Anomalous Quark Chromomagnetic Moment and Dynamics of Elastic Scattering

Nikolai Kochelev\textsuperscript{1} and Nikolai Korchagin\textsuperscript{2}

\textit{Bogoliubov Laboratory of Theoretical Physics, Institute for Nuclear Research,}
\textit{Dubna, Moscow region, 141980 Russia}

\textbf{Abstract}

We estimate the contribution of nonperturbative quark-gluon chromomagnetic interaction to the high energy elastic proton-proton cross section at large momentum transfer. It is shown that this contribution is very large in the accessible kinematic region of the present experiments. We argue that Odderon which is the $P = C = -1$ partner of Pomeron, is governed by the spin-flip component related to the nonperturbative three-gluon exchange induced by the anomalous quark chromomagnetic moment. We discuss the possible spin effects in the elastic proton-proton and proton-antiproton scattering coming from the interference of spin-flip nonperturbative Odderon and nonspin-flip Pomeron exchanges.

\textsuperscript{1}kochelev@theor.jinr.ru
\textsuperscript{2}korchagin@theor.jinr.ru
1 Introduction

High energy elastic proton-proton and proton-antiproton cross sections reveal very complicated dynamics which is rather difficult to explain within the framework of Quantum Chromodynamics (QCD) (see the discussion in [1–9]). In a conventional approach at small transfer momentum experimental data can be described quite well by the diffractive scattering induced by Pomeron exchange between hadrons. At large $-t \gg 1 \text{ GeV}^2$ in the popular Donnachie-Landshoff (DL) model the dominant contribution comes from the exchange by Odderon which is the $P = C = -1$ partner of Pomeron. It was suggested that this effective exchange originated from the perturbative three gluon exchange in the proton-proton and proton-antiproton scattering [10]. The experimental support for the existence of such exchange comes from high energy ISR data on the difference in the dip structure around $|t| \approx 1.4 \text{ GeV}^2$ in the proton-proton and proton-antiproton differential cross sections at $\sqrt{s} = 53 \text{ GeV}$ [11]. However, there is no any signal for Odderon at very small transfer momentum. We would like to emphasize that one cannot expect the perturbative QCD DL approach to be valid even at the largest transfer momentum $-t \sim 14 \text{ GeV}^2$ accessible at ISR energies. This is related to the fact that in the three-gluon exchange model, which is applied to describe elastic cross sections in the interval $-t = 3 - 14 \text{ GeV}^2$, the average virtuality of exchanged gluons $\hat{t} \approx t/9$ is quite small $\hat{t} = 0.3 - 1.6 \text{ GeV}^2$. Therefore, in this kinematic region nonperturbative QCD effects should be taken into account.

The attempt to include some of the nonperturbative effects into the DL model was made in [12]. In that paper the strength of three-gluon exchange with perturbative quark-gluon vertices was considered as a free parameter and its value was found from the fit of the data. Therefore, a good description of the large $-t$ cross sections in the paper is not the result of calculation but rather of the fine tuning to experimental data.

One of the successful models of nonperturbative effects is the instanton liquid model for QCD vacuum [13,14]. Instantons describe nontrivial topological gluon field excitations in vacuum and their existence leads to the spontaneous chiral symmetry breaking in QCD. One of the manifestations of this phenomenon is the appearance of dynamical quark mass and nonperturbative helicity-flip quark-gluon interaction [14,15]. Such new interaction can be treated as a nonperturbative anomalous quark chromomagnetic moment (AQCM). It was shown that AQCM gives a very important contribution the to quark-quark scattering at large energies for both polarized and unpolarized cases [14,18]. One of the applications of these results is a new model for Pomeron based on AQCM and nonperturbative two gluon exchange between hadrons suggested in [14,17].

In this paper, we extend this model to the case of the three gluon colorless exchange between nucleons. It will be shown that a nonperturbative version of the Donnachie-Landshoff Odderon model based on AQCM describes well high energy data for the elastic proton-proton, proton-antiproton cross sections at large transfer momentum. The spin effects in elastic scattering are also under discussion.
2 Anomalous quark chromomagnetic moment and Odderon exchange

The interaction vertex of a massive quark with a gluon can be written in the following form:

\[ V_\mu(k_1^2, k_2^2, q^2)t^a = -g_s t^a \left[ \gamma_\mu F_1(k_1^2, k_2^2, q^2) - \frac{\sigma_{\mu\nu} q^\nu}{2M_q} F_2(k_1^2, k_2^2, q^2) \right], \tag{1} \]

where the form factors \( F_{1,2} \) describe nonlocality of the interaction, \( k_{1,2} \) is the momentum of incoming and outgoing quarks, respectively, \( q = k_1 - k_2 \), \( M_q \) is the quark mass, and \( \sigma_{\mu\nu} = (\gamma_\mu \gamma_\nu - \gamma_\nu \gamma_\mu)/2 \). Within the instanton model the shape of the form factor \( F_2(k_1^2, k_2^2, q^2) \) is

\[ F_2(k_1^2, k_2^2, q^2) = \mu_a \Phi_q(|k_1| \rho/2) \Phi_q(|k_2| \rho/2) F_g(|q| \rho), \tag{2} \]

where

\[ \Phi_q(z) = -z \frac{d}{dz}(I_0(z)K_0(z) - I_1(z)K_1(z)), \]
\[ F_g(z) = \frac{4}{z^2} - 2K_2(z) \tag{3} \]

are the Fourier-transformed quark zero-mode and instanton fields, respectively, \( I_\nu(z) \) and \( K_\nu(z) \) are the modified Bessel functions and \( \rho \) is the instanton size.

AQCM is defined by formula

\[ \mu_a = F_2(0, 0, 0). \tag{4} \]

For our estimation below we will use the value of AQCM \( \mu_a = -1 \) which is within the interval \(-\mu_a \sim 0.4 - 1.6\) given by the instanton model \[17\]. This value is also supported by hadron spectroscopy (see \[19\] and references therein). Recently, a similar value of AQCM was also obtained within the Dyson-Schwinger equation approach with nonperturbative quark and gluon propagators \[20\]. In Fig.1, the Donnachie-Landshoff perturbative QCD (pQCD) and nonperturbative AQCM-induced three gluon exchange between two nucleons are presented.

Figure 1: The left panel is the Donnachie-Landshoff mechanism for the large \(-t\) proton-proton scattering. The right panels are the example of the AQCM contribution induced by the second term in Eq.1.

Within the DL model the differential cross-section of the proton-proton and proton-antiproton scattering is given by the formula

\[ \frac{d\sigma}{dt} \approx \frac{244P^4}{s^{6}t^{2}R^{12}} |M_{qq}(\theta)|^6 \tag{5} \]
where $M_{qq}$ is the matrix element at the quark level, $\theta$ is the scattering angle in the center of mass, $P$ is the probability of the three quark configuration in a proton, and $R$ is the proton radius. In the pQCD DL approach at the quark level

$$\left| M_{qq}^{pQCD}(\theta) \right|^2 = \frac{128\pi^2\alpha_s^2 s^2}{9\hat{t}^2},$$

where $\hat{s} \approx s/9$, at $\hat{s} \gg -\hat{t}$, $\hat{t}/\hat{s} \sim -\sin^2\theta/4$, and the following values of the parameters were taken \textit{ad hoc}:

$$P = 1/10, \quad \alpha_s = 0.3, \quad R = 0.3 \text{ fm}.$$  \hfill (7)

We should emphasize that DL assumed a very small proton radius which is far away from the real proton size $R \approx 1$ fm. For more suitable values $P = 1$ and $R = 1$ fm, we got $d\sigma/dt \sim 8 \cdot 10^{-4}/t^8$ mb/GeV$^2$. It is about two orders of magnitude less than high energy data $d\sigma/dt \approx 9 \cdot 10^{-2}/t^8$ mb/GeV$^2$ at large $-t$, Fig.2. For the AQCM contribution at

![Graph showing the contribution of pQCD exchange (dashed line) and AQCM contribution (solid line) to the elastic proton-proton scattering at large energy and large momentum transfer in comparison with data [21].](image)

the quark level we have

$$\left| M_{qq}^{AQCM}(\hat{s}, \hat{t}) \right|^2 = \frac{16\pi^3}{3}\alpha_s(\hat{t}) \left| \mu_a \rho_c^2 F_9^2(\sqrt{\hat{t}} | \rho_c) \frac{s^2}{\hat{t}} \right| + \frac{\pi^2}{2}\mu_a^2 \rho_c^4 F_9^4(1 | \hat{t} | \rho_c) s^2$$  \hfill (8)
For estimation, we use $R = 1$ fm, $P = 1$ dynamical quark mass $M_q = 280$ MeV, average instanton size $\rho_c = 1/3$ fm and the strong coupling constant

$$\alpha_s(q^2) = \frac{4\pi}{9\ln((q^2 + m_g^2)/\Lambda_{QCD}^2)},$$

(9)

with $\Lambda_{QCD} = 0.280$ GeV and $m_g = 0.88$ GeV \[17\]. To get Eq.8 the approximation $F_1(k_1^2, k_2^2, q^2) \approx 1$ was used and we neglected nonzero virtuality of quarks in a proton. The final result for the AQCM contribution to the proton-proton and proton-antiproton cross section is presented by the solid line in Fig.2. We should mention that the AQCM contribution asymptotically decays as $1/t^{11}$ due to the form factor Eq.3. Therefore, asymptotically at very large transfer momentum perturbative $1/t^8$ should give the dominating contribution. However, in the kinematic region accessible at the present time in experiments $-t \leq 14$ GeV$^2$, the nonperturbative AQCM contribution describes the available large $-t$ data very well, Fig.2. Finally, some part of the difference between the struc-

\[\text{Figure 3: The interference between a) DL-type AQCM diagram and b) Pomeron spin-flip induced by AQCM.}\]

\[\text{ture of the dip around } -t \approx 1 - 2 \text{ GeV}^2 \text{ in the proton-proton and proton-antiproton elastic scattering at ISR energies might be related to the difference in the sign of the interference between the AQCM Odderon and Pomeron spin-flip amplitudes, Fig.3.}\]

\[\text{In our approach the spin-flip component, which is proportional to } t, \text{ gives the dominating contribution to the negative charge parity Odderon amplitude. In the region of small transfer momentum this contribution to the amplitude of the } PP \text{ and } P\bar{P} \text{ scattering has the dependence}\]

$$M \sim \frac{\sqrt{-t}}{(m_g^2 - t)^3},$$

(10)

due to quark spin-flip induced by AQCM. In Eq.10 $m_g \approx 0.4$ GeV is the dynamical gluon mass \[22\]. Therefore, the difference in the $PP$ and $P\bar{P}$ differential cross sections at small $-t$ and the difference in the total $PP$ and $P\bar{P}$ cross sections should be very small at high energies. This is in agreement with experimental data.

\[\text{Of course, one can describe } PP \text{ and } P\bar{P} \text{ large } -t > 3.5 \text{ GeV}^2 \text{ data by using the assumption about a specific } t \text{ dependence of the Pomeron trajectory (see, for example}\]

\[1\text{The value of the strong proton radius } R \approx 1 \text{ fm is related to the confinement scale. The probability of the three quark configuration in the proton } P = 1 \text{ is a natural assumption in our three quarks on three quarks scattering model for large } -t.\]
However, in anyway, it is necessary to introduce the additional $C = -1$ exchange with high intercept to describe the difference in the $PP$ and $\bar{P}P$ elastic cross sections at $\sqrt{s} = 53$ GeV. A natural candidate for such exchange is the nonperturbative three gluon DL-type exchange. We would like to mention that the sizable contribution from the conventional Pomeron exchange at large $-t > 3.5$ GeV$^2$ is not expected due to the huge suppression factor at large energies, $(s/s_0)^{2\alpha_P t}$, which comes from the nonzero slope of the Pomeron trajectory $\alpha_P \approx 0.25$ GeV$^{-2}$.

In the estimation above we assume, as in the DL model, that momenta of exchanged gluons are approximately equal. The justification of this assumption is quite clear. To keep a proton as a bound state of three quarks at large transfer momentum, all quarks in the proton should scatter approximately to the same angle. In fact, one can also consider more complicated multigluon contributions to elastic scattering, but we believe that such contribution will be suppressed by either additional factors $\alpha_s$ or by extra factors $1/t^n$ coming from gluon propagators and/or from form factors in the quark-gluon vertices.

### 3 Single-spin asymmetry $A_N$ in $PP$ and $\bar{P}P$ elastic scattering

One of the long-standing problems of QCD is the understanding of the large spin effects observed in the different high energy reactions \[1\], \[24\]. Recently, we have shown that the AQCM contribution leads to very large single-spin asymmetry (SSA) in the quark-quark scattering \[16\] and, therefore, it can be considered as a fundamental mechanism for explanation of anomalously large SSA observed in different inclusive and exclusive reactions at high energy. In elastic scattering, large SSA was found in the proton-proton scattering at AGS energies at large transfer momentum, Fig.4. In the bases of the c.m.

![Figure 4: Single-spin asymmetry in the elastic $PP \rightarrow PP$ scattering at large momentum transfer at AGS \[25\].](image)
helicity amplitudes SSA is given by the formula

\[ A_N = -\frac{2Im[\Phi_5^*(\Phi_1 + \Phi_2 + \Phi_3 - \Phi_4)]}{|\Phi_1|^2 + |\Phi_2|^2 + |\Phi_3|^2 + |\Phi_4|^2 + 4|\Phi_5|^2}, \quad (11) \]

where the helicity amplitudes \( \Phi_1 = \langle ++ | ++ \rangle \), \( \Phi_2 = \langle ++ | -- \rangle \), \( \Phi_3 = \langle +-- | -- \rangle \), \( \Phi_4 = \langle ++ | -- \rangle \), and \( \Phi_5 = \langle ++ | ++ \rangle \). It is evident that due to the negative charge parity Odderon contribution, the helicity-flip amplitude \( \Phi_5 \) should have a different sign for the proton-proton and proton-antiproton scattering. Therefore, SSA in the case of the elastic proton-antiproton scattering flips the sign in comparison with the proton-proton scattering. This prediction can be tested by the PAX Collaboration at HESR [26].

Due to the dominance of spin one \( t \)-channel gluon exchanges in the structure of Pomeron and Odderon, we can also expect that single-spin asymmetry at large \(-t\) should have a weak energy dependence. This prediction can be checked in the polarized proton-proton elastic scattering in the pp2pp experiment at RHIC in case of extending their kinematics to the large transfer momentum region [29]. However, the calculation of absolute value of SSA in the elastic \( PP \) and \( PP \) scattering at large transfer momenta is a very difficult task, because one needs to know spin-flip and non-spin flip components of both Odderon and Pomeron exchanges. Furthermore, in the region of small transfer momenta and low energies it is needed to include the effects of secondary Reggeon exchanges as well.

4 Conclusion

In summary, it is shown that the anomalous quark-gluon nonperturbative vertex gives a large contribution to the elastic proton-proton and proton-antiproton scattering at large momentum transfer. One can treat three-gluon exchange induced by this vertex as effective Odderon exchange with the spin-flip dominance in its amplitude. We should mention that the anomalous quark chromomagnetic moment is proportional to \( 1/\alpha_s \) [15]. Therefore, non-spin flip component in Odderon due to perturbative vertex should be suppressed by \( \alpha_s \) factor. We argue that a strong spin dependence of the Odderon amplitude might lead to the large spin effects in the proton-proton and proton-antiproton scattering at large momentum transfer.

Our approach is based on the existence of two quite different scales in hadron physics. One of them is related to the confinement radius \( R \approx 1 \) fm and it is consistent, as well, with an average distance between instanton and anti-instanton within the instanton liquid model, \( R_{II} \approx 1 \) fm [13, 14]. This scale is responsible for the diffractive type scattering at small momentum transfer. Another one is fixed by the scale of spontaneous symmetry breaking. Within the instanton model it is given by an average instanton size in QCD vacuum \( \rho_c \approx 1/3 \) fm. This scale leads to the appearance of a large dynamical quark mass and large anomalous quark chromomagnetic moment and is responsible for the dynamics of the hadron-hadron elastic scattering at large momentum transfer. We would like to mention that the two scale model for the hadron structure was discussed in different aspects in the papers [27, 28].

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