Quasiparticles and Phase Fluctuations in High Tc Superconductors

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We argue based on theoretical considerations and analysis of experimental data that quasiparticle excitations near the nodes determine the low temperature properties in the superconducting state of cuprates. Quantum effects of phase fluctuations are shown to be quantitatively important, but thermal effects are small for $T \ll T_c$. An anisotropic superfluid Fermi liquid phenomenology is presented for the effect of quasiparticle interactions on the temperature and doping dependence of the low $T$ penetration depth.

1. INTRODUCTION:

The superconducting (SC) state of the high Tc cuprates differs from conventional SCs in several ways: a d-wave gap with low energy quasiparticle excitations near the nodes, a small phase stiffness and a short coherence length. There is some controversy about the importance of quasiparticles versus phase fluctuations in determining the low temperature properties. In this paper, we discuss this problem focusing mainly on the doping and temperature dependence of the in-plane superfluid stiffness $\lambda_{\parallel}$ which is related to the penetration depth $\lambda_{\parallel}^{-2} = 4\pi e^2 D_{\parallel}/\hbar^2 c^2 d_c$ where $d_c$ is the mean interlayer spacing; we will set $\hbar = c = e = 1$ below.

We first review experimental evidence for quasiparticle excitations at optimal doping. Transport data in the SC state in YBCO shows a scattering rate decreasing sharply below $T_c$ implying long lived quasiparticle excitations for $T \ll T_c$. Direct evidence from ARPES in Bi2212 shows the presence of sharp quasiparticle peaks over the entire Fermi surface for $T \ll T_c$. Thermal conductivity data in YBCO and Bi2212 shows $\kappa \sim T$ at low $T$. The slope predicted by quasiparticle theory is in good agreement with this $\kappa$ data on Bi2212, using ARPES estimates for the Fermi velocity $v_F$ and the gap slope $v_\Delta = (2\hbar k_F)^{-1}d\Delta/d\phi$ at the node. Experimentally, it thus seems that quasiparticle excitations exist and are important at low temperature.

It then seems natural to interpret the linear $T$ dependence of $\lambda_{\parallel}(T)$ as arising from nodal quasiparticles (QP). Ignoring QP interactions the layer stiffness $D_{\parallel}(T) = D_{\parallel}(0) - A_0 T$ with $A_0 = (k_a \ln 2/\pi)v_F/v_\Delta$. ARPES estimates in Bi2212 for $v_F$ and $v_\Delta$ give $A_0 \sim 0.8 meV/K$, whereas experiments at optimality measure a slope $\sim 0.3 - 0.4 meV/K$. Thus there is at least a factor of two discrepancy which needs to be understood.

Alternatively, it has been suggested that this linear $T$ behavior could arise entirely from thermal phase fluctuations without invoking nodal quasiparticles. However, there are two reasons to believe that thermal phase fluctuations are unimportant at low $T$ in the cuprates. (1) An effective action calculation for charged $d$-wave SCs, summarized below, shows that thermal phase fluctuations become important only near $T_c$ for Bi2212 at optimality. On the other hand, quantum phase fluctuations are important at low $T$ and suppress both $D_{\parallel}(0)$ and the slope. For Bi2212, the resulting renormalized $D_{\parallel}$ is about 30% smaller while the renormalized slope is about 25% smaller than the bare values. (2) Further, with underdoping, $D_{\parallel}(0)$ decreases and the slope of $D_{\parallel}(T)$ also shows evidence of decreasing in Bi2212 and La214, although some YBCO data is consistent with a doping-independent slope (see the compilation in refs.). Insofar as the data indicate a doping dependent slope for $D_{\parallel}(T)$, they independently rule out classical thermal phase fluctuations as the explanation.
for the linear $T$ dependence, since the slope of $D_{\parallel}$ in such theories is insensitive to doping. Finally there is another proposal for understanding $D_{\parallel}(x; T)$ invoking incoherent pair excitations \cite{12} which, in our opinion, does not properly include the effect of Coulomb interactions.

We next summarize our phase action calculation, and then describe how quasiparticle interactions could account for the difference between the free QP value and the measured slope and its doping dependence.

2. PHASE FLUCTUATIONS:

We have recently derived, by appropriate coarse-graining, a quantum XY model describing phase fluctuations in charged, layered $d$-wave SCs, starting with a lattice model of fermions. See ref. \cite{13} for details of this derivation and some of the discussion in this Section.

The phase action for layered SCs with in-plane lattice spacing $a = 1$ takes the form

$$S[\theta] = \frac{1}{8T} \sum_{\mathbf{q}, \omega_n} \frac{\omega_n^2}{V_{\mathbf{q}}} |\theta(\mathbf{q}, \omega_n)|^2$$

$$+ \frac{1}{4} \int_0^{1/T} d\tau \sum_{\mathbf{r}, \alpha} D_{\parallel}^F [1 - \cos(\theta_{\mathbf{r}, \tau} - \theta_{\mathbf{r} + \alpha, \tau})]$$

where $\xi_0$ is the in-plane coherence length, and $D_{\parallel}^F$ refers to the layer stiffness without phase fluctuation effects, but including possible renormalizations due to quasiparticle interactions. Here $V_{\mathbf{q}} = V(\mathbf{q} / \xi_0, q_\perp)$ with $V(\mathbf{q}) = (2\pi c^2 / q || \epsilon_b) \sinh(q_\parallel d_e) / [\cosh(q_\parallel d_e) - \cos(q_\perp d_e)]$, is the Coulomb interaction for layered systems, $\epsilon_b$ is the background dielectric constant, $d_e$ the interlayer spacing, and $\mathbf{q}_\parallel, \mathbf{q}_\perp$ refer to in-plane and $c$-axis momentum components. The prime on the sum indicates a Matsubara frequency cutoff since the energy of the fluctuations should not exceed the condensation energy $E_{\text{cond}} = \frac{1}{2} D_{\parallel}^F (\pi / \xi_0)^2$.

The form of the first term of (1), which arises from coarse-graining up to a scale $\xi_0$, and the importance of cutoffs have not been appreciated earlier. The (typically very small) $c$-axis stiffness $D_{\perp}^F$ can be ignored for in-plane properties since it was found not to lead to qualitative or quantitative changes.

We ignore vortices (transverse phase fluctuations) which are suppressed at low $T$ by their finite core energy. Analyzing longitudinal phase fluctuations for (1) within a self-consistent harmonic approximation (SCHA) \cite{10} leads to the renormalized stiffness $D_{\parallel} = D_{\parallel}^F \exp(-\langle \delta \theta^2 \rangle / 2)$ where $\delta \theta^2 = \langle \theta_{\mathbf{r}, \tau} - \theta_{\mathbf{r} + \alpha, \tau} \rangle^2$. Our numerical results can be simply understood as follows: $\langle \delta \theta^2 \rangle(T = 0) \sim \sqrt{(c^2 / \epsilon_b \xi_0) / D_{\parallel}(0)}$ is a measure of zero point quantum fluctuations, while classical thermal phase fluctuations become important near a crossover scale $T_\times \sim \min [T_c, T_0]$ where $T_0 = \sqrt{D_{\parallel}(0)(c^2 / \epsilon_b \xi_0)}$ is the $T = 0$ oscillator level spacing in the renormalized harmonic theory.

It is easy to see that phase fluctuation effects are negligible in the BCS limit, except very close to $T_c$. With $c^2 / \epsilon_b a \sim D_{\parallel}^F \sim E_p$ and $\xi_0 \sim v_F / \Delta$, one obtains the standard result $E_{\text{cond}} \sim \Delta^2 / E_p$ per unit cell, and $\langle \delta \theta^2 \rangle(T = 0) \sim \sqrt{\Delta / E_p} \ll 1$ and $T_\times \sim \min [T_c, \sqrt{E_p \Delta}] = T_c$.

For the cuprates, the small $\xi_0$ and small $D_{\parallel}^F$ act together to increase $\langle \delta \theta^2 \rangle$, but they push $T_\times$ in opposite directions. For optimal Bi2212 we use $c^2 / \epsilon_b a \approx 0.33 eV$ with $\epsilon_b \approx 10$, $\xi_0 / a \approx 10$, and $d_e / a \approx 4$. Assuming that the two layers within a bilayer are phase-locked, we get the bilayer stiffness $D_{\parallel}(0) \approx 80 meV$, from experimental data which shows $\lambda_1(0) \approx 2000 A$. This leads to $E_{\text{cond}} \approx 6 K$/unitcell and $T_0 \approx 600 K$. Since the bare stiffness $D_{\parallel}^F$ actually decreases with temperature due to quasiparticle excitations, an estimate of the crossover scale $T_\times$ can be obtained from $T_\times \sim \sqrt{D_{\parallel}(T_\times)(c^2 / \epsilon_b \xi_0)}$. Assuming a linearly decreasing $D_{\parallel}(T)$, this leads to $T_\times \sim T_c$. Thus longitudinal thermal fluctuations are clearly unimportant at low temperatures. Quantum fluctuations are important at low temperatures since $\langle \delta \theta^2 \rangle(T = 0) \sim 1$ at optimality and detailed calculations \cite{12} lead to a $30\%$ decrease of $D_{\parallel}^F(0)$ and a $25\%$ decrease in the slope.

While it might appear that there could be a low
to thermal phase fluctuations due to the low energy c-axis plasmon (∼ 7K for Bi2212) in the anisotropic layered SCs, the phase space for these low lying fluctuations is too small to lead to a linear T behavior \[ ∼ T \]. Even in a purely 2D system with a low lying \( \sqrt{t} \) plasmon, the decrease in the phase stiffness due to phase fluctuations only goes as a large power law \( (∼ T^5) \). Quasiparticles are thus crucial in obtaining the observed linear temperature dependence.

We next turn to the doping dependence of phase fluctuations. The (amplitude) coherence length \( \xi_0 \) is crucial in determining the effect of phase fluctuations. Since \( \xi_0 \) is determined by the pairing gap which appears to remain finite as we underdope, we do not expect singular behavior in the phase fluctuations arising from the doping dependence of \( \xi_0 \). In this case, the dominant doping dependence to phase fluctuations arises only from the singular behavior of the bare parameters in the phase action on underdoping. This singular \( x \)-dependence in \( D(x) \) is most naturally explained by quasiparticle interaction effects as discussed below.

To estimate the doping dependence of phase fluctuations, we note that the core energy will lead to vortices being exponentially suppressed at low \( T \), even as we underdope. Using the experimental input \( T_c ∼ D(x; T = 0) ∼ x \) [13] and assuming a doping independent \( \xi_0 \), longitudinal fluctuations within the SCHA lead to \( T_\parallel^0 ∼ \sqrt{t} \) and hence \( T_\parallel^0 ≫ T_c \). Thus, \( T_\parallel ∼ \min \{ T_c, T_\parallel^0 \} ∼ T_c \), which implies that thermal phase fluctuations are unimportant at low \( T \) as one underdopes. Further, within the SCHA, \( ⟨\delta^2⟩ ∼ x^{-1/2} \), which would lead to a destruction of superconductivity at small enough \( x \). However, we do not expect the SCHA to be valid close to this transition.

3. QUASIPARTICLE INTERACTIONS:

The increasing importance of interactions with underdoping is evident: \( D(x; 0) ∼ x \) [13] and the quasiparticle weight diminishes [16] as one approaches the Mott insulator. We thus explore the possibility that residual interactions between the quasiparticles in the SC state can account for the value and doping dependence of the slope of \( D(x; T) \). To this end we use a phenomenological superfluid Fermi liquid theory (SFLT) [13,18].

All available experimental evidence on the ground state and low lying excitations suggests that the correlated SC state in the cuprates is induced by turning on a pairing interaction. We thus feel that SFLT may be a reasonable description of QP interactions in the SC state at low \( T \), even though this formulation makes reference to a (hypothetical) \( T = 0 \) normal Fermi liquid in which SC is induced by turning on a pairing interaction. (One could argue that approaching the superconducting phase from the overdoped side at \( T = 0 \), one obtains a normal Fermi liquid to SC transition.)

The bilayer stiffness after including QP interaction effects is given by \( D^\parallel(T) = \beta_p D^\parallel(0) - \alpha_p 2(k_p T \ln 2/\pi) v_F / v_\Delta \) where \( \alpha_p, \beta_p \) are Fermi liquid renormalizations. We will constrain the Landau QP interaction function by demanding \( \beta_p ∼ x \), consistent with experiments and then determine the doping trends in \( \alpha_p \).

To compute \( \alpha_p, \beta_p \), using a standard Kubo formula in the quasiparticle basis, it is convenient to shift the origin of the Brillouin zone to the \((\pi, \pi)\) point and describe the hole-like Fermi surface of Bi2212 in terms of an angle \( \phi \). The Landau f-function is denoted by \( f(\phi, \phi') \). We define \( \langle O \rangle_\phi \equiv \int_{-\pi}^{\pi} d\phi k_p(\phi)O(\phi)/[2\pi |v_F(\phi)|] \). We get \( \beta_p = 1 + 4π \langle (v_Fx(\phi)v_Fx(\phi')f(\phi, \phi'))\phi/|v_Fx(\phi)| \rangle \) from the diamagnetic response of the free energy \( \partial^2 F / \partial A_\parallel^2 \) to an applied vector potential. The current carried by nodal quasiparticles is then renormalized by the factor \( \sqrt{\alpha_p} = 1 + \langle v_Fx(\phi)f(\phi_n, \phi)\phi/|v_Fx(\phi_n)| \rangle \), relative to its non-interacting value, where the nodes are at \( \phi_n = (2n - 1)\pi/4 \) with \( n = 1 \ldots 4 \).

We expand \( f(\phi, \phi') = \sum_{m,m'} F_{m,m'} [\cos(m\phi + m'\phi') + \cos(m\phi + m\phi')] \) in a set of complete basis functions [13], where \( m, m' = 0, ±1, ±2, \ldots \) with square lattice symmetry imposing \( m + m' = 4p \) with \( p = 0, 1, 2, \ldots \). In an isotropic system only \( p = 0 \) survives and \( k_p \) and \( v_p \) are \( \phi \)-independent. However, as emphasized in ref. [18] one then obtains \( \alpha_p = \beta_p ∼ x^2 \) in disagreement with experiments [19].

To illustrate how anisotropy can qualitatively
Figure 1. Doping dependence of renormalizations $\alpha_F$ and $\beta_F$ plotted for anisotropic model discussed in the text. For the isotropic case one gets $\alpha_F = \beta_F^2$.

change this scaling we keep only the leading $p = 0$ term: $f(\phi, \phi') = 2F_{1,1}\cos(\phi - \phi')$, but retain the full anisotropy of the dispersion seen in ARPES [20]. We make a reasonable choice of $F_{1,1}(x)$ such that $\beta_F \sim x$ as $x \to 0$, and $Q$ such that $\beta_F = 0$ at $x = 0$. This leads to $\alpha_F(x)$ shown in Fig.1, which is a weak function of doping. In general there are too many free parameters in the anisotropic case (an infinite set $F_{m,m'}$) for the theory to have predictive power; nevertheless the simple example above shows how the $T = 0$ value and slope of $D_\parallel$ can easily exhibit rather different $x$-dependences, and account for the experimentally observed $D_\parallel(x;T)$.

We thus arrive at the following picture for the doping and temperature dependence of $D_\parallel$. The bare stiffness arising from non-interacting quasiparticles is renormalized by both QP interactions and quantum phase fluctuations at low $T$ leading to the measured stiffness, $D_\parallel(x;T)$. Its doping dependence, $D_\parallel(x;0) \sim x$, is determined by QP interactions while its linear $T$ behavior is governed by nodal QPs, with its slope renormalized by both QP interactions and quantum phase fluctuations.

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