Research on position inverse solution of electric-driven Stewart platform based on Simulink

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Abstract: The structural characteristics and modes of the Stewart platform driven by electric cylinders are briefly introduced. Based on Simulink, the modelling method of the platform is analysed. To obtain the coordinate transformation of Stewart platform, the rotation matrix and homogeneous transformation are resolved, and the mathematical model of the electric-driven Stewart platform is established related to the structural characteristics. The simulation model of input and output signals is constructed by using graphical user interface (GUI) module provided by Simulink. The motion simulation curves of six electric actuators under different position and posture are obtained, which gives benefit to understand and control the different motion states of the electric-driven Stewart platform.

1 Introduction

As a kind of parallel robot, Stewart parallel mechanism has been widely studied and applied in parallel machine tool [1], flight simulator [2], and ship simulator [3] since 1960s. Compared with serial mechanism, parallel mechanism has some characteristics of compact structure, high bearing capacity, high stiffness, high control precision, and low dead weight load ratio [4, 5]. The correct kinematics solution is the basis of Stewart platform control. As Stewart platform parallel is a closed mechanism, the kinematics forward solution needs to solve a group of non-linear equations, and the solution is often unique, so that the platform motion control is unsteady. In order to obtain the better platform control effect, kinematics inverse solution with high accuracy needs to be obtained [6]. The position inverse solution of Stewart platform needs to establish the mathematical model and to calculate the motion variation of the actuators [7].

Hydraulic, pneumatic, electric, and other driving modes are used in Stewart parallel mechanism [8, 9]. The hydraulic-driven Stewart parallel mechanism is suitable for large loads simulation with rapid response. The pneumatic-driven Stewart parallel mechanism is suitable for light load simulation with rapid response because of air compressibility. The electric-driven Stewart parallel mechanism uses the electric cylinders as the actuators, which is suitable for the motion simulation with low load and high control precision. Research on Stewart platform parallel with electric cylinder driven is in new stage [10], and the motion analysis and the dynamic response of the platform system need to be investigated.

The position inverse solution of Stewart parallel mechanism can be implemented by Simulink [11]. Simulink based on modular graphics input provides graphical user interface (GUI) user interface, simplifies user programming, and provides a convenient simulation calculation platform.

2 Analysis of position inverse solution

2.1 Mathematical model

The Stewart platform is composed of an upper table, a lower base, six upper hooke hinges, six lower hooke hinges, and six electric actuators. In order to describe the spatial position relationship of the Stewart platform, two coordinate systems are established: the dynamic coordinates system O₁-X₁Y₁Z₁ as abbreviation {L} coordinate system and the static coordinate system O₈-X₈Y₈Z₈ as abbreviation {W} coordinate system, and its model is shown in Fig. 1.

2.1.1 Coordinate conversion: The Stewart platform has three translation freedom degrees and three rotation freedom degrees in space. Translation transformation refers to the movement of the upper table along the x, y, and z-axes as shown in Fig. 2. The translation displacements are x_p, y_p, and z_p. The corresponding comprehensive transformation matrix is shown in (1):

$$P = \begin{bmatrix}
1 & 0 & 0 & x_p \\
0 & 1 & 0 & y_p \\
0 & 0 & 1 & z_p \\
0 & 0 & 0 & 1
\end{bmatrix}$$

The rotation transformation refers to the upper table rotating on the x-, y-, and z-axes with rotation angles ϕ, θ, and ψ, respectively, as shown in Fig. 3.

The corresponding rotation matrix is obtained in (2):

$$W = R(z, f)R(y, θ)R(x, ϕ) = \begin{bmatrix}
cωcθ - sωcθf - sωsf + cωθsf & sωcθf & sωsf + cωθsf & -sωθsf - cωθcf \\
-sωcθ & cωcθf & sωsf & cωθsf \\
-sωθf & cωθsf & cωθcf & sωθsf - cωθcf
\end{bmatrix}$$

Fig. 1 Diagram of 6-universal joint & pusher & universal joint (UPU) Stewart platform
The rotation transformation is obtained from the rotation matrix in (3):

$$
WP = WR_1^T P
$$

(3)

Then, the homogeneous coordinate form of the transformation matrix in (4):

$$
T = \begin{bmatrix}
R_{x \times 3} & P_{x \times 1} \\
0_{1 \times 3} & 1_{1 \times 1}
\end{bmatrix}
$$

(4)

2.1.2 Position inverse solution: There are six upper hooke hinges A1, A2, A3, A4, A5, and A6, respectively, installing on the upper table, and six lower hooke hinges B1, B2, B3, B4, B5, and B6, respectively, installing on the lower base. The hexagon diagram of upper hooke hinges and lower hooke hinges is shown in Fig. 4. At initial state, the Z coordinate axis of upper table overlaps the lower base, and the X coordinate axes and Y coordinate axes parallel, respectively, the lower base. The radius of the circle formed by the upper hooke hinges is $R_p$, and the short side lengths $A1A3$, $A2A4$, and $A5A6$ are equal to $d_p$. The radius of the circle formed by the lower hooke hinges is $R_b$, and the short side lengths $B2B3$, $B4B5$, and $B6B1$ are equal to $d_b$.

Equation (5) shows the space coordinates of the upper hooke hinges, and (6) shows the space coordinates of the lower hooke hinges, where $A$ is an upper hooke hinge coordinate matrix, $a$ is the central angle corresponding to the long side of the upper hooke hinge hexagon, and $h_a$ is the distance between the upper hooke hinge and the upper table; $b$ is the central angle corresponding to the long side of the lower hooke hinge hexagon, and $h_b$ is the distance between the lower hooke hinge and the upper table. The transformation relationship between the length variation $\Delta l_i$ and the coordinates of the upper hooke hinge and lower hooke hinge is shown as follows:

$$
\Delta l_i = l_i - l_o = \left\| T \cdot A - B \right\|
$$

(7)

2.2 Simulation model

Based on analysis of kinematics inverse solution of the Stewart platform, the simulation model is established in Simulink. The Simulink global model of the inverse solution of the Stewart platform is shown in Fig. 5. It is composed of the translation position setting module ‘Translation_path’, the rotation position setting module ‘Angle_path’, the ‘Rotation matrix module’, the ‘Matrix concatenate module’, the ‘Matrix Multiply module’, the ‘Lower hooke hinge vector module’, the ‘Upper hooke hinge vector module’, the ‘Leg length calculation module’, the dimension transformation module of matrix ‘Reshape’, the every actuator moving parameters curve module ‘Scope’, and so on.

The ‘Translation path’ module is mainly used for input translation trajectory. The expected position and pose parameters of the upper table are set up in this module with translation transformation of the dynamic coordinate system relative to the static coordinate system. When the input is as follows: $X$ input, $Y$ input, and $Z$ input; and the output is as follows: $X$ output, $Y$ output, and $Z$ output. The ‘Angle_path’ module is mainly used for input rotating motion trajectory. The expected motion parameters of the upper table are set up in this module with rotation transformation of the dynamic coordinate system relative to the static coordinate system, and the input and output are similar to the translation module.

In the ‘Upper hooke hinge vector’ module, the $R_p$ is the upper hooke hinge table radius input, and $n$ is the short side length of the

$$
A = \begin{bmatrix}
R_p \cos(a) & R_p \cos\left(\frac{2}{3} \pi - a\right) & R_p \cos\left(\frac{2}{3} \pi + a\right) & R_p \cos\left(\frac{4}{3} \pi - a\right) & R_p \cos\left(\frac{4}{3} \pi + a\right) & R_p \cos(-a) \\
R_p \sin(a) & R_p \sin\left(\frac{2}{3} \pi - a\right) & R_p \sin\left(\frac{2}{3} \pi + a\right) & R_p \sin\left(\frac{4}{3} \pi - a\right) & R_p \sin\left(\frac{4}{3} \pi + a\right) & R_p \sin(-a) \\
h_a & h_a & h_a & h_a & h_a & h_a
\end{bmatrix}
$$

(5)

$$
B = \begin{bmatrix}
R_b \cos(b) & R_b \cos\left(\frac{2}{3} \pi - b\right) & R_b \cos\left(\frac{2}{3} \pi + b\right) & R_b \cos\left(\frac{4}{3} \pi - b\right) & R_b \cos\left(\frac{4}{3} \pi + b\right) & R_b \cos(-b) \\
R_b \sin(b) & R_b \sin\left(\frac{2}{3} \pi - b\right) & R_b \sin\left(\frac{2}{3} \pi + b\right) & R_b \sin\left(\frac{4}{3} \pi - b\right) & R_b \sin\left(\frac{4}{3} \pi + b\right) & R_b \sin(-b) \\
h_b & h_b & h_b & h_b & h_b & h_b
\end{bmatrix}
$$

(6)
upper hooke hinge hexagon. The function ‘fcn’ establishes the matrix $A$ in the ‘m’ file, and the output results are multiplied with the rotation matrix and added with the translation vector after transformed. The upper hooke hinge vector in the dynamic coordinate system is obtained, as shown in Fig. 6a.

In the ‘lower hooke hinge vector’ module, the $R_b$ is the lower hooke hinge base radius input, and $m$ is the short side length of the lower hooke hinge hexagon. The function ‘fcn’ establishes the formula of matrix $B$ in ‘m’ file. The output result is subtracted from the previous result, as shown in Fig. 6b.

The ‘Rotation matrix’ module is mainly used to calculate the rotation matrix and transform the one-dimensional matrix into two-dimensional matrix, and output by the matrix ‘Reshape’ module, as shown in Fig. 7, where $Q$ is the matrix input and $R$ is the matrix output.

In the ‘Leg length calculation’ module, the motion actuator leg length vector is given to port A, and the initial actuator leg length is given to port B, and two groups of values are assigned to six actuator legs for calculation, as shown in Fig. 8.

3 Simulation results

The Stewart platform has six freedom degrees. The motion simulation can be carried out, respectively. In the case of single freedom degree translation, it moves 30 mm along $X$-axis, 20 mm along $Y$-axis, and 30 mm along $Z$-axis, and the results are shown in Fig. 9. In the case of single freedom degree rotation, 20° rotating on $X$-axis, 20° rotating on $Y$-axis, and 20° rotating on $Z$-axis, and the results are shown in Fig. 10.

Sometimes multiple freedom degrees motion needs to be simulated, so the simulation curves are shown in Fig. 11. Fig. 11a shows the simulation curves of 10 mm moving along the axis of $X$, $Y$, and $Z$ at the same time. Fig. 11b shows the simulation curves of the $5°$ rotating on $X$, $Y$, and $Z$-axis at the
Fig. 9  Single degree of freedom simulation of Stewart platform translation
(a) Translation along X-axis, \(x = 30\sin(t)\), (b) Translation along Y-axis, \(y = 20\sin(t)\), (c) Translation along Z-axis, \(z = 30\sin(t)\)

Fig. 10  Single freedom degree simulation of Stewart platform rotation
(a) Rotation on X-axis, \(\phi = 20^\circ\sin(t)\), (b) Rotation on Y-axis, \(\theta = 20^\circ\sin(t)\), (c) Rotation on Z-axis, \(\psi = 20^\circ\sin(t)\)

Fig. 11  Multiple freedom degree simulation of Stewart platform
(a) Translation along X-, Y-, Z-axis, \(x = y = z = 10\sin(t)\), (b) Rotation on X-, Y-, Z-axis, \(\phi = \theta = \psi = 5^\circ\sin(t)\), (c) Translation along Y-axis, \(y = 10\sin(t)\), rotation on Y-axis, \(\theta = 5^\circ\sin(t)\)
same time. Fig. 11c shows the simulation curves of translating 10 mm along Y-axis and rotating 5° rotating on Y-axis at the same time.

From the above simulation results, the simulation diagram shows that each actuator position and posture change is obvious and easy to modify the process parameters under a given condition. It can make the simulation process fast and convenient.

4 Conclusions
Stewart platform is a non-linear and strongly coupled complex system. The coordinate transformation system and inverse solution equation are established, which can be used to investigate the kinematic relationship between input and output of Stewart platform. The simulation module is established based on Simulink, which can be used to clearly observe the motion state of each actuator in a predetermined position and pose. The motion simulation curves are obtained, respectively, under single freedom degree and the multiple freedom degree with translation or rotation case, which give some motion rules for correctly control the Stewart platform.

5 References
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