Revisiting hidden-charm pentaquarks from QCD sum rules

Jia-Bing Xiang,1 Hua-Xing Chen,1 Wei Chen,† Xiao-Bo Li1, Xing-Qun Yao,1 and Shi-Lin Zhu1,2,†‡
1 School of Physics and Beijing Key Laboratory of Advanced Nuclear Materials and Physics, Beihang University, Beijing 100191, China
2 School of Physics, Sun Yat-Sen University, Guangzhou 510275, China
3 School of Physics and State Key Laboratory of Nuclear Physics and Technology, Peking University, Beijing 100871, China
4 Collaborative Innovation Center of Quantum Matter, Beijing 100871, China
5 Center of High Energy Physics, Peking University, Beijing 100871, China

We revisit the hidden-charm pentaquark states $P_c(3830)$ and $P_c(4450)$ using the method of QCD sum rules by requiring the pole contribution to be larger than or around 30% to better insure the one-pole parametrization to be valid. We find two mixing currents and our results suggest that the $P_c(3830)$ and $P_c(4450)$ can be interpreted as hidden-charm pentaquark states having $J^{P} = 3/2^-$ and $5/2^+$, respectively, while there still exist other possible spin-parity assignments, such as $J^{P} = 3/2^+$ and $J^{P} = 5/2^-$, which needs to be clarified in further theoretical and experimental studies.

PACS numbers: 12.39.Mk, 12.38.Lg

Keywords: pentaquark states, QCD sum rule

I. INTRODUCTION

Many exotic hadrons have been discovered in the past decade due to significant experimental progresses [1], such as the two hidden-charm pentaquark resonances $P_c(3830)$ and $P_c(4450)$ discovered by the LHCb Collaboration [2]. Besides them, more exotic hadrons are likely to be observed in the future BaBar, Belle, BESIII, CMS and LHCb experiments, etc. They are new blocks of QCD matter, providing important hints to deepen our understanding of the non-perturbative QCD, and their relevant theoretical and experimental studies are opening a new page for the hadron physics [3].

In the past year, the $P_c(3830)$ and $P_c(4450)$ have been studied by various methods and models in order to explain their nature. There are many possible interpretations, such as meson-baryon molecules [4], compact diquark-diquark-antiquark pentaquarks [5], compact diquark-triquark pentaquarks [6], genuine multi-quark states other than molecules [7], and kinematical effects related to thresholds and triangle singularity [8], etc. Their productions and decay properties are also interesting [9]. More extensive discussions can be found in Refs. [10].

The preferred spin-parity assignments for the $P_c(3830)$ and $P_c(4450)$ states were suggested to be $(3/2^-, 5/2^+)$, while some other assignments were also suggested to be possible by the LHCb Collaboration, such as $(3/2^+, 5/2^-)$ and $(5/2^+, 3/2^-)$ [2]. It is useful to study all these possible assignments theoretically in order to better understand their properties.

In this paper we shall use the method of QCD sum rule to study the possible spin-parity assignments of the $P_c(3830)$ and $P_c(4450)$. Before doing this, we shall reinvestigate our previous studies on the $P_c(3830)$ and $P_c(4450)$ [11] by requiring the pole contribution to be larger than or around 30% to better insure the one-pole parametrization to be valid. This paper is organized as follows: the above reinvestigation will be done in Sec. II, numerical analyses will be done in Sec. III, the investigation of hidden-charm pentaquark states of $J^{P} = 3/2^+$ and $J^{P} = 5/2^-$ will be done in Sec. IV, and the results will be discussed and summarized in Sec. V. This paper has a supplementary file “OPE.nb” containing all the spectral densities.

II. QCD SUM RULES ANALYSES

All the local hidden-charm pentaquark interpolating currents have been systematically constructed in Refs. [11], and some of them were selected to perform QCD sum rule analyses. The results suggest that the $P_c(3830)$ and $P_c(4450)$ can be interpreted as hidden-charm pentaquark states composed of anticharmed mesons and charmed baryons. However, the analyses therein use one criterion which is not optimized, that is to require the pole contribution to be larger than 10% to insure the one-pole parametrization to be valid. This value is not so significant, and accordingly, the question arises whether we can find a larger pole contribution to better insure the one-pole parametrization?

In the present study we try to answer this question in order to find better (more reliable) QCD sum rule results.
especially, we find the following two mixing currents:
\begin{align}
J_{\mu,3/2^{-}} &= \cos \theta_1 \times \xi_{3\mu} + \sin \theta_1 \times \psi_{\mu} \\
&= \cos \theta_1 \times \left[ e^{abc} (u^T_0 C^\gamma \gamma_5 d_b) \gamma_\mu \gamma_5 c_c \bar{c}_d \gamma_\mu \gamma_5 u_d \right] + \sin \theta_1 \times \left[ e^{abc} (u^T_0 C^\gamma \gamma_5 d_b) \gamma_\mu \gamma_5 c_c \bar{c}_d \gamma_\mu \gamma_5 u_d \right], \\
J_{\mu,5/2^{+}} &= \cos \theta_2 \times \xi_{15\mu} + \sin \theta_2 \times \psi_{4\mu} \\
&= \cos \theta_2 \times \left[ e^{abc} (u^T_0 C^\gamma \gamma_5 d_b) \gamma_\mu \gamma_5 c_c \bar{c}_d \gamma_\mu \gamma_5 u_d \right] + \sin \theta_2 \times \left[ e^{abc} (u^T_0 C^\gamma \gamma_5 d_b) \gamma_\mu \gamma_5 c_c \bar{c}_d \gamma_\mu \gamma_5 u_d \right] + \langle \mu \leftrightarrow \nu \rangle,
\end{align}
where a \cdots d are color indices; \theta_{1/2} are two mixing angles; \( J_{\mu,3/2^{-}} \) and \( J_{\mu,5/2^{+}} \) have the spin-parity \( J^P = 3/2^- \) and \( 5/2^+ \), respectively. The four single currents, \( \xi_{3\mu}, \psi_{\mu}, \xi_{15\mu} \) and \( \psi_{4\mu} \), were first constructed in Refs. [11]. We can verify:

1. The current \( \xi_{3\mu} \) well couples to the \( S \)-wave \( [\Lambda_c(1P)\bar{D}_1] \), \( P \)-wave \( [\Lambda_c(1P)\bar{D}_3] \), \( P \)-wave \( [\Lambda_c\bar{D}_1] \), and \( D \)-wave \( [\Lambda_c\bar{D}] \) channels, etc. Here the \( \Lambda_c(1P) \) denotes the \( \Lambda_c(2593) \) of \( J^P = 1/2^- \) and \( \Lambda_c(2625) \) of \( J^P = 3/2^- \).

2. The current \( \psi_{\mu} \) well couples to the \( S \)-wave \( [\Sigma_c \bar{D}^*] \) channel, etc.

3. The current \( \xi_{15\mu} \) well couples to the \( S \)-wave \( [\Lambda_c(1P)\bar{D}^*] \) and \( P \)-wave \( [\Lambda_c\bar{D}^*] \) channels, etc.

4. The current \( \psi_{4\mu} \) well couples to the \( S \)-wave \( [\Sigma_c \bar{D}_1] \) and \( P \)-wave \( [\Sigma_c \bar{D}] \) channels, etc.

We shall use the above two mixing currents, \( J_{\mu,3/2^{-}} \) and \( J_{\mu,5/2^{+}} \), to perform QCD sum rule analyses, and the results will be given in the next section. Before doing that we briefly introduce our approach here, and we refer interested readers to read Refs. [11, 12] for details. More sum rule studies based on the heavy quark effective theory (HQET) can be found in Refs. [13].

Firstly, we assume \( J_{\mu,3/2^{-}} \) and \( J_{\mu,5/2^{+}} \) couple to physical states through
\begin{align}
\langle 0 | J_{\mu,3/2^{-}} | X_{3/2^{-}} \rangle &= f_{X_{3/2^{-}}} \mu(p) , \\
\langle 0 | J_{\mu,5/2^{+}} | X_{5/2^{+}} \rangle &= f_{X_{5/2^{+}}} \mu(p) ,
\end{align}
and we write the two-point correlation functions as
\begin{align}
\Pi_{\mu,3/2^{-}}(q^2) &= i \int d^4 x e^{i q \cdot x} \langle 0 | T [ J_{\mu,3/2^{-}}(x) \bar{J}_{\mu,3/2^{-}}(0) ] | 0 \rangle \\
&= \left( \frac{4 \mu q}{q^2} - \frac{\mu q}{q^2} \right) (\vec{q} + M_{X_{3/2^{-}}}) \Pi_{3/2^{-}}(q^2) + \cdots ,
\end{align}
where \( \Pi_{\mu,5/2^{+}}(q^2) \) contains non-relevant spin components.

We note that if the physical state has the opposite parity, the \( \gamma_5 \)-coupling should be used [14], for example, if
\begin{equation}
\langle 0 | J_{\mu,3/2^{-}} | X_{3/2^{+}} \rangle = f_{X_{3/2^{+}}} \mu(p) ,
\end{equation}
then
\begin{align}
\Pi_{\mu,3/2^{+}}(q^2) &= i \int d^4 x e^{i q \cdot x} \langle 0 | T [ J_{\mu,3/2^{-}}(x) \bar{J}_{\mu,3/2^{-}}(0) ] | 0 \rangle \\
&= \left( \frac{4 \mu q}{q^2} - \frac{\mu q}{q^2} \right) (\vec{q} - M_{X_{3/2^{+}}}) \Pi_{3/2^{+}}(q^2) + \cdots .
\end{align}

Hence, we can compare terms proportional to \( 1 \times g_{\mu\nu} \) and \( \vec{q} \times g_{\mu\nu} \) to determine the parity of \( X_{3/2^{+}} \). Accordingly, in the present study we shall use the terms proportional to \( 1 \times g_{\mu\nu} \) and \( 1 \times g_{\mu\nu} g_{\sigma\rho} \) to evaluate masses of \( X \)'s, which are then compared with those proportional to \( \vec{q} \times g_{\mu\nu} \) and \( \vec{q} \times g_{\mu\nu} g_{\sigma\rho} \) to determine their parity.

At the hadron level, we use the dispersion relation to rewrite the two-point correlation function as
\begin{equation}
\Pi(q^2) = \frac{1}{\pi} \int_{s_<}^{\infty} \frac{\text{Im} \Pi(s)}{s - q^2 - i\varepsilon} ds ,
\end{equation}
where \( s_< \) is the physical threshold. Its imaginary part is defined as the spectral function, which can be evaluated by inserting intermediate hadron states \( \sum_n | n \rangle \langle n | \), but adopting the usual parametrization of one-pole dominance for the ground state \( X \) together with a continuum contribution:
\begin{align}
\rho(s) &= \frac{1}{\pi} \text{Im} \Pi(s) = \sum_n \delta(s - M_n^2) \langle 0 | J_n | n \rangle \langle n | \bar{J} | 0 \rangle \\
&= f_X^2 \delta(s - m_X^2) + \text{continuum} .
\end{align}

At the quark and gluon level, we insert Eqs. (1–2) into the two-point correlation functions (5–6), and calculate them using the method of operator product expansion (OPE). In the present study we evaluate \( \rho(s) \) at the leading order on \( \alpha_s \) and up to dimension eight. To do this we have calculated the perturbative term, the quark condensate \( \langle \bar{q} q \rangle \), the gluon condensate \( \langle g^2 G G \rangle \), the quark-gluon condensate \( \langle g \bar{q} \sigma G q \rangle \), and their combinations \( \langle \bar{q} q \rangle^2 \) and \( \langle \bar{q} q \rangle \langle q g \bar{q} G q \rangle \). We find that the \( D = 4 \) term \( m_c \langle \bar{q} q \rangle \) and the \( D = 6 \) term \( m_g \langle g \bar{q} \sigma G q \rangle \) are important power corrections to the correlation functions.

Finally, we perform the Borel transform at both the hadron and quark-gluon levels, and express the two-point correlation function as
\begin{equation}
\Pi^{\text{all}}(M_B^2) \equiv B_{M_B^2} \Pi(p^2) = \int_{s_<}^{\infty} e^{-s/M_B^2} \rho(s) ds .
\end{equation}
After assuming that the continuum contribution can be well approximated by the OPE spectral density above a threshold value \( s_0 \), we obtain the sum rule relation
\begin{equation}
M_X^2(s_0, M_B) = \int_{s_<}^{s_0} e^{-s/M_B^2} \rho(s) ds .
\end{equation}
We use the mixing current $J_{µ,3/2-}$ defined in Eq. (1) to perform sum rule analyses, and the terms proportional to $1 \times g_{µ}$ are shown in Eq. (13), where $t_1 = \cos \theta_1$ and $t_2 = \sin \theta_1$. We do not list those proportional to $\tilde{g} \times g_{µ}$ for simplicity, but note that they are almost the same as the former ones, suggesting that the state coupled by $J_{µ,3/2-}$ has the spin-parity $J^P = 3/2^-$. Similarly, we use $J_{µ,5/2+}$ defined in Eq. (2) to perform sum rule analyses, and find its relevant state has the spin-parity $J^P = 5/2^+$. We show the terms proportional to $1 \times g_{µ}g_{σ}$ in the supplementary file “OPE.nb”. These two sum rules will be used to perform numerical analyses in the next section.

III. NUMERICAL ANALYSES

In this section we use the sum rules for $J_{µ,3/2-}$ and $J_{µ,5/2+}$ to perform numerical analyses. Various condensates inside these equations take the following values [1, 15]:

$$\langle \tilde{q}q \rangle = -(0.24 \pm 0.01)^3 \text{GeV}^3, \quad \langle g_{q}^2 G G \rangle = (0.48 \pm 0.14) \text{GeV}^4, \quad \langle g_{q} \tilde{q}σGq \rangle = M_0^2 \times \langle \tilde{q}q \rangle, \quad M_0^2 = -0.8 \text{GeV}^2. \quad (14)$$

We also need the charm and bottom quark masses, for which we use the running mass in the $\overline{MS}$ scheme [1, 15]:

$$m_c = 1.275 \pm 0.025 \text{GeV}, \quad m_b = 4.18^{+0.04}_{-0.03} \text{GeV}. \quad (15)$$

There are altogether three free parameters in Eq. (12): the mixing angles $θ_1/2$, the Borel mass $M_B$, and the threshold value $s_0$. We find that after fine-tuning the two mixing angles to be $θ_1 = -42^\circ$ and $θ_2 = -45^\circ$, the following three criteria can be satisfied so that reliable sum rule results can be achieved:

1. The first criterion is used to insure the convergence of the OPE series, i.e., we require the dimension eight to be less than 10%, which can be used to determine the lower limit of the Borel mass:

$$\text{CVG} \equiv \left| \frac{\Pi(\langle \tilde{q}q \rangle, \langle g_{q} \tilde{q}σGq \rangle, (\infty, M_B))}{\Pi(\infty, M_B)} \right| \leq 10%. \quad (16)$$

2. The second criterion is used to insure the one-pole parametrization to be valid, i.e., we require the pole contribution (PC) to be larger than or around 30%, which can be used to determine the upper limit of the Borel mass:

$$\text{PC}(s_0, M_B) \equiv \frac{\Pi(s_0, M_B)}{\Pi(\infty, M_B)} \geq 30%. \quad (17)$$

This criterion better insures the one-pole parametrization than that used in Refs. [11] which only requires PC $\geq 10%$.

3. The third criterion is to require that both the $s_0$ and the $M_B$ dependence of the mass prediction be the weakest in order to obtain reliable mass predictions.

We use the sum rules (13) for the current $J_{µ,3/2-}$ as an example. Firstly, we fix $θ_1 = -42^\circ$ and $s_0 = 23 \text{ GeV}^2$, and show CVG as a function of $M_B$ in the left panel of Fig. 1. We find that the OPE convergence improves with the increase of $M_B$, and the first criterion requires that $M_B^2 \geq 2.89 \text{ GeV}^2$. We also show the relative contribution of each term in the middle panel of Fig. 1, again we find that a good convergence can be achieved in the same region $M_B^2 \geq 2.89 \text{ GeV}^2$. Secondly, we still fix $θ_1 = -42^\circ$ and $s_0 = 23 \text{ GeV}^2$, and show PC as a function of $M_B$ in the right panel of Fig. 1. We find that PC decreases with the increase of $M_B$, and PC $= 32\%$ when $M_B^2 = 2.89 \text{ GeV}^2$. Accordingly, we fix the Borel mass to be $M_B^2 = 2.89 \text{ GeV}^2$ and choose $2.59 \text{ GeV}^2 < M_B^2 < 3.19 \text{ GeV}^2$ as our working region. We show variations of $M_X$ with respect to $M_B$ in the left panel of Fig. 2, and find that the mass curves are quite stable around $M_B^2 = 2.89 \text{ GeV}^2$ as well as inside the Borel window $2.59 \text{ GeV}^2 < M_B^2 < 3.19 \text{ GeV}^2$.

To use the third criterion to determine $s_0$, we show variations of $M_X$ with respect to $s_0$ in the middle panel of Fig. 2 when fixing $θ_1 = -42^\circ$. The mass curves have a minimum against $s_0$ when $s_0$ is around $17 \text{ GeV}^2$, so the $s_0$ dependence of the mass prediction is the weakest at this point. However, the pole contribution at this point is quite small (just 8%). We find that PC $= 32\%$ at $s_0 = 23 \text{ GeV}^2$. Moreover, the $M_B$ dependence is the weakest at this point. Accordingly, we fix the threshold value to be $s_0 = 23 \text{ GeV}^2$ and choose $21 \text{ GeV}^2 \leq s_0 \leq 25 \text{ GeV}^2$ as our working region.

Finally, we change $θ_1$ and redo the above processes. We show variations of $M_X$ with respect to $θ_1$ in the right panel of Fig. 2 when fixing $s_0 = 23 \text{ GeV}^2$ and choosing $M_B$ to satisfy CVG $= 10\%$. We find that the $θ_1$ dependence of the mass prediction is weak when $θ_1 \geq 40^\circ$. Accordingly, we fix the mixing angle $θ_1$ to be $-42^\circ$ and choose $θ_2 = -42 \pm 5^\circ$ as our working region.

Altogether for the current $J_{µ,3/2-}$, we fine-tune the mixing angle $θ_1$ to be $-42^\circ$, and the working regions are found to be $21 \text{ GeV}^2 \leq s_0 \leq 25 \text{ GeV}^2$ and $2.59 \text{ GeV}^2 < M_B^2 < 3.19 \text{ GeV}^2$. We assume the uncertainty of $θ_1$ to be $-42 \pm 5^\circ$, and obtain the following numerical results:

$$M_{3/2-} = 4.40^{+0.17}_{-0.22} \text{ GeV}, \quad (18)$$
$$f_{3/2-} = (2.4^{+0.9}_{-0.3}) \times 10^{-5} \text{ GeV}^6,$$

where the central value corresponds to $θ_1 = -42^\circ$, $s_0 = 23 \text{ GeV}^2$ and $M_B^2 = 2.89 \text{ GeV}^2$. The mass uncertainty is due to the mixing angle $θ_1$, the Borel mass $M_B$, the threshold value $s_0$, the charm quark mass $m_c$, and various condensates [1, 15]. We note that: a) when calculating the mass uncertainty due to the mixing angle $θ_1$, we have fixed $s_0$ and $M_B$; and b) when plotting the
mass variation as a function of $\theta_1$ as shown in the right panel of Fig. 2, we have fixed $s_0$ but choosing $M_B$ to satisfy CVG = 10%. The above mass value is consistent with the experimental mass of the $P_c(4380)$ [2], and supports it to be a hidden-charm pentaquark having $J^P = 3/2^+$. The current $J_{\mu,3/2^-}$ consists of $\xi_{36\mu}$ and $\psi_{4\mu}$, suggesting that the $P_c(4380)$ may contain the $S$-wave $[\Lambda_c(1P)D_1]$, $P$-wave $[\Lambda_c(1P)D_1]$, $P$-wave $[\Lambda_c(1P)D_1]$, $D$-wave $[\Lambda_cD]$, and $S$-wave $[\Sigma_cD^*]$ components, etc. 

Similarly, we investigate the current $J_{\mu,5/2^+}$ of $J^P = 5/2^+$. We fine-tune the mixing angle $\theta_2$ to be $-45 \pm 5^\circ$, and the working regions are found to be $21$ GeV$^2 \leq s_0 \leq 25$ GeV$^2$ and $2.31$ GeV$^2 < M_B^2 < 2.91$ GeV$^2$. We show variations of $M_X$ with respect to $M_B$, $s_0$, and $\theta_2$ in Fig. 3, and obtain the following numerical results:

$$M_{5/2^+} = 4.50_{-0.26}^{+0.26} \text{ GeV},$$

$$f_{5/2^+} = (1.3_{-0.3}^{+0.6}) \times 10^{-5} \text{ GeV}^6,$$

where the central value corresponds to $\theta_2 = -45^\circ$, $s_0 = 23$ GeV$^2$ and $M_B^2 = 2.61$ GeV$^2$. The above mass value is consistent with the experimental mass of the $P_c(4450)$ [2], and supports it to be a hidden-charm pentaquark having $J^P = 5/2^+$. The current $J_{\mu,5/2^+}$ consists of $\xi_{15\mu}$ and $\psi_{4\mu}$, suggesting that the $P_c(4450)$ may contain the $S$-wave $[\Lambda_c(1P)D^*]$, $P$-wave $[\Lambda_cD^*]$, $S$-wave $[\Sigma_cD_1]$, and $P$-wave $[\Sigma_cD^*]$ components, etc.
FIG. 1: In the left panel we show CVG, defined in Eq. (16), as a function of the Borel mass $M_B$. In the middle panel we show the relative contribution of each term on the OPE expansion, as a function of the Borel mass $M_B$. In the right panel we show the variation of PC, defined in Eq. (17), as a function of the Borel mass $M_B$. Here we use the current $J_{\mu, 3/2^-}$ of $J^P = 3/2^-$, and choose $\theta_1 = -42^\circ$ and $s_0 = 23$ GeV$^2$.

FIG. 2: Variations of $M_{3/2^-}$ with respect to the Borel mass $M_B$ (left), the threshold value $s_0$ (middle) and the mixing angle $\theta_1$ (right), calculated using the current $J_{\mu, 3/2^-}$ of $J^P = 3/2^-$. In the left figure, the long-dashed, solid and short-dashed curves are obtained with $\theta_1 = -42^\circ$ and for $s_0 = 21, 23$ and 25 GeV$^2$, respectively. In the middle figure, the curve is obtained with $\theta_1 = -42^\circ$ and $M_B^2 = 2.89$ GeV$^2$. In the right figure, the curve is obtained for $s_0 = 23$ GeV$^2$ and with $M_B$ satisfying CVG$= 10\%$.

FIG. 3: Variations of $M_{5/2^+}$ with respect to the Borel mass $M_B$ (left), the threshold value $s_0$ (middle) and the mixing angle $\theta_2$ (right), calculated using the current $J_{\mu, 5/2^+}$ of $J^P = 5/2^+$. In the left figure, the long-dashed, solid and short-dashed curves are obtained with $\theta_2 = -45^\circ$ and for $s_0 = 21, 23$ and 25 GeV$^2$, respectively. In the middle figure, the curve is obtained with $\theta_2 = -45^\circ$ and $M_B^2 = 2.61$ GeV$^2$. In the right figure, the curve is obtained for $s_0 = 23$ GeV$^2$ and with $M_B$ satisfying CVG$= 10\%$.

IV. OTHER SPIN-PARITY ASSIGNMENTS

In this section we follow the same approach to study the hidden-charm pentaquark states of $J^P = 3/2^+$ and $J^P = 5/2^+$. We find the following two currents

\[ J_{\mu, 3/2^+} = \cos \theta_3 \times \xi_{35\mu} + \sin \theta_3 \times \psi_{10\mu} \tag{20} \]
\[ = \cos \theta_3 \times [\epsilon^{abc}(u^T a C\gamma_\mu \gamma_5 d_b)\gamma_\nu \gamma_5 c_c][\bar{c}_d \gamma_\mu u_d] \]
\[ + \sin \theta_3 \times [\epsilon^{abc}(u^T a C\gamma_\mu u_b)\gamma_\nu \gamma_5 c_c][\bar{c}_d \gamma_\mu \gamma_5 d_d] \]

\[ J_{\mu, 5/2^-} = \cos \theta_4 \times \xi_{16\mu} + \sin \theta_4 \times \psi_{34\mu} \tag{21} \]
\[ = \cos \theta_4 \times [\epsilon^{abc}(u^T a C\gamma_\mu \gamma_5 d_b)\gamma_\nu \gamma_5 c_c][\bar{c}_d \gamma_\mu u_d] \]
\[ + \sin \theta_4 \times [\epsilon^{abc}(u^T a C\gamma_\mu u_b)\gamma_\nu \gamma_5 c_c][\bar{c}_d \gamma_\mu \gamma_5 d_d] \]
\[ + \{\mu \leftrightarrow \nu\}, \]
which have structures similar to $J_{\mu,3/2}^-$ and $J_{\mu,5/2}^+$, respectively. The extracted transition densities are also similar to previous ones:

$$
\rho_{3/2}^+(s) = \rho_{3/2}^{pert}(s) - \rho_{3/2}^{(qq)}(s) + \rho_{3/2}^{(GG)}(s)
$$

$$
\rho_{5/2}^+(s) = -\rho_{5/2}^{(qq)}(s) + \rho_{5/2}^{(GG)}(s) \quad (22)
$$

$$
\rho_{3/2}^-(s) = \rho_{3/2}^{pert}(s) - \rho_{3/2}^{(qq)}(s) + \rho_{3/2}^{(GG)}(s)
$$

$$
\rho_{5/2}^-(s) = -\rho_{5/2}^{(qq)}(s) + \rho_{5/2}^{(GG)}(s) \quad (23)
$$

where $\rho_{3/2}^{pert}(s)$ and others have been given in the sum rules (13) as well as the supplementary file “OPE.nb”.

Firstly, we study the current $J_{\mu,3/2}^+$ of $J^P = 3/2^+$. With the same mixing angle as $\theta_1$, i.e., $\theta_3 = \theta_1 = -42 \pm 5^\circ$, the working regions are found to be $21 \mathrm{GeV}^2 \leq s_0 \leq 25 \mathrm{GeV}^2$ and $2.58 \mathrm{GeV}^2 < M_B^2 < 3.18 \mathrm{GeV}^2$. We show variations of $M_X$ with respect to $s_0$ in the left panel of Fig. 4 with $\theta_3 = -42^\circ$, where the mass is extracted to be

$$
M_{3/2}^+ = 4.40^{+0.14}_{-0.16} \mathrm{GeV} \quad (24)
$$

Then we study the current $J_{\mu,5/2}^-$ of $J^P = 5/2^-$. With the same mixing angle as $\theta_2$, i.e., $\theta_1 = \theta_2 = -45 \pm 5^\circ$, the working regions are found to be $21 \mathrm{GeV}^2 \leq s_0 \leq 25 \mathrm{GeV}^2$ and $2.20 \mathrm{GeV}^2 < M_B^2 < 2.80 \mathrm{GeV}^2$. We show variations of $M_X$ with respect to $s_0$ in the right panel of Fig. 4 with $\theta_4 = -45^\circ$, where the mass is extracted to be

$$
M_{5/2}^- = 4.43^{+0.26}_{-0.28} \mathrm{GeV} \quad (25)
$$

The above two values are both consistent with the experimental masses of the $P_c(4380)$ and $P_c(4450)$ [2], suggesting that their spin-parity assignments can be different from $J^P = 3/2^-$ and $5/2^-$, and further theoretical and experimental efforts are required to clarify their properties.

V. RESULTS AND DISCUSSIONS

In this paper we use the method of QCD sum rules to study the hidden-charm pentaquark states $P_c(4380)$ and $P_c(4450)$. We achieve better QCD sum rule results by requiring the pole contribution to be larger than or around 30% to insure the one-pole parametrization to be valid, which criterion is more strict than that used in our previous studies [11]. We find two mixing currents, $J_{\mu,3/2}^-$ of $J^P = 3/2^-$ and $J_{\mu,5/2}^+$ of $J^P = 5/2^+$. We use them to perform sum rule analyses, and the masses are extracted to be

$$
M_{3/2}^- = 4.40^{+0.36}_{-0.23} \mathrm{GeV},
$$

$$
M_{5/2}^+ = 4.50^{+0.26}_{-0.23} \mathrm{GeV}.
$$

These values are consistent with the experimental masses of the $P_c(4380)$ and $P_c(4450)$, suggesting that they can be identified as hidden-charm pentaquark states composed of anti-charmed mesons and charmed baryons: the $P_c(4380)$ has $J^P = 3/2^-$ and may contain the $S$-wave $[\Lambda_c(1P)\bar{D}_1]$, $P$-wave $[\Lambda_c(1P)\bar{D}]$, $P$-wave $[\Lambda_c\bar{D}_1]$, $D$-wave $[\Lambda_c\bar{D}]$, and $S$-wave $[\Sigma_c\bar{D}_1]$, components, etc; the $P_c(4450)$ has $J^P = 5/2^+$ and may contain the $S$-wave $[\Lambda_c(1P)\bar{D}^*]$, $P$-wave $[\Lambda_c\bar{D}^*]$, $S$-wave $[\Sigma_c\bar{D}_1]$, and $P$-wave $[\Sigma_c\bar{D}]$ components, etc.

We follow the same approach to study the hidden-charm pentaquark states of $J^P = 3/2^+$ and $J^P = 5/2^-$, and extract their masses to be

$$
M_{3/2}^+ = 4.40^{+0.14}_{-0.16} \mathrm{GeV},
$$

$$
M_{5/2}^- = 4.43^{+0.26}_{-0.28} \mathrm{GeV}.
$$

These values are also consistent with the experimental masses of the $P_c(4380)$ and $P_c(4450)$ [2], suggesting that there still exist other possible spin-parity assignments for them, which needs to be clarified in further theoretical and experimental studies.

To end this paper, we have also investigated the bottom partners of the $P_c(4380)$ and $P_c(4450)$, i.e., the hidden-bottom pentaquark states $(b\bar{b}uud)$ of $J^P = 3/2^-$ and $J^P = 5/2^+$. As shown in Fig. 5, their masses are extracted to be

$$
M_{Pb}(3/2^-) = 10.83^{+0.26}_{-0.29} \mathrm{GeV},
$$

$$
M_{Pb}(5/2^+) = 10.85^{+0.24}_{-0.27} \mathrm{GeV}.
$$

We propose to search for them in the future LHCb and BelleII experiments.

Acknowledgments

We thank Professor Nikolai Kochelev for helpful discussions. This project is supported by the National Natural Science Foundation of China under Grants No. 11722540, No. 11475015, and No. 11261130311, the Fundamental Research Funds for the Central Universities.

[1] C. Patrignani et al. [Particle Data Group], Chin. Phys. C 40, 100001 (2016).
[2] R. Aaij et al. [LHCb Collaboration], Phys. Rev. Lett. 115, 072001 (2015); R. Aaij et al. [LHCb Collaboration], Phys. Rev. Lett. 117, 082003 (2016) Addendum: [Phys. Rev. Lett. 117, 109002 (2016)] Addendum: [Phys. Rev. Lett. 118, 119901 (2017)]; R. Aaij et al. [LHCb Collaboration], Phys. Rev. Lett. 119, 062001 (2017).
[3] H. X. Chen, W. Chen, X. Liu and S. L. Zhu, Phys. Rept. 639, 1 (2016); H. X. Chen, W. Chen, X. Liu, Y. R. Liu
FIG. 4: Variations of $M_{3/2^+}$ (left) and $M_{5/2^-}$ (right) with respect to the threshold value $s_0$, calculated using the current $J_{\mu,3/2^+}$ with $\theta = -42^\circ$ and $J_{\mu,5/2^-}$ with $\theta = -45^\circ$, respectively.

FIG. 5: Variations of $M_{P,(3/2^-)}$ (left) and $M_{P,(5/2^+)}$ (right) with respect to the threshold value $s_0$, calculated using the current $J_{\mu,3/2^-}$ with $\theta = -42^\circ$ and $J_{\mu,5/2^+}$ with $\theta = -45^\circ$, respectively.

and S. L. Zhu, Rept. Prog. Phys. 80, 076201 (2017); A. Esposito, A. Pilloni and A. D. Polosa, Phys. Rept. 668, 1 (2016); F. K. Guo, C. Hanhart, U. G. Meissner, Q. Wang, Q. Zhao and B. S. Zou, arXiv:1705.00141 [hep-ph]; A. Ali, J. S. Lange and S. Stone, Prog. Part. Nucl. Phys. 97, 123 (2017); S. L. Olsen, T. Skwarnicki and D. Zieminska, arXiv:1708.04012 [hep-ph].

[4] J. J. Wu, R. Molina, E. Oset and B. S. Zou, Phys. Rev. Lett. 105, 232001 (2010); Z. C. Yang, Z. F. Sun, J. He, X. Liu and S. L. Zhu, Chin. Phys. C 36, 6 (2012); W. L. Wang, F. Huang, Z. Y. Zhang and B. S. Zou, Phys. Rev. C 84, 015203 (2011); C. W. Xiao, J. Nieves and E. Oset, Phys. Rev. D 88, 056012 (2013); M. Karliner and J. L. Rosner, Phys. Rev. Lett. 115, 122001 (2015); R. Chen, X. Liu, X. Q. Li and S. L. Zou, Phys. Rev. Lett. 115, 132002 (2015); C. W. Xiao and U-G. Meissner, Phys. Rev. D 92, 114002 (2015); J. He, Phys. Lett. B 753, 547 (2016); T. F. Carames, C. E. Fontoura, G. Krein, K. Tsushima, J. Vijande and A. Valcarce, Phys. Rev. D 94, 034009 (2016); Q. F. Lu and Y. B. Dong, Phys. Rev. D 93, 074020 (2016); D. Samart, C. Nualchimplee and Y. Yan, Phys. Rev. D 93, 116004 (2016); Y. Shimizu, D. Suenaga and M. Harada, Phys. Rev. D 93, 114003 (2016); C. W. Shen, F. K. Guo, J. J. Xie and B. S. Zou, Nucl. Phys. A 954, 393 (2016); Y. Yamaguchi and E. Santopinto, Phys. Rev. D 96, 014018 (2017); K. Azizi, Y. Sarac and H. Sundu, Phys. Rev. D 95, 094016 (2017); Y. Dong, A. Faessler and V. E. Lyubovitskij, Prog. Part. Nucl. Phys. 94, 282 (2017); R. Chen, X. Liu and A. Hosaka, arXiv:1707.08306 [hep-ph]; C. W. Shen, D. Ronchen, U. G. Meissner and B. S. Zou, arXiv:1710.03885 [hep-ph].

[5] L. Maiani, A. D. Polosa and V. Riquer, Phys. Lett. B 749, 289 (2015); R. F. Lebed, Phys. Lett. B 749, 454 (2015); Z. G. Wang, Eur. Phys. J. C 76, 70 (2016); Z. G. Wang, Nucl. Phys. B 913, 163 (2016); V. R. Debastiani and F. S. Navarra, arXiv:1706.07553 [hep-ph].

[6] R. Zhu and C. F. Qiao, Phys. Lett. B 756, 259 (2016); J.-M. Richard, A. Valcarce and J. Vijande, arXiv:1710.08239 [hep-ph].

[7] A. Mironov and A. Morozov, JETP Lett. 102, 271 (2015); C. Deng, J. Ping, H. Huang and F. Wang, Phys. Rev. D 95, 014031 (2017); S. Takeuchi and M. Takizawa, Phys. Lett. B 764, 254 (2017); J. Wu, Y. R. Liu, K. Chen, X. Liu and S. L. Zhu, Phys. Rev. D 95, 034002 (2017); W. Park, A. Park, S. Cho and S. H. Lee, Phys. Rev. D 95, 054027 (2017).

[8] F. K. Guo, Ulf-G. Meissner, W. Wang and Z. Yang, Phys. Rev. D 92, 071502 (2015); X. H. Liu, Q. Wang and Q. Zhao, Phys. Lett. B 757, 231 (2016); M. Bayar,
F. Aceti, F. K. Guo and E. Oset, Phys. Rev. D 94, 074039 (2016); T. F. Carams and A. Valcarce, Phys. Lett. B 758, 244 (2016); J. J. Xie and F. K. Guo, Phys. Lett. B 774, 108 (2017).

[9] Y. Huang, J. He, H. F. Zhang and X. R. Chen, J. Phys. G 41, 115004 (2014); G. N. Li, X. G. He and M. He, JHEP 1512, 128 (2015); Q. Wang, X. H. Liu and Q. Zhao, Phys. Rev. D 92, 034022 (2015) V. Kubarovsky and M. B. Voloshin, Phys. Rev. D 92, 031502 (2015); Q. F. Lu, X. Y. Wang, J. J. Xie, X. R. Chen and Y. B. Dong, Phys. Rev. D 93, 034009 (2016); G. J. Wang, L. Ma, X. Liu and S. L. Zhu, Phys. Rev. D 93, 034031 (2016); Y. K. Hsiao and C. Q. Geng, Phys. Lett. B 751, 572 (2015); H. X. Chen, L. S. Geng, W. H. Liang, E. Oset, E. Wang and J. J. Xie, Phys. Rev. C 93, 065203 (2016); E. Wang, H. X. Chen, L. S. Geng, D. M. Li and E. Oset, Phys. Rev. D 93, 094001 (2016); I. Schmidt and M. Siddikov, Phys. Rev. D 93, 094005 (2016); R. Q. Wang, J. Song, K. J. Sun, L. W. Chen, G. Li and F. L. Shao, Phys. Rev. C 94, 044913 (2016); A. Feijoo, V. K. Magas, A. Ramos and E. Oset, Eur. Phys. J. C 76, 446 (2016); S. H. Kim, H. C. Kim and A. Hosaka, Phys. Lett. B 763, 358 (2016); A. N. H. Blin, C. Fernandez-Ramirez, A. Jackura, V. Mathieu, V. I. Mokeev, A. Pilloni and A. P. Szczepaniak, Phys. Rev. D 94, 034002 (2016); C. W. Xiao, Phys. Rev. D 95, 014006 (2017); T. Gutsche, M. A. Ivanov, J. G. Körner, V. E. Lyuboshitskii, V. V. Lyubushkin and P. Santorelli, Phys. Rev. D 96, 013003 (2017); S. Y. Li, Y. R. Liu, Y. N. Liu, Z. G. Si and X. F. Zhang, arXiv:1706.04765 [hep-ph].

[10] T. J. Burns, Eur. Phys. J. A 51, 152 (2015); N. N. Scoccola, D. O. Riska and M. Rho, Phys. Rev. D 92, 051501 (2015); E. Oset et al., Int. J. Mod. Phys. E 25, 1630011 (2016); Z. H. Guo and J. A. Oller, Phys. Rev. D 93, 096001 (2016).

[11] H. X. Chen, W. Chen, X. Liu, T. G. Steele and S. L. Zhu, Phys. Rev. Lett. 115, 172001 (2015); H. X. Chen, E. L. Cui, W. Chen, X. Liu, T. G. Steele and S. L. Zhu, Eur. Phys. J. C 76, 572 (2016).

[12] M. A. Shifman, A. I. Vainshtein and V. I. Zakharov, Nucl. Phys. B 147, 385 (1979); L. J. Reinders, H. Rubinstein and S. Yazaki, Phys. Rept. 127, 1 (1985); M. Nielsen, F. S. Navarra and S. H. Lee, Phys. Rept. 497, 41 (2010). H. X. Chen, A. Hosaka and S. L. Zhu, Phys. Rev. D 74, 054001 (2006); W. Chen and S. L. Zhu, Phys. Rev. D 83, 034010 (2011); W. Chen, T. G. Steele, H. X. Chen and S. L. Zhu, Phys. Rev. D 92, 054002 (2015); H. X. Chen, Q. Mao, W. Chen, X. Liu and S. L. Zhu, Phys. Rev. D 96, 031501 (2017).

[13] H. X. Chen, W. Chen, Q. Mao, A. Hosaka, X. Liu and S. L. Zhu, Phys. Rev. D 91, 054034 (2015); H. X. Chen, Q. Mao, A. Hosaka, X. Liu and S. L. Zhu, Phys. Rev. D 94, 114016 (2016); H. X. Chen, Q. Mao, W. Chen, A. Hosaka, X. Liu and S. L. Zhu, Phys. Rev. D 95, 094008 (2017).

[14] Y. Chung, H. G. Dosch, M. Kremer and D. Schall, Nucl. Phys. B 197, 55 (1982); D. Jido, N. Kodama and M. Oka, Phys. Rev. D 54, 4532 (1996); Y. Kondo, O. Morimatsu and T. Nishikawa, Nucl. Phys. A 764, 303 (2006); K. Ohtani, P. Gabrielli and M. Oka, Phys. Rev. D 87, 034027 (2013).

[15] K. C. Yang, W. Y. P. Hwang, E. M. Henley and L. S. Kisslinger, Phys. Rev. D 47, 3901 (1993); S. Narison, Nucl. Phys. B 509, 312 (1998); S. Narison, Camb. Monogr. Part. Phys. Nucl. Phys. Cosmol. 17, 1 (2002); M. Eidesmuller and M. Jamin, Phys. Lett. B 498, 203 (2001); V. Gimenez, V. Lubicz, F. Mescia, V. Porretti and J. Reyes, Eur. Phys. J. C 41, 535 (2005); M. Jamin, Phys. Lett. B 538, 71 (2002); B. L. Ioffe and K. N. Zyablyuk, Eur. Phys. J. C 27, 229 (2003); A. A. Ovchinnikov and A. A. Pivovarov, Sov. J. Nucl. Phys. 48, 721 (1988) [Yad. Fiz. 48, 1135 (1988)]; P. Colangelo and A. Khodjamirian, “At the Frontier of Particle Physics/Handbook of QCD” (World Scientific, Singapore, 2001), Volume 3, 1495.