FCT-GAN: Enhancing Table Synthesis via Fourier Transform

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Abstract—Synthetic tabular data emerges as an alternative for sharing knowledge while adhering to restrictive data access regulations, e.g., European General Data Protection Regulation (GDPR). Mainstream state-of-the-art tabular data synthesizers draw methodologies from Generative Adversarial Networks (GANs), which are composed of a generator and a discriminator. While convolution neural networks are shown to be a better architecture than fully connected networks for tabular data synthesizing, two key properties of tabular data are overlooked: (i) the global correlation across columns, and (ii) invariant synthesizing to column permutations of input data. To address the above problems, we propose a Fourier conditional tabular generative adversarial network (FCT-GAN). We introduce feature tokenization and Fourier networks to construct a transformer-style generator and discriminator, and capture both local and global dependencies across columns. The tokenizer captures local spatial features and transforms original data into tokens. Fourier networks transform tokens to frequency domains and element-wisely multiply a learnable filter. Extensive evaluation on benchmarks and real-world data shows that FCT-GAN can synthesize tabular data with high machine learning utility (up to 27.8% better than state-of-the-art baselines) and high statistical similarity to the original data (up to 26.5% better), while maintaining the global correlation across columns, especially on high dimensional dataset.

I. INTRODUCTION

While data sharing is crucial for knowledge development, privacy concerns and strict regulations (e.g., European General Data Protection Regulation (GDPR)) limit its full effectiveness. An emerging solution is to leverage synthetic data generated by machine learning models. Synthetic data has been powered by generative adversarial networks (GAN) [6] for various types of data, e.g., image [9], text to image [19] and table [24].

Synthetic tabular data emerges as a prominent research direction because of its ample application scenarios in areas such as medicine [4] and finance [1]. Compared to image data, one key difference of tabular data is that it is composed of different types of columns such as continuous, categorical or mixed variables. Therefore, GANs designed for image synthesis cannot be directly applied for tabular data. Previous works [24], [26], [27] propose feature engineering solutions for different types of data such as using one-hot encoding for categorical variable. One-hot encoding is shown [24] to better recover the categorical variable distribution for tabular GANs and capture inter-dependency across all the columns. However, one-hot encoding inevitably increases the data dimensions. High dimensional data is challenging for tabular GANs to learn global relations. Prior studies [24], [26], [27] show that the tabular GAN algorithms, which adopt CNNs as generator and discriminator, achieve better synthesis quality than using purely fully-connected neural networks. This is due to the fact that CNNs can extract local spatial features well. The first limitation of directly adopting CNN to model tabular data is that it may overlook global relations between columns due to the size of the convolution filter. This limitation exacerbates when one-hot encoding is applied for categorical variables. Secondly, while permuting columns, e.g., reordering the columns by their types, does not have any semantic meaning, the local feature presentation extracted by convolution layers is distorted. When using CNN for tabular GANs, one table row is transformed into one fixed-size image by mapping each column value to a pixel. The relationship between highly distant pixels, e.g. the pixel in the upper left corner and the pixel in the lower right corner, in a real image may not influence image classification. But for the tabular data wrapped as an image, these two pixels can represent highly correlated columns.

To address the above two limitations, we propose a conditional tabular GAN with Fourier Network blocks (FNBs). The objective of the FNBs is to learn the interactions among spatial locations in the frequency domain. We use FNBs for both the discriminator and generator with different designs. The Fourier layer, which is the key part of an FNB, contains three operations: (i) 2D discrete Fourier transform, (ii) element-wise multiplication between frequency-domain features and learnable weights and (iii) 2D inverse discrete Fourier transform. Furthermore, we process input data in a transformer-style tokenization way. A CNN-based filter is applied to original data to capture local spatial features and transform them into feature tokens. Fourier layers transform tokens into frequency domain, then the learnable weights are applied to all the frequencies to learn the global relations. Our results show that FCT-GAN outperforms state-of-the-art (SOTA) up to 27.8% in machine learning utility and 26.5% in statistical similarity on 7 datasets. Thanks to Fourier blocks ability to capture local and global relations, our results also show that among three different column orders, FCT-GAN has the least variation in synthesis quality among all comparisons. The experiment, with one high dimensional dataset, which 3 SOTA algorithms fail

1In this paper, dimension refers to the number of columns
to train due to the data dimension issue, shows that FCT-GAN still maintains its performance and stability above all comparisons.

The main contributions of this study can be summarized as follows: (1) We introduce the Fourier transform into tabular GAN training and design a generator and discriminator architecture. (2) Combining with a transformer-style input tokenizer, the novel architecture can capture both local and global relations of tabular data, leading to the desirable property of column permutation invariance. (3) We extensively evaluate FCT-GAN on 8 datasets against 5 state-of-the-art synthesizers, with a special focus on high dimensional real-world data.

II. RELATED WORK

We introduce various tabular data synthesizing methods and Fourier networks.

A. Tabular Data Synthesizers

There are various approaches for synthesizing tabular data. Probabilistic models such as Copulas [17] use Copula function to model multivariate distributions. But categorical data cannot be modeled by Gaussian Copula. Synthpop [15] works on a variable by variable basis by fitting a sequence of regression models and drawing synthetic values from the corresponding predictive distributions. Since it is variable by variable, the training process is computationally intense. Bayesian networks [2], [25] are used to synthesize categorical variables. It lacks of the ability to generate continuous variables.

Current state-of-the-art introduces several tabular GAN algorithms. Table-GAN [16] introduces an auxiliary classification model along with discriminator training to enhance column dependency in the synthetic data. CT-GAN [24] and CTAB-GAN [26] improve data synthesis by introducing several preprocessing steps for categorical, continuous or mixed data types which encode data columns into suitable form for GAN training. The conditional vector designed by CT-GAN and later improved by CTAB-GAN also helps the GAN training to reduce mode-collapse on minority categories. CTAB-GAN+ [27], PATE-GAN [8], and IT-GAN [10] generate tabular data without risking privacy of original data by either adopting differential privacy or controlling the negative log-density of real records during the GAN training. However, as far as we know, there is no previous work studying tabular GAN algorithm which focuses on countering the influence of training data column permutation on final synthetic data quality.

B. Fourier Networks

The Fourier transform has played an important role in image processing for decades, e.g. JPEG compression [23]. Incorporating Fourier transform into the neural network architecture design has been studied in many vision works [3], [14], [21]. Recent work also leverages the Fourier transform to design deep neural networks to solve partial differential equations (PDE) [12] and NLP tasks [11]. Our Fourier network block architecture design is mainly inspired by the Global Filter Network (GFNet) [20]. We take the design of the input tokenization and global filter layer from GFNet and use it in our design of the generator and discriminator for tabular GAN.

III. ANALYSIS ON PERMUTATION INVARIANCE

We empirically demonstrate the instability of prior SOTA methods to permutations of the columns in the training data. Details on the datasets and evaluation metrics are provided in Sec. Experiment.

We consider three column orders: (i) Original: as the name suggests, maintains the order as in the data downloaded from dataset source. (ii) Order by data type: puts all the continuous columns at the beginning and all categorical columns at the back. (iii) Order by data correlation: first calculates the pairwise correlations between all columns. Then it sorts columns based on the absolute correlation value with highly correlated pairs in front and less correlated pairs later. Duplicate columns are skipped. We evaluate the statistical dissimilarity between real and synthetic data using Average Jensen–Shannon divergence (Avg-JSD) for all categorical columns and Average Wasserstein distance (Avg-WD) for all continuous columns. Finally, Diff. Corr. denotes the averaged distance between the correlation matrix of the real and the synthetic data. The lower these metrics, the better the synthetic data quality. For metric $\mathcal{M}$ and dataset $D$, $\mathcal{L} = \{V_{DM}^N, V_{DM}^N, ..., V_{DM}^N\}$ denotes

![Fig. 1: Normalized maximal absolute variation (MAV) of difference in statistical similarity between original and synthetic data among different column orders](image)

| Method      | Avg-JSD | Avg-WD | Diff. Corr. |
|-------------|---------|--------|-------------|
| CTAB-GAN+   | 0.040   | 0.012  | 2.21        |
| CTAB-GAN    | 0.040   | 0.013  | 2.21        |
| Table-GAN   | 0.20    | 0.017  | 3.57        |
| TVAE        | 0.10    | 0.231  | 2.79        |
| CT-GAN      | 0.065   | 0.038  | 3.17        |

TABLE I: Average statistical similarity difference between original and synthetic data among three different column orders.
Maximal Absolute Variation (MAV) as follows:

\[ \text{MAV} = \max(\mathcal{L}) - \min(\mathcal{L}) \]

MAV denotes the largest pair-wise difference among all order results. Note that different evaluation metrics can be on different scales. To have an easier comparison across all metrics for different algorithms, we further define the normalized MAV as follows:

\[ \text{normalized MAV (\%)} = \frac{\text{MAV}}{\min(\mathcal{L})} \times 100 \]

Fig. 1 presents the Normalized MAV of synthesis quality for 5 SOTA algorithms on 5 classification datasets (i.e., Loan, Adult, Intrusion, Credit and Covertype). CTAB-GAN, CTAB-GAN+ and Table-GAN use CNNs for generator and discriminator while TVAE and CT-GAN use fully-connected networks. Table-GAN is the only method that uses min-max normalization to encode all column types. The remaining four algorithms use one-hot encoding for categorical columns. This increases significantly the dimensions of the feature space making it more difficult to learn global relations. Using CNN plus one-hot encoding makes CTAB-GAN and CTAB-GAN+ more vulnerable to column permutations. Fully-connected networks reduce this impact, but the variation is still non-negligible. Table-GAN is the most stable across all metrics, but its absolute performance is the worst among all SOTAs. Tab. I presents the absolute results for each metric averaged over the same three column orderings. One can see that even though CTAB-GAN and CTAB-GAN+ experience some instability in their results, their performance ranges are still better than other algorithms. None of the SOTAs is able to provide tabular data synthesis both of high quality and invariant to column permutations.

IV. FCT-GAN

In this section, we first present the primer of Fourier Network Blocks (FNB), and then describe the design of integrating them into the tabular GAN.

A. Primer on Fourier Network Blocks

The input tokenization process and Fourier network block [20] are explored for image classification and can be combined in the following architecture, illustrated in Fig. 2. This architecture is the base for constructing the generator and discriminator of FCT-GAN. Each row of encoded tabular data is divided into tokens by a patch embedding layer. Then repeated layers of FNBs extract features from the tokens. In the generator the extracted features directly represent the generated data. In the discriminator they are fed into a final layer for classification.

For input data tokenization (i.e., patch embedding in Fig. 2), we first break the original image into small parts to extract local spatial features (in the same vein as the ViT [5], MLP-Mixer [22] and GFNet [20]). We define a CNN layer with a \( k \times k \) kernel and stride the same as the kernel size. Therefore, for an image of size \( N \times N \), after embedding, we have \( N \times N \) tokens.

The Fourier Network Block consists of two parts: (1) Fourier layer and (2) Feed Forward Network. As suggested by GFNet, we use real fast Fourier transform (rFFT) (i.e., 2D FFT stage) in the Fourier layer since our input is a real tensor with no imaginary component. Since the FFT of a real tensor is conjugate symmetric, we can cut off half of the frequency domain features without losing information. This helps to half the number of learnable weights in the frequency domain, hence accelerating computation.

The input tokenization and the FNB design can mix tokens representing different spatial locations allowing this architecture to take into account both local and global relations.

B. Design of FCT-GAN

Fig. 3 presents the overall structure of FCT-GAN. We first introduce the data feature engineer method, then the network architectures of the generator and discriminator and how FNB is integrated. Finally, we discuss the training procedure and their respective loss functions.

1) Data feature engineering: Before feeding tabular data into FCT-GAN, we adopt the data encoders from CTAB-GAN+: one-hot encoding for categorical variables, and variational Gaussian mixtures (VGM) for continuous ones. For
variables with specific distributions, e.g., uniform or normal, and for high dimensional categorical variables, min-max normalization is applied. Finally columns with missing values use a Mixed-type encoder \[26\].

2) Conditional GAN: Due to the constraint on controlling generated data via GAN, conditional GAN is increasingly used. For each training batch, the same conditional vector is attached to noise latent as the input for generator, and also concatenate to real and generated data as the input for discriminator. Then the total input dimension for the generator and discriminator in conditional GANs is the sum of the original input plus conditional vector dimension. The design of conditional vector of FCT-GAN is also adopted from CTAB-GAN+, which indicates a specific category in the chosen categorical column or an exact Gaussian mode in the chosen continuous column if it uses variational Gaussian mixture to approximate the distribution.

3) Generator: The objective of the generator is to capture the joint probability distribution of all columns to synthesize high fidelity data. In FCT-GAN, we leverage the FNB to achieve the above goal. We opt for CNN-based GAN design which iteratively upscales the resolution at different stages. Hence our generator gradually increases the input sizes and reduces the embedding dimension at each stage.

Fig. 3 depicts an example generator design where the target resolution is 32 × 32 on the left. The noise latent and conditional vector are fed into an embedding layer (i.e., a Multi-Layer Perceptron which the input dimension is the same as the noise latent and conditional vector combined, the output dimension is \[N = H_0 \times W_0 \times C_0\] to convert them to a \[H_0 \times W_0 \times C_0\] (by default we use \[H_0=W_0=4, C_0=256\]) vector. The vector is then reshaped into a \[H_0 \times W_0\] resolution feature map with each point being a \[C_0\]-dimensional embedding. The result is fed into the first Fourier Network Block.

After each FNB, we insert an upsampling module. There are several choices to achieve resolution-upscaling, such as bicubic interpolation or transpose convolutional operation. To mitigate memory usage and computation, we use the PIXELSHUFFLE function. This upscales the resolution of feature maps by a factor of 2 × while reducing embedding dimension to a quarter. We repeat the FNB and UpScale stages until we reach the target resolution. The final linear unflatten layer is used to project the embedding into 1-dimension.

4) Discriminator: Fig. 3 shows our discriminator architecture on the right. The objective of the discriminator is to distinguish real and fake data. We leverage repeated layers of Fourier network blocks from Fig. 2 to extract the features. Since FCT-GAN adopts the training structure of wasserstein GAN plus gradient penalty (WGAN+GP) \[7\], our discriminator outputs one single value instead of probability vector. This is achieved by a final group of normalization, dropout and linear layers.

The generated data from the generator and the conditional vector are concatenated, wrapped as an image (padding missing values with zeros), and fed to the patch embedding layer of the discriminator. Within patch embedding, there is a CNN filter with a \(k \times k\) kernel (we set \(k = 8\) by default), \(C_1 = 256\) by default) output channels, and stride same as the kernel size. Different from the generator, the inputs and outputs of the FNBs use constant dimensions. After 4 successive FNBs, the extracted features are flattened and downsampled to one single value.

5) GAN training loss: To improve the stability of GAN training, FCT-GAN adopts WGAN+GP \[7\] loss. And to elevate the synthesizing performance, three extra training losses are added for generator: (1) downstream loss, (2) informa-
tion loss and (3) generator loss. FCT-GAN incorporates an auxiliary classifier/regressor as suggested in [16], [26], [27]. For each synthesized data item the classifier/regressor outputs a predicted value using the synthesized features. The downstream loss quantifies the discrepancy between the synthesized and predicted value. This helps to increase the semantic integrity of synthetic records. The information loss matches the first-order (i.e., mean) and second-order (i.e., standard deviation) statistics of synthesized and real records. This leads to synthetic records with the same statistical characteristics as real records. The generator loss measures the difference between the given condition and the output class of the generator. This loss helps the generator learn to produce the exact same class as the given conditions.

V. EXPERIMENT

In this section, we evaluate the effectiveness of synthetic data generated by FCT-GAN in maintaining the global correlation and high statistical similarity to the original data which results in high quality downstream machine learning analyses.

A. Experiment Setup

Datasets. All algorithms are tested on 8 machine learning datasets. Intrusion, Adult and Covertype are from the UCI machine learning repository. Credit and Loan are from Kaggle. These five tabular datasets are defined to have a categorical variable as the target for conducting classification tasks. To include regression tasks we use two more datasets Insurance and King from Kaggle where the target variable is continuous. Finally, Youth is a dataset from the Dutch government. It does not contain target variable. We include Youth because it contains many high dimensional categorical variables which lead to dimension explosion under one-hot encoding. This dataset is specially selected for testing algorithms’ ability to capture global relations in high dimensional data and their stability to different training data column permutations.

Due to computing resource limitations, 50K rows of data are sampled randomly in a stratified manner with respect to the target variable for Covertype, Credit and Intrusion datasets. The Adult, Loan, Insurance, King and Youth datasets are taken in their entirety. The details of each dataset are shown in Tab. II. We assume that the data type of each variable is known before training.

For stability on column permutation analysis, three column orders are considered: (1) original, (2) order by data type and (3) order by data correlation which are already defined in section Analysis on Permutation Invariance.

| Dataset | Problem | Train/Test Split | Con. | Cat. | Total Cat. |
|---------|---------|------------------|------|------|------------|
| Adult   | C.      | 39k/10k          | 5    | 9    | 104        |
| Covertype | C.     | 40k/10k          | 10   | 45   | 94         |
| Credit  | C.      | 40k/10k          | 1    | 30   | 2          |
| Intrusion | C.     | 40k/10k          | 22   | 20   | 175        |
| Loan    | C.      | 4k/1k            | 6    | 7    | 17         |
| Insurance | R.     | 1k/300           | 3    | 4    | 14         |
| King    | R.      | 17k/4k           | 13   | 7    | 179        |
| Youth   | -       | 19k/5k           | 17   | 23   | 657/74     |

B. Experiment Metrics

Baselines. FCT-GAN is compared with 5 other SOTA tabular data synthesizing algorithms: CTAB-GAN, CTAB-GAN+, Table-GAN, CT-GAN and TVAE. To separately show the utility of generator and discriminator, we also test two variants of FCT-GAN: FCT-GAN\(_G\) and FCT-GAN\(_D\). FCT-GAN\(_G\)/FCT-GAN\(_D\) only keeps the generator/discriminator of FCT-GAN using as counterpart the CNN-based discriminator/generator from CTAB-GAN.\(^6\) To have fair comparisons, all algorithms are implemented using Pytorch with hyper-parameters and network structures as set in the original papers. To ensure all algorithms converge, we train for 150 epochs on all datasets except Loan and Insurance trained for 300 and 500 epochs due to their small size. Each experiment is repeated 3 times and the average result is reported.

Environment. Experiments run on a machine equipped with 32 GB memory, a GeForce RTX 2080 Ti GPU and a 10-core Intel i9 CPU under Ubuntu 20.04.

C. Evaluation Metrics

The synthetic data is evaluated on two aspects: (1) machine learning utility and (2) statistical similarity.

1) Machine learning utility: Classification and regression datasets are quantified using different metrics, but they share the same evaluation process. We first train each algorithm on the training data and use the trained model to generate synthetic data of the same size as the training data. Then we use the training data and synthetic data to train same set of ML algorithms. For classification dataset, we choose decision tree classifier, linear support-vector-machine (SVM), random forest classifier, multinomial logistic regression and MLP. For regression dataset, we choose linear regression, ridge regression, lasso regression and Bayesian ridge regression model. Finally, we use the test set to separately test the two sets of ML models trained on the original and synthetic data. We use accuracy, F1-score and AUC as evaluation metrics for classification, and mean absolute percentage error (MAPE), explained variance score (EVS) and \(R^2\) score as the metrics for regression. In the end, we calculate the difference between the results of the two sets of ML models for each metric.

\(^6\)These architectures are extracted from their official github https://github.com/Team-TUD/CTAB-GAN-plus.git.
Since we report differences, the lower the result, the better the synthesis quality.

2) Statistical similarity: Three metrics are used to quantify the statistical similarity between real and synthetic data. 

Average Jensen-Shannon divergence (Avg-JSD). The JSD [13] provides a measure to quantify the difference between the probability mass distributions of individual categorical variables belonging to real and synthetic data. This metric is bounded between 0 and 1 and is symmetric allowing for an easy interpretation of results.

Average Wasserstein distance (Avg-WD). In a similar vein, the Wasserstein distance [13] is used to capture how well the distributions of individual continuous variables are emulated by synthetically produced data in correspondence to real data.

We use WD because JSD metric is numerically unstable for evaluating the quality of continuous variables, especially when there is no overlap between the synthetic and original data. Hence, we resort to the more stable Wasserstein distance.

Difference in pair-wise correlation (Diff. Corr.). To evaluate the preservation of column dependency in synthetic data, we first compute the pair-wise correlation matrix for the columns within real and synthetic datasets individually. Pearson correlation coefficient is used between any two continuous variables. It ranges between $[-1,+1]$. Similarly, the uncertainty coefficient is used to measure the correlation between any two categorical features. It ranges between $[0,1]$. And the correlation ratio between categorical and continuous variables is used. It also ranges between $[0,1]$. Note that the dython[7] library is used to compute these metrics. Finally, the difference between pair-wise correlation matrices for real and synthetic datasets is computed.

D. Result Analysis

1) Quality of synthetic data: We first analyze the algorithm performance on the classification and regression datasets. Then, we present results on Youth, a larger real world dataset.

Tab. III shows the ML utility and statistical similarity results of 8 algorithms on 7 datasets. It is worth noting that as results summarize the difference between real and synthetic data, the lower the value the better the result. The table is grouped into three sections: averaged ML Utility results across regression datasets, averaged ML utility for classification datasets and averaged statistical similarity on all 7 datasets. Best results are highlighted in bold. One can see that FCT-GAN outperforms all the baselines on all the metrics. Excluding its direct own variants, FCT-GAN also significantly outperforms the second best algorithm, i.e., CTAB-GAN+, in most metrics. Note that many of our classification datasets have an uneven class distribution, then F1-score is a more appropriate metric than accuracy. The results shows that FCT-GAN improves CTAB-GAN+ by 27.8% on F1-score, which shows that the new architecture enhances the ability to capture global column dependency. Among FCT-GAN and its variants, FCT-GAN$_D$ shows the performance relatively close to FCT-GAN. FCT-GAN$_C$ is still better than most of SOTAs except CTAB-GAN+, but it degrades more from FCT-GAN compared to FCT-GAN$_D$. This shows that the generator can not fully demonstrate its ability without a well-designed discriminator, but a stronger discriminator can level up the synthesizing ability of the generator by better identifying fake data and hence forcing the generator to improve the fidelity of the generated data.

2) Stability on column permutation: We show the maximal absolute variation of each metric on all algorithms among three column orders in Tab. IV. FCT-GAN achieves the smallest MVA in 7 out of 9 metrics and is the second best on the other 2 metrics. This means that no matter which column order is used, FCT-GAN can always capture the (global and local) column dependencies better than any other baseline. As TVAE uses fully-connected network layers only for both generator and discriminator, it also shows better stability. The same conclusion does not hold for CTAB-GAN even if it also uses only fully-connected networks. We can conclude that fully-connected networks can help some of the generative models to counter the influence of column permutations on the quality of the synthesized data, but it is not guaranteed. Moreover one can note that changing training data column order generally influences more ML utility than statistical similarity. This means that for most of the tabular GAN algorithms, it is more difficult to capture the, especially global, column dependencies rather than to model the distribution of every single variable.

3) Results on Youth dataset: The analysis on the Youth dataset is singled out because it contains many categorical columns with large number of categories. Encoding these columns with one-hot encoding translates into a high number of dimensions, i.e., 65774 (see Tab. II). Moreover, the algorithms such as CT-GAN and CTAB-GAN are conditional GANs which require an additional conditional vector. This exacerbates even more the dimensionality issue. To construct the conditional vector for these categories, we need another 65774 dimension vector so that we can indicate each of the category. This stresses the resource demands and we were not able to train TVAE, CT-GAN and CTAB-GAN on Youth dataset even with batch size equal to 1. in our compute environment. Using our feature engineering, we can encode some high dimensional categorical columns by min-max normalization lowering the resource footprint.

Tab. V summarizes the result on all algorithms that can still train on the Youth dataset. The ED Dim. is the encoded data dimension of training data. This is also the dimension of the data generated at generator output. The CV Dim. represents the conditional vector dimension. The first 4 algorithms in Tab. V generate data that is $32 \times 32$ (i.e., upscale from 376 to $k \times k$ where k is an integer power of 2) and the input dimension of the discriminator is $32 \times 32$ (i.e., upscale from $376+343=719$ to $k \times k$). Results show that FCT-GAN outperforms all baselines on both synthesis quality and stability. The difference between FCT-GAN and CTAB-GAN+ on Diff. Corr. demonstrates that CNN-based GANs can not

[http://shakedzy.xyz/dython/modules/nominal/#compute_associations]
TABLE III: Difference of ML Utility and Statistical Similarity between original and synthetic data. ML Utility Difference R. represents the results averaged on 2 regression datasets. ML Utility Difference C. represents the results averaged on 5 classification datasets. Statistical Similarity Difference represents the results averaged on above 7 datasets. Im. to 2nd Best shows the improvement of FCT-GAN to the 2nd best result in the column excluding its own variants. Best results are on bold.

| Method       | MAPE  | EVS   | $R^2$ | Accuracy | F1-score | AUC   | Statistical Similarity Difference |
|--------------|-------|-------|-------|----------|----------|-------|-----------------------------------|
| FCT-GAN      | 0.037 | 0.022 | 0.043 | 4.92%    | 0.065    | 0.039 | 0.010                             |
| FCT-GAN_D    | 0.042 | 0.041 | 0.065 | 4.98%    | 0.066    | 0.053 | 0.014                             |
| FCT-GAN_C    | 0.131 | 0.041 | 0.109 | 9.24%    | 0.140    | 0.066 | 0.014                             |
| CTAB-GAN     | 0.037 | 0.025 | 0.043 | 5.23%    | 0.090    | 0.041 | 0.011                             |
| CTAB-GAN_D   | 0.059 | 0.594 | 0.707 | 8.90%    | 0.107    | 0.094 | 0.021                             |
| CTAB-GAN_C   | 0.871 | 0.594 | 0.709 | 21.51%   | 0.274    | 0.253 | 0.037                             |
| TVAE         | 0.243 | 0.078 | 0.215 | 11.11%   | 0.100    | 0.230 | 0.025                             |
| Table-GAN    | 0.338 | 0.434 | 0.479 | 11.40%   | 0.130    | 0.169 | 0.028                             |

Im. to 2nd Best 0% 13.6% 0% 6.3% 27.8% 5.1% 10% 26.5% 5.1%

TABLE IV: Maximal absolute variation (MAV) of the difference of ML Utility and Statistical Similarity on three type of column orders between original and synthetic data. ML Utility Difference R. represents the results averaged on 2 regression datasets. ML Utility Difference C. represents the results averaged on 5 classification datasets. Statistical Similarity Difference represents the results averaged on above 7 datasets. Best result are on bold.

| Method       | MAV   | EV-S | $R^2$ | Accuracy | F1-score | AUC   | MAV   | EV-S | $R^2$ | Accuracy | F1-score | AUC   |
|--------------|-------|------|-------|----------|----------|-------|-------|------|-------|----------|----------|-------|
| FCT-GAN      | 0.04  | 0.02 | 0.03  | 0.02     | 0.01     | 0.01  | 3e-3  | 2e-3 | 0.11  |
| FCT-GAN_D    | 0.11  | 0.03 | 0.2   | 0.02     | 0.03     | 0.02  | 7e-3  | 5e-3 | 0.2   |
| FCT-GAN_C    | 0.08  | 0.11 | 0.08  | 0.03     | 0.05     | 0.03  | 9e-3  | 1e-3 | 0.2   |
| CTAB-GAN     | 0.28  | 0.26 | 0.35  | 0.02     | 0.03     | 0.03  | 5e-3  | 4e-3 | 0.32  |
| CTAB-GAN_D   | 0.05  | 0.56 | 0.67  | 0.07     | 0.17     | 0.12  | 6e-3  | 7e-3 | 0.3   |
| CTAB-GAN_C   | 0.57  | 0.25 | 0.38  | 0.09     | 0.06     | 0.09  | 0.01  | 0.01 | 0.17  |
| TVAE         | 0.15  | 0.03 | 0.05  | 0.04     | 0.05     | 0.05  | 2e-3  | 3e-3 | 0.15  |
| Table-GAN    | 0.09  | 0.3  | 0.16  | 0.14     | 0.24     | 0.1   | 0.01  | 2e-3 | 0.28  |

TABLE V: Statistical similarity difference on Youth dataset. MAV in the parenthesis. CTAB-GAN+C and TVAE can not train on this dataset due to the huge dimension of encoded data. Though Table-GAN is a CNN-based solution, its min-max normalization does not change the data dimension and outperforms CTAB-GAN+ on Diff. Corr. That is because it operates on a smaller scale image (8 × 8) which makes it easier to capture the pixel (column) dependencies. Still, its overall performance trails far behind FCT-GAN.

| Method       | MAV   | EV-S | $R^2$ | Accuracy | F1-score | AUC   | MAV   | EV-S | $R^2$ | Accuracy | F1-score | AUC   |
|--------------|-------|------|-------|----------|----------|-------|-------|------|-------|----------|----------|-------|
| FCT-GAN      | 0.42  | 0.011| 0.874 | 0.21     | 0.76     | 243   | 3e-3  | 2e-3 | 0.11  |
| FCT-GAN_D    | 0.02  | 0.25  | 0.911 | 0.33     | 0.76     | 243   | 7e-3  | 5e-3 | 0.2   |
| FCT-GAN_C    | 0.04  | 0.66  | 0.946 | 0.34     | 0.76     | 243   | 9e-3  | 1e-3 | 0.2   |
| CTAB-GAN     | 0.01  | 0.66  | 0.946 | 0.34     | 0.76     | 243   | 5e-3  | 4e-3 | 0.32  |
| CTAB-GAN_D   | 0.02  | 0.66  | 0.946 | 0.34     | 0.76     | 243   | 6e-3  | 7e-3 | 0.3   |
| CTAB-GAN_C   | 0.04  | 0.66  | 0.946 | 0.34     | 0.76     | 243   | 2e-3  | 3e-3 | 0.15  |
| TBAE-GAN     | 0.48  | 0.013| 0.950 | 0.24     | 0.11     | 1e-3  | 0.01  | 2e-3 | 0.28  |

TABLE VI: Statistical similarity difference on Youth dataset. MAV in the parenthesis. CTAB-GAN+C and TVAE can not train on this dataset due to the huge dimension of encoded data. Though Table-GAN is a CNN-based solution, its min-max normalization does not change the data dimension and outperforms CTAB-GAN+ on Diff. Corr. That is because it operates on a smaller scale image (8 × 8) which makes it easier to capture the pixel (column) dependencies. Still, its overall performance trails far behind FCT-GAN.

VI. CONCLUSION

In this paper, we present a novel tabular GAN algorithm – FCT-GAN. FCT-GAN leverages the transformer-style tokenizer and Fourier network blocks to design the generator and discriminator. For discriminator, the tokenizer uses a CNN-based filter to extract local spatial features from original input data and tokenizes them for Fourier network blocks. The key component of Fourier network blocks – the Fourier layer uses 2D FFT/IFFT to transform input tokens into frequency domain and applies learnable filters on the frequency features to ease learning global relations. This design takes both local and global features into account. The generator imitates the design of CNN-based GANs, which piles up Fourier network blocks and upscales feature dimensions at each layer by 2× until reaching the target size. The proposed method surpasses state-of-the-art performance, especially on high dimensional data. It also shows brilliant stability to counter the impact of training data column permutations on the synthetic data quality.

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