Recurrence Analysis Applied to Ultrasonic Absorptive Coating

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Abstract.
Metasurfaces are, at the actual state-of-the-art, a constant topic for scientific community as they find widespread applications in several fields, including acoustics and aeroacoustics. In this paper, the potential of recurrence analysis was evaluated by applying it to some metasurfaces. These devices effectiveness has been previously demonstrated with a reflection coefficients analysis in the Fourier and Wavelet domains. A novel strategy based on the Recurrence Analysis, which is a no man’s land applied on the metasurfaces, was performed. The evaluation of the recurrence plot and phase space attractors led to interesting results. The results show an introduction of non linearities after the forcing wave interacts with the metasurfaces. In light of that, there is the evidence that the chaotic analysis principles may be a powerful tool to characterize the performance of metasurfaces in particular through the calculation of the chaotic indexes. This method could be integrated with previous ones for the evaluation of more complex and multiscale geometries that nowadays show huge potential in too many fields of application.

1. Introduction
Metasurfaces, firstly introduced in the electromagnetic realm, are artificially engineered surfaces composed of subwave–length unit cells which manipulate the reflected and (or) transmitted wave fronts in a preferred, predefined, and unconventional manner [1]. This devices manipulate waves through resonant effects and add non–linear components in the phenomenon that are tunable by patterning the metasurfaces unit cells. Thanks to their unique ability, the metasurfaces found a lot of application in several fields of science: electromagnetic [2], acoustics, aeroacoustics and fluid dynamics. In the field of electromagnetism, metasurfaces are adopted to define the shape of antennas [3, 4] or absorbers [5], for cloaking purposes [6] or scattering reduction [7]. Metasurfaces are also implied to control optical phenomenon [8, 9, 10]. Acoustics is another field in which the metasurfaces are deeply investigated. Acoustic properties of the metasurfaces, called UAC (Ultrasonically Absorptive Coatings), are the topic of many studies aimed to the reduction of heat transfer and skin friction in hypersonic vehicles [11]. Delaying the transition of the boundary layer is one of the key to reduce the issues mentioned above [12] and some UACs with different subwave–length structures has been proposed and investigated with very promising results [11],[13].
The contribution of this work shows how recurrence analysis is an efficient method to study metasurface behaviour. By way of case a database already analyzed for a previous work [14] as been processed. Despite the wavelet analysis already proposed by [14] the database has been processed by means of Recurrence Analysis. By implementing such analysis techniques for UACs performance characterization some meaningful information about the interaction between forcing wave and metasurfaces can be achieved. Results show a switching of the forcing wave from something strongly periodically and ordered to something less coherent, that is strongly depending on the starting condition, through a loss of coherence. In addition, the Recurrence analysis can identify the efficiency of the UAC since an introduction of non linearity could bring to a better performance of the metasurface. Since, chaotic analysis can be a very interesting approach to study the acoustic metasurfaces. This method were introduced in many different disciplines [15] and represent a powerful tool for the description of dynamical system. In the present study a never used approach, from our best knowledge, for studying a UAC is implemented. The representation of the state in a phase space has been produced through the trajectories described by the magnitude of interest. Then, the recurrence can be identified when the trajectory evolves through a region in the phase space it passed before. Since, the forcing wave is periodic it is possible to expect a periodic reflected wave and the phase space portrait should be an attractor with several recurrences. In addition, by evaluating the recurrence plot (RP) it is possible to generalize some important information about determinism and divergence associated with the lengths of the diagonal line structures in the RP.

The manuscript is organized as follow, a brief description of the test cases and of the experimental setup is given in Sec.2, the post-processing technique is described in Sec.3. Then, Sec.4 includes a description of the main outcomes of the research activity. Finally, summary of the results, conclusions and future perspectives are given in Sec.5.

2. Test Cases and Experimental Setup

In this section, a description of the test cases and of the experimental setup employed is reported. Experimental parametric investigation of a solid wall (as a reference of total reflection) has been carried out, according to Fedorov et al. [16]. The measurements were performed on a porous UAC (manufactured as [11]) and 2 custom-made metasurfaces (i.e. a flat surface with equally spaced slots of triangular section). Fig.1 reports a scheme of the porous surface in comparison with one of the designed metasurface.

2.1. Test Cases

Porous UAC are characterized by 3 main geometrical parameters: i) cavity depth \( H \) equal to 0.5 mm, ii) pores diameter \( 2b \) of 0.1 mm and iii) hole spacing \( d \) of 0.4 mm. A representation of the geometrical parameters is reported in Fig.2.

Moreover, both slotted UACs are characterized by \( H = 0.5 \) mm, a transversal cavity section length of \( 2b = 0.41 \) mm, and a sample longitudinal length \( a = 50 \) mm. They differ in cavity spacing (1.00 and 8.45 mm), resulting in a different porosity. UACs porosity \( \phi \) plays a fundamental role in this kind of studies and is defined as:

\[
\phi = \frac{S_c}{S}
\]  

where \( S \) is the total UAC surface and \( S_c \) is the surface of top section of the cavities (white area in Fig.1 (b),(e)).

The possible existence of a directivity in the metasurface acoustic response was investigated for two angular configurations, corresponding to \( \theta = 0^\circ \) and \( \theta = 90^\circ \) shown in Fig.1 with respect to the acoustic incident ray [14].

The geometrical parameters values characteristic of the investigated UACs are listed in tab.1.
Figure 1: Comparison between common porous metasurface with cylindrical blind cavities (top row) and an example of our custom-made metasurface with sharp slots (bottom row). The figure shows: the top-view picture of the investigated metasurface (a, d), a schematic sketch of the sample (b, e), and the transversal section of the UAC (c, f). Reference for the angle of rotation of the slotted UACs is shown in (g). In the schematic sketches (b,c,e,f,g) cavities are rendered in white, while the UAC solid substrate is in grey.

Figure 2: Sketch of the transversal section of a slotted and drilled UACs with representation of geometrical parameters.

for all the test cases employed.

UACs design needs to be grounded in the evaluation of the cavity section aspect ratio, commonly defined, as $AR_1 = 2b/H$ for rectangular geometries (see among many [17]). However, since our slotted UACs presents cavities of triangular section (instead of rectangular), a more general definition of $AR_1$ is proposed:

$$AR_2 = \frac{A}{H^2}$$

where $A$ is the transversal section area of the cavity.

For the present study, the cavity aspect ratio $AR_2$ is fixed and equal to 0.2 (nearly a deep
Table 1: Geometrical parameters of the test cases investigated.

| Case | Type  | \(2b\) (mm) | \(d\) (mm) | \(\phi\), \(\%\) | \(\theta\), deg | \(H\) (mm) | \(AR_2\) (−) |
|------|-------|-------------|------------|----------------|---------------|------------|-------------|
| 1    | Solid | -           | -          | -              | -             | -          | -           |
| 2    | Slot  | 0.41        | 1.00       | 41.4           | 0             | 0.5        | 0.4         |
| 3    | Slot  | 0.41        | 1.00       | 41.4           | 90            | 0.5        | 0.4         |
| 4    | Slot  | 0.41        | 8.45       | 4.9            | 0             | 0.5        | 0.4         |
| 5    | Slot  | 0.41        | 8.45       | 4.9            | 90            | 0.5        | 0.4         |
| 6    | Pore  | 0.10        | 0.40       | 4.9            | -             | 0.5        | 0.2         |

cavity) for porous UAC and to 0.4 (closer to a shallow cavity) for slotted UACs. This choice ensured to work far from \(AR_2 = 0.3\), which was proved to be the condition fostering the so called R–mode [17].

2.2. Experimental setup
The experimental activity was carried out in the laboratory of the Italian National Agency for New Technologies, Energy and Sustainable Economic Development (ENEA), equipped with a couple of ultrasonic transducers Ultran model NCG 200-D25, characterized by a diameter of 50 mm and a bandwidth of 35 kHz with center frequency 100 kHz. One of the transducers was used as emitter and the other as receiver, as reported in [14]. The forcing wave generated in order to excite the UAC response was a wave packet in the time domain, this kind of forcing is commonly adopted to artificially force the second–mode instability (see, among many, [18, 19]) since it is representative of the disturbances that naturally propagate in the hypersonic boundary layer. Then, the reflected signal was acquired by the receiving probe with a sampling frequency of 50 MHz. The first test was performed using a solid wall to provide a reference signal, in the following labelled as \(x_{ref}(t)\). Fig.3 reports the reference signal time history. According to [16], under the hypothesis of a total reflection by solid wall, the signal in Fig.3 can be regarded as a quite good approximation of the forcing wave.

Figure 3: Example of forcing/reflected wave packet: dimensionless pressure time series reflected by the solid wall and acquired by the receiver.

3. Post-processing techniques
3.1. Recurrence Plot
Intermittency describes the state in which the dynamical system switches between two different behavior and the residence time in each of them is unpredictable [20]. The type of intermittency
can be identified on the basis of the classification by Berg\textsuperscript{\textregistered} et al. [21]. To this aim, the analysis of events in terms of recurrence plots results to be a suitable and powerful mathematical tool. First of all, the pseudo–phase space is reconstructed using the Time Delay Embedding (TDE) method [22, 23]. The estimation of the optimal time delay $\tau_{opt}$ and the proper embedding dimension $d_0$, is determined by the Average Mutual Information (AMI) and the Average False Nearest Neighbor (AFNN) algorithm, respectively introduced in [24, 25, 26]. Once $\tau_{opt}$ and $d_0$ are obtained, the pseudo-phase space is reconstructed creating a $d_0$–dimensional time series $y(t)$ from the original one–dimensional $x(t)$:

$$y(t) = (x(t), x(t + \tau_{opt}), \ldots, x(t + (d_0 - 1)\tau_{opt})) \quad (3)$$

Starting from Eq. 3, trajectories in the pseudo–phase space are then reconstructed, point by point, by computing the distance between any two points of the trajectory $y(t)$:

$$R(t_i, t_j) = \|y(t_i) - y(t_j)\| \quad (4)$$

being $\| \cdot \|$ a norm representing the Euclidean distance, $y(t_i)$ and $y(t_j)$ two generic points along the $y$ trajectory, and $t_i, t_j \in [0, Nf_s^{-1} - (d_0 - 1)\tau_{opt}]$. Since RPs are usually represented in binary color maps, a binarization of Eq. 4 can be given as follows:

$$\tilde{R}(t_i, t_j) = \mathcal{H}(\epsilon - R(t_i, t_j)) \quad (5)$$

where $\epsilon$ is a proper threshold distance and $\mathcal{H}(\cdot)$ the Heaviside function. Whether a recurrence is found, then $\tilde{R} = 1$ (the distance between two points does not exceed the threshold $\epsilon$) and it is plotted as a black dot; otherwise $\tilde{R} = 0$ and it is plotted as a white dot.

### 3.2. Reflection Coefficient

It is a well-established approach to characterize the acoustic behaviour of metasurfaces by evaluating the reflection coefficient, defined as the ratio between the amplitude of the reflected and the incident signal [16].

In this section, two different definitions of the reflection coefficient $R$ are discussed: $R^F$ and one calculated in the time domain; $R^T$. Each algorithm provides different information closely related to the underlying mathematical approach.

#### 3.2.1. Reflection coefficient in time domain

Usually, it may be interesting to investigate the UAC acoustic behaviour from an energetic point-of-view. The reflection coefficient calculated in time domain answers this specific demand, being it defined on the basis of an energy criterion. The energy associated to a given time series $x(t)$ can be written as:

$$E = \int_0^T |x(t)|^2 dt \quad (6)$$

where $[0; T]$ is the time window of the acquired signal. By Using this definition for time series energy, the energy reflection coefficient can be defined:

$$R^T = \sqrt{\frac{E}{E_{ref}}} \quad (7)$$

thus $R^T$ has the same physical meaning as a transmission loss in duct acoustics [27]. In agreement with [28, 29], dissipative effects related to the acoustic boundary layer formation are negligible, as discussed more in detail in Sec.4. Therefore, any variation in $R^T$ is ascribable to negative energy fluxes or scattering effects.
3.2.2. Reflection coefficient in Fourier domain  The reflection coefficient in Fourier domain was calculated as defined in [30]:

\[ R_F(f) = \frac{\tilde{x}(f)}{\tilde{x}_{ref}(f)} \] (8)

where \( \tilde{x}(f) \) and \( \tilde{x}_{ref}(f) \) are the fast Fourier transform of the signal reflected by the UAC and by the solid wall, respectively. In this context, \( R_F \) is representative of the UAC frequency response function.

Fourier analysis is the optimal tool when dealing with sinusoidal signals, but it introduces some relevant shortcomings in presence of periodic non-sinusoidal signals. One of the primary limit is the occurrence of several harmonics with no physical meaning. In order to overcome such a drawback a powerful strategy is offered by introducing a wavelet-based reflection coefficient proposed by [31].

4. Results

4.1. Recurrence Plot
Based on the results reached by [14] can be very interesting to evaluate the performance of UACs from another point of view. For this purpose, recurrence analysis may represent a fundamental tool for characterizing the properties of dynamical systems. Thanks to this approach it is possible to identify periodic and intermittent phenomenon of system states. Intermittency is a common feature of chaos and occurs when a dynamical system switches between irregularly spaced alternating intervals of chaotic-burst and stationary-periodic states. Several studies provide many criteria for classifying intermittency, for example [32] categorized intermittency into three classes: i) type I, related to a laminar signal with a monotonic increase in intermittency; ii) type II related to a quasi-periodic state onset; iii) type III associated with inverse-periodic-double bifurcation intermittency. Each type of intermittency is represented by a specific pattern in the recurrence diagram [20]. As reported in Fig. 4 type I generates a uniform black square, type II has a similar black square with some typical structures in the upper right corner, and type III has the same square with a rounded upper right corner within a non-uniformly black background.

In the case of metasurfaces it is possible to figure out if the efficiency of the UACs modifies coherence of the interaction phenomenon. Then, by means of the recurrence plot analysis it is possible to visualize if the signal trajectory visits the same area on the mathematically reconstructed phase space and investigate the presence of intermittent characteristic of the data set.

Firstly, by using the phase space reconstruction method [33] the dynamic variables involved in the system dynamics describe the signal trajectory in the phase space. The structures generated by the evolution of trajectory are named attractors and take on different shapes according to the phenomenon described. Then, for the acquired signal, different attractor can be generated from the one representative of the reference test case (flat surface) fig.5 (b).

Fig.5(b) reports the attractor in the pseudo-phase space obtained for the reference case. The attractor shape shows topological feature similar to the ones of dissipative dynamical system of the damped phenomenon describable by the Birman-Williams knot holder [34] with a central point of convergence. For a flat wall a total reflection of the forcing wave was expected, so it is possible to assume that the acquired data set corresponds to the forcing signal. Thus, such a preliminary analysis represents a chaotic characterization of the forcing wave and, in this sense, it can be employed as a reference result. The recurrence plot for the reference signal of a plane wall is given in Fig.6.

The recurrence plot evaluation shed light on some features that give us important information about the data set. Following the illustration method of the recurrence squares and rectangles
Figure 4: Pattern of the recurrence plot associated to each type of intermittences, Type 1 (a), type 2 (b), type 3 (c).

Figure 5: Acquired signal of the total reflected wave (a); phase space plot of the forcing wave (case1) (b).

described in [20] the RP show some pattern confined in various zone of the matrix. Starting from the zone 1 to 4, shown in Fig.6, there is a square that incorporates a pattern of some diagonal structures that become more evident in the zone 5. In this zone the recurrence increase and some small black square are plotted. Moving on along the signal samples there is zone 6 that shows a typical pattern of transition to a quasi-periodic state shown in Fig.4 (b). Others zones of interest, zone 7 and 8, show a strong recurrence between the first and last parts of the signal. Based on these consideration, the evidence is that there is a transition from a more chaotic characteristic to a quasi-periodic feature in the last part of the signal. This results can be observed in Fig.5 (a) that shows how the higher oscillation in the signal become smaller and the central component, with a sinusoidal shape, remain the only wave.

As a result, the generated forcing wave seems a very complex signal composed of a quasi-periodic laminar phase but also of an intermittent components. Consequently, this signal allow to keep
knowledge of the UACs microstructure effect of damping of the pressure fluctuations and which forcing wave components is most affected by the phenomenon.

Fig.7 reports the normalized time series for the 6 test cases. An amplitude variation of the reflected wave by the UACs can be observed compared to the reference case of a flat wall (Fig.7 (a)). As found by Pagliaroli et al.[14], the reflection coefficient of the UACs (\( R_T \) and \( R_F \)) show values less than unity especially for test case 3, the most performing specimen. Such effect is confirmed by the time series (Fig.7(c)) that display a mitigation in signal amplitude of almost 30%. The destructive interference between the forcing wave and the dispersed component by the micro structures of the UAC is very strong.

Implementing chaotic analysis in signal processing produced attractors in the phase space associated with the acquired samples. The attractors in the reconstructed pseudo phase-space for all test cases are reported in Fig.8. Attractors give other information about the nature of the interaction phenomenon and maybe also some answers about the efficiency of the metasurfaces. The trajectories described by the different cases are very similar but there are some different features that deserve to be analyzed. Fig.8(b)-(e), correspond relatively to the test case 4 (\( \phi = 4\% \) and \( \theta = 0^\circ \)) and test case 5 (\( \phi = 4\% \) and \( \theta = 90^\circ \)), these structures show the same behaviour suggesting a uniform damping rate. On the other hand, the trajectories of test cases 3,4 and 6 (Fig.8(c)-(d)-(f)), respectively the UAC with 41\% porosity and \( \theta = 90^\circ \), UAC with 4\% of porosity and \( \theta = 0^\circ \) and pore UAC, present a different distribution. In fact, these structures are mainly clustered in the central convergence point and the distance between the external orbits is variable but characterized by a higher dimension than the other examples. These different distributions on phase space trajectories can be related to different interaction of the metasurfaces with the forcing wave. Thus, this feature can demonstrates a higher or lower damping rate which reduces the amplitude of pressure fluctuation in the acoustic field. This qualitative information does not imply a better performance but ensures that the metasurfaces are actually causing an interaction phenomenon with the forcing wave.

In order to obtain further information a comparison between the recurrence diagrams is reported in Fig.9. As already described (Sec.3) the RPs may represent a powerful tool to evaluate the UACs performance. This analysis enable to characterize the properties of dynamical system through the identification of periodic and intermittent phenomenon. Comparing the solid wall result (Fig.9(a)) with the other configurations, the first evidence is that the forcing

![Figure 6](image-url)

Figure 6: Recurrence plot of case 1 (flat wall), forcing wave totally reflected by flat wall.
wave maintains its nature, described in Fig.6, after the interaction with the metasurfaces. Furthermore, a loss in the recurrence can be observed for all the test cases. Such effect is probably related to the reflected signal interaction with the UACs micro-structures. In particular, UACs present a different transition velocity along the signal samples from a more chaotic feature to a quasi-periodic wave. Such effect of reduction in the recurrence appears graphically more pronounced for case 6, relative to the UAC with 41% porosity and $\theta = 90^\circ$ (see Fig.9(f)), which has been found as the most effective UAC by Pagliaroli et al.[14]. For this metasurface there is an evidence of the loss of recurrence, indeed the structure in the zone 1,2,3 and 4 (see fig.6) appear separated and partially destroyed. In addition, there is an anticipated transition to the quasi periodic pattern described in Fig.4(b). Such results suggest that the loss in recurrence may be indicative of metasurfaces performance, indeed results show an evident bound between the efficiency of the metasurfaces and the introduction of non linearity. Analyzing the other RP (fig.9) and comparing them with the reference case (Fig.9(a)) the first evidence is a lowering of the square generated by the zones 1,2,3,4 and 5 (see Fig.6) in respect to the case of flat wall. Moreover, a faster transition to the quasi-periodic pattern (upper right corner of every RPs) can be observed, such effect is characteristic of the loss in recurrence. This transition describes the damping of the oscillation. This phenomenon, visible in the time series (fig.7), dissipates the oscillation amplitude before the semi-sinusoidal components of the waveform remain the only structure.

Based on this preliminary results, recurrence analysis shows its usefulness in order to characterize the performance of metasurfaces. Quantifying the performance of UACs and establish a relation

Figure 7: Acquired signal of each case. Case 1 (a) of flat wall, case 2 and case 3 relative to slotted UAC with 41% porosity positioned with $\theta = 0^\circ$ (b) and $90^\circ$ (c), case 4 and case 5 about slotted UACs with 4% of porosity positioned with $\theta = 0^\circ$ (d) and $90^\circ$ (e) and case 6 of pore UAC (f).
between the recurrence and the reflection coefficient could be an interesting topic to carry on in a future work that analyze this phenomenon from a deeper point of view.

5. Summary, Conclusions and Future Perspectives
In conclusion, an experimental analysis on different custom–made metasurfaces has been performed and an innovative procedure to characterize their performance was implemented. Starting from a previous work on some metasurfaces called UACs, that has produced reflection coefficient in time, Fourier and wavelet domains, recurrence analysis is implemented to the same database. Results demonstrate that the usage of Recurrence Analysis is an effective and convenient method to evaluate the reflection properties of metasurfaces.

A preliminary Recurrence Analysis of the reflected forcing signal by a flat wall has been processed in order to characterize from a chaotic point of view the forcing wave. The analysis is applied to the wave reflected by the metasurfaces, phase space plot and recurrence matrix are compared with the ones associated to the reference case (flat wall). The surface treatments seems to affect the attractor in the pseudophase space by inducing the trajectories to describe orbits with different distribution. This results in a qualitative loss in the recurrence. Recurrence reduction is confirmed by the recurrence plots. RPs show that the reflected wave by UACs maintains its original nature but the porosity of the metasurfaces increase the presence of non linearities along the signal samples. The main effect can be observed for the UAC with 41% of porosity and $\theta = 90^\circ$.

In light of these results, the Recurrence Analysis applied to metasurfaces definitely deserves more
Figure 9: Recurrence plot of each case. Case 1 (a) of flat wall, case 2 and case 3 relative to slotted UAC with 41% porosity positioned with $\theta = 0^\circ$ (b) and $90^\circ$ (c), case 4 and case 5 about slotted UACs with 4% of porosity positioned with $\theta = 0^\circ$ (d) and $90^\circ$ (e) and case 6 of pore UAC (f).

... attentions and quantify recurrence loss with chaotic indexes like the $RR_\tau$ and $DET$ have to be the topic of future developments. This indexes allow to identify numerically the introduction of non linearities by the metasurfaces in a dynamical aeroacoustic phenomenon. This method can be very helpful as the increasing complexity of metasurface geometries and the increasing fields of applications in which these devices potentially can be used requires more and more analysis methods for their characterization.

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