Experimental investigation of small scale geometries in a turbulent round jet

Markus Gampert, Philip Schaefer and Norbert Peters
Institute for Combustion Technology, RWTH Aachen
Templergraben 64, 52056 Aachen, Germany
E-mail: mgampert@itv.rwth-aachen.de

Abstract. In the present work, we present a method to gather highly accurate three-dimensional measurements of a scalar field in order to experimentally validate the theory of dissipation elements as developed by Wang & Peters (2006, 2008). Combining a two-dimensional high-speed Rayleigh scattering technique with Taylor’s hypothesis allows to resolve the concentration field of gaseous propane discharging into ambient air from a turbulent round jet at a Reynolds number (based on nozzle diameter and exit velocity) of 2,800 down to the Kolmogorov scale in every spatial direction. Based on the acquired data, the normalized probability density function of the length of dissipation elements $\tilde{P}(\tilde{l})$ is investigated at various downstream positions $x/d = 15 - 40$ and an excellent agreement with the theoretically derived model equation is obtained.

1. Introduction

One of the many approaches in turbulence research is to study geometrical structures or characteristic points in the flow field, which allow the extraction of representative information to describe and statistically reconstruct the whole field. Gibson (1968) analyzed the behaviour at the smallest scales of turbulent scalar fields in terms of the properties of zero gradient points and minimal gradient surfaces. He concluded that these regions of the field are of physical importance to the problem of turbulent mixing. Based on the extreme points of turbulent scalar fields, i.e. points of vanishing scalar gradient, Wang & Peters (2006, 2008) developed the theory of dissipation elements, which arise as natural geometries in turbulent scalar fields, when these are analyzed by means of gradient trajectories. Starting from every grid point, trajectories along the ascending and descending gradient directions can be calculated, which inevitably end in extreme points. All points that share the same two ending points define a finite volume which is called a dissipation element. These elements are parameterized by two values, namely the linear length $l$ between and the scalar difference $\Delta \theta$ at the extreme points. Based on this theory, space filling and non-arbitrary elements are identified, which allow the reconstruction of statistical properties of the field as a whole in terms of conditional statistics within the elements. Examples of such analysis can be found in Wang & Peters (2008), Schaefer et al. (2010b), Schaefer et al. (2011) and Gampert et al. (2011). In addition, the latter work analyzed dissipation elements numerically in five different flow types and confirmed the theoretically predicted independence from Reynolds number and type of turbulent flow. From the definition of dissipation elements it follows that their temporal evolution in turbulent fields is inherently connected to the evolution
of their ending points, which are separated by a mean linear distance \( l_m \) of the order of the Taylor microscale \( \lambda \), see Wang & Peters (2006). In addition, direct numerical simulations of homogeneous shear turbulence revealed that a resolution of the order of the Kolmogorov scale \( \eta \) is needed to obtain grid independent statistics. As dissipation elements have mostly been analyzed in simulations so far, a detailed experimental verification is desirable. Due to their corrugated three-dimensional geometry in combination with the required resolution, such an experimental validation is challenging. For a first attempt using three-dimensional measurements of the velocity field in a channel flow obtained via tomographic PIV see Schaefer et al. (2010a).

We study dissipation elements in a passive scalar field \( \theta \), which is governed by the convection-diffusion equation

\[
\frac{\partial \theta}{\partial t} + u_i (\frac{\partial \theta}{\partial x_i}) = D \frac{\partial^2 \theta}{\partial x_i^2},
\]

where \( D \) is the diffusion coefficient and \( u_i \) denotes the velocity component in \( i \)-direction, while repeated indices imply summation. A wide range of experimental investigations of such a scalar field can be found in the literature, three-dimensional data however is limited as often single- or multi-point measurements in combination with Taylor’s hypothesis are conducted, see for instance Antonia et al. (1984) and Mydlarski & Warhaft (1998), which for obvious reasons are of limited use in the context of dissipation element analysis. The development of advanced laser optical techniques with a high pulse energy at a high repetition rate has facilitated the experimental investigation of spatially three-dimensional conserved scalar quantities. In such measurements, the three-dimensional information is found either by imaging in parallel, spatially distinct two-dimensional planes or via a sweeping of a single two-dimensional laser sheet in sheet normal direction, see for instance Su & Clemens (1999) for an overview. For the present purpose however, both approaches are impractical as in the first one the minimal distance, i.e. the maximal resolution, is limited by the minimal distance between the two planes at which the signals do not interfere. This restriction is not only of importance for the analysis of dissipation elements, but also creates a severe restriction, when three-dimensional gradient quantities in scalar turbulence such as the scalar dissipation rate \( \chi \) are considered. The latter is for any dynamically passive, conserved scalar \( \theta \) defined as the scalar gradient magnitude squared times the diffusion coefficient \( D \), yielding

\[
\chi = 2D (\frac{\partial \theta}{\partial x_i})^2 = 2D \left[ (\frac{\partial \theta}{\partial x})^2 + (\frac{\partial \theta}{\partial y})^2 + (\frac{\partial \theta}{\partial z})^2 \right].
\]

The second approach has successfully been used for measurements in water, see for instance Miller et al. (2008), but proves to be difficult in the gas-phase as the Schmidt number \( Sc = \nu / D \), where \( \nu \) is the kinematic viscosity), is in liquids roughly three orders of magnitude larger than in the gas-phase. In chapter two, we will therefore present a method, which combines a high-speed Rayleigh scattering technique with Taylor’s hypothesis to resolve the Kolmogorov scale \( \eta \) in all three spatial directions, though at moderate Reynolds numbers \( Re_\lambda = u\lambda / \nu \), where \( u \) denotes the longitudinal r.m.s. velocity). In chapter three, we present results for the normalized probability density function(pdf) of the length of dissipation elements \( P(\tilde{l}) \), before the paper is concluded in chapter four.

2. Experimental investigation

In the course of this chapter we present the experimental arrangement which has been realized, give some background information regarding Rayleigh scattering and Taylor’s hypothesis and conclude with the description of data reduction, post-processing and evaluation process.
In the present study, a turbulent round propane jet discharging from a nozzle with a diameter \( d = 6 \text{mm} \) into surrounding air has been chosen as the core of the experimental set-up. The scalar field, i.e. the concentration of propane, is visualized via Rayleigh scattering of a diode pumped double cavity Nd:YLF laser (Litron Lasers LDY303HE-PIV) at the molecules. The laser emits frequency-doubled light at a wavelength of 527nm, has a pulse energy of 2x22.5mJ with a pulse width of 150ns at 1kHz and can operate at up to 10kHz, see LitronLasers (2010) for further information. To account for energy fluctuations, the signal is corrected on a shot by shot basis by a 12bit energy monitor (LaVision Online Energy Monitor).

Laser-Rayleigh scattering is used to determine the instantaneous concentration of the binary mixture of jet and reservoir gas in a small focal plane within the turbulent core of the jet around the centre line. Laser-Rayleigh scattering has been used and documented in many previous studies, see for instance Dowling & Dimotakis (1990), Su & Clemens (2003) and Talbot et al. (2009), and is therefore only described briefly here. The technique makes use of the fact that gas molecules elastically scatter photons, and that different molecules have different Rayleigh-scattering cross-sections. In the present study for instance, the cross-section of propane is roughly thirteen times higher than the one of the surrounding air. The Rayleigh scattering light intensity \( I \) in the perpendicular direction to the light source from a binary gas mixture is directly proportional to the Rayleigh cross section \( \sigma_i \) as well as the concentration \( \theta_i \) of the i-th component in the mixture, yielding

\[
I \propto \sigma_1 \theta_1 + \sigma_2 \theta_2 = \sigma_1 \theta_1 + \sigma_2 (1 - \theta_1),
\]

where \( \sigma_1 \) and \( \sigma_2 \) are the coefficients of the pure gases which compose the binary mixture.

**Figure 1.** Experimental arrangement of the high-speed Rayleigh system
under consideration and \( \theta_1 \) denotes the concentration of the gas exiting from the jet. Hence, the magnitude of the detected signal is related in a linear manner to \( \theta_1 \). Therefore, the two end points \( \theta_1 = 0 \) and \( \theta_1 = 1 \) of this linear relation are recorded for calibration purposes, before the conversion from signal to concentration is simply accomplished by linear interpolation, see Eckbreth (1996) and Tropea et al. (2007) for further details.

For the illumination of a two-dimensional plane, a sheet optic for thin, collimated sheets of 130\( \mu \)m diameter and 25mm height is installed behind the laser and energy monitor, thereby illuminating a plane perpendicular to the jet centerline, see fig. 1 for an overview of the full experimental set-up. The resulting signal is recorded with a 12bit LaVision high speed CMOS-camera HighspeedStar6 with a full resolution up to 5.4kHz and 8GB internal memory in combination with a two step high speed intensified relay optic (LaVision HighSpeed IRO). This image intensifier is an electronic shutter device with a maximal repetition rate of 2MHz and an extremely variable exposure time. In contrast to a standard CMOS or CCD, which usually has an exposure time in the ms range, the IRO can be operated in the ns range, thereby allowing time resolved analysis of shortest light pulses as they are produced by pulsed laser sources. In addition, this IRO has an extremely reduced vignetting, as the light is focused to the image intensifier entrance window, converted to electrons and amplified. It is in the following reconverted to light at the exit window, which is focused onto the chip. For further information regarding camera and IRO sensitivity and signal as well as noise considerations see LaVision (2007, 2008) and Weber et al. (2011).

In order to observe the Rayleigh signal without interaction between the optical arrangement and turbulent flow, a mirror is installed in some distance to the laser sheet, which has a thin coating of enhanced aluminium reflecting above 95\% of the incoming light at a wavelength of 527nm, thereby minimizing signal losses. To protect the propane jet from exterior influences such as dust particles, a mild co-flow of clean, dry air discharges from a surrounding tube with a diameter of 150mm and a length of 450mm, in which a honeycomb is installed in the lower third of the tube to guarantee a uniform velocity profile.

Based on the raw images recorded by the camera, several corrections of the data have to take place before a proper analysis can be performed. In a first step, noise stemming from dark current and background are subtracted before the intensity of each image is corrected on a shot-by-shot basis to compensate for fluctuations in the laser energy using the energy monitor data as well as for an inhomogeneous illumination in the laser sheet. The size of the images is then reduced from 1024\(^2\) to 850\(^2\) pixels to remove areas without any signal due to vignetting between IRO and camera. Afterwards, a Mie-filter consisting of an mixed intensity threshold and particle size approach is applied to remove undesirable effects originating from dust particles in the test region. Due to the relatively low jet velocity in combination with the high-speed recording, the signal of a dust particle is captured on several images, once it enters the area of interest. Based on these corrected images, the signals corresponding to pure air and pure propane respectively are calibrated and used to convert the recorded photon counts to propane concentration.

In a next step, the recorded time series of the plane at a fixed downstream location \( x/d \) is transformed into a spatial signal in streamwise direction with \( \Delta x = U \cdot \Delta t \) based on Taylor’s hypothesis, see Taylor (1983), so that we obtain a frozen three-dimensional concentration field. This approximation estimates the spatial derivative in the streamwise \( x \)-direction from the local instantaneous value of the time derivative from a single-point or planar measurement, when the required three-dimensional multipoint measurements are impractical or unavailable. In the limit of low turbulence intensities, the motion of gradients relative to the local mean flow can be approximated as one of pure convection.

Due to the importance of two-point statistics and spatial gradient quantities in turbulence, it is common to use Taylor’s hypothesis to estimate spatial derivatives, see Dahm & Sutherland (1997). Even in multipoint probe measurements of velocity gradients, cf. Tsinober et al. (1992)
As briefly discussed in chapter 1, the motivation for dissipation elements is the reconstruction of the entire three-dimensional scalar field by means of an adequate description of an element’s characteristics, those being the linear length of the entire three-dimensional scalar field by means of an adequate description of an element’s length. The joint probability density function (jpdf) \( P(l, \Delta \theta) \) is expected to contain most of the information needed for a statistical reconstruction.

Based on a trajectory search algorithm, the passive scalar field has been analyzed for the different experimental cases and the resulting joint pdf for case 2 is shown in figure 2 (to relate the values of \( l \) and \( \Delta \theta \) given in this figure to other characteristic flow quantities see table 1). In this illustration, different physical effects are illustrated. Besides a distinct maximum, one observes a decrease at the origin, corresponding to the annihilation of small elements due to molecular diffusion. The region in the upper right hand area of the jpdf is dominated by extensive strain, as large elements are exposed to large velocity differences. The jpdf can be described by a model equation - based on Bayes theorem, it is decomposed into a marginal pdf \( P(l) \) of the linear distance and a conditional pdf \( P(\Delta \theta \mid l) \) of the scalar difference, yielding

\[
P(l, \Delta \theta) = P(l) \, P(\Delta \theta \mid l),
\]

3. Experimental investigation of dissipation element analysis

As briefly discussed in chapter 1, the motivation for dissipation elements is the reconstruction of the entire three-dimensional scalar field by means of an adequate description of an element’s characteristics, those being the linear length \( l \) and the scalar difference \( \Delta \theta \). The corresponding joint probability density function (jpdf) \( P(l, \Delta \theta) \) is expected to contain most of the information needed for a statistical reconstruction.

Based on a trajectory search algorithm, the passive scalar field has been analyzed for the different experimental cases and the resulting joint pdf for case 2 is shown in figure 2 (to relate the values of \( l \) and \( \Delta \theta \) given in this figure to other characteristic flow quantities see table 1). In this illustration, different physical effects are illustrated. Besides a distinct maximum, one observes a decrease at the origin, corresponding to the annihilation of small elements due to molecular diffusion. The region in the upper right hand area of the jpdf is dominated by extensive strain, as large elements are exposed to large velocity differences. The jpdf can be described by a model equation - based on Bayes theorem, it is decomposed into a marginal pdf \( P(l) \) of the linear distance and a conditional pdf \( P(\Delta \theta \mid l) \) of the scalar difference, yielding

\[
P(l, \Delta \theta) = P(l) \, P(\Delta \theta \mid l),
\]
Figure 2. Joint probability density function $P(l, \Delta \theta)$ for case 2 at $x/d=30$.

where the marginal pdf $P(l)$ is defined by

$$P(l) = \int_0^\infty P(\Delta \theta, l) d\Delta \theta. \quad (5)$$

For this pdf in its normalized form $\tilde{P}(\tilde{l})$, with $\tilde{P} = P \ l_m$ and $\tilde{l} = l/l_m$, the following model equation was derived by Wang & Peters (2008)

$$\frac{\partial \tilde{P}(\tilde{l}, \tilde{t})}{\partial \tilde{t}} + \frac{\partial}{\partial \tilde{l}} (\tilde{P}(\tilde{l}, \tilde{t}) [\tilde{v}_D(\tilde{l}) + \tilde{a}(\tilde{l})\tilde{l}]) = \Lambda_s \int_0^\infty \tilde{P}(\tilde{z}, \tilde{t}) d\tilde{z} - \Lambda_a \tilde{P}(\tilde{l}, \tilde{t}). \quad (6)$$

In this equation, $\tilde{a}$ represents the conditional mean strain rate $a$ of the elements of length $l$

$$a = \frac{\langle \Delta u_n | l \rangle}{l}, \quad (7)$$

where $\Delta u_n$ denotes the velocity difference at the ending points projected in direction of the linear connecting line, normalized by its asymptotic value $a_\infty$, which is approached for $l \to \infty$. Furthermore, $\tilde{v}_D$ is defined as

$$\tilde{v}_D = v_D/(l_m a_\infty) = -4D/l \left( c \tilde{l} \exp(-\tilde{l}) \right) / (l_m a_\infty), \quad (8)$$

and denotes the normalized drift velocity due to molecular diffusion in eq. 6.

It is responsible for the linear decrease of $\tilde{P}(\tilde{l})$ for $\tilde{l} \to 0$ as will be shown below. The constant $c$ in eq. 8 is determined from the condition that the total length of the array must not change, cf. Wang & Peters (2008), and $D$ is the molecular diffusion coefficient. In addition in eq. 6 the two non-dimensionalized numbers $\Lambda_s$ and $\Lambda_a$ appear. These describe the splitting (respectively reconnection) of larger (smaller) elements into smaller (larger) ones and are determined from the normalization and the first moment during the solution of the equation as eigenvalues of the problem. Eq. 6 can be solved numerically and will be compared to the experimental results in the following as it is considered to be independent of the Reynolds number and type of turbulent
Figures 3-6 depict the results for the normalized pdf of the length distribution $\tilde{P}(\tilde{l})$ obtained at the different downstream positions. In general, one observes a very good qualitative agreement of the shape of the experimental results with the solution of the theoretically derived model, which is solved using $D_e = 0.6$ as this is considered to be the optimal value for passive scalar fields, cf. Wang & Peters (2008). Note in addition that the dissipation element analysis has been restricted in radial direction. Thereby, the entrainment zone in which external intermittency plays a fundamental role, cf. Mellado et al. (2009), is excluded and the focus is on the purely turbulent core of the jet for which the model has been developed.

The quality of the pdfs can be evaluated based on the agreement with the model in three different regions. A diffusion controlled linear increase from the origin can be observed for all
cases though the slope is slightly steeper for x/d=15, 20 and 40, while at x/d=30 experimental
data and theory agree very good. Due to this small difference of the slope, the position of the
maximum of the pdf is slightly tilted to the left, although its value lies closely to the predicted
value. At x/d=30 however, the maximum is located in agreement with theory but its value is
slightly underpredicted. All pdfs exhibit an exponential tail behind the maximum, whose slope
follows the model solution closely. Overall, the agreement between the experimental data and
the model is very good at all measurement locations so that a first experimental validation of
the whole pdf can be considered successfully completed.

Figure 5. Marginal pdf $\tilde{P}(\tilde{l})$ at x/d=30.

Figure 6. Marginal pdf $\tilde{P}(\tilde{l})$ at x/d=40.
4. Conclusion

We have presented a technique to gather highly resolved three-dimensional data of the concentration field of propane discharging from a round jet into quiescent air. The normalized probability density function of the length of dissipation elements is investigated at different downstream positions and an excellent agreement with the theoretically derived model equation for $P(l)$ is obtained as the measurements confirm a diffusion controlled linear scaling at the origin, position and value of the pdfs maximum as well as the exponential tail, which is modelled by a Poisson process in the theoretical equation.

Acknowledgments

This work was funded by the Deutsche Forschungsgemeinschaft under grant Pe 241/30-3, the NRW-Research School ”BrenaRo” and the Cluster of Excellence ”Tailor-Made Fuels from Biomass”, which is funded by the Excellence Initiative of the German federal state governments to promote science and research at German universities.

References

ANTONIA, R. A., HOPFINGER, E.J., GAGNE, Y. & ANSELMET, F. 1984 Temperature structure functions in turbulent shear flows. *Physical Review A* **30**, 2704–2707.

DAHM, W. J. A. & SOUTHERLAND, K. B. 1997 Experimental assessment of taylor’s hypothesis and its applicability to dissipation estimates in turbulent flows. *Phys. Fluids* **9**, 2101–2107.

DOWLING, D. R. & DIMOTAKIS, P. E. 1990 Similarity of the concentration field of gas-phase turbulent jets. *J. Fluid. Mech.* **218**, 109–141.

ECKBRETH, A.C. 1996 *Laser Diagnostics for Combustion Temperature and Species*, 2nd edn. Informa Healthcare.

FRIEHE, C. A., ATTA, C. W. VAN & GIBSON, C. H. 1971 Jet turbulence dissipation rate measurements and correlations. *AGARD Turbulent Shear Flows CP-93*, 18.1–18.7.

GAMPERT, M., GOEBBERT, J. H., SCHAEFER, P., GAUDING, M., PETERS, N., ALDUDAK, F. & OBERLACK, M. 2011 Extensive strain along gradient trajectories in the turbulent kinetic energy field. *New J. of Physics* **13**, 043012.

GIBSON, C. H. 1968 Fine structure of scalar fields mixed by turbulence i. zero gradient points and minimal gradient surfaces. *Phys. Fluids* **11**, 2305–2315.

KHOLMYANSKY, M. & TSINOBER, A. 2009 On an alternative explanation of anomalous scaling and how well-defined is the concept of inertial range. *Phys. Letters A* **373**, 2364–2367.

LAVISION 2007 Product manual highspeed iro. *LaVision GmbH, Göttingen* .

LAVISION 2008 Product manual highspeedstar6. *LaVision GmbH, Göttingen* .

LIDE, D. 2007-2008 *Handbook of Chemistry and Physics*, 88th edn. CRC Press.

LITRONLASERS 2010 Ldy300piv. *Litron Lasers Ltd; Warwickshire, England* .

MELLADO, J. P., WANG, L. & PETERS, N. 2009 Gradient trajectory analysis of a scalar field with internal intermittency. *J. Fluid Mech.* **626**, 333–365.

MILLER, R.J., DASI, L.P. & WEBSTER, D.R. 2008 Multipoint correlations of concentration fluctuations in a turbulent passive scalar. *Exp Fluids* **44**, 719–732.

MYDLARSKI, L. & WARHAFT, Z. 1998 Passive scalar statistics in high-péclet-number grid turbulence. *J. Fluid Mech.* **358**, 135–175.

PEINKE, J., RENNER, C. & R. FRIEDRICH 2001 Experimental indications for markov properties of small-scale turbulence. *J. Fluid Mech.* **433**, 383–409.
Schaefer, L., Dierksheide, U., Klaas, M. & Schroeder, W. 2010a Investigation of dissipation elements in a fully developed turbulent channel flow by tomographic particle-image velocimetry. Phys. Fluids 23, 035106.

Schaefer, P., Gampert, M., Gauding, M., Peters, N. & Treviño, C. 2011 The secondary splitting of zero gradient points in a turbulent scalar field. J. Eng. Math. pp. DOI 10.1007/s10665–011–9452–x.

Schaefer, P., Gampert, M., Goebbert, J. H., Wang, L. & Peters, N. 2010b Testing of different model equations for the mean dissipation using Kolmogorov flows. Flow, Turbulence and Combustion 85, 225–243.

Su, L. K. & Clemens, N. T. 1999 Planar measurements of the full three-dimensional scalar dissipation rate in gas-phase turbulent flows. Exp. Fluids 27, 507–521.

Su, L. K. & Clemens, N. T. 2003 The structure of fine-scale scalar mixing in gas-phase planar turbulent jets. J. Fluid Mech. 488, 1–29.

Talbot, B., Mazellier, N., Renou, B., Danaila, L. & Boukhalfa, M. 2009 Time-resolved velocity and concentration measurements in variable-viscosity turbulent jet flow. Exp. Fluids 47, 769–787.

Taylor, G. I. 1983 The spectrum of turbulence. Proc. R. Soc. London Ser. A 164, 476.

Tropea, C., Yarin, A.L. & Foss, J.F. 2007 Springer Handbook of Experimental Fluid Mechanics. Springer, Berlin.

Tsinober, A., Kit, E. & Dracaos, T. 1992 Experimental investigation of the field of velocity gradients in turbulent flows. J. Fluid Mech. 242, 169.

Wang, L. & Peters, N. 2006 The length scale distribution function of the distance between extremal points in passive scalar turbulence. J. Fluid Mech. 554, 457–475.

Wang, L. & Peters, N. 2008 Length scale distribution functions and conditional means for various fields in turbulence. J. Fluid Mech. 608, 113–138.

Weber, V., Brbach, J., Gordon, R.L. & Dreizler, A. 2011 Pixel-based characterisation of cmos high-speed camera systems. Appl Phys B pp. DOI 10.1007/s00340–011–4443–1.