STAROBINSKY INFLATION: FROM non-SUSY to SUGRA REALIZATIONS

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ABSTRACT

We review the realization of Starobinsky-type inflation within induced-gravity Supersymmetric (SUSY) and non-SUSY models. In both cases, inflation is in agreement with the current data and can be attained for subplanckian values of the inflaton. The corresponding effective theories retain perturbative unitarity up to the Planck scale and the inflaton mass is predicted to be $3 \cdot 10^{13}$ GeV. The supergravity embedding of these models is achieved by employing two gauge singlet chiral superfields, a superpotential that is uniquely determined by a continuous $R$ and a discrete $Z_n$ symmetry, and several (semi)logarithmic Kähler potentials that respect these symmetries. Checking various functional forms for the non-inflaton accompanying field in the Kähler potentials, we identify four cases which stabilize it without invoking higher order terms.

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1 INTRODUCTION

The idea that the universe underwent a period of exponential expansion, called inflation [1], has proven useful not only for solving the horizon and flatness problems of standard cosmology, but also for providing an explanation for the scale invariant perturbations, which are responsible for generating the observed anisotropies in the Cosmic Microwave Background (CMB). One of the first incarnations of inflation is due to Starobinsky. To date, this attractive scenario remains predictive, since it passes successfully all the observational tests [3, 4]. Starobinsky considered adding an \( R^2 \) term, where \( R \) is the Ricci scalar, to the standard Einstein action in order to source inflation. Recall that gravity theories based on higher powers of \( R \) are equivalent to standard gravity theories with one additional scalar degree of freedom – see e.g. [5]. As a result, Starobinsky inflation is equivalent to inflation driven by a scalar field with a suitable potential and so, it admits several interesting realizations [6–15].

Following this route, we show in this work that induced-gravity inflation (IGI) [16–20] is effectively Starobinsky-like, reproducing the structure and the predictions of the original model. Within IGI, the inflaton exhibits a strong coupling to \( R \) and the reduced Planck scale is dynamically generated through the vacuum expectation value (v.e.v.) of the inflaton at the end of inflation. Therefore, the inflaton acquires a higgs-like behavior as in theories of induced gravity [20–22]. Apart from being compatible with data, the resulting theory respects perturbative unitarity up to the Planck scale [15–17]. Therefore, no concerns about the validity of the corresponding effective theory arise. This is to be contrasted with models of non-minimal inflation (nMI) [23–28] based on a \( \phi^n \) potential with negligible v.e.v. for the inflaton \( \phi \). Although these models yield similar observational predictions with the Starobinsky model, they admit an ultraviolet (UV) scale well below \( m_P \) for \( n > 2 \), leading to complications with naturalness [29, 30].
1 Introduction

Nonetheless, IGI allows us to embed Starobinsky inflation within $\mathcal{N} = 1$ Supergravity (SUGRA) in an elegant way. The embedding is achieved by incorporating two chiral superfields, a modulus-like field $T$ and a matter-like field $S$ appearing in the superpotential, $W$, as well as various Kähler potentials, $K$, consistent with an $R$ and discrete $\mathbb{Z}_n$ symmetries [15, 17, 31] – see also Refs. [9–11, 14, 18]. In some cases [9, 15, 17, 31], the employed $K$’s parameterize specific Kähler manifolds, which appear in no-scale models [32, 33]. Moreover, this scheme ensures naturally a low enough reheating temperature, potentially consistent with the gravitino constraint [15, 34, 35] if connected with a version of the Minimal SUSY Standard Model (MSSM).

An important issue in embedding IGI in SUGRA is the stabilization of the matter-like field $S$. Indeed, when $K$ parameterizes the $SU(2, 1)/(SU(2) \times U(1))$ Kähler manifold [9, 10], the inflationary trajectory turns out to be unstable with respect to (w.r.t.) the fluctuations of $S$. This difficulty can be overcome by adding a sufficiently large term $k_S |S|^4$, with $k_S > 0$ and $|k_S| \sim 1$, in the logarithmic function appearing in $K$, as suggested in Ref. [36] for models of non-minimal (chaotic) inflation [26] and applied in Refs. [27, 28, 38, 39]. This solution, however, deforms slightly the Kähler manifold [40]. More importantly, it violates the predictability of Starobinsky inflation, since mixed terms $k_{ST} |S|^2 |T|^2$ with $k_{ST} \geq 0.01$, which can not be ignored (without tuning), have an estimable impact [17, 37, 41] on the dynamics and the observables. Moreover, this solution becomes complicated when more than two fields are considered, since all quartic terms allowed by symmetries have to be considered, and the analysis of the stabilization mechanism becomes tedious – see e.g. Refs. [17, 37, 41]. Alternatively, it was suggested to use a nilpotent superfield $S$ [42], or a charged field under a gauged R symmetry [40].

In this review, we revisit the issue of stabilizing $S$, disallowing terms of the form $|S|^{2m}$, $m > 1$, without caring much about the structure of the Kähler manifold. Namely, we investigate systematically several functions $h_i(|S|^2)$ (with $i = 1, ..., 11$) that appear in the choices for $K$, and we find four acceptable forms that lead to the stabilization of $S$ during and after IGI. The output of this analysis is new, providing results that did not appear in the literature before. More specifically, we consider two principal classes of $K$’s, $K_{3i}$ and $K_{2i}$, distinguished by whether $h_i$ and $T$ appear in the same logarithmic function. The resulting inflationary scenarios are almost indistinguishable. The case considered in Ref. [31] is included as one of the viable choices in the $K_{2i}$ class. Contrary to Ref. [31], though, we impose here the same $\mathbb{Z}_n$ symmetry on $W$ and $K$. Consequently, the relevant expressions for the mass spectrum and the inflationary observables get simplified considerably compared to those displayed in Ref. [31]. As in the non-SUSY case, IGI may be realized using subPlanckian values for the initial (non-canonically normalized) inflaton field. The radiative corrections remain under control and perturbative unitarity is not violated up to $m_P$ [2, 17, 31], consistently with the consideration of SUGRA as an effective theory.

Throughout this review we focus on the standard $\Lambda$CDM cosmological model [3]. An alternative framework is provided by the running vacuum models [43] which turn out to yield a quality fit to observations, significantly better than that of $\Lambda$CDM. In this case, the acceleration of the universe, either during inflation or at late times, is not attributed to a scalar field but rather arises from the modification of the vacuum itself, which is dynamical. A SUGRA realization of Starobinsky inflation within this setting is obtained in the last paper of Ref. [7].

The plan of this paper is as follows. In Sec. 2, we establish the realiztion of Starobinsky inflation as IGI in a non-SUSY framework. In Sec. 3 we introduce the formulation of IGI in SUGRA and revisit the issue of stabilizing the matter-like field $S$. The emerging inflationary models are analyzed in Sec. 4. Our conclusions are summarized in Sec. 5. Throughout, charge conjugation is denoted by a star ($\ast$), the symbol $\cdot_z$ as subscript denotes derivation w.r.t. $z$, and we use units where the reduced Planck scale, $m_P = 2.43 \times 10^{18}$ GeV, is set equal to unity.
2 Starobinsky Inflation From Induced Gravity

We begin our presentation demonstrating the connection between $R^2$ inflation and IGI. We first review the formulation of nMI in Sec. 2.1, and then proceed to describe the inflationary analysis in Sec. 2.2. Armed with these prerequisites, we present $R^2$ inflation as a type of nMI in Sec. 2.3, and exhibit its connection with IGI in Sec. 2.4.

2.1 Coupling non-Minimally the Inflaton to Gravity

We consider an inflaton $\phi$ that is non-minimally coupled to the Ricci scalar $\mathcal{R}$, via a coupling function $f_R(\phi)$. We denote the inflaton potential by $V_I(\phi)$ and allow for a general kinetic function $f_K(\phi)$ – in the cases of pure nMI [19, 24, 25] $f_K = 1$. The Jordan Frame (JF) action takes the form

$$S = \int d^4x \sqrt{-\tilde{g}} \left( -\frac{1}{2} f_R \mathcal{R} + \frac{1}{2} f_K g^{\mu \nu} \partial_{\mu} \phi \partial_{\nu} \phi - V_I(\phi) \right),$$  \hspace{1cm} (2.1)

where $\tilde{g}$ is the determinant of the Friedmann-Robertson-Walker metric, $g_{\mu \nu}$, with signature $(+,-,-,-)$. We require $\langle f_R \rangle \simeq 1$ to ensure ordinary Einstein gravity at low energies.

By performing a conformal transformation [24] to the Einstein frame (EF), we write the action

$$S = \int d^4x \sqrt{-\hat{g}} \left( -\frac{1}{2} \hat{\mathcal{R}} + \frac{1}{2} \hat{g}^{\mu \nu} \partial_{\mu} \hat{\phi} \partial_{\nu} \hat{\phi} - \hat{V}_I(\hat{\phi}) \right),$$ \hspace{1cm} (2.2)

where a hat denotes an EF quantity. The EF metric is given by $\hat{g}_{\mu \nu} = f_R g_{\mu \nu}$, and the canonically normalized field, $\hat{\phi}$, and its potential, $\hat{V}_I$, are defined as follows:

(a) $\frac{d\hat{\phi}}{d\phi} = J = \sqrt{\frac{f_K}{f_R} + \frac{3}{2} \left( \frac{f_R,\phi}{f_R} \right)^2}$ and (b) $\hat{V}_I = \frac{V_I}{f_R^2}.$ \hspace{1cm} (2.3)

For $f_R \gg f_K$, the coupling function $f_R$ acquires a twofold role. On one hand, it determines the relation between $\hat{\phi}$ and $\phi$. On the other hand, it controls the shape of $\hat{V}_I$, thus affecting the observational predictions – see below. The analysis of nMI can be performed in the EF, using the standard slow-roll approximation. It is [19] completely equivalent with the analysis in the JF. We just have to keep track the relation between $\hat{\phi}$ and $\phi$.

2.2 Observational and Theoretical Constraints

A viable model of nMI must be compatible with a number of observational and theoretical requirements summarized in the following – cf. Ref. [44].

1.2.1 The number of e-foldings $\hat{N}_e$ that the scale $k_e = 0.05/\text{Mpc}$ experiences during inflation must to be large enough for the resolution of the horizon and flatness problems of the standard hot Big Bang model, i.e. [3, 24],

$$\hat{N}_e = \int_{\phi_i}^{\phi_e} d\phi \frac{\hat{V}_I}{\hat{V}_{I,\phi}} \simeq 61.7 + \frac{1}{2} \ln \frac{\hat{V}_I(\phi_e)}{\hat{V}_I(\phi_i)^{1/3}} + \frac{1}{3} \ln T_{\text{th}} + \frac{1}{2} \ln \frac{f_R(\phi_e)}{f_R(\phi_i)^{1/3}},$$ \hspace{1cm} (2.4)

where $\phi_e$ [$\hat{\phi}_e$] is the value of $\phi$ [$\hat{\phi}$] when $k_e$ crosses the inflationary horizon. In deriving the formula above – cf. Ref. [38] – we take into account an equation-of-state with parameter $w_{\text{th}} = 0$ [45], since $\hat{V}_I$ can be well approximated by a quadratic potential for low values of $\phi$ – see Eqs. (2.2b), (2.32b) and (4.13b) below. Also $T_{\text{th}}$ is the reheating temperature after nMI. We take a representative value
$T_{\text{rh}} = 4.1 \cdot 10^{-10}$ throughout, which results to $N_* \simeq 53$. The effective number of relativistic degrees of freedom at temperature $T_{\text{rh}}$ is taken $g_{\text{rh}} = 107.75$ in accordance with the Standard model spectrum. Lastly, $\phi_f [\dot{\phi}_f]$ is the value of $\phi [\dot{\phi}]$ at the end of nMI, which in the slow-roll approximation can be obtained via the condition

$$\max\{\tilde{\epsilon}(\phi_f), |\tilde{\eta}(\phi_f)|\} = 1,$$

where

$$\tilde{\epsilon} = \frac{1}{2} \left( \frac{\hat{V}_{1,\phi}}{\hat{V}_1} \right)^2 = \frac{1}{2J^2} \left( \frac{\hat{V}_{1,\phi}}{\hat{V}_1} \right)^2 \quad \text{and} \quad \tilde{\eta} = \frac{\hat{V}_{1,\phi\phi}}{\hat{V}_1} = \frac{1}{J^2} \left( \frac{\hat{V}_{1,\phi\phi}}{\hat{V}_1} - \frac{\hat{V}_{1,\phi}}{\hat{V}_1} \frac{J}{J} \right). \quad (2.5)$$

Evidently non trivial modifications of $f_R$, and thus of $J$, may have a significant effect on the parameters above, modifying the inflationary observables.

1.2.2 The amplitude $A_s$ of the power spectrum of the curvature perturbation generated by $\phi$ at $k_*$ has to be consistent with the data [46], i.e.,

$$\sqrt{A_s} = \frac{1}{2\sqrt{3 \pi}} \frac{\hat{V}_1(\hat{\phi}_*)^{3/2}}{|\hat{V}_{1,\phi}(\hat{\phi}_*)|} = \frac{|J(\phi_*)|}{2\sqrt{3 \pi}} \frac{\hat{V}_1(\phi_*)^{3/2}}{|\hat{V}_{1,\phi}(\phi_*)|} \simeq 4.627 \cdot 10^{-5}. \quad (2.6)$$

As shown in Sec. 3.4, the remaining scalars in the SUGRA versions of nMI may be rendered heavy enough and so, they do not contribute to $A_s$.

1.2.3 The remaining inflationary observables (the spectral index $n_s$, its running $\alpha_s$, and the tensor-to-scalar ratio $r$) must be in agreement with the fitting of the Planck, Baryon Acoustic Oscillations (BAO) and BICEP2/Keck Array data [3, 4] with the $\Lambda$CDM+$r$ model, i.e.,

(a) $n_s = 0.968 \pm 0.009$ and (b) $r \leq 0.07$, \quad (2.7)

at the 95% confidence level (c.l.) with $|\alpha_s| \ll 0.01$. Although compatible with Eq. (2.7b), all data taken by the BICEP2/Keck Array CMB polarization experiments, up to the 2014 observational season (BK14) [4], seem to favor $r$’s of the order of 0.01, as the reported value is $0.028^{+0.026}_{-0.025}$ at the 68% c.l.. These inflationary observables are estimated through the relations:

(a) $n_s = 1 - 6\hat{\epsilon}_* + 2\hat{\eta}_*$, (b) $\alpha_s = \frac{2}{3} \left( 4\hat{\xi}_*^2 - (n_s - 1)^2 \right) - 2\hat{\xi}_*$ and (c) $r = 16\hat{\epsilon}_*$, \quad (2.8)

where $\hat{\xi} = \hat{V}_{1,\phi} \hat{V}_{1,\phi\phi\phi} / \hat{V}_1^2$ and the variables with subscript $*$ are evaluated at $\phi_*$. 

1.2.4 The effective theory describing nMI remains valid up to a UV cutoff scale $\Lambda_{UV}$, which has to be large enough to ensure the stability of our inflationary solutions, i.e.,

(a) $\hat{V}_1(\phi_*)^{1/4} \leq \Lambda_{UV}$ and (b) $\phi_* \leq \Lambda_{UV}$. \quad (2.9)

As we show below, $\Lambda_{UV} \simeq 1$ for the models analyzed in this work, contrary to the cases of pure nMI with large $f_R$, where $\Lambda_{UV} \ll 1$. The determination of $\Lambda_{UV}$ is achieved expanding $S$ in Eq. (2.2) about $\langle \phi \rangle$. Although these expansions are not strictly valid [30] during inflation, we take the $\Lambda_{UV}$ extracted this way to be the overall UV cut-off scale, since the reheating phase – realized via oscillations about $\langle \phi \rangle$ – is a necessary stage of the inflationary dynamics.
2.3 FROM NON-MINIMAL TO $R^2$ INFLATION

The $R^2$ inflation can be viewed as a type of nMI, if we employ an auxiliary field $\phi$ with the following input ingredients

$$f_K = 0, \quad f_R = 1 + 4c_R\phi \quad \text{and} \quad \hat{V}_I = \phi^2. \quad (2.10)$$

Using the equation of motion for the auxiliary field, $\phi = c_R R$, we obtain the action of the original Starobinsky model (see e.g. Ref. [40]):

$$S = \int d^4x \sqrt{-g} \left( -\frac{1}{2} R + c_R^2 R^2 \right). \quad (2.11)$$

As we can see from Eq. (2.10), the model has only one free parameter ($c_R$), enough to render it consistent with the observational data, ensuring at the same time perturbative unitarity up to the Planck scale. Using Eq. (2.10) and Eq. (2.3), we obtain the EF quantities

(a) $J = 2\sqrt{6c_R} f_R$ and (b) $\hat{V}_I = \frac{\phi^2}{f_R^2} \simeq \frac{1}{16c_R^2}$. \quad (2.12)

For $c_R \gg 1$, the plot of $\hat{V}_I$ versus $\phi$ is depicted in Fig. 1-(a). An inflationary era can be supported since $\hat{V}_I$ becomes flat enough. To examine further this possibility, we calculate the slow-roll parameters. Plugging Eq. (2.12) into Eq. (2.5) yields

$$\hat{\epsilon} = \frac{1}{12c_R^2 \phi^2} \quad \text{and} \quad \hat{\eta} = \frac{1 - 4c_R \phi}{12c_R^2 \phi^2}. \quad (2.13)$$

Notice that $\eta < 0$ since $\hat{V}_I$ is slightly concave downwards, as shown in Fig. 1-(a). The value of $\phi$ at the end of nMI is determined via Eq. (2.5), giving

$$\phi_f = \max \left( \frac{1}{2\sqrt{3c_R}}, \frac{1}{6c_R} \right) \Rightarrow \phi_f = \frac{1}{2\sqrt{3c_R}}. \quad (2.14)$$

Under the assumption that $\phi_f \ll \phi_\star$, we can obtain a relation between $\hat{N}_\star$ and $\phi_\star$ via Eq. (2.4)

$$\hat{N}_\star \simeq 3c_R\phi_\star. \quad (2.15)$$

The precise value of $c_R$ can be determined enforcing Eq. (2.6). Recalling that $\hat{N}_\star \simeq 53$, we get

$$A_s^{1/2} \simeq \frac{\hat{N}_\star}{12\sqrt{2\pi c_R}} = 4.627 \cdot 10^{-5} \Rightarrow c_R \simeq 2.3 \cdot 10^4. \quad (2.16)$$

The resulting value of $c_R$ is large enough so that

$$\phi_\star \simeq \frac{\hat{N}_\star}{3c_R} \simeq 8.3 \cdot 10^{-4} \ll 1 \quad (2.17)$$

consistently with Eq. (2.9b) – see Fig. 1-(a). Impressively, the remaining observables turn out to be compatible with the observational data of Eq. (2.7). Indeed, inserting the above value of $\phi_\star$ into Eq. (2.8) ($\hat{N}_\star = 53$), we get

$$n_s \simeq \frac{(\hat{N}_\star - 3)(\hat{N}_\star - 1)}{N_\star^2} \simeq 1 - \frac{2}{\hat{N}_\star} - \frac{3}{N_\star^2} \simeq 0.961; \quad (2.18a)$$

$$\alpha_s \simeq -\frac{(\hat{N}_\star - 3)(4\hat{N}_\star + 3)}{2N_\star^4} \simeq -\frac{2}{N_\star^2} - \frac{15}{2N_\star^3} \simeq -7.6 \cdot 10^{-4}; \quad (2.18b)$$

$$r \simeq \frac{12}{N_\star^2} \simeq 4.2 \cdot 10^{-3}. \quad (2.18c)$$
Without the simplification of Eq. (2.15), we obtain numerically \( n_s = 0.964 \), \( \alpha_s = -6.7 \cdot 10^{-4} \) and \( r = 3.7 \cdot 10^{-3} \). We see that \( n_s \) turns out to be appreciably lower than unity thanks to the negative values of \( \eta \) – see Eq. (2.13). The mass of the inflaton at the vacuum is

\[
\hat{m}_{\delta \phi} = \left( \hat{V}_{\phi \phi} \right)^{1/2} = \left( \hat{V}_{I, \phi \phi} / J^2 \right)^{1/2} = 1/2\sqrt{3c_R} \simeq 1.25 \cdot 10^{-5} \text{ (i.e. } 3 \cdot 10^{13} \text{ GeV).} \tag{2.19}
\]

As we show below this value is a salient feature in all models of Starobinsky inflation.

Furthermore, the model provides an elegant solution to the unitarity problem [29, 30], which plagues models of nMI with \( f_R \sim \phi^n \gg f_K \), \( n \gg 2 \) and \( f_K = 1 \). This stems from the fact that \( \hat{\phi} \) and \( \phi \) do not coincide at the vacuum, as Eq. (2.12a) implies \( \hat{\phi} = \langle J \rangle \phi = 2\sqrt{3c_R} \hat{\phi} \). In fact, if we expand the second term in the right-hand side (r.h.s.) of Eq. (2.2) about \( \langle \phi \rangle = 0 \), we find

\[
J^2 \phi^2 = \left( 1 - 2\sqrt{2/3} \hat{\phi} + 2 \hat{\phi}^2 - \cdots \right) \hat{\phi}^2. \tag{2.20a}
\]

Similarly, expanding \( \hat{V}_I \) in Eq. (2.12b), we obtain

\[
\hat{V}_I = \frac{\hat{\phi}^2}{24c_R^2} \left( 1 - 2\sqrt{2/3} \hat{\phi} + 2 \hat{\phi}^2 - \cdots \right). \tag{2.20b}
\]

Since the coefficients of the above series are of order unity, independent of \( c_R \), we infer that the model does not face any problem with perturbative unitarity up to the Planck scale.

### 2.4 Induced-Gravity Inflation

It would be certainly beneficial to realize the structure and the predictions of \( R^2 \) inflation in a framework that deviates minimally from Einstein gravity, at least in the present cosmological era.

To this extent, we incorporate the idea of induced gravity, according to which \( m_P \) is generated dynamically [22] via the v.e.v. of a scalar field \( \phi \), driving a phase transition in the early universe. The simplest way to implement this scheme is to employ a double-well potential for \( \phi \) – for scale invariant realizations of this idea see Ref. [21]. On the other hand, an inflationary stage requires a sufficiently flat potential, as in Eq. (2.10). This can be achieved at large field values if we introduce a quadratic \( f_R \) [19,20]. More explicitly, IGI may be defined as a nMI with the following input ingredients:

\[
f_K = 1, \quad f_R = c_R \phi^2 \quad \text{and} \quad V_I = \lambda \left( \phi^2 - M^2 \right)^2 / 4. \tag{2.21}
\]
Given that $\langle \phi \rangle = M$, we recover Einstein gravity at the vacuum if
$$f_R(\langle \phi \rangle) = 1 \Rightarrow M = 1/\sqrt{c_R}. \quad (2.22)$$

We see that in this model there is one additional free parameter, namely $\lambda$ appearing in the potential, as compared to the $R^2$ model.

Eq. (2.3) and Eq. (2.21) imply
$$J \simeq 6/\phi \quad \text{and} \quad \hat{V}_I = \frac{\lambda f_\phi^2}{4c_R^4 \phi^4} \simeq \frac{\lambda}{4c_R^2 \phi^2} \quad \text{with} \quad f_\phi = 1 - c_R \phi^2. \quad (2.23)$$

For $c_R \gg 1$, the plot of $\hat{V}_I$ versus $\phi$ is shown in Fig. 1-(b). As in the $R^2$ model, $\hat{V}_I$ develops a plateau and so, an inflationary stage can be realized. To check its robustness, we compute the slow-roll parameters. Eq. (2.5) and Eq. (2.23) give
$$\hat{c} = \frac{4}{3f_\phi^2} \quad \text{and} \quad \hat{\eta} = \frac{4(1 + f_\phi)}{3f_\phi^2}. \quad (2.24)$$

IGI is terminated when $\phi = \phi_I$, determined by the condition
$$\phi_I = \max \left( \sqrt{\frac{1 + 2/\sqrt{3}}{c_R}}, \sqrt{\frac{5}{3c_R}} \right) \Rightarrow \phi_I = \sqrt{\frac{1 + 2/\sqrt{3}}{c_R}}. \quad (2.25)$$

Under the assumption that $\phi_I \ll \phi_*$, Eq. (2.4) implies the following relation between $\hat{N}_*$ and $\phi_*$
$$\hat{N}_* \simeq 3c_R \phi_*^2/4 \Rightarrow \phi_* \simeq 2\sqrt{\frac{\hat{N}_*}{3c_R}} \Rightarrow \phi_I.$$

Imposing Eq. (2.9b) and setting $\hat{N}_* \simeq 53$, we derive a lower bound on $c_R$:
$$\phi_* \leq 1 \Rightarrow c_R \geq 4\hat{N}_*/3 \simeq 71. \quad (2.27)$$

Contrary to $R^2$ inflation, $c_R$ does not control exclusively the normalization of Eq. (2.6), thanks to the presence of an extra factor of $\sqrt{\lambda}$. This is constrained to scale with $c_R$. Indeed, we have
$$A_*^{1/2} \simeq \frac{\sqrt{\lambda} \hat{N}_*}{6\sqrt{2\pi}c_R} = 4.627 \cdot 10^{-5} \Rightarrow c_R \simeq 42969 \sqrt{\lambda} \quad \text{for} \quad \hat{N}_* \simeq 53. \quad (2.28)$$

If, in addition, we impose the perturbative bound $\lambda \leq 3.5$, we end-up with following ranges:
$$77 \lesssim c_R \lesssim 8.5 \cdot 10^4 \quad \text{and} \quad 2.8 \cdot 10^{-6} \lesssim \lambda \lesssim 3.5, \quad (2.29)$$

where the lower bounds on $c_T$ and $\lambda$ correspond to $\phi_* = 1$ – see Fig. 1-(b). Within the allowed ranges, $\hat{m}_{\delta \phi}$ remains constant, by virtue of Eq. (2.28). The mass turns out to be
$$\hat{m}_{\delta \phi} = \sqrt{\lambda}/\sqrt{3c_R} \simeq 1.25 \cdot 10^{-5}, \quad (2.30)$$

essentially equal to that estimated in Eq. (2.19). Moreover, using Eq. (2.26) and Eq. (2.8), we extract the remaining observables

$$n_s = \frac{(4\hat{N}_* - 15)(4\hat{N}_* + 1)}{(3 - 4\hat{N}_*)^2} \simeq 1 - \frac{2}{2\hat{N}_*} - \frac{9}{2\hat{N}_*^2} \simeq 0.961; \quad (2.31a)$$

$$\alpha_s = -\frac{128\hat{N}_*(4\hat{N}_* + 9)}{(3 - 4\hat{N}_*)^4} \simeq -\frac{2}{2\hat{N}_*^2} - \frac{21}{2\hat{N}_*^3} \simeq -7.7 \cdot 10^{-4}; \quad (2.31b)$$

$$r = \frac{192}{(3 - 4\hat{N}_*)^2} \simeq \frac{12}{\hat{N}_*^2} \simeq 4.4 \cdot 10^{-3}. \quad (2.31c)$$
Without making the approximation of Eq. (2.26), we obtain numerically \((n_s, \alpha_s, r) = (0.964, -6.6 \cdot 10^{-4}, 3.7 \cdot 10^{-3})\). These results practically coincide with those of \(R^2\) inflation, given in Eqs. (2.18a) – (2.18e), and they are in excellent agreement with the observational data presented in Eq. (2.7).

As in the previous section, the model retains perturbative unitarity up to \(m_P\). To verify this, we first expand the second term in the r.h.s. of Eq. (2.1) about \(S\) parameter, (2.18)

The \(K\) SUGRA potential, given in terms of the \(K\)ähler potential \(z\) this work, the complex scalar fields \(z\) Without making the approximation of Eq. (2.26), we obtain numerically \(\Lambda_{UV} = 1\) as for \(R^2\) inflation. Practically identical results can be obtained if we replace the quadratic exponents in Eq. (2.21) with \(n \geq 3\) as first pointed out in Ref. [16]. This generalization can be elegantly performed [17, 18] within SUGRA, as we review below.

3 **Induced-Gravity Inflation in SUGRA**

In Sec. 3.1, we present the general SUGRA setting, where IGI is embedded. Then, in Sec. 3.2, we examine a variety of \(K\)ähler potentials, which lead to the desired inflationary potential – see Sec. 3.3. We check the stability of the inflationary trajectory in Sec. 3.4.

3.1 **The General Set-Up**

To realize IGI within SUGRA [15, 17, 18, 31], we must use of two gauge singlet chiral superfields \(z^\alpha\), with \(z^1 = T\) and \(z^2 = S\) being the inflaton and a “stabilizer” superfield respectively. Throughout this work, the complex scalar fields \(z^\alpha\) are denoted by the same superfield symbol. The EF effective action is written as follows [26]

\[
S = \int d^4 x \sqrt{-g} \left( -\frac{1}{2} \hat{R} + K_{\alpha\beta}\partial^{\mu}\partial^{\nu}z^{\alpha}\partial^{\nu}z^{\beta} - \tilde{V} \right),
\]

where \(K_{\alpha\beta} = K_{z^\alpha z^\beta}\) is the \(K\)ähler metric and \(K^{\alpha\beta}\) its inverse \((K^{\alpha\beta} K_{\beta\gamma} = \delta^{\alpha}_\gamma)\). \(\tilde{V}\) is the EF F-term SUGRA potential, given in terms of the \(K\)ähler potential \(K\) and the superpotential \(W\) by the following expression

\[
\tilde{V} = e^K \left( K^{\alpha\beta} D_\alpha W D_\beta W^* - 3|W|^2 \right) \quad \text{with} \quad D_\alpha W = W_{,z^\alpha} + K_{z^\alpha W}.
\]

Conformally transforming to the JF with \(f_\mathcal{R} = -\Omega/N\), where \(N\) is a dimensionless positive parameter, \(S\) takes the form

\[
S = \int d^4 x \sqrt{-g} \left( \frac{\Omega}{2N} \mathcal{R} + \frac{3}{4N\Omega} \partial_\mu \partial^{\mu} \Omega - \frac{1}{N} \Omega K_{\alpha\beta} \partial_\mu z^{\alpha} \partial^{\nu} z^{\beta} - V \right) \quad \text{with} \quad V = \frac{\Omega^2}{N^2} \tilde{V}.
\]

Note that \(N = 3\) reproduces the standard set-up [26]. Let us also relate \(\Omega\) and \(K\) by

\[
-\Omega/N = e^{-K/N} \Rightarrow K = -N \ln (-\Omega/N).
\]
Then taking into account the definition [26] of the purely bosonic part of the auxiliary field when on shell,
\[ A_\mu = i \left( K_\alpha \partial_\mu z^\alpha - K_\alpha \partial_\mu z^{*\alpha} \right) / 6, \]  
we arrive at the following action
\[ S = \int d^4x \sqrt{-g} \left( \frac{\Omega}{2N} R + \left( \Omega_\alpha \beta + \frac{3 - N}{N} \Omega_\alpha \nu \beta \right) \partial_\mu z^\alpha \partial_\mu z^{*\beta} - \frac{27}{N^3} \Omega A_\mu A^\mu - V \right). \]  

By virtue of Eq. (3.3), \( A_\mu \) takes the form
\[ A_\mu = -iN \left( \Omega_\alpha \partial_\mu z^\alpha - \Omega_\alpha \partial_\mu z^{*\alpha} \right) / 6\Omega \]  
with \( \Omega_\alpha = \Omega_{z^\alpha} \) and \( \Omega_\alpha = \Omega_{z^{*\alpha}} \). As can be seen from Eq. (3.5a), \( -\Omega/N \) introduces a non-minimal coupling of the scalar fields to gravity. Ordinary Einstein gravity is recovered at the vacuum when
\[ -(\Omega)/N \approx 1. \]

Starting with the JF action in Eq. (3.3), we seek to realize IGI, postulating the invariance of \( \Omega \) under the action of a global \( \mathbb{Z}_n \) discrete symmetry. With \( S \) stabilized at the origin, we write
\[ -\Omega/N = \Omega_H(T) + \Omega_H(T^*) \quad \text{with} \quad \Omega_H(T) = c_T T^n + \sum_{k=2}^{\infty} \lambda_k T^{kn}, \]  
where \( k \) is a positive integer. If \( T \leq 1 \) during IGI and assuming that \( \lambda_k \)'s are relatively small, the contributions of the higher powers of \( T \) in the expression above are small, and these can be dropped. As we verify later, this can be achieved when the coefficient \( c_T \) is large enough. Equivalently, we may rescale the inflaton, setting \( T \rightarrow \tilde{T} = c_T^{1/n} T \). Then the coefficients \( \lambda_k \) of the higher powers in the expression of \( \Omega \) get suppressed by factors of \( c_T \). Thus, \( \mathbb{Z}_n \) and the requirement \( T \leq 1 \) determine the form of \( \Omega \), avoiding a severe tuning of the coefficients \( \lambda_k \). Under these assumptions, \( K \) in Eq. (3.3) takes the form
\[ K_0 = -N \ln \left( f(T) + f^*(T^*) \right) \quad \text{with} \quad f(T) \approx c_T T^n, \]  
where \( S \) is assumed to be stabilized at the origin.

Eqs. (3.3) and (3.6) require that \( T \) and \( S \) acquire the following v.e.v.s
\[ \langle T \rangle \approx 1/(2c_T)^{1/n} \quad \text{and} \quad \langle S \rangle = 0. \]  
These v.e.v.s can be achieved, if we choose the following superpotential [17, 18]:
\[ W = \lambda S (T^n - 1/2c_T). \]  
Indeed the corresponding F-term SUSY potential, \( V_{\text{SUSY}} \), is found to be
\[ V_{\text{SUSY}} = \lambda^2 \left[ T^n - 1/2c_T \right]^2 + \lambda^2 n^2 \left| ST^{n-1} \right|^2 \]  
and is minimized by the field configuration in Eq. (2.21).

As emphasized in Refs. [15, 17, 31], the forms of \( W \) and \( \Omega_H \) can be uniquely determined if we limit ourselves to integer values for \( n \) (with \( n > 1 \)) and \( T \leq 1 \), and impose two symmetries:
(i) An R symmetry under which \( S \) and \( T \) have charges 1 and 0 respectively;
(ii) A discrete symmetry \( \mathbb{Z}_n \) under which only \( T \) is charged.

For simplicity we assume here that both \( W \) and \( \Omega_H \) respect the same \( \mathbb{Z}_n \), contrary to the situation in Ref. [31]. This assumption simplifies significantly the formulae in Secs. 3.3 and 3.4. Note, finally, that the selected \( \Omega \) in Eq. (3.7) does not contribute in the kinetic term involving \( \Omega_{TT^*} \) in Eq. (3.5a). We expect that our findings are essentially unaltered even if we include in the r.h.s. of Eq. (3.7) a term \(- (T - T^*)^2/2N \) [18] or \(- |T|^2/N \) [17] which yields \( \Omega_{TT^*} = 1 \ll c_T \) — the former choice, though, violates the \( \mathbb{Z}_n \) symmetry above.
3.2 Proposed Kähler Potentials

It is obvious from the considerations above, that the stabilization of $S$ at zero during and after IGI is of crucial importance for the viability of our scenario. This key issue can be addressed if we specify the dependence of the Kähler potential on $S$. We distinguish the following basic cases:

$$K_{3i} = -n_3 \ln \left( f(T) + f^*(T^*) + h_i(X) \right) \quad \text{and} \quad K_{2i} = -n_2 \ln \left( f(T) + f^*(T^*) \right) + h_i(X), \quad (3.12)$$

where the various choices $h_i, i = 1, ..., 11,$ are specified in Table 1, and $X$ is defined as follows

$$X = \begin{cases} 
-|S|^2/n_3 & \text{for } K = K_{3i} \\
|S|^2 & \text{for } K = K_{2i}.
\end{cases} \quad (3.13)$$

As shown in Table 1 we consider exponential, logarithmic, trigonometric and hyperbolic functions. Note that $K_{3i}$ and $K_{2i}$ parameterize the $SU(2,1)/SU(2) \times U(1)$ and $SU(1,1)/U(1) \times U(1)$ Kähler manifolds respectively, whereas $K_{2i}$ parameterizes the $SU(1,1)/U(1) \times SU(2)/U(1)$ Kähler manifold – see Ref. [31].

To show that the proposed $K$’s are suitable for IGI, we have to verify that they reproduce $\hat{V}_I$ in Eq. (2.23b) when $n = 2$, and they ensure the stability of $S$ at zero. These requirements are checked in the following two sections.

3.3 Derivation of the Inflationary Potential

Substituting $W$ of Eq. (3.10) and a choice for $K$ in Eq. (3.12) (with the $h_i$’s given in Table 1) into Eq. (3.1b), we obtain a potential suitable for IGI. The inflationary trajectory is defined by the

| $i$ | $h_i(X)$ | $h''_i(0)$ | $\frac{\bar{m}_s^2}{\bar{H}_s^2}$ | $K = K_{3i}$ | $K = K_{2i}$ |
|-----|---------|-----------|----------------|--------------|--------------|
| 1   | $X$     | 0         | $-2 + 2^n/f_0^2$ | $3 \cdot 2^{n-1}/f_0^2$ |              |
| 2   | $e^X - 1$ | 1         | $2^{3-n} c_T \phi^n - 2 + 2^n/f_0^2$ | $-6 + 3 \cdot 2^{n-1}/f_0^2$ | $6(1 + 2^{n-1}/f_0^2)$ |
| 3   | $\ln(X + 1)$ | $-1$ | $-2(1 + 2^{1-n}c_T \phi^n)$ | $3 \cdot 2^{n-1}/f_0^2$ |              |
| 4   | $-\cos(\arcsin 1 + X)$ | 0 | $-2(1 - 2^{n-1}/f_0^2)$ | $3 \cdot 2^{n-1}/f_0^2$ |              |
| 5   | $\sin(\arccos 1 + X)$ | 0 | $-2(1 - 2^{n-1}/f_0^2)$ | $3 \cdot 2^{n-1}/f_0^2$ |              |
| 6   | $\tan(X)$ | 0 | $-2(1 - 2^{n-1}/f_0^2)$ | $3 \cdot 2^{n-1}/f_0^2$ |              |
| 7   | $-\cot(\arcsin 1 + X)$ | 0 | $-2(1 - 2^{n-1}/f_0^2)$ | $3 \cdot 2^{n-1}/f_0^2$ |              |
| 8   | $\cosh(\arcsin 1 + X) - \sqrt{2}$ | $\sqrt{2}$ | $2^{\frac{3-n}{2}} c_T \phi^n - 2 + 2^n/f_0^2$ | $3 \cdot 2^{n-1}/f_0^2 - 6\sqrt{2}$ |              |
| 9   | $\sinh(X)$ | 0 | $-2(1 - 2^{n-1}/f_0^2)$ | $3 \cdot 2^{n-1}/f_0^2$ |              |
| 10  | $\tanh(X)$ | 0 | $-2(1 - 2^{n-1}/f_0^2)$ | $3 \cdot 2^{n-1}/f_0^2$ |              |
| 11  | $-\coth(\arcsinh 1 + X) + \sqrt{2}$ | $-2\sqrt{2}$ | $2^n/f_0^2 - 2^{\frac{3-n}{2}} c_T \phi^n - 2$ | $3 \cdot 2^{n-1}/f_0^2 + 12\sqrt{2}$ |              |

Table 1: Definition of the various $h_i(X)$’s, $h''_i(0) = d^2h_i(0)/dX^2$ and masses squared of the fluctuations of $s$ and $\bar{s}$ along the inflationary trajectory in Eq. (3.14) for $K = K_{3i}$ and $K_{2i}$.
where a prime denotes a derivative w. r. t. \( X \)

The field \( S \) where we take into account that 

\[ f = 1 \]

we have expanded \( f \) in real and imaginary parts as follows

\[ T = \frac{\phi}{\sqrt{2}} e^{i\theta} \text{ and } S = \frac{s + is}{\sqrt{2}}. \]  

(3.15)

Along the path of Eq. (3.14), \( \hat{V} \) reads

\[ \hat{V}_1 = \hat{V}(\theta = s = \bar{s} = 0) = e^{K^S S^* } |W, S|^2. \]  

(3.16)

From Eq. (3.10) we get \( W, S = f - 1/2 \). Also, Eq. (3.12) yields

\[ e^K = \begin{cases} (2f + h_i(0))^{-\frac{n_3}{2}} & \text{for } K = K_{3i} \\ \frac{e^{h_i(0)} / (2f)^{n_2}}{h_i'(0)} & \text{for } K = K_{2i}, \end{cases} \]  

(3.17)

where we take into account that \( f(T) = f^*(T^*) \) along the path of Eq. (3.14). Moreover, \( K^S S^* = 1/K_{SS} \) can be obtained from the Kähler metric, which is given by

\[ (K^{\alpha \bar{\beta}}) = \text{diag}(K_{T^*}, K_{S^*}) = \begin{cases} \text{diag}(n_3 n^2 / 2 f^2, h_i'(0) / (2 f + h_i(0))) & \text{for } K = K_{3i} \\ \text{diag}(n_2 n^2 / 2 f^2, h_i'(0)) & \text{for } K = K_{2i}, \end{cases} \]  

(3.18)

where a prime denotes a derivative w. r. t. \( X \). Note that \( K_{T^*} \), for \( K = K_{2i} \) (and \( S \neq 0 \)) does not involve the field \( S \) in its denominator, and so no geometric destabilization \([48]\) can be activated, contrary to the \( K = K_{3i} \) case. Inserting \( W, S \), and the results of Eqs. (3.17) and (3.18) into Eq. (3.11), we obtain

\[ \hat{V}_1 = \lambda^2(1 - 2 f^2) \frac{c^2}{e^{3f}} \cdot \begin{cases} (2f + h_i(0))^{1-n_3} / h_i'(0) & \text{for } K = K_{3i} \\ \frac{e^{h_i(0)} / (2f)^{n_2} h_i'(0)}{h_i'(0)} & \text{for } K = K_{2i}. \end{cases} \]  

(3.19)

Recall that \( f \sim \phi^n \) — see Eq. (3.8). Then \( \hat{V}_1 \) develops a plateau, with almost constant potential energy density, for \( c_T \gg 1 \) and \( \phi < 1 \) (or \( c_T = 1 \) and \( \phi \gg 1 \)), if we impose the following conditions:

\[ 2n = \begin{cases} n(n_3 - 1) & \text{for } K = K_{3i} \\ nn_2 & \text{for } K = K_{2i} \end{cases} \Rightarrow \begin{cases} n_3 = 3 & \text{for } K = K_{3i} \\ n_2 = 2 & \text{for } K = K_{2i}. \end{cases} \]  

(3.20)

This empirical criterion is very precise since the data on \( n_s \) allows only tiny (of order 0.001) deviations [14]. Actually, the requirement \( c_T \gg 1 \) and the synergy between the exponents in \( W \) and \( K \)’s assist us to tame the well-known \( \eta \) problem within SUGRA with a mild tuning. If we insert Eq. (3.20) into Eq. (3.19) and compare the result for \( n = 2 \) with Eq. (2.23b) (replacing also \( \lambda^2 \) with \( \lambda \)), we see that the two expressions coincide, if we set

\[ h_i(0) = 0 \text{ and } h_i'(0) = 1. \]  

(3.21)

As we can easily verify the selected \( h_i \) in Table 1 satisfy these conditions. Consequently, \( \hat{V}_1 \) in Eq. (3.19) and the corresponding Hubble parameter \( \hat{H}_1 \) take their final form:

\[ \text{a) } \hat{V}_1 = \lambda^2 f_\phi^2 / 4c_T^2 \phi^{2n} \text{ and (b) } \hat{H}_1 = \lambda f_\phi / \sqrt{3} = \frac{\lambda f_\phi}{2\sqrt{3}c_T^2 \phi^n}, \]  

(3.22)

with \( f_\phi = 2^{n/2} - c_T \phi^n < 0 \) reducing to that defined in Eq. (2.23b). Based on these expressions, we investigate in Sec. 4 the dynamics and predictions of IGI.
Our results are listed in Table 1 together with the masses squared normalizing – see Eq. (3.25) – does not affect their dynamics.

\[
\begin{array}{|c|c|c|c|}
\hline
\text{FIELDS} & \text{EIGEN-} & \text{MASSES Squared} \\
 & \text{STATES} & K = K_{3i} & K = K_{2i} \\
\hline
1 \text{ real scalar} & \hat{\theta} & \tilde{m}_{\tilde{\theta}}^2/\tilde{H}_1^2 & 4(2^{n-2} - c_T \phi^n f_\phi)/f_\phi^2 \\
 & & & 6(2^{n-2} - c_T \phi^n f_\phi)/f_\phi^2 \\
1 \text{ complex} & \tilde{s}, \hat{s} & \tilde{m}_{\tilde{s}}^2/\tilde{H}_1^2 & 2^n/f_\phi^2 - 2 \\
\text{scalar} & & & 3 \cdot 2^{n-1}/f_\phi^2 \\
2 \text{ Weyl spinors} & \hat{\psi}_\pm & \tilde{m}_{\psi\pm}^2/\tilde{H}_1^2 & 2^n/f_\phi^2 \\
& & & 6 \cdot 2^{n-3}/f_\phi^2 \\
\hline
\end{array}
\]

Table 2: Mass-squared spectrum for \( K = K_{3i} \) and \( K_{2i} \) along the inflationary trajectory in Eq. (3.14).

### 3.4 Stability of The Inflationary Trajectory

We proceed to check the stability of the direction in Eq. (3.14) w.r.t. the fluctuations of the various fields. To this end, we have to examine the validity of the extremum and minimum conditions, i.e.,

\[
(a) \quad \left. \frac{\partial \tilde{V}_I}{\partial \bar{z}^\alpha} \right|_{s=\bar{s}=\theta=0} = 0 \quad \text{and} \quad (b) \quad \tilde{m}_{z\alpha}^2 > 0 \quad \text{with} \quad \bar{z}^\alpha = \theta, s, \bar{s}.
\]

(3.23)

Here \( \tilde{m}_{z\alpha}^2 \) are the eigenvalues of the mass matrix with elements

\[
\tilde{M}_{\alpha\beta}^2 = \left. \frac{\partial^2 \tilde{V}_I}{\partial \bar{z}^\alpha \partial \bar{z}^\beta} \right|_{s=\bar{s}=\theta=0}
\]

and a hat denotes the EF canonically normalized field. The canonically normalized fields can be determined if bring the kinetic terms of the various scalars in Eq. (3.1a) into the following form

\[
K_{\alpha\beta} \ddot{z}^\alpha \ddot{z}^\beta = \frac{1}{2} \left( \dot{s}^2 + \dot{\bar{s}}^2 \right) + \frac{1}{2} \left( \dot{\theta}^2 + \dot{\bar{\theta}}^2 \right),
\]

(3.25a)

where a dot denotes a derivative w.r.t. the JF cosmic time. Then the hatted fields are defined as follows

\[
\frac{d\hat{\phi}}{d\phi} = J = \sqrt{K_{TT}}, \quad \hat{\theta} = J \theta/\phi, \quad \text{and} \quad (\hat{s}, \hat{\bar{s}}) = \sqrt{K_{SS}}(s, \bar{s}),
\]

(3.25b)

where by virtue of Eqs. (3.20) and (3.21), the Kähler metric of Eq. (3.18) reads

\[
(K_{\alpha\beta}) = \text{diag}(K_{TT}, K_{SS}) = \begin{cases} 
(3n^2/2^2 \phi^2, 2^{n-2}/c_T \phi^n) & \text{for } K = K_{3i} \\
(n^2/\phi^2, 1) & \text{for } K = K_{2i}.
\end{cases}
\]

(3.25c)

Note that the spinor components \( \psi_T \) and \( \psi_S \) of the superfields \( T \) and \( S \) are normalized in a similar way, i.e., \( \hat{\psi}_T = \sqrt{K_{TT}} \psi_\phi \) and \( \hat{\psi}_S = \sqrt{K_{SS}} \psi_S \). In practice, we have to make sure that all the \( \tilde{m}_{z\alpha}^2 \)'s are not only positive, but also greater than \( \tilde{H}_1^2 \) during the last 50 – 60 e-foldings of IGI. This guarantees that the observed curvature perturbation is generated solely by \( \phi_s \), as assumed in Eq. (2.6). Nonetheless, two-field inflationary models which interpolate between the Starobinsky and the quadratic model have been analyzed in Ref. [47]. Due to the large effective masses that the scalars acquire during IGI, they enter a phase of damped oscillations about zero. As a consequence, the \( \phi \) dependence in their normalization – see Eq. (3.25b) – does not affect their dynamics.

We can readily verify that Eq. (3.23a) is satisfied for all the three \( \bar{z}^\alpha \)'s. Regarding Eq. (3.23b), we diagonalize \( \tilde{M}_{\alpha\beta}^2 \), Eq. (3.24), and we obtain the scalar mass spectrum along the trajectory of Eq. (3.14). Our results are listed in Table 1 together with the masses squared \( \tilde{m}_{\psi\pm}^2 \) of the chiral fermion eigenstates \( \hat{\psi}_\pm = (\hat{\psi}_T \pm \hat{\psi}_S)/\sqrt{2} \). From these results, we deduce the following:
For both classes of $K$’s in Eq. (3.12), Eq. (3.23b) is satisfied for the fluctuations of $\hat{\theta}$, i.e. $\hat{m}_s^2 > 0$, since $f_\phi < 0$. Moreover, $\hat{m}_s^2 \gg \hat{H}_1^2$ because $c_T \gg 1$.

- When $K = K_{3i}$ and $h_i''(0) = 0$, we obtain $\hat{m}_s^2 < 0$. This occurs for $i = 1, 4, \ldots, 7, 9$ and 10, as shown in Table 1. For $i = 1$, our result reproduces those of similar models Refs. [17, 26, 27, 39]. The stability problem can be cured if we include in $K_{3i}$ a higher order term of the form $k_S|S|^4$ with $k_S \sim 1$, or assuming that $S^2 = 0$ [42]. However, a probably simpler solution arises if we take into account the results accumulated in Table 2. It is clear that the condition $\hat{m}_s^2 > \hat{H}_1^2$ can be satisfied when $h_i''(0) > 0$ with $|h_i''(0)| \geq 1$. From Table 1, we see that this is the case for $i = 2$ and 8.

- When $K = K_{2i}$ and $h_i''(0) = 0$, we obtain $\hat{m}_s^2 > 0$, but $\hat{m}_s^2 < \hat{H}_1^2$. Therefore, $S$ may seed inflationary perturbations, leading possibly to large non-gaussianities in the CMB, contrary to observations. From the results listed in Table 2, we see that the condition $\hat{m}_s^2 \gg \hat{H}_1^2$ requires $h_i''(0) < 0$ with $|h_i''(0)| \geq 1$. This occurs for $i = 3$ and 11. The former case was examined in Ref. [31].

To highlight further the stabilization of $S$ during and after IGI we present in Fig. 2 $\hat{m}_s^2/\hat{H}_1^2$ as a function of $\phi$ for the various acceptable $K$’s identified above. In particular, we fix $n = 2$ and $\phi_* = 1$, setting $K = K_{32}$ or $K = K_{38}$ in Fig. 2-(a) and $K = K_{23}$ or $K = K_{2,11}$ in Fig. 2-(b). The parameters of the models ($\Lambda$ and $c_T$) corresponding to these choices are listed in third and fifth rows of Table 3. Evidently $\hat{m}_s^2/\hat{H}_1^2$ remain larger than unity for $\phi_I \leq \phi \leq \phi_*$, where $\phi_*$ and $\phi_I$ are also depicted. However, in Fig. 2-(b) $\hat{m}_s^2/\hat{H}_1^2$ exhibits a constant behavior and increases sharply as $\phi$ decreases below 0.2. On the contrary, $\hat{m}_s^2/\hat{H}_1^2$ in Fig. 2-(a) is an increasing function of $\phi$ for $\phi \gtrsim 0.2$, with a clear minimum at $\phi \simeq 0.2$.

Employing the well-known Coleman-Weinberg formula [49], we find from the derived mass spectrum – see Table 1 – the one-loop radiative corrections, $\Delta\hat{V}_I$, to $\hat{V}_I$, depending on renormalization group mass scale $\Lambda$. It can be verified that our results are insensitive to $\Delta\hat{V}_I$, provided that $\Lambda$ is determined by requiring $\Delta\hat{V}_I(\phi_*) = 0$ or $\Delta\hat{V}_I(\phi_I) = 0$. A possible dependence of the results on the choice of $\Lambda$ is totally avoided [17] thanks to the smallness of $\Delta\hat{V}_I$, for $\Lambda \simeq (1 - 1.8) \cdot 10^{-5}$ – see Sec. 4.2 too. These conclusions hold even for $\phi > 1$. Therefore, our results can be accurately reproduced by using exclusively $\hat{V}_I$ in Eq. (3.22a).

**Figure 2**: The ratio $\hat{m}_s^2/\hat{H}_1^2$ as a function of $\phi$ for $n = 2$ and $\phi_* = 1$. We set (a) $K = K_{32}$ or $K = K_{38}$ and (b) $K = K_{23}$ or $K = K_{2,11}$. The values corresponding to $\phi_*$ and $\phi_I$ are also depicted.
4 Analysis of SUGRA Inflation

Keeping in mind that for $K = K_{3i}$ [$K = K_{2i}$] the values $i = 2$ and $8$ [$i = 3$ and $11$] lead to the stabilization of $S$ during and after IGI, we proceed with the computation of the inflationary observables for the SUGRA models considered above. Since the precise choice of the index $i$ does not influence our outputs, here we do not specify henceforth the allowed $i$ values. We first present, in Sec. 4.1, analytic results which are in good agreement with our numerical results displayed in Sec. 4.2. Finally we investigate the UV behavior of the models in Sec. 4.3.

4.1 Analytical Results

The duration of the IGI is controlled by the slow-roll parameters, which are calculated to be

$$\tilde{c}, \tilde{\eta} = \begin{cases} \left( \frac{2^n}{3f_0^2}, \frac{2^{1+n/2}(2^{n/2} - c_T\phi^n)}{3f_\phi^2} \right) & \text{for } K = K_{3i} \\ \left( \frac{2^n}{f_0^2}, \frac{2^{1+n/2}(2^{n/2} - c_T\phi^n)}{f_\phi^2} \right) & \text{for } K = K_{2i} \end{cases}$$

(4.1)

The end of inflation is triggered by the violation of the $\tilde{c}$ condition when $\phi = \phi_f$ given by

$$\tilde{c}(\phi_f) = 1 \Rightarrow \phi_f \simeq \sqrt{2} \cdot \begin{cases} \left( (1 + 2/\sqrt{3})/2c_T \right)^{1/n} & \text{for } K = K_{3i} \\ \left( (1 + \sqrt{2})/2c_T \right)^{1/n} & \text{for } K = K_{2i} \end{cases}$$

(4.2a)

The violation of the $\tilde{\eta}$ condition occurs when $\phi = \tilde{\phi}_f < \phi_f$:

$$\tilde{\eta}(\tilde{\phi}_f) = 1 \Rightarrow \tilde{\phi}_f \simeq \sqrt{2} \cdot \begin{cases} \left( (5/6c_T)\right)^{1/n} & \text{for } K = K_{3i} \\ \left( (\sqrt{3}/2c_T)\right)^{1/n} & \text{for } K = K_{2i} \end{cases}$$

(4.2b)

Given $\phi_f$, we can compute $\tilde{N}_*^i$ via Eq. (2.4):

$$\tilde{N}_* = \frac{K}{2} \left( 2^{1-n/2}c_T (\phi_* - \phi_f) - n \ln \frac{\phi_*}{\phi_f} \right) \quad \text{with } \kappa = \begin{cases} 3/2 & \text{for } K = K_{3i} \\ 1 & \text{for } K = K_{2i} \end{cases}$$

(4.3)

Ignoring the logarithmic term and taking into account that $\phi_f \ll \phi_*$, we obtain a relation between $\phi_*$ and $\tilde{N}_*$:

$$\phi_* \simeq n^{\frac{n}{2}}/\sqrt{2\tilde{N}_*/\kappa c_T}.$$  

(4.4a)

Obviously, IGI, consistent with Eq. (2.9b), can be achieved if

$$\phi_* \leq 1 \Rightarrow c_T \geq 2^{n/2}\tilde{N}_*/\kappa.$$  

(4.4b)

Therefore, we need relatively large $c_T$'s, which increase with $n$. On the other hand, $\tilde{\phi}_*$ remains transplanckian, since solving the first relation in Eq. (3.25b) w.r.t. $\phi$ and inserting Eq. (4.4a), we find

$$\tilde{\phi}_* \simeq \tilde{\phi}_c + \sqrt{\kappa} \ln(2\tilde{N}_*/\kappa) \simeq \begin{cases} 5.2 & \text{for } K = K_{3i} \\ 4.6 & \text{for } K = K_{2i} \end{cases}$$

(4.5)

where the integration constant $\tilde{\phi}_c = 0$ and, as in the previous cases, we set $\tilde{N}_* \simeq 53$. Despite this fact, our construction remains stable under possible corrections from higher order terms in $f_{kc}$, since when these are expressed in terms of initial field $T$, they can be seen to be harmless for $|T| \leq 1$. 


Upon substitution of Eqs. (3.22) and (4.4a) into Eq. (2.6), we find
\[ A_s^{1/2} \approx \begin{cases} \lambda(3 - 4\hat{N}_\ast)^2/96\sqrt{2\pi c_T\hat{N}_\ast} & \text{for } K = K_{3i} \\ \lambda(1 - 2\hat{N}_\ast)^2/16\sqrt{3\pi c_T\hat{N}_\ast} & \text{for } K = K_{2i}. \end{cases} \] (4.6)

Enforcing Eq. (2.6), we obtain a relation between \( \lambda \) and \( c_T \), which turns out to be independent of \( n \). Indeed we have
\[ \lambda \approx \begin{cases} 6\pi \sqrt{2A_\ast c_T/\hat{N}_\ast} \Rightarrow c_T \approx 42969\lambda & \text{for } K = K_{3i} \\ 4\pi \sqrt{3A_\ast c_T/\hat{N}_\ast} \Rightarrow c_T \approx 52627\lambda & \text{for } K = K_{2i}. \end{cases} \] (4.7)

Finally, substituting the value of \( \phi_\ast \) given in Eq. (4.4a) into Eq. (2.8), we estimate the inflationary observables. For \( K = K_{3i} \) the results are given in Eqs. (2.31a) – (2.31c). For \( K = K_{2i} \) we obtain the relations:
\[ n_s = \frac{4\hat{N}_\ast(\hat{N}_\ast - 3) - 3}{(1 - 2\hat{N}_\ast)^2} \approx 1 - \frac{2}{\hat{N}_\ast} - \frac{3}{N_\ast^2} \approx 0.961; \] (4.8a)
\[ \alpha_s \approx \frac{16\hat{N}_\ast(3 + 2\hat{N}_\ast)}{(2\hat{N}_\ast - 1)^4} \approx -\frac{2}{\hat{N}_\ast^2} - \frac{7}{\hat{N}_\ast^3} \approx -0.00075; \] (4.8b)
\[ r \approx \frac{32}{(1 - 2\hat{N}_\ast)^2} \approx \frac{8}{\hat{N}_\ast^2} + \frac{8}{\hat{N}_\ast^3} \approx 0.0028. \] (4.8c)

These outputs are consistent with our results in Ref. [31] for \( m = n \) and \( n_{11} = n_2 = 2 \) (in the notation of that reference).

### 4.2 Numerical Results

The analytical results presented above can be verified numerically. The inflationary scenario depends on the following parameters – see Eqs. (3.10) and (3.12):
\[ n, c_T \text{ and } \lambda. \]

Note that the stabilization of \( S \) with one of \( K_{32}, K_{34}, K_{23} \) and \( K_{2,11} \) does not require any additional parameter. Recall that we use \( T_{11} = 4.1 \cdot 10^{-9} \) throughout and \( \hat{N}_\ast \) is computed self-consistently for any \( n \) via Eq. (2.4). Our result is \( \hat{N}_\ast \approx 53.2 \). For given \( n \), the parameters above together with \( \phi_\ast \) can be determined by imposing the observational constraints in Eqs. (2.4) and (2.6). In our code we find \( \phi_\ast \) numerically, without the simplifying assumptions used for deriving Eq. (4.4a). Inserting it into Eq. (2.8), we extract the predictions of the models.

The variation of \( \tilde{V}_1 \) as a function of \( \phi \) for two different values of \( n \) can be easily inferred from Fig. 3. In particular, we plot \( \tilde{V}_1 \) versus \( \phi \) for \( \phi_\ast = 1, n = 2 \) or \( n = 6 \), setting \( K = K_{3i} \) in Fig. 3-(a) and \( K = K_{2i} \) in Fig. 3-(b). Imposing \( \phi_\ast = 1 \) for \( n = 2 \) amounts to \( (\lambda, c_T) = (0.0017, 77) \) for \( K = K_{3i} \) and \( (\lambda, c_T) = (0.0017, 113) \) for \( K = K_{2i} \). Also, \( \phi_\ast = 1 \) for \( n = 6 \) is obtained for \( (\lambda, c_T) = (0.0068, 310) \) for \( K = K_{3i} \) and \( (\lambda, c_T) = (0.0082, 459) \) for \( K = K_{2i} \). In accordance with our findings in Eq. (4.4b), we conclude that increasing \( n \) (i) requires larger \( c_T \)'s and therefore, lower \( \tilde{V}_1 \)'s to obtain \( \phi \leq 1 \); (ii) larger \( \phi_1 \) and \( \langle \phi \rangle \) are obtained – see Sec. 4.3. Combining Eqs. (4.2a) and (4.7) with Eq. (3.22a), we can conclude that \( \tilde{V}_1(\phi_1) \) is independent of \( c_T \) and to a considerable degree of \( n \).

Our numerical findings for \( n = 1, 2 \) and 3 and \( K = K_{3i} \) or \( K = K_{2i} \) are presented in Table 2. In the two first rows, we present results associated with Ceccoti-like models [50], which are defined by \( c_T = n = 1 \) and can not be made consistent with the imposed \( \mathbb{Z}_n \) symmetry or with Eq. (2.9). We see that selecting \( \phi_\ast \gg 1 \), we attain solutions that satisfy all the remaining constraints in Sec. 2.2. For the
other cases, we choose a $c_T$ value so that $\phi_* = 1$. Therefore, the presented $c_T$ is the minimal one, in agreement with Eq. (4.4b).

In all cases shown in Table 2, the model’s predictions for $n_s$, $\alpha_s$ and $r$ are independent of the input parameters. This is due to the attractor behavior [16–18] that these models exhibit, provided that $c_T$ is large enough. Moreover, these outputs are in good agreement with the analytical findings of Eqs. (2.31a) – (2.31c) for $K = K_{3i}$ or Eqs. (4.8a) – (4.8c) for $K = K_{2i}$. On the other hand, the presented $c_T$, $\lambda$, $\phi_*$ and $\phi_I$ values depend on $n$ for every selected $K$. The resulting $n_s \approx 0.964$ is close to its observationally central value; $r$ is of the order of 0.001, and $|\alpha_s|$ is negligible. Although the values of $r$ lie one order of magnitude below the central value of the present combined BICEP2/Keck Array and Planck results [4], these are perfectly consistent with the 95\% c.l. margin in Eq. (2.7). The values of $r$ for $K = K_{3i}$ or $K = K_{2i}$ distinguish the two cases. The difference is small, at the level of $10^{-3}$. However, it is possibly reachable by the next-generation experiments. E.g., the CMBPol experiment [51] is expected to achieve a precision for $r$ of the order of $10^{-3}$ or even $0.5 \cdot 10^{-3}$. Finally, the renormalization scale $\Lambda$ of the Coleman-Weinberg formula, found by imposing $\Delta \hat{V}_I(\phi_s) = 0$, takes the values $7.8 \cdot 10^{-5}$, $9.3 \cdot 10^{-5}$, $1.3 \cdot 10^{-5}$, $2.1 \cdot 10^{-5}$ for $K_{32}$, $K_{38}$, $K_{23}$ and $K_{2,11}$ respectively. As a consequence, $\Lambda$ depends explicitly on the specific choice of $i$ used for $K_{3i}$ or $K_{2i}$.

The overall allowed parameter space of the model for $n = 2, 3$ and 6 is correspondingly

$$
77, 105, 310 \lesssim c_T \lesssim 1.6 \cdot 10^5 \quad \text{and} \quad (1.7, 2.4, 6.8) \cdot 10^{-3} \lesssim \lambda \lesssim 3.54 \quad \text{for} \quad K = K_{3i} \quad ; \quad (4.9a)
$$

$$
113, 159, 453 \lesssim c_T \lesssim 1.93 \cdot 10^5 \quad \text{and} \quad (2, 2.9, 8.2) \cdot 10^{-3} \lesssim \lambda \lesssim 3.54 \quad \text{for} \quad K = K_{2i} \quad ; \quad (4.9b)
$$

where the parameters are bounded from above as in Eq. (2.29). Letting $\lambda$ or $c_T$ vary within its allowed region above, we obtain the values of $n_s$, $\alpha_s$ and $r$ listed in Table 3 for $K = K_{3i}$ and $K_{2i}$ independently of $n$. Therefore, the inclusion of the variant exponent $n > 2$, compared to the non-SUSY model in Sec. 2.4, does not affect the successful predictions of model.

### 4.3 UV Behavior

Following the approach described in Sec. 2.2, we can verify that the SUGRA realizations of IGI retain perturbative unitarity up to $m_P$. To this end, we analyze the small-field behavior of the theory, expanding $S$ in Eq. (2.1) about

$$
\langle \phi \rangle = 2^{(n-2)/2n} c_T^{-1/n} , \quad (4.10)
$$

**Figure 3:** The inflationary potential $\hat{V}_I$ as a function of $\phi$ for $\phi_* = 1$ and $n = 2$ or $n = 6$. We set (a) $K = K_{3i}$ and (b) $K = K_{2i}$. The values corresponding to $\phi_*$ and $\phi_I$ are also depicted.
which is confined in the ranges $(0.0026 - 0.1)$, $(0.021 - 0.24)$ and $(0.17 - 0.48)$ for the margins of the parameters in Eqs. (4.9a) and (4.9b).

The expansion of $S$ is performed in terms of $\delta \phi$ which is found to be

$$\tilde{\delta} \phi = \langle J \rangle \delta \phi$$

with

$$\langle J \rangle \simeq \sqrt{\kappa n}/\langle \phi \rangle = 2^{(2-n)/2n} \sqrt{\kappa n c_T^{1/n}}, \quad (4.11)$$

where $\kappa$ is defined in Eq. (4.3). Note, in passing, that the mass of $\tilde{\delta} \phi$ at the SUSY vacuum in Eq. (3.9) is given by

$$\tilde{m}_{\delta \phi} = \left( \frac{\tilde{V}_{1,\tilde{\delta} \phi}}{} \right)^{1/2} \simeq \frac{\lambda}{\sqrt{2 \kappa c_T}} \simeq \frac{2\sqrt{6\Lambda_\pi}}{N_*} \simeq 1.25 \cdot 10^{-5}, \quad (4.12)$$

precisely equal to that found in Eqs. (2.19) and (2.30). We observe that $\tilde{m}_{\delta \phi}$ is essentially independent of $n$ and $\kappa$, thanks to the relation between $\lambda$ and $c_T$ in Eq. (4.7).

Expanding the second term in the r.h.s. of Eq. (3.1a) about $\langle \phi \rangle$ with $J$ given by the first relation in Eq. (3.25b), we obtain

$$J^2 \tilde{\phi}^2 = \left( 1 - \frac{2}{n} \sqrt{\kappa} \tilde{\delta} \phi + \frac{3}{n^2 \kappa} \tilde{\delta} \phi^2 - \frac{4}{n^3 \kappa} \tilde{\delta} \phi^3 + \cdots \right) \tilde{\delta} \phi^2. \quad (4.13a)$$

On the other hand, $\tilde{V}_1$ in Eq. (3.22a) can be expanded about $\langle \phi \rangle$ as follows

$$\tilde{V}_1 \simeq \frac{\lambda^2 \tilde{\phi}^2}{4 \kappa c_T^2} \left( 1 - \frac{n + 1}{\sqrt{\kappa n}} \tilde{\delta} \phi + (1 + n) \frac{11 + 7n}{12n^2 \kappa} \tilde{\delta} \phi^2 - \cdots \right). \quad (4.13b)$$

Since the expansions above are $c_T$ independent, we infer that $\Lambda_{UV} = 1$ as in the other versions of Starobinsky-like inflation. The expansions above for $K = K_{3i}$ and $n = 2$ reduce to those in Eqs. (2.32a) and (2.32b). Moreover, these are compatible with the ones presented in Ref. [17] for $K = K_{3i}$ and those in Ref. [31] for $K = K_{2i}$ and $n_{11} = 2$. Our overall conclusion is that our models do not face any problem with perturbative unitarity up to $m_p$. 

\begin{table}[h]
\centering
\begin{tabular}{|c|c|c|c|c|c|c|c|c|}
\hline
\textbf{Kähler Potential} & \textbf{Input} & \textbf{Parameters} & \textbf{Output} \\
\hline
$K$ & $n$ & $c_T$ & $\phi_*$ & $\lambda (10^{-3})$ & $\phi_I$ & $n_s$ & $\alpha_s (10^{-4})$ & $r(10^{-3})$ \\
\hline
$K_{3i}$ & 1 & 1 & 54.5 & 0.022 & 1.5 & 0.964 & -6.3 & 3.6 \\
$K_{2i}$ & 1 & 1 & 80 & 0.028 & 1.7 & 0.964 & -6.6 & 2.5 \\
$K_{3i}$ & 2 & 77 & 1 & 1.7 & 0.17 & 0.964 & -6.7 & 3.7 \\
$K_{3i}$ & 3 & 109 & 1 & 2.4 & 0.3 & 0.964 & -6.5 & 3.7 \\
$K_{2i}$ & 2 & 113 & 1 & 2 & 0.15 & 0.964 & -6.7 & 2.5 \\
$K_{2i}$ & 3 & 159 & 1 & 3 & 0.3 & 0.964 & -6.7 & 2.6 \\
\hline
\end{tabular}
\caption{Input and output parameters of the models which are compatible with Eq. (2.4) for $\tilde{N}_* = 53.2$, Eq. (2.6) and Eq. (2.7).}
\end{table}
5 Conclusions and Perspectives

In this review we revisited the realization of Induced Gravity Inflation (IGI) in both a non-supersymmetric and a Supergravity (SUGRA) framework. In both cases the inflationary predictions exhibit an attractor behavior towards those of Starobinsky model. Namely, we obtained a spectral index $n_S \simeq (0.960 - 0.965)$ with negligible running $\alpha_\nu$ and a tensor-to-scalar ratio $0.001 \lesssim r \lesssim 0.005$. The mass of the inflaton turns out to be close to $3 \cdot 10^{13}$ GeV. It is gratifying that IGI can be achieved for subplanckian values of the initial (non-canonically normalized) inflaton, and the corresponding effective theories are trustable up to Planck scale, although a parameter has to take relatively high values. Moreover, the one-loop radiative corrections can be kept under control.

In the SUGRA context this type of inflation can be incarnated using two chiral superfields, $T$ and $S$, the superpotential in Eq. (3.10), which realizes easily the idea of induced gravity, and several (semi)logarithmic Kähler potentials $K_{3i}$ or $K_{2i}$ – see Eq. (3.12). The models are pretty much constrained upon imposing two global symmetries – a continuous $R$ and a discrete $Z_N$ symmetry – in conjunction with the requirement that the original inflaton, $T$, takes subplanckian values. We paid special attention to the issue of $S$ stabilization during IGI, and worked out its dependence on the functional form of the selected $K$’s with respect to $|S|^2$. More specifically, we tested the functions $h_i(|S|^2)$, which appear in $K_{3i}$ or $K_{2i}$ – see Table 1. We singled out $h_2(|S|^2)$ and $h_8(|S|^2)$ for $K = K_{3i}$ or $h_3(|S|^2)$ and $h_{11}(|S|^2)$ for $K = K_{2i}$, which ensure that $S$ is heavy enough, and so well stabilized during and after inflation. This analysis provides us with new results that do not appear elsewhere in the literature. Therefore, Starobinsky inflation realized within this SUGRA set-up preserves its original predictive power, since no mixing between $|T|^2$ and $|S|^2$ is needed for consistency in the considered $K$’s – cf. Ref. [17, 37].

It is worth emphasizing that the $S$-stabilization mechanisms proposed in this paper can be also employed in other models of ordinary [26] or kinetically modified [38] non-minimal chaotic (and Higgs) inflation driven by a gauge singlet [26, 28, 38] or non-singlet [27, 39] inflaton, without causing any essential alteration to their predictions. The necessary modifications involve replacing the $|S|^2$ part of $K$ with $h_2(|S|^2)$, or $h_8(|S|^2)$ if we have a purely logarithmic Kähler potential. Otherwise, the $|S|^2$ part can be replaced by $h_3(|S|^2)$ or $h_{11}(|S|^2)$. Obviously, the last case can be employed for logarithmic or polynomial $K$’s as regards to the inflaton terms.

Let us, finally, remark that a complete inflationary scenario should specify a transition to the radiation dominated era. This transition could be facilitated in our setting [15, 34, 35] via the process of perturbative reheating, according to which the inflaton after inflation experiences an oscillatory phase about the vacuum, given by Eq. (2.22) for the non-SUSY case or Eq. (3.9) for the SUGRA case. During this phase, the inflaton can safely decay, provided that it couples to light degrees of freedom in the lagrangian of the full theory. This process is independent of the inflationary observables and the stabilization mechanism of the non-inflaton field. It depends only on the inflaton mass and the strength of the relevant couplings. This scheme may also explain the origin of the observed baryon asymmetry through non-thermal leptogenesis, consistently with the data from the neutrino oscillations [15]. It would be nice to obtain a complete and predictable transition to the radiation dominated era. An alternative graceful exit can be achieved in the running vacuum models, as described in the fourth paper of Ref. [43].

Disclaimer

The authors declare that there is no conflict of interest regarding the publication of this paper.
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