Reduction of Register Pushdown Systems with Freshness Property to Pushdown Systems in LTL Model Checking

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Abstract

Pushdown systems (PDS) are known as an abstract model of recursive programs, and model checking methods for PDS have been studied. Register PDS (RPDS) are PDS augmented by registers to deal with data values from an infinite domain in a restricted way. A linear temporal logic (LTL) model checking method for RPDS with regular valuations has been proposed; however, the method requires the register automata (RA) used for representing a regular valuation to be backward-deterministic. This paper proposes another approach to the same problem, in which the model checking problem for RPDS is reduced to that problem for PDS by constructing a PDS bisimulation equivalent to a given RPDS. The construction in the proposed method is simpler than the previous model checking method and does not require RAs deterministic or backward-deterministic, and the bisimulation equivalence clearly guarantees the correctness of this reduction. On the other hand, the proposed method requires every RPDS (and RA) to have the freshness property, in which whenever the RPDS updates a register with a data value not stored in any register or the stack top, the value should be fresh. This paper also shows that this model checking problem with regular valuations defined by general RA is undecidable, and thus the freshness constraint is essential in the proposed method.

Keywords: model checking, register pushdown system, regular valuation

1. Introduction

A pushdown system (PDS) is a pushdown automaton without input and is well known as an abstract model of recursive programs \cite{1,2}. The model checking problem for a PDS \(\mathcal{P}\), i.e. testing whether all runs of \(\mathcal{P}\) conforms to a given specification \(\varphi\), has been studied for various logics such as linear temporal logic (LTL) and branching-time temporal logic \cite{1,2,3}. In \cite{4}, the LTL model checking of PDS with regular valuations was investigated and shown to be decidable. A valuation is a function that labels each configuration in a run with a subset of atomic propositions. A regular valuation is a valuation that labels each configuration \(c\) of a PDS with atomic propositions depending on whether the stack contents in \(c\) match a given regular pattern.

Although PDS is a natural model of recursive programs, it cannot deal with data values directly. Register automata (RA) were introduced as an extension of finite automata (FA) by adding the capability of dealing with data values in a restricted way \cite{5}. RA has attracted attention as a formal model of navigational queries concerning data values to structured data such as XML documents \cite{6}. RA has also been considered as a formal model of software systems with unbounded resources in e.g. runtime verification \cite{7} and reactive synthesis \cite{8}. Similarly to extending FA to RA, PDS were extended to pushdown register systems (PDRS) \cite{9} and register pushdown systems (RPDS) \cite{10}, both of them are equivalent each other, and the reachability problem for them has been shown to be EXPTIME-complete.

In previous work \cite{11}, we have investigated the LTL model checking problem with regular valuations for RPDS and proposed a method for solving the problem. This method is a natural ex-
tension of the model checking method for PDS in [4]. Similarly to the method in [4] that reduces the model checking problem to the emptiness problem for Büchi pushdown systems, in [11] we introduced Büchi register pushdown systems (BRPDS) and showed a reduction of the model checking problem to the emptiness problem for BRPDS. Note that the regular valuations in [11] were defined in terms of backward-deterministic RA, in which every configuration has a unique predecessor configuration for each input data value. This constraint is essential because unlike FA, determinization is not possible for general RA.

In this paper, we work on the same problem in a different approach. We reduce the model checking problem for RPDS to that problem for PDS by constructing a PDS $P'$ bisimulation equivalent [12] to a given RPDS $P$. In the same way, we also construct an FA $A'$ bisimulation equivalent to an RA $A$ used for a regular valuation. The bisimulation equivalences between $P$ and $P'$ and between $A$ and $A'$ guarantee the correctness of this reduction. The construction of a PDS bisimulation equivalent to an RPDS is basically the same as the one used in our recent work [13] on reactive synthesis from specifications given by deterministic register pushdown automata (DRPDA). In [12], we construct a pushdown automaton (PDA) simulating a DRPDA for reducing the realizability problem for DRPDA to the same problem for PDA. The proposed method in this paper is an application of this construction to the model checking problem, where not only RPDS but also RA used for a regular valuation can be reduced to models without data values in a uniform way, and the correctness of the reduction of the model checking problem can be proved easily based on bisimulation equivalence.

Another feature of the proposed method is that it does not require RA used for regular valuations to be deterministic or backward-deterministic. However, the proposed method requires every RPDS (and RA) to have the freshness property instead, in which whenever the RPDS updates a register with a data value not stored in any register or the stack top, the value should be fresh, i.e., not used before. This paper also shows that the LTL model checking problem for RPDS with regular valuations defined by general RA is undecidable (Theorem 1), and this fact compels the proposed method to require the freshness property instead of requiring RA to be deterministic or backward-deterministic.

Advantages of the proposed method are summarized as follows: (i) The construction in the proposed method is much simpler than the method in [11]. The bisimulation equivalence between an RPDS and a PDS clearly shows the correctness of the reduction. We have proved this bisimulation equivalence using a proof assistant software Coq [14]. (ii) By the proposed method, we can apply existing model checking tools for PDS to solving the model checking problem for RPDS. (iii) The method does not require RA used for regular valuations to be deterministic or backward-deterministic, though it requires the freshness property instead. Showing the undecidability of the model checking problem in the general case is another contribution of the paper.

The rest of the paper is organized as follows. We define basic notions in Section 2, RPDS and RA in Section 3, and the LTL model checking problem for RPDS with regular valuations in Section 4. In Section 4, we also show that this problem is undecidable in general. In Section 5, we introduce the freshness property and redefine the semantics of RPDS so that every RPDS has this property. In Section 6, we show the construction of a PDS bisimulation equivalent to a given RPDS, which is the main part of the proposed method. We conclude the paper in Section 7.

2. Preliminaries

Let $\mathbb{N} = \{1, 2, \ldots\}$, $\mathbb{N}_0 = \{0\} \cup \mathbb{N}$, and $[n] = \{1, 2, \ldots, n\}$ for $n \in \mathbb{N}$. We assume a countable set $D$ of data values. For a given $k \in \mathbb{N}_0$, a mapping $\theta : [k] \rightarrow D$ is called an assignment (of data values to $k$ registers). Let $\Theta_k$ be the set of assignments to $k$ registers. Sometimes we consider an assignment $\theta \in \Theta_k$ as the set of assigned data values; e.g., $d \in \theta$ means $d = \theta(i)$ for some $i \in [k]$.

For a set $A$, let $A^*$ and $A^\omega$ be the sets of finite and infinite words over $A$, respectively. Let $A^\infty = A^* \cup A^\omega$. For a word $\alpha \in A^\infty$, let $\alpha(i) \in A$ be the $i$-th element of $\alpha$ and $\alpha(i) = \alpha(i)\alpha(i+1) \ldots$ for $i \geq 0$. Let $\text{snd}$ be the function over pairs that gives the second element of a pair; i.e., $\text{snd}((a, b)) = b$. For a word $w = (a_0, b_0)(a_1, b_1) \ldots$ over pairs, let $\text{snd}(w) = b_0b_1 \ldots$.

For a relation $\Rightarrow$, let $\Rightarrow^*$ be the reflexive transitive closure of $\Rightarrow$.

2.1. Linear Temporal Logic (LTL)

The definition of LTL formulas we used is the same as [4]. Let $A(t)$ be a finite set of atomic propo-
sitions, and let $\Sigma = 2^A$. An LTL formula over $A$ is given by the following syntax:

$$
\varphi ::= \top | A | \neg \varphi | \varphi_1 \land \varphi_2 | X \varphi | \varphi_1 \cup \varphi_2
$$

where $A \in A_t$. For an infinite word $w \in \Sigma^*$, the satisfaction relation $\models$ is defined as follows:

$$
w \models \top, \quad w \models A \iff A \in w(0), \quad w \models \neg \varphi \iff w \not\models \varphi,
$$

$$
w \models \varphi_1 \land \varphi_2 \iff w \models \varphi_1 \text{ and } w \models \varphi_2,
$$

$$
w \models X \varphi \iff w(1) \models \varphi,
$$

$$
w \models \varphi_1 \cup \varphi_2 \iff \exists j : (w(j) \models \varphi_2) \land (\forall i < j : w(i) \models \varphi_1).
$$

We also define $\bigcup \varphi = \top \cup \varphi$ and $\bigcap \varphi = \top \cap \neg \neg \varphi$.

2.2. Pushdown systems and finite automata

For a finite set $\Gamma$, we define the set $\text{Com}(\Gamma)$ of commands over $\Gamma$ as $\text{Com}(\Gamma) = \{\text{pop, skip} \cup \{\text{push}(\gamma) \mid \gamma \in \Gamma\}$.

Definition 1. A pushdown system (PDS) $P$ over a stack alphabet $\Gamma$ is a pair $(P, \Delta)$, where $P$ is a finite set of states and $\Delta \subseteq P \times \Gamma \times P \times \text{Com}(\Gamma)$ is a set of transition rules. We write an element $(p, \gamma, q, \text{com}) \in \Delta$ as $(p, \gamma) \rightarrow (q, \text{com})$ for readability.

Let $ID_P = P \times \Gamma^*$ and call each element of $ID_P$ an instantaneous description (ID) of $P$. The transition relation $\rightarrow_P$ of $P$ is the smallest relation over $ID_P$ satisfying the following inference rule, where $\text{upds}'(\gamma v, \text{com}') = v, \gamma v$, or $\gamma'/\gamma v$ if $\text{com}' = \text{pop, skip, or push}(\gamma)$, respectively.

$$
(p, \gamma) \rightarrow (q, \text{com}') \in \Delta
$$

A run of $P$ is a sequence $\rho \in ID_P^*$ such that $\rho(i) \rightarrow_P \rho(i + 1)$ for $i \geq 0$.

We define nondeterministic finite automata as follows, which is used for representing a (regular) subset of IDs of some PDS.

Definition 2. A nondeterministic finite automaton (NFA) $A$ over an alphabet $\Gamma$ is a quadruple $(Q, I, F, \delta)$, where $(Q, \delta)$ is a PDS over $\Gamma$ where $\delta$ consists of pop rules only, $I \subseteq Q$ is a set of initial states, and $F \subseteq Q$ is a set of final states. We call $(Q, \delta)$ the base PDS of $A$. The set $ID_A$ of IDs and the transition relation of $A$ are the same as those of its base PDS. We write the transition relation of $A$ as $\vdash_A$. The language $L(A)$ of $A$ is a subset of $ID_A$ defined as $L(A) = \{(p, w) \in I \times \Gamma^+ \mid (p, w) \vdash_A (q, \varepsilon) \text{ for some } q \in F\}$.

When we use an NFA $A$ for representing a subset of IDs of a PDS $P = (P, \Delta)$, we let the set of the initial states of $A$ be $P$.

Definition 3. For a PDS $P = (P, \Delta)$, we call a subset $C \subseteq ID_P$ regular if there exists an NFA $A = (Q, P, F, \delta)$ that satisfies $C = L(A)$.

2.3. Equivalence relations over registers

Let $\Phi_k$ be the set of equivalence relations over the set of $2k + 1$ symbols $X_k = \{x_1, \ldots, x_k, x'_1, \ldots, x'_k, x_{\top}\}$. We write $a \equiv_\phi b$ and $a \not\equiv_\phi b$ to mean $(a, b) \in \phi$ and $(a, b) \not\in \phi$, respectively, for $a, b \in X_k$ and $\phi \in \Phi_k$. Intuitively, each $\phi \in \Phi_k$ represents the equality and inequality among the data values in the registers and the stack top, as well as the transfer of the values in the registers between two assignments. Two assignments $\theta, \theta' \in \Theta_k$ and a value $d$ at the stack top satisfy $\phi$, denoted as $\theta =_\phi \theta' =_\phi \phi$, if $i, j \in [k]$:

$$
x_i \equiv_\phi x_j \iff \theta(i) = \theta(j), \quad x_i \equiv_\phi x_{\top} \iff \theta(i) = d,
$$

$$
x_i \equiv_\phi x'_j \iff \theta(i) = \theta'(j), \quad x'_i \equiv_\phi x_{\top} \iff \theta'(j) = d,
$$

$$
x_i \equiv_\phi x'_j \iff \theta'(i) = \theta'(j).
$$

We will use elements of $\Phi_k$ to specify transition rules of a register pushdown system with $k$ registers ($k$-RPDS), defined in the next section.

Let $\Phi_k'$ be the set of equivalence relations over the $k$ symbols $\{x_1, \ldots, x_k\}$. An assignment $\theta \in \Theta_k$ satisfies $\phi' \in \Phi_k'$, denoted as $\theta \models \phi'$, if $i, j \in [k]$, $x_i \equiv_\phi x_j \iff \theta(i) = \theta(j)$. Elements of $\Phi_k'$ will be used to specify accepting conditions of a register automaton with $k$ registers ($k$-RAA).

Let $\text{lat} : \Phi_k \rightarrow \Phi_k'$ be the function defined as:

$$
\forall i, j \in [k] : x_i \equiv_{\text{lat}(\phi)} x_j \iff x_i' \equiv_{\phi'} x_j'.
$$

3. Register pushdown systems and register automata

Definition 4. A register pushdown system with $k$ registers ($k$-RPDS) $P$ is a pair $(P, \Delta)$, where $P$ is a finite set of states and $\Delta \subseteq P \times P \times \text{Com}(k)$ is a set of transition rules. We write an element $(p, \phi, q, \text{com}) \in \Delta$ as $(p, \phi) \rightarrow (q, \text{com})$ for readability.
Let $ID_P = P \times \Theta_k \times D^*$ and call each element of $ID_P$ an instantaneous description (ID) of $P$. The transition relation $\Rightarrow_P$ of $P$ is the smallest relation over $ID_P$ satisfying the following inference rule, where $\text{upds}(du, \theta', \text{com}) = u, d_u, \text{or } \theta'(j)du$ if $\text{com} = \text{pop}, \text{skip}, \text{or } \text{push}(j)$, respectively.

$$(p, \phi) \Rightarrow (q, com) \in \Delta \quad (q, d_u, \theta') \models \phi$$

$$(p, \theta, du) \Rightarrow_P (q, \theta', \text{upds}(du, \theta', \text{com}))$$

A run of $P$ is a sequence $\rho \in ID^*_P$ such that $\rho(i) \Rightarrow_P \rho(i+1)$ for $i \geq 0$.

**Example 1.** Let us consider 2-RPDS $P = \{p_0, p_1, p_2\}, \{r_1, r_2, r_3, r_4, r_5\}$ where

$$
\begin{align*}
r_1 &= (p_0, \phi_0) \rightarrow (p_1, \text{push}(1)), \\
r_2 &= (p_1, \phi_1) \rightarrow (p_1, \text{push}(1)), \\
r_3 &= (p_1, \phi_1) \rightarrow (p_1, \text{pop}), \\
r_4 &= (p_1, \phi_2) \rightarrow (p_1, \text{pop}), \\
r_5 &= (p_1, \phi_3) \rightarrow (p_2, \text{push}(2)),
\end{align*}
$$

and $\phi_0, \ldots, \phi_3 \in \Phi_2$ are defined by the following quotient sets:

$$
\begin{align*}
X_2/\phi_0 &= \{\{x_1\}, \{x_2, x'_2, x_{\text{top}}\}, \{x'_1\}\}, \\
X_2/\phi_1 &= \{\{x_1, x_{\text{top}}\}, \{x_2, x'_2\}, \{x'_1\}\}, \\
X_2/\phi_2 &= \{\{x_1\}, \{x_2, x'_2\}, \{x_1, x_{\text{top}}\}\}, \\
X_2/\phi_3 &= \{\{x_1, x'_1\}, \{x_2, x_{\text{top}}\}, \{x'_2\}\}.
\end{align*}
$$

In this example, we let $[d_1, d_2]$ for $d_1, d_2 \in D$ denote the assignment $\theta \in \Theta_2$ such that $\theta(1) = d_1$ and $\theta(2) = d_2$. Let $d_0, d_1, \ldots \in D$ represent distinct data values. Figure 1 shows a transition sequence of $P$ starting from an ID $(p_0, [d_1, d_0], d_0)$. For example, we can apply $r_1$ to this starting ID and obtain $(p_1, [d_2, d_0], d_0)$, because $\phi_0$ requires that the value of the second register before the transition is the same as the stack top, the second register is not changed by the transition, and the value of the first register after the transition is not equal to the value of any register before the transition.

**Definition 5.** A register automaton with $k$ registers (k-RA) $A$ is a quadruple $(Q, I, \xi, \delta)$, where $(Q, \delta)$ is a $k$-RPDS where $\delta$ consists of pop rules only, $I \subseteq Q$ is a set of initial states, and $\xi \subseteq Q \times \Phi_k'$ is a set of accepting conditions. We call $(Q, \delta)$ the base RPDS of $A$. The set $ID_A$ of IDs and the transition relation of $A$ are the same as those of its base RPDS. We write the transition relation of $A$ as $\Rightarrow_A$. Let $\text{Acc}_A = \{(p, \theta, \varepsilon) \in ID_A \mid \theta \models \psi$ for some $(p, \psi) \in \xi\}$. The language $L(A)$ of $A$ is a subset of $ID_A$ defined as $L(A) = \{(p, \theta, w) \in ID_A \mid p \in I \text{ and } (p, \theta, w) \Rightarrow_A (q, \theta', \varepsilon)$ for some $(q, \theta', \varepsilon) \in \text{Acc}_A\}$.

We write a transition rule $(q_1, \phi) \rightarrow (q_2, \text{pop})$ of an RA as $(q_1, \phi) \rightarrow q_2$ for readability.

**Definition 6.** For a $k$-RPDS $P = (P, \Delta)$, we call a subset $C \subseteq ID_P$ regular if there exists a $k$-RA $A = (Q, P, \xi, \delta)$ that satisfies $C = L(A)$.

**Example 2.** Let us consider 2-RA $A = \{(p_1, q_1), (p_1), (q_2, \psi)\}, \{r_6, r_7, r_8\}$ where

$$
\begin{align*}
r_6 &= (p_1, \phi_1) \rightarrow q_1, \\
r_7 &= (q_1, \phi_4) \rightarrow q_2, \\
r_8 &= (q_1, \phi_3) \rightarrow r_2,
\end{align*}
$$

$\phi_1$ and $\phi_3$ are the same as Example 1. $\phi_4 \in \Phi_2$ is defined by the quotient set

$$
\begin{align*}
X_2/\phi_4 &= \{\{x_1, x'_1\}, \{x_2, x'_2\}, \{x_{\text{top}}\}\},
\end{align*}
$$

and $\psi \in \Phi_2'$ is the equivalence relation such that $x_1 \not\equiv x_2$. Figure 2 shows a transition sequence of $A$ starting from an ID $(p_1, [d_3, d_0], d_3d_2d_0)$. As shown in the figure, $(p_1, [d_3, d_0], d_3d_2d_0) \Rightarrow_A (q_2, [d_4, d_3], \varepsilon)$. Since $\text{Acc}_A = \{(q_2, [d_4, d_3], \varepsilon) \in L(A)\}$.  

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*Figure 1: A transition sequence of $P$ from $(p_0, [d_1, d_0], d_0)$. 
Figure 2: A transition sequence of $A$ starting from an ID $(p_1, [d_3, d_0], d_3d_2d_0)$.  

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*4*
4. LTL model checking problem and valuations

We fix a finite set \( At \) of atomic propositions, and let \( \Sigma = 2^{At} \).

A valuation for a k-RPDS \( \mathcal{P} = (P, \Delta) \) is a function \( \Lambda : IDP \rightarrow \Sigma \), which labels each ID of \( \mathcal{P} \) with a subset of atomic propositions. We extend the domain of \( \Lambda \) to \((IDP)^\infty\) in the usual way; i.e., \( \Lambda(0c_1 \ldots) = \Lambda(0)\Lambda(c_1) \ldots \).

**Definition 7.** The LTL model checking problem for RPDS is defined as:

**Instance:** a k-RPDS \( \mathcal{P} = (P, \Delta) \), an LTL formula \( \varphi \) over \( At \), a valuation \( \Lambda : IDP \rightarrow \Sigma \), and an ID \( c_0 \in IDP \).

**Question:** Does every run \( r \in (IDP)^\omega \) of \( \mathcal{P} \) with \( r(0) = c_0 \) satisfy \( \Lambda(r) \models \varphi \)?

In the rest of the paper, we fix a k-RPDS \( \mathcal{P} = (P, \Delta) \), a valuation \( \Lambda : IDP \rightarrow \Sigma \), and a starting ID \( c_0 \).

**Definition 8.** We call \( \Lambda : IDP \rightarrow \Sigma \) a regular valuation if the set \( \{ c \in IDP \mid A \in \Lambda(c) \} \) is regular for every \( A \in At \).

We assume that \( \Lambda \) is a regular valuation and a k-RPDS \( \mathcal{A} = (Q_A, P, \xi_A, Q_A, \delta_A) \) for each \( A \in At \) satisfying \( L(Q_A) = \{ c \in IDP \mid A \in \Lambda(c) \} \) is given.

We also define the model checking problem and regular valuations for PDS in the same way. It is known that the LTL model checking problem with regular valuations for PDS is decidable [4, Theorem 3]. The main objective of this paper is to show a reduction of that problem for RPDS to the one for PDS.

However, that problem for RPDS is undecidable in general, as shown below.

**Theorem 1.** The LTL model checking problem with regular valuations for RPDS is undecidable.

**Proof.** The universality problem for RA stated as follows is known to be undecidable [15, Theorem 5.1]:

**Instance:** a k-RA \( \mathcal{A} \), an initial state \( q_0 \), and an initial assignment \( \theta_0 \). **Question:** Does \( (q_0, \theta_0, w) \in L(\mathcal{A}) \) for every \( w \in D^* \)? We can reduce this problem to the model checking problem with regular valuations for RPDS: for given \( \mathcal{A}, q_0, \) and \( \theta_0 \), we can construct a \((k + 1)\)-RPDS \( \mathcal{P} \) and a \((k + 1)\)-RA \( \mathcal{A}' \) that satisfy the following: Let \( S \in D \) be an arbitrary data value, which is used as the stack bottom. \( \mathcal{P} \) has \( q_0 \) as its only state. \( \mathcal{P} \) does not alter the first \( k \) registers, and in every state transition, it loads an arbitrary data value to the \((k + 1)\)-th register and pushes it into the stack. Therefore, from any starting ID \((q_0, \theta_0, \$)\), \( \mathcal{P} \) can reach an ID \((q_0, \theta_0, w\$)\) for every \( w \in D^* \), where \( \theta'(i) = \theta(i) \) for \( i \in [k] \), \( \mathcal{A}' \) is a modified version of \( \mathcal{A} \) that does not use the \((k + 1)\)-th register and satisfies for any \( \theta_0 \) with \( \theta'_0(i) = \theta(i) \) for \( i \in [k] \), \((q_0, \theta_0, w) \in L(\mathcal{A}) \) iff \((q_0, \theta'_0, w\$) \in L(\mathcal{A}') \). Let \( At = \{ A \} \) and \( \Lambda \) be the regular valuation such that \( \Lambda(c) = \{ A \} \) if \( c \in L(\mathcal{A}') \) and \( \Lambda(c) = \emptyset \) otherwise. Let \( c_0 = (q_0, \theta_0, \$) \) for some \( \theta'_0 \) with \( \theta'_0(i) = \theta(i) \) for \( i \in [k] \). Let \( \varphi = \Box A \). Then, the answer to the model checking problem on \( \mathcal{P}, \Lambda, \varphi, \) and \( c_0 \) coincides with the answer to the universality problem on \( \mathcal{A}, q_0, \) and \( \theta_0 \).

5. Freshness property

The method proposed in this paper requires that the transition relation \( \Rightarrow_R \) of every k-RPDS (including k-RA) \( \mathcal{P} \) should have the freshness property stated as follows: When \( r \) and the updated assignment \( \theta' \) contains a data value \( d' \) not in \( \theta \cup \{ d \} \), then \( d' \) must be “fresh”; i.e., \( d' \) should have never appeared in the computation from a starting configuration to \((p, \theta, du) \). Since the set \( D \) of data values is infinite, such a fresh data value \( d' \) always exists whenever the rule \( r \) can be applied to \((p, \theta, du) \) above.

To define the freshness property formally, we slightly modify the semantics of k-RPDS so that each stack cell keeps the assignment at the time when the cell is pushed into the stack. The assignments “saved” in the stack are used only for choosing a fresh data value and do not affect the behavior of an RPDS in other ways. We redefine the set \( IDP \) of IDs of a k-RPDS \( \mathcal{P} = (P, \Delta) \) as

\[
IDP = P \times \Theta_k \times (D \times \Theta_k)^* 
\]

and the transition relation \( \Rightarrow_R \) as follows:

\[
(p, \phi) \rightarrow (q, com) \in \Delta \quad (\theta, d, \theta') \models \phi \\
frsp(\theta', d, \theta, \theta'' \text{and} (u))
\]

\[
(p, \theta, (d, \theta''u)) \Rightarrow_R (q, \theta', \text{upsd}((d, \theta''u, \theta', \text{com}))
\]

where \( \text{upsd}((d, \theta''u, \theta', \text{com}) = u, (d, \theta''u), \theta', \) and \( \theta'' \) if \( \text{com} = \text{pop}, \text{skip}, \text{or push}(j) \), respectively, and \( \text{frsp}(\theta', d, \theta, \theta_n \ldots \theta_1) \) is a predicate that is true iff for each \( i \in [k] \), \( \theta'(i) \in \theta \cup \{ d \} \) or \( \theta'(i) \notin \theta_1 \cup \ldots \cup \theta_n \). That is, each value \( \theta'(i) \)
in the updated assignment is either the value \( \theta(l) \) of some register, the value \( d \) at the stack top, or a fresh value that has not appeared in \( \theta_n \ldots \theta_1 \).

We say an ID \((p, \theta_n, (d_{n-1}, \theta_{n-1}) \ldots (d_1, \theta_1)) \) in \( ID_P \) is proper if for all \( i, j, l \in [n] \) with \( i < j \leq l \), \( d_i, \theta_i, \theta_j, \theta_l \) satisfy \( d_i = \theta_i \) and \( \forall m \in [k] : \theta_i(m) \notin \theta_j \) implies \( \theta_i(m) \notin \theta_j \) and \( (d_i \notin \theta_j \) implies \( d_i \notin \theta_j \). Under the assumption of the freshness property, every ID reachable from a proper starting ID is also proper. We assume the starting ID \( c_0 \) given to the model checking problem is proper.

Note that RA with the freshness property is similar to session automata (SA) \cite{16}, which is a special case of fresh-register automata (FRA) \cite{17}. An SA has the same structure as RA but requires an input data value to be either a value stored in some register or a value not used before. On the other hand, RA in this paper can update registers with data values not given as input but chosen arbitrary from values satisfying a guard condition, and the freshness constraint is imposed only on values not given as input.

6. PDS simulating RPDS

6.1. Bisimulation relation between an RPDS and a PDS

The bisimulation equivalence \cite{12} is a basic notion to capture the equivalence of behaviors of two state transition systems. We review the definition of the notion in this subsection. We will show the construction of a PDS bisimulation equivalent to a given RPDS in Section \cite{6.3}.

**Definition 9.** For an RPDS \( \mathcal{P} = (P, \Sigma) \) and a PDS \( \mathcal{P}' = (P', \Sigma') \), we call a relation \( R \subseteq ID_P \times ID_{P'} \) a bisimulation relation between \( \mathcal{P} \) and \( \mathcal{P}' \) if \( R \) satisfies the following:

1. For every \( c_1, c_2 \in ID_P \) and \( c_1' \in ID_{P'} \), if \( c_1 \Rightarrow c_2 \) and \( (c_1, c_1') \in R \), then \( \exists c_2' \in ID_{P'} : c_1' \Rightarrow c_2' \) and \( (c_2, c_2') \in R \).

2. For every \( c_1 \in ID_P \) and \( c_1', c_2' \in ID_{P'} \), if \( c_1' \Rightarrow c_2' \) and \( (c_1, c_1') \in R \), then \( \exists c_2 \in ID_P : c_1 \Rightarrow c_2 \) and \( (c_2, c_2') \in R \).

For an RA \( \mathcal{A} = (Q, I, \xi, \delta) \) and an NFA \( \mathcal{A}' = (Q', I', F, \delta') \), we call a relation \( R \subseteq ID_A \times ID_{A'} \) a bisimulation relation between \( \mathcal{A} \) and \( \mathcal{A}' \) if \( R \) is a bisimulation relation between \( (Q, \delta) \) and \( (Q', \delta') \) and also satisfies:

3. If \( (c, c') \in R \), then \( c \in Acc_A \) iff \( c' = (q, c) \) and \( q \in F \).

An RPDS \( \mathcal{P} \) and a PDS \( \mathcal{P}' \) (or an RA \( \mathcal{A} \) and an NFA \( \mathcal{A}' \)) are bisimulation equivalent if there is a bisimulation relation between them.

By definition, we obtain the following propositions.

**Proposition 2.** If there is a bisimulation relation \( R \) between an RA \( \mathcal{A} \) and an NFA \( \mathcal{A}' \), then for any pair \( (c, c') \in R \), \( c \in L(\mathcal{A}) \) iff \( c' \in L(\mathcal{A}') \).

**Proposition 3.** Let \( \mathcal{P} \) be an RPDS and \( \mathcal{P}' \) be a PDS. Let \( \Lambda : ID_P \rightarrow \Sigma \) and \( \Lambda' : ID_{P'} \rightarrow \Sigma' \) be their valuations and \( c_0 \in ID_P \) and \( c_0' \in ID_{P'} \) be their IDs. If there is a bisimulation relation \( R \) between \( \mathcal{P} \) and \( \mathcal{P}' \) such that \( (c_0, c_0') \in R \) and \( \Lambda(c) = \Lambda'(c') \) for every pair \( (c, c') \in R \), then:

1. For every run \( \rho \) of \( \mathcal{P} \) with \( \rho(0) = c_0 \), there exists a run \( \rho' \) of \( \mathcal{P}' \) with \( \rho'(0) = c_0' \) such that \( \Lambda(\rho) = \Lambda'(\rho') \).

2. For every run \( \rho' \) of \( \mathcal{P}' \) with \( \rho'(0) = c_0' \), there exists a run \( \rho \) of \( \mathcal{P} \) with \( \rho(0) = c_0 \) such that \( \Lambda(\rho) = \Lambda'(\rho') \).

6.2. Composition of equivalence relations

Let \( \odot \) and \( \odot_T \) be the binary predicates over \( \Phi_k \) defined as:

\[ \phi_1 \odot \phi_2 : \Leftrightarrow (x'_i \equiv \phi_1, x'_j \equiv \phi_2, x_i \equiv \phi_2, x_j \text{ for } i, j \in [k]). \]

\[ \phi_1 \odot_T \phi_2 : \Leftrightarrow (\phi_1 \odot \phi_2 \text{ and } x'_i \equiv \phi_1, x_{\text{top}} \equiv \phi_2) \text{ for } i \in [k]). \]

Intuitively, \( \phi_1 \odot \phi_2 \) represents the composability of \( \phi_1 \) and \( \phi_2 \); for \( \phi_1 \) and \( \phi_2 \) satisfying \( (\theta_1, d_1, \theta_2) \models \phi_1 \) and \( (\theta_2, d_2, \theta_3) \models \phi_2 \) for some \( \theta_1, d_1, \theta_2, \theta_3 \), we will define (after Example \[3\]) the composition \( \phi_1 \odot \phi_2 \) that satisfies \( (\theta_1, d_1, \theta_3) \models \phi_1 \odot \phi_2 \) (under the assumption on the freshness property), and \( \phi_1 \odot \phi_2 \) represents the condition “\( (\theta_1, d_1, \theta_2) \models \phi_1 \) and \( (\theta_2, d_2, \theta_3) \models \phi_2 \) for some \( \theta_1, d_1, \theta_2, \theta_3 \).” Similarly, \( \phi_1 \odot_T \phi_2 \) represents the condition “\( (\theta_1, d, \theta_2) \models \phi_1 \) and \( (\theta_2, d, \theta_3) \models \phi_2 \) for some \( \theta_1, d, \theta_2, \theta_3 \).”

**Example 3.** The equivalence relations \( \phi_0 \) and \( \phi_1 \) shown in Example \[1\] satisfy \( \phi_0 \odot \phi_1 \) because \( x'_1 \neq x'_0, x'_2 \neq x'_0 \) and \( x_1 \neq x_2 \). However, \( \phi_0 \odot_T \phi_1 \) does not hold because \( x'_2 \equiv x_{\text{top}} \), but \( x_2 \neq x_{\text{top}} \). For \( \phi_2 \) and \( \phi_3 \) also shown in Example \[1\] both \( \phi_0 \odot_T \phi_3 \) and \( \phi_1 \odot_T \phi_2 \) hold.
For \( \phi_1, \phi_2 \in \Phi_k \) with \( \phi_1 \circ \phi_2 \) of them is the equivalence relation in \( \Phi_k \) that satisfies the following:

\[
x_i \equiv_{\phi_1 \circ \phi_2} x_j :\Leftrightarrow x_i \equiv_{\phi_1} x_j \quad \text{for } i, j \in [k] \cup \{ \text{top} \},
\]

\[
x'_i \equiv_{\phi_1 \circ \phi_2} x'_j :\Leftrightarrow x'_i \equiv_{\phi_2} x'_j \quad \text{for } i, j \in [k],
\]

\[
x_i \equiv_{\phi_1 \circ \phi_2} x'_j :\Leftrightarrow (\exists l \in [k] : x_i \equiv_{\phi_1} x'_l \land x_l \equiv_{\phi_2} x'_j) \quad \text{for } i \in [k] \cup \{ \text{top} \}, j \in [k].
\]  

(1) (2) (3)

By definition, if an ID \((p, \theta_1, (d_2, \theta_2)(d_1, \theta_1)u)\) is proper, \((\theta_1, d_1, \theta_2) \models \phi_1\) and \((\theta_2, d_2, \theta_3) \models \phi_2\), then \((\theta_1, d_1, \theta_3) \models \phi_1 \circ \phi_2\). Guaranteeing this property is the main purpose of the freshness property and the properness of IDs: If the properness of the above ID is not assumed, and \(d_1 \notin \theta_2\) and \(\theta_1(j) \notin \theta_2 \cup \{d_2\}\) for some \(j \in [k]\), then either \(d_1 = \theta_3(j)\) or \(d_1 \neq \theta_3(j)\) holds (and only the latter satisfies \((\theta_1, d_1, \theta_3) \models \phi_1 \circ \phi_2\) ). This uncertainty prevents a PDS from simulating an RPDS: When a PDS \(\mathcal{P}'\) simulating an RPDS is popping off the stack top in an ID corresponding to the above ID, without assuming the properness of the ID, \(\mathcal{P}'\) cannot know whether or not the data value \(d_1\) in the new stack top of the RPDS is contained in the current register assignment \(\theta_3\).

Similarly to the composition \(\circ\), for \(\phi_1, \phi_2 \in \Phi_k\) with \(\phi_1 \circ \phi_2\) as the same as \(\phi_1 \circ \phi_2\) except that the Equation (3) is replaced with the following (4):

\[
x_i \equiv_{\phi_1 \circ \phi_2} x'_j :\Leftrightarrow (\exists l \in [k] : x_i \equiv_{\phi_1} x'_l \land x_l \equiv_{\phi_2} x'_j) \quad \text{for } i \in [k] \cup \{ \text{top} \}, j \in [k].
\]

By definition, \(\circ\) and \(\circ_T\) are associative.

Let \((\phi)_{\equiv}\) for \(\phi \in \Phi_k\) and \(j \in [k]\) be the equivalence relation defined as follows: \(\forall i, l \in [k] : (x_i \equiv_{(\phi)_{\equiv}} x_l)\) if \(x_i \equiv_{\phi} x_l\) \land \((x_i \equiv_{(\phi)_{\equiv}} x_l)\) if \(x_i \equiv_{\phi} x_l\) \land \((x_i \equiv_{(\phi)_{\equiv}} x_l)\) \land \((x_i \equiv_{(\phi)_{\equiv}} x_l)\). The intention of the above definition is to guarantee that \((\theta', \theta'(j), \theta') \models (\phi)_{\equiv}\) whenever \((\theta, \theta, \theta) \models \phi\).

6.3. Construction of PDS simulating RPDS

For a \(k\)-RPDS \(\mathcal{P} = (P, \Delta)\), we construct a PDS \(\mathcal{P}' = (P', \Delta')\) bisimulation equivalent to \(\mathcal{P}\). The set of states of \(\mathcal{P}'\) is \(P' = P \times \Phi_k\), and the stack alphabet of \(\mathcal{P}'\) is \(\Phi_k\). \(\mathcal{P}'\) must simulate \(\mathcal{P}\) without keeping data values in the stack. When popping off the stack top, \(\mathcal{P}'\) must know whether or not the data value in the new stack top of \(\mathcal{P}\) equals the current value of each register. For this purpose, \(\mathcal{P}'\) keeps an abstract “history” of the register assignments represented by a sequence of equivalence relations in the stack, which tells whether each of the data values in the stack of \(\mathcal{P}\) equals the current value of each register. The second component of each state of \(\mathcal{P}'\) is “the last element” of the abstract history, which represents the accumulated updates since the current stack top has been pushed into the stack. Because a PDS cannot replace the symbol at the new stack top when \(\text{pop}\) or \(\text{skip}\) is performed, \(\mathcal{P}'\) keeps the last element of the history in its finite state and updates it in every transition.

For example, configuration \((q, \theta_2, (d_1, \theta_1)(d_0, \theta_0))\) of \(\mathcal{P}\) is simulated by configuration \(((q, \phi_2), (\theta_1 \circ \phi_3))\) of \(\mathcal{P}'\), where \((\theta_0, d_0, \theta_1) \models \phi_1\) and \((\theta_1, d_1, \theta_2) \models \phi_2\). Equivalence relation \(\phi_1\) abstractly represents the updates of assignments between when \(d_0\) has been pushed and when \(d_1\) has been pushed. Similarly, \(\phi_2\) represents the updates of assignments since \(d_1\) has been pushed.

The set \(\Delta'\) of transition rules of \(\mathcal{P}'\) is the smallest set satisfying the following inference rules:

\[
(q, \phi_3) \rightarrow (q', \text{skip}) \in \Delta' \quad \phi_1 \circ \phi_2 = (\phi_2 \circ_T \phi_3) \\
((q, \phi_2), \phi_1) \rightarrow ((q', \phi_2 \circ_T \phi_3), \text{skip}) \in \Delta'
\]

\[
(q, \phi_3) \rightarrow (q', \text{pop}) \in \Delta' \quad \phi_1 \circ \phi_2 = (\phi_2 \circ_T \phi_3) \\
((q, \phi_2), \phi_1) \rightarrow ((q', \phi_1 \circ (\phi_2 \circ_T \phi_3)), \text{pop}) \in \Delta'
\]

\[
(q, \phi_3) \rightarrow (q', \text{push}(j)) \in \Delta' \quad \phi_1 \circ \phi_2 = (\phi_2 \circ_T \phi_3) \\
((q, \phi_2), \phi_1) \rightarrow ((q', \text{push}(j)), \text{push}(\phi_2 \circ_T \phi_3)) \in \Delta'
\]

In the above inference rules, \(\phi_2 \circ_T \phi_3\) represents the accumulation of the update of registers by \(\phi_3\) into \(\phi_2\). When \(\text{pop}\) is performed, \(\phi_1\) at the stack top is composed into \(\phi_2 \circ_T \phi_3\), which then represents the accumulated updates since the new stack top has been pushed into the stack. When \(\text{push}\) is performed, \(\phi_2 \circ_T \phi_3\) is pushed into the stack because the current assignment is “the assignment when the stack top was pushed into the stack.” In this case, \(\mathcal{P}'\) sets the second component of the state to \((\phi_3)_{\equiv}\), which represents the current assignment (which is the result of the update by \(\phi_3\)) equals the assignment saved in the stack top.

In the same way, we construct an NFA \(A'_A\), as a PDS with pop rules only, from \(A_A = (Q_A, P, \xi_A, \delta_A)\) for each \(A \in At\). Moreover, let the set of initial states and the set of final states of \(A'_A\) be \(P \times \Phi_k\) and \(\{(p, \phi) \mid (p, \text{last}(\phi)) \in \xi_A\}\), respectively,
for each $A \in At$. Let $ID_A = \bigcup_{A \in At} ID_{A \mathcal{A}}$ and $ID_{\mathcal{A}} = \bigcup_{A \in At} ID_{A \mathcal{A}}$.

We assume that the stack $u_0$ in the given starting ID $c_0 = (p_0, \theta_0, u_0)$ is not empty, and let $(d_0, \theta_0)$ be the last (bottom-most) element of $u_0$. Note that the last element of the stack of every ID reachable from $c_0$ equals $(d_0, \theta_0)$, because an ID with empty stack has no successor and thus any computation cannot alter the stack bottom.

Let $R \subseteq ID_{\mathcal{A}} \times ID_{\mathcal{A}}$ be the smallest relation satisfying the following: $((p, \theta, u), ((p, \phi_n), v)) \in R$ for $u = (d_{n-1}, \theta_{n-1}) \ldots (d_1, \theta_1)$ and $v = \phi_n \ldots \phi_1$ if $(p, \theta, u)$ is proper, $(d_1, \theta_1) = (d_0, \theta_0)$ (or $u = \varepsilon$), and $\forall i \in [n]: (\theta_{i-1}, d_{i-1}, \theta_i) \models \phi_i$. (Remember that $(d_0, \theta_0)$ is the stack bottom in the starting ID $c_0$.)

By definition, $R$ is functional; that is, each $(p, \theta, u, \phi_n) \in ID_{\mathcal{A}}$ has exactly one $((p, \phi_n), v) \in ID_{\mathcal{A}}$ that satisfies $((p, \theta, u), ((p, \phi_n), v)) \in R$. Let $R(c)$ for $c \in ID_{\mathcal{A}}$ denote the unique ID $c' \in ID_{\mathcal{A}}$ that satisfies $(c, c') \in R$.

Let $R_{\mathcal{A}} = R \cap (ID_{\mathcal{A}} \times ID_{\mathcal{A}})$ and $R_{A \mathcal{A}} = R \cap (ID_{A \mathcal{A}} \times ID_{A \mathcal{A}})$ for each $A \in At$. We have proved the following proposition using the Coq proof assistant:

**Proposition 4.** $R_{\mathcal{A}}$ is a bisimulation relation between $\mathcal{P}$ and $\mathcal{P}'$. $R_{A \mathcal{A}}$ is a bisimulation relation between $A \mathcal{A}$ and $A' \mathcal{A}$ for each $A \in At$.

The reduction is completed by letting $c'_0 = R(c_0)$.

**Example 4.** Let $\mathcal{P}'$ be the PDS obtained from the 2-RPDS $\mathcal{P}$ shown in Example 1. Let $\phi_0, \ldots, \phi_3 \in \Phi_2$ be the equivalence rules that are also shown in Example 1. By the above inference rules, $\mathcal{P}'$ has rules $((p_1, \phi'), \phi') \rightarrow ((p_1, \phi_1), \text{push}(\phi' \circ \phi_1))$ (obtained from $r_2$) and $((p_1, \phi'), \phi') \rightarrow ((p_1, \phi' \circ \phi' \circ \phi_1), \text{pop})$ (obtained from $r_3$) for each $\phi'$ such that $\phi' \circ \phi'$ and $\phi' \circ \phi_1$. Let $\phi_2, \phi_0 \in \Phi_2$ be the equivalence relations defined by the following quotient sets:

$X_2/\phi_0 = \{\{x_1, x'_1, x_{top}\}, \{x_2, x'_2\}\}$

$X_2/\phi_0 = \{\{x_1, x'_1\}, \{x_2, x'_2, x_{top}\}\}$

Because $\phi_0 \circ \phi_2, \phi_3 \circ \phi_1, \phi_5 \circ \phi_1 = \phi_1$, and $\phi_1 \circ \phi_5, \mathcal{P}'$ has rule $r'_2 = ((p_1, \phi_5), \phi_1) \rightarrow ((p_1, \phi_5), \text{push}(\phi_1))$. Similarly, because $\phi_0 \circ \phi_2$ and $\phi_1 \circ \phi_5$ have rule $r'_3 = ((p_1, \phi_0), \phi_1) \rightarrow ((p_1, \phi_5), \text{push}(\phi_1))$. By these two rules, $\mathcal{P}'$ has the following transition sequence:

$$
((p_1, \phi_0), \phi_0 \phi_0) \Rightarrow \mathcal{P}' ((p_1, \phi_5), \phi_1) \Rightarrow \mathcal{P}' ((p_1, \phi_5), \phi_0 \phi_0).
$$

The following is a part of the transition sequence of $\mathcal{P}$ in Figure 1 in which each stack cell is augmented by the assignment at the time when the cell has been pushed. (See Section 5)

$$(p_1, [d_2, d_0], [d_2, [d_2, d_0]]([d_0, [d_1, d_0]])) \Rightarrow \mathcal{P} (p_1, [d_3, d_0], [d_3, [d_3, d_0]]([d_2, [d_2, d_0]]([d_0, [d_1, d_0]])) \Rightarrow \mathcal{P} (p_1, [d_4, d_0], [d_2, [d_2, d_0]]([d_0, [d_1, d_0]]).$$

Let $c_1, c_2, c_3$ be the IDs in the sequence (5), respectively. We can see that the IDs in the sequence are $R(c_1), R(c_2)$, and $R(c_3)$, respectively. (See the paragraphs before Proposition 4 for the definition of $R$.)

**6.4. Time complexity**

Consider the LTL model checking problem on $\mathcal{P} = (\mathcal{P}, \Delta)$, $\varphi$, $\Lambda$, and $c_0$, where $\mathcal{P}$ is a regular valuation and is represented by $k$-RA $A_A = (Q_A, P, \xi_A, \delta_A)$ for $A \in At$. Let $\mathcal{P}' = (\mathcal{P}', \Delta')$ and $A'_A = (Q'A, P', F_A, \delta_A')$ be the PDS and NFA obtained from $\mathcal{P}$ and $A_A$ in the last subsection. Applying the LTL model checking method for PDS in 3 to $\mathcal{P}'$, we can solve the model checking problem on $\mathcal{P}, \varphi, \Lambda, c_0$ in $O(|\mathcal{P}'|^2 \cdot |\Delta'| \cdot \prod_{A \in At} |Q_A| \cdot 2^{O(|\varphi|)})$ time, if $A'_A$ for each $A \in At$ is backward-deterministic. By the construction, $|\mathcal{P}'| = |\mathcal{P}| \cdot |\Phi_k|$ and $|Q'A| = |Q_A| \cdot |\Phi_k|$. Moreover, $|\Delta'| \leq |\Delta| \cdot |\Phi_k|^2$, because in the inference rules defining $\Delta'$, we choose two equivalence relations $\phi_1$ and $\phi_2$ for each transition rule in $\Delta$. $|\Phi_k|$ equals the $(2k + 1)$-th Bell number and thus $|\Phi_k| = 2^{O(k \log k)}$. We can assume that $|At| \leq |\varphi|$ and thus $\prod_{A \in At} |Q_A| \leq \prod_{A \in At} |Q_A| \cdot |\Phi_k||\varphi|$. Therefore the time complexity of the proposed method is exponential in $k$ and $|\varphi|$ and polynomial in $|\mathcal{P}|$ and $|\Delta|$ and $\prod_{A \in At} |Q_A|$. Note that if the NFA $A'_A$ for some $A \in At$ is not backward-deterministic, we have to apply backward-determinization to $A'_A$, which

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1 The proof scripts are available at [https://github.com/ytakata69/rpds-to-pds-proof](https://github.com/ytakata69/rpds-to-pds-proof)
may increase the time complexity of the proposed method. Also note that $A'$ is not necessarily backward-deterministic even when $A$ is backward-deterministic in the sense defined in [11].

7. Conclusion

This paper proposed a method for solving the LTL model checking problem for RPDS with regular valuations, in which the problem is reduced to the same problem for PDS. In contrast to the method for the same problem proposed in [11], the method in this paper does not require RA used for a regular valuation to be deterministic or backward-deterministic. On the other hand, the method in this paper requires every RPDS and RA has the freshness property instead. This paper also showed that the LTL model checking problem for RPDS with regular valuations defined by general RA is undecidable, and thus the freshness constraint is essential in this method.

References

[1] R. Alur, A. Bouajjani, J. Esparza, Model checking procedural programs, in: Handbook of Model Checking, Springer, 2018, Ch. 17, pp. 541–572. doi:10.1007/978-3-319-10575-8_17
[2] A. Bouajjani, J. Esparza, O. Maler, Reachability analysis of pushdown automata: Application to model-checking, in: 8th Int. Conf. on Concurrency Theory, CONCUR ’97, Vol. 1243 of Lecture Notes in Computer Science, Springer, 1997, pp. 135–150. doi:10.1007/3-540-61474-5_58
[3] I. Walukiewicz, Pushdown processes: Games and model checking, in: Computer Aided Verification, 8th Int. Conf., CAV ’96, Vol. 1102 of Lecture Notes in Computer Science, Springer, 1996, pp. 62–74. doi:10.1007/3-540-61474-5_58
[4] J. Esparza, A. Kučera, S. Schwoon, Model checking LTL with regular valuations for pushdown systems, Inf. Comput. 186 (2) (2003) 355–376. doi:10.1016/S0890-5401(03)00139-1
[5] M. Kaminski, N. Francez, Finite-memory automata, Theor. Comput. Sci. 134 (2) (1994) 329–363. doi:10.1016/0304-3975(94)90242-9
[6] L. Libkin, D. Vrgoč, Regular path queries on graphs with data, in: 15th Int. Conf. on Database Theory, ICDT ’12, ACM, 2012, pp. 74–85. doi:10.1145/2274576.2274588
[7] R. Grigore, D. Distefano, R. L. Petersen, N. Tzevelekos, Runtime verification based on register automata, in: TACAS 2013, Vol. 7795 of Lecture Notes in Computer Science, Springer, 2013, pp. 260–276. doi:10.1007/978-3-642-36742-7_19
[8] L. Exibard, E. Filiot, P. Reynier, Synthesis of data word transducers, Log. Methods Comput. Sci. 17 (1) (2021). URL https://lmcs.episciences.org/7279
[9] A. S. Murawski, S. J. Ramsay, N. Tzevelekos, Reachability in pushdown register automata, J. Comput. Syst. Sci. 87 (2017) 58–83. doi:10.1016/j.jcss.2016.11.008
[10] R. Senda, Y. Takata, H. Seki, Forward regularity preservation property of register pushdown systems, IEICE Trans. Inf. Syst. 104-D (3) (2021) 370–380. doi:10.1587/transinf.2020TP0008
[11] R. Senda, Y. Takata, H. Seki, LTL model checking for register pushdown systems, IEICE Trans. Inf. Syst. 104-D (12) (2021) 2131–2144. doi:10.1587/transinf.2020EDP7265
[12] E. M. Clarke, O. Grumberg, D. Kroening, D. Peled, H. Veith, Model checking, 2nd Edition, MIT Press, 2018, Ch. 11, pp. 177–182.
[13] R. Senda, Y. Takata, H. Seki, Reactive synthesis from visibly register pushdown automata, in: ICTAC 2021, Vol. 12819 of Lecture Notes in Computer Science, Springer, 2021, pp. 334–353. doi:10.1007/978-3-030-85315-0_19
[14] Y. Bertot, P. Castéran, Interactive Theorem Proving and Program Development, Coq’Art: The Calculus of Inductive Constructions, Texts in Theoretical Computer Science. An EATCS Series, Springer, 2004. doi:10.1007/978-3-662-07964-5
[15] F. Neven, T. Schwentick, V. Vianu, Finite state machines for strings over infinite alphabets, ACM Trans. Comput. Log. 5 (3) (2004) 403–435. doi:10.1145/1013560.1013562
[16] B. Bollig, P. Habermehl, M. Leucker, B. Monmege, A robust class of data languages and an application to learning, Log. Methods Comput. Sci. 10 (4) (2014) 1–23. doi:10.2168/LMCS-10(4:19)2014
[17] N. Tzevelekos, Fresh-register automata, in: POPL ’11, ACM, 2011, pp. 295–306. doi:10.1145/1926385.1926420