Precision Constraints on Extra Fermion Generations

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There has been renewed interest in the possibility of additional fermion generations. At the same time there have been significant changes in the relevant electroweak precision constraints, in particular, in the interpretation of several of the low energy experiments. We summarize the various motivations for extra families and analyze them in view of the latest electroweak precision data.

In the electroweak (EW) standard model (SM) and most extensions, the number of fermion generations is arbitrary. It is thus fair to ask whether there may be additional families of quarks and leptons. There are interesting theoretical considerations supporting this idea, though most of them arise in the context of scenarios that hypothesize rather drastic departures from the SM.

So far, there is no direct experimental evidence either supporting or conflicting with a fourth generation (or anti-generation). In view of only three observed (nearly) massless neutrinos, however, it is difficult to maintain the notion of sequential families of new fermions, although there are examples where the appearance of a heavy ($m_{\nu'} \gtrsim M_Z/2$) fourth neutrino, $\nu'$, does not appear entirely unnatural. On the upside, there is a number of experimental conflicts with the SM expectations at the level of several standard deviations (too small to be seen as uncontroversial evidence for new physics, yet too large to be ignored) and some of them could be interpreted as quantum loop effects by the fourth generation states.

The main point of this letter is the reconsideration of EW precision data in the presence of extra families. There are new experimental results from low energy measurements, and there are shifts that occurred due to changes in the interpretation of previous ones driven in turn by recent progress on the theory side.

One possibility to put a fourth family of quarks, $t'$ and $b'$, to work is within models of extended technicolor. Another one is to replace the top quark condensation mechanism by $t'$ condensation, since the top is too light for the scenario to work. The strongly coupled and condensing fourth generation can also be embedded into a warped extra dimension, where a heavy $\nu'$ can be arranged for by constructing it as a Dirac fermion while the three standard neutrinos are Majorana.

Extra fermions when strongly coupled to the standard Higgs boson may help to generate a strongly first-order EW phase transition as needed for baryogenesis. Models of dynamical EW symmetry breaking due to fourth-family quarks and leptons may then also succeed in this. The extra quarks could introduce the needed CP violation, which may be enhanced relative to the SM by as much as a factor of $10^{13}$ or more.

Finally, string theory vacua typically and easily give rise to even numbers of generations, while it is usually cumbersome to construct three generation models. This has been noted, e.g., for both free fermionic and orbifold string constructions of grand unified theories.

Of course, the Yukawa couplings associated with the new fermions are large. This may help to achieve non-supersymmetric grand unification but may also potentially destabilize the Higgs potential or lead to Landau poles below the Planck scale.

A rough bound on the $t'$ mass, $m_{t'}$, is obtained if one assumes unitarity of the partial S-wave amplitude for color-singlet elastic same-helicity $t'b'$ scattering already at the tree level, which for large energies yields

$$\frac{m_{t'}^2}{v^2} < \frac{4\pi}{3}, \quad m_{t'} < 504 \text{ GeV}. \quad (1)$$

The CDF Collaboration set the very recent bound, $m_{t'} > 338 \text{ GeV}$, from $b' \to tW^\mp$, complementing and helped by their previous limits, $m_{t'} > 268 \text{ GeV}$ from $b' \to qZ^0$ and $m_{t'} > 311 \text{ GeV}$ from $q' \to qW^\mp$, by bypassing the points raised in Ref.

The CP violating decay rate asymmetry,

$$A_{K^{\mp}\pi^0} \equiv \frac{\Gamma(B^+ \to K^+\pi^0) - \Gamma(B^- \to K^-\pi^0)}{\Gamma(B^+ \to K^+\pi^0) + \Gamma(B^- \to K^-\pi^0)}, \quad (2)$$

was determined by the BaBar, Belle, and CLEO Collaborations to an average of $A_{K^{\mp}\pi^0} = +0.051 \pm 0.025$. The analogously defined isospin rotated asymmetry, $A_{K^{\mp}\pi^+} = -0.098 \pm 0.013$, differs from $A_{K^{\mp}\pi^0}$ by 5.3 $\sigma$, strongly contradicting the naive expectation $A_{K^{\mp}\pi^0} \approx A_{K^{\mp}\pi^+}$. The Yukawa matrices for the four family case may be a remedy since $Z$ boson diagrams and the parameter choice, $m_{t'} \approx 300 \text{ GeV}$ and $V_{tb}^*V_{tb} \approx 0.03 e^{-i\delta_{tb}}$, can move $A_{K^{\mp}\pi^0}$ (but not $A_{K^{\mp}\pi^+}$) to basically zero, explaining the larger part of the effect. Based on this, a large time-dependent CP violation in the $B_d^0$ system was predicted. Subsequently the CDF and DØ Collaborations measured this asymmetry in $B_d^0 \to J/\Psi\phi$ and found good agreement with this prediction and with each other, but disagreement with the...
SM, albeit only at the 2.4 $\sigma$ level when the results are combined [23]. Measurements of other time-dependent CP asymmetries give qualitatively similar results.

Overall the experimental situation is not conclusive and in flux, and so is the optimal parameter choice. For recent accounts of flavor physics in view of a fourth family, see Refs. [24, 27]. For more details on both the theoretical and experimental situation and for statements about physics beyond the SM with four families, see Ref. [28].

The main purpose of this letter is to address the question whether the EW data add to the hints that are perhaps implied by the flavor sector. We employ the oblique parameters, $S$, $T$, and $U$ [29], which parametrize effects of heavy new physics, i.e., $M_{new} \gg M_Z$, contributing to the $W$ and $Z$ self-energies without coupling directly to the ordinary fermions. For what follows, it is important to recall that new physics models usually come with additional free parameters, $N_{\text{par}}^{\text{new}}$, relative to those in the SM, $N_{\text{par}}^{\text{SM}}$, and this decreases the number of effective degrees of freedom used in a fit, $N_{\text{eff}} = N_{\text{obs}} - N_{\text{SM}} - N_{\text{par}}^{\text{new}}$, where $N_{\text{obs}}$ is the number of observables.

We start our discussion with a case for which $N_{\text{par}}^{\text{new}} = 0$, so the $\chi^2$ minimum, $\chi^2_{\text{min}}$, for three and four families can be compared directly. This occurs when the new quarks and leptons form degenerate doublets and corresponds to $S = 2/3\alpha = 0.2122$, $T = U = 0$. For the Higgs boson mass, $M_H = 112$ GeV (we fix $M_H$ at its 95% CL lower limit [30] from LEP 2 whenever otherwise it would be driven below it), we obtain $\chi^2_{\text{min}} = 75.54$ compared to $\chi^2_{\text{min}}(\text{SM}) = 43.84$ in the SM ($S = T = U = 0$ by our definition), so this case is excluded at the 5.6 $\sigma$ level (we have $N_{\text{eff}} = 44$). Equivalently, one can interpret a fit to $S$ as a fit to the number of degenerate generations and one obtains $N_F = 2.86 \pm 0.20$. This agrees with a fit to the number of active neutrinos, $N_F = 2.995 \pm 0.007$ (for the same $M_H$) when interpreted as the generation number. One concludes from $N_F$ that $m_{\nu'} \gtrsim M_Z/2$, and from the $S$ parameter fit (which is applicable to the heavy $\nu'$ case) that the good agreement of $N_F$ with the SM value $N_F = 3$ would be coincidental if a fourth family existed.

This restriction can be relaxed drastically by allowing $T$ to vary, since $T > 0$ is predicted by nondegenerate extra doublets. Fixing $S = 2/3\alpha$, the global fit favors a contribution to $T$ of 0.21 $\pm$ 0.04 (for $M_H = 112$ GeV) with $\chi^2_{\text{min}}/N_{\text{eff}} = 46.90/43$. This is due to the strong correlation (87%) of $S = 0.03 \pm 0.09$ and $T = 0.07 \pm 0.08$. The central values move to $S = -0.03 \pm 0.10$ and $T = 0.14 \pm 0.29$ when $M_H$ is increased to 246 (800) GeV. Thus generically, the data favor small or negative values of $S$ and $T > 0$. For example, this is the case for nonchiral (vector-like) extra doublets ($S = 0$) which are most consistent with a moderate $T = \mathcal{O}(0.1)$. The goodness of the fit, $\chi^2_{\text{min}}/N_{\text{eff}} = 42.66/43$, is very similar to that of the SM. If, moreover, the nonchiral matter is also degenerate as predicted in many grand unified theories and other extensions of the SM, it does not contribute to any of the oblique parameters and does not require large coupling constants. Such multiplets may occur in partial families, as in $E_6$ models, or as complete vector-like families [31].

But for chiral fermions, $S$ cannot be made that small. To elucidate the parameter space we define the 90% C.L. by the 90% C.L. allowed region in $(S, T)$ [cf. Fig. 1], and assume in what follows that $m_{\nu'} = 101$ GeV [32] and $m_{\nu} = 338$ GeV are fixed at their lower limits. Then we find $S > 0.107$, where the smallest $S$ occurs in a corner of parameter space simultaneously saturating the limits, $M_H < 475$ GeV and $T < 0.38$. In addition, this case has the new charged lepton, $\nu'$, strongly split from the $\nu$, $m_{\nu'} - m_{\nu} = 140$ GeV, while we find for the quarks, $m_{u'} - m_u = 28$ GeV. Our $M_H$ bound is at best only marginally consistent with extra family models which have a strongly interacting Higgs boson (assuming the absence of other contributions to $S$, $T$, and $U$). There is a larger allowed parameter space for a light Higgs boson mass, $M_H = 112$ GeV. It is bounded by $T < 0.24$ (saturated for $S = 0.19$) and $S < 0.216$ (for $T = 0.218$), and contains the smallest possible $T = 0.099$ which is reached...
for \( m_{\ell'} = m_{\ell} \) and \( m_{\ell'} - m_{\ell} = 75 \text{ GeV} \). It also contains
the best fit which we find for \( S = 0.137, T = 0.157, m_{\ell'} - m_{\ell} = 91 \text{ GeV} \), and \( m_{\ell'} - m_{\ell} = 14 \text{ GeV} \). Thus
the data prefer the leptons to be more split than the quarks, although near the global minimum \( \chi^2 \) is quite
shallow along the direction with \( m_{\ell'} + m_{\ell} \) approximately constant, so our splittings are not inconsistent with those
found in Ref. [32]. The best fit has \( \chi^2_{\text{min}}/N_{\text{eff}} = 43.98/40 \),
so that contrary to statements made occasionally in the
literature, there is no choice fitting the four family hy-
pothesis better than the SM, even though \( N_{\text{post}} = 4 \) pa-
rameters have been added. The important exception is
the tuned scenario of a stable \( \nu' \) with mass very close to
\( M_2/2 \) [34, 35]. We conclude that a fourth family is dis-
favored but we find that there is more allowed parameter
space than with earlier data sets [22], which only allowed
rather tuned scenarios even at the 90\% C.L. The reasons
can be found mainly in developments in the low energy
precision physics, of which we now briefly discuss the two
most important ones.

For decades, measurements of \( Z \) induced atomic parity
violation (APV) in cesium [39] implied \( S < 0 \), at times
at the 2 \( \sigma \) level. Several improvements in the atomic
theory [37, 38] — needed to extract the EW physics —
have now moved \( S \) to values well consistent with zero.
In addition, the NuTeV result [39] for \( \nu \)-nucleus deep
inelastic scattering in terms of the on-shell weak mixing
angle, \( s_W^2 = 0.2277 \pm 0.0016 \), was initially 3 \( \sigma \) higher
than the SM prediction, \( s_W^2 = 0.2292 \pm 0.0028 \). Since
then a number of experimental and theoretical develop-
ments shifted the extracted \( s_W^2 \), most of them towards
the SM: (i) NuTeV also measured [40] a non-vanishing
strange quark asymmetry, shifting \( s_W^2 \) by about
-0.0007. (ii) The measured branching ratio for \( K_{e3} \) decays
enters in the determination of the \( \nu_e(\bar{\nu}_e) \) contamination
of the \( \nu_\mu(\bar{\nu}_\mu) \) beam. Since the time of Ref. [39]
\( s_W^2 \) has changed by more than 4 \( \sigma \) and the corresponding \( s_W^2 \) by
+0.0016. (iii) Parton density functions seem to violate
isospin symmetry much stronger than expected, imply-
ing a shift, \( \Delta s_W^2 = -0.0026 \) [41, 42]. (iv) The isovector
EMC effect [41] reduces \( s_W^2 \) by -0.0019 [42]. With these
corrections we find \( s_W^2 = 0.2242 \pm 0.0018 \) (we also in-
creased the error). The contributions of these and other
data sets to \( S \) and \( T \) are illustrated in Fig. 1.

We have assumed \( U = 0 \), since we have verified that
in most of the relevant parameter space \( U < 0.03 \), and
where it exceeds this we find \( U < 0.11T \). In any case,
allowing \( U \neq 0 \) decreases \( S \) and \( T \) (it is negatively corre-
lated with them) which is disfavored. Similarly, we set
the small non-linear oblique parameters, \( V, W, \) and
\( X \) to zero. This is currently a sufficiently accurate
approximation but we point out (i) that exact one-loop
results are complete only after their inclusion and their
determination from low energy data; (ii) that the dif-
ference between the use of differences and derivatives in
the definitions for \( S, T, \) and \( U \) is formally of the order
of ignoring \( V, W, \) and \( X \); and (iii) that at that level of
precision one should employ \( M_{\text{SM}} \) rather than pole quark
masses which reduces the \( T \) parameter by \( O(10\%) \).

We were so far considering situations with \( m_{\ell'} \) and \( m_{\ell'} \)
at their direct lower bounds. One can scale the lepton or
quark masses without affecting \( S \) and \( U \) (in our approx-
imation) while \( T \) scales with the square of the masses.
This would increase \( \chi^2_{\text{min}} \) and strengthen our \( M_H \) bound,
but can bring some mass combinations into play [points
strictly below the allowed contour in Fig. 2].

We also assumed that generation mixing is absent. A
nonzero mixing angle, \( \theta_{34} \), between the third and fourth
families [50] give positive and negative definite contribu-
tions to \( T \) and the \( Z \to b\bar{b} \) decay rate, respectively,
both worsening the fits. The \( T \) effect can be eased by
allowing larger \( M_H \) but at the expense of aggravating
the \( S \) constraint. In fact, we exclude the scenario with
\( M_H = 810 \text{ GeV} \) and \( (S,T) = (0.15,0.48) \) [50] for which
we find \( \chi^2_{\text{min}} = 53.34 \) after allowing yet another parame-
ter (\( \theta_{34} \)). We traced most of the disagreement with
Ref. [50], where a much milder increase in \( \chi^2 \) was found,
in about equal parts to the low energy and more recent
high energy data, an increase in the hadronic vacuum po-
larization contribution (and decrease in its uncertainty)
due to more complete and up-to-date experimental and
theoretical results [51, 52], and the implementation of
radiative corrections [53]. Thus, the "three prong com-
posite solution" [54] with Cabbibo-sized mixing, and the
Higgs boson as well as the \( t' \) and \( b' \) quarks all close to
their unitarity bounds, is strongly conflicting with EW
data. Furthermore, the aforementioned parameters [22]
to address the asymmetry are no longer viable and have to be adjusted to smaller mixing, and the flavor sector considerations become less convincing (there are also constraints from flavor changing neutral currents). The remaining parameter space is also difficult to reconcile with gauge coupling unification.

As always, loopholes remain. Since the three prong composite solution is really a theory of dynamical symmetry breaking with a composite Higgs sector (and not just a four-generation extension of the SM), it comes with all the complications of this kind of scenario. Then the discussion of EW constraints becomes less quantitative for the lack of precise predictions for $S$ and $T$. A more detailed analysis is required if the $\nu'$ is not a Dirac fermion or only couples to the $\nu_\tau$, in which cases the L3 $m_{\nu'}$ bounds are weaker and slightly negative $S$ and $T$, and $U$ contributions are possible.

We conclude that while the EW precision constraints have eased somewhat, a fourth family remains disfavored given that adding up to five new parameters to the SM still deteriorates the global fit. The part of the parameter space which passes the oblique parameter space at the 90\% C.L. is at odds with large $M_H$ scenarios as in technicolor-type models. It also implies smaller mixing than one would like in face of the flavor physics issues. To truly address the latter, we encourage a global EW plus flavor analysis with all sectors, loopholes, and refinements considered and with a critical view of how the favored parameter space compares with the expectations from the various motivations discussed earlier.

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