Interplay between interaction and (un)correlated disorder in one-dimensional many-particle systems: delocalization and global entanglement

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Introduction
● Interplay between disorder and interaction
● Uncorrelated and long range correlated disorder
● Chaos

Outline
● Model
● Effect on half filled chain
● Comparison with dilute chain
# Heisenberg Spin-1/2 Model

One dimensional Hamiltonian

\[
H = H_0 + H_{\text{int}} + H_{XY}
\]

On Site Energy

\[
H_0 = \frac{1}{2} \sum_{n=1}^{L} \Omega_n \sigma_n^z \\
\Omega_n = \omega + \omega_n \\
\omega_n = d \varepsilon_n
\]

Pauli matrices

\[
\sigma_n^{x,y,z}
\]

Zeeman splitting

\[
\alpha = 0 \quad \text{Uncorrelated disorder}
\]

\[
\alpha > 0 \quad \text{Correlated disorder}
\]

\[
\phi \in [0, 2\pi] \quad \text{Uniform random numbers}
\]

Interaction Term

\[
H_{\text{int}} = \sum_{n=1}^{L-1} \frac{J \Delta}{4} \sigma_n^z \sigma_{n+1}^z
\]

Hopping Term

\[
H_{XY} = \sum_{n=1}^{L-1} \frac{J}{4} \left( \sigma_n^x \sigma_{n+1}^x + \sigma_n^y \sigma_{n+1}^y \right)
\]
Delocalization & Entanglement

**Spatial Delocalization**

\[ |\Psi_j\rangle = \sum_{k=1}^{N} c_j^k |\phi^k\rangle \] => \[ NPC_j = \frac{1}{\sum_{k=1}^{N} |c_j^k|^4} \]

Basis: eigenstates of \[ H_0 = \frac{1}{2} \sum_{n=1}^{L} \Omega_n \sigma_n^z \]

**Chaoticity**

Integrated difference between Poisson distribution and Wigner-Dyson distribution:

\[ \eta = \frac{\int_0^{s_0} [P_s(s) - P_{WD}(s)]ds}{\int_{s_0}^{0} [P_P(s) - P_{WD}(s)]ds} \]

- Poisson distribution: \[ P_P(s) = \exp(-s) \]
- Wigner-Dyson distribution: \[ P_{WD}(s) = \pi s / 2 \exp\left(-\frac{\pi s^2}{4}\right) \]

**Quantum Correlation - Global Entanglement**

\[ Q_j = 2 - \frac{2}{L} \sum_{n=1}^{L} Tr(\rho_n^2) \Rightarrow Q_j = 1 - \sum_{n=1}^{L} \left| \langle \Psi_j | \sigma_n^z | \Psi_j \rangle \right|^2 \]

**Integrable**

\[ \eta \rightarrow 1 \] Integrable

**Chaos**

\[ \eta \rightarrow 0 \] Chaos

Q=1: maximum global entanglement
Half Filled Chains: Uncorrelated Disorder

- In the gapless phase, $\Delta < 1$
- NPC peaks in the chaotic region
- Interaction is essential for entanglement

Average over 20 realizations
Long Range Correlated Disorder

\[ \frac{d}{J} = \frac{1}{4} \quad \text{Chaotic Region} \]

\[ \alpha \geq 0 \]

L=12 sites, M=6 excitations

\[ \Delta < 1 \text{: Delocalization and entanglement increase with correlated disorder} \]

\[ \Delta > 1 \text{: Delocalization and entanglement decrease with correlated disorder} \]

Average over 20 realizations
Dilute Limit

\begin{align*}
\langle Q \rangle & \quad \langle NPC \rangle \\
\Delta = 0 & \quad \Delta = 0.25 & \quad \Delta = 1 & \quad \Delta = 1.35 \\
\alpha = 3, 4, 5 & \quad \alpha = 2 & \quad \alpha = 1 & \quad \alpha = 0
\end{align*}

Average over 20 realizations
Conclusions

Half Filled Chains
- Disorder + Interaction $\rightarrow$ Chaos
  - More Delocalization
- Entanglement more affected by interaction than chaos
- Correlated disorder $\begin{cases} \Delta \leq 1 & \text{Increase of NPC, Q} \\ \Delta > 1 & \text{Decrease of NPC, Q} \end{cases}$

Dilute Chains
- Correlated disorder is more efficient in increasing NPC, Q
- No sign of Anderson localization for finite chains with $M=2$

Future
- Transport behavior
- To include long-range interactions
- Experimental verification

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How histograms help understanding the results

$\Delta = 2$

$N_t$: number of states with a given diagonal energy
How histograms help understanding the results

Black: uncorrelated disorder

Red: correlated disorder $\alpha = 10$

$N_t$: number of states with a given diagonal energy
NPC computed in different basis

FP: basis consisting of states of the clean Hamiltonian with no Ising interaction
IP: basis consisting of states of the clean Hamiltonian