Exploration-RRT: A multi-objective Path Planning and Exploration Framework for Unknown and Unstructured Environments.

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Abstract—This article establishes the Exploration-RRT algorithm: A novel general-purpose combined exploration and path planning algorithm, based on a multi-goal Rapidly-Exploring Random Trees (RRT) framework. Exploration-RRT (ERRT) has been specifically designed for utilization in 3D exploration missions, with partially or completely unknown and unstructured environments. The novel proposed ERRT is based on a multi-objective optimization framework and it is able to take under consideration the potential information gain, the distance travelled, and the actuation costs, along trajectories to pseudo-random goals, generated from considering the on-board sensor model and the non-linear model of the utilized platform. In this article, the algorithmic pipeline of the ERRT will be established and the overall applicability and efficiency of the proposed scheme will be presented on an application with an Unmanned Aerial Vehicle (UAV) model, equipped with a 3D lidar, in a simulated operating environment, with the goal of exploring a completely unknown area as efficiently and quickly as possible.

I. INTRODUCTION AND BACKGROUND

The exploration and mapping of unknown and complex structures has been always one of the main application areas of autonomous robots [1], including the inspection of large-scale infrastructures [2], search-and-rescue missions [3], and subterranean exploration [4]. Specifically, the area of subterranean environments, and in general harsh and GPS-denied ones, has been in focus for autonomous exploration missions with the utilization of autonomous robots. This area has gained a lot of attention in the latest years, especially due to the DARPA subterranean challenge [5], where teams of robots are tasked to explore and identify artifacts in complex and unstructured environments. Together with Simultaneous Localization and Mapping (SLAM), the problem of path planning and exploration behavior [6], [7] is one of the most fundamental problems in such applications, which was the main inspiration for this work as part of the NEBULA autonomy developments [8].

Both path planning and exploration have been cornerstone research areas in the field of robotics and have been massively studied as for example in [9], [10], [11], [12]. At the same time, it should be mentioned that there is a very large range of solutions to the problem of total exploration of unknown areas, including pattern-based solutions [13], frontier [14] or entropy [15] approaches that have been also extensively demonstrated. Occupancy-based path planning (e.g. path planning that relies on a map of occupied and free space) are fundamentally based either on applications of Dijkstra’s Algorithm [16], with modern examples including improved versions of A* [17] or Jump-point-search [18], or on the Rapidly-Exploring Random Tree (RRT) algorithm [19], that also has numerous modern improved versions [20] [21] and is the core component of the proposed path planner as well. The advantage of RRT is its computational efficiency and the ease of adding functionalities [22] on top of the central planning problem, while its downside is that for limited iterations there is no guarantee of finding the shortest path to the goal.

In many applications, the two problems of exploration and planning are solved separately. In [23] a Deep Reinforcement model selects optimal frontiers and an A* algorithm is applied to find the shortest path. In [24] stochastic differential equations are used for identifying optimal frontiers, while a RRT* path planner computes the path.

Incorporating an exploration behavior into the central planning problem is a research area that gained significant attention lately, with multiple proposed solutions [25] [26], where the Next-Best-View planners [27] established the fundamental concept. The flagship modern Next-Best-View planners have seen multiple application scenarios as local path planners [28] [29] to name a few. The core concepts of Exploration-RRT (ERRT) are in the same direction as Next-Best-View planners, while its novelty comes from the difference in the algorithmic implementation where ERRT is explicitly solving and evaluating paths to pseudo-random goals versus
an iterative evaluation of RRT-branches until a sufficiently good branch is found. ERRT novelty is also supported from the ability to compute the optimal actuation required to track the generated trajectory, which is done via solving a receding horizon NMPC problem. In the proposed method the NMPC problem is solved by the Optimization Engine\[30\] an open-source and very fast Rust-based optimization software.

II. Contributions

This article proposes a novel solution to the combined path planning and exploration problem, based on a minimum-cost formulation problem with pseudo-random generated goals combined with multi-path planning and evaluation. The proposed method evaluates the model-based actuation between quickly discovering more space, limiting actuation

optimal trajectory for the exploration task, in a compromise and choosing the minimal-cost solution. An overview of this process is found in Figure II-1 Each component will be explained in more detail in III-C. The process of generating many goals, solving the path to each of them, and then computing the total costs associated with each trajectory, turns \[1\] from a true optimization problem into:

\[
\text{arg min} (J_a(u_i) + J_d(x_i) + J_e(\nu_i)), i = 1, \ldots, n_{goals} \tag{2}
\]

that considers the finding of \(x_{\min} \in V_{\text{free}}\) that is the \(x_i\) trajectory having the lowest cost associated to it, and \(n_{goal} \in \mathbb{N}\) is the overall number of path planner goals. This process will converge towards approximating the complete problem in \(1\), as we increase \(n_{goal}\) and optimize the trajectory-generating algorithm (or simply by increasing the number of iterations of the RRT\(^*\)), as more and more possible solutions are being investigated.

C. The Algorithm

For the sake of notation, let us directly define desired points to explore, or unknown voxels, as \{\(U\}\}, and a binary 3D grid map \(G\) where occupied voxels, \{\(O\)\} and unknown voxels \(\{U\}\) are set to 1, and the resulting \(V_{\text{free}}\) is set to 0. Let us also define the measured vehicle state as \(\hat{x}\).

1) Goal Generation: The fundamental part of ERRT that allows for the exploration behavior, is the generation of pseudo-random goals \(\rho^r\) within \(V_{\text{map}}\). At each call to the algorithm, \(n_{goal}\) goals are generated, under the conditions of being inside the sensor range of at least one unknown voxel \(U \in \{U\}\). Other conditions are \(\rho^r \in V_{\text{free}}\), and the straight path from \(\rho^r\) to the center of \(U\) being obstacle-free (including being blocked by other unknown voxels). While many works focus on onboard cameras \[28\] ERRT considers an onboard 3D lidar, with the advantage in terms of mapping being a 360\(^\circ\) vision in the x-y plane and a long sensor range, and with the drawback of a narrow field-of-view in z close to the platform, thus making narrow 3D spaces a challenge. A simplified sensor model of a 3D lidar is shown in Figure II-3 assuming the lidar is placed flat on the platform, considering only three parameters: the sensor range \(r_3 \in \mathbb{R}\), the field-of-view \(\theta_s \in \mathbb{R}\) and \(\ell_s \in \mathbb{R}\), describing the size of the sensor array.

Assuming a randomly generated point \(\rho^r = [\rho^r_x, \rho^r_y, \rho^r_z] \in \mathbb{R}^3\) and a center-position of an unknown voxel \(U\) as \(p^u = [p^u_x, p^u_y, p^u_z] \in \mathbb{R}^3\), we can evaluate, by assuming small roll/pitch angles in the UAV case, if \(\rho^r\) is inside the space seen by the sensor model if \(r_s \geq \sqrt{(\rho^r_x - p^u_x)^2 + (\rho^r_y - p^u_y)^2}\) and \(| \rho^r_z - p^u_z | \leq \sqrt{(\rho^r_x - p^u_x)^2 + (\rho^r_y - p^u_y)^2 \tan(\frac{\theta_s}{2})} + \frac{1}{2}\).
Fig. 2: Exploration-RRT framework and pipeline (left) and simulation architecture (right).

**Algorithm 1: Goal generation**

**Inputs:** $G$, $\{U\}, n_{\text{goal}}, r_s, \theta_s, l_s$

**Result:** Generate random goals within sensor range

while $i \leq n_{\text{goal}}$

| Provided by Algorithm 1 |
|--------------------------|
| Algorithm 1 where $n_{\text{goal}} \in \mathbb{N}$ denotes the number of unknown voxels, and $\rho^r \in \mathbb{R}^{n_{\text{goal}} \times 3}$ is the list of random goals. The result of this selection process is a list of candidate goals that the multi-goal RRT* can be tasked to plan potential paths to. In the evaluation of the algorithm we will set $r_s$ to 6 m, a very conservative choice as high-end 3D lidar can have ranges of up to 100 m, but it is generally good to under-value sensor ranges in mapping missions to guarantee detection hits. Moreover, $\theta_s$ and $l_s$ are set to 32$^\circ$ and 0.1 m approximating a Velodyne VLP16 Puck LITE.

2) Multi-goal RRT*: While there are numerous RRT implementations, ERRT uses a 3D RRT* structure. Random points $\rho^r$ are generated within $V_{\text{map}}$ and added as an end point $N_{\text{end}}$ to extend the closest graph vertex $N_{\text{closest}}$ in a graph network $\mathcal{N}$, if $\rho^r \in V_{\text{free}}$ and there is no collision/obstacles between the $p_r$ and $N_{\text{closest}}$. In RRT, this process continues until the goal $\rho$ is reached, generally as $| | \rho - N_{\text{end}} | | \leq d$, with $d$ denoting some specified distance. In RRT* the process is instead run for a pre-defined number of iterations, and the shortest generated graph that reached the goal (by the goal condition being satisfied), $N_{\text{goal}} \in \mathcal{N}$ is selected. The ERRT addition to this baseline framework of RRT algorithms, is the consideration of multiple goals, $\rho^r$ provided by Algorithm 1.

This results in Algorithm 1 where $n_{\text{goal}} \in \mathbb{N}$ denotes the number of unknown voxels, and $\rho^r \in \mathbb{R}^{n_{\text{goal}} \times 3}$ is the list of random goals. The result of this selection process is a list of candidate goals that the multi-goal RRT* can be tasked to plan potential paths to. In the evaluation of the algorithm we will set $r_s$ to 6 m, a very conservative choice as high-end 3D lidar can have ranges of up to 100 m, but it is generally good to under-value sensor ranges in mapping missions to guarantee detection hits. Moreover, $\theta_s$ and $l_s$ are set to 32$^\circ$ and 0.1 m approximating a Velodyne VLP16 Puck LITE.

3) Trajectory Actuation via NMPC: Many exploration-planning frameworks are lacking a consideration of the predicted actuation a vehicle will use, while following a generated path. Minimizing actuation is a key component

\[ x(t) = A x(t) + B u(t) \]

where $x(t)$ is the state of the vehicle at time $t$, $A$ is the state transition matrix, $B$ is the input matrix, and $u(t)$ is the control input. The NMPC controller is designed to minimize a cost function that includes a term for the control effort $u(t)$.

Fig. 3: Simplified sensor model of a 3D lidar. $r_s$ denotes the sensor range and $\theta_s$ the field-of-view. $l_s$ denotes the size of the inner scanner array.
in limiting the energy utilization, while providing easier-to-follow (or more feasible) trajectories. Especially when the ERRT is considered for an UAV case a lower-actuation trajectory implies more stable flying behavior, which stresses localization, mapping, or detection software less as there are less rapid movements. Also, rovers or legged robots often traverse slowly or need extra maneuvering to make tight turns that is not at all included in the path selection if the predicted model-based actuation along the path is not considered. In short: only evaluating information gain and the length of the trajectory (ex. [28]) in trajectory selection misses key aspects that are more properly considered by also evaluating the predicted actuation along the trajectory.

In general, there will be differences between the predicted actuation and the real live actuation of the vehicle, but for comparison any computation of information gain is similarly just a gauge for the approximate information gain as there is no way to know, for example, if more unknown voxels are right behind the frontier and would also be discovered. Similarly, solving a receding horizon NMPC problem where the reference set-points along the prediction horizon are the points in the generated trajectory, will result in the optimal actuation based on the provided nonlinear dynamic model of the system with added constraints, and provides an approximation of the required actuation of the real vehicle (or rather the minimal required actuation). The flexibility in defining the NMPC cost function and constraints also allow platform-specific penalties when computing the actuation cost $J_{nmpc}(u)$.

The NMPC approach used in ERRT follows previous works in the literature closely, while the same nonlinear UAV model is used[31], [32], [30], with inputs as $u = [T, \theta_{ref}, \phi_{ref}]$ being references in thrust, roll, and pitch angles, but with a different application in this case. Thus, instead of solving the NMPC problem, with a very short sampling time to compute real-time control signals to a platform, we use a longer sampling time, combined with the interpolation length of trajectory $x_*$, to match a desired predicted behavior and velocity of the vehicle. Let us denote $n_p \in \mathbb{N}^+$ as the prediction horizon, and $k+j|k$ the predicted time step $k+j$ produced at time step $k \in \mathbb{N}^+$. Similarly let $j \in \mathbb{N}^+$ also index the first $n_p$ entries in the path $x$. As an example and based on the UAV case, the cost function, to make each predicted state reach the desired reference set as the entries in trajectory $x_*$, and at each predicted time step, is:

$$J_{nmpc}(x_{p,k}, u_k, u_{k-1}|k) = \sum_{j=0}^{N-1} \left( \|x_j - x_{p,k+j|k}\|^2_{Q_x} + \|u_j - u_{k+j|k}\|^2_{Q_u} + \|u_{k+j|k} - u_{k+j-1|k}\|^2_{Q_{\Delta u}} \right)$$

(3)

where $Q_x, Q_t \in \mathbb{R}^{8 \times 8}$, $Q_u, Q_{\Delta u} \in \mathbb{R}^{1 \times 3}$ are positive definite weight matrices for the states, inputs and input change respectively, $x_{p,k+j|k}$ is the predicted states and $u_{ref}$ is the reference input, commonly set as a steady-state input of the platform. Let us also define input constraints as $u_{min} \leq u_{k+j|k} \leq u_{max}$. This leads to the following optimization problem:

Minimize $J_{nmpc}(x_{p,k}, u_k, u_{k-1}|k)$

subject to: $x_{p,k+j+1|k} = \zeta(x_{p,k+j|k}, u_k, u_{k-1}|k), j \in \mathbb{N}[0,n_p-1]$, $u_{min} \leq u_{k+j|k} \leq u_{max}, j \in \mathbb{N}[0,n_p-1]$, $x_{p,k} = \hat{x}_k$,

with $\zeta(x_{p,k+j|k}, u_k, u_{k-1}|k)$ defining the discrete state model of the platform. The optimization problem is solved, for each generated trajectory from the multi-goal RRT*, to compute $u_i$ with the Optimization Engine [30], following a single-shooting approach. In the following evaluation we solve the problem with $n_p = 50$. Since the actuation is solved as a receding horizon problem, only $n_p$ predicted time steps can be considered and as such, we are solving for the actuation $u$ only for the $n_p$ first entries in $x$. In the case where the trajectory length is lower, the last entry is simply repeated. How far into the trajectory this limit of only considering $n_p$ entries is, depends on the interpolation of $x_*$ and the sampling time $T_s$ of the NMPC. For the following evaluation case we use a sampling time of 0.5 s and an interpolation length 0.75 m, meaning that the desired platform’s velocity to predict and optimize actuation for is 1.5 $\frac{m}{s}$. We should also note that based on the optimal actuation vector $u$ and the initial measured state vector $\hat{x}$, we can compute the full-state dynamic trajectory $x_p$ up until the $N$:th entry, although despite the NMPC trajectory reference tracking, there is no guarantee that $x_p$ is obstacle-free until the obstacle avoidance, based on the known map, is integrated into the NMPC framework. As such, in the following evaluation, we shall stick with a position-trajectory $x$ to guarantee completely obstacle-free paths and only use the NMPC module solution to calculate the predicted actuation cost $J_d(u)$ along the trajectories, defined by the last two terms in [5] the input cost and the input rate cost.

4) Cost Calculation: The final step of the ERRT algorithm is to evaluate each computed path in accordance with [4]. By denoting the number of entries in $x_*$ as $n_i$, the distance cost of each trajectory is easily computed as the sum:

$$J_d(x_i)_k = K_d \sum_{j=1}^{n_i} \| p_i,j - p_i,j-1 \| .$$

(5)

Based on the predicted actuation $u_i$ along the trajectory, the actuation cost can be computed by feeding the actuation vector back into the relevant parts of the NMPC cost function. For the presented case it is

$$J_d(u_i)_j = \sum_{j=1}^{n_p} \| u_{ref} - u_{i,j}\|^2_{Q_u} + \| u_{i,j} - u_{i,j-1}\|^2_{Q_{\Delta u}}$$

(6)

with $u_{ref} = [9.81, 0, 0]$, the input that describes no movement for the UAV, with the thrust value of 9.81 compensating for gravity. The total information gain $\nu$ is calculated by evaluating the information gain, at each point in the trajectory,
as each unknown voxel within sensor view, and without obstructed line-of-sight and removing any duplicates (voxels seen at multiple points in the trajectory). For clarity the process is seen in Algorithm 2.

**Algorithm 2: Information Gain**

**Inputs:** $G, \{U\}, x_i, n_{goal}, r_s, \theta_s, l_s$

**Result:** Information gain along the trajectory $x_i$

For $j = 0, n_i$ do

For $k = 0, n_u$

If in_sensor_view then

If not collision_check then

$\nu = \text{length(remove_duplicates(seen_unknown))}$

Output: $\nu$

Once $\nu$ is computed, as the total number of unknowns that will be in sensor range by following $x_i$, the exploration cost is then computed as:

$$J_e(\nu) = -K_{\nu}\nu$$

(7)

with $K_{\nu}$ denoting a gain representing the relative emphasis on maximizing the information gain. With $J_u(u_i), J_s(s_i), J_d(d_i)$ calculated for $i = 0, \ldots, n_{goal}$ (2) can be evaluated and the $x_i$ related to the minimum-cost solution and denoted by $x_{min}$ is selected as the final result of the algorithm.

5) ERRT input structure: The overall ERRT framework has been implemented in ROS [33], with a custom message including all input parameters to the algorithm, which are summarized in Table I.

| Occupied voxels | \{O\} |
| Unknown voxels | \{U\} |
| Vehicle state | $\bar{x}$ |
| Number of goals | $n_{goal}$ |
| Sensor model parameters | $r_s, \theta_s, l_s$ |
| Cost parameters | $K_d, K_r, Q_u, Q_{\Delta u}$ |
| RRT$^*$ iterations | $iter$ |
| Grid resolution | $g_{res}$ |

**TABLE I: ERRT Initialization Parameters.**

At every call to the algorithm the grid map $G$ is initialized based on point clouds of occupied and unknown voxels, and the user is free to change resolution, size of the map, or any other parameter at every call to the algorithm to meet the desired mission specifications. After calculation is complete the node publishes the computed trajectory. The very general input model was one of the motivations for developing ERRT, allowing for example high-resolution exploration of the local area, and low-resolution computation of what area to explore next, by the same planner with different inputs. The process described in Section III can be summarized in Algorithm 3.

**Algorithm 3: The ERRT algorithm.**

**Inputs:** $G, \{U\}, n_{goal}, r_s, \theta_s, l_s, \hat{x}$

**Result:** Minimum-cost trajectory $x_{min}$

$$\rho^* = \text{generate_goals}(G, \{U\}, n_{goal}, r_s, \theta_s, l_s)$$

$$N_{goal} = \text{multigoal_rrt}(\rho^*, G)$$

$$x_i = \text{trajectory_improvement}(\rho^*, N_{goal}, G)$$

$$u_i = \text{NMPC_module}(x_i, \hat{x})$$

$$(J_a, J_d, J_e) = \text{cost_calc}(\{U\}, G, x_i, u_i, K_d, K_r, Q_u, Q_{\Delta u})$$

$$x_{min} = \text{path_selection}((J_a, J_d, J_e), x_i)$$

Output: $x_{min}$

The proposed method is evaluated in a simulated environment separated from other mapping or frontier-generating software, that considers a nonlinear dynamic UAV model with added small magnitude localization noise, and a sensor model that mimics the model described in III-C.1 with a 1 m longer sensor range for a voxel to be considered discovered than what is considered in the ERRT. The map is initialized with the full space as undiscovered, except a small area around the starting location, and as the center of a voxel comes into sensor range, it is set either as free space or as occupied. As the planner is ROS-integrated, the exploration process can be easily visualized in Rviz[34]. It should be noted that for the sake of visualization, the occupied voxels are always depicted. A full-state reference tracking controller, tuned to approximately match the desired predicted behavior as stated in III-C.3 follows the generated path, until the goal is reached and the path is recalculated.

The considered evaluation environment has been generated to mimic a subterranean cave area with multiple rooms, a larger void area, and several small hard-to-reach nooks and passages, that encompasses a total of 27x27x4 m$^3$, with a grid size of 1 m. The ERRT algorithm was executed for a total of twenty times in the simulated area, ten times with a greedy tuning prioritizing information gain over minimizing distance and actuation and a more conservative tuning with higher costs on distance and actuation as compared to information-gain. Figures 7 and 8 present time sampled images through the evolution of the mission from one of the greedy runs, displaying all the generated paths at critical times during the simulation. We evaluate the simulations in terms of the total distance travelled, and the time required to complete the task, considering both at 90% of voxels discovered as a gauge for the effective volumetric gain, and at full completion.

The results for the greedy tuning can be found in Figure 4 and for the conservative in Figure 5 As it can be observed, a majority of the exploration time is devoted to cleaning up voxels that were ignored along the way due to maximizing the information gain, and exploring the final hard to reach areas to achieve complete coverage of the area. In Figure 6 the volumetric gain (or information gain) from one of the runs is depicted, which also represents the behavior of maximizing the total explored volume as quickly as possible. Both configurations show very consistent results over multiple executions, despite the very different behavior for each tuning, with the greedy tuning over-all
performing better. This is naturally the case in a completely unknown area, as maximizing the information gain is of paramount importance and the exploration missions are often time-constrained by the platform that further promotes the greedy behavior. The average translation speed of the platform was 1.2 m/s, controlled by the interpolation of the trajectories, and the tuning of the reference tracking controller, while mimicking the slower pace of a fully autonomous UAV in a subterranean field application. A video compilation of different exploration executions in the evaluation environment can be found here: https://drive.google.com/file/d/1v3yg3Z9iB2DR-Oec3MxWUG_39d3IjI/view?usp=sharing.

The ERRT algorithm was running 1500 RRT* iterations and the $n_{goa}$ was set to 40, which resulted in solver times of 0.3-0.9 s, averaging at 0.6 s, of which around 0.1 s was consumed by the NMPC module (around 2.5 ms per goal). Although the framework is set up as to optimize over the full trajectory, it needs to be speed up to enable an online implementation where trajectories are recalculated at each execution step, mainly related to the multi-goal RRT*.

V. FUTURE WORK

From the algorithmic perspective, there are two main directions of future work: speeding up the core multi-goal RRT* framework which would expand the planner’s application areas, and adding integrated collision avoidance that considers the size-radius of the platform both in terms of obstacle enlargement in the main RRT* planner and in the NMPC optimization problem to be able to guarantee the actuator-based paths $x_p$ are completely obstacle free. Another interesting direction is adding negative costs related to moving through areas with well-defined features, which are commonly tracked by SLAM software [35], as doing so could improve localization accuracy. This would allow the planner to also consider the localization cost, and can be implemented into the framework in a very straight-forward way assuming that the locations of such features is provided by the SLAM software. From the implementation perspective we aim to release the framework as an open-source ROS package, and evaluate it with mapping, frontier-generation, and reactive obstacle avoidance software in the loop.

VI. CONCLUSIONS

This paper has presented a novel algorithm for combined exploration and path planning behavior towards the goal of considering the path planning and information-gain (exploration) as a coupled problem. In the selected initial evaluation environment, the algorithm quickly and efficiently explored all unknown areas of the map, showing a consistent behavior over multiple exploration runs and without failure to complete the task of complete coverage.

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Selected point during exploration. Unknown areas (green voxels), all generated paths (white), selected (lowest cost) path (purple), total exploration path (grey). a) greedily maximizing information gain, b) selecting to ignore some unexplored voxels to go to the more information-dense area, c) cleaning up after exploring high-information open-areas.

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