Transport properties of one-dimensional interacting fermions in aperiodic potentials

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Abstract

Motivated by the existence of metal-insulator transition in one-dimensional non-interacting fermions in quasiperiodic and pseudorandom potentials, we studied interacting spinless fermion models using exact many-body Lanczos diagonalization techniques. Our main focus was to understand the effect of the fermion-fermion interaction on the transport properties of aperiodic systems. We calculated the ground state energy and the Kohn charge stiffness $D_c$. Our numerical results indicate that there exists a region in the interaction strength parameter space where the system may behave differently from the metallic and insulating phases. This intermediate phase may be characterized by a power law scaling of the charge stiffness constant in contrast to the localized phase where $D_c$ scales exponentially with the size of the system.

I. INTRODUCTION

Understanding the transport properties of interacting many-fermion disordered systems has been one of the most challenging problems in recent years. Non-perturbative effects
are introduced by strong fermion-fermion correlations occurring in these systems and the 
available numerical tools are plagued by the exponential increase of the Hilbert space as 
larger and larger systems are studied.

One-dimensional (1D) interacting fermion systems have emerged as useful models since 
exact solutions exist for some of them. Additionally, non-interacting 1D models, in aperiodic 
(such as quasiperiodic or random) potentials are known to exhibit interesting phase dia-
grams. In case of random potentials, the non-interacting fermion systems exhibit Anderson 
localization. Anderson insulator is characterized by a gapless point spectrum. This phase 
should be distinguished from the one occurring in a Mott insulator which is characterized by 
a finite gap originated in fermion-fermion interaction effects. The quasiperiodic and deter-
ministic aperiodic models exhibit metal-insulator transition. In addition, these aperiodic 
models exhibit a phase which is intermediate between the metallic and the insulating phase 
and has been christened as critical. Critical behavior is characterized by multifractal quan-
tum states and energy spectrum. The transport properties for this phase, for example, 
resistance, are known to be oscillating. Unlike the Anderson localized phase characterized 
by the exponential decay of the single particle fermion wave function, the critical states 
exhibit at most power law decay.

The possible existence of a phase which is in-between metallic and insulating in an 
aperiodic many-body fermion systems is a completely open problem. In this paper we present 
some preliminary results regarding this interesting problem in spinless many-body fermion 
model. We compute the Kohn stiffness constant describing the ground state conductivity 
at zero temperature by exact diagonalization on finite chains of various sizes. The main 
question that we address here is how the metal-insulator transition and the critical phase of 
aperiodic non-interacting models are affected by the presence of many-fermion interaction.

The significance of our studies can be also viewed from a different perspective. Recently, 
there have been many theoretical investigations of the effects of fermion-fermion interaction 
on the problem of persistent currents in mesoscopic rings. These studies were moti-
vated by the fact that the free fermion theory underestimates the magnitude of the observed
persistent current compared to the observed experimental value. The studies on spinless models showed that the repulsive interaction always decrease the amplitude of the current. As pointed out by Giamarchi et al., both attractive and repulsive ground states have charge density fluctuations which are easily pinned by the disorder. However, in the case of attractive ground state, superconducting fluctuations screen the disorder resulting in the enhancement of current. Our numerical results on aperiodic interacting systems are consistent with the above picture. Novel aspect of our results is the existence of a peak in the stiffness constant at a characteristic value of the attractive interaction suggesting the existence of a new type of phase in strongly correlated disordered fermion systems.

In section II, we describe the basic model under investigation and give a short description of the method utilized to compute the charge stiffness $D_c$. Section III describes the results obtained from our numerical simulations. Finally, section IV contains our conclusions and a discussion of the possible implications of this research.

II. MODEL SYSTEM AND CHARGE STIFFNESS CALCULATION

We studied an interacting spinless fermion model on a 1D ring in an aperiodic potential,

$$H = -\sum_{i=1}^{N} (c_{i}^\dagger c_{i+1} + c_{i+1}^\dagger c_{i}) + V \sum_{i=1}^{N} n_{i} n_{i+1} + \sum_{i=1}^{N} h_{i} n_{i}.$$  \hspace{1cm} (2.1)

The site dependent potential is chosen to be of the form, $h_{i} = \lambda \cos(2\pi \sigma i \nu)$. Here, $\lambda$ represents the strength of the potential and $\sigma$ is an irrational number which is chosen for convenience to be the Golden Mean ($\frac{\sqrt{5} - 1}{2}$). The parameter $\nu$, determines the nature of aperiodicity: for $\nu = 1$ the potential is quasiperiodic while for $\nu > 3$ it has been shown to generate pseudorandom terms. Therefore, this particular form of potential facilitates the study of both quasiperiodic as well as pseudorandom cases by varying $\nu$.

For $\lambda = 0$, the interacting spinless fermion problem could be mapped to the Heisenberg-Ising XXZ spin problem. This is an old problem that have been extensively studied and for which a closed Bethe’s anzast solution exists. In the non-interacting limit ($V = 0$), the
quasiperiodic case ($\nu = 1$) can be reduced to the famous Harper equation\textsuperscript{16}. The Harper equation exhibits a metal-insulator transition in one dimension\textsuperscript{4,5}. At the onset of transition $\lambda_c = 1$, the quantum states are neither extended nor localized but instead exhibit fractal characters and have been termed as \textit{critical}. The spectrum contains an infinite number of gaps and is believed to be a Cantor set of zero measure. These interesting aspects of the wave function and the spectra have been shown to be reflected in the transport properties such as Launder resistance\textsuperscript{8,9}.

For finite values of fermion interaction $V$, the problem is more complicated due to the many-body nature of the wave function. Unlike the non-interacting case, where the behavior of the system can be described by studying the single particle wave function and the associated eigenvalues, in many-body problem, one needs an alternative method to characterize the nature of many-body state. Recently, the Kohn stiffness constant $D_c$ has been introduced to characterize the difference between the metallic and insulating phases as it gives a direct quantitative measure of the electronic conductivity of the system\textsuperscript{17}. In this paper, we will use $D_c$ to determine the nature of the phase of the aperiodic system.

To compute the Kohn stiffness constant $D_c$, we assumed periodic boundary conditions for the fermion model described by equation (2.1). We are interested in the \textit{persistent} current response to a vector potential of magnitude $|\vec{A}| = \frac{\Phi}{N}$ in the $x$ direction, where $\Phi$ is the flux threading the 1D ring and $N$ is the number of sites in the chain. We used the Lanczos diagonalization method\textsuperscript{18} to obtain the ground state energy of the system $E_0(\Phi)$ as a function of the flux. The Kohn stiffness constant is then given by the equation\textsuperscript{10,17},

$$D_c = \frac{N}{2} \frac{d^2 E_0(\Phi)}{d\Phi^2} |_{\Phi = \Phi_{\text{min}}}.$$  \hspace{1cm} (2.2)

The numerical calculations were done using several sizes for the 1D ring up to maximum size of $N = 14$. We studied the behavior of the Kohn stiffness constant $D_c$ as a function of the parameters $V$ and $\lambda$ describing the strength of the interaction and the strength of the aperiodic potential respectively.
III. SIMULATION RESULTS

We did simulations of the aperiodic spinless system for various sizes \( N \), and the electronic densities \( \rho = \frac{N}{N_e} \), for many values of the parameters \( V \) and \( \lambda \). To simulate golden mean quasiperiodicity into the model, we used Lanczos methods for systems of various Fibonacci sizes. Furthermore, in order to keep the fermion density \( \rho \) almost a constant, we worked with densities which are the rational approximants to the golden mean \( \sigma \) or the square of the golden mean \( \sigma^2 \). This procedure provides several possible sizes (5, 8, 13) for which the Lanczos diagonalization can be done at almost constant density. Therefore, our studies are for systems away from half-filling where the umklapp processes become irrelevant and the system in absence of disorder is metallic. By studying few different sizes, we were able to monitor finite size effects that could be present.

Figure 1, shows the results for \( D_c \) describing the interplay between aperiodicity (with \( \nu = 1 \)) and fermion-fermion interaction. The ground state transport properties are obtained for both the repulsive as well as for the attractive many-body interaction. The values of \( D_c \) decrease as \( \lambda \) increases for all values of the fermion interaction. Consistent with the previous results on disordered systems, the repulsive interaction is found to decreases the \( D_c \) while the attractive interaction is found to increase the stiffness. As seen from the figure, an interesting aspect of the attractive ground state is the existence of a very prominent peak around \( V = -2.5 \) that survives even in the regime where the non-interacting system is an insulator. The location of the peak is insensitive to the value of \( \lambda \), however the peak gets narrower with increase of \( \lambda \). Similar effects were also observed for other densities which are rational approximants of \( \sigma^2 \).

The figures indicate the possible existence of a region where \( D_c \) may takes intermediate values: between those of a metallic and those corresponding to the Anderson localized insulating phase. In order to study this effect in more detail, we performed further simulations to study the behavior of \( D_c \) versus \( \lambda \) for different values of \( V \) (see figure 1c). The figure clearly shows that in spite of the general decreasing behavior of \( D_c \) with \( \lambda \) for any value of
for $V = -2.5$ the values of $D_c$ are relatively larger than those for $V = -1$ and $V = 0$. This confirms the special behavior of $D_c$ around $V = -2.5$.

Figure 2 shows the corresponding results for the pseudorandom case ($\nu > 3$). For $\lambda = 0$ case we observe the well known transition from an insulating to the metallic phase as the interaction strength parameter $V$ is varied. In analogy with the results of the quasiperiodic case, the attractive ground state in the pseudorandom case also exhibits the novel characteristics namely the existence of a peak in $D_c$ as $V$ is varied. However, unlike the quasiperiodic case, the effect is less dominant in the pseudorandom. For the repulsive ground state, the interaction decreases the $D_c$ value. For larger values of $\lambda$ a slight increase is observed for small values of $V$.

The intriguing behavior of the conductivity manifested by a characteristic peak as the attractive interaction $V$ is varied, for various strengths of the deterministic disorder $\lambda$ was further analyzed by studying the dependence of $D_c$ on the size of the system. We notice that even though the height of the peak in $D_c$ decreases with $N$, the size of the system, this decrease is rather slow, particularly in comparison with the variation in $D_c$ with $N$ in the regime far from the peak, i.e. in the insulating phase. We conjecture that in the insulating phase, the $D_c$ decays exponentially with the size of the system, while in the regime near the peak, the charge stiffness decays as a power law. This conjecture was verified at a special point $V = 0$. For $\lambda = 2$ where the system is known to be in a critical state, the $D_c$ value was found to exhibit a power law decay with the size of the system. On the other hand, in the localized phase, the $D_c$ value was shown to decay exponentially with $N$.

We would like to mention that the reasoning for the above conjecture regarding the variation in $D_c$ with $N$ is in our observation that figure 1 describing the dependence of $D_c$ with $V$ is reminiscent of the plots of total band width (TBW) in the models of non-interacting fermions in a quasiperiodic field. It is known, for example, that the TBW for a critical non-interacting fermion model decays like a power law with the size of the systems. This is in clear contrast to the behavior at the localized phase where the TBW decays exponentially. This fact offers a powerful criterion to distinguish both phases. Since the
TBW is a measure of transport properties, it is conceivable that its scaling properties are similar to those of the $D_c$ constant.

**IV. CONCLUSIONS AND DISCUSSION**

The purpose of this work has been to gain some insight in understanding of the transport properties of a 1D chain of spinless fermions under the concurrent presence of strong interaction and aperiodicity. The numerical Lanczos diagonalization method used for these simulations is in essence an exact method. Unfortunately, its major limitation is due to practical aspects; even for relatively small sizes of the systems, large amounts of RAM memory and fast CPUs are required to obtain accurate values of the ground state energy.

In spite of these difficulties, we think that it is possible to extract some valuable information from our numerical simulations as our results are independent of various parameters such as the amount of disorder, fermion density and the system sizes. First of all, our calculations confirm the known fact that the effect of disorder in the interacting system is the localization of the metallic phase. However, the simulations show that for aperiodic potentials, we may have a phase that is not completely localized as in an Anderson insulator, nor completely metallic as in a Mott conductor. This fact seems to be independent of the density. Comparison of quasiperiodic and pseudorandom cases suggest that in the quasiperiodic case this intermediate phase may exist in a finite parameter interval while in the pseudorandom case this phase may exist at a single point marking the boundary between the metallic and Anderson insulating phase. This distinction between the behavior in the quasiperiodic and the pseudorandom cases is reminiscent of the analogous distinctions that is known to exist in some non-interacting models studied previously.

Our preliminary results are on systems of rather small sizes, and therefore, it is hard to reach any definite conclusion regarding the the nature of the new phase proposed here. On the other hand, the existence of a region with a characteristic peak is seen on systems of various sizes as well as of various electron densities. The clarity with which this peak
appears seems to hint some new mechanism involving some sort of competition other than the known screening of the disorder due to superconducting fluctuations. Therefore, it is tempting to speculate the possibility of a new type of phase in aperiodic strongly correlated systems. We hope that our preliminary studies will simulate further research in this area.

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FIGURES

FIG. 1. Quasiperiodic case ($\nu = 1$). (a) Charge stiffness versus $V$ for $\rho = \frac{3}{8}$ and $\sigma = \frac{5}{8}$. (b) Charge stiffness versus $V$ for $\rho = \frac{5}{13}$ and $\sigma = \frac{8}{13}$. Different curves represent different values of $\lambda$, namely: $\lambda = 0.0$ (solid line), $\lambda = 0.5$ (dotted line), $\lambda = 1.0$ (short-dash line), $\lambda = 1.5$ (long-dash line) and $\lambda = 2.0$ (dot-dash line). (c) Charge stiffness versus $\lambda$ for $\rho = \frac{5}{13}$ and $\sigma = \frac{8}{13}$. Different curves represent different values of $V$, namely: $V = -2.5$ (solid line), $V = -1.0$ (dotted line) and $V = 0.0$ (short-dash line).

FIG. 2. Pseudorandom case ($\nu > 3$). (a) Charge stiffness versus $V$ for $\rho = \frac{3}{8}$. (b) Charge stiffness versus $V$ for $\rho = \frac{5}{13}$. Different curves represent different values of $\lambda$, namely: $\lambda = 0.0$ (solid line), $\lambda = 0.5$ (dotted line), $\lambda = 1.0$ (short-dash line), $\lambda = 1.5$ (long-dash line) and $\lambda = 2.0$ (dot-dash line).
Chaves: fig. 1b
Chaves: fig. 2a

![Graph](image_url)
