Accuracy of view factor calculations for digital terrain models of comets and asteroids

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ABSTRACT

Context. Detailed shape and topographic models coupled with sophisticated thermal physics are critical elements to proper characterization of surfaces of small bodies in our solar system. Calculations of self-heating effects are especially important in the context of thermal evolution of non-convex surfaces, including craters, cracks, or openings between “rocks”.

Aims. Our aim is to provide quantitative comparisons of multiple numerical methods for computing view factors for concave geometries and provide a more rigorous criteria for the validity of their application.

Methods. We contrasted five methods of estimating the view factors. First, we studied specific geometries, including shared-edge facets for a reduced two-facet problem. Then, we applied these methods to the shape model of 67P/Churyumov-Gerasimenko. Nevertheless, the presented results are general and could be extended to shape models of other bodies as well.

Results. The close loop transformation of the double area integration method for evaluating view factors of nearby or shared-edge facets is the most accurate, although computationally expensive. Two methods of facet subdivision we evaluate in this work provide reasonably accurate results for modest facet subdivision numbers, however, may result in a degraded performance for specific facet geometries. Increasing the number of subdivisions improves their accuracy, but also increases their computational burden. In practical applications, a trade-off between accuracy and computational speed has to be found, therefore, we propose a combined method based on a simple metric that incorporates a conditional application of various methods and an adaptive number of subdivisions. In our study case of a pit on 67P/CG, this method can reach average accuracy of 2-3% while being about an order of magnitude faster than the (most accurate) line integral method.

Key words. comets:general, methods: data analysis, methods: numerical

1. Introduction

The main energy input into a cometary nucleus is solar irradiation, which is then redistributed via energy sink processes such as sublimation of ices, conduction into the subsurface, or infrared surface emissions. Thermal re-emission from a surface element may be absorbed at another distinct surface element, becoming its secondary source of energy. This is the so-called self-heating effect, which is most pronounced for certain topographic environments such as pits, craters, cracks, and cliffs (Colwell et al. 1990; Lagerros 1997; Ivanova & Shulman 2006). In this work, we ignore the reflection of visible light as a secondary source of energy, as considered for example in Rozitis & Green (2011). The importance of the self-heating effect is well recognized and nowadays many thermostatic models incorporate these calculations in the context of high-resolution shape models of comets (e.g., 67P/Churyumov-Gerasimenko) and asteroids (e.g., Ryugu, Bennu, etc) (Gutiérrez, P. J. et al. 2001; Rozitis & Green 2011; Hu et al. 2017; Shi, X. et al. 2016; Pelivan 2018; Hamm et al. 2018). We should add that the complexity of calculations of the net radiation exchange among surface elements (facets) can be made less complex by assuming that each surface is diffusive and has homogeneous emissive power (Lambertian surface). This work is also done within this framework.

The mathematical formalism for including the self-heating effect requires calculations of the so-called view factor(s). A vast body of literature in the field of computer graphics and engineering simulations deals with the complexities of these calculations (Siegel & Howell 2002; Modest 2013). There is still ongoing research into improving these algorithms (Francisco et al. 2014; Kramer et al. 2015; Narayanawamy 2015). By definition, the view factor represents a fraction of radiation from a surface element, $i$, directly reaching a facet, $j$. Its differential form is be expressed as

$$dF_{ij} = \frac{\delta_{ij} dA_i \cos \phi_{ij} \cos \phi_{ji}}{\pi d_{ij}^2}. \quad (1)$$

In this form and under stated assumptions a view factor is purely a geometric quantity independent of surface properties and temperature. This expression relates cosines of angles $(\phi_{ij}, \phi_{ji})$ between the normal direction of each facet and the ray connecting the two differential areas $dA_i$ and $dA_j$, on facet $i$ and $j$, respectively (Fig. 1), and the square of the distance between the differential areas, $d_{ij}$. The binary flag, $\delta_{ij}$, is a convenient parameter indicating whether there is a direct visibility between
facets. View factors possess an important reciprocity relation, \(dA_i dF_{ij} = dA_j dF_{ji}\).

This straight forward approach is often applied in thermophysical studies of comets and asteroids (Davidsson & Rickman 2014; Delbo et al. 2015). It is also acknowledged that this approach is only valid when \(\sqrt{A_j} \ll d_{ij}\), and that it can reach a singularity as \(d_{ij}\) approaches zero.

In search of a simple analytical solution for small \(d_{ij}\) cases, we found an expression in the documentation of the Abaqus® commercial package (Abaqus 2020), which aims to address this issue, as follows:

\[
F_{ij} = \frac{4 \sqrt{A_i A_j}}{\pi^2 A_i \pi \frac{\cos \phi_{ij}}{2d_{ij}} \arctan\left(\frac{\sqrt{\pi A_j \cos \phi_{ij}}}{2d_{ij}}\right) \arctan\left(\frac{\sqrt{\pi A_j \cos \phi_{ij}}}{2d_{ij}}\right)}.
\]

We designate this method as M2. The limit of Eq. 4 is Eq. 3 when \(d_{ij}\) is large.

Generally, the Monte Carlo method is convenient for computing the four-dimensional integral in Eq. 2 (Modest 1978; Yarbrough & Lee 1986; Modest 2003; Howell et al. 2010; Maltby & Burns 1991; Baranoski et al. 2001; Vujičić et al. 2006; Mirhosseini & Saboonchi 2011). However, it may be very computationally expensive for high-resolution digital terrain models (DTM) of comets, which makes other methods more practical for the view factor evaluation.

In particular, the subdivision approach which splits facets into smaller ones (Rozitis & Green 2011; Davidsson & Rickman 2014; Hu et al. 2017; Pelivan 2018; Hamm et al. 2018), offers a reasonable way to fulfill the crucial condition for Eq. 3 and avoid the risk of grossly overestimating view factors. The strategy is to divide facets into smaller triangles \(A_{im}\) or/and \(A_{jm}\) and apply Eq. 3 to determine the view factors between the smaller facets. The expression of \(F_{ij}\) is written as

\[
F_{ij} = \frac{1}{A_i} \sum_{m=1}^{N_{d_i}} \sum_{n=1}^{N_{d_j}} A_{im} A_{jm} \cos \phi_{mn} \cos \phi_{jm} \cos \phi_{jm,im} \frac{\pi \delta^2_{mn}}{\pi \delta^2_{im}},
\]

where \(N_{d_i}\) and \(N_{d_j}\) are the number of subdivisions. This method, adopts a linear subdivision\(^1\) algorithm to split the triangular facets, labeled as M3-Linear. This works well for nonadjacent facets, but may prove to be inaccurate when applied to touching (shared-edge) facets. In practice, the approach to the singularity may still be avoided because the contribution of the border region to \(F_{ij}\) is scaled down by the fractional area (which acts as a weight in the sum), however, the cost in accuracy has not been yet quantified for typical applications to cometary surface models.

The derivative approach of the facet subdivision method, which we call M3-R2, is also analyzed in this work. The goal is to avoid the explicit numerical calculation of splitting facets into smaller triangular elements. Instead, it relies on an algorithm efficiently generating an even distribution of points over an arbitrary shaped triangle using a quasi-random sequence (Roberts 2018) (see appendix A for the R2 algorithm description). Our motivation for testing this approach is twofold. First, the M3-Linear can subdivide triangles only in multiples of four by definition, which induces limits for optimization as to the number of subtriangles. In addition, the actual (linear) subdivision algorithm (Warren & Weimer 2001) may be computationally costly for repeated calculations as required for applications when the shape model itself is evolving with time (Zhao et al. 2020).

Another approach to solve Eq. 2, first proposed by Sparrow (1963), is to transform the area integrals into two contour integrals via the Stoke’s theorem, which could be expressed as

\(^1\) https://vtk.org/doc/nightly/html/classvtkLinearSubdivisionFilter.html#details
of the five dimensions aim to study the accuracy of each method from a different perspective. First we investigate the accuracy of methods described above.

3. Results

The main goal is to learn about the numerical accuracy and efficiency of the methods. As already noted, the M4 method is taken as a reference value because of its high accuracy, especially for shared-edge and close-by facets (Mazumder & Ravishankar 2012).

In Fig. 3, a parallel facets case is considered. For both panels of the figure, the abscissa shows the \( N_{\parallel} \) metric (Eq. 9), while the ordinate indicates the view factor \( F_{ij} \) (top panel), and relative differences with respect to \( M_2 \) in the bottom panel. In this scenario, the \( N_{\parallel} \) only varies due to changes in the distance between facets, while areas of the two triangles remain constant. We can immediately see (bottom panel) that the \( M_1 \) method, as expected, is the least accurate for close distances (small \( N_{\parallel} \)), however, this approach improves in accuracy as we increase the distance between the facets. The \( M_2 \) method demonstrates improved accuracy in comparison with \( M_1 \) in this setup. The subdivision techniques are superior in accuracy to both \( M_1 \) and \( M_2 \) for the entire range of \( N_{\parallel} \) values. Even at the smallest \( N_{\parallel} \) both \( M_3 \) approaches reach a sub-percent difference relative to the \( M_4 \) reference. We should point out that this very high accuracy is contingent on the number of subdivisions, which is taken in this work as \( N_{\parallel} = N_{dj} = 256 \). This means that the accuracy of \( M_3 \) methods can be further improved or degraded by increasing/decreasing \( N_{\parallel} \), respectively. The improvements in the view factor accuracy come at a computational cost, which are discussed later in this work in connection with a more practical example.

Figure 4 also has the same two panels as the previous figure. However, these results correspond to the case in which one of the facets is inclined at 45° as illustrated in panel B) in Fig. 2. Naturally, the absolute value of the view factor is reduced compared to the parallel facet example. Here, the relative accuracy between \( M_1 \) and \( M_2 \) does not show any difference, and they only reach the 1% mark of accuracy (with regard to \( M_4 \)) for large \( N_{\parallel} \) values of about 500-600. As in the previous case study the two subdivision methods show an accuracy of < 1% for the entire range of \( N_{\parallel} \) values. We can also observe a discontinuity in the relative accuracy curve for the \( M_1 \) method (also visible in the previous example). This feature, manifesting as a rapid increase in accuracy over a narrow range of \( N_{\parallel} \) values, is purely a numerical artifact of the method. It is related to the mapping of the point via the parallelogram algorithm (see Appendix A). However, its dependence on \( N_{\parallel} \) is not trivial. From these and other tests we see that it depends on the exact shape of the two facets, as well as the angle between them, and the number of sampling points.
Fig. 3. (Top) Comparison of multiple methods for calculating the view factor $F_{21}$ for two parallel facets (the case A) in Fig. 2) as a function of $N_s$ (Eq.9). (Bottom) Relative deviation from $M_4$ as a function of $N_s$ for different methods. The standard approach, $M_1$, is accurate to within 1% when $N_s$ is larger than about 100, while the subdivision methods, $M_3$-Linear and $M_3$-R2, are in accurate within 1% for the entire range of $N_s$. Therefore, these methods are clearly superior to the $M_1$ and $M_2$ methods.

Fig. 4. (Top) Value of view factor $F_{21}$ as a function of $N_s$ (Eq.9) for a case when one facet is inclined at 45° (the case B) in Fig. 2). The different curves correspond to different methods as labeled. (Bottom) Relative deviation from $M_4$. As in the previous case, even in this case the $M_3$-Linear and $M_3$-R2 methods remain accurate within 1% even for very low $N_s$ values. However, the $M_3$-R2 method does not asymptotically approach the reference value with increasing $N_s$ in this setup. The $M_3$ and $M_4$ agreement shown in this figure is just a coincidence due to facet parameters.

In connection with the previous figure (Fig. 5), additional numerical experiments show that when we increase the number of sampling points the $M_3$-R2 method reaches an accuracy of 2-3%. On the other hand, the $M_3$-Linear result improves at slower rate with increasing number of subdivisions. This is demonstrated, in a more general way, in Fig. 6. This is a case of two touching (shared-edge) facets that have 60° opening angle between them. The abscissa shows the different number of subdivisions and sampling points relevant to the two $M_3$ methods, and the ordinate corresponds to the relative deviation in $F_{21}$ view factor from the reference value, $M_4$. $M_1$ and $M_2$ methods do not depend on subdivision, therefore they keep a constant value. The first thing to note is that the $M_3$-Linear method with a subdivision of zero recovers the $M_1$ view factor value. Then, as the number of subdivisions increases the value approaches the $M_4$ reference, however, this value can reach below about 5% accuracy only for 256 sub-facets. We note that the $M_3$-Linear can increase subdivisions only in steps of $4^n$ for $n = 0, 1, ...$, hence the segmented curve in Fig. 6. The $M_3$-R2 method, as it depends on quasi-random sequence of points, does not behave deterministically for a small number of subdivisions and does not recover the $M_1$ value. In this, as well as other numerical experiments, we observe that a reliable accuracy is reached when using 50-80 subdivisions applied. For instance, increasing the $N_{i1}$ and $N_{i2}$ over 300 seems to completely eliminate this feature for this test case.

Fig. 5 shows calculations of the view factor $F_{21}$ as a function of the opening angle between the two facets sharing an edge (panel C) in Fig. 2). The top panel shows the absolute value of $F_{21}$, while the relative deviation with regards to the $M_4$ method is plotted in the bottom panel. This geometrical setup is challenging for all methods, but in particular for $M_1$ and $M_2$ for small opening angles. In addition, both $M_1$ and $M_2$ methods do not reach better than 10-20% accuracy even for large angles, although the $M_3$ method is not monotonic in its behavior. On the other hand, the two $M_3$ methods show a trend of increasing accuracy for angles larger than 25° until the value of about 170°, where there is a small spike of worsening. The curve for $M_3$-R2 stays more flat with an increasing angle going from about 8% (at 25°) to 5% at 170°. The $M_3$-Linear method starts at ~10% accuracy reaching a value 1-2% for large opening angles. This result is important especially for modeling cracks, pits, and other concave surface structures. Nevertheless, a physical shape model of a comet or an asteroid is not expected to have two touching facets with an opening angle smaller than perhaps 30-40°, however, there are likely to be many cases of large opening angles (>120°).

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Fig. 5. (Top) Dependence of view factor, $F_{21}$, in the case of shared-edge facets as a function of the opening angle between these facets (case C) in Fig. 2). As in previous cases, $M_4$ is the reference value. (Bottom) Relative deviation from $M_4$. The $M_3$-Linear and $M_3$-R2 methods show a reasonable accuracy even for small angles (around 10%). Both methods become more accurate with larger opening angles, although never get to less than 1%.

Fig. 6. Relative deviation of view factor $F_{21}$ with respect to $M_4$ for two facets sharing an edge with a 60° opening angle between them, shown as a function of subdivisions of the two triangles. Only the two $M_3$ methods are dependent on the number of subdivision. Both $M_3$-Linear and $M_3$-R2 become more accurate with larger number of subdivisions. We find that around 100 subdivisions the deviation from the reference is about 10% for $M_3$-Linear and about 5% for the $M_3$-R2. However, the $M_3$-R2 is unpredictable for very small number of subdivisions (see text for details).

Fig. 7. Small subset of SHAP7 shape model of 67P/CG featuring a large pit to be used for performance inter-comparison of the view factor calculation method. The pit can be found in the northern hemisphere of the comet (border of Hapi and Seth region). The color scale corresponds to the total $F_{12}$ view factor for each facet evaluated with the $M_4$ method, which is used as a reference (true value). In this calculation the effects of the surrounding shape model are not taken into account. However, this should not carry importance for facets inside the pit, which is our main focus in the following comparisons. The bright blue colored facet indicates total view factor below the minimum value of $10^{-8}$.
Fig. 8. Inter-comparison of the various methods of estimating the view factors for a 1000 facet segment dominated by a large pit. In each of the four panels the color scale represents absolute value percent differences in total view factor of a facet with respect to the $M_4$ method. The limits on the color scale are 0 to 5%, and facets above this limit are shown in yellow. This includes a couple of facets at the edges of the extracted surface that do not have the visibility of any other facets. The labels in each panel indicate a particular method to which the results belong. The numbers in parentheses indicate the number of sub-facets for the $M_3$ methods, for example, $M_3$-Lin(64) means that $M_3$-Linear with each facet split into 64 sub-facets. A comparison of the “combined” method aimed to optimize speed and accuracy is presented in the bottom right panel, which is detailed in Appendix B.

4. Conclusions

The goal of this paper is to present quantitative comparisons of several numerical algorithms for calculating view factors. These are routinely required for accurate thermophysical modeling of cometary and asteroid surfaces. In particular, we want to investigate challenging cases of shared-edge facets as a function of the angle between them. Having the accuracy of these calculations under control becomes especially important when modeling energy dissipation inside cracks, pits, and/or fractures. As a benchmark of accuracy we provide the double-line integral method, $M_4$, which does not suffer shortcomings of assumptions inherent in the more typical technique of a facet subdivision. The five methods investigated in this work include the “standard” approaches ($M_1$ and $M_3$-Linear); an analytical solution, $M_2$, which should improve upon $M_1$ for conditions of close distance; and

| Computation method | Time (hours) |
|--------------------|--------------|
| $M_1$              | 0.2          |
| $M_2$              | 0.2          |
| $M_3$-Lin(64), (256) | 37, 540      |
| $M_3$-R2(64), (256) | 30, 463      |
| $M_4$              | 333          |
| Combined           | 25           |
a derivative of the subdivision method based on quasi-random point distribution, M₃-R2.

First, we reduced the problem to a two-facet system to clearly demonstrate the performance of each approach as a function of a single parameter, Nₛ, that combines the distance and areas of the two facets (Eq. 9). Second, we present a view factor accuracy as a function of opening angle between the triangles with a shared edge, and we also show how the performance depends on number of subdivisions and samples for the two M₃ methods.

Finally, we also applied the main algorithms to a subset of 67P/CG surface and discussed the accuracy and computational burden for each method. The main conclusions of this work can be summarized as follows:

1. The view factor calculations transformed into closed loop integral, M₁, is the most accurate and it is especially suitable when high accuracy calculations are required for touching or very nearby facets. This is, however, very time consuming, more than two orders of magnitude compared to the most simple solution, M₁.

2. The applicability of the simple approach, M₁, method, depends on the Nₛ value and to some degree on the relative orientation of the facets in question. For example, an accuracy of 1% for parallel facets seems to require Nₛ parameter larger than about 100. However, for facets with smaller opening angles (45° in our case) in our setup these conditions could reach values of 500-600 (see Fig. 4). Nevertheless, if an accuracy of 10-20% is acceptable the M₁ method appears to be generally proper for conditions of Nₛ > 100, provided that facets do not share an edge.

3. We find the accuracy of M₃ approach not significantly different from M₁ except in cases of few special geometries. We do not see an advantage of using this method for general application to comet or asteroid shape models.

4. The M₃-Linear approach, often used in applications to comets or asteroids (e.g., (Hu et al. 2017) and references therein), is a viable option for view factor calculations of nearby and/or adjacent facets. However, the number of facet subdivisions used to improve the accuracy plays a crucial role and depends on facet configurations, as shown in Fig. 5 and 6. If the number of sub-facets needs to be increased beyond 64 we found that the combined method presented in appendix B offers a better choice in terms of accuracy and computational speed.

5. The M₃-R2 method also turns out to perform well, being able to deal with nearby or shared-edge facets. For the same number of sampling points it is very close in accuracy to the M₃-Linear approach and even may outperform it for selected cases of geometry. This method has the advantage that the number of sampling points can be optimized easily, it is simpler to implement, and the computational burden is about 20% less than the M₃-Linear. However, this approach relies on a quasi-random sequence of points distributed over a facet, hence, it cannot be used for number of sampling points smaller than about 60 (an empirical finding). In addition, this method does not behave predictably for low number of sampling points.

6. In larger scale applications where the view factor accuracy is a crucial aspect we suggest combining the methods M₁, (M₃-Linear or, M₃-R2) with M₄ to strike a good balance between accuracy and speed. We present an algorithm in appendix B, which we used in this work to reach an average accuracy inside the pit better than 2% (Fig. 8). The algorithm is robust in that it relies on the Nₛ metric to decide which method to apply and how many subdivisions to use. However, there are still several tuning parameters (the constants used in the algorithm) that allow a user the freedom to adjust to trade between accuracy and speed, depending on the nature of the problem at hand.

As a final remark, we acknowledge that our numerical implementation of the discussed algorithms is not optimized for calculation speed. The absolute, perhaps even the relative times, might be improved with a more rigorous approach (taking advantage of implementation in compiled programming language, hardware acceleration, and/or better parallelization). This work made use of the open source software PyVista (Sullivan & Kaszynski 2019) to produce Fig. 8 and for efficient M₃-Linear facet subdivision implementation.

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Appendix A: M₃-R2: Distribution of quasi-random points over triangle

An algorithm for calculating the low discrepancy quasi-random sequence, and mapping them over an arbitrary shaped triangle in described in this appendix. We provide this algorithm for convenience since, as pointed out, the original work is only available in online form (Roberts 2018).

In the first step, we generate the two-dimensional R2 quasi-random sequence with n points following the given algorithm:

Algorithm 1: R2 quasi-random sequence in 2D

```plaintext
1 g = 1.32471795724474602596
2 α = [1/g, 1/g²]
3 seed = 0.5
4 z = 0
5 for i = 0 to n do
6   z = (seed + α * (i + 1)) mod 1
7 Return z
```

What remains to be pointed out is that the R2 sequence is sensitive to the seed value and should remain as set in this appendix.

We follow recommended parallelogram method to map the R2 sequence onto a triangle. This is a simple yet effective way in preserving low discrepancy point distribution in a triangle without aliasing.

Algorithm 2: R2 mapping onto a triangle

```plaintext
1 AB: a vector along an edge of a triangle ABC
2 AC: a vector along an edge of a triangle ABC
3 z: the R2 sequence
4 n: Number of points for mapping s = z[; 0] + z[; 1]
5 p = 0
6 for i = 0 to n do
7   if s[i] > 1 then
8     p = (1.0 - s[i, 0]) * AB + (1.0 - s[i, 1]) * AC
9   else
10     p = s[i, 0] * AB + s[i, 1] * AC
11 Return p
```

An example of a quasi-random, low discrepancy distribution of points via the M₃-R2 method is contrasted with “random” point distribution in Fig. A.1. The random distribution of points seems inefficient in evenly covering the area of a triangle and would require a large number of sampling points for required accuracy. Within an acceptable error (see results, section 4) the M₃-R2 method is more evenly distributed (low discrepancy) and allows for a fast area subdivision of a facet.

Appendix B: Combined algorithm for view factor calculations

We present an algorithm for view factor calculation combining the M₁, M₁-Linear (M₃-R2 could be substituted ), and M₄ method. The algorithm relies on the Nᵣ metric (Eq. 9) and the

Algorithm combined

```plaintext
1 Calculate Nᵣ for a facet pair (Eq. 9)
2 Calculate, cosNᵣ dot product of normal vectors
3 Nᵣ = 300/Nᵣ
4 result = 0.0
5 if Nᵣ < 1 then
6   results = M₁
7 else if Nᵣ <= 64 AND cosNᵣ < 0.9848 then
8   result = M₁-Linear(Nᵣ)
9 else
10   result = M₄
11 Return result
```

Appendix C: Inter-comparisons of methods for view factor computations

Figure C.1 summarizes the accuracy and precision of the presented algorithms for view factor calculations. In this comparison we selected facets with Fᵢⱼ > 0.01, which includes all the facets inside the large and small pits (Fig. 7). The circle marker represents the mean percent difference with respect to the M₄ reference and error bars correspond to the 1 sigma standard deviation. Although on average the method M₄ shows acceptable accuracy (~2.5%) the size of the error bars indicates rather large deviations among the facets; this is also seen in Fig. 8, where many facets show significantly larger deviation. Increasing subdivision number of M₁-Linear method from 64 to 256 marginally improves the overall accuracy and reduces the deviations among
individual facets. A similar conclusion is reached for the R2 method, however, it has noticeably larger deviations compared to the M3-Linear for the same number of subdivisions. The combined method, as given in the algorithm in appendix B, provides excellent agreement and very small deviations (which are about the size of the marker). This plots highlights the advantage of using the combined method, which balances speed and accuracy.

![Fig. C.1. Summary of mean accuracy and overall precision of the main methods of view factor calculations studied in this paper (see text for description).](image)

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