Operational interpretation of weight-based resource quantifiers in convex quantum resource theories

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Extended Abstract

The framework of Quantum Resource Theories (QRTs) [1] has proven to be a powerful framework within quantum information [2–20]. Within the the language of QRTs, properties of different objects deemed as resources can be addressed under the same umbrella and this has consequently led to the cross fertilisation of ideas amongst different quantum phenomena; results in a particular QRT with a particular resource has led to insights into different resources and QRTs of different objects. One of the main goals of QRTs is to define resource quantifiers in order to properly quantify the amount of a resource present in an object, as well as to devise operational tasks explicitly harnessing these resources.

In this work, we introduce the resource quantifier of weight of resource for convex quantum resource theories of states and measurements with arbitrary resources. We show that it captures the advantage that a resourceful state (measurement) offers over all possible free states (measurements), in the operational task of exclusion of subchannels (states). Furthermore, we introduce information-theoretic quantities related to exclusion for quantum channels, and find a connection between the weight of resource of a measurement, and the exclusion-type information of quantum-to-classical channels. The results found in this article apply to the resource theory of entanglement, in which the weight of resource is known as the best-separable approximation or Lewenstein-Sanpera decomposition, introduced in 1998. Consequently, the results found here provide an operational interpretation to this 21 year-old entanglement quantifier.

![Figure 1: Three-way correspondence between: resource quantifiers, operational tasks, and single-shot information-theoretic quantities, for QRTs of measurements with arbitrary convex resources.](image-url)
References

[1] E. Chitambar and G. Gour, “Quantum resource theories,” Rev. Mod. Phys., vol. 91, p. 025001, Apr 2019. [Online]. Available: https://link.aps.org/doi/10.1103/RevModPhys.91.025001

[2] M. Oszmaniec, L. Guerini, P. Wittek, and A. Acín, “Simulating positive-operator-valued measures with projective measurements,” Phys. Rev. Lett., vol. 119, p. 190501, Nov 2017. [Online]. Available: https://link.aps.org/doi/10.1103/PhysRevLett.119.190501

[3] M. Oszmaniec and T. Biswas, “Operational relevance of resource theories of quantum measurements,” Quantum, vol. 3, p. 133, Apr. 2019. [Online]. Available: https://doi.org/10.22331/q-2019-04-26-133

[4] T. Guff, N. A. McMahon, Y. R. Sanders, and A. Gilchrist, “A resource theory of quantum measurements,” 2019.

[5] J. I. de Vicente, “On nonlocality as a resource theory and nonlocality measures,” Journal of Physics A: Mathematical and Theoretical, vol. 47, no. 42, p. 424017, oct 2014. [Online]. Available: https://doi.org/10.1088%2F1751-8113%2F47%2F42%2F424017

[6] B. Amaral, “Resource theory of contextuality,” 2019.

[7] R. Gallego and L. Aolita, “Resource theory of steering,” Phys. Rev. X, vol. 5, p. 041008, Oct 2015. [Online]. Available: https://link.aps.org/doi/10.1103/PhysRevX.5.041008

[8] I. Šupić, P. Skrzypczyk, and D. Cavalcanti, “Methods to estimate entanglement in teleportation experiments,” Phys. Rev. A, vol. 99, p. 032334, Mar 2019. [Online]. Available: https://link.aps.org/doi/10.1103/PhysRevA.99.032334

[9] Z.-W. Liu and A. Winter, “Resource theories of quantum channels and the universal role of resource erasure,” 2019.

[10] Y. Liu and X. Yuan, “Operational resource theory of quantum channels,” 2019.

[11] G. Vidal and R. Tarrach, “Robustness of entanglement,” Phys. Rev. A, vol. 59, pp. 141–155, Jan 1999. [Online]. Available: https://link.aps.org/doi/10.1103/PhysRevA.59.141

[12] D. Cavalcanti and P. Skrzypczyk, “Quantitative relations between measurement incompatibility, quantum steering, and nonlocality,” Phys. Rev. A, vol. 93, p. 052112, May 2016. [Online]. Available: https://link.aps.org/doi/10.1103/PhysRevA.93.052112

[13] M. Piani and J. Watrous, “Necessary and sufficient quantum information characterization of einstein-podolsky-rosen steering,” Phys. Rev. Lett., vol. 114, p. 060404, Feb 2015. [Online]. Available: https://link.aps.org/doi/10.1103/PhysRevLett.114.060404

[14] M. Piani, M. Cianciaruso, T. R. Bromley, C. Napoli, N. Johnston, and G. Adesso, “Robustness of asymmetry and coherence of quantum states,” Phys. Rev. A, vol. 93, p. 042107, Apr 2016. [Online]. Available: https://link.aps.org/doi/10.1103/PhysRevA.93.042107

[15] C. Napoli, T. R. Bromley, M. Cianciaruso, M. Piani, N. Johnston, and G. Adesso, “Robustness of coherence: An operational and observable measure of quantum coherence,” Phys. Rev. Lett., vol. 116, p. 150502, Apr 2016. [Online]. Available: https://link.aps.org/doi/10.1103/PhysRevLett.116.150502

[16] P. Skrzypczyk and N. Linden, “Robustness of measurement, discrimination games, and accessible information,” Phys. Rev. Lett., vol. 122, p. 140403, Apr 2019. [Online]. Available: https://link.aps.org/doi/10.1103/PhysRevLett.122.140403

[17] L. Guerini, J. BavareSCO, M. T. Cunha, and A. Acín, “Operational framework for quantum measurement simulability,” Journal of Mathematical Physics, vol. 58, no. 9, p. 092102, Sep. 2017. [Online]. Available: https://doi.org/10.1063/1.4994303

[18] R. Uola, T. Kraft, J. Shang, X.-D. Yu, and O. Gühne, “Quantifying quantum resources with conic programming,” Phys. Rev. Lett., vol. 122, p. 130404, Apr 2019. [Online]. Available: https://link.aps.org/doi/10.1103/PhysRevLett.122.130404

[19] C. Carmeli, T. Heinosaari, T. Miyadera, and A. Toigo, “Witnessing incompatibility of quantum channels,” 2019.

[20] T. Theurer, D. Egloff, L. Zhang, and M. B. Plenio, “Quantifying operations with an application to coherence,” Phys. Rev. Lett., vol. 122, p. 190405, May 2019. [Online]. Available: https://link.aps.org/doi/10.1103/PhysRevLett.122.190405