Hydro-mechanical coupling simulation of hydraulic fracturing considering stress-induced damage

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Abstract. The essence of hydraulic fracturing is rock fracture and rock damage under the fluid pressure effect, and the rock fracture mechanism is the main research field all the time. However, under the effect of induced stress and fluid pressure caused by hydraulic fracturing, the rock properties damage exits in accompany with fracture propagation, which has a great impact on the hydro-mechanical responses. In order to seek the effects of stress-induced damage on hydraulic fracturing, a hydro-mechanical coupling model simulating hydraulic fracturing is developed in this study. And as for the stress-induced damage during hydraulic fracturing, the stress-induced porosity and permeability model is taken into account. Then the hydro-mechanical responses during hydraulic fracturing and fracture interaction with natural crack and void are investigated. It is found that the stress-induced damage of permeability and porosity will contribution to the leak-off of the fracturing fluid, which can weaken the effectiveness of hydraulic fracturing. Besides, the stress-induced damage has some extent effect on the interaction patterns of hydraulic fracture with natural crack and void.

Keywords: Hydro-mechanical coupling; Hydraulic fracturing; Stress induced damage; Natural crack; Void

1 Introduction

In hydraulic fracturing, highly viscous fluid is injected to achieve stratum fracturing. And after propagation, hydraulic fracture with effective conductivity, which can be connected to natural heterogeneous reservoirs, was developed. Thus, a sharp increase of oil and gas is realized. Hence, hydraulic fracturing is a key technology in oil and gas stimulation. Recent years, simulations of hydraulic fracturing have been rapidly developed. In virtue of developments in computer technology and numerical calculation method, models for hydraulic fracture propagation have evolved from analytical models (KGD model [1,2], PKN model [3], pseudo-3D model [4]), semi-analytical models (liner network model [5,6], UFM model [7,8]) to numerical models (classical finite element method [9],
displacement discontinuous method [10-11], discrete element methods [12,13], extended finite element method [14-15]), and the theory has evolved from double wing seam theory to multi-factor theory, which considers the effects of natural fracture, stress shadow, hydro-mechanical coupling, thermal strain.

As for the numerical simulations of hydraulic fracture interacting with natural crack and void, there are many researches around natural crack [16-17], while the studies about void is rarely reported. Dong Joon Youn [18] established a two-dimensional model to simulate the effect of void on the propagation path of dry crack. Cheng et al [19] developed a numerical model of hydraulic fracture interacting with natural cave with the extended finite element method. Among the studies, it can be found that there are few about hydraulic fracture interaction with void. Besides, with the fracturing fluid injecting, as the pressure increases the effective stress of formation will be changed due to the poroelastic effect, which will cause a stress induced damage effect. The stress induced damage has a high effect on the matrix permeability and porosity, which will conversely affect hydraulic fracture propagation. Therefore, the mechanism of hydraulic fracture interaction with natural crack and void in consideration of the stress induced damage effect should be investigated.

In this paper, a hydro-mechanical coupling model is established to simulate hydraulic fracture interacting with natural crack and void. And the stress induced damage effect is taken into account, and the evolution equations of matrix permeability and porosity are developed. And the extended finite element method is used to solve the hydro-mechanical coupling model. In extended finite element method, the discontinuity of fracture and void is characterized using enrichment functions, and fracture is independent from the calculation mesh so that remeshing and re-modelling of rock mechanics parameters random field during fracture propagation are avoided. And with the proposed model, some numerical results are simulated and analyzed about the effects of stress induced damage on hydraulic fracture interaction with natural crack and void.

2 Governing model

2.1 Rock deformation

It is assumed that rock deformation is small strain in plane strain, and the stress equilibrium equation of porous medium is described as follow:

$$\nabla \cdot (\sigma' - \alpha I_m p_m) + f = 0$$  \hfill (1)

where $\sigma'$ refers to the effective stress tensor, $f$ refers to the body force vector, $\alpha$ is the Biot coefficient.

The boundary conditions consist of displacement boundary condition and stress boundary condition:

$$\sigma \cdot n = t \quad \text{in } \Gamma_s$$  \hfill (2)

$$u = \bar{u} \quad \text{in } \Gamma_u$$  \hfill (3)

The fluid pressure in fracture will act on the fracture surface.
\[ \sigma \cdot n_{r_f} = -p_f^+ \text{ in } \Gamma_f^+ \] (4)

\[ \sigma \cdot n_{r_f} = -p_f^- \text{ in } \Gamma_f^- \] (5)

2.2 Fluid flow

It is assumed that the injected fluid is incompressible, and fluid flow in matrix is described by Darcy’s law. The continuity equation of fluid seepage in matrix is written as:

\[ \frac{\phi_m}{K_m} \frac{\partial p_m}{\partial t} - \frac{k_m}{\mu} \nabla^2 p_m = q_f \] (6)

where \( v_m \) refers to the seepage velocity in matrix, \( k_m \) refers to the permeability of matrix, \( p_m \) refers to the fluid pressure in matrix, \( \mu \) refers to the fluid viscosity, \( \rho \) refers to the fluid density, \( \phi_m \) refers to the matrix porosity, \( q_f \) refers to the leak-off term, \( K_m \) refers to the bulk modulus of fluid.

The continuity equation of fluid flow in fracture can be given as:

\[ \frac{\phi_f}{K_c} \frac{\partial p_f}{\partial t} + \nabla \left( \rho \cdot v_f \right) + q_f = 0 \] (7)

where \( v_f \) refers to the seepage velocity in fracture, \( k_f \) refers to the permeability of fracture, \( p_f \) refers to the fluid pressure in fracture, \( \phi_f \) refers to the porosity of fracture system.

And in consideration the effects of stress induced damage on the matrix permeability and porosity, the permeability and porosity of matrix evolution is given as:

\[ \phi_m = \left( \phi_{m0} - \phi_0 \right) \exp \left( \alpha_i \sigma_i \right) + \phi_m \] (8)

\[ k_m = k_{m0} \exp \left[ -a_i \left( \sigma_{\text{eff}} - \alpha p_m \right) \right] \] (9)

where, \( \phi_{m0} \) is the initial porosity. \( \alpha_i \) is stress sensitivity coefficient. \( \sigma_i \) is the effective mean stress, \( \sigma_{\text{eff}} = (\sigma_x + \sigma_y)/2 + \alpha p_m \). \( k_{m0} \) is the initial permeability. \( a_1 \) is the constant.

2.3 XFEM discretization

The XFEM displacement approximation consists of continuous and discontinuous parts. The continuous part is consistent with that in conventional finite element, while the discontinuous part is added enrichment functions.

\[ u(x) = \sum_{i \in N^c} N_i(x) u_i + \sum_{i \in N^{\text{dis}}} N_i(x) \left[ H(x) - H(x_i) \right] a_i \]

\[ + \sum_{i \in N^{\text{tip}}} \sum_{a=1}^{4} N_i(x) b^a \left[ B_a(x) - B_a(x_i) \right] + \sum_{i \in N^{\text{void}}} N_i(x) H^{\text{vo}}(x) c_i \] (10)

where \( N^c \) refers to the standard nodes set; \( N^{\text{dis}} \) refers to nodes enriched by Heaviside function; \( N^{\text{tip}} \) refers to the nodes enriched by fracturing tip enrichment function; \( N^{\text{void}} \) refers to the nodes enriched by void enrichment function; \( N_i \) refers to the finite element shape function; \( R^a(x) \) refers to the ramp function, which can effectively improve the accuracy reduction problem due to the blending element; \( H^{\text{vo}} \) refers to the void enrichment function; \( u_i \) refers to the nodal displacement; \( a_i, b^a \) and \( c_i \) refer to the
additional freedom degrees of the enrichment nodes.

With the divergence law, the weak form of stress field can be written as:

\[
\int_{\Omega} \delta \varepsilon_{ij} \sigma_{ij} d\Omega - \int_{\Gamma} \delta \varepsilon_{ij} \alpha I_m p_i d\Gamma - \int_{\Gamma_f} \delta u_i \tilde{p}_f \cdot \mathbf{n}_i d\Gamma - \int_{\Gamma_f} \delta u_i p_f \cdot \mathbf{n}_i d\Gamma = 0
\]  

(11)

The pressure approximation and temperature approximation is expressed as.

\[
p(x) = \sum_{i \in N^e} \bar{N}_i(x)p_i + \sum_{j \in N^f} \bar{N}_j(x)\left[H(x) - H(x_i)\right]d_j
\]

\[+ \sum_{i \in N^e} \bar{N}_i(x)\sum_{a=1}^n [\bar{B}_a(x) - \bar{B}_a(x_i)]f_i^a\]

(12)

The weak form of fluid flow in fracture and matrix can be written as follow:

\[
\int_{\Omega} k_m \mu \left[\delta \left(\frac{\partial^2 p_m}{\partial x^2}\right) + \delta \left(\frac{\partial^2 p_m}{\partial x^2}\right)\right] d\Omega + \int_{\Omega_f} k_f \mu \left[\delta \left(\frac{\partial^2 p_f}{\partial x^2}\right) + \delta \left(\frac{\partial^2 p_f}{\partial y^2}\right)\right] d\Omega
\]

\[+ \int_{\Omega_f} \delta \left[p \frac{\phi_f}{K_f} \frac{\partial p_f}{\partial t}\right] d\Omega + \int_{\Omega_f} \delta \left[p \frac{\phi_m}{K_m} \frac{\partial p_m}{\partial t}\right] d\Omega + \int_{\Gamma_f} \delta p q_f d\Gamma = 0
\]

(13)

By substituting XFEM approximations into the weak forms, and the discrete XFEM equations are given as follows.

\[Ku = F\]

(14)

\[
\left[\Delta t (H_m + H_f) + (D_m + D_f)\right]p^{n+1} = \Delta t (Q + (D_m + D_f)p^n)
\]

(15)

where \(K\) refers to the global stiffness matrix, \(F\) refers to the global force vector, \(H_m\) and \(H_f\) refer to global seepage matrix of the matrix system and the fracture system, respectively; \(D_m\) and \(D_f\) refer to global compressibility matrix of the matrix system and the fracture system, respectively; \(FQ\) refers to the global flow vector. \(\Delta t\) refers to the time step; \(p^{n+1}\) refers to the fluid pressure at the step of \(n+1\); \(p^n\) refers to the reservoir pressure at the step of \(n\). As for the stress induced damage on matrix permeability and porosity, the detailed algorithm is set as follows.

I. Assume the initial pressure and other simulation parameters.

II. Solve Eq (14) to obtain the displacement, stress and fracture width distributions by using the pressure at last step times.

III. Update the matrix permeability and porosity evolution in consideration of stress induced damage with Eq (8) and Eq (9).

IV. Solve Eq (15) with the displacement field and updated permeability and porosity evolution.

V. Determine the hydraulic fracture propagation and update the fracture enrichment types.

2.4 Fracture propagation criterion

In this paper, maximum energy release rate criterion was employed to determine the fracture propagation. For maximum energy release rate criterion, fracture can only propagate when the energy release rate at fracture tip exceeds the critical energy release rate of rocks, and the fracture propagation direction is along the direction of the maximum circumferential stress.
\[
\theta_c = 2 \arctan \left( \frac{K_I}{4K_{II} - \text{sign}(K_{II})\sqrt{\left(\frac{K_I}{K_{II}}\right)^2 + 8}} \right)
\]

where \( \theta_c \) refers to propagation direction in polar coordinate system at fracture tip; \( K_I \) refers to the stress intensity factor of mode I; \( K_{II} \) refers to the stress intensity factor of mode II; \( \text{sign} \) refers to the sign function. In order to test the proposed mode, KGD model is used to contrast. The simulation parameters are: Young’s modulus 25GPa, Poisson's ratio 0.2, injection rate 0.00045 m²/s, fluid viscosity 0.1 mPa·s. And the comparison of the fracture length and width of two models is shown in Figure 1 and Figure 2. It can be seen that calculation results of two models show a great agreement.

3 Numerical results

With the proposed model, this section simulated hydraulic fracture interaction with natural crack and void when considering the stress induced damage effect. And the effects of stress induced damage on interaction patterns of hydraulic fracture intersecting with natural crack and void were discussed.

3.1 Hydraulic fracture interaction with natural crack

Firstly, hydraulic fracture intersecting with natural crack is studied. And the simulation parameters are fixed as follows: the maximum horizontal stress 3MPa, the minimum horizontal stress 1MPa, Young’s modulus 2GPa, Poisson's ratio 0.23, injection rate of fracturing fluid 0.0015m/s, fluid viscosity 0.1 mPa·s. As shown in Figure 2, the model is subjected to the maximum horizontal stress and minimum horizontal stress. And the left boundary of the domain prevents from rolling and right boundary is constrained. And hydraulic fracture is located in the center of the left boundary.
Figure 2. The diagrammatic sketch of hydraulic fracture interacting with natural crack.

The stress distributions at different orientation of natural crack \((\theta=90^\circ, 45^\circ)\) when uncoupling and coupling stress induced damage are presented in Figure 3 and Figure 4, respectively. It can be found that the stress induced effect has a great impact on the interaction patterns between hydraulic fracture and natural crack. Hydraulic fracture is arrested by natural crack and stops propagating when uncoupling stress induced effect, while natural crack is activated and hydraulic fracture will propagate along natural crack when coupling stress induced effect. And for the case that couples stress induced effect, as the pressure constantly rises with the injection of fracturing fluid, the volumetric strain shall vary due to the poroelastic effect, which can contribute to the increase of matrix permeability and porosity. The increase of permeability and porosity can promote fracturing fluid leak-off and the further rise of pore pressure, and the increase of pore pressure around natural crack as well as the induced stress of hydraulic fracture tip can contribute to the activation of natural crack. Therefore, it is of great significance take the stress induced damage effect into account when investigate hydraulic fracture intersecting with natural crack.

(a) Uncoupling stress induced damage   (b) Coupling stress induced damage

Figure 3. The stress distributions for uncoupling and coupling stress induced damage \((\theta=90^\circ)\).

(The unit of legend: MPa)
3.2 Hydraulic fracture interaction with void

The effects of stress induced damage on hydraulic fracture interaction with void were analyzed, and the simulation parameters are set as: the horizontal stress difference 5MPa, Young’s modulus 10GPa, Poisson's ratio 0.23, injection rate of fracturing fluid 0.01m/s, fluid viscosity 1mPa·s. The physics model is shown in Figure 5, and size of the model is 50m×50m. The model is subjected to the maximum horizontal stress and minimum horizontal stress. The left boundary of the domain prevents from rolling and right boundary is constrained. And hydraulic fracture is located in the center of the left boundary.

The stress distribution and pressure at different void location are displayed in Figure 6 and Figure 7, respectively. One can see that the stress induced damage effect will highly affect interaction patterns of hydraulic fracture and void. The further improvement of fracturing fluid leak-off and pore pressure would weaken the fracture pressure. And for the horizontal void in Figure 6, the stress induced damage effect will lead to the diversion of hydraulic fracture, but hydraulic fracture is connected to the void for two cases. However, as for the deviated void in Figure 7, the stress induced damage effect alters the interaction patterns. The leak-off region for coupling stress induced damage is much larger than the case of uncoupling stress induced damage, while the pressure in fracture is smaller. And it can be inferred that the stress induced damage can weaken the stress interference of void, then hydraulic fracture propagates along the void finally when coupling stress induced damage.
Figure 6. The stress distributions for uncoupling and coupling stress induced damage for the horizontal void. (The unit of legend: MPa)

Figure 7. The pressure distributions for uncoupling and coupling stress induced damage for the deviated void. (The unit of legend: MPa)

4 Conclusions
This paper established a hydro-mechanical coupling model to simulate hydraulic fracture interaction with natural crack and void in consideration of the stress induced damage effect. The extended finite element method is used to describe the discontinuity of fracture and void, and the hydro-mechanical coupling model is solved by extended finite element method as well. And some numerical simulations are studied and discussed.

The simulations present that the stress induced damage can improve the matrix permeability and porosity around fracture, and promote the leak-off of the fracturing fluid. The stress induced damage effect has a high impact on the interaction patterns of hydraulic fracture with void and natural crack. The stress induced damage effect will contribute to the activation of natural crack, and hydraulic fracture is inclined to propagate along natural crack. Also, hydraulic fracture maybe turns around and fails to connect to void when coupling stress induced damage effect.

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