Broadband field enhancement and giant nonlinear effects in terminated unidirectional plasmonic waveguides

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Unidirectional wave propagation in nonreciprocal structures enables exciting opportunities to control and enhance wave-matter interactions in extreme ways. Within this context, here we investigate the possibility of using terminated unidirectional plasmonic waveguides to enhance typically weak nonlinear effects by orders of magnitude. We theoretically demonstrate remarkable levels of electric field enhancement and confinement (field hot-spots) when the unidirectional waveguiding structure is terminated with a suitable boundary that fully stops the one-way mode. Such a large field enhancement, originating from a nonresonant effect, is fundamentally different from the narrow-band field concentration effects in resonant plasmonic structures. Instead, it is analogous to the broadband response of plasmonic tapers, but without the need for any adiabatic impedance matching. We show that this effect can indeed lead to a substantial boosting of nonlinear light-matter interactions, exemplified by an improvement of several orders of magnitude in the third-harmonic-generation efficiency, which is of large significance for several applications. More broadly, our findings show the potential of extreme nonreciprocal configurations for enhanced wave-matter interactions.

I. INTRODUCTION

Nonlinear light-matter interactions are at the basis of a large variety of classical and quantum optical devices, and are used by scientists and engineers to generate new light frequencies, perform laser diagnostics, and advance quantum computing, among many other applications [1]. Nonlinear effects depend on the powers of the local electric field $E$, with the nonlinear polarization density, in time domain, given by: $P(t) = \varepsilon_0 (\chi(E(t)) + \chi^{(2)}E^2(t) + \chi^{(3)}E^3(t) + \ldots)$, where $\chi$ is the linear susceptibility and $\chi^{(n)}$ the nonlinear susceptibilities. Since $\chi^{(n)} \ll \chi$ in natural materials, extremely high incident light intensities, enhanced local fields in narrow-band cavities, or long propagation distances in bulky nonlinear crystals are required to produce detectable optical nonlinear effects. The realization of compact broadband nonlinear devices (wavelength-scale or smaller) is indeed a fundamental challenge in modern photonics and nano-optics.

Two general strategies are typically used to locally enhance electromagnetic fields: (i) localized resonances and (ii) slow-light effects accompanied by adiabatic impedance matching. Resonances, for example localized surface plasmon resonances in plasmonic nanostructures and metasurfaces, or more complex Fano resonances and bound states in the continuum, can dramatically increase the local field, producing field hot-spots that may be used to boost linear and nonlinear effects [2–12]. However, the resonant nature of these platforms makes them very sensitive to dissipative processes and, more importantly, reduces the bandwidth over which the desired effect is obtained. An alternative approach is to use elongated plasmonic tapers [13–15], which support surface plasmon-polariton (SPP) modes with decreasing wavelength and group velocity as they propagate toward the taper tip. For long adiabatic tapers, the energy carried by the SPP tends to accumulate at the tip, producing an intense field hot-spot. Despite the change in geometry seen by the propagating SPP, resulting in a change in wave impedance, reflections are minimized over a broad bandwidth (broadband impedance matching) due to the adiabatic transition. In this way, a broadband signal can be focused at the tip of a plasmonic taper, producing a ultra-large field hot-spot without the need for a local resonance. However, this effect is possible only if impedance matching is ensured at any distance from the tip, which requires very long adiabatically tapered structures.

Here, we propose a novel strategy to fundamentally break these conventional trade-offs between field enhancement, bandwidth, and size – with the final goal of realizing boosted nonlinear effects – based on exploiting the extreme response of nonreciprocal plasmonic platforms supporting inherently unidirectional modes. Nonreciprocal plasmas and plasmonic materials, obtained by breaking Lorentz reciprocity via an external magnetic field (gyrotropic materials), have been the subject of extensive research for decades. A distinctive effect of nonreciprocity in this context is the existence of frequency ranges where surface plasmon-polaritons are unidirectional in the plane orthogonal to the bias (Voigt configuration). In other words, the SPP dispersion diagram is markedly asymmetric and, in certain frequency windows, SPPs are allowed to propagate along a certain direction but not in the opposite direction. Although this
effect has been known for several decades [16,18], only recently it has been shown that unidirectional SPPs on nonreciprocal plasmonic structures can be divided in two classes with distinct topological properties: (i) Topological SPPs, whose unidirectionality is an intrinsic property arising from the topological nature of the bulk modes [19–28]. These SPPs are supported by interfaces between a biased plasmonic material and an opaque medium (e.g., a metal) and exist within the upper bulk-mode bandgap. (ii) Unidirectional surface magnetoo-plasmons, which are supported by interfaces between a biased plasmonic material and a transparent medium, and exist within the lower bulk-mode bandgap. The unidirectionality of these SPPs is a manifestation of strong nonreciprocity and is not a topological property [31,32]. Indeed, it has been recently recognized that these SPPs lose their strict unidirectionality if nonlocal effects are properly included in the material model [31,33], while they remain strongly asymmetrical.

Unidirectional SPPs of both classes can be used to achieve significant field enhancements in configurations where the unidirectional mode is fully stopped at a suitable termination. Since the SPP cannot reflect back (and radiation is forbidden), its energy accumulates at the termination, forming an intense field hot-spot, and is eventually dissipated in the form of heat. Terminated unidirectional wave-guiding structures – and their counterintuitive electromagnetic response – were originally studied by Barzilai and Ishimaru, among others, in the 1960s [17,35], and are now the subject of significant research interest [31,36–38]. Despite the huge potential of such nonreciprocity-induced hot-spots for boosting weak nonlinear effects, all studies on this topic so far have been focused on the linear response of terminated one-way channels, whereas, to the best of our knowledge, no attention has been devoted to their interactions with material nonlinearities. In the following, we propose and discuss engineered nonreciprocal plasmonic platforms to maximally enhance the field intensity at a suitable termination, while fully taking into account, for the first time, the unavoidable impact of dissipation and nonlocality. We then show that this effect can indeed lead to an improvement of several orders of magnitude in non-linear light-matter interactions, exemplified by a giant enhancement in the efficiency of third-harmonic generation. These findings may open new directions in nonlinear electromagnetics and photonics.

II. GIANT FIELD ENHANCEMENT IN TERMINATED ONE-WAY CHANNELS

To gain more physical insight into the response of terminated one-way channels, we first consider an ideal configuration that is amenable to theoretical analysis. As shown in Fig. 1(a), the structure under consideration consists of a nonreciprocal (magnetized) plasma bounded by dual hard boundaries, i.e., a perfect electric conductor (PEC) and a perfect magnetic conductor (PMC). The plasma region is biased normal to the plane, along the z-axis, as indicated in the figure. As discussed in [16,17,25,27,34], if the operational frequency lies within the upper bulk-mode bandgap, the interface between magnetized plasma and PEC (or an opaque medium) supports a unidirectional and topological SPP mode propagating toward right or left, depending on the bias direction. This one-way propagation channel is terminated by a PMC boundary, such that a corner of arbitrary angle φ is formed between the PEC and PMC walls, as illustrated in Fig. 1(a). No surface mode is supported on the PMC-plasma interface, since the PMC boundary “shorts” the tangential magnetic field of the transverse-magnetic (TM) SPP mode, as recognized in [17] (this is strictly true only in the local case; if plasma nonlocalities are considered, an extremely confined surface mode does exist on interface, but is very rapidly attenuated by any physical level of dissipation, as discussed in [39]). The supported one-way surface mode, therefore, cannot “escape” the termination since all other propagation channels – backward propagation, radiation into the bulk, and surface-wave propagation on the PMC interface – are forbidden. As a result, the energy carried by this mode can only accumulate at the corner, leading to a dramatic field enhancement. Indeed, the only escape channel is provided by absorption losses, which ultimately dissipate all the incident energy even in the limit of vanishing loss [17]. If the loss rate is not too large, the field intensity is expected to exhibit a sharp peak (a field hot-spot) near the corner.

To better understand this field enhancement mechanism, we theoretically analyze the behavior of the surface mode as it approaches the corner. This analysis concerns TM modes, i.e., with $E_\parallel = 0$, and time-harmonic dependence $e^{-i\omega t}$. The magnetized plasmonic medium can be modeled by a non-symmetric permittivity tensor $\epsilon = \epsilon_0 [\epsilon_{11} \mathbf{I}_3 + \epsilon_{33} \mathbf{z}\mathbf{z}^T - i \epsilon_{13} \mathbf{z} \times \mathbf{I}_3]$, where the z-axis (bias direction) is supposed to be normal to the plane of propagation, $\mathbf{I}_3 = \mathbf{I} - \mathbf{z}\mathbf{z}^T$, and $\epsilon_{12}$ is the magnitude of the gyration pseudovector. The frequency dispersion function of $\epsilon_{11}$, $\epsilon_{12}$ and $\epsilon_{33}$ can be found in, e.g., [40]. As a relevant example of solid-state magnetized plasma, we consider a magnetized semiconductor in the local THz regime, e.g., n-type InSb with plasma frequency $\omega_p = 2$ THz, electron density $N_e = 1.1 \times 10^{22}/\text{m}^3$, and dielectric constant due to bound charges $\epsilon_\infty = 15.6$. By expanding Maxwell’s equations, $\nabla \times \mathbf{E} = i\omega \mu_0 \mathbf{H}$, $\nabla \times \mathbf{H} = -i\omega \epsilon_\infty \mathbf{E}$ in cylindrical coordinates, it can be shown that the most general solution of the magnetic field $H_z$ takes the following form,

$$H_z = (a_n J_n(k_n \rho) + b_n Y_n(k_n \rho)) (a_n \cos(n\phi) + b_n \sin(n\phi)),$$

where $a_n$, $b_n$, $c_n$, $d_n$ are unknown modal constants, and $J_n$ and $Y_n$ are solutions of the Bessel equation of order $n$. The geometry contains the termination point $\rho = 0$, and the PMC and PEC boundaries are located at $\phi = 0$ and $\phi = \phi_0$, respectively. The dispersion equation of the supported surface mode can be found by applying these
boundary conditions (see also Appendix A), which gives,

\[ \frac{nJ_n(k_s \rho)}{k_s \rho} + i \frac{\epsilon_{12}}{\epsilon_{11}} J'_{n}(k_s \rho) \tan(n \phi_0) = 0 \]  \hspace{1cm} (2)

where \( k_s = \sqrt{\epsilon_{eff}} \omega / c \) and \( \epsilon_{eff} = (\epsilon_{11}^2 - \epsilon_{12}^2) / \epsilon_{11} \), with \( c \) being the speed of light in vacuum.

The allowed modal index \( n \) that satisfies the dispersion equation depends on frequency and on the distance, \( \rho \), from the corner. In other words, the surface mode transforms as it approaches the corner, and its amplitude, wavelength, and wave impedance depend on the allowed modal index \( n \) at each distance \( \rho \). Although Eq. (2) is a complex, nonlinear equation and should be solved numerically, it can be simplified in the region very close to the the termination, \( \rho \to 0 \), by replacing the Bessel function with its small argument approximation, \( J_n(x) \approx \frac{1}{\pi} \left( \frac{x}{2} \right)^n \). This converts Eq. (2) to \( \tan(n \phi_0) - i \epsilon_{12} / \epsilon_{11} = 0 \), which matches the result reported in [17]. The new dispersion equation is independent of \( \rho \) since it is valid only in the extreme vicinity of the corner, and it can be solved as \( n = i \tanh^{-1}(\epsilon_{11} / \epsilon_{12}) / \phi_0 \), which reveals that \( n \) is purely imaginary in the lossless case (since \( \epsilon_{11} \) and \( \epsilon_{12} \) are real).

The above assumption also simplifies the expression of the SPP electric field as follows,

\[ E_\phi \approx \frac{c_n}{-i \omega \epsilon_0 \epsilon_{eff}} n \rho^{n-1} \left[ \frac{\epsilon_{12}}{\epsilon_{11}} \cos(n \phi) - \sin(n \phi) \right] \]  \hspace{1cm} (3)

In a lossy structure, the modal index becomes a complex number, \( n = n_r + i n_i \), with \( n_r > 0 \) in a passive system. In this case, Eq. (3) shows that, as the surface wave approaches the corner, it oscillates as \( \rho^{n_r} \) and its amplitude increases/decreases as \( 1/\rho^{1-n_r} \), depending on the value of \( n_r \), which in turn depends on the level of loss. Specifically, if \( 0 \leq n_r < 1 \) the amplitude diverges at \( \rho = 0 \), whereas if \( n_r > 1 \) it decays to zero. Thus, this simplified analysis allows making qualitative predictions about the underdamped or overdamped behavior of surface waves in a terminated one-way channel with loss; however, the approximated dispersion equation and the associated field expressions do not fully capture the correct physics of a surface mode in this configuration. Specifically, the divergent behavior at \( \rho = 0 \) for a lossy structure is nonphysical since it would correspond to infinite absorbed energy. Instead, we expect the presence of a peak in the field amplitude near the termination, whose maximum value and location should depend on the level of loss and the geometry.

FIG. 1: Giant field enhancement in an idealized terminated one-way channel. (a) The geometry under consideration consists of a magnetized plasmonic wedge of angle \( \phi_0 \) between perfect-electric-conducting (PEC) and perfect-magnetic-conducting (PMC) walls. (b) Time-snapshot of the normal component of the electric field at the plasma-PEC boundary, as a function of distance \( \rho \) from the corner (corresponding to the dashed box in the inset). Inset: Zoomed-in view of the electric field distribution in the plasmonic wedge, launched by a point source (black arrow). The wedge angle is \( \phi_0 = 5 \) degrees, the operational frequency is \( \omega / \omega_p = 1.05 \), and the magnetized plasma is InSb with parameters given in the text, cyclotron frequency \( \Gamma / \omega_p = 0.01 \). (c) Corresponding distribution of the electric field magnitude. (d) Same as panel (c), but for a right-angle wedge with \( \phi_0 = 90 \) degrees. All the field values are normalized to \( |E_0| \), the magnitude of the electric field of a surface wave propagating along an unbounded, lossless, plasma-PEC interface.
wave propagation is supported, which can be quite wide entire frequency window in which unidirectional surface termination, such a field enhancement effect occurs over the case – forming a clear field hot-spot as shown in the inset. For a 90 degrees corner. The field distribution for this extreme scenario is shown in Fig. 1(d). Again, we observe that the field intensity increases dramatically close to the termination – to even higher values than in the tapered case – forming a clear field hot-spot as shown in the inset. We also note that, despite the abrupt, non-adiabatic termination, such a field enhancement effect occurs over the entire frequency window in which unidirectional surface wave propagation is supported, which can be quite wide depending on the magnetic bias intensity 18, 33, 34.

III. BROADBAND ENHANCED NONLINEAR EFFECTS

The broadband giant hot-spots supported by terminated one-way channels appear ideal to enhance weak light-matter interactions, especially nonlinear effects since they depend on the powers of the local field. However, the idealized configuration in Fig. 1, which uses PEC and PMC boundaries, is not practical, especially at frequencies above the microwave range. Fortunately, as mentioned in the Introduction, unidirectional surface waves known as surface magneto-plasmons also exist on an interface between a magnetized plasma and a dielectric material, for frequencies below the plasma frequency 15, 31, 33, 34. An important advantage of these surface waves is that there is no need to impose impractical PMC boundaries to create a termination. In fact, surface magneto-plasmons can be stopped by a PEC wall or by an interface with a conventional opaque medium, as discussed in 31, 33, 36. A disadvantage of this configuration is that a backward mode may be excited if nonlocal effects are included in the material model, and the impact of this additional propagation channel must be assessed carefully, as discussed below.

We first consider a single interface between biased InSb and silicon (Si) with relative permittivity ε = 11.68 [32]. The SPP dispersion diagram for this structure is shown in Fig. 2(a). The unidirectional frequency window of surface magneto-plasmons, indicated by the shaded white area in the figure, is defined by the following upper and lower bounds: ω± = (±ωc + √(2ωp + ωc2))/2 [32, 41]. When ωc = 0, the unidirectional frequency window closes and the interface supports symmetric and bidirectional SPPs.

As mentioned above, unidirectional surface magneto-plasmons can be stopped using a conventional opaque medium, for example an isotropic metallic wall. However, in this configuration, the termination would also scatter energy into the transparent dielectric, which would limit the field enhancement at the termination. To reduce and control this radiation channel, we consider a modified configuration in which the silicon layer is sandwiched between two oppositely magnetized plasmonic materials (alternative configurations are also possible, with, for example, the silicon layer sandwiched between the magnetized plasma and the same metal used for the termination, similar to the structure in 33). The SPP dispersion diagram for this new configuration is shown in Fig. 2(b), which reveals that two one-way surface modes now exist within the unidirectional frequency window. These two bands originate from the coupling between the SPPs supported by the individual plasma-silicon interfaces. While large radiation loss is to be avoided in order to maximize the field enhancement at the termination, the structure should not be completely closed. Indeed, it would be beneficial, and more practical, if these unidirectional surface modes could be excited by an external propagating wave, e.g., a laser beam, rather than by localized sources inside the structure. Fortunately, an inspection of the dispersion diagram in Fig. 2(b) reveals that the surface mode dispersion overlaps with the light cone for plane waves propagating in free-space. Thus, if one of the plasma layers is sufficiently thin and interfaced with free space, as shown in Fig. 3(a), the unidirectional surface magneto-plasmon becomes a leaky wave that can be excited by, and coupled to, free-space propagating waves [29, 30]. The radiation leakage can be controlled by the thickness of the plasma layer.

The opaque termination that is introduced to stop the excited surface magneto-plasmons is indicated by the black region in Fig. 3(a). To boost the field enhancement further, a resonant termination may be designed, for example in the form of a non-magnetized plas-
is rapidly attenuated for moderate levels of dissipation. Propagating mode exists for large wavenumber values and the dispersion remains strongly asymmetrical, as the backward-propagation channel deviates from the unidirectional frequency window. While this makes the system no longer strictly unidirectional, the dispersion diagram remains strongly asymmetrical, as the backward-propagating mode exists for large wavenumber values and is rapidly attenuated for moderate levels of dissipation. Thus, the relative impact of dissipation and nonlocal effects should be carefully assessed in order to make correct predictions regarding the maximum field intensity at the termination. Nonlocal effects can be included in the InSb material model by writing Ampere’s Law as \( \nabla \times \mathbf{H} = -i\omega \epsilon_0 \epsilon_{\infty} \mathbf{E} + \mathbf{J} \), where \( \mathbf{J} \) is the induced free-electron current governed by an hydrodynamic equation of motion \[ \frac{\beta^2}{\omega^2} \nabla (\nabla \cdot \mathbf{J}) + \omega (\omega + i\Gamma) \mathbf{J} = i\omega (\omega^2 \epsilon_0 \epsilon_{\infty} \mathbf{E} - \mathbf{J} \times \hat{z}) \] (4) where \( \beta = 1.07 \times 10^6 \text{ m/s} \) is the nonlocal parameter and \( \Gamma \) is the damping rate due to absorption losses. The first term of the equation is a pressure term determining the convective currents that are responsible for nonlocal effects. The silicon layer is instead assumed to be local, as usually done for dielectric materials. The field distribution everywhere can then be calculated numerically using the finite element method, solving Maxwell’s equation and the hydrodynamic equation simultaneously, with suitable boundary conditions. In particular, neglecting electron spill-over, the free-electron current normal to the surface is required to vanish at the plasma-dielectric interfaces.

To understand how the presence of dissipation and nonlocality affects the field hot-spots, we investigate the field enhancement at the termination for different scenarios. Specifically, Fig. 3(c) compares the field enhancement for reciprocal/nonreciprocal and local/nonlocal InSb as a function of the loss rate, \( \Gamma \), in the magnetized plasmonic material, at a frequency within the unidirectional frequency window, \( \omega/\omega_p = 0.7 \). These results confirm that the field enhancement achievable in the nonreciprocal structure is an order of magnitude larger than in the reciprocal case (\( \omega_c = 0 \)), even for large values of dissipation. In the reciprocal (nonbiased) configuration, a simple standing wave forms along the channel, which only produces a moderate field enhancement. Most importantly, we see that nonlocal effects do reduce the enhancement due to the emergence of a new backward-propagation channel; however, the surface mode dispersion remains strongly asymmetrical and only SPPs with very large wavenumbers can escape through this channel. As a result, despite the presence of spatial dispersion, the field enhancement remains an order of magnitude larger than in the reciprocal case, even in the structures with substantial optical losses. In Fig. 3(d), we also show this enhancement over a wide range of frequencies, for both the reciprocal and nonreciprocal nonlocal cases. These results demonstrate that, in sharp contrast to the conventional enhancement methods based on resonances, a very large electric field intensity is achievable here over the entire unidirectional frequency window, and especially at frequencies for which the surface mode is overlapped to the free-space light cone (see Fig. 2(b)). The oscillations in Fig. 3(d) are mostly due to the dispersive nature of the involved materials. The physical reason for this broadband effect is that no resonance is required in the proposed process (although a resonant termination may further increase the field enhancement).

FIG. 2: Dispersion diagrams of the surface modes supported by the configurations in the insets. The dispersion diagrams are plotted as density plots of the inverse determinant of the boundary-condition matrix. The bright bands correspond to the SPP poles. The magnetized plasma is InSb with parameters given in the text, and cyclotron frequency \( \omega_c \) is set to 0.4. The green dashed lines represent the light lines for plane waves propagating in free space. The shaded white area indicates the frequency window where unidirectional surface modes are supported.
This behavior is clearly ideal to boost light-matter interactions and nonlinear effects in a broadband fashion. Next, we demonstrate the potential of these ideas, for a specific nonlinear process, by investigating the third-harmonic generation (THG) efficiency in the proposed waveguiding platform, considering the natural $\chi^{(3)}$ nonlinear properties of silicon.

In materials with non-negligible third-order nonlinear susceptibility, a third-harmonic (TH) wave is generated by the nonlinear polarization density $P_{TH} = \varepsilon_0 \chi^{(3)} E^3_{FF}$, where $E_{FF}$ is the electric field at the fundamental frequency, equal to the frequency of the Gaussian beam illuminating the structure. The nonlinear polarization is included as a new nonlinear source term in the wave equation describing the silicon region $\nabla \times \nabla \times \mathbf{E} - \frac{n_d^2}{\varepsilon_0} \frac{\partial^2 \mathbf{E}}{\partial t^2} = -\frac{1}{\varepsilon_0} \frac{\partial^2 P_{TH}}{\partial t^2}$. Silicon is an indirect bandgap semiconductor with a pronounced third-order nonlinear susceptibility on the order of $\chi^{(3)} = 2.8 \times 10^{-18}$ m$^2$/V$^2$. Instead, the nonlinear properties of the remaining materials are neglected since the nonlinear susceptibility of InSb is much lower compared to silicon. In addition to these nonlinear effects in silicon, we still fully include nonlocalities and absorption losses in the InSb regions, making the system under study nonlinear, nonlocal, nonreciprocal, dissipative, and dispersive.

The strength of the THG process is evaluated by computing the conversion efficiency defined as $CE = P_{out,TH}/P_{in,FF}$, that is, as the ratio between the output power at the third-harmonic frequency and the input power at the fundamental frequency. The input power of the incident Gaussian beam can be calculated as $P_{in,FF} = 0.5 \pi I_0 w_0^2 \cos(\theta_0)$, where $I_0 = H_0^2 \eta/2$ is the maximum beam intensity with $H_0$ being the magnitude of the incident magnetic field and $\eta = 377$ $\Omega$ the free space impedance. The total output power is computed by integrating the outgoing Poynting vector on the outer boundaries of the entire computational domain at the third-harmonic frequency. While the structure supports guided unidirectional modes at the fundamental frequency, the system operating at the third harmonic has different properties due to the dispersive nature of the materials involved. In particular, since the third-harmonic frequency is much larger than the plasma frequency of the nonreciprocal plasmonic materials, these media are mostly transparent to the third-harmonic field, which is therefore able to exit the structure, while the fundamental-frequency field is trapped in the waveguiding structure.

FIG. 3: Broadband field enhancement, and impact of dissipation and nonlocality. (a) Illustration of the terminated one-way waveguiding geometry under consideration, where a coupling region exists to allow the incident wave to excite the one-way surface modes. The termination region (black) is composed of an isotropic metal with $\varepsilon_m = -\varepsilon_d$, at $\omega/\omega_p = 0.7$. (b) Distribution of the electric field magnitude for the geometry in panel (a), with the same InSb parameters as in Fig. 1 at a frequency $\omega/\omega_p = 0.7$. The Si layer and the top InSb layer have thickness $d = 0.1\lambda_0$ and $t = 0.03\lambda_0$, respectively, where $\lambda_0$ is the free space wavelength of the incident wave. (c) Field enhancement at the termination for different scenarios (reciprocal/nonreciprocal and local/nonlocal) as a function of the loss rate, $\Gamma$, at a fixed frequency, $\omega/\omega_p = 0.7$. For the nonlocal cases, $\beta = 1.07 \times 10^6$ m/s. (d) Field enhancement as a function of frequency, for a fixed level of loss, $\Gamma/\omega_p = 0.03$. Vertical dashed lines indicate the unidirectional frequency window, consistent with Fig. 2. The field is normalized at each frequency with respect to the electric field $|E_0|$ of a surface wave propagating along a low loss ($\Gamma/\omega_p = 0.01$), local, non-terminated biased structure. For all panels, the structure is illuminated by a Gaussian beam with waist $w_0 = 100$ $\mu$m and incident angle of $\theta = 45$ degrees with respect to the interface.
The computed THG CE results are reported in Fig. 4(a) for the proposed terminated one-way waveguiding structure. Rather remarkably, the THG conversion efficiency takes very high values, in the order of few percent, by using relatively low input intensities. This is indeed due to the giant field enhancement and confinement at the termination for the fundamental frequency. In comparison, the CE is five orders of magnitude lower in the nonreciprocal case without termination, and in the reciprocal cases with or without termination, as seen in Fig. 3(b). We also note that input intensity values comparable to the ones considered here have been experimentally obtained at infrared frequencies by using different excitation configurations emitting terahertz pulses and quantum cascade lasers. The low input intensity values used here also ensure that we operate within the undepleted pump regime. Even for much lower input intensities, our results show a huge enhancement in conversion efficiency compared to the reciprocal case. Finally, we stress that much larger TH fields can be generated, with similar efficiencies, using arrays of terminated one-way channels forming large-scale interfaces, as shown in the Supplementary Material.

The computed field distribution at the third-harmonic frequency is shown in Fig. 4(c) for the same one-way terminated waveguide. At this frequency, the waveguiding structure becomes asymmetrically bidirectional and the TH wave is allowed to propagate backward from the termination, and escape the structure in the form of radiation from both sides. The field distributions for all the different configurations considered in Fig. 4(b) – reciprocal/nonreciprocal and with/without termination – are provided in Supplementary Material. Most importantly, Fig. 4(d) shows the THG conversion efficiency over a wide frequency range, for the terminated reciprocal and nonreciprocal waveguides, demonstrating a broadband orders-of-magnitude improvement in the nonreciprocal case. A comparison with Fig. 3(d) shows that such a broadband nonlinear effect indeed originates from the broadband field enhancement in our structure.

These results clearly demonstrate the potential of the proposed terminated one-way plasmonic waveguides to achieve extreme levels of field enhancement and giant nonlinear effects over a broad range of frequencies.

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**Appendix A: SPP modal analysis of a terminated one-way channel**

Consider the geometry in Fig. 1(a), with the nonreciprocal plasmonic material (magnetized n-type InSb) characterized by a gyrotropic permittivity tensor. Starting from Maxwell’s equation in cylindrical coordinates, we obtain the following equations for the different components of the supported transverse magnetic (TM) mode at the InSb-PEC interface

\[
\frac{1}{\rho} \left[ \frac{\partial (\rho E_\rho)}{\partial \rho} - \frac{\partial E_\phi}{\partial \phi} \right] = i \omega \mu_0 H_z
\]

\[
\frac{1}{\rho} \frac{\partial H_z}{\partial \phi} = -i \omega \epsilon_0 \left[\epsilon_1 E_\rho + i \epsilon_2 E_\phi\right]
\]

\[
- \frac{\partial H_z}{\partial \rho} = -i \omega \epsilon_0 \left[-i \epsilon_2 E_\rho + \epsilon_1 E_\phi\right]. \quad (A1)
\]

Using these equations, \( H_z \) can be obtained in terms of the other field components,

\[
\frac{1}{\rho} \frac{\partial^2 H_z}{\partial \phi^2} + \frac{\partial}{\partial \rho} \left[\rho \frac{\partial H_z}{\partial \rho}\right] =
\]

\[
- i \omega \epsilon_0 \left\{ \epsilon_1 \left(\frac{\partial E_\rho}{\partial \phi} - \frac{\partial (\rho E_\phi)}{\partial \rho}\right) + i \epsilon_2 \left(\frac{\partial E_\phi}{\partial \phi} + \frac{\partial (\rho E_\rho)}{\partial \rho}\right) \right\}. \quad (A2)
\]

The transversality condition (Gauss Law), \( \nabla \cdot D = \)
FIG. 4: Third-harmonic-generation (THG) conversion efficiency for (a) the nonreciprocal plasmonic waveguide with metallic termination considered in Fig. 3 (red solid line); and (b) the same nonreciprocal waveguide, but without termination (blue dashed line), and the same waveguide, but with bias, with termination (purple solid line), and without termination (black dotted line). The THG conversion efficiency of the terminated nonreciprocal waveguide is five orders of magnitude compared to all the other cases. (c) Distribution of the electric field magnitude at the third-harmonic frequency for the terminated nonreciprocal waveguide. Due to the dispersive nature of the materials, the waveguide becomes only weakly nonreciprocal at the third-harmonic frequency and the THG field can leak out of the structure. In this panel, $E_0 = 3.88\times10^7$ V/m, which is the input electric field peak amplitude for an input intensity of 200 MW/cm$^2$. Panels (a)-(c) are calculated at a fundamental frequency of $\omega/\omega_p = 0.7$. (d) THG conversion efficiency as a function of frequency, for a fixed input intensity $I_0 = 200$ MW/cm$^2$. As in Fig. 3(d), vertical dashed lines indicate the unidirectional frequency window, consistent with Fig. 2.

The first equation in (A1) can be written as
\begin{equation}
\frac{\partial}{\partial \rho} \left[ \rho \frac{\partial A}{\partial \rho} \right] - \frac{\partial E_0}{\partial \phi} = \frac{\epsilon_{12}}{\epsilon_{11}} \omega_0 \rho H_z.
\end{equation}

which gives the following ODEs for the radial and angular terms,
\begin{equation}
\frac{1}{B} \frac{\partial^2 B}{\partial \phi^2} = -n^2
\end{equation}
\begin{equation}
\rho \frac{\partial}{\partial \rho} \left[ \rho \frac{\partial A}{\partial \rho} \right] + (k_s^2 \rho^2 - n^2) A = 0,
\end{equation}

with the following general solutions
\begin{equation}
B = a_n \cos(n\phi) + b_n \sin(n\phi)
A = c_n J_n(k_s \rho) + d_n Y_n(k_s \rho).
\end{equation}

Thus, $H_z$ takes the following form,
\begin{equation}
H_z = (c_n J_n(k_s \rho) + d_n Y_n(k_s \rho)) (a_n \cos(n\phi) + b_n \sin(n\phi)).
\end{equation}

The other field components can be written in terms of $H_z$ as,
\begin{equation}
E_\rho = \frac{1}{-i\omega \epsilon_0 \epsilon_{eff}} \left[ \frac{1}{B} \frac{\partial H_z}{\partial \phi} + i \frac{\epsilon_{12}}{\epsilon_{11}} \frac{\partial H_z}{\partial \rho} \right]
\end{equation}
\begin{equation}
E_\phi = \frac{1}{-i\omega \epsilon_0 \epsilon_{eff}} \left[ i \frac{\epsilon_{12}}{\epsilon_{11}} \frac{\partial H_z}{\partial \rho} - \frac{\partial H_z}{\partial \phi} \right].
\end{equation}
The geometry contains the point \( \rho = 0 \) (wedge apex), which implies that \( a_n = 0 \), otherwise the field solution would diverge in all cases. In addition, the PMC boundary is at \( \phi = 0 \), which implies that \( a_e = 0 \) (vanishing tangential magnetic field on the PMC boundary). Applying these conditions simplifies the field components as follows,

\[
H_z = c_n J_n(k_s \rho) \sin(n \phi) \quad (A12)
\]

\[
E_\rho = \frac{c_n}{-i\omega \varepsilon_{eff}} \left[ \frac{n J_n(k_s \rho)}{k_s \rho} \cos(n \phi) + i \frac{\varepsilon_{12}}{\varepsilon_{11}} J'_n(k_s \rho) \sin(n \phi) \right]
\]

\[
E_\phi = \frac{c_n}{-i\omega \varepsilon_{eff}} \left[ i \frac{\varepsilon_{12}}{\varepsilon_{11}} n J_n(k_s \rho) \cos(n \phi) - J'_n(k_s \rho) \sin(n \phi) \right] \quad (A13)
\]

Finally, since the PEC boundary is located at \( \phi = \phi_0 \), the tangential electric field vanishes on this boundary, \( E_\phi|_{\phi=\phi_0} = 0 \), which leads to the following dispersion equation for the TM modes supported by the wedge-like structure in Fig. 1(a)

\[
n J_n(k_s \rho) - \frac{i\varepsilon_{12}}{\varepsilon_{11}} \frac{\partial J_n(k_s \rho)}{\partial(k_s \rho)} \tan(n \phi_0) = 0, \quad (A14)
\]

which corresponds to Eq. (2) of the main text.

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[50] Supplementary material includes the third-harmonic field distribution for all the different configurations considered in Fig. 3(b), i.e., reciprocal or nonreciprocal plasmonic waveguides, with or without termination, as well as the third-harmonic-generation conversion efficiencies for a periodic arrangement of such plasmonic waveguides.