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Static cosmic strings in space–time with torsion, and strings with electromagnetism

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Abstract
Cosmic strings are considered in space–time with torsion derived from a two-form potential, and in Minkowski space–time with electromagnetism. Using the curvature scalar as the geometrical part of the Lagrangian density, and the natural coupling to strings, the field equations are derived. It is shown a static string loop may satisfy the string equations of motion in the presence of an external torsion field. The gravitational field of the static string are derived in the weak field limit, and it is shown the gravitational field is repulsive exterior to the string. In Minkowski space–time, it is shown an external magnetic field can also give rise to a static loop.

1. Introduction
Cosmic strings, long singular threads weaving their way through the cosmos, are perhaps the most unique objects in the Universe. Although the interest in cosmic strings waned after it was learned they could not fully account for inflation, interest has since been rekindled. This is partly because it is now believed cosmic strings and fundamental (string theory) strings may be related [1], and even more importantly, there is some observational evidence of cosmic strings [2, 3]. In Ref. 2 CSL-1 (Capodimonte Sternberg Lens Candidate) is considered. Arguing undistorted double images of a background source can only be produced by a cosmic string, it is concluded that this indeed the case. The following reference, using the data collected on 355 quasars [4], points to partial alignment in optical polarization. It is argued the random chance of that is 0.1%, and it is likely produced by lensing from a cosmic string.

Theoretical evidence now points to other observations that may lead to the detection of cosmic strings. For example, curl-like distortions from weak lensing of galaxies may indicate the presence of a string [5]. The discovery of intense radio frequency bursts called sparks [6] has not been shown to be described by conventional astrophysical sources, and may be the signal of a cosmic string [7]. In addition, the only model to describe high-redshift gamma-ray bursts may be due to cosmic strings [8].

Cosmic strings, if they indeed exist, are difficult to find due to their properties distinguishing themselves from ordinary matter. Consider a static string. The static solution to the homogeneous string wave equation yields a straight line. However, outside the static string the geometry is locally flat, allowing local objects to go unaffected by the string. However, there is a global effect, and thus, excepting the observations and speculations above, the primary method of detecting the straight string is by measuring the deficit angle by its lensing effects on galaxies. Time dependent strings, say loops, expand and contract, losing their mass to gravitational radiation, which is severe near the turning points [9]. So far, this has not been observed. It has also been conjectured strong gravitational emission can arise from string cusps, although this has not been observed so far [10]. Rigid strings were first analyzed by Polyakov [11] and their statistical properties studied later [12]. Stable cosmic strings were shown to exist if the electroweak bosons are coupled to fermions [13].

In this article it is shown static string loops can exist in the presence of an external field. It is shown to be true for the case of an external torsion field, and for the situation in which there exists an external magnetic field. The existence of static loops can open a new window of potential observations of cosmic strings.
Static strings that are coupled to other fields have been considered in the past [14]. Superconducting strings in a Kaluza–Klein geometry were considered, and their coupling to an external magnetic field was also studied [15]. Witten showed that spontaneously broken gauge theories can produce stable strings [16], and the effect of current flowing in a superconducting string was also studied [17].

Cosmic strings with torsion have been considered over the years. Torsion, an integral part of string theory, has been reviewed [18–20]. The torsion field arises due to the spin of elementary particles. The string was coupled to torsion with rigid strings and gives the mass to the gauge field [21]. Baker found a torsion field that goes like $1/r$ from an infinite string [22]. He did not present a source for torsion and the torsion is of a very special kind, that made up only from the trace of the torsion tensor. One way to endow a string with torsion is to give it an interior containing a torsion source density. Various interior solutions have been investigated by Krisch [23]. Spinning strings that may contain torsion have been studied by Anandan [24], and the effect of screwed superconducting cosmic strings in scalar-tensor theories have also been investigated [25]. In this case the torsion was assumed to arise from a scalar field. The role of torsion in cosmology has also been considered [26].

However, in space–time with torsion it will be shown static loops can indeed exist. The torsion is assumed to be of the string theory type so that torsion is derived from a potential according to $S = d \wedge \psi$. Gravity with torsion derived from a potential has been reviewed [20], and string theory torsion is also called the Kalb–Ramond field. Using the curvature scalar in the geometrical action, it produces dynamic torsion with no need to add quadratic terms.

One may ask why consider torsion as the field to stabilize the cosmic string. First, the connection between strings and torsion is an ongoing area of research, as indicated above. More important though, torsion couples most naturally to a string. As shown below the action is simple the torsion potential times the string area. For other fields, such as electromagnetism, more complicated couples must be arranged.

### 2. The action

Some of the salient features of space–time with torsion have been collected in the appendix. In gravitation there are two natural string couplings to the source. Consider the induced metric on the two-dimensional world sheet of the string

$$\gamma_{ab} = x^a_{,a} x^b_{,b} \ g_{\mu\nu},$$

and its determinant $\gamma$. Lower case Latin indices take on the values $(0, 1)$, representing the string coordinates. With this we have an action given by $I_m = -\mu \int d^2 \zeta \sqrt{-\gamma}$, which is a direct generalization of the conventional point mass term, $-m \int d\tau$ if we interpret $\mu$ as the mass per unit length of the string. However string theory is much richer and has another term, something coupled to the string area $d\sigma^{\mu\nu} = x^a_{,a} x^b_{,b} \ e^{ab}$, where $e^{ab}$ is the antisymmetric tensor in two dimensions. Since $d\sigma^{\mu\nu}$ is antisymmetric it must couple to an antisymmetric potential, which is taken here to be the torsion potential $\psi^{\mu\nu}$.

It follows the most natural action coupling the string and gravitation is

$$I = I_k + I_m,$$

where the geometrical part is

$$I = -\frac{1}{2k} \int \sqrt{-g} R,$$

where $R$ is the curvature scalar of space–time with torsion, $k = 8\pi G$ and the source term consists of those just described

$$I_m = -\mu \int \sqrt{-\gamma} d^2 \zeta + \frac{\eta}{k} \int \psi^{\mu\nu} d\sigma^{\mu\nu},$$

where $\eta$ is a dimensionless coupling constant.

This formalism has been used to provide the material energy momentum tensor for a particle with intrinsic spin. It was shown in detail that the string gives rise to intrinsic spin of an elementary particle due to geometry: the spin does not arise from any rotations of the string. With this it was shown the correct conservation laws for total angular momentum plus spin result from the equations of motion provided the source of torsion is taken to be intrinsic spin [28].

With the above, the variational principle is

$$\delta \left( -\frac{1}{2k} \int d^2 \sqrt{-g} R - \mu \int \sqrt{-\gamma} d^2 \zeta + \frac{\eta}{k} \int \psi^{\mu\nu} d\sigma^{\mu\nu} \right) = 0.$$  

This is a very simple and therefore compelling starting point.
To proceed, we consider variations with respect to the metric tensor \( g_{\mu \nu} \), the torsion potential \( \gamma_{\mu \nu} \), and \( x^\mu \), which give

\[
G^{(\mu \nu)} = 2S_{\alpha \beta}S^{\nu \alpha} - kT^{\mu \nu},
\]

where

\[
T^{\mu \nu} = \mu \int d^4 \delta^4 (x - x(\zeta)) \sqrt{-\gamma} \gamma^{\alpha \beta} x^\mu_{,\alpha} x^\nu_{,\beta},
\]

where \( x(\zeta) = x(\zeta^0, \zeta^1) \) and the parentheses around the indices imply taking the symmetric part. The torsional field equations are

\[
S_{\mu \nu}^{\alpha \beta} = -\gamma_{\mu \nu}^{\alpha \beta},
\]

where

\[
\gamma_{\mu \nu}^{\alpha \beta} = \int d^4 \zeta \sqrt{-\gamma} \gamma^{\alpha \beta} (x - x(\zeta)) x^\mu_{,\alpha} x^\nu_{,\beta} e^{ab}.
\]

and finally we have

\[
\mu \Box_2 x^\nu = \frac{3\eta}{k} S_{\alpha \beta} x^\alpha_{,\beta} x^\beta_{,\nu} e^{ab},
\]

where \( \Box_2 x^\nu = (x^\nu)^{\mu}_{,\mu} \).

It is enlightening to write the Einstein tensor of \( U_4 \) space–time \( G^{\mu \nu} \) in terms of the Einstein tensor of \( V_4 \) space–time \( \tilde{G}^{\mu \nu} \)

\[
G^{(\mu \nu)} = \tilde{G}^{\mu \nu} - S_{\alpha \beta} S^{\nu \alpha} + \frac{1}{2} S_{\alpha \beta} S^{\nu \alpha},
\]

so that the field equations become

\[
\tilde{G}^{\mu \nu} = kT^{\mu \nu} + 3S_{\alpha \beta} S^{\nu \alpha} - \frac{1}{2} S_{\alpha \beta} S^{\nu \alpha}.
\]

Finally, it is also helpful to introduce the dual torsion vector \( b_\alpha = \epsilon_{\mu \alpha \beta} S^{\nu \beta} \) so that (12) in vacuum becomes

\[
\tilde{G}^{\mu \nu} = \frac{1}{6} \left( b^{\mu \nu} - \frac{1}{2} \gamma^{\mu \nu} b_\alpha b^\alpha \right).
\]

It is evident the torsion acts like an energy momentum tensor. In fact, it is the force from this 'stress tensor' that can maintain a string loop from collapsing, as is now shown. This will be shown by an example of a constant, external torsion field assumed to be in the \( z \)-direction and a string loop in the \( x - y \) plane. In the following we also adopt the conformal gauge for the two-dimensional string space.

Assuming the torsion field points in the \( z \)-direction, \( b = b\hat{z} \), gives \( b = b_3 = 6S^{012} \). With this (10) becomes, seeking a static solution and using \( q \equiv 6\eta/k\mu \)

\[
\frac{d^2 x^\lambda}{d\zeta^2} = q b_x x^\lambda_{,\zeta}^2
\]

and

\[
\frac{d^2 x^2}{d\zeta^2} = -q b_x^2.
\]

For a string of length \( L \) (\( L \) is the string length throughout) one may show, for constant \( b \), a solution to these equations consistent with the conformal gauge is

\[
x = \frac{L}{2\pi} \left( \sin \left( \frac{2\pi \zeta}{L} \right) \hat{x} - \cos \left( \frac{2\pi \zeta}{L} \right) \hat{y} \right)
\]

provided \( b = -4k\mu\pi/\eta L \).

This shows a static string loop solution exists in the presence of a torsion field. There are at least two mechanisms that would yield the constant torsion field used above. One source is other strings, as (8) shows. Of course the field may not be exactly constant, but if it is nearly uniform across the size of the string then the above would be at least approximately true. The other source of torsion arises from the galactic magnetic field. This magnetic field will partially align spins in the colder regions of the Galaxy. It shown elsewhere [20] that for particles, the source of torsion is the elementary spin of the particle. So, consider particles with a magnetic dipole moment. The galactic magnetic field will, to some degree depending on the temperature, align these particles through the magnetic dipole interaction. But now there is a net alignment of spin, which creates a galactic torsion field.
Now, one is led to ask about the effects of a static cosmic string. Let us consider the weak field approximation, $g_{\mu\nu} = h_{\mu\nu} + \eta_{\mu\nu}$, so that
\begin{equation}
\square h_{\mu\nu} = k S_{\mu\nu},
\end{equation}
where, with $T \equiv T_\sigma^\sigma$
\begin{equation}
S_{\mu\nu} = -2(T_{\mu\nu} - g_{\mu\nu} T).
\end{equation}

Here we are considering only the effect of the string so that we have
\begin{equation}
h_{\mu\nu} = \int G(x - x')S^{\mu\nu}dv'.
\end{equation}
and using the Green’s function $G(x - x') = 1/[x - x']$, we have
\begin{equation}
h_{\mu\nu} = -k \mu \int d\xi d\eta \frac{\sqrt{\gamma - \eta}}{\sqrt{x^2 + x'^2 - 2xx' \sin \theta \cos \phi'}}.
\end{equation}

The geometry is such that the string is centered on the $z$-axis and the observation point is located in the $\phi' = 0$ plane. This may be written
\begin{equation}
h_{\mu\nu} = 2GMaI_{\mu\nu}
\end{equation}
and performing the integrations we have $I^{11} = I^{22} = I^{33} \equiv I$, where
\begin{equation}
I = \frac{8\sqrt{\alpha - \beta}}{3\pi\beta^2} \left(\alpha E\left(\frac{\pi}{4}, \gamma\right) + \alpha E\left(\frac{3\pi}{4}, \gamma\right) - (\alpha + \beta) F\left(\frac{\pi}{4}, \gamma\right) + F\left(\frac{3\pi}{4}, \gamma\right)\right)
\end{equation}
and where $F$ and $E$ are the elliptic integrals of the first and second kind respectively and
\begin{equation}
\gamma = \frac{2\beta}{\beta - \alpha} \quad \text{and} \quad \alpha = 1 + \xi^2, \quad \beta = 2\xi \sin \theta,
\end{equation}
where $\xi = r/a$ is the dimensionless radial variable.

For $\xi \gg 1$ this becomes
\begin{equation}
h^{11} = \frac{2GM}{a^2},
\end{equation}
where $M = \mu L$. This looks like the Newtonian potential, but it does not act that way! The equation of motion of a particle of mass $m$ is given by
\begin{equation}
\frac{d\nu^\sigma}{d\tau} + \{\nu^\sigma\} \nu^\alpha \nu^\nu = 0
\end{equation}
and in the low velocity the following term dominates:
\begin{equation}
\frac{d\nu^\sigma}{d\tau} + \{\nu^\sigma\} = 0,
\end{equation}
but using the $h_{\mu\nu}$ given above, $\{\nu^\sigma\} = 0$. The only forces produced by the string are velocity dependent. In fact, this shows $d\nu^\alpha/d\tau = 0$ which implies $\nu_\alpha d\nu^\alpha/d\tau = 0$ so that the acceleration is perpendicular to the velocity. In other words, the kinetic energy of the test mass does not change.

Nevertheless, it is now shown the string has gravitational effects. As an example, consider the region near the center of string, in which case we imagine a large string around the Galaxy, in the plane of the Galaxy.

The effect of the conventional mass and dark matter will be ignored. Of course, these have the major effect on the test object, but in order to clearly separate the effect of the string that mass will be ignored. Consider motion in the $\theta = \pi/2$ plane. The equations of motion are
\begin{equation}
\frac{dv^1}{d\tau} + \{v^1\} v^1 + \{v^2\} v^2 + 2\{v^1\} v^1 v^2 = 0
\end{equation}
and
\begin{equation}
\frac{dv^2}{d\tau} + \{v^2\} v^1 + \{v^2\} v^2 + 2\{v^2\} v^1 v^2 = 0.
\end{equation}
For $\xi \ll 1$ (near the center of the loop) we have
\begin{equation}
\{v^1\} = -\{v^3\} = \{v^3\} = -\frac{GM}{a^2} \xi.
\end{equation}
With this, from (27) to (28) we plot the results. In the figures, initial conditions are taken for convenience, \( v_0 \), the initial \( x \) component of the velocity, is given, but the others are found by assuming they are compatible with the condition noted above that the velocity is orthogonal to the acceleration.

In a similar way we find, far from the cosmic string, the results displayed in figure 2. A remarkable feature is the loop gives rise to a repulsive gravitational field exterior to it.

### 3. Electromagnetic coupling

We will now show the same kind of solution exists in the electromagnetic case. We consider the minimally coupled string in Minkowski space–time [29]

\[
I = - \mu \int \sqrt{-\gamma} d^2 \zeta - \frac{1}{16 \pi} \int d^4 x \sqrt{g} F_{\mu \nu} F^{\mu \nu} \\
- \int d^2 \zeta \sqrt{-\gamma} \left( A_{\mu} \phi_{\mu} \right) (x^{\mu} x^{\mu} - 4 e^{ab} \phi_{ab} \gamma^{ab}).
\]

(30)

Setting

\[
\delta I = 0
\]

(31)
gives the field equations. Variations with respect to \( x^\mu \) give

\[
\Box x^\nu + (t_{ab} x^{\nu}_{,ab})_b = -F_{\nu} ^{\mu} x^\mu_{,ab} j^a,
\]

(32)

where \( j^a = e^{ab} \phi_{,b} \). Variations with respect to \( A_{\mu} \) yield

\[
F_{\mu \nu} = 4 \pi j^\nu,
\]

(33)

where

\[
j^\nu = \frac{1}{\sqrt{-g}} \int \sqrt{-\gamma} d^2 \zeta \delta^{\nu} j\phi x^{\mu}_{,ab},
\]

(34)
where \( \delta^4 = \delta^4(x - x(\zeta)) \). Variations with respect to \( \phi \) give

\[
\Box_2 \phi = -\frac{1}{2} F_{\mu\nu} x'^{\mu
u} \epsilon \epsilon^{ab}.
\]  

(35)

The scalar field \( \phi \) has units of charge (this differs from [29]).

Now we consider an external magnetic field taken to be in the \( z \)-direction so \( F_{12} = -B \) and therefore

\[
\Box_2 \phi = -F_{\mu\nu} x'^{\mu
u} = 0
\]

in the case of a static string a solution is \( \phi = a \zeta + nt \), where \( a \) and \( n \) are constants of integration. (It is noted even though \( \phi \) is a function of \( t \), the physical observables are not.) In this case we find

\[
t_1 = t_2 = \frac{a^2 + n^2}{2}.
\]

(37)

Now we consider (32). With the above this becomes

\[
A \frac{d^2x^i}{d\zeta^2} + C \frac{dx^i}{d\zeta} = 0
\]

(38)

and

\[
A \frac{d^2x^i}{d\zeta^2} - C \frac{dx^i}{d\zeta} = 0,
\]

(39)

where

\[
A = \mu - \frac{a^2 + b^2}{2} \quad \text{and} \quad C = \frac{nB}{\mu}.
\]

(40)

A solution to these equations is given by (16) provided

\[
\frac{A}{C} = \frac{L}{2\pi}.
\]

(41)

Similar to the case with torsion, an external magnetic field can maintain a static electrically charged string. One may also note, from (34),

\[
j^0 = e \delta^3
\]

(42)

with \( a \equiv e/L \), where \( e \) is the total charge of the string. For the static loop derived above, the current density vanishes.

4. Summary

Starting from a simple and natural coupling of a string source to gravitation with torsion that is derived from a two-form potential, it was shown a static solution exists if there is an external torsion field. The gravitational field of such a string was found and numerical solutions to the equation of motion were given. The gravitational force is velocity dependent, and the perhaps surprising result was found that, far from a closed string, the gravitational field was repulsive. This was shown in the non-relativistic case with all motion in the plane of the string. In addition, it was shown a magnetic field can maintain a static string with electric charge. It is interesting to conjecture about the repulsive effect of such strings on the expansion of the Universe. As the Universe expands, the effects of being external to such strings could drive the expansion, although much more work is needed here. Other fields may also lead to stabilizing effects such as a scalar, and in addition, may be extended to higher dimensions [30]. It would also be useful to know if these configurations are stable under perturbations, an area of current investigation.

5. Appendix on \( U_4 \) space–time

Some basic concepts of space–time with torsion, \( U_4 \) space–time, are briefly described in this appendix. Reviews may be found in the literature [18–20], while a definitive mathematical treatment may also be noted [27]. The covariant derivative of a vector \( A^\nu \) is defined by

\[
\nabla_\nu A^\nu = A^\nu_{\nu^\rho} + \Gamma_{\nu\mu}^\rho A^\rho
\]

(43)

and the curvature tensor is defined by parallel transportation of a vector \( A^\nu \) around an infinitesimal closed path \( \xi^\mu \).
\[ \Delta A' = \frac{1}{2} A' \partial_{\beta} R_{\alpha}^{\sigma} \oint \xi^\mu \mathrm{d}x^3. \]  

(44)

From this the curvature tensor is given by

\[ R_{\beta \mu \nu} = \Gamma_{\beta \mu, \nu} - \Gamma_{\beta \nu, \mu} + \Gamma_{\beta \sigma} \Gamma_{\sigma \nu}^{\alpha} - \Gamma_{\beta \sigma} \Gamma_{\sigma \mu}^{\alpha}. \]  

(45)

As in \( V_4 \) space–time (no torsion) there a set of identities the curvature tensor obeys, which may be found in Schouten [27]. The Ricci tensor of \( U_4 \) space–time is defined as

\[ R_{\mu \nu} = R_{\alpha \mu \nu}, \]  

(46)

which has both a symmetric and antisymmetric part in \( U_4 \) space–time. The antisymmetric part of the Ricci tensor is

\[ R^{[\mu \nu]} = (\Delta_\alpha + 2\zeta_0)(S^{[\mu \nu \sigma \tau]} + S^{[\mu \nu \sigma \tau]}, \]  

(47)

where \( S^{\nu} = S^{[\mu \nu \sigma]} \) is called the torsion trace and sometimes the torsion vector (but see below, for torsion dual vector).

We assume the metric tensor is symmetric, but in the above, it has not been introduced into the affine geometry. The question is, how is the metric tensor related to the affine connection? There are two procedures, one is to adopt the Palatini formalism, or first order approach, and consider variations with respect to the metric tensor and connection independently. In this case those field equations establish the relationship. The other approach is to introduce the nonmetricity tensor \( Q_{\alpha \mu \nu} \) which is defined by

\[ \nabla_{\nu} g^{\mu \alpha} \equiv Q_{\mu \alpha}, \]  

(48)

which gives

\[ \Gamma_{\alpha \nu} = \{ \nu \} + S_{\alpha \nu} + S^{\nu} + S_{\nu}^{\alpha} + \frac{1}{2}(Q_{\alpha \nu} + Q_{\alpha \nu}^{\varphi} - Q_{\alpha \nu}^{\varphi}), \]  

(49)

where the symmetric Christoffel symbol is

\[ \{ \nu \} = \frac{1}{2} g^{\nu \nu} (g_{\alpha \mu, \sigma} + g_{\alpha \mu, \nu} - g_{\alpha \mu, \nu}), \]  

(50)

and the torsion is defined as

\[ S^{\alpha \nu} \equiv \Gamma_{\mu \nu}^{\alpha} = \frac{1}{2}(\Gamma_{\mu \nu}^{\alpha} - \Gamma_{\nu \mu}^{\alpha}). \]  

(51)

To fix a relation between the metric tensor and the connection we set the non-metricity tensor to zero. This ensures the length of a vector upon parallel transport around a closed path is invariant, and therefore has great physical merit.

The Einstein tensor is defined by

\[ G^{\mu \nu} = R^{\mu \nu} - \frac{1}{2} g^{\mu \nu} R \]  

(52)

and the curvature scalar is defined by \( R = R_{\nu}^{\nu}. \)

One should note in \( U_4 \) space–time there are other tensors one may define that vanish in Riemannian geometry, for a full exposition one may consult the literature [27]. The contracted Bianchi identities are given by

\[ \nabla_{\nu} G^{\mu \nu} = 2S^{[\mu \nu \lambda] \rho} R_{\lambda \rho} - S_{[\lambda \rho] \sigma} R^{[\lambda \rho \sigma]}, \]  

(53)

In this paper, as mentioned above, we are considering torsion derived from an antisymmetric potential according to

\[ S_{\mu \nu \rho} = \psi_{[\mu \nu, \rho]} = \frac{1}{3}(\psi_{[\mu, \nu, \sigma]} + \psi_{[\mu, \nu, \nu]} + \psi_{[\nu, \nu, \mu]}). \]  

(54)

In this case we have some useful results:

\[ \Gamma_{\mu \nu}^{\sigma} = \{ \sigma \} + S_{\mu \nu}^{\sigma}, \]  

(55)

and

\[ G^{[\mu \nu]} = S^{[\mu \nu \sigma]} \]  

(56)

where the semicolon represents the Levi–Civita, or Christoffel, covariant derivative.

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