Thomas precession, persistent spin currents and quantum forces

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\textbf{Abstract} – We consider $T$-invariant spin currents induced by spin-orbit interactions which originate from the confined motion of spin carriers in nanostructures. The resulting Thomas spin precession is a fundamental and purely kinematic relativistic effect occurring when the acceleration of carriers is not parallel to their velocity. In the case where the carriers (e.g. electrons) have magnetic moment, the forces due to the electric field of the spin current can, under certain conditions, exceed the van der Waals-Casimir forces by several orders of magnitude. We also discuss a possible experimental set-up tailored to use these forces for checking the existence of a non-zero anomalous magnetic moment of the photon.

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\textbf{Introduction.} – The studies of the electronic spin degrees of freedom, spintronics, are an active branch of solid-state physics. In particular, spintronics of nanostructures, or nanospintronic, has developed quite rapidly in recent years [1–3], motivated by a number of basic questions on the nature of nanophenomena as well as by its potential impact on information technology. The underlying physical mechanism is the spin-orbit interaction of conducting electrons, which couples their spin degree of freedom to their orbital dynamics.

One distinguishes the extrinsic phenomena, the result of spin-dependent scatterings on (external) impurities, from the intrinsic phenomena, arising because of a certain spin-orbit coupled band structure. There are two basic dynamical mechanisms underlying intrinsic phenomena in semiconductors: i) the Rashba [4] coupling due to the combined effects of atomic spin-orbit coupling and structured inversion asymmetry; ii) the Dresselhaus [5] coupling due to the bulk crystal inversion asymmetry.

The intrinsic spin-orbit interactions are essential for many potential applications like spin-polarized currents without magnetism [6], spin field-effect transistor [7], topological insulator states [8,9], etc. One of the important spin-orbit effects in nanostructures is the persistent spin currents arising from the above dynamical sources of asymmetry, see, e.g., [10,11].

In this article we consider yet another mechanism for spin-orbit coupling whose origin is the Thomas spin precession [12–16], a fundamental relativistic effect. It occurs when the particles acceleration is not parallel to their velocity, \textit{i.e.} for any motion of relativistic particles with curving or winding trajectories. The precession has a purely kinematical origin since it results solely from the confined motion of the particles in a sufficiently small volume. It is not necessarily related to impurity scatterings or other extrinsic \cite{17,18} spin-orbit mechanisms. Likewise, the Thomas precession can occur even for ideal metallic nanoparticles without Rashba or Dresselhaus couplings.

As we will show below, the Thomas precession will induce persistent spin currents whose dependence on the spin degree of freedoms and on the quantum spectrum is different from those resulting from the Rashba-Dresselhaus couplings.

The inverse asymmetry (the broken $P$-invariance) is the cause of spin persistent currents due to spin-orbital coupling. However, time-reversal invariance ($T$-invariance) is not broken in this case. This is in complete agreement with the symmetry of pure spin (or other rotation) equilibrium currents, which are obviously not $P$-invariant but are $T$-invariant. Indeed, any geometric constraint breaks $P$-invariance.

A confining strip as an example of geometry in which there exist the winding persistent spin currents due to the
Thomas precession has been already considered in ref. [20] in the case of classical Brownian motion.

In this paper we discuss a quantum phenomenon in nanostructures, whose origin is also the Thomas precession. In this case the $P$-invariance with respect to winding and spin is also absent because of the confined motion of carriers.

**Thomas precession in nanostructures.** – We consider an ideal and neutral metallic wire as an example of Thomas precession due to the confinement of particles. Clearly if the wire were straight, the motion of free electrons along the wire would have no relation to precession and spin-orbit interactions. If, however, the wire is curved into a closed loop, then the motion of electrons along the loop will inevitably produce the Thomas precession of their spins. Note that there is no electric field in this case and the wire is neutral and equipotential. We mean here an external field except for the confining field, which, we assume, is included in the definition of the corresponding quasiparticles.

Let us estimate the order of magnitude of spin-orbit effects resulting from the Thomas precession in a loop of nanosize assuming for simplicity that the loop is a ring. In the leading relativistic approximation the spin-orbit energy from the Thomas precession is [12,14]

$$E_{s,o} \approx \frac{s \cdot (\vec{v} \times \dot{\vec{r}})}{2c^2} = \frac{v^2}{2c^2} \hat{s} \cdot \hat{\theta}. \quad (1)$$

Here the over-dot denotes the time derivative, $\times$ the vector product, $c$ the speed of light, $\hat{s}$ the particle spin in angular momentum units, $\vec{v}$ its velocity and $\hat{\theta}$ its winding angle defined as

$$\hat{\theta} = \int_0^t \frac{\vec{v} \times \dot{\vec{r}}}{v^2} \, dt.$$  

It is the vector sum of all the windings of the particle trajectory (see [20]). The acceleration of the winding modes in a ring at constant angular velocity is simply $\ddot{\vec{r}} = \vec{v}^2/r$, where $r$ is the radius of the ring. Comparing (1) with the commonly used estimate $e\hat{s} \cdot (\vec{v} \times E_{\text{eff}})/2mc^2$ of the spin-orbit energy in an electric field $E_{\text{eff}}$, where $e$ and $m$ are, respectively, the charge and the mass of the particle [21], we conclude that the Thomas precession of a free electron inside a metallic nanoring corresponds to a velocity-dependent dependent effective electric field,

$$E_{\text{eff}} \sim \frac{mv^2}{er}. \quad (2)$$

Typically for electrons in metals the Fermi energy $\varepsilon_F$ is of the order of several electronvolts (e.g. $\varepsilon_F \sim 6$ eV for Au), hence the Fermi velocity is $v_F \sim 10^6$ m/s. Thus, in the case of nanoring, where $r \sim 10^{-9} - 10^{-8}$ m and the particle velocity is of the order of the Fermi velocity $v \sim v_F$, the effective electric field in (2) can be quite large, i.e., $E_{\text{eff}} \sim 10^{10}$ V/m. We conclude that the spin-orbit effects can be well pronounced in nanorings. Note again that the electric field is just an “effective” field arising from the trajectory windings and there is no gradient of an electrical potential along a metallic nanoring.

Let us now show that the spin-orbit coupling (1) due to the Thomas precession leads to persistent spin currents. Note that we are using here a 1d ring and non-interacting fermions to obtain simple estimates, keeping in mind that we are interested in 2d or 3d nanostructures.

The effective classical Lagrangian of a free particle with spin, which takes into account the Thomas precession, is [13,16]

$$L = -mc^2 \sqrt{1 - \frac{v^2}{c^2}} - \hat{s} \cdot \left( \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} - 1 \right) \frac{\vec{v} \times \dot{\vec{v}}}{v^2}. \quad (3)$$

Note that the low-velocity expansion of the second term in the r.h.s. of (3) rightly reproduces (1). In the simple ring geometry we have in general

$$\ddot{\vec{r}} = r \hat{\theta} \hat{e}_\theta, \quad (4a)$$

$$\ddot{\vec{v}} = -r^{-1} \vec{v} \times \dot{\vec{e}}_\theta + r \hat{\theta} \hat{e}_\theta, \quad (4b)$$

where $\theta$ is the polar angle, $\hat{e}_\theta$ and $\hat{e}_\theta$ are the polar unit vectors in the plane of the ring. It follows that for the ring geometry

$$\frac{\vec{v} \times \dot{\vec{v}}}{v^2} = \hat{\theta} \hat{e}_z, \quad (5)$$

where $\hat{e}_z$ is the unit vector of the $z$-axis perpendicular to the ring, so that the winding angle and the polar angle are equal, thus denoted by the same symbol $\theta$. We obtain from (3)

$$L = -mc^2 \sqrt{1 - \frac{v^2 \hat{\theta}^2}{c^2}} - \left( \frac{1}{\sqrt{1 - \frac{v^2 \hat{\theta}^2}{c^2}}} - 1 \right) s_z \hat{\theta}. \quad (6)$$

where $s_z$ is the spin projection on $\hat{e}_z$. Note that the double derivative $\ddot{\theta}$ in (4b) does not contribute to the r.h.s. of (6). Nevertheless the quantization of (6) is quite non-trivial. Fortunately, in the case of non-interacting fermions, we can neglect states above the Fermi energy assuming for simplicity zero temperature, i.e. $r\hat{\theta} = v \leq v_F \ll c$. Thus, expanding the r.h.s. of (6) in the small parameter $vF/c \sim 10^{-2}$ and dropping the constant energy $-mc^2$, we obtain

$$L \approx \frac{mr^2 \hat{\theta}^2}{2} - s_z \frac{r^2 \hat{\theta}^3}{2c^2}, \quad r\hat{\theta} \ll c. \quad (7)$$

The angular momentum $p_\theta$ plays for (6) the role of a generalized momentum,

$$p_\theta = \frac{\partial L}{\partial \dot{\theta}} \approx mr^2 \hat{\theta} - \frac{3}{2} s_z r^2 \hat{\theta}^2. \quad (8)$$

Taking into account that $p_\theta \ll mv_F \approx m^2 r^2 c^2 / \hbar$ (the second inequality means that the quantum uncertainty of
the velocity $\hbar/mr$ has to be much smaller than $c$, i.e., $\theta \ll c/r \ll mc^2/\hbar$, we can express $\theta$ as

$$\theta \approx \frac{p_0}{m r} + \frac{3}{2} \frac{s \sigma z^2}{m^2 r^4 c^2} p_0^2. \quad (9)$$

Thus, using (7) and (9), the Hamiltonian corresponding to (6) is

$$H = p_0 \theta - L \approx \frac{p_0^2}{2mr^2} + \frac{1}{2} \frac{s \sigma z^2}{m^2 r^4 c^2}. \quad (10)$$

Now the standard quantization procedure

$$p_0 \rightarrow \hat{p}_0 = -i\hbar \frac{\partial}{\partial \theta}, \quad s_z \rightarrow \hat{s}_z = \frac{\hbar}{2} \hat{\sigma}_z. \quad (11)$$

yields the quantum Hamiltonian

$$\hat{H} = -\frac{\hbar^2}{2mr^2} \frac{\partial^2}{\partial \theta^2} + \frac{1}{4} \frac{\hbar^4}{m^3 r^6 c^2} \frac{\partial^3 \Psi}{\partial \theta^3}. \quad (12)$$

Its spectrum has doubly degenerate energy levels,

$$\epsilon_{n\sigma_z} = \frac{\hbar^2 n^2}{2mr^2} + \frac{\hbar^4 \sigma_z n^3}{4m^3 r^6 c^2}, \quad (13)$$

$$\sigma_z = \pm 1, \quad n = 0, \pm 1, \pm 2, \ldots,$$

since $\epsilon_{n\sigma_z} = \epsilon_{-n,-\sigma_z}$, and two-component states

$$\Psi_{n\sigma_z} = \frac{1}{4\pi} e^{in\theta} (1 + \sigma_z) \quad \Psi_{n\sigma_z}^\dagger \sim \hat{\Psi}_{n\sigma_z} \quad (14)$$

Equations (13), (14) can be used only for the winding numbers $n$ satisfying (cf. (7) and (9))

$$|n| < N \ll \frac{mc r}{\hbar}, \quad (15)$$

where

$$N \approx \frac{r \sqrt{2m e F}}{\hbar} \quad (16)$$

is a natural cutoff fixed by the Fermi energy, i.e. $\epsilon_{N\sigma_z} \lesssim \epsilon_F < \epsilon_{N+1\sigma_z}$. With $r \sim 1 \text{nm}$ we have roughly $N \sim 10$–100. Hence, $N$ is large but still much smaller than $mc r/\hbar \sim 10^3$, and (15) holds. In addition, the spacing between levels $\sim 0.1 \text{eV}$, i.e. $\sim 1000 \text{kelvin}$, is such that the zero-temperature approximation is still valid at room temperature.

A conventional method to describe the electron spin-orbit interaction in condensed-matter theory [22] as well as in a more general context of geometry of spin-orbit interaction [23] is based on the Dirac equation. However, we believe that the Thomas precession provides a rather general mechanism of interaction of spin and orbit of curvilinear motion, hence it can be of interest in the studies of other particles and quasiparticles: spinons in the Luttinger liquid, magnons, ions, photons, etc. This is why we believe that a semi-classical quantization of the classical “quasi-relativistic” Lagrangian (3) to obtain the quantum Hamiltonian (12) is quite appropriate and provides a wider range of applicability than a more involved argument based on the Dirac equation.

To find the operator of the spin current we use again the standard procedure based on the time derivative of the spin density observable $\hat{s}$, see, e.g., [24–26] (other approaches are also possible, see, e.g., [27,28])

$$\frac{d}{dt} \hat{s} = \frac{d}{dt} \hat{\Psi}^\dagger \hat{\sigma} \hat{\Psi} = \frac{1}{i\hbar} \left[ \hat{\Psi}^\dagger \hat{\sigma} \hat{H} \hat{\Psi} - (\hat{H} \hat{\Psi})^\dagger \hat{\sigma} \hat{\Psi} \right], \quad (17)$$

where $\hat{\Psi}$ is the wave function. Using (10)–(12), we obtain in the coordinate representation (recall that in our case the coordinate is the winding angle $\theta$) the continuity equation,

$$\frac{d}{dt} \hat{s}_i = -\frac{1}{r} \frac{\partial}{\partial x} \left( \hat{\Psi}^\dagger \hat{J}_{\theta,i} \hat{\Psi} \right), \quad i = x, y, z, \quad (18)$$

where

$$\hat{J}_{\theta,i} = \frac{1}{2} \left\{ \frac{\hat{p}_0}{mr} - \frac{\hat{\theta}}{m^3 r^5 c^2}, \hat{s}_i \right\} = \left\{ \hat{\sigma}_i, \hat{s}_i \right\} \quad (19)$$

are the components of the operator of the 1d spin current circulating in the ring. Here $\{,\}$ is the anticommutator and $\hat{\Psi}$ can be viewed as the operator of spin velocity.

Using the spectrum (13) and the eigenstates (14), we find that in the leading $N^{-1}$ approximation the only non-vanishing component of the spin current density is that in the $z$-direction,

$${\hat{s}_z} = \frac{1}{r} \sum_{n=-N}^{N} \sum_{\sigma_z=\pm 1} \hat{\Psi}_{n\sigma_z}^\dagger \hat{J}_{\theta,z} \hat{\Psi}_{n\sigma_z} \approx \sum_{n=-N}^{N} \sum_{\sigma_z=\pm 1} \frac{\hbar^2}{4m^3 r^6 c^2} \frac{\partial^3 \Psi_{n\sigma_z}}{\partial \theta^3} \Psi_{n\sigma_z} \approx \frac{1}{3} \frac{\hbar^4}{m^3 r^6 c^2} \left( \frac{r \sqrt{2m e F}}{\hbar} \right)^3. \quad (20)$$

Note that while all the components of the spin current operator (19) are non-zero, the $\hat{J}_{\theta,x}$ and $\hat{J}_{\theta,y}$ components vanish after the quantum-mechanical averaging. This is because of the symmetry of the problem: the plane ring with constant curvature, thus the circular symmetry. Hence the spin precession is symmetric around the $z$-axis.

The current is protected by the $T$-invariance. Indeed, the current dissipation requires the backscatterings without spin flip, which are impossible since the higher winding numbers are too distant (remember that the energy level spacings are $\sim 100$–1000K). Besides, all the states below the Fermi energy are occupied. As a result, the backscatterings with spin flip contribute only to the same spin current. Thus, the current is dissipationless and in equilibrium, similarly to known persistent currents (charge currents in normal metallic mesorings, surface currents in topological insulators, etc.). Because of the asymmetry of spectrum (13) (proportional to the quantum uncertainty due to the spatial confinement of particle motion), the current appears when the particles fill the sufficient amount.
of non-zero levels. This is similar to the case of persistent spin currents along the edges in topological insulators, where the spectrum of gapless edge states with a given spin is asymmetric with respect to the quasi-momentum reflections relative to the edges.

An important fact is that the whole spectrum (13)–(16) contributes to (20) (except for the levels with zero winding number). Note that one can rewrite (20) as

\[ j \sim -\frac{8\pi}{3m \hbar} N \frac{\varepsilon_F h}{mc^2} \frac{1}{2\pi r} \sim \hbar \left( \frac{v_F}{c} \right)^2 \frac{v_F}{r}. \]  

One obtains an estimate for \( j \) as a constant times the velocity quantum uncertainty \( \delta v \sim h/mr \), the Fermi level number \( N \), the relativistic factor \( \varepsilon_F/(mc^2) \), the spin momentum \( h/2 \), divided by the length of the ring \( 2\pi r \). Thus, the spin current results from the combination of geometric, quantum and relativistic effects.

It follows also from (21) that the current density decays in \( r \), so that it could become negligible at a macroscopic scale. If, however, we consider a “metamaterial”, i.e., a macroscopic sample paved by a high number of nanorings, then the resulting spin current can be quite large (a sample area of the order of 1 mm\(^2\) can contain up to \( 10^{12} \) such nanorings), thereby facilitating the experimental detection.

It is known that the current of the magnetic moment can produce an electric field decaying as \( R^{-1} \) in the distance \( R \) from the current or as \( R^{-2} \) in the case of spin rotation \[26,29\]. This field \( \vec{E} \) can be estimated by using a Lorentz force in the rest frame of the spin, i.e., via the transformation rules for a magnetic field in the rotating reference frame,

\[ \vec{E} \approx -\vec{\theta} \times \vec{R} \times \vec{B}. \]  

Here \( \vec{B} \) is the magnetic field induced by the electron magnetic moment in the rest frame and it is assumed that \( R \gg r \) and \( \vec{\theta} \ll c/R \), i.e., that the distance from the ring is much larger than its size and the rotation is slow enough. In fact the validity of the formula is a rather delicate issue (see, e.g., the review [30] and references therein), but we believe that the formula provides a correct order of magnitude in our estimates. Since the magnetic moment of an electron with spin \( s_z \) is \( g\mu_B s_z/\hbar \), where \( g \approx 2 \) is the gyromagnetic factor and \( \mu_B = e\hbar/(2m) \) is the Bohr magneton, the magnetic field induced by \( s_z \) in the rest frame is (cf. [29])

\[ \vec{B} \approx -\frac{g\mu_B \hat{z}}{2\pi \hbar} \frac{\partial Z}{\partial \vec{R}} \frac{\vec{R}}{R^3}, \quad Z = \hat{e}_z \cdot \vec{R}. \]  

Replacing here \( \hat{z} \) by \( 2\pi \vec{\theta} \) and using (20)–(23), we obtain the electric field in the laboratory reference frame,

\[ \vec{E} \sim \frac{\mu_0 g\mu_B}{3m c^2} \left( \frac{2\varepsilon_F}{m} \right)^\frac{\hat{z}}{3} \left( \frac{\vec{R} + 2Z\hat{e}_z}{R^3} - \frac{3Z^2\vec{R}}{R^5} \right). \]  

This leads to

\[ \mathcal{E} \sim \frac{4\mu_0 \mu_B \varepsilon_F}{3\pi R^2 mc^2 v_F} \]  

so that \( \mathcal{E} \sim 10^{-2} \) V m\(^{-1}\) at the nanoscale (\( R \sim 1-10 \) nm), i.e., it is comparable to or bigger than the electric field \( \sim 10^{-2} \) V m\(^{-1}\) obtained [10] at a distance 5 nm from a Rashba ring. It is also worth noting that the electric field due to the Thomas precession may exist in any (not necessarily metallic) nanoparticle which confines magnetic moments in motion. Note also that (18)–(25) have been derived for a constant curvature: we believe, however, that the same conclusions can essentially be reached in the case of a non-constant but sufficiently slowly varying curvature (similarly to the model of slowly varying bands in the semiconductor theory). If, however, a wire loop contains segments with strongly varying curvature, then we have not only to carry out the standard averaging over the corresponding quantum modes, but also to take into account an additional non-divergent term due to the non-uniform spin rotation in the continuity equation for spin current (see, e.g., [24]). This could make the corresponding theory much more involved.

Let us finally compare the electric force \( F_T = e\mathcal{E} \) acting on the charge carrier due to Thomas spin precession and the electric force \( F_C \) due to the van der Waals-Casimir effect, the only known so far electric force in metallic devices under the conditions of equilibrium and neutrality. According to [31]

\[ F_C \sim \frac{7c_0}{\pi} \frac{\varepsilon_F}{R^3}, \quad c_0 = \frac{143}{16} R \gg r. \]  

It follows then from (25) and (26)

\[ \frac{F_T}{F_C} \sim \frac{21c_0}{4\pi} \frac{\varepsilon_F}{\varepsilon_F^2} \frac{R^6}{mc^2 v_F^2} \]  

Let \( R_0 \) be the distance where the Thomas force and the van der Waals-Casimir force are equal: from (27) one has

\[ R_0 \sim \left( \frac{mc^2}{\varepsilon_F^2} \frac{h\varepsilon}{\mu_0 \mu_B v_F} \right)^\frac{1}{3} r, \]  

so that taking \( r \sim 1 \) nm one gets \( R_0 \sim 10^2 \) nm. In other words, the Thomas precession forces may dominate the van der Waals-Casimir forces for distances from \( 10^2 \) nm and larger. Therefore, the measurements of the above more slowly decaying Thomas forces can be carried out in experiments similar to but even simpler than those of works [32] dealing with the Casimir forces. It has already been mentioned above that by using “metamaterials” (pavements of macroscopic devices by nanoparticles) one can obtain forces proportional to the area of the macroscopic sample.

On a possible experiment on the anomalous magnetic moment of the photon. – We have so far considered electrons because they possess a non-zero magnetic
moment. On the other hand, photons have zero magnetic moment despite their spin being 1. Nevertheless, there are theoretical arguments (see, e.g., [33,34] and references therein) according to which the photon has an anomalous magnetic moment $\mu_\gamma$. It is clear, however, that any experimental proof of this fact has to be fairly sophisticated. For instance, the use of inhomogeneous magnetic fields for direct measurements would require extremely strong fields in view of a very large magnetic moment $\mu_\gamma$. It has also been suggested that the electric forces stemming from other sources, since the unique difference which does not contain $\mu_\gamma$. Hence, the effective magnetic field

$$B \sim \frac{E_{s-o}}{\mu_\gamma} \sim \frac{\hbar c}{2n^3r\mu_B} \frac{\mu_B}{\mu_\gamma} \sim \frac{10^5}{2} \frac{\mu_B}{\mu_\gamma} \text{ tesla}$$

acting on $\mu_\gamma$ is quite strong.

It follows that the Thomas precession could be used in order to detect $\mu_\gamma$ even if it is very small. Indeed, the resulting photonic spin current $j_\gamma$ can be estimated as

$$j_\gamma \sim \rho_\gamma \frac{E_{s-o}}{p} \sim \rho_\gamma \frac{\lambda c}{n^3r},$$

where $p = \hbar/\lambda$ is the photon momentum and $\rho_\gamma$ is the 1D photon density. The electric field due to the Thomas precession follows by repeating the same steps as in (24) with $g\mu_B$ replaced by $\mu_\gamma$ and $j$ by $j_\gamma$,

$$\mathcal{E}_\gamma \sim \frac{\mu_0\mu_\gamma j_\gamma}{\hbar R^2} \sim \frac{\mu_0\rho_\gamma \mu_\gamma \lambda c}{n^3 R^2}.$$

Therefore, $\mu_\gamma$ can be detected from the measurements of $\mathcal{E}_\gamma$ for a sufficiently dense array of curved light circuits.

We can compare the electric field $\mathcal{E}_\gamma$ obtained here and the electric field $\mathcal{E}$ derived above for electrons in a metal nanoring of the same radius,

$$\frac{\mathcal{E}_\gamma}{\mathcal{E}} \sim \frac{\rho_\gamma \lambda}{\rho \mu_B} \left( \frac{\epsilon}{v_F} \right)^3 \frac{1}{n^3}.$$

We see that a very small value of $\mu_\gamma/\mu_B$ can be in part compensated by the cube of the inverse relativistic factor $v_F/c$.

We have presented above estimates for winding effects in light guide experiments. Similar effects could as well show up in the winding of diffuse light and the associated magnetic moment edge currents for the strip geometry considered in [20].

**Conclusion.** – We have shown that the Thomas precession resulting from the confined motion of spins carriers can generate some specific persistent spin currents. For the sake of concreteness, we have considered conducting electrons in metallic nanorings. It was pointed out, however, that similar effects can be expected for other spin carriers like magnons, spinons, ions in gases, etc. The Thomas precession of magnetic moments can generate electric forces that are stronger than the usual van der Waals-Casimir forces in metallic samples. Hence, it seems certainly appropriate to take the Thomas precession effects into account when dealing with metallic nanoparticles. It has also been suggested that the electric forces due to the Thomas precession can be a possible tool for experimentally testing the existence (see [33,34] and references therein) of a non-zero anomalous magnetic moment for the photon. We have not discussed the influence of the Thomas precession, our results in particular, on transport phenomena. However, the obtained deformation of the carrier spectrum, resulting from their curved motion, can

\[ \boxed{18001-p5} \]
produce contributions of the “non-extrinsic” Thomas pre-
cession to various transport characteristics. We will ad-
dress this important question elsewhere.

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