A study of the tunnelling-charging Hamiltonian of a Cooper pair pump

M. Aunola
Dept. of Physics, University of Jyväskylä, P.O. Box 35 (Y5), FIN-40351 Jyväskylä, Finland
Email: Matias.Aunola@phys.jyu.fi

(October 24, 2018)

General properties of the tunnelling-charging Hamiltonian of a Cooper pair pump are examined with emphasis on the symmetries of the model. An efficient block-diagonalisation scheme and a compatible Fourier expansion of the eigenstates is constructed and applied in order to gather information on important observables. Systematics of the adiabatic pumping with respect to all of the model parameters are obtained and the link to the geometrical Berry’s phase is identified.

Adiabatic transport of single electrons in arrays of small metallic tunnel junctions has been widely studied in recent years. In the Coulomb blockade regime phase-shifted gate voltages have been used to induce a dc current \( I = -nef \), where \( n \) is the number of carried electrons and \( f \) is the gating frequency. Normal-state pumps transporting single electrons have reached accuracy that can be considered for metrological applications. Pumping of Cooper pairs has gained interest due to new ideas in quantum measuring and computing.

A quantitative theory of pumping Cooper pairs in gated arrays of Josephson junctions when the environment has negligible impedance has been presented. The leading order pumped current is \( I \approx -2ef[1 - a_N(\varepsilon_j)^{N-2}\cos \phi] \), where \( a_N \) is a constant and \( \varepsilon_j := E_1/E_C \) is the coupling strength. Here \( E_1 \) and \( E_C := (2\varepsilon)^2/2C \) are the Josephson coupling energy and the charging energy, respectively. According to recent calculations the \( \cos \phi \)-dependent inaccuracy should be experimentally observable at least in a certain frequency range. In this article the model for adiabatic pumping from Refs. 3 and 4 is examined in a thorough manner, concentrating on the symmetries and general features of the simplified system, thus extending the treatment to strong couplings and long arrays. The overall behaviour of the pumped current is explained and systematised, but not rigorously proven.

A schematic view of a Cooper pair pump and the ideal operation of the gate voltages \( V_{\ell,k} \) are shown in Fig. 1. On any of the \( N \) legs of a cycle at most two gate voltages are changed. The tunnelling-charging Hamiltonian

\[
H = H_C + H_3, \tag{1}
\]

neglects the quasiparticle tunnelling and other degrees of freedom. The full set of model parameters is \( \varepsilon_j \), total phase difference over the array \( \phi \), the relative junction capacitances \( c_k \), where \( c_k := C_k/C \) and \( \sum_{k=1}^{N} C_k = N/C \), and the (normalised) gate charges \( \tilde{q}_k := \{ q_1, \ldots, q_{N-1} \} \), where \( q_k := -C_{g,k}V_{g,k}/2e \). For homogeneous arrays \( c_k := 1 \) and the inhomogeneity is quantified by \( X_{\text{inh}} = \sqrt{\sum_{k}(c_k^{-1} - 1/n)^2/N}^{1/2} \). The model Hamiltonian is diagonal with respect to \( \phi \), which is assumed to be fixed, but could be controlled by an external bias voltage according to \( d\phi/dt = -2eV/h \) and is subject to any voltage fluctuations. The conjugate variable \( \dot{\phi} \), the average number of tunnelled Cooper pairs, is undetermined in the present model.

The matrix elements of the charging Hamiltonian \( H_C \) are given by the capacitive charging energy

\[
\langle \tilde{n}\rangle H_C(\tilde{q})|\tilde{n}\rangle = EC \left[ \sum_{k=1}^{N} \frac{v_k^2}{c_k} - \frac{1}{N} \left( \sum_{k=1}^{N} v_k c_k \right)^2 \right], \tag{2}
\]

where the number of Cooper pairs on each island is given by \( \tilde{n} \). The quantities \( v_k, k = 1, \ldots, N \), are a solution of

\[
v_k - v_{k+1} = n_k - q_k. \tag{3}
\]

Tunnelling of one Cooper pair through the \( k \)th junction changes \( |\tilde{n}\rangle \) by \( \delta_k \), where the non-zero components are (if applicable) \( (\delta_k)_{k} = 1 \) and \( (\delta_k)_{k-1} = -1 \). The tunnelling Hamiltonian then reads

\[
H_3 = -\sum_{\tilde{n},k=1}^{N} \frac{c_k E_1}{2} \langle |\tilde{n} + \delta_k\rangle |\tilde{n}\rangle e^{i\phi/N} + \text{H.c.}. \tag{4}
\]

The supercurrent flowing through the array is determined by the supercurrent operator

\[
\begin{align*}
\langle \tilde{n}\rangle H_C(\tilde{q})|\tilde{n}\rangle = EC \left[ \sum_{k=1}^{N} \frac{v_k^2}{c_k} - \frac{1}{N} \left( \sum_{k=1}^{N} v_k c_k \right)^2 \right],
\end{align*}
\]

where the number of Cooper pairs on each island is given by \( \tilde{n} \). The quantities \( v_k, k = 1, \ldots, N \), are a solution of

\[
v_k - v_{k+1} = n_k - q_k.
\]

Tunnelling of one Cooper pair through the \( k \)th junction changes \( |\tilde{n}\rangle \) by \( \delta_k \), where the non-zero components are (if applicable) \( (\delta_k)_{k} = 1 \) and \( (\delta_k)_{k-1} = -1 \). The tunnelling Hamiltonian then reads

\[
H_3 = -\sum_{\tilde{n},k=1}^{N} \frac{c_k E_1}{2} \langle |\tilde{n} + \delta_k\rangle |\tilde{n}\rangle e^{i\phi/N} + \text{H.c.},
\]

The supercurrent flowing through the array is determined by the supercurrent operator.
\[ I_5 = (-2e/h)(\partial H/\partial \phi), \] (5)

These terms represent the interaction of the charge due to direct supercurrent, \( Q \), and the change of the gate voltages, \( \Delta V \), with properties \( \tilde{m} \). The canonical representation \( \hat{M} = -i\partial \hat{H}/\partial \phi \) connects the pumped charge to the average number of tunnelled Cooper pairs by

\[ Q_p = 2 \oint_{\Gamma} \text{Re} \left[ \langle m | M | dm \rangle \right]. \] (6)

A single eigenstate for phase differences \( \phi \) and \( \phi + d\phi \) is required for each integration point. This expression identifies \( \hat{M} \) as the link between \( Q_p \) and the geometrical Berry’s phase, \( \gamma_m(\Gamma) = i \oint_{\Gamma} \langle m | dm \rangle \). The connection was given without an explicit identification in Ref. 3 and was mentioned in Ref. 11. For \( \varepsilon \to 0 \) only two charges are of importance during each leg. From the wave function \( |m\rangle = \left( \begin{array}{c} 1 - a^2 \alpha e^{i\phi}/N \end{array} \right)^T \), where \( a : 0 \to 1 \), one obtains \( \langle m | M | dm \rangle = d(a^2)/2N \) and \( Q_p = 1 \) for a full cycle.

A reference state \( |\tilde{n}\rangle \) conveniences a better labelling of the charge states. Each state is denoted by integers \( \{y_k\}_{k=1}^{N} \), \( 0 \leq Y_k := \sum_k y_k < N \), such that \( \tilde{n} = n_0 + \sum_k y_k \delta_k \). The numbers \( y_k \) tabulate the number and direction of tunnelings from \( n_0 \) to \( \tilde{n} \). The distance between charge states is defined by \( d(\tilde{n}_1, \tilde{n}_2) := \min \left( \sum_{k=1}^{N} |y_k(1) - y_k(2)| + l : l \in \mathbb{Z} \right) \). The \( d(\tilde{n}_1, \tilde{n}_2) = 1 \) \( (l = 1) \), then \( |\tilde{n}_1\rangle \) and \( |\tilde{n}_2\rangle \) are (th) nearest neighbours.

A change of basis \( \{ |\tilde{n}\rangle \} \rightarrow \{ e^{i\phi Y_k}/\sqrt{N} |\tilde{n}\rangle \} \) yields a Hamiltonian matrix \( \hat{H} = UHU^{-1} \) with properties \( \hat{H}(\phi + 2\pi) = \hat{H}(\phi) \) and \( \hat{H}(-\phi) = (\hat{H}(\phi))^* \). Thus eigenstates and eigenvalues of \( \hat{H} \) are periodic under \( 2\pi \), or state labels change cyclically. Usually the former happens, so the supercurrent in a stationary state is described by a Fourier sine series

\[ \langle I_5 \rangle_{\{N, \varepsilon, \phi, \varepsilon, \phi\}} := \sum_{l=\pm\infty} \alpha_l \sin(l\phi), \] (9)

The ground state supercurrent behaves differently only at the so-called resonance points, where the ground state becomes degenerate for \( \phi = \pi + 2l\pi, l \in \mathbb{Z} \). For homogeneous arrays resonance points are located at \( \tilde{q} = n \pm (1, \ldots, 1)/N \), where \( n \) is arbitrary. The corresponding poles in the Berry’s phase give raise to \( Q_p \), and \( \varepsilon \) determines which poles are important.

Due to symmetry of the representation \( \hat{H} \) the original amplitudes in \( 2\pi \)-periodic states are given by

\[ a_{\tilde{n}}^q = \sum_{l=-\infty}^{\infty} a_{\tilde{n},l}^q e^{i\phi(l+Y_\tilde{n}/N)}, \] (10)

where real Fourier coefficients \( a_{\tilde{n},l}^q \) are fixed by the gauge condition \( a_{\tilde{n},l}^{q=0} := |a_{\tilde{n}}^q|^e^{i\phi Y_\tilde{n}/N} \) for the charge state \( |\tilde{n}\rangle \). The averaged number of tunnelled Cooper pairs,

\[ \mathcal{M} := \sum_{\tilde{n},l} (l + Y_\tilde{n}/N)(a_{\tilde{n},l}^q)^2, \] (11)

is unique up to a gauge-dependent integer. The apparent contradiction between sharp phase difference combined with sharp value of \( \mathcal{M} \) is an artefact due to gauge-fixing. Stronger coupling increases the variance of \( \mathcal{M} \). Discontinuities in gauges certainly occur on any closed path encircling an odd number of resonance points. A gauge is unstable near a discontinuity, but everywhere away from resonance points many valid gauges exist. Especially on the gating path depicted in Fig. 1, the dominant charge states on each leg give very stable gauges.

The pumped charge, \( Q_p \), can be evaluated using a gauge-independent differential expression

\[ dQ_p(\phi) = \sum_{l=0}^{\infty} \sum_{\tilde{n},l} \left[ 2(l + Y_\tilde{n}/N)/1 + \delta(l\phi + 2\pi) d(a_{\tilde{n},l}^q a_{\tilde{n},l+\nu}^q + \text{c.c.}) \right] \cos(l\phi), \] (12)

where the \( \phi \)-independent average is simply \( d\mathcal{M} \). Due to the normalisation of \( |m\rangle_{\phi} \), the coefficients are orthonormal, i.e. \( \sum_{\tilde{n},l} a_{\tilde{n},l}^q a_{\tilde{n},l+\nu}^q = \delta(\nu) \), which cancels the terms multiplying the full differential by \( l \). Expression (12) indicates that the averaged charge transfer for a full cycle is exactly \(-2e\), regardless of the inhomogeneity of the array or reasonable deformations of the gating path.

The tunnelling-charging Hamiltonian can often be block diagonalised with the following transformation. Let an orthonormal basis \( \{|s\rangle\} \) span the Hilbert space \( \mathcal{H} \) and the matrix elements of a Hamiltonian \( H \) be \( h_{s,s'} := \langle s | H | s' \rangle \). Choose projection operators \( \{ P_i \} \) by \( P_i = \sum_{k=1}^{d_i} \delta_{k} \langle i_k | i_k \rangle \) and require that \( \sum_{i} P_i P_j = \sum_{i} P_i = 1 \). If all of the row sums

\[ W_{ij,k} := \sum_{k'=1}^{d_j} h_{ik,jk'}, \quad k = 1, \ldots, d_i, \] (13)

are independent of \( k \), the Hamiltonian commutes with the projection operator \( P := \sum_{i} |\psi_i\rangle \langle \psi_i| \), where \( |\psi_i\rangle = d_i^{-1/2} \sum_{k=1}^{d_i} |i_k\rangle \). Thus \( H \) can be written as a direct sum \( H = H_P \oplus H_{1-P} \) with matrix elements of \( H_P \) given as

\[ h_{ij} := \langle \psi_i | H | \psi_j \rangle = W_{ij}(d_j/d_i)^{1/2} = W_{ji}^*(d_j/d_i)^{1/2}. \] (14)
The block-diagonalising transformation is not specific to the present problem, as it amounts to extracting states with a specific symmetry from all of the system’s eigenstates. The ground state of $H_C + H_I$ is always an eigenstate of $H_P$, because $\langle \hat{\epsilon}_l^q \phi = 0 \rangle > 0$ for all $\hat{\epsilon}_l$ and $\phi$. The Fourier expansion \( \langle \hat{\epsilon}_l^q \rangle \) can be used simultaneously if each subspace label, $i$, corresponds to a single value of $\hat{\epsilon}_l^q$.

For homogeneous arrays at $\hat{\epsilon}_l = \hat{n}_0$ all junctions are indistinguishable in terms of the charging energy. The subsystem labels are of the form $\hat{n}_z$ with $d \hat{n}_z = N!/(\prod_{j=1}^{m_{\text{max}}} m_j!)$. This subspace contains eigenstates carrying the label
\[
\vec{y} = (z_1^{(1)}, \ldots, z_1^{(m_1)}, z_2^{(1)}, \ldots, z_2^{(m_2)}, \ldots, z_{\text{max}}^{(m_{\text{max}})}),
\]
where $z_1 < \cdots < z_{\text{max}}$ and $m_1 + \cdots + m_{\text{max}} = N$, or any distinct permutation of $\vec{y}$. Thus only the number and multiplicity of tunnellings is of importance.

On the gating path the gate charges can be written as $\vec{q} = (1 - x) \hat{n}_0 + x \bar{r}_1$, where $\hat{n}_0$ ($\bar{r}_1$) is the initial (final) optimal charge state for the given leg. Thus one junction becomes distinguishable and subsystem labels can be chosen as $\hat{n}_0$; $\bar{r}_1$, where $\bar{r}_1$ refers to the remaining $N - 1$ components, $0 \leq \hat{n}_0 + \sum_{i=1}^{N-1} m_i z_i < N$ and $d_{\hat{n}_0; \bar{r}_1} = (N - 1)!/(\prod_{j=1}^{m_{\text{max}}} m_j!)$. The diagonal and non-zero off-diagonal matrix elements of $H_P$ are the common charging energies and
\[
- (\varepsilon_m)_{\hat{n}_0; \bar{r}_1} / e^{+i \varepsilon_m / N} (d_{\hat{n}_0; \bar{r}_1}) d_{\hat{n}_0; \bar{r}_1} / e^{-i \varepsilon_m / N} (d_{\hat{n}_0; \bar{r}_1})^{1/2}
\]
with $n_0 = 1$ when applicable, respectively. The block-diagonalised matrices are sparser than the original matrices and the amplitudes in eigenvectors are multiplied by $(d_{\hat{n}_0; \bar{r}_1})^{1/2}$ due to combining of several amplitudes into one. A further block diagonalisation is possible at $\vec{q} = \hat{n}_0$ and $\vec{q} = (\hat{n}_0 + \bar{r}_1)/2$ if $\phi$ is a multiple of $\pi$.

A truncation picks the charge states required for reliable evaluation of eigenstates and observables. First an initial truncation, a set of states $B_1 = \{|\bar{r}_j\rangle\}$, is chosen. Its extensions, bases $B_l^{(i)}$, contain all neighbours up to and including $l$th nearest neighbours of each and every $|\bar{r}_j\rangle$. For non-ideal cycles and/or inhomogeneous arrays the initial truncation is the “b basis” of Ref. [3] which reproduces leading order supercurrent and inaccuracy. A “c basis” is just the first extension of a “b basis”, i.e. “b basis$^{(1)}$”.

For ideal cycles and homogeneous arrays the optimal $B_0$ truncation corresponds to $3N - 2$ labels, that is $0; (j, N - 1 - j)_{(0,1)}$, “1; (j, N - 1 - j)_{(-1,0)}”, $j = 0, \ldots, N - 1$, and $1; (j, N - 1 - j)_{(0,1)}$, $j = 1, \ldots, N - 2$. These $3N - 1 - 2$ charge states actually contribute to the leading order supercurrent and inaccuracy. More restricted, few-state truncations $B_1 = \{|\bar{r}_j\rangle\}$ and $B_2 = \{|\bar{r}_j\rangle, |\bar{r}_{j+1}\rangle\}$ are of use especially when the ground state energy is sought. The efficiency of the block diagonalisation for bases $B_0^{(i)}$ is shown in Table 1 where the number of labels is compared against the number of included charge eigenstates. Examples of bases $B_1^{(i)}$ and $B_2^{(i)}$ are also given.

**Table I.** The number of subspace labels (sl) and included charge states (ch) for bases $B_0^{(i)}$ with several $N$ and examples of bases $B_1^{(i)}$ and $B_2^{(i)}$.

| $N_{0/1/2}$ | sl | ch | $sl_{1/2}$ | ch |
|-------------|----|----|----------|----|
| 40          | 522| 182| 1872,14  | 9572|
| 50          | 822| 626| 3082,1   | 46400|
| 60          | 1142| 1918| 3468,9 | 1.56 · 10^5 |
| 70          | 1462| 5428| 3941,4 | 5.22 · 10^5 |
| 80          | 1782| 14498| 3648,7 | 1.30 · 10^6 |
| 90          | 2102| 37082| 2890,6 | 2.44 · 10^6 |
| 81          | 69,5| 12331| 14727,29 | 6.15 · 10^7 |
| 72          | 2485, | 9452| 12842,14 | 2.19 · 10^6 |

As a concrete example, take basis $B_1^{(i)}$ and the subspace labels $N_0, (N - 1, 1)_{(0,1)}$ and $(1, N - 1)_{(0,1)}$, standing for $2N + 1$ charge eigenstates at $\vec{q} = \hat{n}_0$. The block-diagonalised Hamiltonian obtained from Eqs. (13) and (14) is given by
\[
H_{\text{bd}} = \begin{pmatrix}
0 & K_N^* & K_N
K_N & (N - 1)/N & 2\delta_{N3}K_N/\sqrt{N}
K_N^* & 2\delta_{N3}K_N/\sqrt{N} & (N - 1)/N
\end{pmatrix}
\]
where $K_N := - (\varepsilon_m/2)e^{i \phi / \sqrt{N}}$. For $N = 3$ the leading order supercurrent for $\varepsilon_m \ll 1$ is already reproduced. Note that labels $(N - 1, 1)_{(0,1)}$ and $(1, N - 1)_{(0,1)}$ can be joined, if $\phi$ is a multiple of $\pi$.

The discussion has now lead to the main results of the paper, the general systematics of the pumped charge. For ideal gating sequences $dM = 1/N$ for each leg due to symmetry. Corresponding integrated pumped charge reads
\[
Q_p(N, \varepsilon, \phi, \bar{c}, \text{leg}) := 1/N + \sum_{l=1}^{\infty} b_l, \text{leg} \cos(\phi),
\]
which is always positive. Replace $1/N$ by $dM$ for non-ideal cycles. When performing numerical calculations, the basis must contain all of the important charge states and the number of angles used in fast Fourier transform, $2^l$, must be large enough not to allow misidentification of non-negligible components $a_{\bar{r}_l}$ and $a_{\bar{r}_{l+1}}$ as a single component. Failure to satisfy these requirements causes systematical error due to incorrect amplitudes and spurious interference between Fourier components in Eq. (15), respectively. The integration points should be chosen so that the magnitude of norms $\| dm \|$ is neither too large nor varies too much.

The rate of convergence for each $\cos(l\phi)$-dependent term in Eq. (15) behaves as $1/(\#\text{steps})^2$. For a reasonable choice of integration points 300–500 steps per leg, much less than used in Refs. [3] and [1], usually gives relative precision of the order of $10^{-5}$ (disregarding the systematical error) for coefficients with magnitude greater than $10^{-8}$. This greatly diminishes convergence problems due
to smallness of $da_{i,l}$ for large bases and large number of angles. The expression $2l' da_{i,l}da_{i,l+1}$ has been used as the latter part of Eq. (12) for many of the data points. In this case the rate of convergence behaves as $1/#$ steps. Bases $B_2^{(l)}$ have also been used when calculating $Q_p$.

The sequence of the Fourier coefficients \{$b_{l,\text{leg}}$\}$_{l=0}^{\infty}$, where $b_0 = 2/N$, is alternating and decreasing in magnitude. Furthermore, the ratios $\bar{\beta}_l := |b_l/b_{l-1}|$, $l = 1, 2, \ldots$ form a decreasing sequence with limiting value $\bar{\beta} := \lim_{l \to \infty} \bar{\beta}_l \geq b_0\bar{\beta}_1$. Because $\partial Q_p/\partial \phi > 0$ in the range $\phi \in (0, \pi)$, $Q_p$ is bounded from above by $[2(1 - \bar{\beta})^{-1} - 1]/N$ and from below by $1/N - 2\bar{\beta}_l/(N + 2\bar{\beta}_l)$. For finite values of $\varepsilon_j$ the ratios $\bar{\beta}_l$ and $\bar{\beta}/\beta_1$ are monotonously increasing functions of $\varepsilon_j$ with limiting value of unity, because $Q_p(\phi = 0, \varepsilon_j \to \infty) \searrow 0$, which enforces a stricter limit for the ratio $\bar{\beta}/\beta_1$. This behaviour is explained by increasing long-range (high-$l'$) correlations in Eq. (12) or actually in the state itself. On the other hand, the ratios are monotonously decreasing functions of $N$ for fixed $\varepsilon_j$ as correlations are weakened in longer arrays. All of the above-mentioned features are clearly shown in Table II, where $\beta_1$ for the homogeneous case are depicted as function of $N$ at $\varepsilon_j = 0.4$ and as function of $\varepsilon_j$ for $N = 4.4$. These properties of the pumped charge are rather robust against relatively small systematical errors.

For small values of $\varepsilon_j \ll 1$ one has $\beta_{j+1} \approx b_0\beta_1$ and $\beta_1 \sim (N\varepsilon_j/2)^{N-2}[N(N - 1)/2(N - 2)]!$. The effects due to inhomogeneity, that is the relative sizes of $b_{l,\text{leg}}$, can estimated by fixing the the leg index $r$ in the inhomogeneity prediction, Eq. (37) of Ref. 3. The corresponding results work quite well, only slightly overestimating the ratio between largest and smallest coefficients, even for large inhomogeneities $X_{\text{inh}}$. The total transferred charge $Q_p = \sum_{\text{leg}} Q_{p,\text{leg}}$ for inhomogeneous arrays does not have to satisfy $\beta_l \geq \beta_{l+1}$, although always $-b_{l+1}/b_l \leq 1$. This symmetry is also broken by non-ideal gating sequences, even for single legs.

In order to conclude, the properties of the tunnelling-charging Hamiltonian of a Cooper pair pump have been examined using an efficient block-diagonalisation scheme and a compatible Fourier expansion of the eigenstates. Explicit enforcement of the model symmetries produces strong systematics of the pumped charge, even if the structure of the Fourier coefficients $\{b_{l,\text{leg}}\}^{\infty}_{l=0}$ was not exhaustively proven. These properties are possibly related to the dynamical algebra of single and coupled Josephson junctions described in Ref. 3, which offers a complementary view of the present problem in case of a superconducting loop.

**ACKNOWLEDGMENTS**

This work has been supported by the Academy of Finland under the Finnish Centre of Excellence Programme 2000-2005 (Project No. 44875, Nuclear and Condensed Matter Programme at JYFL). The author thanks Mr. J. J. Toppari for insightful comments.

1. H. Pothier, P. Lafarge, C. Urbina, D. Esteve and M.H. Devoret, Europhys. Lett. 17, 249 (1992).
2. M. W. Keller, J. M. Martinis, N. M. Zimmerman and A.H. Steinbach, Appl. Phys. Lett. 69, 1804 (1996); M.W. Keller, J.M. Martinis and R.L. Kautz, Phys. Rev. Lett. 80, 4530 (1998); M. W. Keller, A. L. Eichenberger, J. M. Martinis and N. M. Zimmerman, Science 285, 1716 (1999).
3. D. V. Averin, Solid State Commun. 105, 659 (1998).
4. A. Shnirman and G. Schön, Phys. Rev. B 57, 15400 (1998).
5. J. P. Pekola, J. J. Toppari, M. Aunola, M. T. Savolainen and D. V. Averin, Phys. Rev. B 60, R9931 (1999).
6. M. Aunola, J. J. Toppari and J. P. Pekola, Phys. Rev. B 62, 1296 (2000).
7. J. P. Pekola and J. J. Toppari, submitted to Phys. Rev. Lett. (2000).
8. G.-L. Ingold and Yu. V. Nazarov, in *Single Charge Tunneling, Coulomb Blockade Phenomena in Nanostructures*, eds. H. Grabert and M.L. Devoret, (Plenum Press, New York, 1992).
9. W. Rudin, *Functional Analysis*, (McGraw Hill, 1973).
10. M. V. Berry, Proc. R. Soc. Lond. A 392, 45 (1984).
11. G. Falcì, R. Fazio, G.M. Palma, J. Siewert, and V. Vedral, Nature 407, 355 (2000).
12. To some degree already in matrix elements $\langle m|\hat{M}|dm\rangle$.
13. The full data set and some tools for reproducing the data is available at [http://www.jyu.fi/~mimau/Cooper/](http://www.jyu.fi/~mimau/Cooper/).
14. E. Celeghini, L. Faoro, and M. Rasetti, Phys. Rev. B 62, 3054 (2000).