Weak redshift discretisation in the Local Group of Galaxies?

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Abstract. We discuss the distribution of radial velocities of galaxies belonging to the Local Group. Two independent samples of galaxies as well as several methods of reduction from the heliocentric to the galactocentric radial velocities are explored. We applied the power spectrum analysis using the Hann function as a weighting method, together with the jackknife error estimation. We performed a detailed analysis of this approach. The distribution of galaxy redshifts seems to be non-random. An excess of galaxies with radial velocities of $\sim 24 \, \text{km} \cdot \text{s}^{-1}$ and $\sim 36 \, \text{km} \cdot \text{s}^{-1}$ is detected, but the effect is statistically weak. Only one peak for radial velocities of $\sim 24 \, \text{km} \cdot \text{s}^{-1}$ seems to be confirmed at the confidence level of 95%.

Key words: galaxies: distances and redshift, — (galaxies): Local Group
1. Introduction

In the large-scale Universe, one aspect of the search for regularities is connected by testing the radial velocity of galaxies. These velocities can be observed as having arbitrary values, regular patterns regarded as periodisation, discretisation or quantization of galaxy redshifts. The discretisation of redshift for astronomical objects can be discussed independently for three cases, namely galaxies, quasars and large-scale periodicity (120 Mpc). The latter studies have been discussed (Bajan et al. 2003), so we will not repeat these investigations here. In our previous paper (Bajan et al. 2004), the quest for quasar redshift periodicity is described. Here, only the main points from its history, together with some recent results, will be mentioned.

The story of redshift periodisation started when Burbidge (1968) noted the existence of sharp peaks in redshift distribution, grouped around the values of $z = 0.01$ and $z = 1.95$. He also found periodicity in the redshift distribution, which can be described by the formula $z = 0.01 \cdot n$.

The existence of periodicity in the $\log(1 + z)$ scale was established by Karlsson (1971). In early stage of these investigations, association between bright low-redshift galaxies and quasars was sought in order to check for the non-cosmological origin of QSO redshifts, as well as to test for the possibility of ejection of QSO from parent galaxies, a problem investigated even now (Bell 2004). The link between GRBs and massive star formation is also suggested. Over the next years, the effect of periodisation was either confirmed or denied on the basis of incorrectly applied statistics and/or possible selection effect. The selection effects have been discussed by many authors. Some of them, (e.g. Karlsson 1971), claimed that the observed redshift distribution is not due to the selection effects, while other authors (e.g. Basu 2005) expressed the opposite opinion.

Burbidge and Napier (2001) took into account a new sample of high redshift quasars located in projection on celestial sphere close ($\leq 10''$) to the low redshift galaxies. The existence of periodicity in the $\log(1 + z)$ scale, was confirmed. The 2dF QSO Redshift Survey containing over 10000 objects and the 2dF Galaxy Redshift Survey with over 100000 galaxies served as an objective basis in the periodicity search for quasar-galaxy pairs. Having investigated 1647 objects, Hawkins, Maddox & Merrifield (2002) found no periodicity. Bell (2004) used two quasar samples, one with high redshifts of $2.4 - 4.8$ and the other with low redshifts of $0.02 - 0.2$. He showed that all peaks in these two redshift distributions occur at the previously predicted preferred values.

Napier & Burbidge (2001) re-examined the redshift distribution in the 2dF QSO Redshift Survey, detecting periodicity. Arp, Roscoe & Fulton (2005) showed that the maxima in redshift distribution in the 2dF and DSS surveys fit well into the formula. Bell’s (2004) investigation showed clear periodicity in the redshift distribution of QSO. The latest work on redshift periodicity was done by Basu (2005). He investigated the same 33 objects (GRBs, QSO and active...
galaxy) as Burbidge (2003) and contrary to Burbidge, he claimed that all existing peaks are due to observational and analytical selection effects.

The discussion of galaxy redshift quantization started with the work of Tifft (1976). He claimed that the redshifts of galaxies in the Coma cluster were preferentially offset from each other in multiples of $72.46 \text{ km} \cdot \text{s}^{-1}$. A few years later, the existence of global periodicity was reported (Tifft & Cocke 1984); however, the period was not $72 \text{ km} \cdot \text{s}^{-1}$, but $36 \text{ km} \cdot \text{s}^{-1}$ or possibly $24 \text{ km} \cdot \text{s}^{-1}$. It should also be noted that the values of $\sim 24 \text{ km} \cdot \text{s}^{-1}$, as reported by Tifft & Cocke (1984), were not confirmed by the later investigations.

We know that this subject is not popular and usually very suspicious at first glance. However, on the basis of the claimed results of redshift periodisation, several theoretical papers pointed out the necessity for so-called new physics were published.

Therefore, we decided to check if the discretisation does occur. We share the opinion expressed by Hawkins et al. (2002) that all these effects should be carefully checked. They claimed:

“The criticism usually leveled at this kind of study is that the samples of redshifts have tended to be rather small and selected in a heterogeneous manner, which makes it hard to assess their significance. The more cynical critics also point out that the results tend to come from a relatively small group of astronomers who have a strong prejudice in favour of detecting such unconventional phenomena. This small group of astronomers, not unreasonably, responds by pointing out that adherents to the conventional cosmological paradigm have at least as strong a prejudice towards denying such results.

We have attempted to carry out this analysis without prejudice. Indeed, we would have been happy with either outcome: if the periodicity were detected, then there would be some fascinating new astrophysics for us to explore; if it were not detected, then we would have the reassurance that our existing work on redshift surveys, etc, has not been based on false premises.”

Iwanowska (1989) claimed that the spatial distribution of galaxies belonging to the Local Group and globular clusters located close to our Galaxy is linear, i.e., these objects form long chains. For galaxies located in such lines, Zabierowski (1993) distinguished 5 different redshift groups finding that the observed periodicity is consistent with the Tifft’s (1976) value of $72 \text{ km} \cdot \text{s}^{-1}$. He counted mean velocities in each group, using old data (RC3 (de Vaucouleurs et al. 1991). Rudnicki, Godłowski & Magdziarz (2001) addressed the same problem, using better observational data. They considered 40 galaxies as well as globular clusters and performed a simple statistical analysis based on the calculation of the mean values of redshifts in bins and their dispersions. They tested strict quantization, that is precise multiplication of the value of $36 \text{ km} \cdot \text{s}^{-1}$, finding no effect. However, with the result of the power spectrum analysis, the weak effect of periodisation, i.e. the grouping around some values of galaxy radial velocities, was noted for galaxies situated in two of the Iwanowska’s lines.
2 OBSERVATIONAL DATA

The expected interpretation of galaxy redshift quantisation is that redshifts are not cosmological. Moreover, there are suggestions that clusters of quasars evolved into clusters of galaxies, so the galaxy redshift distribution should reflect this fact.

Tifft (1996) claimed that galaxy redshift distribution is quantised, because time is quantised. If this effect global, it means that it should be observed in all galaxy structures.

The number of galaxies in the Local Group is small but in the present work we consider all galaxies regarded as members of the Local Group (hereafter referred to as the LG) (Irwin, 2000; van den Bergh 1999). We applied the power spectrum analysis as the statistical tool in our investigation.

We used the standard method, i.e. power spectrum analysis, corrected for its possible disadvantages when applied to this problem. Moreover, we adopted a totally new approach. We use the Hann function as a weighting factor together with the jackknife error estimation, first used by Hawkins et al. (2002) (who investigated quasars, not galaxies). Because it is quite a new approach, we are enclosing several plots showing not only the results but also detailed analysing properties of the method. Such detailed analysis was not performed in the Hawkins et al. (2002) paper. Our result is that a weak effect of periodisation is observed for galactocentric velocities, while for heliocentric and LG-centric radial velocities no effect was observed.

The paper is organized in the following manner. The second section presents observational data, and the third section describes the method of analysis applied. The fourth section consists of our results. In the last section we give our conclusions.

2. Observational Data

There is no common agreement as to which galaxies belong to the dynamical aggregate called the Local Group (LG). We considered 55 objects (see Tables 1 and 2) in our vicinity taken from the Irwin’s list (Irwin 2000) (based on Mateo (1998)) together with 7 galaxies, mostly within the Maffei group, which could also probably be regarded as the LG members (Iwanowska, 1989) (marked in Tables 1 and 2 as M). Considering various parameters, van den Bergh (1999) concluded that only 32 objects can be LG members, while 3 further objects can be regarded as possible LG members. We decided to perform all calculations using these two sets of data. Sets based on the Irwin List were denoted as A, while those based on the van den Bergh’s (1999) were denoted as B.

It should be noted that in the van den Bergh list there are 7 galaxies (not including Phoenix) without redshift, while the Irwin’s list contains 2 more such objects (Cetus and Cam A). In this manner, 46 and 28 galaxies remain to be analysed in each sample respectively. We denoted these samples as I and II. Separately, we analysed pure Irwin data (39 galaxies) as sample III.

The search for any systematic effects in the distribution of redshifts requires precise knowledge of redshifts. There are some discrepancies among various redshift determinations. In order to avoid the influence of this factor on the results, we analysed separately samples with
radial velocities taken from the Irwin list (2000) (sample I) from those from van den Bergh’s (1999) (sample II). Whenever no data was available for sample I, we decided to use the data from RC3 (de Vaucouleurs et al. 1991).

The latest data allows us to find redshifts for 8 of the total 9 galaxies (marked as \( N \) in Tables 1 and 2), these redshifts were not known prior to these analysis. Redshifts for 7 of them are noted by Karachentsev et al. (2002). We repeated the analysis with these objects considered. However, we decided to exclude Tucana because its radial velocity measurement is uncertain. It should be noted that for the 6 remaining galaxies redshifts are also included in the new version of the Irwin’s list (sample IV). Moreover, for two galaxies, the new version of the Irwin’s list replaces the old redshift (NGC 147 157 km \( \cdot \) s\(^{-1} \) and NGC 221 190 km \( \cdot \) s\(^{-1} \)). In our analysis we do not include, Cam A either (noted in NED as uncertain, with no errors determined). Finally, adding 6 galaxies to the Irwin’s and the van den Bergh’s lists we obtain samples of 45 galaxies (sample IV) and 34 galaxies (sample V) respectively. A detailed list of galaxies is given in Tables 1 and 2. Column (10) gives heliocentric radial velocity for set A (samples I, III and IV), while column (11) for set B (samples II and V). As seen from the data, errors of measuring radial velocities are really small for the majority of galaxies.

The heliocentric radial velocities of galaxies should be corrected for the motion of the Sun relative to the center of our Galaxy and/or LG. This is usually done by applying the correction to the center of our Galaxy only. There are several prescriptions for how to perform this reduction. In this paper, we applied various galactocentric corrections known from literature. We analysed the following solar motions:

a): velocity \( v = 232 \text{ km} \cdot \text{s}^{-1} \) in the direction of \( l = 88^\circ, b = 2^\circ \) as proposed by Guthrie & Napier [1991] denoted as \( a \),

b): or \( v = 233 \pm 7 \text{ km} \cdot \text{s}^{-1} \) in the direction of \( l = 93^\circ \pm 10^\circ, b = 2^\circ \pm 5^\circ \) also proposed by Clube & Waddington [1989] denoted as \( b \),

c): and \( v = 213 \pm 10 \text{ km} \cdot \text{s}^{-1} \) in the direction of \( l = 93^\circ \pm 3^\circ, b = 2^\circ \pm 5^\circ \) according to Guthrie & Napier [1996] denoted as \( c \).

There was also a suggestion that the local standard of rest has an additional radial component directed outward our Galaxy (Clube & Waddington 1989). We decided to add this component into correction \( a \). In such a way, we obtained a new correction denoted as \( d \) with the following values: \( v = 234 \text{ km} \cdot \text{s}^{-1}, l = 98^\circ, b = 2^\circ \) (Guthrie & Napier 1991). Pure heliocentric data that is without any corrections hereafter denoted as \( e \) were analysed, too.

Moreover, the correction of the Sun’s motion with respect to the LG centroid was included (Coutreau & van den Bergh 1999). This gave the velocity of the motion of the Sun relative to the LG of \( v = 306 \pm 18 \text{ km} \cdot \text{s}^{-1} \) toward an apex at \( l = 99^\circ \pm 5^\circ \) and \( b = -3.5^\circ \pm 4^\circ \) (\( f \)). Two additional corrections for the motion of the Sun relative to the LG were also taken into account. The correction denoted as \( g \) with the value of \( v = 305 \pm 136 \text{ km} \cdot \text{s}^{-1} \) toward \( l = 94^\circ \pm 48^\circ, b = -34^\circ \pm 29 \) calculated (Rauzy & Gurzadyan 1998) from two subgroups (ours and that of M31) in the LG, and "the historical" velocity (Yahil, Tammann & Sandage...
1977), denoted as \( v = 308 \pm 23 \) km \( \cdot \) \( s^{-1} \) directed toward \( l = 105^\circ \pm 5 \) and \( b = -7^\circ \pm 4^\circ \) were used.

It is interesting to note that the spatial structure of the Galaxy is flat. Therefore, the correction to galactocentric radial velocities lies in the Galactic plane. Therefore, we checked how the situation changes when a fictitious perpendicular component of the Sun velocity vector is assumed. We considered a velocity of \( v = 224 \) km \( \cdot \) \( s^{-1} \) directed toward \( l = 109^\circ \) and \( b = 65^\circ \) as correction \( i \). Finally, the new proposed correction of the Sun’s motion (Karachentsev et al. 2002) in respect to the LG of \( v = 316 \) km \( \cdot \) \( s^{-1} \) toward the apex at \( l = 93^\circ \) and \( b = -4^\circ \), denoted as \( j \), was used. All of the corrections are summarized in Table 3.

3. Method of analysis

The power spectrum analysis (PSA) (Yu & Peebles 1969; Webster 1976; Guthrie & Napier 1990), together with the Rayleigh test (Mardia 1972; Batschelet 1981), has been used as a statistical tool. It was shown (Newman, Haynes & Terzian 1994) that this method is very useful for finding periodicity among irregularly distributed points. The Rayleigh’s test (Mardia, 1972; Batschelet 1981) is a simple test of uniformity which allows one to detect periodicities in irregularly distributed points. For a given frequency, the Rayleigh power spectrum corresponds to the Fourier power spectrum, as well as measuring the probability of the existence of a sinusoidal component.

Let us assume that \( m \) points are distributed along a finite line with coordinates \( x_i \), where \( i = 1, \ldots, m \). We can define the phase with respect to the period \( P \):

\[
\Phi_i = \frac{2\pi x_i}{P}.
\]  

(1)

The phase \( \Phi_i \) which corresponds to the \( i \)-th \( x \) coordinate unambiguously describes the radius vector \( I_i \).

The length of the vector \( R \), constituting the sum of all \( I_i \) vectors, can be used for testing isotropy of the distribution of points \( x_i \) with respect to the period \( P \)

\[
R(P) = \sum_{i=1}^{N} I_i(P).
\]  

(2)

From the formal point of view, the vector \( R \) describes a random walk in the plane after \( m \) steps.

We use the statistic \( R^2 \) defined by:

\[
R^2(P) = \left( \sum_{i=1}^{N} \cos \Phi_i \right)^2 + \left( \sum_{i=1}^{N} \sin \Phi_i \right)^2.
\]  

(3)

The probability distribution of \( R^2 \) can be calculated from the null hypothesis:

\[
p_i d\Phi = d\Phi/2\pi \quad \Phi \in (0, 2\pi),
\]  

(4)
where \( p_i(\Phi) d\Phi \) is the probability that phase \( \Phi_i \) corresponding to point \( x_i \) is located in the interval \((\Phi, \Phi + d\Phi)\). The distribution of \( R^2 \) corresponds to the Fourier power spectrum for function \( f(\Phi) = \sum \delta(\Phi - \Phi_i) \).

It is known that the variable:

\[
s(P) = 2R^2/m,
\]

has the following distribution (Webster 1976):

\[
p(s, m) ds = ds(m/4) \int_0^{\infty} J^m_0(\omega)J_0(\omega \sqrt{ms}/2) \omega d\omega,
\]

where \( J_0 \) is a Bessel function. This formula is also valid for a small number of objects, which is the case of the LG.

The distribution \( p(s, m) \) is calculated numerically by integrating approximations of the Bessel function using the Romberg method (Press et al. 1992). For large \( m \), it could be also approximated as a \( \chi^2 \) distribution with 2 degrees of freedom.

Error bars of \( s(P) \) can be estimated using the "jackknife" technique of drawing all possible samples of \( N - 1 \) values from \( N \) data points, repeating the power spectrum analysis on these samplings. Such a procedure allowed us to calculate the standard deviation in the derived values of \( s \sigma_j(P) \). The best estimator for the standard errors in the value of \( s \) is then just \( \sqrt{N - 1} \sigma_j \) (Hawkins et al. 2002).

The simulation of the power spectrum for random uniformly distributed data is presented in Figure 1. The diagrams showing the values of \( s \)-statistics versus \( 1/P \) present the results of the power spectrum analysis. There are several peaks in each diagram. These peaks allow one to find each possible period, as well as to investigate the significance of each particular peak in the power spectrum. The level of significance of each peak is given by \( C = 1 - (1 - p)^{n_t} \), where \( p \) is the probability of obtaining, from the theoretical, random distribution the value of the \( s \)-statistics equal to or greater than the observed value of the \( s \)-statistics, while \( n_t \) is the number of independent peaks within the analysed frequency range (Lake & Roeder 1972; Guthrie & Napier 1991).

The additional test increasing the efficiency of the test for weak clustering, following (Webster 1976; Scott 1991), is based on the summation over the whole power spectrum. This sum gives the value of \( SI = \sum s_i \), with a \( \chi^2 \) distribution with \( 2n_t \) degrees of freedom. Thus, the expected value of the \( SI \) statistic is \( 2n_t \). The \( SI \)-test can be used for testing the randomness of the distribution.

The clustering statistics \( Q \) is equal to the value of the \( SI \)-statistics over its expected value. The expected value for \( Q \) statistics is calculated in the case of a random walk \((2n_t)\). For a random distribution, the expected value of \( Q \) is equal to 1, with the error of: \( \sigma(Q) = 1/\sqrt{n_t} \) (Webster 1976). We tested the hypothesis that the value of \( Q \) is greater than unity rather than equal to this value. In our analysis, we decided to consider the first 50 peaks. In this case, at the significance level of 0.05, the critical value of the \( SI \) statistics is 124.3, with 135.8 for the significance level of 0.01, also note that \( \sigma(Q) = 0.14 \).
Newman et al. (1994) pointed out that Yu and Peebles’ version of PSA can be applied correctly only when a uniform distribution function is tested. As seen from Figure 2, such an approximation could be accepted for raw data but not when a correction for the solar motion is included.

Hawkins et al. (2002) discussed the power spectrum method when the data is not uniformly distributed. They proposed to use the window function and showed that the power spectrum method works well in that case. Now, the power of $s$ at the period of $P$ is given via the formulae (Hawkins et al. 2002):

$$s(P) = 2R^2 / \sum_{i=1}^{N} w_i^2,$$

(7)

where

$$R^2(P) = (\sum_{i=1}^{N} w_i \cos \Phi_i)^2 + (\sum_{i=1}^{N} w_i \sin \Phi_i)^2,$$

(8)

Following Hawkins et al. (2002), we repeat our analysis using the Hann’s function as a weight:

$$w_i = \frac{1}{2} \left[ 1 - \cos \left( \frac{2\pi x_i}{L} \right) \right]$$

where $L$ is chosen to cover the whole range over which the data is selected.

It should be pointed that now the expected value of the clustering statistics $Q$ is not necessarily equal to 1, especially for a small number of points. Therefore, we decided to test how the PSA with the Hann’s weighting function is working. The observed distribution of galactocentric redshift in the LG is normal (Fig. 2). Using the K-S tests, we checked that the distributions denoted as I and II and containing 46 and 28 galaxies respectively, are normal at the significance level of $\alpha = 0.05$. Thus, in the future analysis we use the Gaussian redshift distribution, as theoretically expected distribution.

We run 1000 simulations for data distributed normally, and with variance taken from the real data with correction $a$. We find that in the case of sample I (46 galaxies), the mean value of $Q$ is equal to 1, but $\sigma(Q) = 0.195$. For sample II (28 galaxies), the mean value of $Q$ decreases and is equal to 0.90, but $\sigma(Q) = 0.177$. This means that the noise value of $s = 2$ (see Hawkins et al. 2002) is slightly changed. The example of the simulation of the power spectrum for normally distributed data weighted using the Hann’s function is presented in Figure 3. One can see that when the data are apodized, the expected height of the peak decreases.

4. Result

We started our investigations with a classical power spectrum analysis (Yu & Peebles 1969; Webster 1976). The two basic samples of galaxies (based on sets $A$ and $B$), containing 46 and 28 objects each, denoted as sample I and II respectively, were analysed. For each sample, 10 different reductions of heliocentric radial velocities to galactocentric velocities or a LG
centroid, were performed. In Figure 4 the power spectra of sample I analysed as $s$ versus $1/P$ diagrams in the range of $1/P \in (0.0025, 0.05)$ is shown. The visual impression is that some peaks are clearly distinguishable, which can be regarded as concentration around some particular values of the period. These values are close to 36 km·s$^{-1}$, and 24 km·s$^{-1}$. However, this impression is not confirmed by statistical analysis.

The result of our analysis is given in Table 4 (sample I) (and Table 5 for sample II respectively). The first column of both tables describes the analysed sample, while the next two give the values of the velocity $v$ at which the most prominent peak was observed, together with the probability $P(s)$ that the peak is generated by a random distribution (when we restrict for period $P \in (20, 100)$ km·s$^{-1}$).

From Table 4, it is easy to see that the most prominent peaks are statistically not significant, at the significance level of 95%. Therefore, we concluded that in random distribution such peaks can also appear. The power spectrum analysis shows that the peaks observed in the period distributions are consistent with the assumption of randomness.

The second aim of the power spectrum analysis was to check for the possibility of weak clustering using $SI$ and $Q$-statistics. The result of our analysis is also given in Table 4. The last two columns of the table contain the values of $SI$ and $Q$ statistics. It can be easily seen that in the case of galactocentric correction prescriptions $a$, $b$, $c$, both $SI$ and $Q$-statistics confirm the hypothesis that the redshift distributions are non-random at the confidence level of 95%. For prescriptions $a$ and $b$ this seems to be confirmed even at the level of 99%.

For the prescription $d$ computed using the probable velocity of the local rest frame, we do not find any departure from isotropy. A comparison with the previous cases allows us to state that this vector does not correspond to reality. The reduction to the LG centroid, using any of our vectors, did not reveal any statistically significant individual preferred values of velocities and showed that the distributions are random.

The high value of $s$ given by the PSA is expected at scales greater than the clustering scale (Yu & Peebles 1969; Scott 1991). We deal with one structure (Local Group) only, so this is not the cause of the reported non-randomness. Nevertheless, we noted that from the theoretical point of view the non-random distribution detection by the PSA can be due to the inclusion of some external objects. In order to examine this possibility, we performed three tests:

i) we consider sample II, from which all objects with untypical values of redshift were excluded (van den Bergh 1999),

ii) we repeat our analysis of sample I without the Argo galaxy, which is its most pronounced external member,

iii) because the artificially high values of $s$ given by the PSA are observed mainly in the first mode (Webster 1976), for the period we perform the PSA also in the interval $i = 2, \ldots, 51$ instead of the first 50 peaks (this is also the reason why we restrict our analysis to $P \in (20, 100)$ km·s$^{-1}$).
All these changes alter the statistics values, but for the prescriptions $a$ and $b$, the PSA distribution remains non-random.

Karachentsev et al. (2002) showed that, for galaxies belonging to the Local Group, errors in heliocentric velocities are generally $\pm 5 \text{ km} \cdot \text{s}^{-1}$ or smaller. This is true for galaxies belonging to sample II. However, it should be noted that some of the galaxies belonging to sample I have large errors in their reported radial velocities. This is the case for the sample of 7 galaxies (marked in Tables 1 and 2 as $M$) which Iwanowska (1989) regarded as the probable LG members (although this is still very questionable). It should be pointed out that large errors in the data can mask any existing periodisation (however, this cannot be a source of a fictitious signal in PSA). Now we repeated our analysis for the pure Irwin’s list, without those 7 galaxies (sample III). The result (Table 5) is similar to that obtained for sample I but the value of statistics $SI$ for reduction $a$, $b$ and $c$ is even larger than in sample I. This confirms that these 7 galaxies denoted as $M$ in Tables 1 and 2 should be rejected from further analysis.

When we add 6 new galaxies (marked in Tables 1 and 2 as $N$), we obtain (for samples IV and V analysed now) that the PSA distribution clearly remains non-random for prescriptions $a$ only; however a weak effect survives for prescriptions $b$ and $c$ as well (see Table 5).

Still, to be sure that the obtained effect is real, we should take into account the fact that our data are not uniformly distributed. In that case (Hawkins et al. 2002), the PSA must take into account the data window function. Therefore, we repeated our analysis using the Hann’s function as a weighting function (Hawkins et al. 2002). For sample II, we clearly obtain that the distribution of the PSA remains non-random for reduction $a$ (Table 6, statistic $Q$). Figure 5 also shows that, for prescriptions $c$, the most prominent peak is significant at about $2\sigma$ level above the noise value. For prescriptions $a$ and $b$, the significance of the most prominent peak is reduced below the $2\sigma$ level. A similar situation is seen in Figure 5 for sample V, where for prescriptions $a$ and $c$ the peak is significant at about $2\sigma$ level. It should be noted that the PSA with an appropriate window function does not show any signs of periodicity in other cases. This suggests significant periodisation close to $24 \text{ km} \cdot \text{s}^{-1}$.

Moreover, it should be pointed out that using the window function we change not only the expected value of the clustering statistics $Q$, but also the expected height of the peak. As a result, the probability $P(s)$ formally obtained on the basis of Equation 6 (presented in Table 6) is not valid any longer. Again we run 1000 simulations for the normally distributed data with variance taken from the real data. The histogram for the distribution of the height of the most significant peak with data weighted using the Hann’s function for sample II (28 galaxies) with variance taken from the real data with correction $a$, is presented in the left panel of Figure 6, the cumulated distribution is presented in the right panel. We can see that peaks with $2R^2/m \geq 8.2$ are significant at the 95% level. Please note that, at least for prescription $c$, for samples II and V the most pronounced peaks are significant at the level of 95%. A similar situation occurs for sample V, prescription $a$. This confirms weak periodisation close
5 CONCLUSIONS

to $24 \text{ km} \cdot \text{s}^{-1}$ and is in contrast with the previous cases (without using the data window function), where all peaks were not significant.

5. Conclusions

In our analysis, we considered the nearby galaxies which are widely accepted to be LG members. Moreover, we defined a sample of galaxies according to several physical properties and not only to spatial distribution, as belonging to the LG. We took into account several possible ways of correcting the heliocentric velocity to the galactocentric velocity. The correction to the LG centroid was also considered. We used the best data currently available.

The preliminary statistical analysis excluded the possibility of strict redshift quantization in the LG (Rudnicki et al. 2001). The lack of strict multiplicity of the value of $36 \text{ km} \cdot \text{s}^{-1}$ based on good data does not confirm Zabierowski’s claim (Zabierowski 1993) to this effect.

We obtained that distributions of galaxy redshift seem to be non-random when the correction for the motion of the Sun relative to the center of our Galaxy is taken into account. The argument revealing a possible significance of the result is the fact that it is practically independent of the samples. There is some periodisation though which in some cases is close to $36 \text{ km} \cdot \text{s}^{-1}$, while in other cases it is close to $24 \text{ km} \cdot \text{s}^{-1}$. However, this effect is statistically weak. The statistical significance of our result as seen from the Tables is rather low.

The most important argument is that the result survives the PSA when the data window function is taken into account. This provides that the effect found is real. However, only periodisation close to $24 \text{ km} \cdot \text{s}^{-1}$ seems to be confirmed at the significance level of 95%. One can see that the strongest effect is obtained for galaxies from the van den Bergh’s list. Van den Bergh (1999) concluded that only 32 objects can be LG members, while 3 further objects can be regarded as possible LG members. The Irwin’s list contains more galaxies for whom membership in the LG is questionable.

With correction for the Sun’s motion with respect to the LG centroid, we obtained no periodisation. We did not obtain any statistically significant preferred values of velocities in this case. It is interesting to note that the spatial structure of the Galaxy is flat. Therefore, the correction to the galactocentric radial velocities lies in the galactic plane. We also checked how the situation changes when a fictitious perpendicular component of the Sun’s velocity vector is assumed. In this case there is no periodisation. This result, together with the analysis with prescription $e$ (pure heliocentric velocity) shows that the effect is diluted for an artificial value of this vector. Thus, this provides evidence that the observed weak periodisation is not an artifact of the galactocentric correction.

The latest data allows us to include redshifts for 6 galaxies determined by Karachentsev et al (2002). When we repeated the analysis with these objects, the effect decreased. However, it should be pointed out that these objects are connected with M31 (and they are probably M31
satellites). Thus, any possible periodisation could be stronger than for other galaxies masked by the local influence.

The interpretation of this result is neither unique nor clear. We would like to point at the following possibilities:

1. The effect of periodisation of galaxies in the LG can be connected artificially with the manner of galactocentric correction due to either velocity determination or to the data itself but the above-mentioned diminishing of non-randomness when artificial velocities are used does not seem to support this conclusion.

2. Periodisation is found only when galactocentric velocities are included, and it is not connected with the LG centroid. This is possible, for example, when the effect is global as claimed by Tifft and it is not connected with the LG.

3. Periodisation is a relic of quantum effects in the early Universe (Tifft 1996). Thus, the effect is not connected with the LG, which was formed during much later epoch.

4. Periodisation is a real phenomenon, also connected with the LG, but the correct LG structure is not known at present.

Clearly, power spectrum analysis is a good tool for studying non-randomness in velocity data. The main result of the paper is that the distributions of galaxy redshift seem to be non-random, but each specific individual particular value of periodisation is statistically marginal. Only periodisation close to $24 \, \text{km} \cdot \text{s}^{-1}$ seems to be confirmed, and only at the significance level of 95%. One of the reasons is that errors in the data are large. However, it should be noted that in the version of the PSA considered, non-systematic errors can not produce "false periodisation" but only "destroy" any real existing effect. Measurement errors will be important if explicitly taken into account. In such a version of PSA, we analysed the sample from the point of view of the objects likelihood rather than using its best values. However, such a procedure changes the initial assumptions of the method as well as the analysed statistics, therefore it is not considered in our present paper. It will be analysed in our future work. Further investigation should concern such large-scale structures as the Local Supercluster, Coma/A1367, the Perseus or Hercules Superclusters, when more accurate HI velocity data become available.

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**References**

Arp, H., Roscoe, D., Fulton C., 2005. astro-ph 0501090
REFERENCES

Bajan, K., Biernacka, M., Flin, P., Godlowski, W., Piervushin, V., Zorin, P., 2003 Spacetime & Substance 4, 225
Bajan, K., Flin, P., Godlowski, W., Piervushin, 2004 Physics of Particles and Nuclei 35, 1
Basu, D., 2005. ApJ 618 L71
Batschelet, E., 1981. Circular Statistics in Biology, Academic Press, London
Bell, M.B., 2004. ApJ 616, 738
van den Bergh, S., 1999. A&Arv 9, 273
Burbidge, G., 1968. ApJ 154 L41
Burbidge, G., 2003. ApJ 585 112
Burbidge, G., Napier, W.M., 2001 AJ 121 21
Clube, S.V.M., Waddington, W.G., 1989. MNRAS 237, 7
Coutreau, S., van den Bergh, S., 1999. AJ 118, 337
Guthrie, B.N.G., Napier, V.M., 1990. MNRAS 243, 431
Guthrie, B.N.G. Napier, W.M., 1991. MNRAS 253, 533
Guthrie, B.N.G. Napier, W.M., 1996. Astr.Astroph. 310, 353
Hawkins, E., Maddox, S.J., Merrifield M, R., 2002 MNRAS 336 L13
Irwin, M., 2000. [http://www.ast.cam.ac.uk/~mike/local_members.html](http://www.ast.cam.ac.uk/~mike/local_members.html)
Iwanowska, W., 1989. In: From Stars to Quasars, eds. S. Grudzinska & B.Krygier, Nicolaus Copernicus University Press, Torun, p. 159
Karachentsev, I.D., et al. 2002. A&A 389, 812
Karlsson, K.G., 1971 Astrn. Astroph 13, 333
Karlsson, K.G., 1990 Astrn. Astroph 239, 50
Lake, R.G., Roeder, R.C., 1972. Jour. RASC 66, 111
Mardia, K.V. 1972. Statistics of Directional Data, Academic Press, London
Mateo, M., 1998. A&ARv 36, 435
Napier, W.M., Burbidge, G., 2003. MNRAS 342, 601
Newman, W.I., Haynes, M.P., Terzian, Y., 1994. ApJ 431, 147
Press, W.H., Teukolsky, S.A., Vetterling, W.T., Flannery, B.P., 1992. Numerical recipes, Cambridge University Press., Cambridge, UK
Rauzy, S., Gurzadyan, V.G., 1998. MNRAS 298, 114
Rudnicki, K., Godlowski, W., Magdziarz, A., 2001. In: Gravitation, Electromagnetism and Cosmology. Toward a New Synthesis, ed. K.Rudnicki, Montreal, Apeiron, p. 133
Scott, D., 1991. Astr.Astroph. 242, 1
Tifft, W.G., 1976. ApJ 206, 38
Tifft, W.G., Cocke, W.J., 1984. ApJ 287, 492
Tifft, W.G., 1996. ApJ 468, 491
de Vaucouleurs, G., de Vaucouleurs, A., Corwin, H.G., Buta, R.J., Paturel, G., Fouque, P., 1991. Third Reference Catalogue of Bright Galaxies, Springer Verlag, New York (RC3)
Webster, A., 1976. MNRAS 175, 61
Yahil, A., Tammann, G.A., Sandage, A., 1977. ApJ 217, 903
Yu, J.T., Peebles, P.J.E., 1969. ApJ 158, 103
Zabierowski, M., 1993. In: Progress in New Cosmologies: Beyond the Big Band eds H. Arp, C.R. Keys, K. Rudnicki, Plenum Press, New York, p.71
**Fig. 1.** Example of simulation of the raw power spectrum of a random uniform distribution — sample II (28 galaxies). Dashed lines show errors derived by applying the jackknife estimator.

**Table 3.** Motion of the Sun.

| Sample | $V_h$ | $l$ | $b$ |
|--------|-------|-----|-----|
| $a$    | 232   | 88  | 2   |
| $b$    | 233   | 93  | 2   |
| $c$    | 213   | 93  | 2   |
| $d$    | 234   | 98  | 2   |
| $e$    | 0     | 0   | 0   |
| $f$    | 306   | 99  | -3.5|
| $g$    | 305   | 94  | -34 |
| $h$    | 308   | 105 | -7  |
| $i$    | 224   | 109 | 65  |
| $j$    | 316   | 93  | -4  |

**Fig. 3.** Example of simulation of the power spectrum for normal distribution derived with data weighted using the Hann’s function — sample II (28 galaxies). Dashed lines show errors derived by applying the jackknife estimator.

**Table 4.** Statistical analysis of samples investigated in the Local Group (Sample I)

| Sample | $V$ | $P(s)$ | $SI$ | $Q$ |
|--------|-----|--------|------|-----|
| $1a$   | 23.8| 0.232  | 139.4| 1.394|
| $1b$   | 25.7| 0.151  | 135.9| 1.359|
| $1c$   | 31.2| 0.351  | 131.2| 1.312|
| $1d$   | 25.6| 0.477  | 113.8| 1.138|
| $1e$   | 26.8| 0.081  | 110.3| 1.103|
| $1f$   | 37.0| 0.350  | 88.6 | .886 |
| $1g$   | 37.2| 0.561  | 108.1| 1.081|
| $1h$   | 56.5| 0.922  | 101.1| 1.011|
| $1i$   | 36.2| 0.439  | 113.6| 1.136|
| $1j$   | 41.5| 0.809  | 109.9| 1.099|
Fig. 2. Redshift distribution histogram for raw data (left side), and those corrected for the solar motion (correction a).
Fig. 4. Raw power spectrum of sample I (46 galaxies). Dashed lines show errors derived by applying the jackknife estimator.
Fig. 5. Power spectrum of sample II (28 galaxies), III (39 galaxies) and V (34 galaxies) after apodization with the Hann’s function.
Fig. 6. Histogram for distribution of the height of the most significant peak with data weighted using the Hann’s function — sample II (28 galaxies) (left panel), and cumulated distribution of the height of the most significant peak with data weighted using the Hann’s function — sample II (28 galaxies) (right panel).
| Nr | PGC | NGC | name | morf. | $\alpha_{1950.0}$ | $\delta_{1950.0}$ | $l$  | $b$ | $v_l$ | $v_B$ |
|----|-----|-----|------|-------|------------------|------------------|-----|-----|------|------|
| 1  | 1305|      | IC 10| Ir IV | 001742           | +590052          | 118.97|−03.34|−344±4|−344±5|
| 2  |     |      |      |       | 002336           | −111900          | 101.40|−72.80|       |       |
| 3  | 2004| 147 | DDO 3| dE5  | 003027           | +481356          | 119.82|−14.25|−193±3|−193±3|
| 4  | 2121|      | And III| Dsph | 003242           | +361400          | 119.34|−26.25|−355±10|−355±10|
| 5  | 2329| 185 |      | dE3p | 003612           | +480350          | 120.79|−14.48|−208±7|−202±7|
| 6  | 2555| 221 | M32  | E2   | 003658           | +403529          | 121.15|−21.98|−205±3|−205±3|
| 7  | 2429| 205 |      | E6p  | 003739           | +412444          | 120.72|−21.14|−239±3|−244±3|
| 8  | 2557| 224 | M31  | Sb I-II| 004000       | +405943          | 121.17|−21.57|−299±1|−301±1|
| 9  | 2666|      | And I| Dsph | 004248           | +374600          | 121.65|−24.82|−380±2|−380±2|
| 10 | 3085|      | SMC  | Ir IV-V| 005053     | −730418          | 302.81|−44.33|163±4 |148±4 |
| 11 | 3589|      | Scl  | Dsph | 005747           | −335842          | 287.53|−83.16|107±3 |110±3 |
| 12 | 3792|      | LGS 3| DIf  | 010112           | +213700          | 126.75|−40.89|−277±5|−286±5|
| 13 | 3844|      | IC 1613| Ir V  | 010220           | +015156          | 129.79|−60.56|−236±1|−232±1|
| 14 | 4126| 404 |      | S0   | 010639           | +352706          | 127.03|−27.01|−45±9 |       |
| 15 |     |     |      |      | 010718           | +472200          | 126.20|−15.10|−403±4|−403±4|
| 16 | 4601|      | And II| Dsph | 011336           | +331100          | 128.89|−29.14|−188±3|−188±3|
| 17 | 5818| 598 | M33  | Sc II-III| 031302     | +302415          | 133.61|−31.33|−180±1|−181±1|
| 18 |     |     | Phoenix| Ir  | 014900           | −444200          | 272.19|−68.95|56±3 |56±6 |
| 19 | 9892|      | Maffei 1| E | 023236           | +592600          | 135.83|−00.57|2±7  |       |
| 20 | 10093|     | Fornax| Dsph | 023755           | −343948          | 237.10|−65.65|53±3 |53±2 |
| 21 | 10217|     | Maffei 2| Sbc | 023808           | +592324          | 136.50|−00.33|−1±6 |       |
| 22 | 15345| 1569|      | Ir  | 042606           | +644418          | 143.68|+11.24|−77±6|       |
| 23 | 15488| 1560|      | Sd  | 042708           | +714629          | 138.37|+16.02|−40±7|       |
| 24 | 15439|     | UGCA 92| Ir | 042724           | +633000          | 144.71|+10.51|−105±5|       |
| 25 |     |     | Cam A| Ir  | 043130           | +712500          | 138.88|+16.04|−127±7|       |
| 26 | 17223|     | LMC  | Ir III-IV| 052400     | −694800          | 280.47|−32.89|272±8 |275±2 |
| 27 | 19441|     | Carina| Dsph | 064024           | −505500          | 260.11|−22.22|223±3 |223±3 |
| 28 |     |     | ARGO | Ir  | 070430           | −582700          | 268.96|−21.15|554±10|       |
### Table 2. List of galaxies.

| Nr | PGC | NGC | name          | morf | α<sub>1950.0</sub> | δ<sub>1950.0</sub> | l   | b   | v<sub>i</sub> | v<sub>B</sub> |
|----|-----|-----|---------------|------|---------------------|---------------------|-----|-----|-------------|-------------|
| 29 | 21600 |     | DDO 47 Ir     |      | 073900              | +165500             | 203.10 | +18.54 | 270 ± 4 |             |
| 30 | 28868 |     | Leo A Ir V    |      | 095624              | +305900             | 196.90 | +52.41 | 26 ± 2  | 24 ± 4     |
| 31 | 28913 |     | Sextans B     |      | 095723              | +053422             | 233.20 | +43.78 | 301 ± 2 |             |
| 32 | 29128 | 3109 | DDO 236 Ir    |      | 100049              | −255500             | 262.10 | +23.07 | 403 ± 1 |             |
| 33 |      |     | Antlia dE3    |      | 100148              | −270500             | 263.10 | +22.32 | 361 ± 2 |             |
| 34 | 29488 |     | Leo I Dsph    |      | 100547              | +123310             | 225.98 | +49.11 | 285 ± 2 | 287 ± 5    |
| 35 | 29653 |     | Sextans A     |      | 100830              | −042800             | 246.17 | +39.86 | 325 ± 3 |             |
| 36 |      |     | Sextans dE4   |      | 101049              | −012400             | 243.55 | +42.27 | 224 ± 2 | 226 ± 1    |
| 37 | 34176 |     | Leo II Dsph   |      | 111050              | +22532              | 220.17 | +67.23 | 76 ± 2  | 76 ± 1     |
| 38 | 35286 |     | UGC 6456 P    |      | 112436              | +791600             | 127.84 | +37.33 | −92 ± 5 |             |
| 39 | 39346 | 4236 | Sdm           |      | 121422              | +694436             | 127.43 | +47.36 | 0 ± 4   |             |
| 40 | 44491 |     | Gr 8 Ir       |      | 125606              | +142900             | 310.72 | +76.98 | 216 ± 3 |             |
| 41 | 50961 |     | DDO 187 Ir    |      | 141336              | +231700             | 25.57  | +70.47 | 154 ± 4 |             |
| 42 | 54074 |     | UMi Dsph     |      | 150812              | +672300             | 104.97 | +44.84 | −250 ± 2 | −247 ± 1   |
| 43 | 60095 |     | Draco Dsph    |      | 171924              | +575750             | 86.37  | +34.72 | −289 ± 2 | −293 ± 1   |
| 44 |      |     | Galaxy Sbc    |      | 174224              | −285550             | 0.00   | +00.00 | 0 ± 10  | 16 ± 0     |
| 45 |      |     | Sagittar dE7  |      | 185154              | −303000             | 5.65   | −14.08 | 140 ± 5 | 142 ± 1    |
| 46 | 63287 |     | Sgr Ir        |      | 192706              | −174700             | 21.06  | −16.29 | −79 ± 2 | −79 ± 4    |
| 47 | 63613 | 6822 | DDO 209 Ir IV-V|     | 194208              | −145529             | 25.34  | −18.40 | −49 ± 6 | −56 ± 2    |
| 48 | 65367 |     | Aqr Ir        |      | 204406              | −130200             | 34.05  | −31.35 | −131 ± 5 | −131 ± 5   |
| 49 | 67908 |     | IC 5152 Ir IV |      | 215926              | −513218             | 343.92 | −50.19 | 121 ± 4 |             |
| 50 |      |     | Tucana dE5    |      | 223830              | −644100             | 322.90 | −47.37 | 130 ± 2 | 130 ± 2 N  |
| 51 |      |     | UKS2323-     |      | 231438              | −324000             | 11.86  | −70.86 | 62 ± 6  |             |
| 52 |      |     | AND VII dE3  |      | 232412              | +302500             | 109.50 | −09.90 | −307 ± 2 | −307 ± 2 N |
| 53 | 71538 |     | Peg Ir        |      | 232603              | +142816             | 94.77  | −43.55 | −181 ± 2 | −182 ± 2   |
| 54 |      |     | AND VI dE3   |      | 234912              | +241800             | 106.00 | −36.30 | −354 ± 3 | −354 ± 3 N |
| 55 | 143  |     | WLM Ir IV-V   |      | 235923              | −154343             | 75.87  | −73.61 | −116 ± 2 | −116 ± 2   |
Table 5. Statistical analysis of samples investigated in the Local Group (Samples II - V)

| Sample | V   | $P(s)$ | $S_I$ | Q    | Sample | V   | $P(s)$ | $S_I$ | Q    |
|--------|-----|--------|-------|------|--------|-----|--------|-------|------|
| 2a     | 33.3| 0.219  | 143.1 | 1.431| 4a     | 28.4| 0.054  | 141.9 | 1.419|
| 2b     | 36.8| 0.229  | 142.2 | 1.422| 4b     | 28.4| 0.076  | 118.2 | 1.182|
| 2c     | 20.6| 0.508  | 115.3 | 1.153| 4c     | 31.3| 0.323  | 126.9 | 1.269|
| 2d     | 37.5| 0.907  | 99.8  | .998 | 4d     | 20.1| 0.584  | 105.0 | 1.050|
| 2e     | 51.8| 0.363  | 98.7  | .987 | 4e     | 27.2| 0.154  | 95.0  | .950 |
| 2f     | 56.3| 0.595  | 103.8 | 1.038| 4f     | 37.2| 0.354  | 107.9 | 1.079|
| 2g     | 43.3| 0.125  | 95.5  | .955 | 4g     | 37.8| 0.472  | 114.3 | 1.143|
| 2h     | 55.7| 0.595  | 96.4  | .964 | 4h     | 52.6| 0.996  | 104.2 | 1.042|
| 2i     | 31.2| 0.235  | 115.4 | 1.154| 4i     | 29.6| 0.053  | 114.3 | 1.143|
| 2j     | 20.0| 0.482  | 95.7  | .957 | 4j     | 25.7| 0.217  | 115.4 | 1.154|
| 3a     | 23.6| 0.156  | 147.4 | 1.474| 5a     | 23.5| 0.558  | 130.0 | 1.300|
| 3b     | 36.8| 0.241  | 135.7 | 1.357| 5b     | 27.6| 0.159  | 119.8 | 1.198|
| 3c     | 23.2| 0.203  | 145.3 | 1.453| 5c     | 20.4| 0.375  | 105.5 | 1.055|
| 3d     | 25.4| 0.697  | 106.5 | 1.065| 5d     | 29.9| 0.766  | 99.0  | .990 |
| 3e     | 26.9| 0.100  | 88.5  | .885 | 5e     | 57.8| 0.168  | 91.3  | .913 |
| 3f     | 48.0| 0.135  | 103.0 | 1.030| 5f     | 29.5| 0.517  | 104.5 | 1.045|
| 3g     | 37.3| 0.370  | 104.9 | 1.049| 5g     | 43.7| 0.105  | 91.4  | .914 |
| 3h     | 52.5| 0.885  | 107.1 | 1.071| 5h     | 55.7| 0.839  | 100.2 | 1.002|
| 3i     | 29.7| 0.075  | 116.8 | 1.168| 5i     | 47.6| 0.588  | 115.7 | 1.157|
| 3j     | 25.6| 0.809  | 108.5 | 1.085| 5j     | 20.0| 0.548  | 94.4  | .944 |
Table 6. Statistical analysis of samples investigated in the Local Group (with data weighted using the Hann’s function).

| Sample | V    | $P(s)$ | SI   | Q    |
|--------|------|--------|------|------|
| 2a     | 32.9 | 0.353  | 138.1| 1.381|
| 2b     | 36.9 | 0.367  | 98.3 | .983 |
| 2c     | 20.6 | 0.195  | 116.7| 1.167|
| 2d     | 33.5 | 0.802  | 75.9 | .759 |
| 2e     | 51.9 | 0.230  | 96.4 | .964 |
| 2f     | 30.5 | 0.985  | 67.7 | .677 |
| 2g     | 43.4 | 0.341  | 70.2 | .702 |
| 2h     | 77.8 | 0.913  | 90.8 | .908 |
| 2i     | 31.8 | 0.551  | 86.9 | .869 |
| 2j     | 35.1 | 0.943  | 66.4 | .664 |
| 3a     | 81.3 | 0.697  | 73.9 | .739 |
| 3b     | 100.0| 0.990  | 67.5 | .675 |
| 3c     | 84.7 | 0.968  | 75.1 | .751 |
| 3d     | 23.1 | 0.492  | 85.3 | .853 |
| 4a     | 28.5 | 0.639  | 80.9 | .809 |
| 4b     | 20.4 | 0.768  | 68.3 | .683 |
| 4c     | 20.1 | 0.816  | 75.8 | .758 |
| 4d     | 22.9 | 0.733  | 71.6 | .716 |
| 5a     | 27.4 | 0.257  | 107.3| 1.073|
| 5b     | 36.5 | 0.729  | 87.3 | .873 |
| 5c     | 20.4 | 0.219  | 101.5| 1.015|
| 5d     | 21.3 | 0.834  | 82.0 | .820 |