A Hazy Image Restoration Algorithm via JND Based Histogram Equalization and Weighted DCP Transmission Factor

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Abstract. This paper proposed a three-step hazy image restoration method. Firstly, in HSI color space, we proposed a novel histogram equalization algorithm based on just noticeable difference (JND) to enhance the hazy image. Secondly, we apply the dark channel prior (DCP) to remove the image haze and compute the weighted transmission factor map. Finally, we integrated the enhancement results and dehazing results by the weighted transmission factor maps adaptively, and then the restoration result is obtained. Since the proposed method takes account of visual perception, and well balance image enhancement and image dehazing. Therefore, it results in a more natural haze-free scene for the human visual system (HVS). Experiment results demonstrate that our method achieves better restorations compared with some other state-of-the-art approaches.

1. Introduction
Outdoor Images are often degraded by the haze or fog in the atmosphere. The degraded hazy or fog images usually present a poor view of the scene contents. For example, the haze may reduce image contrasts, degrade image details, and blur image colors. However, most computer vision systems require high-quality image input. Consequently, it is of practical significance to develop a haze removal technique [1, 2].

Enhancing the local contrast or global contrast of a blurred image is a kind of traditional haze removal strategy [3-9]. Mostly, these methods neglect the degrading mechanism of a hazy image, thereby limiting the restoration performance. Especially, in a dark or a dense hazy image, these approaches hard to achieve better dehazing performances. To address this problem, another kind of image haze removal method based on the atmospheric scattering model is proposed [10-31]. Guiding by the above physical model, these methods follow a similar methodology: (1) estimating the transmission map from a hazy image; (2) estimating the global atmospheric light; (3) recovering the clean image. Among the three steps, the majority of attention is paid to the first step. The image prior knowledge is a widely used strategy to estimate the transmission in the first step. They can be further divided into patch-wise [11-26] and pixel-wise [27-31] methods.
The patch-wise strategy computes the transmission map by extracting local prior information from each patch. He’s method [11] is an efficient patch-wise strategy for single image dehazing. It proposes the patch-based dark channel prior (DCP) to detect the haze thickness, then refines the initial transmission map by soft matting, using the refined transmission map the realistic haze-free image is obtained. This method can remove image haze and meanwhile preserve image color. However, it often degrades the image brightness, saturation, and fail to achieve ideal image contrast. In [12], a color ellipsoid prior based is proposed. This method can estimate the transmission map by maximizing the contrast of restored pixels while avoiding the over-saturation. But it always regards the high-bright close-range areas as dense haze. Based on an assumption that a hazy image contains approximated color or repeated patches, Berman [13] proposed nonlocal prior. This method presents a good performance. But it cannot handle the heavy haze image effectively. Most learning-based dehazing methods [14-19] are patch-wise strategies. In [14], Cai proposed a trainable end-to-end system employing CNN to estimate the transmission map. In [15], Golts proposed a trainable end-to-end system applying the Context Aggregation Network (CAN) to estimate the transmission map. Overall, these learning-based methods can relieve the limitations of the traditional patch-wise strategy, they also fail to deal with dense haze and need a large number of training samples.

The pixel-wise strategy [27-31] estimates the transmission map by singling out the minimum channel information from each pixel in a hazy image. In [27], Lu estimates the transmission map by using the image saturation and brightness in HSV color space. It can remove the haze while discouraging the resulting image from becoming over-saturation. In [28], Tarel estimates the atmospheric veil associated with the minimum channel to realize the haze removal. This algorithm is good at recovering the scenic details, but it often fails to remove the mist in the discontinuous areas. In [29], Ju proposes a gamma correction prior based transmission for whole image pixels. It can achieve a better restoration of object color and target details. But it fails to preserve image brightness and saturation. On the whole, these pixel-wise methods have high implementation efficiency. However, they extract the prior just from a single pixel, thereby, tend to miss local information that may have a significant impact on the accuracy of the transmission map. Overall, most aforementioned methods can remove image haze and preserve the image color effectively. But a few of them take account of visual perception. They also cannot well balance the enhancement and the dehazing. Therefore, the results of these methods may seem unnatural and hard to conform to the HVS.

In this paper, we proposed a novel three-step hazy image restoration method. In the first step, we proposed a JND based [32] histogram equalization in HSI color space and applied it to enhance the hazy image. Compared to traditional histogram equalization, our work takes account of human visual sensitivity for each image pixels, the enhancement results more conform to the HVS. In the second step, we use He’s method [11] to remove the image haze and compute a novel weighted transmission factor for each pixel. Finally, we integrate the enhancement result of the first step and the dehazing result of the second step together with the weighted transmission factor. The enhancement results of the first step focus on removing the close-range area haze, enhancing the contrast of the whole image, preserving the image brightness and saturation. He’s results of the second phase can effectively remove the image haze and preserve the image color and details even the long-range area. Thereby, the fusion results by the weighted transmission factor of the third step can realize that: (1) remove image haze, (2) preserve image color, brightness, saturation, and details, (3) enhance image contrast. The proposed weighted transmission factor can well balance enhancement results between the dehazing results. For example, the close-range pixels are paid more attention to enhancement for the larger weighted transmission factor; and long-range pixels are paid more attention to dehazing for the smaller weighted transmission factor. The experimental results of this paper demonstrate that our method achieves better restoration compared with some other state-of-the-art dehazing method.
2. Related Works

2.1. The JND Threshold

The JND threshold uses background luminance, spatial masking, and the pixels disorder to explain the visual concealment for each pixel [32]. It reveals the distinguishability of the human visual system. In this section, we briefly review this JND model. Then in section 3.1, we introduce a JND data distribution function and propose a novel image enhancement algorithm.

The JND threshold consists of luminance adaptation, spatial masking, and disorderly concealment effect. We can compute it as,

\[ JND(i) = JND_p(i) + JND_d(i) \]

Where \( i \) is a pixel at \((i_x, i_y)\) whose gray value is \( x \), \( JND_p \) is the JND threshold of orderly part of the original image which is predicted by the free-energy principle, \( JND_d \) is the JND threshold of disorderly part of the original image. Since the Bayesian brain theory and the free-energy principle can be united under a uniform cognitive frame of the brain [33], paper [34] adopts the Bayesian brain theory [35] to predict the orderly image. The luminance adaptation (LA) and spatial masking (SM) are used to compute \( JND_d \)

\[ JND_d(i) = LA(i) + SM(i) - C_{\text{exp}} \cdot \min(LA(i), SM(i)) \]

Where \( C_{\text{exp}} \) is an experience value, set as 0.3.

\[ LA(i) = \begin{cases} 17 \times \left(1 - \frac{B(i)}{127}\right) & \text{if } B(i) \leq 127 \\ \frac{3}{128} (B(i) - 127) + 3, & \text{else} \end{cases} \]

\[ SM(i) = [0.1B(i) + 11.5][0.01G(i) - 1] - 12 \]

Where \( G(i) \) is the maximum edge height of its 5x5 neighborhood. The residue between the original image \( x \) and its prediction \( x' \) is the disorderly image \( \mathcal{D} \). According to the residue image, the JND\(_d\) can be computed as

\[ JND_d(i) = 1.125 \cdot \mathcal{D}(i) \]

\[ \mathcal{D} = |x - x'| \]

Set the mutual information between the central pixel and its surrounding pixels \( I(i; i_k) \) as the autoregressive coefficient, and then the predicted image pixel \( x'_i \) can be computed as

\[ x'_i = \sum_{i_k \in \Omega_i} c_k x_{i_k} \]

\[ c_k = \sum_{i_p \in \Omega_i} \frac{I(i_k; i)}{I(i_p)} \]

The above JND model fully considers the pixel luminance, the pixel contrast, and the pixel disorder. All of them are important factors for the human visual system to understand or percept an image. Therefore, it has always been a useful tool in the field of computer vision. In this paper, we use this model to construct a novel histogram. Then combine it with a weighted transmission factor map to restore the single hazy image.

2.2. Image Dazing Based on Dark Channel Prior and Guilder Filtering

The dark channel prior-based algorithm is regarded as the classical haze removal method. He’s method is the simplest and most effective one among them. It describes the hazy image as

\[ x_i = y_i \cdot t_i + A(1 - t_i) \]

Where \( x_i \) is the hazy image, \( y_i \) is the clear image, \( A \) is the global atmospheric light and \( t_i \) is the medium transmission. The dark channel is defined as,

\[ y_i^{\text{dark}} = \min_{j \in \Omega_i} \left( \min_{c \in \{r, g, b\}} y_j^c \right) \]

Where, \( y_i^c \) is the \( \{r, g, b\} \) channel of \( y_i \), \( \Omega_i \) is a local patch centered at \( i \). For a clear natural image, \( y_i^{\text{dark}} \) tends to be zero. Denote the transmission in the local window \( \Omega_i \) as \( \tilde{t}_i \), it can be computed as
\[ \tilde{t}_i = 1 - \omega \cdot \min_{j \in S} \left( \min_{c \in \{r, g, b\}} \frac{x^c_j}{A^c} \right) \]  

(11)

Where, \( x^c_j \) is the \( \{r, g, b\} \) channel of \( x_j \), \( \omega = 0.95 \) leaves a little of haze for natural-looking results. The atmospheric light \( A \) is taken as the mean of the brightest 0.1% pixel in the dark channel. By using the guelder filtering, the transmission \( \tilde{t}_i \) is refined to be \( \tilde{t}_i \) \[36\].

Finally, the haze image is restored as:

\[ y_i = \frac{x_i - A}{\max(\tilde{t}_i, 0.1)} + A \]  

(12)

The dark channel prior-based method has achieved superb results, it has an ideal dehazing effect on local detail and color degradation. Therefore, it has been widely applied in single image dehazing and underwater image enhancement. However, it also has some weaknesses: (1) the brightness of dehazing result decreases obviously, and the contrast of dehazing result is hard to achieve the ideal visual effect; (2) the white object, which does not conform to the dark channel prior, will present unnatural results.

3. Our Work

In this section, we propose a novel hazy image restoration algorithm with a free energy-based JND model and a weighted transmission factor. So as to achieve that the restoration image can preserve image brightness, color, saturation, and meanwhile enhance image the local and global contrast.

3.1. The JND-based Image Enhancement in HSI Color Space

Mostly, the tasks of image enhancement highlight the areas concealing rich information and meanwhile suppress the areas with pool information. In general, there are much information in detail, edge, texture, and disorderly area. By contrast, there is less information in the flatness area. The JND model consists of pixel luminance concealment, pixel contrast concealment, and pixel disorderly concealment. According to the data structure of the JND model, the large JND value pixel easily conceals image information. In other words, the low or high-density pixel easily conceals image information, the strong contrast pixel easily conceals image information, and the high disorderly pixels also easily conceal image information.

![Image](image_url)

Figure 1. The free-energy based JND data: (a) haze image, (b) LA map, (c) SM map, (d) disorder map, (e) JND map

In Figure 1(b), the high (or low) gray level pixels have larger LA value (corresponding to the brightness pixels). Figure 1(c) shows the strong contrast pixels have larger SM values. Figure 1(d) presents us with the disorder map. It demonstrates that the pixels in the high disorderly area have larger values. According to the internal generative mechanism (IGM) \[37\] of the human visual system (HVS), those brightness pixels in Figure 1(b)-(d) are the foundation for us to understand the haze image Figure 1(a). The JND data constructed by the LA, SM, and disorder map is shown in Figure 1(e). Obviously, introduce the JND data to the histogram can ensure that the distribution probability of each gray level both counting the pixel numbers and perceptions. The enhancement results by this novel histogram should more conform to the HVS.

HSI color space widely uses in image processing and computer vision. Paper \[38\] reveals that image haze leads to I component value increasing and saturation decreasing while leaving the Hue
component unchanged. Thereby, we just focus on extending the I component and increasing the S component in our image enhancement algorithm. The details of our approach are as follows:

Step 1. Convert the observed haze image from the RGB color space to the HSI color space. Three channels are H, S, and I.

Step 2. Calculate the pixel JND threshold according to section 2.1 in the I component.

\[ JND^I(i) = JND^K(i) + JND_D(i) \]  

(13)

Step 3. Calculate the sum JND value \( JND^I_r \) for each gray level throughout the I component image.

\[ JND^I_r = \sum_{a \in \Omega_r} JND^I(a) \]  

(14)

Where \( r \) is a gray level of the value component image. \( r \) locates in the range \([0, L-1]\) (e.g. 0 and 255 for an 8-bit image). \( r = 0 \) represents black. \( r = L - 1 \) represents white. \( \Omega_r \) is the location pixel set. In this set, all pixels have the same gray level \( r \).

Step 4. Construct the JND density distribution function for each gray level.

\[ P_r^{IND} = \frac{JND^I_r}{T^{IND}} \]  

(15)

Where \( T^{IND} \) is the sum JND value of all pixels in the I component image. Equation (15) and (13) demonstrate that any gray level with more pixels, the higher or lower gray value, the stronger contrast, and the more disorderly construction has the higher probability density. When we use this density distribution function to redistribute the gray level, the enhancement results will more conform to our visual perception.

Step 5. Set a transformation function to redistribute the gray level

\[ I^*_k = (L - 1) \sum_{r=0}^{k} P_r^{IND}, \ k = 0,1,2,\cdots,L - 1 \]  

(16)

Where \( I^* \) is the enhancement value component image.

Step 6. Calculate the pixel JND threshold according to section 2.1 in the saturation component.

\[ JND^S(i) = JND^K_S(i) + JND_D_S(i) \]  

(17)

Step 7. To remedy the saturation, we utilize a nonlinear dynamic range adjustment model. It’s a logarithm model since the human brain processing the received image signal can approximate logarithmic mapping [39]. The saturation is at the range of \([0, 1]\), the new saturation is therefore defined as:

\[ S'_i = \log_2(S_i + 1), \]  

(18)

Where \( S' \) is the Remedied saturation component.

Step 8. Convert the image composed by H, S', and I' back to RGB color space.

Figure 2 presents the JND-based histogram, ordinary gray histogram, and the corresponding enhancement results. In Figure 2(a), the trees are at the lower gray levels and the buildings are at the higher gray levels, so they have larger JND values. The distribution densities of those gray levels are higher in our histogram. The sky areas with medium gray levels have smaller JND values. The distribution densities of those gray levels are lower in our histogram. According to the histogram equalization theory, the higher distribution densities of gray levels will assign the wider the gray range. As shown in Figure 2(b) (c), our enhancement results more conform to human visual perception.
Figure 2. Comparison of JND-based histogram equalization and ordinary histogram equalization for the haze image “City”: (a) hazy image, (b) ordinary histogram equalization, (c) JND-based histogram equalization, (d) gray histogram of I component, (e) JND-based histogram of I component, (f) the difference between (e) and (f).

3.2. The Hazy Image Restoration Algorithm Based on The Weight Transmission Factor

The hazy image restoration usually requires not only haze removal but also local and global contrast enhancement. In this section, we integrate the JND-based image enhancement with DCP image dehazing to ensure that fusion results can highlight the advantages of the two methods, weaken their defects. Generally, in a haze image, the haze density is low on the close-range pixels and high on the long-range. That means that we should mainly focus on the enhancement for the close-range pixels and the dehazing for the long-range pixels. The transmission, which is approximately in the range of [0, 1], describes the depth of image pixels. Therefore, taking the transmission as the weighted factor in our fusion algorithm can perfectly balance the dehazing and the enhancement. The details of our fusion algorithm are as follows:

(1). Normalized transmission weighted factor

\[ \tilde{t}_{i} = \frac{t_{i} - \text{min}(t)}{\text{max}(t) - \text{min}(t)} \]  

(19)

Where \( \tilde{t}_{i} \) is a transmission weighted factor of the pixel \( i \). To keep the minimum influence of the enhancement and dehazing on the restoration image, the weighted factor is defined as:

\[ t_{i}^* = \begin{cases} 0.1, & t_{i}^* \leq 0.1 \\ 0.9, & t_{i}^* \geq 0.9 \\ t_{i}^*, & \text{else} \end{cases} \]  

(20)

(2). Dehaze the haze image by Equation (12), then we get the dehazing image \( y \).

(3). According to the JND-based image enhancement, we get the enhancement image \( y_{\text{enh}} \).

(4). Integrate the image \( y \) with \( y_{\text{enh}} \), we get the final output image \( y_{\text{out}} \).

The weight factor in equation (19) is directly constructed from the transmission \( t^* \). For the pixels with high transmission, the weight of the dehazing image \( y_i \) is smaller, while the weight of the enhancement image \( y_i^* \) is larger. Otherwise, the pixels with the low transmission, the weight of \( y_i^* \) is
larger and that of $y_i$ is smaller. Therefore, this approach can fuse the dehazing and enhancement adaptively by the transmission value of each pixel.

Figure 3. The restoration results of hazy image “Mountain”: (a) haze image, (b) JND-based enhancement results, (c) DCP dehazing result, (d) transmission map, (e) our approach

Figure 3 presents the restoration image for a natural haze image “Mountain” by our method, JND-based enhancement, and He’s DCP dehazing [11]. Fig.3 (d) is the weighted transmission factor map. The pixels in the red circle are close-range pixels whose transmission factors are bigger. According to Equation (21), we mainly focus on enhancement. The pixels in the blue circle are long-range pixels with low transmission factors. In those areas, we mainly focus on image dehazing. The result of our method Fig.3 (e), which is obtained by integrating JND-based image enhancement Fig.3 (b) and DCP dehazing Fig.3 (c), demonstrates that our method can adaptively select the advantages of both methods and discard their disadvantages. The result of our way more conforms to human visual perception.

4. Experiment Results
In this section, we assess the effectiveness of our method against some state-of-the-art dehazing methods. He’s method [11], Ju’s method [29], Berman’s method [13], Golts’s method [15], Cai’s method [14], and Lu’s method [27] are chosen for comparison. The experiments consist of two parts: (1) qualitative analysis (2) quantitative evaluation. Six natural hazy images are processed in our experiments, as shown in Figure 4. All algorithms are implemented in the Matlab R2018b on a PC with 2.50GHz Intel i5-3210M.

Figure 4. The test hazy image (a) Mountain, (b) Traffic, (c) Tricycle, (d) Bridge, (e) City, (f) Road

4.1. Qualitative Analysis
Figure 5-7 shows our haze removal results compared with He’s method, Ju’s method, Berman’s method, Golts’s method, Cai’s method, and Lu’s results. In Figure 5, He’s method, Berman’s method and Golts’s method can remove the haze effectively, while He’s results Figure 5(b) decrease the image brightness, Berman’s results Figure 5(d) and Golts’s results make the results over-statured. Lu’s
method Fig 5(g) and Cai’s method Fig 5(g) can prevent the dehazing results from being over-statured, but it fails to remove the haze on the long-range pixel effectively. In contrast, our method can remove the haze effectively and make the result more natural, as shown in Figure 5(h).

Figure 5. Dehazed results of the image “Mountain” compared with the state-of-the-art methods (a) original image (b) He’s method (c) Ju’s method, (d) Berman’s method, (e) Golts’s method, (f) Cai’s method, (g) Lu’s method, (h) Our method

In Figure 6, He’s method and Berman’s method had better performances on removing the image haze, especially the long-range pixels (for instance, the long-range trees). But He’s recovery result, as shown in Figure 6 (b), is too dark, and Berman’s result, as shown in Figure 6(d), makes the trees and the buildings oversaturated. Ju’ result, as shown in Figure 6(c), Golts’s result, as shown in Figure 6(e), Cai’s result, as shown in Figure 6(f), and Lu’s result, as shown in Figure 6(g), can restore the close-range pixels effectively, but they fail to restore the long-range pixels. Compared with the above state-
of-the-art methods, our method achieved the best performance on removing image haze, as shown in Figure 6(h).

![Image](a) ![Image](b) ![Image](c) ![Image](d) 

![Image](e) ![Image](f) ![Image](g) ![Image](h) 

Figure 7. Dehazed results of the image “Bridge” compared with the state-of-the-art methods (a) original image (b) He’s method (c) Ju’s method, (d) Berman’s method, (e) Golts’s method, (f) Cai’s method, (g) Lu’s method, (h) Our method

Figure 7(a) is a natural image with a dense fog. We can see that Berman’s method makes the color distortion, and He’s method makes the brightness distortion. Ju’s method, Golts’s method, and Cai’s method cannot remove the dense fog effectively. Our method and Lu’s method achieved better performance on image fog removal. Compared with Lu’s method, our method can well handle the close range scene, for instance, the trees and leaves. Besides, our method can preserve the colors effectively.

Form the above three experiments, our method can remove the haze effectively and make the results more natural. Therefore, the results of our method more conform to the HVS.

4.2. Quantitative Comparisons

In this section, some well-known evaluation indexes [39] [40] are used to quantitatively assess the algorithms. The Fog Aware Density Evaluator (FADE) [39] predicts the visibility on a hazy scene with nonreference to the corresponding haze-free image, and it correlates well with human judgments of haze density. According to [40], we compute (1) a new apparent edge ratio $e$, (2) a normalized gradient of the visible edge $\bar{r}$, and (3) the proportion of saturated white or black pixels $\sigma$ to measure the contrast of dehazing results. Generally, the smaller value of FADE implies the lower perceptual haze density; the higher value of $e$ and the value of $\sigma$ closer to zero imply better performance. And a higher value of $\bar{r}$ implies the robust recovery of the local contrast. As shown in Table 1, our algorithm usually achieves the smallest value of FADE, the highest value of $\bar{r}$ and $e$, and the saturation $\sigma$ is nearly close to zero.

Overall, the qualitative and quantitative comparison results in Figures 5, 6, 7, and Table 1 show that our algorithm has advantages over the compared state-of-the-art dehazing methods in terms of haze removal, visibility improvement, visual perception, and illumination compensation while avoiding most of the negative effects.

| method | indicator | He’s | Ju’s | Berman’s | Golts’s | Cai’s | Lu’s | Ours |
|--------|-----------|------|------|----------|---------|------|------|------|
| Mountain | $e$ | 0.17 | 0.05 | 0.01 | 0.09 | 0.04 | 0.06 | **0.18** |
| Traffic | 1.21 | 0.31 | **1.37** | 0.47 | 0.72 | 0.94 | 1.30 |
| Bridge | 1.91 | 0.36 | 1.78 | 0.18 | 1.17 | 1.49 | 1.95 |
| Tricycle | 1.09 | 0.14 | 1.03 | 0.43 | 0.23 | 0.49 | **1.17** |
| City | 0.30 | 0.05 | 0.29 | 0.21 | 0.05 | 0.20 | **0.33** |
5. Conclusions

In this paper, we proposed a novel three-step image haze removal algorithm. In the first step, we introduce a free-energy based JND histogram and then apply this histogram in HSI color space, we get an enhancement result. Since our enhancement way takes account of the sensitivity of our eyes, the enhancement image more conforms to our visual perception. In the second step, we use He’s method to remove the image haze and estimate the weighted transmission factor map. Finally, we integrate the enhancement result and the dehazing result by the transmission factor map to obtain the haze removal image. According to the transmission factor, we can realize that the long-range pixels are mainly focused on dehazing and the close-range pixels are mainly focused on enhancement. In general, the transmission of a haze image is continuous. Therefore, the fusion results can effectively removal image haze, preserve image brightness, color, saturation, and meanwhile enhance image contrast. The experiments also show that our results seem more natural and more conform to the HVS when we compare it with some state-of-the-art dehazing methods.

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