REIONIZATION IN THE WARM DARK MATTER MODEL

Bin Yue$^{1,2}$ and Xuelei Chen$^{1,3}$

$^1$ National Astronomical Observatories, Chinese Academy of Sciences, 20A Datun Road, Chaoyang, Beijing 100012, China
$^2$ Graduate University of Chinese Academy of Sciences, Beijing 100049, China
$^3$ Center of High Energy Physics, Peking University, Beijing 100871, China

Received 2011 November 10; accepted 2012 January 17; published 2012 February 23

ABSTRACT

Compared with the cold dark matter (CDM) model, the formation of the small-scale structure of the universe is suppressed in the warm dark matter (WDM) model. It is often thought that this would delay the reionization of the intergalactic medium (IGM) because the star formation rate during the epoch of reionization (EOR) would be lowered. However, during the later stage of the EOR, a large portion of the ionizing photons is consumed by recombination inside the minihalos, which is where the gas has higher density and recombination rates than that in the IGM. The suppression of the small-scale structure would therefore reduce the recombination rate and could potentially shorten the reionization process. This effect is investigated here by using the analytical “bubble model” of reionization. We find that, in some cases, although the initiation of the EOR is delayed in the WDM model, its completion could be even earlier than the CDM case, but the effect is generally small. We obtain limits on the mass of WDM particles for different reionization redshifts.

Key words: dark matter – large-scale structure of universe

Online-only material: color figures

1. INTRODUCTION

The nature of dark matter is presently unknown. The cold dark matter (CDM) model has been very successful in explaining the observed properties of large-scale structures of the universe, but there are still some discrepancies in small scales, because the abundance of satellite galaxies falls far short of the number of subhalos predicted (Klypin et al. 1999; Moore et al. 1999; Springel et al. 2008; see also Simon & Geha 2007 and Primack 2009 for a different view). An interesting alternative is the warm dark matter (WDM) scenario, where a dark matter particle has a smaller (a few keV) mass (Blumenthal et al. 1982; Pagels & Primack 1982; Melott & Schramm 1985; Bardeen et al. 1986; Dodelson & Widrow 1994; Colombi et al. 1996; Colín et al. 2000; Sommer-Larsen & Dolgov 2001). As the growth of the structure below the free-streaming scale of the WDM is quenched, such a model predicts much fewer small halos, but its prediction is similar to the CDM case on large scales and could still match the large-scale structure observations quite well (Bode et al. 2001; Jing 2001; Abazajian 2006; Knebe et al. 2008; Tikhonov et al. 2009; Macciò & Fontanot 2010; Smith & Markovic 2011; Polisensky & Ricotti 2011).

The abundance of small halos may also greatly affect the reionization of the universe. As the stars and galaxies form at the end of the cosmic dark age, star light ionizes intergalactic medium (IGM), causing the latter to ionize. Eventually, all of the hydrogen gas in the IGM is ionized; this is the so-called (hydrogen) reionization (Barkana & Loeb 2001; Shapiro et al. 2004; Iliev et al. 2007), although the magnitude and significance of this effect is highly uncertain (Ripamonti et al. 2007a, 2007b). In this paper, we shall limit ourselves to the passive WDM model and shall not consider such effects.

However, the time it took the reionization process to finish depends not only on the supply rate but also on the consumption rate of the ionizing photons. After being ionized, the atoms of the IGM gas will recombine, and, to keep the gas ionized, more ionizing photons must be consumed. Because the recombination rate is proportional to the density squared, the minihalos could potentially consume the majority of the ionizing photons at the later stage of the EOR (Haiman et al. 2001; Benson et al. 2001; Barkana & Loeb 2002; Shapiro et al. 2004; Ciardi et al. 2004, 2005; Ciardi et al. 2006; Yue et al. 2009; Alvarez & Abel 2009), indicating that we are approaching the end stage of the epoch of reionization (EOR) at $z > 6$ (Becker et al. 2001; Fan et al. 2002). The cosmic microwave background (CMB) anisotropy observation is another important source of information about the EOR (Zaldarriaga 1997; Kogut et al. 2003; Spergel et al. 2007).

The current Wilkinson Microwave Anisotropy Probe (WMAP) analysis gives the redshift of completion of reionization as $z_{\text{re}} = 10.5 \pm 1.2$ (Larson et al. 2011). This result can be used to constrain the properties of dark matter particles. In the WDM model, halos of a given mass would typically form later than in the CDM model, so the initiation of the EOR would be delayed. Also, since the formation of small halos, which are most common during the EOR, is suppressed, fewer ionizing photons are produced, and it was believed that this would significantly delay the completion of the reionization (Barkana et al. 2001; Yoshida et al. 2003; Somerville et al. 2003).

In some WDM models, it should be noted that the WDM particle may be able to decay, which would ionize and heat up the IGM (Abazajian et al. 2001; Mapelli & Ferrara 2005; Mapelli et al. 2006). This (and the annihilation of some CDM particles) is an interesting problem that involves rich physics (see, e.g., Chen & Kamionkowski 2004; Furlanetto et al. 2006; Chuzhoy 2008; Yuan et al. 2010; Ripamonti et al. 2010). The effect on reionization is fairly complicated; for example, the free electrons produced in the dark age could help catalyze the formation of molecular hydrogen, making it possible to form the first stars and begin reionization early in this case (Biermann & Kusenko 2006; Kusenko 2007; Stasielak et al. 2007), although the magnitude and significance of this effect is highly uncertain (Ripamonti et al. 2007a, 2007b).
2010). In the WDM scenario, since the formation of small-scale structures is suppressed, the number of minihalos is fewer than that in the CDM, so the global recombination rate is much lower. When this is considered, it is not obvious whether the WDM would delay or advance the completion of the EOR.

We use the “bubble model” of reionization (Furlanetto et al. 2004a) to investigate the reionization process in the WDM scenario and take into account both the reduction in the photon production rate and the consumption rate due to the suppression of halo formation in WDM models. Here we adopt the WMAP five-year cosmology parameters (the WMAP seven-year parameters are almost identical): \((\Omega_m, \Omega_b, \Omega_{\Lambda}, h, \sigma_8, n_s) = (0.274, 0.726, 0.0456, 0.705, 0.812, 0.95)\) (Komatsu et al. 2009). Note that this set of parameters may not be the best-fit values for the WDM model, but, to illustrate the physical effect of different dark matter masses, we have used the same parameters for all models.

2. METHOD OF CALCULATION

We analytically calculate the halo mass function in the WDM model by following the prescription of Smith & Markovic (2011). In the WDM model, the free-streaming comoving scale is given by

\[
\lambda_{fs} \approx 0.11 \left( \frac{\Omega_{WDM} h^2}{0.15} \right)^{1/3} \left( \frac{m_{WDM}}{\text{keV}} \right)^{-4/3} \text{Mpc},
\]

and the corresponding mass is \(M_{fs} = 4\pi/3(\lambda_{fs}/2)^3 \rho_0\). The halo mass function is then given by

\[
\frac{dn}{dM}(M, z) = \frac{1}{2} \left( 1 + \text{erf} \left[ \frac{\log_{10}(M/M_{fs})}{\sigma_{\log M}} \right] \right) \left[ \frac{dn}{dM} \right]_{PS},
\]

where \(\sigma_{\log M} = 0.5\), and \([dn/dM]_{PS}\) can be calculated with the usual Press–Schechter (PS) prescription (Press & Schechter 1974; Bond et al. 1991), with the matter power spectrum for the WDM case given by the fit (Bode et al. 2001; Viel et al. 2005):

\[
P_{WDM}(k) = P_{CDM}(k) \left[ 1 + (ak)^{\mu_1} \left( \frac{\nu_1}{\nu_2} \right)^{-5/\mu_2} \right]^2,
\]

where \(\mu = 1.12\), and

\[
\alpha = 0.049 \left( \frac{m_{WDM}}{\text{keV}} \right)^{-1.11} \left( \frac{\Omega_{WDM}}{0.25} \right)^{0.15} \left( \frac{h}{0.7} \right)^{1.22} \text{Mpc}.
\]

The halo mass functions for the CDM and WDM models with different particle masses at redshift 15 are plotted in Figure 1. We see that the abundance of dark matter halos on small scales is significantly suppressed, while on large scales the halo abundance is almost the same as the CDM model. On the same plot, we also mark the mass range of which we call the “minihalos,” i.e., halos that are sufficiently massive to accrete the gas, but are not massive enough for the gas to cool by atomic hydrogen (\(T_{\text{vir}} > 10^4\) K; Barkana & Loeb 2001; Bromm & Yoshida 2011). In either case, the stars and galaxies tend to form more abundantly in overdense regions, so the ionized regions first appeared as “bubbles” in such regions. As more and more bubbles appeared and grew in size, the ionized regions eventually overlapped, thus completing the reionization.

Inspired by numerical simulation results, a popular analytical model of the process called the “bubble model” has been developed (Zaldarriaga et al. 2004; Furlanetto et al. 2004a, 2004b). According to this model, the reionization proceeds by first forming ionized bubbles. The number and size of these bubbles continue to grow until they overlap with each other, thus completing the reionization process. The formation of the ionized bubbles is determined by the condition that the number of ionizing photons produced within the given region exceeds the total number of atoms to be ionized in the same region. If we assume that each collapsed baryon, on average, contributes \(\xi\) ionizing photons, with each photon ionizing one atom once, and the average number of recombination per atom during that time is \(n_{\text{rec}}\), then the condition for the region to be ionized at a given redshift can be written as

\[
\xi f_{\text{coll}} > 1 + n_{\text{rec}},
\]

where \(f_{\text{coll}}\) is the fraction of baryons collapsed into star-forming halos. This treatment of the effect of recombination differs from Furlanetto & Oh (2005). They considered the limiting case, where the ionizing photon production is required to counteract the recombination at that instant. Our model in Equation (5) is more in line with the original bubble model, in which the total integrated number of ionizing photons is considered.

In such a model, one assumes that halos with the virial masses above a certain threshold value could form stars. The collapse fraction can then be calculated with the extended PS method (Bond et al. 1991; Lacey & Cole 1993; Mo & White 1996). During the early EOR, the Population III stars that formed in the molecular hydrogen-cooled halos may have played an important role.
role. These halos are much less massive than the typical star-forming halos during the EOR, which are cooled by atomic hydrogen or, at a somewhat later time, by metals. To account for this, we may split the contribution into the part from molecular hydrogen-cooled halos and the part from more massive halos. We denote the first type by the subscript “mol,” and, to avoid confusion, the second type is denoted without any subscript. Then, Equation (5) can be rewritten as

$$\xi_{f_{\text{mol}}} + f > 1 + n_{\text{rec}}.$$  \hspace{1cm} (6)

A molecular hydrogen-cooled halo should have a virial temperature of at least $10^4$ K for this mechanism to work. Furthermore, the formation of the molecules is strongly modulated by the Lyman–Werner (LW) radiation background. For the molecules to form, the mass of the halo should be greater than (Trenti & Stiavelli 2009)

$$M_{\text{H}_2, \text{cool}}(J_{21}, z) = 6.44 \times 10^6 J_{21}^{0.457} (\frac{1 + z}{31})^{-3.557} M_\odot.$$  \hspace{1cm} (7)

The mass threshold is then given by

$$M_{\text{mol}}(z) = \max[M_{\text{vir}}(10^3 \text{ K}, z), M_{\text{H}_2, \text{cool}}(J_{21}, z)].$$  \hspace{1cm} (8)

We model the LW intensity as

$$J_{21}(z) = 10^{-3} \left(\frac{\xi_{\text{mol}}}{50}\right) + 0.28 \left(\frac{\xi}{40}\right) (1 + z)^3 f_{\text{coll}}(z).$$  \hspace{1cm} (9)

The first term represents a slowly evolving component due to Population III stars whose formation is self-regulated, while the second term comes from the more massive, atomically cooled halos. Strictly speaking, the formation of Population III stars is suppressed in the WDM model, so the coefficient of the first term would be smaller. But, this change would not affect the final result because the contribution of the few stars in the WDM model is already very small. The molecule-cooled halo collapse fraction is then calculated with

$$f_{\text{mol}} = \frac{1}{\rho_m} \int dz' \int_{M_{\text{mol}}(z')}^{M_{\text{mol}}(10^3 \text{ K}, z')} M d^2 n / d M d z'.$$  \hspace{1cm} (10)

The values of $\xi$ and $\xi_{\text{mol}}$ depend on a number of factors, e.g., the fraction of baryons in the halo that ended up in stars, the energy released by the star during its lifetime, and the fraction of the photons that escaped to the IGM from the cloud surrounding the star. There are currently a number of uncertainties in this parameter. Here we adopt $\xi = 40$ (Barkana & Loeb 2001) and $\xi = 50$; the latter parameter value is in agreement with the properties of massive metal-free stars given in Schaerer (2002). With these parameter values, $J_{21} \approx 10^{-3}$ during the Population III stars dominated stage in the CDM model, which is in good agreement with the value widely adopted in analytical and numerical studies of the first stars (Wise & Abel 2007), while, at $z = 10 J_{21} \approx 7$, it is very close to the value given in Trenti & Stiavelli (2009).

At high redshifts, the clustering in the IGM is relatively low. We assume that only one photon is needed to ionize an IGM atom, while, for the atoms in the minihalo, the number of recombinations is given by

$$n_{\text{rec}, \text{MH}} = \frac{1}{\rho_m} \int_{M_{f(z)}}^{M_{\text{mol}}(z)} \xi M d^2 n / d M d M.$$  \hspace{1cm} (11)

Here, $\xi$ is the average number of recombinations per atom of the minihalo and $M_f$ is the Jeans mass. The upper limit of the integration $M_{\text{mol}}$ is the minimum mass of halos that could host radiation sources, i.e., Population III stars or galaxies, that is $\min[M_{\text{mol}}(z), M_{\text{mol}}(10^4 \text{ K}, z)]$. The recombination inside the galaxies should be relegated to the net photon production number $\zeta$ and should not be included again here. Earlier analytical estimates typically gave $\xi \sim 10^2$ (Haiman et al. 2001), in which case the recombination of the minihalos would consume the majority of photons at the late stage of the EOR. However, numerical simulation of the ionizing front passing through minihalos shows that the number of actual recombinations may be far smaller, e.g., at $z = 15, \xi < 8$ for halos with mass below $10^7 M_\odot$ and irradiated by typical ionizing flux (Shapiro et al. 2004; Iliev et al. 2005). As the halo is ionized, it is heated and the gas expands, causing the density to decrease and the recombination rate to be quickly lowered; thus, the recombination rate is much lower than the earlier estimates. We adopt the $\xi$ value derived from Shapiro et al. (2004) and Iliev et al. (2005).

In Equation (6), for a bubble with mass $m$, $f$ and $f_{\text{mol}}$ are both functions of the linear overdensity of this bubble. By solving the equation $\xi_{f_{\text{mol}}} f(z, \delta_x) + f(z, \delta_x) = 1 + n_{\text{rec}}$, a critical overdensity $\delta_x(m, z)$ is determined. This is the barrier in the excursion set; regions with linear overdensity above this barrier should be ionized (Furlanetto et al. 2004a). We obtain this barrier by solving the above equation with numerical iterations, and we found that it is still well approximated by a linear function of the squared variance of density fluctuation $\delta^2(m)$ (the following process is the same as in Furlanetto et al. 2004a): $\delta_x(m, z) \approx B(m, z) = B_0 + B_1 \sigma^2(m)$, where $B_0 = \delta_x(m \rightarrow \infty, z)$ and $B_1 = \delta_x / \partial \sigma^2(m \rightarrow \infty, z)$. With this linear barrier, the bubble mass function is expressed analytically by

$$\frac{dn_b}{dm} = \sqrt{\frac{2}{\pi m^2}} \left| \frac{d\sigma}{dm} \right| B_0 \left( \frac{B^2(m, z)}{2\sigma^5 m} \right)^{1/2},$$  \hspace{1cm} (12)

where $n_b$ is the number density of bubbles. Finally, the volume-filling factor of all bubbles $Q_V$ is directly calculated by integrating over the bubble mass function:

$$Q_V = \int V(m) \frac{dn_b}{dm} dm.$$  \hspace{1cm} (13)

3. RESULTS AND DISCUSSIONS

We plot the redshift evolution of the ionization volume-filling factor in Figure 2. Here we considered four different cases: the reionization in the ΛCDM model with and without minihalo recombinations, and the reionization in the AWDM model with minihalo recombinations for $m_{\text{minihalo}} = 10$ keV and 2 keV.

First, we note that, as illustrated in the ΛCDM case, in the absence of minihalos, the reionization would be completed earlier by as much as $\Delta z = 1$. This shows how much the minihalo recombinations could have on the reionization. This is obtained with $\xi$ derived from Iliev et al. (2005). If we adopt the higher values of $\xi$ given in the earlier literature (Haiman et al. 2001), the impact would be even stronger.

In the AWDM case, the reionization starts later than in the ΛCDM cases because the first stars would form later. In these

\footnote{Our definition of $\xi$ differs by the one from Iliev et al. (2005): $\xi = \xi_{\text{liiev}} - 1$, where $\xi_{\text{liiev}}$ is the value given in Iliev et al. (2005).}
models, the small-scale power is suppressed. This difference is most obvious at $z > 20$, when the first stars begin to form in large numbers. Because small-scale powers are suppressed in the ΛWDM model, there are very few halos in which the gas could be cooled by molecular hydrogen and form Population III stars (O’Shea & Norman 2006). Instead, in ΛWDM, most first stars only form in atomically cooled halos; this causes a significant difference in the initiation of reionization.

However, it is generally believed that the reionization is primarily due not to the stars formed in the H$_2$ cooled halos, but to stars formed in the more massive halos at lower redshifts. For the more massive halos, the difference between the 10 keV WDM and the CDM is not very large. In fact, from Figure 2 we can see that in this case because the bubble-filling factor of the 10 keV WDM model catches up with the CDM model at $z \sim 13$, and later it even exceeds that of the CDM model; thus, the reionization is actually completed earlier than the CDM case. This is because, at this stage, the ionizing photons are mainly produced in the more massive halos, which is about equally abundant in the 10 keV WDM and CDM cases; however, the suppression of minihalos reduced the global recombination rate in the WDM model, making the bubbles overlap earlier. Thus, we observe the interesting result that the universe is reionized earlier in the WDM model. This effect is relatively small compared with the current theoretical and observational uncertainties, especially the mean number of ionizing photons produced by a halo of given mass. However, it could potentially be useful if these uncertainties are greatly reduced by improvements in observations and theoretical modeling of the reionization process.

For models with still lower WDM masses, the more massive halos begin to be affected. In the 2 keV WDM model, the star formation associated with the more massive halos is also suppressed, and the reionization is delayed in that model. The reduction of photon supply and consumption would be balanced somewhere between these two cases, but the exact value would depend on the parameters we adopt, especially the values of the parameters $\xi$ and $\zeta$. The values we adopted here are plausible and non extreme, with the reionization happening at $z \sim 10$, which is in agreement with the WMAP constraint. Nevertheless, there are large uncertainties within these parameters. In particular, if the $\xi$ value is greater for the minihalos, as was used in some earlier papers, the effect would be still stronger.

We plot the redshift of the reionization completion as a function of the WDM particle mass in Figure 3. For reference, the corresponding cutoff mass scale $M_{\text{cut}}$, where the mass function $dn/dM$ is suppressed by a factor of $e$ when compared with the ΛCDM model, is also plotted on the upper abscissa. We see that, below 11 keV the reionization redshift is very sensitive to the WDM particle mass. When $m_{\text{WDM}}$ increases, the reionization redshift also rises quickly, which is usually assumed for WDM particles. However, a peak of reionization redshift of $z_{\text{re}} \approx 9$ is reached at $m_{\text{WDM}} \approx 11$ keV. Above this mass, the reionization redshift begins to decrease slowly as the suppression of the number of star-forming halos becomes relatively insignificant, while the suppression of minihalos reduces the global recombination rate. As a comparison, we also plot $z_{\text{re}}$ in the CDM model with and without minihalo recombinations using filled and open circles, respectively. Quasar absorption line studies show that the universe has been reionized before redshift 6 (Fan et al. 2006). This gives the constraint that $m_{\text{WDM}} > 1.3$ keV.

This limit on the WDM mass can be compared with recent WDM mass limits obtained from other observations. For example, Narayanan et al. (2000) obtained $m_{\text{WDM}} > 0.75$ keV from the Lyα forest observations, while Viel et al. (2005) obtained a...
lower limit of 0.55 keV from CMB (WMAP) and the Lyα forest data. Barkana et al. (2001) gave $m_{\text{WDM}} > 1.2$ keV with the requirement $z_{\text{re}} > 5.8$ for their fiducial model, while Polisensky & Ricotti (2011) obtained a constraint of $m_{\text{WDM}} > 2.3$ keV from the number of Milky Way satellites.

From the above example, we have adopted the same value of $\zeta$ as used in the fiducial model of Barkana et al. (2001), which was derived from the observations of $z \sim 3-4$ and present-day galaxies. However, there are still a number of uncertainties regarding the properties of sources in the EOR. If these sources are stars with metallicity $Z = 5 \times 10^{-4} Z_\odot$, and if their initial mass function is the Salpeter form (Salpeter 1955) with the mass range $1 M_\odot < M < 100 M_\odot$, the number of ionizing photons produced per stellar atom is $\approx 13,000$ (Schaefer 2003) in the starburst model. Assuming a star formation efficiency of 0.05 and an escape fraction of 0.5, $\zeta$ could be as high as $\approx 300$. Considering these uncertainties, we also calculated the reionization redshift for different $\zeta$ values. In Figure 4, we plot the contours of $z_{\text{re}}$ with both $m_{\text{WDM}}$ and $\zeta$ varying. We see that WDM models with particle masses below 0.5 keV have already been excluded, otherwise the reionization could not be completed before redshift 6 for reasonable values of $\zeta$. On the other hand, for particles with $m > 8$ keV, $\zeta$ should be less than $\approx 180$, otherwise the completion of reionization would be too early, and this would be in conflict with the WMAP observations (Larson et al. 2011).

Besides the halo abundance, the distribution of gas within the halo is also affected by the replacement of CDM with WDM. The collapse of halos in the WDM model is generally later than in the CDM model; hence, the halo concentration is typically smaller (Barkana et al. 2001; Smith & Markovic 2011) because the dark matter halos formed later would reach smaller average densities (Navarro et al. 1997). In addition to the delayed collapse redshift, Smith & Markovic (2011) pointed out that, due to the relic velocities of WDM particles, the core of the halo would be smoother in the WDM case. These effects could, in principle, reduce the photon consumption rate of the minihalos. We have calculated this effect and found that it should only have a very slight influence on our results. For example, in the $m_{\text{WDM}} = 10$ keV case, for a halo with $10^7 M_\odot$ at $z = 10$, the density profile of the halo within $4 \times 10^{-2} R_{\text{vir}}$ would be flattened. But, outside this core radius, the density profile hardly changes. The gas density profile changed even less, since, even in the CDM model, the baryonic gas has pressure and, therefore, a smoother distribution. The larger minihalos contribute more to photon consumption in the WDM model because the abundance of smaller minihalos is significantly reduced. In the end, we find that the change in density profile does not significantly affect the total recombination rate of minihalos.

In the present calculation, we have used the bubble model to investigate the reionization process. The bubble model is, of course, only an approximate model, although it does reproduce more elaborate simulations (Zahn et al. 2007; McQuinn et al. 2007; Zahn et al. 2011). Furthermore, the bubble model only provides a distribution of bubbles at a given redshift; it does not tell us how each individual bubble has grown, so we have treated the recombination process statically: we calculated the volume of the ionized bubble at a certain redshift, then counted the number of minihalos in this region at that time, and then we calculated the number of recombinations for these minihalos until they were all photoevaporated. This treatment might result in a slightly overestimate in the number of recombinations because some of the bubbles are formed by the growth and merger of smaller bubbles that were formed at earlier time. In these regions, the minihalo formation has been suppressed. To check whether neglecting the bubble-growth history would alter our results, we make a simplifying ansatz of bubble growth. We assume that, for a given bubble at redshift $z$, the different parts of its volume were acquired (i.e., first ionized) at different redshifts $z' > z$, and the contribution at $z'$ is proportional to $df_{\text{coll}}/dz(z')$. With this assumption of bubble-growth history, the suppression of minihalo formation in a previously ionized region can be taken into account, and the average number of recombinations within the bubbles is given by

$$\bar{n}_{\text{rec, MH}} = \frac{1}{f_{\text{coll}}(z)} \int n_{\text{rec, MH}}(z') \frac{df_{\text{coll}}(z')}{dz'} dz',$$

where $n_{\text{rec, MH}}$ on the right-hand side is given by Equation (11). We then calculate the evolution of $Q_V$ with this new recombination number. However, we find that the resulting difference is very small. This is not surprising since $df_{\text{coll}}/dz$ rapidly increases as the redshift decreases in the calculation of $n_{\text{rec, MH}}(z)$; $n_{\text{rec, MH}}(z' \sim z)$ contributes the most to the integration.

Finally, Gao & Theuns (2007) suggested that, in the WDM models, unlike the case of the CDM models, the first stars do not necessarily form in halos, but they may instead form in filaments. They also argued that, with this new way of star formation, the first stars may have smaller typical mass ($\sim M_\odot$) and the global star formation rate could be even higher. This also raises the interesting possibility that the reionization could occur earlier in the WDM model than in the CDM model. However, whether this new formation mechanism would indeed work as they proposed is still not completely clear, because, presently, it is very difficult to model the star formation process at a sufficient resolution to really check the outcome of the filament star formation process. Here, we have maintained the standard view and assumed that the stars only formed in dark matter halos. We note that, if the stars indeed form in the way suggested by Gao & Theuns (2007), the reionization in the WDM model would occur at...
even higher redshifts, and there is nothing incompatible with the recombination effects we discussed.

In summary, we find that the completion of reionization is not always delayed in WDM models. In some cases, when the WDM mass is not too low, it could even be advanced due to the reduction of recombination rates. This effect is relatively small compared with the uncertainties in present observations and theoretical models. We also find that, for $\zeta = 40$ to be consistent with the observations of the Gunn–Peterson trough in $z \sim 6$ quasar spectra, the mass of dark matter particles should be greater than $1.3 \text{ keV}$. However, if more ionizing photons were produced and escaped into the IGM, the dark matter could be warmer, i.e., have a smaller mass. However, for dark matter particles that are less than 0.5 keV, $\zeta > 500$ is needed. This would only be possible if the reionization photons are primarily contributed by metal-free or massive extremely metal-poor stars.

We thank Yidong Xu and Liang Gao for helpful discussions. This work is supported by the 973 project under grant 2007CB815401, NSFC grant 11073024, the John Templeton Foundation, and the CAS knowledge innovation program.

REFERENCES

Abazajian, K. 2006, Phys. Rev. D, 73, 063513
Abazajian, K., Fuller, G. M., & Tucker, W. H. 2001, ApJ, 562, 593
Alvarez, M. A., & Abel, T. 2010, arXiv: 1003.6132
Bardeen, J. M., Bond, J. R., Kaiser, N., & Szalay, A. S. 1986, ApJ, 304, 15
Barkana, R., Haiman, Z., & Ostriker, J. P. 2001, ApJ, 558, 482
Barkana, R., & Loeb, A. 1994, Phys. Rev. Lett., 72, 17
Barkana, R., & Loeb, A. 2001, Phys. Rep., 349, 125
Becker, R. H., Fan, X., White, R. L., et al. 2001, AJ, 122, 2850
Benson, A. J., Nusser, A., Sugiyama, N., & Lacey, C. G. 2001, MNRAS, 320, 153
Biermann, P. L., & Kusenko, A. 2006, Phys. Rev. Lett., 96, 091301
Blumenthal, G. R., Pagels, H., & Primack, J. R. 1982, Nature, 299, 37
Bode, P., Ostriker, J. P., & Turok, N. 2001, ApJ, 556, 93
Bond, J. R., Cole, S., Efstathiou, G., & Kaiser, N. 1991, ApJ, 379, 440
Bromm, V., & Yoshida, N. 2011, ARA&A, 49, 373
Chen, X., & Kamionkowski, M. 2004, Phys. Rev. D, 70, 043502
Chuzhoy, L. 2008, ApJ, 679, L65
Ciardi, B., Scannapieco, E., Stoehr, F., et al. 2006, MNRAS, 366, 689
Colín, P., Avila-Reese, V., & Valenzuela, O. 2000, ApJ, 542, 622
Colombi, S., Dodelson, S., & Widrow, L. M. 1996, ApJ, 458, 1
Dodelson, S., & Widrow, L. M. 1994, Phys. Rev. Lett., 72, 17
Fan, X., Carilli, C. L., & Keating, B. 2006, ARA&A, 44, 415
Fan, X., Narayanan, V. K., Strauss, M. A., et al. 2002, AJ, 123, 1247
Furlanetto, S. R., & Oh, S. P. 2005, MNRAS, 363, 1031
Furlanetto, S. R., Oh, S. P., & Pierpaoli, E. 2006, Phys. Rev. D, 74, 103502
Furlanetto, S. R., Zaldarriaga, M., & Hernquist, L. 2004a, ApJ, 613, 1
Furlanetto, S. R., Zaldarriaga, M., & Hernquist, L. 2004b, ApJ, 613, 16