Dark Matter Caustics in Galaxy Clusters

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We interpret the recent gravitational lensing observations of Jee et al. \cite{1} as first evidence for a caustic ring of dark matter in a galaxy cluster. A caustic ring unavoidably forms when a cold collisionless flow falls with net overall rotation in and out of a gravitational potential well. Evidence for caustic rings of dark matter was previously found in the Milky Way and other isolated spiral galaxies. We argue that galaxy clusters have at least one and possibly two or three caustic rings. We calculate the column density profile of a caustic ring in a cluster and show that it is consistent with the observations of Jee et al.

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INTRODUCTION

Using strong and weak gravitational lensing methods, Jee et al. \cite{1} constructed a column density map of the central region of the galaxy cluster Cl 0024+1654. The map shows a ring of dark matter of radius \( \approx 400 \) kpc, width \( \approx 150 \) kpc and maximum column density \( \approx 58 \, \text{M}_{\odot} \, \text{pc}^{-2} \). Remarkably, the overdensity in dark matter is not accompanied by an analogous structure in x-ray emitting gas or luminous matter. In this regard, Jee et al. discovered a new instance where dark and ordinary matter have dramatically different spatial distributions. The well-known observations of the “Bullet Cluster” 1E0657-56 \cite{2} provided an earlier example.

Jee et al. interpret the dark matter ring in Cl 0024+1654 as the product of a near head-on collision, along the line of sight, of two subclusters. They performed a simulation of the response of the dark matter particles to the time-varying gravitational field and found that, after the collision has occurred, the dark matter particles move outward and form shell-like structures which appear as a ring when projected along the collision axis \cite{1}. The interpretation of Jee et al. fits with independent lines of evidence that Cl 0024+1654, an apparently relaxed cluster, is a collision of two subclusters. However, it is shown in ref. \cite{3} that the observed dark matter ring is reproduced only for highly fine-tuned, and hence unlikely, initial velocity distributions.

The purpose of our paper is to propose an alternative interpretation, to wit that Jee et al. have observed a caustic ring formed by the in and out flow of dark matter particles falling onto the cluster for the first time. We show that the observed ring is explained assuming only that the dark matter falling onto the cluster has net overall rotation, with angular momentum vector close to the line of sight, and velocity dispersion less than 60 km/s.

When cold collisionless dark matter falls from all directions into a smooth gravitational potential well, the phase space distribution of the dark matter particles is characterized everywhere by a set of discrete flows \cite{4}. The flows form outer and inner caustics. The outer caustics are formed by outflows where they turn around before falling back in. Each outer caustic is a fold catastrophe \((A_2)\) located on a topological sphere surrounding the potential well. The inner caustics \cite{5, 6} are formed near where the particles with the most angular momentum in an inflow reach their closest approach to the center before going back out. The catastrophe structure of the inner caustics \cite{7} depends on the angular momentum distribution of the infalling particles. If that angular momentum distribution is characterized by net overall rotation, the inner caustics are rings (closed tubes) whose cross-section is a section of the elliptic umbilic catastrophe \((D_{-4})\) \cite{8}. These statements are valid independently of any assumptions of symmetry, self-similarity, or anything else.

It is argued in ref. \cite{8} that discrete flows and caustics are a generic and robust property of galactic halos if the dark matter is collisionless and cold. The radii of the outer caustic spheres are predicted by the self-similar infall model \cite{9, 10} of halo formation. The radii of the inner caustic rings are predicted \cite{5}, in terms of a single parameter \( j_{\text{max}} \), after the model is generalized \cite{11} to allow angular momentum for the infalling particles. Evidence for inner caustic rings distributed according to the predictions of the self-similar infall model has been found in the Milky Way \cite{5, 12, 13} and in other isolated spiral galaxies \cite{5, 14}.

The resolution of most present numerical simulations is inadequate to see discrete flows and caustics. However such features are seen in dedicated simulations which increase the number of particles in the relevant regions of phase space \cite{15, 16}. They should also become apparent in fully general simulations of structure formation through the use of special techniques \cite{17}.

DARK MATTER CAUSTICS IN GALAXY CLUSTERS

With regard to our proposal that Jee et al. have observed a caustic ring of dark matter, the first question...
that arises is whether discrete flows and therefore caustics should be expected in galaxy clusters, in view of the fact that gravitational scattering by the galaxies in the cluster tends to diffuse the flows. A flow of particles passing through a region populated by a class of objects of mass $M$ and number density $n$ gets diffused by gravitational scattering over a cone of opening angle $\Delta \theta$ whose square [4]

\[
(\Delta \theta)^2 \sim \int dt \int_{b_{\text{min}}}^{b_{\text{max}}} \frac{4G^2M^2}{b^2v^4} n v^2 \pi b db
\]

\[
\sim 4.7 \times 10^{-3} \left( \frac{10^3 \text{ km/s}}{v} \right)^3 \left( \frac{M}{10^{12}M_\odot} \right)^2 \cdot \ln \left( \frac{b_{\text{max}}}{b_{\text{min}}} \right) \left( \frac{t}{10 \text{ Gyr}} \right) \left( \frac{n}{\text{Mpc}^{-3}} \right),
\]

where $b$ is impact parameter, $v$ is the velocity of the flow and $t$ is the time over which it encountered the objects in question. Because the density $n$ falls off with distance $r$ to the cluster center as a power law $\frac{1}{r^\gamma}$ with $\gamma \geq 2$, the integral in Eq. (1) is dominated by contributions received while passing through the central parts of the cluster. To obtain an estimate, we consider a cluster of 200 galaxies of mass $M \sim 2 \cdot 10^{12}M_\odot$ each, within a region of radius 1 Mpc, and with velocity dispersion $\sqrt{\langle v^2 \rangle} \simeq 1300$ km/s. These properties are descriptive of Cl 0024+1654 [18]. The velocity of the flow going through the central parts for the first time is approximately $v \sim 2\sqrt{\langle v^2 \rangle}$. The time to fall through the central region is approximately $t \sim \frac{2 \text{ Mpc}}{v} \sim 7 \cdot 10^8$ year. Since $\ln \left( \frac{b_{\text{max}}}{b_{\text{min}}} \right) \sim 1$, we find $\Delta \theta \sim 6 \times 10^{-2}$, indicating that the flows and caustics associated with the first throughfall ($n = 1$) are only partially diffused by gravitational scattering. It appears unlikely that the caustics associated with flows that have passed several times ($n \sim \text{few}$) through the central parts of the cluster survive. On the other hand, we expect discrete flows and associated caustics for $n = 1$, and possibly for $n = 2, 3$.

Mahdavi et al. have already presented evidence for caustics in galaxy clusters. When plotted as a function of position on the sky, the velocity dispersion and surface density of the galaxies in the group around NGC 5846 has a sharp drop at 840 kpc = $R_1$ from the group’s center, consistent with the occurrence of the outer caustic at that radius [19]. Furthermore, the observations of Mahdavi et al. allow a test of the self-similar model prediction for the relationship between velocity dispersion and the radius $R_1$ of the first outer caustic. The model depends on a parameter $\epsilon$ [4] which is related to the slope of the power spectrum of density perturbations [20]. $\epsilon$ is a slowly increasing function of the object size. On the scale of galaxies, $\epsilon$ is in the range 0.25 to 0.30, whereas for clusters it is in the range 0.35 to 0.45 [11]. Because NGC 5846 is a small cluster, $\epsilon$ is near 0.35. For $\epsilon = 0.3$, 0.35 and 0.40, the model predicts respectively $\sqrt{\langle v^2 \rangle} = 620, 590$ and 580 km/s Mpc, where $\sqrt{\langle v^2 \rangle}$ is the velocity dispersion measured at the center of the cluster. The result $\sqrt{\langle v^2 \rangle} = 425$ km/s of Mahdavi et al. [10] for NGC 5846 agrees with the self-similar model at the 15% level.

Since the flow of infalling dark matter forms the first inner caustic before it forms the first outer caustic, the evidence of ref. [10] for the first outer caustic of the NGC 5846 group is also evidence for its first inner caustic. The former cannot exist without the latter.

We mentioned that the catastrophe structure of the inner caustics depends on the angular momentum distribution of the infalling dark matter. If that distribution is dominated by net overall rotation, the inner caustics are rings whose cross-section is an elliptic umbilic catastrophe. Evidence has been found for caustic rings of dark matter in galaxies [2, 12, 13, 14], implying that the dark matter accreting onto galactic halos has net overall rotation. Assuming that this is so, there is no reason to expect differently on the only somewhat larger length scale of galactic clusters. Thus we are led to expect that clusters have one or more caustic rings of dark matter.

### GRAVITATIONAL LENSING BY CAUSTICS IN GALAXY CLUSTERS

Weak lensing responds to the column density distribution $\Sigma(x, y)$ along lines of sight in the direction $\hat{z}$. Caustics have precisely defined density profiles and hence precisely defined column density profiles. Thus one may be able to answer unambiguously whether a feature seen in a column density map is due to a caustic or not.

For an outer caustic, the density is

\[
d(r) = A \sqrt{R - r} \Theta(R - r)
\]

where $r$ is the radial coordinate, $R$ is the radius of the caustic, $A$ is a constant called the fold coefficient, and $\Theta$ is the step function. The column density is

\[
\Sigma(x) = \pi A \sqrt{2R} \Theta(R - x)
\]

for $x \ll R$. $x$ is the projected radial coordinate. For the cluster Cl 0024+1654, appropriate values for the self-similar infall model input parameters are: age $t = 9.4$ Gyr, velocity dispersion $\sqrt{\langle v^2 \rangle} = 1300$ km/s and $\epsilon = 0.4$. The model predicts then

\[
\{R_n : n = 1, 2, \ldots 6\} = \{1.6, 0.9, 0.7, 0.55, 0.45, 0.40\} \text{ Mpc}
\]

and

\[
\{\Sigma_n \equiv \pi A \sqrt{2R_n} : n = 1, 2, \ldots 6\} = \{12, 11, 10, 9.5, 9.5\} \frac{M_\odot}{\text{pc}^2}
\]
Let us briefly consider the proposal that the feature seen by Jee et al. is due to an outer caustic in Cl 0024+1654. An advantage of this interpretation is that the circular appearance of the feature does not require an accident. Since outer caustics are approximately spherically symmetric, they appear approximately axially symmetric from any vantage point. However, there are serious difficulties with this hypothesis. First, for the radius $R_n$ to fit the observed radius of the feature (400 kpc), $n$ should be of order 6; see Eq. (1). But, as we discussed above, the caustics with such high $n$ are almost entirely degraded by gravitational scattering off the galaxies in the cluster. Second, the feature identified by Jee et al. has, at its maximum, a column density of order $58 \frac{M_\odot}{pc^2}$, which is a factor five or so larger than the column density contrasts $\Sigma_n$ produced by outer caustics; see Eq. (3). Finally, the predicted profile does not fit the observed profile. As a function of projected distance $x$ to the cluster center, the observed $\Sigma(x)$ shows a bump at $x \approx 400$ kpc, whereas the $\Sigma(x)$ due to outer caustics is a step function.

Caustic rings of dark matter were described in detail in ref. [6]. The properties of an axially symmetric caustic ring are determined entirely in terms of six parameters: the caustic ring radius $a$, the sizes $p$ and $q$ of its cross-section in the directions parallel and perpendicular to the plane of the ring, the speed $v$ and the mass infall rate per unit solid angle $\frac{dM}{d\Omega dt}$ of the particles constituting the caustic, and a parameter $s$ which is of order $a$. Let $z$ be the coordinate perpendicular to the plane of the ring and $x$ the radial coordinate at a particular location on the ring. $\hat{y}$ is in the direction tangent to the ring. The density $d(x, z)$ has a unique expression in terms of the stated parameters [6]. It is straightforward to integrate $d(x, z)$ along any line in the $xz$ plane. For the lines of sight perpendicular to the plane of the ring, the result is

$$\Sigma(x) = \int dz \, d(x, z) = \frac{4}{v} \sqrt{\frac{a}{s}} \frac{dM}{d\Omega dt} \frac{1}{x} I \left( \frac{2(x-a)}{s} \right)$$

(6)

where

$$I(\Delta) = \int_0^\pi d\alpha \Theta(\alpha^2 + \Delta) \frac{\cos \alpha}{\sqrt{\alpha^2 + \Delta}}$$

(7)

$I(\Delta)$ has a logarithmic singularity

$$I(\Delta) \sim \frac{1}{2} \ln \left( \frac{3.2}{|\Delta|} \right)$$

(8)

when $\Delta \rightarrow 0$. The finite velocity dispersion of the particles in the flow forming the caustic smoothes the caustic and therefore also the logarithmic singularity of Eq. (8). Fig. 1 shows the function $I(\Delta)$, as well as $I(\Delta)$ averaged over two different scales. The column density does not depend sharply on the angle of view. One can show in particular that from any vantage point in the $xz$ plane there is a line of sight for which the column density is logarithmically divergent in the cold flow limit.

**FIG. 1:** The function $I(\Delta)$ defined by Eq. (7) (solid line), the function $I(\Delta)$ convoluted with a Gaussian of FWHM 0.07 (dot-dashed), and FWHM 0.133 (dashed).

**COMPARISON WITH OBSERVATION**

The self-similar model predicts $v$ and $\frac{dM}{d\Omega dt}$ in terms of the age and velocity dispersion of the object. For Cl 0024+1654, using the previously mentioned input parameters, this implies

$$\Sigma(x) = 31 \frac{M_\odot}{pc^2} \frac{\sqrt{a}}{s} \frac{v}{x} I \left( \frac{2(x-a)}{s} \right).$$

(9)

To determine $\frac{a}{s}$, information must be provided about the dependence of specific angular momentum on declination $\alpha$ near $\alpha = 0$ [6]. In the absence of such information, it is only possible to state that $\frac{a}{s}$ is of order one, i.e. that it is likely between 0.5 and 2.

As was mentioned, the logarithmic singularity in the column density profile at $x = a$ is smoothed by the velocity dispersion of the dark matter flow which makes the caustic. We now discuss two sources of velocity dispersion: the presence of small scale structure in the dark matter falling onto the cluster, and gravitational scattering by inhomogeneities in the cluster.

In the hierarchical clustering scenario of structure formation, the dark matter falling onto a galaxy cluster has previously formed smaller scale objects. The infalling flow of dark matter has effective velocity dispersion equal to the velocity dispersion of the small scale objects in the flow. If this effective velocity dispersion is $\delta v$, the spread in specific angular momentum is $\delta \ell \sim \delta v R$ where $R$ is the turnaround radius. Hence the spread in the caustic ring radius is $\delta a \sim \frac{\delta \ell}{v} \sim \frac{\delta v R}{s}$. For Cl 0024+1654, $R \sim 6.7$ Mpc. Since $v \sim 2\sqrt{v^2} \sim 2600$ km/s, the requirement that the caustic ring not spread in radius more than over the observed width ($\sim 150$ kpc) of the
ring implies $\delta v \lesssim 60$ km/s. This places an upper limit on the size of the small scale structures composing the ring. They cannot be larger than small galaxies. Since no small scale structures are observed in the ring, $\delta v$ may be much less than 60 km/s.

The flow of dark matter falling onto a cluster for the first time is diffused through gravitational scattering at least somewhat by the time it forms its first inner caustic. Using Eq. (11), we estimate that the resulting velocity dispersion in Cl 0024+1654 is of order $\delta v \sim 0.035$ km/s. The associated spread in specific angular momentum is of order $\delta \ell \sim a \delta v$. Hence the first inner caustic is smoothed over a minimum distance scale of order 0.035 (400 kpc) = 14 kpc. This corresponds to a smoothing scale of 0.07 in $\Delta = \frac{2(x-a)}{s}$ if $s = a$. Fig.1 shows $I(\Delta)$ defined by Eq. (7), and $I(\Delta)$ averaged over smoothing scales of 0.07 and 0.133 in $\Delta$, by convoluting with Gaussians with those full widths at half maximum (FWHM).

The observed dark matter ring is also smoothed by the resolution of the Cl 0024+1654 mass reconstruction, which is limited by the number density of background galaxies and the finite grid size. If the actual ring column density were a delta function in the radial coordinate, it would be observed as a Gaussian bump with FWHM of order 27 kpc [24]. Let us call $I_{av}(\Delta)$ the function $I(\Delta)$ smoothed over the corresponding scale (0.133) in $\Delta$. $I_{av}(\Delta)$ is shown by the dashed curve in Fig.1 for the case $s = a$.

The dark matter ring in Cl 0024+1654 appears as a bump in the graph of the azimuthally averaged column density plotted as a function of distance $x$ to the center of the ring [1]. Fig.2 shows the data in the region of the bump. Each data point is the measured average column density at the corresponding radius with the $1 \sigma$ error bar [1]. The step in $x$ between successive data points is the bin size (21 kpc). The significance of the bump over an assumed constant background is approximately $8 \sigma$ [1].

The data points in Fig. 2 were fitted with the function

$$F(x) = B \frac{1}{x} + C + A \sqrt{\frac{a}{s}} I_{av}\left(\frac{2(x-a)}{s}\right), \quad (10)$$

where $A = 31 \frac{M_{\odot}}{\text{pc}}$, $I_{av}\left(\frac{2(x-a)}{s}\right)$ is $I\left(\frac{2(x-a)}{s}\right)$ averaged over 27 kpc in $x$, and $B$, $C$, $a$ and $s$ are fitting parameters. The first two terms describe the background column density due to the cluster as a whole and due to other matter that is smoothly distributed on the scale of the ring thickness. The last term represents the contribution of the caustic ring of dark matter smoothed over the resolution of the observations. The least squares fit returned $a = 394$ kpc, $s = 171$ kpc, $B = 6.2 \times 10^6 \frac{M_{\odot}}{\text{pc}}$, and $C = 1.26 \times 10^3 \frac{M_{\odot}}{\text{pc}}$. The $\chi^2$ value was 2.2 for 8 degrees of freedom, indicating an acceptable fit. The $\chi^2$ value is low because the data points do not fluctuate relative to one another as much as independent data points with the stated error bars would. The fitted function is plotted in Fig.2. The value 1.5 of $\sqrt{\frac{\chi^2}{n}}$, although slightly high, is acceptable in the context of the model since $\frac{\chi^2}{n}$ is expected to be of order one. We also fitted the data with $F(x)$ defined in Eq. (10) imposing the constraint $s = a$, and using $A$, $B$, $C$ and $a$ as fitting parameters. In this case, the least squares fit returned $a = 395$ kpc, $A = 46 \frac{M_{\odot}}{\text{pc}}$, $B = -6.6 \times 10^6 \frac{M_{\odot}}{\text{pc}}$, and $C = 1.28 \times 10^3 \frac{M_{\odot}}{\text{pc}}$, with $\chi^2 = 2.0$ for 8 degrees of freedom. The fitted function is almost the same as the one plotted in Fig. 2. Thus, if one imposes $s = a$ as a constraint, the measured column density is 50% higher than the column density predicted by the self-similar infall model.

The caustic ring radius $a$ is determined in the self-similar infall model in terms of a free parameter called $J_{max}$ which is the specific angular momentum of the particles in the plane of the ring. For $a$ to equal the observed radius (395 kpc) of the dark matter ring, one must require $J_{max} \simeq 0.45$.

**CONCLUSIONS**

We conclude that the dark matter ring observed by Jee et al. has properties consistent with a caustic ring of dark matter. The column density agrees both in shape and overall amplitude. At present the data are too imprecise to infer with confidence the nature of the observed ring. However, our proposal makes a distinct prediction for the column density profile across the ring, as illustrated in Fig.1. This may be tested by future observations. In this regard, let us emphasize that the $\frac{1}{2} I_{av}\left(\frac{2(x-a)}{s}\right)$ profile applies to each azimuth. If the caustic ring interpretation
is confirmed, an important corollary is that dark matter falls onto Cl 0024+1654 with net overall rotation.

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