Fragmentation contributions to $J/\psi$ production at the Tevatron and the LHC

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We compute leading-power fragmentation corrections to $J/\psi$ production at the Tevatron and the LHC. We find that, when these corrections are combined with perturbative corrections through next-to-leading order in the strong coupling constant $\alpha_s$, we obtain a good fit to high-$p_T$ cross section data from the CDF and CMS collaborations. The fitted long-distance matrix elements lead to predictions of near-zero $J/\psi$ polarization in the helicity frame at large $p_T$.

Much of the current phenomenology of heavy-quarkonium production in high-energy collisions is based on the effective field theory nonrelativistic QCD (NRQCD) [1]. Specifically, calculations are based on the conjecture known as NRQCD factorization [2], which states that the inclusive cross section to produce a quarkonium state $H$ at large momentum transfer in a collision of particles $A$ and $B$ can be expressed as

$$d\sigma_{A+B\to H+X} = \sum_n d\sigma_{A+B\to Q\bar{Q}(n)+X} \langle \mathcal{O}^H(n) \rangle.$$  \hspace{1cm} (1)

Here, the $d\sigma_{A+B\to Q\bar{Q}(n)+X}$ are short-distance coefficients (SDCs) which are, essentially, the partonic cross sections to produce a heavy-quark-antiquark ($Q\bar{Q}$) pair in a particular color and angular-momentum state $n$, convolved with parton distributions. The $\langle \mathcal{O}^H(n) \rangle$ are long-distance matrix elements (LDMEs) of NRQCD operators and are, essentially, the probabilities for a $Q\bar{Q}$ pair in a particular state $n$ to evolve into the quarkonium $H$. The partonic cross sections can be calculated in QCD perturbation theory. The LDMEs are nonperturbative in nature, but are conjectured to be universal, i.e., process independent. This conjecture implies that information that is gained about the LDMEs by studying one quarkonium production process can be used to make predictions about another.

The LDMEs have a well-defined scaling with the relative velocity $v$ of the $Q$ and the $\bar{Q}$ in the quarkonium center-of-momentum frame. Consequently, the sum over $n$ in Eq. (1) is actually an expansion in powers of $v$, where $v^2 \approx 0.25$ for the $J/\psi$ charm-anticharm ($c\bar{c}$) state. In present-day quarkonium-production phenomenology, the sum over $n$ is usually truncated at relative order $v^4$.

Four $Q\bar{Q}$ states appear in this truncation: $Q\bar{Q}(3S_1^1)$, $Q\bar{Q}(3S_1^8)$, $Q\bar{Q}(3P_1^8)$, and $Q\bar{Q}^3 P_j^8$, where we use standard spectroscopic notation for the angular momentum, and the superscripts 1 and 8 denote color-singlet and color-octet states, respectively.

Three groups have now completed the formidable task of calculating the SDCs that appear in the four-LDME truncation through next-to-leading order (NLO) in the QCD coupling $\alpha_s$. Generally, the NLO calculations, combined with the four-LDME phenomenology, lead to reasonable agreement with a wide range of inclusive $J/\psi$-production measurements that have been made at the Tevatron, the LHC, and the B factories [10, 11]. Problematic exceptions to this agreement arise from NLO predictions, which are based on fits to $J/\psi$ cross sections, that the $J/\psi$ polarization in the helicity frame is substantially transverse at large $J/\psi$ transverse momentum $p_T$ [3, 11, 12]. Measurements of the $J/\psi$ polarization at the Tevatron [13, 14] and the LHC [15, 16] are in contradiction with these predictions.\footnote{The authors of Ref. [17] found that it is possible to obtain a reasonable fit to both the $J/\psi$ cross section and the $J/\psi$ polarization data at the Tevatron. However, a fit of that NLO calculation [8] to data with $p_T \geq 7$ GeV from the CDF Collaboration [18] yields a polarization prediction that is too uncertain to be compared meaningfully with data.}

In this Letter we make use of the leading-power (LP) factorization formalism to compute fragmentation contributions to $J/\psi$ production beyond NLO that appear in the leading power of $m_{J/\psi}^2/p_T^2$ for $p_T \gg m_{J/\psi} \approx 2m_c$, where $m_{J/\psi}$ is the $J/\psi$ mass and $m_c$ is the charm-mass. Specifically, we compute contributions that arise from NLO parton scattering convolved with order-$\alpha_s^2$ fragmentation functions (FFs) for a single parton to fragment into a $J/\psi$ and contributions that arise from LO and NLO parton scattering convolved with all-orders summations of logarithms of $p_T^2/(2m_c)^2$ in the single-parton FFs. Through NLO in $\alpha_s$, it has been found that the contribution from the color-singlet channel is negligible in the phenomenology of $J/\psi$ production [3–9]. Therefore, we focus on the color-octet channels. We find that the fragmentation contributions of higher order in
\(\alpha_s\) have important effects on the shapes of the SDCs as functions of \(p_T\) and on the relative contributions of the various angular-momentum channels in fits to the experimental differential cross sections. We are able to obtain good fits to the data of the CDF Collaboration \[15\] and the CMS Collaboration \[16\] for prompt \(J/\psi\) production for \(p_T \geq 10\) GeV. The resulting LDMEs lead to a prediction that the \(J/\psi\) polarization in the helicity frame in direct production at both the Tevatron and the LHC is near zero at high \(p_T\), in good agreement with the CMS data \[15\] and in greatly improved agreement with the CDF data \[13\,14\].

The LP factorization states that the contribution to the inclusive cross section to produce a hadron \(H\) at LP in \(1/p_T^2\) (\(d\sigma/dp_T^2 \sim 1/p_T^4\)) can be written as

\[
d\sigma_{A+B\to H+X} = \sum_i d\hat{\sigma}_{A+B\to i+X} \otimes D_{i\to H}, \tag{2}
\]

where the \(d\hat{\sigma}_{A+B\to i+X}\) are inclusive SDCs to produce a single parton \(i\) (gluon, quark, or antiquark), the \(D_{i\to H}\) are single-parton FFs \[20\] for the parton \(i\) to fragment into the hadron \(H\), and \(\otimes\) denotes a convolution with respect to the longitudinal momentum fraction \(z = p^+/k^+\), where \(p^+\) and \(k^+\) are the + light-cone momenta of the hadron and the fragmenting parton, respectively. The SDCs can be computed in QCD perturbation theory; the FFs are generally nonperturbative in nature and must be determined phenomenologically. The LP factorization formula in Eq. (2) was proven for \(e^+e^-\) annihilation to a light hadron in Ref. \[21\]. The proof of Eq. (2) for production of a heavy quarkonium in hadronic collisions was sketched in Ref. \[21\]. In this case, the corrections to the formula in Eq. (2) are of relative order \(m_Q^2/p_T^2\), where \(m_Q\) is the heavy-quark mass. Expressions for next-to-leading-power (NLP) factorization, which involves two-parton fragmentation, have been derived for quarkonium production in Refs. \[22\,23\].

One can apply the NRQCD factorization conjecture to express the FFs in Eq. (2) as sums of products of perturbatively calculable SDCs with NRQCD LDMEs \[22\,23\]. Alternatively, one can apply LP factorization to the SDCs in Eq. (1). The result is

\[
d\sigma_{A+B\to Q\bar{Q}(n)+X} = \sum_i d\hat{\sigma}_{A+B\to i+X} \otimes D_{i\to Q\bar{Q}(n)}, \tag{3}
\]

where now both the partonic SDCs \(d\hat{\sigma}_{A+B\to i+X}\) and the FFs \(D_{i\to Q\bar{Q}(n)}\) for a parton into a \(Q\bar{Q}\) pair can be calculated in QCD perturbation theory. In this Letter, we use the LP factorization formula (3) for the SDCs \(d\hat{\sigma}_{A+B\to Q\bar{Q}(n)+X}\) to compute contributions that augment the NLO calculations of the SDCs. Specifically, we compute the partonic cross sections \(d\hat{\sigma}_{A+B\to i+X}\) through order \(\alpha_s^3\) (NLO), and we compute the FFs \(D_{i\to Q\bar{Q}(n)}\) through order \(\alpha_s^2\) and, for leading logarithms of \(p_T^2/(2m_c)^2\), to all orders in \(\alpha_s\).

The partonic cross sections through order \(\alpha_s^3\) were computed in Refs. \[24\,25\]. We evaluate them by making use of a computer code that was provided by the authors of Ref. \[24\]. The FFs for a gluon to fragment into a \(QQ\) pair in Eq. (3) are given for the \(1s_0^{[8]}\) channel at order \(\alpha_s^2\) (LO) in Refs. \[26\,27\], for the \(3S_1^{[8]}\) channel at order \(\alpha_s\) (LO) in Ref. \[28\] and at order \(\alpha_s^2\) (NLO) in Refs. \[24\,30\], and for the \(3P_j^{[8]}\) channels at order \(\alpha_s^2\) (LO) in Refs. \[27\,28\]. The FF for a light quark to fragment into a \(QQ\) pair in the \(3S_1^{[8]}\) channel is given at order \(\alpha_s^2\) (LO) in Ref. \[31\]. Light-quark fragmentation in the other color-octet channels vanishes through order \(\alpha_s^2\) and will be ignored here.

We calculate, to all orders in \(\alpha_s\), contributions to the FFs from leading logarithms of \(p_T^2/(2m_c)^2\) by making use of the LO DGLAP evolution equation \[32\,33\]:

\[
\frac{d}{d\log \mu_f^2} \left( \frac{D_S}{D_g} \right) = \frac{\alpha_s(\mu_f)}{2\pi} \left( \frac{P_{qg}}{P_{gg}} \frac{2\eta_f P_{gq}}{P_{gg}} \right) \otimes \left( \frac{D_S}{D_g} \right), \tag{4}
\]

where \(D_S = D_{g\to Q\bar{Q}(n)}\), \(D_S = \sum_f \left( D_{q\to Q\bar{Q}(n)} + D_{\bar{q}\to Q\bar{Q}(n)} \right)\), \(f\) is the light-quark or light-antiquark flavor, \(n_f = 3\) is the number of active quark flavors, the \(P_{ij}\)'s are the DGLAP splitting functions for FFs, and \(\mu_f\) is the factorization scale. \(D_{i\to Q\bar{Q}(n)}\) and \(d\hat{\sigma}_{A+B\to i+X}\) both depend on \(\mu_f\), but that dependence cancels in \(d\hat{\sigma}_{A+B\to Q\bar{Q}(n)+X}\) when the calculation is extended to infinite order in \(\alpha_s\). We solve Eq. (4) by taking a Mellin transform with respect to \(z\) in order to convert the convolution into a multiplication, integrating \(\mu_f\) from the scale \(2m_c\) to the scale \(m_T = \sqrt{p_T^2 + 4m_c^2}\) in order to incorporate the logarithms of \(m_T^2/(2m_c)^2 \approx p_T^2/(2m_c)^2\), and taking an inverse Mellin transform in order to obtain a \(z\)-space expression.

In order to avoid double counting, we must subtract from \(d\hat{\sigma}_{A+B\to Q\bar{Q}(n)+X}\) in Eq. (3) contributions through order \(\alpha_s^3\), which also appear into the LO and NLO calculations of the SDCs. We denote these contributions by \(d\sigma_{\text{NLO}}^{F}/dp_T\), and we denote the sum of the LO and NLO contributions to the SDCs by \(d\sigma_{\text{NLO}}^{E}/dp_T\). The contributions of \(d\sigma_{\text{NLO}}^{E}/dp_T\) to \(J/\psi\) production at the LHC at the center-of-momentum energy \(\sqrt{s} = 7\) TeV are compared with \(d\sigma_{\text{NLO}}^{E}/dp_T\) in Fig. 1. We have taken \(d\sigma_{\text{NLO}}^{E}/dp_T\) from the calculation of Refs. \[7\,8\]. In order to maintain compatibility with that calculation, we have made the following choices: we have taken \(m_c = 1.5 \pm 0.1\) GeV; we have used the CTEQ6M parton distributions \[36\] and the two-loop expression for \(\alpha_s\), with \(n_f = 5\) active flavors and \(A_\text{QCD}^{(5)} = 226\) MeV; we have set the renormalization, factorization, and the NRQCD scales to \(\mu = m_T, \mu_f = m_T, \) and \(\mu_A = m_c\), respectively; and we have dropped contributions involving more than one heavy-quark-antiquark pair in the final state.

As can be seen from Fig. 1 in the \(3S_1^{[8]}\) and \(3P_j^{[8]}\) channels, \(d\sigma_{\text{NLO}}^{E}/dp_T\) accounts well for the full fixed-order cross section \(d\hat{\sigma}_{\text{NLO}}^{E}/dp_T\) for \(p_T\) greater than \(10\text{–}20\) GeV.
However, in the $1S_0^{[8]}$ channel, $d\sigma_{NLO}/dp_T$ does not approach $d\sigma_{NLO}/dp_T$ until much larger values of $p_T$. This can be understood, as is explained in Ref. [27], to be a consequence of the fact that the LP FFs in the $3S_1^{[8]}$ and $3P_3^{[8]}$ channels contain $\delta$ functions and plus distributions, which are remnants of canceled infrared divergences, that are strongly peaked near $z = 1$, while the LP FF in the $1S_0^{[8]}$ channel contains no such peaking. The NLO correction in the $3S_1^{[8]}$ channel is small relative to the LO contribution because of a cancellation between the NLO parton-scattering contribution and the NLO FF contribution, which contribute about $-50\%$ and $+100\%$, respectively, relative to the LO contribution at $p_T = 52.7$ GeV.

Our result for the LO plus NLO cross section, augmented by the LP fragmentation contributions that we have computed, is given by

$$\frac{d\sigma_{LPNLO}}{dp_T} = \frac{d\sigma_{LP}}{dp_T} - \frac{d\sigma_{NLO}}{dp_T} + \frac{d\sigma_{NLO}}{dp_T}.$$  

Here, $d\sigma_{LP}/dp_T$ is the LP fragmentation contribution in which the partonic cross sections have been computed through order $\alpha_s^3$ and the FFs have been computed exactly through order $\alpha_s^2$ and, at the level of the leading logarithms of $p_T^2/(2m_c)^2$, to all orders in $\alpha_s$. In Fig. 2 we compare $d\sigma_{LPNLO}/dp_T$ with $d\sigma_{NLO}/dp_T$ in each channel. The LP corrections are modest in the $3S_1^{[8]}$ channel, larger in the $1S_0^{[8]}$ channel, and quite dramatic in the $3P_3^{[8]}$ channel, largely because the LO and NLO contributions tend to cancel at low $p_T$ in that channel.

The contributions that we have calculated are dominated by $gg$ initial states, which account for about $70\%$ of the cross section at $p_T = 52.7$ GeV. Contributions from light-quark fragmentation are small, as are contributions from $q\bar{q}$ mixing in the DGLAP equation, amounting to about $1\%$ and less than $1\%$ of the cross section, respectively, at $p_T = 52.7$ GeV.

At $p_T = 52.7$ GeV, the all-orders resummation of leading logarithms contributes about $-43\%$ in the $1S_0^{[8]}$ channel relative to the LO fragmentation contribution, but $-44\%$ out of that $-43\%$ is already contained in the logarithm of $m_T^2/m_c^2$ in the NLO calculation. In the $1S_0^{[8]}$ and $3P_3^{[8]}$ channels, the all-orders resummations contribute only $2\%$ and $5\%$, respectively, relative to the NLO fragmentation contribution, owing to an accidental cancellation between contributions from the running of $\alpha_s$ and contributions from the DGLAP splitting. Hence, in each channel, $d\sigma_{LP}/dp_T - d\sigma_{NLO}/dp_T$ is given to good approximation by the contribution from NLO parton scattering convolved with the order-$\alpha_s^2$ contribution to the FF.

In each channel, the ratio of the contribution from NLO parton scattering to the contribution from LO parton scattering is typically about a factor of 1.5, suggesting that corrections of still higher orders in $\alpha_s$ may be large. However, if we take $\mu_r = \mu_f = 2m_T$ or $\mu_r = \mu_f = m_T/2$, then the average effect of these scale variations on the SDCs is only about $25\%$ at each value of $p_T$ over the range that we consider. Overall factors of $m_c$ in the SDCs can be absorbed into redefinitions of the LDMEs, and, hence, the uncertainty in $m_c$ from these factors does not affect fits to the cross sections or the polarization predictions that we make. The residual $p_T$-dependent effects from the uncertainty in $m_c$ are largest in the $1S_0^{[8]}$ channel, and amount to only about $5\%$ in that channel. Therefore, in fitting the data, we assume that the theoretical uncertainty is $25\%$. This value is also typical of the uncertainty that one would expect from corrections of higher order in $v$.

In Fig. 3 we show a combined fit of our cross section predictions to CDF [13] and CMS [19] data for prompt $J/\psi$ production. In obtaining these fits, we have included only data with $p_T \geq 10$ GeV in order to suppress possible power corrections to NRQCD factorization. The resulting fit is quite good, with $\chi^2$/d.o.f. = 0.085, suggesting that higher-order corrections do not affect the

![FIG. 1: The ratio $(d\sigma_{NLO}/dp_T)/(d\sigma_{NLO}/dp_T)$ for the $1S_0^{[8]}$, $3S_1^{[8]}$, and $3P_3^{[8]}$ channels in $pp \to J/\psi + X$ at $\sqrt{s} = 7$ GeV.](image1)

![FIG. 2: The ratio $(d\sigma_{LPNLO}/dp_T)/(d\sigma_{NLO}/dp_T)$ for the $1S_0^{[8]}$, $3P_3^{[8]}$, and $3S_1^{[8]}$ channels in $pp \to J/\psi + X$ at $\sqrt{s} = 7$ GeV.](image2)
use only differential-cross-section data as inputs. 

we have described in this Letter result in substan-
tial changes to the predictions of NRQCD factorization for $J/\psi$ production. This initial investigation suggests that these corrections might resolve the long-standing conflict between NRQCD factorization predictions for quarkonium polarizations and the polarization measurements that have been made in collider experiments. Several caveats should be mentioned. First, we are comparing theoretical predictions for direct $J/\psi$ production with prompt $J/\psi$ production data that includes feeddown from the $\chi_{cJ}$ and $\psi(2S)$ states. Collider experiments have yet to determine whether feeddown effects substantially alter shapes of differential cross sections or measured polarizations. Second, the large sizes of the corrections that arise from the parton-scattering cross sections at NLO suggest that the perturbation expansion may not yet be under good control. Investigations of higher-order corrections to the parton-scattering cross sections should be pursued, as should NLP fragmentation corrections. It would also be useful to compute higher-order corrections to the quarkonium FFs. Finally, the approach that has been presented in this Letter should be tested for additional quarkonium states, such as the $\chi_{cJ}$, $\Upsilon(nS)$, and $\chi_{bJ}$ states, and for additional production processes.
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