Electron affinity of the hydrogen atom and a resonance state of the hydrogen negative ion embedded in Debye plasmas

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Abstract. The 2s2 1S0 autoionization resonance state of the hydrogen negative ion embedded in Debye plasmas is determined by calculating the density of resonance states using the stabilization method. The electron affinity of the hydrogen atom is also estimated for various Debye lengths. A screened Coulomb potential obtained from the Debye model is used to represent the interaction between the charged particles. A correlated wave function consisting of a generalized exponential expansion has been used to represent the correlation effect between the three charged particles. The screening effect is taken care of for all pairs of the charged particles. The calculated resonance energies and widths for various Debye parameters ranging from infinity to a small value along with the electron affinity are reported.

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1. Introduction

In recent years, the ground state and the resonance state of two-electron atoms embedded in Debye plasma environments have been interesting topics of research. The energy levels for one-electron systems have been reported in several studies with screened Coulomb potentials. Two relevant studies for such systems are the calculations of the bound eigenstates of the static screened Coulomb potential by Rogers et al [1] and the variational solutions of Schrödinger’s equation for the static screened Coulomb potential by Roussel and O’Connell [2]. Investigations on two-electron systems in plasma networks play an important role because correlation effects between the charged particles can be studied. In such environments, the interaction between the electrons and the atomic nucleus is screened, as well as that between the electrons themselves. Several theoretical studies have been reported on the ground state of two-electron atoms for various Debye lengths [3]–[8]. Dai et al [9] performed a calculation to investigate the properties of the screened He-like systems using correlated wave functions. In a study on the influence of Debye screening on the spectra of two electrons, Mukherjee et al reported the ground and first three excited states of He-like atoms [10]. Very recently, the resonance state of H− [11], He [8] and Ps− [12] in the Debye plasma environments have been investigated for the first time by Kar and Ho. It is worth mentioning in this context that an investigation of the analytic method for the calculation of shape resonances for a model problem has been performed by Wang and Winkler [13]. The electron affinity of Ps in plasma environments has also been reported by Saha et al [14], and by Kar and Ho [12]. The works of Stein et al [15] have focused on the screened Coulomb potentials appropriate for the scattering in the field of an atom.

In the present work, an investigation has been made on the ground state and the resonance state of H−. The ground state energies of H− in Debye plasmas have been obtained in an accurate variational calculation in the framework of the Rayleigh–Ritz variation principle. The 2s2 1S0 autoionization resonance of H− in Debye plasma environments has also been investigated. In [11], the screening effect was applied only to electron–nucleus pairs, but not explicitly to the electron–electron pair. Instead of considering the screening on the electron–electron interactions [11], a multiplicative factor \( b \) was used with the interaction terms of the electron–nucleus pairs to reproduce the ground-state energies reported by Winkler [5] in which the screening effect on the electron–electron pair was also included. In the present work, the screening effect is applied to all pairs of charged particles. Also, in [11], a correlated Hylleraas basis was used. In the present work, correlated wave functions consisting of exponential expansions are used. Using such an exponential basis enables us to work out the necessary integrals for the screened Coulomb potentials in a straightforward manner. The inclusion of electron–electron screening produces results that are lower than those reported in [11] for various Debye lengths. In the present work, we have also been able to extract resonance parameters for stronger screening with the inclusion of the electron–electron screening effects. We use a model of screened Coulomb potential [5, 8] of Debye-type, and employ the stabilization method proposed by Mandelshtam et al [16] to calculate the resonance energy \( E_r \) and the width \( \Gamma \) of the 2s2 1S0 autoionization state of H− embedded in Debye plasmas. The details of this simple and powerful method to calculate resonance parameters have been discussed in [11, 17]. Here, the correlated wave function expanded in terms of product basis sets involving inter-particle coordinates have been used to represent the correlation effect between the three charged particles. The convergence of our calculations has been examined with an increasing number of terms in the basis expansion. The electron affinity of the hydrogen atom is also
estimated for the various Debye parameters. Throughout the present work, the atomic unit (au) has been used, and all calculations are performed in quadruple precision (32 significant figures) on DEC-ALPHA workstations.

2. The method

The non-relativistic Hamiltonian describing the hydrogen negative ion embedded in Debye plasmas characterized by the parameter $D$ is given by [5]

$$H = -\frac{1}{2} \nabla_1^2 - \frac{1}{2} \nabla_2^2 - \left[ \frac{\exp(-r_1/D)}{r_1} + \frac{\exp(-r_2/D)}{r_2} \right] + \frac{\exp(-r_{12}/D)}{r_{12}},$$

(1)

where $r_1$ and $r_2$ are the radial coordinates of the two electrons and $r_{12}$ is their relative distance. A particular value of the screening parameter $D$ corresponds to the range of plasma conditions, as the Debye parameter is a function of electron density and electron temperature. The smaller values of $D$ are associated with stronger screening. A parametrized screening potential approximated the effects of the plasma charges on the interaction between the bound electron and the atomic nuclei.

For the $^1S^o$ states of $H^-$, we have considered the wave function

$$\Psi = (1 + P_{12}) \sum_{i=1}^{N} C_i \exp\left[(-\alpha_i r_1 - \beta_i r_2 - \gamma_i r_{12})\omega\right],$$

(2)

where $\alpha_i, \beta_i, \gamma_i$ are the nonlinear variational parameters, $C_i (i = 1, \ldots, N)$ are the linear expansion coefficients, $\omega$ is a scaling constant to be discussed later in the text, and $P_{12}$ is the permutation operator defined by $P_{12} f(r_1, r_2, r_{12}) = f(r_2, r_1, r_{12})$. To obtain the ground-state energy of $H^-$ we first set $\omega = 1$. The wave functions of equation (2) have been widely used in several bound-state calculations of two-electron systems in model plasma environments (see [5, 7, 8], [10]–[12], [14] and references therein). The nonlinear variational parameters $\alpha_i$, $\beta_i$ and $\gamma_i$ in the wave function (2) have been selected by using a quasi-random process suggested by Frolov [18]. Following the multi-box strategy of Frolov [18], the parameters $\alpha_i$, $\beta_i$ and $\gamma_i$ are chosen from the three positive intervals $[A(k)_1, A(k)_2]$, $[B(k)_1, B(k)_2]$ and $[C(k)_1, C(k)_2]$, where $k = \mod(i, 3) + 1, 1 \leq i \leq N$.

$$\alpha_i = \eta^{(k)}_{1}\left[\langle\frac{1}{2}i(i+1)\sqrt{2}\rangle\right] (A^{(k)}_2 - A^{(k)}_1) + A^{(k)}_1,$$

$$\beta_i = \eta^{(k)}_{2}\left[\langle\frac{1}{2}i(i+1)\sqrt{3}\rangle\right] (B^{(k)}_2 - B^{(k)}_1) + B^{(k)}_1,$$

$$\gamma_i = \eta^{(k)}_{1}\left[\langle\frac{1}{2}i(i+1)\sqrt{5}\rangle\right] (C^{(k)}_2 - C^{(k)}_1) + C^{(k)}_1,$$

(3)

where the symbol $\langle \cdots \rangle$ designates the fractional part of a real number. The positive scaling factors $\eta^{(k)}_{1}$, $\eta^{(k)}_{2}$ and $\eta^{(k)}_{3}$ are equal to 1 in the first stage and in the second stage will be varied. But for the present problem, we have observed that better optimization will be possible by setting $A^{(k)}_1 = 0, A^{(k)}_2 = a; B^{(k)}_1 = 0, B^{(k)}_2 = b; C^{(k)}_1 = 0, C^{(k)}_2 = c$ and $\eta^{(k)}_{1} = 1, \eta^{(k)}_{2} = 1, \eta^{(k)}_{3} = 1.$

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Ultimately, three variation parameters $a$, $b$ and $c$ are used in the entire calculations. The ground-state energies of $\text{H}^-$ are obtained by diagonalizing the Hamiltonian (1) using the basis expansion (2) with $\omega = 1$. The optimization has been done to obtain the minimum energy for a particular value of $D$ by varying the nonlinear parameters following the rule (3). The ground-state and excited-state energies of the neutral hydrogen atom have been calculated using the standard Slater-type orbital.

3. Results and discussions

The stabilization method [16] has been used to extract resonance energies and widths by calculating the density of resonance states. After diagonalization of the Hamiltonian (1) using the basis functions (2) with different $\omega$ values, we obtained the energy levels $E(\omega)$ which leads to a stabilization plot from which a resonance position can be identified. The scaling parameter $\omega$ in the wave function (equation (2)) can be considered as the reciprocal range of a ‘soft’ wall [11, 17]. Detailed discussions are available in the works of Ho and co-workers [17]. Varying the Debye length $D$ from infinity to small values, different resonance parameters (energy and width) have been obtained.

To extract the resonance energy $E_r$ and the resonance width $\Gamma$, we have calculated the density of resonance states for a single energy level with the help of the following formula:

$$
\rho_n(E) = \frac{\left| E_n(\omega_{i+1}) - E_n(\omega_{i-1}) \right|^{-1}}{\omega_{i+1} - \omega_{i-1}},
$$

where the index $i$ is the $i$th value for $\omega$ and the index $n$ is for the $n$th resonance. After calculating the density of resonance states $\rho_n(E)$ with the above formula (4), we fit it to the following Lorentzian form that yields the resonance energy $E_r$ and a total width $\Gamma$, with

$$
\rho_n(E) = y_0 + \frac{A}{\pi} \frac{\Gamma/2}{(E - E_r)^2 + (\Gamma/2)^2},
$$

where $y_0$ is the baseline offset, $A$ is the total area under the curve from the baseline, $E_r$ is the centre of the peak and $\Gamma$ denotes the full width of the peak of the curve at half height.

To construct the stabilization plot, we have used an expansion length of $N = 700$ in the basis function (2). The stabilization diagram (in figure 1(a)) corresponding to the Debye length $D \rightarrow \infty$ (i.e., for unscreened case) in the range of $\omega = 0.4–1.0$ shows the stabilization character near $E = -0.15$ au. We use 301 points to cover the range of $\omega$ from 0.4–1.0 in the mesh size of 0.002. We have calculated the density of resonance states for the individual energy levels in the range $\omega = 0.4–1.0$, with one energy level at a time. The calculated density of resonance states from the single energy eigenvalue is then fitted to equation (5), and the one that gives the best fit (with the least chi-square) to the Lorentzian form is considered as the desired results for that particular resonance. Figure 1(b) shows the fitting of the density of the resonance states for the 18th eigenvalue of the stabilization plot. From the fit, we obtain the resonance energy $E_r = -0.14876$ au and the corresponding width as $\Gamma = 0.0017328$ au. The resonance energy and width are nicely comparable with the results of Ho et al [19] and with that reported in [11]. The circles are the results of the actual calculations of the density of resonance states using formula (4), and the solid line is the fitted Lorentzian form of the corresponding $\rho_n(E)$. Similarly, the
Figure 1. (a) Stabilization plots of the $2s^21S_e$ state of $H^-$. (b) Calculated density (○) and the fitted Lorentzian (——) corresponding to the $2s^21S_e$ state of $H^-$. 

Figure 2. (a) Stabilization plots of the $2s^21S_e$ state of $H^-$ in the Debye plasma environments for $D = 30$. (b) Calculated density (○) and the fitted Lorentzian (——) corresponding to the $2s^21S_e$ state of $H^-$ for $D = 30$. 

stabilization plots in figures 2(a), 3(a) and 4(a) corresponding to the Debye length $D = 30$, 10 and 5 respectively exhibit the stabilization characters near $E = -0.12$, -0.06 and -0.01. Figures 2(b), 3(b) and 4(b) show the fittings of the density of states corresponding to $D = 30$, 10 and 5 respectively. From the figures 2(b), 3(b) and 4(b), we obtain the resonance energies $E_r = -0.11668, -0.063110$ and $-0.014167$ au respectively and the corresponding widths are $\Gamma = 0.0016950, 0.0013089$ and $0.0003415$ au.

Table 1 shows the convergence of the ground state energies, resonance energies and the widths for $N = 500$, 600 and 700 basis terms calculated for various Debye lengths $D$. For $N = 500$ and 600 terms we have used the same scheme to determine the parameters in equation (3).
as that for the 700-term basis functions. In the present work, we are interested in calculating resonances for H⁻ in the framework of Debye plasmas and so we have first obtained an accurate ground-state energy of H⁻. Our value of \(-0.52775101654\) au reported in table 2 is nicely comparable to the best results of \(-0.52775101654\) au available in the literature [20]. All other ground-state energies obtained from our calculations for various Debye parameters ranging from 0.86–100 are presented in table 2 and these results are nicely comparable with the results of Winkler [5]. Our calculated ground-state eigenenergy values presented in table 2 for \(D = 4, \ldots, 100\) are estimated for the first time up to 11 decimal places and are lower than

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Table 1. Convergence of the ground-state energies ($E_{\text{H}^-}$), the resonance energies ($E_r$) and widths ($\Gamma$) for the $2s^2 \, 1S^e$ resonance using the basis expansion 500, 600 and 700 for the various Debye lengths.

| $D$ (au) | Ground-state and resonance energies, and widths (au) | $N$ |
|---------|--------------------------------------------------|-----|
|         | $-E_{\text{H}^-}$ | $E_r$ | $\Gamma$ | 500 | 600 | 700 |
| $\infty$ | 0.5277510165 | 0.5277510165 | 0.5277510165 | 0.5277510165 | 0.5277510165 | 0.5277510165 |
| 20       | 0.14876 | 0.14876 | 0.14876 | 0.001737 | 0.001729 | 0.001733 |
| 6        | 0.10183 | 0.10182 | 0.10182 | 0.001643 | 0.001646 | 0.001638 |
| 5        | 0.02620 | 0.02621 | 0.02621 | 0.000662 | 0.000678 | 0.000642 |
| 4        | 0.1415  | 0.1418  | 0.1417  | 0.000365 | 0.000338 | 0.000345 |
| 3        | 0.31075 | 0.31075 | 0.31075 | 0.310758 | 0.310758 | 0.310758 |
| 2        | 0.25298 | 0.25298 | 0.25298 | 0.252985 | 0.252982 | 0.252982 |
| 1        | 0.15782 | 0.15782 | 0.15782 | 0.157826 | 0.157824 | 0.157824 |
| 0.9      | 0.01095 | 0.01095 | 0.01095 | 0.010951 | 0.010957 | 0.010957 |
| 0.87     | 0.00052 | 0.00052 | 0.00052 | 0.000523 | 0.000523 | 0.000523 |
| 0.86     | 0.00024 | 0.00024 | 0.00024 | 0.000242 | 0.000244 | 0.000245 |
| 0.85     | 0.00006 | 0.00006 | 0.00006 | 0.000064 | 0.000064 | 0.000064 |

The reported results [5] for some Debye lengths. It is evident from table 2 that our calculated ground-state energies of $\text{H}^-$ corresponding to $D = 8$ and 20 are not lower than those reported by Winkler [5] probably because Winkler rounded off the last digit of his results to lower values. It is interesting to mention here that the same set of nonlinear parameters are needed to obtain the ground-state energies for Debye lengths $D = 4$ to $\infty$. In this range of the Debye length, the best value of the nonlinear parameters are $a = 1.84$, $b = 1.53$ and $c = 1.24$. For $1 \leq D \leq 3$, we need to vary the nonlinear parameters $b$ and $c$, keeping the other parameter $a = 1.84$ fixed. The best value of the parameters in this range of $D$ are $b = 1.46$, $c = 1.0$ for $D = 3$; $b = 1.36$, $c = 0.08$ for $D = 2$ and $b = 1.26$, $c = 0.12$ for $D = 1$. For $0.85 \leq D \leq 0.9$, variation is required for all the three variational parameters. The best values are $a = 1.18$, $b = 0.28$ and $c = 0.072$ for $D = 0.9$; $a = 1.08$, $b = 0.14$ and $c = 0.062$ for $D = 0.87$; $a = 0.96$, $b = 0.11$ and $c = 0.05$ for $D = 0.86$; $a = 0.82$, $b = 0.03$ and $c = 0.05$ for $D = 0.85$. All the results for the ground-state energies of $\text{H}^-$ presented in table 2 are for the $N = 700$ basis function of equation (2). It seems from table 1 that the convergence of the resonance energies and the widths are quite good. Table 2 also presents the ground-state energies and the electron affinity of the neutral hydrogen atom. The ground-state energies of the neutral hydrogen atom have been calculated by diagonalizing the atomic Hamiltonian with the standard Slater-type orbitals for the different Debye parameters. Our calculated H(1S) threshold energies for various Debye lengths presented
Table 2. The ground-state energy of neutral hydrogen ($-E_H$), the ground-state energy ($-E_{H^-}$) of hydrogen negative ion and the electron affinity ($E_H - E_{H^-}$) for various Debye lengths. The quoted results are in atomic units.

| $D$   | $-E_H$   | $-E_{H^-}$ | $E_H - E_{H^-}$ |
|-------|----------|------------|----------------|
| $\infty$ | 0.50000000000$^a$ | 0.52775101654 | 0.02775101654 |
|       | 0.49007450675 | 0.51780162125 | 0.02772711450 |
|       | 0.49007$^c$ | 0.51780$^c$ | 0.027731$^c$ |
| 70    | 0.48586591709 | 0.51356876180 | 0.02770284471 |
| 50    | 0.48029610598 | 0.50795426332 | 0.02765815732 |
|       | 0.48030$^c$ | 0.50795$^c$ | 0.027654$^c$ |
| 40    | 0.47546119393 | 0.50306920329 | 0.02760800936 |
| 30    | 0.46748228005 | 0.49495805224 | 0.02750280219 |
|       | 0.46748$^c$ | 0.49495$^c$ | 0.027505$^c$ |
| 20    | 0.45181642853 | 0.47903478451 | 0.02721835598 |
|       | 0.45182$^c$ | 0.47904$^c$ | 0.027215$^c$ |
| 15    | 0.43653059675 | 0.46337750444 | 0.02684716369 |
|       | 0.43653$^c$ | 0.46338$^c$ | 0.026848$^c$ |
| 10    | 0.40705803061 | 0.43925219929 | 0.02589416868 |
|       | 0.40706$^c$ | 0.43925$^c$ | 0.025892$^c$ |
| 8     | 0.38587872028 | 0.41091499024 | 0.02503626996 |
|       | 0.38588$^c$ | 0.41092$^c$ | 0.025035$^c$ |
| 7     | 0.37121833907 | 0.39558509793 | 0.02436657588 |
| 6     | 0.35225907126 | 0.37567695260 | 0.02341788134 |
|       | 0.35226$^c$ | 0.37568$^c$ | 0.023417$^c$ |
| 5     | 0.32680851137 | 0.34882135187 | 0.02201284050 |
| 4     | 0.29091958752 | 0.31073581900 | 0.01981623148 |
|       | 0.29092$^c$ | 0.31074$^c$ | 0.019816$^c$ |
| 3     | 0.236832670 | 0.252985283 | 0.016152613 |
| 2     | 0.148117022 | 0.157826419 | 0.009719397 |
|       | 0.14812$^c$ | 0.15783$^c$ | 0.009706$^c$ |
| 1     | 0.01028579 | 0.01095180 | 0.00066601 |
|       | 0.01029$^c$ | 0.01095$^c$ | 0.000655$^c$ |
| 0.9   | 0.001757 | 0.00191387 | 0.00015687 |
|       | 0.00175$^c$ | 0.00189$^c$ | 0.000137$^c$ |
| 0.87  | 0.00045 | 0.0005235 | 0.0000735 |
| 0.86  | 0.0002 | 0.000242 | 0.000042 |

$^a$Exact non-relativistic energy. $^b$[20], $^c$[5].

The ground-state energies of $H^-$ and the neutral hydrogen are plotted in figure 5 for different values of $D$ and $1/D$. It is evident from figure 5 and table 2 that the increase in screening decreases the ground-state energies. Such changes in the ground-state energies are more rapid in table 2 are nicely comparable with the results reported in the literature [1, 2, 5, 21]. It is interesting to mention here that Zhao and Ho [21] have reported the energy levels of the neutral hydrogen atom for various Debye lengths during their investigation on the influence of plasma environments on photoionization of atoms.
in the strong plasma environments. Thus the system will be destabilized with the increase in screening and ultimately ionizes the system. From table 2 it is clear that the electron affinity decreases monotonically with the increase in plasma strength.

Table 3 shows the resonance energies and the widths for various Debye lengths ranging from infinity (corresponding to no screening) to a small value 5 (corresponding to strong screening) along with the H(2S) threshold energy obtained from our calculations. The H(2S) threshold energies obtained from our calculations are comparable with the reported results of Rogers et al [1] and Roussel et al [2]. The results for the 2s2 1Se resonant state of H\(^-\) reported in [11] are also presented in table 3 to compare the results with the unscreened case.

When the density of the resonance states \( \rho_n(E) \) is fitted to the Lorentzian form, it has been observed that the value of \( \chi^2 \) for each fitting is much less than 0.1. All the results shown in figures 1–7 and tables 2 and 3 are obtained using the 700-term wave functions. Our calculated resonance energies associated with the \( N = 2 \) hydrogenic threshold are shown in figures 6(a) and 7(a) along with the H(2S) threshold energies for different values of \( D \) and \( 1/D \), respectively, and the corresponding widths are plotted in figures 6(b) and 7(b). In figures 6 and 7, we have included the results given in [11] for the case when the screening between the two electrons was not included. It is apparent from table 3 and figures 6 and 7 that the resonance energies and widths reported in [11] are different from our present results. Such differences increase for increasing \( 1/D \). In the present work, it is possible to determine the resonance positions and widths for strong screening for the inclusion of the screening in the electron–electron pair.

From figure 6(b) and table 3, it is seen that the resonance width \( \Gamma \) decreases with decreasing values of \( D \). The situation can be explained in the following way: the 2s2 1Se state in H\(^-\) is a ‘+’ state, and the two electrons are located on the opposite sides of the nucleus. The movements of the two electrons are moving towards the nucleus ‘in phase.’ The autoionization of such a state is through momentum transfer, as one of the electrons is ‘knocked out’ by the other

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**Figure 5.** (a) The ground state energy of H\(^-\) for various Debye lengths \( D \). (b) The ground-state energy of H\(^-\) for different values of \( 1/D \). The solid line denotes the present results and dashed line denotes the H(1S) threshold energy in both the figures.
Table 3. The resonance energy ($E_r$) and width ($\Gamma$) of the $2s^2 \, ^1S^e$ resonance in hydrogen negative ion for various Debye parameters along with the calculated H(2S) threshold energy ($E_{H(2S)}$). The quoted results are in atomic units.

| D   | $-E_r$  | $\Gamma$    | $-E_{H(2S)}$ |
|-----|---------|-------------|--------------|
| ∞   | 0.14876 | 0.001733    | 0.12500000000a |
|     | 0.148775b | 0.001735b   |              |
| 100 | 0.13885 | 0.001730    | 0.11529328517 |
|     | 0.129495b | 0.001715b   |              |
| 70  | 0.13467 | 0.001727    | 0.11130728211 |
|     | 0.111635b | 0.001655b  |              |
| 50  | 0.12917 | 0.001721    | 0.10614832025 |
|     | 0.111635b | 0.001655b  |              |
| 40  | 0.12442 | 0.001713    | 0.10177590310 |
|     | 0.08983b | 0.00153b   |              |
| 30  | 0.11668 | 0.001695    | 0.09477191199 |
|     | 0.08983b | 0.00153b   |              |
| 20  | 0.10182 | 0.001638    | 0.08177119580 |
|     | 0.065625b | 0.001295b  |              |
| 15  | 0.08791 | 0.001554    | 0.07002356058 |
|     | 0.044635b | 0.00112b   |              |
| 10  | 0.063110 | 0.001309    | 0.04992827133 |
|     | 0.047264 | 0.001074    | 0.03754069403 |
| 8   | 0.02621 | 0.00064     | 0.0214063134 |
|     | 0.01417 | 0.00034     | 0.01210786383 |

aExact non-relativistic energy. b[11].

via the nucleus. Apparently, when the electron-nucleus screening is increased (decreasing $D$, increasing $1/D$), the movement of the electrons will be slowed down. As a result, the lifetime of the autoionization process will be prolonged, leading to the narrowing of the resonance width, a consequence of the uncertainty principle. Also, in comparing the present widths with those in [11] where the screening for the electron–electron pair was not included, the present widths are larger than those in [11]. Such a finding can be explained as follows: when the screening for the electron–electron pair is also included, the repulsive force between the electron–electron pair is decreased. The resonance energy of the system is therefore decreased, as substantiated by our actual calculations. In autoionization processes for a Feshbach resonance to which the $2s^2 \, ^1S^e$ state belongs, the resonance width is proportional to the overlap between the closed channel wave functions ($Q/\Psi$ in the Feshbach projection formalism) and the open channel wave functions ($P/\Psi$). By lowering energy to $Q/\Psi$, the overlap between the $P/\Psi$ and $Q/\Psi$ usually becomes larger. As a result, the present resonance widths with the inclusion of the electron–electron screening are larger than those in [11] where the screening for the electron–electron pair was not included.

It should be mentioned here that we have not found any resonance for $D \leq 4$. From table 3 and figures 6 and 7, it is clear that, for $D \leq 4$, the resonance, if it exists, would be located very near the H(2S) threshold. However, calculations of such resonances would require a more extensive basis set of wave functions, and no attempt is made to carry out large-scale calculations using more extensive wave functions than those used here.
Figure 6. (a) The $2s^2 \ ^1S^e$ resonance energy $E_r$ for different values of the Debye length $D$. The dashed line denotes the H(2S) threshold energy. (b) Resonance width $\Gamma$ corresponding to the resonance energy in (a) for different values of the Debye length $D$. The solid line denotes the present results and dash-dotted line denotes the resonance energies in (a) and widths in (b) of [11], in which the electron–electron screening was not included.

Figure 7. (a) The $2s^2 \ ^1S^e$ resonance energy $E_r$ as a function of $1/D$. Dashed line denotes the H (2S) threshold energy. (b) Resonance width $\Gamma$ corresponding to the resonance energy in (a) as a function of $1/D$. Solid line denotes the present results and dash-dotted line denotes the resonance energies in (a) and widths in (b) of [11], in which the electron–electron screening was not included.

4. Conclusions

This work presents a calculation to estimate the accurate ground-state energy and the $2s^2 \ ^1S^e$ autoionization resonance for the hydrogen negative ion embedded in Debye plasma environments. The resonance energies and widths for various Debye parameters ranging from
infinity to small values (up to 5) have been reported along with the electron affinity of the hydrogen atom. The ground-state energies of H\(^-\) for various Debye lengths are an improvement over those in the literature. In the earlier work on the autoionization state of H\(^-\) [11], the screening on the electron–electron correlation was not included, whereas in the present work such a screening effect is included explicitly. Furthermore, we have made a comparison for the effect of the electron–electron screening with the unscreened case. The stabilization method is used to extract resonance energies and widths. This method is a practical method to calculate resonance parameters (\(E_r, \Gamma\)). Our present work will provide useful information to the plasma physics research community.

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