Abstract—Cosmic ray radiation is mostly composed, at sea level, by high energy muons, which are highly penetrating particles capable of crossing kilometers of rock. The ubiquitous and steady presence at the Earth’s surface and the high penetration capability have motivated the use of cosmic ray radiation also in fields beyond particle physics, from geology, archeology, speleology to industrial applications and homeland security.

In particular, in recent years, the novel technique of muon tomography has been proposed, with the aim of performing non invasive inspection of large non accessible volumes, material atomic number $Z$ and density discrimination, and three dimension image reconstruction of the inspected volume.

In the present paper, after a short recall of the physical principles and mathematical formalism on which muon tomography is based, a number of examples of application of the novel technique in industry and homeland security issues is given.

Moreover, a new application of cosmic rays detection techniques in the field of civil engineering is proposed. The aim is the monitoring of the stability of large structures, in particular the static monitoring of historical buildings, where conservation constraints are more severe and the time evolution of the deformation phenomena under study may be of the order of months or years. The new technique may be seen, in some way, as the reverse problem of muon tomography.

As a significant case study, the monitoring of the wooden vaulted roof of the Palazzo della Loggia in the town of Brescia, in Italy, has been considered. The feasibility as well as the performances and limitations of a monitoring system based on cosmic ray muons have been studied by Monte Carlo simulation and discussed in comparison with more traditional monitoring systems.

I. INTRODUCTION

WHEN primary cosmic rays, mainly composed of high energy protons coming from the sun and from the outer Galaxy, strike the Earth’s atmosphere, a cascade of many types of subatomic particles is created [1]. Hadronic particles produced in the shower either interact or decay, and electrons and photons lose energy rapidly through pair production and bremsstrahlung, so that, by the time the charged component of this particle shower reaches the Earth surface, it comprises primarily positive and negative muons. The flux reaching the surface of the Earth is about $10,000 \mu /\text{min} m^2$ and the mean muon energy is 3 to 4 GeV. Since muons are heavy particles and do not undergo nuclear interactions, they are highly penetrating in matter and their average energy is sufficient to penetrate tens of meters of rock.

Cosmic radiation has been known since the first decades of the 20th century and, until the construction of the first particle accelerators, it constituted the best source of projectiles to investigate the fundamental structure of matter and the fundamental interactions between elementary particles. Nowadays, cosmic rays are largely exploited in nuclear and elementary particle physics for detector testing and calibration and for the alignment of detectors in the very complex apparatuses used in this field.

Making practical use of this natural flux of highly penetrating particles, continuous, free and ubiquitously present on the entire Earth surface, has always been an attractive idea. As the spectrum of cosmic ray muons is continuous and the average range is long, differential attenuation can be used to produce radiographies of large and dense objects. E.P. George [2] measured, in 1955, the depth of rock above an underground tunnel by making use of the attenuation of cosmic ray muons. With the same technique, L.W. Alvarez performed the radiography of the Second Pyramid of Giza [3] seeking for the possible presence of hidden chambers.

Over the following years, muon radiography has been used to perform inspection of large inaccessible systems or in the fields of geology, archeology, speleology [4]-[9]. Several groups are actively working in the imaging of the interior of volcanoes and in the prediction of volcanic eruptions [10]-[17], [18]-[21], [22]-[23], [24]-[25].

Proposals have been presented to obtain radiographic images of the interior of large vessels with dimensions over many tens of meters, where storage or long-term structural integrity is an important issue [26], [27]. Potential uses of cosmic ray muon radiography in industrial applications have been explored [28]-[30], including the inspection of nuclear waste containers [31] and of the inner structure of a blast furnace [32]-[35].

In 2003 a new method has been proposed [36]-[39], the muon tomography, in which the angular scattering that every muon undergoes when crossing matter is exploited. The scattering angles have a Gaussian distribution, with variance proportional to the traversed thickness and to the average “scattering density” of the material crossed by the muons. The scattering density is roughly proportional to the product of the material mass density times its atomic number.

This technique needs a more complex apparatus. While the absorption technique requires the measurement of the muon position and direction only downstream of the object to be inspected, the technique based on muon scattering requires the measurement of muon position and direction both upstream and downstream, to measure the angular deviation of every single muon. Considering the increasing interest for
the application of the muon tomography technique to industry and homeland security issues, in Section II a brief illustration of the muon tomography physical principles and formalism is given, with few examples of possible applications.

In 2007 cosmic ray muon detection techniques were assessed [40] for measurement application in civil and industrial engineering, for the monitoring of alignment and stability of large civil and mechanical structures. A Monte Carlo analysis was developed concerning the case of monitoring of the mechanical alignment of parts of an industrial press.

This specific application is, in some way, the reverse problem of muon tomography. Indeed, in the latter case, from the known positions of the upstream and downstream detectors, the content of the inspected volume is inferred; in the former case, the relative displacement of the upstream and downstream detectors is continuously monitored, and the information on the content of the inspected volume in terms of geometry and material composition may improve the precision of the measurement of their displacement.

In the present paper, the same general idea of exploiting the cosmic ray natural source of radiation for the monitoring of the alignment and stability of large structures is applied, with an improved detector scheme, to the case of civil buildings. The study is devoted especially to historical buildings, whose cultural and artistic value often puts severe constraints of non-invasiveness to the monitoring techniques that may be employed.

The study of the application of the muon stability monitoring method to historical buildings was performed with a Monte Carlo technique and was applied to a realistic situation, the exemplary case of the “Palazzo della Loggia”, seat of the Mayor, in the town of Brescia, Italy [41], [42]. The “Palazzo della Loggia” was built in 1574 by the Venetian Government of the town and suffered, since the first years after the construction, of several structural problems. In recent years, from 1990 to 2001, a campaign of measurements was performed to monitor the stability and progressive deformation of its wooden vaulted roof, completely reconstructed in 1914, by means of a mechanical monitoring system based on the elongation of metallic wires [43], [44].

In Section III, the application of the method of muon stability monitoring to the case of the wooden vaulted roof of the “Palazzo della Loggia” is described. The performance of the designed measurement system is evaluated by means of a Monte Carlo program based on the GEANT4 toolkit [45], in terms of precision of the measurement of the relative displacements and observation time needed. This performance is compared with the features of the mechanical methods adopted in [44]. In Section IV results are discussed and conclusions are drawn.

II. MUON TOMOGRAPHY APPLICATIONS

A. Tomographic reconstruction with cosmic rays

The main difference between X-ray tomography and muon tomography lies in the nature of the information used for the reconstruction process. In X-ray tomography several projections of the density distribution are combined. Each projection is obtained by the differential adsorption of a beam of parallel X-rays. Given the large intensity of the X-ray beam, in general the images do not suffer from statistical fluctuations.

In muon tomography, the three dimensional material “scattering density” distribution (see Section II-B) is obtained by the measurement of the scattering of cosmic ray muons. The relatively small number and the randomness of cosmic tracks traversing the volume generate sparse and unequal sampling in the data space, making not adequate any procedure based on histogram distributions obtained combining similar events. So called List-Mode strategies, based on the processing of each track one at a time, are preferable. However, such procedures require an iterative approach that may become an issue when calculation time is a crucial concern.

A further difference must be mentioned with respect to the X-ray tomography. The absorption of an X-ray photon is a well defined event. The photon travels on a straight line and the absorption takes place along this line. In the muon scattering process, the difference between the final and initial muon directions is the overall result of a hidden chain of many small, uncorrelated, individual scatterings in the crossed material. The exact trajectory inside the scanned volume of each individual muon is therefore not known, and can only be approximated. Any difference between real and approximated trajectory will produce a blur in the reconstructed image.

For each muon track crossing the inspected volume two quantities are collected, the scattering angle and the displacement vector from the expected muon exit point in absence of scattering. Those two quantities are then projected on two orthogonal planes passing through the incoming muon direction. The scattering angle and the displacement are the overall result of a hidden chain of individual scatterings in the crossed elementary volumes (voxels), usually cubes, in which the full inspected volume is subdivided.

The problem formulation has a suitable generic solution in a hidden Markov model theory, where an Expectation Maximization estimator of the voxel densities is iterated. Technically, the reconstruction algorithm consists of a 3D tomographic projector-back-projector pair applied to a Maximum Log-Likelihood functional. The whole procedure is effectively based on a List-Mode strategy described in detail in [39] and shortly outlined in the following.

The reconstruction of the three dimensional distribution of scattering density is generally performed as follows. First the approximated muon track path inside the reconstruction volume is calculated. Entering and exiting directions and positions of each incoming muon are extrapolated inside the inspection volume. In general, the two straight lines do not intersect. To approximate the muon trajectory, the Point of Closest Approach (PoCA) between the two lines is computed and a broken line from the entry point to the PoCA and from PoCA to the exit point is assumed as best approximation.

Inside each voxel the scattering density is assumed to be constant with a proper initialization value. The likelihood function of each event is computed as the product of the probability of individual scatterings in each voxel. To build the likelihood, the path length of every muon inside each crossed voxel is needed. A well profiled ray tracing function evaluates
the trajectory from the entry point to the exit point through the PoCA for each muon. This information, along with the scattering angle and position displacement for each muon, are finally passed in input to the likelihood maximization algorithm, based on an Expectation Maximization technique. The algorithm produces, by successive iterations, the best guess of the scattering density values in each voxel compatible with the measured quantities of all collected muons. The algorithm is described more formally in next Section.

The statistical fluctuations in the quantities used for the tomographic reconstruction will unavoidably imply the presence of noise in the image. To avoid false interpretations of the tomographic reconstruction, this noise will have to be filtered, either by processing the image after the reconstruction phase, or by inserting a filtering prior function in the image reconstruction phase.

B. Muon tomography formalism

Muon tomography is based on Multiple Coulomb Scattering of cosmic ray muons crossing the volume to be inspected. When a charged particle crosses a material, its trajectory is deflected from the direction of incidence. The distribution of the scattering angle \( \delta \theta \) projected on a plane containing the incident particle trajectory is approximately Gaussian with zero mean value and variance related to the properties of the incident particle trajectory is approximately Gaussian with zero mean value and variance related to the properties of the incident particle.

Suppose now that a muon tomography of a large volume is performed placing two muon detectors above and below the inspection volume, as sketched in Fig. 1. We divide the full volume in volume elements, voxels, small enough to allow considering homogeneous the material distribution inside a single voxel. Let call \( \lambda_i \) the unknown scattering density of the voxel \( i \).

When a muon crosses the volume, muon \( \mu_1 \) in Fig. 1 for instance, the incoming and outgoing trajectories are measured. This is not sufficient to know the exact path of the muon inside the unknown volume. The true path is approximated with the broken line from entry point to the exit point through the PoCA. Within this approximation, the voxels traversed by the muon can be identified. The total scattering angle of the muon will be determined by the scattering density of all the voxels crossed by that muon, the ones marked in yellow.

By the measurement of many muons crossing the inspected volume in different positions and at different angles, the scattering density of each individual voxel can be disentangled, as long as the individual voxel has been crossed by a sufficient number of muons.

More formally, the total variance of the scattering of the \( i \)-th muon through \( N \) voxels can be written as a function of the voxel scattering density and path length \( L_{ij} \) of the \( i \)-th muon inside the \( j \)-th voxel:

\[
\sigma_i^2 = \frac{(13.6 \text{ MeV}/c)^2}{\beta^2} \sum_{j=1}^{N} L_{ij} \lambda_j
\]

Fig. 1. Hypothetic paths of two different muons inside the inspected volume. Muon \( \mu_1 \) is supposed to cross yellow voxels and muon \( \mu_2 \) the green ones.

Since in general the measurement of the muon momentum is impossible or very difficult, and the muon momentum spectrum is very broad, the previous equation is averaged over the muon momentum spectrum and the \( p \) value is substituted by an “effective” momentum value \( p_{\text{eff}} \). This approximation
will lead to a degree of noise of the reconstructed image.

To build a useful 3D map, the contribution of the largest possible set of muons must be collected, covering different positions and trajectory angles in the reconstructed volume. From (3) with the fixed $p_{\text{eff}}$ value of the momentum, one can write the likelihood of the occurrence of the scattering angles measured in the sample of $M$ muons as a function of the scattering density profile in the $N$ voxels: $\lambda = \lambda_1, \lambda_2, \ldots, \lambda_N$:

$$P(\lambda) = \prod_{i=1}^{M} \frac{1}{\sigma_i(\lambda) \sqrt{2\pi}} \exp \left( -\frac{\delta \theta_i^2}{2\sigma_i^2(\lambda)} \right).$$

(4)

Thus it is possible to build the density profile $\lambda$ in the volume by maximizing the logarithmic likelihood:

$$\ln P(\lambda) = -\sum_{i=1}^{M} \left[ \frac{\delta \theta_i^2}{2\sigma_i^2(\lambda)} + \ln (\sigma_i(\lambda)) \right]$$

(5)
in the set of $N$ variables $\lambda$. This is achieved through an iterative Expectation-Maximization algorithm [38], [39].

C. Examples of application of cosmic ray muon tomography

In the seminal work reported in [38], the Los Alamos group has given a proof of principle of the possibility of performing non-destructive inspection, material Z discrimination and image reconstruction by means of cosmic ray muon tomography. A small experimental apparatus was built with a set of four position sensitive drift chambers delimiting a volume of few dm$^3$. After this work, a number of projects for applications of cosmic ray muon tomography to industry and homeland security issues was proposed.

One of the most appealing application regards the inspection of commercial cargoes in ports, seeking for “special nuclear materials” or even nuclear weapons. Indeed, the technique is particularly suited to recognize the presence of compact high-Z objects in the presence of medium-Z or low-Z background matrix, considering the dependence of the scattering density on the Z and density of the material.

The technique offers important advantages relative to other commonly used ones, like detection of radiation emitted by the nuclear material or X-ray radiography by means of an intense X-ray source. The nuclear material can be identified even in case it is heavily screened, preventing direct detection; a natural source of radiation is utilized, avoiding both the complex and costly installation of an artificial source of X or gamma rays and demanding radiation protection measures; a three dimensional density profile of the inspected volume can be obtained.

The same Los Alamos group [49] has constructed a large muon tracker with muon detectors based on 12 planes of drift tubes, 5.0 cm diameter and 3.65 m long, placed above and below a sample volume with dimensions 1.5 m x 1.5 m x 1.0 m. With this demonstrator, a 10 cm x 10 cm x 10 cm cube of lead, representing the threat, placed inside an automobile engine, was recognized after 160 m data taking. A full scale portal has been developed as a spin-off in collaboration with Decision Sciences Corporation [50].

A cosmic ray muon tomography demonstrator with sensitive volume 3.0 m x 3.0 m x 1.5 m has been constructed at IHEP Protvino [51] using, as muon detectors, planes of 3.0 cm diameter drift tubes. Muon tomography of high-Z objects has been performed successfully. Other groups have constructed small scale muon tomography prototypes, with scanning volumes of tens of dm$^3$, using high resolution resistive plate chambers HRPC [52], or GEM detectors [53]. Promising results have been obtained with these techniques.

In the Mu Portal Project [54]-[57], a collaboration between research centers and industrial partners has started a project for the construction a full-scale prototype for the scanning of traveling containers. Due to the large inspection volumes needed, of the order of several tens of cubic meters, large tracking detectors are needed. In the final detector design, four tracking planes of plastic scintillator bars 1 cm x 1 cm x 300 cm with embedded WLS fibers with SiPM readout are foreseen, two placed above and two placed below the investigated volume. The detector covers an overall area of 18 m$^2$ and is suitable for a full inspection of a 20’ container.

The same technical solution has been adopted in [58], [59], where a project for the construction of a prototype of a muon tomography system for inspection of shipping containers for aircrafts, with 2.0 m x 2.0 m x 1.6 m inspection volume, has been funded by the Canadian Safety and Security Program (CSSP) and supported by private and public sectors. The “proof of principle” prototype has already been built and tests are in progress.

The muon tomography technique has also been proposed for the detection of radioactive “orphan” sources hidden in scrap metal containers [60]-[64]. Indeed, one of the major concerns of the metal recycling industry is the accidental melting of radioactive sources, which may involve extreme environmental and economical consequences. To avoid accidental melting of radioactive sources, radiation portal are usually employed at the entrance gates of steel mill factories, but they can’t detect sources completely screened by heavy metal shields.

The Mu-Steel European project [61] studied the possibility to detect shielded radioactive sources hidden in truck containers filled with scrap metals entering still mills foundries by means of muon tomography. The designed muon portal provides an inspection volume with dimensions 5.0 m height, 4.0 m largeness and 18.0 m length. It is surrounded by four large area muon detectors upper, lower and two lateral, built by layers of aluminum drift tubes, 5.0 cm diameter and 4.0 m and 6.0 m length. It was demonstrated that the designed portal is able to localize a radioactive source with a 5 liter lead shield, immersed in a scrap metal container, in less than 4 min observation.

The simulation and reconstruction software developed to evaluate the Mu-Steel muon portal performance, was tuned on a muon tomography prototype [60], based on large drift chambers providing a large inspection volume of about 11 m$^3$.

Recently, the same groups have started another European project Mu-Blast [65] aimed at studying the possibility of obtaining images of the interior of a blast furnace and a precise measurement of the linear scattering density of its content by means of cosmic ray muon tomography and radiography.

Finally, proposals have been presented to exploit muon...
tomography to inspect legacy nuclear waste containers [66]-[69],[70]. A small scale prototype tracker for cosmic ray muon tomography based on arrays of 2 mm scintillating fiber modules has been constructed and successfully operated [69] for the discrimination of low, medium and high-Z materials. The method was also proposed to perform a diagnosis of the damaged cores of the Fukushima reactors [71] and, recently, the fuel melt at Daiichi 1 has been assessed by muon data [72].

III. MONITORING OF STATIC ANOMALIES IN THE “PALAZZO DELLA LOGgia”

Since its completion in 1574, the “Palazzo della Loggia” has cumulated a long sequence of injuries, transformations, repairing interventions, some of which have generated considerable problems of structural stability of the building. The grandiose wooden vaulted roof was completely reconstructed in 1914, with the same architectural shape and construction techniques of the original one, destroyed by a fire one year after the completion of the building. The shape of the dome is like an upside down ship which reaches in elevation a maximum of 16 m, having the planar rectangular sides of about 25 and 50 m respectively. The structural architecture of the vault consists of principal truss wooden arches and simple secondary arches, both connected at the top by a truss made wooden beam.

Immediately after its construction, the present wooden vaulted roof structure exhibited a progressive deformation of the longitudinal top beam and of the key points of the connected arches. The progressive deflection of the top beam was measured to be 190 mm in 1923, 520 mm in 1945, 800 mm in 1980. Starting from 1990, a systematic campaign of investigation and monitoring of the different stability problems of the Palace has been committed by the Brescia municipality to the University of Brescia [43], [44]. In particular, the progressive deformations of the principal arches of the wooden vault have been studied with a specifically designed mechanical measurement system.

On four out of the seven principal truss wooden arches, three couples of wires 2 mm in diameter, one made of ordinary steel and the other made of invar, were stretched between symmetric points at three different levels, as seen in Fig. 2: A1-B1, at the point of connection of the arches with the building structure, A2-B2 and A3-B3 on the arch reins. The wire tension was maintained by means of a system of pulleys and balance weights.

The relative displacements of the symmetric points were continuously monitored through the differential elongation of the two wires; the different thermal dilatation coefficients of the two materials made possible to depurate the deformation of the monitoring system itself, subject to the considerable daily and seasonal thermal variations under the roof covered by lead plates. The elongation of the two wires was measured by an electronic system based on clip-gages, with a sensitivity of 1/100 mm, and recorded every six hours. Besides the electronic system, a mechanical measurement system was also employed based on a vernier with a sensitivity of 1/10 mm, as a check of reliability of the electronic system and recovery for possible failures.

In Fig. 3 the elongation of the ordinary steel wire, of the invar wire and the effective deformation of the structure are shown as a function of the monitoring time in days, for the couple of wires connecting the arch reins at middle height (points A2-B2). The effective deformation is practically coincident with the elongation of the invar wire. The general trend shows a progressive collapse of the wooden structure of the arch of about 1 mm per year.

A. Application of the muon stability monitoring system to the case of “Palazzo della Loggia”

The main component of the proposed monitoring system is the “muon telescope”. It is constituted of a set of three muon detector modules supported by an appropriate mechanical structure and axially aligned at a distance of 50 cm one from
the other. Each module is composed by two orthogonal layers of 120 scintillating optical fibers with 3 mm $\times$ 3 mm square cross section and 400 mm length. The scintillating fibers are read at the ends by silicon photomultipliers SiPM.

The two layers of orthogonal scintillating fibers provide the measurement of the crossing position of an incident muon in the $x$ and $y$ coordinates on the module plane, with a pitch of 3 mm. Considering a flat detection efficiency over the entire surface of the scintillating fiber, the expected spatial resolution on the hit coordinate is about 0.9 mm.

As seen in Fig. 2, the “muon telescope” is mechanically fixed to a structural element of the building, that constitutes the reference system, with its axis aligned in the direction corresponding to the part of the structure whose displacements should be monitored: points B1, B2 or B3 of the wooden arches in the case considered. A fourth muon detector module, with the same geometry and structure of the previous ones, is positioned as “muon target” on the point to be monitored.

Thanks to their high penetration capability, cosmic ray muons are able to cross the system of four detectors as well as the interposed building structures and make possible to continuously monitor the horizontal relative displacements of the “muon target” relative to the “muon telescope”, fixed on the masonry structure of the building.

Indeed, the trajectory of a cosmic ray muon crossing the system of four detectors can be extrapolated from the “muon telescope” to the plane of the “muon target” detector, in the hypothesis that it is a perfect straight line. The difference between the effective muon crossing point on the “muon target” and the extrapolated one from the “muon telescope” allows the position of the “muon target” relative to the “muon telescope” to be measured. Possible displacements of the position of the “muon target” relative to a reference position previously determined can be monitored.

In crossing the interposed materials, the trajectories of cosmic ray muons suffer multiple scattering angular deviations. At fixed momentum and at low deviation angles, these deviations follow a Gaussian law with variance depending on the inverse square of the muon momentum [46]-[48].

These stochastic effects are largely dominant over intrinsic detector resolution in the examined geometrical conditions. Therefore, statistical distributions of the difference between measured crossing coordinates in the “muon target” and the predicted crossing coordinates determined by extrapolation from the “muon telescope” are necessary to reduce the stochastic effects by statistical inference methods. As shown in [40], efficient unbiased estimators of the systematic displacement can be extracted from these distributions.

B. Simulation of the muon stability monitoring system in the “Palazzo della Loggia”

The features and expected performances of the proposed measurement system were studied by Monte Carlo simulations using the GEANT4 package [45]. A cosmic ray muon generator based on experimental data was implemented in the code in order to simulate as realistically as possible the momentum, the angular distribution and the charge composition of the cosmic ray radiation at the sea level [73].

In order to study the performance of the proposed monitoring system, the structure and composing materials of the “muon telescope” and “muon target” were modeled as well as the relevant structures of the “Palazzo della Loggia” building.

Three configurations were considered: the first with the “muon target” in position B1 of Fig. 2, located 0.50 m above the wooden ceiling of the “Salone Vanvitelliano”, at the first floor of the Palace; the second with the “muon target” in position B2, the third with the “muon target” in position B3, located respectively 5.8 m and 10.0 m above the wooden ceiling. In the tree different conditions the “muon telescope” was located on the vertical of the corresponding “muon target”, 3.0 m below the wooden ceiling. The ceiling of the large “Salone Vanvitelliano” was modeled as a bulky 15.0 cm thick wooden layer.

C. Position measurement uncertainty of the stability monitoring system versus data taking time

Simulation campaigns of populations of cosmic ray muons crossing the measurement system were performed for the three configurations described above. The distributions of the differences $\Delta x$ and $\Delta y$ between the muon crossing point coordinates measured by the “muon target” and the crossing point coordinates extrapolated from the “muon telescope” in the “muon target” plane were calculated. Due to the symmetry of the system, $\Delta x$ and $\Delta y$ are statistically identical and only $\Delta x$ distributions will be considered in the following.

As the “muon target” and the “muon telescope” are exactly coaxial in the simulation, the $\Delta x$ distributions are symmetric and centered at zero. The shape of the distributions exhibits a central narrow peak with very long tails on both sides. This shape is due both to the intrinsic uncertainty of the “muon telescope” in measuring the direction of the cosmic ray muon and to multiple scattering angular deviations of the muon trajectories traversing the interposed materials. The latter effect dominates for large distances of the “muon target” from the “muon telescope”. The long tails of the distributions are mostly due to low momentum muons, suffering larger deviations.

The mean value of the sample distributions represents an unbiased estimator of the position of the “muon target” relative to the “muon telescope” axis. The root mean square of the sample distribution represents the uncertainty in the measurement of the position of the “muon target” relative to the “muon telescope”.

The uncertainty on the mean value of the sample distribution is given by the well known statistical relation:

$$\sigma_{\text{mean}} = \sigma_{\text{distr}}/\sqrt{N_{\text{ev}}}$$

where $N_{\text{ev}}$ is the number of events in the distribution. Since, in the same geometrical conditions, the number of events in the sample distribution is proportional to the data taking time, the measurement standard uncertainty depends only on the inverse of the square root of the data taking time.

In Fig. 4 the relation of the position measurement standard uncertainty versus the data taking time for the three examined conditions is plotted up to a data taking time of one month. The plots are fitted with the general relation $\sigma_{\text{mean}} = C/\sqrt{t}$, where
Fig. 4. Standard uncertainty on the mean value of the $\Delta x$ sample distributions versus data taking time, for the “muon target” in position B1 (open circles), B2 (full squares) and B3 (open triangles). Fitting curves are given by the general relation $\sigma_{\text{mean}} = C/\sqrt{t}$; $C$ is a constant and $t$ is the data taking time.

$C$ is a constant depending on the geometry and materials interposed and $t$ is the data taking time. As time increases, the measurement standard uncertainty decreases.

As an example, in one month of data taking in position B3, where “muon target” and “muon telescope” are positioned 13.0 m far apart, a measurement standard uncertainty of the order of 0.5 mm may be achieved. The same standard uncertainty may be achieved in a week of data taking in position B2, whereas a 0.1 mm uncertainty may be measured in just one day in the position B1.

In the three geometrical conditions the expected acquisition rates of cosmic ray muons crossing the full system are respectively: $7.1 \mu/\text{min}$ in position B1, $1.2 \mu/\text{min}$ in position B2 and $0.55 \mu/\text{min}$ in position B3.

As expected, the standard uncertainty of the measurement system depends on the geometrical configuration considered, since both the root mean square of the $\Delta x$ distributions and the rate of useful events collected are strongly dependent on the geometry of the system and on the amount of materials interposed.

Nevertheless, the position monitoring of all the three inspected points by a cosmic ray tracking system could provide a performance compatible with the requested precisions and with the time scale characteristic of the inspected deformation phenomenon. Typical time scales, in the case of “Palazzo della Loggia” and, in general, for historical buildings, may span over several years.

Furthermore, a stability monitoring system based on tracking of cosmic ray muons can efficiently operate also when the points of the building to be monitored are not reciprocally visible and are separated by solid masonry structures.

D. Measurement of seasonal deformations of the wooden vaulted roof of the “Palazzo della Loggia”

Due to the low cosmic ray rate, a monitoring system based on cosmic ray muon tracking can be competitive with other monitoring techniques only when the deformation under study develops over periods of months or years and the requirements for the monitoring system is to track the slow deformation with time.

This is the case of the cyclic seasonal deformations of the wooden vaulted roof of the “Palazzo della Loggia”, which have been simulated with the Monte Carlo program for points B1, B2 and B3 of the roof structure. As a realistic model of the seasonal deformation, the measured displacement in point B2 on the arch reins, reported in Fig. 3 was adopted.

In Fig. 5 the curve corresponding to the assumed seasonal deformation (the same for the three points) is shown as a continuous line. The results of the simulated measurements of the position of the “muon target” displaced following the assumed structure deformation are shown; sampling rates of one week, two weeks and one month respectively for points B1, B2 and B3 have been used. It is evident the ability of the proposed measurement system to follow seasonal structural displacements of few millimeters and, even more so, also systematic ones.

Finally, it is worth recalling that the overall monitoring system standard uncertainty and, consequently, data collection time could be significantly improved by a factor 2 to 3, either
by improving the data analysis, exploiting information on the geometry and material composition of the interposed building structures, or by modifying some geometrical parameters of the proposed measurement system [42], [74].

IV. DISCUSSION AND CONCLUSIONS

Cosmic ray muon detection techniques for stability monitoring of historical buildings have to deal with the low rate of muon events and with the stochastic nature of the deviations of the muon trajectories due to multiple scattering in crossing materials. However, due to the very slow evolution of the deformation phenomena that may characterize the steady behavior of historical building structures, these constraints do not really constitute a severe limitation for the employment of the proposed method.

Conversely, the ability of muons to penetrate large thicknesses of material suffering only small deviations of the trajectories offers a new possibility to perform the stability monitoring of parts of the building physically and optically separated by solid structures, as walls or floors.

Appealing features of the proposed monitoring system are: (i) the use of a natural and ubiquitous source of radiation avoiding any problem of radiation protection; (ii) its applicability also in presence of horizontal and/or vertical building structures interposed between the reference system and the building parts to be monitored; (iii) the limited invasiveness, and the flexibility and ease of installation of the monitoring system devices; (iv) the possibility to design a global monitoring system, where the position of different points of the building may be simultaneously monitored relative to the same reference system; (v) the use of well-known physical principles and established technologies in the field of nuclear and particle physics to build up the cosmic ray muon detectors featuring the characteristics suitable to satisfy the requirements of any specific application.

For particular applications, these performances may be competitive in respect to the ones of monitoring systems today widely employed as laser scanner and theodolites, which make use of visible light, or as global position system based methods, hardly applicable to monitor with high resolution internal parts of the building. In addition, whereas the performances of monitoring techniques based on pendulums, inclinometers and extensometers provide measurements of deformation or strain in specific point positions, a global and simultaneous muon monitoring system may be constituted exploiting compact muon detectors distributed in different positions inside the building.

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