Deterministic Entanglement via Molecular Dissociation in Integrated Atom Optics

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Deterministic entanglement of neutral cold atoms can be achieved by combining several already available techniques like the creation/dissociation of neutral diatomic molecules, manipulating atoms with micro fabricated structures (atom chips) and detecting single atoms with almost 100% efficiency. Manipulating this entanglement with integrated/linear atom optics will open a new perspective for quantum information processing with neutral atoms.

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The central task to most applications in quantum information processing (QIP) and fundamental tests of quantum mechanics is the manipulation of entanglement. Currently, the most widely used reliable source of bi-particle entanglement is the polarization entanglement of photons, which is created via parametric down-conversion in a nonlinear optical crystal [1]. The down-conversion sources are probabilistic, which is the major deficiency of current entangled-photon sources. The usefulness of the photon sources also suffers from the drawbacks of low coincidence count rate and low detection efficiency of single-photon detectors. Entanglement creation for neutral atoms has also been proposed, which requires controllable interactions [2, 3] or four-wave-mixing-type atomic collision processes [4, 5].

In this Letter we present a scheme for deterministic generation and detection of entanglement of neutral atoms. Our scheme integrates several currently available technologies on detecting single atoms with almost 100% efficiency [2, 5], molecular Bose-Einstein condensates [6, 7], dissociation of diatomic molecules [10, 11, 12], and manipulating, trapping and guiding matter waves with microfabricated structures [13]. The atom entanglement can then be manipulated by “linear atom optics elements” [14], which can be integrated on the Atom Chips like atomic beam splitters (BS) [15, 16], phase shifters and interferometers [17]. For entanglement creation we exploit the perfect correlations inherent in an appropriately prepared two-atom state and as such, no controlled interaction is demanded; the entanglement can be created in either path or internal (“spin”) (or in both path and spin) degrees of freedom of atoms, depending on one’s demand.

Let us start by considering the free-space decay of a two-atom system (e.g., diatomic molecule) with zero total momentum. In the spirit of the original EPR protocol, the two atoms will freely propagate along correlated directions due to momentum conservation: If one of the two atoms leaves along a specific direction, say from the left side at 1 and with momentum k1, the remaining atom will certainly leave along the corresponding direction at 2 (opposite to 1) and with momentum −k1. However the decay in free space leads to freely propagating atoms along many pairs of correlated directions such that the probability for the two atoms moving along any specified pair is small.

Fortunately, one can overcome this drawback with the help of integrated, miniaturized atom optical devices on atom chips [18] based on microfabricated guiding structures using current carrying wires [18], electric charged microstructures [19], and RF induced adiabatic potentials [19]. By restricting the decay to a limited phase space given by the atom optical microstructure one can reduce the available decay modes significantly to only the few desired modes. If there are only two directions the two atoms will move along, one can deterministically obtain the path-entangled state

$$|\Phi\rangle_{path} = \alpha |a_1 a_2\rangle + \beta |b_1 b_2\rangle,$$  \hspace{1cm} (1)

where $|\alpha|^2 + |\beta|^2 = 1$; $|a\rangle$ and $|b\rangle$ are two orthonormal spatial states of atoms. The two atoms are in a coherent superposition; the probability amplitude $\alpha$ ($\beta$) determines the probability for the two atoms to move along the correlated directions $a_1$ and $a_2$ ($b_1$ and $b_2$). We mention that the free-space dissociation of molecules may lead to continuous-variable entanglement and squeezing [20].

A schematic drawing of the setup for our entanglement creation protocol is shown in Fig. 1(a). In such an experimental arrangement, a diatomic molecule is guided into a molecule BS, which can split the molecule into either path a or path b. In each of the one-dimensional (1D) guide, the molecule can be dissociated (for details, see below) into correlated atoms, which will propagate along the two pairs of correlated directions.
If the released decay energy for each atom is smaller than the transverse level spacing in the guides, the decay can only occur in the lowest energy state of the transverse modes, and is restricted into only one mode per path. In this case each two-atom correlated decay leads to an atom pair entangled in the specified paths (spatial modes). By contrast, the parametric down-conversion for photonic entanglement creation is stochastic, populates many modes and has a very low efficiency.

If one uses a 50-50 molecule BS in Fig. 1(a), then \( \alpha \) and \( \beta \) in state \( \Phi_{\text{path}} \) satisfy \( |\alpha|^2 = |\beta|^2 = \frac{1}{2} \) and the two-atom state \( \Phi_{\text{path}} \) is a maximally entangled state \( \Phi_{\text{ME}} \) for spatial modes. For definiteness, in the following we assume one of the Bell states \( \Phi_{-\text{path}} = \frac{1}{\sqrt{2}}(|a_1a_2) - |b_1b_2) \) has been successfully created.

After the above protocol is finished, one has to verify if the two-atom entanglement is successfully generated. To this end, one can combine the modes \( a_1 \) and \( b_1 \) (\( a_2 \) and \( b_2 \)) on the 50-50 atomic BS (see Fig. 1(b)). Adding a phase shift \( \phi \) in one of the two branches, e.g., by adjusting the depth of the guiding potentials adiabatically, the Bell state \( \Phi_{\text{path}} \) generated above becomes \( \Phi_{\text{path}} = \frac{1}{2\sqrt{2}}(e^{i\phi}(i|a_1 \rangle + |b_1 \rangle)(i|a_2 \rangle + |b_2 \rangle) - (|a_1 \rangle + i|b_1 \rangle)(|a_2 \rangle + i|b_2 \rangle)) \) at the outgoing ports. Thus, the probabilities of the coincidence detections of single atoms at any pair of the outgoing ports \( 1' \) (\( a_1',b_1' \)) and \( 2' \) (\( a_2',b_2' \)) are 

\[
C_{1'2'}(\phi) = \frac{1}{4}|1 \pm e^{i\phi}|^2 = \frac{1}{4}|1 \pm \cos \phi|^2
\]

satisfying 

\[
C_{a_1'a_2'}(\phi) + C_{b_1'b_2'}(\phi) + C_{a_1'b_2'}(\phi) + C_{b_1'a_2'}(\phi) = 1.
\]

Then by observing two-atom interference fringes one can verify the successful creation of the desired atom entanglement as the interference fringe will be vanishing if the two probability amplitudes in \( |\Phi_{-\text{path}}\rangle \) are not in a coherent superposition.

The entangled pair is created deterministically, since there are only two correlated decay channels. It is easy to see that the fidelity of the two atom path-entanglement depends on only two factors: the precision of linear atom optical devices, i.e., the BS, which coherently manipulates the external state of a single molecule; and the efficiency of the dissociation of the molecule into two separated atoms, which determines whether we will find the two atoms in the same output port. We will show below that one can get a path entangled pair with fidelity of up to 97% within our protocol using realistic parameters.

One can also deterministically entangle internal states of two neutral fermionic atoms by the above mechanism. Suppose one prepares a ground-state bosonic molecule (of total spin 0) built from two bound fermionic atoms of spin \( \frac{1}{2} \). The spin states may correspond to, e.g., two Zeeman sub-levels of a spin-\( \frac{1}{2} \) hyperfine ground states: |\( F, m_F \rangle = |\frac{1}{2}, \pm \frac{1}{2} \rangle \). Such a system can easily be implemented using \( ^6\text{Li}_2 \) molecules formed from two fermionic \(^6\text{Li} \) atoms \([21]\).

Detection of the entanglement is analogous to that for polarization entanglement of photons. A projection to the +45/−45 bases can be accomplished by applying a \( \pi/4 \) RF pulse which transforms |\( |\rangle \rightarrow +45 \rangle = \frac{1}{\sqrt{2}}(|\uparrow \rangle + |\downarrow \rangle) \) and \( |\downarrow \rangle \rightarrow -45 \rangle = \frac{1}{\sqrt{2}}(|\uparrow \rangle - |\downarrow \rangle) \). Two-particle interferometry can be done using separated oscillatory field techniques borrowed from the Ramsey-type interferometer \([22]\).

Interestingly, by using the structure in Fig. 1(a), one can create spin-path entanglement as following: For definiteness, suppose that the spin state is \( \Phi_{\text{spin}} \) after the free decay of the two dissociated fermionic atoms guided along either path \( a_1-a_2 \) or path \( b_1-b_2 \). As the two possibilities for the two atoms decaying along path \( a_1-a_2 \) or path \( b_1-b_2 \) are indistinguishable, the two fermionic atoms must be in a state \( \Phi_{\text{ME}} \otimes \Phi_{\text{spin}} \) which is maximally entangled both in path and in spin degrees of freedom. This kind of “double entanglement” may have important applications, e.g., testing two-party all-versus-nothing refutation of local realism against quantum mechanics \([23]\).

Now let us consider factors that are essential for a practical implementation of the present scheme. The required diatomic molecules can be created either by photoassociation \([24]\) or by ramping through Feshbach resonances \([25]\) using a time-dependent magnetic field. The Feshbach resonance technique has recently been exploited to produce large, quantum degenerate assemblies of diatomic molecules, starting from either an atomic Bose-Einstein condensate \([8]\) or a quantum degenerate gas of fermionic atoms \([9]\). For our scheme we need a single molecule. This can be prepared either by a quantum phase transition in an optical lattice \([26]\) and taking one potential well, or by extracting one molecule using a “quantum tweezer” \([27]\) technique, which can keep the extracted molecule remaining in the motional ground state during the operation.

After having successfully obtained the molecule in the motional ground state, the molecule needs to be prepared.
in a spatial superposition state, which can be achieved by a BS splitting it to either path $a$ or path $b$ [see Fig. 1(a)]. The RF induced potentials for coherent manipulation of matter waves, which can dynamically split a single trap into a double-well potential with two local minima, are good candidates for BS in our protocol [28]. They allow a much better control of the external motional trapped states than the usual BS, as demonstrated by recent experiments on atom chips [14].

The correlated decay of the atom pairs, under the confined geometry in Fig. 1(a), can be achieved either by photo-dissociating the molecule by a bound-free transition, detuned from the zero-energy threshold, or by sweeping the magnetic field fast across a Feshbach resonance, as reported by recent experiments [10, 11, 12]. If the sweep across the resonance to a final field $B_{\text{final}}$ is fast enough, one can achieve nearly mono-energetic low energy single-mode scattering can be described into a double-well potential with two local minima, are

The RF induced potentials for coherent manipulation of

$H_1D$ goes from negative to positive value and

$\psi(x)$ goes from the bound state of $H_1D$ to a scattered wave packet. The typical wave-packet evolution is depicted in Fig. 2. It is easy to see that the diatomic molecule is dissociated into two well separated wave packets after sweeping the magnetic field across the Feshbach resonance.

Taking into account the center-of-mass (CM), we can estimate the fidelity of the path entanglement. Suppose the BS is a perfect 50-50 one, then after dissociation the entangled state can be written as $|\Psi_{l-\text{path}}\rangle = 1/\sqrt{2}|\langle \Psi_{a_1a_2} | - | \Psi_{b_1b_2} \rangle\rangle$, where $|\Psi_{a_1a_2}\rangle, |\Psi_{b_1b_2}\rangle$ are the overall wave function in the two branches respectively. The two branches are symmetric, so we can consider the decay only in one branch, i.e., branch $a$. We have $\Psi(x_1, x_2, t) = \psi(x, t)\varphi(X, t)$, where $\psi(x, t)$ is the relative-motion wave function and $\varphi(X, t)$ is the CM wave function in the ground state, which can be approximated as a Gaussian wave packet. One has $|\varphi(X, t)|^2 = \frac{1}{2\pi\Delta X_0} \exp \left[-\frac{x^2}{2\Delta X_0^2}\right]$, with $\Delta X_0$ being the initial width of the Gaussian wave packet and $\Delta V_0 = \frac{h}{4\pi\Delta X_0}$ the CM velocity spreading. In this case the fidelity $F$ can be estimated by $F = 1/(1 + \kappa)$, where $\kappa$ is the ratio of two second-order correlation function $\kappa = G^{(2)}(a_1a_1)/G^{(2)}(a_2a_2)$, and $G^{(2)}(a_1a_1)$ (or $G^{(2)}(a_2a_2)$) is just the probability of finding the two atoms in the same output (or opposite output). When the molecule is fully dissociated at a certain time $t_0$, the probability can be approximated by $G^{(2)}(a_1a_1) \approx \int_{-\infty}^{t_0} dx_1 \int_{-\infty}^{t_0} dx_2 |\Psi(x_1, x_2, t_0)|^2$ and $G^{(2)}(a_2a_2) \approx \int_{-\infty}^{t_0} dx_1 \int_{-\infty}^{t_0} dx_2 |\Psi(x_1, x_2, t_0)|^2$, where $|\Psi(x_1, x_2, t_0)\rangle$ is the overall two atom wave function. We numerically calculate the fidelity for different magnetic field sweeping speeds $B$ and initial widths of CM wave packet. The time $t_0$ is chosen 13ms so that the molecule is fully disso-
FIG. 3: Fidelity of the path entangled pair for different magnetic sweeping speeds $\dot{B}$ and widths of CM wave packet, with $R \equiv \Delta X_0/a_{1D}(0)$.

uated. The results are reported in Fig. 3. We find that the fidelity is insensitive to the sweeping speeds of magnetic field, as long as the molecule is fully dissociated. The fidelity mainly depends on the initial width of the CM wave packet, with $R \geq 0.5$ for fidelity as high as 97% by properly manipulating the CM wave packet.

In summary, we have proposed a novel scheme for generating, detecting and manipulating entanglement of neutral atoms with integrated linear atom optics and single-atom detection. Our scheme entails the advantages of the usual linear optics QIP and opens up a new avenue to QIP with neutral atoms by means of integrated linear atom optics. The fact that the atom entanglement can be manipulated by linear atom optics elements is interesting in its own right and opens the exciting possibility for linear optics QIP and for experimental test of fundamental problems in the atomic domain.

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