The concept of a supersolid state combines the crystallization of a many-body system with dissipationless flow of the atoms from which it is built. This quantum phase requires the breaking of two continuous symmetries: the phase invariance of a superfluid and the continuous translational invariance to form the crystal\(^1\,^2\). Despite having been proposed for helium almost 50 years ago\(^3\,^4\), experimental verification of supersolidity remains elusive\(^5\,^6\). A variant with only discrete translational symmetry breaking on a preimposed lattice structure—the ‘lattice supersolid’\(^7\)—has been realized, based on self-organization of a Bose–Einstein condensate\(^8\,^9\). However, lattice supersolids do not feature the continuous ground-state degeneracy that characterizes the supersolid state as originally proposed. Here we report the realization of a supersolid with continuous translational symmetry breaking along one direction in a quantum gas. The continuous symmetry that is broken emerges from two discrete spatial symmetries by symmetrically coupling a Bose–Einstein condensate to the modes of two optical cavities. We establish the phase coherence of the supersolid and find a high ground-state degeneracy by measuring the crystal position over many realizations through the light fields that leak from the cavities. These light fields are also used to monitor the position fluctuations in real time. Our concept provides a route to creating and studying glassy many-body systems with controllably lifted ground-state degeneracies, such as supersolids in the presence of disorder.

At a phase transition, the type of symmetry that is broken has fundamental consequences on the system. Whereas a discrete symmetry results in robust states with gapped excitations, a continuous symmetry leads to an infinite number of degenerate ground states that can evolve from one to another without energy cost, making the system highly susceptible to fluctuations. In quantum gas experiments, several phase transitions have been explored and periodic structures have been studied with imposed optical lattices\(^10\). Creating self-ordered crystalline structures by continuous spatial symmetry breaking in a superfluid has remained a challenge. Efforts are focusing on long-range interacting dipolar\(^11\,^14\) and spin–orbit-coupled quantum gases\(^15\,^16\), and on the interaction of matter with a continuum of modes\(^17\,^18\), for which either the absence of superfluidity\(^19\) or the number of modes\(^20\) have posed a limitation so far.

Continuous symmetries can also emerge through symmetry enhancement with competing order parameters—a concept that recently attracted interest in the context of high-temperature superconductors, high-energy physics and cosmology\(^21\,^22\). Here, the underlying symmetry groups of two order parameters are combined into a single group with higher symmetry by fine-tuning the parameters of the Hamiltonian. We introduce this concept to quantum gases and create an equivalent situation in a Bose–Einstein condensate (BEC) that is dispersively coupled with two optical cavities and illuminated by a one-dimensional transverse pump lattice (Fig. 1a). Increasing the coupling to each cavity induces a phase transition to self-organized states, at which atomic ordering is accompanied by intracavity light fields with parity symmetry\(^23\). These two parity symmetries can be combined to form one U(1) symmetry, similarly to constructing the XY model from two Ising models with equal couplings, for which the scalar magnetizations form a vector order parameter that exhibits rotational symmetry. Because the position of the atomic ordering along the x axis determines the combination of the cavity-field amplitudes (Fig. 1b), the crystal position and the cavity fields are directly connected (see Methods for the full interference potential). The crystallization is defect-free and homogeneous, because all atoms couple equally to the cavity modes, equivalent to an effective atomic interaction of global continuous translational symmetry. Because the position of the atomic ordering along the x axis determines the combination of the cavity-field amplitudes (Fig. 1b), the crystal position and the cavity fields are directly connected (see Methods for the full interference potential). The crystallization is defect-free and homogeneous, because all atoms couple equally to the cavity modes, equivalent to an effective atomic interaction of global continuous translational symmetry.

**Figure 1** | **Breaking continuous translational symmetry in a superfluid quantum gas.** a, A BEC (blue) is trapped at the intersection of two optical cavities that cross at an angle of 60° and exposed to a one-dimensional optical lattice formed by a transverse pump beam (red stripes). Photons scattered by the atoms populate the cavity mode 1 (2), shaded in red (yellow), with coherent field amplitude \(\alpha_1\) (\(\alpha_2\)) that can be detected when leaking from the cavity. b, The two BEC-cavity systems individually exhibit parity symmetry and combined have U(1) symmetry. The resulting ground-state manifold in terms of the order parameters \(\alpha_1\) and \(\alpha_2\) forms a circle. The interference potential of the cavity fields with the transverse pump field is \(d\)-periodic and moves continuously along the x axis when changing the angle in the \(\alpha_1\)–\(\alpha_2\) plane. c, The atomic momentum states associated with Raman processes describing the coherent scattering of a photon with momentum \(\hbar k\) from the transverse pump into cavity 1 (red) and 2 (yellow) are illustrated, all starting from the atomic ground state with zero momentum (blue). For each cavity, solid (dashed) lines describe these two-photon excitation processes involving the creation (annihilation) of a cavity photon and the annihilation (creation) of a pump photon. The inverse processes are equally driven, but are not shown for clarity. The momentum axes are denoted by \(p_x\), \(p_y\) and \(p_z\).

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range\textsuperscript{24}. This results in a perfectly rigid crystal structure, which inhibits the presence of phonons at non-zero wavenumbers.

A microscopic picture of the coupled system is obtained by considering Raman processes between transverse pump and cavity modes that coherently transfer atoms between the motional ground state and excited momentum states (Fig. 1c\textsuperscript{23,25}). The energies of the excited momentum states split into \( h\omega_{\kappa} = 3h\omega_{\text{rec}} \) and \( h\omega_{\kappa} = h\omega_{\text{rec}} \) owing to the angle of 60° between the transverse pump and the cavities, with \( \omega_{\text{rec}} \) the single photon recoil frequency and \( \hbar \) the reduced Planck constant. This description results in the following effective Hamiltonian (see Methods):

\[
\hat{H} = \sum_{i=1,2} \left[ -\hbar\Delta_i \hat{a}_i^\dagger \hat{a}_i + h\omega_i \hat{c}_i^\dagger \hat{c}_i + h \omega \hat{c}_i^\dagger \hat{c}_i - \hbar \Delta_i \right] + \frac{\hbar \lambda}{\sqrt{N}} (\hat{a}_i^\dagger + \hat{a}_i) (\hat{c}_i^\dagger \hat{c}_0 + \hat{c}_0^\dagger \hat{c}_i + \text{h.c.})
\]

where \( N \) is the atom number and h.c. is the Hermitian conjugate. The atoms are described by creation (annihilation) operators \( \hat{c}_i^\dagger (\hat{c}_i) \) for the motional ground state, and for the high- and low-energy states by \( \hat{c}_i^\dagger \) (\( \hat{c}_i \)), respectively, associated with cavity \( i \in \{1, 2\} \). The photon fields are denoted by \( \hat{a}_i^\dagger (\hat{a}_i) \), with detuning \( \Delta_i = \omega_p - \omega_i < 0 \) between the resonance frequency \( \omega \) and the transverse pump laser frequency \( \omega_p \).

The interplay of the atoms with each cavity is governed by competition between interaction and kinetic energy scales. Although the Raman coupling energy \( \hbar \lambda \) favours atomic ordering and the presence of an intracavity field, this comes at the cost of kinetic energy \( h\omega_{\kappa} \) and photonic energy \( h\Delta \). Experimentally, we can access \( \lambda \) through the transverse pump lattice depth, and each \( \Delta_i \) by adjusting \( \omega_i \). The driven-dissipative character of the experiment only minimally influences the position of the phase boundaries and affects criticalities\textsuperscript{26,27}. Because the decay rates and vacuum Rabi frequencies of the cavities are similar but not equal, the phase boundaries are expected to be displaced by a different amount (see Methods).

We first explore the phase diagram of the system to characterize the competition between the energy terms (see Fig. 2a, b). The starting point of the experiment is an optically trapped BEC of 87Rb atoms exposed to an attractive transverse pump lattice potential with wave-number \( \lambda_p = 785.3 \text{ nm} \) and recoil frequency \( \omega_{\text{rec}} = 2\pi \times 3.7 \text{ kHz} \), which restricts the motion of the atoms to the \( x-z \) plane (for experimental details, see Methods). While leaving \( \lambda \) constant and independently changing the detunings \( \Delta_1 \) and \( \Delta_2 \), we simultaneously record the photons leaking from both cavities, giving real-time access to the intracavity light fields. Owing to the concurrence of photonic and atomic excitations, the intracavity light fields allow us to access the degree of atomic ordering.

For each cavity, we observe a build-up of the cavity field at a critical point (Fig. 2c), which indicates the transition to a self-organized state. Three regions are immediately visible: one normal phase without an intracavity field, and two self-organized phases of cavity 1 (SO1) and 2 (SO2). We compare the data with the expected phase boundaries from a mean-field approximation of the Hamiltonian that includes cavity decay (see Methods) and find good agreement. The largest displacement of a phase boundary due to cavity decay amounts to 0.15 MHz. The phase boundary between the self-organized phases is shifted slightly off-diagonal owing to the mismatch in the decay rates of the cavities. We therefore achieve symmetric coupling to both cavities when cavity 2 is detuned slightly closer to the transverse pump frequency than cavity 1.

To probe the superfluidity of the atomic cloud, we measure its phase coherence. We suddenly turn off all trapping potentials and allow the atomic wavefunction to expand freely. Subsequently, we perform absorption imaging perpendicular to the cavity plane (see Fig. 2e–g). The presence of narrow interference maxima that reflect the initial atomic wavefunction to expand freely. Subsequently, we perform absorption imaging perpendicular to the cavity plane (see Fig. 2e–g).

The presence of narrow interference maxima that reflect the initial atomic wavefunction to expand freely. Subsequently, we perform absorption imaging perpendicular to the cavity plane (see Fig. 2e–g).
Emergence of a doubly self-organized phase. a, Mean intracavity photon numbers (binned in intervals of 0.5 ms) for the frequency ramp shown in b for cavity 1 (red) and 2 (yellow) at a constant transverse pump lattice depth of 38(1)\(\hbar\omega_{\text{sec}}\). The simultaneous presence of photons in both cavities signals the transition to a doubly self-organized phase. b, Detuning ramp through the phase diagram. At constant \(\Delta_2/(2\pi) = -2.2\text{ MHz}\), \(\Delta_1\) is ramped from far-detuned to reaching the phase boundary, held there for 25 ms and subsequently ramped closer to resonance. c, The mean-field energy as a function of cavity field amplitudes \(\alpha_1\) and \(\alpha_2\) is qualitatively displayed for three different regions in the phase diagram: inside the self-organized phase to cavity 2 (I) and 1 (II), and on the phase boundary in between (II).

For \(\hat{c}_{1,i}\) and \(\hat{c}_{2,i}\), for each cavity \(i \in \{1, 2\}\). The presence of a lattice supersolid for cavity 1 (2) is accompanied by atomic density modulation along the x axis forming at discrete positions 0 or d/2 (d/4 or 3d/4) for even or odd parity.

The crucial feature of the phase diagram is the discontinuity of the order parameters between the SO1 and the SO2 phase (Fig. 2d). We investigate this situation more closely by probing the system on the phase boundary (Fig. 3a, b). After preparing the atoms in the SO2 phase, we change the detuning \(\Delta_1\) until the phase boundary is reached. There we keep the detunings fixed and monitor the evolution of both cavity light fields. We observe finite mean intracavity photon numbers in both cavities, signalling a new type of self-organization. This additional phase is visible from the Hamiltonian, which is invariant under the unitary transformation \(\hat{a}_i \rightarrow (\hat{a}_1 \cos \theta + \hat{a}_2 \sin \theta, -\hat{a}_1 \sin \theta + \hat{a}_2 \cos \theta)\) and accordingly for \((\hat{c}_{1,i}, \hat{c}_{2,i})\) for any rotation angle \(\theta\) (see Methods). A low-energy evolution on the ground-state manifold results in the observed anticorrelated signals in the mean intracavity photon numbers \(\bar{n}_i = |\alpha_i|^2\), corresponding to a spatial displacement of the density modulation along the x axis.

We use the connection between crystal position and intracavity fields to further characterize the ground-state symmetry of the doubly self-organized phase. To this end, we ramp up the transverse pump power with symmetric couplings to the two cavities, resulting in a more adiabatic preparation of the system. When entering the doubly self-organized phase, we observe finite intracavity photon numbers in both cavities (Fig. 4a), and interference maxima at momenta associated with scattering processes for both cavities are visible in absorption imaging (Fig. 4b). Repeating this experiment, we observe that the cavity-field amplitudes distribute on a quarter circle (Fig. 4c), revealing a high ground-state degeneracy of the system (see Methods). Although the U(1) symmetry includes the sign of the cavity-field amplitudes, our photon detection allows us to measure only their magnitude. Therefore, the full circular symmetry is folded to the positive quadrant. Because the combination of the intracavity field amplitudes determines the position of the atomic density modulation along the x axis, we conclude that in each realization the superfluid crystallizes to a different position. This realizes a supersolid phase, because it exhibits both superfluid phase-coherence and broken continuous translational symmetry.

An interesting extension to our work would be to project a speckle potential on the atomic cloud, which would lift the ground-state degeneracy in a controlled way. This would permit the realization of supersolidity in the presence of disorder and the study of the dynamics in a quasi-degenerate ground-state manifold\(^{18,28}\). Beyond supersolidity, our
set-up can also be interpreted as identical two-level systems coupled with two quantized light fields, thus providing access to a new class of quantum optical models that break a continuous symmetry.  

Online Content Methods, along with any additional Extended Data display items and Source Data, are available in the online version of the paper; references unique to these sections appear only in the online paper.

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Author Information Reprints and permissions information is available at www.nature.com/reprints. The authors declare no competing financial interests. Readers are welcome to comment on the online version of the paper. Correspondence and requests for materials should be addressed to T.E. (esslinger@phys.ethz.ch).

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METHODS
Preparation of the Bose–Einstein condensate (BEC). After optically transporting a cold thermal cloud of $^{87}$Rb atoms along the x axis into the cavity set-up, we optically evaporate to an almost pure BEC with $N = 1.05(3) \times 10^7$ (here and in the following the value in parentheses denotes one standard deviation) atoms in a dipole trap, which is formed by two orthogonal laser beams at a wavelength of 1,064 nm along the x and y axes. The final trapping frequencies are $(\omega_x, \omega_y, \omega_z) = 2 \pi \times (881(1), 763(1), 154(1))$ Hz, resulting in Thomas–Fermi radii of $(R_x, R_y, R_z) = (7.9(1), 9.2(1), 4.5(1))$ μm. We subsequently expose the atoms to an attractive one-dimensional lattice potential along the y direction by linearly ramping up the transverse pump beam to a lattice depth of 38(1)ℏωc, with 50 ms. The mirror retroreflecting the transverse pump beam is positioned in vacuum at a distance of 8.6 mm from the atomic position. The beam has a 1/e² radius of $(\omega_w, \omega_p) = (35(3), 45(3))$ μm and is polarized along $\hat{z}$, parallel to gravity. The wave-length is set to $\lambda_p = 785.3$ nm, far red-detuned with respect to the atomic D$_1$ line.

Two crossed optical cavities. We mounted the mirrors forming the Fabry–Perot cavities as close as possible to each other to achieve large vacuum Rabi rates. For two crossing cavities, this required us to specifically machine the substrates before gluing them in place. The two cavities have comparable single-atom Rabi frequencies of $(g_i, g_p) = 2 \pi \times (1.95(1), 1.77(1))$ MHz and decay rates $(\kappa_i, \kappa_p) = 2 \pi \times (147(4), 800(1))$ kHz. The cavity modes intersect at an angle of 60° and have 1/e² radii of (49(1), 50(1)) μm. We position the atoms vertically in between the two mode axes, so that they are at a distance of 8(2) μm from each mode centre. For each cavity, we individually set the frequency of a longitudinal mode closely detuned with respect to the transverse pump frequency by stabilizing the cavity lengths with a weak stabilization laser beam at 830.4 nm. The resulting additional intracavity lattice potential is incommensurate with the transverse pump wavelength and has a depth of 0.11ℏωc, which is negligible compared to the self-organization lattice depth in the organized phases of typically (2–4)ℏωc. Long-term frequency stability between the transverse pump laser and the stabilization laser of around 50 kHz is achieved by locking all of them simultaneously to a passively stable transfer cavity. Single-photon counting modules are used to detect photons leaving the cavities.

Lattice calibrations. The lattice depths of the transverse pump and the intracavity lasers are calibrated by performing Raman–Nath diffraction on the cloud. The calibrated intracavity photon number can then be deduced from the vacuum Rabi frequencies of the cavities. We extract overall intracavity photon detection efficiencies of $(\eta_x, \eta_y) = (9.7(4), 2.0(1))$, with relative systematic uncertainties of 8%.

Cavity-induced spin transitions. The BEC is prepared in the $|F_m, m\rangle = |1, -1\rangle$ state with respect to the quantization axis along $\hat{z}$. The birefringences for the two cavities between the H and V eigenmodes are (4.5, 4.8) MHz. Whenever the resonance condition for a two-photon process involving pump and cavities is met, we observe collective Raman transitions between different Zeeman sublevels accompanied by macroscopic occupation of the cavity modes. To suppress such spin–changing scattering processes, all data were taken at a large offset field of $B_L = 34$ G, creating a Zeeman level splitting that is large compared to $\Delta_0$ and the birefringences. We can therefore neglect the presence of the H polarized eigenmode and solely couple to the V-polarized mode pointing along the z direction.

Effective Hamiltonian. We start with the following many-body Hamiltonian in a frame rotating at the pump laser frequency $\omega_p$:

$$\hat{H}_{\text{lab}} = \sum_{i=1,2} \left( -\hbar \Delta_i \hat{a}_i^+ \hat{a}_i \right) + \int \mathcal{D}(x,y) \left( \frac{\hat{p}_x^2 + \hat{p}_y^2}{2m} + \int \hat{\psi}(x,y) \hat{\psi}^\dagger(x,y) \right) + \int \mathcal{D}(x,y) \left( \gamma \hat{c} \hat{c} \hat{c}^\dagger \hat{c}^\dagger + \hbar \Delta_0 \hat{c}^\dagger \hat{c} \right) \delta(x,y) dx dy \tag{1}$$

where the index $i \in \{1, 2\}$ labels the two cavities, $\hbar \omega_p$ is the reduced Planck constant, $\hat{c}$ is the spatial mode of the transverse pump lattice, $m$ is the mass of one Rb atom and $(\mu_x, \mu_y)$ its momentum. The integration runs over the area $A$ of the Wignier–Seitz cell with the spatial coordinate $r = (x,y)$, $\eta = -\omega_p \epsilon / \Delta_0$ is the two-photon Rabi frequency for cavity $i$, the potential depths for cavity i and the transverse pump are given by $U_i = \hbar \eta^2 / \Delta_0$ and $U_p = \hbar \eta^2 / \Delta_0$, respectively. The vacuum Rabi frequencies of the cavities are denoted by $g_i$. The standing-wave transverse pump beam at frequency $\omega_p$ with Rabi frequency $\Omega_p$ is red-detuned by $\Delta_0 = \omega_p - \omega_0$ from the atomic resonance frequency $\omega_0$. It is oriented along the y axis with wavevector $\mathbf{k}_p = \mathbf{k}_y$. The cavities $i \in \{1, 2\}$ at frequencies $\omega_i$ are detuned by $\Delta_i = \omega_i - \omega_0$ from the transverse pump frequency and are described by modes $\hat{a}_i$ and wavevectors $\mathbf{k}_i = \sin(\theta)^2 \mathbf{k} + (1 - \sin(\theta))^2 \mathbf{k}_y$. This effective Hamiltonian that creates (annihilates) a particle at position $(x,y)$. We neglect any cavity decay rate in this part of our formalism as well as the cavity–cavity interference term, which is negligible for the choice of our experimental parameters.

The first term of the Hamiltonian describes the energy of the photon fields in the cavities and the kinetic energy of the atoms. The remaining terms take into account the cavity–pump interference terms for the two cavities, the cavity lattices and the pump lattice. The transverse pump lattice beam at wavelength $\lambda_p = 2\pi/k_p$ is far red-detuned with respect to atomic resonance, but closely detuned by $\Delta_0$ to the two cavity resonances. Therefore, photons from the transverse pump can be scattered into a cavity mode and back via off-resonant Raman processes. These two-photon processes coherently couple the zero–momentum state of the BEC $|\psi_k\rangle = |0, 0\rangle$ to the eight momentum states $|\psi_{k\pm \pm}\rangle = \pm |\pm k\rangle \pm \pm h \kappa \rangle$, which are sketched in Fig. 1c. We neglect scattering processes between the two cavities, and between excited atomic momentum states, because their amplitudes are negligible for our experimental parameters. Because the cavities and the transverse pump are not orthogonal, these eight momentum states group in high- and low-energy states with energies $\hbar \omega_i = \pm 2 \hbar \omega_{\pm \pm} \pm \pm h \kappa$ and $\hbar \omega_i = \pm 2 \hbar \omega_{\pm \pm} \pm \pm h \kappa$, which is negligible for the choice of our experimental parameters. Neglecting a difference between the two Raman couplings, as in the Hamiltonian in the main text, leads to only a small correction because the vacuum Rabi frequencies $\gamma_i$ are similar.

Critical couplings. Self-organization to a single cavity occurs when the coupling $\lambda$ crosses the critical coupling $\lambda^c = -\hbar \Delta_0 \gamma / \Delta_0$ with $\sigma^+ = \sigma_+ - \sigma_i$ and $\sigma_i$.

The phase boundary between the SO1 and the SO2 phase is identified by the condition $\lambda^c / \lambda^c = \lambda^c / \lambda^c$, where the coupling to both cavities is symmetric. Including dissipation rates $\gamma_i$ for the cavity fields changes the critical couplings into $\lambda^c = -\hbar \Delta^c \gamma^c / \Delta_0$ with $\sigma^+ = \sigma_+ - \sigma_i$.

Symmetries of the Hamiltonian. The Hamiltonian can be read as the sum of two formally identical Hamiltonians for the two cavities, $\hat{H}_i = \sum_{i=1,2} \hat{H}_i$.

Both Hamiltonians individually manifest parity symmetry, generated by the operator $\hat{G}_i = \hat{a}_i^\dagger \hat{a}_i + \sum_{o=x,y} \hbar \gamma o \hat{c}^\dagger \hat{c}$. They stay unchanged upon the simultaneous transformation $(\hat{a}_i, \hat{c}_i) \rightarrow (\hat{a}_i, \hat{c}_i)$ on the photonic and atomic field operators. This symmetry is broken at the phase transition. The choice of sign of the field operators now corresponds to a choice of $0$ or $\pi$ for the phase of the light field in cavity i, which is equivalent to atoms crystalizing on odd or even sites of a checkerboard lattice with rhomboid symmetry.

For $\lambda = \lambda_1$ and $\lambda_2 = \lambda_2$, the Hamiltonian $\hat{H}_1$ exhibits a U(1) symmetry instead: it is possible to perform a simultaneous rotation by an arbitrary angle $\theta$ in the space of the cavity field and atomic field operators that leaves the Hamiltonian $\hat{H}_2$ unchanged. The transformation acts in the following way on the operators:

$$\hat{a}_1 \rightarrow \hat{a}_1 \cos \theta - \hat{a}_1 \sin \theta$$
$$\hat{a}_2 \rightarrow \hat{a}_2 \sin \theta + \hat{a}_2 \cos \theta$$
$$\hat{c}_1 \rightarrow \hat{c}_1 \cos \theta - \hat{c}_1 \sin \theta$$
$$\hat{c}_2 \rightarrow \hat{c}_2 \sin \theta + \hat{c}_2 \cos \theta$$

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It shifts photons between the two cavities while simultaneously redistributing the momentum excitations accordingly. This produces the circle on the $\alpha_1$-$\alpha_2$ plot in Fig. 4c. The corresponding generator $\hat{C}$ of the symmetry $\hat{U}(\theta) = e^{i \theta \hat{C}}$ is the Hermitian operator

$$\hat{C} = -i \left[ \hat{a}_1^\dagger \hat{a}_2 - \hat{a}_2^\dagger \hat{a}_1 + \sum_{n=\pm} \left( \hat{c}_n^\dagger \hat{c}_n - \hat{c}_n^\dagger \hat{c}_n \right) \right]$$

It satisfies $[\hat{C}, \hat{H}] = 0$ and, as a direct consequence, the Hamiltonian $\hat{H}$ stays unchanged for any $\theta$ under the symmetry $\hat{U}_1$; that is, $\hat{U}_1 \hat{H} \hat{U}_1^\dagger = \hat{H}$. This symmetry is spontaneously broken at the phase transition.

**Mean-field solution.** We can obtain the ground state of the effective Hamiltonian by performing a mean-field expansion around the expectation value of each operator and numerically solving the resulting mean-field Hamiltonian. To get a direct comparison with the experimentally measured data, we plot the order parameters $\alpha_1$ and $\alpha_2$ as a function of the two detunings $\Delta_i$ (see Extended Data Fig. 1). The calculation includes cavity decay, the influence of the transverse pump potential and atom–atom contact interactions.

**Supersolid potential.** The self-consistent potential in the supersolid phase is formed from the interference between the transverse pump field and the two cavity fields. Here we derive the relation between the spatial position of the optical lattice structure and the ratio between the coherent fields $\alpha_1$ and $\alpha_2$ in each cavity.

The full potential landscape for the atoms is given by the coherent superposition of the transverse pump field and the two cavity fields,

$$U(r) = \sum_{i \in \{0,1,2\}} \Omega_i \cos \left( k \cdot r + \phi_i \right)^2$$

where $\Omega_i^2 = \mu_i$ are the potential depths created by each field. We set $\phi_0 = \phi_2 = 0$ by choosing the origin of the coordinate system appropriately. The atomic spatial distribution is then determined by the phase $\phi_0 \equiv \phi$ of the transverse pump standing wave, which we can change via a piezo-electric actuator attached to the retroreflecting transverse pump mirror. For our experimental parameters, $\mu_0 \gg \mu_1, \mu_2$ such that the atoms are separated into two-dimensional layers in the $x$-$z$ plane at $k y + \phi = \pi n, \ n \in \mathbb{Z}$, where $k = 2 \pi / \lambda_p$. Tuning the spatial phase $\phi$ of the transverse pump standing wave results in triangular ($\phi = 0$) and hexagonal ($\phi = \pi / 2$) lattice geometries (Extended Data Fig. 2).

Within one layer we get

$$U(x) = (\Omega_1 \cos(2\phi) + \Omega_2 [(\cos \theta + \sin \theta) \cos(\phi/2) \cos(\sqrt{3} \pi x) + (\cos \theta - \sin \theta) \sin(\phi/2) \sin(\sqrt{3} \pi x)])^2$$

where $\Omega_1 = \Omega_2 \cos \theta$ and $\Omega_2 = \Omega_2 \sin \theta$, with $\theta$ corresponding to the position on the $\alpha_1$-$\alpha_2$ circle as before. This describes a lattice whose position depends on $\theta$, unless $\phi = 0$, in which case only the lattice depth is modified. The lattice depth has a $\theta$ modulation that disappears as $\phi$ approaches $\pi / 2$, in which case equation (3) simplifies to $U(x) = [- \Omega_2 + \Omega_2 \cos(\theta + \sqrt{3} \pi x - \pi / 4)]^2$. We choose $\phi \approx \pi / 2$ in our experiments such that in the broken U(1) symmetry each realization of cavity fields corresponds to a different translation, as shown in Extended Data Fig. 3. Neighbouring layers move in opposite directions so that the translation is staggered.

**Finite width and ground-state degeneracy of the supersolid phase.** Although the supersolid phase theoretically extends over only a line in the phase diagram, we experimentally observe a finite width of around 100 kHz. We attribute this to two reasons. First, our experimental preparation of a point in the phase diagram has a resolution of several percent. Second, the chemical potential of the cloud limits the resolution with which we can probe the ground state of the system. Close to the U(1)-symmetric line, the two minima of the parity symmetry are only very weakly pronounced. As the chemical potential increases compared to the depth of the minima, the ground-state manifold approaches a U(1) symmetry.

To quantify the homogeneity of the U(1) symmetry we analyse the distribution of the obtained angles. Extended Data Fig. 4 shows the histogram of the observed angles in the positive quadrant of the U(1) manifold for three different points across the supersolid phase. Despite the limited sample sizes, a qualitative difference between the histograms is visible. Whereas the data taken in the centre of the supersolid phase show an almost homogeneous distribution, a clear trend towards the effectively more strongly coupled cavity is visible for a positive or negative change in the detuning $\Delta_i$.

**Data availability.** All data files are available from the corresponding author on request.

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Extended Data Figure 1 | Mean-field solution of the phase diagram.
Order parameters $\alpha_1$ and $\alpha_2$ as a function of the pump–cavity detunings $\Delta_1$ and $\Delta_2$. The Raman coupling $\lambda$ is fixed by the experimental value of the transverse pump lattice depth of $38\hbar\omega_{rec}$. The mean-field model includes cavity decay, different vacuum Rabi frequencies $g_i$ and the transverse pump potential.
Extended Data Figure 2 | Lattice geometries for different choices of the phase $\phi$ of the transverse pump field for balanced cavity fields. The atoms are cut into one-dimensional lines by the strong transverse pump field. On top, spatial distributions between triangular ($\phi = 0$) and hexagonal ($\phi = \pi/2$) can form through the interference between the cavity light fields and the transverse pump, depending on the phase $\phi$ of the latter.
Extended Data Figure 3 | Dependence of the lattice structure on the cavity field amplitudes. The ground-state manifold for equal couplings and detunings in $H$ is a circle in the space of the cavity fields $\alpha_1$ and $\alpha_2$. For each combination of fields, the interference potential in equation (3) between the transverse pump and cavity fields for $\phi = \pi/2$ will have its minima at different positions. Following the circle clockwise, every second line moves left (top highlighted line) while the others move right (bottom highlighted line).
Extended Data Figure 4 | Transition between discrete and continuous symmetries. a–c, Histograms (normalized to unity area) of the angles $\theta$ describing the position of the crystal lattice, measured in the same way as the data in Fig. 4, but at $\Delta_1 = -2.1$ MHz (a), $\Delta_1 = -2.2$ MHz (b) and $\Delta_1 = -2.3$ MHz (c). The grey lines show kernel density estimation analyses with a Gaussian kernel, whose bandwidth of $0.13\pi/2$ was determined from a cross-validation maximum likelihood reconstruction. Each histogram consists of 19–49 realizations.