Karl Stein (1913-2000)

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Karl Stein was born on the first of January 1913 in Hamm in Westfalen, grew up there, received his Abitur in 1932 and immediately thereafter began his studies in Münster. Just four years later, under the guidance of Heinrich Behnke, he passed his Staatsexam, received his promotion and became Behnke’s assistant.

Throughout his life, complex analysis, primarily in higher dimensions (“mehrere Veränderliche”), was the leitmotif of Stein’s mathematics. As a fresh Ph.D. in Münster in 1936, under the leadership of the master Behnke, he had already been exposed to the fascinating developments in this area. The brilliant young Peter Thullen was proving fundamental theorems, Henri Cartan had visited Münster, and Behnke and Thullen had just written the book on the subject. It must have been clear to Stein that this was the way to go.

Indeed it was! The amazing phenomenon of analytic continuation in higher dimensions had already been exemplified more than 20 years before in the works of Hartogs and E. E. Levi. Thullen’s recent work had gone much further. In the opposite direction, Cartan and Thullen had proved their characterization of domains in $\mathbb{C}^n$ which admit a holomorphic function which can not be continued any further. Behnke himself was also an active participant in mathematics research, always bringing new ideas to Münster. This was indeed an exciting time for the young researcher, Karl Stein.

Even though the pest of the Third Reich was already invading academia, Behnke kept things going for as long as possible. But this phase of the Münster school of complex analysis could not go on forever. Although
Stein was taken into the army, during a brief stay at home he was able to prepare and submit the paper which contained the results from his Habilitationssarbeit which was accepted in 1940. At a certain point he was sent to the eastern front. Luckily, however, the authorities were informed of his mathematical abilities, and he was called back to Berlin to work until the end of the war in some form of cryptology. Stein told me he was not very good at this.

Almost immediately after the war, in a setting of total destruction, Behnke began to rebuild his group, and very soon Stein became the mathematics guru in Münster. At the time there were only two professor positions in pure mathematics, those of Behnke and F. K. Schmidt. Although it must have been very difficult, Behnke somehow found a position for Stein which he held from 1946 und 1955.

In 1955 Stein took a chair of mathematics at the Ludwigs-Maximilian-Universität in München where he stayed for the remainder of his academic career. There he continued his mathematics and built his own group in complex analysis. A number of his doctoral students later became professors at universities here in Germany. One of the most exciting periods in München was certainly that in the late-1960s with the young Otto Forster, who received his doctorate in 1961, leading a group of up-and-coming researchers.

Not only being an outstanding researcher and teacher, Karl Stein worked tirelessly on all sides of academia. Among other activities he was managing editor of Manuscripta Mathematica from 1969 until 1983, and in 1966 he was president of the DMV. He was awarded numerous honors, including membership in the Bavarian and the Austrian Academies of Sciences, and corresponding membership of the Göttingen Academy of Sciences. In 1973 he received an honorary doctor’s degree from the faculty of mathematics in Münster, and in 1990, on the occasion of the 100th anniversary of the founding of the DMV, he was awarded the inaugural Cantor-Medaille.

Up until a few years before his death in October of 2000 Stein was still actively thinking about and even doing mathematics. I remember his talk in
Bochum in the fall of 1992, just before his 80th birthday. He still radiated his intense interest in discovery and the joy of being involved with something so beautiful. Even the youngest of students who heard that talk were mesmerized, knowing they had experienced the real thing!

As the reader has certainly noticed we have barely touched upon the mathematics that so fascinated Stein and his contributions as a researcher and teacher. Let us devote the remainder of this article to a chronological sketch of some of the high points.

Although Stein’s thesis does not reflect his later work, it does reflect one of the main directions of that time, namely “analytic continuation”, and it also shows that even at this beginning stage he was ahead of his time. It was already known that a function which is holomorphic in a neighborhood of the standard Euclidean sphere in \( \mathbb{C}^n, n > 1 \), extends holomorphically to the full Euclidean ball. In his thesis (see [S1]), under assumptions, e.g., on dimension, which we now know to be inessential, Stein shows that such results are in fact local in nature. For example, a function which is holomorphic in a neighborhood of a piece of the sphere extends to an open set which only depends on that piece. He even realized that such results are possible for functions holomorphic in neighborhoods of higher-codimensional real manifolds. These results, which represent a change in viewpoint, are precursors to the highly developed modern theory of Cauchy-Riemann manifolds.

One group of leading problems of that period revolved around the question of whether or not holomorphic or meromorphic functions could be constructed with certain prescribed properties. The model situations were the theorem of Mittag-Leffler and the Weierstrass-theory of infinite product expansions on the complex plane. In the former case, at each point of a divergent sequence \( \{z_n\} \) a finite negative part \( P_n \) of a Laurent series is given and one asks if there is a meromorphic function \( f \) on the complex plane which is holomorphic everywhere except at points of the sequence with \( f - P_n \) being holomorphic near each \( z_n \). Formulated without the details, one asks if one can arbitrarily prescribe the principal parts of a meromorphic function.

In the original Weierstrass-theory one prescribes an positive integer \( m_n \) at
each of the points $z_n$ and asks for the existence of a holomorphic function $f$ whose zeros only occur at points of the sequence and the orders of the zeros $f$ at these points should be the given integers. More generally one allows $m_n$ to be an arbitrary integer and asks for a meromorphic function with prescribed zeros and poles. In this case the “principal part” $P_n$ is replaced by $D_n = (z - z_n)^{m_n}$ and the requirement is that $\frac{f}{D_n}$ is holomorphic near $z_n$. Briefly stated, one asks if the “divisor” of a meromorphic function can be arbitrarily prescribed.

Due to the early work of P. Cousin ([C]) one referred to the higher-dimensional versions of these as the additive and multiplicative Cousin problems or simply Cousin I. and II.

As Stein was starting out, it was well-known that the appropriate domains for solving the interesting problems of the time, such as the Cousin problems, were the “Regularitätsbereiche”. Precisely speaking, they can be defined as domains $D$ in $\mathbb{C}^n$ so that given any divergent sequence $\{x_n\}$ in $D$ there exists a function $f$ holomorphic on $D$ with $\lim |f(x_n)| = \infty$. In fact such a domain possesses a holomorphic function which cannot be continued across any boundary point. In other words $D$ is the “region of regularity” for that function or its “domain of holomorphy”. In the mid-1930s Cartan ([Ca]) and Oka ([O]) had already proved definitive results for Cousin I for domains in $\mathbb{C}^n$: If $D$ is a domain of holomorphy, then every Cousin I problem on $D$ is solvable!

Immediately after his thesis Stein turned to the Cousin problems. Later he discovered the correct abstract setting for solving these and many other problems, e.g., on complex manifolds or even complex spaces, but at this point his attention was focused on Cousin II for domains in $\mathbb{C}^n$.

The situation at the time of Stein’s entry into the subject is beautifully described in ([S2]). There were already a number of fascinating examples which showed that solving this multiplicative problem on $D$ required more than $D$ just being a domain of holomorphy. There was a natural way to logarithmically change this to the additive problem, i.e., to Cousin I, but in the process problems of well-definedness arise. This was not unknown in complex analysis. Monodromy, something in the fundamental group
or first homology, was well-known, but the obstruction to Cousin II was
clearly higher order. Nowadays we know that this is the Chern class of the
line bundle associated to the divisor and, at least when the ambient mani-
fold is compact, we can regard it as the Poincaré dual of the divisor itself.
But in those days these concepts were not available. Furthermore, even
had they been on hand, in the noncompact setting which is appropriate
for Cousin II, relating a deRham- or Čech-class to something geometric is
not a simple matter.

In the late 1930s, without modern topological methods, but armed with
strong geometric insight, this is exactly what Stein had in mind: understand-
ing this geometric obstruction. Being able to spend the year 1938
with Seifert in Heidelberg was in this regard certainly his good fortune or
maybe even fate. In any case he returned to Münster being one of the few
(perhaps the only) complex analyst who was in the position of applying
“modern” topological methods to problems such as Cousin II.

In the work ([S10]), which should be regarded as one of the most im-
portant in this early phase of several complex variables, Stein completely
solved Cousin II and the related Poincaré problem using methods which
opened doors to important new directions. The Oka principle, that a well-
formulated problem in the complex analytic setting has a holomorphic
solution on a domain of holomorphy if and only if it has a topological so-
lution, could be seen in precise form in the hands of Stein. In brief, modulo
details which are now well-understood, here is what Stein did.

In its simplest form Cousin II amounts to the following: On a domain of
holomorphy $D$ we are given a 1-codimensional subvariety $M$, i.e., a closed
subset which is locally defined as the 0-set of a holomorphic function. We
ask for a function which is globally defined and holomorphic on $D$, which
vanishes exactly on $M$ and vanishes there exactly of order one. Carefully
worrying about triangulations, orientations and all other matters that were
known to be delicate in the infantile state of the topology of the days, he
developed a theory which led to well-defined intersection numbers $M.K$, where $M$ is as above, or more generally a divisor in $D$, and $K$ runs through
the 2-dimensional homology cycles. Under minor technical conditions,
even for domains finitely spread over domains of holomorphy, he showed
that a given divisor is the divisor of a meromorphic function if and only if all of these (topologically defined!) intersection numbers vanish. Not only did Stein prove this, he could see the topological obstruction! — — I was fortunate to talk with him about this on a number of occasions. As was mentioned above, nowadays we often only mouth something about the Chern class, either deRahm or Cech, of the associated bundle, and maybe we are not nearly seeing as much as Stein did in the late 1930s!

Stein’s, and also Behnke’s, interests in Cousin type problems were not only restricted to the higher-dimensional setting. Although the questions they were discussing for domains in \( \mathbb{C}^n, n \geq 2 \), had long before been completely handled for domains in the complex plane, not much was known for general noncompact Riemann surfaces. On the one hand, that situation was simpler, because there were no higher order topological obstructions. On the other hand, the complex analysis looked quite difficult: Why should a noncompact Riemann surface possess even one nonconstant holomorphic function? In fact, the likes of Koebe and Carathéodory had attempted without success to construct such functions!

From their experience with higher-dimensional domains, and knowledge of proofs of theorems of Mittag-Leffler type for plane domains, Behnke and Stein at least knew what to try to do: Extend the Runge approximation theorem to noncompact Riemann surfaces and show that a noncompact Riemann surface possesses a Runge exhaustion! The Runge condition can be described as follows: Let \( \{U_n\} \) be an increasing sequence of open, relatively compact subsets which exhaust the Riemann surface \( X \). Denote by \( K_n \) the topological closure of \( U_n \). The exhaustion is said to be Runge if for every \( n \) every function holomorphic in a neighborhood of \( K_n \) can be arbitrarily well approximated in the sup-norm of \( K_n \) by functions which are holomorphic on \( U_{n+1} \). At the time it was well-known that, e.g., for plane domains the condition that \( U_n \) is Runge in \( U_{n+1} \) is equivalent to the topological condition that the \( U_n \) is relatively simply-connected in \( U_{n+1} \). In (S11) Behnke and Stein succeeded in proving this in the more general setting, thus proving that a noncompact Riemann surface possesses a Runge exhaustion and as a consequence it follows that both Cousin I and II (S14) have positive answers in that context. Due to the war-time conditions this work was published long after its completion.
Up until the early 1950s Stein was still focused on the Cousin problems, particularly Cousin II. His last work in this direction ([S15]) may have turned out to be his most famous. From this work one sees that Stein has studied the deep and perhaps mysterious work of Oka, whom he credits with the theorem that on a domain of holomorphy a Cousin II problem is holomorphically solvable if and only if it is topologically solvable.

As mentioned above, under a certain assumption which would seem only to be technical, Stein had made this precise in terms of his intersection numbers. This assumption is that the first homology group of the domain should have a basis. Here Stein observes that (believe it or not!) this is really an assumption, and in order to do away with it he must refine his topological condition. Underway he even proves several new results for countable Abelian groups!

Of course ([S15]) is a basic work, but the reason that it may be one of Stein’s most famous is that, without pursuing matters much further, he noted that most results of the type he had been considering are true for, in Stein’s words and notation, domains $\mathcal{G}$ in complex manifolds $\mathbb{M}^{2n}$ which satisfy the following three axioms:

1. **(Holomorphic convexity)** For every compact subset $\mathcal{G}_0$ of $\mathcal{G}$ there is a compact subset $\mathcal{G}_1$ which contains it so that for every point $P$ in $\mathcal{G}$ which is not in $\mathcal{G}_1$ there is a holomorphic function $f_P$ on $\mathcal{G}$ with
   \[ |f_P(P)| > \Max |f_P(K_0)|. \]

2. **(Point separation)** For any two different points $P_1$ and $P_2$ in $\mathcal{G}$ there is a function $f_{P_1,P_2}$ which is holomorphic on $\mathcal{G}$ and which takes on different values at $P_1$ and $P_2$.

3. **(Coordinates)** For every $Q$ in $\mathcal{G}$ there is a system of $n$ holomorphic functions on $\mathcal{G}$ whose functional determinant at $Q$ is nonzero.

The Cartan-Serre theory, in particular the vanishing theorems for cohomology defined by coherent sheaves on spaces which satisfy these axioms, was announced by Cartan at the famous *Colloque sur les fonctions de plusieurs variables* in Brussels in 1953. There he baptized these spaces
Variété de Stein, a notation that is still used today. During my very first seminar talk where Stein was present, his manifolds arose and, noticing my nervousness, without prompting, he said, “I like to call them holomorphically complete”.

Returning to Münster after participating in the Brussels Colloquium where he announced his own fundamental work on analytic decompositions, Stein lamented, “Die Franzosen haben Panzer, wir nur Pfeile und Bogen”\(^1\) To a certain extent this analogy might fit, but in appearance only. Looking back one sees that these “Bows and Arrows” were really quite sophisticated and that the accomplishments of the Münsteraner were truly extraordinary!

The most well-known names associated with the early days of the post-war Münster school of Heinrich Behnke are Hirzebruch, Grauert, Remmert and Stein. Hirzebruch, who was one of the first doctoral students after the war, went on to prove numerous important results in complex geometry, primarily for compact manifolds. Certain of his fundamental works utilize topological methods which go well beyond those employed by Stein, but which are of a similar basic spirit in that invariants such as characteristic classes or intersection numbers are fundamental topological obstructions to solving problems of analytic or algebraic geometric interest. In the early days he and Stein often commuted together from Hamm (Hirzebruch also grew up there), sometimes having to ride on the outside running board of the train, but nevertheless discussing mathematics. I can imagine that Stein’s animated expositions about his intersection numbers, or whether or not the first Betti group has a basis, made a lasting impression on the young Hirzebruch!

Certain of Grauert’s early works, e.g., his Oka principle, can be regarded as taking Stein’s prewar mathematics to another universe (see, e.g., our article, *Hans Grauert: Mathematician Pur*, *Mitteilung of the DMV*, 2008, for a brief summary of Grauert’s work). Later on (Stein had been retired for a number of years) they had close common interests in understanding the conditions under which the quotient of a complex space by an analytic

\(^1\)Oral communication from R. Remmert. See ([R]) for other recollections of the spirit of those times.
or meromorphic equivalence relation is again a complex space. I recall several very animated discussions in Oberwolfach!

In any account of Stein’s mathematics after his period of intense interest in the Cousin problems, in particular in the topological obstructions, his work with Reinhold Remmert must have center stage. This turned the page to a completely new direction!

Very early in Remmert’s studies, Behnke sent him to Stein, who at the time had an idea that analytic continuation was something that applied not only to functions. Maybe Thullen’s result in the 1-codimensional case could be proved for general analytic sets! Stein had in mind that the appropriate elimination theory could be found in Osgood’s book and Remmert should check this. What a daunting task for someone just starting out! As it turned out, nothing of this sort could be found in Osgood, and work could be started toward what would be the Remmert-Stein extension theorem ([S18]).

Here is a statement of the simplest version of that result: Let \( E \) be an analytic set in a domain \( D \) in \( \mathbb{C}^n \), i.e., a closed subset which is locally defined as the common 0-set of finitely many holomorphic functions, and suppose that \( A \) is an analytic set in the complement \( D \setminus E \) which is everywhere of larger dimension than \( E \). Then the topological closure \( \bar{A} \) of \( A \) in \( D \) is an analytic subset of \( D \) and what one adds to \( A \) to obtain this closure is just the lower-dimensional analytic subset \( \bar{A} \cap E \).

To the ear of the nonspecialist the above may sound overly complicated. However, considering the following example, which was a starting point for the Remmert-Stein discussions, should allay any doubts about its importance. Let \( D \) be \( \mathbb{C}^n \) itself and \( E \) just be the origin. Assuming that \( A \) is everywhere at least 1-dimensional, in this case the theorem just says that \( \bar{A} = A \cup \{0\} \) is an analytic subset of \( \mathbb{C}^n \) and, using results that were already known at the time, \( \bar{A} \) is the common 0-set of finitely many holomorphic functions which are globally defined on \( \mathbb{C}^n \), i.e., convergent power series.

Preimages \( A = \pi^{-1}(V) \) via the standard projection \( \pi : \mathbb{C}^n \setminus \{0\} \rightarrow \mathbb{P}_{n-1}(\mathbb{C}) \) of analytic sets \( V \) in projective space are examples of analytic
sets where the Remmert-Stein theorem can be applied. In this case $A$ is invariant by the $\mathbb{C}^*$-action defined by scalar multiplication. Thus, writing the defining power series $A$ as sums of homogeneous terms, one shows that $A$ is also the common 0-set of finitely many of these homogeneous polynomials. Consequently the original variety $V$ is the common 0-set of the same polynomials and is therefore an algebraic variety.

The above proof of Chow’s theorem was given ahead of time by Cartan in his lecture at the International Congress of Mathematicians in Boston in 1950! This result is a first example of a general principle which states that in many algebraic geometric settings there is no difference between algebraic and analytic phenomena. The Remmert-Stein theorem is certainly one of the guiding forces behind this principle!

The theme of holomorphic and meromorphic maps was one of Stein’s favorites and throughout this area the Remmert-Stein theorem plays a key role. The idea, e.g., for analyzing a holomorphic map $F : X \to Y$, is to throw out the analytic subsets (images and preimages) where $F$ degenerates, prove a good result for the restricted map, and then obtain the desired result by Remmert-Stein continuation. In several complex variables, meromorphic maps have indeterminacies and thus it is necessary to define such via their graphs. In any theory for these set valued maps the Remmert-Stein result is used at many steps along the way. Remmert developed this theory for (generically single-valued) meromorphic maps, and Stein later generalized this to correspondences which are not necessarily generically single-valued (see, e.g. [S34, S35]).

Remmert’s mapping theorem, Images of analytic sets under proper holomorphic maps are analytic sets, is very much in the spirit of the times. Of course this result is extremely useful. However, it is perhaps just as important that it calls our attention to the concept “proper”, i.e., inverse images of compact sets are compact. Its role had already been emphasized by Henri Cartan in 1935 in the context of actions automorphism groups on bounded domains and some basic results were proved in Bourbaki, but the proper mapping theorem and Stein’s fundamental paper on analytic decompositions ([S23]) cemented the position of properness in complex analysis.

Stein’s paper contains a wealth of interesting and useful results, some even
at the general topological level (see for example Satz 9), but due to lack of space we will only extract the most well-known one. For this it should be recalled that, in Münster, complex spaces were defined as topological spaces which could be locally realized as finite ramified covers (with obvious topological assumptions) over domains in $\mathbb{C}^n$. Stein had in fact shown that unramified (even infinite) covers of holomorphically complete spaces are holomorphically complete ([S24]), but he had really focused his interests on situations where some sort of properness is available.

Let us state an example of a result which is an important special case of those in ([S23]). Suppose $F : X \to Y$ is a proper holomorphic mapping of complex spaces. The domain space $X$ is assumed to be normal — for our purposes here it is enough to consider the smooth case. In order to analyze $F$, first apply Remmert’s theorem so that it may be assumed that it is surjective. Then define an equivalence relation $\sim$ on $X$ with two points being equivalent whenever they are in the same connected component of an $F$-fiber. The decomposition of $X$ into equivalence classes is a special case of what Stein called an “analytic decomposition”. In this case at hand, he shows that $X / \sim =: X^*$ carries a unique structure of a normal complex space such that the quotient map $\Phi : X \to X^*$ is holomorphic and every other holomorphic map which is constant on the equivalence classes of $\sim$ factors through it. In particular, this induces a holomorphic map $f : X^* \to Y$ which is a finite ramified cover! The factorization $F = f \circ \Phi$ is what is now called the Stein factorization of $F$.

A number of Stein’s last published works are devoted to understanding more general situations where it is possible to construct a universal quotient of the above type. The works ([S29, S30]) are typical of this. One exception is ([S27]). In this jewel, given two (concrete) domains in $\mathbb{C}^n$, Remmert and Stein study the possibilities for proper holomorphic maps between them. For two polyhedral domains $A$ and $A^*$ with sufficient structure coming from the affine structure of $\mathbb{C}^n$, they show that proper holomorphic maps which respect this structure are in fact affine. In particular, for domains in $\mathbb{C}^2$ this leads to strong nonexistence (rigidity) results, e.g., that certain very simple explicitly given domains have only the identity as proper holomorphic self-maps. Their methods even shed new light on situations which were classically “understood”. For example, Poincaré showed that the Euclidean ball $B_2 := \{(z,w) \in \mathbb{C}^2; |z|^2 + |w|^2 < 1\}$ and
the polydisk $\Delta_2 := \{(z, w) \in \mathbb{C}^2; |z| < 1 \text{ and } |w| < 1\}$ are not equivalent by a biholomorphic map, because their automorphism groups don’t have the same dimensions. Remmert and Stein show that, just as the beginner would like to believe, the reason for the inequivalence of these domains is that the boundary of $B_2$ is round and most of the boundary of $\Delta$ is flat!

We have now come to the end of our tour of what we find to be the highest points of Karl Stein’s mathematical works and would like to close this note by expressing our greatest respect and admiration, not only for the science of the man, but equally for the man behind the science!

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