On daisy and superdaisy resummation of
the effective potential at finite
temperature∗

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Abstract

We describe in detail, in the context of the simplest scalar $\phi^4$ theory, the prescription for resummation of daisy and superdaisy diagrams in the effective potential using the solution of the gap equations in the infrared limit. We find that the latter procedure is consistent provided we neglect logarithmic terms from the finite-temperature self energies and from the integration of overlapping momenta. This amounts to dressing only the zero-mode contribution to the finite-temperature effective potential. Improving also the non-zero modes, would require exactly solving (not in the IR limit) the gap equations. In general this can only be done in a theory where all self-energies are momentum independent (e.g. in the scalar theory at the symmetric phase $\phi = 0$). However some partial dressing procedures are still possible in general.

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It was recently realized that Sakharov’s conditions for baryogenesis could be accomplished (in particular that the rate of anomalous $B$-violating processes is unsuppressed) at high temperatures in the electroweak theory \[1\]. This fact revived the interest in the study of the electroweak phase transition, mainly because the out-of-equilibrium condition usually requires the phase transition to be strongly first-order. The theoretical foundation for a quantitative treatment of symmetry restoration and phase transitions at high temperatures is already twenty years old \[2, 3\]. Soon after it was realized \[3\] that the perturbative expansion fails at temperatures close to the critical temperature, because of infrared (IR) divergences, and it was proposed to solve the IR problem by the resummation of an infinite set of the most IR divergent diagrams: *i.e.* those belonging to the daisy and superdaisy classes. Improving the effective potential in different theories by the inclusion of daisy and superdaisy diagrams has produced a lot of activity in the field during the last years \[4]-[17]. Since there has been some controversy about the correct resummation procedure concerning the leading infrared divergent graphs \[6, 7, 10, 12, 16, 17\], I will develop in this note the formalism which was followed in \[11, 14\] as well as will compare it with other different approaches recently used by different authors.

We will consider, for simplicity, the theory of a real scalar field $\phi$, with a tree level potential,

$$V_{\text{eff}}^{(0)}(\phi) = -\frac{\mu^2}{2}\phi^2 + \frac{\lambda}{4}\phi^4$$

with positive $\lambda$ and $\mu^2$. At the tree level, the field-dependent mass of the scalar field is $m^2(\phi) = 3\lambda\phi^2 - \mu^2$, and the minimum of $V_{\text{eff}}^{(0)}$ corresponds to $\nu^2 = \mu^2/\lambda$, so that $m^2(\nu) = 2\lambda\nu^2 = 2\mu^2$.

At finite temperature, the one-loop effective potential can be written diagrammatically as $\square$

$$V_{\text{eff}}^{(1)} = \sum_{n=0}^{\infty} V_{[n]}^{(1)} = \frac{1}{2} \quad \bigcirc$$

where $n$ are the bosonic Matsubara frequencies and $V_{[n]}^{(1)}$ are the contributions to the one-loop effective potential from the different frequencies. They can be written to lowest order in $m/T$ as

$$V_{[0]}^{(1)} = \frac{1}{2} \quad \bigcirc = -\frac{1}{12\pi} T m^3$$

$$\sum_{n=1}^{\infty} V_{[n]}^{(1)} = \frac{1}{2} \quad \bigcirc = \frac{1}{24} T^2 m^2 + \cdots$$

\[1\]There is an overall negative sign in front of all diagrams contributing to the effective potential and self-energies that (for simplicity) will be dropped systematically from the figures, but will be taken into account in the calculation.
where big bubbles denote the contribution from zero modes and small bubbles the one from all non-zero modes. The contribution from all modes will be denoted by a big dotted bubble, \( i.e. \)

\[
\bullet = \bigcirc + \bigcirc
\]  

(5)

For the zero modes (\( n = 0 \)) there is a severe infrared problem in the loop expansion for values of \( \phi \) such that \( m(\phi) \ll \lambda T \) at \( \vec{p} = 0 \). At one-loop the potential (3) is non-analytic at \( m = 0 \), while the validity of the perturbative expansion breaks down at higher-loop order, which contribute powers of \( \alpha \) and \( \beta \) [3, 5]

\[
\alpha = \lambda \frac{T^2}{m^2}, \quad \beta = \lambda \frac{T}{m}
\]  

(6)

The usual way out is dressing the zero-modes with daisy and super-daisy diagrams [3]. This can be done by solving the gap equations in the IR limit (\( n = 0, \vec{p} \to 0 \)). For the theory defined by Eq.(1), and neglecting the terms represented by the ellipsis in (4), the gap equation can be diagrammatically written as,

\[
\bigcirc + \bigcirc + \bigcirc = \bigcirc + \bigcirc + \bigcirc + \bigcirc
\]  

(7)

where a double line represents a dressed zero-mode propagator. Using the approximation in Eq.(4) the self-energies can be written as

\[
\bigcirc = \frac{\lambda}{4} T^2 + \cdots
\]  

(8)

\[
\bigcirc = -\frac{3\lambda T m}{4\pi}
\]  

(9)

\[
\equiv 0 + \cdots
\]  

(10)

\[
\bigcirc = -\frac{9\lambda^2 \phi^2 T}{4\pi m}
\]  

(11)

and the gap equation (7) as

\[
M^2 = m^2 + \frac{\lambda T^2}{4} - \frac{3\lambda T M}{4\pi} - \frac{9\lambda^2 \phi^2 T}{4\pi M}
\]  

(12)

where \( M \) is the solution to (12). In the approximation of Eq.(4) the small bubbles are constant proportional to \( T^2 \) [3] or zero [10] (the ellipsis is neglected), and so they do not have to be dressed. Going beyond this approximation, also small bubbles would need to be dressed. However for bubbles of the kind [10] and [11] the IR limit would no longer be justified at all.
Now we will see how the daisy and superdaisy diagrams amount to a resummation in the loop expansion of the effective potential which can therefore be written in terms of the solution to the gap equation (12). In the order of approximation we are working only the zero modes need to be dressed, and only $V_{[0]}^{(1)}$ in Eq.(3) is improved, while $\sum_{n \neq 0} V_{[n]}^{(1)}$ in Eq.(4) does not have any IR problem and can be considered as a good estimate. We will prove the resummation to four-loop order though also functional methods [18] can be used [13].

The loop expansion of the effective potential will be written as

$$V_{\text{eff}} = \sum_{\ell=0}^{\infty} V_{\text{eff}}^{(\ell)} + \text{non-\text{(super)}daisies}$$  \hspace{1cm} (13)

where $V_{\text{eff}}^{(\ell)}$ indicates the contribution to the effective potential from $\ell$-loop daisy and superdaisy diagrams. Non-(super)daisy diagrams contribute to the effective potential to $O(\beta^2)$ [5, 11]. At least, to $O(\beta)$ it is consistent to keep only diagrams of daisy and superdaisy classes. $V_{\text{eff}}^{(1)}$ was given in Eq.(2), while $V_{\text{eff}}^{(2)}$ and $V_{\text{eff}}^{(3)}$ can be written as

$$V_{\text{eff}}^{(2)} = \frac{1}{8} \; \includegraphics[width=0.1\textwidth]{daisy2} + \frac{1}{12} \; \includegraphics[width=0.1\textwidth]{daisy3}$$  \hspace{1cm} (14)

$$V_{\text{eff}}^{(3)} = \frac{1}{16} \; \includegraphics[width=0.1\textwidth]{daisy4} + \frac{1}{8} \; \includegraphics[width=0.1\textwidth]{daisy5} + \frac{1}{16} \; \includegraphics[width=0.1\textwidth]{daisy6}$$  \hspace{1cm} (15)

where we are putting dots everywhere to remember that all modes (zero and non-zero modes) are contributing in the loop propagators, and the numerical pre-factors in front of (14) and (15) are the symmetry factors of the corresponding diagrams.

Using the approximation in (8), (10), we can rearrange the loop expansion in (2), (14) and (15) as

$$V_{\text{eff}}^{(\ell)} = V_{\text{daisy}}^{(\ell)} + V_{\text{superdaisy}}^{(\ell)} + \Delta^{(\ell)} \left[ -\frac{1}{8} \; \includegraphics[width=0.1\textwidth]{daisy7} - \frac{1}{6} \; \includegraphics[width=0.1\textwidth]{daisy8} \right]$$  \hspace{1cm} (16)

where $\Delta^{(\ell)}[\cdots]$ means the contribution to $[\cdots]$ from $\ell$-loop diagrams and the double line is given by Eq.(5). This decomposition is well defined for $\ell \geq 2$. Next we give the results for two and three-loop diagrams.
Two-loop

\[ V^{(2)}_{\text{daisy}} = \frac{1}{4} \bigcirc + \frac{1}{4} \bigcirc + \frac{1}{4} \square \]  

\[ V^{(2)}_{\text{superdaisy}} = 0 \]  

\[ \Delta^{(2)} = -\frac{1}{8} \bigcirc - \frac{1}{6} \square \]  

Three-loop

\[ V^{(3)}_{\text{daisy}} = \frac{1}{16} \bigcirc + \frac{1}{8} \bigcirc + \frac{1}{16} \bigcirc + \frac{1}{8} \bigcirc + \frac{1}{8} \bigcirc + \frac{1}{16} \square \]  

\[ V^{(3)}_{\text{superdaisy}} = \frac{1}{8} \bigcirc + \frac{1}{8} \bigcirc + \frac{1}{4} \bigcirc + \frac{3}{8} \bigcirc + \frac{1}{4} \square \]  

\[ \Delta^{(3)} = -\frac{1}{8} \bigcirc - \frac{1}{8} \bigcirc - \frac{1}{4} \bigcirc - \frac{3}{8} \bigcirc - \frac{1}{4} \square \]  

We can see from (14) and (17) that the symmetry factors for \( \ell = 2 \) do not match the combinatorics for daisy resummation. However including (19) the matching is accomplished as can be seen from the coefficients in (14) and the last two terms in (17) and (19)

\[ \frac{1}{4} - \frac{1}{8} = \frac{1}{8} \]

\[ \frac{1}{4} - \frac{1}{6} = \frac{1}{12} \]

On the other hand, we have seen from (21,22) that \( V^{(3)}_{\text{superdaisy}} + \Delta^{(3)} = 0 \). The reason being that all the diagrams in (15) could be interpreted either as daisies or as superdaisies. Therefore we have considered all of them as daisies, because their coefficients match the correct combinatorics for resummation. This can be seen by comparison with the corresponding coefficients in (21) and it is a general feature of daisy diagrams for \( \ell \geq 3 \). However, for \( \ell \geq 4 \) there are diagrams that can never
be considered as daisies. In that case the previous cancellation does not hold, but still the equation (14) is satisfied. As an example we will consider the theory at the origin (i.e. at $\phi = 0$) to four-loop order.

**Four-loop**

The contributions to (13) and (16) can be written as

$$V_{\text{eff}}^{(4)}(0) = \frac{1}{48} \bullet \bullet \bullet + \frac{1}{32} \bullet \bullet$$

(24)

$$V_{\text{daisy}}^{(4)}(0) = \frac{1}{48} \circ \circ + \frac{3}{48} \bullet \circ \circ + \frac{3}{48} \circ \bullet \circ + \frac{1}{48} \circ \circ \circ$$

(25)

$$V_{\text{superdaisy}}^{(4)}(0) = \frac{1}{16} \bullet \circ \circ + \frac{1}{8} \bullet \circ \circ + \frac{1}{16} \bullet \circ \circ + \frac{3}{16} \circ \bullet \circ + \frac{2}{16} \circ \circ \circ$$

(26)

$$\Delta^{(4)}(0) = -\frac{1}{16} \circ \circ \circ - \frac{1}{8} \bullet \circ \circ \circ - \frac{1}{16} \bullet \circ \circ \circ - \frac{1}{32} \circ \bullet \circ \circ - \frac{1}{8} \circ \bullet \circ \circ - \frac{3}{32} \circ \circ \bullet$$

(27)

The first diagram in (24) can be (and it is) considered as a daisy diagram in (25). For that reason the coefficients of the first three terms in (26) and (27) are equal and opposite in sign. The second diagram in (24) can never be considered as a daisy diagram. So it contributes to the last three terms of (26) and (27) in such a way that their coefficients match to those of the second term of (24). In particular

$$\frac{1}{16} - \frac{3}{32} = \frac{1}{32}$$

$$\frac{3}{16} - \frac{1}{8} = 2 \times \frac{1}{32}$$

$$\frac{2}{16} - \frac{3}{32} = \frac{1}{32}$$

(28)
for the last three terms, respectively.

**All-loop**

Summarizing the above results, we can write the final equation:

\[
V_{\text{eff}} = \frac{1}{2} \begin{array}{c}
\circledast \\
\circledast \\
\circledast
\end{array} - \frac{1}{8} \begin{array}{c}
\circledast \\
\circledast
\end{array} - \frac{1}{6} \begin{array}{c}
\circledast
\end{array}
\] (29)

where the double line indicates the solution of the gap equation (7) and (12), in the approximation of Eqs.(8) and (10). Using the explicit solution to (12) we can write (29) as

\[
V_{\text{eff}} = -\frac{\mu^2}{2} \phi^2 + \frac{\lambda}{4} \phi^4 + \frac{1}{24} T^2 m^2 - \frac{1}{12\pi} T M^3 + \cdots - \frac{3\lambda}{64\pi^2} T^2 M^2 - \frac{6}{32\pi^2} \lambda^2 \phi^2 T^2
\] (30)

which agrees with the result of Amelino-Camelia and Pi [13], who used functional methods [18] and computed (5) and (12) to zeroth order in \(\gamma\)

\[
\gamma \equiv \frac{\phi^2}{T^2}
\] (31)

For that reason the last term in (30) was missing from Eq.(3.25) in Ref. [13]. The others agree. However we will see that, though numerically unimportant in the scalar case because \(\gamma \ll 1\), all terms in (12) should be considered in the \(\beta\)-expansion.

In the improved theory of zero-modes defined by (12) and (30), the expansion parameters \(\alpha\) and \(\beta\) in (8) become

\[
\alpha = \lambda \frac{T^2}{M^2} \sim 1
\] (32)

which is summed to all orders, and

\[
\beta = \lambda \frac{T}{M} \sim \lambda \frac{1}{2}
\] (33)

which remains as the only expansion parameter, where \(M\) is the Debye mass

\[
M^2 = m^2(\phi) + \frac{\lambda}{4} T^2
\] (34)

By expanding the solution of Eq.(12) to different orders in \(\beta\) we can obtain the effective potential (30) to the corresponding order of approximation. To illustrate the procedure we will first obtain the solution to \(O(\beta^0)\). In that case we have

\[
M^2 = M^2
\] (35)

\footnote{We will keep for notational simplicity the same names \(\alpha\) and \(\beta\) for the expansion parameters in the improved (as in the unimproved) theory.}
and the effective potential is given by

\[ V_{\text{eff}} = -\frac{\mu^2}{2} \phi^2 + \frac{\lambda}{4} \phi^4 + \frac{1}{24} T^2 m^2 - \frac{1}{12\pi} T M^3 + \cdots \]  

(36)

This approximation has been worked out in \[3\]. The last two terms in (29) and (30) do not contribute to this order\(^3\) since they start to \(O(\beta)\). The solution to \(O(\beta)\) is equally easy to be worked out. From (12) one can write

\[ M^2 = M^2 - \frac{3\lambda}{4\pi} M T - \frac{9\lambda^2}{4\pi} \phi^2 T \]  

(37)

where the first term is the leading order result, Eq.(35), the second term is \(O(\beta)\) and the third term is \(O(\alpha\beta\gamma)\). As we said above, the last term cannot be neglected in the \(\beta\)-expansion, though it can be numerically unimportant. Replacing (37) in (30) and expanding again to \(O(\beta)\) we can obtain the corresponding approximation to the effective potential, given by

\[ V_{\text{eff}} = -\frac{\mu^2}{2} \phi^2 + \frac{\lambda}{4} \phi^4 + \frac{1}{24} T^2 m^2 - \frac{1}{12\pi} T M^3 + \cdots + \frac{(6-3)}{64\pi^2} \lambda T^2 M^2 + \frac{(9-6)}{32\pi^2} \lambda^2 \phi^2 T^2 \]  

(38)

This solution was presented in \[14\]. We can easily check that the last two terms in (38) are \(O(\beta)\) corrections to the fourth term. They come partly from the expansion (37) and partly from the last two terms in (29).

We have already compared our results with those of Ref. \[13\]. There is another, recent, proposal by P. Arnold and O. Espinosa \[16\] who have advocated a hybrid method using a partial resummation of the leading contribution of \(n \neq 0\) bubbles followed by an ordinary loop expansion \[19\]. One can define a partially dressed propagator as

\[ = \bigcirc + \bigcirc \bigcirc + \bigcirc \bigcirc + \cdots \]  

(39)

where the tiny bubble propagator is defined as,

\[ \bigcirc = \frac{\lambda}{4} T^2, \]  

(40)

and a partially resummed loop expansion as

\[ V_{\text{eff}} = V_{\text{eff}}^{(0)} + \frac{1}{2} \bigcirc + \frac{1}{8} \bigcirc \bigcirc + \frac{1}{12} \bigcirc \bigcirc + \cdots \]  

(41)

\(^3\)In the language of Ref. \[14\] there is no combinatorial term to this order.
It is easy to check that using the approximation of Eqs. (8)-(11), and ignoring overlapping momenta, one recovers, to $O(\beta)$, the same result as that of Eq. (38).

Other authors [9, 12] have proposed computing tadpoles, instead of vacuum diagrams, to exhibit some features of the improved theory, e.g. resummation properties and the absence of a linear term in $m(\phi)$ in the final effective potential. Since the tadpole is nothing else than the $\phi$-derivative of the effective potential, there can be no difference between both formalisms. In fact, by comparison between the contents of this note and those in [17] one can easily see that the resummation properties of the tadpole diagrams are inherited from the corresponding ones in vacuum diagrams. However, in our opinion, the tadpole formalism has two practical drawbacks: i) The classification of tadpole diagrams is much more involved than the classification of vacuum diagrams; ii) To obtain the effective potential the tadpole has to be integrated, which can be a non-trivial operation since it depends on the solution of the gap equation. However, at the end, both methods should yield the same result.

In our approximation of Eqs. (4), (8) and (10) we have been neglecting all logarithmic terms, which amounted to not dressing non-zero modes. Including them would amount to write the previous equations as

$$\frac{1}{2} \bigcirc = \frac{1}{24} T^2 m^2 - \frac{m^4}{64\pi^2} \log \frac{Q^2}{c_B T^2} + \cdots$$

where $Q$ is the renormalization scale in the $\overline{\text{DR}}$ scheme and $\log(c_B) = 3.9076$,

$$\bigcirc = \frac{\lambda}{4} T^2 - \frac{3\lambda m^2}{16\pi^2} \log \frac{Q^2}{c_B T^2} + \cdots$$

$$\overline{\bigcirc} = -\frac{9\lambda^2 \phi^2}{8\pi^2} \log \frac{Q^2}{c_B T^2} + \cdots$$

In that case the gap equation should be written as

$$= + \overline{\bigcirc} + \overline{\bigcirc}$$

and other diagrams should be added to those in (17)-(27). In particular small bubbles dressed by insertions of self-energies of the kind (11). But the latter are computed in the IR limit, which is not justified at all if the external mode has $n \neq 0$. If we insist in keeping the logarithmic terms (which in principle are expected to constitute small corrections to the leading contribution) we should give up resummation in the sense of the gap equation (45). A possibility is making a $\gamma$ expansion for the gap equation, and defining an $O(\gamma^0)$ gap equation as

$$= + \overline{\bigcirc}$$

(46)
where the dotted double line means the solution of the truncated gap equation\(^4\), i.e.

\[
M^2 = m^2 + \frac{\lambda T^2}{4} - \frac{3\lambda T M}{4\pi} - \frac{3\lambda M^2}{16\pi^2} \log \frac{Q^2}{c_B T^2} + \cdots 
\]

(47)

However, we should pay attention to the fact that the last term in the full gap equation (12) is \(\mathcal{O}(\alpha \beta \gamma)\) and solving it, and giving the improved effective potential to some order in \(\beta\) implies that we should consider the same order in \(\gamma\) by adding loop diagrams. In this way repeating the whole above procedure we would find that to \(\mathcal{O}(\beta)\) one can write the effective potential as

\[
V_{\text{eff}} = \frac{1}{2} \begin{array}{c}
\circ \circ \\
\circ 
\end{array} - \frac{1}{8} \begin{array}{c}
\circ \circ \\
\circ 
\end{array} + \frac{1}{12} \begin{array}{c}
\circ \circ \circ \\
\circ 
\end{array} + \mathcal{O}(\beta^2) 
\]

(48)

where the equation (47) is solved to \(\mathcal{O}(\beta)\). This solution coincides at \(\phi = 0\) with that in Ref. [13] and for all values of \(\phi\) with that in Ref. [14]. One can check that the logarithmic corrections which appear are both from the logarithms in (42), (43), (44), and from the overlapping momenta whose integral is explicitly considered. To higher order in \(\beta\) more terms should be added to (48) but care should be taken not to commit overcounting and non-(super)daisy diagrams should be considered.

In conclusion we have reviewed the prescription for resummation of daisy and superdaisy diagrams in the effective potential using the solution of the gap equations in the IR limit. We have found that the latter procedure is consistent provided we neglect logarithmic terms in the finite-temperature self-energies. This amounts to improving only the zero-mode finite temperature contribution to the potential. Trying to go beyond this approximation, and improving also the non-zero modes, would be inconsistent with the usual daisy and superdaisy resummation, based on the solution of the gap equations in the IR limit. It would require exact (not in the IR limit) calculation of self-energy insertions and exact solutions of the gap equations. This can only be done in a theory where all self-energies are momentum independent (e.g. in a scalar theory at the origin \(\phi = 0\)). The corrections due to the dressing of non-zero modes are proportional to logarithmic terms arising from the self-energies and from integration of overlapping momenta in calculation of diagrams. In a theory with fermions, the corrections due to dressing of bosonic non-zero modes should be similar to corrections due to fermion dressing which presumably should also be considered. If we insist in keeping these corrections we should give up the usual daisy and superdaisy resummation. Some partial dressing procedures are still possible.

\(^4\)Of course the gap equation (13), and (17), is exact at the origin \(\phi = 0\), where \(\gamma \equiv 0\).
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