Supplementary Information for

Recurrent ancient geomagnetic field anomalies shed light on future evolution of the South Atlantic Anomaly

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Here we describe methods and additional results for the study "Recurrent ancient geomagnetic field anomalies shed light on future evolution of the South Atlantic Anomaly". The method used to construct the new geomagnetic field model is described in detail in (1) but we here provide a short summary of the model structure (see also table S1).

The geomagnetic field is represented using a spherical harmonic decomposition of the scalar potential for the field \( B = -\nabla \phi \), with each Gauss coefficient expanded in time using Gaussian processes (e.g. 2). The spherical harmonic expansion of the field is truncated at \( L = 5 \). The Gauss coefficients \( \{ g_l^m(t), h_l^m(t) \} \) are sampled every \( \Delta t = 50 \) years from \( t_1 = -7500 \) to \( t_N = 2000 \) (\( N = 191 \)) but the model is only intended to be valid from 7000 BCE. To include the information about geomagnetic field variations from the past 180 years, during which there are full vector observations, the Gauss coefficients are anchored to the COV-OBS.x2 mean model (3) from 1850 CE to 2000 CE field through Gaussian process regression. The Gauss coefficients are stored in a column vector \( \mathbf{x} \) of size \( NL(L+2) \).

The age of archeomagnetic and sedimentary data are parameterized and co-estimated along with the Gauss coefficients in the model. The archeomagnetic ages (\( \beta \)) are assumed to be normally distributed with prior distributions defined by the published age (\( \mu_\beta \)) and age uncertainties (\( \sigma_\beta \)) obtained from GEOMAGIA50.v3 (4). Sediment age-depth models were constructed using a modified version of the BACON age-depth model (5), which estimates variations in accumulation rate \( \alpha(d) \) as a function of depth \( d \) of the sediment sequence and the age \( \theta \) of the top of the sediment core. For computational reasons the sediment sequence is divided into \( D \) equally spaced discrete sections with thickness \( \Delta d = 20 \) cm. By assuming a log-normal prior distribution for \( \alpha(d) \), the logarithm of the accumulations rates \( c(d) = \log(\alpha(d)) \) can conveniently be expressed using Gaussian processes. The “memory” in the system (or the smoothness of the age-depth curves) is determined by a squared exponential covariance function with a characteristic depth-scale \( \tau_c \). Unless otherwise stated, we assign \( \tau_c = 100/\bar{\alpha} \) cm where \( \bar{\alpha} \) is prior estimate of the mean accumulation rate. This choice of \( \tau_c \) corresponds to a characteristic timescale of 100 years and is based on comparisons with published age-depth models. For the age \( \theta \) at \( d_0 \) we assume a truncated normal prior distribution \( \theta \sim N(t_0, \sigma_\theta^2, -\infty, t_0) \) where (unless otherwise stated in the original publication) \( t_0, t_0 \) is the year the sediment cores were retrieved and \( \sigma_\theta = 10 \) years is the associated uncertainty (related to the identification of the sediment/water interface). To facilitate sampling, we first construct a set of initial age-depth models based on independent chronologic constraints and then use these to update the prior distribution in the final geomagnetic field model, where the age-depth models are an integrated part (see Fig. S2-S11).

To account for post-depositional remanent magnetization (pDRM) lock-in effects associated with the gradual processes by which sediments acquire a magnetization, we follow closely the method of (6). The magnetic field signal recorded by the sediments at a specific depth is assumed to correspond to an integrated response over a certain time period after the initial deposition. The response, modelled using a lock-in filter function, depends on two parameters; the lock-in shape factor \( \beta \sim \Gamma(3, 2) \) and the half lock-in depth \( \gamma \sim \Gamma(2, 0.2) \), i.e. the depth at which 50% of the pDRM is acquired (7). To account for potential systematic errors that affect declination and/or inclination data from the same sediment record we introduce site-specific declination \( \delta_d \sim N(0, \sigma_{d,\beta}^2) \) and inclination \( \delta_i \sim N(0, \sigma_{i,\beta}^2) \) corrections, where \( \sigma_{d,\beta} = 5^\circ \) and \( \sigma_{i,\beta} = 1^\circ \). Contrary to (1), we consider both positive and negative corrections for inclination data and therefore do not use a truncated prior distribution.

Finally, we use a Student’s t error distribution to account for the presence of data outliers (8). The length of the tails of the Student’s t-distribution is determined by \( \nu \), which we assign as hyperparameters to five subcategories of data that are assumed to have similar error distributions: archeomagnetic declinations (\( \nu_1 \)), inclinations (\( \nu_2 \)), intensities (\( \nu_3 \)) and sedimentary declinations (\( \nu_4 \)) and inclinations (\( \nu_5 \)), stored in vector \( \nu \). For each of the five parameters we assign a weakly-informative prior distribution \( (\nu - 1) \sim \Gamma(2, 0.1) \).

All in all, the model consists of 16,577 parameters (see table S1 for an overview). We use a Markov Chain Monte Carlo (MCMC) approach to sample from the posterior distribution. More specifically we use the No-U-Turn sampler (NUTS), which is an adaptive form of Hamiltonian Monte Carlo sampler, implemented through Stan 2.21.1 (a statistical programming language) (9). For the final model (pfm9k.2), the MCMC sampler is run on four parallel chains on separate cpu’s. For each chain, we run 1000 burn-in iterations, which are discarded, and 500 sampling iterations from which we retain every \( M \) sample, leading to a total sample size of \( M = 1000 \). In addition, two models with alternative priors for the Gauss coefficients were run on two parallel chains each, with a total sample size of \( M = 500 \) for each.

Built in diagnostics in Stan provide one way to check the performance of the MCMC. The effective sample size \( N_{eff} \) is a measure of the number of independent draws from the posterior distribution, determined by the ability of the draws to estimate the true mean value of a parameter (see 10). Overall, the sampling efficiency is mostly higher than 0.5, which is the motivation for the thinning the final sample (i.e. only retaining every \( 2^{nd} \) draw). The potential scale reduction (\( \hat{R} \)) convergence diagnostic (10) compares between-chain and within-chain estimates of model parameters and checks if the chains are well-mixed or not. In our model runs 99.8% of the parameters (including all Gauss coefficients) are associated with \( \hat{R} \leq 1.1 \), which indicates that the chains have converged to a common distribution (1).

Data selection

Archeomagnetic data. The Archeomagnetic (archaeological artefacts and volcanic rocks) data used in this study were obtained from the GEOMAGIA50.v3 database (4) in June 2021. Following (1), we initially selected all data with ages, provided in the database, between 7000 BCE and 2000 CE. Since data with large age uncertainties frequently end up with complex multimodal posterior probability distributions, all data with age uncertainties in excess of \( \pm 200 \) years as well as data with ages prior to
1700 without specified age uncertainties were removed. We found that this had little impact on the final model as such data provide only limited information about the geomagnetic field in any case. All data uncertainties (age, magnetic direction and magnetic intensity) were assigned according to (1). The final archaeomagnetic dataset is summarized in table S2 and Fig. S1.

**Sedimentary data.** Modelling the age-depth relationship of sediment records is the most computationally demanding part of our new model approach. The age-depth models are frequently associated with complex posterior distributions and we therefore only include sedimentary data from 10 sites, carefully selected based on their (i) geographic location, (ii) available chronologic information and (iii) quality of paleomagnetic data. The paleomagnetic data were binned in 5 cm intervals with uncertainties assigned according to (1). Only directional paleomagnetic data were considered due to the additional and largely unknown uncertainties associated with relative paleointensity data (e.g. 11). Below we provide a detailed description of the chronologic and paleomagnetic data from each record. All data are, or will be, available on GEOMAGIA50.v3 (12). The final dataset is summarized in table S2.

1. **Beaufort Sea.** The sediment core 2004-804-803 was obtained from the Beaufort Sea (BEA) in the Canadian Arctic (Fig. S2). The paleomagnetic data consist of measurements on u-channel samples from the 7 m long single core sequence. The chronology is based on accelerator mass spectrometry (AMS) $^{14}$C measurements on four bivalve shells (13). The radiocarbon dates were calibrated using the Marine13 calibration curve (14) with a regional reservoir correction $\Delta R = 400$ years, originally assigned by (13).

2. **Chilean Margin.** The sediments from ODP site 1233 from the Chilean Margin (CHM) were originally cored in 2002 (Fig. S3). The shipboard paleomagnetic data are published in the cruise report (15), but we here use data from u-channel samples measured in the University of Florida Paleo and Environmental Magnetism Laboratory (16, 17). The declination data from the individual core sections were aligned with respect to each other according to (15). The inclination data were adjusted with $-6^\circ$ to correct for suspected inclination shallowing (15). We note that the model produced a similar correction if the data were not pre-adjusted, but that the MCMC sampling was less efficient as a result. In addition, the paleomagnetic data from the top 50 cm were removed due to suspected coreing induced deformation of the sediments.

The Holocene part of the chronology is constrained by two AMS $^{14}$C measurements on a mixed assemblage of planktonic foraminifera from ODP site 1233 in the early Holocene (18) and seven $^{14}$C measurements on mixed planktonic foraminifera from core geoB 3313-1 from the same area (19), transferred to ODP site 1233 depth using magnetic susceptibility and relative Ca concentration records (20). All radiocarbon dates were calibrated using the Marine13 calibration curve (14). Several different marine reservoir corrections have been suggested in previous studies on the same sediment cores, ranging from $\Delta R \approx 0$ years (19) to $\Delta R = 400 \pm 100$ years (21). A recent study based on $^{14}$C measurements on museum shells of known age along the Chilean Margin recommend a $\Delta R = 141 \pm 43$ years for the region (22). Based on these different estimates we have here chosen to adopt a $\Delta R = 150 \pm 100$ years. To account for the additional uncertainty introduced by transferring the seven radiocarbon dates from core geoB 3313-1 to ODP site 1233, an additional 100 years were added in quadrature to the total $^{14}$C error budget of these measurements. The large uncertainties coupled with the weak additional constraints provided by the regional geomagnetic field (sparse data coverage) leads to multimodality in the posterior distribution (see e.g. at 200 cm depth in Fig. S3), which is difficult to sample using the MCMC approach. To reduce the complexity, we increase the characteristic depth-scale of the accumulation rates to $\tau_c = 200/\alpha$, i.e. corresponding to characteristic timescale of 200 years, which leads to a smoother age-depth model with less degrees of freedom and a less complicated posterior distribution.

3. **Lake Biwa, Japan.** The paleomagnetic data from Lake Biwa (BIW), Japan (Fig. S4), consist of measurements on discrete samples from three parallel cores (23). We were unable to obtain data at specimen level data and therefore used the stacked and smoothed data from the original publication, available in GEOMAGIA50.v3 (12). The steps taken to make the available data as consistent as possible with the binned specimen data is described in detail in (1). The chronology is constrained by two independently dated tephra horizons.

4. **Lake Eilandvlei, South Africa.** The paleomagnetic data from coastal Lake Eilandvlei (EIL), South Africa (Fig. S5), consist of measurements on u-channel samples from one 1.5 m long core (24). The chronology of the entire 30 m long core sequence is constrained by seven $^{14}$C measurements on wood fragments, calibrated using the SHCal13 calibration curve (25), and an additional 16 $^{14}$C measurements on marine influenced bulk sediment samples, calibrated using the Marine13 calibration curve (14). Marine reservoir corrections, varying with depth, were applied following (24).

5. **Fish Lake, USA.** The paleomagnetic data from Fish Lake (FIS), USA (Fig. S6), consist of measurements on discrete samples from a 9 m long core sequence (26). The chronology is based on 18 $^{14}$C measurements on bulk sediment samples that were digitized from the original publication, but are also available in (27). To account for the increased uncertainty due to unknown dead carbon influence, and additional 200 years were added in quadrature to the total $^{14}$C error budget. All radiocarbon ages were calibrated using the IntCal13 calibration curve (14). The tephra ages Fig. S6 are shown for comparison only, since the ages were partially based on the radiocarbon dates from Fish Lake (27).

The large age uncertainties of the record coupled with relatively weak regional geomagnetic field constraints prior to 4000 BCE, leads to a problematic bimodality in the age-depth posterior distribution around 550 cm. This bimodality is not efficiently sampled by the MCMC algorithm, resulting in individual sampling chains getting stuck in one or the other state. Contrary to similar features associated with some archaeomagnetic ages (1) and in CHM (Fig. S3), the problematic posterior distribution in
the FIS chronology does have a noticeable impact on the regional geomagnetic field prediction. To avoid ambiguities in the final model we rely on comparisons to the semi-independent tephra ages and select only chains that sample the most probable state (i.e. Fig. S6). We note that this decision does not have any impact on the main observations of the paper, which remain more or less unchanged even if we remove the problematic Fish Lake data (below 400 cm depth) altogether.

6. Gyltigesjön, Sweden. The paleomagnetic data from Gyltigesjön (GYL), a lake in Sweden (Fig. S7), consist of measurements on discrete samples from four partially overlapping cores over a total of 8.5 m (28). As noted in the original publication, core rotation can be a problem in the top meter of the cores. After further inspection of the data, we suspect this may extend down to at least 150 cm of each core and, therefore, exclude the declination data from these parts in our analysis. We also exclude inclination data from the top 1 m in core GP4 (28). In addition, 2 out of 908 data were removed as outliers. The chronology is constrained by seven $^{14}$C measurements on terrestrial macrofossils and nine $^{14}$C measurements on bulk sediment samples, all calibrated using the IntCal13 calibration curve (14). Additional constraints are provided by a floating varve chronology (873 varves, $\approx 1$ m of sediment), wiggle-match dated to the IntCal09 calibration curve (29) using 15 densely spaced bulk radiocarbon ages (30). Based on the results of (30) we apply a "dead carbon reservoir correction" (treated equivalently to $\Delta R$) of 200 ± 100 years to the nine bulk radiocarbon samples.

7. Iceland / North Atlantic. The paleomagnetic data from core MD99-2269, retrieved north of Iceland (ICE) (Fig. S8), consist of measurements on u-channel samples from the 25 m long core covering the Holocene (31). The chronology is constrained by 25 $^{14}$C AMS measurements on mollusks, benthic and planktic foraminifera and gastropods. Additional constrains are provided by eight identified Icelandic tephra layers with ages based on (32, 33): KOL-1372 (1372 ± 10), Hekla 1104 (1104 ± 10), Snæfellsnes 1 (1790 ± 100 cal. years BP), Hekla 3 (2970 ± 100 cal. years BP), Hekla 4 (4230 ± 100 cal. years BP), TV 5 (6860 ± 100 cal. years BP), Suduroy (8070 ± 100 cal. years BP) and Saksunavatn (10182 ± 60 cal. years BP). Based on comparisons to the tephra ages we apply a marine reservoir correction of $\Delta R = 150 ± 50$ years (34). The uncertainty of the top age of the core was set to $\sigma_{t_{0}} = 50$ to reflect the possibility that the most recent sediments might be missing.

8. Lake Keilambete, Australia. The record from Lake Keilambete (KEI), Australia (Fig. S9), consists of paleomagnetic measurements on discrete samples from eight parallel or partially overlapping cores (ca. 4 m long sediment sequence) covering the Holocene (35). The sediments are dated based on a combination of radiocarbon ages (36–41) and optically stimulated luminescence ages (42), which have previously been compiled and made available on GEOMAGIA50.v3 (12). For the construction of the age-depth model we only included dates that have been assigned with a composite depth and that were accepted by the original authors.

9. Kälksjön, Sweden. The record from Kälksjön (KLK), a lake in Sweden (Fig. S10), is based on paleomagnetic measurements on discrete samples from six partially overlapping cores (42–44). As previously noted (6), the composite depth-scale of KLK does not have zero depth associated with the sediment-water interface and to adjust for this we therefore add 6 cm to all paleomagnetic (and chronologic) data. Similar to GYL, we exclude declination data from the uppermost 75 cm of each core from our analyses, due to suspected core rotation. In addition, 11 out of 1731 data were removed as outliers. The chronology is constrained by four $^{14}$C measurements on terrestrial macrofossils (44) and 12 $^{14}$C measurements on bulk sediment samples (42), all calibrated using the IntCal13 calibration curve (14). Additional constraints are provided by two $^{14}$C wiggle matched sequences based on 19 (44) and nine (45) densely spaced bulk radiocarbon ages. Based on the results of (42, 44) we apply a "dead carbon reservoir correction" (treated equivalently to $\Delta R$) of 350 ± 200 years to the 12 bulk radiocarbon samples. The sediments from KLK are varved (annually laminated) which have been counted and could potentially be used to add further constraints to the chronology (42). Indirectly, the varve chronology is already incorporated in the wiggle match dated sequences. However, due to complication related to unknown number of missing varves (42) we only show the varve chronology for reference here (Fig. S10).

10. Lake Potrok Aike, Argentina. The record from Lake Potrok Aike (POT), Argentina (Fig. S11), consists of paleomagnetic measurements on u-channel samples from the 2CP sediment cores obtained during the Potrok Aike maar lake sediment archive drilling project (PASADO) (17). The individual core sections were not azimuthally oriented and due to the generally steep inclinations of the directions it is not clear how to best align the data. The additional rotations of individual core sections could potentially be incorporated in the model, but we found that this was not a practical solution due the increased complexity of the posterior distribution. Therefore, since we are unable to incorporate this additional uncertainty in the model, we decided not to include the declination data from POT. In addition, 11 out of 1406 inclination data were removed as outliers and the paleomagnetic data from the top 20 cm were removed due to suspected coring induced deformation of the sediments.

The chronology for the top 13 m, covering the Holocene, is based on 13 $^{14}$C measurements on carbonates precipitated from the water column, bulk sediment and different organic macro remains from cores PTA03/12 and PTA03/13 (46) calibrated using, the SHCal13 calibration curve (25) correlated to 2CP. To account for the additional uncertainty introduced by transferring the radiocarbon dates, 100 years were added in quadrature to the total $^{14}$C error budget of these measurements. To reduce the complexity of the posterior distribution, similarly to CHM, we increase the characteristic depth-scale of the accumulation rates to $\tau_c = 200/\alpha$, i.e. corresponding to characteristic timescale of 200 years.

Impact of different geomagnetic field priors - non-dipole field

To construct the model, we assume that the Gauss coefficients result from a stationary stochastic process, that the covariance between different coefficients is zero and that the auto-covariance for coefficients with same degree $l$ is identical (47, 48). The
time dependent Gauss coefficients are described using Gaussian processes (2, 49, 50)

\[ g_l^m(t) \sim \mathcal{G}(\hat{g}_l^m(t), k_l(t, t')) \]  

where \( \hat{g}_l^m(t) \) is the mean function and \( k_l(t, t') \) is the covariance (or kernel) function, evaluated at time \( t \) and \( t' \) (with corresponding notation for \( h_l^m(t) \)). For all Gauss coefficients of degree \( l \) (except for \( g_0^0 \), see below), a priori we assume a background field with zero mean and we use a covariance function of the form (51)

\[ k_l(r) = \sigma_l^2 (1 + \omega_l r) \exp(-\omega_l r) \]  

where \( r = |t - t'| \). This corresponds to a continuous time autoregressive (AR) process of order 2 characterised by two parameters; the variance \( \sigma_l^2 \) and the characteristic timescale \( \tau_l \). For all non-dipole coefficients, a priori estimates of these parameters \((\sigma_l^2 \text{ and } \tau_l)\) were determined from the coefficients of the COV-OBS.x2 model (3), using 300 different model realisations \((j)\) evaluated at every year \((t)\) from 1840CE to 2020CE

\[ \sigma_l^2 = \sqrt{ \frac{1}{2l+1} \sum_{m=0}^{l} \left( \left( \frac{\hat{g}_l^m(t)}{\sigma_l^2} \right)^2 + \left( \frac{\hat{h}_l^m(t)}{\sigma_l^2} \right)^2 \right)_{j=j^*, t=t^*} } \]  

and

\[ \omega_l^{-1} = \frac{1}{\sigma_l^2} \]  

where \( \sigma_l^2 \) is defined as in equation Eq. (3), replacing the coefficients \((\hat{g}_l^m, \hat{h}_l^m)\) with the corresponding time derivatives \((\partial_t \hat{g}_l^m, \partial_t \hat{h}_l^m)\). Finally we obtain the a priori values for \( \sigma_l \) and \( \tau_l \) by taking the average of all the different estimates \( \sigma_l^2 \) and \( \omega_l^{-1} \) (see Table S3 and Fig. S12A).

Using COV-OBS.x2 to determine the prior, rather than a single epoch (2005 CE) from a satellite era model (\textit{gufm-sat-E3} (52)) as previously done (1, 2), primarily results in less power in spherical harmonic degrees \( l = 2 \) and \( l = 3 \) (Fig. S13). This difference is partially related to the growth of the South Atlantic Anomaly (and associated growth of the quadrupole field) over the past 180 years. Since the aim of the study is to investigate similar features back in time we here choose the conservative option based on COV-OBS.x2. We note that the posterior distribution of the new model is more consistent with the COV-OBS.x2 prior, even if \textit{gufm-sat-E3} was used to define the prior.

As shown in Fig. S13, the new model pfm9k.2 has more power in spherical harmonic degrees \( l = 4 \) and \( l = 5 \) than what would be expected from the prior distribution and COV-OBS.x2, which the prior is based on. Similar effects have previously been noted for the COV-ARCH and COV-LAKE model (2), particularly for the latter which includes sedimentary data. This is probably related to errors in the data that are not properly accounted for in the models. Most likely it is due to unresolved chronologic errors that lead to data inconsistencies, which are more easily explained with higher moments of the field. This would explain why the effect is less pronounced in our new model which is able to explain the same variations with lower spherical harmonic degrees by simultaneously synchronizing the timescales of the data (see e.g. Fig. S14). The time-averaged power spectrum of the posterior mean model, which provides an estimate of the variations which are robustly resolved, does not show similarly high power at \( l = 4 \) and \( l = 5 \), but is similar to previously published regularized models.

**Impact of different geomagnetic field priors - dipole field**

The priors for the dipole Gauss coefficients are defined differently to the non-dipole coefficients. For the equatorial dipole coefficients \((g_1^0, h_1^0)\) we assign \( \sigma_1 = 3,500 \text{ nT} \) and \( \omega_1^{-1} = 200 \text{ years} \) based on (1) and for the axial dipole \( g_0^0 \) we consider three different parameterizations (Table. S3 and Fig. S12). In the case of \( g_0^0 \) we assume a non-zero constant background field \( \bar{g}_0^0 \) and either the covariance function given in Eq. (2) or an alternative three parameter covariance function, defined by the variance \( \sigma^2 \) and two timescales \( \chi^{-1} \) and \( \omega^{-1} (51) \). For \( \chi > \omega \), this function is

\[ k(r) = \frac{\sigma^2}{2\chi} \left( \frac{\chi + \xi}{\xi} e^{-(\chi - \xi)r} - \frac{\chi - \xi}{\xi} e^{-(\chi + \xi)r} \right) \]  

with \( \xi^2 = \chi^2 - \omega^2 \). The frequency spectrum exhibits \( f^{-4} \), \( f^{-2} \) and \( f^0 \) dependence at respectively high, intermediate and low frequencies. The transition periods between frequency ranges with 0 and 2 spectral indices is \( T_s = 2\pi(\chi / \xi) / \omega \) and that between 2 and 4 spectral indices is \( T_j = 2\pi(\chi - \xi) / \omega^2 \).

For pfm9k.2A we use the same parameterization used for COV.OBS.x2 (3), which can be considered a conservative prior. The background field value \( g_0^0 = -24,000 \text{ nT} \) and RMS \( \sigma_{g_0^0} = 7,700 \text{ nT} \) are based on the average and standard deviation of the axial dipole moment over the past 2 Myr (3). The timescales \( \omega^{-1} = 770 \text{ years} \) and \( \chi^{-1} = 20 \text{ years} \) are determined by fixing RMS secular variation (SV) \( \sigma_{g_0^0} = 10 \text{ nT/year} \) and \( T_s = 100,000 \text{ years} \).

For pfm9k.2B we use the same parameterization as (1), i.e. with the two parameter covariance function defined in Eq. (2), which amounts to setting \( \chi = \omega \). In this case the background field is only meant to reflect the average dipole over the past 9000 years and we therefore assign \( g_0^0 = -32,500 \text{ nT} \) based on the mean value calculated from the model pfm9k.1a (53). The RMS \( \sigma_{g_0^0} = 3524 \text{ nT} \) and the timescale \( \omega_{g_0^0}^{-1} = 206 \text{ years} \) are determined from the dipole coefficients of \textit{gufm-sat-E3} at 2005 CE.
Eq. (3) and Eq. (4) after first subtracting the background field. This parameterization represents a more flexible prior with \( \sigma_{g_i} = 17.1 \text{ nT/year} \).

For the final model (pfm9k.2) we choose an intermediate parameterization similar to the one used for pfm9k.2A, i.e. using the covariance function in Eq. (5). To allow for more variability on millennial timescales while preserving the power at low and high frequencies, we set \( \sigma_{g_i} = 10,000 \text{ nT, } \sigma_{g_i} = 13.5 \text{ nT/year} \) and the \( T_c = 50,000 \text{ years} \). This amounts to the timescales \( \omega^{-1} = 741 \text{ years} \) and \( \chi^{-1} = 138 \text{ years} \). The background field value \( g_0^B = -23,000 \text{ nT} \) is based on the mean axial dipole over the Brunhes chron (2). Based on the comparison to the \( g_i^B \) power spectra of independent geomagnetic field models (Fig. S12B), we note that our preferred parameterization works acceptably well on Holocene timescales, but could potentially allow for too much power on periods of \( 10^7 - 10^5 \text{ years} \).

In Fig. S15 we compare time series of \( |g_i^B| \) from our new models with a selection of already published models. There are large disagreements between the different published models, but with the exception of a few isolated time periods (e.g. around 700 BCE) we find that pfm9k.2 is consistent with the majority of previous reconstructions within the uncertainties. Comparisons of the average rate of change the axial dipole (\( g_i^B \)), calculated over 50-year intervals for a more consistent comparison, suggests that all three parameterizations of the axial dipole prior proposed here are rather on the conservative side. Overall, published models that include sedimentary data have the lowest RMS rate of change, consistent with pfm9k.2A, whilst models based exclusively on archeomagnetic data have significantly higher RMS rate of change values, more consistent with pfm9k.2B.

The minimum in the axial dipole field \( |g_i^B| \) around 1000 BCE to 500 BCE is notably different from most previous reconstructions (Fig. S15). This is related to the strong intensities in Western Eurasia around this time (e.g. 54) that coincide with previously unresolved shallow inclinations in the Arctic, e.g. Beaufort Sea in the Canadian Arctic (Fig. S2). The difference is largest between pfm9k.2 and models (such as SHA.DIF.14k and ARCH3k..cst.1) that do not include sedimentary data and therefore lack geomagnetic field information from high latitudes. These archeomagnetic models therefore tend to map a large part of the strong intensities in Western Eurasia into the axial dipole moment (resulting in steep inclinations in the Arctic). The pfm9k.2 model, which is able for the first time to capture the full range of variability in these Arctic records, shows that the northern hemisphere geomagnetic field was highly asymmetric around this time and that higher moments therefore have to be invoked to describe the strong intensities in Western Eurasia. This is clearly demonstrated by the model-data comparison to the independent paleomagnetic record from Lake Hajeren, Svalbard (55) (Fig. S16).

**Comparison to cosmogenic radionuclide data**

To investigate if the observed 1300-year and 650-year periodic signals in the dipole moment (Fig. S12C) are geomagnetic (and dipole) in origin we use the independent record of atmospheric \(^{14}\text{C} \) concentrations inferred from tree ring measurements compiled in IntCal20 \(^{14}\text{C} \) calibration curve for the northern hemisphere (56). Cosmogenic radionuclides, such as \(^{14}\text{C} \), are produced in the atmosphere through interactions with cosmic rays, at a rate which is inversely related to variations in the dipole moment and variations in solar activity (e.g. 57). The \(^{14}\text{C} \) production rates were calculated using a box-diffusion carbon cycle model (58) assuming that there were no changes in the carbon cycle during the Holocene. Data after 1950 CE are heavily influenced by anthropogenic \(^{14}\text{C} \) produced during the nuclear bomb tests and are therefore not used.

In Fig. S17 we compare the global \(^{14}\text{C} \) production rates inferred from IntCal20 with the production rates calculated (59) based on the dipole moment variations of pfm9k.2B (which shows the periodogram most clearly) and assuming a constant solar activity of 500 MeV. The \(^{14}\text{C} \) production-rate record based on IntCal20 was normalised to obtain the same average dipole moment and solar-modulation function over the period from 1937 to 1948 (covering one average solar cycle), as indicated by COV-OB8.x2 (3) and the extended neutron-monitor data (60). The variations in \(^{14}\text{C} \) production rates on timescales of 650 and 1300 years, which we are interested in, are dominated by solar modulation. Therefore, to remove as much of the solar modulation signal as possible, we detrend and bandpass filter the inferred production rates using narrow windows with cutoff periods \( T_c = 1300 \pm 100 \text{ years} \) (Fig. S17B) and \( T_c = 650 \pm 100 \text{ years} \) (Fig. S17C). The RMS variation of the original radionuclide derived production rates (\( P \)) captured by the residual signal (\( RP \)) is calculated as follows

\[
\frac{\sum_i (|RP(i)| - |RP|)^2}{\sum_i (|P(i)| - |P|)^2}
\]

where \( i \) runs over all time steps. The heavy filtering means that the residual data only captures 5\% (\( T_c = 1300 \pm 100 \text{ years} \)) and 16\% (\( T_c = 650 \pm 100 \text{ years} \)) of the variation in the original data. As shown in Fig. S17B-C the residual signals of both the pfm9k.2B and IntCal20 derived production rates are mostly in phase and of similar amplitude. We note that the pfm9k.2B production rates show larger amplitude variations in the \( T_c = 1300 \pm 100 \text{ years} \) band and smaller amplitude variations in the \( T_c = 650 \pm 100 \text{ years} \) band prior to 2000 BCE, as noted in the main manuscript as well. This could potentially be due to resolution issues in pfm9k.2B leading to variations on the shorter periods being mapped into the \( T_c = 1300 \pm 100 \text{ years} \) band. Overall, the similarity between the two independent records supports the hypothesis that the observed periodic variations are geomagnetic in origin. However, the relatively strong influence of solar modulation on the production rates on these timescales makes it challenging to draw any firm conclusions.

**High-latitude weak/reverse flux at the CMB and dipole tilt**

Fig. S18 compares the (A) longitudinal variations in the eccentric dipole axis location with (B) the occurrence of weak/reverse flux at northern high latitudes at the CMB (note that the longitude scale is shifted by \( 180^\circ \)), variations in (C) the dipole tilt

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and (D) the dipole moment of model pfm9k.2. The comparison highlights the similar eastward propagation of both the eccentric dipole axis location and the occurrence high-latitude weak/reverse flux patches in the northern hemisphere of the CMB. The high latitude weak/reverse flux appears towards the end of each so-called eccentric phase (delimited by the horizontal dotted lines), roughly 180° away from the eccentric dipole location. As previously noted (61) the occurrence of high northern-latitude weak/reverse flux during the past 4000 years is also linked to a strong equatorial dipole moment (resulting in large dipole tilts).

**High-latitude westward drift at the CMB**

To investigate eastward and westward drift of the geomagnetic field at the CMB we follow the approach of (61–63). First we remove the time-averaged axisymmetric part of the field and then high-pass filter the Gauss coefficients with a cut-off frequency of 1/2500 year$^{-1}$. The cut-off frequency was found to be enough to filter out quasi-stationary field structures without removing too much of the original signal. Fig. S19A shows a time-longitude plot of the mean radial field (based on 1000 samples) at 60°N of the CMB of the filtered pfm9k.2 model. As previously noted (61), westward drift dominates at high northern latitudes of the CMB over the past 9000 years. There is noticeable drop in signal to noise ratio prior to 1000 BCE (Fig. S19B) related to the low density of available archaeomagnetic data in this period (see Fig. S1).

Exceptionally rapid westward drift is observed in the Atlantic hemisphere between 700 BCE and 300 BCE (Fig. S19A), with drift rates initially as high as 0.5-1°/year. The rapid westward drift is related to an intense flux patch beneath Europe that moves rapidly towards North America. This fast movement, which is not resolved by previous generation models (e.g. pfm9k.1a), can be traced to the characteristic westward swing in declination data from Europe, defined by paleosecular variation (PSV) features ‘f’ to ‘e’ using the nomenclature of (64). This is illustrated by the model-data comparison of declination and inclination at the coordinates of the PSV record MD99-2269 (ICE) from north of Iceland (31, 34), which is included in both pfm9k.1a and pfm9k.2 (Fig. S19C-D). As shown by the comparison, the rapid swing in declination and associated large variations in inclination are both poorly resolved by the previous generation model (pfm9k.1a).
Fig. S1. Spatial and temporal distribution of archeomagnetic (archaeological artefacts and volcanics) and sedimentary data used in the model. For the temporal distribution of declinations, inclinations and intensities, the data are grouped in 500 year bins.
Fig. S2. Summary of the model predictions for the Beaufort Sea (BEA) sediment record. (A) The 95% interval of the initial age-depth model (dashed black lines), the calibrated radiocarbon ages: mode and associated 95% uncertainty range (blue circles), the posterior mean (thick red line) and 95% credible interval (red lines) as well as 100 samples drawn from the posterior (grey lines). (B-C) Model-data comparison of declination and inclination: the posterior 95% credible interval (red lines), 100 samples drawn from the posterior (grey lines), the 95% credible interval of the pDRM filtered model predictions (green lines) and the paleomagnetic data from BEA with 1σ errors (black circles). The paleomagnetic data are plotted on the mean posterior age and corrected using the mean posterior $\delta_D = -10.4^\circ$ and $\delta_I = 2.8^\circ$. (D-G) Prior (dashed black lines) and posterior (grey bars) distributions of $\beta$, $\gamma$, $\delta_D$ and $\delta_I$. 
Fig. S3. Summary of the model predictions for the ODP site 1233 sediment record from the Chilian margin (CHM). (A) The 95% interval of the initial age-depth model (dashed black lines), the calibrated radiocarbon ages: mode and associated 95% uncertainty range (blue circles), the posterior mean (thick red line) and 95% credible interval (red lines) as well as 100 samples drawn from the posterior (grey lines). (B-C) Model-data comparison of declination and inclination: the posterior 95% credible interval (red lines), 100 samples drawn from the posterior (grey lines), the 95% credible interval of the pDRM filtered model predictions (green lines) and the paleomagnetic data from CHM with 1σ errors (black circles). The paleomagnetic data are plotted on the mean posterior age and corrected using the mean posterior $\delta_D = -2.4^\circ$ and $\delta_I = -0.9^\circ$. (D-G) Prior (dashed black lines) and posterior (grey bars) distributions of $\beta$, $\gamma$, $\delta_D$, and $\delta_I$. 

Fig. S4. Summary of the model predictions for Lake Biwa (BIW) sediment record. (A) The 95% interval of the initial age-depth model (dashed black lines), the calibrated radiocarbon ages: mode and associated 95% uncertainty range (blue circles), the posterior mean (thick red line) and 95% credible interval (red lines) as well as 100 samples drawn from the posterior (grey lines). (B-C) Model-data comparison of declination and inclination: the posterior 95% credible interval (red lines), 100 samples drawn from the posterior (grey lines), the 95% credible interval of the pDRM filtered model predictions (green lines) and the paleomagnetic data from BIW with 1σ errors (black circles). The paleomagnetic data are plotted on the mean posterior age and corrected using the mean posterior $\delta_D = -3.5^\circ$ and $\delta_I = 0.2^\circ$. (D-G) Prior (dashed black lines) and posterior (grey bars) distributions of $\beta$, $\gamma$, $\delta_D$ and $\delta_I$. 
Fig. S5. Summary of the model predictions for Lake Eilandvlei (EIL) sediment record. (A) The 95% interval of the initial age-depth model (dashed black lines), the calibrated radiocarbon ages: mode and associated 95% uncertainty range (blue circles), the posterior mean (thick red line) and 95% credible interval (red lines) as well as 100 samples drawn from the posterior (grey lines). (B-C) Model-data comparison of declination and inclination: the posterior 95% credible interval (red lines), 100 samples drawn from the posterior (grey lines), the 95% credible interval of the pDRM filtered model predictions (green lines) and the paleomagnetic data from EIL with 1σ errors (black circles). The paleomagnetic data are plotted on the mean posterior age and corrected using the mean posterior $\delta_D = -0.9^\circ$ and $\delta_I = 1.9^\circ$. (D-G) Prior (dashed black lines) and posterior (grey bars) distributions of $\beta$, $\gamma$, $\delta_D$ and $\delta_I$. 

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Fig. S6. Summary of the model predictions for Fish Lake (FIS) sediment record. (A) The 95% interval of the initial age-depth model (dashed black lines), the calibrated radiocarbon ages: mode and associated 95% uncertainty range (blue circles), the posterior mean (thick red line) and 95% credible interval (red lines) as well as 100 samples drawn from the posterior (grey lines). Tephra ages with 2σ uncertainties from (27), partially based on 14C dates from Fish Lake, are shown for reference (green circles). (B-C) Model-data comparison of declination and inclination: the posterior 95% credible interval (red lines), 100 samples drawn from the posterior (grey lines), the 95% credible interval of the pDRM filtered model predictions (green lines) and the paleomagnetic data from FIS with 1σ errors (black circles). The paleomagnetic data are plotted on the mean posterior age and corrected using the mean posterior δD = 0.5° and δI = 1.7°. (D-G) Prior (dashed black lines) and posterior (grey bars) distributions of β, γ, δD, and δI.
Fig. S7. Summary of the model predictions for Gyllingesjön (GYL) sediment record. (A) The 95% interval of the initial age-depth model (dashed black lines), the calibrated radiocarbon ages: mode and associated 95% uncertainty range (blue circles), the posterior mean (thick red line) and 95% credible interval (red lines) as well as 100 samples drawn from the posterior (grey lines). (B-C) Model-data comparison of declination and inclination: the posterior 95% credible interval (red lines), 100 samples drawn from the posterior (grey lines), the 95% credible interval of the pDRM filtered model predictions (green lines) and the paleomagnetic data from GYL with 1σ errors (black circles). The paleomagnetic data are plotted on the mean posterior age and corrected using the mean posterior $\delta D = 4.6^\circ$ and $\delta I = 2.0^\circ$. (D-G) Prior (dashed black lines) and posterior (grey bars) distributions of $\beta$, $\gamma$, $\delta D$, and $\delta I$. 
Fig. S8. Summary of the model predictions for MD99-2269 (ICE) sediment record. (A) The 95% interval of the initial age-depth model (dashed black lines), the calibrated radiocarbon ages: mode and associated 95% uncertainty range (blue circles), the posterior mean (thick red line) and 95% credible interval (red lines) as well as 100 samples drawn from the posterior (grey lines). (B–C) Model-data comparison of declination and inclination: the posterior 95% credible interval (red lines), 100 samples drawn from the posterior (grey lines), the 95% credible interval of the pDRM filtered model predictions (green lines) and the paleomagnetic data from ICE with 1σ errors (black circles). The paleomagnetic data are plotted on the mean posterior age and corrected using the mean posterior $\delta_D = -2.2^\circ$ and $\delta_I = -2.2^\circ$. (D–G) Prior (dashed black lines) and posterior (grey bars) distributions of $\beta$, $\gamma$, $\delta_D$ and $\delta_I$. 

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Fig. S9. Summary of the model predictions for Lake Keilambete (KEI) sediment record. (A) The 95% interval of the initial age-depth model (dashed black lines), the calibrated radiocarbon ages: mode and associated 95% uncertainty range (blue circles), the posterior mean (thick red line) and 95% credible interval (red lines) as well as 100 samples drawn from the posterior (grey lines). (B-C) Model-data comparison of declination and inclination: the posterior 95% credible interval (red lines), 100 samples drawn from the posterior (grey lines), the 95% credible interval of the pDRM filtered model predictions (green lines) and the paleomagnetic data from KEI with 1σ errors (black circles). The paleomagnetic data are plotted on the mean posterior age and corrected using the mean posterior \( \delta_D = -2.2^\circ \) and \( \delta_I = -1.6^\circ \). (D-G) Prior (dashed black lines) and posterior (grey bars) distributions of \( \beta, \gamma, \delta_D \) and \( \delta_I \).
Fig. S10. Summary of the model predictions for Kälksjön (KLK) lake sediment record. (A) The 95% interval of the initial age-depth model (dashed black lines), the calibrated radiocarbon ages: mode and associated 95% uncertainty range (blue circles), the posterior mean (thick red line) and 95% credible interval (red lines) as well as 100 samples drawn from the posterior (grey lines). The original varve chronology (42), i.e. without corrections for missing/duplicate varves, is shown for reference (green line). (B-C) Model-data comparison of declination and inclination: the posterior 95% credible interval (red lines), 100 samples drawn from the posterior (grey lines), the 95% credible interval of the pDRM filtered model predictions (green lines) and the paleomagnetic data from KLK with 1σ errors (black circles). The paleomagnetic data are plotted on the mean posterior age and corrected using the mean posterior δD = 2.4° and δI = 2.6°. (D-G) Prior (dashed black lines) and posterior (grey bars) distributions of β, γ, δD and δI.
Fig. S11. Summary of the model predictions for Lake Potrok Aike (POT) sediment record. (A) The 95% interval of the initial age-depth model (dashed black lines), the calibrated radiocarbon ages: mode and associated 95% uncertainty range (blue circles), the posterior mean (thick red line) and 95% credible interval (red lines) as well as 100 samples drawn from the posterior (grey lines). (B-C) Model-data comparison of declination and inclination: the posterior 95% credible interval (red lines), 100 samples drawn from the posterior (grey lines), the 95% credible interval of the pDRM filtered model predictions (green lines) and the paleomagnetic data from POT with $1\sigma$ errors (black circles). The paleomagnetic data are plotted on the mean posterior age and corrected using the mean posterior $\delta_I = -1.0^\circ$. (D-G) Prior (dashed black lines) and posterior (grey bars) distributions of $\beta$, $\gamma$, $\delta_D$ and $\delta_I$. 
Fig. S12. (A) Theoretical power spectra of the covariance functions used for pfm9k.2 for the axial dipole coefficient ($g_0^1$), the equatorial dipole coefficients ($g_1^1, h_1^1$) and the non-dipole coefficients ($g_m^l, h_m^l$) for different spherical harmonic degrees $l \geq 2$. (B-C) The theoretical power spectra of $g_0^1$ used for models pfm9k.2A, pfm9k.2B and pfm9k.2 compared to the power spectra calculated from (B) different published models (3, 53, 65–73) and (C) our new models, calculated using a multitaper approach. Individual samples in (C) are shown as transparent lines and the mean as thick line. The light grey shaded area highlights the frequencies, between $10^{-4}$ to $10^{-2}$ year$^{-1}$, of interest in this study. Dark shaded areas highlights the frequencies around the observed quasi-periodic signal in the dipole moment, with periods of 1300 ± 100 years and 650 ± 100 years.
Fig. S13. (A) Model comparison of the time-averaged main field power spectra at the CMB. pfm9k.2 (7000BCE - 2000CE): 100 samples drawn from the posterior (grey lines), the 95% credible interval (thin red lines), the posterior mean model (red circles), the prior 95% interval (dashed black lines). In addition, the power spectra of gufm-Sat-E3 (52) (green triangles), COV-OBS.x2 (3) (blue squares; average spectrum based on 300 realizations), COV-LAKE (2) (Magenta crosses; average spectrum based on 50 realizations) and pfm9k.1a (53) (cyan diamonds) are shown for reference.
Fig. S14. A selection of Gauss coefficient time series of pfm9k.2: 1000 samples drawn from the posterior (transparent black lines) and the mean (thick black line) as well as the 95% interval of the prior distribution (dashed black lines). Note that the prior distribution corresponds to the updated prior after anchoring the model to COV-OBS.x2 from 1850 CE to 2000 CE through Gaussian process regression. Gauss coefficients from pfm9k.1a (53) (blue lines), COV-LAKE (2) (red lines) and COV-ARCH (2) (green lines) are shown for reference.
Fig. S15. (A) Model comparison of the absolute axial dipole $|g_0^1|$ over the past 9000 years. The shaded area represents the 95\% interval of the preferred model pfm9k.2. (B) Model comparison of the average rate of change of the axial dipole $\langle \dot{g}_0^1 \rangle$ calculated over 50-year bins (for a consistent comparison) over the past 9000 years and (C) the calculated RMS values $\sigma_{\langle \dot{g}_0^1 \rangle}$. Models that include sedimentary data (2, 53, 69) show lower RMS rate of change than models based only on archeomagnetic data (70–73). The exception is COV-ARCH (2), which is constrained by the choice of prior (similar to pfm9k.2A) to have low secular variation. Note that, due to the 50-year binning, the RMS rate of change values are lower than the theoretical values.
Fig. S16. Model-data comparison of (A) inclination and (B) declination at the coordinates of Lake Hajeren, Svalbard (yellow star in maps in D-F). Lake Hajeren data (55) (black), pfm9k.2: 1000 samples drawn from the posterior (transparent blue lines) and the posterior mean (solid blue line), COV-ARCH mean model (2) (red line) and pfm9k.1a (53) (yellow line). (C) Model comparison of axial dipole $|g_1^x|$ (same colours as in A-B). Maps of (D) inclination anomaly ($I - I_{GAD}$, where $I_{GAD}$ is the inclination predicted by an axial dipole field) and (E) intensity at Earth’s surface based on the total field (i.e. including all Gauss coefficients) of pfm9k.2 and COV-ARCH at 600 BCE. (F) Maps of the surface intensity based only on the axial dipole field of the same models at 600 BCE.
Fig. S17. (A) Global mean $^{14}$C production rates, normalized to the contemporary production over the last solar cycle, inferred from tree-ring $^{14}$C data compiled in the IntCal20 calibration curve (56) (black line) and calculated based on the dipole moment of pfm9k.2B using a constant solar modulation of $\phi = 500$ MeV (59). 500 samples drawn from the posterior (transparent blue lines) and the posterior mean (thick blue line). (B-C) Same data, but detrended and band-filtered to preserve only variations with periods around the observed quasi-periodic signals in the dipole moment; (B) $T_c = 1300 \pm 100$ years and (C) $T_c = 650 \pm 100$ years (see Fig. S12). It is likely that solar peaks in the production (e.g. related to the Maunder minimum 1645-1715 CE) will influence the timing of the filtered variations, which could explain why the two records are not perfectly in phase.
Fig. S18. (A) Time-longitude density plot (0 to 360° E) of the eccentric dipole location projected onto the equatorial plane, (B) time-longitude plot (-180 to 180° E) of the posterior mean radial field at 60° N of the CMB, (C) latitude of the North Geomagnetic Pole (NGP) and (D) the dipole moment, all based on 1000 samples from pfm9k2. In (C) and (D) both the individual samples (grey transparent lines) and the mean (thick black line) are shown. Horizontal dotted black lines are added to illustrate the identified 1300-year quasi-periodic variations in dipole moment and the eccentric dipole location.
Fig. S19. Time-longitude plot of the posterior mean and standard deviation of the radial field at latitude 60°N at the CMB based on pf9k.2, filtered to remove quasi-stationary features (see 61). The dashed black lines in (A) denote 0.75°/yr westward drift, shown for reference. Also shown are the model-data comparison of declination and inclination for the MD99-2269 marine record from north of Iceland (ICE) (31). The original u-channel data (black dots) are plotted on the mean posterior age-depth model and corrected for systematic errors using the mean posterior $\Delta D_2$ and $\delta I_p$ (see Fig. S8). 1000 samples of the pf9k.2 model predictions are shown as blue transparent lines and the mean as thick blue lines. The prediction from an older generation model, pf9k.1a (53) (green lines), is shown for reference.
Table S1. Summary of model parameters and their prior distributions

| Parameter description                  | Prior distribution                        | Dimension | Size  |
|---------------------------------------|-------------------------------------------|-----------|-------|
| Gauss coefficients                     | \( x \sim \mathcal{N}(\mu_x, K_{xx}) \)  | \( NL(L + 2) \) | 6685   |
| Archaeomagnetic ages                   | \( a \sim \mathcal{N}(\mu_a, \text{diag } \sigma_a^2) \) | \( T_{ARC} \) | 9440   |
| Sediment ages at \( d_0 \)            | \( \theta \sim \mathcal{T}\mathcal{N}(\theta_0, \text{diag } \sigma_{\theta_0}^2, -\infty, \theta_0) \) | \( S \) | 10     |
| Log. accumulation rates                | \( c \sim \mathcal{N}(\mu_c, K_{cc}) \)  | \( T_{SED} \) | 398    |
| Sediment ages at section breaks        | \( q \sim \mathcal{N}(\mu_q, \text{diag } \sigma_q^2) \) | \( T_{SED} \) | \( ^a \) |
| Half lock-in depth                     | \( \gamma \sim \Gamma(2, 0.2) \)         | \( S \) | 10     |
| Lock-in shape factor                   | \( \beta \sim \Gamma(3, 2) \)            | \( S \) | 10     |
| Declination correction                 | \( \delta_D \sim \mathcal{N}(0, \sigma_D^2) \) | \( S \) | 9      |
| Inclination correction                 | \( \delta_I \sim \mathcal{N}(0, \sigma_I^2) \) | \( S \) | 10     |
| Degrees of freedom                     | \( \nu \sim \Gamma(2, 0.1) \)            | \( . \)  | 5      |
| **Total**                              |                                           |           | 16577  |

\( ^a \text{Transformed parameter constructed from } c \text{ and } \theta \)
| Abb. \(^a\) | Location/description | Time-span\(^b\) | Data type | Declinations | Inclinations | Intensities |
|---|---|---|---|---|---|---|
| ARC | Archaeomagnetic data | 9000 - 4539 | 5 | 5851 | 210 | 2.79° | 5,83 μT |
| BEA | Beaufort Sea | 4200 | 1/U | 120 | 35.24° | 32 | 3.21° | 0 | - |
| BIW | Lake Biwa, Japan | 8300 | 2/D | 194 | 3.64° | 194 | 2.30° | 0 | - |
| CHM | Chilean Margin | 8500 | 1/U | 201 | 7.12° | 201 | 3.34° | 0 | - |
| EIL | Lake Eilandvlei, S. Africa | 1100 | 1/D | 32 | 7.30° | 32 | 3.66° | 0 | - |
| FIS | Fish Lake, USA | 8200 | 1/D | 122 | 6.97° | 122 | 3.41° | 0 | - |
| GYL | Gyltigesjön, Sweden | 6200 | 1/D | 123 | 8.32° | 153 | 2.73° | 0 | - |
| ICE | Iceland/N. Atlantic | 8500 | 1/U | 302 | 34.09° | 301 | 3.18° | 0 | - |
| KEI | Lake Keilambete, Australia | 7700 | 1/D | 65 | 4.46° | 65 | 1.90° | 0 | - |
| KLK | Källsjön, Sweden | 8600 | 1/D | 96 | 5.73° | 109 | 1.78° | 0 | - |
| POT | Lake Potrok Aike | 8500 | 1/U | 0 | - | 178 | 4.36° | 0 | - |
| **Total** | | | | 5794 | 7326 | 4848 |

Data type refers to sedimentary data format (1=Specimen data and 2=Processed data) and type of sample (D=discrete and U=u-channel). \(N_D\), \(N_I\) and \(N_F\) stands for number of declination, inclination and intensity data respectively. The \(\sigma_{RMS}\) value represents RMS uncertainties of the different categories as described in the text. \(^a\)References associated with the different abbreviations (Abb.): ARC (4), BEA (74), BIW (23), CHM (16, 17), EIL (24), FIS (26), GYL (28, 44), ICE (31, 34), KEI (35), KLK (42–44), POT (17). \(^b\)Approximate time-span (in years) of the data set/record included in the model based on the mean prior ages.

Table S2. Summary of different data types and their respective RMS uncertainties.
Table S3. Summary of parameters used for the AR-2 processes describing the statistics of the Gauss coefficients

| Gauss coefficients | $\sigma$ (nT) | $\dot{\sigma}$ (nT/year) | $\omega^{-1}$ (year) | $\chi^{-1}$ (year) | $T_s$ (year) | $T_f$ (year) |
|--------------------|---------------|--------------------------|---------------------|------------------|--------------|--------------|
| $g_0^1$ (pfm9k.2)  | 10,000        | 13.5                     | 741                 | 138              | 50,000       | 433          |
| $g_1^1$, $h_1^1$   | 3,500         | 17.5                     | 200                 | 200              | 1,257        | 1,257        |
| $g_2^{m}$, $h_2^{m}$ | 1,765        | 13.3                     | 133                 | 133              | 836          | 836          |
| $g_3^{m}$, $h_3^{m}$ | 1,011       | 5.8                      | 174                 | 174              | 1,093        | 1,093        |
| $g_4^{m}$, $h_4^{m}$ | 455          | 3.3                      | 138                 | 138              | 867          | 867          |
| $g_5^{m}$, $h_5^{m}$ | 177          | 1.9                      | 95                  | 95               | 597          | 597          |
| $g_0^1$ (pfm9k.2A) | 7,700         | 10.0                     | 770                 | 75               | 100,000      | 236          |
| $g_1^1$ (pfm9k.2B) | 3,524         | 17.1                     | 206                 | 206              | 1,294        | 1,294        |
Movie S1. Time varying geomagnetic field intensity (F) at Earth’s surface over the past 9000 years based on pfm9k2: Mean (upper panel) and standard deviation (lower panel) of 1000 samples drawn from the posterior.

Movie S2. Time-varying radial field ($B_r$) at the core-mantle boundary (CMB) over the past 9000 years based on pfm9k2: Mean (upper panel) and standard deviation (lower panel) of 1000 samples drawn from the posterior. The dashed white lines shows the CMB surface expression of the inner core tangent cylinder.

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