Estimating the intrinsic dissipation using the second harmonic of the temperature signal in tension-compression fatigue. Part II: Experiments

Giovanni Meneghetti | Mauro Ricotta

Abstract
The theoretical approach developed in the companion paper is applied to the temperature signals acquired by using an infrared camera, during step-load fatigue tests carried out on AISI 304L cold-drawn bars. The intrinsic dissipation, calculated according to the analytical model and by means of the discrete Fourier transform of the material temperature, is compared with that measured by using a well-established experimental approach based on the measurement of the cooling gradient of the material temperature, after the fatigue test has been suddenly stopped. The results of this study indicate that the intrinsic dissipation calculated according to the two approaches become increasingly closer to one another for progressively higher applied stress amplitudes.

KEYWORDS
discrete Fourier transform, dissipated energy, energy approaches, thermal methods

1 | INTRODUCTION

In Part I of this paper, an analytical frame has been developed to evaluate the intrinsic dissipation of a metallic material under fatigue averaged over one loading cycle, $Q_p$, by using the Fourier series of the material temperature signal. The theoretical model is based on the first law of thermodynamics, refers to zero-mean stress fatigue, and was developed under the hypothesis that the plastic strain energy density, $W$, is completely converted into heat and that uniform stress and temperature fields exist in the specimen. In this framework, two different material models were considered, namely, an elastic-perfectly plastic and an elastic-plastic material obeying a Ramberg–Osgood law. Since in actual laboratory test conditions, the average material temperature stabilizes after achieving thermal equilibrium with the surroundings, a heat sink was introduced to simulate heat extraction from the material in a simplified and idealized fashion. The intrinsic dissipation has been linked to the second harmonic of the material temperature, $\Delta T_2$, as follows (eq. 2 of Part I):

$$W = Q_p = 2 \cdot \rho \cdot c \cdot \Delta T_2 \cdot \beta$$  (1)

where $\beta$ is a parameter greater than 1. In more detail, in the case of an elastic-perfectly plastic material, $\beta$ depends on the ratio between the applied strain amplitude and the yield strain, while for an elastic-plastic material behavior, $\beta$ depends on the cyclic strain hardening exponent, $n'$. Moreover, $\beta$ was generally found to be...
dependent on the thermal boundary conditions (BCs). However, by assuming standard laboratory test conditions, the sensitivity of $\beta$ to the thermal BCs is negligible (see the appendix of Part I1).

It has been also shown that when the material temperature stabilizes, the intrinsic dissipation $Q_p$ is equal to the heat energy exchanged with the surroundings by a unit volume of material per cycle, the $Q$ parameter.2 Previously, the heat energy per cycle (the $Q$ parameter) was proposed as a fatigue damage index because it is independent of the mechanical and thermal BCs2,3 for a given load cycle and stress state.4 $Q$ can be easily evaluated by stopping the fatigue test at $t = t^*$ after thermal equilibrium has been achieved and by measuring the cooling gradient immediately after $t^*$,2 according to Equation 2:

$$Q = \frac{1}{f_L} \cdot \rho \cdot c \cdot \left. \frac{dT}{dt} \right|_{t = (t^*)}$$

where $T(t)$ is the time-variant temperature of the material, $\rho$ is the material density, $c$ is the material-specific heat, and $f_L$ is the load test frequency applied before the test interruption.

In this Part II paper, the theoretical approach presented in Meneghetti and Ricotta1 is applied to material temperature signals recorded during fully reversed, stress controlled, axial step-load fatigue tests performed on AISI 304L cold-drawn bars. $Q$ calculated according to Equation 1, hereafter called the second harmonic approach, has been compared with that evaluated by means of Equation 2, hereafter called the cooling gradient approach, as reported in Figure 1. Figure 1 shows also the experimental procedure adopted in the present paper, which will be presented in the relevant section.

The paper is organized as follows. First, the adopted experimental setup is described, and then the results are presented by comparing the $Q$ values obtained using Equations 1 and 2. Afterwards, thermal finite element (FE) analyses are conducted to compare the second harmonic approach and the cooling gradient approach by considering virtual experiments to avoid the influence of noise. A discussion section, which compares the second harmonic and the cooling gradient approaches, concludes the paper.

2 | MATERIAL, SPECIMEN GEOMETRY, AND TEST METHODS

Based on the second harmonic approach, the $Q$ parameter was evaluated using the fatigue data published by the authors previously,5 dealing with different approaches for the rapid determination of the fatigue limit. In view of this, in previous study,5 step-load, zero mean stress, axial fatigue tests were conducted on specimens made of cold-drawn AISI 304L stainless steel, having the elastic modulus $E = 192,200$ MPa, the tensile strength $R_m = 691$ MPa, the proof strength $R_p02 = 468$ MPa and the percent elongation after fracture $A = 43%$. The analyzed material had density $\rho = 7940$ kg/m$^3$, specific heat $c = 507$ J/(kg K)6 and isotropic coefficient of thermal expansion $\alpha = 15.1 \times 10^{-6}$/K.4 The fatigue limit of the material was evaluated to be equal to $\sigma_{A, -1} = 330$ MPa by
performing a short stair-case procedure at 10 million cycles and using eight specimens having their geometry machined according to Figure 2A, while Figure 2B shows the specimen geometry defined for the step-load fatigue tests. Shorter specimens were adopted in the stair-case procedure to speed up long life tests thanks to their higher stiffness and the load test frequency was set in the range of 7 to 23 Hz, in order to keep the material temperature below 60°C. The specimen surface was covered with a black paint having an emissivity \( \kappa = 0.93 \). The material temperature measurements were performed using an infrared camera, as described later. The fatigue tests were conducted by using a servo-hydraulic Schenck Hydropuls PSA 100 machine equipped with a 100 kN load cell and a TRIO Sistemi RT3 digital controller.

During step-load fatigue tests, the applied stress amplitude ranged from 270 to 405 MPa by imposing a step load equal to 15 MPa. After some preliminary fatigue tests, the number of cycles spent during each step was set equal to 10,000 cycles in order to guarantee the stabilization of the material temperature \( T_s \) (see Figure 1) below 60°C, for all the applied step-loads. During the step-load fatigue tests, the material temperature was measured using an FLIR SC7600 infrared camera with 50-mm focal lens, a 1.5–5.1-μm spectral response range, a noise equivalent temperature difference (NETD)<25 mK, an overall accuracy of 0.05°C and operating with ALTAIR 5.90.002 commercial software. The spatial resolution was equal to 0.12 mm/px. The infrared camera was equipped with an analogue input interface, which was used to synchronize the force signal of the load cell with the temperature signal and eventually perform motion compensation. To compare the second harmonic and the cooling gradient approach, two consecutive temperature acquisitions were performed using a 10-s-long sampling window with \( n_{acq} = 2048 \) frames. In particular, the first acquisition consisted of exactly 10s of running test \( n_{acq} = 2048 \) frames between \( t_s \) and \( t^* \), in Figure 1), followed by the second one capturing the machine stop at time \( t^* \) to measure the cooling gradient. The temperature maps recorded during the first acquisition were first processed by using the FLIR MotionByInterpolation tool to allow for relative motion compensation between the fixed camera lens and the moving specimen subject to cyclic loads. Afterwards, the mean temperature signal taken from a small area, \( A \), \( (12 \times 7 \text{px}^2 \text{ in size, i.e., } 1.44 \times 0.84 \text{ mm}^2) \) located in the center of the specimen net-section was processed to evaluate \( \Delta T_2 \) (see Figure 1). It is worth noting that in this paper, the analytical approach presented in the companion paper\(^1\) has been applied to the data acquired experimentally; therefore, the discrete Fourier transform (DFT) was applied to the temperature data points. In view of this approach, Eq. (20) and Eq. (21) of Meneghetti and Ricotta\(^1\) must be modified as follows:

\[
T_i = \bar{T}_s + \sum_{j=1}^{n_{acq}-1} \left\{ \sum_{i=0}^{n_{acq}-1} \left[ A_j \cdot \cos \left( \frac{2\pi}{n_{acq}} \cdot i \cdot j \right) + B_j \cdot \sin \left( \frac{2\pi}{n_{acq}} \cdot i \cdot j \right) \right] \right\}
\]

\[\bar{T}_s = \frac{1}{n_{acq}} \sum_{i=0}^{n_{acq}-1} T_i\]  \hspace{1cm} (3)

\[A_j = \frac{2}{n_{acq}} \sum_{i=0}^{n_{acq}-1} T_i \cdot \cos \left( \frac{2\pi}{n_{acq}} \cdot i \cdot j \right)\]  \hspace{1cm} (4a)

\[B_j = \frac{2}{n_{acq}} \sum_{i=0}^{n_{acq}-1} T_i \cdot \sin \left( \frac{2\pi}{n_{acq}} \cdot i \cdot j \right)\]  \hspace{1cm} (4b)

\[\Delta T_j = 2 \cdot \sqrt{A_j^2 + B_j^2}\]  \hspace{1cm} (4c)

\[\Delta T_2 = 2 \cdot \sqrt{A_j^2 + B_j^2}\]  \hspace{1cm} (4d)

\[
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\]

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\]

**FIGURE 2** Geometry specimen for (A) stair case fatigue tests and (B) step-load fatigue tests
where \( T_i \) is the temperature data at a sampling rate of \( f_{\text{acq}} \), \( n_{\text{acq}} \) is the number of picked-up samples and \( j^* \) is the value assumed by the \( j \)-index relevant to the second harmonic. The DFT was performed by using the dedicated fast Fourier transform (FFT) routine available in the MATLAB 2016 commercial software.

Concerning the cooling gradient approach (see the second line of Figure 1), \( Q \) was evaluated according to Equation 2, and the cooling gradient was evaluated considering the average temperature in the area \( A \), as shown in Figure 1.

3 | EXPERIMENTAL RESULTS

The hysteresis loops measured during the step-load fatigue tests are shown in Figure 3.\(^5\) Having the area of the hysteresis loops for each applied stress amplitude, the \( W_{\text{hyst}} - \sigma_a \) results are reported in Figure 4 and were fitted using Equation 5,\(^7\) which assumes a Masing-material behavior\(^8\):

\[
W_{\text{hyst}} = W_L + W_U - 2W_C \to \frac{1-n'}{1+n'} \cdot 4\sigma_a \left( \frac{\sigma_a}{K} \right)^{\frac{n}{n'}} 
\]

where \( W_L \), \( W_U \), and \( W_C \) were defined in Part I of the companion paper\(^1\) and are recalled in Figure 3A for ease of reading.

Figure 4 shows the resulting \( K' \) and \( n' \) parameters of the fitting procedure, which was conducted separately for the data, both clearly below and above the characteristic stress knee-point, \( \sigma_{A-1,\text{W_{hyst}}} \), respectively, as suggested in Ricotta et al.\(^5\) In Ricotta et al.,\(^5\) the correlation of \( \sigma_{A-1,\text{W_{hyst}}} \) was discussed with the material fatigue limit obtained by a short stair-case procedure. The \( n' \) and \( K' \) values fitted on the data reported in Figure 4 and the relevant \( \beta \) parameter, evaluated according to Equation 6\(^1\):

\[
\beta = 16.627 \cdot (n')^3 - 8.282 \cdot (n')^2 + 2.738 \cdot n' + 1.572 
\]

are listed in Table 1.
Figure 4 shows that for applied stress amplitudes higher than $\sigma_A, \sigma_{0,1}, \sigma_{0,2}$, the $W_{\text{hyst}}$ values were not stable for a given applied stress amplitude, but increased with the elapsed number of cycles, due to the material softening, as reported in the literature for the AISI 304L stainless steel.9–11 Strain hardening usually takes place after the softening phase owing to the austenitic-martensitic plastic-induced transformation, and it is revealed by a reduction of the strain amplitude.10,11 However, a limited number of cycles was spent at each stress amplitude ($N=10,000$ cycles); therefore, strain hardening was never detected according to the monitoring of the strain amplitude which was performed for all applied stress levels. In light of this experimental outcome, the hypothesis formed in the Part I paper1 was supported that plasticity-induced phase transformations did not take place.

To verify the validity of the Masing behavior assumption, first the origin of the measured hysteresis cycles was set at their lower tip and then the branches of the hysteresis loops were described by using the Ramberg–Osgood law, magnified by a factor of two:

![Figure 4](image-url)
\[ \Delta e^* = \frac{\Delta \sigma^*}{E} + 2 \left( \frac{\Delta \sigma^*}{2 \cdot K'} \right)^{1/n'} \]  

Figure 5 shows the comparison among the measured hysteresis cycles and Equation 7, which was evaluated with material parameters \( n' \) and \( K' \), reported in Table 1. It is seen that, although the Masing behavior is not fulfilled in the entire range of applied stress \( \Delta \sigma \); nevertheless, Equation 7 fits the apexes of the hysteresis loops with a good level of accuracy for both groups, that is, for \( \sigma_a < \sigma_{A_1,W_{hyst}} \) and for \( \sigma_a > \sigma_{A_1,W_{hyst}} \).

Figure 6 reports a characteristic example of the temperature versus time trends measured during the step-load fatigue tests of the P_11 specimen (Figure 6a), the details of the second harmonic of the temperature signal evaluated with Equation (4) (Figure 6b and d) and the cooling gradient evaluated according to Equation 2 (Figure 6C,E), measured for two load steps. Figure 6A shows that \( T_s \) achieved stabilization for all applied stress amplitudes and that the temperature increase with respect to the ambient temperature ranged from 10°C to 34°C, while the temperature oscillations due to the thermoelastic effect was on the order of tenth of a Celsius degree. Concerning the regularity of the distribution of both dissipation and the thermoelastic heat sources, careful analyses are reported in the works of Boulanger et al.\(^{12}\) and Giancane et al.\(^{13}\) Figure 6B,D shows that \( \Delta T_2 \) is of the order of thousandths and hundredths of a Celsius degree for \( \sigma_a = 300 \text{ MPa} \) and \( \sigma_a = 345 \text{ MPa} \), respectively, while the temperature drop measured during the cooling gradient is at least one order of magnitude higher in both cases (see Figure 6C,E). Therefore, the noise can affect the measurement of \( \Delta T_2 \) to a greater extent than the cooling gradient, making in some cases its evaluation impossible. In light of the temperature signals reported in Figure 6, a discussion concerning the accuracy of the second harmonic and the cooling gradient approaches will be reported in the relevant section.

Figure 7 shows the DFT results relevant to the temperature measured for the P_9 specimen, when the applied stress amplitude was equal to \( \sigma_a = 270 \text{ MPa} \) (Figure 7A) and \( \sigma_a = 330 \text{ MPa} \) (Figure 7B) and for the P_11 sample, in the case of \( \sigma_a = 300 \text{ MPa} \) (Figure 7C) and \( \sigma_a = 390 \text{ MPa} \) (Figure 7D). It is worth noting that in the \( x \)-axis of Figure 7, the \( j \) index was translated into discrete frequencies, \( f_j \), for ease of reading. The maximum value of the vertical axis was set to highlight the range of the second harmonic, \( \Delta T_2 \). As a consequence, the \( T_s \) value is out of scale. For both specimens, it is seen that when the applied stress amplitude is lower than \( \sigma_a = 270 \text{ MPa} \) (Figure 7A) and \( \sigma_a = 300 \text{ MPa} \) (Figure 7C), \( \Delta T_2 \) does not arise clearly, while for \( \sigma_a = 330 \text{ MPa} \) (Figure 7B) and \( \sigma_a = 390 \text{ MPa} \) (Figure 7D), a sharp peak of \( \Delta T_2 \) is evident. Moreover, Figure 7 shows that harmonics higher than the second are also present, as highlighted by the theoretical model presented in the companion paper\(^1\) and according to the experimental results reported in Bar et al.\(^{14,15}\)
Figure 8 compares the $Q$ parameter evaluated according to the second harmonic approach of Equation 1 and to the cooling gradient approach of Equation 2 and shows that they are in closer agreement at progressively higher applied stress amplitudes, since the peak relevant to the second harmonic becomes increasingly higher, as shown in Figure 7.

To verify the hypothesis that the disagreement observed between the two approaches (particularly for the lowest applied stress amplitudes) is due to the presence of noise, idealized conditions have been simulated by carrying out dedicated thermal FE analyses, where the second harmonic and the cooling gradient approaches were applied to the numerical temperatures, as explained in the next section.
FIGURE 7  DFT of the temperature signal measured in the case of P_9 specimen for (A) $\sigma_a = 270$ MPa and (B) $\sigma_a = 330$ MPa and in the case of P_11 for (C) $\sigma_a = 300$ MPa and for (D) $\sigma_a = 390$ MPa.

FIGURE 8  Step load-fatigue tests reanalyzed in terms of the $Q$ parameter calculated according to Equations 1 and 2.
3.1 | Steady-state FE analysis

Steady-state conditions imply that \( \dot{T} = 0 \) in Equation 8; moreover, the contribution of the thermoelastic heat source vanishes because it consists of a reversible exchange between mechanical and thermal energy,
which does not produce a net energy dissipation or absorption over one loading cycle,\textsuperscript{17–19} therefore \(Q_{\text{th}} = 0\). Since \(\dot{Q} = \dot{W}\), the plastic strain power density per cycle was calculated averaging \(\dot{W}(t)\) (see eq. 46 of Part I and Figure 3A) over one loading cycle, as follows:

\[
\dot{W} = \frac{1}{T} \int_0^T \dot{W}(t) \cdot dt
\]

\[
= \left[ 1 - n' \cdot \frac{4\sigma_a}{K} \cdot \left( 1 + \frac{2n'}{2n' - (1 - n')} \right) \right] \cdot f_L \cdot f_L
\]

\[
(10)
\]

According to Figure 3A, one can see that the plastic strain energy per cycle evaluated according to Equation 10 is equal to the area of the hysteresis loop \(W_{\text{hyst}}\) evaluated according to Equation 5, when the material experiences a purely elastic behavior during the unloading phase, that is, when \(W_C = 0\) in Figure 3A. Considering the \(n'\) values listed in Table 1, the difference between the plastic strain energy per cycle from Equations 5 and 10 ranges from approximately 0.0% for \(n' = 0.0646\) to 15.4% when \(n' = 0.354\).

The plastic strain energy per cycle resulting from Equation 10 is \(W = 0.135 \text{MJ}/(\text{m}^3 \text{ cycle})\) and \(W = 0.490 \text{MJ}/(\text{m}^3 \text{ cycle})\) for \(\sigma_a = 300 \text{ MPa}\) and \(\sigma_a = 360 \text{ MPa}\), respectively. Since the experimental tests were performed at \(f_L = 1 \text{ Hz}\) and recalling the hypothesis that \(Q_0 = \dot{W}\), the thermal loads were input in the FE model as \(\dot{W} = 0.135 \text{ MW/m}^3\) for \(\sigma_a = 300 \text{ MPa}\) and \(\dot{W} = 0.490 \text{ MW/m}^3\) for \(\sigma_a = 360 \text{ MPa}\).

The stabilized temperature obtained by the steady-state analyses was compared with the experimental results. Figure 10A,B report the comparison for \(\sigma_a = 300 \text{ MPa}\) and Figure 10C,D for \(\sigma_a = 360 \text{ MPa}\). It is seen that there is a very good agreement between the numerical and experimental results. It is worth noting that the temperature of the lower grip of the fatigue test machine is higher than the upper one due to its proximity to the hydraulic power pack. This is more evident in the case of \(\sigma_a = 300 \text{ MPa}\), due to the weaker effect of the material-self heating caused by plasticity.

Figure 10B,D show the influence of the adopted coefficient by natural convection, \(\alpha_{\text{conv}}\). By changing \(\alpha_{\text{conv}}\) from 10 to 6 \(\text{W/(m}^2 \text{ K)}\), there is a maximum variation of the stabilized temperature profile equal to 0.10% and

![Figure 10](https://example.com/figure10.png)

**Figure 10** Comparison between experimental and steady-state FE results, in the case of P_11 specimen for \(\sigma_a = 300 \text{ MPa}\) (A,B) and \(\sigma_a = 360 \text{ MPa}\) (C,B) [Colour figure can be viewed at wileyonlinelibrary.com]
0.39% for $\sigma_a = 300$ MPa and $\sigma_a = 360$ MPa, respectively. In light of this, it can be concluded that the $\alpha_{\text{conv}}$ values taken in a range consistent with standard laboratory conditions have a negligible influence on the numerical results. It is interesting to note that the hypothesis that all plastic strain power density $\dot{W}$ is fully converted into specific heat energy rate $\dot{Q}_p$ leads to a good agreement between experimental and numerical temperature distributions along the specimen’s axis reported in Figure 10B,D.

4.2 | Transient FE analyses

In the case of transient FE analyses, all the terms in Equation 8 are different from zero. The thermoelastic heat source was calculated according to Equation 9. The plastic strain power density, $\dot{W}(t)$, recalling that $\dot{Q}_p(t) = \dot{W}(t)$, was evaluated according to eq. (46) of Part I. However, in this paper, $\dot{W}(t)$ was shifted by a factor of $\frac{1}{2}$ to refer the plastic strain power density to the unstressed state $\sigma = 0$; in fact eq. (46) of Part I has as starting point $t = 0$ at the lower tip of the hysteresis loop. Figure 11 shows the input heat power adopted for the transient thermal analyses for $\sigma_a = 300$ MPa and $\sigma_a = 360$ MPa. The figure reports also the stress signal in a non-dimensional form to appreciate the timing between the stress cycle and the input heat power.

The results, obtained by adopting a time increment equivalent to a numerical sampling rate, $f_{\text{FE}} = 128$ Hz, are plotted in Figure 12A,B, where it is seen that temperature stabilization is achieved for $t > 100$ s. The FE results are compared with the relevant experimental data in the close-up view reported inside Figure 12A,B, and a satisfactory agreement can be appreciated, in that a maximum difference equal to $0.1^\circ$C between the numerical and stabilized temperatures $T_s$ is seen.

Next, the FFT algorithm was applied to the temperature data within the time window $\Delta t = 32$ s in Figure 12A,B. The results are reported in Figure 13, which highlights the range of the second harmonic $\Delta T_2 = 7.43 \times 10^{-3}\,^\circ$C and $\Delta T_2 = 3.53 \times 10^{-2}\,^\circ$C for $\sigma_a = 300$ MPa and $\sigma_a = 360$ MPa, respectively. Clearly, the sharp peaks corresponding to the harmonics are now much more visible compared to the experimental data reported in Figure 7, due to the absence of noise. From experiments, it was found $\Delta T_2 = 9.68 \times 10^{-3}\,^\circ$C and $\Delta T_2 = 2.22 \times 10^{-2}\,^\circ$C for $\sigma_a = 300$ MPa and $\sigma_a = 360$ MPa, respectively.

FIGURE 11  Input heat power for transient thermal analyses in the case of P_11 specimen for $\sigma_a = 300$ MPa and $\sigma_a = 360$ MPa

FIGURE 12  Temperature evolution evaluated by means of a transient thermal analysis in the case of P_11 specimen for (A) $\sigma_a = 300$ MPa and (B) $\sigma_a = 360$ MPa [Colour figure can be viewed at wileyonlinelibrary.com]
The $Q$ values calculated according to Equation 1 are listed in the second column of Table 2. Moreover, recalling that $Q_p = W$, the $Q_p$ values adopted as input in the FE analyses are also reported in Table 2. It is worth noting that $Q_{2\text{nd harmonic}}^\text{FE}$ from FE analyses reported in Table 2 has been estimated by using $\beta$ values found in the companion Part I paper, where the material diffusivity has been neglected. The applicability of such $\beta$ values is demonstrated by the excellent level of agreement between assigned ($Q_p$) and calculated ($Q_{2\text{nd harmonic}}$) heat generation per cycle. However, the applicability may not be guaranteed for materials having higher diffusivity than the stainless steel material adopted here.

Finally, the cooling gradient approach was applied to the numerical temperature trends reported in Figure 14. The cooling gradients were calculated at $t^* = 300$ s when the heat generation was suddenly stopped, and it was found $\hat{T}_{|t=t^*} = -3.43 \times 10^{-2} \degree C/s$ for $\sigma_a = 300$ MPa and $\hat{T}_{|t=t^*} = -1.12 \times 10^{-2} \degree C/s$ for $\sigma_a = 360$ MPa. Then, the relevant $Q$ values were calculated according to Equation 2 and reported in Table 2. The relevant experimental values are also reported in the table. Table 2 shows that (i) the $Q$ values evaluated according to the second harmonic and the cooling gradient approaches are in excellent agreement with the input values of the FE analysis (consequently, the second harmonic and cooling gradient approaches are consistent with one another); (ii) the cooling gradient approach applied to the experimental data is in a better agreement with the hypothesis that the plastic strain energy is completely converted into heat.

### Table 2

| $\sigma_a$ (MPa) | $Q_p$ (MJ/[m$^3$ cycle]) | $Q_{2\text{nd harmonic}}$ Equation 1 | $Q_{\text{cooling}}$ Equation 2 | $Q_{2\text{nd harmonic}}$ Equation 1 | $Q_{\text{cooling}}$ Equation 2 |
|----------------|---------------------|-------------------------------|---------------------|---------------------|---------------------|
| 300           | 0.135               | 0.134                         | 0.134               | 0.175               | 0.138               |
| $n' = 0.354$  |                     |                               |                     |                     |                     |
| $\beta = 2.245$ |                    |                               |                     |                     |                     |
| 360           | 0.490               | 0.488                         | 0.488               | 0.308               | 0.450               |
| $n' = 0.0646$ |                     |                               |                     |                     |                     |
| $\beta = 1.723$ |                    |                               |                     |                     |                     |

*From finite element analyses.

bFrom experiments.

The $Q$ values calculated according to Equation 1 are listed in the second column of Table 2. Moreover, recalling that $Q_p = W$, the $Q_p$ values adopted as input in the FE analyses are also reported in Table 2. It is worth noting that $Q_{2\text{nd harmonic}}^\text{FE}$ from FE analyses reported in Table 2 has been estimated by using $\beta$ values found in the companion Part I paper, where the material diffusivity has been neglected. The applicability of such $\beta$ values is demonstrated by the excellent level of agreement between assigned ($Q_p$) and calculated ($Q_{2\text{nd harmonic}}^\text{FE}$) heat generation per cycle. However, the applicability may not be guaranteed for materials having higher diffusivity than the stainless steel material adopted here.

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## | DISCUSSION

In this paper, the intrinsic dissipation per cycle $Q_p$ has been evaluated experimentally, starting from the second
harmonic of the temperature signal. It has also been evaluated in parallel by using the well-established cooling gradient approach proposed by Meneghetti.\(^2\) Some pros and cons of the two approaches are reported in the following.

The cooling gradient approach requires suddenly stopping the test machine. However, the machine takes some time to completely stop cycling (for example some tenths of a second); therefore, the material cooling starts slightly before the time \(t^*\) when the cooling gradient can actually be measured. This aspect may restrict the use of this approach when the time window to be adopted for the cooling gradient evaluation is on the same order of the time required by the machine to completely stop cycling. This situation typically occurs in presence of severe stress raisers, as discussed in\(^3,20\) and/or when the stabilized temperature \(T_s\) is close to room temperature, that is, when the heat power dissipated by the material due to plasticity is limited. Conversely, the second harmonic approach does not suffer these drawbacks because it is based on the measurement of the material temperature during a running fatigue test. However, the present paper shows that evaluating the intrinsic dissipation by means of the DFT of the temperature signal is not trivial, since the magnitude of \(\Delta T_2\) ranged from thousandths to hundredths of a Celsius degree. Therefore, an average value of \(W_{hyst}\) was considered for a given \(\sigma_{a}\) as described above. On the contrary, when material stabilization is not reached in terms of \(W\) along a fatigue test, the \(n^\prime\) evolution is required to evaluate \(\beta\). Conversely, the cooling gradient approach does not require any hypothesis regarding the elastic–plastic material behavior, because it relies solely on the density and the specific heat of the material. Moreover, strictly speaking the \(\beta\) parameter generally depends on thermal BCs,\(^1\) while the cooling gradient approach does not require any control of the thermal boundary conditions. However, it must be highlighted that for practical applications, \(\beta\) proved to be quite independent of the thermal boundary conditions. In fact, dedicated thermal FE analyses were carried out, and reduced variations of \(\beta\) were found, despite the significant variations of the convective heat transfer coefficient \(\alpha_{conv}\). More precisely, \(\beta\) was seen to vary by only 0.41% and 2.49% when \(\alpha_{conv}\) was increased from 10 W/(m\(^2\) K) to 100 W/(m\(^2\) K) and 1000 W/(m\(^2\) K), respectively.

The cooling gradient approach (Equation 2) enables one to estimate the specific heat loss per cycle regardless the mean stress and stress state of the fatigue loading cycle, as it relies solely on the energy balance of the first principle of thermodynamics. As opposed to the cooling gradient approach, the second harmonic approach requires that the mean stress dependence of the relationship between \(\Delta T_2\) and the specific heat loss per cycle is investigated.

Equation 1 requires the \(\beta\) parameter is evaluated, which depends on the assumed elastic–plastic material behavior. In this paper, the Ramberg–Osgood law and the Masing behavior have been assumed. Thanks to the reduced number of cycles spent at each step-load \((N = 10,000\) cycles\)), a limited cyclic evolution was observed for the tested material at each applied stress amplitude. Therefore, an average value of \(W_{hyst}\) was considered for a given \(\sigma_{a}\) as described above. On the contrary, when material stabilization is not reached in terms of \(W\) along a fatigue test, the \(n^\prime\) evolution is required to evaluate \(\beta\). Conversely, the cooling gradient approach does not require any hypothesis regarding the elastic–plastic material behavior, because it relies solely on the density and the specific heat of the material. Moreover, strictly speaking the \(\beta\) parameter generally depends on thermal BCs,\(^1\) while the cooling gradient approach does not require any control of the thermal boundary conditions. However, it must be highlighted that for practical applications, \(\beta\) proved to be quite independent of the thermal boundary conditions. In fact, dedicated thermal FE analyses were carried out, and reduced variations of \(\beta\) were found, despite the significant variations of the convective heat transfer coefficient \(\alpha_{conv}\). More precisely, \(\beta\) was seen to vary by only 0.41% and 2.49% when \(\alpha_{conv}\) was increased from 10 W/(m\(^2\) K) to 100 W/(m\(^2\) K) and 1000 W/(m\(^2\) K), respectively.

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6 | CONCLUSIONS

The intrinsic dissipation per cycle (the $Q_p$ parameter) was evaluated by means of the range of the second harmonic, $\Delta T_2$, of the temperature signal measured during a fatigue test (the second harmonic approach). The relationship between $\Delta T_2$ and $Q_p$ was derived analytically and discussed in the companion paper. Step-load fatigue tests were performed on AISI 304L cold-drawn bars and the material temperature was acquired by using an infrared camera. Utilizing the DFT, the temperature signal was processed, and the $Q_p$ values were calculated according to the analytical model. In parallel, $Q_p$ was also measured during the same tests by using the well-established experimental approach proposed in the past by one of the authors, based on the measurement of the cooling gradient of the material temperature after having suddenly stopped the fatigue test (the cooling gradient approach).

Considering the material and test conditions analyzed in the present paper, it was determined that the results obtained by following the two approaches become increasingly closer for progressively higher applied stress amplitudes. In fact, the higher the applied stress is, the higher the $\Delta T_2$ values are. As a consequence, the signal to noise ratio becomes increasingly favorable.

To verify the hypothesis that the disagreement observed between the two approaches is due to noise, virtual experiments were performed by means of dedicated thermal FE analyses, where the second harmonic and the cooling gradient approaches were applied to the numerical temperatures. In the noise-free virtual experiments, excellent agreement was observed between the second harmonic and the cooling gradient approaches.

DATA AVAILABILITY STATEMENT

The data that support the findings of this study are available from the corresponding author upon reasonable request.

NOMENCLATURE

- $A_j, B_j$: coefficients of the discrete Fourier transform of the temperature signal (K)
- $E$: material elastic modulus (MPa)
- $f_{acq}$: sampling rate of the infrared camera (Hz)
- $f_{FE}$: sampling rate adopted in finite element analyses (Hz)
- $f_L$: load test frequency (Hz)
- $K'$: cyclic material strength coefficient (MPa)
- $n'$: cyclic material strain hardening exponent
- $n_{acq}$: number of frames acquired by the infrared camera
- $n_{FE}$: number of picked-up temperature samples in finite element analyses
- $\dot{Q}$: heat energy rate exchanged by a unit volume of material per cycle, by conduction, convection, and radiation ($W/m^3$)
- $\dot{Q}$: heat energy exchanged by a unit volume of material per cycle, by conduction, convection and radiation ($J/m^3\cdot cycle$)
- $Q_p$: intrinsic dissipation per unit volume ($J/m^3$)
- $Q_{p, av}$: intrinsic dissipation per unit volume of material per cycle ($Q_p$ averaged over one loading cycle) ($J/m^3\cdot cycle$)
- $\dot{Q}_{the}$: rate of thermoelastic energy per unit volume ($W/m^3$)
- $R$: nominal stress ratio (ratio between the minimum and the maximum applied nominal stress)
- $t^*$: time at which the fatigue test is suddenly stopped (s)
- $T$: material temperature (K)
- $T_s$: material temperature averaged over one loading cycle (K)
- $\bar{T}_{s, FE}$: $T_s$ evaluated from the finite element analyses (K)
- $\bar{T}_{s, exp}$: $T_s$ evaluated from experiments (K)
- $W$: plastic strain energy density ($J/m^3$)
- $\dot{W}$: plastic strain energy density per cycle ($J/m^3\cdot cycle$)
- $\bar{W}$: plastic strain power density per cycle ($W$ averaged over one loading cycle) ($W/m^3$)
- $\alpha$: isotropic coefficient of thermal expansion ($K^{-1}$)
- $\alpha_{conv}$: coefficient of convective heat transfer ($W/m^2\cdot K$)
- $\beta$: parameter correlating the intrinsic dissipation and the second harmonic of the temperature signal ($/\cdot$)
- $\kappa$: material emissivity
- $\lambda$: material thermal conductivity ($W/m\cdot K$)
- $\rho$: material density ($kg/m^3$)
- $\sigma(t)$, $\sigma_a$: applied stress function, its amplitude (MPa)
- $\sigma_{A_{n-1}}$: characteristic stress knee-point (MPa)
- $\sigma_{A_{n-1}}$: material fatigue limit evaluated by short stair case procedure (MPa)
- $\Delta T_2$: range of the second harmonic of the discrete Fourier transform of the temperature signal (K)
- $\Delta\varepsilon^{*}(t)$: strain function, evaluated starting from the lower tip of the hysteresis cycle ($m/m$)
Δσ*(t) applied stress function, evaluated starting from the lower tip of the hysteresis cycle (MPa)

ORCID
Giovanni Meneghetti https://orcid.org/0000-0002-4212-2618
Mauro Ricotta https://orcid.org/0000-0002-3517-9464

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