A Population’s Feasible Posterior Beliefs

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We consider a population of Bayesian agents who share a common prior over some finite state space and each agent is exposed to some information about the state. We ask which distributions over empirical distributions of posteriors beliefs in the population are feasible. We provide a necessary and sufficient condition for feasibility. We apply this result in several domains. First, we study the problem of maximizing the polarization of beliefs in a population. Second, we provide a characterization of the feasible agent-symmetric product distributions of posteriors. Finally, we study an instance of a private Bayesian persuasion problem and provide a clean formula for the sender’s optimal value.

CCS Concepts: • Theory of computation → Social networks.

Additional Key Words and Phrases: Feasible Posterior Distribution, information design, mean preserving contraction

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Economics is deeply concerned with the question of whether agents’ behaviors can be rationalized in the sense that they can be explained as an equilibrium behavior in a Bayesian model with rational agents. One such related question is under what conditions agents’ beliefs, i.e., posterior distribution, are compatible with Bayes’s rule? This question is well understood for the single agent where by the splitting lemma ([3, 4]) feasibility simply means that the expectation of the posterior equals the prior. But the characterization of feasible posterior distribution for two or more agents is trickier.

The first paper to address the feasibility of posterior distributions for more than one agent is [5] who characterize feasible distributions for two agents. Recently a growing number of papers have examined the feasibility of posterior distributions and their connection to information design problems. [1] connect the notion of feasibility to the agreement theorem of [2] and to the related no-trade theorem of [7]. A follow-up paper by [9] generalizes the result in [1] to an arbitrary finite state space. Despite these elegant connections, in general, determining whether a given distribution of posterior beliefs is feasible is a tough question.

In this work we study an anonymous variant of the feasibility question. The object whose feasibility we are willing to determine is a distribution over the empirical distributions of posterior beliefs in the population rather than a distribution over profiles of posteriors. The difference between these two variants is that a profile of posteriors indicates which agent holds which posterior, while

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the anonymous version of the problem only reveals how many agents hold each posterior without revealing their identity.

Surprisingly, we show that determining feasibility for the anonymous variant of the problem is relatively easy, especially in the case where the number of different posteriors for an individual in the population is low. Our characterization provides a necessary and sufficient condition for a distribution over empirical samples of a population to be feasible. The characterization relies on the observation that conditional on a state the frequencies of posterior beliefs in the population can be identified. We show that every combination of spreads of these expected frequencies (one for each state) is feasible. Unlike the feasibility characterization by [1] and [9], our characterization does not become more complex as the number of agents in the population increases. Furthermore, the complexity of our characterization does not increase with the number of states either. However, the parameter of the number of possible posteriors in the population has an effect on the complexity of our characterization.

As was mentioned above, the feasibility problem is an intriguing instance of rationalization in the case where an econometrician observes the distribution over empirical distributions of posteriors in the population. In the context of rationalization, it is natural to consider another variant where the econometrician observes a single sample; i.e., she observes a single empirical distribution of posteriors and wants to determine whether it is possible that the agents are Bayesian.¹ [8] shows that every empirical distribution of posteriors can be rationalized. This question is equivalent to understanding which empirical distributions of posteriors might belong to the support of a feasible distribution. Our results can be viewed as an extension of [8]; however, we go a step further and determine the entire set of feasible distributions rather than focus on points that might belong to the support.

We apply our main theorem in three different domains: Polarization, private private information, and private Bayesian Persuasion.

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¹[6, 10] have studied Bayesian rationalization of the behavior of a single agent in a repeated environment.