Revelations of the U(1)-Extended Supersymmetric Standard Model

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Abstract. If an extra supersymmetric U(1) gauge factor exists at the TeV energy scale, which is then broken together with the supersymmetry, there will be several interesting and important phenomenological consequences, not only at the TeV scale, but also at the 100 GeV scale. For one, the generic two-doublet Higgs structure will involve 3 additional parameters beyond that of the Minimal Supersymmetric Standard Model (MSSM), thereby raising the upper bound on the mass of the lighter of the two neutral Higgs scalars. For another, the supersymmetric scalar quarks and leptons receive new contributions to their masses from the spontaneous breaking of this extra U(1). Assuming a grand-unified $E_6$ gauge symmetry and universal soft supersymmetry-breaking terms at the grand-unification energy scale, we find solutions relating the U(1) breaking scale and the ratio of the vacuum expectation values of the two electroweak Higgs doublets.

TWO-DOUBLET HIGGS STRUCTURE

The possible existence of a supersymmetric U(1) gauge factor [1–4] which is broken together with the supersymmetry at the TeV scale will affect the physics at the electroweak scale, through tree-level nondecoupling effects in the Higgs sector. [5] Specific examples have been given previously. [6–8] Here we consider the most general case. [9] We assume the Higgs sector to consist of two doublets and a singlet. Under $SU(3)_C \times SU(2)_L \times U(1)_Y \times U(1)_X$,

\[
\tilde{\Phi}_1 = \begin{pmatrix} \tilde{\phi}_1^0 \\ -\tilde{\phi}_1^- \end{pmatrix} \sim (1, 2, -1/2; -a),
\]

\[
\Phi_2 = \begin{pmatrix} \phi_2^+ \\ \phi_2^0 \end{pmatrix} \sim (1, 2, 1/2; 1 + a),
\]

\[
\chi = \chi^0 \sim (1, 1, 0; 1),
\]

where each last entry is the arbitrary assignment of that scalar multiplet under the extra $U(1)_X$ with coupling $g_x$ such that a term in the superpotential exists linking
all three superfields with coupling \( f \). Consequently, the quartic terms of the scalar potential are given by

\[
V = f^2[(\Phi_1^+ \Phi_2)(\Phi_2^+ \Phi_1) + (\Phi_1^+ \Phi_1 + \Phi_2^+ \Phi_2)\bar{\chi}\chi]
+ \frac{1}{8}g_2^2[(\Phi_1^+ \Phi_1)^2 + (\Phi_2^+ \Phi_2)^2 + 2(\Phi_1^+ \Phi_1)(\Phi_2^+ \Phi_2) - 4(\Phi_1^+ \Phi_2)(\Phi_2^+ \Phi_1)]
+ \frac{1}{8}g_1^2[\Phi_1^+ \Phi_1 + \Phi_2^+ \Phi_2]^2 + \frac{1}{2}g_x^2[-a\Phi_1^+ \Phi_1 - (1 - a)\Phi_2^+ \Phi_2 + \chi\chi]^2,
\]

(4)

Let \( \langle \chi \rangle = u \), then \( \sqrt{2} \text{Re} \chi \) is a physical scalar boson with \( m^2 = 2g_x^2u^2 \), and the \( (\Phi_1^+ \Phi_1)\sqrt{2} \text{Re} \chi \) coupling is \( \sqrt{2}u(f^2 - g_x^2a) \). Hence the effective \( (\Phi_1^+ \Phi_1)^2 \) coupling \( \lambda_1 \) is given by

\[
\lambda_1 = \frac{1}{4}(g_1^2 + g_2^2) + g_x^2a - \frac{2(f^2 - g_x^2a)^2}{2g_x^2} = \frac{1}{4}(g_1^2 + g_2^2) + 2af^2 - \frac{f^4}{g_x^2},
\]

(5)

Similarly,

\[
\lambda_2 = \frac{1}{4}(g_1^2 + g_2^2) + 2(1 - a)f^2 - \frac{f^4}{g_x^2},
\]

(6)

\[
\lambda_3 = -\frac{1}{4}g_1^2 + \frac{1}{4}g_2^2 + f^2 - \frac{f^4}{g_x^2},
\]

(7)

\[
\lambda_4 = -\frac{1}{2}g_x^2 + f^2,
\]

(8)

where the two-doublet Higgs potential has the generic form

\[
V = m_1^2\Phi_1^+ \Phi_1 + m_2^2\Phi_2^+ \Phi_2 + m_{12}^2(\Phi_1^+ \Phi_2 + \Phi_2^+ \Phi_1) + \frac{1}{2}\lambda_1(\Phi_1^+ \Phi_1)^2
+ \frac{1}{2}\lambda_2(\Phi_2^+ \Phi_2)^2 + \lambda_3(\Phi_1^+ \Phi_1)(\Phi_2^+ \Phi_2) + \lambda_4(\Phi_1^+ \Phi_2)(\Phi_2^+ \Phi_1).
\]

(9)

In the above, we have assumed that the soft supersymmetry-breaking term \( fA_f\Phi_1^+ \Phi_2\chi + h.c. \) (from which we obtain \( m_{12}^2 = fA_fu \)) is small, otherwise the electroweak Higgs sector reduces effectively to just one light doublet. Note also that the Minimal Supersymmetric Standard Model (MSSM) is recovered in the limit \( f = 0 \), independent of \( g_x^2 \) and \( a \).

Let \( \langle \phi_{1,2}^+ \rangle \equiv v_{1,2} \), \( \tan \beta \equiv v_2/v_1 \), and \( v^2 \equiv v_1^2 + v_2^2 \), then the above \( V \) has an upper bound on the lighter of the two neutral scalar bosons given by

\[
(m_h^2)_{max} = 2v^2[\lambda_1 \cos^4 \beta + \lambda_2 \sin^4 \beta + 2(\lambda_3 + \lambda_4) \sin^2 \beta \cos^2 \beta] + \epsilon,
\]

(10)

where we have added the radiative correction due to the \( t \) quark and its supersymmetric scalar partners, \( i.e. \)

\[
\epsilon = \frac{3g_2^2m_t^4}{8\pi^2M_W^2} \ln \left( 1 + \frac{m_{\tilde{g}}^2}{m_t^2} \right).
\]

(11)
The existence of a supersymmetric U(1) gauge factor thus implies
\[
(m_h^2)_{\text{max}} = M_Z^2 \cos^2 2\beta + \epsilon + \frac{f^2}{\sqrt{2}G_F} \left[ A - \frac{f^2}{g_x^2} \right], \tag{12}
\]
where
\[
A = \frac{3}{2} + (2a - 1) \cos 2\beta - \frac{1}{2} \cos^2 2\beta. \tag{13}
\]
If \( A > 0 \), then the MSSM bound of 128 GeV can be exceeded. However, \( f^2 \) is still constrained because \( V \) of Eq. (9) has to be bounded from below. For a given \( g_x^2 \), we can vary \( f^2, a, \) and \( \cos 2\beta \) to find the largest \( m_h \), which increases with increasing \( g_x^2 \). We find that for \( g_x^2 = 0.5 \), \( m_h < 190 \) GeV. On the other hand, for specific models [3,4,6] where \( a \) and \( g_x^2 \) are known, the largest \( m_h \) is only between 140 and 145 GeV. [9]

**SUPERSYMMETRIC SCALAR MASSES**

Let us specify the extra U(1) to be that which comes from the reduction of \( E_6 \) to the standard \( SU(3)_C \times SU(2)_L \times U(1)_Y \). In the chain \( E_6 \rightarrow SO(10) \times U(1)_\psi \), then \( SO(10) \rightarrow SU(5) \times U(1)_\chi \), we assume that
\[
U(1)_\psi \times U(1)_\chi \rightarrow U(1)_\alpha, \tag{14}
\]
which then survives down to the TeV energy scale. Under \( E_6 \rightarrow SU(5) \times U(1)_\psi \times U(1)_\chi \), the fundamental \( 27 \) representation of \( E_6 \) is decomposed as follows:
\[
\begin{align*}
27 &= (10; 1, -1) + (5^*; 1, 3) + (1; 1, -5) \\
&+ (5; -2, 2) + (5^*; -2, -2) + (1; 4, 0),
\end{align*}
\tag{15}
\]
where \( 2\sqrt{6}Q_\psi \) and \( 2\sqrt{10}Q_\chi \) are denoted. The usual quarks and leptons belong to \((10; 1, -1)\) and \((5^*; 1, 3)\), whereas the two Higgs doublets are in \((5; -2, 2)\) and \((5^*; -2, -2)\). The \((1; 1, -5)\) is identifiable with the right-handed neutrino \( N \) and the \((1; 4, 0)\) is the singlet \( S \) whose scalar component is \( \chi \). Let
\[
Q_\alpha = Q_\psi \cos \alpha - Q_\chi \sin \alpha, \tag{16}
\]
then the so-called \( \eta \)-model [1,3] is obtained with \( \tan \alpha = \sqrt{3/5} \) and we have
\[
\begin{align*}
27 &= (10; 2) + (5^*; -1) + (1; 5) + (5; -4) + (5^*; -1) + (1; 5),
\end{align*}
\tag{17}
\]
where \( 2\sqrt{15}Q_\eta \) is denoted; and the \( N \)-model [4,8] is obtained with \( \tan \alpha = -1/\sqrt{15} \) resulting in
\[
\begin{align*}
27 &= (10; 1) + (5^*; 2) + (1; 0) + (5; -2) + (5^*; -3) + (1; 5),
\end{align*}
\tag{18}
where $2\sqrt{10}Q_N$ is denoted.

Consider now the masses of the supersymmetric scalar partners of the quarks and leptons:

$$m_B^2 = m_0^2 + m_R^2 + m_F^2 + m_D^2,$$

(19)

where $m_0$ is a universal soft supersymmetry breaking mass at the grand-unification scale, $m_R^2$ is a correction generated by the renormalization-group equations running from the grand-unification scale down to the TeV scale, $m_F$ is the explicit mass of the fermion partner, and $m_D^2$ is a term induced by gauge-boson masses. In the MSSM, $m_D^2$ is of order $M_Z^2$ and does not change $m_B$ significantly. In the $U(1)_{\alpha}$-extended model, $m_D^2$ is of order $M_{Z'}^2 = (4/3)\cos^2\alpha g^2 u^2$ and will affect $m_B$ in a nontrivial way. Specifically,

$$\Delta m_D^2(10; 1, -1) = \frac{1}{8} M_{Z'}^2 \left(1 + \sqrt{\frac{3}{5}} \tan \alpha\right),$$

(20)

$$\Delta m_D^2(5^*; 1, 3) = \frac{1}{8} M_{Z'}^2 \left(1 - 3 \sqrt{\frac{3}{5}} \tan \alpha\right),$$

(21)

$$\Delta m_D^2(1; 1, -5) = \frac{1}{8} M_{Z'}^2 \left(1 + \sqrt{15} \tan \alpha\right),$$

(22)

$$\Delta m_D^2(5; -2, 2) = -\frac{1}{4} M_{Z'}^2 \left(1 + \sqrt{\frac{3}{5}} \tan \alpha\right),$$

(23)

$$\Delta m_D^2(5^*; -2, -2) = -\frac{1}{4} M_{Z'}^2 \left(1 - \sqrt{\frac{3}{5}} \tan \alpha\right),$$

(24)

$$\Delta m_D^2(1; 4, 0) = \frac{1}{2} M_{Z'}^2.$$  

(25)

This will have important consequences on the experimental search of supersymmetric particles. In fact, depending on $m_F$, it is possible for exotic scalars to be lighter than the usual scalar quarks and leptons. [10] It may explain why a scalar “leptoquark” can be as light as 200 GeV, as a possible interpretation of the recent HERA data. [11]

**U(1) AND ELECTROWEAK BREAKING**

Another important outcome of Eq. (19) is that the $U(1)_{\alpha}$ and electroweak symmetry breakings are related [9]. To see this, go back to the two-doublet Higgs potential $V$ of Eq. (9). Using Eqs. (5) to (8), we can express the parameters $m_{12}^2$, $m_1^2$, and $m_2^2$ in terms of the mass of the pseudoscalar boson, $m_A$, and $\tan \beta$. 


\[ m_{12}^2 = -m_A^2 \sin \beta \cos \beta, \]  
\[ m_1^2 = m_A^2 \sin^2 \beta - \frac{1}{2} M_Z^2 \cos 2\beta \]
\[ -\frac{2f^2}{g_Z^2} M_Z^2 \left[ 2 \sin^2 \beta + \left( 1 - \sqrt{\frac{3}{5}} \tan \alpha \right) \cos^2 \beta - \frac{3f^2}{2 \cos^2 \alpha g_\alpha^2} \right], \]
\[ m_2^2 = m_A^2 \cos^2 \beta + \frac{1}{2} M_Z^2 \cos 2\beta \]
\[ -\frac{2f^2}{g_Z^2} M_Z^2 \left[ 2 \cos^2 \beta + \left( 1 + \sqrt{\frac{3}{5}} \tan \alpha \right) \sin^2 \beta - \frac{3f^2}{2 \cos^2 \alpha g_\alpha^2} \right]. \]

On the other hand, using Eq. (19), we have
\[ m_{12}^2 = fA_f u, \]
\[ m_1^2 = m_0^2 + m_R^2(\tilde{g}, f) + f^2 u^2 - \frac{1}{4} \left( 1 - \sqrt{\frac{3}{5}} \tan \alpha \right) M_Z^2, \]
\[ m_2^2 = m_0^2 + m_R^2(\tilde{g}, f) + f^2 u^2 - \frac{1}{4} \left( 1 + \sqrt{\frac{3}{5}} \tan \alpha \right) M_Z^2 + m_R^2(\lambda_t), \]

where \( fA_f \) is the coupling of the soft supersymmetry-breaking \( \tilde{\Phi}_1 \tilde{\Phi}_2 \chi \) scalar term, \( \tilde{g} \) is the gluino, and \( \lambda_t \) is the Yukawa coupling of \( \Phi_2 \) to the \( t \) quark. Matching Eqs. (26) to (28) with Eqs. (29) to (31) allows us to determine \( u \) and \( \tan \beta \) as a function of \( f \) for a given set of parameters at the grand-unification scale.

In the MSSM assuming Eq. (19),
\[ m_1^2 - m_2^2 = -m_R^2(\lambda_t) = -(m_A^2 + M_Z^2) \cos 2\beta. \]

Since \( m_R^2(\lambda_t) < 0 \), we must have \( \tan \beta > 1 \). In the \( U(1)_\alpha \)-extended model, because of the extra D-term contribution, \( \tan \beta < 1 \) becomes possible. Another consequence is that because of Eq. (20), a light scalar \( t \) quark is not possible unless \( \tan \alpha < -\sqrt{3/5} \).

To obtain the spontaneous breaking of \( U(1)_\alpha \), i.e. \( \langle \chi \rangle = u \), we need \( m_\chi^2 \) to be negative. This may be achieved radiatively in analogy to electroweak symmetry breaking by having a relevant large Yukawa coupling drive \( m_\chi^2 \) from its universal positive value at the grand-unification energy scale to a negative one at the TeV scale. Consider the superpotential
\[ W = f H_1 H_2 S + f' hh^c S + \lambda_t H_2 Q_3 t^c, \]

where \( h \) and \( h^c \) are the exotic color triplets belonging to the \((5; -2, 2)\) and \((5^*; -2, -2)\) representations, as well as the corresponding trilinear soft supersymmetry-breaking terms in the scalar potential:
along with all the soft supersymmetry breaking scalar masses. We have verified
that for $f'$ of order $\lambda_t$, it is indeed possible to break $U(1)_{\alpha}$ at the TeV scale. [2,12]

However, it is highly nontrivial to find solutions which can satisfy Eqs. (26) to (31)
simultaneously. A typical solution has $f = 0.345$, $m_{\tilde{g}} = 300$ GeV, $m_0 = A_0 = 950$
GeV, for which $|u| \sim 2$ TeV and $\tan \beta \sim 3.5$. That would predict $M_{Z'} \sim 1$ TeV
and the $h-h^c$ fermion mass to be about 2 TeV. Details are given in Ref. [9].

CONCLUSIONS

(1) Supersymmetric $U(1)_{\alpha}$ from $E_6$ is a good possibility at the TeV scale. (2)
The two-doublet Higgs structure at around 100 GeV will be different from that of
the MSSM. (3) Supersymmetric scalar masses depend crucially on $U(1)_{\alpha}$. (4) The
$U(1)_{\alpha}$ breaking scale and $\tan \beta$ are closely related.

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