Research Article

Neural Network-Based Nonlinear Fixed-Time Adaptive Practical Tracking Control for Quadrotor Unmanned Aerial Vehicles

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1. Introduction

During the last decades, more attention has been paid to UAVs by the engineering communities because of their potential for commercial, military, and academic platforms [1–3]. To support military and civilian applications, the ability of UAVs to hover stably, maneuver sharply, and operate safely is considerably important, especially in take-off and landing stages [4–6]. However, tracking control design for UAVs in complex environments is challenging because of the nonlinear dynamics of UAVs. In order to meet various application requirements, especially military requirements, UAVs are required to have better robustness, stability, and fast maneuverability [7]. Drones have a wealth of applications in various fields, such as monitoring, spraying pesticides, and delivering couriers.

Since the advent of the first rotorcraft in the twentieth century, in order to solve the impact of the actual situation and better complete the motion control of the four-rotor UAV, scholars have published numerous documents to solve a series of problems. With the advancement of science and the development of NNs [8–10], NNs have been widely used and verified in the quadrotor intelligent control. As we all know, the neural network has been questioned by some scholars in the long history since it was proposed, but it is finally proved after research by scholars in many fields that the neural network is an advantageous control algorithm [11–13]. Although neural networks have long been used in the quadrotor intelligent control and have achieved good results in many research studies, the research is not comprehensive. In recent years, the study of finite time adaptive neural networks has been carried out by scholars, and a fixed-time control is proposed to solve the dependence of convergence on the initial state. In this paper, the nonlinear fixed-time adaptive neural network control of the quadcopter UAV is studied.

Although the quadrotor has many advantages, due to the nonlinearity, coupling, underdrive, and susceptibility to interference of the dynamics of the UAV, it is necessary to develop tracking control in uncertain environments. In the past few years, the authors studied the design of the dynamic mathematic model of the UAV in [14], as well as dynamic control of the UAV. In [15–17], the authors proposed a PID/PD control method to maintain flight stability, but due to the shortcomings of the algorithm, the authors designed a neural network adaptive PID algorithm in [18]. In order to
solve the underdrive of the quadrotor, the authors proposed four additional control inputs in [19] to represent a fully driven aircraft. In [20], the authors used sliding film control. The quadrotor is susceptible to a variety of unknown disturbances during flight. In [21–23], the authors designed a variety of controllers to stabilize the disturbances, but they were not perfect. Backstepping control design has been widely used [24, 25] for a class of nonlinear system control. The four-rotor aircraft has obtained relevant research results during take-off, hover, and landing [26–28]. The interference caused by machine failure cannot be avoided during flight. In [29], the authors proposed a method to solve the loss of control effectiveness of the aircraft through adaptive control and quantum logic. It is well known that neural network algorithms have advanced control performance. In [30, 31], the authors proposed neural network algorithms to control flight attitude and position. Some researchers also used NNs to model a quadrotor aircraft in [32], which can more accurately describe the quadrotor model. In order to ensure the tracking error, the inversion controller was used in [33] to study the adaptive flight control neural network and dynamic inversion control. Quadrotor flight control researchers have proposed a variety of control algorithms. In [34], the authors proposed dynamic feedback linearization of a four-axis aircraft for chart learning. In order to enhance the stability of the UAV, the author applied the feedforward control method in [35]. Finite-time control [36, 37] plays an important role in solving the convergence time problem.

In this paper, practical fixed-time adaptive NN control for the UAV system is proposed. The closed-loop UAV position and attitude tracking error system is practically fixed-time stable, whereas the convergence time is independent of initial states and tracking error is convergence to a small neighborhood of the origin point. The major contributions of this brief can be stated as follows:

1. Based on NNs adaptive control and practical fixed-time stability theory, the controller parameters for the UAVs position and attitude tracking are designed.
2. The fixed-time adaptive law is designed for NNs, and error weight is practically stable in finite time. Therefore, the NNs can approximate the nonlinear system in finite time independent of ideal weight and initial value of estimated weight.
3. To avoid the finite-time control singular problem in the high-order system, a switch virtual control law is proposed to guarantee the feasibility of the fixed-time control.

### 2. Preliminary Problem Formulation

A dynamic model [30, 38] is established according to the motion laws of the four-rotor power system; the body coordinate system\(B: O_x Y_x Z_x\) and the ground coordinate system\(E: O_Y X_Y Z_Y\) are selected, as shown in Figure 1. In the Newton–Euler formula, \([x, y, z]^T\) indicates the Euclidean position and \([\phi, \theta, \psi]^T\) indicates the Euler angles of UAVs. When the four-rotor drone changes its motion, the body coordinate system will change continuously with the drone and the ground coordinate system will remain constant. Due to the need to analyze the problem in the ground coordinate system, the spatial rotation of the body coordinate system relative to the geographic coordinate system is usually expressed in the order of yaw angle, pitch angle, and roll angle. The matrix is used to describe the yaw angle \(\psi\), pitch angle \(\theta\), and roll angle \(\phi\). Based on the relationship between the angles in the coordinate system \(B\) and the coordinate system \(E\), the coordinate transformation matrix can be obtained.

The transformation from body coordinates to ground coordinates is performed by the rotation matrix:

\[
R = \begin{bmatrix}
\cos \theta \cos \psi & \cos \psi \sin \theta - \sin \phi \sin \theta \cos \psi & \cos \phi \cos \psi \sin \theta + \sin \phi \sin \theta \cos \psi \\
\cos \theta \sin \psi & \sin \theta \sin \phi + \cos \phi \cos \theta \sin \phi & \cos \phi \sin \theta \sin \phi - \cos \phi \cos \theta \cos \psi \\
-\sin \theta & \cos \theta \cos \phi & \cos \theta \sin \phi
\end{bmatrix}
\]

(1)

When modeling the quadrotor as a rigid body with uniform mass distribution, symmetry, and constant, according to Newton's second law:

\[
F_{\text{all}} = m \begin{bmatrix} \ddot{x} \\ \ddot{y} \\ \ddot{z} \end{bmatrix},
\]

(2)

where \(m\) is the weight of the quadrotor drone and \(\ddot{x}, \ddot{y}, \ddot{z}\) are the acceleration in three directions, and the force \(F_{\text{all}}\) in three directions is obtained.

According to the force analysis of the drone in motion, it can be known that the external force received has its own gravity \(G\), the lift force \(F_{\text{at}}\) generated by the rotor, and the rising resistance \(f_i\):

\[
G = mg,
\]

\[
F_{\text{at}} = k \omega^2 (j = 1, 2, 3, 4),
\]

(3)

\[
\begin{bmatrix} f_x \\ f_y \\ f_z \end{bmatrix} = \begin{bmatrix} \xi_x \ddot{x} \\ \xi_y \ddot{y} \\ \xi_z \ddot{z} \end{bmatrix}.
\]

According to equation (3), the centroid motion equation of the quadrotor UAV with the positive Z axis in the ground coordinate system can be obtained:
the quadrotor UAV as

\begin{align*}
    m \begin{bmatrix}
        \ddot{x} \\
        \ddot{y} \\
        \ddot{z}
    \end{bmatrix} &= R F_{at} \begin{bmatrix}
        0 \\
        0 \\
        1
    \end{bmatrix} - mg \begin{bmatrix}
        0 \\
        0 \\
        1
    \end{bmatrix} - \frac{\xi_x}{m} \dot{x} \\
    \dot{y} \\
    \dot{z}
\end{align*}

(4)

To sum up, we can get the centroid motion equations of the quadrotor UAV as

\begin{align*}
    x &= \frac{F_{at}}{m} (\sin \psi \sin \phi + \cos \psi \sin \theta \cos \phi) - \frac{\xi_x}{m} \dot{x}, \\
    y &= \frac{F_{at}}{m} (-\cos \psi \sin \phi + \sin \phi \sin \theta \cos \phi) - \frac{\xi_y}{m} \dot{y}, \\
    z &= \frac{F_{at}}{m} (\cos \theta \cos \phi) - g - \frac{\xi_z}{m} \dot{z},
\end{align*}

where $k$ is the motor pulling force coefficient, $\omega_j$ ($j = 1, 2, 3, 4$) is the motor speed, and $\xi_i$ ($i = x, y, z$) is the air resistance coefficient.

Four-rotor drones are divided into cross-shaped structure and X-shaped structure. Due to the difference in structure, the rotation dynamic equation of the drone will also be different. The cross-shaped structure is selected to build a model based on the Newton–Euler equation:

\begin{align*}
    \overline{M} &= I \tau + \overline{w} \times \overline{I} \overline{w} \\
    &= \begin{bmatrix}
        I_x & 0 & 0 \\
        0 & I_y & 0 \\
        0 & 0 & I_z
    \end{bmatrix} \begin{bmatrix}
        \dot{\rho} \\
        \dot{\varphi} \\
        \dot{\psi}
    \end{bmatrix} + \begin{bmatrix}
        (I_z - I_y) q r \\
        (I_x - I_z) r p \\
        (I_y - I_x) p q
    \end{bmatrix},
\end{align*}

(6)

and then

\begin{align*}
    \overline{M} &= \begin{bmatrix}
        M_x \\
        M_y \\
        M_z
    \end{bmatrix} = \begin{bmatrix}
        \dot{\rho} I_x + (I_z - I_y) q r \\
        \dot{\varphi} I_y + (I_x - I_z) r p \\
        \dot{\psi} I_z + (I_y - I_x) p q
    \end{bmatrix}.
\end{align*}

(7)

Among them, $\overline{M}$ is the combined external moment on the body, $I = \text{diag}(I_x, I_y, I_z)$ is the moment of inertia of the three coordinate axes in the body coordinate system, and $\overline{w} = \text{diag}(p, q, r)$ is the rotational angular velocity in the body coordinate system. The force analysis of the moving four-rotor UAV shows that there are torque generated by the rotor, air reaction force, and gyro effect.

The moment produced by the rotor is as follows:

\begin{align*}
    u_1 &= F_{at} = k(\omega_1^2 + \omega_2^2 + \omega_3^2 + \omega_4^2), \\
    u_2 &= \tau_{\varphi} = kl(\omega_3^2 - \omega_1^2), \\
    u_3 &= \tau_{\varphi} = kl(\omega_2^2 - \omega_1^2), \\
    u_4 &= \tau_{\psi} = b(\omega_2^2 - \omega_1^2 + \omega_3^2 - \omega_4^2),
\end{align*}

(8)

where $l$ is the distance between the center of mass and the rotor axis, $b$ is the drag coefficient, $k$ is the reverse torque coefficient, and $\omega$ is the motor speed.

The air reaction force is as follows:

\begin{align*}
    \tau_f &= \xi_k \omega,
\end{align*}

(9)

where $\xi_k = \text{diag}(\xi_\theta, \xi_\phi, \xi_\psi)$ is the air drag moment coefficient in the airframe coordinate system and $\omega$ is the rotor angular velocity.

The torque of the quadcopter is as follows:

\begin{align*}
    E_f &= I_r \dot{\omega} \omega,
\end{align*}

(10)

where $I_r$ is the rotor inertia, $\omega$ is the rotor angular velocity, and $\dot{\omega} = \omega_1 + \omega_2 - \omega_3 - \omega_4$ is the overall remaining rotor angle.

From Newton mechanical analysis and equations (8)–(10), we get
In summary, when the rotation angle is small, the dynamic model of the quadrotor flight can be obtained as

\[
\begin{bmatrix}
\dot{x} \\
\dot{y} \\
\dot{z} \\
\dot{\phi} \\
\dot{\theta} \\
\dot{\psi}
\end{bmatrix}
= \begin{bmatrix}
\frac{F_x}{m} (\sin \psi \sin \phi + \cos \psi \sin \theta \cos \phi) - \frac{\xi_x}{m} \dot{x} \\
\frac{F_y}{m} (-\cos \psi \sin \phi + \sin \psi \sin \theta \cos \phi) - \frac{\xi_y}{m} \dot{y} \\
\frac{F_z}{m} (\cos \theta \cos \phi) - g - \frac{\xi_z}{m} \dot{z} \\
\frac{I_y}{I_x} \frac{\tau_{\phi\theta}}{I_x} - \frac{\xi_y}{I_x} \dot{\phi} \\
\frac{I_z}{I_y} \frac{\tau_{\theta\psi}}{I_y} - \frac{\xi_z}{I_y} \dot{\theta} \\
\frac{I_z}{I_x} \frac{\tau_{\psi\phi}}{I_x} - \frac{\xi_z}{I_x} \dot{\psi}
\end{bmatrix}
\]

(12)

\[
\begin{bmatrix}
\dot{x} \\
\dot{y} \\
\dot{z} \\
\dot{\phi} \\
\dot{\theta} \\
\dot{\psi}
\end{bmatrix}
= \begin{bmatrix}
\frac{1}{I_x} 0 0 \\
0 \frac{1}{I_y} 0 \\
0 0 \frac{1}{I_z} \\
\frac{I_y}{I_x} \frac{I_z}{I_x} \frac{\xi_y}{I_x} - \frac{\xi_y}{I_x} \\
\frac{I_z}{I_y} \frac{I_x}{I_y} \frac{\xi_z}{I_y} - \frac{\xi_z}{I_y} \\
\frac{I_x}{I_z} \frac{I_y}{I_z} \frac{\xi_x}{I_z} - \frac{\xi_x}{I_z}
\end{bmatrix}
\begin{bmatrix}
f_1(x, t) \\
f_2(x, t)
\end{bmatrix}
\]

(15)

In order to facilitate the design of the controller, the dynamic model of the quadrotor flight was rewritten into a compact dynamic equation, \( d_a(\cdot) = \text{diag} \left[ d_x(\cdot) \ d_y(\cdot) \ d_z(\cdot) \right] \) and \( d_p(\cdot) = \text{diag} \left[ d_{\phi}(\cdot) \ d_{\theta}(\cdot) \ d_{\psi}(\cdot) \right] \) take into account the external disturbance of the position and attitude of the quadrotor in motion.

(1) Translational dynamics:

\[
\ddot{x} = A u_x(t) + f_1(\cdot) + d_a(x, t).
\]

(13)

(2) Rotational dynamics:

\[
\ddot{p} = B u_r(t) + f_2(\cdot) + d_p(p, t),
\]

(14)

where

\[
A = \begin{bmatrix}
\frac{1}{m} & 0 & 0 \\
0 & \frac{1}{m} & 0 \\
0 & 0 & \frac{1}{m}
\end{bmatrix},
\]

\[
B = \begin{bmatrix}
0 \\
0 \\
0 \\
\frac{I_y}{I_x} \frac{\xi_y}{I_x} - \frac{\xi_y}{I_x} \\
\frac{I_z}{I_y} \frac{\xi_z}{I_y} - \frac{\xi_z}{I_y} \\
\frac{I_x}{I_z} \frac{\xi_x}{I_z} - \frac{\xi_x}{I_z}
\end{bmatrix},
\]

\[
B = \begin{bmatrix}
\frac{1}{I_x} 0 0 \\
0 \frac{1}{I_y} 0 \\
0 0 \frac{1}{I_z} \\
\frac{I_y}{I_x} \frac{I_z}{I_x} \frac{\xi_y}{I_x} - \frac{\xi_y}{I_x} \\
\frac{I_z}{I_y} \frac{I_x}{I_y} \frac{\xi_z}{I_y} - \frac{\xi_z}{I_y} \\
\frac{I_x}{I_z} \frac{I_y}{I_z} \frac{\xi_x}{I_z} - \frac{\xi_x}{I_z}
\end{bmatrix},
\]

\[
f_1(x, t) = \begin{bmatrix}
f_{11}(x, t) \\
f_{12}(x, t)
\end{bmatrix},
\]

\[
f_2(x, t) = \begin{bmatrix}
f_{21}(x, t) \\
f_{22}(x, t)
\end{bmatrix},
\]

\[
\begin{bmatrix}
u_1(t) \\
u_2(t) \\
u_3(t) \\
u_4(t)
\end{bmatrix} = \begin{bmatrix}
\frac{(\sin \psi \sin \phi + \cos \psi \sin \theta \cos \phi)}{m} \\
\frac{(\cos \psi \sin \phi + \sin \psi \sin \theta \cos \phi)}{m} \\
\frac{(\cos \phi)}{m} \\
\frac{g}{m}
\end{bmatrix}.
\]

(13)
Lemma 1 (see [30, 39, 40]). Suppose that $V(\cdot) : \mathbb{R}^n \rightarrow \mathbb{R}_+ \cup \{0\}$ is a continuous radically unbounded function and the following two conditions hold

$$V(x) = 0 \iff x = 0,$$

$$V(x) \leq -aV(x)^p - bV(x)^q + c,$$  

(16)

where $a, b, p, q$ are the positive real numbers with $p \in (0, 1)$ and $q \in (1, \infty)$. Then, the origin $x = 0$ of the system $\dot{x} = f(t, x)$ and $x(0) = x_0$ is practically fixed-time stable. Furthermore, the following inequality holds:

$$V(x, t) \leq \xi, t \geq T_{\text{max}},$$

(17)

where $\xi$ is the equation roots, and

$$2^{p-1}a\xi^p + b\xi^q = c,$$

$$T_{\text{max}} = \frac{1}{2^{p-1}a(1-p)} + \frac{1}{b(q-1)}.$$  

(18)

Lemma 2 (see [39]). For $x_i \in R$ and $x_i \geq 0$, $i = 1, 2, \ldots, n$, $0 < p < 1$, and $q > 1$, and then

$$\left( \sum_{i=1}^{n} x_i \right)^p \leq \left( \sum_{i=1}^{n} x_i^p \right) \leq n^{1-p} \left( \sum_{i=1}^{n} x_i \right)^p,$$  

(19)

$$n^{-q} \left( \sum_{i=1}^{n} x_i \right)^q \leq \left( \sum_{i=1}^{n} x_i^q \right) \leq \left( \sum_{i=1}^{n} x_i \right)^q.$$  

(20)

Proof. Inequalities (19) and (20) hold trivially for

$$x_1 = x_2 = \ldots = x_n = 0.$$  

(21)

For inequality (19), using the fact $x^p \geq x$ for $\forall x \in (0, 1)$ and $0 < p < 1$, we have

$$\frac{n}{\left( \sum_{i=1}^{n} x_i \right)^p} \leq \left( \sum_{i=1}^{n} x_i^p \right) \leq n^{1-p} \left( \sum_{i=1}^{n} x_i \right)^p = 1,$$  

which proves the inequality

$$\left( \sum_{i=1}^{n} x_i \right)^p \leq \left( \sum_{i=1}^{n} x_i^p \right) \leq \left( \sum_{i=1}^{n} x_i \right)^p.$$  

(23)

Based on the Holder inequality,

$$\sum_{i=1}^{n} x_i^p \eta_i \leq \left( \sum_{i=1}^{n} (x_i^p)^{(1/p)} \right)^{1-p} \left( \sum_{i=1}^{n} (\eta_i)^{(1/p)} \right)^{1-p} \left( \sum_{i=1}^{n} \eta_i \right)^{1-p},$$  

(24)

for $\forall x_i, \eta_i > 0$, and let $\eta_i = 1, i = 1, 2, \ldots, n$,

$$\sum_{i=1}^{n} x_i^p \leq n^{1-p} \left( \sum_{i=1}^{n} x_i \right)^p,$$  

(25)

which proved inequality (19).

Based on the Holder inequality,
\[-\tilde{\omega}_i \tilde{\omega}_i^{(1/3)} \leq -\frac{1}{2} \tilde{\omega}_i^{(4/3)} + \omega_i^{(4/3)}. \]  

Therefore,

\[-\sum_{i=1}^{m} \tilde{\omega}_i \tilde{\omega}_i^{(1/3)} \leq -\frac{1}{2} \sum_{i=1}^{m} \tilde{\omega}_i^{(4/3)} + \sum_{i=1}^{m} \omega_i^{(4/3)}, \]  

and then the following inequation holds:

\[-\tilde{W}^T \tilde{W}^{(1/3)} \leq -\frac{1}{2} \tilde{W}^{(2/3)} \tilde{W}^{(2/3)} + W^* (2/3)^T W^* (2/3). \]  

This is the proof. \(\Box\)

**Notation 1.** To avoid control singularity problem in virtual control, if \(p < 1\), then design \(\xi^p\) is chosen as

\[\xi^p = \begin{cases} \xi^p, & |\xi| \geq \varepsilon, \\ \xi^{p-1}, & |\xi| < \varepsilon. \end{cases} \]

Therefore, \(\xi^p\) is continuous and differentiable.

Let rational number \(\eta\), matrix \(W \in \mathbb{R}^{m \times n}\), and matrix \(W\eta\) denote element-by-element powers, where

\[W = \begin{bmatrix} w_{11} & w_{12} & \cdots & w_{1n} \\ w_{21} & w_{22} & \cdots & w_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ w_{mn} & w_{m2} & \cdots & w_{m1} \end{bmatrix}, \]

\[W\eta = \begin{bmatrix} w_{11}^{\eta} & w_{12}^{\eta} & \cdots & w_{1n}^{\eta} \\ w_{21}^{\eta} & w_{22}^{\eta} & \cdots & w_{2n}^{\eta} \\ \vdots & \vdots & \ddots & \vdots \\ w_{mn}^{\eta} & w_{m2}^{\eta} & \cdots & w_{m1}^{\eta} \end{bmatrix}. \]

\(\tilde{W}^n\) denotes the transposition of matrix \(\tilde{W}\), where

\[\tilde{W}^n = (\tilde{W}^\eta)^T. \]

### 3. Neural Network Control

Based on the dynamic model of the quadrotor (12), the translational and rotational tracking errors are defined as

\[e_x = x - x_d, \quad e_p = p - p_d. \]

To facilitate the control design, the derivation of translational and rotational tracking errors is as follows:

\[s_1 = \dot{x} - \dot{x}_d, \quad s_2 = \dot{p} - \dot{p}_d. \]

Therefore, the translational and rotational tracking error control system is as follows:

\[\begin{align} \dot{e}_x & = s_1, \\
\dot{s}_1 & = Au_s (t) + f_1 (\cdot) + d_n (t) - \dot{x}_d, \end{align} \]  

\[\begin{align} \dot{\omega}_p & = s_2, \\
\dot{s}_2 & = Bu_r (t) + f_2 (\cdot) + d_p (t) - \dot{p}_d. \end{align} \]

**Theorem 1.** Considering the quadrotor UAV system (12), the translational dynamic system (13), and the translational error control system (44), the fixed-time control parameters \(p = (1/3)\) and \(q = 3\) and the virtual control are designed as

\[a_\xi = -a_\xi_0 p - h_1 \xi_1, \quad a_1 > 0, b_1 > 0. \]

The fixed-time neural network adaptive control can be designed as

\[u_s = A^{-1} (\tilde{W}^n_1 \Psi_s - \tilde{x}_d + \xi_0 - a_\xi_0 \dot{\xi}_0 - b_\xi_0 \xi_1), \quad a_2 > 0, b_2 > 0. \]

With the neural fixed-time adaptive law,

\[\tilde{W}_1 = \Gamma_1 \left( \Psi_s \tilde{\xi}_2 - \sigma_{ax} \tilde{W}_1 - \sigma_{px} \tilde{W}_1 \right), \quad \Gamma_1 > 0, \sigma_{ax} > 0, \sigma_{px} > 0. \]

The translational dynamic system is fixed-time stable and the fixed time is \(T_{x_{\text{max}}} = (2^{1-p} - p^2)/(1-p) + (2/(q-1)b_\xi^2). \)

For the rotational dynamic system (14), the desired roll and pitch references are obtained from

\[\begin{align} \phi_d & = \arcsin \left( \frac{Q_x \sin (\psi_d) - Q_y \cos (\psi_d)}{u_d} \right), \\
\theta_d & = \arctan \left( \frac{Q_x \cos (\psi_d) + Q_y \sin (\psi_d)}{Q_z} \right). \end{align} \]

Based on the translational errors control system (45), the virtual control is designed as

\[a_\xi = -a_\xi \dot{\xi}_0 - b_3 \xi_1, \]

and the fixed-time neural network adaptive control can be designed as

\[u_r = B^{-1} (\tilde{W}_2^T \Psi - \dot{p}_d + \xi_0 - a_\xi \dot{\xi}_0 - b_4 \xi_1), \]

With the neural fixed-time adaptive law,

\[\tilde{W}_2 = \Gamma_2 \left( \Psi_p \dot{\xi}_2 - \sigma_{hp} \tilde{W}_2 - \sigma_{bp} \tilde{W}_2 \right). \]

The rotational dynamic system is fixed-time stable and the fixed time is \(T_{p_{\text{max}}} = (2^{1-p} - p^2)/(1-p) + (2/(q-1)b_\xi^2). \)

Then, the error translational and rotational dynamic closed-loop systems are practically fixed-time stable; the output tracking error and error of estimate weights can converge to the origin in finite time, and all the signals in the closed-loop system are bounded.
Proof. The proof of the sufficiency of Theorem 1 is divided into two parts. The first part gives translational dynamic system controller design and stability analysis, and the second one proposes rotational dynamic system controller design and stability analysis.

Step 1. According to the translational tracking error control system, let

\[ \xi_1 = e_x, \]

and then based on (44), we have

\[ \dot{\xi}_1 = -a_1 \xi_1^p - b_1 \xi_1^q + \xi_2, \]

where

\[ \xi_2 = s_1 - \alpha_t. \]

and then we have

\[ \dot{\xi}_2 = \dot{\xi}_x + d_a - \xi_2 - \alpha_t. \]

Neural networks approximate the nonlinear system as

\[ f_1 + d_a = W_1^T \Psi_x + \xi_x. \]

Based on the neural networks adaptive law (48) and the control design (47), we have

\[ V_1 = 1/2 \xi_1^2. \]

If we choose the Lyapunov candidate function

\[ V_1 = 1/2 \xi_1^2, \]

then we have

\[ V_1 = -a_1 \xi_1^{p+1} - b_1 \xi_1^{q+1} + \xi_1 \xi_2. \]

If we choose the Lyapunov candidate function

\[ V_2 = 1/2 \xi_2^2, \]

then we have

\[ V_2 = -\xi_2 \dot{W}_1^T \Psi_x + \xi_x \xi_2 - \alpha_t \xi_2^{p+1} - b_2 \xi_2^{q+1}. \]

If we choose the Lyapunov candidate function

\[ V_{NNX} = 1/2 \dot{W}_1^T \dot{W}_1, \]

then we have

\[ V_{NNX} = \dot{W}_1^T \dot{W}_1 = \dot{W}_1^T \Psi_x \xi_2 - \sigma_{xx} \dot{W}_1^p - \sigma_{xx} \dot{W}_1^q. \]

If we choose the Lyapunov candidate function

\[ V = V_1 + V_2 + V_{NNX}, \]

then we have

\[ V = -a_1 \xi_1^{p+1} - b_1 \xi_1^{q+1} + e_x \xi_2 - a_2 \xi_2^{p+1} - b_2 \xi_2^{q+1} - \sigma_{xx} \dot{W}_1^T \dot{W}_1. \]

Based on Lemmas 3 and 4, the following inequalities hold

\[ \xi_1^2 \leq 1/2 \xi_1^{p+1} + q \xi_1^{q+1} - \sigma_{xx} \dot{W}_1^T \dot{W}_1. \]

Then, based on Lemma 3, we have

\[ V \leq -a_1 \xi_1^{p+1} - b_1 \xi_1^{q+1} - a_2 \xi_2^{p+1} - b_2 \xi_2^{q+1} + 1/2 \xi_2^{q+1} \]

\[ + \frac{q}{q+1} \xi_1^{q+1}, \]

\[ + b_{p1} \dot{W}_1 \dot{W}_1^T \dot{W}_1 \]

\[ - \sigma_{bx} \dot{W}_1^T \dot{W}_1 \xi_1, \]

\[ + b_{q1} \dot{W}_1 \dot{W}_1^T \dot{W}_1 \xi_1. \]

Then, based on Lemma 2, we have

\[ V \leq 2(q+1/2) \left( 2 - \frac{1}{q+1} \right) V_{NNX} - c_{p1} V_{NNX}^2 - c_{q1} V_{NNX}^{q+1} \]

\[ + \frac{q}{q+1} \xi_1^{q+1} + b_{p1} \dot{W}_1 \dot{W}_1^T \dot{W}_1 \]

\[ + b_{q1} \dot{W}_1 \dot{W}_1^T \dot{W}_1 \xi_1 \]

\[ \leq -a_1 V_{NNX}^{p+1} - b_1 V_{NNX}^{q+1} + c_{xx}. \]

where

\[ a_x = \min \left( 2(q+1/2) a_1, 2(q+1/2) a_2, 2(q+1/2) c_q \right), \]

\[ b_x = \min \left( 2(q+1/2) a_1, 2(q+1/2) a_2, 2(q+1/2) c_q \right) \]

\[ c_x = \frac{q}{q+1} \xi_1^{q+1} + b_{p1} \dot{W}_1 \dot{W}_1^T \dot{W}_1 \xi_1. \]
Therefore, based on Lemma 1, the closed-loop system is practically fixed-time stable, and convergence time $T_{x_{max}} = (2^{-p_{1}^{-1}})/(1 - p)a_s + (2/\sigma b_p)$.

**Step 2.** There are six degrees of freedom, namely, $Q_x$, $Q_y$, and $Q_z$ and angles $\theta$, $\phi$, and $\psi$, where $Q_x$, $Q_y$, and $Q_z$ can be obtained by using $u_c$ which was designed by using (47) and $u_1 = \sqrt{Q_x^2 + Q_y^2 + Q_z^2}$. The arbitrary desired yaw references $\psi_d$ can be designed in advance, and then based on the mathematical model (12), the desired roll and pitch references $\phi_d$ and $\theta_d$ can be generated as (49) and (50). Then, the rotational tracking errors control system can be controlled in fixed-time neural network adaptive control.

Therefore, neural network control for the rotational tracking errors control system (45) is designed as follows:

$$\ddot{\zeta}_2 = \frac{1}{2} \zeta_1^2,$$

(72)

Then, based on (45), we have

$$\ddot{\zeta}_2 = \frac{1}{2} \zeta_1^2 - \zeta_2 + \zeta_1 \dot{\zeta}_2,$$

(73)

where

$$\zeta_2 = \zeta_2 - \alpha \zeta,$$

(74)

and then we have

$$\ddot{\zeta}_2 = Bu_r + f_2 + d_p - \alpha \zeta,$$

(75)

Neural networks approximate the nonlinear system as

$$f_2 + d_p = W_2^T \Psi_p + \epsilon_p,$$

(76)

Based on the neural networks adaptive law (53) and the control design (52), we have

$$\ddot{\zeta}_2 = -\dot{\zeta}_2 W_2^T \Psi_p + \epsilon_p - \zeta_2 - \alpha \zeta,$$

(77)

If we choose the Lyapunov candidate function

$$V_3 = \frac{1}{2} \zeta_1^2,$$

(78)

then we have

$$\dot{V}_3 = -a_3 \zeta_1^{p+1} - b_3 \zeta_1^{p+1} + \zeta_1 \dot{\zeta}_2,$$

(79)

If we choose the Lyapunov candidate function

$$V_4 = \frac{1}{2} \zeta_2^2,$$

(80)

then we have

$$\dot{V}_4 = -\dot{\zeta}_2 W_2^T \Psi_p + \epsilon_p \zeta_2 - \alpha \zeta - a_3 \zeta_2^{p+1} + b_3 \zeta_2^{p+1},$$

(81)

If we choose the Lyapunov candidate function

$$V_{NN} = \frac{1}{2} \zeta_2^2,$$

(82)

then we have

$$\dot{V}_{NN} = \frac{1}{2} \dot{\zeta}_2 W_2^T \Gamma_2^{-1} W_2,$$

(83)

If we choose the Lyapunov candidate function

$$V = V_3 + V_4 + V_{NN},$$

(84)

then we have

$$\dot{V} = -a_3 \zeta_1^{p+1} - b_3 \zeta_1^{p+1} + \epsilon_p \zeta_2 - a_4 \zeta_2^{p+1} - b_4 \zeta_2^{p+1} - \alpha \zeta_2 - a_3 \zeta_1^{p+1} - b_3 \zeta_1^{p+1} + \epsilon_p \zeta_2 - a_4 \zeta_2^{p+1} - b_4 \zeta_2^{p+1} - \alpha \zeta_2 + b_p W_2^T \Gamma_2^{-1} W_2,$$

(85)

Based on Lemmas 3 and 4, the following inequalities hold:

$$\dot{V} \leq -c_{pp} \left( \frac{1}{2} W_2^T \Gamma_2^{-1} W_2 \right)^{p+(1/2)} + b_p W_2^*(p+(1/2)) W_2^* \left( q+(1/2) \right).$$

(86)

Then, based on Lemma 3, we have

$$\dot{V} \leq -a_3 \zeta_1^{p+1} - b_3 \zeta_1^{p+1} - a_4 \zeta_2^{p+1} - b_4 \zeta_2^{p+1} + \frac{1}{q+1} \zeta_2^{p+1}$$

$$+ \frac{q}{q+1} \epsilon_p \zeta_2^{q+(1/2)} + c_{pp} \frac{1}{2} W_2^T \Gamma_2^{-1} W_2 \left( q+(1/2) \right) + b_p W_2^*(p+(1/2)) W_2^* \left( q+(1/2) \right).$$

(87)

Then, based on Lemma 2, we have

$$\dot{V} \leq -2 \left( \frac{p+1}{2} \right) a_3 V_3^{p+1/2} - 2 \left( q+(1/2) \right) b_p V_3^{q+1/2}$$

$$+ b_p W_2^*(p+(1/2)) W_2^* \left( q+(1/2) \right) \leq -a_p V^{p+1/2} - b_p V^{q+1/2} + c_p,$$

(88)
\[ a_p = \min \left( 2^{(p+1/2)} a_3, 2 \left( \frac{p+1/2}{q} \right)^{q+1/2} a_4 \right), \]
\[ b_p = \min \left( 2^{(q+1/2)} 3^{q-1} b_3, 2^{(p+1/2)} 3^{q-1} q \left( \frac{1}{q+1} \right)^{q+1/2} \right), \]
\[ c_p = -\frac{q}{q + T_p} \beta_p + b_p W_2 \gamma \left( \frac{p+1/2}{q} \right)^{q+1/2} W_2^* \gamma \left( \frac{p+1/2}{q} \right)^{q+1/2}. \]

(89)

Therefore, based on Lemma 1, the closed-loop system is practically fixed-time stable, and convergence time \( T_{p\text{max}} = \frac{2}{(p+2/3)(1-q)a_p} + \frac{2}{(q-1)b_p} \).

The proof for the theorem is completed.

4. Simulation

In this section, simulation is conducted to verify the effectiveness of the presented fixed-time adaptive neural network adaptive control for UAVs.

Example 1. Consider the UAV dynamic system (12) with the parameters as
\[ m = 2, l = 0.2, g = 9.8, \]
\[ \xi_x = \xi_y = \xi_z = 1.2, \]
\[ I_x = 1.25, I_y = 1.25, I_z = 2.5. \]

The reference trajectory is selected as follows:
\[ \chi_d(t) = \left[ 5 \left( 1 - \cos \left( \frac{\pi}{10} t \right) \right), 5 \sin \left( \frac{\pi}{10} t \right), 10 \left( 1 - e^{-0.3t} \right) \right]^T. \]

(91)

In addition, the desired yaw angle is \( \psi_d = 0 \).

Choose the parameters of controllers and NNs as
\[ a_1 = a_3 = b_1 = b_3 = 1.1, \]
\[ a_2 = a_4 = b_2 = b_4 = 1, \]
\[ \sigma_{ax} = \sigma_{bx} = \sigma_{ap} = \sigma_{bp} = 1, \]
\[ \Gamma_1 = \Gamma_2 = I, p = \frac{1}{3}, q = 3. \]

(92)

Based on such parameters and the fixed-time control result in Theorem 1, \( T_{x\text{max}} = T_{p\text{max}} = 5.3811 \), which is independent with system initial conditions. Then, the virtual control, control input, and adaptive law of NNs are given as follows:
\[ \alpha = -1.1 \xi \left( 1/2 \right), -1.1 I \]
\[ u_e = 2 \left( -\hat{W}_1 \xi - \hat{\xi}_2 \hat{\xi}_2 - \hat{\xi}_1 \hat{\xi}_2 - \hat{\xi}_2^2 \right), \]
\[ \hat{W}_1 = \Psi \hat{\xi}_2 - \hat{W}_1 \left( 1/2 \right) - \hat{W}_1. \]

(93)

The initialization of the variable are selected as
\[ x(0) = y(0) = z(0) = \phi(0) = \theta(0) = \psi(0) = 1. \]

(94)

The simulation results of the neural network controller are shown in Figures 2–9. The trajectories of positions \( x, y, \) and \( z \) can be seen in Figures 2–4. The position tracking errors show that the proposed neural fixed-time control has good tracking performance, and based on parameters selected, the convergence time is \( T_{x\text{max}} = 5.3811 \). Figure 5 shows the 3-D moving trajectory by neural network control. The trajectories of attitudes \( \phi, \theta, \) and \( \psi \) can be seen in Figures 6–8, which indicate better tracking performance of neural network control. The trajectories of NNs can be seen in Figure 9. The attitude tracking errors show that the
Figure 4: Trajectories of position $z$, ideal position $z_d$, and error $e_z$.

Figure 5: 3-D space state tracking result.

Figure 6: Trajectories of attitude $\phi$, ideal position $\phi_d$, and error $e_\phi$. 

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proposed neural fixed-time control has good tracking performance, and based on parameters selected, the convergence time is $T_{p\,\text{max}} = 5.3811$.

5. Conclusion

In this paper, based on the backstepping algorithm, fixed-time Lyapunov practical stable theory, and neural network adaptive control technology, a new fixed-time adaptive practical tracking control method is developed for quadrotor UAVs for the first time. The states of close-loop systems which contain error states and weights of the NNs are practically fixed-time stable, which are independent of states. Simulation result shows that the adaptive neural network controller ensures stable position and attitude tracking of UAVs in finite time, which only depends on controller parameters. Bench simulation results showed the effectiveness of the adaptive fixed-time NN control method.

Data Availability

No data were used to support this study.

Conflicts of Interest

The funding did not lead to any conflicts of interest regarding the publication of this manuscript. There are no conflicts of interest in this study.
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