Higher Derivative Terms from Threebranes in F Theory

Robert de Mello Koch and Radu Tatar

Department of Physics,
Brown University
Providence RI, 02912, USA
robert,tatar@het.brown.edu

The computation of higher derivative corrections to the low energy effective actions of $\mathcal{N} = 2$ gauge theories is considered. In particular, higher derivative corrections are computed for four dimensional $\mathcal{N} = 2$ super Yang-Mills theory with gauge group $SU(2)$ and $N_f = 4$ hypermultiplets in the fundamental representation. The four derivative terms computed in an approach which realizes the gauge theory as the world volume theory of three branes in F theory are in agreement with the field theory result.

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1. Introduction

A particularly efficient way to construct the low energy effective action of a super Yang-Mills theory is to realize the field theory of interest as the worldvolume theory of a suitable brane. In this approach, gauge theories with a reduced number of supersymmetries can be obtained by considering a web of intersecting branes in type IIA string theory. After lifting to M theory, the type IIA web can be realized in terms of a single M theory fivebrane wrapping a Riemann surface. The Riemann surface is the Seiberg-Witten curve.

The limit in which the field theory is realized on the brane world volume is a low energy limit in which one decouples bulk gravity from the world volume theory. In addition, the string tension must be taken to be large in order to decouple open string oscillator excitations. Finally, Kaluza-Klein modes associated with the brane geometry and the compact eleventh (strong coupling) dimension have to be decoupled. Quantities in the low energy effective action which are constrained by supersymmetry are not sensitive to the limit in which they are computed. The more supersymmetry a theory has, the more the low energy effective action is constrained. For the case of $\mathcal{N} = 4$ supersymmetry in four dimensions, the constraints are so severe that they restrict the form of four derivative terms in the low energy effective action. For $\mathcal{N} = 2$ theories in four dimensions, the constraints due to supersymmetry imply that the leading low energy effective action can be written as an $\mathcal{N} = 2$ superspace chiral integral of a holomorphic prepotential. For $\mathcal{N} = 1$ supersymmetry in four dimensions, supersymmetry constrains the superpotential to be a holomorphic function of a chiral superfield. There is an impressive collection of holomorphic (BPS) quantities that have been computed using the brane approach.

The computation of non-holomorphic quantities is more delicate though, and they are sensitive to the limit in which they are computed. Interesting non-holomorphic quantities include the higher derivative corrections to the $\mathcal{N} = 2$ super Yang-Mills theory and the Kähler potential of the $\mathcal{N} = 1$ theory. In these quantities were computed using the M theory fivebrane. The results obtained show a clear quantitative disagreement with what is expected from the four dimensional gauge theories. This would seem to suggest that although the brane approach is a useful tool for computing holomorphic quantities, it can not be used to compute quantities that are not protected by supersymmetry. This is unfortunate, since ultimately one would like to get insights into QCD which is not a supersymmetric theory.

In a recent paper, $\mathcal{N} = 2$ and $\mathcal{N} = 1$ field theories were realized as worldvolume theories of Dirichlet threebranes moving near sevenbranes, i.e. threebranes in F theory.
The authors of [5] showed that if the number of threebranes is large, the geometry can be trusted in the field theory limit, suggesting that one could compute non-BPS quantities. The aim of this work is to test this exciting suggestion in some simple cases.

Specifically, in this article we consider the calculation of higher derivative corrections to the low energy effective action of $\mathcal{N} = 2$ supersymmetric Yang-Mills field theories in four dimensions. In section two, we begin by reviewing what is known from field theory about these corrections. In section three, the computation of these quantities using the Dirichlet fivebrane is performed for the finite theory with gauge group $SU(2)$ and four massless hypermultiplets in the fundamental representation. The computation in this case is particularly simple and both the low energy effective action and the first higher derivative corrections can be computed exactly. The fivebrane result disagrees with the field theory result. In section four, we compute the higher derivative corrections using threebranes in F theory. The supergravity solution is known, and the higher derivative corrections can simply be read from an expansion of the Born-Infeld action. The result is in perfect agreement with the field theory result. Section five contains a discussion of our results.

2. Field Theory Results

The low energy effective action of $\mathcal{N} = 2$ super Yang-Mills theory, when written in $\mathcal{N} = 2$ superspace, has the form

$$S = \int d^4xd^4 \theta \mathcal{F}(A_i) + \int d^4xd^4 \bar{\theta} \bar{\mathcal{F}}(\bar{A}^i) + \int d^4xd^4 \theta d^4 \bar{\theta} \mathcal{H}(A_i, \bar{A}^i). \quad (2.1)$$

The prepotential $\mathcal{F}$ is a holomorphic function of the abelian $\mathcal{N} = 2$ chiral vector superfields. This quantity can be computed directly in field theory using Seiberg-Witten theory[6]. The real function $\mathcal{H}(A, \bar{A})$ gives the first non-holomorphic corrections to the low energy effective action. In general, the exact form of $\mathcal{H}$ is not known although several contributions to $\mathcal{H}$ are known explicitly. These are the one loop contribution[7], the two loop contribution[8], the one instanton contribution[9] and the two instanton contribution[10]. We will be most interested in the gauge theory with gauge group $SU(2)$ and $N_f = 4$ massless hypermultiplets in the fundamental representation, which is a finite and scale invariant gauge theory. In this case, scale invariance forbids a normalization scale $\Lambda$ and hence one may be tempted to conclude that there are no higher loop or instanton corrections. In this case, because $\mathcal{H}$ would be one loop exact, there are claims that[7]
\[ H(A, \bar{A}) = \frac{3}{256\pi^2} \ln^2 \left( \frac{A\bar{A}}{(A)(\bar{A})} \right), \] (2.2)

in an exact formula. \( H \) is invariant under the Kähler gauge transformations

\[ H(A, \bar{A}) \rightarrow H(A, \bar{A}) + f(A) + \bar{f}(A), \] (2.3)

so that (2.2) is explicitly scale invariant. At this point a comment is in order. The absence of a normalization scale \( \Lambda \) has been used to argue that the leading low energy effective action itself does not receive quantum corrections. Explicit instanton corrections show that this is not the case. Thus, the claim that (2.2) is exact is doubtful.

The results we wish to compare with will be expressed in terms of components so that we need to find the component expansion of (2.1). This is most easily done using an \( \mathcal{N} = 1 \) superspace notation. The \( \mathcal{N} = 1 \) chiral superfield contained in \( A^i \) is denoted by \( \Phi^i \); the \( \mathcal{N} = 1 \) field strength contained in \( A^i \) is denoted by \( W_{\alpha}^i \). The complex scalar appearing in \( \Phi^i \) is denoted by \( \phi^i \). Using the \( \mathcal{N} = 1 \) expansion of [11], we find the following four derivative terms for the scalars \( \phi^i \)[12]

\[ S_4 = \int d^4x \left[ 2 \frac{\partial^2 H}{\partial \phi_i \partial \phi_j} (\partial^m \phi^i)(\partial^n \phi^j) + \frac{\partial^3 H}{\partial \phi_i \partial \phi_j \partial \phi_k} (\partial^m \phi^i)(\partial^m \phi^j)(\partial^n \phi^k) + \frac{\partial^3 H}{\partial \phi_i \partial \phi_j \partial \phi_k \partial \phi_l} (\partial^m \phi^i)(\partial^m \phi^j)(\partial^n \phi^k)(\partial^n \phi^l) \right]. \] (2.4)

Similarly, the kinetic term for the \( \phi^i \) is

\[ S = \int d^4x \partial_m \phi^i \partial^m \phi^j I m \left( \frac{\partial^2 F}{\partial \phi_i \partial \phi^j} \right) = \int d^4x \partial_m \phi^i \partial^m \phi^j K_{ij}. \] (2.5)

In the remaining two sections we will see that the branes provide a form for the four-derivative term that is only consistent with the \( \mathcal{N} = 2 \) field theory result after we make certain field redefinitions. The need for these field redefinitions has been interpreted in [12] as a consequence of the fact that the \( \mathcal{N} = 2 \) supersymmetry in field theory is realized differently than it is in the fivebrane field theory. The field equation for \( \phi^i \) reads

\[ \partial^m \partial_m \phi^i = -(K_{ij})^{-1} \partial K_{jk}^l (\partial^m \phi^k)(\partial_m \phi^l). \] (2.6)

The field redefinitions that are needed correspond to replacing \( \partial_m \phi^m \phi \) in (2.4) with the right hand side of (2.4). This leads to the following expression[12]
\[ S_4 = \int d^4x \tilde{H}_{ijkl}(\partial^m \phi^i)(\partial_m \phi^j)(\partial^n \phi^k)(\partial_n \phi^l), \quad (2.7) \]

where

\[ \tilde{H}_{ijkl} = \frac{\partial^4 \mathcal{H}}{\partial \phi_i \partial \phi_j \partial \phi_k \partial \phi_l} - \frac{\partial^3 \mathcal{H}}{\partial \phi_i \partial \phi_j \partial \phi_p} (K_{pq})^{-1} \frac{\partial K_{qk}}{\partial \phi^l} - \frac{\partial^3 \mathcal{H}}{\partial \phi_i \partial \phi_l} (K_{pq})^{-1} \frac{\partial K_{jk}}{\partial \phi^p} \]

\[ + 2 \frac{\partial K_{ji}}{\partial \phi^p} (K_{pq})^{-1} \frac{\partial^2 \mathcal{H}}{\partial \phi_q \partial \phi_r} (K_{rs})^{-1} \frac{\partial K_{sk}}{\partial \phi^l}. \quad (2.8) \]

Using the explicit expressions (valid for \( N_c = 2 \) and \( N_f = 4 \) massless hypermultiplets in the fundamental representation)

\[ K_{u\bar{u}} = \frac{\text{Im}(\tau)}{8\sqrt{u\bar{u}}}, \quad u = \frac{1}{2} A^2, \quad \tau = \frac{\theta}{\pi} + \frac{8\pi i}{g^2}, \quad (2.9) \]

and the formula (2.2) for \( \mathcal{H} \), we finally find

\[ S = \int d^4x (\partial^m u \partial_m u)(\partial^n \bar{u} \partial_n \bar{u}) \frac{3}{28 \pi^2 u^2 \bar{u}^2}. \quad (2.10) \]

The formulas (2.9) and (2.10) do not include instanton corrections. Before leaving this section, we note that in the pure gauge case, the one loop results for \( SU(2) \) are

\[ K_{u\bar{u}} \sim \log(16u\bar{u}/\Lambda^4), \quad u = \frac{1}{2} A^2, \quad H(A, \bar{A}) \sim \log\left(\frac{A}{\Lambda}\right) \log\left(\frac{\bar{A}}{\Lambda}\right). \quad (2.11) \]

Thus, the semiclassical four derivative term reads\[12\]

\[ S = \int d^4x (\partial^m u \partial_m u)(\partial^n \bar{u} \partial_n \bar{u}) \frac{8 + 4\log\left(\frac{16u\bar{u}}{\Lambda^4}\right) + \left[\log\left(\frac{16u\bar{u}}{\Lambda^4}\right)\right]^2}{u^2 \bar{u}^2 \left[\log\left(\frac{16u\bar{u}}{\Lambda^4}\right)\right]^2}. \quad (2.12) \]

Thus, in the large \( u \) (semiclassical) region, the fall off of the four derivative correction is again \( |u|^{-4} \).
3. The Fivebrane Description

In this section we describe the fivebrane description of the $\mathcal{N} = 2$ super Yang-Mills theory with gauge group $SU(2)$ and $N_f = 4$ massless hypermultiplets in the fundamental representation. The relevant brane configuration is realized in type IIA string theory. It consists of two parallel Neveu-Schwarz fivebranes, with world volume coordinates $x^0, x^1, x^2, x^3, x^4, x^5$. These two fivebranes are separated by a finite distance in the $x^6$ direction and two Dirichlet fourbranes with world volume coordinates $x^0, x^1, x^2, x^3, x^6$ are suspended between the two fivebranes. There are four semi infinite fourbranes with world volume coordinates $x^0, x^1, x^2, x^3, x^6$. Two semi infinite fourbranes stretch from $x^6 = -\infty$ to the left most fivebrane and another two semi infinite fourbranes stretch from $x^6 = \infty$ to the right most fivebrane. We will take the $x^1$ and $x^7$ directions to be finite. Using the arguments given in [13], this type IIA brane configuration can be mapped into a single Dirichlet fivebrane in IIB string theory as follows: Lifting this type IIA configuration to M theory, we obtain a single M theory fivebrane wrapped on the Seiberg-Witten curve[14]. If we now return to IIA string theory, interpreting $x^1$ as the direction which grows at strong coupling, we obtain a single Dirichlet fourbrane wrapping the Seiberg-Witten curve. Finally, performing a T duality along the $x^7$ direction, we obtain a single Dirichlet fivebrane in type IIB string theory. The Seiberg-Witten curve for the above brane configuration takes the form[14]

\[ v^2 t^2 - 2B(v)t + ev^2 = 0, \quad B(v) = v^2 + u, \]

\[ t = \exp(-s/R_7) = \exp(-(x^6 + ix^7)/R_7), \quad v = x^4 + ix^5. \]  

(3.1)

The Dirichlet fivebrane has world volume coordinates $x^0, x^1, x^2, x^3, x^6, x^7$. The low energy world volume description of this Dirichlet fivebrane is given by the following 5 + 1 dimensional Yang-Mills theory

\[ \mathcal{L} = Tr \left( F_{\mu\nu} F^{\mu\nu} + D_\mu X^I D^\mu X^I + [X^I, X^J]^2 \right), \]  

(3.2)

where $I = 4, 5, 8, 9$, $\mu, \nu = 0, 1, 2, 3, 6, 7$ and only the bosonic part of the Lagrangian is shown. The $X^I$ are $2 \times 2$ dimensional matrices. The classical fivebrane solution[13] is given by taking $X^4$ and $X^5$ diagonal and setting all other fields to zero. It is convenient to assemble the eigenvalues $x^4_i$ and $x^5_i$ of $X^4$ and $X^5$ into the single complex number $v_i = x^4_i + ix^5_i$. The complex numbers $v_i$ are now identified with the roots of the Seiberg-Witten curve (3.1). In this way, the Higgs fields trace out the curve described in (3.1) as
the worldvolume coordinates vary so that we do indeed obtain a fivebrane wrapped on the Seiberg-Witten curve. For the case that we study here, the roots \( v \) are given by

\[
\begin{aligned}
    v_{1, 2} &= \pm \sqrt{2u} \sqrt{\frac{t}{t^2 - 2t + e}}.
    \end{aligned}
\]  

(3.3)

Notice that the sum of roots vanishes so that the Higgs fields can be expanded in the Lie algebra of \( SU(2) \) as expected. The terms in the action (3.2) which give rise to the scalar kinetic term of the four dimensional field theory are \( (m = 0, 1, 2, 3, Y = X^4 + iX^5) \)

\[
\mathcal{L}_{\text{kin}} = \int d^2 s Tr \left( \partial_m Y \partial^m Y^\dagger \right) = \int d^2 s \partial_m v_i \partial^m \bar{v}_i,
\]  

(3.4)

The \( u \) dependence of the action can be extracted without performing any explicit integrals

\[
S = \int d^4 x \frac{\partial_m u \partial^m \bar{u}}{8\sqrt{u\bar{u}}} \text{Im}(\tau), \quad \text{Im}(\tau) = 4 \int d^2 s \sqrt{\frac{\bar{t}t}{(t^2 - 2t + e)(t^2 - 2t + e)}}.
\]  

(3.5)

A few comments are in order. The above \( u \) dependence of the effective action shows that \( a \sim \sqrt{u} \). This is the expected result. It would be wrong to conclude that the effective action has not received any perturbative or instanton corrections. In the case of finite gauge theories, there are both loop and instanton corrections\[15\]. These corrections enter in the relation between the parameters in the fivebrane curve and parameters in the field theory. Note however, that independently of this relation, \( \tau \) is a constant. The ease with which we evaluated the \( u \) dependence of the low energy effective action is a direct consequence of this.

The higher derivative corrections to the super Yang-Mills theory are expected to arise from a non-Abelian Born-Infeld action. An explicit form for this action has been suggested by Tseytlin \[16\]. Although there have been some questions regarding the validity of this action \[17\], our solutions are diagonal matrices and we do not expect further corrections, which presumably probe the non-Abelian structure of the solution, to affect our result. For that reason, we will consider the action

\[
S_p = T_p \int d^{p+1} x ST r \left[ \sqrt{- \det(\eta_{rs} + D_r X_a (\delta_{ab} - iT [X_a, X_b])^{-1} D_s X_b + T^{-1} F_{mn})} \right. \\
\times \sqrt{\det(\delta_{ab} - iT [X_a, X_b])},
\]  

(3.6)
where $T_p$ is the $p$-brane tension, $T^{-1} = 2\pi\alpha'$ and the symmetrized $STr$ is defined by

$$STr(A_1...A_n) = \frac{1}{n!} Tr\left(A_1...A_n + \text{all permutations}\right). \quad (3.7)$$

At low energy, we regain the super Yang-Mills description from this action. The above action can be expanded as follows

$$S = Tr(L) + \frac{1}{2} Tr(M^r_rL) + \frac{1}{8} Tr(M^r_rM^s_sL) - \frac{1}{4} Tr(M_{rs}M^{sr}L)$$

$$L = \sqrt{det(\delta_{ab} - iT[X_a, X_b])}$$

$$M_{rs} = D_rX_a(\delta_{ab} - iT[X_a, X_b])^{-1}D_sX_b + T^{-1}F_{rs}). \quad (3.8)$$

Although we have only considered the bosonic piece of the fivebrane action, it is interesting to note that a supersymmetric extension of (3.8) has been constructed in [18]. Inserting the classical solution, the higher derivative corrections take the form

$$S \sim \int d^4x Tr(\partial_m Y \partial^m Y \partial_n \bar{Y} \partial^n \bar{Y}) \sim \int d^4x \partial_m u \partial^m u \partial_n \bar{u} \partial^n \bar{u} \frac{1}{u \bar{u}}. \quad (3.9)$$

Notice that the higher derivative corrections obtained from the fivebrane have the same structure as the higher derivative corrections computed in field theory. It is clear however that the $u$ dependence of the four derivative terms disagree with the field theory result. The $u$ dependence of the above result is in perfect agreement with the $u$ dependence obtained in [19], where the higher derivative corrections from a fivebrane wrapping the Seiberg-Witten curve corresponding to pure $SU(2)$ $\mathcal{N} = 2$ gauge theory were estimated.

This discrepancy between the field theory result and the fivebrane result is not unexpected, as we now explain. The $\mathcal{N} = 2$ super Yang-Mills theory is expected to arise from the IIA brane configuration at low energy and weak string coupling. The analysis we have performed for the Dirichlet fivebrane is valid at weak string coupling and low energy. Thus, for the analysis of this section to be applicable to the field theory, we need to verify that the weak coupling low energy description of the Dirichlet fivebrane is dual to the weak coupling low energy description of the IIA configuration. The results of [13] show that the low energy weakly coupled type IIA description is dual to a strong coupling low energy description of the type IIB Dirichlet fivebrane. Thus, there is no reason to expect that the higher derivative corrections computed using the fivebrane should be related to the higher derivative corrections of the $\mathcal{N} = 2$ super Yang-Mills theory. This is a good example showing that non-holomorphic corrections are sensitive to the limit in which they are computed.
4. Threebrane in F Theory

We begin by reviewing the supergravity solution for threebranes moving in a sevenbrane background given in \[5\]. We start from a solution for the sevenbranes by themselves. The NSNS two form and RR two and four forms are set to zero. This leaves the metric and the dilaton from the NSNS sector and the axion from the RR sector. It is convenient to combine the dilaton and axion into a single complex coupling
\[
\tau = \tau_1 + i\tau_2 = \chi + ie^{-\phi}.
\]
The parameter \(\tau\) is the modular parameter of the elliptic fiber of the F theory [22] compactification. Introduce the complex coordinate
\[
z = x^8 + ix^9.
\]
In terms of \(z\) we take the following ansatz for the metric
\[
ds^2 = e^{\phi(z,\bar{z})}dzd\bar{z} + dx_2^2 + ... + dx_7^2 - dx_0^2.
\]
(4.1)
This ansatz is for a sevenbrane with worldvolume coordinates \(x^0, x^1, x^2, x^3, x^4, x^5, x^6, x^7\). With this ansatz, the type IIB supergravity equations reduce to [20]
\[
\partial\bar{\partial}\tau = \frac{2\partial\tau\bar{\partial}\bar{\tau}}{\bar{\tau} - \tau},
\]
\[
\partial\bar{\partial}\phi = \frac{\partial\tau\bar{\partial}\bar{\tau}}{(\bar{\tau} - \tau)^2}.
\]
(4.2)
The sevenbrane background of relevance for the \(\mathcal{N} = 2\) field theory is obtained by identifying \(\tau\) with the effective gauge coupling constant. This implies that \(\tau = \tau(z)\) so that the first equation in (4.2) is automatically satisfied. The general solution to the second equation in (4.2) is
\[
\phi(z, \bar{z}) = \log\tau_2 + F(z) + \bar{F}(\bar{z}).
\]
(4.3)
The functions \(F(z)\) and \(\bar{F}(\bar{z})\) should be chosen in order that (4.1) yields a sensible metric. For the case that we are considering (i.e. constant \(\tau\)), the explicit form for the metric transverse to the sevenbranes is [20]
\[
ds^2 = e^{\phi(z,\bar{z})}dzd\bar{z} = \tau_2|da|^2,
\]
(4.4)
where \(a\) is the quantity that appears in the Seiberg-Witten solution [3]. This specifies the solution for the sevenbranes by themselves.

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\[1\] This solution has also appeared in [20]. For additional work on the large \(N\) limit of field theory from threebranes in F theory see [21].
Next, following \[5\] we introduce threebranes into the problem. The world volume coordinates of the threebranes are \(x^0, x^1, x^2, x^3\). One obtains a valid solution by making the following ansatz for the metric

\[ ds^2 = f^{-1/2} dx_0^2 + f^{1/2} g_{ij} dx^i dx^j \]  

(4.5)

and the following ansatz for the self-dual 5-form field strength

\[ F_{0123i} = -\frac{1}{4} \partial_i f^{-1}. \]  

(4.6)

This solution corresponds to introducing \(N\) coincident threebranes. The complex field \(\tau\) is unchanged. Inserting the above ansatz into the IIB supergravity equations of motion, one finds that \(f\) satisfies the following equation of motion \[5\]

\[ \frac{1}{\sqrt{g}} \partial_t (\sqrt{g} g^{ij} \partial_j f) = -(2\pi)^4 N \frac{\delta^6(x - x^0)}{\sqrt{g}}. \]  

(4.7)

In the limit that \(N \to \infty\) the curvature becomes small almost everywhere and the supergravity solution can be used to reliably compute quantities in the field theory limit as explained in \[5\]. A sensitive test of this claim is the computation of higher derivative corrections performed below.

To obtain information about the \(\mathcal{N} = 2\) super Yang-Mills theory, we now consider a threebrane separated from the rest of the threebranes. The dynamics of this threebrane probe is given by a Born-Infeld action in the above supergravity background. The leading low energy effective action plus four derivative terms for the scalars are thus obtained by expanding the action \[24\]

\[ S = \frac{T_3}{2} \int d^4 x \left[ \sqrt{\det(G_{mn} + e^{-\frac{1}{2} \phi} F_{mn})} + \chi F \wedge F \right] \]

\[ = \frac{T_3}{2} \int \left( \tau_2 F^2 + \tau_1 F \wedge F + e^{\phi(z, \bar{z})} \partial_m z \partial^m \bar{z} + f e^{2\phi(z, \bar{z})} \partial_m z \partial^m \bar{z} \partial_n \bar{z} \partial^n \bar{z} \right). \]  

(4.8)

where \(z = x^8 + ix^9\) and \(x^i\) with \(i = 4, 5, 6, 7\) have been set to zero. It is clear that the low energy effective action of the threebrane probe is the same as the exact solution of the corresponding low-energy field theories \[25\]. Note once again that the brane answer for the

\footnote{See also \[23\] where this solution was independently discovered.}
four derivative terms has the same general structure as the four derivative terms computed in field theory.

We are now ready to return to the super Yang-Mills theory with \( N_c = 2 \) and \( N_f = 4 \). In this case, the supergravity solution can be determined exactly\(^5\). The metric transverse to the sevenbranes takes the form given in (2.2). In terms of \( a \) the solution of (4.7) reads

\[
\tau = \frac{\theta}{\pi} + \frac{8\pi i}{g^2}, \quad f = \frac{Nc_1}{(\tau_2 |a|^2 + y^2)^2}.
\]

with \( c_1 \) a constant which can be fixed using (4.7). The coordinate \( y \) is transverse to the threebranes but parallel to the sevenbranes. The coordinate \( a \) is transverse to both the seven branes and the three branes. The \( N \) threebranes are at \( y = a = 0 \). The probe threebrane is at \( y = 0 \) and at some \( a \neq 0 \). Moving the probe in the \( a \) direction corresponds to moving in the moduli space of the \( \mathcal{N} = 2 \) field theory. Evaluating the probe action (4.8) at this solution, we find

\[
S = \frac{T_3}{2} \int d^4x \left( \tau_2 \partial_n a \partial^n \bar{a} + Nc_1 \frac{1}{a^2 \bar{a}^2} \partial_m a \partial^m a \partial_n \bar{a} \partial^n \bar{a} \right)
\]

for the scalar fields in the probe action. This is in perfect quantitative agreement with the field theory results. Note that the present computation does seem to test the coefficient in front of the four derivative term, as we now explain. The relation between the Higgs expectation value of the field theory and the corresponding threebrane coordinate allows the introduction of one multiplicative constant \( a = ca_{SW} \) for any constant \( c \). Since the two terms in the low energy effective action scale with different powers of \( c \), their relative normalization can be fixed to the field theory prediction by a judicious choice of \( c \). The tension of the threebrane introduces an overall constant which can then be fixed so that the overall normalisation of the probe action and the field theory action agree.

5. Discussion

In this letter we have considered the computation of non-holomorphic quantities using the Dirichlet fivebrane and threebranes in F theory. The results obtained using the fivebrane disagree with the field theory results. This disagreement could be traced back to the fact that the description of the Dirichlet fivebrane was not valid in the limit in which field theory is expected to emerge. This clearly illustrates the fact that the four derivative terms are not constrained by supersymmetry. The results obtained using threebranes in
F theory are in perfect agreement with field theory. The good agreement in this case is due to the fact that the supergravity solution is valid in the field theory limit, if one takes a large number of three branes. To the best of our knowledge, this is the first time that corrections to a field theory which are not protected by supersymmetry, have been computed using a brane approach. This suggests that the method derived in [5] provides a reliable approach to the computation of non-holomorphic corrections. This is an important result because these quantities can not, at present, be computed directly in the field theory.

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