A mechanism for air induced fluidization in vibrated granular beds

L. I. Reyes, I. Sánchez, and G. Gutiérrez
Departamento de Física, Universidad Simón Bolívar, Apartado 89000, Caracas 1080-A, Venezuela
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We present a new mechanism for the fluidization of a vibrated granular bed in the presence of interstitial air. We show that the air flow induced across the bed, as the gap in the bottom of the cell evolves, can fluidize the bed in a similar way as in gas-fluidized static beds. We use the model of Kroll to quantify the relevant variables suggested by the mechanism proposed. The relevance of the above fluidization for segregation phenomena is discussed.

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A forced collection of grains can exhibit very rich and striking behavior that sometimes resembles the properties of fluids and solids [1, 2]. Fluidlike behavior can be induced, among other means, by shaking [2, 3] or by injecting gas through the grains [4, 5, 6]. Due to its particular transport properties gas-fluidized granular beds have found many applications in industry [4]. Vibration commonly induces undesirable segregation in many industrial processes which involve the handling of large collections of grains [1, 2]. From a fundamental point of view, forced granular beds offers a plethora of interesting collective behavior. Thus a basic understanding of phase changes as a response of these dissipative systems to a change in forcing conditions, grains properties and of any relevant parameter is of practical and fundamental interest.

The influence of interstitial air in phenomena related to vibrated granular media have been studied in different contexts [2, 3, 10, 11, 12, 13, 14]. Under certain conditions, interstitial air has proved to be a crucial ingredient in segregation phenomena [10, 11, 12, 13, 14], and in this line some efforts to include air in molecular dynamics simulations have appeared recently [13]. However, a coherent description that would allow us to understand the role played by the air in vibrated granular systems is still lacking.

In this article we present a mechanism by which a vibrated bed of grains can become fluidized in the presence of interstitial air. The key ingredient is not just the presence of air but the way it flows through the vibrated bed. We discuss the consequences of this kind of fluidization in terms of a previously proposed quantitative model [15] for reverse buoyancy [16].

We assume that the vibrated grain pack acts like a porous piston so the acceleration of the container induces a pressure difference between its ends as the pack moves relative to the bottom of the container, which in turn produces an air flow across the bed. We use Kroll’s model [7] to describe the evolution of the gap under the grain pack, within one period of vibration. This model takes into account a force that arises as a consequence of the pressure difference across the bed that develops as the gap in the bottom evolves. We propose that the flow of air induced by this pressure difference can fluidize the medium in much the same way as it occurs in gas-fluidized static beds, where gas is injected against gravity. The model of Kroll will allow us to present explicit relations between the flow of air through the bed, the pressure difference between its ends and the gap formed at the bottom of the bed. From these relations we develop a quantitative description of the proposed mechanism of fluidization.

**Kroll’s Model**

In the Kroll’s model [7] the granular bed its treated as a porous piston. When the effective gravity starts to point upward the piston takes off. According to Kroll’s model, in the noninertial reference frame placed on the container, only two forces are taken into account during the flight: the effective weight of the granular bed and the force due to the pressure difference across the piston. The following evolution equation for the gap at the bottom of the bed is obtained (see figure 1):

\[
\frac{d^2 s}{dt^2} + \left(p_a - p_i\right) \frac{A}{m} = -\frac{d^2 w}{dt^2} - g,
\]

where \(s\) is the size of the gap, \(p_a\) is the atmospheric pressure, \(p_i\) is the time dependent pressure in the gap, \(A\) is the transversal area of the container, \(m\) is the mass of the piston, \(g\) is the acceleration of gravity and \(w\) is the position as a function of time of the container.

We can use an equation of state to relate the gap, and hence the volume of the air in the gap, with the pressure \(p_i\). The air is assumed to behave as an incompressible fluid [6, 8]. This results in the additional equation

\[
\frac{ds}{dt} = \frac{1}{\rho_a A} \frac{dG}{dt},
\]

where \(G\) is the mass of air in the gap and \(\rho_a\) is the density of air. It is assumed that the flow of air through the bed is given by Darcy’s law [6, 17] which gives us the following equation

\[
\frac{dG}{dt} = \rho_a A \left[ \frac{k (p_a - p_i)}{\mu h} \right],
\]

where \(k\) is the permeability of the piston, \(h\) it’s height and \(\mu\) is the viscosity of air. Equations [11, 31] can be
\[
\Gamma = a\omega
\]

the adimensional gap \(s\), then the gap will be small. The adimensional number \(\tau\) depends on the grains diameter through the permeability \(K\). We introduce the adimensional quantities \(s = s/a\) and \(t' = \omega t\), where \(s = s/a\) and \(\tau = \omega t_0\) and the adimensional number \(\tau_K\) is given by

\[
\tau_K = \frac{\omega k \rho_m}{\mu}.
\]

We see that in this model the effect of air in the piston movement is a usual frictional factor, with a friction coefficient given by \(1/\tau_K\). We expect that if \(\tau_K\) is small then the gap will be small. The adimensional number \(\tau_K\) depends on the grains diameter through the permeability \(k\) of the medium \(\tau_K\). An interesting prediction that arises is that the gap’s dynamics depends only on the adimensional numbers \(\Gamma\) and \(\tau_K\). Integrating equation (4) with initial conditions \(s'(0) = s'(0) = 0\) we obtain the evolution of the adimensional gap \(s'(t')\):

\[
s' = \frac{\tau_K}{\Gamma}[C - t' - \tau_K(1 + \lambda \Gamma \sin \delta)e^{-t'/\tau_K} - \lambda \Gamma \cos(t' - \delta)],
\]

where \(C = \tau_K + \lambda \Gamma (\tau_K \sin \delta + \cos \delta), \lambda = (1 + \tau_K)^{-1/2},\) and \(\delta = \arctan \tau_K - \arcsin 1/\Gamma\). Equation (6) is valid while \(s' > 0\).

The gap is small when a large pressure difference between the ends of the bed is induced. From equations (2) and (3) we see that in this model the pressure difference across the bed is proportional to the first derivative of \(s\) and is given by

\[
\frac{p_a - p_i}{gh \rho_m} = \frac{\Gamma}{\tau_K} s'.
\]

From Darcy’s law, the flow of air through the grains is proportional to this pressure difference.

**Gas-Fluidized beds and cyclic fluidization in vibrated granular beds**

In the technique of fluidizing beds by injecting gas against gravity a fluidized state is achieved when the force due to the pressure difference across the bed is high enough to cancel the weight of the bed. An important control parameter in this technique is the velocity \(U\) at which gas is injected upward at the bottom of the static bed. As the gas flows up through the granular medium it induces a pressure difference that results in an effective force on the bed that point upward. This force rises as \(U\) is increased from zero and when it equals the weight of the bed fluidlike behavior starts to be observed. This fluidization condition can be written as

\[
\Delta PA \geq mg,
\]

where \(\Delta P\) is the pressure difference between the ends of the bed.

In the Kroll’s model the pressure difference and the effective gravitational field are time dependent. This can be seen if we rewrite the evolution equation (10) for the gap as

\[
m \frac{d^2 s}{dt^2} = mg_{ef}(t) - [p_a - p_i(t)] A,
\]

where \(g_{ef}(t) = g(\Gamma \sin[\omega(t + t_0)] - 1)\) is the effective gravitational field. Before taking off, when \(g_{ef}\) points downward and the bed is in contact with the bottom of the container, there is no significant air flow, the bed is being pushed downward against the bottom of the container and there is no relative movement between the grains; under these conditions we consider the bed as being in a solid state. When \(g_{ef}\) starts to point upward the bed takes off, the size of the gap and the air flow through the grains starts to increase. Following condition (5), for this case we can write the fluidization condition as follows \((g_{ef} \neq 0)\):

\[
\frac{[p_a - p_i(t)] A}{mg_{ef}(t)} \geq 1.
\]

Combining equations (9) and (10), the fluidization condition (10) is initially satisfied at the inflection point of the curve \(s' vs t'\) shown in figure 2. This result is independent of the validity of equations (2) and (3) and, consequently, of the assumptions made in deriving them. If we assume that equations (2) and (3) hold, the beginning of the fluidized state occurs at the adimensional
time \( t_f' \) (measured from the time when \( g_{ef} \) starts to point upwards) which is given by

\[
\cos(t_f' - \delta) e^{t_f' / \tau_K} = \frac{1 + \lambda \Gamma \sin \delta}{\Lambda \Gamma \tau_K}.
\]

(11)

Within the Kroll’s assumptions, at \( t_f' \) the pressure difference across the bed is maximum. From equations (4) or (7) and the discussion above, it is expected that the mechanism proposed will be effective if \( \tau_K \) is small. Equation (11) gives, to our knowledge, a new result. With this equation we can estimate when the new mechanism of fluidization is initiated. This can be experimentally verified by measuring what happens to the granular medium at the inflection point of the curve that represents the evolution of the gap. This, for example, predicts for the case of reverse buoyancy, a change in the motion of an intruder at time \( t_f' \).

In figure 2 we show the evolution in time of the gap (from eq. (6)), the pressure difference across the bed (from eq. (7)), the effective gravitational field acting on the bed and the position of the container for the experimental conditions of reference [15]. Within the fraction of the cycle that runs from \( t_f' \) until the bed hits the bottom of the cell at the end of its flight, we can identify four zones depending on the direction and intensity of the effective gravity \( g_{ef} \) and of the air flow. The fluidization condition (10) is satisfied in zones I and IV (see figure). If the flight time of the bed is small compared with the period of vibration, then we have a granular bed that alternates cyclically between a solid and a fluidized state.

Cyclic Fluidization and Reverse Buoyancy

We are now going to explore the consequences of the above mechanism of fluidization in the context of the phenomenon of reverse buoyancy. Under certain conditions large heavy objects immersed in a vibrated granular medium rise and similar light ones sink to the bottom. This was called reverse buoyancy [16]. In reference [16] it was proposed that this phenomenon could be explained by assuming that the granular bed behaves as a fluid in only a fraction of the cycle of vibration. The necessary feature is that during most of the time in which the bed is in a fluidized state the effective gravity points upward.

In the fluidization condition explained in the previous section, the air flow points downward just after the bed takes off, while the effective gravity points upward [19]. If a buoyancy force arises in the above fluidized state, large heavy objects immersed in it may sink upward while similar light ones may float downward. This case is portrayed in zone I shown in figure 2. In zone IV the fluidization condition (10) is also satisfied, but now \( g_{ef} \) is pointing downwards and we expect to observe regular buoyancy in this zone. Further work is needed to anticipate how the bed would behave in zones II and III, but it is expected that relative movement between the grains is possible in these zones. In particular, we can ask ourselves: is there a buoyancy force in zones II and III? In this line, valuable information can be obtained by resolving the motion of an intruder immersed in the bed within a period of vibration and comparing it with the evolution of the measured pressure difference across the bed. Other mechanisms can play a role in these zones [14].

The time that the bed would spend in each of the zones shown in figure 2 will depend on experimental conditions. Within the model of Kroll, it would depend on \( \Gamma \) and \( \tau_K \). If reverse buoyancy is observed, then it could be a sign that zone I prevails. In figure 3 we show the time that the bed would spend in zones I (\( T_I \)) and IV (\( T_{IV} \)) as predicted by the model of Kroll. It can be seen that if \( \tau_K \) is small then we have an scenario that favors reverse buoyancy. The model predicts that there is a particular value \( \tau_K,c(\Gamma) \) for which \( T_I = T_{IV} \). For \( \tau_K = \tau_K,c \) the average motion of an intruder immersed in the bed would depend on how the bed behaves in zones II and III.

In reference [12] reverse buoyancy was supressed by evacuating the air of the system. If we quench the flow of air through the bed for the experimental conditions of reference [15] by keeping the top of the container open and introducing a filter permeable to air in it’s bottom, as was made in [11], we do not observe reverse buoyancy: light and heavy intruders rise. In both procedures the
FIG. 3: (Color online) Time that the bed will spend in zone I (\(T_I\)) and in zone IV (\(T_{IV}\)) of figure \(\text{E}\) as a function of \(\tau_K\) calculated from the model of Kroll, for \(\Gamma = 2\) and \(\Gamma = 5\).

\(T_I\) and \(T_{IV}\) are reported as its percentage fraction with respect to \(T_I + T_{III} + T_{IV}\), that is the time in which an intruder immersed in the bed can move with respect to the small grains. For \(\tau_K > 3\) the flight time of the bed is close to the period of vibration.

In this article we have presented a mechanism by which a vibrated granular bed can be fluidized in the presence of interstitial air in much the same way as it occurs in gas-fluidized static beds. This mechanism results from an interplay between the flow of air through the bed and the effective gravitational field acting on the bed. In order to quantify the relevant variables of our model, we presented explicit results for the model of Kroll. Since the mechanism proposed is an extension of an static one, and since the bed cannot switch between a fluidized state and a solid state instantly, we expect that this results are applicable when the period of vibration is large. Under this condition, we can predict when a vibrated granular system initially becomes gas fluidized during each cycle of oscillation. The proposed model constitutes a promising quantitative framework that gives some relevant information about the process of cyclic fluidization that occurs in a vibrated granular bed. It would be important to monitor experimentally the evolution of the granular medium, within one period of oscillation of the system.

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[18] If the granular packing undergoes ballistic flight it can be shown that for \(\Gamma > \Gamma_c \approx 1.81\) the maximum gap would be greater than the amplitude of vibration \(a\). So for \(\Gamma > \Gamma_c\) if \(s_{max}\) is small then we can say that the gap is small. If \(\tau_K\) is small and \(\Gamma\) is large we find that the maximum adimensional gap is given by \(s_{max} \approx \frac{\gamma}{\Gamma c} \tau_K\).
[19] It can be shown that within the model of Kroll the fluidization condition \(\text{E}\) always occurs when the effective gravity \(g_{eff}\) is still pointing upwards.