Regularities in frequency spacings of δ Scuti stars: the Kepler star KIC 9700322

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ABSTRACT

In the faint star KIC 9700322 observed by the Kepler satellite, 76 frequencies with amplitudes from 14 to 29 000 ppm were detected. The two dominant frequencies at 9.79 and 12.57 d$^{-1}$ (113.3 and 145.5 µHz), interpreted to be radial modes, are accompanied by a large number of combination frequencies. A small additional modulation with a 0.16 d$^{-1}$ frequency is also seen; this is interpreted to be the rotation frequency of the star. The corresponding prediction of slow rotation is confirmed by a spectrum from which $v \sin i = 19 \pm 1$ km s$^{-1}$ is obtained. The analysis of the spectrum shows that the star is one of the coolest δ Sct variables. We also determine $T_{\text{eff}} = 6700 \pm 100$ K and $\log g = 3.7 \pm 0.1$, compatible with the observed frequencies of the radial modes. Normal solar abundances are found. An $\ell = 2$ frequency quintuplet is also detected with a frequency separation consistent with predictions from the measured rotation rate. A remarkable result is the absence of additional independent frequencies down to an amplitude limit near 14 ppm, suggesting that the star is stable against most forms of non-radial pulsation. A low-frequency peak at 2.7763 d$^{-1}$ in KIC 9700322 is the frequency difference between the two dominant modes and is repeated over and over in various frequency combinations involving the two dominant modes. The relative phases of the combination frequencies show a strong correlation with frequency, but the physical significance of this result is not clear.

Key words: stars: abundances – stars: individual: δ Sct – stars: individual: KIC 9700322 – stars: oscillations – stars: rotation.

1 INTRODUCTION

The Kepler mission is designed to detect Earth-like planets around solar-type stars (Koch et al. 2010). To achieve that goal, Kepler is continuously monitoring the brightness of over 150,000 stars for at least 3.5 yr in a 105 deg$^2$ fixed field of view. Photometric results show that after 1 year of almost continuous observations,
pulsation amplitudes of 5 ppm are easily detected in the periodogram for stars brighter than $V = 10$ mag, while at $V = 14$ mag the amplitude limit is about 30 ppm. Two modes of observation are available: long cadence (29.4-min exposures) and short-cadence (1-min exposures). With short-cadence exposures (Gilliland et al. 2010), it is possible to observe the whole frequency range seen in δ Sct stars.

Many hundreds of δ Sct stars have now been detected in Kepler short-cadence observations. This is an extremely valuable homogeneous data set which allows for the exploration of effects never seen from the ground. Ground-based observations of δ Sct stars have long indicated that the many observed frequencies, which typically span the range 5–50 d$^{-1}$, are mostly μ modes driven by the κ-mechanism operating in the He ii ionization zone. The closely related γ Dor stars lie on the cool side of the δ Sct instability strip and have frequencies below about 5 d$^{-1}$. These are g modes driven by the convection-blocking mechanism. Several stars exhibit frequencies in both the δ Sct and γ Dor ranges and are known as hybrids. Dupret et al. (2005) have discussed how the κ and convective blocking mechanisms can work together to drive the pulsations seen in the hybrids.

The nice separation in frequencies between δ Sct and γ Dor stars disappears as the amplitude limit is lowered. Kepler observations have shown that frequencies in both the δ Sct and γ Dor regions are present in almost all of the stars in the δ Sct instability strip (Grigahcène et al. 2010). In other words, practically all stars in the δ Sct instability strip are hybrids when the photometric detection level is sufficiently low.

Statistical analyses of several δ Sct stars observed from the ground have already shown that the photometrically observed frequencies are not distributed at random, but that the excited non-radial modes cluster around the frequencies of the radial modes over many radial orders. The observed regularities can be partly explained by modes trapped in the stellar envelope (Breger, Lenz & Pamyatnykh 2009). This leads to regularities in the observed frequency spectra, but not to exact equidistance.

In examining the Kepler data for δ Sct stars we noticed several stars in which many exactly equally spaced frequency components are present. There are natural explanations for nearly equally spaced frequency multiplets such as harmonics and non-linear combination frequencies. In some of these stars, however, these mechanisms do not explain the spacings. In these stars there is often more than one exact frequency spacing and these are interleaved in a way which so far defies any explanation.

Some examples of equally spaced frequency components which remain unexplained are known from ground-based observations. The δ Sct star 1 Mon has a frequency triplet where the departure from equidistance is extremely small: only 0.000079 ± 0.000001 d$^{-1}$ (or 0.91 ± 0.01 Hz), yet the frequency splitting cannot be due to rotation because $\ell = 0$ for the central component and $\ell = 1$ for the other two modes (Balona et al. 2001; Breger & Kolenben 2006). In the β Cep star 12 Lac, there is a triplet with side peaks spaced by 0.1558 and 0.1553 d$^{-1}$. The probability that this is a chance occurrence is very small, yet photometric mode identification shows that two of these modes are $\ell = 2$ and the third is $\ell = 1$. This is therefore not a rotationally split triplet either (Handler et al. 2006).

One solution to these puzzling equally spaced frequencies could be non-linear mode interaction through frequency locking. Buchler, Goupil & Hansen (1997) show that frequency locking within a rotationally split multiplet of a rapidly rotating star (150 to 200 km s$^{-1}$) could yield equally spaced frequency splitting, which is to be contrasted to the prediction of linear theory where strong departures from equal splitting are expected.

In this paper, we present a study of the δ Sct star KIC 9700322 (RA = 19:07:51, Dec. = 46:29:12 J2000, $K_P = 12.685$). There are two modes with amplitudes exceeding 2000 ppm and several more than 1000 ppm. The equal frequency spacing is already evident in these large amplitude modes. This star does not fall in the unexplained category discussed above. It is, however, a remarkable example of a star in which combination frequencies are dominant.

The star has a large pulsational amplitude which can easily be observed from the ground. It was found to be variable in the 'All Sky Automated Survey' (Pigulski et al. 2009), where it is given the designation ASAS 190751+4629.2. It is classified as a periodic variable (PER) with a frequency of 7.79 d$^{-1}$. This is the 2 d$^{-1}$ alias of the main frequency (9.79 d$^{-1}$), which is determined below from the Kepler data. The Kepler data are, of course, not affected by daily aliasing. It was also examined during the 'Northern Sky Variability Survey' (Wozniak et al. 2004) with up to two measurements per night. Due to the short periods of the star, the 109 points of NSVS 5575265 were not suitable for a comparison with our results.

## 2 NEW OBSERVATIONS OF KIC 9700322

This star was observed with the Kepler satellite for 30.3 d during quarter 3 (BJD 245 5093.21 − 245 5123.56) with short cadence. An overview of the Kepler Science Processing Pipeline can be found in Jenkins et al. (2010). The field crowding factor given in the KAC is 0.016, which is about the average for the Kepler field. The data were filtered by us for obvious outliers. After prewhitening the dominant modes, a number of additional points were rejected with a four-sigma filter as determined from the final multi-frequency solution. 42990 out of 43103 data points could be used. We emphasize that most rejected points are extreme outliers and that the present conclusions do not change if no editing is performed. As can be expected from near-continuous set of observations with one measurement per minute, the spectral window is very clean with the second highest peak at 0.046 d$^{-1}$ and a height of 22 per cent relative to the main peak.

A small, typical sample of the Kepler measurements is shown in Fig. 1. Inspection of the whole light curve indicates that the pattern shown in Fig. 1 is repeated every 0.72 d. The repetition, however, is not perfect. This simple inspection already suggests, but does not prove, that most of the variability is caused by a few dominant modes and that additional, more complex effects are also present.

The Wozniak et al. (2004) Input Catalogue also does not list any photometry for this star, but some information on the spectral energy distribution is available. The spectral energy distribution was constructed using literature photometry: 2MASS (Skrutskie et al. 2006), GSC2.3 B and R (Lasker et al. 2008), TASS V and I (Droege et al. 2006) and CMC14 r′ (Evans, Irwin & Helmer 2002) magnitudes. Interstellar Na D lines present in the spectrum have equivalent widths of $60 ± 15$ mA and $115 ± 20$ mA for the D1 and D2 lines, respectively. The calibration of Munari & Zwitter (1997) gives $E(B − V) = 0.03 ± 0.01$.

The dereddened spectral energy distribution was fitted using solar-composition (Kurucz 1993a) model fluxes. The model fluxes were convolved with photometric filter response functions. A weighted Levenberg–Marquardt non-linear least-squares fitting procedure was used to find the solution that minimized the difference between the observed and model fluxes. Since the surface
3 CHARACTERIZATION OF THE STELLAR ATMOSPHERE

In order to classify the star with higher precision and to test the very low rotational velocity predicted by our interpretation of the pulsation spectrum in later sections, a high-dispersion spectrum is needed. KIC 9700322 was observed on 2010 August 12 with the High Resolution Spectrograph on the Hobby-Eberly Telescope at McDonald Observatory. The spectrum was taken at $R \sim 30,000$ using the 316g cross-disperser setting, spanning a wavelength region from 4120 to 7850 Å. The exposure time was 1800 s. A signal-to-noise ratio (S/N) of 194 was found at 593.6 nm. We reduced the data using standard techniques with IRAF\textsuperscript{1} routines in the echelle package. These included overscan removal, bias subtraction, flat-fielding, order extraction and wavelength calibration. The cosmic ray effects were removed with the L.A. COSMIC package (van Dokkum 2001).

The effective temperature, $T_{\text{eff}}$, and surface gravity, $\log g$, can be obtained by minimizing the difference between the observed and synthetic spectra. We used a fit to the H$\beta$ line to obtain an estimate of the effective temperature. For stars with $T_{\text{eff}} < 7000$ K the Balmer lines are no longer sensitive to gravity, so we used the Mg I triplet at 5167.321, 5172.684 and 5183.604 Å for this purpose.

The goodness-of-fit parameter, $\chi^2$, is defined as

$$\chi^2 = \frac{1}{N} \sum \left( \frac{I_{\text{obs}} - I_{\text{th}}}{\delta I_{\text{obs}}} \right)^2,$$

where $N$ is the total number of points and $I_{\text{obs}}$ and $I_{\text{th}}$ are the intensities of the observed and computed profiles, respectively. $\delta I_{\text{obs}}$ is the photon noise. The error in a parameter was estimated by the variation required to change $\chi^2$ by unity. The projected rotational velocity and the microturbulence were determined by matching the metal lines in the range 5160–5200 Å.

From this procedure we obtained $T_{\text{eff}} = 6700 \pm 100$ K, $\log g = 3.7 \pm 0.1$, $v \sin i = 19 \pm 1$ km s$^{-1}$, $\xi = 2.0 \pm 0.5$ km s$^{-1}$. In Fig. 2, we show the match to observed spectrum. The theoretical profiles were computed with SYNTH (Kurucz & Avrett 1981) using ATLAS9 atmospheric models (Kurucz 1993b). The solar opacity distribution function was used in these calculations. The effective temperature calculated from the spectrum is somewhat lower than that obtained by matching the spectral energy distribution discussed in the previous section. The difference is within the statistical uncertainties. We note that the spectrum showed no evidence for the presence of a companion.

Because of problems of line blending, we decided to use direct matching of rotationally broadened synthetic spectra to the observations in order to determine the projected rotational velocity. For this purpose, we divided the spectrum into several 100 Å segments. We derived the abundances in each segment using SYNTH minimization. We used the line lists and atomic parameters in Kurucz & Bell (1995) as updated by Castelli & Hubrig (2004).

Table 1 shows the abundances expressed in the usual logarithmic form relative to the total number of atoms $N_{\text{tot}}$. To more easily compare the chemical abundance pattern in KIC 9700322, Fig. 3 shows the stellar abundances relative to the solar values (Grevesse et al. 2010) as a function of atomic number. The error in abundance for a particular element which is shown in Table 1 is the standard error of the mean abundance computed from all the wavelength segments. This analysis shows that the chemical abundance in KIC 9700322 is the normal solar abundance.

The effective temperature determined for KIC 9700322 makes the star one of the cooler δ Sct stars. Other pulsators with similar gravity is poorly constrained by our spectral energy distribution, fits were performed for $\log g = 4.5$ and $\log g = 2$ to assess the uncertainty due to unconstrained $\log g$. A final value of $T_{\text{eff}} = 7140 \pm 310$ K was found. The uncertainties in $T_{\text{eff}}$ include the formal least-squares error and that from the uncertainties in $E(B - V)$ and $\log g$. We note here that in the next section values with considerably smaller uncertainties are determined from high-dispersion spectroscopy.

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Figure 1. A sample of the Kepler light curve covering 0.8 d. The multi-frequency fit is shown as a solid curve. The pattern shown here roughly repeats every 0.72 d.

\overfullrule=1em
\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{kepler_light_curve.png}
\caption{A sample of the Kepler light curve covering 0.8 d. The multi-frequency fit is shown as a solid curve. The pattern shown here roughly repeats every 0.72 d.}
\end{figure}
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Two portions of observed spectrum matched to a model with $T_{\text{eff}} = 6700$, $\log g = 3.7$ (red line). In the left-hand panel, we show the region around H$\beta$ that is sensitive to temperature, and in the right-hand panel the region around Mg I triplet sensitive to gravity.

**Table 1.** Abundances derived for KIC 9700322 expressed in term of $\log N(\text{el})/N_{\text{tot}}$.

| Element | $\log N(\text{el})/N_{\text{tot}}$ |
|---------|----------------------------------|
| C       | $-3.6 \pm 0.3$                   |
| O       | $-3.1 \pm 0.2$                   |
| Na      | $-5.8 \pm 0.3$                   |
| Mg      | $-4.4 \pm 0.4$                   |
| Si      | $-4.6 \pm 0.1$                   |
| Ca      | $-5.8 \pm 0.4$                   |
| Sc      | $-9.0 \pm 0.3$                   |
| Ti      | $-7.2 \pm 0.3$                   |
| V       | $-7.7 \pm 0.3$                   |
| Cr      | $-6.5 \pm 0.2$                   |
| Mn      | $-6.8 \pm 0.2$                   |
| Fe      | $-4.6 \pm 0.2$                   |
| Co      | $-6.8 \pm 0.2$                   |
| Ni      | $-5.8 \pm 0.3$                   |
| Sr      | $-8.6 \pm 0.1$                   |
| Y       | $-9.4 \pm 0.2$                   |
| Ba      | $-9.2 \pm 0.4$                   |
| Sr      | $-8.6 \pm 0.1$                   |
| Fe      | $-4.6 \pm 0.2$                   |

**Figure 2.** Two portions of observed spectrum matched to a model with $T_{\text{eff}} = 6700$, $\log g = 3.7$ (red line). In the left-hand panel, we show the region around H$\beta$ that is sensitive to temperature, and in the right-hand panel the region around Mg I triplet sensitive to gravity.

**Figure 3.** Abundance pattern derived for KIC 9700322.

temperatures are known, e.g. 6700 K for $\rho$ Pup (Netopil et al. 2008) and 6900 K for 44 Tau (Lenz et al. 2010).

## 4 FREQUENCY ANALYSIS

The *Kepler* data of KIC 9700322 were analysed with the statistical package *PERIOD04* (Lenz & Breger 2005). This package carries out multi-frequency analyses with Fourier as well as least-squares algorithms and does not rely on the assumption of white noise. Previous comparisons of multi-frequency analyses of satellite data with other techniques such as *SIGSPEC* (Reegen 2007) have shown that *PERIOD04* is more conservative in assigning statistical significances, leads to fewer (Poretti et al. 2009), and hopefully also fewer erroneous, pulsation frequencies, but may consequently also miss some valid frequencies.

We did not concern ourselves with small instrumental zero-point changes in the data since we have no method to separate these from intrinsic pulsation. Consequently, our solution contains several low frequencies in the region below 1 d$^{-1}$ which may only be mathematical artefacts of instrumental effects. The suspicion concerning the unreliable low frequencies is confirmed when comparing the present *PERIOD04* results with those from other period search programs and different data editing.

Following the standard procedures for examining the peaks with *PERIOD04*, we have determined the amplitude S/N values for every promising peak in the amplitude spectrum and adopted a limit of S/N of 3.5. The value of 3.5 (rather than 4) could be adopted because most low peaks do not have random frequency values due to their origin as combinations. This standard technique is modified for all our analyses of accurate satellite photometry: the noise is calculated from pre-whitened data because of the huge range in amplitudes of three orders of magnitudes.

After pre-whitening 76 frequencies, the average residual per point was 430 ppm. The large number of measurements (42990) lead to very low noise levels in the Fourier diagrams as computed by *PERIOD04*: 7 ppm (0–10 d$^{-1}$), 4.7 ppm (10–20 d$^{-1}$), 3.9 ppm (20–40 d$^{-1}$) and 3.6 ppm (40–200 d$^{-1}$). At low frequencies the assumption of white noise is not realistic.

Our analysis was performed using intensity units (ppm). The analysis was repeated with the logarithmic units of magnitudes, which are commonly used in astronomy. The differences in the results were, as expected, minor and have no astrophysical implications. The only small difference beyond the scaling factor of 1.0857 involved neighbouring peaks with large intensity differences, in which the weaker peak was in an extended ‘wing’ of the dominant peak: the effects are numerical from the multiple-least-squares solutions.
Regularities in frequency spacings

Figure 4. Amplitude spectra of KIC 9700322. The amplitudes (Table 2) are shown both linearly and logarithmically. The top-panel illustrates the dominance of the two excited frequencies and that standard ground-based photometry would only detect these and a few combinations.

Fig. 4 shows the results of the frequency search. KIC 9700322 has only six frequencies with amplitudes larger than 1000 ppm, of which only the two main frequencies are independent. Although a few ground-based campaigns lasting several years have succeeded in detecting statistically significant modes with smaller amplitudes, 1000 ppm can be regarded as a good general limit. Observed with standard ground-based techniques, the star would show few frequencies. In all, we find 76 statistically significant frequencies.

4.1 The observed frequency combinations

Most of the detected frequencies can be identified as parts of regular patterns (see Fig. 5). Visual inspection shows that the most obvious pattern is the exact spacing of $\delta f = 2.7763 \text{ d}^{-1}$. This is confirmed by statistical analyses of all possible frequency differences present in the data. However, this pattern is not continued over the whole spectrum, but is present as different patterns, repeated and interleaved several times. Consequently, a simple explanation in terms of a Fourier series (e.g. of a non-sinusoidal light curve) is not applicable.

Fig. 6 shows the echelle diagram using $2.7763 \text{ d}^{-1}$, which demonstrates the presence of remarkable patterns. Investigation of these patterns reveals that they originate in very simple frequency combinations and that the $2.7763 \text{ d}^{-1}$ is only a marker of the true explanation: combinations of the two dominant modes $f_1$ and $f_2$, as shown in Table 2. In fact, $2.7763 \text{ d}^{-1} = (f_2 - f_1)$.

The frequencies shown in the top panel of Fig. 5 can be expressed in a very simple way through the equation $f = mf_1 \pm nf_2$, where $m$ and $n$ are small integers. The fact that $f_1$ and $f_2$ are the two modes with the highest amplitudes makes this approach also physically reasonable (see below).

We also detect a frequency at $0.1597 \text{ d}^{-1}$ (called $f_3$). This frequency is important, since additional patterns are also seen: a number of peaks are separated by exactly the value of $f_3$ (see the middle panel of Fig. 5).

Altogether, 57 frequencies can be identified as numerical combinations and multiples involving $f_1$, $f_2$ and $f_3$ by comparing the observed to the predicted frequencies. We can essentially rule out accidental agreements. Let us consider the combination frequencies at frequencies larger than $3 \text{ d}^{-1}$, where the noise figures in the amplitude spectrum are reliable. For our identifications the average deviation between the observed and predicted frequency value is only 0.00021 $\text{d}^{-1}$. Such agreement is remarkable if one considers that the Kepler measurements used a time base of only 30 d and that $1/T = 0.03 \text{ d}^{-1}$ where $T$ is the time-span between the last and first observation. The present result is typical for Kepler satellite data.

If we use the least-squares frequency uncertainties calculated by PERIOD04, on average the observed agreement is 44 per cent better than predicted. However, such calculations assume white noise, which is not warranted. We can adopt the formulae given in Kallinger, Reegen & Weiss (2008) for the upper limit of the frequency uncertainty to include frequency-dependent noise. We calculated S/N in $5 \text{ d}^{-1}$ bins centred on each frequency with PERIOD04 using the pre-whitened spectrum. With this more realistic approach, the observed deviation of 0.00021 $\text{d}^{-1}$ is exactly a factor of 2 lower than the statistical upper limit. This supports our identifications.
4.2 The quintuplet

Five almost equidistant frequencies in the 11–12 d\(^{-1}\) range are also present together with various combinations of these frequencies with \(f_1\) and \(f_2\). This is shown in the bottom panel of Fig. 5.

4.3 Explanation of the echelle diagram

We can now explain the patterns seen in the echelle diagram (Fig. 6) in a simple manner. The vertical structures are the combination frequencies involving \(f_1\) and/or \(f_2\). They are displaced from each other because different low integers of \(m\) and \(n\) (in the equation \(f = mf_1 \pm nf_2\)) are involved. The horizontal structures with a slight incline correspond to the frequencies separated by \(\sim 0.13\) and \(0.16\) d\(^{-1}\), which are connected with the rotational frequency of 0.1597 d\(^{-1}\) through rotational splitting and modulation. The incline occurs because the small frequency differences between adjacent frequency values must show up in both the \(x\) and \(y\) directions of the diagram. Details on the values and identifications of the individual frequencies are listed in Table 2.

4.4 Additional frequencies

A few additional peaks have been identified, which are not related to \(f_1 - f_8\) in an obvious or unique manner. The lowest frequencies were already discussed earlier as probable zero-point drift and their values were dependent on how the data were reduced. The non-combination frequency at 51.75 d\(^{-1}\) has an amplitude of only 14 ppm. Calculation of the noise around the frequency gives a S/N of 3.6, which makes it a very marginal detection.

Fig. 7 shows the amplitude spectrum of the residuals after pre-whitening of the 76 frequencies. No peak is statistically significant and the overall distribution of amplitudes is typical of noise. Nevertheless, we have examined the highest (not significant) peaks, since a few of these may be real. Three peaks can be identified with expected values of additional combination frequencies, e.g. a peak at 8.129 d\(^{-1}\) can be fit by 2\(f_1\) - \(f_5\) at an amplitude S/N of 3.0.

5 DISCUSSION

Although this star was selected because of its very clear exactly equal frequency spacing, it turns out that the frequency spacing is explained as simple combination frequencies arising from non-linearities of the oscillation. This is different from another class of \(\delta\) Sct stars in the Kepler data base which also show exact frequency spacings, but in a manner which is not at present understood. Examples of this strange class will be presented in a separate paper.

What makes KIC 9700322 interesting is the remarkable way in which the large number of frequencies are related to the two main frequencies, \(f_1\) and \(f_2\). This behaviour is very similar to the high-amplitude \(\delta\) Sct star KIC 9408694, also discovered in the Kepler data base. The frequency patterns together with their amplitudes permit us to identify the different frequencies and to provide physical interpretations.
plume ratios tend to favour the interpretation in terms of amplitude modulation.

Based on this mode identification assumption we investigated representative asteroseismic models of the star. We have used two independent numerical packages: the first package consisted of the current versions of the Warsaw-New Jersey stellar evolution code and the Dziembowski pulsation code (Dziembowski 1977; Dziembowski & Goode 1992). The second package is composed by the evolutionary code CESAM (Morel 1997), and the oscillation code FILOU (Suárez 2002; Suárez, Goupil & Morel 2006b). Both pulsation codes consider second-order effects of rotation including near degeneracy effects.

The period ratio between the first radial overtone and fundamental mode mainly depends on metallicity, rotation and stellar mass. Moreover, the radial period ratio also allows for inferences on Rosseland mean opacities as shown in Lenz et al. (2010).

Indeed, an attempt to reproduce the radial fundamental and first overtone mode at the observed frequencies with the first modelling package revealed a strong dependence on the choice of the chemical composition and the OPAL versus OP opacity data (Iglesias & Rogers 1996; Seaton 2005). The best model found in this investigation was obtained with OP opacities and increased helium and metal abundances. Unfortunately, this model (\(T_{\text{eff}} = 7400\) K, log \(L/L_\odot = 1.27\), log \(g = 3.87\), 2M\(\odot\)) is much hotter than observations indicate. The disagreement in effective temperature indicates that this model is not correct despite the good fit of the radial modes.

As an additional test, by adopting the radial linear non-adiabatic models developed by Marconi & Palla (1998) and Marconi et al. (2004), we are able to reproduce the values of the two dominant frequencies with pulsation in the fundamental and first overtone modes, but with a lower period ratio (0.770) than observed. The best-fitting solution obtained with these models, for an effective temperature consistent with the spectroscopic determination and assuming solar chemical composition, corresponds to: \(M = 1.65\) M\(\odot\), log \(L/L_\odot = 1.1\), \(T_{\text{eff}} = 6700\) K, log \(g = 3.83\). We notice that for this combination of stellar parameters, both the fundamental and the first overtone mode are unstable in these models. Moreover, looking at the Main Sequence and post-Main-Sequence evolutionary tracks in the gravity versus effective temperature plane, as reported in fig. 4 of Catanzaro et al. (2011), the solution \(T_{\text{eff}} = 6700\) K, log \(g = 3.83\) is consistent with a 1.65 M\(\odot\) stellar mass.

However, as already noted, the period ratio in our models is lower than the observed value. To resolve this discrepancy, the possibility of low metallicity and rotation effects was examined in more detail with the second modelling package. Models between \(T_{\text{eff}} = 6200\) K and 8600 K with masses between 1.2 and 1.76 M\(\odot\) were found to represent a good fit of \(f_1\) and \(f_2\) as radial fundamental and first overtone, respectively. The best fit with the observations was found for \(M = 1.2\) M\(\odot\) models computed with \(\alpha_{\text{MLT}} = 0.5\), \(\delta_m = 0.1\) and a metallicity of [Fe/H] = −0.5 dex. Such a low value for the convection efficiency is in good agreement with the predictions by Casas et al. (2006) for \(\delta\) Sct stars, based on their non-adiabatic asteroseismic analysis. All these parameters roughly match the general characteristics of the \(\delta\) Sct stars with dominant radial modes and large amplitudes, despite being in the limit in metallicity.

The \(P_1/P_0\) period ratios predicted by these models (which simultaneously fit \(P_0\)) are near 0.775, which is lower than the observed ratio, 0.779. A period ratio of 0.775 is also obtained by adopting the radial linear non-adiabatic models by Marconi et al. (2004) at \(Z = 0.006\), according to which the best-fitting solution with effective temperature consistent with the spectroscopic
Table 2. Multi-frequency solution of KIC 9700322 and identifications. Frequencies are given in cycles d\(^{-1}\) and also in µHz. Amplitudes are in parts per million (ppm).

| Frequency (d\(^{-1}\)) | Amplitude (µHz) | Identification | Comment | Frequency (d\(^{-1}\)) | Amplitude (ppm) | Identification |
|-------------------------|-----------------|----------------|----------|-------------------------|-----------------|----------------|
| ±0.0001\(^1\)          | ±0.0001\(^1\)   | ±3\(^2\)        |          |                         |                 |                |

Main frequencies

| Frequency (d\(^{-1}\)) | Amplitude (µHz) | Identification | Comment |
|------------------------|-----------------|----------------|---------|
| 9.7925                 | 113.339         | \(f_1\)        | Dominant mode |
| 12.5688                | 145.472         | \(f_2\)        | Dominant mode |
| 0.1597                 | 1.848           | \(f_3\)        | Rotation, causes combinations |
| 11.3163                | 130.975         | \(f_4\)        | Quintuplet |
| 11.4561                | 132.593         | \(f_5\)        | Quintuplet |
| 11.5940                | 134.190         | \(f_6\)        | Quintuplet |
| 11.7200                | 135.648         | \(f_7\)        | Quintuplet |
| 11.8593                | 137.261         | \(f_8\)        | Quintuplet |

Combination frequencies

| Frequency (d\(^{-1}\)) | Amplitude (µHz) | Identification |
|------------------------|-----------------|----------------|
| 0.3194                 | 3.697           | \(2f_3\)       |
| 0.4791                 | 5.545           | \(3f_3\)       |
| 0.6388                 | 7.394           | \(4f_3\)       |
| 0.7095                 | 8.211           | \(f_2 - f_8\)  |
| 0.9748                 | 11.282          | \(f_2 - f_6\)  |
| 1.1127                 | 12.879          | \(f_2 - f_5\)  |
| 1.6636                 | 19.254          | \(f_3 - f_1\)  |
| 2.7763                 | 32.131          | \(f_3 - f_1\)  |
| 2.9360                 | 33.981          | \(f_2 - f_1 + f_3\) |
| 5.5526                 | 64.266          | \(f_2 - 2f_1\) |
| 7.0162                 | 81.206          | \(2f_1 - f_2\) |
| 9.4731                 | 109.643         | \(f_1 - 2f_3\) |
| 9.6328                 | 111.491         | \(f_1 - f_3\)  |
| 9.9522                 | 115.188         | \(f_1 + f_3\)  |
| 10.1119                | 117.036         | \(f_1 + f_2\)  |
| 12.2494                | 141.775         | \(f_2 - 2f_3\) |
| 12.4091                | 143.624         | \(f_2 - f_3\)  |
| 12.7285                | 147.321         | \(f_2 + f_3\)  |
| 12.8882                | 149.169         | \(f_2 + 2f_3\) |
| 15.3451                | 177.605         | \(2f_2 - f_1\) |
| 16.8088                | 194.546         | \(3f_2 - f_1\) |
| 19.5850                | 226.679         | \(2f_1\)       |
| 21.2486                | 245.933         | \(f_3 + f_1\)  |
| 21.3865                | 247.529         | \(f_3 + f_1\)  |
| 21.5125                | 248.987         | \(f_5 + f_1\)  |
| 21.6519                | 250.600         | \(f_8 + f_1\)  |
| 22.2016                | 256.963         | \(f_2 + f_3 - f_1\) |
| 22.3613                | 258.812         | \(f_1 + f_2\)  |
| 22.5210                | 260.660         | \(f_1 + f_2 + f_3\) |

Other peaks in the amplitude spectrum

| Frequency (d\(^{-1}\)) | Amplitude (µHz) | Identification |
|------------------------|-----------------|----------------|
| 0.0221                 | 0.256           | \(f_{66}\)  |
| 0.0555                 | 0.642           | \(f_{67}\)  |
| 0.1346                 | 1.558           | \(f_{68}\)  |
| 0.3542                 | 4.100           | \(f_{69}\)  |
| 12.5347                | 145.077         | \(f_{70}\)  |
| 12.5837                | 145.645         | \(f_{71}\)  |

\(^1\)Accuracy of frequencies determined experimentally (see Section 4.1), independent of amplitude. The numbers apply only to unblended frequency peaks. Because of the high quality of the Kepler data, the frequency accuracy is much better than the resolution calculated from the length of a 30.3 d run.

\(^2\)Determined by a multiple-frequency least-squares solution.

determination corresponds to \(Z = 0.006, Y = 0.25, M = 1.5, \log L/L_\odot = 1.04, T_e = 6700\ K, \log g = 3.83\). Again, the fundamental and first overtone modes are predicted to be simultaneously unstable for this parameter combination. We explored the possibility that such a discrepancy might be due to rotation effects, particularly second-order distortion effects, as discussed by Suárez, Garrido & Goupil (2006a) and Suárez, Garrido & Moya (2007). These investigations analyse theoretical Petersen Diagrams including rotation effects (Rotational Petersen Diagrams, hereafter RPDs), and show that \(P_1/P_0\) ratios increase as stellar surface rotation increases. The rotation rate derived from observations is slightly below 25 km s\(^{-1}\) (see Section 5.3). At such rotation rates near degeneracy effects on the period ratio are small (less than 0.001 in \(P_1/P_0\)). However, when non-spherically symmetric components of the centrifugal force are
considered, near-degeneracy effects may be larger, around 0.0025, causing the presence of wriggles in the RPDs [see fig. 5 in Suárez et al. (2007) and fig. 6 in Pamyatnykh (2003)]. Such effects are even more significant for rotational velocities higher than 40–50 km s⁻¹. Consequently, near-degeneracy effects may help to decrease the discrepancy between the observed period ratio and the slightly lower values predicted by the models.

If the star had a low metal abundance (close to Pop. II), a detailed analysis of RPDs might have provided an independent estimate of the true rotational velocity (and thereby of the angle of inclination). However, the spectroscopic analysis indicates that the star has a solar abundance. KIC 9700322 therefore represents a challenge to asteroseismic modelling, since it appears impossible to reproduce all observables simultaneously with standard models.

5.2 The combination frequencies

We have already shown that the 50+ detected frequency peaks can be explained by simple combinations of the two dominant modes and the rotational frequency. Several different non-linear mechanisms may be responsible for generating combination frequencies between two independent frequencies, \( f_1 \) and \( f_2 \). For example, any non-linear transformation, such as the dependence of emergent flux variation on the temperature variation \( (L = \sigma T^4) \), will lead to cross terms involving frequencies \( f_1 + f_2 \) and \( f_1 - f_2 \) and other combinations. The inability of the stellar medium to respond linearly to variations in the star.

The interest in the combination frequencies derives from the fact that their amplitudes and phases may allow indirect mode identification. For non-radial modes, some combination frequencies are not allowed depending on the parity of the modes (Buchler et al. 1997) which could lead to useful constraints on mode identification. Since \( f_1 \) and \( f_2 \) in KIC 9700322 are both presumably radial, there are no such constraints.

The identification of \( f_1 \) and \( f_2 \) with radial modes allows us to investigate the properties of the Fourier parameters of the combination modes with the aim to disentangle less obvious cases and/or solutions with a smaller number of combination terms. Buchler et al. (1997) show that a resonance of the type \( f_1 = nf_1 + nrf_2 \) leads to a phase \( \phi_r = \phi_1 - (n\phi_1 + r\phi_2) \). In the same way we may define the amplitude ratios \( A_r = A_r / (A_r A_1) \). To investigate how \( \phi_r \) and \( A_r \) behave with frequency, we first need the best estimate of the parent frequencies. We obtained these by non-linear minimization of a truncated Fourier fit involving \( f_1 \), \( f_2 \) and all combination frequencies up to the fourth order. The best values are \( f_1 = 9.792514 \) and \( f_2 = 12.568811 \) d⁻¹. The resulting amplitudes and phases are shown in Table 3 together with the values of \( \phi_r \) and \( A_r \). The phases were calculated relative to BJD 245 5108.3849 which corresponds to the mid-point of the observations.

Fig. 8 shows how \( A_r \) and \( \phi_r \) vary with frequency. From the figure we note that \( A_r \) is largest for \( f_1 + f_2 \), \( 2f_1 \), \( 2f_2 \) and \( f_2 - f_1 \) and very small for the rest. It is also interesting that \( \phi_r \) is a relatively smooth function of frequency, being practically zero in the vicinity of the parent frequencies, decreasing towards smaller frequencies and increasing towards higher frequencies. This result is almost independent of the choice of \( f_1 \) and \( f_2 \). The standard deviation of \( f_1 \) and \( f_2 \) is 0.0001 d⁻¹ using the Montgomery & O’Donoghue (1999) formula. One may arbitrarily adjust \( f_1 \) and \( f_2 \) in opposite directions by this value, and using the corresponding calculated values of the combination frequencies, fit the data to obtain new phases. The resulting \( \phi_r \) versus frequency remains monotonic, but the slope does change. The smooth monotonic nature of the \( \phi_r \) versus frequency diagram remains even for a change of 10 times the standard deviation in opposite directions for \( f_1 \) and \( f_2 \) and for arbitrary changes in epoch of phase zero. The result is clearly robust to observational errors, but it is not clear what physical conclusions may be derived from this result. The behaviour is certainly not random and must have a physical basis. Note that for simple trigonometric products, \( \phi_r \) will always be zero.

Finally, we note that the amplitudes of the combination modes relative to the amplitudes of their parents can be compared with values detected in the star 44 Tau (Breger & Lenz 2008). They agree to a factor of 2 or better, suggesting that KIC 9700322 is not unusual in this regard, just more accurately studied because of the Kepler data.

5.3 The quintuplet

In addition to the quintuplet structure around the two dominant modes another quintuplet with different properties is present in KIC 9700322 (see the listing of \( f_3 \) to \( f_8 \) in Table 4). The average spacing between the frequencies in this quintuplet is slightly smaller than the rotational frequency (0.1338 d⁻¹ versus 0.1597 d⁻¹). This
The location of the quintuplet near the centre in between the radial fundamental and first overtone mode rules out pure acoustic modes. For combination modes the frequency spacing must be appreciably larger than the characteristic frequency separation of the quintuplet as $f = 2$ mode. Moreover, this quintuplet is 0.164. For quadrupole modes the gravity-mode cavity. For such modes theory predicts a smaller oscillations of these modes in the deep interior leads to severe radiative damping. As a result, non-radial modes are increasingly damped for more massive $\delta$ Sct stars, which explains why high-amplitude $\delta$ Sct stars pulsate in mostly radial modes and why in even more massive classical Cepheids non-radial modes are no longer visible.

In general, we do not expect the frequencies in the $\delta$ Sct stars observed by Kepler to be regularly spaced because, unlike ground-based photometry, the observed pulsation modes are not limited to small spherical harmonic degree, $l$. For the very low amplitudes detected by Kepler we may expect to see a large number of small-amplitude modes with high $l$. The observed amplitudes decrease very slowly with $l$ and, all things being equal, a large number of modes with high $l$ might be expected to be seen in $\delta$ Sct and other stars (Balona & Dziembowski 1999). The $\delta$ Sct stars HD 50844 (Poretti et al. 2009) and HD 174936 (García Hernández et al. 2009) observed by CoRoT show many hundreds of closely spaced frequencies and may be examples of high-degree modes. Curiously, no counterparts of these two stars are known from Kepler. The relatively small number of independent frequencies detected in KIC9700322 stands in strong contrast to the two stars observed by CoRoT.

It should be noted that, unlike many $\delta$ Sct stars observed by Kepler, KIC9700322 does not have any frequencies in the range normally seen in $\gamma$ Dor stars. The only strong frequencies in this range are a few combination frequencies. Although we have identified significant frequencies below 0.5 d$^{-1}$, it is not possible at this stage to verify whether these are due to the star or instrumental artefacts. At present, we do not understand why low frequencies are present in so many $\delta$ Sct stars.

Regularities in the frequency spacing due to combination modes have already been observed from the ground even in low-amplitude $\delta$ Sct stars. An example is the star 44 Tau (Breger & Lenz 2008). Fig. 2 of Breger et al. (2009) demonstrates that all the observed regularities outside the 5–13 d$^{-1}$ range are caused by combination modes. For combination modes the frequency spacing must be absolutely regular within the limits of measurability. This is found for KIC9700322.

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