ZERO-CURVATURE REPRESENTATION FOR HARMONIC-SUPERSPACE EQUATIONS OF MOTION IN $N = 1, D = 6$ SUPERSYMMETRIC GAUGE THEORY

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Abstract

We consider the $SYM^1_6$ harmonic-superspace system of equations that contains superfield constraints and equations of motion for the simplest six-dimensional supersymmetric gauge theory. A special $A$-frame of the analytic basis is introduced where a kinematic equation for the harmonic connection $A^{--}$ can be solved. A dynamical equation in this frame is equivalent to the zero-curvature equation corresponding to the covariant conservation of analyticity. Using a simple harmonic gauge condition for the gauge group $SU(2)$ we derive the superfield equations that produce the general $SYM^1_6$ solution. An analogous approach for the analysis of integrability conditions for the $SYM^2_4$-theory and $SYM$-supergravity-matter systems in harmonic superspace is discussed briefly.

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1 Introduction

Harmonic superspace ($HS$) was introduced in Refs [1, 2] in order to consistently describe supergravity, supersymmetric gauge and matter theories with $N = 2, D = 4$ supersymmetry. The harmonic approach is a covariant version of the twistor method and it has been intensively used for the construction of self-dual solutions in ordinary and supersymmetric Yang-Mills and gravity theories [3, 4].

We shall use the standard notation $SYM^N_D$ for supersymmetric gauge theories with $(D, N)$-supersymmetry in the space-time of dimension $D$.

It should be noted that twistor-harmonic methods have been applied for the integrability interpretation of the non-self-dual theories $SYM^3_4$ [5]-[11], $SYM^4_5$ [12], however, the corresponding harmonic superspaces are very complicated. The simple harmonic $SU(2)/U(1)$ formalism allows the construction of a general solution to the 3-dimensional $SYM^3_3$ equations [15].

The notion of $HS$-integrability is connected with a reformulation of the $SYM$-equations as conditions of zero-harmonic-superfield curvatures constructed by means of covariant harmonic derivatives and harmonized spinor or vector covariant derivatives. These conditions can be interpreted as the covariant conservation of harmonic analyticity [1]. The harmonic coordinates in the superfield formalism of $HS$-theories are the analogues of the auxiliary (spectral) parameters. The final construction of $HS$-solutions can be reformulated in terms of the ordinary coordinates.

We propose the $HS$-integrability interpretation of the supersymmetric $N = 1, D = 6$ gauge theory $SYM^1_6$ connected via a dimensional reduction with the $SYM^2_4$-theory. The $HS$-formalism of $SYM^1_6$ has been considered in Refs [14, 15, 17] by analogy with [1]. A review of the standard harmonic approach is presented in Section 2. We call this version of the harmonic formalism the $V$-frame of the analytic basis, because it uses the analytic prepotential $V^{++}$ [1] as a basic field variable. Section 3 contains a new version of the harmonic formalism ($A$-frame) using the nonanalytic harmonic connection $A^{--}$ as an independent variable. It will be shown that the $SYM^1_6$ superfield constraints and equations of motion can be reformulated as a dynamical zero-curvature equation plus a linear solvable constraint in this frame. The $HS$-approach produces also an infinite number of conservation laws and equations for the Bäcklund-transformation matrix in $SYM^1_6$. Section 4 is devoted to the analysis of $SYM^1_6$-solutions in the $V$-frame for the gauge group $SU(2)$. We use a special harmonic representation of the $SU(2)$-prepotential and the simplest harmonic gauge [15]. This gauge simplifies the study of the spontaneously broken phase of $SYM^1_6$. The $A$-frame analysis of $SU(2)$-solutions is considered in Section 5.

In the conclusion the $HS$-integrability of the $SYM^2_4$-theory is discussed briefly. In particular, self-dual and anti-self-dual solutions and a duality transformation have simple representations in the $A$-frame. We consider also a possible modifications of the superfield $SYM^2_4$-action. We hope that the $HS$-integrability of $SYM^2_4$ can help to understand the remarkable quantum properties of this theory [18].

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2 Harmonic formalism of $SYM^1_6$

Several versions of superfield formalism produce manifestly supersymmetric descriptions of the off-shell $SYM^1_6$-theory \cite{14, 17, 21, 22}. Consider $N = 1, D = 6$ superspace $M(6,8)$ \cite{21, 22} with the 6 even vector coordinates $x^{ab}$ and 8 odd spinor coordinates $\theta^a$ where $a, b \ldots$ are 4-spinor indices of the Lorentz group $SU^*(4) \sim SO(5,1)$ and $i, k \ldots$ are 2-spinor indices of $SU(2)$ group. Let $z = (x^{ab}, \theta^a)$ be the short notation for the coordinates in this superspace.

The (plane) spinor derivatives in $M(6,8)$ satisfy the basic relation

$$\{ D^k_a, D^l_b \} = i \varepsilon^{kl} \partial_{ab} \quad (2.1)$$

where $\partial_{ab} = \partial / \partial x^{ab}$. We shall use the following combinations of the spinor derivatives \cite{21}

$$D^2_{ab} = (1/2) \varepsilon_{ik} D^i_a D^k_b \quad (2.2)$$

$$D^4_{iklm} = (1/24) \varepsilon^{abcd} D^a_i D^k_b D^l_c D^m_d \quad (2.3)$$

where parentheses denote symmetrization of the indices. These satisfy the useful identities \cite{21}:

$$D^a_i (D^4_{iklm}) = 0, \quad (D^2_{ab})(D^4_{iklm}) = 0 \quad (2.4)$$

By analogy with the $SYM^2_4$-theory \cite{20} the superfield constraints of $SYM^1_6$ can be written in the following form \cite{21, 22}:

$$\{ \nabla^i_a, \nabla^k_b \} + \{ \nabla^k_a, \nabla^i_b \} = 0 \quad (2.5)$$

where $\nabla^i_a = D^i_a + A^i_a(z)$ is the covariant spinor derivative and $A^i_a$ is the spinor gauge superfield in a central basis $(CB)$.

The $SYM^1_6$ superfield equation of motion has dimension $d = -2$ in units of length

$$\nabla^i_a W^{ak} + \nabla^k_a W^{ai} = 0 \quad (2.6)$$

where $W^{ai}$ is the covariant superfield-strength of $SYM^1_6$

$$W^{ak} = (i/12) \varepsilon^{abcd} \varepsilon_{jl} \{ \nabla^b_j, \{ \nabla^l_c, \nabla^i_d \} \} \quad (2.7)$$

The integrable superfield constraints (2.5) can be solved in the harmonic approach, and this solution generates a covariant off-shell description of the $SYM^1_6$ theory. The integrability of the whole $SYM^1_6$-system including Eqs (2.3) and (2.4) will be discussed below.

We shall use the standard notation for $SU(2)/U(1)$ harmonics $u_i^\pm$ and the partial harmonic derivatives

$$[\partial^{++}, \partial^{--}] = \partial^0 \quad (2.8)$$

$$\partial^{++} u^+_i = 0, \quad \partial^{++} u^-_i = u^+_i$$

$$\partial^{--} u^-_i = 0, \quad \partial^{--} u^+_i = u^-_i$$

where $\partial^0$ is the operator corresponding to $U(1)$-charge $q$. These harmonics and derivatives have simple representations in terms of the real $U(1)$-variable $\varphi$ and the complex spectral variable $\lambda$ (see e.g.\cite{1, 25})

$$\begin{pmatrix} u^-_1 & u^+_1 \\ u^-_2 & u^+_2 \end{pmatrix} = \frac{1}{\eta(\lambda)} \begin{pmatrix} 1 & -\lambda \\ \lambda & 1 \end{pmatrix} \begin{pmatrix} e^{-i\varphi} & 0 \\ 0 & e^{i\varphi} \end{pmatrix} \quad (2.9)$$
where $\eta(\lambda) = \sqrt{1 + \lambda \bar{\lambda}}$. The convenient representation of the harmonic derivatives has the following form:

$$
\partial^{++} = e^{2i\varphi}[\eta^2(\lambda)\bar{\partial}_\lambda - (i/2)\lambda \partial_\varphi] \\
\partial^{--} = -e^{-2i\varphi}[\eta^2(\lambda)\partial_\lambda + (i/2)\bar{\lambda} \partial_\varphi], \quad \partial^0 = -i\partial_\varphi
$$

(2.10) (2.11)

where corresponding partial derivatives are introduced.

Consider the harmonic (twistor) transform from the central basis of $SYM^6_b$ to the analytic basis $(AB)$

$$
u_i^+ \nabla_a^i = u_i^+ h^{-1} D_a^i h
$$

(2.12)

where $h(z, u)$ is a bridge matrix satisfying the basic harmonic equation [1]

$$(\partial^{++} + V^{++})h(z, u) = \nabla^{++} h = 0
$$

(2.13)

We now discuss briefly the terminology of the harmonic approach used in this paper. The notion of the basis ($CB$ or $AB$) includes the choice of the gauge group representation ($\tau$-group or $\Lambda$-group [1]) and the complete set of relations between covariant derivatives. We use also the notion of the frame in the analytic basis. This means the choice of independent field variables and basic equations generating the complete system of equations.

The analytic connection $V^{++}$ with $q = +2$ (prepotential) determines the off-shell structure of $SYM^6_b$ in the $V$-frame. It 'lives' in an analytic harmonic superspace with the coordinates $\zeta = (x^{ab}_A, \theta^a_+)$

$$
x^{ab}_A = x^{ab} + (i/4)(\theta_+^a \theta_-^b - \theta_+^b \theta_-^a) \\
\theta^a_+ = u_i^+ \theta^a_i, \quad \theta^a_- = u_-^i \theta^a_i
$$

(2.14) (2.15)

The differential operators in the analytic coordinates $(\zeta, \theta^a_-)$ have the following form:

$$
\partial^{++}_A = \partial^{++} + (i/2)\theta^a_+ \theta^a_- \partial_{ab} + \theta^a_+ \partial^a_+ \\
\partial^{--}_A = \partial^{--} + (i/2)\theta^a_- \theta^a_- \partial_{ab} + \theta^a_- \partial^a_- \\
D^+_a = \partial^+_a = \partial / \partial \theta^a_+ = u^+_i D^i_a \\
D^-_a = -\partial^-_a - i\theta^b \partial_{ab} = u^-_i D^i_a
$$

(2.16) (2.17) (2.18) (2.19)

Note that the $AB$-superfields can be described in terms of the central coordinates $z, u$, too. We have the useful relations and definitions for spinor derivatives:

$$
\{ D_a^-, D_b^+ \} = i\partial_{ab}, \quad (D_2)_{ab} = D^+_a D^-_b \\
(D^+)^4 = u^+_i u^+_k u^+_l u^+_m (D_4)^{iklm} = (1/24)\varepsilon^{abcd} D^+_a D^+_b D^+_c D^+_d \\
(D^{++})^q = (1/6)\varepsilon^{abcd} D^+_b D^+_c D^+_d
$$

(2.20) (2.21) (2.22)

The $V$-system of equations in the $SYM^6_b$-theory contains off-shell constraints and an equation of motion. The basic $V$-frame constraints are:

1) The harmonic zero-curvature ($HZC$) equation [14, 23]

$$
[V^{++}, \nabla^{--}] - \partial^0 = \partial^{++}_A A^{--} - \partial^{--}_A V^{++} + [V^{++}, A^{--}] = 0
$$

where $A^{--}(\zeta, \theta^a_-, u)$ is the harmonic connection with $q = -2$. The harmonic connections can be expressed via the bridge matrix $h$ but we treat the $HZC$-equation as an independent
basic equation. The general perturbative and non-perturbative solutions of the basic harmonic equations (2.13) and (2.23) have been discussed in Refs. [2, 14, 15, 23].

2) The 'kinematic' $V$-analyticity condition ( $VZC$-equation ) [1]

\[ [\nabla_a^+, \nabla^{++}] = D_a^{++}V^{++} = 0 \] (2.24)

3) The conventional spinor constraint [14, 23]

\[ [\nabla^- , \nabla^+] = \nabla^- = D^- + A^- = D^- - D_a^+ A^- \] (2.25)

This constraint allows us to write the spinor connection $A^-_a$ in terms of the harmonic connection $A^-_a$.

4) The initial $CB$-integrability condition is solved trivially by the transition to $AB$ [1]

\[ u_i^+ u_k^+ \{\nabla_i^a, \nabla_k^b\} = 0 \Rightarrow \{\nabla^+_a, \nabla^+_b\} = \{D^+_a, D^+_b\} = 0 \] (2.26)

Secondary constraints follow from the basic constraints 1)-4)

\[ [\nabla^- , \nabla^-_a] = 0, \quad \{\nabla^- , \nabla^-_b\} = 0 \] (2.27)

\[ \{\nabla^+_a, \nabla^+_b\} + \{\nabla^- , \nabla^+_b\} = 0 \] (2.28)

The $V$-frame $SYM_{16}^1$-equation of motion has been obtained in Ref [14] by the use of a corresponding nonpolynomial action

\[ F^{++} = (1/4)D_a^{++} W^{a+}(V) = (D^+_a)^4 A^{--}(V) = 0 \] (2.29)

A perturbative solution for $A^{--}$ has the following form

\[ A^{--}(V) = \sum_{n=1}^{\infty} (-1)^n \int du_{1} \ldots du_{n} \frac{V^{++}(z,u_1) \ldots V^{++}(z,u_n)}{(u^+_1u^+_1) \ldots (u^+_nu^+_n)} \] (2.30)

where the harmonic distributions $(u^+_1u^+_2)^{-1}$ [2] are used. Eq(2.29) is equivalent to the analyticity condition on the $AB$-superfield-strength $W^{a+}(V)$.

The harmonic distributions have a simple complex representation

\[ \frac{1}{(u^+_1u^+_2)} = e^{-i(\varphi_1+\varphi_2)} \frac{\eta(\lambda_1) \eta(\lambda_2)}{\lambda_1 - \lambda_2} \] (2.31)

Using this representation, Eq(2.10) and the known formula for complex distributions

\[ \frac{\partial}{\partial \lambda_1} \frac{1}{\lambda_1 - \lambda_2} = \pi \delta(\lambda_1 - \lambda_2) \] (2.32)

one can reproduce the differential relation [2]

\[ \partial^{1+} \frac{1}{(u^+_1u^+_2)} = \delta^{(1,-1)}(u_1, u_2) = \pi e^{i(\varphi_1 - \varphi_2)} \eta^4(\lambda_1) \delta(\lambda_1 - \lambda_2) \] (2.33)
The equation of motion simplifies in the normal $V$-gauge \[1\] which is an analogue of the $WZ$-gauge of the simplest superfield theories. The prepotential of the normal gauge $V^++$ is nilpotent and does not contain pure gauge harmonic component fields

\[ V^+ = \frac{1}{2}\theta^a_+ \theta^b_+ A_{ab}(x_A) + (\theta^+)^4 u^- u^- D^{ik}(x_A) \]  

(2.34)

\[
(\theta^+)^4 = \frac{1}{6} \varepsilon_{abcd} \theta^a_+ \theta^b_+ \theta^c_+ \theta^d_+;
\]

where $A_{ab_1}$, $\psi_a^{i_1}$, and $D^{ik}$ are the component fields of a gauge supermultiplet.

$SYM^1_6$-action in the normal gauge is the 4-th order polynomial

\[ S_N = \sum_{n=2}^{4} \int d^{12}z u_1 \ldots u_n \frac{\text{Tr} V^+_{1}(z, u_1) \ldots V^+_{n}(z, u_n)}{(u_1^1 u_2^1) \ldots (u_n^1 u_1^1)} \]  

(2.35)

This action generates the superfield $V$-equation of motion equivalent to the component $SYM^1_6$ equations of motion. An analysis of the nonlinear equation (2.29) is a difficult problem even in the normal gauge. Thus, the $V$-frame is useful for the solution of the off-shell constraints (2.5) and quantization \[2\] but is not very convenient for the search of the classical solutions.

The original works on harmonic superspaces \[1, 2\] and Refs \[14, 15\] use the regular harmonic functions $V^+$ and $h(z, u)$ treated as the convergent or formal harmonic series. Regular harmonic functions $f(u)$ correspond to globally defined functions on the sphere $S^2$, and irregular functions can contain poles and other singularities. The assumption of regularity is natural for the perturbation theory (e.g. in the normal gauge) but it leads to unreasonable restrictions on the nonperturbative solutions. We shall discuss the irregular bridge functions in Section 3.

### 3 New harmonic frame for the $SYM^1_6$ equations of motion

Now we shall consider a new harmonic representation of the $SYM^1_6$ equations which allows to prove the $HS$-integrability of this theory and to solve the equation with a dimension $d = -2$. Only the complete system of covariant equations in the analytic basis has an invariant meaning, however, one can change the choice of field variables and independent equations. A basic field variable of the $V$-frame is $V^+ = h \partial^+ h^{-1}$. It is clear that one can use other functions of the bridge $h$ as the field variables of $AB$.

Let us treat the harmonic connection $A^- = h \partial^- h^{-1}$ as a basic superfield of the classical $SYM^1_6$ theory in the $A$-frame. The complete $A$-system of $SYM^1_6$-equations for covariant derivatives with $d \geq -2$ is identical to the corresponding $V$-system, however we change the interpretation, the basic set and the order of the dynamical equations and the auxiliary field structure of the harmonic formalism in the new frame. The $HZC$-equation (2.23) in this frame is treated as an integrable equation for the connection $V^+(A^-)$. A basic $A$-bridge equation contains the covariant derivative $\nabla^-$, and Eq(2.13) becomes a secondary equation. Harmonic equations with $d = 0$ do not guarantee the conservation of analyticity. We shall preserve the standard transform between $CB$ and $AB$ (2.12), the basic $AB$ constraints (2.23), (2.24) and (2.26) but treat analyticity in the $A$-frame as a new dynamic zero-curvature equation instead of the 'kinematic' analyticity constraint of the $V$-frame (2.24).
It should be underlined that the nonlinear in $V^{++}$ equation (2.29) transforms to a linear kinematic constraint of the new $A$-frame:

$$(D^+)^4 A^-(z, u) = 0$$

(3.1)

Using a nilpotency of $D^+_a$ we can obtain the following general solution of this constraint:

$$A^-(z, u) = D^+_a A^{a(-3)}(z, u)$$

(3.2)

where $A^{a(-3)}$ is the on-shell $SYM^1$ prepotential.

Now the whole $SYM^1$-system reduces to the dynamic analyticity (zero-spinor-curvature) condition which we shall call AZC-equation

$$[\nabla^-, \nabla^-] = D^- A^- + \partial^{-} D^+_a A^- - [D^+_a A^-, A^-] = 0$$

(3.3)

where the constraint (2.25) is used.

This condition and the representation (3.2) generate a nonlinear equation for the superfield $A^{a(-3)}$

$$D^-_a D^+_b A^{b(-3)} + \partial^- D^+_a D^+_b A^{b(-3)} - [D^+_a D^+_b A^{b(-3)}, D^+_c A^{c(-3)}] = 0$$

(3.4)

This equation has the following gauge invariance:

$$\delta A^{a(-3)} = R^{a(-3)} \Lambda + [\Lambda, A^{a(-3)}] + D^+_a \Lambda^{ab(-4)}$$

(3.5)

where a general symmetrical spinor $\Lambda^{ab(-4)}$ and an analytic scalar $\Lambda$ are the Lie-algebra valued superfield gauge parameters. The spinor derivative of $\delta A^{a(-3)}$ produces the standard $AB$-gauge transformation $\delta A^- = \nabla^- \Lambda$

$$\{D^+_a, R^{a(-3)}\} = \partial^{-} \Lambda$$

(3.6)

$$R^{a(-3)} = \theta^a \partial^- + \frac{i}{4} \theta^a \theta^b \theta^c \partial_{bc} + \frac{1}{2} \theta^a \theta^b \partial^-$$

(3.7)

Let us consider a regular harmonic functions $A^{a(-3)}(z, u)$ and choose a normal $A$-gauge for the on-shell superfield $A^- = D^+_a A^{a(-3)}$

$$A^-_N = \theta^a \beta^a \Lambda + \frac{1}{2} \theta^a \theta^b \alpha_{ab} \Lambda + (\theta^{-3})_a \psi^a \Lambda$$

(3.8)

$$\theta^{-3} = D^+_a (\partial^-)^4 + (1/6) \epsilon_{abcd} \theta^b \theta^c \theta^d$$

where $\beta$, $\alpha$ and $\psi$ are the analytic functions. This gauge has a residual gauge invariance with restricted parameters $\partial^- \Lambda = 0$

$$\delta \beta^a = \partial^a \Lambda + \ldots, \quad \delta \alpha_{ab} = \partial_{ab} \Lambda + \ldots$$

(3.9)

Note that $(\theta_-)^4 \text{term vanishes due to the constraint (3.2)}$.

The superfield $A^-_N$ contains a physical part $A^-_P$ and an auxiliary-field part $H^-$. All auxiliary harmonic component fields vanish as a consequence of Eq(3.4) so the physical harmonic connection be

$$A^-_P = (1/2) \theta^a \theta^b [A_{ab} + \epsilon_{abcd} \theta^c u^i \psi^d_i] + (\theta^{-3})_a [u^i \psi^a_i + \theta^b F^a_b]$$

(3.10)

6
where \(A_{ab}(x_A)\) and \(\psi^a_i(x_A)\) are the vector and spinor fields and \(F^a_b(x_A)\) is an independent field-strength.

Eq(3.4) generates the usual connection between \(F^a_b\) and \(A_{ab}\) and the component \(SYM^1_0\) equations. Thus, the \(A\)-frame corresponds to the first-order component \(SYM^1_0\) formalism. It is evident that all frames of \(AB\) are equivalent on-shell and have identical component solutions for the physical fields.

It should be underlined that an alternative equivalent form of the \(HS\)-integrability condition in the \(A\)-frame can be written as a dynamical \(VZC\)-equation

\[
[\nabla^+_a, \nabla^++] = D^+_a V^{++}(A^{--}) = 0
\]  (3.11)

where \(V^{++}(A^{--})\) is a solution of Eq(2.23). A perturbative form of this solution is an analogue of the solution (2.30) but contains the new harmonic distribution

\[
\frac{1}{(u^1 u^2)} = e^{i(\varphi_1 + \varphi_2)} \frac{\eta(\lambda_1) \eta(\lambda_2)}{\lambda_1 - \lambda_2}
\]  (3.12)

satisfying the relation

\[
\partial^{--}_1 \frac{1}{(u^1 u^2)} = \delta^{(-1,1)}(u_1, u_2)
\]  (3.13)

The third equivalent form of the dynamical \(A\)-frame equation can be written as

\[
[\nabla^{++}, \nabla^{--}_a] = \nabla^{--}_a \Rightarrow [\nabla^{++}, [\nabla^{--}, \nabla^{--}_a]] = 0
\]  (3.14)

The bridge matrix \(h_A = h(A^{--})\) of the \(A\)-frame is a solution of the following harmonic equation

\[
\nabla^{--}_a h_A = (\partial^{--} + D^+_a A^{u(-3)}) h_A = 0
\]  (3.15)

This equation on the sphere \(SU(2)/U(1)\) is the harmonic part of the linear problem for the \(HS\)-integrable \(SYM^1_0\)-system. A key point of the harmonic approach is the integrability of the bridge harmonic equation. If we restrict ourselves by regular solutions for \(h_A\), then the consistency conditions on the regular harmonic connections appear \([24, 15]\). The explicit solutions for the \(SU(2)\) gauge group be considered in sections 5 and 6.

Consider a typical example of the linear harmonic differential equation that arises in an analysis of the bridge equation

\[
\partial^{++} f^{(-2)} = f^0(u) = c + c^{(ik)}u^+_i u^-_k + \ldots
\]  (3.16)

where \(f^0\) is a regular harmonic function. For \(c \neq 0\) this equation has no regular solution in terms of the harmonic expansion. However, Eq(3.10) in the complex coordinates (2.9) has the integral solution with a simple complex pole kernel. The harmonic analogue of this integral representation is

\[
f^{(-2)}(u) = \int du_i G^{(-2,0)}(u, u_i) f^0(u_1)
\]  (3.17)

\[
\partial^{++} G^{(-2,0)}(u, u_i) = \delta^{(0,0)}(u, u_1) = \pi \eta^4(\lambda) \delta(\lambda - \lambda_i)
\]  (3.18)

In contrast to the standard harmonic distributions \([2]\) \(G^{(-2,0)}(u, u_i)\) has not any illustrative harmonic expansion, but it has the simple complex representation

\[
G^{(-2,0)}(\lambda, \lambda_1) = e^{-2i\varphi} \frac{\eta^2(\lambda)}{(\lambda - \lambda_1)}
\]  (3.19)
Thus, one can admit the appearance of isolated harmonic singularities in the bridge function and even in the harmonic connections. As a rule we shall use regular initial data and choose the gauge freedom to obtain the solutions with a minimal number of singularities.

Using irregular harmonic fields one should remember the following simple general rule [1]:
The physical component fields are defined naturally in the central basis.

The $CB$-gauge superfield does not depend on the harmonics

$$A^i_a(z) = h^{-1} D^i_a h - u^{+i} h^{-1} (D^+_a A^{-}) h$$

This superfield satisfies the relations $\partial^{\pm} A^i_a(z) = 0$ and also the equations (2.7) and (2.8) which are equivalent to the component $SYM_6^1$-equations.

4 Conservation laws and Bäcklund transformations in $SYM_6^1$

The most attractive feature of integrable field theories is an infinite number of conservation laws. The explicit construction of the conserved quantities follows immediately from the zero-curvature representation and has a clear geometric interpretation in terms of the contour variables [29]. Analogous constructions arise also for the integrable $SYM_4^1$ equation [10].

The $HS$-integrable theories possess the specific properties. The corresponding zero-curvature equations contain covariant spinor and harmonic derivatives and mean a conservation of the analyticity in $HS$ [1]. Now we shall try to show that ordinary conservation laws follow from the dynamic harmonic-spinor analyticity equation of $SYM_6^1$-theory. Consider a vector covariant derivative in the $A$-frame

$$\nabla_{ab} = -i \left\{ \nabla^+_a, \nabla^-_b \right\} = \partial_{ab} + iD^+_a D^+_b A^{-}$$

The basic equation (3.4) generates the relation

$$[\nabla^{-}, \nabla_{ab}] = 0$$

Let us choose a time variable $t = x^{12}$

$$\nabla_t = \nabla_{12} = \partial_t + A_{12}, \quad A_{12}(z, u) = iD^+_1 D^+_2 A^{-}$$

It is evident that $\nabla_t$ commutes on-shell with the covariant harmonic derivatives (4.2).

It should be stressed that the bridge is a natural harmonic analogue of the contour variables of integrable theories in the zero-curvature representation. The transformation law of the bridge has the following form:

$$\delta h_A = \Lambda(\zeta, u) h_A - h_A \tau(z)$$

where $\Lambda$ and $\tau$ are the gauge parameters in $AB$ and $CB$, correspondingly. The covariant constancy of the bridge in the all spinor and vector directions is a consistency condition for the dynamic analyticity equations (3.4) or (3.11), for instance

$$\nabla_t h_A = \partial_t h_A + A_{12} h_A - h_A A_t(z) = 0$$

The importance of this rule for the harmonic method was remarked by V.I.Ogievetsky
where $A_t(z)$ is a time component of the gauge $CB$-superfield.

One can choose a special $\tau$-gauge for the $SYM_{1}^6$-theory

$$A_t(z) = 0, \quad \partial_t \tau(z) = 0$$

The $A$-frame covariant derivative $\nabla_{12}$ commutes with $D_1^+$ and $D_2^+$. The simplest conserved quantities in the $\tau$-gauge can be constructed as $\Lambda$-invariant functions of $h_A$, for example

$$C^{++}(z, u) = \text{Tr}(D_1^+ h_A D_2^+ h_A^{-1}), \quad \partial_t C^{++} = 0$$

It is not difficult to build the conserved quantities invariant under the $\tau$- and $\Lambda$-gauge transformations

$$P_{ab(\pm)} = \text{Tr} W^{a\pm} W^{b\pm}, \quad \partial_{ab} P_{ab(\pm)} = 0$$

where $W^{a\pm}$ are components of the on-shell superfield-strength

$$W^{a+} = (D_1^+)^a A^{--}, \quad W^{a-} = \nabla^{-+} W^{a+}$$

$$\nabla_{ab} W^{a\pm} = 0, \quad \nabla_{ab} W^{a\pm} W^{b\pm} = 0$$

Note that the last equation is not valid off-shell.

The Bäcklund transformations (BT) play an important role for integrable theories as transformations in the spaces of solutions. For the $SDYM$ and $SDSYM$ solutions these transformations have been considered in Refs[30, 31]. We shall discuss BT in the HS-formalism of $SYM_{1}^6$.

Let $A^{--}$ and $\hat{A}^{--}$ be two different solutions of the $SYM_{1}^6$-system (3.4). Consider the corresponding bridges $h_A$ and $\hat{h}_A$. Then the Bäcklund transformation between these solutions has the following form:

$$\hat{A}^{--} = D_1^+ \hat{A}^{a(-3)} = B^{-1} A^{--} B + B^{-1} \partial^{--} B$$

where the $B$-matrix can be written in terms of two bridges

$$B(A, \hat{A}) = h_A \hat{h}_A^{-1}$$

It is easy to derive the equations for the matrix $B$ in terms of the background solution $A^{--}$, $h_A$. Formally the new superfield variable $\hat{A}^{--}$ has an independent $\hat{A}$ transformation, and it is 'invariant' under the $\Lambda$-transformation of a background superfield. Eq(4.11) can be treated as a harmonic equation for $B$ in terms of the background solution $A^{--}$, $h_A$ and the second prepotential $\hat{A}^{a(-3)}$. The analyticity equations (3.4, 3.11) for the second solution $\hat{A}^{--}$ produce the following $\Lambda$-covariant equations

$$\nabla^{++}(A) \beta_a^+ = 0, \quad \nabla^{--}(A) \nabla^{--}(A) \beta_a^+ = 0$$

where $\beta_a^+ = D_a^+ B B^{-1}$. Validity of these equations is evident in the representation (1.12)

$$h_A^{-1} \beta_a^+ h_A = h_A^{-1} D^+_a h_A - \hat{h}_A^{-1} D^+_a \hat{h}_A = u_i^+[A_t^i(z) - \hat{A}_t^i(z)]$$

This representation is equivalent to the following form of the Bäcklund transformation of the spinor $CB$ gauge superfield:

$$\hat{A}_a^i(z) = A_a^i(z) + h_A^{-1} [u_+^i \beta_a^+ - u_-^i \nabla^{--}(A) \beta_a^+] h_A$$

The equations for $B$ are simplified in the case of infinitesimal Bäcklund transformations $B = I + \delta B$

$$\delta B = D_{(ab}^+ \nabla_{b)} B^{ab}$$

The analyticity produces an additional restriction on $B^{ab}$.
5 V-frame analysis of SU(2) solutions in the simplest harmonic gauge

The HS-integrability interpretation allows us to analyze the explicit constructions of the SYM4-solutions by analogy with the harmonic formalism of SDYM [3,4] or SYM6 [13]. Let us go back to the V-frame and consider the case of the gauge group SU(2). We shall use a harmonic representation of the general SU(2) prepotential \( \Lambda \). Let

\[
V^{++} = (U^{+2}) b^{0}(\zeta, u) + (U^{0}) b^{(+2)}(\zeta, u) + (U^{-2}) b^{(+4)}(\zeta, u)
\]

where \( b^{0}, b^{(+2)}, b^{(+4)} \) are arbitrary real analytic superfields and \( (U^{q}) \) are matrix generators of the Lie algebra SU(2) in a harmonic representation

\[
(U_{k}^{\pm 2})_{i}^{\pm} = u_{k}^{\pm} u_{i}^{\pm}, \quad (U_{k}^{0})_{i}^{\pm} = u_{k}^{\pm} u_{i}^{\pm} + u_{i}^{\pm} u_{k}^{\pm}
\]

An analogous representation of the prepotential was used as a special Ansatz for instanton and monopole solutions in the harmonic formalism [27, 28].

Consider the infinitesimal gauge transformations of the harmonic components \( b^{(q)} \)

\[
\delta b^{0} = \partial^{++}_{A} \Lambda^{(-2)} + 2 \Lambda^{0} + 2 b^{0} \Lambda^{0} - 2 b^{(+2)} \Lambda^{(-2)}
\]

\[
\delta b^{(+2)} = \partial^{++}_{A} \Lambda^{0} + \Lambda^{(+2)} + b^{(+4)} \Lambda^{(-2)} - b^{0} \Lambda^{(+2)}
\]

\[
\delta b^{(+4)} = \partial^{++}_{A} \Lambda^{(+2)} + 2 b^{(+2)} \Lambda^{(+2)} - 2 b^{(+4)} \Lambda^{0}
\]

where \( \Lambda^{(q)} \) are the real harmonic components of the analytic SU(2)-gauge matrix \( \Lambda \). Remark that the \( (\theta_{+})^{4} \)-component in (5.5) contains the term \( \partial^{ab} \Lambda_{ab}(x) \) with a total derivative of vector function from \( \Lambda^{(+2)} \).

The simplest general gauge for \( SU(2) \)-prepotential is

\[
V^{++}(b^{0}, \rho) = (U^{+2}) b^{0}(\zeta, u) + (U^{-2}) (\theta_{+})^{4} \rho
\]

where \( b^{0} \) is an arbitrary analytic function and \( \rho \) is a constant part of the trace of the auxiliary scalar matrix field with \( d = -2 \) in \( b^{(+4)} \) that can be written as \( D_{ik}^{\pm}(x) = \rho + \partial^{ab} f_{ab}(x) \). The \( \rho \neq 0 \) solutions characterize the phase of the SYM4-theory with the spontaneous breaking of symmetry.

Stress that this \( (b^{0}, \rho) \)-gauge has the residual gauge invariance with \( \Lambda^{(+2)} = 0, \ \Lambda^{0} = const \) and an arbitrary parameter \( \Lambda^{(-2)} \). The additional condition \( (\partial^{+})^{5} b^{0} = 0 \) fix the \( \Lambda \)-gauge and results in the vanishing of harmonic components with isospin \( T > 4 \) in \( b^{0} \) [24]

\[
b^{0}(V_{iklm}, u) = (D_{4})^{iklm} V_{iklm} + 4(u^{+} u^{-})_{ik}(D_{4})^{lmn(i} V_{kmn}^{k)} + (60/7)(u^{+} u^{-}) V_{ijkl}^{mn} V_{mn}^{kl} + (100/9)(u^{+} u^{-})_{i_{1} \ldots i_{6}}(D_{4})^{a(i_{1} \ldots i_{6}} V_{n}^{a)} + (50/9)(u^{+} u^{-})_{i_{1} \ldots i_{8}}(D_{4})^{(i_{1} \ldots i_{8}} V_{n})
\]

where an analogue of the Mezinchescu prepotential with \( d = 2 \) [22] and the irreducible symmetrical combinations of harmonics \( (u^{+q} u^{-q})_{i_{1} \ldots i_{2q}} \) are used. The analyticity of this representation follows from the identity (2.4).
The phase of \( SYM_6^1 \) and \( SYM_7^1 \) with \( \rho = 0 \) was considered in Refs\cite{13, 25, 20}. The \( HZC \)-equation (2.23) has the following solution in the \((b^0, 0)\)-gauge

\[
A^{--}(b^0, 0) = (U^{+2}) a^{(-4)}_0 + (U^0) a^{(-2)}_0 + (U^{-2}) a^{(0)}_0
\]

where \( a^{(0)}_0 \) are harmonic-quadrature functions of the prepotential \( b^0 \)

\[
a^{(0)}_0 = \frac{b(z)}{1 + b(z)}, \quad b(z) = \int du b^0(z, u)
\]

\[
a^{(-2)}_0(z, u) = \int du \frac{(u - u_1^2)(b^0(z, u_1) - b(z))}{(u + u_1^2)}
\]

\[
a^{(-4)}_0(z, u) = [1 + b(z)] \left[ \partial^- a^{(-2)}_0 - a^{(-2)}_0 a^{(-2)}_0 \right]
\]

Note that this solution has a singular point \( b(z) = -1 \).

The classical action of \( SYM_6^1 \) in the \((b^0, 0)\)-gauge has the following form \cite{25, 26}:

\[
S(b) = \int d^{14}z [\ln(1 + b(z)) - b(z)]
\]

(5.12)

where \( b(z) = (D_4)^{iklm} V_{iklm}(z) \) is a constrained potential.

The \( SYM_6^1 \)-equation of motion in the \((b^0, 0)\)-gauge has only one independent component

\[
(D^+)^4 \left[ \frac{b(z)}{1 + b(z)} \right] = 0
\]

(5.13)

A spinor part of the gauge \( CB \)-superfield can be written in terms of the single superfield \( b(z) \) \cite{26}

\[
[A^i_a(z)]^k_i = \frac{1}{1 + b(z)} \left[ \delta_i^k D_0^a b(z) - (1/2) \delta_i^k D_0^a b(z) \right]
\]

(5.14)

Note that the \( SYM_6^1 \)-constraints (2.3) in this representation follow from the identity

\[
(D_2)_{ab} b(z) = 0
\]

(5.15)

The harmonic equations (2.13) and (2.23) with the prepotential \( V^{++}(b^0, \rho) \) (5.14) can be integrated in quadratures. The integration procedure uses a nilpotency of the term \( \rho(\theta_+)^4 \).

Eq(2.23) has the following harmonic components in the \((b^0, \rho)\)-gauge:

\[
\partial^{++} a^{(-4)}_\rho + 2(1 + b^0) a^{(-2)}_\rho - \partial^- b^0 = 0
\]

(5.16)

\[
\partial^{++} a^{(-2)}_\rho + (1 + b^0) a^{(0)}_\rho - b^0 - \rho(\theta_+)^4 a^{(-4)}_\rho = 0
\]

(5.17)

\[
\partial^{++} a^{(0)}_\rho - 4\rho(\theta_+^3)(\theta_+^3) a - 2\rho(\theta_+^4) a^{(-2)}_\rho = 0
\]

(5.18)

Note that it is convenient to analyze harmonic equations in the central coordinates \( z, u \).

Consider the harmonic equation for \( a^{(0)}_\rho \) which follows from these equations

\[
(\partial^{++})^2 a^{(0)}_\rho = 2\rho(\theta_+)^4 [2 + b^0 - a^{(0)}_\rho - b^0 a^{(0)}_\rho]
\]

(5.19)

Using (5.9) as a zero approximation one can obtain an exact solution for \( a^{(0)}_\rho \) by two iterations and then the other harmonic components can be calculated.

The classical action in the \((b^0, \rho)\)-gauge has the following form:

\[
S(b^0, \rho) = \int d^{14}z du b^0 \int_0^1 ds a^{(0)}(sb^0, \rho)
\]

(5.20)

where \( s \) is an auxiliary parameter.
6 The A-frame analysis of $SU(2)$-solutions

Now we shall discuss properties of the $SU(2)$-solution in the alternative $A$-frame. The first step of this approach is a solution of harmonic equations in the representation (3.2) and then the dynamical analyticity equation should be used.

Consider the harmonic ($U^q$)-components of the AZC-equation (3.3)

$$D_a^+ a^{(0)} + \partial^+ D_a^- a^{(0)} + 2D_a^+ a^{(-2)} + 2a^{(-2)} D_a^+ a^{(0)} - 2a^{(0)} D_a^+ a^{(-2)} = 0 \quad (6.1)$$

$$D_a^- a^{(-2)} + \partial^- D_a^+ a^{(-2)} + D_a^+ a^{(-4)} + a^{(-4)} D_a^+ a^{(0)} - a^{(0)} D_a^+ a^{(-4)} = 0 \quad (6.2)$$

$$D_a^- a^{(-4)} + \partial^- D_a^+ a^{(-4)} + 2a^{(-4)} D_a^+ a^{(-2)} - 2a^{(-2)} D_a^+ a^{(-4)} = 0 \quad (6.3)$$

These equations are equivalent to the dynamical equations $D_a^+ b^0(A) = 0$ for the harmonic components of $V^{++}$ (5.1) in the $A$-frame.

The analyticity equations imply the following condition

$$\nabla^{++} W^{+a} = \nabla^{++} (D^{+4}) a^{+-} = 0 \quad (6.4)$$

producing the relations between harmonic components of $W^{+a}$. Remark that the additional conditions $D_a^+ a^{(0)} = 0$ or $(D^{+4}) a^{(-2)} = 0$ correspond to pure gauge solutions $W^{+a} = 0$.

By analogy with (6.4) one can obtain the general relations between the harmonic components of Eq(3.1):

$$(D^+) 4 a^{(-4)} = 0 \Leftrightarrow (D^+) 4 a^{(0,-2)} = 0 \quad (6.5)$$

The on-shell dependence of the superfields $a^{(q)}$ allow us to simplify the $SYM^4_6$-equations.

Now the convenient 'hybrid' choice of the field variables will be considered. Let $a^{(0)}, a^{(-2)}, b^{(+2)}$ and $b^{(+4)}$ be the independent variables and $b^0$ and $a^{(-4)}$ be treated as the functions of these variables. We can use the gauge (5.3) and Eqs (5.16-5.18) in this frame, too.

Using Eq(5.17) one can obtain the relation for the dependent function of the hybrid frame

$$b^0(A) = \frac{1}{1 - a^{(0)}_\rho} [\partial^+ a^{(0)}_\rho + a^{(0)}_\rho - \rho(\theta_+)^4 a^{(-4)}_\rho] \quad (6.6)$$

The analyticity condition $D_a^+ b^0(A) = 0$ is a single dynamical equation in this approach. It should be stressed that this equation describe the general $SU(2)$ solution.

Consider a solution of the harmonic bridge equation (3.14) for the case $\rho = 0$

$$h_A = \exp[(1/2)(U^0)\ln(1 - a^{(0)}_0)[1 - (U^{+2})a^{(-2)}_0] \quad (6.7)$$

This solution has only one singular point $a^{(0)}_0 = 1$. More general solution can contain additional singularities. An arbitrariness in the bridge solution is connected with the gauge freedom of Eq (3.13). Eq (6.7) produces a relation for $a^{(-4)}_0$ analogous to (5.11).

The polynomial form of the corresponding dynamical equation is

$$(1 + \partial^+ a^{(-2)}_0) D_a^+ a^{(0)}_0 + (1 - a^{(0)}_0) D_a^+ \partial^+ a^{(-2)}_0 = 0 \quad (6.8)$$

$$a^{(0)}_0(z) = (D_2)_{ab} A^{ab}(z), \quad a^{(-2)}_0(z,u) = D_a^+ A^{a(-3)}(z,u) \quad (6.9)$$
Remark that this one-component equation is covariant under the residual gauge transformations of the \((b^0,0)\)-gauge. The consistency condition for this equation follows from the restriction (5.15)

\[
(D_2)_{ab} \int du b^0(z, u) = (D_2)_{ab} \left[ \frac{a_0^{(0)}}{1 - a_0^{(0)}} \right] = 0
\]  

(6.10)

One can try to solve these equations in superfields or in components and then use the \(b^0\)-solution for the construction of the bridge to the central basis.

Thus, the \(SYM^1_6\)-system reduces to Eqs(5.13) or (6.8) in the \((b^0,0)\)-gauge. This reduction simplifies significantly the initial \(SYM^1_6\)-system and gives the hope to obtain the explicit solutions of this problem.

7 Conclusion

The harmonic-superspace integrability of \(SYM^1_6\)-theory guarantees the analogous property of its \(N = 2, D = 4\) subsystem \(SYM^2_4\). Consider the representation (3.2) in the Euclidean version of \(SYM^2_4\)

\[
A^-(z, u) = D^+_\alpha A^{\alpha(-3)} + \bar{D}^+_\dot{\alpha} \bar{A}^{\dot{\alpha}(-3)}
\]  

(7.1)

where two-component spinors are used.

The case \(A^{\alpha(-3)} = 0\) corresponds to the general self-dual solution of \(SYM^2_4\)

\[
W(A) = (\bar{D}^+)^2 A^- = 0
\]  

(7.2)

The self-dual prepotential \(\bar{A}^{\dot{\alpha}(-3)}\) satisfies also the nonlinear AZC-equation (3.3).

Note that \(SYM^2_4\)-equations in \(HS\) are covariant under the discrete transformation

\[
\theta_\alpha^i \leftrightarrow \bar{\theta}_\dot{\alpha}^i \quad A^{\alpha(-3)} \leftrightarrow \bar{A}^{\dot{\alpha}(-3)}
\]  

(7.3)

that is a residual form of the Lorentz transformation in \(D = 6\). This discrete transformation corresponds to the duality transformation between self-dual and anti-self-dual solutions. Note that other discrete transformations exist in the 1-st order formalism of \(SYM^2_4\) with the independent field-strengths \(F^\alpha_\beta, \bar{F}^\dot{\alpha}_{\dot{\beta}}\) and \(F^\alpha_\dot{\beta}\).

It is interesting to discuss possible alternative forms of the \(SYM^2_4\)-action. A simple possibility is the action of a gauge-invariant harmonic interaction of the \textit{independent unconstrained} harmonic superfields \(h, V^{++}, A^{\alpha(-3)}\) and \(L^{\alpha(-3)}\)

\[
S(h, V, A, L) = \int dz \, du \text{Tr}[(D^+_\alpha A^{\alpha(-3)} + \partial^- hh^{-1})(V^{++) + \partial^{++} hh^{-1}) + L^{\alpha(-3)} D^+_\alpha \partial^{++} hh^{-1}]
\]  

(7.4)

where \(a = \alpha, \dot{\beta}\). The first part of \(S\) is a product of two covariant terms in the framework of the above-mentioned \(\Lambda\)-transformations of \(h, V, A\), and a choice of the \(L\)-transformation is evident.

This \(\sigma\)-model-type action produces the standard on-shell analyticity and other dynamical equations for the bridge \(h\), however, it contains additional degrees of freedom. Specific features of \(SYM^2_4\)-solutions will be discussed elsewhere.

It seems natural that the effective quantum action of \(SYM^2_4\) can be rewritten in terms of \(N = 2\) superfields. Note that the simplest harmonic gauge for the gauge group \(SU(3)\)
contains analytic components $b_3^0$ and $b_8^{(+2)}$ corresponding to the Cartan generators of $SU(3)$ \cite{25,26}. Analogous harmonic gauges can be found for any gauge group.

The integrable theory $SYM_4^3$ can be described in the framework of $SYM_4^2$ with the special hypermultiplet interactions \cite{[2]}. An analogous construction exists for the integrable $SYM_6^3$-theory in terms of $HS_6^3$-superfields. It seems natural to consider the $A$-frame $HS$-equations of more general interacting $SYM$-supergravity-matter systems. Any $HS$-integrable system can be reduced to the dynamical analyticity conditions and some solvable linear constraints. This formulation may help to build the explicit classical solutions and to study quantum solutions.

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