SCALAR OR VECTOR TETRAQUARK STATE CANDIDATE: $Z_+(4100)$

Zhī-Gāng Wang

Department of Physics, North China Electric Power University, Baoding 071003, P. R. China

Abstract

In this article, we separate the vector and axialvector components of the tensor diquark operators explicitly, construct the axialvector-axialvector type and vector-vector type scalar tetraquark currents and scalar-tensor type tensor tetraquark current to study the scalar, vector and axialvector tetraquark states with the QCD sum rules in a consistent way. The present calculations do not support assigning the $Z_+(4100)$ to be a scalar or vector tetraquark state. If the $Z_+(4100)$ is a scalar tetraquark state, if should have a mass about $3.9 \pm 0.18$ GeV, on the other hand, if the $Z_+(4100)$ is a vector tetraquark state, if should have a mass about $4.2 \pm 0.18$ GeV. As a byproduct, we obtain an axialvector tetraquark candidate for the $Z_+(4020)$.

PACS number: 12.39.Mk, 12.38.Lg

Key words: Tetraquark state, QCD sum rules

1 Introduction

The attractive interactions of one-gluon exchange favor formation of the diquarks in color antitriplet [1]. The diquarks (or diquark operators) $e^{abc} q^T_a C T_a'$ in color antitriplet have five structures in Dirac spinor space, where $CT = C \gamma_5$, $C \gamma_\mu \gamma_5$, $C \gamma_\mu$ and $C \sigma_{\mu \nu}$ for the scalar ($S$), pseudoscalar ($P$), vector ($V$), axialvector ($A$) and tensor ($T$) diquarks, respectively. The scalar, pseudoscalar, vector and axialvector diquarks have been studied with the QCD sum rules in details, which indicate that the favored configurations are the scalar and axialvector diquark states $2\bar{3} \bar{3}$. The scalar and axialvector diquark operators have been applied extensively to construct the tetraquark currents to study the lowest tetraquark states $[5, 6, 7, 8, 9, 10, 11, 12]$. In Refs. [11], we tentatively assign the $Z_+(4050)$ to be a scalar tetraquark state $[12, 13, 14]$, or charge conjugation of the $Z_+^+(4050)$? In Refs. [10], we study the $[sc]_S [\bar{s}c]_S$-type, $[sc]_A [\bar{s}c]_A$-type, $[sc]_P [\bar{s}c]_P$-type and $[sc]_V [\bar{s}c]_V$-type scalar tetraquark states with the QCD sum rules systematically, and obtain the ground state masses $M_{SS} = 3.89 \pm 0.05$ GeV ($3.85^{+0.18}_{-017}$ GeV), $M_{AA} = 3.92^{+0.19}_{-0.18}$ GeV, $M_{PP} = 5.48 \pm 0.10$ GeV and $M_{VV} = 4.70^{+0.05}_{-0.09}$ GeV for the $SS$, $AA$, $PP$ and $VV$ diquark-antidiquark type tetraquark states, respectively. The larger uncertainties $\pm 0.18$ GeV and $\pm 0.19$ GeV originate from the QCD sum rules, where both the ground state and the first excited state are taken into account at the hadron side. If only the ground states are taken into account in the QCD sum rules, the uncertainties $|\delta M| \leq 0.10$ GeV.

In Ref. [11], we tentatively assign the $X^+(3860)$ to be the $[qc]_S [\bar{q}c]_S$-type scalar tetraquark state, study its mass and width with the QCD sum rules in details, and obtain the mass $M_{SS} = 3.86 \pm 0.09$ GeV. Now we can see that the $SU(3)$ breaking effects of the masses of the $[sc]_S$ and $[qc]_S$ tetraquark states from the QCD sum rules are rather small, roughly speaking, they have degenerate masses. If we take the larger uncertainty, the predicted mass $M_{AA} = 3.92^{+0.19}_{-0.18}$ GeV has overlap with the experimental data $M_{Z_+} = 4096 \pm 20^{+25}_{-18}$ MeV marginally [13], and favors assigning the $Z_+(4100)$ to be the $AA$-type scalar tetraquark state [14]. On the other hand, if we take the smaller uncertainty, the predicted mass $M_{AA} = 3.92 \pm 0.10$ GeV has no overlap with the

1E-mail: zgwang@aliyun.com.
experimental data $M_{Z'} = 4096 \pm 20^{+18}_{-22}$ MeV, and disfavors assigning the $Z_c(4100)$ to be the $AA$-type scalar tetraquark state. In a word, it is not robust assigning the $Z_c(4100)$ to be the $AA$-type scalar tetraquark state.

In Ref. [12], Sundu, Agaev and Azizi obtain the mass $M_{SS} = 4.08 \pm 0.15$ GeV, which differs from the prediction $M_{SS} = 3.86 \pm 0.09$ GeV greatly [11]. The differences originate from the different input parameters at the QCD side and different pole contributions at the hadron side. In Ref. [11], the pole contribution is about $(46 - 70)\%$, which is much larger than the pole contribution in Ref. [12]. While in early works, we took the pole contributions $> 50\%$ and chose the energy scales of the QCD spectral densities to be $\mu = 1$ GeV, and obtained almost degenerate masses $M_{SS} \approx M_{AA} \approx 4.4$ GeV [6], which is much larger than the value $M_{SS} = 4.08 \pm 0.15$ GeV [12]. In Ref. [8], we observe that the energy scale $\mu = 1$ GeV is not the optimal energy scale for the hidden-charm tetraquark states.

In summary, the QCD sum rules do not support assigning the $Z_c(4100)$ to be the $[q\bar{c}]_S[q\bar{c}]_S$-type, $[q\bar{c}]_A[q\bar{c}]_A$-type, $[q\bar{c}]_P[q\bar{c}]_P$-type and $[q\bar{c}]_V[q\bar{c}]_V$-type scalar tetraquark states.

In Refs. [14] [19], we take the scalar and axialvector diquark operators as basic constituents, introduce an explicit P-wave between the diquark and antidiquark operators to construct the vector tetraquark currents, and study the vector tetraquark states with the QCD sum rules systematically, and obtain the lowest vector tetraquark masses up to now, $M_V = 4.24 \pm 0.10$ GeV, 4.28 $\pm 0.10$ GeV, 4.31 $\pm 0.10$ GeV and 4.33 $\pm 0.10$ GeV for the tetraquark states $|0, 0; 1, 1\rangle$, $|1, 1; 0, 1\rangle$, $|1, 1; 1, 1\rangle$ and $|1, 1; 1, 1\rangle$, respectively, where the tetraquark states are defined by $|S[qc], S[q\bar{c}]; S, L; J\rangle$, the $S$, $L$ and $J$ are the diquark spin, angular momentum and total angular momentum, respectively. For other QCD sum rules with the $[q\bar{c}]_S \partial_\mu [q\bar{c}]_S$-type interpolating currents, one can consult Ref. [20]. In fact, if we take the pseudoscalar and vector diquark operators as the basic constituents, even (or much) larger tetraquark masses are obtained [21][22][23]. Up to now, it is obvious that the QCD sum rules do not support assigning the $Z_c(4100)$ to be the vector tetraquark state.

It is interesting to analyze the properties of the tensor diquark states, and take the tensor diquark operators as the basic constituents to construct the tetraquark currents to study the $Z_c(4100)$.

Under parity transform $\hat{P}$, the tensor diquark operators have the properties,

$$
\hat{P} \varepsilon^{abc} q^T(x) C \sigma_{\mu\nu} \gamma_5 Q^c(x) \hat{P}^{-1} = \varepsilon^{abc} q^T(\bar{x}) C \sigma_{\mu\nu} \gamma_5 Q^c(\bar{x}),
$$
$$
\hat{P} \varepsilon^{abc} q^T(x) C \sigma_{\mu\nu} Q^c(x) \hat{P}^{-1} = -\varepsilon^{abc} q^T(\bar{x}) C \sigma_{\mu\nu} Q^c(\bar{x}),
$$

(1)

where the four vectors $x^\mu = (t, \vec{x})$ and $\bar{x}^\mu = (t, -\bar{x})$. The tensor diquark states have both $J^P = 1^+$ and $1^-$ components,

$$
\hat{P} \varepsilon^{abc} q^T(x) C \sigma_{jk} \gamma_5 Q^c(x) \hat{P}^{-1} = +\varepsilon^{abc} q^T(\bar{x}) C \sigma_{jk} \gamma_5 Q^c(\bar{x}),
$$
$$
\hat{P} \varepsilon^{abc} q^T(x) C \sigma_{0j} Q^c(x) \hat{P}^{-1} = +\varepsilon^{abc} q^T(\bar{x}) C \sigma_{0j} Q^c(\bar{x}),
$$
$$
\hat{P} \varepsilon^{abc} q^T(x) C \sigma_{0j} \gamma_5 Q^c(x) \hat{P}^{-1} = -\varepsilon^{abc} q^T(\bar{x}) C \sigma_{0j} \gamma_5 Q^c(\bar{x}),
$$
$$
\hat{P} \varepsilon^{abc} q^T(x) C \sigma_{jk} Q^c(x) \hat{P}^{-1} = -\varepsilon^{abc} q^T(\bar{x}) C \sigma_{jk} Q^c(\bar{x}),
$$

(2)

where $j, k = 1, 2, 3$. Now we introduce the four vector $t^\mu = (1, \vec{0})$ and project out the $1^+$ and $1^-$ components explicitly,

$$
\hat{P} \varepsilon^{abc} q^T(x) C \sigma_{\mu\nu} \gamma_5 Q^c(x) \hat{P}^{-1} = +\varepsilon^{abc} q^T(\bar{x}) C \sigma_{\mu\nu} \gamma_5 Q^c(\bar{x}),
$$
$$
\hat{P} \varepsilon^{abc} q^T(x) C \sigma_{\mu\nu} Q^c(x) \hat{P}^{-1} = +\varepsilon^{abc} q^T(\bar{x}) C \sigma_{\mu\nu} Q^c(\bar{x}),
$$
$$
\hat{P} \varepsilon^{abc} q^T(x) C \sigma_{\mu\nu} \gamma_5 Q^c(x) \hat{P}^{-1} = -\varepsilon^{abc} q^T(\bar{x}) C \sigma_{\mu\nu} \gamma_5 Q^c(\bar{x}),
$$
$$
\hat{P} \varepsilon^{abc} q^T(x) C \sigma_{\mu\nu} Q^c(x) \hat{P}^{-1} = -\varepsilon^{abc} q^T(\bar{x}) C \sigma_{\mu\nu} Q^c(\bar{x}),
$$

(3)
tetraquark states, while the current J tetraquark states, properties, where we choose the axialvector diquark operator $\hat{\epsilon}$ degenerate masses. Under charge conjugation transform the Z operators explicitly, construct the $\tilde{\epsilon}$ quark operator

Firstly, we write down the two-point correlation functions in the QCD sum rules, 2 QCD sum rules for the scalar and vector tetraquark states of the tetraquark states in section 2; in section 3, we present the numerical results and discussions; section 4 is reserved for our conclusion.

In this article, we choose the axialvector diquark operator $\hat{\epsilon}$ quark operators, we can obtain the same predictions.

The article is arranged as follows: we derive the QCD sum rules for the masses and pole residues of the tetraquark states in section 2; in section 3, we present the numerical results and discussions; section 4 is reserved for our conclusion.

## 2 QCD sum rules for the scalar and vector tetraquark states

Firstly, we write down the two-point correlation functions in the QCD sum rules,

$$
\Pi(p) = i \int d^4x e^{ip.x} \langle 0 | T \{ J(x) J^\dagger(0) \} | 0 \rangle,
$$

$$
\Pi_{\mu\nu\alpha\beta}(p) = i \int d^4x e^{ip.x} \langle 0 | T \{ J_{\mu\nu}(x) J^\dagger_{\alpha\beta}(0) \} | 0 \rangle,
$$

where $J(x) = J_{\hat{A}\hat{A}}(x), J_{\hat{V}\hat{V}}(x)$,

$$
J_{\hat{A}\hat{A}}(x) = \hat{\epsilon}^{ijk}\hat{\epsilon}^{imn}ue^{T}(x)C\sigma^\nu_{\mu\nu}e^k(x)d^m(x)\sigma^\nu_{\mu\nu}C\tilde{e}^Tn(x),
$$

$$
J_{\hat{V}\hat{V}}(x) = \hat{\epsilon}^{ijk}\hat{\epsilon}^{imn}ue^{T}(x)C\sigma^t_{\mu\nu}e^k(x)d^m(x)\sigma^t_{\mu\nu}C\tilde{e}^Tn(x),
$$

$$
J_{\mu\nu}(x) = \frac{\hat{\epsilon}^{ijk}\hat{\epsilon}^{imn}}{\sqrt{2}} \left[ u^{Tn}(x)C\gamma_5\epsilon^k(x)d^m(x)\sigma_{\mu\nu}\tilde{C}\tilde{e}^Tn(x) - u^{T}(x)C\sigma_{\mu\nu}\epsilon^k(x)d^m(x)\gamma_5\tilde{C}\tilde{e}^Tn(x) \right],
$$

the $i, j, k, m, n$ are color indexes. We take the isospin limit by assuming the u and d quarks have degenerate masses. Under charge conjugation transform $\hat{C}$, the currents $J(x)$ and $J_{\mu\nu}(x)$ have the properties,

$$
\hat{C}J(x)\hat{C}^{-1} = +J(x),
$$

$$
\hat{C}J_{\mu\nu}(x)\hat{C}^{-1} = -J_{\mu\nu}(x),
$$

the currents have definite charge conjugation. The currents $J(x)$ couple potentially to the scalar tetraquark states, while the current $J_{\mu\nu}(x)$ couples potentially to both the $J^{PC} = 1^{+-}$ and $1^{--}$ tetraquark states,
the $\varepsilon_{\mu/\alpha}$ are the polarization vectors of the tetraquark states, the superscripts $^\pm$ denote the positive parity and negative parity, respectively, the $M_{Z^\pm}$ and $\lambda_{Z^\pm}$ are the masses and pole residues, respectively.

We insert a complete set of intermediate hadronic states with the same quantum numbers as the current operators into the correlation functions to obtain the hadronic representation [24, 25], and isolate the ground state tetraquark contributions, the current operators into the correlation functions to obtain the hadronic representation [24, 25], and isolate the ground state tetraquark contributions,

$$
\Pi(p) = \frac{\lambda_{Z^+}^2}{M_{Z^+}^2 - p^2} + \cdots
= \Pi(p^2),
$$

$$
\Pi_{\mu\nu\alpha\beta}(p) = \frac{\lambda_{Z^-}^2}{M_{Z^-}^2 (M_{Z^-}^2 - p^2)} \left( p^2 g_{\mu\alpha} g_{\nu\beta} - p^2 g_{\mu\beta} g_{\nu\alpha} - g_{\mu\alpha} p_{\nu\beta} p_{\mu\beta} - g_{\mu\beta} p_{\nu\alpha} p_{\mu\beta} + g_{\nu\alpha} p_{\mu\beta} p_{\nu\beta} \right) + \cdots
+ \Pi_-(p^2) \left( -g_{\mu\alpha} p_{\nu\beta} - g_{\nu\beta} p_{\mu\alpha} + g_{\mu\beta} p_{\nu\alpha} + g_{\nu\alpha} p_{\mu\beta} \right) + \cdots
$$

$$
\Pi_+(p^2) \left( -g_{\mu\alpha} p_{\nu\beta} - g_{\nu\beta} p_{\mu\alpha} + g_{\mu\beta} p_{\nu\alpha} + g_{\nu\alpha} p_{\mu\beta} \right).
$$

(9)

We can project out the components $\Pi_{\pm}(p^2)$ explicitly by introducing the operators $P_{Z^\pm}^{\mu\nu\alpha\beta}$,

$$
\Pi_{\pm}(p^2) = p^2 \Pi_{\pm}(p^2) = P_{Z^\pm}^{\mu\nu\alpha\beta},
$$

(10)

where

$$
P_{Z^-}^{\mu\nu\alpha\beta} = \frac{1}{6} \left( g_{\mu\alpha} - \frac{p^\mu p^\alpha}{p^2} \right) \left( g_{\nu\beta} - \frac{p^\nu p^\beta}{p^2} \right),
$$

$$
P_{Z^+}^{\mu\nu\alpha\beta} = \frac{1}{6} \left( g_{\mu\alpha} - \frac{p^\mu p^\alpha}{p^2} \right) \left( g_{\nu\beta} - \frac{p^\nu p^\beta}{p^2} \right) - \frac{1}{6} g_{\mu\alpha} g_{\nu\beta}.
$$

(11)

In this article, we choose the correlation functions $\Pi(p^2)$, $\Pi_-(p^2)$ and $\Pi_+(p^2)$ to study the scalar, vector and axial-vector tetraquark states, respectively.

If we cannot project out the components $\Pi_{\pm}(p^2)$ explicitly, we have to choose the currents $J_{\mu\nu}^V(x)$ and $J_{\mu\nu}^A(x)$,

$$
J_{\mu\nu}^V(x) = \frac{\varepsilon_{ijkl} \varepsilon_{immn}}{\sqrt{2}} \left[ u^{Tj}(x) C_\gamma e^k(x) \bar{d}^m(x) \sigma_{\mu\nu}^i C \bar{c}^{Tn}(x) - u^{Tj}(x) C \sigma_{\mu\nu}^i e^k(x) \bar{d}^m(x) \gamma_5 C \bar{c}^{Tn}(x) \right],
$$

$$
J_{\mu\nu}^A(x) = \frac{\varepsilon_{ijkl} \varepsilon_{immn}}{\sqrt{2}} \left[ u^{Tj}(x) C \gamma_5 e^k(x) \bar{d}^m(x) \sigma_{\mu\nu}^i C \bar{c}^{Tn}(x) - u^{Tj}(x) C \sigma_{\mu\nu}^i \gamma_5 e^k(x) \bar{d}^m(x) C \bar{c}^{Tn}(x) \right],
$$

(12)

which couple to the $J^P = 1^-$ and $1^+$ tetraquark states respectively,

$$
\langle 0 | J_{\mu\nu}^V(0) | Z^-_c(p) \rangle = \frac{\lambda_{Z^-}}{M_{Z^-}} (\varepsilon_{\mu\nu} - \varepsilon_{\nu\mu}) ,
$$

$$
\langle 0 | J_{\mu\nu}^A(0) | Z^+_c(p) \rangle = \frac{\lambda_{Z^+_c}}{M_{Z^+_c}} (\varepsilon_{\mu\nu} - \varepsilon_{\nu\mu}) ,
$$

(13)

as the current operators $J_{\mu\nu}^V(x)$ and $J_{\mu\nu}^A(x)$ have the properties,

$$
\hat{P} J_{\mu\nu}^V(x) \hat{P}^{-1} = -J_{\mu\nu}^V(\hat{x}) ,
$$

$$
\hat{P} J_{\mu\nu}^A(x) \hat{P}^{-1} = +J_{\mu\nu}^A(\hat{x}) ,
$$

(14)
under parity transform $\tilde{P}$.

It is more easier to carry out the operator product expansion for the current $J_{\mu\nu}(x)$ than for the currents $J^\mu_{\nu}(x)$ and $J^\nu_{\mu}(x)$. In this article, we prefer the current $J_{\mu\nu}(x)$, and denote the corresponding vector and axialvector tetraquark states as $SV$ type and $SA$ type, respectively.

We carry out the operator product expansion for the correlation functions up to the vacuum condensates of dimension 10 in a consistent way, then obtain the QCD spectral densities through dispersion relation, take the quark-hadron duality below the continuum threshold condensates of dimension 10 in a consistent way, then obtain the QCD spectral densities through a ratio,

$$\lambda^2 \exp \left(-\frac{M^2}{T^2}\right) = \int^{s_0}_{4m^2} ds \, \rho(s) \exp \left(-\frac{s}{T^2}\right),$$

the $\rho(s)$ are the QCD spectral densities. The explicit expressions of the QCD spectral densities are available upon request by contacting me with E-mail. For the technical details, one can consult Refs. [8, 23]. In carrying out the operator product expansion for the correlation functions $\Pi(p)$, we encounter the terms $(p \cdot t)^2$, and set $(p \cdot t)^2 = p^2$, just like in the QCD sum rules for the baryon states separating the contributions of the positive parity and negative parity, where we take the four vector $p^\mu = (p_0, \vec{0})$.

We derive Eq. (15) with respect to $\tau = \frac{1}{T}$, and obtain the QCD sum rules for the masses of the scalar, vector and axialvector tetraquark states $Z_c$ through a ratio,

$$M^2_Z = -\frac{\int^{s_0}_{4m^2} ds \, \frac{d}{ds} \rho(s) \exp (-\tau s)}{\int^{s_0}_{4m^2} ds \rho(s) \exp (-\tau s)}.$$

### 3 Numerical results and discussions

We choose the standard values of the vacuum condensates $\langle \bar{q}q \rangle = -(0.24 \pm 0.01 \text{ GeV})^3$, $\langle \bar{q}q, \sigma Gq \rangle = m^2_0(\bar{q}q)$, $m^2_0 = (0.8 \pm 0.1) \text{ GeV}^2$, $\langle \frac{\alpha_s G^2}{\pi} \rangle = (0.33 \text{ GeV})^4$ at the energy scale $\mu = 1 \text{ GeV}$ [24, 25, 27], and choose the $\overline{MS}$ mass $m_c(m_c) = (1.275 \pm 0.025) \text{ GeV}$ from the Particle Data Group [28], and set $m_u = m_d = 0$. Moreover, we take into account the energy-scale dependence of the input parameters at the QCD side,

$$\langle \bar{q}q \rangle(\mu) = \langle \bar{q}q \rangle(1\text{ GeV}) \left[ \frac{\alpha_s(1\text{ GeV})}{\alpha_s(\mu)} \right]^\frac{28}{25},$$

$$\langle \bar{q}q, \sigma Gq \rangle(\mu) = \langle \bar{q}q, \sigma Gq \rangle(1\text{ GeV}) \left[ \frac{\alpha_s(1\text{ GeV})}{\alpha_s(\mu)} \right]^\frac{28}{25},$$

$$m_c(\mu) = m_c(m_c) \left[ \frac{\alpha_s(\mu)}{\alpha_s(m_c)} \right]^\frac{28}{25},$$

$$\alpha_s(\mu) = \frac{1}{b_0 t} \left[ 1 - \frac{b_1 \log t}{b_0^2} + \frac{b_2^2 (\log^2 t - \log t - 1) + b_0 b_2}{b_0^2 t^2} \right],$$

where $t = \log \frac{s}{\Lambda^2}$, $b_0 = \frac{33 - 2n_f}{12\pi}$, $b_1 = \frac{153 - 19n_f}{12\pi}$, $b_2 = -\frac{2857 - 2033n_f + 347n_f^2}{12\pi^2}$, $\Lambda = 210 \text{ MeV}$, $292 \text{ MeV}$ and $332 \text{ MeV}$ for the flavors $n_f = 5$, 4 and 3, respectively [28, 29], and evolve all the input parameters to the ideal energy scales $\mu$ to extract the tetraquark masses.

Now we search for the ideal Borel parameters $T^2$ and continuum threshold parameters $s_0$ to satisfy the four criteria:

1. Pole dominance at the hadron side;
2. Convergence of the operator product expansion;
3. Appearance of the Borel platforms;
4. Satisfying the energy scale formula, via try and error. The resulting Borel parameters, continuum threshold parameters, energy scales of the QCD spectral densities and pole contributions are shown explicitly in Table 1. From the Table, we can see that the pole contributions are about \((40 - 60)\%\), the pole dominance condition at the hadron side is well satisfied. In calculations, we observe that the contributions of the vacuum condensates of dimension 10 are \(< 1\%\), the operator product expansion is well convergent.

We take into account the uncertainties of the input parameters and obtain the masses and pole residues of the tetraquark states, which are shown explicitly in Table 2 and in Figs. 1–2. From Tables 1–2, we can see that the energy scale formula \(\mu = \sqrt{M_{X/Y}^2 - (2M_c)^2}\) is well satisfied, where we take the updated value of the effective \(c\)-quark mass \(M_c = 1.82 \text{ GeV}\) \[22\]. In Figs. 1–2, we plot the masses and pole residues of the tetraquark states with variations of the Borel parameters at much larger ranges than the Borel widows, the regions between the two perpendicular lines are the Borel windows. In the Borel windows, the uncertainties induced by the Borel parameters are \(< 1\%\) for the masses and \(< 2\%\) for the pole residues, there appear Borel platforms. Now the four criteria are all satisfied, we expect to make reliable predictions.

In Ref. [30], we study the tensor-tensor type scalar hidden-charm tetraquark states with currents 

\[
\eta(x) = \varepsilon^{ijk} \varepsilon^{imn} q^j(x) C\sigma_{\mu
u} k^m(x) q^n(x) \sigma^{\mu\nu} C \bar{q} T^n(x),
\]

via the QCD sum rules by taking into account both the ground state contributions and the first radial excited state contributions, where \(\sigma_{\mu\nu} = \frac{1}{2} [\gamma_\mu, \gamma_\nu] = \sigma^{\mu\nu}_v + 2\sigma^v_{\mu\nu}\), and obtain masses \(M_{TT,1S} = 3.82 \pm 0.16 \text{ GeV}\) and \(M_{TT,2S} = 4.38 \pm 0.09 \text{ GeV}\). We can rewrite the current \(\eta(x)\) as a linear superposition \(\eta(x) = J_\bar{V} \bar{V}(x) + 4J_{\bar{A}A}(x)\), the tensor-tensor type scalar hidden-charm tetraquark states have both the \(V V\) and \(A \bar{A}\) components. Compared to the \(A \bar{A}\) \((VV)\) type tetraquark mass \(M_{\bar{A}A} = 3.98 \pm 0.08 \text{ GeV}\) \(M_{VV} = 5.35 \pm 0.09 \text{ GeV}\), the ground state mass \(M_{TT,1S} = 3.82 \pm 0.16 \text{ GeV}\) is (much) lower. In the QCD sum rules for the tensor-tensor type scalar tetraquark states, the terms \(m_c(q\bar{q})\) and \(m_c(\bar{q}q, \sigma Gq)\) disappear due to the special superposition \(\eta(x) = J_\bar{V} \bar{V}(x) + 4J_{\bar{A}A}(x)\), the contamination from the \(V V\) component is large.

In Fig. 1, we also present the experimental values of the masses of the \(Z_c(4100)\) and \(Z_c(4020)\) \[13, 28, 31, 32\]. From the figure, we can see that the predicted mass for the \([uc]_S [\bar{d}c]_A - [uc]_A [\bar{d}c]_S\) type axialvector tetraquark state is in excellent agreement with the experimental data in the Borel window, and supports assigning the \(Z_c(4020)\) to be the \([uc]_S [\bar{d}c]_A - [uc]_A [\bar{d}c]_S\) type axialvector tetraquark state, while the \(Z_c(4100)\) lies above the \(A \bar{A}\) type scalar tetraquark state, and much below the \(V V\)-type scalar tetraquark state and \(SV\)-type vector tetraquark state in the Borel windows, the present QCD sum rules do not support assigning the \(Z_c(4100)\) to be the scalar or vector tetraquark state. The masses of the scalar hidden-charm tetraquark states have the hierarchy \[9, 10, 11\],

\[
M_{SS} \leq M_{AA} \leq M_{\bar{A}A} < M_{Z_c(4100)} \ll M_{VV} \ll M_{\bar{V}V} \leq M_{PP},
\]

the QCD sum rules disfavor assigning the \(Z_c(4100)\) to be a scalar tetraquark state.

The masses extracted from the QCD sum rules depend on the Borel windows, different Borel windows lead to different predicted masses. From Fig. 1, we can see that if we choose larger Borel parameter for the \(A \bar{A}\)-type scalar tetraquark state, for example, choose \(T^2 > 4.2 \text{ GeV}^2\) rather than choose \(T^2 = (3.1 - 3.5) \text{ GeV}^2\), we can obtain a mass about 4.1 GeV, which is compatible with the experimental value of the mass of the \(Z_c(4100)\). If we choose the Borel window \(T^2 = (4.9 - 5.3) \text{ GeV}^2\), the predicted mass \(M_{\bar{A}A} = 4.09 \pm 0.08 \text{ GeV}\), which is in excellent agreement with the experimental data \(M_{Z_c} = 4096 \pm 20\) MeV \[13\]. However, the pole contribution is about \((14 - 24)\%\), the prediction is not robust.

The mass \(M_{\bar{A}A}\) extracted from the QCD sum rules decreases monotonously with increase of the energy scales of the QCD spectral density. If we choose \(\mu = 1.4 \text{ GeV}\), \(T^2 = (3.3 - 3.7) \text{ GeV}^2\), \(\sqrt{s_0} = 4.70 \pm 0.10 \text{ GeV}\), the pole contribution is \((41 - 61)\%\) and the operator product expansion
is well convergent, we obtain the tetraquark mass $M_{\tilde{A}\tilde{A}} = 4.12 \pm 0.08$ GeV, which is in excellent agreement with the experimental data $M_{Z_c} = 4096 \pm 20^{+18}_{-22}$ MeV \[13\], see Fig.3. In Fig.3, we plot the mass $M_{\tilde{A}\tilde{A}}$ extracted at the energy scale $\mu = 1.4$ GeV with variation of the Borel parameter, again the region between the two perpendicular lines is the Borel windows. However, the energy scale formula $\mu = \sqrt{\int_{s_{0}}^{\mu} \frac{ds}{\lambda_{\tilde{A}}(\mu)}} = \frac{M_{X/Y/Z}^2 - (2M_{\tilde{Q}})^2}{\lambda_{\tilde{A}}(\mu)}$ is not satisfied. In the QCD sum rules for the hidden-charm (or hidden-bottom) tetraquark states and molecular states, the integrals

$$\int_{4m_{\tilde{Q}}^2(\mu)}^{s_0} ds p_{QCD}(s, \mu) \exp \left( -\frac{s}{\lambda_{\tilde{A}}(\mu)} \right),$$

are sensitive to the energy scales $\mu$. We suggest an energy scale formula $\mu = \sqrt{\int_{s_{0}}^{\mu} \frac{ds}{\lambda_{\tilde{A}}(\mu)}} = \frac{M_{X/Y/Z}^2 - (2M_{\tilde{Q}})^2}{\lambda_{\tilde{A}}(\mu)}$ with the effective $Q$-quark mass $M_{\tilde{Q}}$ to determine the ideal energy scales of the QCD spectral densities in a consistent way \[23\]. The energy scale formula works well for the tetraquark states $[8, 9, 10, 11, 22, 23, 33, 34]$, tetraquark molecular states \[35\], and even for the hidden-charm pentaquark states \[36\]. For example, in Refs.\[8, 22, 33\] and the present work, we observe that there exist one axialvector tetraquark candidate $[uc]_S[\bar{d}\bar{c}]_A - [uc]_A[\bar{d}\bar{c}]_S$ for the $Z_c(3900)$, three axialvector tetraquark candidates $[uc]_A[\bar{d}\bar{c}]_A, [uc]_A[\bar{d}\bar{c}]_S - [uc]_A[\bar{d}\bar{c}]_S$ and $[uc]_A[\bar{d}\bar{c}]_S - [uc]_A[\bar{d}\bar{c}]_S$ for the $Z_c(4020)$, which is consistent with the almost degenerate scalar and axialvector diquark masses from the QCD sum rules \[3\]. Furthermore, the $Z_c(4430)$ can be assigned to be the first radial excited state of the $Z_c(3900)$. If the $Z_c(4100)$ is a diquark-antidiquark type scalar tetraquark state, the energy scale formula should be satisfied, as the $Z_c(4100)$ has no reason to be an odd or special tetraquark state. The masses of the scalar tetraquark states $SS, AA$ and $\tilde{A}\tilde{A}$ are estimated to be $3.9 - 4.0$ GeV, if the spin-spin interactions are neglected, the QCD sum rules support this estimation.

In Ref.\[12\], Sundu, Agaev and Azizi choose the $SS$ type scalar current to study the mass and width of the $Z_c(4100)$, and obtain the mass $M_{SS} = 4.08 \pm 0.15$ GeV with the pole contribution (19–54)%. In Ref.\[11\], we tentatively assign the $X^+(3860)$ to be the $[qc][\bar{q}\bar{c}]_S$-type scalar tetraquark state, study its mass and width with the QCD sum rules, and obtain the mass $M_{SS} = 3.86 \pm 0.09$ GeV with the pole contribution (46 – 70)%, which is much larger than the pole contribution (19 – 54)% in Ref.\[12\]. In Ref.\[11\], just like in the present work, we use the energy scale formula $\mu = \sqrt{\int_{s_{0}}^{\mu} \frac{ds}{\lambda_{\tilde{A}}(\mu)}} = \frac{M_{X/Y/Z}^2 - (2M_{\tilde{Q}})^2}{\lambda_{\tilde{A}}(\mu)}$ to enhance the pole contribution. In the QCD sum rules, we prefer larger pole contributions to obtain more robust predictions.

The predicted mass of the $[uc]_S[\bar{d}\bar{c}]_S - [uc]_A[\bar{d}\bar{c}]_S$ type vector tetraquark state $M_{SV} = 4.61 \pm 0.08$ GeV is much larger than the mass of the $Z_c(4100)$, see Table \[2\]. Furthermore, the mass of the $Z_c(4100)$ is even smaller than the lowest vector hidden-charm tetraquark mass from the QCD sum rules \[14, 19\],

$$M_{Z_c(4100)} < M_Y(4260/4220) = 4.24 \pm 0.10 \text{ GeV} < M_{SV} = 4.61 \pm 0.08 \text{ GeV},$$

the QCD sum rules also disfavor assigning the $Z_c(4100)$ to be a vector tetraquark state.

If the $Z_c(4100)$ is a $SS, AA$ or $\tilde{A}\tilde{A}$ type scalar hidden-charm tetraquark state, it should have a mass about 3.9 GeV or 4.0 GeV, on the other hand, if the $Z_c(4100)$ is a vector hidden-charm tetraquark state, it should have a mass about 4.2 GeV rather than 4.1 GeV.

### 4 Conclusion

In this article, we separate the vector and axialvector components of the tensor diquark operators explicitly, construct the axialvector-axialvector-type and vector-vector type scalar tetraquark currents and scalar-tensor type tensor tetraquark current to study the scalar, vector and axialvector tetraquark states with the QCD sum rules by carrying out the operator product expansion up to vacuum condensates of dimension 10 in a consistent way. In calculation, we use the energy scale
Figure 1: The masses with variations of the Borel parameters $T^2$ for the tetraquark states, the (I), (II), (III) and (IV) denote the $[uc]_A[\bar{d}\bar{c}]_\bar{A}$, $[uc]_V[\bar{d}\bar{c}]_\bar{V}$, $[uc]_S[\bar{d}\bar{c}]_\bar{V}$ $- [uc]_V[\bar{d}\bar{c}]_S$ and $[uc]_S[\bar{d}\bar{c}]_A - [uc]_A[\bar{d}\bar{c}]_S$ tetraquark states, respectively, the regions between the two perpendicular lines are the Borel windows.
Figure 2: The pole residues with variations of the Borel parameters $T^2$ for the tetraquark states, the (I), (II), (III) and (IV) denote the $\left[ uc \right]_A \bar{d} \bar{c} \bar{A}$, $\left[ uc \right]_V \bar{d} \bar{c} \bar{V}$, $\left[ uc \right]_S \bar{d} \bar{c} \bar{V} - \left[ uc \right]_V \bar{d} \bar{c} \bar{S}$ and $\left[ uc \right]_A \bar{d} \bar{c} \bar{S}$ tetraquark states, respectively, the regions between the two perpendicular lines are the Borel windows.
Table 1: The Borel parameters, continuum threshold parameters, energy scales of the QCD spectral densities and pole contributions of the ground state tetraquark states, where the superscript * denotes the energy scale formula is not satisfied.

| $Z_c$ | $T^2(\text{GeV}^2)$ | $\sqrt{s_0}(\text{GeV})$ | $\mu(\text{GeV})$ | pole |
|-------|---------------------|--------------------------|-------------------|------|
| $[uc]_A[dc]_\bar{A}$ | 3.1 – 3.5 | 4.55 ± 0.10 | 1.6 | (42 – 62)% |
| $[uc]_C[dc]_\bar{C}$ | 4.9 – 5.5 | 5.90 ± 0.10 | 3.9 | (43 – 61)% |
| $[uc]_S[dc]_\bar{V} - [uc]_V[dc]_S$ | 3.8 – 4.2 | 5.18 ± 0.10 | 2.8 | (43 – 61)% |
| $[uc]_S[dc]_\bar{A} - [uc]_A[dc]_S$ | 3.1 – 3.5 | 4.56 ± 0.10 | 1.6 | (42 – 62)% |
| $[uc]_A[dc]_\bar{A}^*$ | 3.3 – 3.7 | 4.70 ± 0.10 | 1.4 | (41 – 61)% |

Table 2: The masses and pole residues of the ground state tetraquark states, where the superscript * denotes the energy scale formula is not satisfied.

| $Z_c$ | $M_Z(\text{GeV})$ | $\lambda_Z(\text{GeV}^2)$ |
|-------|-----------------|-----------------|
| $[uc]_A[dc]_\bar{A}$ | 3.98 ± 0.08 | (4.30 ± 0.63) × 10^{-2} |
| $[uc]_V[dc]_\bar{V}$ | 5.35 ± 0.09 | (4.86 ± 0.50) × 10^{-4} |
| $[uc]_S[dc]_\bar{V} - [uc]_V[dc]_S$ | 4.61 ± 0.08 | (6.15 ± 0.80) × 10^{-2} |
| $[uc]_S[dc]_\bar{A} - [uc]_A[dc]_S$ | 3.99 ± 0.09 | (2.73 ± 0.41) × 10^{-2} |
| $[uc]_A[dc]_\bar{A}^*$ | 4.12 ± 0.08 | (4.84 ± 0.71) × 10^{-2} |

Figure 3: The mass with variation of the Borel parameter $T^2$ for the $[uc]_A[dc]_\bar{A}$ tetraquark state at the energy scale $\mu = 1.4\text{ GeV}$, the region between the two perpendicular line is the Borel window.
formula to determine the ideal energy scales of the QCD spectral densities to extract the masses from the QCD sum rules with the pole contributions about \((40 - 60)\%\). The present calculations do not support assigning the \(Z_c(4100)\) to be the scalar or vector tetraquark state. If the \(Z_c(4100)\) is a scalar tetraquark state, it should have a mass about 3.9 GeV or 4.0 GeV, on the other hand, if the \(Z_c(4100)\) is a vector tetraquark state, it should have a mass about 4.2 GeV. More precise measurements of the mass, width and quantum numbers are still needed. As a byproduct, we obtain an axialvector tetraquark candidate for the \(Z_c(4020)\).

Acknowledgements

This work is supported by National Natural Science Foundation, Grant Number 11775079.

References

[1] A. De Rujula, H. Georgi and S. L. Glashow, Phys. Rev. D12 (1975) 147; T. DeGrand, R. L. Jaffe, K. Johnson and J. E. Kiskis, Phys. Rev. D12 (1975) 2060.
[2] H. G. Dosch, M. Jamin and B. Stech, Z. Phys. C42 (1989) 167; M. Jamin and M. Neubert, Phys. Lett. B238 (1990) 387.
[3] Z. G. Wang, Eur. Phys. J. C71 (2011) 1524; R. T. Kleiv, T. G. Steele and A. Zhang, Phys. Rev. D87 (2013) 125018.
[4] Z. G. Wang, Commun. Theor. Phys. 59 (2013) 451.
[5] R. D. Matheus, S. Narison, M. Nielsen and J. M. Richard, Phys. Rev. D75 (2007) 014005; F. S. Navarra, M. Nielsen and S. H. Lee, Phys. Lett. B649 (2007) 166; C. F. Qiao and L. Tang, Eur. Phys. J. C74 (2014) 3122.
[6] Z. G. Wang, Phys. Rev. D79 (2009) 094027; Z. G. Wang, Eur. Phys. J. C67 (2010) 411.
[7] Z. G. Wang, Eur. Phys. J. C70 (2010) 139.
[8] Z. G. Wang and T. Huang, Phys. Rev. D89 (2014) 054019.
[9] Z. G. Wang, Eur. Phys. J. C77 (2017) 78.
[10] Z. G. Wang, Eur. Phys. J. A53 (2017) 19.
[11] Z. G. Wang, Eur. Phys. J. A53 (2017) 192.
[12] H. Sundu, S. S. Agaev and K. Azizi, [arXiv:1812.10094](https://arxiv.org/abs/1812.10094).
[13] R. Aaij et al, Eur. Phys. J. C78 (2018) 1019.
[14] Z. G. Wang, Eur. Phys. J. C78 (2018) 933.
[15] J. Wu, X. Liu, Y. R. Liu and S. L. Zhu, Phys. Rev. D99 (2019) 014037.
[16] M. B. Voloshin, Phys. Rev. D98 (2018) 094028.
[17] Q. Zhao, [arXiv:1811.05357](https://arxiv.org/abs/1811.05357).
[18] X. Cao and J. P. Dai, [arXiv:1811.06434](https://arxiv.org/abs/1811.06434).
[19] Z. G. Wang, Eur. Phys. J. C79 (2019) 29.
[20] J. R. Zhang and M. Q. Huang, JHEP 1011 (2010) 057; J. R. Zhang and M. Q. Huang, Phys. Rev. D83 (2011) 036005.

[21] R. M. Albuquerque and M. Nielsen, Nucl. Phys. A815 (2009) 532009; Erratum-ibid. A857 (2011) 48; W. Chen and S. L. Zhu, Phys. Rev. D83 (2011) 034010; Z. G. Wang, Eur. Phys. J. C78 (2018) 518; H. Sundu, S. S. Agaev and K. Azizi, Phys. Rev. D98 (2018) 054021.

[22] Z. G. Wang, Eur. Phys. J. C76 (2016) 387.

[23] Z. G. Wang, Eur. Phys. J. C74 (2014) 2874.

[24] M. A. Shifman, A. I. Vainshtein and V. I. Zakharov, Nucl. Phys. B147 (1979) 385; Nucl. Phys. B147 (1979) 448.

[25] L. J. Reinders, H. Rubinstein and S. Yazaki, Phys. Rept. 127 (1985) 1.

[26] D. Jido, N. Kodama and M. Oka, Phys. Rev. D54 (1996) 4532; Z. G. Wang, Eur. Phys. J. A45 (2010) 267; Z. G. Wang, Eur. Phys. J. C68 (2010) 459; Z. G. Wang, Eur. Phys. J. A47 (2011) 81.

[27] P. Colangelo and A. Khodjamirian, [hep-ph/0010175]

[28] M. Tanabashi et al, Phys. Rev. D98 (2018) 030001.

[29] S. Narison and R. Tarrach, Phys. Lett. 125 B (1983) 217; S. Narison, “QCD as a theory of hadrons from partons to confinement”, Camb. Monogr. Part. Phys. Nucl. Phys. Cosmol. 17 (2007) 1.

[30] Z. G. Wang and J. X. Zhang, Eur. Phys. J. C76 (2016) 650.

[31] M. Ablikim et al, Phys. Rev. Lett. 112 (2014) 132001.

[32] M. Ablikim et al, Phys. Rev. Lett. 111 (2013) 242001.

[33] Z. G. Wang, [arXiv:1901.10741]

[34] Z. G. Wang and Y. F. Tian, Int. J. Mod. Phys. A30 (2015) 1550004; Z. G. Wang, Commun. Theor. Phys. 63 (2015) 325.

[35] Z. G. Wang and T. Huang, Eur. Phys. J. C74 (2014) 2891; Z. G. Wang, Eur. Phys. J. C74 (2014) 2963.

[36] Z. G. Wang, Eur. Phys. J. C76 (2016) 70; Z. G. Wang and T. Huang, Eur. Phys. J. C76 (2016) 43.