Generic Formula of Soft Scalar Masses in String Models

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Abstract

We derive formula of soft supersymmetry breaking scalar masses from 4-dimensional string models within a more generic framework. We consider effects of extra gauge symmetry breakings including an anomalous $U(1)$ breaking through flat directions, that is, $D$-term and $F$-term contributions, particle mixing effects and heavy-light mass mixing effects. Some phenomenological implications are discussed based on our mass formula.
1 Introduction

Superstring theories (SSTs) are powerful candidates for the unification theory of all forces including gravity. There are various approaches to explore 4-dimensional (4-D) string models, for example, the compactification on Calabi-Yau manifolds [1], the construction of orbifold models [2, 3] and so on. The effective supergravity theories (SUGRAs) have been derived based on the above approaches [1, 2, 3]. The structure of SUGRA [7] is constrained by considering field theoretical non-perturbative effects such as a gaugino condensation [8] and stringy symmetries such as duality [9] besides of perturbative results.

Effective theories, however, have several problems. First, there are thousands of effective theories corresponding to 4-D string models. They have, in general, large gauge groups and many matter multiplets compared with those of the minimal supersymmetric standard model (MSSM). We have not known how to select a realistic model among them from stringy theoretical point of view yet. Another serious problem is that the mechanism of supersymmetry (SUSY) breaking is unknown. To solve these problems, non-perturbative effects in SSTs and SUSY field theories should be fully understood.

At the present circumstance, the following approaches and/or standpoints have been taken. For the first problem, study on flat directions is important [12]. Because effective theories have, in general, flat directions in the SUSY limit. Large gauge symmetries can break into smaller ones and extra matter fields can get massive through flat directions. Further flat directions could relate different models in string vacua. Actually some models with realistic gauge groups and matter contents have been constructed based on $Z_3$ orbifold models [13]. Recently generic features of flat directions in $Z_{2n}$ orbifold models have been also investigated [14].

The flat directions based on $Z_3$ orbifold models have been analyzed considering the existence of anomalous $U(1)$ symmetry ($U(1)_A$) because 4-D string models, in general, have the $U(1)_A$ symmetry. Some interesting features are pointed out in those models. For example, Fayet-Iliopoulos $D$-term [13] is induced at one-loop level for $U(1)_A$ [15]. As a result, some scalar fields necessarily develop vacuum expectation values (VEVs) and some gauge

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* Recently there have been various remarkable developments in study on non-perturbative aspects of SSTs and SUSY models [1, 2].
† Some conditions for absence of anomalous $U(1)$ are discussed in ref. [17].
Symmetries can break down \([12, 13]\).

For the second problem, some researches have been done from the standpoint that the origin of SUSY breaking is unspecified. That is, soft SUSY breaking terms have been derived under the assumption that SUSY is broken by \(F\)-term condensations of the dilaton field \(S\) and/or moduli fields \(T\) \([18, 19, 20]\). Some phenomenologically interesting features are predicted from the structure of soft SUSY breaking terms which are parameterized by a few number of parameters, for example, only two parameters such as a goldstino angle \(\theta\) and the gravitino mass \(m_{3/2}\) in the case with the overall moduli and the vanishing vacuum energy \([21]\). The cases with multimoduli fields are also discussed in Refs.\([22]\). Recently study on soft scalar masses has been extended in the presence of an anomalous \(U(1)\) symmetry \([23, 24, 25]\).

This strategy for string phenomenology is quite interesting since the soft SUSY breaking parameters can be powerful probes for physics beyond the MSSM such as SUSY-grand unified theories (SUSY-GUTs), SUGRAs and SSTs. We give two examples. The pattern of gauge symmetry breakdown can be specified by checking certain sum rules among scalar masses. The specific mass relations are derived for \(SO(10)\) breakings \([26, 27]\) and for \(E_6\) breakings \([28]\). String models with the SUSY breaking due to the dilaton \(F\)-term lead to the highly restricted pattern such as \([19, 21]\)

\[
|A| = |M_{1/2}| = \sqrt{3}|m_{3/2}|
\]  

(1)

where \(A\) is a universal \(A\)-parameter, and gauginos and scalars get masses with common values \(M_{1/2}\) and \(m_{3/2}\), respectively. In this way, soft SUSY breaking parameters can play important roles to probe a new physics.

The above two approaches are attractive to explore particle phenomenology beyond the standard model based on SST. Hence it is important to examine what features soft SUSY breaking terms can show at low energy when we construct a realistic model through flat direction breakings starting from 4-D string models with extra gauge symmetries including \(U(1)_A\).

In this paper, we derive formula of soft SUSY breaking scalar masses from 4-D string models within a more generic framework. We consider effects of extra gauge symmetry breakings, that is, \(D\)-term and \(F\)-term contributions, particle mixing effects and heavy-light mass mixing effects. Some phenomenological implications are discussed based on our mass formula. In particular, we study the degeneracy and the positivity of squared scalar masses.
in special cases. In addition, we examine specific features of scalar potential and scalar masses taking an explicit model.

This paper is organized as follows. In the next section, we explain our starting point reviewing the structure of effective SUGRA derived from SST in a field theory limit. In section 3, we derive formula of soft SUSY breaking scalar masses and discuss the phenomenological implications. In subsection 3.1, flat directions in the SUSY limit are discussed in the framework of SUGRA. In subsection 3.2, we discuss classification of scalar fields. In subsection 3.3, we examine the existence of heavy-light mass mixing. In subsection 3.4, a generic formula of soft scalar masses is given. In subsection 3.5, phenomenological implications are discussed. In section 4, the results in section 3 are applied to an explicit model. In section 5, we remark some extensions. Section 6 is devoted to conclusions and discussions. In Appendix A, formulae of the Kähler metric and its inverse which we use are summarized.

2 Effective SUGRA as a Field Theory Limit of String Models

The effective SUGRAs are derived from $Z_N$ orbifold models taking a field theory limit. Here we assume the existence of a realistic effective SUGRA, that is, our starting theory has the following excellent features.

The gauge group is $G = G'_\text{SM} \times U(1)^n \times U(1)_A \times H'$ where $G'_\text{SM}$ is a group which contains the standard model gauge group $G_\text{SM} = SU(3)_C \times SU(2)_L \times U(1)_Y$ as a subgroup, $U(1)^n$ are non-anomalous $U(1)$ symmetries, $U(1)_A$ is an anomalous $U(1)$ symmetry and $H'$ is a direct product of some non-abelian symmetries. The anomalies related to $U(1)_A$ are canceled by the Green-Schwarz mechanism [29]. When gauginos of $H'$ condense, they can trigger SUSY breaking [8]. Or $H'$ might be broken by VEVs of some scalar fields at a higher energy scale. We take a standpoint that an origin of SUSY breaking is unspecified.

Chiral multiplets $\Phi^i$ are classified into two categories. One is a set of chiral multiplets whose scalar components $\phi^i$ have large VEVs of $O(M)$. Here $M$ is the gravitational scale defined as $M \equiv M_{Pl}/\sqrt{8\pi}$ and $M_{Pl}$ is the Planck scale. The dilaton field $S$ and the moduli fields $T_{ij}$ belong to $\{\Phi^i\}$. For the present, we treat only the overall moduli field $T$ ($T = T_1 = T_2 = T_3$,
\( T_{ij} = 0 \) for \( i \neq j \) and also neglect moduli fields \( U_i \) corresponding to complex structure. Further we neglect effects of threshold corrections and an \( S-T \) mixing. Later we will discuss the case with several moduli fields \( T_i \) and \( U_i \) and the case that Kähler potential has an \( S-T \) mixing term. The other is a set of matter multiplets denoted as \( \Phi^\kappa \) which contains the MSSM matter multiplets and Higgs multiplets. Some of them have non-zero \( U(1)_A (U(1)^n, H') \) charges and can induce to the \( U(1)_A (U(1)^n, H') \) breaking at high energy scales by getting VEVs. We denote the above two types of multiplet as \( \Phi^I \) together. The matter multiplets correspond to massless string states one to one.

We suppose the following situations related to extra gauge symmetry breakings.

1. The \( U(1)_A \) symmetry is broken by VEVs of \( S \) and some chiral matter multiplets.

2. Some parts of \( U(1)^n \) and \( H' \) are broken at much higher energy scales than the weak scale by VEVs of some chiral matter multiplets. Those VEVs are smaller than those of \( S \) and \( T \), i.e.

\[
\langle \phi^\kappa \rangle \ll \langle S \rangle, \langle T \rangle = O(M).
\]

This condition is justified from the fact that a \( D \)-term condensation of \( U(1)_A \) vanishes up to \( O(m_{3/2}^2) \) as will be shown. Here \( m_{3/2} \) is the gravitino mass defined later.

3. The rest extra gauge symmetries are broken spontaneously or radiatively by the SUSY breaking effects at some lower scales.

It is straightforward to apply our method to more complicated situations.

We give a comment here. Such a symmetry breaking generates an intermediate scale \( M_I \), which is defined as the magnitude of VEVs of scalar fields, below the Planck scale \( M_{Pl} \). Using the ratio \( M_I/M_{Pl} \), higher dimensional couplings could explain hierarchical structures in particle physics like the fermion masses and their mixing angles. Recently much attention has been paid to such a study on the fermion mass matrices [30, 31]. In Refs. [30], \( U(1) \) symmetries are used to generate realistic fermion mass matrices and some of them are anomalous, while stringy selection rules on nonrenormalizable couplings are used in Refs. [31].
Next let us explain the three constituents, the Kähler potential $K$, the superpotential $W$ and the gauge kinetic function $f_{\alpha\beta}$, in effective SUGRAs derived from SSTs. Orbifold models lead to the following Kähler potential $K$: \[ K = -\ln(S + S^* + \delta^A_{GS} V_A) - 3\ln(T + T^*) + \sum_\kappa (T + T^*)^{n_\kappa} |\phi^\kappa|^2 + \cdots \] where $\delta^A_{GS}$ is a coefficient of the Green-Schwarz mechanism to cancel the $U(1)_A$ anomaly and $V_A$ is a vector superfield of $U(1)_A$. Here and hereafter we take $M = 1$ according to circumstances. The dilaton field $S$ transforms nontrivially as $S \to S - i\delta^A_{GS} \theta(x)$ under $U(1)_A$ with the transformation parameter $\theta(x)$. The coefficient $\delta^A_{GS}$ is given as \[ \delta^A_{GS} = \frac{1}{96\pi^2} Tr Q^A \] where $Q^A$ is a $U(1)_A$ charge operator. Further $n_\kappa$’s are modular weights of matter fields $\phi^\kappa$. The formulae of $n_\kappa$ are given in Ref.[3, 18]. The same Kähler potential is derived from Calabi-Yau models with the large $T$ limit up to twisted sector field’s contributions. If the VEV of $\phi^\kappa$ is comparable with one of $T$, we should replace the second and third terms in Eq. (3) as \[ -3\ln(T + T^* - \sum_\kappa |\phi^\kappa|^2) \] for the untwisted sector and \[ -\ln[(T + T^*)^3 - \sum_\kappa (T + T^*)^{n_\kappa + 3} |\phi^\kappa|^2] \] for the twisted sector.

The superpotential $W$ consists of the following two parts, \[ W = W_{NP} + W_{Pert}. \] Here $W_{NP}$ is a superpotential induced by some non-perturbative effects, and it is expected that VEVs of $S$ and $T$ are fixed and SUSY is broken by this part. The other part $W_{Pert}$ is a superpotential at the tree level and starts from trilinear couplings for massless fields \[ W_{Pert} = \sum_{\kappa,\lambda,\mu} f_{\kappa\lambda\mu} \phi^\kappa \phi^\lambda \phi^\mu + \cdots \]
where Yukawa couplings $f_{\kappa\lambda\mu}$ generally depend on the moduli fields $T$ and the ellipsis stands for terms of higher orders in $\phi^\kappa$. Note that if the above superpotential includes mass terms as $m_{\kappa\lambda}\phi^\kappa\phi^\lambda$, a natural order of these masses is of $O(M)$. Thus we do not include these fields with mass terms at the tree level. The total Kähler potential $G$ is defined as $G \equiv K + \ln|W|^2$. The gauge kinetic function $f_{\alpha\beta}$ is given as $f_{\alpha\beta} = S\delta_{\alpha\beta}$. For simplicity, here we assume that Kac-Moody levels satisfy $k_\alpha = 1$ because our results on soft terms are independent of a value of $k_\alpha$. The scalar potential is given as

$$
V = V^{(F)} + V^{(D)},
$$

$$
V^{(F)} \equiv e^G(G^I(G^{-1})^I J G - 3),
$$

$$
V^{(D)} \equiv \frac{1}{2} (Re f^{-1})_{\alpha\beta} D^\alpha D^\beta
$$

$$
= \frac{1}{S+S^*}(K_\kappa(T^a\phi^\kappa)^2 + \frac{1}{S+S^*}(\frac{\delta_{GS}}{S+S^*} + K_\kappa(Q^A\phi^\kappa)^2
+ \frac{1}{S+S^*}(K_\kappa(Q^B\phi^\kappa)^2 + \frac{1}{S+S^*}(K_\kappa(T^C\phi^\kappa)^2)
$$

(10)

where $G_I = \partial G/\partial \phi^I$ and $G^J = \partial G/\partial \phi^*_J$, and $(Re f^{-1})_{\alpha\beta}$ and $(G^{-1})^I_J$ are the inverse matrices of $Re f_{\alpha\beta}$ and $G^I_J$, respectively. And the indices $I, J,...$ run all scalar species, the index $a \ (B, C)$ runs generators of the $G'_{SM} \ (U(1)^n, H')$ gauge group and $Q^B$'s are $U(1)^n$ charge operators. Note that the Fayet-Iliopoulos $D$-term appears in $V^{(D)}$ for $U(1)_A$ if we replace $S$ by its VEV $\text{[16, 12, 13]}$.

By the use of Kähler potential (3), $D$-terms for $U(1)_A$ and $U(1)^n$ are given as

$$
D^A = \frac{\delta_{GS}}{S+S^*} + \sum_\kappa (T + T^*)^{n_\kappa}q^A_\kappa |\phi^\kappa|^2
$$

(11)

and

$$
D^B = \sum_\kappa (T + T^*)^{n_\kappa}q^B_\kappa |\phi^\kappa|^2
$$

(12)

where $q^A_\kappa(B)$ is the $U(1)_A(U(1)^n)$ charge of scalar field $\phi^\kappa$ and we use the relation $(Q^A(B))_\lambda^\kappa = q^A(B)^\kappa_\lambda$. 

6
Finally let us give our assumption on the SUSY breaking. The gravitino mass $m_{3/2}$ is given by

$$m_{3/2} = \langle e^{K/2M^2} W \rangle$$  \hspace{1cm} (13)

where $\langle \cdots \rangle$ denotes the VEV of the quantity. In the next section, it will be often taken to be real as a phase convention. The $F$-auxiliary fields of the chiral multiplets $\Phi^I$ are defined as

$$F^I \equiv M e^{G/2M^2} (G^{-1})^I_J G^J.$$  \hspace{1cm} (14)

It is assumed that SUSY is broken by the $F$-term condensations of $\phi^i$ such that

$$\langle F^i \rangle = O(m_{3/2} M).$$  \hspace{1cm} (15)

In this case, stationary conditions of $V$ by $\phi^i$ require that VEVs of $D$-auxiliary fields should be very small, i.e. $\langle D^a \rangle \leq O(m_{3/2}^2)$ and $\langle V^{(D)} \rangle$ should vanish up to $O(m_{3/2}^4)$, i.e. $\langle V^{(D)} \rangle = O(m_{3/2}^4)$ \cite{32,33}.

3 Derivation of Soft Scalar Mass Formula

3.1 On flat directions

The effective theories derived from SSTs have, in general, flat directions in the SUSY limit, which can be a source to break gauge symmetries \cite{12}. In this subsection, we discuss such flat directions in the framework of SUGRA with $U(1)_A$. The reasons are as follows. First we should classify scalar fields in a well-defined manner to derive the low-energy effective theory. That is, we need to specify light fields which appear in a low-energy spectrum. Second there is a possibility that breaking scales of extra gauge symmetries can be determined by the existence of $U(1)_A$ and the introduction of SUSY breaking effects. It is known that some of them are fixed from $D$-flatness condition

\footnote{It is also applicable to the case of SUSY breaking by gaugino condensations \cite{8} because the dynamics are effectively described by a non-perturbative superpotential for $\phi^i$ after integrating out gauginos.}
of $U(1)_A$ in the SUSY limit \[12\]. We discuss this possibility from a general viewpoint of scalar potential in SUGRA.

Let us discuss the classification of scalar fields using mass spectra. The conditions that SUSY is not spontaneously broken in the sector related to matter multiplets are simply expressed as

$$\frac{\partial \hat{W}}{\partial \phi^\kappa} = 0, \quad \hat{W} \equiv \langle e^{\frac{K}{2M^2}}W \rangle,$$

(16)

$$D^\alpha = 0.$$  

(17)

Here $\phi^i$'s are replaced by their VEVs in $\hat{W}$ and $D^\alpha$. We denote solutions of the above conditions as $\phi^k = \phi^k_0$. There, in general, exist several flat directions and then the magnitudes of $\phi^k_0$ are not fixed along such flat directions.

In the presence of SUSY breaking, the vacuum $\langle \phi^I \rangle$ is obtained by solving the stationary condition $\partial V/\partial \phi^I = 0$.

We have the following two kinds of classification of scalar fields using $\phi^k_0$ or $\langle \phi^k \rangle$ and mass matrices.

1. In global SUSY models, the supersymmetric fermion mass $\mu_{k\lambda}$ is given as

$$\mu_{k\lambda} = \frac{\partial^2 \hat{W}}{\partial \phi^k \partial \phi^\lambda}|_0$$

(18)

where $\cdots |_0$ denotes the value of the quantity in the SUSY limit. By using a basis of $\phi^k$ to diagonalize $\mu_{k\lambda}$, we can classify scalar fields.

2. In SUGRA, the supersymmetric fermion mass $M_{IJ}$ is given as

$$M_{IJ} = \langle Me^{G/2M^2}(G_{IJ} + \frac{G_IG_J}{M^2} - G_L(G^{-1})^L_JG_{IJ}) \rangle$$

(19)

where the VEV is estimated at the minimum $\langle \phi^I \rangle$ of $V$ in the presence of SUSY breaking. We take a basis of $\phi^I$ to diagonalize the SUSY fermion mass matrix $M_{IJ}$ and can classify $\phi^I$'s using them.

\[\text{§ We assume that VEVs of } \phi^I \text{ are determined by solving stationary conditions } \partial V/\partial \phi^I = 0.\]
If we know relations among the above two classifications in advance, it is enough to use the most convenient one for our purpose. Thus let us discuss the relation between $\phi_0^\kappa$ and $\langle \phi^\kappa \rangle$. We estimate the order of $\hat{\delta}\phi^\kappa$ where $\langle \phi^\kappa \rangle = \phi_0^\kappa + \hat{\delta}\phi^\kappa$. The expansion of $V$ around $\phi_0^\kappa$ is given as

$$
V = V|_0 + \frac{\partial V}{\partial \phi^\kappa}|_0 \hat{\delta}\phi^\kappa + \frac{\partial^2 V}{\partial \phi^\kappa \partial \phi^\lambda}|_0 \hat{\delta}\phi^\kappa \hat{\delta}\phi^\lambda + \frac{1}{2} \frac{\partial^2 V}{\partial \phi^\kappa \partial \phi^\lambda}|_0 \hat{\delta}\phi^\kappa \hat{\delta}\phi^\lambda + \text{H.c.} + \cdots.
$$

(20)

Magnitudes of coefficients are estimated as

$$
\frac{\partial V}{\partial \phi^\kappa}|_0 = O(m_{3/2}\Lambda^2),
$$

(21)

$$
\frac{\partial^2 V}{\partial \phi^\kappa \partial \phi^\lambda}|_0, \quad \frac{\partial^2 V}{\partial \phi^\kappa \partial \phi^\lambda}|_0 = O(\Lambda^2)
$$

(22)

where we assume that both VEVs of some scalar fields and masses of heavy fields are of $O(\Lambda)$, that is, $\phi_0 = O(\Lambda)$, $m_{3/2}G_{KL}|_0 = O(\Lambda)$. In string models, this assumption holds for heavy scalar masses whose origin is Higgs mechanism through VEVs of some scalar fields. From the stationary condition $\partial V/\partial \langle \phi^\kappa \rangle = 0$, we find that $\hat{\delta}\phi^\kappa = O(m_{3/2})$.

In this way, we have the following conclusion. If there exists a local minimum solution $\langle \phi^\kappa \rangle$, its position is very near to $\phi_0^\kappa$, i.e., $\langle \phi^\kappa \rangle = \phi_0^\kappa + O(m_{3/2})$. By the use of this relation, the following relations are derived

$$
\langle W_{\kappa\lambda} \rangle = W_{\kappa\lambda}|_0 + O(m_{3/2}),
$$

(23)

$$
M_{\kappa\lambda} = \mu_{\kappa\lambda} + O(m_{3/2}) \quad \text{up to phase factor}.
$$

(24)

Hence scalar fields are classified into “heavy” fields and “light” ones using $\mu_{\kappa\lambda}$ in the next subsection.

Let us study SUSY breaking effects on flat directions in effective SUGRAs derived from string models. There are several flat directions before SUSY breaking. After SUSY breaking, the vacuum energy $V_0 \equiv \langle V \rangle$, in general, includes VEVs corresponding to flat directions. Hence such flat directions can

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There can be several solutions to satisfy the stationary condition. Here we pick out the solution such that $\langle \phi^\kappa \rangle = \phi_0^\kappa + O(m_{3/2})$.

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We owe this estimation to the discussion with I. Joichi and M. Yamaguchi.
be lifted due to SUSY breaking and corresponding VEVs can be determined
so that a minimum of $V_0$ is realized.

The gravitino mass $m_{3/2}$ is written as

$$m_{3/2}^2 = \langle e^G \rangle = \langle e^{K(S,T)}W(S,T)\exp[\sum_\kappa(T + T^*)n_\kappa|\phi_\kappa|^2]\rangle$$

(25)

where $K(S,T)$ represents the Kähler potential only for $S$ and $T$. Here we
rescale the gravitino mass as

$$m_{3/2}^2 \to m_{3/2}^2e^{f(1)}.$$  (26)

In the above Eq. (26), $a_\kappa$ is unity.

For a type of Kähler potential such as Eq. (3), we can write $V_0$ up to
$O(m_{3/2}^4)$ as

$$V_0 = V_0(S,T) + V_0^{(M)},$$

(28)

where $(K^I_J)$ is a reciprocal of $(K^{-1})_J^I$. Here from Eqs. (13), (14) and (15),
the magnitude of $V_0(S,T)$ is estimated as $O(m_{3/2}^2M^2)$. Recall that $D$-term
contributions to $V_0$ are of $O(m_{3/2}^4)$. Using the notation of (27) and the results
in Appendix A, we can write $V_0^{(M)}$ as

$$V_0^{(M)} = \frac{m_{3/2}^2e^{f(1)}}{(T + T^*)^2}v_0^{(M)},$$

(31)

$$v_0^{(M)} = \frac{f(n_\kappa^2)[9 - 3f(n_\kappa) - 3f(n_\kappa^2) - f^2(n_\kappa^2) + f(n_\kappa)f(n_\kappa^2)]}{(3 - f(n_\kappa))^2}$$

(32)

up to $O(m_{3/2}^4)$. The vacuum energy $V_0$ depends on VEVs through $f(1)$,
$f(n_\kappa)$ and $f(n_\kappa^2)$. Hence SUSY breaking lifts a potential along flat directions
corresponding VEVs appearing in \( f(1) \), \( f(n_\kappa) \) and \( f(n_\kappa^2) \). Note that when we derive the above results, we use the Kähler potential \( \mathcal{K} \), which is available for \( \langle \phi^\kappa \rangle \ll \langle T \rangle \). Thus Eqs.(31) and (32) are available for \( f(n_\kappa) \), \( f(n_\kappa^2) \ll 1 \). In this limit, we have \( v_0^{(M)} \approx f(n_\kappa^2) \).

In general, \( f(1) \), \( f(n_\kappa) \) and \( f(n_\kappa^2) \) are independent linear combinations of VEVs. By the definition, we have \( f(n_\kappa^2) \geq 0 \). It is obvious that the minimum of \( f(n_\kappa^2) \) corresponds to the minimum of \( v_0^{(M)} \). At first, we study the case without an anomalous \( U(1) \) symmetry. In this case, \( v_0^{(M)} \) as well as \( V_0 \) takes the minimum \( v_0^{(M)} = 0 \) up to \( O(m_{3/2}^2) \) at \( f(n_\kappa^2) = 0 \) for \( f(n_\kappa^2) \ll 1 \). This minimum point means \( |\langle \phi^\kappa \rangle|^2 = 0 \) up to \( O(m_{3/2}^2) \) for any fields with \( n_\kappa \neq 0 \), because \( n_\kappa^2 \geq 0 \). In this case, the vanishing cosmological constant \( V_0 = 0 \) requires \( V_0(S, T) = 0 \).

In Ref.\([14]\), generic flat directions of \( Z_{2n} \) orbifold models are discussed. In these flat directions, pairs of fields with \( R \) and \( \overline{R} \) representations in the same twisted sector, \( n_R = n_{\overline{R}} \), develop their VEVs as \( \langle R \rangle = \langle \overline{R} \rangle \neq 0 \). The SUSY breaking effects lift these flat directions determining these VEVs \( \langle R \rangle = \langle \overline{R} \rangle = 0 \) up to \( O(m_{3/2}) \). Note that these VEVs do not exactly vanish and they could lead to symmetry breaking even though these VEVs are very small.

Next we consider the case with an anomalous \( U(1) \) symmetry. In this case, some parts of VEVs are related with \( \langle \delta_{GS}/(S + S^*) \rangle \) because of the \( D \)-flatness due to the anomalous \( U(1) \). Thus VEVs are written as

\[
\langle (T + T^*)^n | \phi^\kappa \rangle = \sum_i a_{i\kappa} v_i + a_{X\kappa} \langle \frac{\delta_{GS}}{S + S^*} \rangle \tag{33}
\]

where \( v_i \)'s are independent degrees of freedom of VEVs and \( v_i \geq 0 \). Note that some of the coefficients \( a_{i\kappa} \), in general, can be negative, although \( \sum_i a_{i\kappa} v_i + a_{X\kappa} \langle \delta_{GS}/(S + S^*) \rangle \) should satisfy

\[
\sum_i a_{i\kappa} v_i + a_{X\kappa} \langle \frac{\delta_{GS}}{S + S^*} \rangle \geq 0. \tag{34}
\]

Here we have

\[
f(n_\kappa^2) = \sum_i \sum_\kappa a_{i\kappa} n_\kappa^2 v_i + \sum_\kappa a_{X\kappa} n_\kappa^2 \langle \frac{\delta_{GS}}{S + S^*} \rangle. \tag{35}
\]
If $\sum_\kappa a_{i\kappa}n_\kappa^2 > 0$, the minimum of $f(n_\kappa^2)$ is obtained at $v_i = 0$ for a finite value of $\langle S \rangle$. On the other hand, the VEVs $v_i$ corresponding to $\sum_\kappa a_{i\kappa}n_\kappa^2 < 0$ take the largest values at the minimum of $f(n_\kappa^2)$ satisfying Eq.\((34)\). Their VEVs $v_i$ are at most of $O((\delta_{GS}/(S + S^*))$. Thus values of $f(n_\kappa^2)$ and $v_0^{(M)}$ are of $O((\delta_{GS}/(S + S^*))$ at the minimum point. The vanishing cosmological constant $V_0 = 0$ requires $V_0(S, T) = 0$ up to $O((\delta_{GS}/(S + S^*))$.

In some cases, we have $\sum_\kappa a_{i\kappa}n_\kappa^2 = 0$, and either of $f(n_\kappa^2)$ and $v_0^{(M)}$ does not include corresponding VEVs $v_i$. Thus the directions along $v_i$ with $\sum_\kappa a_{i\kappa}n_\kappa^2 = 0$ are still flat directions of $V_0$ up to $O(m_{3/2}^2)$. As will be shown, soft scalar masses depend on $f(q_\kappa)$, $f(q_\kappa,n_\kappa)$ and $f(q_\kappa^2)$. If the VEVs $v_i$ with $\sum_\kappa a_{i\kappa}n_\kappa^2 = 0$ appear in these functions, $f(q_\kappa)$, $f(q_\kappa,n_\kappa)$ and $f(q_\kappa^2)$, a potential along flat directions of $v_i$ are lifted at the level of $O(m_{3/2}^2)$. In models with anomalous $U(1)$ symmetries, $D$-term contributions on soft masses are dominantly obtained by VEVs including $\langle \delta_{GS}/(S + S^*) \rangle$ and $v_i$ with $\sum_\kappa a_{i\kappa}n_\kappa^2 < 0$, because these VEVs are of $O((\delta_{GS}/(S + S^*))$ and other VEVs $v_i$ with $\sum_\kappa a_{i\kappa}n_\kappa^2 > 0$ are of $O(m_{3/2}^2)$.

In this way, a breaking scenario we supposed in section 2 can be realized. Then the symmetry breakings at a very large scale are induced by $D$-flatness of $U(1)_A$ and the order is given as $O((\delta_{GS}/(S + S^*))^{1/2})$. We denote it by $M_I$ and it is estimated as $O(10^{-1}M_I) \sim O(10^{-3}M)$ by using explicit models. Other symmetry breakings can occur by the SUSY breaking effects spontaneously or radiatively at $O(m_{3/2})$.

### 3.2 Classification of scalar fields

We take the basis $\hat{\phi}^\kappa$ that the supersymmetric fermion mass $\mu_{\kappa\lambda}$ is diagonalized in the SUSY limit. Then we assume that scalar fields are classified into “heavy” fields $\hat{\phi}^K, \hat{\phi}^L, \cdots$ and “light” fields $\hat{\phi}^k, \hat{\phi}^l, \cdots$, such as $\mu_{KL} = O(M_I)$ and $\mu_{kl} = O(m_{3/2})$, respectively. In string models, all light fields are massless, i.e. $\mu_{kl} = 0$, in the SUSY limit. The diagonalized fields $\hat{\phi}^\kappa$ are given as linear combinations of original ones $\phi^\kappa$ (string states) such as

$$\hat{\phi}^\kappa = R^\kappa_\lambda \phi^\lambda. \tag{36}$$

The Kähler potential of matter parts $K^{(M)}$ is, in general, written as

$$K^{(M)} = K^\lambda_\kappa \phi^\kappa \phi^*_\lambda + H_{\kappa\lambda} \phi^\kappa \phi^\lambda + \text{H.c.}$$
\[= \hat{K}^{\lambda}_{\kappa} \hat{\phi}^{\kappa} \hat{\phi}_{\lambda} + \hat{H}_{\kappa\lambda} \hat{\phi}^{\kappa} \hat{\phi}_{\lambda} + \text{H.c.,} \quad (37)\]
\[
\hat{K}^{\lambda}_{\kappa} \equiv (R^{-1})^\mu_\kappa K^\nu_\mu R^\lambda_\nu, \quad (38)
\]
\[
\hat{H}_{\kappa\lambda} \equiv H_{\mu\nu} (R^{-1})^\mu_\kappa (R^{-1})^\nu_\lambda, \quad (39)
\]

where the second (third) term in RHS is a holomorphic (anti-holomorphic) part on matter fields. The $D$-terms are also written as

\[
D^\alpha = K_\kappa (T^\alpha)^\kappa_\mu \hat{\phi}^{\mu} = \hat{K}_\kappa (T^\alpha)^\kappa_\mu \hat{\phi}^{\mu}, \quad (40)
\]
\[
\hat{K}_\kappa \equiv (R^{-1})^\mu_\kappa K_\mu, \quad (41)
\]
\[
(\hat{T}^\alpha)^\kappa_\lambda \equiv R^\kappa_\mu (T^\alpha)^\mu_\nu (R^{-1})^\nu_\lambda, \quad (42)
\]

for a linear-realization of gauge symmetries.

Next we discuss the case of the effective SUGRA derived from $Z_N$ orbifold models. The Kähler potential of matter parts $K^{(M)}$ is given as

\[
K^{(M)} = \sum_{\kappa, \lambda, \mu} (R^{-1})^\mu_\kappa (T + T^*)^\nu_\mu R^\lambda_\mu \hat{\phi}^{\kappa} \hat{\phi}^{*}_{\lambda} \quad (43)
\]

where we neglect the contribution of moduli fields $U_i$. $D$-terms for $U(1)$ symmetries are given as

\[
D^A = \frac{\delta_{GS}}{S + S^*} + \sum_{\kappa, \lambda, \mu} (R^{-1})^\mu_\kappa (T + T^*)^\nu_\mu q^A_\mu R^\lambda_\mu \hat{\phi}^{\kappa} \hat{\phi}^{*}_{\lambda}, \quad (44)
\]
\[
D^B = \sum_{\kappa, \lambda, \mu} (R^{-1})^\mu_\kappa (T + T^*)^\nu_\mu q^B_\mu R^\lambda_\mu \hat{\phi}^{\kappa} \hat{\phi}^{*}_{\lambda}. \quad (45)
\]

As discussed in section 2, Yukawa couplings has a moduli-dependence, so $\mu_{\kappa\lambda}$ and $R^\kappa_\mu$, in general, depend on the VEV of the moduli field $T$.

### 3.3 Heavy-light mixing terms

In this subsection, we estimate magnitudes of heavy-light mixing mass terms $\langle V^k_H \rangle \equiv \langle \partial^2 V / \partial \hat{\phi}^H \partial \hat{\phi}^k \rangle$ and $\langle V_{Hk} \rangle \equiv \langle \partial^2 V / \partial \hat{\phi}^H \partial \hat{\phi}^k \rangle$ including both the SUSY part and the soft SUSY breaking part. We use the vacuum $\langle \hat{\phi}^k \rangle$ in place of $\hat{\phi}^k_0$ since the difference between the estimation by $\langle \hat{\phi}^k \rangle$ and that by $\hat{\phi}^k_0$ is the quantity of $O(m^2_{3/2})$. After some calculations, $\langle V^k_H \rangle$ is expressed as

\[
\langle V^k_H \rangle = M_{HH'} \langle (\hat{K}^{-1})^H_1 \rangle M^{Hk} - M_{HH'} \langle (\hat{K}^{-1})^H_1 \rangle \langle (\hat{K})^{1k}_I \rangle \langle \hat{F}^I \rangle + O(m^2_{3/2}) \quad (46)
\]
where \((\hat{K}^{-1})_J^I\) is the inverse matrix of \((\hat{K})_J^I\). Carets represent functions of \(\hat{\phi}^I\) and \(\hat{\phi}^*_J\). If there are heavy-light mixing terms of \(O(1)\) in the Kähler potential, the order of the first term in the RHS of Eq. (46) can be \(O(\frac{m_3}{2}M_I)\). This contribution is SUSY one and is negligible except the stop and Higgs doublets in the MSSM since \(M_{kl}\) is very small compared with the weak scale. The second term in the RHS of Eq. (46) can be of \(O(\frac{m_3}{2}M_I)\) if the Kähler potential has heavy-light mixing terms of \(O(1)\) and a holomorphic part.

The chirality flipped scalar mass \(\langle V_{Hk} \rangle\) is written as

\[
\langle V_{Hk} \rangle = m_{3/2} \langle \hat{G}_{HkI} \rangle \langle \hat{F}^I \rangle - M_{HH'} \langle \langle \hat{K}^{-1} \rangle^I_J \rangle \langle \langle \hat{K} \rangle^I_J \rangle \langle \hat{F}^I \rangle + O(\frac{m_3^2}{2}).
\] (47)

If there are Yukawa couplings of \(O(1)\) among heavy, light and moduli fields in the superpotential, the order of the first term in the RHS of Eq. (47) can be \(O(\frac{m_3}{2}M)\). The second term in the RHS of Eq. (47) can be of \(O(\frac{m_3}{2}M_I)\) if the Kähler potential has heavy-light mixing terms of \(O(1)\). If there are Yukawa couplings of \(O(1)\) among light and moduli fields in the superpotential, that is, \(m_{3/2} \langle \hat{G}_{klI} \rangle = O(1)\), the order of the scalar masses for light fields can be \(O(\frac{m_3}{2}M)\) and the weak scale can be destabilized in the presence of weak scale Higgs doublets with such intermediate masses. This is so called “gauge hierarchy problem”. Only when \(m_{3/2} \langle \hat{G}_{IJr} \rangle \langle \hat{F}^r \rangle\)'s meet some requirements, the hierarchy survives. In many cases, we require the following condition,

\[
m_{3/2} \langle \hat{G}_{IJr} \rangle \langle \hat{F}^r \rangle \leq O(\frac{m_3^2}{2})
\] (48)

for light fields \(\hat{\phi}^I\) and \(\hat{\phi}^r\). If we impose the same condition to the light fields and the heavy fields, we neglect the effect of the first term in the RHS of Eq. (47). In such a case, unless there exist heavy-light mixing terms in the Kähler potential, there appear no heavy-light mass mixing terms of \(O(\frac{m_3}{2}M_I)\). In string models, whether there exist heavy-light mixing terms in the Kähler potential or not is model-dependent.

### 3.4 Soft scalar masses
3.4.1 Soft scalar mass terms

For convenience, we introduce new notations related to the classification of scalar fields,

\[ \hat{\phi}^H \equiv \left( \Delta \hat{\phi}^H \right), \quad \hat{\phi}^L \equiv \left( \hat{\phi}^I \right) \]

(49)

where \( \hat{\phi}^H \)'s are heavy scalar fields and \( \hat{\phi}^I \)'s are light scalar fields. We assume that there are no light fields whose VEVs are of \( O(M_I) \). The light fields can get VEVs of \( O\left( m_{3/2}^3 / 2 \right) \) and induce extra gauge symmetry breakings, but we treat them as a sum of the VEVs and fluctuations since both have a same order and our goal is to derive soft scalar mass formula at \( M_I \). We can take effects of symmetry breakings at \( O\left( m_{3/2}^3 / 2 \right) \) in the same way.

Scalar mass terms are written as

\[ V^{\text{Mass}} = \frac{1}{2} \{ \hat{\phi}^H \hat{H}^\dagger \hat{\phi}^H + \hat{\phi}^L M^H \hat{\phi}^i + \hat{\phi}^L M^L \hat{\phi}^i \}
\]

where

\[ H \equiv H^H = \left( \begin{array}{cc} \langle V_H^H \rangle & \langle V_H^{HH} \rangle \\ \langle V_H^{HH} \rangle & \langle V_H^H \rangle \end{array} \right), \]

(51)

\[ M \equiv M^L = \left( \begin{array}{cc} \langle V_L^i \rangle & \langle V_L^{ii} \rangle \\ \langle V_L^{ii} \rangle & \langle V_L^i \rangle \end{array} \right), \]

(52)

\[ L \equiv L^L = \left( \begin{array}{cc} \langle V_L^i \rangle & \langle V_L^{ii} \rangle \\ \langle V_L^{ii} \rangle & \langle V_L^i \rangle \end{array} \right). \]

(53)

The order of the above mass matrices are estimated as \( H = O(M_I^2) \), \( M = O(m_{3/2}^2 M_I) \) and \( L = O(m_{3/2}^2) \). Scalar mass terms are rewritten as

\[ V^{\text{Mass}} = \frac{1}{2} \{ \hat{\phi}^H \hat{H}^\dagger \hat{\phi}^H + \hat{\phi}^L M^H \hat{\phi}^i \}
\]

(54)

We discuss implication of each term in Eq.(54). The first term is the mass term among heavy fields. After the integration of the heavy fields \( \hat{\phi}^H + \)
there appear $D$-term contributions and extra $F$-term contributions to scalar masses which will be discussed later. The second term is new contributions which appear after the diagonalization of scalar mass terms. This contribution can be sizable, i.e., $O\left(\frac{m_3^2}{2M_I}\right)$, if the heavy-light mass mixing is $O\left(\frac{m_3}{M_I}\right)$. The last term is a mass term among light fields. Note that the heavy fields $\hat{\phi}_H^\dagger H^{-1} M \hat{\phi}^c_L$ and the light fields $\hat{\phi}^c_L$ used here are different from properly diagonalized fields up to $O\left(\frac{m_3}{2M_I}\right)$ terms. The final expressions of scalar masses of $O\left(\frac{m_3^2}{2}\right)$ for light fields are same whichever we use as a definition of the scalar fields. We give a more fully expression for extra contribution due to the existence of heavy-light mass mixing terms as

$$V_{\text{Mix Soft Mass}}^{\phi^c} = -\frac{1}{2} \hat{\phi}^c_L M^H H^{-1} M \hat{\phi}^c_L^\dagger$$

$$= (V_{\text{Mix Soft Mass}}^{\phi^c})^k_l \hat{\phi}_k^c \hat{\phi}_l^c + \frac{1}{2} (V_{\text{Mix Soft Mass}}^{\phi^c})^{kl} \hat{\phi}_k^c \hat{\phi}_l^c + \text{H.c.} \quad (55)$$

where

$$ (V_{\text{Mix Soft Mass}}^{\phi^c})^k_l = -\langle V_H (V^{-1})^H H' V^H_k \rangle - \langle V_H (V^{-1})^H H' V^H_k \rangle$$

$$\quad - \langle V_H V_H (V^{-1})^H V_H' \rangle - \langle V_H V_H (V^{-1})^H V_H' \rangle, \quad (56)$$

$$ (V_{\text{Mix Soft Mass}}^{\phi^c})^{kl} = -\langle V_H (V^{-1})^H V_H' \rangle - \langle V_H (V^{-1})^H V_H' \rangle$$

$$\quad - \langle V_H (V^{-1})^H V_H' \rangle - \langle V_H (V^{-1})^H V_H' \rangle. \quad (57)$$

### 3.4.2 Parametrization

For the analysis of soft SUSY breaking parameters, it is convenient to introduce the following parameterization

$$\langle e^{G/2} (K_S^T)^{-1/2} G \rangle = \sqrt{3} C m_{3/2} e^{i\alpha_S} \sin \theta, \quad (58)$$

$$\langle e^{G/2} (K_T^T)^{-1/2} (G + (K_T^T)(K^{-1})^T \kappa G^\kappa) \rangle = \sqrt{3} C m_{3/2} e^{i\alpha_T} \cos \theta \quad (59)$$

where $(K_T^T)$ is a reciprocal of $(K^{-1})^T$. Using Eq. [29], the vacuum energy $V_0$ is written as

$$V_0 = \langle e^G (G' (G^{-1})^T G_J - 3) \rangle$$

$$= 3(C^2 - 1)m_{3/2}^2 + V_0^{(M)} \quad (60)$$
up to $O(m_{3/2}^4)$. Since $C^2$ should be positive or zero, we have a constraint $V_0^{(M)} \leq 3m_{3/2}^2 + V_0$ from Eq. (60). In the case with $V_0 = 0$, it becomes as

$$
\langle G^\kappa((G^{-1})_\kappa^\lambda (G^{-1})_\kappa^T (K_T^T) \langle G^{-1}\rangle_\kappa^\lambda )G_\lambda \rangle \leq 3.
$$

(61)

It gives a constraint on VEVs of $\phi^\kappa$ and $T$. Further a larger value of $V_0^{(M)}$ in the above region means $C \ll 1$. Such a limit as $C \to 0$ corresponds to the “moduli-dominated” breaking, that is, $\langle F^S \rangle \ll 1$ and $\langle F^T \rangle$ and $\langle F^* \rangle$ contribute to the SUSY-breaking. Note that this situation does not agree with the case of the moduli-dominated breaking without extra gauge symmetry breakings $\sin \theta \to 0$. The relation (2) implies the relation $\langle G^\kappa \rangle \ll \langle G^S \rangle, \langle G^T \rangle$ because we discuss the vacuum solution near to the flat direction which leads to the relations $\langle W_\kappa \rangle \equiv \langle \partial W/\partial \phi^\kappa \rangle = O(m_{3/2}^2)$ and $\langle G_\kappa \rangle = \langle K_\kappa \rangle + O(m_{3/2})$. In this case, the parameterization (33) becomes a simpler one such as

$$
\langle e^{G/2}(K_T^T)^{-1/2}G^T \rangle = \sqrt{3}Cm_{3/2} e^{i\alpha T} \cos \theta.
$$

(62)

Further the discussion in 3.1 means that $V_0$ is dominated by $V_0(S, T)$ because of $V_0(S, T) \gg V_0^{(M)}$. Thus the vacuum energy $V_0$ becomes as

$$
V_0 = 3(C^2 - 1)m_{3/2}^2.
$$

(63)

Soft SUSY breaking scalar mass terms are given as

$$
V_{\text{Soft Mass}}^{(0)} = (m_{3/2}^2 + V_0) \sum_\kappa \langle (T + T^*)^{n_\kappa} \rangle |\phi^\kappa|^2 + \sum_\kappa \langle F^I \rangle \langle K^I_{\kappa} (K^{-1})^J_{\mu} K_J^{\mu} - K^I_{\kappa} K^J_{\mu} \rangle \langle F^J \rangle \langle (T + T^*)^{n_\kappa} \rangle |\phi^\kappa|^2.
$$

(64)

before heavy fields are integrated out. By the use of the parametrization and the diagonalized fields $\hat{\phi}^\kappa$, $V_{\text{Soft Mass}}^{(0)}$ is rewritten as

$$
V_{\text{Soft Mass}}^{(0)} = \sum_{\kappa, \lambda}(m_{3/2}^2 + V_0) \langle \hat{K}_\kappa^\lambda \rangle \hat{\phi}_\lambda \hat{\phi}_\lambda^* + \sum_{\kappa, \lambda} m_{3/2}^2 C^2 \cos^2 \theta \hat{N}_\kappa^\lambda \hat{\phi}_\kappa \hat{\phi}_\lambda^*.
$$

(65)

where

$$
\hat{K}_\kappa^\lambda \equiv (R^{-1})^\mu_\kappa (T + T^*)^{n_\kappa} R_\mu, \\
\hat{N}_\kappa^\lambda \equiv \langle \hat{K}_\kappa^\mu \rangle \hat{\eta}_\mu, \\
\hat{\eta}_\kappa \equiv (R^{-1})^\nu_\kappa n_\nu R_\nu.
$$

(66, 67)
After heavy fields are integrated out, we have the following mass terms for light fields at the energy scale $M_I$,

$$V_{\text{Soft Mass}} = \sum_{k,l} (m_{3/2}^2 + V_0) \langle \hat{K}^l_k \rangle \hat{\phi}^k \hat{\phi}^*_l + \sum_{k,l} m_{3/2}^2 C^2 \cos^2 \theta \langle \hat{N}^l_k \hat{\phi}^k \hat{\phi}^*_l \rangle$$

$$+ V_{\text{Soft Mass}}^D + V_{\text{Soft Mass}}^\text{Extra F} + V_{\text{Soft Mass}}^\text{Mix} + V_{\text{Soft Mass}}^\text{Ren} \quad (68)$$

where $V_{\text{Soft Mass}}^D$, $V_{\text{Soft Mass}}^\text{Extra F}$, $V_{\text{Soft Mass}}^\text{Mix}$ and $V_{\text{Soft Mass}}^\text{Ren}$ are $D$-term contributions, extra $F$-term contributions which will be discussed in the following subsections, the contributions due to the existence of heavy-light mass mixing discussed in the previous section and contributions of renormalization effects from $M$ to $M_I$, respectively.

### 3.4.3 $D$-term contributions

The $D$-term contributions are given as \cite{34, 27, 33},

$$V_{\text{Soft Mass}}^D = \sum_{k,l} \sum_{\alpha} g_\alpha^2 \langle D^{\alpha} \rangle (\hat{Q}^\alpha)_k \hat{\phi}^k \hat{\phi}^*_l \quad (69)$$

where

$$(\hat{Q}^\alpha)_k \equiv \langle \hat{K}^\mu_k \rangle (\hat{q}^\alpha)_\mu, \quad (\hat{q}^\alpha)_\kappa \equiv (R^{-1})^\nu_\kappa q^\alpha_\nu. \quad (70)$$

Here $g_\alpha$’s are gauge coupling constants and we use the relation $\langle R e S \rangle = 1/g_\alpha^2$. We omit the terms whose magnitudes are less than $O(m_{3/2}^4)$.  

Next we rewrite $V_{\text{Soft Mass}}^D$ using the parametrization introduced before. For this purpose, it is useful to adapt to the following relation of $D$-term condensations \cite{33},

$$g_\alpha \langle D^{\alpha} \rangle = 2(M_V^{-2})^{\alpha\beta} g_\beta \langle F^I \rangle \langle F^J \rangle \langle (D^{\beta})^I_J \rangle \quad (71)$$

where $(M_V^{-2})^{\alpha\beta}$ is the inverse matrix of gauge boson mass matrix $(M_V^2)^{\alpha\beta}$ given as

$$(M_V^2)^{\alpha\beta} = 2g_\alpha g_\beta \langle (T^{\beta}(\phi^I))_I K^I_J (T^{\alpha}(\phi))^J \rangle. \quad (72)$$

Here the gauge transformation of $\phi^I$ is given as $\delta \phi^I = ig_\alpha (T^{\alpha}(\phi^I))$ up to space-time dependent infinitesimal parameters.
After some straightforward calculations, \( D \)-term condensations are written as
\[
g_\lambda^2 \langle D^\lambda \rangle = 2 g_\lambda m_3^{2/3} \left\{ (M_V^{-2}) \hat{\gamma}_A g_A (1 - 6 C^2 \sin^2 \theta) \left( \sum_\kappa q_\kappa A (T + T^*) n_\kappa | \phi^\kappa |^2 \right) \right. \\
- \sum_\beta (M_V^{-2}) \hat{\alpha}_A \hat{\beta}_A g_\beta C^2 \cos^2 \theta \left( \sum_\kappa q_\kappa \hat{\beta}_A n_\kappa (T + T^*) n_\kappa | \phi^\kappa |^2 \right) \right\} 
\]
where \( \hat{\alpha} \) and \( \hat{\beta} \) run over broken generators.

We give some comments. We need to introduce three kinds of model-dependent quantities such as
\[
\langle \sum_\kappa q_\kappa \hat{\alpha}_A n_\kappa (T + T^*) n_\kappa | \phi^\kappa |^2 \rangle, 
\]
\[
\langle \sum_\kappa q_\kappa \hat{\alpha}_A \hat{\beta}_A n_\kappa (T + T^*) n_\kappa | \phi^\kappa |^2 \rangle, 
\]
\[
\langle \sum_\kappa q_\kappa \hat{\alpha}_A n_\kappa (T + T^*) n_\kappa | \phi^\kappa |^2 \rangle. 
\]
Their magnitudes are of order \( O(M_2^2) \). Note that \( \langle \sum_\kappa q_\kappa \hat{\alpha}' A n_\kappa (T + T^*) n_\kappa | \phi^\kappa |^2 \rangle = O(m_3^2/2) \) due to the \( D \)-flatness condition in the presence of SUSY breaking. Here \( \hat{\alpha}' \) runs over only non-anomalous diagonal broken generators.

In the case that there are no mixing elements between \( U(1)_A \) and other symmetries in \( (M_V^2)^{\alpha \beta} \), the mass of \( U(1)_A \) gauge boson is given as
\[
\langle (M_V^2)^{A} \rangle = 2 g_A^2 \left\{ \langle \sum_\kappa q_\kappa \hat{\alpha}_A n_\kappa (T + T^*) n_\kappa | \phi^\kappa |^2 \rangle \right. \\
+ \left. \langle \sum_\kappa q_\kappa \hat{\alpha}_A n_\kappa (T + T^*) n_\kappa | \phi^\kappa |^2 \rangle \right\} 
\]
by the use of the relation \( Q^A S = \delta_{GS} \) and \( (Q^A \phi)^\lambda = q_\lambda A \phi^\lambda \). Under the assumption that \( | \phi^\kappa | \ll M, (M_V^2)^{A} \) is simplified as
\[
\langle (M_V^2)^{A} \rangle = 2 g_A^2 \left\{ \langle \sum_\kappa q_\kappa \hat{\alpha}_A n_\kappa (T + T^*) n_\kappa | \phi^\kappa |^2 \rangle \right\} 
\]
and \( D \)-term condensation of \( U(1)_A \) is written as
\[
g_\Lambda^2 \langle D^A \rangle = \frac{m_3^{2/3}}{\langle \sum_\kappa (q_\kappa A)^2 (T + T^*) n_\kappa | \phi^\kappa |^2 \rangle} \times \\
\left\{ (1 - 6 C^2 \sin^2 \theta) \langle \sum_\kappa q_\kappa A (T + T^*) n_\kappa | \phi^\kappa |^2 \rangle \\
- C^2 \cos^2 \theta \langle \sum_\kappa q_\kappa A n_\kappa (T + T^*) n_\kappa | \phi^\kappa |^2 \rangle \right\}. 
\]
Furthermore, at that time, $D$-term condensations of non-anomalous symmetries are given as

$$g_{\alpha'}^2 \langle D^{\hat{\alpha}'} \rangle = -m_{3/2}^2 C^2 \cos^2 \theta \frac{\sum_{\kappa} q_{\kappa}^2 n_{\kappa} (T + T^*) n_{\kappa} |\phi_{\kappa}|^2}{\sum_{\nu} (q_{\nu}^2)^2 (T + T^*) n_{\nu} |\phi_{\nu}|^2}$$

(80)

where broken charges are re-defined by the use of diagonalization of $(M_I^2)^{\hat{\alpha}' \hat{\beta}'}$. Using the expression (80), we can show that there appears no sizable $D$-term contribution to scalar masses if a broken symmetry is non-anomalous and SUSY is broken by the dilaton $F$-term, i.e., $\cos^2 \theta = 0$.

In a simple case that only one field $X$, which has no charges except the $U(1)_A$ charge, gets VEV to cancel the contribution of $S$ in $D^A$, the above result is reduced to the previous one obtained in Ref. [24]. ** Note that our result is not reduced to that obtained from the theory with the Fayet-Iliopoulos $D$-term, which is derived from the effective SUGRA by taking the flat limit first [23], even in the limit that $|\delta_{GS}^A/q_{X}^A| \ll 1$. This disagreement originates from the fact that we regard $S$ and $T$ as dynamical fields, that is, we use the stationary conditions $\partial V/\partial \phi^I = 0$ to calculate $D$-term condensations.

The dominant $D$-term contributions to mixing mass terms are obtained as

$$\sum_{\alpha} g_{\alpha}^2 \langle D^{\alpha} \rangle \langle K_{i \lambda}^\dagger \phi^*_{\lambda} \phi^k (T^\alpha \phi)^l \rangle + \text{H.c.}$$

(81)

The magnitude of them are estimated as $O(m_{3/2}^4/M_I)$ and so they are neglected in the case that $M_I \ll M$. Note that the contribution of $O(m_{3/2}^4)$ such as $g_{\alpha}^2 \langle D^{\alpha} \rangle \langle H_{i \lambda} \phi^\dagger (T^\alpha \phi)^l \rangle$ vanishes from gauge invariance of the holomorphic part $H$ of the Kähler potential.

### 3.4.4 Extra $F$-term contributions

After the integration of complex heavy fields $\hat{\phi}^K$ and Nambu-Goldstone multiplets $\hat{\phi}^\alpha$, the following $F$-term contributions appear in the low-energy effective scalar potential [27, 33],

$$V^{\text{Extra } F} = V^{\text{Extra } F}_0 + V^{\text{Extra } F}_1 + V^{\text{Extra } F}_2$$

(82)

** In the formula obtained in Ref. [24], there is a sign error: $+6C^2 \sin^2 \theta$ should be $-6C^2 \sin^2 \theta$. 

20
\[ V_{(0)}^{\text{Extra F}} \equiv E \left\{ -\delta^2 \left( \frac{\hat{W}^*}{M^2} \hat{G}^K \right) \langle (\hat{K}^{-1})^L_K \rangle \delta^2 \left( \frac{\hat{W}}{M^2} \hat{G}_L \right) + \delta^2 \left( \frac{\hat{W}^*}{M^2} \hat{G}^\delta \right) \langle (\hat{K}^{-1})^{\delta}_\alpha \rangle \delta^2 \left( \frac{\hat{W}}{M^2} \hat{G}_\beta \right) + \delta \left( \frac{\hat{W}^*}{M^2} \hat{G}^\delta \right) \langle (\hat{K}^{-1})^{\delta}_\alpha \rangle \delta^3 \left( \frac{\hat{W}}{M^2} \hat{G}_\beta \right) + \text{H.c.} \right. \]
\[ + \left. \delta^2 \left( \frac{\hat{W}^*}{M^2} \hat{G}^K \right) \langle (\hat{K}^{-1})^L_K \rangle \left( \langle \hat{W}_{Lk} \rangle \hat{\phi}^k + \frac{1}{2} \langle \hat{W}_{Lkl} \rangle \hat{\phi}^k \hat{\phi}^l + \cdots \right) + \text{H.c.} \right\}, \quad (83) \]

\[ V_{(1)}^{\text{Extra F}} \equiv E \left\{ \delta \left( \frac{\hat{W}^*}{M^2} \hat{G}^K \right) \langle (\hat{K}^{-1})^{\delta}_\alpha \rangle \delta^2 \left( \frac{\hat{W}}{M^2} \hat{G}_\beta \right) + \text{H.c.} \right. \]
\[ + \delta \left( \frac{\hat{W}^*}{M^2} \hat{G}^\delta \right) \delta^2 \left( \frac{\hat{W}}{M^2} \hat{G}_L \right) + \text{H.c.} \]
\[ + \delta \left( \frac{\hat{W}^*}{M^2} \hat{G}^\delta \right) \langle \delta^2 \left( \hat{K}^{-1} \right)^L_k \rangle \delta \left( \frac{\hat{W}}{M^2} \hat{G}_\lambda \right) \right\}; \quad (84) \]

\[ V_{(2)}^{\text{Extra F}} \equiv E \left\{ \delta^2 \left( \frac{\hat{W}^*}{M^2} \hat{G}^\delta \right) \langle (\hat{K}^{-1})^{\delta}_k \rangle \delta^2 \left( \frac{\hat{W}}{M^2} \hat{G}_k \right) + \text{H.c.} \right. \]
\[ + \delta \left( \frac{\hat{W}^*}{M^2} \hat{G}^\delta \right) \langle \delta^2 \left( \hat{K}^{-1} \right)^L_k \rangle \delta \left( \frac{\hat{W}}{M^2} \hat{G}_\lambda \right) + \text{H.c.} \right\}. \quad (85) \]

We explain our notations in the above Eqs. (83)–(85). Carets represent functions of \( \hat{\phi}^l \) and \( \hat{\phi}^*_j \). The quantity \( \hat{G}_\lambda \) is defined as
\[ \hat{G}_\lambda \equiv \hat{G}_\lambda + (\hat{G})^\kappa_\lambda (\hat{G}^{-1})^j_\kappa \hat{G}_j \quad (86) \]
and \( E \) is defined as \( E \equiv \langle \exp(\hat{K}/M^2) \rangle \). Any quantity \( A \) is expanded in powers of \( m_{3/2} \) such as
\[ A = \langle A \rangle + \delta A + \delta^2 A + \cdots. \quad (87) \]

For example,
\[ \delta \left( \frac{\hat{W}}{M^2} \hat{G}_K \right) = \langle \hat{W}_K \rangle + \langle \hat{W}_{KL} \rangle \delta \hat{\phi}^L + \langle \hat{W} \rangle \langle \hat{K} \rangle \]
\[ + \langle (\hat{K})^L_K \rangle \langle \delta^2 \left( \hat{K}^{-1} \right)^L_k \rangle \delta \left( \frac{\hat{W}}{M^2} \hat{G}_j \right). \quad (88) \]
\[
\delta^2 \left( \frac{\tilde{W}}{M^2} \hat{G}_K \right) = \langle \tilde{W}_{KL} \rangle \delta^2 \hat{\phi}^L + \langle \tilde{W}_{Kk} \rangle \delta \hat{\phi}^k + \langle \tilde{W}_{K\beta} \rangle \delta \hat{\phi}^\beta + \frac{1}{2} \langle \tilde{W}_{K\lambda\mu} \rangle \delta \hat{\phi}^\lambda \delta \hat{\phi}^\mu \\
+ \frac{1}{M^2} \left( \frac{1}{2} \langle \tilde{W}_{LM} \rangle \delta \hat{\phi}^L \delta \hat{\phi}^M \langle \tilde{K}_K \rangle + \langle \tilde{W} \rangle \langle \tilde{K}_K \rangle \delta \hat{\phi}^J \right) \\
+ \langle \tilde{W} \rangle \langle \tilde{K}_K \rangle \delta \hat{\phi}^J \right)
\]

When there exist flat directions, we have \( \langle \tilde{W}_\lambda \rangle = O(m_{3/2}^2) \). Quantities with a prime such as \( \delta^3 \left( \frac{\tilde{W}}{M^2} \hat{G}_\beta \right) \) mean that the terms proportional to \( \delta^2 \hat{\phi}^I \) are omitted. The ellipsis in Eq. (89) represents other terms in \( \delta^2 \left( \frac{\tilde{W}}{M^2} \hat{G}_K \right) - \langle \tilde{W}_{KL} \rangle \delta^2 \hat{\phi}^L \).

The first and second terms in (84) are estimated as \( O(m_{3/2}^4 I_1/M) \) and the last one is \( O(m_{3/2}^4 (M_1/M)^2) \). They are negligible in the case that \( M_1 \ll M \).

Scalar masses of \( O(m_{3/2}^2) \) are given as

\[
\begin{align*}
V_{\text{Extra F Soft Mass}}^l & = (V_{\text{Extra F Soft Mass}}^k \hat{\phi}^k \hat{\phi}^*), \\
(V_{\text{Extra F Soft Mass}}^l)_k & = E \left\{ - \langle \tilde{W}^{-1} L^M \rangle \delta \left( \frac{\tilde{W}^*}{M^2} \hat{G}^* \right) \langle \langle \tilde{K}^{-1} \rangle^\alpha_k \rangle \langle \tilde{W}_{\alpha M K} \rangle \langle \tilde{K}^K \rangle \right. \\
& \quad \times \left. \langle \tilde{W}^{* \beta N I} \rangle \langle \tilde{K}^{-1} \rangle^{\beta} \delta \left( \frac{\tilde{W}}{M^2} \hat{G}_\lambda \right) \langle \langle \tilde{W}^{* -1} \rangle_{N K} \rangle \right. \\
& \quad + \left( \langle \tilde{W}_{\alpha k} \rangle + \langle \tilde{W} \rangle \frac{M^2}{M^2} \langle \langle \tilde{K} \rangle_{\alpha k} \rangle \right) \langle \langle \tilde{K}^{-1} \rangle^{\alpha}_k \rangle \\
& \quad \times \left( \langle \tilde{W}^{* \beta I} \rangle + \langle \tilde{W}^* \rangle \frac{M^2}{M^2} \langle \langle \tilde{K} \rangle^{\beta I} \rangle \right) \\
& \quad + \langle \tilde{W} \rangle \frac{M^2}{M^2} \langle \langle \tilde{K} \rangle^{\alpha I}_k \rangle \langle \langle \tilde{K}^{-1} \rangle^{\beta}_k \rangle \frac{\tilde{W}^*}{M^2} \langle \langle \tilde{K}^\beta \rangle \rangle \\
& \quad - \langle \langle \tilde{W}^{-1} L^M \rangle \delta \left( \frac{\tilde{W}^*}{M^2} \hat{G}^* \right) \langle \langle \tilde{K}^{-1} \rangle^{\alpha}_k \rangle \langle \tilde{W}_{\alpha M K} \rangle \langle \tilde{W} \rangle \frac{M^2}{M^2} \langle \tilde{K}^l \rangle \rangle \\
& \quad - \langle \langle \tilde{W}^* \rangle \langle \tilde{K}^L \rangle \langle \tilde{W}^{* \beta N I} \rangle \langle \langle \tilde{K}^{-1} \rangle^{\beta}_k \rangle \delta \left( \frac{\tilde{W}}{M^2} \hat{G}_\lambda \right) \langle \langle \tilde{W}^{* -1} \rangle_{N K} \rangle \rangle 
\end{align*}
\]

We discuss conditions that \( (V_{\text{Extra F Soft Mass}}^l)_k \) is neglected. If Yukawa couplings among Nambu-Goldstone, heavy and light fields are small enough, the first
term and the last two terms are neglected. If we impose $R$-parity conservation, the second and third terms in $(V^{\text{Extra F Soft Mass}}_k)^l$ are forbidden since bilinear couplings between Nambu-Goldstone and light fields are $R$-parity odd. Here we define the $R$-parity of $\hat{\phi}^\kappa$ as follows,

$$R(\hat{\phi}^\alpha) = +1, \quad R(\hat{\phi}^K) = R(\hat{\phi}^R) = -1.$$  

### 3.4.5 Formula of soft scalar masses

Using scalar mass terms (68) and diagonalizing of the Kähler potential, i.e., $\langle \hat{K}_k \rangle = \delta^l_k$, we have the following mass formula for light scalar fields at the energy scale $M_I$,

$$m^2_k|_{M_I} = (m^2_{3/2} + V_0)\delta^l_k + m^2_{3/2}C^2\cos^2\theta \hat{N}_k^l + \sum_\alpha g_\alpha^2 (D^{\hat{\alpha}})(\hat{\theta}^{\hat{\alpha}})_k^l + (V^{\text{Extra F Soft Mass}}_k)^l_k + (V^{\text{Mix Soft Mass}}_k)^l_k + (V^{\text{Ren Soft Mass}}_k)^l_k$$  

where $(V^{\text{Ren Soft Mass}}_k)^l_k$ is a sum of contributions related to renormalization effects from $M$ to $M_I$ and consists of the following two parts. One is a radiative correction between $M$ and $M_I$. This contribution $(\Delta m^2)^\lambda_k|_{M_I}$ is given as [33],

$$\Delta m^2_k|_{M_I} = -\sum_{\alpha} b_\alpha C_2(R_\alpha^\kappa)(M_\alpha^2(M_I) - M_\alpha^2(M))\delta^\lambda_k$$

$$+ \sum_{B} \frac{1}{b_B} (Q^{B}_{R_\kappa}(S_B(M_I) - S_B(M))\delta^\lambda_k,$$  

$$S_B(M_I) = \frac{\alpha_B(M_I)}{\alpha_B(M)} S_B(M), \quad S_B(M) = \sum_{R_\kappa} Q^{B}_{R_\kappa} n_{R_\kappa} (m^2)^\kappa_\kappa(\mu)$$

where $\alpha$ runs all the gauge groups but $B$ runs only non-anomalous $U(1)$ gauge groups whose charges are $Q^{[\alpha]}_R$, $C_2(R_\alpha^\kappa)$'s are the second order Casimir invariants, $M_\alpha$'s are gaugino masses and $n_{R_\kappa}$ is the multiplicity. Here we neglect effects of Yukawa couplings. It is straightforward to generalize our results to the case with large Yukawa couplings. Here we use the anomaly cancellation condition $\sum_{R_\kappa} C_2(R_\alpha^\kappa)Q^{B}_{R_\kappa} n_{R_\kappa} = 0$ and the relation of orthogonality $\sum_{R_\kappa} Q^{B}_{R_\kappa} Q^{B'}_{R_\kappa} n_{R_\kappa} = b_B \delta_{BB'}$. Note that there is no contribution related to $U(1)_A$ symmetry since it is broken at $M$.  

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The other is $D$-term contribution due to mass splitting which is induced by mass renormalization. We denote it by $(\Delta m^2_D)_k|_{M_I}$. This contribution is given as

$$(\Delta m^2_D)_k|_{M_I} = \sum_{\hat{\alpha}} g^2_{\hat{\alpha}} (\Delta D^{\hat{\alpha}})(\hat{Q}^{\hat{\alpha}})_k$$  \hspace{1cm} (96)

where

$$g_{\hat{\alpha}} (\Delta D^{\hat{\alpha}}) \equiv -2(M_v^{-2})^{\hat{\alpha}\hat{\beta}} g_{\hat{\beta}} \langle \sum_{\kappa,\lambda,\mu} \hat{\phi}_\lambda^{\kappa} (\Delta m^2)_\lambda^\mu|_{M_I} (\hat{Q}^{\hat{\beta}})_{\mu}^{\kappa} \hat{\phi}_\kappa \rangle. \hspace{1cm} (97)$$

### 3.5 Phenomenological implications of soft scalar mass

Here we discuss phenomenological implications of our soft scalar mass formula, especially $D$-term contributions, considering simple cases. In general, $D$-term contributions are comparable with $F$-term contributions. Our formula could lead to a strong non-universality of soft scalar masses. Recently much work is devoted to phenomenological implications of the non-universality \cite{26, 27, 36}. In addition, various researches of soft scalar masses have been done in the presence of anomalous $U(1)$ symmetry \cite{23, 24, 25, 37}. We examine the universality, the degeneracy and the positivity of squared soft scalar masses in the case that there are neither particle mixing in the Kähler potential, heavy-light mass mixing effects nor extra $F$-term contributions. Here we take $V_0 = 0$, i.e. $C^2 = 1$ and consider the case that there are no mixing elements between $U(1)_A$ and other symmetries in $(M^2_0)^{\alpha\beta}$ for simplicity.

#### 3.5.1 Anomaly-free symmetry case

Here we consider models with an anomaly-free symmetry. In this case, soft scalar masses are obtained as

$$m^2_k = m^2_{3/2} \left[ 1 + \cos^2 \theta \left( n_k - q_k \frac{\langle \sum_{\lambda} q^{\alpha}_{\lambda} n_{\lambda} (T + T^*)^{n_{\lambda}} |\phi^\lambda|^2 \rangle}{\langle \sum_{\kappa} (q^{\alpha}_{\lambda})^2 (T + T^*)^{n_{\lambda}} |\phi^\lambda|^2 \rangle} \right) \right]. \hspace{1cm} (98)$$

If $q_k \langle \sum_{\lambda} q^{\alpha}_{\lambda} n_{\lambda} (T + T^*)^{n_{\lambda}} |\phi^\lambda|^2 \rangle > 0$, squared soft masses $m^2_k$ can easily become negative, especially for larger value of $\cos \theta$.

Experiments for the process of flavor changing neutral current (FCNC) require that $\Delta m^2/m_{3/2} \lesssim 10^{-2}$ for the first and the second families in the
case with \( m^2_0 \sim O(1) \text{TeV} \)\(^3\). Hence we should derive \( \Delta m^2/m^2_{3/2} \approx 0 \) within the level of \( O(10^{-2}) \). Hereafter \( a \approx 0 \) denotes such a meaning.

In the limit that \( \cos^2 \theta \to 0 \), i.e. the dilaton dominant SUSY breaking, we obtain universal soft scalar masses, \( m^2_k = m^2_{3/2} \)\(^3\). In order to realize degenerate soft scalar masses in the other values of \( \cos \theta \), one needs a “fine-tuning” condition for differences of \( n_k \) and \( q_k \), \( \Delta n \) and \( \Delta q \), as

\[
\Delta n = \Delta q \left\langle \frac{\sum_{\lambda} q_\lambda^0 n_\lambda (T + T^*)^n |\phi^\lambda|^2}{\sum_{\lambda} (q_\lambda^0)^2 (T + T^*)^n |\phi^\lambda|^2} \right\rangle. \tag{99}
\]

If fields with non-vanishing VEVs have the same modular weight, i.e. the same Kähler metric, the \( D \)-term contributions on soft scalar masses vanish due to the \( D \)-flatness condition. This fact is important. This situation can happen in some cases. In these cases, degeneracy of soft masses is realized for fields with the same values of modular weights. One example for the vanishing \( D \)-term contribution is shown in the next section.

Another interesting example is the case where enhanced gauge symmetries break by VEVs of moduli fields in orbifold models. Gauge symmetries are enhanced at specific points of moduli spaces, where some massless states \( \eta_i \) also appear in the untwisted sector. For example, \( Z_3 \) orbifold models have enhanced \( U(1)^6 \) symmetries. Here we expand moduli fields \( \tau_i \) around these points so that vanishing or nonvanishing VEVs \( \langle \tau_i \rangle \) correspond to unbroken or broken enhanced symmetries. Neither \( \tau_i \) nor \( \eta_i \) has well-defined charges under the \( U(1) \)'s and we take linear combinations \( s_i \), which have definite \( U(1) \) charges\(^4\). These fields \( s_i \) have the same Kähler metric. If only these fields \( s_i \) develop VEVs and no symmetry other than enhanced symmetries break, \( D \)-term contributions on soft scalar masses vanish. Because enhanced symmetries are anomaly-free and fields developing VEVs have the same Kähler metric. These models will be studied in detail elsewhere.

\[3.5.2 \text{ Anomalous } U(1) \text{ case}\]

Here we study models with an anomalous \( U(1) \) symmetry. In this case, soft scalar masses are obtained as\(^4\)

\[
m^2_k = m^2_{3/2}[1 + n_k \cos^2 \theta + \frac{q_k}{f(q_3)} (f(q_\lambda)(1 - 6 \sin^2 \theta) - f(q_\lambda n_\lambda \cos^2 \theta)], \tag{100}\]

\(^4\) Throughout this subsection, we omit the superscript \( A \) in the \( U(1)_A \) charge \( q^A_k \).
where \( f(a_\lambda) \) denotes Eq. (27). Here \( f(q_\lambda) \) does not vanish for a finite value of \( \langle S \rangle \) because of the \( D \)-flatness of \( U(1)_A \). It is remarkable that even if \( \cos \theta = 0 \), the \( D \)-term contribution does not vanish. That is different from \( D \)-term contributions due to the breakdown of anomaly-free \( U(1) \) symmetries. In general, non-universal soft scalar masses are obtained even if \( \cos \theta = 0 \).

The \( D \)-term contribution vanishes if the following fine-tuning condition is satisfied

\[
(6 f(q_\lambda) - f(q_\lambda n_\lambda)) \sin^2 \theta = f(q_\lambda) - f(q_\lambda n_\lambda).
\]

(101)

We have \( f(q_\lambda n_\lambda) = n_\lambda f(q_\lambda) \) in the case where the fields in the summation \( f(q_\lambda n_\lambda) \) have the same modular weight \( n_\lambda \). In this case Eq. (101) reduces as

\[
(6 - n_\lambda) \sin^2 \theta = 1 - n_\lambda.
\]

(102)

For \( n_\lambda = 1 \), the moduli dominant SUSY breaking, i.e., \( \sin \theta = 0 \), satisfies this condition although this modular weight \( n_\lambda = 1 \) is not naturally obtained \[13, 41\]. The modular weight satisfying \( n_\lambda \leq 0 \) leads to \( 0 < \sin^2 \theta < 1 \) for Eq. (102).

Degenerate soft scalar masses are obtained for differences of modular weights and \( U(1)_A \) charges, \( \Delta n \) and \( \Delta q \), in the case where the following fine-tuning condition is satisfied

\[
(6 f(q_\lambda)^2 \Delta n + 6 f(q_\lambda) \Delta q - f(q_\lambda n_\lambda)) \cos^2 \theta = 5 f(q_\lambda) \Delta q.
\]

(103)

Soft scalar masses are written for two extreme cases of the SUSY breaking, i.e. \( \cos \theta = 0 \) and 1 as

\[
\frac{m_k^2}{m_{3/2}^2} = 1 - 5 q_k \frac{f(q_\lambda)}{f(q_\lambda^2)} \quad \text{for} \quad \cos \theta = 0,
\]

(104)

\[
\frac{m_k^2}{m_{3/2}^2} = 1 + n_k + q_k \frac{f(q_\lambda) - f(q_\lambda n_\lambda)}{f(q_\lambda^2)} \quad \text{for} \quad \cos \theta = 1.
\]

(105)

Squared soft scalar masses become easily negative for \( q_k f(q_\lambda) > 0 \) in the former case. On the other hand, we obtain likely negative \( m_k^2 \) for \( q_k \{ f(q_\lambda) - f(q_\lambda n_\lambda) \} < 0 \) in the latter case.

As a concrete example, we discuss a simple model where only one field \( X \) develops its VEV. Its modular weight and anomalous \( U(1) \) charge are denoted as \( n_X \) and \( q_X \). In this case, the soft scalar mass is given as

\[
m_k^2 = m_{3/2}^2 [1 + n_X \cos^2 \theta + \frac{q_k}{q_X} ((6 - n_X) \cos^2 \theta - 5)].
\]

(106)
Note that the coefficient of $q_k/q_X$ in Eq. (106) is sizable. We obtain the difference of the soft masses as

$$\frac{\Delta m^2}{m_{3/2}^2} = \Delta n \cos^2 \theta + \frac{\Delta q}{q_X}((6 - n_X) \cos^2 \theta - 5)$$  \hspace{1cm} (107)$$

by using Eq. (106). If $\Delta q/q_X \approx 0$, we have $\Delta m^2/m_{3/2}^2 = \Delta n \cos^2 \theta$. In this case, the limit $\cos^2 \theta \to 0$ leads to $\Delta m \to 0$. It corresponds to the dilaton-dominated breaking, where soft masses are universal \[39, 21\]. Unless $\Delta q/q_X \approx 0$, we need “fine-tuning” on the value of $\cos \theta$ as

$$\cos^2 \theta \approx \frac{5}{6 - n_X + q_X \Delta n/\Delta q}.$$  \hspace{1cm} (108)$$

This “fine-tuning” is possible only in the case where

$$n_X \leq 1 + \frac{\Delta n}{\Delta q} q_X.$$  \hspace{1cm} (109)$$

If Eq. (108) is satisfied, the soft scalar mass is written as

$$m_k^2 = m_{3/2}^2[1 + \frac{5(n_k - \Delta n q_k/\Delta q)}{6 - n_X + \Delta n q_X/\Delta q}].$$  \hspace{1cm} (110)$$

The condition for the positivity of $m_k^2$ is written as

$$-n_k + \frac{q_k}{q_X}(n_X - 6) \leq (1 - 5 \frac{q_k}{q_X}) \cos^2 \theta.$$  \hspace{1cm} (111)$$

If $1 - 5q_k/q_X$ is positive, we can find a solution $\cos \theta$ of the above constraint for any $n_k, n_X, q_k$ and $q_X$. On the other hand, if $1 - 5q_k/q_X$ is negative, it leads to the following constraint:

$$1 + n_k \geq \frac{q_k}{q_X}(n_X - 1),$$  \hspace{1cm} (112)$$

because $\cos^2 \theta \geq 1$.

Let us consider two extreme examples for the SUSY-breaking, i.e. $\cos \theta = 0$ and 1. We have

$$m_k^2 = m_{3/2}^2(1 - 5 \frac{q_k}{q_X}) \text{ for } \cos \theta = 0,$$

$$m_k^2 = m_{3/2}^2[1 + n_k + \frac{q_k}{q_X}(1 - n_X)] \text{ for } \cos \theta = 1.$$  \hspace{1cm} (113) (114)
Matter fields usually have modular weights $n_k \leq 0$. Thus the fields with $q_k/q_X > 0$ for Eq. (113) and $q_k/q_X < 0$ for Eq. (114) can easily have negative squared scalar mass of $O(m^2/2)$ at the Planck scale. That implies that several fields could develop VEVs and they could trigger symmetry breakings. We can show that there exist fields with $q_k/q_X < 0$ for each gauge group other than $U(1)_A$. The reason is as follows. Let us assume the gauge group is $U(1)_A \times \prod G_\ell$. The Green-Schwarz anomaly cancellation mechanism requires that $C_{G_\ell} = \delta_A^{G_\ell} k_\ell$ for any $\ell$, where $C_{G_\ell}$ is a coefficient of $U(1)_A \times G_\ell^2$ anomaly and $k_\ell$ is a Kac-Moody level of $G_\ell$. Through the $U(1)_A$ breaking due to the Fayet-Iliopoulos $D$-term, the field $X$ develops its VEV. Here its charge should satisfy $q_X Tr Q^A < 0$ and $q_X C_{G_\ell} < 0$ to satisfy the $D$-flatness of the anomalous $U(1)$. Each gauge group $G_\ell$ always has fields $\phi^\alpha$ which corresponds nontrivial representation on its group and whose $U(1)_A$ charges satisfy $q_X q_k < 0$ because of $q_X C_{G_\ell} < 0$. The $D$-term contribution on soft terms is very sizable. That could naturally lead to $m^2_k < 0$ except a narrow region and cause $G_\ell$ breaking.

4 Analysis on Explicit Model

4.1 Flat direction

In this section, we study $U(1)_A$ breaking effects on flat directions and derive specific scalar mass relations by using an explicit model \[42\]. The model we study is the $Z_3$ orbifold model with a shift vector $V$ and Wilson lines $a_1$ and $a_3$ such as

$$V = \frac{1}{3}(1,1,1,2,0,0,0)(2,0,0,0,0,0,0,0,0)'$$

$$a_1 = \frac{1}{3}(0,0,0,0,0,0,2)(0,0,1,1,0,0,0,0,0)'$$

$$a_3 = \frac{1}{3}(1,1,1,2,1,1,1,0)(1,1,0,0,0,0,0,0,0)'$$

This model has a gauge group as

$$G = SU(3)_C \times SU(2)_L \times SU(2)_R \times U(1)^7 \times SO(8)' \times SU(2)'$$

One of $U(1)^7$ is anomalous. This model has matter multiplets as

$$U - \text{sec. : } 3[(3,2,1)_0 + (\bar{3},1,2)_0 + (1,2,2)_0]$$
$$+3[(8,2)_{\nu} + (1,1)'_{-12}] .$$

$$T - \text{sec.}: \quad 9[(3,1,1)_{4} + (3,1,1)_{4}] + 15[(1,2,1)_{4} + (1,1,2)_{4}]$$

$$(N_{OSC} = 0) \quad +3(1,2,2)_{4} + 3[(1,2,1)(1,2)_{-2} + (1,1,2)(1,2)_{-2}]$$

$$24(1,2)_{-2} + 60(1,1,1)_{4} + 3(1,1,1)_{-8} ,$$

$$T - \text{sec.}(N_{OSC} = -1/3): \quad 9(1,1,1)_{4}$$

where the number of suffix denotes the anomalous $U(1)$ charge defined as $Q^{A} \equiv Q_{5} - Q_{6}$ and $N_{OSC}$ is the oscillator number. This model has $TrQ^{A} = 864$. The $U(1)$ charge generators of $U(1)^{7}$ are defined in Table 1.

This model has many $SU(3)_{C} \times SU(2)_{L} \times SU(2)_{R}$-singlets as shown above. These fields are important for flat directions leading to realistic vacua. For example, this model includes the following $SU(3)_{C} \times SU(2)_{L} \times SU(2)_{R}$-singlets

$$u: \quad Q_{a} = (0,0,0,0,-6,6,0) ,$$
$$Y: \quad Q_{a} = (0,-4,-4,0,2,-2,0) ,$$
$$S_{1}: \quad Q_{a} = (0,-4,-4,0,-4,4,0) ,$$
$$D_{3}' : \quad Q_{a} = (6,4,0,-2,-2,0,-2) ,$$
$$D_{4}' : \quad Q_{a} = (6,4,0,2,-2,0,2) ,$$
$$D_{5}' : \quad Q_{a} = (-6,0,4,-2,0,2,-2) ,$$
$$D_{6}' : \quad Q_{a} = (-6,0,4,2,0,2,2)$$

where $U(1)^{7}$ charges $Q_{a} (a = 1, 2, \cdots , 7)$ are represented in the basis of Table 1. Here we follow the notation of the fields in Ref. [42] except the $D_{i}'$ fields. These $D_{i}'$ fields are $SU(2)'$-doublets in the non-oscillated twisted sector with $n_{k} = -2$, corresponding to $T_{i}$ fields in Ref. [42]. The others are singlets under any non-abelian group. The $u$ fields corresponds to the untwisted sector with $n_{u} = -1$. In addition, $S_{1}$ corresponds to the non-oscillated twisted sector with $n_{k} = -2$ and $Y$ corresponds to the twisted sector with a nonvanishing oscillator number. Thus the field $Y$ has the modular weight $n_{Y} = -3$. There exist the following flat directions [42]

$$\langle (T + T^{*})^{-1}|u|^{2} \rangle = v_{1} ,$$

$$\langle (T + T^{*})^{-3}|Y|^{2} \rangle = \langle (T + T^{*})^{-2}|D_{3}'|^{2} \rangle = \langle (T + T^{*})^{-2}|D_{4}'|^{2} \rangle = \langle (T + T^{*})^{-2}|D_{5}'|^{2} \rangle = v_{2} ,$$

$$\langle (T + T^{*})^{-2}|S_{1}|^{2} \rangle = \langle (T + T^{*})^{-2}|D_{4}'|^{2} \rangle = \langle (T + T^{*})^{-2}|D_{5}'|^{2} \rangle = v_{3} .$$

29
where \(v_i \geq 0\). Along this flat direction, the gauge symmetries break as \(U(1)^7 \times SU(2)' \rightarrow U(1)^3\). One of unbroken \(U(1)^3\) charges corresponds to \(Q_{B-L}\). Here we define the broken charges as follows,

\[
Q'_1 \equiv \frac{1}{3}(2Q_1 + Q_2 - Q_3 - Q_5 - Q_6),
\]

\[
Q'_2 \equiv \frac{1}{\sqrt{3}}(2Q_4 + Q_7),
\]

\[
Q'_3 \equiv \frac{1}{\sqrt{2}}(Q_2 + Q_3),
\]

\[
Q^A \equiv Q_5 - Q_6, \quad T'^3
\]

where \(T'^3\) is a third component of generators of \(SU(2)\). Note that the gauge boson mass matrix is not diagonalized in this definition of charges. The modular weights and broken charges of the light scalar fields and the fields with VEVs are given in Table 2. For the light fields, we follow the notation of fields in Ref.[42]. Chiral multiplets are denoted as \(Q_L\) for left-handed quarks, \(Q_R\) right-handed quarks, \(L\) for left-handed leptons and \(R\) for right-handed leptons. In addition, \(H\) are Higgs doublets.

The \(D\)-flatness condition for \(U(1)_A\) requires

\[
\langle \frac{\delta_{GS}}{S + S^*} \rangle - 12v_1 - 12v_3 = 0. \tag{116}
\]

On the top of that, we have

\[
f(n^2_\lambda) = v_1 + 17v_2 + 12v_3. \tag{117}
\]

Under the constraint (116), the minimum of \(f(n^2_\lambda)\) is obtained at the following point:

\[
v_1 = \frac{1}{12} \langle \frac{\delta_{GS}}{S + S^*} \rangle, \quad v_2 \sim O(m^2_{3/2}), \quad v_3 \sim O(m^2_{3/2}). \tag{118}
\]

Using \(TrQ^A = 864\) and \(\langle ReS \rangle \sim 2\), we estimate \(v_1 \sim M^2/53\). From Eqs.(118), the breaking scale of \(U(1)'_i\)’s \((i = 1, 2, 3)\) and \(SU(2)'\) is estimated as \(O(m_{3/2})\).
4.2 Soft mass relations

We derive specific relations among soft scalar masses. The basic idea and the strategy are the same as those in Ref. [26, 27, 28]. The SUSY spectrum at the weak scale, which is expected to be measured in the near future, is translated into the soft SUSY breaking parameters. The values of these parameters at higher energy scales are obtained by using the renormalization group equations (RGEs) [35]. In many cases, there exist some relations among these parameters. They reflect the structure of high-energy physics. Hence we can specify the high-energy physics by checking these relations.

The generic formula of scalar mass is given as Eq.(92). We have the same number of observable scalar masses as that of species of scalar fields, e.g., 17 observables in the MSSM. There are several model-dependent parameters in the RHS of Eq.(92) such as $m_3^2/2 + V_0$, $\cos \theta$ and so on. If the number of independent equations is more than that of unknown parameters, non-trivial relations exist among scalar masses.

In our model, the breaking scale $M_I$ is estimated as $O(10^{-1} M)$ and so renormalization effects from $M$ to $M_I$ are neglected. We assume that Yukawa couplings among heavy and light fields are small enough and the $R$-parity is conserved. In such a case, we can neglect the effect of extra $F$-term contributions. In this model, the light fields $\hat{\phi}^k$ equal to just string states and so there are no mixing terms among heavy and light fields in the Kähler potential. As discussed in subsection 3.3, there appear no heavy-light mixing terms of $O(m_{3/2}/M_I)$ if Yukawa couplings among heavy, light and moduli fields are suppressed sufficiently, i.e., $(\hat{W}_{Hkl}) = O(m_{3/2}/M)$. At that time, the quantities $\hat{N}_k^l$ and $(\hat{Q}^\alpha)_k^l$ are simplified as

$$\hat{N}_k^l = n_k \delta_k^l, \quad (\hat{Q}^\alpha)_k^l = q_k^\alpha \delta_k^l. \quad (119)$$

Under the above assumptions and excellent features, our soft scalar mass formula is written in a simple form such as,

$$(m^2)_k|_{M_I} = m_{3/2}^2 + m_{3/2}^2 n_k \cos^2 \theta + \sum_\alpha g_\alpha^2 \langle D^{\hat{\alpha}} \rangle q_k^\alpha$$

$$= m_{3/2}^2 \left\{ 1 + n_k \cos^2 \theta + \frac{q_k^A}{12} (5 - 7 \cos^2 \theta) \right\} \quad (120)$$

where we take $V_0 = 0$, i.e., $C = 1$. Here we use the formula of $D$-term
condensation \( \text{(79)} \) and the following values,
\[
f(q^A) = -12v_1, \quad f(q^An_\kappa) = 12v_1, \quad f((q^A)^2) = 144v_1.
\]
In this model, the gauge boson mass matrix is diagonalized for the components of \( U(1)_A \) and \( U(1)'_3 \) up to \( m_{3/2}^2/M_I^2 \).

In Table 3, we give a ratio \( m_\tilde{\kappa}^2/m_{3/2}^2 \) at \( M_I \) for all light species except \( G'_{SM} \) singlets in two extreme cases, \( \cos^2 \theta = 0 \) and \( \cos^2 \theta = 1 \). For \( \cos^2 \theta = 1 \), \( L_i (i = 3, 4, 5) \) and \( R_j (j = 1, 4, 5) \) fields acquire negative squared masses and they could trigger a “larger” symmetry breaking including the dangerous charge symmetry breaking. In addition, we have a strong non-universality of soft masses. However, in this model, soft masses are degenerate for squarks and sleptons with same quantum numbers under \( G_{SM} \) because they have same quantum numbers under the gauge group \( G \) and same modular weights.

We have the following relations by eliminating model-dependent parameters,
\[
\begin{align*}
 m_{\tilde{Q}_L}^2 &= m_{\tilde{Q}_R}^2 = m_H^2, \\
 m_{\tilde{L}}^2 &= m_{\tilde{R}}^2, \\
 13m_{\tilde{Q}_L}^2 &= 3m_L^2 + 5m_{3/2}^2
\end{align*}
\]
(121)
where the tilde represents scalar components.

On the top of that, the gaugino mass \( M_{1/2} \) is obtained as \( \text{(21)} \)
\[
 M_{1/2}^2 = 3m_{3/2}^2 \sin^2 \theta.
\]
(122)
We can use this gaugino mass to obtain a relation not including \( m_{3/2} \) as
\[
 3m_{\tilde{Q}_L}^2 = M_{1/2}^2.
\]
(123)
In the case that the SUSY breaking is induced by the dilaton \( F \)-term, there are no modular weight dependence. Hence we have a more specific relation such that
\[
 8m_{\tilde{Q}_L}^2 = 3m_L^2.
\]
(124)
Further various contributions should be added at lower energy scales. For example, the following \( D \)-term contribution appears after \( U(1)'_i \)'s and \( SU(2)' \) breakings at \( O(m_{3/2}) \),
\[
 (m_D^2)_k |_{m_{3/2}} = -m_{3/2}^2 \frac{q_k^2}{6\sqrt{2}} \rho \cos^2 \theta
\]
(125)
where \( \rho = v_2/(v_2 + v_3) \). The ratio \( \rho \) takes a value as \( \rho \leq 1 \) because \( v_i \geq 0 \).

To derive Eq. (125), we use the formula of \( D \)-term condensation (80) and the following values,

\[
f(q^3 n_\kappa) = 4\sqrt{2} v_2, \quad f((q^3)^2) = 48(v_2 + v_3).
\]

Note that the \( D \)-terms of \( Q'_1 \), \( Q'_2 \) and \( T'_3 \) do not contribute soft scalar masses up to renormalization effects because \( f(q^1 n_\kappa) = f(q^2 n_\kappa) = f(q^3 n_\kappa) = 0 \). Here \( Q'_1 (q^1), Q'_2 (q^2) \) and \( T'_3 (q^3) \) are the diagonal charge operators (charges) where the gauge boson mass matrix is diagonalized and they are constructed as linear combinations of \( Q'^1, Q'^2 \) and \( T'^3 \).

In general, original string states are different from the MSSM fields in string models including \( G_{SM} \). The coefficients \( R_\lambda^\kappa \) of linear combinations depend on the VEVs of moduli fields. A study of flat directions and soft masses in such a situation has been progressed by using explicit models [43].

5 Remarks on extension of Kähler potential

Here we discuss extensions of our soft mass formula for different types of Kähler potentials. At the one-loop level, the dilaton field \( S \) and the moduli field \( T \) are mixed in the Kähler potential as

\[
- \ln(S + S^* + \Delta(T + T^*)) - 3\ln(T + T^*).
\]  

(126)

In this case we can obtain the same parametrization of soft scalar masses as the case without the mixing, i.e. \( \Delta(T + T^*) = 0 \), except replacing \( \cos^2 \theta \) as

\[
\cos^2 \theta \to \left[ 1 - \frac{(T + T^*)^2 \Delta''(T + T^*)}{3(S + S^* + \Delta(T + T^*))} \right] \cos^2 \theta
\]

(127)

where \( \Delta''(T + T^*) \) is the second derivative of \( \Delta(T + T^*) \) by \( T \).

In general, string models have several moduli fields other than one overall moduli field \( T \). In this case, their \( F \)-terms could contribute on the SUSY breaking and one needs more goldstino angles to parametrize these \( F \)-terms.

For example, we discuss the models with three diagonal moduli fields \( T_i \) \((i = 1, 2, 3)\). These moduli fields have the following Kähler potential:

\[
- \sum_i \ln(T_i + T_i^*)
\]

(128)
instead of $-3\ln(T + T^*)$ in the case of the overall moduli field. Here we parametrize their contributions on the SUSY breaking as 

$$\left< e^{G/2}(K^{T_i} - 1/2G^{T_i}) \right> = \sqrt{3} C m_{3/2} e^{i\alpha_{T_i}} \cos \theta \Theta_i$$

(129)

where $\sum_i \Theta_i^2 = 1$. Using these parameters, $F$-term contributions on soft scalar masses are written as

$$m^2_{3/2} + V_0 + 3m^2_{3/2} C^2 \sum_i n_{i\kappa} \cos^2 \theta \Theta_i^2$$

(130)

where $n_{i\kappa}$ is a modular weight of $\phi^\kappa$ for the $i$-th moduli field $T_i$. Similarly $D$-term contributions can be written by the use of these parameters. For example, the $D$-term condensations (80) are extended as

$$g_{d_i}' \left< D^{d_i'} \right> = -3m^2_{3/2} C^2 \cos^2 \theta \sum_i \Theta_i^2 \left( \frac{\sum_k q_{d_i}^k n_{i\kappa} (T + T^*)^{n_{\kappa}} |\phi^\kappa|^2}{(\sum_k (q_{d_i}^k)^2 (T + T^*)^{n_{i\kappa}} |\phi^\kappa|^2)} \right)$$

(131)

where $(T + T^*)^{n_{\kappa}}$ means $\prod_{i=1}^3 (T_i + T_i^*)^{n_{i\kappa}}$.

Some orbifold models have complex structure moduli fields $U_i$. In such models, a Kähler potential includes holomorphic parts as

$$\frac{1}{(T_i + T_i^*) (U_i + U_i^*)} \phi \phi'$$

(132)

We can extend our formula into these models. These holomorphic parts are important for mixing of fields. Further they could originate the $\mu$-term with a suitable order naturally.

The Kähler potential can receive radiative corrections and be modified by non-perturbative effects. Our approach is generic and basically available to other types of Kähler potential although one might need more complicated parametrization than \(58\) and \(59\).

### 6 Conclusions and Discussions

We have derived the formula of soft SUSY breaking scalar masses from the effective SUGRA derived from 4-D string models within a more generic framework. The gauge group contains extra gauge symmetries including the
anomalous $U(1)$ some of which are broken at a higher energy scale. The breakings are related to the flat direction breakings in the SUSY limit. It is supposed that there are two types of matter multiplets classified by supersymmetric fermion mass, i.e., heavy fields and light ones. The physical scalar fields are, in general, linear combinations of original fields corresponding to massless states in string models.

The mass formula contains the effects of extra gauge symmetry breakings, i.e., $D$-term and extra $F$-term contributions, particle mixing effects and heavy-light mass mixing effects. The $D$-term contributions to soft scalar masses are parameterized in terms of three types of new parameter in addition to the goldstino angle, gravitino mass and vacuum energy. These contributions, in general, are sizable. In particular, $D$-term contribution of $U(1)_A$ survives even in the case of the dilaton dominant SUSY breaking. The $D$-term contributions for anomaly-free $U(1)$ symmetries vanish if the fields developing VEVs have the same modular weight. Extra $F$-term contributions are neglected in the case that Yukawa couplings among Nambu-Goldstone, heavy and light fields are suppressed and the $R$-parity is conserved. In the case that there exist heavy-light mixing terms in the Kähler potential, the extra contributions can appear after the diagonalization of scalar mass terms in the presence of heavy-light mass mixings of $O(m_3/2M_I)$.

We have discussed the degeneracy and the positivity of squared scalar masses in special cases that there are neither particle mixing in the Kähler potential, heavy-light mass mixing effects nor extra $F$-term contributions. We find that the $F$-term contribution from the difference among modular weights and the $D$-term contribution to scalar masses can destroy universality among scalar masses at $M$. This non-degeneracy endangers the discussion of the suppression of FCNC process. On the other hand, the difference among $U(1)$ charges is crucial for the generation of fermion mass hierarchy. It seems to be difficult to make two discussions compatible. As a byway, we can take a model that the fermion mass hierarchy is generated due to non-anomalous $U(1)$ symmetries and SUSY is broken by the dilaton $F$-term condensation. For example, it is supposed that anomalies from contributions of the MSSM matter fields are canceled out by those of extra matter fields in such a model. Further “stringy” symmetries are also useful for fermion mass generation leading to degenerate soft scalar masses [31], because these symmetries do not induce $D$-terms.

Many fields could acquire negative squared masses and they could trigger
a “larger” symmetry breaking including the dangerous color and/or charge symmetry breaking. This type of symmetry breaking might be favorable in the case where $G'_{SM}$ is a large group like a grand unified group. These results might be useful for model building.

We have derived specific scalar mass relations by taking an explicit string model. It is expected that such relations can be novel probes to select a realistic string model since they are model-dependent.

The moduli fields have a problem in string cosmology because their masses are estimated as of $O(m_{3/2})$ and they weakly couple with the observable matter fields, i.e. through the gravitational couplings [15]. They decay slowly to the observable matter fields. That makes the standard nucleosynthesis dangerous. In our model, some linear combinations of $S$, $T$ and other fields like $X$ remain light whose $F$-terms are of $O(m_{3/2}M)$ and break the SUSY. It is supposed that the couplings between such fields and observable fields are strongly suppressed to guarantee the stability of the weak scale. Such a problem have to be considered for the light linear combinations, too.

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**A Kähler Potential and its Derivatives in String Models**

The Kähler potential $K$ in $Z_N$ orbifold models is given as [4, 5, 6]

$$K = -\ln(S + S^*) - 3\ln(T + T^*) + \sum_\kappa (T + T^*)^{n_\kappa} |\phi^\kappa|^2 + \cdots$$

(133)

in the case of overall moduli. The derivatives of $K$ are given as

$$K_S^S = \frac{1}{(S + S^*)^2}, \quad K_S^T = 0, \quad K_S^\phi = 0,$$

$$K_T^T = \frac{3}{(T + T^*)^2} + \sum_\kappa n_\kappa (n_\kappa - 1)(T + T^*)^{n_\kappa - 2} |\phi^\kappa|^2,$$

36
\[ K_I^\lambda = n_\lambda (T + T^*)^{n_\lambda -1} \phi^\lambda, \quad K_\kappa^\lambda = (T + T^*)^{n_\kappa -1} \delta_\kappa^\lambda. \]

The determinant \( K_I^J \) is calculated as
\[
\Delta \equiv \det K_I^J = \frac{3 \prod_\lambda (T + T^*)^{n_\lambda}}{(S + S^*)^2 (T + T^*)^2} \{ 1 - \sum_\kappa \frac{n_\kappa}{3} (T + T^*)^{n_\kappa} |\phi^\kappa|^2 \}. \tag{134}
\]

The inverses \( (K_I^J)^{-1} \) are given as
\[
(K_S^S)^{-1} = (S + S^*)^2, \quad (K_T^T)^{-1} = 0, \quad (K_S^\kappa)^{-1} = 0,
\]
\[
(K_T^T)^{-1} = \frac{\prod_\kappa (T + T^*)^{n_\kappa}}{(S + S^*)^2 \Delta} \left\{ 1 + \sum_\kappa \frac{n_\kappa}{3} (T + T^*)^{n_\kappa} |\phi^\kappa|^2 \right\} + O(|\phi|^4),
\]
\[
(K_T^\kappa)^{-1} = \left\{ \frac{n_\kappa (T + T^*)^{n_\kappa} \phi^\kappa}{3 - \sum_\lambda n_\lambda (T + T^*)^{n_\lambda} |\phi^\lambda|^2} \right\} \left\{ 1 + \sum_\lambda \frac{n_\lambda}{3} (T + T^*)^{n_\lambda} |\phi^\lambda|^2 \right\} + O(\phi^* |\phi|^4),
\]
\[
(K_\kappa^\lambda)^{-1} = \frac{3(T + T^*)^{-n_\kappa} \delta_\kappa^\lambda}{3 - \sum_\lambda n_\lambda (T + T^*)^{n_\lambda} |\phi^\lambda|^2} \left\{ 1 + \sum_\lambda \frac{n_\lambda}{3} (T + T^*)^{n_\lambda} |\phi^\lambda|^2 \right\} + O(|\phi|^2)
\]

where \( \phi \) represents scalar field of matter multiplet.

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Table Captions

Table 1 U(1) charge generators in terms of $E_8 \times E_8'$ lattice vectors. An anomalous U(1) charge $Q^A$ is defined as $Q^A \equiv Q_5 - Q_6$. We denote the third component of generators of SU(2)' as $T^3'$ and the number in the seventh column represents the eigenvalue of $2T^{3'}$ for the field component with VEV.

Table 2 The modular weights and broken U(1) charges for the light scalar fields and the scalar fields with large VEVs.

Table 3 The particle contents and the ratios of $m_k^2/m_3^2/2$. We refer to the chiral multiplets as $Q_L$ for left-handed quarks, $Q_R$ right-handed quarks, $H$ Higgs doublets, $L$ for left-handed leptons and $R$ for right-handed leptons. The fields $L'$, $L'$, and $R'$, $R'$ are extra $SU(2)_L$ and $SU(2)_R$ doublets, respectively.
Table 1

\[ Q_1 = 6(1, 1, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0) \]
\[ Q_2 = 6(0, 0, 0, 1, -1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0) \]
\[ Q_3 = 6(0, 0, 0, 0, 0, 1, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0) \]
\[ Q_4 = 6(0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0) \]
\[ Q_5 = 6(0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0) \]
\[ Q_6 = 6(0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0) \]
\[ Q_7 = 6(0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0) \]

Table 2

| String states | \(n_k\) | \(Q_1^4\) | \(\sqrt{3}Q_2^2\) | \(\sqrt{2}Q_3^3\) | \(Q_4^4\) | 2T^3 |
|--------------|--------|-----------|----------------|----------------|---------|------|
| \(Q_L\)      | -1     | 6         | 0              | -6             | 0       | 0    |
| \(Q_R\)      | -1     | -6        | 0              | -6             | 0       | 0    |
| \(H\)        | -1     | 0         | 0              | 12             | 0       | 0    |
| \(L (L_4)\)  | -2     | -2        | 0              | -2             | 4       | 0    |
| \(R (R_5)\)  | -2     | 2         | 0              | -2             | 4       | 0    |
| \(L' (L_3)\) | -2     | -2        | 0              | -2             | 4       | 0    |
| \(R' (L_5)\) | -2     | -2        | 0              | -2             | 4       | 0    |
| \(R' (R_4)\) | -2     | 2         | 0              | -2             | 4       | 0    |
| \(\bar{R}' (R_1)\) | -2 | 2 | 0 | -2 | 4 | 0 |
| \(u\)        | -1     | 0         | 0              | 0              | -12     | 0    |
| \(Y\)        | -3     | 0         | 0              | -8             | 4       | 0    |
| \(S_1\)      | -2     | 0         | 0              | -8             | -8      | 0    |
| \(D'_3\)     | -2     | 6         | -6             | 4              | -2      | 1    |
| \(D'_4\)     | -2     | 6         | 6              | 4              | -2      | -1   |
| \(D'_5\)     | -2     | -6        | -6             | 4              | -2      | 1    |
| \(D'_6\)     | -2     | -6        | 6              | 4              | -2      | -1   |
### Table 3

| Rep. | \( q_k^A \) | \( m_{2/3}^2/m_{1/2}^2 | M_i \) |
|------|-------------|----------------------------------|
|      | \( \cos^2 \theta = 0 \) | \( \cos^2 \theta = 1 \) |
| U-sec. | \( Q_L \ (3, 2, 1) \) | 0 | 1 | 0 |
|       | \( Q_R \ (3, 1, 2) \) | 0 | 1 | 0 |
|       | \( H \ (1, 2, 2) \) | 0 | 1 | 0 |
| T-sec. | \( L \ (1, 2, 1) \) | 4 | 8/3 | -5/3 |
| \( N_{OSC} = 0 \) | \( R \ (1, 1, 2) \) | 4 | 8/3 | -5/3 |
|       | \( L' \ (1, 2, 1) \) | 4 | 8/3 | -5/3 |
|       | \( L' \ (1, 2, 1) \) | 4 | 8/3 | -5/3 |
|       | \( R' \ (1, 1, 2) \) | 4 | 8/3 | -5/3 |
|       | \( R' \ (1, 1, 2) \) | 4 | 8/3 | -5/3 |