Azimuthally symmetric theory of gravitation – I. On the perihelion precession of planetary orbits

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ABSTRACT

From a purely non-general relativistic standpoint, we solve the empty space Poisson equation ($\nabla^2 \Phi = 0$) for an azimuthally symmetric setting (i.e. for a spinning gravitational system like the Sun). We seek the general solution of the form $\Phi = \Phi(r, \theta)$. This general solution is constrained such that in the zeroth-order approximation it reduces to Newton’s well-known inverse square law of gravitation. For this general solution, it is seen that it has implications on the orbits of test bodies in the gravitational field of this spinning body. We show that to second-order approximation, this azimuthally symmetric gravitational field is capable of explaining at least two things: (i) the observed perihelion shift of solar planets; (ii) the fact that the mean Earth–Sun distance must be increasing (this resonates with the observations of two independent groups of astronomers, who have measured that the mean Earth–Sun distance must be increasing at a rate between about 7.0 ± 0.2 m century$^{-1}$ and 15.0 ± 0.3 m cy$^{-1}$).

In principle, we are able to explain this result as a consequence of the loss of orbital angular momentum; this loss of orbital angular momentum is a direct prediction of the theory. Further, we show that the theory is able to explain at a satisfactory level the observed secular increase in the Earth year (1.70 ± 0.05 ms yr$^{-1}$). Furthermore, we show that the theory makes a significant and testable prediction to the effect that the period of the solar spin must be decreasing at a rate of at least 8.00 ± 2.00 s cy$^{-1}$.

Key words: ephemerides – planetary systems.

1 INTRODUCTION

Since the 1850s, it has been known that the orbit of the planet Mercury exhibits a peculiar motion of its perihelion; specifically, the perihelion of Mercury advances by 43.1 ± 0.5 arcsec century$^{-1}$. When Newton’s theory of gravitation was applied to try and explain this [by making use of the oblateness of the planets, because when the Sun’s gravitational force acts on the oblate planets, the oblateness causes torque (on the planets) and this torque is thought to give rise to the anomalous motion of the planets], it was found first by Leverrier in 1859 (see, for example, Kenyon 1990) that it predicted a precession of 532 arcsec cy$^{-1}$, which is larger than that observed (Kenyon 1990). With the failure of Newton’s theory to explain this, it was proposed that a small undetected planet was the cause. Careful scrutiny of the terrestrial heavens by telescopes and space probes revealed no such object – which meant that the cause could very well be a hitherto unknown gravitational phenomenon. Einstein was to demonstrate that this was the case, that there existed a hitherto unknown gravitational phenomenon that was the cause of this peculiar motion.

With the herald of Einstein’s general theory of relativity (GTR) in 1915, Einstein immediately applied his GTR to this problem. Much to his elation, which caused him heart palpitations, he obtained the unprecedented value of 43.0 arcsec cy$^{-1}$. This was (and is still) hailed as one of the greatest triumphs for the GTR, and this led to its quick acceptance. Venus, the Earth and other planets show such peculiar motion of their perihelion – observations reveal shifts of 8.40 ± 4.80 and 5.00 ± 1.00 arcsec cy$^{-1}$, respectively (see, for example, Kenyon 1990). Einstein’s theory is able to explain the perihelion shift of other planets well, so much that it is now a well-accepted paradigm that the perihelion shift of planetary orbits is a general relativistic phenomenon.

Einstein’s GTR explains the perihelion shift of planetary orbits as a result of the curvature of space–time around the Sun. It does not take into account the spin of the Sun, and at the same time it assumes that all the planets lie on the same plane. The assumption that the planets lie on the same plane is in the GTR solution only taken as a first-order approximation; in reality, planets do not lie on the same plane. In this paper, we set forth what we believe is a new paradigm. We have called this paradigm the azimuthally symmetric theory of gravitation (ASTG). This is derived from Poisson’s well-accepted equation for empty space: $\nabla^2 \Phi = 0$. The Poisson law is a differential form of Newton’s law of gravitation. We explain the perihelion shift of the orbits of planets as a consequence of
the spin of the Sun (i.e. solar spin). It is well known that the Sun does exhibit some spin angular momentum; specifically, the Sun undergoes differential rotation. On average, it spins on its spin axis about once every ~25.38 d (see, for example, Miura et al. 2009). Its spin axis makes an angle of about 83° with the ecliptic plane. It is important that we state clearly here that by no means have we discovered a new theory or a set of new equations; we have merely applied Poisson’s well-known azimuthally symmetric solution to gravity for a spinning gravitating body.

Furthermore, in its solution to the problem of the perihelion shift of planetary orbits, the GTR assumes the traditional Newtonian gravitational potential, \( \Phi(r) = -\frac{G M}{r} \), where \( G = 6.667 \times 10^{-11} \) kg\(^{-1}\) m\(^2\) s\(^{-2}\) is Newton’s universal constant of gravitation, \( M \) is the mass of the central gravitating body and \( r \) is the radial distance from this gravitating body. Einstein’s GTR, which is embodied in Einstein’s law of gravitation,

\[
R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} = \kappa T_{\mu\nu} + \Lambda g_{\mu\nu},
\]

(1)

is designed such that in the low-energy limit and low space–time curvature, such as in the Solar system, this equation reduces directly to the Poisson equation. In Einstein’s law above, \( R_{\mu\nu} \) is the Ricci tensor, \( R \) is the Ricci scalar and \( g_{\mu\nu} \) is the metric of space–time. \( \Lambda \) is Einstein’s controversial cosmological constant, which at best can be taken to be zero unless making computations of a cosmological nature where dark energy is involved. \( \kappa = 8\pi G/c^4 \), where \( c = 2.99792458 \times 10^8 \) ms\(^{-1}\) is the speed of light in vacuum. The Poisson equation is given by

\[
\nabla^2 \Phi = 4\pi G \rho ,
\]

(2)

where \( \rho \) is the density of matter and the operator \( \nabla^2 \) written for the spherical coordinate system (see Fig. 1 for the coordinate set-up) is given by

\[
\nabla^2 = \frac{1}{r^2} \frac{\partial}{\partial r} \left( r^2 \frac{\partial}{\partial r} \right) + \frac{1}{r^2 \sin \theta} \frac{\partial}{\partial \theta} \left( \sin \theta \frac{\partial}{\partial \theta} \right) + \frac{1}{r^2 \sin^2 \theta} \frac{\partial^2}{\partial \phi^2} .
\]

(3)

As has already been said, our solution or paradigm comes directly from the Poisson equation, which in itself is a first-order approximate solution to Einstein’s GTR, albeit with the important difference that we have taken into account solar spin. The fact that our paradigm explains reasonably well (within the confines of its error margins) the precession of planetary orbits as a consequence of solar spin and at the same time the GTR explains this same phenomenon well as a consequence of the curvature of space–time raises the question: ‘is the precession of the perihelion of solar orbits a result of (i) solar spin or (ii) of the curvature of space–time?’

If anything, this is the question that seems to be raised in this paper, and an answer to it will only come once the meaning of the ASTG is fully understood.

Above, we say that the ASTG ‘explains reasonably well (within the confines of its error margins)’. Here, what immediately comes to mind is whether a theory can have error margins. Is it not experiments that have error margins? As can be seen, certain undetermined constants (\( \lambda \)) in the theory emerge and, at present, we have to infer these from observations. It is here that the error margins of the ASTG come into play.

Furthermore, we show that, in principle, the ASTG does explain (i) the increase in the mean Earth–Sun distance and (ii) the increase in the mean Earth–Moon distance, etc. These emerge as a consequence of the fact that from the ASTG, the orbital angular momentum is not a conserved quantity as is the case in Newtonian gravitational theory and Einstein’s GTR. The fact that the orbital angular momentum is not a conserved quantity may lead us to think that the ASTG violates the law of conservation of angular momentum; however, this is not the case. The lost angular momentum is transferred to the spin of the orbiting body, as well as the Sun.

2 THEORY

For empty space, \( \nabla^2 \Phi = 0 \), and for a spherically symmetric setting we have \( \Phi = \Phi(r) \); this leads directly to Newtonian gravitation. For a scenario or setting that exhibits azimuthal symmetry (e.g. a spinning gravitating body such as the Sun) we must have \( \Phi = \Phi(r, \theta) \). Thus, we solve the Poisson equation: \( \nabla^2 \Phi(r, \theta) = 0 \). The Poisson equation for this setting can be easily solved, and its solution can be readily found, for example, in most good textbooks of electrodynamics and quantum mechanics; it is instructive that we present this solution here.

We solve the Poisson equation for empty space (\( \nabla^2 \Phi = 0 \)) exactly, by means of separation of variables; that is, we set \( \Phi(r, \theta) = \Phi(r) \Phi(\theta) \). Inserting this into the Poisson equation, after some basic algebraic operations, we have

\[
\frac{1}{\Phi(r)} \frac{\partial}{\partial r} \left[ r^2 \frac{\partial \Phi(r)}{\partial r} \right] + \frac{1}{\Phi(\theta)} \frac{1}{\sin \theta} \frac{\partial}{\partial \theta} \left[ \sin \theta \frac{\partial \Phi(\theta)}{\partial \theta} \right] = 0 .
\]

(4)

The radial and angular portions of this equation must equal some constant as they are independent of each other. Following tradition, we must set

\[
\frac{1}{\Phi(r)} \frac{\partial}{\partial r} \left[ r^2 \frac{\partial \Phi(r)}{\partial r} \right] = \ell(\ell + 1) ,
\]

(5)

The solution to this is

\[
\Phi_\ell(r) = A_\ell r^\ell + \frac{B_\ell}{r^{\ell+1}} ,
\]

(6)
where $A_\ell$ and $B_\ell$ are constants and $\ell = 0, 1, 2, 3, \ldots$ If we set the boundary conditions, $\Phi_\ell(r = \infty) = 0$, then $A_\ell = 0$ for all $\ell$. Just as Einstein demanded of his GTR to reduce to the well-known Poisson equation in the low-energy regime of minute curvature, we must demand that for $\Phi(r)$, in its zeroth-order approximation (where $\ell = 0$ and the terms $\ell \geq 1$ are so small that they can be neglected), the theory must reduce to Newton’s inverse square law. For this to be so, we must have

$$B_\ell = -\lambda_c c^2 \left( \frac{GM}{r^2} \right)^{\ell+1},$$

(7)

where $\lambda_c$ is an infinite set of dimensionless parameters such that $\lambda_0 = 1$ and the rest of the parameters $\lambda_c$ for $\ell > 1$ will take values different from unity. For now, until such a time that we are able to deduce them directly from theory, these constants will have to be determined from observations. In Section 7, we hint at our current thinking on the nature of these constants. This means we have

$$\Phi_\ell(r) = -\lambda_c c^2 \left( \frac{GM}{r^2} \right)^{\ell+1} \cdot$$

(8)

Now, moving on to the angular part, we have

$$\sin \theta \frac{\partial}{\partial \theta} \left[ \sin \theta \frac{\partial \Phi(\theta)}{\partial \theta} \right] + \left[ \ell (\ell + 1) \right] \sin^2 \theta = 0.$$  

(9)

A solution to this is a little complicated. It is given by the spherical harmonic function

$$\Phi(\theta) = P_\ell(\cos \theta),$$

(10)

of degree $\ell$, and $P_\ell(\cos \theta)$ is the associated Legendre polynomial. As already mentioned, the derivation of $\Phi(\ell, \theta)$ just presented can be found in most good standard textbooks of quantum mechanics and classical electrodynamics. Because equation (9) is a second-order differential equation, we would naturally expect there to exist two independent solutions for every $\ell$. It so happens that the other solutions give infinity at $\theta = (0, \pi)$, which is physically meaningless (see, for example, Griffiths 2008). Now, putting all these things together, the most general solution is given:

$$\Phi(\ell, \theta) = -\sum_{\ell=0}^{\infty} \left[ \lambda_c c^2 \left( \frac{GM}{r^2} \right)^{\ell+1} \cdot P_\ell(\cos \theta) \right].$$

(11)

This is a linear combination of all the solutions for $\ell$. In the case of ordinary bodies such as the Sun, the higher-order terms [i.e. $\ell > 1$, of the term $(GM/r^2)^{\ell+1}$] will be small. In these cases, the gravitational field will tend to Newton’s gravitational theory. Equation (11) is the embodiment of the ASTG, and from this, we show that we are able to explain the precession of the perihelion of planetary orbits.

The value of the Sun’s spin ($T_{25} \simeq 25.38$ d) does not enter into equation (11). The question arises: “where has this been taken into account?” To answer this, it is important to note that if the potential is a function of $r$ only [i.e. $\Phi = \Phi(r)$]. Then, it is technically a function of $r$ and $\theta$ as well (with the $\theta$-dependence being trivial). This means that spherical symmetry implies an azimuthal symmetry around any arbitrarily chosen axis. If a specific axis is singled out (e.g. by the spin of a body about the spin axis), then the spherical symmetry of the static body is broken, and only an azimuthal symmetry remains. This azimuthal symmetry is only about the plane cutting the body into hemispheres such that this plane is normal to the spin axis. For any other plane cutting the body into hemispheres, the two hemispheres are asymmetric. From this, we see that the azimuthally symmetric solution is a consequence of the breaking of the spherical symmetry by the introduction of a spin axis. Hence, we are automatically led to consider the solutions for which $\Phi = \Phi(r, \theta)$. In this way, the spin has been taken into account.

2.1 Equations of motion

Here, we derive the equations of motion for the azimuthally symmetric gravitational field, $\Phi(r, \theta)$. We know that the force per unit mass [or the acceleration, i.e. $g = -\nabla \Phi(r, \theta)$] is given by $a = (\ddot{r} - r \dot{\theta}^2) \hat{r} + (r \dot{\theta}^2 + 2r \dot{\theta} \dot{\phi}) \hat{\theta}$ (see any good textbook on classical mechanics). Here, a single dot represents the time derivative $\mathrm{d}/\mathrm{d}t$ and likewise a double dot represents the second time derivative $\mathrm{d}^2/\mathrm{d}t^2$. A comparison of $a = (\ddot{r} - r \dot{\theta}^2) \hat{r} + (r \dot{\theta}^2 + 2r \dot{\theta} \dot{\phi}) \hat{\theta}$ with $(g)$ (i.e. $a \equiv g$) leads to the equations

$$\frac{d^2r}{dt^2} = r \left( \frac{d\psi}{d\theta} \right)^2 = \frac{d\Phi}{dr},$$

(12)

for the $\hat{r}$-component and

$$\frac{d^2\psi}{dt^2} + 2 \frac{d\psi}{dt} \frac{d\phi}{dt} = -\frac{1}{r} \frac{d\Phi}{d\theta}$$

(13)

for the $\hat{\theta}$-component.

Now, taking equation (13), dividing throughout by $r \dot{\phi}$ and remembering that the specific angular momentum $J = r^2 \dot{\phi}$, we have

$$\frac{1}{r} \dot{\phi} = \frac{2}{r} \frac{d\psi}{dt} = -\frac{1}{J} \frac{d\Phi}{d\theta} \frac{1}{J} \frac{dJ}{dt} = -\frac{1}{J} \frac{d\Phi}{d\theta}$$

(14)

and thus

$$\frac{dJ}{dt} = -\frac{d\Phi}{d\theta}$$

(15)

The specific orbital angular momentum is the orbital angular momentum per unit mass. Unless otherwise specified, we refer to this as angular momentum.

Digressing a little, what equation (15) means is that the orbital angular momentum of a planet around the Sun is not a conserved quantity. If it is not conserved, then the sum of the orbital and spin angular momentum must be a conserved quantity (if this angular momentum is not, say, transferred to the Sun or other solar bodies). This means that at different $r$-positions, the spin of a planet about its own axis must vary. This could mean the length of the day must vary depending on the radial position away from the Sun. We come to this later. All we simply want to do is to emphasize this, as it points to the possibility of a secular change in the mean length of the day.

Moving on, if we make the transformation $u = 1/r$, then for $\dot{r}$ and $\ddot{r}$ we have

$$\frac{d\dot{r}}{dt} = -\frac{1}{J^2} \frac{d\Phi(u, \theta)}{du} \frac{du}{d\psi} + \frac{1}{J^2} \frac{d\Phi(u, \theta)}{du} \frac{d\Phi(u, \theta)}{d\psi},$$

(16)

respectively. Inserting these into equation (12), then dividing the resultant equation by $-u^2 \dot{\phi}$ and remembering equation (15) and also that $\dot{r} = -du/dr^2$, we are led to

$$\frac{d^2u}{du^2} = \left[ \frac{1}{J^2 u^2} \frac{d\Phi(u, \theta)}{du} \right] \frac{du}{d\psi} + u = \frac{1}{J^2} \frac{d\Phi(u, \theta)}{du} \frac{d\Phi(u, \theta)}{d\psi}.$$  

(17)

The solutions we consider are those where $\theta$ is a time constant [i.e. $r = r(\psi)$]. For convenience, we write $\theta$ with subscript ‘p’ (i.e. $\theta_p$). This is just to remind us that $\theta$ is not a variable in the equations of motion as this is a constant for a particular planet ‘p’. Hence

$$\frac{d^2u}{du^2} = \left[ \frac{1}{J^2 u^2} \frac{d\Phi(u, \theta_p)}{du} \right] \frac{du}{d\psi} + u = \frac{1}{J^2} \frac{d\Phi(u, \theta_p)}{du}.$$  

(18)
3 EINSTEIN’S SOLUTION

When Einstein applied his newly discovered GTR to the problem of the precession of the perihelion of the planet Mercury, he obtained that the trajectory of solar planets must be described by the equation

\[ \frac{d^2 u}{d\phi^2} + u - \frac{GM}{J^2} = \frac{3GMu^2}{c^2}, \]

(20)

where again \( u = 1/r \). To obtain a solution to this equation, we note that the left-hand side is the usual Newtonian equation for the orbit of planets:

\[ \frac{d^2 u}{d\phi^2} + u - \frac{GM}{J^2} = 0. \]

(21)

The solution to this equation is \( u = (1 + \epsilon \cos \varphi)/l \) where \( \epsilon \) is the eccentricity of the orbit and \( l = (1 - \epsilon^2)R \), where \( R \) is half the size of the major axis of the ellipse. Written in a different form, this solution is

\[ r = \left( \frac{1 + \epsilon}{1 + \epsilon \cos \varphi} \right) R_{\text{min}}, \]

(22)

where \( R_{\text{min}} \) is the planet’s distance of closest approach to the Sun (see Fig. 2 for an illustration). This solution is a good approximate solution to equation (20) because the orbit of Mercury is nearly Newtonian. Consequently, we can rewrite the small term on the right-hand side of equation (20) as \( 3GM(1 + \epsilon \cos \varphi)^2/l^2c^2 \). In so doing, we make an entirely negligible error. With this substitution, equation (20) becomes

\[ \frac{d^2 u}{d\phi^2} + u - \frac{GM}{J^2} = \frac{3GM}{l^2c^2} (1 + 2\epsilon \cos \varphi + \epsilon^2 \cos^2 \varphi). \]

(23)

The solution to this equation is

\[ u = \frac{1 + \epsilon \cos \varphi}{l} + \frac{3GM}{l^2c^2} \left( \frac{\epsilon^2}{2} + \frac{\epsilon^2 \cos 2\varphi}{6} + \epsilon \sin \varphi \right). \]

(24)

Of the additional terms, the first [i.e. \((1 + \epsilon^2/2)\)] is a constant and the second oscillates through two cycles on each orbit; both terms are immeasurably small. However, the last term increases steadily in amplitude with \( \varphi \), and hence with time, whilst oscillating through one cycle per orbit. Clearly, this term is responsible for the precession of the perihelion. Dropping all unimportant terms, we have

\[ u = \frac{1 + \epsilon \cos \varphi + \epsilon \eta \sin \varphi}{l}, \]

(25)

where \( \eta = 3GM/lc^2 \) is extremely small. Thus, all this leads us to

\[ u = \frac{1 + \epsilon \cos (\beta_E \varphi)}{l}. \]

(26)

where \( \beta_E = (1 - \eta) \). At the perihelion, we have \( \beta_E \varphi = 2\pi \), which implies \( \varphi = 2\pi + 6\pi GM/lc^2 \). Essentially, this means that the perihelion advances by \( \Delta \varphi = 6\pi GM/lc^2 \) per revolution. The resultant equation for the orbit is

\[ r = \frac{l}{1 + \epsilon \cos (\varphi + \Delta \varphi)}. \]

(27)

Thus, the rate of precession of the perihelion is given by

\[ \left\langle \frac{\Delta \varphi}{\tau} \right\rangle_{E} = \frac{6\pi GM}{\tau c^2 (1 - \epsilon^2) R}. \]

(28)

This is Einstein’s formula derived in 1916 soon after he discovered the GTR. In the paper containing this formula, Einstein concluded: ‘...Calculation gives for the planet Mercury a rotation of the orbit of 43° per century, corresponding exactly to the astronomical observation (Leverrier); for the astronomers have discovered in the motion of the perihelion of this planet, after allowing for disturbances by the other planets, an inexplicable remainder of this magnitude.’

4 SOLUTION FROM THE ASTG

For the present, we take the second-order approximation of the potential \( \Phi(r, \theta) \) in order to make our calculation for the precession of the perihelion of planetary orbits. This potential is

\[ \Phi(u, \theta) = -GMu \left[ 1 + \lambda_1 \left( \frac{GMu}{c^2} \right) \cos \theta \right. \]

\[ \left. + \lambda_2 \left( \frac{GMu}{c^2} \right)^2 \left( \frac{3\cos^2 \theta - 1}{2} \right) \right]. \]

(29)

As has already been said, we consider only those solutions for which \( \theta \) is a time constant [i.e. \( r = r(\varphi) \)]. So that we do not think of \( \theta \) as a variable, we have set \( \theta := \theta_p \). The solutions \( r = r(\varphi) \) are those solutions for which the orbit of a planet remains in the same \( \theta \)-plane. Now, from potential (29) we have

\[ \frac{d^2 u}{d\phi^2} + \left( \frac{J}{J^2 u^3} \right) \frac{du}{d\phi} + u = -GMu^2 \left[ 1 + \lambda_1 \left( \frac{2GMu \cos \theta}{c^2} \right) \right. \]

\[ \left. + \lambda_2 \left( \frac{3GMu}{c^2} \right)^2 \left( \frac{3\cos^2 \theta - 1}{2} \right) \right]. \]

(30)

Making the transformation \( r = 1/u \), the first term on the left-hand side of equation (30) transforms to

\[ \frac{d^2 u}{d\phi^2} + u - \frac{GM}{J^2} = \beta_1 u + \beta_2 u^2. \]

(32)
where
\[ \beta_1 = \left( \frac{GM}{J} \right)^2 \left( \frac{2\lambda_1 \cos \theta_p}{c^2} \right), \] (33)
and
\[ \beta_2 l = \lambda_2 \left( \frac{3GM}{c^4} \right) \left( \frac{GM}{J} \right)^2 \left( \frac{3 \cos^2 \theta_p - 1}{2} \right). \] (34)

The left-hand side of equation (32) is what we obtain from pure Newtonian theory and the term on the right is the new term due to the first-order term in the corrected Newtonian potential. Likewise, the second term on the right is a new term due to the second-order term in the corrected Newtonian potential.

Taking the term \( \beta_1 u \) in equation (32) to the right-hand side, we have
\[ \frac{d^2 u}{d\phi^2} + (1 - \beta_1) u - \frac{GM}{J^2} = \beta_2 l u^2. \] (35)

We know that the solution of the right-hand side of the above equation when set to zero, that is
\[ \frac{d^2 u}{d\phi^2} + (1 - \beta_1) u - \frac{GM}{J^2} = 0, \] (36)
is given by
\[ r = \frac{1}{1 + \epsilon \cos(\eta_1 \phi)}, \] (37)
where
\[ \eta_1 = \sqrt{1 - \beta_1} = \sqrt{1 - \left( \frac{GM}{J} \right)^2 \left( \frac{2\lambda_1 \cos \theta_p}{c^2} \right)}. \] (38)

To obtain a solution to equation (35) to first-order approximation, we note that the left-hand side has solution (37) and that for nearly Newtonian orbits this solution, \( u = (1 + \epsilon \cos \phi)/l \), is a good approximation to equation (35) for nearly Newtonian orbits, such as Mercury. Consequently, we can rewrite the small term on the right-hand side of equation (35) as \( 3 \frac{GM(1 + \epsilon \cos \phi)^2}{l^2} \). We make an entirely negligible error (see, for example, Kenyon 1990).

With this substitution, equation (35) becomes
\[ \frac{d^2 u}{d\phi^2} + \eta_1 u - \frac{GM}{J^2} = \frac{\beta_2}{l} (1 + 2\epsilon \cos \phi + \epsilon^2 \cos^2 \phi). \] (39)
The solution to this equation is
\[ u = \frac{1 + \epsilon \cos \eta_1 \phi + \epsilon \eta_2 \phi \sin \eta_1 \phi}{l} \left[ \left( 1 + \frac{\epsilon^2}{2} \right) + \frac{\epsilon^2 \cos 2\phi}{6} + \epsilon \phi \sin \phi \right]. \] (40)

As before (i.e. as in the steps leading to Einstein’s solution), of the additional terms, the first is a constant and the second oscillates through two cycles on each orbit; both terms are immeasurably small. However, the last term increases steadily in amplitude with \( \phi \), and hence with time, whilst oscillating through one cycle per orbit. Clearly, this term is responsible for the precession of the perihelion. Now, dropping all the unimportant terms, we are led to
\[ u = \frac{1 + \epsilon \cos \eta_1 \phi + \epsilon \eta_2 \phi \sin \eta_1 \phi}{l}. \] (41)

For convenience, here we have set \( \eta_2 = \beta_2 \). This quantity is extremely small, in which case \( \cos \eta_2 \phi \approx 1 \) and \( \sin \eta_2 \phi \approx \eta_2 \phi \). Using these approximations, in the cosine addition formula
\[
\cos \eta_1 \phi + \eta_2 \phi \sin \eta_1 \phi \\
\approx \cos \eta_1 \phi \cos \eta_1 \phi + \sin \eta_2 \phi \sin \eta_1 \phi \\
= \cos [(\eta_1 + \eta_2) \phi],
\]
we have
\[ u = \frac{1 + \epsilon \cos [(\eta_1 + \eta_2) \phi]}{l}. \] (42)

Now, at the perihelion, we have \( (\eta_1 + \eta_2) \phi = 2n \pi \) where \( n = 1, 2, 3, \ldots \), which implies \( \phi = 2n \pi (\eta_1 + \eta_2)^{-1} = 2n \pi [\sqrt{1 - \beta_1} + \beta_2]^{-1} \approx 2n \pi (1 - (\beta_1 - 2\beta_2)/2)^{-1} = 2n \pi (1 + (\beta_1/2 - \beta_2) + \ldots) \).

Hence, \( \phi \approx 2 \pi n + n \lambda_1 h_1 + n \lambda_2 h_2 \), where
\[ h_1 = \left( \frac{6\pi GM}{l c^2} \right) \left( \frac{\cos \theta_p}{2} \right), \] (43)
and
\[ h_2 = -\left( \frac{3 \cos^2 \theta_p - 1}{12\pi} \right) \left( \frac{6\pi GM}{l c^2} \right)^2. \] (44)

This shows that for every revolution, the perihelion advances by
\[ \frac{\Delta \phi}{\tau} = \left( \frac{6\pi GM}{l c^2} \right) \left( \frac{\lambda_1 \cos \theta_p}{3} \right) \left[ \lambda_2 \left( \frac{3 \cos^2 \theta_p - 1}{12\pi} \right) \left( \frac{6\pi GM}{l c^2} \right)^2 \right]. \] (45)

This formula, which is a second-order approximation, tells us of the perihelion shift of the planets. In the next section, we use this to deduce an estimate of the values of \( \lambda_1 \) and \( \lambda_2 \) for the Solar system. Thereafter, we proceed to calculate the predicted values of the perihelion shift. As a way of showing that these are solar values, let us denote \( \lambda_1 \) and \( \lambda_2 \) as \( \lambda_1^\odot \) and \( \lambda_2^\odot \), respectively.

5 AN ESTIMATE FOR \( \lambda_1^\odot \) AND \( \lambda_2^\odot \)

If \( P_p \) is the precession per century of the perihelion of planet ‘p’, that is
\[ P_p = \left( \frac{\Delta \phi}{\tau} \right) \left( \frac{1}{\phi_p} \right), \] (46)
then equation (45) can be written as
\[ P_p = A \phi_p^\odot + B \phi_p^\odot, \] (47)
where
\[ A_p = \left( \frac{\Delta \phi}{\tau} \right) \left( \cos \theta_p \right) \left( \frac{1}{3} \right), \] (48)
and
\[ B_p = -\left( \frac{3 \cos^2 \theta_p - 1}{12\pi \tau - 1} \right) \left( \left( \frac{\Delta \phi}{\tau} \right) \left( \cos \theta_p \right) \left( \frac{1}{3} \right) \right)^2. \] (49)

Given a set of observed values for the size (\( l_p \)), the period of revolution (\( \tau_p \)), the tilt (\( \theta_p \)) and the known precessional values of the perihelion of planets (\( P_p^\phi \)) – these values are listed in columns 2, 3, 4 and 8 of Table 1, respectively – we can solve for \( \lambda_1^\odot \) and \( \lambda_2^\odot \) since \( P_p, A_p \) and \( B_p \) are all known. Thus, we simply have to solve equation (47) for any pair of planets as a simultaneous equation.

The values of \( A_p \) and \( B_p \) for all the solar planets are listed in columns 6 and 7 of Table 1, respectively. It is important that we state that the values of the inclination listed in column 4 of Table 1 are the inclinations of the planetary orbits relative to the ecliptic plane. In order to compute the inclinations of these orbits relative to
the solar equator, we have to add $7$ to this, because the ecliptic plane and the solar equator are subtended at this angle. The solar equator is here defined as the plane cutting the Sun into hemispheres, and this plane is normal to the spin axis of the Sun.

Now, having calculated the values of $\lambda_1$ and $\lambda_2$, we have to use these values ($\lambda_1$ and $\lambda_2$) to check the predictions for the precession of the perihelion of the other seven planets. If the predictions of our theory are in agreement with the observed precession of the perihelion of these seven planets, then our theory is correct. If the predictions are otherwise, then our theory cannot be correct, and it must be wrong.

For the present, we have calculated $\lambda_1$ and $\lambda_2$ for the different planet pairs where we have all the information to do so. These values are displayed in Table 2. The final adopted values are

$$\lambda_1 = 24.0 \pm 7.0 \quad \text{and} \quad \lambda_2 = -0.200 \pm 0.100.$$  (50)

These values are the mean and the standard deviation; there is a 27 per cent error in $\lambda_1$ and about twice (50 per cent) that error margin in $\lambda_2$. From the values given in equation (50), the predicted values of the precession of the perihelion of the other seven planets (i.e. Earth, Mars, . . . , Pluto) were computed and are listed in column 10 of Table 1. The equivalent predictions of these values from Einstein’s theory are listed in column 9 of the same table.

An inspection of the predictions of our theory reveals that our predicted values are (as Einstein predicted) in good agreement with observations. We believe that this does not mean the theory is correct, but merely that it contains an element of truth. This means we have reason to believe in it and also reason to persevere it further from the present exploration to its furthest reaches, if this is at all possible.

It should be noted that, in our derivation, we have assumed as a first-order approximation the Newtonian result, namely that the angular momentum is a time constant. From the preceding section, this clearly is not the case. We have only assumed this as a starting point for our exploration. It is hoped that taking into account the fact arising from the ASTG, that orbital angular momentum is not a conserved quantity, should lead to improved results that hopefully come closer to the observed values.

## 6 Non-conserved Orbital Angular Momentum and Its Implications

Through equation (31), which clearly states that the orbital angular momentum of a planet must change with time, there are three immediate consequences: (i) a change in the mean Sun–planet distance; (ii) a change in the length of a planet’s day; (iii) a secular change in solar spin. In the following subsection, we discuss these implied phenomena.

### 6.1 Increase in mean Sun–planet distance

One of the most accurately determined physical parameters in astronomy is the mean Earth–Sun distance, which is about the size
of the astronomical unit (au) where \( 1 \text{ au} = 149597870696.1 \pm 0.1 \text{ m} \) (Pitjeva 2005). This is known to an accuracy of 10 cm (Pitjeva 2005). The astronomical unit, according to the International Astronomical Union (Resolution No. 10, 1976\(^1\)) is defined as the radius of an unperturbed circular orbit that a massless body would revolve about the Sun in \( 2\pi /k \) d, where \( k = 0.1720209895 \text{ au}^{3/2} \text{ d}^{-1} \) is the Gauss constant. This definition is such that there is an equivalence between the astronomical unit and the mass of the Sun \( M_\odot \), which is given by \( GM_\odot = k^2 A^3 \). So, if \( M_\odot \) is fixed, it is technically incorrect to speak of a changing astronomical unit.

Before it was noticed that the mean Earth–Sun distance was changing, it made perfect sense to refer to the mean Earth–Sun distance as the astronomical unit. Now, because units must not change, and because the mean Earth–Sun distance is changing, then, until such a time that the astronomical unit is correctly defined so that it is a true constant (as a physical unit must be), it makes sense to talk only of the mean Earth–Sun distance instead of the astronomical unit.

The change in the mean Earth–Sun distance has been measured by Krasinsky & Brumberg (2004) and Standish (2005). Krasinsky & Brumberg (2004) find 15.0 \( \pm \) 4.0 m cy\(^{-1} \), which in SI units is \((4.75 \pm 1.27) \times 10^{-9} \text{ m s}^{-1}\). Standish (2005) finds 7.00 \( \pm \) 0.20 m cy\(^{-1} \), which in SI units is \((2.22 \pm 0.06) \times 10^{-9} \text{ m s}^{-1}\), where 1 cy = 100 yr.

With this surprising result (i.e. the apparent secular change in the mean Earth–Sun distance), Iorio (2005) states that the secular increase in the mean Earth–Sun distance cannot be explained within the realm of classical physics. Contrary to this, we believe and hold that the ASTG can, in principle, explain this result. The ASTG is well within the provinces of classical physics, and thus this result can be explained from within the domains and confines of classical physics. Iorio (2005) argues that the Dvali–Gabadadze–Porrati braneworld scenario (a non-classical theory, which is a multidimensional model of gravity aimed at explaining the observed cosmic acceleration without dark energy) predicts, among other things, a perihelion secular shift, due to the Lue–Starkman effect of \( 5 \times 10^{-4} \text{ arcsec cy}^{-1} \) for all the planets of the Solar system (Lue & Starkmann 2003). It yields a variation of about 6 m cy\(^{-1} \) for the increase in mean Earth–Sun distance. This is compatible with the observed time rate of change of the mean Earth–Sun distance, hence giving some weight to the Dvali–Gabadadze–Porrati braneworld theory.

Iorio (2005) goes on to say that the recently measured corrections to the secular motions of the perihelia of the inner planets of the Solar system are in agreement with the predicted value of the Lue–Starkman effect for Mercury, Mars and, at a slightly worse level, the Earth. We show that, in principle, the ASTG can explain this result as a consequence of the secular increase of orbital angular momentum of planets in this azimuthally symmetric gravitational setting. The non-conservation of the orbital angular momentum leads directly to a time variation in the eccentricity of planetary orbits. This makes the secular change a purely classical result.

Now, given the definition of the eccentricity of an orbit,

\[
\epsilon^2 = 1 - \left( \frac{R_{\text{min}}}{R_{\text{max}}} \right)^2, \tag{51}
\]

where \( R_{\text{min}} \) and \( R_{\text{max}} \) are the spatial extent of the minor and major axes, respectively, and then differentiating this with respect to time, we find

\[
\frac{d\epsilon}{dt} = -\frac{R_{\text{min}}}{R_{\text{max}}^2} \left( \frac{dR_{\text{min}}}{dt} - \frac{R_{\text{min}}}{R_{\text{max}}} \frac{dR_{\text{max}}}{dt} \right). \tag{52}
\]

There is no reason to assume that the rate of change of the minor and major axes are the same. Thus, we must set

\[
\frac{dR_{\text{min}}}{dt} = (\gamma + 1) \frac{dR_{\text{min}}}{dt}, \tag{53}
\]

From this, it follows that

\[
\frac{d\epsilon}{dt} = -\left( \frac{R_{\text{min}}}{R_{\text{max}}^2} \right) \left[ 1 - (\gamma + 1) \frac{R_{\text{min}}}{R_{\text{max}}} \right] \frac{dR_{\text{min}}}{dt}. \tag{54}
\]

Multiplying by \( R_{\text{min}} \) both sides, and thereafter substituting \( R_{\text{min}}/R_{\text{max}} \) on the right-hand side, we have

\[
\epsilon R_{\text{min}} \frac{d\epsilon}{dt} = -\left( 1 - \epsilon^2 \right) \left[ 1 - (\gamma + 1) \sqrt{1 - \epsilon^2} \right] \frac{dR_{\text{min}}}{dt}. \tag{55}
\]

Therefore

\[
\frac{dR_{\text{min}}}{dt} = -\left( 1 - \epsilon^2 \right) \left[ 1 - (\gamma + 1) \sqrt{1 - \epsilon^2} \right] \left( \frac{d\epsilon}{dt} \right). \tag{56}
\]

On average, the rate of change of the minor axis must, to a large extent, be a good measure of the rate of change of the average distance \( \langle R \rangle \) between the planet and the Sun. Hence

\[
\frac{d \langle R \rangle}{dt} = \frac{\langle R \rangle}{1 - \epsilon^2} \left[ (\gamma + 1) \sqrt{1 - \epsilon^2} - 1 \right] \left( \frac{d\epsilon}{dt} \right). \tag{57}
\]

In the realm of Newtonian gravitation where spherical symmetry is assumed, thus producing equations only dependent on the radial distance \( r \), the eccentricity is an absolute time constant (i.e. \( d\epsilon/dt = 0 \)). This leads directly to \( d\langle R \rangle/dt = 0 \). Thus, when we find that the mean Earth–Sun distance is increasing, it comes more as a surprise. If we consider azimuthal symmetry in the Poisson equation, as has been done here, the result emerges naturally because the eccentricity is expected to increase with the passage of time; this we demonstrate shortly.

In Section 4, against the clear message from the ASTG, we have assumed that the orbital angular momentum of a planet is a conserved quantity. It turns out that taking this into account leads us to two types of orbit: (i) spiral orbits; (ii) normal elliptical orbits with the important difference that the eccentricity of these orbits varies with time (it is this variation of eccentricity in which we believe the secular increase of the mean Earth–Sun distance is rooted).

Taking into account the predicted change in the angular momentum, equation (35) will be

\[
\frac{d^2u}{d\phi^2} + \left( \frac{1}{J^2u^2} \frac{dJ}{d\phi} \right) \frac{du}{d\phi} + \frac{GM}{c} = \beta_2lu^2. \tag{58}
\]

Taking the change of angular momentum to first-order approximation from equation (31), we have

\[
\frac{dJ}{dr} = -\left[ \lambda_1 \left( \frac{GM}{c} \right)^2 \sin \theta_p \right] u^2 = -2\alpha u^2, \tag{59}
\]

where \( \alpha \) is clearly defined from this equation:

\[
\alpha = \frac{1}{2} \left[ \lambda_1 \left( \frac{GM}{c} \right)^2 \sin \theta_p \right]. \tag{60}
\]

It therefore follows that

\[
\frac{d^2u}{d\phi^2} - \frac{2\alpha u}{J^2} \frac{du}{d\phi} + \frac{GM}{c} = \beta_2lu^2. \tag{61}
\]
Writing $k = \alpha/J^2$, which is

$$k = \frac{\lambda_1}{2} \left( \frac{GM}{c^2} \right)^2 \sin \theta_p = \frac{\lambda_1}{2} \left( \frac{GM}{c^2} \right) \sin \theta_p, \quad (62)$$

where the Newtonian approximation $J^2 = GMJ^2$ has been used and $K = GM/J^2$, the above becomes

$$\frac{d^2 u}{d\psi^2} - k \frac{du}{d\psi} + \eta_1^2 u - K = \beta_2 l u^2. \quad (63)$$

If the orbital angular momentum varies constantly with time, then $J = Jt + J_0$ where $J_0$ is the angular momentum at time $t = 0$ and $J$ is a time constant, then $k = k(t)$ and $K = K(t)$. This means that $k(t)$ and $K(t)$ will be dependent not on the coordinates $r, \theta, \psi$ but only on time. Hence, in solving the above equation we can treat these as constants as they do not depend on $r, \theta, \psi$. We believe the assumption that $J = constant$ is justified because if this was not the case, there would be an accelerated increase in the orbital angular momentum, and this would have been noticed by now. With this assumption that $J = constant$, we must have $J$ being so small that it is not easily noticeable. This appears to be the case, as we have had to rely on delicate observations to deduce the secular increase of the mean Earth–Sun distance.

To obtain a solution to equation (63), first we need to obtain a solution to

$$\frac{d^2 u}{d\psi^2} - 2k \frac{du}{d\psi} + \eta_1^2 u - \frac{GM}{J^2} = 0. \quad (64)$$

To obtain a solution to this, first we need to solve

$$\frac{d^2 u}{d\psi^2} - 2k \frac{du}{d\psi} + \eta_1^2 u = 0, \quad (65)$$

and to its solution we add $GM/J^2$. The auxiliary differential equation to this differential equation is $X^2 - 2kX + \eta_1^2 = 0$ and the solutions to equation (65) are

$$X = k \pm \sqrt{k^2 - \eta_1^2} = k \pm i\eta_3, \quad (66)$$

where $\eta_3 = \sqrt{\eta_1^2 - k^2}$. If $|\eta_1|^2 \leq 0$ the solution is $u = A e^{i(k + \eta_3)\psi} + B e^{i(k - \eta_3)\psi}$ where $A$ and $B$ are constants. Thus, adding $GM/J^2$ we have

$$u = A e^{i(k + \eta_3)\psi} + B e^{i(k - \eta_3)\psi} + \frac{GM}{J^2}. \quad (67)$$

If $|\eta_1|^2 > 0$, the solution is $u = (A\psi + B) e^{i\psi}$. Thus, adding $GM/J^2$ we have

$$u = (A\psi + B) e^{i\psi} + \frac{GM}{J^2}. \quad (68)$$

Solutions (67) and (68) are clearly spiral orbits. These solutions are obviously very interesting. However, because our focus is not on these but on the solutions giving elliptical orbits in which the eccentricity varies, we do not look into these spiral orbit solutions any further than we have already done.

In the event that $|\eta_1|^2 > 0$, the solution to equation (64) is

$$u = \frac{1 + e^{i\psi} \cos(\eta_1\psi)}{1}. \quad (69)$$

Using the same strategy as that used in Sections 3 and 4 to solve equations (20) and (35), respectively, we find that the resultant orbit equation is

$$r = \frac{1}{1 + e^{i\psi} \cos[(\eta_2 + \eta_3)\psi]}. \quad (70)$$

As before, at the perihelion we have $(\eta_2 + \eta_3)\psi = 2\pi n$, which implies $\psi = 2\pi n (\eta_2 + \eta_3)^{-1} \simeq 2\pi n [\beta_2 + \sqrt{\eta_1^2 - k^2}]^{-1} = 2\pi n |\beta_2 + \sqrt{1 - \beta_1^2 - k^2}|^{-1} \simeq 2\pi n [1 + (2\beta_2 - \beta_1)/2 - k^2/2]^{-1}$. Taking only first-order terms we have $\psi \simeq 2\pi n [1 + (\beta_1 - 2\beta_2)/2 + k^2/2]$, which shows that the perihelion will precess by an amount $\Delta \psi = 2\pi [(\beta_1/2 - \beta_2) + k^2/2]$. In comparison with $\Delta \psi \simeq 2\pi [\beta_1/2 - \beta_2]$ obtained without taking into account the changing angular momentum, there is an additional precession of $\Delta \psi \simeq \pi k^2$. The value of $k^2$ for the Solar system is so small that, in practice, we can neglect it. Thus, we have not missed out much in our calculation, in which we have assumed a constant orbital angular momentum.

While this result is important, our main thrust is to deduce the variation of the eccentricity of elliptical orbits (we postpone any deliberations on this result for a later paper).

In equation (70), the term $e^{i\psi}$ in the denominator is the eccentricity. Let us write this as $e_\epsilon = e^{i\psi}$. From this we see that the eccentricity varies with time (i.e. as the orbital angular momentum changes with the passage of time, so does the eccentricity). Substituting this into equation (57), we can determine the variation of the mean Earth–Sun distance if we have knowledge of $\gamma$. Unfortunately we do not have this. However, if we are to reproduce the observed variation of the Earth–Sun distance, we find that if we set $\gamma_E = 1.48 \times 10^{-4}$, which practically means that the orbit grows evenly at every point, we are able to explain the secular increase of the mean Earth–Sun distance.

It should be said that if the ASTG is to stand on its own (i.e. independent of observations), then it must be able to explain the result $\gamma_E = 1.48 \times 10^{-4}$ from within its own provinces. It is for this reason that we say that, in principle, the ASTG is able to explain the secular increase in the mean Earth–Sun distance. Only until such a time when we are able to derive the value $\gamma_E = 1.48 \times 10^{-4}$ from within the theory itself, will we be able to say the ASTG explains the secular increase in the Earth–Sun distance.

Other than the secular increase in the mean Earth–Sun distance, there is also the increase in the mean Earth–Moon distance. This has been measured by Williams & Boggs (2009) to be $\sim 3.50 \times 10^{-3}$ m yr$^{-1}$, which in SI units is $1.11 \times 10^{-12}$ m s$^{-1}$. This provides a test for the ASTG, but unfortunately we do not have the value of $\lambda_3$ in order to check what the ASTG says about this. We believe we cannot use the same $\lambda$ values obtained for the Sun, because these values must be specific to the gravitating body and may very well be connected to the spin or the gravitating body in question. We are working on these ideas to improve the ASTG and at present we can only say it is prudent to assume that the $\lambda$ values are specific to the body in question, and hence we have to calculate these from observational data. For the Earth, this increase in the Earth–Moon distance is but the only observation we have in order for us to deduce $\lambda_3^E$. Hence, the ASTG is unable to make any predictions on this as it stands at present. We hope in the future we will be able to deduce a general form of the $\lambda$ values, thus placing the ASTG on a level where it is able to make predictions that are independent of observations.

It is important to note from $\epsilon_\epsilon = e^{i\psi}$ that, as $\psi \longrightarrow -\infty$, the eccentricity will decrease and the reverse is that the eccentricity will increase as $\psi \longrightarrow +\infty$ decreases. An increasing eccentricity leads to a secular decrease in the planet–Sun distance, and a decreasing eccentricity leads to a secular increase in the planet–Sun distance. This means that the sense in which the planet orbits the Sun is important. Because we believe from Krasinsky & Brumberg (2004) and Standish (2005) that there is a secular increase in the Earth–Sun distance, this means the current direction of rotation of the Earth around the Sun must be such that $\psi \longrightarrow -\infty$. This must be true for other planets rotating in the same sense as the Earth. For any object in the Solar system that rotates in a direction opposite to this,
this body will experience a secular decrease in its distance from the Sun.

6.2 Secular increase in the orbital period of planets

Given that the mean Earth–Sun distance (a distance supposed to be a sacrosanct parameter) is changing over time, and given that the rate of change of the specific orbital angular momentum is given by

\[ J = 2r\dot{r} + r^2 \dot{\theta} \] (i.e. the sum total of the spin angular momentum, \( S \), and the orbital angular momentum), then \( \dot{S} = -2r\dot{r} + r^2 \dot{\theta} \) becomes \( \dot{S} = -2r\dot{r} \). Inserting the relevant values for the Earth, we find \( T_1 \dot{Y} = 2.97 \text{ ms} \). Because \( T_1 \dot{Y} = 365.25T_0 \) (where \( T_0 \) is the period of an Earth day), it follows that \( T_1 \dot{Y} = 365.25T_0 \). Further, it follows that we must have \( T_1 \dot{Y} = 8.13 \mu s \), a value that is at odds with physical reality. Records held over for 2700 yr indicate that the Earth day changes by \( T_1 \dot{Y} = +1.70 \pm 0.05 \mu s \) (see, for example, Miura et al. 2009), which is about 200 times that expected if the orbital angular momentum were a conserved quantity, as in Newtonian gravitation. Clearly, this suggests that the orbital angular momentum may not be conserved.

If, say, the conserved quantity were the total angular momentum of a planet (i.e. the sum total of the spin angular momentum, \( S \), and the orbital angular momentum), then \( J = -\dot{S} \). Also, if the radius of the planet does not change over time, then \( T_1 \dot{Y} = -2\pi^2 J T_0^2 \).

For the Earth, we find that \( T_1 \dot{Y} = -5.18 \text{ s} \), which is about 3000 times the observed value. This cannot be possible, surely something must be wrong. We explain this observational value \( T_1 \dot{Y} = 1.70 \pm 0.05 \mu s \) using the ASTG.

Using the ASTG, we have

\[ \left( \frac{J}{J}_\Theta \right) = -(6.00 \pm 2.00) \times 10^{-12} \text{ s}^{-1}. \] (71)

We know that

\[ \left( \frac{J}{J}_P \right) = 2 \left( \frac{\dot{R}}{R} \right) \left( \frac{\dot{Y}}{Y} \right) \] (72)

Hence, inserting the observed values and remembering that for the Earth \( T_1 \dot{Y} = 365.25T_1^2 \), then we have

\[ \left( \frac{J}{J}_\Theta \right) = -(2.28 \pm 0.07) \times 10^{-15} \text{ s}^{-1}. \] (73)

The order of magnitude of this value is in good agreement with observations. We take this as further indication that the ASTG contains a grain of truth.

6.3 Secular increase in solar spin

We know that angular momentum must be conserved, but according to equation (61) it is not conserved. This lost orbital angular momentum must go somewhere, it cannot just disappear into the thin interstices of space–time or into the wilderness of space–time thereof. Let \( L_{\text{tot}} \) be the sum total angular momentum of the Solar system, where we consider that the Solar system is composed of the planets. If the sum total of the angular momentum of a planet and its system of satellites is \( J_{p}^{\text{tot}} \), then \( L_{\text{tot}} = M_{\odot} \Theta_{\odot} + \sum_i M_i J_{i}^{\text{tot}} \). We would expect the total angular momentum of the Solar system to be conserved (i.e. \( dL_{\text{tot}}/dt = 0 \)). From this, we must have

\[ \frac{S_{\odot}}{S_{\odot}} = \frac{M_{\odot}}{M_{\odot}} - \frac{1}{S_{\odot}} \sum_i \left[ \frac{M_i}{M_{\odot}} \left( \frac{dJ_{i}^{\text{tot}}}{dt} \right) \right], \] (74)

and \( dJ_{p}^{\text{tot}}/dt = dJ_{p}/dt \), and thus

\[ \frac{T_{\odot}}{T_{\odot}} = \frac{2R_{\odot}}{R_{\odot}} \frac{M_{\odot}}{M_{\odot}} + \frac{2\pi R_{\odot}^2}{\sum_i M_i} \frac{dJ}{dt}. \] (75)

This means that the orbital period of the Sun must be changing. If we assume that the Sun’s radius has remained constant over time (i.e. \( R_{\odot} = 0 \); which is certainly not true), then what we obtain from the above is a minimum value for the secular change in the Sun’s spin. The reason for invoking this assumption is because there is currently no information on the secular change of the Sun’s radius (see, for example, Miura et al. 2009). Hence, we make this assumption so that we can proceed with our calculation. As already said, what we obtain is not the exact secular change in the Sun’s spin but a lower limit of this.

The second term in equation (75) (i.e. \( M_{\odot}/M_{\odot} \)) represents the effect of solar mass loss, which can be evaluated in the following way. The Sun has a luminosity of at least \( 3.939 \times 10^{26} \text{ W} \), or \( 4.382 \times 10^{19} \text{ kg s}^{-1} \); this includes electromagnetic radiation and the contribution from neutrinos (Noerdlinger 2008). The particle mass-loss rate as a result of solar wind is \( \sim 1.374 \times 10^{10} \text{ kg s}^{-1} \) (see, for example, Noerdlinger 2008). From this information, it follows that \( M_{\odot}/M_{\odot} \approx 3.9.10^{-12} \text{ cm}^{-1} \).

The last term in equation (75) can be evaluated from the ASTG as \( J \) is known and we find that it is equal to \( \sim (4.00 \pm 1.00) \times 10^{-6} \text{ cm}^{-1} \); this implies \( T_{\odot} = 8.00 \pm 2.00 \text{ cm}^{-1} \). This result is a significant 10 times larger than the term emerging from the solar mass loss, so much that we can neglect this altogether and consider only the last term in equation (75). Thus, \( T_{\odot} = 8.00 \pm 2.00 \text{ cm}^{-1} \). This value is significantly larger than that calculated by Miura et al. (2009), who find a value of \( 21.0 \mu s \). Although the secular change in the period of the solar spin is not yet been measured exactly, it should however be possible and would settle the disagreement between the results of the ASTG and Miura et al. (2009).

Furthermore, Miura et al. (2009) propose that the Sun and the Earth are literally pushing each other away (leading to the increase in the Earth–Sun distance) as a result of their tidal interaction. They believe that this same process is what is gradually driving the Moon’s orbit outwards. They say: ‘Tides raised by the moon in our oceans are gradually transferring Earth’s rotational energy to lunar motion. As a consequence, each year the Moon’s orbit expands by about 4 cm and Earth’s rotation slows by about \( 30 \mu s \).’ Furthermore, Miura et al. (2009) assume that our planet’s mass is raising a tiny but sustained tidal bulge in the Sun. They calculate that, because of the Earth, the Sun’s rotation rate is slowing by \( 30 \mu s \). Thus, according to their explanation, the distance between the Earth and Sun is growing because the Sun is losing its angular momentum. The ASTG gives a different explanation altogether and this is, in our opinion, very interesting.

7 DISCUSSION AND CONCLUSIONS

We have considered the Poisson equation for empty space and we have solved this for an azimuthally symmetric setting, calling this new theory ASTG. From this solution, we have shown that the ASTG is capable of explaining certain observed (and yet to be observed) anomalies:
(i) the precession of the perihelion of planets;
(ii) the secular increase in the Earth–Sun distance;
(iii) the secular increase in the Eary year;
(iv) the secular decrease in solar spin;
(v) the fact that spiral orbits must exist.

One of the drawbacks of the ASTG as it currently stands is that it is heavily dependent on observations, as the values of $\lambda_e$ need to be determined from observations. With no knowledge of $\lambda_e$, we are unable to produce the numbers required to make any quantifications. Clearly, a theory incapable of making any numerical quantifications is useless, and this must be rectified. We shall make use of the solar values of $\lambda_e$ to shed some light on our current thinking on this (i.e. finding a general form for the constants $\lambda_e$). In the following, we make what we believe are reasonable suggestions and give our current view of the general form for these constants.

(i) First, if the constants $\lambda_e$ were all independent of each other, then the theory would clearly be very complicated. If we take as a guide the principle of Occam’s razor (which, in most if not all cases, leads to the simplest theory), then these constants must be dependent on each other in some way so as to reduce the labyrinth of complications. The simplest imaginable dependence is $\lambda_e = F(\ell)\lambda_1$. In this way, the entire system of constants $\lambda_e$ is dependent on just the one constant $\lambda_1$. This idea (that the system of constants be dependent on just one constant) is drawn from the theory of polynomial functions, where for a polynomial function $F(x) = \sum_{n=0}^{\infty} c_n x^n$, we can have ‘well-behaved’ polynomial functions for which the constants $c_n$ have a general form (i.e. where they depend on $n$, for example $e^x = \sum_{n=0}^{\infty} x^n / n!$). We envisage the function $\Phi(\ell, \theta)$ to be a ‘well-behaved’ function. By ‘well-behaved’, we simply mean that its system of constants, $\lambda_e$, is critically dependent on $\ell$, just as the constants $c_n$ depend on $n$.

(ii) Secondly, we could like that on a practical level, only the second-order approximation of the theory must suffice. This means that the terms $\ell > 3$ must be practically negligible. We have already shown that the second-order approximation of the ASTG is able to explain a large amount of anomalous observations. With the ASTG written in its second-order approximation (and as will be shown in a second paper, which we hope will be published in the present journal), we are able, without too much difficulty, to explain from this second-order approximation the emergence of molecular bipolar outflows in star-forming systems, as a gravitational phenomenon. If the other terms beyond the second-order approximation become practically significant, we will have difficulty explaining the outflows. So, in a way, we would like to state clearly that we want (albeit with a priori and a posteriori justification) to fine tune the theory so that it is able to explain the emergence of bipolar outflow. This is the strongest reason for wanting the terms for which $\ell > 3$ to be so small that in practice we can neglect them entirely.

(iii) Thirdly, and most importantly, the only data points we have of these constants are the determined values for the Sun (i.e. $\lambda_2^\odot = 24.0 \pm 7$ and $\lambda_2 = -0.2 \pm 0.1$). If logic is to hold (as it must), then our suggestion, $\lambda_e = F(\ell)\lambda_1$, must be able to explain this. We find that the following proposal

$$\lambda_e = \left[ \frac{(-1)^{\ell+1}}{((\ell)!)^2} \right] \lambda_1,$$  

meets (i), (ii) and (iii). We assume this result until such a time that evidence to the contrary is found. Checking on (iii) we see that within the error margins $\lambda_2^\odot \simeq \{(-1)^{\ell+1}/[(\ell^2)!(\ell^2)]\} \lambda_2^\odot$. Further checking on (ii), from equation (76) we have $\lambda_4 = 3.40 \times 10^{-30} \lambda_1$, which is practically small. This means that all terms for which $\ell > 3$ can, in practice, be neglected entirely.

If the above proposal proves itself to be correct, then the resultant theory will have just one undetermined parameter, $\lambda_1$. We are not going to try and deduce what this parameter depends on, but simply hint at our current thinking. We believe this parameter must depend on the angular frequency of the spin of the gravitating body in question. If we can find the correct dependence, then the ASTG will stand on its own, thus positioning itself to make testable predictions. We have left the task of making this deduction for our second paper.

The fact the we have deduced the crucial parameters $\lambda_2^\odot$ and $\lambda_2^\odot$ from experience means that we have in this paper carried out some reverse engineering. Normally, a theory must give these values and make clear predictions, just as when Einstein wrote down his equations. He found that his theory predicted a factor of 2 difference when compared to Newton’s theory when it came to the bending of light by the Sun. When applied to the Sun–Mercury system, it accounted very well for the then unexplained 43.0 arcsec per century for the precession of the perihelion of this planet; it just came out right. There were no free parameters that needed fitting, as is the case of the ASTG. As argued above, once a general form for $\lambda_e$ is found, this problem with the ASTG will be solved. Because we are able to obtain the values $\lambda_1^\odot$ and $\lambda_2^\odot$, which lead to acceptable values for the perihelion precession, this means that the values $A$ and $B$ are not random but systematic. If the theory were all wrong, then only luck would make the obtained values for $A$ and $B$ give values of $\lambda_1^\odot$ and $\lambda_2^\odot$ such that equation (47) gives, in general, acceptable values for the precession of the perihelion of the planets.

The values obtained from the ASTG for the precession of the perihelion of solar planets (as shown in column 10 of Table 1) are acceptable when weighed against the observational values listed in column 8 of Table 1. Given that we have taken into account the fact that the orbits of these planets do not lie in the same plane, this can hardly be a coincidence. Changing their inclination by just 1° will alter the predicted values of the precession of their perihelion.

Iorio (2005) states that the secular increase in the mean Earth–Sun distance cannot be explained within the realm of classical physics. Contrary to this, we believe that we have shown from within the provinces of classical physics that this result can be explained from within the domains and confines of classical physics. Previously, this observation appeared perhaps beyond the reach of classical physics because classical physics has not really considered gravitation as an azimuthally symmetric phenomenon, as in this paper. This suggests that the ideas presented here need to be explored further as they contain a grain of truth.

One interesting outcome, which has not been explored in this paper for fear of digression, is that the ASTG has a provision for spiral orbits (equations 67 and 68). These orbits occur when $(n_1)^2 \geq 0$. This condition implies the existence of a region $(r < R_{crit})$ in which spiral orbits will occur. Evaluating the inequality $(n_1)^2 \geq 0$ leads to $R_{crit} = (2\lambda_1 GM/c^2) \cos^2(\theta/2)$. From this, it is easy for us to deduce that spiral orbits are unlikely in the Solar system, as these will have to occur inside the Sun, because $R_{crit} \leq R_\odot$.

At this point, as we approach the end of this paper, we feel strongly that we must address the question: ‘does the spin along the azimuthal axis of a gravitating body induce an azimuthal symmetry into the gravitational field for this spinning body?’ To answer this, we must ask the question: ‘will a contracting non-spinning cloud of gas experience any bulge along its equator?’ First, we know that the equatorial bulge will occur on a plane perpendicular to the spin.
axis. As a non-spinning gas cloud will have no spin axis, there will be no spin axis about which the equatorial bulge will occur. If the material in the cloud is randomly and uniformly distributed, the cloud will exhibit a spherically symmetric distribution of mass and its gravitational field is expected to be spherically symmetric. A spherically spherically symmetric gravitational field is one that only has a radial dependence [i.e. \( \varphi = \varphi(r) \)].

If the gas cloud is spinning, the centrifugal forces will cause there to exist a disc and the material distribution will have an azimuthal symmetry [i.e. \( \rho = \rho(r, \theta) \)]. Should not this azimuthal symmetric distribution of matter induce an azimuthal gravitational field? From the Poisson equation (2), \( \rho = \rho(r, \theta) \) implies \( \Phi = \Phi(r, \theta) \). Should not \( \Phi = \Phi(r, \theta) \) also hold for a spinning gravitating body in a vacuum? From this, clearly, a spinning gravitating body ought to exhibit azimuthal symmetry. We argued in the last paragraph of Section 2 that the spin of a gravitating body breaks the existing spherical symmetry of the non-spinning body, and the above argument is an extension to this. It is from this that the paper's title, 'Azimuthally symmetric theory of gravitation', finds its justification.

If the ASTG turns out to be correct (as we believe it will), then we have an important question to ask: ‘what is the speed of light doing in a theory of gravitation because from equation (7) we see that the constants \( G \) and \( c \) are intimately connected? This is a similar (if not a congruent) question to that asked by Nieto & Anderson (2009) in their expository work on Earth Flyby Anomalies (AFAs). The empirical formula deduced to quantify EFAs contains in it the speed of light, \( c \). So, in their exposition of the phenomenon of AFAs, Nieto & Anderson (2009) have asked the eternal question: ‘what is the speed of light doing there?’ EFAs are thought to be a gravitational phenomenon, so what does the speed of light have to do with gravitation? If there is an intimate relationship between the speed of light and gravitation, then we must be forgiven for thinking that this suggests a link between gravitation and the theory of light – electromagnetism. The speed of light, \( c \), appears to be detrimental to the ASTG. Why not another value but the speed of light, \( c \)? We leave these matters unanswered.

In relation to the question, ‘what is the speed of light doing in a theory of gravitation’, we note that Newtonian gravitation (which requires instantaneous interaction as a postulate) does not imply the dependence of the gravitational potential on the azimuthal angle for a spinning body (as is the case in the ASTG), because at any instant \( t \), the gravitating body appears spherically symmetric. Here we have the speed of light \( c \) because of the azimuthal symmetry. Does this speed of light \( c \) link (or not) the propagation of the gravitational phenomenon to the speed of light? At present, we can only pose this as a question, for we still have further work to carry out on these ideas.

In closing, allow me to say that we find it hard to call what has been presented here ‘a new theory of gravitation’. When told that someone has come up with a new theory of gravitation, what immediately comes to mind is that a new principle has been discovered upon which gravitation can further be understood from the present understanding. The ASTG is not founded on any new physical principle but on the well-known Poisson equation. What we have done is simply to take the azimuthally symmetric equations of this equation and to apply them to gravitation. Based on this understanding, it is difficult to call it a new theory. However, the azimuthally symmetric equations of Poisson have brought new and exciting physics. Perhaps only because of this, the title of this paper is justified.

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