On the Origin of Nuclear Superfluidity

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ABSTRACT

The induced pairing interaction arising from the exchange of collective surface vibrations among nucleons moving in time reversal states close to the Fermi energy is found to lead to values of the pairing gap which are similar to those experimentally observed.

It is well established that nucleons moving close to the Fermi energy in time reversal states have the tendency to form Cooper pairs which eventually condense. This phenomenon, which parallels that which is at the basis of low temperature superconductivity, modifies in an important way the nuclear properties, in particular the occupation number of single-particle levels around the Fermi energy, the moment of inertia of deformed nuclei, the lifetime of cluster decay and of fission process, the depopulation of superdeformed configurations, the cross section of two-nucleon transfer reactions, etc. (cf. e.g. refs. and refs. therein).

While in the case of superconductivity the attraction among the electrons is generated by the exchange of phonons, in the nuclear case the origin of the pairing interaction is related to the $^1S_0$ phase shift of the free nucleons, which is attractive at low relative momenta. In keeping with the fact that the free nucleon-nucleon interaction is strongly renormalized in nuclei, Cooper pair for-
Information can benefit from the exchange of low-lying collective surface vibrations between pairs of nucleons moving in time reversal states close to the Fermi energy (cf. ref. [2], p. 432 and refs. [17, 18]). In fact, we shall show below that this mechanism gives rise to pairing gaps which are similar to those observed experimentally.

Calculations have been carried out for a number of isotopic chains: \(^{40}\)Ca, \(^{42}\)Ti and \(^{50}\)Sn. The results provide insight into the role the induced interaction plays in neutron and proton pairing correlations in nuclei. Calculations have also been carried out for the case of \(^{42}\)Sc and found to lead to strong proton-neutron pairing correlations.

The properties of the collective surface vibrations were determined within the framework of the random-phase approximation and of the particle-vibration coupling model (cf. e.g. refs. [2, 19]). The induced interaction (cf. Fig. 1 (inset))

\[
\langle \nu' \bar{\nu} | v | \nu \bar{\nu} \rangle = \sum_{\lambda n} \frac{< \nu | R_o \partial U/\partial r | \nu' >^2}{\sqrt{(2j_\nu + 1)(2j_{\nu'} + 1)}} \frac{2(2\lambda + 1)\Lambda^2_\lambda(n)}{E_\nu - (\epsilon_\nu + \epsilon_{\nu'} + \hbar \omega_\lambda(n))},
\]

was then calculated, and the state dependent pairing gap

\[
\Delta_\nu = - \sum_{\nu'} \frac{\sqrt{2j_{\nu'} + 1}}{2E_{\nu'}} \Delta_{\nu'} < \nu' \bar{\nu} | v | \nu \bar{\nu} >,
\]
determined. In Eq.(1), the quantum numbers of the single-particle states are labeled by \(\nu(=n_\nu, l_\nu, j_\nu)\). The states \(|\nu \bar{\nu} >\) are coupled to angular momentum zero, and \(\bar{\nu}\) denotes the state time reversed to \(\nu\). The corresponding single-particle energies are denoted \(\epsilon_\nu\) and \(\epsilon_{\nu'}\). The quantity \(E_o\) is the energy of the resulting ground state, in keeping with Bloch-Horowitz perturbation theory [20]. The quantity inside the reduced matrix element is the particle-vibration formfactor (deformation potential [19], see also ref. [21]), product of the nuclear radius \(R_o\), of the derivative \(\partial U/\partial r\) of the single-particle potential and of a spherical harmonic of multipolarity \(\lambda\). The potential \(U\) is assumed to have Woods-Saxon shape, and is parametrized according to ref. [22]. The quantities \(E_{\nu'}\) in Eq.(2) denote the quasiparticle energies.

The quantity \(\Lambda_\lambda(n) = \frac{\beta_\lambda(n)}{2n+1}\) is the particle-vibration coupling strength associated with the n-th vibration (in order of increasing energy) of multipolarity \(\lambda\) and frequency \(\omega_\lambda(n)\). The values of the coupling strength for the even- and odd-multipolarities have been determined so as to reproduce the experimental transition probabilities of the low-lying 2\(^+\) and 3\(^-\) vibrational states. The calculations have then been carried out making use of vibrational states of multipolarities and parities \(\lambda^\pi = 2^+, 3^-, 4^+\) and \(5^-\).

In Fig. 1 we show the calculated state dependent pairing gap for the nucleus \(^{120}\)Sn. For levels close to the Fermi energy, this quantity is of the order of 1 MeV. This result is to be compared with the empirical value of 1.5 MeV, obtained making use of the relation
$$\Delta = \frac{1}{2} [B(N - 2, Z) + B(N, Z) - 2B(N - 1, Z)], \quad (3)$$

where $B(N, Z)$ is the binding energy of the nucleus with $N$ neutrons and $Z$ protons.

In Fig. 2, we show the value of the state dependent pairing gap averaged over levels lying within an energy interval of the order of $\pm 2\Delta$ around the Fermi energy, for a number of Sn-isotopes in comparison with the corresponding values obtained from Eq. (3). In all cases, theory accounts for a consistent fraction of the empirical values of the pairing gap.

In Fig. 3 we display the results of calculations carried out for the isotopes $^{42}\text{Ca}$ and $^{42}\text{Ti}$, in comparison with the corresponding results of Eq.(3). As in the previous case, the induced interaction leads to pairing gaps which account for a large fraction of the empirical value, and which furthermore display a similar dependence with $A$, a behaviour which reflects the fluctuations of the nuclear surface. In particular, the low predicted value of $\Delta$ in $^{50}\text{Ca}$ as compared to $^{42}\text{Ca}$ is due to the fact that the ”core” $^{48}\text{Ca}$ is more rigid than the ”core” $^{40}\text{Ca}$. We have also determined the induced proton-neutron pairing interaction in $^{42}\text{Sc}$, arising from the exchange of the low-lying collective surface vibrations of the core $^{40}\text{Ca}$. The calculated value of 1.5 MeV (cf. also the result obtained for $^{42}\text{Ca}$, Fig. 2) essentially coincides with the empirical value (1.6 MeV) obtained making use of Eq. (3).

We conclude that the exchange of low-lying surface vibrations among nucleons moving in time reversal states close to the Fermi energy, gives rise to an induced pairing interaction which leads to pairing gaps of similar magnitude to those experimentally observed. This result will force us to review our present understanding of the pairing phenomenon in nuclei, in particular its description in terms of $\omega$–independent effective interactions. It will also have consequences in the analysis of phenomena like the quenching of the pairing gap taking place as a function of the angular momentum and of the energy (temperature) content of the nuclear system. In keeping with these results, and because collective vibrations couple democratically to all nucleons, regardless of the isospin quantum number, the induced interaction mechanism is expected to lead to a conspicuous proton-neutron pairing correlation.

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Captions to the figures

Fig. 1
State-dependent pairing gap $\Delta_\nu$ (cf. Eq.(2)) for the nucleus $^{120}$Sn, calculated making use of the induced interaction defined in Eq.(1) (cf. inset, where particles are denoted by arrowed lines and phonons by a wavy line).

Fig. 2
Average value of the state dependent pairing gap associated with levels lying close to the Fermi energy of $^{40}$Sn-isotopes, calculated as discussed in the text, making use of the pairing gap defined in Eq.(2), in comparison with the empirical pairing gap (cf. Eq.(3)). The results of two calculations are shown, associated with RPA solutions which fit two different sets of transition probabilities connecting the lowest lying quadrupole and octupole vibrations with the ground state. The first set (also used in the calculation shown Fig. 1 for $^{120}$Sn) was taken from ref. [23], while the second set from ref. [24]. The corresponding theoretical results are denoted by solid squares and triangles respectively.

Fig. 3
Average value of the neutron pairing gap of the $^{40}$Ca-isotopes and of the proton pairing gap of the $^{48}$Ti-isotopes, in comparison with the empirical pairing gap (cf. Eq.(3)). In both cases the pairing matrix elements have been obtained from Eq. (1), making use of the vibrational states of the “core” $^{40}$Ca, and using the values of the experimental transition probabilities given in ref. [25]. The gap of the Ca-isotopes has been calculated making use of Eq.(2). In the case of the Ti isotopes, the matrix of the induced interaction was diagonalized. The pairing gap was then calculated making use of the relation $\Delta = \frac{1}{2} [B(N, Z - 2) + B(N, Z) - 2B(N, Z - 1)]$, the proton analogous to the expression given in Eq.(3) and used to calculate the neutron pairing gap.
