Examination of $T/CP$ Invariance in the $e^+e^- \rightarrow \tau^+\tau^-$ Reaction

T. Ohshima, S. Suito, A. Sugiyama, S. Suzuki
Nagoya University, Chikusa, Nagoya 464
and
N. Haba
Mie University, Tsu, Mie 514

Abstract
We propose a method to examine the $T/CP$ invariance of the heaviest lepton, $\tau$, by means of a triple-momentum correlation for the reaction $e^+e^- \rightarrow \tau^+\tau^-$, $\tau \rightarrow e/\mu\nu\bar{\nu}$ during the course of the $B$-Factory experimental program. An unprecedented high sensitivity could be obtained without requiring a high integrated luminosity.

1. Introduction

Time reflection ($T$) is equivalent to the space-charge conjugation transformation ($CP$) under the $CPT$ invariant theorem. To date, only the neutral kaon has exhibited the phenomenon of $CP$ violation at the $O(10^{-3})$ level; the $B$-factory aims to scrutinize the violation mechanism, expecting its appearance in $B$-meson decay at the $O(10^{-1})$ level through the so-called $CKM$ complex coupling in the Standard Model. On the other hand, $T$ invariance has been examined in many different processes, such as the electric-dipole moments of the neutron, electron, and atoms, neutron $\beta$-decay, triple correlations among initial- and final-state particles in nuclear-decays [1]. However, no search has been made for pure leptonic transitions of leptons. The only exception is a measurement of the triple correlation among the spin and momenta in $\mu \rightarrow e\nu\bar{\nu}$ decay, which found no violation with a sensitivity of 2.3% [2].
In a way, it is natural to expect the existence of CP violation in the lepton sector as well as the quark sector; in particular, the leptonic decay of the heaviest lepton, $\tau$, could exhibit a larger violation than others, just like what happened in heavy quark transitions, $b \to u$ and $t \to d$. CP violation in the lepton sector would appear beyond the Standard Model. For instance, the three-Higgs doublet model [4], and $R$-parity conserving [5] and violating [6] SUSY models; also, Dirac or/and Majorana neutrino masses could provide one or more CP phases. An examination of the $T$ and $CP$ invariances in any possible leptonic reactions should therefore be performed as accurately as experiments allow.

T.D. Lee [3] recommended a study of $T$ and $CP$ violation in the $e^+e^- \to \tau^+\tau^-$ reaction, where the $\tau$'s decay pure-leptonically to both $(\mu/e)\nu\bar{\nu}$, in terms of a triple-momentum correlation as

$$<A> \equiv \left< \hat{p}_1 \cdot (\hat{p}_2 \times \hat{p}_3) \right>, \quad (1)$$

where $\hat{p}_1$ is the unit vector of incident $e^-$ (or $e^+$) momentum and $\hat{p}_2$ and $\hat{p}_3$ are the unit vectors of the outgoing $\mu$ and $e$ momenta, respectively. $<A>$ means an average quantity comprised by these unit vectors, and yields a non-zero value if the $T/CP$ invariance does not hold. This triple correlation, $A$, is odd under both $P$ and $T$ transformations, while the correlations in most experiments include the spin vector instead of the momentum vector of a respective particle; thus, $P=$even, but $T=$odd. Since the radiative correction would have an effect on the order of $\mathcal{O}(\alpha/2\pi)$, $T/CP$ violation can be simply established if $<A>$ is larger [3].

We consider here the experimental feasibility of testing the above mentioned lepton’s $T/CP$ violation in the $e^+e^- \to \tau^+\tau^-$ reaction by the BELLE detector [7,8] at the KEK – B Factory. Although, for the purpose of simplifying the arguments, we form a ratio $R$, instead of $<A>$, between the numbers of data samples with positive and negative $A$ values, all of the following issues are in essence also valid for $<A>$. In order to control the systematic uncertainty we take the product of two $R$’s with opposite charge configurations of two leptons. The methods to repeal the dominant background effects arose from two-photon process and particle mis-identification are presented. Also, a simulation study is performed to evaluate the achievable sensitivity by the BELLE experiment. The first goal of this study is to aim for a sensitivity of $\mathcal{O}(10^{-3})$ with an integrated luminosity of $10 fb^{-1}$.

2. $A$ vs. $R$

In this text we assign the unit vectors of the momenta of positive- and negative-charged particles in the final state to $\hat{p}_2$ and $\hat{p}_3$, respectively. It is obvious that the $T$ and $CP$ reflected states are different. Although both transformations change the sign of $A$, $CP$ reverses the particle’s charges while $T$ keeps them unchanged.
Let us denote the number of samples with \( A > 0 \) and \( A < 0 \) as \( N(l_2^+ l_3^-; >) \) and \( N(l_2^+ l_3^-; <) \), respectively, where \( l_2 \) and \( l_3 \) are either an electron or a muon of the final states, and \( A > 0 \) and \( A < 0 \) are denoted simply as \( > \) and \( < \), and form the following four ratios, \( R \)’s, as violation parameters instead of the asymmetry, \( < A > \):

\[
\begin{align*}
R_{\mu^+e^-}^T &\equiv \frac{N(\mu^+e^-; >)}{N(\mu^+e^-; <)} = \frac{N_o(1 + \delta_{\mu e}^T)}{N_o(1 - \delta_{\mu e}^T)} = 1 + 2\delta_{\mu e}^T, \\
R_{e^+\mu^-}^T &\equiv \frac{N(e^+\mu^-; >)}{N(e^+\mu^-; <)} = 1 + 2\delta_{e\mu}^T, \\
R_{\mu^+e^-}^{CP} &\equiv \frac{N(\mu^+e^-; >)}{N(\mu^+e^-; <)} = \frac{N_o(1 + \delta_{\mu e}^{CP})}{N_o(1 - \delta_{\mu e}^{CP})} = 1 + 2\delta_{\mu e}^{CP}, \\
R_{e^+\mu^-}^{CP} &\equiv \frac{N(e^+\mu^-; >)}{N(e^+\mu^-; <)} = 1 + 2\delta_{e\mu}^{CP},
\end{align*}
\]

where the total number of \( \mu^+e^- \) samples, \( N(\mu^+e^-; >) + N(\mu^+e^-; <) \), is normalized to be equal to those of the \( e^+\mu^- \) samples, \( N(e^+\mu^-; >) + N(e^+\mu^-; <) \), and is expressed as \( 2N_o \). \( \delta_{l_2 l_3}^T \) and \( \delta_{l_2 l_3}^{CP} \) are the portions of \( T \) and \( CP \) violations, respectively, with a subscript of \( l_2 l_3 \).

If the \( CPT \) invariance does not hold, the \( \delta_{l_2 l_3}^{T/CP} \)’s should have different non-zero values. With \( \delta \) and \( \Delta \) being \( T \) and \( CPT \)-violating portions in the number of samples, respectively, as can be seen in Fig.1, the above \( R \)’s can be expressed as

\[
\begin{align*}
R_{\mu^+e^-}^T &= R_{e^+\mu^-}^T = 1 + 2\delta, \\
R_{\mu^+e^-}^{CP} &= 1 + 2(\delta + \Delta); \quad R_{e^+\mu^-}^{CP} = 1 + 2(\delta - \Delta),
\end{align*}
\]

and the \( CPT \) violation parameters are also formed as

\[
\begin{align*}
R_{\mu^+e^-; >}^{CP} &\equiv \frac{N(\mu^+e^-; >)}{N(e^+\mu^-; >)} = R_{\mu^+e^-; <0}^{CP} = \frac{N(\mu^+e^-; <)}{N(e^+\mu^-; <)} = 1 + 2\Delta.
\end{align*}
\]

The \( CP \)-violation parameters, \( R_{\mu^+e^-}^{CP} \) and \( R_{e^+\mu^-}^{CP} \), are obviously not equal. Therefore, we can in principle test the \( T, CP \) and \( CPT \) invariances by examining the above \( R \) ratios. When \( CPT \) holds, but \( T/CP \) is violated, all four \( \delta_{l_2 l_3}^{T/CP} \)’s have the same non-vanishing value, but the \( R^{CP \prime} \)’s are unity. In the following, we assume that \( CPT \) invariance holds.

The systematic uncertainty is controlled with high precision by forming the following \( \tilde{R} \), a product of two \( R^{T/CP} \)’s:

\[
\tilde{R} \equiv R_{\mu^+e^-}^T \cdot R_{e^+\mu^-}^T = R_{\mu^+e^-}^{CP} \cdot R_{e^+\mu^-}^{CP}
\]
By factorizing the whole detection efficiency, \( \eta \), comprising the geometrical acceptances, detection and reconstruction efficiencies, as a product of the efficiency, \( \eta_1 \), independent of lepton-charge configuration and a factor \( \eta_2 \), reflecting the efficiency difference due to different charge configurations: \( \eta(l_2^+l_3^-) \equiv \eta_1(l_2^+l_3^-) \cdot \eta_2(l_2^+l_3^-) \), the \( \hat{R} \) is written as

\[
\hat{R} = \frac{N(\mu^+e^-;>)}{N(\mu^+e^-;<)} \times \frac{N(e^+\mu^-;>)}{N(e^+\mu^-;<)}
\]

\[
= \frac{[N_{\text{orig}}^{\mu^+e^-}(1 + \delta)\eta_1(\mu e^+;>)\eta_2(\mu^+e^-)] [N_{\text{orig}}^{e^+\mu^-}(1 + \delta)\eta_1(e\mu^-;>)\eta_2(e^+\mu^-)]}{[N_{\text{orig}}^{\mu^+e^-}(1 - \delta)\eta_1(\mu e^-;>)\eta_2(\mu^+e^-)] [N_{\text{orig}}^{e^+\mu^-}(1 - \delta)\eta_1(e\mu^-;>)\eta_2(e^+\mu^-)],} \tag{10}
\]

where \( N_{\text{orig}}^{l_2^+l_3^-} \) is the original number of samples produced. Since the sample of \( \mu^\pm e^\mp > 0 \) has the same geometrical configuration, but opposite charge configuration, to the sample of \( e^\pm \mu^\mp < 0 \), \( \eta_1(\mu e^\mp;>) \approx \eta_2(e\mu^\mp;>) \) could be valid. On the other hand, the \( \eta_2(\mu^\mp e^\pm) \)'s cancel each other out in the same reaction. Therefore, a high degree of accurate cancellation is expected in the form of \( \hat{R} \). The normalization between the charge-conjugated reactions used in eqs.(4) and (5) is not necessary in this case. Also, long-term instabilities of the experimental situations, such as the beam conditions and detector performances, do not affect \( \hat{R} \).

3. Sensitivity vs. Background

Including the background, \( N_{BG} \), the \( \hat{R} \) ratio is expressed as

\[
\hat{R} = 1 + 4\delta + \{\varepsilon(\mu^+ e^-;>) - \varepsilon(\mu^+ e^-;<) + \varepsilon(e^+ \mu^-;>) - \varepsilon(e^+ \mu^-;<)\}, \tag{11}
\]

where the third term is the background contributions, and each \( \varepsilon \) is the ratio of \( N_{BG} \) corresponding to the signal samples. The statistical sensitivity is approximated as

\[
(\frac{\Delta \hat{R}}{\hat{R}})^2 = 4 \left[ (\frac{\Delta N_o}{N_o})^2 + (\frac{\Delta N_{BG}}{N_o})^2 \right], \tag{12}
\]

where \( \Delta N_{BG} \) is the uncertainty of \( N_{BG} \), and is assumed to be the same in the four different kinds of samples. The background does not affect the statistical sensitivity as long as \( \Delta N_{BG} \ll \Delta N_o \) is satisfied: \( \Delta \delta = 1/(2\sqrt{N_o}) \). (Hereafter, the subscript of \( l_2^3 \) of the single \( \mathcal{R} \) parameter, \( \mathcal{R}^T \) and \( \mathcal{R}^{CP} \), is omitted unless it is essential.)

The dominant physics background arises from a two-photon process, especially, \( e^+ e^- \rightarrow e^+ e^- \mu^+ \mu^- \), whose contamination could be a few percent of the signal samples. Since the distributions of the \( \mu^+ \) and \( \mu^- \) of the process is symmetric, the two-photon backgrounds, \( N^{\gamma\gamma} \), in \( N(\mu^+ \mu^-) \) and \( N(\mu^+ e^-) \) samples can be estimated just
by \(N(e^+\mu^+)\) and \(N(\mu^-e^-)\) of two-photon samples, respectively. Therefore, \(\Delta N^{\gamma\gamma}\) is the statistical accuracy of the observed \(N(e^+\mu^\pm)\) samples, so that the second term on the right-hand side in eq.(12) can be ignored compared to the first one, as evaluated in the next section. Furthermore, since the observed \(\mathcal{R}^T\) ratio is expressed as

\[
\mathcal{R}^{T: obs} = \frac{(N^{\text{obs}}; >)}{(N^{\text{obs}}; <)} = \frac{(N^{\tau\tau}; >)}{(N^{\tau\tau}; <)} + \frac{(N^{\gamma\gamma}; >)}{(N^{\gamma\gamma}; <)}
\]

\[
= \mathcal{R}^{T: true} (1 + \zeta_{\gamma\gamma}),
\]

\[\zeta_{\gamma\gamma} \equiv \alpha_{\gamma\gamma}(1 - \frac{\mathcal{R}^{T: obs}}{\beta_{\gamma\gamma}^{T: obs}}) \]  \hspace{1cm} (13)

where the signal sample is here denoted as \(N^{\tau\tau}\) to distinguish it from \(N^{\gamma\gamma}\). Here, \(\alpha_{\gamma\gamma} = (N^{\gamma\gamma}; >)/(N^{\tau\tau}; >))\) is the contamination rate, and \(\mathcal{R}^{T: true}, \mathcal{R}^{T: obs}\) and \(\beta_{\gamma\gamma}^{T: obs}\) are the \(\mathcal{R}^T\) ratios formed by \(N^{\tau\tau}\), \(N^{\text{obs}}\) and \(N^{\gamma\gamma}\) samples, respectively. Any artificial background asymmetry, if it exists, appears to reduce its size by \(\alpha_{\gamma\gamma}\) times, and then becomes ineffectual. The statistical uncertainty of \(\zeta_{\gamma\gamma}\) is \(\Delta \zeta_{\gamma\gamma} \approx \sqrt{2\alpha_{\gamma\gamma}/\sqrt{N^{\tau\tau}}}\).

Another dominant background sample comprises, as will be shown later when discussing a simulation study, an electron from \(\tau \rightarrow e\nu\pi\) decay and a pion predominantly from \(\tau \rightarrow \pi\nu\). This is because a small fraction of hadrons would be mis-identified as muons by a muon counter due to their punch through, while electrons would be correctly identified with high purity by an electromagnetic calorimeter. With this knowledge in mind, we argue on how to control the effect of the mis-identified samples on \(\mathcal{R}^T\). Let us first suppose to have prepared an \(e + X\) sample set, \(N^{\text{obs}}_{e+X}\), by applying some kinematical conditions and electron identification on collected data samples, as is done in the next section. The sample comprises an electron and an opposite-charged particle, \(X\), being a muon or hadron. Next, relying upon a muon identification, classify the samples into \(e + \mu\) samples, \(N^{\text{obs}}_\mu\), or \(e + \Pi\) samples, \(N^{\text{obs}}_\Pi\), where \(\Pi\) means a hadron. With \(\kappa_\mu\) and \(\kappa_\Pi\) being the probabilities of a muon and a hadron to be classified as a muon, respectively, and \(N^{true}_\mu\) and \(N^{true}_\Pi\) being the true numbers of \(e + \mu\) and \(e + \Pi\) samples in \(e + X\), these samples relate among them as \(N^{\text{obs}}_\mu = \kappa_\mu N^{true}_\mu + \kappa_\Pi N^{true}_\Pi\), \(N^{\text{obs}}_\Pi = (1 - \kappa_\mu)N^{true}_\mu + (1 - \kappa_\Pi)N^{true}_\Pi\), and \(N^{true}_X = N^{true}_\mu + N^{true}_\Pi\). The observed \(\mathcal{R}^T\)-ratio is then expressed as

\[
\mathcal{R}^{T: obs} = \frac{(N^{\text{obs}}_\mu; >)}{(N^{\text{obs}}_\mu; <)} = \frac{\kappa_\mu (N^{true}_\mu; >)}{\kappa_\mu (N^{true}_\mu; <)} + \frac{\kappa_\Pi (N^{true}_\Pi; >)}{\kappa_\Pi (N^{true}_\Pi; <)}
\]

\[
= \mathcal{R}^{T: true} (1 + \zeta_\Pi),
\]

\[\zeta_\Pi \equiv \frac{\kappa_\Pi}{\kappa_\mu} \left[ \frac{(N^{true}_\Pi; >)}{(N^{true}_\mu; >)} - \frac{(N^{true}_\Pi; <)}{(N^{true}_\mu; <)} \right], \]  \hspace{1cm} (15)

where the \(\mathcal{R}^{T: true}\) is formed by \(N^{true}_\mu\) samples, and \(\kappa_\mu\) and \(\kappa_\Pi\) are irrelevant of the sign of \(A\). The \(\zeta_\Pi\) is approximated in terms of the observed ratios as

\[
\zeta_\Pi \approx \kappa_\Pi \alpha_X (1 - \frac{\mathcal{R}^{T: obs}}{\beta_X^{T: obs}}), \]  \hspace{1cm} (17)
where \( \alpha_X = (N_{\text{obs}; X;} >) / (N_{\mu; X;} >) \) and \( \beta_X^{T; \text{obs}} \) is a \( R^{T; \text{obs}} \) ratio formed by the \( e + X \) samples. Since \( \kappa_\mu \) and \( \kappa_\Pi \) are in the most cases about 90% and 1%, respectively, any higher order term in an approximation of eq. [17] can be disregarded. The \( \zeta_\Pi \) is thus a product of two elements. One is the detector performance of the hadron mis-identification probability, and the other is \( \alpha_X \) times the \( R^{T; \text{obs}} \) and \( \beta_X^{T; \text{obs}} \) difference. The magnitude of the former element is on an order of \( O(10^{-2}) \), and the latter would be zero within the statistical accuracy under \( T/CP \) invariance. By taking into account the correlation between \( R^{T; \text{obs}} \) and \( \beta_X^{T; \text{obs}} \), the statistical uncertainty of \( \zeta_\Pi \) is 
\[
\Delta \zeta_\Pi \approx \kappa_\Pi \sqrt{2 \alpha_X (1 + \alpha_X)}/\sqrt{N_{\text{obs}} \mu}.
\]

It is worth mentioning that since both statistical uncertainties of \( \zeta_{\gamma\gamma} \) and \( \zeta_\Pi \) are inversely proportional to the squareroot of the signal samples, the achievable sensitivity on \( \delta \) is improved with increased integrated luminosity.

4. Simulation Study

Using a fast BELLE detector-simulator [8] for \( p_T >0.1 \) GeV/c tracks, the \( QQ \) generator [9] and KORALB [10] are used for \( e^+e^-\mu^+\mu^- \) two-photon process, \( BB \), continuum, and lepton-pair productions, and \( \tau \)-pair production, respectively, in the collision of a 3.5-GeV positron and an 8-GeV electron for an integrated luminosity of 10\( fb^{-1} \). The following selection criteria are imposed on generated samples, where kinematical variables are evaluated in the lab-frame:

1. The sample should comprise only 2 charged tracks with opposite charges and no photon. The photon is defined as having an energy of \( E >100 \) MeV (cut-1).

2. In order to remove lepton-pair productions, four conditions are required: Each track should have a momentum of \( p <6 \) GeV/c; their sum should be \( p_{\text{sum}} <9 \) GeV/c; the missing transverse momentum squared would be \( (\Sigma p_t)^2 >0.02 \) (GeV/c)\(^2 \); the direction of the missing momentum should be \( \cos \theta_{\text{missing}} <0.990 \) (cut-2).

3. The polar-angle and momentum apartures for tracks are set to fit those of both an electromagnetic calorimeter (ECL) and a muon counter (KLM: \( \kappa_\mu = 90\% \) and \( \kappa_\Pi = 1\% \)) of the barrel region as \(-0.642 < \cos \theta < 0.860 \) and \( P >0.5 \) GeV/c (cut-3).

4. A track which meets the following conditions is assigned as an electron: ECL counts it as an electron, but KLM does not count it as a muon, and the electron probability evaluated by a set of the central tracking chamber (CDC), the aerogel Cerenkov counter (AER), and the time-of-flight counter (TOF) is >1%. A track with \( p >1.2 \) GeV/c is assigned as \( X \) when it is not regarded as an electron by ECL, and its evaluated probability by the CDC/AER/TOF to be either a muon or any hadron is >1%. The requirement \( p >1.2 \) GeV/c is imposed due to
the $KLM$ performance for particle identification.

The classification of $X$ into $\mu$ or $\Pi$ is performed according to $KLM$ information. The resulting samples which pass the above criteria are summarized in Table 1.

Concerning the two-photon process, the equivalent photon approximation $QQ$ generator employed might not be accurate enough in the case an electron or a positron scattered at a large angle. Therefore, their numbers given in Table 1 should be considered to be a brief estimate. However, the arguments discussed in the previous section are indeed valid. According to Table 1, $N^{\tau\tau} (=N_o) =$34.8K and $N^{\gamma\gamma} \approx 700$: $\alpha_{\gamma\gamma}$ is 2% and the second term of eq.(12) is $1/50$-times smaller than the first one, and thus can be disregarded. $\Delta \zeta_{\gamma\gamma}$ is $\approx 1 \times 10^{-3}$. The two-photon $e^+e^-\mu^+\mu^-$ background thus does not yield an appreciable contribution.

Since $B\bar{B}$ and continuum samples produce high multiplicities of both charged tracks and photons, they are reduced to a negligible level. Although $\mu$-pair samples cannot be kinematically removed by cut-2, their contaminations in the $e\mu$ samples are sufficiently rejected by the particle-identification performance.

For $\tau$-pair production, the selected and classified sample rates are listed in Table 2. The acceptance for the $e\mu$ reaction is 1% to the generated $\tau$-pair samples, and the expected total number of samples is $N_{e\mu}=139K$, among which mis-identified samples amount to 1%. The $\alpha_X$ is about 2 in this case, and the $\Delta \zeta_\Pi$ is $\approx 2 \times 10^{-4}$. The expected statistical sensitivity is $\Delta \mathcal{R}=0.01$ or $\delta=0.0027$. The $A$ asymmetry parameters calculated in the CM-frame are given in Fig.2. The dip structure at $A=0$ is due to an effect of the $(\Sigma p_t)^2 >0.02$ (GeV/c)$^2$ and $\cos \theta^{missing}$ cuts. Removing these cuts yields a 10% increase of the $N_o$ samples, and makes $N^{\gamma\gamma}$ samples twice but enhances $\Delta \zeta_{\gamma\gamma}$ only by $\sqrt{2}$ times.

Among the $e\mu$ samples, the backgrounds whose tracks are correctly particle-identified

| Table 1: Selected sample rates satisfying the selection criteria discussed in the text. $B\bar{B}$, continuum, and lepton pair samples are generated for an integrated luminosity of $10fb^{-1}$. For $e\overline{\nu}\mu\overline{\nu}$ two-photon reaction, the resulting numbers for a generated 10M samples are normalized to 700M. |
|-----------------|---------|---------|---------|---------|---------|---------|
| Mode            | $B^oB^\bar{o}$ | $B^+B^-$ | conti.  | $\tau\tau$ | $\mu\mu$ | $e\overline{\nu}$ | $e\overline{\nu}\mu\mu$ |
| Generated samples | 7.5M    | 7.5M    | 45M     | 13.5M   | 13.5M   | 13.5M   | 700M       |
| Selected samples |          |         |         |          |         |         |            |
| $e^+\mu^-$ and $e^-\mu^+$ | 3       | 0       | 5       | 139.3K  | 0       | 0       | 2.8K       |
| $e^+e^-$        | 2       | 0       | 4       | 78.3K   | 0       | 434     | 0          |
| $\mu^+\mu^-$   | 2       | 1       | 5       | 60.1K   | 32.1K   | 0       | 70.5K      |
Table 2: Selected and classified sample rates for 13.5M $\tau$-pairs. The right-most column in (b) indicates the contamination rate due to particle mis-identification.

| (a) Samples passed cut-1: | 2,472K |
| cut-2: | 2,051K |
| cut-3: | 1,124K |

| (b) Classified mode: | Samples | Accepted rates(%) | mis-PID’ed rates(%) |
| $e^\pm\mu^\pm$: | 139.3K | 2×0.52% | 1.1% |
| $e^+e^-$: | 78.3K | 0.58% | 0.04% |
| $\mu^+\mu^-$: | 60.1K | 0.45% | 2.1% |

| (c) Contents of $N_X$ | $N_{e^+X}$ | $N_{e^-X}$ | $N_{e^+\pi^0}$ | $N_{e^-\pi^0}$ |
| $N_{e^+X}$: | 145.1K | 48.2% | 51.8% |
| $N_{e^-X}$: | 144.5K | 48.0% | 52.0% |

comprise a muon from $\tau \rightarrow \mu \nu \bar{\nu}$ and an electron from a single Dalitz mode of $\pi^0$ decay following the decay sequence of $\tau \rightarrow \rho \nu$ and $\rho \rightarrow \pi^\pm \pi^0$. The rate of this background consists of only 9 samples for 13.5M generated $\tau$-pairs. We can therefore disregard it.

The above fact causes us to consider another background. Namely, an electron produced through pair conversion or Compton scattering of photons by detector materials could play a similar role as the electron from single Dalitz decay. In order to briefly evaluate this background, we calculated the rate of (single $\mu$-track) + (single photon) samples for which the momentum and polar angle apartures are set to be the same as those of cut-3. It is $4 \times 10^{-4}$ to the generated $\tau$-pairs, or 7% to the final $e\mu$ samples. The material thickness around the collision point is 2% of the radiation length for the BELLE detector [7,8]. This yields $\approx 2\%$ of the pair conversion and $\approx 0.2\%$ of the Compton-scattering rates for photons with $E > 500\text{MeV}$. Presuming that the probability for only one of the pair-conversion electrons detected is less than 10%, a photon would yield an objective electron by 0.5% at most. As a result, the background rate to $e\mu$ samples could be $3 \times 10^{-4}$ or less, so that its effect on $\Delta R$ can be totally disregarded.

Consequently, with an integrated luminosity of 10$\text{fb}^{-1}$, we can reach a $\delta$ sensitivity of 0.003, for which the background effects are at most 1-2 orders of magnitude small. Furthermore, a 100$\text{fb}^{-1}$ accumulated luminosity allows us to achieve a sensitivity of 0.001.

5. Conclusion

In addition to studying the $CP$ violation of $B$ mesons, the $B$-Factory would also provide a good playground for testing the $T/CP$ invariance of the $\tau$ lepton in the re-
action of $e^+e^- \rightarrow \tau^+ \tau^-$ with $\tau \rightarrow \mu/e\nu \bar{\nu}$. This experiment is rather simple and does not require any spin-related information, such as a polarized beam or a polarization measurement of the final state leptons. With a $10 fb^{-1}$ of integrated luminosity, the $T/CP$ invariance can be tested with an accuracy of $O(10^{-3})$. Furthermore, both the statistic and systematic sensitivities could be improved with additional data samples.

Other $\bar{\mathcal{R}}$ ratios, such as $\mathcal{R}_{e^+e^-}$, $\mathcal{R}_{\mu^+\mu^-}$, $\mathcal{R}_{e\pi}$, $\mathcal{R}_{\mu\pi}$, and $\mathcal{R}_{\pi\pi}$, can also be constructed with a $\pi$ from $\tau \rightarrow \pi\nu$ decay, in addition to a muon and an electron from $\tau \rightarrow \mu/e\nu \bar{\nu}$. For those cases, contamination of $\mu^+\mu^-$ pair production should be brought down to a sufficiently small level; also, high purity of pion identification is indispensable.

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**Figure Captions**

**Fig.1:** Schematic relation among $T$, $CP$, and $CPT$ reflections. $l_2^+ l_3^- > 0$ denotes the reactions which the positive and negative charged particles, $l_2$ and $l_3$, form either $A > 0$ or $A < 0$. The $T$ transformation changes the sign of $A$, and $\delta$ denotes its violation portion in the data samples. $CP$ changes the particles to anti-particles and reverses the sign of $A$ according to our convention. $CPT$, whose violation portion is denoted as $\Delta$, also changes the particles to anti-particles, but not the sign of $A$. Any triangle route results in a vanishing violation, as it must.

**Fig.2:** Resulting $A$ distributions for 13.5 M $\tau$-pairs, corresponding to an integrated luminosity of $10 fb^{-1}$. (a) is for the samples selected as $\mu e$ according to the criteria mentioned in the text. (b) is for the particle mis-identified background contamination in the sample of (a).
Fig. 1: Schematic relation among $T$, $CP$, and $CPT$ reflections. $l_2^+ l_3^-$ or $l_3^+ l_2^-$ denotes the reactions which the positive and negative charged particles, $l_2$ and $l_3$, form either $A > 0$ or $A < 0$. The $T$ transformation changes the sign of $A$, and $\delta$ denotes its violation portion in the data samples. $CP$ changes the particles to anti-particles and reverses the sign of $A$ according to our convention. $CPT$, whose violation portion is denoted as $\Delta$, also changes the particles to anti-particles, but not the sign of $A$. Any triangle route results in a vanishing violation, as it must.
Fig. 2: Resulting $A$ distributions for 13.5 M $\tau$-pairs, corresponding to an integrated luminosity of $10 fb^{-1}$. (a) is for the samples selected as $\mu e$ according to the criteria mentioned in the text. (b) is for the particle mis-identified background contamination in the sample of (a).