Discovering Non-Abelian Weak Couplings and an Anomalous Magnetic Dipole Moment of the $W^\pm$ at LEP 2

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Abstract

In view of the forthcoming results on $W^\pm$ pair production at LEP 2, we emphasize that direct empirical evidence on non-trivial properties of the weak vector bosons can be obtained with relatively limited integrated luminosity. An integrated luminosity of $10 \text{ pb}^{-1}$ at $\sqrt{s} = 175 \text{ GeV}$ will be sufficient to provide direct experimental evidence for non-vanishing self-couplings of non-Abelian type among the weak vector bosons. An integrated luminosity of $100 \text{ pb}^{-1}$ at $\sqrt{s} = 175 \text{ GeV}$ will provide direct evidence for the existence of an anomalous magnetic dipole moment of the charged vector bosons $W^\pm$.

With respect to properties of the electroweak vector bosons, there are two salient predictions of the standard electroweak theory which lack direct experimental confirmation so far: the non-Abelian coupling between the members of the $SU(2)$ triplet of vector bosons, and, closely connected to it, the anomalous magnetic dipole moment coupling of the charged vector bosons, $W^\pm$. From the detailed analysis of the LEP 1 precision data there is strong indirect evidence that indeed the standard couplings are realized in nature. The production of $W^+W^-$ at LEP 2 will nevertheless open up a new domain of investigations by providing direct tests of the non-Abelian structure and the anomalous magnetic dipole moment prediction.

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Limited luminosity will restrict the possibility of precision measurements of the trilinear $\gamma W^+ W^-$ and the $Z^0 W^+ W^-$ couplings at LEP 2. It is the purpose of the present note to point out that a rather small integrated luminosity at LEP 2, however, will nevertheless be sufficient to provide empirical evidence for the existence of a non-Abelian trilinear coupling among the massive vector bosons and for the existence of an anomalous magnetic dipole moment of the $W^\pm$.

We start from the phenomenological Lagrangian for trilinear vector boson couplings widely used in the simulation of future data \cite{3}.

\[
\mathcal{L}_{\text{int}} = -ie[A_\mu(W^{-\mu\nu}W^\nu - W^{+\mu\nu}W^-_\nu) + F_{\mu\nu}W^{+\mu\nu}W^-] \\
-ix_\gamma F_{\mu\nu}W^{+\mu\nu}W^- \\
-ix(c^W + \delta_Z)[Z_\mu(W^{-\mu\nu}W^\nu - W^{+\mu\nu}W^-) + Z_{\mu\nu}W^{+\mu\nu}W^-] \\
-ix_Z Z_{\mu\nu}W^{+\mu\nu}W^-.
\]

(1)

It contains the three arbitrary parameters $x_\gamma$, $\delta_Z$ and $x_Z$ and reduces to the standard form for $x_\gamma = x_Z = \delta_Z = 0$. The parameter $x_\gamma$ is directly related to the anomalous magnetic dipole moment $\kappa_\gamma$ of the $W^\pm$ via

\[
x_\gamma \equiv \kappa_\gamma - 1,
\]

where $\kappa_\gamma = 1$ is the value in the standard model. This value follows from the linear realization of the $SU(2) \times U(1)$ symmetry in conjunction with the restriction to dimension-4 terms as embodied in the standard electroweak theory \cite{1}. Realizing $SU(2) \times U(1)$ symmetry non-linearly, or else removing the restriction to dimension-4 terms in the linearized form, removes the restriction $\kappa_\gamma = 1$, e.g. \cite{4}. We also note that $\kappa_\gamma = 1$ corresponds to a gyromagnetic ratio, $g$, of the $W^\pm$ of magnitude $g = 2$, in units of the particle’s Bohr-magneton $e/(2M_W)$, while $\kappa_\gamma = 0$ corresponds to $g = 1$ as obtained for a classical rotating charge distribution. In general $g = 1 + \kappa_\gamma$. The value of $g = 2$ puts the electromagnetic properties of the $W^\pm$ in close analogy to the $g = 2$ value of the spin-1/2 Dirac theory.

In order to relate $\delta_Z$ and $x_Z$ to the parameters in the Lagrangian before diagonalization of the neutral sector, it is advantageous to rewrite (1) in terms of unmixed neutral vector boson fields. For the present purpose it is useful to describe mixing in terms of current-mixing \cite{5}, i.e a mixing term between the third component of the weak isotriplet, $\tilde{W}_3^{\mu\nu}$, and the photon field tensor, which is denoted by $\tilde{F}^{\mu\nu} \equiv \partial^\mu \tilde{A}^\nu - \partial^\nu \tilde{A}^\mu$ in order to discriminate $\tilde{F}^{\mu\nu}$ from the physical photon field tensor $F^{\mu\nu}$ emerging upon diagonalization. For details we refer to \cite{6}. Carrying out the transformation to the unmixed fields,

\[
\begin{pmatrix}
  A^\mu \\
  Z^\mu
\end{pmatrix} = \begin{pmatrix}
  1 & s_W \\
  0 & c_W
\end{pmatrix} \begin{pmatrix}
  \tilde{A}_3^\mu \\
  \tilde{W}_3^\mu
\end{pmatrix},
\]

(3)

the Lagrangian \cite{1} becomes

\footnote{We restrict ourselves to dimension-four, P- and C-conserving interactions.}
\[ \mathcal{L}_{\text{int}} = \frac{\hat{g}}{2} \tilde{W}_{\mu \nu} \cdot (\tilde{W}^\mu \times \tilde{W}^\nu) \\
- i \hat{f} \tilde{W}^3_{\mu \nu} W^+ \nu W^- \nu \\
- i e \left[ \tilde{A}_\mu (W^{- \mu \nu} W^+ \nu - W^{+ \mu \nu} W^- \nu) + \kappa_\gamma \tilde{F}_{\mu \nu} W^{+ \mu} W^{- \nu} \right], \tag{4} \]

where the third component of \( \tilde{W} \) is understood to be \( \tilde{W}_3 \). The relations between \( \hat{g}, \hat{f}, \kappa_\gamma \) and \( x_\gamma, x_Z, \delta_Z \) are given by \( \text{(2)} \) and

\[
\delta_Z = \frac{\hat{g}}{e c_W} - \frac{1}{s_W c_W}, \\
x_Z = \frac{\hat{f}}{e c_W} - (\kappa_\gamma - 1) \frac{s_W}{c_W}. \tag{5} \]

The standard model thus corresponds to \( \hat{g} = \frac{e}{s_W}, \hat{f} = 0 \) and \( \kappa_\gamma = 1 \).

Let us consider the Lagrangian \( \text{(4)} \) in some detail. It contains [6]:

- An \( SU(2) \) symmetric interaction term of non-Abelian form with arbitrary strength \( \hat{g} \) which coincides with the standard term for the special choice \( \hat{g} = \frac{e}{s_W} \). Note that this term is contained in the kinetic term for the \( \tilde{W} \) fields,

\[
- \frac{1}{4} \tilde{W}^{\mu \nu} \tilde{W}_{\mu \nu} = - \frac{1}{2} W^{- \mu \nu} W^+ \nu - \frac{1}{4} \tilde{W}^{\mu \nu} \tilde{W}_{\mu \nu}, \tag{6} \]

provided we use the non-Abelian field tensor,

\[ \tilde{W}^{\mu \nu} = \partial^\mu \tilde{W}^\nu - \partial^\nu \tilde{W}^\mu - \hat{g} \tilde{W}^\mu \times \tilde{W}^\nu. \tag{7} \]

For \( \hat{g} = 0 \), the kinetic term reduces to a triplet of Abelian vector boson fields\[6\]. Accordingly, the case of \( \hat{g} = 0 \) may be referred to as the Abelian triplet model.

- A term of strength \( \hat{f} \) which violates \( SU(2) \) symmetry independently of the presence of the photon field. Imposing the constraint of no intrinsic \( SU(2) \) violation\[4\], i.e. \( SU(2) \) symmetry, when electromagnetism in \( \text{(4)} \) is absent, we have to require \( \hat{f} = 0 \), i.e.

\[ x_Z = - \frac{s_W}{c_W} x_\gamma. \tag{8} \]

This requirement is abstracted from its empirical validity in the vector boson mass term and is sometimes called custodial \( SU(2)_c \) symmetry.

- The two terms describing the electromagnetic interactions. The term containing \( \tilde{A}_\mu \) in \( \text{(4)} \) is simply obtained by applying the minimal substitution principle, \( \partial^\mu \rightarrow \partial^\mu + ie \tilde{A}_\mu Q, Q \) being the charge operator, \( Q W^\pm = \pm W^\pm \) and \( Q W_3 = 0 \), to the derivatives in the kinetic term \( \text{(3)} \) of the \( \tilde{W} \) triplet. The anomalous magnetic moment term \( \kappa_\gamma \), \( \kappa_\gamma \neq 0 \) in \( \text{(4)} \), on the other hand, need not necessarily be present, even though it is not excluded by the principle of

\[ \text{The local Abelian symmetry, } W^i_\mu \rightarrow W^i_\mu + \partial_\mu \alpha^i(x), \text{ is obviously broken by the mass terms for the vector bosons.} \]
minimal substitution \[4\]. In fact, adding the \(SU(2)\)-invariant total derivative term
\[
\mathcal{L}_{tot} = -\frac{\kappa_\gamma}{2} \frac{\partial_\mu}{W^{+\nu}} \partial_\nu W^{-\mu} - W^{+\mu} \partial_\nu W^{-\nu}
\]
\[
+ W^{-\nu} \partial_\mu W^{+\mu} - W^{-\mu} \partial_\nu W^{+\nu}
\]
\[
+ W^\nu_3 \partial_\nu W^\mu_3 - \tilde{W}_3^\nu \partial_\mu \tilde{W}_3^\nu
\]
(9)
to the kinetic term (8) and carrying out the minimal substitution prescription implies the electromagnetic interaction (4) with an arbitrary value of \(\kappa_\gamma\).

We note that both the charge coupling, \(e\), as well as the magnetic moment coupling, \(\kappa_\gamma\), appearing in connection with the unmixed field \(\tilde{A}_\mu\), \(\tilde{F}_{\mu\nu}\) in (4), are identical to the couplings in the Lagrangian (1) for the physical fields, \(A_\mu\), \(F_{\mu\nu}\). This is in contrast to the case of the massive neutral vector boson, where the third component of the triplet in (4) couples with a strength \(\hat{g}\) different from the strength of the \(Z\) coupling, \(e\left(\frac{cW}{sW} + \delta Z\right)\), in (1).

We note in passing that the mentioned \(SU(2)\)-symmetry properties of the terms multiplied by \(\hat{g}\) and \(\hat{f}\) in (4) are not a unique feature of the \(\tilde{A}W_3\) basis. The physical interpretation of the parameters \(\hat{g}\) and \(\hat{f}\) is thus not only valid in this basis. Indeed, expressing (1) in terms of the more conventional \(BW_3\) basis via
\[
\begin{pmatrix} A_\mu \\ Z_\mu \end{pmatrix} = \left( \begin{array}{c} cW \\ sW \\ -sW \\ cW \end{array} \right) \begin{pmatrix} B_\mu \\ W^3_\mu \end{pmatrix},
\]
(10)
one obtains the Lagrangian in the form
\[
\mathcal{L}_{int} = \frac{\hat{g}}{2} \tilde{W}_{\mu\nu} \cdot (\tilde{W}^\mu \times \tilde{W}^\nu)
\]
\[
- i \hat{f} \tilde{W}^3_{\mu\nu} W^{+\mu} W^{-\nu}
\]
\[
- i \left[ \left( \frac{e}{cW} - \hat{g} \frac{sW}{cW} \right) B_\mu (W^{-\mu\nu} W^{+\nu} - W^{+\mu\nu} W^{-\nu})
\right.
\]
\[
+ \left( \frac{e}{cW} \kappa_\gamma - (\hat{g} + \hat{f}) \frac{sW}{cW} \right) B_{\mu\nu} W^{+\mu\nu} W^{-\nu} \right].
\]
(11)
This form of the Lagrangian coincides with (4) in the first two terms apart from \(\tilde{W}^\mu_3\) being replaced by \(W^\mu_3\). This is a consequence of the fact that the second columns of the matrices in (8) and (10) are identical. Obviously, however, the \(B\)-dependent terms in (11) differ from the \(A\) terms in (4).

In summary, it is a characteristic feature of a non-Abelian theory to contain a coupling \(\hat{g} \neq 0\). Experimental evidence for \(\hat{g} \neq 0\) thus provides evidence for a non-Abelian structure of the interactions among the members of the \(\tilde{W}\) triplet and rules out a theory based on three Abelian vector boson fields.

It is a second characteristic feature of the non-Abelian nature of the standard model interactions to contain an anomalous magnetic dipole moment term of definite strength, \(\kappa_\gamma = 1\). While a precision measurement of \(\kappa_\gamma\) will be difficult, empirical evidence for \(\kappa_\gamma > 0\) will nevertheless provide first direct experimental evidence for the existence of an anomalous magnetic dipole moment of the \(W^\pm\) vector bosons.

We turn to the experimental search for a non-Abelian coupling, \(\hat{g} \neq 0\), and the search for an anomalous magnetic dipole moment, \(\kappa_\gamma \neq 0\).
We note that the present most stringent direct bound to the $WW\gamma$-coupling $x_\gamma$ \cite{8} is $-1.6 < x_\gamma < 1.8$ at 95\% CL\footnote{These bounds depend on the assumption of a form factor in the $WW\gamma$ coupling. The bounds to the $WWZ$ couplings reported in \cite{3} are based on specific model assumptions which are inconsistent with the present work.}, corresponding to $-0.6 < \kappa_\gamma < 2.8$.

We consider a measurement of the total cross section of $e^+e^- \rightarrow W^+W^-$ at LEP 2 with a cut $|\cos \theta| < 0.98$ on the scattering angle. Formulae for the cross section in terms of the parameters $x_\gamma, x_Z$ and $\delta_Z$ have been given in \cite{3}. We assume that the decay mode
\[ e^+e^- \rightarrow W^+W^- \rightarrow l^\pm \nu_l + 2 \text{ jets}, \tag{12} \]
will be detected at LEP 2, where the lepton $l^\pm$ can be either an electron (positron) or a muon (anti-muon). The branching ratio for this decay mode is 29.6\%. We assume that future data are identical to the standard model predictions and calculate the lines of constant $\chi^2 = 4$ (86.5\% CL) and $\chi^2 = 1$ (39.4\% CL) in the $x_\gamma-\delta_Z$-plane according to\footnote{We assume that the measurement is the outcome of a random sample of a Gaussian distribution with mean value $N(x_\gamma, \delta_Z)$ such that the statistical error is given by $\sigma = \sqrt{N(x_\gamma, \delta_Z)}$. If one uses the error estimated from the experiment, $\sigma = \sqrt{N_{SM}}$, instead of the statistical error in the denominator of (13), the values of the parameter pair $(x_\gamma, \delta_Z)$ which have $\chi^2 = 4$ or $\chi^2 = 1$ change only little provided the number of measured events, $N_{SM}$, is much greater than one, $N_{SM} \gg 1$.}
\[
\chi^2 \equiv \frac{(N_{SM} - N(x_\gamma, \delta_Z))^2}{N(x_\gamma, \delta_Z)}. \tag{13}
\]
In (13), $N_{SM}$ and $N(\delta_Z, x_\gamma)$ denote the number of events in the SM and in the alternative model with the $SU(2)$ constraint \cite{3} and $\delta_Z, x_\gamma \neq 0$, respectively.

In Figure 1 we show the $1\sigma$ and $2\sigma$ contours in the $(\delta_Z, x_\gamma)$ plane for an integrated luminosity of $L = 8$ pb$^{-1}$, at $\sqrt{s} = 175$ GeV. We see that this very small value of $L$, corresponding to a few weeks of running at LEP 2, is sufficient to detect a genuine non-zero non-Abelian coupling, $\hat{g} \neq 0$, at the $2\sigma$ level. Likewise, a vanishing anomalous magnetic moment, $\kappa_\gamma = 0$, in conjunction with an Abelian $\hat{W}$ triplet, $\hat{g} = 0$, is excluded. We note that similar results can be obtained with $L = 20$ pb$^{-1}$ at $\sqrt{s} = 170$ GeV and with $L = 2.2$ pb$^{-1}$ at $\sqrt{s} = 190$ GeV.

To detect a non-vanishing magnetic dipole moment, $\kappa_\gamma \neq 0$, in the presence of a non-vanishing non-Abelian coupling, $\hat{g} \neq 0$, needs a somewhat higher integrated luminosity. The result in Figure 2 corresponds to a luminosity of $L = 100$ pb$^{-1}$ at $\sqrt{s} = 175$ GeV, thus providing direct evidence for a non-vanishing anomalous magnetic dipole moment of the $W^\pm$, $\kappa_\gamma > 0$. Since a sufficient number of standard events, $N_{SM} = 519$, is expected in this case, the events can be arranged in 6 bins equidistant over the scattering angle $\cos \theta$, leading to the result also presented in Figure 2. In the same Figure we also show the theoretical prediction corresponding to a vanishing $ZW^+W^-$ coupling, $g_{ZWW}$, corresponding to $\delta_Z = -\frac{cw}{sW}$ in (1), and $\hat{g} = esW$, which is similarly ruled out.

In conclusion, after a few weeks of running at $\sqrt{s} = 175$ GeV at LEP 2, definite direct evidence for the existence of a genuine, non-vanishing coupling among the members of the $W^\pm, W^0$ triplet, characteristic of a non-Abelian structure, can be
obtained. Likewise, after 7 months of running at LEP 2, definite evidence for a non-
vanishing anomalous magnetic dipole moment of the charged vector bosons may be
expected.

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Figure 1: Detecting a non-Abelian vector-boson coupling, $\hat{g} \neq 0$, at LEP 2.
Figure 2: Detecting a non-zero anomalous magnetic dipole moment, $\kappa_\gamma \neq 0$, of the $W^\pm$ at LEP 2.