Synthetic Schlieren: Determination of the density gradient generated by internal waves propagating in a stratified fluid

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Abstract. The synthetic schlieren technique and a convenient image processing are employed to attain a new insight into the visualization of internal waves and to make quantitative precise measurements of the bidimensional distributions of very small density gradient variations, harmonics distributions patterns, etc., in a stratified fluid. The laboratory implementation of this technique is much simpler than that of the optical methods as classical schlieren and interferometry, and makes possible to obtain useful information in situations in which shadowgraph gives only qualitative results. In addition, much greater domains than those with optical methods may be analyzed, and high optical quality windows are not required. Its application is illustrated by means of the visualization of the internal gravity waves generated by an oscillating body and thermals.

1. Introduction
The information related to density fluctuations in a stratified fluid obtained in many situations has contributed to improve the understanding of the flows produced there. The available techniques range from the introduction of passive tracers, such as non-Bouyant, to shadowgraph, schlieren and interferometric techniques which depend on the refractive index variation. Examples of how to perform these visualizations [1-3] and how to extract the quantitative information of interest from them [4-5] may be found within the literature.

In particular Dalziel et al. [6] reported a number of techniques for obtaining both high quality visualizations and accurate measurements of the density field, based on the same optical principles as the classical schlieren method but with the advantage of being easier to setup and adaptable to much larger domains. They outlined four related techniques: qualitative synthetic schlieren, line refractometry, dot tracking refractometry and pattern matching refractometry. The first one has two operational modes: “line mode” and “dot mode”, and allow the basic qualitative visualization of the density variations with respect to an unperturbed initial state of the fluid. Together with the other three quantitative techniques, it conforms the “synthetic schlieren” diagnostic, that is relatively cheap to setup and provides a powerful method to visualize, for example, bidimensional internal waves of small magnitude and measure the associated density fields.

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Synthetic schlieren allows analyzing the density fluctuations that are too small to be visualized by applying the techniques related to the concentration of a passive dyeing (e.g. light induced by fluorescein or light absorption by dyeing), and those in which the curvature of the beams due to density variations is out of the range of application of the common interferometry. The name reflects the relationship with classical schlieren, although it must be noted that the digital processing is used to form the “synthetized” schlieren image, instead of the knife edge in the focus of a lens or a convergent mirror.

A particular experimental challenge is the visualization of the internal waves generated in a stratified medium. Basically, if a fluid element is perturbed vertically in a stratified fluid, it experiences a buoyancy force directed to restore it to its original position. This buoyancy force, in combination with the inertia of the fluid element, acts as a simple harmonic oscillator with a frequency (the buoyancy or Brunt-Väisälä frequency)

\[ N^2 = -\frac{g}{\rho_0} \frac{d\rho}{dz}, \]

where \( \rho \) is the fluid density, \( \rho_0 \) is a reference density, \( g \) is the gravity and \( z \) is the coordinate oriented vertically upwards. If the element oscillation occurs in a direction at an angle \( \phi \) to the vertical, the restoring force is reduced, allowing the formation of waves of angular frequency

\[ \omega = \pm N \cos \phi. \]

This is the dispersion relationship for small amplitude waves in the Boussinesq approximation. The fluid motion and group velocity occurs at an angle \( \phi \) to the vertical, while the phase velocity is perpendicular to them (the sign of the vertical component of the phase velocity is opposite to that of the group velocity).

Therefore, a linearly stratified fluid \( (N: constant) \) is able to transmit waves of frequencies \( 0 \leq \omega \leq N \) corresponding to \( 0 < \phi < \pi/2 \). Mowbray & Rarity [7] demonstrated theoretically and experimentally (using classical schlieren) that for the waves forced by an oscillating cylinder, the fluid motion was confined within a region of fluid resembling a Saint Andrew’s Cross. Other authors have subsequently used different experimental techniques to analyze these waves (for example Peters [8] and Merzkirch and Peters [9] used interferometry). A recent work by Hurley & Keady [10] has provided additional theoretical details of the structure of this wave field for oscillating cylinders of a finite size in a viscous fluid. The first experimental results confirming their prediction were provided by Sutherland et al. [11] by means of the vertical oscillation of a circular cylinder in a linear stratification.

In this work we present the results obtained with an optimized application of the synthetic schlieren technique and the subsequent image processing to determine the density gradient field in two different situations. In the following we review briefly others schlieren techniques in order to distinguish the comparative advantages of the synthetic schlieren, and describe the experimental setup and the methodology that introduces a new procedure to process the images using the specialized software DigiFlow. As an example, the results corresponding to the bidimensional internal waves and their harmonics generated by the periodical oscillation of a cylindrical body in a stratified medium are presented in Section 3. Up to our knowledge, the intensities and phases of the internal waves corresponding to the higher harmonics have not been yet quantified. In Section 4 another interesting application of the synthetic schlieren diagnostic is presented, which arises when the fluctuations field of the density gradient produced by the displacement of a thermal in a stratified fluid is looked for.

2. Experimental description

2.1. Classic schlieren techniques

The schlieren techniques ([7], [12]) employ the same optical principle, namely that a beam of light is bent in regions with refractive index changes. As the refractive index of a stratified fluid varies with density, the beams of light passing through it to angles close to the horizontal are bent depending on the density gradient. When internal gravity waves propagate in a stratified fluid, isopycnal (constant
density) surfaces are displaced resulting in regions where the local density gradient changes with respect to the background gradient and the path of a nearly horizontal light beam passing through the medium is deflected [11].

**Figure 1.** Optical setup for applying classical schlieren method.

In the classical schlieren technique, whose typical setup is illustrated in figure 1, a convergent lens or a parabolic mirror is employed to create beams parallel to those emitted from a light source. After passing through the test section of the tank containing the stratified fluid the rays are focused by a second lens or a parabolic mirror, while the edge of a “knife” in the focus stops the rays deviated from the parallelism in the test section. This technique allows determining the first derivative of the density variations with respect to (for example) a state with constant initial gradient [12]. Practical difficulties with this technique are associated with the careful setting up and high cost of the lens and parabolic mirrors. In addition, high quality images can only formed when the apparatus itself is optically good, and the need for a point source of illumination limits the intensity of illumination available.

**Figure 2:** Moiré fringe method optical setup.

The Moiré fringe method [13], illustrated in figure 2, operates in a similar manner to classical schlieren but the pair of parabolic mirrors and the knife edge are replaced for a pair of accurately aligned masks that consist of a set of parallel lines. They are normally aligned so that 50% of the light passing through the first mask located at a side of the test section is stopped by the second “analyzer” mask located on the opposite side of the test section. Due to camera parallax, the lines on the mask in front of the tank are more closely spaced than those at the rear of the tank. The Moiré fringe method is cheaper to implement and may be scaled up to cover larger domains more easily than classical schlieren because expensive parabolic mirrors are not required. The main difficulty of this method is that the alignment between the masks is critical and non-trivial, especially if light entering the camera is not parallel or if the stratification is non-linear so that the lines spacing of the mask are not related by a constant scale factor.
2.2. Synthetic schlieren

The synthetic schlieren method overcomes the above mentioned difficulties. It needs neither lens nor good quality mirrors, the positioning and alignment of the elements are no critical, and it may be scaled to any dimension without raising the cost or making difficult the laboratory implementation. An analyzer digital “virtual” mask is used, which allows measuring quantitatively the density variations with respect to an arbitrary equilibrium state. Measurements are made in two directions simultaneously, so that the knife edge (classical schlieren) or the masks (Moiré fringe) must not be rotated to obtain the gradients in the perpendicular direction.

The figure 3 illustrates the typical arrangement of the experimental setup for applying the synthetic schlieren technique. The beams of light generated in a diffuse light source pass through a mask and the tank containing the stratified fluid forming small angles to the horizontal, and are captured by a video camera located at a distance that is longer than the size of the test section. The solid line represents the path of the beam deviated when the density gradient is greater than the equilibrium gradient.

![Figure 3: Synthetic schlieren experimental setup.](image)

A white and black camera JAI CVM4+MCL, with resolution 1372×1024 pixels of 8 bits operating up to 24 images/s, is employed to acquire the images that are stored directly in a PC for processing digitally later. The camera is placed at 3.00 m far away from the tank to minimize the angle (< 3°) at which the rays are captured by the camera. A fabric with small circular holes of 1 mm diameter is used as a mask. A reference images sequence of the initial situation, that replaces the mask of the Moiré fringe method, is saved. Then any small change in the refraction index gradient inside the environment produces the deflection of the light beams. During each experiment, the deviations of the intensities with respect to the reference image corresponding to each mask hole are detected and quantified. DigiFlow software [14] allows the images to be processed easily obtaining qualitative information online. Quantitative results are obtained with an additional processing in which the light intensity deviations are converted to changes of the vertical and horizontal density gradients, provided the density and the refraction index of the stratified fluid follow an almost linear relationship [15].

The particular features related to the implementation of this diagnostic are introduced en the following two sections showing concrete applications.

3. Internal waves generated by an oscillating object

By using the double-bucket method, a 0.30 m deep, 0.91 m long and 0.08 m wide tank is filled with 0.275 m deep uniformly stratified salt water. The maximum density of the fluid, measured with a Reichert-TS refractometer and a 5500 Antor-Paar densimeter, varies between 1.0645 and 1.0225. Then the buoyancy frequency is

$$N^2 = 1.43 \pm 0.07 \text{ s}^{-2}.$$  

A cylindrical body of radius 0.02 m with its centre at 0.15 m over the tank bottom is immersed with its axis aligned horizontally and spanning the full width of the tank. It is suspended from a thin metallic supporting arm which is driven by an adjustable speed.
eccentric cam, forcing the body to undergo sinusoidal vertical oscillations (see figure 4). The amplitude of the body oscillation (that is, the maximum distance between the lower and higher positions of the body axis) is $0.0081 \pm 0.0001$ m. The flow around the body is laminar for the oscillation frequencies and amplitudes generated.

![Figure 4: Experimental setup to generate internal waves by means of an oscillating body.](image)

After the completion of an experiment, a segment of a vertical column of pixels passing through the lowest point of the oscillating cylinder was extracted from the images sequence to form a time series image as that shown in figure 5. The upper and lower zones clearly distinguished in the figure correspond to the body and the fluid, respectively. The vertical position of the lowest part of the body, \( h = h(t) \), is obtained from the contour between these zones. The fundamental frequency \( \omega_0 \) is determined by means of: (a) the best fit sinusoidal curve in the table \((t,h)\), and (b) the spectral analysis of frequencies of the time series. The uncertainty in each case and the difference between both values are smaller than 1%.

![Figure 5: Time series of a fraction of a column of pixels of the images. The interface between the two colours indicates the height of a point belonging to the lower contour of the oscillating body as a function of time](image)

3.1. Internal waves visualization

Figure 6 shows the perturbations at a given time generated by a cylinder oscillating with a frequency $\omega_0 \equiv 0.298 \pm 0.003$ s$^{-1}$. Applying the synthetic schlieren technique, quantitative information of the variation of the instantaneous density gradient field $\rho^{-1} \partial \rho / \partial y$ is obtained starting from the intensity distribution of the images.

As mentioned in Section 1, internal waves are evidenced by means of fluid elements oscillations
Figure 6: Intensity of the vertical component of the density gradient in arbitrary units for a cylinder oscillating in a stratified fluid. The colour scale (below) ranges between black (negative values) and white (positive values).

whose direction forms an angle $\phi = \cos^{-1} \omega / N$ to the vertical. In the case reported here, four beams at angle $\phi$ are seen far from the body. Each of these beams is composed by two thinner beams of opposite phases forming the known St. Andrew Cross. Due to the symmetry of the body cross-section and since the motion axis coincides with the direction of stratification, the intensity distribution looks symmetrical with respect to vertical and horizontal axis passing through the centre of the oscillation. This suggests that the used experimental setup avoids spurious horizontal motions that might give place to flow asymmetries.

The beams evolve as expected [7], that is, they appear above (below) the cylinder with increasing intensities, then move down (up) following a direction that is perpendicular to the respective beam while their intensities decrease, and finally vanish. The angle $\phi$ and the direction of the motion of the beams are related to the oscillation frequency as anticipated by theory.

3.2 Higher harmonics visualization

The angular fundamental frequency $\omega_0$ of the oscillations generates distributions varying in time as those showed in figure 6. An important experimental difficulty arises when the deviations from these distributions are wanted to visualize. The internal waves corresponding to the harmonics are also difficult to visualize because of their very small intensities (typically of one order of magnitude smaller than those of the amplitude of waves with main frequency $\omega_0$) and their superposition with the waves of frequency $\omega_0$. We use the fact that the density gradient in each point of the region of interest varies in time with a given frequency. This information is found in the time variations of the intensity distributions that appear in the sequences of images registered during $n$ periods of oscillation.

Therefore, for each pixel located at $(x,y)$ in the image, we calculate the complex parameter

$$
\Re(x,y) = 2 \int_0^{nT} \left( \frac{1}{\rho} \frac{\partial \rho}{\partial y} \right)_{(x,y)} e^{i \omega t} dt,
$$

where $\omega = \omega_0$, $2 \omega_0$, $3 \omega_0$, … are the fundamental frequency and its harmonics. Thus the value of $|\Re|$ is maximum if the frequency of the variations of the pixel intensity coincides with $\omega_0$ and is minimum if it is different from $\omega_0$.

Figures 7 and 8 show the images corresponding to the modulus and phase of $\Re(x,y)$, respectively, calculated with the corresponding sequence of images of the instantaneous density gradient as those shown in figure 6. The white regions in figure 7 are associated with the maximum amplitude of the
time variations of the density gradient, while the light blue ones, for which $|\Re| \approx 0$, do not vary significantly with time at the fundamental frequency.

![Figure 7](image1.png)  
**Figure 7.** Modulus of $\Re$ at frequency $\omega_0$ for the same case of figure 6. The false colour scale is also the same.

![Figure 8](image2.png)  
**Figure 8.** Phase of $\Re$ at frequency $\omega_0$ for the same case of figure 6. The false colour scale is also the same.

![Figure 9](image3.png)  
**Figure 9.** Modulus of $\Re$ at frequency $2\omega_0$ for the same case of figure 6. The intensity scale is amplified by a factor 5 with respect to the values of figure 7.

![Figure 10](image4.png)  
**Figure 10.** Phase of $\Re$ at frequency $2\omega_0$ for the same case of figure 6.

Figure 8 shows a 180º change of phase at the horizontal axis of symmetry resulting that the inferior half-plane is nearly a mirror reflection of the superior one but with an opposite phase. Then the density gradients at points located below the oscillating objects are in the opposite direction with respect to the mirror points located above. This may also be observed in figure 6.

Thus, from figures 7 and 8 it is possible to determine the intensities and phases corresponding to oscillations with frequency $\omega_0$ of each point $(x,y)$ located in the plane of observation. This information is more useful and contains major precision than that obtained from figure 6, because only the oscillations with the frequency of interest is involved, and a sequence of more than 100 images is used to obtain it.

Figure 9 show the modulus of $\Re$ for the first harmonics ($\omega = 2\omega_0$) obtained using the same images sequence that in figure 6. The beams now form a greater angle $\phi_1 = \arccos(2\omega_0 / N)$ to the vertical when compared with those of figure 6. It is found that the harmonic waves are generated in the same regions around the body where the waves of frequency $\omega_0$ are produced. The amplitude of the beams is
about one fifth of that corresponding to the beams formed at the main frequency. The distribution of the phase of the function $\Re$ for the first harmonics is presented in figure 10 and shows the same main features of figure 8.

Figures 11 and 12 show the modulus and phase of $\Re$, respectively, for the second harmonic ($\omega = 3\omega_0$) obtained using the same images sequence than that employed to obtain the distribution in figure 6. The intensity scale is amplified 5 times with respect to that of figure 9. Although tenuous, the beams corresponding to this harmonics may be distinguished forming an angle $\phi_2 = \arccos(3\omega_0 / N)$ to the vertical, superposed to the beams belonging to the main and first harmonic frequencies. This superposition comes from an incomplete filtering applied to the calculus of $\Re$ in the digital processing. Then it may be inferred the very small intensity of the oscillations associated with the second harmonic and the practical limit of the method for visualizing them. Better results would require images with a better space and time resolutions, and a sequence of a greater number of images.

4. Internal waves generated by thermals

The experiments were performed in a 75 cm long, 29.5 cm wide and 38 cm high tank of transparent walls. It is filled up to a deep of 34 cm with linearly stratified salt water prepared by means of the double-bucket method, with a density $\rho$ decreasing from the bottom to the fluid free surface where $\rho_0 \approx 0.998 \text{ g/cm}^3$. The value of $1/\rho_0 \Delta \rho / \Delta z$, with $z$ as the vertical coordinate, ranged in the experiments from 0 (no stratified fluid) up to 0.002 cm$^{-1}$.

The thermals were released from a Perspex cylinder with an interior diameter of 3.7 cm as shown in figure 13. The rounded lower end of the tube was covered with a stretched latex membrane. The tube was then immersed completely in the stratified medium slowly with its upper end at about 3 cm below the water free surface. A saline solution of known volume and density is added inside the tube using a pipette, minimizing the mixing during this process. The experiment starts when the membrane is broken using a needle, releasing the salt water in a negligible time. A series of experiments was carried out combining different initial volumes $3 < V_i < 10 \text{ ml}$ and densities $1.09 < \rho_i < 1.2 \text{ g/cm}^3$ (saturated salt water) of the dense fluid released, and the stratification of the environment.

Figure 14 shows a series of images corresponding to a run. A diagnostic based on the attenuation of the light [16] by the presence of a calibrated quantity of dye in the salt water provides visualization, and synthetic schlieren technique allows observing the perturbations generated by the descending motion of the thermal in the stratified surrounding. The tank was illuminated from behind using a fluorescent strip lights and a diffusing sheet to provide nearly uniform back-lighting. The light attenuation due to the dyed water allows the fluid density variations to be observed. The thermal descends from rest under the unique influence of density difference between the fluids while its

Figure 11. Modulus of $\Re$ for the frequency $3\omega_0$ (or second harmonics) for the same case of figure 6.

Figure 12. Phase of $\Re$ for the frequency $3\omega_0$ for the same case of figure 6.
volume increases continuously due to mixing between the fluids. The total vertical momentum increases progressively under gravity action but density and velocity decrease due to the entrainment of external fluid. Since the total excess of buoyancy is reduced as the thermal descends, the dense fluid density may balance that of the surrounding fluid at an intermediate height level of the tank, stopping its vertical motion and then extending axially at that height.

![Figure 13. Experimental device for releasing thermals.](image)

Figure 15 shows the distribution of the variations of the density gradient in the surrounding fluid obtained by synthetic schlieren. It is observed that, from the release of the thermal, internal waves forming beams with different angles to vertical are produced. The horizontal beams correspond to internal waves of very small frequency. As the thermal evolves, the angle of the others beams increases indicating the decreasing of the frequency of the waves excited in the medium. This effect is associated with the decreasing of the thermal velocity. Eventually new vertical beams are generated implying the formation of new internal waves. The symmetry of the dense blob, and also that of the internal waves, is lost gradually giving place to the formation of a great number of beams at different angles at the end of the run.

As appreciated, the synthetic schlieren technique allows measuring without inconvenient the small density fluctuations produced when a dense transparent fluid moves and mixes in another transparent fluid. The difference between figures 14 and 15 makes evident the extreme sensitivity of this diagnostic.

5. Conclusions
A non-intrusive optical technique suitable for determining the density gradient fluctuations in two-dimensional unsteady flows is employed to visualize and measure quantitatively the amplitude of the bidimensional internal waves generated in a stratified medium by an oscillating body and the displacement of a thermal. Combining the application of this diagnostic with the image processing employing the specialized software DigiFlow, the intensity changes of small lit circles are related to the deviation of light beams, and density gradient \( \rho^{-1} \frac{dp}{dx} \) and \( \rho^{-1} \frac{dp}{dy} \) is determined with respect to an initial reference image.

The methodology is more powerful, useful and versatile than other available techniques of visualization due to its low cost of implementation, and because it allows working in a larger spatial domain and provides non-intrusive measurements of the amplitude of the quasi-bidimensional wave field. The resolution of the experimental setup depends on the distances between the mask and the tank and between the camera and the tank (see figure 3), the characteristics of the camera employed and the density of points in the mask. Thus, in the arrangement described in Section 2.2 it is possible to measure changes of the density gradient so small as 1% of the ambient density gradient.

A key point of the images processing, integrated to an adequate software, is the on-line visualization of the results in a qualitative way, the later estimation of the density gradients in two
directions and the possibility of performing more sophisticated processing as that described in Section 3.2. Therefore, the methodology introduced allows obtaining results that are hard to acquire using the traditional techniques (as optical schlieren, interferometry, etc.). The results may be transformed directly in a matrix of data to be compared with results of theoretical models and to detect experimentally new aspects of the physical process involved.

Figure 14. Sequence of images in false color obtained by using the diagnostics based on the light attenuation for a thermal with $V_i = 5$ ml and $\rho_i = 1.2$ g/cm$^3$ in a stratified medium ($\Delta\rho/\Delta z(1/\rho_0) = 0.0005$).

Figure 15. Variations of $(1/\rho_0)\Delta\rho/\Delta z$ in the ambient fluid generated by the thermal of figure 14 and visualized using synthetic schlieren.

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