Resonant decay of flat directions

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We study preheating, i.e., non-perturbative resonant decay, of flat direction fields, concentrating on MSSM flat directions and the right handed sneutrino. The difference between inflaton preheating and flaton preheating, is that the potential is more constraint in the latter case. The effects of a complex driving field, quartic couplings in the potential, and the presence of a thermal bath are important and cannot be neglected.

Preheating of MSSM flat directions is typically delayed due to out-of-phase oscillations of the real and imaginary components and may be preceded by perturbative decay or $Q$-ball formation. Particle production due to the violation of adiabaticity is expected to be inefficient due to back reaction effects. For a small initial sneutrino VEV, $\langle \mathcal{N} \rangle \lesssim m_N/h$ with $m_N$ the mass of the right handed sneutrino and $h$ a yukawa coupling, there are tachyonic instabilities. The $D$-term quartic couplings do not generate an effective mass for the tachyonic modes, making it an efficient decay channel. It is unclear how thermal scattering affects the resonance.

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I. INTRODUCTION

During inflation the scale of quantum fluctuations is set by the Hubble constant. Scalar fields which are light with respect to the Hubble constant fluctuate freely, resulting in condensate formation. Such condensates can play an important role in the evolution of the universe. A condensate can affect the thermal history of the universe, for example if its decay is accompanied by a large entropy production $^{1}$. The baryon asymmetry in the universe may originate from a condensate: Affleck-Dine baryogenesis utilizes a scalar condensate along a flat direction of the supersymmetric standard model $^{2}$, whereas Refs. $^{3}$ discuss non-thermal leptogenesis from a decaying right-handed sneutrino condensate. Condensates can fragment into non-topological solitons, called $Q$-balls, which may have implications for dark matter $^{4,5}$. In the curvaton scenario, quantum fluctuations of a condensate generate the density perturbations responsible for the observed CMB temperature anisotropy $^{4,6,8}$.

Field directions along which the effective mass vanishes, so-called flat directions, are a generic
feature of supersymmetric theories. There are many directions in field space along which the potential vanishes identically at tree level. The classical degeneracy along the flat directions is protected from perturbative quantum corrections in the supersymmetric limit by the non-renormalization theorem \[9\]. The flatness is lifted by soft terms from supersymmetry breaking. If the soft terms are sufficiently small during inflation the fields along the flat directions will be excited. Examples of flat directions in the MSSM are the $H_uL$ and $\bar{u}\bar{d}d$ directions \[2, 10, 11\]. Other examples of flat directions are the right-handed sneutrino and moduli fields. The masses of pseudo-Goldstone bosons are protected by an approximate global symmetry, and therefore can be kept naturally light during inflation. The Peccei-Quinn axion is an example of a pseudo-Goldstone boson that might condense during inflation. We will generically refer to fields parameterizing flat directions as flaton fields, or simply flatons.

Inflation erases all inhomogeneities along the flat direction, leaving only the zero-mode condensate. The vacuum expectation value (VEV) of the homogeneous mode can become large during inflation. In the post-inflationary epoch the field amplitude is initially damped by the expansion of the universe and remains essentially fixed. This stage ends when the Hubble constant becomes of the order of the flaton mass, at which point the field starts oscillating around the minimum of the potential. This is similar to what happens in chaotic inflation models, where at the end of inflation the inflaton field starts oscillating around the minimum of the potential \[12\].

Reheating of the universe through inflaton decay is a well studied problem \[13\]. It is now understood that in many models non-perturbative processes, collectively known as preheating, can lead to rapid decay of the inflaton field \[14, 15, 16, 17, 18\]. In preheating the decay of the inflaton occurs resonantly, leading to a rapid amplification of one or more bosons to exponentially large occupation numbers. This process is eventually halted by the expansion of the universe, and by the back reaction of the produced quanta. The decay products scatter off the inflaton field leading to further decay of the oscillating inflaton zero-mode \[19, 20\].

Flat direction condensates with large initial amplitudes may also undergo resonant decay. There is almost no mention of this possibility in the existing literature on cosmological scenarios based upon flat direction condensates. In this paper we analyze the possible occurrence of flaton preheating. We start our discussion in the next section with a review of preheating. We will highlight the differences between inflaton and flaton preheating. We restrict a detailed discussion of flaton preheating to two specific examples: Preheating of MSSM flat directions is the subject of section III, whereas section IV is devoted to resonant decay of the right-handed sneutrino. We conclude in section V.
II. PREHEATING

Preheating with a real driving field has been discussed in detail in [14, 15]. Numerical studies of preheating including the subsequent thermalization phase can be found in [19, 20]. The generalization to complex fields is described in [16, 17]. We will summarize the main results.

A. Real driving fields

The flat direction condensate starts oscillating in its potential when the Hubble parameter becomes of the order of the flaton mass. Higher order terms in the potential rapidly become negligible, and the scalar potential can be written as

\[ V(\Phi, \chi) = \frac{1}{2} m^2_\phi \Phi^2 + \frac{1}{2} m^2_\chi \chi^2 + \frac{1}{2} h^2 \Phi^2 \chi^2, \]

with \( \Phi \) the field parameterizing the flat direction, and \( \chi \) some other scalar field it couples to. For the moment we take \( \Phi \) real, postponing the discussion of preheating with a complex driving field to the next subsection. In supersymmetric theories the quartic coupling can arise from either a Yukawa coupling in the superpotential, or from gauge interactions.

The equation of motion for the rescaled modes \( \chi_k = a^{-3/2} X_k \) in an expanding FRW universe is

\[ \ddot{X}_k + \omega^2_k X_k = 0, \]

with \( k \) the comoving momentum, and a dot denotes differentiation with respect to time. Further, \( \omega^2_k = k^2/a(t)^2 + m^2_\chi + h^2 \Phi^2(t) + \Delta \), with \( \Delta = -\frac{3}{4}(\dot{a}/a)^2 - \frac{3}{2}(\ddot{a}/a) \). The scale factor is \( a \propto t^p \) with \( p = 1/2 \) (\( p = 2/3 \)) in a radiation (matter) dominated universe. Then \( \Delta = \frac{3}{4}(\frac{p-1}{p} - \frac{1}{3})H^2 + \xi R \). Soon after the onset of flaton oscillations \( \Delta \) becomes negligible small. The flaton zero-mode is \( \Phi = \phi \cos(m_\phi t) \) with the amplitude red shifting as \( \phi \propto a^{-1/p} \).

The mode equation can be brought in the form of a Mathieu equation [21]

\[ X''_k + (A_k - 2q \cos 2z)X_k = 0, \]

with \( z = m_\phi t \), and prime denotes differentiation with respect to \( z \). Further

\[ A_k = \frac{k^2/a^2 + m^2_\chi}{m^2_\phi} + 2q, \]

\[ q = \frac{h^2 \phi^2}{4m^2_\phi}. \]
An important feature of the solutions to the Mathieu equation is the existence of exponential instabilities. For \( q > 1 \), many resonance bands are excited. Preheating occurs in the broad resonance regime where particle production is efficient for modes with momenta \( k^2 \leq A - 2q \). The occupation numbers of quantum fluctuations grows exponentially: \( n_k \propto \exp(\mu_k z) \) with significant exponent \( \mu_k \sim 0.1 \). The typical momenta \( k_* \) of the particles produced is \( k_* = k_{\text{max}}/\sqrt{2} = m_\phi q^{1/4}/\sqrt{2} \), where we have assumed \( m_\chi^2 \ll \sqrt{q} m_\phi^2 \).

In an expanding universe preheating attains a stochastic character. However the net result is still an exponential growth of \( \chi \)-fluctuations. Preheating is halted by the expansion of the universe when \( q \) falls below unity.

Effective preheating rests on two principles, violation of adiabaticity and Bose enhancement. Particle production results from non-adiabatic changes in the effective frequency of the \( X_k \) modes. Adiabaticity is violated when

\[
|\dot{\omega}_k| \gtrsim \omega_k^2, \tag{6}
\]

which happens each time the flaton zero-mode goes through the minimum of the potential and changes rapidly. The occupation numbers of the decay quanta grow exponentially fast due to Bose-enhancement. As a result, preheating is robust. Resonant production occurs as long as the non-adiabaticity condition Eq. (6) is fulfilled. Adding additional fields or couplings (e.g. \( h m \Phi_2 \chi^2 \)) has little or no effect on the resonant period. However, the back reaction effects are very model dependent; when and how they become important depends on the specifics of the potential.

**B. Complex Driving fields**

Supersymmetric theories inherently involve complex scalar fields. Phase-dependent terms can arise naturally in the potential through soft SUSY-breaking terms. A relative phase between the oscillations of the real and imaginary components of the flaton field leads to a trajectory that is elliptic. The minimum amplitude is no longer \( |\Phi|_{\text{min}} = 0 \) as it is for a real driving field, but instead \( |\Phi|_{\text{min}} = b \) with \( b \) the semi-minor axis of the ellipse. This may prevent adiabaticity violation from occurring.

We can decompose the complex driving field into real and imaginary components: \( \Phi = \Phi_R + i \Phi_I \). By a phase rotation the largest amplitude component of oscillation can be put in the real piece.
Then
\[
\Phi_R = \phi \sin(m_\phi t), \\
\Phi_I = f\phi \cos(m_\phi t),
\]
with \( f = b/a \), the ratio of the semi-minor and semi-major axis of the elliptic trajectory. We will refer to \( f \) as the ellipticity of the orbit; note that \( f = \sqrt{1-e^2} \) with \( e \) the eccentricity of the ellipse. As before, the mode equation for the \( \chi \)-quanta in the time-dependent \( \Phi \)-background can be mapped into a Mathieu equation, with now
\[
A_k(f) = \frac{k^2/a^2 + m_\chi^2 + f^2 h^2 \phi^2}{m_\phi^2} + 2q(f), \\
q(f) = (1 - f^2) \frac{g^2 \phi^2}{4m_\phi^2}.
\]
(8)
(9)
The \( q \)-parameter for an elliptic trajectory is reduced by a factor \((1 - f^2)\), with respect to the case of a pure oscillatory trajectory.

Adiabaticity violation as defined in Eq. (6) occurs when
\[
\frac{k^2}{a^2} + m_\chi^2 + h^2 f^2 \phi^2 + (1 - f^2) h^2 \Phi_R^2 \lesssim (1 - f^2) h^2 \Phi_R \Phi m_\phi)^{2/3},
\]
(10)
The ellipticity is negligible small when for typical momenta \( k \sim k_* \), the \( h^2 f^2 \phi^2 \) term in the above equation can be neglected. This is the case for
\[
f \lesssim f_R \equiv \frac{1}{2q(0)^{1/4}},
\]
(11)
and preheating proceeds as for a real driving field. For larger ellipticities the term proportional to \( f^2 \) in Eq. (10) dominates. This leads to an upper bound on \( q(0) \) for which broad resonance is effective:
\[
q(0) \lesssim q_c \equiv \frac{1 - f^2}{16f^4}.
\]
(12)
The reason is that for larger values of \( q(0) \) the semi-minor axis of the ellipse is large, \( b \propto f \sqrt{q(0)} \), and adiabaticity violation does not occur. For large initial \( q \)-values resonant decay is delayed until the expansion of the universe red shifts the \( q \)-parameter below the critical value. As discussed below, preheating can be very efficient if at its onset \( q(f) \gtrsim 10^3 \), which requires \( f \lesssim 0.1 \). Note that for large ellipticities \( f = 1 - e > 0.5 \) the upper bound requires \( q \lesssim \epsilon \), and preheating never takes place.
In supersymmetric theories bosonic preheating is generically accompanied by fermionic preheating, since a Yukawa coupling in the superpotential leads also to a fermion coupling of the form

$$\mathcal{L} \ni (m_f + h\phi)\bar{\psi}\psi,$$

(13)

with $m_f$ the fermion mass. Resonant production of fermions has been studied in [18]. As long as the flaton amplitude is larger than $m_f/h$ (i.e., $q \gtrsim 1$), and the ellipticity is sufficiently small, there is an instant during each flaton oscillation that the effective mass of the fermion vanishes, and fermions are produced. Within about ten oscillations the fermion occupation number is saturated at a time-averaged value $n_\psi \sim 1/2$ for momenta within a Fermi sphere of radius $k_f \sim m_\phi q^{1/4}$. The Pauli-exclusion principle forbids occupation numbers beyond one. The back reaction of fermions can catalyze bosonic preheating [18].

Another interesting property of supersymmetric theories with regard to preheating was observed in [17], who studied a superpotential of the form $W = \frac{1}{2} S^2 + \frac{1}{2} (m_\chi + h\phi)\chi^2$. After diagonalizing the mass matrix for the real and imaginary component of $\chi$, it turns out that one of the masses can become tachyonic during part of the oscillation period of $\phi$. As a result, quantum fluctuations of $\chi_k$ grow exponentially. We will refer to this kind of instability as tachyonic preheating. The occurrence of tachyonic modes is quite model dependent.

\section*{C. Back reaction effects}

Which back reaction effects play an important rôle, and at what stage, is rather model dependent. We will list here four different back reaction effects for bosonic preheating.

- When the quantum fluctuations of $\chi$ grow exponentially large, the effective flaton mass $m_{\phi,\text{eff}}^2 = m_\phi^2 + g^2 \langle \chi^2 \rangle$ becomes dominated by the variance term. The oscillation frequency of $\Phi$ rapidly increases, and energy is rapidly dumped into $\chi$-particles until $q \sim 1$ and preheating is halted by the expansion of the universe. At the end of this phase $\langle \chi^2 \rangle \sim \phi^2$, and occupation numbers are enormously large $n_k \sim 10^2 h^{-2}$. Initial values $q \gtrsim 10^3$ are needed for this rapid energy transfer to occur; for smaller values preheating is halted by the expansion of the universe before the variance term comes to dominate the flaton mass [15].

- The non-zero variance $\langle \chi^2 \rangle$ can also induce an effective mass for the $\chi$-field itself if quartic couplings $\lambda \chi^4$ are present in the potential. Preheating of particles with masses $m_\chi \gg m_\phi$
is generically inefficient. This may halt preheating before the stage of rapid energy transfer, rendering preheating inefficient.

- The decay quanta scatter with the zero mode, leading to exponential amplification of flaton quanta. However, scattering only becomes important when the occupation numbers of the $\chi_k$ quanta become exponentially large, in the very last stages of efficient preheating. Scattering leads to further decay of the zero-mode.

- Flaton preheating can occur in the presence of a thermal bath. If the interaction rate for scattering of $\chi$ quanta with the particles in the thermal bath is much greater than the flaton oscillation frequency, resonance modes are depopulated rapidly on the relevant time scale. There is no Bose enhancement, and resonant production is inefficient.

### D. Flaton vs. Inflaton

In this subsection we consider the differences between resonant decay of generic flat direction fields and the usual considered case of resonant decay of the inflaton field.

- The energy density stored in the inflaton field dominates the energy density in the universe during the epoch of preheating. As a result, the expansion rate of the universe is set by the inflaton field itself, and is initially that of matter domination. The flaton field on the other hand is generically subdominant at decay. It evolves in a fixed background, which can be either radiation or matter dominated. This difference in universe evolution leads to only small changes in the various quantities, leaving the order of magnitude estimates unchanged.

- The universe is reheated by the decay of the inflaton field. Thus inflaton preheating occurs in an empty universe. This is not necessarily the case for flaton preheating. Note in this respect that even before the reheating process of inflation is completed there is a dilute plasma with temperature $T \sim (T_R^2 M_{\text{pl}}^2 H)^{1/4}$, where $T_R$ is the reheat temperature of the universe. The effects of the thermal bath should be taken into account for flaton preheating.

- The inflaton is usually chosen to be a gauge singlet of the standard model, its properties largely unconstrained by particle physics or experimental data. Moreover, preheating is mostly studied in the simplest setting: the inflaton is a real field, with only a quartic interaction term. Flat direction fields naturally appear in SUSY theories. This means that for flaton preheating we can no longer ignore the complexity of the driving field. Furthermore,
we restrict our attention to two specific examples, namely the right handed sneutrino and MSSM flat directions. For these flatons the potential is constrained by supersymmetry and particle physics, and there is no freedom in ignoring/adding unwanted/wanted terms.

III. MSSM FLAT DIRECTIONS

Directions in field space along which the potential vanishes identically are called flat directions. In supersymmetric theories there are generically many flat directions at the classical level. The classical degeneracy along flat directions is protected from perturbative quantum corrections in the supersymmetric limit by the non-renormalization theorem. The flatness is lifted by soft terms from supersymmetry breaking and non-renormalizable operators. During inflation the inflaton potential dominates the energy density in the universe. The non-zero energy density breaks supersymmetry leading to soft masses and $A$-terms for scalar fields \cite{22}. In this section we will discuss MSSM flat directions in detail, whereas the right handed sneutrino field is the subject of the next section.

The MSSM flat directions can be parametrized by gauge invariant operators, $X$, formed from the product of $m$ chiral superfields making up the flat direction. Defining $X = c\phi^m$, the effective potential for the MSSM flat direction is of the form \cite{2,11}

$$V = V_m(\Phi) + c_H H^2|\Phi|^2 + \left( \frac{Am_{3/2} + aH}{nM^{n-3}}\lambda\Phi^n + \text{h.c.} \right) + \frac{|\lambda|^2|\Phi|^{2n-2}}{M^{2n-6}}. \quad (14)$$

Here $M$ is the cutoff scale, typically the GUT or the Planck scale. In the MSSM with parity conservation, most flat direction are lifted by $n = 4, 5$ or $6$ non-renormalizable operators. The flattest one is lifted by $n = 9$. The Hubble induced terms proportional to $H$ are the soft terms from SUSY breaking by the finite energy density in the universe. $V_m$ and the term proportional to the gravitino mass $m_{3/2}$ are the MSSM soft mass and $A$-term respectively. The mass term depends on the SUSY breaking scheme. For gravity mediated SUSY breaking $V_m = m_{3/2}^2|\Phi|^2$ at tree level. In the case of gauge mediation $V_m = m_{3/2}^4\log(1 + |\Phi|^2/m_{3/2}^2)$, where $m_{3/2} \sim 1 - 100$ TeV.

Condensation along the flat direction occurs during inflation, if (1) the flaton mass is much smaller than the Hubble constant and quantum fluctuations can lead to large initial field values; or if (2) the effective flaton mass squared is negative and the field settles at a minimum away from the origin. The natural size for the soft masses during inflation is of the order of the Hubble constant \cite{2,22}. Non-minimal Kähler potentials can induce negative soft (mass)$^2$ terms, with $c_H < 0$, realizing possibility (2). Scenario (1) can be realized in the context of e.g. $D$-term inflation \cite{23} or no-scale type supergravity models \cite{24}, where symmetries forbid soft mass terms.
FIG. 1: The ellipticity $f$ as a function of $\theta_A - \theta_a$ for different Hubble induced masses $m^2 = cH^2$. Solid lines correspond to $c = -5$, dashed lines to $c = -1$, short dashed lines to $c = -0.25$ and dotted lines to $c = 10^{-2}$. All plots is for $n = 4$ in a radiation dominated background.

at tree level and $c_H, a \ll 1$ naturally.

Writing $\Phi = \frac{1}{\sqrt{2}} \phi e^{i\theta}$, the $\theta$ dependent part of the potential reads

$$V_A \sim \frac{|A|^2}{M^{n-3}} (|a| H \cos(\theta_a + \theta_\lambda + n\theta) + |A|m_{3/2} \cos(\theta_A + \theta_\lambda + n\theta)),$$

with $\theta_a, \theta_A$ and $\theta_\lambda$ the phase of $a, A$ and $\lambda$ respectively. During inflation the $\theta$-field will settle in one of the minima of the $a$-term if $a \sim \mathcal{O}(1)$. For $a \ll 1$ the radial motion is dominated by quantum fluctuations.

In the post-inflationary epoch the evolution of the flaton field is given by its equations of motion: $\ddot{\phi} + 3H \dot{\phi} + \partial V/\partial \phi = 0$. The $\phi$ field is damped as long as $H > m_{3/2}$. For negative (mass)$^2$ ($c_H < 0$) the damping is critical, and the field closely tracks its instantaneous minimum

$$\phi_{\text{min}}(H) \sim \left( \frac{\beta H M^{n-3}}{|\lambda|} \right)^{1/n-2},$$

with $\beta$ some numerical constant depending on $a, c_H$ and $n$. Quantum fluctuations of fields with a positive (mass)$^2$ ($c_H \geq 0$) saturate at $V(\phi) \sim H^4 \square[25]$, and initial amplitudes $\phi_0 \sim H_I^2/m$ are expected, with $H_I$ the Hubble constant during inflation. The field is over damped at the end of inflation, and remains essentially fixed until $\phi_0 \sim \phi_{\text{min}}$, and it starts slow rolling in the non-perturbative potential.

Eventually the Hubble induced mass becomes equal to the soft mass $H \sim \sqrt{m_{3/2}}$, and the flaton starts to oscillate in the potential well. This is the moment ellipticity is created. In gravity mediated SUSY breaking this phase starts when $H \sim m_{3/2}$. At this time the Hubble induced
and MSSM $A$-terms are of comparable magnitude and there is a torque in the angular direction if $\theta_A \neq \theta_a$. We expect ellipticities of order unity. In the gauge mediated case $m_{3/2} < \text{GeV}$ and the MSSM $A$-term is small compared to the Hubble induced one at the onset of oscillations. Therefore smaller ellipticities are expected. The ellipticity asymptotes to a constant when $H \sim 0.1m_\phi$.

This is confirmed by numerical calculations [26]. In gravity mediated SUSY breaking the ellipticity is $f \lesssim 0.5 - 0.2$ for $n = 4 - 6$. In more than half of the parameter space $(\theta_a - \theta_A) \in [0, 2\pi]$ the ellipticity is $f < 0.1$. The parameters chosen in these simulations are $M = M_{\text{pl}}$ and $|A| = |a| = -|c_H| = 1$. The results are independent of the gravitino mass and the cutoff scale. The ellipticities obtained in the gauge mediated case are smaller $f \lesssim 0.1 - 0.01$ for $n = 4 - 6$. These results are for $m_{3/2} = 10^{-5}, 10^{-9}m_\phi$; for much larger values the behavior as in the gravity mediated case.

We have extended the numerical calculations of [26] to study the elliptical trajectory for more general parameters, focusing on a mass term of the form $V_m = m_\phi^2|\Phi|^2$. We found that the ellipticity has the same order of magnitude independent of the sign of the Hubble induced masses, i.e., it is independent of $c_H$, as shown in Fig. 1. Further, the same results are obtained in both $F$-term and $D$-term inflation, see Fig. 2. In the latter no Hubble induced terms are generated, and $a = 0$.

Ellipticities do depend sensitively on the ratio of the gravitino to flaton mass, and also on the initial amplitude of the flaton field. The ellipticity is suppressed by a factor $m_{3/2}/m_\phi$. This is as expected, as in the limit $m_{3/2}/m_\phi \to 0$ no potential in the angular direction is generated. This is shown in Fig. 3. For negative (mass)$^2$ the field is trapped in the minimum of the potential,
FIG. 3: Ellipticity for different ratios of the gravitino to flaton mass. Solid lines correspond to $m_{3/2}/m = 3$, dashed lines to $m_{3/2}/m = 1$, and dotted lines to $m_{3/2}/m = 0.1$. For all plots $c = 0$.

given by Eq. (16). But for positive (mass)$^2$ it is possible to have an initial amplitude differing from $\phi_{\text{min}}$. Ellipticities decrease rapidly for $\phi_0 < \phi_{\text{min}}$, as shown in Fig. 4. For the parameter values $m_{3/2} \sim \text{TeV}$ and $M = M_{\text{pl}}$, $\phi_{\text{min}} = 10^{11} - 10^{14} \text{GeV}$ for $n = 4 - 6$ at the onset of oscillations. Quantum fluctuations saturate at a field value lower than $\phi_{\text{min}}$ for Hubble constants during inflation $H_I \lesssim 10^7 - 10^9 \text{GeV}$ for $n = 4 - 6$.

The flaton has $D$-term couplings to the fields making up the polynomial $X$, which are orthogonal to the direction which acquires the VEV. In addition, the flaton has $F$-term couplings to fields not present in $X$. The interaction term is of the form $h^2 \phi^2 \chi^2$, with $h$ either a gauge or yukawa coupling. The $q$-parameter is $q(f) = (1 - f^2)h^2 \phi^2/(4m_\chi^2)$. The gauge and Yukawa couplings in the MSSM range between $h \sim 1 - 10^{-6}$, $m_\chi = m_{3/2} \sim 1 \text{TeV}$, and the field amplitude at $H \sim m_\phi$ is given by Eq. (16). Typically $q(0) \gg 1$. We can then distinguish the following possibilities.

1. The ellipticity is negligible small. Preheating starts when $H \sim m_\phi$ and can be effective, depending on the back reaction effects. Such small ellipticities are obtained for fine-tuned parameters $\theta_A \approx \theta_a$, for $\phi_0 \ll \phi_{\text{min}}$ as is possible for positive (mass)$^2$ and low scale inflation, or in the absence of $A$-terms. Generically, supergravity corrections will induce non-zero $A$-terms.

2. The ellipticity is small $f \lesssim 0.1$. This is the case in gauge mediated SUSY breaking, and also in most of the parameter space for gravity mediation. Moreover, ellipticities are suppressed if $m_{3/2}/m_\phi < 1$ and/or $\phi_0/\phi_{\text{min}} < 1$. Preheating starts off in the broad-band regime. However, its onset is delayed until $q(0) \lesssim q_c \approx 1/(16f^4)$, and therefore might not occur if
the condensate decays perturbatively, via thermal scattering or through fragmentation into Q-balls before this time. Note that the decay quanta that will be first exited are the fields corresponding to the smallest $q$-parameter, i.e., the fields with the smallest coupling to the flaton.

Denote the decay width of the condensate by $\Gamma_\phi \sim \beta m_\phi$. For perturbative decay $\beta = h^2$ with $h$ a Yukawa or gauge coupling. For temperatures higher than the effective mass of the particles coupling to the flaton, i.e., $T \gtrsim (m_\chi)_{\text{eff}} \sim h \phi$, there is a thermal bath of $\chi$-particles. Thermal scattering can lead to decay of the condensate; in this case $\beta \sim h^2 \alpha (T/m_\phi)$ with $\alpha = g^2/4\pi$. Note that in the presence of a thermal bath the flaton mass should be corrected by the thermal contribution in all formulas $\delta m_\phi^2 \sim h^2 T^2$. If it dominates, the thermal mass induces early oscillations. Now the field amplitude, and therefore $q$, not only decreases because of the red shift, but also because the effective mass decreases. This will speed up the onset of preheating, since $q(0) < q_c$ earlier. Thermal effects are particularly important for $n = 4$ directions with small Yukawa couplings $h \lesssim 10^{-3}$ \cite{27}. Since the thermal history is rather model dependent, we will not pursue this issue further. Finally, if the flaton potential grows less than $\phi^2$ the condensate is unstable against fragmentation into Q-balls. Numerical simulations indicate that Q-ball formation takes place at $H \sim 10^{-4} - 10^{-6} m_\phi$ in gravity and gauge mediated scenarios respectively \cite{5}.

If we denote the Hubble constant at the moment $q(0) = q_c$ with $H_c$, then preheating can play a rôle if $H_c \gtrsim \Gamma_\phi$. In a matter dominated universe this translates into $\alpha^{3/4} f^2 h \lesssim m_\phi/\phi_{\text{min}}$, with $\phi_{\text{min}}$ evaluated at $H \sim m_\phi$. The universe is matter (radiation) dominated before (after) inflaton decay. For example, for $m_\phi \sim \text{TeV}$ and $M \sim M_{\text{pl}}$ preheating occurs before perturbative decay if $h^{5/2} f^2 < 10^{-8}, (10^{-12})$ for $n = 4, (6)$. Preheating precedes Q-ball formation if $f^2 h \lesssim 10^{-6} - 10^{-9}$ for $n = 4 - 6$ and gravity mediated SUSY breaking, and $f^2 h \lesssim 10^{-4} - 10^{-8}$ for $n = 4 - 6$ and gauge mediated SUSY breaking.

3. The ellipticity is appreciable $0.2 < f < 1$. Preheating is delayed. If it takes place at all, it is shut off by the expansion of the universe before the back reaction becomes important. Large, order one, ellipticities are possible in gravity mediated SUSY breaking with $n = 4$ non-renormalizable operators.
Quartic self-couplings of the decay mode $V \ni h^2 \chi^4$ will induce an effective mass term $m_\chi \sim h^2 \langle \chi^2 \rangle$ as the variance grows. All MSSM bosons (except for the right handed sneutrino) are charged under the standard model gauge group, and have a quartic interaction with $h$ the gauge coupling. In addition, the superpotential term $W \ni h \phi \chi^2$ leads to a quartic interaction in the $F$-term potential. It has been found that for a potential of the form $V(\phi) + m_\chi^2 \chi^2 + h^2 \phi^2 \chi^2$ resonant production is only effective for couplings $h^2 \gtrsim 10^{-7} (m_\chi/m_\phi)^4$ \cite{19}. Therefore, bosonic preheating shuts off when $(m_\chi)_{\text{eff}} \sim m_\phi$. From $n_\chi = g \phi_0 \langle \chi^2 \rangle$ \cite{15}, it follows $\rho_\chi \sim m_\phi^3 \phi_0/h \ll \rho_\phi \sim m_\phi^2 \phi_0^2$. Thus at the of preheating, only a small fraction of the energy stored in the flaton zero mode will be transferred to the decay products.

No effective mass term for the fermion superpartners of $\chi$ are generated. However, the Pauli exclusion principle forbids occupation numbers exceeding unity. The typical energy transferred to the fermions is $k_F^4 \sim h^2 m_\phi^2 \phi_0^2$, which is only significant for large couplings.

The typical interaction rate for MSSM particles in a thermal bath is of the order $\sigma \sim \alpha^2 T$. The relevant time scale in preheating is $m_\phi^{-1}$. It follows that thermal scattering is important for $\alpha^2 \gtrsim m_\phi/\sqrt{H M_{\text{pl}}}$ for $T < T_R$, and $\alpha^2 \gtrsim m_\phi/(T_R^2 H M_{\text{pl}})^{1/4}$ for $T > T_R$, with $T_R$ the reheat temperature of the universe. Effective scattering kills Bose-Enhancement, but can increase the efficiency of fermionic preheating, since in between each moment of particle production the decay products are scattered out of the resonance bands.
IV. RIGHT HANDED SNEUTRINO FIELD

The right-handed neutrino and its scalar partner appear in grand unified theories (GUT) based upon $SO(10)$. At first sight it appears the sneutrino direction is lifted by a $D$-term potential due to the $SO(10)$ gauge interactions, preventing a large VEV for the sneutrino field $\tilde{N}$. The crucial observation made in \cite{3} however is that the $D$-term decouples from the potential of $\tilde{N}$ as long as the value of $\tilde{N}$ is smaller than the $SO(10)$ breaking scale. The sneutrino field condenses during inflation if $m_N^2 \ll H^2$, with a maximum amplitude set by the GUT scale.

The superpotential containing the three right-handed sneutrino fields (after $B-L$ breaking) is

$$W = \frac{3}{2} \sum_{i=1}^{3} (m_N)_i N_i N_i + \sum_{i,\alpha=1}^{3} h_{i\alpha} N_i L\alpha H,$$

(17)

here $N_i$ are the three sneutrino superfields, $L\alpha$ the three left-handed lepton doublets, $H$ is shorthand for $H_u$ the down Higgs superfield, and $h_{i\alpha}$ are the Yukawa couplings. We start our discussion of preheating by considering a single generation. Writing the scalar potential in terms of the component fields $L^T = (L^0 L^-)$ and $H^T = (H^+ H^0)$

$$V_F = m_N^2 N^2 + |h|^2 |N|^2 (|L|^2 + |H|^2) + h^* m_N N (L^0 H^0 - L^- H^+) + c.c.$$  

(18)

$$V_D = \frac{g^2}{8} (|L^0|^2 - |H^0|^2)^2 + \frac{g_W^2}{2} (L^0 L^- + H^+ H^0)^2 + c.c.$$  

(19)

$$V_S = m_L |L|^2 + m_H |H|^2 + (B m_{3/2}^2 + b H^2) N^2 + \frac{A m_{3/2} + a H}{n M^{n-3}} N^n + c.c.$$  

(20)

with $|L|^2 \equiv |L^0|^2 + |L^-|^2$ and $|H|^2 \equiv |H^+|^2 + |H^0|^2$. $V_F$ and $V_D$ are the $F$ and $D$-term potential respectively, and $V_S$ is the soft SUSY breaking potential. The coefficients $B, b, A, a \sim O(1)$, $g_W = e/\sin \theta_W$ and $g_Z = e/\sin \theta_W \cos \theta_W$. The $a, b$-terms are from SUSY breaking by the finite energy density in the universe, whereas the $A, B$-terms arise from low energy SUSY breaking. $\Lambda$ is the ultraviolet cutoff scale, typically $\Lambda \sim M_{GUT}$ or $\Lambda \sim M_{pl}$. If R-parity is conserved the lowest order $A$-term for the sneutrino has $n = 4$. Furthermore, we have assumed that non-renormalizable operators are sub-dominant. An inclusion of these operators will not affect the results in any essential way.

Since the $B$-terms do not contain a coupling between the real and imaginary parts of $N$, it will not induce an ellipticity. The sole source of ellipticity are the $A$-terms. During inflation the Hubble induced $A$-term is dominant and the the $\theta$ field quickly settles in one of the minima of $\cos(\theta_A + \theta_H + n \theta)$. At the onset of oscillations, when $H \sim m_N$, the low energy $A$-term is negligible small for gravitino masses $m_{3/2} \ll m_N$, and the field remains essentially fixed in its $\theta$-minimum. The field amplitude red shift with time and all $A$-terms quickly become sub-dominant; hereafter
no further ellipticity is generated. We expect therefore that for sneutrino masses $m_N \gg m^3/2$ the ellipticity is negligible small, and we can use the theory of preheating for a real driving field. This is confirmed by our numerical calculations which give $f \sim 0.1(m^3/2m_N)$, see Fig. 4. Therefore we take $N$ real. For now we also take the yukawa coupling $h$ real, deferring a discussion of a complex coupling to section IV B.

We decompose the slepton and Higgs fields in its real and imaginary component. Diagonalizing the mass matrix gives the following mass eigenvalues and eigenstates:

$$M_{R \pm}^2 = hN(hN \pm m_N), \quad \text{for } X_{R \pm} = L^0_R \pm H^0_R,$$
$$M_{I \pm}^2 = hN(hN \pm m_N), \quad \text{for } X_{I \pm} = L^0_I \mp H^0_I,$$
$$\tilde{M}_{R \pm}^2 = hN(hN \pm m_N), \quad \text{for } \tilde{X}_{R \pm} = L^{-}_R \mp H^+_R,$$
$$\tilde{M}_{I \pm}^2 = hN(hN \pm m_N), \quad \text{for } \tilde{X}_{I \pm} = L^{-}_I \pm H^+_I,$$

where the unbarred (barred) quantities correspond to the neutral (charged) fields.

Due to both the linear and quadratic term in $N$ we cannot map the mode equation for the decay products in terms of a Mathieu equation. However, adiabaticity violation and thus particle production occurs when $N$ goes through zero, and

$$|\dot{\omega}_k| \gtrsim \omega_k^2.$$  \hspace{1cm} (25)

During most of the resonance time the quadratic term dominates over the linear term. Hence, the physics is well captured by this term alone, and the problem can be rewritten in terms of a Mathieu equation with $q = h^2N_0^2/(4m^2_N)$. Particle production occurs in a time interval $\delta t_{\text{non-ad}} \sim (hm_N N_0)^{-1/2}$.

The (mass)$^2$ eigenvalues are not positive definite. During part of the $N$-oscillation the masses $M^2$ can go negative, and quantum fluctuations of the $X_-$ fields grow exponentially due to the tachyonic instability. For $hN_0 \lesssim m_n$ or equivalently $q \lesssim 1$, the various mass terms become tachyonic during half of the sneutrino oscillation, and tachyonic preheating is very efficient. On the other hand, for $q \gtrsim 1$ the mass terms only become tachyonic during the small time interval $\delta t_{\text{tach}} \sim 1/(hN_0)$. Since $(\delta t)_{\text{non-ad}} \gg (\delta t)_{\text{tach}}$, it is expected that in this regime tachyonic particle production is sub-dominant. Particles with masses up to $m^2_N/4$ can be produced during tachyonic preheating.

The $q$-parameter for sneutrino decay is $q^2 = h^2N_0^2/4m^2_N$. In the seesaw mechanism the neutrino masses are related to the Yukawa couplings and sneutrino masses through $m_\nu = \frac{h^2\langle H_u \rangle^2}{m_N}$, where
⟨H_u⟩ = 174 GeV × sin β and tan β = ⟨H_u⟩/⟨H_d⟩. The solar and atmospheric neutrinos have masses in the range 10^{-1} – 10^{-3} eV. Typical values of the q-parameter are

\[ q \sim 10^5 \left( \frac{m_\nu}{10^{-2} \text{eV}} \right) \left( \frac{10^{12} \text{GeV}}{m_\nu} \right) \left( \frac{N_0}{10^{16} \text{GeV}} \right)^2. \] (26)

Typically \( q \gg 1 \), and we expect non-adiabatic preheating to happen generically. Note however that there is no lower bound on the lightest neutrino mass, and therefore some of the Yukawa couplings might be arbitrarily small. Moreover it might be that \( N \) gives a negligible contribution to the neutrino mass, which are dominated by their couplings to the the sneutrinos in the other two families. In those cases the q-parameter can be lowered to arbitrary low values.

### A. Back reaction effects

The quartic interaction terms can be written in terms of the mass eigenstates:

\[ V_D = \frac{g_2^2}{8} (X_{I+}X_{I-} + X_{R+}X_{R-})^2 - \frac{g_2^2}{4} (X_{R+}\tilde{X}_{I-} + X_{R-}\tilde{X}_{I+} - X_{I+}\tilde{X}_{R-} - X_{I-}\tilde{X}_{R+})^2 \]

\[ + \frac{g_2^2}{4} (X_{I+}\tilde{X}_{I-} + X_{I-}\tilde{X}_{I+} + X_{R+}\tilde{X}_{R-} + X_{R-}\tilde{X}_{R+})^2 \] (27)

\[ V_F \ni \frac{h^2}{16} ((X_{2-} - X_{2+})^2 + (X_{1+} - X_{1-})^2) \left( (X_{2-} + X_{2+})^2 + (X_{1+} + X_{1-})^2 \right) \] (28)

Consider first the case \( q \gg 1 \). Then tachyonic instabilities are ineffective, and the eigenstates \( X_i \) all grow with the same rate. The D-term potential generates effective mass terms for all modes \((m_X)_{\text{eff}} \sim g^2\langle X^2 \rangle \). Just as in the MSSM case, this effective mass will halt preheating before a significant amount of energy is transferred from the sneutrino zero mode to the sleptons and Higgses. This conclusion can be avoided if one of the \( X \)-fields has a large VEV before the start of preheating. For example, if \( \langle X_{I+} \rangle \) is initially large, it gives a large mass to the \( X_{I-} \) and \( \tilde{X}_{I-} \) fields. As a result, these eigenstates are not exited during preheating, and no effective mass for the \( X_{I+} \) field is generated.

The case \( q \lesssim 1 \) is quite different. Tachyonic preheating is effective, and only the tachyonic eigenstates \( X_- \) are produced. The D-term will not generate an effective mass state for these fields. The F-term potential does lead to an effective mass term \( m_{X_-} \sim h^2\langle X_-^2 \rangle \). Preheating halts when the effective mass term becomes of the order of the right handed sneutrino mass. Before this time a significant amount of energy can be transferred from the sneutrino zero mode to the tachyonic fields.
The last back reaction effect to be considered is thermal scattering. The typical interaction rate for Higgs, selectron and left handed sneutrino particles in a thermal bath is of the order $\Gamma \sim T$, $\Gamma \sim \alpha^2 T$ and $\Gamma \sim G_F^2 T^5$ respectively, with $G_F$ the fermi constant. The relevant time scale in preheating is $m_{\phi}^{-1}$. It follows that scattering is important, both for the Higgs and slepton fields, even before inflaton decay when there is a dilute plasma with temperature $T \sim (T_R^2 H M_{pl})^{1/4}$. Decay quanta are scattered rapidly out of the resonance bands. It requires further studies to determine how effective thermal scattering is in killing the resononance. The reason is that tachyonic preheating can be very rapidly itself: In the absence of a thermal bath the zero mode typically decays within one or two oscillations [28].

B. CP violation

To study CP violation during preheating we allow the coupling constant $h$ to be complex. Particle production due to violation of non-adiabaticity is dominated by the quartic CP conserving term $h^2 N^2 \chi^2$. Hence we expect CP violation to be small.

To study CP violation for the tachyonic decay mode, we decompose $h = h_R + i h_I$ and diagonalize the mass matrix. The mass eigenvalues and eigenstates are:

$$M_{R\pm}^2 = |h| N (|h| N \pm M) \quad \text{for } X_{R\pm} = L_R^0 \pm \frac{h_R}{|h|} H_R^0 \mp \frac{h_I}{|h|} H_I^0 \quad (29)$$
$$M_{I\pm}^2 = |h| N (|h| N \pm M) \quad \text{for } X_{I\pm} = L_I^0 \mp \frac{h_R}{|h|} H_R^0 \pm \frac{h_I}{|h|} H_I^0 \quad (30)$$

Similar expression holds for the charged lepton and charged higgs fields. It follows that tachyonic preheating will not generate a lepton asymmetry.

Finally we consider the fermionic decay modes. The mass matrix for the neutrino and higgsino, in the basis $(\nu_L \tilde{H}^0)$, is of the form:

$$\begin{pmatrix} 0 & hN \\ h^* N & 0 \end{pmatrix} \quad (32)$$

Although the fermion mass matrix is hermitian, CP is broken since for a complex coupling since $M \neq M^*$. A lepton asymmetry is expected in fermionic preheating, as discussed in reference [29].
V. DISCUSSION

We studied non-perturbative, resonant decay of flat direction fields, concentrating on MSSM flat directions and the right handed sneutrino. The difference between inflaton preheating and flaton preheating, is that the potential is more constraint in the latter case. The effects of a complex driving field, and of quartic couplings in the potential are important and cannot be neglected. Moreover, preheating occurs in the presence of a thermal bath. Effective scattering can kill Bose-enhancement effects.

Ellipticities of MSSM flat directions condensate are generically of order one, thereby delaying the onset of preheating. Preheating may be preceded by perturbative or thermal decay, or by the formation of $Q$-balls. If preheating does occur it is generically ineffective due to the quartic self-couplings of the decay modes, and due to the presence of the thermal bath.

The ellipticity for the right handed sneutrino field is negligible small. Particle production due to the violation of adiabaticity is generically expected to occur. However, just as in the MSSM case, it is expected to be inefficient due to back reaction effects. The new feature in the sneutrino potential is that for small values $q = h^2N_0^2/(4m_N) \lesssim 1$, there are tachyonic instabilities. The $D$-term quartic couplings do not generate an effective mass for the tachyonic modes, making it an efficient decay channel. It is unclear how thermal scattering affects the resonance.

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