Semileptonic Decays of Heavy Mesons with the Fat Clover Action

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1. OBJECTIVES

We are studying the semileptonic decays $B \rightarrow \pi \ell \nu$, $B \rightarrow D \ell \nu$, $B \rightarrow \rho \ell \nu$, $B \rightarrow D^* \ell \nu$, and $B \rightarrow K^* \gamma$ and the corresponding decays with a strange spectator quark. For a companion study of purely leptonic decays, see [1]. The CKM matrix element $V_{ub}$, for example, is obtained from the differential semileptonic decay rate for $B \rightarrow \pi \ell \nu$ at total leptonic four-momentum $q$:

$$\frac{d\Gamma}{dq^2} = \frac{G_F^2 q^3}{24\pi^3} |V_{ub}|^2 |f^+(q^2)|^2.$$

The unknown hadronic form factor $f^+(q^2)$ is to be determined in lattice gauge theory from the matrix element of the weak vector current $V_\mu$:

$$\langle \pi(k)|V_\mu|B(p)\rangle = (p_\mu + k_\mu - q_\mu \frac{m_B^2 - m_\pi^2}{q^2}) f^+(q^2) + q_\mu \frac{m_B^2 - m_\pi^2}{q^2} f^0(q^2).$$

2. FAT CLOVER ACTION

Since the heavy-light meson decays involve light quarks, it is important to choose an $O(a^2)$ lattice fermion implementation with good chiral properties. To this end we have been experimenting with an action proposed by DeGrand, Hasenfratz, and Kovács which introduces, in effect, a cutoff-dependent form factor at the quark-gluon vertex to suppress lattice artifacts at the level of the cutoff. The action is the usual clover action but with a gauge background constructed by replacing the usual gauge links by APE-smoothed links with coefficient $1 - c$ for the forward link and $c/6$ for the sum of staples.
link is projected back to SU(3). This smoothing process is repeated $N$ times. For the present experiment we take $c = 0.45$ and $N = 10$. These values are to be kept constant in the continuum limit, thus giving the local continuum fermion action. This “fattening” process reduces problems with “exceptional” configurations that obstruct extrapolations to light quark mass [5,6].

3. PARAMETERS IN THE STUDY

Calculations were done on an archive of 200 $24^3 \times 64$ gauge configurations, generated with two flavors of dynamical staggered quarks of mass $am_q = 0.01$ at the one-plaquette coupling $6/g^2 = 5.6$, corresponding to a lattice spacing (from the rho mass) of about 0.11 fm. The fat clover propagator was generated for three “light” (spectator and recoiling) quarks and five “heavy” (decaying and recoiling) quarks over a mass range $0.5m_s < m < 1.1m_b$. The coefficient of the clover term $c_{SW}$ was set to 1. The mass of the lightest fat clover quark was adjusted to give the same pion mass as the staggered fermion Goldstone boson.

We use the Fermilab program through $O(a)$ for the quark wave function normalization, including the three-dimensional rotation [7] with coefficient $d_1$. The light meson source is placed at $t = 0$ and the heavy-light meson at $t = 32$, with antiperiodic boundary conditions in $t$. We treat three values of the heavy-light-meson momentum and 21 values of the three-momentum transfer at the current vertex. Computations are in progress. Results are presented for a subset of about half of the 200 configurations including only the two lightest spectator quark masses.

4. SELECTED RESULTS

An example of the meson dispersion relation is shown in Fig. 1. It is quite satisfactory.

The form factor is extracted by amputating the external meson legs — at present, by dividing by $\exp[(E_B - E_M)t]$, where the $B$ meson energy $E_B$ and recoil meson energy $E_M$ are taken from central values of a fit to the corresponding two-point dispersion relations. The diagonal vector form factor at zero three-momentum transfer gives the vector current renormalization factor $Z_V$. It is shown as a function of the inverse meson mass in Fig. 2 for the two currently available choices of the spectator quark mass. We see that this non-perturbative renormalization constant is within $10-15\%$ of unity.

We test the soft pion theorem [8] which states that in the chiral limit $f_0^2(q_{max}^2) = f_B/\langle f_\pi \rangle$. The
same action and configurations are used to get \( f_B \) \[1\]. Both spectator and recoil quark masses \((m \text{ and } m')\) are extrapolated to zero. If we use \( f_0(q_{\text{max}}^2, m, m') = a + bm + cm' \) we obtain Fig. 3, a disagreement similar to that found by JLQCD \[9\]. If we include an extra term \( d\sqrt{m + m'} \) as advocated by Maynard \[10\] the theorem is satisfied, but with large extrapolated errors. We hope our eventual full data sample will help resolve these complexities \[11, 12\].

Sample form factors for the process \( B_s \to K\ell\nu \) are shown in Fig. 4.

5. DISCUSSION

Fattening has allowed us to obtain results for an ostensibly \( O(a^2) \) action on unquenched lattices for quark masses at least as low as 0.5\( m_s \) with no noticeable trouble from exceptional configurations. Our experiment raises a number of important questions: Will a one-loop-perturbative determination of current renormalization factors be adequate? How much fattening is good? Does fattening push us farther from the continuum limit for some quantities? Work is in progress.

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