NbO$_2$ Memristive Neurons for Burst-Based Perceptron

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Neuromorphic computing using spike-based learning has broad prospects in reducing computing power. Memristive neurons composed with two locally active memristors have been used to mimic the dynamical behaviors of biological neurons. Herein, the dynamic operating conditions of NbO$_2$-based memristive neurons and their transformation boundaries between the spiking and the bursting are comprehensively investigated. Furthermore, the underlying mechanism of bursting is analyzed, and the controllability of the number of spikes during each burst period is demonstrated. Finally, pattern classification and information transmitting in a perceptron neural network by using the number of spikes per bursting period to encode information is proposed. The results show a promising approach for the practical implementation of neuristor in spiking neural networks.

1. Introduction

Neuromorphic computing based on artificial neural network (ANN) has received extensive attention due to its low energy consumption. The high-power consumption in the conventional complementary metal–oxide–semiconductor (CMOS) hardware based on the von Neumann framework limits the use for data-driven machine learning tasks.[11] From a software perspective, the energy consumed for training an ANN can be reduced by optimizing the algorithm. However, brain-inspired neuromorphic hardware has demonstrated promising potential for energy-efficient computation. Replacing a portion of the CMOS components with emerging devices can implement an ANN circuit using fewer components and lower power consumption than conventional CMOS.[2,3]

Artificial neuron is one of the most important elements in ANNs.[4] Memristor is one of the promising candidates because of its low-power consumption, plasticity, and compatibility with conventional CMOS.[5–7] Pickett et al.[8,9] demonstrated a memristive neuron by using two Mott memristors and realized the four basic neuronal functions, including all-or-nothing spiking of an action potential, a bifurcation threshold to a continuous spiking regime, signal gain, and a refractory period.

Yi et al.[10] achieved 23 types of biological neuronal behaviors in memristive neurons, which possessed most of the known biological neuronal dynamics. Furthermore, Cassidy et al.[11] demonstrated the potential of achieving thousands of logic gates in neurons. Comparing with the traditional CMOS-based artificial neurons, memristive neurons greatly reduce the power consumption and the number of components. However, there is still a lack of research on the state dynamics and operational window of the neuronal behaviors in relation to the input signals which limits the use of the rich dynamics of the memristive neuron. Comprehensive understanding of the memristive neurons is crucial for the practical implementation in ANNs.

In this work, we report the spiking dynamics of NbO$_2$-based memristive devices that exhibit insulator–metal transition (IMT). The transformation conditions and operational boundary of NbO$_2$-based memristive neurons are investigated. The effect of input resistance and capacitance on the bursting behavior of the memristive neuron is studied. Furthermore, the memristive neurons are incorporated into a 9 × 1 array perceptron to demonstrate the potential for neuromorphic computing. Potential information spreading between neurons with different layers is also demonstrated.

2. Bursting Behavior of Memristive Neuron

2.1. Circuit and Model for Memristive Neuron

Figure 1a shows a biological neuron that generates an action potential in the direction of an output synapse after receiving sufficient stimulus from dendrites. Figure 1b shows the circuit diagram of NbO$_2$-based memristive neuron. The ion (K$^+$ or Na$^+$) channel consists of an NbO$_2$-based memristor and an opposite voltage source. Capacitors $C_1$ and $C_2$ in parallel with the ion...
channel are membrane capacitors. The resistor $R_2$ couples the two channels together with the input resistor to form a NbO$_2$-based memristive neuron. Memristive neuron dynamics of the output voltage $V_{\text{out}}$ will change subject to different input signals by adjusting the input resistance $R_{\text{in}}$ and input voltage $V_{\text{in}}$.

Lim et al.\cite{12} approximated the resistance of the memristor to a hard switching between two preset resistance values of $R_{\text{on}}$ and $R_{\text{off}}$, and calculated the boundaries A and B of the memristive neuron that can generate spike when one of the memristors $X_1$ or $X_2$ is in the critical state of switching, as shown in Figure 1c,d. However, memristor is a nonlinear resistor, and the memristor resistance at the critical state is different from $R_{\text{on}}$ and $R_{\text{off}}$, which results in inaccurate theoretical boundaries.

A resistor $R_{\text{th}}$ at the threshold voltage and a resistor $R_{\text{h}}$ at the hold voltage are introduced, as shown in Figure 1e. Figure 1f shows the operational window $R_{\text{in}}$-$V_{\text{in}}$ of the simulated (see the parameters used for simulation and calculation are shown in Table S1 and S2, Supporting Information 1). The operational window diagram is divided into three main areas: failure to fire (white), continuous spike (blue), and bursting spike (green). The theoretical boundaries after the introduction of $R_{\text{th}}$ and $R_{\text{h}}$ are optimized from A (or B) to A$'$ (or B$'$), which are consistent with the simulated window diagram (see Equation S(3), S(4), S(7), and S(8), Supporting Information 2).

Interestingly, a new boundary C$'$ for the two dynamic transformations between continuous spike and bursting spike is theoretically introduced considering that $X_1$ and $X_2$ are simultaneously in the critical state of transition (see Equation S(5) and S(6), Supporting Information 2), and the calculated boundary is consistent with the simulated boundary. According to the calculation formula of the three boundary lines, we can design the desired window size of the continuous spike and the bursting spike.

Figure 1. a) Schematic structure of a biological neuron. b) Circuit diagram of memristive neuron composed by two NbO$_2$ active memristors. c) Current-Voltage ($I$-$V$) curve simulated by the simplest model with fixed values of $R_{\text{on}}$ and $R_{\text{off}}$ of the memristor and d) the corresponding operation window diagram calculated by memristive neurons circuit. The two boundaries A and B marked by solid lines divide the window into two areas: failure to fire (white) and oscillation (shaded). e) $I$-$V$ curve simulated by the IMT model of memristor and f) the corresponding operation window diagram calculated by memristive neurons circuit. The three boundaries A$'$, B$'$, and C$'$ marked by solid lines divide the window into three areas: failure to fire (white), continuous spike (blue), and bursting spike (green). The simulation results of the optimized calculation verified the three boundaries, which is close to the dynamic behavior of the real memristive neurons system. Note that the dashed line is the boundary B marked in part (d).
2.2. Controllable Bursting Behaviors under the DC Input Voltage

Generally, every bursting spike possesses two oscillations components: one is the interspike oscillation which is a fast spiking oscillation within a single burst; the other is interburst oscillation which is modulated by a slow oscillation between the bursts (see Figure S2a, Supporting Information 3).[13] In memristive neuron, $V_K$ represents fast spiking oscillation and $V_Na$ represents slow oscillation. Figure 2a–e shows the waveforms of $V_Na$ and $V_K$ when the capacitance $C_1$ is in the range between 2 and 5 nF and $C_2$ is between 0.2 and 0.5 nF. Equation (1) and (2) describe the charging and discharging speed of $V_Na$ and $V_K$, respectively.

\[
\frac{dV_{Na}}{dt} = \frac{1}{C_1} \left( \frac{V_{in} - V_{Na}}{R_1} + \frac{V_{K} - V_{Na}}{R_2} - \frac{V_{Na} - V_{1}}{R_{X1}} \right) \\
\frac{dV_{K}}{dt} = \frac{1}{C_2} \left( \frac{V_{Na} - V_{K}}{R_2} - \frac{V_{K} - V_{2}}{R_{X2}} \right)
\]

Equation (1) and (2) can also be used to control the number of spikes.

The number of spikes per period can be regulated by adjusting the $V_{Na}$ and $R_{in}$. To demonstrate the bursting spike behaviors in details, the $R_{in}$–$V_{in}$ phase diagram is plotted with $C_1 = 5$ nF and $C_2 = 0.5$ nF, as shown in Figure 3a. The phase diagram is divided into three main areas: failure to fire (white), continuous spike (blue), and bursting spike (other colors). In the bursting spike area, two kinds of spikes are observed with different conditions of $V_{Na}$ and $R_{in}$, i.e., three-spike (olive) and four-spike (pink) responses during each burst period.

When $C_1$ is increased to 10 nF with a fixed $C_2$ of 0.5 nF, the similar three-area $R_{in}$–$V_{in}$ phase diagram is obtained, as shown in Figure 3b. In the bursting spike area, there are five kinds of spikes from five-spike (green) to nine-spike (red) responses during each burst period. Figure 3c shows the effect of $C_1$ on the maximum number range of spikes in each burst period, where the $V_{in}$ is fixed at

![Figure 2](image)

Figure 2. a–e) $K^+$ and $Na^+$ ion channel voltage waveforms. a) $C_1 = 2$ nF, $C_2 = 0.2$ nF. b) $C_1 = 2$ nF, $C_2 = 0.3$ nF. c) $C_1 = 2$ nF, $C_2 = 0.5$ nF. d) $C_1 = 3.5$ nF, $C_2 = 0.5$ nF. e) $C_1 = 5$ nF, $C_2 = 0.5$ nF. f) Effect of $C_1/C_2$ on the number of spikes in each burst period.
400 mV and \( R_{in} \) varies within the proper range. When \( C_1 \) is increased from 5 to 25 nF, the number range of spikes in each burst period increases from (3–4) to (14–22). Nine kinds of spikes can be obtained when \( C_1 = 25 \text{nF} \). Larger the number range of spikes can carry more information, which is very important in pattern classification and neuromorphic computing.\(^{[14]}\)

2.4. Nonlinear Relationship between Spikes Number and DC Input Current

The memristive neurons have exhibited the diversity of spiking behaviors under the constant current-source configuration.\(^{[8]}\) In this work, the input current \( I_{in} \) is also used to activate the bursting spikes instead of using the \( V_{in} \) and \( R_{in} \). To get the bursting signal, the input current amplitude should be larger than a critical current value \( I_{\text{critical}} \), as shown in Figure 3d. When \( I_{in} \) is larger than \( I_{\text{critical}} \), the number of spikes per period decreases with increasing the input current amplitude for both cases of \( C_1 = 25 \text{nF} \) and \( C_1 = 50 \text{nF} \). According to the circuit analysis, \( I_{\text{critical}} = \frac{V_{in}}{R_{in} + \frac{1}{C_1 \cdot f(t)}} \), for case of \( R_{x1} = R_{in} \) and \( R_{x2} = R_{off} \). Here, the \( I_{\text{critical}} \) is about 10.7 \text{\mu A}.

For the application in the spiking neural networks, the nonlinear relationship between input current and spiking numbers can be constructed for the proposed devices. As shown in Figure 3d, the activation function of the burst-based device can be used as the Exponential Units (EU) function

\[
f(x) = \begin{cases} 
0, & \text{if } x < m \\
 a + b e^{-kx}, & \text{otherwise}
\end{cases}
\]

where \( f(x) \) represents the number spikes, \( x \) is the input current, and \( m \) is the critical input current for activating bursting spikes. The other parameters \( a, b, \) and \( k \) are decided by the fitting results.

3. Exploring Potential Application for Bursting Spikes

3.1. Burst-Based Perceptron for Pattern Classification

A perceptron is an algorithm that produces an output by applying the weighted sum of the input values through an activation function.\(^{[15]}\) The activation function in ANNs loosely represents the firing rate of biological neurons, where there is a nonlinear relationship between the firing rate and the input. To facilitate the construction of the burst-based perceptron, the memristive neuron circuit is symbolically represented as ‘N’ element, as shown in Figure 4a. We used resistors as input synapses and memristive neurons as neurons to build a 9 × 1 array, as shown in Figure 4b. The simulated parameters of the memristive neurons are taken from Table S1, Supporting Information 1, except for \( C_1 = 25 \text{nF} \).

To prove that bursting behavior can be used for pattern classification, synaptic resistances are programmed with different weights. Three letter patterns, ‘n’, ‘z’, and ‘v’ are used as inputs,
as shown in Figure 4c. Nine squares correspond to the inputs of each synapse. A gray square means a 0.6 V input and a white square means a 0.1 V input. The number of spikes in each burst period of memristive neurons corresponding to the three different modes of 'n', 'z', and 'v' are 16, 17, and 18, respectively.

In addition, there are two other types of information embedded within each bursting which includes the interburst frequency and the duty cycle. Both of these features can be programmed as a function of input voltage (see Figure S2b and S2c, Supporting Information 3).

3.2. Information Transition by Bursting Spikes

The burst-based perceptron is potential to make the multilayer neuron network. The information can be transformed between the adjacent layers. To prove that the bursting behavior of the memristive neurons can be transmitted from the upper neuron to the lower neuron, three memristive neurons are connected in series with two resistors, as shown in Figure 5a.

When $C_1 = 10 \text{nF}$ and $C_2 = 1 \text{nF}$ in neuron $N_1$, $C_1 = 3 \text{nF}$ and $C_2 = 0.3 \text{nF}$ in neuron $N_2$, $C_1 = 1 \text{nF}$ and $C_2 = 0.08 \text{nF}$ in neuron $N_3$, $R_m = 10 \text{k}\Omega$, $R_n = 10 \text{k}\Omega$, and $R_q = 15 \text{k}\Omega$, bursting behavior can stably propagate from the neuron in the previous layer to the neuron in the next layer, as shown in Figure 5b. Neuron $N_1$ generates bursting signals at the input of a DC voltage source.
Subsequently, a bursting signal with the number of five spikes in each burst period generated by neuron N1 will excite the next neuron N2, and neuron N2 will generate the same number of spikes in each burst period after a short delay after N1 excitation. Compared with the bursting of neuron N1, the number of spikes in each burst period of neuron N2 is the same, but interspike period of neuron N2 is shorter. Similarly, neuron N3 can receive the bursting signal generated by neuron N2, thereby generating a bursting signal of the same number of spikes in each burst period but with a shorter interspike period.

When $C_1 = 8 \text{nF}$ and $C_2 = 4 \text{nF}$ in neuron N1, $C_1 = 3.5 \text{nF}$ and $C_2 = 2 \text{nF}$ in neuron N2, $C_1 = 2 \text{nF}$ and $C_2 = 1 \text{nF}$ in neuron N3, $R_m = 10 \text{k}\Omega$, $R_0 = 10 \text{k}\Omega$, and $R_q = 15 \text{k}\Omega$, a single spike can also spread between neurons, as shown in Figure 5c. This shows that the bursting signal can keep the number of spikes in each burst period transmitted from the memristive neuron in the previous layer to the memristive neuron in the next layer.

4. Conclusion

In conclusion, we investigated the rich dynamics and the transition conditions between different dynamic behaviors of NbO$_2$-based memristive neuron. A memristor model is used to calculate three theoretical boundaries of the memristive neuron for transformation of the dynamics. The three boundaries are consistent with the simulation. More specifically, NbO$_2$ devices with dynamics in different timescales are coupled to emulate the slow and fast dynamics of Na$^+$ and K$^+$ ionic channels in biological membranes during the creation of action potentials. The impact of $C_1$ and $C_2$ on the bursting behaviors was explored. The rise in $C_1/C_2$ will lead to an increase in the number of spikes in each burst period. When $C_1$ becomes larger, the number range of the spikes per period increases, then the bursting can carry more information. A burst-based perceptron for pattern classification by using memristive neurons was proposed. Bursting behaviors are shown to propagate the signals from the upper memristive neuron to the lower memristive neuron. The results in this work propose a promising approach to implement the rich dynamics of the memristive neurons for neuromorphic computing.

Supporting Information

Supporting Information is available from the Wiley Online Library or from the author.

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Conflict of Interest

The authors declare no conflict of interest.

Keywords

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