Electromagnetically induced switching of ferroelectric thin films

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\textbf{Abstract}

We analyze the interaction of an electromagnetic spike (one cycle) with a thin layer of ferroelectric medium with two equilibrium states. The model is the set of Maxwell equations coupled to the undamped Landau-Khalatnikov equation, where we do not assume slowly varying envelopes.
From linear scattering theory, we show that low amplitude pulses can be completely reflected by the medium. Large amplitude pulses can switch the ferroelectric. Using numerical simulations and analysis, we study this switching for long and short pulses, estimate the switching times and provide useful information for experiments.

1 Introduction

Recently there has been a revival of interest in the study of phenomena occurring during the propagation of electromagnetic waves in different media having a long-range order. The propagation of electromagnetic pulses (in particular, of optical range) as stable solitary waves, sometimes called solitons, occupies a special place in such investigations. As an example, we can point out to studies of electromagnetic solitons propagating in ferro(antiferro)magnetic media [1, 2, 3]. Based on the Landau-Lifshitz equations for magnetization and the Maxwell equations for the electromagnetic field, solitary waves (supersonic, as distinct from magnetic solitons) were found. These can be approximately described by the modified Korteweg-de Vries (mKdV) equation. It was shown that they are stable against structural perturbations of the mKdV equation caused by energy dissipation. Using the multiscale perturbation theory, electromagnetic solitons were considered in antiferromagnetic [4] and anisotropic ferromagnetic media [5]. It was shown in [6] that the modulation of an electromagnetic wave in a ferromagnet in an external field can be described by the nonlinear Schrödinger equation. This opens the way to study the modulational instability and the formation of electromagnetic solitons in magneto-ordered media.

Dielectrics having a permanent polarization in the absence of an external electric field, called pyroelectrics, are, in many respects, similar to magneto-ordered media. If the phase transition between the pyroelectric and the nonpyroelectric states is of second-order, such pyroelectrics are called ferroelectrics. The Landau phenomenological theory of phase transitions is based on the assumption of an order parameter that is zero in one phase and is nonzero in the other phase. A uniaxial ferroelectric provides a simple example of this theory of phase transitions. Due to the nonlinearity of the free energy of the ferroelectric relative to the order parameter, there can exist nonlinear waves of spontaneous polarization both in the form of solitons [7] - [10] and in the form of domain
walls [11]. Thin films of ferroelectrics [12, 13] and liquid crystals having ferroelectric properties [14, 15, 16] are of interest for practical use. This is because such media, having a permanent dipole moment, can be used to generate efficiently optical harmonics [14]. Ferroelectrics can also be used to create memory devices and optically controlled switches [13, 16, 17, 18]. Many recent results are collected in the review on two dimensional ferroelectrics and thin polymer ferroelectric films represents in [19].

Recently [24] we investigated the traveling wave solutions of the Maxwell-Duffing homogeneous system describing the interaction of short electromagnetic pulses with a bulk ferroelectric. The description of the fast switching and ultra-short pulse propagation requires that the duration of these pulses be shorter than the relaxation time of the nonequilibrium polarization, which, for ferroelectrics, is equal to several nanoseconds [13]. In this study, we consider the interaction of an extremely short light pulse with a thin film of dielectric medium having a spontaneous polarization (a ferroelectric). Using the phenomenological Landau-Khalatnikov model [20, 21, 22, 23] describing a uniaxial ferroelectric and the Maxwell equation for an electromagnetic wave, we consider the reflections and refractions of a short electromagnetic spike (i.e., a pulse without a carrier wave) through a ferroelectric thin film. We assumed that the film width is less than the spatial size of the spike, but is greater then the critical length \( L_c \) below which no ferroelectricity exists [17, 23, 19]. According to theoretical estimations and \( L_c = 0.5 \text{nm} \) for \( \text{BaTiO}_3 \), \( L_c = 20 \text{nm} \) for \( \text{PbTiO}_3 \). For experimental films of two or five monolayers of polyvinylidene fluoride (PVDF) and copolymer P(VDF-TrFE) \( L_c \approx 7 \text{nm} \) [19].

The article is organized in the following way. Section 2 presents the model together with a solution of the Maxwell equations for a localized ferroelectric medium. We apply these results to a thin film in section 3. Section 4 is devoted to the switching caused by large amplitude electromagnetic pulses. We conclude in section 5.

2 Phenomenology of ferroelectricity

A phenomenological description of ferroelectricity due to Landau and Khalatnikov [20, 21, 25] gives the following Lagrangian density for the interaction of
an electromagnetic field and a dielectric medium

\[
L = \frac{1}{8\pi c^2} \left( \frac{\partial A}{\partial t} \right)^2 - \frac{1}{8\pi} (\text{rot} A)^2 + I(x) \left[ \frac{1}{2g} \left( \frac{\partial P}{\partial t} \right)^2 - \frac{1}{g} \Phi(P) - \frac{1}{c} \frac{\partial A}{\partial t} \right]
\]

(1)

where \( A \) is the vector potential, \( P \) is the polarization of the medium, \( c \) the speed of light, \( g \) is coupling constant, \( \Phi(P) \) is the thermodynamic potential, the last term describes the coupling between \( A \) and \( P \). The ferroelectric medium can exist only in some region and therefore we have introduced in (1) \( I(x) \), the characteristic function of the medium, i.e., \( I(x) = 1 \) inside the medium and \( I(x) = 0 \) outside. The Lagrangian approach guarantees that we take into account the correct couplings.

### 2.1 Homogeneous case

We first consider the homogeneous situation i.e. we assume \( I(x) \equiv 1 \). The variation of the action functional yields the equations for \( A \) and \( P \)

\[
\text{rot} \text{rot} A + \frac{1}{c^2} \frac{\partial^2 A}{\partial t^2} = -\frac{4\pi}{c^2} \frac{\partial P}{\partial t},
\]

\[
\frac{\partial^2 P}{\partial t^2} + \frac{\delta \Phi(P)}{\delta P} = -\frac{g}{c^2} \frac{\partial A}{\partial t},
\]

which when written in terms of the electric field \( E = -(1/c)\partial A/\partial t \) are

\[
\text{rot} \text{rot} E + \frac{1}{c^2} \frac{\partial^2 E}{\partial t^2} = -\frac{4\pi}{c^2} \frac{\partial^2 P}{\partial t^2},
\]

\[
\frac{\partial^2 P}{\partial t^2} + \frac{\delta \Phi(P)}{\delta P} = gE.
\]

(2)

(3)

We can introduce a phenomenological relaxation of the medium polarization by adding to the right hand side of (3) a linear damping term and obtain the well-known Landau-Khalatnikov equation.

To simplify the problem we will consider an electric field polarized along the propagation variable \( x \) so that \( E = Ee_x \) and a polarization along \( x \) \( P = Pe_x \). Following Landau the potential can be chosen as

\[
\Phi(P) = \Phi_0 + \alpha P^2 + \frac{1}{4} \beta P^4 + \frac{1}{2} D \left( \frac{\partial P}{\partial x} \right)^2.
\]

(4)

In the theory of Landau \( \alpha = \alpha_0(T - T_c) \) depends on the temperature and not \( \beta \). If \( \alpha > 0 \) the potential is minimum for \( P = 0 \) and this corresponds
to a (disordered) paraelectric phase. On the contrary if $\alpha < 0$ and $\beta > 0$ there are two minima located at $P = \pm \sqrt{-\alpha/\beta}$ corresponding to two opposite orientations of the polarization, this is the ferroelectric phase. Then the Euler-Lagrange equations for $E, P$ are

$$\frac{1}{c^2} \frac{\partial^2 E}{\partial t^2} - \frac{\partial^2 E}{\partial x^2} = -\frac{4\pi}{c^2} \frac{\partial^2 P}{\partial t^2}$$

$$\frac{\partial^2 P}{\partial t^2} - D \frac{\partial^2 P}{\partial x^2} + \alpha P + \beta P^3 = gE.$$ (5)

Traveling wave solutions for the system [24] were found in [24], they include a one-parameter family of solitons, a one-parameter family of kinks (domain wall) solutions and a one-parameter family of periodic (cnoidal) waves. In [24] we found numerically that the bright solitons on a zero and nonzero polarization background are stable while the dark solitons are stable only on a zero background.

### 2.2 Inhomogeneous case

We will now normalize the fields and variables as

$$t' = t/\sqrt{\alpha}, \quad x' = x\sqrt{\alpha}/c, \quad A = 2c \sqrt{\frac{\pi|\alpha|}{g\beta}} a, \quad P = \frac{|\alpha|}{\beta} q.$$ (6)

where the polarization $P$ is normalized by the saturation value. The Lagrangian density (1) becomes

$$L = \frac{1}{2} \left( \frac{\partial a}{\partial t} \right)^2 - \frac{1}{2} (\text{rot} a)^2 + I(x) \left[ \frac{1}{2} \left( \frac{\partial q}{\partial t} \right)^2 + m q^2 - \frac{1}{4} q^4 - \gamma q \frac{\partial a}{\partial t} \right]$$ (7)

where the primes have been dropped. The parameters are

$$\gamma = 2 \sqrt{\frac{\pi g}{|\alpha|}}, \quad m = \alpha/|\alpha|.$$ (8)

where $m$ is the "mass" of the excitations which can be negative for ferroelectrics and positive for paraelectrics and $\gamma$ is the polarisability of the medium. In the theory of Landau-Khalatnikov, the parameter $g$ is the susceptibility of the material so $g = 1/(4|\alpha|)$ in the ferroelectric phase. From [8] we have

$$\gamma = \sqrt{\frac{\pi}{2} \frac{1}{|\alpha|}} = \sqrt{\frac{\pi}{2} \frac{1}{|\alpha_0|} |T_c - T|^{-1}}.$$
Consider for example a crystal $\text{BaTiO}_3$, then $\alpha_0 = 6 \times 10^{-6} K^{-1}$, $\beta = 2 \times 10^{-15} m^3 J^{-1}$ so that $\gamma = 2 \times 10^5 |T_c - T|^{-1}$.

The simplified Lagrangian density \[26\] corresponding to a one-dimensional plane wave:

$$\mathcal{L} = \frac{a_t^2}{2} - \frac{a_x^2}{2} + I(x) \left( \frac{q_t^2}{2} + m \frac{q_x^2}{2} + \frac{q^4}{4} - \gamma q a_t \right),$$

where $a$ is the analog of vector potential. The Euler-Lagrange equations are

$$a_{tt} - a_{xx} = -\gamma I(x) q_t,$$ (10)

$$q_{tt} + m q + q^3 = \gamma a_t$$ (11)

The equations for the electric field $e = -a_t$ and medium variable can then be obtained

$$e_{tt} - e_{xx} = -\gamma I(x) q_{tt},$$ (12)

$$q_{tt} + m q + q^3 = \gamma e,$$ (13)

where the coupling between the fields $e$ and $q$ only occurs in the medium i.e. on the support of $I(x)$.

We now consider the general scattering formalism assuming a localized electromagnetic wave impinging on the medium from the left like is shown in Fig. \[1\] The general problem can only be treated numerically so we simplify it and consider two limiting cases, an array of thin films and a single thin film.

### 2.3 Scattering of an electromagnetic wave by a ferroelectric slab

We assume that the electromagnetic wave is incident from the left $x < 0$ on a medium whose position is given by the indicator function $I(x)$. Then the wave equation reads

$$e_{tt} - e_{xx} = g(x,t)$$ (14)

$$g(x,t) = -\gamma q_{tt} I(x),$$ (15)

where the boundary conditions are

$$e(t,x = \pm \infty) = 0, \quad e_t(t,x = \pm \infty) = 0,$$ (16)
Figure 1: Schematic of an incident electromagnetic pulse on a thin ferroelectric film panel a (left). The b and c panels show respectively the case of switching and non switching of the polarization.

and the initial conditions

\[ e(t = 0, x) = e_0(x), \quad e_t(t = 0, x) = e_1(x) = -\frac{\partial}{\partial x}e_0(x). \]  

(17)

We suppose that the initial pulse \( e_0(x) \) is located far to the left of the medium. The equation for the field is linear so to solve we introduce the Fourier transform of \( e \) and similarly for \( g \)

\[ e(x, t) = \int_{-\infty}^{+\infty} \frac{dk}{2\pi} \hat{e}(k, t) \exp(ikx), \quad \hat{e}(k, t) = \int_{-\infty}^{+\infty} e(k, t) \exp(-ikx) \, dx, \]  

(18)

to obtain the initial value problem

\[ \hat{e}_{tt} + k^2 \hat{e} = \hat{g}(k, t), \]  

(19)

with \( \hat{e}(k, t = 0) = \hat{e}_0(k), \quad \hat{e}_t(k, t = 0) = \hat{e}_1(k). \)

Following the general approach [28] we write \( \hat{e} = \hat{e}^0 + \hat{e}^1 \) where \( \hat{e}^0 \) solves the homogeneous equation

\[ \hat{e}_{tt}^0 + k^2 \hat{e}^0 = 0, \]  

(20)

with the initial conditions \( \hat{e}^0(k, t = 0) = \hat{e}_0(k), \quad \hat{e}_t^0(k, t = 0) = \hat{e}_1(k). \) and \( \hat{e}^1 \) solves the inhomogeneous equation

\[ \hat{e}_{tt}^1 + k^2 \hat{e}^1 = \hat{g}(k, t), \]  

(21)
with \( \hat{e}_1(k, t = 0) = 0, \hat{\gamma}_1(k, t = 0) = 0 \).

The general solution of (20) is 
\[
\hat{e}_0 = \frac{1}{2} \left( \hat{e}_0(k) + \frac{i}{k} \hat{e}_1(k) \right) \exp(-ikt) + \frac{1}{2} \left( \hat{e}_0(k) - \frac{i}{k} \hat{e}_1(k) \right) \exp(+ikt),
\]
Using for example a Green’s function approach one easily sees that the solution of the problem (21) is
\[
\hat{e}_1 = t \int_0^\infty \sin k(t - \tau) g(k, \tau) d\tau,
\]
The solution of the homogeneous problem can be transformed into the following expression
\[
e^0(x, t) = \frac{1}{2} \left[ e_0(x - t) + e_0(x + t) + \int_{x-t}^{x+t} e_1(y) dy \right] \equiv e_0(x - t)
\]
the first part of which is D’Alembert’s formula. The inhomogeneous solution can be rewritten as
\[
e^1(x, t) = \frac{1}{2} \int_{-\infty}^{+\infty} \int_0^t \theta(x - y - t + \tau) - \theta(x - y + t - \tau) g(y, \tau) d\tau dy,
\]
where we use the step-function \( \theta(z) = \int_{-\infty}^z \delta(x) dx \). So we have the general solution of the scattering problem under consideration.

\[
e(x, t) = e_0(x - t) + \frac{1}{2} \int_{-\infty}^{+\infty} \int_0^t \theta(x - y - t + \tau) - \theta(x - y + t - \tau) g(y, \tau) d\tau dy
\]
(22)

As an example we give the result for an array of thin films, useful for applications and theory. The indicator function is
\[
I(x) = \sum_{i=1}^N \delta(x - x_i),
\]
and the final result is
\[
e(x, t) = e_0(x - t) + \frac{\gamma}{2} \sum_{i=1}^N \int_0^t q_t(x_i, \tau) \left[ \delta(t + x - x_i) + \delta(t - x + x_i) \right] d\tau,
\]
(24)
where the evolution of the state of every film at point \( x = x_i \) is defined by the equation

\[
q_{tt}(x_i, t) + mq(x_i, t) + q(x_i, t)^3 = \gamma e(x = x_i, t).
\]

This problem is still complicated to analyze because of the delays and advances in equation (24). We therefore simplify drastically the situation by considering a single thin film only.

3 A single thin film

There is the simplest not trivial case when the nonlinear medium is represented by a single thin film of anharmonical oscillators. Here we can calculate the integral in expression (24) exactly. The electric field is defined by the following expression

\[
e(x, t) = e_0(x - t) + \frac{\gamma}{2} \int_0^t q_t(0, \tau) \left[ \delta(\tau - t + x) + \delta(\tau - t - x) \right] d\tau. \tag{25}
\]

At the point where the film is placed \( (x = 0) \) we have

\[
e(0, t) = e_0(-t) + \frac{\gamma}{2} \int_0^t q_t(0, \tau) \left[ \delta(\tau - t) + \delta(\tau - t) \right] d\tau
= e_0(-t) - \frac{\gamma}{2} q_t(0, t).
\]

Substituting this expression into the equation of the oscillator results in the following equation

\[
q_{tt} + mq + q^3 = \gamma e_0(-t) - \frac{\gamma^2}{2} q_t. \tag{26}
\]

This model represents the evolution of the nonlinear Duffing oscillator with both damping and forcing.

3.1 Linear considerations

In the homogeneous case \( g \equiv 1 \) we can compute the dispersion relation for the system (10) by assuming \( a = a_0 e^{i(kx - \omega t)}, q = q_0 e^{i(kx - \omega t)} \) and obtain

\[
k^2 = \frac{\omega^2 + \omega^2}{1 - \omega^2} + \omega^2
\]

The dispersion relation \( \omega(k) \) presents the well known polaritonic gap.
When the medium is localized like for the thin film case, the picture changes and we need to make a scattering experiment to understand the linear behavior of the device. The equations (12) become for the paraelectric case

\begin{equation}
\varepsilon_{tt} - \varepsilon_{xx} = -\gamma q_{tt} \delta(x),
\end{equation}

\begin{equation}
q_{tt} + q = \gamma e \ (x = 0, t).
\end{equation}

We compute the reflection and transmission coefficients of an incident linear wave on such a medium. For that we introduce the harmonic dependence $e(x, t) = e^{i\omega t} E(x), \quad q(t) = Q e^{i\omega t}$. Plugging these into the previous system of equations yields the Schroedinger equation with delta function potential

\begin{equation}
E_{xx} + \left[ \omega^2 + \frac{\gamma \omega^2}{1-\omega^2} \delta(x) \right] E = 0.
\end{equation}

For the scattering we assume a wave incident from the left $x < 0 \ E = e^{-ikx} + Re^{ikx}$ and a transmitted wave $x > 0 \ E = Te^{-ikx}$. The continuity of $E$ at $x = 0$ and the jump of the derivative $[E_x]_{0}^{+} = -\gamma \omega^2 E(0)/(1-\omega^2)$ give the transmission and reflection coefficients

\begin{equation}
T = \frac{2i(1-\omega^2)}{2i(1-\omega^2) - \gamma \omega},
\end{equation}

\begin{equation}
R = T - 1 = \frac{\gamma \omega}{2i(1-\omega^2) - \gamma \omega},
\end{equation}

where we used the dispersion relation $k = \omega$.

Several remarks can be made. First transparency $R = 0$ is obtained only for $\omega = 0$ and total reflection occurs for $\omega^2 = 1$ as expected \cite{29}. We also have two bound states corresponding to the poles of $R$ and $T$

\begin{equation}
\omega = -\frac{i\gamma^2}{4} \pm \sqrt{1 - \frac{\gamma^4}{16}},
\end{equation}

which are located in the lower half complex plane.

In the ferroelectric case the medium will oscillate around one of the equilibria $\bar{q} = \pm 1$ so that the linearized equations are

\begin{equation}
\varepsilon_{tt} - \varepsilon_{xx} = -\gamma q_{tt} \delta(x),
\end{equation}

\begin{equation}
q_{tt} + 2q = \gamma e \ (x = 0, t).
\end{equation}

The reflection and transmission coefficients become

\begin{equation}
R = T - 1 = \frac{\gamma \omega}{2i(2-\omega^2) - \gamma \omega}.
\end{equation}
In this case the poles are given by
\[ \omega = -\frac{i\gamma^2}{4} \pm \sqrt{2 - \frac{\gamma^4}{16}}, \] (36)

In Fig. 2 we show the reflection coefficient \(|R(\omega, \gamma^2)|^2\). It is close to 1 for the resonant frequency \(\omega = 1\) (resp. \(\sqrt{2}\)) in the paraelectric (resp. ferroelectric) case. The width of the resonance is the same for both cases and increases when the coupling \(\gamma\) grows. The coefficient \(\gamma\) is the coupling between the electromagnetic field and the medium. It is appears as a damping of the medium polarization in (26). The mechanism of relaxation is radiative.

4 Effect of anharmonicity: switching

When the amplitude of the incident pulse is large enough the anharmonic term in (26) needs to be taken into account. If the system is paraelectric \((m > 0)\) then it gets kicked out of the equilibrium state \(q = 0\) and relieves back to it. More interesting is the ferroelectric system \((m < 0)\) which has two stable equilibria \(\bar{q} = \pm 1\) and one unstable \(\bar{q} = 0\) so that switching between them is possible. We will solve
\[ q_{tt} - q + q^3 = \gamma e_0(-t) - \frac{\gamma^2}{2} q_t, \] (37)
for an incoming electromagnetic pulse \(e_0(-t)\) assuming the ferroelectric medium is in its equilibrium position i.e. with the initial condition \(q_t(t = -\infty) = 0\).
0, \( q(t = -\infty) = \bar{q} = -1 \). For a harmonic perturbation, chaotic behavior can occur, see for example [27]. Here we have a force of finite duration so we expect transient chaos which will cause irregular switching.

From the experimental point of view it is easier to work with pulses whose profiles are gaussian or plateau-like form. We chose the plateau-like form

\[
e_0(-t) = A_0 \left( \tanh \frac{-t - t_1}{t_f} - \tanh \frac{-t - t_2}{t_f} \right),
\]

(38)

The initial polarization of the ferroelectric medium is defined by the parameter \( q(t = -\infty) = \bar{q} \).

4.1 Short electromagnetic pulse

For a short electromagnetic pulse the equilibrium positions can be considered as fixed. After the interaction, the system evolves as if free. Figure 3 shows the evolution of the medium polarization and the corresponding phase plane under the action of an ultrashort electric field. The amplitude is not enough for switching and the system relaxes to \( \bar{q} = -1 \) following the linearized behavior \( \exp(\omega t) \) where \( \omega \) is given by (36). If the amplitude is increased as in Fig. 4 the system switches to the fixed point \( \bar{q} = 1 \) following again the linearized behavior (36).

4.2 Slowdown of switching

Now we consider the special cases where the incoming pulse brings the system on a trajectory that goes near the unstable fixed point \( \bar{q} = 0 \). Fig. 5 shows such a case below threshold. The left panel of Fig. 5 shows the slowing down of \( q(t) \) around \( q = 0 \) because there \( dq/dt \approx 0 \). As seen from the plots, the system remains "frozen" near the unstable equilibrium state \( q = 0 \) for a certain time \( T_{del} \). In Fig. 6 we present the results for a slightly duration where the system has switched.

To investigate this slowing down, we introduced a gaussian initial electric field

\[
e_0(-t) = A_0 \exp[-t^2/a^2].
\]

In Fig. 7 we plot a typical evolution \( q(t) \) indicating \( T_{del} \) on the left panel. On the right panel we give \( \log(T_{del}) vs \log(P) \) where \( P \) is the total power in the
Figure 3: Plot of $q(t)$ (left panel) and phase portrait $(q, dq/dt)$ (right panel) for an incident pulse below threshold so that the medium does not switch. The parameters are $A_0 = 0.2$, $t_1 = -20$, $t_2 = -22$, $t_f = 0.5$, $\gamma = 1$.

Figure 4: Same as fig. 3 but with a larger amplitude $A_0 = 0.4$ so that switching occurs.
Figure 5: Plot of $q(t)$ (left panel) and phase portrait $(q, dq/dt)$ (right panel) for an incident pulse below threshold so that the medium does not switch. The parameters are $A_0 = 1$, $t_1 = -20$, $t_2 = -22.988$, $\gamma = 1$.

Figure 6: Same as fig. 5 but with a longer pulse $t_2 = -22.989$ so that switching occurs.
Figure 7: Plot of $q(t)$ (left panel) showing the slowing down as the system approaches the unstable fixed point $\bar{q} = 0$. The right panel shows the duration of the plateau $T_{\text{del}}$ vs the power $P$ of the incident pulse in log-log scale.

The electromagnetic pulse

$$P \equiv \int_{-\infty}^{+\infty} dt e^2(t) = A_0^2 a \sqrt{\frac{\pi}{2}}.$$  

We present three different amplitudes $A_0 = 1, 0.75$ and 0.5 and vary $P$ by varying the width $a$. Fig. 7 shows that there are critical values of power where the time delay becomes very large. There does not seem to be a single critical index for the description of the singular behavior of the time delay.

4.3 Free evolution of the oscillator

We consider the limit when the external electromagnetic field is so short that it just gives an impulse to the oscillator which then evolves as free. Then $e_0(t) = A_0 \delta(t)$. Plugging this into (37), integrating in a small interval around $t = 0$ and assuming continuity of $q$ we obtain $q_{\mid t=0^+} = \gamma A_0$. The free evolution of the oscillator is then governed by the equations

$$\frac{dq}{dt} = p,$$

$$\frac{dp}{dt} = q - q^3 - 0.5\gamma^2 p,$$
Figure 8: Phase portrait \((q, \dot{q})\) showing free evolution of the system for a delta function like electromagnetic pulse so that \(q(t = 0) = -1, \; \dot{q}(t = 0) = -0.75\). The long dash corresponds to \(\gamma = 0.2\) and the short dash to \(\gamma = 0.5\). The continuous curve corresponds to \(\gamma = 0\).

with the initial conditions \(q(t = 0) = -1, \; q(t = 0) = \gamma A_0\). The phase trajectories are solution of the equation

\[
p \left(\frac{dp}{dq}\right) = q - q^3 - 0.5\gamma^2 p.
\]

The phase planes corresponding to this free evolution of the Duffing oscillator for \(\gamma = 0.5\) and 0.2 are presented in Fig. 8 together with the nondamped situation. For \(\gamma = 0.5\) the damping is strong and prevents switching while for \(\gamma = 0.2\) the system can escape to the other equilibrium and slowly converge to it. The pictures on Fig. 8 show that dissipation leads to damping of oscillation around equilibrium position and supports the switching from one to another equilibrium state. This dissipation results from the radiation of the electromagnetic waves out of the thin film.

### 4.4 Long electromagnetic pulse

When the electromagnetic pulse has the form of a plateau with a sharp front and a sharp tail, transient steady states of the ferroelectric are created. We can
find them by considering the static solutions of equation (37):

\[ q - q^3 + \gamma e_0 = 0. \]  

(39)

Let \( e_0 \) be positive. If \( 0 \leq \gamma e_0 < 2/(3\sqrt{3}) \) there are three roots, corresponding each to a fixed point, two stable and one unstable. These can be calculated by perturbation when \( \gamma e_0 << 2/(3\sqrt{3}) \), we have

\[ \bar{q}_0(e_0) \approx -\gamma e_0, \quad \bar{q}_{1,2}(e_0) \approx \pm 1 + \gamma e_0/2. \]  

(40)

When \( \gamma e_0 = 2/(3\sqrt{3}) \) the unstable and left stable equilibrium points merge together and we have only one stable point

\[ \bar{q}_2(e_0) = 2/\sqrt{3}. \]  

(41)

In the limit of very big amplitude of electromagnetic pulse, when \( \gamma e_0 >> 2/(3\sqrt{3}) \), there is only one fixed point which is stable and is defined by the approximate formula

\[ \bar{q}_2(e_0) \approx (\gamma e_0)^{1/3} \left[ 1 + \frac{1}{2} (\gamma e_0)^{-2/3} \right]. \]  

(42)

The numerical simulation of switching under the influence of long plateau-like pulse demonstrates the damping nutations near the stable points \( \bar{q}_{1,2}(e_0) \). The kinetics of switching can be considered by using the linearized equation near the stable fixed points:

\[ \delta q_{tt} + \gamma^2/2 \delta q_t + [3\bar{q}_{1,2}(e_0) - 1] \delta q = 0, \]

where \( \delta q = q - \bar{q}_{1,2}(e_0) \). The associated characteristic equation give the decrement and frequency of nutations

\[ \Gamma_0 = \gamma^2/4, \quad \Omega_0 = \sqrt{3\bar{q}_{1,2}^2(e_0) - 1 - \gamma^4/16}. \]  

(43)

The decrement is independent of the initial pulse amplitude while the frequency of nutation depends on it.

The following evolution of the ferroelectric polarization depends on the area of the external field. In this section we consider long pulses that create transient equilibria in the ferroelectric. Fig. 9 shows in the left panel such a long pulse, in the middle panel the response \( q(t) \) of the medium and in the right panel the
Figure 9: Plot of a long incident pulse $e_0(x)$ (left panel), $q(t)$ (middle panel) and phase portrait $(q, dq/dt)$ (right panel). The parameters are the same as in Fig. except $A_0 = 0.1$, $t_1 = -70$, $t_2 = -20$.

Figure 10: Plot of $q(t)$ (left panel) and phase portrait $(q, dq/dt)$ for an incoming long pulse similar to the one in Fig. except that $A_0 = 0.5$
Figure 11: Parameter plane (amplitude, duration) of incident pulse detailing the final state of the film switched or non switched. The + symbols correspond to no switching and the × to switching.

associated phase plane. The ferroelectric is moved into a transient equilibrium and returns to its previous stable polarization state $\bar{q} = -1$. Here we chose $A_0 = 0.1$ so that $\gamma e_0 = 0.2$ in the plateau region. The system goes to the transient steady state $\bar{q}_1 \approx -0.88$ value that is in good agreement with (40), which gives $-0.9$. The decrement $\Gamma_0 \approx 0.25$ and nutation frequency $\Omega_0 \approx 2$ is of the motion to this transient fixed point is predicted correctly by the estimates (43). When the system returns to its natural fixed point, the estimates are again correct and given by (42). Notice that the nutation frequency around the transient fixed point is twice as big as the one around the natural fixed point.

In Fig. 10 we observe the same phenomenon except that the transient equilibrium is close to the other stable polarization state $\bar{q} = +1$ so that when the field returns to zero, the system relaxes to that state. Here $A_0 = 0.5, \gamma = 1$ so that in the plateau region $e_0 = 1$. We are in the region above the critical $e_0$ so that there is only one fixed point. The estimate (42) gives $\bar{q} \approx 1.5$ which is in excellent agreement with the numerical value 1.4.
5 Concluding remarks

We considered the simplest model of interaction of a short electromagnetic pulse with a thin ferroelectric medium where the polarisation can be considered as uniform. The duration of the electromagnetic pulse is much shorter than the relaxation time of the medium so that only radiative decay occurs.

The linear scattering formalism predicts that low amplitude pulses can be completely reflected by the medium, in a frequency band that grows with the coupling $\gamma$. On the contrary strong electromagnetic fields can switch the medium from one state to another. We have studied such switching phenomena for both short and long pulses. For the latter we characterized the transient states they create.

We define the switching time $t_s$ as the time interval for the system to go from one fixed point to the other. From the Duffing system the typical (normalized) damping time is $\gamma^2/4$. If $\gamma^2 > 4$ switching occurs during the front of the pulse, otherwise the switching time is of the order of $\gamma^2/4$ because the system circles around the fixed point before reaching it. In any case the system will always switch in a time smaller or of the order of $\gamma^2/4$. In physical units this is about

$$t_s = 4|\alpha_0|^{-3/2}|T_c - T|^{-3/2}.$$

In some cases, the switching time can be longer, in particular if the field drives the ferroelectric near the unstable fixed point causing considerable slowing down. Switching is also irregular as shown by Fig. 11 which gives the events in the plane (duration,amplitude) of the incoming pulse. There one sees that a threshold amplitude is needed for switching. Above that the system switches or not depending on the pulse duration. For a given duration there seems to be windows where the system switches. All this information can be used by experimentalists to estimate parameters. Finally we believe the model due to its simplicity and generality can be transposed to other electromagnetic systems.

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References

[1] N. N. Nasonov, Sov. Phys. Solid State 25, 1631 (1983).
[2] N. N. Nasonov and V. V. Chernyshev, Sov. Phys. Tech. Phys. 31, 25 (1986).
[3] I. Nakata, J. Phys. Soc. Jpn. 60, 3976, (1991).
[4] M. Daniel and V. Veerakumar, Phys. Lett. A 302, 77 (2002).
[5] V. Veerakumar, Phys. Lett. A 278, 331 (2001).
[6] H. Leblong and M. Manna, J. Phys. A 27, 3245 (1994).
[7] V. V. Gladkiov, V. A. Kirikov, and E. S. Ivanova, JETP 83, 161 (1996).
[8] J. Pouget and G. A. Maugin, Phys. Rev. B 30, 5306, (1984).
[9] A. Gordon, Phys. Lett. A 154, 79, (1991).
[10] G. Benedek, A. Bussmann-Holder, and H. Bilz, Phys.Rev. B 36, 630, (1987).
[11] A. R. Bishop, E. Domany, and J. A. Krumhansl, Phys.Rev. B 14, 2966, (1976).
[12] G. Vizdrik, S. Ducharme, V. M. Fridkin, and S. G. Yudin,Phys. Rev. B 68 (9), 094113, 2003.
[13] E. D. Mishina, N. E. Sherstyuk, V. I. Stadnichuk, et al., Appl. Phys. Lett. 83, 2402, (2003).
[14] J. E. Macleman, M. A. Handschy, and N. A. Clark, Phys. Rev. A34, 3554, (1986).
[15] I. W. Stewart and E. Momoniat, Phys. Rev. E69, 061714, (2004).
[16] G. Ntogari, D. Tsipouridou, E.E. Kriezis, J. Opt. A: Pure Appl. Opt. 7, 82-87 (2005).
[17] A. Picinin, M. H. Lente, J. A. Eiras, J. P. Rino, Phys Rev B69, 064117 (2004).

[18] D.V. Isakov, T.R. Volk, L.I.Ivleva, K.Betzler, C. David, A.Tunyagi, M. Wohleck, Pis’ma v ZhETF 80, 289 (2004).

[19] L.M. Blinov, V.M.Fridkin, C.P. Palto, A.V. Bune, P.A. Dowben, C. Ducharme, Uspehki Fiz.Nauk. 170, 247 (2000).

[20] J. Osman, Y. Ishibashi, and D. R. Tilley, Jpn. J. Appl. Phys. 37, A, 4887, 1998.

[21] A. I. Larkin and D. E. Khmel’nitskii, Sov. Phys. JETP 29, 1123 (1969).

[22] B. Westwanski, A. Ogaza, and B. Fugiel, Phys. Rev. B 45, 2699, (1992).

[23] Tianquan Lu, Wenwu Cao, Rev. B66, 024102 (2002).

[24] E. V. Kazantseva, A. I. Maimistov, Optics and Spectroscopy. 99, 91, (2005).

[25] C. Kittel, Quantum theory of solids, (New York, 1963)

[26] A. Maimistov et J. G. Caputo, Physica D 189, 107-114, (2003).

[27] J. Guckenheimer and P. Holmes, Dynamical systems and bifurcations of vector fields, Springer (1983).

[28] A.N. Tikhonov, A.A. Samarski, Equations of mathematical physics, (Dover, 1983).

[29] H. Lamb, Elements of soliton theory, (Wiley, 1983).