Tests of discrete symmetries in K systems

Antonio Di Domenico
Dipartimento di Fisica, Sapienza Università di Roma,
and I.N.F.N. Sezione di Roma, P.le A. Moro, 2, I-00185 Rome, Italy
E-mail: antonio.didomenico@roma1.infn.it

Abstract. The status of present experiments and future projects with kaons is reviewed,
 focusing on prospects for discrete symmetries tests.

1. Introduction
In 2014 there are several anniversaries to be celebrated, as reminded by Prof. Mavromatos
in the opening of the DISCRETE 2014 symposium: the 150th anniversary of the publication
of Maxwell’s equations, the 60th anniversary of the discovery of Yang-Mills Gauge theories,
the 50th anniversary of the discovery of CP Violation, and the 50th anniversary of the proposal
of Bell’s Inequalities in Quantum Physics.

Among them there is also the 70th anniversary of the first publication reporting an
indication of a new charged particle with mass \( \sim 0.5 \, \text{GeV}/c^2 \) in cosmic rays
(Leprince-Ringuet, L’héririt [1]), i.e. the first experimental evidence for a \( K^+ \) meson,
remarkably even before the discovery of the pion. Since then, K mesons, or kaons, turned out
out to be one of the most interesting and promising system in particle physics, especially
in the study of discrete symmetries. The discovery in 1957 of the \( \tau - \theta \) anomaly due to
parity non-conservation in weak decays [2], the discovery in 1964 of CP violation in the
\( K^0 - \bar{K}^0 \) mixing [3], and of direct CP violation almost 40 years later [4, 5], are bright
examples of the outcomes of these studies.

After 70 years from the first observation one might wonder whether investigations on kaons
have exhausted the information that such relatively simple but rich system can provide.
The present paper, trying to convince the reader that this is not the case, focuses on the most recent
progresses in the field, updating a previous report on the status of experiments and future
projects [6]; for general and historical reviews the reader is referred to several excellent papers
and books in the literature [7].

2. Kaon experiments at hadron machines
2.1. The intensity frontier for New Physics searches
Precise measurements in the Flavor sector can probe extremely high energy scales, in a
complementary way with respect to the LHC high energy frontier. In fact, the present LHC
searches at high transverse momenta exclude wide regions of the parameter space for New
Physics (NP), thus increasing the importance of the study of rare processes sensitive to very
high energy. This is true, in particular, for the \( K \rightarrow \pi \nu \bar{\nu} \) decays, whose SM predictions are
known with very high precision [8]. The branching ratio expectation of the CP-violating
\( K_L \rightarrow \pi^0 \nu \bar{\nu} \) channel is: \( \text{BR}(K_L \rightarrow \pi^0 \nu \bar{\nu}) = (0.243 \pm 0.039 \pm 0.006) \times 10^{-10} \), while for the charged mode is:
BR(K^+ → π^+ν̅ν) = (0.781±0.075±0.029) × 10^{-10}. In both cases, the first error summarizes the parametric uncertainties, while the second one the remaining theoretical error. The extraction of the hadronic part of the amplitude is obtained from the very well measured K_{E3} decay. The main uncertainties are due to the knowledge of the CKM matrix elements.

Processes mediated by FCNC are suppressed in the SM by the GIM mechanisms [9]. Further suppression can originate from hierarchies in CKM matrix or helicity. In some models of NP large enhancement with respect to SM rates are possible. In this sense the K → πν̅ν̅ modes play a key role in seeking NP beyond the SM. This is clear from Fig.1, which reproduces the BR values predicted by the SM and by some models of NP [10]. Apart from establishing a direct signal of NP, the correlation of the BR of the two modes can be exploited to probe the flavour structure of NP theories and, therefore, to distinguish among different classes of NP scenarios. The exclusion regions given by the Grossman-Nir consistency condition [11], limiting the BR ratio of the two decay modes, and by the only measurement of the charged mode [12] are also reported in Fig.1.

**Figure 1.** Correlation between the branching ratios of the charged and neutral K → πν̅ν̅ decays showing the predictions of SM and some NP models [10].

The measurement BR(K^+ → π^+ν̅ν) = (1.73 ± 1.15) × 10^{-10} is based on 7 events observed by E959 and E787 experiments at BNL [12], and is compatible with the SM prediction. The CP-violating neutral K_L → π^0ν̅ν̅ decay has not been observed yet. An upper limit on the BR was set by the E391a collaboration at KEK [13]: BR(K_L → π^0ν̅ν) < 260 × 10^{-10} (90% C.L.).

### 2.2. NA62 experiment at CERN

The main goal of the NA62 experiment at CERN [14] is to collect about 100 K^+ → π^+ν̅ν̅ events with 10% of background in two years of data taking. This implies to collect more than 10^{13} K
decays with a background rejection factor of at least $10^{12}$, assuming a signal acceptance of 10%.

The experimental method relies on exploiting a decay-in-flight technique with an intense charged kaon beam at high energy. This requires a beam of unseparated charged hadrons (with 6% $K^+$), a long decay region with an extended detector and the event-by-event measurement of kaon momenta. Positive charged kaons will be used in order to get, at the same proton energy and flux, higher kaon fluxes and lower pion background.

The incoming kaon is measured by the Gigatrack system in the beam. The charged decay particle is measured by the straw-chamber spectrometer and is identified by the Ring Imaging Cerenkov (RICH) detector and the muon-veto sampling calorimeter. The LKr calorimeter, originally built for the NA48 experiment, is used as a veto for forward photons. Photons at large angles are intercepted by a series of 12 ring-shaped veto counters constructed using lead-glass blocks from the OPAL electromagnetic barrel calorimeter.

The NA62 analysis strategy is based on the accurate kinematic reconstruction of all the particles detected in the event to disentangle the signal from the huge amount of background processes, a precise timing to associate correctly the $\pi^+$ with the parent $K^+$, a system of efficient vetoes to reject events with $\gamma$ and $\mu$ in the final state, effective particle identification systems to identify $K^+$ among other particles of the intense hadron beam and to distinguish $\pi^+$ from $\mu^+$ and $e^+$ in the final states.

The construction and installation of the experimental apparatus have been completed in May 2014, as shown in the pictures of Fig.2 (see Ref.[15] for more details). The first physics run has successfully started at the end of 2014 at a lower intensity beam, while completing the commissioning of hardware and readout, with the aim of reaching the SM sensitivity. The nominal intensity beam is foreseen for the full physics run in 2015-2017.

### 2.3. KOTO experiment at J-PARC

The KOTO experiment [16] at the high-intensity proton accelerator facility J-PARC (Japan Proton Accelerator Research Complex) [17] aims to observe for the first time the rare decay $K_L \to \pi^0 \nu \bar{\nu}$, which is rather challenging because both initial state and final decay products are completely neutral.

A decay volume for $K_L$ is surrounded by particle detectors. The signature of a $K_L \to \pi^0 \nu \bar{\nu}$ decay is that there are two photons from a $\pi^0$ decay and no other visible particles in the final state. An electromagnetic calorimeter is placed downstream of the decay volume to detect the two photons. All the $K_L$ decay modes except $K_L \to \pi^0 \nu \bar{\nu}$ and $K_L \to \gamma \gamma$ have at least two charged particles, or two or more extra photons in the final state. These decays can be rejected by detecting additional particles with the surrounding detectors. The $K_L \to \gamma \gamma$ decays can be rejected by requiring a finite transverse momentum for the two photon system. In the case of $K_L \to \pi^0 \nu \bar{\nu}$ decay, the two photon system has a finite transverse momentum, because the undetected two neutrinos take some momentum away. The decay vertex is calculated and the $K_L \to \pi^0 \nu \bar{\nu}$ decay reconstructed from two photons in the calorimeter, with the assumption that the two photons come from a $\pi^0$ decay on the line of flight.

The new neutral beam-line has been built at the Hadron Experimental Hall of J-PARC. The KOTO detector, consisting mainly of the CsI crystal calorimeter, the charged-particle veto counters (CV) in front of it, and the main-barrel photon veto counters (MB) surrounding the decay volume in the vacuum vessel, have been built and installed by May 2013.

After the major Earthquake in Japan on 11 March 2011 (no damages to the beam line and the KOTO detector), the KOTO first physics run started on May 17 2013 with a beam power of 24 kW (10% of design intensity). Unfortunately, the data taking was terminated after only 100 hours due to a radiation accident in J-PARC Hadron hall (a gold target partially melted and radioactive material diffused in the atmosphere).

The analysis of the 100 hours run data [18], after determining all the selection criteria and
estimating background levels, showed one observed event in the signal region, as shown in Fig.3, which was statistically consistent with the number of expected background events. Based on the number of simultaneously measured $K_L \rightarrow 2\pi^0$ events, the evaluated single event sensitivity (S.E.S) for the $K_L \rightarrow \pi^0\nu\bar{\nu}$ decay results to be $1.29 \times 10^{-8}$, while the S.E.S of the E391a experiment was $1.11 \times 10^{-8}$. Although data taking time was only 100 hours, about the same sensitivity as E391a, which took data for 1100 hours, has been achieved.

The next data taking is planned to start in 2015 improving the experimental sensitivity by preparing new detectors to reduce the background events and improving the analysis methods to remove hadron interaction events.

2.4. TREK experiment at J-PARC
In the $K^+ \rightarrow \pi^0\mu^+\nu$ decay, the transverse muon polarization $p_t$ (the perpendicular component of the muon spin vector relative to the decay plane determined by the momentum vectors of the muon and the pion in the $K^+$ rest frame) is a T-odd quantity. The SM prediction of $p_t$ is almost vanishing ($\sim 10^{-7}$) and a non-zero value for such quantity at a level above $10^{-4}$ would be an indication of time-reversal violation in NP, still allowed by the current limit ($p_t < 5 \times 10^{-3}$) set by the KEK E246 experiment [19].

A successor to E246 is another new kaon experiment at J-PARC, E06 TREK [20], aiming at a $p_t$ sensitivity of $10^{-4}$. This experiment is conducted in conjunction with the E36 one aiming to measure the $R_K = \Gamma(K^+ \rightarrow e\nu\bar{\nu})/\Gamma(K^+ \rightarrow \mu\nu\bar{\nu})$ ratio with a relative uncertainty of $\sim 2.5 \times 10^{-3}$, which is about half of the current world record. The physics run of E36 is expected in 2015.
2.5. Searches for rare kaon decays at LHCb

It’s worth mentioning that searches for rare kaon decays are also possible at LHC, at least for the cleanest decay channels with a closed kinematics and a controllable background. This holds for the $K_S \rightarrow \mu^+\mu^-$ decay, which is a Flavour Changing Neutral Current (FCNC) transition that has not yet been observed. This decay is suppressed in the SM, with an expected branching fraction $\text{BR}(K_S \rightarrow \mu^+\mu^-) = (5.0 \pm 1.5) \times 10^{-12}$, while the current experimental upper limit is $3.2 \times 10^{-7}$ at 90% confidence level (C.L.) [22]. Although the dimuon decay of the $K_L$ meson is known to be $\text{BR}(K_L \rightarrow \mu^+\mu^-) = (6.84 \pm 0.11) \times 10^{-9}$ [23], in agreement with the SM, effects of new particles can still be observed in $K_S \rightarrow \mu^+\mu^-$ decays.

The LHCb detector, described in detail in Ref.[24], is a single-arm forward spectrometer covering the pseudorapidity range $2 < \eta < 5$. A search for the decay $K_L \rightarrow \mu^+\mu^-$ is performed, based on a data sample of 1.0 fb$^{-1}$ of pp collisions at $\sqrt{s} = 7$ TeV collected by the LHCb experiment at the Large Hadron Collider. The observed number of candidates is consistent with the background only hypothesis, yielding an upper limit of $\text{BR}(K_S \rightarrow \mu^+\mu^-) < 11(9) \times 10^{-9}$ at 95 (90)% confidence level [25]. This limit is a factor of thirty below the previous measurement.

In the future, probing the interesting region of $\text{BR}(K_S \rightarrow \mu^+\mu^-) < 10^{-10}$ will require dedicated trigger studies and improvements [26]. The LHCb collaboration is also considering to study other rare kaon decays as $K_S \rightarrow e^+e^-\mu^+\mu^-$, $K_S \rightarrow e^+e^-e^+e^-$, and $K_S \rightarrow \pi^0\mu^+\mu^-$, thanks to the copious production of kaons at LHC and the excellent LHCb detector reconstruction capabilities, even though the presence of a huge background to fight, and trigger limitations still remain the major concerns for these kind of studies.

2.6. Other projects

The KLOD R&D program for a $K_L$ measurement at Protvino should also be mentioned, as well as the OKA project, in the same laboratory, using a separated charged kaon beam. Unfortunately the ORKA experiment [27], and more broadly kaon physics research, have been terminated at Fermilab for the foreseeable future.
3. Kaon experiments at $e^+e^-$ collider

3.1. The KLOE-2 experiment at DAΦNE

DAΦNE, the Frascati φ-factory is an $e^+e^-$ collider working at a center of mass energy of $\sqrt{s} \sim 1020$ MeV, corresponding to the peak of the φ resonance. The φ-meson production cross section is $\sim 3\mu b$, and its decay into $K^0\bar{K}^0$ has a branching fraction of 34%, yielding $\sim 10^6 K^0\bar{K}^0$ pairs per $pb^{-1}$ of integrated luminosity.

The KLOE detector is a 4π detector setup, which is able to measure both charged and neutral particles. It consists of a large volume drift chamber [28], which provides excellent momentum and vertex reconstruction for charged particles, and a barrel shaped electromagnetic calorimeter with two end-caps [29], made from lead and scintillating fibers, which surrounds the drift chamber. The energy deposits of charged and neutral particles in the calorimeter are measured with very good time resolution, which allows for the identification of charged particles based on their time of flight. Drift chamber and calorimeter are enclosed in a superconducting solenoid, providing an axial 0.52 T magnetic field.

The KLOE experiment at DAΦNE completed its first data taking campaign in March 2006 with a total integrated luminosity of $\sim 2.5$ $fb^{-1}$, corresponding to a production of $\sim 7.5 \times 10^9$ φ-mesons and $\sim 2.5 \times 10^9 K^0\bar{K}^0$ pairs.

After an experimental test [30], DAΦNE has been upgraded implementing an innovative collision scheme based on a crab-waist configuration, providing an improvement in the peak luminosity of a factor $\sim 3$.

The KLOE-2 experiment aims to continue and extend the physics program of its predecessor by collecting $O(10$ $fb^{-1})$ of data at the upgraded DAΦNE with an improved KLOE detector. The KLOE-2 physics program has been described in detail in Ref.[31], and among the main issues includes neutral kaon interferometry and tests of discrete symmetries and quantum mechanics. Improvements of about one order of magnitude in almost all present limits on CPT violation and decoherence parameters are expected [32, 33, 34, 31], thanks to the increased luminosity and the better quality of reconstructed data.

The upgrade of the KLOE detector consists of the addition of (i) an inner tracker [35, 36] based on cylindrical GEM technology for the improvement of tracking and decay vertex resolution close to the interaction point (IP), (ii) a $e^\pm$ tagging system [37, 38, 39] for the $\gamma\gamma$ physics, and (iii) two calorimeters [40, 41, 42] in the final focusing region to improve acceptance and efficiency for photons coming from the IP and neutral kaon decays inside the detector volume.

The installation of the new KLOE-2 sub-detectors has been completed in 2013, and a large consolidation and maintenance program of the DAΦNE complex has been carried out. During the commissioning phase, it emerged that the advantages of the new collision scheme appear more difficult to get, due to the strong perturbation of the KLOE solenoid on the machine optics – absent in the test experiment described in Ref.[30] – and to the increased machine background. A new data taking campaign has started in November 2014 with the aim of collecting $\sim 1$ $fb^{-1}$ of integrated luminosity by mid 2015, and to assess the real DAFNE/KLOE-2 capabilities of integrating the required luminosity in a running period of 2-3 years.

In the following only the most recent published result will be briefly reported, while a novel method to directly test discrete symmetries with entangled neutral kaon pairs at KLOE-2 will be discussed in some details. For a review of previous KLOE results about tests of discrete symmetries and quantum mechanics the reader is referred to the existing literature[32, 33, 34, 31].

3.1.1. CPT and Lorentz symmetry test

CPT invariance holds for any realistic Lorentz-invariant quantum field theory. However a very general theoretical possibility for CPT violation is based on spontaneous breaking of Lorentz symmetry, as developed by Kostelecký [43, 44, 45], which appears to be compatible with the basic tenets of quantum field theory and retains the property
of gauge invariance and renormalizability (Standard Model Extensions - SME). In SME for neutral kaons, CPT violation manifests to lowest order only in the mixing parameter \( \delta \), (e.g. vanishes at first order in the decay amplitudes), and exhibits a dependence on the 4-momentum of the kaon:

\[
\delta \approx i \sin \phi_{SW} e^{i\phi_{SW}} \gamma_K (\Delta a_0 - \beta_K^* \cdot \Delta \vec{a}) / \Delta m
\]

(1)

where \( \gamma_K \) and \( \beta_K^* \) are the kaon boost factor and velocity in the observer frame, \( \phi_{SW} \) is the so called superweak phase, \( \Delta m = m_L - m_S \), and \( \Delta a_\mu \) are four CPT- and Lorentz-violating coefficients for the two valence quarks in the kaon.

Using 1.7 fb\(^{-1}\) of data collected at KLOE and studying the interference pattern of the entangled neutral kaon pairs in the \( \phi \to K^0\bar{K}^0 \to \pi^+\pi^-\pi^+\pi^- \) final state, as a function of sidereal time and particle direction in celestial coordinates, the following results have been obtained [46]:

\[
\begin{align*}
\Delta a_0 & = (-6.0 \pm 7.7_{\text{stat}} \pm 3.1_{\text{syst}}) \times 10^{-18} \text{ GeV} \\
\Delta a_x & = (0.9 \pm 1.5_{\text{stat}} \pm 0.6_{\text{syst}}) \times 10^{-18} \text{ GeV} \\
\Delta a_y & = (-2.0 \pm 1.5_{\text{stat}} \pm 0.5_{\text{syst}}) \times 10^{-18} \text{ GeV} \\
\Delta a_z & = (3.1 \pm 1.7_{\text{stat}} \pm 0.5_{\text{syst}}) \times 10^{-18} \text{ GeV}
\end{align*}
\]

These results [46] constitute the most sensitive measurements in the quark sector of SME, and can be compared to similar results obtained in the B and D meson systems, where an accuracy of \( \mathcal{O}(10^{-13}\text{GeV}) \) has been reached [47].

3.1.2. Direct tests of discrete symmetries with kaons In general any theoretical connection among the parameters describing T, CP or CPT violation, as given by the CPT theorem, does not imply a corresponding connection among the experimental results of the T, CP or CPT tests. A direct test means a test whose outcome is independent from the result of any other discrete symmetry test, as discussed in detail in Refs.[48, 49, 50, 51, 52].

In order to implement a direct test of the T and CPT symmetries, which requires in both cases the inversion of in and out states, it has been suggested to exploit the Einstein-Podolsky-Rosen (EPR) entanglement of neutral mesons produced at a \( \phi \)-factory (or B-factory\(^1\)) [49, 50, 51, 52]. In fact in this case the initial state of the neutral kaon pair produced in \( \phi \to K^0\bar{K}^0 \) decay can be rewritten in terms of any pair of orthogonal states \(|K_+\rangle\) and \(|K_-\rangle\):

\[
|i\rangle = \frac{1}{\sqrt{2}} \left( |K^0\rangle|\bar{K}^0\rangle - |\bar{K}^0\rangle|K^0\rangle \right) = \frac{1}{\sqrt{2}} \left( |K_+\rangle|K_-\rangle - |K_-\rangle|K_+\rangle \right).
\]

(2)

Here one can consider the states \(|K_+\rangle\), \(|K_-\rangle\) defined as follows: \(|K_+\rangle\) is the state filtered by the decay into \( \pi\pi \) (\( \pi^+\pi^- \) or \( \pi^0\pi^0 \)), a pure \( CP = +1 \) state; analogously \(|K_-\rangle\) is the state filtered by the decay into \( 3\pi^0 \), a pure \( CP = -1 \) state. Their orthogonal states correspond to the states which cannot decay into \( \pi\pi \) or \( 3\pi^0 \), defined, respectively, as

\[
\begin{align*}
|\bar{K}_-\rangle & \propto \left( |K_L\rangle - \eta_{\pi\pi}|K_S\rangle \right) \\
|\bar{K}_+\rangle & \propto \left( |K_S\rangle - \eta_{3\pi^0}|K_L\rangle \right)
\end{align*}
\]

(3)

with \( \eta_{\pi\pi} = \langle \pi\pi|T|K_L\rangle/\langle \pi\pi|T|K_S\rangle \) and \( \eta_{3\pi^0} = \langle 3\pi^0|T|K_S\rangle/\langle 3\pi^0|T|K_L\rangle \). With these definitions of states, it can be shown that the condition of orthogonality \( \langle K_-|K_+\rangle = 0 \), (i.e. \(|K_+\rangle \equiv |\bar{K}_-\rangle \) and \(|K_-\rangle \equiv |\bar{K}_+\rangle \)) corresponds to assume negligible direct CP (or CPT) violation contributions, assumption quite well satisfied for neutral kaons (see detailed discussion in Appendix A of Ref.

\(^1\) It’s worth mentioning that in the neutral B meson system a direct T test has been already accomplished [53], exploiting a similar methodology as discussed in the following.
The measurement of any deviation from the prediction $\Delta S = \Delta Q$ rule is also assumed, so that the two flavor orthogonal eigenstates $|K^0\rangle$ and $|\bar{K}^0\rangle$ are identified by the charge of the lepton in semileptonic decays, i.e. a $|K^0\rangle$ can decay into $\pi^-\ell^+\nu$ and not into $\pi^+\ell^-\bar{\nu}$, and vice-versa for a $|\bar{K}^0\rangle$.

Thus, exploiting the perfect anticorrelation of the states implied by Eq. (2), it is possible to have a “flavor-tag” or a “CP-tag”, i.e. to infer the flavor ($K^0$ or $\bar{K}^0$) or the CP ($K_+\text{ or } K_-\text{ state of the still alive kaon by observing a specific flavor decay } (\pi^+\ell^-\nu \text{ or } \pi^-\ell^+\bar{\nu}) \text{ or CP decay } (\pi\pi \text{ or } \pi^0\pi^0\pi^0)$ of the other (and first decaying) kaon in the pair.

In this way one can experimentally access – for instance – the transition $K^0 \to K_+$, taken as reference, and the $K_+ \to K^0$, $\bar{K}^0 \to K_+$ and $K_+ \to \bar{K}^0$ transitions, i.e. the T, CP and CPT conjugated transitions of the reference one, respectively.

For the direct T symmetry test one can define the following ratios of probabilities:

$$R_1(\Delta t) = \frac{P[K^0(0) \to K_+(\Delta t)]}{P[K_+(0) \to K^0(\Delta t)]}$$

$$R_2(\Delta t) = \frac{P[K^0(0) \to K_-(\Delta t)]}{P[K_-(0) \to K^0(\Delta t)]}$$

$$R_3(\Delta t) = \frac{P[\bar{K}^0(0) \to K_+(\Delta t)]}{P[K_+(0) \to \bar{K}^0(\Delta t)]}$$

$$R_4(\Delta t) = \frac{P[\bar{K}^0(0) \to K_-(\Delta t)]}{P[K_-(0) \to \bar{K}^0(\Delta t)]}.$$  \hspace{1cm} (4)

The measurement of any deviation from the prediction $R_i(\Delta t) = 1$ imposed by T invariance is a signal of T violation.

At a $\phi$-factory the corresponding observable quantities are two ratios, $R_2^{\exp}(\Delta t)$ and $R_4^{\exp}(\Delta t)$ of double decay rates $I(f_1, f_2; \Delta t)$ into decay products $f_1$ and $f_2$ as a function of the difference of kaon decay times $\Delta t$ [52, 32], with $f_1$ occurring before $f_2$ decay for $\Delta t > 0$, and vice-versa for $\Delta t < 0$. For $\Delta t > 0$ one has:

$$R_2^{\exp}(\Delta t) \equiv \frac{I(\ell^-, 3\pi^0; \Delta t)}{I(\pi^-, \ell^+; \Delta t)} = R_2(\Delta t) \times D$$  \hspace{1cm} (5)

$$R_4^{\exp}(\Delta t) \equiv \frac{I(\ell^+, 3\pi^0; \Delta t)}{I(\pi^+, \ell^-; \Delta t)} = R_4(\Delta t) \times D$$  \hspace{1cm} (6)

while for $\Delta t < 0$:

$$R_2^{\exp}(\Delta t) = \frac{D}{R_3(|\Delta t|)}$$  \hspace{1cm} (7)

$$R_4^{\exp}(\Delta t) = \frac{D}{R_1(|\Delta t|)}.$$  \hspace{1cm} (8)

Here the normalization constant $D$, assuming no CPT violation in semileptonic decays, is $D = \{\text{BR}(K_L \to 3\pi^0) \cdot \Gamma_L\} / \{\text{BR}(K_S \to \pi\pi) \cdot \Gamma_S\}$.

For the direct CPT symmetry test one can define the following ratios of probabilities, similarly as for the T test:

$$R_{1,\text{CPT}}(\Delta t) = \frac{P[K^0(0) \to K_+(\Delta t)]}{P[K_+(0) \to \bar{K}^0(\Delta t)]}$$

$$R_{2,\text{CPT}}(\Delta t) = \frac{P[K^0(0) \to K_-(\Delta t)]}{P[K_-(0) \to \bar{K}^0(\Delta t)]}$$

$$R_{3,\text{CPT}}(\Delta t) = \frac{P[\bar{K}^0(0) \to K_+(\Delta t)]}{P[K_+(0) \to K^0(\Delta t)]}$$

$$R_{4,\text{CPT}}(\Delta t) = \frac{P[\bar{K}^0(0) \to K_-(\Delta t)]}{P[K_-(0) \to K^0(\Delta t)]}.$$  \hspace{1cm} (9)

The measurement of any deviation from the prediction $R_{i,\text{CPT}}(\Delta t) = 1$ imposed by CPT invariance is a signal of CPT violation.
At a $\phi$-factory the corresponding observable quantities are, for $\Delta t > 0$:

$$R_{2,CPT}^{\exp}(\Delta t) = \frac{I(\ell^-, 3\pi^0; \Delta t)}{I(\pi\pi, \ell^-; \Delta t)} = R_{2,CPT}(\Delta t) \times D_{CPT}$$ (10)

$$R_{4,CPT}^{\exp}(\Delta t) = \frac{I(\ell^+, 3\pi^0; \Delta t)}{I(\pi\pi, \ell^+; \Delta t)} = R_{4,CPT}(\Delta t) \times D_{CPT}$$ (11)

while for $\Delta t < 0$:

$$R_{2,CPT}^{\exp}(\Delta t) = \frac{D_{CPT}}{R_{1,CPT}(|\Delta t|)}$$ (12)

$$R_{4,CPT}^{\exp}(\Delta t) = \frac{D_{CPT}}{R_{3,CPT}(|\Delta t|)}.$$ (13)

Here the normalization constant $D_{CPT}$ is

$$D_{CPT} = \{\text{BR} \left( K_L \rightarrow 3\pi^0 \right) \cdot \Gamma_L \}/\{\text{BR} \left( K_S \rightarrow \pi\pi \right) \cdot \Gamma_S \}$$

without any assumption on CPT violation in semileptonic decays.

The KLOE-2 experiment at DAΦNE with an integrated luminosity of $O(10 \text{ fb}^{-1})$ [31] could make statistically significant T and CPT tests, measuring the ratios $R_{2,CPT}^{\exp}(\Delta t)$, $R_{4,CPT}^{\exp}(\Delta t)$, $R_{2,CPT}(\Delta t)$ and $R_{4,CPT}(\Delta t)$ integrated in the statistically most populated $\Delta t$ region, $0 \leq \Delta t \leq 300 \tau_S$ [52]. In this region these ratios are expected to be constant and a precise knowledge of the normalization constants $D$ and $D_{CPT}$ is needed in order to not weaken the significance of the test. A precise reconstruction of $K_S \rightarrow \pi\ell\nu$ and $K_L \rightarrow 3\pi^0$ decays is needed, and specific studies are in progress at KLOE-2 [54, 55].

4. Conclusions
The new generation of kaon experiments at hadron machines (NA62 and KOTO) are operational, and very soon will allow us to definitely enter a new precision era in probing the SM, and in searching for New Physics in a complementary way to LHC.

Some specific issues in the field, e.g. the transverse muon polarization in $K^+$ decays or the detection of very rare kaon decays, will be addressed by dedicated experiments or by general purpose experiments, as LHCb.

The neutral kaon system still constitutes an excellent laboratory for the study of discrete symmetries, and a $\phi$-factory represents a unique opportunity to push forward these studies. CPT symmetry and quantum mechanics tests will be one of the key issues at KLOE-2, and their precision will be further improved, while new ideas - e.g. the direct T and CPT symmetry tests - will be implemented.

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