Simulation and Modelling of a Spatially-Efficient 3D Wireless Power Transfer System for Multi-User Charging

Han-Wei Wang¹, Nelson Xuntuo Wang, and Jeffrey H. Lang²
¹Tsinghua University, China and ²Massachusetts Institute of Technology, USA

Abstract. This paper presents the design, modelling and simulation of a wireless power transfer system with improved 3D spatial efficiency. It is shown that a rotational field driven by balanced magnetic coils carrying phase-shifted currents can achieve almost uniform efficiency in 3-D space. Both 2-coil and 3-coil transmitting systems are designed and studied. Effective mutual inductance is proposed to visualize the magnetic fields from a multiple transmitter system, and efficiency distributions are simulated. Different excitation modes including phase-shifted and non-phase-shifted currents are analysed and compared. The results provide an approach to the design and excitation of the 3D multiple-transmitting-coil system.

1. Introduction
The development of wireless power transfer (WPT) systems has continued for over a hundred years since its first demonstration by Nikola Tesla, with rapid developments occurring over the past decade. Building on the demonstration of effective mid-range WPT using resonance coupling in [1], related papers [2-5] focused on development for various applications such as electric vehicles [2-3], high-voltage transmitting systems [4] and autonomous sensors [5].

Mobile devices are widely used today, and WPT is often employed to conveniently charge them. To improve user freedom, magnetic multi-input-multi-output WPT was invented for the position-free and multi-user charging scenario [6-7]. Further, omnidirectional WPT has been studied to offer full coverage over all user orientations [8]. However, to the best of our knowledge, the uniformity of power distribution over space has not been studied in detail. The concern is that magnetic blind spots in space may result in the inability to transfer power to some users.

To create uniformly efficient power distribution over all space, this paper studies both 2-coil and 3-coil 3-D transmitting coil sets. Further, with analogy to the rotating field in an AC electric motor, excitation using phase-shifted transmitter currents is proposed and compared to in-phase excitation. The comparisons are based on theoretical models and simulation.

2. Modelling of Transmitting System
A WPT transmitting system can be modelled through a mutual coupling circuit. For an m-transmitter and n-receiver system such as shown in Figure 1, the relationship between the voltages and currents of the transmitters and receivers can be expressed as
where: $v_t, v_r, i_t,$ and $i_r$ are voltages and currents; $L_t$ and $L_r$ are self-inductances; $M_t$ and $M_r$ are mutual inductances among transmitters and receivers alone; and $M$ are mutual inductances between transmitters and receivers.

In the sinusoidal steady state, the receiver voltages and currents are related according to
\[
\begin{bmatrix}
v_t \\
v_r \\
v_{t_i} \\
v_{r_i}
\end{bmatrix}
= j\omega \cdot
\begin{bmatrix}
L_t & \cdots & M_{t_1} & \cdots & M_{t_m} \\
\vdots & \ddots & \vdots & \ddots & \vdots \\
M_{t_1} & \cdots & L_{r_1} & \cdots & M_{r_1} \\
\vdots & \ddots & \vdots & \ddots & \vdots \\
M_{t_m} & \cdots & M_{r_m} & \cdots & L_{r_m}
\end{bmatrix}
\begin{bmatrix}
i_t \\
i_r \\
i_{t_i} \\
i_{r_i}
\end{bmatrix}
\tag{1}
\]

Let the complex vectors $V_t$ and $V_r$ be the transmitter and receiver voltages, respectively; similarly let $I_t$ and $I_r$ be the currents. Also, let the matrices $L_t, L_r$ and $M$ be the block transmitter self inductances, receiver self inductances, and transmitter-receiver mutual inductances, respectively, all from (1).

Substitution of (2) into (1) then yields the relation between $I_t$ and $I_r$ as
\[
I_r = j\omega X_r^{-1} \cdot M \cdot I_t
\tag{3}
\]

where
\[
X_r =
\begin{bmatrix}
R_t + R_{r_1} + j(\omega L_{r_1}) (\omega C_{r_1})^{-1} & \cdots & 0 \\
0 & \ddots & \vdots \\
0 & \ddots & R_s + R_{r_m} + j(\omega L_{r_m}) (\omega C_{r_m})^{-1}
\end{bmatrix}
\tag{4}
\]

Finally, each user’s power ratio (receiving power over total power input to the transmitting system) is
Substituting (3) into (5), one can calculate the power ratio of each user as well as overall efficiency.

Treating the transmitter as a whole, the excitation of the transmitter current vector is used to represent the overall current. The effective mutual inductance $M_{\text{eff}}$ defined by $M_{\text{eff}} = M \cdot \mathbf{I}_t \cdot |\mathbf{I}_t|^{-1}$ is used to represent the coupling between the transmitting coils and the receivers. In this case, (3) becomes

$$\mathbf{I}_t = j\omega X_{\text{r}}^{-1} \cdot M_{\text{eff}}$$

When the coupling between receivers can be ignored, (5) can be simplified to

$$\frac{P_{i}}{P_j} = \text{Re}\left(\frac{R_{i}}{R_{i} + R_{\text{r}}} \cdot \frac{R_{j}}{R_{j} + R_{\text{r}}}\right)$$

where

$$R_{i} = \frac{\omega^2 M_{\text{eff}}(i)M_{\text{eff}}(i)^*}{R_{i} + R_{\text{r}} + j[\omega L_{\text{r}} - (\omega C_{\text{r}})^{-1}]}$$

For the single receiver scenario,

$$\eta = \text{Re}\left(\frac{\omega^2 M_{\text{eff}} M_{\text{eff}}^*}{\omega^2 M_{\text{eff}} M_{\text{eff}}^* + R_{i} (R_{i} + R_{\text{r}} + j[\omega L_{\text{r}} - (\omega C_{\text{r}})^{-1}]) \cdot \frac{R_{j}}{R_{j} + R_{\text{r}}}\right)$$

which shows that the efficiency distribution is determined by the distribution of effective mutual inductance, which is influenced by the transmitter structure and current ratio of each transmitter.

### 3. Simulation of Two Transmitter Structures

Two balanced transmitter structures are designed to achieve a uniform efficiency distribution as shown in Figures 2(a) and 2(b). The first structure uses two vertically-oriented orthogonal coils, and the second structure uses three orthogonal coils. The receivers are put on the table plane as shown in Figure 2. The diameter for all transmitting coils is 15 cm, and for the receiving coils it is 10 cm. All coils have 10 turns.

Two transmitter excitations are discussed: equal currents with no phase-shift and with proper phase-shift (90 degrees for the 2-coil structure and 120 degrees for the 3-coil structure). Further, it is assumed that all receivers point optimally towards the transmitters. The mutual inductances are then calculated by the Biot-Savart Law. The effective mutual inductance distributions on the table for the two excitations of the two structures are shown in Figures 3-4, which shows that phase shift can achieve almost uniform effective mutual inductance distribution.
Figure 2. (a) 2-Coil Structure (b) 3-Coil Structure.

(a) 2-Coil Structure (b) 3-Coil Structure.

Figure 3. $M_{\text{eff}}$ of the 2-Coil Structure Driven by a) In-Phase Current. b) Phase-Shifted Current.

(a) 2-Coil Structure (b) 3-Coil Structure.

Figure 4. $M_{\text{eff}}$ of the 3-Coil Structure Driven by a) In-Phase Current. b) Phase-Shifted Current.

(a) 2-Coil Structure (b) 3-Coil Structure.

Figure 5. Efficiency Distribution of the 2-Coil Structure.

Figure 6. Efficiency Distribution of the 3-Coil Structure.
Driven by the phase-shifted current, the simulated efficiency distribution for a single user on the table plane is shown in Figures 5 and 6, which shows good uniformity all around.

To test the uniformity of power distribution among multiple users, the scenario of 8 users equally spaced around the transmitter every 45 degrees, and 15 cm away from the center, is considered. Table 1 shows the minimum power ratio of all users and overall efficiency of the 2-coil and 3-coil structures. Both structures show good uniformity of power distribution, and the 3-coil structure performs better than the 2-coil structure.

| Table 1. Power and Efficiency for Multiple Users |
|-----------------------------------------------|
| 2-Coil Structure                               | 3-Coil Structure                               |
| Minimum power ratio                           | Minimum power ratio                           |
| η                                               | η                                               |
| 4.7%                                           | 70.08%                                          |
| 6.77%                                          | 72.89%                                          |

4. Conclusion
A method is proposed to create a uniform efficiency distribution all around the WPT transmitter. Effective mutual inductance is proposed to visualize coupling in 3-D space around the transmitting system. The efficiency distributions of a single user and multiple users are studied, and the power ratio and overall efficiency of multiple receivers are analysed. The results show that an almost-uniform efficiency distribution is achieved by driving the balanced transmitting coil structure with phase-shifted currents, and that the 3-coil structure performs better than the 2-coil structure.

5. References
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