Polarization Spin-Tensors in Two-Spinor Formalism and Behrends–Fronsdal Spin Projection Operator for \( D \)-Dimensional Case

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Abstract—In the work, the recurrent differential relations that connecting the polarization spin-tensor of the wave function of a free massive particle of an arbitrary spin for \( D \) and new formula of the \( D \)-dimensional Behrends–Fronsdal spin projection operator are found.

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1. INTRODUCTION

This work is a continuation of the article [1]. The [1] is devoted to a two-spinor description of free massive particles of arbitrary spin and to the Berends–Fronsdal projection operators – a projector onto irreducible completely symmetric representations of the \( D \)-dimensional Poincaré group. Each of these two sections received a small addition in the present work.

We briefly recall the main results of the [1]. We use the Wigner unitary representations of the group \( ISL(2, C) \), which covers the Poincaré group. These representations are irreducible and one can reformulate them in such a way that these irreps act in the space of spin-tensor wave functions of a special type. The construction of the functions \( \psi^{(\alpha)} \) is carried out with the help of Wigner operators, which translate the unitary massive representation of the group \( ISL(2, C) \) (induced from the irreducible representation of the stability subgroup \( SU(2) \)) acting in the space of Wigner wave functions \( \phi \) to a representation of the group \( ISL(2, C) \), acting in the space of special spin-tensor fields \( \psi^{(\alpha)} \) of massive particles. In addition, the generalization on arbitrary dimension \( D \) of the four-dimensional Behrends–Fronsdal spin projection operator was found.

In the first section of this article, for fixing the notation and material consistency, we present the definitions of the spin-tensor wave function \( \psi^{(\alpha)} \) the polarization spin-tensors \( (m) e(k) \) and the expansion formula for \( \psi^{(\alpha)} \) to the sum over the polarization spin-tensors. Further, using a special parametrization of Wigner operators in terms of two Weyl spinors, we prove the main Proposition 1 of the first section. On the existence of differential recurrence relations connecting various polarization spin-tensors \( j \). These relations allow us to write out explicit expressions for the polarization spin-tensors \( (m) e(k) \) in terms of two spinors.

In the second part of the paper, we describe a new method for constructing of the Berends–Fronsdal spin projection operator for \( D \)-dimensional case.

2. POLARIZATION SPIN-TENSORS FOR THE FIELD OF ARBITRARY SPIN

Based on the Wigner construction of massive unitary and irreducible representations of the covering Poincaré group \( ISL(2, C) \), one can show (see [1]) that the space of the unitary representation of the group \( ISL(2, C) \) with spin \( j \) is transformed to the space of the spin-tensor wave functions of \( \left( \frac{p}{2}, \frac{r}{2} \right) \)-type depending on four-momentum \( k = (k_0, k_1, k_2, k_3) \):

\[
\psi^{(\alpha_1, \ldots, \alpha_j)}(k) = \frac{1}{4!} \prod_{r=1}^{n} (A(k))^{\delta}_{\alpha_r} \times \left( \prod_{j=1}^{r} (A^{-1})_{\beta_j}^{\delta_j} (q^n)^{\xi_{\delta_j}} (\bar{\sigma}_n) \right) \phi^{(\delta_1, \ldots, \delta_{2j-1}, \delta_{2j})}(k).
\]

Here \( (p + r) = 2j \), \( \phi^{(\delta_1, \ldots, \delta_{2j-1}, \delta_{2j})}(k) \) is an arbitrary symmetric tensor of rank \( 2j \) (the Wigner wave function), \( q^n \) are components of the test four-momentum \( q = (q^0, q^1, q^2, q^3) \):

\[
q^0 q_n = q_n = q_0^2 - q_1^2 - q_2^2 - q_3^2 = m^2,
\]

\[
\eta_{kn} = \text{diag}(+1, -1, -1, -1),
\]

parameter \( m > 0 \) is the mass, \( \bar{\sigma}_n = (\sigma_0, -\sigma_1, -\sigma_2, -\sigma_3) \) and \( \sigma_1, \sigma_2, \sigma_3 \) are Pauli matrices while \( \sigma_0 \) — is the unit
\[ (A_k^{\alpha})_{\gamma}^\beta = (k^\alpha \sigma_{\gamma})_{\alpha}(A_k^{\alpha})_{\gamma}^\beta = (k^\alpha \sigma_{\gamma})_{\alpha}. \]  

The matrix \( A_k \) parametrizes coset space \( SL(2,\mathbb{C})/SU(2) \). The upper index \( r \) of the spin-tensors \( \psi^{(r)} \) in (2.1) distinguishes these spin-tensors with respect to the number of dotted indices. In Eq. (2.1) we use operators \( A_k^{(p)} @ (A_k \psi) ) \) to translate the Wigner wave functions \( \psi \) into spin-tensor functions \( \psi^{(r)} \) of \( \left( \frac{p}{2}, \frac{r}{2} \right) \)-type. These operators are called the Wigner operators.

According to [1], the spin-tensor wave functions \( \psi^{(r)}(k) \) can be represented as the following sum over the polarization spin-tensors \( e^{(m)}(k) \):

\[ \psi^{(r)}(m, \ldots, \beta, \alpha)_{\alpha, \ldots, \beta}(k) = \frac{1}{\sqrt{(2j)!}} \sum_{j=0}^{j=2} \phi_m(k) e^{(m, \ldots, \beta, \alpha)}_{\alpha, \ldots, \beta}(k). \]  

The explicit form of the coefficients \( \phi_m(k) \) is not needed here (you can see it in [1]), if the test momentum \( q \) is fixed as \( q = (m, 0, 0) \) polarizations \( e^{(m)}(k) \) are given by

\[ e^{(m\beta, \ldots, \beta)}_{\alpha, \ldots, \beta}(k) = \frac{1}{\sqrt{(2j)!}} \prod_{i=1}^{p} (A_k^{(p)})_{\beta, \ldots, \beta}(A_k^{(p)})_{\alpha, \ldots, \alpha}(k), \]  

where we introduced

\[ e^{(m\beta, \ldots, \beta)}_{\alpha, \ldots, \beta}(k) = \partial^{(m\beta)}_{\alpha} \cdots \partial^{(m\beta)}_{\alpha} (v^1, \ldots, v^2), \]

\[ T^i_j(v) = \sqrt{(j + m)(j - m)!}, \quad \partial^{(m\beta)}_{\alpha} = \frac{\partial}{\partial v^\beta}, \]

where \( v^1, v^2 \) the components of the auxiliary Weyl Spinor.

**Proposition 1.** The spin-tensors \( e^{(m\beta, \ldots, \beta)}_{\alpha, \ldots, \beta}, \) defined in (2.4) satisfy the relations:

\[ \left( \begin{array}{c} \mu \frac{\partial}{\partial \lambda} - \tilde{\lambda} \frac{\partial}{\partial \mu} \\ \tilde{\mu} \frac{\partial}{\partial \mu} - \mu \frac{\partial}{\partial \lambda} \end{array} \right) e^{(m\beta, \ldots, \beta)}_{\alpha, \ldots, \beta} = \sqrt{(j + m)(j + m + 1)} e^{(m\beta, \ldots, \beta)}_{\alpha, \ldots, \beta}, \]

\[ \left( \begin{array}{c} \mu \frac{\partial}{\partial \lambda} - \tilde{\lambda} \frac{\partial}{\partial \mu} \\ \tilde{\mu} \frac{\partial}{\partial \mu} - \mu \frac{\partial}{\partial \lambda} \end{array} \right) e^{(m\beta, \ldots, \beta)}_{\alpha, \ldots, \beta} = \sqrt{(j + m)(j - m + 1)} e^{(m\beta, \ldots, \beta)}_{\alpha, \ldots, \beta}, \]

\[ \frac{1}{2} \left( \begin{array}{c} \mu \frac{\partial}{\partial \mu} - \tilde{\lambda} \frac{\partial}{\partial \mu} - \tilde{\lambda} \frac{\partial}{\partial \mu} - \mu \frac{\partial}{\partial \lambda} \end{array} \right) e^{(m\beta, \ldots, \beta)}_{\alpha, \ldots, \beta} = m e^{(m\beta, \ldots, \beta)}_{\alpha, \ldots, \beta}, \]

where \( \mu, \lambda, \tilde{\lambda}, \tilde{\mu} \) — Weyl spinors.

**Proof.** The proof is based on the use the representation of matrices \( A_k, A_k^{(p)} \in SL(2,\mathbb{C}) \) in terms of Weyl spinors \( \lambda, \mu \)

\[ (A_k^{(p)})_{\alpha}^{\beta} = \frac{1}{z} \left( \begin{array}{c} \mu \lambda \\ \lambda \lambda \end{array} \right), \quad (A_k^{(p)})_{\alpha}^{\beta} = \frac{1}{z^*} \left( \begin{array}{c} \tilde{\lambda} \tilde{\lambda} \\ -\tilde{\lambda} \tilde{\lambda} \end{array} \right), \]

\[ (z)^2 = \mu^2 \lambda, (z^*)^2 = \mu^* \tilde{\lambda}, \]

\[ \lambda = (\lambda), \quad \mu = (\mu), \quad \tilde{\lambda} = \frac{\tilde{\lambda}}{\lambda}, \quad \tilde{\lambda} = \frac{\tilde{\lambda}}{\lambda}, \]

proposed in paper [1]. Let us show, for example, how one can prove of relation (2.7). The proofs of relations (2.6), (2.8) are similar. First of all we consider the obvious identity which follows from definitions (2.5):

\[ e^{(m-1)}_{\rho_1, \ldots, \rho_2} = \sqrt{(j + m)(j - m + 1)} e^{(m-1)}_{\rho_1, \ldots, \rho_2}. \]

Now we expand the numerator of the right-hand side of the formula (2.11)

\[ \partial^{(m\beta)}_{\rho_1} \cdots \partial^{(m\beta)}_{\rho_2} (v^2 \partial^{(m\beta)}_j T^j_i (v)) \]

\[ = \delta^2_{\rho_1} \partial^{(m\beta)}_{\rho_1} \cdots \partial^{(m\beta)}_{\rho_2} + \cdots \]

\[ = \delta^2_{\rho_1} \partial^{(m\beta)}_{\rho_1} \cdots \partial^{(m\beta)}_{\rho_2} \cdots \partial^{(m\beta)}_{\rho_j} + \cdots "]
which follow from (2.9) and (2.10). Note that the relations are holds
\[
\lambda_\gamma \frac{\partial z}{\partial \mu_\gamma} = \mu_\gamma \frac{\partial z^*}{\partial \lambda_\gamma} = 0. \tag{2.15}
\]

From the identities (2.14) and the relations (2.15) we can get formulas
\[
\frac{\lambda_\gamma}{z} = \lambda_\gamma \frac{\partial}{\partial \mu_\gamma} (A_{(k)})^{j}_{\alpha} \frac{\partial^*}{\partial \lambda_\gamma} = -\mu_\gamma \frac{\partial}{\partial \lambda_\gamma} (A_{(k)})^{j}_{\alpha} \frac{\partial^*}{\partial \mu_\gamma}.
\]  

(2.16)

Using first identities (2.14) and then (2.16), the right-hand side of formula (2.13) (without numeric factor and monomial \(T_d^j\)) can be rewritten as follows
\[
\left(\sum_{d=1}^{n} \lambda_\gamma \frac{\partial}{\partial \mu_\gamma} (A_{(k)})^{j}_{\alpha} \right) \frac{\partial^*}{\partial \lambda_\gamma} \prod_{(\gamma \rho \alpha)} \left( (A_{(k)})^{\gamma}_{\rho \alpha} \partial^*_{\lambda_\rho} \right)
\times \prod_{(\gamma \rho \alpha)} \left( (A_{(k)})^{\gamma}_{\rho \alpha} \partial^*_{\lambda_\rho} \right)
+ \sum_{d=p=1}^{n} \left( \frac{\partial^*}{\partial \lambda_\gamma} (A_{(k)})^{j}_{\alpha} \right) \partial^*_{\lambda_\rho}.
\]

(2.17)

Now we add to (2.17) the following zero terms
\[
\left(\sum_{d=1}^{n} \lambda_\gamma \frac{\partial}{\partial \mu_\gamma} (A_{(k)})^{j}_{\alpha} \right) \frac{\partial^*}{\partial \lambda_\gamma} \prod_{(\gamma \rho \alpha)} \left( (A_{(k)})^{\gamma}_{\rho \alpha} \partial^*_{\lambda_\rho} \right)
\times \prod_{(\gamma \rho \alpha)} \left( (A_{(k)})^{\gamma}_{\rho \alpha} \partial^*_{\lambda_\rho} \right)
+ \sum_{d=p=1}^{n} \left( \frac{\partial^*}{\partial \lambda_\gamma} (A_{(k)})^{j}_{\alpha} \right) \partial^*_{\lambda_\rho}.
\]

(2.18)

And as a result the sum of (2.17) and (2.18) has the form
\[
\left(\sum_{d=1}^{n} \lambda_\gamma \frac{\partial}{\partial \mu_\gamma} (A_{(k)})^{j}_{\alpha} \right) \frac{\partial^*}{\partial \lambda_\gamma} \prod_{(\gamma \rho \alpha)} \left( (A_{(k)})^{\gamma}_{\rho \alpha} \partial^*_{\lambda_\rho} \right)
\times \prod_{(\gamma \rho \alpha)} \left( (A_{(k)})^{\gamma}_{\rho \alpha} \partial^*_{\lambda_\rho} \right)
- \sum_{d=p=1}^{n} \left( \frac{\partial^*}{\partial \lambda_\gamma} (A_{(k)})^{j}_{\alpha} \right) \partial^*_{\lambda_\rho}.
\]

(2.19)

where to obtain equality we used the product rule. Further, substituting the right-hand side (2.19) in (2.13), we are convinced of the validity of the relation (2.7).

**Remark 1.** Formulas from Proposition 1 can be used to construct tensors of arbitrary polarization \(m\), expressed in terms of Weyl spinors \(\mu, \lambda\). We first construct the polarization tensor \(e^{(m)}\) for \(m = j\), using the parametrization of the Wigner operators (2.4) in terms of parameterization (2.9) of operators \(A_{(k)}, A_{(k)}^{-1}\)
\[
\Theta^{(j)}(\mu_{\alpha}, \lambda_{\gamma}) = \frac{(-1)^j}{(z^*)^j} \frac{1}{\sqrt{2j}} (z^*)^j \mu_{\alpha} \cdots \mu_{\alpha'} \lambda_{\beta} \cdots \lambda_{\beta'}. \tag{2.20}
\]

Now, using recurrence formula (2.7) we can write the polarization tensor for \(m = j - 1\)
\[
\Theta^{(j)}(\mu_{\alpha}, \lambda_{\gamma}) = \frac{1}{\sqrt{j}} (z^*)^j \mu_{\alpha} \cdots \mu_{\alpha'} \lambda_{\beta} \cdots \lambda_{\beta'}.
\]

(2.21)

Further, applying the formula (2.7), one can obtain all the polarization tensors.

### 3. BEHERNDS–FRONSDAL OPERATOR FOR D-DIMENSIONAL CASE

**Definition 1.** The Behrends–Fronsdal projection operator \(\Theta(k)\) uniquely determined by the following conditions

1. **projective property and reality:** \(\Theta^2 = \Theta, \Theta^* = \Theta\);
2. **symmetry:** \(\Theta_{\alpha_{\gamma_1} \beta_{\gamma_2} \cdots} = 0, \Theta_{\alpha_{\gamma_2} \beta_{\gamma_1} \cdots} = 0\);
3. **transversality:** \(k^\gamma \Theta_{\gamma_{\gamma_1} \cdots} = 0, k_{\alpha} \Theta_{\alpha_{\gamma_1} \cdots} = 0\);
4. **tracelessness:** \(\eta^{\gamma_{\gamma_1} \cdots} = 0\).

For the four-dimensional space-time \(D = 4\), the Behrends–Fronsdal projection operator \(\Theta(k)\) for any spin \(j\) was explicitly constructed in [2, 3]. In [1, 4, 5] the generalization of the Behrends–Fronsdal operator to the case of an arbitrary number of dimensions \(D > 2\) was found. The construction was based on the properties of this operator, which are listed in Definition 1.

Instead of the tensor \(\Theta^{(\gamma_{\gamma_1} \cdots)}(k)\) symmetrized in the upper and lower indices, was considered the generating function
\[
\Theta^{(j)}(x, y) = x^\alpha \cdots x^\gamma \Theta^{(\gamma_{\gamma_1} \cdots)}(k) y^\alpha \cdots y^\gamma.
\]

(3.1)

For concreteness, we assume that the tensor \(\Theta(k)\) with components \(\Theta^{(\gamma_{\gamma_1} \cdots)}(k)\) is defined in the pseudo-
Euclidean $D$-dimensional space $\mathbb{R}^{t, d}$ ($s + t = D$) with an arbitrary metric $\eta = [\eta_{\mu\nu}]$, having the signature $(s, t)$. Indices $n$ and $i$ in (3.1) run through values $0, 1, \ldots, D - 1$ and $(x_0, \ldots, x_{D-1}), (y_0, \ldots, y_{D-1}) \in \mathbb{R}^{t, d}$.

**Proposition 2.** (See [1]) The generating function (3.1) of the covariant projection operator $\Theta^{\eta_{i,j}}_{\tilde{\eta}_{i,j}}$ (in $D$-dimensional space-time), satisfying properties (1)–(4), in Definition 1, has the form

$$a_{(j)}^0 = \frac{1}{2}! \frac{j!}{(j - 2A)!} A^2 (2j + D - 5)(2j + D - 7) \cdots (2j + D - 2A - 3),$$

where $\frac{j}{2}$ is the integer part of $j/2$, the coefficients $a_{(j)}^0$ satisfy the recurrence relation

$$a_{(j+1)}^0 = -\frac{1}{2} \left( j - 2A + 2 \right) \left( j - 2A + 1 \right) \frac{A}{2} A (2j - 2A + D - 3) a_{(j)}^0,$$

The solution of Eq. (3.3) has the form

$$x^n \cdots x^n \left( \prod \frac{\partial}{\partial \tau^{\ell_1}} \cdots \frac{\partial}{\partial \tau^{\ell_{j-1}}} \Theta^{(j)} \right) \Theta(t, y).$$

Now we will prove some new statement about the generation function of the operator $\Theta^{(j)}$.

**Proposition 3.** For the generation function (3.1) the following recurrence formula is hold

$$\Theta^{(j)}(x, y) = \frac{1}{j!} x^n \cdots x^n \left( \prod \frac{\partial}{\partial \tau^{\ell_1}} \cdots \frac{\partial}{\partial \tau^{\ell_{j-1}}} \Theta^{(j)}(z, y) \right) \Theta(t, y).$$

Here we defined element

$$\prod_{j=1} \left( \tilde{\eta}_{i,j} + \tilde{\tau}_{j-1} \right) + \tilde{\tau}_{j-2} \tilde{\tau}_{j-1} + \cdots + \tilde{\tau}_{j-1} \tilde{\tau}_{j-1},$$

where $\tilde{\eta} = \tilde{\Theta}^{\eta}_{\tilde{\eta}}$ and element $\tilde{\tau}$ have the form

$$\left( \tilde{\tau}_{j} \right)_{\mu, n_{j}} = \left( \Theta^{\eta}_{\mu} \cdots \Theta^{\eta}_{n_{j}} \right),$$

where $\omega = D - 1$.

**Proof.** First we simplify the formula (3.6). Note, we will need it later, that the following new form is hold.

$$\prod_{j=1} \left( \tilde{\eta}_{i,j} + \tilde{\tau}_{j-1} \right).$$

Consider the differential operator

$$\tilde{\Theta}^{\eta}_{\tilde{\eta}} \Theta^{\eta}_{\eta} \Theta^{\eta}_{\eta} - \Theta^{\eta}_{\eta} \Theta^{\eta}_{\eta} \Theta^{\eta}_{\eta} - \frac{2}{\omega} \Theta^{\eta}_{\eta} \Theta^{\eta}_{\eta}.$$

We make a contraction similar to (3.10) for $j = 2$, using the formula (3.13) and the definition of the generating function $\Theta^{(x)}$ (3.5), as a result we have

$$\frac{\partial}{\partial \tau^{\ell_{j-1}}} \frac{\partial}{\partial \tau^{\ell_{j-1}}} \Theta^{(j)} = 2 \Theta^{(x)} \Theta^{(x)} - \frac{2}{\omega} \Theta^{(x)} \Theta^{(x)}.$$
In the proof, we will consider the case when \( j - 1 \) is an even number, the proof for odd \( j - 1 \) is carried out in a similar way. Let’s make in each line (3.17) some transformations

\[
a_0 (\Theta^{(y)})^j + \sum_{A=1}^{[j/2]} a^{(j-1)}_A (\Theta^{(y)})(\Theta^{(x)}) A (\Theta^{(y)})^{j-2A}
\]

\[
\times \frac{1}{(\omega + 2(j - 2))} \sum_{A=1}^{[j/2]} (j + 1 - 2A) 
\times (\Theta^{(x)})^{j-2A}
\]

In the first line we excluded the first term from the total sum and we took into account the fact that for even \( j - 1 \) the relation \( j - 1 / 2 = \lfloor j / 2 \rfloor \) is hold. In the second line, we again used the equality \( j - 1 / 2 = \lfloor j / 2 \rfloor \) and eliminated the obviously zero first term. In the third line, we eliminated the last term, then made a shift of the summation parameter \( A \rightarrow A - 1 \), and used \( j - 1 / 2 = \lfloor j / 2 \rfloor \). Now we add all terms (3.18) as a result we get

\[
a_0 (\Theta^{(y)})^j + \sum_{A=1}^{[j/2]} a^{(j-1)}_A (\Theta^{(y)})(\Theta^{(x)}) A (\Theta^{(y)})^{j-2A} \times \frac{1}{(\omega + 2(j - 2))} \sum_{A=1}^{[j/2]} (j + 1 - 2A) \times (\Theta^{(x)})^{j-2A},
\]

where the coefficient \( B_A \) is determined by the following chain of equalities

\[
B_A = a^{(j-1)}_A - \frac{2A}{(\omega + 2(j - 2))} a^{(j-3)}_A - \frac{(j - 2A + 1)}{(\omega + 2(j - 2))} a^{(j-1)}_{A-1}
\]

\[
= a^{(j-1)}_A \left( \frac{2A}{(\omega + 2(j - 2))} \right) - \frac{1}{(\omega + 2(j - 2))} \times \frac{1}{(\omega + 2(j - 2))} \sum_{A=1}^{[j/2]} (j - 2A + 1)(j - 2A) + (j - 2A + 1) \times a^{(j-1)}_{A-1},
\]

\[
\text{here we used the recurrence relation (3.3) for the coefficients } a^{(j-1)}_A. \text{ Substituting now the explicit expression for } a^{(j-1)}_A, \text{ we see that } B_A \text{ exactly coincides with } a^{(j)}_A. \text{ As a result, we can write}
\]

\[
\Theta^{(j)}(x, y) = \sum_{A=0}^{[j/2]} a^{(j)}_A (\Theta^{(y)})(\Theta^{(x)}) A (\Theta^{(y)})^{j-2A}.
\]
4. CONCLUSIONS

We hope that the formalism considered in this paper for describing massive particles of arbitrary spin will be useful in the construction of scattering amplitudes of massive particles in a similar way to the construction of spinor-helicity scattering amplitudes for massless particles [6, 7]. Some steps in this direction have already been done in papers [8–10] where the analogous formalism and its special generalization were used. We also think that using the methods from [11] the formulas (3.6)–(3.8) can be generalized for projection operators of any type of symmetry (corresponding to arbitrary Young diagrams).

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