Decentralized Event-Driven Algorithms for Multi-Agent Persistent Monitoring

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Abstract—We address the issue of identifying conditions under which the centralized solution to the optimal multi-agent persistent monitoring problem can be recovered in a decentralized event-driven manner. In this problem, multiple agents interact with a finite number of targets and the objective is to control their movement in order to minimize an uncertainty metric associated with the targets. In a one-dimensional setting, it has been shown that the optimal solution can be reduced to a simpler parametric optimization problem and that the behavior of agents under optimal control is described by a hybrid system. This hybrid system can be analyzed using Infinitesimal Perturbation Analysis (IPA) to obtain a complete on-line solution through an event-driven centralized gradient-based algorithm. We show that the IPA gradient can be recovered in a distributed manner in which each agent optimizes its trajectory based on local information, except for one event requiring communication from a non-neighbor agent. Simulation examples are included to illustrate the effectiveness of this “almost decentralized” algorithm and its fully decentralized counterpart where the aforementioned non-local event is ignored.

I. INTRODUCTION

Systems consisting of cooperating mobile agents are often used to perform tasks such as coverage [1], surveillance [2], or environmental sampling [3]. A persistent monitoring task is one where agents must cooperatively monitor a dynamically changing environment that cannot be fully covered by a stationary team of agents (as in coverage control) [4]. Once the exploration process leads to the discovery of various “points of interest”, these become “data sources” or “targets” which need to be perpetually monitored. Thus, in contrast to sweep coverage and patrolling where every point in a mission space is of interest [5], [6], the problem we address here focuses on a finite number of data sources or “targets” (typically larger than the number of agents).

In this setting, the agents interact with targets through their sensing capabilities which are normally dependent upon their physical distance from the target. The uncertainty state of a target increases when no agent is visiting it and decreases when it is being monitored by one or more agents (i.e., it is within their sensing range). The objective is to minimize an overall measure of target uncertainty states by controlling the movement of all agents in a cooperative manner. Unlike many other multi-agent systems modeled solely through a network of interconnected agents, here we have two networks, one whose nodes are agents and one whose nodes are targets. Since agents interact with targets, this interaction is modeled by establishing links between nodes belonging to the two different networks. Moreover, since agents are mobile, the overall graph topology in such systems is time-varying. Thus, the resulting complexity of this class of problems is significant. This has motivated approaches where rather than viewing these as agent-to-target assignment problems [7], [8] (which are computationally intensive and do not scale well in the number of targets and agents), one treats them as trajectory design and optimization problems [4], [9].

In [10], we studied the persistent monitoring problem in a one-dimensional (1D) mission space and showed that it can be formulated as an optimal control problem whose solution is parametric, i.e., the optimal control problem is reduced to a parametric optimization one. In particular, every optimal agent trajectory is characterized by a finite number of points where the agent switches direction and by a dwell time at each such point. As a result, the behavior of agents under optimal control is described by a hybrid system. This allows us to make use of Infinitesimal Perturbation Analysis (IPA) [11], [12] to determine on-line the gradient of the objective function with respect to these parameters and to obtain a (possibly local) optimal trajectory. Our approach exploits IPA’s event-driven nature to render it scalable in the number of events in the system and not its state space.

The optimal controller developed in [10] is established based on the assumption that agents are all connected under a centralized controller which can provide information and coordinate all agents. Similar centralized controllers for such problems can be found in [3], [4], [13]. Clearly, a centralized controller can be energy-consuming due to communication costs [14] and unreliable in adversarial environments. In this paper, we address the question of whether it is possible to develop decentralized controllers for persistent monitoring problems with a finite numbers of targets to be monitored.

Decentralization aims to achieve the same global objective as a central controller by distributing functionality to the agents so that each one acts based on local information or by communicating with only a set of neighbors. Such distributed
algorithms have been derived and applied to coverage control [1], formation control [15], and consensus problems [16] where we usually assume a static fully connected network environment. On the other hand, decentralization in a persistent monitoring setting is particularly challenging due to the time-varying nature of the agent network and the fact that agents take actions depending on interactions with the environment (targets) which cannot be easily shared through the agent network.

The contribution of this paper consists of identifying explicit conditions under which the centralized solution to the optimal persistent monitoring problem studied in [10] can be recovered through an “almost decentralized” and entirely event-driven manner. In particular, each agent uses (i) its own local information (to be precisely defined later), (ii) information (in the form of observable events) from agents that happen to be its neighbors at the time such events occur, and (iii) a single specific event type communicated by a non-neighbor agent when it occurs. It is the latter that prevents a completely decentralized control scheme, although, as we will see, ignoring this non-local event results in little loss of accuracy. In addition, we develop such an “almost decentralized” algorithm which, compared to the centralized solution in [10], significantly reduces communication costs while yielding the same performance. The main decentralization result exploits the structure of the IPA gradient of the objective function: the gradient component associated with an agent turns out to depend only on a limited number of events, all of which are local or observed by (time-varying) neighbors except for one event requiring communication with a non-neighbor when it occurs. Moreover, this IPA gradient structure is not limited to the 1D problem considered in this paper, but extends to its 2D version as well, a direction we are pursuing in ongoing research.

The paper is organized as follows. In Section II we present the persistent monitoring problem formulation and introduce different neighborhood concepts that capture the interaction between agents and targets. In Section III, we review the optimal control solution of the problem in the 1D setting and in Section IV carry out IPA for the resulting hybrid system. In Section V, we show that the optimal control solution can be decentralized with only one non-neighbor event needed by an agent to derive it in a distributed manner. We provide simulation examples in Section VI to illustrate the resulting algorithm and its performance.

II. PROBLEM FORMULATION

We begin by reviewing the persistent monitoring model and problem formulation introduced in [10].

Agent dynamics. We consider $N$ agents moving in a one-dimensional (1D) mission space $[0, L] \subseteq \mathbb{R}$. Each agent can control its speed and direction. The speed input is scaled and bounded in $[-1, 1]$. The position of each agent $j$ is represented as $s_j(t) \in [0, L]$ with the dynamics:

$$
\dot{s}_j(t) = u_j(t), \quad |u_j(t)| \leq 1, \quad \forall j = 1, 2, \ldots, N \quad (1)
$$

Agent sensing model. The ability of an agent to sense its environment is modeled by a function $p_j(x, s_j)$ that measures the probability that an event at location $x \in [0, L]$ is detected by agent $j$ at $s_j(t)$. We assume that $p_j(x, s_j) = 1$ if $x = s_j$, and that $p_j(x, s_j)$ is monotonically non-increasing in the distance $|x - s_j|$, thus capturing the reduced effectiveness of a sensor over its range. We consider this range to be finite and denoted by $r_j$. Although our analysis is not affected by the precise sensing model $p_j(x, s_j)$, we will limit ourselves to a linear decay model as follows:

$$
p_j(x, s_j) = \max \left\{ 0, 1 - \frac{\|x - s_j\|}{r_j} \right\} \quad (2)
$$

Unlike the sweep coverage problem, here we consider a known finite set of targets located at $x_i \in [0, L], \ i = 1, \ldots, M$. We then set $p_j(x_i, s_j(t)) \equiv p_{ij}(s_j(t))$ for simplicity. For $N$ agents sensing simultaneously, assuming detection independence, the sensing capability of agent $i$ on target $s$ can be captured by the joint detection probability function

$$
P_i(s(t)) = 1 - \prod_{j=1}^{N} (1 - p_{ij}(s_j(t))) \quad (3)
$$

where we set $s(t) = [s_1(t), \ldots, s_N(t)]^T$.

Target dynamics. We define uncertainty functions $R_i(t)$ associated with targets $i = 1, \ldots, M$, so that they have the following properties: (i) $R_i(t)$ increases with a prespecified rate $A_i$ if $P_i(s(t)) = 0$ (as shown in [10]), this can be allowed to be a random process $\{A_i(t)\}$. (ii) $R_i(t)$ decreases with a fixed rate $B_i$ if $P_i(s(t)) = 1$ and (iii) $R_i(t) \geq 0$ for all $t$. It is then natural to model uncertainty dynamics associated with each target as follows:

$$
\dot{R}_i(t) = \begin{cases} 
0 & \text{if } R_i(t) = 0 \text{ and } A_i \leq B_i P_i(s(t)) \\
A_i - B_i P_i(s(t)) & \text{otherwise}
\end{cases} \quad (4)
$$

where we assume that initial conditions $R_i(0), i = 1, \ldots, M$ are given and that $B_i > A_i > 0$ to ensure a strict decrease in $R_i(t)$ when $P_i(s(t)) = 1$.

Optimal control problem. Our goal is to control the movement of the $N$ agents through $u_i(t)$ in (1) so that the cumulative average uncertainty over all targets $i = 1, \ldots, M$ is minimized over a fixed time horizon $T$. Thus, setting $u(t) = [u_1(t), \ldots, u_N(t)]^T$ we aim to solve the following optimal control problem:

$$
P1: \quad \min_{u(t)} J = \frac{1}{T} \int_0^T \sum_{i=1}^{M} R_i(t) dt \quad (5)
$$

subject to the agent dynamics (1) and target uncertainty dynamics (4). Generally, the classical solution of (5) involves solving a Two Point Boundary Value Problem (TPBVP) which requires global information of all agents and targets. In this paper, we will limit the information of each agent to itself and its neighbors and study whether this objective function can be optimized in a distributed manner.

Limited information model for decentralization. In our model, an agent is capable of observing information within
its sensing range, specifically the state $R_i(t)$ of all targets $i$ such that $p_{ij}(s_j(t)) > 0$. Moreover, agents can communicate with their neighboring agents to acquire information such as agent positions, speeds, and the states of targets which are within their own sensing ranges. In contrast to traditional multi-agent systems modeled through a network of agents, in the persistent monitoring setting agents move to interact with targets as shown in Fig. 1. Therefore, the network model includes both agents and targets and we need to revisit the concept of neighborhood, accounting as well for the fact that neighborhoods are time-varying. We begin with the observation that agents have two types of neighbors: nearby agents and nearby targets. On the other hand, the neighborhood of a target consists of just nearby agents. We do not explicitly model any possible connectivity among targets; however, if the target topology is fully connected, then it is possible for an agent near one target to acquire information about all targets.

**Definition 1.** The agent neighborhood of agent $j$ is the set $A_j(t) = \{k : \|s_k(t) - s_j(t)\| \leq r_c, k \neq j, k = 1, \ldots, N\}$.

This is a conventional definition of neighbors in multi-agent systems, where $r_c$ is a communication range, but we point out that it is time-dependent since agents are generally moving. As an example, in Fig. 1 $A_1 = \{A_2, A_3, A_5\}$.

**Definition 2.** The target neighborhood of agent $j$ is the set $T_j(t) = \{i : |x_i - s_j(t)| \leq r_j, i = 1, \ldots, M\}$.

This includes all targets which are within agent $j$’s sensing range. In Fig. 1 $T_3 = \{T_1, T_2, T_3\}$. Assuming the agents are homogeneous with a common sensing range $r$, we require that $r_c \geq 2r$ in order to establish communication among agents that are sensing the same target.

**Definition 3.** The agent neighborhood of target $i$ is the set $B_i(t) = \{j : |s_j(t) - x_i| \leq r_j, j = 1, \ldots, N\}$.

This set captures all the neighbor agents of target $i$. In Fig. 1 $B_2 = \{A_1, A_2, A_3\}$. Using Definition 3 the joint sensing probability in (3) can be rewritten as:

$$P_i(s(t)) = 1 - \prod_{j \in B_i(t)} (1 - p_{ij}(s_j(t)))$$

where $B_i(t) \subseteq \{1, \ldots, N\}$. We further define

$$N_{ij}(t) = B_i(t) \setminus \{j\}$$

to indicate the “collaborators” of agent $j$ in sensing target $i$. Note that $N_{ij}(t) = \{k : k \in A_j(t) \text{ and } k \in B_i(t)\}$, thus capturing a neighbor of agent $j$ and target $i$ at the same time.

Our limited information model restricts observations of each agent to the agent’s sensing range. However, any agent $j$ is allowed to communicate with its neighbors in $A_j(t)$. Therefore, the local information of an agent is the union of the observations of agent $j$ and the observations of agents $k \in A_j(t)$. In Section IV-A we will explicitly define the precise meaning of “information” above to consist of observable events such as “agent stops” or “target state becomes $R_i(t) = 0$”. In Section V we will show how $P_1$ can be solved by each agent under this limited information model as opposed to the centralized one in [10].

### III. FROM OPTIMAL CONTROL TO PARAMETRIC OPTIMIZATION

In this section, we review properties of the centralized optimal control solution of $P_1$ which allow it to be reduced to a parametric optimization problem [10]. This leads to the use of the Infinitesimal Perturbation Analysis (IPA) gradient estimation approach [11] to find an explicit solution through a gradient-based algorithm. We begin by defining the state vector $x(t) = [R_1(t), \ldots R_M(t), s_1(t), \ldots s_N(t)]$ and associated costate vector $\lambda = [\lambda_1(t), \ldots, \lambda_M(t), \lambda_{s_1}(t), \ldots, \lambda_{s_N}(t)]$. Due to the discontinuity in the dynamics of $R_i(t)$ in (4), the optimal state trajectory may contain a boundary arc when $R_i(t) = 0$ for some $i$; otherwise, the state evolves in an interior arc. Using (1) and (4), the Hamiltonian is

$$H(x, \lambda, u) = \sum_{i=1}^{M} R_i(t) + \sum_{i=1}^{M} \lambda_i(t) \dot{R}_i(t) + \sum_{j=1}^{N} \lambda_{s_j}(t) u_j(t)$$

(8)

Applying the Pontryagin Minimum Principle to (8) with $u^*(t), t \in [0, T]$, denoting an optimal control, a necessary condition for optimality is

$$u_{ji}^*(t) = \begin{cases} 1 & \text{if } \lambda_{s_j}(t) < 0 \\ -1 & \text{if } \lambda_{s_j}(t) > 0 \end{cases}$$

(9)

Note that there exists a possibility that $\lambda_{s_j}(t) = 0$ over some finite singular intervals [17], in which case $u_{ji}^*(t)$ may take values in $\{-1, 0, 1\}$.

A complete solution of the optimal control problem requires solving a TPBVP. However, this is unnecessary, since the optimal control structure is fully characterized by $u_{ji}^*(t) \in \{1, 0, -1\}$, it follows that we can parameterize the optimal trajectory (illustrated in Fig. 2) so as to determine (i) control switching points in $[0, L]$, where an agent switches its control from $\pm 1$ to $\mp 1$ or possibly 0 and (ii) corresponding dwell

![Fig. 1: Agent-target network. Red triangles are targets and blue squares are agents. Blue lines indicate the neighbor targets of an agent and red lines indicate the neighbor targets of an agent.](image-url)
The overall cost function (5) can be parametrically expressed as

\[ R_j = \int_{t_j}^{t_j+1} dw \ \text{for} \ \text{all agent and event time derivatives.} \]

This allows us to apply IPA to determine a gradient \( \nabla J(\theta, w) \) with respect to those parameters of the agent trajectories and apply any standard gradient descent algorithm to obtain an optimal solution.

Fig. 3: A simple example of a hybrid system consisting of one agent and one target. The system has six modes in which switches are triggered by events.

IV. INFINITESIMAL PERTURBATION ANALYSIS

We briefly review the IPA framework for general stochastic hybrid systems as presented in [11]. Let \( \{ \tau_k(\theta) \}, k = 1, \ldots, K \), denote the occurrence times of all events in the state trajectory of a hybrid system with dynamics \( \dot{x} = f_k(x, \theta, t) \) over an interval \([\tau_k(\theta), \tau_{k+1}(\theta)]\), where \( \theta \in \Theta \) is some parameter vector and \( \Theta \) is a given compact, convex set. For convenience, we set \( \tau_0 = 0 \) and \( \tau_{K+1} = T \). We use the Jacobian matrix notation: \( x'(t) = \frac{\partial f_k(\theta)}{\partial x} \) and \( \tau'_k = \frac{\partial \tau_k(\theta)}{\partial \theta} \), for all state and event time derivatives. It is shown in [11] that

\[ \frac{d}{dt} x'(t) = \frac{\partial f_k(\theta)}{\partial x} x'(t) + \frac{\partial f_k(\theta)}{\partial \theta}, \quad (11) \]

for \( t \in [\tau_k, \tau_{k+1}) \) with boundary condition:

\[ x'((\tau_k)^+) = x'(\tau_k) + [f_k-1(\tau_k^+) - f_k(\tau_k)] \tau'_k \quad (12) \]

for \( k = 1, \ldots, K \). In order to complete the evaluation of \( x'((\tau_k)^+) \) in (12), we need to determine \( \tau'_k \). If the event at \( \tau_k \) is exogenous (i.e., independent of \( \theta \)), \( \tau'_k = 0 \). However, if the event is endogenous, there exists a continuously differentiable function \( g_k : \mathbb{R}^n \times \Theta \to \mathbb{R} \) such that \( \tau_k = \min \{ t > \tau_{k-1} : g_k(x(\theta, t), \theta) = 0 \} \) and

\[ \tau'_k = -\left[ \frac{\partial g_k}{\partial x} f_k(\tau_k) \right]^{-1} \left[ \frac{\partial g_k}{\partial \theta} \right], \quad (13) \]

as long as \( \frac{\partial g_k}{\partial x} f_k(\tau_k) \neq 0 \) (details may be found in [11]).

In our setting, the cost along a given trajectory is

\[ \frac{1}{T} \int_0^T \sum_{i=1}^M R_i(t) dt. \]

Following (10), the gradient for each agent \( j \) denoted by \( \nabla J(\theta, w) = [\partial J(\theta, w)/\partial \theta_j, \partial J(\theta, w)/\partial w_i]^T \) is

\[ \nabla J(\theta, w) = \frac{1}{T} \int_{\tau_k}^{\tau_{k+1}} \nabla R_i(t) dt \quad (14) \]

where \( \nabla R_i(t) = [\partial R_i(t)/\partial \theta_j, \partial R_i(t)/\partial w_i]^T \).

We begin by deriving the gradient above within any inter-event interval \([\tau_k, \tau_{k+1})\) when the dynamics of both agent \( j \) and target \( i \) remain unchanged. Then, in Section IV-A, we will define all events involved in switching these dynamics, hence affecting the gradient evaluation possibly through discontinuities characterized by (12). We proceed with the derivation of \( \partial R_i(t)/\partial \theta_j \), since \( \partial R_i(t)/\partial w_i \) can be derived in a similar way.

It follows from (11), observing that the first term vanishes since \( f_k(t) = \dot{R}_i(t) \) is not an explicit function of \( R_i(t) \), that

\[ \frac{d}{dt} \frac{\partial R_i(t)}{\partial \theta_j} = \frac{\partial R_i(\tau_k)}{\partial \theta_j} \quad \text{if} \ \dot{R}_i(t) = 0 \quad (15) \]

and

\[ \frac{\partial R_i(t)}{\partial \theta_j} = \frac{\partial R_i(\tau_k)}{\partial \theta_j} - B_i \int_{\tau_k}^{t} \frac{\partial P_i(s(\tau))}{\partial \theta_j} d\tau \quad \text{if} \ \dot{R}_i(t) = A_i - B_i P_i(s(t)) \quad (16) \]

The integrand in (16) is obtained from (6):

\[ \frac{\partial P_i(s(\tau))}{\partial \theta_j} = \frac{\partial p_{i j}(s_j(\tau))}{\partial s_j} \frac{\partial s_j(\tau)}{\partial \theta_j} \prod_{g \neq j} [1 - p_{i g}(s_g(\tau))] \quad (17) \]
Note that \( \frac{dp_{ij}(s_j(\tau))}{\partial s_j} \) is piece-wise constant and takes values in \( \{0, \pm 1\} \) depending on \( s_j(t) - x_i \) and \( r_j \) (see agent sensing mode (2)). We can, therefore, factor the constant \( \frac{dp_{ij}(s_j(\tau))}{\partial s_j} \) out of the integral in (16). As for the term \( \frac{\partial s_j(\tau)}{\partial t} \), we apply (11) and (1) to obtain
\[
\frac{d}{dt} \frac{\partial s_j(\tau)}{\partial t} = 0
\] (18)

Therefore, \( \frac{\partial s_j(\tau)}{\partial \tau} = \frac{\partial s_j(\tau^+)}{\partial \tau} \) which is also a constant. The product term in (17) captures the contributions from all agents other than \( j \) in monitoring target \( i \). Using the definition of \( N_{ij}(\tau) \) in (7), it can be restricted to this set, since for any agent \( g \notin N_{ij}(\tau) \), \( p_{ig}(s_g(\tau)) = 0 \). For notational simplicity, we define the integral of this term over \([\tau_k, \tau], t < \tau_{k+1}\), as:
\[
G_{ij}(t) = \int_{\tau_k}^{t} \prod_{g \in N_{ij}(\tau)} [1 - p_{ig}(s_g(\tau))] d\tau
\] (19)

which can be interpreted as a “collaboration factor” involving all agents in \( N_{ij}(\tau) \). Clearly, this is affected by an agent leaving or joining the neighbor set \( N_{ij}(\tau) \) which motivates defining an event associated with such changes (see Section IV-A).

When we combine (15) and (16), the derivative \( \frac{\partial R_i(\tau)}{\partial \theta_j} \), \( i = 1, \ldots, M \), over any inter-event interval \([\tau_k, \tau_{k+1})\) becomes:
\[
\frac{\partial R_i(\tau)}{\partial \theta_j} = \left\{ \begin{array}{ll}
0 & \text{if } R_i(t) = 0, \ A_i \leq B_i P_i(s(t)) \\
B_i \frac{\partial p_{ij}(s_j(\tau^+))}{\partial s_j} \frac{\partial s_j(\tau^+)}{\partial \tau} G_{ij}(t) & \text{otherwise}
\end{array} \right.
\] (20)

A similar derivation can be applied to the derivative \( \frac{\partial R_i(\tau)}{\partial \omega_j} \) and gives:
\[
\frac{\partial R_i(\tau)}{\partial \omega_j} = \left\{ \begin{array}{ll}
0 & \text{if } R_i(t) = 0, \ A_i \leq B_i P_i(s(t)) \\
B_i \frac{\partial p_{ij}(s_j(\tau^+))}{\partial s_j} \frac{\partial s_j(\tau^+)}{\partial \tau} G_{ij}(t) & \text{otherwise}
\end{array} \right.
\] (21)

A. Events in the hybrid system

We are now in a position to define as “events” all switches in the hybrid system which can result in changes in the derivatives in (20) and (21), so we can apply (12) to determine the initial conditions \( \frac{\partial R_i(\tau^+)}{\partial \theta_j} \) and \( \frac{\partial R_i(\tau^+)}{\partial \omega_j} \) at \( t = \tau_k \), as well as the terms \( \frac{\partial s_j(\tau^+)}{\partial \theta_j} \) and \( \frac{\partial s_j(\tau^+)}{\partial \omega_j} \).

We classify events into four categories depending on the effect they have on target dynamics (type I), agent sensing relative to a target (type II), agent dynamics (type III), and neighbor set \( N_{ij}(\tau) \) (type IV). Referring to Fig. 3, observe that only event types I and III (red and blue arrows) affect the dynamics of the corresponding target and agent. Event types II and IV do not change the system dynamics but still affect the derivative values in (20) and (21). In what follows, we define all events types and their corresponding effects on (20) and (21) and summarize them in Table I.

**Event type I: switches in target dynamics \( \dot{R}_i(t) \).**

Referring to (4), when \( \dot{R}_i(t) \) either reaches zero or leaves zero, the IPA derivative switches between (15) and (16). We denote the former event as \( \rho_i^0 \) and the latter as \( \rho_i^+ \) for all \( i = 1, \ldots, M \) (see Table I). When such events occur, the dynamics of \( s_j(t) \) in (1) remain unchanged, so it follows from (12) that \( \nabla_j s_j(\tau^+) = \nabla_j s_j(\tau^+) \). However, the target dynamics switch between \( \dot{R}_i = A_i - B_i P_i(s(t)) \) and \( \dot{R}_i = 0 \) and cause discontinuities in \( \nabla_j R_i(t) \) as follows.

**Event \( \rho_i^0 \): This event causes a transition from \( \dot{R}_i(t) = A_i - B_i P_i(s(t)) \) to \( \dot{R}_i(t) = 0 \) at \( \tau_k \).**

Then we apply (12) and (13) to obtain
\[
\nabla_j R_i(\tau^+) = \nabla_j R_i(\tau^-) + [A_i - B_i P_i(s(\tau^-))] \tau_k
\] (22)

Combining (22) and (23), we get
\[
\nabla_j R_i(\tau^+) = 0 \quad \text{if event } \rho_i^0 \text{ occurs at } \tau_k
\] (24)

**Event \( \rho_i^+ \):** This event causes a transition from \( \dot{R}_i(t) = 0 \), \( t < \tau_k \) to \( \dot{R}_i(t) = A_i - B_i P_i(s(t)) \), \( t > \tau_k \). It is easy to see that the dynamics in both (1) and (4) are continuous when this happens and since \( A_i - B_i P_i(s(\tau^-)) = 0 \) we have \( \dot{R}_i(\tau^-) = \dot{R}_i(\tau^-) = 0 \). It follows from (12) that \( \nabla_j R_i(\tau^-) = \nabla_j R_i(\tau^-) \). Moreover, since \( R_i(t) = 0 \), \( R_i(t) = 0, t < \tau_k \), we have \( \nabla_j R_i(\tau^-) = 0 \) and we get
\[
\nabla_j R_i(\tau^+) = 0 \quad \text{if event } \rho_i^+ \text{ happens at } \tau_k
\] (25)

**Remark 1:** Combining (24) and (25) with (20) and (21), we conclude that a \( \rho_i^0 \) event occurring at \( t = \tau_k \) resets the value of \( \nabla_j R_i(t) \) to \( 0 \) for all \( j = 1, \ldots, N \) regardless of the value \( \nabla_j R_i(\tau^-) \) and the state of the agents. Moreover, \( R_i(t) = 0 \) and \( \nabla_j R_i(t) = 0 \) for \( t > \tau_k \) until the next \( \rho_i^+ \) event occurs.

**Event type II: switches in agent sensing \( p_{ij}(s_j(t)) \).**

These events trigger a switch in \( \frac{\partial p_{ij}(s_j(t))}{\partial s_j} \) from \( \pm 1 \) to 0 or vice versa in (20) and (21). We denote the former event as \( \pi_i^0 \) and the latter as \( \pi_i^+ \). These events trigger a switch

| Event Name | Description |
|------------|-------------|
| \( \rho_i^0 \) | \( R_i(t) \) hits 0 |
| \( \rho_i^+ \) | \( R_i(t) \) leaves 0 |
| \( \pi_i^0 \) | \( p_{ij}(s_j(t)) \) hits 0 |
| \( \pi_i^+ \) | \( p_{ij}(s_j(t)) \) leaves 0 |
| \( \nu_i(-1,0) \) | \( u_j(t) \) switches from \( 1 \) to \( 0 \) |
| \( \nu_i(0,1) \) | \( u_j(t) \) switches from \( -1 \) to \( 0 \) |
| \( \nu_i(-1,1) \) | \( u_j(t) \) switches from \( 0 \) to \( 1 \) |
| \( \nu_i(1,1) \) | \( u_j(t) \) switches from \( 1 \) to \( 1 \) |
| \( \Delta_j^\pi_i \) | \( N_{ij}(\tau^+) = N_{ij}(\tau^-) \cup \{k\}, k \notin N_{ij}(\tau^-) \) |
| \( \Delta_j \) | \( N_{ij}(\tau^+) = N_{ij}(\tau^-) \setminus \{k\}, k \in N_{ij}(\tau^-) \) |
of $\frac{\partial s_j}{\partial t_j}(s_j(t)) = 0$ at $t = \tau_k$ from $\frac{\partial s_j}{\partial t_j}$ to 0 or vice versa in (20) and (21). However, the dynamics in both (1) and (2) remain unchanged when this happens (due to the continuity of the sensing function $p_{ij}(s_j(t))$) and it follows from (12) that $\nabla_j R_i(\tau_k^+) = \nabla_j R_i(\tau_k^-)$ and $\nabla_j s_j(\tau_k^+) = \nabla_j s_j(\tau_k^-)$.

**Event type III: switches in agent dynamics** $s_j(t)$. Referring to (1), these are events that cause a switch in the optimal control values $u_j^+(t_k)$: (i) $\pm 1 \to 0$, (ii) $0 \to \pm 1$, and (iii) $\pm 1 \to \mp 1$. We denote these events as $\nu^{(1,0)}_{j}, \nu^{(-1,0)}_{j}, \nu^{(0,1)}_{j}, \nu^{(-1,1)}_{j}, \nu^{(1,-1)}_{j}$ using the general notation $\nu^{*,*}_{j}$ with the superscript corresponding to the six total possible control switches. The effect of these events in (20) and (21) is through possible discontinuities in the terms $\frac{\partial \nu_j}{\partial \theta_j}$ and $\frac{\partial \nu_j}{\partial w_j}$ at $t = \tau_k$. Clearly, the gradient cannot be affected by future events, so we consider all prior and current control switches indexed by $l = 1, 2, \ldots, \xi$ where the $l$ is the current control switch and $\theta_{j,l}$, $w_{j,l}$ are the $l$-th switching point and dwelling time respectively. These agent control switches are endogenous events with switching functions $g_k(s_j(t), t) = s_j - \theta_{j,l} = 0$. We can now apply (12) and (13) to (1), similar to the derivation for type I events. We omit the details (which can be found in [4]) and present the final results.

**Events** $\nu^{(1,0)}_{j}, \nu^{(-1,0)}_{j}$. These are switches such that $u_j(\tau_k^+)=\pm 1$, $u_j(\tau_k^-)=0$ and we get

$$\frac{\partial s_j}{\partial \theta_j}(\tau_k^+) = \begin{cases} 1 & \text{if } l = \xi \\ 0 & \text{if } l < \xi \end{cases} \quad (26)$$

$$\frac{\partial s_j}{\partial w_j}(\tau_k^+) = 0 \quad \text{for all } l \leq \xi \quad (27)$$

**Events** $\nu^{(0,1)}_{j}, \nu^{(0,-1)}_{j}$. These are switches such that $u_j(\tau_k^+)=0$, $u_j(\tau_k^-)=\pm 1$ and we get

$$\frac{\partial s_j}{\partial \theta_j}(\tau_k^+) = \begin{cases} \frac{\partial s_j}{\partial \theta_j}(\tau_k^-) - u_j(\tau_k^+) sgn(\theta_{j,l} - \theta_{j,(l-1)}) & \text{if } l = \xi \\ \frac{\partial s_j}{\partial \theta_j}(\tau_k^-) - u_j(\tau_k^-) sgn(\theta_{j,l+1} - \theta_{j,(l+1)}) & \text{if } l < \xi \end{cases} \quad (28)$$

$$\frac{\partial s_j}{\partial w_j}(\tau_k^+) = -u_j(\tau_k^-) \quad \text{for all } l \leq \xi \quad (29)$$

**Events** $\nu^{(-1,1)}_{j}, \nu^{(1,-1)}_{j}$. These are switches such that $u_j(\tau_k^+)=\pm 1$, $u_j(\tau_k^-)=\mp 1$ so that a dwell time is not involved and we get

$$\frac{\partial s_j}{\partial \theta_j}(\tau_k^+) = \begin{cases} 2 & \text{if } l = \xi \\ \frac{\partial s_j}{\partial \theta_j}(\tau_k^-) & \text{if } l < \xi \end{cases} \quad (30)$$

**Remark 2:** Observe that $\nabla_j s_j(t)$ is independent of the states of other agents $k \neq j$. This follows from the fact that $\nabla_j s_j(t)$ is constant over inter-event intervals $[\tau_k, \tau_{k+1})$ as shown in (18) and only depends on parameter and control values known to agent $j$ as seen in (20) - (30). Moreover, if $k \neq j$, $\nabla_j s_j(t) = 0$.

**Event type IV: changes in neighbor sets** $N_{ij}(t)$. These events change the topology of the agent-target network by altering the neighbors of agent $j$, hence affecting the value of $G_{ij}(t)$ in (19) which in turn affects (20) and (21). We denote by $\Delta_{ij}^+$ the event causing the addition of an agent to the neighbor set $N_{ij}(t)$ and by $\Delta_{ij}^-$ the event causing the removal of an agent from the neighbor set $N_{ij}(t)$. However, the dynamics of both $R_i(t)$ and $s_j(t)$ remain unchanged when these events occur. Due to the continuity of the sensing function $p_{ij}(s_j(t))$ in (12), the addition/removal of an agent $g$ to/from the set $N_{ij}(t)$ does not affect the continuity of $G_{ij}(t)$, which implies $\nabla_j R_i(\tau_k^+) = \nabla_j R_i(\tau_k^-)$ as well as $\nabla_j s_j(\tau_k^+) = \nabla_j s_j(\tau_k^-)$.

The set of all events defined above and summarized in Table I is denoted by $E$. Furthermore, we define the set of all type III events of the form $\nu^{*,*}_{j}$ as the *agent event set* $E^A$ and the set of all other events (type I, III, and IV) as the *target event set* $E^T$. The subset of $E^A$ that contains only events related to agent $j$ is denoted by $E^A_j$. Similarly, the subset of $E^T$ that contains only events related to target $i$ is denoted by $E^T_i$. We then have:

**Definition 4.** The local event set of any agent $j$ is the union of agent events $E^A_j$ and target events $E^T_i$ for all $i \in T_j(t)$:

$$E_j(t) = E^A_j \cup \bigcup_{i \in T_j(t)} E^T_i \quad (31)$$

In contrast, the global event set for agent $j$ includes all non-neighboring target events in $E^T_i$ for all $i \notin T_j$ and non-neighboring agent events $E^A_k$, for all $k \notin A_j$. Based on the limited information model of Section III, we define the local information set of agent $j$, denoted by $I_j(t)$, as follows:

**Definition 5.** The local information set of any agent $j$ is the union of its local event set and those of its neighbors in $N_{ij}(t)$ for all $i \in T_j(t)$:

$$I_j(t) = E_j(t) \bigcup_{k \in N_{ij}(t), i \in T_j(t)} E_k(t) \quad (32)$$

This includes all local information necessary for agent $j$ to evaluate the IPA gradient $\nabla_j R_i(t)$ for $i \in T_j(t)$ (observe that agent $j$ does not need to communicate with all its neighbors in $A_j(t)$, but only a subset which includes those neighbors who are sharing the same target(s) as $j$ at time $t$ since $\bigcup_{i \in T_j(t)} N_{ij}(t) \subseteq A_j(t)$).

**Remark 3:** It is clear from the analysis thus far, that IPA is entirely event-driven, since all gradient updates happen exclusively at events occurring at times $\tau_k(\theta, w)$, $k = 1, 2, \ldots$. Thus, this approach scales with the number of events characterizing the hybrid system, and not its (generally much larger) state space.

**V. EVENT-DRIVEN DECENTRALIZED GRADIENT EVALUATION AND OPTIMIZATION**

Our main results are presented in this section. In particular, we show in Theorem 1 that each agent can evaluate the gradient of the objective function in (10) with respect to its own controllable parameters $\theta_j$ and $w_j$ based on its local information set (32) and only one non-local event. We begin with the following lemma which asserts that the gradient
\( \nabla_j R_j(t) \) takes a very simple form as long as \( i \notin T_j(t) \), i.e., while target \( i \) cannot be sensed by agent \( j \).

**Lemma 1.** Let \( t \in [t_1, t_2] \) such that \( i \notin T_j(t) \). Then,

1. If \( R_i(t) > 0 \) for all \( t \in [t_1, t_2] \), then
   \[
   \nabla_j R_i(t) = \nabla_j R_i(t_1^+) \tag{33}
   \]
2. If there exists an event \( \rho_i^0 \) at \( \tau \in (t_1, t_2) \), then
   \[
   \nabla_j R_i(t) = \begin{cases} 
   \nabla_j R_i(t_1^+) & t \in [t_1, \tau) \\
   0 & t \in [\tau, t_2] 
   \end{cases} \tag{34}
   \]

**Proof:** By the definition of \( T_j(t) \), when \( i \notin T_j(t) \) we have \( \|s_i(t) - x_i\| > r_j \) and \( \frac{\partial s_i}{\partial s_j} \) for all \( t \in [t_1, t_2] \). If \( R_i(t) > 0 \) for all \( t \in [t_1, t_2] \), it follows directly from (20) and (21) that \( \nabla_j R_i(t) = \nabla_j R_i(t_1^+) \). Otherwise, there exists an event \( \rho_i^0 \) at \( \tau \in (t_1, t_2) \) which results in \( R_i(\tau) = 0 \). The previous argument applies to \((t_1, \tau)\) giving \( \nabla_j R_i(t) = \nabla_j R_i(t_1^+) \) for \( t \in [t_1, \tau) \). According to (24), event \( \rho_i^0 \) resets the gradient to \( \nabla_j R_i(0) \). Subsequently, over \([\tau, t_2]\), regardless of which of the cases in (20) and (21) applies, it holds that \( \nabla_j R_i(t) = 0 \).

**Corollary 1.** \( \nabla_j R_i(t) \) is independent of events \( \rho_i^\pm \) for \( i \notin T_j(t) \).

**Proof:** Note that the \( \rho_i^\pm \) event can only occur after a \( \rho_i^0 \) event. The proof is self-evident following Lemma 1. We have \( \nabla_j R_i(t) = 0 \) for \( t > \tau \) until target \( i \) joins the target neighborhood of agent \( j \). Therefore, any non-local \( \rho_i^\pm \) event that may occur cannot affect \( \nabla_j R_i(t) \).

Lemma 1 and its Corollary imply that agent \( j \) does not need any knowledge of non-neighboring target events except for \( \rho_i^0 \) with \( i \notin T_j(t) \) in order to evaluate its gradient. We can further establish that the gradient \( \nabla_j J(\theta, w) \) along the agent trajectory is affected by only local events in \( I_j(t) \), as defined in (32), and a small subset of global events.

**Lemma 2.** A sufficient event set to evaluate \( \nabla_j J(\theta, w) \) is \( I_j(t) \cup \{ \rho_i^0 : i \notin T_j(t) \} \).

**Proof:** Let \( \tau_k \) be any event time when \( T_j(\tau_k) \) is altered, i.e., a new target is added to the target neighborhood of agent \( j \) or one is removed from it. From Lemma 1 if \( i \notin T_j(t) \), then either \( \nabla_j R_i(t) = \nabla_j R_i(\tau_k) \) and remains constant at this value or \( \nabla_j R_i(t) = 0 \), depending on whether an event \( \rho_i^0 \) takes place. It follows from (14) that the objective function gradient can be rewritten as

\[
\nabla_j J(\theta, w) = \sum_{k=0}^{K} \sum_{i=1}^{M} \int_{\tau_k}^{\tau_{k+1}} \nabla_j R_i(t) dt
\]

\[
= \sum_{k=0}^{K} \left( \sum_{i \notin T_j(\tau_k)} \nabla_j R_i(\tau_k)(\tau_{k+1} - \tau_k) + \sum_{i \in T_j(\tau_k)} \int_{\tau_k}^{\tau_{k+1}} \nabla_j R_i(t) dt \right) \tag{35}
\]

The value of \( \nabla_j R_i(\tau_k) \) in the first term of (35) depends on \( \{ \rho_i^0 : i \notin T_j(t) \} \) which is a subset of events non-local to agent \( j \). The second term of (35) depends only on the local information set events \( I_j(t) \) since target \( i \in T_j(t) \) is local to agent \( j \). Therefore, \( I_j(t) \cup \{ \rho_i^0 : i \notin T_j(t) \} \) is a sufficient event set to evaluate \( \nabla_j J(\theta, w) \).

**Remark 4:** Although an event \( \rho_i^0 \) for \( i \notin T_j(t) \) is non-local to agent \( j \), it must be observed by at least one agent \( k \neq j \) such that \( i \in T_k(t) \). This is because \( \rho_i^0 \) at some time \( \tau_k \) can only take place if one or more agents in its neighborhood cause a transition from \( R_i(\tau_k) > 0 \) to \( R_i(\tau_k) = 0 \) in (4). Therefore, such events can be communicated to agent \( j \) through the agent network, possibly with some delays. The implication of Lemma 2 is an “almost decentralized” algorithm in which each agent optimizes its trajectory through the gradient \( \nabla_j J(\theta, w) \) using only agent local information; the only exception is occasional target uncertainty depletion events transmitted to it from other agents.

Returning to the parametric optimization problem (10), a centralized solution was obtained in (10) using the IPA gradients in (20) and (21) and a standard gradient descent scheme to optimize the parameter vector \( [\theta, w]^T \) as follows:

\[
[\theta^{l+1}, w^{l+1}]^T = [\theta^l, w^l]^T - [\alpha_\theta, \alpha_w] \nabla_j J(\theta, w) \tag{36}
\]

where \( l = 0, 1, \ldots \) is the iteration index and \( \alpha_\theta \) and \( \alpha_w \) are diminishing step-size sequences satisfying \( \sum_{l=0}^{\infty} \alpha_\theta = \infty, \lim_{l \to \infty} \alpha_\theta = 0 \) and \( \sum_{l=0}^{\infty} \alpha_w = \infty, \lim_{l \to \infty} \alpha_w = 0 \). A decentralized version of (36) is

\[
[\theta_j^{l+1}, w_j^{l+1}]^T = [\theta_j^l, w_j^l]^T - [\alpha_\theta, \alpha_w] \nabla_j J(\theta, w) \tag{37}
\]

where \( \theta \) and \( w \) are agent \( j \)'s estimates based on the limited information provided in Lemma 2.

**Theorem 1.** Any centralized solution of (10) through (36) can be recovered by (37) in which each agent \( j \) optimizes its trajectory given the following conditions:

1. Initial parameters \( [\theta_0, w_0]^T \);
2. The local information set \( I_j(t) \);
3. The subset of the global information set \( \{ \rho_i^0 : i \notin T_j(t) \} \).

**Proof:** The proof is immediate from Lemma 2. The gradient \( \nabla_j J(\theta, w) \) can be evaluated by each agent given conditions 2 and 3. Condition 1 provides initial parameters for each agent trajectory in order to execute (37).

Note that condition 3 involves only a small subset of global events. As shown in our simulation results in Section VI, ignoring such non-local events will affect the cooperation among agents and increase the final cost. Thus, it can be interpreted as the “price of anarchy” commonly associated with decentralization limiting agent actions to only local information.

It is important to point out that the method of Theorem 1 relies on the gradient \( \nabla_j R_i(t) \) for \( i \notin T_j(t) \) and not on \( R_i(t) \). In fact, there is no attempt by agent \( j \) to reconstruct or estimate the states of targets \( i \notin T_j(t) \); the only information from such targets is provided through the occasional \( \rho_i^0 \) events.

We briefly discuss next some open issues defining ongoing research directions. While the event-driven nature of IPA
has several computational advantages (see Remark 3), the optimization process depends on these events being observed so as to “excite” algorithms such as (36) and (10). To resolve this event excitation issue, potential field methods were proposed in [18] and [10]. However, these methods generally require global information such as target states. In this paper, we have assumed that initial trajectories have been selected so that all necessary events are excited. It is also possible to address this issue by having each agent create an initial estimated potential field, until all necessary events are excited. In addition, here we have also assumed that all agent communications are without delays, in particular when non-local events $\rho_i^j$ for $i \notin T_j(t)$ are communicated to agent $j$ through multiple hops. The presence of delays generally affects (10). However, asynchronous versions of (10) can still guarantee convergence (to the same local optima) under certain mild conditions (see [19] and [14]).

Algorithm 1 IPA-driven gradient descent for each agent

1: Initialize parameters $\theta_j, w_j$
2: Select an error tolerance $\epsilon > 0$ and a maximum number of iterations $n_0$
3: repeat:
4: Compute the IPA gradient $\nabla_j J(\hat{\theta}, \hat{w})$
5: Update $\theta_j, w_j$ using (37)
6: until $\|\nabla_j J(\theta, w)\| < \epsilon$ or number of iterations exceeds $n_0$
7: Set the optimized parameter $\theta_j^* = \theta_j, w_j^* = w_j$

VI. SIMULATION EXAMPLES

We present two simulation examples to demonstrate the performance of the decentralized scheme described in Theorem 1.

In the first example, three homogeneous agents are allocated to persistently monitor seven targets in the 1D mission space for $T = 300$ seconds. The targets are located at $x_i = 5i$ for $i = 1, \ldots, 7$. The uncertainty dynamics in (4) are defined by the parameters $A_i = 1, B_i = 5$, with initial values $R_i(0) = 1$ for $i = 1, \ldots, 7$. Each agent has a sensing range of $r = 3$ and is initialized with $s_j(0) = 0.5(j-1)$, $u_j(0) = 1$, $\theta_j^1 = [5, 10, 15, 10, 5, \ldots]$, $\theta_j^2 = [15, 20, 25, 20, 15, \ldots]$, $\theta_j^3 = [25, 30, 35, 30, 25, \ldots]$, and $w_j^0 = [0.5, 0.5, 0.5, \ldots]$ for all $j = 1, \ldots, 3$. Results of the method in Theorem 1 are shown in Fig. 4. The top plot depicts the optimal trajectories of each agent determined after 200 iterations of (37), while the bottom plot shows the overall cost $J(\theta, w)$ as a function of iteration number. The final cost is $J^* = 37.38$. The exact same results (not shown here) as in Fig. 4 were also obtained through the centralized scheme (36) where all information is available to every agent. This shows the effectiveness of the method in Theorem 1.

As pointed out earlier, the method of Theorem 1 does not involve any knowledge by agent $j$ of the states of targets $i \notin T_j(t)$. This is illustrated in Fig. 5 which shows (in blue) the fraction of time that agent 1 has any information on the state of target 3 because it happens that $3 \in T_1(t)$. The rest of the time (shown in red) agent 1 is unable to accurately estimate the state of this target, but such information is unnecessary. The agent only needs a small subset of non-local information, as illustrated by the green dots in Fig. 5.

The second example uses the same environment as the first one and agents start with the same initial trajectories. However, we eliminate the non-local information (condition 3 in Theorem 1) and each agent calculates its own IPA-based gradient using only local information in the set $I_j(t)$. Figure 6 shows the results after 200 iterations of (37). Note that without non-local information, each agent tends to spend more time dwelling on the local targets instead of better coordinating with the other agents. Therefore, the final cost after convergence increases from $37.38$ to $41.66$. Even though the gradient estimate for agent $j$ is no longer accurate without the $\rho_i^j$ event information when $i \notin T_j(t)$, the cost still decreases and converges as shown in Fig. 6 illustrating the robustness of the IPA-based gradient descent method.

VII. CONCLUSIONS AND FUTURE WORK

The decentralization of multi-agent systems that involve the interaction of agents with “points of interest” (targets) in their mission space is particularly challenging. We have shown that in 1D persistent monitoring problems an optimal centralized solution can be recovered by an event-driven “almost decentralized” algorithm which significantly reduces communication costs while yielding the same performance as the centralized algorithm. In particular, each agent uses only local information except for one event requiring communication with a non-neighbor agent when it occurs.

In addition to the event excitation issue mentioned following Theorem 1 and incorporating communication delays in the algorithm we have developed, the extension of this approach to the 2D case is the subject of ongoing research. The derivations in this paper that lead to the “almost de-
centralized” IPA gradient evaluation apply to 2D-trajectories as long as these trajectories have a parametric form and the number of parameters is finite. Moreover, the derivation holds if agents move in straight-lines under graph-limited mobility constraints as shown in [20]. If an agent trajectory in 2D is not limited to straight lines, the constant terms in the 1D gradient derivation in [17] will be time-varying. The decentralization then requires agents to share part of their trajectories with their neighboring agents when they are in the same target neighborhood. However, this additional requirement only involves local agent information exchanges, therefore it will not affect the framework of our “almost decentralized” solution.

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