Numerical analysis of corrugated shells rotation

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Abstract. In the paper, we’ve made an attempt to numerically analyse a rotational motion of the fluted shells collapsing under dynamic loading of expanding gases. It was shown that the primary cause of the rotation is a gas oblique shock wave interaction with the fluted surface of a shell. Also, we evaluate the influence of a shell material properties and cross-section dimensions on jet angular velocity. Obtained results are in a good agreement with known experimental data.

1. Introduction
There is a well-known effect of corrugated conical shell rotation under expanding gas dynamic loading [1-10]. Under gas pressure, a liner is collapsing and producing a rotating jet. Direction and magnitude of angular velocity depend on a corrugated cone geometry. All corrugated liners types are divided into five groups by the type and presence of the rifling on a shell two lateral surfaces. The most interesting type of a corrugated shell is the one which has rifling on both internal and external surfaces. Its cross-section can be described by five design parameters (Figure 1): $a$ – corrugation depth; $R$ – radius of the external lateral surface; $T$ – wall thickness; $\delta$ – angle of indexing (angular of overlap between rifling on external and internal cone surfaces); $\psi$ – angle between radial direction and the shortest wave side; $n$ – total number of corrugation waves in circumferential direction.

![Figure 1. Corrugated shell cross-section [3].](image-url)
Such fluted shells demonstrate several non-trivial effects. High-speed jet, which is produced from these shells can change its spinning direction. For example, if we monotonically increase index angle $\delta$ then, surprisingly, jet angular velocity can increase/decrease or even change the sign to the opposite one. The same non-monotonic behavior is observed when number of flutes $n$ is changed.

There were several ideas explaining these effects [3-9]. The most prominent one is a combination of two mechanisms – “thick-thin” and “transport” [3]. These hypotheses are mainly qualitative and cannot help formulate any quantitative relations about jet angular velocity. Besides that, explanation of the angular velocity sign change with these ideas is a bit cloudy. It is unclear how exactly “thick-thin” and “transport” mechanisms can reverse angular velocity. These ideas limitations were the reason to study fluted liners rotational movement in a more detailed manner based on the numerical calculations technique.

2. Numerical calculations

Numerical modelling was performed in ANSYS LS-DYNA. We used 2D Euler multi-material method with plain symmetry to study dynamics of a corrugated shell cross-section motion. Cross-section dimensions were chosen similar to the paper [3]: $a = 0.25 \text{ mm}$; $T = 1.0 \text{ mm}$; $\psi = 30^\circ$; $n = 16$; $R = 15.0 \text{ mm}$, $\delta = 6^\circ$.

Multi-material Eulerian mesh region was a square with dimensions 84 x 84 mm filled by void. Total number of elements is 705,600 (size of an element 0.1 x 0.1 mm). On the region boundaries, free flow condition was applied. The detonation was initiated at the outer circular boundary of the explosive.

Two group of material considered in simulations – explosives, with JWL equation of state and elastic perfectly plastic medium, which is modeled copper in the corrugated shell and aluminum in a case. Geometrical model of the simulation shown in Figure 2.

![Geometrical model of the cross-section](image)

Figure 2. Geometrical model of the cross-section, which is used in simulations (blue – void; gray – case; green – explosive; yellow – corrugated shell).

First of all, we tried to understand jet angular velocity sign change phenomenon. Figure 3 represents angular velocity sign distribution in the corrugated liner cross-section. As we can see from Figure 3a, there are two group of regions in the shell – rotating counterclockwise and clockwise. This is due to the presence of the rifling on the shell outer surface. Expanding gas pushes each side of every corrugation
wave to the direction perpendicular its surface. So, in Fig. 3a, each wave left side is accelerating clockwise and right side – counterclockwise.

After collapsing process is almost complete (Figure 3b) some parts of the shell turn out to be in the vicinity of the symmetry axis. If they have predominantly counterclockwise rotation (Figure 3b) then a jet will also rotate in a counterclockwise direction and vice versa.

**Figure 3.** Angular velocity sign distribution in the corrugated shell cross-section: a) beginning of the collapsing process; b) ending of the collapsing process (gray regions – counterclockwise rotation, black regions – clockwise rotation).

Which part of the liner will be near the symmetry axis and produce a jet? It depends on their radial velocity during collapsing process. Parts with maximum radial velocity will be the first near the axis of symmetry, produce a jet and, consequently, define its angular velocity. According to the Gurney equations collapsing velocity increasing if a liner thickness decreasing [1]. So, parts of a liner with minimal thickness will have the maximum radial velocity and, consequently, produce a jet. This simple analysis is confirmed by Figure 3. and can be used to explain the non-monotonic dependence of a jet angular velocity on index angle.

If we calculate angular velocity distribution and define shell regions which are producing a jet for two value of index angle: 6 and 18 degrees (Figure 4) then we can clearly see the legitimacy of the performed analysis.

**Figure 4.** Correspondence between angular velocity distribution in the corrugated liner cross-section and jet-producing regions for different index angle values.
The left side of Figure 4 corresponds to \( \delta = 6^\circ \), right side corresponds to \( \delta = 18^\circ \). If we impose bottom row of Figure 4 on an upper row then we can clearly see that different values of \( \delta \) make different regions of a shell thinner, so they move faster and produce jet with opposite rotation direction.

After obtaining a qualitative understanding of corrugated shell rotation we tried to calculate dependencies of a shell angular velocity from every key parameter (shell cross-section dimensions, properties of a shell materials) and build general dimensionless formula.

Corrugation depth \( a \) is the first parameter which was analyzed. It can be shown on a simple analytical model that a shell angular velocity must be proportional to \( a \). The same result was obtained from numerical solution and it is in a good agreement with data from [3].

Dependencies from a radius of the external lateral surface \( R \) and wall thickness \( T \) are trickier and can’t be obtained from the simple analysis. This happens because of additional factors come into a play, such as dependence of \( n \) from \( R \) etc. Numerical calculation and experimental results [3] are shown that shell angular velocity is proportional to \( R \) and inversely proportional to \( T \).

Influence of an angle between radial direction and the shortest corrugation side \( \psi \) on a shell angular velocity can be derived from oblique shock waves theory [13]. For the values of \( \psi \) up to 30 degrees there is no significant influence on shell rotation. This result is also confirmed by [3] and numerical calculations.

The most interesting design parameters are an angle of indexing \( \delta \) and a total number of corrugation waves \( n \). As we mentioned above, shell angular velocity depends on them in a non-monotonic way which is quite difficult to describe in a simple analytic manner. That is why numerical calculation results were approximated by constructed dependencies and implement in a general formula.

Shell material was modeled by an elastic perfectly plastic model which is using three parameters – material density \( \rho \), Young’s modulus \( E \) and yield strength \( \sigma_t \). Numerical calculations show that influence of material density \( \rho \) on a fluted shell angular velocity is small and proportional to the \( \rho^{-\frac{3}{4}} \). Young’s modulus is a more extreme case. It has no effect on shell spinning. Yield strength \( \sigma_t \) influence is more or less predictable. With an increase of \( \sigma_t \) shell angular velocity decreasing in such a manner that at \( \sigma_t = 1200 \text{ MPa} \) shell angular velocity is equal to 0 rad/s. In the report [3] there is no data about an influence of material parameters on fluted shell rotation.

Summing up all numerical calculation results, we made an attempt to create a general dimensionless formula which describes which angular velocity can be compensated by a fluted shell:

\[
\omega R \sqrt{\frac{\sigma_t}{\rho} \frac{\Delta p(\alpha, \beta)}{\rho \cdot V^2}} = \left( \frac{114}{4 \cdot \pi} \right) \left( \frac{\Delta p(\alpha, \beta)}{\rho \cdot V^2} \right) \left( \frac{\sigma_t}{\rho} \right) \left( \frac{a}{R} \right) \left( \frac{R}{T} \right) \cdot \delta_m^{0.1} \times \\
(0.25 - j_1(21.7 \cdot \delta)) \cdot (j_1(0.25 \cdot n^{0.80})) \cdot \left( \exp \left( -295 \cdot \left( \frac{\sigma_t}{\Delta p(\alpha, \beta)} \right)^2 \right) \right)
\]

where \( \omega \) – angular velocity of a corrugated shell, which can be compensated; \( \Delta p(\alpha, \beta) \) – pressure difference between adjacent sides of a wave (\( \alpha \) and \( \beta \) are wave sides inclination angles to the circumferential direction); \( V \) – shell velocity in a radial direction according to the Gurney formula; \( j_1 \) – spherical Bessel function of the first order, \( j_1(x) = \frac{\sin(x)}{x^2} - \frac{\cos(x)}{x} \); \( \delta_m \) – jet relative mass.

General formula is valid in the following ranges of dimensionless quantities:

\[
\frac{\Delta p(\alpha, \beta)}{\rho \cdot V^2} = 0...0.66; \quad \sqrt{\frac{\sigma_t}{\rho}} = 47...\infty; \quad \frac{a}{R} = 0.017...0.05; \quad \frac{R}{T} = 1.67...30; \quad \delta_m = 0...1;
\]

\( n = 5...80; \quad \delta = 0...2 \cdot \pi / n \); \( \frac{\sigma_t}{\Delta p(\alpha, \beta)} = 0...3. \)
3. Conclusion
Numerical analysis of a corrugated shell rotation under loading of expanding gases was performed. We investigate the influence of different design parameters and shell material properties on shell rotation. Non-monotonic dependencies of shell angular velocity on index angle $\delta$ and number of corrugation waves $n$ were explained. Based on the results of this analysis general dimensionless formula of an angular velocity which can be compensated by a fluted liner was obtained.

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