Encounter rates and transit time distributions for surfaces moving in turbulent flows

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Abstract. The motions of point particles and virtual surfaces are studied by use of data from direct numerical solutions of the Navier-Stokes equation. A selected particle within a reference volume with given form and size is moving self-consistently with the local flow velocity, thereby determining the motion of the entire surface. We estimate the encounter rates between particles in the surrounding flow and such surfaces. We find also the probability distribution of the transit times of particles through the reference volumes. The transit time is defined as the interval between entrance and exit times of surrounding particles convected through the volume by the turbulent motions. Scale sizes in the inertial as well as in the viscous subrange of the turbulence are considered. Simple, and seemingly universal, scaling laws are obtained for the encounter rate as well as the probability density of the transit times in terms of the basic properties of the turbulent flow and the geometry.

Reaction rates as relevant for many chemical processes as well as a number of biological applications depend on encounter rates of reacting elements and the probability of a reaction occurring given an encounter. Turbulent mixing can enhance the encounter rate, but it can also have adverse effects by reducing the reaction probability. We model the probability for a reaction by letting it depend on the time available, i.e. the time spent by of reacting particles within some given interaction volume. We study the problems by numerical simulations where we analyze the statistics of the motion of passively convected point particles in turbulent flows. The database used is obtained by direct numerical solution of the Navier-Stokes equation. We estimate the encounter rates and also the probability distribution of the transit times of particles through reference volumes with given forms and sizes. A selected position within the reference volume is moving self-consistently with the local flow velocity, thus determining the motion of the entire surface. The transit time is defined as the interval between entrance and exit times of surrounding particles convected through the volume by the turbulent motions. Scale sizes in the inertial as well as in the viscous subrange of the turbulence are considered. Simple, and seemingly universal, scaling laws are obtained for the encounter rate as well as the probability density of the transit times in terms of the basic properties of the turbulent flow and the geometry. In the present formulation, the results of the analysis are relevant for chemical reactions, but also for understanding details of the feeding rate of micro-organisms in turbulent waters.
1. Introduction

Reaction rates in turbulent flows are here modeled by following the motion of passively advected particles by numerical simulations of homogeneous and isotropic turbulent flows. The reaction rate $J_{\text{react}}$ contains two elements: encounter between particles and a subsequent reaction, written as

$$J_{\text{react}} = J_{\text{enc}} P(\text{react}),$$

where $J_{\text{enc}}$ is the encounter rate. The reaction probability $P(\text{react})$ depends basically on the time the two particles spend within a reaction volume. The accuracy of the postulated relation (1) can be tested numerically. The analysis outlined here has general interest, for chemical reactions, for instance. Here we emphasize a different application for understanding details of the feeding rate of micro-organisms in turbulent waters (Kiørboe, 2008). Analytical results (Rothschild & Osborn, 1988; Osborn, 1996) supported by observations (Sundby & Fossum, 1990; MacKenzie et al., 1994; Kiørboe & MacKenzie, 1995; MacKenzie & Kiørboe, 2000) indicate that turbulent mixing can be important for the encounter rate and thus for the feeding process of planktonic organisms (Mauchline, 1998). Strong turbulence will, on the other hand, have adverse effects by reducing the interaction time, in this case the time available for capture (Granata & Dickey, 1991; MacKenzie et al., 1994; Caparroy et al., 2000; Lewis & Pedley, 2001; Fiksen & MacKenzie, 2002).

As an illustration, we show in Figure 1 a diagram for the field of view for a copepod with a) being the standard spherical model with range $R$, and b) a more realistic conical view with opening angle $\theta$, where a hemispherical model with $\theta = \pi/2$ has received particular attention. The encounter volume is assumed to move due to the motion of the copepod. Encounter rates are obtained by counting prey crossing the surface of the volume. In the present study we assume both predators and prey to be moving passively with the local flow, but self induced motions can be considered as well (Pécseli et al., 2010).

We model the interaction probability, given an encounter, by use of the time available for interaction, here taken to be the transit time of a particle through the reference volume (Pécseli & Trulsen, 2010). An essential parameter for studies of this problem is consequently the probability distribution of the transit times $P_\tau(\tau)$ of particles through the prescribed volume. The transit time $\tau$ is defined as the difference between entrance and first exit times of surrounding particles advected through this volume by the turbulent motions. Empirical models for the transit time probability densities are obtained in terms of a universal scaling for inertial as well as viscous subrange for two different shapes of the reference volume, spheres and hemispheres. A simple model suggests that capture or interaction requires a minimum time $\Delta \tau$. This implies that $P(\text{react})$ is proportional to the integral $\int_{\Delta \tau}^{\infty} P_\tau(\tau) d\tau$ of the transit time probability density $P_\tau(\tau)$. This integral is related to the cumulative distribution of transit times.
2. Dimensional arguments

Results for the encounter rates in turbulent environments were obtained by dimensional reasoning (Mann et al., 2005; Boffetta et al., 2006; Pécseli & Trulsen, 2007) and the same type of analysis can be applied with confidence also to the transit time distributions, $P_\tau(\tau)$, in terms of the relevant physical parameters. In Table 1 we show the parameters entering the relevant version of Buckingham’s II theorem for the transit time probability density $P_\tau(\tau)$.

Table 1. Buckingham’s II matrix for discussing transit-time probabilities, with the exponents for the physical dimensions of characteristic quantities in terms of length $L$ and time $T$ (Buckingham, 1914).

| $\nu$ | $\epsilon$ | $P_\tau(\tau)$ | $R$ | $\tau$ |
|-------|-------------|-----------------|-----|-------|
| $L$   | 2           | 2               | 0   | 1     |
| $T$   | -1          | -3              | -1  | 0     |

2.1. Inertial limit

First, we formally write an expression

$$f_\tau(\nu, \epsilon, P_\tau(\tau), R, \tau) = 0,$$

(2)

relating the specified variables already defined. Since we have not said anything concerning the functional form of $f_\tau$ this statements is close to trivial, except for the fact that we have indicated which parameters may be relevant. There may be other relevant parameters, the size of the system for instance, but we assume that these outer scales $L_o$ are so large that a range of $R$ exists where $P_\tau$ is independent of $L_o$. The point is now that we can express these variables (here $\epsilon, P_\tau(\tau), R, \tau$ and $\nu$) in any units we want, lengths can be measured in meters, inches, or if we like a quantity characterizing the turbulence. The form of the unknown function $f_\tau$ must be such as to accommodate any choice of units. These arguments are made systematic by use of Buckingham’s II-theorem (Buckingham, 1914; Kurth, 1972).

In Table 1 we emphasize the physical dimensions of relevant quantities: we have, for instance, for the kinematic viscosity the physical dimension $[\nu] = L^2 T^{-1}$, giving the corresponding numbers for the exponents as 2 and -1 in Table 1. Here $L$ and $T$ represent the length and time units chosen. The table represents a $n \times m = 2 \times 5$ matrix with rank $\kappa = 2$ (Cramér, 1946). The dimension mass is redundant in the matrix, since we for incompressible fluid dynamics introduce the kinematic viscosity $\nu$ and the energy dissipation rate $\epsilon$ per unit mass of fluid.

For $f_\tau$ to be physically acceptable, it must be possible to express it in terms of dimensionless products of the variables involved, that is, products of the form $\pi = \nu^{\alpha_1} \times \epsilon^{\alpha_2} \times P_\tau^{\alpha_3} \times R^{\alpha_4} \times \tau^{\alpha_5}$, where the exponents $\alpha_k$, with $k = 1, \ldots, 5$, are chosen to make $\pi$ dimensionless. For instance, if the expression for $f_\tau$ were to contain a nonlinear function, like exp, then the argument of this function has to be dimensionless. For the present case, $m - \kappa = 3$ linearly independent products can be formed. In terms of these “Buckingham $\pi_j$’s”, with $j = 1, \ldots, m - \kappa$ we can write an expression equivalent to $f_\tau$ as $g_\tau(\pi_1, \pi_2, \pi_3) = 0$, with $g_\tau$ unspecified so far. The terminology using the notation $\pi_j$ is universally adopted in this context, and should not be confused with the number $\pi \approx 3.14\ldots$. By Table 1 we have that $(L^2/T)^{\alpha_1} \times (L^2/T^3)^{\alpha_2} \times (1/T)^{\alpha_3} \times L^{\alpha_4} \times T^{\alpha_5} = 1$, leading to two constraints on the choice of the exponents $\alpha_k$, i.e. $2\alpha_1 + 2\alpha_2 + \alpha_4 = 0$ and $-\alpha_1 - 3\alpha_2 - \alpha_3 + \alpha_5 = 0$. We make use of the freedom of choice to make sure that the desired
variable, here $P_\tau$, only appears in one of the three dimensionless products. We choose $\alpha_3 = 1$ in one of the combinations, and $\alpha_3 = 0$ in the others. We then find
\[ g_\tau \left( \frac{R^{2/3} P_\tau(\tau)}{\epsilon^{1/3}}, \frac{\tau^{1/3}}{R^{2/3}}, \frac{Re^{1/4}}{\nu^{3/4}} \right) = 0, \tag{3} \]
in terms of linearly independent products. For the inertial subrange considered here, the kinematic viscosity does not enter the problem, so we can discard the last dimensionless product in (3). We can then determine the dimensionless parameter combinations to find the transit time probability density in the form
\[ P_\tau(\tau) = \frac{\epsilon^{1/3}}{R^{2/3}} F_\tau \left( \frac{\epsilon^{1/3}}{R^{2/3}} \right), \tag{4} \]
with $F_\tau$ being an unknown dimensionless function. The parameter scaling suggested by (4) is confirmed by analysis of data from laboratory experiments (Jørgensen et al., 2005) as well as numerical simulations (Pécseli & Trulsen, 2010). In particular, the expression given in (4) is not restricted to spherical forms of the interception volume: the shape enters through the functional form of $F_\tau$, but the parameter combination is the same for all shapes with scale size $R$ in the inertial subrange.

2.2. Viscous limit
For the transit time probability density appropriate for the viscous subrange we found a scaling in the form
\[ P_\tau(\tau) = \sqrt{\frac{\epsilon}{\nu}} F_3 \left( \tau \sqrt{\frac{\epsilon}{\nu}} \right), \tag{5} \]
being independent of $R$. This somewhat counter-intuitive result can be justified on dimensional grounds by taking the transit time to be the ratio of the characteristic length $\ell$ and a typical velocity estimated by the root-mean-square of the second order structure function, corresponding to the same length scale. For the inertial subrange this argument gives $\ell/(\epsilon \ell^{1/3}) = \ell^{2/3}/\epsilon^{1/3}$, in agreement with (4). With the second order structure function being $C_\nu \tau^2 (\epsilon/\nu)$ in the viscous subrange (Davidson, 2004), the same arguments give the characteristic time (apart from a numerical constant) for transit time normalizations given by $\ell/\sqrt{\ell^2 \epsilon/\nu} = \sqrt{\nu/\ell}$ independent of the length-scale as given in (5).

3. Numerical results
Our analysis follows the simultaneous trajectories of up to 400,000 passively moving point-particles (Biferale et al., 2004, 2005), and addresses two separate problems: encounter rates and transit time distributions. Encounter rates are obtained by following virtual surfaces that move self-consistently with the flow. For a Lagrangian description, we select one of the particles to represent the motion of a moving closed surface. The surface determines a reference volume, which may be a sphere, for instance. For spherical surfaces, the reference particle is at the center. We subsequently determine estimates for the encounter rate by obtaining the statistical average of the particle flux into this volume. The analysis is here relevant only for a Lagrangian version. For illustration, we show in Figures 2 and 3 the relative motion of a small group of particles. The figure allows three dimensional stereoscopic views (Pécseli et al., 2010; Pécseli & Trulsen, 2007). Figures 2 and 3 allow a three dimensional, stereoscopic view by focusing the eyes approximately 20 cm behind the plane of the paper or computer screen. It requires a little exercise. In our experience, the distance to the eyes is not so critical provided it is sufficiently
large, but it is essential that the figure is kept plane and horizontally aligned with the observers eyes.

The variation of the flux with the scale size of the surface of interception, as well as the variation with basic flow parameters is described by a simple model for radii $R$ smaller than a characteristic large length scale, the outer scale, for the turbulence. For the inertial subrange (Pécseli & Trulsen, 2007) we find $J_{enc} = C_M n_0 \epsilon^{1/3} R^{7/3}$ where $n_0$ is the reference concentration of particles, $\epsilon$ is the average specific turbulent energy dissipation rate, while $C_M \approx 6$ is a numerical coefficient determined empirically. The scaling law summarized here has received solid experimental and numerical support (Mann et al., 2005; Boffetta et al., 2006; Pécseli & Trulsen, 2007). For the viscous subrange we find $J_{enc} = C_V n_0 R^{3} \sqrt{\epsilon/\nu}$ giving an $R^{9/3}$ scaling of the turbulent flux for the viscous subrange rather than the $R^{7/3}$ found in the inertial subrange: this is a non-trivial difference. For the viscous subrange, the numerical coefficient is $C_V \approx 1.1$.

### 3.1. Numerical results for encounter rates

In Figure 4 we show numerically obtained results for the encounter rate covering radii $R$ in the inertial as well as the viscous subranges. We note a very good agreement (over several decades) with the results in the inertial as well as viscous subranges, as obtained by the dimensional analysis.

**Figure 2.** Trajectories for 20 point-particles in the Eulerian or rest frame. The particles are initially confined to a sphere of radius $R/\eta = 20$. Units on axes are in computational units. The heavy line shows the reference predator. The figure is representative for $R$ being in the universal subrange. For comparison we have the scale size of the largest energy containing eddies to $\sim 3$ in the present computational units.

**Figure 3.** Trajectories for the 20 point-particles in Figure 2, now shown in the Lagrangian or co-moving frame for the particle in the center. The reference predator is represented by a point in this co-moving frame, and therefore not noticeable.
In the numerical analysis, we select first a reference sphere of interception with given radius $R$, centered at a moving particle. At each time step, we remove all particles entering this fixed sphere in that time interval from the database, and thus obtain an estimate for the particle flux. The analysis is then repeated for a new position of the reference sphere, in order to improve the signal-to-noise ratio, or for a new choice of radius in the sphere, all particles are reintroduced in the database. The analysis can be carried out in a similar manner for non-spherical volumes, hemispherical or conical ones, in particular. Empirical corrections factors for varying opening angles $\theta$ can be obtained (see Figures 1 and 5) to account for the reduction in encounter rates when the volumes are changed from spherical to conical surfaces (Pécseli et al., 2010). Results similar to those shown in Figure 4 were obtained also using data from laboratory experiments (Mann et al., 2006). In this case, however, the analysis had to be restricted to the inertial range, since the viscous range was poorly resolved. For the case where the interaction volume is conical, we have one more parameter as compared to the spherical case, namely the angle between the cone-axis and the flow velocity vector at the reference position, here taken to be the predator position. It is conceivable that aquatic organisms can sense the local strain rate, for instance, but it seems implausible that this information can be used to orient the predator with respect to the local fluid flow vector. In the data analysis we let the angle between the cone-axis and the flow velocity vector to be randomly distributed over the full solid angle $\{0; 4\pi\}$.

Figure 4. Numerically obtained time asymptotic prey fluxes to a predator with conical fields of view, for the case with $Re_\lambda = 286$. The full line gives a model-fit for the spherical case. Circles are for $\theta = 180^\circ$, triangles for $\theta = 45^\circ$ and crosses for $\theta = 10^\circ$. The results are shown on a double-logarithmic scale.

Figure 5. Empirical results for a $\theta$-depending multiplicative correction factor that accounts for the reduction in encounter rate for conical encounter volume taken in comparison with reference spherical volumes. The two data sets (shown with full and open circles) refer to $R$ being in the inertial and viscous subranges. The correction factor has negligible $R$-variation within the two respective subranges.
We proposed a simple analytical approximation for the multiplicative correction factor in the form

$$\chi(\theta) = 0.31 \theta + 0.19 \theta^2 - 0.06 \theta^3,$$

where the conical opening angle $\theta$ is given in radians. The approximation (6) is good for angles up to a hemisphere, i.e. $0 < \theta < \pi/2$. We obtained also better approximations than (6) but these give polynomials of higher degree.

3.2. Numerical results for transit times

For the inertial subrange we found $P_\tau(\tau) = (\epsilon^{1/3}/R^{2/3})F(\tau \epsilon^{1/3}/R^{2/3})$ where $F$ depends on the shape of the reference volume. Figure 6 shows illustrative results for spherical volumes with different values of $R$. The numerical results are compared to similar observations from laboratory experiments, and good agreement is demonstrated. In Figure 7 we show results corresponding to Figure 6 but now for a hemispherical capture volume. For this case we have no experimental results available for comparison. The numerical fits for the cumulative distributions are summarized in Table 2, where we present the numerical coefficients used in an algebraic model

$$G_\tau(x) = 1 - \frac{1}{1 + a[1]x + a[2]x^2 + a[3]x^3 + a[4]x^4},$$

where $x = \tau \epsilon^{1/3}/R^{2/3}$ or $x = \tau \epsilon^{1/2}/\nu^{1/2}$ in the inertial and viscous limits, respectively. The probability density is obtained by differentiation of $G_\tau$. A shortcoming of the model (7) is that higher order averages will be divergent. Practical applications will, however, only involve the lowest order averages, so we believe this shortcoming to be of minor consequence.

Results like those summarized here are obtained also for $R$ in the viscous subrange, with data summarized in Table 2.

4. Conclusions

The simplest analysis of reaction probabilities assumes that a finite time $\Delta \tau$ is needed for a reaction, so that particles spending times shorter than some prescribed time-interval will not interact with the reference particle, but leave the reference volume with the possibility of interacting at some later time. The simplest model fulfilling these assumptions takes $P(\text{react}) = \int_{\Delta \tau}^{\infty} P_\tau(\tau)d\tau$, but also more general models have been studied. The results of
Table 2. Numerical coefficients in the best fits for the cumulative transit time distributions. We use the analytical form $G_\tau(x) = 1 - 1/(1 + a[1]x + a[2]x^2 + a[3]x^3 + a[4]x^4)$. The results account for spherical, hemispherical and conical (with $\theta = \pi/4$ opening angle) capture volumes, with radii in the inertial as well as the viscous subranges.

| Volume Type                  | $a[1]$ | $a[2]$ | $a[3]$ | $a[4]$ |
|------------------------------|--------|--------|--------|--------|
| Sphere / viscous subr.       | 0.02   | 0.195  | -0.0095| 0.00035|
| Hemisphere / viscous subr.  | 0.20   | 0.25   | -0.015 | 0.0017 |
| Conical ($\theta = \pi/4$) / viscous subr. | 0.25   | 0.50   | -0.05  | 0.01   |
| Sphere / inertial subr.     | 0.0    | 6.2    | -3.5   | 2.2    |
| Hemisphere / inertial subr. | 0.80   | 8.4    | -4.7   | 8.2    |
| Conical ($\theta = \pi/4$) / inertial subr. | 1.25   | 13.0   | -6.0   | 45.0   |

Figure 7. The cumulative distribution for transit times through a hemispherical reference volume. Note that in this case $P_\tau(\tau)$ assumes a finite value for $\tau \to 0$ in contrast to the spherical case, where $P_\tau(\tau \to 0) \to 0$.

Our analysis are relevant for describing some features of chemical reactions, but also for understanding details in the feeding rate of micro-organisms in turbulent waters (MacKenzie et al., 1994), for instance.

Our studies were covering cases where characteristic length scales $R$ are in the inertial as well as in the viscous ranges of the turbulence. In studies of encounter rates on very small scales in, for instance, coagulation processes, the standard applications usually assume that the particle motion is Brownian, as for instance relevant for aerosols etc. (Chandrasekhar, 1954; Harris, 1982). If we want to include larger particles or droplets in the analysis, the question obviously arises: at what particle size is turbulent transport more important than molecular, or Brownian, diffusion. The answer can readily be found by comparing the turbulent flux to the absorbing surface with what is found for the molecular diffusion. The latter result is readily found by solutions of the diffusion equation in three spatial dimensions (Chandrasekhar, 1954; Pécseli, 2000). We have the Brownian flux to a spherical absorbing surface with radius $R$ as $J_B = 8\pi DRn_0$, where $D$ is the diffusion coefficient. For the small particles relevant here, we use the result for the viscous subrange $J_0 \approx (C_M/\sqrt{15C_K}) n_0 R^2 \sqrt{\epsilon/\nu}$, and find the limiting length scale $R_l \approx (64\pi^2(15C_K)/C_M^2)^{1/4} (D^2\nu/\epsilon)^{1/4} \approx 4.2 \eta/\sqrt{Sc}$, in terms of the Schmidt number $Sc \equiv \nu/D$, where $\eta$ is the Kolmogorov length-scale (Pécseli & Trulsen, 2007). The flux due to Brownian motion will dominate if $R < R_l$. These results will apply for turbulent incompressible
flows, in particular also for turbulent transport in the atmosphere when the conditions can be taken to be locally homogeneous and isotropic. The values of the diffusion coefficient $D$ are, however, very different in water and air: at around room temperature in water we have $D \approx 10^{-5} - 10^{-4} \text{mm}^2\text{s}^{-1}$, while typical values for air can be $D \approx 10 \text{mm}^2\text{s}^{-1}$ (Schroeder, 2000). For instance for water we then have $R_\ell \approx 10^{-3} \epsilon^{-1/4} \text{mm}$, giving $R_\ell \approx 5 \times 10^{-2} \text{mm}$ for $\epsilon \approx 1 \text{mm}^2\text{s}^{-3}$. The limit where Brownian motion is important on the same level as turbulent diffusion is poorly understood.

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