Issues with rounding in the GCC implementation of the ISO 18037:2008 standard fixed-point arithmetic

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Abstract—We describe various issues caused by the lack of rounding in the gcc compiler implementation of the fixed-point arithmetic data types and operations. We demonstrate that there is no rounding in the conversion of constants, conversion from one numerical type to a less precise type and results of multiplications. Furthermore, we show that mixed-precision operations of fixed-point arithmetic lose precision on arguments, even before carrying out arithmetic operations. The ISO 18037:2008 standard was created to standardize C language extensions, including fixed-point arithmetic, for embedded systems. Embedded systems are usually based on ARM processors, of which approximately 100 billion were manufactured by now. Therefore, the observations about numerical issues that we show in this paper can be rather dangerous and are important to address, given a wide ranging types of applications that these embedded systems are running.

Index Terms—fixed-point arithmetic, rounding, ISO18037:2008

I. INTRODUCTION

The ISO 18037:2008 standard [1] defines C programming language extensions to support various unconventional features of embedded processors. Embedded processors are usually low power/performance processors found in trains, planes, fabrication equipment and communication devices [2]. Another notable example is battery-powered medical devices using integer processors such as the ARM Cortex-M3 [3]. One of the main features that the ISO 18037:2008 standard addresses is fixed-point arithmetic and numerical data types for embedded processors. The standard aims to move away from embedded software designed in assembly languages to a more portable and reusable C programming language, since code is getting bigger and new platforms are rapidly being developed with each new one requiring assembly level changes.

Since these processors need to be extremely low power, floating-point hardware support is not affordable and either hardware fixed-point support is provided, or more commonly integer arithmetic instructions are used to simulate fixed-point arithmetic. However, as the standard states, the C programming language does not provide support for any fixed-point arithmetic types which leads to a common solution of handcrafted arithmetic libraries in assembly languages. The standard aims to improve this situation by defining numerical types and operations that C compilers can support.

In this paper we describe some issues that arise in the gcc compiler implementation of fixed-point arithmetic defined by this standard. Section II gives some background on fixed-point arithmetic. Section III describes the issues with rounding decimal constants to fixed-point data types. Section IV describes lack of rounding in conversions between different types. In Section V we address mixed-format operations and issues with bit truncation of the arguments, due to limited support for mixed-format operations by gcc. Finally Section VI shows that gcc does not support rounding of the results of fixed-point multiplication and that the pragma that should turn on rounding, as defined by the standard, does not work.

All of the experiments are run on an ARM968 processor with the latest gcc compiler version, 9.2.1, cross-compiling the binaries on a macOS 10.15.1 using the flag -O2.

II. FIXED-POINT ARITHMETIC

The standard defines multiple numerical types for fixed-point arithmetic in the form \( \{s, u\} \cdot X.Y\), where \( \{s, u\}\) defines whether it is a signed or unsigned format (if signed, 2's complement representation is used), \( X\) defines the number of integer bits and \( Y\) defines the number of fractional bits. Machine epsilon, or unit roundoff, of a fixed-point type is defined as \( \epsilon_{\{s, u\} \cdot X.Y} = 2^{-Y}\), which is the gap between any two neighbouring fixed-point values and is absolute across the dynamic range. Some notable fixed-point numerical formats supported by GCC are: s16.15, u16.16, s0.31, u0.32, s8.7, u8.8, s0.15, u0.16. Here is one example in s16.15. We have a real value 1.5 represented as s16.15 data type, where the binary value of it has 14th and 15th bits set to 1, while the other bits set to 0, i.e. 0xC000 in hex, or \( 2^{15} + 2^{14} = 49152\) as an integer. Then this can be converted to a decimal value by multiplying it with \( \epsilon_{s16.15} = 2^{-15}\). 49152 \( \times 2^{-15} = 1.5\).

Table I shows examples of some decimal values of the three main numerical types explored in this paper.

| Property                | s16.15  | u0.32   | s0.11   |
|-------------------------|---------|---------|---------|
| Accuracy                | \(2^{-15}\) (abs.) | \(2^{-32}\) (abs.) | \(2^{-31}\) (abs.) |
| Min (exact)             | \(2^{-15}\)      | \(2^{-32}\)      | \(2^{-31}\)      |
| Min (approx.)           | 0.00000305     | \(2.32 \times 10^{-10}\) | \(4.65 \times 10^{-10}\) |
| Max (exact)             | \(2^{16} - 2^{-15}\) | \(1 - 2^{-32}\) | \(1 - 2^{-31}\) |
| Max (approx.)           | 65535.9999969  | 0.99...   | 0.99...  |
III. Rounding of Constants

The following three quotes can be found in Section 4 and Annex A of the ISO standard [1], dealing with fixed-point number rounding:

**Quote 1:** Conversion of a real numeric value to a fixed-point type may require rounding and/or may overflow. If the source value cannot be represented exactly by the fixed-point type, the source value is rounded to either the closest fixed-point value greater than the source value (rounded up) or to the closest fixed-point value less than the source value (rounded down).

Note that Quote 1 can be interpreted to state that one way rounding is suitable, either round-up or round-down, since it does not mention that the decision has to be done based on the round-off bits.

**Quote 2:** The FX_FULL_PRECISION pragma provides a means to inform the implementation when a program requires full precision for these operations (the state of the FX_FULL_PRECISION pragma is "on"), or when the relaxed requirements are allowed (the state of the FX_FULL_PRECISION pragma is "off"). For more discussion on this topic see A.4. Whether rounding is up or down is implementation-defined and may differ for different values and different situations; an implementation may specify that the rounding is indeterminable.

Quote 2 talks about a pragma that can be set in order to improve the accuracy of arithmetic operations. However, the standard does not mention half unit roundoff accuracy, which could be obtained with round-to-nearest.

**Quote 3:** Generally it is required that if a value cannot be represented exactly by the fixed-point type, it should be rounded up or down to the nearest representable value in either direction. It was chosen not to specify this further as there is no common path chosen for this in hardware implementations, so it was decided to leave this implementation defined.

Quote 3 seems to indicate that rounding should be to one of the two directions, rather than any direction which will give the nearest value.

Firstly, we address rounding of constants. This was commented by us previously in [4] and also noticed by [5]. A constant, for example 0.04, cannot be represented exactly in a finite-precision arithmetic (Table [III] and has to be rounded to the nearest value of the numerical data type. For example, the two nearest values in the integer representation in s16.15 are $[0.04] = 1310$ and $[0.04] = 1311$, round-down and round-up respectively. This corresponds to the real values of 0.039978... and 0.040008.... However, since $0.04 = 1310.72$, it makes most sense to represent 0.04 as $[0.04] = 1311$, since it is closer to the real value of 0.04. That is, round 0.04 to the nearest s16.15 value (or any other given fixed-point format that is being used to store the constant). This operation is done on compilation, when the constant is written into the memory by the compiler, and therefore there is no run-time performance penalty. Unfortunately we found that this was not done by the gcc compiler, which resulted in large total errors due to magnification of these small errors in the constants, for example in ODE solvers [5], [6]. The code for this is

```
accu a = 0.04k;
```

where the letter k is used to indicate that this constant is in s16.15 format (not necessary to use in this context since the destination format is known but we chose to use it for demonstration). Accum data type is another name in C for s16.15 data type.

We believe this to be an issue due to Quote 2 - the pragma that is defined should only be applied to control run-time performance, that is, rounding of various values that come up at run time, not on compilation. And in general, we found that the pragma FX_FULL_PRECISION does not have any effect in gcc and does not turn on rounding neither on compilation nor run time.

| TABLE II |
|---|
| VALUES OF A CONSTANT 0.04 IN DIFFERENT DATA TYPES |

| Data type | round-to-nearest | next nearest |
|---|---|---|
| s16.15 | 0.0400085449218755 | 0.039978027343755 |
| s0.31 | 0.0400000000372513 | 0.0399999995715017 |
| u0.32 | 0.0400000000372513 | 0.0399999998044776 |
| fp32 | 0.0399999991059326 | 0.040000000283125 |

IV. Rounding on Conversion

Here we show that there is no rounding when converting to a fixed-point type a numerical value that is held in a more precise data type. First, we try to convert a value held in s0.31 to s16.15. We choose a value that is smaller than the smallest value representable in s16.15: $2^{-16} + 2^{-17} = 2.288818359375E-5 = 0.75 \epsilon_{s16.15}$, where $\epsilon_{s16.15} = 2^{-15}$.

In C code we write:

```
long fract a = 2.288818359375E-5lr ;
accum b = a ;
```

Here long fract is another name for s0.31 and accum for s16.15. Letters lr next to the constant tell the compiler that this is a s0.31 constant, as defined in the ISO standard. Once this code is executed, b evaluates to 0, rather than the nearest representable value of $2^{-15}$, therefore there is no rounding on conversion.

Another test that demonstrates this involves conversion between fp32 and s16.15

```
float a = 0.04 ;
accum b = a ;
```

This uses the same constant that we have used in the Section [III] which we know is not rounded to the nearest value when specified as a decimal value 0.04 in the source code. In
this case single precision floating-point value of 0.04 is more accurate than s16.15, so the value of 0.04 held as fp32 (which is also not exact) should be rounded to the nearest value of s16.15. However, b still evaluates to the value lower than 0.04, meaning that round-down from fp32 to s16.15 is done rather round-to-nearest.

Therefore, conversion of fixed-point values does not follow the standards definition in Quote 1 or Quote 3.

V. Rounding of Arguments in Mixed-Format Operations

We have observed multiple fixed-point arithmetic routines in the gcc generated assembly with some loss of precision and speed, for example the multiplication of a value in s16.15 by u0.32 is done as the multiplication of two s32.31 values, or the multiplication of s16.15 by u0.16 is done as the multiplication of two s16.15 values. This causes loss of precision on conversion (in the arguments, even before multiplication is performed) and the main reason is that GCC does not support mixed-format multipliers directly, as indicated by a list of internal compiler functions for performing fixed-point arithmetic operations [7]. A test for this is as follows:

\[
\text{unsigned long } \text{frac } a = \text{pow}(2, -32); \\
\text{accum } b = 65535k; \\
\text{unsigned long } \text{frac } c = a * b;
\]

Here we chose \(a = \epsilon_{0.32} = 2^{-32}\) since that is the smallest value representable by u0.32 (only the least significant bit set) and \(b = 65535\) the largest integer value representable by s16.15. Here we expect to get \(c = 65535a\), however we get \(c = 0\) because the last bit of \(a\) is dropped before the multiplication takes place, causing \(a = 0\). Same issue happens irrespective of what \(b\) is set to. Furthermore, we can enclose this code in a conditional execution that checks the values of \(a\) and \(b\) and it executes the conditional code and incorrectly updates \(c\) to 0:

\[
\text{unsigned long } \text{frac } a = \text{pow}(2, -32); \\
\text{accum } b = 65535k; \\
\text{unsigned long } \text{frac } c = a * b; \\
\text{if } (a > 0 \&\& b > 1) \\
\text{c} = a * b;
\]

Lastly, if we modify the code as follows:

\[
\text{long } \text{frac } a = \text{pow}(2, -31); \\
\text{long } \text{frac } b = -11r; \\
\text{long } \text{frac } c = a * b;
\]

(now using signed fractional type s0.31 so we can represent minus one) we do not get \(c = 0\) and instead get a correct multiplication result of \(c = -2^{-31}\). This leads to a major problem: we know that \(a\) is not zero, but multiplying it by a non-zero value with a magnitude larger than 1 sometimes can give an answer of 0 and sometimes a correct answer, depending on the numerical types - for most of the users who do not necessarily think about how exactly arithmetic is performed at the lowest level, this behaviour would be and potentially is very puzzling.

VI. Rounding of Multiplication Results

Here we show that there is no rounding in arithmetic operations with fixed-point numbers, specifically, multiplication. The pragma that is described by Quote 2 does not turn on rounding in gcc. A simple test is with the two s16.15 values, \(a = 3\epsilon_{s16.15} = 0.000091552734375\) and \(b = 0.25\). This should give us \(0.25 \times 3\epsilon_{s16.15} = \frac{3}{4}\epsilon_{s16.15}\) which should round to a nearest value of \(\epsilon_{s16.15}\). The code for this is:

\[
\text{accum } a = 0.000091552734375k; \\
\text{accum } b = 0.25k; \\
\text{accum } c = a \times b;
\]

This piece of code evaluates \(c\) to 0, which means that the result \(\frac{3}{4}\epsilon_{s16.15}\) is rounded down to 0 rather than the closest value of \(\epsilon_{s16.15}\).

VII. Conclusion

We have shown various numerical accuracy issues in the gcc compiler implementation of the standardized fixed-point arithmetic by the embedded C standard [1]. The main issue is lack of rounding in decimal to fixed-point conversion, generally any format to fixed-point conversion and arithmetic operations such as multiplication. Furthermore, there is accuracy loss in the arguments in mixed-format arithmetic operations. In our understanding, the issue happened both because of the vague definitions in the standard and lack of full support of the fixed-point arithmetic in gcc. The arithmetic in gcc should be carefully reimplemented taking care of various edge cases and mixed-format combinations to support the embedded systems community. Our previous work [3] suggests various fast and numerically accurate ways to do mixed-format multiplications. This paper should inform the embedded systems community about the numerical accuracy problems in gcc fixed-point arithmetic as well as help identify numerical problems in their codes.

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