Numerical Modelling of Fluid Flow and Heat Transfer of (TiO₂-Water) Nanofluids in Wavy duct

Safaa A. Ghadhban¹, Salah Haji Abid Aun² and Kadhum Audaa Jehhef²
¹Department of Control & Automation Techniques, Electrical Engineering Technical College, Middle Technical University, Baghdad, Iraq.
²Department of Mechanical power, Institute of Technology, Middle Technical University, Baghdad, Iraq.
Email: safaaabid@mtu.edu.iq

Abstract. This paper investigates numerically pressure drop and forced convection heat transfer of TiO₂-water nanofluids laminar flow through a horizontal curvilinear form or wavy duct with using four baffle height ratio h/H=0.15, 0.25, 0.35 and 0.45. This flow has been investigated assuming constant wall heat flux boundary condition by using ANSYS-Fluent with the finite volume method to discretize the nanofluids. The study has aimed to show the possibility of intensification of heat transfer by adding nanoparticles to the main coolant. The model employed in this study is a single phase (homogenous and dispersion). The effects of various factors, such as Reynolds number (Re) and nanoparticle concentration (φ), on the flow field and thermal distribution of the Nanofluids, have been analysed. The present results show that nanoparticle concentration and Reynolds number play a prevalent role in the horizontal wavy duct. The Nusselt number has increased by 54 % when using high nanoparticle concentration of (0.4 vol. %) at high Reynolds number of (1250), also the skin friction factor increased by (32%) in the same conditions. The results provide good predictions to enhancement the heat transfer. Predictably, as nanoparticle volume fraction and/or the Reynolds number increases, the heat transfer increases. However, the flow is accompanied by high friction factor and consequently, higher pressure drop.

1. Introduction
In recent years, progress in electronic and optical devices, power generation and transport new generation equipment has sharpened the problem of efficient removal of heat currents of considerable density. For instance, the first Intel processors generated orders of 0.3W/cm², while modern computer processors generate over of 200 W/cm². Such significant increases in thermal stress led to the situation that the cooling systems based on the convection and finned surface are no longer meet the requirements on the heat sink [1].

Promising ways to increase heat cooling characteristics of the electronic include: the use of new heat transfer fluids and the use of more effective cooling schemes. The field of new coolants of many promising directions is applied nanofluids. Adding a few percent particles of Al₂O₃, CuO or some others leads to the sharp change in thermophysical properties of the water solution. Moreover, the thermal conductivity coefficient will be enhanced. Consequently, dramatic reductions in power needed for heat exchanger [2].

Unlike ordinary mixtures, nanofluid is less stable in erosion, clogging and channeling when using it. Moreover, an effective way of increasing heat transfer is application of new tech-solutions such as using curvilinear channels. Well known the fact that when a fluid flows along a curved secondary
surface is generated on the surface currents (Dean's whirlwinds) that lead to improved mixing process of the nanofluid and as a result, improved heat transfer [3]. Channels of this form are constructive solutions that allow achieving improved heat transfer conditions by generating a secondary flow and destabilization of the main fluid flow. Induction of vortex out of the structures in the bend of the channel causes fluid movement from wall of channel to the center of main flow and leads to the destruction and thinning boundary layer. This kind of mechanism was investigated in many papers Gyves, et al., [4]. It was Dean's eddies and more complex vertical flows significantly improve heat transfer compared to direct channels. In present paper, the considered problem includes both promising technologies of increasing the heat transfer: the use of curved channels with nanofluids. Explaining a sharp increase in the thermal conductivity related to nanofluids, many studies are conducted Das, et al. [5] in which various physical mechanisms had been proposed such as the taking into account the boundary layer around the rotational particles, the Brownian movements and thermophysical properties in the complex, including viscosity and heat capacity. This is because the coefficient of heat has an effect other than the heat conductivity of and the boundary layer thickness. Enhancement of heat transfer using available nanofluids was to be in the range 15–40% Yu, et al., [6].

As already mentioned, in this work, it is considered a forced flow of a nanofluid in a curved channel. Many studies of forced convection were devoted to several papers Kakac and Pramuanjaroenkij [7] and Lotfi et al., [8], in which there were an increase in the coefficient of heat transfer. It was considered the related studies should deal with the special nature of the flow, and include the formation of secondary currents. In this regard, the heat transfer characteristics other than directly thermal conductivity may be affected by a change in viscosity nanofluids. Albojamal, et al., [9] investigated pressure drop and heat transfer as a result of laminar forced convection of (Al2O3/water) and (CuO/water) nanofluids laminar flow. The numerical investigation was implemented considering constant wall temperature. Their results showed that variable property assumptions play a prevalent role in horizontal tubes. Morteza [10] investigated numerically the flow of Al2O3–H2O nanofluid through the wavy channel including different wave lengths. The wave length changes had important effects on the wavy channel performance. The study resulted correlations for the non-uniform wavy channels. The correlations were functions of nanofluid concentration, Prandtl number, Reynolds number, and geometrical parameters. Aboukazempour, et al., [11] investigated the flow nanofluid flow in a 2-D channel with wavy walls by studying the effects of wavelengths and the phase difference between waves of opposite walls. The result demonstrated that at 135 or 225 degrees, the angle difference would maximize the heat transfer. Another way to enhance heat transfer was decreasing the wavelength. Tokgoz et al. [12] studied heat transfer enhancement of channel flow by using alumina-water nanofluid and different corrugated duct. The results showed that the using of corrugated duct increased turbulent intensity, consequently, enhanced the forced convective heat transfer. Zahid et. al. [13] inspected experimentally the heat transfer of TiO2 nanofluid flowing in wavy channel under laminar regime for three different channel configurations and different heating powers. Results indicated that thermal performance of TiO2 nanofluid was decreased with increasing in heating power. Moreover, change in channel wavelength had dominating effect on performance of heat transfer in comparison with the channel width. Hatamia, et al., [14] studied effects of convergent-divergent passages on heat transfer of nano-fluid flow by using the software (ANSYS). The turbulent flow was simulated using enhanced wall function with the k-ε model. Two different cases were studied; a venturi with different nano particles (Al2O3, CuO, and SiO2), and a wavy tube for different wall functions. The results demonstrated that the surface Nusselt number of SiO2-water was the most. Moreover, the sub-wall temperatures increased for the smallest wavelength (sin (2x) function wall).

The objective of the present work is to study the effect of wave duct usage on the thermal and hydrodynamic performance when using the coolant type of TiO2-water nanofluid with different concentrations at a range of Reynolds numbers and using different height ratio of baffle.

2. Formulation of the Problem
Consider the movement of the liquid in the waveform shown in the (Fig. 1). Despite the presence of relative nanoparticle scaling in relation to the main fluid, the calculation works involves consideration
of nanofluids as a homogeneous mixture Maiga, et al., [15], while the influence itself of relative speeds and the associated Brownian motion is taken into account by empirical coefficients for the properties of nanofluids. Accounting, concentration influence is simple recalculation the average properties of a liquid without an accounting of the real concentration distribution of nanoparticles. The system of equations corresponding to the nanofluid in a homogeneous approximation has the form:

Continuity equation:

\[ \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0. \]  

Momentum equation:

\[ u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = -\frac{1}{\rho_{nf}} \frac{\partial p}{\partial x} + \nu_{nf} \Delta^2 u \]  
\[ u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} = -\frac{1}{\rho_{nf}} \frac{\partial p}{\partial y} + \nu_{nf} \Delta^2 v \]  

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\[ u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \alpha_{nf} \Delta^2 T \]  

Consider the problem in periodic graphs. The boundary conditions are in the input and output sections:

The boundary conditions at the inlet are given by:

\[ \int_0^H u(0, y) \, dy = U_{av} \]  
\[ \int_0^H T(0, y) \, dy = T_{in} \]  

And on the side walls, the boundary conditions are:

\[ u = 0, v = 0, T = k \text{ at } y = S(x) \]  

where \( S(x) \) describes the channel boundary. Equations (1) must be supplemented by tangent relations for thermophysical properties Maiga, et al., [15]. By defining the volume fraction of nanofluid (\( \phi \)), equations of its thermophysical properties can be summarized as in Table (1), Jehhef, et al. [16].

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**Figure 1.** The computational domain of the present problem \( L=100 \text{ mm}, S=20 \text{ mm} \) and \( A=10 \text{ mm} \).

**Table 1.** Equations of the thermophysical properties are used in numerical modelling

| Thermo-physical property | Equation | References |
|--------------------------|----------|------------|
| Density                  | \( \rho_{nf} = \phi \rho_p (1 - \phi) \rho_f \) | Pak and Cho [17] |
|                          | \( \frac{\rho_p C_p + (1 - \phi) \rho_f C_p}{\phi} \) | Xuan et al., [18] |
| Specific heat            | \( \mu_{nf} = \frac{\rho_{nf} \mu_f}{(1 - \phi)^2} \) | Einstein [19] |
| Dynamic viscosity        | \( k_{nf} = \frac{\left[ (k_p + 2k_f) - 2\phi(k_f - k_p) \right]}{(k_p + 2k_f) + \phi(k_f - k_p)} k_f \) | Maxwell [20] |
In the present study, the nanoparticles were (TiO$_2$). Table 2 detailed all the needed thermophysical properties of (TiO$_2$) and (H$_2$O) needed in numerical calculations Sui, et al., [21].

Table 2. Specifications of thermophysical properties of Al$_2$O$_3$ nanoparticles and H$_2$O at T=293 k [21]

| Material | $C_p$ (J/kg K) | $\rho$(kg/m$^3$) | $\mu$(Pa.s) | $\beta$(1/k) | $\lambda$(w/m k) |
|----------|----------------|------------------|-------------|-------------|-----------------|
| TiO$_2$  | 686.2          | 4250             | //          | //          | 8.95            |
| Water    | 4182           | 998.2            | 993x10$^{-6}$ | 2.1x10$^{-4}$ | 0.597           |

The heat transfer and fluid flow parameter where the Nusselt number is given by [22]

$$Nu_s = \frac{aH}{\bar{f}}$$  \hspace{1cm} (8)

$$Nu = \frac{1}{L} \int_0^L Nu_s ds,$$  \hspace{1cm} (9)

$$\tau_w = \mu_n f (\partial u/\partial x + \partial v/\partial y)_{y=S(x)}$$  \hspace{1cm} (10)

And the Reynolds number is given by:

$$Re = \frac{u_{av}H}{\bar{f}},$$  \hspace{1cm} (11)

And skin friction factor calculated by:

$$C_f = \int_0^L \frac{2\tau_w}{\rho_n u_{av}^2} ds$$  \hspace{1cm} (12)

3. Numerical Solution

Finite Volume Method - FVM- is used for solving and discretization the physical governing equations along the computational domain of horizontal wavy duct with specific boundary conditions. In order to couple the system of pressure-velocity, a SIMPLE algorithm is utilized. The selected scheme for the convective terms is second order upwind scheme in order to achieve a more precise numerical solution. The suitable convergence criteria are acquired. The criteria of numerical solution convergence for the governing equations of continuity, energy and momentum equations are given by $10^{-6}$, $10^{-8}$, and $10^{-6}$, respectively.

4. Results and Discussions

The system of governing equations of (1-4) and boundary conditions (5-7) were modeled by using the Fluent package. Calculations were made for Re numbers range [0 ... 1250] and nanoparticles concentrations in the range zone [0 ... 0.4] with using four baffle height ratio $h/H=0.15$, 0.25, 0.35 and 0.45. The main objective of the work is studying the effects of concentration changes of nanoparticles on the thermal characteristics as follows from the formula for Nusselt number (Nu) and fluid flow characteristics as follows from the formula for skin friction factor ($C_f$), on which heat transfer effect on the transfer coefficient itself and temperature profile curvature (and wall gradient). Similarly, for friction factor, such factors are viscosity and velocity gradient. Fig.2 shows the dependence of the coefficient of thermal conductivity, density and dynamic viscosity upon the volume concentration of nanoparticles. Fig. 3 shows the dependence of the normalized temperatures ($\theta$) in relation to the clean liquid gradient temperature ($\theta$) and gradient speeds (U) (based on calculations). The relation between the nanoparticle concentration and the Nusselt number is plotted in Fig. 4. The results showed that the
Nusselt number is increased as the nanoparticle concentration increased, due to increasing the thermal conductivity of the fluids (water) and this will lead to increasing the rate of heat transfer. Moreover, Fig.4 revealed Nusselt number is increased with increasing the Reynolds number. However, the effect of adding the nanoparticles on the skin friction factor is plotted in Fig. 5. The figure concludes that the increasing the nanoparticle concentration will lead to increase the skin friction factor at constant Reynolds number, due to increasing the fluid viscosity by adding more nanoparticles to the fluid. In this study, the Ansys-fluent is used in order to simulate the fluid flow and heat transfer in the wavy duct as a contour of the velocity and temperatures. Fig. 6 presented the temperatures and velocity contours for two selected Reynolds number in order to assess the effect of increasing the Reynolds number. The results show that the increasing the Reynolds number will lead to increasing the thermal boundary layer along the upper wall of the duct and decreasing this layer along the lower wall of the duct due to increasing the secondary flow intensity with increasing the Reynolds number from Re=500 to 750. Also, the velocity circulation zone will increase as increasing the Reynolds number. The effect of increasing the nanoparticle concentration is discussed in Fig. 7. The result shows that the maximum temperature will decrease from 416.197 K to 415.754 K as increasing the nanoparticle concentration from 0.1 to 0.3 vol. %.

In the present study, the effect of using variable height baffle inserted in the wavy channel is presented in the Figures 8 to 12. The results show that the fluid flow and heat transfer is significantly affected by using the inserted baffle in the wavy channel, as shown in Fig. 8. Increasing of the baffle height ratio from 0.15 to 0.45, increases the maximum temperatures near the wall in front of the baffle region and grows the vortex near the baffle which lead to increasing the heat transfer in the wavy channel. On the other hand, the pressure drop increases rapidly with increasing the baffle height as plotted in Fig. 9. In the case of using TiO2 nanofluid with increasing the nanoparticles concentrations from the 0 to 0.3 vol.% as shown in Fig. 10 to 12. The results indicate that increasing of the nanoparticles concentrations leads to increase of the pressure drop in the wavy channel and there is a little effect on the temperatures contours, due to using the single-phase approximation instead of using the two phase nanofluid flow in the numerical analysis of the nanofluid flow. Finally, the secondary flow in caused of the baffle can be shown in the Fig. 13 which demonstrates the velocity vectors near the baffle inside the wavy channel.

![Figure 2. Dependence of nanofluid properties on the concentration of nanoparticles.](image-url)
Figure 3. Normalized gradient temperatures and velocity for $Re = 500$ and $Re = 1000$ at baffle height ratio ($h/H=0$).

Figure 4. The dependence of the average $Nu$ on the concentration of nanoparticles at baffle height ratio ($h/H=0$).

Figure 5. The dependence of average $Cf$ on the concentration of nanoparticles at baffle height ratio ($h/H=0$).

a) Temperatures distribution at $Re=500$

b) Temperatures distribution at $Re=750$
Figure 6. Velocity and temperatures contours distribution with different inlet Reynolds number at baffle height ratio h/H=0.

Figure 7. Velocity and temperatures contours distribution with different nanoparticles concentrations at baffle height ratio h/H=0.
Figure 8. Velocity and temperatures contours distribution with different baffle height ratio (h/H).
**Figure 9.** Effect of baffle height ratio on the pressure contours distribution.
Figure 10. Effect of nanoparticles concentrations and baffle height ratio on the pressure contours distribution.
Figure 11. Effect of nanoparticles concentrations and baffle height ratio on the temperatures contours distribution.
Figure 12. Effect of nanoparticles concentrations and baffle height ratio on the velocity contours distribution.
5. Conclusion

The present article is performed to analyze the effect of using of TiO$_2$-water nanofluids with different nanoparticles concentrations as a coolant fluid flow in a curved wavy duct. It is from marked that besides the direct influence on heat transfer coefficients (Nu) and skin friction factor (C$_f$) by influenced by the addition of nanoparticles leads to a change in the steepness wall temperature and speed profile. The results demonstrated that the increasing of the nanoparticles concentration will lead to increase the average number of Nu and less a significant increase in the coefficient of C$_f$. The heat transfer between nanofluid and wavy channel increases by inserting a baffle. The rate of heat transfer increase with increasing of height ratio of the baffle.

Nomenclature

| Symbol | Description |
|--------|-------------|
| A      | amplitude function |
| C$_f$  | friction coefficient |
| H      | channel height |
| h      | baffle height |
| L      | channel bending step |
| Pr     | Prandtl number |
| Re     | Reynolds number |
| T$_{in}$ | average temperature in the inlet section |
| T$_w$  | channel wall temperature |
| U$_{av}$ | average flow velocity in the channel |
| x, y   | coordinates |
| ϕ      | volume concentration of nanoparticles |
| ψ      | spatial basis function |

Subscript

| Symbol | Description |
|--------|-------------|
| f      | clear liquid |
| n$_f$  | nanofluid |
| p      | nanoparticles |
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