Signatures of spin-orbit coupling in scanning gate conductance images of electron flow from quantum point contacts

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Electron flow through a quantum point contact in presence of spin orbit coupling is investigated theoretically in the context of the scanning gate microscopy (SGM) conductance mapping. Although in the absence of the floating gate the spin-orbit coupling does not significantly alter the conductance, we find that the angular dependence of the SGM images of the electron flow at the conductance plateaux is significantly altered as the spin-orbit interaction mixes the orbital modes that enter the quantum point contact. The radial interference fringes that are obtained in the SGM maps at conductance steps are essentially preserved by the spin-orbit interaction although the effects of the mode mixing are visible.

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I. INTRODUCTION

Scanning gate microscopy (SGM) has become a widely used technique that allows for mapping the current flow and charge densities in nanoscopic structures. The perturbation induced by the floating gate was used to map scarred wave functions in quantum billiards12,13 local density of states in quantum rings,4,5,33 magnetic focusing of electrons24,25 or for demonstration of a mesoscopic analogue to the Braess paradox.2 Mapping of conductance of quantum point contacts (QPCs) allowed for observation of spatial maps of the coherent electron flow24,25 and signatures of interference involving the presence of the tip.15,16 Moreover it has been found that the electron current after leaving the QPC propagates in narrow branches17,18 and that checkerboard interference patterns19 are observed in the maps of the electron flow.20 Recent experiments measured nonequilibrium transport phenomena in QPCs,21 and demonstrated the control of the edge channel trajectories in the quantum Hall regime.22 Recent theoretical studies on SGM in QPC systems delivered a perturbative description23 of the transport and a temperature-induced amplification of the interference fringes.16

There is a growing interest in spin phenomena in two-dimensional electron gas (2DEG). A particular attention is paid to spin-orbit (SO) interaction that results in an effective magnetic field for propagating electrons and allows for control of the electron spin by the electric fields.24,25 The majority of SGM experiments probing electron flow from QPCs focus on structures based on GaAs where the SO interaction is usually weak. On the other hand materials such as InGaAs provide much stronger SO coupling.26 QPCs in these structures can be used to generate spin-polarized currents in the absence of external magnetic field.27–30 Nevertheless, the number of SGM studies of InGaAs based QPCs is still limited.26–33 The present work describes the SO coupling effects on SGM imaging of electron flow from InGaAs QPCs in conditions of strong SO coupling. Previously, the impact of SO interaction on SGM maps of electron flow has been studied in the context of spin-dependent magnetic focusing.24 We find that the SO interaction barely modifies the conductance of the unperturbed system. Nevertheless, we demonstrate that the SO coupling leaves a distinct signature on the QPC conductance response to the perturbation introduced by the tip.

In the present work we focus on two types of QPC work points. The first one is set by the conductance plateaux $G = 1, 2, 3$ in units of $G_0 = [2e^2/h]$ where the experiments detect the angular patterns of the electron flow from the QPC.2 We find that the angular patterns are modified by the SO interaction in such a way that the double and triple branches are smeared out due to mixing of the orbital modes of the propagating electron by the SO coupling. A similar effect is observed for the QPC tuned to the conductance steps. We find that in the latter case that the radial interference fringes that are resolved by the the SGM25 preserve their oscillation pattern.

II. THEORY

A. Model

We consider a two-dimensional system described by the Hamiltonian

$$H = \left[ \frac{\hbar^2 k^2_x}{2m^*} + \frac{\hbar^2 k^2_y}{2m^*} + V(x,y) \right] \mathbf{1} + \alpha (\sigma_x k_y - \sigma_y k_x), \quad (1)$$

where the latter term corresponds to SO coupling of Rashba type34 with the strength controlled by the parameter $\alpha$ and $\mathbf{1}$ is identity matrix. We consider the system which is schematically shown in Fig. I with a channel opening to an infinite space through a restriction introduced by the QPC. The total potential in the
system $V(x, y)$ is taken in the following form,

$$V(x, y) = V_{ch}(x, y) + \frac{V_{tip}}{(x - x_{tip})^2/d^2 + (y - y_{tip})^2/d^2 + 1} + \exp \left\{ - \frac{(y - Y_{qpe})/(\sqrt{2}w)}{m^*\omega^2x^2/2} \right\}$$

(2)

where the first term describes the input channel of width $W$

$$V_{ch}(x, y) = \begin{cases} 0 & |x| \leq W \\ 500 \text{ meV} & |x| > W \text{ and } < 1600 \text{ nm} \end{cases}$$

(3)

with the potential of a QPC with the center at $2300$ nm. In these conditions there are $N = 8$ subbands at the Fermi level in the input channel including the spin degree of freedom – see the dispersion relation in the input channel that is plotted in Fig. 2(a). The conductance of the system is calculated using the Landauer formula

$$G = \frac{e^2}{h} \sum_{i} T_i,$$

(4)

where the transmission of each $i$'th channel transport mode is calculated from the reflection probability $T_i = N - \sum_{j} R_{i\rightarrow j}$ – the sum goes over $N$ modes propagating in the $-y$ direction.

For determination of the transport probability we first calculate the available transport modes in the input channel. Since $([-i\hbar\partial/\partial y, H] = 0)$ the wave vector $k$ is a good quantum number and the spinor of the electron in the input channel can be written in a separated form

$$\psi^k(x, y) = e^{iky} \left( \psi^k_x(x) \psi^k_y(x) \right).$$

(5)

We use this form of the spinor for calculation of the $N$ Fermi wave vectors in the input channel ($y = 0$) for each of the transport modes at a given Fermi energy $E_F$. We repeat the procedure for the output channel ($y = 4000$ nm) that is $1368$ nm wide – where for $E_F = 3$ meV there are 52 subbands [see Fig. 2(b) for the dispersion relation] – obtaining $M$ spinors with the corresponding wave vectors.

In order to determine $R_{i\rightarrow j}$ we solve the Schrödinger equation $H\psi = E\psi$ for the electron incident from $i$'th subband with wavevector $k_i$. The boundary conditions for the ends of the computational box $y = 0, y = 4000$ nm are adopted from Ref. 35 upon generalization to the SO coupling case. In the input lead the electron wave function is a superposition of an incoming wave with $k_i$ and $N$ backscattered waves (with negative currents)

$$\Psi(x, y) = c_i\psi^{k_i}(x, y) + \sum_{j=1}^{N} d_j\psi^{-k_j}(x, y).$$

(6)

Its derivative has the form,

$$\frac{\partial \Psi(x, y)}{\partial y} = ic_i k_i \psi^{k_i}(x, y) - \sum_{j=1}^{N} id_j k_j \psi^{-k_j}(x, y).$$

(7)

We add to the both sides of the above equation $ik_i\Psi(x, y)$ and replace $\frac{\partial \Psi(x, y)}{\partial y}$ by its central finite difference formula obtaining the boundary condition at the low end of the computational box

$$\Psi(x, y - \Delta y) = \Psi(x, y + \Delta y) + 2\Delta y i k_i \Psi(x, y)$$

$$-2i\Delta y \left( 2k_i c_i \psi^{k_i} + \sum_{j=1}^{N} (k_i - k_j) d_j \psi^{-k_j} \right)$$

(8)

FIG. 1. (color online) Sketch of the system under consideration. The grey area depicts the confinement potential of the input channel. The contours show the potential of a QPC for which we obtain the first plateau of conductance response. The grey area is located at $x = 100$ nm and $y = 2300$ nm. The crossed parts at the input and output channels denote the regions used for resolution of the transmitted and reflected plane waves. The red lines at the edges depict the regions where transparent boundary conditions are introduced.

The grey area in Fig. 1 shows the confinement potential of the input channel of Eq. (3). The second term in Eq. (2) accounts for the effective potential of the tip localized above point $(x_{tip}, y_{tip})$. This potential is a result of interaction of the Coulomb charge at the tip and the 2DEG, which has a form close to a Lorentzian. We adopt $d = 20$ nm, $V_{tip} = 7$ meV as the parameters of the scanning probe, for which the conductance response that we obtain is comparable to the experimental values of Ref. 26. The third term in Eq. (2) describes a smooth potential of the QPC with the center at $Y_{qpe}$ with $w$ responsible for the length of the constriction (we take $w = 100$ nm). We consider that the QPC is parabolic in the $x$ direction and described by the energy $\hbar \omega$.

B. Calculation of the conductance

We take $E_F = 3$ meV and for most of the calculations we consider the input channel width of $W = 240$ nm. In order to determine $R_{i\rightarrow j}$ we solve the Schrödinger equation $H\psi = E\psi$ for the electron incident from $i$'th subband with wavevector $k_i$. The boundary conditions for the ends of the computational box $y = 0, y = 4000$ nm are adopted from Ref. 35 upon generalization to the SO coupling case. In the input lead the electron wave function is a superposition of an incoming wave with $k_i$ and $N$ backscattered waves (with negative currents)
At the other end of the computational box there are no backscattered waves, the wave function has the form

\[ \Psi(x, y) = \sum_{j=1}^{N} a_j \psi^{k_j}(x, y), \tag{9} \]

and its derivative reads,

\[ \frac{\partial \Psi(x, y)}{\partial y} = \sum_{j=1}^{N} i a_j k_j \psi^{k_j}(x, y), \tag{10} \]

We follow the procedure applied to the wave function in the incoming lead but now we subtract \( ik_l \Psi(x, y) \) obtaining the boundary condition at the top of the computational box

\[ \Psi(x, y + \Delta y) = \Psi(x, y - \Delta y) + 2 \Delta y i k_l \Psi(x, y) + 2 i \Delta y \sum_{j=1}^{N} (k_j - k_l) a_j \psi^{k_j}(x, y). \tag{11} \]

The expressions for \( \Psi(x, y - \Delta y) \) and \( \Psi(x, y + \Delta y) \) are introduced to the linear system of equations that is given by the finite difference form of the Schrödinger equation.

We use a finite computational box in our calculations. In order to simulate an infinite semi-plane at the output from QPC we introduce transparent boundary conditions for the electron waves. For that purpose at the \( x \) edges of the computational box we assume the following boundary conditions for \( y > 1600 \text{ nm} \) [see the red lines in Fig. 1],

\[ \Psi(x \pm \Delta x, y) = \Psi(x, y) \exp[i k_b \Delta x], \tag{12} \]

where (−) is for the left boundary and (+) for the right one. The value of the wave vector \( k_b \) was set to remove the scattering from the edges. We found that the scattering is nearly removed for \( k_b \) which corresponds to a unique wave vector that appears at the considered Fermi energy \( E_F = 3 \text{ meV} \) for a thin channel of \( W_b = 80 \text{ nm} \). These boundary conditions allow the electron to flow out freely through the left and right edges of the computational box after QPC leaving the region where the system is scanned by the probe unperturbed – see the wave function plotted in Fig. 1 with the color map.

We discretize the Hamiltonian Eq. (1) [taking \( \Delta x = \Delta y = 8 \text{ nm} \)] and solve the resulting system of linear equations using LU method for sparse matrices. The scattering amplitudes \( a_j, c_j, d_j \) are determined in a self-consistent manner, with an initial guess \( a_j = c_j = 0 \) and \( d_j = 0 \). The solution to the linear system of equation provides us with the scattering spinor wave function \( \Psi(x, y) \) for a given energy \( E_F \). Before the convergence is reached this wave function contains contributions of all the sub-bands at a given energy, and the asymptotic form of Eq. (6) and (9) is only obtained at the self-consistence conditions. We extract then new values of the scattering amplitudes by projection of wave function \( \Psi(x, y) \) on the eigenmodes \( \psi^{k_l} \) given by Eq. (5), for all transport sub-bands at the Fermi level. In particular in the input lead we take

\[ \sum_{j=1}^{N} c_j |\psi^{k_j}|^2 + \sum_{j=1}^{N} d_j |\psi^{k_j}|^2 = |\psi^{k_l}|^2. \tag{13} \]

The scalar products in Eq. (11) are evaluated by integration that is performed on 160 nm strips at the beginning and at the end of the channel [see the crossed regions in Fig. 1]. The new values of the scattering amplitudes are used for the subsequent iteration of the solution of the Schrödinger equation and the procedure is repeated until the convergence is reached, i.e. in the input channel there is only one incoming wave corresponding to \( k_l \) and the amplitudes of backscattered waves at the output channel vanish.

Finally, the backscattering probabilities for the Landauer formula are calculated from the probability currents for given wave vectors,

\[ R_{i \rightarrow j} = \left| \frac{d_j}{c_i} \right|^2 \cdot \frac{j_y}{j_x}. \tag{14} \]

For numerical calculations we adopt parameters for for In_{0.5}Ga_{0.5}As, i.e. \( m^* = 0.0465 m_0 \) and Rashba SO coupling constat that is comparable to the experimentally measured value i.e. \( m^* \) and \( m^* \) and otherwise) which stems \( (\alpha = \alpha_{SO} F_z) \) from the material constant \( \alpha_{SO} \) and electric field in the growth direction \( F_z = 200 \text{ kV/cm} \). The computational box consist of \( 171 \times 501 \times 2 \) points.

III. RESULTS

A. Conductance of an unperturbed system

Let us start with the conductance obtained in the absence of the scanning probe. The narrowing introduced to the channel by the QPC limits the number of the conducting modes. In Figure 2(c) we plotted with the black curve the conductance versus the QPC potential for \( \alpha = 0 \). In the absence of the QPC potential – for \( \hbar \omega = 0 \) [see Eq. (2)] – we obtain no backscattering. As the QPC is introduced to the system (\( \hbar \omega \neq 0 \)) the conductance is reduced and the characteristic \( G \) plateaux appear. The conductance obtained in the presence of SO interaction is plotted by the green curve in Fig. 2(c). We observe that SO coupling does not induce any qualitative changes in the dependence of \( G \) on QPC potential, and the quantitative modification is also very weak. The green curve is only shifted on the two last steps to higher energies as compared to the dependence obtained in the absence of SO interaction.
are displayed in Fig. 3(d–f). For the QPC tuned to $G_0$, namely for $G = 4 \, e^2/h$, the conductance does not change. On the other hand there is a significant flow on the conductance of QPC – as observed in the clear signature of the angular dependence of the electron density and the current is approximately constant along the $x$-axis. This can be precisely observed in the cross sections of the results obtained for $y = 1896$ nm that are presented in Fig. 4(a) in the absence and in Fig. 4(b) in the presence of SO interaction.

To explain this observation let us focus on the charge densities of the modes that enter the QPC. In Fig. 5(c) we plotted the charge densities of the eigenmodes across the input channel. Each curve corresponds to the density of a spin-degenerate mode. For the QPC tuned to $G = 4 \, e^2/h$ the transfer probabilities of the subsequent modes are $T_1 = T_2 = 0.97$, $T_3 = T_4 = 0.99$, $T_5 = T_6 = 0.03$, and $T_7 = T_8 = 0$. Figure 5(e) presents the probability densities multiplied by the corresponding transfer probabilities. We observe that there are two main modes – the one with a single and two maxima. Figure 5(g) presents the sum of charge densities from Fig. 5(e). We find that the latter resembles the two-maxima character observed previously at the cross section of the charge density obtained after QPC in Fig. 4(a) so it is the transmission through this two modes that results in the two paths observed in Fig. 4(b).

In the map obtained without SO interaction Fig. 4(a) we observe two maxima whereas in the case with SO interaction present depicted in Fig. 4(b) the probability density and the current is approximately constant along the $x$-axis. This can be precisely observed in the cross sections of the results obtained for $y = 1896$ nm that are presented in Fig. 5(a) in the absence and in Fig. 5(b) in the presence of SO interaction.

Let us now discuss the maps of the $\Delta G$ – the difference between the conductance obtained in the presence of the scanning gate tip potential and the unperturbed result – for the QPC tuned to conductance plateaux. In Figs. 6(a–c) we present maps for $\hbar \omega = 3 \, \text{meV}$, $\hbar \omega = 1.6 \, \text{meV}$, and $\hbar \omega = 1 \, \text{meV}$ – which correspond to $\bullet$, ■, ▲ symbols in Fig. 6(c) respectively. In the absence of SO interaction a clear signature of the angular dependence of the electron flow on the conductance of QPC – as observed in the experiment performed for GaAs – emerges in the maps. The number of paths – a single one [Fig. 6(a)], two [Fig. 6(b)], and three [Fig. 6(c)] – corresponds to the number of quantized spin-degenerate orbital modes in the QPC that conduct. Moreover a strong checkerboard pattern is present in the results which is due to interference between the paths backscattered to the QPC and to its gates. 

The maps obtained in the presence of SO interaction are displayed in Fig. 6(d–f). For the QPC tuned to $G = 2 \, e^2/h$ – Fig. 6(d) – the character of the measured flow does not change. On the other hand there is a significant modification of the maps obtained for more open QPC, namely for $G = 4 \, e^2/h$ and $G = 6 \, e^2/h$. Comparing the maps obtained without SO interaction and with SO coupling included we observe that SO interaction results in smearing out the pattern with double and triple paths.

In order to inspect closer the above finding we focus on $G = 4 \, e^2/h$. In Fig. 6(a,b) we plot with the color maps the probability densities, with the arrows the current and with the isolines the amplitude of the current. Comparing Fig. 6(b) and Fig. 6(a) we notice that with the exception of the checkerboard pattern that is due to the interference introduced by the tip, the SGM conductance maps are well correlated to the current amplitude and probability density of the transmitted electron. This is also the case when the SO interaction is present as plotted in Figs. 6(c) and 6(b).

B. Branched electron flow on the conductance plateaux

When SO interaction is included the eigenmodes in the input channel are mixed and the spin degeneracy is lifted. Now each mode is split into two and we marked their charge densities with the solid and dashed curves in Fig. 6(d) where the colors of the curves correspond to the ones from Fig. 6(c). The transfer probabilities of these modes are $T_1 = 0.98, T_2 = 0.98, T_3 = 0.63, T_4 = 0.97, T_5 = 0.38, T_6 = 0.04, T_7 = 0.02, T_8 = 0$. Fig. 6(f) presents the Fermi electron densities multiplied by the corresponding transfer probabilities. We observe that the contribution of the modes with maximum of the charge density in $x = 0$ is increased as compared to the case without SO interaction of Fig. 6(e). The sum of the charge densities is presented in Fig. 6(h) and it corresponds to the cross section of the charge density obtained from the solution of the transport problem plotted in Fig. 6(b). The central local minimum present without SO coupling is replaced by a shallow local maximum. We conclude that the changes in the conductance maps – the vanishing of
FIG. 3. (color online) Maps of the conductance changes $\Delta G$ as a function of the SGM tip position for the QPC tuned to three plateaux – the symbols correspond to the ones in Fig. 2. The upper (bottom) row corresponds to the results obtained without (with) SO interaction.

FIG. 4. (color online) Color maps show the probability density for the transmitted electron. The arrows present the probability current and the contours present amplitude of the probability current. Results obtained for the plateau $G = 4 \frac{e^2}{h}$ for (a) $\alpha = 0$ and for (b) $\alpha = 11.44$ meVnm.

The distinct branches as observed without SO interaction is a result of mixing of the orbital states in the input channel by the SO interaction.

C. Channel width and SO coupling strength

The width of the input channel $W$ modifies the number of the eigenstates for a given $E_F$. In the experimental situation the QPCs usually separate two 2DEG semiplanes so $W$ is very large. In Fig. 6 we show the SGM maps for the QPC tuned to $G = 4 \frac{e^2}{h}$ for two widths of the input channel – $W = 180$ nm and $W = 400$ nm. We obtain results that are nearly identical with the ones presented above in Figs. 3(b,c) for $W = 240$ nm. Therefore, the discussed effects of smearing of the current flow as dis-
cussed above is not an effect specific to a particular width of the input channel.

Comparing the plots of the input channel above is not an effect specific to a particular width for (a) α

FIG. 7. (color online) Maps of the conductance changes ∆G for varied width W of the input channel for the QPC tuned to G = 4 e²/h.

FIG. 6. (color online) Maps of the conductance changes ∆G for varied width W = 180 nm W = 400 nm of the input channel for the QPC tuned to G = 4 e²/h.

For QPC tuned to the conductance step below the last plateaux G < 2 e²/h there are no quantitative changes between the results obtained without SO coupling Fig. S(a) and with SO interaction present Fig. S(d). On the other hand the difference is visible comparing the maps for the QPC tuned to the conductance step between G = 2 e²/h and G = 4 e²/h plotted in Figs. S(b,e) and for the QPC tuned to the conductance step between G = 4 e²/h and G = 6 e²/h displayed in Figs. S(c,f). Here, we observe that the ∆G maps bear the signature of mode mixing as there is nonzero flow present along the symmetry axis of the QPC in Fig. S(e) and five electron flow paths are present in Fig. S(f).

The interference fringes that are present in the maps are separated by the half of Fermi wavelength (l) for unconfined 2DEG. The energy reads

\[ E = \frac{\hbar^2 k_F^2}{2m^*}, \]  

then \( l = \lambda_F = \pi/k_F \) and we obtain

\[ l = \frac{\pi\hbar}{\sqrt{2m^*E_F}}, \]  

which for \( E_F = 3 \) meV gives \( l = 51.92 \) nm. In Fig. D with the black curve we show the cross section of the conductance corresponding to map of Fig. S(a) obtained for \( x = 0 \). We mark the oscillation period \( l \) with the solid black vertical lines.

In the presence of SO interaction the single parabola in the dispersion relation is split into two, each one corresponding to the opposite spin polarization. In the present case for the electron propagating along the y-direction in presence of the Rashba coupling the spin is polarized in the x-direction and the dispersion relation consists of two branches,

\[ E_- = \frac{\hbar^2 k^2}{2m^*} - \alpha k, \]

\[ E_+ = \frac{\hbar^2 k^2}{2m^*} + \alpha k, \]  

of the coherent transport. The oscillations are most pronounced at the conductance steps – where the checkerboard pattern – characteristic to G plateaux are replaced by radial features.

Figure S presents the maps of conductance changes ∆G for the QPC tuned to the conductance steps for \( \hbar\omega = 6.1 \) meV, \( \hbar\omega = 2 \) meV and \( \hbar\omega = 1.25 \) meV that are marked in Fig. E with \( \nabla, \bigtriangleup \) and \( \triangleleft \) respectively. The results of Fig. S(a) reproduce the radial fringes obtained in the experimental SGM maps of Ref. 20. We notice that the calculated conductance amplitude is two times larger as compared with the maps obtained at the G plateaux [see Fig. S]. Moreover, the tip now induces also an increase in conductance far from the QPC – the positive changes are denoted with the brown colors in the maps of Figs. S.

D. Interference fringes on conductance steps

The characteristic feature of the conductance maps obtained in the SGM experiments on QPC is that the interference fringes are separated by the half of the Fermi wavelength. These conductance oscillations appear due to interference between the wave function flowing from the constriction and backscattered from the tip. These oscillations in the experiments is treated as a signature of the Fermi wavelength in the dispersion relation is split into two, each one corresponding to the opposite spin polarization. In the present case for the electron propagating along the y-direction in presence of the Rashba coupling the spin is polarized in the x-direction and the dispersion relation consists of two branches,

\[ E_- = \frac{\hbar^2 k^2}{2m^*} - \alpha k, \]

\[ E_+ = \frac{\hbar^2 k^2}{2m^*} + \alpha k, \]  

for each parabola in the Fermi surface. The oscillations are most pronounced at the conductance steps – where the checkerboard pattern – characteristic to G plateaux are replaced by radial features.
FIG. 8. (color online) Maps of conductance changes obtained with the scanning gate tip potential for the QPC tuned to three conductance steps – the symbols correspond to the ones in Fig. 2. The upper (bottom) row corresponds to the results obtained without (with) SO interaction.

FIG. 9. (color online) Conductance as a function of scanning gate position along $x = 0$ axis obtained at the last step for $\hbar \omega = 6.1$ meV without (black curve) and with SO interaction (red curve). The vertical solid and dashed lines depict $l$ and $l_{SO}$ respectively [see text].

which gives four possible values of the wavevector:

$$k_1^+ = \pm \sqrt{\alpha^2 m^* + 2E_F m^* - \alpha m^*},$$
$$k_2^+ = \pm \sqrt{\alpha^2 m^* + 2E_F m^* + \alpha m^*}. \quad (18)$$

There are two positive wave vectors: $k_1^+ = \sqrt{\alpha^2 m^* + 2E_F m^* - \alpha m^*}$ and $k_2^+ = \sqrt{\alpha^2 m^* + 2E_F m^* + \alpha m^*}$. Therefore one might expect that the two close frequencies of the oscillations should disturb the interference fringes by forming a beating pattern. In Fig. 8 with the red curve we present the cross section of the conductance corresponding to the map of Fig. 8(d). Nevertheless, we find an oscillation with a single frequency as in the case of absent SO coupling. We find that the backscattering by the tip does not induce significant spin flips. As the electron is backscattered by the tip its wave vector changes not only sign but also the value. The fringes observed in the SGM map are due to formation of standing waves between the tip and the QPC. Since on one way the electron travels with wave vector $k_2^+$ and on the other with $k_1^-$ the entire phase shift on the back and forth travel can be accompanied to the average wave vector $k_a = (k_1^- + k_2^+)/2$. The conductance oscillations have period that corresponds to the value of $l_{SO} = \pi/k_a$, i.e. half of the wavelength of the average $k$. In the present case $l_{SO} = 51.58$ nm and we mark this period with the red vertical lines in Fig. 9. We conclude that in contrast to the changes of the angular flow the SO interaction does not significantly alter the interference fringes and the obtained oscillations have period that is similar to the one obtained in the absence of SO coupling.

IV. CONCLUSIONS

In summary we described the impact of the spin-orbit interaction on scanning gate microscopy of the electron flow from a QPC. We demonstrated that SO coupling induces mixing of the orbital states that enter the QPC and that the SGM maps gathered at the conductance plateaux for $G > 2e^2/h$ loose their distinct angular pattern. We found that this result is independent of the
input channel width and increases as the SO interaction becomes stronger. The maps measured at the conductance steps also bear signatures of the mode mixing but the distinct radial fringe pattern is not destroyed by the SO interaction despite the presence of two different Fermi wavelengths. Moreover we find that when SO interaction is present the fringes are separated by a length that corresponds to the mean value of the two Fermi wave vectors.

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1. R. Crook, C. G. Smith, A. C. Graham, I. Farrer, H. E. Beere, and D. A. Ritchie, Phys. Rev. Lett. 91, 246803 (2003).
2. A. M. Burke, R. Akis, T. E. Day, Gil Speyer, D. K. Ferry, and B. R. Bennett, Phys. Rev. Lett. 104, 176801 (2010).
3. F. Martins, B. Hackens, M. G. Pala, T. Ouisse, H. Sellier, X. Wallart, S. Bollaert, A. Cappy, J. Chevrier, V. Bayot, and S. Huant, Phys. Rev. Lett. 99, 136807 (2007).
4. M. G. Pala, B. Hackens, F. Martins, H. Sellier, V. Bayot, S. Huant, and T. Ouisse, Phys. Rev. B 77, 125310 (2008).
5. M. G. Pala, S. Baltazar, F. Martins, B. Hackens, H. Sellier, T. Ouisse, V. Bayot, and S. Huant, Nanotechnology 20, 264021 (2009).
6. K. E. Aida, R. E. Parrott, T. Kramer, E. J. Heller, R. M. Westervelt, M. P. Hanson, and A. C. Gossard, Nat. Phys. 3, 464 (2007).
7. M. G. Pala, S. Baltazar, P. Liu, H. Sellier, B. Hackens, F. Martins, V. Bayot, X. Wallart, L. Desplanque, and S. Huant, Phys. Rev. Lett. 108, 076802 (2012).
8. M. A. Topinka, B. J. LeRoy, S. E. J. Shaw, E. J. Heller, R. M. Westervelt, K. D. Maranowski, and A. C. Gossard, Science 289, 2323 (2000).
9. B. J. LeRoy, M. A. Topinka, R. M. Westervelt, K. D. Maranowski and A. C. Gossard, Appl. Phys. Lett. 80, 4431 (2002).
10. R. Crook, C. G. Smith, M. Y. Simmons, and D. A. Ritchie, Phys. Rev. B 62, 5174 (2000).
11. N. Aoki, C. R. Da, C. R. Akis, D. K. Ferry, and Y. Ochiai, Appl. Phys. Lett. 87, 223501 (2005).
12. A. Pioda, S. Kicin, D. Brunner, T. Ihn, M. Sigrist, K. Ensslin, M. Reinwald, and W. Wegscheider, Phys. Rev. B 75, 045433 (2007).
13. S. Schnez, C. Rössler, T. Ihn, K. Ensslin, C. Reichl, and W. Wegscheider, Phys. Rev. B 84, 195322 (2011).
14. A. A. Kozikov, D. Weinmann, C. Rössler, T. Ihn, K. Ensslin, C. Reichl, and W. Wegscheider, New J. Phys. 15, 083005, (2009).
15. A. A. Kozikov, C. Rössler, T. Ihn, K. Ensslin, C. Reichl, New J. Phys. 15, 013056 (2013).
16. A. Abbout, G. Lemarié, and J. L. Pichard, Phys. Rev. Lett. 106, 156810 (2011).
17. M. P. Jura, M. A. Topinka, L. Urban, A. Yazdani, H. Shtrikman, L. N. Pfeiffer, K. W. West, and D. Goldhaber-Gordon, Nat. Phys. 3, 841 (2007).
18. M. A. Topinka, B. J. LeRoy, R. M. Westervelt, S. E. J. Shaw, R. Fleischmann, E. J. Heller, K. D. Maranowski, and A. C. Gossard, Nature 410, 183 (2001).
19. B. J. LeRoy, A. C. Blozynski, K. E. Aida, R. M. Westervelt, A. Kalben, E. J. Heller, S. E. J. Shaw, K. D. Maranowski, and A. C. Gossard, Phys. Rev. Lett. 94, 126801 (2005).
20. M. P. Jura, M. A. Topinka, M. Grobis, L. N. Pfeiffer, K. W. West, and D. Goldhaber-Gordon, Phys. Rev. B 80, 041303(R) (2009).
21. M. P. Jura, M. Grobis, M. A. Topinka, L. N. Pfeiffer, K. W. West, and D. Goldhaber-Gordon, Phys. Rev. B 82, 155328 (2010).
22. N. Paradiso, S. Heun, S. Roddaro, L. N. Pfeiffer, K. W. West, L. Sorba, G. Biasiol, F. Beltram, Physica E, 42, 1038 (2010).
23. R. A. Jalabert, W. Szewe, S. Tomsovic, and D. Weimann, Phys. Rev. Lett. 105, 166802 (2010); C. Gorini, R. A. Jalabert, W. Szewe, S. Tomsovic, and D. Weimann, Phys. Rev. B 88, 035406 (2013).
24. K. C. Nowack, F. H. L. Koppens, Yu, V. Nazarov and L. M. K. Vandersypen, Science 318, 1430 (2007).
25. S. Nadj-Perge, S. M. Frolov, E. P. A. M. Bakkers and L. P. Kouwenhoven, Nature (London) 468, 1084 (2010).
26. Y. H. Park, H. Kim, J. Chang, S. H. Han, J. Eom, H.-J. Choi and H. C. Koo, Appl. Phys. Lett. 103, 252407 (2013).
27. M. Eto, T. Hayashi and Y. Kurotani, J. Phys. Soc. Jpn. 74, 1934 (2005).
28. V. A. Sablikov, Phys. Rev. B 82, 115301 (2010).
29. S. W. Kim, Y. Hashimoto, Y. Iye and S. Katsumoto, J. Phys. Soc. Jpn. 81, 054706 (2012).
30. M. P. Nowak, B. Szafran, Appl. Phys. Lett. 103, 202404 (2013).
31. N. Aoki, C. R. Cunha, T. Morimoto, R. Akis, D. K. Ferry, Y. Ochiai, AIP Conference Proceedings, 893, 715 (2007).
32. U. Zülicke, J. Bolte, and R. Winkler, New. J. Phys. 9, 355 (2007).
33. F. Martins, S. Faniel, B. Rosenow, H. Sellier, S. Huant, M. G. Pala, L. Desplanque, X. Wallart, V. Bayot and B. Hackens, Sci. Rep. 3, 1416 (2013).
34. Y. A. Bychkov and E. I. Rashba, J. Phys. C 17, 6039 (1984).
35. B. Szafran, Phys. Rev. B 84, 075336 (2011).
36. X. S. Li, J. W. Demmel, J. R. Gilbert, IL, Grigori, M. Shao and I. Yamazaki, Lawrence Berkeley National Laboratory, LBNL-44289, SuperLU Users’ Guide, September 1999; J. W. Demmel, S. C. Eisenstat, J. R. Gilbert, X. S. Li and J. W. H. Liu, SIAM J. Matrix Analysis and Applications, 3, 720 (1999).
37. E. A. de Andrada e Silva, G. C. La Rocca, and F. Bassani, Phys. Rev. B 55, 16293 (1997).