Effect of pre-existing baryon inhomogeneities on the dynamics of quark-hadron transition.

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Abstract

Baryon number inhomogeneities may be generated during the epoch when the baryon asymmetry of the universe is produced, e.g. at the electroweak phase transition. The regions with excess baryon number will have a lower temperature than the background temperature of the universe. Also the value of the quark hadron transition temperature $T_c$ will be different in these regions as compared to the background region. Since a first-order quark hadron transition is very susceptible to small changes in temperature, we investigate the effect of the presence of such baryonic lumps on the dynamics of quark-hadron transition. We find that the phase transition is delayed in these lumps for significant overdensities. Consequently, we argue that baryon concentration in these regions grows by the end of the transition. We briefly discuss some models which may give rise to such high overdensities at the onset of the quark-hadron transition.

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I. INTRODUCTION

Phase transitions are supposed to have happened at different epochs in the early universe and one of the many possible consequences of these transitions is the generation of baryonic inhomogeneities. Most discussions in the literature are about baryon inhomogeneities generated during the quark-hadron transition [1–4]. One reason for this is that baryon inhomogeneities naturally develop during the quark-hadron transition [1]. Another reason is that the main importance of these inhomogeneities lies in the fact that if they survive till the nucleosynthesis epoch, then they will affect the calculated abundances of the light elements, leading to an inhomogeneous big bang nucleosynthesis scenario. Since it is more probable that inhomogeneities generated during the QCD transition would survive until nucleosynthesis, that is why they are the ones which are mostly studied. However, baryon inhomogeneities can be produced at earlier stages as well. For example, there are certain baryogenesis scenarios utilizing the electroweak transition which leave an inhomogeneous distribution of baryons [5]. These are scenarios where the baryogenesis occurs through strongly non-equilibrium processes. For example ref. [6] discusses the effect of baryon inhomogeneities generated by a first order electroweak transition on the nucleosynthesis epoch. Apart from these there are also defect-mediated baryogenesis models where the baryons generated are usually localized [7–11]. These inhomogeneities are expected to get homogenized by the effects of neutrino inflation and baryon diffusion by the nucleosynthesis epoch [12]. So they usually do not affect the big-bang nucleosynthesis (BBN) calculations.

However, even if these inhomogeneities do not survive until the stage of nucleosynthesis, they may still survive until the stage of quark-hadron phase transition. For example, in a detailed study by Jedamzik and Fuller [13], it was shown that overdensities with large amplitude (of the order of \(n_b' > 10^3\), where \(n_b'\) and \(n_b\) are the baryon densities in the overdense and underdense regions respectively) and lengthscales of the order of \(10^{-3}\) cm (comoving at 100 MeV), are dissipated very little (by the neutrino inflation process) and may survive relatively undamped up to the stage when the temperature of the universe is of order of 100 MeV, which is the scale of the quark-hadron transition. At a later stage the mechanism for the damping of baryon inhomogeneities becomes dominated by baryon diffusion which may completely wipe out inhomogeneities which have a lengthscale less than the baryon diffusion length which is of the order of \(10^{-1}\) cm at the nucleosynthesis epoch. However we will argue in this paper that if the quark-hadron transition is of first-order, and proceeds via bubble nucleation, then the baryon inhomogeneities which are already present during that time may affect the dynamics of the phase transition. This is because the bubble nucleation process in a first-order phase transition can depend crucially on very small temperature changes. In this work we study the effect of any pre-existing baryon inhomogeneities on the dynamics of the quark-hadron transition. Since here the baryon inhomogeneity has to be present before the quark-hadron transition begins, we only consider the inhomogeneities generated at earlier stages, e.g., those generated during the electroweak epoch.

We consider the quark-hadron transition to be a first order transition. The presence of the baryon inhomogeneities causes small temperature fluctuations throughout the universe. The temperature of the inhomogeneities being less than the surrounding temperature, neutrinos passing through them (for relatively small scale inhomogeneities) will tend to deposit heat...
in them. The lumps of inhomogeneities then inflate to achieve pressure equilibrium, thereby reducing the amplitude of the inhomogeneity. At the same time, the inflation of the lump decreases its temperature so the whole process is repeated again until the inhomogeneity is wiped out. This is the process of neutrino inflation, which can efficiently erase baryon inhomogeneities depending on the amplitude and the scale of the inhomogeneity. As was shown by Jedamzik and Fuller [13], only for high overdensities the inhomogeneities may survive for a large time. For example, the inhomogeneities produced via certain models of baryogenesis at the electroweak scale may survive until the quark-hadron transition. [6]

In the lumps with a higher baryon number density than the background, the value of the critical temperature for quark hadron transition within the lump will be lower than that for the background [14]. Now bubbles of hadronic phase only nucleate when there is sufficient supercooling. We show that the combination of the effects of lower critical temperature and the heat deposited by neutrinos is such that there is no nucleation of bubbles in the baryonic dense regions while nucleation of bubbles gets completed in the surrounding region. So ultimately, we do not have a homogeneous bubble nucleation scenario throughout the universe. Since at the site of the lumps the process of bubble nucleation is delayed, the bubbles nucleated outside the lump start expanding and reheat the universe to $T_c$ even before nucleation has started in the baryon overdense regions. This region which already had an overdensity remains in the quark-gluon plasma (QGP) phase. As the baryon number tends to remain in the QGP phase rather than in the hadronic phase [1], this will lead to an increase in the overdensity in the already existing lump. This will cause a further increase in pressure inside the lump which will lead to the expansion of the size of the lump. Thus even though the seed inhomogeneity was small in size, it will give rise to a larger inhomogeneity as the phase transition is completed. Basically the temperature fluctuation due to the baryonic inhomogeneities prevents homogeneous nucleation of bubbles throughout the universe. We show that due to this inhomogeneous nucleation of bubbles, pre-existing baryon inhomogeneities grow in size as well as in amplitude as the phase transition proceeds. (We mention here that it is likely that the bubble nucleation rate itself may be different in the two regions, even with similar features of supercooling etc. If bubble nucleation rate increases strongly with baryon density then one cannot rule out a reverse situation where phase transition in the baryonic overdense regions happen first. This will then cause a rapid decay of the baryonic inhomogeneities, instead of it’s growth. However in this paper we ignore the dependance of bubble nucleation rate on baryon density.)

In section II we present a detailed calculation of the characteristics of the baryonic lumps, their temperature difference with the background and their critical temperature. In section III, we briefly review the time and temperature scales involved in a first order QCD phase transition. Section IV then describes the effect of the baryonic inhomogeneities on the phase transition. In section V we briefly discuss some models in which such inhomogeneities may be generated before the quark hadron transition. Conclusions are presented in section VI.

II. CHARACTERISTICS OF THE BARYONIC LUMPS.

In the early universe, any small scale density inhomogeneity that is created achieves pressure equilibrium rapidly (typically with the speed of sound) with its surroundings. Before
the QCD phase transition, the universe is in the QGP phase, with the pressure in a given region of space due to quarks being,

\[ P_q = \frac{7}{4} N_q a T^4 + 9 N_q^{-1} \frac{n_b^2}{T^2} \]  (1)

where the second term on the right gives the contribution of the baryon number to the pressure, \( N_q \) is the number of relativistic quark flavors at temperature \( T \), and \( a = \frac{\pi^2}{30} \). A region of baryon inhomogeneity achieves pressure equilibrium rapidly with its surroundings. The condition for pressure equilibrium between the outside and inside the lump is given by,

\[ \frac{1}{3} \epsilon + 9 N_q^{-1} \frac{n_b^2}{T^2} = \frac{1}{3} \epsilon' + 9 N_q^{-1} \frac{n_b'^2}{T'^2} \]  (2)

Here \( \xi \) and \( \xi' \) are the radiation pressures due to all the relativistic particles including leptons and photons in the two regions (with \( \epsilon = g_{eff} a T^4 \), and \( g_{eff}(\approx 51) \) is the effective degrees of freedom in the quark-gluon plasma phase). For the relevant values of baryon number, one can see from Eq.(2) that among the two regions whichever has a higher baryon number \( (n_b') \) must have a lower temperature. Replacing \( T' = T + \delta T \) in Eq.(2) one can calculate the temperature difference \( \frac{\delta T}{T} \) between the baryon over-dense and under-dense regions [13]. It is given by,

\[ \frac{\delta T}{T} = -\frac{27}{4 g_{eff} a T^4} \frac{n_b^2}{T^2} \frac{1}{N_q} \left[ \left( \frac{n_b'}{n_b} \right)^2 - 1 \right] \]  (3)

For \( T \sim 170 \) MeV, and for \( \left( \frac{n_b'}{n_b} \right)^2 >> 1 \) we get,

\[ \Delta T' = \frac{\delta T}{T} = -3 \times 10^{-19} \times \left( \frac{n_b'}{n_b} \right)^2 \]  (4)

This equation gives the dependence of the temperature difference of the overdense region from the background on the magnitude of the overdensity in the region. We will see that for sufficiently large values of \( \left( \frac{n_b'}{n_b} \right)^2 \) the difference in temperature between the inside of the lump and its surroundings can be significant enough so that heat deposition in these regions by neutrinos can disrupt the usual dynamics of a first order quark-hadron phase transition. This is because the dynamics of a first order phase transition depends crucially on temperature differences of even very small scales. As we will see below, this happens if we have \( \left( \frac{n_b'}{n_b} \right) > 10^7 \).

Even though the temperature in the baryonic overdense lump is lower than the background temperature, this temperature difference is relatively small (as we will see below). Thus, with higher baryon density inside the lump, the baryon chemical potential \( \mu \) inside the lump is also larger than the corresponding value in the background region. As the critical temperature for the quark-hadron phase transition depends on the chemical potential, the difference in the chemical potentials will also cause a difference in the values of the critical temperature, with \( T'_c \) and \( T_c \) denoting the values of critical temperature for the overdense and the background regions respectively. Unless a region supercools by a certain amount below the critical temperature (suitable for the region under consideration), bubble nucleation
will not start. As the critical temperature is different in the two regions, bubble nucleation may also start in the two regions at two different times, unless the temperature difference between the two regions exactly compensates for the effect of difference between $T'_c$ and $T_c$. We will express the value of the chemical potentials in a given region in terms of $\eta = \frac{n_b}{s}$ where $n_b$ is the baryon number density and $s = \frac{2\pi^2}{45}g_{eff}T^3$ is the entropy density. $T$ is the temperature of the region. We get [15],

$$\frac{\mu}{T} \sim 12 \times \eta,$$

(5)

Using Gibb's criterion for a first order phase transition, we equate the pressure in the quark-gluon phase to the pressure in the hadronic phase to determine the corresponding critical temperature [16]. With non-zero chemical potential we get [16],

$$P_q = P_\pi + P_N$$

(6)

where

$$P_q = \frac{37}{90}\pi^2 T^4 + \left(\frac{\mu}{3}\right)^2 T^2 + \frac{1}{2\pi^2} \left(\frac{\mu}{3}\right)^4 - B,$$

(7)

$$P_\pi = \frac{3m^2 T^2}{2\pi^2} \sum_{k=1}^{\infty} \frac{K_2\left(\frac{km}{T}\right)}{k^2},$$

(8)

and

$$P_N = \frac{2M^4}{3\pi^2} \int_0^1 \frac{u^4 du}{(1-u^2)^3} \left[f(u; T, \mu) + f(u; T, -\mu)\right].$$

(9)

Here $P_q$ is the pressure in the quark-gluon phase, while the right hand side of Eq.(6) gives the total pressure in the hadronic phase (taken as the pressure of pions and nucleons). Note that here we consider only the QCD degrees of freedom, as contribution from other particle species cancels out. $K_2\left(\frac{km}{T}\right)$ is the modified Bessel function of the second kind and the function $f(u; T, \mu) = \left[\exp\left(M/(1-u^2)\right)^{1/2} - 1\right]^{-1}$. $M = 940$ MeV is the nucleon mass and $m = 140$ MeV is the pion mass. Solving Eq.(6) we can get the value of $T_c$ for a given value of $\mu$.

We mention here that we use Eqs.(6)-(9) only to get an order of magnitude estimate of the shift in the transition temperature as a function of $\mu$. This is primarily determined by $\mu$ dependant terms in Eqs.(6)-(9). As lattice results are not available for the range of values of $\mu$ relevant to us which could constrain these terms, we use Eqs.(6)-(9) for our order of magnitude estimates. For $\mu = 0$, lattice results indicate small values of surface tension. With that, supercooling will be even smaller than used in our paper. As we have discussed below, this does not affect our conclusions (as long as the transition still remains first order), as with smaller supercooling, our results should apply even for baryon inhomogeneities of smaller magnitude.
III. FIRST ORDER QCD PHASE TRANSITION.

We now briefly describe the time and temperature scales involved in a first order QCD phase transition in the early universe as has been discussed extensively in the literature. A first order phase transition proceeds with the formation of bubbles of hadronic phase within the QGP phase. These bubbles then expand and gradually convert all the QGP phase to the hadronic phase. Since the critical size of the bubbles is too large near $T_c$, the universe has to supercool slightly below the critical temperature for the rate of nucleation of bubbles to be adequate. Essentially, the rate of bubble nucleation should become significant compared to the expansion rate of the universe. The amount of supercooling required and its duration depends on quantities such as the latent heat of the transition $L$, the surface tension $\sigma$ of the interface etc. Using results from lattice calculations, $[3,4,17]$ ($L \sim 4T_c^4$ and $\sigma \sim 0.015T_c^3$) we can estimate that the amount of supercooling $\Delta T_{sc} \equiv \frac{T_c - T_{sc}}{T_c}$ required for the nucleation of bubbles is $[3]$ of the order of $\sim 10^{-4}$. We mention here that it has been argued in the literature that the amount of supercooling may be smaller by many orders of magnitude $[18]$. As we will see later, for smaller supercooling our results imply that the quark-hadron transition can be affected by baryon inhomogeneities of even much smaller magnitudes. After nucleation, the bubbles begin to expand releasing latent heat which reheats the universe back to the critical temperature. This happens in a very short time and temperature interval and bubble nucleation is shut off after that. The time scale for this is given by $\delta t \sim 10^{-5}t_H$ (where $t_H \sim 10^{-6}$ sec is the Hubble time at QCD transition) and the temperature interval is given by $\Delta T_n \sim 10^{-6}$ (see also ref. $[3,4]$). No nucleation of bubbles can happen after this and the transition proceeds by the very slow expansion of the already nucleated bubbles as the universe expands and cools, gradually transforming the QGP phase to the hadronic phase. This phase of slow expansion is usually referred to as the “slow combustion phase” $[1]$. It is this phase which is very different in models where there is an inhomogeneous nucleation of bubbles. Evidently some parts of the universe enter the slow combustion phase before other parts. If a large portion of the universe is in the slow combustion phase, then the large amount of latent heat generated by the expansion of bubbles prevent nucleation of bubbles in the other parts (which have not entered the slow combustion phase as yet). This modifies the nature of the phase transition drastically.

We again emphasize that it is likely that the rate of bubble nucleation in the baryon overdense region will itself be different from the rate of bubble nucleation in the outside regions even with similar factor of supercooling etc. This in principle could even reverse the sequence of transitions in the two regions. However, for simplicity we ignore this possibility in the present work, and only focus on the temperature differences between the two regions and resulting differences in the onset of bubble nucleation. We also assume that the supercooling required for starting off the phase transition in the QGP phase is of the same order of magnitude for different chemical potentials in our case. A more thorough investigation will have to include both these considerations.
IV. EFFECT OF THE INHOMOGENEITIES ON THE PHASE TRANSITION.

Now we discuss in detail how exactly the phase transition is affected by the presence of the baryon inhomogeneities. We consider the stage when the background region surrounding the inhomogeneities reaches sufficient supercooling for bubble nucleation to start. For bubble nucleation to start in a baryon dense region, it should also achieve sufficient supercooling. We consider the situation when the baryon inhomogeneities are at least of order, $\frac{n'_b}{n_b} = 10^7$. This is because only for such magnitudes of baryon inhomogeneities, we find a significant effect on the dynamics of quark-hadron transition. (Note that this is for $\Delta T_{sc} \sim 10^{-4} T_c$. For smaller values of $\Delta T_{sc}$, as in ref. [18], similar effects will be found for even smaller values of $\frac{n'_b}{n_b}$.) We will later briefly discuss how such inhomogeneities could possibly arise. With $T \sim 170 \text{MeV}$, the value of $\mu$ (using Eq.(5)), for $\frac{n'_b}{n_b} = 10^7$ comes out to be $\mu \sim 14 \text{MeV}$. Compared to this, the value of $\mu$ outside is $\sim 10^{-6} \text{MeV}$. We have taken the background value of $\eta \sim 7 \times 10^{-10}$. The value of critical temperature $T_c$ at zero chemical potential is $T_c \simeq 172 \text{MeV}$ (From Eqs.(6)-(9)).

Using the values of $\mu$ corresponding to the values of $n_b$ and $n'_b$, we can calculate the corresponding values of the critical temperatures using Eq.(6). For $\mu = 10^{-6} \text{MeV}$ corresponding to the background, the fractional change in the value of critical temperature (compared to the zero chemical potential case) is $\Delta T_c / T_c \ll 10^{-5}$. As we have mentioned before, values of fractional temperature differences which are relevant for the phase transition are of order $10^{-6} - 10^{-4}$ Therefore, the change in the critical temperature for the background chemical potential is negligible and the process of phase transition in the regions outside the baryonic lumps remains unaffected. As the background regions supercool sufficiently to the required temperature $T_{sc}$ (with $\frac{T - T_{sc}}{T_c} \simeq 10^{-4}$), the region inside the baryonic lump also cools by approximately same factor. At any stage, the difference between the background temperature $T$ and the temperature $T'$ inside the lump is given by Eq.(4). With the values of the chemical potential given above, and for temperatures close to $T_c$, we get,

$$
\frac{T' - T}{T} \equiv \Delta T' \simeq -4 \times 10^{-5}.
$$

(10)

At the same time the critical temperature for the phase transition to occur in such a lump is given by $T'_c$ where, using Eq.(6),

$$
\frac{T'_c - T_c}{T_c} \equiv \Delta T'_c \simeq -4 \times 10^{-5}
$$

(11)

Thus the temperature difference between the lump and the background is essentially the same as the difference in the values of $T_c$.

Assuming similar supercooling factor of $10^{-4}$ for the baryonic lump, bubble nucleation there cannot start until the temperature drops to a value $T'_{sc} \simeq (1 - 4 \times 10^{-5} - 10^{-4}) \times T_c$. Here, the factor of $4 \times 10^{-5}$ comes from Eq.(11), while the other factor of $10^{-4}$ arises from the requirement of sufficient supercooling for bubble nucleation.

Using the relationship between $T_c$ and $T'_{sc}$ we get,

$$
T'_{sc} = (1 - 4 \times 10^{-5}) \times T_{sc}
$$

(12)
Thus the difference between $T_{sc}'$ and $T_{sc}$ is of the same order as the temperature difference between the lump and the background given by Eq.(10). For simplicity we assume that nucleation rates are not significantly affected by the relatively large value of $\mu$ inside the lump. Thus, one will conclude that when background temperature reaches the value $T_{sc}$, the temperature in the lump may also be close to the corresponding supercooling temperature $T_{sc}'$ for the lump. We mention here again that it is entirely possible that nucleation rates change significantly [19] as a function of $\mu$, which can strongly affect our discussion. We hope to discuss this issue in a future work.

The discussion above neglects one important factor, whose effect is to delay the phase transition process in the baryonic lumps as explained below. Since the dense region has a lower temperature than the outside, neutrinos will be constantly pumping in heat which will keep on raising the temperature of the lump. This will temporarily increase the temperature of the lump until the pressure equilibrium relaxes the lump, thereby decreasing its temperature again in accordance with Eq.(3). However, the lump will require a certain amount of time to regain its pressure equilibrium. When the outside temperature reaches $T_{sc}$ then even though we are allowing for the possibility that the lump also could reach the corresponding supercooling temperature $T_{sc}'$, it will be possible only when the lump maintains pressure equilibrium with the surroundings. Until pressure equilibrium is achieved, the temperature of the lump will rise above the value $T_{sc}'$. This implies that during a period until pressure equilibrium is re-established, no nucleation of bubbles will be possible in the overdense region.

The timescale for the lump to attain pressure equilibrium will be of order $\Delta t_p = R/c_s$, where $c_s$ is the sound velocity and $R$ is the size of the lump. If we take the size of the lumps $R \sim 1$ cm, then with the sound speed $c_s = \frac{1}{\sqrt{3}}$, the time for attaining pressure equilibrium will be $\Delta t_p \sim 6 \times 10^{-11}$ sec. This is larger than the time taken to complete the bubble nucleation process in the outside region $(\Delta t_n \sim 10^{-5}t_H = 10^{-11}$ sec). Since the temperature of the lump will be temporarily increased during this time period, the bubble nucleation process may not start inside the lump, depending on the magnitude of this temperature increase. Note that as we are interested in the temperatures very close to the critical temperature, the relevant sound speed will typically be much smaller than the value $\frac{1}{\sqrt{3}}$. The duration $\Delta t_p$ for which the bubble nucleation will be delayed inside the lump should therefore be much longer than the value given above. In fact as shown in ref. [20], the sound speed near $T_c$ can become very small (e.g. even smaller than 0.2), whereby the time for attaining pressure equilibrium will become at least $10^{-10}$ sec, which is much larger than the time taken to complete the bubble nucleation process in the region outside. Also note that heat deposited in the lump is carried by relativistic neutrinoes so the timescale for neutrinoes to pass through the lump always remains much shorter than the time scales discussed here [13].

We now obtain the temperature rise due to the heat deposited in the lump for a time duration $\Delta t_n$ which, as we have discussed above, is the timescale of bubble nucleation in the surrounding region $(\Delta t_n = 10^{-5}t_H)$.

The heat deposited by the neutrinos in a given volume depends on the size of the lump $R$ compared to the neutrino mean free path $\lambda$ at that temperature. Since the neutrino mean free path around the quark-hadron transition is few cms, hence in this case $R \sim \lambda$. For
$R \sim \lambda$, the neutrino radiation can be approximately considered to be perfectly absorbed throughout the lump [21]. Hence we have,

$$\frac{dE}{dt} \simeq 4\pi R^2 \Phi$$  \hspace{1cm} (13)$$

where $\Phi$ is the net energy flux into the lump.

$$\Phi = \sum_i \rho_i(T) \frac{\delta T}{T},$$  \hspace{1cm} (14)

$\rho_i(T)$ being the energy density of the neutrinos, ($i$ representing the type of neutrinos) and $\frac{\delta T}{T}$ is given by equation (4). Keeping the volume ($V = \frac{4}{3}\pi R^3$) constant, the temperature rise is given by,

$$V g_{eff} a^4 T^3 \frac{dT}{dt} = 4\pi R^2 g_{\nu} a T^4 \times 3 \times 10^{-19} \left(\frac{n_b'}{n_b}\right)^2$$  \hspace{1cm} (15)$$

Here $g_{\nu}$ is the effective degrees of freedom for the neutrinos and is taken to be $6 \times \frac{7}{8}$. On the right hand side of equation (15), we have contribution only from the neutrinos because they are the only particles depositing heat in the lumps. However this heat is absorbed by all the particle species in the lump, hence on the left hand side of Eq.(15) we have considered all the particle species. Substituting all the values we get,

$$\frac{dT}{T} = 0.4 \times 10^{-19} \left(\frac{n_b'}{n_b}\right)^2 \times \frac{dt}{R}$$  \hspace{1cm} (16)$$

For $R = 1$ cm and $dt = \Delta t_n = 10^{-11}$ sec we get,

$$\frac{dT}{T} \sim 10^{-20} \left(\frac{n_b'}{n_b}\right)^2 > 10^{-6} \quad \text{for} \quad \frac{n_b'}{n_b} > 10^7$$  \hspace{1cm} (17)$$

We note that the temperature rise due to heat deposited in a given baryonic lump during the time interval $\Delta t_n$ is of the same order of magnitude as $\Delta T_n$ which is the temperature interval below $T_{sc}$ in which bubble nucleation process shuts off. This implies that while outside region reaches it’s lowest temperature ($\Delta T_{sc} - \Delta T_n$) before it starts reheating due to latent heat release , the temperature inside the lump can barely reach down to $T'_{sc}$. We, therefore, conclude that inside a lump of size $R \simeq 1$ cm, and an overdensity corresponding to $\frac{n_b'}{n_b}$ of the order of $10^{-3}$, it is not possible to have any bubble nucleation when the surrounding region undergoes bubble nucleation. The region in the lump cannot reach its respective nucleation temperature while nucleation starts and completely shuts off outside. Expanding bubbles in the outside region will release latent heat, thereby raising the temperature of the outside region back to the critical temperature $T_c$. Importantly, heat transport by neutrinos will also further raise the temperature inside the lumps, implying that bubble nucleation will remain shut-off inside the lump while the outside region undergoes the slow combustion phase [1]. It is important to mention here that if smaller values of sound velociy are taken into account then even with smaller size lumps these conditions hold. With a smaller size lump (say $\sim 0.1$ cm) and smaller velocity of sound ($\sim 0.1$), $\Delta t_p$ remains of the same order,
so that the region will not be able to achieve pressure equilibrium in the interval $\Delta t_n$. But for a smaller size lump ($R \lt \lambda$), Eq.(16) will be modified by a factor of $\frac{R}{\lambda}$ on the right hand side as only this much fraction of neutrino energy will get deposited in the lump. However this does not change the value of $dT$ in Eq.(17) as can be seen by multiplying the R.H.S. of Eq.(16) by $\frac{R}{\lambda}$. It will depend on $\lambda$ instead, and for temperatures around 100 MeV we have $\lambda \approx 1$ cm. So for lumps with $R < \lambda$, we can reach the same conclusion as before.

There only remains the case of $R > \lambda$. For this case neutrino heat conduction is small [21], but the size of the lump being quite large, $\Delta t_p$ will be much greater than $\Delta t_n$. So the lump will be able to come to pressure equilibrium in a time which will be much larger than the time in which nucleation of bubbles is complete in the outside region. So we may conclude that essentially for any size lump there is no nucleation of bubbles possible within the overdense region.

Baryonic lumps having $n'_b$ of the order of $10^{-3}$ is the lower limit for lumps which would affect the QCD phase transition (again, for $\Delta T_{sc} \sim 10^{-4}$). If we consider lumps with higher overdensities, $\frac{\delta T}{T}$ in Eq.(3) will be larger. At the same time the critical temperature will also go down. Important thing is that due to larger $\delta T$, the heat deposition by the neutrinoes will increase (see Eq.(14)). However for the background region the amount of supercooling required and the temperature interval below $T_{sc}$ for bubble nucleation to shut off (i.e $\Delta T_{sc}$ and $\Delta T_n$) will remain the same. Therefore due to larger value of $dT$ in Eq.(17), it will be even more difficult for bubble nucleation to take place within these lumps. For example, if we have $n'_b = 10^8$, then $\frac{\delta T}{T} = -4 \times 10^{-3}$. But the main factor responsible for suppressing bubble nucleation in the overdense region is the heat deposited by the neutrinoes. This will now be much greater than the temperature interval of bubble nucleation ($\Delta T_n$). In this case the temperature increase due to the heat deposited by the neutrinoes ($\frac{dT}{T}$) is of the order of $10^{-4}$ which is greater than $\Delta T_n$ by two orders of magnitude. Hence there is no way that bubble nucleation can start in these overdense lumps. So if there are inhomogeneities present which have $n'_b \geq 10^{-3}$, there is every possibility that the whole phase transition will proceed by the inhomogeneous nucleation of bubbles in the various regions of different baryon densities. So the lumps tend to remain in the QGP phase while the rest of the universe undergoes the phase transition to the hadronic phase.

As first discussed in ref. [1] (see, also, ref. [2,22]), baryon number tends to remain in the quark-gluon phase rather than the hadronic phase where they are carried by the more massive hadrons. Since the baryon overdense region will be in the QGP phase while the outside region hadronizes, baryon number will tend to concentrate in this region. Thus, in our model, the inhomogeneity in the lump is not depleted as long as the quark-hadron transition continues because the baryon number keeps getting concentrated inside the remaining QGP regions which are the regions of baryonic lumps. The total amount of baryons which get concentrated inside such lumps will depend on the detailed geometry of bubble collisions and coalescence. For example, even in the regions outside the lumps, spherical QGP regions will form with increased baryon concentration due to QGP regions getting trapped in between coalescing bubbles. [1] It has been discussed in the literature that the largest separation [4,6] between these inhomogeneities which are formed during the quark - hadron transition is of the order of few centimetres for homogeneous nucleation of bubbles and of the order of a metre if inhomogeneous nucleosynthesis (due to temperature fluctua-
tions) is considered. [4] One thing that we have to consider is the fact that in our model, the pre-existing baryon inhomogeneities have a very high $n_b$. Hence the fraction of volume occupied by these inhomogeneities must be very small, otherwise most of the baryon number would anyway be concentrated in these regions. But again if the number of these pre-existing inhomogeneities be very few and far between then during the transition they would focuss very little of the total baryon number. To check whether it is okay to have such high inhomogeneities and also focuss a fairly large percentage of baryon number due to the previously discussed phenomenon, we make an estimate of the distance of separation $l$ and radius $R$ of the pre-existing inhomogeneities such that only a small fraction of the baryon number in the universe before the quark-hadron transition is concentrated in these regions.

The fraction of volume occupied by the high density regions is roughly given by $f_H = (\frac{R}{l})^3$. We may therefore write,

$$f_H n_H + (1 - f_H) n_L = n_b$$

(18)

Here, $n_H$ and $n_L$ are the baryon densities in the high density regions and low density regions respectively, while $n_b$ gives the total average baryon density. For $R \ll L$, we will have $f_H \ll 1$, hence, $n_L \simeq n_b - f_H n_H$. Since we have $n_b = 10^{-10} \times s$ and $n_H = 10^{-3} \times s$, if we want to have $n_L \sim 10^{-10} \times s$ we must have, $f_H < 10^{-7}$. So the condition for the pre-existing inhomogeneities to have a negligible effect on the total baryon number turns out to be,

$$\left(\frac{R}{l}\right)^3 \leq 10^{-7}.$$  

(19)

As we have discussed before, the size of the inhomogeneity can be taken to be as small as 0.1 cm, thus we get $l \geq 20$ cm. So it is possible to have pre-existing inhomogeneities with radius 0.1 cm and separated by a distance scale of 20 cm at the onset of the quark-hadron transition. Since there will be two processes going on simultaneously, focussing of baryon number in pre-existing inhomogeneities and generation of new regions of baryon overdense regions in the inter-bubble spacings, we compare the lengthscales involved in the two cases. In homogeneous bubble nucleation it is known that [4,6] the separation of baryon over-dense regions formed as the bubbles coalesce is few cms. This is not too small compared to the length scale $l \sim 20$ cm of pre-existing baryonic lumps. Further, with an even smaller velocity of sound (near $T_c$), we can even have smaller values of $R$ and smaller $l$. For example with $R \sim 0.01$ cm, we will get $l \sim 2$ cm. This is the same as the expected separation between the QGP droplets generated during the phase transition. In such situations at least 50% of the baryon number can get concentrated in these pre-existing inhomogeneities. Also note that the actual fraction of baryon concentration in the pre-existing inhomogeneities will depend on detailed geometry of bubble collision in the background region. It is possible that for larger values of $l$ also, a good fraction of baryons may get concentrated in these pre-existing lumps.

One more important thing to keep in mind is that the proton diffusion length scale (which is the dominant mechanism for erasing out inhomogeneities after neutrino decoupling) depends crucially on the amplitude of the overdensities. For amplitudes greater than 100 the proton diffusion length scale decreases rapidly, [6] going down to even 0.001 cm for amplitudes of the order of $10^6$. So even though the size of our pre-existing inhomogeneities will be much smaller compared to the the length of the newly generated inhomogeneities ($\sim 2$ cms) they will not be erased by proton diffusion because of their high amplitudes.
All this will only be possible if there are such large overdensities present just before the QCD phase transition. We now discuss some processes which may generate such overdensities before the QCD transition. However we mention here that the principal aim of our work was to show how large pre-existing baryon inhomogeneities may affect the dynamics of a first order quark-hadron transition. Whether such high overdensities exist or not is still not clear. As we have mentioned earlier, the inhomogeneities generated at an early stage may or may not survive until nucleosynthesis, but as shown in ref. [13], sufficiently high overdensities may remain more-or-less unchanged between the electroweak epoch and the quark-hadron transition. Most of these inhomogeneities usually do not survive up to the nucleosynthesis epoch if their lengthscale at any stage is less than the proton diffusion length. [6,13]. If such inhomogeneities are present then our work shows that they will definitely affect the dynamics of the QCD phase transition. We now suggest some possibilities as to how such inhomogeneities may be generated.

One of the processes through which one may be able generate such inhomogeneities is by electroweak strings. Electroweak strings, which may be formed during the electroweak phase transition, are unstable and are expected to decay rapidly. There is a lot of literature which discusses the generation of baryon numbers by electroweak strings [8–10]. The changing helicity of an electroweak string network can generate a baryon to entropy ratio of order $10^{-4}$, where $\epsilon$ is the CP violating factor [9]. Though this asymmetry is generated when the network consists of a large number of loops and gives the average over the horizon volume, there may be a possibility that over certain regions the baryon number concentration may be larger than at other places. This possibility will definitely arise when the electroweak strings are metastable and manage to survive to lower temperatures. Recently several authors have indeed discussed such a possibility. [23]. If these strings are stable up to around 1 GeV, then strings, which decay late, will typically be of large size and will generate overdensities of larger sizes, well separated and far from each other. However since the number of such large strings will be very small at later stages, the baryon number they generate will not affect the overall baryon-to-photon ratio. (The bulk of baryons will still be generated by small strings decaying right after the electroweak transition.) The larger strings decaying late will generate sharp peaks of overdensities in localized regions.

Another way in which electroweak strings may generate baryons is by their decay, especially in a magnetic field [10]. Large overdensities may be generated by these strings over small length scales ($\sim 10^{-10}$cms), in the presence of a magnetic field if they decay at the electroweak epoch itself. These densities will generally diffuse out to some extent but as shown in [13] they may not be wiped out completely. Again if such strings decay later in the presence of strong magnetic fields they may generate larger inhomogeneities locally. As we are interested only in local baryon inhomogeneities the volume suppression factor will not be of concern here.

There is also the possibility of large superconducting string loops (formed around the TeV scale) decaying into vortons and generating baryon number when these vortons subsequently decay. In these models the baryon asymmetry generated earlier is protected from sphaleron wash-out and released later when sphaleron processes fall out of equilibrium [11](i.e after the electroweak transition). Apart from strings, other defects like the domain walls also
generate baryon number [24]. Black holes evaporating between the EW epoch and the QCD phase transition are another source of generation of such baryon inhomogeneities [25]. Even a first order electroweak phase transition seeded by impurities can generate inhomogeneities of lengthscale $10^{-3}$ cm at the electroweak epoch, (which translates to about 1 cm at the QCD epoch) [6] but the amplitude of these inhomogeneities are only as high as $10^4$. Altogether we see that there are several possibilities in which large baryon inhomogeneities may be generated before the quark-hadron transition and we will postpone the details of the generation of such large inhomogeneities for a later work.

VI. CONCLUSION

In conclusion, we have studied the effect of pre-existing baryon inhomogeneities on the dynamics of a first order quark-hadron transition. Our studies show that though the temperature in the lumps of high baryon densities is lower than the outside temperature, the bubble nucleation temperature at these high density lumps is also lower, and due to heat deposition by neutrinos, the process of phase transition is delayed in these regions. To demonstrate this we have estimated the difference in the critical temperature between the outside region and the baryon rich region as also the temperature rise due to neutrino heat deposition. We show that the baryonic regions do not achieve enough supercooling for bubble nucleations to effectively start in these regions. (We mention here again that if smaller supercooling is required for nucleation of bubbles, e.g. as discussed in ref. [18], our scenario remains unchanged. In fact if supercooling required is much less than $10^{-4}$, our mechanism will also work for overdensities much smaller than $10^7$.)

These regions do not reach sufficient supercooling due to heat being continuously deposited in the lumps by neutrinos and the time scale for the lumps to inflate and achieve pressure equilibrium is at least of the same order of magnitude as the time required for bubble nucleation to be completely shut off in the outside region. Once bubble nucleation is completely shut off and the outside region has reached the slow combustion phase the latent heat released by the expanding bubbles prevents further nucleation of bubbles everywhere, including the baryon dense regions. The final result is that the bubble nucleation never takes place in the regions of baryonic lumps. Since baryon number tends to stay in the quark phase, the baryon number in the already overdense region increases as the region outside gets converted to the hadronic phase, and since the size of the inhomogeneity also increases due to neutrino inflation, we may get large baryon overdense regions at the end of the quark hadron transition. Thus we see that a smaller baryon inhomogeneity may lead ultimately to a much larger baryon inhomogeneity. This is very different from the conventional picture of the evolution of baryon inhomogeneities in the early universe where the region of inhomogeneity will anyway increase in size while the amplitude of the inhomogeneity decreases [13]. In our work we have shown that during the quark-hadron transition the inhomogeneity will still increase in size but its amplitude will not decrease, on the other hand its amplitude may increase substantially by the end of the transition. In our calculation we have neglected the difference in nucleation rates because of the difference in chemical potential in the baryon overdense regions. For a more complete study both the difference in nucleation rate and the amount of supercooling in the two regions need to be taken into
account. If the nucleation rate depends strongly on the chemical potential, (say, it increases with chemical potential) then as we have mentioned earlier, it may so happen that phase transition occurs in the inhomogeneities first and then in the rest of the region; in which case the inhomogeneities will not grow but will dissipate away. Recently there has been some lattice calculations of the QCD phase transition at finite $\mu$ and finite $T$. [26] Though for small values of $\mu$ and finite $T$ the transition is expected to be a crossover, however as $\mu$ increases there happens to be a critical $\mu$ beyond which it becomes a first order transition. The endpoint is still at too large a value of $\mu$ but it is expected to move closer to the $\mu = 0$ value for more realistic quark masses. Also we mention that it is possible that the value of $\mu$ in the baryon inhomogeneities may be greater for large overdensities.

We have also briefly mentioned various possibilities about the generation of such high overdensities. There are many ways by which localized regions of high densities may be generated between the electroweak epoch and the quark hadron transition. We have not given any specific estimates but have commented on a variety of models dealing mostly with topological and non-topological defects. In a later work we hope to address this issue in more detail.

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