Magnetization of Nuclear-Spin-Polarization-Induced Quantum Ring

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Properties of a Nuclear-Spin-Polarization-Induced Quantum Ring (NSPI QR) are studied theoretically. In the proposed system a local nuclear spin polarization creates an effective hyperfine field which confines the electrons with the spins opposite to the hyperfine field to the regions of maximal nuclear spin polarization. We investigate the electron energy spectrum and the magnetic response of NSPI QR and their evolution in time due to the nuclear spin diffusion and relaxation.

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Nuclear spin polarization in semiconductor heterostructures has several intriguing properties such as the large value of effective nuclear hyperfine field, experienced by electrons, and long longitudinal nuclear spin relaxation time [1–4]. Nuclear spins embedded into mesoscopic systems were proposed to be used in spintronic devices and for quantum computation (see [5,6] and references therein). There are several techniques of polarizing nuclear spins. The most extensively employed methods use the optical pumping scheme [7] or transport polarization scheme [8]. These techniques are developed to allow local polarization and controllability of nuclear spins [9].

It has been recently suggested that locally polarized nuclei can be used to confine the electrons of two-dimensional electron gas (2DEG) into different so-called nuclear-spin-polarization-induced structures [10]-[13]. In these papers the electronic states in NSPI quantum wire [10], quantum dot [11]-[12], and periodic structure [13] were studied. The idea of producing NSPI structures was as following. The local nuclear spin polarization can be described by the effective hyperfine field $B_{hf}$, which acts at electronic spins and enters in electronic Hamiltonian through Zeeman-type potential $\frac{g}{2} \mu_B \sigma B_{hf}(r,t)$ [14]. (Here $g$ is the electron $g$-factor, $\mu_B$ is the Bohr magneton and $\sigma$ is the Pauli-matrix vector corresponding to the electron spin.) The Zeeman splitting results in that the potential is attractive for electrons with one spin projection and repulsive for others. The energy is proposed to be shifted by the constant potential of the order of the 2DEG Fermi energy ($eV_{gate} \simeq E_{F}^{2DEG}$) by the means of the gate located under the 2DEG. Then the potential landscape, created by the inhomogeneous nuclear spin polarization, can be considered as the confining potential

$$U_{conf}(r,t) = -\frac{|g|}{2} \mu_B B_{hf}(r,t). \quad (1)$$
It is assumed that \( B_{hf}(r, t) > 0 \) in Eq. (1). The maximum nuclear field in GaAs can be as high as \( B_{hf} = 5.3 \text{T} \) in the limit that all nuclear spins are fully polarized [1]. This high level of nuclear spin polarization has been achieved experimentally. For example, the optical pumping of nuclear spins in 2DEG has demonstrated nuclear spin polarization of the order of 90\%, [2]. A similarly high polarization has been created by quantum Hall edge states (85\%) [3].

In this paper we extend the study of NSPI systems by considering the electronic structure of the NSPI QR. This means that we assume the hyperfine field in Eq. (1) to be axially symmetric and to have maximum at \( r \approx r_0 \) (which can be considered as the characteristic radius of the ring). The energy profile of the system is schematically presented in Fig. 1. The system is placed into an external magnetic field \( B \) perpendicular to the ring plane, i.e. along \( z \)-axis. It is well known that in such geometry an equilibrium persistent current appears as a direct manifestation of Aharonov-Bohm effect (for a review see [15], [16]). The persistent current yields the orbital magnetic moment, which is an oscillating function of the Aharonov-Bohm flux. In the present paper we calculate the magnetic moment of NSPI QR, which may be directly measured in experiments.

We would like to underline the difference of the present investigation from previous works in this field. In Ref. [17] the interplay of nuclear magnetization and persistent current was considered. The authors of [17] have shown that the Aharonov-Bohm-like oscillations of the persistent current in mesoscopic rings could exist at zero magnetic field as the result of combined action of the nonequilibrium nuclear spin population and charge carrier spin-orbit interaction. The role of the nuclear spin magnetization in Ref. [17] and in the present paper is different. In NSPI QR the local nuclear spin polarization is used only to confine electrons into a ring and the oscillations of the magnetic moment are because of the evolution of the confining potential, while in Ref. [17] a time-dependent non-trivial profile of nuclear spin polarization enters into the phase of the electron wave function and causes Aharonov-Bohm-like oscillations.

Local nuclear spin polarization is not static. Processes of nuclear spin diffusion and relaxation change the hyperfine field in time making the electron confining potential (Eq. (1)) time-dependent. To find the hyperfine magnetic field \( B_{hf}(r, t) \) we should solve the diffusion equation with relaxation. We do this as in Refs. [10] and [12] assuming that the nuclear hyperfine field has Gaussian distribution at the initial moment of time \( t = 0 \). The solution of the diffusion equation with relaxation has a form

\[
B_{hf}(r, t) = B_{hf}(r_0, 0) e^{-\frac{r^2}{2d_0^2}} \left( 1 + \frac{t}{t_0} \right)^{-1} e^{-\frac{r^2}{2d_0^2(1 + t/t_0)}} I_0 \left( \frac{rr_0}{d_0^2}(1 + \frac{t}{t_0}) \right),
\]

(2)

where \( t_0 = \frac{d_0^2}{D} \) is a characteristic diffusion time; \( D \) is the nuclear spin diffusion coefficient; \( T_1 \) is the nuclear spin relaxation time; \( r_0, d_0 \), and \( B_{hf}(r_0, 0) \) define the radius, the half-width and the amplitude of the distribution of the hyperfine field at \( t = 0 \), respectively; \( I_0(z) \) is the modified Bessel function. When the width of the ring is smaller than its radius (\( 2d_0\sqrt{1 + t/t_0} < r_0 \)), the approximate expression for the nuclear hyperfine field is:

\[
B_{hf}(r, t) \approx B_{hf}(r_0, 0) \frac{\exp(-t/T_1)}{\sqrt{1 + t/t_0}} \frac{r_0}{r} \exp \left( -\frac{(r - r_0)^2}{2d_0^2(1 + t/t_0)} \right).
\]

(3)

This time-dependent hyperfine magnetic field corresponds (due to Eq. (1)) to the confining potential with minimum at \( r \approx r_0 \), the half-width
and the depth

$$V_0(t) = -\frac{|g|}{2}\mu_B B_{hf} (r_0, t) \simeq -\frac{|g|}{2}\mu_B B_{hf} (r_0, 0) \frac{\exp(-t/T_1)}{\sqrt{1+t/t_0}}.$$  

Since the characteristic time of nuclear spin evolution is much larger than the time scale of electron motion in the mesoscopic ring, we can consider the electrons in the quasi-time-independent confining potential $U_{\text{conf}}(r, t)$ at each moment of time. To proceed, we approximate the exact hyperfine field potential (Eq.(1)) by the following potential

$$\tilde{U}_{\text{conf}}(r) = a_1 r^{-2} + a_2 r^2 + a_3$$  

(4)

where $a_i$ are time-dependent parameters as described below. Such a potential was a success to model the electronic states in both quantum dots and rings [16]. To make this model potential (Eq.(4)) corresponding to the confining potential of NSPI QR, given by Eqs. (1) and (3), we adjust the position of the minimum, depth and width of these potentials. The following dependence of the coefficients $a_i$ on the parameters of the latter potential is obtained: $a_1 = -V_0 r_0^4/8d^2$, $a_2 = -V_0/8d^2$, and $a_3 = V_0 (1 + r_0^2/4d^2)$.

The solution of the Schrödinger equation with approximating potential (4) can be expressed in terms of the hypergeometric function [18], [16] and the energy spectrum is

$$E_{n,m} = \left(n + \frac{1}{2} + \frac{M}{2}\right) \hbar \omega + \frac{m}{2} \hbar \omega_c + V_0 \left(1 + \frac{r_0^2}{4d^2}\right),$$  

(5)

where we used the following notations: $M = \sqrt{m^2 - \frac{\hbar^2 \omega^2}{2 e V_0}}$, $\omega = \sqrt{\omega_c^2 + \omega_0^2}$, $E_0 = \frac{\hbar^2}{2m^* r_0}$, $\omega_c = |e| B/m^* c$ is the cyclotron frequency, $m^*$ and $e$ are the electron effective mass and charge, $n = 0, 1, 2, ...$ and $m = 0, \pm 1, \pm 2, ...$, $\omega_0 = (-\frac{V_0}{m^* c^2})^{1/2}$ stands for the frequency of the parabolic potential to which $\tilde{U}_{\text{conf}}$ tends at $r \sim r_0$ ($\frac{d^2 \tilde{U}_{\text{conf}}}{dr^2} |_{r=r_0} = m^* \omega_0^2$). In the limit of one-dimensional ring ($\hbar \omega_0 \gg \hbar \omega_c$, $E_0$) we obtain from Eq.(5) for $n = 0$: $E_{n,m} \simeq V_0 + \frac{\hbar \omega_0}{2} + E_0 \left(m + \frac{\Phi}{\omega_c}\right)^2$, where $\Phi$ is the flux of external magnetic field, $\Phi_0 = h\omega/e$. The spectrum of NSPI quantum dot can be obtained from Eq.(5) in the limit when the inner ring radius tends to zero. For example in the weak magnetic field, i.e. assuming $E_0 \gg \hbar \omega_0 \gg \hbar \omega_c$, we get: $E_{n,m} \simeq 2V_0 + \hbar \omega_0 (n + 1) + \frac{\hbar \omega_0}{2} \left(|m| + \frac{\omega_c}{\omega_0} m\right)$.

Further we study the magnetic moment of the electronic system which is given by the standard formula:

$$\mathcal{M} = \sum_{n,m} f_F(E_{n,m}) \mathcal{M}_{n,m},$$  

(6)

where $f_F$ is the Fermi distribution function and the magnetic moment of $(n,m)$– state is

$$\mathcal{M}_{n,m} = -\frac{\partial E_{n,m}}{\partial B} = M_0 \left(m + (M + 2n + 1) \frac{\omega_c}{\omega}\right),$$  

(7)

where $M_0 = -\frac{eh}{2m^* c}$.

If we are interested in the magnetic field dependence of the magnetic moment, we would obtain the results thoroughly presented in Ref. [16]. Besides, in NSPI QR there is the time evolution of the nuclear spin polarization due to diffusion.
and relaxation. The time evolution of the electronic structure of NSPI QR depends upon the relation between two characteristic times, \( T_1 \) and \( t_0 \). Correspondingly three characteristic time-regimes can be discussed as in Ref. [10]. The illustrations of the obtained results are shown on Fig. 2 for the case of "relaxation regime" when \( T_1 < 2t_0 \). We present the time evolution of the energy spectrum \( E_{n,m} \) (Fig. 2a), number of electrons \( N = \sum_{n,m} f_r(E_{n,m}) \) (Fig. 2b), and the magnetic moment \( \mathcal{M} \) (Fig. 2c) in NSPI QR. We have considered GaAs-AlGaAs heterostructure based NSPI QR assuming ring radius \( r_0 = 1\mu m \), initial NSPI QR half-width \( d_0 = 0.1\mu m \) and initial value of hyperfine field \( B_{hf}(r_0,0) = 2.65T \) (50\% of nuclear spins are polarized). The corresponding energies are \( E_0 = 0.57 \cdot 10^{-3}meV \) and \( V_0(0) = -3.4 \cdot 10^{-2}meV \).

Let us consider the results of our calculations in more detail. In the relaxation regime presented here, the change of the confining potential is described by the first exponential factor in Eq. (3). The amplitude of the confining potential exponentially decreases in time, while the shape only slightly changes due to the diffusion. Consequently, the depth of the energy levels and their number decreases with time (Fig. 2a,b). As the result, the magnetization of electrons in NSPI QR changes its value and sign with time (Fig. 2c). Each sign change of the magnetization occurs when the number of the electron in systems decreases by one. The dependence of the energy spectrum \( E_{n,m} \), number of electrons \( N \), and the magnetic moment \( \mathcal{M} \) on time is characterized by the parameters, which define the value and the profile of the initial nuclear polarization, by the value of external magnetic flux, and by the parameters of the electronic subsystem.

In conclusion, we have studied the electronic spectrum and magnetization of NSPI QR. In this structure the spin-polarized electrons are confined by locally polarized nuclear spins. The most interesting feature of NSPI QR is that its properties are time-dependent at fixed values of external parameters, such as, for example, the applied magnetic field. This effect is due to the nuclear spin diffusion and relaxation processes. Experimental realization of NSPI QR will demonstrate a new kind of mesoscopic phenomenon and can be useful in future spintronic devices. On the other hand, investigation of magnetic moment could be used to extract information on nuclear spin relaxation and diffusion mechanisms.

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FIG. 1. Schematic representation of NSPI QR energy structure. The shaded regions are occupied by electrons.
FIG. 2. Time evolution of energy levels $E_{0,n}$ (a), the number of levels in NSPI QR $N$ (b), and the magnetic moment $M$ of the NSPI QR (c) for $T_1/t_0 = 0.1$, $kT/E_0 = 0.1$, $\hbar\omega_c/E_0 = 1$, $V_0(t = 0)/E_0 = -60$, $r_0 = 1\mu m$, $d = 0.1\mu m$. 