MHD flow of Carreau nanofluid over a stretching surface with suction/injection and slip effects by using Haar wavelet quasilinearization method

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Abstract. Carreau nanofluid model is presented for the magnetohydrodynamic (MHD) flow over a stretching surface with velocity slip boundary and suction/injection parameters. Transformation procedure is used to reduce from partial differential equations (PDEs) to ordinary differential equations (ODEs). The Haar wavelet quasilinearization method (HWQM) is utilized for solving ODEs. The behavior of various pertinent parameters are computed and analyzed through graphs and tables. Comparison with published results in the literature is made under some special and limited cases and an excellent agreement is found.

Keywords: Carreau nanofluid, velocity slip, suction/injection, Haar wavelet quasilinearization method

1. Introduction
The study of nanofluids have received remarkable attention in recent years by several researchers due to the enlargement in thermal conductivity and important role of nanofluid flows in science and engineering fields [1]. The great thought of the distinctive capability to set up thermal conductivity over nanoparticles is firstly introduced by Choi [2]. Later, Buongiorno [3] studied the convective transport in nanofluid with Brownian diffusion and thermophoresis parameters, in which these parameters contribute to the higher thermal conductivity rather than base fluid. Khan and Pop [4] investigated the nanofluids on a stretching sheet. In their work, they concluded that the values of the thermophoresis decreases with the decrease of the Nusselt number, but Brownian motion and Prandtl number increase with the reduction of Sherwood number. Noghrehabadi et al. [5] investigated the effect of partial slip boundary on the flow and heat transfer of nanofluids.

The study of three-dimensional MHD boundary flow and heat transfer of a viscoelastic fluid has been carried out by Eswaramoorthi et al. [6] with radiation’s effect. They also studied the effects of Soret and Dufour in the presence of chemical reaction and radiation [7]. Kasmani et al. [8] investigated the impact of chemical reaction on heat transfer past a wedge along with heat generation/absorption and suction. By using a shooting iteration technique and fourth-order Runge-Kutta integration scheme, Karthikeyen et al. [9] investigated the performance of MHD mixed convection stagnation point flow near the vertical plate in porous medium. Bhuvaneswari et al. [10] utilized the Lie symmetry method to investigate the typical feature of the radiation natural convection flow in a porous medium. Furthermore, the recent research of the nanofluid can be found in Refs. [11-15].

Many researches are dedicated on non-Newtonian fluids especially for power law and Sisko fluids, including the work by Postelnicu and Pop [16], where they investigated the Falkner-Skan flow over a stretching wedge by using power law fluid. The Sisko fluid’s behavior near the stagnation point has been discussed by Khan and Shahzad [17]. Khan et al. [18] studied the heat transfer flow of Sisko
nanofluid past a stretching sheet. Malik et al. [19] investigated the Sisko fluid flow past a permeable stretching cylinder by using Cattaneo-Christov heat flux model. Recently, Malik et al. [20] extended their previous work to analyze the Sisko fluid flow over a flat stretching sheet alongside velocity slip and thermal radiation. The Carreau nanofluid is one of the non-Newtonian fluids that frequently used in chemical engineering. However, this type of fluid has received less attention compared to other non-Newtonian fluids. In several flow problems, the Carreau fluid is suitable for the suspensions of polymer behavior, where the viscosity changes with the deformation rate.

Previous studies showed that there is no attempt has been made so far to the study of two-dimensional Carreau nanofluid with velocity slip boundary and suction/injection by using HWQM [21]. It is noticed that, the most common numerical algorithms used in solving boundary layer flow problems are homotopy analysis method, shooting method and Runge-Kutta method. To fill this gap, the governing problems are computed and analyzed through HWQM. Hence, this work may help practitioners in science and engineering for finding an alternative formulation to solve problem in boundary value problems.

2. Problem statement

Consider a MHD two-dimensional of an incompressible Carreau nanofluid induced by a stretching surface. The two-dimensional boundary layer equations governing the flow are the same as that in Ref. [22]. Similarly, the boundary conditions are extended by including the parameter of suction/injection and velocity slip boundary. Hence, they can be written as

\[ u = U(x) + ZV \frac{\partial u}{\partial y}, \quad v = -V(x), \quad -l \frac{\partial T}{\partial y} = k_f (T_f - T), \quad D_p \frac{\partial C}{\partial y} + D_v \frac{\partial T}{\partial y} = 0 \quad \text{at} \quad y = 0, \quad (2.1) \]

\[ u \to 0, \quad T \to T_\infty, \quad C \to C_\infty \quad \text{as} \quad y \to \infty. \quad (2.2) \]

Here, \( U(x) = U_0 e^{x/N} \) is the stretching velocity, \( Z(x) = Z_0 e^{x/N} \) is the velocity slip, \( V(x) = V_0 e^{x/N} \) is the suction/injection and \( k_f = ke^{x/2N} \) is the heat transfer coefficient. After employing the similarity transformations [22], the governing equations can be reduced to

\[ \left(1 + n W_0^2 f \right) \left(1 + We^2 f \right) \left(1 + n W_0^2 f \right)^{n-3} f'' + f f'' - 2 f'^2 - 2 \frac{Ha^2}{f'^2} = 0, \quad (2.3) \]

\[ \left(1 + \frac{4}{3} Rd \right) \theta'' + Pr \left(f f' + \sum_{i=1}^{m-3} a_i f' \right) + \frac{N_b g'}{N_i} \theta = 0, \quad (2.4) \]

\[ g'' + \text{Scf} g'' + \frac{N_i}{\text{Scf}} \theta = 0, \quad (2.5) \]

boundary conditions transformed to

\[ \eta = 0 : \quad f = F_\eta, \quad f' = 1 + \beta f'', \quad \theta' = -\gamma' (1 - \theta), \quad N_b g' + N_i \theta = 0, \quad (2.6) \]

\[ \eta \to \infty: \quad f' \to 0, \quad \theta \to 0, \quad g \to 0. \quad (2.7) \]

All the mathematical expressions for skin friction coefficient, \( C_f \) and local Nusselt number, \( Nu_\lambda \) are defined as in Ref. [22].

3. Computational strategy

In this section, the Eqs. (2.3) - (2.7) are solved numerically by using HWQM [21]. By using the collocation points on the interval [11], we obtain the following systems

\[ \sum_{i=0}^{m-1} a_i K_i = L_i, \quad (3.1) \]
\[
\sum_{i=0}^{m-1} a_i K_i + \sum_{i=0}^{m-1} b_i K_i + \sum_{i=0}^{m-1} c_i K_i = L_2, \tag{3.2}
\]
\[
\sum_{i=0}^{m-1} a_i K_i + \sum_{i=0}^{m-1} b_i K_i + \sum_{i=0}^{m-1} c_i K_i = L_3, \tag{3.3}
\]

where \(a_i, b_i\) and \(c_i\) are the Haar coefficients, \(K_1, K_2, K_3, \ldots, K_7\) consist of the iteration of quasilinearization technique and Haar wavelet, including the integration of Haar wavelet and \(L_4, L_2\) and \(L_3\) consists of the value of boundary conditions and initial approximations.

4. Results and discussion

The nonlinear ODEs (2.3) - (2.5) subjected to the boundary conditions (2.6) - (2.7) were numerically solved by means of HWQM. All the computations are carried out by using MATLAB. Figure 1(a) describes the behavior suction/injection parameter, \(F_w\) at \(\beta = 0.3\) and \(Ha = We = 1.0\) on the velocity profile. We observed that the velocity and associated boundary layer thickness decrease with the increase of \(F_w\). The influence of velocity slip boundary, \(\beta\) on the velocity distribution is delineated through Fig. 1(b). It shows that the effects of \(\beta\) leads to decrease in velocity distribution. It means that the sheet’s stretching can lead to a fall in the flow of the fluid, hence the fluid velocity becomes smaller with an increase of \(\beta\).

Figure 1(c) and Fig. 3(c) show the effects of the Weissenberg number, \(We\) on the velocity and nanoparticle concentration distributions, respectively. Figure 1(c) depicts that the velocity profile decreases for the fluid of shear thinning increases, but increases in fluid of shear thickening as we increase the value of \(We\). Meanwhile a contrast behavior can be seen in the nanoparticle profile. The effect of Hartman number, \(Ha\) is reported in Fig. 1(d). From this figure, it turns out that increase the value of \(Ha\) will decrease in velocity profile. This is because of the Lorenz force that arises in magnetic parameter becomes higher when \(Ha\) is increased.

The behavior of the temperature profile is plotted in Fig. 2(a) for different value of thermal Biot number, \(\gamma\). This figure reveals that increase in \(\gamma\) increases the temperature profile. Figure 2(b) and Fig. 3(b) describe the role of thermophoresis parameter in analyzing its influences to the temperature and concentration particles. It is noted that temperature profile enhances along with the increase of \(N_t\). It means that the rise of the temperature caused by more nanoparticles that are moved away from the heated surface. Similarly to Fig. 3(b), it is obvious that the concentration profile enhance with increase in the thermophoresis parameter. This is due to the thermophoretic force effects an increase in the boundary layer thickness of concentration.

Figure 2(c) is drawn to elaborate the effects of radiative heat flux parameter, \(Rd\) on temperature profile. It is noted that the radiation parameter and boundary layer thickness increase with increase in \(Rd\). Physically, the larger the values of \(Rd\), the more heat will produce to the operation fluid which shows an improvement in the temperature and thermal boundary layer thickness. The feature Eckert number, \(Ec\) on temperature profile is addressed in Fig. 2(d). It observed that temperature increases with the increase in \(Ec\). It means that the forces of friction between the fluid particles lead to the production of heat, so that the temperature gradient is increased. Figure 3(a) exhibits the effect of Brownian motion parameter, \(N_b\) on concentration profile. It is found that the concentration profile and the boundary layer thickness reduce as \(N_b\) increases.
The computational result obtained by the present method is compared with the published results on the surface drag force for limited condition. It is concluded that a favourable agreement is found between our employed technique with the methods use in [22] and [23]. This result also convince us the reliability of the present numerical technique in handling the nonlinear boundary value problems.

Table 2 is presented to display the effect of $Ha$ and $We$ on surface drag force for shear thinning ($n < 1$) and shear thickening ($n > 1$) fluids. Clearly, the surface drag force decreases with the increase in $Ha$, whereas it tends to increase as the Weissenberg number is enhanced. The influences of heat transfer rate is tabulated in Table 3. It is observed that heat transfer rate enhances for higher estimation of $Pr$ and $\gamma$, for both shears. In contrary, it is noticed that an increase in the values of $N_t$, $Sc$ and $Ec$ decrease the heat transfer rate for both shear fluids.

Fig. 1. The velocity profile for (a) $F_w$ when $L = 6$, $\beta = 0.3$, $Ha = We = 1$ (b) $\beta$ when $L = 8$, $F_w = Ha = We = 1$ (c) $We$ when $L = 8$, $F_w = Ha = 1$, $\beta = 0.3$ and (d) $Ha$ when $L = 8$, $F_w = 1$, $\beta = 0.3$, $We = 3$, with $m = 2^9$. 
Fig. 2. The temperature profile for (a) $\gamma$ when $m = 2^{10}$, $F_w = 1$, $N_t = 0.3$, $Pr = 2$, $Rd = Ec = 0.5$ (b) $N_t$ when $m = 2^7$, $F_w = 0.3$, $Pr = 2$, $\gamma = Rd = Ec = 0.5$ (c) $Rd$ when $m = 2^8$, $F_w = Pr = 1$, $N_t = 0.3$, $\gamma = Ec = 0.5$ (d) $Ec$ when $m = 2^8$, $F_w = Pr = 1$, $N_t = 0.3$, $\gamma = Rd = 0.5$, with $L = 4$, $\beta = 0.3$, $N_b = Ha = 0.5$, $We = 3$ and $Sc = 1$.

Table 1. Comparison of surface drag force when $n = We = 0$.  

| $Ha$ | Waqas et al. [22] | Hayat et al. [23] | Present |
|------|------------------|------------------|---------|
| 0.0  | 1.281828         | 1.2818           | 1.28184420 |
| 0.2  | 1.313294         | 1.3133           | 1.31328916 |
| 0.4  | 1.403028         | 1.4030           | 1.40299923 |
Fig. 3. The concentration profile for (a) $N_b$ when $F_w = 0.3$, $We = 3$, $Sc = Pr = 1$, $N_t = 0.5$ (b) $N_t$ when $F_w = Sc = Pr = 1$, $We = 3$, $N_b = 0.5$ and (c) $We$ when $F_w = N_t = 0.3$, $Sc = 1$, $Pr = 2$, $N_b = 0.5$.

Table 2. Surface drag force $-C_f Re^{1/2}_x$ for $Ha$ and $We$ values when $Pr = 2.0$, $Sc = 1.0$, $n = N_b = Rd = Ec = 0.5$, $N_t = 0.3$, $m = 256$, $L = F_w = \beta = 1$.

| $Ha$ | $We$ | $n = 0.5$  | $n = 1.5$  |
|------|------|------------|------------|
| 0.0  | 0.5  | 0.62848629 | 0.65156956 |
| 0.6  | 0.5  | 0.65046272 | 0.67633196 |
| 1.2  | 0.5  | 0.69671302 | 0.72983059 |
| 0.5  | 1.5  | 0.57321690 | 0.74106372 |
| 2.0  | 1.5  | 0.53447076 | 0.78435116 |
| 2.5  | 1.5  | 0.49966737 | 0.82787314 |
Table 3. Heat transfer rate $Nu_xRe_x^{1/2}$ for Pr, $N_t$, $Sc$, $\gamma$ and Ec values when $N_b = Rd = Ha = 0.5$, $We = 3.0$, $m = 256$, $L = F_w = \beta = 1$.

| $Pr$ | $N_t$ | $Sc$ | $\gamma$ | $Ec$ | $n = 0.5$ | $n = 1.5$ |
|------|-------|------|----------|-----|-----------|-----------|
| 2.0  | 0.3   | 1.0  | 0.5      | 0.5 | 0.45726780 | 0.45592430 |
| 2.5  | 0.4   | 0.5  | 0.5      | 0.5 | 0.4686471 | 0.46712637 |
| 3.0  | 0.6   | 1.2  | 0.5      | 0.5 | 0.45616379 | 0.45479887 |
| 0.3  | 1.5   | 0.6  | 0.5      | 0.5 | 0.45713761 | 0.45579224 |
| 1.7  | 2.0   | 0.8  | 0.5      | 0.5 | 0.45696363 | 0.45561586 |
| 1.0  | 0.6   | 1.0  | 0.5      | 0.5 | 0.45686004 | 0.45551089 |
| 0.7  | 0.5   | 0.8  | 0.5      | 0.5 | 0.52566103 | 0.52411427 |
| 1.1  | 0.9   | 1.0  | 0.5      | 0.5 | 0.45304222 | 0.45008996 |

5. Conclusions
The MHD stretchable flow of Carreau nanofluid model is numerically studied with emphasizing effect of velocity slip boundary and suction/injection. The governing nonlinear equations are solved by using HWQM. The influence of different crucial parameters is investigated. Furthermore, the effect of surface drag force and heat transfer rate is evaluated quantitatively. The main findings of this study are summarized as follows:

a) The increase of Hartman number slow down the flow of both shear fluids.
b) The velocity field decreases as $We$ increase for shear thinning liquid, in which contrary to the effect for shear thickening.
c) The increase in thermophoretic forces lead to enhancement in thickness of thermal and concentration boundary layers.
d) The temperature profile increases for large values of Biot number. When Biot number is incremented, the heat transfer rate will improve.
e) The surface drag force decreases with the increase of $Ha$, whereas it tends to increase as the Weissenberg number is enhanced.
f) Heat transfer rate enhances for larger estimation of Prandtl number and Biot number, for both shear fluids.
g) The varies of suction/injection parameter affected the velocity field. It decreases with the increasing values of suction/injection.
h) The velocity field decreases as the velocity slip boundary increase.

Acknowledgement
The authors would like to thank University of Malaya Research Grant (RG397-17AFR).

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