Effects of Surface Roughness on Conical Squeeze Film Bearings with Micropolar fluid

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Abstract: In the current paper, a hypothetical analysis of the impact of surface roughness on squeeze film lubrication of rough conical bearing using Micropolar fluid is examined using Eringen’s Micropolar fluid model. The generalized averaged Reynolds type equation for roughness has been determined analytically using the Christensen’s stochastic theory of roughness effects and the closed form expressions are obtained for the fluid film pressure, load carrying capacity and squeezing time. Further, the impacts of surface roughness using micropolar fluids on the squeeze film lubrication of rough conical bearings has been discussed and according to the outcomes arrived, pressure, load carrying capacity and squeezing time increases for azimuthal roughness pattern and decreases for radial roughness patterns comparatively to the smooth case.

1. Introduction

In present days, the investigation of squeeze film phenomena between two approaching surfaces is essential in many engineering applications and other allied areas. With the growth of recent instrumental developments, the prominence of utilizing many fluids as lubricants under distinct conditions has been highlighted. The usage of added substances in lubricant increases the performance of the lubricants and displays non-Newtonian behaviour. To define the additive effects, a number of Microcontinuum theories have been proposed [1-3].Eringen’s [4] suggested the study of fluids with suspension in nature.

In several studies [5,6] it is explored that usage of base oil in long chain added substances improves the properties of lubricant and to minimize the friction and surface destruction. Zaheeruddin et al.[7] in their study found that the load carrying capacity increases with reducing the friction due to the presence of base oil. Naduvinamani and Santosh et al.[8] found that, effect of micropolar fluid significantly enhances the characteristics of bearings. Tsai-Wang Huang et al.[9] observed that, the impact of Micropolar fluid improves load carrying capacity and reduces the friction. Naduvinamani et al.[10] found that, impact of viscosity variation of Micropolar fluid on short journal bearings improves the load carrying capacity.

In recent years, enormous studies have been devoted to the investigation of impact of surface roughness. Various experts have considered the impact of surface roughness on hydrodynamic lubrication of bearings, mainly on the grounds that, in mechanical practice, the various part of the bearing has rough surfaces. Hence, several investigations have been done to understand the impact of surface roughness on the performance quality of hydrodynamic lubrications. Naduvinamani et al.[11] found that, influence of Micropolar fluid significantly enhances the pressure and load capacity associated to the related Newtonian case. Naduvinamani et al.[12] studied that, impact of viscosity
variation in Micropolar fluid of rough journal bearings maximizes the load carrying capacity and pressure and squeezing time reduces because of viscosity variation.

In the current paper, an effort has been done to study the impact of surface roughness on conical squeeze film bearings with Micropolar fluid.

2. **Mathematical Analysis**

Figure-1 demonstrates the physical configuration of geometry between conical bearings lubricated with micropolar fluid having normal velocity $V = \frac{dH}{dt}$, in which $a$ is radius of the cone having an angle $2\theta$ and the film height in the direction of cone axis is $H$.

![Figure 1: Schematic diagram of the problem](image)

The flow of Micropolar lubricant under the standard consideration of lubrication theory are governed by the following basic equations,

Conservation of linear momentum:

$$\left(\mu + \frac{\chi}{2}\right) \frac{\partial^2 u}{\partial y^2} + \chi \frac{\partial v_1}{\partial y} = \frac{\partial p}{\partial x}$$

(1)

Conservation of annular momentum:

$$\gamma \frac{\partial^2 v_1}{\partial y^2} - \chi \frac{\partial u}{\partial y} - 2\chi v_1 = 0$$

(2)

Conservation of mass:

$$\frac{1}{x} \frac{\partial (xu)}{\partial x} + \frac{\partial v}{\partial y} = 0$$

(3)

Where $p$ is film pressure, $(u,v)$ are the velocity components in $x$ and $y$ directions, $v_1$ is the Microrotational velocity component, $\mu$ is the Newtonian viscosity coefficient, $\gamma$ and $\chi$ are the viscosity coefficient for Micropolar fluids.
The significant boundary conditions are given as:

\[ u = 0 \] at \( y = 0 \) and at \( y = H \sin \theta \) (4a)

\[ v = 0 \] at \( y = 0 \) (4b)

\[ v = \sin \theta \frac{dH}{dy} \] at \( y = H \sin \theta \) (4c)

Integrating equation (1) and (3) and using the boundary conditions given in (4a), (4b) and (4c) the solution is obtained in the form

\[
\{ \begin{array}{l}
\frac{p}{\mu} \left[ \frac{y^2}{2} - \frac{N^2 H \sin \theta (\cosh m y - 1)}{m \sinh H \sin \theta} \right] + \\
\frac{D_i}{(1 - N^2)} \left[ y - \frac{N^2}{m} \left( \sinh m y - (\cosh m y - 1) \frac{\cosh m H \sin \theta - 1}{\sinh H \sin \theta} \right) \right]
\end{array} \}
\] (5)

and

\[
v_i = \frac{D_i}{2(1 - N^2)} (\cosh m y - 1) + \\
\frac{\sinh m y}{\sinh H \sin \theta} \left[ \frac{H \sin \theta p^1}{2 \mu} - \frac{D_i}{2(1 - N^2)} (\cosh m H \sin \theta - 1) \right] - \frac{y}{2 \mu} p^1
\] (6)

Where

\[
D_i = \frac{(1 - N^2)}{2} \left[ \frac{H \sin \theta p^1}{2 \mu} \right] \quad \text{and} \quad m = \frac{N}{k}, \quad k = \left( \frac{\gamma}{4 \mu} \right)^{\frac{1}{2}}
\]

Integrating the equation (3) with respect to \( y \) over \( H \) between 0 to \( H \sin \theta \), the modified Reynolds equation is derived as

\[
\frac{1}{x} \frac{\partial}{\partial x} \left[ f(N,k,H,\theta) x \frac{dp}{dx} \right] = 12 \mu \sin \theta \frac{dH}{dy}
\] (7)

where \( f(N,k,H,\theta) = H \sin \theta + 12k^2 H \sin \theta - 6NH^2 \sin \theta \cosh \left( \frac{NH \sin \theta}{2k} \right) \)

Let \( f(h_s) \) be the probability density function of the stochastic film thickness \( h_s \). Taking the stochastic average of equation (7) with respect to \( f(h_s) \), the averaged modified Reynolds equation derived as

\[
\frac{1}{x} \frac{\partial}{\partial x} \left[ E[f(N,k,H,\theta)] x \frac{dE(p)}{dx} \right] = 12 \mu \sin \theta \frac{dH}{dy}
\] (8)

where \( E(*) = \int_{-\infty}^{\infty} (*) f(h_s) dh_s \) (9)

Christensen [13], has given that

\[
f(h_s) = \begin{cases} 
\frac{35}{32c^2} (c^2 - h_s^2)^{\frac{3}{2}}, & -c < h_s < c \\
0, & \text{otherwise}
\end{cases}
\] (10)

where \( \sigma = c/3 \) is the standard deviation.
In perspective of Christensen stochastic approach of rough surfaces, two roughness striations are of interest in the present study namely, radial roughness pattern and azimuthal roughness pattern.

**Radial one-dimensional roughness**

For radial one-dimensional roughness, the structure of bearing surface has the long narrow ridges and valleys running in the radial direction. Hence the film thickness is having two parts \( H = h + h_1(\theta, \xi) \) and equation (8) assumes as

\[
\frac{1}{x} \frac{\partial}{\partial x} \left[ E \left( f (N, k, H, \theta) \right) x \frac{dE(p)}{dx} \right] = 12 \mu \sin \theta \frac{dH}{dt} \tag{11}
\]

**Azimuthal one-dimensional roughness**

For azimuthal one-dimensional roughness, the structure of bearing surface has the long narrow ridges and valleys running in the azimuthal direction. Hence the film thickness is having two parts \( H = h + h_1(x, \xi) \) and equation (8) assumes as

\[
\frac{1}{x} \frac{\partial}{\partial x} \left[ E \left( f (N, k, H, \theta) \right) x \frac{dE(p)}{dx} \right] = 12 \mu \sin \theta \frac{dH}{dt} \tag{12}
\]

Combining equations (11) and (12), we get

\[
\frac{1}{x} \frac{\partial}{\partial x} \left[ G(N, k, c, H, \theta) x \frac{dE(p)}{dx} \right] = 12 \mu \sin \theta \frac{dH}{dt} \tag{13}
\]

Where \( G(N, k, c, H, \theta) = \left\{ E \left( f (N, k, H, \theta) \right) \right\} \) radial roughness

\[
E \left[ f (N, k, H, \theta) \right] = \frac{35}{32c} \int_{-c}^{c} f (N, k, H, \theta) (c^2 - h_1^2)^{3} dh_1, \tag{14a}
\]

\[
E \left[ \frac{1}{f (N, k, H, \theta)} \right] = \frac{35}{32c} \int_{-c}^{c} \frac{(c^2 - h_1^2)^{3}}{f (N, k, H, \theta)} dh_1, \tag{14b}
\]

Introducing non-dimensional quantities

\[
H^* = \frac{h}{h_0}, k^* = \frac{k}{h_0}, x^* = \frac{x}{\text{acosec} \theta}, p^* = -\frac{E(p) h_0^3}{\mu a^2 \left( -\frac{dH}{dt} \right) \cos \theta}, C^* = \frac{c}{h_0}
\]

Equation (13) takes the form

\[
\frac{1}{x} \frac{\partial}{\partial x} \left[ G^*(N, k^*, C^*, H^*, \theta) x^* \frac{dp^*}{dx} \right] = -12 \tag{15}
\]

where

\[
G^*(N, k^*, C^*, H^*, \theta) = \left\{ E \left( f^* (N, k^*, H^*, \theta) \right) \right\} \text{ radial roughness}
\]

\[
E \left[ f^* (N, k^*, H^*, \theta) \right] = \frac{35}{32C^*} \int_{-c^*}^{c^*} f^* (N, k^*, H^*, \theta) (C^* - h_1^2)^{3} dh_1, \tag{16a}
\]

\[
E \left[ \frac{1}{f^* (N, k^*, H^*, \theta)} \right] = \frac{35}{32C^*} \int_{-c^*}^{c^*} \frac{(C^* - h_1^2)^{3}}{f^* (N, k^*, H^*, \theta)} dh_1, \tag{16b}
\]

\[
f^* (N, k^*, H^*, \theta) = H^* \sin \theta + 12k^* h_1 \sin \theta - 6Nk^* H^* \sin \theta \cot \phi \left( \frac{NH \sin \theta}{2k^*} \right) \tag{17a}
\]

On integrating equation(15) and utilizing the appropriate boundary conditions
At $x^* = 0$ we have $\frac{dp^*}{dx} = 0$  
(18a) 
At $x^* = 1$ we have $p^* = 0$  
(18b) 
We get, film pressure $p^*$ in the form 

$$p^* = -\frac{3(x^*-1)}{G(N,k^*,C^*,H^*,\theta)}$$  
(19) 

The load carrying capacity can be found by integrating the film pressure 

$$E(W) = \int_0^1 2\pi x E(p) dx$$  
(20) 

Equation (20) takes the non-dimensional form 

$$W^* = \int_0^1 p^* dx$$  
(21) 

Substituting the expression of the film pressure and integrating the equation (21), one can obtain the $W^*$ as 

$$W^* = \frac{E(W) h_0^2}{\mu a^3 (dH^*/dt) \csc^2 \theta} = -\frac{4\pi}{G(N,k^*,H^*,\theta)}$$  
(22) 

The squeezing time can be calculated by integrating $W^*$ with respect to $H^*$ as follows 

$$t^* = \int_{h_0}^{h_1} \frac{4\pi}{G(N,k^*,H^*,\theta)} dH^*$$  
(23) 

Where $t^* = \frac{E(W) h_0^2}{\mu a^3 (dH^*/dt) \csc^2 \theta}$ 

3. Results and discussions
In the current investigation and solutions, the behaviour of squeeze film between rough conical bearings lubricated using Micropolar fluid is analysed. The results are analysed for various non-dimensional parameters. The impact of two roughness pattern on pressure, load and squeezing time are discussed graphically.

3.1. Squeeze film pressure
The variation of $p^*$ versus $x^*$ for various values of $k^*$ with $C^* = 0.3$, $H^* = 0.6$, $N = 0.8$ and $\theta = \pi/3$ for both roughness patterns is illustrated in Figure-2 and observed that non-dimensional pressure decreases with growing values of $x^*$ and increases for both roughness patterns along increasing values of $k^*$. Figure 3 illustrates the deviation of $p^*$ versus $x^*$ for different values of $C^*$ along $\theta = \pi/3$, $N= 0.8$, $H^* = 0.6$ and $k^* = 0.3$ for two types of roughness patterns. One can see that with increasing values of $C^*$, the non-dimensional pressure reduces for radial roughness and increases for azimuthal roughness. Figure 4 depicts variation of $p^*$ versus $x^*$ using various values of $\theta$ and $H^* = 0.6$, $k^* = 0.3$, $N= 0.8$ and $C^* = 0.3$ for both roughness patterns. From the figure it is clear that, the non-dimensional pressure reduces for rising values of $\theta$ for both form of roughness. The deviation of $p^*$ versus $x^*$ is seen in figure 5 for distinct measures of $N$ with $H^* = 0.6$, $\theta = \pi/3$, $k^* = 0.3$and $C^* = 0.3$ for both roughness patterns. One can see the non-dimensional pressure increasing along growing values of $N$ for two form of roughness.

3.2. Load carrying capacity
The variation of $W^*$ versus $H^*$ for distinct measures of $k^*$ along $C^* = 0.3$, $N = 0.8$ and $\theta = \pi/3$ is as shown in Figure 6 for two roughness patterns. It is seen that, $W^*$ reduces along rising values of $H^*$ and increases for both radial and azimuthal roughness patterns. Figure 7 describe the deviation of $W^*$ versus $H^*$ for different measures of roughness parameter $C^*$ along $k^* = 0.3$, $N = 0.8$ and $\theta = \pi/3$.In both type of roughness, it is seen that, with increasing measures of $C$, $W^*$ decreases for radial roughness and increases for azimuthal roughness patterns.Figure8 illustrates the variation of $W^*$ versus $H^*$ for various
values of $\theta$ along fixed roughness parameter $C^*=0.3, k^*=0.3, N=0.8$ and $\theta=\pi/3$ for both sorts of roughness. One can observe $W^*$ reducing for both roughness along growing values of $\theta$. Figure 9 displays the deviation of capacity $W^*$ versus $H^*$ for distinct values of $N$ with $\theta=\pi/3, k^*=0.3, C^*=0.3$ for both roughness patterns. It is clear from the figure that, the non-dimensional pressure enhances along growing values of $N$ for both patterns.

3.3. Squeeze film pressure

Figure 10 demonstrates the deviation of $t^*$ versus $h_f^*$ using various measures $k^*$ with $N=0.8, C^*=0.3$ and $\theta=\pi/3$. In both roughness patterns, one can see the response time $t^*$ reducing for growing values of $h_f^*$ and increasing for increasing values of $k^*$. Figure 11 depicts the deviation of $t^*$ versus $h_f^*$ for various measures of $C^*$ with $k^*=0.3, N=0.8$ and $\theta=\pi/3$ for both roughness patterns. It is seen that the squeeze film time decreases for radial roughness and increases for azimuthal roughness pattern with increasing values of $C^*$. The variation of $t^*$ versus $h_f^*$ for various values of $\theta$ with $k^*=0.3, N=0.8$ and $C^*=0.3$ is as shown in the Figure 12. It is seen that the $t^*$ reduces for both roughness pattern with increasing values of $\theta$. Figure 13 displays the variation of $t^*$ versus $h_f^*$ for various values of $N$ with $k^*=0.3, C^*=0.3$ and $\theta=\pi/3$. In both roughness patterns it is seen that, the $t^*$ increases with growing values of $N$.

Conclusions

The influence of surface roughness and its effects on squeeze film among rough conical bearings using Micropolar fluid is presented in this paper. For both roughness patterns, the averaged modified Reynolds equations are derived. According to the above analysis and the results discussed, the following conclusions have been considered.

- It is viewed that, with the increasing value of $C^*$ for the rough conical bearings, the pressure, load and squeezing time decreases for radial roughness and increases for azimuthal roughness pattern.
- It is seen that for the rough conical bearings, the pressure, load and squeezing time decreases along increasing value of $\theta$ and increases with growing value of $k^*$ and $N$ for two sorts of roughness patterns.
- With the existence of Micropolar fluid, azimuthal surface roughness pattern on the rough conical bearings improves the squeezing characteristics and the radial surface roughness pattern, reduces the performance of the squeeze film.
Figure 3: Variation of $p^*$ versus $x^*$ for various values of $C^*$ with $H^* = 0.6$, $N = 0.8$, $\theta = \pi/3$ and $k^* = 0.3$

Figure 4: Variation of $p^*$ versus $x^*$ for various values of $\theta$ with $H^* = 0.6$, $k^* = 0.3$, $N = 0.8$ and $C^* = 0.3$

Figure 5: Variation of $p^*$ versus $x^*$ for various values of $N$ with $H^* = 0.6$, $\theta = \pi/3$, $k^* = 0.3$ and $C^* = 0.3$
Figure 6: Variation of $W^*$ versus $H^*$ for different values of $k^*$ with $C^*=0.3$, $N=0.8$ and $\theta = \pi/3$.

Radial roughness
- $k^*=0$
- $k^*=0.2$
- $k^*=0.4$
- $k^*=0.6$

Azimuthal roughness
- $\theta = \pi/3$

Figure 7: Variation of $W^*$ versus $H^*$ for various values of $C^*$ with $k^*=0.3$, $N=0.8$ and $\theta = \pi/3$.

Radial roughness
- $C^*=0$
- $C^*=0.1$
- $C^*=0.2$
- $C^*=0.3$

Azimuthal roughness
- $\theta = \pi/3$
- $\theta = \pi/4$

Figure 8: Variation of $W^*$ versus $H^*$ for various values of $\theta$ with $N=0.8$, $k^*=0.3$ and $C^*=0.3$. 

Radial Roughness
- $\theta = \pi/4$
- $\theta = \pi/3$
- $\theta = \pi/2$

Azimuthal Roughness
- $N=0.8$
- $k^*=0.3$
- $C^*=0.3$
Figure 9: Variation of $W^*$ versus $H^*$ for various values of $N$ with $k^* = 0.3$, $C^* = 0.3$ and $\theta = \pi/3$

Figure 10: Variation of $t^*$ versus $h_f^*$ for various values of $k^*$ with $N = 0.8$, $C^* = 0.3$ and $\theta = \pi/3$
Figure 11: Variation of $t^*$ versus $h_f^*$ for various values of $C^*$ with $k^* = 0.3$, $N = 0.8$ and $\theta = \pi/3$

Figure 12: Variation of $t^*$ versus $h_f^*$ for various values of $\theta$ with $k^* = 0.3$, $N = 0.8$ and $C^* = 0.3$
Figure 13: Variation of $t^*$ versus $h^*$ for various values of $N$ and $k^* = 0.3, C^* = 0.3$ and $\theta = \pi/3$.

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