Cosmological Constraints on Invisible Decay of Dark Matter

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The cold dark matter may be in a meta-stable state and decays to other particles with a very long lifetime. If the decaying products of the dark matter are weakly interacting, e.g. neutrinos, then it would have little impact on astrophysical processes and is therefore difficult to observe. However, such a decay would affect the expansion history of the Universe because of the change of the equation of state. We utilize a high-quality type Ia supernovae (SN Ia) data set selected from several recent observations and the position of the first peak of the Cosmic Microwave Background (CMB) angular spectrum given by the WMAP three-year data to constrain the dark matter decay-to-neutrino rate \( \Gamma = \alpha \Gamma_\chi \), where \( \alpha \) is the fraction of the rest mass which gets converted to neutrinos, and \( \Gamma_\chi \) is the decay width. We find that \( \Gamma^{-1} > 0.7 \times 10^3 \) Gyr at 95.5% confidence level.

I. INTRODUCTION

The nature of dark matter is a great mystery of the Universe. Although it constitutes about a quarter of the total cosmic density, its many properties are still not known. Various kinds of dark matter candidates have been proposed, for instance, the weakly interacting massive particle (WIMP) is one of the most popular candidates \(^1\). However, the standard Cold Dark Matter (CDM) still have some problems at small scales, namely the halo cuspy profile problem and the missing satellite problem \(^2\), \(^3\), which motivated researchers to study other possibilities \(^4\).

Decaying cold dark matter model (DCDM) is one of such possibility. It has been suggested that DCDM may help resolve the small scale problems of the standard CDM \(^5\). Furthermore, from the particle physics perspective, it is also natural to think that the dark matter particle may be a meta-stable particle, which could decay into more stable particles. Such possibility could be realized if the dark matter is not the true Lightest Supersymmetric Particle (LSP) \(^6\), or if the R-parity of the LSP is not exactly conserved \(^7\).

If the decaying products are photons, electrons, quarks or other particles which are involved in the strong or electromagnetic interactions, then it is relatively easy to be observed. Stringent limit on radiative particle decay have been derived, e.g., from the reionization history of the Universe \(^8\), \(^9\), \(^10\). However, if the decaying products include only weakly interacting particles, e.g. neutrinos or other WIMPs, then it is more difficult to obtain such constraint.

We consider a generic unstable cold dark matter particle which decays invisibly, producing only weakly interacting particles. Within the standard model of particle physics, the only such weakly interacting particle is neutrino. The unstable decaying particle may also produce another weakly interacting particle which is beyond the standard model. For example, if the sneutrino is a meta-stable particle, it may decay to a neutralino and a neutrino, conserving both lepton number and R-parity.

Since neutrinos have very small masses, those produced in such decays will be highly relativistic. We assume here that the recoil on the heavier decaying product is small so that it could still be treated as part of CDM. As the non-relativistic DCDM decays to relativistic neutrinos, the equation of state changes from 0 to 1/3, and this will affect the expansion rate \( H \). Observational constraint on relativistic component were discussed previously, see e.g. Ref. \(^11\), \(^12\), \(^13\).

Here, we constrain the invisible decay of dark matter using two observations which probe the cosmic expansion rate. The observations we use include (1) type Ia supernovae (SN Ia) data selected from several recent observations (involving Gold06 \(^14\), SNLS \(^15\) and ESSENCE \(^16\)); (2) the position of the first peak of the Cosmic Microwave Background (CMB) angular spectrum \( \ell_1 \) given by the three-year WMAP observations \(^17\), \(^18\). These two observations cover a large redshift range \( (z_{\text{SN}} \rightarrow 1.7 \) and \( z_{\text{CMB}} \rightarrow 1048 \) and could give a tight and reliable constraint on the DCDM. Throughout the paper, we assume the dark energy is a cosmological constant and the geometry of the Universe is flat.

II. CONSTRAINTS

The evolution equations for the energy density of DCDM particle \( \rho_\chi \) and its decay products \( \rho_\nu \) are given by

\[
\dot{\rho}_\chi + 3H \rho_\chi = -\Gamma \rho_\chi ,
\]

\[
\dot{\rho}_\nu + 4H \rho_\nu = \Gamma \rho_\chi ,
\]

where \( \Gamma = \alpha \Gamma_\chi \), \( \Gamma_\chi \) is the decay width of the particle, \( 0 < \alpha < 1 \) is the fraction of the rest mass of the dark matter particle which goes to relativistic neutrinos during each decay. The Hubble expansion rate \( H \) is given by

\[
H^2 = \frac{8\pi G}{3} \left( \rho_\chi + \rho_b + \rho_\nu + \rho_\Lambda \right) ,
\]

here \( \rho_b \) is the baryon energy density which evolves as \((1+z)^3\), \( \rho_\nu \) is the relativistic particle energy density, which in the absence of decaying contribution would
evolves as \((1 + z)^4\), and \(\rho_{\Lambda}\) is the energy density of cosmological constant. Substitute \(- \frac{d}{dz} \frac{H}{H(z)} = \frac{H(z)}{H_{0}} = 1 - \frac{\delta}{a}\) into Eq. \(23\), the redshift evolution of the energy density for different components can be solved numerically. We denote \(\Omega_{\gamma} = \rho_{\gamma}/\rho_{0}\), where \(\rho_{0}\) is the critical density at \(z = 0\), and take the radiation background (photon and neutrinos produced in the early universe) in the absence of decaying contribution as \(\Omega_{\gamma} = 4.183 \times 10^{-5} h_{0}^{-2}\) where \(h_{0} = H_{0}/100 \text{ km s}^{-1} \text{ Mpc}^{-1}\) is the reduced Hubble constant. The baryon density is given by the CMB \([17, 18]\) and big bang nucleosynthesis measurements \([19]\), though the actual value does not significantly affect our result. The cosmological model is then described by the parameter set

\[
\theta = (\Omega_{\Lambda}, \Omega_{\gamma}, \Gamma, h_{0}).
\]

The type Ia supernovae can be used as “standard candles”. We have selected a sample of 182 high quality SN Ia data from several recent observations, including 30 HST supernovae, 47 SNLS supernovae from the Gold06 data set, 60 ESSENCE supernovae, and 45 nearby supernovae from WVN7 \([20]\). The MLCS2K2 light-curve fitter \([21, 22]\) is used to process this data set. The \(\chi^{2}\) for SN Ia data is

\[
\chi^{2}_{\text{SN Ia}}(\theta) = \sum_{i=1}^{N} \frac{(\bar{\mu}_{\text{obs}}(z_{i}) - \mu_{\text{th}}(z_{i}))^{2}}{\sigma_{i}^{2}},
\]

where \(\bar{\mu}_{\text{obs}}\) and \(\sigma_{i}\) are the observational modulus and its error, and \(\mu_{\text{th}}\) is the theoretical modulus given by

\[
\mu_{\text{th}}(z) = 5 \log_{10} d_{L}(z) + 25,
\]

where the luminosity distance can be computed as

\[
d_{L}(z; \theta) = (1 + z) \int_{0}^{z} \frac{c dz'}{H(z')}.
\]

In order to constrain the evolution at high redshift, we also use the position of the first peak of the CMB angular power spectrum \(\ell_{1}\) as measured by the WMAP three-year data. An alternative way of using CMB to constrain cosmic expansion history without invoking a full CMB calculation is to use the so-called shift parameter \(R\) \([23]\), which is a geometric test which assumes that the size of the sound horizon at recombination is fixed \([24, 25]\). However, the shift parameter was derived for assuming constant CDM density, which might not apply in our case \([26]\).

The angular scale of the sound horizon at last scattering, \(\ell_{A} = \pi c r(z_{ls})/r_{s}(z_{ls})\), is the ratio of the comoving angular diameter distance of the last scattering surface \(r(z_{ls})\) to the comoving size of the acoustic horizon \(r_{s}(z_{ls})\) at decoupling, which denotes both the size of sound horizon and the geometrical property of the Universe. Relating to the cosmic expansion rate, this acoustic scale can be written as \([27]\)

\[
\ell_{A} = \frac{\pi c}{h_{0}} \int_{0}^{z_{ls}} c dz' \frac{dz}{H(z')} \sqrt{\frac{H(z)}{H_{0} a}} \ell_{A}.
\]

where \(z_{ls}\) is the redshift of last scattering, \(a\) is the scale factor and

\[
c_{s} = c \left( 3 + \frac{9 \Omega_{\Theta} \Omega_{\Lambda}}{4 \Omega_{\Theta}} \right)^{-1/2}
\]

is the sound speed \([23]\). Here, \(\Omega_{\mu} = \Omega_{\Theta}/1.681\) is the energy density of photon today. The value of \(\ell_{A}\) is related to \(\ell_{1}\) \([27]\), the first peak in the CMB angular power spectrum. Following Hu et al. \([28]\), we use the relation

\[
\ell_{1} = \ell_{A}(1 - \phi_{1}),
\]

where \(\phi_{1}\) is a phase factor, and if we set the spectral index \(n = 1\) and \(\Omega_{\Theta}h_{0}^{2} = 0.02\), then it is given by the fitting formula:

\[
\phi_{1} = 0.267 \left( \frac{r_{s}}{0.3} \right)^{0.1}.
\]

Here \(r_{s} \equiv \rho_{\gamma}(z_{ls})/\rho_{m}(z_{ls})\) and \(\rho_{m}(z_{ls}) = \rho_{s}(z_{ls}) + \rho_{b}(z_{ls})\). This fitting formula is consistent with the other forms \([29]\), and is a good approximation for our model, especially for the long lifetime DCDM.

According to the measurement of the three-year WMAP data \([17, 18]\), we have \(\ell_{1} = 220.7 \pm 0.7\) and finally

\[
\chi_{1}^{2} = \frac{(\ell_{1} - 220.7)^{2}}{0.7^{2}}.
\]

For the combined SN Ia and CMB data set, the \(\chi^{2}\) is given by

\[
\chi^{2} = \chi_{\text{SN Ia}}^{2} + \chi_{1}^{2}.
\]

We employ the Markov Chain Monte Carlo (MCMC) technique to simulate the probability distributions of the parameters in the model. The following priors on the parameters are adopted: \(\Omega_{\Theta} \in (0, 1), \Omega_{\mu} \in (0, 1), \log_{10} H_{0} (\text{km s}^{-1} \text{ Mpc}^{-1}) \in (-7.5, 6.5)\), and \(h_{0} \in (0.2, 0.9)\). We also assume that the energy density of each component is positive, \(\Omega_{\Theta} + \Omega_{\mu} + \Omega_{\Lambda} + \Omega_{b} = 1\). We generate six MCMC chains, and each chain contains about one hundred thousand points after convergence is reached. Finally, these chains are thinned to produce about ten thousand points in the parameter space. The details of our MCMC code can be found in our earlier paper \([30]\).

The marginalized one-dimensional probability distribution function (PDF) for the the decay rate \(\Gamma\) is shown in Fig. 1 and the contour map of \(\Omega_{\Theta}\) vs. \(\Gamma\) is plotted in Fig. 2. We find that the data favors a cold dark matter density around \(\Omega_{\Theta} = 0.186\), and it is consistent with a stable (not decaying) dark matter. The limit on the decay width \(\Gamma\) are

\[
\Gamma < 0.8 \times 10^{-20} \text{ s}^{-1}, 68.3\text{ % C.L.}
\]

and

\[
\Gamma < 4.5 \times 10^{-29} \text{ s}^{-1}, 95.5\text{ % C.L.}
\]

respectively. Thus, the decay-to-neutrino rate of the DCDM must satisfy \(\Gamma^{-1} > 0.7 \times 10^{3}\) Gyr at 95.5 % C.L. This is a much tighter constraint than previous ones \([31]\).
We considered a generic decaying cold dark matter (DCDM) model which decays invisibly, producing relativistic neutrinos in the process. As the produced neutrinos has an equation of state of $1/3$ whereas the original cold dark matter has an equation of state of (almost) 0, the cosmic expansion rate is affected. We constrain the decay-to-neutrino rate $\Gamma$ by analyzing the expansion history of the Universe. Using a high-quality SN Ia data set selected from several recent observations (including Gold06, SNLS and ESSENCE) at low and intermediate redshift, and the position of the first peak of the CMB angular spectrum given by the WMAP three-year data at high redshift, we obtain a model-independent constraint on the decay width $\Gamma$ by employing the MCMC technique. We find $\Gamma < 4.5 \times 10^{-20} \text{s}^{-1}$ at 95.5% C.L. i.e., $\Gamma^{-1} > 0.7 \times 10^5 \text{Gyr}$. One could put further constraint on the decay life time of the particle if $\alpha$ is known. For example, if $\alpha = 0.05$, then the decay life time $\Gamma^{-1} > 14 \text{Gyr}$, i.e. greater than the current age of the Universe.

Recently the terrestrial atmospheric neutrino spectra observed by several neutrino detectors such as Frejus [32], AMANDA [33, 34, 35] and Super-Kamiokande [36] has been used to constrain the annihilation [37, 38, 39] or decay [40] properties of CDM. It is assumed that the annihilations or decays could produce very energetic neutrinos, which would produce a peak in the observed atmospheric neutrino background. These studies generally yields tighter constraint on the decay width of the dark matter than obtained here. For example, for $E_\nu \sim 1-10\text{GeV}$, the atmospheric neutrinos yields a constraint which is several order of magnitudes stronger than ours. However, there are certain limitations on the applicability of such method. If the neutrinos produced in the decay is of relatively low energy, below the threshold energy of the detector, then they would not be detected in atmospheric neutrino experiments. Also, if the decay process involve multiple particles and produce neutrinos with a spread of energy distribution, then one may not always be able to identify a sharp peak in the atmospheric neutrino experiment data, although in some analyses, e.g. that of Ref. [40], this effect is not obvious due to the large bin size used. Our method is not affected by these problems and is therefore a more conservative limit which complements their studies.

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