Equations, chromopermittivity and instabilities of the quark-gluon medium

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Abstract

The quark-gluon medium described by QCD equations is considered at high energies. Within the assumptions of the linear response theory the chromopermittivity of the medium is introduced and it is argued that it exceeds 1 at TeV energies. The dispersion equations show that the proper modes of the medium reveal instability and the parton currents traversing it induce the emission of Cherenkov gluons. Their distributions at LHC can differ from those typical for lower energies of RHIC because they are determined by the high energy dependence of the chromopermittivity while the latter ones arise due to collective resonance excitations.

1 Introduction

The quark-gluon medium formed in the collisions of high energy hadrons and nuclei can possess some collective properties. At the classical level QCD equations are similar to those of QED. Therefore it is quite natural to use the analogy with electrodynamical processes in matter. This approach is widely discussed (for recent review papers see, e.g., [1, 2]). The energy losses of charged particles in electrodynamics may be classified as elastic-collisional, radiative, Cherenkov, transition and wakes not to say about the ionization losses at low energies and nonlinear effects. The inelastic processes play especially important role in hadron collisions.

A deflection of particle trajectory from straight line is typical for the first two processes. Cherenkov and transition radiation as well as wakes behind the particle are described as proceeding at its (almost) constant velocity. The particle charge induces polarization of the medium (the permittivity different from 1). Namely this collective effect using the energy stored in the medium leads to Cherenkov radiation and wakes arising mostly from the matter response induced by the polarization. We consider them here for hadronic medium and, first of all, formulate the relevant equations.
The in-medium equations of gluodynamics were proposed in \cite{3} (see also \cite{4}). They differ from the in-vacuum equations by introducing a chromopermittivity of the quark-gluon medium. Similar to the dielectric permittivity in electrodynamics it describes the linear response of the matter to passing partons. At the classical level the equations are completely analogous to electrodynamical ones with chromopermittivity just replacing the dielectric permittivity. However the energy behavior (dispersion) of these permittivities may be different because the collective quasi-particle excitations differ. Therefore the dispersion equations and their predictions for spectra and instabilities differ. As an example, we consider a specific model for high energy dependence of the chromopermittivity inspired by experimental data on hadronic elastic scattering amplitudes. It is shown that in this model the quark-gluon medium is unstable and high energy partons moving in it emit Cherenkov gluons due to collective polarization effects. At very high energies their angular and energy distributions may differ from those of Cherenkov photons in electrodynamics. These conclusions can be verified at LHC.

2 The in-medium equations and the Cherenkov gluons

In this section we briefly remind the main results of the paper \cite{3}.

The classical in-vacuum Yang-Mills equations are

\begin{equation}
D_\mu F^{\mu\nu} = J^\nu, \tag{1}
\end{equation}

\begin{equation}
F^{\mu\nu} = \partial^\mu A^\nu - \partial^\nu A^\mu - ig[A^\mu, A^\nu], \tag{2}
\end{equation}

where $A^\mu = A^\mu_a T_a$; $A_a(A_a^0 \equiv \Phi_a, A_a)$ are the gauge field (scalar and vector) potentials, the color matrices $T_a$ satisfy the relation $[T_a, T_b] = i f_{abc} T_c$. $D_\mu = \partial_\mu - ig[A_\mu, \cdot]$, $J^\nu(\rho, j)$ is a classical source current, $\hbar = c = 1$ and the metric tensor is $g^{\mu\nu} = \text{diag}(+, -, -, -)$.

In the covariant gauge $\partial_\mu A^\mu = 0$ they are written as

\begin{equation}
\Box A^\mu = J^\mu + ig[A_\mu, \partial^\nu A^\mu + F^{\mu\nu}], \tag{3}
\end{equation}

where $\Box$ is the d’Alembertian operator. The classical gluon field is given by the solution of the corresponding abelian problem.
The chromoelectric and chromomagnetic fields are

\[ E^\mu = F^\mu_\nu, \]  
\[ B^\mu = -\frac{1}{2} \epsilon^{\mu ij} F_{ij}, \]  

or, as functions of the gauge potentials in vector notation,

\[ E_a = \nabla \Phi_a - \frac{\partial A_a}{\partial t} + g f_{abc} A_b \Phi_c, \]  
\[ B_a = \text{curl} A_a - \frac{1}{2} g f_{abc} [A_b A_c]. \]

The equations of motion (1) in vector form are written as

\[ \text{div} E_a - g f_{abc} A_b E_c = \rho_a, \]  
\[ \text{curl} B_a - \frac{\partial E_a}{\partial t} - g f_{abc} (\Phi_b E_c + [A_b B_c]) = j_a. \]

Analogously to electrodynamics, the medium is accounted for if \( E \) is replaced by \( D = \epsilon E \) in \( F^\mu_\nu \), i.e. in Eq. (4). Therefore (8), (9) in vector form are most suitable for generalization to the in-medium case.

In terms of potentials the equations of in-medium gluodynamics are cast in the form

\[ \triangle A_a - \epsilon \frac{\partial^2 A_a}{\partial t^2} = -j_a - g f_{abc} \left( \frac{1}{2} \text{curl} [A_b, A_c] + \epsilon \frac{\partial}{\partial t} (A_b \Phi_c) + \frac{1}{2} [A_b \text{curl} A_c] \right) - \epsilon \Phi_b \frac{\partial A_c}{\partial t} - \epsilon \Phi_b \text{grad} \Phi_c - \frac{1}{2} g f_{cmn} [A_b [A_m A_n]] + g \epsilon f_{cmn} \Phi_b A_m \Phi_n), \]  
\[ \triangle \Phi_a - \epsilon \frac{\partial^2 \Phi_a}{\partial t^2} = -\frac{\rho_a}{\epsilon} + g f_{abc} (-2 A_b \text{grad} \Phi_c + A_b \frac{\partial A_c}{\partial t} - \epsilon \frac{\partial \Phi_b}{\partial t} \Phi_c) + g^2 f_{amn} f_{nb} A_m A_n \Phi_b. \]

If the terms with explicitly shown coupling constant \( g \) are omitted, one gets the set of abelian equations, which differ from electrodynamical equations by the color index \( a \) only. We omit it in what follows. The most important property of the solutions of these equations is that while the in-vacuum (\( \epsilon = 1 \)) equations do not admit any radiation processes, it happens for \( \epsilon \neq 1 \) that
there are solutions of these equations with non-zero Poynting vector even in the classical approach. Here $j_a$ is treated as an external current ascribed to partons moving fast relative to the other partons "at rest". It is proportional to $g$ as seen from Eq. (12). Thus the terms with explicitly shown $g$ in Eqs (10), (11) are of the order $g^3$. The higher order corrections may be calculated (preliminary results are published in [5]) but we postpone their consideration for further publications. An impact of internal currents is phenomenologically taken into account by the chromopermittivity $\epsilon$. Its spatio-temporal dependence may be included similar to electrodynamics. It is well known in electrodynamics that the magnetic permeability is automatically taken into account if the proper dependence of $\epsilon$ (in the Fourier representation) on the frequency ($\omega$) and the wave vector ($k$) is used. However it is premature now to discuss these complications.

For the current with velocity $v$ along the $z$-axis:

$$j(r, t) = v j_\rho(r, t) = 4\pi g v \delta(r - vt)$$  \hspace{1cm} (12)

the classical lowest order solution of in-medium gluodynamics can be cast in the form [3, 6]

$$\Phi^{(1)}(r, t) = \frac{2g}{\epsilon} \frac{\theta(vt - z - r_\perp \sqrt{v^2 - 1})}{\sqrt{(vt - z)^2 - r_\perp^2(v^2 - 1)}}, \hspace{1cm} (13)$$

and

$$A^{(1)}(r, t) = \epsilon v \Phi^{(1)}(r, t), \hspace{1cm} (14)$$

where the superscript (1) indicates the solutions of order $g$, $r_\perp = \sqrt{x^2 + y^2}$ is the cylindrical coordinate; $z$ is the symmetry axis.

This solution describes the emission of Cherenkov gluons at the typical angle

$$\cos \theta = \frac{1}{v \sqrt{\epsilon}}, \hspace{1cm} (15)$$

It is constant for constant $\epsilon > 1$.

First experimental data about ringlike (at constant $\theta$) emission of particles in hadron collisions at high energies were published in [7]. They were interpreted as an effect due to Cherenkov gluons in [8, 9].

The expression for the intensity of the radiation is given by the Tamm-Frank formula (up to Casimir operators)

$$\frac{dE}{dl} = 4\pi \alpha_S \int \omega d\omega (1 - \frac{1}{v^2 \epsilon(\omega)}) \theta(v^2 \epsilon(\omega) - 1). \hspace{1cm} (16)$$
For absorbing media $\epsilon$ acquires the imaginary part. The sharp front edge of the shock wave (13) is smoothed. The angular distribution of Cherenkov radiation widens. The $\delta$-function at the angle (15) is replaced by the Breit-Wigner shape [10, 11] with maximum at the same angle (but $|\epsilon|$ in place of $\epsilon$) and the width proportional to the imaginary part. This has been used for fits of RHIC data as explained in the next section.

3 The chromopermittivity

In in-medium electrodynamics the permittivity of real substances depends on $\omega$. Moreover, it has an imaginary part determining the absorption. E.g., $\text{Re}\, \epsilon$ for water [12] is approximately constant in the visible light region $(\sqrt{\epsilon} \approx 1.34)$, increases at low $\omega$ (up to $\epsilon \approx 80$) and becomes smaller than 1 at high energies, tending to 1 asymptotically as

$$\text{Re}\, \epsilon_{ED} = 1 - \frac{\omega_L^2}{\omega^2},$$

where $\omega_L$ is the Langmuir (plasma) frequency. The absorption $(\text{Im}\, \epsilon_{ED})$ is very small for visible light but dramatically increases in nearby regions both at low and high frequencies. Theoretically this behavior is ascribed to various collective excitations in water relevant to its response to radiation with different frequencies. Among them the resonance excitations are quite prominent (see, e.g., [13]). Even in electrodynamics, the quantitative theory of this behavior is still lacking, however. Moreover the formula (17) is purely electrodynamical one and does not take into account hadronic processes at extremely high energies.

Then, what can we say about the chromopermittivity?

Up to now, the attempts to calculate the chromopermittivity from first principles are not very convincing. It can be obtained from the polarization operator. The corresponding dispersion branches have been computed in the lowest order perturbation theory [14, 15, 16] and in the framework of the thermal field theories [17, 18, 19]. The results with an additional phenomenological ad hoc assumption about the role of resonances were used in a simplified model of scalar fields [20] to show that the chromopermittivity can be larger than 1, which admits Cherenkov gluons. Extensive studies were performed in [4].
In view of this situation, we prefer to use the general formulae of the scattering theory [21] to estimate the chromopermittivity. It is related to the refractive index \( n \) of the medium:

\[
\epsilon = n^2
\]  

(18)

and is expressed [12, 21, 22] through the real part of the forward scattering amplitude \( \text{Re} F_0(\omega) \) of the refracted quanta:

\[
\text{Re}\Delta \epsilon = \text{Re}\epsilon(\omega) - 1 = \frac{4\pi N_s \text{Re} F_0(\omega)}{\omega^2} = \frac{N_s \sigma(\omega) \rho(\omega)}{\omega}
\]  

(19)

with

\[
\text{Im} F_0(\omega) = \frac{\omega}{4\pi} \sigma(\omega).
\]  

(20)

Here \( \omega \) denotes the energy, \( N_s \) is the density of scattering centers, \( \sigma(\omega) \) the cross section and \( \rho(\omega) \) the ratio of real to imaginary parts of the forward scattering amplitude \( F_0(\omega) \). Thus the emission of Cherenkov gluons is possible only for processes with positive \( \text{Re} F_0(\omega) \) or \( \rho(\omega) \). Unfortunately, we are unable to calculate directly in QCD these characteristics of gluons and have to rely on analogies and our knowledge of the properties of hadron interactions.

The experimental facts we get for this medium are brought about only by particles registered at the final stage. They have some features in common, which (one may hope!) are also relevant for gluons as the carriers of the strong forces. Those are the resonant behavior of amplitudes at rather low energies and the positive real part of the forward scattering amplitudes at very high energies for hadron-hadron (pp, Kp, \( \pi p \)) and photon-hadron (\( \gamma p \)) processes as measured from the interference of the Coulomb and hadronic parts of the amplitudes. This shows that the necessary condition for Cherenkov effects might be satisfied at least within these two energy intervals. This fact was used to describe experimental observations at SPS, RHIC and cosmic ray energies.

The first region is typical for the comparatively low energies of secondary particles registered in SPS and RHIC experiments. \( \text{Re} F_0(\omega) \) is always positive (i.e., \( \epsilon > 1 \)) within the low-mass wings of the Breit-Wigner resonances. Therefore, Cherenkov gluons can be emitted in these energy intervals.

The asymmetry of the \( \rho \)-meson mass shape observed in leptonic decays of \( \rho \)-mesons created in nuclei collisions at SPS [23] was explained by emission of low-energy Cherenkov gluons [24, 25] inside the left (low mass) wing.\(^3\)

\(^3\)In electrodynamics these quanta are photons, in QCD they are gluons.
of the Breit-Wigner resonance. It is predicted that this feature should be common for all resonances traversing the nuclear medium. Some preliminary experimental indications which favor this conclusion have appeared for other resonances as well [26, 27, 28, 29, 30].

The experimental data of STAR and PHENIX collaborations at RHIC [31, 32, 33, 34, 35] on two- and three-particle azimuthal correlations also deal with rather low energies of secondary particles. The ringlike distribution of the particles around the (away-side) jet traversing the quark-gluon medium was observed in the form of two humps when projected on the diameter of the ring. That is completely analogous to what was done by Cherenkov in his original publications (see, e.g., [36]).

Similar to electrodynamics [10], it is possible to get the energy-angular spectrum of emitted gluons [37] per the unit length

$$\frac{dN^{(1)}}{d\Omega d\omega} = \frac{\alpha_s C}{2\pi} \left[ \frac{(1 - x)\Gamma_t}{(x - x_0)^2 + (\Gamma_t)^2/4} + \frac{\Gamma_l}{x} \right],$$  \hspace{1cm} (21)

where

$$x = \cos^2 \theta, \quad x_0 = \epsilon_{1t}/|\epsilon_t|^2 v^2, \quad \Gamma_j = 2\epsilon_{2j}/|\epsilon_j|^2 v^2, \quad \epsilon_j = \epsilon_{1j} + i\epsilon_{2j}. \hspace{1cm} (22)$$

The first term in the brackets corresponds to the transverse gluon Cherenkov radiation (index t at \(\epsilon_t\)) and the second term to the radiation due to the longitudinal wake (index l at \(\epsilon_l\)). The transverse and longitudinal components of the chromopermittivity tensor are explicitly indicated here even though they are equal in any homogeneous medium. The real (\(\epsilon_1\)) and imaginary (\(\epsilon_2\)) parts of \(\epsilon\) are taken into account. The angle \(\theta\) is the polar angle if the away-side jet axis would be chosen as \(z\)-axis. The ringlike structure around this axis is clearly exhibited in (21). The angle \(\theta\) is related to the laboratory polar (\(\theta_L\)) and azimuthal (\(\phi_L\)) angles in RHIC experiments as

$$x = \cos^2 \theta = \sin^2 \theta_L \cos^2 \phi_L. \hspace{1cm} (23)$$

Integrating (21) over \(\theta_L\) one gets the final formula to compare with the two-hump structure of azimuthal (\(\phi_L\)) correlations observed at RHIC. It is quite lengthy and not reproduced here. It is seen already from (21) and (23) that this projection of the two-dimensional ring on its diameter is symmetrical about \(\phi_L = \pi\) and exhibits humps whose positions are mostly determined by \(\epsilon_1\) and widths determined by \(\epsilon_2\). The first term in (21) clearly demonstrates the a’la Breit-Wigner angular hump which replaces the \(\delta\)-functional.
angular dependence characteristic for real $\epsilon$. For two-particle correlations in central Au-Au collisions at 200 GeV measured by STAR [31] and PHENIX [32] it was found in [11] that $\epsilon_1$ and $\epsilon_2$ are ranging, correspondingly, from 5.4 to 9 and from 0.7 to 2. The results differed because of disagreement in peaks positions and widths in the old data of these collaborations with peaks at $\pi \pm 1.04$ and $\pi \pm 1.27$, correspondingly. The new data [38] agree that maxima are positioned at $\pi \pm 1.1$ so that $\epsilon_1 \approx 6$ and $\epsilon_2 \approx 1$ would be good estimates $^2$. The main conclusion about the large values of $\epsilon_1$ indicating on the non-gaseous matter (large $N_s$ in (19)) is supported in any case. For three-particle correlations the peak is shifted to larger angles in the earlier data [35] and agrees with above estimates in the new data [38]. The in-plane and out-of-plane structures of two- and three-particle correlations in semi-central events [38] can be described in the same way. More important is the fact that the wake contribution described by the second term in (21) leads to the shift of the maxima [37] observed in the data [38] obtained with triggers positioned in-between (at $\pi/4$).

In principle, the two-hump structure may arise not only due to Cherenkov gluons but also as the Mach cone. However, there exists the clear distinctive feature which favors the QCD interpretation compared with relativistic hydrodynamics. This is the property of the corresponding wakes. The hydrodynamical Mach cone interpretation was not yet used for quantitative fits of the data because the particle yield from the wake $^3$ moving opposite to the trigger jet largely overwhelms the weak Mach cone signal $^4$ for relativistic particles. It should produce a jet of particles in this direction (the strong away-side jet) that is not observed in experiment.

4 Instabilities in the high energy region

The specifics of RHIC experiments is that triggers register trigger jets with energies of several GeV emitted at large angles (close to $\pi/2$) to the collision axis. In its turn, cosmic ray data [7] at energies corresponding to LHC energies require that beside effects induced by the comparatively low energy (several GeV) partons there should be very high-energy Cherenkov gluons emitted by the ultrarelativistic partons moving along the collision axis as

$^2$The constant values (no dispersion) are considered because the range of pion energies within the ring is comparatively small.

$^3$It is transverse in hydrodynamics (see [39, 40, 41]).
was first proposed in \cite{8, 9}. Let us note the important difference from pure
electrodynamics, where $\epsilon < 1$ at high frequencies (see (17)). In hadron
physics, $\text{Re} F_0(\omega)$ is positive (and consequently $\epsilon > 1$ according to (19))
above some threshold as measured in experiment. The dispersion relations
for particle scattering amplitudes explain it as the corollary of the cross
sections increase at high energy\cite{4}. This could be of crucial importance for
experiments at the LHC opening a new channel for the medium polarization
effects.

In what follows, we concentrate on the high energy region. The analogy
of the two colliding nuclei considered as clouds of partons to the two colliding
bunches of plasma (see, e.g., \cite{44}) will be used. However, the permittivities
differ in these cases. It leads to different conclusions.

The energy dependence of the factors in (19) is not precisely known.
The total cross section increases but not faster than $\ln^2 \omega$, the density of
scatterers must saturate and the real part of the forward scattering amplitude
is positive and, probably, decreasing. The increase of total cross sections leads
to positive $\text{Re} F_0(\omega)$ according to the dispersion relations \cite{45}. Therefore,
we will study the properties of the quark-gluon medium in the model with
chromopermittivity behaving above some threshold as

$$\text{Re} \epsilon = 1 + \frac{\omega_0^2}{\omega^2}, \quad (24)$$

where $\omega_0$ is some real free parameter. In principle, it may depend on energy
but at the beginning we consider it to be a constant. This is the simplest
formula for the even (as required by the Kramers-Kronig relations) function
of $\omega$ exceeding 1 and tending to 1 at $\omega \to \infty$ satisfying the general analytical
requirements imposed on $\epsilon$. It reduces to electrodynamics with imaginary
$\omega_0$ and is somewhat different from treatment in \cite{8, 9} where $\omega_0^2 = a\omega$ with
constant $a$ was used\cite{5}. In hadronic processes the threshold of positiveness of
$\text{Re} F_0(\omega)$ is near 100 GeV in the rest system of the target proton in the above
mentioned processes. If the experimental values of $\sigma(\omega)\rho(\omega)/\omega$ are plotted,
they show the increase at the threshold, the maximum slightly below 1 TeV
in the rest system and the subsequent decrease. Thus (24) is also motivated

\footnote{Surely, taking into account the hadronic channels of photon interactions at high ener-
gies one would come to the similar conclusions and the formula (17) will change.}

\footnote{This contradicts to the requirement for $\text{Re} \epsilon$ to be the even function of $\omega$ but can be
considered as a purely phenomenological fit in some restricted region of $\omega$.}
by Eq. (19). It is premature now for our purposes to consider more elaborate forms of the relation (21).

For the dispersion law (24) the classical (no recoil) coherence length is

\[ l_{coh} = \frac{2\gamma^2}{\omega} \left( 1 - \frac{\omega_0^2 \gamma^2 + \theta^2 \gamma^2}{\omega^2} \right)^{-1}, \tag{25} \]

where \( \gamma \) is the \( \gamma \)-factor of the colliding hadrons. This length is larger than for negative \( \omega_0^2 \) and becomes especially large at Cherenkov angles quadratically increasing with energy. The requirement of positive \( l_{coh} \) restricts the transverse momenta of emitted gluons

\[ k_t^2 \geq \omega_0^2 - \frac{\omega^2}{\gamma^2} \geq \omega_0^2 - x^2 m^2 \geq 0, \tag{26} \]

where \( m \) is the proton mass and \( x \) is the share of its momentum carried by the parton. For these inequalities to be valid at any \( x \), one needs \( \omega_0 \geq m \approx 1 \text{ GeV} \).

Let us mention that the relation of the current with the field is determined by the conductivity \( \sigma_{i,j}(\omega, k) \) (in the tensor notation) as

\[ j_i(\omega, k) = \sigma_{i,j}(\omega, k)E_j(\omega, k). \tag{27} \]

The permittivity is related to the conductivity as

\[ \epsilon_{i,j} = 1 + \frac{4\pi i}{\omega} \sigma_{i,j}. \tag{28} \]

Thus, the positive sign in (24) implies the negative inductive conductivity, i.e. the damped induced current opposite to the direction of the field. In electrodynamics, the negative conductivity arises due to the presence of magnetic fields [46]. It is actively studied now [47]. Whether the dispersion law (24) in the case of chromodynamics is related to internal chromomagnetic fields must be studied.

The dielectric permittivity of the macroscopic matter (e.g., as given by (17)) is usually considered in its rest system which is well defined. For collisions of two nuclei (or hadrons) such a system requires special definition (see [3]). In particular, for fast forward moving partons the spectators (the

\[ \text{Sometimes it is called as the formation length while the term coherence length is ascribed to the factor } 2\gamma^2/\omega \text{ in front of the brackets in (25).} \]
medium) are formed by the partons of another (target) nucleus at rest. Thus we consider a problem of the system of the quark-gluon medium impinged by a bunch of fast partons similar to the problem of plasma physics, namely, that of the interaction of the electron bunch with plasma [48]. This differs from RHIC conditions when partons scattered at $\pi/2$ were considered with the rest system of the medium coinciding with the nuclei center of mass system.

Two complications are to be considered. First, there are selected directions of the current ($z$-axis) and its radiation. This is cured by introducing the permittivity tensor as

$$D_i(\omega, k) = \epsilon_{ij}(\omega, k)E_j(\omega, k).$$

(29)

Second, the impinging bunch content is similar to that of the target (plasma-plasma collisions!) and its permittivity must also be accounted for. In the projectile system it is the same as the permittivity of the target in its rest system, i.e. given by (24). Then one takes into account that the total induced current is the sum of currents induced in the target and in the projectile and Lorentz-transforms the projectile internal fields and currents to the target rest system where we consider the whole process as it is done in [44, 48, 49, 50]. Using the current conservation and classical field equations (8), (9) with $g = 0$ one gets for non-zero components of the chrom permittivity tensor

$$\epsilon_{xx} = \epsilon_{yy} = 1 + \frac{\omega_0^2}{\omega^2}(1 + \frac{1}{\gamma}),$$

$$\epsilon_{xz} = -\epsilon_{zx} = \frac{\omega_0^2k_t}{\omega^2(\omega - k_z)\gamma},$$

$$\epsilon_{zz} = 1 + \frac{\omega_0^2}{\omega^2}\left(1 + \frac{k_t^2}{(\omega - k_z)^2\gamma}\right).$$

(30)

Here $k_t$ and $k_z$ are the transverse and longitudinal components of $k$. We use the approximation of high energies (large $\gamma \gg 1$ factor, i.e. $v \approx 1$). The terms depending on $\gamma$ are due to the impinging partons. They can be omitted everywhere except the terms which determine the Cherenkov gluon radiation at $\omega - k_z \approx 0$.

The classical equations derived from (10), (11) and written in the momentum space have solution if the following dispersion equation is valid

$$\det(\omega, k) = \mid k^2\delta_{ij} - k_i k_j - \omega^2\epsilon_{ij} \mid = 0.$$ 

(31)
It is of the sixth order in momenta dimension. However, the sixth order terms cancel and (31) leads to two equations (of the second order):

\[ k^2 - \omega^2 - \omega_0^2 = 0, \]  
(32)

\[ (k^2 - \omega^2 - \omega_0^2)(1 + \frac{\omega_0^2}{\omega^2}) = \frac{\omega_0^4 k_t^2}{\omega^2(\omega - k_z)^2 \gamma} = 0. \]  
(33)

They determine the internal modes of the medium and the bunch propagation through the medium, correspondingly.

The equation (32) shows that the quark-gluon medium is unstable because there exists the branch with \( \text{Im} \omega > 0 \) for modes \( k^2 < \omega_0^2 \). Thus the energy increase of the total cross sections is related to the instability of the quark-gluon medium by the positiveness of \( \text{Re} F_0(\omega) \) at high energies.

The equation (33) has solutions corresponding to Cherenkov gluons emitted by the impinging bunch and determined by the last term in (33). They can be found at \( \omega = k_z + \delta \) (\( \delta \ll \omega \)). For \( k_t = \omega_0 \) one gets the solutions with

\[ \text{Im} \delta_1 = \frac{\omega_0^2}{2 k_z (2 \gamma (1 + \omega_0^2/k_z^2))^{1/3}}. \]  
(34)

For \( k_t \neq \omega_0 \) there is the solution with

\[ \text{Im} \delta_2 = \frac{\omega_0^2 k_t}{k_z [\gamma |k_t^2 - \omega_0^2| (1 + \omega_0^2/k_z^2)]^{1/2}}. \]  
(35)

It is well known (see [6]) that the solutions of the disperion equation (31) determine the Green function of the system equations

\[ G(t, z) = \frac{1}{2 \pi^2} \int_{-\infty}^{\infty} \int_{C(\omega)} \frac{1}{\text{det}(\omega, k)} \exp(-i \omega t + ik z) d\omega, \]  
(36)

where the contour \( C(\omega) \) passes above all singularities in the integral. Therefore, the positive \( \text{Im} \delta \) in (34) and (35) correspond to the absolute instability of the system. Let us note that the instability at \( k_t = \omega_0 \) is stronger than at \( k_t \neq \omega_0 \) approximately by the factor \( \gamma^{1/6} \) (this factor is about 4 times larger at LHC compared to RHIC). The instability exponent (31) decreases as \( \gamma^{-1/3} \) and is about 16 times smaller at LHC compared to RHIC. It tends to zero asymptotically.

Thus Cherenkov gluons are emitted with constant transverse momentum \( k_t = \omega_0 \) and their number is proportional to \( d\omega/(\omega)^2 \theta(\omega - \omega_{thr}) \) for \( \epsilon(\omega) \).
given by Eq. (24) with account of the threshold above which this equation is applicable. It differs from the traditional folklore of constant emission angle of Cherenkov radiation and the number of gluons $\propto d\omega$ (or the total energy loss proportional to $\omega d\omega$). This difference is easily explained by Eqs (15), (16) which give $\cos \theta = \text{const}$ and the energy loss $\omega d\omega$ for $\epsilon = \text{const}$ and $k_t \approx \omega_0$ and $d\omega/\omega$ energy loss for $\epsilon = 1 + \omega_0^2/\omega^2$. The electrodynamical formulae for transition radiation (see, e.g., [43]) also get the singularity in the spectra at $k_t \approx \omega_0$ for such $\epsilon$.

Therefore, the conclusions strongly depend on the realistic shape of $\epsilon(\omega)$ (and according to (18) on the refraction index $n$) at high energies. It was shown [8, 9] that according to experimental data on $pp$-scattering and eq. (19) $\Delta\epsilon(\omega)$ increases above the threshold, has a maximum about $10^{-5}$ at $\omega \approx 1$ TeV and then decreases. One easily gets this rough estimate if inserts in Eq. (19) the total $pp$ cross section about 60 mb, $\rho_{pp} \approx 0.1$ at $\omega \approx 1$ TeV and $N_s \approx 3m^3/4\pi$ (considering the proton as a single scattering center). Then $\omega_0 \approx 3$ GeV. From inequalities (26) it follows that the coherence length is positive for $x < \omega_0/m \approx 3$, i.e. for all partons inside the impinging nucleon (nucleus). These inequalities must be valid for all gluons radiated with extremely high energies. If optimistic, one might expect some excess at transverse momenta from 1 to 10 GeV due to these processes. This intuitive estimate tries to account for variation of the above parameters and possible role of the imaginary part of $\epsilon$.

The rate of the decrease of $\Delta\epsilon(\omega)$ is not well determined (in [8, 9] it was used as $\omega^{-1}$). Thus the step-like approximation with almost constant $\epsilon$ in between the threshold and some higher energy is not excluded. Unfortunately, the data about $\rho(\omega)$ are obtained from so small transferred momenta that at LHC we hardly get any. Experimentally, different possibilities can be verified by measuring the transverse momenta and energy spectra of particles in the humps.

With increments of the increase of the process (34) known, it is possible to estimate the corresponding effective time in (36). Using $\Delta\epsilon_{\text{eff}} \approx 10^{-5}$ one gets it about $10^4$ times larger than the typical hadronic scale. This implies that usual hadronization effects proceed much earlier than the full instability develops, i.e. a characteristic time of instability growth in the high energy region is much larger than other time scales of the parton system evolution. It is known [51, 52] that instabilities at lower frequencies may play more important role. Their experimental verification is still awaited.

The roots $k_m(\omega)$ of Eqs. (32), (33) or the poles in Eq. (36) are found...
at Im(ω) → +∞ both in upper and lower halves of the k-plane. According to Starrock criteria [6] it implies the absolute instability both for proper oscillations and Cherenkov radiation.

5 Conclusion

It is argued that the dispersion law of the quark-gluon medium at high energies differs from that in the electromagnetism. Its main feature is the excess of the chromopermittivity over 1. The definite model is considered. The dispersion equation of in-medium gluodynamics is solved. It is shown that the quark-gluon medium is unstable and responses to external high energy partons by creation of Cherenkov gluons with specific properties.

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