The Determination of Production and Distribution Policy in Push-Pull Production Chain with Supply Hub as the Junction Point

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Abstract. The development of technology causes the needs of products and services become increasingly complex, diverse, and fluctuating. This causes the level of inter-company dependencies within a production chains increased. To be able to compete, efficiency improvements need to be done collaboratively in the production chain network. One of the efforts to increase efficiency is to harmonize production and distribution activities in the production chain network. This paper describes the harmonization of production and distribution activities by applying the use of push-pull system and supply hub in the production chain between two companies. The research methodology begins with conducting empirical and literature studies, formulating research questions, developing mathematical models, conducting trials and analyses, and taking conclusions. The relationship between the two companies is described in the MINLP mathematical model with the total cost of production chain as the objective function. Decisions generated by the mathematical models are the size of production lot, size of delivery lot, number of kanban, frequency of delivery, and the number of understock and overstock lot.

1. Introduction
An efficient production chain can be achieved by harmonizing production and distribution activities [12]. Distribution activities involve more than a company. Therefore, the management of intercompany interactions within the production chain is needed to improve the efficiency of the production chain. Improved efficiency results are generally seen in decreasing cost of production chains and increasing end-customer satisfaction [4]. Improved efficiency is positively correlated with increased company competitiveness. Increased competitiveness of a company causes the company to maintain and improve its position in competition.

The simplest production chain is a production chain that consists of 2 (two) companies. One characteristic of inefficient production chains is when companies in the production chain are still optimizing individually. Individual optimization can be beneficial to a party and bring harm to the other party. In addition, the production chain is said to be inefficient if the distribution activities between two companies are not harmonious. This disharmony can occur because the two companies implement different production systems. One of the symptoms of disharmonization of production and distribution processes are the incidence of excess inventory or shortage of products.

The main objective of this research is to improve the efficiency of the production chain between two companies so that both companies can maintain and improve their competitive ability. The main
proposaof this research is to change the contract-based interaction into a partnership based. With partnership interaction, optimization between 2 (two) companies is done together. The optimization goal used in this research is to minimize the cost of production chain.

Harmonization of production and distribution activities between 2 (two) companies is done by applying the push-pull system. The push-pull system is chosen to accommodate the different production systems used by both companies. Push-pull system is a system that aims to combine the advantages possessed by the push system and pull system. The development done in this research is to involve supply hub as junction point between 2 (two) companies in a production chain. The use of a supply hub can help a company to receive just-in-time raw materials, especially if the company is located far away from its suppliers [7]. Under these circumstances, suppliers can deliver large quantities of products to the supply hub to minimize transportation costs, while customers still get the desired product at a smaller and precise batch size and just when the products are needed.

2. Literature Review
The early studies about push-pull systems summarized in Table 1.

| Reference | Year | Research Model Characteristics | Algorithm |
|-----------|------|--------------------------------|-----------|
| [9] 1996  | Four Production line | Single Work station | Inventory minimization | DES |
| [3] 1998  | Multi Production line | Single Work station | Production cost minimization | DES, SA |
| [1] 2004  | Two Production chain | Single Company | Production chain cost minimization | Dynamic Prog. |
| [12] 2004  | Multi Production line | Single Work station | Product variance minimization | DES |
| [13] 2004  | Single Production chain | Single - | Production chain cost minimization | B&B |
| [11] 2006  | Single Production chain | Single - | Production chain cost minimization | B&B |
| [2] 2008  | Multi Production line | Multi Work station | Production cost minimization | DES, GA |
| [5] 2009  | Multi Production line | Single Work station | Production cost minimization | DES, GA |
| [6] 2012  | Multi Production chain | Single Company | Production chain cost minimization | SA |
| [8] 2012  | Three Production line | Single Work station | Inventory minimization | Simulation |
| [10] 2013  | Six Production line | Single Work station | Inventory minimization and number of lost customers | DES |

From research on push-pull systems that have been done earlier, research is generally done on production lines, using near optimal algorithms, and using work station or company as the junction point. Research on the scope of the production line reviews the performance of the push-pull system on the interaction between machines or between work stations. Previous research has generally shown that push-pull systems are superior to push and pull systems in terms of cost and inventory.

3. Mathematical Model
The mathematical model is the development of Wang and Sarker [13] and Shah and Goh [11]. Implementation of push-pull production chain is done by involving supply hub as junction point, which is where switching of production system from push production system become pull production system. Research is limited to the relationship between one supplier, one supply hub, one customer, and one product. Supplier has real time data inventory in the supply hub to be able to react to the sudden change in customer’s demand. The products in the supply hub become the supplier.
responsibility. After the product is transferred to the customer's location, the ownership of the product is held by the customer. At the supply hub is applied the lower limit and upper limit of inventory. Supplier is charged by penalty fee if the product in the supply hub exceeds a predetermined limit. The assumptions used in this research are:

- Demand is known and deterministic
- Customer production rate is known and bigger than demand rate
- Production facility is not deformed
- Raw material accepted by supplier in time so the fulfillment of customer need is not disturbed
- Supplier production and supply hub replenishment lead time are zero

Variables and parameters notation used in this research listed in Table 2 and Table 3.

### Table 2. Decision variables

| Notation | Definition                                      | Unit of Measurement |
|----------|------------------------------------------------|---------------------|
| $Q_d$    | Supplier’s dispatch lot size                   | unit                |
| $x$      | Integer to determine the supplier’s production lot size | unit                |
| $y$      | Number of understock lot at the supply hub     | unitless            |
| $z$      | Number of overstock lot at the supply hub      | unitless            |
| $k$      | Number of kanban                               | unitless            |
| $n$      | Number of shipments placed at customer         | unitless            |
| $Q$      | Total quantity of finished goods produced over specific period | unit                |

### Table 3. Parameters

| Notation | Definition                                      | Unit of Measurement |
|----------|------------------------------------------------|---------------------|
| $I_b$    | Minimum inventory limit at the supply hub      | unit                |
| $I_a$    | Maximum inventory limit at the supply hub      | unit                |
| $D$      | Demand rate                                    | unit/ month         |
| $p$      | Customer’s production rate                     | unit/ month         |
| $A_p$    | Supplier’s production cost                     | rupiah/ batch       |
| $A_d$    | Supplier’s dispatching cost                    | rupiah/ batch       |
| $h_p$    | Inventory carrying cost at supplier’s warehouse | rupiah/unit/ month  |
| $h$      | Inventory carrying cost at supply hub          | rupiah/unit/ month  |
| $p_u$    | Penalty cost for under stocking at supply hub  | rupiah/unit/ month  |
| $p_o$    | Penalty cost for over stocking at supply hub   | rupiah/unit/ month  |
| $A_w$    | Customer’s ordering cost                       | rupiah/ batch       |
| $A_s$    | Customer’s production cost                     | rupiah/ batch       |
| $H_f$    | Inventory carrying cost at customer’s warehouse | rupiah/unit/ month  |
| $A_f$    | Customer’s dispatching cost                    | rupiah/ batch       |

There are 10 cost components, 6 (six) supplier-related costs and 4 (four) customer-related costs. 6 (six) supplier costs consist of 3 (three) costs related to the supplier’s plant and 3 (three) costs related to the supply hub.

#### 3.1 Supplier’s plant cost

Production lot size at supplier’s plant are always integer multiples of the dispatch lot size. Therefore, supplier’s production and dispatching costs are shown in Equation (1) and Equation (2).

\[
\text{Supplier’s production cost} = A_p \times \left( \frac{D}{Q_p} \right) = A_p \times \left( \frac{D}{xQ_d} \right) \tag{1}
\]

\[
\text{Supplier’s dispatching cost} = A_d \times \left( \frac{D}{Q_d} \right) \tag{2}
\]
As shown in Figure 1, inventory at the supplier’s warehouse follows a step function. After one production cycle, supplier would hold \((x-1)Q_d\) products for the first period \(T\), \((x-2)Q_d\) products for the second period \(T\), and so on. Supplier’s inventory cost shown in Equation (3).

\[
\text{Supplier’s inventory cost} = \left\{ \frac{(x-1)Q_d^2}{D} + \frac{(x-2)Q_d^2}{D} + \cdots + \frac{Q_d^2}{D} \right\} \times \frac{D}{xQ_d} \times h_p
\]

\[
= \frac{(x-1)Q_d}{2} \times h_p \tag{3}
\]

**Figure 1.** Inventory at supplier’s warehouse

### 3.2 Supply hub cost

Products in the supply hub are delivered to the customer with the lot size \(Q_w\). \(Q_w\) is defined as the lot size \(Q_d\) divided by the \(k\) kanban used. Understock products in supply hub are represented by \(s\) at Figure 2. The number of understock product is always integer multiples of the supply hub’s dispatch lot size. Therefore, number of understock product follows Equation (4).

\[
s = yQ_w = \frac{yQ_d}{k} \tag{4}
\]

**Figure 2.** Inventory at supply hub

Supply hub’s inventory cost follows Equation (5).

\[
\text{Supply hub’s inventory cost} = \left\{ \frac{1}{2} \frac{Q_w}{k} (1 + 2 + \cdots + k) \right\} \times \frac{Q_d}{D} \times \frac{D}{Q_d} \times h
\]

\[
= \frac{Q_w(k+1)}{2k} + (I_b-s) \times h
\]

\[
= \frac{Q_d(k+1)}{2k} + \left( I_b - \frac{yQ_d}{k} \right) \times h \tag{5}
\]

Similar to \(s\), number of overstock product is always integer multiples of supply hub’s dispatch lot size. Therefore, number of overstock product follows Equation (6).

\[
\text{Overstock product} = zQ_w = z\frac{Q_d}{k} \tag{6}
\]

Supply hub’s understocking and overstocking penalty costs shown in Equation (7) and Equation (8).

\[
\text{Supply hub’s understocking penalty cost} = \left\{ \left[ \frac{1}{2} \frac{y(1+y)}{k} \right] \times \frac{Q_d}{kD} \times \frac{T}{k} \times \frac{D}{Q_d} \times p_s \right\}
\]

\[
= \left\{ \left[ \frac{1}{2} \frac{y(1+y)}{kD} \right] \times \frac{Q_d}{kD} \times D \times \frac{p_s}{Q_d} \right\}
\]
Therefore, total objective function in this research is shown at Equation (13).

\[
\begin{align*}
&= A_p \times \left( \frac{D}{xQ_d} \right) + A_d \times \left( \frac{D}{Q_d} \right) + \frac{(x-1)Q_d h_p}{2} + \left[ \frac{Q_d(k + 1)}{2k} \right] h + \left[ \frac{\left[ \frac{1}{2}(1 + y) \right] Q_d}{k} \right] p_s + \left\{ \left[ \frac{1}{2}(1 + z) \right] \left[ \frac{Q_d}{k} \right] p_e \right\}_{\text{if } (Q_d - s) \leq (l_a - l_b)} + \left\{ \left[ \frac{1}{2}(1 + z) \right] \left[ \frac{Q_d}{k} \right] p_e \right\}_{\text{if } (Q_d - s) > (l_a - l_b)}
\end{align*}
\]

\[
\times \left( \frac{D_{1n}}{Q} \right) + H_f \frac{Q}{2} \left( \frac{1}{n} + \left( 1 - \frac{D}{p} \right) \right)
\]

\( (13) \)

Subject to: \( s \leq I_b; Q_d \geq s; y, z < k; s, Q \geq 0; k, n \geq 1; k, n \in \text{integer} \)

4. Numerical Example

From Equation (13) it is known that the objective function has if else condition. Therefore, this research has 2 (two) models, that is model without considering overstocking and model by considering overstocking. The model chosen to apply is a model that has a lower cost. Parameters that used for numerical example listed in Table 4.
Table 4. Numerical Example

| Demand and production rate (unit/month) | Dispatch and order cost (rupiah/setup) | Production cost (rupiah/setup) | Inventory carrying cost (rupiah/month/unit) | Penalty cost (rupiah/month/unit) | Inventory limit at supply hub (unit) |
|----------------------------------------|---------------------------------------|-------------------------------|--------------------------------------------|----------------------------------|-----------------------------------|
|                                        | \(A_d = 13.09\)                        | \(A_p = 96\)                 | \(h_p = 0.55\)                            | \(p_s = 5.46\)                   | \(I_a = 1000\)                   |
|                                        | \(A_w = 3.11\)                        | \(A_s = 129\)                | \(h = 10\)                                 | \(p_e = 10.91\)                  | \(I_b = 40\)                     |
|                                        | \(A_f = 13.38\)                       | \(H_f = 35\)                 |                                            |                                  |                                   |

Solution for Equation (13) reported in Table 5.

Table 5. Best solution

| Dec. variables | Without Overstock | With Overstock |
|----------------|-------------------|----------------|
| \(Q_d\)        | 573 unit          | 733 unit       |
| \(x\)          | 3                 | 2              |
| \(y\)          | 0                 | 1              |
| \(z\)          | -                 | 1              |
| \(k\)          | 73                | 11             |
| \(n\)          | 5                 | 5              |
| \(Q\)          | 1442 unit         | 1442 unit      |
| Total Cost     | Rp199,253,000.00  | Rp214,359,000.00 |

5. Sensitivity Analysis

Sensitivity analysis is done by changing the model’s parameters. The summary of sensitivity analysis shown in Table 6.

Table 6. One parameter sensitivity analysis

| No  | Parameters                           | Without Overstock |                      | With Overstock |                      |
|-----|--------------------------------------|--------------------|----------------------|----------------|----------------------|
|     |                                      | Significant       | Sensitive Decision  | Robustness Range | Significant       | Sensitive Decision  | Robustness Range |
|     |                                      | Change in         | Variable             |                 | Change in         | Variable             |                 |
|     |                                      | Objective         |                      |                 | Function Value     |                      |                 |
| 1   | Demand rate                          | Lowered over 30%  | \(Q\)                | -5% – 20%       | Lowered over 30%  | \(Q\)                | -15% – 0%        |
| 2   | Minimum inventory at the supply hub  | Lowered over 40%  | \(Q_d\)              | 0% – 75%        | Lowered over 50%  | \(Q_d\)              | -5% – 20%        |
| 3   | Maximum inventory at the supply hub  | Lowered over 70%  | \(Q_d\)              | -75% – 5%       | Lowered over 70%  | \(Q_d\)              | -15% – 0%        |
| 4   | Customer’s production rate            | Lowered over 30%  | \(Q\)                | -15% – 5%       | Lowered over 30%  | \(Q\)                | -15% – 5%        |
| 5   | Supplier’s production cost            | Lowered over 70%  | \(Q_d\)              | -20% – 40%      | Lowered over 70%  | \(Q_d\)              | -60% – 10%       |
| 6   | Customer’s production cost            | Lowered over 40%  | \(Q\)                | -5% – 40%       | Lowered over 50%  | \(Q\)                | -5% – 40%        |
|     |                                      | Lowered over 40%  |                      |                 | Raised over 70%   |                      |                 |

As shown in Table 6, demand, maximum inventory at the supply hub, and customer’s production rate are the parameters that causing significant change in objective function value. Therefore, two parameters sensitivity analysis is conducted. It is summarized in Table 7.
As shown in **Figure 4**, it is known that the total cost increase due to the increasing of the demand and decreasing of maximum inventory at the supply hub. The optimum total cost is Rp160,709,300. This condition occurs when the demand decreases by 50% and the maximum inventory limit is in the range -10% to +50%.

As shown in **Figure 5**, it is known that the total cost increase due to the increasing of the demand and customer’s production rate. The optimum total cost is Rp134,402,300. This condition occurs when the demand and the customer’s production rate decrease by 10%.

As shown in **Figure 6**, it is known that the total cost increase due to the increasing of customer’s production rate and decreasing of maximum inventory at the supply hub. The optimum total cost is Rp159,436,500. This condition occurs when customer’s production rate decrease by 40% and maximum inventory at the supply hub increase by 40%.

### Table 7. Two parameters sensitivity analysis

| Demand | Maximum Inventory Limit |
|--------|-------------------------|
| 1000   | 500                     |
| 2000   | 1000                    |
| 3000   | 1500                    |
| 4000   | 2000                    |
| 5000   | 2500                    |
| 6000   | 3000                    |
| 7000   | 3500                    |
| 8000   | 4000                    |
| 9000   | 4500                    |

6. Conclusion
The conclusions of this research are consolidated below:
- Improved efficiency of the production chain can be done by collaborative optimization and harmonization of production and distribution activities.
- Harmonization of production and distribution activities in this research is done by applying push-pull system and supply hub on production chain relation between 2 companies.
- The purpose of this research is to do supply chain optimization collaboratively by determining the size of production lot, lot size of shipment, number of kanban, and frequency of delivery done by company in production chain, and the number of understock and overstock lot.
- Demand, maximum inventory limit at the supply hub, and customer’s production rate are the parameters that causing significant change in objective function value.
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References

[1] Ahn, H.-S. & Kaminsky, P., 2004. Push, pull, or hybrid control in supply chain management. *IIE Transactions*, 37(1), pp. 609-621.
[2] Cochran, J. K. & Kaylani, H. A., 2008. Optimal design of a hybrid push/pull serial manufacturing system with multiple part types. *International Journal of Production Research*, 46(4), p. 949–965.
[3] Cochran, J. K. & Kim, S., 1998. Optimum junction point location and inventory levels in serial hybrid push/pull production systems. *International Journal of Production Research*, 36(4), pp. 1141-1155.
[4] Fugate, B., Sahin, F. & Mentzer, J. T., 2006. Supply Chain Management Coordination Mechanisms. *Journal of Business Logistics*, 27(2), pp. 129-161.
[5] Ghrayeb, O., Phojanamongkolkij, N. & Tan, B. A., 2008. A hybrid push/pull system in assemble-to-order manufacturing environment. *J Intell Manuf*, 20(1), pp. 379-387.
[6] Kim, S.-H., Fowler, J., Shunk, D. & Pfund, M., 2012. Improving the push–pull strategy in a serial supply chain by a hybrid push–pull control with multiple pulling points. *International Journal of Production Research*, 50(1), p. 5651–5668.
[7] Liu, M., Li, G. & Gao, T., 2014. Superiority of Collaborative Replenishment with Supply-hub in Assembly System. *Information Technology Journal*, 13(1), pp. 1062-1069.
[8] Mahapratata, S., Yu, D. & Mahmoodi, F., 2012. Impact of the pull and push-pull policies on the performance of a three-stage supply chain. *International Journal of Production Research*, 50(16), pp. 4699-4717.
[9] Pandey, P. C. & Khokhajaikiat, P., 1996. Performance modeling of multistage production systems operating. *Int. J. Production Economics*, 43(1), pp. 17-28.
[10] Phonsuwan, S. & Kachitvichyanukul, V., 2013. Simulation for OTOP supply chain with hybrid push-pull flow control. *Int. J. Services and Operations Management*, 15(2), pp. 196-214.
[11] Shah, J. & Goh, M., 2006. Setting Operating Policies for Supply Hubs. *International Journal of Production Economics*, 100(1), pp. 239-252.
[12] Takahashi, K. & Nakamura, N., 2004. Push, pull, or hybrid control in supply chain management. *International Journal of Computer Integrated*, 17(2), pp. 126-140.
[13] Wang, S. & Sarker, B. R., 2004. A Single-stage Supply Chain System Controlled by Kanban Under Just-in-time Philosophy. *Journal of the Operational Research Society*, 55(5), pp. 485-494.