Spin filter using a semiconductor quantum ring side-coupled to a quantum wire

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We introduce a new spin filter based on spin-resolved Fano resonances due to spin-split levels in a quantum ring (QR) side-coupled to a quantum wire (QW). Spin-orbit coupling inside the QR, together with external magnetic fields, induces spin splitting, and the Fano resonances due to the spin-split levels result in perfect or considerable suppression of the transport of either spin direction. Using the numerical renormalization group method, we find that the Coulomb interaction in the QR enhances the spin filter operation by widening the separation between dips in conductances for different spins and by allowing perfect blocking for one spin direction and perfect transmission for the other. The spin-filter effect persists as long as the temperature is less than the broadening of QR levels due to the QW-QR coupling. We discuss realistic conditions for the QR-based spin filter and its advantages to other similar devices.

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Introduction.— Spintronics \cite{1} that utilizes the electron’s spin degree of freedom rather than its charge for information processing and storage has been a subject of intense interest in recent decades. The practical realization of spin-based electronic circuits requires the development of efficient means to generate spin-polarized currents, and to manipulate and detect spins. Spin filters that block the transport of one spin direction have been proposed as a device to generate and detect spin currents.\cite{2} The basic scheme of a spin filter exploits spin-dependent transport through systems lacking time-reversal symmetry or having nontrivial geometric structures with spin-dependent interactions; such systems include ferromagnetic junctions\cite{2}, and nanostructures like quantum dots (QD’s) and rings\cite{1,2,3}.

A simple but effective spin-filter implementation without coupling to magnetic materials has been suggested to exploit the spin-dependent resonance through a QD with Zeeman splitting that is embedded in, or side-coupled to a quantum wire (QW). While in both cases the spin-dependent transport is based on scattering from spin-split QD levels and can be tuned by varying the gate voltage or the external magnetic field, the side-coupled configuration is more effective because the Fano resonance in this case can lead to perfect blocking of one spin direction and almost total transmission of the other. Side-coupled QD systems show two dips corresponding to the total suppression in the conductance of spin up and down\cite{1,2,3}. Since such spin filtering deteriorates rapidly with increasing temperature $T$, however, an ideal operation of the device requires large magnetic fields $B$ or high $g$-factors such that $g\mu_B B \gg k_B T, k_B T_K$ where $\mu_B$ is the Bohr magneton and $T_K$ is the Kondo temperature.\cite{2}

Recently, quantum rings (QR’s) with Rashba spin-orbit coupling have been proposed for spin injection devices\cite{2,3}. Spin precession due to the momentum-dependent effective magnetic field and the following spin interference of two quantum states propagating in opposite directions can not only modulate the charge conductance but also induce spin currents through leads attached to the ring.\cite{2,4} When an unpolarized charge current is injected through one of leads, quantum interference can produce pure spin currents through one of other leads. However, this interference-based spin-filter operation requires more than two leads to be linked to different positions of the QR.

In this Letter we propose another kind of spin filter that consists of a QR side-coupled to a QW; see Fig.\textsuperscript{1}a. Spin-resolved Fano resonances due to spin-split levels formed in the QR in the presence of Rashba spin-orbit coupling and external magnetic fields lead to a complete suppression of transport of either spin component at a set of gate voltage values, resulting in a series of valleys in the spin-resolved conductance. The separations between valleys are observed to be of the order of the Coulomb interaction energy. This QR-based spin filter has three advantages: (1) It does not require strong magnetic fields and high $g$-factors like the QD-based system. In the presence of spin-orbit coupling, a weak magnetic field applied to a small QR can induce a large energy splitting between spin levels because it is the magnetic flux that causes the level splitting in the ring geometry. (2) Only a single contact of the QR to the external circuit such as leads or wires is necessary. (3) Finally, on-board tuning of polarization direction of spin currents is possible via the control of spin-orbit coupling strength.

Model.— First, we examine the energy level structure of a non-interacting QR with Rashba spin-orbit coupling in the presence of an external magnetic field. In the ideal one-dimensional limit where the radial width is much smaller than the radius $R$, only the lowest radial subband is occupied, and the effective Hamiltonian projected to the lowest radial mode can be written in polar...
with the polarization angle $\theta = \arctan[−N_{so}]$, where $N_{so} = 2π/R,t_{so}$ is the number of spin flips around the ring. The resulting energy spectrum is periodic not only in $f$ but also in $1/2\cos \theta$ (excluding the overall shift due to the last term in Eq. (3)). Moreover, the energy gaps between neighboring spin-split levels reach their maxima whenever $1/2\cos \theta = (2l + 1)/4$, whereas the spin-splitting disappears at $1/2\cos \theta = 1/2$, for integer $l > 1$. Throughout our Letter $2/\cos \theta$ is assumed to be an odd integer to maximize the spin-splitting. Figure (b) shows the energy spectrum for realistic material parameters for GaAs. The ring size is taken to be $R = 120$nm, which is feasible using current fabrication technology [10]. The spectrum shows that a small magnetic field < 50mT is enough to induce a spin-splitting energy gap comparable to 1 to 3K. This large splitting that exists even in the absence of a strong external magnetic field definitely makes the QG a good candidate for ideal spin filter operation.

To take into account the electron-electron Coulomb interaction in the small QG, we adopt a simple capacitive model where the Coulomb interaction depends only on the total number of electrons: $\mathcal{H}_{\text{RI}} = (U/2)\left[N^2 - 2N_gN\right]$ with $N \equiv \sum_{\mu \nu} d_{\mu \nu}^d d_{\mu \nu}$. Here $U \equiv \epsilon^2/(C + C_g)$ and $N_g \equiv C_g V/|\epsilon|$ denote the interaction strength and the gate charge (in units of $|\epsilon|$), respectively, in terms of self and gate capacitances, $C$ and $C_g$. The total Hamiltonian for a QG side-coupled to a QW can then be written as

$$\mathcal{H} = \mathcal{H}_{\text{RN}} + \mathcal{H}_{\text{RI}} + \mathcal{H}_W + \mathcal{H}_{WR}$$

with $\mathcal{H}_W = -t_w \sum_{\mu \nu} (\epsilon^\dagger_{\nu + 1\mu} c_{\nu \mu} + h.c.)$ and $\mathcal{H}_{WR} = t_w \sum_{\mu} \left(\epsilon^\dagger_{\mu} a_{N \mu} + h.c.\right)$, where the operators $c_{\nu \mu} (\epsilon^\dagger_{\nu \mu})$ destroys (creates) an electron with spin index $\mu$ at site $\nu$ of the wire. $\mathcal{H}_W$ models the QW as an infinite tight-binding chain with a hopping energy $t_w$ between neighboring sites, and $\mathcal{H}_{WR}$ a spin-independent tunneling with strength $t_w$ between site 0 of the wire and site $N_R$ of the ring. Note that the spin quantization axis for the QW has been rotated to align with the spin axis at site $N_R$ of the QG, $\hat{n}$ at $\hat{r} = \hat{x}$.

Spin filter.—We have calculated the zero-bias conductance $G_\mu$ for spin $\mu$ at the Fermi level $\epsilon_F = 0$ under the assumption that two electron reservoirs with nearly the same chemical potentials are attached at both ends of the QW [13]. The non-equilibrium scattering formalism [14] enables us to express the conductance in terms of the Green’s function $G^R_\mu(\epsilon)$ for a spin-$\mu$ electron at site 0 of the QW:

$$G_\mu = \frac{\epsilon^2}{h} \int \frac{d\epsilon}{\pi} \frac{\partial f(\epsilon)}{\partial \epsilon} \text{Im} \Gamma(\epsilon) G^R_\mu(\epsilon).$$

Here, $f(\epsilon)$ is the Fermi distribution function with $\epsilon_F = 0$ and the symmetric coupling $\Gamma(\epsilon)$ of site 0 to the left and right sides of the QW is given by $\Gamma(\epsilon) = (2t_w/h) \sin \chi_\epsilon(\epsilon)$ with $\chi_\epsilon(\epsilon) \equiv \arccos[-e/2t_w]$. In the non-interacting case
(C \ll C_g \text{ and } U \approx 0), by solving the Dyson equation for \( g^R_{\mu} \), we obtain the spin-dependent transmission probability \( T_{\mu}(\epsilon) = -\text{Im} \Gamma(\epsilon)g^R_{\mu}(\epsilon) = 1/(1 + |Q_{\mu}(\epsilon)|^2) \) with \( Q_{\mu}(\epsilon) = \Gamma(\epsilon)^{-1}(t_{wr}/\hbar)^2\sum_m g^R_{m\mu}(\epsilon) = \Delta(\epsilon)\sum_m 1/(\epsilon - \epsilon_{m\mu}) \), where \( g^R_{m\mu} \) is the Green’s function for the uncoupled QR and \( \Delta(\epsilon) \equiv t_{wr}^2/\hbar \Gamma(\epsilon) \) is the level broadening due to the QW-QR coupling. Since \( Q_{\mu} \) diverges at \( \epsilon = \epsilon_{m\mu} \), the transmission probability \( T_{\mu} \) vanishes whenever a resonant state with spin \( \mu \) is formed in the QR, giving rise to perfect suppression of the transport of spin-\( \mu \) electrons. Note that this blocking condition is independent of any characteristics of the wire. Figure 2 shows the formation of a series of spin-split dips in the zero-bias conductances \( G_{\mu} \) as functions of the gate voltage at zero temperature. The width of the valleys is restricted by the minimum of the energy splitting between neighboring levels and the level broadening \( \Delta(\epsilon_F) \). The spin-dependent conductance can also be controlled by varying the flux \( f \). The condition for total transmission (\( T_n = 1 \)), \( \sin 2\pi \sqrt{(\epsilon_F - UN_g)/E_0} = 0 \), does not depend on spin, thus the peak positions in \( G_{\mu} \) are the same for both spins.

This spin-dependent transmission generates a net spin flow through the wire: \( \Delta G \equiv G_+ - G_- = (e^2/h)|[Q_+^2 - |Q_+|^2]/[(1 + |Q_+|^2)(1 + |Q_-|^2)] \) at zero temperature. The net spin conductance \( \Delta G \) has local maxima or minima whenever one of the \( Q_{\mu} \) diverges; see Fig. 2. The peak height in \( |\Delta G| \) reaches almost the maximum value \( e^2/h \) if the spin splitting \( \delta \epsilon \) is larger than the broadening \( \Delta(\epsilon_F) \), in which case the unblocked states with opposite spin are transmitted almost completely. The ideal operation of the spin filter, therefore, requires \( \delta \epsilon \gg \Delta(\epsilon_F) \). In addition, to avoid temperature-induced broadening through Eq. 6, both the spin splitting and the broadening should be larger than the temperature \( T \) as well. Interestingly, at \( f = 1/4 \), \( |\Delta G| \) reaches \( e^2/h \) at its peaks regardless of \( UN_g \), implying perfect blocking for one spin direction and perfect transmission for the other. Also, the peak widths are maximal at \( f = 1/4 \). This is related to the appearance of degenerate levels with the same spin at \( f = 1/4 \) [see Fig. 1(b)], which merges two peaks separated at \( f \neq 1/4 \) into one peak and strengthens the Fano resonances with broader width. This observation indicates that the best performance of the spin filter can be achieved at \( f = 1/4 \).

Coulomb interaction.— We now turn on the self-charging interaction in the QR with moderate values of \( U \) and investigate its effect on the transport at finite temperatures. The numerical renormalization group method, proven to be an excellent numerical tool for Anderson-type impurity systems, was applied to calculate the spin-resolved local density of states \( \rho_{\mu}(\epsilon) \) on site \( N_R \) of the ring. The transmission amplitude can then be calculated using the Dyson equation: \( T_n(\epsilon) = 1 - \pi \Delta(\epsilon)\rho_{\mu}(\epsilon) \).

Figure 3 shows the dependence of the zero-temperature conductances \( G_{\mu} \) and \( \Delta G \) on magnetic flux and gate voltage which has been tuned to such large values that high
FIG. 4: (Color online) Finite-temperature conductance $G_{\mu}$ for $f = 1/4$. The temperatures are given by $k_B T_1 = 0.13\Delta(\epsilon_F)$, $k_B T_2 = \Delta(\epsilon_F)$, and $k_B T_3 = 8.1\Delta(\epsilon_F)$. (a) $t_{wr} = 0.4\text{meV}$ and $\Delta(\epsilon_F) = 0.016\text{meV}$. (b) $t_{w} = 0.8\text{meV}$ and $\Delta(\epsilon_F) = 0.064\text{meV}$. Other parameters as in Fig. 3.

QR levels with $\delta \epsilon \gg \Delta(\epsilon_F)$ contribute to the transport: the dips in conductance correspond to the QR levels with $m = 13$ and 14. At $f = 0$ the correlation between spin-degenerate QR levels and QW conduction electrons induces the Kondo effect whenever the QR contains an odd number of electrons. As a consequence, the Fano resonances due to the spin-split levels in the QR that is side-coupled to a QW with one conduction channel. We predict perfect or considerable suppression of the transport of either spin direction under real experimental conditions that are accessible using current technology.

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