QUARK MATTER SYMMETRY ENERGY AND QUARK STARS

PENG-CHENG CHU and LIE-WEN CHEN

1 Department of Physics and Astronomy and Shanghai Key Laboratory for Particle Physics and Cosmology, Shanghai Jiao Tong University, Shanghai 200240, China; lwchen@sjtu.edu.cn
2 Center of Theoretical Nuclear Physics, National Laboratory of Heavy Ion Accelerator, Lanzhou 730000, China

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ABSTRACT

We extend the confined-density-dependent-mass (CDDM) model to include isospin dependence of the equivalent quark mass. Within the confined-isospin-density-dependent-mass (CIDDM) model, we study the quark matter symmetry energy, the stability of strange quark matter, and the properties of quark stars. We find that including isospin dependence of the equivalent quark mass can significantly influence the quark matter symmetry energy as well as the properties of strange quark matter and quark stars. While the recently discovered large mass pulsars PSR J1614−2230 and PSR J0348+0432 with masses around 2 $M_{\odot}$ cannot be quark stars within the CDDM model, they can be well described by quark stars in the CIDDM model. In particular, our results indicate that the two-flavor $u$–$d$ quark matter symmetry energy should be at least about twice that of a free quark gas or normal quark matter within the conventional Nambu–Jona-Lasinio model in order to describe PSR J1614−2230 and PSR J0348+0432 as quark stars.

Key words: dense matter – equation of state – stars: neutron

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1. INTRODUCTION

One of the fundamental issues in contemporary nuclear physics, astrophysics, and cosmology is the investigation of the properties of strong interaction matter, especially its equation of state (EOS), which plays a central role in understanding the nuclear structures and reactions, many critical issues in astrophysics, and the matter state at early universe. Quantum chromodynamics (QCD) is believed to be the fundamental theory for the strong interaction. Although the perturbative QCD (pQCD) has achieved impressive success in describing high energy processes, the direct application of QCD to lower temperature regime with zero baryon density (baryon chemical potential). However, the regime of finite baryon density is still inaccessible by Monte Carlo because of the Fermion sign problem (Barbour et al. 1986).

In a terrestrial laboratory, heavy ion collisions (HICs) provide a unique tool to explore the properties of strong interaction matter. The experiments of high energy HICs performed (or being performed) in the Relativistic Heavy Ion Collider (RHIC) at BNL and the Large Hadron Collider (LHC) at CERN have revealed many interesting features of strong interaction matter at zero baryon density and high temperature. Instead of the original picture of a hot ideal gas of noninteracting deconfined quarks and gluons at zero baryon density and high temperature, the experimental data support a new picture of quarks and gluons forming a strongly interacting system, just like a perfect liquid, in which nonperturbative physics plays an important role (Tang 2009). On the other hand, the properties of strong interaction matter at higher baryon density regions can be explored by the beam-energy scan program at RHIC, which aims to give a detailed picture of the QCD phase structure, especially to locate the so-called QCD critical point (Stephanov et al. 1998). Knowledge of strong interaction matter at high baryon density regions can be further complemented by future experiments planned in the Facility for Antiproton and Ion Research (FAIR) at GSI and the Nuclotron-based Ion Collider Facility (NICA) at JINR.

In nature, the compact stars provide another way to explore the properties of strong interaction matter at high baryon density (and low temperature). Neutron stars (NSs) have been shown to provide natural testing grounds of our knowledge about the EOS of neutron-rich nuclear matter (Lattimer & Prakash 2004; Steiner et al. 2005). In the interior (or core) of NSs, there may exist hyperons, meson condensations, and even quark matter. Theoretically, NSs may be converted to (strange) quark stars (QSs; Bombaci et al. 2004; Staff et al. 2007; Herzog & Röpke 2011), which is made purely of absolutely stable deconfined $u$, $d$, and $s$ quark matter (with some leptons), i.e., strange quark matter (SQM). Although most observations related to compact stars can be explained by the conventional NS models, the QS hypothesis cannot be conclusively ruled out. One important feature of QSs is that for a fixed mass (especially for a lighter mass), QSs usually tend to have smaller radii than NSs (Kapoor & Shukre 2001). It has been argued (Weber 2005) that the unusual small radii exacted from observational data support that the compact objects SAX J1808.4C3658, 4U 1728C34, 4U 1820C30, RX J1856.5C3754, and Her X-1 are QSs rather than NSs. The possible existence of QSs is one of the most intriguing aspects of modern astrophysics and has important implications for astrophysics and the strong interaction physics, especially the properties of SQM that essentially determine the structure of QSs (Ivanenko & Kurdgelaidze 1969; Itoh 1970; Bodmer 1971; Witten 1984; Farhi & Jaffe 1984; Alcock et al. 1986; Weber 2005).

The EOS of dense quark matter is usually soft because of the asymptotic freedom of QCD for quark–quark interactions at extremely high density. In addition, the EOS of SQM will be further softened because of the addition of the $s$ quark,
which contributes a new degree of freedom. Therefore, most of quark matter models predict relatively smaller maximum mass of QSs. Recently, by using the general relativistic Shapiro delay, the mass of PSR J1614−2230 was precisely measured to be 1.97 ± 0.04 \( M_\odot \) (Demorest et al. 2010). This high mass seems to rule out conventional QS models (whose EOS’s are soft), although some other models of pulsar-like stars with quark matter can still describe the large mass pulsar (Alford & Reddy 2003; Baldo et al. 2003; Rüster & Rischke 2004; Alford et al. 2005, 2007; Klähn et al. 2007; Ippolito et al. 2008; Lai & Xu 2011; Weissborn et al. 2011; de Avellar et al. 2011; Bonanno & Sedrakian 2012). All these models seem to indicate that to obtain a large mass (about 2 \( M_\odot \)) pulsar-like star with quark matter, the interaction between quarks should be very strong, which is remarkably consistent with the finding that quarks and gluons form a strongly interacting system in high energy HICs.

In QSs, the \( u-d \) quark asymmetry (isospin asymmetry) could be large; thus, the isovector properties of SQM may play an important role. Furthermore, the quark matter formed in high energy HICs at RHIC/LHC (and future FAIR/NICA) generally also has unequal \( u \) and \( d \) (\( u \) and \( d \)) quark numbers, i.e., it is isospin asymmetric. In recent years, some interesting features of the QCD phase diagram at finite isospin have been revealed on the basis of LQCD and some phenomenological models (Son & Stephanov 2001; Frank et al. 2003; Toublan & Kogut 2003; Kogut & Sinclair 2004; He & Zhuang 2005; He et al. 2005; Di Toro et al. 2006; Zhang & Liu 2007; Pagliara & Schaffner-Bielich 2010; Shao et al. 2012). These studies are all related to the isovector properties of quark matter, which is poorly known, especially at finite baryon density. Therefore, it is of great interest and critical importance to explore the isovector properties of quark matter, which is useful for understanding the properties of QSs, the isospin dependence of hadron-quark phase transition as well as the QCD phase diagram, and the isospin effects of partonic dynamics in high energy HICs.

In the present work, we show that QSs provide an excellent astrophysical laboratory to explore the isovector properties of quark matter at high baryon density. Through extending the confined-density-dependent-mass (CDDM) model to include isospin dependence of the equivalent quark mass, we investigate the quark matter symmetry energy and the properties of SQM and QSs. We find that although the maximum mass of QSs within the original CDDM model is significantly smaller than 2 \( M_\odot \), the isospin dependence of the equivalent quark mass introduced in the extended confined-isospin-density-dependent-mass (CIDDM) model can significantly influence the properties of SQM, and the large mass pulsar with a mass of 2 \( M_\odot \) can be well described by a QS if appropriate isospin dependence of the equivalent quark mass is applied.

2. THE THEORETICAL FORMULISM

2.1. The Confined Isospin and Density Dependent Mass Model

According to the Bodmer–Witten–Terazawa hypothesis (Witten 1984; Weber 2005), SQM might be the true ground state of QCD matter (i.e., the strong interaction matter) and is absolutely stable. Furthermore, Farhi and Jaffe found that SQM is stable near nuclear saturation density for a large model parameter space (Farhi & Jaffe 1984). The properties of SQM generally cannot be calculated directly from pQCD or LQCD, because SQM has finite baryon density and its energy scale is not very high. To understand the properties of SQM, people have built some QCD-inspired effective phenomenological models, such as the MIT bag model (Chodos et al. 1974; Farhi & Jaffe 1984; Alcock et al. 1986; Alford et al. 2005; Weber 2005), the Nambu–Jona-Lasinio (NJL) model (Rehberg et al. 1996; Hanauske et al. 2001; Rüster & Rischke 2004; Menezes et al. 2006), the pQCD approach (Freedman & McLerran 1977, 1978; Fraga et al. 2001, 2002; Fraga & Romatschke 2005; Fraga 2006; Kurkela et al. 2010), the Dyson–Schwinger approach (Roberts & Williams 1994; Zong et al. 2005; Qin et al. 2011; Li et al. 2011b), the CDDM model (Fowler et al. 1981; Chakrabarty et al. 1989; Chakrabarti 1991, 1993, 1996; Benvenuto & Lugones 1995; Peng et al. 1999, 2000, 2008; Zhang & Su 2002; Wen et al. 2005; Mao et al. 2006; Wu et al. 2008; Yin & Su 2008), and the quasi-particle model (Schertler et al. 1997, 1998; Pesher et al. 2000; Horvath & Lugones 2004; Alford et al. 2007). At extremely high baryon density, SQM could be in a color-flavor-locked (CFL) state (Rajagopal & Wilczek 2000), in which the current masses of \( u \), \( d \), and \( s \) quarks become less important compared with their chemical potentials and the quarks have equal fractions with the lepton number density being zero according to charge neutrality.

In quark matter models, one of most important things is to treat quark confinement. The MIT bag model and its density dependent versions provide a popular way to treat quark confinement. Another popular way to treat quark confinement is to vary the interaction part of quark mass, such as the CDDM model and the quasi-particle model. In the present work, we focus on the CDDM model in which the quark confinement is modeled by the density dependence of the interaction part of quark mass, i.e., the density dependent equivalent quark mass. In the CDDM model, the (equivalent) quark mass in quark matter with baryon density \( n_\text{B} \) is usually parameterized as

\[
m_q = m_{q0} + m_I = m_{q0} + \frac{D}{n_\text{B}^z},
\]

where \( m_{q0} \) is the quark current mass, \( m_I = D/n_\text{B}^z \) reflects the quark interactions in quark matter that is assumed to be density dependent, \( z \) is the quark mass scaling parameter, and \( D \) is a parameter determined by stability arguments of SQM. In the original CDDM model used to study two-flavor \( u-d \) quark matter (Fowler et al. 1981), an inversely linear quark mass scaling (i.e., \( z = 1 \)) was assumed and the parameter \( D \) was taken to be three times the famous MIT bag constant. The CDDM model was later extended to include \( s \) quarks to investigate the properties of SQM (Chakrabarti et al. 1989; Chakrabarti 1991, 1993, 1996; Benvenuto & Lugones 1995). Obviously, the CDDM model satisfies two basic features of QCD, i.e., the asymptotic freedom and quark confinement through density dependence of the equivalent quark mass (i.e., \( \lim_{n_\text{B} \to \infty} m_I = 0 \) and \( \lim_{n_\text{B} \to 0} m_I = \infty \)). For two-flavor \( u-d \) quark matter, the chiral symmetry is restored at high density because of \( \lim_{n_\text{B} \to \infty} m_q = 0 \) if the current masses of \( u \) and \( d \) quarks are neglected.

The density dependence of the interaction part of the quark mass (i.e., \( m_I = D/n_\text{B}^z \)) is phenomenological in the CDDM model, and in principle it should be determined by nonperturbative QCD calculations. Instead of the inversely linear density dependence for \( m_I \), which is based on the bag model argument, a quark mass scaling parameter of \( z = 1/3 \) was derived on the basis of the in-medium chiral condensates and linear confinement (Peng et al. 1999) and has been widely used for exploring the properties of SQM and QSs since then (Lugones & Horvath 2003; Zheng et al. 2004; Peng et al. 2006, 2008; Wen et al. 2007; Li et al. 2011a). In a recent work (Li et al. 2010),
Li A. et al. investigated the stability of SQM and the properties of the corresponding QSs for a wide range of quark mass scalings. Their results indicate that the resulting maximum mass of QSs always lies between 1.5 $M_{\odot}$ and 1.8 $M_{\odot}$ for all the scalings chosen there. This implies that the large mass pulsar PSR J1614–2230 with a mass of 1.97 ± 0.04 $M_{\odot}$ cannot be a QS within the CDDM model. In particular, the maximum mass with scaling parameter $z = 1/3$ is only about 1.65 $M_{\odot}$, which is significantly less than 1.97 ± 0.04 $M_{\odot}$.

Physically, the quark–quark effective interaction in quark matter should be isospin dependent. On the basis of chiral perturbation theory, it has been shown recently (Kaiser & Weise 2009) that the in-medium density dependent chiral condensates are significantly dependent on the isospin. The isospin dependence of the in-medium chiral condensates can also be seen from the QCD sum rules (Drukarev et al. 2004; Jeong & Lee 2013). In addition, the quark–quark interaction in quark matter will be screened because of pair creation and infrared divergence, and the (Debye) screening length is also isospin dependent (Dey et al. 1998). These features imply that the equivalent quark mass in Equation (1) should be isospin dependent, which is neglected in the CDDM model. However, the detailed form of isospin dependence of the equivalent quark mass is unknown, and in principle it should be determined by nonperturbative QCD calculations. In the present exploratory work, we extend the CDDM model to include the isospin dependence of the quark–quark effective interactions by assuming a phenomenological parameterization form that takes the basic features of QCD into consideration, such as asymptotic freedom, quark confinement, and isospin symmetry, for the equivalent quark mass in isospin asymmetric quark matter with isospin asymmetry $\delta$, i.e.,

$$m_q = m_{q_0} + m_t + m_{\text{iso}} = m_{q_0} + \frac{D}{n_B} - \tau_q \delta D_q n_B e^{-\beta n_B},$$

where $D_q$, $\alpha$, and $\beta$ are parameters determining isospin dependence of the quark–quark effective interactions in quark matter, and $\tau_q$ is the isospin quantum number of quarks — here we set $\tau_u = 1$ for $q = u$ (u quarks), $\tau_d = -1$ for $q = d$ (d quarks), and $\tau_s = 0$ for $q = s$ (s quarks). The isospin asymmetry is defined as

$$\delta = \frac{n_d - n_u}{n_d + n_u},$$

which equals $-n_3/n_B$ with isospin densities of $n_3 = n_u - n_d$ and $n_B = (n_u + n_d)/3$ for two-flavor $u$–$d$ quark matter. The above definition of $\delta$ for quark matter has been extensively used in the literature (Di Toro et al. 2006, 2010; Pagliara & Schaffner-Bielich 2010; Shao et al. 2012). We note that one has $\delta = 1$ ($-1$) for quark matter converted by pure neutron (proton) matter according to the nucleon constituent quark structure, which is consistent with the conventional definition for nuclear matter, i.e., $(\rho_n - \rho_p)/(\rho_n + \rho_p) = -n_3/n_B$.

In Equation (2), the last term $m_{\text{iso}}$ provides a simple and convenient phenomenological parameterization form for isospin dependence of the equivalent quark mass, which respects the asymptotic freedom and quark confinement. Indeed, one can see that the quark confinement condition $\lim_{n_B \to \infty} m_q = \infty$ can be guaranteed if $\alpha > 0$ or $\alpha = 0$. For $\alpha = 0$ in particular, a finite isospin splitting of the equivalent quark mass will appear even at zero baryon density. In addition, if $\beta > 0$, then $\lim_{n_B \to \infty} m_{\text{iso}} = 0$; thus, the asymptotic freedom $\lim_{n_B \to \infty} m_q = m_{\text{iso}}$ is satisfied. Therefore, in general, the parameter $\alpha$ should be nonnegative and the parameter $\beta$ should be positive in Equation (2). Using different values of $\alpha$ and $\beta$ can flexibly mimic different density dependences for the isospin dependent equivalent quark mass and thus different density dependences for the quark matter symmetry energy. In this exploratory work, we determine $\alpha$ and $\beta$ by assuming the density dependence of the quark matter symmetry energy has some well-known empirical forms. The parameter $D_q$ can be used to adjust the strength of the isospin dependent equivalent quark mass and thus the strength of the quark matter symmetry energy. In addition, for the quark mass scaling parameter $z$, in this work we mainly focus on $z = 1/3$ since it can be derived on the basis of the in-medium chiral condensates and linear confinement (Peng et al. 1999). However, we also study how our main results change if the quark mass scaling parameter $z$ can be varied freely. Obviously, in the extended CDDM model the equivalent quark mass in Equation (2) satisfies the exchange symmetry between $u$ and $d$ quarks, which is required by isospin symmetry of the strong interaction. Therefore, the phenomenological parameterization form of the isospin dependent equivalent quark mass in Equation (2) is quite general and satisfies the basic features of QCD. Although other functional forms could be used to describe the isospin dependence of the equivalent quark mass, the exact form of $m_{\text{iso}}$ is not crucial so long as the functional and its parameters are chosen to be consistent with the asymptotic freedom, quark confinement, and isospin asymmetry, as well as some empirical forms of the symmetry energy.

### 2.2. The Quark Matter Symmetry Energy

Similar to the case of nuclear matter (see, e.g., Li et al. 2008), the EOS of quark matter consisting of $u$, $d$, and $s$ quarks, defined by its binding energy per baryon number, can be expanded in isospin asymmetry $\delta$ as

$$E(n_B, \delta, n_s) = E_0(n_B, n_s) + E_{\text{sym}}(n_B, n_s)\delta^2 + O(\delta^4),$$

where $E_0(n_B, n_s) = E(n_B, \delta = 0, n_s)$ is the binding energy per baryon number in three-flavor $u$–$d$–$s$ quark matter with an equal fraction of $u$ and $d$ quarks; the quark matter symmetry energy $E_{\text{sym}}(n_B, n_s)$ is expressed as

$$E_{\text{sym}}(n_B, n_s) = \frac{1}{2} \left. \frac{\partial^2 E(n_B, \delta, n_s)}{\partial \delta^2} \right|_{\delta = 0}.$$

In Equation (4), the absence of odd-order terms in $\delta$ is due to the exchange symmetry between $u$ and $d$ quarks in quark matter when one neglects the Coulomb interaction among quarks. The higher-order coefficients in $\delta$ are usually very small, and this has been verified by the calculations with the model parameters in the present work. Neglecting the contribution from higher-order terms in Equation (4) leads to the empirical parabolic law, i.e., $E(n_B, \delta, n_s) \simeq E_0(n_B, n_s) + E_{\text{sym}}(n_B, n_s)\delta^2$ for the EOS of isospin asymmetric quark matter, and the quark matter symmetry energy can thus be extracted approximately from the following expression:

$$E_{\text{sym}}(n_B, n_s) \simeq \frac{1}{9} \{ E(n_B, \delta = 3, n_s) - E(n_B, \delta = 0, n_s) \}.$$

On the basis of the model parameters used in the present work, we have checked that the above expression is a pretty
good approximation. However, we have still used the exact analytical expression of the quark matter symmetry energy in all calculations in the following.

In quark matter consisting of $u$, $d$, and $s$ quarks, the baryon number density is given by $n_B = (n_u + n_d + n_s)/3$, and the quark number density can be expressed as

$$n_i = \frac{8}{2\pi^2} \int_0^{v_i} \frac{k^2 dk}{\pi^2} = \frac{v_i^3}{\pi^2},$$

(7)

where $g_i = 6$ is the degeneracy factor of quarks, and $v_i$ is the Fermi momentum of different quarks ($i = u, d, s$). Furthermore, the Fermi momenta of $u$ and $d$ quarks can be expressed, respectively, as

$$v_u = (1 - \delta/3)^{1/2} v, \quad v_d = (1 + \delta/3)^{1/2} v,$$

(8)

where $v$ is the quark Fermi momentum of symmetric $u$-$d$ quark matter at quark number density $n = 2n_u = 2n_d$. The total energy density of the $u$-$d$-$s$ quark matter can then be expressed as

$$\epsilon_{uds} = \frac{g}{2\pi^2} \left[ \int_0^{(1-\delta)/3} \sqrt{k^2 + m_u^2 k^2} dk + \int_0^{(1+\delta)/3} \sqrt{k^2 + m_d^2 k^2} dk + \int_{v_d}^{v} \sqrt{k^2 + m_s^2 k^2} dk \right].$$

(9)

Using the isospin and density dependent equivalent quark masses as in Equation (2), one can obtain analytically the quark matter symmetry energy as

$$E_{sym}(n_B, n_s) = \frac{1}{2} \left[ \frac{\varepsilon_{uds}}{n_B} \right]_{\delta=0} = \frac{g}{2\pi^2} \left[ \sqrt{v^2 + 18mD_1n_B^a} e^{-\beta_{aB}} \right] \frac{18\sqrt{v^2 + m^2}}{18\sqrt{v^2 + m^2}} + A + B \frac{3n_B - n_s}{3n_B},$$

(10)

with

$$A = \frac{9m^2}{2v^2\sqrt{v^2 + m^2}} (D_1n_B^{a} e^{-\beta_{aB}})^2,$$

(11)

$$B = \frac{9}{4v^3} \left[ \sqrt{v^2 + m^2} - 3m^2 \ln \left( \frac{v\sqrt{v^2 + m^2}}{m} \right) \right] \times (D_1n_B^{a} e^{-\beta_{aB}})^2,$$

(12)

and $m = m_{u0}$ (or $m_{d0}$) + $(D/n_B^{2})$. In the present work, we assume $m_{u0} = m_{d0} = 5.5$ MeV and $m_{s0} = 80$ MeV. In the CDDM model, the quark matter symmetry energy is reduced to

$$E_{sym}(n_B, n_s) = \frac{1}{18} \frac{v^2}{\sqrt{v^2 + m^2}} \frac{3n_B - n_s}{3n_B}.$$  

(13)

It should be noted that the quark matter symmetry energy generally depends on the fraction of $s$ quarks in quark matter since $s$ quarks contribute to the baryon density $n_B$. For two-flavor $u$-$d$ quark matter, the quark matter symmetry energy is reduced to the well-known expression $E_{sym}(n_B) = (1/18)(v^2/\sqrt{v^2 + m^2})$.

### 2.3. Properties of Strange Quark Matter

For SQM, we assume it is neutrino-free and composed of $u$, $d$, and $s$ quarks and $e^-$ in beta-equilibrium with electric charge neutrality. The weak beta-equilibrium condition can then be expressed as

$$\mu_u + \mu_e = \mu_d = \mu_s,$$

(14)

where $\mu_i$ ($i = u, d, s$, and $e^-$) is the chemical potential of the particles in SQM. Furthermore, the electric charge neutrality condition can be written as

$$\frac{2}{3} n_u = \frac{1}{3} n_d + \frac{1}{3} n_s + n_e.$$  

(15)

The chemical potential of particles in SQM can be obtained as

$$\mu_i = \frac{d\epsilon}{dn_i} = \sqrt{v_i^2 + m_i^2} + \sum_j n_j \frac{\partial m_j}{\partial n_i} f \left( \frac{v_j}{m_j} \right) + \sum_j n_j \frac{\partial m_j}{\partial \delta} \frac{\partial \delta}{\partial n_i} f \left( \frac{v_j}{m_j} \right),$$

(16)

with

$$f(x) = \frac{3}{2\chi^3} \left[ x\sqrt{(x^2 + 1)} + \ln(x + \sqrt{x^2 + 1}) \right].$$

(17)

and $\epsilon$ is the total energy density of SQM. One can see clearly from Equation (16) that the chemical potential of quarks in SQM has two additional parts compared with the case of free Fermi gas, because of the density and isospin dependence of the equivalent quark mass, respectively. In particular, the $u$ quark chemical potential can be expressed analytically as

$$\mu_u = \sqrt{v_u^2 + m_u^2} + \frac{1}{3} \sum_{j=u,d,s} n_j \frac{f(v_j)}{m_j}$$

$$\times \left[ \frac{-zD}{n_B^{u+e}} - \tau_j D_1 \delta(a \eta_{u-1} - \beta n_{uB}) e^{-\beta_{aB}} \right]$$

$$+ D_1 n_B^{uB} e^{-\beta_{aB}} \left[ n_u f \left( \frac{v_u}{m_u} \right) - n_d f \left( \frac{v_d}{m_d} \right) \right]$$

$$\times \frac{6n_d}{(n_u + n_d)^2},$$

(18)

For $d$ and $s$ quarks, we have, respectively,

$$\mu_d = \sqrt{v_d^2 + m_d^2} + \frac{1}{3} \sum_{j=u,d,s} n_j \frac{f(v_j)}{m_j}$$

$$\times \left[ \frac{-zD}{n_B^{u+e}} - \tau_j D_1 \delta(a \eta_{u-1} - \beta n_{uB}) e^{-\beta_{aB}} \right]$$

$$+ D_1 n_B^{uB} e^{-\beta_{aB}} \left[ n_d f \left( \frac{v_d}{m_d} \right) - n_u f \left( \frac{v_u}{m_u} \right) \right]$$

$$\times \frac{6n_u}{(n_u + n_d)^2},$$

(19)

and

$$\mu_s = \sqrt{v_s^2 + m_s^2} + \frac{1}{3} \sum_{j=u,d,s} n_j \frac{f(v_j)}{m_j}$$

$$\times \left[ \frac{-zD}{n_B^{u+e}} - \tau_j D_1 \delta(a \eta_{u-1} - \beta n_{uB}) e^{-\beta_{aB}} \right].$$

(20)
Figure 1. Quark matter symmetry energy as a function of baryon number density in the CIDDM model with three parameter sets, i.e., DI-0, DI-300, and DI-2500. The two-flavor u–d quark matter with \( n_s = 0 \) (left window) and the u–d–s quark matter with \( n_s = n_B \) (right window) are considered. The nuclear matter symmetry energy from the RMF model with interaction NL\( \rho \delta \) (Liu et al. 2002), which includes the isovector-scalar \( \delta \) meson field, is obtained by fitting the empirical properties of asymmetric nuclear matter, and describes reasonably well the binding energies and charge radii of a large number of nuclei (Gaitanos et al. 2004). Although the density dependence of the nuclear matter symmetry energy is still largely uncertain, especially at supersaturation density (for a recent review, see, e.g., Chen 2012), the NL\( \rho \delta \) result nevertheless represents a typical prediction for the nuclear matter symmetry energy.

For all the parameter sets, DI-0, DI-300, and DI-2500, we have fixed \( z = 1/3 \). In particular, for the parameter set DI-0 we have \( D_I = 0, \alpha = 0, \beta = 0, \) and \( D = 125.328 \text{ MeV fm}^{-3} \), which corresponds to a typical parameter set in the CDDM model and is used here mainly for comparison purposes. For the parameter set DI-300, we have \( D_I = 300 \text{ MeV fm}^{-3}, \alpha = 1, \beta = 0.1 \text{ fm}^{-3}, \) and \( D = 115.549 \text{ MeV fm}^{-3} \), with the values of \( D_I, \alpha, \) and \( \beta \) having been obtained so that the quark matter symmetry energy has roughly the same strength and density dependence as the nuclear matter symmetry energy predicted by NL\( \rho \delta \), while the value of \( D \) was obtained to guarantee the stability of SQM. Comparing the results in DI-300 to those in DI-0, one can see how the properties of SQM and QSs will be influenced if the quark matter symmetry energy has a similar strength as that of nuclear matter symmetry energy. For the parameter set DI-2500, we have \( D_I = 2500 \text{ MeV fm}^{-3}, \alpha = 0.8, \beta = 0.1 \text{ fm}^{-3}, \) and \( D = 105.084 \text{ MeV fm}^{-3} \), and as shown in the following these parameter values have been obtained by searching for the minimum \( D_I \) value so that the maximum mass of a QS can reach 1.93 \( M_\odot \) to be consistent with the recently discovered large mass pulsar PSR J1614–2230 with a mass of 1.97 ± 0.04 \( M_\odot \).

It is seen from Figure 1 that the three parameter sets, DI-0, DI-300, and DI-2500, give very different predictions for the density dependent quark matter symmetry energy; thus, the three parameter sets allow us to explore the quark matter symmetry energy effects. In particular, one can see that the symmetry energy of two-flavor u–d quark matter predicted by DI-300 is nicely in agreement with that of nuclear matter with NL\( \rho \delta \), while the u–d quark matter symmetry energy predicted by DI-2500 is about 50 times the nuclear matter symmetry energy. On the other hand, the amplitude of the quark matter symmetry energy predicted by DI-0 is much smaller than that of the nuclear matter symmetry energy. As we will show later, the parameter set DI-0 predicts roughly the same quark matter symmetry energy as that of a free quark gas or normal quark matter within the conventional NL\( \delta \) model. In addition, one can see from Figure 1 that increasing the \( z \) parameter in u–d–s quark matter reduces the quark matter symmetry energy as expected since s quarks contribute to the baryon density \( n_B \), while the symmetry energy is defined by per baryon number.

In the CIDDM model, we can generally increase the amplitude of quark matter symmetry energy by increasing the \( D_I \) value. It should be mentioned that when the value of \( D_I \) parameter is varied, the other three parameters, \( \alpha, \beta, \) and \( D \) (the parameter \( D \) if \( \alpha \) and \( \beta \) have been fixed), usually need corresponding readjustments to guarantee the stability of SQM. As we will see in the following, the three parameter sets, DI-0, DI-300, and DI-2500, all satisfy the stability conditions of SQM;
thus, they can be used to study the quark matter symmetry energy effects on the properties of SQM and QSs.

### 3.2. The Stability of SQM

Following Farhi and Jaffe (Farhi & Jaffe 1984), the absolute stability of SQM requires that the minimum energy per baryon of SQM should be less than the minimum energy per baryon of observed stable nuclei, i.e., $M(^{56}\text{Fe})c^2/56 = 930$ MeV, and at the same time the minimum energy per baryon of the beta-equilibrium two-flavor $u$–$d$ quark matter should be larger than 930 MeV to be consistent with standard nuclear physics. These stability conditions usually put very strong constraints on the value of the parameters in quark matter models.

Figure 2 shows the energy per baryon and the corresponding pressure as functions of the baryon density for SQM and two-flavor $u$–$d$ quark matter in $\beta$-equilibrium within the CIDDM model with DI-0, DI-300, and DI-2500. One can see that for all the three parameter sets, DI-0, DI-300, and DI-2500, the minimum energy per baryon of the beta-equilibrium two-flavor $u$–$d$ quark matter is larger than 930 MeV, while that of SQM is less than 930 MeV, satisfying the absolute stable conditions of SQM. Furthermore, it is seen from Figure 2 that in all cases, the baryon density at the minimum energy per baryon is exactly the zero-pressure density, which is consistent with the requirement of thermodynamical self-consistency. In particular, we note that the zero-pressure density of SQM is $0.24$ fm$^{-3}$, $0.23$ fm$^{-3}$, and $0.21$ fm$^{-3}$ for DI-0, DI-300, and DI-2500, respectively, which are not so far from the nuclear matter normal density of about $0.16$ fm$^{-3}$. Moreover, one can see from Figure 2 that the stiffness of SQM increases with the $D_I$ parameter (i.e., the quark matter symmetry energy). In addition, we have checked the sound speed in the quark matter on the basis of the calculated pressure and energy density with DI-0, DI-300, and DI-2500, and we find that the sound speed in all cases is less than the speed of light in a vacuum, thus satisfying the causality condition. We would like to note here that the causality condition is also satisfied for all other EOS’s used in this work for the calculations of QSs.

In Figure 3, we show the quark fraction as a function of the baryon density in SQM within the CIDDM model with DI-0, DI-300, and DI-2500. It is interesting to see that the difference among $u$, $d$, and $s$ quark fractions becomes smaller when the quark matter symmetry energy is increased (i.e., from DI-0 to DI-300 and then to DI-2500). When the quark matter symmetry energy is not so large (i.e., in the cases of DI-0 and DI-300), the $u$, $d$, and $s$ quark fractions are significantly different, especially at lower baryon densities, which leads to a larger isospin asymmetry in SQM. On the other hand, a large quark matter symmetry energy (i.e., DI-2500) significantly reduces the difference among $u$, $d$, and $s$ quark fractions. In particular, for DI-2500, it is remarkable to see that the $u$, $d$, and $s$ quark fractions become essentially equal and approach a value of about 0.33 for $n_B \gtrsim 0.4$ fm$^{-3}$, similar to the results from the picture of the CFL state. In NS matter, the similar symmetry energy effect has also been observed, i.e., a larger nuclear matter symmetry energy will give a larger proton fraction and thus reduce the difference between neutron and proton fractions in the beta-equilibrium NS matter (see, e.g., Xu et al. 2009).

Figure 4 shows the equivalent quark mass as a function of the baryon density in SQM within the CIDDM model with DI-0, DI-300, and DI-2500. It is seen that in all cases, the equivalent quark mass increases drastically with decreasing baryon density, reflecting the feature of quark confinement. Furthermore, interestingly one can see a clear isospin splitting of the $u$ and $d$ equivalent quark masses in SQM for the parameter sets DI-300 and DI-2500, with $d$ quarks having a larger equivalent mass than $u$ quarks. These features reflect the isospin dependence of quark–quark effective interactions in isospin asymmetric quark matter in the present CIDDM model. It should be mentioned that the isospin splitting of $u$ and $d$ equivalent quark masses in SQM with DI-2500 is smaller than that with DI-300, although the former has a much larger symmetry energy (and $D_I$ value) than the latter. This is due...
to the fact that the isospin splitting of $u$ and $d$ equivalent quark masses in SQM also depends on the isospin asymmetry, which is much smaller for DI-2500 than for DI-300 as seen in Figure 3. In particular, one can see from Figure 4 that the isospin splitting of $u$ and $d$ equivalent quark masses in SQM becomes extremely small at higher baryon densities for DI-2500 because of the extremely small isospin asymmetry in SQM as shown in Figure 3.

3.3. Quark Stars

Using the EOS’s of SQM as shown in Figure 2, one can obtain the Mass–radius relation of static QSs by solving the Tolman–Oppenheimer–Volkov equation. Shown in Figure 5 is the mass–radius relation for static QSs within the CIDDM model with DI-0, DI-300, and DI-2500. Indicated by the shaded band in Figure 5 is the measured pulsar mass of $1.79 \pm 0.04 M_\odot$ from PSR J1614–2230 (Demorest et al. 2010). (A color version of this figure is available in the online journal.)

Figure 5. Mass–radius relation for static quark stars within the CIDDM model with DI-0, DI-300, and DI-2500. The result for rotating quark stars with a spin period of 3.15 ms is also shown for the case of DI-2500 with the radius at the equator. The shaded band represents the pulsar mass of $1.79 \pm 0.04 M_\odot$ from PSR J1614–2230 (Demorest et al. 2010).

The maximum mass, the corresponding radius, and the central baryon number density of the static quark stars and the maximum rotational frequency $f_{\text{max}}$ for the maximum-mass static quark stars, as well as the corresponding gravitational mass and equatorial radius at $f_{\text{max}}$, within the CIDDM model with DI-0, DI-300, and DI-2500.

| Table 1 | Properties of Quark Stars in the CIDDM Model with DI-0, DI-300, and DI-2500 |
|---------|-------------------------------------------------|
| $M/M_\odot$ (static) | DI-0 | DI-300 | DI-2500 |
| R(km)(static) | 9.60 | 10.40 | 11.12 |
| Central density(fm$^{-3}$) | 1.31 | 1.11 | 1.06 |
| $f_{\text{max}}$(Hz) | 1680 | 1547 | 1458 |
| $M/M_\odot$ (at $f_{\text{max}}$) | 1.78 | 2.12 | 2.43 |
| R(km)(equator at $f_{\text{max}}$) | 9.93 | 11.6 | 14.2 |

Notes. The maximum mass, the corresponding radius, and the central baryon number density of the static quark stars and the maximum rotational frequency $f_{\text{max}}$ for the maximum-mass static quark stars, as well as the corresponding gravitational mass and equatorial radius at $f_{\text{max}}$, within the CIDDM model with DI-0, DI-300, and DI-2500.
corresponding radius, and the central baryon number density of the static QSs. From Table 1, one can see the maximum rotational frequency \( f_{\text{max}} \) decreases with \( D_I \), while the corresponding mass and equatorial radius increase with \( D_I \). In particular, for the parameter set DI-2500, we obtain \( f_{\text{max}} = 1458 \) Hz, and the corresponding mass is 2.43 \( M_\odot \) with a radius of 14.2 km, which essentially corresponds to the maximum mass configuration of rotating QSs that the parameter set DI-2500 can support.

3.4. Effects of the Quark Mass Scaling Parameter

As mentioned before, the quark mass scaling parameter \( z \) is phenomenological in the CDDM model, and in principle it should be determined by nonperturbative QCD calculations. In the original CDDM model (Fowler et al. 1981), an inversely linear quark mass scaling of \( z = 1 \) was assumed on the basis of the bag model argument, while a quark mass scaling parameter of \( z = 1/3 \) was derived on the basis of the in-medium chiral condensates and linear confinement (Peng et al. 1999) and \( z = 1/3 \) has been used in the calculations above. As pointed out by Li A. et al. (Li et al. 2010), however, the derivation in Peng et al. (1999) is still not well justified since only the first-order approximation of in-medium chiral condensates was considered and higher orders of the approximation could nontrivially complicate the quark mass scaling parameter. Actually, there are also some other quark mass scalings in the literature (Dey et al. 1998; Wang 2000; Zhang & Su 2002; Li et al. 2010). Therefore, it is interesting to see how the calculated results above will change if the quark mass scaling parameter \( z \) can be varied freely. As pointed out before, the CDDM model (i.e., the CDDM model with \( D_I = 0 \)) cannot describe PSR J1614–2230 as a QS even though the \( z \) parameter can be varied freely (Li et al. 2010). In the following, we look for the minimum value of \( D_I \) and thus the smallest quark matter symmetry energy that is necessary to support a QS with a mass of 1.93 \( M_\odot \) in the CDDM model.

To check the effects of the quark mass scaling parameter \( z \) and search for the smallest quark matter symmetry energy to support a QS with a mass of 1.93 \( M_\odot \), we assume the quark matter symmetry energy has similar density dependence as that predicted by the conventional NJL model or that of a free quark gas at higher baryon densities (here we consider the density region from 0.25 \( \text{fm}^{-3} \) to 1.5 \( \text{fm}^{-3} \)) since the lower density EOS will not affect the results of QSs; thus, we obtain \( \alpha = 0.7 \) and \( \beta = 0.1 \text{ fm}^3 \). Furthermore, for fixed values of the parameters \( D \) and \( D_I \), varying the scaling parameter \( z \) can significantly change the maximum mass of QSs, and we find that \( z = 1.8 \) generally gives rise to the largest QS maximum mass. For fixed parameters of \( \alpha = 0.7 \), \( \beta = 0.1 \text{ fm}^3 \), and \( z = 1.8 \), we then search for the minimum value of \( D_I \) that can support a QS with mass of 1.93 \( M_\odot \) by varying \( D_I \) and \( D \). Shown in the left panel of Figure 6 is the \( D_I \) dependence of the maximum mass of static QSs. The value of the \( D \) parameter at different \( D_I \) shown in Figure 6 corresponds to the value at which the QS maximum mass becomes the largest. It is seen from the left panel of Figure 6 that the maximum mass of QSs is sensitive to the \( D_I \) parameter and increases with \( D_I \). To obtain a QS with a mass larger than 1.93 \( M_\odot \), we find that the minimum value of \( D_I \) should be 70 MeV fm\(^{-3}\), and the corresponding parameter set is denoted as DI-70 (\( z = 1.8 \)). For DI-70 (\( z = 1.8 \)), we thus have \( D_I = 70 \text{ MeV fm}^3 \), \( \alpha = 0.7 \), \( \beta = 0.1 \text{ fm}^3 \), \( D = 24.181 \text{ MeV fm}^{-3} \), and \( z = 1.8 \). The corresponding radius of the maximum mass configuration of the QS with DI-70 (\( z = 1.8 \)) is 9.69 km, and the central baryon number density is 1.3 \( \text{fm}^{-3} \), while the surface (zero-pressure point) baryon number density is 0.48 \( \text{fm}^{-3} \).

Shown in the right panel of Figure 6 is the density dependence of the two-flavor \( u-d \) quark matter symmetry energy in the CDDM model with DI-70 (\( z = 1.8 \)). For comparison, we also include the results of DI-0 as well as the symmetry energy of a free quark gas (with current mass of 5.5 MeV) and normal quark matter within the conventional NJL model (Rehberg et al. 1996). One can see that the DI-0 parameter set and the NJL model predict a very similar quark matter symmetry energy as that of the free quark gas, while the DI-70 (\( z = 1.8 \)) parameter set predicts a two times larger quark matter symmetry energy than the free quark gas but is still significantly smaller than the nuclear matter symmetry energy predicted by NL\( \rho \delta \). Therefore, our results indicate that if the \( z \) parameter can be varied freely in the CDDM model, the quark matter symmetry energy could be smaller than the nuclear matter symmetry energy, but its strength should be at least about twice that of a free quark gas or normal quark matter within the conventional NJL model in order to describe the PSR J1614–2230 as a QS. It is interesting to mention that the symmetry energy of the two-flavor color superconductivity (2SC) phase has been shown to be about three times that of the normal quark matter phase (Pagliara & Schaffner-Bielich 2010) and thus is close to the symmetry energy predicted by DI-70 (\( z = 1.8 \)).

3.5. The Maximum Mass of Quark Stars

Very recently, a new pulsar PSR J0348+0432 with a mass of 2.01 ± 0.04 \( M_\odot \) was discovered (Antoniadis et al. 2013). This pulsar is only the second pulsar with a precisely determined mass around 2 \( M_\odot \) after PSR J1614–2230 (Demorest et al. 2010) and sets a new record for the maximum mass of pulsars. Thus, it is interesting to examine whether the new pulsar PSR J0348+0432 can be described as a QS within the CDDM model. In addition, it is also interesting to see if the CDDM model can predict even heavier QSs, which may provide useful implications of future mass measurements for pulsars.

As shown above, the maximum mass of static QSs is sensitive to both the quark matter symmetry energy (via the \( D_I \) parameter) and the quark mass scaling parameter \( z \) in the CDDM model. For \( z = 1/3 \), by increasing the value of the \( D_I \) parameter, we find that the maximum mass of static QSs will saturate at a value of 2.43 \( M_\odot \). This work was supported in part by the National Natural Science Foundation of China (Grants 11373022, 11375021, and 11075012).
of about 1.96 $M_\odot$ when $D_I$ is larger than about 3000 MeV fm$^{-3}$, and further increasing the value of the $D_I$ parameter essentially does not change the maximum mass of static QSs. Furthermore, we note that when the value of the $D_I$ parameter is very large (e.g., $D_I = 3000$ MeV fm$^{-3}$), varying the values of $\alpha$ and $\beta$ essentially has no effects on the maximum mass of static QSs (see also Figure 8 and the related discussions in the following). These interesting features can be easily understood from the fact that a very large value of $D_I$ gives an extremely large quark matter symmetry energy, which leads to almost an equal fraction of $u$, $d$, and $s$ quarks in the SQM with a very small isospin asymmetry as shown in Figure 3, and consequently the symmetry energy (and the $D_I$ parameter) effects are strongly suppressed. These results indicate that the CIDDM model with the $z$ parameter fixed at 1/3 cannot describe a 2 $M_\odot$ pulsar (e.g., the pulsar PSR J0348+0432) as a static QS. We note here that the rotation of QSs with a spin period of 39 ms as measured for PSR J0348+0432 (Antoniadis et al. 2013) essentially has no influence on the maximum mass of QSs.

To further enhance the maximum mass of static QSs, we have to vary the value of the quark mass scaling parameter $z$ in the CIDDM model. As demonstrated in Section 3.4, the quark mass scaling parameter $z$ may significantly affect the maximum mass of QSs, and a value of $z = 1.8$ generally gives the largest maximum mass of static QSs. As shown in the left panel of Figure 6, for $z = 1.8$ with $\alpha = 0.7$ and $\beta = 0.1$ fm$^3$, which leads to a similar density dependence of the quark matter symmetry energy as that predicted by the conventional NJL model (see also Figure 8 and the related discussions in the following). The parameter set and the resulting strength of the symmetry energy is still essentially has no effects on the maximum mass of static QSs. From the right panel of Figure 7, one can see that the two-flavor $u$–$d$ quark matter symmetry energy is much larger than the nuclear matter symmetry energy when $D_I$ is larger than about 2000 MeV fm$^{-3}$, and similar results are also observed in the case of DI-2500 with $z = 1/3$ as shown in the left panel of Figure 1.

For the results shown in Figure 7, we have fixed $\alpha = 0.7$ and $\beta = 0.1$ fm$^3$ to follow the density dependence of the quark matter symmetry energy of a free quark gas or that predicted by the conventional NJL model. For $D_I = 3500$ MeV fm$^{-3}$, we find that varying the values of $\alpha$ and $\beta$ only has a very small influence on the maximum mass of static QSs. In particular, when $\beta$ is fixed at 0.1 fm$^3$, the maximum mass of static QSs will become 2.39 $M_\odot$ and 2.40 $M_\odot$ for $\alpha = 0.8$ and $\alpha = 0$, respectively. Moreover, when $\alpha$ is fixed at 0, the maximum mass of static QSs will stay at the same value of 2.40 $M_\odot$ for both $\beta = 1$ and $2$ fm$^3$. In order to see how different the quark matter symmetry energy becomes with these various values of $\alpha$ and $\beta$, we plot in Figure 8 the density dependence of the two-flavor $u$–$d$ quark matter symmetry energy in the CIDDM model using $z = 1.8$ and $D_I = 3500$ MeV fm$^{-3}$ with different values of $\alpha$ and $\beta$. Similar to the left panel of Figure 6, the value of the $D_I$ parameter for different values of $\alpha$ and $\beta$ is obtained so that the QS maximum mass becomes the largest. As expected for very large $D_I$ values, one can see that although the various values of $\alpha$ and $\beta$ indeed give very different predictions for the two-flavor $u$–$d$ quark matter, all of them predict almost the same maximum mass of static QSs. We have also checked the maximum mass of static QSs with the parameter set $DI-85$ ($z = 1.8$) by keeping $D_I$ and $D$ fixed but varying $\alpha$ and $\beta$ as in Figure 8, and our results indicate a similar tiny effect on the maximum mass of QSs as in the case of $z = 1.8$ and $D_I = 3500$ MeV fm$^{-3}$ (the variation is only about 0.01 $M_\odot$). Because of the finite isospin splitting of the equivalent quark mass in asymmetric quark matter at $n_s = 0$ for $\alpha = 0$, one can see that the symmetry energy is also finite at zero baryon number density for $\alpha = 0$. From the above results and discussions,

![Figure 7](image_url)

(a) $z=1.8$
(b) $n_s=0$ (u-d QM)

| $D_I$ (MeV fm$^{-3}$) | $n_s$ (fm$^{-3}$) | $E_{\text{sym}}$ (MeV) |
|---|---|---|
| 0.00 | 0.00 | 0.00 |
| 1.00 | 1.00 | 1.00 |
| 2.00 | 2.00 | 2.00 |

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2000 MeV fm$^{-3}$. On the other hand, when $D_I$ is larger than about 2000 MeV fm$^{-3}$(which predicts a value of 2.38 $M_\odot$ for the maximum mass of static QSs), the maximum mass of static QSs becomes insensitive to the $D_I$ parameter, and similar to the case of $z = 1/3$, the maximum mass of static QSs saturates at a value of about 2.39 $M_\odot$ when $D_I$ is larger than 3500 MeV fm$^{-3}$, and further increasing the value of the $D_I$ parameter essentially has no effect on the maximum mass of static QSs. From the right panel of Figure 7, one can see that the two-flavor $u$–$d$ quark matter symmetry energy is much larger than the nuclear matter symmetry energy when $D_I$ is larger than about 2000 MeV fm$^{-3}$, and similar results are also observed in the case of DI-2500 with $z = 1/3$ as shown in the left panel of Figure 1.
therefore, we conclude that the maximum value of the static QS maximum mass is about 2.40 M⊙ within the CDDM model if the z parameter and the strength of the quark matter symmetry energy can be varied freely.

From the right panel of Figure 7, one can see that an extremely large two-flavor u–d quark matter symmetry energy with its amplitude about 100 times larger than that of nuclear matter symmetry energy is necessary to describe a static QS with a mass of 2.40 M⊙ within the CDDM model with z = 1.8. For z = 1/3, a similar large amplitude of the two-flavor u–d quark matter symmetry energy is also necessary to describe a static QS with a mass of 1.96 M⊙. Such a large two-flavor u–d quark matter symmetry energy is definitely surprising, and it will be very interesting to investigate its observable effects in experiments. At this point, we would like to point out that the absolute stability condition of SQM is still satisfied although the two-flavor u–d quark matter symmetry energy is extremely large, and this usually leads to an almost equal fraction of u, d, and s quarks in the SQM with a very small isospin asymmetry, consistent with the picture of CFL state. On the other hand, the extremely large two-flavor u–d quark matter symmetry energy with very high values of Dz may lead to a negative equivalent quark mass for the u quark in isospin asymmetric quark matter, and this will indicate a breakdown of the model. For example, for D1-2500, the equivalent quark mass of the u quark will become negative when the baryon density is larger than 0.98 fm⁻³ if the isospin asymmetry is fixed at 0.05. The corresponding critical density will reduce to 0.48 fm⁻³ if the isospin asymmetry is fixed at 0.1. These features imply that the possible existence of quark matter with high baryon density and large isospin asymmetry may put important limitations on the amplitude of the two-flavor u–d quark matter symmetry energy or the model parameters in the CIDDM model. In addition, an extremely large two-flavor u–d quark matter symmetry energy may significantly affect the partonic dynamics in ultra-relativistic HICs induced by neutron-rich nuclei, e.g., Pb + Pb at LHC/CERN or Au + Au at RHIC/BNL, and in principle the symmetry energy effects in these collisions can be studied within partonic transport models in which the parton potentials have been considered (Song et al. 2013; Xu et al. 2013). In this case, combining the constraints from astrophysical observations of heavy QSs and the quark matter symmetry energy effects in ultra-relativistic HICs may provide important information on both the amplitude of quark matter symmetry energy and the z parameter in the CIDDM model. Our results presented here indicate that z = 1/3 is ruled out in the CIDDM model if the new pulsar PSR J0348+0432 is a QS.

4. CONCLUSION AND OUTLOOK

We have extended the CDDM model, in which the quark confinement is modeled by the density-dependent quark masses, to include isospin dependence of the equivalent quark mass. Within the CIDDM model, we have explored the quark matter symmetry energy, the stability of SQM, and the properties of QSs, and we found that including isospin dependence of the equivalent quark mass can significantly change the quark matter symmetry energy as well as the properties of SQM and QSs. We have demonstrated that although the recently discovered large mass pulsar PSR J1614−2230 with a mass of 1.97 ± 0.04 M⊙ cannot be a QS within the original isospin-independent CDDM model, it can be well described by a QS in the CIDDM model if appropriate isospin dependence of the equivalent quark mass is applied. In particular, if the density dependent quark mass scaling parameter z is fixed at z = 1/3 according to the argument of first-order in-medium chiral condensates and linear confinement, the equivalent quark mass should be strongly isospin dependent so as to describe PSR J1614−2230 as a QS, indicating that the two-flavor u–d quark matter symmetry energy should be much larger than the nuclear matter symmetry energy. On the other hand, if the mass scaling parameter z can be varied freely, the two-flavor u–d quark matter symmetry energy could be smaller than the nuclear matter symmetry energy, but its strength should be at least about twice that of a free quark gas or normal quark matter within the conventional Nambu–Jona-Lasinio (NJL) model in order to describe PSR J1614−2230 as a QS. In addition, the most recently discovered large mass pulsar PSR J0348+0432 with a mass of 2.01 ± 0.04 M⊙ can also be described as a QS within the CIDDM model if the z parameter can be varied freely and the two-flavor u–d quark matter symmetry energy is larger than about twice that of a free quark gas or normal quark matter within the conventional NJL model. Our results have further indicated that z = 1/3 is ruled out in the CIDDM model if the new pulsar PSR J0348+0432 is a QS.

We have further studied the maximum possible mass of static QSs within the CIDDM model, and we have found it could be as large as 2.40 M⊙ if the z parameter can be varied freely and the strength of the two-flavor u–d quark matter symmetry energy is allowed to be extremely large.

Therefore, our results have demonstrated that the isovector properties of quark matter may play an important role in understanding the properties of SQM and QSs. If PSR J1614−2230 and PSR J0348+0432 were indeed QSs, they can put important constraints on the isovector properties of quark matter, especially the quark matter symmetry energy. In particular, our results have shown that the two-flavor u–d quark matter symmetry energy should be at least about twice that of a free quark gas or normal quark matter within the conventional NJL model in order to describe PSR J1614−2230 and PSR J0348+0432 as QSs.

In the present work, we have mainly focused on the quark matter symmetry energy and the properties of QSs within the CIDDM model. In future, it will be interesting to see how the
present results change if other quark matter models are used and how the isovector properties of quark matter, especially the quark matter symmetry energy, will affect other issues such as the quark-hadron phase transition at finite isospin density, the partonic dynamics in high energy HICs induced by neutron-rich nuclei, and so on. These works are in progress.

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