Emergent dual holographic description for interacting Dirac fermions in the large $N$ limit

Ki-Seok Kim

Department of Physics, POSTECH, Pohang, Gyeongbuk 37673, Korea

(Dated: March 3, 2020)

We derive a dual holographic Einstein-Maxwell theory, applying renormalization group transformations to interacting Dirac fermions in a recursive way. In particular, we show how both dynamical metric-tensor and U(1) gauge fields appear to describe the renormalization group flows of coupling functions and order parameter fields of the corresponding quantum field theory along the direction of the emergent extra dimension, respectively. Finally, we propose a prescription on how to calculate correlation functions in a non-perturbative way, where quantum fluctuations of both metric and gauge fields are frozen to cause a classical field theory of the Einstein-Maxwell type in the large $N$ limit. Here, $N$ is the flavor degeneracy of Dirac fermions. This prescription turns out to coincide with that of the holographic duality conjecture.

PACS numbers:

I. INTRODUCTION

Quasiparticles are fundamental building blocks for the perturbative theoretical framework. Their existence is allowed by sufficient number of emergent symmetries and protected from decaying into bunch of quasiparticle-quasihole excitations as long as such emergent symmetries are preserved. However, strong correlations between quasiparticles can arise through renormalization group flows. These effective interactions may give rise to predominantly fast relaxation and result in local equilibrium through transferring their energies into each other and mixing good quantum numbers between them. As a result, only minimal number of symmetries are preserved and dynamics of these non-quasiparticle systems are described by remaining conserved currents such as charge (particle number), momentum, and energy [1–4].

Recently, this effective hydrodynamics has been observed in the system of quasi two-dimensional Dirac fermions [5–7]. More concretely, experiments measured typical hydrodynamic behaviors such as negative longitudinal resistivity, violation of the Wiedemann-Franz law, and fluid viscosity, respectively, in the hydrodynamic regime. In spite of these clear features on effective hydrodynamics, we would like to point out that the physical origin is not completely clarified. More precisely, it is not clear how the relaxation rate due to inelastic collisions between electrons prevails over either that between electrons and phonons or the elastic scattering rate between electrons and nonmagnetic impurities, responsible for such emergent hydrodynamic phenomena. One has to investigate the renormalization group flows of these three time scales in a self-consistent way. However, validity of the renormalization group analysis may be beyond the perturbative regime.

Besides possible phenomenological approaches for effective hydrodynamics, where transport coefficients are taken to be fitting parameters for experiments [8], it is natural to consider the dual holographic approach as a non-perturbative theoretical framework [9–15]. The holographic dual description gives rise to effective hydrodynamic phenomena in $D$ spacetime dimensions, formulated as an effective classical field theory on an emergent curved spacetime in $D + 1$ dimensions and regarded to be dual to strongly coupled quantum field theories in $D$ spacetime dimensions. For example [16], solving the Maxwell equation coupled to the Einstein equation, referred to as Einstein-Maxwell theory in the holographic duality conjecture, those classical solutions describe not only diffusive dynamics in transverse modes but also sound dynamics in longitudinal modes as those modes in the hydrodynamics [1–4]. In particular, the effective hydrodynamics in the holographic approach turns out to give a universally small value of the $\eta/s$ ratio, where $\eta$ is shear viscosity and $s$ is entropy [1]. One may call this emergent fluid holographic liquid. This holographic-liquid description could explain the violation of the Wiedemann-Franz law in graphene rather remarkably, which fits both electrical and thermal transport coefficients near the charge-neutral point in the regime of intermediate temperatures based on two fitting parameters [17].

In the present study, we propose how to derive the holographic-liquid dual description, more precisely, an effective Einstein-Maxwell theory from an effective quantum field theory of interacting Dirac fermions. We take
Wilsonian renormalization group transformations \[18\] recursively not in momentum space but in real space a la Polchinski \[19\], developed by Sung-Sik Lee in his emergent holographic construction \[20\--\[22\]. In particular, we show that both background static metric and U(1) electromagnetic gauge fields turn into dynamical quantum objects through the evolution along the emergent extra dimension, where the infinitesimal distance in the extra dimensional space is identified with an energy scale for renormalization group transformations. In other words, global symmetries are uplifted into gauge symmetries through this extra dimension. It turns out that quantum fluctuations of both metric and gauge fields are frozen to cause a classical field theory of the Einstein-Maxwell type in the large \(N\) limit, where \(N\) is the flavor degeneracy of Dirac fermions. We claim that the Einstein equation describe the renormalization group flows of coupling functions along the extra dimension in the corresponding quantum field theory, which implies that Einstein equations may be decomposed into renormalization group \(\beta\)–functions \[23\--\[25\]. On the other hand, we point out that the Maxwell equation gives the evolution of the U(1) conserved current along the extra dimension, which may be identified with the Callan-Symanzik equation of an order parameter field in the dual description. Finally, we propose a prescription on how to calculate correlation functions in a non-perturbative way \[26\], solving coupled differential equations of the Einstein-Maxwell theory with boundary conditions. This prescription coincides with that of the dual holographic description \[3\--\[15\].

II. EFFECTIVE GEOMETRIC DESCRIPTION FOR A FREE DIRAC THEORY

A. Free Dirac theory in a static background metric and an external U(1) electromagnetic gauge field

We first consider a free Dirac theory in a static background metric and an external U(1) electromagnetic gauge field \[27\]

\[
Z = \int D\psi_\alpha \exp \left[ -\int d^Dx \sqrt{g} \left\{ \bar{\psi}_\alpha \gamma^a \tilde{c}_a^\mu \left( \partial_\mu - ie_\psi A_\mu \right) - \frac{i}{4} \tilde{\omega}_a^{\mu \nu} \sigma_{\mu \nu} \psi_\alpha + m \bar{\psi}_\alpha \psi_\alpha \right\} \right],
\]

(1)

Here, \(\psi_\alpha\) is a Dirac spinor at \(x\) in \(D\) spacetime dimensions. \(\alpha\) runs from 1 to \(N\), denoting the flavor degeneracy of Dirac fermions. \(\gamma^a\) is a Dirac gamma matrix, defined in a local rest frame at \(x\) and satisfying the Clifford algebra \(\{\gamma^a, \gamma^b\} = 2\delta^{ab}\) with the Euclidean signature. \(\tilde{c}_a^\mu\) defines the local rest frame given by the tangent manifold at \(x\), called vierbein. The corresponding background metric is given by the vierbein as follows \(\hat{g}_{\mu \nu} = \tilde{e}_a^\mu \tilde{e}_b^\nu \delta_{ab}\). \(A_\mu\) is an external electromagnetic U(1) gauge field and \(e_\psi\) is a unit electromagnetic charge of \(\psi_\alpha\). \(\tilde{\omega}_a^{\mu \nu} = \tilde{e}_a^\mu \partial_\mu \tilde{e}_b^\nu + \frac{1}{2} \tilde{\Gamma}_{a \mu \nu} \tilde{e}^{\sigma b}\) is a background spin connection and \(\sigma_{\mu \nu} = \frac{i}{2} [\gamma^a, \gamma^b]\) is a commutator of Dirac gamma matrices in the local rest frame. \(m\) represents a mass of Dirac fermions. Here, \(\tilde{\Gamma}_a^\mu = \frac{i}{2} \hat{g}^{\rho \sigma} (\partial_\rho \tilde{g}_{\mu \nu} + \partial_\mu \tilde{g}_{\sigma \nu} - \partial_\nu \tilde{g}_{\sigma \mu})\) is the Christoffel symbol.

B. Introduction of dynamical metric as a coupling function and dynamical U(1) gauge field as a dual order parameter field

We reformulate the above expression as follows

\[
Z = \int D\psi_\alpha D\theta_\alpha D\rho_{\mu \nu} D\theta_{\mu \nu} \exp \left[ -\int d^Dx \sqrt{g} \left\{ \bar{\psi}_\alpha \gamma^a \tilde{c}_a^\mu \left( \partial_\mu - ie_\psi a_\mu - \frac{i}{4} \omega_{a \mu}^{\nu \rho} \sigma_{\nu \rho} \right) \psi_\alpha + m \bar{\psi}_\alpha \psi_\alpha \right\} \right] - N j^\mu (a_\mu - A_\mu) - N \theta_\mu^a (\tilde{c}_a^\mu - \tilde{e}^\mu_a)\right]\}
\]

(2)

Here, \(\tilde{e}_a^\mu\) is a dynamically introduced vierbein and its dual tensor field \(\theta_\mu^a\) is a Lagrange multiplier field to impose the constraint given by \(\delta(\tilde{e}_a^\mu - \tilde{e}^\mu_a)\). \(a_\mu\) is also a dynamically introduced U(1) gauge field and its dual current field \(j^\mu\) is a Lagrange multiplier field to impose the constraint given by \(\delta(a_\mu - A_\mu)\). Frankly speaking, this reformulation is not meaningful at this stage. It turns out that both metric tensor and U(1) gauge fields become fully dynamical when their corresponding interactions are taken into account. After renormalization group transformations are performed in the presence of correlations, evolution equations for both dynamical metric and U(1) gauge fields appear. We will argue that the evolution equations of the vierbeins describe renormalization group flows of coupling constants \[23\--\[25\] such as \(e_\psi\) and \(m\) while those of the U(1) gauge fields express Callan-Symanzik equations of U(1) currents in the dual description \[24\--\[26\].
C. Real space renormalization group transformation a la Polchinski

We perform a renormalization group transformation in real space a la Polchinski \[19\], further developed by Sung-Sik Lee in his emergent holographic construction \[20–22\]. We introduce an auxiliary Dirac spinor $\Phi_\alpha$ with its mass $M$ as follows

$$Z = \int D\psi_\alpha D\Phi_\alpha D\bar{\psi}_\alpha Dj^\mu \, D\bar{e}_\alpha^\mu \, D\theta_\mu^\alpha \exp \left[ - \int d^D x \sqrt{g} \left\{ \bar{\psi}_\alpha \gamma^\mu \epsilon_\mu^\alpha \left( \partial_\mu - i e_\psi a_\mu - \frac{i}{4} \omega_\mu^{\alpha' \gamma} \sigma_{\alpha' \gamma} \right) \psi_\alpha \\
+ m \bar{\psi}_\alpha \psi_\alpha + M \bar{\Phi}_\alpha \Phi_\alpha - N j^\mu (a_\mu - A_\mu) - N \theta_\mu^\alpha (\epsilon_\mu^\alpha - \tilde{\epsilon}_\mu^\alpha) \right\} \right].$$

(3)

We emphasize that the introduction of the auxiliary Dirac spinor in this way does not alter the original physics at all except for the normalization constant in the partition function, here omitted for simplicity.

Now, we decompose these Dirac spinor fields in the following way

$$\psi_\alpha \mapsto \psi_\alpha + \Psi_\alpha, \quad \Phi_\alpha \mapsto c_\phi \psi_\alpha + c_\Psi \Psi_\alpha.$$

(4)

$\psi_\alpha$ is a Dirac spinor field with a light mass and $\Psi_\alpha$ is that with a heavy mass, to be discussed below in more details. Two positive coefficients of $c_\phi$ and $c_\Psi$ are determined by the fact that all mixing terms between $\psi_\alpha$ and $\Psi_\alpha$ do not appear in the two mass terms. In other words, taking these two coefficients in the following way

$$c_\phi = \frac{m}{\sqrt{\mu M}}, \quad c_\Psi = -\sqrt{\frac{\mu}{M}},$$

(5)

we rewrite the mass terms with light and heavy Dirac spinor fields as

$$m \bar{\psi}_\alpha \psi_\alpha + M \bar{\Phi}_\alpha \Phi_\alpha \mapsto e^{2\alpha dz} m \bar{\psi}_\alpha \psi_\alpha + e^{2\alpha dz} \mu \bar{\Psi}_\alpha \Psi_\alpha.$$

(6)

Here, the heavy mass $\mu$ is given by

$$\mu = \frac{m}{e^{2\alpha dz} - 1},$$

(7)

where $dz$ is an infinitesimal parameter and $\alpha$ is a local speed of coarse graining, both of which control the path integral for heavy Dirac spinor fields.

Taking rescaling of both Dirac spinor fields to return the above mass terms into the almost original expression as follows

$$\psi_\alpha \mapsto e^{-\alpha dz} \psi_\alpha, \quad \Psi_\alpha \mapsto e^{-\alpha dz} \Psi_\alpha,$$

(8)

we finish our setup for a renormalization group transformation, given by

$$Z = \int D\psi_{\alpha x} D\bar{\psi}_{\alpha x} D a_\mu D j^\mu \, D\bar{e}_{\alpha x}^\mu D\theta_{\mu x}^\alpha \exp \left[ - \int d^D x \sqrt{g} \left\{ \bar{\psi}_{\alpha x} \gamma^\mu \epsilon_\mu_{\alpha x} \left( \partial_\mu - i e_\psi (a_\mu - A_\mu) - \frac{i}{4} \omega_\mu^{\alpha' \gamma} \sigma_{\alpha' \gamma} \right) \psi_{\alpha x} \\
- N j^\mu (a_\mu - A_\mu) - N \theta_{\mu x}^\alpha (\epsilon_{\mu x}^\alpha - \tilde{\epsilon}_{\mu x}^\alpha) \right\} \right].$$

(9)

It is straightforward to perform the Gaussian integral for heavy Dirac spinor fields formally and find the following partition function

$$Z = \int D\psi_{\alpha x} D a_\mu D j^\mu \, D\bar{e}_{\alpha x}^\mu D\theta_{\mu x}^\alpha \exp \left[ - \frac{S_\psi^{(0)}}{2} - \Delta S_\psi - S_\Psi^{(0)} \\
+ N \int d^D x \sqrt{g} \left\{ j^\mu_x \left( a_\mu + \frac{i}{e_\psi} dz \partial_\mu \alpha_x - A_\mu x \right) + \theta_{\mu x}^\alpha (\epsilon_{\mu x}^\alpha - \tilde{\epsilon}_{\mu x}^\alpha) \right\} \right].$$

(10)
Here, $S^{(0)}_{\psi}$ is a free Dirac theory with background metric and U(1) gauge fields in the presence of rescaling, given by

$$S^{(0)}_{\psi} = \int d^Dx \sqrt{g_x} \left\{ e^{-2\sigma_x dz} \tilde{\psi}_ax \gamma^a \epsilon^\mu_{ax} \left( \partial_{\mu x} - ie_\psi a_{\mu x} - \frac{i}{4} \omega^a_{\mu x} \gamma^a \sigma_{d' b'} \right) \psi ax + m \tilde{\psi}_ax \psi ax \right\}. \tag{11}$$

$\Delta S_\psi$ is a correction from the Gaussian integral for heavy Dirac spinor fields, which serves as renormalization for the kinetic-energy term. Below, we revisit this issue for renormalization of the kinetic-energy term. $S^{(0)}_{\psi}$ serves as vacuum energy from fluctuating heavy Dirac spinor fields, given by

$$\Delta S_\psi = - \int d^Dx \sqrt{g_x} e^{-2\sigma_x dz} \int d^Dx' \sqrt{g_x} e^{-2\sigma_x dz} \tilde{\psi}_ax \gamma^a \epsilon^\mu_{ax} \left( \partial_{\mu x} - ie_\psi a_{\mu x} - \frac{i}{4} \omega^a_{\mu x} \gamma^a \sigma_{d' b'} \right) \left[ 1 - \frac{1}{2} \delta^D(x - x') \right] \psi ax'. \tag{12}$$

The second line expresses the Green’s function of heavy Dirac spinor fields. We point out that this correction is nonlocal in nature, clarified in the subscripts $x$ and $x'$. However, the heavy-mass nature given by $\mu_x \sim (2\sigma_x dz)^{-1}$ allows us to perform the gradient expansion, where the propagator is exponentially decaying. Below, we revisit this issue for renormalization of the kinetic-energy term. $S^{(0)}_{\psi}$ serves as vacuum energy from fluctuating heavy Dirac spinor fields, given by

$$S^{(0)}_{\psi} = -N \text{tr} \ln \sqrt{g_x} \left\{ e^{-2\sigma_x dz} \gamma^a \epsilon^\mu_{ax} \left( \partial_{\mu x} - ie_\psi a_{\mu x} - \frac{i}{4} \omega^a_{\mu x} \gamma^a \sigma_{d' b'} \right) \left[ 1 + \frac{1}{2} \delta^D(x - x') \right] \psi ax \right\}. \tag{13}$$

We point out that the following gauge transformation

$$a_\mu \mapsto a_\mu + \frac{i}{e_\psi} dz \partial_\mu \alpha \tag{14}$$

has been performed before the Gaussian integral for heavy Dirac spinor fields.

To simplify further calculations, we choose the local speed of coarse graining as follows

$$\partial_\mu \alpha x = 0. \tag{15}$$

Physically, this gauge choice implies that the mass of heavy Dirac spinor fields is uniform in spacetime and the renormalization group transformation is performed with the same rate independent of spacetime. This gauge fixing leads us to lose general covariance. It turns out that this component plays the role of $g_{zz}$ in the emergent gravity description $[20, 22]$, where $z$ is the coordinate of an emergent extra-dimensional space and $g_{zz}$ is the metric tensor of this emergent space.

Now, we perform the gradient expansion in $S^{(0)}_{\psi} = -N \text{tr} \ln \sqrt{g_x} \left\{ e^{-2\sigma_x dz} \gamma^a \epsilon^\mu_{ax} \left( \partial_{\mu x} - ie_\psi a_{\mu x} - \frac{i}{4} \omega^a_{\mu x} \gamma^a \sigma_{d' b'} \right) + \mu \right\}$ for both background metric and U(1) gauge fields with respect to the uniform mass $\mu$ $[28, 29]$. Then, we obtain their effective kinetic-energy terms in the partition function as follows

$$Z = \int D\psi x D\theta x D\psi x D\theta x D\psi x D\theta x \exp \left[ - \int d^Dx \sqrt{g_x} \left\{ e^{-2\sigma_x dz} \tilde{\psi}_ax \gamma^a \epsilon^\mu_{ax} \left( \partial_{\mu x} - ie_\psi a_{\mu x} - \frac{i}{4} \omega^a_{\mu x} \gamma^a \sigma_{d' b'} \right) \psi ax + m \tilde{\psi}_ax \psi ax \right\} \right.$$  

$$- e^{-2\sigma_x dz} \int d^Dx' \sqrt{g_x} \tilde{\psi}_ax' \gamma^a \epsilon^\mu_{ax'} \left( \partial_{\mu x'} - ie_\psi a_{\mu x'} - \frac{i}{4} \omega^a_{\mu x'} \gamma^a \sigma_{d' b'} \right) \psi ax' + m \tilde{\psi}_ax' \psi ax'.$$

$$+2\sigma_x dz N \left( -C_A + C_R R_x + \tilde{C}_F F_{\mu x} F_{\mu x} + 2\sigma_x dz \left( A_{x}^{RR} + A_{x}^{FF} + A_{x}^{FR} \right) \right)$$

$$- N j^\mu_{\psi} (a_{\mu x} - A_{\mu x}) - N \theta^\mu_{\psi} (e^\mu_{\mu x} - \tilde{e}^\mu_{\mu x}) \right]. \tag{16}$$

Here, $C_A$ denotes vacuum energy, identified with a cosmological constant. $R_x$ is Ricci scalar to give the kinetic energy for the metric tensor and $C_R$ is a positive constant. $-C_A + C_R R_x$ is nothing but the Einstein-Hilbert
action in $D$ spacetime dimensions, known to be the notion of induced gravity \[28, 29\]. $F_{\mu \nu} F^{\mu \nu}$ is the Maxwell action for the U(1) gauge field $a_{\mu}$ and $C_{\alpha}$ is a positive constant. $C_{\alpha}$, $C_{R}$, and $C_{F}$ decrease as the bare mass $m$ of Dirac spinor fields increases. $A_{x}^{RR}$, $A_{x}^{EF}$, and $A_{x}^{FR}$ represent possible anomaly terms involved with gravitational curvature, electromagnetic field strength, and their mixing, respectively. Here, we show symbolic expressions only to point out the possibility of their appearances. Definitely, more detailed investigations are required in separate publications. $G_{xx'}$ is the Green’s function of the heavy Dirac spinor, given by

$$G_{xx'} = \frac{1}{\sqrt{g_{x}}^{(D)}(x-x')}.$$  \hspace{1cm} (17)

The above partition function is the result of the first renormalization group transformation.

**D. Recursive renormalization group transformations**

To perform the second renormalization group transformation, we keep all terms up to the linear order in $dz$, given by

$$Z = \int D\psi_{\alpha} D\bar{c}_{\alpha} D\psi_{\alpha} Dc_{\alpha} D\theta_{\alpha} D\bar{\theta}_{\alpha} \exp \left[ -\int d^{D}x \sqrt{g(x)} \left\{ \left( 1 - 2\alpha(x) dz \right) \bar{\psi}_{\alpha} \gamma^{\mu}(0) \left( \partial_{\mu} - i e_{\mu}(0) a_{\mu} - \frac{i}{4} \omega_{\mu}^{\nu} \sigma_{\nu} \right) \psi_{\alpha} + m\bar{\psi}_{\alpha} \psi_{\alpha} ight\} + 2\alpha(x) dz N \left( -C_{\alpha} + C_{R} R_{x}^{(0)} + C_{F} F_{\mu x}^{(0)} F^{\mu x}(0) \right) + 2\alpha(x) dz N \left( A_{x}^{RR} + A_{x}^{EF} + A_{x}^{FR} \right) $$

$$-N j_{\mu}^{(0)}(a_{\mu}(0) - A_{\mu}) - N \theta_{\mu}^{(0)}(c_{\mu}(0) - \bar{c}_{\mu}) \right\}].$$ \hspace{1cm} (18)

Here, the superscript $(0)$ has been introduced into all dynamic field variables in the partition function, meaning "unrenormalized" quantum fields. Accordingly, the Green’s function is given by

$$G_{xx'} = \frac{1}{\sqrt{g_{x}}^{(D)}(x-x')}.$$ \hspace{1cm} (19)

which is the order of $dz$.

To implement renormalization group transformations in a recursive way, it is necessary to rewrite the above effective action in the same form as the original expression of the Dirac Lagrangian, where both vierbein and U(1) gauge fields have to be updated, clarified soon. First of all, we take the leading order in the gradient expansion for the Green’s function, where higher order derivatives are with higher powers of $dz$, thus safely neglected in the $dz \to 0$ limit. We call this locality approximation. Within the locality approximation, the partition function is given by

$$Z = \int D\psi_{\alpha} D\bar{c}_{\alpha} D\psi_{\alpha} Dc_{\alpha} D\theta_{\alpha} D\bar{\theta}_{\alpha} \exp \left[ -\int d^{D}x \sqrt{g(x)} \left\{ \left( 1 - 2\alpha(x) dz \right) \bar{\psi}_{\alpha} \gamma^{\mu}(0) \left( \partial_{\mu} - i e_{\mu}(0) a_{\mu} - \frac{i}{4} \omega_{\mu}^{\nu} \sigma_{\nu} \right) \psi_{\alpha} + m\bar{\psi}_{\alpha} \psi_{\alpha} ight\} + 2\alpha(x) dz N \left( -C_{\alpha} + C_{R} R_{x}^{(0)} + C_{F} F_{\mu x}^{(0)} F^{\mu x}(0) \right) + 2\alpha(x) dz N \left( A_{x}^{RR} + A_{x}^{EF} + A_{x}^{FR} \right) $$

$$-N j_{\mu}^{(0)}(a_{\mu}(0) - A_{\mu}) - N \theta_{\mu}^{(0)}(c_{\mu}(0) - \bar{c}_{\mu}) \right\}],$$ \hspace{1cm} (20)

where the subscript of $x$ has been omitted without confusion.
Focusing on the linear order of $dz$ in the kinetic-energy term of Dirac spinor fields, our object is to reformulate the kinetic-energy term as follows

$$
\bar{\psi}_\alpha \left[ \gamma^a e^{\mu(0)}_a \left( \partial_\mu - i e_\psi a^{\mu(0)}_\mu - \frac{i}{4} \omega^{\nu(0)}_\mu \gamma^{\nu(0)} \sigma_\nu \right) \right] + \frac{1}{2\alpha(0)dz} \left\{ \gamma^c e^{\mu(0)}_c \left( \partial_\mu - i e_\psi a^{\mu(0)}_\mu - \frac{i}{4} \omega^{\nu(0)}_\mu \gamma^{\nu(0)} \sigma_\nu \right) G^{(0)}_{xx'} \right\}_{x' \to x} x' \times \gamma^f e^{\mu(0)}_f \left( \partial_\mu - i e_\psi a^{\mu(0)}_\mu - \frac{i}{4} \omega^{\nu(0)}_\mu \gamma^{\nu(0)} \sigma_\nu \right) \psi_\alpha \rightarrow \bar{\psi}_\alpha \gamma^a e^{\mu(1)}_a \left( \partial_\mu - i e_\psi a^{\mu(1)}_\mu - \frac{i}{4} \omega^{\nu(1)}_\mu \gamma^{\nu(1)} \sigma_\nu \right) \psi_\alpha. \tag{21}
$$

It turns out to be straightforward finding the updated vierbein and U(1) gauge fields in the leading order of $dz$. They are given by

$$
e^{\mu(1)}_a = e^{\mu(0)}_a + \frac{1}{2\alpha(0)dz} \gamma_a \left\{ \gamma^c e^{\mu(0)}_c \left( \partial_\mu - i e_\psi a^{\mu(0)}_\mu - \frac{i}{4} \omega^{\nu(0)}_\mu \gamma^{\nu(0)} \sigma_\nu \right) G^{(0)}_{xx'} \right\}_{x' \to x} x' \gamma^a e^{\mu(0)}_a \tag{22}
$$

and

$$a^{(1)}_\mu = a^{(0)}_\mu + \frac{1}{2\alpha(0)dz} \gamma_a \left\{ \gamma^c e_x^{\mu(0)} \left( \partial_\mu - i e_\psi a^{\mu(0)}_\mu - \frac{i}{4} \omega^{\nu(0)}_\nu \gamma^{\nu(0)} \sigma_\nu \right) G^{(0)}_{xx'} \right\}_{x' \to x} x' \gamma^a e^{\mu(0)}_a \tag{23}
$$

respectively. These solutions are verified in the $dz \to 0$ limit. As a result, we reach the following expression of the partition function after the first renormalization group transformation

$$Z = \int D\psi_a D\bar{\psi}_\alpha D\psi^{(0)}_\alpha D\bar{\psi}^{(0)}_a D\psi^{(1)}_\alpha D\bar{\psi}^{(1)}_a D\theta^{(0)}_\mu D\theta^{(0)}_a D\theta^{(0)}_\alpha \exp \left\{ - \int d^D x x \left[ \bar{\psi}_\alpha \gamma^a e^{\mu(0)}_a \left( \partial_\mu - i e_\psi a^{\mu(0)}_\mu - \frac{i}{4} \omega^{\nu(0)}_\mu \gamma^{\nu(0)} \sigma_\nu \right) \psi_\alpha + m \bar{\psi}_\alpha \psi_\alpha \right] - \int d^D x x \left[ 2\alpha(0)dz \left( - C_L + C_R F^{(1)} F^{(0)} + C_F F^{(0)} F^{(0)} + 2\alpha(0)dz \left( A^{RR(0)} + A^{FF(0)} + A^{FFR(0)} \right) \right) \right] - N j^{(0)}(a^{(0)}_\mu - A_\mu) - N \theta^{(0)}_\mu (\bar{e}^{(0)}_a - e^{(0)}_a) \right\} \right\}

- \int d^D x x \left[ - N \theta^{(0)}_\mu \left[ e^{(1)}_a - e^{(0)}_a - \gamma_a \left\{ \gamma^c e^{\mu(0)}_c \left( \partial_\mu - i e_\psi a^{\mu(0)}_\mu - \frac{i}{4} \omega^{\nu(0)}_\mu \gamma^{\nu(0)} \sigma_\nu \right) G^{(0)}_{xx'} \right\}_{x' \to x} x' \gamma^a e^{\mu(0)}_a \right] \right] \right\}

- N j^{(1)}(a^{(1)}_\mu - a^{(0)}_\mu - e^{(1)}_a - \gamma_a \left\{ \gamma^c e^{\nu(0)}_c \left( \partial_\mu - i e_\psi a^{\nu(0)}_\mu - \frac{i}{4} \omega^{\nu(0)}_\mu \gamma^{\nu(0)} \sigma_\nu \right) G^{(0)}_{xx'} \right\}_{x' \to x} x' \gamma^a e^{\mu(0)}_a \right) \right\} \right]. \tag{24}
$$

Here, $\theta^{(1)}_\mu$ and $j^{(1)}$ are Lagrange multiplier fields to impose each constraint of Eq. \[\text{Eq.} \tag{24}\] respectively.

Based on the above discussion, it is now straightforward to implement renormalization group transformations in a recursive way. After the $(f-1)$th renormalization group transformation, we obtain the following expression
for the partition function

\[
Z = \int D\psi_d \Pi_{k=0}^f Da^{(k)}_\mu Dj^{\mu(k)}_D e^{\mu(k)}_a D\theta^\mu_a(k)
\]

\[
\exp \left[ -\int d^D x \sqrt{g^{(k)}} \left\{ \bar{\psi}_a \gamma^\alpha \delta^\mu_a(f) \left( \partial_\mu - ie_\mu a_\mu(f) - \frac{i}{4} \omega^\mu_{\alpha\beta \gamma} a^{\beta}(f) \sigma_{\alpha\beta} \right) \psi_a + m \bar{\psi}_a \psi_a \right\} \right]
\]

\[
- N \int d^D x \left\{ dz \sum_{k=1}^f \alpha^{(k-1)} \sqrt{g^{(k-1)}} \left( -C_A + C_R R^{(k-1)} + C_{FR} F^{(k-1)} F^{\mu\nu(k-1)} \right) \right. \\
+ dz \sum_{k=1}^f \alpha^{(k-1)} \sqrt{g^{(k-1)}} \left( A^{RR(k-1)} + A^{FF(k-1)} + A^{FR(k-1)} \right) - \sqrt{g^{(0)}} j^{\mu(0)} \left( a^{(0)}_\mu - A_\mu \right) - \sqrt{g^{(0)}} \theta^\mu(0) (e^{\mu(0)}_a - \bar{e}^{\mu}_a) \\
- \sum_{k=1}^f \sqrt{g^{(k-1)}} \theta^{\mu(0)}_a \left[ e^{\mu(0)}_a - e^{\mu(0)}_a(k-1) - \gamma a \left\{ \gamma^\alpha e^\mu_a(k-1) \left( \partial_\mu - ie_\mu a_\mu(k-1) - \frac{i}{4} \omega^\mu_{\alpha\beta \gamma} a^{\beta}(k-1) \sigma_{\alpha\beta} \right) C^{(k-1)}_{\mu x} \right\} \right] \\
- \sum_{k=1}^f \sqrt{g^{(k-1)}} j^{\mu(0)}_a \left[ a^{(0)}_\mu - a^{(0)}_\mu(k-1) - \gamma a \left\{ \gamma^\alpha e^\mu_a(k-1) \left( \partial_\mu - ie_\mu a_\mu(k-1) - \frac{i}{4} \omega^\mu_{\alpha\beta \gamma} a^{\beta}(k-1) \sigma_{\alpha\beta} \right) C^{(k-1)}_{\mu x} \right\} \right] \left. \right| ^{x' - x} \\
\times \gamma^b e_b^{(k-1)} a^{(k-1)}_\nu \left\} \right].
\]

(25)

We emphasize again that this reformulation is verified in the \( dz \to 0 \) limit. Hereafter, we set the local speed of coarse graining as

\[
\alpha^{(k)} = 1
\]

for simplicity.

E. Emergent geometric description for a free Dirac theory: Extra dimension, "dynamical" metric tensor, and "dynamical" U(1) gauge field

The final task is to rewrite the above partition function Eq. \((25)\) in the continuous coordinate representation instead of the discrete variable \(k\). Resorting to

\[
\sum_{k=1}^f \sqrt{g^{(k-1)}} \theta^{\mu(0)}_a \left( e^{\mu(0)}_a - e^{\mu(0)}_a(k-1) \right) \right\|_{x,z}^{z_j} dz \sqrt{g(x,z)} \theta^{\mu}_a(x,z) \partial_\mu e^{\mu}_a(x,z)
\]

(27)
with \( dz \sum_{k=1}^{f} \rightarrow \int_{0}^{z} dz \) and \((e_{a}^{(k)} - e_{a}^{(k-1)_{1}})/dz \rightarrow \partial_{a}e_{a}^{(k)}(x, z)\), we obtain

\[
\begin{align*}
Z = \int D\bar{\psi}_{\alpha}(x) Da_{\mu}(x, z) D j^{\mu}(x, z) D\psi_{\alpha}(x, z) D\theta_{\alpha}^{\mu}(x, z) \\
\exp \left[ -\int d^{D}x \sqrt{g(x, z)} \left\{ \bar{\psi}_{\alpha}(x) \gamma^{\alpha}e_{a}^{\mu}(x, z) \left( \partial_{\mu} - ie_{\psi}a_{\mu}(x, z) - \frac{i}{4} \omega_{\mu}^{a\nu}(x, z) \sigma_{a\nu} \right) \psi_{\alpha}(x) + m\bar{\psi}_{\alpha}(x)\psi_{\alpha}(x) \right\} \right] \\
+ N \int d^{D}x \sqrt{g(x, z)} \left\{ j^{\mu}(x, 0) \left( a_{\mu}(x, 0) - A_{\mu}(x) \right) + \theta_{\alpha}^{\mu}(x, 0) \left( e_{a}^{\mu}(x, 0) - e_{a}^{\mu}(x) \right) \right\} \\
- N \int_{0}^{z} dz \int d^{D}x \sqrt{g(x, z)} \left\{ \frac{1}{2\kappa} \left( R(x, z) - 2\Lambda \right) + \frac{C_{A}}{4} F_{\mu\nu}(x, z) F^{\mu\nu}(x, z) + \left( A_{R}^{RR}(x, z) + A_{FF}(x, z) + A_{FR}(x, z) \right) \right\} \\
- \theta_{\mu}^{\alpha}(x, z) \left[ \bar{\psi}_{\alpha}(x, z) \gamma^{\mu}e_{a}^{\mu}(x, z) \left( \partial_{\mu} - ie_{\psi}a_{\mu}(x, z) - \frac{i}{4} \omega_{\mu}^{a\nu}(x, z) \sigma_{a\nu} \right) \psi_{\alpha}(x, z) \right] \\
- j^{\mu}(x, z) \left[ \bar{\psi}_{\alpha}(x, z) \gamma^{\mu}e_{a}^{\mu}(x, z) \left( \partial_{\mu} - ie_{\psi}a_{\mu}(x, z) - \frac{i}{4} \omega_{\mu}^{a\nu}(x, z) \sigma_{a\nu} \right) \psi_{\alpha}(x, z) \right]
\end{align*}
\]

(28)

Here, we introduced changes in notations as

\[
C_{R} = \frac{1}{2\kappa}, \quad C_{A} = 2\Lambda, \quad C_{FR} = \frac{C_{A}}{4}.
\]

(29)

Physical ingredients in this partition function are clear. The background vierbein \( e_{a}^{\mu}(x, 0) = \tilde{e}_{a}^{\mu}(x) \) and electromagnetic U(1) gauge field \( a_{\mu}(x, 0) = A_{\mu}(x) \) evolve along the extra dimensional space, governed by

\[
\partial_{z}e_{a}^{\mu}(x, z) = \frac{1}{2dz} \gamma_{\alpha} \left\{ \gamma^{c}e_{c}^{\mu}(x, z) \left( \partial_{\mu} - ie_{\psi}a_{\mu}(x, z) - \frac{i}{4} \omega_{\mu}^{c\nu}(x, z) \sigma_{c\nu} \right) G_{x'x}[a_{\mu}, e_{\mu}^{x}]) \right\}_{x'\rightarrow x} \gamma^{a'}e_{a'}^{\mu}(x, z)
\]

(30)

and

\[
\partial_{z}a_{\mu}(x, z) = \frac{1}{2dz} e_{a}^{\mu}(x, z) \gamma_{\alpha} \left\{ \gamma^{c}e_{c}^{\mu}(x, z) \left( \partial_{\mu} - ie_{\psi}a_{\mu}(x, z) - \frac{i}{4} \omega_{\mu}^{c\nu}(x, z) \sigma_{c\nu} \right) G_{x'x}[a_{\mu}, e_{\mu}^{x}]) \right\}_{x'\rightarrow x} \gamma^{b}e_{b}^{\mu}(x, z)a_{\nu}(x, z),
\]

(31)

respectively, and resulting from renormalizations by matter fluctuations, here Dirac fermions. The bulk Green’s function is

\[
\left\{ \gamma^{a}e_{a}^{\mu}(x, z) \left( \partial_{\mu} - ie_{\psi}a_{\mu}(x, z) - \frac{i}{4} \omega_{\mu}^{c\nu}(x, z) \sigma_{c\nu} \right) + m \right\} G[x, x'; a_{\mu}, e_{\mu}^{x}] = \frac{1}{\sqrt{g(x, z)}} \delta^{(D)}(x - x').
\]

(32)

As a result, a fully renormalized vierbein field appears into the IR effective action \( S_{IR} = \int d^{D}x \sqrt{g(x, z)} \left\{ \bar{\psi}_{\alpha}(x) \gamma^{a}e_{a}^{\mu}(x, z) \left( \partial_{\mu} - ie_{\psi}a_{\mu}(x, z) - \frac{i}{4} \omega_{\mu}^{a\nu}(x, z) \sigma_{a\nu} \right) \psi_{\alpha}(x) + m\bar{\psi}_{\alpha}(x)\psi_{\alpha}(x) \right\} \) to describe all possible renormalizations such as field renormalization, mass renormalization, and interaction renormalization (not here but below). This evolution equation for the vierbein field plays essentially the same role as renormalization group \( \beta \)–functions. On the other hand, the evolution equation of the U(1) gauge field may be regarded as a Callan-Symanzik equation for the conserved U(1) current in the dual description.
III. EMERGENT DUAL HOLOGRAPHIC DESCRIPTION IN THE PRESENCE OF INTERACTIONS

We consider correlations between Dirac fermions in the following way

$$Z(0) = \int D\psi_\alpha \exp \left[ - \int d^Dx \sqrt{g} \left\{ \bar{\psi}_\alpha \gamma^\mu e^\mu_\alpha (\partial_\mu - i e_\psi A_\mu - \frac{i}{4} \omega_\mu^a b^a \sigma_{ab} \right) \psi_\alpha + m \bar{\psi}_\alpha \psi_\alpha + \frac{\lambda_1}{2} j_\mu j^\mu + \frac{\lambda_\theta}{2} \hat{\theta}_\mu \hat{\theta}^\mu \right\} \right].$$

(33)

Here, $j_\mu$ and $\hat{\theta}_\mu$ are conserved electromagnetic U(1) current and energy-momentum tensor, minimally coupled to background electromagnetic U(1) gauge and vierbein tensor fields, respectively. $\lambda_1$ and $\lambda_\theta$ are coupling constants for such local interactions.

Repeating essentially the same procedure [24, 25] as discussed in the previous section, we obtain a dual holographic Einstein-Maxwell theory

$$Z(z_f) = \int D\psi_\alpha(x) D\psi_\beta(x) D\psi_\alpha(x) \exp \left\{ -S_{UV} - S_{IR} - S_{Bulk} \right\}.$$  

(34)

Here, the UV action is

$$S_{UV} = N \int d^Dx \sqrt{g(x,0)} \left\{ \frac{1}{2\lambda_\gamma} (a_\mu(x,0) - A_\mu(x))^2 + \frac{1}{2\lambda_\theta} (e^\mu_\alpha(x,0) - \bar{e}^\mu_\alpha(x))^2 \right\}.$$  

(35)

where both quadratic-order terms result from Gaussian integrals of two conserved currents, $j_\mu$ and $\theta_\mu$, dual to U(1) gauge and vierbein fields, respectively. The IR boundary action is given by

$$S_{IR} = \int d^Dx \sqrt{g(z,f)} \left\{ \bar{\psi}_\alpha(x) \gamma^\mu e^\mu_\alpha \left( \partial_\mu - i e_\psi a_\mu(x) - i \frac{1}{4} \omega_\mu^a b^a \sigma_{ab} \right) \psi_\alpha(x) + m \bar{\psi}_\alpha(x) - i e_\psi a_\mu(x) - i \frac{1}{4} \omega_\mu^a b^a \sigma_{ab} \right\}.$$  

(36)

where renormalized values of both U(1) gauge and vierbein fields enter the Dirac Lagrangian in the large $N$ limit, to be discussed below in more details. In this case renormalization group $\beta$-functions to determine such renormalized values are given by IR boundary conditions, where the linear derivative in $z$ appears from the bulk effective action by the technique of integration-by-parts. For the gravity sector, this boundary term is referred to as a Gibbons-Hawking-York term [30, 31]. The bulk effective action is an Einstein-Maxwell type theory

$$S_{Bulk} = N \int_0^{z_f} dz \int d^Dx \sqrt{g(x,z)} \left\{ \frac{1}{2 \lambda_\gamma} \left( \partial_\mu a_\mu(x,z) - \frac{1}{2d_\gamma} e^\mu_\alpha(x,z) \gamma_a \left( \gamma^\nu e^\nu_\alpha(x,z) \left( \partial_\nu - i e_\psi a_\nu(x) \right) G_{x\alpha}(a_\mu, e^\mu_\alpha) \right) \right)^2 + \frac{C}{4} F_{\mu\nu}(x,z) F^{\mu\nu}(x,z) ight\}$$

$$+ \frac{1}{2 \lambda_\theta} \left( \partial_\mu e^\mu_\alpha(x,z) - \frac{1}{2d_\theta} \gamma_a \left( \gamma^\nu e^\nu_\alpha(x,z) \left( \partial_\nu - i e_\psi a_\nu(x) \right) - i \frac{1}{4} \omega_\mu^a b^a \sigma_{ab} \right) G_{\mu
u}(a_\mu, e^\mu_\alpha) \right)^2$$

$$+ \frac{1}{2 \lambda_\theta} \left( R(x,z) - 2\Lambda \right) + \left( A^{RR}(x,z) + A^{FF}(x,z) + A^{FR}(x,z) \right).$$

(37)

The crucial difference from the case of the free Dirac theory is that derivatives in the extra dimensional space are in the second order, also resulting from interaction-induced quantum fluctuations. As a result, both vierbein and U(1) gauge fields are now fully dynamical. In other words, we may say that global symmetries are uplifted into local ones, consistent with the holographic duality conjecture, even if this bulk effective action is a gauge-fixed version and those gauge symmetries are explicitly broken by gauge fixing.

We recall $g_{zz} = \epsilon_a^\mu \epsilon^b_\mu \delta_{ab} = 1$, given by the globally uniform speed of coarse graining, i.e., $\alpha(x,z) = 1$. We point out that there is an additional gauge choice, giving rise to $g_{zz} = \epsilon_a^\mu \epsilon^b_\mu \delta_{ab} = 0$ with $\mu = 0, ..., D - 1$ in
this effective bulk action. This gauge freedom originates from the invariance of the partition function with respect to \( D \)-dimensional diffeomorphism after the renormalization group transformation with \( dz \). Furthermore, we have the gauge choice of \( a_z = 0 \) in the Maxwell dynamics. A fully covariant formulation has been constructed in the absence of U(1) gauge fields, where \( D \)-dimensional Einstein-Hilbert action is uplifted into \( (D + 1) \)-dimensional Einstein-Hilbert one via recursive renormalization group transformations.

It is interesting to observe that quantum fluctuations are suppressed for both vierbein and U(1) gauge fields in the large \( N \) limit. Integrating over the Dirac spinor fields in the IR boundary action and performing the gradient expansion for both IR-boundary vierbein and U(1) gauge fields with respect to the mass of Dirac fermions, we obtain an effective free-energy functional, given by

\[
\mathcal{W}[\epsilon_a^\mu(x,z), a_\mu(x,z); \epsilon_a^\mu, A_\mu, z_f] \equiv -\ln Z[\epsilon_a^\mu(x,z), a_\mu(x,z); \epsilon_a^\mu, A_\mu, z_f]
\]

\[
= N \int d^D x \sqrt{g(x,0)} \left\{ \frac{1}{2\lambda_0} \left( a_\mu(x,0) - A_\mu(x) \right)^2 + \frac{1}{2\lambda_0} \left( \epsilon_a^\mu(x,0) - \tilde{\epsilon}_a^\mu(x) \right)^2 \right\} 
\]

\[
+ N \int_0^{z_f} dz \int d^D x \sqrt{g(x,z)} \left\{ \frac{1}{2\lambda_0} \left( \partial_\mu a_\mu(x,z) - \frac{1}{2d z} \epsilon_a^\mu(x,z) \gamma_\alpha \{ \gamma^\nu \epsilon_a^\nu \}(x,z) \left( \partial_\mu - i e_\psi a_\mu(x,z) \right) \right)^2 + \mathcal{C}_F F_\mu(\epsilon_a^\mu, A_\mu) \right\}
\]

\[
+ \frac{1}{2\kappa} \left( R(x,z) - 2\Lambda \right) + \left( \mathcal{A}^{RR}(x,z) + \mathcal{A}^{FF}(x,z) + \mathcal{A}^{FR}(x,z) \right)
\]

\[
+ N \int d^D x \sqrt{g(x,z_f)} \left\{ \frac{C_F}{4} F_\mu(\epsilon_a^\mu, z_f) F_\nu(\epsilon_a^\nu, z_f) + \frac{1}{2\kappa_f} \left( R(x,z_f) - 2\Lambda_f \right) + \left( \mathcal{A}^{RR}(x,z_f) + \mathcal{A}^{FF}(x,z_f) + \mathcal{A}^{FR}(x,z_f) \right) \right\}.
\]

Here, \( C_f, \kappa_f, \) and \( \Lambda_f \) are defined in the same way as the previous section. This concludes our derivation for a dual holographic Einstein-Maxwell theory from the quantum field theory of interacting Dirac fermions.

One may ask the origin of non-perturbness in this emergent dual holographic description. Although the renormalization group flows for both vierbein and U(1) gauge fields, given by the above effective free-energy functional, look similar to those of the free Dirac theory, these evolution equations have a complex intertwining structure between the vierbein and U(1) gauge fields. An essential ingredient seems to be that interactions promote both vierbein and U(1) gauge fields to be fully dynamical, where their dynamics are described by the second-order differential equations in the extra dimensional space. In particular, the Green’s function is determined by both the vierbein and U(1) gauge fields beyond the free Dirac theory, and it is introduced into the second-order differential equations as a kernel for the dynamics. This nonlinear intertwining renormalization would be responsible for the non-perturbative physics.

The remaining task is to solve the modified Maxwell equations coupled to the modified Einstein equations with both UV and IR boundary conditions, all of which are derived from this effective free-energy functional. Certainly, this is not an easy work. First of all, we have to determine the background geometry under effects of U(1) electromagnetic currents, given by the Maxwell-type equations and resulting from effective interactions between conserved U(1) currents. Then, we should consider weak perturbations around the resulting background solution and express such fluctuations of both vierbein and U(1) gauge fields in terms of external electromagnetic fields and temperature gradients. Finally, we obtain an effective on-shell action in terms of such external source fields. Taking derivatives for the effective on-shell action with respect to external electromagnetic fields and temperature gradients, we find correlation functions of electromagnetic U(1) currents and energy-momentum tensors, which correspond to electrical, thermal, and thermo-electrical transport coefficients. Here, the question is how we can understand the emergence of effective hydrodynamics in this gravity reformulation with several modifications. This requires further investigations near future.
IV. CONCLUSION

The holographic duality conjecture describes the holographic liquid in terms of both Maxwell and Einstein coupled equations with an extra dimension. These coupled equations in the extra dimension may be regarded to be a dual description for conserved U(1) currents and energy-momentum tensors in effective hydrodynamics, where the disappearance of quasiparticles would be realized by strong renormalization effects along the extra dimension. In this study, we tried to construct a microscopic foundation for this holographic-liquid phenomenology.

We applied renormalization group transformations a la Polchinski in real space and in a recursive way a la Sung-Sik Lee to interacting Dirac fermions in the presence of both background metric or vierbein and external electromagnetic U(1) gauge fields. Besides detailed mathematical derivations, it turns out that both external source fields minimally coupled to conserved currents are uplifted to be dynamical and are forced to renormalization-group flow along the emergent extra dimension. In other words, global symmetries of quantum field theories turn into gauge symmetries through the renormalization group flows of the boundary source fields in the extra dimensional space. As a result, we obtain an effective Einstein-Maxwell theory in \((D + 1)\) spacetime dimensions from the quantum field theory of interacting Dirac fermions in \(D\) spacetime dimensions. IR boundary conditions self-consistently determined within this effective field theory serve as renormalization group \(\beta\)–functions for all types of interaction vertices including field renormalization.

It is natural to expect that our microscopically derived Einstein-Maxwell theory would show the holographic-liquid phenomenology. However, we point out that there exist additional evolution terms to modify that of the conventional holographic setup. In this respect it is necessary to look at possible solutions more carefully and compare these results with recent experiments near future. In addition, we emphasize that our derivation or construction has not been performed in a covariant way. It is also important to develop a covariant formulation in our opinion.

Acknowledgement

K.-S. Kim was supported by the Ministry of Education, Science, and Technology (No. 2011-0030046) of the National Research Foundation of Korea (NRF) and by TJ Park Science Fellowship of the POSCO TJ Park Foundation. K.-S. Kim appreciates fruitful discussions with Shinsei Ryu and his hospitality during the sabbatical leave. K.-S. Kim thanks Sung-Sik Lee for his comments on the manuscript.

[1] G. Policastro, D. T. Son, and A. O. Starinets, J. High Energy Phys. 09 (2002) 043.
[2] G. Policastro, D. T. Son, and A. O. Starinets, J. High Energy Phys. 12 (2002) 054.
[3] Pavel Kovtun, Dam T. Son, and Andrei O. Starinets, J. High Energy Phys. 10 (2003) 064.
[4] P. Kovtun, D. T. Son, and A. O. Starinets, Phys. Rev. Lett. 94, 111601 (2005).
[5] A. Bandurin, I. Torre, R. Krishna Kumar, M. Ben Shalom, A. Tomadin, A. Principi, G. H. Auton, E. Khestanova, K. S. Novoselov, I. V. Grigorieva, L. A. Ponomarenko, A. K. Geim, and M. Polini, Science 351, 1055 (2016).
[6] J. Crossno, J. K. Shi, K. Wang, X. Liu, A. Harzheim, A. Lucas, S. Sachdev, P. Kim, T. Taniguchi, K. Watanabe, and T. A. Ohki, Science 351, 1058 (2016).
[7] P. J.W. Moll, P. Kushwaha, N. Nandi, B. Schmidt, and A. P. Mackenzie, Science 351, 1061 (2016).
[8] David Tong, Lectures on Kinetic Theory, unpublished, http://www.damtp.cam.ac.uk/user/tong/kinetic.html.
[9] J. M. Maldacena, Int. J. Theor. Phys. 38, 1113 (1999).
[10] S. S. Gubser, I. R. Klebanov, and A. M. Polyakov, Phys. Lett. B 428, 105 (1998).
[11] E. Witten, Adv. Theor. Math. Phys. 2, 253 (1998).
[12] M. Bianchi, D. Z. Freedman, and K. Skenderis, Nucl. Phys. B 631, 159 (2002).
[13] J. de Boer, E. P. Verlinde, and H. L. Verlinde, J. High Energy Phys. 08 (2000) 003.
[14] E. P. Verlinde and H. L. Verlinde, J. High Energy Phys. 05 (2000) 034.
[15] O. Aharony, S. S. Gubser, J. Maldacena, H. Ooguri, and Y. Oz, Phys. Rep. 323, 183 (2000).
[16] Ki-Seok Kim and Takuya Tsukioka, Phys. Rev. D 85, 045011 (2012).
[17] Y. Seo, G. Song, P. Kim, S. Sachdev, and S.-J. Sin, Phys. Rev. Lett. 118, 036601 (2017).
[18] M. E. Peskin and D. V. Schroeder, An Introduction to quantum field theory (Westview, Boulder, 1995).
[19] J. Polchinski, Nucl. Phys. B 231 (1984) 269.
[20] Sung-Sik Lee, J. High Energy Phys. 10 (2012) 160.
[21] Sung-Sik Lee, J. High Energy Phys. 01 (2014) 076.
[22] Peter Lunts, Subhro Bhattacharjee, Jonah Miller, Erik Schnetter, Yong Baek Kim, and Sung-Sik Lee, J. High Energy Phys. 08 (2015) 107.
[23] K.-S. Kim, M. Park, J. Cho, and C. Park, Phys. Rev. D 96, 086015 (2017).
[24] Ki-Seok Kim and Shinsei Ryu, Entanglement transfer from quantum matter to classical geometry in an emergent holographic dual description of a scalar field theory, in preparation.
[25] Ki-Seok Kim, Geometric encoding of renormalization group \(\beta\)-functions in an emergent holographic dual description, in preparation.
[26] Ki-Seok Kim, Suk Bum Chung, Chanyong Park, and Jae-Ho Han, Phys. Rev. D 99, 105012 (2019).
[27] N. D. Birrell and P. C. W. Davies, Quantum Fields in Curved Space (Cambridge University Press, New York, 1982).
[28] A. Sakharov, Sov. Phys. Dokl. 12, 1040 (1968), and reprinted in Gen. Rel. Grav. 32, 365 (2000).
[29] M. Visser, Mod. Phys. Lett. A 17, 977 (2002), and references therein.
[30] James W. York, Jr., Phys. Rev. Lett. 28, 1082 (1972).
[31] G. W. Gibbons and S. W. Hawking, Phys. Rev. D 15, 2752 (1977).