An extensive study of stylized facts displayed by Bitcoin returns

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Abstract. In this paper, we explore some stylized facts in the Bitcoin market using the BTC-USD exchange rate time series of historical intraday data from 2013 to 2018. Despite Bitcoin presents some very peculiar idiosyncrasies, like the absence of macroeconomic fundamentals or connections with underlying asset or benchmark, a clear asymmetry between demand and supply and the presence of inefficiency in the form of very strong arbitrage opportunity, all these elements seem to be marginal in the definition of the structural statistical properties of this virtual financial asset, which result to be analogous to general individual stocks or indices. In contrast, we find some clear differences, compared to fiat money exchange rates time series, in the values of the linear autocorrelation and, more surprisingly, in the presence of the leverage effect. We also explore the dynamics of correlations, monitoring the shifts in the evolution of the Bitcoin market. This analysis is able to distinguish between two different regimes: a stochastic process with weaker memory signatures and closer to Gaussianity between the Mt. Gox incident and the late 2015, and a dynamics with relevant correlations and strong deviations from Gaussianity before and after this interval.

1. Introduction

A cryptocurrency is a digital currency that can perform all the three functions of money, namely, medium of exchange, unit of account and store of value. However, until now, the spread of this type of money is still in its infancy, despite its considerable numbers: it is estimated that today there are about 5,000 cryptocurrencies with a market capitalization of the order of US$240 billions [1]. The first and most important cryptocurrency to date is Bitcoin. It was developed in 2008 and diffused in a paper assigned using the pseudonym of Satoshi Nakamoto [2]. Bitcoin is an open source, peer-to-peer currency, and does not depend on monetary or governmental authority. Its invention is revolutionary given that there is no need to have an agent like a bank to effect the transaction, only the two parties involved: the payer and the receiver of the debt. All transactions carried out are stored in a public register called blockchain so that future transactions are unable to use the previously spent Bitcoins. Note that transactions on the Bitcoin network are not made in another fiat currency but they are made in Bitcoins. Thus, in addition to being a completely decentralized network, it is a virtual currency, where its value is defined in a market, such as the dollar, euro, swiss franc, etc. As its use has been quite restricted, many economists still do not consider it as currency, as it lacks at least one of its typical functions. In fact, Baur et al. [3] notes that, for the period from 2011 to 2013, most Bitcoins were used as a portfolio asset (value store) and not as a currency (medium of exchange).

Empirical studies [3, 4, 5, 6] show that Bitcoin and other cryptocurrencies generally present time series characterized by the following simple descriptive statistics: high returns, high volatility, important skewness and high kurtosis. Moreover, some return autocorrelations and a changing behavior of the Hurst exponent has been detected.

The study of autocorrelations and its connection with the presence of short or long range memory is a particularly interesting topic because of its relation with the efficient
markets hypothesis (EMH). This idea implies determining whether asset prices fully reflect all available information [7]. If so, it means that any new information is revealed entirely in its price. The question here is to know what kind of information might be taken into consideration for a market to be considered efficient. The most widely used version of EMH is the semi-strong: prices accurately reflect all publicly known information. Under this version, there is no cheap or expensive asset: the current price is always its best estimate. This definition implies that the knowledge of past prices is useless to predict future prices, which leads us to the weak version of EMH: historical prices have no relevance to predict prices. From a statistical viewpoint, this means that asset prices or returns cannot present long range memories, since they would allow a riskless profitable trading strategy. Finally, there is the strong version of EMH: the prices accurately reflect all the information (public and private). While the first two forms of market efficiency, weak and semi-strong, have numerous advocates, there is a general perception that the strong version is difficult to empirically validate.

Since its foundation, EMH has been questioned and the analysis of the presence of long memory in financial time series has become an important topic, with papers presenting some empirical evidences of long memory [8], and others challenging it [9]. Several studies have been done to draw some conclusion about the efficiency of Bitcoin, however without a definitive conclusion (see [5] [10] [11] [12] [13] [14] among others). Bartos [10] analyses the effects of public announcements on Bitcoin price concluding for a positive answer. Therefore, its price seems to follow the EMH. Urquhart [11], through several tests, finds that Bitcoin market is inefficient, but when he splits into two subsample periods, he finds that the Bitcoin may be in a process towards efficiency. Nadarajah and Chu [12] show, through eight different tests, that Bitcoin satisfies the EMH. Dimitrova et al. [13] conclude that, although the self-similarity exponent of the BTC-USD price series is different than 0.5, this result is not due to the presence of significant memory but to its underlying distribution. Bariviera et al. [5] analyzed the behavior of long memory of returns from 2011 until 2017, using the Hurst’s exponent. They showed a persistent behavior from 2011 until 2014, whereas the series is more informational efficient since 2014. Finally, Nan and Kaizoji [14] study the Bitcoin market efficiency in terms of the Bitcoin exchange rate and conclude that the weak and semi-strong form market efficiency of the USD/EUR Bitcoin exchange rate holds in the long run concerning the spot, futures, and forward FX markets.

These analysis are of particular relevance since the common sense speaks about inefficiency of the cryptocurrency market. Arbitrage opportunity are evident, as the price arrived to present an enormous spread among the different platforms used for buying and selling the cryptocurrency. Moreover, Bitcoin price is not driven by macro-financial indicators, a fact that can generate more sensitivity on information flows in market affecting the supply and demand interaction. For these reasons, a natural question that emerges is whether these inefficiencies can be tracked in the Bitcoin historical time series, either in the form of a clear long memory, or in more subtle temporal structures present in the data.
With this aim, our work will try to systematically characterize the empirical properties of the returns of the Bitcoin time series, highlighting the most relevant stylized facts [15, 16] present in these data. Among them, we will describe the heavy tails of the distribution of returns, and the autocorrelations of some nonlinear functions of returns. In particular, for the first time, we will characterize the presence of the leverage effect. A comparison of the obtained results with well known features already detected in the time series of the exchange rate of fiat currencies, which are among the most liquid assets in the world, could shed a light on the real impact that inefficiencies can have on the statistical properties of the temporal dynamics of a general financial asset. This approach will be used also for characterizing the time varying behavior of the evolution of Bitcoin returns. In this way we will explore the possibility of using some of these statistical features as empirical indicators or signals for monitoring the shifts in the evolution of the Bitcoin market. This analysis will show some interesting features related to how important events and publicly announced information can affect the prices of cryptocurrencies.

2. Data and Methods

We examine the time series of the price of Bitcoin, expressed in Dollars (exchange rate), from 31/3/2013 to 27/4/2018, with a sampling time interval of 5 hours, which corresponds to a series of 8894 elements (see Fig. 1). We leave out earlier periods due to low market liquidity. Data are extracted from the site Bit Coin Charts [17] and are representing the Bitcoin price from the Bitstamp exchange market platform. Note that Bitcoin price, in the considered period, can depend strongly on the platform used for trading. At high frequency, data downloaded from BitcoinCharts do not present regular time intervals of sampling. A resampling of the series with intervals of 5 hours make it possible to obtain a time series of good quality.

As usual, the analysis are realised using the return of the price series \( r(t) \), defined as:

\[
r(t) = \ln(P(t)) - \ln(P(t - \Delta t))
\]

where \( P(t) \) is the price at time \( t \) and \( \Delta t \) is the considered sampling time interval (for our series, \( \Delta t \) corresponds to 5 hours).

As a first step, we study the distribution of returns, with the aim of characterising the expected presence of heavy tails [15, 16]. The non-Gaussian shape of the distribution of price changes is a well known character when sufficient small sampling time intervals are considered. It can be quantified looking at the kurtosis of the distribution. More interesting is the description of the behaviour of the tail of the distribution. Based on some general empirical results for foreign exchange markets and for stock markets [18, 19], the tail can be characterised using a power-law distribution. For a better comparison with similar works present in the literature, the distribution of the price
change is defined using a normalised return: $r_N(t) = \frac{r(t) - \langle r \rangle}{\sigma}$, where $\langle r \rangle$ is the mean value of the returns and $\sigma$ the standard deviation estimated over the whole time series. We estimate the power-law exponent $k$ (tail index) using the Maximum Likelihood estimate (see, for example [20]). As usual, we use only the data larger than the smallest value for which the power-law behaviour holds ($|r_N^{\text{min}}|$). This lower bound on the power-law behaviour is estimated following an approach proposed by Clauset et al. [27]: we select
the value that makes the probability distribution of the measured data and the best-fit power-law model as similar as possible above $r_{min}^N$.

We continue our analysis studying the dependence and memory properties of the time series. We analyse the autocorrelation function:

$$A(\tau) = \text{corr}(r(t+\tau), r(t))$$

(2)

where $\text{corr}()$ is the Pearson’s correlation between the two variables. In liquid markets this linear autocorrelation function is expected to reach zero in a few minutes, with really faster decays in the most liquid ones, like the foreign exchange markets [22, 23]. This fact has been often cited for supporting the efficient market hypothesis [7]. In addition, we tested the autocorrelation present in our time series with a non-parametric measure, by using the Spearman’s rank-order correlation [24]. The Spearman’s correlation between two variables is equal to the Pearson’s correlation between the rank values $R$ of those two variables. In our case, the Spearman’s autocorrelation function corresponds to: $A_S(\tau) = \text{corr}(R_{r(t+\tau)}, R_{r(t)})$. More details can be found in [25].

For assuring the independence of the elements of the time series not only the linear autocorrelation, but any nonlinear function of returns should present no autocorrelation. This property is not satisfied by financial time series. Absolute and squared returns show important positive autocorrelations, a fact generated by the tendency of large price variations to be followed by large price variations (volatility clustering). For this reason we will analyse the autocorrelation functions of powers of the returns:

$$A_\alpha(\tau) = \text{corr}(|r(t+\tau)|^\alpha, |r(t)|^\alpha)$$

(3)

c Considering the case with $\alpha = 1$ and $\alpha = 2$. The first one generally presents the highest correlations, and the second one is commonly used for measuring volatility clustering.

The decay of these autocorrelation functions are usually well described by a power law (see [26]): $A_\alpha(\tau) \propto \tau^{-\beta}$. For $\alpha = 1$ and $\alpha = 2$ the coefficient $\beta \in [0.2, 0.4]$, as reported in [27].

Finally, we measure the correlation of returns with subsequent squared returns:

$$L(\tau) = \text{corr}(|r(t+\tau)|^2, r(t)).$$

(4)

It was shown empirically that this measure generally starts from a negative value for $\tau \approx 0$ and it grows towards zero [22, 28], suggesting that negative returns generate a rise in volatility. This negative correlation of volatility with returns is usually named “leverage effect”.

We introduce a simple way for characterising the presence of the leverage effect in our time series, defining a critical correlation time $\tau_0$, which estimates the temporal scale over which the leverage effect (anticorrelation) is relevant. This critical time corresponds to the smallest value of $\tau$ where $L(\tau)$ crosses zero. We can easily estimate $\tau_0$ calculating the cumulative leverage $L_c(\tau) = \sum_{i=0}^{\tau} L(i)$. Note that, if the time series presents the
leverage effect, $L(0)$ is negative and $L_c(\tau)$ is a convex function with a well defined minimum. In this case $\tau_0$ is the $\tau$ value where $L_c(\tau)$ reaches the minimum [20]. In fact, at this lag time the leverage vanishes and the volatility no more exercises a negative influence on the return. If $L(0)$ is not clearly negative and $L_c(\tau)$ does not present a convex shape the leverage effect is not present.

3. Results

3.1. General results

An inspection of simple descriptive statistics of the considered time series gives a Standard Deviation value of 0.035, a Skewness of -2.92 and a Kurtosis of 68.16. These results are consistent with previous results appeared in the literature [3] which, in relation to other traditional currencies, pointed out an extreme high value of the volatility, with a difference of one order of magnitude and comparable values for higher moments.

In Figure 2 we present the Bitcoin distribution of the normalised returns $r_N$, with its obvious non-Gaussian shape. The estimation of the tail index of the cumulated distribution of the absolute value of the normalised returns gives a value of $k = 3.42 \pm 0.02$, in good correspondence with the ones measured for the spot intra-daily foreign exchange markets [18] and for stock markets [19], where the power law estimation presented exponents close to 3.

![Figure 2](image)

**Figure 2.** On the left: Normalised returns distribution of Bitcoin. A Gaussian fitting is represented by the red continuous line. On the right: Log-log plot of the cumulated distribution of the absolute value of the normalised returns. The red continuous line is the power-law fitting of the higher values of the distribution ($|r_N^{\text{min}}| = 2.18$).

In Figure 3 we displayed the results relative to the values of the linear autocorrelations for our dataset. For the Pearson’s autocorrelation, as the empirical
observations are clearly not Gaussian, the significance levels are estimated comparing the correlation function to the 1-/99-percentiles of a generated ensemble of randomised data. In this case, the results show that the linear autocorrelations are close to be negligible. Anyway, we can detect some values slightly above the significance level. Also the Spearman’s autocorrelations present statistical significant values, in particular for $\tau \leq 6$. Spearman’s autocorrelations are smaller than the Pearson’s ones. This is probably due to important observations present in the tails of the distributions which positively impact the Pearson’s correlation. Our general findings in the measure of the autocorrelation function are consonant with some previous studies [5], in particular with the work of Urquhart [11] which showed some antipersistent behaviours dependent on the sampling period.

Figure 3. Autocorrelation for returns of Bitcoin. On the left: Pearson’s autocorrelation; the light blue bands represent the 1-/99-percentiles of the distribution of the linear autocorrelation of randomised data estimated over 1000 realisations. On the right: Spearman’s autocorrelation. Red points correspond to autocorrelation values with a $p$-value less than 0.05 (the null hypothesis corresponds to absence of autocorrelation).

The analysis of the behaviour of the absolute and squared returns for the Bitcoin time series follow the behaviour already recorded for other financial assets. In fact, it presents important positive autocorrelations which decay with a power-law. The exponents of the best fitted functions are $0.32 \pm 0.02$ for $\alpha = 1$ and $0.71 \pm 0.09$ for $\alpha = 2$. This second value is larger than expected, which corresponds to a faster decay in relation to the behaviour reported in [27].

The measurement of the volatility-return correlation shows that the values are not significantly different from zero for $\tau < 0$, whereas they are significant and negative for $\tau > 0$. This behaviour is consistent with the so-called leverage effect (see Figure 5). It follows that the cumulated function $L_c(\tau)$ is a clear convex function, and $L_c(\tau)$ reaches the minimum for $\tau_0 = 10$, which delineates a correlation with a decay time slightly longer than 2 days. Leverage effect has been previously detected for individual stocks and stock indices [28, 30], with a typical decay time of about 50 days for stocks and 10
days for indices. To our knowledge, leverage effect has never been detected before in exchange rates time series.

3.2. Running windows

In order to describe the dynamics of correlations all along the evolution of the Bitcoin time series, we partition our dataset using sliding windows. We consider windows of 1600 data points, which correspond to less than one year. The starting points of the windows are considered as the time stamp.

By looking at the behaviour of the moments of the distribution (see Figure 6) we can note that the volatility decreases monotonically until close to 2/2016, when it starts to grow. Around the same period there is a drop towards strong negative values of the skewness and, conversely, a peak in the value of the kurtosis.

We estimate also the values of the $\beta$ exponents for characterising the autocorrelation functions of powers of the returns and the characteristic time $\tau_0$ for describing the presence of the leverage effect (see Figure 6). Looking at these quantities it is possible to detect two clear critical moments in the time evolution of Bitcoin market: the early 2014, which presents an abrupt rise in the leverage effect, which reaches a characteristic time of more than 7 days, followed by the disappearance of the same effect. In the same
Figure 5. The leverage effect for the Bitcoin time series. A negligible correlation exists for $\tau < 0$, which becomes a relevant anticorrelation for $\tau > 0$. In the inset the cumulated function $L_c(\tau)$, which is a convex function. In this case, the time series presents the leverage effect and $L_c(\tau)$ reaches the minimum for $\tau_0 = 10$.

period a strong fall in the exponents of the non-linear correlations is recorded. This period follows the Mt. Gox incident which happened in February 2014.

The second critical moment is in the late 2015, and it is characterised by high values of the $\beta$ exponent for $\alpha = 1$ and the reappearance of the leverage effect ($\tau_0 > 0$). It can be associated with the unsuccessful fork attempt of Bitcoin into Bitcoin XT in August 2015. In between these two important events, the Bitcoin history is characterised by relative small Kurtosis and Skewness.

4. Discussion

In this paper, we explore some stylized facts of the Bitcoin market from 2013 to 2018, using the BTC-USD exchange rate time series with a time lag of 5 hours.

We detect heavy tails in the distribution of returns, which can be characterised by
Figure 6. Running windows analysis for Bitcoin. From top to bottom: the dynamics of the moments of the distribution (a,b,c). The values of the $\beta$ exponent for characterising the autocorrelation functions of powers of the returns (d) and the leverage effect (e), described using the characteristic time $\tau_0$. Note that $\tau_0 = 0$ corresponds to data which do not present the leverage effect. The analysis is obtained using overlapping windows of 1600 data points, moving forward by 200 datapoints. The first dashed vertical line represents the Mt. Gox incident (2/2014) and the second one the unsuccessful fork attempt of Bitcoin into Bitcoin XT (8/2015).

A power-law distribution with a finite tail index close to 3, a value comparable to the one displayed by foreign exchange markets and stock markets [18, 19]. It is interesting to note that using higher frequencies data Bitcoin returns distributions with heavier tails.
(k < 2.5) have been recorded. Our result suggests that, at larger temporal scales, the expected exponent close to 3 is recovered.

The analysis of the behaviour of the absolute and squared returns shows important positive autocorrelations, which decay slowly as a function of the time lag, following a power-law with the exponents 0.32, for the absolute returns, and 0.71 for the squared returns. These results are homologous with the behaviour found in the time series of exchange rates of fiat money. Looking at the linear autocorrelations we have observed that even if they are very close to be always negligible, as expected for the exchange rates of fiat money, we can detect some values above the significance level. This fact is consonant with some previous studies, in particular with the work of Urquhart, which shows some antipersistent behaviours dependent on the sampling period. More marked differences can be found in the measurement of the volatility-return correlation, which shows a negative correlation (leverage effect) with a decay time slightly longer than 2 days. To our knowledge, leverage effect has been detected previously for individual stocks and stock indices but not for exchange rates time series. Moreover, our result is particularly visible. In fact, it does not need to rely on an averaging procedure, as for individual stocks, and the negative correlation is larger than the one usually seen in stock indices. Important differences are also present in the extreme high value of the volatility, with a difference of one order of magnitude in relation to other fiat money, and the presence of a gain/loss asymmetry, a property usually present in stock prices and stock index values, but not in exchange rates, where there is a higher symmetry in up/down movements.

Bitcoin market is characterised by some very peculiar idiosyncrasies. There are no macroeconomic fundamentals for digital currencies, nor their values can be derived from an underlying asset or benchmark. The absence of macro-financial indicators should generate more sensitivity on information flows in market and affect the supply and demand interaction. Moreover, there is a clear asymmetry between a demand, strongly influenced by speculative interests, and its rigid supply. The presence of inefficiency in the form of very strong arbitrage opportunity can be easily tracked looking at the important spread among the different platforms used for buying and selling this cryptocurrency. Anyway, all these elements seem to be marginal in the definition of the structural statistical properties of this virtual financial asset, which result to be analogous to general individual stocks or indices. This fact suggests conjecturing about what determines the statistical properties of the considered stylized facts for a general financial asset. As these properties do not change in the case of the Bitcoin, they should be related more to the market and the social component shaping the price, and much less to macroeconomic aspects, or connections with underlying assets or benchmarks.

If the comparison is restricted to fiat money exchange rates time series, we can outline some clear differences in the linear autocorrelation and, most importantly, in the presence of the leverage effect, gain/loss asymmetry and the extreme high value of the volatility. Foreign exchange rates, with their high liquidity, can be considered as a market which better supports the efficient market hypothesis. In the particular case of
the Bitcoin market, the evident detection of positive autocorrelations in the absolute and squared returns and, most importantly, the presence of a clear leverage effect which can be interpreted as a sign of long-range dependence, are clear evidence against the efficient market hypothesis.

The second part of our analysis explores the dynamics of correlations all along the evolution of the Bitcoin time series, by partitioning it with sliding windows of 1600 data points, which correspond to less than one year. We look at the behaviour of the moments of the distribution, the character of the slow decay of the absolute and squared returns autocorrelation functions (using the $\beta$ exponent) and the leverage effect (using the characteristic time $\tau_0$). These indicators of specific statistical characters of the time series behaves particularly well as empirical signals for monitoring the shifts in the evolution of the Bitcoin market. In fact, their changes are clearly correlated to two important critical moments of the Bitcoin dynamics: the Mt. Gox incident, when, in February 2014, this Bitcoin exchange suspended trading and closed its service [31], and the unsuccessful fork attempt of Bitcoin into Bitcoin XT in August 2015.

In the interval between these two important events, the Bitcoin history is characterised by relative small kurtosis and skewness, with absence of the leverage effect and a strong fall in the exponents of the absolute return correlations. These features generally correspond to a stochastic process with weaker memory signatures and closer to Gaussianity. In contrast, outside this region, deviation from Gaussianity (higher skewness and kurtosis) and correlations (leverage and a slower decay in the absolute return correlation) become more relevant. We can note how Mt. Gox incident was a huge strike to the Bitcoin’s credibility and reputation, which affected its price generating a shock in a moment of euphoria, which led to a stable decreasing trend. An inversion to this tendency appeared in the late 2015, with a new rise associated with an increasing popular attention. Following the work of Gerlach et al., this interval corresponds to the correction regime which follows the second Bitcoin’s long bubble, and the neutral period before the rise of the third long bubble [33]. From these considerations, we can suppose that periods of greater interest associated to speculative activities generate stronger deviation from Gaussianity and more relevant memory effects.

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