Research on Ratio Consensus of Flywheel Energy Storage System Based on Hamiltonian Theory

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Abstract. A control strategy based on Hamiltonian energy theory is proposed for the wind farm with flywheel energy storage system (FESS). The control of the ratio consensus of the flywheel energy storage system is realized by adjusting the speed of the flywheel energy storage unit. First, the port-controlled Hamilton (PCH) model of the flywheel energy storage unit is established, and the port-controlled Hamiltonian with dissipation (PCH-D) model is obtained by using the feedback stabilization principle of the PCH system. Then, the problem of the ratio consensus control of the flywheel energy storage system is investigated. In order to realize that all flywheel energy storage units can store and release energy at the same rate, the control strategy of Hamilton energy shaping is applied to adjust the speed of flywheel energy storage units. Finally, the effectiveness of the proposed control strategy is verified by simulations.

1. Introduction

Due to the intermittent and volatile nature of renewable energy, if connected directly to the grid, it will seriously affect the power quality of the grid [1-3]. Energy storage is an important way to solve this problem. As a new energy storage technology, flywheel energy storage system (FESS) is widely used in power smoothing, power quality regulation and voltage recovery of wind turbines [4].

For a large number of units equipped with FESS, how to distribute the power reasonably is also a problem worthy of study. From the point of view of Hamilton energy theory, the upper power distribution command is converted into the speed command of flywheel energy storage unit by the power expression, and then the rotational speed of flywheel energy storage unit is adjusted by the control strategy of Hamilton energy shaping to achieve the uniform ratio of flywheel energy storage unit.

In this paper, the port-controlled Hamilton (PCH) system of flywheel energy storage unit is transformed into a port-controlled Hamilton (PCH-D) system by feedback stabilization principle. Based on the method of energy shaping and damping injection in Hamilton energy theory [5], the speed regulation of flywheel energy storage unit is realized by changing the energy function.

Based on the control strategy of Hamilton energy shaping, the ratio factor is synchronized by adjusting the speed of flywheel energy storage unit, so that each flywheel energy storage unit can store and release energy according to the same ratio. Firstly, the port-controlled Hamilton (PCH) model of the flywheel energy storage unit is established, which is the main model studied in this paper. Then, the upper power dispatching instruction is transformed into the speed regulation of flywheel energy storage unit, and the control strategy of Hamilton energy shaping is applied to achieve the ratio factor synchronization to ensure that all flywheel energy storage units are in the same ratio. Finally, based on
the simulation results, the validity of the energy shaping control strategy of the flywheel energy storage system is verified, and the synchronous energy storage and release is realized by the ratio regulation.

2. Problem description
It can be connected to large energy storage devices in wind farms to smooth the power of wind farms. With the continuous expansion of the scale of wind farms, the capacity of a single flywheel energy storage unit has been unable to satisfy the needs of wind farms, so it is necessary to include multiple flywheel energy storage system to cooperate with the wind farm. The power relationship between wind farm and flywheel energy storage system is introduced below:

\[ \Delta P = P^* - P_w \]  

(1)

where, \( P^* \) is upper level power command, \( P_w \) is actual power released from wind farms. When \( \Delta P > 0 \), FESS needs to charge to smooth the peak of wind farms, when \( \Delta P < 0 \), FESS needs to discharge to supplement the power shortage of wind farms.

2.1 Definition of the ratio factor
Because the charging and discharging process of flywheel energy storage system is similar, the charging model is mainly discussed. For a flywheel energy storage unit, the ratio factor is defined as the ratio of charging margin to power [6]. First, define the charging margin of each unit is:

\[ \bar{y}_{\text{char},i} = \frac{E_i - E_{i0}}{\tau_e} \]  

(2)

where, \( E_i \) and \( E_{i0} \), the maximum and minimum storage capacity of flywheel energy storage unit; \( E_{i0} \), current storage energy of flywheel energy storage unit which satisfy \( E_i < E_{i0} < \bar{E}_i \), \( y_{\text{char},i} \), the charging margin of the flywheel energy storage unit.

The charging power expression of flywheel energy storage unit is

\[ P = \tau_e \Delta \omega \]  

(3)

In addition, the expression of the energy stored in the flywheel energy storage unit is

\[ E_i = \frac{1}{2} J \omega_i^2 \]  

(4)

Based on the definition of ratio factor, the initial speed of flywheel energy storage unit is defined as \( \omega_{i0} \), energy stored in the charging process is recorded as \( E_{ic} \), the corresponding speed is \( \omega_{ic} \) and the ratio factor of charging each flywheel energy storage unit in charging is \( r_{\text{char},i} \), Which is as follow:

\[ r_{\text{char},i} = \frac{y_{\text{char},i}}{P_{\text{char},i}} = \frac{E_i - E_{i0}}{\tau_e \Delta \omega} = \frac{1}{2} J_f (\omega_{ic}^2 - \omega_{i0}^2) = \frac{1}{2} J_f (\omega_{ic} + \omega_{i0}) \]  

(5)

2.2 Ratio consensus
For the flywheel energy storage system, the flywheel energy storage unit stores energy at the same rate, which not only completes the task of power distribution within the flywheel energy storage system, but also prolongs the life of the flywheel energy storage unit.

For the flywheel energy storage unit, the conditions to achieve ratio consistency are required as follows:

\[ \lim_{t \to \infty} \| r_{\text{char},i} - r_{\text{char},j} \| = 0 \]  

(6)

The ratio factor of charging is

\[ r_{\text{char},i} = \frac{1}{2} J_f (\omega_{i0} + \omega_{ic}) \]  

\( i = 1, 2, \ldots, N \)  

(7)
For a flywheel energy storage unit with the same rotational inertia and load torque, the $\tau_L$ is constant. To achieve the same ratio of energy storage in the flywheel energy storage unit, the conditions satisfy

$$\omega_{i0} + \omega_c = T_1$$

(8)

Where, $T_1$ is a constant value, indicates the constant sum of the initial speed and desired speed when the flywheel energy storage unit charges.

The PCH-D model of the flywheel energy storage system is established by the feedback stabilization principle and the energy shaping control strategy is designed to adjust the speed of FESS.

3. Model establishment of flywheel energy storage system

The mathematical model of charging flywheel energy storage unit in d-q axis be expressed as [7]:

$$\begin{cases}
L_{fd} \frac{di_{fd}}{dt} = -R_s i_{fd} + n_p \omega L_{fd} i_{fq} + u_{fd} \\
L_{fq} \frac{di_{fq}}{dt} = -R_s i_{fq} - n_p \omega L_{fd} i_{fd} - n_p \omega \Phi + u_{fq} \\
J_f \frac{d\omega}{dt} = \tau_e - \tau_L = 1.5 n_p \Phi i_{fq} - \tau_L
\end{cases}$$

(9)

where, the main parameters of flywheel energy storage unit are: $R_s$ is stator resistance; $L_{fd}$ and $L_{fq}$ is d and q axis inductance; $\Phi$ is flux linkage; $n_p$ is pole number; $J_f$ is moment of inertia; $e\tau$ is electromagnetic torque; $\tau_L$ is load torque; $i_{fd}$, $i_{fq}$ and $\omega$ are state variable; $u_{fd}$ and $u_{fq}$ are input variable.

3.1 Hamilton model for flywheel energy storage unit in charging

For the system (9), the Hamilton energy function of flywheel energy storage unit is designed as

$$H(x) = \frac{1}{2} x^T D^{-1} x = \frac{1}{2} \left[ \begin{array}{c} x_1 \frac{2}{L_{fd}} + x_2 \frac{2}{L_{fq}} + x_3 J_f \end{array} \right]$$

(10)

Based on the Hamilton energy function, the system (9) can be represented as PCH system, which is

$$\dot{x} = (J - R) \frac{\partial H(x)}{\partial x} + Gu_c$$

(11)

where,

$$J = \begin{bmatrix}
0 & 0 & n_p x_2 \\
0 & 0 & -n_p (x_1 + \Phi) \\
-n_p x_2 & n_p (x_1 + \Phi) & 0
\end{bmatrix}, \quad R = \begin{bmatrix}
R_s & 0 & 0 \\
0 & R_s & 0 \\
0 & 0 & 0
\end{bmatrix}, \quad G(x) = \begin{bmatrix}
0 & 1 & 0 \\
0 & 0 & 0 \\
0 & 0 & 1
\end{bmatrix}, \quad u_c = \begin{bmatrix}
u_{fd} \\
u_{fq} \\
-\tau_L
\end{bmatrix}, \quad \frac{\partial H}{\partial x} = \begin{bmatrix}
i_{fd} \\
i_{fq} \\
\omega
\end{bmatrix}.$$

The design control law is as follows:

$$u_c = \alpha_c(x) + \mu_c(x)$$

(12)

The control law is divided into two parts: $\alpha_c(x)$ is the controller for feedback during charging to ensure that the system satisfies the PCH-D system; $\mu_c$ is the output feedback controller of charging to adjust the output speed of flywheel energy storage unit. Where, the feedback control strategy is [8]:

$$\alpha_c(x) = u_{jo} = -r_i i_{fd} + (k_j - n_p L_{fq} i_{jq} \omega - \frac{k_j \tau_L}{n_p \Phi} \omega)$$

$$\mu_c(x) = \frac{u_{jq}}{-r_2 i_{jq} + (n_p - k_j) i_{jq} \omega + n_p \Phi \theta_0 + (R_s + \tau_L) \frac{\tau_L}{n_p \Phi} - \tau_L}$$

(13)

Lately, the output feedback controller is designed for the PCH-D model of the flywheel energy storage unit, and the speed regulation of the FESS is realized by the energy shaping control strategy.
3.2 Energy shaping control strategy of FESS

The FESS is converted into the rotational speed command of the flywheel energy storage unit according to the upper power distribution command, and the required command is achieved by the energy shaping control strategy.

Considering the PCH-D model of flywheel energy storage unit charging

\[
\dot{x} = [J - R] \frac{\partial H(x)}{\partial x} + Gu_c
\]

\[
y = G^T \frac{\partial H(x)}{\partial x}
\]

(14)

Thanks to \( u_c = \alpha_c(x) + \mu_c(x) \), where, formula (13) is the feedback stabilization controller; \( \mu_c(x) \) is a control strategy for energy shaping. After the feedback controller, the structural parameters of the PCH-D model are respectively:

\[
J_d = \begin{bmatrix}
0 & k_5 & 0 \\
-k_5 & 0 & -n_p \Phi \\
0 & n_p \Phi & 0 
\end{bmatrix}
\]

\[
R_d = \begin{bmatrix}
R_5 + r_1 & 0 & 0 \\
0 & R_5 + r_2 & 0 \\
0 & 0 & 0 
\end{bmatrix}
\]

\[
\frac{\partial H_d}{\partial x} = \begin{bmatrix}
\frac{i_{f_d}}{i_{f_q}} \\
\frac{\tau_L}{n_p \Phi} \\
\omega - \omega_0 
\end{bmatrix}
\]

The main idea of energy shaping of flywheel energy storage unit is that the current Hamilton function is known as \( H_d(x) \) and the corresponding output is \( y_d = G^T \frac{\partial H_d(x)}{\partial x} \), while the expected Hamilton function is \( H_r(x) \) and the corresponding output is \( y_r = G^T \frac{\partial H_r(x)}{\partial x} \). The control strategy of energy shaping needs to be satisfied as:

\[
y_r = G^T \frac{\partial H_r(x)}{\partial x} = y_d + d
\]

(16)

The constant vector \( d = [d_1 \quad d_2 \quad d_3]^T \) represents the adjustment output of the flywheel energy storage unit after receiving the upper instructions; where \( d_1 \), \( d_2 \) and \( d_3 \) correspond to the adjustment parameters of \( i_{f_d} \), \( i_{f_q} \) and \( \omega \) respectively; for the flywheel energy storage unit speed adjustment, set \( d_1 = d_2 = 0 \), by adjusting the value of \( d_3 \) to achieve speed adjustment. Where, \( y_d \) is the output of flywheel energy storage system based on equilibrium point, and the output of the corresponding flywheel energy storage unit is

\[
\begin{bmatrix}
i_{f_d} \\
i_{f_q} \\
\omega
\end{bmatrix}
= \begin{bmatrix}
i_{f_d0} \\
i_{f_q0} \\
\omega_0
\end{bmatrix}
\]

\[
\begin{bmatrix}
i_{f_d} \\
i_{f_q} \\
\omega
\end{bmatrix}
= \begin{bmatrix}
0 \\
\tau_L/n_p \Phi \\
\omega_0
\end{bmatrix}
\]

(17)

In addition, \( y_{d1} \) is the expected output of the system and the corresponding flywheel energy storage unit output is

\[
\begin{bmatrix}
i_{f_d} \\
i_{f_q} \\
\omega
\end{bmatrix}
= \begin{bmatrix}
i_{f_d0} - d_1 \\
i_{f_q0} - d_2 \\
\omega_0 - d_3
\end{bmatrix}
\]

(18)

The following through the energy shaping control strategy to adjust the flywheel unit speed to achieve the goal of ratio consistency. The energy shaping control strategy is \( \mu_c(x) \), under which system (15) can be changed as:
\[
\left(J_d - R_d\right) \frac{\partial H_d(x)}{\partial x} + G \mu_c = \left(J_d - R_d\right) \frac{\partial H(x)}{\partial x} \quad (19)
\]

Noting that \( G \) is a full rank matrix, and the energy shaping control strategy can be expressed as:
\[
\mu_c = \left(G^T G\right)^{-1} G^T \left(\frac{\partial H_d(x)}{\partial x} - \frac{\partial H(x)}{\partial x}\right) \quad (20)
\]

**Theorem 1** Based on the method of energy shaping, the PCH-D model (31) for flywheel energy storage unit is considered. The Hamilton energy control strategy is assumed to be
\[
\mu_c = \begin{bmatrix}
-(R_x + n_2)d_1 + kx_3d_2 \\
-kx_3d_1 - (R_x + n_2)d_2 - n_p\Phi d_3 \\
n_p\Phi d_2
\end{bmatrix} \quad (21)
\]

The system (15) satisfies formula (16) under the new Hamilton function, and the output of the flywheel energy storage unit reaches the reference value given by the instruction.

### 4. Simulation Implementation

MATLAB simulation is carried out for a flywheel energy storage system connected to a wind farm. The flywheel energy storage system consists of six units, where \( \tau_L = 5N \cdot m \).

**4.1 Ratio consensus simulation under energy shaping control strategy**

FESS has two models of operation. The difference between the actual power and the reference power of the wind farm is respectively \( \Delta P = 3250kW \) and \( \Delta P = -3200kW \), FESS operates in charging and discharging model respectively. When \( \Delta P > 0 \), the ratio factors of 6 converge to a positive number, which means that FESS store energy at the same rate. When \( \Delta P < 0 \), ratio factors of 6 converge to a negative number, which means that FESS release energy at the same rate.

![Figure 1. The speed of FESS](image1.png) ![Figure 2. The rotational speed of FESS](image2.png)

**4.2 Ratio consensus of FESS under charging model**

When the state of FESS is storing energy, that is \( \Delta P = 3250kW \). When the initial rotational speeds of six flywheel energy storage units are 100, 80, 90, 120, 110 and 130 rad/s respectively, the output speed of the flywheel energy storage system is shown in Figure 1 under the action of the energy shaping control strategy (21) of the flywheel energy storage system.

From the simulations, it can be concluded that the initial speed of the flywheel energy storage unit is output within 20s, and after 20s, the FESS operates under the energy shaping control strategy and takes \( d_3 = [-100 -140 -120 -60 -80 -40]^T \), the output of the ratio factor is shown in figure 2.

From the simulations, it can be concluded that the FESS works in a state of maintenance in 0~20s, and the ratio factor of the flywheel energy storage unit is 0; after 20s, the FESS works in a state of charging, and after the speed adjustment, the system charges at the same ratio factor.

**4.3 Ratio consensus of FESS under discharging model**
When the state of FESS is releasing energy, that is \( \Delta P = -3200kW \). When the initial rotational speeds of six flywheel energy storage units are 200, 220, 180, 210, 160 and 190 rad/s respectively, the output speed of the flywheel energy storage system is shown in figure 3 under the action of the energy shaping control strategy.

From the simulations, it can be concluded that the initial speed of the flywheel energy storage unit is output within 20s, and after 20s, the FESS operates under the energy shaping control and takes \( d_s = [120 \ 160 \ 80 \ 140 \ 40 \ 100]^T \). The output simulation diagram of ratio factor is shown in figure 4.

From the simulations, it can be concluded that the FESS works in a state of maintenance within 20s, and the ratio factor of the flywheel energy storage unit is 0; after 20s, the FESS works in a state of discharging, and after the speed adjustment, the system discharges at the same ratio factor.

5. Conclusion
In this paper, the power coordination problem of flywheel energy storage system in wind farm is studied. Based on the control strategy of energy shaping in Hamilton energy theory, the ratio is uniform by adjusting the rotational speed of flywheel energy storage unit. Firstly, the Hamilton model of the flywheel energy storage unit is established based on the expression of the ratio factor defined during charging. Then, the control strategy of Hamilton energy shaping is applied to change the ratio consensus control into the rotational speed regulation of the flywheel energy storage system to realize the same ratio under charging and discharging model. Simulation results show that the flywheel energy storage system can be allocated and controlled effectively under the ratio consensus control.

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