On the extraction of skewed parton distributions from experiment

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Abstract

In this paper we will discuss algorithms for extracting skewed parton distributions from experiment as well as the relevant process and experimental observable suitable for the extraction procedure.

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I. INTRODUCTION

Skewed parton distributions\(^1\) which appear in exclusive, hard diffractive processes like deeply virtual Compton scattering (DVCS)\(^2\) or vector meson production with a rapidity gap, to name just a few, have attracted a lot of theoretical and experimental interest over the last few years as a hot bed for interesting new QCD physics \(^2\). The list of references is probably far from being complete and thus we apologize beforehand to everybody not mentioned.

\(^1\)This is the unified terminology since the Regensburg conference of ’98, finally eradicating the many terms like non-diagonal, off-diagonal, non-forward and off-forward which have populated the literature on this subject over the last few years. However recent publications have, alas, again fallen back upon the old terminology!

\(^2\)First discussed in Ref. \[1\].
The basic concept of skewed parton distributions is illustrated in Fig. 1 with the lowest order graph of DVCS in which a quark of momentum fraction $x_1$ leaves the proton and is returned to the proton with momentum fraction $x_2$. The two momentum fractions not being equal is due to the fact that an on-shell photon is produced which necessitates a change in the $+$ momentum in going from the virtual space-like photon with $+$ momentum usually taken to be $-xp_+$, where $p_+$ is the appropriate light cone momentum of the proton and $x$ is the usual Bjorken $x$, to basically zero $+$ momentum of the real photon. This sets $x_2 = x_1 - x$ and thus the skewedness parameter to $x$. (See [3] for more details on the kinematics.)

Thus one has a nonzero momentum transfer onto the proton and the parton distributions which enter the process are non longer the regular parton distributions of inclusive reactions since the matrix element of the appropriate quark and gluon operators is now taken between states of unequal momentum rather than equal momentum as in the inclusive case (see for example [2]). These parton distributions still obey DGLAP-type evolution equations but of a generalized form (see for example Radyushkin’s references in [2]).

The above mentioned kinematical situation is not the only one possible. One can also have the situation where $x_2$ becomes negative. In this case not a quark is returned to the proton but rather an anti-quark is emitted. In this situation one does not deal with parton distributions any more but rather distributional amplitudes obeying now Efremov-Radyushkin-Brodsky-LePage (ERBL) type evolution equations (again see, for example, Radyushkin’s
references in [2]). Furthermore, both momentum fractions could be negative in which case one is dealing with anti-quark distributions which again obey DGLAP-type evolution equations.

After having answered the question how these skewed parton distributions arise, the next question is which of the exclusive, hard diffractive processes is most suitable for extracting these skewed parton distributions and how can this be achieved. This question will be answered in the following sections, where we discuss the most promising process and the appropriate experimental observable in Sec. II in Sec. III we will explain the algorithm and the problems associated with it and finally in Sec. IV we will give an outlook on further research in this area.

II. APPROPRIATE PROCESS AND EXPERIMENTAL OBSERVABLE

The most desirable process for extracting skewed parton distributions is the one with the least theoretical uncertainty, the least singular $Q^2$ behavior so as to be accessible over a broad range of $Q^2$ and with a proven factorization formula. The last requirement is actually the most important one since without a factorization theorem one has no reliable theoretical basis for extracting parton distributions.

The process which fulfills all the above criteria is DVCS since it is least suppressed in $Q^2$ of all known exclusive, hard diffraction processes, in fact it is only down by an additional factor of $Q^2$ in the differential cross section as compared to DIS\(^3\), the theoretical uncertainty is minimal since we are dealing with an elementary particle in the final state as compared to, for example, vector meson production where one also has to deal with the vector meson wavefunction in the factorization formula as an additional uncertainty and there exists a proven factorization formula [3].

\(^3\)Compare this to the $1/Q^8$ behavior of vector meson production, di-muon production or di-jet production.
Furthermore it was shown in Ref. [4] that there will be sufficient DVCS events at HERA as compared to DIS, albeit only at small $x$ between $10^{-4} - 10^{-2}$, to allow an analysis with enough statistics.

The experimental observable which allows direct access to the skewed parton distributions is the azimuthal angle asymmetry $A$ of the combined DVCS and Bethe-Heitler (BH) differential cross section, where the azimuthal angle is between the final state proton - $\gamma^*$ plane and the electron scattering plane. $A$ is defined as [4]:

$$A = \frac{\int_{-\pi/2}^{\pi/2} d\phi \; d\sigma_{DVCS+BH} - \int_{3\pi/2}^{-\pi/2} d\phi \; d\sigma_{DVCS+BH}}{\int_0^{2\pi} d\phi \; d\sigma_{DVCS+BH}}. \tag{1}$$

In other words one counts the events where electron and photon are in the same hemisphere of the detector and subtracts the number of events where they are in opposite hemispheres and normalizes this expression with the total number of events.

The reason why this asymmetry is not 0 is due to the interference term between BH and DVCS which is proportional not only to $\cos(\phi)$, as compared to the pure DVCS and BH differential cross sections which are constant in $\phi$, but also to the real part of the DVCS amplitude. The factorized expression for the real part of the amplitude takes the following form [3]:

$$Re \; T(x, Q^2) = \int_{-1+x}^1 dy \; \frac{dy}{y} Re \; C_i(x/y, Q^2) f_i(y, x, Q^2). \tag{2}$$

$Re \; C_i$ is the real part of the hard scattering coefficient and $f_i$ are the skewed parton distributions. The sum over the parton index $i$ is implied and $y$ is defined to be the parent momentum fraction in the parton distribution. As mentioned above, one is mainly restricted to the small-$x$ region where gluons dominate and thus $i$ in Eq. (2) will be only $g$ to a very good accuracy. Note that since the parton distributions are purely real, the real part of the amplitude in its factorized form has to contain the real part of the hard scattering coefficient.

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4In the Bethe-Heitler process the incoming electron exchanges a coulomb photon with the proton and radiates off a real photon, either before or after the interaction with the proton.
Thus Eq. (1) contains only measurable or directly computable quantities except Eq. (2) in the interference part of the differential cross section for real photon production which is isolated in Eq. (1). Therefore, one would now be able to extract the skewed parton distributions from experimental information on $A$, the directly computable part of the interference term and the knowledge about the hard scattering coefficient if one could deconvolute Eq. (2).

As we will see in the next section this direct deconvolution is not possible, however there is a way around the deconvolution problem.

III. ALGORITHMS FOR EXTRACTING SKEWED PARTON DISTRIBUTIONS

A. The Deconvolution Problem in DIS

The deconvolution problem in inclusive DIS presents itself in a similar way as in Eq. (2). For the structure function $F_2(x, Q^2)$, for example, one has the following factorization equation (in a general form):

$$F_2(x, Q^2) = \int_x^1 \frac{dy}{y} C_i(x/y, Q^2) f_i(y, Q^2),$$

(3)

where one has the same situation as in Eq. (2) except, of course, that the hard scattering coefficient $C_i$ and the parton distributions $f_i$ are now different from the DVCS case. Also notice that the parton distributions depends now only on $y$ rather than $y$ and $x$. In this case one can now easily deconvolute Eq. (3) by taking moments in $x$ via $\int_0^1 dx x^N$. It is an easy exercise to show that the convolution integral turns into a product in moment space:

$$\tilde{F}_2(N, Q^2) = \tilde{C}_i(N, Q^2) \tilde{f}_i(N, Q^2).$$

(4)

Thus, after having calculated the hard scattering coefficient to the appropriate order and having measured $F_2$ such that the moment integral can be taken numerically, one can directly extract the parton distribution. What remains to be done is to perform the inverse Mellin transform to obtain the parton distribution in terms of $x$ and $Q^2$. Of course, we have
simplified the actual procedure and the inverse Mellin transform is also not easy to perform but this example serves more as a pedagogical exercise to illustrate the basic concept of deconvolution and extraction of parton distributions.

In the case of interest to us, however, life is not that ”simple”, since the skewed parton distributions depend on two rather than one variable. Furthermore, the hard scattering coefficient depends on the same variables as the parton distribution. This makes the deconvolution of Eq. (2), at least to the best knowledge of the author, impossible because both the hard scattering coefficient and the parton distribution have two rather than one variable in common, one of which is even fixed, thus one does not have enough information to perform a deconvolution.

This seems like an intractable problem but there is a way out. For the purpose of as simple a presentation as possible the following discussion will only be done in LO but the same principles also apply in NLO. However, the precision of the data in the foreseeable future, will be such that a leading order analysis will be sufficient. The following two discussions rest heavily on the methods in Ref. [5,6].

**B. The First Principle Extraction Algorithms**

It is always possible to expand the parton distributions or any smooth function for that matter, with respect to a complete set of orthogonal polynomials \( P_j^{(\alpha_P)}(t) \). In this particular case we need the orthogonality of the polynomials of our choice to be on the interval \(-1 \leq t \leq 1\) with \( t = \frac{2y-x}{2-x} \) which translates to an interval in \( y \) of \(-1 + x \leq y \leq 1\) as found as the upper and lower bounds of the convolution integral in Eq. (2). One can then write the following expansion:

\[
f^{q,g}(y, x, Q^2) = \frac{2}{2 - x} \sum_{j=0}^{\infty} \frac{w(t|\alpha_P)}{n_j(\alpha_P)} P_j^{q,g}(t) M_j^{q,g}(x, Q^2)
\]  

with \( w(t|\alpha_P) \) and \( n_j(\alpha_P) \) being weight and normalization factors determined by the choice of the orthogonal polynomial used. The labels \( q, g \) for quarks and gluons are necessary since
the $j$ label will be different for quarks and gluons. $\alpha_P$ is a label which depends on the orthogonal polynomials used\footnote{$\alpha_P = \alpha, \beta$, in other words two labels, if Jacobi polynomials are used or $\alpha_P = \mu - 1/2$ if Gegenbauer polynomials are used.}: $M^{q,g}_j(x, Q^2)$ is given by:

$$M^{q,g}_j(x, Q^2) = \sum_{k=0}^{\infty} E^{q,g}_{jk}(x) f^{q,g}_k(x, Q^2),$$  \hspace{1cm} (6)$$

where

$$f^{q,g}_j(x, Q^2) = \sum_{k=0}^{j} x^{j-k} B^{q,g}_{jk} \tilde{f}^{q,g}_k(x, Q^2).$$  \hspace{1cm} (7)$$

$B^{q,g}_{jk}$ is an operator transformation matrix which fixes the NLO corrections to the eigenfunctions of the kernels and is thus just the identity matrix in LO. The explicit form of the transformation matrix $B^{q,g}_{jk}$ is highly non-trivial but computable from the evolution equations in NLO. The upper limit in Eq. (6) is given by the constraint $\theta$-functions\footnote{$\theta_{jk} = 1$, if $k \leq j$; 0, if $j < k$} present in the expansion coefficients, which are generally defined by

$$E_{jk}(\nu; \alpha_P|x) = \frac{\theta_{jk}}{(2\pi)^k} \frac{\Gamma(\nu) \Gamma(\nu + k)}{\Gamma(\nu + k + \frac{1}{2})} \int_{-1}^{1} dt (1 - t^2)^{k+\nu-\frac{1}{2}} \frac{d^k}{dt^k} P^{\alpha_P}_j \left( \frac{x t}{2 - x} \right).$$  \hspace{1cm} (8)$$

The moments of the parton distributions evolve according to

$$\tilde{f}^{q,g}_j(x, Q^2) = \tilde{E}_j(\alpha_s(Q^2), \alpha_s(Q^2_0)) f^{q,g}_j(x, Q^2_0)$$  \hspace{1cm} (10)$$
where the evolution operator is a matrix of functions in the singlet case (and just a function in the non-singlet case) taking account of quark and gluon mixing and depending on the order in the strong coupling constant.

Finally, the Gegenbauer moments of the initial parton distributions at $Q_0^2$ are defined in the following way

\begin{align*}
\tilde{f}_j^q(x, Q_0^2) &= \int_{-1}^{1} dt \left( \frac{x}{2-x} \right)^j C_{j/2}^3 \left( \frac{tx}{2-x} \right) f^q(t, x, Q_0^2) \\
\tilde{f}_j^g(x, Q_0^2) &= \int_{-1}^{1} dt \left( \frac{x}{2-x} \right)^{j-1} C_{j-1/2}^5 \left( \frac{tx}{2-x} \right) f^g(t, x, Q_0^2). \quad (11)
\end{align*}

In LO order and at small $x$, the above formalism which looks tremendously complicated, simplifies considerably. Owing to the conformal properties of the operators involved in the definition of the parton distributions (see the papers by Müller and Belitsky in [2] for exhaustive treatment of this subject), one finds the following simple expansion in terms of partial conformal waves

\begin{align*}
f^q(y_1, x, Q^2) &= \frac{2}{2-x} \sum_{j=0}^{\infty} \sum_{k=0}^{\infty} w(t|3/2) w(t|5/2) N_j(3/2) E_{jk}(x) C_{j/2}^3(t) \tilde{f}_k^q(x, Q^2) \quad (12) \\
f^g(y_1, x, Q^2) &= \frac{2}{2-x} \sum_{j=0}^{\infty} \sum_{k=1}^{\infty} w(t|5/2) w(t|7/2) N_{j-1}(5/2) E_{j-1,k-1}(x) C_{j-1/2}^5(t) \tilde{f}_{k-1}^g(x, Q^2), \quad (13)
\end{align*}

with $w(t|\nu) = (t(1-t))^{\nu-1/2}$ and the $C_{j/2}$ are Gegenbauer polynomials which in LO diagonalize the operators of the parton distributions. The multiplicatively renormalizable moments evolve as above but with the explicit evolution operator:

\begin{equation}
\tilde{E}_{ij}^k(\alpha_s(Q^2), \alpha_s(Q_0^2)) = T exp \left( -\frac{1}{2} \int_{Q_0^2}^{Q^2} d\tau \gamma_{i,j}^k(\alpha_s(\tau)) \right) \quad (14)
\end{equation}

where $T$ orders the matrices of LO anomalous dimensions ($i,k = q,g$) along the integration path. Note that there is a slight difference in the anomalous dimensions in the skewed case to the anomalous dimensions in the non-skewed, i.e. inclusive, case due to the particular definition of the conformal operators used in the definition of the parton distributions:

$\tilde{\gamma}_{j}^{qq} = 6 \tilde{\gamma}_{j}^{qq, incl.}$ and $\tilde{\gamma}_{j}^{gq} = \tilde{\gamma}_{j}^{gq, incl.}$.

\footnote{This is true in LO, in NLO, however, the anomalous dimensions obtain, besides the NLO...}
Now we have all the ingredients to proceed. Inserting Eq. (13) in Eq. (4) one obtains for small $x$:

$$\text{Re } T(x, Q^2) = 2 \sum_{j=0}^{\infty} \sum_{k=1}^{\infty} \tilde{E}_{k-1}(\alpha_s(Q^2), \alpha_s(Q_0^2)) \tilde{f}_{k-1}^g(x, Q_0^2) E_{j-1}^g(x)$$

$$\int_{-1}^{1} \frac{dt}{2t + x} \frac{w(t|5/2)}{N_j(5/2)} \text{Re } C_g \left( \frac{1}{2} + \frac{t}{x} \right) C_{j-1}^{5/2}(t),$$

(15)

where we chose the factorization scale to be equal to the renormalization scale, which we chose to be equal to $Q^2$. As one can see the integral in the sum is now only over known functions and will yield, for fixed $x$, a function of $j$ as will also do the expansion coefficients for fixed $x$. The evolution operator can also be evaluated and will yield for fixed $Q^2$ also just a function of $j$, which leaves the coefficients $\tilde{f}_{k-1}^g(x, Q_0^2)$ as the only unknowns, albeit an infinite number of them. Since the lefthand side will be known from experiment for fixed $x$ and $Q^2$, we are still in the unfortunate situation that a number is determined by the sum over $j$ of an infinite number of coefficients. Thus, if one had measured the real part of the DVCS amplitude through the asymmetry $A$ at fixed $x$ and at an infinite number of $Q$, one would have an infinite dimensional column vector on the lefthand side namely the real part of the amplitude at fixed $x$ but at an infinite number of $Q$ and on the right hand side one would have a square matrix times another column vector of coefficients of which the dimension is determined by the number of $j$. Since all the entries in the matrix are real and positive $\gamma_j$’s of the inclusive case, additional anomalous dimensions due to non-diagonal elements in the renormalization matrix of the conformal operators entering the skewed parton distributions (see Ref. [7]).

8 This fixes the undetermined coefficients up to the $j$ index.

9 The number of $Q$ values determines the column dimension and the number of the index $j$ determines the row dimension. The matrix is square since we can choose the number of $Q$ values to be equal to the number of $j$ values!
definite, it can be inverted, using the well known linear algebra theorems on inversion of infinite dimensional square matrices, provided that there are no zero eigenvalues in other words no physical zero modes in the problem which would imply that the real part of the DVCS amplitude would have to be zero which is, of course, never the case. After having found the inverse, we can directly compute the moments of our initial parton distributions which are needed to reconstruct the skewed gluon distribution at small $x$ from Eq. (5).

The drawback of the above procedure is that this process has to be repeated anew for each $x$. Nothing, however prevents us from doing so, in principle. Even for a finite number of $j$'s and $Q$'s, the task seems formidable, however, this is not as problematic as it seems, since experiment will only render information for small $x$, at least in the beginning, and not the whole range of $x$, thus one does not need an infinite number of coefficients and thus an infinite number of $Q$ for each $x$ to get a good approximation. Unfortunately a $j_{\text{max}}$ of $50-100$ will be necessary, therefore, if the lefthand side is known for each $x$ at 50 values of $Q^2$, Eq. (15) reduces to a system of 50 equations with 50 unknowns for $j_{\text{max}} = 50$. This system can readily be solved as explained above. Experimentally speaking, of course, this procedure is not feasible, though theoretically very attractive, since one will never be able to measure the any experimental observable for fixed $x$ at 50 different values of $Q^2$. Nevertheless, there

\[10\] The evolution operator will, of course, always yield a positive number, the integrals in the sum, are integrals over positive definite functions in the integration interval and the expansion coefficients are also positive definite as can be seen from Eq. (9).

\[11\] The DGLAP part of the amplitude will be zero at $x = 1$ but the contribution from the ERBL region will not be!

\[12\] Note that the same moments of the initial parton distribution will appear for different values of $j$, since the sum over $k$ runs up to $j$, for fixed $Q$, such that each unknown moment is just multiplied by a number determined from known functions in Eq. (15).

\[13\] The author’s thanks go to Andrei Belitsky for pointing this out.
may be ways using constraints on SPD’s to reduce this number of 50 – 100 polynomials as can be done in the forward case, however this has to be further explored.

Notwithstanding the above, let us give a toy example of the above extraction algorithm. Take \( x \) discrete and fix \( Q^2 \) then one can write a factorized expression for a cross section:

\[
\sigma_a = \sum_j H_{j,a} f_{j,a}.
\]  

(16)

The index \( j \) corresponds to the parton fractional momentum, and \( a \) to the \( x \) variable. Obviously, it is not possible to obtain \( f_{j,a} \) from \( \sigma_a \). If one now puts in an index for \( Q \), the parton densities will now be \( f(Q)_{j,a} \), and the solution of the evolution equation has the form

\[
f(Q)_{j,a} = \sum_k U(Q)_{j,k,a} f(Q_0)_{k,a}.
\]  

(17)

Here \( f(Q_0)_{k,a} \) is the initial parton density at the value \( Q = Q_0 \) which is left implicit. The cross section as a function of \( Q \) takes now the form

\[
\sigma_{Q,a} = \sum_{j,k} H_{j,a} U(Q)_{j,k,a} f(Q_0)_{k,a}
\]

\[= \sum_k A(Q)_{j,k} f(Q_0)_{k,a},
\]  

(18)

for a suitable matrix \( A \). The \( Q \) dependence of the hard scattering function \( H \) can be ignored for our present purpose.

As a next step, take enough values of \( Q \) such that the matrix is square. The most trivial example is to have two values of \( Q \): the initial value and one other:

\[
f_{1;j} = f(Q_0)_j
\]

\[
f_{2;j} = \sum_k U(Q)_{j,k} f(Q_0)_k.
\]  

(19)

One can take \( U \) to be triangular, as is appropriate for DGLAP evolution.

\[^{14}\] The author would like to thank John Collins for suggesting such an example to clarify the problem at hand.
\[ U = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix} , \] 

(20)

the hard scattering cross section to be

\[ H = (1, 1) \] 

(21)

and the parton distributions to be a two dimensional column vector:

\[ f_0 = \begin{pmatrix} f(Q_0)_1 \\ f(Q_0)_2 \end{pmatrix} . \] 

(22)

This then yields

\[ \begin{pmatrix} \sigma(Q) \\ \sigma(Q_0) \end{pmatrix} = \begin{pmatrix} 1 & 2 \\ 1 & 1 \end{pmatrix} \begin{pmatrix} f(Q_0)_1 \\ f(Q_0)_2 \end{pmatrix} . \] 

(23)

Clearly one has an invertible matrix in Eq. (23) and can thus compute \( f(Q_0)_1 \) and \( f(Q_0)_2 \).

C. The Practical Extraction Algorithm

A practical way out of the polynomial predicament is by making a simple minded ansatz for the skewed gluon distribution, since we are still at small \( x \), in the different ER-BL and DGLAP regions of the convolution integral Eq. (2). An example of such an Ansatz could be \( A_0 z^{-A_1} (1 - z)^A_3 \) for the DGLAP region where \( z \) is now just a dummy variable. If one inserts this Ansatz in Eq. (2) and can fit the coefficients to the data of the real part of the DVCS amplitude for fixed \( x \) and \( Q^2 \). One can then repeat this procedure for different values of \( Q^2 \) and then interpolate between the different coefficients to obtain a functional form of the coefficients in \( Q^2 \). Alternatively, after having extracted the values of the coefficients for different values of \( x \) at the same \( Q^2 \), use an evolution program with the ansatz and the fitted coefficients as input and check whether one can reproduce the data for the real part at higher \( Q^2 \), thus checking the viability of the model ansatz.

To obtain an ansatz fullfilling the various constraints for SPD’s (see Ji’s and Radyushkin’s references in [2,6]), one should start from the double distributions (DD) (see Redyushkin’s
references in [2,6]) which yield the skewed gluon distribution in the various regions of the convolution integral

\[ g(y, x) = \theta(y \geq x) \int_{0}^{1-x} dz G(y - xz, z) + \theta(y \leq x) \int_{0}^{y} dz G(y - xz, z). \] (24)

Due to the fact that there are no anti-gluons, the above formula is enough to cover the whole region of interest \(-1 + x \leq y \leq 1\). What remains is to choose an appropriate model ansatz for \(G\), for example,

\[ G(z_1, z) = \frac{h(z_1, z)}{h(z_1)} f(z_1) \] (25)

with \(f(z_1)\) being taken from a diagonal parametrization with its coefficients now being left as variants in the skewed case and the normalization condition \(h(z_1) = \int_{0}^{1-z_1} dz h(z_1, z)\) such that, in the diagonal limit, the DD just gives the diagonal distribution. The choice for \(h(z_1, z)\) is a matter of taste but should be kept as simple as possible. The drawback of this algorithm as compared to the previous one is that it is model dependent and thus not a first principle methods, which theoretically speaking, is not as satisfying but from the practical side this method is much simpler and thus experimentally much more feasible.

Thus, one has solved the problem of extracting the parton distributions from the factorization equation, at least for small-\(x\). The remaining problem is an experimental one.

**IV. CONCLUSIONS AND OUTLOOK**

After having showed, that the extraction of skewed parton distributions from DVCS experiments is both principally and practically possible given the high enough statistics data on the asymmetry, one should now get a more accurate model description of the asymmetry. This will be done elsewhere.

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