Quantifying the harmonic impact for multi-harmonic sources

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Abstract. Accurately dividing the harmonic impact of each harmonic source on the PCC bus is an important prerequisite for implementing the 'reward and punishment scheme'. At first, the equivalent circuit model of multi-linear load harmonic source in PCC bus is obtained by using circuit theory, and the method of quantifying the harmonic impact for multi-harmonic sources is derived. Secondly, the M-estimated robust regression method is introduced into quantifying the harmonic impact for multi-harmonic sources. The method uses the iteratively weighted least squares iterative method to solve the regression coefficients, and calculates the harmonic impact of each harmonic source in the multi-harmonic source system according to the regression coefficient. In order to verify the effectiveness of the method, the simulation analysis is carried out in the power system analysis software ETAP. The results show that the method can effectively evaluate quantifying the harmonic impact for multi-harmonic sources.

1. Introduction

As the nonlinear load in the power grid increases, plenty of grid harmonic problems arise. With the continuous advancement of building smart grids, more and more sensitive loads put forward higher requirements for power quality. It is expected that there will be further requirements for the division of responsibility for harmonic pollution, that is, the multiple harmonic sources on the user side each bear the responsibility of harmonic pollution. To avoid power quality disputes and effectively suppress harmonics in the power grid, the ‘reward and punishment’ supervision scheme has been proposed internationally [1-12]. The prerequisite for implementing this scheme is quantifying the harmonic impact for multi-harmonic sources.

At present, the main focus of scholars is to use the ‘intervention’ or ‘non-intervention’ method to estimate the harmonic impedance, so as to evaluate the harmonic emission level of a single harmonic source [1-4]. However, the harmonic voltage of the point of common coupling (PCC) bus in the grid is the result of the interaction of all harmonic sources. The literature [5] theoretically deduced that the two indicators of harmonic voltage responsibility and harmonic current responsibility are not equivalent. For the users, the responsibility of the grid is to provide qualified voltage. Therefore, the harmonic voltage responsibility is used as the measurement indicator of harmonic impact. The reference [6] firstly proposed quantifying the harmonic impact for multi-harmonic sources, which uses the least squares method to estimate the harmonic impact. In [7, 8], multivariate linear regression is used to identify multi-harmonic sources and to divide responsibility. This method decomposes the voltage phasors into real and imaginary parts, and uses the least squares to perform regression fitting in the real number domain, but the accuracy is low. Literature [9] introduced the IGG weight function.
Based on the multivariate linear regression mathematical model of the complex domain, the IGG weight function was repeatedly referenced in each time period to determine the regression coefficient. In document [10], the traditional least squares method is improved, and the harmonic impact evaluation method based on M-estimated robust regression is proposed, which overcomes the shortcomings of low accuracy of traditional least squares method. However, it only targets decentralized harmonic sources.

Aiming at quantifying the harmonic impact for multi-harmonic sources, this paper uses the M-estimated robust regression method to evaluate the multi-harmonic source responsibility. Firstly, the mathematical model of harmonic impact division of multi-harmonic source system is established. According to the mathematical model, the harmonic impact index of each nonlinear harmonic source is derived. The weighting factor is introduced, and the regression coefficient is solved by the least squares method of iterative weighted iteration. The harmonic impact of each harmonic source in the multi-harmonic source system is calculated according to the regression coefficient. The calculation results of the ETAP simulation model verify the effectiveness of the method used.

2. Establish the mathematical model of harmonic impact
The PCC point in the power grid is connected with linear loads and non-linear loads. As shown in Figure 1, if a serious harmonic voltage is detected on the PCC bus, it is necessary to quantitatively evaluate the harmonic voltage responsibility of each harmonic source generated on the PCC bus. According to the relevant theory of harmonic power flow calculation, the fundamental current and harmonic current can be calculated separately. When calculating the fundamental current, the load node injects power into the node. At harmonic frequencies, the nonlinear load can be equivalent to a harmonic current source that injects harmonic current into the grid, while other linear loads can be equivalent to equivalent impedance.

\[ u_s^h, z_s^h \text{ are the equivalent voltage source and equivalent impedance of the grid side respectively, } i_k^h \text{ and } z_k^h (k=1, 2,...,n) \text{ are the Norton equivalent current and equivalent impedance of the } k\text{th harmonic current source respectively, } i_s^h \text{ is the } h\text{th harmonic current flowing into the PCC point.} \]

When the nonlinear load \( k \) acts alone, the \( h\text{th harmonic voltage } U_{\text{PCC},kh} \text{ generated on the PCC bus and the } h\text{th harmonic current } I_{kh}' \text{ flowing through the nonlinear load } k \text{ are} \]

\[ U_{\text{PCC},kh} = z_{h,k} I_{kh}' \]

\[ I_{kh}' = \frac{U_{kh}^h}{z_k^h + z_{h,k}} \]

\[ z_{h,k} = \frac{z_{h,k}}{z_{h,k}} \]
where $\hat{U}_h^k = Z_h^k \hat{i}_h^k$, $\hat{i}_h^k$ is the equivalent $h$th harmonic current phasor in Figure 2, and $\hat{U}_h^k$ is the equivalent $h$th harmonic voltage phasor, $Z_{h,k}$ is the parallel impedance ($h$th equivalent harmonic impedance) of the portion other than the linear load $k$. According to the above analysis, $\hat{i}_{kh}$ is the harmonic current emitted by the nonlinear load $k$ itself, that is, the ‘theoretical current’. In reality, the harmonic current flowing through the nonlinear load $k$ is the harmonic current that the harmonic sources work together (‘Actual current’), and the ‘theoretical current’ of each harmonic source is difficult to obtain, while the ‘actual current’ can be obtained by the actual measurement on the secondary side of the substation. Therefore, the article uses ‘actual current’ instead. According to [11], the actual current can effectively assess the division of harmonic impact. So the harmonic currents in this paper are all ‘actual currents’.

$$Z_{h,k} = \frac{1}{\frac{1}{Z_{h}^k} + \sum_{j=1,j\neq k}^{n} \frac{1}{Z_{j}^h}}$$  \hspace{1cm} (3)

The background harmonic voltage generated by the grid side equivalent harmonic voltage source on the PCC bus is

$$\hat{U}_{PCC_{-sh}} = \frac{Z_n^h}{Z_n^h + Z_{n}^h} \hat{U}_s^h$$  \hspace{1cm} (4)

where $Z_n^h$ is the parallel impedance of all nonlinear loads.

$$Z_n^h = \frac{1}{\sum_{k=1}^{n} \frac{1}{Z_k^h}}$$  \hspace{1cm} (5)

According to the superposition theorem, the $h$th harmonic voltage $\hat{U}_{PCC_{h}}$ on the PCC bus is

$$\hat{U}_{PCC_{h}} = \sum_{k=1}^{n} \hat{U}_{PCC_{kh}} + \hat{U}_{PCC_{sh}}$$  \hspace{1cm} (6)

When the harmonic order is $h$, the phase angle of the total harmonic voltage $\hat{U}_{PCC_{h}}$ of the PCC bus is taken as the reference phase angle. The contribution of the harmonic load $\hat{U}_{PCC_{kh}}$ generated by the nonlinear load $k$ on the PCC bus and the background harmonic voltage $\hat{U}_{PCC_{sh}}$ to the total harmonic voltage $\hat{U}_{PCC_{h}}$ of the PCC bus can be represented by its projection on $\hat{U}_{PCC_{h}}$ (as shown in Figure 3). Where $\theta_{kh}(k = 1, 2)$ and $\theta_{sh}$ are the angles between $\hat{U}_{PCC_{kh}}$, $\hat{U}_{PCC_{sh}}$ and $\hat{U}_{PCC_{h}}$ respectively.

When the harmonic order is $h$, the harmonic impact $H_{kh}$ corresponding to the harmonic source $k$ can be defined as the projection of $\hat{U}_{PCC_{kh}}$ on $\hat{U}_{PCC_{h}}$ divided by the modulus of $\hat{U}_{PCC_{h}}$, then

$$H_{kh} = \frac{\left| \hat{U}_{PCC_{kh}} \right| \cos \theta_k}{\left| \hat{U}_{PCC_{h}} \right|} \times 100\%$$  \hspace{1cm} (7)

Substituting equation (1) into equation (7), then
\[ H_{kh} = \left[ Z_{h,k} \left| i_{kh} \right| \cos \theta_k \right] \frac{\cos \theta_k}{\left| U_{PCC,h} \right|} \times 100\% \quad (8) \]

In equation (8), the \( h \)th harmonic voltage \( \dot{U}_{PCC,h} \) on the PCC bus and the \( h \)th harmonic current \( \dot{i}_{kh} \) flowing through the nonlinear load \( k \) can be obtained by using the power quality monitor. It is necessary to distinguish the harmonic sources on the bus PCC. The key to quantifying harmonic impact is to accurately calculate the impedance \( h_k Z \).

**Figure 3.** Schematic diagram for the projection of harmonic voltage.

### 3. Quantifying the harmonic

Taking the nonlinear load harmonic source \( k \) as an example, equation (6) can be written as

\[ \left| \dot{U}_{PCC,h} \right| = \sum_{k=1}^{n} Z_{h,k} \left| i_{kh} \right| \cos \theta_k + \dot{U}_{PCC_{sh}} \cos \theta_{sh} \quad (9) \]

The general model of regression estimation is

\[ y_i = \sum_{k=1}^{n} \beta_{kh} x_{ki} + \epsilon_i \quad (10) \]

where \( i(i=1, \ldots, m) \) is the number of sampling tests, \( x_{ki} \) is the independent variable, \( y_i \) is the dependent variable, \( \beta_{kh} \) is the regression coefficient; \( \epsilon_i \) is the error, and each sampling test error is independently and identically distributed, and the average value is 0. The least squares estimate can be expressed by equation (11).

\[ \hat{\beta} = \left( X^T X \right)^{-1} X^T Y \quad (11) \]

where \( X \) is an independent variable matrix and \( Y \) is a dependent variable matrix. The purpose of equation (11) is to find the best \( \beta_{kh} \) to minimize the sum of squared residuals \( \sum_{i=1}^{n} \sum_{k=1}^{n} (y_i - \sum_{k=1}^{n} \beta_{kh} x_{ki})^2 \).
On this basis, using M-estimates robust regression method, the data with larger residuals are given smaller weights, the data with small residuals are given larger weights, the least squares after weighting is solved, and the weighting coefficients are repeatedly superimposed. To make the calculations more accurate. The optimization objective function in the text is

$$\sum_{i=1}^{n} \eta_i (y_i - \sum_{k=1}^{n} \beta_{k_i} x_{ik})^2 = \min$$

(12)

where $\eta_i$ is

$$\eta_i = \begin{cases} 
1, & |u_i| \leq c \\
1/c |u_i|, & |u_i| > c 
\end{cases}$$

(13)

where $c$ is a constant, generally taking 1.345, $u_i = \varepsilon_i / s$ is the standardized residual indicator, and $s$ is the residual scale.

$$s = \frac{\text{Median}|\varepsilon_i - \text{Median}(\varepsilon_1, \varepsilon_2, \ldots, \varepsilon_m)|}{0.6745}$$

(14)

where Median represents the median of the variable. Define the argument matrix $X$

$$X = \begin{bmatrix}
I_{1h}(t_1) & I_{2h}(t_1) & \cdots & I_{nh}(t_1) \\
I_{1h}(t_2) & I_{2h}(t_2) & \cdots & I_{nh}(t_2) \\
I_{1h}(t_3) & I_{2h}(t_3) & \cdots & I_{nh}(t_3) \\
\vdots & \vdots & \cdots & \vdots \\
I_{1h}(t_n) & I_{2h}(t_n) & \cdots & I_{nh}(t_n)
\end{bmatrix}$$

(15)

where $I_{kh}(t_i)$ is the $h$th harmonic current generated by the $k$th nonlinear load at the $i$th sampling test, and the dependent variable matrix $Y$ is

$$Y = \begin{bmatrix}
U_{PCC,h}(t_1) \\
U_{PCC,h}(t_2) \\
\vdots \\
U_{PCC,h}(t_n)
\end{bmatrix}^T$$

(16)

where $U_{PCC,h}(t_i)$ is the $h$th harmonic voltage of the PCC bus when the $i$th sampling test is performed.

$|\hat{U}_{PCC,h}| \cos \theta_{sh}$ and $|Z_{h,k}| \cos \theta_{kh}$ in equation (9) can be obtained by partial least squares regression estimation, then

$$|\hat{U}_{PCC,h}| = \sum_{k=1}^{n} \beta_{kh} |\hat{I}_{kh}| + \beta_{sh} + \varepsilon$$

(17)

where $\varepsilon$ is the error introduced by the regression estimation process, $\beta_{sh}$ is the regression coefficient, then the harmonic impact of the nonlinear load $k$ can be written as

$$H_{kh} = \frac{\beta_{kh} |\hat{I}_{kh}|}{|\hat{U}_{PCC,h}|} \times 100\%$$

(18)
4. Simulation calculation

The ETAP simulation software (ETAP 16.2.0 Chinese) is used to model a 220kV power grid in a certain area, which includes five non-linear loads access, as shown in Figure 4. It shows the capacity of the nonlinear loads (MVA) and the length of the lines. In order to ensure the reliability of the nonlinear load supply, each non-linear load is powered by a double-circuit line. In this paper, the harmonic voltage of SD substation is taken as the research focus, and the harmonic impact of each nonlinear harmonic source load is studied.

During simulation, the active and reactive power of all substations varies randomly from 95% to 105% of the rated value. All non-linear loads are regarded as constant loads with known apparent power. A total of 1000 tests are performed using Newton-Raphson. The method calculates the fundamental wave current of the power grid and generates the basic data needed for the division of responsibility. In the actual power grid, the nonlinear harmonic sources are far apart. In the simulation, harmonic currents are injected into the five nonlinear load nodes. The harmonic current injected by each nonlinear load node comes from the actual test of the nonlinear load in the region. The typical spectrum of data is injected into the nonlinear load nodes by harmonic currents with random volatility. Taking the 3rd, 5th, 7th, and 11th harmonics as examples, quantifying harmonic impact of the harmonic current generated by each nonlinear load to the node of interest are calculated separately. The results of the calculations in ETAP are used as an accurate result of harmonic impact and are used to compare the estimates derived from the methods used in this paper, as shown in Figure 5-Figure 8.

![Figure 4. Multiple non-linear loads connect to the bus A.](image)

![Figure 5. Harmonic impact of each nonlinear harmonic source to PCC bus at 3rd harmonic.](image)
Figure 6. Harmonic impact of each nonlinear harmonic source to PCC bus at 5th harmonic.

Figure 7. Harmonic impact of each nonlinear harmonic source to PCC bus at 7th harmonic.

Figure 8. Harmonic impact of each nonlinear harmonic source to PCC bus at 11th harmonic.
Figure 5- Figure 8 show that:

- M-estimated robust regression method results are closer to the calculation results in ETAP, which verifies the validation of M-estimated robust regression method in estimation of quantifying the harmonic impact for multi-harmonic sources;
- As the injected distance of uncharacteristic harmonics increases, the impact on transformer substation harmonic voltage is less.

5. Conclusions
The M-estimated robust regression method is used to quantify the harmonic impact. By repeating weighting in least squares iterative estimation to solve the regression coefficient, the harmonic impacts of different harmonic in multi-harmonic sources system can be calculated from the regression coefficient. The calculation conclusion in ETAP verifies the validation of M-estimated robust regression method in estimation of quantifying the harmonic impact for multi-harmonic sources;

When the injected distance of uncharacteristic harmonics increases, the impact on transformer substation harmonic voltage is smaller.

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