Sudakov expansions at one loop and beyond
for charged scalar and fermion pair production in SUSY models
at future Linear Colliders *

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* Partially supported by EU contract HPRN-CT-2000-00149
Abstract

We consider the high energy behaviour of the amplitudes for pair production of charged leptons, quarks, Higgs bosons, sleptons, squarks and charginos at lepton colliders. We give the general expressions of the leading quadratic and subleading linear logarithms that appear at the one loop level, and derive the corresponding resummed expansions to subleading logarithmic order accuracy. Under the assumption of a relatively light SUSY scenario and choosing the MSSM as a specific model, we compare the predictions of the one-loop and of the resummed expansions at variable energy. We show that the two predictions are very close in the one TeV regime, but drastically differ in the few (2, 3) TeV range.

PACS numbers: 12.15.-y, 12.15.Lk, 14.80.Ly, 14.80.Cp

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I. INTRODUCTION

It is by now well-known that the electroweak radiative corrections to Standard Model pair production processes increase strongly with the center of mass energy $\sqrt{s}$ at the one-loop level. This is due to the presence of large double and single logarithms $\simeq \frac{\alpha}{\pi} \log^2 \frac{s}{M_W^2}$, $\frac{\alpha}{\pi} \log \frac{s}{M_W^2}$, [1, 2, 3, 4, 5]. In the TeV range such terms reach the several percent level, which should be easily observable (and measurable) at future lepton colliders [6, 7]. Actually, for energies beyond the few TeV range, the numerical size of their effect begins to be too large [5] (beyond the relative ten percent level), and the validity of the simple one-loop approximation must be seriously questioned. This has led a number of theoretical groups to propose resummation prescriptions [8, 9, 10, 11]. Without entering the details of the different calculations, we shall accept the conclusion that, for massive final pairs, an agreement seems to exist that a full resummation can be given, but only to subleading logarithmic level.

In the case of supersymmetric extensions of the SM, similar studies have been recently performed for the processes of production of fermion pairs at the one-loop level [12] and of scalar pairs [13] in the MSSM. In ref. [13] both the one-loop approximation and the resummed expansions have been computed. Under the qualitative assumption of a relatively light SUSY scenario, with all the relevant masses of the process "adequately" smaller than e.g. one TeV, it has been shown that the two approximations are "essentially" (i.e. at the expected relative one percent level of experimental accuracy) identical in the $\simeq 1$ TeV region (final possible reach of the proposed LC [6]), but differ drastically in the few (2, 3) TeV range (aim of the proposed CLIC [7]). Thus, for the spin zero production case, to subleading logarithmic accuracy, a simple one-loop approximation would seem definitely inadequate at CLIC energies, but valid in the LC regime.

The aim of this paper is that of performing a general investigation, analogous to that already carried through for scalar production, to include also the processes of charged spin one-half (fermions, charginos) pair production. For all these cases we shall give both the one-loop level and the resummed logarithmic expansions to subleading logarithmic accuracy. Under the assumption of a relatively light SUSY scenario, we shall then compare the two approximations at variable c.m. energies for a number of experimental observables. We shall show that the same conclusions obtained for scalar production are apparently valid for the extended case, making the validity to subleading logarithmic order of a one-
loop approximation for spin zero and one-half charged pair production in the MSSM at future colliders to only depend on the chosen c.m. energy, and not on the specific process. Our analysis has been performed and our conclusions have been drawn in the particular case of the MSSM, but they could be easily adapted or modified to treat different supersymmetric models, although we believe that this is beyond the purposes of this first, necessarily preliminary work.

Technically speaking, the plan of this paper is the following: Section 2 contains the one-loop expansion of all the considered processes. The resummed expansions are shown in Section 3, and a comparison between the two approximations is given in Section 4. A final discussion and some possible conclusions are given in Section 5. The definitions of the observables for final fermionic or scalar pairs are given in Appendix A. Appendices B and C contain a summary of several long (but necessary) analytic formulae for the asymptotic expansions at the one loop level and with resummation.

II. ASYMPTOTIC LOGARITHMIC EXPANSION AT ONE-LOOP

A. Generalities

As illustrated and discussed in several previous papers [12, 13, 14], at the one-loop level, the logarithmic terms appearing in $e^+e^-$ annihilation processes can be separated into three categories, Renormalization Group (RG) terms, Universal terms and Non Universal (angular dependent) terms, so that the invariant scattering amplitude for the pair production process

$$e^+e^- \rightarrow f_1 f_2$$

($\alpha$ representing the electron chirality) can be written in the following form

$$A^{1 \text{ loop}} = A^{RG} + A^{univ} + A^{non \text{ univ}}$$

The RG contribution represents the linear logarithms [15] generated by the ”running” of the gauge coupling constants, that are known and calculable in a straightforward way. It is obtained by introducing in $A^{\text{Born}}$ the running $SU(2) \times U(1)$ couplings $g, g'$ according to the asymptotic MSSM $\tilde{\beta}_0, \tilde{\beta}'_0$ functions:

$$\tilde{\beta}_0 = \frac{3}{4} C_A - \frac{n_g}{2} - \frac{n_h}{8}, \quad \tilde{\beta}'_0 = -\frac{5}{6} n_g - \frac{n_h}{8}$$
\[ g^2(s) = \frac{g^2(\mu^2)}{1 + \beta_0 g^2(\mu^2) \log \frac{s}{\mu^2}} \quad \text{and} \quad g'^2(s) = \frac{g'^2(\mu^2)}{1 + \beta'_0 g'^2(\mu^2) \log \frac{s}{\mu^2}} \quad (2.4) \]

where \( C_A = 2 \), \( n_g = 3 \), \( n_h = 2 \) and \( g \sin \vartheta_w = g' \cos \vartheta_w = e \).

At one loop, this quite general procedure gives single logarithms that do not factorize, but can be obtained from \( A^{\text{Born}} \) as:

\[ A^{\text{RG}} = -\frac{1}{4\pi^2} \left( g^4 \beta_0 \frac{dA^{\text{Born}}}{dg^2} + g'^4 \beta'_0 \frac{dA^{\text{Born}}}{dg'^2} \right) \log \frac{s}{\mu^2} \quad (2.5) \]

where \( \mu \) is a reference scale defining the numerical values of \( g, g' \).

The universal contribution can be of quadratic and of linear kind and in a covariant gauge is produced by diagrams of vertex (initial and final triangles) and of box type; it only depends on the quantum numbers of the initial \( e^+, e^- \) and final \( f_1, f_2 \) lines and can be written as:

\[ A^{\text{univ}} = A^{\text{Born}} \cdot \left( c_{\alpha}^{\text{in}} + c_{\alpha}^{\text{fin}} \right) \quad (2.6) \]

with the correction to the initial \( e^+e^- \) lines

\[ c_{\alpha}^{\text{in}} = \frac{1}{16\pi^2} \left( g^2 I_{e_\alpha} (I_{e_\alpha} + 1) + g'^2 Y_{e_\alpha}^2 \right) \left( 2 \log \frac{s}{M_V^2} - \log^2 \frac{s}{M_V^2} \right) \quad (2.7) \]

where \( M_V \) is a common gauge boson mass (we use \( M_Z \approx M_W \) for simplicity and we have separated the infrared part of the photon contribution, keeping here only the ultraviolet one by putting \( M_\gamma = M_Z \)), \( \alpha \) is the chirality and \( I, Y \) the isospin and hypercharge of the initial electron \( e^- \). The expression for \( c_{\alpha}^{\text{fin}} \) will be given separately for each of the examples listed below.

The non universal (angular dependent) contribution can be written

\[ A^{\text{non univ}} = A^{\text{Born}} \cdot c^{\text{ang}} \quad (2.8) \]

It only consists in residual terms arising from the quadratic logarithms \( \log^2 t, \log^2 u \) (from which the \( \log^2 s \) part has been subtracted and put in the universal contribution) generated by box diagrams containing \( W, Z, \gamma \) gauge boson internal lines where \( t = -\frac{\mu}{2} (1 - \cos \vartheta) \) and \( u = -\frac{\mu}{2} (1 + \cos \vartheta) \), \( \vartheta \) being the scattering angle. There are only few such diagrams.
and they have been all explicitly computed.

To subleading logarithmic accuracy, these three types of contributions have been calculated exactly. We shall now summarize, trying to make the review as short as possible but reasonably complete and self-contained, the results of our effort for the charged final pairs of the MSSM, including those for scalar production and fermion production that were already given in other papers. The list of considered final states is now given in the following subsections.

B. Chiral Lepton or Quark Pair $\overline{f}_\beta f_\beta$

The Born amplitude is given in Appendix B1, using the notations given in Appendix A; for more details see previous papers [12]. For a final fermion pair with chirality $\beta$, the logarithmic part of the one loop amplitude (2.2) is obtained by adding eq. (2.5), (2.6), (2.8) with:

$$c_{\text{fin}}^\beta = c_{\text{fin gauge}}^\beta + c_{\text{fin Yukawa}}^\beta$$  

$$c_{\text{fin gauge}}^\beta = \frac{1}{16\pi^2} \left( g^2 I_\beta (I_\beta + 1) + g'^2 \frac{Y_\beta^2}{M_V^2} \right) \left( 2 \log \frac{s}{M_V^2} - \log \frac{s}{M_V^2} \right)$$  

$$c_{\text{fin Yukawa}}^\beta = -\frac{g^2}{16\pi^2} \left( \frac{1 + \delta_{\beta,R}}{2} \frac{\hat{m}_f^2}{M_V^2} + \frac{\delta_{\beta,L}}{2M_V^2} \right) \log \frac{s}{M_S^2}$$

and

$$c_{\text{ang}}^\alpha = -\frac{g^2}{16\pi^2} \log \frac{s}{M^2} \left[ \left( \tan^2 \vartheta_w Y_{e\alpha} Y_{f\beta} + 4 I_{e\alpha}^3 I_{f\beta}^3 \right) \log \frac{t}{u} \right. \left. + \frac{\delta_{\alpha,L} \delta_{\beta,L}}{\tan^2 \vartheta_w Y_{e\alpha} Y_{f\beta} + 4 I_{e\alpha}^3 I_{f\beta}^3} \left( \delta_{d,f} \log \frac{-t}{s} - \delta_{u,f} \log \frac{-u}{s} \right) \right]$$

We denote $\hat{m}_f = m_f / \sin \beta$ if $f = t$ and $\hat{m}_f = m_b / \cos \beta$ if $f = b$; $f'$ denotes the corresponding isopartner of $f$, and $\beta$ (not to be confused with the chirality index) is the mixing angle between the vacuum expectation values of the up and down Higgs chiral superfield (in standard notation $\tan \beta = v_u / v_d$).

For particles other than those belonging to the third family of quarks/squarks, the Yukawa terms are negligible.
C. Slepton or Squark Pair $\tilde{f}_\beta \tilde{f}_\beta$

The results for this case are quite similar to those valid for fermion production, see Appendix B2 and ref. [13]. The logarithmic part of the one loop contribution for the process

$$e_\alpha^+ e_\alpha^- \rightarrow \tilde{f}_\beta \tilde{f}_\beta$$  \hspace{1cm} (2.13)

($\beta = L, R$) can be written in the same form as for $e_\alpha^+ e_\alpha^- \rightarrow \tilde{f}_\beta \tilde{f}_\beta$ producing a fermion pair with chirality $\beta$, with the same expressions for $c^\text{fin}_{\alpha}, c^\text{fin gauge}_{\beta}, c^\text{fin Yukawa}_{\beta}$ and $c^\text{ang}_{\alpha\beta}$.

D. Charged Higgs Bosons $H^\pm$ or charged Goldstones $G^\pm$

In the MSSM the charged Higgs bosons and Goldstones are produced in pairs through the same diagrams that appear in the case of sfermion production. At $M_W^2/s$ accuracy the amplitudes for $G^+G^-$ production are equivalent to the physical amplitudes for longitudinal $W^+_L W^-_L$ states. The previous equations, (2.10-2.12) which concern the gauge parts give the correct result for the process

$$e_\alpha^+ e_\alpha^- \rightarrow H^+H^-$$  \hspace{1cm} (2.14)

provided that $f_\beta$ is replaced by $H^-$ with the following quantum numbers

$$Q(H^-) = -1, \quad I^3(H^-) = -\frac{1}{2}$$  \hspace{1cm} (2.15)

For what concerns the Yukawa part, one has

$$c^\text{fin Yukawa}_{\text{Yukawa}} = -\frac{3}{32\pi^2} g^2 \left( \frac{m^2 t \cot^2 \beta}{M^2_W} + \frac{m^2 b \tan^2 \beta}{M^2_W} \right) \log \frac{s}{M^2_S}$$  \hspace{1cm} (2.16)

These formulae then apply also to

$$e_\alpha^+ e_\alpha^- \rightarrow G^+G^-$$  \hspace{1cm} (2.17)

without any change concerning the Born term and the one loop gauge terms, but the Yukawa part has to be simply modified with $m^2 b \tan^2 \beta \rightarrow m^2_t$ and $m^2 t \cot^2 \beta \rightarrow m^2_b$.

E. Charginos $\chi_\pm_i \chi_j$?

Charginos are mixtures of gaugino(Wino) and Higgsino components selected respectively by the mixing matrix elements $Z^\pm_{1i}$, $Z^\pm_{2i}$ using the notations of ref. [16]. The expressions of
the Born terms and the one-loop contributions written in the Appendix B3 reflect clearly the properties of these two types of components.

The Born amplitude involves both Wino components \( (Z_{1k}^\pm Z_{1j}^\pm) \) and Higgsino components \( (Z_{2i}^\pm Z_{2j}^\pm) \) produced in s-channel through \( \gamma, Z \) formation, but it only involves Wino components produced through sneutrino exchange in the u-channel (the contribution of the Higgsino component vanishes like the electron mass).

The RG amplitude is computed using Eq. (2.5). We have written separately its Higgsino part. Its Wino part is regrouped with the other one loop Wino contributions because of remarkable cancellation properties explained below.

The universal terms \( c^{\text{fin}} \) generated by the \( \chi_i^\pm \chi_j^\mp \) lines contain Higgsino and Wino components that factorize the corresponding Born term components. We have computed them both through one loop diagrams in the ’t Hooft-Feynman gauge and through the splitting function formalism [17] with addition of Parameter Renormalization terms obtaining an agreement that we consider, given the not simple structure of the related formulae, encouraging. We observed the following properties. The s-channel terms contain universal corrections from both Higgsino and Wino components; their Wino part can be identified with the sum of a \( \chi^\pm \) splitting function contribution and of a Wino Parameter Renormalization term (the RG Wino part computed through eq. (2.5), see details in Appendix B3); its Higgsino part contains both a ”gauge” and a ”Yukawa” part. The u-channel terms only contain a Wino part also identifiable with the sum of a \( \chi^\pm \) splitting function contribution and of a Wino Parameter Renormalization term. These properties of the Wino and of the Higgsino parts are similar to those observed in the cases of \( W^+W^- \) and \( H^+H^- \) production, respectively; in particular at one loop we get only a DL term (the SL terms cancel) for the universal ”gauge” parts both in \( W^+W^- \) and in Wino pair production.

The non universal (angular dependent) terms \( c^{\text{ang}} \) arise from residual terms of \( \gamma\gamma, \gamma Z, ZZ \) and \( WW \) boxes in the s channel and of single \( \gamma, Z, \) and \( W \) boxes in the sneutrino exchange channel leading to both \( 1/u \) and \( 1/t \) terms (see Appendix B3).

A point that must be noticed is the fact that an important difference exists between the chargino pair production and the previous considered processes. This is due to the fact that, in the chargino case, the Born approximation already contains typical SUSY parameters in the mixing coefficients. Being by definition ”bare” quantities, a suitable extra renormalization is required. This is a well-known fact, already discussed in previous papers.
(see e.g. \[18\]), and the choice of a convenient renormalization scheme is essential. In our case, at the chosen level of logarithmic accuracy, we can neglect this complication since the difference between the bare parameters and the physical ones will be in any case a constant term of order $\alpha$, and will not find place in our expansion (it should be, though, properly retained in a more complete next-to next to leading order- analysis). This means that in our one-loop expansions one can systematically assume, for the values of these parameters, those of the corresponding physical ones in the suitable renormalization scheme that has been adopted (in our case, we are using systematically the minimal reduction scheme).

### III. RESUMMATION OF SUBLEADING LOGARITHMS IN GENERAL SUSY-PROCESSES

The size of the one loop SUSY Sudakov corrections, investigated in Refs.\[12, 13, 14, 19\], indicates that at TeV energies one must in principle also include the higher order contributions as mentioned in the introduction. While for SM processes a lot of attention has been devoted to this particular problem in Refs. \[3, 4, 9, 10, 12, 20, 21, 22, 23, 24, 25, 26, 27\], for SUSY Sudakov logarithms only higher order corrections to scalar production in $e^+e^-$ collisions are known \[13\].

We therefore discuss the situation in the MSSM for final fermion production in some more detail in order to clarify the arguments. As in Ref. \[13\] we assume a “light mass” SUSY scenario (with the SUSY mass scale $m_s \sim m_t \sim m_H \equiv M$ to logarithmic accuracy) for all particles involved in the loop corrections.

For what concerns the DL and angular dependent logarithmic corrections, since they are only mediated by the exchange of SM gauge bosons, their treatment will be identical with that already given in the Standard Model case \[9, 28, 29\], so that their exponentiation will be granted at the same subleading logarithmic order accuracy, and does not need to be rediscussed here.

For the subleading Sudakov corrections of the universal, i.e. process independent type, we have now novel contributions in the MSSM, both from particles with electroweak gauge couplings and from particles with coupling of Yukawa type. Since the former where already
discussed in Ref. \[13\], we will only discuss here the novel Yukawa type Sudakov terms for third family quarks which are of particular interest since they contain a strong \(\tan \beta\) dependence. For the purposes of this paper we shall only consider the asymptotic corrections above the scale of electroweak symmetry breaking since the mass-gap contributions originate only from QED and are therefore in principle known \[9, 11\]. For realistic collider simulations they must, though, carefully be included via matching at the weak scale.

In order to establish the “exponentiation” of the one loop Yukawa corrections, i.e. the fact that we only need to consider the situation depicted in Fig. (1), we need to show that the diagrams of the type shown in Fig. (2) cancel each-other.

As we argued in Ref. \[11, 13, 30, 31, 32\] using the symmetric basis, this feature is ensured by the gauge invariance of the Yukawa sector. In order to demonstrate the cancellation in the physical basis let us first consider the case of right handed external top quarks.

In the following we only need to consider soft gauge boson insertions (with loop momentum \(l\)) since we want to generate three large logarithms at the two loop level. This means that the \(l\)-dependence of the loops with Yukawa couplings can be neglected to SL accuracy since the on-shell self energy and vertex diagrams don’t produce an infrared-type contribution in the limit \(l \to 0, k \to 0, p_i^2 = m_i^2\). Note that to logarithmic accuracy we can set \(p_i^2 = m_s^2\) since all \(m_i \leq m_s \sim M\) and since, for now, we consider only the case of a heavy photon \((\lambda = M)\). The QED type corrections are included via matching \[9, 11\] as indicated above. We thus need to show that the UV-logarithms from the sum of the self energy diagrams cancel precisely the UV-logarithms in the sum of the vertex corrections.

The generic one-loop diagrams (modulo the couplings and with common mass scale \(m_s \sim m_t \sim M\), corresponding to the inner loop insertions depicted in Fig. (2), are given by:

\[
S^\Sigma \equiv \int \frac{d^n k}{(2\pi)^n} \frac{\not k}{(k^2 - m_s^2)((k - p_1)^2 - m_s^2)}
\]

\[
= \int \frac{d^n k}{(2\pi)^n} \frac{\not (k + p_1)}{(k^2 - m_s^2)((k + p_1)^2 - m_s^2)}
\]

for the self energy insertion and for the vertex diagrams we have for zero momentum transfer (“soft gauge bosons”, i.e. \(l = 0\)):  

\[
S^1_{\Lambda\mu} \equiv \int \frac{d^n k}{(2\pi)^n} \frac{\not k (2p_1 - 2k)\mu}{(k^2 - m_s^2)((k - p_1)^2 - m_s^2)^2}
\]

\[
S^2_{\Lambda\mu} \equiv \int \frac{d^n k}{(2\pi)^n} \frac{\not (p_1 + k)\gamma\mu (p_1 + \not k)}{(k^2 - m_s^2)((k + p_1)^2 - m_s^2)^2}
\]
Now it is straightforward to obtain from Eq. (3.1):

\[ \frac{\partial S_\Sigma}{\partial p_{1\mu}} = -S_{\Lambda_{1\mu}} \]  

(3.5)

and analogously from Eq. (3.2):

\[ \frac{\partial S_\Sigma}{\partial p_{1\mu}} = -S_{\Lambda_{2\mu}} \]  

(3.6)

Thus, for identical Yukawa couplings, we have to show that the sum of the couplings to the various contributing diagrams, multiplying the same UV-divergence, cancel each other!

Indeed this is what happens when one makes on the one hand the sum of the two loop diagrams with the vertex contributions for the Yukawa terms (diagrams on the left of Fig. (2)) and one the other hand the sum of the two loop diagrams with the self energy insertions (diagrams on the right of Fig. (2)). This is easy to check in the case of external right handed top quarks with the exchange of a photon and a Z-boson. For each type of (scalar, fermion) virtual contribution like \((G^\pm, b), \ (H^\pm, b), \ (\tilde{b}, \chi^\pm), \ (G^0, t), \ ... \ (\tilde{t}, \chi^0)\) one sees that the same contribution \(e^2 Q_t^2/c_w^2\) due to the sum of the photon and of the Z-boson exchanges factorizes the Yukawa couplings in the case of the self energy insertions and in the sum of the two types of vertex contributions. In the left handed top quark case, an analogous result is obtained by considering all the left handed diagrams of the process.

As mentioned above, this cancellation is a consequence of the fact that in spontaneously broken gauge theories also the Yukawa sector is gauged and that softly broken supersymmetry preserves the gauge structure. We can therefore employ the non-Abelian generalization of the Gribov theorem \[33\] (in the context of the infrared evolution equation method \[9, 34\]) as indicated in Fig. (1). As a result one obtains as a solution of the evolution equation, for all orders, to SL accuracy, the exponential of the universal (DL and SL) and of the non universal (angular dependent) SL one loop contributions listed in Section II. In addition, there appears a contribution arising from the implementation of the RG effect in the couplings of the exchanged gauge bosons. This is explicitly discussed in ref. \[11\] and the result was already reproduced in eq.(3.6) and (3.9) of ref. \[13\]. Using the running expressions, eq.(2.34), of the gauge couplings and expanding to subleading accuracy the rather involved combinations appearing in the equations of ref. \[11, 13\], one obtains the additional \(\text{log}^3\) term, with the coefficients \(\tilde{b}\), in the following equation.
The coefficients $b$ are extracted from the coefficients $c$ in $c_{\text{fin}}, c_{\text{Yukawa}}, c_{\text{ang}}$ in Sect.II:

\[
\begin{align*}
\bar{b}_a &= \frac{g^4 I_a (I_a + 1) \tilde{\beta}_0}{64\pi^4} + \frac{g'^4 Y_a^2 \tilde{\beta}_0'}{256\pi^4} \quad a = \alpha, \beta \\
\end{align*}
\]

The coefficients $b$ are extracted from the coefficients $c^{\text{in}}, c^{\text{fin}}, c_{\text{Yukawa}}, c_{\text{ang}}$ in Sect.II:

\[
\begin{align*}
\bar{b}_a &= \frac{1}{16\pi^2} \left( g^2 I_a (I_a + 1) + g'^2 Y_a^2 \frac{1}{4} \right) \quad a = \alpha, \beta \\
b_{Yuk}^\beta &= -\frac{g^2}{16\pi^2} \left( \frac{1 + \delta_{\beta,R}}{2} \frac{\hat{m}_f^2}{M_V^2} + \delta_{\beta,L} \frac{\hat{m}_{f'}^2}{2M_V^2} \right) \\
\end{align*}
\]

and

\[
\begin{align*}
b_{\text{ang}}^{\alpha\beta} &= \frac{-g^2}{16\pi^2} \left[ \left( \frac{\tan^2 \vartheta \varphi_{\alpha\beta} Y_{\alpha\beta} + 4 I_{\alpha\beta}^2 I_{\beta\beta}^3}{4} \right) \log \frac{t}{u} \\
&+ \frac{\delta_{\alpha,L} \delta_{\beta,L}}{\tan^2 \vartheta \varphi_{\alpha\beta} Y_{\alpha\beta} / 4 + I_{\alpha\beta}^2 I_{\beta\beta}^3} \left( \delta_{d,f} \log \frac{-t}{s} - \delta_{u,f} \log \frac{-u}{s} \right) \right] \right] \\
\end{align*}
\]

where $I_a$ denotes the total weak isospin of the particle with chirality $a$, $Y_a$ its weak hypercharge, $\hat{m}_f^2 = \text{and } \hat{m}_{f'}^2 =$. It should be noted that the one-loop RG corrections do not exponentiate and are omitted in the above expressions. They are, however, completely determined by the renormalization group in softly broken supersymmetric theories such as the MSSM and sub-subleading at the higher than one loop order. The one loop value is the already mentioned RG term Eq. (2.5).

In Eq. (3.7) we use the SUSY-mass scale $m_s$ in the Yukawa and SL-RG terms, but not in the gauge terms. Using the wrong scale in the DL-type corrections would unavoidably lead to wrong SL contributions.

The already existing explicit results for sfermion production can be obtained in the straightforward way by using the corresponding expressions of the various coefficients $b$.

In order to generalize the above results to arbitrary “light” SUSY processes it is convenient to work in the symmetric basis, i.e. in terms of the symmetry eigenstates. This is particularly important for the chargino production that we discussed at one loop in the previous section. In the general case let us denote physical particles (fields) by $f$ and particles (fields) of the unbroken theory by $u$. Let the connection between them be denoted by
\[ f = \sum_u C^{f\mu \nu} u, \] where the sum is performed over appropriate particles (fields) of the unbroken theory. Note that, in general, physical particles, having definite masses, don’t belong to irreducible representations of the symmetry group of the unbroken theory (for example, the photon and Z bosons have no definite isospin). On the other hand, particles of the unbroken theory, belonging to irreducible representations of the gauge group, have no definite masses. Then for the amplitude \( M^{f_1 \cdots f_n} (\{p_k\}, \{m_i\}; M, \lambda) \) with \( n \) physical particles \( f_i \) with momenta \( p_i \) and infrared cut-off \( \lambda \), the general case for virtual corrections is given by

\[ M^{f_1 \cdots f_n} (\{p_k\}, \{m_i\}; M, \lambda) = \sum_{u_1 \cdots u_n} \prod_{j=1}^n C^{f_j \mu_j} M^{u_1 \cdots u_n} (\{p_k\}, \{m_i\}; M, \lambda) \quad (3.12) \]

In the following we give only the corrections for a light SUSY mass scale \( m_s \sim M \) and for a heavy photon (\( \lambda = M \)) with all \( |2p_i p_j| \gg m_s^2, M^2 \). In this case, we can easily work in the symmetric basis and give the results for these amplitudes. As discussed above and described in detail in Refs. [9, 11, 13, 31], the soft virtual and real QED corrections must be added by matching at the weak scale \( M \). It should be mentioned, however, that the Yukawa terms are independent of the matching terms and that the Ward identities of the type (3.5) and (3.6) now apply to the amplitudes \( M^{u_1 \cdots u_n} (\{p_k\}; M) \).

Under these assumptions, we have for general on-shell matrix elements with \( n \)-arbitrary external lines the following resummed SL corrections in the symmetric basis:

\[ M_{\text{SL}}^{u_1 \cdots u_n} (\{p_k\}; m_s; M) = \exp \left\{ \sum_{k=1}^n -\frac{1}{2} \left( b_k (\log^2 \frac{s}{M^2} - 2 \log \frac{s}{M^2}) \right)_{i_k \neq \{W, \tilde{W}, B, \tilde{B}\}} - \frac{1}{2} (\bar{b}_k \log^2 \frac{s}{M^2})_{i_k = \{W, \tilde{W}, B, \tilde{B}\}} + \frac{1}{6} (\bar{b}_k \log^3 \frac{s}{m_s^2}) + b_k^{PR} \log \frac{s}{m_s^2} |_{i_k = \{W, \tilde{W}, B, \tilde{B}\}} \right\} M_{\text{Born}}^{u_1 \cdots u_n} (\{p_k\}) \quad (3.13) \]

with \( b_k, \bar{b}_k, b_k^{Yuk} \) defined in eqs. (3.8-3.10) and

\[ b_k^{PR} = \frac{g^2 \beta_0}{8 \pi^2} (\delta_{i_k, B} + \delta_{i_k, \tilde{B}}) + \frac{g^2 \tilde{\beta}_0}{8 \pi^2} (\delta_{i_k, W} + \delta_{i_k, \tilde{W}}) \quad (3.14) \]

\[ b_k^{\text{ang}} = \frac{1}{8 \pi^2} \sum_{l < k} \sum_{Y_{a,B,W}} \tilde{I}^{\nu_a}_{i_k, i_l} \tilde{I}^{\nu_a}_{i_k, i_l} \log \frac{2p_i p_k}{s} \quad (3.15) \]

The fields \( u \) have a well defined isospin, but for angular dependent terms involving CKM mixing effects, one has to include the extended isospin mixing appropriately in the corresponding couplings \( \tilde{I}^{\nu}_{i_k, i_l} \) of the symmetric basis. If some of the sparticles should be heavy, additional corrections of the form \( \log^2 \frac{m_s^2}{M^2} \) etc. would be important.
The result in Eq. (3.13), is valid for arbitrary softly broken supersymmetric extensions of the SM with the appropriate changes in the $\beta$-functions. Taking the SUSY-QCD limit
\[ \frac{g^2}{4\pi^2} \to \alpha_s, g' \to 0, I_g(I_g + 1) = I_{\bar{g}}(I_{\bar{g}} + 1) \to C_A = 3, I_q(I_q + 1) = I_{\bar{q}}(I_{\bar{q}} + 1) \to C_F = \frac{4}{3}, n_h = 0, M = \lambda_g = m_{\tilde{g}}, C_{i_h}^{\gamma\nu} = 0 \]
of the various terms, Eq. (3.13) is also valid for the virtual SUSY-QCD results. It should be emphasized, however, that in this case the virtual corrections are not physical in the sense that the gluon mass is zero and thus we would need to add the virtual matching and real contributions before we could make predictions for collider experiments, while in the SM soft QED energy cuts can define an observable and the heavy gauge boson masses are physical. In any case, the form of the operator exponentiation in color space agrees with the dimensionally regularized terms in Ref. [35] for non-SUSY QCD.

The general result in Eq. (3.13) agrees on the cross section level with the specific cross section expressions for fermion in Eqs. (3.7) and similar ones for sfermions and charged Higgses, as well as with the one loop results for chargino production presented in the previous section. The appropriate mixing matrices $C^{f_u}$ and RG-terms must be included for this comparison at one loop. They are sub-subleading at higher orders.

We have now at our disposal the sub-leading expressions for all the considered final states, both at the one-loop level and completely resummed. Our next goal is that of comparing the two approximations at variable energy and verify whether and where they can be considered as "essentially" (i.e. at the expected one percent experimental accuracy level) equivalent or, in the same spirit, "drastically" different. This will be done in the forthcoming Section 4.

IV. COMPARISON OF THE TWO APPROXIMATIONS AT SUBLEADING ORDER ACCURACY

In this Section we evaluate numerically the basic observables (cross section, forward-backward and left-right asymmetries), whose expression is given in Appendix A, when the final state is $t\bar{t}, b\bar{b}$ and when it is a pair of charged "genuinely" supersymmetric partners, i.e., sfermions, charged Higgs and charginos. For simplicity, we do not report the analysis for production of leptons or light quarks. In these cases, the previous analytical expressions apply, with the simplification that the Yukawa terms can be neglected. The numerical
results are comparable with the cases considered here. We compute the full effect in the
observables in two approximate asymptotic approaches. First, we consider the complete
set of one loop Sudakov contributions. As we have explained, these are terms growing
like $\log^2 \frac{s}{m_W}$ or $\log \frac{s}{m_W}$. Then, we compare this result with the one that is obtained by
resumming to all orders at subleading logarithmic accuracy. We remind that this corresponds
to using an expression that predicts rigorously all the terms of the form $\alpha s \log^{2L} (s/M_W^2)$
and $\alpha s \log^{2L-1} (s/M_W^2)$.

The deviation on the total cross section, the forward-backward asymmetry and the left-
right asymmetry are defined as

$$\Delta \sigma_{\text{tot}} = \frac{\sigma_{\text{tot}}}{\sigma_{\text{Born}}} - 1 \quad (4.1)$$
$$\Delta A_{\text{FB}} = \frac{\sigma_{\text{FB}}}{\sigma_{\text{tot}}} - \frac{\sigma_{\text{FB}}^{\text{Born}}}{\sigma_{\text{tot}}^{\text{Born}}} \quad (4.2)$$
$$\Delta A_{\text{LR}} = \frac{\sigma_{\text{LR}}}{\sigma_{\text{tot}}} - \frac{\sigma_{\text{LR}}^{\text{Born}}}{\sigma_{\text{tot}}^{\text{Born}}} \quad (4.3)$$

where $\sigma_{\text{tot}}, A_{\text{FB}}, A_{\text{LR}}$ are the radiatively corrected observables. If we consider for instance
the total cross section, in the perturbative one-loop scheme we have

$$\sigma_{\text{tot}} = \sigma_{\text{tot}}^{\text{Born}} + \sigma_{\text{tot}}^{\text{one loop}} \quad (4.4)$$

and in the resummation scheme we must use the expressions that we have described in the
previous Section.

For each final state and for both the one-loop scheme and the resummed scheme we
consider two values of the important parameter $\tan \beta$ that we choose to be $\tan \beta = 10$ and
$\tan \beta = 40$. This will allow a discussion of the role of the phenomenologically important
Yukawa terms and of the validity of the two approaches.

The one-loop Sudakov terms are quadratic and linear (and the resummed expressions are
based on them). At subleading accuracy we need not specify the scale of the linear logs. The
scale of the quadratic ones is conversely important at this level of accuracy. In conclusion,
the choice of scales can be numerically relevant and we explain how we fixed it.

**quadratic logarithms**: these are terms of purely gauge origin and their scale is pre-
dicted unambiguously by expanding the full one loop calculation. It is $M_W$ or $M_Z$
or the infrared photon mass regulator $M_\gamma$ according to which of the various diagrams
originating them is considered. For the aim of our discussion (mainly the comparison with the resummation approach) we have used $M_W$ (called $M_V$ in the Appendices) in all such terms.

**linear logs of gauge origin:** these are the logarithms that combine with the quadratic ones in order to reconstruct the $2 \log - \log^2$ combination. Here, we take $M_V = M_W$ for the same previous reasons.

**non universal linear logs:** these are single logarithms multiplying non-trivial functions of the scattering angle. Again, these are originated by diagrams where the correct scale can be taken to be $M_V = M_W$.

**linear RG logs:** here $M_V = M_W$.

**SUSY logs:** these are single logarithms of Yukawa type originated by diagrams with exchange of SUSY partners. We use a common scale $M_{\text{SUSY}}$ fixed at $M_{\text{SUSY}} = 300$ GeV. Here, the choice of $M_{\text{SUSY}}$ is rather arbitrary, the difference being a sub-subleading term constant with respect to the energy. Our choice is motivated by the recent investigation [36] where it is shown that this value is a typical one for scenarios where the Sudakov expansion can represent accurately the MSSM effects at energies above 1 TeV.

After these preliminary remarks, we turn to the discussion of the numerical results and in particular, of their reliability. With this purpose, we try to sketch a preliminary qualitative evaluation of the expectable accuracy of the resummed expressions. For simplicity, let us consider just the gauge logarithms, that is the combination $2 \log - \log^2$. In the one loop approximation, the correction is of the form:

$$\text{relative effect at one loop} = 1 + \delta_{\text{one loop}}, \quad \delta_{\text{one loop}} = c_{1L} \cdot \alpha (2 \log \frac{s}{M^2_W} - \log^2 \frac{s}{M^2_W}), \quad (4.5)$$

where $c_{1L}$ is a numerical constant. Exponentiating this term would produce, at two loops, the extra correction $\frac{1}{2} \delta_{\text{one loop}}^2$. Since the resummation is able to identify correctly only the leading and subleading logs at all orders, but not the sub-subleading terms, we would have for the complete two-loop correction an expression of the kind:

$$\text{real relative resummed effect} = 1 + \delta_{\text{one loop}} + \frac{1}{2} \delta_{\text{one loop}}^2 \pm c_{R} \alpha^2 \log^2 \frac{s}{M^2_W} + \cdots \quad (4.6)$$
where we have used in the theoretical error the smallest scale that we have. The dominant theoretical uncertainty of the resummation procedure can then be estimated at 3 TeV as

$$\pm c_R^2 c_{\text{one loop}}^2 \log^2 \frac{s}{M_W^2} = \pm \frac{c_R^2}{c_{\text{1L}}^2} \delta_{\text{one loop}}^2 \cdot \frac{\log^2 \frac{s}{M_W^2}}{(2 \log \frac{s}{M_W^2} - \log^2 \frac{s}{M_W^2})^2} \simeq \pm \frac{1}{28} \frac{c_R^2}{c_{\text{1L}}^2} \delta_{\text{one loop}}^2$$  \hspace{1cm} (4.7)

Hence, the dominant theoretical uncertainty of the resummation expressions (at subleading accuracy) would be below 1% if the one loop effect were below 50%, 25% or 10% for $c_R/c_{\text{1L}} = 1.1, 2.1, 5.3$ respectively. Since potentially large contributions to $c_R$ can in principle appear due to terms that are already present at one loop, (e.g. relatively large non logarithmic - constant - terms like those found in [31] for charged Higgs production) and since the above analysis is admittedly naive, our conservative attitude would be for the moment that of not considering the resummation expressions as the final word for a high accuracy prediction when the one loop effect is beyond, say, 10%, in which case we feel that a (tough!) partial two-loop calculation would be highly desirable.

In the remaining part of this Section, for each final states, we shall plot the effects in the two asymptotic approximate approaches as function of the energy for two values of $\tan \beta = 10, 40$. We also collect in Tab. (I) the results for $\tan \beta = 10$ at 1 and 3 TeV. The list of the considered case is now following.

**final top and bottom**

The effects are shown in Figs. (3,4). At 1 TeV, the one-loop effects in $\sigma_t$ and $\sigma_b$ are still, essentially, within the assumed limit of 10%, and their differences with respect to the resummed expansions are of approximately one percent. Similar conclusions apply for the two considered asymmetries. To the subleading logarithmic accuracy, a one-loop calculation seems therefore sufficient for all the considered observables at that energy, i.e. it does not “practically” change after (the corresponding) resummation. At 3 TeV, the one-loop effect on $\sigma_t$ is definitely beyond the ten percent value, the difference with respect to the resummed expressions reaches a ten percent size (note the reduction of the size of the resummed effect). Also the two asymmetries show visible differences between the two approaches (and $A_{LR}$ is particularly large at one-loop) at that energy, as one sees from the Figure. The situation appears definitely better for bottom production, where both the one-loop effects and the differences with respect to the resummed expressions remain reasonably small.
(also, it is conceivable that in this case the available experimental accuracy is worse than in the top case).

**final sfermions and charged Higgs**

The effects are shown in Figs. (5-9). At 1 TeV all the observables are "under control" with quite small correction from the resummation procedure. At 3 TeV, there are problematic large corrections to \( \sigma_{\tilde{t}_L}, A_{FB,\tilde{t}_L}, \sigma_{\tilde{b}_L}, A_{FB,\tilde{b}_L}, \sigma_{H}, A_{FB,H} \). In particular, for the cross sections \( \sigma_{\tilde{t}_L}, \sigma_{\tilde{t}_R}, \sigma_{\tilde{b}_L}, \) and \( \sigma_H \) the effects are around 20% and raise, in our opinion, serious doubts about the reliability of the resummation at this level of accuracy.

**final charginos**

If the final state is a pair of chargino and antichargino, we can in principle consider three separate cases according to which charginos are produced. In other words we consider the process

\[
e^+e^- \rightarrow \chi^+_a \chi^-_b
\]

in the three cases \((a,b) = (1,1), (1,2), (2,2)\). In this case the observables depend on the mixing between the Higgsino and the Wino component in the mass eigenstates charginos. The chargino mixing matrix depend on the MSSM standard parameters \(M_2, \mu\) and \(\tan \beta\). We have chosen to evaluate the observables at \(M_2 = 200\) GeV and \(\mu = 300\) GeV as a representative light scenario. In a forthcoming paper, we shall discuss in full details the dependence on the mixing focusing on the constraints that virtual correction impose on the determination of the chargino system parameters. The effects are shown in Figs. (10-12). In the phenomenological analysis, it is sometimes reasonable to assume that only the lightest chargino has been produced asking for what information can be gained in this case. However, especially in a light SUSY scenario, one can also consider a more favorable situation where both charginos can be produced. Then, one can study the inclusive observables like, for instance,

\[
\sigma_{\text{inc}} = \sigma_{11} + 2\sigma_{12} + \sigma_{22}
\]

for which the Sudakov expansion is completely independent on the mixing and there is no dependence on both \(M_2\) and \(\mu\). The effects in this representative example are shown in Fig. (13). Looking at Tab. (1) we see that at 1 TeV the asymmetries have corrections below 10% and the cross section is just above that value. The effect of resummation is in the range
of 1-2 percents for $\sigma$ and $A_{FB}$ and one order of magnitude smaller in $A_{LR}$. At 3 TeV, on the contrary, both the cross section and the forward-backward asymmetry are large and the resummation introduces large shifts, particularly in the cross-section ($+10\%$). A detailed analysis reveals that the large effects are mainly due to the Wino component where, at one loop, there is not single logarithm compensating the leading squared logarithm.

V. CONCLUSIONS

In this paper we have considered all the processes of production of pairs of charged particles with spin zero and one-half from electron-positron annihilation at TeV energies in the framework of a given (MSSM) supersymmetric model. Assuming a (relatively) light SUSY scenario, with typical values for the relevant masses below a few hundred GeV, we have concentrated our attention on the asymptotic Sudakov logarithmic expansion of the electroweak component of the invariant scattering amplitude. We have computed at one-loop all the leading Double logarithms and the subleading Linear logarithms of the expansions, also including the linear logarithms of RG origin. To subleading logarithmic accuracy, we have also computed, for the same processes, all the resummed exponentiated expressions. To our knowledge, this is the first complete calculation of this kind, and we do not have at disposal calculations of different authors with which to compare our expressions. We have, though, performed an internal self-consistency check of our results for the chargino case, and we hope that there are no mistakes in our several formulae.

Given the two approximate asymptotic expressions, we have verified that, for energies in the one TeV range (final goal of a future LC), there are practically never visible (in our working assumptions, beyond a one percent level) differences between the two results for a set of realistic experimental observables. On the contrary, rather strong discrepancies appear almost systematically as soon as one approaches the few (2,3) TeV regime (goal of a future CLIC).

A first conclusion is therefore that, in the one TeV range, an asymptotic Sudakov expansion at one loop has a subleading logarithmic accuracy that does not require extra resummation, for the considered processes. The same conclusion cannot evidently be drawn in the higher energy considered regime.

We stress at this point two important facts. The first one is that the validity to sublead-
ing accuracy of the one-loop expansion does not necessarily mean that it is completely accurate when one moves beyond that level of accuracy. Extra terms, in particular energy-independent ones, might well be relevant at the one percent level in the expansion. The second one is that the validity of the expansion itself, i.e. the fact that it gives indeed an adequate description of the real (complete) effect even after inclusion of possible extra e.g. constant terms, remains to be demonstrated.

It is rather simple to realize that the two points are related, and one possible solution would be represented by the preparation of a full one-loop program. This would allow to check the validity of a logarithmic expansion, at the same time allowing to determine by a proper fit to the complete result the value of a possible extra constant term.

In the special case of charged Higgs production, this task has been actually carried through [31]. One can see therefore that for that process an asymptotic Sudakov expansion at one loop, with an additional constant term to be realistically estimated, is suitable at one TeV. The benefit of this conclusion is represented by the fact that, in such an improved expansion, the coefficients of the different asymptotic terms depend each one on special reduced subsets of the supersymmetric parameters. This would allow, via identification of the various terms, stringent and simple tests of the model, of which one must assume the previous experimental confirmation. In fact, a declared goal of future colliders will be also that of performing precision tests of the (assumedly discovered ) supersymmetric model.

In the case of charged Higgs production, the coefficient of the linear Yukawa logarithm was only dependent on tan $\beta$. For the latter, particularly if its value were large (beyond, say, ten), the precise experimental determination is not at the moment completely clean, and we proposed in previous papers [13, 19, 36] to measure tan $\beta$ from the slope of the final pair cross section. In the case of e.g. chargino production, other SUSY parameters e.g. of mixing type would enter the coefficients of the subleading logarithms, and different ones will appear in next-to subleading terms, so that another independent relevant test of the model would be available. To perform the test in a rigorous way would require the preparation of a complete one-loop program, analogous to the one completed for Higgs production. From what shown in this paper, we would hope that, in a relatively light SUSY scenario, a logarithmic one-loop expansion (with a possible addition of an extra constant term) were able to provide an adequate description of chargino production in the one TeV range, and no need of hard two-loop calculations were advocated. We are already working along that direction.
APPENDIX A: OBSERVABLES FOR PRODUCTION OF FERMIONIC AND OF SCALAR PAIRS

1. Fermionic pairs

For any type of fermionic pair (leptons, quarks, charginos, neutralinos) we can write the complete amplitude as:

\[ A(e^+e^- \rightarrow ij) \equiv \sum_{ab} A^{ab}(\gamma^\mu P_a)^{ee}(\gamma^\mu P_b)^{ij} \]  \hspace{1cm} (A1)

with \( a, b = L \) or \( R \) and

\[ (\gamma^\mu P_a)^{ee} = \bar{v}(e^+) \gamma^\mu P_a u(e^-) \quad (\gamma^\mu P_b)^{ij} = \bar{u}(i) \gamma^\mu P_a v(j) \]  \hspace{1cm} (A2)

The unpolarized angular distribution is given by

\[ \frac{d\sigma}{d\cos \vartheta} = \frac{1}{32\pi s} \left[ u^2 \left( |A^{RR}|^2 + |A^{LL}|^2 \right) + t^2 \left( |A^{LR}|^2 + |A^{RL}|^2 \right) \right] \]  \hspace{1cm} (A3)

and the Left-Right polarization asymmetry is obtained as

\[ A_{LR}(s, \vartheta) = \left[ \frac{d\sigma^{LR}}{d\cos \vartheta} \right] / \left[ \frac{d\sigma}{d\cos \vartheta} \right] \]  \hspace{1cm} (A4)

with

\[ \frac{d\sigma^{LR}}{d\cos \vartheta} = \frac{1}{32\pi s} \left[ u^2 (|A^{LL}|^2 - |A^{RR}|^2) + t^2 (|A^{LR}|^2 - |A^{RL}|^2) \right] \]  \hspace{1cm} (A5)

The Forward-Backward asymmetry is

\[ A_{FB} = \frac{(f_F - f_B) \frac{d\sigma}{d\cos \vartheta}}{(f_F + f_B) \frac{d\sigma}{d\cos \vartheta}} \]  \hspace{1cm} (A6)

where \( f_F = \int_0^1 d\cos \vartheta \) and \( f_B = \int_{-1}^0 d\cos \vartheta \).

2. Scalar pairs

Following the notations of ref.[13] we write the amplitude as:

\[ A \equiv \frac{2e^2}{s} \bar{v}(e^+) \gamma^\mu p_\mu (a_L P_L + a_R P_R) u(e^-) \]  \hspace{1cm} (A7)
The unpolarized cross section is

\[ \frac{d\sigma}{d\cos \vartheta} = N_{\text{col}} \frac{\pi \alpha^2}{8s} \sin^2 \vartheta \left( |a_L|^2 + |a_R|^2 \right) \tag{A8} \]

whereas the Left-Right polarization asymmetry is obtained from eq. (A4) with

\[ \frac{d\sigma_{LR}}{d\cos \vartheta} = N_{\text{col}} \frac{\pi \alpha^2}{8s} \sin^2 \vartheta \left( |a_L|^2 - |a_R|^2 \right) \tag{A9} \]

and the Forward-Backward asymmetry is still given by (A6).
APPENDIX B: AMPLITUDES AT ONE LOOP

1. Leptons or quarks

The Born amplitudes for a final fermion pair $f\bar{f}$, with the notations of Appendix A are

$$A_{LL}^{\text{Born}} = \frac{e^2}{4s_w^2c_w^2s}[(2s_w^2 - 1)(2I_f^3) - 2s_w^2 Q_f]$$

$$A_{LR}^{\text{Born}} = -\frac{e^2}{2c_w^2s}(Q_f)$$

$$A_{RL}^{\text{Born}} = \frac{e^2}{c_w^2s}[I_f - Q_f]$$

$$A_{RR}^{\text{Born}} = -\frac{e^2}{c_w^2s}(Q_f)$$

(B1)

The one loop terms, factorizing these Born terms are given in Section II.

2. Sleptons, squarks or charged Higgs bosons

With the notations of Appendix A and ref. [13], the Born amplitudes for a final sfermion pair $\tilde{f}\bar{\tilde{f}}$ are

$$a_L^{\text{Born}} = -Q_f + \frac{(I_f^3 - s_w^2 Q_f)g_{eL}}{2s_w^2c_w^2} = -\frac{s_w^2 Q_f + (1 - 2s_w^2)I_f^3}{2s_w^2c_w^2}$$

$$a_R^{\text{Born}} = -Q_f + \frac{(I_f^3 - s_w^2 Q_f)g_{eR}}{c_w^2} = \frac{I_f^3 - Q_f}{c_w^2}$$

(B3)

This applies to $H^+H^-$ using $Q_f = -1$, $I_f^3 = -\frac{1}{2}$ and $Y_f = -1$, i.e.

$$a_L^{\text{Born}} = \frac{1}{4s_w^2c_w^2}$$

$$a_R^{\text{Born}} = \frac{1}{2c_w^2}$$

(B4)

(B5)

The one loop terms, factorizing these Born terms are given in Section II.

3. Charginos $\chi_i^+\chi_j^-$

At one loop, following the notations of Appendix A, the amplitudes $A_{ij}^{ab}$, where $ab$ refer to $LL, LR, RL, RR$, originate from $s, u, t$ channel contributions:

$$A_{ij}^{ab} \equiv \frac{e^2}{s}S_{ij}^{ab} + \frac{e^2}{u}U_{ij}^{ab} + \frac{e^2}{t}T_{ij}^{ab}$$

(B6)

with

$$S_{ij}^{ab} = S_{ij}^{ab, \text{Born}} + S_{ij}^{ab, \text{Born, fin}} c_a + \sum_k S_{ik}^{ab, \text{Born, fin}} c_k + S_{ij}^{ab, \text{ang}} + S_{ij}^{ab, \text{RG}}$$

(B7)
\[ U_{ij}^{LL} = U_{ij}^{LL, \text{Born}} + \sum_k U_{ik}^{LL, \text{Born}} c_{kj}^{\text{fin}} + U_{ij}^{LL, \text{ang}} \]  

\[ T_{ij}^{LR} = T_{ij}^{LR, \text{ang}} \]

in which we have specified the contributions of the Born terms (photon and Z exchange in the s channel, sneutrino exchange in the u channel), the universal corrections from initial and final lines, the angular dependent corrections and the RG corrections. To make them explicit, it is convenient to separate the Higgsino and the Wino parts:

\[ A_{ij}^{ab} = A_{ij}^{ab, \text{Hig}} + A_{ij}^{ab, \text{Win}} \]  

At Born level, the only non vanishing terms are:

\[ S_{ij}^{LL, \text{Hig Born}} = -\frac{1}{4s_w^2c_w^2}Z_{2i}^{++}Z_{2j}^+ \]

\[ S_{ij}^{LL, \text{Win Born}} = -\frac{1}{2s_w^2}Z_{1i}^{++}Z_{1j}^+ \]  

\[ S_{ij}^{LR, \text{Hig Born}} = -\frac{1}{4s_w^2c_w^2}Z_{2i}^-Z_{2j}^- \]

\[ S_{ij}^{LR, \text{Win Born}} = -\frac{1}{2s_w^2}Z_{1i}^-Z_{1j}^- \]  

\[ S_{ij}^{RR, \text{Hig Born}} = -\frac{1}{2c_w^2}Z_{2i}^-Z_{2j}^- \]

\[ S_{ij}^{RR, \text{Win Born}} = -\frac{1}{2c_w^2}Z_{1i}^-Z_{1j}^- \]

such that, at one loop, one can write:

\[ A_{ij}^{ab, \text{Hig}} = \frac{e^2}{s} S_{ij}^{ab, \text{Hig Born}} \left\{ 1 + (b_{in}^{\text{Hig}} + b_{fin}^{\text{Hig}})(2\log \frac{s}{M^2} - \log^2 \frac{s}{M_V^2}) + b_{Yuk}^{\text{Hig}}(\log \frac{s}{M_V^2}) + b_{ang}^{\text{Hig, RG}} \right\} \]  

\[ A_{ij}^{ab, \text{Win}} = \frac{e^2}{s} S_{ij}^{ab, \text{Win Born}} \left\{ 1 + (b_{in}^{\text{Win}})(2\log \frac{s}{M^2} - \log^2 \frac{s}{M_V^2}) + b_{fin}^{\text{Win}}(-\log^2 \frac{s}{M_V^2} + b_{ang}^{\text{Win}}(\log \frac{s}{M^2}) \right\} \]

with the initial and final "gauge" coefficients

\[ b_{in}^{\text{Hig}} = b_{in, \text{Hig}} = \frac{\alpha}{16\pi s_w^2 c_w^2} (1 + 2c_w^2) \]

\[ b_{in}^{\text{Win}} = \frac{\alpha}{4\pi c_w^2} \]  

\[ b_{fin}^{\text{Hig}} = \frac{\alpha}{16\pi s_w^2 c_w^2} \]

\[ b_{fin}^{\text{Win}} = \frac{\alpha}{4\pi c_w^2} \]  

\[ b_{ang, \text{Hig}} = \frac{\alpha}{8\pi s_w^2 c_w^2} \]

\[ b_{ang, \text{Win}} = \frac{\alpha}{2\pi c_w^2} \]  

\[ b_{Yuk}^{\text{Hig}} = \frac{\alpha}{4\pi c_w^2} \]  

\[ b_{ang}^{\text{Hig, RG}} = \frac{\alpha}{8\pi s_w^2 c_w^2} \]  

\[ b_{ang}^{\text{Win}} = \frac{\alpha}{2\pi c_w^2} \]  

\[ b_{Yuk}^{\text{Win}} = \frac{\alpha}{4\pi c_w^2} \]  

\[ b_{fin}^{\text{Hig}} = \frac{\alpha}{16\pi s_w^2 c_w^2} \]  

\[ b_{fin}^{\text{Win}} = \frac{\alpha}{4\pi c_w^2} \]  

\[ b_{ang}^{\text{Hig, RG}} = \frac{\alpha}{8\pi s_w^2 c_w^2} \]  

\[ b_{ang}^{\text{Win}} = \frac{\alpha}{2\pi c_w^2} \]  

\[ b_{Yuk}^{\text{Win}} = \frac{\alpha}{4\pi c_w^2} \]
the final Yukawa corrections

\[ b^{Yuk}_{L} = -\frac{3\alpha}{8\pi s_w^2} \frac{m_t^2}{M_b^2} (1 + \cot^2 \beta) \quad b^{Yuk}_{R} = -\frac{3\alpha}{8\pi s_w^2} \frac{m_t^2}{M_b^2} (1 + \tan^2 \beta) \]

(B20)

the angular dependent terms

\[ b^{ang, Hig}_{LL} = b^{ang, Hig}_{LR} = -\frac{\alpha}{4\pi s_w^2} \log \left( \frac{u}{t} \right) \]

(B21)

\[ b^{ang, Hig}_{RL} = b^{ang, Hig}_{RR} = -\frac{\alpha}{2\pi s_w^2} \log \left( \frac{u}{t} \right) \]

(B22)

\[ b^{ang, Wino}_{LL} = b^{ang, Wino}_{LR} = -\frac{\alpha}{2\pi s_w^2} \left\{ \log \left( \frac{u}{t} \right) + (1 - \frac{u}{t}) \log \left( \frac{-u}{s} \right) \right\} \]

(B23)

and the RG terms (from the Higgsino components)

\[ S^{LL, RG} = -\frac{\alpha}{4\pi} \left[ Z_{2i} Z_{2j} \right] \left( \frac{1}{s_w} \right) \left( \frac{3 - 6s_w^2 + 8s_w^4}{6} N_{fam} - \frac{5 - 10s_w^2 + 4s_w^4}{4} \right) \left\{ \log \left( \frac{s}{M^2} \right) \right\} \]

(B24)

\[ S^{LR, RG} = -\frac{\alpha}{4\pi} \left[ Z_{2i} Z_{2j} \right] \left( \frac{1}{s_w} \right) \left( \frac{3 - 6s_w^2 + 8s_w^4}{6} N_{fam} - \frac{5 - 10s_w^2 + 4s_w^4}{4} \right) \left\{ \log \left( \frac{s}{M^2} \right) \right\} \]

(B25)

\[ S^{RL, RG} = -\frac{\alpha}{4\pi} \left[ Z_{2i} Z_{2j} \right] \left( \frac{1}{s_w} \right) \left( \frac{3 - 6s_w^2 + 8s_w^4}{6} N_{fam} - \frac{5 - 10s_w^2 + 4s_w^4}{4} \right) \left\{ \log \left( \frac{s}{M^2} \right) \right\} \]

(B26)

\[ S^{RR, RG} = -\frac{\alpha}{4\pi} \left[ Z_{2i} Z_{2j} \right] \left( \frac{1}{s_w} \right) \left( \frac{3 - 6s_w^2 + 8s_w^4}{6} N_{fam} - \frac{5 - 10s_w^2 + 4s_w^4}{4} \right) \left\{ \log \left( \frac{s}{M^2} \right) \right\} \]

(B27)

It is interesting to check the expression of the "gauge" Wino contribution eq.(B19), by making the sum of the \( \chi^\pm \) splitting function term and of the Wino Parameter Renormalization term, ref.[14]:

\[ 2\epsilon^{split}_{kj} = \frac{\alpha}{2\pi s_w^2} \left\{ -\log \left( \frac{s}{M^2} \right) \right\} - \left( N_{fam} - \frac{5}{2} \right) \left\{ \log \left( \frac{s}{M^2} \right) \right\} \left[ Z^+ Z L + Z^- Z R \right] \]

(B28)

\[ 2\epsilon^{PR}_{kj} = \left( \frac{\alpha}{2\pi s_w^2} \right) \left\{ \left( N_{fam} - \frac{5}{2} \right) \left\{ \log \left( \frac{s}{M^2} \right) \right\} \left[ Z^+ Z L + Z^- Z R \right] \right\} \]

(B29)

and observing that all single log cancel and that the total Wino part is a pure quadratic log, exactly like in \( W^+ W^- \) production.
APPENDIX C: RESUMMED AMPLITUDES

The case of ordinary fermions has already been given in Section III, eq.(3.7). The extension to sfermions and Higgses is straightforward using the corresponding expressions for the various coefficients \( b \). We make now explicit the case of Charginos which is more delicate.

In correspondence with eq.(B10), following eq.(3.13), the resummed amplitude called \( B_{ij}^{ab} \), with \( ab = LL, LR, RL, RR \), is written:

\[
B_{ij}^{ab} = B_{ij}^{ab,Hig} + B_{ij}^{ab,Win}
\]

\[
B_{ij}^{ab,Hig} = \frac{e^2}{s} S_{ij}^{ab,Hig} \text{Born} \exp \left\{ (\bar{b}_{in}^{Hig} + \bar{b}^{fin,Hig}) \left[ \frac{1}{3} \log^3 \left( \frac{s}{M^2} \right) \right] + (b_{in}^{Hig} + b^{fin,Hig})(2 \log \frac{s}{M^2} - \log \frac{M^2}{M_V^2}) + b_{out}^{ang,Hig} (2 \log \frac{s}{M_V}) \right\}
\]

\[
B_{ij}^{ab,Win} = \left[ \frac{e^2}{s} S_{ij}^{ab,Win} \text{Born} + \frac{e^2}{u} U_{ij}^{ab,Win} \text{Born} \right] \exp \left\{ (\bar{b}_{in}^{Win} + \bar{b}^{fin,Win}) \left[ \frac{1}{3} \log^3 \left( \frac{s}{M^2} \right) \right] + (b_{in}^{Win} - b^{fin,Win}(- \log \frac{s}{M_V})^{\text{PR}} \log \left( \frac{s}{M^2} \right)) + b_{out}^{ang,Win} (2 \log \frac{s}{M_V}) \right\}
\]

with the new quantities (not defined in the one loop expression):

\[
\bar{b}_{L}^{Hig} = \bar{b}^{fin, Hig} = \frac{3\alpha^2 \tilde{\beta}_0}{16\pi^2 s^4} + \frac{\alpha^2 \tilde{\gamma}_0}{16\pi^2 c^4} \quad \bar{b}_{R}^{Hig} = \frac{\alpha^2 \tilde{\gamma}_0}{4\pi^2 c^4}
\]

\[
\bar{b}_{L}^{Win} = \frac{\alpha^2 \tilde{\beta}_0}{2\pi^2 s^4} \quad b^{Win \text{ PR}} = \frac{\alpha \tilde{\beta}_0}{\pi s^2}
\]

\[
B^{LL, Hig, RG} = -\left\{ \frac{e^2(s)}{4s^2_w(s)c^2_w(s)} - \left[ \frac{e^2}{4s^2_w c^2_w} \text{Born} \right] \right\} [Z_{2i}^+ Z_{2j}^+]
\]

\[
B^{LR, Hig, RG} = -\left\{ \frac{e^2(s)}{4s^2_w(s)c^2_w(s)} - \left[ \frac{e^2}{4s^2_w c^2_w} \text{Born} \right] \right\} [Z_{2i}^- Z_{2j}^-]
\]
\[ B^{RL, Hig, RG} = -\left\{ \frac{e^2(s)}{2s\cot^2(s)} - \left[ \frac{e^2}{2\cot^2(s)} \right]_{\text{Born}} \right\} [Z_{2i}^{+} Z_{2j}^{+}] \]  
\quad (C8)

\[ B^{RR, Hig, RG} = -\left\{ \frac{e^2(s)}{2s\cot^2(s)} - \left[ \frac{e^2}{2\cot^2(s)} \right]_{\text{Born}} \right\} [Z_{2i}^{-} Z_{2j}^{-*}] \]  
\quad (C9)

\[ B^{LL, Win, RG} = -\left\{ \frac{e^2(s)}{2s^2\cot^2(s)} + \frac{e^2(s)}{2us\cot^2(s)} - \left[ \frac{e^2}{2s^2\cot^2(s)} + \frac{e^2}{2us\cot^2(s)} \right]_{\text{Born}} \right\} [Z_{1i}^{+} Z_{1j}^{+}] \]  
\quad (C10)

\[ B^{LR, Win, RG} = -\left\{ \frac{e^2(s)}{2s^2\cot^2(s)} - \left[ \frac{e^2}{2s^2\cot^2(s)} \right]_{\text{Born}} \right\} [Z_{1i}^{-} Z_{1j}^{-*}] \]  
\quad (C11)

\[ B^{RL, Win, RG} = B^{RL, Win, RG} = 0 \]  
\quad (C12)

where the running expressions depending of \( s \) are defined in eq.(2.4).

One can check that with the expansion \( \exp(X) = 1 + X \) one recovers the one loop contribution apart from a two loop \( \alpha^2 \log^3 \) term and a reshuffling of the Wino PR terms.
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FIG. 1: Higher order corrections to the original on-shell vertex diagrams with Yukawa couplings. The non-Abelian generalization of the Gribov theorem can be applied as is shown in the text. The figure is only schematic since in principle the gauge bosons couple to all external legs.
FIG. 2: Two loop Feynman diagrams involving Yukawa couplings. In the text it is shown that the sum of all such contributions with self energy subloops are canceled by the sum of the corresponding vertex diagrams to SL accuracy.
| Observable | 1\text{loop} (1 \text{TeV}) \text{ res.} (1 \text{TeV}) | $\Delta$ | 1\text{loop} (3 \text{TeV}) \text{ res.} (3 \text{TeV}) | $\Delta$ |
|------------|----------------------------------|------|----------------------------------|------|
| $\sigma_t$ | -10 | -8.8 | 1.2 | -26 | -21 | 5 |
| $A_{FB,t}$ | -3.5 | -3.1 | 0.4 | -6.4 | -4.5 | 1.9 |
| $A_{LR,t}$ | -4.2 | -3.8 | 0.4 | -12 | -8.9 | 3.1 |
| $\sigma_b$ | 9.6 | 11 | 1.4 | 1.3 | 4.0 | 2.7 |
| $A_{FB,b}$ | 4.8 | 5.1 | 0.3 | 7.1 | 6.7 | -0.4 |
| $A_{LR,b}$ | 1.5 | 2.0 | 0.5 | -0.54 | 0.18 | 0.72 |
| $\sigma_{\tilde{t}L}$ | -4.7 | -4.0 | 0.7 | -21 | -17 | 4 |
| $A_{FB,\tilde{t}L}$ | -7.0 | -6.5 | 0.5 | -12 | -9.2 | 2.8 |
| $A_{LR,\tilde{t}L}$ | -0.086 | -0.083 | 0.003 | -0.47 | -0.42 | 0.05 |
| $\sigma_{\tilde{t}R}$ | -1.9 | -1.4 | 0.5 | -9.9 | -7.6 | 2.3 |
| $A_{FB,\tilde{t}R}$ | -1.6 | -1.5 | 0.1 | -2.6 | -2.0 | 0.6 |
| $A_{LR,\tilde{t}R}$ | -1.4 | -1.2 | 0.2 | -3.8 | -2.9 | 0.9 |
| $\sigma_{\tilde{b}L}$ | -4.1 | -3.4 | 0.7 | -20 | -17 | 3 |
| $A_{FB,\tilde{b}L}$ | 7.2 | 6.9 | -0.3 | 13 | 9.8 | -3.2 |
| $A_{LR,\tilde{b}L}$ | -0.071 | -0.071 | 0 | -0.58 | -0.55 | 0.03 |
| $\sigma_{\tilde{b}R}$ | 4.5 | 4.6 | 0.1 | 2.7 | 3.2 | 0.5 |
| $A_{FB,\tilde{b}R}$ | 0.77 | 0.74 | -0.03 | 1.1 | 1.0 | -0.1 |
| $A_{LR,\tilde{b}R}$ | -1.3 | -1.2 | 0.1 | -3.3 | -2.9 | 0.4 |
| $\sigma_H$ | -2.8 | -2.2 | 0.6 | -17 | -14 | 3 |
| $A_{FB,H}$ | 5.9 | 5.6 | -0.3 | 10 | 7.7 | -2.3 |
| $A_{LR,H}$ | -0.53 | -0.48 | 0.05 | -2.4 | -2.0 | 0.4 |
| $\sigma_{\text{charginos}}$ | -13 | -11 | 2 | -40 | -30 | 10 |
| $A_{FB,\text{charginos}}$ | -6.3 | -5.4 | 0.9 | -11 | -6.7 | 4.3 |
| $A_{LR,\text{charginos}}$ | -1.0 | -0.79 | 0.21 | -3.8 | -2.2 | 1.6 |

**TABLE I**: Summary table for the effect at $\tan \beta = 10$. The numbers in boldface are effects larger than about 10%.
FIG. 3: Top production. Comparison between the one-loop calculation and the resummation at subleading accuracy. The mass scales in the logarithms are $M_V = M_W = M_{\text{SUSY}} = 300$ GeV.

- $\tan \beta = 10$, one loop
- $\tan \beta = 40$, one loop
- $\tan \beta = 10$, resummed
- $\tan \beta = 40$, resummed
FIG. 4: Bottom production. Same caption as in Fig. 3.

$e^+e^- \rightarrow b\bar{b}$

- $\tan\beta = 10$, one loop
- $\tan\beta = 10$, resummed
- $\tan\beta = 40$, one loop
- $\tan\beta = 40$, resummed
FIG. 5: Left handed stop production. Same caption as in Fig. (3).
FIG. 6: Right handed stop production. Same caption as in Fig.

$100 \Delta A_{LR}$ vs $\sqrt{s}$ [TeV]

$100 \Delta \sigma/\sigma$ vs $\sqrt{s}$ [TeV]

$\tan \beta = 10$, one loop

$\tan \beta = 40$, one loop

$\tan \beta = 10$, resummed

$\tan \beta = 40$, resummed

$e^+ e^- \rightarrow \tilde{t}_R \tilde{t}_R^*$
FIG. 7: Left handed sbottom production. Same caption as in Fig. (3).

\[ e^+ e^- \rightarrow \tilde{b}_L \tilde{b}_L^* \]

\[
\Delta A/\sigma = 10, \text{ one loop}
\]

\[
\Delta A/\sigma = 40, \text{ one loop}
\]

\[
\Delta A/\sigma = 10, \text{ resummed}
\]

\[
\Delta A/\sigma = 40, \text{ resummed}
\]
FIG. 8: Right handed sbottom production. Same caption as in Fig. 3.

\[ \tan\beta = 10, \text{one loop} \]
\[ \tan\beta = 40, \text{one loop} \]
\[ \tan\beta = 10, \text{resummed} \]
\[ \tan\beta = 40, \text{resummed} \]
FIG. 9: Charged Higgs production. Same caption as in Fig. (3).

\[ e^+ e^- \rightarrow H^+ H^- \]

- \( \tan \beta = 10 \), one loop
- \( \tan \beta = 10 \), resummed
- \( \tan \beta = 40 \), one loop
- \( \tan \beta = 40 \), resummed
$N^S_{\text{ SUSY}} = 300$ GeV, $N^W_{\text{ SUSY}} = 200$ GeV, the MSSM mixing parameters are $N^2_{\chi^0} = N^2_{\chi^1}$.

The resummation at subleading accuracy. The mass scales in the logarithms are $N^W_{\text{ SUSY}}$.

FIG. 10: Chargino production $e^+e^-\rightarrow\chi^+_1\chi^-_1$. Comparison between the one-loop calculation and the resummation at subleading accuracy. The mass scales in the logarithms are $N^W_{\text{ SUSY}}$. The MSSM mixing parameters are $M^2_{\text{ SUSY}} = 200$ GeV, $\mu = 300$ GeV.

$e^+e^-\rightarrow\chi^+_1\chi^-_1$

$\tan\beta = 10$, one loop

$\tan\beta = 40$, resummed
FIG. 11: Chargino production $e^+e^- \rightarrow \chi^+\chi^-$. Same caption as in Fig. (10).

$e^+e^- \rightarrow \chi^+_1\chi^-_2$

$\sqrt{s} [\text{TeV}]$

$100 \Delta A_{\log}$

$100 \Delta A_{FB}$

$\tan\beta = 10$, one loop
$\tan\beta = 10$, resummed
$\tan\beta = 40$, one loop
$\tan\beta = 40$, resummed

$M_2 = 200 \text{ GeV}, \mu = 300 \text{ GeV}$
FIG. 12: Chargino production $e^+ e^- \to \chi^+_1 \chi^-_2$. Same caption as in Fig. 10.

\[ \Delta \sigma/\sigma = 10, \text{one loop} \]
\[ \tan \beta = 10, \text{resummed} \]
\[ \Delta A_{LR} \]

\[ e^+ e^- \to \chi^+ \chi^- \]
\[ M^2 = 200 \text{ GeV}, \mu = 300 \text{ GeV} \]
FIG. 13: Inclusive chargino production (see Eq. (4.9)). Same caption as in Fig. (10).

Inclusive production

\[ M_2 = 200 \text{ GeV}, \mu = 300 \text{ GeV} \]

- \( \tan \beta = 10, \) one loop
- \( \tan \beta = 10, \) resummed
- \( \tan \beta = 40, \) one loop
- \( \tan \beta = 40, \) resummed