Robust antenna design through a hybrid inversion strategy combining interval analysis and nature-inspired optimization

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Abstract. The problem of designing robust antenna configurations is cast as the solution of an inverse problem that is addressed by means of a hybrid approach based on Interval Analysis (IA) and Particle Swarm Optimization (PSO). The admissible tolerance error on the surface of reflector antennas or on the element excitations of antenna array is maximized while guaranteeing the upper and lower pattern bounds, including all power patterns potentially generated by the antenna, satisfying user-defined mask constraints. The pattern bounds are analytically computed through the IA for each trial antenna configuration defined by means of the PSO. Representative numerical examples are shown to validate the effectiveness of the proposed approach.

1. Introduction
When designing antennas for some critical applications like advanced communications and radar, there is the need to generate directive radiation patterns with low secondary lobes in order to effectively suppress the undesired signals/interferences and guarantee the system reliability. In the literature, a variety of synthesis techniques have been proposed to define the values of the antenna parameters such to generate the desired radiation pattern in case of both continuous (e.g., reflector antennas) and discrete (e.g., phased array) apertures. However, it has been also verified that the antenna side-lobes are greatly influenced by the accuracy with which one can build the reflector dish and implement the synthesized excitations [1, 2].

To address this problem, robust synthesis techniques have been introduced aimed to achieve the desired performance with a high degree of reliability of the antenna configuration. In this framework, probabilistic approaches considering simplified problem formulations and small errors, approaches considering the power pattern as the superposition of the pattern generated by the nominal antenna plus a “background” power level proportional to the antenna random errors, or Monte Carlo (MC) methods allowing the definition of the maximum tolerance errors guaranteeing the fitting of the desired mask constraints have been proposed [3]-[5].

The limits of the MC analysis, which are the unavoidably high computational burden and consequently the fact that only a subset of all possible error combinations can be contemplated, have been overcome in [6]-[8] for the case of robust phased array design where approaches based on the Interval Analysis [11]-[18] has allowed to analytically compute inclusive power pattern bounds, namely bounds robust to the uncertainties of the array control points. The values of the nominal amplitude coefficients have been optimized by means of the Particle Swarm Optimizer [6] and a
deterministic convex minimization procedure [7] for a-priori fixed tolerance errors in order to obtain an interval power pattern lying within pre-imposed mask constraints.

By following the guidelines of [8], this work is aimed to present a method for designing robust beamforming weights of linear antenna arrays as well as, to the first time to the best of the authors’ knowledge, reflector antennas where the direct maximization of the tolerance errors on the excitation weights or the dish surface is addressed without any a-priori assumptions on their nominal values or error probability. More in details, assuming the errors be defined as real-valued intervals, the widths of the intervals, namely the admitted tolerance, are maximized while guaranteeing that the bounds of the interval power pattern, computed by means of the IA, satisfy user-defined constraints.

As for the organization of the paper, the problem is mathematically formulated in Sect. II where the integrated PSO-IA-based robust design antenna approach is presented. Representative numerical examples dealing with the design of both discrete array and reflector antennas are shown in Sect. III. Eventually, some conclusions are given in Sect. IV.

2. PSO-based Robust Synthesis Method
Let us consider either the amplitude weights of a phased array or the surface of a reflector antenna characterized by tolerance errors such that their values can be represented through intervals

\[ [\xi_n] = [\xi_{n_{\text{inf}}}, \xi_{n_{\text{sup}}}] \]  

(1)

where \( \xi_{n_{\text{inf}}} \) and \( \xi_{n_{\text{sup}}} \) are the corresponding left and right interval end-points.

The expression of the field radiated by the antenna turns out to be a complex interval function

\[ [E(\theta, \phi)] = \mathbb{R}([\xi_n], n = 0, ..., N - 1) \]  

(2)

defined in terms of the antenna interval parameters which is represented by means of a complex interval value for each angular direction \((\theta, \phi)\) and computed according to [11] or [13] in case of discrete or continuous antenna aperture, respectively.

The interval power pattern is computed as

\[ [P(\theta, \phi)] = [E(\theta, \phi)]^* \]  

(3)

and the analytical expressions for the efficient evaluation of its lower \( P_{\text{inf}}(\theta, \phi) \) and upper \( P_{\text{sup}}(\theta, \phi) \) bounds are given in [11, 13]. In order to design robust antenna configurations, the end-points of the antenna interval parameters \( \xi_{n_{\text{inf}}} \) and \( \xi_{n_{\text{sup}}} \), \( n = 0, ..., N - 1 \) are optimized by means of the PSO in order to minimize the following cost function

\[ \Phi([\xi_n]) = \Phi_{\text{sup}}([\xi_n]) + \Phi_{\text{inf}}([\xi_n]) + \Phi_{\text{adv}}([\xi_n]) \]  

(4)

where

\[ \Phi_{\text{sup}}([\xi_n]) = H_{\text{sup}}(\theta, \phi) \int_{\phi_0}^{\phi_0 + 2\pi/2} \int_{\theta_0}^{\theta_0 + 2\pi/2} d\theta d\phi \]  

(5a)

\[ \Phi_{\text{inf}}([\xi_n]) = H_{\text{inf}}(\theta, \phi) \int_{\phi_0}^{\phi_0 + 2\pi/2} \int_{\theta_0}^{\theta_0 + 2\pi/2} d\theta d\phi \]  

(5b)

being \( L(\theta, \phi) \) and \( U(\theta, \phi) \) the lower and upper user-defined power pattern constraints and

\[ H_{\text{sup}}(\theta, \phi) = 1 \quad (H_{\text{inf}}(\theta, \phi) = 1) \quad \text{if} \quad P_{\text{sup}}(\theta, \phi) > U(\theta, \phi) \quad (P_{\text{inf}}(\theta, \phi) < U(\theta, \phi)) \quad \text{and} \quad H_{\text{sup}}(\theta, \phi) = 0 \quad (H_{\text{inf}}(\theta, \phi) = 0) \quad \text{otherwise}. \]

The term \( \Phi_{\text{adv}}([\xi_n]) \) of (4) is defined such to maximize the robustness of the antenna configuration, expressed as maximum admissible tolerance on the amplitude weights in case of phased arrays or maximum surface distortion in case of reflector antennas.
3. Numerical Results
In order to validate the proposed PSO-IA-based robust design method, two representative numerical results dealing with the design of a phased array and a reflector antenna are shown in the following. As for the PSO, the following setting of the control parameters has been taken into account in all simulations [19]: dimension of the swarm equal to 20, inertial weight set to 0.4, and cognitive and social acceleration coefficients to 2.0.

3.1. Phased Antenna Array
Let us consider a linear array with 10 elements end inter-element distance equal to half-wavelength. The mask constraints on the power pattern are shown in Figure 1(a) where \( u = \sin \theta \). The values of the interval amplitude configuration of the element weights obtained for the best solution at the end of the PSO-based optimization are shown in Figure 1(b).

![Figure 1](image.png)

**Figure 1.** Plot of (a) the optimized interval amplitude coefficients and (b) the upper and lower bounds of the interval power pattern and of the mask constraints.

As it can be observed from Fig 1(a), the interval power pattern fits the user-defined constraints while the tolerance of the amplitude coefficients have been maximized through the optimization of (4). The simulation converged after \( K = 100 \) iterations with a total computational time of 135 [sec] by using a standard CPU (2.4 GHz PC with 2 GB of RAM) and a non-optimized source code.
3.2. Reflector Antennas

In case of continuous aperture antennas, a circular parabolic reflector with diameter equal to $100\lambda$ and focal distance $70\lambda$ is considered. Like in [13], the far-field pattern is computed by discretizing the aperture into 50 annular sectors and the number of cells on each ring has been determined by imposing a cell area equal to $0.45\lambda^2$.

The power pattern bounds computed by means of the proposed PSO-IA-based method together with the user-defined mask constraints are shown in Figure 2. Such a solution has been obtained through a maximization of the admissible surface error which is characterized by an equivalent root-mean-square error equal to 0.03$\lambda$ for the best solution of the PSO at convergence. The simulation required $K = 500$ iterations with a total computational time of 734 [sec].

![Figure 2. Plot of the upper and lower bounds of the interval power pattern and of the mask constraints.](image)

4. Conclusions

The design of robust antenna configurations generating a reliable power pattern regardless the uncertainties on the antenna amplitude weights in case of phased arrays or on the dish surface in case of reflector antennas has been addressed in this work. The tolerances have been maximized without assuming a-priori error distributions while forcing the bounds of the arising power pattern to satisfy user-defined mask-power constraints. A hybrid approach integrating the PSO for optimizing the endpoints of the real-valued interval antenna parameters and the IA-based analysis for analytically defining the interval bounds containing the possibly arising power patterns has been presented and assessed.

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