GeoFlow: 3D numerical simulation of supercritical thermal convective states

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Abstract. ‘GeoFlow’ is a thermal convection experiment in rotating spherical shell geometry, which is going to take place in microgravity environment of International Space Station. We present numerical preliminary studies of the spherical Rayleigh-Bénard problem under an artificial central force field. Numerical simulation is done with a pseudospectral method. Special focus here is the simulation of flow states at selected parameter points of Rayleigh and Taylor number of a defined plan for experimental runs on ISS. One loop will contain thermal convection without rotation, i.e. rising temperature gradient between inner and outer sphere. Another loop investigates convection superimposed by rotation, i.e. fixing temperature gradient and then rising rotation rate. In such cases different transitions are expected to be observed. Just rising Rayleigh number shows different stable states depending on initial conditions. Fixing Rayleigh number and then rising up Taylor number leads to traverse of different convective states showing rich dynamics of the system.

1. Introduction
Thermal convection in spherical shells is a fundamental model in geophysical fluid dynamics. Instabilities provide details for understanding large scale geophysical motions as convective transport phenomena in the Earth’s liquid outer core. We face this hydrodynamical problem with the ‘GeoFlow’ experiment on thermal convection in rotating spherical shells influenced by a central force field [1]-[2]. The experiment will take place at International Space Station in European Columbus Modul inside Fluid Science Laboratory. Set up of central force is done by establishing a high voltage potential between inner and outer sphere using the dielectrophoretic effect. Resulting acceleration due to radial buoyancy force achieves \( g_e |_o \approx 10^{-1} \text{ m/s}^2 \) at outer shell. Compared to acceleration due to gravity with \( g \approx 10 \text{ m/s}^2 \), one can see that natural gravity would always be dominant if experiments are done in a laboratory. Hence microgravity conditions are necessary to perform the ‘GeoFlow’ experiment successfully.

Running ‘GeoFlow’ in Fluid Science Laboratory environment requires automation of the experiment. For this purpose an experiment flow plan has been set up. Main parts of parameter variation include set-up of temperature gradient between inner and outer sphere (corresponding to Rayleigh number variation) and rotation of spherical system (corresp. to Taylor number variation). Work schedule is splitted into two parts, a non-rotating (loop 1) and a rotating case (loop 2). During first loop rotation rate will be set to zero while temperature difference will be
Table 1. Range of parameter values of GeoFlow experiment. Relevant properties of experimental fluid Baysilone silicone oil are described to be thermal diffusivity $\kappa = 7.735 \cdot 10^{-8} \text{m/s}^2$, relative permittivity $\epsilon_r = 2.7$, thermal coefficient of expansion for relative permittivity $\gamma = 1.065 \cdot 10^{-3} \text{1/K}$, density $\rho = 920 \text{kg/m}^3$.

| Parameter                  | Value               |
|----------------------------|---------------------|
| Gap width                  | $r_o - r_i$ [mm]    |
| Viscosity                  | $\nu$ [m/s²]       |
| High voltage               | $V_{\text{rms}}$ [V]|
| Temperature gradient       | $\Delta T$ [K]     |
| Rotation rate              | $n$ [Hz]            |

- $r_o - r_i$: gap width
- $\nu$: viscosity
- $V_{\text{rms}}$: high voltage
- $\Delta T$: temperature gradient
- $n$: rotation rate

increased stepwise. During second loop temperature difference will be set to preferred values, which will be kept fixed, while rotation rate will be increased to maximum, though passing different rotational regimes gradually from zero to high rotation. This experiment scenario is supported by numerical simulation, presented in the following.

2. Equations

For governing equations of spherical Rayleigh-Bénard problem under a central dielectrophoretic force in microgravity environment we scale length by outer sphere radius $r_o$, time by thermal diffusive timescale $\tau_{\text{th}} = r_o/\kappa$ and temperature by imposed temperature difference $\Delta T = T_o - T_i$. Then equations in their non-dimensional Boussinesq approximation become

$$\nabla \cdot U = 0,$$

$$Pr^{-1} \left[ \frac{\partial U}{\partial t} + (U \cdot \nabla)U \right] = -\nabla p + \nabla^2 U + \frac{Ra_{\text{centr}}}{\beta^2 r_o^5} \hat{e}_r + \sqrt{Ta} \hat{e}_z \times U + \tilde{Ra} T r \sin \theta \hat{e}_q,$$

$$\frac{\partial T}{\partial t} + U \cdot \nabla T = \nabla^2 T,$$

with no-slip boundary conditions for velocity $U(\eta) = U(1) = 0$ and temperature fixed by $T(\eta) = 1, T'(1) = 0$. Arising parameters considering different experimental aspects are radius ratio of inner and outer shell $\eta = r_i/r_o$ (considering geometry, with aspect ratio $\beta = 1/\eta - 1$) and Prandtl number $Pr = \nu/\kappa$ (considering physical properties of fluid such as viscosity and thermal diffusivity). Temperature difference determines Rayleigh number, i.e. buoyancy to central gravity with

$$Ra_{\text{centr}} = \frac{2\epsilon_0 \epsilon_r \gamma}{\rho \nu \kappa} V_{\text{rms}}^2 \Delta T.$$

Rotation rate gives Taylor number

$$Ta = \left( \frac{2\Omega r_o^2}{\nu} \right)^2.$$
$Ra = 2 \cdot 10^3$

basic state

$5 \cdot 10^3$

$1 \cdot 10^4$

← steady state →

$2 \cdot 10^4$

$5 \cdot 10^4$

irregular

**Figure 1.** Expected convective flows analog automatic experiment run during loop 1, i.e. increasing Rayleigh number, setting Taylor number to zero. Transition from basic via steady to time-dependent flow is observed. Temperature field is visualized over hemispherical shell, sphere is cut in azimuthal direction. Bright color denotes outward flow of hot fluid. Random isosurface is highlighted.

### 3. Numerical Results

#### 3.1. Non-rotating case

Increasing Rayleigh number corresponds to strength thermal impulse of the system. Transition from basic state to steady and further to irregular time-dependent convective states are found for simulation of selected parameter values for $Ra$ (Fig. 1).

While our direct numerical simulation shows axisymmetric solutions, [4] showed that $m \neq 0$ is the stable state. These convective states can be calculated by longterm simulations or by starting from zero (see section rotating case below), but can also be found by use of path following methods, presented by [5]. Here we realized numerical simulation lasting several multiples of thermal diffusive time $\tau_{th} = d^2/\kappa$. During experiment runs scientific wait-up time for stable states is set to one thermal diffusive time $\tau_{th}$. For prediction of experiment scenario that means, that just increasing temperature difference and waiting only one $\tau_{th}$ as a minimum requirement for expecting stable states, during this loop 1 only axisymmetric state is expected to be investigated.

#### 3.2. Rotating case

Second loop of experimental run starts with setting Rayleigh number, Taylor number is set to zero at the beginning. Solutions for this show, that starting with zero start solution (corresp. to increasing Rayleigh number from zero to chosen value), $m \neq 0$ mode is the stable state immediately (Fig. 2). Thermal impulse seems to be more effective than the small perturbations by increasing $Ra$ with small step size as in loop 1. So during loop 2 convective states as described in [4] are observable.

Regarding stability curves for $Ra = f(Ta)$, already shown also in [6], superposition of rotation corresponds to special traverse of these curves from irregular to steady convective states (Fig. 3, from left to right). Simulation in low and medium region of rotation confirm this behaviour (Fig. 4).

### 4. Conclusion

We presented numerical preliminary studies for microgravity experiment 'GeoFlow' on thermal convection in rotating spherical shells. Analog to an automatic experimental run simulations are done for non-rotating and rotating case. Rich dynamics of this spherical Rayleigh-Bénard problem under a central dielectrophoretic force in microgravity environment can be investigated.

During loop 1 (non-rotating case) transition from basic via steady to irregular flow is shown. Onset of time-dependence still has to be resolved with much more dense steps in Rayleigh
Figure 2. Convective states for Taylor number $Ta = 0$. Upper row shows axisymmetric results from reaching Rayleigh number analog to loop 1, thus increasing it stepwise. Lower row shows results with $m \neq 0$ for parameter set-up analog to loop 2, where $Ra$ is set, starting from zero. Temperature field visualized over sphere cut in radial direction, view from side. Bright color denotes outward flow of hot fluid.

number $Ra$. As well time-dependence has to be investigated with non-linear methods.

During loop 2 (rotating case) influence of initial conditions shows different stable states for Taylor number $Ta = 0$ as starting point of superposition of rotation. Traverse from irregular to regular flow can be visualized for increasing $Ta$.

Further parameter simulations has to be done to track also transition to turbulence.

Acknowledgment
This work was supported by the German Federal Ministry of Education and Research, through the German Aerospace Center e.V. (DLR), grant number 50 WM0122.

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Figure 3. Overview of convective states in spherical shell of aspect ratio $\eta = 0.5$ for $Pr = 64.64$: Visualized is $Ra$ versus $Ta$. Besides types of numerical solution diagram show linear stability (solid) and critical line (dashed), where time-dependent solution gets irregular. Additionally region of most stable mode $m$ is marked.

Figure 4. Convective states for Taylor number $Ta \neq 0$ for slightly supercritical Rayleigh number $Ra = 5 \cdot 10^3$. Top row shows temperature field visualized over sphere cut in radial direction, view from side. Bright color denotes outward flow of hot fluid. Bottom row shows temperature field in equatorial plane.