Examples of cosmological rips in the scalar-tensor cosmology

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Abstract. Plenitude of models incorporating a dynamical dark energy equation of state are proposed and the scalar-tensor gravity is certainly one possible candidate. In this paper we examine whether a cosmological rip scenarios are possible in the scalar-tensor gravity by parametrizing the equation of state of dark energy \( w_{de}(z) \) in an appropriate way. The parameters of the model are constrained by the observational data of type Ia supernovas collected to \textit{Union 2.1} compilation.

1. Introduction

According to the contemporary observational data the expansion of the universe has been accelerating for about five billion years. Adding a cosmological constant \( \Lambda \) into the dynamical equations (derived in the framework of general relativity (GR)) of the cosmological model accounts for the accelerating expansion. This is equivalent to an additional matter component with an equation of state (EoS) parameter \( w_{\Lambda} = p_{\Lambda}/\rho_{\Lambda} \), \( w_{\Lambda} \equiv -1 \). However, recent analyses point out the possibility that dark energy is dynamical, in which case the EoS parameter is not constant, for a review see [1]. Moreover, observationally it cannot be excluded [2] that the EoS parameter of dark energy may cross the so-called phantom divide line (PDL) \( w_{de} = -1 \) [3, 4]. Phenomenological models where the EoS parameter of dark energy is assumed to be oscillatory [5, 6, 7] are also analysed and they are not completely contradicted by recent data.

If the energy density of dark energy \( \rho \) is growing and the EoS parameter \( w_{de} \) is crossing the PDL the consequences on the future dynamics of the universe are significant. In the most dramatic case, called a big rip [8], the energy density of dark energy will grow uncontrollably and will quickly become infinite. The acceleration of the universe is fast enough to break apart all bounded structures before the evolution ends in a spacetime singularity. According to a more moderate scenario, known as a little rip [4], the energy density approaches infinity as \( t \to \infty \). Recently, a pseudo-rip [10] and a quasi-rip scenarios [11] were proposed. Neither of them contain spacetime singularities, for a classification of future singularities, see [12, 13], but the energy density of dark energy still becomes sufficiently high to rip apart certain bound systems while leaving the other systems bounded. The key difference is that in the case of the pseudo-rip systems that were once disintegrated remain unbounded, whereas in the case of the quasi-rip once disintegrated systems may recover and form bounded systems again [11]. The latter scenario can be realized only in the case where the energy density of dark energy is not monotonically increasing but is increasing and decreasing in turns or even oscillating steadily. A similar behaviour occurs also in the quintom scenario where the EoS parameter is able to evolve across the cosmological constant boundary, for a review see [14].
The unknown nature of dark energy encourages us to look for extensions of the GR. Among the others, scalar-tensor gravity (STG) [15] where a tensor and a scalar degrees of freedom are non-minimally coupled is a substantial candidate. In STG the gravitational coupling depends on scalar field $\Psi$, and free functions $\omega(\Psi)$ and $V(\Psi)$ provide additional freedom to incorporate a number of competitive theories of gravity. Crossing the PDL in the framework of STG is possible [16, 17, 18] and the question about the future dynamics of dark energy is relevant. In this paper we examine whether a big rip, a little rip, a pseudo-rip and a quasi-rip scenarios are possible in the general STG by parametrizing the EoS parameter $w_{de}(z)$ of dark energy in an appropriate way. For comparable approaches see also [19, 20, 21].

The outline of the paper is as follows. We start in Sec. 2 with the equations for the STG cosmology and define the energy density and the pressure of dark energy so that the usual conservation equation is satisfied. In Sec. 3 we list some parametrizations to deal with the unknown EoS parameter $w_{de}(z)$ of dark energy. In Sec. 4 we use the observational data of type Ia supernova (SN Ia) from the Union 2.1 compilation [22] the parameters of the EoS $w_{de}$. We propose a modified parametrization of the effective gravitational constant $G_{\text{eff}}$ and calculate the best fit parameters for different parametrizations of $w_{de}$ (see Table 1), but using only the Union 2.1 compilation. In Sec. 5 we present a general condition for cosmological rips to happen and give some examples of rip scenarios which are possible in the framework of STG and are consistent with the constraints found in section 4. Finally, Sec. 6 provides a brief summary.

2. General scalar-tensor cosmology

In what follows we consider a general STG in the Brans-Dicke-Bergmann-Wagoner (BDBW) parametrization in the Jordan frame representation. The action of the STG is given by

$$S = \frac{1}{2\kappa^2} \int d^4x \sqrt{-g} \left[ \Psi R(g) - \frac{\omega(\Psi)}{\Psi} \nabla^\rho \Psi \nabla_\rho \Psi - 2\kappa^2 V(\Psi) \right] + S_m[g_{\mu\nu}, \chi_m]$$

(2.1)

where $\omega(\Psi)$ is the coupling function, $V(\Psi) > 0$ is the scalar field self-interaction, $\kappa^2$ is the constant part of the effective gravitational constant $\kappa^2/\Psi$ and $S_m[g_{\mu\nu}, \chi_m]$ is the matter contribution term. Since we require that the gravitational constant remains positive it follows that $\Psi > 0$.

Assuming that the geometry of the universe is described by the flat ($k = 0$) Friedmann-Lemaître-Robertson-Walker (FLRW) metric and the ordinary matter is modelled by a perfect barotropic fluid we can write the field equations derived from the action (2.1) as follows

$$H^2 = -\frac{\dot{\Psi}}{\Psi} + \frac{1}{6} \frac{\dot{\Psi}^2}{\Psi^2} \omega(\Psi) + \frac{\kappa^2}{3} \frac{\rho_m}{\Psi^3} + \frac{\kappa^2}{3} \frac{V(\Psi)}{\Psi}$$

(2.2a)

$$2\dot{H} + 3H^2 = -2\frac{\dot{\Psi}}{\Psi} - \frac{1}{2} \frac{\dot{\Psi}^2}{\Psi^2} \omega(\Psi) - \frac{\Psi}{\Psi^3} \dot{\Psi} - \frac{\kappa^2}{3} \left( \rho_m - \frac{\Psi}{\Psi^3} \dot{V}(\Psi) \right)$$

(2.2b)

$$\ddot{\Psi} + 3H \dot{\Psi} = -\frac{1}{2\omega(\Psi) + 3} \left[ \frac{d\omega(\Psi)}{d\Psi} \dot{\Psi}^2 - \kappa^2 (1 - 3w) \rho_m - 2\kappa^2 \left( 2V(\Psi) - \frac{\Psi}{\Psi^3} \frac{dV(\Psi)}{d\Psi} \right) \right]$$

(2.2c)

Since in the Jordan frame representation the matter part of the action does not contain a scalar field $\Psi$ the usual conservation law holds

$$\dot{\rho}_m + 3H (\rho_m + p_m) = 0$$

(2.3)

It turns out that defining the energy density of dark energy $\rho_{de}$ and the pressure of dark energy $p_{de}$ are as follows (for a discussion, see [23, 17]):

$$\rho_{de} = \frac{3}{\kappa^2} \left( -\frac{\dot{\Psi}}{\Psi} + \frac{1}{6} \frac{\dot{\Psi}^2}{\Psi^2} \omega - H^2 (\Psi - \Psi_0) \right) + V$$

(2.4a)
The constraint equation (2.2a) and the dynamical equation (2.2b) can be written in a similar way as in the case of the standard Friedmann model

\[ H^2 = \frac{\kappa^2}{3\Psi_0} (\rho_m + \rho_{de}) \]  

(2.5a)

\[ 2\dot{H} + 3H^2 = -\frac{\kappa^2}{\Psi_0} (\rho_m + p_{de}) \]  

(2.5b)

Here \( \Psi_0 \) corresponds to the value of the scalar field \( \Psi \) at the present moment \( t_0 \). Moreover, definitions (2.4a) and (2.4b) provide that the usual conservation law holds also for the dark energy

\[ \dot{\rho}_{de} + 3H(\rho_{de} + p_{de}) = 0 \]  

(2.6)

Note that equation (2.4) is nothing but the scalar field equation (2.2c). Using the relation between the scale factor \( a \) and the redshift \( z \)

\[ \frac{da}{a} = -dz/(1+z) \]  

(2.7)

we can write the solution of equation (2.4) as an integral

\[ \rho_{de} = \rho_{de,0} \exp \left[ \int_0^z \frac{3(1 + w_{de}(\tilde{z}))/\tilde{z} + a_{de}(\tilde{z})}{1 + \tilde{z}} d\tilde{z} \right] \]  

(2.8)

where the EoS parameter of dynamical dark energy \( w_{de}(z) = p_{de}/\rho_{de} \) is an unknown function in general.

3. Different parametrizations

In this section we will list some parametrizations for the dark energy EoS parameter \( w_{de}(z) \). An appropriate parametrization allows us to complete the integration (2.8) and represent the Hubble parameter as \( H(z; p_i) \) where parameters \( p_i \) should be determined from observations. All parametrizations employed below contain two free parameters \( (w_0, w_1) \) and can be represented in a general form

\[ w_{de}(z) = w_0 + w_1 f(z) \]  

(3.1)

The first free parameter \( w_0 \) corresponds to the EoS parameter at redshift \( z = 0 \) and the second free parameter \( w_1 \) describes the dynamics of EoS parameter. By construction the functional form of \( f(z) \) describes the asymptotic behaviour of parametrization.

3.1. The Chevallier-Polanski-Linder (CPL) parametrization

The well known parametrization proposed by Chevallier and Polanski [24] and by Linder [25] is represented as follows

\[ w_{de}(z) = w_0 + w_1 (1 - a) = w_0 + w_1 z/(1 + z) \]  

(3.2)

The parametrization (3.2) works very well at high redshifts and has the limiting value at the limit \( z \to \infty \). However, the EoS described by the parametrization (3.2) blows up quickly if we
approach to $z \to -1$ and is singular at this limit. Therefore, the CPL parametrization can be used in the case if negative redshifts are small enough. Integrating equation (2.8) we get for the energy density of dark energy

$$\rho_{de} = \rho_{de,0} (1 + z)^{3(1+w_0+w_1)} \exp\left[-3w_1 z/(1 + z)\right]$$

(3.3)

The dimensionless density parameters are defined as

$$\Omega_m = \frac{\kappa^2 \rho_m}{3H^2 \Psi_0}$$

(3.4a)

$$\Omega_{de} = \frac{\kappa^2 \rho_{de}}{3H^2 \Psi_0}$$

(3.4b)

and the constraint equation (2.5a) can be written as (in the case of flat universe)

$$\Omega_m + \Omega_{de} = 1$$

(3.5)

Now the Hubble parameter $H(z; p_i)$ can be represented as

$$H^2(z; \Omega_{m,0}, w_0, w_1) = H_0^2 \left[ \Omega_{m,0}(1+z)^3 + (1-\Omega_{m,0})(1+z)^{3(1+w_0+w_1)} \exp\left[-\frac{3w_1 z}{1 + z}\right] \right]$$

(3.6)

3.2. The Ma-Zhang (MZ) parametrization: type I and type II

Ma and Zhang proposed two types of divergence-free parametrizations

$$w_{de}(z) = w_0 + w_1 \left\{ \frac{\ln(2 + z)}{1 + z} - \ln 2 \right\} \quad \text{(MZ type I)}$$

(3.7a)

$$w_{de}(z) = w_0 + w_1 \left\{ \frac{\sin(1 + z)}{1 + z} - \sin(1) \right\} \quad \text{(MZ type II)}$$

(3.7b)

Note that the parametrization of type II allows the oscillating behaviour of EoS parameter. Both parametrizations (3.7a) and (3.7b) are bounded at high redshifts as well as negative redshifts. MZ type I approaches to the finite limiting value $w(z) \to w(-1) = w_0 + w_1(1 - \ln 2)$ and MZ type II approaches to the finite limiting value $w(z) \to w(-1) = w_0 + w_1(1 - \sin(1))$ as $z \to -1$.

3.2.1. MZ type I From equation (2.8) we get for the energy density

$$\rho_{de} = \rho_{de,0} \left\{ 2^{6w_1} (1 + z)^{3(1+w_0+w_1(1-\ln 2))} \exp\left[-3w_1 \left( \frac{z + 2}{z + 1} \right) \ln(z + 2)\right] \right\}$$

(3.8)

and using dimensionless density parameters (3.4a) we can write $H(z; p_i)$ as

$$H^2(z; \Omega_{m,0}, w_0, w_1) = H_0^2 \left[ \Omega_{m,0}(1+z)^3 \right] + H_0^2 (1-\Omega_{m,0}) 2^{6w_1} (1 + z)^{3(1+w_0+w_1(1-\ln 2))} \times \exp\left[-3w_1 \left( \frac{z + 2}{z + 1} \right) \ln(z + 2)\right]$$

(3.9)
3.2.2. MZ type II Similarly to the previous case we get relations for the energy density and the Hubble parameter

\[ \rho_{de} = \rho_{de,0} \left[ (1 + z)^{3(1+w_0-w_1 \sin(1))} \right] \times \]
\[ \times \exp \left\{ 3w_1 \left[ (\text{Ci}(1+z) - \text{Ci}(1)) - \left( \frac{\sin(z+1)}{z+1} - \sin(1) \right) \right] \right\} \] (3.10a)
\[ H^2(z; \Omega_{m,0}, w_0, w_1) = H_0^2 \left( \Omega_{m,0}(1+z)^3 \right) + \left( 1 - \Omega_{m,0} \right) \left[ (1+z)^{3(1+w_0-w_1 \sin(1))} \right] \times \]
\[ \times \exp \left\{ 3w_1 \left[ (\text{Ci}(1+z) - \text{Ci}(1)) - \left( \frac{\sin(z+1)}{z+1} - \sin(1) \right) \right] \right\} \] (3.10b)

where the cosine integral \( \text{Ci}(z) \) is defined by

\[ \text{Ci}(z) = - \int_{z}^{\infty} \frac{\cos z}{z} \, dz \] (3.11)

3.3. The Li-Zhang (LZ) parametrization

In order to take into account possible oscillations of the dark energy EoS parameter between the redshift range 0 to 2 Li and Zhang have proposed [27] to use the parametrization

\[ w_{de}(z) = w_0 - w_1 \sin [A \ln(1+z)] \] (3.12)

Here an additional parameter \( A \) determines the frequency of oscillations and was chosen to be \( A = \frac{3}{2} \pi \) in [27]. The parametrization (3.12) is bounded at the limit \( z \to -1 \) but the frequency of oscillations becomes infinite (the amplitude remains constant). Different parametrizations for the oscillating EoS parameter of dark energy is also considered by Kurek et al [28].

As previously the expressions for the energy density and for the Hubble parameter are presented as follows:

\[ \rho_{de} = \rho_{de,0} \left\{ (1 + z)^{3(1+w_0)} \exp \left[ \frac{3w_1}{A} \left[ \cos[A \ln(1+z)] - 1 \right] \right] \right\} \] (3.13a)
\[ H^2(z; \Omega_{m,0}, w_0, w_1) = H_0^2 \left( \Omega_{m,0}(1+z)^3 \right) + \left( 1 - \Omega_{m,0} \right) \left\{ (1+z)^{3(1+w_0)} \exp \left[ \frac{3w_1}{A} \left[ \cos[A \ln(1+z)] - 1 \right] \right] \right\} \] (3.13b)

The constant EoS parameter \( (w_{de} = -w_0) \) can be considered as a special case of parametrizations (3.2), (3.7a), (3.7b) and (3.12) if \( w_1 = 0 \), i.e the dark energy is not dynamical.

4. Constraints from observations

At the present paper we will use the observations of SN Ia to constrain the free parameters of models presented above. Specifically, we apply the Bayesian analysis to Union 2.1 compilation which contains data sets for 580 supernova. Since the likelihood function \( \mathcal{L} \) is defined to be proportional to \( e^{-\chi^2/2} \) it follows that we are looking for the values of parameters which are minimizing the value of \( \chi^2 \). For Union 2.1 compilation \( \chi^2_{SN} \) is given as follows

\[ \chi^2_{SN} = \sum_{i=1}^{580} \left[ \mu_i^{\text{the}}(z; p_i) - \mu_i^{\text{obs}}(z) \right]^2 / \sigma_i^2(z) \] (4.1)

Here \( \mu_i^{\text{obs}} \) is the observed distance modulus and \( \mu_i^{\text{the}}(z; p_i) \) is the theoretical distance modulus, \( p_i \) stands for the parameters of the model and \( \sigma_i \) is the 1σ uncertainty of the ith data point.
Distance modulus $\mu$ is defined as the difference between an apparent magnitude $m$ and an absolute magnitude $M$ and can be expressed by means of the luminosity distance $D_L$ [20, 30]

$$\mu = 5 \log_{10} [D_L(z)] + \mu_0$$  \hspace{1cm} (4.2)

where the nuisance parameter $\mu_0 = -5 \log_{10} h + 42.38$ \hspace{1cm} (h = 100/H_0 \text{ km s}^{-1} \text{ Mpc}^{-1}) is independent of observed data points. The luminosity distance $D_L(z)$ in the case of the flat universe is given by

$$D_L(z) = (1 + z)/H_0 \cdot \int_0^z d\tilde{z}/H(\tilde{z})$$  \hspace{1cm} (4.3)

Since we consider the dynamics of the universe in the framework of STG it is necessary to take into account an additional effect. Namely, the effective gravitational constant $G_{eff}$ depends on the scalar field (i.e. cosmological time) and is different from the value $G_N$ due to contribution from the interaction of the scalar field and a test particle. The effective gravitational constant $G_{eff}$ measured (on scales where the scalar field is effectively massless) between two test masses in the Cavendish-type experiment is given in the BDBW parametrization by [31, 32]

$$8\pi G_{eff} = \frac{\kappa^2}{2}\left(\frac{2\omega(\Psi)}{\Psi} + 3\right)$$  \hspace{1cm} (4.4)

Using the observational data of type Ia supernovas as a probe to fix the free parameters of the model $\rho_t$ we should take into account that the Chandrasekhar mass depends on $G_{eff}$ given by (4.4) and not on $G_N$. This changes the peak luminosity of supernovas [33, 34, 35] see also [Preprint astro-ph/9907222] and the expression of the distance modulus (4.2) should be modified by an additional term

$$\mu = 5 \log_{10} [D_L(z)] + \mu_0 + \frac{15}{4} \log_{10} \frac{G_{eff}(z)}{G_{eff,0}}$$  \hspace{1cm} (4.5)

where $G_{eff,0}$ is the effective gravitational constant at the redshift $z = 0$. As in the case of $w_{de}(z)$ the functional form of $G_{eff}(z)$ is unknown in general and using some appropriate parametrization is essential. From different considerations it is known that the effective gravitational constant should be nearly constant starting from the early universe until today [30]. The weak field observations give also a strong constraint for the present variation of the effective gravitational constant $|G_{eff}/G_{eff,0}| \leq 10^{-12}$ [37].

Nesseris and Perivolaropoulos have proposed [38, 39] to use the parametrization for $G_{eff}$ as

$$G_{eff}(z) = G_{eff,0} \left[ 1 + \alpha (1 - a)^2 \right]$$  \hspace{1cm} (4.6)

where $\alpha$ is the parameter of order $\alpha \sim 0.2$ [39]. The parametrization [14] has a singularity at the limit $z = -1$ and grows fast if we approach to this limit. Inspired by the parametrization of MZ type I (6.7a) for the $w_{de}(z)$ we will propose a divergence-free ansatz for $G_{eff}(z)$

$$G_{eff}(z) = G_{eff,0} \left[ 1 + G_1 (\frac{\ln(2 + z)}{1 + z} - \ln 2) \right]$$  \hspace{1cm} (4.7)

which has the asymptotic values as follows:

$$G_{eff}(z) = \begin{cases} G_{eff,0} & \text{if } z = 0 \\
G_{eff,0} \left[ 1 - G_1 \ln 2 \right] & \text{if } z \to \infty \\
G_{eff,0} \left[ 1 + G_1 (1 - \ln 2) \right] & \text{if } z \to -1 \end{cases}$$  \hspace{1cm} (4.8)
Figure 1. Hubble diagram for the LCDM (red dotted line) with $\Omega_{m,0} = 0.277$ and for the STG with Li-Zhang parametrization $w_0 = -1.10$, $w_1 = -0.25$, $\Omega_{m,0} = 0.289$ (blue line).

Note that negative $G_1$, i.e. decreasing of the effective gravitational constant supports the accelerated expansion of the universe.

Next, adopting the idea of the Li-Zang parametrization (3.12) we propose the following ansatz for $G_{_{\text{eff}}}$

$$G_{_{\text{eff}}}(z) = G_{_{\text{eff}},0} \left[ 1 - \tilde{G}_1 \sin \left( \tilde{A} \ln (1 + z) \right) \right]$$

(4.9)

As in the case of EoS parameter, at the limit $z \to -1$ the frequency of oscillations becomes infinite but the amplitude remains constant. Here we take $\tilde{A} = \frac{2\pi}{ln2}$, i.e. the frequency of oscillation of $G_{_{\text{eff}}}$ is smaller than for the EoS parameter.

Next we use the data from SN Ia observations collected to Union 2.1 compilation to restrict the parameter space of selected models. The Hubble diagram for the Union 2.1 sample is presented in Figure 1. The best fit values of parameters for the parametrizations described above are presented in Table 1 and 1$\sigma$ and 2$\sigma$ confidence level contours in the $(w_0, w_1)$ parameter space are presented in Figure 2.

| Table 1. Best fit values of parameters for different parametrizations |
|-------------------|---|---|---|---|---|
| Parametrization   | $\Omega_{m,0}$ | $w_0$  | $w_1$  | $\tilde{G}_1$ | $\chi^2_{SN}/\text{dof}$ |
| CPL               | 0.280         | -1.008 | -0.016 | 0.001         | 0.9761                      |
| MZ I              | 0.279         | -1.007 | 0.011  | 0.001         | 0.9761                      |
| MZ II             | 0.281         | -1.012 | 0.015  | 0.001         | 0.9761                      |
| LZ                | 0.289         | -0.970 | 0.158  | 0.003         | 0.9760                      |
5. Classification of cosmological rips and rip scenarios

As mentioned in the introduction there is at least four different scenarios for which the evolution of the universe results in the dissolution (eternally or temporarily) of bound systems: a big rip, a little rip, a pseudo-rip and a quasi-rip. It was argued by Frampton et al. [10] that if the dark energy density is either constant or monotonically increasing, then there are three possible rip scenarios. Using the time asymptotic of the Hubble parameter, rip conditions can be classified as

- **Big rip:** $H(t) \to \infty$, $t \to t_{\text{rip}} < \infty$
- **Little rip:** $H(t) \to \infty$, $t \to \infty$
- **Pseudo-rip:** $H(t) \to H_{\text{max}}$, $t \to \infty$

Here $H_{\text{max}}, t_{\text{rip}}$ are finite constants. Monotonic evolution of the dark energy is certainly not the only possibility (for a review, see [11]). As pointed out by Wei et al. [11] most interesting is the case where energy density of dark energy is monotonically increasing, crossing the PDL and then monotonically decreasing in the second stage. This being so, it is possible that some
structures dissolve in the first stage and then form again a bounded structure in the second stage. The scenario is known as a quasi-rip [11].

The situation where the energy density of dark energy behaves non-monotonically seem to be possible in the framework of STG. Even in the so-called limit of GR the STG contains oscillating solutions as shown in [40, 41, 42, 43] and hence the oscillating EoS parameter of dark energy.

In order to find a boundary condition for massive structures to be dissolved Nesseris and Perivolaropoulos [44] proposed to use in the Newtonian limit a hybrid metric which interpolates between Schwarzschild and FLRW metrics

\[ ds^2 = -\left(1 - \frac{2G_NM}{a(t)b}\right)dt^2 + a(t)^2 \left[db^2 + b^2(d\theta^2 + \sin^2\theta d\phi^2)\right] \]  

(5.1)

Here \( M \) is the characteristic mass of the bounded system, \( b \) is the comoving radial coordinate and \( G_N \) is the Newton gravitational constant. Using the definition \( r = a(t) \cdot b \) and inserting the metric (5.1) into the geodesic equation it reads

\[ \ddot{r} = \frac{\ddot{a}}{a} r - \frac{L^2}{r^3} - \frac{G_NM}{r^2} \equiv -\frac{dV_{eff}}{dr} \]  

(5.2)

where \( L \) is the constant angular momentum per unit mass. Using Kepler’s third law the square of constant angular momentum can be written as \( L^2 = G_NM r_0^2 \). The equation for the radius of an initially stable (\( \ddot{r} = 0, \dot{r} = 0 \)) circular bound system reads (effective potential \( V_{eff} \) has a minimum)

\[ \frac{\ddot{a}}{a} r^4 - G_NM r + G_NM r_0 = 0 \]  

(5.3)

The fourth order equation (5.3) has a solution only if the following condition is fulfilled [11]

\[ \frac{\ddot{a}}{a} \equiv H(t) + H(t)^2 \leq \frac{27}{256} \frac{G_NM}{r_0^3} \]  

(5.4)

where \( r_0 \) is the characteristic scale of corresponding system. The condition (5.4) can be considered as a criterion for systems to stay bounded. If the expansion rate exceeds this limit the minimum of effective potential will vanish and systems start to break apart. It is worth to note, that the criterion (5.4) contains due to construction a constant gravitational coupling \( G_N \) which is not accurate in the framework of STG. At present we do not consider this question as a serious problem and assume that condition (5.4) is sufficiently accurate in STG and even if we are looking for the redshift values approaching \( z \to 1 \).

Finally, taking into account the relation between the cosmic time \( t \) and the redshift \( z \)

\[ \frac{d}{dt} = -H(1+z) \frac{d}{dz} \]  

(5.5)

we can write the condition (5.4) in a dimensionless form as follows

\[ F(z; p_i) \equiv \dot{h} + h^2 = -(1+z)h(z) h'(z) + h^2(z) \leq \frac{27}{256} \frac{G_NM}{r_0^3} H_0^{-2} \]  

(5.6)

where \( \dot{t} = d/dz, \ h(z) = H(z)/H_0 \) and \( p_i \) refers to parameters of the model. The derivative of \( F(z; p_i) \) with respect to redshift \( z \) determines the redshift value \( z_{ext} \) when the expansion rate is extremal. However, since the interesting dynamics takes place at the close limit \( z \to -1 \) it is more convenient to use the variable \( N = \ln a \). Upon using the relation between the cosmic time \( t \) and the number of e-folds \( N \)

\[ \frac{d}{dt} = H \frac{d}{dN} \]  

(5.7)
Figure 3. The quasi-rip realized by CPL (left) with parameter values $h = 0.6989$, $w_0 = -1.108$, $w_1 = -8 \cdot 10^{-6}$, $\Omega_{m,0} = 0.280$ and little rip realized by LZ (right) with parameter values $h = 0.6989$, $w_0 = -1.10$, $w_1 = -0.25$, $\Omega_{m,0} = 0.289$. Parameters $(w_0, w_1)$ are chosen to be inside $1\sigma$ likelihood contour (see Figure 2) and $\Omega_{m,0}$ is chosen as the best-fit value. The dashed line corresponds to the disintegration limit of the Coma cluster.

we can represent the rip condition (5.4) by

$$F(N, p_i) \equiv h(N) h'(N) + h^2(N) \leq \frac{27}{256} \frac{G_N M}{r_0^3} H_0^{-2}$$

(5.8)

where $' = d/dN$. In Figure 3 we present two examples to illustrate the dynamics allowed by parametrizations CPL (5.2) and LZ (5.12).

6. Summary

We considered different parametrizations of the EoS parameter $w_{de}(z)$ of dark energy and the oscillating parametrization of the effective gravitational constant $G_{eff}$ in the framework of general STG. Parameters of the model are fitted with the observational data of type Ia supernovas collected to Union 2.1 compilation. We also notice the following.

- Not surprisingly the data from SN Ia observation do not constrain the parameter space sufficiently to make significant conclusion about the future dynamics of the universe.

- As expected the results depend strongly on the parametrizations applied. The $1\sigma$ and $2\sigma$ likelihood contours for different parametrizations allow various dynamics, including the quasi-rip scenario. Moreover, it was shown [Frantskjavitšius A 2011 Cosmological rips in the general relativity limit (Tartu: BSc Theses)] that quasi-rip scenario is possible (without fitting the parameters of the model) in the framework of potential dominated STG cosmology at the limit of GR [41]. However, the best fit values of the parameters $(w_0, w_1)$ are in a range where the little rip scenario can be realized.

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