A dynamical study of the Kugo-Ojima function

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Abstract. As has been recently realized, a certain two-point function \( \Lambda_{\mu\nu} \) – and its associated form factors \( G \) and \( L \) – play a prominent role in the PT-BFM formulation of the Schwinger-Dyson equations used to study gauge-invariantly the gluon and ghost propagators. After showing that in the (background) Landau gauge \( \Lambda_{\mu\nu} \) fully constrains the QCD ghost sector, we show that \( G \) coincides with the Kugo-Ojima function \( u \), whose infrared behavior has traditionally served as the standard criterion for the realization of the Kugo-Ojima confinement mechanism. The determination of the behavior of \( G \) for all momenta through a combination of the available lattice data on the gluon and ghost propagators, as well as the dynamical equation \( G \) satisfies, will be then discussed. In particular we will show that in the deep infrared the function deviates considerably from the value associated with the realization of the Kugo-Ojima confinement scenario; the dependence on the renormalization point of \( u \), and especially of its value at \( q^2 = 0 \), will be also briefly discussed.

Keywords: Pinch technique, Background field method, Non-perturbative QCD

PACS: 11.15.Tk 12.38.Lg, 12.38.Aw

Over the last few years large volumes ab-initio lattice gauge theory computations have established beyond any reasonable doubt that the gluon propagator and the ghost dressing function of pure Yang-Mills theories in the Landau gauge saturates in the deep infrared (IR) at a finite, non-vanishing value both for SU(2) [1] and SU(3) [2] gauge groups. Specifically choosing the \( R_\xi \) Landau gauge and defining the gluon propagator cofactor \( \Delta \), and the ghost dressing function \( F \) as

\[
\Delta_{\mu\nu}(q) = -i P_{\mu\nu}(q) \Delta(q^2), \quad D(q^2) = \frac{F(q^2)}{q^2}, \quad (1)
\]

where \( P_{\mu\nu}(q) = g_{\mu\nu} - q_\mu q_\nu / q^2 \) is the transverse projector, and \( D(q^2) \) the ghost propagator, the aforementioned lattice results tell us that (in Euclidean space)

\[
\Delta^{-1}(0) > 0, \quad \text{and} \quad F(0) > 0. \quad (2)
\]

In the continuum formulation, the only way of obtaining these so-called massive solutions in a gauge invariant way and without breaking (either explicitly or softly) the BRST symmetry of the original Yang-Mills action, is within the PT-BFM framework [3], where a truncation scheme that respects gauge invariance at every level of the dressed-loop expansion has been developed in [4] using the systematic rearrangement of the entire Schwinger-Dyson series allowed by the pinch technique [5, 6, 7].

In the PT-BFM construction one studies the PT-BFM propagator \( \Delta \) which is related to the conventional propagator of Eq. (1) through the background-quantum identity [8]

\[
\hat{\Delta}(q^2) = [1 + G(q^2)]^2 \Delta(q^2), \quad (3)
\]

\( \Lambda_{\mu\nu}(q) \)

where \( G \) is the \( g_{\mu\nu} \) form factor appearing in the Lorentz decomposition of the auxiliary Green’s function \( \Lambda_{\mu\nu} \) defined as (see Fig. 1)

\[
\Lambda_{\mu\nu}(q) = \int k D(k + q) \Delta_{\mu\nu}(k) H_{\sigma\nu}(k, q)
\]

\[
= g_{\mu\nu} G(q^2) + q_\mu q_\nu L(q^2). \quad (4)
\]

The function \( H_{\mu\nu} \) appearing above (Fig. 1 again) is in fact a familiar object, since it appears in the all-order Slavnov-Taylor identity satisfied by the standard three-gluon vertex. It is also related to the full gluon-ghost vertex \( \Gamma_{\mu} \) by the identity \( q^2 \Delta(\mu, q) = -i \Gamma_{\mu}(k, q) \), at tree-level, \( H^{(0)}_{\mu\nu} = i g_{\mu\nu} \) and \( \Gamma^{(0)}_{\mu}(k, q) = \Gamma_{\mu}(k, q) = -q_\mu \).

The \( G \) and \( L \) form factors play a prominent role in the (background) Landau gauge, where the presence of an extra local functional equation (the so-called antighost

FIGURE 1. The auxiliary functions \( \Lambda_{\mu\nu} \) and \( H_{\sigma\nu} \) appearing in the PT-BFM framework.

\[
\Lambda_{\mu\nu}(q) = \int k D(k + q) \Delta_{\mu\nu}(k) H_{\sigma\nu}(k, q)
\]

\[
= g_{\mu\nu} G(q^2) + q_\mu q_\nu L(q^2). \quad (4)
\]
a relation that is valid also in the conventional Landau gauge [10]. Since, under very general conditions on the
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gluon and ghost propagators, \( L(q^2) \to 0 \) when \( q^2 \to 0 \)
one has the IR relation \( F^{-1}(0) = 1 + G(0) \). Thus, we
see that a divergent – or enhanced [11] – dressing function
requires the condition \( G(0) = -1 \). To the practitioners,
this latter condition will look suspiciously similar to
the Kugo-Ojima (KO) confinement criterion which de-
mands (as a necessary condition for confinement through
the so-called \textit{quartet mechanism}) that a certain func-
tion \( u(q^2) \) (the KO function) acquires the IR value
\( u(0) = -1 \) [13]. Indeed, it is possible to show that \( G \) is
nothing but the KO function [9, 12]
\[
u(q^2) \equiv G(q^2).
\]
Therefore the form factor \( G \) encodes practically all rele-

cant information on the IR dynamics of the ghost sector, and,
at least partially, the gluon sector as well [through the
the identity (3)].

Approximating the three point functions \( \Gamma_\mu \) and \( H_{\mu\nu} \)
with their tree-level value (the first approximation on the
ghost-gluon vertex being supported by lattice studies), the
dynamical equations satisfied by \( G \) and \( L \) read
\[
G(q^2) = \frac{g^2 C_A}{3} \int k \left[ \frac{2 + (k \cdot q)^2}{k^2 q^2} \right] \Delta(k) D(k + q),
\]
\[
L(q^2) = \frac{g^2 C_A}{3} \int k \left[ 1 - 3 \frac{(k \cdot q)^2}{k^2 q^2} \right] \Delta(k) D(k + q).
\]

Then, since within our approximation scheme, in the

equations above only \( \Delta, F \) (through the ghost propagator
\( D \) ) and \( g \) appears, in order to determine the behavior of \( G 
and \( L \) one can fully exploit the available lattice data on \( \Delta \)
and \( F \), through the following general strategy [12, 14].

One starts by using the lattice gluon propagator as an input
for the ghost SDE; then solves for the ghost dressing
function, tuning the coupling constant \( g \) such that
the solution gives the best possible approximation to the
lattice results. Obviously one must check that the cou-
pling so obtained (at the renormalization scale used for
the computation) is fully consistent with known perturba-
tive results (obtained in the MOM scheme, which is the
scheme used in our computations); this is indeed what
happens [12, 14]. At this point the one has the three
building blocks \( \Delta, F \) and \( g \) fully determined, and can start
analyzing other quantities constructed from them such as
the \( G \) and \( L \) form factors above [12] or the renormaliza-
tion group invariant effective charge [14, 15].

Before solving numerically the equations (7), there is
one last issue that needs to be addressed. Specifically,
one needs to identify a renormalization procedure for \( G 
and \( L \) that does not break the identity (5), which, due to
its BRST origin, should not be deformed (within the PT-
BFM scheme) by the renormalization process. Note in
fact that Eq. (5) constrains the cutoff-dependence of the
unrenormalized quantities involved; specifically, denoting by \( Z_\tau \) the ghost wave-function renormalization con-
stant \( (Z_\tau F_0^{-1} = F^{-1}) \) and with \( Z_A \) the (yet unspecified)
renormalization constant of the function \( \Lambda_{\mu\nu}(q) \),
with \( Z_A \{ \delta^{\mu\nu} + \Lambda_0^{\mu\nu} \} = \delta^{\mu\nu} + \Lambda^{\mu\nu} \), one finds that (5) is pre-
served iff \( Z_A = Z_\tau \) [15]; as a result, one finds the relation
\[
Z_\tau (\Lambda^2, \mu^2)[1 + G_0(q^2, \Lambda^2) + L_0(q^2, \Lambda^2)]
= 1 + G(q^2, \mu^2) + L(q^2, \mu^2).
\]

Imposing then the renormalization condition \( F(\mu^2) = 1 \),
going to Euclidean space, setting \( q^2 = x \), \( k^2 = y \) and \( \alpha_s = g^2/4\pi \), and implementing the standard angular approxi-

\[\text{FIGURE 2. Left panel: The form factor } -G(q^2) \text{ determined from Eq. (24) at different renormalization points } \mu \text{ through the procedure described in the text. Right panel: Same as in the previous panel but this time for the } L(q^2) \text{ form factor.}\]
mation, one finds the renormalized equations \cite{12,15}

\begin{align}
1 + G(x) &= Z_c - \frac{\alpha_s C_A}{16\pi} \left[ F(x) \int_0^x \frac{dy}{y} \left( 3 + \frac{x}{3y} \right) \Delta(y) \right. \\
&\quad + \left. \int_x^\infty dy \left( 3 + \frac{x}{3y} \right) \Delta(y)F(y) \right], \\
L(x) &= \frac{\alpha_s C_A}{12\pi} \left[ F(x) \int_0^x \frac{dy}{y^2} \Delta(y) \right. \\
&\quad + \left. \int_x^\infty dy \frac{\Delta(y)F(y)}{y} \right].
\end{align}

Notice that \( L \) is finite, as expected from power counting; in addition, we see (by means of the change of variables \( y = zx \)) that if \( \Delta \) and \( F \) are IR finite, then \( L(0) = 0 \), as mentioned before (notice however that the same result is obtained for scaling solutions \cite{11}, where \( \Delta(y) \sim y^a \) and \( F(x) \sim x^b \), provided that \( a + b > -1 \)).

At this point, all necessary ingredients for determining the functions \( G \) and \( L \) are available. Substituting them into the corresponding equations given in (9), we obtain the solutions shown in Fig. 2, where we see that \( L \) is subdominant, and that indeed it vanishes in the deep IR. Also, the \( \mu \) dependence of the KO function and the KO parameter are clearly shown; for the range of renormalization points \( \mu \) chosen, the KO function saturates in the deep IR to the value \( G(0) \approx -0.6 \) which deviates irremediably from the value \( G(0) = -1 \) required for the realization of the KO confinement scenario.

The curves plotted for \( G \) should finally be compared with those obtained on the lattice in \cite{16}, where the KO function \( u \) was studied in terms of Monte Carlo averages, and its asymptotic behavior inferred from the identity (5). Even though in \cite{16} the extrapolation towards the zero limit was problematic, due to a lack of knowledge of the function \( L \) (our analysis does not suffer from such a limitation, given that \( L \) is completely determined by its own equation) and that, for essentially the same reason, the renormalization procedures employed are different, we clearly see in Fig. 3 the same behavior emerging, and in particular the saturation of the KO function in the deep IR to a value sensibly different from the critical \( -1 \).

In conclusion, we have shown that the massive gluon propagator \( \Delta \) and ghost dressing function \( F \) found as solutions to the SDE and confirmed by all large volume lattice simulations up to now, do not support a confinement scenario based on the original KO mechanism/criterion. However, due to the equality (6) between the KO function \( u \) and the auxiliary function \( G \), and the central role that the latter function has in the PT-BFM scheme, it would still be very interesting to carry a thorough study of such function on large volume lattices (for different space-time dimensions and gauge groups).

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure3.png}
\caption{Comparison between our results and direct lattice calculation of the KO function \( -u(q^2) \) at \( \mu = 4 \) GeV.}
\end{figure}

\section*{Acknowledgments}

The author thanks the organizers of Quark Confinement IX International Conference for the hospitality and the very stimulating conference.

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