Mathematical Modeling of the Thermal Regime of Lip Seal

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Abstract. In this article, on the basis of mathematical modeling, simplified models of the thermal process of the lip seal are presented. Comparison of the computational results revealed that for the model, taking into account the reciprocating motion of the rod (the "with moving source" model), the simplified model with a fixed uniform distribution of the heat flux taking into account convective heat transfer from the free friction surface is the closest model - the "to lack" heat source. It is established that the traditionally used simplified model with a fixed uniform distribution of the heat flux, which does not take into account convective heat transfer from the friction surface (the "with excess" model), deviates over time from the "with moving source" model. Methods are proposed for determining the oscillation frequencies of the rod at which the simplified model describes the temperature field in the seal with suitable accuracy for practical use.

1. Introduction

One of the main parameters that limit the performance of seals made of polymer composite materials is temperature. When calculating the thermal regime of the rod seal making reciprocating motion, in most works the rod is considered as stationary and the assumption is made that the heat flux is evenly distributed over the friction area [1]. In this case, the convective heat transfer of the free surface of the friction zone with the environment is not taken into account. In this regard, such a model gives overestimated temperature values. We will call such a model a “with excess” model of a heat source. It is proposed to consider a simplified model with the assumption of a uniform distribution of the heat flux over the friction area, which takes into account convective heat transfer from the free surface of the friction zone. In this case, the averaged coefficient of heat transfer to the environment from different sides of the seal is used.

It can be assumed that simplified models will describe the thermal process in rod seal with accuracy acceptable for practical calculations when certain conditions for the frequency of reciprocating motion are met. At the same time, the determination of such “boundary” values of oscillation frequencies for a particular seal with reciprocating motion of the rod is complicated by the complexity of the mathematical description of the heat process with a moving heat source for bodies with finite dimensions. In the classical approach, the friction of a body with half-space is considered, which allows the body motion during friction to be taken into account by introducing a convective term in the heat equation [2]. However, this technique can be applied when one of the bodies is representable in the form of a half-space or the cyclic condition is true for it, or there is a heat sink at the ends of the body [3]. In seals with reciprocating rod movement, such conditions are not met. In this case, when a convective term is introduced into the heat equation, the heat balance is violated.
2. The mathematical model
This article proposes a thermal process model in rod seals that takes into account the reciprocating motion of a heat source without introducing a convective term into the heat equation. We call it the "with source motion" model. A stationary rod was considered, on the surface of which a polymer seal of length \( d \) performs a reciprocating motion. The seal stroke length is \( l \).

In early studies, it was shown that with friction of a polymer seal with a shaft, heat transfer through the polymer seal is minimal [4]. This situation allows us to make assumptions about the removal of all the heat released as a result of friction into the rod. Therefore, when calculating the thermal state of the seal, we can consider the movable heat flux acting on the rod. Figure 1 shows the design diagram of the rod seal. The heat equation in cylindrical coordinates is written as:

\[
C \frac{\partial T}{\partial t} = \lambda \left( \frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial T}{\partial r} \right) + \frac{\partial^2 T}{\partial z^2} \right) + \frac{fPV(t)}{2\pi r \rho n} \delta(r - r_0) \sum_{i=1}^{n} \delta(z - z_i(t)), \quad 0 < r < r_0, \quad 0 < z < L, \quad 0 < t \leq t_m
\]

\( T \) – the temperature; \( t \) – the time; \( t_m \) – the test time; \( r, z \) – cylindrical coordinates; \( r_0, L \) – radius and length of the rod; \( C, \lambda \) – volumetric heat capacity, thermal conductivity coefficient of the rod material; \( f \) – coefficient of friction; \( P \) – the load; \( V \) – the rod velocity; \( \delta \) – the Dirac delta function; \( z_i(t) \) – the coordinate of the moving \( i \)-th point of the contact zone of the seal with the rod at time \( t \), \( n \) – the number of concentrated heat sources in the calculations.

\[\text{Figure 1. The design diagram of the rod seal: 1- rod; 2- friction zone; 3 - zone of contact with the seal.}\]

On free surfaces, conditions for convective heat exchange with environments are specified. To the left of the seal, the lateral and end surfaces of the rod interact with the air, and to the right, with the working fluid. The heat transfer coefficient of the rod with air is \( \alpha \) \( \text{air} \), with a liquid \( \alpha \) \( \text{fluid} \). When the rod moves, the areas of surfaces interacting with air and the working fluid change, which is taken into account when calculating the temperature field. According to equation (1), concentrated heat sources from friction act at the contact points of the rod and seal lip.

3. Computational experiments
The problem was solved by the finite element method using the FEniCS computing package library. Computational experiments were carried out with the following initial data: the rod length was 0.21; seal – 0.01; friction zone – 0.05 m. Rod diameter – 0.03 m; \( \lambda = 30.98 \text{ W/(m} \cdot \text{°C)} \), \( C = 3.7 \cdot 10^6 \text{ J/(m}^3 \cdot \text{°C)} \). The temperature at the initial time was 20 °C. The grid step in the region of the friction zone is \( h = 0.001 \) m. To ensure the stability of the solution, the movement of the contact points was implemented in software with a time step \( \tau \) satisfying the Courant condition \( \tau = h / V \).

For a comparative analysis of the dynamics of temperature fields at various oscillation frequency, the calculations were carried out at a constant power of frictional heat \( fVP = 20 \text{ W} \). With a constant stroke length, an increase in the oscillation frequency leads to an increase (decrease) in sliding speed and, accordingly, to an increase (decrease) in heat transfer to the environment, as well as a decrease.
(increase) in the load. The heat transfer coefficients $\alpha$ from the rod surface during longitudinal flow were calculated using the calculation formula for heat transfer of bodies of any shape:

$$Nu = 0.662 \Pr^{0.3} \Re^{0.5}, \quad \alpha = \frac{Nu \lambda_{sc}}{2r_s},$$

(2)

where the Prandtl $\Pr = \frac{\nu \rho_c}{\lambda_{sc}}$ and Reynolds $Re = \frac{2rV}{\nu}$ criterion, $\nu$, $\lambda_{sc}$, $\rho_c$ - kinematic viscosity, thermal conductivity, specific heat and density of the environment, $r_s$ - rod radius. In the calculations, the hydraulic fluid AMG-10 was taken as the working medium, the temperature of which is unchanged. On the other side of the seal is air.

Figure 2 shows the temperature distribution in the rod at various points in time at a frequency of 1 Hz. The temperature in the friction zone of the rod and seal is higher at the end of the contact, which is natural, since subsequent points of the cuff are pushed on the heated parts of the rod.

![Figure 2](image)

**Figure 2.** The temperature distribution in the rod at different phases of one cycle at a frequency of 1 Hz: a) $t = 5.25$; b) $t = 5.75$; c) $t = 6.0$ s.

Figure 3 shows the temperature distribution in the friction zone of the rod and seal, calculated by different models at different frequencies of reciprocating motion at time $t = 1$ hour.

![Figure 3](image)

**Figure 3.** Temperature distributions along the length of the friction zone of the rod obtained by the models “with a moving source” (curves with a solid line), “with excess” (curves with a dash-dot line) and “to lack” (curves with a dash line) at time $t = 1$ hour at different frequencies $\nu$ of the reciprocating motion of the rod: curves 1-3 at $\nu = 0.05$; 4-6 - $\nu = 0.1$; 7-9 - $\nu = 0.5$; 10-12 - $\nu = 1.0$ Hz.
Calculations show that on the surface of the rod in contact with the working fluid, the temperature is lower than on a portion of the surface interacting with the air. This pattern is due to higher heat transfer to the working fluid. A comparative analysis of temperatures in the friction zone, calculated using different models shows that simplified models with a uniform distribution of heat flux more accurately describe the temperature field at low frequencies of rod oscillations. With a decrease in the oscillation frequency, heat transfer to the environment decreases and the influence of accounting or not accounting for convective heat transfer from the friction surface becomes insignificant. It is proposed to determine the oscillation frequency of the rod, below which it is possible to use simplified models, as follows. The calculated temperature dependences according to the “with moving source” model are accepted as accurate. In the numerical solution of the model with a moving source, the Peclet condition is taken into account, which, with an increase in the frequency of movement of the rod, reduces the time step, thereby increasing the time it takes to solve a computer, which is not advantageous for calculations in a large time interval. By comparing the temperature dependences obtained by the “moving source” model and the simplified model, the rod oscillation frequency is determined at which the accuracy of the solutions of the simplified model acceptable for practical use is achieved. The choice of a simplified “with excess” or “to lack” model depends on the purpose of the calculations. For example, to determine the permissible load-speed parameters of polymer rod seals according to the temperature limiting condition, it is preferable to use the “with excess” model, since the temperature values are determined with some margin. For evaluative calculations, it is possible to use the “to lack” model.

4. Conclusion
The developed methodology for calculating the dynamics of the temperature field in the rod seals can be used to determine the permissible load-speed parameters of the polymer rod seals according to the temperature limiting condition.

5. References
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