Particle Physics I: The SM and the MSSM

R. D. Peccei

Department of Physics and Astronomy, UCLA, Los Angeles, CA 90095-1547

Abstract

After discussing alternative scenarios for the origin of the electroweak symmetry breaking, I briefly review the experimental status of the Standard Model. I explore further both the hints for, and constraints on, supposing that a supersymmetric extension of the Standard Model exists, with supersymmetry broken at the weak scale. I end with a few comments on the theoretical implications of the recent evidence for neutrino oscillations.

It is a sad honor to be able to speak at Inner Space/Outer Space II, a symposium in memory of David Schramm. Dave was an old friend, whose exuberance and enthusiasm I greatly miss. It was from him that I first realized that indeed the cosmos could tell us some things of importance for particle physics. It is a testament to his influence and vision that now no one doubts that much of what is interesting in high energy physics is writ large in the history of the Universe. A measure of the changes that have occurred since David first entered the field in the early 1970’s is that now cosmological data is often one of the few weapons that we have to exclude or constrain new ideas in particle physics. In this respect, Schramm’s famous limit on the allowed number of neutrino species, coming from Nucleosynthesis, has proven particularly effective as a theory “sorter”!

1 Theoretical Issues in the Standard Model

The Standard Model (SM), based on the gauge group $SU(3) \times SU(2) \times U(1)$, has proven very robust, with all precision electroweak data in excellent agreement with the predictions of the theory. Nevertheless, there remain important open questions in the SM. Chief among them is the mechanism which causes the spontaneous breakdown of $SU(2) \times U(1)$ to $U(1)_{em}$ and the nature of the symmetry breaking parameter $v_F$—the Fermi scale. Although the size of $v_F$ ($v_F \sim 250$ GeV) is known, its precise origin is yet unclear.

To understand some of the issues involved, it proves useful to examine the simplest example of symmetry breakdown in which the symmetry breaking is effected by just one complex Higgs doublet $\Phi$ in $\mathcal{L}_{SB}$. In this case, the Fermi scale $v_F$ enters directly as a scale parameter in the Higgs potential

$$V = \lambda \left[ \Phi^\dagger \Phi - \frac{1}{2} v_F^2 \right]^2. \quad (1)$$

The sign of the $v_F^2$ term is chosen to guarantee that $V$ will be asymmetric, with a minimum at a non-zero value for $\Phi^\dagger \Phi$. This triggers the breakdown of $SU(2) \times U(1)$ to $U(1)_{em}$, since it forces $\Phi$ to develop a non-zero VEV:

$$\langle \Phi \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} v_F \\ 0 \end{pmatrix}. \quad (2)$$

\[\text{With only one Higgs doublet one can always choose } U(1)_{em} \text{ as the surviving } U(1) \text{ in the breakdown.}\]
Because $v_F$ is an internal scale in the potential $V$, in isolation it clearly makes no sense to ask what physics fixes the scale of $v_F$. This question, however, can be asked if one considers the SM as an effective theory valid up to some very high cut-off scale $\Lambda$, where new physics comes in. In this broader context it makes sense to ask what is the relation of $v_F$ to the cut-off $\Lambda$. In fact, because the $\lambda\Phi^4$ theory is trivial, with the only consistent theory being one where $\lambda_{\text{ren}} \to 0$, considering the scalar interactions in $L_{SB}$ without some high energy cut-off is not sensible.

To appreciate this point, let’s compute the evolution of the coupling constant $\lambda$ from the Renormalization Group equation (RGE)

$$\frac{d\lambda}{d\ln q^2} = + \frac{3}{4\pi^2} \lambda^2 + \ldots$$

(3)

This equation, in contrast to what happens in QCD, has a positive rather than a negative sign in front of its first term, so $\lambda$ grows with $q^2$. As a result, if one solves the RGE, including only this first term, one finds a singularity at large $q^2$ which is a reflection of this growth

$$\lambda(q^2) = \frac{\lambda(\Lambda_0^2)}{1 - \frac{3\lambda(\Lambda^2_0)}{4\pi^2} \ln \frac{q^2}{\Lambda_0^2}}.$$ 

(4)

Of course, one cannot trust the location of the, so called, Landau pole derived from Eq. (3) $\Lambda_c^2 = \Lambda_0^2 \exp \left[ \frac{4\pi^2}{3\lambda(\Lambda_0^2)} \right]$, since Eq. (3) stops being valid when $\lambda$ gets too large. Nevertheless, for any given cut-off $\Lambda_c$, one can predict $\lambda(q^2)$ for scales $q^2$ sufficiently below this cut-off. Indeed, the $\lambda\Phi^4$ theory is perfectly sensible as long as one restricts oneself to $q^2 \ll \Lambda_c^2$. If one wants to push the cut-off to infinity, however, one sees that $\lambda(\Lambda_0^2) \to 0$. This is the statement of triviality, within this simplified context.

In the case of the SM, one can “measure” where the cut-off $\Lambda_c$ is in $L_{SB}$ from the value of the Higgs mass. Using the potential (2) one finds that

$$M_{H}^2 = 2\lambda(M_{H}^2)v_F^2.$$ 

(6)

Physics is rather different depending on whether the Higgs mass is light or is heavy with respect to $v_F$. If $M_{H}$ is light the effective theory described by $L_{SB}$ is very reliable, and weakly coupled, with $\lambda \leq 0.3$ up to very high scales. In these circumstances it is meaningful to ask whether the large hierarchy $v_F \ll \Lambda_c$ is a stable condition. This question, following ‘t Hooft, is often called the problem of naturalness.

If, on the other hand, the Higgs mass is heavy, of order of the cut-off ($M_{H} \sim \Lambda_c$), then it is pretty clear that $L_{SB}$ as an effective theory stops making sense. The coupling $\lambda$ is so strong that one cannot separate the particle-like excitations from the cut-off itself. Numerical investigations on the lattice have indicated that this occurs when

$$M_{H} \sim \Lambda_c \sim 700 \text{ GeV}.$$ 

(7)

In this case, it is clear that $\langle \Phi \rangle$, as the order parameter of the symmetry breakdown, must be replaced by something else.

The Planck scale $M_P$ is clearly a natural physical cutoff. So, in the weak coupling case, one has to worry whether the hierarchy $v_F \ll M_{P}$ is stable. It turns out that this is not the case, since radiative
effects in a theory with a cutoff destabilize any pre-existing hierarchy. Indeed, this was ’t Hooft’s original argument. Quantities like the Higgs mass that are not protected by symmetries suffer quadratic mass shifts. Schematically, the Higgs mass shifts from the value given in Eq. (6) to

\[ M_H^2 = 2\lambda v_F^2 + \alpha \Lambda_c^2 . \] (8)

It follows from Eq. (8) that if \( \Lambda_c \sim M_P \gg v_F \), the Higgs boson cannot remain light. If one wants the Higgs to remain light one is invited to look for some protective symmetry to guarantee that the hierarchy \( v_F \ll M_P \) is stable. Such a protective symmetry exists—it is supersymmetry (SUSY).

SUSY is a boson-fermion symmetry in which bosonic degrees of freedom are paired with fermionic degrees of freedom. If supersymmetry is exact then the masses of the fermions and of their bosonic partners are the same. In a supersymmetric version of the Standard Model all quadratic divergences cancel. Thus parameters like the Higgs boson mass will not be sensitive to a high energy cut-off. Via supersymmetry the Higgs boson mass is kept light since its fermionic partner has a mass protected by a chiral symmetry.

Because one has not seen any of the SUSY partners of the states in the SM yet, it is clear that if a supersymmetric extension of the SM exists then the associated supersymmetry must be broken. Remarkably, even if SUSY is broken the naturalness problem in the SM is resolved, provided that the splitting between the fermion-boson SUSY partners is itself of \( O(v_F) \). For example, the quadratic divergence of the Higgs mass due to a \( W \)-loop is moderated into only a logarithmic divergence by the presence of a loop of Winos, the spin-1/2 partners of the \( W \) bosons. Schematically, in the SUSY case, Eq. (8) gets replaced by

\[ M_H^2 = 2\lambda v_F^2 + \alpha(\tilde{M}_W^2 - M_W^2) \ln \Lambda_c/v_F . \] (9)

So, as long as the masses of the SUSY partners (denoted by a tilde) are themselves not split away by much more than \( v_F \), radiative corrections will not destabilize the hierarchy \( v_F \ll \Lambda_c \).

Let me recapitulate. Theoretical considerations regarding the nature of the Fermi scale have suggested two alternatives for new physics associated with the \( SU(2) \times U(1) \to U(1)_{em} \) breakdown:

i) \( \mathcal{L}_{SB} \) is the Lagrangian of some elementary scalar fields interacting together via an asymmetric potential, whose minimum is set by the Fermi scale \( v_F \). The presence of non-vanishing VEVs triggers the electroweak breakdown. However, to guarantee the naturalness of the hierarchy \( v_F \ll M_P \), both \( \mathcal{L}_{SB} \) and the whole SM Lagrangian must be augmented by other fields and interactions so as to be (at least approximately) supersymmetric. Obviously, if this alternative is true, there is plenty of new physics to be discovered, since all particles have superpartners of mass \( \tilde{m} \approx m + O(v_F) \).

ii) The symmetry breaking sector of the SM has itself a dynamical cut-off of \( O(v_F) \). In this case, it makes no sense to describe \( \mathcal{L}_{SB} \) in terms of strongly coupled scalar fields. Rather, \( \mathcal{L}_{SB} \) describes a dynamical theory of some new strongly interacting fermions \( F \), whose condensates cause the \( SU(2) \times U(1) \to U(1)_{em} \) breakdown. The strong interactions which form the condensates \( \langle \bar{F}F \rangle \sim v_F^3 \) also identify the Fermi scale as the dynamical scale of the underlying theory, very much analogous to \( \Lambda_{QCD} \). If this alternative turns out to be true, then one expects also to see lots of new physics, connected with these new strong interactions, when one probes them at energies of \( O(v_F) \).

\(^6\)Note that a stable hierarchy \( v_F \ll M_P \) does not explain why such a hierarchy exists.
2 Experimental Tests of the SM

The expectations of the SM, assuming the simplest form of symmetry breaking, have been confronted experimentally to high accuracy. These results provide already some important indications on the nature of the electroweak symmetry breakdown, which I review here. In practice, since all fermions but the top are quite light compared to the scale of the $W$ and $Z$-bosons, all quantities in this simplest version of the SM are specified as functions of 5 parameters: $g'$, $g_2$, $v_F$, $M_H$ and $m_t$. It proves convenient to trade the first three of these for another triplet of quantities which are better measured: $\alpha$, $M_Z$ and $G_F$. Once one has adopted a set of standard parameters then all physical measurable quantities can be expressed as a function of this "standard set". Because $\alpha$, $M_Z$, and $G_F$ as well as $m_t$ are rather accurately known, all SM fits essentially constrain only one unknown—the Higgs mass $M_H$. This constraint, however, is not particularly strong because all radiative effects depends on $M_H$ only logarithmically.

The result of the SM fit of all precision data gives for the Higgs mass

$$M_H = (98^{+57}_{-38}) \text{ GeV} \quad (10)$$

and the 95% C.L. upper bound: $M_H < 235$ GeV. It is particularly gratifying that this fit indicates the need for a light Higgs boson, since this “solution” is what is internally consistent. Furthermore, this result is also compatible with the limit on $M_H$ coming from direct searches for the Higgs boson in the process $e^+ e^- \rightarrow ZH$ at LEP 200. The limit given at the 1999 Lepton Photon Conference at SLAC is, at 95% C.L.,

$$M_H > 95.2 \text{ GeV} \quad (11)$$

By running LEP 200 at $\sqrt{s} = 200$ GeV in the coming year one opens up another 10 GeV of discovery potential for the Higgs boson. The present Tevatron bounds for $M_H$ are weaker, being roughly a factor of 20-50 too insensitive for $M_H = 95$ GeV. However, with the substantial luminosity increased planned, the Tevatron can explore a Higgs window up to $M_H = 110$-130 GeV before the turn-on of the LHC. The LHC, of course, has the capability of exploring the full range for $M_H$, well beyond the upper bound estimate (7).

The physical lower bound (11) suffices to rule out the possibility of electroweak baryogenesis within the context of this simplest version of the SM. To allow for electroweak baryogenesis it is necessary that the $SU(2) \times U(1)$ phase transition be strongly first order. Only in this case can one prevent having the (B+L)-violating interactions in the SM going back into equilibrium after the electroweak phase transition, thereby erasing any matter asymmetry established during the phase transition. One can show that to prevent erasing the established asymmetry one needs the order parameter at the phase transition to have a value $\langle \Phi(T^*) \rangle / T^* \geq 1$. Such a large jump in the Higgs VEV, however, only occurs for relatively light Higgs boson masses—typically $M_H \lesssim 50$ GeV with $\langle \Phi(T^*) \rangle / T^*$ decreasing rather rapidly as $M_H$ increases.

Within the context of this simplest version of the SM one expects $M_H$ to be larger than the bound (11) from the requirement of vacuum stability. The argument is rather simple. Because top is rather heavy, in the RGE for the Higgs coupling $\lambda$ one cannot neglect the effect of the top Yukawa coupling. Thus, instead of Eq. (3) one has

$$\frac{d\lambda}{d\ln \mu^2} = \frac{3}{4 \pi^2} \left[ \lambda^2 - \frac{1}{4} \lambda_t^4 \right] + \ldots \quad (12)$$

9The top mass is quite accurately determined now. The combined value obtained by the CDF and DO collaborations fixes $m_t$ to better than 3%: $m_t = (173.8 \pm 5.0)$ GeV.

dIn fact, for the minimal SM, the matter asymmetry established at the electroweak phase transition (before its erasure) is also much below what is needed because there is not enough CP violation, due to GIM suppression factors.
Because the top contribution comes with a negative sign, it will slow down and can actually reverse the growth of $\lambda$. Indeed, if the Higgs coupling $\lambda(M_H)$ is not large enough, because the Higgs boson is light, the contribution coming from the $\lambda^4 t$ term can drive $\lambda$ negative at some scale $\mu$. This cannot happen physically, because for $\lambda < 0$ the Higgs potential is unbounded!

To avoid this vacuum instability below some cut-off $\Lambda_c$ one needs to have $\lambda(M_H)$, and therefore the Higgs mass, sufficiently large. Hence, these considerations give a lower bound for the Higgs mass. Taking $\Lambda_c = M_P$, this lower bound is

$$M_H \geq 134 \text{ GeV}.$$  

(13)

Lowering the cut-off $\Lambda_c$ weakens the bound on $M_H$. Interestingly, to have a SM Higgs as light as 100 GeV—which is the region accessible to LEP 200 and the Tevatron—requires a very low cut-off, of order $\Lambda_c \sim 100$ TeV.

Of course, a good fit of the data with the minimal SM does not necessarily exclude possible extensions of the SM involving either new particles or new interactions, provided that these new particles and/or interactions give only small effects. Typically, the effects of new physics are small if the excitations associated with this new physics have mass scales several times the W-mass. One can quantify the above discussion in a more precise way by introducing a general parametrization for the vacuum polarization tensors of the gauge bosons and the $Z\bar{b}b$ vertex. These are the places where the dominant electroweak radiative corrections occur and therefore are the quantities which are most sensitive to new physics.

As an illustration, I will discuss an example which has a bearing on the nature of the electroweak symmetry breaking.

There are four distinct vacuum polarization contributions $\Sigma_{AB}(q^2)$, where the pairs $AB = \{ZZ, WW, \gamma\gamma, \gamma Z\}$. For sufficiently low values of the momentum transfer $q^2 (q^2 \approx M^2_W)$ it obviously suffices to expand $\Sigma_{AB}(q^2)$ only up to $O(q^2)$. Thus, approximately, one needs to consider 8 different parameters associated with these contributions:

$$\Sigma_{AB}(q^2) = \Sigma_{AB}(0) + q^2 \Sigma'_{AB}(0) + \ldots .$$  

(14)

In fact, there are not really 8 independent parameters since electromagnetic gauge invariance requires that $\Sigma_{\gamma\gamma}(0) = \Sigma_{\gamma Z}(0) = 0$. Of the 6 remaining parameters one can fix 3 combinations of coefficients in terms of $G_F$, $\alpha$ and $M_Z$. Hence, in a most general analysis, the gauge field vacuum polarization tensors (for $q^2 \lesssim M^2_W$) only involve 3 arbitrary parameters. The usual choice is to have one of these contain the main quadratic $m_t$-dependence, leaving the other two essentially independent of $m_t$. In the notation of Altarelli and Barbieri, the parameter that depends on $m_t$ is called $\epsilon_3$, with $\epsilon_2$ and $\epsilon_3$ being at most logarithmically dependent on this mass. For our purpose, the interesting parameter is $\epsilon_3$, whose value, obtained from a fit of all precision electroweak data, turns out to be

$$\epsilon_3 = (3.9 \pm 1.1) \times 10^{-3}.$$  

(15)

Given some assumption of how $SU(2) \times U(1)$ is broken down, one can estimate the various $\epsilon_i$ parameters. This is somewhat harder to do in theories where the spontaneous breakdown occurs dynamically, since these involve strong interactions in the symmetry breaking sector. Nevertheless, one can estimate $\epsilon_3$ in a dynamical symmetry breaking theory, if one assumes that the spectrum of such a theory, and its dynamics, is QCD-like. From its definition $\epsilon_3$ involves the difference between the spectral functions of vector and axial vector currents. This difference has two components in a dynamical symmetry breaking theory. There is a contribution from a heavy Higgs boson ($M_H \sim$ TeV) characteristic of such theories, plus a term detailing the differences between the vector and axial vector spectral functions. This second component reflects the spectrum of the
underlying theory which causes the symmetry breakdown. The first piece is readily computed from the SM expression, using $M_H \sim \text{TeV}$. The second piece, in a QCD-like theory, can be estimated, modulo some counting factors. One finds
\[ \epsilon_3 = \left[ 6.65 \pm 3.4 N_D \left( \frac{N_{TC}}{4} \right) \right] \times 10^{-3}. \tag{16} \]

The second term follows if the underlying theory is QCD-like, so that the resonance spectrum is saturated by $\rho$-like and $A_1$-like, resonances. Here $N_D$ is the number of doublets entering in the underlying theory and $N_{TC}$ is the number of “Technicolors” in this theory. Using Eq. (16) and taking $N_{TC} = 4$, as is usually assumed, one sees that
\[ \epsilon_3 = \begin{cases} 
10.05 \times 10^{-3} & N_D = 1 \\
20.25 \times 10^{-3} & N_D = 4 
\end{cases} \tag{17} \]

These values for $\epsilon_3$ are, respectively, 5.5$\sigma$ and 15$\sigma$ away from the best fit value of $\epsilon_3$. Obviously, one cannot countenance anymore a dynamical symmetry breaking theory which is QCD-like!

Nothing as disastrous occurs instead if one considers a supersymmetric extension of the SM, provided the superpartners are not too light. Fig. 1, taken from a recent analysis of Altarelli, Barbieri, and Caravaglios, shows a typical fit, scanning over a range of parameters in the MSSM—the minimal supersymmetric extension of the SM. Although the MSSM improves the $\chi^2$ of the fit over that for the SM (which is already very good!), these improvements are small and one cannot use this as evidence for low energy supersymmetry.

3 Searching for Supersymmetry

3.1 Unification of Couplings

An indirect piece of evidence favoring the existence of supersymmetry at the weak scale is the way the $SU(3) \times SU(2) \times U(1)$ coupling constants evolve with energy. Although these couplings are quite different at low energy, they evolve differently with $q^2$. In leading order, the RGE
\[ \frac{d\alpha_i(\mu^2)}{d\ln \mu^2} = -\frac{b_i}{4\pi} \alpha_i^2(\mu^2). \tag{18} \]

imply a logarithmic change for the inverse couplings. The rate of change of the coupling constants with energy is governed by the coefficients $b_i$ which enter in the RGE. In turn, these coefficients depend on the matter content of the theory—which matter states are “active” at the scale one is probing. Remarkably, with ordinary matter, one gets near unification of couplings at high energy. However, assuming that there are supersymmetric partners of ordinary matter present above the Fermi scale, the three SM couplings really unify, as shown in Fig. 2!

The unification of the couplings in the SUSY SM case is quite spectacular. However, per se, this is only suggestive. It is neither a “proof” that a low energy supersymmetry exists, nor does it mean that there exists some high energy Grand Unified Theory (GUT) which breaks down to the SM at a high scale. The proof of the former requires the discovery of the predicted SUSY partners, while for GUTs one must find typical phenomena which are associated with these theories—like proton decay. Nevertheless, if such a GUT exists, one learns that the unification scale is rather high in the supersymmetric case [$M_X \simeq 2 \times 10^{16}$ GeV]. Such

\footnote{For QCD, of course, $N_D = 1$ and $N_{TC} = 3$.}
Figure 1: Comparison of SM and MSSM fits in the $\epsilon_2 - \epsilon_3$ plane, from Ref.19. The ellipse is the 1-$\sigma$ range determined by the data. The shaded region is the result of a scan over a range of SUSY parameters, with the star marking the lowest $\chi^2$ point.

high scales gives unobservedly long lifetimes for proton decay arising from $d = 6 \, qq\ell\ell$ operators, since this lifetime scales as $\tau_p \sim M_X^4$. In SUSY GUTs, however, one has dangerous $d = 5$ operators where two quarks are replaced by squarks resulting in a $\tilde{q}q\ell\ell$ operator. These terms lead to rather rapid proton decay, unless they are suitably suppressed. As a result, the predictions of SUSY GUT models for proton decay are rather model-dependent, with modes involving strangeness in the final state, like $p \to \nu K$, dominating. For these reasons the existing bounds on proton decay only serve to constrain parameters and cannot be adduced either for or against supersymmetry.

Much the same comments can be made regarding more careful calculations of the evolution of the SM couplings, including both 2-loop effects and more detailed SUSY thresholds. It turns out that the results of these more refined calculations, assuming a high scale unification of couplings, do not quite give the correct value of $\alpha_3(M_Z^2)$, unless one assumes a rather high average SUSY threshold $T_{\text{SUSY}} \sim 1 \text{ TeV}$. However, this is probably not a serious problem, since one could well imagine being able to lower $T_{\text{SUSY}}$ as a result of a few percent correction to the evolution equations coming from, difficult to pin down, GUT thresholds.
3.2 SUSY Higgs Sector

Supersymmetry associates bosonic partners to fermions and vice versa. However, it also requires two Higgs doublets, since the superpotential which describes a SUSY extension of the Yukawa interactions in the SM can only contain chiral superfields and not their adjoint. Although $H_u^*$ has the same quantum number as $H_d$, supersymmetry does not allow this more parsimonious choice. As a result, all supersymmetric extensions of the SM necessarily imply the presence of 5 physical Higgs states. Three of these states are neutral ($h, H, A$) and two are charged ($H^\pm$), with $h$ and $H$ being scalar and $A$ pseudoscalar.

The minimal set of Higgs states which appear in a supersymmetric extension of the SM has another remarkable property. Their quartic interactions are entirely fixed by supersymmetry, since they arise from the structure of the gauge interactions dictated by supersymmetry—the, so called, $D$-terms. No other quartic terms can be induced by supersymmetry breaking, if one wants to have supersymmetry be the solution to the hierarchy problem, since such $d = 4$ terms would trigger a hard breaking of supersymmetry. However, supersymmetry breaking can affect the $d = 2$ terms in the Higgs potential. As a result, in this minimal supersymmetric extension of the SM—the, so called, MSSM—one can write down a quite specific Higgs potential

$$V(H_u, H_d) = (H_u^\dagger H_d^\dagger)M^2 \left( \begin{array}{cc} H_u \\
H_d \end{array} \right) + \frac{1}{8}g'^2[H_u^\dagger H_u - H_d^\dagger H_d]^2$$

$$+ \frac{1}{8}g^2[H_u^\dagger \vec{\tau} H_u + H_d^\dagger \vec{\tau} H_d \cdot [H_u^\dagger \vec{\tau} H_u + H_d^\dagger \vec{\tau} H_d].$$

(19)

Here the mass squared $M^2$ contains both SUSY preserving terms ($\mu$) and SUSY breaking terms ($B$ and $\mu_{ij}$):

$$M^2 = \left( \begin{array}{cc}
\mu^2 + \mu_{11}^2 & -B\mu + \mu_{12}^2 \\
-B\mu + \mu_{12}^2 & \mu^2 + \mu_{22}^2 \end{array} \right) = \left( \begin{array}{cc}
m_1^2 & m_2^2 \\
m_2^2 & m_3^2 \end{array} \right).$$

(20)

Obviously, a breakdown of $SU(2) \times U(1) \rightarrow U(1)_{em}$ requires that $\det M^2 < 0$. 

8
The Higgs mass spectrum arising from Eq. (19) can be parametrized as a function of one of the masses, usually taken to be $M_A$, and the ratio of the two Higgs fields VEVs: $\tan \beta = \langle H_u \rangle / \langle H_d \rangle$. One finds:

\[
M^2_{H^\pm} = M_A^2 + M_W^2
\]
\[
M^2_{H,h} = \frac{1}{2} (M_A^2 + M_Z^2) \pm \frac{1}{2} \left[ (M_A^2 + M_Z^2)^2 - 4M_Z^2M_A^2 \cos^2 2\beta \right]^{1/2}.
\] (21)

It is easy to see from Eq. (21) that there is always one light Higgs in the spectrum:

\[
M_h \leq M_Z |\cos 2\beta| \leq M_Z.
\] (22)

However, the bound of Eq. (22) is not trustworthy, as it is quite sensitive to radiative effects which are enhanced by the large top mass. Fortunately, the magnitude of the radiative shifts for $M_h^2$ can be well estimated, by either direct computation or via the renormalization group.

As an example, in the case where $M_A \to \infty$ and $|\cos 2\beta| \to 1$, one finds:

\[
M_h^2 = M_Z^2 + \frac{3\alpha}{2\pi \sin^2 \theta_W} \left[ m_t^4 \right] \ln(M_{\text{SUSY}}^2/M_W^2),
\] (23)

where $M_{\text{SUSY}}$ is an assumed common scale for all the SUSY partners. As one can appreciate from the above formula, this is quite a large shift since, for $M_{\text{SUSY}} \approx 1$ TeV, one finds $\Delta M_h \approx 20$ GeV. Eq. (23) was obtained in a particular limit ($|\cos 2\beta| \to 1$), but an analogous result can be obtained for all $\tan \beta$. For small $\tan \beta$ the shifts are even larger than those indicated in Eq. (23). However, for these values of $\tan \beta$ the tree level contribution is also smaller, since $M_h|_{\text{tree}} < M_Z \cos 2\beta$. The actual details of the SUSY spectrum are in general not very important. The biggest effect of the SUSY spectrum for $\Delta M_h$ arises if there is an incomplete cancellation between the top and the stop contributions, due to large $\tilde{t}_L - \tilde{t}_R$ mixing. At their maximum these effects cause a further shift of order $(\Delta M_h)_{\text{mixing}} \approx 10$ GeV.

One can contrast these predictions with experiment. At LEP 200, the four LEP collaborations have looked both for the process $e^+e^- \to hZ$ and $e^+e^- \to hA$. The first process is analogous to that used for searching for the SM Higgs, while $hA$ production is peculiar to models with two (or more) Higgs doublets. One can show that these two processes are complementary, with one dominating in a region of parameter space where the other is small, and vice versa. LEP 200 has established already rather strong bounds for $M_h$ and $M_A$ setting the 95% C.L. bounds (for $\tan \beta > 0.4$):

\[
M_h > 80.7 \text{ GeV} \quad ; \quad M_A > 80.9 \text{ GeV}
\] (24)

As Fig. 3 shows, if there is not much $\tilde{t}_L - \tilde{t}_R$ mixing the low $\tan \beta$ region $[0.8 < \tan \beta < 2.1]$ is also already excluded.

It is apparent that the available window for $M_h$ is tantalizing small even for larger $\tan \beta$. LEP 200, running at its maximum energy of 200 GeV and the upgraded Tevatron with more luminosity can explore a good deal more still, providing even more stringent tests for the MSSM. In fact, because more complicated supersymmetric extensions of the SM (e.g. those obtained by including additional gauge singlet Higgs superfields) retain the same qualitative features, probing in detail the Higgs spectrum is a very effective way to test the whole notion of the existence of an approximate supersymmetry at the weak scale.
Figure 3: LEP200 limits for $M_h$ and $M_A$ as a function of $\tan \beta$, from Ref. 30.
3.3 Sparticle Searches

Although the SUSY SM is rather predictive when it comes to the Higgs sector, beyond this sector the spectrum of SUSY partners and possible allowed interactions is quite model dependent. Most supersymmetric extensions of the SM considered are assumed to contain a discrete symmetry, \( R \)-parity, which is conserved. This assumption restricts the form of the possible interactions allowed. In fact, \( R \)-parity conservation provides an essentially unique way to generalize the SM since \( R \), defined by \( R = (-1)^{Q + L + 2J} \), with \( Q \) being the quark number, \( L \) the lepton number and \( J \) the spin, turns out simply to be +1 for all particles and -1 for all sparticles.

Obviously, \( R \) parity conservation implies that SUSY particles enter in vertices always in pairs, and hence sparticles are always pair-produced. This last fact implies, in turn, the stability of the lightest supersymmetric particle (LSP), even in the presence of supersymmetry breaking interactions, since these SUSY breaking interactions can be arranged to preserve \( R \)-parity. Because the LSP has interactions of weak scale strength, this particle is an excellent dark matter candidate. This point is discussed in much more detail by John Ellis in this conference.

The decay chains of sparticles is predicated both on the modality of supersymmetric symmetry breaking and on the nature of the LSP. Of course, these two are intimately connected. Let me discuss the issue of SUSY breaking in a little more detail, since the manner in which one breaks supersymmetry is the principal source of model-dependence for the SUSY SM. In general, one assumes that SUSY is spontaneously broken at some scale \( \Lambda \) in some hidden sector of the theory. This sector is coupled to ordinary matter by some messenger states of mass \( M \), with \( M \gg \Lambda \), and all that obtains in the visible sector is a set of soft SUSY breaking terms—terms of dimension \( d < 4 \) in the Lagrangian of the theory. Ordinary matter contains supersymmetric states with masses \( \tilde{m} \sim \) TeV, with \( \tilde{m} \) given generically by \( \tilde{m} \sim \Lambda^2/M \).

Within this general framework, two distinct scenarios have been suggested which differ by what one assumes are the messengers that connect the hidden SUSY breaking sector with the visible sector. In supergravity models (SUGRA), the messengers are gravitational interactions, so that \( M \sim M_P \). Then the demand that \( \tilde{m} \sim \) TeV fixes the scale of SUSY breaking in the hidden sector to be of order \( \Lambda \sim 10^{11} \) GeV. In contrast, in models where the messengers are gauge interactions (Gauge Mediated Models) with \( M \sim 10^6 \) TeV, then the scale of spontaneous breaking of supersymmetry is around \( \Lambda \sim 10^3 \) TeV.

In both cases one assumes that the supersymmetry is a local symmetry, gauged by gravity. Then the massless fermion which originates from spontaneous SUSY breaking, the goldstino, is absorbed and serves to give mass to the spin-3/2 gravitino—the SUSY partner of the graviton, whose mass is of order \( m_{3/2} \sim \Lambda^2/M_P \). Obviously, in SUGRA models the gravitino has a mass of the same order as all the other SUSY partners (\( \tilde{m} \sim \) TeV), but there is no reason why the gravitino should be the LSP. However, in Gauge Mediated Models, since \( \Lambda \ll 10^{11} \) GeV, the gravitino is definitely the LSP.

Besides the above difference, the other principal difference between SUGRA and Gauge Mediated Models of supersymmetry breaking is the assumed form of the soft breaking terms. In SUGRA models, to avoid FCNC problems, one needs to assume that the soft breaking terms are universal. This assumption is unnecessary in Gauge Mediated Models, where in fact one can explicitly compute the form of the soft breaking terms and show that they do not lead to FCNC.

\[ \text{Recall that terms of } d = 4 \text{ would re-introduce the hierarchy problem.} \]
The search strategies and the resulting bounds on sparticles are quite model-dependent. For instance, in Gauge Mediated Models, in general the lightest chargino and neutralino have related masses \( m_{\tilde{\chi}^+} \simeq 2m_{\tilde{\chi}^0_1} \). Thus the lightest chargino decays always to the lowest neutralino and this state in turn radiatively decays to the gravitino LSP. At the Tevatron, therefore, if one produces charginos, these would typically produce decays with two photons and missing energy from the chain \( \tilde{\chi}^+_1 \to W^+ \tilde{\chi}^0_1 \to W^+ \gamma \tilde{G} \).

Unfortunately, no signals of SUSY states have been found yet. To illustrate the nature of the present bounds, I will describe these bounds for the MSSM, when the SUSY symmetry breaking is assumed to have a SUGRA origin. This scenario is characterized by just a few universal parameters, which include a common scalar mass \( m_o \) and a universal gaugino mass \( m_{1/2} \), at scales of order \( M_X \). Given these inputs, one can then derive the spectrum of the SUSY states, essentially by using the RGE evolution from \( M_X \) to the weak scale.

In this model at low energy the ratio of gaugino masses follows the ratio of the gauge coupling constants, so that
\[
M_1 : M_2 : M_3 = \alpha_1 : \alpha_2 : \alpha_3 = 1 : 2.45 : 8.62 .
\]

This pattern is quite different from that which obtains if the supersymmetry breaking at large scales is mediated by scale anomalies, as suggested recently by Randall and Sundrum. In this latter case \( M_i = -b_i a_i M_{SUSY} \) and thus
\[
M_1 : M_2 : M_3 = 3.3 : 1 : -10 .
\]

Obviously the phenomenology is also quite different, since now \( m_{\tilde{\chi}^+_1} \simeq m_{\tilde{\chi}^0_1} \).

For the MSSM with the simplest universal SUGRA breaking, not surprisingly the best bounds for the strongly interacting sparticles [squarks, \( \tilde{q} \), and gluinos, \( \tilde{g} \)] come from the Tevatron, while LEP 200 gives the best bounds for weakly interacting sparticles [sleptons, \( \tilde{\ell} \), and weak gauginos, both \( \tilde{\chi}^\pm_1 \) and \( \tilde{\chi}^0_1 \)]. Typically, for \( m_{\tilde{g}} \sim m_{\tilde{q}} \) the mass limits are above 250 GeV for squarks and gluinos. For the sleptons, the LEP limits are near half the CM energy, with the most recent analysis giving, at the 95% C.L.
\[
m_{\tilde{\ell}_e} > 89 \text{GeV} ; m_{\tilde{\mu}} > 81 \text{GeV} ; m_{\tilde{\tau}} > 71 \text{GeV} .
\]

The stop limits are a special case. Because of the large top mass there can be sizable \( \tilde{t}_L - \tilde{t}_R \) mixing, so that the stop eigenvalues can have large splittings. The lightest stop, \( \tilde{t}_1 \), can be searched for at LEP 200, as well as at the Tevatron where it can either be pair produced, or can originate from top decay: \( t \to \tilde{t}_1 \tilde{\chi}^0_1 \). The LEP 200 stop bounds are (slightly) dependent on the \( \tilde{t}_L - \tilde{t}_R \) mixing angle. The Tevatron bound, on the other hand, is quite strong provided that the LSP is light (\( M_{\text{LSP}} \leq 50 \text{ GeV} \)), and one finds, at 95% C.L.
\[
\tilde{m}_{t_1} > 122 \text{ GeV} .
\]

Both this bound, as well as the LEP 200 bounds, are shown in Fig. 4.

### 3.4 Electroweak Baryogenesis

The bounds on \( M_h \) and on the lightest stop state are quite relevant to the whole issue of electroweak baryogenesis. This is because if one has a rather light stop this helps make the electroweak phase transition more strongly first order, thereby ameliorating the need for having a very light Higgs. One can understand this qualitatively as follows. A light stop modifies significantly the coefficient of the cubic term in the Higgs
Figure 4: Tevatron and LEP200 limits for $\tilde{m}_t$, from Ref. 30.
effective potential

\[ V_{\text{eff}} = -m^2(T)\phi^2 + \lambda(T)\phi^4 - \mathcal{E}(T)\phi^3. \]  

Because

\[ \frac{\langle \phi(T^*) \rangle}{T^*} \simeq \frac{\mathcal{E}(T^*)}{\sqrt{2}\lambda(T^*)}, \]

if \( \mathcal{E}(T^*) \) is larger this allows \( \lambda(T^*) \), and hence the Higgs mass, to be larger for a given jump in the Higgs VEV at the phase transition.

In practice, however, one must be careful with what goes on with the rest of the Higgs potential. In particular, one must make sure that the relaxation of the Higgs bound occurs in a region of parameter space where there is no charge or color breaking minima. Fig. 5, adapted from a paper by Carena, Quirós and Wagner, shows the region in the \( M_h - \tilde{m}_{t} \) plane for which \( \langle \phi(T^*) \rangle \geq 1 \). Given the present bounds on \( M_h \) and \( \tilde{m}_{t} \), there appears to be little phase space still allowed, particularly if one excludes the cosmologically troubling possibility of having a two-step electroweak phase transition. Of course, this graph is somewhat misleading because it projects all the existing parameter space onto the \( M_h - \tilde{m}_{t} \) plane. In addition, one should remember that the \( \tilde{m}_{t} \) bounds where obtained under some assumptions– e.g. that \( m_{\tilde{\chi}_1^0} < 50 \text{ GeV} \) and that the lightest stop decayed 100\% of the time via \( \tilde{t}_1 \rightarrow c\tilde{\chi}_1^0 \). Nevertheless, taking Fig. 5 at face value, the allowed region is rather small. Furthermore, particularly the large \( \tilde{m}_{t} \) piece slice has other problems, since large \( \tilde{m}_{t} \) goes hand in hand with low \( \tan \beta \), which is not favored by the SUSY Higgs searches. Given the above, it is not inconceivable to me that, even before the turn-on of the LHC, one may shut the window for electroweak baryogenesis also in the MSSM.

Of course, the non erasure of the matter asymmetry produced at the electroweak phase transition is only a necessary condition for electroweak baryogenesis. One needs also to be able to produce a sufficient asymmetry during the phase transition (\( \eta \sim 5 \times 10^{-10} \)). The calculation of \( \eta \) depends crucially on the strength of the CP violation associated with Baryon-violating processes. In this respect, supersymmetric theories have an advantage, since it is possible to have CP-violating phases which are flavor conserving. An example of such a phase is that associated with the parameter \( \mu = |\mu|e^{i\alpha_{\mu}} \) in the SUSY-preserving Higgs superpotential \( W = \mu H_u H_d \). Another example is the phase entering in the SUSY-breaking gaugino mass \( m_{1/2} = |m_{1/2}|e^{i\alpha_{1/2}} \). The calculation of \( \eta \) in SUSY models in the literature typically requires that the CP-violating phase which enters in establishing the asymmetry is not too small–typically, \( \alpha_{\text{asym}} \sim 10^{-1} - 10^{-2} \).\[42\]

It is not clear, however, whether such largish SUSY CP phases are in contradiction with the bounds one obtains on these phases from the neutron dipole moment, \( \alpha_{\text{edm}} \leq 10^{-2} \), since \( \alpha_{\text{asym}} \) and \( \alpha_{\text{edm}} \) are not simply related to each other.

4 What are Neutrinos Telling Us?

Precision measurements of the Z-line shape now determine the number of light neutrinos species to a very high accuracy– much greater that that which can be obtained from Nucleosynthesis. The value of \( N_{\nu} \) extracted from the combined data of all four LEP experiments,\[44\] \( N_{\nu} = 2.9835 \pm 0.0083 \), gives very strong evidence that there exist only three families of quarks and leptons.

\[9\] Here \( \phi \) is the effective scalar field describing the electroweak phase transition in the model. It is an open question whether, for the purposes of discussing the electroweak phase transition, it really suffices to reduce the multi-Higgs SUSY potential in this manner.
Figure 5: Allowed region for electroweak baryogenesis in the $M_h - \tilde{m}_t$ plane, from Ref. 41. The short-dashed lines demark the region where a two-step phase transition may occur.

Although the result on $N_\nu$ is remarkable, the most exciting news from neutrinos in the last year is the evidence coming from SuperKamiokande for neutrino oscillations. In a simple 2-neutrino description, the SuperKamiokande results are consistent with maximum mixing and a mass-squared difference in the milli-eV$^2$ range:

$$\sin^2 2\theta \simeq 1 ; \quad \Delta m^2 \simeq 3 \times 10^{-3} \text{eV}^2.$$ \hfill (30)

The SuperKamiokande evidence for neutrino masses already has important implications since it gives a lower bound on some neutrino mass: $m_3 \geq \sqrt{\Delta m^2} \geq 5 \times 10^{-2} \text{eV}$. This mass value, in turn, gives a lower bound for the cosmological contribution of neutrinos to the Universe’s energy density:

$$\Omega_\nu \geq \frac{m_3}{92 \text{eV} \cdot h^2} \sim 1.5 \times 10^{-3}.$$ \hfill (31)

Although this number is far from that needed for closure of the Universe, the contribution of neutrinos to $\Omega$ is comparable to that of luminous matter, $\Omega_{\text{luminous}} \sim (3 - 7) \times 10^{-3}.$

For particle physics, a value of $m_3 \sim 5 \times 10^{-2} \text{eV}$ is also quite interesting, since it provides the best evidence for “new physics” to date. Because neutrinos are neutral, one can write different type of mass terms for them:
L_{\text{mass}}^\nu = - \left[ \nu_R m_D \nu_L + \nu_L m_D^\dagger \nu_R \right] - \frac{1}{2} \nu_R \tilde{C} m_S \nu_R^T + \nu_R^T \tilde{C} m_S^\dagger \nu_R^T + \frac{1}{2} \nu_L^T \tilde{C} m_T \nu_L + \nu_L^T \tilde{C} m_T^\dagger \nu_L^T \right]. \tag{32}

Here $\tilde{C}$ is a charge conjugation matrix and the mass matrices $m_D, m_S, m_T$ are Lorentz scalars. However, their presence is only possible as a result of different symmetry breakdowns. Specifically, $m_D$, often called a Dirac mass, conserves fermion number, but violates $SU(2) \times U(1)$ since it does not transform as an $SU(2)$ doublet. Clearly $m_D$ is proportional to the Fermi scale $v_F$ and is similar to the mass terms that gets induced for quarks and charged leptons, after the electroweak breakdown. Thus the eigenvalues of this matrix should be of the same order as those of the quarks and charged leptons. Both $m_S$ and $m_T$ violate fermion number by two units and are known as Majorana masses. Because $m_S$ couples $\nu_R$ with itself, clearly it is an $SU(2) \times U(1)$ invariant. Thus the eigenstates of $m_S$ are totally unconstrained and are new parameters in the theory. This is not the case for $m_T$, which violates $SU(2) \times U(1)$ because it does not transform as an $SU(2)$ triplet. Naively, because of its transformation law under $SU(2) \times U(1)$, one would expect the eigenvalues of this matrix to scale as $v_F^2/\Lambda$, with $\Lambda$ again being a new parameter in the theory.

Because the neutrino masses inferred from the SuperKamiokande experiment are in the sub-eV range, and hence much less than $m_\ell$ and $m_q$, it is clear that the neutrino Majorana mass terms must play a role. Hence one learns that not only individual lepton number, but also total lepton number must be violated. If one assumes that there are no right handed neutrinos, then the neutrino mass matrix is only $m_T$ and one can write for $m_3$ the formula $m_3 \sim v_F^2/\Lambda$. Using the SuperKamiokande result, $\Lambda$ is clearly a very high scale: $\Lambda \sim 10^{15}$ GeV—a scale of the order of the GUT scale!

Alternatively, if $\nu_R$ exists (and one neglects $m_T$), then the neutrino mass matrix reads simply:

$$M = \begin{pmatrix} 0 & m_D^T \\ m_D & m_S \end{pmatrix}.$$ \tag{33}

If the eigenvalues of $m_S$ are large, then M has two eigenmatrices, given approximately by $m_S$ and $-m_D^T m_D/m_S$. In this case, the spectrum splits into a very heavy neutrino sector and a very light neutrino sector. This, so called, see-saw mechanism is very suggestive. For any neutrino, it is natural to expect that the eigenvalues of $m_D$ should be of the order of the corresponding charged lepton mass. Hence one expects $m_3 \sim m_\tau^2/m_S$. Again, to fit the SuperKamiokande result requires that there be a large mass scale, now associated with the right-handed neutrinos: $m_S \sim 10^{11}$ GeV.

These considerations clearly point to new physics at very large scales, of order $10^{11} - 10^{15}$ GeV, associated with broken lepton number. This has suggested alternative scenarios for establishing the matter-antimatter asymmetry in the Universe. This asymmetry could originate from a primordial Lepton asymmetry established at temperatures of the order of the scale of Lepton number violation. Since $(B+L)$ processes eventually come into equilibrium in the early Universe, this primordial Lepton asymmetry can get transmuted into a Baryon asymmetry. Remarkably, in this scenario, the CP-violating phases in the neutrino sector are the root cause of our existence!

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5 Concluding Remarks

Eventhough the SM continues to give an excellent description of the precision electroweak data, the question of the origin of the Fermi scale argues for new physics at, or below, the TeV scale. The most likely form of this new physics, in the view of many, is the presence of an approximate supersymmetry, with spartners which could be as light as 100 GeV. \[\text{[86]}\] Perhaps most interestingly, such a low energy supersymmetry has important implications for cosmology. It provides both an interesting candidate for dark matter in the LSP and it may actually make possible baryogenesis at the electroweak scale.

That there is physics beyond the SM has been made clear by the observation of neutrino oscillations. However, this new physics appears to be associated with scales much above the Fermi scale, involving the breakdown of Lepton number. Nevertheless, these phenomena could also have important implications for cosmology. At any rate they indicate already that neutrinos, although they probably do not dominate the Universe’s energy density, give a non negligible contribution to this density.

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