Variety of c-axis collective excitations in layered multigap superconductors

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We present a dynamical theory for the phase differences along a stacked direction of intrinsic Josephson junctions (IJJ’s) in layered multigap superconductors, motivated by the discovery of highly-anisotropic iron-based superconductors with thick perovskite-type blocking layers. The dynamical equations describing AC and DC intrinsic Josephson effects peculiar to multigap IJJ’s are derived, and collective Leggett mode excitations in addition to the Josephson plasma established in single-gap IJJ’s are predicted. The dispersion relations of their collective modes are explicitly displayed, and the remarkable peculiarity of the Leggett mode is demonstrated.

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Highly-anisotropic layered High-$T_c$ superconductors are natural nano-scale stacks of Josephson junctions, i.e., intrinsic Josephson junction (IJJ) arrays, since superconducting and insulating layers with atomic thickness regularly alternate along the crystalline c-axis. Their high-quality single crystals clearly exhibit Josephson effects only in c-axis electromagnetic response, which are called intrinsic Josephson effects (IJE’s). IJE’s have been experimentally confirmed in various layered High-$T_c$ superconductors, such as Bi$_2$Sr$_2$CaCu$_2$O$_8$ [9,10]. An intriguing feature in IJE’s is unique dynamics arising from couplings between the stacked junctions. Two types of inter-Junction couplings due to inductive [4,5] and capacitive [6,7] origins have been mainly proposed. The inductive coupling in IJJ’s is very strong since the charge screening length is comparable to the layer thickness. However, the capacitive one is not so strong since the inter-plane magnetic penetration depth characterizing the magnetic field screening range extends over several hundred junctions. Meanwhile, the capacitive one is not so strong since the charge screening length is comparable to the layer thickness. However, it has a significant role on IJE’s due to the atomic-scale structure [8].

Most of High-$T_c$ cuprate materials are identified as single-band superconductor. One then defines just a single phase difference between consecutive superconducting layers. The dynamics of the phase difference in single-gap IJJ’s has been intensively studied after the discovery of High-$T_c$ cuprate IJJ’s [9]. In this paper, we extend the dynamical theory for the phase difference to multi-gap IJJ’s, in which more than one phase differences are active through stacked all junctions. Our motivation comes from the discovery of highly-anisotropic iron-based superconductors, such as (Fe$_2$As$_2$)(Sr$_3$V$_2$O$_8$), (Fe$_2$P$_2$)(Sr$_4$SeyO$_8$) and (Fe$_2$As$_2$)(Sr$_4$(Mg, Ti)$_2$O$_6$) [10,11]. These compounds contain thick perovskite-type blocking layers Sr$_4$Mg$_2$O$_6$ (M = Sc, Cr, V) with a thickness of $\sim 15 \AA$, which clearly remind us of Bi$_2$Sr$_2$CaCu$_2$O$_8$. The first principles calculations on these materials indicate that they are multiband systems with strong two-dimensional character whose anisotropy is estimated to be comparable to Bi$_2$Sr$_2$CaCu$_2$O$_8$. In fact, the experimentally observed resistivity of their polycrystalline samples [10] exhibits a broad superconducting transition in the presence of external magnetic field, which is a clear sign of high anisotropy. We also note a direct report that single crystals of an iron-based superconductor, PrFeAsO$_{1-x}$, show the I-V characteristics peculiar to Josephson junctions in the c-axis [12].

What is the most fundamental issue in multi-gap IJJ’s? Since the tunneling channel is also multiple, the number of collective modes in the phase oscillation is simply expected to be multiplied. Confining ourselves to the simplest two-gap systems, we study the multiple collective modes. First, we derive coupled equations of motion for the phase differences describing AC and DC multigap IJE’s. Then, a mode analysis on them clarifies that the in-phase mode corresponding to the Josephson plasma is not significantly altered while the out-of-phase one, i.e., Leggett mode suggested in the presence of two superfluid orders by Leggett [10], emerges as a unique mode. An intriguing focus in this paper is that the c-axis Leggett mode is weakly dispersive and favors synchronous oscillations along c-axis. Such a behavior is striking contrast to the Josephson plasma.

Consider the two-gap IJJ’s composed of $N$ junctions as shown in Fig. 1(a). We assume the pairing interaction as $-\sum_{ij=1,2} g_{ij} \hat{v}_{li\uparrow} \hat{v}_{l'j\downarrow} + \hat{v}_{li\downarrow} \hat{v}_{l'j\uparrow}$ on each superconducting layer, where $v_{l\sigma}$ is the electron field operator with spin $\sigma$ in the $i$th band on the $l$th superconducting layer. The coupling constants $g_{11}$ and $g_{22}$ ($g_{12} = g_{21}$) denote the intra-band (inter-band) pairing interaction constants. The inter-band pairing interaction generates the inter-band Josephson coupling energy, $v_{in} \cos(\phi_i^{(1)} - \phi_i^{(2)})$, in the effective action of superconducting phases $\phi_i^{(1)}$. Here,
Consider only the $z$-component of the electric field and only the $y$-component of the magnetic field, which are expressed as $E_{z,l+1}^y = -(1/c)\partial_t A_{l+1, t} - (A_{l+1}^y - A_l^y)/d$ and $B_{l+1, t}^y = (A_{l+1}^y - A_l^y)/d - \partial_x A_{l+1, t}$. As in the single-gap IJJ's one can define the inductive $C$ and capacitive $G$ coupling constants in the dimensionless form as $\eta = \lambda_{ab, t}/sd$ and $\alpha_l = \epsilon_l \mu_l^2/\epsilon$ for each channel in this system. The effective action $\Sigma$ describes low energy dynamics of the superconducting phases in the two-gap IJJ's. In the derivation of Eq. (4), all junction parameters (e.g., $j_{c,i}$ and $\lambda_{ab,i}$) are approximated as local quantities for brevity, although they are originally nonlocal ones. As for the nonlocal electromagnetic effects, see Ref. \cite{18}.

Now, let us derive the coupled equations of motion of the superconducting phase differences. First, one obtains the so-called Josephson relations associated with time and spatial variations of the superconducting phase differences as,

\[ \partial_t \psi_{l+1, t} - \frac{\xi}{2} \partial_t \psi_{l+1, t} = e^* d/\hbar (1 - \tilde{\alpha} \Delta^2) E_{l+1, t}, \tag{2a} \]

\[ \partial_x \psi_{l+1, t} - \frac{\zeta}{2} \partial_x \psi_{l+1, t} = 2e d/\Phi_0 (1 - \tilde{\eta} \Delta^2) B_{l+1, t}^y, \tag{2b} \]

where $\tilde{\alpha}$ and $\tilde{\eta}$ are, respectively, the reduced capacitive and inductive coupling constants given as $\tilde{\alpha}^{-1} = \alpha_1^{-1} + \alpha_2^{-1}$ and $\tilde{\eta}^{-1} = \eta_1^{-1} + \eta_2^{-1}$, $\Delta^2$ the second-order finite difference defined as $\Delta^2 f_{l+1, t} = f_{l+2, t} - 2f_{l+1, t} + f_{l-1, t}$ (for $f_{l+1, t}$), $\Phi_0(=2\pi\hbar/e^*)$ the unit flux, and $\xi = (\alpha_1 + \alpha_2)/(\alpha_1 + \alpha_2)$ and $\zeta = (\eta_1 - \eta_2)/(\eta_1 - \eta_2)$. Here, we introduce $\theta_{l+1, t} = (\theta_{l+1, t} - \theta_{l+1, t})/2$ and $\psi_{l+1, t} = \psi_{l+1, t} - \psi_{l+1, t}$. Equations (2a) and (2b) are interpreted as the generalized Josephson relations in the two-gap IJJ's. These relations are reduced to the conventional ones in the single-gap IJJ's when $\xi = \zeta = 0$. On the variation of $A_{l+1, t}$, the Lagrangian [11] derives the Maxwell equation as

\[ \partial_x B_{l+1, t}^y - \frac{\zeta}{2} \partial_t E_{l+1, t}^y = \frac{4\pi}{c} (j_{l+1, t} + j_{l+1, t}^{\text{QP}}), \tag{3} \]

where $\bar{j}_{l+1, t} = \sum_{i=1}^2 j_{c,i} \sin \theta_{l+1, t}^{0(i)}$ and $\bar{j}_{l+1, t}^{\text{QP}} = \sum_{i=1}^2 j_{l+1, t}^{\text{QP}(i)}$. Here, we add the quasi-particle tunneling current $j_{l+1, t}^{\text{QP}}$, which can be derived microscopically \cite{9]. Furthermore, we have the continuity equations, which can be derived by the variation with respect to $\phi_{l+1, t}^{0(i)}$. From the continuity equations with Eq. (2) we also have the “pseudo” Maxwell equation, which describe the motion of the relative phase differences $\psi_{l+1, t}$, as

\[ \partial_t \bar{E}_{l+1, t}^y = \frac{\xi}{2} \partial_t \bar{E}_{l+1, t}^y = \frac{4\pi}{c} (j_{l+1, t} \sin \psi_{l+1, t} - \sin \psi_{l+1, t}) + \frac{4\pi}{c} \Delta^2 (\bar{j}_{l+1, t} + \bar{j}_{l+1, t}^{\text{QP}}), \tag{4} \]

where $d_{l+1, t}^{\text{QP}(1,2)} = -j_{c,1} \sin \theta_{l+1, t}^{(1,2)} + j_{c,2} \sin \theta_{l+1, t}^{(2,2)}$, $d_{l+1, t}^{\text{QP}(1,2)} = -j_{l+1, t}^{\text{QP}(1,2)}$. The “pseudo” electromagnetic fields

\[ \bar{E}_{l+1, t}^y = \frac{4\pi}{c} (j_{l+1, t} \sin \psi_{l+1, t} - \sin \psi_{l+1, t}) + \frac{4\pi}{c} \Delta^2 (\bar{j}_{l+1, t} + \bar{j}_{l+1, t}^{\text{QP}}), \tag{4} \]
\begin{equation}
\tilde{E}_{l+1,l}^2 = \frac{\hbar}{e^*} \frac{1 - \xi^2}{\Delta d} \partial \psi_{l+1,l} + \xi \Delta^{(2)} E_{l+1,l}^2,
\end{equation}
\begin{equation}
\tilde{B}_{l+1,l}^y = \frac{\Phi_0}{2\pi} \frac{1 - e^2}{\eta d} \partial \psi_{l+1,l} + \xi \Delta^{(2)} B_{l+1,l}^y.
\end{equation}

Equations (2), (3), and (4) provide a set of equations of motion for the phase differences and the electromagnetic fields in the two-gap IJJ’s, that is, the DC and AC Josephson effects in the two-gap IJJ’s can be described by these coupled equations. To solve these equations it is convenient to use the relation defined as \( \psi = \sum_{m=1}^{l} \psi_{m,m-1} + \psi_0 \), where the value of \( \psi_0 \) is specified as the boundary condition [Figs. 1(b) and 1(c)].

Let us focus on the collective phase oscillation modes in the two-gap IJJ’s. Consider the N junction system under the periodic boundary condition along the c-axis. For simplicity, the case of \( \xi = \zeta = 0 \) and \( j_{c,1} = j_{c,2} \) is examined in the following. More general cases will be published elsewhere. Assuming small oscillations, we linearize Eqs. (3) and (4) around \( \theta_{l+1,l} = 0 \) and \( \psi_l = 0 \) with neglecting the dissipation currents \( J_{QP}^{+1,l} \) and \( d_{QP}^{+1,l} \) for the standard mode analysis. The dynamical simulation taking into account the quasiparticle contributions was performed in Ref. 20. Eliminating the electric and magnetic fields from the coupled linearized equations, we can derive the decoupled equations for \( \theta_{l+1,l} \) and \( \psi_{l+1,l} \) as follows,

\begin{equation}
C \partial^2 \bar{\theta} - \frac{e}{c^2} L \partial^2 \bar{\theta} = \frac{1}{\lambda_e^2} C L \bar{\theta},
\end{equation}
\begin{equation}
\frac{1}{2\alpha_{in}} I \partial^2 \bar{\psi} - \frac{1}{2\alpha_d} I \partial^2 \bar{\psi} = \frac{2}{\lambda_{in}} N \bar{\psi},
\end{equation}

where \( \lambda_e^{-2} = \lambda_{c,1}^{-2} + \lambda_{c,2}^{-2} \), \( \lambda_{c,1}^{-2} = 4\pi e^* d J_{in}/\hbar c^2 \), \( \lambda_{c,2}^{-2} = 4\pi e^* d J_{in}/\hbar c^2 \), \( \bar{\theta} = \bar{t}(\theta_{2,1}, \theta_{2,2}, \ldots, \theta_{N-1,N}) \), and \( \bar{\psi} = \bar{t}(\psi_{2,1}, \psi_{3,2}, \ldots, \psi_{N-1,N}) \). The coefficients, \( C \) and \( L \), are \( N \times N \) matrices given as \( C = (1 + 2\tilde{\alpha})\mathbf{I} - \tilde{\alpha S} \) and \( L = (1 + 2\tilde{\eta})\mathbf{I} - \tilde{\eta S} \), where \( S \) is an \( N \times N \) tridiagonal matrix with the elements, \( S_{l,l} = 0 \) and \( S_{l,l,\pm 1} = 1 \), and \( \mathbf{I} \) is the \( N \times N \) unit matrix. We note that the matrices \( C \) and \( L \) represent, respectively, the capacitive and inductive couplings between junctions, which are the same as those in the single-gap IJJ’s. Thus, the collective motion of the mean phase differences, which is described by Eq. (5a), is understood to be the Josephson plasma. Moreover, it is clearly found that its dispersion is brought about by the inductive and capacitive couplings between junctions. On the other hand, due to two-gap IJJ’s, we have another collective oscillation mode in the relative phase channel, which is described by Eq. (5b). In the new mode, its origin, i.e., the coupling between junctions is found to be induced by the off-diagonal components of the matrix \( \mathbf{N} = (1 + 2\nu)\mathbf{I} - \nu \mathbf{S} \) with

\begin{equation}
\nu = \frac{1}{4\gamma_{in}^2}, \quad \gamma_{in} = \frac{\lambda_e}{\lambda_{in}} = \sqrt{\frac{|J_{in}|}{j_{c,1} + j_{c,2}}},
\end{equation}

We note that the coupling constant \( \gamma_{in} \) (or \( \nu \)) depends on not the inductive and capacitive coupling constants but just the inter-band Josephson coupling \( J_{in} \). Thus, this mode has its origin only in the inter-band pairing interaction. Hence, one understands that Eq. (5b) describes the Leggett mode in the two-gap IJJ’s. From these results, it is concluded that the Josephson plasma mode is originated from the inductive and capacitive coupling arising from the electromagnetic field screening, while the Leggett mode is brought about by the intra-layer inter-band coupling.

The dispersion relations of these two eigen-modes are obtained from Eqs. (5a) and (5b), which are specified in terms of the wave numbers \( k_x \) (in-plane direction) and \( k_z = l\pi/(N+1)d \) (c-direction) as

\begin{equation}
\omega_{\mathbf{P}}(k_x,l) = \omega_p(0,l) \sqrt{1 + \frac{k_x^2 \lambda_e^2}{1 + 2\eta(1 - s_l)}},
\end{equation}

with \( \omega_p(0,l) = \omega_{pl}\sqrt{1 + 2\tilde{\alpha}(1 - s_l)} \) for the longitudinal Josephson plasma and

\begin{equation}
\omega_{\mathbf{L}}(k_x,l) = \omega_{\mathbf{L}}(0,l) \sqrt{1 + \frac{k_x^2 \lambda_e^2_{\mathbf{L}}}{1 + 2\nu(1 - s_l)}},
\end{equation}

FIG. 2: (color online) (a) Dispersion relations for the Josephson-plasma (solid lines) and the Leggett’s modes (dash lines) when \( N = 5 \) and \( \gamma_{in} = 1.0 \). We set \( \alpha_1 = \alpha_2 = 0.1 \), \( \eta_1 = \eta_2 = 10^3 \), and \( j_{c,1} = j_{c,2} \). Enlarged views of the dispersion relations at small in-plane wavenumbers for the five eigenmodes of the Josephson-plasma (b) and the Leggett’s modes (c).
as seen in the figures, since the excitation mode is as
in the Leggett mode does not show such level crossing
perconductors \[22\]. On the other hand, the dispersions
zero and weak field in layered High-
electromagnetic-wave emission is observed only at the
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in the Josephson plasma mode under the presence of the
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ble that both modes are closely located in the low en-
ues applicable to the cuprate IJJ’s. If the Leggett mode
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FIG. 3: (color online) (a) Dispersion relations for the
Josephson-plasma (solid lines) and the Leggett’s modes (dash
lines) when \(\gamma_{in} = 3.0\). The other parameter values are the
same as in Fig. Enlarged viewe at small wave numbers are
shown in (b) and (c).

with \(\lambda_{Leg} = 2\sqrt{\pi}\lambda_c/\sqrt{\eta_1 + \eta_2}\) and \(\omega_L(0, l) = \omega_{Leg}(l, l) - \omega_{pl}(1 + 2\pi(1 - s_j))\) for the longitudinal Leggett mode, where \(s_j = \cos(\pi/(N + 1))\) and \(\omega_{pl}\) and \(\omega_{Leg}\) are, respectively, the Josephson plasma and the Leggett mode frequencies, i.e., \(\omega_{pl} = c/\sqrt{\pi}\lambda_c\) and \(\omega_{Leg} = c/\alpha_1 + \alpha_2/\sqrt{\pi}\lambda_c\). Here, it is clearly found that the origin of the Leggett mode is a fluctuation between two
superfluids which is essential to neutral multiple super-
fluids.

We plot the dispersion relations of these eigen modes in the case of \(N = 5\) with \(j_{c,1} = j_{c,2}\) for \(\gamma_{in} = 1.0\) and
3.0, respectively, in Figs. Enlarged view at small wave numbers are
shown in (b) and (c).

\[\alpha_1 = \alpha_2 = 0.1\] and \(\eta_1 = \eta_2 = 10^3\), which are the values applicable to the cuprate IJJ’s. If the Leggett mode
is a low-energy excitation mode and can lie inside the
energy gaps as the Josephson plasma, then it is possible
that both modes are closely located in the low en-

\(c\)-axis wave number, i.e., \(l = 5\), is the lowest energy one
close to \(k_x = 0\), but this mode changes to the highest
one for larger values of \(k_x\). This is because the large
inductive coupling, which is predominant in a wide \(k_x\)
range, favors \(\pi\) phase shift in the phase differences be-
tween consecutive junctions \[21\]. This discussion clearly
leads to that \(\pi\) antiphase synchronization is preferable
in the Josephson plasma mode under the presence of the
layer parallel magnetic field. In fact, strong synchronous
electromagnetic-wave emission is observed only at the
zero and weak field in layered High-\(T_c\) copper oxide superconductors \[22\]. On the other hand, the dispersions
in the Leggett mode does not show such level crossing
as seen in the figures, since the excitation mode is as-
sociated with only the density channel. This indicates
that the Leggett excitation always prefers synchronous
oscillations along junction stacked direction even in the
presence of the magnetic field. If the Leggett mode is
excited by the charge injection or other ways, then the
synchronized Leggett oscillation emerges and a conver-
sion into the synchronized Josephson plasma excitation
due to inherent nonlinearity may occur.

Finally, we mention that when the difference between
the two tunneling channels exist (i.e., \(j_{c,1} \neq j_{c,2}\), \(\alpha_1 \neq \alpha_2\), and \(\eta_1 \neq \eta_2\)) a mode coupling between the Josephson-
plasma and the Leggett modes can occur. Such a cou-
pling effect is an interesting future task.

In summary, we derived the coupled dynamical equa-
tions for the phase differences which can be utilized for
the analysis of AC and DC Josephson effects in the multi-
gap IJJ’s. The equations revealed that multi-gap IJJ’s
have two collective phase oscillation modes, the Joseph-
son plasma and the Leggett mode whose origins are dif-
ferent. Moreover, it is found the Josephson plasma and
Leggett modes favor \(\pi\) anti-phase and in-phase synchro-
nization along the junction stacking , respectively, in a
wide wave-number range.

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