Hierarchical Neutrino Masses and Large Mixing Angles from the Fritzsch Texture of Lepton Mass Matrices

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Abstract

We show that the Fritzsch texture of lepton mass matrices can naturally lead to the bi-large flavor mixing pattern, if three neutrinos have a normal but weak mass hierarchy (typically, $m_1 : m_2 : m_3 \sim 1 : 3 : 10$). The effective mass of the tritium beta decay and that of the neutrinoless double beta decay are too small to be observable in this ansatz, but CP violation at the percent level is allowed and could be measured in long-baseline neutrino oscillations.
The solar and atmospheric neutrino anomalies established recently in SNO and Super-Kamiokande experiments are most likely due to neutrino oscillations, a quantum phenomenon which can naturally happen if neutrinos are massive and lepton flavors are mixed. In the framework of three-neutrino oscillations, the mixing angles associated with both solar and atmospheric neutrino conversions are found to be surprisingly large ($\theta_{\text{sun}} \sim 30^\circ$ and $\theta_{\text{atm}} > 37^\circ$). To understand the smallness of neutrino masses and the largeness of flavor mixing angles, many phenomenological ansätze of lepton mass matrices have been proposed. An interesting category of the ansätze take account of texture zeros of charged lepton and neutrino mass matrices in a given flavor basis, from which some nontrivial relations between flavor mixing angles and lepton mass ratios can be derived.

The present paper follows a similar idea to study the Fritzsch texture of lepton mass matrices and its consequences on neutrino masses, flavor mixing and CP violation. In the quark sector, the Fritzsch ansatz is partly successful to interpret the strong hierarchy of quark masses and that of flavor mixing angles. A number of authors have applied the same ansatz to the lepton sector, in order to calculate the angles of lepton flavor mixing in terms of the masses of charged leptons and neutrinos. In most of those works, however, the unknown spectrum of neutrino masses was assumed to be strongly hierarchical, leading consequently to a small mixing angle for the solar neutrino oscillation.

Can the Fritzsch texture of lepton mass matrices be incorporated with the bi-large flavor mixing pattern, which is remarkably favored by current data of solar and atmospheric neutrino oscillations? The answer may certainly be affirmative, if one gives up the assumption that neutrino masses perform a strong hierarchy as charged lepton masses. Nevertheless, this interesting possibility has not been carefully examined in the literature.

We carry out a careful analysis of the Fritzsch texture of lepton mass matrices, and find that it is able to predict the bi-large flavor mixing pattern if three neutrinos have a normal but weak mass hierarchy (typically, $m_1 : m_2 : m_3 \sim 1 : 3 : 10$). The absolute values of three neutrino masses can then be determined from the experimental data of atmospheric and solar neutrino oscillations. Both the effective mass of the tritium beta decay and that of the neutrinoless double beta decay are too small to be observable in this ansatz, but CP violation at the percent level is allowed and could be measured in the upcoming long-baseline neutrino oscillation experiments.

Let us consider a simplest extension of the standard electroweak model, in which the effective mass term of charged leptons ($M_l$) and Majorana neutrinos ($M_\nu$) can be written as

$$-\mathcal{L}_{\text{mass}} = (e, \mu, \tau)_L M_l \begin{pmatrix} e \\ \mu \\ \tau \end{pmatrix}_R + \frac{1}{2} (\nu_e, \nu_\mu, \nu_\tau)_L M_\nu \begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix}_R + \text{h.c.},$$

where $\nu_\alpha^c \equiv C \nu_\alpha^T$ (for $\alpha = e, \mu, \tau$) with $C$ being the charge-conjugation operator. While $M_\nu$ must be symmetric, $M_l$ is in general arbitrary. Without loss of generality, one may arrange $M_l$ to be Hermitian through a suitable redefinition of the right-handed fields of charged leptons. The concrete structures of $M_l$ and $M_\nu$ are unfortunately unknown. As pointed out in Ref. [12], an appropriate weak-basis transformation allows us to put $M_l$ in the “nearest-neighbor” mixing form or the Fritzsch texture within the left-right symmetric

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1 It is also plausible to take $M_l$ to be symmetric in some extensions of the standard model. For instance, all fermion mass matrices are dictated to be symmetric in the SO(10) grand unified models.
models, but \( M_\nu \) cannot simultaneously have the same texture. It is therefore a phenomenological assumption that both \( M_l \) and \( M_\nu \) are of the Fritzsch texture in a specific flavor basis:

\[
M_l = \begin{pmatrix}
0 & C_l & 0 \\
C_l^* & 0 & B_l \\
0 & B_l^* & A_l
\end{pmatrix},
\]

\[
M_\nu = \begin{pmatrix}
0 & C_\nu & 0 \\
C_\nu^* & 0 & B_\nu \\
0 & B_\nu^* & A_\nu
\end{pmatrix}.
\]

(2)

We remark that the texture zeros of \( M_l \) are just a special choice of the weak basis, but those of \( M_\nu \) are nontrivial and may stem from some underlying flavor symmetries (see, e.g., Refs. [5, 6, 7] for detailed discussions). Taking account of the observed hierarchy of charged lepton masses, \( m_\tau \gg m_\mu \gg m_e \), one naturally expects that \( |A_l| \gg |B_l| \gg |C_l| \) holds in \( M_l \). There is no unique limitation on the parameters of \( M_\nu \), on the other hand, because the mass spectrum of neutrinos has not been definitely determined from the present experimental data.\(^\S\)

Without loss of generality, one may take \( A_l \) and \( A_\nu \) to be real and positive. Then only the off-diagonal elements of \( M_l \) and \( M_\nu \) are complex. It is possible to express \( M_l \) and \( M_\nu \) as

\[
M_l = P_l^T \overline{M}_l P_l, \quad M_\nu = P_\nu^T \overline{M}_\nu P_\nu,
\]

(3)

where

\[
\overline{M}_{l,\nu} = \begin{pmatrix}
0 & |C_{l,\nu}| & 0 \\
|C_{l,\nu}|^* & 0 & |B_{l,\nu}| \\
0 & |B_{l,\nu}|^* & A_{l,\nu}
\end{pmatrix},
\]

(4)

and

\[
P_l = \begin{pmatrix}
1 & 0 & 0 \\
0 & e^{i\varphi_l} & 0 \\
0 & 0 & e^{i(\varphi_l + \phi_l)}
\end{pmatrix},
\]

\[
P_\nu = \begin{pmatrix}
e^{i(\varphi_\nu - \phi_\nu)} & 0 & 0 \\
0 & e^{i\phi_\nu} & 0 \\
0 & 0 & 1
\end{pmatrix},
\]

(5)

with \( \phi_{l,\nu} \equiv \arg(B_{l,\nu}) \) and \( \varphi_{l,\nu} \equiv \arg(C_{l,\nu}) \). The real symmetric matrices \( \overline{M}_l \) and \( \overline{M}_\nu \) can be diagonalized by use of the following unitary transformations:

\[
U_l^T \overline{M}_l U_l = \begin{pmatrix}
m_e & 0 & 0 \\
0 & m_\mu & 0 \\
0 & 0 & m_\tau
\end{pmatrix},
\]

\[
U_\nu^T \overline{M}_\nu U_\nu = \begin{pmatrix}
m_1 & 0 & 0 \\
0 & m_2 & 0 \\
0 & 0 & m_3
\end{pmatrix},
\]

(6)

\(^\S\)The mass-squared differences extracted from solar and atmospheric neutrino oscillation data hint at three possible patterns of the neutrino mass spectrum: (a) normal hierarchy: \( m_1, m_2 \ll m_3 \); (b) inverted hierarchy: \( m_1 \approx m_2 \gg m_3 \); (c) approximate degeneracy: \( m_1 \approx m_2 \approx m_3 \).
where \((m_e, m_\mu, m_\tau)\) and \((m_1, m_2, m_3)\) are the physical masses of charged leptons and neutrinos, respectively. The matrix elements of \(U_l\) and \(U_\nu\) depend on four mass ratios:

\[
x_l \equiv \frac{m_e}{m_\mu}, \quad x_\nu \equiv \frac{m_1}{m_2}; \quad y_l \equiv \frac{m_\mu}{m_\tau}, \quad y_\nu \equiv \frac{m_2}{m_3}.
\]

As the values of \(m_e, m_\mu\) and \(m_\tau\) have been determined to a good degree of accuracy [13], we obtain \(x_l \approx 0.00484\) and \(y_l \approx 0.0594\). The analytically exact results for nine elements of \(U_l\) or \(U_\nu\) are found to be

\[
U_{11} = + \left[ \frac{1 - y}{(1 + x)(1 - xy)(1 - y + xy)} \right]^{1/2}, \\
U_{12} = -i \left[ \frac{x(1 + xy)}{(1 + x)(1 + y)(1 - y + xy)} \right]^{1/2}, \\
U_{13} = + \left[ \frac{xy^2(1 - x)}{(1 - xy)(1 + y)(1 - y + xy)} \right]^{1/2}, \\
U_{21} = + \left[ \frac{x(1 - y)}{(1 + x)(1 - xy)} \right]^{1/2}, \\
U_{22} = +i \left[ \frac{1 + xy}{(1 + x)(1 + y)} \right]^{1/2}, \\
U_{23} = + \left[ \frac{y(1 - x)}{(1 - xy)(1 + y)} \right]^{1/2}, \\
U_{31} = - \left[ \frac{xy(1 - x)(1 + xy)}{(1 + x)(1 - xy)(1 - y + xy)} \right]^{1/2}, \\
U_{32} = -i \left[ \frac{y(1 - x)(1 - y)}{(1 + x)(1 + y)(1 - y + xy)} \right]^{1/2}, \\
U_{33} = + \left[ \frac{(1 - y)(1 + xy)}{(1 - xy)(1 + y)(1 - y + xy)} \right]^{1/2},
\]

where we have omitted the index “\(l\)” or “\(\nu\)” for simplicity. Note that \(U_{i2}\) (for \(i = 1, 2, 3\)) are imaginary, and their nontrivial phases are due to the negative determinant of \(M_{l,\nu}\). Note also that two possibilities are allowed for the parameter space of \(x_\nu\) and \(y_\nu\) in Eq. (8): (a) \(0 < x_\nu < 1\) and \(0 < y_\nu < 1\); (b) \(x_\nu > 1\) and \(y_\nu > 1\). Now that \(x_\nu > 1\) is disfavored in respect of the large-mixing-angle MSW solution to the solar neutrino problem [3], we focus only on possibility (a) in the following (namely, \(m_1 < m_2 < m_3\)).

The lepton flavor mixing matrix \(V\) arises from the mismatch between the diagonalization of the charged lepton mass matrix \(M_l\) and that of the neutrino mass matrix \(M_\nu\). In view of Eqs. (3) – (6), we obtain \(V = U_{l}^T (P_l P_\nu) U_\nu^*\), whose nine matrix elements read explicitly as

\[
V_{pq} = U_{1p}^l U_{1q}^\nu e^{i\alpha} + U_{2p}^l U_{2q}^\nu e^{i\beta} + U_{3p}^l U_{3q}^\nu,
\]

The extreme cases such as \(x_\nu = 0\) or 1 and (or) \(y_\nu = 0\) or 1 are not taken into account, because they are incompatible with current experimental data on solar and atmospheric neutrino oscillations.
where the subscripts \( p \) and \( q \) run respectively over \((e, \mu, \tau)\) and \((1, 2, 3)\), and two phase parameters \( \alpha \) and \( \beta \) are defined as \( \alpha \equiv (\varphi_\nu - \varphi_\ell) - (\phi_\nu + \phi_\ell) \) and \( \beta \equiv (\varphi_\nu - \varphi_\ell) \). Note that an overall phase factor \( e^{i(\varphi_\nu + \phi_\ell)} \) has been omitted from the right-hand side of Eq. (9), since it has no contribution to lepton flavor mixing and CP violation. It is obvious that \( V \) consists of six parameters: \( x_1, y_1, x_2, y_2, \alpha \) and \( \beta \). Among them, \( x_1 \) and \( y_1 \) are already known. The other four free parameters can be constrained by current experimental data on neutrino oscillations.

In the framework of three-neutrino oscillations, the solar and atmospheric neutrino anomalies are approximately decoupled: they are attributed, respectively, to \( \nu_e \to \nu_\mu \) and \( \nu_\mu \to \nu_\tau \) conversions. The corresponding neutrino mass-squared differences \( \Delta m^2_{\text{sun}} \) and \( \Delta m^2_{\text{atm}} \) are given by

\[
\Delta m^2_{\text{sun}} = m_2^2 - m_1^2 = m_2^2 \left( 1 - x_\nu^2 \right), \\
\Delta m^2_{\text{atm}} = m_3^2 - m_2^2 = m_3^2 \left( 1 - y_\nu^2 \right). \tag{10}
\]

The analyses of current SNO [3] and Super-Kamiokande [4] data yield \( \Delta m^2_{\text{sun}} = (3.3 - 17) \times 10^{-5} \text{eV}^2 \) [3] and \( \Delta m^2_{\text{atm}} = (1.6 - 3.9) \times 10^{-3} \text{eV}^2 \) [4] at the 90\% confidence level. Then the ratio

\[
R_\nu \equiv \frac{\Delta m^2_{\text{sun}}}{\Delta m^2_{\text{atm}}} = y_\nu^2 \frac{1 - x_\nu^2}{1 - y_\nu^2} \tag{11}
\]

lies in the range \((0.85 - 10) \times 10^{-2}\). On the other hand, the mixing factors of solar and atmospheric neutrino oscillations are related to the relevant matrix elements of \( V \) in the following way:

\[
\sin^2 2\theta_{\text{sun}} = 4|V_{e1}|^2|V_{e2}|^2, \\
\sin^2 2\theta_{\text{atm}} = 4|V_{\mu3}|^2 \left( 1 - |V_{\mu3}|^2 \right). \tag{12}
\]

At the 90\% confidence level, \( \tan^2 \theta_{\text{sun}} = 0.30 - 0.58 \) [3] and \( \sin^2 2\theta_{\text{atm}} > 0.92 \) [4] have been extracted from current data. Note that the magnitude of \( V_{e3} \) is well restricted by the CHOOZ reactor experiment on neutrino oscillations [14], whose mixing factor is given by

\[
\sin^2 2\theta_{\text{chz}} \approx 4|V_{e3}|^2 \left( 1 - |V_{e3}|^2 \right). \tag{13}
\]

The present upper bound is \( \sin^2 2\theta_{\text{chz}} < 0.2 \) or \( \theta_{\text{chz}} < 13^\circ \) [14], corresponding to the mass scale of atmospheric neutrino oscillations.

With the help of Eqs. (11) – (13), the four unknown parameters \( x_\nu, y_\nu, \alpha \) and \( \beta \) in the Fritzsch texture of lepton mass matrices can be determined from current experimental constraints on \( R_\nu, \sin^2 2\theta_{\text{sun}}, \sin^2 2\theta_{\text{atm}} \) and \( \sin^2 2\theta_{\text{chz}} \). A careful numerical analysis shows that the phase parameter \( \beta \) must be around 180\° to assure \( \sin^2 2\theta_{\text{atm}} > 0.92 \). In comparison, the phase parameter \( \alpha \) is not very restrictive. Hence we simply fix \( \beta = 180^\circ \) in our calculations, and typically take \( \alpha = (0^\circ, 90^\circ, 180^\circ) \) to illustrate the quantitative dependence of four observables on \( \alpha \). We present the allowed regions of \((x_\nu, y_\nu), (\tan^2 \theta_{\text{sun}}, \sin^2 2\theta_{\text{atm}})\) and \((R_\nu, \sin^2 2\theta_{\text{chz}})\) in Figs. 1, 2 and 3, respectively. Some comments are in order.

(1) We demonstrate that the Fritzsch texture of lepton mass matrices is actually compatible with the present experimental data on solar and atmospheric neutrino oscillations. The allowed parameter space decreases with the change of \( \alpha \) from 0\° to 180\°. Typically, we obtain \( x_\nu \sim 1/3 \) and \( y_\nu \sim (1/4 - 1/3) \) or

\[
m_1 : m_2 : m_3 \sim 1 : 3 : 10. \tag{14}
\]
This result indicates that three neutrino masses are weakly hierarchical. In contrast, three charged lepton masses \( m_e : m_\mu : m_\tau \approx 1 : 207 : 3478 \) are strongly hierarchical.

(2) Given the numerical ranges of \( x_\nu \) and \( y_\nu \), the absolute values of three neutrino masses can be calculated by use of Eq. (10):

\[
\begin{align*}
\langle m \rangle_e & = \sum_{i=1}^{3} (m_i |V_{ei}|^2) \\
& = m_3 \left( x_\nu y_\nu |V_{e1}|^2 + y_\nu |V_{e2}|^2 + |V_{e3}|^2 \right), \\
\langle m \rangle_{ee} & = \sum_{i=1}^{3} (m_i V_{ei}^2) \\
& = m_3 \left| x_\nu y_\nu V_{e1}^2 + y_\nu V_{e2}^2 + V_{e3}^2 \right|.
\end{align*}
\]

We present the allowed ranges of \( \langle m \rangle_e \) and \( \langle m \rangle_{ee} \), normalized by \( m_3 \), in Fig. 4. Given \( m_3 \approx 5 \times 10^{-2} \) eV, \( \langle m \rangle_e \approx 10^{-2} \) eV and \( \langle m \rangle_{ee} \approx 10^{-3} \) eV are typically obtained. Note
that $\langle m_{ee} \rangle \ll \langle m_e \rangle$ holds in our ansatz, because the former involves significant cancellation due to the existence of an extra phase (of 90°) associated with $V_{e2}$, as one can see from Eqs. (8) and (9). It should be noted that our prediction for $\langle m_e \rangle$ is far below the proposed sensitivity of the KATRIN experiment ($\sim 0.3$ eV [16]), and that for $\langle m_{ee} \rangle$ is also below the expected sensitivity of next-generation experiments for the neutrinoless double beta decay ($\sim 10$ meV to 50 meV [17]). Therefore it seems hopeless to detect both effects in practice.

How big can the strength of CP violation be in neutrino oscillations? To answer this question, we calculate the Jarlskog invariant of CP violation $J$ [18], which is defined through the following equation:

$$\text{Im} \left( V_{ai} V_{bj} V_{aj}^* V_{bi}^* \right) = J \sum_{c,k} (\epsilon_{abc} \epsilon_{ijk}) ,$$

where the subscripts $(a, b, c)$ and $(i, j, k)$ run respectively over $(e, \mu, \tau)$ and $(1, 2, 3)$. We show the numerical changes of $J$ with $|V_{e3}|$ in Fig. 5, where $\alpha = 90^\circ$ and $\beta = 180^\circ$ have been used. It is clear that the magnitude of $J$ increases with that of $V_{e3}$. We obtain $J \sim 1\%$ as a typical result in the allowed parameter space of $x_\nu$ and $y_\nu$. Thus leptonic CP-violating effects could be measured in a variety of long-baseline neutrino oscillation experiments [19].

We have shown that the Fritzsch texture of lepton mass matrices is actually able to yield the bi-large flavor mixing pattern, if three neutrinos perform a normal but weak mass hierarchy. The absolute values of three neutrino masses are approximately $m_1 \sim \sqrt{\Delta m^2_{\text{sun}}}/3$, $m_2 \sim \sqrt{\Delta m^2_{\text{sun}}}$ and $m_3 \sim \sqrt{\Delta m^2_{\text{atm}}}$. The smallness of $m_i$ implies that there is little chance to observe the effective mass of the tritium beta decay and that of the neutrinoless double beta decay. On the other hand, our ansatz predicts that the strength of leptonic CP violation can be at the percent level. It is therefore possible to detect CP-violating effects in the long-baseline experiments of neutrino oscillations.

The weakly hierarchical spectrum of three neutrino masses must be a consequence of the weakly hierarchical texture of the neutrino mass matrix $M_\nu$. To see this point more clearly, we calculate the nonvanishing matrix elements of $M_\nu$ in Eq. (4):

$$A_\nu = m_1 - m_2 + m_3 ,$$

$$|B_\nu| = \left| \frac{(m_1 - m_2)(m_2 - m_3)(m_1 + m_3)}{m_1 - m_2 + m_3} \right|^{1/2} ,$$

$$|C_\nu| = \left( \frac{m_1 m_2 m_3}{m_1 - m_2 + m_3} \right)^{1/2} .$$

(19)

Taking the typical mass spectrum given in Eq. (14) as well as $m_3 = 5 \times 10^{-2}$ eV, we explicitly obtain

$$\overline{M}_\nu \sim 4 \times 10^{-2} \text{ eV} \times \begin{pmatrix} 0 & 0.24 & 0 \\ 0.24 & 0 & 0.55 \\ 0 & 0.55 & 1 \end{pmatrix} .$$

(20)

In comparison, the real charged lepton mass matrix $\overline{M}_l$ reads

$$\overline{M}_l \approx 1.67 \text{ GeV} \times \begin{pmatrix} 0 & 0.0045 & 0 \\ 0.0045 & 0 & 0.26 \\ 0 & 0.26 & 1 \end{pmatrix} .$$

(21)

It becomes obvious that the mixing angle $\theta_{\text{sun}}$ is dominated by the $(1, 2)$ subsector of $\overline{M}_\nu$, while the mixing angle $\theta_{\text{atm}}$ gets comparable contributions from the $(2, 3)$ subsector of $\overline{M}_\nu$. 
and the \((\mu, \tau)\) subsector of \(\mathcal{M}_1\). This observation would be useful for model building, from which some deeper understanding of the Fritzsch texture or its variations could be gained.

To conclude, the Fritzsch texture of lepton mass matrices is predictive and its predictions are compatible with current experimental data on solar and atmospheric neutrino oscillations. A stringent test of this ansatz will soon be available, in particular, in a variety of long-baseline neutrino oscillation experiments.

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Figure 1: Allowed regions of $x_\nu$ and $y_\nu$ for chosen values of $\alpha$ and $\beta$. 
Figure 2: Allowed regions of $\tan^2 \theta_{\text{sun}}$ and $\sin^2 2\theta_{\text{atm}}$ for chosen values of $\alpha$ and $\beta$. 
Figure 3: Allowed regions of $R_{\nu}$ and $\sin^{2}2\theta_{\text{cha}}$ for chosen values of $\alpha$ and $\beta$. 
Figure 4: Allowed regions of $\langle m \rangle_e/m_3$ and $\langle m \rangle_{ee}/m_3$ for chosen values of $\alpha$ and $\beta$. 
Figure 5: Allowed regions of $|V_{e3}|$ and $\mathcal{J}$ for the inputs $\alpha = 90^\circ$ and $\beta = 180^\circ$. 