Coupled channel analysis of molecule picture of $P_c(4380)$

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We construct a potential obtained by one-pion exchange for the coupled channel $\Sigma_c^* D - \Sigma_c D^*$, and solve the coupled Schrödinger equations to determine the binding energy. We find that there exists one or two bound states with the binding energy of several MeV below the threshold of $\Sigma_c^*$ and $\bar{D}$, dominantly made from a $\Sigma_c^*$ baryon and a $\bar{D}$ meson, with the size of about 1.5 fm for a wide parameter region. We also study the pentaquark states including a $b$ quark and/or an anti-$b$ quark. We show that there exist pentaquarks including $\bar{b}c$, $bc$, and $bb$, all of which lie at about 10 MeV below the corresponding threshold and have size of about 1.5 fm.

I. INTRODUCTION

Hadrons made of more than three quarks are interesting objects to study. In the summer of 2015, the LHCb announced the discovery of the hidden charm pentaquark [1]: one has a mass of $4380 \pm 8 \pm 29$ MeV and a width of $205 \pm 18 \pm 86$ MeV, while the second is narrower, with a mass of $4449.8 \pm 1.7 \pm 2.5$ MeV and a width of $39 \pm 5 \pm 19$ MeV. Soon after the announcement, there appeared many theoretical analyses on the pentaquark based on the molecular picture [2–10], the rescattering effects [17–20], the diquark-diquark-antiquark (or diquark-triquark) picture [21–32], and so on [33–47], in addition to some relevant works [48–53] done before the LHCb result.

There are many analyses for the molecule picture. There are several different molecule structures. In Ref. [2], the pentaquarks are regarded as the bound states of the $D^*$ meson and the $\Sigma_c$ baryon by using the potential made by the one-pion exchange. The contributions from the $K$ and $\omega$ mesons are further included in the potential [3], which shows that $P_c(4380)$ can be understood as a bound state of $\Sigma_c^*$ and $\bar{D}$. In Ref. [4], the QCD sum rule is used to show that $P_c(4380)$ is a bound state of $\Sigma_c$ and $D^*$, and that $P_c(4450)$ is a bound state of a mixture of $\Sigma_c D^*$ and $\Sigma_c^* \bar{D}$. An analysis based on a quark model was performed [50] before the LHCb result, which showed that there exists a bound state of $\Sigma_c$ and $D$ with the threshold being about 4.3 GeV. There are many other analyses as those in Refs. [5 7 9 12 14 15] showing several different molecule structures.

The recently observed $P_c(4380)$ lies below the $\Sigma_c^* \bar{D}$ threshold in several MeV, so that this new state can be naturally regarded as a molecular state of $\Sigma_c^* \bar{D}$. However, it is impossible to construct a $\Sigma_c^* \bar{D}$ molecular state by a potential made by just one-pion exchange because $DD\pi$ vertex is prohibited by the parity invariance. Then, we need to take into account effects of coupled channels to study the existence of the molecular state mainly made from $\Sigma_c^* \bar{D}$ by the one-pion exchange. The most likely channel coupled to $\Sigma_c^* \bar{D}$ through the one-pion exchange is the $\Sigma_c D^*$ channel, since sum of their masses is closer to the sum of masses of $\Sigma_c^*$ and $\bar{D}$ than the other channels. Thus in this paper, we investigate the coupled channel effect of $\Sigma_c^* D - \Sigma_c D^*$ to molecular states. As pointed out in Ref. [54], this coupled channel effect was not yet studied. In the present analysis, we construct a one-pion exchange potential following the procedure explained in Ref. [55] and solve the Schrödinger type equation of motion. Our results show that the binding energy of the ground state is about several MeV below the sum of $\Sigma_c^*$ and $\bar{D}$ masses of 4385.3 MeV in the wide range of the relevant parameters, and that the percentage of the $\Sigma_c D$ component is more than 99%. This implies that the observed $P_c(4380)$ can be reasonably understood as a molecular state dominantly made from the $\Sigma_c$ baryon and the $\bar{D}$ meson.

This paper is organized as follows. In Sec. II we construct a potential by one-pion exchange. Then, we make a numerical analysis in Sec. III. We extend the analysis by replacing the charm quark with the bottom quark in Sec. IV. Finally, a summary and discussions are given in Sec. V.

II. ONE-PION EXCHANGE POTENTIAL FOR $\Sigma_c^* D - \Sigma_c D^*$ CHANNELS

In this section, we construct a potential for $\Sigma_c^* D - \Sigma_c D^*$ channels generated by one-pion exchange.

Here, we first specify interactions of relevant hadrons with the pions based on the heavy quark symmetry and the chiral symmetry. The pion field is introduced into our model within the framework of the chiral Lagrangian based on the spontaneous chiral symmetry breaking of SU(2)$_R \times$ SU(2)$_L \rightarrow$ SU(2)$_V$. The basic quantity is

$$\alpha_{\perp \mu} = \frac{1}{2i} \left[ \partial_\mu \xi \cdot \xi^\dagger - \partial_\mu \xi^\dagger \cdot \xi \right], \quad (1)$$

where $\xi = e^{i \pi / f_\pi}$ with $\pi = \pi_a T_a$ ($a = 1, 2, 3$) and $f_\pi = 92.4$ MeV being the pion fields and the pion decay constant. The quantity $\alpha_{\perp \mu}$ transforms as

$$\alpha_{\perp \mu} \rightarrow h \alpha_{\perp \mu} h^\dagger, \quad (2)$$

where $h$ is an element of SU(2)$_V$. 

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We include the $\bar{D}$ and $\bar{D}^*$ fields through the standard heavy meson effective field expressed as

$$\bar{H} = [\bar{D}^\mu \gamma_\mu - \bar{D} \gamma_5] \frac{1 + \gamma_\perp}{2},$$

where $v^\mu$ denotes the velocity of the heavy meson, $\bar{D}$ and $\bar{D}^*$ are the isodoublet fields for the fluctuation of the heavy mesons, $\bar{D}^{+,0}$ and $\bar{D}^{+,0}$. Under the chiral transformation, $\bar{H}$ transforms as

$$\bar{H} \rightarrow h\bar{H}.$$

By using this together with $\alpha_{\perp \mu}$ for the pion fields, an interaction for heavy mesons with pions with least derivatives is written as $^{56,58}$

$$\mathcal{L}_{\text{int}} = g \text{Tr} \left[ H \gamma_\mu \alpha_{\perp \mu} \bar{H} \right],$$

where $g$ is a dimensionless coupling constant. Expanding the $H$ fields and $\alpha_{\perp \mu}$, the one-pion interaction terms of the heavy mesons are expressed as

$$\mathcal{L}_{\text{int}} = \left( \frac{2g}{f_\pi} \bar{D}^\dagger \partial^\mu \pi \bar{D} + \text{h.c.} \right) + \frac{2ig}{f_\pi} \epsilon^{\mu\nu\rho\sigma} v_\nu D^\dagger \partial_\rho \pi D^\sigma.$$

The relevant baryons $\Sigma_c$ and $\Sigma_c^*$ are included through an isotriplet heavy-quark doublet field $S_\mu$ as

$$S_\mu = -\sqrt{\frac{2}{3}} (\gamma_\mu + v_\mu) \gamma_5 \Sigma_c + \Sigma_c^* \mu.$$

These two fields are expressed in the isospin space as

$$\Sigma_c = \left( \begin{array}{c} \Sigma_{c}^{++} \\ \Sigma_{c}^+ \\ \Sigma_{c}^0 \end{array} \right), \quad \Sigma_c^* = \left( \begin{array}{c} \Sigma_{c}^{*++} \\ \Sigma_{c}^{*+} \\ \Sigma_{c}^{*0} \end{array} \right).$$

The $S_\mu$ field transforms under the SU(2)$_R \times$SU(2)$_L$ chiral transformation as

$$S_\mu \rightarrow h S_\mu h^T.$$

An interaction Lagrangian with least derivative is expressed as $^{57,59}$

$$\mathcal{L}_{\text{int}} = -\frac{3}{2} ig_1 \epsilon^{\mu\nu\rho\sigma} v_\sigma \text{Tr} \left[ S_\mu \alpha_{\perp \nu} S_\rho \right],$$

which leads to the following one-pion interaction terms:

$$\mathcal{L}_{\text{int}} = \frac{3ig_1}{2f_\pi} \epsilon^{\mu\nu\rho\sigma} v_\sigma \text{Tr} \left[ \Sigma_{c} \gamma_\mu \gamma_\rho \partial_\pi \Sigma_c \right]$$

$$\quad + \frac{3ig_1}{2f_\pi} \epsilon^{\mu\nu\rho\sigma} v_\sigma \text{Tr} \left[ \Sigma_{c}^* \gamma_\mu \gamma_\rho \partial_\pi \Sigma_c^* \right]$$

$$\quad + \left( \frac{\sqrt{3} ig_1}{2f_\pi} \epsilon^{\mu\nu\rho\sigma} v_\sigma \text{Tr} \left[ \Sigma_{c}^* \gamma_\mu \gamma_\rho \gamma_5 \Sigma_c \right] + \text{H.c.} \right).$$

We construct a one-pion exchange potential (OPEP) between $(\bar{D}, \bar{D}^*)$ mesons and $(\Sigma_c, \Sigma_c^*)$ baryons from the above interaction terms. Following the procedure explained in Ref. $^{55}$, we introduce the monopole-type form factor at each vertex given by

$$F(q) = \frac{\Lambda^2 - m^2}{\Lambda^2 + |\vec{q}|^2}$$

where $m_\pi$ is the pion mass, $\vec{q}$ is the momentum of the pion, and $\Lambda$ is a cutoff parameter. Although the cutoff $\Lambda$ for the meson-pion vertex may not be the same as that for the baryon-pion vertex, we use the same parameter in the present analysis for simplicity. By including this form factor, the OPEPs for the S-wave channels of $\Sigma_c^* \bar{D}$, $\Sigma_c^* \bar{D}^*$, $\Sigma_c \bar{D}^* \Sigma_c \bar{D}$, and $\Sigma_c^* \bar{D}^* \Sigma_c \bar{D}$ with $I(J^P) = \frac{1}{2}(\frac{1}{2}^-)$ are obtained as

$$V_{\Sigma_c^* \bar{D} - \Sigma_c^* \bar{D}^*}(r) = 0$$

$$V_{\Sigma_c \bar{D}^* - \Sigma_c \bar{D}}(r) = -\frac{1}{3} \times \frac{g_1 g_2 m_\pi^2}{8\pi f_\pi^2} Y_1(m_\pi, \Lambda, r)$$

$$V_{\Sigma_c^* \bar{D}^* - \Sigma_c^* \bar{D}}(r) = -\frac{1}{2\sqrt{3}} \times \frac{g_1 g_2 m_\pi^2}{8\pi f_\pi^2} Y_1(m_\pi, \Lambda, r),$$

where $Y_1(m_\pi, \Lambda, r)$ is defined as

$$Y_1(m_\pi, \Lambda, r) = Y(m_\pi r) - \frac{\Lambda}{m_\pi} Y(\Lambda r) - \frac{\Lambda^2 - m^2_\pi}{2m_\pi \Lambda} e^{-\Lambda r},$$

with $Y(x) = \frac{e^{-x}}{x}$. It should be noted that the OPEP for the $\Sigma_c^* \bar{D}^* - \Sigma_c^* \bar{D}$ channel is zero because the $\bar{D} \bar{D} \pi$ vertex vanishes by parity.

### III. NUMERICAL RESULTS FOR THE BINDING ENERGY AND THE MIXING STRUCTURE

The relevant Schrödinger equation is expressed as

$$\left[ -\frac{1}{2m} \vec{\nabla}^2 + V(r) \right] \Psi(\vec{r}) = E \Psi(\vec{r}),$$

where $m$ is the reduced mass, $E$ is the energy eigenvalue, $V(r)$ is the potential matrix obtained from the OPEPs in the previous section as

$$V(r) = \begin{pmatrix} V_{\Sigma_c^* \bar{D} - \Sigma_c^* \bar{D}^*}(r) & V_{\Sigma_c^* \bar{D} - \Sigma_c \bar{D}}(r) \\ V_{\Sigma_c^* \bar{D}^* - \Sigma_c^* \bar{D}}(r) & V_{\Sigma_c \bar{D}^* - \Sigma_c \bar{D}}(r) \end{pmatrix}$$

$$= \begin{pmatrix} 0 & -\frac{1}{2\sqrt{3}} \times \frac{g_1 g_2 m_\pi^2}{8\pi f_\pi^2} Y_1(m_\pi, \Lambda, r) \\ \frac{1}{2\sqrt{3}} \times \frac{g_1 g_2 m_\pi^2}{8\pi f_\pi^2} Y_1(m_\pi, \Lambda, r) & 0 \end{pmatrix}.$$

The wave function $\Psi(\vec{r})$ has two components for the $\Sigma_c^* \bar{D}$ and $\Sigma_c \bar{D}^*$ states:

$$\Psi = \begin{pmatrix} \psi_{\Sigma_c^* \bar{D}} \\ \psi_{\Sigma_c \bar{D}^*} \end{pmatrix}. \quad (20)$$
Solving the above Schrödinger equation, we determine the binding energy of the bound states and the mixing structure. We use \( m_π = 137.2 \text{MeV}, m_{Σ_c} = 2453.5 \text{MeV}, m_{Σ_c^∗} = 2518.1 \text{MeV}, m_\bar{D} = 1867.2 \text{MeV}, m_{D_c} = 2008.6 \text{MeV} \) for the hadron masses. For the one-pion coupling of charmed baryons we use the binding energy of the bound states and the mixing structure. We use the one adopted in Ref. [2].

For studying the mixing structure of these bound states, we first show the dependence of two potentials \( V_{Σ_c D^∗} - Σ_c^∗ D(\mathbf{r}) \) and \( V_{Σ_c D^∗} - Σ_c^∗ D(\mathbf{r}) \) in Fig. 1 for several choices of the cutoff parameter \( \Lambda \) with fixed value of \( g_1 = 0.95 \) as an example. We note that the shape of the potential for \( Σ_c D^∗ - Σ_c^∗ D \) shown in Fig. 1(a) is different from the one shown in Ref. [2]. This may be since our regularization method following Ref. [55] is different from the one adopted in Ref. [2].

\[
V_{Σ_c D^∗ - Σ_c^∗ D}(\mathbf{r}) \quad g_1 = 0.95
\]

\[
V_{Σ_c D^∗ - Σ_c^∗ D}(\mathbf{r}) \quad g_1 = 0.95
\]

Next, we plot the resultant values of the binding energy against the cutoff parameter \( \Lambda \) for fixed values of \( g_1 = 0.75, 0.95 \) and 1.95 in Fig. 2. In this plot, we measure the binding energy from the \( Σ_c^∗ D \) threshold of 4385.3 MeV. For studying the mixing structure of these bound states, we plot the percentage of the \( Σ_c^∗ D \) component of the wave function defined as

\[
R_{Σ_c^∗ D} = \frac{\int d^3 \mathbf{r} |ψ_{Σ_c^∗ D}(\mathbf{r})|^2}{\int d^3 \mathbf{r} \left[ |ψ_{Σ_c^∗ D}(\mathbf{r})|^2 + |ψ_{Σ_c D^∗}(\mathbf{r})|^2 \right]^2}
\]  

(21)

in Fig. 3. To see the size of the bound states, we show the mean square radius (MSR) for the bound states defined by \( \langle r^2 \rangle \), where

\[
\langle r^2 \rangle = \frac{\int d^3 \mathbf{r} r^2 \left[ |ψ_{Σ_c^∗ D}(\mathbf{r})|^2 + |ψ_{Σ_c D^∗}(\mathbf{r})|^2 \right]}{\int d^3 \mathbf{r} \left[ |ψ_{Σ_c^∗ D}(\mathbf{r})|^2 + |ψ_{Σ_c D^∗}(\mathbf{r})|^2 \right]^2}
\]

(22)

in Fig. 4. We also plot the \( r \) dependence of the wave functions of the \( Σ_c^∗ D \) and \( Σ_c D^∗ \) component with the fixed values of \( \Lambda = 1600 \text{MeV} \) and \( g_1 = 0.95 \) in Fig. 5.
the cutoff \( \Lambda \) is increased, the shape of the potential is
the shape of the potential and the kinetic energy. When
binding energy and the size (MSR) are determined by
dashed curves. As a result, there are two bound states
to 4 MeV. But the values before the jump are smoothly
g their values at a certain cutoff, e.g., at \( \Lambda = 2200 \) MeV for
the ground state shown by solid curves suddenly change
functions for \( \Sigma^c \). While the percentage
changes, i.e., the depth becomes deep. On the other
hand, the kinetic energy by the quantum fluctuation is
stable since the reduced mass is unchanged. Therefore,
when the \( \Lambda \) reaches a certain value, the potential energy
exceeds the value for which the first excited state can
exist. Then, there is a jump of three quantities.

From the above analysis, we conclude that there are
one or two bound states in the coupled channel of \( \Sigma^c \bar{D} \)
and \( \Sigma_c \bar{D}^* \) with the binding energy of several MeV and
the size of about 1.5 fm dominantly made from a \( \Sigma_c \)
baryon and a \( D \) meson. Since the sum of the masses of
\( \Sigma_c \) and \( D \) is 4885.3 MeV, and the observed mass of
\( P_c(4380) \) is 4380 \( \pm 8 \pm 29 \) MeV, then the obtained
binding energy is just suitable for considering \( P_c(4380) \) as
a molecular state existing in the coupled channel of \( \Sigma_c \bar{D} \)
and \( \Sigma_c \bar{D}^* \). Furthermore, for some parameter region, there ex-
st two molecular states within a few MeV range.

### IV. PENTAQUARKS INCLUDING A \( b \) QUARK
AND/OR A \( b \) QUARK

In this section, we extend our analysis in the previ-
ous section to pentaquarks including a \( b \) quark and/or
a \( \bar{b} \) quark. As in the case of the charmed baryons and
mesons, we use the heavy-quark spin symmetry to relate
the \( B^* \bar{B} \pi \) coupling to \( B^* \bar{B}^* \) coupling as well as the
\( \Sigma_c \bar{D} \pi \) coupling to the \( \Sigma_c \bar{D}^* \pi \) coupling. The heavy-quark
flavor symmetry further relates these couplings to the
ones for the charmed hadrons. Then, in the present anal-
ysis, we fix \( |g_{B^* B^* \pi}| = |g_{B^* B \pi}| = |g_{D^* D^* \pi}| = |g_{D^* D \pi}| = 0.60 \)
and vary the value of \( g_{\Sigma^c \bar{D} \pi} = g_{\Sigma^c \bar{D}^* \pi \pi} = 0.75 \) to
1.95. As in the previous section, we introduce one com-
mon cutoff parameter \( \Lambda \) for two form factors, and study
the dependence of the results.

We first study the molecular state in the coupled chan-
el of \( \Sigma^c \bar{D} \) system, using \( m_{\Sigma^c} = 5813.4 \) MeV, \( m_{\Sigma^c} \bar{D} =
5833.6 \) MeV, \( m_B = 5279.4 \) MeV, \( m_{B^*} = 5324.8 \) MeV. In
Fig. 6, we show the binding energy measured from the
\( \Sigma^c \bar{D} \) threshold of 11113.0 MeV, together with the per-
centage of the \( \Sigma^c \bar{B} \) component and the mean square ra-
dius. This shows that the values of the binding energy
are larger than those for the \( \Sigma^c \bar{D} \Sigma^c \bar{D} \) molecular state.
The percentage of the \( \Sigma^c \bar{B} \) component is slightly smaller
for some parameter range, but still more than 99% in
most region. The value of the mean square radius takes
about 1.5-1.7 fm, some of which are slightly larger than
those for the \( \Sigma^c \bar{D} \Sigma^c \bar{D} \) molecular state. Our results
summarized in Fig. 6 indicate that there exists a hidden
bottom pentaquark with mass of about 11080-11110 MeV
and quantum number of \( J^P = \frac{3}{2}^- \). Furthermore, similar
to the case for \( P_c(4380) \), there may exist two or three
molecular states within a few 10 MeV range.

We next study the molecular states in the coupled chan-
el of \( \Sigma^c \bar{D} \Sigma^c \bar{D} \), and that of \( \Sigma^c \bar{D} \Sigma^c \bar{D} \), which
carry the pure exotic flavor quantum numbers. In Figs. 7
and 8, we show the result values of the binding energy,
the mixing structure and the mean square radius. These
FIG. 6. (color online) (a) Binding energy (B.E.) for the $\Sigma_y^b B^*$-$\Sigma_y^b B$ molecular state measured from the $\Sigma_y^b B$ threshold of 11113.0 MeV, (b) the percentage of the $\Sigma_y^b B$ component and (c) the mean square radius. The values for the ground states, first excited states and second excited states are shown by solid, dashed and dotted curves, respectively. The red, blue and green curves are for $g_1 = 0.75$, 0.95, and 1.95.

show that there exist molecular states several MeV below the thresholds, dominantly made from $\Sigma_y^b \bar{D}$ or $\Sigma_c \bar{B}$, with the size of about 1.5 fm.

The results for the binding energy in Figs. 6-8 combined with those in Fig. 2 indicate that the binding energy is larger for the bound state including heavier components. However, the binding energy cannot keep growing with increasing reduced mass, since the depth of the potential is fixed by the values of the cutoff $\Lambda$ and the coupling $g_1$. Then, the binding energy is expected to be saturated to a certain value with increasing reduced mass. To check this, we show the dependence of the binding energy on the reduced mass with fixed values of the cutoff $\Lambda = 1600$ MeV in Fig. 9. This shows that the binding energy is actually saturated at a certain value of the reduced mass.

V. A SUMMARY AND DISCUSSIONS

We investigated the coupled channel effect of $\Sigma_y^c \bar{D}$-$\Sigma_y^c \bar{D}$ to the molecular states. We constructed a one-pion exchange potential following the procedure explained in Ref. [55], and solved the Schrödinger-type equation of motion. Our results showed that the binding energy of the ground state is about several MeV below the thrash-
FIG. 8. (color online) (a) Binding energy (B.E.) for the \( \Sigma_c^{*}B \) molecular state measured from the \( \Sigma_c^{*} \) threshold of 7778 MeV, (b) the percentage of the \( \Sigma_c^{*}B \) component and (c) the mean square radius. The values for the ground states, first excited states, and second excited states are shown by solid, dashed, and dotted curves, respectively. The red, blue and green curves are for \( g_1 = 0.75, 0.95, \) and 1.95.

old of \( \Sigma_c^{*} \bar{D} \), 4385.3 MeV, in wide range of the cutoff \( \Lambda \) for the form factor and the unknown coupling constant of \( \Sigma_c^{*}\Sigma_c \pi \). Furthermore, for some parameter region, there exist two molecular states within a few MeV range. This value is quite similar to the one in Ref. [4], where the attractive force in a single \( \Sigma_c^{*} \bar{D} \) channel is obtained by the \( \sigma \) exchange. We would like to stress that, although the one-pion exchange does not provide attractive force in a single \( \Sigma_c^{*} \bar{D} \) channel, coupled channel effect of \( \Sigma_c^{*} \bar{D} \) and \( \Sigma_c \bar{D}^{*} \) makes \( \Sigma_c^{*} \bar{D} \) bound. We also note the value of the binding energy obtained here is smaller compared with the one in a single \( \Sigma_c \bar{D}^{*} \) channel obtained in Ref. [2]. This may originate from the difference between our regularization of the potential following Ref. [55] and the one in Ref. [2]. We also studied the size and the mixing structure of the molecular states. We found that the size of the molecule is about 1.5 fm and the percentage of the \( \Sigma_c^{*} \bar{D} \) component is more than 99%. These results indicate that the observed \( P_c(4380) \) can be reasonably understood as a loosely bound molecular state dominantly made from the \( \Sigma_c^{*} \) baryon and the \( \bar{D} \) meson. We would like to stress that the \( \Sigma_c^{*} \) baryon and the \( \bar{D} \) meson can form a molecular state mediated by one-pion exchange because the coupled channel effects are included.

We further extended our analysis to the pentaquarks including a \( b \) quark and/or an anti-\( b \) quark. Our results showed that there exists a loosely bound molecular state dominantly made from one of the \( \Sigma_c^{*}, \Sigma_b^{*} \) baryons and one of the \( (\bar{D}, B) \) mesons, and that the size is always about 1.5 fm. We expect that the existence of these pentaquarks will be tested in future experiments.

In the present analysis, we focus on the \( S \)-wave bound states, and we do not include the effects of the tensor force by the one-pion exchange. We expect that inclusion of the tensor force by considering the mixing to the \( D \)-wave states makes the binding energy larger. In addition, inclusion of other channels may modify the properties of the bound states.

The present analysis can be extended to the \( P \)-wave and \( F \)-wave state of the \( \Sigma_c^{*} \bar{D}-\Sigma_c \bar{D}^{*} \) channel which can be expected to give some explanations of the recently observed \( P_c(4450) \). In this case, \( P_c(4450) \) can be regarded as the Feshbach resonance state since the mass of \( P_c(4450) \) is greater than the value of the \( \Sigma_c^{*} \bar{D} \) threshold and smaller than that of the \( \Sigma_c \bar{D}^{*} \) threshold.

It will be also very interesting to study the decays of the molecular states obtained in this analysis. One possible way is to apply the complex scaling method adopted in, e.g., Ref. [61].

We leave the above analyses for future publications.

FIG. 9. (color online) Reduced mass dependence of binding energy (B.E.) with \( \Lambda = 1600 \) MeV. The values of B.E. for the ground states, first excited states and second excited states are shown by solid, dashed and dotted curves. The red, blue and green curves are for \( g_1 = 0.75, 0.95, \) and 1.95, respectively.
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