Prediction for $5^{++}$ mesons

Cheng-Qun Pang$^{1,*}$, Ya-Rong Wang$^1$, and Chao-Hui Wang$^1$

$^1$College of Physics and Electronic Information Engineering, Qinghai Normal University, Xining 810000, China

In this paper, we study the spectrum and decay behavior of the $5^{++}$ meson family which is still missing in experiment. By the modified Godfrey-Isgur model with a color screening effect, we obtain the mass spectrum of $a_5$, $f_5$ and $f_5'$ mesons. And we predict their two-body strong decays by means of a phenomenology quark pair creation model. This study is crucial to establishing the $J^{PC} = 5^{++}$ light meson family and helpful for searching for these states in the future.

PACS numbers: 14.40.Be, 12.38.Lg, 13.25.Jx

I. INTRODUCTION

As an important part of hadron, light meson family is phenomenologically studied by many works, such as vector mesons associated with $Y(2175)$ [1, 2], $\rho$ and $\rho_1$ mesons [3], axial vector mesons [4], tensor mesons [5, 6], pseudotensor mesons [7], kaons [8], and higher spin mesons [9]. When checking the experimental status of light mesons [10], we notice an interesting phenomenon, i.e., the light meson family with $J^{PC} = 5^{++}$ are still absent, yet the other families for $a$ and $f$ meson (such as $a_0$, $a_1$, $a_2$, $a_3$, $a_4$, and $a_6$) are all reported by PDG [10]. This phenomenon stimulates our interest in exploring where are the light mesons with $J^{PC} = 5^{++}$.

According to the isospin, the light mesons with $J^{PC} = 5^{++}$ can be categorized into three groups, which are isovector $a_5$ meson family, and isoscalar $a_5$, $f_5(n\bar{n})$ and $f_5'(s\bar{s})$ meson families. In this work, we firstly study the properties of these states of $a_5$, $f_5$ and $f_5'$ meson families. Since they are missing in experiment, we predict the spectrum and the Okuba-Zweig-Iizuka (OZI) allowed two-body strong decay behavior, which is a key information of experimental search for them.

We hope that our effort will be helpful to establish $a_5$, $f_5$ and $f_5'$ meson families.

This paper is organized as follows. In Sec.II, the mass spectrum analysis of the light meson family with $J^{PC} = 5^{++}$ will be performed. In Sec.III, we further study the two-body OZI-allowed strong decay behavior of these discussed states. The paper ends with a discussion and conclusion in Sec.IV.

II. THE MASS SPECTRUM ANALYSIS

In this work, the modified GI quark model is utilized to calculate the mass spectrum and wave functions of the light meson family, and relativistic effects is considered.

In the following, this model will be illustrated in details.

A. The modified GI model

In 1985, Godfrey and Isgur raise the GI model for describing relativistic meson spectra with great success, exactly in low-lying mesons[11]. As for the excited states, the screening potential must be taken into account for coupled-channel effect.

The interaction between quark and antiquark is depicted by the Hamiltonian of potential model including kinetic energy pieces and effective potential piece,

$$\hat{H} = \sqrt{m_1^2 + p^2} + \sqrt{m_2^2 + p^2} + \tilde{V}_{\text{eff}}(p, r), (2.1)$$

where $m_1$ and $m_2$ denote the mass of quark and antiquark respectively, and effective potential $\tilde{V}_{\text{eff}}$ contains two ingredients, a short-range $\gamma^\mu \gamma_\nu$ one-gluon-exchange interaction and a $1 \otimes 1$ linear confinement interaction. The meaning of tilde will explain later.

In the nonrelativistic limit, effective potential has familiar format[11, 12]

$$V_{\text{eff}}(r) = H_{\text{conf}} + H_{\text{hyp}} + H_{\text{so}}, (2.2)$$

with

$$H_{\text{conf}} = \left[ -\frac{3}{4}(c + br) + \frac{a_1(r)}{r} \right] (F_1 \cdot F_2),$$

$$= S(r) + G(r) (2.3)$$

$$H_{\text{hyp}} = \frac{a_2(r)}{m_1 m_2} \left[ \frac{8\pi}{3} S_1 \cdot S_2 \delta^3(r) + \frac{1}{r^3} \left( \frac{3S_1 \cdot r S_2 \cdot r}{r^2} \right. - S_1 \cdot S_2 \right] (F_1 \cdot F_2), (2.4)$$

$$H_{\text{so}} = H_{\text{so(cm)}} + H_{\text{so(hyp)}}, (2.5)$$

where $H_{\text{conf}}$ includes the spin-independent linear confinement piece $S(r)$ and Coulomb-like potential from one-gluon-exchange $G(r)$, $H_{\text{hyp}}$ denotes the color-hyperfine interaction consists tensor and contact terms, and $H_{\text{so}}$ is the spin-orbit interaction with

$$H_{\text{so(cm)}} = \frac{-a_3(r)}{r^3} \left( \frac{1}{m_1} + \frac{1}{m_2} \right) \left( \frac{S_1}{m_1} + \frac{S_2}{m_2} \right) \cdot L(F_1 \cdot F_2), (2.6)$$
colour magnetic term causing of one-gluon-exchange and

\[ H^{\text{conf}(p)} = \frac{1}{2r} \frac{\partial H^{\text{conf}}}{\partial r} \left( \frac{S_1}{m_1^2} + \frac{S_2}{m_2^2} \right) \cdot \mathbf{L}, \]  

(2.7)

the Thomas precession term.

For above formulas, \( S_1/S_2 \) indicates the spin of quark/antiquark and \( \mathbf{L} \) the orbital momentum between them. \( F \) is relevant to the Gell-Mann matrix, i.e., \( F_1 = \lambda_1/2 \) and \( F_2 = -\lambda_2/2 \), and for a meson, \( (F_1 \cdot F_2) = -4/3 \).

Now relativistic effects of distinguish influence must be considered especially in light meson system, which is embedded in two ways. Firstly, based on the nonlocal interactions and new \( r \) dependence, a smearing function is introduced for a meson \( \bar{q}q \)

\[ \rho(r-r') = \frac{\sigma^3}{\pi^{3/2}} e^{-s^2} (r-r')^2, \]  

(2.8)

which is applied to \( S(r) \) and \( G(r) \) to obtain smeared potentials \( \tilde{S}(r) \) and \( \tilde{G}(r) \) by

\[ f'(r) = \int d^3r' \rho(r-r') f(r'), \]  

(2.9)

with

\[ \sigma_{12}^2 = \sigma_0^2 \left[ \frac{1}{2} + \frac{1}{2} \left( \frac{4m_1m_2}{(m_1 + m_2)^2} \right)^4 \right] + s^2 \left( \frac{2m_1m_2}{m_1 + m_2} \right)^2, \]  

(2.10)

where \( \sigma_0 \) and \( s \) are defined as [11].

Secondly, owing to relativistic effects, a general potential should rely on the mass-of-center of interacting quarks. Momentum-dependent factors which will be unity in the non-relativistic limit are applied as

\[ \tilde{G}(r) \rightarrow \left( 1 + \frac{p^2}{E_1E_2} \right)^{1/2} \tilde{G}(r) \left( 1 + \frac{p^2}{E_1E_2} \right)^{1/2}, \]  

(2.11)

and

\[ \tilde{V}_r(r) \rightarrow \frac{m_1m_2}{m_1m_2} \tilde{V}_r(r) \left( \frac{m_1m_2}{E_1E_2} \right)^{1/2+\epsilon}, \]  

(2.12)

where \( \tilde{V}_r(r) \) delegate the contact, tensor, vector spin-orbit and scalar spin-orbit terms, and \( \epsilon \) the relevant modification parameters.

Diagonalizing and solving the Hamiltonian in Eq. (2.1) by exploiting a simple harmonic oscillator (SHO) basis, we obtain the mass spectrum and wave functions. In configuration and momentum space, SHO wavefunctions have explicit form respectively

\[ \Psi_{\text{nl}_lM_l}(\mathbf{r}) = R_{\text{nl}}(r, \beta) Y_{\text{nl}M_l}(\Omega_r), \]  

\[ \Psi_{\text{nl}_lM_l}(\mathbf{p}) = R_{\text{nl}}(p, \beta) Y_{\text{nl}M_l}(\Omega_p), \]  

(2.13)

with

\[ R_{\text{nl}}(r, \beta) = \beta^{3/2} \sqrt{\frac{2n!}{\Gamma(n+L+3/2)}} \beta^{L} e^{-\beta^2 r^2} \]  

\times L_n^{L+1/2}(\beta^2 r^2), \]  

(2.14)

\[ R_{\text{nl}}(p, \beta) = \frac{(-1)^{n+1/2}}{\beta^{3/2}} e^{-\beta^2 p^2} \sqrt{\frac{2n!}{\Gamma(n+L+3/2)}} \left( \frac{p}{\beta} \right)^L \]  

\times L_n^{L+1/2}(\beta^2 p^2), \]  

(2.15)

where \( Y_{LM}(\Omega) \) is spherical harmonic function, and \( L_n^{L+1/2}(x) \) is the associated Laguerre polynomial, and \( \beta = 0.4 \) GeV for our calculation.

Although the GI model has obtained great success in describing the meson spectrum, there are still exist differences between the predicted values of GI models and the experimental observations. In the previous work [13], a modified GI model was proposed, and the prediction results for the charm-strange mesons are consistent with the experimental data. For higher excitation states, the screen effect is considered to be very important by the authors of Ref. [13]. It could be introduced by the transformation \( br + c \rightarrow \frac{b(1-e^{-\mu r})}{\mu} + c \), where \( \mu \) is screened parameter whose particular value is needed to be fixed by the comparisons between theory and experiment. Modified confinement potential also requires similar relativistic correction, which has been mentioned in the GI model. Then, we further write \( \tilde{V}^{\text{sc}}(r) \) as the way given in Eq. (2.16),

\[ \tilde{V}^{\text{sc}}(r) = \int \frac{d^3 r' \rho(r-r') b(1-e^{-\mu r'})}{\mu}, \]  

(2.16)

By inserting the form of \( \rho(r-r') \) in Eq. (2.9) into the above expression and finishing this integration, the concrete expression for \( \tilde{V}^{\text{sc}}(r) \) is given by

\[ \tilde{V}^{\text{sc}}(r) = \frac{b}{\mu r} \left[ r + e^{\frac{e^{2\mu r}+2\mu r^2}{2r^2}} \left( \frac{1}{\sqrt{\pi}} \int_0^{2\mu r^2} e^{-x^2} dx - \frac{1}{2} \right) \right. \]

\[ -e^{\frac{e^{2\mu r}+2\mu r^2}{2r^2}} \left( \frac{1}{\sqrt{\pi}} \int_0^{2\mu r^2} e^{-x^2} dx - \frac{1}{2} \right) \]  

(2.17)

Notability, except converting the confinement potential to the screened potential, the other processing contents and the Hamiltonian matrix elements contained in the original GI model are calculated. In our calculation, we need the spatial wave functions of the discussed the light meson family with \( J^{PC} = 5^{++} \) which can be numerically obtained by the modified GI model.

B. Mass spectrum analysis

Although the GI model has succeeded in describing the mass of ground states of the light mesons, it does not well describe the excited states. Since unquenched effects are important for a heavy-light system, it is better to adopt the modified GI model (MGI) [13, 14] which uses a screening potential with a new parameter \( \mu \). The parameter \( \mu \) describes inverse of the size of screening. In our previous work [8], we calculate the kaon family spectra use MGI model. In this work we will use this MGI model to obtain the mass spectrum of the light meson with \( J^{PC} = 5^{++} \). Beforehand, we need to adjust
the parameters of MGI model by fitting with the experiments data. So we fix the following ten parameters listed in Tab. I by fitting forty one experimental data which is listed in Tab. II. In Tab. II, we select forty one experimental data of light meson listed in PDG and optimize these light meson masses to determine ten parameters in Tab. I. This optimization has $\chi^2/n = 374$ which is smaller than 2638 for the GI model as shown in Tab. II.

Of course, besides the mass spectrum light mesons with $J^{PC} = 5^{++}$ was calculated by the GI model, there is a light meson spectrum with a Nambu-Goldstone Pion $\bar{\pi}$. But the mass values calculated by A. Le Yaouanc etc are larger than those observed in experiments, which are greater than the values of MGI models, so the accuracy is not very high. Finally, we can obtain the mass spectrum of these four $5^{++}$ states by the MGI model list in Tab. III.

So we can conclude that

1. The ground states of the $5^{++}$ states are still missing in experiments, and the predicted mass are 2.469 GeV for $a_1/s_1$, which are smaller than Ref.[11] and close to the result of Ref.[11]. $f_0(1H)$ has the mass 2.660 GeV which is smaller than Ref.[11] and Ref.[11].

2. The first exited states of $a_1/s_1$ and $f_1$ have the mass 2.686 GeV and 2.882 GeV, respectively. For the second exited states, $a_2/s_2(3H)$ and $f_2(3H)$ have the mass 2.885 GeV, 3.08 GeV, respectively, which are also smaller than Ref.[11].

The above conclusions are only from the point of mass spectra view and we will study their strong decays in the next section.

### III. THE DECAY BEHAVIOR ANALYSIS

#### A. QPC Model

The structure information of experimentally observed mesons and predicted family members, the QPC model is used
widely applied to the OZI-allowed two-body strong decay of hadrons. The QPC model is first proposed by Micu and Ebert.

For a decay process \( A \rightarrow B + C \), we can write

\[
\langle BC|T|A\rangle = \delta^3(p_B + p_C)\mathcal{M}^{M_A,M_B,M_C},
\]

where \( p_B(p_C) \) is a three-momentum of a meson \( B(C) \) in the rest frame of a meson \( A \). A superscript \( M_A \) (\( i = A, B, C \)) denotes an orbital magnetic momentum. The transition operator \( T \) is introduced to describe a quark-antiquark pair creation from vacuum, which has the quantum number \( J^{PC} = 0^{++} \), i.e., \( T \) can be expressed as

\[
T = -3\gamma \sum \langle 1m; -m|00 \rangle \int dp_3 dp_4 \delta^3(p_3 + p_4) \times Y_{lm}(p_3, p_4) \times Y_{Im}(p_3 - p_4)
\]

This is completely constructed in the form of a visual representation to reflect the creation of a quark-antiquark pair from vacuum, where the quark and antiquark are denoted by indices 3 and 4, respectively.

A dimensionless parameter \( \gamma \) depicts the strength of the creation of \( \bar{q}q \) from vacuum, where the concrete values of the parameter \( R \) will be discussed in the later section. \( Y_{Im}(p) = |p|^l Y_{lm}(p) \) are the solid harmonics. \( \chi, \phi, \omega \) denote the spin, flavor, and color wave functions respectively, which can be treated separately. Subindices \( i \) and \( j \) denote the color of a \( q\bar{q} \) pair.

By the Jacob-Wick formula [41], the decay amplitude is expressed as

\[
\mathcal{M}^{UL}(P) = \frac{\sqrt{4\pi(2L + 1)}}{2J_A + 1} \sum_{M_B,M_C} \langle L0;JM_A|JM_A,M_B,M_C \rangle \times \langle J_B,M_B;J_C,M_C,J_A,M_J_A \rangle \mathcal{M}^{M_A,M_B,M_C},
\]

and the general decay width reads

\[
\Gamma = \frac{\pi}{4m_A} \sum |\mathcal{M}^{UL}(P)|^2,
\]

where \( m_A \) is the mass of an initial state \( A \). In our calculation, we need the spatial wave functions of the discussed light mesons, which can be numerically obtained by the modified GI model.

In the previous section, we calculate the mass spectrum of the light meson and obtain the wave functions of the light meson. At the same time, we can use QPC model to study the strong decay of the \( J^{PC} = 5^{++} \) meson families. The parameter in the QPC model is determined by fitting with the experiment data. Thus, there is no free parameter in the QPC model. We obtain \( \gamma = 11.6 \) as shown in Tab. IV.

| State | This work | Gl[11] | Ebert[17] |
|-------|-----------|--------|-----------|
| \( a_1(1H) \) | 2.469 | 2.610 | 2.359 |
| \( a_1(2H) \) | 2.686 | 2.941 | – |
| \( a_1(3H) \) | 2.882 | 3.255 | – |
| \( f_1(1H) \) | 2.469 | 2.610 | 2.359 |
| \( f_1(2H) \) | 2.686 | 2.941 | – |
| \( f_1(3H) \) | 2.882 | 3.255 | – |
| \( f_1(3C) \) | 2.660 | 2.771 | 2.720 |
| \( f_1(3T) \) | 2.885 | 3.097 | – |
| \( f_1(3S) \) | 3.08 | 3.406 | – |

To obtain Okubo-Zweig-Iizuka (OZI) allowed hadronic strong decays. The QPC model is first proposed by Micu [18], which is further developed by Orsay group [19–23]. This model was widely applied to the OZI-allowed two-body strong decay of hadrons in Refs. [1, 3, 5–7, 24–40].

In Tab. IV, the mass spectrum of \( 5^{++} \) states. The unit of the mass is GeV.

| Decay channel | Exp(MeV) [5]/[8] | This work |
|---------------|------------------|-----------|
| \( \rho \rightarrow \pi\pi \) | 151.2 ± 1.2 | 68.9 |
| \( b_1 \rightarrow \omega\pi \) | 142±8 | 191 |
| \( \phi \rightarrow KK \) | 2.08±0.02 | 1.53 |
| \( a_2 \rightarrow \eta\pi \) | 15.5±0.7 | 2.65 |
| \( a_2 \rightarrow \rho\pi \) | 75.0±4.5 | 66.2 |
| \( a_2 \rightarrow KK \) | 5.2±0.2 | 4.53 |
| \( \pi_2 \rightarrow f_2(1270)\pi \) | 145.8±5.1 | 48.6 |
| \( \pi_2 \rightarrow \rho\pi \) | 80.3±2.8 | 220 |
| \( \pi_2 \rightarrow K'K \) | 10.1±3.4 | 26.7 |
| \( \rho_1 \rightarrow \pi\pi \) | 38±2.4 | 61.8 |
| \( \rho_1 \rightarrow \omega\pi \) | 25.8±1.6 | 44.3 |
| \( \rho_1 \rightarrow KK \) | 2.5±0.2 | 2.43 |
| \( f_2 \rightarrow \pi\pi \) | 156.8±3.2 | 136 |
| \( f_2 \rightarrow KK \) | 8.6±0.8 s | 8.31 |
| \( f_2(2050) \rightarrow \omega\omega \) | 54±13 | 70.1 |
| \( f_2(2050) \rightarrow \pi\pi \) | 35.4±3.8 | 79 |
| \( f_2(2050) \rightarrow KK \) | 1.4±0.7 | 0.941 |
| \( f_2(1525) \rightarrow K\bar{K} \) | 61±5 | 53.7 |
| \( K^*(892) \rightarrow K\pi \) | 48.7±0.8 | 18.6 |
| \( K^*(1410) \rightarrow K\pi \) | 15.3±1.4 | 62.2 |
| \( K^*(1430) \rightarrow K\pi \) | 251±74 | 291 |
| \( K^*_2(1430) \rightarrow K\pi \) | 54.4±2.5 | 50.2 |
| \( K^*_2(1430) \rightarrow K'\pi \) | 26.9±1.2 | 19.9 |
| \( K^*_2(1430) \rightarrow K\rho \) | 9.5±0.4 | 7.18 |
| \( K^*_2(1430) \rightarrow K\omega \) | 3.16±0.15 | 2.13 |
| \( K^*_2(1780) \rightarrow K\rho \) | 74±10 | 25.8 |
| \( K^*_2(1780) \rightarrow K'\pi \) | 45±7 | 28.3 |
| \( K^*_2(1780) \rightarrow K\pi \) | 31.7±3.7 | 38.1 |
| \( K^*_2(2045) \rightarrow K\pi \) | 19.6±3.8 | 22 |
| \( K^*_2(2045) \rightarrow K'\phi \) | 2.8±1.4 | 34.8 |

Next, we will analyse the strong decay behavior of these \( 5^{++} \) states.
B. The ground states

The ground state $a_5$ which is not observed in experiment is predicted in this work, with the mass 2469 MeV ($a_5$(2469)), and the total width is 367 MeV. $\rho \pi$ is its dominant decay channel, the width is about 130 MeV, and the branch ratio is 0.36. $a_5 \rho$, $\omega \pi$, $\pi \pi$ and $h_1 \rho$, are its important decay channels which have the branch ratio about 0.08 each one, just as shown in Tab. V. The final states $b_1 \omega$, $f_2 \pi$, $f_4(2050) \pi$, $\rho(1450) \pi$ and $a_1 \rho$, also have sizable decay widths, in which $b_1 \omega$ and $f_2 \pi$ almost has same width about 20 MeV.

As the iso-spin partner of $a_5(1H)$, we predict $f_5(1H)$ will have the mass 2.47 GeV and width 300 MeV, respectively. In the final decay channels of $f_5(1H)$, $\rho \rho$ and $b_1 \rho$ will be the most important final states which have the width 67 MeV and 66 MeV respectively, and their branch ratio are about 0.22. $a_2 \pi$ and $a_1 \pi$ are the important decay channels too, with the widths 58 MeV and 49 MeV, respectively. In addition, $h_1 \omega$ and $\pi \pi$ also have visible widths 28 MeV and 25 MeV which is presented in Tab. V. The widths of $K K_2(1780)$ and $\rho(1450) \rho$ are very small (see Tab. V), their branch ratio is about 0.015.

$f_5^*(1H)$ is the $ss$ partner of $f_5(1H)$, has the mass 2.66 GeV and width more than 800 MeV in our prediction. As shown in Tab. V, $f_5^*(1H)$ main decay to two kaon mesons for its $ss$ component. $K K_2^*(1780)$ is the dominant decay mode with the width 159 MeV. $KK^*_{1}(1430)$, $K_1 K^*$ are also the important decay channels which widths are over 100 MeV. Besides, $KK^*, KK^*_{2}(1430), KK^*_{1}(1410), KK^*$ are its sizable final channels with the branch ratio about 0.08. $KK_1$, $KK_2^{*}(2045), KK_1^*, KK_2^{*}(1680), KK_1^{*}(1430)$ are the visible decay channels of $f_5^*(1H)$. Here, we do not consider the mix of the flavor between $f_5(1H)$ and $f_5^*(1H)$.

C. The first exited states

In this section, we will analyse the strong decay behavior of the first exited states of $5^{++}$ family.

$a_5(2H)$ has the mass 2686 MeV and narrow width 133 MeV in our theory result. According to Tab. VI, $\rho \pi$ is the dominant decay channel of $a_5(2H)$ which is similar with $a_5(1H)$, the branch ratio is about 0.56. $\rho(1450) \pi$ and $a_1(1450) \pi$ are its important decay channels which have the branch ratio about 0.1 and 0.06 respectively, just as Tab. VI shown. The final decay modes $\pi \rho$, $\omega \rho$, $a_1 \rho$, $\rho \rho$ and $\pi \omega$, also have sizable decay widths, with the width 4-6 MeV. The other decay information is shown in Tab. VI.

$f_5(2H)$ as the iso-spin partner(I=0) of $a_5(2H)$ has the mass 2.69 GeV and width 91 MeV, respectively. $f_5(2H)$ main decays to $\rho(1450) \rho$ which has the width 33 MeV and the branch ratio 0.36, $\omega(1420) \omega \rho$ and $a_1 \rho$ are the important final states which have the width 13.7 MeV, 9.64 MeV and 9.23 MeV, respectively, and their branch ratio are 0.15, 0.11 and 0.10, respectively. $\rho \omega$, $\omega \omega$, $a_1 \omega$ and $\omega \rho$ also have visible width from 2.3 MeV to 7 MeV which are presented in Tab. VI. The widths of other channels are very small (see Tab. VI) which branch ratios are less than 0.02.

As the $ss$ partner of $f_5(2H)$, $f_5^*(2H)$ has the mass 2.89 GeV and width 578 MeV in our prediction. According to Tab. VI, $f_5^*(2H)$ also main decays into two kaon mesons. $KK_2^{*}(1780)$ and $KK^*(1680)$ are the most important decay modes with the width 94 MeV and 90 MeV, respectively. $K_1 K^*(1410)$,

### Table V: The partial decay widths of the ground states for $5^{++}$ family, the unit of widths is MeV.

| Channel  | Value    | Channel  | Value    | Channel  | Value    |
|----------|----------|----------|----------|----------|----------|
| $a_5(1H)$|          |          |          |          |          |
| Total    | 367      | Total    | 300      | Total    | 813      |
| $\rho \pi$| 131      | $\rho \rho$| 67.1      | $KK_2^{*}(1780)$ | 159      |
| $a_2 \rho$| 48.3      | $h_1 \rho$| 65.7      | $KK_2^{*}(1430)$ | 134      |
| $\omega \rho$| 32.8      | $a_2 \pi$| 57.5      | $K_1 K^*$ | 103      |
| $\rho \pi$| 29.4      | $a_1 \pi$| 48.9      | $K^* K^*$ | 79.9      |
| $h_1 \rho$| 28.4      | $h_1 \omega$| 27.7      | $KK_2^{*}(1430)$ | 74.2      |
| $b_1 \omega$| 21.2      | $a_2 \pi$| 24.5      | $KK^{*}(1410)$ | 69.8      |
| $f_2 \pi$| 21.1      | $KK_2^{*}(1780)$| 4.72      | $K^* K^*$ | 56.9      |
| $f_4(2050) \pi$| 16.9      | $\rho(1450) \pi$| 4.05      | $K_1 K_1$ | 45.7      |
| $\rho(1450) \pi$| 15.7      |          |          |          |          |
| $a_2 \rho$| 15.6      |          |          |          |          |
| $KK_2^{*}(1780)$| 4.55      |          |          |          |          |
| $\omega(1420) \rho$| 2.42      |          |          |          |          |
| $f_1(1425) \eta'$| 1.29      |          |          |          |          |
TABLE VII: The partial decay widths of the second exited states for $S^+\overline{p}$ family, the unit of widths is MeV.

| Channel | Value | Channel | Value | Channel | Value |
|---------|-------|---------|-------|---------|-------|
| Total   |  95   | Total   |  49.8 | Total   |  319  |
| $\rho_1\pi$ | 37.2 | $\rho_3\rho$ | 11.7 | $KK^*(1410)$ | 75.5 |
| $\rho(1450)\pi$ | 12.0 | $\rho_1\pi$ | 11.4 | $KK_1^*(1780)$ | 61.2 |
| $\rho\pi$ | 6.30 | $\rho(1450)\pi$ | 9.44 | $K_1K_2^*(1430)$ | 37.7 |
| $\omega\rho$ | 5.63 | $\omega_0(1420)\omega$ | 4.21 | $KK^*$ | 33.3 |
| $\pi_2\rho$ | 5.06 | $a_1(a_2,a_2)\rho$ | 3.37 | $K^*K_1^*(1780)$ | 24.7 |
| $\omega(1420)\rho$ | 4.83 | $a_0(1450)\rho$ | 2.71 | $K_0(1920)$ | 15.6 |
| $\omega(1420)\omega$ | 4.42 | $\pi(1450)\pi$ | 2.31 | $K_1(1410)$ | 12.3 |
| $\rho(1450)\rho$ | 3.00 | $a_1(a_1,a_1)\omega$ | 1.70 | $K_1K_1^*(1410)$ | 12.1 |
| $a_2\rho$ | 2.82 | $b_1b_1$ | 1.37 | $KK^*(1410)$ | 10.5 |
| $f_2\omega$ | 2.27 | $a_1\pi$ | 1.14 | $K'K_1$ | 7.01 |
| $a_0b_1$ | 2.41 | $K^*K_1^*(1780)$ | 6.90 |
| $f_2\rho$ | 2.09 | $K_0(1920)$ | 6.13 |
| $a_0(1450)\rho$ | 1.12 | $K_1(1410)$ | 4.76 |
| $a_1(1450)\omega$ | 1.11 | $K^*K_1^*(1430)$ | 4.64 |
| $b_1b_1$ | 1.11 | $K^*K_1$ | 4.12 |
| $K^*K_1^*(1430)$ | 2.35 |

$KK^*(1680)$, $KK^*$, $K'K^*$ are also the more important decay channels which widths are in the range of 45-52 MeV. In addition, $K^*K^*(1410)$, $KK_1$, $K_1K_1^*$, $KK_2^*(1430)$, $K_1K_2^*(1430)$, $K^*K^*_1(1780)$ and $K^*K_1^*(1430)$, are its sizable final channels which branch ratio in the range of 0.036-0.066.

D. The second exited states

We also calculate the two body strong decays of the second exited states of $S^+\overline{p}$ family.

As the iso-vector meson of $S^+\overline{p}$ family, $a_3(3H)$ has the mass 2.88 GeV, the total width 95 MeV which is very narrow. $\rho_3\pi$ also is its dominant decay channel as shown in Tab. VII, the width is about 37.2 MeV and the branch ratio is 0.4, $\rho(1450)\pi$ has a large ratio (0.13) in its decay final channels. $\rho\pi$, $\omega_\rho$, $a_1(a_2,a_2)\rho$, $\omega(1420)\rho$ and $\rho(1450)\omega$ also have sizable contribution in the total widths.

$f_3(3H)$ state is the second radial excited state of $f_3$, which with the mass 2.88 GeV and width 50 MeV, $f_3(3H)$ main decays into $\rho_3\rho$, $\rho\rho$ and $\rho(1450)\rho$, whose decay widths are 11.7, 11.4 and 9.44 MeV, respectively, and each channel almost has the branch ratio 0.2. $a_3a_2$ and $\omega(1420)a_1$ are the important decay channels too, with the widths about 5 MeV. In addition, $a_3a_1$, $a_0(1450)a_1$, and $\pi\rho$ also have visible widths which are presented in Tab. VII. The width of other modes are very small(Tab. VII).

$f_3(3H)$ has the $s\bar{s}$ component as the partner of $f_3(3H)$ which has the mass 3.4 GeV. $f_3(3H)$ has the total width 320 MeV in our calculation. Just as shown in Tab. VII, $f_3(3H)$ main decay to $KK^*(1410)$ and $KK^*_1(1780)$ with the width 76 MeV and 61 MeV, $K_1K_1^*(1430)$, $K_1K^*$, $K_1K_2^*(1430)$ and $K_1K^*(1680)$ almost have the same decay widths which are 37.7 MeV and 37.2 MeV, respectively. $KK^*$ and $K^*K_1^*(1780)$ are its sizable final channels with the branch ratio 0.1 and 0.08, respectively. Besides, $KK_1$, $K_1K_1^*(1410)$, $K_1K^*(1410)$ and $KK^*_1(1410)$ have the visible contribution to the total width too. The other modes, such as $K^*K_1$, $K^*K^*(1680)$, $K_1K_1^*(1410)$, $K_1K^*_1$ and $K_1K_1$ have very small width in the final states of $f_3(3H)$.

IV. CONCLUSION

In this paper, we study the spectrum and two body strong decay of the family with $I^{PC} = 5^+$ which is still missing in experiment. By the modified Godfrey-Isgur model with a color screening effect, we analyse the mass spectrum of $a_3$ and $f_3$ mesons, in which we find that the ground states of the $5^+$ states, $a_3$, $f_3$ and $f'_3$ have the mass 2.469 GeV, 2.469 GeV and 2.66 GeV and the width 367 MeV, 300 MeV, and 813 MeV, respectively. The first exited states of $a_3/f_3$ and $f'_3$ have the mass 2.686 GeV and 2.882 GeV and $a_3(3H)/f_3(3H)$ and $f'_3(3H)$ have the mass 2.686 GeV, 2.882 GeV. The total widths are predicted to be 131 MeV($a_3(2H)$), 91 MeV($f_3(2H)$), and 578 MeV($f'_3(2H)$) for the first states. For the second exited states of $5^+$, $a_3(3H)$, $f_3(3H)$ and $f'_3(3H)$ have the widths 95 MeV($a_3$), 50 MeV($f_3$) and 320 MeV ($f'_3$), respectively.

We also predict the detail decay information of $5^+$ family using QPC model which can be helpful for searching the mesons in the future experiments just as BESIII and COMPASS.

V. ACKNOWLEDGMENTS

This work is supported in part by the Fundamental Research Funds for the Central Universities, the High-End Creative Talent Thousand People Plan of Qinghai Province, No. 0042801, Chunhui Plan, No. Z2017054 and the Applied Basic Research Project of Qinghai Province, No. 2017-ZJ-748.
