Fatigue crack-growth predictions for a railway axle under material data variability

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Abstract. This document reports a probabilistic approach for residual fatigue life predictions of a cracked railway axle with allowance for variability of the input data. A NASGRO formulation was used for such a purpose, with its main parameters considered as uncorrelated random variables varying in given ranges. A load spectrum was considered in order to take into account of the actual loading conditions that these components undergo. The presented approach aimed to perform residual fatigue life predictions in terms of both mean value and dispersion around it. The results were also used to drive deterministic approaches in calculating conservative crack-growth assessments.

1. Introduction

Since early studies, it was understood that the crack-growth phenomenon in mechanical components is stochastic in nature. To reduce probability of failures, safety factors were commonly introduced to counterbalance the effects of the multiple sources of uncertainty. Unfortunately, the use of these safety factors generally resulted in conservative or even over-conservative results, with obvious consequences on the final designs.

Deterministic approaches to fracture can generally provide a limited perspective on the phenomenon. Nevertheless, such approaches have been widely demonstrated to be capable of providing accurate predictions even in complex operating conditions [1], with particular load spectra [2-3], etc. These approaches have been the dominating methods in the industry, even though they provide predictions lacking of any substantial information about the reliability of designed structures. The need for designing processes that provide life predictions of mechanical components with well-defined ranges of variability, nowadays can be considered as one of the key aspects of mechanical design.

The aim of this study was to quantify the effects of some sources of uncertainty on the phenomenon of fatigue crack-growth for a railway axle. However, the whole procedure is of general validity and can be easily applied nearly to any kind of engineering structure. The accurate designing process of railway axles is still nowadays of great importance for the railway industry, as their failures may cause serious accidents [4-6]. Advanced deterministic approaches were proposed along the years to tackle the problem of fatigue crack-growth, providing results in terms of residual mileage and reasonable inspection intervals to use for maintenance [7]. Nevertheless, in the safety design of these critical mechanical components, the related sources of uncertainty, including variations in material properties, loading and geometry, have to be taken into account. This stochasticity has been here considered as only coming from one of the primary sources of uncertainty, namely the scattering of crack-growth rates (CGRs) material properties. Further sources of uncertainty, such as loading conditions, fracture algorithm
accuracy, etc., will be a future development of this work so as to eventually achieve a wide-ranging understanding of this argument.

2. Material properties variability

The material under analysis was the high-strength steel 30NiCrMoV12, according to the standard UNI 6787 [8]. This material is one of the most accepted materials for these mechanical components. The related material data, in terms of CGRs and threshold stress intensity range \( \Delta K \) were obtained in [7] by experimental tests on full-scale axles. CGRs experimental data are reported in Figure 1: it is worth noting that the data exhibit a significant scatter, in other words, a given \( \Delta K \) leads to a series of equally acceptable CGRs, eventually resulting in a significant scatter for the life estimates. It is also worth to mention that CGRs should be calculated with the highest possible accuracy since from their values descend the life assessment.

These material data are usually adopted in order to calibrate a fatigue crack-growth law useful to perform life estimates for the actual components made out of the same material. In the following, the NASGRO formulation was selected since it takes into account of:

1) the three distinct parts of the crack-growth diagram (near-threshold, Paris’ range and critical zone);
2) the crack closure effects according to Newman definition [9];
3) the stress ratio influence on crack-growth rates.

The NASGRO curve fitted on the related experimental data available in [7], here referenced as “nominal” curve, are shown in Figure 1, whereas the NASGRO formulation and experimental data available in [7] were here elaborated in a probabilistic way.

Different elements of the NASGRO equation were initially documented in [10] and continuously developed along the years. The main equation is given by:

\[
\frac{da}{dN} = C \left[ \left( \frac{1-f}{1-R} \right) \Delta K \right]^n \frac{(1-\Delta K_{th}/\Delta K)^p}{(1-K_{max}/K_C)^q},
\]

where \( N \) is the number of fatigue cycles, \( a \) is the crack depth and \( da/dN \) the crack-growth rate, \( K_C \) represents the critical \( K \) value under Mode I condition, whereas \( R \) is the stress ratio equals to \( \sigma_{min}/\sigma_{max} \). \( \Delta K \) is the stress intensity factor range equals to \( K_{max} - K_{min} \) and can be considered as the key parameter, which is generally continuously increasing along the propagation.

\( C, n, p \) and \( q \) are empirical constants required by NASGRO formulation to fit the crack-growth rate experimental data. \( K_C \) is the critical \( K \) value corresponding to the final rupture of component. These data were available from full-scale axles tests [7].

The crack opening function, \( f \), allows to consider that the \( \Delta K \) contributes only partially to the crack-growth. The phenomenon related to crack closure was first discovered in [11-12] leading to a widely accepted definition [13] as:

\[
f = \frac{K_{op}}{K_{max}} \left( \max\left(R, A_0 + A_1 R + A_2 R^2 + A_3 R^3\right), R \geq 0 \right) \frac{A_0 + A_1 R}{A_0 + \frac{A_1 R}{2}}, \quad -2 \leq R < 0,
\]

where the coefficients \( A_i \) are given by:

\[
\begin{align*}
A_0 &= (0.825 - 0.34\alpha + 0.05\alpha^2)\left(\cos\left(\frac{\pi S_{max}}{2}\right) / \sigma_0\right)^{1/\alpha} \\
A_1 &= (0.415 - 0.071\alpha)S_{max} / \sigma_0 \\
A_2 &= 1 - A_0 - A_1 - A_3 \\
A_3 &= 2A_0 + A_1 - 1
\end{align*}
\]

where \( \alpha \) is the plane stress/strain constraint factor and assumes, for materials such as the high-strength steel here considered, relatively high values (2.5 or higher). The considered value here was 2.5.
$S_{\text{max}}/\sigma_0$ is the ratio of the maximum applied stress $S_{\text{max}}$ to the flow stress $\sigma_0$, the latter generally defined as an average between yield stress $\sigma_y$ and ultimate tensile stress $\sigma_{UTS}$. Even though such ratio varies through the propagation, especially when considering load spectrum effects, it was assumed to be constant for all the calculations performed here.

The threshold stress intensity factor range ($\Delta K_{th}$) is a function of $R$:

$$\Delta K_{th} = \Delta K_{th,0} = \frac{\sqrt{a/(a+a_0)}}{(\Delta - A_0)(\Delta - R)}^{1+\epsilon R},$$

where $\Delta K_{th,0}$ is the threshold $\Delta K$ for $R = 0$, $C_{th}$ is a fitting parameter with different values for positive or negative $R$ values, whereas $a_0$ is the El-Haddad’s parameter according to [14]. $\Delta K_{th,0}$ represents the value of $\Delta K$ below which no crack-growth would occur for $R = 0$. Finally, the El-Haddad’s parameter $a_0$ allows to take into account of the crack-growth for short cracks; this is generally always assumed to be equal to 0.0381 mm.

All data required by the NASGRO equation were available from literature [7] or ad hoc calculated. These data are referenced here as “nominal” parameters and are listed in table 1.

Table 1. NASGRO nominal parameters.

| $\sigma_{UTS}$ [MPa] | $\sigma_y$ [MPa] | $K_C$ [MPa√m] | $C$ [MPa$m^{1-n/2}$] | $n$ [-] | $p$ [-] |
|---------------------|-----------------|----------------|----------------------|--------|--------|
| 980                 | 880             | 120            | 1.15e-10             | 2.41   | 0.65   |
| $q$ [-]             | $a_0$ [m]       | $C_{th}$ [-]   | $C_{th}$ [-]         | $\Delta K_{th,0}$ [MPa√m] | $\alpha$ [-] |
| 0.001               | 3.81e-5         | 1.4            | -0.07                | 6.13   | 2.5    |

By using data and formulations as the one above described, fatigue life predictions in presence of defects can be performed in a deterministic way. Here, to capture the uncertainty given by the variability of material properties, the main parameters ($C$, $n$, $p$, $\Delta K_{th,0}$, $\alpha$, $S_{\text{max}}/\sigma_0$) required by the NASGRO formulation were considered as uncorrelated random variables varying in given ranges. Figure 1 shows the crack-growth rate curves obtained by varying one of the main NASGRO parameters at a time, while keeping the remaining ones as fixed to their nominal values. With reference to Figure 1c-f, it can be noticed that variation of parameters $\alpha$ and $S_{\text{max}}/\sigma_0$ do not modify the curve significantly, therefore, their values were considered as fixed to the nominal ones for the subsequent analyses. Ranges of variability for variables $C$, $n$, $p$, $\Delta K_{th,0}$ are listed in Table 2. Finally, the Root Mean Square (RMS) error between each single curve and the experimental data was also calculated, allowing to estimate the accuracy of each curve in replicating the experimental data. A RMS error of 3% was considered as cut-off threshold for discarding curves too far from the experimental data.

Table 2. Ranges of variability of the four material data considered as random variables; units are MPa and m for stress and length respectively.

| Log10(C) [-] | Min value | Nominal value | Max value |
|--------------|-----------|---------------|-----------|
| -10.3372     | -9.9393   | -9.5414       |
| $n$ [-]      | 2.169     | 2.410         | 2.678     |
| $p$ [-]      | 0.325     | 0.650         | 1.3       |
| $\Delta K_{th,0}$ [MPa√m] | 4.85 | 6.13 | 6.3 |
Figure 1. NASGRO curve variability produced by its main parameters; experimental data and nominal NASGRO are added to the graphs; “Good” and “Bad” curve refers to a RMS error lower or higher than 3% respectively.

3. Stochastic procedure description

Railway axles are subjected to variable amplitude loading, caused by many different regimes of train operation. Beside the nominal loading condition, additional loads can be generated when train goes over
crossovers, through switches, rail joints, etc. In this work, the Load Spectrum (LS) shown in figure 2 was considered in order to take into account of this aspect. Such LS consisted of a sequence of stress amplitudes $\sigma_j$, each of them associated with a given number of cycles $n_j$, the latter defining the amount of repetitions that the axle undergoes at a given stress level $\sigma_j$ along its operation. The LS was generated so as to correspond to a total mileage of $N_{LS}=2000$ km on railway lines, considering an average railway wheel diameter equal to 880 mm.

![Figure 2. Considered load spectrum for railway axle.](image)

Fatigue crack-growth of mechanical components is generally modelled by using material parameters derived in a deterministic way, therefore, the related outcomes are generally associated with relatively large safety factors. On the other hand, the presented probabilistic approach is capable of providing outcomes both mean value and dispersion around it.

The procedure for the probabilistic assessment of fatigue crack-growth phenomenon in a railway axle has been fully performed with a MATLAB routine [15]. The routine considers as input data the random variables presented previously and provides as outputs the residual fatigue lives estimated for the axle. Such a procedure starts by reading the input data required by the analysis, namely: nominal NASGRO parameters, $K_i$ values, LS data, etc., performs the calculations to derive the residual fatigue lives. Then, it re-iterates the procedure until a user-defined amount of iterations is reached.

For each $k$-th iteration, a set of material parameters $(C_k, n_k, p_k, \Delta K_{th.0,k})$ are separately and randomly sampled in their given admissible ranges of variability (Table 2). These data are used to generate a NASGRO curve to calculate the $CGRS_k$; comparing the $CGRS_k$ with the experimental data, a $RMS_k$ error is computed in such a way to evaluate whether or not the $k$-th NASGRO curve well fits the material data (a maximum $RMS$ error equal to 3% was considered). Only for the accepted data sets $(C_k, n_k, p_k, \Delta K_{th.0,k})$, the $K$ values and the LS data are used to obtain the related $k$-th fatigue life prediction.

$K$ values represent the only driving force that is used to predict the crack-growth and, in particular, $K_i$ values were the only contributors to crack-growth since $K_{II}$ and $K_{III}$ were null. Since the geometry under analysis was very simple, an analytical formulation to calculate such $K_i$ values was already available in literature [16] and was here reported (equation 5). In equation 5, $\sigma$ is the maximum bending stress, $a$ is the crack depth, $D$ is the diameter of the cylinder. Since the study case involved a simple geometry, no dedicated numerical investigations were performed to produce $K$ vs. crack depth. Numerical simulations are generally required for these kind of calculations and, as an example, numerical investigations for similar cases can be found from literature based on finite elements [17-19].
boundary elements [19-21] and combined approaches [22], whereas some examples of fracture phenomena tackled numerically for more complex geometries can be found in [23].

\[
K_I = \beta \sigma \sqrt{\pi a} \left[ 0.923 + 0.199 \left( 1 - \sin \frac{na}{2D} \right)^4 \right] \quad \text{and} \quad \beta = \frac{1.84}{\pi} \left[ \frac{n_0}{2D} \frac{\sigma}{2D} \right]^{1/2} \frac{1}{\cos \frac{na}{2D}}.
\]  

(5)

Each fatigue life prediction was performed by means of a user made algorithm, based on a rearrangement of the main NASGRO equation (equation 1), in such a way to calculate the total advancing \( \Delta a \) corresponding to the application of the LS of figure 2, where the number of cycles corresponding to each stress level are indicated by \( n_j \):

\[
\Delta a = \sum_j \left( n_j \times C \left[ \left( \frac{1-f}{1-Rj} \right) \Delta K_j \right] \left[ \frac{1-\Delta K_{th}/\Delta K_j}{(1-\Delta K_j/(1-Rj)\Delta K)} \right]^n \right),
\]  

(6)

\( \Delta a \) was used to calculate the amount of repetitions of the LS in figure 2 that are needed to cause a crack advances \( \Delta a_{i,i+1} = a_{i+1} - a_i \):

\[
\Delta N_{i,i+1} = \frac{\Delta a_{i,i+1}}{\Delta a} N_{LS}.
\]  

(7)

Equation 7 was then used to compute how many mileages \( \Delta N_{i,i+1} \) were travelled during the crack growth from \( a_i \) to \( a_{i+1} \), namely causing a crack advance \( \Delta a_{i,i+1} \). Finally, summing up all the \( \Delta N_{i,i+1} \), starting from the initial crack size of 2 mm and until final rupture, the total fatigue life \( \Delta N \) was obtained (intended as the total mileages travelled due the calculated number of repetitions of the spectrum in figure 2). With such an approach, the distribution of the fatigue life predictions, as depending on uncertainty of material parameters, can be obtained, thus leading to an improved and more robust control over the safety required by these mechanical components.

4. Results and discussion

The presented procedure aims to provide a robust tool capable to perform probabilistic fatigue life predictions for a railway axle with allowance for material variability. By means of this framework, it was possible to quantitatively evaluate the distribution of the residual fatigue life with its mean value and standard deviation. The maximum number of samples to be extracted was set up to 2e6, however, all the figures reported below included results of only 5e4 data sets to improve their readability.

Only 12% of the extracted samples had a time consuming impact on the algorithm, since all the crack-growth rate curves presenting a \( RMS_k > 3\% \) were discarded. The remaining “good” curves are shown in figure 3, superimposed to the experimental data, to the nominal and the best-fit curves (the latter presented the lowest RMS overall error). Furthermore, the “good” curves that provided the 3% lowest residual mileages were separated from all the accepted ones and were highlighted in green in Figure 3. On the same line, also the residual fatigue lives expressed in residual mileages of the axle were plotted in figure 4. Such residual mileages are also reported in figure 5 in terms of their probability density distribution, showing that the calculated data can be adequately described by means of a normal distribution.
Figure 3. Crack-growth rate diagram comprising all the considered valid curves; best-fit curve presenting the minimum RMS error is also highlighted (dashed line); green curves are those that provided the 3% lowest fatigue lives; “Conservative Paris” refer to an equation 8 fitting.

Figure 4. Fatigue life predictions for all the simulations; best-fit curve was obtained with the best-fit of experimental material data; 3% lowest fatigue lives are in green; “Conservative Paris” refer to an equation 8 fitting.
Figure 5. Probability density function for all the predicted residual mileages.

From figures 3-4, it was understood that the lowest fatigue life predictions were obtained with CGR curves that presented the lowest $\Delta K_{th,0}$, a steep slope at low $\Delta K$ (i.e. a low p value), C and n values defined such as to fit the material data. This outcome suggested that conservative predictions can be directly obtained also by considering the following Paris-like crack-growth law:

$$\frac{da}{dN} = C \left[ \frac{(1-f)}{(1-R)} \Delta K \right]^n. \quad (8)$$

The “Conservative Paris” curve in figures 3-4 was obtained on the basis of equation 8, considering the minimum value for $\Delta K_{th,0}$ (Table 2) and considering the C and n values that allowed to get the lowest RMS error against experimental data. The so obtained curve for crack-growth rates (figure 3) presented the highest speed values for the initial part of propagation, namely the fastest crack-growth was expected for the initial (at low $\Delta K$), most critical part of crack-growth. The related residual mileage was consequently the lowest, see figure 4. This outcome turned out to have a practical use since it can be used as reference to drive deterministic fatigue crack-growth methodologies in calculating conservative predictions (figures 3-4 and equation 8).

5. Conclusions
This document reports an approach to evaluate the phenomenon of fatigue crack-growth in a railway axle, considering the variability of material data as a source of uncertainty. The presented framework was based on numerical and experimental data that provided the input parameters and was able to predict the distribution of residual fatigue life of the axles with its mean value and standard deviation. The presented results were also used as a reference tool to drive deterministic fracture calculations in achieving conservative fatigue life assessments.
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