THE MAXIMUM SURPLUS BEFORE RUIN IN A JUMP-DIFFUSION INSURANCE RISK PROCESS WITH DEPENDENCE

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Abstract. We consider a compound Poisson risk process perturbed by a Brownian motion through using a potential measure where the claim sizes depend on inter-claim times via the Farlie-Gumbel-Morgenstern copula. We derive an integro-differential equation with certain boundary conditions for the distribution of the maximum surplus before ruin. This distribution can be calculated through the probability that the surplus process attains a given level from the initial surplus without first falling below zero. The explicit expressions for this distribution are derived when the claim amounts are exponentially distributed.

1. Introduction. The compound Poisson risk model perturbed by a diffusion was proposed by [13]. From then on, perturbed risk models have been considered by many authors. Among them, [14] considered a penalty scheme which is defined by a constant \( w_0 \) and a non-negative function \( w(-y), y > 0 \) based on the same risk model in [13] and obtained a defective renewal equation for the expected discounted value of a penalty at ruin. [23] generalized the defective renewal equation for the expected discounted value of a penalty at the time of ruin in [14]. [17] studied the distribution of the dividend payments in the compound Poisson risk models perturbed by diffusion. [8] considered a perturbed MAP risk model with dividend barrier strategies. [15] investigated dividend payments in a perturbed renewal risk model with multiple thresholds where the inter-claim times are phase-type distributed. The above-mentioned literatures assumed that claim amounts are independent of inter-claim times. Actually, this assumption of independence is too restrictive in real applications. For example, in modeling natural catastrophic events, more often than not, on the occurrence of a catastrophe, the total claim amount might depend on the time elapsed since the previous catastrophe. Under this background, some risk models that allow for specific dependence between the claim amounts and the inter-claim times have been investigated. For example, [10] and [11] extended the classical compound Poisson risk model to a dependence structure in which the claim amounts and claim inter-arrival times are dependent though a (generalized) Farlie-Gumbel-Morgenstern (FGM) copula. [9] studied a generalized Gerber-Shiu...
function by incorporating the surplus immediately after the second last claim before ruin and the minimum surplus level before ruin in the classical Gerber-Shiu function in Sparre Andersen models allowing for possible dependence between claim sizes and interclaim times. [24] considered the Gerber-Shiu function in the compound Poisson risk model perturbed by diffusion with the dependence between claim sizes and interclaim times. For more details about the dependence structure, please refer to [2], [5], [1], [6] and [22] among others.

The maximum surplus before ruin is an important indicator of the assets in insurance institutions. By analyzing the maximum surplus, an insurance company can estimate its ability of withstanding bankruptcy and decide whether to carry out other businesses or not. [19] discussed the maximum surplus before ruin in an Erlang(n) risk process and related problems. [18] considered the time of recovery and the maximum severity of ruin in a Sparre Andersen model. [21] studied the compound Poisson risk process with a threshold dividend strategy. In these models, the results about the maximum surplus before ruin are based on the assumption of independence between the claim amounts and the inter-claim times. Under the framework of dependence, [7] investigated a generalized Gerber-Shiu function by incorporating the maximum surplus before ruin into the penalty function in a MAP risk model where the claim amounts and the inter-claim times form a chain of dependent variables. [16] studied the distribution of the maximum surplus before ruin in a risk model in which the claim sizes and inter-claim times are dependent via a FGM copula. Motivating by [16], we now consider the maximum surplus before ruin in a dependent risk model perturbed by diffusion through using a potential measure.

The rest of this paper is organized as follows. Section 2 describes the dependent risk model perturbed by diffusion. In Section 3, we derive the integro-differential equation with boundary conditions satisfied by the distribution of the maximum surplus before ruin. In Section 4, we utilize the general theory of differential equations to solve the homogenous integro-differential equation satisfied by the probability that the surplus process attains a given level from the initial surplus without first falling below zero. An analytic expression for the distribution of the maximum surplus before ruin is given when claim sizes are exponentially distributed in Section 5. Last, Section 6 concludes.

2. Dependence structure and notation description. The perturbed risk model $R(t)$ is given by

$$R(t) = u + ct - S(t) + \sigma B(t), \quad t \geq 0,$$

where $u \geq 0$ is the initial surplus, $c$ represents the insurer’s premium income per unit time. The aggregate claims process $S(t) = \sum_{i=1}^{N(t)} X_i$ is a compound Poisson process, where $\{X_1, X_2, \ldots\}$ be independent and identically distributed (i.i.d.) positive random variables representing the successive individual claim amounts. These random variables, identically distributed as the canonical r.v. $X$, are assumed to have common cumulative distribution function (c.d.f.) $F(x), x \geq 0, \overline{F}(x) = 1 - F(x)$, with probability density function (p.d.f.) $f(x) = F'(x)$ and Laplace transform $\tilde{f}(s) = \int_0^\infty e^{-sx} f(x) dx$. The ordinary renewal process $\{N(t); t \geq 0\}$ denotes the number of claims up to time $t$ and is defined as $N(t) = \sup \{n : W_1 + W_2 + \cdots + W_n \leq t\}$, where the i.i.d. inter-claim times $\{W_1, W_2, \ldots\}$, identically distributed as the canonical r.v. $W$, have common c.d.f. $K(t) = 1 - e^{-\lambda t}, t \geq 0$, p.d.f. $k(t) = \lambda e^{-\lambda t}, t \geq 0$, and
Laplace transform $\tilde{k}(s) = \frac{1}{s^2 + \lambda}$. Finally, $\{B(t); t \geq 0\}$ independent of the aggregate claims process is a standard Brownian motion and $\sigma > 0$ is the dispersion parameter. Throughout the text, we use the superscript "$\sim$" to designate the Laplace transform of that function.

In this paper, we assume that $\{(X_i, W_i), i \in \mathbb{N}^+\}$ forms a sequence of i.i.d. random vectors distributed as the canonical r.v. $(X, W)$, in which the components may be dependent. Now, we use the FGM copula to define the dependence between the claim size and the inter-claim time. The FGM copula is given by

$$C^\text{FGM}_\theta(u_1, u_2) = u_1 u_2 + \theta u_1 (1 - u_1)(1 - u_2), \quad 0 \leq u_1, u_2 \leq 1,$$

(2)

where $-1 \leq \theta \leq 1$. The FGM copula allows negative and positive dependence, and it also includes the independence copula ($\theta = 0$). The reader may consult e.g. [12], [10] and [11] for the property of the FGM copula.

Under the dependence structure, the bivariate c.d.f. of $(X, W)$ is given by

$$F_{X,W}(x, t) = C^\text{FGM}_\theta(F(x), K(t)), \quad (x, t) \in (0, \infty) \times (0, \infty),$$

and the joint p.d.f. of $(X, W)$ is given by

$$f_{X,W}(x, t) = c^\text{FGM}_\theta(F(x), K(t))f(x)k(t), \quad (x, t) \in (0, \infty) \times (0, \infty),$$

where $c^\text{FGM}_\theta(u_1, u_2) = -\frac{\partial^2}{\partial u_1 \partial u_2} C^\text{FGM}_\theta(u_1, u_2)$. Further, we conclude from (2) that

$$F_{X,W}(x, t) = F(x)K(t) + \theta F(x)K(t)(1 - F(x))(1 - K(t)),$$

(3)

$$f_{X,W}(x, t) = \lambda e^{-\lambda t} f(x) + \theta(2\lambda e^{-2\lambda t} - \lambda e^{-\lambda t})h(x),$$

(4)

where $h(x) = (1 - 2F(x))f(x)$.

From (4), the conditional p.d.f. of the claim size is given by

$$f_{X|W=x}(x) = f(x) + \theta(2e^{-\lambda t} - 1)h(x),$$

(5)

Let $T = \inf \{t \geq 0 : R(t) \leq 0\}$ be the time of ruin of risk model (1) with $T = \infty$ if $R(t) > 0$ for all $t \geq 0$. To ensure that ruin is not a certain event, we suppose that

$$E[e^{W - X}] > 0,$$

(6)

providing a positive safety loading. Define

$$\psi(u) = P(T < \infty | R(0) = u)$$

to be the ultimate ruin probability. Let $\phi(u) = 1 - \psi(u)$ be the non-ruin probability.

For $b > 0, u \geq 0$, define

$$\mathcal{G}(u, b) = P\left(\sup_{0 \leq t \leq T} R(t) < b, T < \infty | R(0) = u\right),$$

which is the probability of the event that the maximum surplus before ruin is less than $b$ starting from initial surplus $u$. Alternatively, $\mathcal{G}(u, b)$ is the probability of ruin in the presence of an absorbing barrier at $b$. Obviously, $\mathcal{G}(u, b) = 0$ for $b \leq u$ and $\mathcal{G}(u, b) = \psi(u)$ for $b \to \infty$. In addition, $\mathcal{G}(u, b)$ can be decomposed according to whether ruin is caused by a claim or oscillation, i.e.

$$\mathcal{G}(u, b) = \mathcal{G}_c(u, b) + \mathcal{G}_o(u, b),$$

(7)

where

$$\mathcal{G}_c(u, b) = P\left(\sup_{0 \leq t \leq T} R(t) < b, T < \infty, R(t) < 0 | R(0) = u\right)$$

and

$$\mathcal{G}_o(u, b) = P\left(\sup_{0 \leq t \leq T} R(t) < b, T < \infty, R(t) > 0 | R(0) = u\right).$$
is the probability of the event that the maximum surplus before ruin is less than \( b \) when ruin is caused by a claim, and

\[
G_b(u, b) = P\left( \sup_{0 \leq t \leq T} R(t) < b, T < \infty, R(t) = 0 | R(0) = u \right)
\]

is the probability of the event that the maximum surplus before ruin is less than \( b \) when ruin is caused by oscillation.

3. Integro-differential equations. In order to obtain the integro-differential equation for the distribution of the maximum surplus before ruin, we firstly introduce some notations and results. Let \( H(t) = -ct - \sigma B(t) \), which is a Brownian motion starting from zero with drift \(-c\) and variance \( \sigma^2 \). \( H(t) = \sup_{0 \leq s \leq t} H(s) \) denotes the running supremum of \( H(t) \). \( \varpi_u = \inf \{ t \geq 0 : H(t) = u \} \) denotes the first hitting time of the value \( u > 0 \). Thanks to [4], we have for \( \delta \geq 0 \),

\[
E[e^{-\delta \varpi_u}] = e^{-\varepsilon u}, \quad (8)
\]

where \( \varepsilon = \frac{c}{\sigma^2} + \sqrt{\frac{2q}{\sigma^2} + \frac{c^2}{\sigma^4}} \). Define the following potential measure for \( u, x > 0, y < u, u < b \),

\[
\vartheta(u, b, dy, dx) = E[I(u - b < \tilde{H}(W) < u, H(W) \in dy, X \in dx)], \quad (9)
\]

where \( I(\cdot) \) is the indicator function. Which plays a key role in analyzing the probability of the maximum surplus before ruin.

Within the rest of this paper, \( \varepsilon_q \) denotes an exponential random variable with rate \( q \) and variance \( \sigma^2 \).

Lemma 3.1. Assume that \( \varepsilon_q \) is independent of \( \{H(t)\} \). Then random variables \( \tilde{H}(\varepsilon_q) \) and \( H(\varepsilon_q) - H(\varepsilon_q) \) are independent and exponentially distributed with rates

\[
q_1 = \frac{c}{\sigma^2} + \sqrt{\frac{2q}{\sigma^2} + \frac{c^2}{\sigma^4}}, \quad q_2 = -\frac{c}{\sigma^2} + \sqrt{\frac{2q}{\sigma^2} + \frac{c^2}{\sigma^4}},
\]

respectively.

We first derive the following measure in order to calculate potential measure (9)

\[
\kappa_q(u, b, dy) = P(u - b < \tilde{H}(\varepsilon_q) < u, H(\varepsilon_q) \in dy), 0 < u < b, u > y.
\]

Let \( \tilde{\kappa}_q(u, dy) = P(\tilde{H}(\varepsilon_q) < u, H(\varepsilon_q) \in dy), 0 < u < b, u > y \), thus,

\[
\kappa_q(u, b, dy) = \tilde{\kappa}_q(u, dy) - \tilde{\kappa}_q(u - b, dy).
\]

Utilizing Lemma 3.1 and from [24], we have

\[
\tilde{\kappa}_q(u, dy) = \frac{q_1 q_2}{q_1 + q_2} (e^{-q_2 y} - e^{-(q_1 + q_2)u + q_2 y})dy, 0 \leq y < u, \quad (10)
\]

and

\[
\tilde{\kappa}_q(u, dy) = \frac{q_1 q_2}{q_1 + q_2} (e^{q_2 y} - e^{-(q_1 + q_2)u + q_2 y})dy, y < 0. \quad (11)
\]

Since random variables \( \tilde{H}(\varepsilon_q) \) is exponentially distributed and \( u - b < 0, \tilde{\kappa}_q(u - b, dy) = 0 \), which derives \( \kappa_q(u, b, dy) = \tilde{\kappa}_q(u, dy) \).

Conditioning on the value of \( W \), it follows that

\[
\begin{align*}
\vartheta(u, b, dy, dx) &= \int_0^\infty \lambda e^{-\lambda t} f_X(w-x)(u-b < \tilde{H}(t) < u, H(t) \in dy)dxdt \\
&= \int_0^\infty \lambda e^{-\lambda t} f_X(w-x)(u-b < \tilde{H}(t) < u, H(t) \in dy)dxdt + \int_0^\infty 2\theta \lambda e^{-2\lambda t} h(x)(u-b < \tilde{H}(t) < u, H(t) \in dy)dxdt \\
&= \int_0^\infty (f(x) - \theta h(x))\kappa_q(u, b, dy)dx + \theta h(x)\kappa_{2\lambda}(u, b, dy)dx.
\end{align*}
\]
Lemma 3.2. The density of potential measure $\vartheta(u,b,dy,dx)$ is given by
\begin{equation}
\begin{aligned}
\varepsilon_1 &= \frac{c}{\sigma^2} + \sqrt{\frac{2\lambda}{\sigma^2} + \frac{c^2}{\sigma^4}}, \\
\varepsilon_2 &= \frac{c}{\sigma^2} + \sqrt{2 \frac{\lambda}{\sigma^2} + \frac{c^2}{\sigma^4}},
\end{aligned}
\end{equation}

and
\begin{equation}
\begin{aligned}
\varepsilon_1 &= \frac{c}{\sigma^2} + \sqrt{\frac{4\lambda}{\sigma^2} + \frac{c^2}{\sigma^4}}, \\
\varepsilon_2 &= \frac{c}{\sigma^2} + \sqrt{\frac{4\lambda}{\sigma^2} + \frac{c^2}{\sigma^4}}.
\end{aligned}
\end{equation}

Throughout the text, $\mathcal{I}$ and $\mathcal{D}$ represent the identity and the differentiation operators with respect to (w.r.t.) $u$, respectively. Let
\begin{equation}
\begin{aligned}
\Delta_1(\mathcal{D}) := & \mathcal{D}^2 + \frac{2c}{\sigma^2} \mathcal{D} - \frac{2\lambda}{\sigma^2} \mathcal{I} = (\mathcal{D} + \varepsilon_1 \mathcal{I})(\mathcal{D} - \varepsilon_2 \mathcal{I}), \\
\Delta_2(\mathcal{D}) := & \mathcal{D}^2 + \frac{2c}{\sigma^2} \mathcal{D} - \frac{4\lambda}{\sigma^2} \mathcal{I} = (\mathcal{D} + \varepsilon_1 \mathcal{I})(\mathcal{D} - \varepsilon_2 \mathcal{I}).
\end{aligned}
\end{equation}

Now, we show that $\mathcal{G}(u,b)$ satisfies an integro-differential equation.

Theorem 3.3. For $0 < u < b$ and $-1 < \theta < 1$, the distribution of the maximum surplus before ruin $\mathcal{G}(u,b)$ satisfies the following equation
\begin{equation}
\begin{aligned}
\Delta_1(\mathcal{D})\Delta_2(\mathcal{D})\mathcal{G}(u,b) + \frac{2\lambda}{\sigma^2} \Delta_2(\mathcal{D})(\gamma_1(u) - \theta \gamma_2(u)) + \frac{4\lambda\theta}{\sigma^2} \Delta_1(\mathcal{D})\gamma_2(u) = 0,
\end{aligned}
\end{equation}

with the boundary conditions
\begin{equation}
\begin{aligned}
\mathcal{G}(0,b) = 1, \\
\mathcal{G}''(0,b) + \frac{2c}{\sigma^2} \mathcal{G}'(0,b) = -\frac{2\lambda}{\sigma^2} + \varepsilon_1^2, \\
\end{aligned}
\end{equation}

where
\begin{equation}
\begin{aligned}
\gamma_1(u) &= \int_0^u \mathcal{G}(u-x,b)f(x)dx + \int_u^\infty f(x)dx, \\
\gamma_2(u) &= \int_0^u \mathcal{G}(u-x,b)h(x)dx + \int_u^\infty h(x)dx.
\end{aligned}
\end{equation}

Proof. By conditioning on the time and the amount of the first claim, we derive the following equations for $\mathcal{G}_c(u,b)$ and $\mathcal{G}_o(u,b)$, respectively,
\begin{equation}
\begin{aligned}
\mathcal{G}_c(u,b) &= \int_0^\infty \int_0^u \mathcal{G}_c(u-y-b)P(u-b < \bar{H}(t) < u, H(t) \in dy) \times \\
& \quad \mathcal{G}_c(u-y-x,b)f_{X,W}(x,t)dxdt + \\
& \quad \int_0^\infty \int_0^u \mathcal{G}_c(u-y-b)P(u-b < \bar{H}(t) < u, H(t) \in dy) f_{X,W}(x,t)dxdt \\
& = \int_0^u \int_0^u \mathcal{G}_c(u-y-x,b)h(u,b,y,x)dxdy + \int_0^\infty \int_0^u \mathcal{G}_c(u-y-b)h(u,b,y,x)dxdy,
\end{aligned}
\end{equation}

and
\begin{equation}
\begin{aligned}
\mathcal{G}_o(u,b) &= \int_0^\infty \int_0^u \mathcal{G}_o(u-y-b)P(u-b < \bar{H}(t) < u, H(t) \in dy) \times \\
& \quad \mathcal{G}_o(u-y-x,b)f_{X,W}(x,t)dxdt + E[I(\varpi_u < W)] \\
& = \int_0^u \int_0^u \mathcal{G}_o(u-y-x,b)h(u,b,y,x)dxdy + E[I(\varpi_u < W)].
\end{aligned}
\end{equation}
Since \( W \) is exponentially distributed with rate \( \lambda \) and independent of \( H(t) \), in view of (1), we have
\[
E[I(\bar{w}_u < W)] = EE[I(\bar{w}_u < W)|H(t)] = e^{-\lambda \bar{w}_u} = e^{-\xi_t u}.
\]

Adding (18) to (19), we obtain
\[
\begin{align*}
G(u, b) &= \int_0^\infty \int_0^u \int_u^\infty P(u - b < H(t) < u, H(t) \in dy) \\
&\quad \times \int_0^\infty \int_u^\infty P(u - b < H(t) < u, H(t) \in dy) f_X(x, t) dx dt \\
&\quad + e^{-\xi_t u}.
\end{align*}
\]

Substituting (13) and (14) into (20) yields
\[
\begin{align*}
G(u, b) &= \frac{\xi_1 \xi_2}{\xi_1 + \xi_2} \int_0^u e^{-\xi_1 u} - e^{-(\xi_1 + \xi_2)u} \gamma_2(u - y) dy \\
&\quad + \frac{\xi_1 \xi_2}{\xi_1 + \xi_2} \int_0^u e^{-\xi_1 u - e^{-\xi_1 + \xi_2}u} \gamma_2(u - y) dy \\
&\quad + \frac{\xi_1 \xi_2}{\xi_1 + \xi_2} \int_0^u e^{-\xi_1 u - e^{-\xi_1 + \xi_2}u} \gamma_2(u - y) dy \\
&\quad + e^{-\xi_t u}.
\end{align*}
\]

Using the change of variable \( s = u - y \) and then rearrange it, which results in
\[
\begin{align*}
G(u, b) &= \frac{\xi_1 \xi_2}{\xi_1 + \xi_2} \left[ \int_0^u e^{-\xi_1 u} \gamma_2(s) ds - \int_0^\infty e^{-\xi_1 u - e^{-\xi_1}u} \gamma_2(s) ds \right] \\
&\quad + \frac{\xi_1 \xi_2}{\xi_1 + \xi_2} \left[ \int_0^u e^{-\xi_1 u} \gamma_2(s) ds + \int_0^\infty e^{-\xi_1 u - e^{-\xi_1}u} \gamma_2(s) ds \right] + e^{-\xi_t u}.
\end{align*}
\]

Since
\[
\begin{align*}
(D + \xi_1 I) \int_0^u e^{-\xi_1 u} \gamma_2(s) ds &= \gamma_2(u) - \gamma_2(u), \\
(D - \xi_2 I) \int_0^u e^{\xi_2 u} \gamma_2(s) ds &= -\gamma_2(u) + \gamma_2(u), \\
(D + \xi_1 I) \int_0^\infty e^{-\xi_1 u - e^{-\xi_1}u} \gamma_2(s) ds &= 0, \\
(D + \xi_1 I) \int_0^\infty e^{-\xi_1 u - e^{-\xi_1}u} \gamma_2(s) ds &= 0, \\
(D - \xi_2 I) \int_0^\infty e^{\xi_2 u} \gamma_2(s) ds &= 0, \\
(D + \xi_1 I) \int_0^\infty e^{-\xi_1 u - e^{-\xi_1}u} \gamma_2(s) ds &= 0.
\end{align*}
\]

Applying the operator \( D_1(D)D_2(D) \) to (21), this leads to (15).

The boundary condition \( G(0, b) = 1 \) can be obtained by setting \( u = 0 \) in (21).

Differentiating (21) w.r.t. \( u \), we obtain
\[
\begin{align*}
G'(u, b) &= \frac{\xi_1 \xi_2}{\xi_1 + \xi_2} \left[ -\xi_1 \int_0^u e^{-\xi_1 u} \gamma_2(s) ds + \xi_1 \int_0^\infty e^{-\xi_1 u - e^{-\xi_1}u} \gamma_2(s) ds \right] \\
&\quad + \frac{\xi_1 \xi_2}{\xi_1 + \xi_2} \left[ -\xi_1 \int_0^u e^{-\xi_1 u} \gamma_2(s) ds + \xi_1 \int_0^\infty e^{-\xi_1 u - e^{-\xi_1}u} \gamma_2(s) ds \right] + \xi_1 \int_0^\infty e^{-\xi_1 u - e^{-\xi_1}u} \gamma_2(s) ds - \xi_1 e^{-\xi_t u}.
\end{align*}
\]

Letting \( u = 0 \) in (22) leads to
\[ G'(0, b) = \frac{2\lambda}{\sigma^2} \int_0^\infty e^{-\xi z s}(\gamma_1(s) - \theta \gamma_2(s))ds + \frac{4\lambda \theta}{\sigma^2} \int_0^\infty e^{-\xi z s} \gamma_2(s)ds - \xi_1. \] 

(23)

Differentiating (22) w.r.t. \( u \) again, then let \( u = 0 \), we have
\[ G''(0, b) = -\frac{4c}{\sigma^2} \int_0^\infty e^{-\xi z s}(\gamma_1(s) - \theta \gamma_2(s))ds - \frac{8mc}{\sigma^2} \int_0^\infty e^{-\xi z s} \gamma_2(s)ds, \]
which together with (23) leads to
\[ G''(0, b) + \frac{2c}{\sigma^2} G'(0, b) = -\frac{2\lambda}{\sigma^2} (\gamma_1(0) + \theta \gamma_2(0)) + \xi_1^2. \]

(25)

Noting that \( \gamma_1(0) = 1 \) and \( \gamma_2(0) = 0 \), we obtain the boundary condition
\[ G''(0, b) + \frac{2c}{\sigma^2} G'(0, b) = -\frac{2\lambda}{\sigma^2} + \xi_1^2. \]

(26)

Next, define
\[ \varrho_b = \inf \{ t > 0 : R(t) \geq b \mid R(0) = u \}, \quad 0 \leq u \leq b \]
to be the first time that the surplus process (1) upcrosses the level \( b \), and
\[ \chi(u, b) = P(T > \varrho_b \mid R(0) = u) \]
to be the probability that the surplus process (1) attains a given level \( b \geq u \) from initial surplus \( u \) without first falling below zero. Specially, \( \chi(u, b) = \phi(u) \) when \( b \to \infty \). Since eventually either ruin occurs without the surplus process attaining \( b \) or the surplus attains level \( b \), we get \( \chi(u, b) = 1 - \chi(u, b) \). It is easy to obtain the following result for \( \chi(u, b) \) from Theorem 3.3.

**Corollary 1.** For \( 0 \leq u < b \) and \(-1 \leq \theta \leq 1\), \( \chi(u, b) \) satisfies the following integro-differential equation
\[ \Delta_1(\mathcal{D}) \Delta_2(\mathcal{D}) \chi(u, b) + \frac{2\lambda}{\sigma^2} \Delta_2(\mathcal{D})(\eta_1(u) - \theta \eta_2(u)) + \frac{4\lambda \theta}{\sigma^2} \Delta_1(\mathcal{D}) \eta_2(u) = 0, \]

(27)

with the boundary conditions
\[ \chi(0, b) = 0, \quad \chi''(0, b) + \frac{2c}{\sigma^2} \chi'(0, b) = \frac{2\lambda}{\sigma^2} - \xi_1^2, \]

where
\[ \eta_1(u) = \int_0^u \chi(u - x, b) f(x)dx \quad \text{and} \quad \eta_2(u) = \int_0^u \chi(u - x, b) h(x)dx. \]

4. Solution for \( \chi(u, b) \).

4.1. Laplace transforms. We relax the constraint \( 0 \leq u < b \) in (27) and consider the case of \( 0 \leq u \). Let \( v(u) = \lim_{b \to +\infty} \chi(u, b) \), it follows that,
\[ \Delta_1(\mathcal{D}) \Delta_2(\mathcal{D}) v(u) + \frac{2\lambda}{\sigma^2} \Delta_2(\mathcal{D})(\kappa_1(u) - \theta \kappa_2(u)) + \frac{4\lambda \theta}{\sigma^2} \Delta_1(\mathcal{D}) \kappa_2(u) = 0, \quad u \geq 0, \]

(28)

where
\[ \kappa_1(u) = \int_0^u v(u - x) f(x)dx \quad \text{and} \quad \kappa_2(u) = \int_0^u v(u - x) h(x)dx. \]
Using the properties of Laplace transform and the boundary conditions, we have

\[
\int_0^\infty e^{-su} \Delta_1(D) \Delta_2(D) v(u) du = \Delta_1(s) \Delta_2(s) \tilde{v}(s) - s^3 - s^2 [v'(0) + \frac{2s}{\sigma^2}] - s [v''(0) + \frac{4s}{\sigma^2} v'(0) + \frac{4s^2}{\sigma^2} - \frac{6\lambda}{\sigma^2}]
\]
\[
- v'''(0) - \frac{4s}{\sigma^2} v''(0) - \left( \frac{4s^2}{\sigma^2} - \frac{6\lambda}{\sigma^2} \right) v'(0) + \frac{12\lambda}{\sigma^2} \theta
\]
\[
= \Delta_1(s) \Delta_2(s) \tilde{v}(s) - s^3 - v'(0) \left( s^2 + \frac{2s}{\sigma^2} \right) - s^2 \frac{4s}{\sigma^2} - s \left( \frac{4s^2}{\sigma^2} - \frac{6\lambda}{\sigma^2} + \xi^2 \right)
\]
\[
- v'''(0) - \frac{4s}{\sigma^2} v''(0) - \left( \frac{4s^2}{\sigma^2} - \frac{6\lambda}{\sigma^2} \right) v'(0) + \frac{12\lambda}{\sigma^2} \theta
\]
and

\[
\int_0^\infty e^{-su} \frac{4\lambda}{\sigma^2} \Delta_1(D) \kappa_2(u) du = \frac{4\lambda}{\sigma^2} \Delta_1(s) \tilde{\kappa}_2(s) - \frac{4\lambda}{\sigma^2} \kappa_2'(0)
\]
\[
= \frac{4\lambda}{\sigma^2} \Delta_1(s) \tilde{h}(s) \tilde{v}(s) - \frac{4\lambda}{\sigma^2} \kappa_2'(0).
\]

Taking the Laplace transform on both sides of (28) and combining (29)-(31) leads to

\[
\tilde{v}(s) = \frac{\tilde{\beta}_1(s) - \tilde{\beta}_2(s)}{\alpha_1(s) - \alpha_2(s)},
\]
where

\[
\tilde{\beta}_1(s) = \frac{\sigma^2}{2} v'(0) \left( \frac{\sigma^2}{2} s^2 + cs \right) + \frac{\sigma^2}{4} v''(0) + c\sigma^2 v'''(0)
\]
\[
+ \left( c^2 - \frac{3\lambda\sigma^2}{2} \right) v'(0) + \frac{\lambda\sigma^2}{2} (\kappa'_1(0) + \theta \kappa'_2(0)) - 2c\lambda,
\]
\[
\tilde{\beta}_2(s) = - \frac{\sigma^4}{4} s^3 - c\sigma^2 s^2 - s \left( c^2 - \frac{3\lambda\sigma^2}{2} + \frac{\sigma^4}{4} \xi^2 \right),
\]
\[
\tilde{\alpha}_1(s) = \left( \frac{\sigma^2}{2} s^2 + cs - \lambda \right) \left( \frac{\sigma^2}{2} s^2 + cs + 2\lambda \right),
\]
and

\[
\tilde{\alpha}_2(s) = \lambda \tilde{f}(s) (2\lambda - \frac{\sigma^2}{2} s^2 + cs) - \lambda \tilde{h}(s) \left( \frac{\sigma^2}{2} s^2 + cs \right).
\]

By virtue of Lemma 1 in [24], when \( \theta \neq 0 \), Lundberg’s equation \( \tilde{\alpha}_1(s) - \tilde{\alpha}_2(s) = 0 \) has exactly one root which has a positive real part, say \( \rho_1 \) and the second root \( \rho_2 = 0 \). Assuming that \( \tilde{v}(s) \) is analytic, \( \rho_1, \rho_2 \) must also be roots of the numerator of (32). In terms of \( \frac{\sigma^2}{2} s^2 + cs \), \( \tilde{\beta}_1(s) \) is a polynomial of degree 1. By the Lagrange interpolating formula, (33) can be rewritten as

\[
\tilde{\beta}_1(s) = \frac{L_2(s)}{L_2(\rho_1)} \tilde{\beta}_2(\rho_1) + \frac{L_1(s)}{L_1(\rho_2)} \tilde{\beta}_2(\rho_2),
\]
where \( L_i(s) = \frac{\sigma^2}{2} s^2 + cs - \frac{\sigma^2}{2} \rho_i^2 - c\rho_i, i = 1, 2. \)
4.2. Renewal equation. In the present subsection, we derive the renewal equation for \( v(u) \). Above all, we recall the Dickson-Hipp operator which will be useful for obtaining the main results. A complex operator of an integrable real-valued function \( f \) is defined as

\[
\mathcal{T}_r f(x) = \int_x^\infty e^{-r(x-u)} f(u) du, \quad r \in \mathbb{C}, \ x \geq 0,
\]

where \( r \) has a non-negative real part, \( \Re(r) \geq 0 \). [20] provide some properties of the operator \( \mathcal{T}_r \), for example,

\[
\mathcal{T}_r f(0) = \int_0^\infty e^{-ru} f(u) du = \tilde{f}(r), \ r \in \mathbb{C},
\]

is the Laplace transform of \( f \), and

\[
\mathcal{T}_r \mathcal{T}_s f(x) = \mathcal{T}_s \mathcal{T}_r f(x) = \frac{\mathcal{T}_s f(x) - \mathcal{T}_r f(x)}{r - s}, \ s \neq r, \ x \geq 0.
\]

Furthermore, we have for \( s \neq r, n \in \mathbb{N} \),

\[
\frac{r^n T_{r} - s^n T_s}{r - s} = \frac{r^n T_{r} - r^n T_r + r^n T_r - s^n T_s}{r - s} = r^n T_s T_r - \sum_{k=1}^{n} s^{n-k} r^k T_s, \quad (\text{define } \sum_{k=1}^{0} \cdot = 0).
\]

Since \( \rho_1, \rho_2 \) are the roots of \( \tilde{\alpha}_1(s) - \tilde{\alpha}_2(s) = 0 \), and \( \tilde{\alpha}_1(s) - L_1(s)L_2(s) \) is a polynomial of \( \frac{s^2}{r} \sigma^2 + cs \) with degree 1, in view of the Lagrange interpolating formula, we get

\[
\tilde{\alpha}_1(s) - L_1(s)L_2(s) = \frac{L_2(s)}{L_2(\rho_1)} \tilde{\alpha}_2(\rho_1) + \frac{L_1(s)}{L_1(\rho_2)} \tilde{\alpha}_2(\rho_2),
\]

which leads to

\[
\begin{align*}
\tilde{\alpha}_1(s) - \tilde{\alpha}_2(s) & = L_1(s)L_2(s) + \frac{L_2(s)}{L_2(\rho_1)} (\tilde{\alpha}_2(\rho_1) - \tilde{\alpha}_2(s)) + \frac{L_1(s)}{L_1(\rho_2)} (\tilde{\alpha}_2(\rho_2) - \tilde{\alpha}_2(s)) \\
& = L_1(s)L_2(s) \left( 1 - \frac{\tilde{\alpha}_2(s) - \tilde{\alpha}_2(\rho_1)}{L_1(s)L_2(\rho_1)} \frac{\tilde{\alpha}_2(s) - \tilde{\alpha}_2(\rho_2)}{L_2(s)L_1(\rho_2)} \right) \\
& = L_1(s)L_2(s) \left( 1 - \frac{\tilde{\alpha}_2(s) - \tilde{\alpha}_2(\rho_1)}{\frac{L_1(s)L_2(\rho_1)}{L_2(s)L_1(\rho_2)}} \right).
\end{align*}
\]

From (36), it follows that for \( i = 1, 2 \)

\[
\begin{align*}
\frac{\tilde{\alpha}_2(s) - \tilde{\alpha}_2(\rho_i)}{s - \rho_i} & = \frac{\sigma^2 s^2 (T_{\rho_i}(f(0) + \theta h(0)) - \rho_i^2 (T_{\rho_i}(f(0) + \theta h(0)))]}{\rho_i - s} \\
& \quad + \frac{c \lambda [s (T_{\rho_i}(f(0) + \theta h(0)) - \rho_i (T_{\rho_i}(f(0) + \theta h(0)))]}{\rho_i - s} \\
& \quad - \frac{2 \lambda^2 (T_{\rho_i}(f(0) - \rho_i f(0))}{\rho_i - s} \\
& = \mathcal{T}_s \Gamma_i(u) - \lambda \left[ \frac{\sigma^2}{2} (s + \rho_i) + c \mathcal{T}_s (f(0) + \theta h(0)) \right],
\end{align*}
\]

in the second step, we use (38), where

\[
\Gamma_i(u) = \left( \frac{\sigma^2}{2} \rho_i^2 + c \rho_i \right) \lambda \mathcal{T}_s (f(0) + \theta h(u)) - 2 \lambda^2 \mathcal{T}_{\rho_i} f(u).
\]

Substituting (40) into (39) and noting that \( L_1(\rho_2) + L_2(\rho_1) = 0 \), we have

\[
\begin{align*}
\tilde{\alpha}_1(s) - \tilde{\alpha}_2(s) & = L_1(s)L_2(s) \left( 1 - \frac{\mathcal{T}_s \Gamma_1(0)}{\frac{L_1(s)L_2(\rho_1)}{L_2(s)L_1(\rho_2)}} - \frac{\mathcal{T}_s \Gamma_2(0)}{\frac{L_1(s)L_2(\rho_2)}{L_2(s)L_1(\rho_2)}} \right).
\end{align*}
\]
At the same time, we get by using (37),
\[
\beta_1(s) - \beta_2(s) = \frac{L_2(s)}{L_2(p_1)} \beta_2(p_1) - \frac{L_1(s)}{L_2(p_2)} \beta_2(p_2) + \frac{L_1(s)}{L_2(p_1)} \beta_2(p_1) - \frac{L_1(s)}{L_2(p_2)} \beta_2(p_2) \quad (43)
\]
Similarly,
\[
\frac{\beta_2(p_1) - \beta_2(p_2)}{s - p_i} = \frac{\sigma^2}{2} \left( \frac{\sigma^2}{2} \rho_i^2 + c \rho_i + \frac{\sigma^2}{2} \xi_1^2 - 3 \lambda \right) + \left( \frac{\sigma^2}{2} (s + \rho_i) + c \right) \left( \frac{\sigma^2}{2} s + c \right).
\]
Substituting (44) into (43) results in
\[
\beta_1(s) - \beta_2(s) = \frac{L_1(s) L_2(s)}{L_2(p_1)} \left( \frac{\sigma^2}{2} \left( \frac{\sigma^2}{2} \rho_1^2 + c \rho_1 + \frac{\sigma^2}{2} \xi_1^2 - 3 \lambda \right) + \left( \frac{\sigma^2}{2} (s + \rho_1) + c \right) \left( \frac{\sigma^2}{2} s + c \right) \right).
\]
By substituting (42) into (32), we have the following result
\[
\text{Theorem 4.1. The Laplace transform of } v(u) \text{ can be expressed as}
\]
\[
\hat{v}(
) = \frac{\sigma^2}{2} \left( \frac{\sigma^2}{2} \rho_1^2 + c \rho_1 + \frac{\sigma^2}{2} \xi_1^2 - 3 \lambda \right) + \left( \frac{\sigma^2}{2} (s + \rho_1) + c \right) \left( \frac{\sigma^2}{2} s + c \right),
\]
where \( \Gamma_i(\cdot), i = 1, 2 \) are given by (41), respectively.

Inverting the Laplace transform (46) gives
\[
\text{Theorem 4.2. The solution } v(u) \text{ to the homogeneous integro-differential equation (28) admits a defective renewal equation representation}
\]
\[
v(u) = \int_0^u v(u - x) \vartheta(x) dx + \mathcal{Z}(u), \quad u \geq 0,
\]
where
\[
\vartheta(x) = \frac{\Gamma_1(x) \ast e^{-(s + \frac{\sigma^2}{2})x}}{\sigma^2 L_2(p_1)} + \frac{\Gamma_2(x) \ast e^{-(s + \frac{\sigma^2}{2})x}}{\sigma^2 L_1(p_2)},
\]
and
\[
\mathcal{Z}(u) = \left( \frac{\sigma^2}{2} \rho_1^2 + c \rho_1 + \frac{\sigma^2}{2} \xi_1^2 - 3 \lambda \right) \frac{e^{-(s + \frac{\sigma^2}{2})u}}{L_2(p_1)}
\]
\[
\lambda \hat{h}(s) \left( \frac{\sigma^2}{2} \rho_1^2 + c \rho_1 + \frac{\sigma^2}{2} \xi_1^2 - 3 \lambda \right) \frac{e^{-(s + \frac{\sigma^2}{2})u}}{L_2(p_1)}
\]
and \( \ast \) represents the convolution operator.

Proof. Inverting Laplace transform on both sides of (46) yields the expression (47).

Next we show (47) is a defective renewal equation.

Considering the following equation for \( \delta \geq 0 \),
\[
\left( \frac{\sigma^2}{2} s^2 + cs - \lambda - \delta \right) \left( \frac{\sigma^2}{2} s^2 + cs - 2 \lambda - \delta \right) -
\lambda f(s) \left( 2 \lambda + \delta - \frac{\sigma^2}{2} s^2 - cs \right) = 0.
\]
When $\delta = 0$, (50) becomes equation $\tilde{\alpha}_1(s) - \tilde{\alpha}_2(s) = 0$. Letting $\rho_2 = \rho_2(\delta)$, we have $\rho_2(\delta) = 0$ when $\delta \to 0$. Setting $s = \rho_2(\delta)$ in (50), then differentiating (50) w.r.t. $\delta$, we get

$$\rho'(0) = \frac{1}{c - \lambda E(X)} > 0,$$

due to the positive safety loading (6). From (42), we have

$$\tilde{\vartheta}(s) = 1 - \frac{\tilde{\alpha}_1(s) - \tilde{\alpha}_2(s)}{L_1(s)L_2(s)}.$$

Thus

$$\int_0^\infty \vartheta(x)dx = \tilde{\vartheta}(0) = 1 - \frac{\tilde{\alpha}_1(0) - \tilde{\alpha}_2(0)}{L_1(0)L_2(0)} = \lim_{\delta \to 0} \left[ 1 - \frac{(2\lambda + \delta)}{(2\lambda)^2 [\rho_2(\delta)^2 + c\rho_2(\delta)]} \right]$$

$$= \lim_{\delta \to 0} \left[ 1 - \frac{2(\lambda - \lambda E(X))}{(2\lambda)^2 c} \right] < 1.$$

The fourth step follows from L’Hospital’s rule. This completes the proof. \qed

5. Exponentially distributed claims. Now, we assume that the claim sizes are exponentially distributed with density function $f(x) = \mu e^{-\mu x}$, then, $f(s) = \frac{\mu}{s + \mu}$. It follows that $h(x) = 2\mu e^{-2\mu x} - \mu e^{-\mu x}, x \geq 0$, and $\tilde{h}(s) = \frac{2\mu}{s + 2\mu} - \frac{\mu}{s + \mu}$.

From (41), we have for $i = 1, 2$

$$\Gamma_i(u) = \Xi_{i,1} e^{-\mu u} + \Xi_{i,2} e^{-2\mu u},$$

where $\Xi_{i,1} = \lambda \mu [(1-\theta)(\frac{\sigma^2}{4} \rho_1^2 + c\rho_1) - 2\lambda]$ and $\Xi_{i,2} = \frac{2 \lambda \theta \mu (\frac{\sigma^2}{4} \rho_1^2 + c\rho_1)}{\rho_1 + 2\mu}$.

Thus

$$\hat{T}_s \Gamma_i(0) = \frac{\Xi_{i,1}}{s + \mu} + \frac{\Xi_{i,2}}{s + 2\mu}, \quad i = 1, 2.$$

Substituting (52) into (46), then multiplying both the numerator and denominator of (46) by $L_2(\rho_1)(s + \mu)(s + 2\mu)[\frac{\sigma^2}{4}(s + \rho_1) + c][\frac{\sigma^2}{4}(s + \rho_2) + c]$ and noting that $L_1(\rho_2) = -L_2(\rho_1)$, which yields

$$\tilde{\vartheta}(s) = \frac{Q_1(s)}{Q_2(s)},$$

where

$$Q_1(s) = \frac{\sigma^2}{4}(s + \mu)(s + 2\mu)[\frac{\sigma^2}{4}(s + \rho_1) + c]\left(\frac{\sigma^2}{4} \rho_1^2 + c\rho_1 + \frac{\sigma^2}{4} \xi_1^2 - 3\lambda\right) - \frac{\sigma^2}{4}(s + \mu)(s + 2\mu)[\frac{\sigma^2}{4}(s + \rho_1) + c]\left(\frac{\sigma^2}{4} \rho_2^2 + c\rho_2 + \frac{\sigma^2}{4} \xi_1^2 - 3\lambda\right),$$

and

$$Q_2(s) = L_2(\rho_1)(s + \mu)(s + 2\mu)[\frac{\sigma^2}{4}(s + \rho_1) + c]\left(\frac{\sigma^2}{4} \rho_1^2 + c\rho_1 + \frac{\sigma^2}{4} \xi_1^2\right) - (s + \mu)(s + 2\mu)[\frac{\sigma^2}{4}(s + \rho_2) + c]\left(\frac{\sigma^2}{4} \rho_2^2 + c\rho_2 + \frac{\sigma^2}{4} \xi_1^2\right) + (s + \mu)(s + 2\mu)[\frac{\sigma^2}{4}(s + \rho_1) + c]\left(\frac{\sigma^2}{4} \rho_1^2 + c\rho_1 + \frac{\sigma^2}{4} \xi_1^2\right).$$

Clearly, the numerator $Q_1(s)$ is a polynomial of degree 3 and the denominator $Q_2(s)$ is a polynomial of degree 4. It is easy to see that the equation $Q_2(s) = 0$ has four
roots with nonnegative real part, consequently, we have

\[ Q_2(s) = \frac{\sigma^4}{4} L_2(\rho_1) \prod_{j=1}^{4} (s + R_j), \]

where \( R_j, j = 1, 2, 3, 4 \) have positive real parts, which are assumed to be different from each other.

By the partial fraction decomposition, we get

\[ \tilde{v}(s) = \frac{Q_1(s)}{Q_2(s)} = \sum_{j=1}^{4} \frac{N_j}{s + R_j}, \]

where

\[ N_j = \frac{\sigma^4}{4} L_2(\rho_1) \frac{Q_1(-R_j)}{\prod_{i=1, i\neq j}^{4} (R_i - R_j)}. \]

By inverting (55), we obtain the following theorem.

**Theorem 5.1.** If the claim amount is exponentially distributed, then \( v(u) \) is given by

\[ v(u) = \sum_{j=1}^{4} N_j e^{-R_j u}, \quad u \geq 0, \]

where \( N_j, j = 1, 2, 3, 4 \) are defined by (56).

Thus,

\[ \chi(u, b) = v(u), \quad 0 \leq u < b. \]

Finally, the explicit result for the distribution of the maximum surplus before ruin is obtained by \( G(u, b) = 1 - \chi(u, b), 0 \leq u < b. \)

**Example 1.** In this example, we assume that the claim sizes are exponentially distributed \( \text{Exp}(\mu) \) with \( \mu = 1 \). Set \( c = 3, \lambda = 1, \sigma = 5 \). For different dependence parameters \( \theta = -1, -0.5, 0, 0.5, 1 \), the explicit expressions for the maximum surplus before ruin \( G(u, b), 0 \leq u \leq b, \) are given by

\[ G_{\theta=-1}(u, b) = 0.5102e^{-0.1341u} + 0.2906e^{-0.4863u} + 0.2411e^{-1.201u} - 0.0419e^{-1.945u}, \]
\[ G_{\theta=-0.5}(u, b) = 0.486e^{-0.1404u} + 0.3427e^{-0.51u} + 0.1899e^{-1.148u} - 0.0189e^{-1.974u}, \]
\[ G_{\theta=0}(u, b) = 0.4169e^{-0.4163u} + 0.1328e^{-1.094u} + 0.4053e^{-0.5376u}, \]
\[ G_{\theta=0.5}(u, b) = 0.4382e^{-0.1518u} + 0.4871e^{-0.571u} + 0.0593e^{-1.036u} + 0.0154e^{-2.024u}, \]
\[ G_{\theta=1}(u, b) = 0.4152e^{-0.1506u} + 0.6156e^{-0.6139u} - 0.0592e^{-0.9713u} + 0.0284e^{-2.046u}. \]

Now, we fix \( b = 10 \), Figure 1 shows the behavior of the maximum surplus before ruin \( G(u, 10) \). We can see that the dependent parameter \( \theta \) has a clear impact on the maximum surplus before ruin \( G(u, 10) \).

From Figure 1, it shows that the probability \( G(u, b) \) increases (decreases) as the dependence parameter \( \theta \) decreases (increases) for a fixed value of initial surplus \( u \). In Example 5.1, we can obtain that the covariance between \( X \) and \( W \) is \( \text{Cov}(W, X) = \frac{\sigma}{\pi} \) by using the joint p.d.f of \( (X, W) \). This manifests that the probability of having an important claim increase (decreases) as the time elapse since the last claim increase (decreases) when \( \theta > 0(< 0) \). Thus, the probability that the insurance company has enough premium income to pay the claim is higher (lower). It indicates that the probability of the event that the maximum surplus before ruin is less
than \( b \) starting from initial surplus \( u \) is lower when \( \theta > 0 \). Furthermore, when the positive (negative) dependence relation becomes stronger the impact on the probability \( \mathcal{G}(u, b) \) is more significant.

6. Conclusions. [16] investigates the distribution of the maximum surplus before ruin in an extensive compound Poisson risk model in which the claim sizes and inter-claim times are dependent via a FGM copula. In this paper, we generalize the corresponding results to the jump-diffusion risk model. By using a potential measure, we obtain the integro-differential equation for the distribution of the maximum surplus before ruin. This distribution can be calculated through the probability that the surplus process attains a given level from the initial surplus without first falling below zero. Some explicit results are given when the claims are exponentially distributed.

The maximum surplus before ruin for other dependence structures can also be considered by the same techniques. For example, we can use the generalized Farlie-Gumbel-Morgenstern copula to model the dependence risk model with perturbed by diffusion. At the same time, some related ruin measures, including the maximum severity of ruin, can be considered in dependence risk model by utilizing the relation between these ruin measures and the maximum surplus before ruin.

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