Regge models of proton diffractive dissociation based on factorization and structure functions

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Abstract

Recent results by the authors on proton diffractive dissociation (single, double and central) in the low-mass resonance region with emphasis on the LHC kinematics are reviewed and updated. Based on the previous ideas that the contribution of the inelastic proton-Pomeron vertex can be described by the proton structure function, the contribution of the inelastic Pomeron-Pomeron vertex appearing in central diffraction is now described by a Pomeron structure function.

1 Introduction

Proton and deuteron diffractive dissociation was intensively studied in the past century at FNAL and at CERN-ISR. The relevant experimental results and their phenomenological interpretation were covered in a large number of papers, see Refs. [1, 2, 3] and references therein. Recent LHC-related developments are discussed e.g. in Refs.[4, 5, 6]. The basic idea behind these and similar studies is the identification of the exchanged Pomeron with a flux emitted by the diffractively scattered proton [7].

A different point of view was taken in Refs. [8, 9, 10, 11], where, following C.A. Jaroszkiewicz and P.V. Landshoff [12], the unknown inelastic proton-Pomeron (pP) vertex was associated with the deep inelastic scattering (DIS) photon-nucleon structure function (SF), known from the experiments at HERA. In doing so, G.A. Jaroszkiewicz and P.V. Landshoff [12] used a high-energy, Regge-behaved formula for the DIS SF, leaving outside the low-energy (missing mass) resonance structure. Resonances were included in this formalism in a series of papers [8, 9, 10, 11], where by duality the high-energy behaviour of the SF was replaced by its low-energy (missing mass) SF, dominated by direct channel non-linear complex Regge trajectories, producing finite-widths resonances. Now we extend the structure function formalism to inelastic Pomeron-Pomeron (PP) vertex to model central diffractive processes.

Diffractive dissociation is interesting and important for many reasons. One is that new experimental data are expected from ongoing the LHC run, especially in the central region, that will help
us to fix the remaining freedom/flexibility of the models. On the other hand, the predictions of the model may guide experimentalists in tuning their detectors. Also, it is important to remember that the high energies, typical of the LHC make possible to neglect in most of the kinematical configurations the contribution from secondary reggeons and allow us to use Regge factorisation and concentrate on the nature of the Pomeron.

The paper is organized as follows: in Sec. 2 models of differential cross sections of the diffractive processes, including elastic scattering as well as single, double and central diffractive dissociation are constructed. In Sec. 3 the treatment of the $pP$ and $PP$ vertices is introduced based on the formalism of structure functions. The calculated integrated cross sections for processes with diffractive dissociation including fits to the available measured data are presented in Sec. 4. The calculated differential cross sections are presented in Sec. 5. Our results and the conclusions are summarised in Sec. 6.

## 2 Differential cross sections

In this section we summarise and update the basic formulae for elastic scattering, single diffractive dissociation and double diffractive dissociation (elaborated in a series of papers \[13, 8, 9, 10, 11\]), also we extend the formalism based on the use of structure functions to central diffractive dissociation and mixed processes. This is an important step on the way towards the elaboration of a unique and adequate language and relevant set of variables and measurables, understandable and convenient both for theorists and experimentalists.

Fig. 1 shows the main topologies appearing in diffractive dissociation under discussion. It may serve also as a guide to relevant equations that follow.

![Diagram of diffractive processes](image)

**Figure 1:** Diffraction: elastic (EL); single (SD), double (DD) and central (CD) dissociation; mixed central and single dissociation (CDS); mixed central and double dissociation (CDD).

The differential cross section of elastic (EL) proton-proton scattering is:

$$\frac{d\sigma_{EL}}{dt} = A_{EL} \beta^2(t) \beta^2(t) |\eta(t)|^2 \left( \frac{s}{s_0} \right)^{2\alpha P(t)-2},$$

where $A_{EL}$ is a free parameter, $s$ and $t$ are the Mandelstam variables. The proton-Pomeron coupling squared is: $\beta^2(t) = e^{b t}$, where $b$ is a free parameter and $b \approx 1.97 \text{ GeV}^{-2}$ determined in Ref. \[13\]. The Pomeron trajectory is $\alpha_P(t) = 1 + \epsilon + \alpha't$, where $\epsilon \approx 0.08$ and $\alpha' \approx 0.3 \text{ GeV}^{-2}$ \[13\]. The signature factor

\footnotesize
\[ A_i \text{ with } i \in \{SD, DD, CD, CDS, CDD\} \text{ are free parameters of dimension } [A_i] = \text{mb/GeV}^2, \text{ including also normalisation constants.} \]
is \( \eta(t) = e^{-i \frac{x}{4} \alpha_p(t)} \); its contribution to the differential cross section is \(|\eta(t)|^2 = 1\), therefore we ignore it in what follows. We set \( s_0 = 1 \text{ GeV}^2 \) for simplicity.

The differential cross section of proton-proton single diffraction (SD) is:

\[
2 \cdot \frac{d^2 \sigma_{SD}}{dt \, dM_p^2} = A_{SD} \beta^2(t) \bar{W}_2^{PP}(M_X^2, t) \left( \frac{s}{M_X^2} \right)^{2 \alpha_p(t)-2},
\]

where \( \bar{W}_2^{PP}(M_X^2, t) \) is related to the proton SF, \( F^p_2(M_X^2, t) \) (see Sec. 3 for details).

From Fig. 1 the differential cross section proton-proton double diffraction (DD) process is:

\[
\frac{d^3 \sigma_{DD}}{dt \, dM_X^2 \, dM_Y^2} = A_{DD} \bar{W}_2^{PP}(M_X^2, t) \bar{W}_2^{PP}(M_Y^2, t) \left( \frac{s_{10}}{M_X^2 M_Y^2} \right)^{2 \alpha_p(t)-2},
\]

where \( \bar{W}_2^{PP}(M_X^2, t) \) is the same function as that used in the SD reaction, with corresponding arguments.

Accordingly, the differential cross sections of proton-proton central diffraction (CD), central diffraction with single diffraction (CDS) and central diffraction with double diffraction (CDD):

\[
\frac{d^4 \sigma_{CD}}{dt_1 \, dt_2 \, d\xi_1 \, d\xi_2} = A_{CD} \beta^2(t_1) \beta^2(t_2) \bar{W}_2^{PP}(M_Z^2, t_1, t_2) \xi_1^{1-2\alpha_p(t_1)} \xi_2^{1-2\alpha_p(t_2)},
\]

\[
2 \cdot \frac{d^5 \sigma_{CDS}}{dt_1 \, dt_2 \, d\xi_1 \, d\xi_2 \, dM_X^2} = A_{CDS} \beta^2(t_2) \bar{W}_2^{PP}(M_X^2, t_1) \bar{W}_2^{PP}(M_Y^2, t_2) \left( \frac{s_{10}}{M_X^2} \right)^{2 \alpha_p(t_1)+2} \xi_2^{1-2\alpha_p(t_2)},
\]

\[
\frac{d^6 \sigma_{CDD}}{dt_1 \, dt_2 \, d\xi_1 \, d\xi_2 \, dM_X^2 \, dM_Y^2} = A_{CDD} \bar{W}_2^{PP}(M_X^2, t_1) \bar{W}_2^{PP}(M_Y^2, t_2) \bar{W}_2^{PP}(M_Z^2, t_1, t_2) \left( \frac{s_{10}}{M_X^2 M_Y^2} \right)^{2 \alpha_p(t_1)+2} \xi_1^{1-2\alpha_p(t_1)} \xi_2^{1-2\alpha_p(t_2)},
\]

where \( M_Z^2 = \xi_1 \xi_2 s \), furthermore \( \bar{W}_2^{PP}(M_X^2, t) \) is the contribution of the inelastic \( PP \) vertex to the differential cross section related to the Pomeron SF, \( F^p_2(M_X^2, t) \) as explained in Sec. 3. Note that \( t_1 \) and \( t_2 \) are connected to the virtualities of the colliding Pomeron: \( Q_1 = -q_1^2 = -t_1 \) and \( Q_2 = -q_2^2 = -t_2 \), where \( Q_1 \) and \( Q_2 \) are the virtualities and \( q_1 \) and \( q_2 \) are the four momenta of the Pomeron.

3 The inelastic \( Pp \) and \( PP \) vertices

Following Refs. [8-10], we write the Pomeron-proton vertices as:

\[
\bar{W}_2^{PP}(M_X^2, t) \equiv \frac{W_2^{PP}(M_X^2, t)}{2m_p},
\]

where

\[
W_2^{PP}(M_X^2, t) = \frac{F^p_2(M_X^2, t)}{\sqrt{v(M_X^2, t)}} \quad F^p_2(M_X^2, t) = -\frac{t(1-x)}{4\pi \alpha(1-4m_p^2x^2/\tau)} \sigma_{pp}^{pp}(M_X^2, t),
\]

The equality \( M_Z^2 = \xi_1 \xi_2 s \) is not exact. If there are two incoming protons with four-momenta \( p_1 \) and \( p_2 \), then \( \xi_1 p_1 \) four-momentum is carried by one of the two Pomeron and \( \xi_2 p_2 \) four-momentum is carried by the other one. Consequently, the squared mass of the centrally produced system is: \( M_Z^2 = (\xi_1 p_1 + \xi_2 p_2)^2 = (\xi_1^2 + \xi_2^2)m_p^2 + 2\xi_1 \xi_2(s/2 - m_p^2) \), where \( m_p \) is the mass of the proton. Using the fact that \( m_p^2 \ll s \), one has: \( M_Z^2 \approx \xi_1 \xi_2 s \).
\( \sigma_{pp}^{Pp} \) is the total Pomeron-proton cross section, \( m_p \) is the mass of the proton, \( \alpha \) is the fine structure constant,

\[
x \equiv x(M_X^2, t) = \frac{-t}{M_X^2 - t - m_p^2},
\]

and

\[
v(M_X^2, t) = \frac{-t}{2m_p x(M_X^2, t)}.
\]

The total \( Pp \) cross section is:

\[
\sigma_{t}^{Pp}(M_X^2, t) = \sigma_{t,0}^{Pp}(M_X^2) + \sigma_{t,\text{res}}^{Pp}(M_X^2, t),
\]

where

\[
\sigma_{t,0}^{Pp}(M_X^2) = \sigma_0 \tau^8(M_X^2) \left( \frac{M_X^2}{8} \right)^e,
\]

and according to the optical theorem

\[
\sigma_{t,\text{res}}^{Pp}(M_X^2, t) = \frac{8\pi}{P_{CM} M_X} \Im A_{\text{res}}^{Pp}(M_X^2, \tilde{t} = 0),
\]

with \( \sigma_0 = 2.82 \text{ mb} \) or 7.249 GeV\(^{-2} \)[8],

\[
\tau(M_X^2) = \frac{e^{-M_X^2/m_0^2} - 1}{e^{-M_X^2/m_0^2} + 1}, \quad m_0^2 = 1 \text{ GeV}^2,
\]

\[
P_{CM} \equiv P_{CM}(M_X^2, t) = \frac{M_X^2 - m_p^2}{2(1-x)} \sqrt{\frac{1 - 4m_p^2 x^2/t}{M_X^2}},
\]

where \( x \) is defined by Eq. (9). Here \( \tau^8(M_X^2) \) is included\(^3\) in \( \sigma_{t,0}^{Pp}(M_X^2) \) to suppress it in the region \( M_X^2 < (m_p + m_{\pi})^2 \) where no dissociation occurs and also in the low \( M_X^2 \) region where dissociation occurs but resonances do not appear.

Note that \( t \neq \tilde{t} \). \( t \) is connected to the virtuality of the radiated particle, the Pomeron, in the \( pp \rightarrow Xp \) process, \( Q^2 = -q^2 = -t \), where \( q \) is the four-momentum of the Pomeron, \( \tilde{t} \) is the squared four-momentum transfer in the \( Pp \rightarrow Pp \) process. Hence, by the optical theorem, \( \sigma_{t,\text{res}}^{Pp} \equiv \Im A_{\text{res}}^{Pp}(M_X^2, \tilde{t} = 0) \) up to normalization, where \( \Im A_{\text{res}}^{Pp} \) is the imaginary part of the \( Pp \) scattering amplitude that includes the resonances. According to Ref. [8,9], the latter is given as:

\[
\Im A_{\text{res}}^{Pp}(M_X^2, \tilde{t}) = \sum_f \frac{|f(\tilde{t})|^4/3 \alpha_{N^*}(M_{\tilde{t}}^2)}{(J - \Re \alpha_{N^*}(M_{\tilde{t}}^2))^2 + (\Im \alpha_{N^*}(M_{\tilde{t}}^2))^2},
\]

where \( \alpha_{N^*} \) is the nucleon trajectory,

\[
f(\tilde{t}) = (1 - \tilde{t} / t_0)^{-2},
\]

and \( t_0 = 0.71 \text{ GeV}^2 \).

The explicit form of the nucleon trajectory is given in Refs. [8,10]. Resonances on this trajectory appear with total spins \( J = 5/2, 9/2, 13/2, \ldots \).

The contribution from the \( PP \) vertex to the differential cross section is:

\[
W_{2,PP}^{Pp}(M_Z^2, t_1, t_2) = \frac{F_2^{PP}(M_Z^2, t_1, t_2)}{y^P(M_Z^2, t_1, t_2)},
\]

where

\[
F_2^{PP}(M_Z^2, t_1, t_2) = \frac{y^P |t_1|}{4\pi^2 \alpha \sqrt{(y^P)^2 - t_1 t_2}} \sigma_t^{PP}(M_Z^2, t_1, t_2),
\]

is the Pomeron structure function based on the structure function of the virtual photon given in

\(^3\)The power of 8 is needed to provide sharp enough suppression in the kinematical region where no dissociation occurs.
\( \eta \) mb/GeV

where

\[ M \]

Experimental data. The theoretical uncertainties in Fig. 2 are correlated with the errors in the data.

where the index \( i \) runs over the states with full spins \( \sum \) coming study.

Ref. [14], while that of the \( f \) meson by the conservation of the quantum numbers. The explicit form of the Pomeron trajectory can be found in Ref. [13], while that of the \( f \) meson trajectories are given in Ref. [15]. At the present stage of research we include only glueballs lying on the Pomeron trajectory. Ordinary mesons will be added in a forthcoming study.

\[ \Delta \]

For CD it is convenient to use the variables \( \Delta \eta = \ln \frac{x}{M_Z^2} \) (rapidity gap) and \( \eta_c \) (the center of the centrally-produced system in \( \eta \)).

\[ \frac{d^3 \sigma_{CD}}{dt_1 dt_2 d\Delta \eta d\eta_c} = A_{CD} \beta^2(t_1) \beta^2(t_2) \mathcal{W}_2 \left( s e^{-\Delta \eta}, t_1, t_2 \right) \]

\[ \times e^{\frac{1}{2} \left[ a_\perp(t_1) - 1 \right] \left[ \Delta \eta + \eta_c \right]} e^{\frac{1}{2} \left[ a_\perp(t_2) - 1 \right] \left[ \Delta \eta - \eta_c \right]} . \]
Figure 2: Integrated SD cross section. The shaded area corresponds to the uncertainty arising from the normalisation parameter $A_{SD}$.

Now the integrated cross section for CD is

$$\sigma_{CD} = \int_{t_{1min}}^{t_{1max}} dt_1 \int_{t_{2min}}^{t_{2max}} dt_2 \int_{\Delta \eta_{min}}^{\Delta \eta_{max}} d\Delta \eta \int_{\eta_{c min}}^{\eta_{c max}} d\eta_c \frac{d^4 \sigma_{CD}}{dt_1 dt_2 d\Delta \eta d\eta_c},$$

(25)

where $t_{1min} = t_{2min} = -\infty$, $t_{1max} = t_{2max} = 0$ GeV$^2$, $\Delta \eta_{min} = 3$, $\Delta \eta_{max} = \ln(s/s_0)$, $s_0 = 1$ GeV$^2$, $\eta_{c min} = -\frac{1}{2} (\Delta \eta - \Delta \eta_{min})$ and $\eta_{c max} = \frac{1}{2} (\Delta \eta - \Delta \eta_{min})$ [6].

The results are shown in Fig. 4 with $A_{CD} = 0.066^{+0.124}_{-0.54}$ mb/GeV$^2$. The value of this normalisation parameter is obtained using the relation $\sigma_{CD} \approx \left(\frac{2\sigma_{SD}}{\sigma_{pp tot}}\right)^2$ based on Regge factorisation. The uncertainty is obtained by the calculated uncertainty of $2\sigma_{SD}$ and the total experimental uncertainty of $\sigma_{pp tot}$ [16] at 7 TeV.

5 Predictions for differential cross sections

This section is devoted to our predictions for SD, DD and CD multiple differential cross sections at $\sqrt{s} = 14$ TeV in the low-mass region.

The $M_X$ dependence of SD double differential cross section is shown in Fig. 5. The visible peaks correspond to nucleon resonances: $N^*(1680)$, $N^*(2220)$, and $N^*(2700)$. Fig. 5 shows the squared momentum transfer dependence of this cross section: a peak at low-$|t|$ followed by the usual exponential decrease. The shaded areas around the curve shows the uncertainty of the calculations following from the uncertainty of the normalisation parameter.

The $M_X$ and $M_Y$ dependence of the DD triple differential cross section is shown in Fig. 6 as a surface. Similar to SD, the peaks correspond to nucleon resonances: $N^*(1680)$, $N^*(2220)$, and $N^*(2700)$. Fig. 6 is a "slice" of Fig. 5 corresponding to a fixed $M_X$ showing the uncertainty of the calculation originating from the uncertainty of the normalisation parameter.

The $M_Z$ dependence of the CD quadruple differential cross section is shown in Fig. 7. The visible peaks correspond to glueball resonances lying on the Pomeron trajectory: $J^{PC} = 2^{++}$, $4^{++}$, and $6^{++}$. Mesons will be included in a forthcoming study.
6 Summary

In this paper we presented updated results on modelling single and double diffraction as well as novel results on modelling central diffraction. The modelling is based on Regge factorisation accompanied by the identification of the contributions of inelastic vertices by structure functions.
We stress that one of the main unknown objects is the inelastic $Pp$ vertex. As mentioned in the Introduction, in most of the papers on the subject, e.g. in Refs. [1-5] one associates (following the ideas of Ref. [7]) the Pomeron with a flux radiated by the incoming proton. The authors of Refs. [8, 9, 10], following [12], take a different viewpoint and identify the inelastic $Pp$ vertex with the proton SF, known from deep-inelastic electron-proton scattering [17]. In Refs. [8, 9, 10, 11] this SF is specified by the direct-channel resonance diagrams dominated by relevant baryon trajectories producing excited nucleon states (mainly $N^*$ resonances).

A completely novel result of this paper is the identification of the inelastic $PP$ vertex with a Pomeron SF. The Pomeron SF is constructed based on the virtual photon SF [14] in a way it can contain mesonic
and glueball resonances. The treatment of the inelastic $PP$ vertex is crucial in central diffractive dissociation (diagrams 4-6 in Fig.1). They contain a subdiagram corresponding to collision of two Pomerons (or, more generally, reggeons). Construction of amplitudes describing scattering of virtual hadrons (by "virtual hadrons" we mean states lying on the Pomeron (or any reggeon) trajectory) is of course an open problem. Our present approach is one possibility although experimental data on central diffraction is needed for justification or for further guide in theoretical developments.

Finally, we highlight that the main part of the dynamics in diffractive dissociation is carried by the Regge trajectories, i.e., nonlinear complex functions. The construction of explicit models of such trajectories is a basic part of this approach, deserving further studies.
\( \eta_c = 0 \)
\( t_1 = -0.01 \text{ GeV}^2 \)
\( t_2 = -0.01 \text{ GeV}^2 \)
\( \sqrt{s} = 14 \text{ TeV} \)

\[ d^4 \sigma_{\text{CD}} / dt_1 dt_2 d\Delta \eta d\eta_c \left[ \text{mb/GeV}^4 \right] \]

Figure 9: Mass dependence of the CD quadruple differential cross section.

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