Sudakov effects in BBNS approach

Dong-Sheng Du\textsuperscript{a,b}, Chao-Shang Huang\textsuperscript{a,c}, Zheng-Tao Wei\textsuperscript{a,c}, Mao-Zhi Yang\textsuperscript{b}

\textsuperscript{a}CCAST(World Laboratory), P.O.Box 8730, Beijing 100080, China
\textsuperscript{b}Institute of High Energy Physics, P.O.Box 918(4), Beijing 100039, China
\textsuperscript{c}Institute of Theoretical Physics, P.O.Box 2735, Beijing 100080, China

Abstract
The end-point singularity is an unsolved problem in BBNS approach. Incorporating the partonic transverse momentum and the Sudakov form factor, this problem can be solved model-independently. We discuss the Sudakov effects in BBNS approach. The BBNS approach is compared with the modified PQCD approach. The main idea of Sudakov form factor is briefly discussed. Our conclusion is that the twist-3 contribution for the hard spectator scattering is numerically not important in $B \to \pi\pi$ decays, compared with the twist-2 contribution.

\textsuperscript{1}Supported in part by National Natural Science Foundation of China

\textsuperscript{2}e-mail address: duds@mail.ihep.ac.cn; huangcs@itp.ac.cn; weizt@itp.ac.cn; yangmz@mail.ihep.ac.cn
1. Introduction

The calculation of exclusive process from perturbative QCD (PQCD) is one of the important problems in hadron physics. Soon after the successes of PQCD application in the deep-inelastic scattering, Drell-Yan process, etc, the application of PQCD in some exclusive process with large momentum transfer has been carried out and is successful in the asymptotic limit ($Q^2 \rightarrow \infty$) \[1, 2\]. The key for using PQCD is factorization, i.e., the separation of long- and short-distance dynamics. It has been shown in the PQCD framework for exclusive processes with large momentum transfer that the long-distance dynamics is involved in the light-cone hadronic wave function, the distribution amplitude, and the physical quantity is the convolution of the distribution amplitudes of the initial and final hadrons and the hard scattering kernel \[2\].

Because of the importance in exploring CP violation and determining the CKM parameters, the exclusive, nonleptonic two-body decays of B meson have got extensive theoretical investigations. However, the complication caused by soft interactions in both initial and final hadrons makes it difficult to analyze the full QCD dynamics. Recently, Beneke et al. proposed a QCD improved factorization formula in exclusive B decays \[3\], which is called “BBNS approach” for simplicity later in this letter. The early BSW model \[4\] is the lowest order approximation of this approach. At the order of $\mathcal{O}(\alpha_s)$, the hadronic matrix element is generally the convolution of the three light-cone distribution amplitudes and the hard scattering kernel. For the vertex correction and the penguin correction, it is assumed that the $B \rightarrow M_1$ ($M_1$ denotes the meson that picks up the spectator quark) form factor is dominated by soft interaction. Under this assumption, the factorization formula is simplified as the multiplicity of the $B \rightarrow M_1$ form factor and the convolution of meson wave function with hard scattering kernel. BBNS approach works well at leading twist level. While for twist-3 case, it will lead to infrared divergence in the one-loop vertex correction and end-point singularity in hard spectator scattering.

The problem of the infrared divergence in the vertex correction is partly solved in \[5\]. The authors used the massive gluon to regulate the infrared divergence. They find that the soft and collinear divergences cancel in the vertex correction for the symmetrical twist-3 distribution amplitude. However, the problem of end-point singularity still remains unsolved. In \[6\], the
The phenomenological treatment is introduced to deal with the end-point contribution. But their
 treatment is artificial and unsatisfactory. To solve this problem is the main purpose of this
 paper.

The appearance of end-point singularity means that the separation of hard and soft dyna-
 mics is not justified, so the factorization of BBNS approach breaks down at twist-3 level.
The solution of the end-point singularity is known for a long time [2]. The method is to retain
the partonic intrinsic transverse momentum and include the mechanism of Sudakov suppres-
sion. The average of the transverse momentum $\sqrt{<k^2_T>} \sim \mathcal{O}(\Lambda_{QCD})$ in hadron is much
smaller than $M_B$. In the region far from the end-point, the effect of transverse momentum is
power suppressed and negligible at tree level. However, in the end-point region, the partonic
transverse momentum $k_T$ is the same order as the longitudinal momentum $xP$, its effect is
important. Neglecting it will lead to singularity. In the hard spectator scattering, the twist-3
contribution gives a factor $\int \frac{du}{u^2} \phi_\sigma(u)$ which diverges when $u \to 0$. The transverse momentum
smears the end-point singularity so that the end-point contribution is not dominant. On the
other hand, the parton with transverse momentum will give rise to soft divergence which
cancels in the collinear limit. However, in the soft region, the Sudakov suppression begins
to take effect. For a quark-antiquark pair separated by a transverse distance $b$, the Sudakov
form factor $e^{-S(Q,b)}$ suppresses the contribution at large $b$, so that the dominant contribution
comes from the region with small separation. This idea has been grouped into a self-consistent
and model-independent PQCD formula. This modified PQCD approach is given clearly in
[7]. Its application in B decays can be found in [8] and the reference therein. As we will show,
the modified PQCD approach enlarges the range of PQCD application.

This paper is devoted to study the Sudakov effects in BBNS approach. Here, the Sudakov
effects include the transverse momentum effects and the Sudakov form factor. We will restrict
our discussion in $B \to \pi\pi$ decays, its extension to other $B \to PP$ decays is straightforward.
The relation of BBNS approach and the modified PQCD approach is given in Sect. 2. In Sect.
3, we briefly discuss the Sudakov double logarithm and its resummation to all orders. Some
general properties of Sudakov form factor is also discussed. In Sect. 4, we study the Sudakov
effects in BBNS approach. The end-point singularity in the hard spectator scattering is solved
in the modified PQCD approach. In Sect. 5, we give our conclusions and discussions.
2. BBNS approach and the modified PQCD approach

The essential problem in $B \to \pi \pi$ decay is to calculate the hadronic matrix elements $<\pi \pi|Q_i|\bar{B}>$. In [3], a factorization formula is given as:

$$<\pi(p')\pi(q)|Q_i|\bar{B}(p)> = F^{B\pi}_0(q^2) \int_0^1 dv T^I(v)\Phi_\pi(v)$$  \hspace{1cm} (1)

$$+ \int_0^1 d\xi dudvT^{II}(\xi,u,v)\Phi_B(\xi)\Phi_\pi(u)\Phi_\pi(v)$$

In BBNS approach, the $B \to \pi$ transition form factor $F^{B\pi}_0$ is assumed to be dominated by soft interactions and treated as a nonperturbative input parameter. In this study, we hold this assumption and leave the discussion about it in the last section.

There is only one scale $m_b$ in the hard kernel of BBNS approach. We neglect the mass difference of b quark and the B meson. The scale $\mu$ in $\alpha_s$ is the renormalization scale which is chosen as $\mu \sim O(m_b)$ to eliminate the large logarithms in the loop calculation. This scale is also the factorization scale which separates the long- and short-distance dynamics. The contribution from the momentum larger than $m_b$ is involved in the hard scattering kernel while the contribution from the momentum lower than $m_b$ is contained in the light-cone distribution amplitude. As we will see, in the modified PQCD approach, the scales become rich.

Before the discussion of the modified PQCD approach, it is necessary to consider the most general case of PQCD method in exclusive process. According to [2], a physical quantity $M$ is given in terms of the hadronic wave function and the hard scattering kernel in general:

$$M = \int [dx][d^2k_T] \prod_i \psi_i(x_i,Q,k_{Ti})T(x,Q,k_T)$$  \hspace{1cm} (2)

where $Q \gg \Lambda_{QCD}$ is the large scale involved in a process. If the transverse momentum $k_T$ can be negligible in $T(x,Q,k_T)$, the above formula can be simplified as

$$M = \int [dx] \prod_i \phi_i(x_i)T(x,Q)$$  \hspace{1cm} (3)

In the above equation, we have used the relation

$$\phi(x,Q) = \int d^2k_T\psi(x,Q,k_T)$$  \hspace{1cm} (4)

The BBNS approach is just the application of the formula (3) in B decays under the assumption that the form factor is soft dominated.
If the end-point singularity can not be removed in the convolution, the transverse momentum in hard kernel cannot be neglected. As we discussed in Sect. 1, a reliable treatment of the transverse momentum effects must consider the mechanism of Sudakov suppression. Once the effects of transverse momentum and Sudakov form factor are taken into account, a transverse b-space factorization formula \[7\] will be obtained. In B decays, this modified PQCD factorization formula is the convolution of both the longitudinal momentum fraction and the transverse impact parameter \(b\):

\[
M(B \to \pi_1 \pi_2) = \int [d\xi][d^2b] P_B(\xi, Q, b_1, \mu) P_{\pi_1}(u, Q, b_2, \mu) \cdot P_{\pi_2}(v, Q, b_3, \mu) T(\xi, u, v, Q, b_1, b_2, b_3)
\]  

where \(b\) is the conjugate variable of the transverse momentum \(k_T\) and \([d\xi] = d\xi du dv\), \([d^2b] = d^2b_1 d^2b_2 d^2b_3\).

In Eq.(5), the function \(P(x, Q, b)\) is:

\[
P(x, Q, b) = \int d^2k_T e^{-ik_T \cdot b} \psi(x, Q, k_T) = e^{-\left[s(x, Q, b) + s(\bar{x}, Q, b)\right]} \phi(x, \frac{1}{b})
\]  

It includes all leading logarithmic enhancement at large \(b\) which has been included in Sudakov form factor. For the light meson, Sudakov form factor suppresses the large \(b\) contribution, so it selects component of the light meson wave function with small spatial extent. Thus, \(\phi(x, \frac{1}{b}) \approx \phi(x, Q)\).

The hard kernel \(T(\xi, Q, b)\) is the Fourier transformation of the hard scattering kernel defined in momentum space

\[
T(\xi, Q, b) = \int [d^2k_T] e^{-ik_T \cdot b} T(\xi, Q, k_T)
\]  

The evolution of the function \(P\) satisfies

\[
\mu \frac{d}{d\mu} P(x, Q, b, \mu) = -2\gamma_q P(x, Q, b, \mu)
\]

where \(\gamma_q\) is the quark anomalous dimension in axial gauge.

Unlike the BBNS approach, there are many scales in the modified PQCD approach, such as \(uvQ^2, \frac{1}{b}\), etc. In the modified PQCD approach, the scale parameter \(\mu\) should take the
largest value of them. The renormalization group equation must be used to eliminate the large logarithm between many scales.

From the above discussion, we may expect that the BBNS approach and the modified PQCD approach should be equivalent if the hard kernel \(T\) is fully hard dominated. It is really so. For the case that the scattering kernel \(T\) in Eq.(5) is concentrated near \(b \sim \frac{1}{Q}\), the Sudakov form factor is unity and the function \(\mathcal{P}(x, Q, b)\) is replaced by distribution amplitude \(\phi(x, Q)\). The modified PQCD formula of Eq.(5) will be reduced into the BBNS factorization formula.

If the contributions of \(O(k_T^2 \sim \Lambda_{QCD} M_B)\) in the scattering kernel \(T(x, Q, k_T)\) in Eq.(5) are important so that the end-point singularity of \(T(x, Q, k_T)\) at \(k_T = 0\) can not be removed in the convolution, the two approach will be different. In \([7]\), an intuitive argument hold that summed to all orders, the two approach are equivalent at leading power in \(1/Q^2\). However, at finite order, their difference is unavoidable. The proof of BBNS approach needs the heavy quark limit. While the modified approach enlarges the range of PQCD down to accessible energies with the help of Sudakov suppression.

3. Sudakov double logarithm and resummation

Sudakov form factor comes from the summation of the double logarithms to all orders. In QED, the vertex correction in Feyman gauge gives rise to Sudakov double logarithm \(\ln^2 \frac{Q^2}{m_e^2}\) where \(Q\) is the large energy scale and \(m_e\) the electron mass. The summation of the Sudakov double logarithms to all orders is the exponential of the one-loop result. However, the non-abelian theory QCD is more complicated than QED. First, there is gluon self interaction in QCD which makes the coupling constant large at low energy, so it is necessary to consider the next-to-leading-log approximation. Second, the light quark mass is smaller than the QCD scale \(\Lambda_{QCD}\), it cannot be taken as the infrared regulator. The technic to perform the summation of double logarithms to all orders in QCD is called resummation. In this case, it is Sudakov resummation \([9, 10]\). Although the resummation technic is fruitful and has been known for more than ten years, its intricacy makes it difficult to understand. To apply the resummation into a new process is more difficult. So, it is necessary to discuss the main idea of the Sudakov form factor without involving the intricate technic.

The Sudakov double logarithm is produced through the overlap of collinear and soft di-
vergence. The transverse momentum degree is used to regulate the infrared divergence. The calculation is performed in the transverse configuration $b$ space instead of the momentum $k_T$ space. The advantage of using the transverse $b$ space is analyzed in [11]. The momentum conservation is automatically maintained in $b$ space and it is not necessary to make any further assumptions about the transverse momentum $k_T$ in higher orders. Moreover, in momentum $k_T$ space, it is difficult to perform the next-to-leading-log approximation.

The collinear divergence in the massless limit depends on the choice of gauge. In axial gauge, or say physical gauge $n \cdot A = 0$, the gluon propagator $D_{\mu\nu}$ satisfies $n^\mu D_{\mu\nu} = 0$. So the non-factorizable collinear divergence diminishes in axial gauge. This simplifies the analysis of factorization and Sudakov form factor. We will choose the axial gauge in this section for discussion. However, the obtained Sudakov form factor is gauge independent. A recent literature about this conclusion can be found in [12].

In axial gauge, the Sudakov double logarithm occurs only in the two-particle reducible diagrams. Thus, the Sudakov form factor is included in each wave function itself, i.e., it is universal, process-independent. The double logarithms at $\mathcal{O}(\alpha_s)$ are given by [10]

$$I = -\frac{C_F}{4\pi^3} \int_{l_T < Q} \frac{d^2l_T}{l_T^2} g_s^2(l_T)(e^{i l_T \cdot b} - 1) \int_Q^{l_T} \frac{dl^+}{l^+}$$

$$\approx -\frac{2C_F}{\beta_1} \ln\left(\frac{Q^2}{\Lambda^2_{QCD}}\right) \ln\left[\frac{\ln(\frac{Q^2}{\Lambda^2_{QCD}})}{\ln(\frac{1}{b^2 \Lambda^2_{QCD}})}\right]$$

In the above equation, we have chosen the light-cone variable and $\beta_1 = \frac{33-2n_f}{12}$. The factor of $e^{i l_T \cdot b}$ comes from the Fourier transformation from the transverse momentum space to $b$-space. The occurrence of double logarithm requires two condition: (1) two scales, $Q \gg \frac{1}{T} \gg \Lambda_{QCD}$; (2) the overlap of collinear and soft regions. In Eq.(9), the lower limit of $l^+$ must be in the soft region. To sum the leading and next-to-leading logarithms to all order, it needs to solve the renormalization group equation below [9, 10]:

$$Q \frac{\partial}{\partial Q} P(x, Q, b) = [K(b\mu) + \frac{1}{2} G(\frac{xQ}{\mu}) + \frac{1}{2} G(\frac{Q}{\mu})]P(x, Q, b)$$

where the functions of $K$ and $G$ satisfies

$$\mu \frac{d}{d\mu} K = -\gamma_K, \quad \mu \frac{d}{d\mu} G = \gamma_K$$

7
where $\gamma_K$ is anomalous dimension. The functions $K$ and $G$ only depend on one scale: $K$ is independent of large scale $Q$ and $G$ is independent of scale $\frac{1}{b}$. The scale $\mu$ in $K$ and $G$ takes different value: $\mu \sim \mathcal{O}(\frac{1}{b})$ in $K$; $\mu \sim \mathcal{O}(Q)$ in $G$. The appearance of different scales can be compared with the one scale case in BBNS approach. Solve the above differential equations, one will obtain a Sudakov form factor in distribution function

$$\mathcal{P}(x,Q,b) = e^{-[s(x,Q,b)+s(\bar{x},Q,b)]\phi(x,\frac{1}{b})}$$ (12)

The definition of $\mathcal{P}$ function is shown in Eq.(6).

The Sudakov form factor $e^{-s}$ falls off quickly in large $b$, or soft region and vanishes as $b > 1/\Lambda_{QCD}$. Therefore it suppresses the long-distance contribution, which is called Sudakov suppression. The behavior of Sudakov form factor with the variable $b$ is plotted in Figure. The physical reason is that an isolated colored parton tends to radiate gluons. As $b$ increases, the color dipole associated with quark and antiquark becomes more isolated, and they would have more tendency to radiate gluons. In exclusive process, however, the gluon radiation is forbidden by definition. So the process with large $b$ separation will be suppressed. Sudakov form factor manifests this phenomena in theory. For small $b$, Sudakov form factor provides no suppression, this region is dominated by hard scattering. In summary, the Sudakov effects make small $b$ contributions dominant. By including the Sudakov effects the effective scale of the subprocess is $\mathcal{O}(\Lambda_{QCD}Q)$. As we have discussed, the Sudakov form factor is universal. This simplifies the application of this effects in exclusive processes.

4. Sudakov effects in BBNS approach

In $B \to \pi\pi$ decays, the two light pions carry the energy of $\frac{m_B}{2}$ and moves fast away from the decay point. In [3], the authors argue that for realistic $b$ quark mass, Sudakov from factor is not sufficiently effective. As discussed in the last section, the Sudakov form factor is a perturbative result. It plays a role in presence of the large scales $Q \gg k_T \gg \Lambda_{QCD}$. The fact that the Heavy Quark Effective Theory works very well and some successful prediction of PQCD in inclusive $B$ decays implies that $m_b$ scale is large enough to ensure the perturbative analysis. Moreover, BBNS approach underlies the assumption of the heavy quark limit. In this limit, the effectiveness of the perturbative Sudakov form factor is obvious.
For light pion meson, the Sudakov form factor is known. Its explicit form can be found in [10, 13]. For simplicity, we do not present it here again. For the heavy meson, such as B meson, the heavy quark carries the most energy while the light quark carries the momentum about $\Lambda_{QCD}$. The wave function of B meson is soft dominated. For $b$ quark, there is no collinear divergence thus the Sudakov form factor is absent for it. For the light quark in B meson, its longitudinal momentum mostly lies in the soft region. It seems that there is no overlap of the collinear and soft regions. In general, the soft dominance does not exclude the possibility that the light quark may have the large longitudinal momentum although this possibility is very small. For the case that the longitudinal momentum of the light quark in B meson is small, i.e., $\xi$ is small, Sudakov form factor contributed by the light quark is $e^{-s(\xi,m_B,b)} \approx 1$, (see Figure 1), here $\xi$ is the momentum fraction of the light quark. Because the large possibility is that the light quark of B meson only carries small momentum which is around the order of $\Lambda_{QCD}$, $\xi$ is dominantly distributed in the small region around $\Lambda_{QCD}/m_B$. The possibility of large $\xi$ is seriously suppressed by the B meson wave function. Thus the Sudakov form factor for B meson only gives small effect (Mostly it approximately equals to 1). In this paper, we give a Sudakov form factor for the light quark in B meson for general consideration as in [8]. The numerical results in our study show that the difference between the cases with and without Sudakov form factor for B meson is less than $10^{-2}$ because of the soft dominance of B meson wave function.

![Figure 1: $b$-$\xi$ dependence of $e^{-s}$](image)

Now it’s time to discuss the Sudakov effects in $B \to \pi \pi$ decays. We restrict our discussion
in the process analyzed in [3]. The contributions to the $B \to \pi$ transition form factor and the annihilation diagram are not discussed here. We will give a detailed study about them in the next research. As in [3], we discuss the vertex correction, penguin correction and the hard spectator scattering, see Figure 2.

Figure 2: Order $\alpha_s$ corrections to the hard scattering kernels $T^I_i$ and $T^{II}_i$. (a)-(d): vertex corrections, (e) and (f): penguin correction, (g) and (h): hard spectator scattering.

For the vertex corrections, the soft and collinear divergences cancel for twist-2 and symmetrical twist-3 distribution amplitudes in the collinear limit. If considering the transverse momentum effects, these non-factorizable radiative corrections contribute subleading logarithms in axial gauge. The leading contribution is double logarithms which have been summed to a Sudakov form factor. The subleading logarithms come from the soft gluon where all the four components of its momentum becomes soft. This soft contributions cancel in the collinear limit required by the factorization theorem. As pointed out before, Sudakov form factor suppresses the large $b$ region and makes the dominate contribution come from the small $b$ region which is near the collinear limit. In [10], the authors study this soft gluon contribution with the transverse momentum effects in hadron-hadron scattering. The non-factorizable soft contributions are summed to all orders by using the renormalization group equation. Their conclusion is that the soft contribution is small and can be neglected. So the Sudakov effects in the vertex corrections is expected to be small, and this conclusion is also
applicable for the penguin corrections.

The hard spectator scattering is depicted in Figure 2 (g) and (h). The partons in meson has the transverse momentum as well as the longitudinal momentum. Compared to the collinear limit, the momentum of partons in meson (with momentum $P_1$) changes to

$$k_1 = uP_1 + k_{T1}, \quad k_2 = \bar{u}P_1 - k_{T1}$$

where $u$ and $\bar{u}$ denote the longitudinal momentum fraction. Our form is slightly different from that in [6]. Our treatment corresponds to the case that the meson is on-shell and the parton is slightly off-shell. The off-shellness of the parton is proportional to $k_{T1}^2$.

For the hard spectator scattering, the contribution of the operator $(S - P) \otimes (S + P)$ insertion vanishes in the total hard scattering. The contribution of the operator $(V - A) \otimes (V + A)$ insertion is equal to that of $(V - A) \otimes (V - A)$. So it only needs to consider the contribution of $(V - A) \otimes (V - A)$ operator insertion.

The twist-3 distribution amplitude contributes power correction. But at the realistic $m_b$ energy scale, the power correction parameter $r_\chi = \frac{2m_b^2}{m_b(m_u + m_d)} \sim O(1)$ is not small. So the twist-3 contribution should be considered in B decays. The twist-2 and twist-3 distribution amplitudes are defined by

$$<\pi^-(P)|\bar{d}_\alpha(x)u_\beta(y)|0> = \frac{iF_\pi}{4N_c} \int_0^1 du e^{i(uP \cdot x + \bar{u}P \cdot y)}[\gamma_5 P\phi_\pi(u)$$

$$+ \mu_\pi \gamma_5 \phi_P(u) - \mu_\pi \sigma^{\mu\nu} \gamma_5 P_\mu (x - y)_\nu \phi_\sigma(u)]_{\beta\alpha}$$

where $\mu_\pi = \frac{m_\pi^2}{(m_u + m_d)}$. $\phi_\pi$, $\phi_P$, and $\phi_\sigma$ are the twist-2 and twist-3 distribution amplitudes, respectively. In the asymptotic limit, $\phi_\pi(u) = 6u\bar{u}$, $\phi_P(u) = 1$ and $\phi_\sigma(u) = 6u\bar{u}$.

First we discuss the hard spectator scattering contribution in BBNS approach. The formula is derived in $k_T$ space and remains the transverse momentum at the beginning. About the coordinate variable $(x - y)_\nu$ in Eq.(14), we make the transformation to project it into the momentum space as adopted in [3]:

$$(x - y)_\nu e^{-i(x - y) \cdot P} = i \frac{\partial}{\partial P^\nu} e^{-i(x - y) \cdot P}$$

With this projection, the hard spectator scattering contribution in Figure 2 (g) and (h) is formulated in transverse momentum space:

$$S_{g+h} = \frac{-iBf_\pi^2}{4N_c^2 g_s^2 C_F} \int [d\xi] [d^2k_T]$$

where
\[
\cdot \left[ \frac{um_B^2 \phi_B(\xi) \phi_\pi(u) \phi_\pi(v)}{[\xi um_B^2 + (k_T - k_{T1})^2][-uv m_B^2 + (k_T - k_{T1} + k_{T2})^2]} \right. \\
\left. + 2 \mu_\pi m_B^5 \frac{uv \phi_B(\xi) \phi_\pi(u) \phi_\pi(v)}{[\xi um_B^2 + (k_T - k_{T1})^2][-uv m_B^2 + (k_T - k_{T1} + k_{T2})^2]} \right]
\]

We have assumed the momentum fraction \( \xi \) in B meson is small and the distribution amplitudes are symmetric. Neglecting the transverse momentum in both the numerator and the denominator will give a simplified formula for the hard spectator scattering:

\[
S_{g+h} = \frac{if_B f_F^2}{4N_c^2} g_s^2 C_F \int d\xi dudv \frac{\phi_B(\xi) \phi_\pi(u) \phi_\pi(v)}{\xi uv} + \frac{2 \mu_\pi \phi_B(\xi) \phi_\pi(u) \phi_\pi(v)}{m_B^2} \frac{\phi_\pi(v)}{\xi u^2 v}
\]  

(17)

This formula is consistent with the corresponding one given in [5]. The scale \( \mu \) in \( g_s \) is chosen as \( m_b \). For twist-2 distribution amplitude, there is no end-point singularity. When \( u \to 0 \), the twist-3 contribution will lead to end-point singularity. The physical reason is that the virtual gluon approaches to the mass shell. This is a soft logarithmic divergence. As discussed in Sect. 1, the occurrence of end-point singularity is the result of neglecting the transverse momentum in the denominator.

In the modified PQCD approach, the partonic transverse momentum is retained without assuming \( k_T^2 \ll \xi um_b^2, uv m_b^2 \). The final formula contains the convolutions of the longitudinal momentum fraction and the transverse impact parameter \( b \),

\[
S_{g+h} = \frac{-if_B f_F^2}{4N_c^2} g_s^2 C_F \int d\xi dudv \int bdbb_2db_2
\]

\[
\cdot \left\{ um_B^4 P_B(\xi, b) P_\pi(u, b_2) P_\pi(v, b_2) K_0(-i \sqrt{uv m_B b_2}) \right. \\
\left. -2uv \mu_\pi m_B^5 P_B(\xi, b) \frac{P_\pi(u, b)}{6} P_\pi(v, b_2) \frac{b_2}{-2i \sqrt{uv m_B}} K_{-1}(-i \sqrt{uv m_B b_2}) \right\}
\]

(18)

where \( K_i \) and \( I_i \) are modified Bessel functions and \( i \) is its order.

This formula is more complicated than the result of BBNS approach. One can check that the result in the right hand side of Eq.(18) is finite, and there is no divergence in it. The modified QCD formula is self-consistent and contains no arbitrary phenomenological parameter except for the input distribution amplitudes.

In the numerical calculation, the distribution amplitudes of pions for twist-2 and twist-3 are taken as their asymptotic limit. The distribution amplitude for B meson is \( \phi_B(x, b) = ... \)
\[ N_B x^2 (1-x)^2 \exp \left( -\frac{M_B^2 x^2}{2\omega_B^2} - \frac{1}{2} (\omega_B b)^2 \right) \] where \( \omega_B = 0.3 \text{GeV} \), \( N_B \) is the normalization constant.

The QCD scale \( \Lambda_{QCD} = 0.3 \text{GeV} \), and the other input parameters are taken as follows: \( f_B = 0.19 \text{GeV} \), \( f_\pi = 0.13 \text{GeV} \), \( F_0^{B\pi}(0) = 0.3 \), \( m_B = 5.27 \text{GeV} \), \( \mu_\pi = 1.2 \text{GeV} \).

In our numerical result, the twist-3 contribution is not important in hard spectator scattering. So, there is only a little improvement in numerical value compared with the former calculations in BBNS approach \cite{3, 8, 14}. We will not present the full calculation of \( B \to \pi \pi \) decays because it is unnecessary. The comparison of the prediction for the hard spectator scattering in both BBNS approach and the modified QCD approach is presented below.

Define \( f \) as the value of the hard spectator scattering contribution divided by the lowest order result \( f \equiv \frac{S_{+h}}{f_B f_\pi F_0^{B\pi} m_B^2} = f_2 + f_3 \) where \( f_2, f_3 \) represent the contribution of twist-2 and twist-3 terms. The numerical result (For \( (V-A) \otimes (V-A) \) operator insertion) is:

**Twist-2:** In BBNS approach, \( f_2 = 0.043 \); In the modified PQCD approach, \( f_2 = 0.057 + i0.0037 \);

**Twist-3:** In BBNS approach, \( f_3 \) cannot be calculated, it is expressed in terms of phenomenological parameters \( \rho_H \) and \( \phi_H \) \cite{4}.

\[
\begin{align*}
\frac{\pi \alpha_s C_F f_B f_\pi}{m_B^2 F_0^{B\pi} N_c^2} \int_0^1 \frac{d\xi}{\xi} \phi_B(\xi) \int_0^1 \frac{dx}{x} \phi_\pi(x) & \frac{2\mu_\pi}{m_b (1 + \rho_H e^{i\phi_H})} \ln \frac{m_B}{\Lambda_h}.
\end{align*}
\]

where \( \rho_H \leq 1, \Lambda_h = 0.5 \text{GeV} \); In the modified PQCD approach, \( f_3 = 0.0166 - i0.0144 \); Compare the two results, we can get \( \rho_H = 0.97 \), which is within the constraint of \( \rho_H \leq 1 \), and the strong phase is very large, it is \( \phi_H = -83.7^\circ \).

5. Conclusions and discussions

Through the exchange of the gluons, the partons in hadron carries the transverse momentum \( k_T \). Its effects is important in the end-point region. Neglecting it will lead to the end-point singularity in BBNS approach. The problem of end-point singularity can be reliably treated in the modified PQCD approach. Retaining the partonic intrinsic transverse momentum and with the help of Sudakov form factor, the modified PQCD approach is a self-consistent, model-independent framework. For the vertex corrections and the penguin corrections, the end-point singularity in the hard kernel is cancelled in the convolution and Sudakov suppression gives little effect. The separation of long- and short-distance dynamics is good enough to ensure the
validity of factorization. BBNS approach provides a successful, easy-to-do framework for these diagrams. For the hard spectator scattering, if neglecting the partonic intrinsic transverse momentum the end-point singularity in the hard scattering kernel can not be cancelled in the twist-3 case, which implies that the contributions from the end-point region are important and such amplitude can not be analyzed at a fixed order in PQCD. In this case one has to include the transverse momenta of partons and Sudakov form factor in order to proceed at a fixed order in PQCD. Sudakov form factor can suppress the soft contribution and make the hard contribution dominant. In this case, Sudakov correction is important. Our numerical results show that Sudakov correction is small at leading twist level and important at twist-3 level. The twist-3 contribution in the hard spectator scattering is non-negligible, but not dominant.

In [3], it is argued that the transverse momentum effect is power suppressed so that it can be neglected. This is valid only in the $m_b \to \infty$ limit. Actually, in the loop corrections, the large logarithms such as $\ln^2 \frac{Q^2}{k_T^2}$, $\ln \frac{Q^2}{k_T^2}$ etc will occur. In the tree level, the hard kernel contains the terms such as $\frac{1}{u + p_B^2 + k_T^2}$, $\frac{1}{u + p_B^2 + k_T^2}$. Dropping the transverse momentum, or set it to zero, will lead to and end-point singularity which will destroy the factorization theorem. This is the reason to incorporate the Sudakov effects. Including the Sudakov effects, the naive power counting in [3] will be modified. The contribution of the end-point region is smeared by the transverse momentum effects. So the assumption that the $B \to \pi$ transition form factor is dominated by soft end-point interaction may be questionable. It can be hard momentum transfer dominant. Recently a complete PQCD method was applied to the study of two-body $B$ meson decays of $B \to \pi\pi$, $K\pi$, and so on, including completely perturbative treatment of $B \to \pi$, $B \to K$ transition form factors and annihilation diagrams [15]. Some interesting results have been obtained. However, this approach has been upgrading continuously. It seems that there is still a bit of long way to go before getting final success. A systematic analysis about the $B \to \pi$ transition form factor, the annihilation diagram and the radiative corrections is still needed. Except these problems, another important subject is to understand the factorization theorem in B decays. Up to now, some works along this direction has been done [3, 16]. More detailed works on the proof of factorization theorem in B decays to all orders is still highly needed. Without the proof of factorization theorem, any formulas can only be regarded as a “model”. In one word, it is desirable to carefully consider the factorization
in B decays.

Acknowledgment

Two of the authors (Z. Wei, M. Yang) would like to thank Prof. Hsiang-Nan Li for his useful discussion and his interesting talks. Wei also expresses his gratitude to Prof. Hai-Yang Cheng for his warmful discussions about the BBNS approach. This work is supported in part by National Natural Science Foundation of China and the Grant of State Commission of Science and Technology of China. M. Yang thanks the partial support of the Research Fund for Returned Overseas Chinese Scholars.

References

[1] F. Farrar and D. Jackson, Phys. Rev. Lett.\textbf{43} (1979) 246; S. Brodsky and G. Lepage, Phys. Rev. Lett.\textbf{43} (1979) 545, Phys. Lett. B\textbf{87} (1979) 359; A. Efremov and A. Radyushkin, Phys. Lett. B\textbf{94} (1980) 245; A. Duncan, A. Mueller, Phys. Rev. D\textbf{21} (1980) 1636.

[2] G. Lepage and S. Brodsky, Phys.Rev.D\textbf{22} (1980) 2157.

[3] M. Beneke, G. Buchalla, M. Neubert, C.T. Sachrajda, Phys.Rev.Lett.\textbf{83} (1999) 1914-1917; Nucl.Phys.B591 (2000) 313-418.

[4] M. Bauer, B. Stech and M. Wirbel, Z.Phys.C \textbf{34}, 103 (1987); M. Wirbel, B. Stech and M. Bauer, Z.Phys.C \textbf{29}, 637 (1985).

[5] D. Du, D. Yang, G. Zhu, Phys.Lett.B\textbf{509} (2001) 263; Phys.Rev.D\textbf{64} (2001) 014036.

[6] M. Beneke, G. Buchalla, M. Neubert, C.T. Sachrajda, hep-ph/0104110.

[7] H. Li, G. Sterman, Nucl.Phys.B\textbf{381} (1992) 129-140; G. Sterman, P.Stoler, hep-ph/9708353.

[8] H. Li, H. Yu, Phys.Rev.Lett.\textbf{74} (1995) 4388; T. Yeh, H. Li, Phys.Rev.D\textbf{56} (1997) 1615-1631.

[9] J. Collins, D. Soper, Nucl.Phys.B\textbf{193} (1981) 381.
[10] J. Botts, G. Sterman, Nucl.Phys.B\textbf{325} (1989) 62.

[11] Y. Dokshitzer, D. D’Yakonov, S. Troyan, Phys.Lett. B\textbf{488} (2000)

[12] H. Li, Phys.Rev.D\textbf{55} (1997) 105.

[13] H. Li, Phys.Rev.D\textbf{52} (1995) 3958.

[14] T. Muta, A. Sugamoto, M. Yang, Y. Yang, Phys.Rev.D\textbf{62} (2000) 094020; D. Du, D. Yang, G. Zhu, Phys.Lett. B\textbf{488} (2000) 46.

[15] Y. Keum, H, Li, A. Sanda, Phys.Lett.B\textbf{504}(2001)6; C. Chen, H, Li, Phys.Rev.D\textbf{63} (2001) 014003; Y. Keum, H, Li, A. Sanda, Phys.Rev.D\textbf{63} (2001) 054008; Y. Keum, H. Li, Phys.Rev.D\textbf{63} (2001) 074006; C. Lu, K. Ukai, M, Yang, Phys.Rev.D\textbf{63} (2001) 074009.

[16] H. Li, Phys.Rev.D\textbf{64} (2001) 014019; C. Bauer, D. Pirjol, I. Stewart, hep-ph/0107002.