Local Interlayer Tunneling Between Two-Dimensional Electron Systems in the Ballistic Regime

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We study a theoretical model of virtual scanning tunneling microscopy (VSTM), a proposed application of interlayer tunneling in a bilayer system to locally probe two-dimensional electron systems (2DES) in semiconductor heterostructures. We focus on tunneling in the ballistic regime, where we show that the zero-bias anomaly is suppressed by extremely efficient screening. Since such an anomaly would complicate the interpretation of data from a VSTM, this result is encouraging for the ongoing experimental effort.

I. INTRODUCTION

The availability of increasingly clean low-density two-dimensional electronic systems (2DES) has allowed access to a regime in which electron-electron interactions play a major role. Evidence is accumulating from transport measurements that the physics of this regime is much richer than was previously appreciated (see Ref. 1 and references therein). In particular, while much is understood about the two limiting cases, $r_s \equiv 1/\sqrt{\pi a_B^2} \to 0$ (Fermi liquid) and $r_s \to \infty$ (Wigner crystal), experiments on systems with intermediate values of $r_s = 10 - 30$ reveal a host of unanticipated anomalies.2,3 (Here, $n$ is the real density of doped electrons or holes and $a_B$ is the effective Bohr radius, $a_B = \hbar^2/e^2/m^*\epsilon^2$, where $m^*$ is the effective mass, $e$ is the electron charge, and $\epsilon$ is the dielectric constant of the host semiconductor.)

Experiments on 2DESs have been based mainly on transport measurements on large (micron to millimeter scale) samples. Information on the local structure of electronic states could powerfully elucidate the physics underlying these transport measurements, possibly including the recently proposed “electronic micromulsion phases.”4,5 Momentum-space probes6 and finite-frequency probes7 have provided some important insights, but the residual spatial inhomogeneity in even the cleanest low-density 2DESs favors use of real-space probes. Over the past decade, important progress has been made in locally probing 2DESs, for example see Refs. 8,9,10,11,12,13. However, the challenge that a 2DES is generally buried of order 100 nanometers deep in a heterostructure prevents use of powerful conventional techniques, such as scanning tunneling microscopy (STM), and a comparable position-sensitive scanning probe that can map out the local density of states at low energy is still lacking for such buried structures. An ongoing effort to develop such a capability – termed virtual scanning tunneling microscopy or VSTM – is based on tunneling into a 2DES not from a scanned metal tip as in STM, but rather from a second “Probe” 2DES grown above the 2DES of interest (henceforth “Sample 2DES”), within the same heterostructure. Since the barrier between the two 2DESs can be made very low by proper design of the layer structure, and since the Probe 2DES is not perfectly compressible, the barrier can be tuned at a particular location by applying a voltage to a sharp metal tip positioned above the heterostructure surface (See Fig. 1). Tunneling between the Probe and Sample 2DESs is then strongly enhanced locally below the tip, and the location of enhanced tunneling can be scanned across the Sample 2DES by scanning the metal tip above the heterostructure.

In this paper, we introduce a simple model of the VSTM – two parallel 2DESs connected by tunneling at a single point – and address the feasibility of the VSTM at its simplest level. For its intended purpose, the VSTM should ideally meet the following criteria: (i) there should be sufficient tunneling near zero bias to allow probing the low energy physics of interest, (ii) the tunneling rate should be sensitive to the local density of states at the location of tunneling.

In the diffusive limit, the well known “zero-bias anomaly” occurs as a consequence of the inefficient screening of charge in 2D. Specifically, in 2D the conductivity has units of velocity and hence the Coulomb energy $E(t) \sim e^2/R(t)$ associated with adding a charge to a 2D system decays with time in proportion to the screening radius $R(t) \sim \sigma t$, leading to an action that logarithmically diverges at small bias for tunneling into such a system. The result is a strong suppression of the tunneling rate near zero-bias,14,15,16 violating criterion (i). Moreover, the tunneling rate has a dominant contribution from long-distance physics, violating criterion (ii).

The central result in the present paper is that in the clean limit, even in 2D, screening is sufficiently efficient that the tunneling action at zero-bias is finite and hence no zero-bias anomaly occurs. In this regime, the tun-
neling rate can be calculated perturbatively and is hence proportional to the local density of states. Our results indicate that using a VSTM to probe low energy local density of states should be feasible, if the 2DES of interest is clean enough.

The outline of this paper is as follows: In section II, we present the model, which treats the tunneling electron as a two state system and the remaining electrons in the 2DES that interact with the tunneling electron as the “bath” degree of freedom. In section III, we calculate the tunneling rate to lowest order in the tunneling matrix element. Finally, in section IV, we discuss the implications of our results.

II. THE MODEL

Our model consists of two 2DESs characterized by 2D Fermi liquids with electron densities \( \rho_1 \) and \( \rho_2 \), respectively, separated by a distance \( a \). A voltage bias \( V_{\text{bias}} \) is applied across a single tunneling center at the origin (see Fig. 2). We treat the tunneling electron as a two-state system and the remaining electrons as a “bath” degree of freedom. In section III, we calculate the tunneling rate to lowest order in the tunneling matrix element. Finally, in section IV, we discuss the implications of our results.

III. RESULTS

We consider a simple model that captures essential aspects of the VSTM setup sketched in Fig. 1. Our model

\[
S[\rho_1, \rho_2, \sigma_z] = S_\sigma + S_{\sigma,\rho}, \quad (2.1)
\]

\[
S_\sigma = \frac{1}{2} V_{\text{bias}} \sigma_z - \frac{1}{2} \hbar \Delta \sigma_x, \quad (2.2)
\]

\[
S_{\sigma,\rho} = \int \frac{d\omega}{(2\pi)} \int \frac{d^2q}{(2\pi)^2} \left[ \rho^1 \mathbf{K} \rho + \sigma^1 \mathbf{V} \rho \right], \quad (2.3)
\]

where \( \sigma = (1/2) \) \( [1 + \sigma_z(q, i\omega)], [1 - \sigma_z(q, i\omega)] \), \( \rho = < \rho_1(q, i\omega), \rho_2(q, i\omega) > \), and

\[
\mathbf{K} = \begin{pmatrix}
\chi_1^{-1}(q, i\omega) & V(q) \\
V(q)^\dagger & \chi_2^{-1}(q, i\omega)
\end{pmatrix}, \quad (2.4)
\]

\[
\mathbf{V} = \begin{pmatrix}
U_1(q) - V(q) & 0 \\
0 & U_2(q) - V(q)
\end{pmatrix}.
\]

Here \( S_\sigma \) is the bare tunneling action in the absence of any interactions, while \( S_{\sigma,\rho} \) is the action for the rest of the (“bath”) electrons, which we treat in the context of linear response theory: \( \chi_i \) denotes the density correlation function in each layer \( i \), \( V(q) \) denotes inter-layer interaction and \( V \) is the coupling between the tunneling electron and the “bath” electrons through the intra-layer (\( U(q) \)) and inter-layer (\( V(q) \)) Coulomb interaction. From here on, for simplicity, we restrict ourselves to the symmetric case \( \chi_1^{-1}(q, i\omega) = \chi_2^{-1}(q, i\omega) = \chi^{-1}(q, i\omega) \) and \( U_1(q) = U_2(q) = U(q) \), although the general case can be treated in an identical fashion.

Since Eq. (2.1) is quadratic, \( \rho_i \)'s can be readily integrated out to yield

\[
S[\sigma_z] = \int \frac{d\omega}{2\pi} \left[ \frac{1}{2} V_{\text{bias}} \sigma_z (i\omega) - \frac{1}{2} \Delta \sigma_x (i\omega) \right] + S_0, \quad (2.5)
\]

where

\[
S_0[\sigma_z] \equiv - \int \frac{d\omega d^2q}{(2\pi)^3} \frac{[(U(q) - V(q))^2 |\sigma_z (i\omega)|^2}{\chi^{-1}(q, i\omega) + V(q)} \cdot \quad (2.6)
\]

Here, the effects of correlations in the 2DESs are encoded in \( \chi(q, i\omega) \). We treat the correlation effects through RPA in the rest of this paper; however, the form of the action in Eqs. (2.5 and 2.6) is more general. (The semiclassical results of Levitov and Shytov\textsuperscript{15} can be reproduced in this formalism if \( \chi \) is taken to be the susceptibility of a diffusive 2DES.)
III. PERTURBATIVE CALCULATION OF THE TUNNELING RATE

A. Evaluation of the Action

We evaluate the action Eqs. (2.5-2.6) using the RPA expression for \( \chi \) in the clean limit:

\[
\chi(q, i\omega) \approx \frac{\chi^2_D(q, i\omega)}{1 - U(q) \chi^2_D(q, i\omega)},
\]

(3.1)

where the bare density correlator at zero temperature is

\[
\chi^2_D(q, i\omega) = -\nu_0 \left( 1 - \frac{|\omega|}{\sqrt{\omega^2 + (v_F q)^2}} \right).
\]

(3.2)

where the dimensionless parameter \( \alpha \equiv 2\pi a^2 \nu_0 \) and the UV frequency cutoff \( 1/\tau_0 = v_F/\alpha \). Applying the analysis of Levitov and Shytov\(^\text{15}\) to this effective action, it is easy to see that the accommodation distribution function, as a collection of Harmonic oscillators. The heat bath of phonons is typically defined in terms of a spectral distribution function, \( J(\omega) \), which is simply the Hilbert transform of the kernel \( \kappa(\omega) \) in Eq. (3.1):

\[
\kappa(\omega) = \frac{1}{\pi} \int_0^\infty d\omega' \frac{\omega'}{\omega^2 + \omega'^2} J(\omega').
\]

(3.4)

Noting that the addition of a frequency independent constant to \( \kappa(\omega) \) results only in an (unimportant) additive correction to the ground-state energy, we find that

\[
J(\omega) = A \omega^2 e^{-\omega/\Omega},
\]

(3.5)

where

\[
A = \frac{1}{4\pi \nu_0 v_F} \left( \frac{\alpha}{1 + \alpha} \right)^3 \Omega = \frac{v_F (1 + \alpha)}{\alpha}.
\]

(3.6)

The low frequency behavior \( J \sim \omega^2 \) is conventionally classified\(^\text{15}\) as “superOhmic” for \( x > 1 \) (the present case),

Here, \( \nu_0 = k_F/\pi v_F \) is the bare density of states per volume at the Fermi surface. Furthermore, we assume that the distance \( a \) between the two 2DESs sets the shortest length scale and hence serves as the UV momentum cutoff, and hence we can make use of the approximate expression \( U(q) - V(q) \approx 2\pi a e^2 + O \left( \left| q/a \right|^2 \right) \) to lowest order in \( |q/a| \). This makes it possible to explicitly perform the \( q \) integral in Eq. (2.6) to yield

\[
S_{\text{eff}}(\tau) = S_\sigma + \frac{1}{2} \int_{1/\tau}^{1/\tau_0} \frac{d\omega}{2\pi} |\sigma_z(\omega)|^2 \kappa(\omega), \quad \kappa(\omega) \equiv \frac{\alpha^2 \omega^2}{4\pi^2 \nu_0 v_F (1 + \alpha)} \left\{ \left( \frac{v_F}{a \omega} \right)^2 - 2 \alpha \log \left( \frac{v_F}{a \omega} \right) \right\},
\]

(3.3)

where perturbation theory is applicable, "Ohmic" for \( x = 1 \) (which is obtained in the diffusive case), and "sub-Ohmic" for \( x < 1 \), in which cases non-perturbative methods are necessary.

With the spectral function in hand, we can straightforwardly calculate the tunneling rate following the steps of Ref. 15, to second order in the tunneling matrix element to obtain

\[
\tau^{-1}(V_{\text{bias}}) = \frac{\pi^2 \tilde{\Delta}^2}{\Omega} \delta \left( \frac{V_{\text{bias}}}{\hbar \Omega} \right) + \frac{\pi \hbar \tilde{\Delta}^2}{\varepsilon} \sqrt{2} V_{\text{bias}} I_1 \left( \frac{V_{\text{bias}}}{\varepsilon} \right) e^{-V_{\text{bias}}/\hbar \Omega}.
\]

(3.7)

Here \( I_1(x) \) is a modified Bessel function of the first kind. \( \varepsilon \equiv \frac{\hbar^2}{2m} \propto v_F \) has the dimension of energy and \( A \) and \( \Omega \) are defined in Eq. (3.3). We note that the effect of Coulomb interaction enters the tunneling rate through the renormalized tunneling matrix element

\[
\tilde{\Delta} = \Delta \exp \left[ -\frac{\sqrt{2} r_s}{2} \left( \frac{1}{\frac{1}{2} \left( \frac{2}{\pi} \right)^{1/2} + 1} \right)^2 \right],
\]

(3.8)

where \( r_s = (1/n\pi)^{1/2} a_B^{-1} \) is the ratio of the Coulomb interaction energy to the kinetic energy, \( n \) is the electron density, and \( a_B \) is the Bohr radius. Tunneling is suppressed for lower density, i.e., for larger \( r_s \). The most notable feature of our results in Eq. (3.7) and Eq. (3.9) is the existence of an “elastic” term proportional to \( \delta (V_{\text{bias}}/\hbar \Omega) \), which dominates the tunneling rate in the \( V_{\text{bias}} \to 0 \) limit. This term is absent in the Ohmic and subOhmic cases, due to the vanishing overlap (infrared
catastrophe) between the $\sigma_z = \pm 1$ unperturbed ground states. This proves the existence of a finite tunneling amplitude at zero bias.

**IV. SUMMARY**

In general, the zero-bias anomaly in tunneling into 2DES’s reflects the qualitative effects of the Coulomb interactions on the tunneling process. While these effects are interesting in their own right, in the context of a VSTM they would represent a barrier to obtaining useful data. Through an explicit calculation, we have shown that in a ballistic system with efficient screening, the tunneling rate in the limit of zero bias is not entirely suppressed by Coulomb effects. This implies that the VSTM will indeed be capable of probing low energy physics of clean 2DESs through tunneling. We note that the main purpose of the current paper was a proof of principle, hence we limited ourselves to the simplest possible application of the VSTM. There are many other systems to which the VSTM might be applied, where other considerations may be necessary, including tunneling in a magnetic field and tunneling into a non-Fermi liquid. These issues will be subjects of future studies.

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