On dynamics of pseudorapidity fluctuations in central C–Cu collisions at 4.5 A GeV/c

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Abstract

Results on dynamical fluctuations of charged particles in the pseudorapidity space of central C–Cu interactions at 4.5 A GeV/c are performed in the transformed variables and using higher order scaled factorial moments modified to remove the bias of infinite statistics in the normalization. The intermittency behavior is found up to eighth order of the moments increasing with the order and leading to the pronounced multifractality. Two differed intermittent-like rises are obtained, one indicating an occurrence of the non-thermal phase transition, and no critical behavior is found to be reached in another case. The observations may be treated to show different regimes of particle production during the cascade. Comparison with some conventional model approximations notes the multiparticle character of the fluctuations. The results presented can be effective in sense of sensitivity of intermittency to the hadronization phase.

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1 Introduction

At present intermittency seems to be a well-founded fact observed in high-energy multiparticle production. This phenomenon, expressing as the power-law behavior,

$$\langle F_q \rangle \propto M^{\varphi_q}, \quad 0 < \varphi_q \leq q - 1,$$

of the $q$-order normalized scaled factorial moments (SFM),

$$\langle F_q \rangle = \frac{1}{M} \sum_{m=1}^{M} \frac{\langle n_m^{[q]} \rangle}{\langle n_m \rangle^q}$$

(“vertical” analysis), identifies dynamical fluctuations. Here $n_m^{[q]}$ is the $q$th power factorial multinomial, $n_m (n_m - 1) \cdots (n_m - q + 1)$, with multiplicity $n_m$ in the $m$th bin from $M$ ones, into which the space of the particles produced are divided. Average is taken over all events.

The intermittency indexes, $\varphi_q$, obtained via (1) are shown to be related to the fractal codimensions $d_q$, and correspondingly reflects the inner structure of fluctuations representing monofractal patterns with the unique $d_q$, or multifractal ones with the hierarchy $d_q > d_p$, $q > p$.

Applied to multihadron production processes, formation of such geometrical structures are pointed out to be a manifestation of one of two possible mechanisms. The first one, leading to the $q$-dependence of $d_q$, is that one connects with self-similar cascade, rather a “non-thermal” (non-equilibrium) phase transition during the cascade than particle creation within one phase, e.g. hadronic. The second scenario assumed at monofractality, is associated with thermal transition, e.g. from a quark-gluon plasma expected to be reached in central collisions of (ultra)relativistic nuclei.

So study of intermittency decodes through fractality the geometrical and then thermodynamical features of high-energy multiparticle production. Note that the random cascading models being widely used as suitable to describe such processes (e.g., negative binomial distribution), lead in the most general form to the scaling, though they seem to be problematic to reproduce intermittency observed. In general, the observations are still far from the qualitative explanation using existed particle-production codes, and origin of intermittency/fractality is a matter of debates.

Meanwhile, intermittency turns out to be very sensitive to the hadronic phase. Since the interpretation is mostly done in the approach based on the parton cascade models, it is rather difficult to explain the fact that partonic local fluctuations survive the hadronization process. In this context it is noticeable that “universality” of multihadron production in different type of reactions (from lepton-hadron to nucleus-nucleus) can be just a reflection of hadronization dynamics properties. Thus, search for dynamical fluctuations and its possible treatment in “soft” process terms attracts considerable interest in reactions at intermediate energies.

Important features of intermittency systematics behavior found in high-energy nuclear interactions are also already found to be manifested at relativistic energies. Besides purely hadronic model calculations showing intermittent structure due to the random cascade, enhancement of intermittency with beam energy decrease and weakening of the effect with increase complexity of reaction are obtained. Recently, processes with low average multiplicity was discussed to dominate in the fractional SFM analysis.

2 Experimental procedure

The present paper deals with the study of the data obtained after processing the pictures of the 2m Streamer Chamber SKM-200 equipped with a copper target. The chamber was installed in a 0.8 T magnetic field and it was exposed to the 4.5 $A$ GeV/c $^{12}$C beam at the JINR Synchrophasotron (Dubna). In data taking, the central collision trigger was used: the chamber was started if there were no charged...
particles with momenta larger than 3 GeV/c in a forward cone of 2.4°. Details of the set-up design and data reduction procedure are described elsewhere \[23\]. Systematic errors related to the trigger effects, low energy pion and proton detection, the admixture of electrons etc. have been considered in detail earlier \[22\] and the total contribution does not exceed 3%.

The scanning and handling of the film data were carried out on special scanning tables of the Lebedev Physical Institute (Moscow), using the method elaborated in ref. \[34\]. The average measurement error in the momentum \((\varepsilon_p/p)\) was about 12%, and that in the polar angle measurements was \(\langle \varepsilon_\theta \rangle \approx 2°\). To search for dynamical fluctuations the charged particles in the pseudorapidity \((\eta = -\ln \tan(\theta/2))\) region \(\Delta \eta = 0.2 - 3.0\) (in the target rest-frame) were used, in which the angular measurement accuracy was not larger than 0.1 in the \(\eta\)-units. The samples of 305 C–Cu events, which meet the above centrality criterion, have been selected with the average multiplicity of 27.2 ± 0.8 in the \(\Delta \eta\) under consideration.

Earlier we have already analyzed fluctuations in the data presented and existence of non-statistical (dynamical) fluctuations was shown \[30\] within intermittency approach and using multifractal analysis \[31\]. It is noteworthy that only applying the methods with statistical background suppression allowed dynamical nature of the fluctuations to be manifested, while the comparison between data and completely uncorrelated particle-production simulation fails to do that \[28\]. It is significant in sense of the above discussed “hadronic origin” of the intermittency that analogous observation have been done at ultra-relativistic energies \[24\] (see also ref. \[39\]).

The study \[30\] also showed multifractality of the particle spectra along with multiparticle character of the fluctuations being important to choice of the real particle-production model, as discussed. Multifractality have been also observed at ultra-relativistic heavy-ion collisions but obliged mainly to the two-particle correlations \[3\]. Note that multiparticle contribution to the very-short-range correlations are directly observed in our data using the method of factorial cumulants \[40\] as found in hadronic interactions recently \[23\].

Before presenting the results, two important technical remarks should be made. First of all, in the previous investigations we have used “horizontal” analysis of the SFM taking into account the non-flat shape of the one-particle pseudorapidity distribution \(\rho(\eta)\) to minimize the difference from “vertical” normalization \[3\]. Note that, though the “corrected” SFM were widely applied to analysis (in particular, at relatively small multiplicities) it implies the fluctuations to be bin-independent (the sum of the quotient in \(2\) transforms into the fraction of independent sums, \(\sum_{m=1}^M \langle n_m^q \rangle / \sum_{m=1}^M \langle n_m \rangle^q\)) that is, generally, a non-trivial assumption \[3\]. In our data the slopes of the vertically-normalized moments are significantly greater the corrected ones at \(\delta \eta < 0.5\) \[41\], while they coincides in collisions of ultra-relativistic ions \[24\].

To overcome the problem and, moreover, to compare the results observed in different experiments a new transformed variable,

\[
\tilde{\eta} (\eta) = \frac{\int_{\eta_{\min}}^{\eta} \rho(\eta') d\eta'}{\int_{\eta_{\max}}^{\eta_{\min}} \rho(\eta') d\eta'},
\]

have been introduced \[42\], so that \(\tilde{\eta}\) is uniformly distributed in the [0;1] interval \(\langle \rho(\tilde{\eta}) \rangle \approx \text{const.}\). Due to the scale properties of the variables \(4\), the one-particle spectrum stretches in its central region (not significantly changing at the \(\Delta \eta\)-edges), eliminating losses from bin-splitting and, thus, allowing to observe higher-order moments.

Another remark regards to the biased estimator of the SFM normalization, being the \(q\)-particle density function for uncorrelated production of particles in assumption of infinite statistics \[1\] \[4\]. This sensitively influence the scaling law \(1\) for small bins. Note that a flattening of the moments for \(M \geq M_0\) is expected to be a reflection of the attainment of the correlation length \(1\). Really, measured SFM go to zero as the bin size aspires to the experimental resolution \[24\], or much before because of the statistics limitations ("empty bin effect" \[14\] \[15\]).

Recently, the modification of the method of the SFM have been proposed \[16\] to remove the bias in the normalization, and then the bias-free moments are defined to be, e.g. at “vertical” analysis,

\[
\langle F_q \rangle = \frac{N^q}{M} \sum_{m=1}^M \frac{\langle n^q_m \rangle}{N_m^q},
\]

3
where \( N_m \) is the number of particles in the \( m \)th bin in all \( N \) events, \( N_m = \sum_{j=1}^{N} (n_m)_j \). The property of the “vertical” and “horizontal” analysis to give the same results if the scaled variables (3) are used \((n_m \approx (n)/M)\) seems to be also valid for the definition (3).

### 3 Results and discussion

In fig.1 we show the log-log plots of the modified SFM (3) vs. number of bins of the space of “pseudorapidity” (1). Though by illustration reasons the dependencies are given only for some orders, viz. \( q = 3, 5, 6, 8 \), they reflect the common peculiarities to be noticed. From the plots and the values of \( \varphi_q \) (see table) one can conclude that, besides considerable intermittent behavior, pronounced up to higher orders, there are the different power-like dependences in different intervals. Since at low orders \((q = 2, 3)\) this expresses as two sharply differed slopes: the values at \( M \leq 22 \) are about seven times less than ones at \( M \geq 23 \), the effect grows weak at \( q = 4 \) and \( 5 \). It is visible also that as the order increase an additional intermittent structure manifests at the range of small bins, but doesn’t in fact survive when five and more particles are required to fill the bin and effect of small statistics becomes sizeable.

However, regardless to the irregularities in the \( M \)-dependence of the SFM, \( \varphi_q \) increase with the order up to higher moments. Moreover, in the \( M \)-interval of [7;17] and especially at \( 10 \leq M \leq 17 \) strong intermittency is seen.

It should be noted that the irregular behavior of the SFM was also observed in our previous investigation [3], but using of the “ordinary” pseudorapidity variable strongly limited the order of the moment. Transforming \( \eta \)-spectra into the uniform ones made this effect more pronounced and allowed to consider the SFM of high orders, where a new sub-structure is revealed. Note that strongly non-linear increasing of the log-log plots as presented have been observed in different interactions, from lepton-hadron to heavy ion collisions [2], particularly at high-orders of the moments (e.g. [22]) and/or at multidimensional analysis [23, 24, 25].

As discussed, intermittency index, or rather codimension (3) dependence on \( q \)-order gives an information on the possible particle-production scenario. In fig.2 we demonstrate \( d_q \) as a function of \( q \) for different \( M \)-intervals, within which a linear fit of the SFM plots (fig.1) is valid; the corresponding \( \varphi_q \)-values are shown in the table. From the behavior of the \( d_q \) obtained one can definitely conclude that very multifractal structure \((d_q > d_p, p > q)\) of multiparticle production is revealed, independent of the interval of \( M \) considered; no evidence for monofractality, and then for a second-order phase transition is seen. Let us to mention that decrease of the \( d_q \) at high order is, in our opinion, connected with relatively small number of particles per event. Meantime, no considerable difference are obtained for some \( M \)-intervals, namely for the couples of \( 7 \leq M \leq 17, 10 \leq M \leq 22 \) and \( 4 \leq M \leq 15, 2 \leq M \leq 22 \).

The multifractality, as noted above, lends support to find the condition for the non-thermal phase, not characterized by a thermodynamical behavior (1). As signal of the transition to this phase the existence of a minimum of the function

\[
\lambda_q = \frac{\varphi_q + 1}{q}
\]

at a certain “critical” value of \( q = q_c \) is required. In such a “spin-glass” phase the events are dominated by a few clots in projected distributions, while the “normal” phase is represented by the events with a bulk of peaks and holes [17, 15]. The minimum of eq. (1), as shown [17], may be a manifestation of the fact that these two different phase mixed. Moreover, the phase transition can accompany (or occur inside) the branching process [13, 16].

In fig.3 the \( \lambda_q \) are displayed as function of the \( q \)-order for different \( M \)-intervals with considerably distinguished \( d_q \) (see fig.2). A clear minimum at \( 7 \leq M \leq 17 \) (\( 10 \leq M \leq 22 \)) and \( 4 \leq M \leq 15 \) (\( 2 \leq M \leq 22 \)) are seen, whilst the fits at \( 2 \leq M \leq 28 \) do not exhibit such a feature. In our opinion, this is very intrinsic finding.

Indeed, while sensitive behavior of the SFM in the region of \( 2 \leq M \leq 22 \) are observing right up to eighth order (fig.1 and the table) such a sharp difference of the dependence of \( \lambda_q \) may be a signal of a non-trivial dynamical effect [19]. Referring to the non-thermal phase transition interpretation [19].

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3 The contribution of the statistics limitations is estimated to be small for these \( M \)-intervals.
one should conclude that fits at different $M$-intervals lead to different cases of the intermittency: “weak” intermittency for monotone function (7) (large $q_c$) and “stronger” intermittency when minimum is reached at $q = 4$ or 5. The stronger intermittency is related to the so-called “peak transition”, i.e. transition at positive index $q_c$, and the weak case contains the absence of the transition also. In our view, such an ambiguity is related to the cascade mechanism description that includes many steps irregardless to the change of the regime of particle production at different bin-averaging scales. This is, that apparently brings indetermination in treating of the intermittency observations e.g. in $e^+e^-$-annihilation [19] and, meanwhile, being manifesting in our data is an extra proof of hadronic nature of intermittency observed [11].

Noteworthily, only using of higher-order moments makes clear the hint to the non-thermal transition found earlier slightly visible up to fifth order [23, 37, 26]. Whether the minimum is taken place, the low-$p_t$ effect in the intermittency manifested at hadronic interactions [1] leads to a clear minimum confirming responsible of hadronization for the effect of intermittency [11]. The $p_t$ range for the stronger intermittency in our case was observed to be of 0.35–0.45 GeV/c like one have been already found searching for the maximum fluctuations [38]. It should be stressed also that since the minimum of the $\lambda_0$ (9) corresponds [2] to zeros of the fractal spectra, the system “frozen” at this point is no longer self-averaging and introducing of new observables is needed [50].

The shown importance of the cascade approach in the multiparticle production and the abovementioned underlying of these processes to describe the hadronization attracts considerable interest to compare the predictions with the observations. Let us to limit oneself what have been earlier studied related to the SFM method, viz. negative binomial distribution (e.g. [17]), gaussian approximation [1] and scale-invariant mass-splitting branching model [13].

If the negative binomial distribution (NBD) is valid then the SFM are determined by the recurrence relation

$$\langle F_q \rangle = (1 + \frac{1}{k}) (1 + \frac{2}{k}) \cdots (1 + \frac{q-1}{k}),$$

(7)

where $k$ is one of two NBD parameters should be independent of the $\delta \eta$. The bin dependence of $k$ expected from the intermittent rise at $q = 2$ (see table) is a reflection of the instability of the NBD [23]. Besides, the NBD is a good fit of the data in the central $\eta$-region only, and does not describe the “tails” of the multiplicity spectra, for which it transforms into the $\Gamma$-distribution being stable [51]. In the previous ref. [22] we have shown that at the values of $\varphi_q$ closed to the experimentally obtained ones, the absolute values of the SFM approach each other when the shape of the $\eta$-spectra is accounted for. From this observation and the noted “central property” of the NBD the satisfactory agreement of the moments based on (6) with the experimentally measured ones in the “flat” spectra over the scaled variable (6) have been predicted. This is a fact well seen now from fig. 1 ($\chi^2/NDF$ for the $\langle F_q \rangle$ are less than one standard deviation in all the $M$-regions), may be excluding the case of $q = 8$ (e.g. $\chi^2/NDF \approx 6$ at $2 \leq M \leq 22$). Meanwhile, the table shows that the NBD calculated slopes $\varphi_q$ yield the values markedly distinguished from the observed ones in the stronger intermittency case with possible non-thermal phase transition ($7 \leq M \leq 17, 10 \leq M \leq 17$), and trend to coincide each other at weak intermittency. In our opinion, this is because of input two-particle character of $k$ calculated from (7): flattening of the NBD prediction with $q$ increase is visible at $q \geq 6$ (fig. 1).

Such a difficult meat when the NBD is applied to the intermittency search makes it problematic to describe the data. Let us note that invalidity of the interpretation of multiparticle production at high energies in the NBD terms was also showed recently [14, 17, 23].

The gaussian (“log-normal”) approximation (GA) predicts the relations for the slopes $\varphi_q$ to be defined as

$$\varphi_q = \frac{\varphi_2}{2} q (q - 1).$$

(8)

In figs. 2 and 3 we show the GA predictions for the $d_q$ (8) and $\lambda_q$ (6) using the $\varphi_2$-indexes at $2 \leq M \leq 28$ only. This is done by force of almost equal quantities of the $d_q$ for all the cases of the $M$-intervals considered. One can conclude the GA seems to be also hard to describe the intermittency effect especially when the phase transition is possible. In this sense it is interesting to note that the GA and the NBD approach both are found [44] to describe the cascade with small number of steps, reflecting
invalidity of these approximations for the multiplicity asymptotics. The non-gaussian character of the correlations in multihadron production was also shown earlier \cite{2, 10}.

The same result though less clearly is seen (fig. 2 and 3) if the simple scale-invariant cascade model proposed by Ochs and Wosiek \cite{13} is applied. In this case a modified power-law,

$$\langle F_q(\delta \eta) \rangle \propto [g(\delta \eta)]^{\phi_q},$$

was showed to be universal dependence for large class of models and of the available data of the one- and multidimensional SFM \cite{13}. Here $g(\delta \eta)$ is the model-dependent function of $\langle F_2 \rangle$, so that $\phi_q = r_q \phi_2$. The conclusion agrees with the earlier observations \cite{37} and shouts to one another obtained in hadronic interactions \cite{11}.

Thus all used model approximations, being based on the second order moments, indicate multiparticle character of the dynamical fluctuations at (possible) non-thermal (peak) phase transition.

4 Conclusions

The study of the intermittency phenomenon up to higher ranks of the scaled factorial moments of the pseudorapidity distributions of charged particles produced in central C–Cu interactions at 4.5 GeV/c per nucleon is performed. To eliminate the problem of the spectrum shape and to reach the higher multiple fluctuations the transformed variables are used. The unbiased modification is applied to the moments normalization to avoid a bias at small bins. The study shows existence of intermittent-like increase of the moments at all the orders considered, leading to a pronounced multifractality. The higher moment analysis allows to reveal two sharply differed increases with bin size, one of which indicates stronger intermittency and then non-thermal phase formation, and another one is usually connected with the weak intermittency when the critical $q$-order is not reached. While this comes from the fitting procedure at different bin intervals we are inclined to consider the fact obtained as a manifestation of different regimes of particle production at different scales of random cascading with a non-thermal phase transition inside. The results are compared to ones given in the assumption of a validity of the negative binomial distribution, gaussian approximation and scale-invariant mass-splitting branching model. The essential multiparticle character of the phase transition is indicated. Accounting possible hadronization influence on the dynamical correlations \cite{52} and search for the manifestations of a new matter formation the investigation of higher multiparticle fluctuations in nuclear collisions at intermediate energies gives evidence of further study of the effects observed.

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Figure captions

Fig. 1. The log-log plots of the modified scaled factorial moments \((\langle n \rangle)\) vs. the number of divisions. The curves present the NBD calculations \((\langle \rangle)\) and the straight lines show the least-squares fits at \(10 \leq M \leq 17\).

Fig. 2. The codimensions \(d_q\) versus \(q\)-order.

Fig. 3. The \(\lambda_q\)-functions \((\langle \rangle)\).

Table 1

The intermittency indexes \(\varphi_q\) compared to the NBD predictions. The errors present the covariance matrix estimators of the linear least-squares fits.

| \(q\) | \(2 \leq M \leq 42\) | \(2 \leq M \leq 28\) | \(2 \leq M \leq 22\) | \(4 \leq M \leq 15\) |
|------|-----------------|-----------------|-----------------|-----------------|
| 2    | 0.017±0.001    | 0.011±0.001    | 0.009±0.001    | 0.009±0.001    |
| 3    | 0.058±0.001    | 0.039±0.001    | 0.037±0.002    | 0.032±0.004    |
| 4    | 0.130±0.002    | 0.087±0.003    | 0.092±0.004    | 0.079±0.009    |
| 5    | 0.121±0.004    | 0.124±0.007    | 0.164±0.008    | 0.153±0.019    |
| 6    | –               | 0.113±0.015    | 0.233±0.013    | 0.234±0.035    |
| 7    | –               | –               | 0.268±0.021    | 0.275±0.059    |
| 8    | –               | –               | 0.207±0.031    | 0.207±0.091    |

| \(q\) | \(10 \leq M \leq 22\) | \(7 \leq M \leq 17\) | \(10 \leq M \leq 17\) |
|------|-----------------|-----------------|-----------------|
| 2    | 0.004±0.002    | 0.012±0.002    | –               |
| 3    | 0.051±0.009    | 0.054±0.006    | 0.020±0.018    |
| 4    | 0.194±0.030    | 0.178±0.015    | 0.149±0.046    |
| 5    | 0.450±0.044    | 0.432±0.031    | 0.489±0.100    |
| 6    | 0.894±0.076    | 0.837±0.058    | 1.187±0.178    |
| 7    | 1.416±0.121    | 1.346±0.101    | 2.239±0.277    |
| 8    | 1.643±0.200    | 1.847±0.171    | 3.490±0.401    |
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$\ln < F_3 >$ vs $\ln M$
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$\ln \langle F_5 \rangle$
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\[ \varphi_q = 0 \]

GA at \( 2 \leq M \leq 28 \)

OWA at \( 2 \leq M \leq 28 \)

\( 10 \leq M \leq 17 \)

\( 7 \leq M \leq 17 \)

\( 4 \leq M \leq 15 \)

\( 2 \leq M \leq 28 \)
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