A quantum model for the magnetic multi-valued recording

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Abstract

We have proposed a quantum model for the magnetic multi-valued recording in this paper. The hysteresis loops of the two-dimensional systems with randomly distributed magnetic atoms have been studied by the quantum theory developed previously. The method has been proved to be exact in this case. We find that the single-ion anisotropies and the densities of the magnetic atoms are mainly responsible for the hysteresis loops. Only if the magnetic atoms contained by the systems are of different (not uniform) anisotropies and their density is low, there may be more sharp steps in the hysteresis loops. Such materials can be used as the recording media for the so-called magnetic multi-valued recording. Our result explained the experimental results qualitatively.
Magnetic thin films with perpendicular “easy-axis” anisotropy have attracted much attention these years both experimentally and theoretically since such systems have good potential to be taken as recording media [1]-[4]. In practice, many efforts have been made to increase the recording density of the devices, and several proposals have been made to achieve this end. However, the recording density has already come to a limit of the conventional scheme so that one must find new approaches. Recently, experimental studies have been made on the magnetic multi-valued (MMV) recording which is believed to be the next strategy of high density recording [1]-[2]. The key point is that there should exist more metastable states which are stable enough to record a message. Thus the media for MMV recording must possess several sharp sub-steps in its hysteresis loop. Experimentally, such a phenomenon had been confirmed in some kind of magnetic layered systems [1] -[2]. However, the theoretical origin is not yet very clear.

On theoretically side, a quantum theory for the coercive force of a magnetic system [5] has been developed based on some previous works [6]-[8]. In such a quantum approach, the concept of the metastable state was adopted, and the magnetic excitation gap was defined to be the order parameter to monitor the stability of such metastable states. The coercive force can be determined by the condition that the gap comes to zero [5]. The so-called “capping effect” in a double-film structure [4] has been explained successfully by the quantum theory [5].

The present work is devoted to proposing a theoretical explanation for the MMV recording which is confirmed by a randomly magnetic thin film [2]. We first propose a model Hamiltonian for such a system in which the magnetic atoms are distributed randomly, then studied its hysteresis loop by the quantum method which is proved to be exact in such case. The results are averaged for samples finally to overcome the fluctuations of the distribution. We show that: 1) When the magnetic atoms contained by the system are of uniform single-ion anisotropy, there should be only one sharp step in the hysteresis loop. 2) When the magnetic atoms are of different anisotropies but the density is higher than a critical value, the hysteresis loop is highly smoothed and there are no obvious steps. 3) only when the magnetic atoms are of different anisotropies and their density is low, more sharp steps can be clearly observed in the hysteresis loop.

The Hamiltonian is given as:

\[
H = \frac{1}{2} \sum_{i,j} J_{ij} \mathbf{S}_i \cdot \mathbf{S}_j - \sum_i D_i (S_{iz})^2 - h \sum_i S_{iz}^z
\]  

(1)

where \{i\} are the lattice sites randomly distributed in the x-y plane in which the magnetic atoms are occupied. \{D_i > 0\} are the single-ion anisotropies. An external field is applied along the z axis. \(J_{ij} = J\) is the exchange constant and only the nearest-neighbor interaction is considered. In real magnetic materials, the single-ion anisotropic constant \(D\) is usually much smaller than the exchange constant \(J\).

Following Refs. [5],[8], a local coordinates (LC) system \((x_i, y_i, z_i)\) can be introduced to the Hamiltonian. The \(x_i\) and \(z_i\) axes in the LC system are rotated by an angle \(\theta_i\) which may be different from site to site, while the \(y_i\) axis is not rotated. It is helpful to
apply a Bose transformation such as Holstein-Primakoff (H-P) [9], Dyson-Maleev (D-M) [10]–[11] and the complete Bose transformation (CBT) [12] to study the spin systems. In a harmonic approximation, these transformations are the same. However, we will use the CBT in this paper because it can present high-order terms correctly.

After the LC transformation and the CBT, the Hamiltonian becomes:

\[
\tilde{H} = U_0 + H_1 + H_2 + \cdots
\]

where

\[
U_0 = \text{const.} - \frac{1}{2} \sum_{i,j} J_{ij} \cos(\theta_i - \theta_j) S_i S_j - h \sum_i \cos \theta_i S_i
\]

\[+ \frac{1}{2} \sum_i D_i (2S_i - 1) S_i \cos^2 \theta_i\]

\[H_2 = \sum_{i,j} F_{i,j}(\theta) a_i^+ a_j + \sum_{i,j} G_{i,j}(\theta) (a_i^+ a_j^* + a_i a_j).\]

The coefficients are

\[F_{ii}(\theta) = -D_i (S_i - \frac{1}{2}) (\sin^2 \theta_i - 2 \cos^2 \theta_i) + \sum_j S_j J_{ij} \cos(\theta_i - \theta_j) + h \cos \theta_i,\]

\[F_{i,j}(\theta) = -\frac{1}{2} J_{i,j} \sqrt{S_i S_j} [1 + \cos(\theta_i - \theta_j)], \quad i \neq j,\]

\[G_{ii}(\theta) = -\frac{1}{4} \sqrt{2S_i (2S_i - 1)} D_i \sin^2 \theta_i,\]

\[G_{i,j}(\theta) = \frac{1}{4} J_{i,j} \sqrt{S_i S_j} [1 - \cos(\theta_i - \theta_j)], \quad i \neq j.\]

\{\theta_i\} can be obtained by minimizing the ground state energy: \(dU_0/d\theta_i = 0\), which yield:

\[\sum_j J_{ij} S_j \sin(\theta_i - \theta_j) + h \sin \theta_i + D_i (2S_i - 1) \sin \theta_i \cos \theta_i = 0.\]

Equations above are the same as the condition of \(H_1 = 0\). \(H_2\) can be diagonalized by a Bogolyubove transformation. So, we have:

\[\tilde{H} \simeq U'_0 + \sum_i \epsilon_i a_i^+ a_i + \cdots\]

In general, the nonlinear equations [9] may have many solutions in a definite external field \(h\). For each solution, the spin state may be a metastable one only when the excitation energy \(\{\epsilon_i\}\) based on such a solution are all positive. In another word, the excitation should have a positive gap \(\Delta(h) > 0\), otherwise, such a spin state is not stable. So, if every metastable states of the systems have been investigated when one altering the external field from positive to negative, the hysteresis loop can be obtained while the system transits from one metastable states to another one.
Before carrying out a general investigation, we will first study some particular systems which are illustrated in figure 1 in order to get some informations. In figure 1 and other figures, we suppose that the magnetic atoms represented by the circles have the anistropy $\tilde{D}_1/J = D_i(2S_i - 1)/J = 0.1$, and those by the triangles have the anistropy $\tilde{D}_2/J = D_i(2S_i - 1)/J = 0.02$. It is also supposed that $S_1 = S_2$ for convenince. It should be noted that such systems have a character that the magnetic atoms are all coupled together since we have supposed that only the nearest-neighbour interaction is considered.

Eqs. (9) may have two kinds of solutions: (1) The trivial solutions: $\{ \theta_i = 0, \text{ or } \pi \}$. (2) The non-trivial solutions: $\{ \theta_i \neq 0, \text{ or } \pi \}$. The number of solutions is certainly very big, and any of them may be a metastable state if the elementary excitations have a positive gap. However, after the numerical calculations for the systems which are illustrated in figure 1, we find that only two solutions among so many ones can be metastable within a definite region of external field. They are: (1) $\{ \theta_i = 0 \}$ (2) $\{ \theta_i = \pi \}$. Other spin configurations are all unstable. Of cause, we can not give a vigorous proof for that in a general case, but one can understand the above numerical results as follows. First, the “easy-axis” of each magnetic atom is along the $z$ axis so that the spins all prefer to paralell or antiparallell the $z$ axis, and there are no such atoms which have “in-plane easy-axis” or “easy-plane” anisotropies, as the result, the non-trivial spin configuration $\{ \theta_i \neq 0, \pi, i = 1, 2, \cdots \}$ is not likely to appear. Second, since the exchange interaction $J$ is usually much larger than the single-ion anistropy $D$ in real material, the spins are willing to paralell with each other. Since the spins are all coupled together in the systems studied (figure 1), such spin configurations that some spins are up while others are down can not be stable. This is an explanation rather than a proof, but at least, one may understand the numerical results that only two metastable states may exist for a system in which the spins are coulped together.

Applying the solution $\{ \theta_i = 0 \}$ into the Hamiltonian, we find that the excitation energy $\{ \epsilon_i \}$ can be obtained by diagonalizing the matrix $\{ F_{ij} \}$ since the matrix $\{ G_{ij} \}$ is zero. In the Appendix, we prove that $\{ \epsilon_i \}$ are the rigorous excitation energies of the Hamiltonian in this case although they seem to be obatained by a harmonic approximation. Thus we get exactly

$$\Delta(h) = Min[\epsilon_i],$$

and the coercive force of the model can be determined by $\Delta(h_c) = 0$.

Following are the numerical results we obtained.

**Case 1.** Only kind of magnetic atoms are distributed in the systems: $S_i = S$ and $(2S_i - 1)D_i = \tilde{D}$. In this case, the matrix $\{ F_{ij} \}$ can be rewritten into the following form:

$$\{ F_{ij} \}_{\theta_i=0} = \tilde{D} + h + \{ F'_{ij} \}$$

One can check at once that the matrix $\{ F'_{ij} \}$ has the lowest eigenvalue 0. Then the gap will be:

$$\Delta(h) = \tilde{D} + h$$
The coercive force of the system can derived: $h_c = -\tilde{D}$.

Thus, no matter how the atoms are distributed in the pattern, if there are only one kind of magnetic atoms, the coercive forces are always the same. In practice, one makes the systems amorphous to minimize the domain’s size to improve the recording density. The above results can warrant that every domains possess the same coercive force so that they can be used in a same way.

**Case 2:** The system contains more than one kind of magnetic atoms. The results for the systems shown in figure 1 are listed in Table 1, from which we can find that the coercive forces $h_c$ are strongly dependent on the distributions of the magnetic atoms. Generally, the coercive force turns smaller when the small-anisotropy magnetic atoms are relatively increased in the system. But the results are different for the systems in which the magnetic atoms are coupled differently although the numbers of the two kinds of atoms are the same.

We will discuss the general behaviour of our random model’s hysteresis loop. Throughout this paper, a $10 \times 10$ lattice is discussed just to illuminate the main physical idea, and the samples are averaged in the end to take account of the fluctuations.

A typical distribution is illustrated in figure 2. One may find that there are many so-called “Isolated Islands” (II) in the patterns. Within each II, the magnetic atoms are coupled together so that they can be treated following the method mentioned above. However, the coupling between those IIs is zero. So, such IIs must be able to be considered independently. Actually, the matrix $F_{ij}$ has the following form:

$$
\begin{bmatrix}
C^{(1)} & \vdots & \vdots & \vdots \\
C^{(2)} & C^{(3)} & \vdots & \vdots \\
\vdots & \vdots & \ddots & \vdots \\
C^{(n)} & \vdots & \vdots & C^{(n)}
\end{bmatrix},
$$

(14)

where $[C^{(i)}]$ are the sub-matrixes for each IIs. They are found not to couple with each other in the matrix. Thus, the eigenvalues of the matrix $\{F_{ij}\}$ must be those of the sub-matrixes $[C^{(i)}]$. Without losing any generality, it can be supposed that the coercive forces $h_c$ of all the IIs have been arranged from small to large: $|h_c^1| \leq |h_c^2|, i < j$.

The hysteresis loop of the entire structure can be obtained as follows. As the external field $h$ reaches $h_c^1$, the minimum eigenvalue of the sub-matrix $[C^{(1)}]$ is zero while that in others are still positive, that means the first II is not stable while others are still stable. It has been discussed that each II only has two metastable configurations: the spin-up and the spin-down configuration. So, the first II will turn to its spin-down configuration in the vicinity of $h_c^1$ while the other IIs will remain in their spin-up configuration. Following the same reason, the $i$th II will turn to its spin-down configuration while other II’s configuration are not changed as the external field $h$ reaches the $i$th coercive force $h_c^i$. If we do that step by step, we can obtain the hysteresis loop finally. For instance, the
hysteresis loop for the system illustrated in figure 2 has been shown in figure 3, where there are sharp decrease in the vicinites of the coercive forces $h_i^c$.

However, we must do the sample avarages to overcome the fluctuations. In our calculations, 1000 samples are averaged for a definite case. The results are presented in figures 4-5. In the case that the density of magnetic atoms is 40% and the two kinds of magnetic atoms are equally sputtered, we average two groups of 1000 smaples to get two final results. They are compared in figure 4. However, one may find that the difference of the two lines even can not be detected which means the number of the samples for average is large enough to overcome the fluctuation. Figures 5a-5d present the hysteresis loops for the systems in different cases. From figures 4, 5a-5c, one may find that more sharp steps can be apperantly found in the hysteresis loop. Futhermore, when the density is lowered, the steps are sharper and more sub-steps in the hysteresis loops may appear (figure 5a). This can be understood as follows. When the density is low, the possibilities of apperaing some definite structures will be high. As a result, there may be a distint decrease of the magnetization in the vicinity of the coercive force for such a structure. Actually, a limit case is that there are only two diferent atoms in the lattice. in this case, the possibility for the distribution that the two atoms are seperated should be greatly larger than that they are coupled together. Thus, they may be a very clear multi-step shaped hysteresis loop.

When the density of the magnetic atoms is high, especially when the density is larger than the percolation value, the hysteresis loop is greatly smoothed and the step is almost undetectable (figure 5d). Actually, near the percolation value, the distributions of the Is are quite complicated, and any pattern is possible. Since the coercive force of each II is strongly dependent on the distribution, any value of the coercive force is then possible to appear. Thus, there should be an infinite number of metastable states which are all different in the system. This is very similar to that in the spin-glass system, although such a system is quite different with that one. So, one should not use such materials for recording.

To summarize, in this letter, we have investigated the hysteresis loops of the 2-d systems with randomly sputtered magnetic atoms. The method is proved to be rigorous in this case. The results show: a multi-step shaped hysteresis loop can be achieved only if the following two conditions can be satisfied: 1) the sputtered magnetic atoms must have different coercive forces. 2) the density of the magnetic atoms must be lower than the percolation value. Such materials have the potential to be considered as the recording media for MMV recording. If the first condition is dissatisfied, there is only one sharp step in the hysteresis loop; if the second condition, there are no sharp step in the hystesis loop and such material can not be used for recording.

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tional Education commission under the grant for training Ph.Ds.

Appendix

In this appendix, we will prove that \( \{ \epsilon_i \} \) are the exact excitation energies of the Hamiltonian (1) in the case of \( \theta_i = 0 \).

Since \( \theta_i = 0 \), the Bose transformation can be applied naively to Hamiltonian (1). Following Ref. [12], the CBT is given as:

\[
\begin{aligned}
S^z_i &= \sum_{l=0}^{\infty} A^l_i a^+_i a^l_i \\
S^+_i &= \sum_{l=0}^{\infty} B^l_i a^+_i a^{l+1}_i \\
S^-_i &= \sum_{l=0}^{\infty} B^l_i a^{l+1}_i a^+_i 
\end{aligned}
\]

\( \{ A^l_i \}, \{ B^l_i \} \) are the coefficients of Bose expansion which are dependent on \( S \). We can also derive the expansions for the single-ion anisotropy terms:

\[
(S^z_i)^2 = \sum_{l=0}^{\infty} G^l_i a^+_i a^l_i
\]

Applying the CBT to the Hamiltonian, we have a transformed Hamiltonian \( \tilde{H} \) which has exactly the same eigenvalues as the Hamiltonian (1):

\[
\tilde{H} = U_0 + H_2 + H_4 + \cdots.
\]

where

\[
H_2 = \sum_{i,j} F_{ij} a^+_i a_j
\]

\[
\cdots
\]

Of course, Hamiltonian \( \tilde{H} \) is still impossible to solve exactly. However, some eigenstates can be obtained exactly. \( H_2 \) can be diagonalized by a orthogonal transformation:

\[
\tilde{a}^+_m = \sum_n P_{mn} a^+_n
\]

After the transformation, we find

\[
H_2 = \sum_n \epsilon_n \tilde{a}^+_n \tilde{a}_n
\]
where \( \{ \epsilon_n \} \) are the eigenvalues of the matrix \( \{ F_{ij} \} \) and \( P_{mn} \) can be found from calculating the eigenvectors of the matrix \( \{ F_{ij} \} \).

From the CBT, we find that every terms in the remainder interaction \( H_I = H_4 + H_6 + \cdots \) contains equivalent numbers of the creation operators \( a^+ \) and annihilation operators \( a \) and contains at least two annihilation operators. For example, \( H_4 \) is found to be

\[
H_4 = \frac{1}{2} \sum_{ij} J_{ij} [A^+_0 A^+_2 a^+_i a^+_j + A^+_0 A^+_2 a^+_j a^+_i + A^+_1 A^+_2 a^+_i a^+_j a_i a_j \\
+ C^+_1 C^+_0 a^+_i a^+_j (a^2_i + a^2_j)] - \sum_i D_i G^+_2 a^+_i a^+_i - h \sum_i A^+_2 a^+_2 a^+_i. \tag{22}
\]

Then, it is easy to prove that

\[
H_I \hat{a}_n^+ |0\rangle = \sum_m P_{n,m} H_I a^+_m |0\rangle \equiv 0 \tag{23}
\]

As the result,

\[
\tilde{H}(\hat{a}_n^+ |0\rangle) = \epsilon_n (\hat{a}_n^+ |0\rangle) \tag{24}
\]

So, such one-magnon eigenstates \( \hat{a}_n |0\rangle \) are the \textbf{exact} eigenstates of the system, and \( \{ \epsilon_i \} \) must be the \textbf{exact} excitation energies of the system.

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Table 1: Coercive forces $h_c$ for some particular patterns illustrated in figure 1

| Pattern | Coercive forces $h_c$          |
|---------|-------------------------------|
| A       | 0.059200319744255659          |
| B       | 0.046196842386699633          |
| C       | 0.045505418405692213          |
| D       | 0.039408095183044223          |
| E       | 0.038938721535082826          |
| F       | 0.079256330941564368          |

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Captions:

Figure 1: Some particular patterns studied in this paper

Figure 2: A typical distribution of the magnetic ions in the system we studied

Figure 3: The hysteresis loop of the system illustrated in figure 2.

Figure 4: Comparison of the hysteresis loops obtained by averaging two groups of 1000 samples separately

Figure 5: Hysteresis loops for the systems in different cases