Momentum dependent mean-fields of (anti)hyperons

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Abstract

We investigate the in-medium properties of hyperons and anti-hyperons in the framework of the Non-Linear Derivative (NLD) model. We focus on the momentum dependence of in-medium strangeness optical potentials. The NLD model is based on the simplicity of the well-established Relativistic Mean-Field (RMF) approximation, but it incorporates an explicit momentum dependence on a field-theoretical level. The extension of the NLD model to the (anti)baryon-octet is formulated in the spirit of SU(6) and G-parity arguments. It is shown that with an appropriate choice of momentum cut-offs the $\Lambda$, $\Sigma$ and $\Xi$ optical potentials are consistent with recent studies of the chiral effective field theory and Lattice-QCD calculations over a wide momentum region. In addition, we present NLD predictions for the in-medium momentum dependence of $\Lambda$-, $\Sigma$- and $\Xi$-hyperons. This work is important for future experimental studies such as CBM, PANDA at the Facility for Antiproton and Ion Research (FAIR). It is relevant for nuclear astrophysics too.

Keywords: Equations of state of hadronic matter, optical potential, in-medium hyperon potentials.

1. Introduction

Astrophysical observations on particularly massive neutron stars [1, 2, 3] have driven the nuclear physics and astrophysics communities to detailed investigations of the nuclear equation of state (EoS) under conditions far beyond the ordinary matter [4]. On one hand, theoretical and experimental studies on heavy-ion collisions over the last few decades concluded a softening of the high-density EoS in agreement with phenomenological and microscopic models [5, 6, 7]. On the other hand, the observations of two-solar mass pulsars [1, 2, 3] together with additional constraints on the high-density limit of the speed of sound [8] gave some controversial insights on the EoS of compressed baryonic matter. They provide an upper limit for the neutron star mass by excluding soft-type hadronic EoS’s at high baryon densities.

Compressed baryonic matter may consist not only of nucleons. It can include fractions of heavier baryons, when their production is energetically allowed. These are the hyperons $\Lambda$, $\Sigma$ and $\Xi$ as a part of the irreducible representations of SU(3). While the nucleon-nucleon (NN) interaction is very well known, the hyperon interactions are still not fully understood. Indeed, there are many experimental data for NN-scattering in free space and inside hadronic media (finite nuclei, heavy-ion collisions, hadron-induced reactions) allowing a precise determination of the
NN-interaction. Concerning the strangeness sector (hyperon-nucleon (YN) or hyperon-hyperon (YY) interactions), there exist phenomenological and microscopic models with predictions for the in-medium hyperon properties at matter densities close to saturation and at higher densities. However, the experimental access to the strangeness sector is still scarce. A common prediction of theoretical models is a considerable softening of the hadronic EoS at high densities by adding to a system more degrees of freedom such as strangeness particles. The inclusion of hyperons into nuclear approaches made many of them, which were successfully applied to nuclear systems (nuclear matter, finite nuclei, nuclear reactions), incompatible with the astrophysical observations of two-solar mass pulsars [1, 2]. This is the so-called hyperon-puzzle [9, 10]. This puzzle has received recently theoretical attraction by a new observation of a quite massive neutron star [3]. A comprehensive theoretical view concerning the microscopic descriptions of in-medium properties of the baryon-octet is given in Ref. [11]. There exist also theoretical reviews based on the RMF approximation, see for instance Refs. [12, 13, 14].

It is thus of great interest to address the in-medium behaviour of hyperons in nuclear matter, as we do in this work. We use an alternative RMF approach based on the fact, that compressed matter consists of particles with high relative momenta. Therefore, not only the density dependence, but the momentum dependence of the in-medium interactions is important too. The reason for doing so is that conventional RMF-models do not explain the empirical saturation of the in-medium interactions of high-momenta (anti)nucleons. In terms of SU(6) this issue appears for high-momenta (anti)hyperons too. This is the Non-Linear Derivative (NLD) model [15]. It retains the basic RMF Lagrangian formulation, but it includes higher-order derivatives in the NN-interaction Lagrangians. It has been demonstrated that this Ansatz corrects the high-momentum behaviour of the interaction, makes the EoS softer at densities just above saturation, but at the same time it reproduces the two-solar mass pulsars at densities far beyond saturation [15]. Here we extend the NLD approach by including strangeness into the nuclear matter and discuss the momentum dependence of the in-medium hyperon potentials.

2. The NLD Model for the baryon octet

In this section we briefly introduce the non-linear derivative (NLD) model and extend it to the baryon octet. A detailed description of the NLD model for nucleons can be found in Ref. [15]. The NLD-Lagrangian is based on the conventional Relativistic Hadro-Dynamics (RHD) [16] and it reads as

\[ \mathcal{L} = \frac{1}{2} \sum_B \left[ \bar{\Psi}_B \gamma_\mu \partial^\mu \Psi_B - \bar{\Psi}_B i \gamma_\mu \partial^\mu \Psi_B \right] - \sum_B m_B \bar{\Psi}_B \Psi_B - \frac{1}{2} m_\sigma^2 \sigma^2 + \frac{1}{2} \partial_\mu \sigma \partial^\mu \sigma - U(\sigma) + \frac{1}{2} m_\omega^2 \omega_\mu \omega^\mu - \frac{1}{4} F_\mu \nu F^{\mu \nu} + \frac{1}{2} m_\rho^2 \rho_\mu \rho^\mu - \frac{1}{4} G_\mu \nu G^{\mu \nu} - \frac{1}{2} m_\delta^2 \delta^2 + \frac{1}{2} \partial_\mu \delta \partial^\mu \delta + \mathcal{L}_\text{int}^\sigma + \mathcal{L}_\text{int}^\omega + \mathcal{L}_\text{int}^\rho + \mathcal{L}_\text{int}^\delta. \]
The sum over $B$ runs over the baryonic octet

$$\Psi_B = (\Psi_N, \Psi_A, \Psi_\Sigma, \Psi_\Xi)^T$$

with

$$\Psi_N = (\psi_p, \psi_n)^T, \quad \Psi_A = \psi_A$$

$$\Psi_\Sigma = (\psi_{\Sigma^+}, \psi_{\Sigma^0}, \psi_{\Sigma^-})^T, \quad \Psi_\Xi = (\psi_{\Xi^0}, \psi_{\Xi^-})^T$$

for the isospin-doublets $\Psi_N$ and $\Psi_\Xi$, isospin-triplet $\Psi_\Sigma$ and the neutral $\Psi_A$. The interactions between the nucleon fields are described by the exchange of meson fields. These are the scalar $\sigma$ and vector $\omega^\mu$ mesons in the isoscalar channel, as well as the scalar $\delta$ and vector $\rho^\mu$ mesons in the isovector channel. Their corresponding Lagrangian densities are of the Klein-Gordon and Proca types, respectively. The term $U(\sigma) = \frac{1}{4} b_\sigma^2 + \frac{1}{4} c_\sigma^4$ contains the usual selfinteractions of the $\sigma$ meson. The notations for the masses of fields in Eq. (1) are obvious. The field strength tensors are defined as

$$F_{\mu\nu} = \partial_\mu \omega_\nu - \partial_\nu \omega_\mu, \quad G_{\mu\nu} = \partial_\mu \rho_\nu - \partial_\nu \rho_\mu$$

for the isoscalar and isovector fields, respectively. In the following we restrict to a minimal set of interaction degrees of freedom. In the iso-scalar sector, the $\sigma$- and $\omega$-fields are obviously considered. In the iso-vector channel, we keep the vector, iso-vector $\rho$-meson field and neglect the $\delta$-field.

The NLD interaction Lagrangians contain the conventional RHD combinations between the bilinear baryon- and linear meson-fields, however, they are extended by the inclusion of non-linear derivative operators $\overrightarrow{D}_B$, $\overleftarrow{D}_B$ for each baryon species $B$:

$$L_{\text{int}}^\sigma = \sum_B \frac{g_B}{2} \left[ \overline{\Psi}_B \overrightarrow{D}_B \Psi_B \sigma + \sigma \overline{\Psi}_B \overrightarrow{D}_B \overleftarrow{D}_B \Psi_B \right],$$

$$L_{\text{int}}^\omega = -\sum_B \frac{g_B}{2} \left[ \overline{\Psi}_B \overrightarrow{D}_B \gamma^\mu \Psi_B \omega_\mu + \omega_\mu \overline{\Psi}_B \gamma^\mu \overrightarrow{D}_B \Psi_B \right],$$

$$L_{\text{int}}^\rho = -\sum_B \frac{g_B}{2} \left[ \overline{\Psi}_B \overrightarrow{D}_B \gamma^\mu \rho_\mu + \rho_\mu \overline{\Psi}_B \gamma^\mu \overrightarrow{D}_B \Psi_B \right],$$

for the isoscalar-scalar, isoscalar-vector and isovector-vector vertices, respectively. The arrows on the non-linear operator $\overrightarrow{D}_B$, $\overleftarrow{D}_B$ indicate the direction of their action. The only difference with respect to the conventional RHD Lagrangian is the presence of additional operator functions $\overrightarrow{D}_B$, $\overleftarrow{D}_B$. As we will see, they will regulate the high momentum component of hyperons. For this reason we will call them as regulators too. The operator functions (or regulators) $\overrightarrow{D}_B$, $\overleftarrow{D}_B$ are hermitian and generic functions of partial derivative operator. That is, $\overrightarrow{D}_B := \mathcal{D} \left( \overrightarrow{\xi}_B \right)$ and $\overleftarrow{D}_B := \mathcal{D} \left( \overleftarrow{\xi}_B \right)$ with the operator arguments $\overrightarrow{\xi}_B = -\zeta_\alpha \overrightarrow{\partial}_\alpha$, $\overleftarrow{\xi}_B = i \overleftarrow{\partial}_\alpha \zeta_\alpha$. The four vector $\zeta_\mu = v_\mu / \Lambda_B$ contains the cut-off $\Lambda_B$ and $v_\mu$ is an auxiliary vector. These regulators are assumed to act on the baryon spinors $\Psi_B$ and $\overline{\Psi}_B$ by a formal Taylor expansion with respect to the operator argument. The functional form of the regulators is constructed such that in the limit $\Lambda_B \to \infty$ the original RHD Lagrangians are recovered, that is, $\overrightarrow{D}_B \to \overrightarrow{D}_B^\dagger \to 1$. 

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The presence of higher-order partial derivatives in the Lagrangian mediate a modification of the field-theoretical prescriptions. As discussed in detail in the original work of Ref. [15], the generalized Euler-Lagrange equations as well as the Noether-currents contain additional infinite terms of higher-order partial derivative contributions. However, the main advantage of the NLD approach relies on the fact that these terms can be resummed to compact expressions.

From the generalized Euler-Lagrange formalism we obtain the equations of motion for the degrees of freedom in the NLD model. The meson field equations of motion read

\[ \partial_\alpha \partial^\alpha \sigma + m_\sigma^2 \sigma + \frac{\partial U}{\partial \sigma} = \frac{1}{2} \sum_B g_{\sigma B} \left( \overline{\Psi}_B \overrightarrow{D}_B \Psi_B + \overline{\Psi}_B \overleftarrow{D}_B \Psi_B \right), \]

\[ \partial_\mu F^{\mu\nu} + m_\omega^2 \omega^\nu = \frac{1}{2} \sum_B g_{\omega B} \left( \overline{\Psi}_B \overrightarrow{D}_B \gamma^\nu \Psi_B + \overline{\Psi}_B \gamma^\nu \overleftarrow{D}_B \Psi_B \right), \]

\[ \partial_\mu G^{\mu\nu} + m_\rho^2 \rho^\nu = \frac{1}{2} \sum_B g_{\rho B} \left( \overline{\Psi}_B \overrightarrow{D}_B \gamma^\nu \tau \Psi_B + \overline{\Psi}_B \gamma^\nu \overleftarrow{D}_B \Psi_B \right), \]

for the isoscalar-scalar, isoscalar-vector and isovector-vector exchange mesons, respectively.

Each baryon-field obeys a Dirac-equation of the following type

\[ [\gamma_\mu (i \partial^\mu - \Sigma^\mu_B) - (m_B - \Sigma_s B)] \psi_B = 0, \]

with the selfenergies \( \Sigma^\mu_B \) and \( \Sigma_s B \) defined as

\[ \Sigma^\mu_B = g_{\omega B} \omega^\mu \overrightarrow{D}_B + g_{\rho B} \rho^\mu \overleftarrow{D}_B, \]

\[ \Sigma_s B = g_{\sigma B} \sigma \overrightarrow{D}_B. \]

Both Lorentz-components of the selfenergy, \( \Sigma^\mu \) and \( \Sigma_s \), show an explicit linear behaviour with respect to the meson fields \( \sigma, \omega^\mu \) and \( \rho^\mu \) as in the standard RHD. However, they contain an additional dependence on the regulators. General expressions for the Noether-current and energy-momentum tensor can also be derived. We give them below in the RMF approximation.

The RMF application of the NLD formalism to static hadronic matter follows the same procedure as in the conventional RHD. The spatial components of the meson fields in Minkowski- and isospin-spaces vanish, \( \omega^\mu \rightarrow (\omega^0, \vec{0}) \) and \( \rho^\mu \rightarrow (\rho^0, \vec{0}) \). For simplicity, we denote in the following the remaining isospin component of the isovector fields as \( \rho^\mu \). The solutions of the RMF equations start with the usual plane wave ansatz \( \psi_B(s, \vec{p}) = u_B(s, \vec{p}) e^{-ip^\mu x_\mu} \) where \( B \) stands for the various isospin states of the baryons and \( p^\mu = (E, \vec{p}) \) is the single baryon 4-momentum. The application of the non-linear derivative operator \( D_B \) to the plane wave Ansatz of the spinor fields results in regulators \( D_B \) which are now functions of the scalar argument \( \xi_B = -\frac{v_0 p^0}{\Lambda_B} \).

That is, they depend explicitly on the single baryon momentum \( p \) (with an appropriate choice of the auxiliary vector \( v^0 \)) and on the cut-off \( \Lambda_B \), which may differ for each baryon type \( B \). Each baryon fulfills a Dirac equation with the same form as in Eq. (11) and with corresponding explicitly momentum dependent scalar and vector selfenergies. Their vector components are given by

\[ \Sigma^\mu_p = g_{\omega N} \omega^\mu D_N + g_{\rho N} \rho^\mu D_N, \]

\[ \Sigma^\mu_n = g_{\omega N} \omega^\mu D_N - g_{\rho N} \rho^\mu D_N, \]
\[ \Sigma^\mu_\Lambda = g_\omega \omega^\mu D_\Lambda , \quad (16) \]

\[ \Sigma^\mu_\Sigma^+ = g_\omega \omega^\mu D_\Sigma^+ + g_\rho \rho^\mu D_\Sigma , \quad (17) \]

\[ \Sigma^\mu_\Sigma^- = g_\omega \omega^\mu D_\Sigma^- - g_\rho \rho^\mu D_\Sigma , \quad (18) \]

\[ \Sigma^\mu_\Sigma^0 = g_\omega \omega^\mu D_\Sigma^0 , \quad (19) \]

\[ \Sigma^\mu_\Xi^- = g_\omega \Xi^\mu D_\Xi^- - g_\rho \Xi^\mu D_\Xi , \quad (20) \]

\[ \Sigma^\mu_\Xi^0 = g_\omega \Xi^\mu D_\Xi^0 + g_\rho \Xi^\mu D_\Xi . \quad (21) \]

Similar expressions result for the scalar selfenergies. In the following the scalar and time-like component of the baryon selfenergy will be denoted as \( S_B \) and \( V_B \), respectively. Note that the selfenergies are explicitly momentum dependent due to the regulators \( D_B = D_B(p) \) as specified below. The solutions of the Dirac equation are the standard Dirac-spinors with a proper normalization \( N_B \)

\[ u_B(s, \vec{p}) = N_B \left( \begin{array}{c} \varphi_s \\ \vec{\sigma} \cdot \vec{p} \\ E_B^* + m_B^* \varphi_s \end{array} \right) , \quad (22) \]

but now for quasi-free baryons \( B \) with an in-medium energy

\[ E_B^* := E_B - V_B(p) , \quad (23) \]

and a Dirac mass

\[ m_B^* := m_B - S_B(p) . \quad (24) \]

At a given momentum the single particle energy \( E \) is obtained from the in-medium on-shell relation (23). These expressions are needed for evaluation of expectation values, for instance, the source terms of the meson-field equations. For the definition of the nuclear matter we need a conserved nucleon density. It is obtained from the time-like component of the Noether-current \( J^\mu \) defined as

\[ J^\mu = \frac{\kappa}{(2\pi)^3} \sum_{B=p,n} \int d^3p \frac{\Pi^\mu_B}{\Pi^0_B} \quad (25) \]

with the generalized 4-momentum

\[ \Pi^\mu_B = p_B^{*\mu} + m_B^* \left( \partial_\mu S_B \right) - \left( \partial_\mu \Sigma_B^\beta \right) p_B^{*\beta} \quad (26) \]

and the usual effective 4-momentum

\[ p_B^{*\mu} = p^\mu - \Sigma_B^\mu . \quad (27) \]
The EoS (Equation of State) is obtained from the time-like components of the energy-momentum tensor. In nuclear matter the resummation procedure of the NLD model results in the following expression

$$T_{\mu\nu} = \sum_{B} \frac{\kappa}{(2\pi)^3} \int_{|\vec{p}| \leq p_{FB}} d^3p \ \frac{\Pi_{B\mu} \Pi_{B\nu}}{\Pi_{B}^0} - g^{\mu\nu} \langle L \rangle, \quad (28)$$

from which the energy density $\varepsilon \equiv T^{00}$ and the pressure $P$ can be calculated, see for details Ref. [15]. Finally, the NLD meson-field equations in the RMF approach to nuclear matter can be resummed to the following forms

$$m_\sigma^2 \sigma + \frac{\partial U}{\partial \sigma} = \sum_{B} g_{\sigma B} \langle \bar{\psi}_B D_B \psi_B \rangle = \sum_{B} g_{\sigma B} \rho_{sB}, \quad (29)$$

$$m_\omega^2 \omega = \sum_{B} g_{\omega B} \langle \bar{\Psi}_B \gamma^0 D_B \Psi_B \rangle = \sum_{B} g_{\omega B} \rho_{0B}, \quad (30)$$

with the scalar and vector density sources

$$\rho_{sB} = \frac{\kappa}{(2\pi)^3} \int_{|\vec{p}| \leq p_{FB}} d^3p \ \frac{m_{B}^s}{\Pi_{B}^0} D_B(p), \quad (31)$$

$$\rho_{0B} = \frac{\kappa}{(2\pi)^3} \int_{|\vec{p}| \leq p_{FB}} d^3p \ \frac{E_{B}^s}{\Pi_{B}^0} D_B(p). \quad (32)$$

The isovector densities are calculated through the standard isospin relations. For a hyperon with a given momentum relative to nuclear matter at rest (at a given nucleon density and isospin asymmetry) the mesonic sources contain only nucleons, that is $B = p, n$.

The meson-field equations of motion show a similar structure as those of the standard RMF approximation. However, the substantial difference between NLD and other conventional RMF models appears in the source terms which now contain in addition the momentum-dependent regulators $D_B$. This is an important feature of the NLD model. The cut-off leads naturally to a particular suppression of the vector field at high densities or high Fermi-momenta in agreement with phenomenology, as discussed in detail in the previous work [15]. This feature is absent in conventional RHD approaches, except if one introduces by hand additional scalar/vector self-interactions.

The key observable for general discussions related to momentum or energy dependencies of in-medium hadronic potentials is the Schroedinger-equivalent optical potential $U_{opt}$, which is a complex quantity. The imaginary part describes the scattering processes of a given particle, e.g., a hyperon, with a nucleon of the nuclear matter. The real part of the optical potential is related to the mean-field that a particle, e.g., a hyperon with a given momentum, experiences in the nuclear medium at a given density and isospin-asymmetry. The imaginary part of $U_{opt}$ cannot be calculated within a conventional RMF prescription. In RMF models one is usually
| NLD parameters | $\Lambda_{sN}$ | $\Lambda_{vN}$ | $g_{\sigma N}$ | $g_{\omega N}$ | $g_{\rho N}$ | $b$ | $c$ |
|----------------|----------------|----------------|----------------|----------------|----------------|-----|-----|
|                | [GeV]          | [GeV]          | [GeV]          | [GeV]          | [GeV]          | [fm] | [fm] |
|                | 0.95           | 1.125          | 10.08          | 10.13          | 3.50           | 15.341 | −14.735 |
| Bulk saturation properties | $\rho_{\text{sat}}$ | $E_b$ | $K$ | $a_{\text{sym}}$ |
|                | [fm$^3$]       | [MeV]         | [MeV]          | [MeV]          | [MeV]          |
|                | 0.156          | −15.30         | 251            | 30             |                |

Table 1: (Top) NLD parameters: meson-nucleon couplings $g_{mN}$, $(m = \sigma, \omega, \rho)$, $\sigma$ self-interaction constants $b, c$, and NLD cut-off for scalar ($\Lambda_{sN}$) and vector ($\Lambda_{vN}$) meson-nucleon isoscalar vertices. The isovector meson-nucleon cut-off is the same as the isoscalar-vector one. (Bottom) Bulk saturation properties of nuclear matter: saturation density $\rho_{\text{sat}}$, binding energy per nucleon $E_b$, compression modulus $K$ and asymmetry parameter $a_{\text{sym}}$ in the NLD model. See Ref. [15] for more details.

interested in the real part of an optical potential that can be then examined in more realistic systems, for instance, in heavy-ion collisions or hadron-induced reactions within a relativistic transport theory. The missing imaginary part is then modelled within a collision term in terms of cross sections for elastic, quasi-elastic and inelastic channels with a proper counting of Pauli-Blocking effects.

In the NLD model one cannot calculate precisely the imaginary part of $U_{\text{opt}}$. However, the NLD approach contains an explicit momentum dependence of the mean-fields, and thus, of the optical potential. This particular feature allow us to give, at least, estimations for the imaginary part of an optical potential too. This will be discussed in the case of the anti-hyperons, and we will mainly focus the study here on the real part of the optical potentials.

The real part of the Schroedinger-equivalent optical potential for hyperons is obtained from a non-relativistic reduction of the Dirac-equation and reads

$$U_{\text{opt}}^B = -S_B + \frac{E_B}{m_B} V_B + \frac{1}{2m_B} \left(S_B^2 - V_B^2\right). \tag{33}$$

It describes the in-medium interaction of a baryon species $B$, e.g., a hyperon, with a momentum $p$ (or single-particle energy $E_B = E_B(p)$, see Eq. (23)) relative to nuclear matter at rest at a given density and isospin asymmetry. We will use Eq. (33) to compare the NLD results with the microscopic calculations from $\chi$-EFT and Lattice-QCD for the hyperon in-medium potentials.

### 3. Results and discussion

#### 3.1. Nucleonic sector

We briefly give the status of the NLD model for the in-medium nucleons, before starting the discussion on the in-medium hyperon potentials. As in detail discussed in [15], a momentum dependent monopole form

$$D(p) = \frac{\Lambda^2}{\Lambda^2 + \bar{p}^2}. \tag{34}$$
for the regulators turned out to be very effective for a simultaneous description of the low and high density nuclear matter properties. An example is shown in table 1 for the extracted saturation properties together with the model parameters. It is seen that the NLD model leads to a very good description of the empirical values at saturation. The NLD EoS is rather soft and similar to the density dependence of Dirac-Brueckner-Hartree-Fock microscopic calculations. At high densities, however, the NLD EoS becomes stiff. This feature makes a prediction of the maximum mass of neutron stars of $2M_\odot$ possible even with a soft compression modulus. Note that the NLD model gives a correct description of the Schrödinger-equivalent optical potential for in-medium protons and antiprotons simultaneously by imposing G-parity only [15].

3.2. Strangeness sector

For the strangeness sector we consider again nuclear matter at rest, at a given density, isospin-asymmetry and at zero temperature, in which hyperons ($\Lambda$, $\Sigma$, $\Xi$) are situated at a given momentum relative to the nuclear matter at rest. The quantity of interest will be the optical potential $U_{opt}$ of the in-medium hyperons, see Eq. (33). Since there is no experimental information on the momentum dependence of the in-medium hyperonic potentials, we use for our comparisons the recent microscopic calculations from Refs. [17] (see also Ref. [18]) and [19] as a guidance. They are based on the $\chi$-EFT approach in Next-To-Leading (NLO) order and to Lattice-QCD.

In the NLD calculations we assume for the in-medium hyperon interactions no additional parameters except of the strangeness cut-off of the hyperons. That is, the various hyperon-nucleon couplings are fixed from the corresponding nucleon-nucleon ones by means of SU(6). The hyperon cut-offs retain their monopole form as in Eq. (34). In particular, they take the form

$$D_Y(p) = \frac{\Lambda_{Y1}^2}{\Lambda_{Y2}^2 + \vec{p}^2},$$

with $\gamma = \sigma, \omega, \rho$ indicating the cut-off values for the hyperon-nucleon $\sigma, \omega$- and $\rho$-vertices, respectively, and $Y = \Lambda, \Sigma, \Xi$ denotes the hyperon type. In principle, one could use a single cut-off $\Lambda_{Y1} = \Lambda_{Y2} = \Lambda$, for each meson-hyperon vertex. However, in order to describe the non-trivial momentum dependence of the microscopic calculations as precise as possible we allow for different cut-off values for the vector-isoscalar $\omega$- and vector-isovector $\rho$-hyperon vertices, as shown in Eq. (35). For the isoscalar meson-hyperon interactions a single cut-off $\Lambda\gamma = \Lambda_{\sigma1} = \Lambda_{\sigma2}$ for each hyperon type is used. This prescription was found to be the most appropriate one when comparing to the microscopic calculations. In fact, the scalar-like interactions are in any case better controlled with increasing density (respectively momentum) by $m^*/E^*$-suppression factors while the vector-like vertices do not include them, besides the NLD-regulators in the source terms of the meson-field equations (31,32). Note that $\Pi^0 = E^*$ for momentum-dependent regulators $\Pi^0 = E^*$ and for each baryon type B. Similar studies concerning the peculiar role of the vector $\omega$-meson exist in the literature. For instance, in Refs. [20, 21, 22] non-linear quadratic $\omega$-field contributions were considered as an alternative approach for the vector-like interaction Lagrangian leading to more complex density dependencies of their mean-fields. In the NLD model all higher-order non-linear terms are summed up into regulators. The novel feature of NLD is that these regulators mediate a non-linear density and, at the same time, a non-linear
momentum dependence of in-medium potentials not only for nucleons, but for hyperons too. This will become clear in the following discussions.

At first, the cut-offs of the hyperons have to be determined. The strangeness-$S = 1$ cut-offs are adjusted to the corresponding hyperonic optical potentials at saturation density of symmetric and cold nuclear matter from $\chi$-EFT calculations. This is shown in Fig. 1 for the optical potential of $\Lambda$-hyperons. The gray bands correspond to the microscopic calculations at different orders in $\chi$-EFT, while the solid curve represents the NLD result. At low momenta the $\Lambda$ in-medium interaction is attractive, but it becomes repulsive at high momenta. The non-trivial momentum dependence in NLD arises from the explicitly momentum dependent regulators which show up twice: in the scalar and vector selfenergies and in the source terms of the meson fields. As a consequence, the cut-off regulates the $\Lambda$-potential not only at zero momentum, but particularly over a wide momentum region. The in-medium $\Lambda$-potential does not diverge with increasing $p$-values (not shown here), but it saturates. Furthermore, the in-medium $\Lambda$-potential at zero kinetic energy leads to a value of $U_{\text{opt}}^\Lambda \simeq -28$ MeV, which is consistent with the NLO-calculations and also consistent with phenomenology. Therefore it exists an appropriate choice of cut-off regulators that do reproduce the microscopic calculations over a wide momentum range up to

Figure 1: Optical potential of $\Lambda$-hyperons as function of their momentum $p$ in symmetric nuclear matter at saturation density. The NLD-results (thick-solid curve) are compared with $\chi$-EFT microscopic calculations (taken from [17]) at different orders LO (band with closed dashed borders) and NLO (band with closed solid borders) [17]. Further microscopic calculations from the Jülich group (dot-dashed curve) are shown too [23].
Table 2: \( \Lambda \), \( \Sigma \) and \( \Xi \) cut-offs for \( \sigma \)- (\( \Lambda_\sigma \)), \( \omega \)- (\( \Lambda_\omega_1, \omega_2 \)) and \( \rho \)-hyperon-nucleon (\( \Lambda_\rho_{1,2} \)) vertices in units of GeV. In the cases for \( \Sigma \) and \( \Xi \) the isospin cut-offs (\( \Lambda_\rho_{1,2} \)) are relevant for the charged particles only. For the \( \Sigma \)-hyperon different cut-off values \( \Lambda_\rho_{1,2} \) are used for \( \Sigma^- \) (upper line) and for \( \Sigma^+ \) (bottom line).

\[
\begin{array}{cccccc}
\text{\( \Lambda \) cut-off} & \text{\( \Sigma \) cut-off} & \text{\( \Xi \) cut-off} \\
\hline
\Lambda_\sigma & \Lambda_\omega_1 & \Lambda_\omega_2 & \Lambda_\rho_1 & \Lambda_\rho_2 & \Lambda_\sigma & \Lambda_\omega_1 & \Lambda_\omega_2 & \Lambda_\rho_1 & \Lambda_\rho_2 \\
0.7 & 0.85 & 0.79 & -- & -- & 0.67 & 0.95 & 0.79 & 0.47 & 0.47 \\
& & & & & & & & 0.63 & 0.5 & 0.6 & 0.8 & 0.71 & 1.3 & 1.2 \\
\end{array}
\]

\( p \simeq 1 \) GeV very well. A similar picture occurs for the in-medium potential of \( \Sigma \)-hyperons, as shown in Fig. 2. The NLD cut-off for the \( \Sigma \)-particles can be regulated in such way to reproduce a repulsive potential at vanishing momentum with a weak momentum dependence at finite \( \Sigma \)-momentum. Again, the NLD calculations are able to describe the microscopic \( \chi \)-EFT results in NLO very well. The corresponding values for the strangeness cut-offs are tabulated in 2. Even if the origin of the cut-offs is different between the NLD model and the microscopic calculations, it may be interesting to note that these NLD cut-off values are close to the region between 500 and 650 GeV used in the \( \chi \)-EFT calculations.

We emphasize again the non-trivial momentum dependence of the in-medium hyperon-potentials, as manifested in the \( \chi \)-EFT calculations at different orders, see for instance Ref. [17]. This prescription modifies the momentum dependencies in such a complex way, which cannot be reproduced in standard RMF models by imposing SU(6) arguments. Furthermore, any standard RMF model leads to a divergent behaviour of optical potentials at high momenta. Note that a weak repulsive character of the \( \Sigma \)-potential, as proposed by the microscopic calculations, cannot be achieved in conventional RMF. The momentum-dependent NLD model resolves these issues effectively through momentum cut-offs of natural hadronic scale. Since we are dealing with hadronic matter, values of hadronic scale in the GeV-regime for the NLD regulators seem to be an adequate choice.

So far we have discussed the momentum dependence of the \( \Lambda \) and \( \Sigma \) hyperons at saturation density (Figs. 1 and 2). These comparisons served also as a guideline for the NLD cut-offs for the \( \Lambda \) and \( \Sigma \) baryons. Now we discuss the predictive power of the NLD approach by comparing in more detail the density and momentum dependence of the NLD formalism with the microscopic \( \chi \)-EFT calculations. This is shown in Figs. 3 and 4, where the momentum dependence of the \( \Lambda \) (Fig. 3) and \( \Sigma \) (Fig. 4) particles is displayed again, but now at various densities of symmetric nuclear matter. At first, the \( \Lambda \) and \( \Sigma \) optical potentials become more repulsive with increasing nuclear matter density in NLD. However, the non-trivial momentum and density dependence, as manifested in the NLD selfenergies and the meson-field sources, weakens the in-medium potentials with increasing momentum. In particular, the NLD model predicts astonishingly well the complex microscopic behaviours in momentum and at various densities of symmetric nuclear matter.

In asymmetric matter besides the standard iso-scalar and iso-vector vertices (\( \sigma \) and \( \omega \) meson fields, respectively) the iso-vector and Lorentz-vector \( \rho \)-meson must be taken into account.
NLD we assume again a monopole form for the $\rho$-meson coupling to the hyperons too by using the coupling constant of table 1 and the cut-off values of table 2 for the isospin sector. Relevant are the cut-off values $\Lambda_{\rho_{1,2}}$ for the charged $\Sigma^{\pm}$-hyperons. They have been fixed from the corresponding $\chi$-EFT calculations for $\Sigma^-$ and $\Sigma^+$ at saturation density. The NLD calculations for the neutral $\Lambda$- and $\Sigma^0$-hyperons are free of parameters here.

The results for pure neutron matter at three different baryon densities are summarized in Fig. 5. The NLD model does predict the general microscopic trends. In particular, in the case of the neutral hyperons ($\Lambda$ and $\Sigma^0$), where within the RMF approximation the $\rho$-meson does not appear at all, one would expect identical results between symmetric and pure neutron matter (at same total baryon density and momentum). This is in fact not the case. There is an inherent isospin dependence in the source terms of the meson-field equations, see Eqs. (30) even for the $\sigma$- and $\omega$-fields. The upper limits in those integrals (31, 32) are different for protons and neutrons between symmetric and asymmetric nuclear matter at the same total density. This leads to a different cut value in the regulators $\mathcal{D}_{p,n}$ and thus to a different result between symmetric and asymmetric matter. This NLD feature induces a hidden isospin dependence which is qualitatively consistent with the microscopic calculations at the three total densities as indicated in Fig. 5 for the "isospin-blind" hyperons. Concerning the charged $\Sigma^{\pm}$-hyperons, the comparison between NLD and $\chi$-EFT calculations is obviously at best for densities close to saturation. In general, the NLD predictions follow satisfactorily the details of the microscopic in-medium potentials as
function of momentum and matter density.

Finally we discuss the in-medium properties of the cascade-hyperons as shown in Figs. 6 and 7 for symmetric nuclear matter (SNM) and pure neutron matter (PNM). Here we apply for comparison the latest microscopic calculations from Lattice-QCD. The same NLD scheme with appropriate monopole-type regulators leads to the results in Fig. 6 for symmetric nuclear matter at saturation density. It is seen that a simple monopole-like regulator with hadronic cut-off values can explain the microscopic Lattice calculations. Indeed, a soft attractive potential for in-medium \( \Xi \)-hyperons is obtained in the NLD model over a wide momentum range. The prediction of NLD is then displayed in Fig. 7 for pure neutron matter but at the same total density at saturation as in the previous figure. The hidden isospin-dependence modifies slightly the momentum dependence of the neutral \( \Xi^0 \)-hyperon. In this case the Lattice calculations are reproduced only qualitatively by the NLD model, while for the charged cascade partner (\( \Xi^- \)) the comparison between NLD and Lattice is very well for pure neutron matter at saturation and over a broad region in single-particle cascade-momentum.

In the future experiments such as those at FAIR the in-medium properties of anti-hadrons will be investigated too. We thus give predictions for anti-hyperon in-medium potentials too. We recall the novel feature of the NLD formalism\[15\], that is, a parameter free predictions for anti-baryon optical potentials in the spirit of G-parity. In fact, once the cut-off parameters are fixed from saturation properties, the application of NLD to anti-matter gave very successful results by imposing G-Parity only. Note that in conventional RMF models one has to introduce by hand additional scaling factors in order to reproduce the weak attractiveness of the anti-proton optical potential at vanishing momenta\[24\]. We therefore use the same NLD formalism for the description of anti-hyperons too and performed additional calculations for the \( \Lambda, \Sigma \) and \( \Xi \) optical potentials as function of momentum and density. These results are shown in Fig. 8 for anti-\( \Lambda \) (left), anti-\( \Sigma \) (middle) and anti-\( \Xi \) optical potentials versus their momentum at three densities of symmetric nuclear matter. Due to the negative sign in the Lorentz-vector component of the hyperon self-energy these potentials are in general attractive over a wide momentum range. Compared to the anti-proton potential at saturation these potentials are less attractive with a similar dependence on single-particle momentum.

Since for anti-hyperons we make predictions and for anti-particles in general one may expect significant contributions to the imaginary part of the optical potential at vanishing momenta\[20\], too, we briefly discuss the imaginary part of the anti-hyperon optical potentials too. An exact treatment of the imaginary part of the optical potential is not possible within an RMF model. However, within the NLD approach one can estimate the strength of \( Im U_{opt} \) from dispersion relations\[15\]. This prescription was successfully applied to the antiproton case in a previous work (see Ref. \[15\]), thus we apply it here for the anti-hyperons too. The results for \( Im U_{opt} \) are shown in the same figure 8 by the thin curves. One generally observes a strong contribution to the in-medium anti-hyperon interactions from the imaginary parts of the optical potentials too. These contributions are quite similar to the imaginary potential of antiprotons with a value around \(-150\) MeV at very low kinetic energies (see for instance in \[15\] the second citation of 2015). However, in the antiproton-case the imaginary potential is rather strong relative to its real part, while for anti-hyperons both parts of the potential are sizeable. Even if the NLD results for the \( Im U_{opt} \) are only estimations, we can give a physical interpretation. In antinucleon-nucleon scattering annihilation can occur through
Figure 3: Optical potential of $\Lambda$-hyperons versus their momentum at various densities of symmetric nuclear matter, as indicated by the Fermi-momenta in units of 1/fm$^{-1}$. The NLD calculations at these three Fermi-momenta (thick-solid, thick-dashed and thick-dot-dashed curves) are compared to the $\chi$-EFT calculations at NLO [17].

the production of light pions. On the other hand, the interaction of anti-hyperons with nucleons can happen via the production of the heavier kaons due to strangeness conservation, which may influence the imaginary potential at low energies. This might be one reason why the imaginary part of the anti-hyperon optical potential is comparable with its corresponding real part particularly at very low energies. These calculations can be applied to anti-hadron induced reactions in the spirit of relativistic transport theory and can be tested in the future experiments at FAIR.

4. Summary

We have investigated the properties of strangeness particles inside nuclear matter in the framework of the NLD approach. The NLD model is based on the simplicity of the relativistic mean-field theory, but it includes the missing momentum dependence in a manifestly covariant fashion. This is realized by the introduction of non-linear derivative series in the interaction Lagrangian. In momentum space this prescription leads to momentum dependent regulators, which are determined by a cut-off. The NLD approach does not only resolve the optical potential issues of protons and antiprotons at high momenta, but it affects the density dependence. That is, the cut-off regulators make the EoS softer at densities close to saturation and stiffer at very high densities relevant for neutron stars.

Because of the successful application of the NLD model to infinite nuclear matter (and to finite nuclei [25]), it is a natural desire to extend this approach to hadronic matter by taking strangeness degrees of freedom into account. This is realized in the spirit of SU(6) symmetry. We applied the NLD model to the description of in-medium hyperon interactions for ordinary
nuclear matter. It was found that the strangeness cut-off regulates the momentum dependence of the optical potentials of hyperons in multiple ways. At first, the optical potentials do not diverge with increasing hyperon momentum. Furthermore, the NLD model predicts an attractive Λ-optical potential at low momenta, which becomes repulsive at high energies and finally saturates. In particular, it is possible to predict a weak and repulsive in-medium interaction for Σ-hyperons inside nuclear matter at saturation density. These results are in consistent agreement with calculations based on the chiral effective field theory. Regarding Ξ-hyperons, the NLD predictions turned out to be in agreement with recent Lattice-QCD calculations. In symmetric nuclear matter the cascade optical potential is attractive and it follows the Lattice-QCD results. In pure neutron matter the isospin-separation as predicted by the NLD model agrees with the Lattice-QCD behaviours qualitatively. While the potential of the neutral cascade particle remains attractive, the Ξ⁻-hyperon shows a weak repulsion in neutron matter. The weak repulsion of those hyperons may likely effect to a stiffer EoS for neutron star matter.

We briefly discussed the imaginary part of $U_{opt}$ of anti-hyperons too. These estimations indicate a significant contribution of the imaginary part to the anti-hyperon dynamics that could be explored in anti-hadron induced reactions. For instance, the present calculations can be tested in anti-proton induced reactions and in reactions with secondary Ξ-beams, as they are planned at FAIR in the future PANDA experiment.

Obviously this study is relevant not only for hadron physics, but also for nuclear astrophysics. The application of the NLD approach to β-equilibrated compressed matter is under progress, in order to investigate the hyperon-puzzle in neutron stars. Another interesting application concerns the dynamics of neutron star binaries. To do so, an extension to hot and compressed hadronic matter is necessary and under progress too. Note that the NLD formalism is fully thermodynam-
Figure 5: Optical potentials for hyperons (as indicated) versus their momentum for pure neutron matter. Solid curves with symbols indicate the NLD calculations while pure curves without symbols are the microscopic $\chi$-EFT results at NLO from Ref. [17]. Green pairs (circles-solid for NLD and solid for $\chi$-EFT) refer to low density of $p_F = 1$ fm$^{-1}$, red pairs (diamonds-dashed for NLD and dashed for $\chi$-EFT) refer to saturation density of $p_F = 1.35$ fm$^{-1}$ and blue pairs (triangles-dot-dashed for NLD and dot-dashed for $\chi$-EFT) refer to a density of $p_F = 1.53$ fm$^{-1}$.

ically consistent, which is an important requirement before applying it to hot and dense systems. In summary, we conclude the relevance of our studies for future experiments at FAIR and for nuclear astrophysics.

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References

References

[1] P. Demorest, T. Pennucci, S. Ransom, M. Roberts, and J. Hessels, Nature 467 (2010) 1081.

[2] J. Antoniadis et al., Science 340 (2013) 6131.

[3] M. Linares, T. Shahbaz and J. Casares, Astrophys. J. 859 (1) (2018) 54
H.T. Cromartie, E. Fonseca, et al., Nature Astronomy 4 (2020) 72.
Figure 6: Optical potential of cascade hyperons versus their momentum for symmetric nuclear matter (SNM) at saturation density. The solid curve indicates the NLD predictions while the dashed curve and the gray band refer to recent Lattice calculations from Refs. [19] (Lattice2016 and Lattice2019).

[4] J.M. Lattimer, A.W. Steiner, *Astrophys. J.* **784** (2014) 123.

[5] T. Klähn, *et al.*, *Phys. Rev.* C **74** (2006) 035802.

[6] C. Fuchs, *Prog. Part. Nucl. Phys.* **56** (2006) 1.

[7] C. Hartnack, *et al.*, *Phys. Rept.* **510** (2012) 119.

[8] Ch. Moustakidis, T. Gaitanos, Ch. Margaritis, G.A. Lalazissis, *Phys. Rev.* C **95** (2017) 045801.

[9] I. Bombaci, arXiv:1601.05339 [nucl-th], D. Chatterjee, I. Vidana, *Eur. Phys. J.* A **52** (2016) 29.

[10] J. Haidenbauer, U. G. Meißner, N. Kaiser and W. Weise, *Eur. Phys. J.* A **53** (2017) no.6, 121.

[11] S. Petschauer, J. Haidenbauer, N. Kaiser, U. G. Meißner and W. Weise, *Front. in Phys.* **8** (2020) 12.
Figure 7: Same as in Fig. 6, but for pure neutron matter (PNM). The curves and bands belonging to $\Xi^-$ and $\Xi^0$ are indicated in this figure.

[12] J. Schaffner and I. N. Mishustin, *Phys. Rev.* C 53 (1996) 1416.

[13] N. Hornick, L. Tolos, A. Zacchi, J. E. Christian and J. Schaffner-Bielich, *Phys. Rev.* C 98 (2018) no.6, 065804.

[14] J. E. Christian and J. Schaffner-Bielich, *Astrophys. J. Lett.* 894 (2020) no.1, L8.

[15] T. Gaitanos, M. Kaskulov, *Nucl. Phys.* A 899 (2013) 133,
    T. Gaitanos, M. Kaskulov, *Nucl. Phys.* A 940 (2015) 181.

[16] H.-P. Duerr, *Phys. Rev.* 103 (1956) 469,
    J.D. Walecka, *Ann. Phys.* 83 (1974) 491,
    J. Boguta, A. Bodmer, *Nucl. Phys.* A 292 (1977) 413.

[17] S. Petschauer, *et al.*, *Eur. Phys. J.* A 52 (2016) 15,
    J. Haidenbauer, private communication.

[18] J. Haidenbauer, U. G. Meißner and A. Nogga, *Eur. Phys. J.* A 56 (2020) no.3, 91.

[19] T. Inoue [LATTICE-HALQCD], *PoS INPC2016* (2016), 277,
    T. Inoue [HAL QCD], *AIP Conf. Proc.* 2130 (2019) no.1, 020002.
Figure 8: Optical potentials for anti-hyperons versus their kinetic energy for symmetric nuclear matter (SNM) at various densities, as indicated. The NLD predictions for saturation density $\rho_0$ (thick-solid) and higher densities of $2\rho_0$ (thick-dashed) and $3\rho_0$ (thick-dashed-dot) are shown. For the anti-hyperons we show estimates for the imaginary part of their optical potentials too at saturation density $\rho_0$ (thin-solid), at $2\rho_0$ (thin-dashed) and at $3\rho_0$ (thin-dashed-dot).

[20] M. Fortin, S. S. Avancini, C. Providência and I. Vidaña, *Phys. Rev.* C 95 (2017) no.6, 065803.

[21] C. Providência and A. Rabhi, *Phys. Rev.* C 87 (2013) 055801.

[22] Y. Sugahara, and H. Toki, *Nucl. Phys.* A 579 (1994) 557.

[23] J. Haidenbauer and U.-G. Meissner, *Phys. Rev.* C 72 (2005) 044005.

[24] A.B. Larionov, *et al.*, *Phys. Rev.* C 80 (2009) 021601(R).

[25] S. Antic and S. Typel, *AIP Conf. Proc.* 1645 (2015) 276.