Chaos of interband cascade lasers in the mid-infrared regime

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Chaos in nonlinear dynamical systems is featured with irregular appearance and with high sensitivity to initial conditions. Near-infrared semiconductor lasers subject to optical feedback from an external reflector are popular chaotic light sources, which have enabled multiple applications. Here, we report the fully-developed chaos in a mid-infrared interband cascade laser with external optical feedback. The chaos leads to significant electrical power enhancement over a frequency span of 500 MHz. In addition, the laser also exhibits periodic oscillations or low-frequency fluctuations before producing chaos, depending on the operation conditions. This work paves the way for extending chaos investigations from the near-infrared regime to the mid-infrared regime, which can stimulate potential applications in this spectral range.
Chaos is a common phenomenon in numerous nonlinear dynamical systems, with features of random appearance and high sensitivity to initial conditions. Since Haken's prediction of chaos in laser systems in 1975, chaotic oscillations have been observed in various types of lasers, including gas lasers, solid-state lasers, fiber lasers, and semiconductor lasers. Among these, semiconductor lasers are the most popular testbed for studying chaotic dynamics, owing to their compactness and the ease of control. Most semiconductor lasers belong to Class-B laser systems, where the carrier lifetime is much longer than the photon lifetime. Consequently, the generation of chaos and other nonlinear dynamics requires some external perturbation, such as external optical or optoelectronic feedback, current or loss modulation, as well as optical injection. However, chaos was also observed in a free-running vertical-cavity surface-emitting laser (i.e., without any external perturbation), and the physical origin was attributed to the nonlinear coupling between the two polarized modes in the vertical cavity. In addition, it has been shown that quantum-dot micropillar lasers operated close to the quantum limit also exhibited chaos, and the synchronization of these coupled lasers were reported as well. The extensive and intensive investigations of chaos in semiconductor lasers have enabled the applications in chaotic secure communications, in random number generations, as well as in light detection and ranging (LIDAR) systems. In recent years, chaos also advances the development of artificial intelligence in the field of optical reservoir computing.

It is worthwhile to point out that chaos produced from the above semiconductor lasers are mostly operated in the near-infrared regime, especially in the O-band (1260-1360 nm) and C-band (1530-1565 nm) communication windows of optical fibers. Very recently, mid-infrared chaos is drawing more and more interests, because it has potential applications in long-reach free-space secure communications and in remote LIDAR systems, owing to the low transmission loss in the atmosphere (3-5 μm and 8-12 μm). In order to produce mid-infrared chaos, a few efforts have been made using quantum cascade lasers (QCLs) as the chaotic light source. However, the QCLs only exhibited low frequency fluctuations (LFFs), while fully-developed chaos has been not observed. This is because the carrier lifetime (around one picosecond) of QCLs is shorter than the photon lifetime, and hence resembles Class-A laser systems, which are much more stable than Class-A laser systems. However, LFFs are of low dimensionality and the electrical bandwidth is reduced, which significantly limit the applications, and hence fully-developed chaos are preferred. In addition to QCLs, interband cascade lasers (ICLs) emit light in the mid-infrared regime as well. Most ICLs are grown on GaSb substrates and the emission wavelengths are usually in range of 3-6 μm. On the other hand, InAs-based ICLs extend the lasing wavelength beyond 10 μm, which are nevertheless less mature than GaSb-based ones. In addition, the power consumption of ICLs is more than one order of magnitude lower than the QCL counterpart. On the other hand, The stimulated emission of ICLs relies on interband transition of carriers in type-II (mostly) quantum wells, and the carrier lifetime (sub-nanosecond) is longer than the photon lifetime. Therefore, ICLs are classified into Class-B laser systems like common...
semiconductor lasers, and hence are more prone for chaos generation. Here, we report the first demonstration of fully-developed chaos in the mid-infrared regime using an ICL. The ICL is perturbed by optical feedback using an external reflection mirror. Beyond a critical feedback level, the ICL produces chaotic oscillations for both low and high pump currents. In addition, the ICL also generates periodic oscillations or LFFs through manipulating the operation conditions.

Results

Experimental setup and laser emission spectra. The ICL under study has an active region of 7 cascading stages of InAs/GaInSb type-II quantum wells, and it is fabricated into a Fabry-Perot laser with a narrow ridge (see Methods). In order to produce chaos, the ICL is perturbed by external optical feedback, which is provided by a gold mirror as illustrated in Fig. 1a. The gold mirror is placed 36 cm away from the laser sample, which results in an external cavity frequency of 417 MHz. Because the light of ICL is transverse electric polarized, the feedback strength can be adjusted by rotating the polarizer. The feedback ratio is defined as the ratio of the mirror reflected power to the laser output power, which is monitored by the power meter. For different feedback ratios, the optical spectra, the time traces, and the electrical spectra are all recorded in the experimental setup (see Methods).

The free-running ICL in Fig. 1b shows four visible longitudinal modes around 3392 nm, at a low pump current $(1.09 \times I_{th})$ slightly above the lasing threshold ($I_{th}$ = 78 mA). When applying optical feedback to the ICL, the number of longitudinal mode increases. This is because the optical feedback reduces the lasing threshold, and hence more modes surpasses the threshold (see Methods). Although chaos can broaden the spectral linewidth of the longitudinal modes, it is hard to be identified from the measured spectra due to the limited resolution (0.1 nm) of the optical spectrum analyzer.

![Experimental setup and laser emission spectra. (a) Experimental setup for the chaos generation in an ICL subject to external optical feedback. The feedback is provided by a gold mirror, and the feedback ratio is controlled by rotating the polarizer. OSA: optical spectrum analyzer. ESA: electrical spectrum analyzer. BS: beam splitter. (b) The optical spectra of the ICL at $1.09 \times I_{th}$ for several feedback ratios. The free-running laser threshold is $I_{th}$ = 78 mA.](image-url)
**Time domain characteristics.** Figure 2a shows the evolution of the time traces of the ICL with increasing feedback strength. The ICL produces continuous wave for weak feedback levels up to a feedback ratio of about -16 dB (example of -21.3 dB). For feedback ratios in the range of -16 to -10 dB (example of -12.7 dB), weak oscillations arise in the time traces. This is because the relaxation oscillation of the ICL becomes less damped owing to the effect of optical feedback. Increasing the feedback ratios to the range of -10 to -8 dB (example of -9.4 dB), the ICL becomes destabilized and exhibits strong oscillations in the time traces.

Both the time trace and the corresponding phase portrait in Fig. 2b prove that the dynamics is period-one oscillation, which shows a single period in the time trace and one cycle in the phase portrait. The physical origin of period-one oscillation is that the relaxation oscillation is undamped by optical feedback through the Hopf bifurcation. Period-one oscillations have been widely investigated in near-infrared semiconductor lasers, which are high-quality photonic microwave sources for applications in radio-over-fiber communications. When the feedback strength exceeds the critical feedback level, which quantifies the onset of fully-developed chaos, the ICL begins to produce chaotic oscillations. The critical feedback level of the ICL is measured to be around -8.0 dB, and fully-developed chaos examples for feedback ratios of -7.9 dB, -6.4 dB, and -4.2 dB are displayed in Fig. 2a. The corresponding phase portraits in Fig. 2b show that the chaotic oscillations become more and more complex with increasing feedback level.

The bifurcation diagram in Fig. 3a describes the power extremes (both maxima and minima) extracted from the time traces. It clearly shows that the Hopf bifurcation point occurs around -10 dB, which leads to the generation of period-one oscillations with one maximum and one minimum. Beyond the critical feedback level of -8 dB, the ICL produces chaotic oscillations with multiple extremes. We did not observe intermediate dynamics between period-one oscillations and chaos in the measured ICL. However, common semiconductor lasers subject to optical feedback usually follow the quasi-periodic route to chaos, whereas it is also possible to follow the period-doubling route under specific conditions. The absence of intermediate dynamics in the tested ICL
might be because the corresponding regime is too small, and the dynamics is concealed by the multimode hopping as well as by the spontaneous emission noise and other technical noise sources. Simulations using a set of coupled ICL rate equations associated with the Lang-Kobayashi model demonstrate that the regime of intermediate dynamics can be enlarged either by reducing the pump current or by shortening the feedback length\textsuperscript{10,41} (see Supplementary Note One). The simulations also unveil that the ICL follows the typical quasi-periodic route to chaos like near-infrared semiconductor lasers. In order to quantify the sensitivity of chaos to the initial conditions, we extract the largest Lyapunov exponent from the time traces using Wolf’s algorithm\textsuperscript{42,43}. A chaotic system at least has one positive Lyapunov exponent, which reflects the average exponential rate of divergence for nearby orbits in phase space, and hence implies the time scale on which the system dynamics become unpredictable. Figure 3b shows that the largest Lyapunov exponent is around 0.55 /ns and has little change for the feedback range of -8 to -6 dB. With increasing feedback strength, it goes up to the maximum value of 2.1 /ns at a feedback ratio of -4.6 dB. Further raising the feedback level reduces the Lyapunov exponent down to 1.6 /ns at the feedback ratio of -4.2 dB. The reduction of the Lyapunov exponent suggests that the laser system may evolve into LFFs or steady state for feedback ratios beyond -4.0 dB\textsuperscript{44}. It is worthwhile to mention that these Lyapunov exponents are more than three orders of magnitude larger than those of LFFs in QCLs\textsuperscript{45}. Consequently, the fully-developed chaos in the ICL is much more sensitive to the initial conditions, and much less predictable than the low-dimensional LFFs in QCLs.

**Figure 3** | Analysis of chaotic time traces. (a) Bifurcation diagram of the power extremes and (b) largest Lyapunov exponent as a function of the feedback ratio.

**Frequency domain characteristics.** In Fig. 4a, the free-running ICL shows a smooth electrical spectrum except the low-frequency part below 100 MHz, where the noisy spikes are mainly due to the technical noise sources (current source noise, thermal noise, and mechanical noise) as well as the mode partition noise\textsuperscript{46,47}. The spectrum does not show any relaxation resonance peak, suggesting that the ICL is strongly damped. This observation is consistent with the measured intensity modulation responses\textsuperscript{48-50}, where no resonance peak occurs either. Therefore, ICLs resemble quantum dot lasers, the relaxation oscillations of which are
usually over-damped\textsuperscript{51,52}. When optical feedback is applied to the ICL with a feedback ratio of -12.7 dB, a small peak appears around 168 MHz, which determines the oscillation period of the corresponding time trace in Fig. 2a. The peak frequency is much smaller than the external cavity frequency of 417 MHz, and therefore the peak must be due to the weakly damped relaxation oscillations. It is known that the oscillation frequency of semiconductor lasers with optical feedback varies around the resonance frequency of the free-running laser\textsuperscript{8}. Therefore, the resonance frequency of the ICL is deduced to be around 168 MHz, which is smaller than the common GHz-level resonance of near-infrared semiconductor lasers. This is because the tested ICL is not designed for high-speed operations, such as the long cavity length as well as the long carrier transport time across the thick separate confinement layer, which limit the relaxation oscillation frequency\textsuperscript{46}. However, ICLs can reach GHz-level resonance through proper optimizations\textsuperscript{48-50}. Increasing the feedback level to -9.4 dB, the ICL exhibits a typical period-one oscillation at 155 MHz, and the oscillation peak amplitude is about 40 dB higher than the background noise level. At feedback ratios of -7.9 dB and -4.2 dB, the ICL produces chaotic oscillations and the electrical power levels are substantially raised in a broad frequency range, up to the bandwidth limit (450 MHz) of the photodetector. The map in Fig. 4b displays the evolution of the electrical power distribution with feedback strengths. It is shown that the frequency of the weak relaxation oscillation slightly increases with increasing feedback level from -16 to -10 dB. At the onset of period-one oscillation (around -10 dB), the oscillation frequency abruptly shifts to a slightly smaller value. Beyond the critical feedback level of -8 dB, the ICL exhibits chaotic oscillations within a wide feedback strength window, up to the feedback limit (-4.2 dB) of the experimental configuration. In order to quantify the bandwidth of the chaotic signals, we employ the definition that the frequency span from DC to the frequency where 80\% of the total power is contained within the power spectrum\textsuperscript{53}. Using this definition, Fig. 4c demonstrates that the chaos bandwidth (dots) firstly declines and then rises with the increasing feedback ratio. The maximum chaotic bandwidth is 269 MHz, which is reached right above the critical feedback level. The chaos bandwidth of the ICL is much higher than the LFF bandwidth (less than 50 MHz) in QCLs\textsuperscript{18,21}, which is very beneficial for high-speed applications. In addition, Fig. 4c proves that the chaos significantly raises the intensity noise (stars) of the ICL by more than 15 dB once beyond the critical feedback level. Besides, the noise level climbs up with increasing feedback strength.

**Chaos at a high pump current.** In the above analysis, the ICL is operated near the lasing threshold. However, we find that the ICL produces chaos when it is operated well above threshold as well. When it is pumped at \(1.35 \times \text{l}_{\text{th}}\), the bifurcation diagram in Fig. 5a and the electric power distribution map in Fig. 5b demonstrate that the ICL does not show any periodic oscillations, which is different to the near-threshold case. Instead, the ICL produces LFFs before transferring to the regime of fully-developed chaos.
Consequently, the ICL undergoes the LFF route to chaos. For feedback levels in range of -16 to -14 dB (example of -14.3 dB) in Fig. 5d, the multimode hopping slightly raises the noise level at frequencies below 200 MHz, which leads to the weak fluctuations in the corresponding time trace in Fig. 5c. In the feedback range of -14 to -8 dB (example of -11.9 dB), the ICL produces LFFs, which are confirmed by the statistical analysis (see Supplementary Note Two). The LFFs in Fig. 5c show irregular power jump-ups with gradual power increase and following drastic power decrease. This is in contrast to typical LFFs observed in
common semiconductor lasers, which exhibits random power dropouts with sudden power decrease followed by gradual power recovery. However, LFFs with power jump-ups have been indeed observed in semiconductor lasers biased well above threshold. In addition, we also observe coexistence of power jump-ups and dropouts in the experiment (not shown). The LFF leads to the power enhancement of the electrical spectra in Figs. 5b, d, and the bandwidth broadens with rising feedback level. Besides, a dominate peak appears around the resonance frequency of the ICL, and the peak frequency increases with the feedback level as well. Further increasing the feedback level to the range of -8 to -6 dB (example of -7.1 dB), both the LFF and the chaos coexists in the time trace of Fig. 5c, and the bandwidth of the power spectrum in Fig. 5d further broadens. The dynamics is finally evolved into fully-developed chaos when the feedback level surpasses -6 dB, and an exampled time trace at -4.2 dB is displayed in Fig. 5c. As illustrated in Figs. 5b, d, the chaos bandwidth is higher than that of LFFs. Detailed comparisons of the chaos and the LFFs are discussed in Supplementary Note Two.

Discussion

We have reported the fully-developed chaos generation from a mid-infrared ICL subject to external optical feedback. The ICL produces chaos when the feedback strength surpasses the critical feedback level around -8 to -6 dB. The chaos bandwidth is more than 200 MHz, whereas the electrical power spectra are significantly raised over a frequency span of 500 MHz. Before producing chaos, the ICL exhibits periodic oscillations when it is operated close to the threshold, while exhibits LFFs when operated well above threshold. In comparison with common quantum well lasers, the ICL is more stable against optical feedback and its critical feedback level is more than 20 dB higher than the former ones. On the other hand, the critical feedback level of the tested ICL is comparable to those of quantum dot lasers. The critical feedback level of semiconductor lasers are primarily determined by the linewidth broadening factor (α-factor) and the damping factor. Therefore, the strong resistance to optical feedback of the ICL can be attributed to its small α-factor (2.2) as well as its large damping factor, which are worthy of more physical investigations. On the other hand, the chaos bandwidth of the ICL is smaller than the common GHz bandwidth of near-infrared lasers. Because the chaotic bandwidth is limited by the relaxation resonance frequency of the ICL, it can be improved by high-speed design and the bandwidth is expected to reach the GHz level in future work.

In comparison with its QCL counterpart, the ICL owns a few advantages for the production of chaos. Firstly, the ICL is proved to show fully-developed chaos in this work, whereas QCLs can only generate low-dimensional LFFs. Secondly, the chaos bandwidth of the ICL is much broader than the LFF bandwidth of QCLs, which is very favorable for high-speed applications.
Thirdly, the generation of LFFs in QCLs requires critical operation conditions, and the LFFs only exist in a narrow parameter window\textsuperscript{18,21}. In contrast, chaos in ICL appears for a broad range of feedback strengths, and for both low and high pump currents. Consequently, we strongly believe that this work can stimulate extensive investigations of chaos in the mid-infrared regime. Its potential applications in long-reach secure free-space communications and in remote chaotic LIDAR systems may emerge in the future.

**Methods**

**ICL sample.** The ICL sample was grown on a GaSb substrate by solid source molecular beam epitaxy. The laser consists of 7 cascading gain stages, which are formed by W-shape InAs/GaInSb type-II quantum wells. The laser has a ridge waveguide with a ridge width of 9.0 μm and a cavity length of 1.5 mm. Both laser facets are as-cleaved without any coatings. At an operation temperature of 20 °C, the lasing threshold is $I_{th}=78$ mA. When the ICL is subject to optical feedback with a feedback ratio of -4.2 dB, the threshold is reduced down to 73 mA. The linewidth broadening factor of the ICL operated above threshold was measured to be around 2.2\textsuperscript{61}.

**Experimental setup.** The ICL sample is mounted on a heat sink and its temperature is maintained at 20 °C by using a thermos-electric controller. The pump current is supplied by a low-noise battery current source (LDX-3620B). The laser output is collimated by an aspherical lens with a focal length of 4.0 mm. The light is split into two paths by a beam splitter (BS). One path provides the optical feedback through a gold mirror, which is placed 36 cm away from the laser sample. The feedback strength is adjusted by rotating the polarizer, and the feedback power is monitored by a power meter. The feedback ratio offered by this configuration reaches up to a maximum value of -4.2 dB. The other optical path is used for characterization. The optical spectra are measured by a grating-based optical spectrum analyzer (OSA, Yokogawa AQ6376) with a resolution of 0.1 nm. The optical signal is converted to the electrical one by a HgCdTe photodetector (Vigo PVI-4TE-6) with a nominal bandwidth of 450 MHz. The electrical spectra are measured by a broad bandwidth (50 GHz) electrical spectrum analyzer (ESA), and the time series are recorded on high speed (59 GHz) oscilloscope.

**Data availability.** All data supporting this study are available from the corresponding author upon request.

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Author contributions

C.W. initiated the study. Y.D. performed the experiment and Z.F. run the simulations. All authors analyzed the results and wrote the manuscript.

Additional information

Supplementary Information accompanies this paper at XXX.

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