Proposition of numerical modelling of BEC

Oleg V. Utyuzh\textsuperscript{1}, Grzegorz Wilk\textsuperscript{2} and Zbigniew W\l{}odarczyk\textsuperscript{3}

\textsuperscript{1}Nuclear Theory Department,  
The Andrzej Sołtan Institute for Nuclear Studies,  
Hoża 69, 00681 Warsaw, Poland,  
Tel: +48 22/ 553 22 28, Fax: +48 22/ 621 60 85,  
e-mail: utyuzh@fuw.edu.pl

\textsuperscript{2}Nuclear Theory Department,  
The Andrzej Sołtan Institute for Nuclear Studies,  
Hoża 69, 00681 Warsaw, Poland,  
Tel: +48 22/ 553 22 26, Fax: +48 22/ 621 60 85,  
e-mail: wilk@fuw.edu.pl

\textsuperscript{3}Institute of Physics,  
Świętokrzyska Academy,  
Świętokrzyska 15, 25-405 Kielce, Poland,  
Tel./Fax: +48 41/ 362 64 52,  
e-mail: wlod@pu.kielce.pl
Abstract

We propose extension of the numerical method to model effect of Bose-Einstein correlations (BEC) observed in hadronization processes which allows for calculations not only correlation functions \( C_2(Q_{\text{inv}}) \) (one-dimensional) but also corresponding to them \( C_2(Q_{x,y,z}) \) (i.e., three-dimensional). The method is based on the bunching of identical bosonic particles in elementary emitting cells (EEC) in phase space in manner leading to proper Bose-Einstein form of distribution of energy (this was enough to calculate \( C_2(Q_{\text{inv}}) \)). To obtain also \( C_2(Q_{x,y,z}) \) one has to add to it also symmetrization of the multiparticle wave function to properly correlate space-time locations of produced particles with their energy-momentum characteristics.

Key words: hadronization • correlations • BEC

Recently we have proposed novel numerical method to model effect of the Bose-Einstein correlations (BEC) observed in hadronization processes (see [1, 2] for details and for other references concerning generalities of BEC). This method allows for calculation of the so called invariant (i.e., one-dimensional) correlation function \( C_2(Q_{\text{inv}}) \). Referring to [1, 2] for motivation and justification and all other details let us list here only the main points proposed so far:

• Bosonic character of produced secondaries demands that they are produced in bunches (named by us Elementary Emitting Cells - EEC’s) in momentum space [3].

• The problem then is how to model production of EEC. It is done by adding to some preselected particle with energy \( E_1 \) (according to some distribution \( f(E) \), which in our numerical calculations was taken in the Bolztmann form,
\[ f(E) \sim \exp(-E/T), \text{ with temperature } T \text{ being a parameter} \] another particles of the same energy with some probability \( P = P_0 \cdot \exp(-E_1/T) \) (where \( P_0 \) is another parameter) up to first failure. After it one starts to build another EEC and continues as long as total energy allows.

- Such procedure results in a number of EEC’s (distributed according to Poisson distribution and follow boltzmannian energy level distribution) with a number of identical particles in each of them distributed according to geometrical (Bose-Einstein) distribution with energy level distribution following Bose-Einstein statistics.

- In this way one gets \( C_2(Q_{inv}) = 2 \) but only in the first bin, i.e., for \( Q_{inv} = 0 \). In order to get the characteristic shape of \( C_2(Q_{inv}) \) function one has to allow for particles in each EEC to have slightly different energies, for example distributed around \( E_1 \) according with gaussian distribution with the width \( \sigma \), which is our next and last parameter on this stage. It should be stressed at this point that in the field theory approach to BEC, as for example that presented in \cite{4}, \( \sigma = 0 \) corresponds to infinite source and commutation relations with Dirac-delta functions, whereas nonzero values of sigma arise for finite space-time extensions of the hadronizing sources.

To obtain also correlation functions \( C_2(Q_{x,y,z}) \) (i.e., three-dimensional, calculated for different components of the differences of particle momenta \( Q \)) one has to proceed further. What we propose here is the following:

- Change of \( Q_{inv} \) to \( \vec{Q} \) means that we shall be now sensitive not only to overall spatial difference between particle production points \( r \) but to the whole vector \( \vec{r} \) as well. We have to then assume that particles are produced from some spatial region and that density of production points is given by some function (our additional input) \( \rho(x, y, z) \)

\footnote{For a time being we are assuming for simplicity that hadronization is instantaneous so \( \rho \) is...}
• Momenta of each particle in a given EEC, \( p_i \), obtained in the first part of algorithm must now be decomposed into their components, \( \left( p_{ix}^{(i)}, p_{iy}^{(i)}, p_{iz}^{(i)} \right) \). To do this let us observe that BE statistics demands that multiparticle wave functions must be symmetrized accordingly and this results in correlations between production points represented by \( \rho(x, y, z) \) and momenta \( \left( p_{ix}^{(i)}, p_{iy}^{(i)}, p_{iz}^{(i)} \right) \). Noting by \( \delta_{i=x,y,z} \) the corresponding differences in position this correlation is given (in the plane wave approximation) by the known \( 1 + \cos(\delta_i \cdot Q_i) \) term.

• We proceed then in the following way. In each EEC we select the \( \left( p_{ix}^{(i)}, p_{iy}^{(i)}, p_{iz}^{(i)} \right) \) for the first, \( i = 1 \) particle in some prescribe way (here isotropically but one can introduce at this moment \( p_T \) cutting or something else as well) and then establish \( \left( p_{ix}^{(i)}, p_{iy}^{(i)}, p_{iz}^{(i)} \right) \) for every one of additional particles, \( i \geq 2 \), in such a way that \( \cos(\delta_i \cdot Q_i) \leq 2 \cdot \text{Rand} - 1 \) where \( \text{Rand} \) is random number uniformly chosen from interval \((0, 1)\).

This leads us to results presented in Figs. 1 and 2. We regard them as very promising but we are aware of the fact that our proposition is still far from being complete. To start with one should allow for time depending emission by including \( \delta E \cdot \delta t \) term in the \( \cos(\ldots) \) above. The other is the problem of Coulomb and other final state interactions. Their inclusion is possible by using some distorted wave function instead of the plane waves used here. Finally, so far only two particle symmetrization effects have been accounted for: in a given EEC all particles are symmetrized with the particle number 1 being its seed, they are not symmetrized between themselves. To account for this one would have to add other terms in addition to the \( \cos(\ldots) \) used above - this, however, would result in dramatic increase of the calculational time.

\text{time-independent.}
Fig. 1 Example of results obtained for $C_2(Q_{mx})$ (left column) and corresponding $C_2(Q_{x,y,z})$ (right column, for parameters used here all $C_2(Q_{x,y,z})$ are the same). Calculations were performed assuming spherical source $\rho(r)$ of radius $R = 1$ fm (spontaneous decay was assumed, therefore there is no time dependence) and spherically symmetric distribution of $p_{x,y,z}$ components of momenta of secondaries $p$. Energies were selected from $f(E) \sim \exp(-E/T)$ distribution. The changes investigated are - from top to bottom: different energies of hadronizing sources, different temperatures $T$, different values of parameter $P_0$ and different spreads $\sigma = \sigma_0 \cdot T$ of the energy in EEC.
To summarize - we propose new method of numerical modelling of hadronization events in such way as to respect the bosonic character of produced secondaries and therefore leading to BEC. It seems to converge in some sense to the proposition presented long time ago in [5], which was, however, in practice impossible to be implemented. The only hope that it could work now is that in our case symmetrization is within given EEC and not for the whole bunch of particles produced. Therefore the number of terms involved is rather limited, whereas in [5] the whole source had to be symmetrized at once. But the effect of including at least terms when symmetrization between, say, particles 2 and 3 are added to the already present symmetrization between 1 and 2 and 1 and 3, must be carefully investigated and estimated before any conclusion is to be reached.

OU is grateful for support and for the warm hospitality extended to him by organizers of the QM2005. Partial support of the Polish State Committee for Scientific Research (KBN) (grant 621/E-78/SPUB/CERN/P-03/DZ4/99 (GW)) is acknowledged.
References

[1] Utyuzh O, Wilk G, Włodarczyk Z (2005) Quantum Clan Model description of Bose-Einstein correlations. Acta Phys. Hung. A (HIP), in press (hep-ph/0503046).

[2] Utyuzh O, Wilk G, Włodarczyk Z (2004) How to model BEC numerically? Nukleonika 49 (Supplement 2): S33-S35.

[3] Purcell E E (1956) Nature 178 : 1447-1448.

[4] Kozlov G A, Utyuzh O, Wilk G (2003) The Bose-Einstein correlation function $C_2(Q)$ from a Quantum Field Theory point of view. Phys. Rev. C68 : 024901-5.

[5] Zajc (1987) W Monte Carlo calculational methods for the generation of the events with Bose-Einstein correlations. Phys. Rev. D53 : 3396-3408.