On QCD RFT corrections to the propagator of reggeized gluons

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Abstract

We calculate one loop QCD Regge Field Theory (RFT) correction to the propagator of reggeized gluons basing on the QCD effective action of Lipatov, [1–6], and results of [7], where Dyson-Schwinger hierarchy of the equations for the correlators of reggeized gluon fields was derived. The correction is calculated entirely in the framework of RFT basing on the obtained expressions of the RFT bare triple Reggeon vertices and propagator of reggeized gluons. As the propagator of the reggeized gluons we consider separately the cases with bare transverse propagator and reggeized propagator calculated to one loop QCD precision. In both results the correction violates the reggeization property of the propagator and additionally some non-linear corrections to the propagator are obtained. Additionally, this RFT one-loop correction changes the structure of the reggeized propagator, a dependence of the propagator on the longitudinal coordinates is arising in the expressions. The further application of the obtained results is also discussed.

1 Introduction

The action for an interaction of the reggeized gluons, introduced in [1,2], see also [3–6], describes quasi-elastic amplitudes of high-energy scattering processes in the multi-Regge kinematics. The applications of this action for the description of high energy processes and calculation of sub-leading, unitarity corrections to the amplitudes and production vertices can be found in [8], see also [9]. The generalization of the formalism for a purpose of the calculation of production amplitudes and impact factors was obtained in [5], where the prescription of the calculation of S-matrix elements of the different processes was given accordingly to an approach of [10]. This effective action formalism, based on the reggeized gluons as main degrees of freedom, see [11], can be considered as reformulation of the RFT (Regge Field Theory) calculus introduced in [12], see also [13–21], for the case of high energy QCD. It was underlined in [12] that the main purposes of the approach is the construction of the S-matrix unitarity in the direct and crossing channels of the scattering processes through the multi-Reggeon dynamics described by the vertices of multi-Reggeon interactions, see other and similar approaches in [22–29]. The unitarity of the Lipatov’s formalism, therefore, is related to the unitary corrections in both RFT and QCD sectors of the theory.

Similarly to the phenomenological theories of interacting Reggeons, see [12–16] and references therein, there is a very natural question to ask: what are the corrections to the amplitudes which come from the pure RFT sector of the formalism. Namely, consider the Lipatov’s effective action
for reggeized gluons $A_\pm$, formulated as RFT (Regge Field Theory) which can be obtained by an integration out the gluon fields $v$ in the generating functional for the $S_{\text{eff}}[v, A]$:

$$e^{\Gamma[A]} = \int Dv e^{S_{\text{eff}}[v,A]}$$

(1)

where

$$S_{\text{eff}} = -\int d^4x \left( \frac{1}{4} G^a_{\mu\nu} G^a_{\mu\nu} + tr \left[ (T_+(v_+) - A_+) j_{\text{reg}}^+ + (T_-(v_-) - A_-) j_{\text{reg}}^- \right] \right),$$

(2)

with

$$T_{\pm}(v_{\pm}) = \frac{1}{g} \partial_{\pm} O(v_{\pm}) = v_{\pm} O(v_{\pm}) , \quad j_{\text{reg}}^{\pm} = \frac{1}{C(R)} \partial_{\pm} A^\pm ,$$

(3)

here $C(R)$ is eigenvalue of Casimir operator in the representation $R$, $tr(T^a T^b) = C(R) \delta^{ab}$ see [1][2][3].

The form of the Lipatov’s operator $O$ (and correspondingly $T$) depends on the particular process of interests, see [4]. We take it in the form of the Wilson line (ordered exponential) for the longitudinal gluon fields in the adjoint representation:

$$O(v_{\pm}) = Pe^{g \int_{-\infty}^{+} dx_{\pm} v_{\pm}(x^+, x^-)} , \quad v_{\pm} = iT^a v^a_{\pm}$$

(4)

see also [30]. There are additional kinematical constraints for the reggeon fields

$$\partial_- A_+ = \partial_+ A_- = 0 ,$$

(5)

corresponding to the strong-ordering of the Sudakov components in the multi-Regge kinematics, see [1][2][6]. The action is constructed by the request that the LO value of the classical gluon fields in the solutions of equations of motion will be fixed as

$$v_{\pm} = A_{\pm} .$$

(6)

In the light-cone gauge $v_- = 0$, the equations of motion can be solved and the general expressions for the gluon fields can be written in the following form:

$$v^a_{\pm} \to v^a_{\pm, cl}(A_{\pm}) + \varepsilon^a_{\pm} , \quad v^a_+ \to v^a_+ + v^a_{+ cl}(A_{\pm}) + \varepsilon^a_{+} .$$

(7)

The integration in respect to the fluctuations around the classical solutions provides QCD loop corrections to the effective vertices of the Lipatov’s action which now can be written as functional of the Reggeon fields only:

$$\Gamma = \sum_{n,m=1} \left( A_{\pm}^{a_1} \cdots A_{\pm}^{a_n} (K_{\pm}^{a_1} \cdots \pm_{b_1 \cdots b_m} A_{\pm}^{b_1} \cdots A_{\pm}^{b_m}) = - A_{\pm}^{a} \partial_{\pm} A_{\pm}^{a} + A_{\pm}^{a} \left( K^{a b}_{\pm} \right)_{\pm}^{b} + \cdots \right) .$$

(8)

in general the summation on the color indexes in the r.h.s of the equation means the integration on the corresponding coordinates as well. Now we see, that the theory have two different sources of any perturbative/unitary corrections. The first one comes from the QCD side of the framework and based on the precision of the effective vertices (kernels) calculated in the pure QCD. Another source of the corrections is described by the diagrams directly from the RFT sector of the theory constructed entirely in terms of the Reggeon fields and Eq. (8) vertices to some given QCD precision. These RFT type of corrections were calculated a lot in the previous phenomenological RFT theories, see [13][14] and references therein for example; there are the QCD RFT corrections to the propagator and vertices of the Eq. (5) action as well. In the paper [7] the Dyson-Schwinger hierarchy of the equations for the correlators of reggeized gluons was derived in the formalism, of that allows to define and calculate

\[^1\]In order to make the notations shorter, we change the position of the color and other indexes of the vertices further in the article, preserving only the overall number of the indexes.
these RFT sector corrections to any correlator of interests. We also note, that formally this hierarchy is similar to the Balitsky hierarchy of equations and BK-JIMWLK approaches, see [22–24, 27], and there is a correspondence between different degrees of freedom such as reggeized gluons and Wilson line operators, see details in [7, 29].

In this paper we calculate a one Reggeon loop correction to the propagator of reggeized gluons. In order to perform the calculations we need the expressions for the vertices of interactions of three Reggeons. These vertices to the bare QCD precision are also calculated and basing on this result we construct the RFT Reggeon loop contribution to the propagator using [7] equation for the correlator of two Reggeon fields. We separately consider two different forms of the Reggeon loop. The first one we construct with the use in the equations of hierarchy only bare QCD vertices, whereas in the second case we account also a leading order one-loop QCD correction to the correlators that leads to the different final expressions in two cases. Therefore, the paper is organized as follows. In the next section we remind some basic definitions from the [4]. The Section 3 is dedicated to the calculation of the one loop QCD propagator of the reggeized gluons. The Section 4 is about the calculations of the bare QCD vertices of the triple Reggeon interactions required for the one RFT loop construction. Sections 5 and 6 are about the derivation of the expression for the two Reggeon fields correlator to one RFT loop precision and it’s calculation. The last Section is the Conclusion of the article, there are also Appendixes where some technical details of the calculations are present.

2 One loop effective action

This section is based mainly on the results from [4] paper, therefore we remind shortly only the some important formulae from there. In the derivation of the QCD RFT action the following representation of the gluon fields is used:

\[ v_i^a \rightarrow v_i^a \delta_i \epsilon^+ \, , \quad v_i^a \rightarrow v_i^a \epsilon^+ \, , \quad (9) \]

at the next step we expand the Lagrangian of the effective action around the classical solutions. Preserving in the expression only terms which are quadratic with respect to the fluctuation fields, we obtain for this part of the action:

\[
S_{\epsilon^\pm} = \frac{1}{2} \int d^4x \left( \epsilon_i^a \left( \delta_{ac} (\delta_{ij} \Box + \partial_i \partial_j) - 2g f_{abc} \left( \delta_{ij} v_k^{bcl} \partial_k - 2v_j^{bcl} \partial_i + v_i^{bcl} \partial_j - \delta_{ij} v_+^{bcl} \partial_- \right) - \right.ight. \\
\left. - g^2 f_{abc} f_{cibj} \left( \delta_{ij} v_k^{bcl} - v_i^{bcl} - v_j^{bcl} - \partial_- v_i^{bcl} \right) \right) \epsilon_j^c + \\
\left. + \epsilon_i^+ \left( -2\delta^{ac} \partial_\mp \partial_\pm - 2g f_{abc} \left( v_i^{bcl} \partial_- - \left( \partial_- v_i^{bcl} \right) \right) \right) \epsilon_j^c + \\
\left. + \epsilon_i^+ \delta_{ac} \partial_\mp \epsilon_j^c - g \epsilon_i^+ \int d^4y \left( U_1^{ab} \right)_1^{+} (\partial_\pm \partial_\mp)_{xy} \epsilon_i^+ \right) = \\
\frac{-1}{2} \epsilon_i^a \left( \left( M_0 \right)^{ac}_{\mu \nu} + \left( M_1 \right)^{ac}_{\mu \nu} + \left( M_2 \right)^{ac}_{\mu \nu} + \left( M_L \right)^{ac}_{\mu \nu} \right) \epsilon_j^c. \quad (10)
\]

Here we defined \( (M_1)^{ac}_{\mu \nu} \propto g^4 \) and note that

\[
(M_1)_{\mu \nu}^- = -g f_{abc} \left( v_i^{bcl} \partial_- - \left( \partial_- v_i^{bcl} \right) \right), \quad (M_1)_{\mu \nu}^- = -g f_{abc} \left( v_+^{bcl} - \left( \partial_- v_i^{bcl} \right) \right) .
\quad (11)
\]

The last term in Eq. (10) expression, denoted as \( (M_L)^{ac}_{\mu \nu} \) represents contribution of the Lipatov’s effective current into the action. This term is defined trough the following function:

\[
\left( U_1^{ab} \right)_1^{+} = \text{tr}[ f_a G_{xy}^+ f_c O_y f_b O_x^T ] + \text{tr}[ f_c G_{yx}^+ f_a O_x f_b O_y^T ] ,
\quad (12)
\]
see [4] and Appendix B. There is also some color density function introduced as

$$\partial_i \partial_- \rho_a^i = -\frac{1}{N} \partial_-^2 A_a^+, \quad (13)$$

or

$$\rho_a^i = \frac{1}{N} \partial_- (\partial^i A_a^+), \quad (14)$$

see [6, 22–24]. Now we can perform the integration obtaining the one loop QCD effective action:

$$\Gamma = \int d^4x \left( L_{YM}(v^c_i, v^c_f) - v^a_i J^a_i(v^c_f) - A_a^+ (\partial_2^2 A_a^+) \right) +$$

$$+ \frac{1}{2} Tr \ln \left( \delta_{\rho \nu} + G_{0\rho\mu} \left( (M_1)_{\mu\nu} + (M_2)_{\mu\nu} + (M_L)_{\mu\nu} \right) \right) +$$

$$+ \frac{1}{2} \int d^4x \int d^4y j^a_{\mu x} G^{ab}_{\mu \nu}(x, y) j^b_{\nu y}. \quad (15)$$

Here we have $G_{0\nu\mu}$ as bare gluon propagator

$$G_{0\nu\rho} = \delta_{\mu\rho}, \quad (16)$$

see Appendix A; the full gluon propagator is defined as

$$G^{ac}_{\mu\nu}(x, y) = G^{ac}_{0\mu\nu}(x, y) - \int d^4z G^{ab}_{0\rho\mu}(x, z) \left( (M_1(z))^{bd}_{\rho\gamma} + (M_2(z))^{bd}_{\rho\gamma} + (M_L(z))^{bd}_{\rho\gamma} \right) G^{dc}_{\gamma\mu}(z, y); \quad (17)$$

and can be written in the form of the following perturbative series:

$$G^{ac}_{\mu\nu}(x, y) = G^{ac}_{0\mu\nu}(x, y) - \int d^4z G^{ab}_{0\rho\mu}(x, z) \left( (M_1(z))^{bd}_{\rho\gamma} + (M_2(z))^{bd}_{\rho\gamma} + (M_L(z))^{bd}_{\rho\gamma} \right) G^{dc}_{\gamma\mu}(z, y); \quad (18)$$

the auxiliary currents $j^a_{\mu x}$ and $j^b_{\nu y}$ are requested for the many-loops calculations of the effective action, in our case of calculation to one loop QCD precision we take them zero from the beginning.

3 Propagator of reggeized gluons

In order to calibrate the calculations and introduce some useful further notations, in this Section we rederive the calculations of the propagator of reggeized gluons to one loop QCD precision done in [4].

The interaction of reggeized gluons $A_+$ and $A_-$ is defined through an effective vertex of the interaction in Eq. (15) as:

$$\left( K^{ab}_{xy} \right)^{+-} = K^{ab}_{xy} = \left( \frac{\delta^2 \Gamma}{\delta A_a^{+x} \delta A_b^{-y}} \right)_{A_+, A_-, v_f \perp = 0}, \quad (19)$$

Due the properties of the Reggeon fields, see Eq. (5), this vertex in the expressions we obtain in the following form:

$$K^{ab}_{xy} = K^{ab}_{xy}(x^+, x^-; y^+, y^-) = \int dx^- dy^+ \tilde{K}^{ab}(x^+, x^-; x_a^+, y^-; y_a^+). \quad (20)$$

At high-energy approximation, the transverse coordinates are factorized from the longitudinal ones in the LO and NLO vertices of $A_-$ and $A_+$ Reggeon fields interactions:

$$\tilde{K}^{ab}(x^+, x^-; y^+, y^-; y_a^+) = \delta(y^- \to x^-) \delta(x^+ \to y^+) \delta^{ab}(x_a^+, y_a^+) \quad (21)$$

Indeed, the LO vertex in the formalism has the following form:

$$K^{ab}_{xy0} = -\delta^{ab} \delta_{x_a^+, y_a^+} \partial_2^2 \quad (22)$$
and the similar property is holding for the NLO vertex, see Appendix C. The bare propagator of the action for the Reggeon can be defined similarly:

\[ D_{ab}^{0} (x^+, x^-, y^+, y^-) = \int dx^- dy^+ \tilde{D}_{ab}^{0} (x^+, x^-, y^+, y^-) \]  

(23)

with

\[ \tilde{D}_{ab}^{0} (x^+, x^-, y^+, y^-) = \delta (y^- - x^-) \delta (x^+ - y^+) D_{ab}^{0} (x_\perp, y_\perp). \]  

(24)

The propagator satisfies the following equation:

\[ \int d^4z \left( \tilde{K}_{ab}^{0} \right)^{+} \left( \tilde{D}_{bc}^{0} \right)_{++} = \delta^{ac} \delta_{xy}^4 \]  

(25)

that provides

\[ D_{ab}^{0} (x_\perp, y_\perp) = D_{ab}^{0} (x_\perp, y_\perp) = \delta^{ab} \int \frac{d^2p}{(2\pi)^2} \frac{e^{-ip_i (x_i - y_i)}}{p_\perp^2}. \]  

(26)

Considering, correspondingly, the perturbative expansion of the kernel

\[ K_{bd}^{zw} = \sum_{k=0}^{\infty} K_{bd}^{zw k}, \]  

(27)

the full propagator of reggeized gluons is defined in the form of the perturbative series as well:

\[ D_{ac}^{xy} = D_{ac}^{xy 0} - \int d^4z \int d^4w D_{ab}^{0} \left( \sum_{k=1}^{\infty} K_{zw k} \right) D_{dc}^{0} \]  

(28)

To the leading order precision we need to calculate the \( K_{zw 1}^{bd} \) kernel, the calculations are presented in Appendix C. Therefore, we obtain to this order:

\[ D_{ac}^{xy} = D_{ac}^{xy 0} - \int d^4z \int d^4w \left( \tilde{D}_{ab}^{0} \right)_{++} \left( \sum_{k=1}^{\infty} K_{zw k} \right) D_{dc}^{0} \]  

(29)

Introducing

\[ D_{ac}^{xy} = \delta^{ac} \delta (y^- - x^-) \delta (x^+ - y^+) \int \frac{d^2p}{(2\pi)^2} \tilde{D} (p_\perp, \eta) e^{-ip_i (x_i - y_i)}, \]  

(30)

we obtain finally:

\[ \tilde{D}_{ab} (p_\perp, \eta) = \frac{\delta^{ab}}{p_\perp^2} + \epsilon (p_\perp^2) \int_{0}^{\eta} d^\eta' \tilde{D}_{ab} (p_\perp, \eta'), \]  

(31)

with

\[ \epsilon (p_\perp^2) = - \frac{\alpha_s N}{4 \pi^2} \int d^2k_\perp \frac{p_\perp^2}{k_\perp^2 (p_\perp - k_\perp)^2} \]  

(32)

as trajectory of the propagator of reggeized gluons. Rewriting this equation as the differential one:

\[ \frac{\partial \tilde{D}_{ab} (p_\perp, \eta)}{\partial \eta} = \tilde{D}_{ab} (p_\perp, \eta) \epsilon (p_\perp^2) \]  

(33)

we obtain the final expression for the propagator:

\[ \tilde{D}_{ab} (p_\perp, \eta) = \frac{\delta^{ab}}{p_\perp^2} e^{\eta \epsilon (p_\perp^2)}, \]  

(34)

with \( \eta \) defined in some rapidity interval \( 0 < \eta < Y = \ln (s/s_0) \) of interest; of course it is the BFKL propagator for reggeized gluons calculated to one loop QCD precision, see [11].
4 Bare vertices of triple Reggeons interactions

The first contribution to any bare (QCD zero-loop) vertex of the three Reggeon fields interactions is coming from the Yang-Mills action and it is defined as following:

\[
\begin{align*}
\left( K^{abc}_{x y z} \right)_{0YM}^{\mu \nu \rho} &= \int d^4 w \left( \frac{\delta^3 L_{YM}(\psi^a_1, \psi^a_2)}{\delta A^a_\mu(x) \delta A^b_\nu(y) \delta A^c_\rho(z)} \right)_{A=0}, \quad \mu \nu \rho = (+, -)
\end{align*}
\]

with the QCD Lagrangian in light-cone gauge

\[
L = -\frac{1}{4} F^a_{ij} F^a_{ij} + F^a_{i+} F^a_{i-} + \frac{1}{2} F^a_{+-} F^a_{+-},
\]

where \( F^a_{i+} F^a_{i-} \) term does not consists transverse fields and \( F^a_{ij} F^a_{ij} \) term does not consist longitudinal fields. Correspondingly, we have for the non-zero contributions in Eq. (35):

\[
\frac{\delta^3 L_{YM}(\psi^a_1, \psi^a_2)}{\delta A^a_\mu(x) \delta A^b_\nu(y) \delta A^c_\rho(z)} = \frac{\delta^3 L_{YM}(\psi^a_1, \psi^a_2)}{\delta A^a_\mu(x) \delta A^b_\nu(y) \delta A^c_\rho(z)} + \frac{\delta^2 L_{YM}}{\delta A^a_\mu(x) \delta A^b_\mu(y) \delta A^c_\rho(z)} \left( \frac{\delta \psi^a_{i1}}{\delta A^a_\mu(x)} \frac{\delta \psi^a_{i2}}{\delta A^b_\nu(y)} \frac{\delta \psi^a_{i3}}{\delta A^c_\rho(z)} \right), \quad \mu_i = (+, i). (37)
\]

The only non-zero first order derivatives of the classical fields in respect to Reggeon fields are the following ones:

\[
\frac{\delta \psi^a_{i1}}{\delta A^a_\mu(x)} = \delta^a a_1 \delta^2(x_\perp - y_\perp) \delta(x^+ - y^+)
\]

and

\[
\frac{\delta \psi^a_{i1}}{\delta A^a_\mu(x)} = \delta^a a_1 \delta^2(x_\perp - y_\perp) \tilde{G}^{-0}_{y^- x^-} \partial y,
\]

see definition of \( \tilde{G}^{-0}_{y^- x^-} \) in Appendix B and calculations in [15]. The second order non-zero derivatives of the classical fields with respect to the Reggeon fields we account only to the \( g \) order, the first second-order derivative is the following one:

\[
\frac{\delta^2 \psi^a_{i1}}{\delta A^a_\mu(x) \delta A^b_\nu(y)} = -2 g f^a_{12b} \square_{z_1}^{-1} \left[ \delta^2(x_\perp - z_\perp) \delta^2(y_\perp - z_\perp) \delta(x^+ - z^+) \tilde{G}^{-0}_{z^- y^-} \right] (40)
\]

also see [3][5]. The second one has the following form

\[
\frac{\delta^2 \psi^a_{i1}}{\delta A^a_\mu(x) \delta A^b_\nu(y)} = g f^a_{12b} \int d^4 z_1 \square_{z_1}^{-1} \delta^2(x_\perp - z_\perp) \delta^2(y_\perp - z_\perp) \left( \tilde{G}^{-0}_{z^- y^-} \tilde{G}^{-0}_{y^- x^-} \tilde{G}^{-0}_{y^- x^-} \right) \partial z_1 \partial^2 z_1
\]

where

\[
\square_{z_1}^{-1} = D_{sc}(z, z_1) = - \int \frac{d^4 k}{(2\pi)^4} \frac{e^{-i(\bar{z}_1 - z_1)k}}{k^2},
\]

and the last one

\[
\frac{\delta^2 \psi^a_{i1}}{\delta A^a_\mu(x) \delta A^b_\nu(y)} = \frac{1}{2} g f^a_{12b} \left( \tilde{G}^{+0}_{z^+ x^+} - \tilde{G}^{+0}_{x^+ z^+} \right) \delta^2(z_\perp - x_\perp) \delta^2(z_\perp - y_\perp) \tilde{G}^{-0}_{z^- y^-} \partial y.
\]

We note, that there are additional contributions to the second order derivatives of higher perturbative orders which we do not consider here.
There is the first triple Reggeon vertex we calculate:

\[
(K_{abc}^{++-})_{0Y_M}^+ = \int d^4 w \left( \frac{\delta^3 L_{YM}(v_i^A, v_i^A)}{\delta A_+^a(x) \delta A_-^b(y) \delta A_-^c(z)} \right)_{A=0} =
\]
\[
= \int d^4 w \int d^4 w_1 \int d^4 w_2 \frac{\delta^2 L_{YM}}{\delta v_i^{cl_a1}(w_1) \delta v_i^{cl_a2}(w_2)} \frac{\delta^2 v_i^{cl_a1}(w_1) \delta v_i^{cl_a2}(w_2)}{\delta A_+^b(y) \delta A_-^c(z) \delta A_-^a(x)} +
\]
\[
+ \frac{\delta^2 L_{YM}}{\delta v_i^{cl_a1}(w_1) \delta v_i^{cl_a2}(w_2)} \frac{\delta^2 v_i^{cl_a1}(w_1) \delta v_i^{cl_a2}(w_2)}{\delta A_+^a(x) \delta A_-^c(z) \delta A_-^a(y)}
\]
(45)

that gives

\[
(K_{abc}^{++-})_{0Y_M}^+ = g f^{abc} \left( \hat{G}_{x^+ y^+}^+ - \hat{G}_{x^+ y^-}^+ \right) \delta^2(z_\perp - x_\perp) \delta^2(z_\perp - y_\perp) \partial_i^2 z.
\]
(46)

The second triple Reggeon vertex of interests reads as:

\[
(K_{abc}^{+-+})_{0Y_M}^- = \int d^4 w \left( \frac{\delta^3 L_{YM}(v_i^A, v_i^A)}{\delta A_+^a(x) \delta A_-^b(y) \delta A_-^c(z)} \right)_{A=0} =
\]
\[
= \int d^4 w \int d^4 w_1 \int d^4 w_2 \frac{\delta^2 L_{YM}}{\delta v_i^{cl_a1}(w_1) \delta v_i^{cl_a2}(w_2)} \frac{\delta^2 v_i^{cl_a1}(w_1) \delta v_i^{cl_a2}(w_2)}{\delta A_+^b(y) \delta A_-^c(z) \delta A_-^a(x)} +
\]
\[
+ \int d^4 w \int d^4 w_1 \int d^4 w_2 \frac{\delta^2 L_{YM}}{\delta v_i^{cl_a1}(w_1) \delta v_i^{cl_a2}(w_2)} \frac{\delta^2 v_i^{cl_a1}(w_1) \delta v_i^{cl_a2}(w_2)}{\delta A_+^a(x) \delta A_-^c(z) \delta A_-^a(y)}
\]
(47)

The first term in the r.h.s. of Eq. (47) provides:

\[
\int d^4 w \int d^4 w_1 \int d^4 w_2 \frac{\delta^2 L_{YM}}{\delta v_i^{cl_a1}(w_1) \delta v_i^{cl_a2}(w_2)} \frac{\delta^2 v_i^{cl_a1}(w_1) \delta v_i^{cl_a2}(w_2)}{\delta A_+^b(y) \delta A_-^c(z) \delta A_-^a(x)} =
\]
\[
= g f^{abc} \delta^2(x_\perp - z_\perp) \int dw^- dz_1^+ (-\partial_i x_\perp \partial_i y) D_{sc}(w^-, x^+, x_\perp; z^-, z_1^+, y_\perp) \hat{G}_{y^- z^-}^- \partial^2_j x - (y \leftrightarrow z)
\]
(48)

Taking into account that

\[
(-\partial_i x_\perp \partial_i y) \int dw^- dz_1^+ D_{sc}(w^-, x^+, x_\perp; z^-, z_1^+, y_\perp) = -\partial_i x_\perp \partial_i y D_0(x_\perp, y_\perp) = -\delta^2(x_\perp - y_\perp)
\]
(49)

see Eq. \ref{26} definition, we obtain finally for this contribution:

\[
\int d^4 w \int d^4 w_1 \int d^4 w_2 \frac{\delta^2 L_{YM}}{\delta v_i^{cl_a1}(w_1) \delta v_i^{cl_a2}(w_2)} \frac{\delta^2 v_i^{cl_a1}(w_1) \delta v_i^{cl_a2}(w_2)}{\delta A_+^b(y) \delta A_-^c(z) \delta A_-^a(x)} =
\]
\[
= g f^{abc} \delta^2(x_\perp - z_\perp) \delta^2(x_\perp - y_\perp) \left( \hat{G}_{y^- z^-}^- - \hat{G}_{y^- z^-}^+ \right) \partial_i^2 z .
\]
(50)

The two last terms in the r.h.s. of Eq. (47) are proportional to

\[
\int dz_1^+ \int dw^- \partial_{-w} \left( \square_{z_1 w}^{-1} \hat{G}_{y^- z^-}^- \right) = 0,
\]
(51)

therefore Eq. (50) expression is the final answer for the second vertex:

\[
(K_{abc}^{+-+})_{0Y_M}^- = g f^{abc} \delta^2(x_\perp - z_\perp) \delta^2(x_\perp - y_\perp) \left( \hat{G}_{y^- z^-}^- - \hat{G}_{y^- z^-}^+ \right) \partial_i^2 z .
\]
(52)
The additional contributions to the vertices are coming from the Lipatov’s effective currents expression in the Lagrangian:

\[ S_{\text{curr}} = -\frac{1}{N} \int d^4w \, \text{tr} \left[ v_i^{\alpha_1} O^{\alpha_1 \alpha_2} (v_i^{\alpha_1})^2 A_{\alpha_2}^{\alpha_2} \right] \]  

(53)

see [3,4]. We obtain correspondingly:

\[ (K^{abc}_{xyz})_{0 \text{curr}}^{+++} = -\frac{1}{2} g f^{abc} \left( \hat{G}^{+0}_{x+y} - \hat{G}^{+0}_{y+x} \right) \delta^2(z_\perp - x_\perp) \delta^2(z_\perp - y_\perp) \partial^2_{z_\perp}, \]  

(54)

and

\[ (K^{abc}_{xyz})_{0 \text{curr}}^{+-+} = -2 g f^{abc} \delta^2(y_\perp - x_\perp) \delta^2(z_\perp - x_\perp) \left( \hat{G}^{-0}_{y-z} - \hat{G}^{-0}_{z-y} \right) \partial^2_{x_\perp}, \]  

(55)

here the Eq. (51) identity was used again.

Finally, summing up all contributions we obtain for the vertices:

\[ (K^{abc}_{xyz})_{0}^{+++} = \frac{1}{2} g f^{abc} \left( \hat{G}^{+0}_{x+y} - \hat{G}^{+0}_{y+x} \right) \delta^2(z_\perp - x_\perp) \delta^2(z_\perp - y_\perp) \partial^2_{z_\perp}, \]  

(56)

and

\[ (K^{abc}_{xyz})_{0}^{+-+} = -g f^{abc} \left( \hat{G}^{-0}_{y-z} - \hat{G}^{-0}_{z-y} \right) \delta^2(y_\perp - x_\perp) \delta^2(z_\perp - x_\perp) \partial^2_{x_\perp}. \]  

(57)

The Fourier transform of the vertices, in turn, provides:

\[ \left( \hat{K}^{abc}_{p_1p_2p_3} \right)_{0}^{+++} = -\frac{i}{2} g f^{abc} (2\pi)^7 p^2_{31} \left( \frac{1}{p_{1+} + i\varepsilon} + \frac{1}{p_{2-} - i\varepsilon} \right) \delta^2(p_{1+} + 2p_2 + 3p_3) \delta(p_{1+} + 2p_2) \delta(p_{2-}) \delta(p_{3-}) \delta(p_{3+}), \]  

(58)

and correspondingly

\[ \left( \hat{K}^{abc}_{p_1p_2p_3} \right)_{0}^{+-+} = i g f^{abc} (2\pi)^7 p^2_{31} \left( \frac{1}{p_{2+} + i\varepsilon} + \frac{1}{p_{2-} - i\varepsilon} \right) \delta^2(p_{1+} + 2p_2 + 3p_3) \delta(p_{2+} + 3p_3) \delta(p_{2+}) \delta(p_{3+}) \delta(p_{1+}). \]  

(59)

Here the \( \hat{G}^{+0}_{x+y} = \theta(x^+ - y^+) \) representation of the \( \hat{G}^0 \) Green’s function was used, see Appendix B for the details.

5 One loop RFT correction to the correlator of two reggeized gluons

In paper [7] the following equation of the correlator of reggeized gluons to leading RFT order was obtained:

\[ \partial^2_{x_\perp} < A^{\alpha}_{\alpha}(x^+, x_\perp) A^{\alpha}_{\alpha}(y^-, y_\perp) > = -i \delta^{\alpha\alpha_1} \delta(x^+) \delta(y^-) \delta^2(x_\perp - y_\perp) + \]  

\[ + \int d^2z_1 \int d^2z_2 \int d^2z_3 \hat{K}^{b_1b_2a}_{x+y}A^{b_1}_{x}A^{b_2}_{y}(z_1^+, z_\perp_1; z_2^+, z_\perp_2; z_3^+, z_\perp_3) + \]  

\[ + \int d^2z_1 \int d^2z_2 \int d^2z_3 \hat{K}^{b_1b_2a}_{y-x}A^{b_1}_{x}A^{b_2}_{y}(z_1^-, z_\perp_1; z_2^-, z_\perp_2; z_3^-, z_\perp_3) = \left. \right. \]  

(60)

The equation was truncated in comparison to the expression in [7] in order to keep in only vertices of the leading QCD order. We note immediately, that the second term in the r.h.s. of the equation depends on \( x^- \) variable, whereas the correlator in the l.h.s. does not. This discrepancy means a violation of kinematical conditions Eq. (5), which in fact were known as not-absolute, see [12,6], and therefore, generalizing the approach, we have to consider the Regeon fields as four dimensional ones:

\[ A_+(x^+, x_\perp) \rightarrow B_+(x^+, x_\perp, x_\perp), \quad D_+(x^+, x^-) = A_+(x^+, x_\perp) + D_+(x^+, x_\perp, x_\perp), \quad D_+(x^+, x^-) = 0, x_\perp = 0 \]  

(61)
and
\[ A_-(x^-,x_\perp) \rightarrow B_-(x^+,x_\perp) = A_-(x^-,x_\perp) + D_-(x^+,x_\perp), \quad D_-(x^+ = 0, x_\perp) = 0. \] (62)

Respectively we can write for the Eq. (63) correlator:
\[ < A_+^a(x^+,x_\perp) A_+^{a'}(y^-,y_\perp) > \rightarrow \]
\[ < B_+^a(x^+,x_\perp) A_+^{a'}(y^-,y_\perp) > = < A_+^a(x^+,x_\perp) A_+^{a'}(y^-,y_\perp) > + < D_+^a(x^+,x_\perp) A_+^{a'}(y^-,y_\perp) > \] that in turn provides:
\[ \partial_{x_\perp}^2 < D_+^a(x^+,x_\perp) A_+^{a'}(y^-,y_\perp) > = \]
\[ = \int d^2 z_\perp \int d^2 z_\perp K_{++}^{b_1 b_2 a}(x^+,z_\perp; z_\perp_1; z_\perp_2; z_\perp_3; z_\perp_4) \quad (64) \]
\[ = \int d^2 z_\perp \int d^2 z_\perp K_{++}^{b_1 b_2 a}(z_\perp; z_\perp_1; z_\perp_2; z_\perp_3; z_\perp_4) < A_+^{b_1} A_+^{b_2} A_+^{a'} > \]

This RFT perturbative correction to the two Reggeon fields correlator violates the \( x_\perp \) dependence of the correlator, some non-trivial dependence on the \( x^- \) coordinate is arising here, see for comparison Eq. (30) expression. We also note, that due the absence of the formulation of the approach in terms of 4-d Reggeon fields\(^3\) we can treat the Eq. (61) correlator only perturbatively taking it as the bare expression for the correlator and inverting in the r.h.s. of it the Eq. (30) expression for the usual Reggeon correlator.

Now, with the help of Eq. (60) function we rewrite the Eq. (61) equation\(^3\) as follow:
\[ < D_+^a(x^+,x_\perp) A_+^{a'}(y^-,y_\perp) > = \]
\[ = -\int d^2 z_\perp \int d^2 z_\perp D_0(x_\perp, z_\perp) K_{++}^{b_1 b_2 a}(x^+, z_\perp; z_\perp_1; z_\perp_2; z_\perp_3; z_\perp_4) \quad (65) \]
\[ = -\int d^2 z_\perp \int d^2 z_\perp D_0(x_\perp, z_\perp) K_{++}^{b_1 b_2 a}(z_\perp; z_\perp_1; z_\perp_2; z_\perp_3; z_\perp_4) < A_+^{b_1} A_+^{b_2} A_+^{a'} > \]

The notation of the vertices are changed here in comparison to Eq. (53)–Eq. (57) notations for the shortness. The expressions of triple Reggeon fields correlators to the leading order precision also were obtained in \([7]\):
\[ \partial_{x_\perp}^2 < A_+^a(x^+,x_\perp) A_+^{a'}(y^-,y_\perp) A_+^{a''}(z^-,z_\perp) > = \]
\[ = \int d^2 w_\perp dw_1 \int d^2 w_\perp K_{+++}^{a a a'}(w_\perp; x^-; x_\perp; w^-_1; w_1) \quad (66) \]
\[ < A_+^a(x^+,w_\perp) A_+^{a'}(y^-,w^-_1) A_+^{a''}(z^-,w_1) > \]

where
\[ < A_+^a(x^+,w_\perp) A_+^{a'}(y^-,w^-_1) A_+^{a''}(z^-,w_1) > = \]
\[ = \int d^2 w_\perp dw_1 \int d^2 w_\perp D_0(w_\perp, w_1) \quad (67) \]
\[ < A_+^a A_+^{a'} A_+^{a''} > \]

see the derivation in \([7]\). Therefore we have for Eq. (66):
\[ < A_+^a(x^+,x_\perp) A_+^{a'}(y^-,y_\perp) A_+^{a''}(z^-,z_\perp) > = \]
\[ = -i \int d^2 w_\perp dw_1 \int d^2 w_\perp D_0(w_\perp, w_1) K_{+++}^{a a a'}(w_\perp; x^-; x_\perp; w^-_1; w_1) \quad (68) \]
\[ < A_+^{a'}(y^-,y_\perp) A_+^{a''}(w^-_1, w_1) > \delta(x^+) \delta(z^-) , \]

\(^3\)Work in progress.

\(^3\)We note, that in comparison with \([7]\) equation, the coefficient 2 is already included in the Eq. (60)–Eq. (67) definitions of the vertices.
we see that there is also dependence on \(x^-\) variable in the r.h.s. of Eq. (67), that explains the presence of two terms in the Eq. (63) expression. Correspondingly, we obtain for the second triple Reggeon correlator of interests:

\[
\partial_{\perp z}^2 < A_{+}^a(x^+, x_{\perp}) A_{-}^{a_1}(y^-, y_{\perp}) A_{-}^{a_2}(z^-, z_{\perp}) > = \\
= \int d^2w_{\perp} dw_{\perp} d^2w_{1\perp} K_{++_{\perp}}^{a_{1}a_{2}}(x^+, w_{\perp}; w_{1\perp}, w_{1\perp}; x_{\perp}) < A_{+}^{a_{1}}(x^+, w_{\perp}) A_{+}^{a_{2}}(w_{1\perp}, w_{1\perp}) A_{-}^{a_1}(y^-, y_{\perp}) A_{-}^{a_2}(z^-, z_{\perp}) >
\]

that with the help of Eq. (67) gives:

\[
< A_{+}^{a_{1}}(x^+, x_{\perp}) A_{-}^{a_1}(y^-, y_{\perp}) A_{-}^{a_2}(z^-, z_{\perp}) > = \\
= -i \int d^2w_{\perp} \int dw_{\perp}^2 d^2w_{1\perp} \int d^2w_{2\perp} D_0(x_{\perp}, w_{\perp}) K_{++_{\perp}}^{a_{1}a_{2}}(x^+, w_{\perp}; w_{1\perp}, w_{1\perp}; w_{2\perp}) D_0(w_{2\perp}, y_{\perp}) \\
< A_{+}^{a_{1}}(w_{\perp}, w_{1\perp}) A_{-}^{a_2}(z^-, z_{\perp}) > \delta(x^+) \delta(y^-) - \\
- i \int d^2w_{\perp} \int dw_{\perp}^2 d^2w_{1\perp} \int d^2w_{2\perp} D_0(x_{\perp}, w_{\perp}) K_{++_{\perp}}^{a_{1}a_{2}}(x^+, w_{\perp}; w_{1\perp}, w_{1\perp}; w_{2\perp}) D_0(w_{2\perp}, z_{\perp}) \\
< A_{+}^{a_{1}}(w_{\perp}, w_{1\perp}) A_{-}^{a_2}(y^-, y_{\perp}) > \delta(x^+) \delta(y^-).
\]

Taking Eq. (65), Eq. (69) and Eq. (70) together, we obtain finally corollor for the correlator of interests:

\[
< D_{+}^{a}(x^+, x_{\perp}) A_{-}^{a_1}(y^-, y_{\perp}) > = \\
= i \int d^2z_{\perp} \int d^2z_{1\perp} \int d^2z_{2\perp} \int d^2w_{\perp} \int dw_{\perp}^1 d^2w_{1\perp} \int d^2w_{2\perp} \\
D_0(x_{\perp}, z_{\perp}) K_{++_{\perp}}^{a_{1}b_{3}}(x^+, z_{\perp}; z_{1\perp}; z_{2\perp}) D_0(z_{1\perp}, w_{\perp}) K_{++_{\perp}}^{b_{3}a_{2}}(w_{\perp}; x^-, w_{\perp}; w_{1\perp}; w_{1\perp}) D_0(w_{2\perp}, y_{\perp}) \\
< A_{+}^{a_{1}}(z_{\perp}, z_{1\perp}) A_{-}^{a_2}(w_{\perp}, w_{1\perp}) > \delta(x^+) \delta(y^-) + \\
+ i \int d^2z_{\perp} \int d^2z_{1\perp} \int d^2z_{2\perp} \int d^2w_{\perp} \int dw_{\perp}^1 d^2w_{1\perp} \int d^2w_{2\perp} \\
D_0(x_{\perp}, z_{\perp}) K_{++_{\perp}}^{a_{1}b_{3}}(z_{\perp}; z_{1\perp}; x^-; z_{2\perp}) D_0(z_{1\perp}, w_{\perp}) K_{++_{\perp}}^{b_{3}a_{2}}(x^+, w_{\perp}; w_{1\perp}; w_{1\perp}) D_0(w_{2\perp}, z_{\perp}) \\
< A_{+}^{a_{1}}(w_{\perp}, w_{1\perp}) A_{-}^{a_2}(y^-, y_{\perp}) > \delta(x^+) \delta(y^-) + \\
+ i \int d^2z_{\perp} \int d^2z_{1\perp} \int d^2z_{2\perp} \int d^2w_{\perp} \int dw_{\perp}^1 d^2w_{1\perp} \int d^2w_{2\perp} \\
D_0(x_{\perp}, z_{\perp}) K_{++_{\perp}}^{a_{1}b_{3}}(z_{\perp}; z_{1\perp}; z_{2\perp}; x^-) D_0(z_{1\perp}, w_{\perp}) K_{++_{\perp}}^{b_{3}a_{2}}(x^+, w_{\perp}; w_{1\perp}; w_{1\perp}) D_0(w_{2\perp}, y_{\perp}) \\
< A_{+}^{a_{1}}(w_{\perp}, w_{1\perp}) A_{-}^{a_2}(z_{\perp}, z_{1\perp}) > \delta(x^+) \delta(y^-).
\]

see Fig. (11) Taking the integrals in these three terms we obtain:

\[
< D_{+}^{a}(x^+, x_{\perp}) A_{-}^{a_1}(y^-, y_{\perp}) > = \\
= -\frac{i}{2} g^a f^{ab_{1}b_{2}} f_{a_{1}b_{3}b_{4}} \int dz^+ \left( \tilde{G}^{+}_{x^+ z^-} - \tilde{G}^{+}_{z^+ x^-} \right) \int dw^- \left( \tilde{G}^{-}_{x^- w^-} - \tilde{G}^{-}_{w^- x^-} \right) D_0(x_{\perp}, y_{\perp}) \\
< A_{+}^{a_{1}}(z^+, x_{\perp}) A_{-}^{a_{1}}(w^-, y_{\perp}) > \delta(x^+) \delta(y^-) - \\
- \frac{i}{2} g^a f^{ab_{1}b_{2}} f_{a_{1}b_{3}b_{4}} \left( \tilde{G}^{-}_{x^+ z^-} - \tilde{G}^{-}_{z^+ x^-} \right) \int dw^+ \left( \tilde{G}^{+}_{x^+ w^+} - \tilde{G}^{+}_{w^+ x^+} \right) \int d^2w_{\perp} \int d^2z_{\perp} \\
D_0(x_{\perp}, z_{\perp}) (\partial_{z_{\perp}}^2 \partial_{w_{\perp}}^2 D_0(z_{\perp}, w_{\perp})) D_0(w_{\perp}, y_{\perp}) < A_{+}^{a_{2}}(w^+, w_{\perp}) A_{-}^{a_{1}}(y^-, y_{\perp}) > \delta(x^+) - \\
- \frac{i}{2} g^a f^{ab_{1}b_{2}} f_{a_{1}b_{3}b_{4}} \int dz^- \left( \tilde{G}^{-}_{x^- z^+} - \tilde{G}^{-}_{z^- x^+} \right) \int dw^+ \left( \tilde{G}^{+}_{x^- w^+} - \tilde{G}^{+}_{w^+ x^-} \right) \int d^2w_{\perp} \int d^2z_{\perp} \\
D_0(x_{\perp}, z_{\perp}) (\partial_{z_{\perp}}^2 \partial_{w_{\perp}}^2 D_0(z_{\perp}, w_{\perp})) D_0(w_{\perp}, y_{\perp}) < A_{+}^{a_{2}}(w^+, w_{\perp}) A_{-}^{a_{2}}(z^-, z_{\perp}) > \delta(x^+) \delta(y^-)
\]

Treating the expression perturbatively and writing the correlators as propagators

\[
< A_{+}^{a_{1}}(x^+, x_{\perp}) A_{-}^{a_{2}}(y^-, y_{\perp}) > = i \delta(x^+) \delta(y^-) \delta^{ab_{1}b_{2}} D(x_{\perp}, y_{\perp})
\]
and
\[
< D_+^a(x^+, x^-, x_\perp; y_\perp) A_0^{a_1}(y_\perp, y_\perp) > = i G_+^{a_1}(x^+, x^-, x_\perp; y_\perp, y_\perp),
\] (75)
where \( D(x_\perp, y_\perp) \) propagator is given by Eq. \( G_{\perp 0} \) - Eq. \( G_{\perp 1} \) expression, see the derivation of Eq. \( G_{\perp 2} \) in \( \gamma \), we obtain for Eq. \( G_{\perp 3} \) expression:

\[
G_+^{a_1}(x^+, x^-, x_\perp; y_\perp, y_\perp) =
\frac{-i}{2} g^2 N \delta^{a_1} \left( \tilde{G}_{x^+ 0}^{0} - \tilde{G}_{x^0}^{+ 0} \right) \left( \tilde{G}_{x^- 0}^{0} - \tilde{G}_{x^0}^{- 0} \right) D_0(x_\perp, y_\perp) D(x_\perp, y_\perp) - \\
\int d^2 w_\perp \int d^2 z_\perp D_0(x_\perp, z_\perp) \left( \partial_{x^+}^2 \partial_{w^+}^2 D_0(z_\perp, w_\perp) \right) D_0(w_\perp, z_\perp) D(w_\perp, y_\perp) - \\
\int d^2 w_\perp \int d^2 z_\perp D_0(x_\perp, z_\perp) \left( \partial_{z^+}^2 \partial_{w^+}^2 D_0(z_\perp, w_\perp) \right) D_0(w_\perp, y_\perp) D(w_\perp, z_\perp) \delta(x^+) \delta(y^-).
\] (76)

Performing Fourier transform of the propagator with respect to \( x^+ \), \( x_\perp - y_\perp \) and \( y^- \) variables we obtain:

\[
\tilde{G}_+^{a_1}(p_+, p_\perp, p_-; x^-; Y) = g^2 N \delta^{a_1} \left( \tilde{G}_{x^+ 0}^{0} - \tilde{G}_{x^0}^{+ 0} \right) \\
\left( \frac{d}{(2\pi)^2} \int \frac{d^2 k_\perp}{(2\pi)^2} \left( e^{\eta} \epsilon(k_\perp^2) - \frac{e^{\eta} \epsilon(k_\perp^2)}{p_\perp^2 (p_\perp - k_\perp)^2} - \frac{e^{\eta} \epsilon(k_\perp^2)}{p_\perp^4} (p_\perp - k_\perp)^2 \right) \right).
\] (77)

The integration with respect to \( k^+ \) can be performed with the help of the rapidity variable \( y = \frac{1}{2} \ln(\Lambda k_+) \) introduced, where a rapidity interval \( \eta \) appears as an analog of an ultraviolet cut-off in the relative longitudinal momenta integration. At the end of the integration it’s limit is taking to \( Y = \ln(s/s_0) \), therefore we have:

\[
\tilde{G}_+^{a_1}(p_+, p_\perp, p_-; x^-; Y) = \frac{g^2 N}{4\pi^3} \delta^{a_1} \left( \tilde{G}_{x^+ 0}^{0} - \tilde{G}_{x^0}^{+ 0} \right) \\
\frac{1}{p_\perp^4} \int d^2 k_\perp \left( \frac{p_\perp^4}{k_\perp^4} \left( \epsilon(k_\perp^2)Y - 1 \right) - \frac{k_\perp^2 \epsilon(p_\perp^2)Y - 1}{(p_\perp - k_\perp)^2 \epsilon(p_\perp^2)} - \frac{(p_\perp - k_\perp)^2 \epsilon(k_\perp^2)Y - 1}{k_\perp^2 \epsilon(k_\perp^2)} \right).
\] (78)

Figure 1: The diagram represents the first term in the r.h.s. of Eq. \( G_5 \) expression.
To the leading order this expression reads as:

$$
\mathcal{G}^{a_{a_{a_1}}}_{+}(p_+,p_\perp,p_-;x^-;Y) = \frac{g^2 N}{4\pi^3} \ln(s/s_0) \delta^{a{a_1}} \left( \tilde{G}^{-0}_{x^0} - \tilde{G}^{-0}_{0x^-} \right)
$$

(79)

and taking into account that two last integrals are zero due to the ’t Hooft-Veltman conjecture in the dimensional regularization scheme, see [31], we correspondingly obtain:

$$
\mathcal{G}^{a_{a_{a_1}}}_{+}(p_+,p_\perp,p_-;x^-;Y) = -\frac{4}{p_\perp^2} \epsilon(p_\perp^2) \ln(s/s_0) \delta^{a{a_1}} \left( \tilde{G}^{-0}_{x^0} - \tilde{G}^{-0}_{0x^-} \right),
$$

(80)

see Eq. (82) definition. In Appendix D the calculations to all perturbative orders of Eq. (78) are performed and we obtain for the full answer:

$$
\mathcal{G}^{a_{a_{a_1}}}_{+}(p_+,p_\perp,p_-;x^-;Y) = \frac{2\delta^{a{a_1}}}{p_\perp^2} \left( 1 - e^{\epsilon(p_\perp^2)} Y - \int_0^{-\epsilon(p_\perp^2)} \frac{dy}{y} (e^{-y} - 1) \right) \left( \tilde{G}^{-0}_{x^0} - \tilde{G}^{-0}_{0x^-} \right).\tag{81}
$$

We notice, that this correlator arises due the presence in the effective action kinetic term of the \( B_+ \partial_{\perp}^2 A_- \) form with the Eq. (81) field’s change applied in some full 4-d effective action. Taking into account also the presences of the \( A_+ \partial_{\perp}^2 B_- \) kinetic term in the same action, we have an additional correction to the two fields correlator which can be obtained from the similar hierarchy after the variation with respect to \( A_+ \) field. This additional correction can be directly obtained from Eq. (81) answer by change of the + sign in the expression on the – sign everywhere and corresponding change of the coordinates:

$$
\mathcal{G}^{a_{a_{a_1}}}_{-}(p_+,p_\perp,p_-;y^+;Y) = \frac{2\delta^{a{a_1}}}{p_\perp^2} \left( 1 - e^{\epsilon(p_\perp^2)} Y - \int_0^{-\epsilon(p_\perp^2)} \frac{dy}{y} (e^{-y} - 1) \right) \left( \tilde{G}^{-0}_{y^0} - \tilde{G}^{-0}_{0y^+} \right).\tag{82}
$$

Therefore, as the propagator reggeized gluons we can consider the following sum of three different functions:

$$
D^{ab}(x^+,x^-;y^+,y^-;y^+,y^-;Y) = D^{ab}(x_+,y_+;Y) + \mathcal{G}^{a_{a_{a_1}}}_{+}(x^+,x_-,y_-,y_+;Y) + \mathcal{G}^{a_{a_{a_1}}}_{-}(x^+,x_-,y_+,y_-,y_+;Y).\tag{83}
$$

We note, that this expression violates the reggeized form of the initial Eq. (30) propagator as well as it’s dependence only on transverse coordinates.

6 Non-linear unitary correction to the propagator of reggeized gluons

In the above equations for the three and four Reggeon fields correlators, Eq. (66) - Eq. (69), we took into account only the bare QCD RFT corrections in the corresponding equations. The additional one-loop QCD correction in the equations will be provided by Eq. (19) vertex, namely the equation for the triple fields correlator with this correction reads as:

$$
\partial_{\perp}^2 x < A^a_+(x^+,x_+)A^{a_1}_+(y^+,y_+)A^{a_2}_+(z^-,z_+)> =
$$

(84)

$$
\int d^2w_1 d\omega_1 d^2w_1 K_{a_{a_{a_1}}a_{a_2}}(w_1;x^-,x_+;w_1,w_1) < A^a_+(x^+,w_+;y^+;y_+)A^{a_1}_+(y^+,y_+)A^{a_2}_+(z^-,z_+)A^{a_1}_+(w_1,w_1)>
$$

$$
+ \int d^2w_1 K_{a_{a_{a_1}}a_{a_2}}(w_1;x_+) < A^a_+(x^+,w_+)A^{a_1}_+(y^+,y_+)A^{a_2}_+(z^-,z_+)>,
$$

(85)

The considered in the paper Dyson-Schwinger hierarchy is derived from the variation of the effective action with respect to the \( A_- \) field only.
see [7]. Formally, this contribution means the following replacement of the operator in the l.h.s. of the equation:
\[ \partial_\perp^2 \rightarrow \partial_\perp^2 - K_{+-}. \] (86)
We note that the propagator Eq. (30) is a Green’s function of the operator:
\[ (\partial_\perp^2 \delta^{ab} - K^{ab}_{+-}) \ D^{bc} = -\delta^{ac}, \] (87)
see Eq. (28) expression. Therefore, account of this one-loop QCD corrections means the replacement
\[ D^{ab}_0(x_\perp, y_\perp) \rightarrow D^{ab}(x_\perp, y_\perp) \] (88)
everywhere in the above expressions for the correlators of three and four Reggeon fields. For example, we will obtain instead Eq. (67) the following expression
\[ \langle A_{+}^{a_1}(x^+, w_{\perp}) A_{+}^{a_2}(y^+, y_{\perp}) A_{-}^{a_3}(z^-, w_{\perp}) \rangle = \int d^2 z_\perp d^2 z_{\perp} \int d^2 w_\perp \int d^2 w_{\perp} \frac{1}{w_1^2 w_2^2} \right\}
\[ D(z_\perp, w_{\perp}) K_{+-}^{a_1 b_1 a_3} w_{\perp} \int d^2 w_{\perp} \frac{1}{w_1^2 w_2^2} \right\}
\[ \langle A_{+}^{a_1}(x^+, z_{\perp}) A_{+}^{a_2}(w_{\perp}, w_{\perp}) \rangle > \delta(x^+) \delta(y^-), \] (91)
here the one loop QCD correction is accounted for the two fields correlator as well, see Fig. (2). The transverse structure of the above integral is not changing in comparison to the obtained answer, the only change is in the rapidity dependence of the integral, namely we have instead Eq. (77):
that gives after the rapidity integration:

\[ \tilde{G}^{aa_1}_{\pm}(p_+, p_\perp, p_-; x^0; Y) = \frac{g^2 N}{4\pi^3} \delta^{aa_1} \left( \tilde{G}^{-0}_{x^0} - \tilde{G}^{-0}_{0x^0} \right) \int d^2k_\perp \frac{e^{2\epsilon(k_\perp^2)Y} - e^{\epsilon(k_\perp^2)Y}}{k_\perp^2 (p_\perp - k_\perp)^2 \epsilon(k_\perp^2)}. \]  

(93)

Formally the first term in the integral is different from the Eq. (78) answer by 2 in the front of the trajectory, therefore, using results of Appendix D, we obtain after the integration on momenta:

\[ \tilde{G}^{aa_1}_{\pm}(p_+, p_\perp, p_-; x^0; Y) = \frac{2 \delta^{aa_1}}{p_\perp^2} \left( e^{\epsilon(p_\perp^2)Y} - e^{2\epsilon(p_\perp^2)Y} - \int_{-\epsilon(p_\perp^2)Y}^{-2\epsilon(p_\perp^2)Y} dy \left( e^{-y} - 1 \right) \left( \tilde{G}^{-0}_{x^0} - \tilde{G}^{-0}_{0x^0} \right) \right) \]  

(94)

with corresponding change of Eq. (82) expression and full answer for the propagator Eq. (83).

7 Conclusion

In the Lipatov’s effective action formalism we have an additional source of perturbative and unitary corrections to high energy QCD amplitudes which is based on the diagrams constructed entirely in terms of Reggeon fields. In this paper we calculated for the first time the one loop RFT contribution to the propagator of reggeized gluons in the framework of Lipatov’s effective action and Dyson-Schwinger hierarchy of the equations for the correlators of reggeized gluon fields. This correction to the propagator is the first and the simplest non-linear RFT correction which combines the effective vertices of the theory to one loop QCD precision and one RFT loop constructed from the propagators of reggeized gluons.

The main results of the article, therefore, are the corrections to the propagator of reggeized gluons given by contributions in Eq. (51)–Eq. (59) to the bare QCD precision. The important properties of the obtained expressions are that the pole structure of the vertices is determined by the Green’s functions operators in the expression and can be different depending on the these operator’s representation, see Appendix B. The knowledge of these vertices, in turn, allows to consider the theory with Eq. (8) effective action as an usual quantum field theory with the interaction of three Reggeon fields included and correspondingly allows to calculate RFT perturbative corrections to any object of interests in the theory. The next step in the further development of the theory is the calculation of these vertices to one loop QCD precision similarly to done for the vertex of two Reggeon interactions\(^5\) in Section 3. The one loop QCD triple vertices will change the rapidity structure of the two reggeized gluons correlator as well and will allow understand better the quantum structure of the theory.

The main results of the article, therefore, are the corrections to the propagator of reggeized gluons given by contributions in Eq. (51)–Eq. (59) and Eq. (91). We constructed the one loop RFT corrections to the Eq. (51) propagator taking into account the Reggeon fields correlator to one QCD loop precision and bare triple Reggeon vertices only. The form of the obtained one-loop contribution, Eq. (60), demonstrated the inconsistency of the RFT based on the Reggeon fields of the Eq. (51) types only. This result was expected, see notes in [1, 4], but surprisingly this modification of the Reggeon field was required already at one RFT loop contribution. In general, therefore, the consistent construction of QCD RFT requires an use of the 4-d Reggeon fields, see Eq. (61)–Eq. (62) expressions. Correspondingly, obtained corrections Eq. (52) and Eq. (91) can be considered as some bare contributions in closed equations for the mutual correlators of \(D_\pm\) and \(A_\pm\) Reggeon fields, similar to the first term in the r.h.s. of Eq. (51) for example, which can be derived in the framework with 4-d \(B_\pm\) Reggeon fields included. This construction of the Lipatov’s effective action in terms \(B_\pm\) Reggeon fields is very interesting and important task which we postpone for the future work.

It turns out that the expressions obtained are depend as well on the longitudinal coordinates, see Eq. (53), that violates the only transverse coordinates dependence of the Eq. (51) propagator. Moreover, there is additional part of the corrections that breaks the propagator’s reggeization. Namely,

\(^5\)Work in progress
there is the dependence on the coupling constant in the expression which is not "sitting" in the reggeized gluon’s trajectory in the exponent. The change of the propagator’s form is more drastic if we account the one loop QCD correction in the expressions for the three and four Reggeon correlators. In this case the rapidity dependence of the corrections even is more complicated, see Eq. (94) expression. Another important future of the both non-linear corrections is that they contribute to the final propagator with the sign opposite to the sign of perturbative terms from the Eq. (34) trajectory expansion at the each perturbative order. It means that these corrections are unitary ones but the unitarity of the propagator is achieved by the adding of some non-linear terms to the usual propagator and not by the perturbative corrections to the gluon’s trajectory, is also a new result of the calculations.

The next important step to be considered is the calculation of the BFKL Pomeron on the base of the Eq. (83) form of the reggeized gluons propagator. Indeed, the infrared divergence of the obtained propagator is different from the divergence of the usual trajectory function, Eq. (32), by only the coefficients in the front of each $1/\varepsilon^n$ term in the corresponding expansion. Therefore, the interesting subject of the future research is the form of the four Reggeon colorless correlator, BFKL Pomeron, obtained on the base of RFT Dyson-Schwinger hierarchy of the equations for the correlators. Namely, it must be checked that the infrared divergences will be absent there as well as in the usual case and also the important question to investigate is about the form and rapidity dependence of this modified Pomeron.

In conclusion we emphasize, that the article is considered as an additional step to the developing of the high energy QCD RFT which will help clarify the non-linear RFT unitary corrections to the amplitudes of high energy processes.
Appendix A: Bare gluon propagator in light-cone gauge

In order to reproduce the expression for the gluon fields bare propagator\(^\text{\textsuperscript{6}}\) in the light-cone gauge we solve the following system of equations

\[
M^{0\mu\nu} G_{0\nu\rho} = \delta^\mu_\rho \tag{A.1}
\]

with

\[
g_\nu^\mu = \begin{pmatrix}
0 & 1 & 0 & 0 \\
1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1 \\
\end{pmatrix}, \quad \mu, \nu = (+, -, \perp), \tag{A.2}
\]

see also Eq. (A.21) below. The expression for the \(M_{0\mu\nu}\) matrix can obtained from the bare gluon’s Lagrangian for the gluon’s fluctuations field, in light-cone gauge it has the following form:

\[
L_0 = -\frac{1}{2} \varepsilon_i^a \delta_{ab} (\delta_{ij} \Box + \partial_i \partial_j) \varepsilon_j^b + \varepsilon_i^a \partial_- \partial_i \varepsilon_i^a - \frac{1}{2} \varepsilon_+^a \partial_+^2 \varepsilon_+^a = -\frac{1}{2} \varepsilon_-^a M_{0\mu\nu} \varepsilon_\nu \delta^{ab}, \tag{A.3}
\]

In the following system of equations

\[
\begin{align*}
M^{i+} 0_{0ij} + M^{i+} 0_{0kj} &= \delta^i_j \\
M^{0+} 0_{0ij} + M^{0+} 0_{0kj} &= \delta^0_+ \\
M^{00} 0_{0ij} + M^{00} 0_{0+j} &= 0 \\
M^{00} 0_{p+} + M^{00} 0_{0++} &= 0,
\end{align*} \tag{A.4}
\]

the last two equations we can consider as definitions of corresponding Green’s functions:

\[
G_{0+i} = -M^{-1}_{0++} M^{+j} 0_{0ji}, \tag{A.5}
\]

and

\[
G_{0i+} = -M^{-1}_{0ij} M^{j+} 0_{0++}. \tag{A.6}
\]

Here for

\[
M_{0pj} = \delta_{pj} \Box + \partial_p \partial_j, \quad M_{0p-} = -\partial_p \partial_-, \quad M_{0-} = \partial_-^2 \tag{A.7}
\]

we have

\[
M_{0ij}^{-1}(x, y) = -\int \frac{d^4p}{(2\pi)^4} \frac{e^{-ip(x-y)}}{p^2} \left( \delta_{ij} - \frac{p_i p_j}{2(p_\perp p_+)} \right) \tag{A.8}
\]

and correspondingly

\[
M_{01}^{-1}(x, y) = -\int \frac{d^4p}{(2\pi)^4} \frac{e^{-ip(x-y)}}{p_\perp^2}. \tag{A.9}
\]

Therefore, for the two remaining Green’s functions we obtain:

\[
\left( M_{00}^{++} - M_{0ij}^{+1} M_{0ij}^{j+} \right) G_{0++} = \delta^+_+, \tag{A.10}
\]

and

\[
\left( M_{00}^{ik} - M_{0ij}^{+1} M_{0ij}^{j+k} \right) G_{0kj} = \delta^i_j. \tag{A.11}
\]

Performing Fourier transform of the functions, we write the Eq. (A.11) in the following form:

\[
-\int \frac{d^4p}{(2\pi)^4} \left( \delta^{ik} p_\perp^2 + p_i^+ p_k^+ \right) e^{-ip(x-y)} \tilde{G}_{0kj}(p) + \int \frac{d^4p}{(2\pi)^4} p_\perp^2 p_i^+ p_k^+ \frac{e^{-ip(x-y)}}{p_\perp^2} \tilde{G}_{0kj}(p) = \delta^i_j \tag{A.12}
\]

\(^6\)We suppress color and coordinate notations in the definition of the propagators below.
that provides:

\[ G_{0ij}(x,y) = - \int \frac{d^4p}{(2\pi)^4} \frac{e^{-ip(x-y)}}{p^2} \delta_{ij}. \]  

(A.13)

Correspondingly, for Eq. (A.10) we have:

\[- \int \frac{d^4p}{(2\pi)^4} p_i^2 e^{-ip(x-y)} \tilde{G}_{0++}(p) + \int \frac{d^4p}{(2\pi)^4} \frac{p^2 p_i p_j}{p^2} \left( \delta_{ij} - \frac{p_i p_j}{2(p_+ p_-)} \right) e^{-ip(x-y)} \tilde{G}_{0++}(p) = \delta^+ \]

(A.14)

that can be rewritten as

\[ \int \frac{d^4p}{(2\pi)^4} e^{-ip(x-y)} \left( -p_+^2 + p_-^2 \frac{p_i p_i}{p^2} - p_-^2 \frac{p_i p_i (p_j p_j)}{2p^2 (p_+ p_-)} \right) \tilde{G}_{0++}(p) = \delta^+. \]  

(A.15)

Writing this expression as

\[ \int \frac{d^4p}{(2\pi)^4} e^{-ip(x-y)} \left( -p_+^2 - p_-^2 \frac{p_i p_i}{p^2} - p_-^2 \frac{p_i p_i (p_j p_j)}{2p^2 (p_+ p_-)} \right) \tilde{G}_{0++}(p) = \delta^+. \]  

(A.16)

we obtain finally for the Green’s function

\[ G_{0++}(x,y) = - \int \frac{d^4p}{(2\pi)^4} \frac{e^{-ip(x-y)}}{p^2} \frac{2p_+}{p_-}. \]  

(A.17)

Inserting Eq. (A.13) and Eq. (A.17) functions in Eq. (A.5)-Eq. (A.6) definitions we obtain for the last two Green’s functions:

\[ G_{0i+} = G_{0+i} = \int \frac{d^4p}{(2\pi)^4} \frac{e^{-ip(x-y)}}{p^2} \frac{p_i}{p_-}. \]  

(A.18)

Now, introducing the following vector in light-cone coordinates

\[ n_\mu^\dagger = (1, 0, 0), \quad \mu = (+, -, \perp) \]  

(A.19)

we can write the whole propagator as

\[ G_{0\mu\nu}(x,y) = \int \frac{d^4p}{(2\pi)^4} \frac{e^{-ip(x-y)}}{p^2} \left( g_{\mu\nu} - \frac{p^\sigma n^\perp + p^\rho n_\rho^\perp}{p^\rho n_\rho^\perp} g_{\mu\sigma} g_{\nu\rho} \right) \]  

(A.20)

with

\[ g_{\mu\nu} = g_{\mu^\dagger \nu^\dagger} = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix} \quad \mu, \nu = (+, -, \perp) \]  

(A.21)

and where Kogut-Soper convention, [], for the light-cone notations and scalar product is used:

\[ px = p_+ x^+ + p_- x^- + p_i x^i. \]  

(A.22)
Appendix B: Lipatov’s effective current

For the arbitrary representation of $v_+ = iT^a v_+^a$ with $D_+ = \partial_+ - g v_+$, we can consider the following representation of $O$ and $O^T$ operators:

$$O_x = \delta^{ab} + g \int d^4 y G_{xy}^{a a_1} (v_+(y))_{a_1 b} = 1 + g G_{xy}^+ v_+ y \quad (B.1)$$

and correspondingly

$$O^T_x = 1 + g v_+ y G_{yx}^+ \quad (B.2)$$

which is redefinition of the operator expansions used in [1] in terms of Green’s function instead integral operators, see Appendix B above. The Green’s function in above equations we understand as Green’s function of the $D_+$ operator and express it in the perturbative sense as:

$$G_{xy}^+ = G_{xy}^{+0} + g G_{xy}^{+0} v_+ z G_{zy}^+ \quad (B.3)$$

and

$$G_{yx}^+ = G_{yx}^{+0} + g G_{yx}^{+0} v_+ z G_{zx}^+ \quad (B.4)$$

with the bare propagators defined as (there is no integration on index $x$ in expressions)

$$\partial_{+x} G_{xy}^{+0} = \delta_{xy}, \quad G_{yx}^{+0} \partial_{+x} = -\delta_{xy} \quad .$$

The following properties of the operators now can be derived:

1. $$\delta G_{xy}^+ = g G_{xz}^{+0} (\delta v_+) G_{zy}^+ + G_{xz}^{+0} v_+ z G_{zy}^{+0} \delta G_{zy}^+ = g G_{xz}^{+0} (\delta v_+) G_{zy}^+ + G_{xz}^{+0} v_+ z (\delta G_{zy}^+) D_+ p G_{py}^+ =$$

$$= g \left( G_{xz}^{+0} (\delta v_+) G_{zy}^+ - G_{xz}^{+0} v_+ z G_{zy}^+ (\delta D_+ p) G_{py}^+ \right) = g \left( G_{zp}^{+0} + G_{zp}^{+0} v_+ z G_{zy}^+ \right) (\delta v_+ p) G_{py}^+ =$$

$$= g G_{zp}^+ \delta v_+ p G_{py}^+ \quad (B.6)$$

2. $$\delta O_x = g G_{xy}^+ (\delta v_+) + g (\delta G_{xy}^+) v_+ y = g G_{zp}^+ (1 + g G_{py}^+ v_+ y) = g G_{zp}^+ \delta v_+ p O_p \quad ; (B.7)$$

3. $$\partial_{+x} \delta O_x = g \left( \partial_{+x} G_{zp}^+ \right) \delta v_+ p O_p = g \left( 1 + g v_+ x G_{zp}^+ \right) \delta v_+ p O_p = g O_x^T \delta v_+ x O_x \quad ; (B.8)$$

4. $$\partial_{+x} O_x = g \left( \partial_{+x} G_{xy}^+ \right) v_+ y = g v_+ x (1 + g G_{xy}^+ v_+ y) = g v_+ x O_x \quad ; (B.9)$$

5. $$O_x^T \partial_{+x} = g v_+ y \left( G_{yx}^+ \partial_{+x} \right) = -g \left( 1 + v_+ y G_{yx}^+ \right) v_+ x = -g O_x^T v_+ x \quad . \quad (B.10)$$

We see, that the operator $O$ and $O^T$ have the properties of ordered exponents. For example, choosing bare propagators as

$$G_{xy}^{+0} = \theta(x^+ - y^+) \delta^3_{xy}, \quad G_{yx}^{+0} = \theta(y^+ - x^+) \delta^3_{xy} \quad , (B.11)$$

$^5$Due the light cone gauge we consider here only $O(x^+)$ operators. The construction of the representation of the $O(x^-)$ operators can be done similarly. We also note, that the integration is assumed for repeating indexes in expressions below if it is not noted otherwise.
we immediately reproduce:

\[ O_x = P e^g \int_{-\infty}^{x^+} dx^+ v_+(x^+) \quad \text{and} \quad O^T_x = P e^g \int_{x^+}^{\infty} dx^+ v_+(x^+) . \tag{B.12} \]

The form of the bare propagator \( G_{xy}^{0} = \frac{1}{2} [\theta(x^+ - y^+) - \theta(y^+ - x^+)] \delta^3_{xy} \) will lead to the more complicated representations of \( O \) and \( O^T \) operators, see in \([11, 12]\) and \([3, 4]\). We note also that the Green's function notation \( G_{xy}^{\pm 0} \) in the paper is used for the designation of the only theta function part of the full \( G_{x^\pm y^\pm} \) Green's function.

Now we consider a variation of the action's full current:

\[
\delta \text{tr} [v_+ O_x \partial_i^2 A^+] = \frac{1}{g} \delta \text{tr} [(\partial_{+x} O_x) \partial_i^2 A^+] = \frac{1}{g} \text{tr} [(\partial_{+x} \delta O_x) \partial_i^2 A^+] = \text{tr} [O^T_x \delta v_+ O_x (\partial_i^2 A^+)] ,
\]

which can be rewritten in the familiar form used in the paper:

\[
\delta (v_+ J^+) = \delta \text{tr} [(v_+ O_x \partial_i^2 A^+)] = - \delta v_+ \text{tr} [T_a O T_b O^T] (\partial_i^2 A^+) . \tag{B.14}\]

We also note, that with the help of Eq. \([A.3]\) representation of the \( O \) operator the full action’s current can we written as follows

\[
\text{tr} [(v_+ O_x - A_+) \partial_i^2 A^+] = \text{tr} [(v_+ - A_+ + v_+ G_{xy}^+ v_y) (\partial_i^2 A^+)] . \tag{B.15}\]
Appendix C: NLO vertex of interactions of reggeized gluons

The NLO one-loop vertex of reggeized gluons interactions is defined in the formalism as

$$- 2i K_{x y 1}^{a b} = \left( \frac{\delta^2 \ln (1 + G_0 M)}{\delta A^a_{+x} \delta A^b_{-y}} \right)_{A_+, A_-, v_f \perp = 0}$$

$$= \left[ G_0 \frac{\delta^2 M}{\delta A^a_{+x} \delta A^b_{-y}} (1 + G_0 M)^{-1} - G_0 \frac{\delta M}{\delta A^b_{-y}} (1 + G_0 M)^{-1} G_0 \frac{\delta M}{\delta A^a_{+x}} (1 + G_0 M)^{-1} \right]_{A_+, A_-, v_f \perp = 0}$$ (C.1)

where the trace of the expression is assumed. With the help of Eq. (15), see also [4], we have correspondingly:

$$- 2i K_{x y 1}^{a b} = \left[ G_0 \frac{\delta^2 M}{\delta A^a_{+x} \delta A^b_{-y}} - G_0 \frac{\delta M}{\delta A^b_{-y}} G_0 \frac{\delta M}{\delta A^a_{+x}} \right]_{A_+, A_-, v_f \perp = 0}.$$ (C.2)

Taking into account the asymptotically leading contributions of $g^2$ order, that means the $M_L$ term presence in the expressions, see [4] [23], we obtain:

$$- 2i K_{x y 1}^{a b} = \left[ G_0 \frac{\delta^2 M_L}{\delta A^a_{+x} \delta A^b_{-y}} - G_0 \frac{\delta M_L}{\delta A^b_{-y}} G_0 \frac{\delta M_L}{\delta A^a_{+x}} \right]_{A_+, A_-, v_f \perp = 0}.$$ (C.3)

For the first term we have:

$$- 2i K_{x y 1, 1}^{a b} = G_0 \frac{\delta^2 M_L}{\delta A^a_{+x} \delta A^b_{-y}} = G_0^{z+} \frac{g}{N} \frac{\delta (U_{1cd}^{(a)})^{+}}{\delta A^a_{+x}} \frac{\delta \partial^2 A^d}{\delta A^a_{-y}}$$ (C.4)

where the following identity was used:

$$\partial_i \partial_- \rho_a^i = - \frac{1}{N} \partial^2_i A_a^i,$$ (C.5)

see Eq. (15) and [4]. Using the following expressions

$$\frac{\delta (U_{1cd}^{(a)})^{+}}{\delta A^a_{+x}} = g (U_{2}^{(cd1)})^{++} \frac{\delta v_{+w}^{a1 cd}}{\delta A^a_{+x}}$$ (C.6)

and

$$\frac{\delta v_{+w}^{a1 cd}}{\delta A^a_{+x}} = \delta^a a_1 (\delta^{2}_{x_{+w}^{+}} \delta_{x^{+}+w^{+}})$$ (C.7)

to requested accuracy, we obtain for Eq. (C.4):

$$- 2i K_{x y 1, 1}^{a b} = \frac{g^2}{N} G_0^{z+} (U_{2}^{(cd1)})^{++} \left( \delta^{a a_1} \delta^{2}_{x_{+w}^{+}} \delta_{x^{+}+w^{+}} \right) \left( \delta^{2}_{y_{+z}^{+}} \delta_{y^{+}+z^{+}} - \delta^{2}_{z_{+w}^{+}} \right),$$ (C.8)

where the NNLO term of the Lipatov’s current series expansion reads as

$$\left( U_{2}^{(cd1)} \right)^{++} = \frac{1}{2} N^2 \delta^{a b} \left[ \left( G^{0+}_{zw} + G^{0+}_{wt} \right) + 2 \left( G^{0+}_{tw} + G^{0+}_{tw} + G^{0+}_{tw} + G^{0+}_{tw} + G^{0+}_{tw} + G^{0+}_{tw} \right) \right].$$ (C.9)

Therefore, writing explicitly all integrations in the expression, we obtain:

$$- 2i K_{x y 1, 1}^{a b} = \frac{1}{2} g^2 N \delta^{a b} \int d^2 z \ d^4 t \ d^4 w \left( \delta^{2}_{x_{+w}^{+}} \delta_{x^{+}+w^{+}} \right) \cdot \left( \left( G^{0+}_{zw} + G^{0+}_{tw} \right) + 2 \left( G^{0+}_{tw} + G^{0+}_{tw} + G^{0+}_{tw} + G^{0+}_{tw} + G^{0+}_{tw} + G^{0+}_{tw} \right) \right).$$ (C.10)
Formally, there are three additional terms are present in Eq. (C.2). The first one
\[ -2i K_{xy,1,2} = -G_{0+} \frac{\delta M_L}{\delta A_y^{-}} G_{0+i} \frac{\delta M_{1-i}}{\delta A_x^{+}} , \]  
(C.11)
the second one
\[ -2i K_{xy,1,3} = -G_{0+i} \frac{\delta M_L}{\delta A_y^{-}} G_{0++} \frac{\delta M_{1-i}}{\delta A_x^{+}} , \]  
(C.12)
and the third one
\[ -2i K_{xy,1,4} = -G_{0+i} \frac{\delta M_L}{\delta A_y^{-}} G_{0+i} \frac{\delta M_{1+i}}{\delta A_x^{+}} . \]  
(C.13)
Nevertheless, only the third one contributes to the kernel in the limit of zero Reggeon fields, we have there:
\[ \frac{\delta M_d^{cd}}{\delta A_x^{+}} = \frac{g}{N} \left( U_1^{c d} \right)_t^{+} (\delta_{y-,t} \delta_{y-t} \partial_{t}^2) \]  
where
\[ \left( U_1^{c d} \right)_t^{+} = \frac{1}{2} N f_{c d b} (G_{l w}^{+0} - G_{l w}^{+0}) . \]  
(C.14)
Also we have:
\[ \frac{\delta M_{j i}^{j c}}{\delta A_x^{+}} = 2 g f_{d a c} \delta_{j i} \delta_{z_+} \delta_{z+} \delta_{x_+} \delta_{x} \partial_{x}^2 . \]  
(C.15)
The final expression for this terms reads, therefore, as:
\[ -2i K_{xy,1,4} = -g^2 N \delta^{ab} \int d^4t d^4w d^4z \left( G_{l w}^{+0} - G_{l w}^{+0} \right) \delta_{z_+} \delta_{z+} \delta_{x_+} \delta_{x} \left( G_{0+i} \partial_{x}^2 \right) . \]  
(C.17)
We notice that both Eq. (C.17) and Eq. (C.10) contributions are precisely the same as obtained in [4] paper. Therefore, we immediately write the full contribution from [4] which is
\[ K_{xy,1} = -N g^2 \frac{8 \pi}{\delta_{ix}} \int \frac{dp_{-}}{p_{-}} \int \frac{d^2p_{i}}{(2\pi)^2} \int \frac{d^2k_{i}}{(2\pi)^2} \frac{k_{i}^2}{p_{-}^2 (p_{-} - k_{i})^2} e^{-i p_{-}(x^i - y^i)} . \]  
(C.18)
We can rewrite this expression redefining the vertex in Eq. (28) as
\[ K_{xy,1} \rightarrow K_{xy,1} \partial_{ix}^2 = \int \frac{d^2p}{(2\pi)^2} \tilde{K}(p) e^{-ip_{-}(x^i - y^i)} \partial_{ix}^2 \]  
(C.19)
with
\[ \tilde{K}(p, \eta) = -N \frac{g^2}{2} \delta(p_{+}) \delta(p_{-}) \int_{0}^{\eta} d\eta' \int \frac{d^2k_{i}}{(2\pi)^2} \frac{k_{i}^2}{k_{i}^2 (p_{-} - k_{i})^2} . \]  
(C.20)
where the physical cut-off \( \eta \) in rapidity space \( y = \frac{1}{2} \ln(A k_{-}) \) is introduced.
Appendix D: calculation of Eq. (78) integral

The Eq. (78) answer can be obtained immediately if we will note that Eq. (67) expression is not symmetrized in respect with \(a_4\) and \(a_1\) color indexes. Indeed, we can use the same expression with the indexes permuted (we also can use fully symmetrized expression) obtaining immediately Eq. (81) answer. Nevertheless, it is instructive to calculate the first r.h.s. term of Eq. (78) directly, we have for the next to leading order terms in the integral:

\[
I = \frac{4^{1+\varepsilon}}{\pi^{1-\varepsilon}} \sum_{n=2}^{\infty} \frac{(-1)^{n-1} Y^n}{n!} \left( \frac{(N \alpha_s)^n}{(4\pi)^{n(1+\varepsilon)}} \right) \Gamma^{n-1}(1-\varepsilon) \left( \frac{2}{\varepsilon} \right)^{n-1} \int \frac{d^2k}{(k^2)^{1-\lambda} (p_\perp - k_\perp)^2}, \tag{D.1}
\]

with \(\lambda = (n-1)\varepsilon\). Now we use standard formulas:

\[
\int d^Dk \frac{1}{k^2(p_\perp - k_\perp)^{2b}} = \frac{(2\pi)^D}{(4\pi)^D/2} \frac{\Gamma(D/2-a) \Gamma(D/2-b) \Gamma(a+b-D/2)}{\Gamma(a) \Gamma(b) \Gamma(D-a-b)} \frac{1}{(p_\perp^2)^{a+b-D/2}}, \tag{D.2}
\]

Taking \(D = 2 + 2\varepsilon_1\) and taking at the end \(\varepsilon_1 \to \varepsilon\) we see that the obtained answer is proportional to

\[
\frac{\Gamma(\varepsilon_1 + \lambda) \Gamma(\varepsilon_1) \Gamma(1 - \varepsilon_1 - \lambda)}{\Gamma(1 - \lambda) \Gamma(2\varepsilon_1 + \lambda)} \propto \frac{1}{\varepsilon} \left( 1 + \frac{1}{n} \right) + \gamma_E \left( 1 + \frac{1}{n} \right). \tag{D.3}
\]

Therefore we obtain for the Eq. (4.1) sum:

\[
I = -\frac{4^{1+\varepsilon}}{p_\perp^2 \pi^{2-2\varepsilon}} \frac{\varepsilon}{2\Gamma(1-\varepsilon)} \sum_{n=2}^{\infty} \frac{(-1)^n Y^n}{n!} \left( \frac{(N \alpha_s)^n}{(4\pi)^{n(1+\varepsilon)}} \right) \Gamma^n(1-\varepsilon) \left( \frac{2}{\varepsilon} \right)^n \left( \frac{1}{\varepsilon} \left( 1 + \frac{1}{n} \right) + \gamma_E \left( 1 + \frac{1}{n} \right) \right) \left( p_\perp^2 \right)^{n\varepsilon}. \tag{D.4}
\]

We note, that the answer for the Eq. (78) propagator reproduces Eq. (80) leading order expression if we will take \(n = 1\) in the sum, therefore the full answer can be obtained by expanding the summation in the expression to \(n = 1\):

\[
\delta^{ab}_{\varepsilon} \propto -\delta^{ab} \frac{4^{1+\varepsilon}}{p_\perp^2 \pi^{2-2\varepsilon}} \frac{\varepsilon}{2\Gamma(1-\varepsilon)} \sum_{n=1}^{\infty} \frac{(-1)^n Y^n}{n!} \left( \frac{(N \alpha_s)^n}{(4\pi)^{n(1+\varepsilon)}} \right) \Gamma^n(1-\varepsilon) \left( \frac{2}{\varepsilon} \right)^n \left( \frac{1}{\varepsilon} \left( 1 + \frac{1}{n} \right) + \gamma_E \left( 1 + \frac{1}{n} \right) \right) \left( p_\perp^2 \right)^{n\varepsilon} =
\]

\[
= -2 \delta^{ab} \frac{\varepsilon}{p_\perp^2} \left( \sum_{n=1}^{\infty} \frac{(\varepsilon(p_\perp^2) Y)^n}{n!} + \sum_{n=1}^{\infty} \frac{(\varepsilon(p_\perp^2) Y)^n}{n^2 n!} \right) = 2 \delta^{ab} \frac{\varepsilon}{p_\perp^2} \left( 1 - e^{\varepsilon(p_\perp^2) Y} - \int_0^{-e^{\varepsilon(p_\perp^2) Y}} dy \left( e^{-y} - 1 \right) \right). \tag{D.5}
\]
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