NON-CLASSICAL BEHAVIOR OF ATOMS IN AN INTERFEROMETER

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Abstract

We have studied the properties of the non-classical behavior of atoms in a double-slit interferometer. An indication of this behavior for metastable helium was reported by Kurtsiefer, Pfau and Mlynek [Nature 386, 150 (1997)] showing distinctive negative values of the Wigner function, which was reconstructed from the measured diffraction data. Our approach to explain this non-classical behavior is based on the de Broglie-Bohm-Vigier-Selleri understanding of the wave-particle duality and compatible statistical interpretation of the atomic wave function. It follows from the results that the atomic motion is non-classical because it does not obey the laws of classical mechanics. However, there is no evidence that this atomic behavior violates the classical probability law of the addition of probabilities.

PACS number: 03.65.Bz, 03.75.Dg, 03.75.*

Key words: atomic interference, compatible statistical interpretation, (non)violation of the classical probability laws, Wigner’s function

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I. Introduction

The wave function $\psi(x, t)$ of the transverse motion of an atom in an atom interferometer is a linear superposition of states with maxima at two spatially separated locations. These states lead to negative values in Wigner’s function $W(x, p_x, t)$ which is the quasi-probability distribution of coordinate $x$ and momentum $p_x$. The negative values of $W(x, p_x, 0)$, reconstructed from measured and evaluated space distribution, were interpreted as a signature of the highly non-classical behavior of atoms in the atom interferometer [1-3].

In this paper we study the properties and the cause of this non-classical behavior, using the compatible statistical interpretation (CSI) of a wave function [4-6]. The aim of this study is to clarify the meaning of the notion “non-classical motion (behavior)" of atoms in a double-slit interferometer. In our opinion, it is necessary to distinguish clearly two aspects of the notion “non-classical motion (behavior)". It may denote a motion (behavior) which does not obey the laws of classical mechanics and/or a motion which does not obey the classical probability laws, in particular the classical law of the addition of probabilities.

We use CSI because the wave and corpuscular features of a wave function are incorporated in a consistent manner into the basic statistical quantity of CSI, which is the de Broglian probability density $P(x, p_x, t)$. $P(x, p_x, t)$ is the probability density for a particle, which is in the quantum state $\psi(x, t)$, to have a momentum $p_x$ and to be at $x$ at the time $t$ [5]. $P(x, p_x, t)$ satisfies both marginal conditions imposed by Wigner upon any joint probability distribution in phase space and it is always positive [6].

The coherence and the characteristic modulation of the momentum distribution found by Kaiser et al. [7] at the exit of a neutron interferometer was explained by Božić and Marić [5], based on the $P(x, p_x, t)$ function. Božić and Arsenović [10] compared the explanation of the same effect, based on Wigner’s function and given by Lerner, Rauch, and Suda [8] and Suda [9], with an explanation based on the de Broglian probability density [5]. In this paper we use an analogous comparison of the time dependent Wigner’s function to the de Broglian probability density. These are two different distributions in phase space, associated with the same wave function of the atomic transverse degree of freedom in the interferometer.

In the following sections of this paper we summarize quantum properties of atomic motion in an interferometer. In Sec. II is written the solution of Schrödinger’s equation for an interferometer in the form of the Fresnel-Kirchhoff diffraction integral, while in Sec. III is derived the time dependent wave function $\psi(x, t)$ of the transverse motion and, for a chosen set of parameters, graphs of the function $|\psi(x, t)|^2$ are presented. The transverse momentum distribution in the state $\psi(x, t)$ is evaluated and presented graphically in Sec. IV. The de Broglian probability density and Wigner’s function in the state $\psi(x, t)$ are evaluated and graphically presented in Sects. V and VI, respectively. The comparison between these two probability distributions is also given in Sec. VI, while in Sec. VII are derived concluding remarks about the properties of non-classical behavior of atoms in an interferometer.
II. The application of the Fresnel-Kirchhoff diffraction formula

We want to determine the wave function of the transverse motion of an atom which travels with velocity \( \vec{v} = \vec{v}_y \) through region I (see Fig. 1), towards the slits and is then sent through the slits to region II. For this reason we shall determine in this section a stationary solution of the time-dependent two dimensional Schrödinger equation

\[
- \frac{\hbar^2}{2m} \left( \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \right) \Psi(x, y, t) = i\hbar \frac{\partial}{\partial t} \Psi(x, y, t).
\] (1)

The stationary solution of Eq. (1) has the form

\[
\Psi(x, y, t) = e^{-i\omega t} \Phi(x, y),
\] (2)

where \( \hbar \omega = p^2/2m \) and \( p = mv = \hbar k \). The space dependent function \( \Phi(x, y) \) satisfies the equation

\[
- \frac{\hbar^2}{2m} \left( \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \right) \Phi(x, y) = \hbar \omega \Phi(x, y).
\] (3)

The solution of Eq. (3) in region I is a spherical wave

\[
\Phi(P') = \Phi(x', y') = \frac{A e^{ikr'}}{r'},
\] (4)

where \( A \) is a constant and \( r' \) is the distance (Fig. 1) from the source \( (P_0) \) to the point \( P' = (x', y') \) in region I. The spherical wave at the slit points \( (x', y' = 0) \) may be approximated by a plane wave, since the distance \( a \) of the double-slit screen from the source \( P_0 \) is very large compared to the width of the slits. Consequently, without a loss of generality, for \( \Phi(x', y' = 0) \) at the border of region I we may choose the function

\[
\phi_1(x', 0) = \begin{cases} 
\frac{1}{\sqrt{2}}, & -\frac{\Delta}{2} \geq x' \geq -\frac{\Delta}{2} - \delta \\
0, & \text{all other values of } x' 
\end{cases}
\] (5a)

for one open slit, and the function

\[
\phi_2(x', 0) = \begin{cases} 
\frac{1}{\sqrt{2}}, & -\frac{\Delta}{2} \geq x' \geq -\frac{\Delta}{2} - \delta \\
1, & \frac{\Delta}{2} + \delta \geq x' \geq \frac{\Delta}{2} \\
0, & \text{all other values of } x' 
\end{cases}
\] (5b)

for two open slits. This means that in region II the solution of Eq. (3) is given by the formula of the Fresnel-Kirchhoff diffraction [11]

\[
\Phi(x, y) = -\frac{iA e^{ikx}}{2\lambda} \int_{A} dx' \frac{e^{iks}}{s} [1 + \cos \chi],
\] (6)

where \( s = \sqrt{y^2 + (x' - x)^2} \), \( \cos \chi = y/s \), \( \lambda = 2\pi/k \), while \( A = \{x'; -(\Delta/2) - \delta < x' < -(\Delta/2)\} \) when the lower slit is open and upper slit is closed, and \( A = \{x'; (\Delta/2) < x' < \} \) when the lower slit is closed and upper slit is open.
$(\Delta/2) + \delta$ or $-(\Delta/2) - \delta < x' < -(\Delta/2)$} when the two slits are open. The constant $A$ will be chosen from the normalization condition.

The spatial distribution of the transverse degree of freedom as a function of evolution time was investigated in a double slit experiment [1,2] with metastable helium atoms. We will apply the Fresnel-Kirchhoff diffraction formula to analyze the experiment of Kurtsiefer, Pfau, and Mlynek [1]. A diagram of the apparatus is shown in Fig. 2. Atoms are emitted from a gas-discharge source operating in the pulse operation mode. The beam is collimated by a $5 - \mu m$-wide slit and then is sent through a double-slit structure with a slit separation $\Delta + \delta = 8 \mu m$ and an opening $\delta = 1 \mu m$. The atoms then propagate for a distance $d$ to a time- and space-resolving detector. Atom beam velocities lie between $1000$ and $3000 \text{ms}^{-1}$. We shall use the parameters of this experimental arrangement for the following calculations.

III. Time-dependent wave function of the transverse motion

Assuming that the motion of an atom along the $y$-axis can be treated classically and that the transverse motion is quantized, one may use the relation $y = vt$ and determine the time dependent function of the transverse motion from the function $\Phi(x, y)$, by the following definition

$$\Phi(x, y) = \Phi(x, vt) \equiv \psi(x, t).$$

(7)

The graphs of the function $|\Phi(x, y)|^2 \equiv |\psi(x, t)|^2$ for $k = 4\pi \cdot 10^{10} \text{m}^{-1}$ and for the chosen set of values of the coordinate $y \ (t = my/\hbar k)$ are presented in Figs. 3 and 4. Very close to the slit on the single-slit graphs (Fig. 3) we see the minima of the wave function for $x = x_c$, where $x_c = -4 \mu m$ is the coordinate of the slit center. But, with increasing $y$, the maximum is present at $x = x_c$ for all $y$. This maximum becomes wider and wider with increasing $y$.

On double-slit graphs (Fig. 4) we clearly see that near the slits the wave function consists of two widely separated Gaussians on which small oscillations are superimposed. With increasing distance from the slits the Gaussian-like maxima spread and start to overlap, so that the third maximum with superimposed oscillations start to develop. This region of $y$ corresponds to Fresnel diffraction. With further increase of $y$ (t), distinct equally spaced oscillations develop, which correspond to the Fraunhofer diffraction limit.

IV. The transverse-momentum distribution

The time dependent function defined by Eq. (7) should be a solution of the one-dimensional time-dependent Schrodinger’s equation. Therefore, we may assume that it can be written in the form

$$\psi(x, t) = \frac{1}{\sqrt{2\pi\hbar}} \int_{-\infty}^{\infty} c(p_x) e^{ip_xx/\hbar} e^{-i\omega_x t} dp_x = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} c'(k_x) e^{ik_xx} e^{-i\omega_x t} dk_x,$$

(8)

where $\int_{-\infty}^{\infty} |c(p_x)|^2 dp_x = \int_{-\infty}^{\infty} |c'(k_x)|^2 dk_x = 1, p_x = \hbar k_x, c'(k_x) = \sqrt{\hbar} c(p_x)$ and $\hbar \omega_x = p_x^2/2m$. From Eq. (8) we may determine the transverse-momentum distribution $|c(p_x)|^2 = |c'(k_x)|^2/\hbar$ in the state $\psi(x, t)$. At first, one determines

$$C(k_x, t) \equiv \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \psi(x, t)e^{-ik_xx} dx$$

(9)
by performing the Fourier-transform of the function $\psi(x,t)$, defined by Eq. (7), taking $t$ as a parameter. If Eq. (8) is valid, then it should be

$$C(k_x,t) = c'(k_x)e^{-i\omega_xt}.$$  \hspace{1cm} (10)

Consequently,

$$|c'(k_x)|^2 = |C(k_x,t)|^2.$$  \hspace{1cm} (11)

The graph of $|c'(k_x)|^2 = \hbar |c(p_x)|^2$ for one slit is given in Fig. 5a and for two slits in Fig. 5b.

Our numerical calculation for various values of $t$, show that $|c'(k_x)|^2$ is independent of $t$. This fact justifies the assumptions of Eqs. (7) and (8) as well as the statement of Kurtsiefer, Pfau, and Mlynek [1] that the longitudinal motion of the atoms at velocities $v$ of several thousand meters per second can be treated completely classically.

We compared also the transverse momentum distribution $|c'(k_x)|^2$ (evaluated as described above and presented in Fig. 4) with the absolute value square of the Fourier transform

$$F_i(k_x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \phi_i(x',0)e^{-ik_xx'}dx'$$  \hspace{1cm} (12)

of the function $\phi_i(x',0)$, $i = 1,2$. After the evaluation of the latter integral one finds

$$F_1(k_x) = \frac{i e^{ik_x \Delta/2}}{k_x \sqrt{2\pi\delta}} \{1 - e^{ik_x \delta}\};$$

$$|F_1(k_x)|^2 = \frac{2 \sin^2(k_x\delta/2)}{\pi k_x^2};$$  \hspace{1cm} (13)

and

$$F_2(k_x) = \frac{2}{k_x \sqrt{\pi\delta}} \sin \frac{k_x\delta}{2} \cos \frac{k_x(\Delta + \delta)}{2};$$

$$|F_2(k_x)|^2 = \frac{4}{k_x^2 \pi \delta} \sin^2 \frac{k_x\delta}{2} \cos^2 \frac{k_x(\Delta + \delta)}{2}. \hspace{1cm} (14)$$

We found that $|c'(k_x)|^2$ for one slit is practically identical to $|F_1(k_x)|^2$ and that $|c'(k_x)|^2$ for two slits is practically identical to $|F_2(k_x)|^2$.

By comparing the spatial distributions for one and two slits shown in Figs. 3 and 4, one must conclude that the presence of the second slit influences the motion of each atom, independent of the slit through which it has passed to region II (see Fig. 1). This influence is also very well seen by comparing the momentum distributions for one and two slits, presented in Fig. 5. Certain values of the particle’s transverse momentum, which are allowed with one slit, are not allowed when both slits are open. This fact is also a signature of a non-classical atomic behavior that can be understood in a similar way to the quantization of the electronic orbits in atom based on de Broglie’s wavelength. It appears
that the atomic matter wave excludes certain values of transverse momentum and favors others, which is an evident quantum effect.

V. The De Broglian probability density

According to the CSI of a wave function, in an ensemble of particles in a pure state presented by Eq. (8), different particles may have different momenta. Recall that the probability density of \( p_x \) is \( |c(p_x)|^2 \). However, each particle is surrounded by the same wave [5]. In other words, a particle and a wave are two different, but compatible, entities.

The de Broglian probability density, \( P(x, p_x, t) \), of a particle in the quantum state \( \psi(x, t) \) is the probability density for the particle to have a momentum \( p_x \) and to be at a position \( x \) at time \( t \) [5,10],

\[
P(x, p_x, t) = |\psi(x, t)|^2 |c(p_x)|^2 = P'(x, k_x, t)/\hbar = |\psi(x, t)|^2 |c'(k_x)|^2 /\hbar. \tag{15}
\]

\( P(x, p_x, t) \) satisfies both marginal conditions

\[
\int P(x, p_x, t) dp_x = |\psi(x, t)|^2, \tag{16}
\]
\[
\int P(x, p_x, t) dx = |c(p_x)|^2 \tag{17}
\]

imposed by Wigner upon a joint probability distribution in phase space [12]. For operators having the form \( F(\hat{x}, \hat{p}_x) = F_1(\hat{x}) + F_2(\hat{p}_x) \), the probability density \( P(x, p_x, t) \) satisfies Wigner’s condition that the quantum mechanical average value of an operator is equal to the classical average value of the corresponding classical function.

Both \( P(x, p_x, t) \) and Wigner’s function are determined by the state \( \psi(x, t) \). Unlike the Wigner function, \( P(x, p_x, t) \) is always positive. Despite this fact, \( P(x, p_x, t) \) also reflects the non-classical behavior of atoms in the state \( \psi(x, t) \).

Since the simultaneous measurement of a coordinate and momentum is not possible, \( P(x, p_x, t) \) cannot be measured in a single experiment. However, one could experimentally determine the probability density of a coordinate \( x \) and momentum \( p_x \) in the state \( \psi(x, t) \), i.e. \( P(x, p_x, t) \), by measuring separately the distributions \( |\psi(x, t)|^2 \) and \( |c(p_x)|^2 \). These distributions reflect the non-classical behavior, as pointed out in the previous section.

VI. The De Broglian probability distribution and Wigner’s function

In Figs. 6 and 7 we present the graphs of the de Broglian probability density of a coordinate \( x \) and transverse momentum \( p_x \), \( P(x, p_x, t) \), for \( y = 120 \text{ mm} \ (t = y/v = 6.01 \times 10^{-5} \text{s}) \) and \( y = 240 \text{ mm} \ (t = y/v = 12.02 \times 10^{-5} \text{s}) \). For the same values of \( y \) we present in Figs. 8 and 9 the plots of the Wigner distribution function, evaluated from the definition [12,13] expression

\[
W(x, p_x, t) = \frac{1}{\hbar \pi} \int d\tilde{x} e^{2ip_x \tilde{x}/\hbar} \psi^*(x + \tilde{x}, t) \psi(x - \tilde{x}, t) = W'(x, k_x, t)/\hbar. \tag{18}
\]
It is clear from Figs. 6-9 that $W(x, p_x, t)$ and $P(x, p_x, t)$ are very different functions. Consequently, from their forms and properties are derived different interpretations of the behavior of quantum particles. It was shown by Janicke and Wilkens [14] and Kurtsiefer, Pfau, and Mlynek [1] that Wigner’s function $W(x, p_x, 0)$ may be reconstructed from evaluated and measured values of $|\psi(x, t)|^2$ for various values of $t$. The negative values of $W(x, p_x, 0)$ were interpreted as a signature of an atom’s non-classical behavior. These negative values are also associated with the requirement of Heisenberg’s uncertainty relationship that a quantum mechanical particle has to be described by an area of uncertainty in phase space no smaller than $\Delta x \Delta p_x = \hbar/2$ [3]. The authors pointed out that the negative values reflect the impossibility of joint measurement of position and momentum.

However, we interpret de Broglian probability density, presented in Figs. 6 and 7, as an objective probability density of particle coordinate and momentum. The impossibility of simultaneous measurements of a particle’s $x$ and $p_x$ does not forbid us from assuming that their joint distribution objectively exists. The important fact is that this assumption does not lead to any contradiction with the facts derived from measurable distributions. One can see that this joint probability density is consistent with the measurable probability density of position and the measurable probability density of momentum. For example, for values of $\tilde{p}_x$ for which $|c(\tilde{p}_x)|^2 = 0$, the joint distribution $P(x, \tilde{p}_x, t)$ is also equal to zero. Thus, if there is no particle with a certain value of momentum $\tilde{p}_x$, this value can not be found anywhere during the measurement of momentum. Similar reasoning is valid for space points $\tilde{x}$ in which $|\psi(\tilde{x}, t)|^2 = 0$, since $P(\tilde{x}, p_x, t)$ is also equal to zero in these space points for any value of momentum. Therefore, at a point $\tilde{x}$ no particle will be detected in the experiment.

One can see in Figs. 8 and 9 that Wigner’s function $W(x, p_x, t)$ may take values different from zero at the points $\tilde{x}$ and $\tilde{p}$ in which either $\psi(\tilde{x}, t) = 0$ or $c(\tilde{p}_x) = 0$. Despite this property, inconsistent with a notion of a joint probability, the Wigner function satisfies the marginal conditions stated by Eqs. (16) and (17). It is well known that Wigner’s function may assume negative values, even though it is a joint probability distribution by definition. Because of this, it is possible to satisfy Eqs. (16) and (17). Thus, two different properties of Wigner’s function, inconsistent with a notion of a joint probability, cancel each other and make it possible to satisfy two marginal conditions. This is clearly seen by comparing results presented in Figs. 6,7 and 8,9. We note in Fig. 8,9 the negative peaks in the $x$-dependence of the Wigner function for those values $\tilde{p}_x$ of momentum for which $|c(\tilde{p}_x)|^2 = 0$.

VII. Conclusion

The properties and cause of non-classical behavior of atoms in the atomic interferometer are studied using the stationary solution $\Phi(x, y)$ of the two-dimensional Schrodinger’s equation. The solution was written in the form of the Fresnel-Kirchhoff diffraction integral. The time dependent wave function of the transverse motion was derived from the Fresnel-Kirchhoff diffraction integral, using the relation $\Phi(x, y) = \Phi(x, vt) \equiv \psi(x, t)$, where $v$ is the initial longitudinal atomic velocity. The latter relation was used in Refs. [1,2], where it was justified by the experimental facts, suggesting that the longitudinal atomic motion was classical and that the transverse motion was quantum. We determined the transverse
momentum distribution in the state $\psi(x, t)$, by evaluating its Fourier transform.

We calculated $|\psi(x, t)|^2$ for one and two-slit interferometers, and presented results in Figs. 3 and 4. From the data one can see that the evolution (and spreading) of waves from different single slits and their interference (overlap) determine Fresnel’s and Fraunhofer’s regimes and the transition from the former to the latter. We conclude that this spatial distribution, which reflects the non-classical atomic motion, is due to a real atomic wave that is associated with each atom and that influences its motion. The obstacle in front of the incoming atoms determines the concrete form of this influence.

Our results show that the de Broglie \cite{15}, the Bohm and Vigier \cite{16} and the Selleri \cite{17} understandings of wave-particle duality is applicable to the explanation of the non-classical motion of atoms in an interferometer. The conclusion that the motion is non-classical means that it is different from the motion of a particle which obeys the laws of classical mechanics. This difference is due to the fact that with a classical particle no wave is associated, whereas the atom is accompanied by its wave. This conclusion follows from the measured atomic distribution \cite{1,2}, and its theoretical explanation, based on the particle's wave function, in this and in the previous works \cite{1,2,18}. Therefore, the application of methods for determination of the amplitude and phase structure of the atomic wave field, like the method of Raymer, Beck, and McAlister \cite{19} would be of great importance.

However, neither from the measured space distribution \cite{1,2}, nor from the transverse momentum distribution evaluated in this paper, does it follow that the motion of atoms in the double-slit interferometer violates the classical probability laws, especially the law of the addition of probabilities. From the fact that Wigner’s function, associated with the state $\psi(x, t)$, takes a negative value if does not follow that atomic motion violates the latter law. It rather seems that the classical probability laws are satisfied and are consistent with the de Broglie-Bohm-Vigier-Selleri \cite{15-17} understanding of the wave-particle duality and the compatible statistical interpretation of a wave function \cite{4-6}.

To the best of our knowledge, the atomic transverse momentum distribution in the interferometer has not been measured. However, as pointed out in this paper, it is very important characteristic of the quantum state. Experimental evidence of the transverse momentum distribution would contribute a lot to our understanding of the quantum nature of atomic motion.

Acknowledgments

We acknowledge the communication with Tilman Pfau, who provided details of his experiment and commented an early version of the manuscript.

D. A. and M. B. acknowledge support by Ministry of science and technology of Republic of Serbia under contract 01M01.
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Figure captions

Figure 1. Illustration of the diffraction formula presented with Eq. (6).

Figure 2. Diagram of apparatus used in Ref. [1] to observe atomic interference patterns.

Figure 3. The function $|\psi(x, t)|^2 \equiv |\Phi(x, y = vt)|^2$ for a single slit evaluated from Eq. (6) with $A$ chosen, from the condition $\int |\Phi(x, y = vt)|^2 dx = 1$ for a given $y$. Other parameters are: $k = 4\pi \cdot 10^{10}$ m$^{-1}$, $v = \hbar k/m = 1995.58$ m/s, $m = 6.64632 \cdot 10^{-27}$ kg is the mass of the Helium atom.

Figure 4. The function $|\psi(x, t)|^2 \equiv |\Phi(x, y = vt)|^2$ for a double-slit evaluated from Eq. (6) with $A$ chosen, from the condition $\int |\Phi(x, y = vt)|^2 dx = 1$, for a given $y$. Other parameters are: $k = 4\pi \cdot 10^{10}$ m$^{-1}$, $v = \hbar k/m = 1995.58$ m/s, $m = 6.64632 \cdot 10^{-27}$ kg is the mass of the Helium atom.

Figure 5. Momentum distribution $|c'(k_x)|^2 = |c(p_{x})|^2 \cdot \hbar$ in the state $\psi(x, t)$ with parameters given in the caption of Figs. 3 and 4. a) One open slit, b) Two open slits.

Figure 6. The de Broglian probability density $P'(x, k_x, t) = \hbar P(x, p_{x}, t)$ in the single slit state $\psi(x, t)$ with parameters given in the caption of Fig. 3.

Figure 7. The de Broglian probability density $P'(x, k_x, t) = \hbar P(x, p_{x}, t)$ in the double-slit state $\psi(x, t)$ with parameters given in the caption of Fig. 4.

Figure 8. Wigner’s function $W'(x, k_x, t)$ associated with the single slit state $\psi(x, t)$ and evaluated from (18). Parameters are given in the caption of Fig. 3.

Figure 9. Wigner’s function $W'(x, k_x, t)$ associated with the double-slit state $\psi(x, t)$ and evaluated from (18). Parameters are given in the caption of Fig. 4.
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