Two-fluid matter-quintessence FLRW models: energy transfer and the equation of state of the universe

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Abstract. Recent observations support the view that the universe is described by a FLRW model with \( \Omega_{0m} \approx 0.3 \), \( \Omega_{0\Lambda} \approx 0.7 \), and \( w \leq -\frac{1}{3} \) at the present epoch. There are several theoretical suggestions for the cosmological \( \Lambda \) component and for the particular form of the energy transfer between this dark energy and matter. This gives a strong motive for a systematic study of general properties of two-fluid FLRW models. We consider a combination of one perfect fluid, which is quintessence with negative pressure (\( p_Q = w\epsilon_Q \)), and another perfect fluid, which is a mixture of radiation and/or matter components with positive pressure (\( p = \beta \epsilon_m \)), which define the associated one-fluid model (\( p = \gamma \epsilon \)). We introduce a useful classification which contains 4 classes of models defined by the presence or absence of energy transfer and by the stationarity (\( w = \text{const.} \) and \( \beta = \text{const.} \)) or non stationarity (\( w \) or \( \beta \) time dependent) of the equations of state. It is shown that, for given \( w \) and \( \beta \), the energy transfer defines \( \gamma \) and, therefore, the total gravitating mass and dynamics of the model. We study important examples of two-fluid FLRW models within the new classification. The behaviour of the energy content, gravitating mass, pressure, and the energy transfer are given as functions of the scale factor. We point out three characteristic scales, \( a_{E} \), \( a_{P} \) and \( a_{M} \), which separate periods of time in which quintessence energy, pressure and gravitating mass dominate. Each sequence of the scales defines one of 6 evolution types.

Key words. Cosmology:theory – cosmology:dark energy

1. Introduction

A number of recent observations reveal the cosmological \( \Lambda \) component. Type Ia Supernovae (Riess et al. 1998, Perlmutter et al. 1999, Riess et al. 2001) and the Boomerang, Maxima and Dasi measurements of the total density parameter \( \Omega \) via the first acoustic peak location in the angular power spectrum of the CBR (de Bernardis et al. 2000, Balbi et al. 2000, Jaffe et al. 2000) show that \( \Omega = \Omega_{0m} + \Omega_{0\Lambda} = 1 \pm 0.05 \) with \( \Omega_{0\Lambda} \approx 0.7 \). Existing cosmological data allow a wide range for the equation of state coefficient \( w \) from \(-1\) to \(-1/3\) (Perlmutter et al. 1999a, Podariu & Ratra 2000, Wang et al. 2000).

Also, the smooth Hubble flow around our Local Group, inside a highly lumpy matter distribution, suggests still other evidence for a dominating \( \Lambda \) component and its variation with time (Chernin et al. 2001, Baryshev et al. 2001, Klypin et al. 2001, Axenides & Perivolaropoulos 2002).

There are several theoretical models for \( \Lambda \)-like cosmological components of the universe with positive energy density and negative pressure, including vacuum with a constant \( \Lambda \), decaying \( \Lambda \), and variable equation of state \( w(t) \) (Peebles & Ratra 1988, Lima & Maia 1994, Wetterich 1993, Ferreira & Joyce 1997, Caldwell et al. 1998, Steinhardt et al. 1998, Zlatev et al. 1999, Bahcall et al. 1999, Mbonye 2002). We use the term “quintessence” for any kind of substance having the equation of state \( p_Q = w\epsilon_Q \) with \(-1 \leq w < 0\), which may be time variable.

Thus observations and theory give strong motivation to study general properties of two-fluid Friedmann–Lemaître–Robertson–Walker (FLRW) models in which
the Λ component dominates at late epochs and there is energy transfer between Λ and matter components.

The energy transfer between dust-like matter and radiation was first studied by Davidson (1962). Some examples of the physics producing the energy transfer were given in Sistero (1971) and McIntosh (1967, 1968). The first exact solution of the equation of motion for a dust+radiation model with no energy transfer was obtained by Chernin (1965), who applied it to the case where the radiation component is the cosmic background radiation or neutrinos.

In the present paper we give a systematic presentation of the properties of two-fluid cosmological models, with and without energy transfer, in the frame of a new classification which naturally arises in the two-fluid problem. In section 2 we briefly summarize the standard two-fluid FLRW model. In section 3 we consider general properties of two-fluid models with matter and quintessence. Section 4 includes three examples of models from different classes of our classification. Section 5 contains the conclusions.

2. The two-fluid model: a summary
The derivation of the FLRW equations contains the following basic elements: 1) The Einstein equations (notations from Landau & Lifshitz 1974)

\[ R^i_k - \frac{1}{2} g^i_k R = \frac{8\pi G}{c^4} T^i_k \]  

2) The Bianchi identity in the form:

\[ T^i_k;_i = 0 \]  

3) The RW line element in spherical comoving space coordinates (\( \chi, \theta, \phi \)) and synchronous time \( t \):

\[ ds^2 = c^2 dt^2 - S^2(t) d\chi^2 - S^2(t) I^2_k(\chi) (d\theta^2 + \sin^2\theta d\phi^2) \]  

4) The total energy momentum tensor in comoving coordinates (ordinary matter, vacuum, quintessence):

\[ T^i_k = diag(\varepsilon, -p, -p, -p) \]  

Here \( I_k(\chi) = (\sin(\chi), \chi, \sinh(\chi)) \) for the curvature constant \( k = (+1, 0, -1) \), respectively. \( S(t) \) is the scale factor, \( \varepsilon = \rho c^2 \) is the energy density, and \( p \) is the pressure.

The proper metric distance \( r \) of a particle with the dimensionless comoving coordinate \( \chi \) from the observer is

\[ r(t, \chi) = S(t) \cdot \chi \]  

Here physical dimensions \([r] = [S] = \text{cm}\). The volume element in metric (3) \( dV = S^3 I^2_k(\chi) \sin^2(\theta) d\chi d\theta d\phi \).

2.1. The FLRW one-fluid model
For the one-fluid model the \((0,0)\) and \((1,1)\) components of Eq. (1) give the FLRW equations

\[ \frac{\dot{S}^2}{S^2} + \frac{kc^2}{S^2} = -\frac{8\pi G}{3c^2} p, \]  

where \( \dot{} = d/dt \). Eq. (2) implies:

\[ 3\frac{\dot{S}}{S} = -\frac{\dot{\varepsilon}}{\varepsilon + p}. \]  

The equation of state for the one-fluid model is

\[ p = \gamma \varepsilon. \]  

The characteristic energy content \( \varepsilon \) of the sphere with the radius equal to the proper metric distance \( r = S(t)\chi \) is:

\[ \varepsilon(r) = \int \varepsilon dV = 4\pi \varepsilon S^3 \sigma_k(\chi), \]  

where for different values of \( k = +1, 0, -1 \), \( \sigma_k(\chi) = \frac{\pi^2}{4} \chi^3, \frac{\sinh(\chi)}{\chi} - \frac{\chi^2}{2} \), respectively.

The gravitating mass \( M_g \) is defined by the equation of motion, which follows from Eqs. (1) - (3):

\[ \ddot{r} = -G \frac{M_g(r)}{r^2}, \]  

where

\[ M_g(r) = \frac{\varepsilon + 3p}{c^2} \int_0^r dV = \frac{\varepsilon}{c^2}(3\gamma + 1). \]  

This definition corresponds to the general expression for the gravitating mass of a self-gravitating body as given by Landau & Lifshitz (1974 §100).

2.2. Time dependence of energy
There are two factors which cause a time dependence of energy of each particular fluid and, hence, of an associated (as termed by Coley & Wainwright 1992) one-fluid model. The first cause is the expansion of the universe. The second is when one fluid is converted into another.

It is known (Lemaître 1933, Davison 1962, Harrison 1994, Peebles 1993, p.139) that Eqs. (3) and (7) imply the non-conservation of the energy \( \varepsilon \) of one-fluid FLRW models filled by a non dust-like matter:

\[ \frac{d\varepsilon}{dt} + p \frac{d}{dt}(R^3) = 0. \]  

For every particular fluid the time dependence of the energy is defined by its pressure. Eq. (13) is a consequence of the Bianchi identity (8). Rewritten for each particular fluid, it has the form of the classical energy conservation law without the conversion of one-fluid to another:

\[ \frac{d\varepsilon_m}{dt} + p_m \frac{d}{dt}(R^3) = 0, \quad \frac{d\varepsilon_Q}{dt} + p_Q \frac{d}{dt}(R^3) = 0. \]
When the one-fluid converts into another (for instance, the matter into radiation in stars), Eqs.(14) become

\[ \frac{1}{R^3} \left[ \frac{d\varepsilon_Q}{dt} + p_Q \frac{d}{dt}(R^3) \right] = u_Q, \]

\[ \frac{1}{R^3} \left[ \frac{d\varepsilon_m}{dt} + p_m \frac{d}{dt}(R^3) \right] = u_m, \]

\[ u_Q + u_m = 0, \quad (15) \]

\[ u_m \text{ and } u_Q \text{ are the rates of energy transfer per unit volume from the quintessence to matter and vice versa (}[u_m] = \text{erg/sec}\cdot\text{cm}^3). \]

The terms \( p_Q \frac{d}{dt}(R^3) \) and \( p_m \frac{d}{dt}(R^3) \) describe the time dependence of the energy due to space expansion.

### 2.3. The two-fluid model for matter and quintessence

In his pioneering work, Lemaître (1933) studied a model containing dust and the cosmological constant. The first application of the energy transfer for two-fluid cosmology was by Davidson (1962). We also make use of the results by Chernin (1965), Thomas & Schulz (2000), Coley & Tupper (1996a), Amendola & Tocchini–Valentini (2001).

We study two groups of substances: 1) a number of perfect fluids with positive pressure; 2) a number of fluids with positive energy and negative pressure. Each group plays the role of a one-fluid in two-fluid models. The equations of state for the first group are:

\[ p_m = \beta \varepsilon_m, \quad (16) \]

where \( p_m = p_{m1} + p_{m2} + ... \), \( \varepsilon_m = \varepsilon_{m1} + \varepsilon_{m2} + ... \), \( 0 \leq \beta_i \leq 1 \) and \( \beta = \frac{\varepsilon_m}{p_m} > 0 \). The second group has:

\[ p_Q = w \varepsilon_Q, \quad (17) \]

where \( p_Q = p_{Q1} + p_{Q2} + ... \), \( \varepsilon_Q = \varepsilon_{Q1} + \varepsilon_{Q2} + ... \), \( -1 \leq w_i < 0 \) and \( w = \frac{\varepsilon_Q}{p_Q} < 0 \).

We define an important function

\[ \alpha(a) = \frac{\varepsilon_Q(a)}{\varepsilon_m(a)}, \quad (18) \]

which characterizes the relative contributions to the energy density from quintessence and matter (see e.g. Wetterich 1995). The relative energy contribution was studied in another way by Coley & Wainwright (1992).

The two-fluid model is defined on two levels. Firstly, on a level of partial one-fluid models, with two kinds of interactions, 1) through the common gravitational field and 2) by the energy transfer from one-fluid to the other. Secondly, on the level of the associated one-fluid model.

For a complete description of a two-fluid model one has to study both levels simultaneously. We assume state equations of unique form for partial models and for the associated one-fluid model. The essential difference is that the equation of state of the associated model is not produced by physical particles and their interactions.

#### 2.3.1. The associated one-fluid model

We specify the partial energies, pressures and the corresponding equations of state:

\[ \varepsilon_Q = \rho_Q c^2, \quad \varepsilon_m = \rho_m c^2, \quad \varepsilon = \varepsilon_Q + \varepsilon_m, \]

\[ p_Q = w \varepsilon_Q, \quad p_m = \beta \varepsilon_m, \quad p = p_Q + p_m, \]

\[ p = \gamma \varepsilon. \quad (19) \]

Thus the equation of state for the associated one-fluid is:

\[ \gamma = \frac{p}{\varepsilon} = \frac{w \alpha + \beta}{\alpha + 1}. \quad (20) \]

For the energy ratio \( \alpha \geq 0 \) one finds that \( w \leq \gamma \leq \beta \). The associated one-fluid description of the two-fluid model includes Eqs.(18) - (19) and Eq.(20). The three functions of the scale factor \( a(w(a), \beta(a) \text{ and } \alpha(a) \text{ are assumed given.}

#### 2.3.2. The two-fluid model

The energy-momentum tensor of the associated one-fluid model is the sum of the partial tensors of matter and quintessence. According to Eq.(2) we have

\[ T_{(m)k;i} = T_{(Q)k;i} = 0; \quad (21) \]

Eq.(8) implies:

\[ 3 \frac{\dot{S}}{S} = - \frac{\dot{\varepsilon}_{tot}}{\varepsilon_{tot} + p_{tot}}, \]

\[ \varepsilon_{tot} = \varepsilon_Q + \varepsilon_m, \quad p_{tot} = p_Q + p_m. \quad (22) \]

There are two subcases within the two-fluid model description. In the first the energy-momentum tensors of the partial fluids are separately conserved:

\[ T_{(m)k;i} = 0 \quad \text{and} \quad T_{(Q)k;i} = 0, \quad (23) \]

so that \( u_Q = 0 \) and \( u_m = 0 \). In the second case we allow the presence of the energy transfer, when \( u_Q \neq 0 \) and \( u_m \neq 0 \) but \( u_Q + u_m = 0 \) which is equivalent to Eqs.(22).

Each fluid has an equation of state in the form of Eq.(8) with different coefficients: \( \beta(a) \) for the matter component and \( w(a) \) for quintessence (see Eqs.(19)).

For two-fluid FLRW models the characteristic energy content within the sphere of a fixed comoving radius \( \chi \) is

\[ \epsilon = 4 \pi \varepsilon S^3 \sigma_k(\chi) = 4 \pi (\alpha + 1) \varepsilon_m S^3 \sigma_k(\chi) \quad (24) \]

The gravitating mass \( M_g \) is the sum of the partial gravitating masses of two-fluids:

\[ M_g = M_g^m + M_g^Q = \]

\[ \frac{1}{c^2} \left[(1 + 3w)\varepsilon_Q + (1 + 3\beta)\varepsilon_m \right] \int_0^r dV = \frac{\epsilon}{c^2} (3 \gamma + 1). \quad (25) \]
2.4. Dimensionless equations

Now we restate the models in terms of dimensionless quantities. We introduce the following characteristic values:

\[ t_0 = \frac{1}{H_0}, \quad l_0 = c t_0 = R_{H_0}, \quad \rho_0 = \frac{3 H_0^2}{8 \pi G}, \]

\[ M_0 = \frac{4 \pi}{3} \rho_0 R_{H_0}^3 = \frac{c^2}{2 H_0 G}, \quad \varepsilon_0 = \rho_0 c^2, \]

and dimensionless variables:

\[ \alpha(t) = \frac{S(t)}{l_0}, \quad \tau = \frac{t}{t_0}, \quad \delta = \frac{\rho}{\rho_0}, \]

\[ \mu(a) = \frac{M_0(t)}{M_0} = 3 \sigma_k(\chi)(3 \gamma + 1)a^3 \mathcal{E}, \]

\[ \mathcal{E} = \frac{\varepsilon}{\varepsilon_0}, \quad \mathcal{E}_Q = \frac{\varepsilon_Q}{\varepsilon_0}, \quad \mathcal{E}_m = \frac{\varepsilon_m}{\varepsilon_0}, \quad \mathcal{E} = \mathcal{E}_Q + \mathcal{E}_m, \]

\[ \frac{\varepsilon}{\varepsilon_0}, \quad \frac{\varepsilon_Q}{\varepsilon_0}, \quad \frac{\varepsilon_m}{\varepsilon_0}, \quad \mathcal{E} = \mathcal{E}_Q + \mathcal{E}_m, \]

\[ P = \frac{p}{\varepsilon_0}, \quad P_Q = \frac{p_Q}{\varepsilon_0}, \quad P_m = \frac{p_m}{\varepsilon_0}, \quad P = P_Q + P_m, \]

\[ E = \frac{\varepsilon}{\varepsilon_0}, \quad E_Q = \frac{\varepsilon_Q}{\varepsilon_0}, \quad E_m = \frac{\varepsilon_m}{\varepsilon_0}, \quad E = E_Q + E_m, \]

\[ U = \frac{u}{\varepsilon_0} \dot{t}_0, \quad U_Q = \frac{u_Q}{\varepsilon_0} \dot{t}_0, \quad U_m = \frac{u_m}{\varepsilon_0} \dot{t}_0, \quad U = U_Q + U_m. \]

Eqs. (3) - (4) and (13) become

\[ \frac{\dot{a}^2}{a^2} + k \frac{a^2}{a^2} = \mathcal{E}, \quad (27) \]

\[ 2 \ddot{a} + \frac{\dot{a}^2}{a^2} + k \frac{a^2}{a^2} = -3 P, \quad (28) \]

\[ 3 \ddot{a} = -\dot{\mathcal{E}} \frac{\dot{\mathcal{E}}}{\dot{\mathcal{E}} + P}, \quad (29) \]

\[ P = \gamma \mathcal{E}, \quad (30) \]

\[ U_Q = \frac{1}{a^3} \left[ \frac{d}{d \tau} \left( \mathcal{E}_Q a^3 \right) + P_Q \frac{d}{d \tau} a^3 \right], \quad (31) \]

\[ U_m = \frac{1}{a^3} \left[ \frac{d}{d \tau} \left( \mathcal{E}_m a^3 \right) + P_m \frac{d}{d \tau} a^3 \right]. \quad (32) \]

Eq. (13) for the scale factor now has the form:

\[ a^2 \ddot{a} = -\Omega_0 \mu(a), \quad (33) \]

where \( \Omega_0 \) is the cosmological density parameter at the moment of the initial conditions for Eq. (33).

Table 1. Classification of two-fluid FLRW models.

| both \( w, \beta \) are constant | NET-SES | ET-SES |
|---------------------------------|---------|--------|
| at least one of \( w, \beta \) is not constant | NET-NSES | ET-NSES |

3. General properties of two-fluid FLRW models with matter and quintessence

3.1. The classification

A two-fluid model with the vacuum or quintessence together with some mixture of matter is very different from the two-fluid model with “ordinary” matter: negative pressure and gravitating mass components give rise to a new behaviour of the total pressure and total gravitating mass. In order to facilitate their study, we divide all two-fluid models into four classes according to two independent properties: 1) both fluids have a stationary (SES) or at least one has a non stationary (NSES) equation of state and 2) the presence (ET, energy transfer) or absence (NET, no energy transfer) of an energy transfer between the components (see also Table 1).

Furthermore, we separate three different kinds of two-fluid models depending on the behaviour of the function \( \alpha(a) = \mathcal{E}_Q / \mathcal{E}_m \): the coherent model, when

\[ \mathcal{E}_Q = \mathcal{E}_m; \quad (34) \]

the asymptotically coherent model, when

\[ \lim_{a \to \infty} \alpha'(a) = 0, \quad (35) \]

and the non coherent model, when neither Eq.(34) nor Eq.(35) are valid.

3.2. The general solution of two-fluid FLRW models: ET-NSES models

The ET-NSES class is the most general class of two-fluid models in our classification. We assume that all fluids (the associated and the two particular ones) have nonstationary equations of state.

In this subsection we completely describe the two-fluid problem in terms of the associated one-fluid model and two particular fluid models simultaneously. This leads us to rewrite the FLRW equations in terms of the coefficients of the three equations of state: \( \gamma, \beta \) and \( w \).
3.2.1. The input equations and their solution

We describe the associated one-fluid model by Eqs.(26)-(30):

$$-\frac{\dot{\alpha}}{\alpha} = \frac{\mathcal{E}}{\mathcal{E} + P}, \quad P = \gamma \mathcal{E}, \quad \gamma = \gamma(a). \quad (36)$$

The solution of Eqs.(34), with the initial conditions stated for the present epoch, is:

$$\mathcal{E} = \mathcal{E}(1) \exp \left( -3 \int_1^\alpha \frac{\gamma + 1}{x} \, dx \right). \quad (37)$$

In this model the characteristic energy content $E$, pressure $P$ and gravitating mass $\mu$ are:

$$E = \mathcal{E} a^3, \quad P = \mathcal{E} \gamma, \quad \mu = 3 \sigma_0(\chi) \mathcal{E} a^3 (3 \gamma + 1). \quad (38)$$

The scale factor satisfies Eq.(33). $\gamma(a)$ is given by Eq.(23) in case of known $w(a)$ and $\beta(a)$.

The two particular fluids are described by Eqs.(31) - (32) and the equations of state:

$$P_Q = w(a) \mathcal{E} Q, \quad P_m = \beta(a) \mathcal{E} m \quad (39)$$

Calculating $a^3 \left( U_m - \frac{U_m}{\alpha^2} \right)$ and keeping in mind that $U_Q + U_m = 0$, we find the equation

$$\frac{U_m \alpha + 1}{E_m} \alpha^3 = 3 \frac{\dot{\alpha}}{\alpha} (\beta - w) - \frac{\ddot{\alpha}}{\alpha}. \quad (40)$$

Eq.(40) leads to a general property of the model: the model can be coherent ($\alpha = \text{constant}$) only if there is energy transfer. Eq.(40) defines the critical energy transfer $U_m^{\text{crit}}$, which keeps $\alpha$ constant (the coherent solution):

$$U_m^{\text{crit}} = 3 \frac{\dot{\alpha}}{\alpha} E (\beta - w) \frac{\alpha}{(\alpha + 1)^2} = U_m^{\text{crit}}(w, \beta, \gamma, a). \quad (41)$$

$U_m^{\text{crit}}$ separates the behaviour of the two-fluid model with different signs of $\dot{\alpha}(a)$: it follows from Eq.(41) that $\dot{\alpha}(a) > 0$ requires $U_m < U_m^{\text{crit}}$, $\dot{\alpha}(a) < 0$ requires $U_m > U_m^{\text{crit}}$. In other words, the rate $\frac{\dot{E}}{\dot{m}}$ increases, if $U_m < U_m^{\text{crit}}$ and decreases if $U_m > U_m^{\text{crit}}$.

In a number of works the energy transfer has been calculated for a given model of interaction between dust and radiation, i.e. for given $w(a)$ and $\beta(a)$ (see, for instance, McIntosh 1967 and 1968, Sistero 1971, Coley & Tupper 1985 and 1986). Now we know that energy transfer is necessarily required for the coherence.

At the same time Eq.(40) gives the condition for no energy transfer, $U_Q = U_m = 0$, in the form:

$$\alpha = \alpha(1) \exp \left( 3 \int_1^\alpha \frac{\beta - w}{x} \, dx \right). \quad (42)$$

Any other function $\alpha(a)$ leads to non zero energy transfer.

Another form of Eq.(40) is obtained by substituting $\frac{\dot{\alpha}}{\alpha}$ from Eq.(27). This shows how the energy transfer depends on the curvature ($k \neq 0$):

$$\frac{U_m (\alpha + 1)^2}{\alpha} = \pm 3 \sqrt{\mathcal{E} - \frac{k}{a^2} (\beta - w) - \frac{\ddot{\alpha}}{\alpha}}. \quad (43)$$

Using the definition of $\alpha$, Eq.(18), and its time derivate, $\dot{\alpha} (\gamma - w)^2 = (\gamma - w) \beta + (\beta - \gamma) \dot{w} - (\beta - w) \dot{\gamma}$, Eq.(40) leads to the expression for the energy transfer:

$$U_m (\gamma, a) = \left[ \pm 3 \frac{(\beta - \gamma)(\gamma - w)}{\beta - w} \right] \sqrt{\mathcal{E}(\gamma, a) - \frac{k}{a^2}}. \quad (44)$$

where $' = \frac{d}{da}$ and $\mathcal{E}(\gamma, a)$ is defined by Eq.(37). The sign $'+$ corresponds to expansion while the sign $'-'$ corresponds to collapse. So, we found that the energy transfer $U_m$ depends on $\gamma$, $\beta$, $w$, their derivatives and, also, on the scale factor $a$ (as on an independent variable) and curvature. For a more realistic situation with given $k$, $\beta(a)$ and $w(a)$, Eq.(44) demonstrates that $\gamma(a)$ is completely defined by $U_m(a)$.

Eqs.(40), (43) and (44) represent in different forms how a conversion of one-fluid into another (i.e. $U_m \neq 0$) influences the properties of particular fields (its equations of state, and its energy transfer rates) and the expansion of space (i.e. $a(t)$). Eq.(44) states the dependence of the energy transfer on $\gamma$ and scale factor as an independent variable. Excluding $\gamma$ from Eqs.(44), (37) and (33) we find that energy transfer completely defines the gravitating mass as a function of the scale factor and, so, the dynamics of the universe (see Eq.(33)). Therefore, the total gravitating mass and, consequently, the dynamics of the expansion, is determined by the energy transfer.

For the popular flat model Eq.(10) becomes:

$$U_m (\alpha + 1)^2 \alpha = 3 \mathcal{E}^{3/2} (\beta - w) - \mathcal{E} \frac{\ddot{\alpha}}{\alpha}. \quad (45)$$

The general (ET-NSES) solution for particular fluids is represented through the function $\alpha(a)$ and two effective coefficients of the equation of state $\beta(a)$ and $w(a)$:

$$E_Q = \frac{\mathcal{E} \alpha}{\alpha + 1}, \quad E_m = \frac{\mathcal{E} \alpha + 1}{\alpha},$$

$$P_Q = w \mathcal{E} Q, \quad P_m = \beta \mathcal{E} m, \quad (46)$$

$$\mu_Q = 3 \sigma_3(\gamma) (3 \beta + 1) a^3 \mathcal{E} Q, \quad \mu_m = 3 \sigma_3(\gamma) (3 w + 1) a^3 \mathcal{E} w.$$
Depending on the initial conditions and the class of a model, there is one (or, possibly, more) moments of time when the one-fluid model is effectively dust, i.e. \( \gamma(a_{ed}) = 0 \); \( a_{ed} \) is the “effective dust scale”. The general expression for this scale is:

\[
\alpha(a_{ed}) = \frac{-\beta}{w}.
\]  

(47)

We now show the complicated character of the evolution of the two-fluid model from initial conditions, defined at \( a_{min} \), to an asymptotical state with \( a \to \infty \). To do this we study the relative contributions of energy, pressure and gravitating mass of each component to the total energy, total pressure and total gravitating mass. Eqs. (46) give the criteria for the relative contributions in the form:

\[
\alpha = \frac{E_Q}{E_m} = \frac{E_Q}{E_m}, \\
\mathcal{P} = \frac{|P_Q|}{P_m} = \frac{\alpha}{\alpha_P}, \\
\mathcal{M} = \frac{\mu_Q}{\mu_m} = \frac{\alpha}{\alpha_M},
\]  

(48)

where

\[
\alpha_P = \alpha_P(a) = \frac{\beta(a)}{|w(a)|},
\]

\[
\alpha_M = \alpha_M(a) = \frac{3\beta(a) + 1}{3|w(a)| + 1}.
\]  

(49)

Three characteristic scales \( a_\xi, a_P \) and \( a_M \) appear so that

\[
a = a_\xi \quad \text{implies} \quad \alpha = 1,
\]

\[
a = a_P \quad \text{implies} \quad \mathcal{P} = 1, \quad a_P = a_{ed},
\]

\[
a = a_M \quad \text{implies} \quad \mathcal{M} = 1.
\]  

(50)

These are the solutions of the equations:

\[
\frac{|w(a_P)|}{\beta(a_P)} = 1, \\
\frac{|3w(a_M)|}{3\beta(a_M) + 1} = 1.
\]  

(51)

\( \beta(a), \ w(a) \) and \( \alpha(a) \) are model-dependent, so there is no possibility to study the general case.

Eq. (46) shows that the energy transfer is required for the coherence (\( \alpha = \text{const.} \)). Now we give two other examples of how to use Eq. (46). Substituting \( \alpha = \alpha_P \mathcal{P} \) and \( \alpha = \alpha_M \mathcal{M} \) into Eq. (46) we find for \( \mathcal{P} = \text{const} \) and \( \mathcal{M} = \text{const} \) two critical values for the energy transfer:

\[
U_m^P = \left[ 3 \frac{\dot{a}}{a} (\beta - w) - \frac{d \ln \alpha_P}{d\tau} \right] \mathcal{P} \alpha_P (\mathcal{P} \alpha_P + 1)^2.
\]  

(52)

and

\[
U_m^M = \left[ 3 \frac{\dot{a}}{a} (\beta - w) - \frac{d \ln \alpha_M}{d\tau} \right] \mathcal{M} \alpha_M (\mathcal{M} \alpha_M + 1)^2.
\]  

(53)

\( U_m^P \) gives \( P_Q \sim P_m \) and \( U_m^M \) gives \( \mu_Q \sim \mu_m \).

### Table 2. Detailed description of the EPM evolution type

| \( a_{min} < a < a_\xi \) | \( E_M > E_Q \) |
| \( a_\xi < a < a_P \) | \( P_M > |P_Q| \) |
| \( a_P < a < a_M \) | \( P_M < |P_Q| \) |
| \( a_M < a < \infty \) | \( E_M < E_Q \) |

3.2.2. The evolution types

In this paper we study only the models which are fully dominated by the matter-component in the very early epoch (i.e. \( \alpha < 1, \mathcal{P} < 1 \) and \( \mathcal{E} < 1 \) ), while fully dominated by quintessence at the limit of large time (i.e. \( \alpha > 1, \mathcal{P} > 1 \) and \( \mathcal{E} > 1 \) ). Under this assumption the characteristic scales \( a_\xi, a_P \) and \( a_M \) allow 6 types of nonequalities:

\[
a_\xi < a_P < a_M, \quad \text{EPM}
\]

\[
a_\xi < a_M < a_P, \quad \text{EMP}
\]

\[
a_P < a_M < a_\xi, \quad \text{PME}
\]

\[
a_P < a_\xi < a_M, \quad \text{PEM}
\]

\[
a_M < a_\xi < a_P, \quad \text{MEP}
\]

\[
a_M < a_P < a_\xi, \quad \text{MPE.}
\]  

(54)

Each system of nonequalities corresponds to an “evolution type”, defined by a unique sequence of the scales \( a_\xi, a_P, \) and \( a_M \), which represents the sequence of time periods when the quintessence-component dominates in total energy, pressure and gravitating mass, respectively. The ratio between two energies \( E_Q \) (actually \( E_\Lambda \)) and \( E_m \) was first considered by Chernin et al. \( (2000) \) to make a division line between the growth and suppression of structure formation. In Table 2 the EPM evolution type is described in detail. Eq. (46) leads to a dependence of \( \alpha \) on \( U_m \) for given \( w(a) \) and \( \beta(a) \). Eqs. (49) - (52) lead to a dependence of the evolution type on \( \alpha \) for given \( w(a) \) and \( \beta(a) \). So, we resume that for given \( w(a) \) and \( \beta(a) \) the evolution type is connected with the energy transfer. On the other hand, energy transfer defines the gravitating mass. So, there exists a correspondence between the dynamics of a model, its evolution type and energy transfer.

Below we study how quintessence influences the behavior of the total gravitating mass, pressure, and the effective equation of state of the associated one-fluid model and we calculate the evolution types for a few examples.
4. Classifying two-fluid FLRW models: examples

4.1. An example of NET-SES models

This class of two-fluid models has been much studied because the input equations are simple. In this subsection we study the evolution types for \( w = -1, -2/3, -1/3 \) and for \( \beta = 0, 1/3, 2/3, 1 \).

4.1.1. Input equations and their solutions

The input equations are given by Eq.(29) for every fluid:
\[
\frac{3}{a} \frac{d}{dt} = -\frac{\dot{E}_Q}{E_Q + P_Q}, \quad P_Q = w \ E_Q, \quad w = \text{const.},
\]
\[
\frac{3}{a} \frac{d}{dt} = -\frac{\dot{E}_m}{E_m + P_m}, \quad P_m = \beta \ E_m, \quad \beta = \text{const.},
\]
\[ U_Q = 0, \quad U_m = 0. \tag{55} \]

Eq.(60) is written in the form:
\[
\frac{3}{a} (\beta - w) - \frac{\dot{\alpha}}{\alpha} = 0 \tag{56}.
\]

\[ \beta = 0) \text{ has a solution} \]
\[ a = \alpha(1) a^3(\beta - w). \tag{57} \]

The solution of Eqs.(52) is well known. Energy density \( \dot{E}(a) \), characteristic energy content \( E(a) \), pressure \( P(a) \) and gravitation mass \( M(a) \) have the form:
\[
\dot{E}_Q(a) = \frac{E_Q(1)}{a^3(\beta + 1)}, \quad E_m(a) = \frac{E_m(1)}{a^3(\beta + 1)},
\]
\[
E_Q(a) = \frac{E_Q(1)}{a^3 w}, \quad E_m(a) = \frac{E_m(1)}{a^3 \beta},
\]
\[ P_Q(a) = w \frac{E_Q(1)}{a^3(\beta + 1)}, \quad P_m(a) = \beta \frac{E_m(1)}{a^3(\beta + 1)}, \tag{58} \]
\[
\mu_Q(a) = 3 \sigma_k(\chi) E_Q(1) \frac{3 w + 1}{a^3 w},
\]
\[
\mu_m(a) = 3 \sigma_k(\chi) E_m(1) \frac{3 \beta + 1}{a^3 \beta}. \]

We comment on two models in this class. First, \( (w = -1/3, \beta = 0) \) has a constant total gravitating mass and total scale-dependent energy content. Second, \( (w = -1, \beta = 0) \) produces a total constant pressure and non constant energy content and gravitating mass (see Table 3).

4.1.2. The condition of coherence

Assuming coherence (see Eq.(54)) Eq.(57) leads to
\[
\frac{\dot{a}}{a} (\beta - w) = 0, \tag{59}
\]

which for the non-static solution reduces to the simplest case of one-fluid dust model:
\[
\beta = w = 0. \tag{60}
\]

Assuming asymptotic coherence (Eq.(53)) Eq.(57) gives
\[
\gamma = 3 \alpha \frac{\beta - w}{a} = 3 \alpha(1) (\beta - w) a^3(\beta - w)^{-1}, \tag{61}
\]
so, \( \lim_{a \to \infty} \alpha = 0 \) only for \( \beta - w < \frac{1}{3} \).

4.1.3. Asymptotic behaviour and evolution type

The effective coefficient \( \gamma(a) \) of the associated one-fluid depends on time. Substituting (57) into (29) one finds \( \gamma(a) \):
\[
\gamma(a) = \beta - \frac{\beta - w}{1 + \frac{1}{\alpha(\beta)}} \tag{62},
\]

which has the following asymptotical properties:
\[
\lim_{a \to 0} \gamma(a) = \beta, \quad \lim_{a \to \infty} \gamma(a) = w, \tag{63}
\]

so, any matter-quintessence NET-SES model is effectively a matter model at an earlier time and quintessence at the limit of large time.

There is a unique moment of time when the total pressure is equal to zero and the associated one-fluid model is dust-like. The corresponding effective dust scale \( a_{ed} \) is:
\[
a_{ed} = \left[ \frac{-w B}{\beta A} \right]^{-\frac{1}{1 - \alpha(\beta - w)}}, \tag{64}
\]

To find the evolution types of NET-SES models, we use the function \( \alpha(a) \) to calculate two functions \( P \) and \( M \):
\[
\alpha(a) = \frac{\dot{E}_Q(1)}{E_m(1)} a^3(\beta - w),
\]
\[
P(a) = \frac{|w|}{\beta} \alpha(a),
\]
\[
M(a) = \frac{|3 w + 1|}{3 \beta + 1} \alpha(a). \tag{65}
\]

The characteristic scales (see Eq.(50)) may be calculated analytically:
\[
a_E = \left( \frac{E_m(1)}{E_Q(1)} \right)^{\frac{1}{1 - \alpha(\beta - w)}}, \tag{66}
\]
\[
a_P = \left( \frac{\beta E_m(1)}{|w| E_Q(1)} \right)^{\frac{1}{1 - \alpha(\beta - w)}}, \tag{66}
\]
\[
a_M = \left( \frac{3 \beta + 1}{|3 w + 1| E_Q(1)} \right)^{\frac{1}{1 - \alpha(\beta - w)}}. \tag{66}
\]

The numbers in Table 4 illustrate the complicated character of the evolution from the matter-dominated model at the earlier time to the quintessence-dominated stage for large time. Different physical quantities pass through 'boundaries' \( (\alpha = 1; P = 1; M = 1) \) between the matter-dominated and quintessence-dominated stages at different times. It is also seen that \( a_{ed} \) cannot completely characterize the evolution type and dynamics of the universe.
Table 3. Two examples of the NET-SES models: \( w = -1 \) and \( \beta = 0 \) provide a constant pressure; \( w = -1/3 \) and \( \beta = 0 \) provide a constant pressure.

| \( w \) | \( \beta \) | \( E \) | \( P \) | \( \mu \) |
|-------|------|------|------|------|
| -1    | 0    | \( A a^3 + B \) | \( -A \) | \( -2 A a^3 \) |
| -1/3  | 0    | \( A a + B \) | \( -A/(3a^2) \) | \( B \) |

Table 4. Evolution types for NET-SES models with \( w = -1, -2/3, -1/3 \) and for \( \beta = 0, 1/3, 2/3, 1 \). These numbers are obtained from Eqs. (30) and (31) with \( E_0(1) = 0.7 \) and \( E_m(1) = 0.3 \). We denote by \( (\text{ME}) \) the case of \( \alpha_M = \alpha_E \) and so on.

| \( w \) | \( \beta = 0 \) | \( \beta = 1/3 \) | \( \beta = 2/3 \) | \( \beta = 1 \) |
|-------|---------------|---------------|---------------|---------------|
|       | \( a_E = 0.754 \) | \( a_E = 0.809 \) | \( a_E = 0.844 \) | \( a_E = 0.868 \) |
|       | \( a_P = 0.00 \) | \( a_E = 0.754 \) | \( a_P = 0.783 \) | \( a_P = 0.888 \) |
|       | \( \alpha_M = 0.598 \) | \( \alpha_M = 0.949 \) | \( \alpha_M = 1.07 \) | \( \alpha_M = 1.11 \) |
|       | \( \text{P(ME)} \) | \( \text{P(ME)} \) | \( \text{PEM} \) | \( \text{PEM} \) |
|       | \( a_{ed} = \infty \) | \( a_{ed} = 1.07 \) | \( a_{ed} = 0.809 \) | \( a_{ed} = 0.868 \) |

4.2. A NET-NSES example

Here we generalize the known solution for mixed radiation and dust, first obtained by Chernin (1965). We use it as the matter component of a two-fluid. For quintessence we assume \( w = \text{const.} \). A new property, produced by the complicated matter-component is that the effective equation of state is non-stationary. So, this model is NET-NSES.

4.2.1. The input equations and their solutions

The input equations are given by Eqs. (29) for each fluid:

\[
3 \frac{\dot{a}}{a} = -\frac{\dot{E}_Q}{E_Q + P_Q}, \quad P_Q = w E_Q, \quad w = \text{const.},
\]

\[
3 \frac{\dot{a}}{a} = -\frac{\dot{E}_R}{E_R + P_R}, \quad P_R = \frac{E_R}{3},
\]

\[
3 \frac{\dot{a}}{a} = -\frac{\dot{E}_D}{E_D}, \quad P_D = 0,
\]

\[
E_m = E_R + E_D, \quad P_m = P_D + P_R = P_R,
\]

\[
U_Q = 0, \quad U_R = 0, \quad U_R = 0.
\]

The solutions of Eqs. (68) are well-known. The energy density \( E(a) \), characteristic energy content \( E(a) \), pressure \( P(a) \) and gravitating mass \( \mu(a) \) have for each component the form:

\[
E_Q = \frac{E_Q(1)}{a^3(w+1)}, \quad E_D = \frac{E_D(1)}{a^3(w+1)}, \quad E_R = \frac{E_R(1)}{a^4},
\]

\[
E_Q = \frac{E_Q(1)}{a^3}, \quad E_D = \frac{E_D(1)}{a^3}, \quad E_R = \frac{E_R(1)}{a},
\]

\[
P_Q = \frac{E_Q(1)}{a^3(w+1)}, \quad P_D = 0, \quad P_R = \frac{E_R(1)}{a^3},
\]

\[
\mu_Q = \frac{E_Q(1)}{3w+1}, \quad \mu_D = \frac{E_D(1)}{a}, \quad \mu_R = 2 \frac{E_R(1)}{a},
\]

For \( \alpha \) and \( \beta \) we find:

\[
\alpha(a) = \frac{E_Q(1)}{E_D(1)} a^{1-3w},
\]

\[
\beta(a) = \frac{P_m}{E_m} - \frac{E_R(1)}{E_D(1) a + E_R(1)},
\]

Eq. (60) has the form:

\[
3 \frac{\dot{a}}{a} (\beta(a) - w) - \frac{\dot{\alpha}}{\dot{\alpha}} = 0.
\]

4.2.2. The conditions of coherence

From Eqs. (55), (60) and (72) we find

\[
\alpha' = \frac{3 C}{a} \frac{a^{1-3w}}{Aa + B},
\]

\[
\lim_{a \to \infty} \alpha' = -3 w \frac{C}{A} a^{-(3w+1)} = 0, \quad \text{for} \quad w > -\frac{1}{3},
\]

\[
= -3 w \frac{C}{A}, \quad \text{for} \quad w = -\frac{1}{3},
\]

\[
= \infty, \quad \text{for} \quad w < -\frac{1}{3}.
\]
Table 5. These numbers are obtained from Eqs. (50) and (78) with $E_Q(1) = 0.7$ and $E_m(1) = 0.3$; we choose here $E_D = 0.05, E_R = 0.25$.

| $w$    | $a_E$ | $a_P$ | $a_M$ | PME  | PEM  |
|--------|-------|-------|-------|------|------|
| $-1$   | 0.803 | 0.743 | 0.634 | PEM  | PEM  |
| $-2/3$ | 0.587 | 0.563 | 0.921 | PEM  | PEM  |
| $-1/3$ | 0.587 | 0.563 | 0.598 | PEM  | PEM  |

4.2.3. The asymptotic behaviour and evolution types of a dust-radiation-quintessence model

The effective coefficient $\gamma(a)$ of the associated one-fluid depends on time. Substituting (11) into (14), one finds:

$$
\gamma(a) = \frac{E_Q(1) w + \frac{E_0(1)}{3} a^3 w^{-1}}{E_Q(1) + E_D(1) a^3 w^{-1} + E_R(1) a^3 w^{-1}}.
$$

(75)

Because $\beta - w > 0$, the associated one-fluid model has the following asymptotical properties:

$$
\lim_{a \to 0} \gamma(a) = \frac{1}{3}, \quad \lim_{a \to \infty} \gamma(a) = w,
$$

(76)

So the one-fluid model, associated with the NET-NSES two-fluid under consideration is effectively radiative at early times and quintessence-like at the limit of large time.

The model is effectively dust on the (unique) scale

$$
a_{sd} = \left[ -3 w E_Q(1) \right]^{\frac{3}{3 w - 1}}.
$$

(77)

The evolution types are characterized by the three functions $\alpha, \beta$ and $\mathcal{M}$:

$$
\mathcal{P} = \frac{3}{E_R(1)} (E_D(1) a + E_R(1)) \alpha(a),
$$

$$
\mathcal{M} = \frac{E_Q(1) (3 w + 1)}{E_D(1) a + 2 E_R(1)} a^{3 - 3 w} = (3 w + 1) \alpha(a).
$$

(78)

In Table 3 three characteristic scales represent two evolution types for this model: PME and PEM.

We note that the two-fluid description coincides with the one-fluid model only asymptotically, at $a \to \infty$.

4.2.4. The time dependence of the gravitating mass

For $-1/3 < w < 0$ the total gravitating mass has a unique minimum at a scale $a^*$

$$
a^* = \left( \frac{3 E_Q(1)}{2 E_R(1)} (-w) (3 w + 1) \right)^{\frac{1}{3 w - 1}}.
$$

(79)

4.3. An ET-SES example

Unlike the NET-SES models, ET-SES models allow coherence. We inspect here an ET-SES example, with the two-fluids described by constant $\beta$ and $w$.

An important class of dark energy (DE) models is the so-called coupled quintessence (see e.g. Wetterich [1995], Amendola [2000]), where the total (matter + DE) energy momentum tensor is conserved, whereas in ordinary quintessence the matter and DE are separately conserved.

For coupled quintessence, a stationary DE model was recently found (Amendola & Tonini-Valentini [2001]) which predicts a coherent behaviour of matter and DE densities at late cosmic epochs. Such a relation was studied by Baryshev et al. [2000] in order to solve the problem of the low velocity dispersion in the local Hubble flow.

The coherence ($\alpha = \text{constant}$) implies $\gamma = \text{constant}$ and the model is greatly simplified. All these properties are produced by the energy transfer.

4.3.1. Input equations and their solution

The associated one-fluid model is described by

$$
3 \frac{\dot{a}}{a} = -\frac{\dot{\mathcal{E}}}{\mathcal{E} + \mathcal{P}}, \quad P = \gamma \mathcal{E}, \quad \gamma = \text{const}.
$$

(80)

which have the solution:

$$
\mathcal{E} = \frac{\mathcal{E}(1)}{a^{\frac{3}{\gamma + 1}}}, \quad E = \frac{\mathcal{E}(1)}{a^{3 \gamma}},
$$

$$
P = \frac{\mathcal{E}(1)}{a^{\frac{3}{\gamma + 1}}} (3 \gamma + 1).
$$

(81)

For the quintessence component we find:

$$
E_Q = \frac{\mathcal{E}(1)}{a^{3 (\gamma + 1)}} \frac{\alpha}{\alpha + 1}, \quad \mathcal{E} = \frac{\mathcal{E}(1)}{a^{3 \gamma}}, \quad \mu_Q = 3 \sigma_k(\chi) \mathcal{E}(1) \frac{\alpha}{\alpha + 1} \frac{3 w + 1}{a^{3 \gamma}}
$$

and for the matter-component:

$$
E_m = \frac{\mathcal{E}(1)}{a^{3 (\gamma + 1)}} \frac{1}{\alpha + 1}, \quad E_m = \frac{\mathcal{E}(1)}{a^{3 \gamma}}, \quad \mu_m = 3 \sigma_k(\chi) \mathcal{E}(1) \frac{1}{\alpha + 1} \frac{3 \beta + 1}{a^{3 \gamma}}.
$$

(82)

(83)

From Eq.(60) we find the energy transfer

$$
U_m = 3 \frac{\alpha}{(\alpha + 1)^2} \frac{\mathcal{E}(1)}{a^{3 (\gamma + 1)}} \frac{\dot{a}}{a} (\beta - w).
$$

(84)
4.3.2. The condition of coherence

The coherence implicitly exists in this model.

4.3.3. The asymptotic behaviour and evolution types

The asymptotic behaviour strongly depends on the sign and value of γ, obtained at the end of the section (4.3.1).

Three functions, generally defining the evolution type,
\[ \alpha = \frac{\mathcal{E}_Q}{\mathcal{E}_m} = \text{constant}, \]
\[ \mathcal{P} = \frac{|\omega|}{\beta} \alpha = \text{constant}, \]
\[ M = \frac{3\omega + 1}{3\beta + 1} \alpha = \text{constant}. \]  

are constant in this model (a fixed ratio between the energy, pressure and gravitating masses of the quintessence and matter components). What component dominates during all the evolution, depends on the value of α.

Let us study now the case of the Amendola & Tocchini-Valentini (2001) solution, which is reached at late times: \( w = -0.7, \beta = 0, \alpha = 7/3 \). The equation of state of the associated one-fluid model is defined by
\[ \gamma = \frac{w\alpha + \beta}{\alpha + 1} = -\frac{49}{100} \approx -\frac{1}{2}, \]  

which gives
\[ \mathcal{E} = \frac{\mathcal{E}(1)}{a^{153/100}} \approx \frac{\mathcal{E}(1)}{a^{3/2}} \]
\[ E = \mathcal{E}(1) a^{147/100} \approx \mathcal{E}(1) a^{3/2} \]
\[ \mathcal{P} = -\frac{49}{100} \frac{\mathcal{E}(1)}{a^{153/100}} \approx -\frac{1}{2} \frac{\mathcal{E}(1)}{a^{3/2}} \]
\[ \mu = -\frac{47}{100} \frac{\mathcal{E}(1)}{a^{147/100}} \approx -\frac{1}{2} \mathcal{E}(1) a^{3/2}. \]  

The energy transfer for the flat (\( k = 0 \)) model is:
\[ U_m = \frac{3}{(\alpha + 1)^2} \frac{\mathcal{E}^{3/2}(1)}{a^{9/2(\gamma + 1)}} \left( \beta - w \right) \]  

This gives for the Amendola & Tocchini-Valentini solution
\[ U_m = -\frac{441}{1000} \frac{\mathcal{E}^{3/2}(1)}{a^{459/200}} \approx \frac{2}{5} \frac{\mathcal{E}^{3/2}(1)}{a^{2.3}}. \]

Thus this model is asymptotically NET-SES.

5. Conclusions

The cosmological view that the universe is described by a FLRW model with \( \Omega_m^0 \approx 0.3, \Omega_\Lambda^0 \approx 0.7, \) and \( w \leq -1/3 \) has initiated many studies of FLRW models with an essential \( \Lambda \) component at late epochs. Usually one has viewed \( \Lambda \) and matter as independent substances so that the energy-momentum tensors of the partial fluids are separately conserved (Eq. (15)). However, there are a number of suggestions in the literature on the particular forms of energy transfer between dark energy and matter. This has motivated us to consider on a phenomenological level the general case when one-fluid converts into another and the equation of state for both components is non-stationary (Section 3). The properties of the model we gave in terms of the equations of state for two partial models (\( \beta \) for a component with positive pressure and \( w \) for a negative pressure one) and a coefficient \( \gamma \) for the associated one-fluid. The energy transfer, when a one-fluid converts into another, was represented via these coefficients and the scale factor (Eqs.(40) - (43)).

We have analyzed four classes of models defined by the presence or absence of the energy transfer and by the stationarity (\( w = \text{const.} \), and \( \beta = \text{const.} \)) or/and non-stationarity (\( w \) or \( \beta \) time dependent) of the equations of state (see Tables 1, 4, 5). It was shown in sect.3 that

* for given \( w \) and \( \beta \), the energy transfer defines \( \gamma \) and, therefore, the total gravitating mass and the dynamics of the model.

* also, the model can be coherent only if there is energy transfer.

The classification was illustrated with interesting examples of two-fluid FLRW models in sections 4.1 – 4.3.

From the behaviour of the energy content, gravitating mass, and pressure as functions of the scale factor we have defined three characteristic scales, \( a_E \), \( a_P \) and \( a_M \). These separate time intervals when quintessence energy, pressure and gravitating mass were dominating (Eqs.(18) - (22)). Any sequence of the scales defines one of 6 evolution types of the model (Eqs.(54)). There is a correspondence between the dynamics of a model, its evolution type and energy transfer.

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