Relativistic effects in proton-induced deuteron break-up at intermediate energies with forward emission of a fast proton pair

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Recent data on the reaction $pd \to (pp)n$ with a fast forward $pp$ pair with very small excitation energy is analyzed within a covariant approach based on the Bethe-Salpeter formalism. It is demonstrated that the minimum non-relativistic amplitude is completely masked by relativistic effects, such as Lorentz boost and the negative-energy $P$ components in the $^{1}S_{0}$ Bethe-Salpeter amplitude of the $pp$ pair.

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The investigation of hadronic processes at high energies, such as reactions of protons scattering off deuterons, provides a refinement of information about strong interaction at short distances. Nowadays, large research programs of experimental studies of processes with polarized particles are in progress. Important are setups with deuteron targets or beams, since the deuteron serves as a unique source of information on the neutron properties at high transferred momenta, the knowledge of which allows, e.g. to check a number of QCD predictions and sum rules. Additionally, the hadron-deuteron processes can be considered as complementary tool in investigating short-distance phenomena and also as a source of information unavailable in electromagnetic reactions. Of interest is the study of nucleon resonances, checking non-relativistic effective models, meson-nucleon theory, $NN$ potentials etc. In this line is the investigation of the deuteron break-up reaction with a fast $pp$ pair at low excitation energy, proposed in [3] and with first results reported in [4].

One motivation for the experiment [4] was the possibility to investigate the off-mass shell

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effects in $NN$ interactions. As predicted in [4, 5], at a certain initial energy of the beam protons, the cross section should exhibit a deep minimum, corresponding to the node of the non-relativistic $^1S_0$ wave function of the two outgoing protons, provided the non-relativistic picture holds and the off-mass shell effects can be neglected. The recent data [4] exhibits, however, a completely different behavior: the cross section is smoothly decreasing; there is no sign of a pronounced minimum. Accounting for corrections beyond the one-nucleon exchange mechanism improve the agreement with data, however a quantitative description have not achieved [4].

It is clear, that the non-relativistic treatment of the process becomes inadequate because of the high virtuality of the proton in the deuteron at the considered kinematics. More realistic approaches which take into account relativistic effects and the off-mass shellness of the interacting nucleons are desired. The Bethe-Salpeter (BS) formalism can serve as an appropriate approach to the problem because the off-mass shellness of the nucleons is an intrinsic feature of the BS equation. Moreover, the solution of the BS equation, being manifestly covariant, incorporate genuine relativistic effects (Lorenz boosts, negative-energy components etc.), hardly accessible within the Schrödinger formalism. In the present note we use the BS approach to analyze the data [4] on deuteron break-up with the emission of a fast forward $pp$ pair [6]. We pay particular attention on relativistic effects in the wave function of two nucleons in the continuum. The model is based on our solution of the BS equation for the deuteron with a realistic one-boson exchange kernel [7]. The final state interaction of the two protons is treated also within the BS formalism by solving the BS equation for the $t$-matrix within the one-iteration approximation [8, 9]. In doing so, a big deal of off-mass shell effects and relativistic corrections are taken into account already within the spectator mechanism.

Let us consider the process

$$ p + d = (p_1p_2)(0^0) + n(180^0) \quad (1) $$

at low excitation energy of the pair ($E_x \sim 0-3$ MeV) and intermediate initial kinetic energies $T_p \sim 0.6 - 2.0$ GeV corresponding to the conditions at the Cooler Synchrotron COSY in the experiment [4]. In the one-nucleon exchange approximation this reaction can be represented by the diagram depicted in Fig. [4], where the following notation is adopted: $p = (E_p, p)$ and $n = (E_n, n)$ are the four-momenta of the incoming (beam) proton and
outgoing (not registered) neutron, $P_f$ is the total four-momentum of the $pp$ pair, which is a sum of the corresponding four-momenta of the detected protons, $p_1 = (E_1, p_1), p_2 = (E_2, p_2)$. The invariant mass of the $pp$-pair is $M_{pp}^2 = P_f^2 = (2m + E_x)^2$, where $m$ stands for the nucleon mass and $E_x$ is the excitation energy. Our calculations are performed in the laboratory system where the deuteron is at rest. For specific purposes, the center of mass of the pair will be considered as well, where all relevant quantities are superscripted with asterisks.

A peculiarity of the processes (1) is that the transferred three-momentum from the initial proton to the second proton in the pair is rather high. Moreover, from the kinematics one finds that the momentum of the neutron is also high ($|n| \sim 0.3 - 0.5 \text{ GeV/c}$), which implies that, since the outgoing neutron is on-mass shell, the proton inside the deuteron was essentially off-mass shell before the interaction. Correspondingly, it becomes clear that the process of $NN$ interaction in the upper part of the diagram is by far more complicated than the elastic interaction. For instance, let us consider a typical kinematical situation, say $|p_1| = 1.22 \text{ GeV/c, } \theta'_1 \sim 4^0$, excitation energy $E_x = 3 \text{ MeV}$ and $|p_1| = 0.765 \text{ GeV/c}$. This means that the neutron three-momentum is $|n| \simeq 0.5 \text{ GeV/c}$, i.e., the four-momentum of the off-mass shell proton was $q = (M_d - E_n, -n)$. Now, if one supposes that in the upper vertex there was an elastic process of two on-mass shell protons, then only one kinematical quantity would be necessary to describe the process, e.g., at given $|p_1| = 0.765 \text{ GeV/c}$ the scattering angle would be $\sim 28^0$ in the elastic kinematics (instead of $4^0$ in the full reaction); or at given scattering angle $4^0$, the momentum of the elastically scattered proton would correspond to $|p_1| = 1.21 \text{ GeV/c}$ ($|p_2| = 0.334 \text{ GeV/c}$) instead of the detected momentum $|p_1| \simeq |p_2| \simeq 0.765 \text{ GeV/c}$. This demonstrates that the $NN$ interaction in the upper part of the diagram has a quite complicated nature. It can be considered as consisting at least of two steps: (i) an inelastic process which puts the target nucleon on mass-shell, and (ii) an elastic interaction in the $pp$ pair in the $^1S_0$ final state. Since a large amount of the transferred energy is needed to locate the second proton on mass-shell, the relativistic corrections can play a crucial role here.

The invariant cross section of the reaction (1) reads

$$d^6\sigma = \frac{1}{(4\pi)^5 \lambda(p,d)} |M_{fi}|^2 \frac{dP_1 dP_2}{E_1 E_2} \delta(E_0 + E_d - E_1 - E_2 - E_n),$$

where $\lambda(p,d)$ is the flux factor, and $M_{fi}$ the invariant amplitude; the statistical factor $1/2$ due to two identical particles (protons) in the final state has been already included. The
covariant matrix element corresponding to the diagram in Fig. 1 can be written in the form

\[ M_{fi} = \bar{u}(s, n)(\hat{n} - m)\Psi_d(n) \left[ \hat{p}_2 + m)\bar{\Psi}_{1S_0}(p)(\hat{p}_1 - m)u(r, p) \right], \quad (3) \]

where \( u(r, p) \) (\( \bar{u}(s, n) \)) stands for the Dirac spinor of the incident proton (outgoing neutron) with the spin projection \( r \) (\( s \)) and 3-momentum \( p \) (\( n \)), \( \Psi_{D(1S_0)} \) denote the BS amplitudes for the deuteron and the \( pp \)-pair in the continuum, respectively. By using the spin angular basis to obtain the partial decomposition of the deuteron BS amplitude and the covariant form for the four partial components for the \( 1S_0 \)-state [9] of the \( NN \) pair [10], the invariant amplitude \( M_{fi} \) is

\[ M_{fi} = (-1)^{1/2 - r} K_{1S_0} \frac{1}{\sqrt{8\pi}(M_d - 2E_n)} \left\{ \sqrt{2}G_{D,1S_0} \left( G_S - \frac{G_D}{\sqrt{2}} \right) + 3\delta_{M,0}\delta_{s,r} \frac{G_D}{\sqrt{2}} \right\}, \quad (4) \]

where the contribution \( K_{1S_0} \) from the upper part of the diagram in Fig. 1 is

\[ K_{1S_0} = \frac{\sqrt{E_p + m}}{E_n + m} \left[ h_1 \left( E_n + m - \frac{|n||p|}{E_p + m} \right) - h_3 \frac{M_d - 2E_n}{m} \left( E_n + m + \frac{|n||p|}{E_p + m} \right) \right]. \quad (5) \]

\( G_{S,D} \) denote the BS vertices for the deuteron, \( h_1, h_3 \) are the non-vanishing covariant partial components of the \( 1S_0 \) configuration in the continuum. The relation of the amplitudes \( h_i \) \((i = 1 \ldots 4)\) to the partial solutions of the BS equation in the \( NN \) center of mass \( 1S_0^{++}, 1S_0^{--}, 3P_0^{+-}, 3P_0^{-+} \) can be found, e.g., in ref. [9]. In what follows we neglect the contribution of the extremely small \( 1S^{--} \) component, keeping only the \( 1S^{++} \) component as the main one and the \( P \) components as the ones providing purely relativistic corrections.

It is easy to check that for the unpolarized particles the cross section factorizes in two independent parts, i.e., \( \frac{1}{6} \sum_{s,r,M} |M_{fi}|^2 \simeq |K_{1S_0}|^2 (u_S(n)^2 + u_D(n)^2) \), as it should be within the spectator mechanism with \( 1S_0 \) \((L_f = 0)\) in the final state (see also discussion in ref. [11]); \( u_S \) and \( u_D \) are the BS deuteron \( S \) and \( D \) waves [8].

In our numerical calculations we use the deuteron BS amplitude from [7]. For \( 1S_0 \) state one should solve the inhomogeneous BS equation in the \( NN \) center of mass to obtain the covariant amplitudes \( h_{1,3} \). Note that in such a way the effects of the Lorentz boost are automatically taken into account (see, e.g., [8]). The partial ”++” components of the BS vertices in the \( NN \) center of mass, for both the bound and the scattering states have a direct analogue with the corresponding non-relativistic wave functions. Moreover, at low intrinsic relative momenta the BS and non-relativistic wave functions basically coincide. Hence, due to the low excitation energy of the \( pp \)-pair, in expressing \( h_{1,3} \) via the partial amplitudes at
rest one may safely replace the "$++" vertex by its non-relativistic analogue. Relativistic
effects are then included in boosting the $^1S_0^{++}$ component to the deuteron center of mass
system and also by taking into account the contributions of $P$ components in $h_{1,3}$. To find
the $P$ waves we solve the BS equation for the pair in its center of mass within the one
iteration approximation \cite{8,12}, with the trial function as the exact solution for the $t$ matrix
within a separable potential \cite{13}. By defining new partial components $g_i$ ($i = 1 \ldots 4$) as
"connected amplitudes", i.e., the partial amplitudes without the free terms, we obtain for
the $P$-waves

$$g_3(k) = i \frac{g_{\pi NN}^2}{\sqrt{\pi}} \left[ V_{31}(k, p^*) - \int_0^\infty \frac{dp \, p^2}{(2\pi)^2} V_{31}(k, p) \frac{g_1(p)}{M_{pp} - 2E_p + i\epsilon} \right], \tag{6}$$

where the partial kernel $V_{31}$ is $V_{31}(k, p^*) = \frac{\pi m}{|p^*||k|E_{pp}E_k} \{ |p^*|Q_1(y) - |k|Q_0(y) \}$ with $Q_L(y)$ as Legendre function with the argument $y = (|p^*|^2 + |k|^2 + \mu_0^2 - k_0^2)/(2|p^*||k|)$ \cite{8,12}. In
the first iteration the trial function $g_1(k)$ is expressed via the non-relativistic $t$ matrix \cite{13}
as

$$g_1(k) = i \frac{(4\pi)^{5/2}}{2} \frac{m}{2} t_{NR}(k, p^*), \tag{7}$$

where the normalization of $t_{NR}$ corresponds to $t_{NR}(p^*, p^*) = -\frac{2}{\pi m|p^*|} e^{i\delta_0} \sin \delta_0$, where $\delta_0$
denotes the experimentally known phase-shift of the elastic $pp$ scattering in the $^1S_0$ state. By
using the Sokhotsky-Weierstrass formula for the Cauchy type integrals one finally obtains

$$g_3(k) = \frac{i g_{\pi NN}^2}{\sqrt{\pi}} \left[ M_{pp} V_{31}(k, p^*) \left\{ 1 - \frac{i\pi m|p^*|}{2} t_{NR}(p^*, p^*) \right\} + m \mathcal{P} \int_0^\infty dp \, \frac{dpp^2 V_{31}(k, p) t_{NR}(p, p^*)}{E_{p^*} - E_p} \right],$$

where, due to the separable stricture of the chosen $t_{NR}$, further calculations of principal
values of the relevant integrals can be carried out analytically.

In Figs. 2 and 3 we present results of numerical calculations of the five-fold cross section
d$\sigma/d\Omega_1d\Omega_2d|p_1|$ and the two-fold cross section $d\sigma/d\Omega_\alpha$ (with $\Omega_\alpha$ as the solid angle of the
momentum of the neutron in the center of mass of the reaction), integrated over the excitation
energy in a range $E_x = 0 - 3$ MeV. The calculations have been performed with our
numerical solution for the deuteron BS amplitude (inclusion of the $P$ components in the
deuteron amplitude leads to negligibly small corrections, therefore here we do not discus
these contributions). The dotted curves are results of non-relativistic calculations, while the
dashed curves include pure Lorentz boost effects, i.e., relativistic calculations with including
the $^{1}S^{0+}$ component only. It is clearly seen that the boost corrections are fairly visible: They cause a shift of the minimum of the cross section. The agreement at low initial energies becomes better, however, the cross section is still too small at large values of $T_p$. Fig. 3 reveals that an account for only ”++” components is not sufficient to describe data. An excellent description is achieved by taking into account all the relativistic effects, including the contribution of the negative-energy $P$ waves. Note, that as demonstrated in refs. [8, 9] in reactions of $pd$ and in threshold-near $ed$ disintegration, the inclusion of $P$ waves exactly recovers the non-relativistic calculation with taking into account the $N\bar{N}$ pair production effects. Hence, in our case this is a hint that covariant calculations within the relativistic spectator mechanism contains already some contributions beyond the one-nucleon exchange mechanism.

In summary we analyze the recent data [4] of the reactions $pd(p,n) \rightarrow (pp)n$ within a covariant approach based on the Bethe-Salpeter formalism. Relativistic effects (Lorentz boost, negative-energy $P$ components) are important and responsible for the smooth decline of the cross section, in contrast to predictions of non-relativistic models.

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FIG. 1: Kinematics of the process (I).
FIG. 2: Five-fold cross section as a function of the kinetic energy $T_p$ of the incident proton. The dotted curve corresponds to a non-relativistic calculation, i.e., to the case where only the ”++” components in the $^1S_0$ state are taken into account and any Lorentz boost effects ignored. The dashed curve depicts results of calculations with all relativistic effects in ”++” components taken into account. The solid curve is for the results of a complete calculation with taking into account all the relativistic effects including the contribution of $P$ waves in the wave function of the $pp$ pair. The two protons are supposed to be detected in the strictly forward direction, i.e., $\theta_1 = \theta_2 = 0^0$. 
FIG. 3: Differential cross section in the center of mass integrated over the excitation energy $E_x$ as a function of the kinetic energy $T_p$ of the incident proton. Notation is as in Fig. 2. Experimental data are from [4].