElements = t then
      3.1 p[l].Value ← grow.value
      3.2 p[l].distancefromSeed ← totalHashElements
      p[l].Position ← p[l].Position − p[l].Move_Increment
    endif
  endif
endwhile
j ← grow.pebble
4. Create pebble p[j] where j ← grow.pebble
   InitializePebble(p, j, seed)
   p[j].distancefromSeed ← 0
   Sort pebbles by Position

Figure 6: Hash Chain Growth Algorithm

4.2.1 The Online Algorithm’s Setup

For j ← 1 to ⌈log n⌉ do

    p[j].destination ← p[j].position

Furthermore, we must also initialize the next lines

    current.position ← totalHashElements
    current.value ← grow.value

The online algorithm’s Main function is performed exactly as Jakobsson’s algorithm with the exception that current.position cannot simply be set to zero during the setup phase, but rather must be determined from the value of the first pebble in the sorted order.
4.3 Characteristics of the Hash Chain Growth Algorithm

To prove the validity of the Hash Chain Growth Algorithm we will show: (1) at each step of the hash chain growth there are always enough pebbles, and (2) the pebbles are always properly placed such that Jakobsson’s algorithm can immediately begin to run on the stored data from the generated hash chain.

Recall, given \( n = 2^t \), for some integer \( t \), then pebbles are placed in hash element positions

\[
t = \log n, \quad t - 1 = \log(n/2), \quad t - 2 = \log(n/4), \quad \ldots, \quad t = 1.
\]

For any initial pebble \( p[j] \) the value \( p[j].\text{Destination} \) never decreases in Jakobsson’s algorithm. Thus, to determine on which move back the initial pebble \( p[j] \) will be output, compute when

\[
p[j].\text{Destination} > n
\]

first occurs.

Assume \( n \) is a power of 2, and the definition of \( D(j, k) \) can be found in the appendix, and given a fixed \( j \) solve for the minimal \( k \) so that

\[
2^j + k \cdot 2^{j+1} > 2^{\log n} \quad \frac{2k}{2} > 2^{\log n - j} - 1.
\]

For instance, for \( j = \log(n/2) \), then \( k = 1 \) gives

\[
2k = 2 > 2^{\log n - (\log n - 1)} - 1 > 2 - 1 = 1,
\]

which holds.

**Fact 2** Let \( n = 2^k \). In Jakobsson’s algorithm, pebbles are eliminated in the reverse order of their original indices \( j = k - 1, \ldots, 1 \) where we consider the seed pebble at \( j = k \) to never be discarded.

**Proof:** This proof follows from Jakobsson’s algorithm in Figure 2. By Fact 17 of the appendix, it must be that only the pebble in initial position \( j = k - 1 = \log(n/2) \) is going to be eliminated immediately after Jakobsson’s algorithm generates and emits \( n/2 \) hash elements.

Now, suppose the hash elements are renumbered where 1 represents hash element \( n/2 + 1 \); 2 represents hash element \( n/2 + 2 \), all the way to \( n/2 \) represents hash element \( n \). Still \( n = 2^k \).

By Fact 19 of the appendix, with this re-numbering, the remaining pebbles will be placed at pebble positions

\[
2^1, 2^2, \ldots, 2^{k-1},
\]

where \( n/2 = 2^{k-1} \). The pebble in position \( p[k - 1] \) is the seed of the hash chain, so it never moves. By Corollary 4 also of the appendix, for each of these remaining pebbles, \( p[j].\text{Position} = p[j].\text{Destination} \).

However, for any initial pebble \( p[j] \) the value \( p[j].\text{Destination} \) never decreases in the algorithm of Figure 2. Thus, we need to determine what initial pebble \( p[j] \) is placed in position
\[2^{k-2} = n/4.\] This new position is one backward move beyond the \(n/2\)nd emitted hash element. Thus, the Dest.Increment of this pebble must be \(2^{k-2} = n/4\), so the initial pebble was \(p_{[k-2]}\). Furthermore, by Fact 17, this pebble is the second pebble to be discarded.

The proof is completed by induction. 

The next facts follow from the online algorithm in Figure 6.

**Fact 3** Consider the online hash growth algorithm (see Figure 6). A new pebble is only created when totalHashElements is such that

\[2^k + 1 = \text{totalHashElements},\]

for some \(k \geq 1\).

**Proof:** This is a direct result of line 1 of the online hash growth algorithm.

Any time \(2^k = \text{totalHashElements} - 1\), then between now and when \(2^{k-1} = \text{totalHashElements} - 1\), all of the pebble’s positions must be shifted by \(2^{k-1}\). This is done in Figure 6 before a new pebble is created.

**Fact 4** Consider the online hash growth algorithm (see Figure 6). Each time an element is added to the chain, totalHashElements is incremented by one.

**Fact 5** Consider the online hash growth algorithm (see Figure 6). Each time a pebble is created it is initialized as,

\[p[j].\text{distancefromSeed} \leftarrow \text{totalHashElements}.\]

This follows directly from section 1 of Figure 6.

**Fact 6** Consider the online hash growth algorithm (see Figure 6). After its initialization, the only time \(p[j].\text{distancefromSeed}\) changes is when pebble \(j\) is moved and \(p[j].\text{distancefromSeed}\) is always updated to maintain the correct distance from pebble \(p[j]\) to the hash chain seed.

**Proof:** After \(p[j].\text{distancefromSeed}\)’s initialization in section 1 of Figure 6, the value \(p[j].\text{distancefromSeed}\) only changes when the next assignment is made in line 3.2:

\[p[j].\text{distancefromSeed} \leftarrow \text{totalHashElements}\]

and this occurs exactly when the condition in line 3 holds true since

\[p[j].\text{distancefromSeed} + p[j].\text{MoveIncrement} = \text{totalHashElements},\]

completing the proof.
Fact 7 In order to reverse Jakobsson’s algorithm pebbles should be assigned indices in ascending order.

Proof: This holds by Fact 2.

Fact 8 For each pebble, to find the current position for the purposes of the following proof we can calculate,

\[ p[j].\text{Position} = \text{totalHashElements} - p[j].\text{distancefromSeed}. \]

Proof: This holds because Position is a measure of how far from the front of the chain a pebble currently is. By Fact 4 totalHashElements is always equal to the total number of elements in the chain and by Facts 5 and 8 distancefromSeed is always the distance the pebble is from the seed including the seed itself. Thus, we know that we can find how far from the front of the hash chain (\( p[j].\text{Position} \)) a pebble is by \( \text{totalHashElements} - p[j].\text{distancefromSeed} \).

Fact 9 Consider the online hash growth algorithm. The only time a pebble is moved is when \( \text{totalHashElements} = t \) where \( t \leftarrow p[j].\text{Move_Increment} + p[j].\text{distancefromSeed} \). When a pebble is moved, then it is moved to the front of the chain.

Fact 10 When the online hash growth algorithm is halted the final pebble is placed at the root with the seed value, and the next available pebble index.

Fact 10 follows immediately from the code starting at line 4 of Figure 6. Jakobsson assumes \( n \) is a power of two for simplicity [14]. In the case of the online algorithm, values of \( n \) other than powers of two are also critical.

Theorem 1 Suppose that the online algorithm in Figure 6 is halted after generating \( n \) total hash elements. The pebbles associated with the hash chain of size \( n \) will be stored at positions equivalent to the positions in Jakobsson’s algorithm.

Proof: The case where \( n = 2^k \), the online and amortized algorithm both have pebbles in the same positions by Fact 11.

The proof follows by induction on the ranges from \([2^k, 2^{k+1})\) to \([2^{k+1}, 2^{k+2})\), for \( k \geq 2 \).

Basis: Take a hash chain with \( k = 2 \), thus \( n = 2^2 = 4 \) elements as stored by the online by the algorithm in Figure 6.

\[
\begin{align*}
p[1].\text{Position} & = 2 \\
p[1].\text{Move_Increment} & = 4.
\end{align*}
\]
As the hash chain is being grown after $n = 4$, the first occurrence of CreatePebble is at $2^2 + 1 = 5$. This pebble is placed at the front of the chain by Fact 5 and labeled $p[2]$ by Fact 7. Furthermore,

$$p[2].\text{Position} = 4$$
$$p[2].\text{Move Increment} = 8.$$ 

since grow.pebble = 2. Suppose the hash chain growth is halted at $n = 5$. Due to Fact 10 we know that the final pebble is placed for the seed. The pebbles are sorted by their Positions.

Growing the hash chain to $n = 6$, prompts the online hash growth algorithm to check each pebble for totalHashElements = $t = 6$ where,

$$t \leftarrow p[j].\text{Move Increment} + p[j].\text{distance from Seed}.$$ 

But, totalHashElements $\neq t$ for all $j$ and $n = 6$.

The algorithm finds $t_j$ where

$$t_1 \leftarrow p[1].\text{Move Increment} + p[1].\text{distance from Seed}$$
$$t_2 \leftarrow p[2].\text{Move Increment} + p[2].\text{distance from Seed}.$$ 

At $n = 5$ no pebble movement occurs. At $n = 6 = t_1$ the algorithm sets

$$p[1].\text{distance from Seed} = p[1].\text{distance from Seed} + p[1].\text{Move Increment},$$

such that

$$p[1].\text{distance from Seed} = \text{totalHashElements} = 6.$$ 

Suppose the hash chain growth is halted at $n = 7$. Again, due to Fact 10 we know that the final pebble is the seed. Thus, for $n = 7$ we have pebbles at $p[1].\text{Position} = 2$, $p[2].\text{Position} = 4$ and $p[3].\text{Position} = 8$ by Fact 8.

**Inductive Hypothesis:** The statement of this Lemma holds for hash chains from $n = 2^k$ to $n = 2^{k+1}$, for some $k \geq 2$.

**Inductive Step:** Consider the case where $n$ is increased from $n = 2^k$ to $n = 2^{k+1}$ where $k \geq 2$. By the inductive hypothesis we suppose the pebbles are all placed in the correct positions for $n \leq 2^k$.

By Jakobsson’s algorithm, for a chain of original size $n$ we know that a pebble is only discarded when the total number of remaining elements is $n/2$. Thus, the hash chain growth algorithm only adds new elements when the total number of pebbles is $(n/2) + 1$.

Recall, a pebble is always placed at the seed starting at line 4 of the online algorithm, this assures we always have the proper number of pebbles.

By Fact 11 we know that in Jakobsson’s algorithm only pebble $p[1]$ is moved. Further, it is always moved a total of $p[1].\text{Dest Increment}$ after being moved such that it reaches its destination. Since

$$p[i].\text{Dest Increment} = p[i].\text{Move Increment}$$

and because pebbles are always moved to the front by the online algorithm by Fact 9 then we know that pebbles are always moved correctly.
For each $n$, there are always enough pebbles to support the total number of elements emitted in the online algorithm. As just shown, these pebbles are always placed correctly completing the proof.

5 Further Directions

It would be interesting to extend the algorithms of Sella [25] or Matias and Porat [19] to work online. That is, suppose we initially have a data structure capable of holding $n$ points of data, then it would be interesting to be able to generate $n + 1$ elements of data and expand this data structure suitably, i.e. not doubling it in size.

We have not addressed the tamper-resistant hardware. Likewise, for very small time granularity, we have not addressed clock drift and related timing issues.

6 Acknowledgements

Thanks to Markus Jakobsson for his comments on this work. Additionally, the referees’ comments were very helpful and insightful. Thanks to Judge Joseph Colquitt for his perspective on chains of custody and providing us with [6].

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Appendix: A Proof of Correctness for Jakobsson’s Algorithm

This appendix gives a correctness proof for Jakobsson’s amortized algorithm. This is done to derive the correctness of the new online algorithm given in this paper. Jakobsson’s paper [14] does not give a complete proof of correctness for his algorithm. However, our online result seems to require certain factual statements from the correctness of Jakobsson’s algorithm.

Ultimately, what we show when Jakobsson’s algorithm runs emitting \( n/2 = 2^k \) hash elements the remaining pebbles contain hash elements of hash-chain positions \( 2^1, 2^2, 2^3, \ldots, 2^k \) which will prove the correctness of Jakobsson’s algorithm.

Jakobsson’s work immediately gives the next observation.

**Observation 1** Suppose a hash chain has \( n \) hash elements and \( n = 2^k \), for some integer \( k \geq 1 \) and Jakobsson’s algorithm is about to start. The \( k \) pebbles initially contain hash elements in hash-chain positions

\[
2^1, 2^2, 2^3, \ldots, 2^k.
\]

Say Jakobsson’s algorithm is about to start on a hash chain of size \( 2n \). Then \( k + 1 \) pebbles initially contain hash elements in hash-chain positions

\[
2^1, 2^2, 2^3, \ldots, 2^k, 2^{k+1}.
\]

A proof of Observation 1 follows directly from Jakobsson’s setup phase where the pebbles are initialized.

**Definition 5** A pebble \( p[j] \) is moved **backward** if \( p[j].Position \) is increased in line 3.1 of the algorithm in Figure [2]. A pebble \( p[j] \) is moved **forward** if \( p[j].Position \) is decreased by 2 in line 2.1 of this algorithm.

**Fact 11** Only pebble \( p[1] \) moves backwards in Jakobsson’s algorithm. Furthermore, it only does so when \( \text{Current.position} \) is even.

**Proof:** This holds since, the major else in section 3 of Figure [2] is the only place Jakobsson’s algorithm increases any pebble position.

**Fact 12** A pebble \( p[j] \) moves two hash elements forward in lines 2.1 and 2.2 of Jakobsson’s algorithm only after \( p[j] \) was moved previously from the front of the chain by Fact 11.

**Proof:** In code section 2, making \( p[j].Position \neq p[j].Destination \) may only be done by code in section 3 of Figure [2].

Recall that Jakobsson’s algorithm initially sets \( p[j].Position = p[j].Destination \), for all pebbles \( j : \sigma \geq j \geq 1 \). Where \( \sigma \) is the number of pebbles.
Fact 13  For any pebble \( p[j] \), before or after any invocation of Jakobsson’s algorithm in Figure 2, the bound holds:

\[
p[j].\text{Position} \geq p[j].\text{Destination},
\]

for all \( j : \sigma \geq j \geq 1 \) where \( p[j].\text{Destination} \neq +\infty \).

Proof:  For each pebble \( j \geq 1 \), the values \( \text{Start\_Increment} = 3 \times 2^j \) and \( \text{Dest\_Increment} = 2^{j+1} \) are both even and never change. Initially in the setup phase

\[
\begin{align*}
p[j].\text{Destination} &= 2^j \\
p[j].\text{Position} &= 2^j.
\end{align*}
\]

In section 3, the bound \( p[j].\text{Position} \geq p[j].\text{Destination} \) is always enforced when \text{Current\_position} is even, due to the values of \( \text{Start\_Increment} \) and \( \text{Dest\_Increment} \) and also the assignments on lines 3.1 and 3.2,

\[
\begin{align*}
p[j].\text{Position} &\leftarrow p[j].\text{Position} + p[j].\text{Start\_Increment} \\
p[j].\text{Destination} &\leftarrow p[j].\text{Destination} + p[j].\text{Dest\_Increment}.
\end{align*}
\]

However, in section 2 of Figure 2 if \( p[j].\text{Destination} \neq p[j].\text{Position} \), then \( p[j].\text{Position} \) is decremented as follows:

\[
p[j].\text{Position} \leftarrow p[j].\text{Position} - 2.
\]

By Fact 12 during a run of Jakobsson’s algorithm, both

\[
p[1].\text{Position} \text{ and } p[1].\text{Destination}
\]

remain even values since lines 3.1 and 3.2 add even values to even values.

Line 3.2 ensures

\[
p[1].\text{Position} \geq p[1].\text{Destination}
\]

and line 2.1 decrements \( p[j].\text{Position} \) by 2, spanning all even numbers going down. Thus \( p[j].\text{Position} \) must eventually equal \( p[j].\text{Destination} \), completing the proof. 

Definition 6  The expression \( p[j] \overset{k}{\rightarrow} i \) indicates the pebble initially setup in position \( j \) is currently in position \( i \) after \( k \) backward moves. The expression \( p[j] \overset{k}{\rightarrow} i \) indicates this pebble was previously in position \( j \), but \( j \) is not necessarily the initial position of this pebble in setup.

The expression \( p[j] \overset{0}{\rightarrow} 1 \) indicates a pebble that was in position \( j \) becomes \( p[1] \) without moving backwards. That is, the pebble labeled \( p[j] \) becomes \( p[1] \) after emission of \( j - 1 \) hash elements.

Note, \( p[j] \overset{k}{\rightarrow} 1 \) indicates that the pebble initially labeled \( p[j] \) has become pebble \( p[1] \) exactly \( k \) times. Also, \( p[1] \overset{1}{\rightarrow} j \) indicates that the unique pebble initially labeled \( p[1] \) is moved backwards exactly once and eventually relabeled pebble \( p[j] \). Where \( p[1] \overset{1}{\rightarrow} j \) is a pebble that was earlier in position 1 and moved back once, but perhaps it was in another position initially at setup.
Definition 7 Let $D(j,k)$ be the value of $p[j \xrightarrow{k} i].\text{Destination}$ immediately after $k$ backward moves of the pebble initially setup as pebble $p[j]$ and became $p[i]$, by the Algorithm in Figure 2.

Thus, by Fact 11, each backward move is initiated when the initial $p[j]$ has become $p[1]$. Next is a bound on $D(j,k)$, for the initial pebble $p[j]$:

$$D(j,k) = \begin{cases} 2^j + 2^{j+1} & \text{if } k = 1 \\ D(j, k-1) + 2^{j+1} & \text{otherwise.} \end{cases}$$

Definition 8 Let $P(j,k)$ be an upper-bound on the value of $p[j \xrightarrow{k} i].\text{Position}$ immediately after $k$ backward moves of the pebble initially setup as pebble $p[j]$ and became $p[i]$, by the Algorithm in Figure 2.

Next is a bound on $P(j,k)$, for the initial pebble $p[j]$:

$$P(j,k) \leq \begin{cases} 2^j + 3 \cdot 2^j & \text{if } k = 1 \\ P(j, k-1) + 3 \cdot 2^j & \text{otherwise.} \end{cases}$$

Fact 14 Immediately after moving pebble $p[1]$ backwards to $p[i]$, the next upper bound holds:

$$p[j \xrightarrow{k} i].\text{Position} - p[j \xrightarrow{k} i].\text{Destination} \leq 2^j + k \left(3 \cdot 2^j\right) - 2^j - k \left(2 \cdot 2^j\right) \leq k \cdot 2^j.$$ 

Equality holds immediately after $p[j \xrightarrow{k} i]$ is placed in position $i$.

This fact holds by Fact 13.

Fact 15 Consider the Algorithm in Figure 2. Then for every pebble $p[j \xrightarrow{k} 1]$, it must be that

$$p[j \xrightarrow{k} 1].\text{Position} = p[j \xrightarrow{k} 1].\text{Destination},$$

where neither $p[j \xrightarrow{k} 1].\text{Position} = +\infty$ nor $p[j \xrightarrow{k} 1].\text{Destination} = +\infty$.

Proof: Consider the next function that describes how $p[j].\text{Destination}$ works. Note, that line 3.2 of Figure 2 is the only place the $\text{Destination}$ field is changed.

Basis: It must be that

$$p[j \xrightarrow{1} 1].\text{Position} = p[j \xrightarrow{1} 1].\text{Destination},$$

since by the upper-bound in Fact 14,

$$p[j \xrightarrow{1} 1].\text{Position} - p[j \xrightarrow{1} 1].\text{Destination} \leq 2^j.$$ 

However, $D(j,1) = 2^j + 2^{j+1}$ and by line 2.1 of the algorithm, we must decrement $p[j \xrightarrow{1} 1].\text{Position}$ by 2 each of $D(j,1)$ times. This is because the pebbles are sorted by $\text{Position}$. So
immediately after the initial pebble \( p[j] \) is moved back, then its destination is \( D(j, 1) \). Thus, after \( D(j, 1) \) more hash elements are emitted, then this pebble will be labeled \( p[1] \).

Thus, since \( 2D(j, 1) > 2^j \), by Fact 13,

\[
p[j \xrightarrow{+1} 1].\text{Position} = p[j \xrightarrow{+1} 1].\text{Destination},
\]

completing the basis.

**Inductive Hypothesis:** Suppose in \( k \geq 1 \) backward moves of the initial pebble \( p[j] \) by the Algorithm in Figure 2, we have

\[
p[j \xrightarrow{k} 1].\text{Position} = p[j \xrightarrow{k} 1].\text{Destination}.
\]

**Inductive Step:** For some \( k \geq 1 \), consider \( k + 1 \) backward moves of the initial pebble \( p[j'] \) by Algorithm in Figure 2 and this step will show:

\[
p[j' \xrightarrow{k+1} 1].\text{Position} = p[j' \xrightarrow{k+1} 1].\text{Destination}.
\]

Start by considering \( k + 1 \) backward moves of the initial pebble \( p[j'] \), for the sake of a contradiction suppose,

\[
p[j' \xrightarrow{1} j \xrightarrow{k} 1].\text{Position} \neq p[j' \xrightarrow{1} j \xrightarrow{k} 1].\text{Destination},
\]

and in any case, by Fact 13 the next bound holds:

\[
p[j' \xrightarrow{1} j].\text{Position} - p[j' \xrightarrow{1} j].\text{Destination} \leq 2^j.
\]

However \( D(j', 1) = 2^{j' + 1} + 2^{j'} \), which means after \( 2^{j' + 1} + 2^{j'} \) hash elements are emitted to take the initial pebble \( p[j'] \) to \( p[1] \) the first time, then

\[
p[j' \xrightarrow{1} j].\text{Position}
\]

must be decremented by 2 in each run of section 2 of Figure 2 decreasing \( p[j' \xrightarrow{1} 1].\text{Position} \) by at least \( 2D(j', 1) > 2^{j'} \).

Now, by the Inductive Hypothesis it must be that,

\[
p[j \xrightarrow{k} 1].\text{Position} = p[j \xrightarrow{k} 1].\text{Destination}.
\]

giving,

\[
p[j' \xrightarrow{k+1} 1].\text{Position} = p[j' \xrightarrow{k+1} 1].\text{Destination},
\]

indicating our supposition was incorrect, which completes the proof.

**Fact 16** Suppose a hash chain starts with \( n = 2^{k+1} \) total hash elements, for some integer \( k \geq 1 \) and Jakobsson’s algorithm runs emitting \( n/2 = 2^k \) hash elements. Then the pebble initially in position \( j < k \) is moved backwards a total of

\[
2^{k-j-1}
\]
times.
\textbf{Proof: } By Fact 11, a pebble only moves back when it is in pebble position 1. By Fact 13,
\[ p[j].\text{Position} \geq p[j].\text{Destination}, \]
so the only concern is how \( p[j].\text{Destination} \) is initialized and how \( p[j].\text{Destination} \) changes.

After \( n/2 \) hash elements are emitted, it takes exactly
\[
\left( \frac{n}{2} - 2^j \right) \frac{1}{p[j].\text{Dest}_{}\text{Increment}} = \left( 2^k - 2^j \right) \frac{1}{2^{j+1}} = 2^{j-k-1} - \frac{1}{2},
\]
increments of \( p[j].\text{Destination} \) by the time the pebble initially at position \( j \) is disposed of. \( \blacksquare \)

A pebble is disposed of when \( p[j].\text{Destination} \leftarrow +\infty \). In terms of Fact 16, the pebble in position \( k + 1 \) is never moved back and the pebble in position \( k \) is disposed of when it is moved back the first time.

\textbf{Fact 17} Given an \( n = 2^k \) element hash chain and take the Algorithm in Figure 2. The initial pebble in position \( j \), for \( j = k - 1 = \log(n/2) \) is the only pebble that is disposed of by the time the first \( n/2 \) hash elements are emitted.

\textbf{Proof: } For each pebble, the values \textbf{Start}_{}\text{Increment} \text{ and } \textbf{Dest}_{}\text{Increment} \text{ never change. Initially in setup } p[j].\text{Destination} \leftarrow 2^j, \text{ for all } j : \log n \geq j \geq 1.

The value \( p[j].\text{Destination} \) is only increased in line 3.2 of Figure 2. Considering line 3.2 gives the bound
\[
p[j].\text{Destination} = 2^j + p[j].\text{Dest}_{}\text{Increment} = 2^j + 2^{j+1} = 3 \cdot 2^j.
\]

Pebble \( p[1] \) is the only one that is moved back by Fact 11. Further,
\[ 3 \cdot 2^j > n, \]
holds for \( j \geq \log(n/2) \). But the only mobile pebble that satisfies this constraint is initially in position \( j = \log(n/2) = k - 1 \). There is a pebble \( j = \log n \) at the seed of the hash chain, but this never moves.

Now, take any initial pebble that started in position \( j \leq \log(n/4) \) then by Fact 16 such a pebble makes \( 2^k - 2^j - 1 \) backward moves by the time \( n/2 \) hash elements are emitted. Thus, by the time the \( n/2 \)-nd pebble is emitted, the initial pebble \( p[j] \) is moved from the starting position \( 2^j \) to position
\[
2^j + \left( 2^{j+1} \cdot 2^{k-j-1} \right) = 2^j + 2^k,
\]
and the largest \( j \) and \( k \) may be is \( \log(n/4) \) and \( \log(n/2) \), respectively, giving
\[
2^j + 2^k \leq 2^{\log n/4} + 2^{\log n/2} \leq \frac{3n}{4} < n,
\]
Fact 18 Suppose a hash chain starts with \( n = 2^{k+1} \) total hash elements, and for some integer \( k \geq 1 \), while Jakobsson’s algorithm runs emitting the first \( n/2 = 2^k \) hash elements, let \( i \) be the first placement of \( p[j \rightarrow i] \) between hash elements \( 2^k \) and \( 2^{k+1} \). Then, Jakobsson’s algorithm will emit a total of

\[
\frac{n}{2} - \left(2^{k-j-1} - 1\right)2^{i+1} - 2^i
\]

hash elements before emitting hash element \( 2^k \) but immediately after \( p[j \rightarrow i] \) is placed at position \( i \).

Proof: After Jakobsson’s algorithm emits hash element \( 2^k \), a total of \( n/2 \) total hash elements have already been emitted.

By Fact 16 the pebble initially at position \( j < k \) is moved backwards \( 2^{k-j-1} \) times. That is, it appears in pebble position 1, a total of \( 2^{k-j-1} \) times.

The last time this pebble moves backward to position \( i \) is the first time it moves to a position behind hash position \( 2^k \). This gives the factor of \( (2^{k-j-1} - 1) \).

Finally, for initial pebble \( j \), since \textit{Dest_Increment} is always \( 2^{j+1} \), each time we move backwards in Jakobsson’s algorithm, we move \textit{Destination} back by \( 2^{j+1} \) elements. That is, we move back a total of

\[
\left(2^{k-j-1} - 1\right)2^{j+1}
\]

hash elements by the time the pebble initially labeled \( j \) passes hash position \( 2^k \).

Furthermore, the initially labeled \( j \)th pebble starts at position \( 2^i \), so we adjust for the number of hash elements emitted it takes to traverse \( p[j \rightarrow 0] \).

Corollary 1 We know that when \( n/2 \) hash elements are emitted that all remaining pebbles will have

\[
p[j].\text{Destination} = p[j].\text{Position}.
\]

Proof: We know by Fact 18 that there are

\[
\frac{n}{2} - \left(2^{k-j-1} - 1\right)2^{i+1} - 2^i
\]

elements that remain to be output before a total of \( n/2 \) elements are emitted after each pebble at original pebble index \( j \) is moved for the last time before \( n/2 \) elements are emitted.

By Jakobsson’s algorithm we know that each time a pebble at original position \( j \) is moved, it will take,

\[
(3 \cdot 2^j - 2^{j+1})/2
\]
hash exposures for \( p[j] \). **Position** = \( p[j] \). **Destination**.

Thus, since

\[
\frac{n}{2} - \left(2^{k-j-1} - 1\right)2^{i+1} - 2^j > \left(3 \cdot 2^i - 2^{j+1}\right)/2
\]

we know that when \( n/2 \) hash elements are emitted that all remaining pebbles will have \( p[j] \). **Position** = \( p[j] \). **Destination**. Thus, completing the proof. 

The next fact, can also serve as an alternate proof-of-correctness for Jakobsson’s algorithm [14]. It is presented here, since it is needed subsequently.

First, the function \( R() \) reindexes the pebble indices from one power of two down to the next lowest power of two. When \( n/2 \) pebbles are emitted, then the total number of remaining hash elements goes from \( n = 2^{k+1} \) down to \( n = 2^k \).

\[
R_k(l_{k+1}) = l_{k+1} - 2^k,
\]

where \( l_{k+1} \) is the hash element \( \geq 2^{k+1} \) contained by pebble index \( k+1 \), and \( R_k(l_{k+1}) \) is the adjusted index of the hash element \( \geq 2^k \).

**Fact 19** Suppose a hash chain starts with \( n = 2^{k+1} \) total hash elements, for some integer \( k \geq 1 \) and Jakobsson’s algorithm runs emitting \( n/2 = 2^k \) hash elements. Then the remaining pebbles contain hash elements of hash-chain positions

\[
2^1, 2^2, 2^3, \ldots, 2^k.
\]

**Proof:** This proof follows by induction on the algorithm in Figure 2. Setting \( k = 2 \) ensures sections 1, 2 and 3 of Jakobsson’s algorithm will run (see Figure 2). The base case will be \( k = 2 \), so \( n = 2^3 = 8 \), exposing \( n/2 \) elements to get \( k - 1, n = 2^{k-1} \) total elements.

**Basis:** Take a hash chain with \( n = 2^3 \) total hash elements. Thus, \( n = 2^{k+1} \), so \( k = 2 \). Then the setup phase of Jakobsson’s algorithm assigns pebbles to hold hash elements in positions \( 2^1, 2^2 \) and \( 2^3 \).

Now, suppose \( n/2 = 4 \) hash elements are exposed, thus the algorithm in Figure 2 is called 4 times.

By Fact 17 only one pebble, the pebble associated with the \( 2^k \) position will be discarded. This leaves \( 2^{k+1} - 1 = 2 \) pebbles remaining. One of these pebbles is associated with the root and will not change. For the remaining pebble associated with the \( 2^1 \) position

\[
P(j, 1) = 2^j + 3 \cdot 2^j = 2^1 + 3 \cdot 2^1 = 8
\]

\[
D(j, 1) = 2^j + 2^{j+1} = 2^1 + 2^2 = 6.
\]

Now, \( n = 2^k = 4 \) elements remain. In order to put position \( l \), which is in terms of base \( 2^{k+1} \) hash chain, in terms of the base \( 2^k \) hash chain we have our two remaining pebbles at \( R_{k=2}(6_{g=3}) = 2 \), and \( R_{k=2}(8_{g=3}) = 4 \), where \( g = k + 1 \). Since \( k = 2 \) our remaining pebbles are
at positions $2^1$ and $2^2$ or, $2^1 \ldots 2^k$.

**Inductive Hypothesis:** Suppose that for a hash chain of size $2^{k+1}$ where $k \geq 2$ after Jakobsson’s algorithm runs emitting $n/2 = 2^k$ hash elements that the remaining pebbles contain hash elements of hash chain positions

$2^1, 2^2, 2^3, \ldots, 2^k$.

**Inductive Step:** Consider $n/2$ hash element exposures for a hash chain with $n = 2^{k+1}$ total elements where $k \geq 2$. This step will show that the pebbles will be placed at positions equivalent to $2^1, 2^2, \ldots, 2^k$.

Again, by Fact [17] only one pebble, the pebble associated with the $2^k$ position will be discarded. This leaves $k-1$ pebbles remaining. One of these pebbles is associated with the root and will not change.

By Fact [15] for all pebbles $p[j \rightarrow 1]$, it must be that

$p[j \rightarrow 1].\text{Position} = p[j \rightarrow 1].\text{Destination}$.

For all remaining pebbles after $n/2$ hash exposures,

$p[j].\text{Position} = P(j, q)$,

$p[j].\text{Destination} = D(j, q)$.

By Corollary [1] we know that for all remaining pebbles $p[j].\text{Position} = p[j].\text{Destination}$ when $n/2$ elements are output and thus all pebbles are inactive.

Further, since we know that $p[j].\text{Position} = p[j].\text{Destination}$ the remaining pebble positions are defined by the destination function $D(j, m)$ as defined earlier where $m$ is the number of times the pebble at original position $j$ is moved back and

$D(j, m) = 2^j + m \cdot 2^j+1$.

By Fact [16] $m = 2^{k-j-1}$ for each pebble at initial position $j$ over the first $n/2$ hash element exposures.

Therefore,

$D(j, 2^{k-j-1}) = 2^j + 2^{k-j-1} \cdot 2^{j+1}$

$D(j, 2^{k-j-1}) = 2^j + 2^{k}$

given in terms of base $n = 2^{k+1}$ total elements. To put this result in terms of our new hash chain $n = 2^k$ we simply apply the $R_k(l_{k+1})$ function to get

$R_k(2^j + 2^k) = 2^j + 2^k - 2^k = 2^j$,

where $j = 1, \ldots, k$.

Thus, by the inductive hypothesis it must be that when Jakobsson’s algorithm runs emitting $n/2 = 2^k$ hash elements the remaining pebbles contain hash elements of hash-chain positions $2^1, 2^2, 2^3, \ldots, 2^k$, thus completing the proof. ■