Large electroweak penguin contribution in $B \to K\pi$ and $\pi\pi$ decay modes

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Abstract

We discuss about a possibility of large electroweak penguin contribution in $B \to K\pi$ and $\pi\pi$ from recent experimental data. The experimental data may be suggesting that there are some discrepancies between the data and theoretical estimation in the branching ratios of them. In $B \to K\pi$ decays, to explain it, a large electroweak penguin contribution and large strong phase differences seem to be needed. The contributions should appear also in $B \to \pi\pi$. We show, as an example, a solution to solve the discrepancies in both $B \to K\pi$ and $B \to \pi\pi$. However the magnitude of the parameters and the strong phase estimated from experimental data are quite large compared with the theoretical estimations. It may be suggesting some new physics effects are including in these processes. We will have to discuss about the dependence of the new physics. To explain both modes at once, we may need large electroweak penguin contribution with new weak phases and some SU(3) breaking effects by new physics in both QCD and electroweak penguin type processes.

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1 Introduction

One of the main targets at the $B$ factories is to determine the all CP angles in the unitarity triangles of the Cabbibo-Kobayashi-Maskawa (CKM) matrix \[1\]. $\phi_1$ \[2\] as one of the angles has already been measured and established the CP violation in the $B$ meson system by Belle \[3\] and BaBar \[4\] collaborations. The next step is to determine the remaining angles and to confirm the unitarity of the CKM matrix. The good decay modes for measuring $\phi_2$ and $\phi_3$ are $B^0 \to \pi^+\pi^-$ and $B^\pm \to DK^\pm$ respectively but their methods have some difficulties to extract cleanly the angles. To avoid the difficulties, isospin relation \[5\] and SU(3) relation including $B \to K\pi$ modes \[6–15\] are being considered as a method to extract the weak phases. However there seems to be anomalies in the recent experimental data of $B \to K\pi$, $\pi\pi$ \[16–18\]. To explain the discrepancies in $B \to K\pi$ modes, a large electroweak (EW) penguin contribution will be requested \[19\]^3 \[20–22\]. In addition, the magnitude of the branching ratios for $B \to \pi\pi$ do not also agree with the theoretical estimations \[21,22\]. In other words, without solving the problems, we can not extract any informations from these $B$ decays so clear.

$B \to K\pi$ modes have already been measured \[16–18\] (and see also the web page by Heavy Flavor Averaging Group \[23\]) and they will be useful informations to understand the CP violation through the Kobayashi-Maskawa (KM) phases. If we can directly solve these modes, it is very elegant way to determine the parameters and the weak phases. To extract the weak phases through this mode, there are several approaches: diagram decomposition \[6–12, 15\], QCD factorization \[24\], PQCD \[25, 26\] and so on. The contributions including the weak phase $\phi_3$ come from tree type diagrams which have a CKM suppression factor and they are usually dealt with small parameter compared with gluonic penguin diagram. If we can deal the contributions except for gluonic penguin with the small parameters, then, there are several relations among the averaged branching ratios of $B \to K\pi$ modes: $Br(K^+\pi^-)/2Br(K^0\pi^0) \approx 2Br(K^+\pi^0)/Br(K^0\pi^+) \[19,24\]$ et al. However, the recent data do not seem to satisfy them so well. To satisfy the data, we find that the role of a color-favored EW penguin may be important. The color favored EW penguin diagram is included in $B^0 \to K^0\pi^0$ and $B^0 \to K^+\pi^0$ and the data of their branching ratio are slightly larger than half of that for $B^0 \to K^+\pi^-$, where the $1/2$ comes from the difference between $\pi^0$ and $\pi^+$ in final state. Thus we need to know the EW penguin contributions in $B \to K\pi$ decay modes to extract the weak phases. The role of the EW penguin was pointed out and their magnitude was estimated in several works \[11–15, 24–26\]. They said that the ratio between gluonic and EW penguins is about 0.14 as the central value, but the experimental data may suggest that the magnitude seems to be slightly larger than the estimation \[19–22\]. Furthermore, one of the most difficult points to explain them is that we will need quite large strong phase difference of EW penguin diagram compared with the other strong phases. To produce the such large strong phase is difficult in the SM. If there is quite large deviation in the contribution from the EW penguin, it may suggest a possibility of new physics in these modes.

Under the flavor SU(3) symmetry, these discussions in $B \to K\pi$ have to relate to $B \to \pi\pi$ modes, too. If the contributions from EW penguin are so large, they have to appear also

\[3\]In the previous work \[19\], one of authors showed several relations for $B \to K\pi$. Some of these relations was not including a cross term. In this work, it is corrected but the discussion is almost same and it do not make any large changes.
in $B \to \pi\pi$. However it is slightly difficult to explain the discrepancies between theoretical estimations and experimental data of the branching ratios by only the EW penguin contribution, because in these modes the leading contribution comes from tree type diagram and the EW penguin is the sub-leading contribution. It seems to need other contributions such as the SU(3) breaking effects. Perhaps, they are induced from some new physics effects. In the usual sense, new physics contributions should be through in some loop effects such as the penguin-type diagram so that there should not be any discrepancies in tree-type diagrams. We put the new physics contributions with weak phase into both gluonic and EW penguins to find the allowed regions for each parameters.

This paper is organized as follows. In Section 2, we review the diagram decomposition approach in $B \to K\pi$ and $\pi\pi$. In Section 3, the large EW penguin contribution in $B \to K\pi$ decay modes is discussed. We find that we do not only need the large magnitude but also the large strong phase differences. In Section 4 and 5, the SU(3) breaking effect and the large EW penguin contributions in $B \to \pi\pi$ is discussed. And Section 6 shows what are needed to explain both modes at once if the new physics contributions were exiting in these modes. Section 7 summarizes our discussions.

2 $B \to K\pi$ and $\pi\pi$ under flavor SU(3) symmetry

Using the diagram decomposition approach [6–12, 15], the decay amplitudes of $B \to K\pi$ and $\pi\pi$ are

$$A_{K}^{0+} \equiv A(B^+ \to K^0\pi^+) = \left[ AV_{ub}^* V_{us} + \sum_{i=u,c,t} (P_i + EP_i - \frac{1}{3} P_{EW_i}^C + \frac{2}{3} EP_{EW_i}^C) V_{ib}^* V_{is} \right],$$

(1)

$$A_{K}^{00} \equiv A(B^0 \to K^0\pi^0) = -\frac{1}{\sqrt{2}} \left[ CV_{ub}^* V_{us} - \sum_{i=u,c,t} (P_i + EP_i - P_{EW_i} - \frac{1}{3} P_{EW_i}^C + \frac{1}{3} EP_{EW_i}^C) V_{ib}^* V_{is} \right],$$

(2)

$$A_{K}^{+-} \equiv A(B^0 \to K^+\pi^-) = - \left[ TV_{ub}^* V_{us} + \sum_{i=u,c,t} (P_i + EP_i + \frac{2}{3} P_{EW_i}^C - \frac{1}{3} EP_{EW_i}^C) V_{ib}^* V_{is} \right],$$

(3)

$$A_{K}^{+0} \equiv A(B^+ \to K^+\pi^0) = -\frac{1}{\sqrt{2}} \left[ (T + C + A)V_{ub}^* V_{us} \right.$$

$$+ \sum_{i=u,c,t} (P_i + EP_i + P_{EW_i} + \frac{2}{3} P_{EW_i}^C + \frac{2}{3} EP_{EW_i}^C) V_{ib}^* V_{is} \right],$$

(4)
where $T$ is a color-favored tree amplitude, $C$ is a color-suppressed tree, $A(E)$ is an annihilation (exchange), $P_i$ ($i = u, c, t$) is a gluonic penguin, $EP_i$ is a penguin exchange, $P_{EWi}$ is a color-favored EW penguin, $P_{EWi}^C$ is a color-suppressed EW penguin and $EP_{EWi}^C$ is a color-suppressed EW penguin exchange. In the following study, for simplicity, we neglect the $u$- and $c$- EW penguins because of their smallness, Here we redefine the each terms as follows:

$$T + P_u + EP_u - P_c - EP_c \rightarrow T,$$

$$C - P_u - EP_u + P_c + EP_c \rightarrow C,$$

$$A + P_u + EP_u - P_c - EP_c \rightarrow A,$$

$$E \rightarrow E,$$

$$P_t + EP_t - P_c - EP_c - \frac{1}{3} P_{EW}^C + \frac{2}{3} EP_{EW}^C \rightarrow P,$$

$$P_{EW} + EP_{EW}^C \rightarrow P_{EW},$$

$$P_{EW}^C - EP_{EW}^C \rightarrow P_{EW}^C.$$

One can reduce the number of complex parameters up to 7. By using the unitarity relation of the CKM matrix, the amplitudes are written as

$$A_{K}^{0+} = [PV_{tb}^*V_{ts} + AV_{ub}^*V_{us}],$$

$$A_{K}^{00} = \frac{1}{\sqrt{2}} [(P - P_{EW})V_{tb}^*V_{ts} - CV_{ub}^*V_{us}],$$

$$A_{K}^{+-} = -[(P + P_{EW}^C)V_{tb}^*V_{ts} + TV_{ub}^*V_{us}],$$

$$A_{K}^{+0} = -\frac{1}{\sqrt{2}} [(P + P_{EW} + P_{EW}^C)V_{tb}^*V_{ts} + (T + C + A)V_{ub}^*V_{us}],$$

$$A_{\pi}^{00} = \frac{1}{\sqrt{2}} [(P - P_{EW})V_{tb}^*V_{td} - (C - E)V_{ub}^*V_{ud}],$$

$$A_{\pi}^{+-} = -[(P + P_{EW}^C)V_{tb}^*V_{td} + (T + E)V_{ub}^*V_{ud}],$$

$$A_{\pi}^{+0} = [PV_{tb}^*V_{td} + AV_{ub}^*V_{ud}],$$

$$A_{\pi}^{0+} = \frac{1}{\sqrt{2}} [(P - P_{EW})V_{tb}^*V_{td} - (C - E)V_{ub}^*V_{ud}],$$

$$A_{\pi}^{00} = -\frac{1}{\sqrt{2}} [(P + P_{EW} + P_{EW}^C)V_{tb}^*V_{td} + (T + C + A)V_{ub}^*V_{ud}],$$

$$A_{\pi}^{+-} = -(P + P_{EW}^C)V_{tb}^*V_{td} + (T + E)V_{ub}^*V_{ud}],$$

$$A_{\pi}^{+0} = [PV_{tb}^*V_{td} + AV_{ub}^*V_{ud}],$$
\[ A^{+0}_{\pi} = -\frac{1}{\sqrt{2}} \left[ (P_{EW} + P_{EW}^C)V_{tb}^{*}V_{td} + (T + C)V_{ub}^{*}V_{ud} \right]. \] (21)

By this diagram decomposition [7], one can easily find the isospin relation among the amplitudes,

\[ \sqrt{2}A^{+0}_{K} + A^{0}_{K}^{-} = \sqrt{2}A^{00}_{K} + A^{--}_{K}, \] (22)
\[ \sqrt{2}A^{+0}_{\pi} = \sqrt{2}A^{00}_{\pi} + A^{--}_{\pi}. \] (23)

The largest contribution in \( B \to K\pi \) is the gluonic penguin and that in \( B \to \pi\pi \) is the color-favored tree so that by factoring out them the amplitudes are rewritten as follows:

\[ A^{0}_{K}^{+} = -P|V_{tb}^{*}V_{ts}| \left[ 1 - r_A e^{\delta_A} e^{i\phi_3} \right], \] (24)
\[ A^{00}_{K} = -\frac{1}{\sqrt{2}} P|V_{tb}^{*}V_{ts}| \left[ 1 - r_{EW} e^{\delta_{EW}} + r_{CE}^{i\delta_{EC}} e^{i\phi_3} \right], \] (25)
\[ A^{-+}_{K} = P|V_{tb}^{*}V_{ts}| \left[ 1 + r_{CE} e^{i\delta_{EC}} - r_{TE} e^{i\phi_3} \right], \] (26)
\[ A^{00}_{K}^{+} = \frac{1}{\sqrt{2}} P|V_{tb}^{*}V_{ts}| \left[ 1 + r_{EW} e^{i\delta_{EW}} + r_{CE} e^{i\delta_{EC}} - (r_{TE} e^{i\phi_3} + r_{CE} e^{i\delta_{EC}} + r_{TE} e^{i\phi_3}) \right], \] (27)

\[ A^{00}_{\pi} = \frac{1}{\sqrt{2}} T|V_{ub}^{*}V_{ud}| \left[ (\tilde{r}_{P} e^{-i\delta_{P}} - \tilde{r}_{EW} e^{(\delta_{EW} - \delta_{T})}) e^{-i\phi_1} - (\tilde{r}_{CE} e^{(\delta_{EC} - \delta_{T})} - \tilde{r}_{TE} e^{(\delta_{TE} - \delta_{T})}) e^{i\phi_3} \right], \] (28)
\[ A^{+-}_{\pi} = -T|V_{ub}^{*}V_{ud}| \left[ (\tilde{r}_{P} e^{-i\delta_{P}} + \tilde{r}_{CE} e^{i(\delta_{EC} - \delta_{T})}) e^{-i\phi_1} + (1 - \tilde{r}_{TE} e^{i(\delta_{TE} - \delta_{T})}) e^{i\phi_3} \right], \] (29)
\[ A^{++}_{\pi} = -\frac{1}{\sqrt{2}} T|V_{ub}^{*}V_{ud}| \left[ (\tilde{r}_{EW} e^{i(\delta_{EW} - \delta_{T})} + \tilde{r}_{CE} e^{i(\delta_{EC} - \delta_{T})}) e^{-i\phi_1} + (1 - \tilde{r}_{CE} e^{i(\delta_{EC} - \delta_{T})}) e^{i\phi_3} \right], \] (30)

where \( \phi_1 \) and \( \phi_3 \) are the weak phases in \( V_{tb}^{*}V_{td} \) and \( V_{ub}^{*}V_{us} \) respectively, \( \delta^X \)'s are the strong phase differences between each diagram and gluonic penguin, and

\[ r_A = \frac{|AV_{ub}^{*}V_{us}|}{|PV_{tb}^{*}V_{ts}|}, \quad r_T = \frac{|TV_{ub}^{*}V_{us}|}{|PV_{tb}^{*}V_{ts}|}, \quad r_C = \frac{|CV_{ub}^{*}V_{us}|}{|PV_{tb}^{*}V_{ts}|}, \] (31)
\[ r_{EW} = \frac{|P_{EW}|}{|P|}, \quad \tilde{r}_{CE} = \frac{|P_{EW}|}{|P|}, \] (32)

\[ \tilde{r}_{C} = \frac{|C|}{|T|} = \frac{r_C}{r_T}, \quad \tilde{r}_{E} = \frac{|E|}{|T|}, \] (33)
\[ \tilde{r}_{P} = \frac{|PV_{tb}^{*}V_{td}|}{|TV_{ub}^{*}V_{ud}|} = \frac{1}{r_T} \frac{|V_{td}V_{us}|}{|V_{ud}V_{ts}|}, \] (34)
\[ \tilde{r}_{EW} = \frac{|P_{EW}V_{tb}^{*}V_{td}|}{|TV_{ub}^{*}V_{ud}|} = r_{EW}\tilde{r}_P, \quad \tilde{r}_{CE} = \frac{|P_{EW}V_{tb}^{*}V_{td}|}{|TV_{ub}^{*}V_{ud}|} = r_{CE}\tilde{r}_P. \] (35)

We assume the hierarchy of the ratios as \( 1 > r_T, r_{EW} > r_C, r_{EW}^{C} > r_A \) and \( 1 > r_P > r_{EW}, \tilde{r}_C > \tilde{r}_{EW}, \tilde{r}_E \) [7]. \( r_T \) can be estimated from the the ratio of \( Br(B^+ \to \pi^0\pi^+) \) to \( Br(B^+ \to \)
$K^{0,\pi^+}$ [27–31], which are almost pure gluonic penguin and pure tree process, respectively. Under the naive factorization approach, the ratio should be proportional to $r_T^2$:

$$
\frac{2Br(B \to \pi^0\pi^+)}{Br(B \to K^0\pi^+)} = \left| \frac{T}{P} \right|^2 \frac{f_\pi^2 |V_{ub}V_{ud}|^2}{f_\pi^2 |V_{cb}^*V_{ts}|^2} \sim r_T^2 \frac{f_\pi^2}{\lambda^2 f_K^2},
$$

(36)

where the difference between the tree diagrams in $B \to \pi\pi$ and $B \to K\pi$ decays comes from the decay constants, $T_\pi \approx T_K f_K$, and $\lambda$ is the Cabbibo angle, so that $r_T \sim 0.2$ with 10% error from recent experimental data. $r_C$ and $r_{EW}^C$ must be suppressed by color factor from $r_T$ and $r_{EW}$. Comparing the Wilson coefficients, which correspond to the diagrams under the factorization method, we can assume that $r_C \sim 0.1 r_T$ and $r_{EW}^C \sim 0.1 r_{EW}$ [24,27]. Here we do not put any assumption for the magnitude of $r_{EW}$. $r_A$ could be negligible because it should have $B$ meson decay constant and it works as a suppression factor $f_B/M_B$. While, by the similar way one obtains $\tilde{r}_P \sim 0.3$, $\tilde{r}_C = 0.1$. Indeed, the estimations for each parameters in the PQCD approach are

$$
r_T = 0.21, \quad r_{EW} = 0.14 : O(0.1)
$$

(37)

$$
r_C = 0.018, \quad r_{EW}^C = 0.012, \quad r_A = 0.0048 : O(0.01)
$$

(38)

According to this assumption, we neglect the $r^2$ terms including $r_C, r_A$ and $r_{EW}^C$ in $B \to K\pi$. In $B \to \pi\pi$ we will neglect $\tilde{r}_{EW}^C$ and $\tilde{r}_E$ for simplicity, but keep $\tilde{r}_{EW}$ to discuss its magnitude in the both modes. Consequently, the averaged branching ratios are

$$
\begin{align*}
B_K^{0+} & \propto \frac{1}{2} \left[ |A^{0+}|^2 + |A^{0-}|^2 \right] \\
& = |P|^2 |V_{ts}^*V_{ts}|^2 \left[ 1 - 2r_A \cos \delta^4 \cos \phi_3 \right],
\end{align*}
$$

(39)

$$
\begin{align*}
2\tilde{B}_K^{00} & \propto \frac{1}{2} \left[ |A^{00}|^2 + |A^{00}|^2 \right] \\
& = |P|^2 |V_{ts}^*V_{ts}|^2 \left[ 1 + r_{EW}^2 - 2r_{EW} \cos \delta_{EW} + 2r_C \cos \delta^C \cos \phi_3 \right],
\end{align*}
$$

(40)

$$
\begin{align*}
\tilde{B}_K^{+0} & \propto \frac{1}{2} \left[ |A^{-0}|^2 + |A^{+0}|^2 \right] \\
& = |P|^2 |V_{ts}^*V_{ts}|^2 \left[ 1 + r_T^2 + 2r_{EW}^{C} \cos \delta_{EW} - 2r_T \cos \delta^T \cos \phi_3 \right],
\end{align*}
$$

(41)

$$
\begin{align*}
2\tilde{B}_K^{-0} & \propto \frac{1}{2} \left[ |A^{+0}|^2 + |A^{-0}|^2 \right] \\
& = |P|^2 |V_{ts}^*V_{ts}|^2 \left[ 1 + 2r_{EW}^{C} \cos \delta_{EW} + 2r_{EW}^{C} \cos \delta_{EW} - (2r_T \cos \delta^T + 2r_C \cos \delta^C + 2r_A \cos \delta^A) \cos \phi_3 \\
& - 2r_{EW} r_T \cos (\delta_{EW} - \delta^T) \cos \phi_3 \right],
\end{align*}
$$

(42)

$$
\begin{align*}
2\tilde{B}_\pi^{00} & \propto \left[ |A^{00}_\pi|^2 + |A^{00}_\pi|^2 \right] \\
& = |T|^2 |V_{ub}V_{ud}|^2 \left[ \tilde{r}_C^2 + \tilde{r}_P^2 + \tilde{r}_{EW}^2 - 2\tilde{r}_P \tilde{r}_{EW} \cos \delta_{EW} \\
& - 2\tilde{r}_C (\tilde{r}_P \cos \delta^C - \tilde{r}_{EW} \cos (\delta_{EW} - \delta^C)) \cos (\phi_1 + \phi_3) \right],
\end{align*}
$$

(43)
\[ \mathcal{B}_\pi^- \propto \frac{1}{2} \left[ |A_{\pi^+}|^2 + |A_{\pi^-}|^2 \right] \]
\[ = |T|^2 |V_{ub}^{*} V_{ud}|^2 \left[ 1 + \bar{r}_P^2 + 2 \bar{r}_P \cos \delta^T \cos(\phi_1 + \phi_3) \right], \]  
\( (44) \)
\[ 2\mathcal{B}_\pi^0 \propto \left[ |A_{\pi^0}|^2 + |A_{\pi^-}|^2 \right] \]
\[ = |T|^2 |V_{ub}^{*} V_{ud}|^2 \left[ 1 + \bar{r}_C^2 + \bar{r}_E^2 + 2 \bar{r}_E \cos(\delta^E - \delta^T) \right] \]
\[ + 2\bar{r}_E \{ \cos(\delta^E - \delta^T) + \bar{r}_C \cos(\delta^E - \delta^C) \} \cos(\phi_1 + \phi_3) \].  
\( (45) \)

When we compare the theoretical predictions with the experimental data, the ratios among the branching ratios are very useful to reduce uncertainties in hadronic parts. One can take several ratios among the branching ratios. From the averaged values of the recent experimental data in Table 1,
\[ \frac{\mathcal{B}_K^{+}}{2\mathcal{B}_K^{0}} = 0.78 \pm 0.10, \quad \frac{2\mathcal{B}_K^{0}}{\mathcal{B}_K^{0+}} = 1.15 \pm 0.12, \]  
\( (46) \)
\[ \frac{\tau^+ \mathcal{B}_K^{+}}{\tau^0 \mathcal{B}_K^{0+}} = 0.90 \pm 0.07, \quad \frac{\tau^0 \mathcal{B}_K^{0}}{\tau^+ \mathcal{B}_K^{0+}} = 0.98 \pm 0.15, \]  
\( (47) \)
\[ \frac{\tau^+ 2\mathcal{B}_K^{0}}{\tau^0 \mathcal{B}_K^{0+}} = 1.17 \pm 0.16, \quad \frac{\tau^0 2\mathcal{B}_K^{0}}{\tau^+ \mathcal{B}_K^{0+}} = 1.26 \pm 0.13, \]  
\( (48) \)
\[ \frac{\tau^0 2\mathcal{B}_\pi^{0}}{\tau^+ \mathcal{B}_\pi^{0+}} = 2.08 \pm 0.37, \quad \frac{2\mathcal{B}_\pi^{0}}{\mathcal{B}_\pi^{0+}} = 0.83 \pm 0.23, \]  
\( (49) \)

where \( \frac{\tau^+}{\tau^0} \) is a lifetime ratio between the charged and the neutral \( B \) mesons and \( \tau(B^+)/\tau(B^0) = 1.086 \pm 0.017 \) [32].
3 EW penguin contribution in $B \to K \pi$

Under the assumption that all $r$ is smaller than 1 and the $r^2$ terms including $r_C$, $r_A$ and $r_{EW}^C$ are neglected, the ratios among the decay rates of $B \to K \pi$ are

$$\frac{\bar{B}_K^{++}}{2B_K^{0+}} = \left\{ 1 + 2r_{EW} \cos \delta_{EW} + 2r_C^C \cos \delta_{EW} - 2(r_T \cos \delta_T + r_C \cos \delta_C) \cos \phi_3 + r_T^2 \right\}$$

$$- r_{EW}^2 + 4r_{EW}^2 \cos^2 \delta_{EW} - 4r_{EW} r_T \cos \delta_{EW} \cos \delta_T \cos \phi_3,$n

$$\frac{2\bar{B}_K^{+0}}{B_K^{0+}} = \left\{ 1 + 2r_{EW} \cos \delta_{EW} + 2r_C^C \cos \delta_{EW} - 2(r_T \cos \delta_T + r_C \cos \delta_C) \cos \phi_3 + r_T^2 \right\}$$

$$+ r_{EW}^2 - 2r_{EW} r_T \cos (\delta_{EW} - \delta_T) \cos \phi_3,$n

$$\frac{\tau^+ \bar{B}_K^{--}}{\tau^0 B_K^{0+}} = 1 + 2r_{EW} \cos \delta_{EW} - 2(r_T \cos \delta_T - r_A \cos \delta_A) \cos \phi_3 + r_T^2,$n

$$\frac{\tau^0 \bar{B}_K^{+0}}{\tau^+ B_K^{0+}} = 1 + 2r_{EW} \cos \delta_{EW} - 2(r_T \cos \delta_T + 2r_C \cos \delta_C + r_A \cos \delta_A) \cos \phi_3 + r_T^2$$

$$+ 4r_{EW} \cos \delta_{EW} + 8r_{EW}^2 \cos^2 \delta_{EW}$$

$$- 2r_{EW} r_T \cos (\delta_{EW} - \delta_T) \cos \phi_3 - 4r_{EW} r_T \cos \delta_{EW} \cos \delta_T \cos \phi_3,$n

$$\frac{\tau^+ 2\bar{B}_K^{00}}{\tau^0 B_K^{0+}} = 1 - 2r_{EW} \cos \delta_{EW} + 2(r_C \cos \delta_C + r_A \cos \delta_A) \cos \phi_3 + r_{EW}^2,$n

$$\frac{\tau^0 2\bar{B}_K^{+0}}{\tau^+ B_K^{0+}} = 1 + 2r_{EW} \cos \delta_{EW} - 2(r_C \cos \delta_C + r_A \cos \delta_A) \cos \phi_3 + r_{EW}^2$$

$$+ 2r_{EW} r_T \cos (\delta_{EW} + \delta_T) \cos \phi_3.$n

If we neglect all $r^2$ terms, then there are a few relations among Eqs. (50)–(55) as following

$$R_c - R_n \equiv \frac{2\bar{B}_K^{+0}}{B_K^{0+}} - \frac{\bar{B}_K^{++}}{2B_K^{0+}} = 0,$n

$$S \equiv \frac{2\bar{B}_K^{+0}}{B_K^{0+}} - \frac{\tau^+ \bar{B}_K^{--}}{\tau^0 B_K^{0+}} + \frac{\tau^+ 2\bar{B}_K^{00}}{\tau^0 B_K^{0+}} - 1 = 0,$n

$$R_+ - 2 \equiv \frac{\tau^0 2\bar{B}_K^{+0}}{\tau^+ B_K^{0+}} + \frac{\tau^+ 2\bar{B}_K^{00}}{\tau^0 B_K^{0+}} - 2 = 0.$n

However, the experimental data listed in Eqs. (16), (17) and (48) do not satisfy these relations so well. According to the experimental data, $\frac{\bar{B}_K^{++}}{2B_K^{0+}}$ seems to be smaller than 1 but $\frac{2\bar{B}_K^{+0}}{B_K^{0+}}$ be larger than 1. Thus it shows there is a discrepancy between them. The equations of $\frac{\bar{B}_K^{++}}{2B_K^{0+}}$ and $\frac{2\bar{B}_K^{+0}}{B_K^{0+}}$ are same up to $r^2$ term and the difference comes from $r^2$ term including $r_{EW}$. The second
Accordingly, to explain the data we may need some contribution from new physics in the theoretical estimation and the strong phase may not be close to zero. The allowed region of Eq. (63) from 0 also comes from $r^{2}$ term including $r_{EW}$. The differences are

$$R_{c} - R_{a} = \frac{2\bar{B}_{K}^{+0}}{B_{K}^{0+}} - \frac{\bar{B}_{K}^{+-}}{2\bar{B}_{K}^{00}}$$

$$= 2r_{EW}^{2} + 2r_{EW}r_{T}\cos(\delta^{EW} + \delta^{T})\cos \phi_{3} - 4r_{EW}^{2}\cos^{2}\delta^{EW} = 0.37 \pm 0.16 \rightarrow (59)$$

$$S = \frac{2\bar{B}_{K}^{+0}}{B_{K}^{0+}} - \frac{\tau^{+} \bar{B}_{K}^{+-}}{\tau^{0} B_{K}^{0+}} + \frac{\tau^{+} 2\bar{B}_{K}^{00}}{\tau^{0} B_{K}^{+0}} - 1$$

$$= 2r_{EW}^{2} - 2r_{EW}r_{T}\cos(\delta^{EW} - \delta^{T})\cos \phi_{3} = 0.43 \pm 0.20 \rightarrow (60)$$

$$R_{+} + 2 = \frac{\tau^{0} 2\bar{B}_{K}^{+0}}{\tau^{+} B_{K}^{0+}} + \frac{\tau^{+} 2\bar{B}_{K}^{00}}{\tau^{0} B_{K}^{+0}} - 2$$

$$= 2r_{EW}^{2} + 2r_{EW}r_{T}\cos(\delta^{EW} + \delta^{T})\cos \phi_{3} = 0.43 \pm 0.21 \rightarrow (61)$$

so that one can find the EW penguin contributions may be large. All terms are including $r_{EW}$ and the deviation of the relation from zero is finite. These deviations may be an evidence that the EW penguin is larger than the estimation we expected within the SM. Here the errors are determined by adding quadratically all errors. We want to solve from these three relations but there are too many parameters. To estimate the magnitude of the EW penguin contribution satisfying the experimental data roughly, we use Eq. (54).

$$\frac{\tau^{+} 2\bar{B}_{K}^{00}}{\tau^{0} B_{K}^{+0}} - 1 \simeq -2r_{EW}\cos\delta^{EW} + r_{EW}^{2} \simeq 0.17 \pm 0.16 \rightarrow (62)$$

where the $r_{C}$ and $r_{A}$ terms were neglected to reduce the number of parameter, because the $r_{EW}$ in Eqs. (59) and (60) should be larger than the usual prediction, therefore, $r_{C}$ and $r_{A}$ must be quite smaller than $r_{EW}$. Using Eqs. (59), (61) and (62), we can solve them in terms of $r_{EW}$ and if we can respect the central values of experimental data, the solution is

$$\left( r_{EW}, \cos\delta^{EW}, r_{T}\cos(\delta^{EW} - \delta^{T})\cos \phi_{3} \right) = (0.64, 0.19, -0.30) \rightarrow (63)$$

This solution shows that the EW penguin contribution is too large compared with the rough theoretical estimation and the strong phase may not be close to zero. The allowed region of $r_{EW}$, $\cos\delta^{EW}$ and $r_{T}\cos(\delta^{EW} - \delta^{T})\cos \phi_{3}$ at 1σ level from the three relations, Eqs. (59), (61) and (62) is shown in Fig. 11. The “x” in these figures shows the central values of the solution, Eq. (63), and the dotted lines in the right figure show the bound of $r_{T}\cos(\delta^{EW} + \delta^{T})\cos \phi_{3}$ for $r_{T} = 0.2$ and $\phi_{3} = 40^\circ$. The theoretical prediction should be in the bound at $r_{EW} = 0.14$ but there is no overlap region with the allowed one at 1σ level.

What we can expect at present are roughly $40^\circ < \phi_{3} < 80^\circ$ from CKM fitting and $r_{T} = 0.2$ with 10% error from the estimation of the ratio $\frac{\bar{B}_{K}^{+0}}{B_{K}^{0+}}$. Hence the figure shows that $r_{EW}$ will be larger than 0.2 while the theoretical prediction of $r_{EW}$ is 0.14, and a large strong phase difference between gluonic and EW penguins will be requested due to $\cos\delta^{EW} < 0.5$ [19]. Accordingly, to explain the data we may need some contribution from new physics in the
EW-penguin-type contribution with a large phase. We have to also consider the possibility of large strong phases. From Eq. (59) one can extract more information about strong phases to satisfy the experimental data. Eq. (59) is

\begin{equation}
R_c - R_n = -2r_{EW}^2 \cos 2\delta^{EW} + 2r_{EW}r_T \cos(\delta^{EW} + \delta^T) \cos \phi_3 = 0.37 \pm 0.16 > 0.
\end{equation}

The first term has negative sign but $R_c - R_n$ should be positive value. For this reason, negative $\cos 2\delta^{EW}$, that is, $45^\circ < |\delta^{EW}| < 135^\circ$ will be favored. Namely, it seems to show the strong phase difference should be large. Furthermore, considering $S$ in Eq. (60), we can obtain stronger constraint for the parameters because the second term has negative sign which is differ from the case of Eq. (59). If $\cos(\delta^{EW} - \delta^T) \cos \phi_3$ was positive value, $r_{EW}$ must be larger values to satisfy the condition for $S$. In Fig. 2 the allowed regions for $r_{EW}, \delta^T$ and $\delta^{EW}$ from three constraints, Eqs. (59), (60) and (61), are plotted. Here we did not use Eq. (62) because it is including an assumption which $r_C$ can be neglected. One can find that to satisfy the experimental data at $1 \sigma$ level, $r_{EW}$ should be larger than about 0.3. Under exact flavor SU(3) symmetry, the strong phase difference between the EW penguin and the color-favored tree, which is called as $\omega$,

\begin{equation}
\omega \equiv \delta^{EW} - \delta^T,
\end{equation}

should be close to zero because the diagrams are topologically same [12] and effectively the difference is whether just only the exchanging weak gauge boson is $W$ or $Z$. If it is correct, the constraint for $\delta^T$ has to influence on $\delta^{EW}$ due to $\delta^{EW} \sim \delta^T$. We consider the direct CP asymmetry to obtain the informations about strong phase.

The direct CP asymmetries under the same assumption which we neglect the terms of $O(0.001)$ are

\begin{equation}
A_{CP}^{0+} \equiv \frac{|A_{K}^{0+}|^2 - |A_{K}^{0-}|^2}{|A_{K}^{0+}|^2 + |A_{K}^{0-}|^2} = -2r_A \sin \delta^A \sin \phi_3 \sim 0.0,
\end{equation}

Figure 1: The allowed region on $(r_{EW}, \cos \delta^{EW})$ and $(r_{EW}, r_T \cos(\delta^{EW} + \delta^T) \cos \phi_3)$ plane at $1 \sigma$ level varying the magnitude of $r_T \cos(\delta^{EW} + \delta^T) \cos \phi_3$ up to 1. The dashed line shows the bound of $r_T \cos(\delta^{EW} + \delta^T) \cos \phi_3$ for $r_T = 0.2$ and $\phi_3 = 40^\circ$. 
Figure 2: From Eqs. (59), (60) and (61) the allowed regions for $r$ and $40^\circ < \phi_3 < 80^\circ$. The solid (dashed) line shows the $1\sigma$ ($2\sigma$) bound.

\[
A^{00}_{\text{CP}} \equiv \frac{|A^{00}_K|^2 - |A^{00}_T|^2}{|A^{00}_K|^2 + |A^{00}_T|^2} = 2r_C \sin \delta^C \sin \phi_3 \sim O(0.01) ,
\]

\[
A^{+-}_{\text{CP}} \equiv \frac{|A^{+-}_K|^2 - |A^{+-}_T|^2}{|A^{+-}_K|^2 + |A^{+-}_T|^2} = -2r_T \sin \delta^T \sin \phi_3 - r_T^2 \sin 2\delta^T \sin 2\phi_3 ,
\]

\[
A^{+0}_{\text{CP}} \equiv \frac{|A^{+0}_K|^2 - |A^{+0}_{K_0}|^2}{|A^{+0}_K|^2 + |A^{+0}_{K_0}|^2} = -2(r_T \sin \delta^T + r_C \sin \delta^C + r_A \sin \delta^A) \sin \phi_3 + 2r_{EW}r_T \sin(\delta^T + \delta^{EW}) \sin \phi_3 - r_T^2 \sin 2\delta^T \sin 2\phi_3 .
\]

Up to the first order of $r$, there is a relation among the CP asymmetries as follows:

\[
A^{+0}_{\text{CP}} - A^{+-}_{\text{CP}} + A^{00}_{\text{CP}} - A^{+0}_{\text{CP}} = 0 .
\]

The discrepancy of this relation is caused from the cross term of $r_T$ and $r_{EW}$.

\[
A^{+0}_{\text{CP}} - A^{+-}_{\text{CP}} + A^{00}_{\text{CP}} - A^{+0}_{\text{CP}} = 2r_T r_{EW} \sin(\delta^T + \delta^{EW}) \sin \phi_3 = 0.18 \pm 0.24 .
\]

This may also give us some useful informations about $r_{EW}$ and the strong phases but the data of $A^{00}_{\text{CP}}$ still has quite large error, as shown in Table. 2 so that one can not extract from it at present time. We need more accurate data to use this relation. For this reason, we use only $A^{+0}_{\text{CP}}$ because it is accurate measurement and will give some constraint to $\delta^T$. In Fig. 3 we plot $A^{+0}_{\text{CP}}$ as a function of $\delta^T$. From this figure, we can find the constraint for $\delta^T$ at $-0.123 < A^{+0}_{\text{CP}} < -0.067$. It tells us that the small $\delta^T$ is favored and $\delta^T$ should be around $15^\circ$ or $160^\circ$.

In Eq. (52), which leads to Fleischer-Mannel bound [10], if $r_{EW}^C$ and $r_A$ are negligible,

\[
R \equiv \frac{r^T_{\tau \tau} B^+_K}{r^0 B^+_{K}^0} = 1 - 2r_T \cos \delta^T \cos \phi_3 + r_T^2 = 0.90 \pm 0.07 ,
\]

and to satisfy $R = 0.90 \pm 0.07 < 1$ we need positive $\cos \delta^T$ so that the range $\delta^T \sim 10^\circ - 20^\circ$ is favored. Taking account of these constraints for $\delta^T$ from $A^{+0}_{\text{CP}}$, we plot the maximum bound
Table 2: The experimental data of the direct CP asymmetry [16, 18, 33] and the averaged values [23].

|          | CLEO            | Belle            | BaBar            | Average  |
|----------|-----------------|------------------|------------------|----------|
| $A^{0+}_{CP}$ | $0.18 \pm 0.24 \pm 0.02$ | $0.05 \pm 0.05 \pm 0.01$ | $-0.05 \pm 0.08 \pm 0.01$ | $0.03 \pm 0.04$ |
| $A^{00}_{CP}$ | -               | $0.16 \pm 0.29 \pm 0.05$ | $0.03 \pm 0.36 \pm 0.11$ | $0.11 \pm 0.23$ |
| $A^{+}_{CP}$ | $-0.04 \pm 0.16 \pm 0.02$ | $-0.088 \pm 0.035 \pm 0.018$ | $-0.107 \pm 0.041 \pm 0.013$ | $-0.095 \pm 0.028$ |
| $A^{+0}_{CP}$ | $-0.29 \pm 0.24 \pm 0.02$ | $0.06 \pm 0.06 \pm 0.02$ | $-0.09 \pm 0.09 \pm 0.01$ | $0.00 \pm 0.05$ |

Figure 3: $A^\pm_{CP}$ as a function of $\delta^T$.

of $R_c - R_n$ as the functions of $\delta_{EW}$ and $r_{EW}$ in Fig. 4 respectively. Then the allowed regions for $\delta_{EW} - \delta^T$ around 0° and 180° disappear. $R_c - R_n$ seems to favor $45° < |\delta_{EW}| < 135°$, but the constraint from $A^{+}_{CP}$ is strongly suggesting that the strong phase, $\delta^T$, should be around $15°$. In consequence, $\delta_{EW} - \delta^T = 0$ as the theoretical prospect is disfavored.

What we found from $B \to K\pi$ decays are that we need larger $r_{EW} > 0.3$ and large strong phase differences, $\delta_{EW} - \delta^T$. The constraint, $\delta^T \sim 10° - 20°$, from direct CP asymmetry is differ from the favored range of the strong phase of EW penguin $\delta_{EW} > 45°$ so that $\omega = \delta_{EW} - \delta^T \simeq 0$, which is favored in theoretically, will not be satisfied. What the quite large strong phase difference is requested may be a serious problem in these modes. If SU(3) symmetry is good one, these properties should also appear in $B \to \pi\pi$. 

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Figure 4: The lines show the maximum bound of $R_c - R_n$ for $r_{EW}$ and $\delta_{EW}$ at $r_T = 0.2$ and $40^\circ < \phi_3 < 80^\circ$ under constraint $-0.123 < A^C_{CP} < -0.067$

4 SU(3) breaking effect in gluonic penguin

When we consider the ratios among the branching ratios for $B \to \pi\pi$ decays,

$$\frac{2\bar{B}_{\pi}^{00}}{B^+_{\pi}^{-}} = \frac{\bar{r}_C^2 + \bar{r}_P^2(1 + r_{EW}^2 - 2r_{EW}\cos\delta_{EW}) - 2\bar{r}_P\bar{r}_C(\cos\delta_T - r_{EW}\cos\omega)\cos(\phi_1 + \phi_3)}{1 + \bar{r}_P^2 + 2\bar{r}_P\cos\delta_T\cos(\phi_1 + \phi_3)}$$,  

$$\frac{\bar{r}_0^2 \bar{B}_{\pi}^{+0}}{\bar{r}^+ B^\tau_{\pi}^-} = \frac{1 + \bar{r}_C^2 + \bar{r}_P^2 r_{EW}^2 + 2\bar{r}_P r_{EW}(\cos\omega + r_C\cos\omega)\cos(\phi_1 + \phi_3)}{1 + \bar{r}_P^2 + 2\bar{r}_P\cos\delta_T\cos(\phi_1 + \phi_3)}$$,  

there is also discrepancy between theoretical expectation and experimental data. In above equations, $\delta^C$ is taken to be equal to $\delta^T$. From a rough estimation, $\bar{r}_P \sim 0.3, \frac{\bar{r}_0^2 \bar{B}_{\pi}^{+0}}{\bar{r}^+ B^\tau_{\pi}^-} \sim 1$, $\frac{\delta_{EW}^{\pi\pi}}{B^\tau_{\pi}^-} \sim 0.1$, but the experimental data (49) are quite large values and are not consistent with them. To explain the discrepancy, the denominator seems to be smaller value so that $\cos\delta_T$ should be negative or $\phi_1 + \phi_3$ should be larger than $90^\circ$ to reduce the denominator. The negative $\cos\delta_T$ case is inconsistent with the condition $R = \frac{\bar{r}^+ B^\tau_{\pi}^-}{\bar{r}_0^2 \bar{B}_{\pi}^{+0}} = 0.90 \pm 0.07 < 1$. As the result, negative $\cos(\phi_1 + \phi_3)$ is favored. However it is not enough to explain the differences and we will also have to take account of SU(3) breaking effect.

The ratio of the direct CP asymmetries may show the SU(3) breaking effect. We consider the following ratio between $B \to K^+\pi^-$ and $B \to \pi^+\pi^-$:

$$\frac{|\bar{A}_{\pi}^{+0}|^2 - |A_\pi^{+0}|^2}{|A_K^{+0}|^2 - |A_K^{+0}|^2} = \frac{-T + E||P + P^C_{EW}||\sin\delta_T}{|T| |P + P^C_{EW}||\sin\delta_T} \sim \frac{f_\pi |P_\pi| \sin\delta_T}{f_K |P_K| \sin\delta_K}$$
\[
\frac{B r^{\pi^+\pi^-} A_{CP}^{\pi^+\pi^-}}{B r^{K^+\pi^-} A_{CP}^{K^+\pi^-}} = \left\{ \begin{array}{l}
- f_\pi^2 / f_K^2 = -0.66 \text{ (factorization)} \\
- 1.54 \pm 0.66 \text{ (Belle [34])} \\
- 0.50 \pm 0.55 \text{ (BaBar [35])} \\
- 1.12 \pm 0.49 \text{ (Average [23])}
\end{array} \right. 
\tag{75}
\]

where each values in Eq. (75) corresponds to the each experimental data of the direct CP asymmetry \( A_{CP}^{\pi^+\pi^-} \) and the other data was used the averaged values. The factors of the CKM matrix elements are completely canceled. If SU(3) is exact symmetry, the ratio must be \(-1\). From the experimental data in Eq. (75), one can find that a possibility of large SU(3) breaking symmetry \( A \) where each values in Eq. (75) corresponds to the each experimental data of the direct CP matrix elements are completely canceled. If SU(3) is exact symmetry, the ratio must be \(-1\). From the experimental data in Eq. (75), one can find that a possibility of large SU(3) breaking effect in gluonic penguin contribution is remaining. When we assume there is no SU(3) breaking effect in tree type diagram except for the difference of decay constants in sense of factorization because it is good agreement in \( B \rightarrow D_{\pi} \rightarrow D_{K} \), the gluonic penguin contribution might have the SU(3) breaking effect like \( P_\pi / P_K \sim 2 \).

## 5 Large EW penguin contribution in \( B \rightarrow \pi\pi \)

We discuss about a role of \( r_{EW} \) in \( B \rightarrow \pi\pi \) decays. To explain the \( B \rightarrow \pi\pi \) modes, we may need SU(3) breaking effect in gluonic penguin as we discussed in previous section. In this section, we assume that the strong phases almost satisfy the SU(3) symmetry as an anzatz. To enhance the both ratios \( \frac{2B_{\pi^-}^{\text{EW}}}{B_{\pi^+}^{\text{EW}}} \) and \( \frac{3B_{\pi^+}^{\text{EW}}}{B_{\pi^-}^{\text{EW}}} \), smaller denominator will be favored so that the cross term \( 2\tilde{r}_P \cos \delta_T \cos (\phi_1 + \phi_3) \) should have negative sign. Since \( \cos \delta_T \) should be positive from the data of \( A_{CP}^{\pi^+\pi^-}, \cos (\phi_1 + \phi_3) \) has to be negative value. As an example, we plot the ratios as the function of \( \tilde{r}_P \) and \( r_{EW} \) in a case with \( \delta_{EW} = 110^\circ, \delta_C = \delta_T = 10^\circ \) and \( \phi_1 + \phi_3 = 110^\circ \) for \( \tilde{r}_C = 0.1, 0.2 \) and 0.3 in Figs. 5. \( \phi_1 + \phi_3 = 110^\circ \) is almost maximum value of allowed region and \( \delta_T = 10^\circ \) to satisfy \( A_{CP}^{\pi^+\pi^-} \). From the figures, we can find that to explain the discrepancy between the theoretical estimation and the experimental data, \( b \rightarrow d \) gluonic penguin contribution \( P_\pi \) should be larger than \( b \rightarrow s \) gluonic penguin \( P_K \) without the CKM factor. It shows SU(3) breaking effect must appear in these decay modes. In addition, large EW penguin contribution also help to enhance the ratios.

From these figures, we find that the ratios are enhanced by \( \tilde{r}_P, \tilde{r}_C \) and \( r_{EW} \). However \( \tilde{r}_C = C/T \) is 0.1 for the naive estimation by factorization and it will be at largest up to \( 1/N_c \sim 0.3 \) because it is the simple ratio of two tree diagrams between color-allowed and color-suppressed types. Large \( \tilde{r}_P \) is an evidence to explain the discrepancies but it also has some constraints from \( B \rightarrow KK \) decays which are pure \( b \rightarrow d \) gluonic penguin, \( (\sim P_\pi) \) processes. The upper bounds of \( B \rightarrow KK \) decays [23] are

\[
\begin{align*}
Br(B^+ \rightarrow K^+ \bar{K}^0) &\propto |P V_{tb}^* V_{td}|^2 < 2.5 \times 10^{-6}, \\
Br(B^0 \rightarrow K^0 \bar{K}^0) &\propto |P V_{tb}^* V_{td}|^2 < 1.5 \times 10^{-6},
\end{align*}
\tag{76}
\tag{77}
\]

where \( P \) is the gluonic penguin contribution without the CKM factor and \( P \sim P_\pi \) under SU(3) symmetry. The constraint to \( P_\pi / P_K \) comes from

\[
\frac{Br(B^0 \rightarrow K^0 \bar{K}^0)}{Br(B^+ \rightarrow K^0 \pi^+)} \sim \frac{|P_\pi V_{tb}^* V_{td}|^2}{|P_K V_{ts}^* V_{td}|^2} < 7.3 \times 10^{-2},
\tag{78}
\]

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Figure 5: $\frac{2B_{00}}{B_z}$ and $\frac{2B_{z}^{+0}}{B_z}$ as the functions of $\tilde{r}_P$ at (a) $\tilde{r}_C = 0.1$, (b) $\tilde{r}_C = 0.2$ and (c) $\tilde{r}_C = 0.3$. 
\[
\frac{P_\pi}{P_K} < 1.5. \tag{79}
\]

Thus \( \tilde{r}_P \) may be allowed up to \( 0.3 \times 1.5 = 0.45 \). In addition, we will also need the help from \( r_{EW} = \frac{P_{EW}}{P} \) to enhance \( B_{\pi^0} \).

It is slightly difficult to get the values within the 1σ region unless larger \( r_C \) is allowed. However we feel that it may be unnatural that such tree diagram obtains the larger contribution than usual estimation. Therefore we consider the case keeping small \( r_C \) and including some new effects in penguin contribution.

## 6 New physics contribution

If the deviations come from new physics contribution, it has to be included in the penguin like contribution with new weak phases because it is very difficult to produce such large strong phase difference as \( \omega = \delta^{EW} - \delta^T \sim 100^\circ \) within the SM. \( B \to K\pi \) decays need large EW penguin contributions so that it may be including the new physics contribution with new weak phase in the EW penguin. Besides, the effect must appear also in the direct CP asymmetries. For example, \( A_{CP}^{K\pi} \propto 2r_{EW} \sin \delta^{EW} \sin \theta^{New} \), so that we will have to check carefully these modes.

We consider a possibility of new physics in the penguin contributions as follows:

\[
P = P^{SM} + P^{New}, \quad P_{EW} = P_{EW}^{SM} + P_{EW}^{New}, \tag{80}
\]

where \( P^{New} \) and \( P_{EW}^{New} \) are gluonic and EW penguin type contributions coming from new physics, respectively. For simplicity, we assume that the strong phase of the penguin diagram within the SM is the same with one from new physics. Here we parameterize the phases as follows:

\[
P V_{tb}^* V_{ts} \equiv -|P V_{tb}^* V_{ts}| e^{-i\theta^P}, \tag{81}
\]
\[
P_{EW} V_{tb}^* V_{ts} \equiv -|P_{EW} V_{tb}^* V_{ts}| e^{i\delta^{EW}} e^{i\theta^{EW}}, \tag{82}
\]

where \( \theta^P \) and \( \theta^{EW} \) are the weak phases coming from new physics contributions. The ratios among the parameters are

\[
\frac{T V_{ub}^* V_{us}}{P V_{tb}^* V_{ts}} = r_T e^{i\delta_T} e^{i(\phi_3 + \theta^P)}, \tag{83}
\]
\[
\frac{P_{EW} V_{tb}^* V_{ts}}{P V_{tb}^* V_{ts}} = r_{EW} e^{i\delta^{EW}} e^{i(\theta^{EW} + \theta^P)}. \tag{84}
\]

Using this parameterization,

\[
R_c - R_n = 2r_{EW}^2 \left(1 - 2 \cos^2 \delta^{EW} \cos^2(\theta^{EW} + \theta^P)\right) - 2r_{EW} \frac{r_T}{r_{EW}} \cos(\delta^{EW} - \delta^T) \cos(\phi_3 - \theta^{EW}) + 4r_{EW} \frac{r_T}{r_{EW}} \cos(\delta^{EW} - \delta^T) \cos(\phi_3 + \theta^P), \tag{85}
\]
\[
S = 2r_{EW}^2 - 2r_{EW} \frac{r_T}{r_{EW}} \cos(\delta^{EW} - \delta^T) \cos(\phi_3 - \theta^{EW}). \tag{86}
\]
Because of the new weak phase $\theta^{EW}$ and $\theta^P$, the constraints for the strong phases is fairly relaxed. To keep the first term to positive value, $\cos^2 \delta^{EW} \cos^2 (\theta^{EW} + \theta^P)$ must be less than 1/2 so that smaller $|\cos (\theta^{EW} + \theta^P)|$ will be favored. In the second term, if $\cos (\hat{\phi}_3 - \theta^{EW})$ was negative, small $\omega = \delta^{EW} - \delta^T$ was not excluded in contrast with the SM case. The constraint for $r^{EW}$ is almost same but one for the strong phases is changed and almost region for the strong phase $\delta^{EW}$ is allowed. Furthermore, small $\omega$ is also allowed in this case. In other words, the constraint for $\delta^{EW}$ is replaced to one for the new weak phase and their magnitude is not negligible value. Therefore, to investigate the direct CP asymmetries for $B \to K\pi$ will become more important to know the information about the new CP phases. Especially, $A_{CP}^{00}$ will be important mode. The CP asymmetries for $B \to K\pi$ decays are

$$A_{CP}^{+} \propto -2r_T \sin \delta^T \sin (\phi_3 + \theta^P) - r_T^2 \sin 2\delta^T \sin 2(\phi_3 + \theta^P), \quad (87)$$

$$A_{CP}^{00} \propto -2r_{EW} \sin \delta^{EW} \sin (\theta^{EW} + \theta^P) - r_{EW}^2 \sin 2\delta^{EW} \sin 2(\theta^{EW} + \theta^P) + 2r_C \sin \delta^C \sin (\phi_3 + \theta^P), \quad (88)$$

$$A_{CP}^{+0} \propto 2r_{EW} \sin \delta^{EW} \sin (\theta^{EW} + \theta^P) - 2r_T \sin \delta^T \sin (\phi_3 + \theta^P) + 2r_T r_{EW} \sin (\delta^{EW} + \delta^T) \sin (\theta^{EW} + \phi_3 + 2\theta^P) - r_{EW}^2 \sin 2\delta^{EW} \sin 2(\theta^{EW} + \theta^P) - r_T^2 \sin 2\delta^T \sin 2(\phi_3 + \theta^P). \quad (89)$$

To explain the deviation of $R_c - R_n$, large $r_{EW}$ and large $\sin \delta^{EW} \sin (\theta^{EW} + \theta^P)$ are favored so that if there is some new physics contributions in $B \to K\pi$, sizable $A_{CP}^{00}$ will appear in the $B$ factory experiment in the near future. At present time, these experimental data have still large uncertainties.

As we used $A_{CP}^{-}$ before, it seems good accurate one so that it is good examples to plot the maximum bound of $R_c - R_n$ and $S$ as the function of $\theta^{EW}$ for each $r_{EW}$ at $\omega = 0^\circ$ and $r_T = 0.2$ under constraint of $A_{CP}^{+}$. Here we take $\theta^P = 0$, for simplicity. Figure shows that the allowed region for both constraints of $R_c - R_n$ and $S$ exists even if $\omega = \delta^{EW} - \delta^T = 0^\circ$ and it is about $240^\circ < \theta^{EW} < 300^\circ$. If the $\theta^P$ is non zero values, the allowed region should be wider. In Fig. the allowed region on the plane of $\theta^{EW}$ and $\theta^P$ are plotted.

As well as $K\pi$, we have to reconsider $B \to \pi\pi$ modes in the case with new physics contributions. Under the same conditions which are $\omega = 0^\circ$ and $\theta^P = 0^\circ$ but $\tilde{r}_C = 0.2$ which is slightly larger than usual estimation, the ratios among the branching fractions are plotted in Fig. The allowed region for $\theta^{EW}$, however, seems to be slightly different between $K\pi$ and $\pi\pi$. Moreover, $\tilde{r}_P = 0.45$, which is almost 1.5 times as large as the SM expectation, and larger $r_{EW}$ will be requested to satisfy $\frac{B^{00}}{B^{+0}} \geq \tilde{r}_P$. In the allowed region of $\theta^{EW}$ for $B \to K\pi$, there is no region satisfying the data of $B \to \pi\pi$ at 1σ level even if $\tilde{r}_C = 0.2$. It may suggest that the other angles as the parameters of new physics are needed. Hence to find the allowed region to explain both discrepancies between $B \to K\pi$ and $B \to \pi\pi$ may be slightly difficult without considering the SU(3) breaking effects and assuming that these parameters should be independent each other between $K\pi$ and $\pi\pi$ even if $\theta^P$ is non-zero value.

In order to solve them, we introduce one more parameter as a phase difference of the new weak phases between $K\pi$ and $\pi\pi$. The phase difference is defined as

$$\theta_X \equiv \theta^P - \theta^P = \theta^{EW} - \theta^{EW}, \quad (90)$$
Figure 6: The lines show the maximum bound of $R_c - R_n$ and $S$ for $\theta^{EW}$ at $\omega = 0^\circ$ and $r_T = 0.2$ under constraint $-0.123 < A_{CP}^+ < -0.067$, $\theta^P = 0^\circ$ and $40^\circ < \phi_3 < 80^\circ$.

Figure 7: The lines show the allowed region for $\theta^{EW}$ and $\theta^P$ satisfying the data of $R_c - R_n$ and $S$ a $1\sigma$ level. The solid line shows that the case of $r_{EW} = 0.40$ and the dashed lines is $r_{EW} = 0.30$ at $\omega = 0^\circ$ and $r_T = 0.2$ under constraint $-0.123 < A_{CP}^+ < -0.067$ and $40^\circ < \phi_3 < 80^\circ$. 
We assume that the SU(3) breaking effect in the strong phase is not large because the final state for the KM phase, where \( \theta^P = 0^\circ \), is almost 1.5 times as large as the usual estimation in the SM. However, if \( \theta^P \) is a large angle, we have to note that the region for \( \delta^T \) is also changed since the constraint from \( A_{CP}^{K+\pi^-} \) is relaxed by \( \theta^P \). Here \( A_{CP}^{K^0\pi^0} \) was taken account and it give a constraint to \( \theta^{EW} \). Hence it may be slightly complicated to understand the allowed region.

From Fig. 8 we can find that satisfying the several experimental data for \( B \to \pi\pi \) and \( B \to K\pi \) at once does not only require the large \( r_{EW} \) but also large new weak phases, \( \theta^{EW} \).
and a phase difference $\theta_X = \theta^{EW} - \theta^{EW}_\pi$, which may suggest SU(3) breaking effects for the penguin diagrams. The right figure shows that there is no solution for $\theta_X = 0^\circ$ if we take the current experimental data seriously.

Figure 9: The allowed region for the new weak phases $\theta^{EW}, \theta^P$ of each penguin diagrams and $\theta_X$ which is the phase difference between $\pi\pi$ and $K\pi$ final state modes, under an assumption that the tree diagrams do not include any new physics and $\tilde{r}_C = 0.1$ and the all strong phase differences do not also so that $\omega = \delta^{EW} - \delta^T = 0^\circ$. The regions are satisfying $R_c - R_n = 0.37 \pm 0.16, S = 0.43 \pm 0.20, A^{K^{\pm}\pi^-}_{CP} = -0.095 \pm 0.028, A^{K^0\pi^0}_{CP} = 0.11 \pm 0.23, 2B^{\pi^0}_{B^+} = 0.83 \pm 0.23$ and $\frac{\phi^0}{\tau^0} \frac{2B^{\pi^0}_{B^+}}{B^+_\pi} = 2.08 \pm 0.37$. And at $2\sigma$ level $R$ is used to put a constraint because we neglected some small terms to get the bound. Here the KM weak phases are used $\phi_1 = 23.7^\circ$ and $40^\circ < \phi_3 < 80^\circ$. The gluonic penguin contribution in $B \rightarrow \pi\pi$ is $0.45$ which is almost 1.5 times of the usual estimation. In both figures, the solid line show the case of $r_{EW} = 0.4$ and the dashed line is $r_{EW} = 0.3$.

In this analysis, we assumed that the tree processes do not have any new physics contributions and there are not so large strong phase differences which satisfy also SU(3) symmetry. If some new physics exists, it is including in the penguin-type diagrams and they should appear as the large EW penguin contribution with large new weak phase and it may cause some SU(3) breaking effects. As a result we can find several allowed regions such like Fig. 9 for the new weak phases of penguin contributions. In $B \rightarrow \pi\pi$ mode, the role of EW penguin is not so important within the SM because its magnitude much smaller than that of tree. However if we consider some new physics contribution to explain both $B \rightarrow K\pi$ and $B \rightarrow \pi\pi$, then the role of EW penguin contribution with the new weak phase will be more important so that it should not be neglected even if we consider new physics effects in $\pi\pi$.

7 Conclusion

In this paper, we discussed a possibility of large EW penguin contribution in $B \rightarrow K\pi$ and $B \rightarrow \pi\pi$ from recent experimental data. To satisfy several relations among the branching
ratios of $B \to K\pi$, the larger EW penguin contribution with non-negligible strong phase differences. It seems to be difficult to explain them in the SM. If the EW penguin estimated from experimental data is quite large compared with the theoretical estimation, which is usually smaller than tree contributions [12,15,24], then it may be including some new physics effects. In addition, to avoid the large strong phase difference, the EW penguin must have new weak phase.

When we respect the allowed region for the parameters in $B \to K\pi$, then they could not satisfy $B \to \pi\pi$ modes under the SU(3) symmetry. To explain the both modes at once, SU(3) breaking effects in gluonic and EW penguin diagrams with new phase will be strongly requested. In consequence, the role of the EW penguin contribution will be more important even in $B \to \pi\pi$ modes. In several recent works discussing about the branching ratios in the $B \to \pi\pi$ modes, EW penguin contribution is usually neglected because of the smallness and suggest the other large contribution such like color-suppressed tree to explain the deviations from experimental data. However it is unnatural that the color-suppressed tree diagram is including such large new contributions so that in this paper we discussed about the explanation by the penguin-type diagrams.

If there is any new physics and the effects appear through the loop effect in these modes, $B \to K\pi$ and $B \to \pi\pi$, will be helpful modes to examine and search for the evidence of new physics. At the present situation, the deviation from the SM in $B \to K\pi$ is still within the $2\sigma$ level if large strong phase difference is allowed. Thus we need more accurate experimental data. In near future, we can use these modes to test the SM [26] or the several new models [36–40]. For this purpose, the project the $B$ factories are upgrade [41] is helpful and important.

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