A minimal thrust vectoring rotor with six degree of freedom: Design and analysis

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1 | INTRODUCTION

In recent years, the multi-rotor unmanned aerial vehicle (UAV) has been greatly extended in both the application and hardware design. Due to its reliable and low-cost flight ability, the multi-rotor UAV has become a preferred flying robotic platform in many practical fields. However, the limited agility of conventional multi-rotor UAV is still an inevitable obstacle that prevents more advanced application scenarios from being available. This is because the under-actuated dynamics, in which the total thrust direction is fixed in the body frame, makes the translation and rotation always coupled, and only 4-DoF motion is left. Therefore, two defects are obviously seen in this structure: (1) the direction of acceleration can only change when the whole vehicle is reoriented, causing a slow response in translation, and (2) the attitude is strictly limited if the vehicle is required to follow a certain trajectory. In order to build a 6-DoF fully operational UAV, the most intuitive and implementable approach is the thrust vectoring or the tilt rotor technique.

By decoupling the thrust direction from the attitude, that is introducing extra control inputs, the control mapping of the vehicle is augmented from under-actuation to full-actuation, and even over-actuation. Successful attempts can be found among the existing literature. In Escareno et al. [1], a novel triple tilting rotor is modelled and tested in autonomous attitude stabilising flight, which yields satisfying results, but further 6-DoF trajectory tracking is not discussed. Aiming at ground tests for 6-DoF motions of spacecraft, Kaufman et al. [2] presents a hexrotor UAV with six variable pitch propellers, which is capable of hovering at any attitude provided that thrust is sufficiently large, and a geometric controller is also developed on the special Euclidean group to track 6-DoF trajectory under unknown disturbances. Naldi et al. [3] presents the design, modelling, and control of a new class of modular aerial robots, obtained by rigidly interconnecting a number of ducted-fan aircraft, and a combination of control allocation and feedback control algorithms is proposed. A comprehensive modelling and control framework for a quadrotor with four tilting propellers is designed in Ryll et al. [4], including the hardware and software specifications of an experimental prototype, and the results of several simulations and real experiments. The FAST-Hex, a novel UAV concept which is able to smoothly change its configuration from under-actuation to fully actuation by using only one additional motor that tilts all propellers simultaneously is introduced in Ryll et al. [5], and a novel full-pose geometric controller that...
outperforms classical inverse dynamics approaches is presented as well. A vehicle with eight ordinary rotors that maximises the vehicle's agility in any direction is proposed in Rresciaini [6], and a control strategy that allows for exploiting the vehicle's decoupled translational and rotational dynamics is also introduced. A SE(3)-based control strategy for 6-DoF trajectory tracking is proven using a Lyapunov technique in Franchi et al. [7], which works well for both under-actuated and fully actuated tilt rotor platforms. Ryll et al. [8] presents a novel paradigm for physical interactive tasks in aerial robotics allowing reliability to be increased, and weight and costs to be reduced compared with state-of-the-art approaches. It is concluded that for the thrust vectoring UAV, most contributions are focused on over-actuated configurations and their optimal allocations of control inputs, such as the tilt-quadrotor and tilt-hexrotor mentioned above. The redundancy advantage of an over-actuated system can also be its disadvantage, because the vehicle can be larger and heavier, less practical, and less elegant from the aspect of industrial design. Although the over-actuated configuration degenerates to a fully actuated configuration if certain control inputs are disabled, most control schemes in the existing literature require all inputs to work properly, meaning that the degeneration to a minimal configuration is not allowed.

For the fully actuated configuration, the tri-rotor design is also frequently studied in the novel concepts of UAV. Mohamed et al. [9] proposes a feedback linearisation associated with $H_\infty$ loop shaping design to synthesise the controller for tilt tri-rotor. A back-stepping non-linear control is implemented in Kulhare et al. [10] for a tri-rotor with only one tiltable propeller. References [11, 12] present a tri-rotor combined with fixed-wing configuration, including the vehicle design, the non-linear dynamics and the control performance. A robust sliding mode with time delay estimation method for attitude control of a tri-rotor UAV in presence of uncertainties and disturbances is discussed in Kali et al. [13]. A coaxial tri-rotor aircraft is proposed in Zeghlache et al. [14], using an interval type-2 fuzzy logic control scheme to eliminate the chattering phenomenon. Xian et al. [15] presents a fault-tolerant control strategy to deal with the servo malfunction together with unknown disturbances of a tri-rotor with tiltable rear propeller. An adaptive flight control is investigated in D’Amato et al. [16], in which a neural network is implemented to mitigate the effects of imprecise inverse dynamics. Wang et al. [17] presents a feedback linearisation method and a non-linear disturbance observer in the controller for a tilt tri-rotor with coaxial rotor configuration.

The fully actuated configuration is not as perfect as it sounds, because the hardware behaviours are not ideal. This article is aimed at exploring the minimal configuration for 6-DoF flight based on the thrust vectoring and tiltable rotors, including geometry optimisation and overall maneuverability, not only because it is concise in the structure, but also a challenge for the flight control. Therefore, in this article, the vehicle is accurately modelled based on actual hardware components, and extensively evaluated in various simulations under strong disturbances, providing reliable results before a prototype is assembled and tested.

2 | GEOMETRY OPTIMISATION

The motion performance of the minimal thrust vectoring rotor (mTVR) depends on its geometry structure, which decides the mapping from the propeller thrusts to the generalised force that consists of the 3-axis force and 3-axis torque. A longitudinally symmetric Y-shaped structure, where each arm is installed with a single-axis tiltable thrust unit, is selected as the initial geometry configuration. The output direction of the thrust unit is always perpendicular to the tilting axis. Since there are only three thrust units, the plane fixed by their pivots should be perpendicular to the tilting gravity when the vehicle is upright, and the tilting axis of the rear thrust unit should be along the corresponding arm. Therefore, only five parameters $L, A, B, \beta, H$ are needed to determine the geometry of the mTVR, as shown in Figure 1a.

![Figure 1](image_url)  
**Figure 1** (a) Given the longitudinal symmetric layout, the geometry of the mTVR is determined by five parameters in total. $L$ denotes the lateral distance from the left or right unit to the symmetry axis. $A > 0$ is the horizontal distance from the left and right units to the centre of mass, and $B > 0$ the tail arm length. $\beta \in (0, \pi/2)$ is the constantly positive angle between the $Y_B$ axis and the vertical rotary plane of the left or right unit. $H$ is the height from the plane of pivots to the centre of mass, which can be positive or negative. (b) The installation of each arm relative to the body frame is denoted by a rotation matrix $A_i(i = 1, 2, 3)$ around $Z_B$ axis. $f_i$ is the total thrust magnitude of $i$th unit, with a tilting angle $\alpha_i$ around $X_A$ axis.
These geometry parameters are optimised for better manipulation of the mTVR, by studying the mapping matrix (namely the Jacobian matrix $J$) from outputs of thrust units to the 6-DoF generalised force that drives the vehicle. The thrust unit uses a coaxial counter-propellers design, which consists of two motor-propeller systems that rotate in the opposite direction for counter-torque cancelling, and is ideally equivalent to a pure thruster without torque output. Thus, the Jacobian matrix $J$ is easily derived based on Figure 1b, by decomposing three thrusters along with the $X_A$ and $Z_A$ axes of each unit, and organising the decompositions as the input vector for $J$

$$
f = [f_1s_a, f_1c_a, f_2s_a, f_2c_a, f_3s_a, f_3c_a]^T \quad (1)$$

where, $s_a$ and $c_a$ denote $\sin \theta$ and $\cos \theta$ for brevity. The conversion from thrust decompositions to the 6-DoF generalised force is written as

$$
\begin{bmatrix}
F_B \\
M_B
\end{bmatrix} = 
\begin{bmatrix}
J_F \\
J_M
\end{bmatrix}
f \triangleq Jf
\quad (2)
$$

in which the mapping matrices for $F_B$ and $M_B$ are separately derived

$$
J_F = \begin{bmatrix}
-s_\beta & 0 & -s_\beta & 0 & 1 & 0 \\
c_\beta & 0 & -c_\beta & 0 & 0 & 0 \\
0 & 1 & 0 & 1 & 0 & 1
\end{bmatrix}
\quad (3)
$$

$$
J_M = \begin{bmatrix}
-Hc_\beta & A & Hc_\beta & A & 0 & -B \\
-Hs_\beta & -L & -Hs_\beta & L & H & 0 \\
Lc_\beta + As_\beta & 0 & Lc_\beta + As_\beta & 0 & B & 0
\end{bmatrix}
\quad (4)
$$

For better manipulation of the mTVR, the mapping from any desired generalised force to the thrust components, which is to be applied in the controller design, is expected to be as accurate as possible within the output range of all thrust units. The condition number represents the accuracy of a matrix in transformation between coordinates. A Jacobian matrix with large condition number may cause significant variations in $F_B$ and $M_B$ to when $f$ varies slightly (or the other way around), just like solving an ill-conditioned linear equation group. A high accuracy in force and torque generating is essential for 6-DoF thrust vectoring operation; therefore, the condition number is selected as the objective in the geometry optimisation. Considering that all the geometry parameters should suit a predefined size of the mTVR, some of them are picked out as the fundamental dimension in this optimisation. This research uses $L$ as the fundamental dimension, since it resembles the wing span of an ordinary aircraft. Meanwhile, $H$ is independent from the geometry optimisation, because the centre of mass depends on the final payload installation. Thus, there are three parameters left to be optimised for any given $L$ and $H$.

The condition number of $J$ evaluated by singular value decomposition is studied first, by solving the following problem

$$
\begin{align*}
\min_{A,B,\beta} & \quad \text{Cond}(J) \\
\text{s.t.} & \quad 0 < A \leq 4L \\
& \quad 0 < B \leq 4L \\
& \quad 0 < \beta < \pi/2
\end{align*}
\quad (5)
$$

where, the upper limit of $A$ and $B$ are set to an abundant value of $4L$. However, multiple attempts on solving Equation (5) with a given combination of $L$ and $H$ yield unstable results, as demonstrated in Figure 2a. Although the solutions lead to different geometry configurations, their corresponding $J$ have the identical condition number that is only related to the specified $L$ and $H$. This phenomenon indicates that Equation (5) has an area rather than a point as the solution, because a lower boundary of $A$ and an upper boundary of $\beta$ can be discovered in the solutions. A pattern of $B = 2A$ is also revealed from the solutions of different $L$ and $H$ combinations, suggesting that the optimal centre of the triangle should always be on the $Z_B$ axis. This pattern allows Equation (5) to be solved in a visualised way by plotting the image of the objective function, as shown in Figure 2(b), which the solution area is clearly demonstrated. In order to determine the complete geometry, same optimisations are carried out independently for $J_F$ and $J_M$, and the results turn out to be quite coincidental. Figure 2c indicates that the optimal $\beta$ is always $30^\circ$ independent from $A$, while Figure 2d indicates the complementary result, that the optimal $A$ and $B$ always make the geometry an equilateral triangle configuration if $\beta = 30^\circ$, that is $L = \sqrt{3}A$.

The final geometry configuration is presented in Figure 2e, with more detailed assembly provided. A high-voltage digital servo that is strong and fast enough to tilt the coaxial counter-propellers accurately is installed at the end of each arm. A relatively wide spacing is reserved between the upper and lower propellers, which is about half of the propeller diameter.

### 3 | DYNAMICS MODELLING

The full dynamics of the mTVR includes the subsystems of the high-voltage digital servo, the thrust unit and the Newton-Euler equations of the multi-rigid body. This section presents all the mathematical model of each subsystem based on actual hardware components that are selected for a mTVR prototype. There are certain assumptions and approximations during the modelling to simplify the non-dominant factors, which are left as random disturbances for the control system to handle.

### 3.1 | Tilting servo

The high-voltage servo is modelled as a double integrator driven by a one-order torque generator with a cascade controller, as shown in Figure 3a. The mass centre of the thrust
unit is assumed to be on its $X_A$ axis, and this assumption is guaranteed in a prototype by using proper counterweights. Thus, no centrifugal pull is applied to the arm during a rapid tilting operation, making the dynamics simpler.

The model performance in Figure 3b is based on an actual high-voltage digital servo product, and this subsystem is applied in the overall-system simulation. Both the tilting angle and its derivatives are provided for the calculation of non-linear torques between the thrust unit and the main body of the mTVR.

### 3.2 Thrust unit

As mentioned before, each thrust unit implements a coaxial counter-propellers design, including an upper and a lower rotor that tilt synchronously. The coaxial counter-propellers mechanism is effective in both the boosting of the total thrust, and the cancelling of the counter torques and the precession moment caused by tilting. The detailed analysis of the counter-rotating coaxial propellers requires specialised approaches related to aerodynamics, such as the computational fluid
dynamics and wind tunnel test. The thrust model in this section is based on available research results, which is generally regarded as a pure thruster with small force and torque perturbations, and several coefficients of this model are left for identification by experiments using a thrust unit hardware.

For the counter-propellers with identical fixed pitch, the total rotational torque can only be balanced by letting the lower propeller spins relatively faster, since it rotates within the fully developed downwash of the upper propeller due to the spacing between propellers (see Figure 4a). The thrust experiment calibrates the upper throttle for any given lower throttle (namely the nominal throttle $u_i$ for a thrust unit) by balancing the counter torques, and then records the equivalent static thrust against $u_i$. In the simulation of this research, the static thrust curve is replaced by the performance of a single propeller amplified by a gain of 1.5, yielding a nominal thrust coefficient $K_{Tf} = 5.742 \times 10^{-7}$ N/rpm². The rotation dynamics of the motor-propeller is modelled as a first-order process with a time constant $\tau_0 = 0.02$s. The complete thrust model consists of the following equations ($i = 1, 2, 3$)

\[
\begin{align*}
\Omega(u_i) &= -1615u_i^2 + 7147u_i + 615 \\
\tau_0 \dot{\omega}_i + \omega_i &= \Omega(u_i) \\
f_i &= K_{Tf} \omega_i^2 
\end{align*}
\]

The static thrust curve $f_i(u_i)$ is plotted in Figure 4b. The torque balance calibration causes a residue in the angular momentum cancelling, due to the different rotation rates of the counter-propellers, and the tilting precession moment is actually non-zero. Since both $\omega_B$ and $\dot{\alpha}_i$ are limited, this residue is regarded as a torque perturbation, which is modelled in the next subsection about mTVR motion dynamics.

### 3.3 Motion dynamics

The precise motion dynamics based on the Newton–Euler equations of the mTVR multi rigid system is complex, which can be derived by analytical dynamics methods such as the Lagrange equations. For both the precision and simplicity of the mTVR motion dynamics, the modelling in this article is similar to that of a multi-rotor UAV, with the consideration of extra torque that relates the tilting mechanism and the main body.

By assuming the mass centre of the thrust unit is on its $X_A$ axis, the tilting momentum is considered independent from the linear acceleration of the main body, and therefore the mass of all thrust units is also integrated with the main body. The rotational dynamics of the mTVR is built upon the Euler equation of the main body, combined with the required total external torque that drives the thrust units, which is approximated as a cylinder in Figure 5. By denoting the absolute angular velocity of each thrust unit in the unit frame as $\omega_{U_i}$, there is

\[
\omega_{U_i} = [\dot{\alpha}_i \ 0 \ 0]^T + U_i^T \omega_B
\]

and its derivative

\[
\dot{\omega}_{U_i} = [\ddot{\alpha}_i \ 0 \ 0]^T + U_i^T \omega_B + U_i^T \dot{\omega}_B
\]

where, $U_i$ is the transformation matrix from the unit frame to the body frame that varies with $\alpha_i$.

\[
U_i = A_i R_X(\alpha_i), \quad \text{and} \quad \dot{U}_i = \dot{\alpha}_i A_i = \begin{bmatrix} 0 & 0 & 0 \\ -s_{\alpha_i} & -c_{\alpha_i} & 0 \\ c_{\alpha_i} & -s_{\alpha_i} & 0 \end{bmatrix}
\]

The Euler equation of the thrust unit is written as

\[
M_i = I_U \ddot{\omega}_{U_i} + \omega_{U_i} \times (I_U \omega_{U_i}) + \begin{bmatrix} 0 & K_{U2} \omega_i & K_{U2} \omega_i \omega_i^T \\ 0 & K_{U2} \omega_i & K_{U2} \omega_i \omega_i^T \end{bmatrix}^T
\]
The thrust unit is approximated as a cylinder with its centre of mass aligned with $X_A$ axis. The frame fixed with the thrust unit is defined as the unit frame in which the perturbation term is attached, including the simplified residual precession torque $K_{Uy} \Omega_i \hat{\alpha}_i$ along $Y_U$ axis, and the $Z_U$ axis perturbation approximated by a uniformly distributed torque within a variant range of $\pm K_{Uz} \Omega_i$ (the uniform distribution within $\pm 1$ is denoted by $\mathbf{U}_{\pm 1}$).

The Euler equation of the main body is formulated by adding up all torques as Equation (10),

$$\begin{align*}
J_M \ddot{\mathbf{f}} &= I \dot{\mathbf{\omega}}_B + \mathbf{\omega}_B \times (I \mathbf{\omega}_B) + \sum_{i=1}^{3} U_i M_i \\
&= \left( I + \sum_{i=1}^{3} U_i I_u U_i^T \right) \mathbf{\omega}_B + \mathbf{\omega}_B \times \left( I + \sum_{i=1}^{3} U_i I_u U_i^T \right) \mathbf{\omega}_B + \sum_{i=1}^{3} A_i \begin{bmatrix} I_{Ux} \hat{\alpha}_i \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \end{bmatrix} \begin{bmatrix} A_i \\ 0 \end{bmatrix} \right) + \\
&+ \sum_{i=1}^{3} U_i I_u U_i^T \mathbf{\omega}_B + A_i \begin{bmatrix} I_{Ux} \hat{\alpha}_i \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \end{bmatrix} \begin{bmatrix} A_i \\ 0 \end{bmatrix} \times \left( U_i I_u U_i^T \mathbf{\omega}_B \right) + U_i \begin{bmatrix} 0 \\ 0 \end{bmatrix}
\end{align*}$$

in which the total inertia matrix varies along with the tilting angles. The quaternion representing the attitude of the mTVR is updated by

$$\dot{\mathbf{q}} = \frac{1}{2} \begin{bmatrix} -q_1 & -q_2 & -q_3 & q_0 \\ q_0 & q_1 & q_2 & -q_3 \\ q_3 & q_0 & q_2 & -q_1 \\ -q_2 & q_3 & q_0 & q_1 \end{bmatrix} \mathbf{\omega}_B$$

and the corresponding Euler angles are converted from $\mathbf{q}$ simultaneously. Compared with the rotation dynamics of an ordinary multi-rotors vehicle, the most significant difference lies in the tilting torque generated by the servos.

The translation of the mTVR described by Newton equations is quite similar to that of a multi-rotors vehicle, which transforms the compound thrust from the body frame to the inertial frame before applying the Newton’s Law

$$CJ_f \mathbf{f} = M \begin{bmatrix} \ddot{x} \\ \ddot{y} \\ \ddot{z} + g \end{bmatrix}$$

where, $C$ denotes the rotation matrix from the body frame to the inertial frame, with the gravitational acceleration attached.

### 4 | 6-DoF MOTION CONTROLLER

The controller for manipulating the mTVR is different from the ordinary multi-rotor vehicles due to the augmented inputs and thrust vectoring non-linearity. From the view of the controller, the mTVR is taken as a 6-DoF actuator that generates force and torque simultaneously, and therefore in this section, the boundary and performance of this actuator is studied first before designing the control system. The controller output is defined as

$$\mathbf{u} = \begin{bmatrix} u_1 & u_2 & u_3 & a^r_1 & a^r_2 & a^r_3 \end{bmatrix}^T,$$

in which $u_i$ are the nominal throttles for thrust units, and $a^r_i$ are the tilting angle references for $\alpha_i$.

#### 4.1 | 6-DoF actuator envelop

Based on the previously specified maximum output of each thrust unit (21.7 N when $u_i = 1.0$) and the geometry parameters in the Appendix, the output envelop of the 6-DoF actuator is defined as the maximum combined force for translation only and the torque for rotation only. The envelop is visualised by a geometry that represents the magnitudes of force and torque vectors pointing to all directions in the body frame or the inertial frame. In this section, the actuator envelop geometry is discussed assuming that the gravity is compensated, which reflects the performance near hovering.
The results in Figure 6 show that the three vectoring thrust units create a non-spherical envelop for both the force and torque, which is also reshaped by the mTVR attitude. Benefiting from the high thrust-weight ratio, the attitude-variant envelop of the 6-DoF output seems sufficient for the regular manipulation of the mTVR with parameters in the Appendix. In general, however, the 6-DoF actuator generates the force and torque simultaneously, which means the output envelop is more complex in the six-dimensional searching space. The above numerical analysis of some special envelop cases indicates that a feed-forward planner, which limits the 6-DoF acceleration of the trajectory reference within a specified sphere or cube, is necessary for safe manipulation, thus the risk of exceeding the output boundary can be reduced.
4.2 6-DoF actuator dynamic response

Another important feature of the 6-DoF actuator is the output dynamics response, which evaluates the performance of the actuator generating the desired force and torque computed by the controller. An ideal performance is accurate and fast in both the magnitude and direction. However, in the case of the mTVR, the magnitude depends on motors, and the direction depends on servos. As derived in Equation (9), the thrust units behave like reaction wheels during the thrust vectoring, applying counter-torques to the main body, of which the magnitudes are dominated by $\vec{\alpha}$, see Figure 7.

The counter-torque can be roughly compensated by solving the following DAE for $\alpha_i$ as the tilting angle references

$$\begin{bmatrix} I_{U_3} \alpha_1 \\ \vdots \\ I_{U_3} \alpha_6 \end{bmatrix} = \mathbf{f} - \begin{bmatrix} \mathbf{F} \\ \mathbf{T} \end{bmatrix}$$

in which $\alpha_i$ denotes the first column of $A_i$ respectively, and the dynamics lag during tilting is omitted. However, this compensation puts extra constraints to the dynamics, making the translation and rotation coupled with each other, let alone the existence of the solution (hard to obtain) is not guaranteed. Another simple feedback compensation by using a filtered estimation of $\vec{\alpha}$ for cancelling the servo counter-torque, yields very limited improvement.

In general, the actuator shows good performance in generating smooth force and torque references, but abrupt changes in $\mathbf{F}$ usually cause obvious spikes in $\mathbf{M}$, while $\mathbf{M}$ is safe to follow step signals without interfering $\mathbf{F}$ significantly. Therefore, in order to reduce such spontaneous and strong disturbances, abrupt translational acceleration should always be avoided if possible.

4.3 Controller design

The main idea of the 6-DoF control is based on the simplified inverse dynamics of the mTVR, including the proposed feedback-forward planner that provides acceleration-limited 6-DoF reference, and the cascade feed-back loops that stabilises the vehicle. The active disturbance rejecting control (ADRC, [18]) is applied in this section, including the tracking differentiator (TD), the typical proportion-integral-differential controller (PI/PD/PID) and the extended state observer (ESO). The translation and the rotation controller are independent from each other, as shown in the overall control block diagram in Figure 8. Since the controllers have similar parameters in different values, these parameters are generally named in a $K_{PID}$ pattern for brevity, unless specified separately.

The TD implements a time-optimal second-order tracker with limited acceleration $a_{\text{max}}$ for each independent channel of the 6-DoF trajectory. For example, TD for $r_{\text{ref}}$ is written as

$$\dot{r}_{\text{ref}} = a_{\text{max}} \text{sgn} \left( r_{\text{ref}} - r_{\text{ref}} - \frac{\dot{r}_{\text{ref}} \odot |\dot{r}_{\text{ref}}|}{2a_{\text{max}}} \right)$$

in which $\odot$ represents the element-wise product of two vectors, and an identical TD is applied for the attitude set-point trajectory. The Euler angles are used rather than the quaternion formulation at this stage, since it is more intuitive in piloting and convenient in calculating the following

$$\omega_{\text{ref}} = \begin{bmatrix} -\cot \phi & 0 & 0 \\ 0 & 0 & 1 \\ \cot \phi & 0 & 0 \end{bmatrix} \begin{bmatrix} \psi_{\text{ref}} \\ \theta_{\text{ref}} \\ \phi_{\text{ref}} \end{bmatrix}$$

The controller accepts the quaternion reference only, and $\{\psi, \theta, \phi\}_{\text{ref}}$ should be converted to $q_{\text{ref}}$ beforehand. Since

![Figure 7](image-url)
Equation (14) exists, the maximum angular acceleration of \( \dot{\mathbf{q}} \) is no longer \( a_{\text{mass}} \) but still limited.

The position PI controller has the following typical structure

\[
\dot{r}_p = \dot{r}_\text{ref} + K_p (r_\text{ref} - r_m) + K_i \int (r_\text{ref} - r_m) dt
\]

in which \( K_p \) and \( K_i \) are diagonal matrices. The attitude PI controller is similar, but has only one channel to adapt to the quaternion formulation

\[
\begin{aligned}
\dot{\mathbf{q}} &= (\mathbf{q}_m)^{-1} \otimes \mathbf{q}_\text{ref} = \cos \frac{\varepsilon}{2} + \mathbf{e} \sin \frac{\varepsilon}{2} \\
\dot{\mathbf{w}} &= \mathbf{w}_\text{ref} + \mathbf{e} (K_p \mathbf{e} + K_i \int \mathbf{e} dt)
\end{aligned}
\]

where, the error input is replaced by an error quaternion \( \dot{\mathbf{q}} \) (\( \otimes \) denotes the quaternion multiplication), and the magnitude of \( \mathbf{w}_\text{ref} \) is regulated by the single channel controller, while the direction is aligned with the spin axis of \( \dot{\mathbf{q}} \). The anti-windup and extra output limitations are applied for both PI controllers.

The ADRC controllers for generating force (inertial frame) and torque (body frame) references have the same 3-channel structure. For each individual channel, the output is regulated by the following PD and ESO

\[
\begin{aligned}
e_\mathbf{w} &= \mathbf{w}_\text{ref} - \mathbf{w}_m \\
\dot{e}_\mathbf{w} &= \frac{1}{r} (K_p e_\mathbf{w} + K_d e_\mathbf{w} - B_w \mathbf{a}_1 - u_w) \\
\dot{\mathbf{a}}_1 &= \begin{bmatrix} -H_1 & 1 \\ -H_2 & 0 \end{bmatrix} \begin{bmatrix} \mathbf{a}_1 \\ \mathbf{a}_2 \end{bmatrix} + \begin{bmatrix} B_w^{-1} \\ 0 \end{bmatrix} u_w + \begin{bmatrix} H_1 \\ H_2 \end{bmatrix} \mathbf{w}_m
\end{aligned}
\]

in which for channel \( \mathbf{w} \in \{x, y, z\} \) and the corresponding mass or inertia \( B_w \in \{M, I_x, I_y, I_z\} \), \( u_w \) denotes the component of \( P_\text{ref} \) or \( T_\text{ref} \), \( \mathbf{a}_1 \) and \( \mathbf{a}_2 \) are the ESO states and the latter is the extended observation of an overall disturbance, which is cancelled in the controller output. An extra low-pass filter with time constant \( r^* \) is attached to the PD for softening the force and torque references, reducing possible rapid changes in the thrust direction. For the PD controllers, the ideal differential term \( e_\mathbf{w} \) is replaced with the filtered derivative, and output limitations are also applied.

The static inverse dynamics is based on Equations (1, 2, 6), with \( \mathbf{u} \) organised as the following

\[
\begin{aligned}
f &= J^{-1} \begin{bmatrix} C^T (P_\text{ref} + [0 \ 0 \ M_g]^T) \\
M_B^f \\
u_i &= \Omega^{-1} \left( \sqrt{f_{[i-1]}^2 + f_{[i]}^2} \right) \\
\mathbf{a}_i^{\text{ref}} &= \arctan_2(f_{[i-1]}, f_{[i]})
\end{aligned}
\]

where, \( f [i] \) represents the \( i \)th element in \( f \), and \( \Omega^{-1} \) is the inverse function of the static thrust in Equation (6). To verify the control performance when the model identification error exists, some of the parameters in the inverse dynamics are intentionally deviated from the true value of the mTVR dynamics, such as all the elements in \( J^{-1} \) and coefficients in \( \Omega^{-1} \), spoiling the mapping and output accuracy of the 6-DoF actuator.

For cases where the 6-DoF trajectory is optimised for specified objective functions with a bounded acceleration, the TD can be weakened or removed from Figure 8, and the setpoint signals are considered as references.

### 5 6-DoF Trajectory Tracking Simulation

The flight simulation of the mTVR is conducted using a specialised software platform (see Figure 9), which integrates the
full dynamics, the numeric solver for differential equations, the three-dimensional (3D) scene visualisation, the auto/manual pilot interface and auxiliary functions such as the data import/export. The upper machine for communication with a mTVR prototype is also embedded in this software.

Extra setup of all the scenarios in this section:

- An external 6-Dof kick disturbance is applied to the mTVR main body all the time. All channels are independent and uniformly distributed \([-1, 1] N\) for the force, and \([-0.1, 0.1] N \cdot m\) for the torque. Each kick has a random interval within 1 s;
- All elements of \(J\) are deviated from the physics model by a random ratio within 5% before calculating \(J^{-1}\) in Equation (18);
- \(\Omega^{-1}(\mu_i)\) deviates from the precise inverse function based on Equation (6), which causes the thrust unit to generate about 17% higher thrust than required;
- A uniformly distributed thrust noise within \([-0.2, 0.2] N\) is added to each thrust unit output;
- Use \(\tau_r = 0.1245\) for the velocity ADRC, and \(\tau_r = 0.0045\) for the angular velocity ADRC. The bandwidth of \(F^{\phi}\) is more restricted for stability, because the tilting angles are more sensitive to the directional change in \(F^{\phi}\);
- A one-order low-pass filter with time constant 0.05 is applied as the sensor lag for each signal of the feedback measurements.

5.1 | Regular set-point trajectory

The model is firstly tested by a regular 6-Dof trajectory, in which the mTVR is supposed to imitate a satellite by following an inclined elliptical orbit, while always pointing the \(Y_B\) axis toward the focal point and keeping \(\varphi = 0\). The shape of the set-point trajectory is an ellipse with 8 m semi-major axis, 0.2 eccentricity and 45° inclination. The velocity simulates the gravitational pull, which slows down when getting away from the focal point.

As shown in Figure 10, the vehicle starts from stillness at the ascending node, and accelerates until it reaches the apogee (max velocity 3.1 m/s), during which the largest position tracking error takes place. Compared with the position tracking, the attitude tracking contains less error because the \(\tau_r\) of the ADRC for \(\alpha_t\) allows faster response. The ADRCs stabilise the mTVR by fast cancelling the estimated external kicks, as shown in the controller outputs. The transient pointing of \(Y_B\) axis is always close to the focal point under the strong external disturbances and deviated model parameters.

5.2 | Critical set-point trajectory

The 6-Dof controller is tested by numerous regular trajectories and realtime manual piloting, and proved to be reliable in ordinary motions. Therefore, the second scenario is focussed on the performance in critical situations, in which the vehicle will track a zigzag trajectory that requires rapid changes in the thrust direction, which is usually caused by either a huge and abrupt acceleration in the translation, or a zero-cross-thrust for one or more thrust units during the rotation. The acceleration limit in TD is increased to generate trajectory with higher agility as the reference in testing the motion controller.

As shown in Figure 11, the 6-Dof zigzag trajectory is constructed with four ramp segments, three sharp turns take place at \(t = 5, 10,\) and \(15\) s. The total thrust reaches the transient envelop at \(t = 10\) s (\(\alpha_2 = 1.0\) for the left arm) to follow the sharp turn. Oscillations in motor speeds and servo angles are easily triggered when any one of the arms is near-perpendicular to the ground: \(t = 7.5\) s for the left arm, \(t = 12.5\) s for the right arm, and \(t = 2.5, 17.5\) s for the tail arm. When an arm is pointing upward or downward, its thrust contribution is possible to drop to the minimum (e.g. \(\alpha_3 = 0\) near \(t = 17.5\) s), which is a very vulnerable state for the vehicle, because the thrust unit under such a zero-cross-thrust state may tilt randomly and rapidly in disturbance rejection, causing a chain of strong counter-torque. The 6-Dof controller is possible to fail in this critical state when the disturbance happens to be strong, meaning that extra measures should be taken to prevent this. Anyway, the mTVR is able to
follow the critical zigzag trajectory with acceptable precision in this simulation.

5.3 Aggressive set-point trajectory

A space-time symmetric 6-DoF trajectory generated by minimum snap [19, 20] with optimal time allocation [21] is selected as the set-point trajectory, of which the translation has an 8-pattern fixed by six waypoints, and the attitude requires the $Y_B$ axis to point ahead while the main body rotates longitudinal. The mTVR is supposed to follow the aggressive trajectory within $8 \text{s}$, therefore, the position PI controller is weakened for this scenario to reduce the overshoot in translation control, that is, the $\dot{r}^{lp}$ mainly relies on the TD. As shown in Figure 12, the mTVR always falls behind the aggressive set-point trajectory, and the attitude tracking performance is better than that of the position tracking. The maximum velocity exceeds 7.0 m/s around $t = 4.0 \text{s}$, and the maximum tilting angle exceeds $400^\circ$ (left arm at the end time). Since the set-point is well-planned, there is no actuator oscillation during the flight.
6 | CONCLUSION

An mTVR with 6-DoF motion ability is introduced in this article. The mTVR is equipped with three tiltable thrust units that consist a pair of coaxial counter-propellers each. By operating the nominal throttles and tilting angles, the mTVR is able to generate any desired combination of 3-axis force and torque within the output envelop, thus perform 6-DoF flight with decoupled translation and rotation. The geometry of the mTVR is optimised first, by studying the condition number of the Jacobian matrix contains five geometry parameters, which proves an intuitive assumption that the equilateral triangle provides the best-balanced force and torque output. The overall system is then carefully modelled based on the actual hardware configuration of the mTVR prototype, including the coaxial counter-propeller thruster for generating pure thrust, the high-voltage servo for thrust tilting, and the detailed rotation dynamics of the vehicle. Next, the combined 6-DoF actuator performance is evaluated from aspects of output envelop and dynamic response. Finally, the 6-DoF ADRC motion controller for the mTVR is designed and tested by three set-point trajectories with regular/critical/aggressive features, using a specialised software platform. The basic controller shows good stability in regular, even aggressive operation of the vehicle, but it is possible to fail in critical states combined with disturbances.

The future work is focused on the prototype which is already under testing (see Figure 13), as well as the improvement of the ADRC controller policy in handling the critical states. The tilting range of the current servo is limited within $\pm \pi/2$ by hardware, which severely restricts the 6-DoF motion performance. Therefore, a substitute servo with similar capability and being able to rotate continuously is under selection, as well as a hardware modification is also planned to upgrade the current servo.

FIGURE 13 The mTVR prototype, that is under testing. Most of the parameters for simulation in this article is based on this prototype

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APPENDIX

Subscripts and superscripts

| Subscript          | Description                              |
|--------------------|------------------------------------------|
| $x_B$              | body frame                               |
| $x_A$              | arm frame                                 |
| $x_{U}$            | thrust unit frame                         |
| $x^p$              | set-point signal                          |
| $f$                | reference signal                          |
| $x^m$              | measurement signal                        |
| $x^c$              | control related parameter                 |

Primary parameters

- $g = 9.81$ [m/s²] Gravitational acceleration
- $M = 1.84$ [kg] Total mass including tiltable parts
- $L = 0.26$ [m], $H = 0.02$ [m] Fundamental geometry parameters
- $A = 0.15$ [m], $B = 0.3$ [m], $\beta = 30^\circ$ free geometry parameters
- $I = \text{diag}(I_x, I_y, I_z)$ Inertia matrix of main body [kg m²] $I_x = 0.0015$, $I_y = 0.0014$, $I_z = 0.0034$
- $J = [J_F \ J_M]^T$ Jacobian matrix for force and torque mapping
- $r = [x \ y \ z]^T$ Position of mass centre [m]
- $\psi, \theta, \phi$ yaw, pitch and roll of nTVR in ZXY Euler angle [rad]
- $q = [q_0 \ q_1 \ q_2 \ q_3]^T$ Attitude in quaternion formulation
- $C$ Transformation from body frame to inertial frame
- $\omega_B = [\omega_x \ \omega_y \ \omega_z]^T$ Angular velocity
- $F$ or $F_B$ Total thrust for vehicle translation [N]
- $M_B$ Total torque for vehicle rotation [N·m]

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Hardware parameters \( (i = 1, 2, 3) \)

\[ I_U = \text{diag}(I_{Ux}, I_{Uy}, I_{Uz}) \ [kg \ m^2] \]

- Inertia matrix of the thrust unit \( I_{Ux} = I_{Uy} = 0.0007, I_{Uz} = 0.0004 \)
- Installation matrix that rotates around \( Z_B \) axis only

\[ A_i \]

- Installation matrix that rotates around \( Z_B \) axis only
- Transformation matrix from unit frame to body frame

\[ U_i \]

- Nominal rotation rate of coaxial counter-propellers [rpm]
- Time constant of motor rotation rate

\[ \Omega_i \]

- Nominal thrust of coaxial counter-propellers [N]
- Nominal throttle of thrust
- Coefficient of nominal thrust

\[ \tau_{\text{cl}} = 0.02 \ [s] \]

- Coefficient of precession torque along \( Y_U \) axis
- Coefficient of perturbation torque along \( Z_U \) axis
- Servo tilting angle [rad]
- Time constant of servo tilting torque [N-m]

\[ u_i \in [0, 1] \]

\[ K_{Uf} = 5.742 \times 10^{-7} \ [N/\text{rpm}^2] \]

\[ K_{Uy} = 0.001 \ [N\cdot m/\text{rpm}] \]

\[ K_{Uz} = 0.01 \ [\text{rpm}^{-1}] \]

\[ \alpha_i \]

\[ \tau_\alpha = 0.01 \ [s] \]