NRQED IN BOUND STATES:
APPLYING RENORMALIZATION TO AN
EFFECTIVE FIELD THEORY

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ABSTRACT

This is a pedagogical introduction to NRQED, a nonrelativistic approximation to quantum electrodynamics. It can be made to reproduce QED to any degree of precision when applied to nonrelativistic systems and is particularly powerful when applied to bound states. We will begin by explaining what makes bound states so special, and so difficult to treat using QED. We will then develop the most naïve nonrelativistic approximation of QED which will be shown to fail when applied to a simple example (hyperfine splitting in positronium). We will see what was missing to make the theory well-defined, and the resulting theory will be, essentially, NRQED. We will conclude by discussing what makes NRQED so much more powerful than QED in nonrelativistic bound states, and what the approach we followed can teach us about “new physics” beyond QED.

1 Introduction

Quantum electrodynamics is without any doubt the most successful theory developed so far to deal with the behavior of elementary particles. It encompasses in an elegant and covariant framework the interactions of charged particles with electromagnetic fields. It is unfortunately overly sophisticated when it comes to dealing with nonrelativistic situations, since it does not take advantage of the simplifications possible under these circumstances. This is true for any nonrelativistic calculation, but is even more striking in the case of bound states calculations.

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Indeed, although most issues of principle concerning nonrelativistic bound states have long been resolved, there remain significant technical problems connected with the study of these systems. Central among these is the problem of too many energy scales. Typically, a nonrelativistic system has at least three important energy scales: the masses $m$ of the constituents, their three-momenta $p \sim mv$, and their kinetic energies $K \sim mv^2$. These scales are widely different in a nonrelativistic system since $v \ll 1$ (where the speed of light $c = 1$), and this complicates any analysis of such a system.

To illustrate the problems that arise, consider a traditional Bethe-Salpeter calculation of the energy levels of positronium. The potential in the Bethe-Salpeter equation is given by a sum of two-particle irreducible Feynman amplitudes. One generally solves the problem for some approximate potential and then incorporates corrections using time-independent perturbation theory. Unfortunately, perturbation theory for a bound state is far more complicated than perturbation theory for, say, the electron $g$-factor. In the latter case a diagram with three photons contributes only in order $\alpha^3$. In positronium a kernel involving the exchange of three photons can also contribute to order $\alpha^3$, but the same kernel will contribute to all higher orders as well:

$$
\langle V_1 \rangle = \alpha^3 m \left( a_0 + a_1 \alpha + a_2 \alpha^2 + \ldots \right)
$$

So in the bound state calculation there is no simple correlation between the importance of an amplitude and the number of photons in it. Such behavior is at the root of the complexities in high-precision analyses of positronium or other QED bound states, and it is a direct consequence of the multiple scales in the problem. Any expectation value like Eq.(1) will be some complicated function of the ratios of the three scales in the atom:

$$
\langle V_1 \rangle = \alpha^3 m \, F(\langle p \rangle / m, \langle K \rangle / m).
$$

Since $\langle p \rangle / m \sim \alpha$ and $\langle K \rangle / m \sim \alpha^2$, a Taylor expansion of $F$ in powers of these ratios generates an infinite series of contributions just as in Eq.(1). A similar series does not occur in the $g$-factor calculation because there is but one scale in that problem, the mass of the electron.

Traditional methods for analyzing these bound states fail to take advantage of the nonrelativistic character of these systems; and atoms like positronium are very nonrelativistic: the probability for finding relative momenta of $O(m)$ or larger is roughly $\alpha^5 \simeq 10^{-11}$!

There exists however an alternative formulation of QED that corresponds closely to the usual nonrelativistic theory, but where relativistic and radiative corrections can be systematically included. This new formulation is called nonrelativistic QED or NRQED and greatly simplifies calculations involving nonrelativistic systems, especially bound states.

Let us also point out that the difficulties associated with the many different energy scales in bound states are not limited to QED. Consider the study of nonrelativistic QCD bound states, like the $\Upsilon$ for example. Even if the means of extracting the properties of such systems, namely simulating the system on
a lattice, are very different than the use of the Bethe-Salpeter equation, the
nonrelativistic nature of the bound state still greatly complicates calculations.
Indeed, the space-time grid used in such a simulation must accommodate wave-
lengths covering all of the scales in the meson, ranging from $1/mv^2$ down to
$1/m$. Given that $v^2 \sim 0.1$ in $\Upsilon$, one might easily need a lattice as large as
100 sites on side to do a good job. Such a lattice could be three times larger
than the largest wavelength, with a grid spacing three times smaller than the
smallest wavelength, thereby limiting the errors caused by the grid. This is a
fantastically large lattice by contemporary standards and is quite impractical.
The strong force equivalent of NRQED, NRQCD\cite{4}, greatly reduces the size of
the lattices required to study heavy quarks systems since it essentially elimi-
nates the mass of the meson as a relevant energy scale in the calculation (we
will soon see how this occurs in NRQED).

In the next section, we will develop a na¨ıve nonrelativistic approxima-
tion of QED, which we will refer to as “NNRQED” to emphasize the fact that this
theory is not NRQED as described in Ref\cite{1}. It is in fact not well defined, as
we will show later. Only when we will make it well-defined with the help of the
concept of renormalization will it become NRQED.

2 NNRQED as a low energy theory

To take advantage of the simplifications associated with nonrelativistic systems,
one must approximate QED in the limit $p/m \ll 1$ (which is equivalent to the
limit $v/c \ll 1$). One way to do this consists of simply expanding the QED
Lagrangian in powers of $p/m$ to obtain NNRQED. However, it is possible to
define NNQRED without prior knowledge of QED. Indeed, one can build up the
Lagrangian of NNRQED by imposing restrictions coming from the symmetry
obeyed by the theory, such as gauge invariance, chiral symmetry in the limit
$m_e \to 0$, and Lorentz invariance for the photon kinetic term. Lorentz invariance
for the rest of the Lagrangian and renormalizability are not necessary.

It is of course possible to write down an infinite number of terms respecting
these symmetries, but this is not a problem since operators of dimension greater
than 4 will be multiplied by coefficients containing inverse powers of $m$, the only
available energy scale in the problem. But since we are considering the limit
$p/m \ll 1$, these operators will yield contributions suppressed by that many
powers of $p/m$ and therefore, for a given accuracy, only a few terms need to be
kept in an actual calculation. The first few terms one obtains are then

$$
\mathcal{L}_{\text{NNRQED}} = -1/4(F^{\mu\nu})^2 + \psi^\dagger \left\{ i\partial_t - eA_0 + D^2/2m + D^4/8m^3 \\
+ c_1 e /2m \sigma \cdot B + c_2 e /8m^2 \nabla \cdot E \\
+ c_3 ie /8m^2 \sigma \cdot (D \times E - E \times D) + \ldots \right\} \psi \\
+ d_1 /m^2 (\psi^\dagger \psi)^2 + d_2 /m^2 (\psi^\dagger \sigma \psi)^2 + \ldots \\
+ \text{positron and positron-electron terms.}
$$

(3)
where $D$ is the gauge-invariant derivative. An example of a positron-electron term (and the one we will focus on later) is $d_3/m^2 (\psi \sigma \chi \cdot (\chi \sigma \psi))$, which represents, to lowest order, the process $e^+ e^- \rightarrow e^+ e^-$ in the $s$ channel. This term, and the other two having coefficients starting with $d$'s are examples of “contact” interactions.

Notice that we have recovered the interactions familiar from nonrelativistic quantum mechanics such as the first order relativistic correction to the kinetic energy ($D^4/8m^3$), the spin-orbit interaction ($c_1 e/2m \sigma \cdot B$), and the Darwin term ($c_2 e/8m^2 \nabla \cdot E$).

We now have to fix the coefficients appearing in $\mathcal{L}_{\text{NNRQED}}$. To do so, we may simply compare tree scattering amplitudes in both QED and NNRQED, at a given kinematic point (which we choose to be at threshold, i.e. with the external particles at rest). This fixes $d_1$ in Eq.(3) to be equal to $-e^2/4m^2$ for example.

3 Bound states diagrams in NNRQED

3.1 Introduction

Now that NNRQED is completely defined, we can apply it to bound state calculations, where its usefulness is most apparent. To be specific, consider the energy levels of positronium. As noted earlier, the expansion in Feynman diagrams of NNRQED reduces to conventional Rayleigh-Schrödinger perturbation theory. In a bound state, the “external” wavefunctions of course don’t correspond to free wavepackets but to bound states. In the traditional Bethe-Salpeter formalism, the form of these wavefunctions depend on which interactions one desires to include exactly, and which interactions one wants to treat perturbatively. Depending on this choice, the external wavefunctions may be as simple as the Schrödinger wavefunctions or as complicated as the Dirac wavefunctions. The most useful gauge for nonrelativistic systems is the Coulomb gauge. That gauge permits us to keep the simplest interaction for the zeroth order kernel, namely the Coulomb interaction, in which case the Bethe-Salpeter equation reduces to the Schrödinger equation and the wavefunctions are the ones found in any textbook on quantum mechanics. In this calculation we will work only with $1S$ wavefunction in positronium (reduced mass $\mu = m/2$) which is given, in momentum space, by

$$\Psi(\vec{p}) = \frac{8\pi^{1/2} \gamma^{5/2}}{(p^2 + \gamma^2)^2}$$  \(4\)

where $\gamma$ is the typical bound state momentum equal to $\mu v = m\alpha/2$ in positronium. The ground state energy is $-\gamma^2/2\mu = -m\alpha^2/4$. Physically, an infinite number of Coulomb interactions is incorporated in the wavefunction.

The reason why at least an infinite number of Coulomb kernels must be taken into account in the wavefunction can be easily understood by considering

\footnote{Had we chosen to expand the QED Lagrangian, these coefficients would already be fixed.}
Fig. [1a], where a generic bound state diagram is shown. $K_1$ and $K_2$ represent some unspecified kernels. Let us now add one Coulomb interaction between the two kernels to yield Fig.[1b]. In conventional scattering theory, one would conclude that this diagram is of higher order than diagram [1a] because of the factor $\alpha$ coming from the vertices. However, this argument does not hold in a bound state, because the typical momentum flowing through the Coulomb interaction is of order $\gamma$. Indeed, consider how the diagram changes when one adds the Coulomb interaction. The integral acquires a new term of the form

$$\int \frac{d^3k}{(2\pi)^3} \frac{-e^2}{|k-l|^2} \frac{1}{(-\gamma^2 - k^2)/m} \simeq \int d^3k \frac{m\alpha}{(\gamma^2 + k^2)(|k-l|^2)},$$

(5)

where $1/|k-l|^2$ is the Coulomb propagator and $m/(-\gamma^2 - \tilde{k}^2)$ is the nonrelativistic propagator for the $e^-e^+$ pair.

We see that the mass factors out (more about this later!), leaving $\gamma$ as the only energy scale in the integral. By dimensional analysis, the final result will be of order $\simeq m\alpha/\gamma \simeq 1$. This shows that in a bound state, adding a Coulomb interaction in fact does not lead to a diagram of higher order, which is why one must include all of them from the start in the wavefunctions. This implies the relation expressed in Fig.[2] which is in fact a diagrammatic representation of the Schrödinger equation.

### 3.2 Counting rules

The argument we used to show that adding a Coulomb interaction does not increase the order of a bound state diagram is an example of a NNQED counting rule. These rules are extremely important since they permit one to estimate the order of contribution of a diagram without calculating it. Let us first consider non-recoil diagrams\(^2\). In that approximation, the counting rules are based on the observation that the mass $m$ of the electron always factors out of the integrals so that it no longer represents a relevant energy scale. This is clear since $m$ factors out of the nonrelativistic fermion propagators, as we saw in Eq.(5), and it enters the rules of the NNQED vertices only as an overall factor. We therefore see that the use of NNQED eliminates the relativistic energy scale of the system, which, in the light of the discussion in the introduction, greatly simplifies the analysis.

Starting from this observation, we can recast the discussion of the Coulomb interaction in the following manner: adding a Coulomb interaction between two kernels as in Fig.[1] leads to an additional factor of $\alpha$ coming from the vertices and a factor of $m$ coming from the additional fermion propagator. The

\(^2\)In the case of a bound state having equal mass constituents, as positronium, there is no distinction possible between “recoil effects” and the so-called “retardation effects”. This is not, for example, the case in muonium or hydrogen where (sometimes confusing) distinctions are made between non-recoil and non-retardation approximations. What we mean here by the non-recoil approximation is the limit in which the transverse photon does not carry any energy.
corresponding overall factor of $m\alpha$ must be cancelled by a factor having the dimensions of energy in order to keep the dimensions straight, but since the only energy scale left in the problem is $\gamma$, we finally obtain a correction of order $m\alpha/\gamma \simeq 1$. It now becomes obvious that the Coulomb interaction is the only kernel having the property of not increasing the order of a bound state diagram as all the other interactions contain additional factors of $1/m$. Each of these $1/m$ factors will have to be cancelled by a corresponding factor of $\gamma$, leading therefore to a result of higher order in $\alpha$.

With these rules, it becomes extremely simple to evaluate the order of the contribution of a given diagram. All we need to know is that the external wavefunctions contribute a factor $m\alpha^2$ to the energy (this can be seen by inserting a simple Coulomb interaction between two wavefunction and noticing that the result is of order $\alpha \gamma$). To obtain the contribution of a given kernel, one has simply to count how many powers of $\alpha$ and of $m$ this kernel has relative to the Coulomb interaction. After cancelling the $1/m$'s with factors of $\gamma$, the overall factor of $\alpha$ gives the order of the diagram. For example consider inserting a transverse photon (in the non-recoil limit) between two wavefunctions as in Fig.[3]. The transverse photon vertex has the same power of $e$ as the Coulomb vertex, but it has an additional factor of $1/m$. Taking the two vertices into account, we can conclude that this diagram will contribute to order $m\alpha^2 \times \gamma^2/m^2 \simeq m\alpha^4$.

The same is true of the annihilation diagram and of the spin-spin interaction as they also contain a factor of $1/m^2$ relative to the Coulomb interaction. Another important observation is that these diagrams will contribute to only one order in $\alpha^3$, in contradistinction with Feynman diagrams in conventional Bethe-Salpeter analysis.

The situation is only slightly more complicated when one takes into account recoil effects. We will not dwell on this issue here, but let us just mention that they make the kinetic energies $K$ enter as a mass scale and that they are at the origin of the appearance of log’s of $\alpha$ which are characteristic of bound states.

4 A calculation that goes wrong

Now let’s do a calculation! We will consider the hyperfine splitting\(^4\)(E (triplet state) - E (singlet state)) of ground state positronium. We first have to find which diagrams will contribute to the lowest order hfs. Using the counting rules described earlier, we easily find that there are only two such diagrams, and that they will contribute to the order $m\alpha^4$. One of these diagrams is, naturally, the spin-spin interaction familiar from quantum mechanics; the second interaction is the annihilation contact interaction which is of course absent in hydrogen. For the sake of simplicity, we will focus our attention on the annihilation diagram. It will simplify the discussion without affecting the conceptual issues we want to address.

As explained earlier, the annihilation interaction is represented by a contact

\(^3\)We are still limiting ourselves to the non-recoil limit.

\(^4\)Which we will denote by “hfs” in the following.
term; it has no momentum dependence and has for Feynman rule $\frac{e^2}{4m^2} \chi^\dagger \vec{\sigma} \psi^\dagger \vec{\sigma} \chi$. The spin average of this expression gives 2 in the triplet state and 0 in the singlet state (as it must since the singlet state cannot decay to one photon, by charge conjugation invariance). Therefore, the contribution to the hfs coming from this interaction is given by

$$\int \frac{d^3p}{(2\pi)^3} \frac{8\pi^{1/2} \gamma^{5/2}}{(p^2 + \gamma^2)^2} \frac{e^2}{4m^2} \int \frac{d^3q}{(2\pi)^3} \frac{8\pi^{1/2} \gamma^{5/2}}{(q^2 + \gamma^2)^2} = \frac{ma^4}{4}$$

which is indeed the correct result.[2]

The first correction beyond the tree approximation is of order $ma^5$. To isolate it, we may apply the counting rules explained earlier. They tell us that we need to join to the annihilation interaction a kernel that will supply one more power of $1/m$ (indeed, this will force the integral to contribute an additional power of $\gamma$, resulting in a contribution to the energy of order $ma^5$). It turns out that there is no such kernel. The smallest power of $1/m$ one can obtain is $1/m^2$, which seems to indicate that our theory predicts the next order correction to be of order $ma^6$. Actually, the situation is even worse. To see this, consider the simplest of these next order diagrams, the one corresponding to two annihilation diagrams sewn together. The corresponding integral is (I write the external wavefunction integrals directly as $|\Psi(0)|^2$, as they decouple from the rest of the diagram):

$$\left( |\Psi(0)|^2 \ Tr(\sigma_i \sigma_j) \left( \frac{e^2}{4m^2} \chi^\dagger \sigma_i \psi_i \psi_j \chi \int \frac{d^3p}{(2\pi)^3} \frac{1}{-\gamma^2/m - p^2/m} \right) \right)_{\text{spin 1}}$$

$$= - \frac{e^2}{4m^2} \int \frac{d^3p}{(2\pi)^3} \frac{m}{p^2 + \gamma^2}$$

where we have used $Tr(\sigma_i \sigma_j) = 2\delta_{ij}$.

We immediately see that this integral is ill-defined, as it has a linear UV divergence. One can of course argue that, since our theory is based on a nonrelativistic expansion, one must restrict the momentum integration to $p \ll m$, but what cutoff should one use? $m/2$, $m/10$, $m/20$? We realize that, as soon as we push the precision beyond the tree approximation, our nonrelativistic theory is not well defined, and it seems that we must bid farewell to all the nice simplifications inherent to our nonrelativistic field theory...

## 5 Renormalization to the rescue

Fortunately, all is not lost as there is a well-known method that dictates us how we can make our theory finite in a well-defined and entirely self-consistent manner. This method is nothing other than renormalization.

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[1] This is the Feynman rule for the amplitude $M$, and not $-i M$, as the usual QED Feynman rules correspond to. Also, to be consistent with the normalization of the wavefunctions, the external spinors have a nonrelativistic normalization, i.e. $\psi^\dagger \psi = 1$ for example.
To understand how renormalization can teach us how to make the nonrelativistic theory well-defined, let us go back to conventional QED. In QED, to avoid infinities, one regulates the theory by putting a cutoff $\Lambda$ on the loops momentum. This throws away some physics but renormalization theory tells us that this physics can be put back in the theory by shifting (“renormalizing”) the values of the QED parameters and by adding new, “nonrenormalizable”, interactions. There is an infinite string of new interactions which, by dimensional analysis, are accompanied by increasing powers of $1/\Lambda$. To fix the parameters, one calculates a given process, at a given kinematic point, and compares with the experimental value. For example, to fix $\alpha$ in QED, one might calculate the form factor of the scattering of an electron by an external electromagnetic field, in the long wavelength limit. Once the parameters have been fixed, the theory can be used to calculate any process, at any energy (as long as it is small compared to the cutoff $\Lambda$ so that only a finite number of the nonrenormalizable interactions must be taken into account for a given accuracy). The conventional procedure is to take the limit $\Lambda \to \infty$ at the end of the calculation so that the nonrenormalizable interactions actually vanish. The price paid is, as we know, that the original (“bare”) parameters of QED become ill-defined in that limit. We will discuss this point in more details in the conclusion.

Standard renormalization is readily adapted to NNRQED. The only difference here is that there is a physical scale, $\approx m_e$, at which NNRQED is no longer applicable and QED takes over. In this context, renormalization is telling us that the physics taking place beyond $p \approx m_e$ can be put back in NNRQED by an appropriate definition of its parameters, and by adding new interactions representing an expansion in powers of $p/m$. In fact, since our nonrelativistic Lagrangian contains already such an expansion, the net effect of adding the cutoff is to change the (infinite) set of NNRQED parameters. As long as we are considering processes in which the external momenta are smaller than the electron mass, we are certain to reproduce the QED result, up to a certain precision, with a finite number of diagrams providing we have defined the NNRQED parameters correctly. To see how this works, let us go back to the annihilation diagram. We must now think of its coefficient as an expansion in $\alpha$. To fix it to the order we are interested in, we must consider one loop corrections to both NNRQED and QED, and require that the scattering amplitudes in both theories agree (see Fig.[4]). This must be done at a given kinematic point, which, as already mentioned, we choose to be at threshold (external particles being at rest). As in QED, once the coefficients are fixed at a given kinematic point, the theory can be used to calculate any process. The sum of the QED one-loop diagrams is calculated to be $-e^4(1/2 + 1/9)/m^2\pi^2$. To match this result, the contact interaction must acquire an additional finite piece to become $\chi^\dagger \vec{\sigma} \psi \cdot \vec{\sigma} \chi \times e^2/4m^2 \{1 - e^2(1 + 2/9)/\pi^2\}$. Then the lowest order calculation

\footnote{For the sake of conciseness, we are side-stepping completely the question of infrared divergences. Since we are working at threshold, there cannot be any bremsstrahlung divergences, but there is a so-called “Coulomb singularity” divergence present. However, the Coulomb diagram of NNRQED takes care completely of this problem, and this does not change anything to our discussion.}
(Eq. (6)) becomes

$$|\Psi(0)|^2 \times \left\{ \frac{e^2}{2m^2} - \frac{e^4}{m^2 \pi^2} \left( \frac{1}{2} + \frac{1}{9} \right) \right\} = \frac{ma^4}{4} - \frac{ma^5}{\pi} \left( 1 + \frac{2}{9} \right)$$

(8)

The $\alpha^5$ correction is exactly what we needed to obtain the correct answer. Moreover, this procedure will take care of the divergence appearing in the $\alpha^6$ terms as well. This can be seen by looking back at Fig. [4]. We see that the coefficient in front of the contact interaction will contain, in addition to the constant term that we have just described and which comes from the QED one-loop diagrams, a series of NRQED one-loop diagrams evaluated at threshold, which are infinite. It will, for example, contain the scattering diagram representing the double annihilation diagram of Eq. [7]. Therefore, to be rigorous, this term should now be included in Eq. [8] and can be seen to play the role of a counterterm which will render Eq. [7] finite.

6 Conclusion

With the coefficients of our nonrelativistic theory renormalized in the way described above, we have finally obtained a truly well-defined nonrelativistic field theory, which is the NRQED described in refs[1].

We would like to conclude this paper by emphasizing two important aspects of our discussion on NRQED. One of these aspects is conceptual in nature, and has relevance far beyond the realm of nonrelativistic systems. The other is practical in nature and is at the heart of the power of NRQED in bound states.

• It is probable that QED is “wrong” in the sense that it must be an approximate “low-energy” realization of a more fundamental theory. The way in which this clearly shows up is the appearance of the well-known UV divergences. Physically, these divergences tell us that we are pushing the integration momenta far beyond the region of applicability of QED, in a region where the “complete” theory would have to be used. In that respect, we see that the situation of QED with respect to the complete theory is analogous to the situation of NRQED with respect to QED. From that point of view, one should not, in QED, take the limit $\Lambda \to \infty$. As in NRQED, there is a physical finite value of the cutoff $\Lambda_{NP}$ signaling the presence of new physics. This implies that the nonrenormalizable interactions cannot be dispensed of. Therefore, at energies of order $\Lambda_{NP}$, one would observe a complete breakdown of QED as the infinite string of new interactions would contribute to $O(1)$. However, experiments at energies as high as $\Lambda_{NP}$ are not necessary to probe the new physics. Low energy, but very accurate, experiments can also teach us something about $\Lambda_{NP}$. For example, there should be an interaction of the form $e m_e \Psi F^\mu_\nu g^\nu_\sigma \Psi / \Lambda_{NP}^2$ which would contribute to the electron anomalous magnetic moment by an amount propor-

7 The precise way this occurs is that the integrand $1/(p^2 + \gamma^2)$ of Eq. [7] gets replaced by $1/(p^2 + \gamma^2) - 1/p^2$ which leads to a finite integral. The counterterm integrand does not contain $\gamma$ as it comes from a scattering amplitude.
tional to $m_e^2/\Lambda_{NP}^2$. The fact that theory agrees with experiment to 12 decimal places indicates that $\Lambda_{NP} > 10^6 m_e \approx 1\text{TeV}$.

• NRQED is an extremely powerful device when applied to nonrelativistic bound states. To understand why, notice that the contributions from relativistic states (i.e. QED diagrams) enter the theory only by renormalizing the coefficients of the nonrelativistic interactions, and this occurs when one matches the scattering amplitudes of NRQED to the ones of QED. At that stage, no bound state analysis enter the problem. It is only when these coefficients are defined that one attacks the bound state part of the calculation, which, therefore, involves only nonrelativistic interactions. We see that NRQED has decoupled the high momentum contribution from the low-momentum one. This is the fundamental ingredient in the usefulness of NRQED.

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