Performance Analysis of Large Intelligent Surfaces (LISs): Uplink Spectral Efficiency and Pilot Training

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Abstract

Large intelligent surfaces (LISs) constitute a new and promising wireless communication paradigm that relies on the integration of a massive number of antenna elements over the entire surfaces of man-made structures. The LIS concept provides many advantages, such as the capability to provide reliable and space-intensive communications by effectively establishing line-of-sight (LOS) channels. In this paper, the system spectral efficiency (SSE) of an uplink LIS system is asymptotically analyzed under a practical LIS environment with a well-defined uplink frame structure. In order to verify the impact on the SSE of pilot contamination, the SSE of a multi-LIS system is asymptotically studied and a theoretical bound on its performance is derived. Given this performance bound, an optimal pilot training length for multi-LIS systems subjected to pilot contamination is characterized and, subsequently, the performance-maximizing number of devices that the LIS system must service is derived. Simulation results show that the derived analyses are in close agreement with the exact mutual information in presence of a large number of antennas, and the achievable SSE is limited by the effect of pilot contamination and intra/inter-LIS interference through the LOS path, even if the LIS is equipped with an infinite number of antennas. Additionally, the SSE obtained with the proposed pilot training length and number of scheduled devices is shown to reach the one obtained via a brute-force search for the optimal solution.

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Index Terms

Large intelligent surface (LIS), large system analysis, performance analysis, pilot contamination, system spectral efficiency,

I. INTRODUCTION

The notion of a large intelligent surface (LIS) that relies on equipping man-made structures, such as buildings, with massive antenna arrays is rapidly emerging as a key enabler of tomorrow’s Internet of Things (IoT) and 6G applications [2]–[15]. An LIS system can potentially provide pervasive and reliable wireless connectivity by exploiting the fact that pervasive city structures, such as buildings, roads, and walls, will become electromagnetically active in the near future. If properly operated and deployed, the entire environment is expected to be active in wireless transmission providing near-field communications. In contrast, conventional massive multiple-input multiple-output (MIMO) systems is essentially regarded as far-field communications generating non-line-of-sight (NLOS) channels with a high probability. Indeed, the wireless channels of an LIS can become nearly line-of-sight (LOS) channels, resulting in several advantages compared to conventional massive MIMO system. First, noise and inter-user interference through a NLOS path become negligible as the number of antenna arrays on LIS increases [8]. Also, the inter-user interference through a LOS path is negligible providing an interference-free environment, when the distances between adjacent devices are larger than half the wavelength [9], [10]. Moreover, an LIS offers more reliable and space-intensive communications compared to conventional massive MIMO systems as clearly explained in [8] and [11].

A. Prior Art

Owing to these advantages, LISs have recently been receiving significant attention in the literature [8]–[15]. In particular, the works in [8] and [9] provided an analysis of the uplink data rate to evaluate LIS performance considering channel estimation errors, and studied the space-normalized capacity achieved by an optimal receiver and a matched filter (MF), respectively. Moreover, in [10] and [11], the authors proposed an optimal user assignment scheme to select the best LIS unit and analyzed the reliability of an LIS system in terms of the outage probability, respectively. Meanwhile, the works in [12] and [13] derived, respectively, the Fisher-information and Cramer–Rao lower bound for user positions exploiting the LIS uplink signal and the uplink
capacity considering hardware impairments. By enabling an LIS to reflect signals from conventional transmitters, such as base stations (BS) and access points, to desired devices, the authors in [14] and [15] designed beamformer and LIS phase shifter that maximize the ergodic rate and the energy efficiency, respectively. However, these recent works in [8]–[15] have not considered the effects on spectral efficiency (SE) resulting from the use of a practical uplink frame structure in which the pilot training and data transmission period are jointly considered. Given that statistical channel state information (CSI) is typically acquired by pilot signaling, and because the channel uses for data transmission are closely related to the length of the pilot sequence [16], [17], an uplink frame structure that includes pilot training strongly impacts the achievable SE of LIS systems. Moreover, this pilot signal will be contaminated by inter-LIS interference, similar to inter-cell interference in multi-cell MIMO environment (e.g., see [17]–[19]). Therefore, accurate CSI estimations with an optimal pilot training lengths constitute an important challenge in multi-LIS systems where the pilot sequences are reused in adjacent LISs. In fact, prior studies on massive MIMO [16]–[19] do not directly apply to LIS, because the channel model of LIS is significantly different from the one used in these prior studies. For densely located LISs, all channels will be modeled by device-specific spatially correlated Rician fading depending on the distance between each LIS and device, however, the massive MIMO works in [16]–[19] rely on a Rayleigh fading channel considering far-field communications. Moreover, in LIS, each area of the large surface constitutes one of the key parameters that determine the performance of an LIS system [8]–[10], however, in existing massive MIMO works [8]–[10], this notion of an area is not applicable.

B. Contributions

The main contribution of this paper is an asymptotic analysis of the uplink system SE (SSE) in a multi-LIS environment that considers a practical uplink frame structure based on the 3GPP model in [20]. The SSE is typically measured as the data rates that can be simultaneously supported by a limited bandwidth in a defined geographic area [21]. Given an LIS serving multiple devices, we define the SSE as the sum of the individual SE of each LIS device. Then, we analyze the asymptotic SSE including its ergodic value, channel hardening effect, and performance bound, under pilot contamination considerations, relying on a scaling law for a large number of antennas. The devised approximation allows for accurate estimations of the SSE,
deterministically, and it also allows verifying the reliability of an LIS system. Subsequently, we analyze the effect of pilot training in a realistic channel fading scenario in which channel states are limited by the channel coherence block and are exclusively static within limited time and frequency blocks. The pilot training analysis provides insights on how the optimal pilot training length scales with the various parameters of LIS systems in a deterministic way. It also reveals a particular operating characteristic of an LIS, whereby the optimal pilot training length converges to the number of devices located within an LIS area, as the number of antennas increases without bound. Given the derived pilot training length, we finally derive the optimal number of devices that each LIS must schedule, to maximize the SSE. Simulation results show that an LIS with the proposed operating parameters, including the pilot training length and the number of scheduled devices, can achieve a maximum SSE performance both in single- and multi-LIS environments, regardless of the effect of pilot contamination and inter-LIS interference. Moreover, the impact on an LIS system of pilot contamination can be negligible when inter-LIS interference channels are generated from spatially correlated Rayleigh fading, which highlights a significant difference from conventional massive MIMO.

The rest of this paper is organized as follows. Section II presents the LIS-based system model. Section III describes the asymptotic analysis of the SSE, and Section IV describes the performance bound and optimal pilot training length based on the results of Section III. The optimal number of scheduled devices is also discussed in Section IV. Simulation results are provided in Section V to support and verify the analyses, and Section VI concludes the paper.

Notations: Hereinafter, boldface upper- and lower-case symbols represent matrices and vectors respectively, and $\mathbf{I}_M$ denotes a size-$M$ identity matrix. $\mu_X$ and $\sigma^2_X$ denote the mean and variance of a random variable $X$, respectively. The conjugate, transpose, and Hermitian transpose operators are denoted by $\langle \cdot \rangle^*, \langle \cdot \rangle^T$, and $\langle \cdot \rangle^H$, respectively. The norm of a vector $\mathbf{a}$ is denoted by $|\mathbf{a}|$ and the Frobenious norm of a matrix $\mathbf{A}$ is $\|\mathbf{A}\|_F$. $\mathbb{E}[\cdot]$ and $\mathcal{O}(\cdot)$ denote the expectation operator and big O notation, respectively. $\mathcal{CN}(m, \sigma^2)$ is a complex Gaussian distribution with mean $m$ and variance $\sigma^2$.

II. **System Model**

Consider an uplink LIS system with $N \geq 1$ LISs sharing the same frequency band. Each LIS is located in two-dimensional Cartesian space along the $xy$-plane, serving $K$ devices, as shown
Fig. 1. Illustrative system model of the uplink LIS under consideration of (a) indoor case with single LIS, (b) outdoor case with multiple LISs, with \( K \) devices per each LIS.

In Fig. 1, each LIS is composed of \( K \) LIS units, each of which serves a single-antenna device occupying a \( 2L \times 2L \) square shaped subarea of the entire LIS. A large number of antennas, \( M \), are deployed on the surface of each LIS unit with \( \Delta L \) spacing, arranged in a rectangular lattice centered on the \((x, y)\) coordinates of the corresponding device. Considering the location of device \( k \) at LIS \( n \) as \((x_{nk}, y_{nk}, z_{nk})\), antenna \( m \) of LIS unit \( k \) at LIS \( n \) will be located at \((x_{nk}^{LIS}, y_{nk}^{LIS}, 0)\) where \( x_{nk}^{LIS} \in [x_{nk} - L, x_{nk} + L] \) and \( y_{nk}^{LIS} \in [y_{nk} - L, y_{nk} + L] \). Fig. 1 illustrates our system model for an indoor case with a single LIS and an outdoor case with multiple LISs. In case of single LIS, as shown in Fig. 1(a), the desired signal is affected, exclusively, by \textit{intra-LIS interference} which is defined as the interference generated by multiple devices located within the same LIS area. On the other hand, for the case of multiple LISs, as shown in Fig. 1(b), the desired signal can be affected by both \textit{intra-LIS} and \textit{inter-LIS interference} simultaneously. Here, \textit{inter-LIS interference} corresponds to the interference generated by devices serviced by
other LISs.

Some LIS units may overlap depending on the locations of their associated devices, resulting in severe performance degradation. To prevent this problem, we assume that each LIS consists of \( K \) non-overlapping LIS units that use an orthogonal multiple access scheme among devices with similar locations. Moreover, each device controls its transmission power toward the center of its LIS unit according to a target signal-to-noise-ratio (SNR), to avoid the near-far problem.

\[ \text{A. Wireless Channel Model} \]

In LIS systems, entire man-made structures are electromagnetically active and can be used for wireless communication. We then consider the LIS channel \( h_{nnkk}^L \in \mathbb{C}^M \) between device \( k \) at LIS \( n \) and LIS unit \( k \) part of LIS \( n \) as a LOS path defined by:

\[
h_{nnkk}^L = \begin{bmatrix} \beta_{nnkk1}Lh_{nnkk1}^L, & \cdots, & \beta_{nnkkM}Lh_{nnkkM}^L \end{bmatrix}^T ,
\]

where \( \beta_{nnkkm}^L = \alpha_{nnkkm}^L h_{nnkkm}^L \) and \( h_{nnkkm}^L = \exp \left( -j2\pi d_{nnkkm}^L / \lambda \right) \) denote a LOS channel gain and state, respectively, between device \( k \) at LIS \( n \) and antenna \( m \) of LIS unit \( k \) part of LIS \( n \) [22]. The terms \( a_{nnkkm}^L = \sqrt{z_{nk} / d_{nnkkm}^L} \) and \( l_{nnkkm}^L = 1 / \sqrt{4\pi d_{nnkkm}^L} \) represent, respectively, the antenna gain and free space path loss attenuation, where \( d_{nnkkm} \) is the distance between device \( k \) at LIS \( n \) and antenna \( m \) of LIS unit \( k \) part of LIS \( n \). \( \lambda \) is the wavelength of a signal. We model the interference channel \( h_{lnjk} \in \mathbb{C}^M \) between device \( j \) at LIS \( l \) and LIS unit \( k \) part of LIS \( n \) as a Rician fading channel with Rician factor \( \kappa_{lnjk} \), given by:

\[
h_{lnjk} = \hat{h}_{lnjk} + \hat{h}_{lnjk} = \sqrt{\frac{\kappa_{lnjk}}{\kappa_{lnjk} + 1}} h_{lnjk}^L + \sqrt{\frac{1}{\kappa_{lnjk} + 1}} h_{lnjk}^NL, \tag{2}
\]

where \( h_{lnjk}^L \in \mathbb{C}^M = \begin{bmatrix} \beta_{lnjkl}^L h_{lnjkl1}, & \cdots, & \beta_{lnjklM}^L h_{lnjklM} \end{bmatrix}^T \) and \( h_{lnjk}^NL \in \mathbb{C}^M = R_{lnjk}^{1/2} g_{lnjk} \) denote the deterministic LOS and the correlated NLOS component, respectively. Here, if \( l = n \) and \( j \neq k \), then \( h_{lnjk} \) indicates the intra-LIS interference channel, otherwise, if \( l \neq n \) \( \forall j, k \), then \( h_{lnjk} \) indicates the inter-LIS interference channel. Considering \( P \) dominant paths among all NLOS paths, we define \( R_{lnjk} \in \mathbb{C}^{M \times P} \) and \( g_{lnjk} = [g_{lnjkl1}, \cdots, g_{lnjklP}]^T \sim \mathcal{CN}(0, I_P) \) to be the deterministic correlation matrix and an independent fast-fading channel vector between device \( j \) at LIS \( l \) and LIS unit \( k \) part of LIS \( n \), respectively. Since the LIS is deployed on the horizontal plane, as shown in Fig. [1], we can model it as a uniform planar array [23]. Then, the correlation matrix can be defined as \( R_{lnjk}^{1/2} = l_{lnjk}^NL D_{lnjk}, \) where \( l_{lnjk}^NL = \text{diag} \left( l_{lnjkl1}^NL, \cdots, l_{lnjklM}^NL \right) \).
is a diagonal matrix that includes the path loss attenuation factors \( l_{nlkm}^\text{NL} = d_{lnjk}^{-\beta_{PL}/2} \) with a path loss exponent \( \beta_{PL} \) and \( D_{lnjk} = [\alpha_{lnjk1}^\text{NL} \mathbf{d}(\phi_{lnjk1}, \phi_{lnjk1})^\text{h}, \cdots, \alpha_{lnjkm}^\text{NL} \mathbf{d}(\phi_{lnjkm}, \phi_{lnjkm})^\text{h}]^\text{T} \).

\( \mathbf{d}(\phi_{lnjkp}^\text{v}, \phi_{lnjkp}^\text{h}) \in \mathbb{C}^M \) is the NLOS path \( p \) at given angles of \( (\phi_{lnjkp}^\text{v}, \phi_{lnjkp}^\text{h}) \) defined as:

\[
\mathbf{d}(\phi_{lnjkp}^\text{v}, \phi_{lnjkp}^\text{h}) = \frac{1}{\sqrt{M}} \mathbf{d}_v(\phi_{lnjkp}^\text{v}) \otimes \mathbf{d}_h(\phi_{lnjkp}^\text{h}),
\]

\[
\mathbf{d}_v(\phi_{lnjkp}^\text{v}) = \begin{bmatrix} 1, e^{j2\pi \Delta_L \phi_{lnjkp}^\text{v}}, \cdots, e^{j2\pi \Delta_L (\sqrt{M}-1) \phi_{lnjkp}^\text{v}} \end{bmatrix}^\text{T},
\]

\[
\mathbf{d}_h(\phi_{lnjkp}^\text{h}) = \begin{bmatrix} 1, e^{j2\pi \Delta_L \phi_{lnjkp}^\text{h}}, \cdots, e^{j2\pi \Delta_L (\sqrt{M}-1) \phi_{lnjkp}^\text{h}} \end{bmatrix}^\text{T},
\]

where \( \phi_{lnjkp}^\text{v} = \sin \theta_{lnjkp}^\text{v} \) and \( \phi_{lnjkp}^\text{h} = \sin \theta_{lnjkp}^\text{h} \cos \theta_{lnjkp}^\text{h} \) when the elevation and azimuth angles of path \( p \) between device \( j \) at LIS \( l \) and LIS unit \( k \) part of LIS \( n \) are \( \theta_{lnjkp}^\text{v} \) and \( \theta_{lnjkp}^\text{h} \), respectively [24]. Further, \( \alpha_{lnjkp}^\text{NL} = \sqrt{\cos \theta_{lnjkp}^\text{v} \cos \theta_{lnjkp}^\text{h}} \) denotes the antenna gain of path \( p \) with \( \theta_{lnjkp} \in [-\frac{\pi}{2}, \frac{\pi}{2}] \) and \( \theta_{lnjkp} \in \{ \theta_{lnjkp}^\text{v}, \theta_{lnjkp}^\text{h} \} \).

**B. Uplink Pilot Training**

We consider that an MF is used at the LIS to amplify the desired signals and suppress interfering signals. This MF receiver requires CSI which can be estimated by pilot signaling with known pilot signals being transmitted from the device to the LIS. The device transmits its data signals immediately after sending the pilot signals within the channel coherence time \( T \) in which the uplink channel is approximately static. We consider the uplink frame structure shown in Fig. 2 in which the total duration of \( T \) channel uses is divided into a \( t \) period used for pilot training and a \( T - t \) period used for data transmission. Every device simultaneously transmits \( t \geq K \) orthogonal pilot sequence over the uplink channel to the LIS, so that the required CSI can be acquired. Given that those \( K \) pilot sequences are pairwise orthogonal to each other, we have \( \Psi^H \Psi = I_K \), where \( \Psi = [\psi_1, ..., \psi_K] \) and \( \psi_k \) is the \( t \times 1 \) pilot sequence for device \( k \).

For the multi-LIS scenario in which the same frequency band is shared by all LISs and adjacent LISs reuse the pilot sequences, the pilot symbols between adjacent LISs are no longer orthogonal...
to each other and this non-orthogonality causes *pilot contamination*. In large antenna-array systems such as massive MIMO and LIS, the performance can be dominantly limited by residual interference from pilot contamination as explained in [17]–[19]. Since LISs will be located more densely than BSs, the LIS channels associated with pilot contamination will be significantly different than those of massive MIMO, and hence, prior studies on pilot contamination for massive MIMO [16]–[19] do not directly apply to LIS. In order to verify the effect of pilot contamination in an LIS system theoretically, we consider such multi-LIS scenario in which a total of \( N \) LISs share the same frequency band and each LIS reuses \( K \) pilot sequences. Moreover, all LISs are assumed to use the same uplink frame structure shown in Fig. 2 whereby a pilot sequence \( k \) is allocated to device \( k \) for all \( 1 \leq k \leq K \). The uplink pilot sequence received from all devices at LIS unit \( k \) part of LIS \( n \) during period \( t \) will be:

\[
Y_{nk}^p = \sqrt{t \rho_{nk}} h_{nkk}^L \psi_k^H + \sum_{j \neq k} \sqrt{t \rho_{nj}} h_{nnjk} \psi_j^H + \sum_{l \neq n} \sum_{j=1}^K \sqrt{t \rho_{lj}} h_{lnjk} \psi_j^H + N_{nk},
\]

where \( \rho_{nk}, \rho_{nj}, \) and \( \rho_{lj} \) are the transmit SNRs for the pilot symbols of device \( k \) at LIS \( n \), device \( j \) at LIS \( n \), device \( j \) at LIS \( l \), respectively, and \( N_{nk} \in \mathbb{C}^{M \times t} \sim \mathcal{CN}(0, I_M) \) is a noise matrix at LIS unit \( k \) part of LIS \( n \). We assume that the target SNR for a pilot symbol is assumed to be equal to \( \rho_p \) and each device controls its pilot power toward the center of the corresponding LIS unit. On the basis of orthogonal characteristic of the pilot sequences, each LIS unit \( k \) multiplies the received pilot signal by \( \psi_k \) for channel estimation. After multiplying \( \psi_k \) at both sides of (6), we have

\[
Y_{nk}^p \psi_k = \sqrt{t \rho_{nk}} h_{nkk}^L + \sum_{l \neq n} \sqrt{t \rho_{nk}} h_{lnkk} + N_{nk} \psi_k
\]

\[
= \sqrt{t \rho_{nk}} h_{mkk}^L + \sum_{l \neq n} \sqrt{t \rho_{nk}} \left( \tilde{h}_{lnkk} + \tilde{h}_{lnkk} \right) + N_{nk} \psi_k.
\]

In most prior research on pilot contamination in large antenna-array systems such as in [18] and [25], the minimum mean square error (MMSE) channel estimator is assumed to estimate a desired channel given that the BS has knowledge of every correlation matrix between itself and interfering users located in adjacent cells. However, this assumption is impractical for LIS systems because a massive number of devices will be connected to an LIS, and thus, processing complexity will increase tremendously when estimating and sharing device information. Therefore, we consider a simple least square (LS) estimator which does not require such information as a practical
alternative \cite{26}. The LS estimate of the deterministic desired channel $\hat{h}_{nnk}^L$ is then obtained as follows:

$$\hat{h}_{nnk} = h_{nnk}^L + e_{nk},$$

(8)

where $e_{nk}$ indicates the estimation error vector given by $e_{nk} = \sum_{l\neq n}^N \sqrt{\rho_{plnk}}\left(h_{lnkk} + \hat{h}_{lnkk}\right) + \frac{1}{\sqrt{\rho_{nk}}} w_{nk}$ and $w_{nk} = [w_{nk1}, \ldots, w_{nkM}]^T \in \mathbb{C}^M \sim \mathcal{CN}(0, I_M)$.

C. Uplink SSE

The uplink signal received from all devices at LIS unit $k$ part of LIS $n$ is given by:

$$y_{nk} = \sqrt{\rho_{nk}} h_{nnkk}^L x_{nk} + \sum_{j\neq k}^K \sqrt{\rho_{nj}} h_{nnjk} x_{nj} + \sum_{l\neq n}^N \sum_{j=1}^K \sqrt{\rho_{lj}} h_{lnjk} x_{lj} + n_{nk},$$

(9)

where $x_{nk}$, $x_{nj}$, and $x_{lj}$ are uplink transmit signals of device $k$ at LIS $n$, device $j$ at LIS $n$, and device $j$ at LIS $l$, respectively, and $\rho_{nk}$, $\rho_{nj}$, and $\rho_{lj}$ are their transmit SNRs. Also, $n_{nk} \in \mathbb{C}^M \sim \mathcal{CN}(0, I_M)$ is the noise vector at LIS unit $k$ part of LIS $n$. Given a linear receiver $f_{nk}^H$ for signal detection, we will have

$$f_{nk}^H y_{nk} = \sqrt{\rho_{nk}} f_{nk}^H h_{nnkk}^L x_{nk} + \sum_{j\neq k}^K \sqrt{\rho_{nj}} f_{nk}^H h_{nnjk} x_{nj} + \sum_{l\neq n}^N \sum_{j=1}^K \sqrt{\rho_{lj}} f_{nk}^H h_{lnjk} x_{lj} + f_{nk}^H n_{nk},$$

(10)

We consider an MF receiver such that $f_{nk} = \hat{h}_{nnkk}$. Under the imperfect CSI results from an LS estimator, $f_{nk}$ can be obtained from (8) as $f_{nk} = h_{nnkk}^L + e_{nk}$ where $e_k$ is the estimation error vector uncorrelated with $n_{nk}$ \cite{27}. Therefore, the received signal-to-interference-plus-noise ratio (SINR) at LIS unit $k$ part of LIS $n$ will be:

$$\gamma_{nk} = \frac{\rho_{nk}|h_{nnkk}^L|^4}{\rho_{nk}|e_{nk}^H h_{nnkk}|^2 + \sum_{j\neq k}^K \rho_{nj}|\hat{h}_{nnkk}^H h_{nnjk}|^2 + \sum_{l\neq n}^N \sum_{j=1}^K \rho_{lj}|\hat{h}_{nnkk}^H h_{lnjk}|^2 + |(h_{nnkk}^L)^H + e_{nk}^H|^2}.$$ 

(11)

For notational simplicity, we define $\gamma_{nk} = \rho_{nk} S_{nk}/I_{nk}$, where $S_{nk} = |h_{nnkk}^L|^4$, and

$$I_{nk} = \rho_{nk} X_{nk} + \sum_{j\neq k}^K \rho_{nj} Y_{njk} + \sum_{l\neq n}^N \sum_{j=1}^K \rho_{lj} Y_{lnjk} + Z_{nk},$$

(12)

where $X_{nk} = |e_{nk}^H h_{nnkk}|^2$, $Y_{njk} = |\hat{h}_{nnkk}^H h_{nnjk}|^2$, $Y_{lnjk} = |\hat{h}_{nnkk}^H h_{lnjk}|^2$, and $Z_{nk} = (h_{nnkk}^L)^H + e_{nk}^H|^2$. Considering $t$ and $T - t$ periods used for pilot training and data transmission, respectively, the
SSE can be obtained as follows:

\[ R_{n}^{SSE} = \left( 1 - \frac{t}{T} \right) \sum_{k=1}^{K} R_{nk} = \left( 1 - \frac{t}{T} \right) \sum_{k=1}^{K} \log (1 + \gamma_{nk}). \] (13)

Given this SSE, we will be able to analyze the asymptotic value of the SSE and devise an optimal pilot training length \( t \) that maximizes the asymptotic SSE as \( M \) increases without bound. Note that in the following sections, we use a generalized value of \( N \geq 1 \) in order to analyze both single- and multi-LIS cases, simultaneously (i.e., \( N = 1 \) and \( N \geq 2 \) indicate a single-LIS and \( N \)-LIS cases, respectively).

III. Asymptotic SSE Analysis

We analyze the asymptotic value of the SSE under consideration of the pilot contamination as \( M \) increases to infinity. In an uplink LIS system with MF receiver, the desired signal power, \( S_{nk} \), converges to a deterministic value as \( M \) increases to infinity as proved in [8] and [9]:

\[ S_{nk} - \bar{p}_{nk} \xrightarrow{M \to \infty} 0, \]

where \( \bar{p}_{nk} = \frac{M^{2}p_{nk}^{2}}{16\pi^{2}L^{4}} \) and \( p_{nk} = \tan^{-1}\left( \frac{L^{2}/(z_{nk}\sqrt{2L^{2} + z_{nk}^{2}})}{z_{nk}\sqrt{2L^{2} + z_{nk}^{2}}} \right) \). Given the definition of \( \gamma_{nk} = \rho_{nk}S_{nk}/I_{nk} \), we have

\[ \tilde{\gamma}_{nk} \xrightarrow{M \to \infty} 0, \]

where

\[ \tilde{\gamma}_{nk} = \frac{\rho_{nk}p_{nk}^{2}}{16\pi^{2}L^{4}I_{nk}/M^{2}}, \] (14)

We can observe from (14) that the distribution of \( \tilde{\gamma}_{nk} \) depends exclusively on the distribution of \( I_{nk} \). In order to analyze the distribution of \( I_{nk} \) theoretically, we derive the following lemmas.

Here, we define \( R_{nnk}^{1/2} = [c_{lnjk1}, \cdots, c_{lnjkP}] = [r_{lnjk1}^{H}, \cdots, r_{lnjkM}^{H}]^{H} \), where \( c_{lnjkp} \in \mathbb{C}^{M \times 1} \) and \( r_{lnjkm} \in \mathbb{C}^{1 \times P} \).

**Lemma 1.** The mean of \( X_{nk} \) is obtained by \( \mu_{X_{nk}} = \sigma_{x_{nk}}^{2} + |\mu_{x_{nk}}|^{2} \), where

\[ \mu_{x_{nk}} = \sum_{l \neq n}^{N} \sqrt{\frac{\rho_{pk}}{\rho_{pnk}}} h_{lnkk}^{H} h_{nnkk}^{L}, \] (15)

\[ \sigma_{x_{nk}}^{2} = \sum_{l \neq n}^{N} \frac{\rho_{pk}}{\rho_{pnk}} (\kappa_{lnkk} + 1) \sum_{p=1}^{P} |c_{lnkpk}^{H} h_{nnkk}^{L}|^{2} + \frac{1}{t\rho_{pk}} \sum_{m=1}^{M} \beta_{nnkkm}^{2}. \] (16)

**Proof:** The detailed proof is presented in Appendix A.

**Lemma 2.** The mean values of \( Y_{njk} \) and \( Y_{lnjk} \) follow \( \mu_{Y_{njk}} - \bar{\mu}_{Y_{njk}} \xrightarrow{M \to \infty} 0 \) and \( \mu_{Y_{lnjk}} - \bar{\mu}_{Y_{lnjk}} \xrightarrow{M \to \infty} 0 \), respectively, where \( \bar{\mu}_{Y_{njk}} \) and \( \bar{\mu}_{Y_{lnjk}} \) are given, respectively, in (52) and (53).

**Proof:** The detailed proof is presented in Appendix B.
Lemma 3. The mean of $Z_{nk}$ is obtained by
\[ \mu_{Z_{nk}} = \frac{\sum_{m=1}^{M} \left( \sigma_{z_{nm}}^{2} + |\mu_{z_{nm}}|^{2} \right)}{M}, \]
where
\[ \sigma_{z_{nm}}^{2} = \sum_{l \neq n} \rho_{pl} |r_{lnkm}|^{2} + \frac{1}{t \rho_{pn}} \]
(17)
and
\[ \sigma_{Z_{nk}}^{2} = \sum_{l \neq n} \rho_{pl} (\kappa_{lnkk} + 1) + \frac{1}{t \rho_{pn}}. \]
(18)

\textbf{Proof:} The detailed proof is presented in Appendix C.

In Lemmas 1–3, the variables, $\mu_{X_{nk}}$, $\bar{\mu}_{Y_{nk}}$, $\bar{\mu}_{Y_{lnjk}}$, and $\mu_{Z_{nk}}$, are obtained by the deterministic information such as the locations of the devices and covariance matrices. On the basis of Lemmas 1–3, we can asymptotically derive the mean of $I_{nk}$ from (12) as follows:
\[ \mu_{I_{nk}} - \bar{\mu}_{I_{nk}} \to 0, \]
where
\[ \bar{\mu}_{I_{nk}} = \rho_{nk} \mu_{X_{nk}} + \sum_{j \neq k} \rho_{nj} \bar{\mu}_{Y_{nk}} + \sum_{l \neq n} \sum_{j=1}^{K} \rho_{lj} \bar{\mu}_{Y_{lnjk}} + \mu_{Z_{nk}}. \]
(19)
Since the variance of $\bar{\gamma}_{nk}$ exclusively depends on the variance of $I_{nk}/M^2$ from (14), the following Lemma 4 is used to obtain $\sigma_{I_{nk}}^{2}/M^4$ based on the scaling law for $M$.

Lemma 4. According to the scaling law for $M$, the variance of $I_{nk}/M^2$ asymptotically follows
\[ \sigma_{I_{nk}}^{2}/M^4 \to 0. \]

\textbf{Proof:} The detailed proof is presented in Appendix D.

Lemma 4 shows that $I_{nk}/M^2$ converges to the deterministic value $\bar{\mu}_{I_{nk}}/M^2$ without any variance, as $M$ increases. Then, $\bar{\gamma}_{nk}$ converges to a deterministic value as $M$ increases, and finally, we have the following Theorem 1 related to the asymptotic convergence of $R_{n}^{SSE}$.

Theorem 1. As $M$ increases to infinity, we have the following asymptotic convergence of SSE:
\[ R_{n}^{SSE} - \bar{\mu}_{n}^{SSE} \to 0, \]
where
\[ \bar{\mu}_{n}^{SSE} = \left( 1 - \frac{t}{T} \right) \sum_{k=1}^{K} \log \left( 1 + \frac{\rho_{nk} \rho_{nk}^{2}}{16\pi^{2}L^{4} \bar{\mu}_{I_{nk}}/M^{2}} \right). \]
(20)

\textbf{Proof:} The detailed proof is presented in Appendix E.

Theorem 1 shows that the multi-LIS system will experience a channel hardening effect resulting in the deterministic SSE. This deterministic SSE provides the improved system reliability and a low latency. Moreover, we can observe from (19) and (20) that the asymptotic SSE can be obtained from the deterministic information such as the locations of the devices and correlation matrices. Therefore, this asymptotic approximation enables accurate estimation of
the SSE without the need for extensive simulations. Next, we use this asymptotic SSE to derive a performance bound on the SSE, asymptotically, by analyzing $\bar{\mu}^{\text{SSE}}_n$ via a scaling law for an infinite $M$.

IV. SSE PERFORMANCE BOUND FOR LISs AND OPTIMAL SYSTEM PARAMETERS

We now analyze the performance bound of the SSE for a large (infinite) value for $M$. As $M$ increases, the SSE converges to $\bar{\mu}^{\text{SSE}}_n$ which depends on the value of $\bar{\mu}_{I_{nk}}/M^2$, as seen from (20). Hence, in an LIS system equipped with an infinite number of antennas, it is important to obtain its limiting value, $\lim_{M \to \infty} \bar{\mu}_{I_{nk}}/M^2$, and this can provide the performance bound of the SSE, asymptotically. In this section, we derive the asymptotic SSE performance bound for an infinite $M$ using a scaling law of $\bar{\mu}^{\text{SSE}}_n$, and propose an optimal pilot training length based on that asymptotic bound. We consider the uplink frame structure shown in Fig. 2 in which all devices simultaneously transmit their orthogonal pilot sequences before transmitting their own data signals like a 3GPP model in [20]. Due to the pilot training overhead $t$, there is a fundamental tradeoff between the received SINR enhancement and loss of uplink channel uses for the data signal. Therefore, it is imperative to optimize the pilot training length to achieve the maximum SSE and define how the optimal pilot training length deterministically scales with the various parameters of LIS system.

In order to derive the performance bound of LIS system, we first determine the scaling law of $\bar{\mu}_{I_{nk}}/M^2$ according to $M$. Then, we have the following result related to the performance bound of SSE.

**Theorem 2.** As $M$ increases, $\bar{\mu}^{\text{SSE}}_n$ asymptotically converges to its performance bound, $\hat{\mu}^{\text{SSE}}_n$, as given by:

$$\hat{\mu}^{\text{SSE}}_n = \mu^{\text{SSE}}_n - \frac{t}{T} \sum_{k=1}^{K} \log \left( 1 + \frac{M^2 \rho_{nk} \rho_{nk}^2}{16 \pi^2 L^4 \bar{\mu}_{I_{nk}}} \right),$$

(21)

$$\hat{\mu}_{I_{nk}} = \rho_{nk} |\mu_{x_{nk}}|^2 + \sum_{j \neq k}^{K} \rho_{nj} |\mu_{y_{nkj}}|^2 + \sum_{l \neq n}^{N} \sum_{j=1}^{K} \rho_{lj} |\mu_{y_{lnjk}}|^2.$$  

(22)

**Proof:** From (19), $\bar{\mu}_{I_{nk}}/M^2$ is obtained as follows:

$$\frac{\bar{\mu}_{I_{nk}}}{M^2} = \rho_{nk} \frac{\mu_{x_{nk}}}{M^2} + \sum_{j \neq k}^{K} \rho_{nj} \frac{\mu_{y_{nkj}}}{M^2} + \sum_{l \neq n}^{N} \sum_{j=1}^{K} \rho_{lj} \frac{\mu_{y_{lnjk}}}{M^2} + \frac{\mu_{Z_{nk}}}{M^2}. $$

(23)
On the basis of Lemmas 1–3, we determine the scaling laws of the terms \( \mu_{X_{nk}} / M^2 \), \( \mu_{Y_{njk}} / M^2 \), \( \mu_{Z_{nk}} / M^2 \), and \( \mu_{Z_{nkm}} / M^2 \) in (23) according to \( M \). As proved in Appendix D, the terms \( \sigma_{x_{nk}}^2 / M^2 \), \( \sigma_{y_{njk}}^2 / M^2 \), \( \sigma_{y_{lnjk}}^2 / M^2 \), and \( \sum_{m=1}^{M} \sigma_{z_{nkm}}^2 / M^2 \) converge to zero as \( M \) increases. Similarly, the terms \( |\mu_{x_{nk}}|^2 / M^2 \), \( |\mu_{y_{njk}}|^2 / M^2 \), and \( |\mu_{y_{lnjk}}|^2 / M^2 \) follow \( O(1) \), and \( \sum_{m=1}^{M} |\mu_{z_{nkm}}|^2 / M^2 \) decreases with \( O(1/M) \) as \( M \) increases, based on the scaling laws for large \( M \). Therefore, we have the following asymptotic convergence:

\[
\frac{\mu_{X_{nk}}}{M^2} \xrightarrow{M \to \infty} 0,
\]

\[
\frac{\mu_{Y_{njk}}}{M^2} \xrightarrow{M \to \infty} 0,
\]

\[
\frac{\mu_{Y_{lnjk}}}{M^2} \xrightarrow{M \to \infty} 0,
\]

\[
\frac{\mu_{Z_{nk}}}{M^2} \xrightarrow{M \to \infty} 0,
\]

which completes the proof.

The terms \( \mu_{x_{nk}} \), \( \mu_{y_{njk}} \), and \( \mu_{y_{lnjk}} \) in (22) are determined by the LOS channels depending on the locations of the devices, as shown, respectively, in (36), (50), and (54). Therefore, the asymptotic SSE performance bound can be obtained, deterministically, and that deterministic bound leads to several important implications when evaluating an LIS system, that significantly differ from conventional massive MIMO. First, an LIS system has a particular operating characteristic whereby the pilot contamination and intra/inter-LIS interference through the NLOS path and noise become negligible as \( M \) increases. If all of the inter-LIS interference is generated from the NLOS path, the pilot contamination and inter-LIS interference will vanish lead to a performance convergence between the SSE of single- and multi-LIS system. Moreover, unlike conventional massive MIMO in which the performance is dominantly limited by pilot contamination, the channel estimation error including pilot contamination gradually loses its effect on the SSE, and eventually, the SSE of a multi-LIS system will reach that of a single-LIS system with perfect CSI. More practically, even if all of the intra/inter-LIS interference channels are generated by device-specific spatially correlated Rician fading, an LIS system also has a particular characteristic whereby its SSE performance is bounded by three factors that include pilot contamination, intra-, and inter-LIS interference through the LOS path,
Next, we formulate an optimization problem whose goal is to maximize the SSE with respect to the pilot training length $t$ by using the asymptotic SSE, $\tilde{\mu}_{n_k}^{SSE}$, from Theorem 1. Since $\tilde{\mu}_{I_{nk}}$ is a function of $t$, we have

$$\max_t \left(1 - \frac{t}{T}\right) \sum_{k=1}^{K} \log \left(1 + \frac{M^2 \rho_{nk} R_{nk}^2}{16\pi^2 L^4 \tilde{\mu}_{I_{nk}}(t)}\right),$$  

(28)

s.t. $K \leq t \leq T$.

(28a)

$$\tilde{\mu}_{I_{nk}}(t) = \rho_{nk} \left(\sigma^2_{x_{nk}}(t) + |\mu_{x_{nk}}|^2\right) + \sum_{j \neq k} \rho_{nj} \left(\sigma^2_{y_{nj}}(t) + |\mu_{y_{nj}}|^2\right) + \sum_{l \neq n} \sum_{j=1}^{K} \rho_{lj} \left(\sigma^2_{y_{lnj}}(t) + |\mu_{y_{lnj}}|^2\right) + \sum_{m=1}^{M} \left(\sigma^2_{z_{nkm}}(t) + |\mu_{z_{nkm}}|^2\right).$$  

(28b)

In (28b), the terms, $\sigma^2_{x_{nk}}(t)$, $\sigma^2_{y_{nj}}(t)$, $\sigma^2_{y_{lnj}}(t)$, and $\sigma^2_{z_{nkm}}(t)$, are monotonically decreasing functions with respect to $t$ as observed from (37), (51), (55), and (60), respectively. Thus, $\tilde{\mu}_{I_{nk}}(t)$ is also a monotonically decreasing function and $\log \left(1 + \frac{M^2 \rho_{nk} R_{nk}^2}{16\pi^2 L^4 \tilde{\mu}_{I_{nk}}(t)}\right)$ is thus a positive concave increasing function with respect to $t$. From the operations that preserve the concavity of functions [28], the product of a positive decreasing function and a positive concave increasing function is concave. Thus, (28) is a convex optimization problem and we can obtain the globally optimal result, $t_{opt}$, using a simple gradient method. Moreover, we can observe from the objective function in (28) that $t_{opt}$ changes according to deterministic values such as the locations of the devices and correlation matrices. From Theorem 2, $\tilde{\mu}_{I_{nk}}(t)$ asymptotically converges to $\hat{\mu}_{I_{nk}}$ as $M$ increases, resulting in the received SINR independent with $t$. Therefore, in the following Corollary 1, we obtain an asymptotic value of $t_{opt}$, independent of the locations of the devices and correlation matrices, using the asymptotic bound of the SSE from Theorem 2.

**Corollary 1.** As $M$ increases, the optimal pilot training length can be asymptotically obtained as follows: $t_{opt} - K \xrightarrow{M \to \infty} 0$.

**Proof:** As proved in Theorem 2, the SSE asymptotically converges to its performance bound $\mu_n^{SSE}$ as $M$ increases, and $\hat{\mu}_{I_{nk}}$ is independent with $t$ as seen from (22). Then, $\hat{\mu}_{I_{nk}}^{SSE}$ is a monotonically decreasing function with respect to $t$, and therefore, the optimal pilot training length asymptotically converges to $K$, which completes the proof.

In conventional massive MIMO system, the pilot training length affects the received SINR and its optimal value is determined by various system parameters such as the number of users, uplink transmission period $T$, and transmit SNR, as proved in [16].
TABLE I
SIMULATION PARAMETERS

| Parameter                                      | Value                  |
|-----------------------------------------------|------------------------|
| Carrier frequency                             | 3 GHz                  |
| Minimum distance between LIS & device         | 1 m                    |
| Target SNR for uplink pilot                   | 0 dB                   |
| Target SNR for uplink data                    | 3 dB                   |
| Coherence block length \((T)\)                | 500 symbols            |
| Length of LIS unit \((2L)\)                   | 0.5 m                  |
| Rician factor \((\kappa[\text{dB}]\) [29]    | 13 – 0.03d[m]          |
| LOS path loss model [22]                      | 11 + 20\log_{10}d[m]  |
| NLOS path loss model [18]                     | 37\log_{10}d[m] \((\beta_{PL} = 3.7)\) |

MIMO, Corollary 1 shows that, for both the single- and multi-LIS cases, the asymptotic bound of the received SINR does not increase with the increase in the pilot training length and the maximum SSE can be achieved through a minimum pilot training length such as \(t = K\) (i.e., one pilot symbol per device), despite the pilot contamination effect. With the proposed pilot training length \(t = K\), we then formulate an optimization problem that asymptotically maximizes the sum of SSE for neighboring \(N\) LISs with respect to \(K\), as follows:

\[
\max_K \left(1 - \frac{K}{T}\right) \sum_{n=1}^{N} \sum_{k=1}^{K} \log(1 + \hat{\gamma}_{nk}(K)),
\]

\(29\)

\[
\text{s.t. } 1 \leq K \leq T,
\]

\(29a\)

\[
\hat{\gamma}_{nk}(K) = \frac{M^2 \rho_{nk} P_{nk}^2}{16\pi^2 L^4 \hat{\mu}_{nk}(K)}.
\]

\(29b\)

Consider that each device has its own prioritization coefficients for scheduling and each LIS schedules devices in order of their priority. According to this priority order, the objective function in \((29)\) can be calculated from \(K = 1\) to \(K = T\), deterministically, given that the value of \(\hat{\gamma}_{nk}(K)\) depends on the locations of the devices. Therefore, by comparing those values over the entire ranges of \(K\), the optimal number of scheduled devices can be asymptotically derived, without
the need for extensive simulations. Note that an LIS has the potential for estimating the locations of serving devices from their uplink signal \[12\] and the Rician factor is calculated according to the distance between the LIS and device \[29\]. By cooperating across adjacent multi-LISs like a network LIS, an LIS is able to share those information without the heavy burden of backhaul load and perform joint scheduling to maximize the network SE (NSE). Therefore, the asymptotically optimal number of scheduled devices, \(K_{opt}\), can be calculated at each LIS, practically, when adjacent multi-LISs cooperate with each other as a network LIS and share the information about the locations of their own serving devices.

V. SIMULATION RESULTS AND ANALYSIS

We present Monte Carlo simulation results for the uplink SSE in an LIS system, and compare them with the results of the asymptotic analyses. All simulations are statistically averaged over a large number of independent runs. The simulation parameters are based on the LTE specifications, presented in Table I. In accordance with the LTE specifications \[30\], the target SNR for uplink power control is semi-statically configured by upper-layer signaling in the LTE system. The range of target SNRs for the uplink data and pilot signals are -8 dB to 7 dB and -8 dB to 23 dB, respectively \[30\]. The uplink target SNRs presented in Table I satisfy the constraints of practical target SNRs. Furthermore, the minimum scheduling unit is defined as 1 ms in the time domain,
and 180 kHz in the frequency domain, which is the so-called physical resource block (PRB) in the LTE specifications [20]. Each PRB consists of a total of 168 symbols (14 orthogonal frequency-division multiplexing (OFDM) symbols in the time domain, and 12 subcarriers in the frequency domain) including the cyclic prefix overhead of the OFDM symbols. Since the value of $T = 500$ presented in Table I corresponds to approximately 3 PRBs ($500 \approx 168 \times 3$), we consider the uplink frame as one of two frames such as 1 ms $\times$ 540 kHz or 3 ms $\times$ 180 kHz. The value of $T = 500$ used in performance evaluation may be regarded as a moderate coherence block length, given that the generalized coherence lengths are $T = 200$ for high-mobility or high-delay spread scenarios, and $T = 5000$ for low-mobility or low-delay spread scenarios [31].

In the simulations, we consider both single- and multi-LIS cases. In both cases, we consider a scenario in which the devices are randomly and uniformly distributed within a three-dimensional space of 4 m $\times$ 4 m $\times$ 2 m in front of each LIS. For the single-LIS case, $N = 1$ and only a single target LIS is located in two-dimensional Cartesian space along the $xy$-plane. For the multi-LIS case, to be able to consider the effect of pilot contamination, we assume a total of $N = 4$ LISs consist of one target LIS and three neighboring LISs, located on both sides and in front of the target LIS, as shown in Fig. 3. The parameters presented in Fig. 3 are such that $x_L = y_L = 4$, $d_x = 4$, and $d_z = 6$. All LISs are assumed to share the same frequency band,
Fig. 5. Uplink ergodic SSE of an LIS system with Rician fading interference as a function of the number of antennas on the LIS unit when \( K = 20 \).

each of which serves \( K \) devices and reuses \( K \) pilot sequences. According to the 3GPP model in [29], the existence of a LOS path depends on the distance from the transmitter and receiver. The probability of a LOS is then given by

\[
P_{\text{LOS}}^{\text{lnjk}} = \begin{cases} 
\frac{(d_C - d_{\text{lnjk}})}{d_C}, & 0 < d_{\text{lnjk}} < d_C, \\
0, & d_{\text{lnjk}} > d_C, 
\end{cases}
\]  

(30)

where \( d_{\text{lnjk}} \) is the distance in meters between device \( j \) at LIS \( l \) and the center of the LIS unit \( k \) part of LIS \( n \), and \( d_C \) denotes the cutoff point, which is assumed to be 10 m, as in [8]. The Rician factor, \( \kappa_{\text{lnjk}} \), is calculated according to \( d_{\text{lnjk}} \), as per Table I.

Fig. 4 shows the channel hardening effect of an LIS system whereby the variances of individual SE in both single- and multi-LIS cases converge to zero as \( M \) increases, despite the pilot contamination effect. Here, the individual SE of device \( K \) at LIS \( n \) is derived from (13) as follows:

\[
R_{nk}^{\text{SE}} = \left(1 - \frac{t}{T}\right) \log \left(1 + \frac{\rho_{nk}S_{nk}}{I_{nk}}\right).
\]  

(31)

The variance convergence of \( R_{nk}^{\text{SE}} \) verifies the asymptotic convergence of \( S_{nk}/I_{nk} \), and Lemma 4 is then verified given that \( S_{nk} \) converges to a deterministic value as \( M \) increases.

In Figs. 5–7, Theorems 1 and 2 are verified in the following scenario. All intra-LIS interference
channels are generated by device-specific spatially correlated Rician fading. In Fig. 5, all inter-
LIS interference channels are also generated by that Rician fading, however, in Figs. 6 and 7, those channels are generated entirely from the NLOS path such as spatially correlated Rayleigh fading. In both Figs. 5 and 6, the asymptotic results from Theorem 1 become close to the results of our simulations and these results gradually approach to their performance bounds obtained from Theorem 2, as $M$ increases. Moreover, those performance bounds also converge to the limiting values resulting from the intra/inter-LIS interference through the LOS path. In Fig. 5, the performance gap between the results of the single- and multi-LIS is roughly 33 bps/Hz at $M = 900$, and it is expected to converge to 36 bps/Hz from the bound gap between the two systems, as $M$ increases. This performance gap between the two systems results from pilot contamination and inter-LIS interference generated from the LOS path, as proved in Theorem 2. In Fig. 6, the performance gap between the results of the single- and multi-LIS increases as $K$ increases from $K = 20$ to $K = 80$ because of the increase in the inter-LIS interference. However, the performance gap between the two systems converges to zero even at $K = 80$, as $M$ increases, and their bounds achieve an equal performance over the entire range of $M$. Since the the pilot contamination and inter-LIS interference generated from the NLOS path become negligible compared to the intra-LIS interference through the LOS path, this results
Fig. 7. Performance comparison of the ergodic SSE resulting from scenarios with perfect CSI and imperfect CSI when $K = 20$ with NLOS inter-LIS interference.

In the performance convergence between the two systems and eventually the multi-LIS system becomes an inter-LIS interference-free environment.

Fig. 7 compares the ergodic SSE resulting from cases with perfect CSI and imperfect CSI, when $K = 20$ and all inter-LIS interference channels are generated by spatially correlated Rayleigh fading. We can observe that all ergodic SSE converge to same value of roughly 110 bps/Hz. Hence, despite the pilot contamination in the multi-LIS case, the ergodic SSE of the multi-LIS system with the imperfect CSI converges to that with the perfect CSI, and it eventually reaches the single-LIS performance with perfect CSI, as $M$ increases. This clearly shows a particular characteristic of LIS systems whereby pilot contamination and inter-LIS interference become negligible, representing a significant difference from conventional massive MIMO.

Fig. 8 compares the ergodic SSE resulting from the optimal training lengths obtained from Corollary 1 and a brute-force search, as a function of $M$. As shown in Fig. 8, the optimal performance obtained by a brute-force search is nearly achieved by the proposed pilot training length, $t = K$, over the entire range of $M$. Although the ergodic SSE of a multi-LIS system is affected by pilot contamination and inter-LIS interference through the LOS path, thus resulting in performance degradation compared with the single-LIS case, the minimum training length
always achieves the optimal performance, regardless of the number of neighboring LISs and devices located within their serving area. This result shows a particular characteristic of an LIS that the accurate CSI is not an important system parameter in both single- and multi-LIS cases, unlike conventional massive MIMO.

Fig. 8 shows the performance comparison between ergodic SSE resulting from the proposed pilot training length and brute-force search as a function of the number of antennas on the LIS unit.

Fig. 9 shows the average NSE with the proposed pilot training length as a function of $K$, when $T = 50$, considering very high-mobility scenarios. The average NSE is defined by the sum of ergodic SSE for $N$ LISs divided by $N$. Since the pilot training length $t$ does not affect the asymptotic received SINR as proved in Corollary 1, the uplink channel uses for the data signal and its SINR decrease as $K$ increases due to the increase in the pilot training overhead and the intra/inter-LIS interference, respectively. Meanwhile, the NSE improves as $K$ increases given that it stems from the sum rate of $NK$ devices located within the serving area of a network LIS. Therefore, a fundamental tradeoff exists in terms of the average NSE according to the value of $K$. Due to the logarithmic nature of the mutual information, the average NSE increases with $K$ when $K$ is small, and starts decreasing with $K$ when $K$ exceeds a given threshold point, as shown in Fig. 9. The maximum NSE can be achieved, statistically, by the optimal value of $K$, which could be obtained experimentally as $K = 20$.

Fig. 10 compares the total NSE with the proposed number of scheduled devices, $K_{\text{opt}}$, and
that with a fixed number of scheduled devices, as a function of $M$. This fixed number is obtained experimentally as $K = 20$ from Fig. 9 and $K_{\text{opt}}$ can be obtained deterministically according to each device distribution. The optimal performance is also presented in Fig. 10. This optimal value is obtained, experimentally, by comparing every NSE over the entire ranges of $K$ for each device distribution. Fig. 10 shows that the NSE with the proposed $K_{\text{opt}}$ is always higher than that with $K = 20$ over the entire ranges of $M$ and their performance gap increases as $M$ increases. Moreover, the NSE with $K_{\text{opt}}$ nearly achieves the optimal performance obtained from our extensive simulations.

VI. CONCLUSIONS

In this paper, we have asymptotically analyzed the performance of an LIS system under practical LIS environments with a well-defined uplink frame structure and the pilot contamination. In particular, we have derived the asymptotic SSE by considering a practical LIS environment in which the interference channels are generated by device-specific spatially correlated Rician fading and channel estimation errors can be caused by pilot contamination based on a practical uplink frame structure. We have shown that the asymptotic results can accurately and analytically determine the performance of an LIS without the need for extensive simulations. Moreover, we
have studied the performance bound of SSE from the derived asymptotic SSE and obtained the optimal pilot training length to maximize the SSE, showing that the maximum SSE can be achieved with a minimum pilot training length of $t = K$, regardless of the pilot contamination effect. Furthermore, we have proved that the SSE of a multi-LIS system is bounded by three factors: pilot contamination, intra-LIS interference, and inter-LIS interference generated from the LOS path. On the other hand, the pilot contamination and intra/inter-LIS interference generated from the NLOS path and noise become negligible as $M$ increases. Simulation results have shown that our analytical results are in close agreement with the results arising from extensive simulations. Our results also show that, unlike conventional massive MIMO system, the effect of pilot contamination has been shown to become negligible when the inter-LIS interference is generated from NLOS path. Moreover, we have observed that the SSE of the proposed pilot training lengths achieve those obtained with the optimal lengths determined by a brute-force search, both in single- and multi-LIS environments. Furthermore, the maximum value of the NSE has been shown to be achievable, practically, by using the proposed number of scheduled devices based on a network LIS. In summary, in order to properly conduct the standardization process for LIS systems, it will be necessary to take into account the need for an adequate frame
structure including the proposed pilot training length and the number of scheduled devices.

**APPENDIX A**

**PROOF OF LEMMA 1**

Given the definition of \( X_{nk} = |e_{nk}^H h_{nnkk}^L|^2 \), we define

\[
x_{nk} = e_{nk}^H h_{nnkk}^L = \sum_{l \neq n}^N \sqrt{\frac{\rho_{pl}}{\rho_{pk}}} (\bar{h}_{lnkk}^H h_{nnkk}^L + \tilde{h}_{lnkk}^H h_{nnkk}^L) + \frac{1}{\sqrt{t \rho_{pk}}} w_{nk}^H h_{nnkk}^L.
\] (32)

Since \( \bar{h}_{lnkk}^H h_{nnkk}^L \) is deterministic value without any variance, the terms \( \tilde{h}_{lnkk}^H h_{nnkk}^L \) and \( w_{nk}^H h_{nnkk}^L \) determine the distribution of \( x_{nk} \). From [8], the terms \( \tilde{h}_{lnkk}^H h_{nnkk}^L \) and \( w_{nk}^H h_{nnkk}^L \) follow a complex Gaussian distribution as follows:

\[
\tilde{h}_{lnkk}^H h_{nnkk}^L \sim \mathcal{CN} \left( 0, \frac{1}{\kappa_{lnkk} + 1} \right),
\] (33)

\[
w_{nk}^H h_{nnkk}^L \sim \mathcal{CN} \left( 0, \sum_{m=1}^M \beta_{nnkm}^2 \right),
\] (34)

Since \( \tilde{h}_{lnkk}^H h_{nnkk}^L \) and \( w_{nk}^H h_{nnkk}^L \) are independent random variables, we have

\[
x_{nk} \sim \mathcal{CN} \left( \mu_{x_{nk}}, \sigma_{x_{nk}}^2 \right),
\] (35)

where

\[
\mu_{x_{nk}} = \sum_{l \neq n}^N \sqrt{\frac{\rho_{pl}}{\rho_{pk}}} \bar{h}_{lnkk}^H h_{nnkk}^L,
\] (36)

\[
\sigma_{x_{nk}}^2 = \sum_{l \neq n}^N \frac{\rho_{pk}}{\rho_{pk}} (\kappa_{lnkk} + 1) \sum_{p=1}^P |c_{lnkpp}^H h_{nnkk}^L|^2 + \frac{1}{t \rho_{pk}} \sum_{m=1}^M \beta_{nnkm}^2.
\] (37)

From the definition of \( X_{nk} = |x_{nk}|^2 \), the mean of \( X_{nk} \) can be obtained by \( \mu X_{nk} = \sigma_{x_{nk}}^2 + |\mu_{x_{nk}}|^2 \), which completes the proof.

**APPENDIX B**

**PROOF OF LEMMA 2**

Given the definition of \( Y_{njk} = |\hat{h}_{nnkk}^H h_{nnjk}^L|^2 \), we define

\[
y_{njk} = \hat{h}_{nnkk}^H h_{nnjk}^L = (h_{nnkk}^L)^H \bar{h}_{mmjk}^H + e_{nk}^H \bar{h}_{mmjk} + (\bar{e}_{nk}^H + (h_{nnkk}^L)^H) \tilde{h}_{mmjk}.
\] (38)
For notational simplicity, we define that \( y_{njk}^{LL} = (h_{nnkk}^L)^H \tilde{h}_{nnjk}, \ y_{njk}^{EL} = e_{nk}^H \tilde{h}_{nnjk}, \) and \( y_{njk}^{EN} = (e_{nk}^H + (h_{nnkk}^L)^H) \tilde{h}_{nnjk}. \) Then, \( y_{njk}^{LL} \) is the deterministic value depending on the locations of the devices. Also, \( y_{njk}^{EL} \) is obtained similarly as (35) as follows: 

\[
y_{njk}^{EL} \sim \mathcal{CN}(\mu_{x_{njk}}^{EL}, \sigma_{x_{njk}}^{EL}),
\]

where \( \mu_{x_{njk}}^{EL} = \sum_{l \neq n} \sqrt{\frac{\rho_{pl}}{\rho_{nl}}} g_{nl}^H \tilde{h}_{lnkk}^H \tilde{h}_{nnjk}, \)

\[
\sigma_{y_{njk}^{EL}}^2 = \sum_{l \neq n} \frac{\rho_{pl}}{\rho_{nl}} (\kappa_{lnkk} + 1) \sum_{p=1}^P |c_{lnkp}^H \tilde{h}_{nnjk}|^2 + \frac{1}{t \rho_{nk}} \sum_{m=1}^M \beta_{mnjm}^2.
\]

Next, a random variable \( y_{njk}^{EN} \) can be expressed as follows:

\[
y_{njk}^{EN} = \tilde{q}_{nk}^H \tilde{h}_{nnjk} + \sum_{l \neq n} N \sqrt{\frac{\rho_{pl}}{\rho_{nl}}} \tilde{h}_{lnkk}^H \tilde{h}_{nnjk} + \frac{\tilde{w}_{nk}^H \tilde{h}_{nnjk}}{\sqrt{t \rho_{nk}}}.
\]

where

\[
\tilde{q}_{nk} = h_{nnkk}^L + \sum_{l \neq n} \sqrt{\frac{\rho_{pl}}{\rho_{nl}}} \tilde{h}_{lnkk}.
\]

The terms \( \tilde{q}_{nk}^H \tilde{h}_{nnjk} \) and \( \tilde{w}_{nk}^H \tilde{h}_{nnjk} \) in (41) can be obtained, respectively, similarly as (33) and using the Lyapunov central limit theorem from [8], as follows:

\[
\tilde{q}_{nk}^H \tilde{h}_{nnjk} \sim \mathcal{CN} \left( 0, \frac{1}{\kappa_{mnjk} + 1} \sum_{p=1}^P |\tilde{q}_{nk}^H c_{mnj kp}|^2 \right),
\]

\[
\sqrt{\frac{Mt \rho_{nk} (\kappa_{mnjk} + 1)}{\sum_{m, p} (c_{mnj kp}^N)^2}} \tilde{w}_{nk}^H \tilde{h}_{nnjk} \xrightarrow{d_{M \rightarrow \infty}} \mathcal{CN} (0, 1),
\]

where “\( \xrightarrow{d_{M \rightarrow \infty}} \)” denotes the convergence in distribution. Also, \( \tilde{h}_{lnkk}^H \tilde{h}_{nnjk} \) in (41) is given by

\[
\tilde{h}_{lnkk}^H \tilde{h}_{nnjk} = \frac{g_{lnk}^H (R_{lnkk}^{1/2})^H R_{nnjk}^{1/2} g_{mnjk}}{\sqrt{(\kappa_{lnkk} + 1) (\kappa_{mnjk} + 1)}}.
\]

Given random vectors \( g_{lnkk} \) and \( g_{mnjk}, \) those elements are independent each other and identically follow \( \mathcal{CN} (0, 1). \) Similarly as (44), we have

\[
\sqrt{\frac{(\kappa_{lnkk} + 1) (\kappa_{mnjk} + 1)}{\left\| (R_{lnkk}^{1/2})^H R_{nnjk}^{1/2} \right\|^2_F}} \tilde{h}_{lnkk}^H \tilde{h}_{nnjk} \xrightarrow{d_{M \rightarrow \infty}} \mathcal{CN} (0, 1).
\]

Since the terms \( \tilde{q}_{nk}^H \tilde{h}_{nnjk}, \tilde{h}_{lnkk}^H \tilde{h}_{nnjk}, \) and \( \tilde{w}_{nk}^H \tilde{h}_{nnjk} \) in (41) are independent of each other, we have the following convergence in distribution:

\[
\frac{1}{\sigma_{y_{njk}^{EN}}} y_{njk}^{EN} \xrightarrow{d_{M \rightarrow \infty}} \mathcal{CN} (0, 1),
\]
where
\[
\sigma^2_{y_{njk}} = \frac{1}{\kappa_{njk} + 1} \left( \sum_{p=1}^{P} |\bar{q}_{nk}^Hc_{mnjkp}|^2 + \sum_{l \neq n}^{N} \rho_{pk} \left( \frac{R_{lnkk}^{1/2}}{R_{lnjk}^{1/2}} \right)^F + \sum_{m,p} M_{\bar{p}_{nk}} \left( \frac{\alpha_{mnjkp}^L \alpha_{mnjkp}^R}{M_{\bar{p}_{nk}}} \right)^2 \right)
\]
(48)

We can observe from (38) that \( y_{njk}^{LL} \), \( y_{njk}^{EL} \), and \( y_{njk}^{EN} \) are independent of each other. Thus, \( y_{njk} \) asymptotically follows
\[
\frac{1}{\sigma_{y_{njk}}} \left( y_{njk} - \mu_{y_{njk}} \right) \xrightarrow{d} \mathcal{N}(0, 1),
\]
(49)

where
\[
\mu_{y_{njk}} = \left( h_{mnkk}^L \right)^H \bar{h}_{nnjk} + \sum_{l \neq n}^{N} \sqrt{\rho_{pk} \bar{h}_{lnkk}^H \bar{h}_{nnjk}},
\]
(50)
\[
\sigma^2_{y_{njk}} = \sigma^2_{y_{njk}}^L + \sigma^2_{y_{njk}}^E.
\]
(51)

From the definition of \( Y_{njk} = |y_{njk}|^2 \), the mean of \( Y_{njk} \) follows \( \mu_{Y_{njk}} - \bar{\mu}_{Y_{njk}} \xrightarrow{M \to \infty} 0 \), where
\[
\bar{\mu}_{Y_{njk}} = \sigma^2_{y_{njk}} + \left| \mu_{y_{njk}} \right|^2.
\]
(52)

Similarly, given that \( Y_{lnjk} = |\bar{h}_{lnkk}^H \bar{h}_{lnjk}|^2 \) and \( y_{lnjk} = \bar{h}_{lnkk}^H \bar{h}_{lnjk} \), the mean of \( Y_{lnjk} \) follows \( \mu_{Y_{lnjk}} - \bar{\mu}_{Y_{lnjk}} \xrightarrow{M \to \infty} 0 \), where
\[
\bar{\mu}_{Y_{lnjk}} = \sigma^2_{y_{lnjk}} + \left| \mu_{y_{lnjk}} \right|^2,
\]
(53)

and
\[
\mu_{y_{lnjk}} = \left( h_{mnkk}^L \right)^H \bar{h}_{lnjk} + \sum_{l \neq n}^{N} \sqrt{\rho_{pk} \bar{h}_{lnkk}^H \bar{h}_{lnjk}},
\]
(54)
\[
\sigma^2_{y_{lnjk}} = \sigma^2_{y_{lnjk}}^L + \sigma^2_{y_{lnjk}}^E,
\]
(55)
\[
\sigma^2_{y_{lnjk}}^L = \sum_{l \neq n}^{N} \frac{\rho_{pk}}{\rho_{pk} + \kappa_{lnkk} + 1} \sum_{p=1}^{P} \left| c_{lnkpp}^H \bar{h}_{lnjk} \right|^2 + \frac{1}{M_{\bar{p}_{nk}}} \sum_{m=1}^{M} \beta_{lnjkm}^2,
\]
(56)
\[
\sigma^2_{y_{lnjk}}^E = \frac{1}{\kappa_{lnjk} + 1} \left( \sum_{p=1}^{P} |\bar{q}_{nk}^Hc_{lnjkp}|^2 + \sum_{l \neq n}^{N} \rho_{pk} \left( \frac{R_{lnkk}^{1/2}}{R_{lnjk}^{1/2}} \right)^F \right) + \sum_{m,p} M_{\bar{p}_{nk}} \left( \frac{\alpha_{lnjkp}^L \alpha_{lnjkp}^R}{M_{\bar{p}_{nk}}} \right)^2,
\]
(57)

which completes the proof.
APPENDIX C

PROOF OF Lemma 3

Given the definition of $Z_{nk} = \left| (h_{nnkk}^L)^H + e_{nk}^H \right|^2$, we define

$$z_{nk} = (h_{nnkk}^L)^H + e_{nk}^H = (h_{nnkk}^L)^H + \sum_{l \neq n} \sqrt{\frac{\rho_{plk}}{\rho_{pnn}}} \left( h_{lnkk}^H + \hat{h}_{lnkk}^H \right) + \frac{1}{\sqrt{\rho_{pnn}}} w_{nk}^H,$$  \hspace{1cm} (58)$$

where $z_{nk} \in \mathbb{C}^M = [z_{nk1}, \ldots, z_{nkM}]$ and

$$z_{nkm} = \beta_{nnkkm}^L h_{nnkkm}^* + \sum_{l \neq n} \sqrt{\frac{\rho_{plk} \kappa_{lnkk}}{\rho_{pnn} (\kappa_{lnkk} + 1)}} \left( \beta_{lnkkm}^L h_{lnkkm}^* + g_{lnkk}^H r_{lnkkm}^H \right) + \frac{1}{\sqrt{\rho_{pnn}}} w_{nkm}^*.$$  \hspace{1cm} (59)$$

Since $g_{lnkk}^H r_{lnkkm}^H$ is calculated by the sum of $P$ independent complex Gaussian random variables, $g_{lnkk}^H r_{lnkkm}^H$ is also a complex Gaussian random variable. Thus, we have $z_{nkm} \sim \mathcal{CN} (\mu_{z_{nkm}}, \sigma_{z_{nkm}}^2)$, where

$$\mu_{z_{nkm}} = \beta_{nnkkm}^L h_{nnkkm}^* + \sum_{l \neq n} \sqrt{\frac{\rho_{plk} \kappa_{lnkk}}{\rho_{pnn} (\kappa_{lnkk} + 1)}} \beta_{lnkkm}^L h_{lnkkm}^*$$  \hspace{1cm} (60)$$

and

$$\sigma_{z_{nkm}}^2 = \sum_{l \neq n} \frac{\rho_{plk} |r_{lnkkm}|^2}{\rho_{pnn} (\kappa_{lnkk} + 1)} + \frac{1}{\rho_{pnn}}.$$  \hspace{1cm} (61)$$

From the definition of $Z_{nk} = |z_{nk}|^2$, we finally have $\mu_{Z_{nk}} = \sum_{m=1}^M \left( \sigma_{z_{nkm}}^2 + |\mu_{z_{nkm}}|^2 \right)$, which completes the proof.

APPENDIX D

PROOF OF Lemma 4

Given the definition of $I_{nk}$ from (12), we have

$$I_{nk} = \frac{M^2}{M} \rho_{nk} \left| \frac{e_{nk}^H h_{nnkk}^L}{M} \right|^2 + \sum_{j \neq k} \rho_{nj} \left| \frac{h_{nnkk}^H h_{nnjk}}{M} \right|^2 + \sum_{l \neq n} \sum_{j=1}^K \rho_{lj} \left| \frac{h_{nnkk}^H h_{lnjk}}{M} \right|^2 + \left| \frac{(h_{nnkk}^L)^H + e_{nk}^H}{M} \right|^2.$$

In order to analyze the scaling law of $\sigma_{I_{nk}}^2 / M^4$, we determine the scaling laws of $\sigma_{z_{nk}}^2 / M^2$, $\sigma_{y_{njk}}^2 / M^2$, $\sigma_{y_{njk}}^2 / M^2$, and $\sigma_{z_{nkm}}^2 / M^2$ according to $M$. First, we determine the scaling law of $\sigma_{z_{nk}}^2 / M^2$ from (37). From (3), the correlation vector $c_{lnkkp}$ is normalized by $\sqrt{M}$ and $c_{lnkkp} h_{nnkk}^L$ in (37) is calculated by the sum of $M$ elements. Then, both $|c_{lnkkp}^H h_{nnkk}^L|^2$ and $\sum_{m=1}^M \beta_{nnkkm}^2$ in (37) increase with $\mathcal{O}(M)$ and thus, $\sigma_{z_{nk}}^2 / M^2$ decreases with $\mathcal{O}(1/M)$ as $M$ increases. Therefore,
we have $\sigma^2_{\eta_{njk}}/M^2 \xrightarrow{M \to \infty} 0$. Next, we analyze $\sigma^2_{\eta_{njk}}/M^2$ from (51) for large $M$. From (40), $\sigma^2_{\eta_{njk}}$ follows $\mathcal{O}(M)$ similarly as $\sigma^2_{\eta_{n}}$, and therefore, $\sigma^2_{\eta_{njk}}/M^2$ follows $\mathcal{O}(1/M)$ as $M$ increases. To determine the scaling law of $\sigma^2_{\eta_{njk}}/M^2$, we analyze the terms $|q^H_{nk}c_{nnjkp}|^2$, $\|R_{\eta_{njk}}^{1/2} R_{\eta_{njk}}^{1/2}\|_F^2$, and $\sum_{m,p} (\alpha_{nnjkp}^{NL} R^{NL}_{nnjkm})^2/M$ in (48) for large $M$. Given that the correlation vector, $c_{nnjkp}$, and matrices, $R_{\eta_{njk}}^{1/2}$ and $R_{\eta_{njk}}^{1/2}$, are normalized by $\sqrt{M}$, the terms $|q^H_{nk}c_{nnjkp}|^2$, $\|R_{\eta_{njk}}^{1/2} R_{\eta_{njk}}^{1/2}\|_F^2$, and $\sum_{m,p} (\alpha_{nnjkp}^{NL} R^{NL}_{nnjkm})^2/M$ follow, respectively, $\mathcal{O}(M)$, $\mathcal{O}(1)$, and $\mathcal{O}(1)$ as $M$ increases. Consequently, $\sigma^2_{\eta_{njk}}/M^2$ decreases with $\mathcal{O}(1/M)$ and we have $\sigma^2_{\eta_{njk}}/M^2 \xrightarrow{M \to \infty} 0$. Similarly, $\sigma^2_{\eta_{njk}}/M^2$ from (55) also converges to zero as $M \to \infty$. Finally, we determine the scaling law of $\sigma^2_{\eta_{nkm}}/M^2$ from (60). Given that $r_{\eta_{nkk}}$ is a correlation vector normalized by $\sqrt{M}$, $|r_{\eta_{nkk}}|^2$ is calculated by the sum of $P$ elements divided by $M$. Hence, $\sigma^2_{\eta_{nkm}}/M^2$ decreases with $\mathcal{O}(1/M^3)$ as $M$ increases, and the variance of $|z_{n}\eta_{nkm}/M^2$ eventually converges to zero as $M \to \infty$. In conclusion, we have $\sigma^2_{\eta_{nk}}/M^4 \xrightarrow{M \to \infty} 0$, which completes the proof.

APPENDIX E

PROOF OF THEOREM 1

We begin with the definition of $R_{nk}$ as follows:

$$R_{nk} = \log \left(1 + \frac{\rho_{nk} S_{nk}}{I_{nk}}\right) = \log (\rho_{nk} S_{nk} + I_{nk}) - \log I_{nk}. \tag{62}$$

Here, $R_{nk}^L$ can be expressed as

$$R_{nk}^L = \log \left(1 + \frac{I_{nk} - \bar{\mu}_{I_{nk}}}{\rho_{nk} S_{nk} + \bar{\mu}_{I_{nk}}}\right) + \log (\rho_{nk} S_{nk} + \bar{\mu}_{I_{nk}})$$

$$= \frac{I_{nk} - \bar{\mu}_{I_{nk}}}{M^2} \log \left(1 + \frac{(I_{nk} - \bar{\mu}_{I_{nk}})/M^2}{(\rho_{nk} S_{nk} + \bar{\mu}_{I_{nk}})/M^2}\right)^{I_{nk}/\bar{\mu}_{I_{nk}}} + \log (\rho_{nk} S_{nk} + \bar{\mu}_{I_{nk}}). \tag{63}$$

Since $I_{nk}/M^2 - \bar{\mu}_{I_{nk}}/M^2 \xrightarrow{M \to \infty} 0$ from Lemma 4 and $S_{nk} - \bar{p}_{nk} \xrightarrow{M \to \infty} 0$ where $\bar{p}_{nk} = M^2 p_{nk}^{\text{nl}}/16\pi^2 L^4$, we have the following asymptotic convergence using the exponential function definition $e^x = \lim_{n \to \infty} (1 + x/n)^n$: $R_{nk}^L - \bar{R}_{nk}^L \xrightarrow{M \to \infty} 0$, where

$$\bar{R}_{nk}^L = \left(\frac{I_{nk} - \bar{\mu}_{I_{nk}}}{\rho_{nk} \bar{p}_{nk} + \bar{\mu}_{I_{nk}}}\right) + \log (\rho_{nk} \bar{p}_{nk} + \bar{\mu}_{I_{nk}}). \tag{64}$$
Similarly, we have $R_{nk}^R - \bar{R}_{nk}^R \xrightarrow{M \to \infty} 0$, where

$$R_{nk}^R = \frac{I_{nk} - \bar{\mu}I_{nk}}{M^2} \log \left(1 + \frac{(I_{nk} - \bar{\mu}I_{nk})/M^2}{\bar{\mu}I_{nk}/M^2} \right) \xrightarrow{I_{nk} - \bar{\mu}I_{nk} \to 0, M \to \infty} \log \bar{\mu}I_{nk},$$

$$\bar{R}_{nk}^R = \frac{I_{nk} - \bar{\mu}I_{nk}}{\bar{\mu}I_{nk}} + \log \bar{\mu}I_{nk}.$$ (65)

From (64) and (65), we thus have

$$\bar{R}_{nk}^{L} - R_{nk}^{R} = \left(1 - \frac{I_{nk}}{\bar{\mu}I_{nk}} \right) \frac{\rho_{nk}\bar{p}_{nk}}{\rho_{nk}\bar{p}_{nk} + \bar{\mu}I_{nk}} + \log \left(1 + \frac{\rho_{nk}\bar{p}_{nk}}{\bar{\mu}I_{nk}} \right).$$ (66)

Given that $I_{nk}/M^2 - \bar{\mu}I_{nk}/M^2 \xrightarrow{M \to \infty} 0$, $\bar{R}_{nk}$ can be derived as follows:

$$R_{nk} = \log \left(1 + \frac{\rho_{nk}\bar{p}_{nk}}{\bar{\mu}I_{nk}} \right) = \log \left(1 + \frac{M^2\rho_{nk}\bar{p}_{nk}^2}{16\pi^2L^4\bar{\mu}I_{nk}} \right),$$ (67)

where $R_{nk} - \bar{R}_{nk} \xrightarrow{M \to \infty} 0$. Since the pilot training period consumes part of uplink channels for channel estimation, the uplink SSE can be ultimately obtained from (13) as

$$R_{n}^{SSE} = \left(1 - \frac{t}{T} \right) \sum_{k=1}^{K} \log \left(1 + \frac{M^2\rho_{nk}\bar{p}_{nk}^2}{16\pi^2L^4\bar{\mu}I_{nk}} \right) \xrightarrow{M \to \infty} 0,$$ (68)

which completes the proof.

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