Remarks on unsharp quantum observables, objectification, and modal interpretations

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1 Introduction

In this contribution I wish to pose a question that emerged from discussions with Dennis Dieks regarding my talk at the Utrecht Workshop. The question is whether a variant of a modal interpretation is conceivable that could accommodate property ascriptions associated with nonorthogonal resolutions of the unity and nonorthogonal families of relative states as they occur in imperfect or genuinely unsharp measurements. To explain this question I will review a recent formulation of the quantum measurement problem in the form of an insolubility theorem that incorporates the case of unsharp object observables as well as certain types of unsharp pointers. In addition to demonstrating the necessity for some modification of quantum mechanics, this allows me to specify the logical position of the modal interpretations as a resolution to the measurement problem and to indicate why I think their current versions are not yet capable of dealing adequately with unsharp quantum observables. The technical tools that will have been explained along this line of reasoning will finally serve to make precise the notion of (unsharp) value ascription that I would find desirable for a modal interpretation to

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ascertain. I will begin with summarising the essence of unsharp quantum observables, pointing out their ubiquity and inevitability. In turn I hope to indicate how unsharp observables might contribute to avoiding some of the difficulties encountered in the present modal interpretations.

2 Unsharp quantum observables

The starting point for the present considerations is the fact that measurable quantities are to be represented in quantum mechanics as *positive operator valued (POV) measures*, which include the more familiar spectral measures and the associated self-adjoint operators as special cases corresponding to fairly idealised situations. The motivations for this extended notion of observables have been reviewed in great detail in [1]. Suffice it to recall here that a POV measure arises naturally as the representation of the totality of statistics of measurement outcomes for all states of the measured system. The idea underlying this characterisation of an observable is this: given an observable, the quantum formalism assigns to every state a probability measure describing the outcome statistics for that observable, if measured on that state. Conversely, if the statistics of a given type of measurement are known for a sufficient number of states (ideally for all states), then it will be possible to infer uniquely the observable that has been measured.

This minimal notion of an observable – and of a measurement – is captured in the so-called *probability reproducibility condition*. The essential elements of a measurement are conveniently summarised in the concept of a *measurement scheme*, represented as a quadruple $\mathcal{M} := \langle \mathcal{H}_A, \rho_A, U, Z \rangle$, where $\mathcal{H}_A$ denotes the Hilbert space of the measuring device (or probe) $A$, $Z$ the pointer observable of $A$, i.e., a POV measure on some measurable space $(\Omega, \Sigma)$, $\rho_A$ a fixed initial state of $A$, and $U$ the unitary measurement coupling serving to establish a correlation between the object system $S$ (with Hilbert space $\mathcal{H}$) and $A$. Any measurement scheme $\mathcal{M}$ fixes a unique observable of $S$, that is, a POV measure $E$ on $(\Omega, \Sigma)$ such that the following condition is fulfilled:

- **probability reproducibility condition:**

$$\text{tr}[I \otimes Z(X) U \rho \otimes \rho_A U^*] = \text{tr}[E(X) \rho] \quad \text{(PR)}$$

for all states $\rho$ of $S$ and all outcome sets $X \in \Sigma$. 

\[2\]
$E$ is the observable measured by means of $\mathcal{M}$. Conversely, if an observable $E$ of $\mathcal{S}$ is given, then this condition determines which measurement schemes $\mathcal{M}$ serve as measurements of $E$.

Given an observable $E$ on $(\mathbb{R}, \mathcal{B}(\mathbb{R}))$ [where $\mathcal{B}(\mathbb{R})$ is the real Borel algebra] and a state $\rho$, one can compute the moments of the statistics, $E^{(n)}_\rho := \int x^n \text{tr}[\rho E(dx)]$ (when they exist), and these may lead to the definition of the moment operators $E^{(n)} := \int x^n E(dx)$ (provided that suitable domains do exist). These operators will be symmetric but not in general self-adjoint. If $E$ is not a PV measure, then one will not have the relation $E^{(n)} = E^{(1)n}$. This shows that a general POV measure cannot be represented in general by a single operator but only (if at all) by an infinity of operators whose expectations would give the moments and thus reproduce the statistics. In other words, the representation of an observable by means of a single (self-adjoint) operator is a rather accidental exceptional case, and in the general case the full information available in a measurement is to be encoded in a POV measure.

Deviations from projection valued (PV) measures occur for two distinct kinds of reasons. First, POV measures may account for the ever-present imperfections of any measurement. Second there are measurement situations for which there exists no ideal background observable representable as a PV measure. Accordingly, POV measures which are not PV measures are referred to as unsharp observables while PV measures correspond to sharp observables. As an example we may mention that there exists a class of POV measures that can be considered as genuine phase space observables in every physically relevant respect. In particular they are covariant under the Heisenberg group, a feature that is lacking, for instance, in von Neumann’s construction (Section V.4 of [2]) of an approximate joint observable for position and momentum which is based on sharp observables. Such phase space observables are genuinely unsharp, a reflection of the fact that the individual measurement outcomes are fuzzy phase space points in accordance with the Heisenberg uncertainty relation. Moreover, the phase space POV measures seem to provide the appropriate tool for describing macroscopic pointers and their quasi-classical measurability (cf. Chapter VI of [1]). It has been argued on general consistency grounds that macroscopic observables displaying permanent definite values are to be described by POV measures that are not PV measures. More precisely, the notion of quantities having definite values at all times, as it is described in classical theories typically applying to
macroscopic objects, can only be represented approximately within a quantum mechanical many-body theory, and the quantum representatives of such quantities are non-PV POV measures.

3 The objectification problem

The probability reproducibility condition specifies what it means that a measurement scheme serves to measure a certain observable. However, this condition does not exhaust the notion of measurement. In fact the reproduction of probabilities in the pointer statistics requires first of all that in each run of a measurement a pointer reading will occur; in other words: it is taken to be part of the notion of measurement that measurements do have definite outcomes. While the concept of a measurement scheme allows one to describe what happens to the object and apparatus when an outcome arises, quantum mechanics is facing severe difficulties to explain the occurrence of such outcomes. This problem arises if one starts with the interpretational idea that an observable has a definite value when the object system in question is in an eigenstate of that observable. If a probe system is coupled to that object, then probability reproducibility requires that the corresponding value is indicated with certainty by the pointer reading after the measurement interaction has ceased. In this way a definite value of the object observable leads deterministically to a definite value of the pointer observable. However, if the object is not in an eigenstate, the observable cannot be ascertained to have a definite value, and by the linearity of the unitary measurement coupling, the compound object plus probe system ends up in a state in which it cannot be ascertained, by virtue of the eigenstate-eigenvalue rule, that the pointer has a definite (sharp) value. This is the measurement problem, or the problem of the objectification of pointer values.

Resolutions to this problem are being sought by changing the rules of the game: either on the side of the formalism – introduction of classical observables, or modified dynamic –, or on the interpretational side – hidden variables theories such as ‘Bohmian mechanics’, or various ‘no-collapse’ interpretations. Before embarking on such radical revisional programmes, it seems fair to make sure that the measurement problem is not merely a consequence of overly idealised assumptions that would disappear in a more realistic account. It turns out, however, that the problem does persist even
when measurements are allowed to be inaccurate and the measuring system is in a mixed rather than a pure state. The development of these arguments is reviewed in [4], where an insolubility theorem is given that pertains to measurements of sharp and unsharp object observables. This result has recently been overtaken by H. Stein who showed that the objectification problem persists for arbitrary measurement schemes even when the measurement is not applicable to all object preparations but only to states in some subspace [5].

A final step can be taken if the pointer observable is also allowed to be unsharp. As long as the pointers are still such that they will assume definite sharp values, the statement of the insolubility theorem still holds true [6].

In order to give the precise statement of the insolubility theorem, let us consider a measurement scheme $\mathcal{M}$. The theorem is based on the following requirements as necessary conditions for the objectification of sharp values of the pointer $Z$ in the postmeasurement state $\rho'_{SA} \equiv U \rho_S \otimes \rho_A U^*$:

- **Pointer mixture condition:**
  \[
  \rho'_{SA} = \sum_i I \otimes Z(X_i)^{1/2} \rho'_{SA} I \otimes Z(X_i)^{1/2} \equiv \sum_i \rho'_{SA}(X_i) \quad \text{(PM)}
  \]
  for some partition $\Omega = \cup X_i$ and all initial object states $\rho$;

- **Pointer value definiteness:**
  \[
  \text{tr}[I \otimes Z(X_i) \rho'_{SA}(X_i)] = \text{tr}[\rho'_{SA}(X_i)] \quad \text{(PVD)}
  \]
  for all $i$ and all initial object states $\rho$.

For a derivation of these conditions, see [4]. The first says that the postmeasurement state should be a mixture of pointer eigenstates, while the second requires that the final states conditional on reading a result in $X_i$ are indeed eigenstates of the pointer for which $X_i$ has probability one to occur again upon immediate repetition of the reading of the pointer observable $Z$.

The insolubility theorem states: if a measurement scheme $\mathcal{M}$ fulfills (PM) and (PVD), then the measured observable $E$ according to (PR) is trivial; that is, $E(X) = \lambda(X)I$ for all $X \in \Sigma$, where $\lambda$ is a state-independent probability measure on $(\Omega, \Sigma)$. Hence if a measurement scheme is to lead to objective pointer values, it will yield no information at all about the object.
There remains then a last resort to genuinely unsharp pointers: at present it is not clear if a notion of ‘unsharp objectification’ (made precise in [8]), obtained by dropping (PVD), would allow to evade the measurement problem. I suspect it would not. Then one would be forced to consider one of the options: giving up the axiom of linear (unitary) dynamics; considering quantum theories allowing for (some) classical observables (for measuring systems); or considering modified notions of the objectivity, or definiteness, of (pointer) values. Here I will consider briefly the last possibility which amounts to questioning (PM) and pursuing instead some ‘no-collapse’ interpretation of quantum mechanics.

4 Modal interpretation for unsharp measurements?

The question I want to raise in this concluding section is whether a viable version of modal interpretation could be developed that would provide an adequate account of what is happening to an object system in the course of an unsharp measurement. In view of the fact that virtually any measurement is imperfect so that the measured observable is unsharp, it appears to me that in the current versions of modal interpretations too much emphasis is put on the idealised case of sharp measurement of discrete observables and the ensuing biorthogonal decompositions. I would imagine the following as a promising alternative scenario: It appears to be the case that some observable of the measuring system is ‘naturally’ singled out as a suitable pointer observable whose readings indicate firstly the value of the measured observable and secondly what can be known about the object after the measurement. As a matter of fact, experimenters seem to be bound (by Nature?) to take resort to certain quantities as indicators of measurement outcomes. Such quantities will on the one hand have definite values at (practically) all times; but on the other hand they will be sufficiently sensitive to small-scale influences so as to be able to probe a microscopic system. So if the state $\rho_{S'A}$ represents the compound system after the coupling, and if (PM) is not stipulated, then – given that this state goes along with the occurrence of a definite pointer reading – a simple and unique prescription for expressing what happened to the object and apparatus (or probe) would be to assign to them the states
conditional to a particular pointer reading:

\[ \rho'_S(X_i) \equiv \text{tr}_A[\rho'_{SA}(X_i)]; \quad \rho'_A(X_i) \equiv \text{tr}_S[\rho'_{SA}(X_i)]. \]

Here \( \text{tr}_A[\cdot] \) denotes the partial trace with respect to the apparatus Hilbert space, etc. The ensuing state pairs \( \rho'_A(X_i) \) and \( \rho'_S(X_i) \) would not in general form a biorthogonal family; but it seems worthwhile to investigate whether they can be consistently regarded as the actual physical states of the probe and object after the measurement interaction. It may be noted that for sharp pointers, \( \text{(PVD)} \) will be automatically satisfied and implies the mutual orthogonality of the conditional pointer states. Then a necessary and sufficient condition for the conditional object states to be orthogonal is the following pointer mixture condition for \( \mathcal{A} \):

\[ \text{tr}_S[\rho'_{SA}] = \sum \rho'_A(X_i) \]

for all \( \rho \). This is to say that the reduced apparatus state assumes the desired form of a mixture of definite pointer states. See Theorem 3.11 of [8]. The “modal rule” formulated above seems to be quite in line with the experimenters’ actual practice: the said conditional states do represent the predictive content of the situation reached after a definite measurement outcome has been recorded. I must leave it open whether the rule given does amount to a proper interpretation in the sense of internal consistency.

I will sketch two simple idealised models involving discrete observables in order to illustrate the state assignment rules proposed here. This will also make it evident that the ensuing kind of modal interpretation that could accommodate these rules will deviate from the existing variants.

The first model describes an imperfect measurement of a sharp observable where the imperfection lies in the unitary coupling not forcing the object system into an eigenstate of the measured observable. Consider an object described by a 2-dimensional Hilbert space \( \mathcal{H} \), and let \( \varphi_1, \varphi_2 \) be the two orthogonal normalised eigenvectors of the observable to be measured. Let \( \xi_1, \xi_2 \) be two other, nonorthogonal unit vectors in \( \mathcal{H} \). Let \( \mathcal{H}_{AE} \) denote the Hilbert space of the apparatus (including microscopic probes) plus environment (including observers). We assume the initial state of \( \mathcal{AE} \) to be represented by a unit vector \( \phi \in \mathcal{H}_{AE} \), while the final state is described in terms of two orthogonal unit vectors \( \phi_1, \phi_2 \in \mathcal{H}_{AE} \). The role of the environment is to ensure the selection of the pointer states which in this example are supposed to be
mutually orthogonal. It is not hard to show that the following allows an 
extension to a unitary map on the compound Hilbert space: for any initial 
state $\psi = c_1\phi_1 + c_2\phi_2 \in \mathcal{H}$, let

$$\Psi := \varphi \otimes \phi = (c_1\phi_1 + c_2\phi_2) \otimes \phi \rightarrow \Psi' := c_1\xi_1 \otimes \phi_1 + c_2\xi_2 \otimes \phi_2.$$  

The reading of outcomes would be carried out by looking at the positions 
of pointers. This may be modelled by two mutually orthogonal projections 
$P_1, P_2$ on $\mathcal{H}_{AE}$ that are such that $\langle \phi_i | P_k \phi_i \rangle = \delta_{ik}$. Then the measured ob-
ervable according to the probability reproducibility condition (PR) is the 
projection valued measure with range $P_{\phi_1}, P_{\phi_2}$. According to the above rule, 
the final object state conditional on a reading $P_k$ is given by $\rho_{S,k} = P_{\xi_k}$. 
It is clear that the biorthogonal decomposition of $\Psi'$ involves quite dif-
cent component states than $\xi_i, \phi_i$. This example shows that even for sharp 
measurements there are situations where one would wish to allow state at-
tributions from a nonorthogonal family of states. It may be noted that this 
model is an example of the statement of Theorem III.2.3.1 in [7].

The second example concerns a genuinely unsharp measurement, where 
the unsharpness arises from the nonorthogonality of probe states. The ex-
ample is modelled after the Stern Gerlach experiment. The object system 
will be described as above, but this time the compound system consisting 
of probe $A$ plus environment $E = B + C$ [screen ($B$) plus recording device ($C$)] will explicitly appear as a compound system, whose initial state will be 
taken to be $\phi \otimes \theta$ for simplicity. We assume the measurement evolution takes 
place in three stages. Stage 1 is

$$\Psi := (c_1\varphi_1 + c_2\varphi_2) \otimes \phi \otimes \theta \rightarrow \Psi' := (c_1\varphi_1 \otimes \phi_1 + c_2\varphi_2 \otimes \phi_2) \otimes \theta.$$  

Unitarity is again easy to ensure. This time the probe states $\phi_1, \phi_2$ are 
assumed to be nonorthogonal. In the Stern Gerlach experiment, these states 
correspond to the spin-$\frac{1}{2}$ particle’s spatial wavefunctions correlated with the 
spin eigenstates $\varphi_1, \varphi_2$, and due to wave packet spreading they will have a 
spatial overlap and generally nonvanishing inner product. Stage 2 consists 
in the production of a scintillation in the upper or lower screen half, thus 
indicating which way the particle appears to have taken; we write this step 
for the two components of $\Psi'$ separately:

$$\varphi_1 \otimes \phi_1 \otimes \theta \rightarrow \Psi_1'' := \varphi_1 \otimes (\alpha_{11}\phi_{11} \otimes \theta_{11} + \alpha_{12}\phi_{12} \otimes \theta_{12}),$$

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\[ \phi_2 \otimes \phi_2 \otimes \theta \rightarrow \Psi'' := \phi_2 \otimes (\alpha_{21}\phi_{21} \otimes \theta_{21} + \alpha_{22}\phi_{22} \otimes \theta_{22}), \]
\[ \Psi'' := c_1\Psi''_1 + c_2\Psi''_2. \]

Here \( \theta_{11}, \theta_{21} \) represent states of the environment corresponding to sensitive molecules localised in the upper half of the screen, while \( \theta_{22}, \theta_{12} \) correspond to molecules localised in the lower half. Thus, if the wave packet \( \phi_1 [\phi_2] \) were strictly localised in the upper [lower] screen region then only the component containing \( \theta_{11} [\theta_{22}] \) would emerge. But realistically, the tail of \( \phi_1 \) evolving to the lower screen half allows for a chance of the particle to interact with molecules of that "wrong" section. We assume the \( \theta_{ij} \) to be normalised and mutually orthogonal:
\[ \langle \theta_{ik} | \theta_{jl} \rangle = \delta_{ij}\delta_{kl}. \]

Further, we require \( \alpha_{ij} > 0, \alpha_{11}^2 + \alpha_{12}^2 = 1, \alpha_{21}^2 + \alpha_{22}^2 = 1 \). Then a unitary extension of this map is guaranteed to exist. We also assume that the vectors \( \phi_{ij} \) are normalised and mutually orthogonal, representing wave packets localised in the appropriate screen halves.

The third stage, the recording, is modelled by taking into account that the states \( \theta, \theta_{ij} \) are product states of the form
\[ \theta = \chi \otimes \psi, \quad \theta_{ij} = \chi_{ij} \otimes \psi, \]
where the first factor represents the states of the sensitive screen molecule, while the second factor is the state of the recording device (observer). Then after the completion of the second stage, the recording takes place in such a way that the states \( \theta_{ij} \) evolve independently of the object plus probe:
\[ \theta_{ij} \rightarrow \chi_{ij} \otimes \psi_j, \]
where \( \psi_1 [\psi_2] \) represents the state that the recording device (observer) has reached after registering a scintillation in the upper [lower] screen half. These states are assumed to be orthogonal. Hence,
\[ \Psi'' \rightarrow \Psi''' := [c_1\varphi_1 \otimes \alpha_{11}\phi_{11} \otimes \chi_{11} + c_2\varphi_2 \otimes \alpha_{21}\phi_{21} \otimes \chi_{21}] \otimes \psi_1 \]
\[ + [c_1\varphi_1 \otimes \alpha_{12}\phi_{12} \otimes \chi_{12} + c_2\varphi_2 \otimes \alpha_{22}\phi_{22} \otimes \chi_{22}] \otimes \psi_2 \]

This is conveniently described by projections \( P_1, P_2 \) acting in the screen Hilbert space, indicating whether a scintillation has occurred on the upper.
or lower screen half. Hence,
\[ \langle \theta_{11} | P_1 \theta_{11} \rangle = 1, \quad \langle \theta_{21} | P_1 \theta_{21} \rangle = 1, \quad \langle \theta_{12} | P_2 \theta_{12} \rangle = 1, \quad \langle \theta_{22} | P_2 \theta_{22} \rangle = 1. \]

Given that the actual state of the observer is one of \( \psi_1, \psi_2 \), the conditional final states of object and probe are given by the following equations:
\[
\langle \Psi^{\prime\prime\prime} | X \otimes I_A \otimes I_B \otimes P_{\psi_i} \Psi^{\prime\prime\prime} \rangle = \langle \Psi^{\prime\prime} | X \otimes I_A \otimes P_i |, \Psi^{\prime\prime} \rangle =: \text{tr}[\rho_{S,i} X],
\]
\[
\langle \Psi^{\prime\prime\prime} | I_S \otimes Y \otimes I_B \otimes P_{\psi_i} \Psi^{\prime\prime\prime} \rangle =: \text{tr}[\rho_{A,i} Y].
\]

Here \( X,Y \) are arbitrary bounded selfadjoint operators on the object and probe Hilbert spaces, respectively. It follows that the actual states of \( S \) and \( A \) conditional on a reading \( i \) are
\[
\rho_{S,i} = \alpha_{1i}^2 P_{\phi_1} + \alpha_{2i}^2 P_{\phi_2},
\]
\[
\rho_{A,i} = |c_1|^2 \alpha_{1i}^2 P_{\phi_1} + |c_2|^2 \alpha_{2i}^2 P_{\phi_2}.
\]

Notice that one would like to have \( \alpha_{11}^2 \approx 1 \approx \alpha_{22}^2 \) and \( \alpha_{12}^2 \approx 0 \approx \alpha_{21}^2 \), so that \( S \) would be nearly in one of the states \( \phi_i \) and \( A \) nearly in the corresponding state \( \phi_i \). But as it stands, both systems are in states which are proper quantum mixtures, that is, they do not allow an ignorance interpretation with respect to their pure components. This is due to the fact that according to our state ascription rule the system \( S + A + B \) is in one of the pure states strictly correlated with the states \( \psi_i \) of the registration device; and in the situation modelled here there is “nothing in the world” that could allow one to perform a pure state ascription to subsystems in these pure entangled states.

Finally we note that the observable measured in this model is given by
\[
\langle \Psi^{\prime\prime\prime} | I_S \otimes I_A \otimes I_B \otimes P_{\psi_i} \Psi^{\prime\prime\prime} \rangle = \langle \Psi^{\prime\prime} | I_S \otimes I_A \otimes P_i \otimes I_c \Psi^{\prime\prime} \rangle =: \langle \phi | E_i \phi \rangle.
\]

This gives the following effect operators:
\[
E_i = \alpha_{1i}^2 P_{\phi_1} + \alpha_{2i}^2 P_{\phi_2}.
\]

Under the above assumptions on the \( \alpha_{ij} \) the \( E_i \) would be nearly equal to \( P_{\phi_i} \). Notice that then \( E_i \) as well as \( P_{\phi_i} \) have high probabilities to occur in the object state \( \rho_{S,i} \); this is to say that operationally these positive operators correspond to some elements of unsharp reality [1].
These examples make it clear that the state ascription rule applied here relies on a hierarchical order of the observables: there need to be mechanisms by which some observables of macroscopic systems are selected by Nature to have definite values throughout and to be able to correlate with observables of microsystems so as to monitor their values and make them inherit their own definiteness. The case of a closed system is included as a limiting case: if an object system does not interact with the given macroscopic device or observer, its reduced state will always be the state that evolved from its original state due to its free evolution.

As a perhaps somewhat more realistic example of a measurement in which the final object states are not mutually orthogonal, one may think of a ‘preparatory’ phase space measurement in which the said reduced conditional object states correspond to situations in which the system is fairly well though only unsharply localised in some phase space cell indicated by the reading. This is to say that the particle in question has a strong tendency (high probability) to ‘show up’ in that region in a subsequent phase space measurement.

The quest for such a variation of a modal interpretation becomes even more urgent if one accepts the idea that macroscopic pointers are intrinsically unsharp observables themselves: in that case the pointer states are not mutually orthogonal either. [Notice that in the second example above the final probe states $\rho_{A,i}$ are not mutually orthogonal either.] One may model such a situation by describing the pointer again as a phase space observable – after all, pointers are objects localised in space and with fairly definite and controllable speeds. These quasi-classical features require unsharpnesses in positions and momenta that are huge compared to the scale given by Planck’s constant.

The kind of nonorthogonal state attribution, along with the unsharp value-attribution proposed here surely needs to be analysed in regard to its internal consistency. Also, one may ask by what mechanisms (if any) some specific unsharp macroscopic observables should be selected by Nature and enabled to serve as pointers. On the other hand it appears to me that the unsharpness involved could offer new flexibility in dealing with some of the difficulties encountered with current modal interpretations. For example, collections of unsharp observables can be coexistent – i.e., have joint distributions – even if they do no commute. This may help to avoid Kochen-Specker-type inconsistencies arising from simultaneous value assignments to
noncommuting quantities. And this could also have implications on the no-
torious degeneracies problem since it would be conceivable that the simulta-
neous (quasi-)diagonality of a mixed state in different bases corresponding to
noncommuting quantities is consistently interpretable as all these quantities
having definite (unsharp) values.

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