Computer-Aided Design of Pocket Elliptical Journal Bearings, Part 1: Theory*

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Abstract
A computer-aided design procedure suitable for pocket elliptical journal bearings for turbomachines is proposed. After the description of their geometry and manufacturing, the driving geometric parameters characteristic of the pocket bearing are identified together with the design variants that influence their choice, like supply groove layout and the axial size of the pocket. The concept of journal mobility plot, capable to ease the post-processing of results and useful in journal vibration monitoring, is adapted to pocket elliptical bearings and, in general, to non-circular bearings. The proposed design evaluation includes a calculation sequence involving geometric parameters, FEM simulation and the calculation of proper performance indicators, from which the global performance of the bearing is finally computed.

Key words: Journal Bearing, Computer Aided Design, Lubrication, Finite Element Method, Simulation, Turbomachinery

1. Introduction
A lot of works about journal bearings are available in literature, where a large number of mathematical models are presented, but few papers focus on their design, by using models as an investigation tool, particularly when special bearing shapes are studied, e.g. pocket bearings. Designers of turbomachines, in order to support rotating shafts, often adopt such types of fixed pad bearings, which are less expensive than tilting pad journal bearings and still achieve good performance and stability.

The “pocket” journal bearings, often employed in turbo-generators, are usually defined as radial supports which load-carrying surface is a combination of offset circular bearing profiles (1).

Particularly, they include “pocket” elliptical bearings, which are obtained from conventional elliptical bearings by means of simple additional manufacturing operations. Indeed, “elliptical” bearings are made up of two circular arcs which centers are not coincident (lemon shape) (2), so that their manufacturing is particularly straightforward. Carving an additional cylindrical bearing surface in the bottom pad of a conventional elliptical bearing by means of further turning operation is not a burdensome task. The added constant clearance region, usually referred to as “pocket”, is centered in the vertical (load) direction. Optionally, a pocket may be sometimes machined in the top pad too.

The purpose of the pocket is both to improve the bearing cooling, as already proved in (3), and to accommodate the journal when the rotor does not move, so that it is centered with reference to the location of hydrostatic lifting grooves.

The present paper is aimed to develop a computer-aided design procedure suitable for
pocket elliptical journal bearings. In a separate companion paper (part 2) an application of such method is reported.

The theoretical aspects about geometry and manufacturing for the bearing in study, required to define the variables involved in the design procedure, have been already outlined in (3). The current paper delves deeper into the matter (some basics are repeated for the sake of completeness and clarity) and it analyzes design variants based on different pocket axial length and relative pad positions. Particularly, the effects of pad arrangement on bearing performance depend on the design of the lubricant feed system (supply grooves and ducts), which is considered as a further variable. Studies about the influence of design and layout of supply grooves on the performance of fixed-pad bearings, i.e. their influence on feed-pressure flow (or zero-speed film flow), can be found elsewhere (4). Therefore, as the present work does not deal with the choice of shape and location of supply grooves, conventional grooves are considered and concepts related to their layout are discussed, i.e. how many grooves are convenient to arrange in predetermined locations and how to connect them with the external lubrication system.

In addition, in order to complete the design technique with an effective post-processing of results, the concept of journal mobility is elaborated in the geometrical sense. It was introduced for computational purposes by J.F. Booker (5) in the early 1960’s. The mobility method allowed a full analysis of dynamically loaded journal bearings by means of analytical formula. Booker also proposed a graphical version of the method based on mobility maps (6). In the eighties, with the advent of digital computers, graphical computations by means of mobility maps soon disappeared. Therefore, the present work takes advantage of the concept of mobility, generalized to non-circular bearing geometries, for the graphical representation of bearing profile, journal location and film thickness, while computations resort to computer-aided analysis by means of the Finite Element Method (FEM) and the state-of-the-art of TermoHydroDynamic (THD) lubrication theory.

THD analyses have been performed by means of a suitable FEM code, referred to as FEMLub, which has been developed by the author and tested in previous works (7), (8). In recent times, FEMLub has been officially adopted by an important firm, Italian manufacturer of turbogenerators, in order to carry out hydrodynamic bearing verification and design.

Although the genetic algorithms and particle swarm optimization can be successfully applied in bearing optimization (9), the aims of the present paper do not include the development/adaption of an automatic optimization tool. Due to the large number of variables involved in the design problem in study, an automatic (computer-aided) optimization is indispensable in order to determine the best design. Hence the purpose of the work is to devise a method able to improve an existing design, without performing a full optimization.

Nomenclature

- $A$ : integration domain, mm$^2$
- $d$ : shim thickness, mm
- $c_b$ : barrier clearance, mm
- $c_e$ : pocket clearance, mm
- $c_L$ : lubricant specific heat, J/(kg °C)
- $c_p$ : pad clearance, mm
- $C$ : relative clearance
- $C_e$ : relative pocket clearance
- $C_p$ : relative pad clearance
- $c_{v1}$ : vertical clearance of bottom pad, mm
- $c_{v2}$ : vertical clearance of top pad, mm
| Symbol | Description |
|--------|-------------|
| $e_x$  | journal x coordinate, mm |
| $e_y$  | journal y coordinate, mm |
| $e_1$  | bottom pad eccentricity, mm |
| $E_1$  | bottom pad ellipticity |
| $e_2$  | top pad eccentricity, mm |
| $E_2$  | top pad ellipticity |
| $g$    | gravitational acceleration, m/s² |
| $h$    | film thickness, μm |
| $h_{min}$ | minimum film thickness, μm |
| $k, k_L$ | mixture and lubricant conductivity, W/(m °C) |
| $K_p, K_z$ | turbulent coefficients |
| $L$    | bearing (axial) length, mm |
| $L_b$  | barrier length, mm |
| $L_s$  | supply groove length, mm |
| $m$    | journal mass, kg |
| $m_1$  | bottom pad preload |
| $m_2$  | top pad preload |
| $M_c$  | non-dimensional critical mass |
| $O_c$  | bearing center |
| $O_i$  | pad i center |
| $O_J$  | journal center |
| $p$    | pressure, MPa |
| $p_{max}$ | peak pressure, MPa |
| $p_s$  | supply pressure, MPa |
| $P$    | heat dissipation, kW |
| $Q$    | supply flow rate, l/s |
| $Q_{in}$ | inlet flow, l/s |
| $Q_{out}$ | outlet flow (side loss), l/s |
| $Q_1$  | upstream groove inlet flow, l/s |
| $Q_2$  | downstream groove inlet flow, l/s |
| $R$    | journal radius, mm |
| $R_c$  | pocket radius, mm |
| $R_p$  | pad radius, mm |
| $s$    | lathe spindle offset (2nd turning operation), mm |
| $t$    | cut depth, mm |
| $T$    | temperature, °C |
| $T_s$  | supply temperature, °C |
| $t_{H}$ | cut depth @ horizontal diameter, mm |
| $t_{V1}$ | cut depth @ bottom pad center, mm |
| $t_{V2}$ | cut depth @ top pad center, mm |
| $u, w$ | average flow speed, m/s |
| $W$    | journal load, N |
| $x$    | vertical coordinate in the bearing reference system, m |
| $y$    | horizontal coordinate in the bearing reference system, m |
| $Y$    | performance indicator |
| $Y_{ref}$ | reference performance indicator |
| $Y_{g}$ | global performance indicator, % |
| $Y_{g}^*$ | global performance indicator including stability, % |
| $z$    | coordinate along journal axis in bearing reference system, m |
| $\alpha$ | pocket half-opening angle, deg |
| $\alpha_t$ | pocket opening angle, deg |
2. Manufacturing

The first operation in pocket bearing machining consists of turning a cylindrical surface within the bearing block, which has been previously divided in two halves. The radius of such machined surface, the pocket, must be

\[ R_c = R + c_c \]  

where \( R \) is the journal radius, \( c_c \) is the clearance of the cylindrical pocket surface.

Then, in order to manufacture the elliptical bearing surfaces in the journal housing, a second turning operation is run, after the two halves of the bearing block have been split up and constrained at a distance \( d \) by means of suitable shims.

Fig. 1 shows the geometry of the two bearing block halves, kept at a distance \( PO_c = d \) (shim thickness) before the second turning operation is performed. In the same figure, the contour of material removed by such operation is represented by dash-dotted lines. In all of the following figures bearing clearance is amplified for sake of clarity.

In Fig. 1, if \( O_1 \) and \( O_2 \) denote respectively the centers of the bottom and top pads of the “elliptical” part of the bearing and \( O_c \) is the center of the cylindrical pocket, the center of the turning operation (the location of spindle center) is coincident with both \( O_1 \) and \( O_2 \). The point \( O \), which is set in the middle of the distance \( PO_c \), is the origin of the reference system used to determine the offset \( s = OO_1 = OO_2 \) of the lathe spindle axis. The shim thickness is divided in two parts \( O_1O_c \) and \( PO_1 \) measured by the parameters \( e_1 \) and \( e_2 \), which are referred to as the eccentricities of the bottom and top pads, respectively.

Such designation is explained by Fig. 2, where the final bearing profile is shown. After the second machining, the two halves are joint together so that the top pad descends until its
center $O_2$ moves below $O_c$ at a distance $e_2$, while $O_1$ remains in the same location, at distance $e_1$ from $O_c$.

The radius of the surface machined during the second operation is

$$R_p = R_c + t = R + c_p$$

where $t$ is the cut depth and $c_p$ the clearance of the elliptical bearing pads.

The eccentricity $e_1$ is an important parameter, as it rules the bottom pad profile that determines the load-carrying capacity of the bearing in nominal conditions (the bottom pad preload $m_1$, as explained in §3). Let $\alpha$ denote the half-opening angle of the pocket. Fig. 1 allows one to calculate $e_1$ by means of simple geometrical considerations, as follows

$$e_1 = O_1O_e = O_1H - O_e = \sqrt{R_c^2 - R_c^2 \sin^2 \alpha - R_c \cos \alpha}$$

or to express $e_1$ as a function of manufacturing parameters by means of the equation

$$e_1 = s + d/2$$

Similarly, the top pad eccentricity is given by

$$e_2 = d/2 - s$$

As the same bottom pad eccentricity $e_1$ can be obtained by means of several (infinite) couples of $s$ and $d$ according to Eq. (4), different bearing designs exist characterized by the same bottom lobe profile. Two limit cases, which will be denoted by the subscripts A and B, can be obtained by assuming $e_2 = 0$ and $e_2 = t$, respectively.

3. Geometry

Lubrication phenomena (and consequently bearing performance) are ruled by the film thickness, which is on turn determined by bearing clearance distribution and journal position.

The clearance distribution is chosen by the designer and it is driven by geometric parameters. The relations among such parameters and variants of the geometry are described in §§3.1 and 3.2, respectively. In §3.3 film thickness is linked to the bearing geometric parameters and journal position.

3.1 Equations

Fig. 2, by showing the journal profile when its center overlaps $O_c$, defines the bearing clearances in horizontal and vertical direction, $c_{H}$ and $c_{V}$, respectively. As in general vertical clearance is different in bottom and top pad (1 and 2), two corresponding parameters $c_{V1}$ and $c_{V2}$ are defined.
If higher order infinitesimals (the lengths AB and A'B') are neglected, by comparing Fig. 1 with Fig. 2 and by using Eqs. (1) and (2), the following assembly constraint equations can be formulated

\[ c_p = c_r + t \]  
(6)

\[ c_{v1} = c_r \]  
(7)

\[ c_{v2} = c_p - e_2 = c_r + t - e_2 \]  
(8)

Similar equations can be written in terms of cut depths, instead of clearances. With reference to Fig. 2, the cut depth in horizontal direction (along the junction plane AA') as well as at the bottom and top pad center (on the load line), \( t_{H} \), \( t_{V1} \), and \( t_{V2} \) respectively, in agreement with Eqs. (2), (6), (7) and (8) are given by

\[ t_{H} \cong t \]  
(9)

\[ t_{V1} = e_1 - t \]  
(10)

\[ t_{V2} = c_{v2} - c_r = t - e_2 \]  
(11)

The gap distribution may be also identified by means of non-dimensional variables. Firstly, relative pad and pocket clearances are defined respectively as

\[ C_p = c_p / R \]  
(12)

\[ C_r = c_r / R \]  
(13)

Let \( m_i \) and \( E_i \) be the preload (also referred to as ellipticity ratio \(^{10})\) and the ellipticity of lobe \( i \), respectively, defined as

\[ m_i = e_i / c_p \]  
(14)

\[ E_i = c_{v1} / c_{v2} \]  
(15)

where \( i = 1 \) or 2.

For the top lobe \( (i = 2) \), substituting Eqs. (6), (8) and (14) into Eq. (15) yields

\[ E_2 = c_{v1} / c_{v2} \cong 1/(1 - m_2) \]  
(16)

which is the usual relationship between ellipticity and preload for elliptical bearings. It holds only for the top lobe, while for the bottom one a more complicated relation between \( m_i \) and \( E_i \) can be found. To this goal, Equations (6) and (7) allow linking ellipticity (Eq. (15)) and cut depth as follows

\[ E_i = c_{v1} / c_{v2} \cong t / c_p + 1 = 1/(1 - t / c_p) \]  
(17)

Combining Eqs. (1), (2), (3), (6), and (14) yields two solutions for \( t \). The reasonable one can be substituted in Eq. (17), so that the following preload-ellipticity relation for the bottom lobe can be found

\[ E_i = \frac{1}{2c_p(m_i^2 - 1)} \left[ \frac{2m_i \cos \alpha_i - 2R + \sqrt{2}}{2} - c_r \left( m_i^2 - 2 \right) (c_r + 2R) \right] - \frac{m_i R \cos \alpha_i - 4R \cos \alpha_i}{2} \]  
(18)

where \( \alpha_i = 2 \alpha \) is the opening angle of the pocket (Fig. 2).

### 3.2 Variants

Variations of the above-described geometry are obtained by setting different values of the top pad eccentricity \( e_2 \) and of the axial size of the “elliptical” region.

As far as the first type of variant is concerned, design may be potentially varied between the limit cases A and B defined in §2. For design A \( (e_2 = 0) \) the resulting vertical clearance on the bottom and top pads according to Eqs. (7) and (8) are \( c_{v1A} = c_r \) and \( c_{v2A} = c_p = c_r + t \), respectively, i.e. clearance is higher in the top pad than in the bottom pad.
By increasing \( e_2 \) up to the limit value corresponding to design B \((e_2 = t)\), the vertical clearance of both the bush halves is the same \((c_{V1B} = c_{V2B} = c_c)\), in agreement with Eqs. (7) and (8)), as the edge of the top pad is tangent to the cylindrical profile machined by the first turning operation (Fig. 2 with \( t_{V2} = 0 \)).

With reference to design variants based on the axial size of the elliptical profile, the second turning operation may be performed either on the whole length \( L \) of the bearing land (in the axial direction) or on a part of it, symmetrically with reference to the load line. In the latter case, schematized in Fig. 3, on each atmospheric side of the bearing a cylindrical surface with axial length \( L_b \) and uniform radial clearance \( c_b = c_c \) is left. \( L_b \) in usual applications amounts roughly to 10% of \( L \). As \( c_c \) is the smaller clearance of the journal housing, the two “barrier” surfaces are supposed to be capable of reducing the lubricant side loss.

The top pad clearance also influences the side loss in a way that depends of lubricant feed system design. Hence different supply groove layouts are considered as further design variants. To this purpose, if a maximum of two rectangular grooves may be positioned symmetrically with reference to the load vector (the vertical line down the center of the bearing), the four supply groove layouts shown by Fig. 4 are possible. The relevant working conditions, referred to as supply modes, are identified as follows.

**Inlet 1** denotes the supply mode characterized by a single active feed groove, located upstream\(^1\) of the load-carrying film region. If either the groove arrangement is like in Fig. 4 (a), or only the upstream groove for layout of Fig. 4 (c) is fed, while the remaining is inactive (its oil pipe is closed) the mode is **inlet 1**.

**Inlet 2** identifies a supply mode with a single downstream feed groove (Fig. 4 (b)).

**Inlet (1)&2** means that the lubricant flows only through the downstream groove of the bearing in Fig. 4 (c), while the upstream groove is not fed.

When both the grooves spread oil, two supply modes are possible, referred to as **inlet 1&2** and **inlet 1+2**, relevant to the groove layouts of Fig. 4 (c) and (d), respectively. In comparison with layout of Fig. 4 (c), the one of Fig. 4 (d) is more complicated by the manufacturing point of view and the relevant supply mode (**inlet 1+2**) does not permit to reverse the rotation of the journal without getting worse performance. On the contrary, supply mode **inlet 1+2** minimizes the “chamfer flow” \((^3)\), i.e. the oil loss due to the feed pressure through the chamfer or, broadly speaking, any type of opening between the bearing halves. However, chamfer loss may become a significant part of the oil flow required by the bearing if any type of gap between its two halves is not very small and/or if feed pressure is

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\(^1\) By adopting the reference system and the journal rotation direction in Fig. 5, the upstream groove is the one located roughly at \( \theta = 90 \) deg, while the remaining, located roughly at \( \theta = 270 \) deg, is referred to as downstream groove.
high. Therefore, feed pressure rarely exceeds 1 atm (gage) and the chamfer opening is accurately limited. By using layout of Fig. 4 (c) and supply mode inlet 1&2, the chamfer flow may further increase if the feed flow in the upstream groove (on the left side) reverses, i.e. the lubricant flows from the bearing gap to the feed groove. This feed flow reversal is possible in some operative condition because of the film hydrodynamics (2), which causes the oil to flow out of the upstream supply groove whenever the flow into the downstream groove is greater than the total side loss. When feed flow reversal occurs, for mode inlet 1&2 the corresponding oil flow is lost and, as such lubricant is expelled by feed pressure, it is usually included in the chamfer loss. By using layout of Fig. 4 (d) (mode inlet 1+2) the potential reverse flow into the upstream recess enters the duct that joins the two supply grooves, then it reaches the downstream groove and it is recirculated.

Fig. 4 Layouts of supply grooves: (a) single upstream groove (supply mode: inlet 1), (b) single downstream groove (supply mode: inlet 2), and (c) independent double supply (supply modes: inlet 1, inlet (1)&2, inlet 1&2) and (d) interconnected double supply (supply mode: inlet 1+2).

3.3 Film thickness

In lubrication analysis and design film thickness distribution $h$ must be known for computational and safety check purposes. In order to determine the film thickness onto each point of the bearing surface, clearance parameters, i.e. gap distribution assessed when the journal is centered, are not sufficient, as the journal center position must be further available.

In the bearing reference system $O_c$, x, y, z (Fig. 5), the locations of $O_j$ (journal center) and $O_i$ (pad $i$ center of curvature, Fig. 2) are determined by means of the cartesian coordinates ($e_x$, $e_y$, $L/2$) and ($e_{pi}$, 0, $L/2$), respectively. Out of the barrier zones ($z > L_b$ and $z < L−L_b$), the bearing is made up by three pads: the “elliptical” part of the bottom lobe with eccentricity $e_{p1} = −e_p$ and clearance $c_t = c_p (\pi/2 < \theta < \pi−\alpha$ or $\pi+\alpha < \theta < 3 \pi/2)$, the top eccentric lobe with eccentricity $e_{p2} = e_p$ and clearance $c_t = c_p (−\pi/2 < \theta < \pi/2)$ and a pocket pad with eccentricity $e_{p3} = 0$ and clearance $c_3 = c_r (\pi−\alpha < \theta < \pi+\alpha)$. The barrier zone ($z < L_b$ or $z > L−L_b$) is a cylindrical pad with eccentricity $e_{p4} = 0$ and clearance $c_4 = c_b = c_r$. 
Fig. 5 Bearing design parameters and reference system

By neglecting higher order infinitesimals \((c_i^2/R^2 \approx 0)\) and journal misalignments, the film (or gap) thickness \(h\) at the polar coordinate \(\vartheta\) (Fig. 5) is given by

\[
h(\vartheta,z) = c_i + (e_x - e_{\mu}) \cos \vartheta + e_y \sin \vartheta
\]  

where the eccentricities \(e_{\mu}\) and clearances \(c_i\) of the four pads \((i=1, 2, 3, 4)\), known after the design phase, are set on the basis of the local values of \(\vartheta\) and \(z\) as explained above.

4. Design method

The geometric design of the pocket bearing is aimed to determine the optimal profile and variants, i.e. the gap distribution and the supply mode that provide the best bearing performance. The total number of the independent variables that control the gap distribution in a bearing cross-section, referred to as driving variables, is 5. Through a suitable sequence of mathematical operations, referred to as geometric design sequence (§4.1), all of the remaining dependent parameters and particularly those required by lubrication analysis are computed.

The proposed geometric design method consists in performing a set of design evaluations. They are made up by the above-mentioned geometric design sequence and a subsequent assessment of the bearing performance by means of FEM lubrication analysis in nominal working conditions. Such simulation is briefly described in §4.2. Afterwards, the post-processing of the analysis results is carried out by computing the performance indicators (§4.3) and by plotting the journal position in a mobility graph, which is an effective representation of how the journal location and bearing profile determine film thickness, as explained in §4.4. Generally speaking, mobility graphs can be used to plot the journal position, which may be numerically computed and/or measured by means of proximity sensors, for bearing verification/design or vibration monitoring purposes. They pinpoint the journal mobility area, i.e. the region of a bearing cross-section where the journal center can move.

The so-called driving variables are the shaft geometry (the journal radius \(R\)), known from a previous machine design phase, and 4 dimensionless geometric variables \((C, E_1, \alpha_e, E_2)\), which optimal or near-optimal values must be determined by geometric design, while the dependent parameters, i.e. the remaining geometric and constructive variables, must be assessed by means of the geometric design sequence. \(C\) may be either the pad or the pocket relative clearance, \(C_{\mu}\) or \(C_{c}\) respectively. In usual design practice, the optimal values of the non-dimensional driving variables either are known from experience or are reasonably assumed and then minutely adjusted to the working conditions at the hand by means of geometric design. In the latter case, the optimization may be carried out by performing
design evaluations for several sets of driving variables and by choosing the set for which the best performance has been found. The performance assessment is based on suitable performance indicators, i.e. the output values of both the static analysis and a relevant linear analysis of the bearing stability.

The above-mentioned FEM analyses, beyond the lubricant and working condition data, e.g. speed, load and rheological parameters, and beyond the cited driving and driven variables, which determine the gap in a bearing cross-section, also require additional geometrical information, e.g. the length $L$ of the bearing and the barrier length $L_b$ as well as the geometric parameters used to model the feed grooves ($\vartheta_s$, $\beta_s$, $L_s$, shown in Fig. 5). Although those are additional independent variables as far as the lubrication problem is concerned, in the present treatise they are not included in the driving variables, as they do not control the bearing profile, i.e. the gap in a cross-section of the bearing.

4.1 Geometric design sequence

The geometric design sequence, which is the first step of design evaluation, is the ordered set of mathematical operations required to compute from the 5 independent geometric parameters ($R$, $C$, $E_1$, $\alpha$, $E_2$), referred to as driving variables, the remaining geometric parameters and particularly the input data for lubrication analysis. In each design evaluation, the current driving variables are determined by altering a starting set of driving variables. If the geometric design procedure consists in the improvement or optimization of a previous design, the starting set of driving variables is retrieved from the existing design, but if the bearing is designed from scratch, such starting set must be reasonably chosen. The present paragraph considers the latter case and, therefore, it explains the geometric design sequence together with the choice of the starting set of driving variables.

According to the usual geometric design sequence, the first driving variable that must be set is the journal radius $R$. It depends on the dimensions of the turbomachine, which is usually designed separately. Such first driving dimension determines the size of the bearing.

Afterwards, the sequence continues with the choice of the relative clearance $C$ (for pads or pocket), which is the second driving variable, in non-dimensional terms. In the case of a pocket bearing without barriers, the pad relative clearance $C_p$ is chosen as second driving variable. If the bearing is designed from scratch, a good nominal value of $C_p$, from which an optimization process might be started, may be obtained by means of Kingsbury formula (11) for practical design. It suggests the following relative clearance ($R$ is expressed in mm)

$$C_p = 0.002 + 0.0508/R$$ (20)

or, alternately, in order to achieve lower heat dissipation

$$C_p = 0.004$$ (21)

Kingsbury formula, which provides the relative clearance for a full bearing with fixed pads, is used to calculate $C_p$ rather than the relative pocket clearance $C_c$. Such choice preserves the load-carrying capacity of the bearing, which is higher if the mobility area of the journal is smaller. Indeed, the actual mobility area of the pocket bearing without barriers is respectively smaller and wider than the one of a circular bearing with relative clearance $C_p$ and $C_c$.

Actually, due to dissipation, subsequent temperature rise and differential thermal expansion, when the bearing does not work and it is kept at ambient temperature ("cold" condition), its clearance is usually higher than when it works ("hot" condition). Such clearance reduction may also yield higher friction and more heat, which aggravates the differential thermal expansion problem and may lead to bearing seizure. Therefore, when the values obtained from Eqs. (20) and (21) are used to assess the "cold clearance", they may be increased for the sake of bearing safety.

Hence the relative pocket clearance $C_p$ may be set between 1.5/1000 and 6/1000,
depending on the journal size and the operating speed \(^{(11)}\), so that \(c_p\) can be calculated by means of Eq. (12). Subsequently, as \(c_p\) and \(R\) are known, the pad radius \(R_p\) can be found by means of Eq. (2).

The choice of the cut depth \(t\) (Fig. 1) complies with the usual elliptical bearing design rule, which assumes that the horizontal clearance \(c_H\) (Fig. 2) is twice over the vertical clearance \(c_{V1}\) of the active lobe. Equivalently, the bottom lobe ellipticity \(E_1\) (the third driving variable, in non-dimensional terms) is usually greater than 2. When \(E_1\) is set, Eq. (17) furnishes the cut depth \(t\) for the given \(c_H\).

When \(t\) is also known, the pocket radius \(R_c\) and clearance \(c_c\) can be found by means of Eqs. (2) and (1), respectively. The relative pocket clearance \(C_c\) is then found by means of Eq. (13). It is usually in the range from 0.8/1000 to 2.2/1000. Load-carrying capacity is very sensitive to this parameter that controls the curvature of the surface where the active film primarily develops.

Differently, for a pocket bearing with barriers, \(C_c\) is chosen as second driving variable instead of \(C_p\). Indeed, in this case the cross-section with the minimum clearance is on the bearing sides (across the barriers), i.e. the actual mobility area of the journal is a circle with radius \(c_c\). The relative clearance \(C_c\) is chosen in a narrow range, between 1.7/1000 and 2.2/1000, as different values easily lead to unwanted mixed lubrication conditions on the barrier surfaces.

After the choice of \(C_c\), the sequence is analogous. The pocket clearance \(c_c\) and radius \(R_c\) can be directly found by means of Eqs. (13) and (1), respectively. As soon as \(E_1\) is chosen, Equation (17) also provides \(t\) as a function of \(c_c\). From \(t\), the elliptical bearing pad radius \(R_p\) and clearance \(c_p\) are found by means of Eq. (2).

Figures 1 and 2 define the opening angle of the pocket \(a_t = 2\alpha\) (the fourth driving variable, in non-dimensional terms). In agreement with the usual manufacturing practice, it is set between 60 and 110 deg (the two limits of the range, 110 or 60 deg, are the most usual values adopted by manufacturers) and it is used to calculate the bottom pad eccentricity \(e_1\) by means of Eq. (3).

Afterwards, the fifth non-dimensional driving variable is the top pad ellipticity \(E_2\) (or, equivalently preload \(m_2\), directly linked to ellipticity by Eq. (16)). As explained in §3, \(E_2\) ranges between 1 \((m_2=0)\) and \(c_p/c_c\) \((m_2=t/c_p)\). The former value yields design A, the latter design B, while intermediate design is possible. As pad clearance \(c_p\) has already been computed, \(m_2\) definition (Eq. (14), \(i=2\)) directly yields the top pad eccentricity \(e_2\). The algebraic system of equations formed by Eqs. (4) and (5), where the pad eccentricities \(e_1\) and \(e_2\) have already been determined, can be solved in the unknowns \(d\) and \(s\), the shim thickness and the offset of the lathe spindle axis required for manufacturing, respectively.

Finally, in order to complete the determination of manufacturing parameters, the cut depths \((t_{H1}, t_{V1}, t_{V2})\) can be obtained by using Eqs. (9), (10) and (11).

### 4.2 Simulation

When geometry is known the next step is to perform a static lubrication THD analysis to assess performance. To this purpose, among the FEM models already proposed in \(^{(7)}-\(^{(10)}\), a model characterized by a good compromise between reliability of predictions and computational cost has been chosen. Such a method is actually an enhancement of the mass- and energy-conserving lubrication analysis devised by Kumar and Booker \(^{(12)}\). While the original version of the method assumes isothermal grooves and adiabatic walls, the present version adds the groove-mixing algorithm as well as simplified heat exchange and turbulence simulations, as explained below.

As usual in mass-conserving lubrication, the lubricant film is simulated by a fictitious biphase gas–liquid mixture, of which the liquid phase dynamic viscosity and volumic mass are \(\mu_L\) and \(\rho_L\), respectively. The ratio between the mixture viscosity \(\mu\) and density \(\rho\) is
assumed to be equal to $\mu_L / \rho_L$. It is also supposed that the Reynolds equation holds in both active $A_a$ and cavitated film regions $A_c$, whereas the fluid density $\rho$, evenly equal to $\rho_L$ in $A_a$, is reduced by cavitation phenomenon in $A_c$. By adopting the reference system in Fig. 5 and by taking account of non-laminar lubrication regimes, mass conservation in the whole domain $A = A_a \cup A_c$ can be expressed by the following thin film mechanics equation

$$
-\rho_L \frac{\partial}{\partial \theta} \left( \frac{h^3}{12 \mu_L K_s} \frac{\partial p}{\partial \theta} \right) - \rho_L \frac{\partial}{\partial z} \left( \frac{h^3}{12 \mu_L K_s} \frac{\partial p}{\partial z} \right) + \frac{\omega}{2} \frac{\partial (\rho h)}{\partial \theta} + \frac{\partial (\rho h)}{\partial \tau} = 0
$$

(22)

where $K_{\theta L}, K_z$ are suitable functions of the local Reynolds number ($Re_l = \omega R h / \mu_L$), $p$ is the hydrodynamic (gauge) pressure, $\omega$ the shaft rotation speed, and $\tau$ the time. The turbulence phenomena are treated with the classic linearized Ng-Pan model (13) and the functions $K_{\theta L}, K_z$ in Eq. (22) are computed in accordance with (14).

The integration of Eq. (22) requires suitable conditions on the boundaries of the region $A$. For quick and easy reference those boundary conditions are categorized into three types: a) atmospheric, b) pressure-type, c) flow-type.

Let $\Gamma_e$ be the contour of the domain $A$ that lies on the sides of the bearing exposed to the ambient pressure. The essential boundary conditions applied on the contour $\Gamma_e$ are the atmospheric ones and they impose that

$$
\rho = 0, \quad p = 0 \quad \text{on} \quad \Gamma_e
$$

(23)

As shaft misalignments are neglected, taking advantage of symmetry allows the size of the integration domain to be reduced to half area of the bearing surface, so that $\Gamma_e$ is made up only by the line $z=0$.

Other essential conditions, referred to as pressure-type, can be applied on the contour $\Gamma_s$ of a groove

$$
\rho = \rho_s, \quad p = p_s \quad \text{on} \quad \Gamma_s
$$

(24)

where $\rho_s$ is a known supply pressure. By applying Eq. (24), in the nodes lying on $\Gamma_s$ the unknown fields (pressure and density) are constrained, so that the residuals of the integral form of Eq. (22) can be computed. The sum of such residuals turns out to be the supply flow rate. Therefore, pressure-type conditions are suitable to model a supply groove through which lubricant is to be fed.

Alternately, a groove which contour is $\Gamma_s$ may be modeled by means of the following flow-type boundary conditions

$$
\rho = \rho_s, \quad -\int_{\Gamma_s} \rho hu du d\Gamma = m_s \quad \text{on} \quad \Gamma_s
$$

(25)

where $m_s$ is the supply flow, $n$ denotes the outward normal to $\Gamma$, $u = \{u, w\}$ is the average flow velocity vector, which components in circumferential and axial directions are respectively

$$
\begin{align*}
\text{circumferential} & : u = -\frac{h^2}{12 \mu_L K_s} \frac{\partial p}{\partial \theta} + \frac{\omega R h}{2} \\
\text{axial} & : w = -\frac{h^2}{12 \mu_L K_s} \frac{\partial p}{\partial z}
\end{align*}
$$

(26)

The flow-type boundary conditions (Eq. (25)) prescribe the total flow through the groove contour. Such prescription, by assuming that pressure distribution into the groove is hydrostatic, provides the equation required to calculate the unknown hydrostatic pressure within $\Gamma_s$.

Finally, the classic Jakobsson, Floberg and Olsson (JFO) conditions must be fulfilled along the cavitation boundaries. As proved in (8), such conditions are naturally verified by adopting a weak integral formulation of the problem.

The energy equation averaged across the film is
\[-\nabla \left(hkVT\right) + \rho cL_h u\cdot \nabla T + \rho cL_h \partial \tau - \nabla \cdot \mathbf{q} = 0 \tag{27}\]

where $V = \left\{1/R \partial \partial \vartheta, \partial \partial \varphi \right\}$, $T$ is the film temperature, $k$ the mixture conductivity (calculated from lubricant conductivity $kL$) ($^{7}$), $cL$ the lubricant specific heat, $\mathbf{q}$ the power dissipation density function, $q_u$ is the heat exchanged at the bearing walls.

The averaged turbulent dissipation density function in Eq. (27) is given by

\[\Phi = \frac{h^2}{12\mu_c} \left[ \frac{1}{R} \left( \frac{\partial p}{\partial \vartheta} \right)^2 + \frac{1}{K_z} \left( \frac{\partial p}{\partial \varphi} \right)^2 \right] + \mu_c \frac{\varphi^2 R^2}{h^2} \tag{28}\]

where $\mu_c$ is the Couette viscosity. In order to consider the influence of non-laminar regimes on friction, the approach reported in ($^{14}$) and ($^{16}$) is adopted. Accordingly, Couette viscosity $\mu_c = \mu_L (1 + 0.0012 \text{Rel}_0.94)$ in non-laminar regime (Constantinescu formula), $\mu_c = \mu_L$ otherwise.

The heat flow exchanged at the bearing walls $q_w$ required in Eq. (27) is computed by means of heat convection/conduction Luke formula ($^{15}$). It links local Nusselt and Reynolds numbers, $Nu_l = 2 \frac{H h}{kL}$ and $Re_l = \frac{\omega R h}{\mu_L}$, respectively, where $H$ is the local heat transfer coefficient and $kL$ the lubricant conductivity. In turbulent lubrication regime ($Re_l > 1000$) Luke formula states

\[\log_{10} Nu_l = -1.5549 + 0.8 \log_{10} Re_l \tag{29}\]

otherwise, in laminar regime, Nusselt number is constant

\[Nu_l = 7 \tag{30}\]

Therefore, the heat transfer coefficient $H$ is found by means of Eqs. (29) or (30), according to the local lubrication regime. Afterwards, by assuming that the (average) shaft and bush temperatures, $T_S$ and $T_B$ respectively, are uniform, the heat exchange is

\[q_w = H (T - T_b) + H (T - T_g) \tag{31}\]

Boundary conditions for thermal problem are also required. They involve the supply temperature $T_s$. In the original version ($^{12}$), Kumar and Booker have adopted natural boundary conditions on the atmospheric boundary $\Gamma_e$ and essential conditions ($T = T_s$) in the groove boundaries $\Gamma_s$. As already proved in ($^{7}$), the latter conditions yield unreliable temperature predictions. Hence the groove mixing theory developed in ($^{7}$) and ($^{8}$) has been used here. According to this theory, the unknown entry temperature $T_g$ of the oil that flows through the contour $\Gamma_s$ of a groove where the lubricant is fed at known temperature $T_s$, can be calculated by means of the following approximated energy balance

\[-\int_{\Gamma_s} \rho cL_h h\nabla T \cdot \mathbf{n} d\Gamma + \int_{\Gamma_s} \Delta q_g dA + cL \int_{\Gamma_s} \rho hu \cdot \mathbf{n} d\Gamma (T_g - T_s) = 0 \tag{32}\]

where $\Delta q_g$ is the groove power dissipation for unit area, which may be calculated by means of Wendt formula ($^{8}$). Equation (32), by assuming that the temperature is uniform throughout the boundary $\Gamma_s$, allows the calculation of the unknown temperature $T_g$.

All of the details for FEM formulation of the differential problem made up by Eqs. (22) and (27) together with their boundary conditions are given in reference ($^{8}$).

As far as the solution algorithm is concerned, in a typical static analysis the external load $W$ is known and the journal position must be found (indirect problem). Therefore, with reference to Fig. 5, the following journal equilibrium equations must also be solved

\[W + \int_A \rho \cos \theta dA = 0 \]
\[\int_A \rho \sin \theta dA = 0 \tag{33}\]

As the solution of Eqs. (22) and (27) requires a time-marching solution method, initial journal position, gas/vapour-oil distribution in the cavitated region, temperature and pressure fields are arbitrarily assumed. Then the integro-differential problem made up by the coupled thin film mechanics (Eq. (22)), energy (Eq. (27)) and journal equilibrium...
equations (Eq. (33)) is numerically solved for several time steps, until journal position is stationary and lubricant inlet and outlet flows are coincident within a certain numerical tolerance.

The supply modes (§3.2) are simulated in FEMLub by means of the above-described pressure-type and flow-type boundary conditions. For inlet 1 supply mode (Fig. 4 (a) and (c)) the active upstream groove is modeled by means of a pressure-type boundary condition. For layout in Fig. 4 (c) the inactive downstream groove is not simulated (it slightly influences the power dissipation solely).

The same modeling expedient is adopted for inlet 2 supply mode (Fig. 4 (b)), where a pressure-type boundary condition is applied in the downstream groove area.

In mode inlet (1)&2 (Fig. 4 (c)), a pressure-type boundary condition simulates the active downstream groove like for inlet 2 and, in addition, the inactive upstream groove is simulated by means of a flow-type boundary. By assuming zero flow across the inactive groove ($m = 0$ in Eq. (25)), mass conservation is ensured and the influence of the inactive groove on film reformation is taken into account.

As far as two-grooves layouts are concerned, while the simulation of the chamfer flow is not currently available, the reversal flow can be actually predicted by the mass-conserving hydrodynamic model implemented in FEMLub. To this purpose, both inlet 1&2 and inlet 1+2 supply modes are modeled by applying two pressure-type boundary conditions in the areas of the grooves projected in the film plane and the same simulation results hold for both modes except for the flow rates, calculated as follows.

Let $Q_1$ and $Q_2$ be the flows through the upstream and downstream grooves, respectively. Each flow may be positive or negative if the oil enters or leaves the bearing gap through the groove, respectively, so that the net inlet flow passing through the grooves is

$$Q_{in} = Q_1 + Q_2$$  \hspace{1cm} (34)

Flow continuity through the bearing gap in steady state conditions implies

$$Q_{in} = Q_{out}$$  \hspace{1cm} (35)

where $Q_{out}$ is the side loss.

As soon as flow does not revert ($Q_1>0$ and $Q_2>0$), the supply flow rate $Q$ required by the bearing is equal to the net inlet flow and, therefore, taking advantage of Eq. (35) yields

$$Q = Q_{in} = Q_{out}$$  \hspace{1cm} (36)

When reversal flow occurs ($Q_1<0$ and $Q_2>0$), for mode inlet 1&2 the oil required by the bearing is the oil entering the downstream groove solely and so, by means of Eqs. (34) and (35), the supply flow rate must be

$$Q = Q_2 = Q_{out} + |Q_1|$$  \hspace{1cm} (37)

In the same conditions, for mode inlet 1+2 flow $Q_1$ is recirculated so that it is recovered and, therefore, Equations (34) and (35) imply

$$Q = Q_2 - |Q_1| = Q_{out}$$  \hspace{1cm} (38)

or, in other words, Equation (36) still holds.

After the static performance is known, a perturbation method and of the free boundary theory allow the calculation of the dynamic coefficients (stiffness and damping matrices).

4.3 Performance indicators

In the post-processing phase, when pressure and temperature fields as well as gas-oil distribution are known, all of the related performance parameters are computed. Among them, the following static performance (and safety) indicators are considered in order to choose the optimal design: the minimum film thickness $h_{min}$, the peak pressure $p_{max}$, the
maximum temperature $T_{\text{max}}$ (of the film, evaluated by assuming a constant temperature in the white metal \(^{(13)}\)), the heat dissipation $P$, the lubricant flow rate $Q$.

Then the dimensionless critical mass $M_c$ of the journal for a symmetrical rigid rotor \(^{(16)}\) is calculated from the dynamic characteristics. It is defined as

$$M_c = \Omega^2 c_p W$$

where $W$ is the bearing load and $\Omega$ a dimensional critical parameter.

$\Omega$ is the product of either the stability threshold speed and the square root of constant journal mass when speed is variable, or the square root of the critical mass and the constant rotational speed when the stability behavior is analyzed for different values of journal mass.

$M_c$ can be expressed as a function of dynamic coefficients, by means of a solution of the linearized equations of motion of the rotor. In bearing design it is commonly used to help determine whether the bearing will run free of half frequency whirl instability \(^{(18)}\). Nevertheless, it can not give precise information about the dynamic behavior in the actual system where the bearing in study will be inserted. In the following, it is employed to compare different bearing designs by the point of view of stability.

In addition, no bearing type has a kind of optimum stability, irrespective of the operative conditions \(^{(16)}\). The assessment of global (dynamic as well as static) bearing performance presented below is limited to the nominal working conditions, the most significant by the operative point of view. Nevertheless, a more in-depth analysis of stability would also require consideration of working conditions characterized by high Sommerfeld number or, in other words, light loads, which are more critical by the stability point of view.

In nominal conditions the actual bearing, independently of its design, operates with constant journal mass $m = W/g$ and rotation speed $\omega$, i.e. the product $\Omega^2 = m \omega^2$ is constant (independent of design), while pad clearance $c_p$ may vary by changing design. Hence, in the present work, for the sake of higher generality the stability margin $\Omega_c^2 - \Omega^2$ is preferred to $M_c$ as stability performance indicator. As soon as $\Omega_c^2 < 0$ (or $\Omega_c$ is imaginary) the bearing is inherently stable and the stability indicator is not computed, otherwise it is stable only if the indicator $\Omega_c^2 - \Omega^2$ is positive.

In order to assess the global optimum, parametric analyses are run by changing one or more driving variables in their design range. Finally a global performance percentage indicator, which provides the percentage variation of performance relative to a reference design, is computed as

$$Y_g = \sum_{i=1}^{N} \pm \gamma_i \frac{Y_{i} - Y_{i}^{\text{ref}}}{Y_{i}^{\text{ref}}} \times 100$$

where $Y_i$ is a performance indicator ($i$ ranges between 1 and $N$), $Y_i^{\text{ref}}$ is its reference value, $\gamma_i$ is its weight ($\gamma_i = 1/N$ if all of the indicators are equally important). The $i^{th}$ term of the summation is added if the optimum $Y_i$ is a maximum, while it is subtracted if $Y_i$ must be minimized.

In the application case reported in the companion paper (part 2) $\gamma_i = 1/N$ is always used in Eq. (40) and $Y_g$ is computed on the basis of $N=5$ performance indicators ($b_{\text{min}}, p_{\text{max}}, T_{\text{max}}, P, Q$) without the contribution of the stability indicator ($\Omega_c^2 - \Omega^2$). In addition, stability is taken into account by means of a global indicator $Y_g$ calculated like $Y_g$ only in the design regions where negative values of $\Omega_c^2$ are found and based on $N=6$ indicators (including the stability margin contribution) elsewhere. Therefore, inherent stability is ignored in the global performance assessment.

4.4 Journal mobility

Equation (19) \((\S3.3)\) provides the relation between journal position and gap thickness. For the sake of bearing design, verification and monitoring, two graphic representations of such relation are introduced: the clearance plot and the mobility graph.
The former is the most straightforward and it is usually adopted in production inspection in order to verify the design requirements. The latter is a well-established representation for cylindrical bearings and the present paper proposes a generalization for the more complex bearing shape at hand, which can be used in post-processing FEM analysis results.

During quality control, a fake journal with reference radius $R+c_c$ is housed in the bearing under inspection. The radial clearance is measured at regular intervals (usually 15 deg) of the circumferential coordinate $\vartheta$ (Fig. 5) by means of a gauge that provides the radial distance from the reference journal to the Babbitt surface. In order to check the conformity to design, measured deviations from the reference diameter are compared with the corresponding design values. To this purpose, such design deviations, often reported into tables of the drawings, are computed by subtracting $c_c$ from the gap thickness obtained by substituting $e_{x}=0, e_{y}=0$ in Eq. (19).

The construction of a graphical version of such tables by means of a polar graph, referred to as clearance deviation plot, is illustrated by Fig. 6, where the fake journal and the radial deviation segments are depicted by the solid blue circle and dotted green lines, respectively.

A similar construction is shown in Fig. 7. It is different in that the real journal (the solid red circle, with actual radius $R$) is considered. Its center is located at the point $O_c$ (Fig. 5), which is assumed to be the center of the bearing. The relevant polar graph, referred to as clearance plot (Fig. 7 (b)), can be used to visualize the bearing clearance. Although in such clearance plot the journal collapses to a single point, it can not be moved everywhere in the clearance area delimited by the plot contour (the thick black line in Fig. 7 (b)), as the contact between journal and bearing does not generally occur in a point lying on the radial line drawn through plot and journal centers, respectively $O_c$ and $O_j$, as in the case of a circular bearing. In other words, in general the clearance plot does not determine the mobility area of the bearing. In addition, the clearance plot contour is arbitrary: its shape depends on the choice of the bearing “center”, which has been arbitrarily considered coincident with $O_c$ in Fig. 7 (a).
According to Booker’s mobility method (introduced in §1) the non-dimensionalized mobility area of a circular bearing was delimited by a unit circle for mapping purposes (6). For a generic bearing shape the mobility area of the journal may be graphically determined as suggested by Fig. 8, where the journal, which consecutive locations are depicted by solid colored circles with radius $R$, rolls without slipping along the bearing profile while its center draws the mobility area contour. For a circular bearing (Fig. 8 (a)) the circular mobility graph is coincident with the clearance plot.

As far as pocket bearings are concerned, a point $(e_x, e_y)$ of the mobility area contour (the thick black line drawn by the journal center in Fig. 8 (c)) for each (constant) anomaly $\theta$ can be numerically computed by finding the zero of Eq. (19). A non-dimensional version of the mobility plot may be obtained in terms of $(e_x/c_p, e_y/c_p)$ coordinates.

Although the pocket elliptical bearing surface exhibits two sharp edges at the leading and trailing circumferential coordinates of the pocket, they are not displayed by the mobility contour. Indeed, the center of the rolling journal describes an arc of circumference when it rotates around the edges (Fig. 8 (c)). As the ratio $c_p/R$ is in the order 1/1000, in actual mobility graphs the arc appears as a straight line.

The representation of the journal in the mobility graph is a point which locates the journal center $O_j$ in the mobility area. It provides a straightforward visualization of the journal position and also of the minimum film thickness, which is the length of the segment drawn along the normal to the mobility contour from $O_j$ to the point of intersection between normal and contour.

5. Conclusion

In the present theoretical part of the work, a method for the design of pocket elliptical bearings has been presented. Such method includes a systematic procedure capable to assess the global performance of the bearing and to display the journal location in a significant domain, the mobility area.

Accordingly, an initial design can be chosen and improved by changing design variables and variants. Each design can be evaluated by calculating the corresponding global performance with the aid of FEM.

Nevertheless, a systematic method for changing the design variables is still needed to perform a substantial enhancement of pocket bearing design. Such a full optimization and the required numerical procedure will be studied in the next future, as soon as the influence of pocket design and bearing shape on performance, which is the subject of the companion paper (part 2), will be understood.
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