Golden-rule capacity allocation for distributed delay management in peer-to-peer networks

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Abstract

We describe a distributed framework for resources management in peer-to-peer networks leading to “golden-rule” reciprocity, a kind of one-versus-rest tit-for-tat, so that the delays experienced by any given peer’s messages in the rest of the network are proportional to those experienced by others’ messages at that peer.

I. INTRODUCTION

Consider a networked group of peers\(^1\) handling queries. In this paper, we consider a cooperative group of peers operating, e.g., in support of a content-distribution system [4], [2] or a social network [6]. Our framework is similar to the cooperative Jackson-network model of [16] whose aim is to minimize delays, however we assume that the peers act in distributed/decentralized fashion [15]. We also assume that the aim of all peers is to achieve “golden-rule” reciprocity where the service capacity they allocate to foreign/transit queries results in delay that is proportionate to that experienced by exogenous (local) queries entering the network at that node.

This note is organized as follows. We motivate and set-up the problem in Section II. Our solution for golden-rule reciprocity is given in Section III. We discuss distributed computation of an eigenvector problem associated with the solution in Section IV. In Section V, a numerical example is given and we conclude with mention of future work in Section VI.

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\(^1\)Alternatively, a peer may be called a user or station, or, in the context of a network graph, a peer may be called a node or vertex.
II. Motivation and Problem Set-Up

We assume queries randomly forwarded so it’s possible that a query may be forwarded back to its originating peer. Random forwarding in this manner may occur under “onion” routing [1] for purposes of privacy protections. For example, suppose that two peers $i$ and $j$ that are connected ($r_{i,j}, r_{j,i} > 0$) share a private, symmetric cryptographic key $k_{i,j}$, e.g., [11]. Suppose when $i$ sends a query directly to $j$, and $j$ forwards the query with $j$’s return address, i.e., $j$ acts as $i$’s proxy - so, it’s possible that a peer later relaying this query will send it to back to $i$ (as a query).

A. Flow-balance for Randomized Forwarding

A peer may have need to match a query to a cache of previously handled queries [6] or using some other type of forwarding-information base; thus queries may take a variable amount of time to process. Each network peer $i \in \{1, 2, ..., N\}$ is assumed to have a limited ability to service queries, herein modeled by i.i.d. exponentially distributed service times with mean $1/\mu_i$.

Peers not only consider their query burden (or those of their clients), for peer $i$ modeled as external arrival process with mean rate $\lambda_{0,i}$, but also relay/resolve queries on behalf of a much larger population of peers.

Each peer can also control to which other peers it forwards queries which it cannot resolve. Assume that peer $i$ can resolve a query with probability $r_{i,0}$.

Suppose that the graph is connected so that the network is stable, i.e., there exists a set of routing parameters $r_{i,j} \geq 0$ such that the solution of flow-balance equations

$$\Lambda_i = \lambda_{0,i} + \sum_{j=1}^{N} \Lambda_j r_{j,i}, \quad \text{with} \quad 1 = \sum_{j=0}^{N} r_{i,j},$$

such that $\Lambda_i < \mu_i$ for all $i$, where $r_{ij} = 0$ if $i$ and $j$ are not directly connected. Defining the $N \times N$ routing matrix $R = [r_{i,j}]$ for $1 \leq i, j \leq N$ and identity matrix $I$, we can collectively write the the flow-balance equations as

$$\Lambda'(I - R) = \lambda_0' \quad \Rightarrow \quad \Lambda' = \lambda_0' B,$$

where

$$B = [b_{i,j}] := (I - R)^{-1}, \quad \text{i.e.,} \quad \Lambda_i = \sum_{j=1}^{N} b_{i,j} \lambda_{0,j} \quad \forall i.$$  

We assume that $R$ is strictly sub-stochastic (not all $r_{i,0}$ are zero) so that $I - R$ is non-singular. Note that since $R \geq 0$ (a non-negative matrix with all entries $r_{i,j} \geq 0$) then also

$$B = (I - R)^{-1} = I + R + R^2 + R^3 + ... \geq 0.$$  

(1)
Note that $b_{i,i} \geq 1$, i.e., $\Lambda_i \geq \lambda_{0,i}$ (again, some queries may return (as queries) to their originating peer).

**B. Queues for Local and Foreign Queries at each Peer**

Each peer $i$ has two queues, one for its local load $b_{i,i} \lambda_{0,i}$ and one for its foreign load $\Lambda_i - b_{i,i} \lambda_{0,i}$, respectively served at rates $\mu_{0,i} > b_{i,i} \lambda_{0,i}$ and $\mu_i - \mu_{0,i} > \Lambda_i - b_{i,i} \lambda_{0,i}$, $\forall i$, (2) where we have made an implicit assumption of queue stability on the available service capacities $\mu_i, \mu_{0,i}$. These two queues are effectively multiplexed before forwarding to peers in $\{0, 1, 2, ..., N\}$ because, for simplicity of exposition herein, we assume the same forwarding probabilities $r$ for both queues of a given peer.

Note that this can be easily generalized to different forwarding probabilities for the local and foreign load of a given peer, with good reason from a modeling perspective; e.g., a peer may be able to resolve (forward to peer 0) a local query with different probability than a foreign one.

**C. The Jackson Network Model**

Assuming Poisson external arrivals (again, at rates $\lambda_{0,i}$) gives a Jackson network model with $2N$ queues. The mean number of local queries queued at peer $i$ is

$$L_{i,i} = \frac{b_{i,i} \lambda_{0,i}}{\mu_{0,i} - b_{i,i} \lambda_{0,i}}$$

The mean number of queries in $j$’s foreign-traffic queue is

$$L_j := \frac{\Lambda_j - b_{j,j} \lambda_{0,j}}{\mu_j - \mu_{0,j} - (\Lambda_j - b_{j,j} \lambda_{0,j})}$$

The mean number of $i$’s jobs in $j$’s foreign-traffic queue is

$$L_{i,j} := \frac{b_{i,j} \lambda_{0,i}}{\Lambda_j - b_{j,j} \lambda_{0,j}} \cdot \frac{\Lambda_j - b_{j,j} \lambda_{0,j}}{\mu_j - \mu_{0,j} - (\Lambda_j - b_{j,j} \lambda_{0,j})} = \frac{b_{i,j} \lambda_{0,i}}{\mu_j - \mu_{0,j} - (\Lambda_j - b_{j,j} \lambda_{0,j})}.$$
The mean delay (disutility) of foreign traffic at peer $i$ is

$$\frac{1}{\mu_i - \mu_{0,i} - (\Lambda_i - b_{i,i}\lambda_{0,i})}.$$  

Using Little’s theorem [17], we can write the disutility of (delay, cost to) peer $i$ as a combination of the total mean delay of peer $i$’s traffic and the mean local delay of foreign traffic at peer $i$ (of all other traffic), the latter modified by an “altruism” (cooperation) factor $\alpha_i > 0$:

$$C_i = \frac{1}{\lambda_{0,i}} \sum_{j=1}^{N} L_{i,j} + \alpha_i \frac{1}{\mu_i - \mu_{0,i} - (\Lambda_i - b_{i,i}\lambda_{0,i})} \times$$

$$\frac{b_{i,i}}{\mu_{0,i} - b_{i,i}\lambda_{0,i}} + \sum_{j \neq i} \frac{b_{i,j}}{\mu_j - \mu_{0,j} - (\Lambda_j - b_{j,j}\lambda_{0,j})} + \alpha_i \frac{1}{\mu_i - \mu_{0,i} - (\Lambda_i - b_{i,i}\lambda_{0,i})}.$$

III. OPTIMAL SERVICE ALLOCATIONS FOR GOLDEN-RULE RECIPROCITY

Fixing $\mu_j$ for all $j$, note how $C_i$ also depends on $\mu_{0,j}$ for all $j \neq i$, subject to (2). Solving the first-order Nash equilibrium conditions with $\mu_{0,i}$ the play actions:

$$\frac{\partial C_i}{\partial \mu_{0,i}} = 0 \quad \forall i,$$

leads to the unique solution

$$\mu_{0,i}^* = \frac{\sqrt{b_{i,i}}}{\sqrt{b_{i,i} + \sqrt{\alpha_i}}} (\mu_i - \Lambda_i) + b_{i,i}\lambda_{0,i}$$

$$\Rightarrow \mu_i - \mu_{0,i}^* = \frac{\sqrt{\alpha_i}}{\sqrt{b_{i,i} + \sqrt{\alpha_i}}} (\mu_i - \Lambda_i) + \Lambda_i - (b_{i,i}\lambda_{0,i}).$$

Since $C_i$ is concave in $\mu_{0,i}$ for all $i$, $\mu_{0,i}^*$ is the unique Nash equilibrium [14]. Thus, (2) holds when all $\alpha_i \in (0, \infty)$, with mean delay diverging in the foreign queue as $\alpha \downarrow 0$ ($i$ increasingly selfish), and mean delay diverging in the local queue as $\alpha \to \infty$ ($i$ increasingly selfless).

A. Golden-Rule Reciprocity

Rather than exploring the efficiency of Nash equilibria according to a global “social welfare” function (as in e.g., [8]), suppose we simply want a Nash equilibrium such that: for all peers $i$, the mean delay experienced by $i$’s queries in the rest of the network is proportional to the the
mean delay in $i$’s foreign-traffic queue. That is, using $\mu_i = \mu^*_i$ for all $i$, there is a constant $\kappa$ (constant across all peers) such that:

$$\forall i, \sum_{j \neq i} \frac{1 + \sqrt{b_{i,j}/\alpha_j}}{\mu_j - \Lambda_j} b_{i,j} = \kappa \frac{1 + \sqrt{b_{i,i}/\alpha_i}}{\mu_i - \Lambda_i}. \quad (4)$$

Recall $B := (I - R)^{-1} \geq 0$ by (1), and define a zero-diagonal version of $B$,

$$\tilde{B} := B - \text{diag}\{B\} \geq 0,$$

where $\text{diag}\{B\}$ is the diagonal matrix with entries $b_{i,i}$ (i.e., those of $B$).

**Lemma 3.1:** If a square matrix $R \geq 0$ is irreducible, then the following matrix is also non-negative and irreducible:

$$(I - R)^{-1} - \text{diag}\{(I - R)^{-1}\} =: \tilde{B}. \quad (5)$$

**Proof:** Clearly $\tilde{B} \geq 0$ by (1), i.e., since $B := (I - R)^{-1} \geq 0$. Irreducibility is a property of the off-diagonal elements of a matrix that are zero. $\tilde{B}$ is irreducible because

$$\forall i \neq j, \quad \tilde{b}_{i,j} = (R + R^2 + R^3 + ...)_{i,j} \geq r_{i,j},$$

and $R$ is irreducible by hypothesis. \hfill \blacksquare

Now since $\tilde{B} \geq 0$ is irreducible, by the Perron-Frobenius theorem [7] (see also Theorem 5, p. 9, of [13]), there is a positive right-eigenvector $\underline{v}$ corresponding to a maximal eigenvalue ($\kappa > 0$) of $\tilde{B}$, i.e., $\tilde{B} \underline{v} = \kappa \underline{v}$ as (4).

Clearly then, to achieve golden-rule reciprocity (4), each peer $i$ should set their altruism parameter such that

$$v_i = \frac{1 + \sqrt{b_{i,i}/\alpha_i}}{\mu_i - \Lambda_i}$$

$$\Rightarrow \forall i, \quad \alpha_i = b_{i,i}(v_i(\mu_i - \Lambda_i) - 1)^{-2}, \quad (5)$$

which is feasible when $\forall i, \quad v_i(\mu_i - \Lambda_i) > 1$, i.e.,

$$\forall i, \quad \mu_i > v_i^{-1} + \Lambda_i. \quad (6)$$
B. Summary

In our Jackson network setting, given the external demand $\lambda_0$ and assuming strictly substochastic $R$, and hence $\bar{B} \geq 0$, are irreducible:

1) the total loads $\Lambda$ are naturally computed in distributed fashion by flow-balance,
2) peers compute the strictly positive eigenvector $\nu > 0$ of $\bar{B}$ in distributed fashion (cf., discussion below),
3) each peer allocates sufficient service capacity $\mu$ to achieve (6) (alternatively, if service capacity $\mu_i$ of peer $i$ is limited, then their demand $\lambda_{0,i}$ is reduced/thinned to achieve (6)),
4) each peer sets their altruism parameter $\alpha$ as (5), and
5) each peer sets their service allocations for their foreign and local queues according to (3).

IV. DISCUSSION: ACHIEVING GOLDEN-RULE RECIPROCITY IN DISTRIBUTED/DECENTRALIZED FASHION

The routing information $R$ could simply be shared among all the peers in the manner of link-state routing based on Dijkstra’s algorithm [11]. Alternatively, Section 2 of [10] describes a method for distributed computation of the principle eigenvector of a routing/graph matrix based on orthogonal iteration and fast-mixing random walks [9]. Though the eigenvectors of $R$ and $B = (I - R)^{-1}$ coincide, those of $R$ and $\bar{B} = B - \text{diag}\{B\}$ necessarily do not. To accommodate $\bar{B}$, we now specify a modification of the orthogonal iteration stated as Algorithm 1 of [10] (where their $Q = \nu > 0$ and their $A = \bar{B}$). Let $\Omega(\nu)$ be the normalization of $\nu$.

0: initialize $B_0 = I$ and $\nu_0$ as a random, non-zero, and non-negative vector.

$k$: For step $k > 0$, do:

k.1: $B_k = I + B_{k-1}R$

k.2: $v_k = \Omega((B_k - \text{diag}B_k)v_{k-1})$

k.3: If the change in $\nu$ and $B$ are not negligible, go to step $k + 1$, else stop.

Note that $B_k$ converges to $B = (I - R)^{-1}$ (again, $R$ is strictly sub-stochastic by assumption). The approach used for decentralized (ortho)normalization of Section 2.2 of [10], again based on fast-mixing random walks [9], can also be used to update the $B$ matrix in step k.1.
V. Numerical Example

For a simple example network of three peers\(^2\) with routing probabilities

\[
R = \frac{1}{6} \begin{bmatrix}
0 & 2 & 3 \\
2 & 0 & 3 \\
3 & 1 & 0 \\
\end{bmatrix}
\Rightarrow B = \begin{bmatrix}
2.062 & 0.937 & 1.500 \\
1.312 & 1.687 & 1.500 \\
1.250 & 0.750 & 2.000 \\
\end{bmatrix},
\]

and maximum eigenvalue of \( \tilde{B} := B - \text{diag}(B) \) is \( \kappa = 2.366 \) with associated positive right-eigenvector

\[
v = [0.576 \ 0.641 \ 0.507]^\prime.
\]

If exogenous loads are \( \lambda_0 = [1 \ 2 \ 1]^\prime \), then by flow-balance, total loads are

\[
\Lambda' := \lambda'_0 B := \lambda'_0 (I - R)^{-1} = [5.9 \ 5.061 \ 6.5].
\]

Taking the service rates \( \mu \) sufficiently large to satisfy (6), e.g., as

\[
\underline{\mu} = [8 \ 7 \ 9]^\prime > [7.636 \ 6.621 \ 8.472]^\prime = v^{-1} + \Lambda,
\]

the golden-rule cooperation parameters \( \alpha \) are computed using (5):

\[
\alpha = [46.9 \ 28.6 \ 28.0]^\prime.
\]

Finally, the bandwidth allocations for the local and foreign queues under golden-rule cooperation according to (3) are, respectively,

\[
\underline{\mu}_0 = [2.43 \ 3.75 \ 2.32]^\prime
\Rightarrow \underline{\mu} - \underline{\mu}_0 = [5.57 \ 3.25 \ 6.68]^\prime
\]

VI. Future Work

Variations of the disutilities \( C_i \) given above can be studied, possibly leading to similar golden-rule reciprocity solution frameworks.

Also, it’s well known how peer-to-peer systems may experience degraded performance due to various types of selfish behavior, see e.g., [5], [3]. Future work on this problem includes consideration of how to police and mitigate defectors from golden-rule reciprocity, particularly

\(^2\)Computed at http://www.bluebit.gr/matrix-calculator
in the process through which routing information $R$ is shared among the peers and the maximal, positive eigenvalue $\kappa$ of is computed. That is, some peers may want their traffic to be treated better by their peers than they themselves treat their peers’ traffic.

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