Transport properties of a quantum dot with superconducting leads

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Abstract

We report a numerical study of transport properties of a quantum dot with superconducting leads. We introduce a general phenomenological model of quantum dot transport, in which electron tunnel rates are computed within the Fermi’s Golden Rule approach. The low temperature current-voltage (I-V) characteristics are in qualitative agreement with experimental observations of Ralph et al. [Phys. Rev. Lett., 74, 3241 (1995)]. At higher temperatures, our results reveal new effects due to the thermal excitation of quasiparticles in the leads as well as the thermal population of excited quantum levels in the dot. We also study the photon-assisted tunneling phenomena in our system and point out its potential for millimeter wave applications.

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Due to the advancement of modern lithographic technologies, electrons can now be confined to a spatial region so small that not only the Coulomb charging energy $E_C$, but also the spacing of the quantum mechanical energy levels $\epsilon$, become accessible to experiments conducted at low temperatures. While the single electron charging effect has been observed in both metal tunnel junction systems and semiconductor quantum dots, the discrete quantum levels have been studied mostly in semiconductor quantum dots. Recently, Ralph, Black and Tinkham conducted experiments in which they directly measured tunneling transport through discrete quantum levels in a small metallic particle.

In this paper we present numerical simulation results based on the system studied by Ralph et al. Their samples were made of a tiny aluminum particle (with diameter $< 10$ nm) connected to aluminum leads through tunnel junctions on both sides. A third electrode can also be added to act as a gate. Since the experiment was conducted at temperatures well below the superconducting transition temperature of aluminum ($T_C = 1.21$ K for their sample), the leads are nominally superconducting unless a magnetic field is applied. The dot itself can have discrete quantum levels with spacing either smaller or larger than the superconducting energy gap of aluminum. In this paper, we will assume that the quantum dot is a normal metal particle, and the leads can be either superconducting or normal.

We introduce a general phenomenological model of transport through a quantum dot. In this model, we assume that the quantum dot is weakly coupled to the two leads by tunnel barriers. When an appropriate bias voltage $V$ is applied to the leads, an electron can tunnel across one barrier into the dot and subsequently tunnel out through the second barrier. According to general tunneling theory, the tunneling rate across a barrier from side “a” to side “b”, can be evaluated using the Fermi’s Golden Rule,

$$\Gamma_{a\rightarrow b}(\mu_a, \mu_b) = \frac{2\pi}{\hbar} \int_{\infty}^{\infty} |T_{ab}|^2 \mathcal{N}_a(E - \mu_a)\mathcal{N}_b(E - \mu_b) \times f(E - \mu_a) [1 - f(E - \mu_b)] dE,$$

(1)

where $T_{ab}$ is the phenomenological tunneling matrix element, and $f(x) = 1/[1+\exp(x/k_B T)]$ is the Fermi function. $\mathcal{N}_a(E)$ and $\mathcal{N}_b(E)$ are the density of states, and $\mu_a$ and $\mu_b$ are the
chemical potentials, on their corresponding sides. For our system, to compute the tunneling rate from one of the leads to the dot, we take the BCS quasiparticle density of states in the lead and assume that the dot itself has an evenly spaced (with spacing $\epsilon$) discrete level spectrum. Thus the density of states in the superconducting leads is,

$$N_S(E) = \mathcal{N}_N \begin{cases} 0 & \text{if } |E| \leq \Delta; \\ \frac{|E|}{\sqrt{E^2 - \Delta^2}} & \text{if } |E| > \Delta, \end{cases}$$

and the density of states in the dot is,

$$N_D(E) = \mathcal{N}_N \epsilon \sum_n \delta(E - n\epsilon).$$

Here, $2\Delta$ is the superconducting energy gap of the leads, and $\mathcal{N}_N$ is the density of states in the bulk normal metal that comprises both the leads and the dot.

Once we have the tunneling rates, we can compute the tunneling current through the dot using a master equation approach. At steady state, the current that flows into the dot from one lead should balance the current that flows out of the dot into the other lead. Therefore the steady state current is given by,

$$I = \sum_k \left[ \Gamma_{L \rightarrow D}(\mu_L, \mu_k)P_{k-1} - \Gamma_{D \rightarrow L}(\mu_k, \mu_L)P_k \right],$$

where $\mu_L$ and $\mu_k$ are, respectively, the chemical potentials of the left lead and the quantum dot when the dot contains $k$ electrons. The probability of the dot having $k$ electrons is denoted by $P_k$, and it is determined from a steady state rate equation. In Eq. (4), the first term in the summation is the contribution to the current from the $k$-th electron on the dot which tunnels in from the left lead, given that the dot already has $k-1$ electrons on it. The second term corresponds to the reverse process. The net total current is the sum over all possible $k$ values, which depends on the applied voltage and the temperature. A more detailed account of our model and its application to various quantum-dot configurations are discussed elsewhere. We now discuss results concerning the particular system of a quantum dot with superconducting leads.
In Fig. 1, we show a typical low temperature current-voltage (I-V) characteristic of our system. Here the temperature $k_B T = 0.02 E_C^* \ (E_C^* \equiv E_C + \epsilon$ is the spacing between chemical potential levels, and $E_C \equiv e^2/C_\Sigma$ is the charging energy), the superconducting energy gap $2\Delta = 0.3 E_C^*$ and the quantum energy level spacing in the dot $\epsilon = 0.2 E_C^*$. When the leads are superconducting (solid curve), the I-V curve consists of a series of sharp peaks spaced $\epsilon$ apart. This is in contrast with the I-V curve of the same dot with normal metal leads (dashed curve), which has only gentle steps with the same spacing $\epsilon$.

The shape of these I-V curves and the various tunneling processes involved can be understood using the illustration shown in the inset, which is a sketch of the energy spectra of the dot and the leads. In this sketch, the narrow vertical lines represent the two tunneling barriers. The quasiparticle density of states for the superconducting leads are shown; the shaded region indicates occupied states and the unshaded, unoccupied states. The chemical potentials of the superconducting leads are shown as dotted lines in the center of the superconducting gap. The energy spectrum for the dot includes both the excitation spectrum (dotted horizontal lines) and the addition spectrum (heavy horizontal lines). The excitation spectrum has spacing $\epsilon$ and can be populated by, for example, thermal excitation of electrons within the dot without changing the electron number $N$. The addition spectrum, on the other hand, has spacing $\mu_{N+1} - \mu_N = E_C^*$, since it includes the Coulomb charging energy that one has to overcome in order to add an electron ($N \to N + 1$) to the dot by means of tunneling. Assume at zero temperature and zero voltage that there are $N$ electrons on the dot and $\mu_L^0$ and $\mu_R^0$ are midway between $\mu_N$ and $\mu_{N+1}$. A dc voltage, $eV = \mu_L - \mu_R$ is applied to the left lead while the right lead remains at fixed voltage. In order for an electron to tunnel into the dot from the left lead, one needs to raise $\mu_L$, such that $\mu_L - \Delta \geq \mu_{N+1}$, or $V \geq V_t = E_C^* / 2 + \Delta$. As soon as this threshold voltage, $V_t$, is reached, an electron on the occupied states of the left lead can tunnel into the dot at level $\mu_{N+1}$ and subsequently tunnel out to the right lead, resulting in a current flow which reflects the density of states of the left lead. As the voltage is increased further, the first excited state of the $(N + 1)$-electron system (first dotted line above $\mu_{N+1}$) is reached. At this point, an electron can tunnel into
the dot either through level $\mu_{N+1}$ or the first excited state above it, before tunneling out into the right lead. This accounts for the increased current at a voltage $\epsilon$ above the threshold voltage and having the shape again reflecting the density of states of the lead. We also see similar current rises corresponding to the second and higher excited states. However, as long as $\mu_L - \Delta < \mu_{N+2}$, the conduction cycle is always $N \rightarrow N+1 \rightarrow N$. When the voltage is high enough, such that $\mu_L - \Delta > \mu_{N+2}$ ($eV > eV_t + E_C^*$), one extra electron can tunnel into the dot (through $\mu_{N+2}$ or the excited states above it) before the $(N+1)$-th electron can tunnel out. Now the electron number on the dot is a random choice among $N$, $N+1$ and $N+2$ (with corresponding probabilities $P_N$, $P_{N+1}$ and $P_{N+2}$). This corresponds to a larger increase in the current at the voltage $V \approx 1.7$. As the voltage is increased further, $N$ will fluctuate among more and more possible configurations, and eventually the I-V will become essentially linear due to these fluctuations.

The different current response with the superconducting and normal leads is simply the reflection of their respective density of states. The superconducting I-V has the sharp current rises and a higher threshold voltage because its density of state contains a gap and is singular at energies $|E| = \Delta$, both of which disappear in the normal leads where $\Delta = 0$. Quantum dots with discrete levels and normal leads have been studied previously by Averin et al.\textsuperscript{16}

The I-V curves become more complicated at higher temperatures due to thermal excitation of quasiparticles in the leads and electron population of the quantum dot excited states. Noting that there are many energy scales in our system and the fact that the superconducting energy gap decreases with increasing temperature,\textsuperscript{12} we will concentrate in the regime, $E_C \gg k_B T \sim [\Delta(T), \epsilon]$. This parameter range is clearly relevant to the experiment in Ref.\textsuperscript{5} where $E_C \approx 12$ meV, $\epsilon \approx 0.5$ meV, $\Delta(0) = 0.18$ and $k_B T_c \approx 0.1$ meV.

In Fig. 2(a) and (b), we show two I-V curves, both at relatively high temperature, such that in (a), $2\Delta(T) > \epsilon$, while in (b), $2\Delta(T) < \epsilon$. One of the intriguing features of these I-V curves is that in Fig. 2(a) the narrow regions where the current is significantly suppressed do not correspond to the superconducting energy gap $2\Delta(T)$. In fact, $2\Delta(T)$ is much larger...
as indicated in the figure. In Fig. 2(b), however, the relatively wide regions of current suppression do correspond to $2\Delta(T)$. This seemingly contradictory phenomena has a simple explanation using the energy diagrams shown in the insets of Fig. 2, which are the finite temperature versions of the diagram that we used previously in discussing Fig. 1.

First of all, since $[k_BT/\Delta(T), k_BT/\epsilon] \sim 1$ (see the caption of Fig. 2), we expect that there will be many quasiparticles that are thermally excited across the gap. Also, quite a few quantum levels in the dot near the chemical potential levels will be partially occupied due to thermal excitations within the dot. Therefore many quantum levels, both below and above the chemical potential positions of the dot, will become available for tunneling. However, due to conservation of energy, a level can contribute to the current only if there are electrons in the left lead that have the same energy. Since there are no electron states within the gap, whenever a quantum level in the dot lies inside the gap of the left lead, that level will be excluded from conduction. When we sweep the voltage, if $2\Delta(T) > \epsilon$ as in Fig. 2(a), there will always be either one or two quantum levels that fall inside the gap of the left lead. When the voltage is such that there are two levels lined up inside the gap and therefore excluded from conduction, the current will naturally drop in comparison with other voltage values where only one level is being excluded. This will explain Fig. 2(a). The situation in Fig. 2(b) is slightly different. Since we now have $2\Delta(T) < \epsilon$, there can only be one or no level being excluded from conduction. When a level falls inside the gap, it has to traverse the entire gap region before it can join the tunneling process again. Thus the current suppression regions in Fig. 2(b) have the same widths as the gap and are centered about every discrete level.

Since the conduction in our system is carried out by tunneling processes, we expect there might be interesting ac properties to be explored when microwave radiation is coupled to the system. In particular, the photon-assisted tunneling phenomena, observed in large area superconductor-insulator-superconductor (SIS) tunnel junctions, and semiconductor quantum dots should also manifest itself in some fashion in the present system. We treat our system in the presence of microwave radiation using the Tien-Gordon formalism.
originally proposed to explain the photon-assisted tunneling phenomena observed in large area SIS junctions. Recently, the Tien-Gordon theory has also been applied to photon-assisted tunneling in semiconductor quantum structures.

We will treat the externally applied RF signal as an oscillating voltage, \( V_{rf} \cos(2\pi vt) \), applied equally across each of the tunnel barriers. Then the Tien-Gordon theory leads to the following simple modification of the tunneling rates,

\[
\Gamma(\mu_a - \mu_b, \alpha) = \sum_{n=-\infty}^{\infty} J_n^2(\alpha) \Gamma_0(\mu_a - \mu_b + n\hbar\nu)
\]

where \( \alpha = eV_{rf}/(\hbar\nu) \), \( J_n(x) \) are the Bessel functions of the first kind, and \( \Gamma_0 \) is the tunneling rate in the absence of RF radiation, given in Eq. (1).

In Fig. 3, we show I-V curves of our quantum dot with superconducting leads in the presence of RF radiation with frequency \( h\nu/\epsilon = 2/3 \) and two different power levels, along with the dark I-V (\( \alpha = 0 \)). Due to the periodic spikes in the dark I-V, its photo-response is considerably more complicated than that of the conventional SIS tunnel junctions, whose dark I-V has a single discontinuous jump at \( 2\Delta \). For example, Fig. 3 illustrates that if the ratio \( h\nu/\epsilon = p/q \) is a rational number, then the I-V, in sufficiently strong RF signal, will have periodic spikes with period \( h\nu/p \), not \( h\nu \). Even more complicated behavior is observed when \( \epsilon \) and \( h\nu \) are incommensurate. We wish to point out that this effect, if confirmed by experiment, might find applications in millimeter wave detection and mixing schemes. In the weak signal limit, \( \alpha \ll 1 \), one expects a single peak below the dark I-V threshold voltage \( V_t \), which will give information about the frequency as well as the amplitude of the RF signal. We found the sharpness of the current peaks in the superconducting-lead system makes it much more robust against thermal noise in comparison with its normal-lead counterpart.

In summary, we have carried out numerical simulations of single electron transport through a quantum dot with superconducting leads. Our current-voltage characteristics calculated at low temperature is in good qualitative agreement with the experimental observation of Ralph et al. In addition our analysis show that at higher temperatures, thermal excitation of quasiparticles in the leads and thermal population of the excited quantum levels
within the quantum dot should lead to interesting changes in the I-V curves. We also predict that when RF radiation is coupled to the system, the photon-assisted tunneling phenomena should manifest itself by producing extra periodic structures in the I-V curves, which might be useful in the millimeter wave detector/mixer applications. Due to the presence of many different characteristic energy scales, the rich dynamical properties of this system demands more exploration.

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14 For convenience, we attributed all of \( C_\Sigma \) to the self-capacitance of the dot, but in a tunnel junction system, \( C_\Sigma = C_L + C_R \). In order to compare the I-V curves in this paper with those measured in an experiment, one needs to multiply all the voltage scales with the “lever arm” factor, \( \eta = C_\Sigma / C_R \), if the voltage on the left lead is being swept.

15 The gap structure and the entire density of states on the right lead have little effect in the
I-V curve when it is held at constant voltage. This is because when an electron tunnels into the dot from the left lead, its energy always corresponds to empty states that are much higher than the gap in the right lead.

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FIGURES

FIG. 1. Low temperature I-V characteristics of a quantum dot with superconducting leads (solid curve) and normal metal leads (dashed curve). The temperature is $k_B T = 0.02 E_C^*$, and the superconducting energy gap in the leads is $2\Delta = 0.3 E_C^*$. The quantum level spacing is, $\epsilon = 0.2 E_C^*$. The inset is a sketch of the energy spectra in the leads and the dot. Note the quantum dot energy spectrum includes the excitation spectrum (with spacing $\epsilon$), and addition spectrum (with spacing $E_C^* = \frac{e^2}{C} + \epsilon$).

FIG. 2. I-V characteristics at higher temperatures for (a) $2\Delta(T) > \epsilon$ and (b) $2\Delta(T) < \epsilon$. The relevant energy ratios are: (a) $T/T_c = 0.815$, $k_B T/E_C^* = 0.07$, $k_B T/2\Delta(T) = 0.311$, $k_B T/\epsilon = 0.35$, $2\Delta(T)/\epsilon = 1.14$; (b) $T/T_c = 0.931$, $k_B T/E_C^* = 0.08$, $k_B T/2\Delta(T) = 0.576$, $k_B T/\epsilon = 0.40$, $2\Delta(T)/\epsilon = 0.70$. The vertical dotted lines are drawn at the positions of the quantum levels ($\epsilon$ apart).

FIG. 3. Photon assisted tunneling in quantum dots with superconducting leads. The dark I-V (solid curve) has the same parameter values as Fig. 1(a). The dotted curve and the dashed curve, respectively correspond to power levels, $\alpha = eV_{rf}/h\nu = 2.5$, and $\alpha = 0.5$, with $h\nu/\epsilon = 2/3$ for both curves.
\[ I \left( E_C^* / eR_\Sigma \right) \]

\[ V \left( E_C^*/e \right) \]

s.c. leads

normal leads
Whan and Orlando Fig. 3

\[ \alpha = 2.5 \]

\[ \alpha = 0.5 \]

\[ \alpha = 0.0 \]