Minimum measurement time: lower bound on the frequency cutoff for collapse models

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The CSL model predicts a progressive breakdown of the quantum superposition principle, with a noise randomly driving the state of the system towards a localized one, thus accounting for the emergence of a classical world within a quantum framework. In the original model the noise is supposed to be white, but since white noises do not exist in nature, it becomes relevant to identify some of its spectral properties. Experimental data set an upper bound on its frequencies, while in this paper we bound it from below. We do so in two ways: by considering a ‘minimal’ measurement setup, requiring that the collapse is completed within the measurement time; and in a measurement modelling-independent way, by requiring that the fluctuations average to zero before the measurement time. We find that the lower bound is comparable to the upper bound coming from bulk heating experiments.

I. INTRODUCTION

Collapse models are a phenomenological solution to the quantum measurement problem, where the Schrödinger dynamics is modified by adding non-linear stochastic terms, which trigger the collapse of the wave function [1, 2]. In the widely popular mass-proportional CSL model [3], two parameters control the collapse: a collapse rate $\lambda$ and the noise space correlator $r_C$. Their numerical values are chosen in such a way that the collapse is negligible for microscopic objects, thus recovering the standard quantum predictions, and become increasingly stronger for larger objects. While there is a generic consensus on the potential value of $r_C$ ($\sim 10^{-7} \text{m}$), the value of $\lambda$ is rather open [4, 5]. If one requires the collapse to be effective at the level of latent image formation [5], then $\lambda \sim 10^{-8} \text{ s}^{-1}$, with an uncertainty of about 2 orders of magnitude. This is the
value we consider in this article.

The original CSL noise is white in time, because it is easier to study. However, since white noises do not exist in nature, it becomes relevant to consider the possibility of a non-white CSL noise, with a frequency cut off $\omega_M$. This has been done in $[6]$. The cutoff $\omega_M$ is a new phenomenological parameter; in this paper we aim at bounding it from below, by the requirement that the collapse be effective in measurement situations. We do so by following two distinct paths. On the one hand we consider the measurement process performed by a minimal experimental setup, consisting of a detector, an amplifier, a recording device, and a Lithium-ion battery. On the other hand, we bound the cutoff values by considerations on the noise fluctuations, which should average to zero over a measurement process.

II. COLLAPSE RATE FOR A NON-WHITE CORRELATION FUNCTION

The non-white CSL reduction factor for the off-diagonal elements of the density matrix $\langle r_1 | \rho(t) | r_2 \rangle$ is given by Eq. (17) of $[8]$: 

$$\Gamma(t) = \frac{\lambda m^2}{m_0} \left( \frac{r_C}{\sqrt{\pi \hbar}} \right)^3 \int d^3 x \int d^3 y \int_0^t ds F(x - y, s) \Delta m(x) \Delta m(y),$$  

(1)

where $\lambda$ and $r_C$ are respectively the collapse rate and the collapse length; $m$ is the system mass and $m_0$ a nucleon mass. $\Delta m(x) = m_1(x) - m_2(x)$ is the difference of the eigenvalues of the mass density operator for the states $| r_{1,2} \rangle$, and $F(x - y, t)$ is the time integral of the noise correlation function $D(x - y, t - s)$:

$$F(x - y, t) = \int_0^t ds D(x - y, t - s).$$  

(2)

We assume that $D(x - y, t - s)$ is factorized and that the noise has a high-frequency cutoff:

$$D(x - y, t - s) = G(x - y) \cdot \delta_\gamma(\omega)(t - s),$$  

(3)

where $G(x - y)$ is the noise spatial correlator, taking a Gaussian form of width $r_C$, and $\delta_\gamma(\omega)(t - s)$ is the time correlator later defined. Accordingly, Eq. (1) can be rewritten as follows

$$\Gamma(t) = \frac{\lambda m^2}{m_0} \left( \frac{r_C}{\sqrt{\pi \hbar}} \right)^3 \Lambda(t) \int d^3 x \int d^3 y G(x - y) \Delta m(x) \Delta m(y),$$  

(4)

where we have defined

$$\Lambda(t) = \int_0^t ds \int_0^s du \delta_\gamma(\omega)(u - s).$$  

(5)

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1 For the extension to other models see e.g. $[9, 12]$
The temporal correlation \( \delta_{\gamma}(\omega) \) is defined in terms of the cutoff function \( \gamma(\omega) \) as follows:

\[
\delta_{\gamma}(\omega)(t - s) = \frac{1}{\pi} \int_{0}^{\infty} d\omega \gamma(\omega) \cos[\omega(t - s)].
\] (6)

Note that when \( \gamma(\omega) \) is constant (no cutoff), one recovers the white noise case (Dirac delta correlation). There exist a number of cutoff functions that allow to evaluate \( \Lambda(t) \) analytically, but their respective values of \( \Lambda(t) \) differ only little (see Appendix A). For simplicity in the following we will use the Lorentzian cutoff function, because the associated \( \Lambda(t) \) has a simpler analytic expression:

\[
\gamma(\omega) = \frac{\omega_M^2}{\omega^2 + \omega_M^2} \quad \rightarrow \quad \Lambda(t) = e^{-\omega_M t} + \omega_M t - 1 \frac{1}{2\omega_M}.
\] (7)

Figure 1 compares \( \Lambda(t) \) for different values of frequency cutoff with the white noise case. One immediately sees that the lower the frequency cutoff, the smaller the collapse effect and the longer it takes to reach the same reduction rate as for the white noise case. The plot further shows that \( \Lambda(t) \) has a “sub-white” behavior for \( t \lesssim \omega_M^{-1} \), while the rate becomes effectively white for \( t \gtrsim \omega_M^{-1} \).

FIG. 1: \( \Lambda(t) \) with Lorentzian cutoff for different values of frequency cutoff. Solid lines denote: (a/blue) \( \omega_M = 10^6 \) s\(^{-1} \), (b/red) \( \omega_M = 10^8 \) s\(^{-1} \), (c/green) \( \omega_M = 4 \times 10^{10} \) s\(^{-1} \). The lower \( \omega_M \), the longer it takes to reach the white noise rate (dashed line).

If the frequency cutoff \( \omega_M \) is too low, the collapse becomes ineffective. Therefore a lower bound exists for \( \omega_M \), below which measurements do not return definite outcomes within their measurement time. Note that the measurement time, i.e. the time after which a single outcome should be produced, might be very different from the resolution time, which quantifies the ability of the detector to distinguish two signals, potentially very close to each other. The second can be much smaller than the first, meaning that two distinct signals remain superimposed till the measurement is completed.
We consider the extreme case of a photon in a superposition state, which is measured by the minimal experimental setup depicted in Fig. 2. This consists of a single photon detector (such as e.g. photomultiplier tubes, avalanche photodiodes, or superconducting nanowires), an amplifier, and a flash drive that records the signal. The setup is powered by a Lithium-ion battery (LIB). A setup of this kind represents a minimal scenario, as it involves the displacement of the least number of nucleons. As such, the analysis of this setup will provide the strongest bounds on the cutoff frequency. In performing this analysis, our essential assumption is that a measurement is completed by the time a permanent record of the event is made, that can be read out at a later time.

Each part of the setup has a different time scale, namely the detection time $t_D$, the amplification time $t_A$, and the recording time $t_R$. We thus identify the measurement time $t_M$ with the sum of these time scales. In our analysis, we consider three different measurement times: $10^{-8}$ s, which can be easily reached with commercial components; $10^{-9}$ s, which is attainable with more advanced setups; $10^{-10}$ s, which may be achievable in a not too far future with dedicated nanodevices\(^2\).

We first try the more stringent requirement that the measurement is completed when the detector produces an output ($t_M = t_D$). In order to compute the number of electrons displaced by more than $r_C$ in the measurement process, we consider a current pulse of 2 mA, corresponding to the typical output current of a detector (100 mV @ 50 Ω). We can realistically assume that the current pulse width $t_P$ is one order of magnitude smaller than the detection time. Thus if we consider a conservative detection time $t_D = 10^{-8}$ s, we have that $t_P = 10^{-9}$ s. The number of

\(^2\)These estimates are based on data taken from [13][17]. We are thankful to V. H. Patel and C. G. Tully for providing these references. These numbers are also similar to those suggested by other people in the acknowledgement list.
electrons displaced by a current of 2 mA in $10^{-9}$ s is $n_e = 1.2 \times 10^7$. Since the circuit is powered by a LIB, to $n_e$ corresponds the same number of Li$^+$ ions moving in the battery.

In order to estimate the collapse effects, we consider the following formula for the collapse factor, which is a simplified version of Eq. (4) [5]:

$$
\Gamma(t) = \lambda n^2 N \Lambda(t),
$$

(8)

where $n$ is the number of displaced nucleons contained in a volume of side $r_C$, and $N$ is the number of such volumes. We thus need to evaluate how many of the Li$^+$ ions displaced in the battery are within a volume $r_C^3$. LIB electrodes are typically made of LiCoO$_2$ (98 amu), with mass density of about 3 g/cm$^3$ [13] (this is an average value, as the literature reports also slightly larger [19] or smaller [20] densities). This corresponds to $1.9 \times 10^7$ LiCoO$_2$/r$^3_C$, and to the same number density of Li atoms. We assume that when the Li$^+$ ions dissociate from the electrode to diffuse in the electrolyte they do so by conserving this number density, which amounts to $7 \times 1.9 \times 10^7 = 1.3 \times 10^8$ nucleons/r$^3_C$. We can thus estimate the total number of displaced nucleons ($7 \times n_e = 8.4 \times 10^7$) to be organized in one volume $r_C^3$ ($N = 1$), containing $n = 8.4 \times 10^7$ nucleons.

If we assume that the reduction is complete when $\Gamma(t) \approx 1$, recalling that for high frequencies $\Lambda(t) \sim t/2$ (white noise), we find that for Adler’s value of $\lambda$, the collapse (or reduction) time is $t_C = 2.8 \times 10^{-8}$ s. Thus reduction occurs on the same time scale as detection.

Although we clearly expect that for shorter time scales reduction would not occur within detection time, for the sake of completeness we provide the analysis also for $t_D = 10^{-9}$ s and $t_D = 10^{-10}$ s, which can be reached with advanced single-photon detectors. For a detection time $t_D = 10^{-9}$ s, the typical pulse width is $t_P = 10^{-10}$ s. Since the measurement time is one order of magnitude shorter, the number of Li$^+$ ions displaced in a battery will be one order of magnitude smaller: $1.2 \times 10^6$, amounting to $8.4 \times 10^6$ displaced nucleons. Following the previous analysis, all these nucleons are contained in one volume of size $r_C^3$ ($N = 1$, $n = 8.4 \times 10^6$). Accordingly, the reduction time is $t_C = 2.8 \times 10^{-6}$. This is two orders of magnitude too large to guarantee the collapse. However, we might argue that it is likely that more electrons/ions are displaced than those we counted, thus collapse should still occur within the measurement time.

A similar analysis applies to a detection time as low as $t_D = 10^{-10}$ s, providing a reduction time $t_C = 2.8 \times 10^{-4}$ s. We can conclude that for detectors with these typical timescales, reduction

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3 Notice that this calculation assumes that the Li$^+$ ions which escape the electrode are packed together as much as possible. If one assumes instead that they are uniformly distributed over the surface of the electrode, $n$ might decrease significantly.
would not occur when an output is produced. Table I summarizes the reduction times associated to each measurement time considered. The last column on the right shows the crossover point, i.e. the minimum number of particles such that \( t_C = t_M \).

TABLE I: Reduction after detection \((t_M = t_D, 2 \text{ mA current})\). Collapse times \((t_C)\) for three different measurement times: \(10^{-8}, 10^{-9}, \text{ and } 10^{-10} \text{ s}\). \(N\) is the number of volumes of size \(r_C^3\), and \(n\) the number of nucleons in each volume. The last column on the right shows the values of \(t_M, n\) and \(N\) such that \(t_C = t_M\).

| \(t_M (\text{s})\) | \(10^{-8}\) | \(10^{-9}\) | \(10^{-10}\) | \(1.4 \times 10^{-8}\) |
|-------------------|-------------|-------------|-------------|----------------|
| \(n\)             | 8.4 \times 10^7 | 8.4 \times 10^6 | 8.4 \times 10^5 | 1.2 \times 10^8 |
| \(N\)             | 1           | 1           | 1           | 1              |
| \(t_C (\text{s})\) | 2.8 \times 10^{-8} | 2.8 \times 10^{-6} | 2.8 \times 10^{-4} | \(= t_M\)       |

Although for shorter detection time scales collapse does not occur, we recall that detector signals are typically amplified before being shown on an oscilloscope or registered on a hard drive. Motivated by these results, we extend our analysis by requiring the collapse to be completed when one bit of information is registered on a flash drive. Such an operation requires a current of about 500 mA, that is the maximum power consumption for a USB2 device\(^4\).

We thus assume that the amplifier is able to provide a gain of 250, and that a flash drive can record, both on the same timescale as the detection process. This allows to perform our analysis by identifying the previous \(t_D\) with the measurement \(t_M\) time for the whole setup. While these two assumptions certainly hold for \(t_M = 10^{-8} \text{ s}\) (commercial components), and \(t_M = 10^{-9} \text{ s}\) (advanced amplifiers and recording devices working on this time scale exist\(^5\)), they do not for the shorter time scale \(t_M = 10^{-10} \text{ s}\). It however seems that amplifiers and recording devices may be able to work on such a time scale in a near future, thus encouraging us to provide an analysis also for this case.

A current of 500 mA displaces \(3.0 \times 10^9\) electrons in \(t_P = 10^{-9} \text{ s}\), and consequently \(2.1 \times 10^{10}\) nucleons in the battery. This implies that \(N = 1.6 \times 10^2\) and \(n = 1.3 \times 10^8\), thus resulting in a reduction time of about \(t_C = 7.4 \times 10^{-11} \text{ s}\). This is faster than before because more particles are displaced.

In \(t_P = 10^{-10} \text{ s}\), \(2.1 \times 10^9\) nucleons are displaced in the battery. This implies that \(N = 1.6 \times 10^1\), thus resulting in a reduction time of about \(t_C = 7.4 \times 10^{-10} \text{ s}\). Reduction thus occurs on a time scale slightly shorter that the measurement one.

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\(^4\) The currently fastest flash recording device has a power consumption of about 2.7 W \([21]\). For a 5 V rail, this makes a current of 540 mA. Since the device is USB2 compatible, we can safely assume the current not to be higher than the maximum for a USB2.

\(^5\) See footnote 2 at page 4.
A 500mA current pulse with width $t_P = 10^{-11}$ s displaces $2.1 \times 10^8$ nucleons in the battery, which are thus organized in $N = 1.6$ volumes of size $r_C^3$ ($n = 1.3 \times 10^8$). The collapse time is thus $t_C = 7.4 \times 10^{-9}$ s. This is almost two orders of magnitude larger than the measurement time, and we might use again the argument that more electrons have been displaced than we have estimated.

Table II summarizes reduction times after recording, for the three measurement times considered, and the values of $n$ and $N$ such that $t_C = t_M$.

| $t_M$ (s) | $10^{-8}$ | $10^{-9}$ | $10^{-10}$ | $8.5 \times 10^{-10}$ |
|-----------|-----------|-----------|------------|-----------------------|
| $n$       | $1.3 \times 10^8$ | $1.3 \times 10^8$ | $1.3 \times 10^8$ | $1.3 \times 10^8$ |
| $N$       | $1.6 \times 10^2$ | $1.6 \times 10^1$ | $1.6$ | $5.5$ |
| $t_C$ (s) | $7.4 \times 10^{-11}$ | $7.4 \times 10^{-10}$ | $7.4 \times 10^{-9}$ | $t_M$ |

TABLE II: Reduction after recording (500 mA current). Collapse time ($t_C$), number of volumes of size $r_C^3$ ($N$), and the number if nucleons in each volume ($n$), for three different measurement time scales. The last column on the right shows the values such that $t_C = t_M$.

We remark that these reduction times were obtained using the value of $\lambda = 10^{-8} \text{ s}^{-1}$ proposed by Adler [5]. If one were using the value proposed by GRW [4], i.e. $\lambda = 10^{-16} \text{ s}^{-1}$, the reduction times would become eight orders of magnitude larger. Accordingly, such a small value of $\lambda$ would not guarantee a definite measurement outcome, not only for the minimal setup here considered, but also for more complicated setups.

Bounds on the frequency cutoff are obtained simply by substituting the values of $n$ and $N$ from the analysis above into Eq. (8), using Eq. (7) for $\Lambda(t)$. Figure 3 shows the values that $t_C$ and $\omega_M$ can take in order to guarantee that the superposition of the incoming photon is suppressed ($\Gamma(t) \approx 1$). Reduction occurs within the measurement time only if the solid line crosses below the corresponding dashed line (i.e. $t_C < t_M$). The intersection between equally colored solid and dashed lines occurs when $\omega_M$ is large enough so that the collapse time $t_C$ becomes equal to the measurement time $t_M$, thus identifying the lower bound for the cutoff frequency $\omega_M$. We thus see that for a measurement time $t_M = 10^{-8}$ s (blue dashed line), a cutoff $\omega_M \gtrsim 10^6 \text{ s}^{-1}$ guarantees that the superposition is reduced within the measurement time, while for a measurement time $t_M = 10^{-9}$ s (red dashed line), a cutoff $\omega_M \gtrsim 5 \times 10^9 \text{ s}^{-1}$ is needed.

In order to overcome the difficulties in providing a precise estimate of the number of nucleons displaced by the measurement process, in the next section we consider a device-independent approach.
FIG. 3: Parameter space for collapse time $t_C$ (s), and cutoff bound $\omega_M$ (s$^{-1}$) such that $\Gamma(t) \approx 1$ (Lorentzian cutoff), for a single detected photon and $\lambda = 10^{-8}$ s$^{-1}$. Dashed lines denote the three measurement time scales considered: (a/blue) $t_M = 10^{-8}$ s, (b/red) $t_M = 10^{-9}$ s, and (c/green) $t_M = 10^{-10}$ s, which imply different numbers of moving charges. Solid lines are the corresponding collapse times as a function of $\omega_M$ (which depend on the number of displaced charges). The intersection between solid and dashed lines (same color) identifies the lower bound for the cutoff frequency $\omega_M$. The vertical gray line corresponds to the cutoff $\omega_M$ upper bound coming from bulk heating experiments [5, 22], and the shaded area denotes the excluded parameter range.

III. DEVICE INDEPENDENT MEASUREMENT TIME

To find a device independent measure of the collapse time, we consider the time it takes for the noise fluctuations to average to zero. This implies that the noise fluctuated enough to produce a different outcome for different individual noise histories. We consider two measures of the noise fluctuations defined by the average of the noise correlation over the time interval:

$$\tilde{I}(t) \equiv \mathbb{E}[\omega(t)\bar{\omega}(t)] = \frac{1}{t} \int_{0}^{t} ds \delta_{\gamma(\omega)}(t - s), \quad (9)$$
$$\tilde{J}(t) \equiv \mathbb{E}[\bar{\omega}(t)^2] = \frac{2}{t^2} \Lambda(t), \quad (10)$$

where $\bar{\omega}(t)$ is the noise average over the time interval:

$$\bar{\omega}(t) \equiv \frac{1}{t} \int_{0}^{t} ds \omega(s). \quad (11)$$

$\tilde{I}(t)$ measures the fluctuations of the noise realisation over a time $t$ with respect to its average, while $\tilde{J}(t)$ measures the fluctuations of the noise average itself. One can easily check that in the
limit $t \to \infty$ both $\tilde{I}(t)$ and $\tilde{J}(t)$ tend to zero (fluctuations average out), while in the limit $t \to 0$, the two functions approach a constant, which depends on the cutoff function. It is convenient to normalize $\tilde{I}(t)$ and $\tilde{J}(t)$ by the respective initial values, obtaining for a Lorentzian cutoff

$$I(t) \equiv \tilde{I}(t)/\tilde{I}(0) = \frac{1}{\omega_M t} \left(1 - e^{-\omega_M t}\right),$$

$$J(t) \equiv \tilde{J}(t)/\tilde{J}(0) = \frac{2}{\omega_M^2 t^2} \left(\omega_M t - 1 + e^{-\omega_M t}\right).$$

$I(t)$ and $J(t)$ thus take values between zero and one. In order to bound $\omega_M$, we need to choose a threshold below which we consider $I(t)$ and $J(t)$ small enough (ideally it should be $I(t) = 0$, $J(t) = 0$). It is reasonable to consider such a threshold to be 0.1, meaning that when $I(t)$ and $J(t)$ go below this value, the noise already fluctuated enough to give different measurement outcomes. Though reasonable, this is an arbitrary value, and if one were to choose a lower threshold, stronger lower bounds on $\omega_M$ would be obtained.

![Diagram](image.png)

**FIG. 4:** Parameter space for $t_C$, $\omega_M$ such that $I(t)$ and $J(t)$ with Lorentzian cutoff are smaller than 0.1 (solid black lines). Dashed lines correspond to the measurement time scales: (a/blue) $t_M = 10^{-8}$ s, (b/red) $t_M = 10^{-9}$ s, (c/green) $t_M = 10^{-10}$ s. The gray solid vertical line shows the frequency upper bound coming from bulk heating experiments ($\sim 4 \times 10^{10}$ s$^{-1}$), and the shaded area denotes the excluded parameter range.

Figure 4 shows the values of $t$ and $\omega_M$ such that $I(t), J(t) = 0.1$ (solid black lines). For each timescale considered, the lower bound on the frequency cutoff is given by the value that $\omega_M$ takes at the intersection between the solid lines with the respective dashed lines. One thus notices that, for the measurement time scale $t_M \leq 10^{-8}$ s, it is possible to have different collapse outcomes, provided
that $\omega_M \gtrsim 10^9$ s$^{-1}$. This bound is three orders of magnitude stronger than the one obtained in the previous section. For a measurement time $t_M \leq 10^{-9}$ s, different measurement outcomes are guaranteed by a cutoff $\omega_M \gtrsim 10^{10}$ s$^{-1}$, that is of the same order as the one obtained previously. For the shortest measurement time we considered, $t_M = 10^{-10}$ s, the cutoff lower bound is about $\omega_M \simeq 10^{11}$ s$^{-1}$, that is slightly larger than the upper bound suggested by bulk heating experiments ($\omega_M \simeq 4 \times 10^{10}$ s$^{-1}$) [5, 22]. However, by assuming that $I(t) \lesssim 0.2$ or $J(t) \lesssim 0.4$ are sufficient to have different measurement outcomes, reduction would be guaranteed also for $t_M = 10^{-10}$ s.

**IV. DISCUSSION AND CONCLUSIONS**

In order to bound the frequency cutoff of the collapse noise of the CSL model, we have considered a minimal experimental setup detecting a single photon in a superposed state. Our analysis concerned three different time scales: $t_M = 10^{-8}$ s, easily achievable with commercial assets; $t_M = 10^{-9}$ s, achievable with more advanced setups; and $t_M = 10^{-10}$ s, which has not been reached yet, but may be in the near future. We first investigated whether the collapse occurs by the time the detector produces an output, and we showed that this is the case only for slower commercial detectors ($t_M = 10^{-8}$ s, with $\omega_M \gtrsim 10^6$ s$^{-1}$), but not for more advanced ones ($t_M \leq 10^{-9}$ s). We thus required the collapse to occur before one bit of information is registered on a flash drive, and we found that this holds true for measurement time scales $t_M \gtrsim 10^{-9}$ s (with $\omega_M \gtrsim 5 \times 10^9$ s$^{-1}$). The futuristic $t_M = 10^{-10}$ s is only few orders of magnitude below the threshold, but we might argue that our ‘minimal’ analysis likely underestimates the number of particles involved in the measurement process.

Since it is difficult to provide a more precise estimate of this kind, we also proposed a device independent method to bound the frequency cutoff of the CSL noise. This method is based on the assumption that noise fluctuations should average to zero, so that the collapse is complete within the measurement time. By defining two fluctuation measures, and by requiring them to be small, we showed that reduction occurs within the measurement times $t_M = 10^{-8}$ s and $t_M = 10^{-9}$ s, provided that the frequency cutoff is respectively larger than $\omega_M \simeq 10^9$ s$^{-1}$ and $\omega_M \simeq 10^{10}$ s$^{-1}$. The minimum frequency that guarantees different reduction outcomes within the measurement time $t_M = 10^{-10}$ s is $\omega_M \simeq 10^{11}$ s$^{-1}$, that is slightly above the upper bound coming from bulk heating experiments ($\omega_M \simeq 4 \times 10^{10}$ s$^{-1}$). Since there is a degree of arbitrariness in defining the fluctuation measure, we think that reduction could occur also within this shorter timescale.

These values are close to the bulk heating upper bound, thus suggesting that this kind of
experiment may show a signal and confront the lower bounds we obtained.

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Appendix A: $\Lambda(t)$ for different cutoff functions.

There exist a number of cutoff functions that allow one to evaluate the time dependent part of the collapse rate analytically. These are some examples (with cutoff frequency $\omega_M$):

\[
g(\omega) = \theta(\omega - \omega_M) \rightarrow \Lambda(t) = \frac{1}{\pi} \left[ \frac{\cos(\omega_M t) - 1}{\omega_M} + t \sin(\omega_M t) \right] 
\]

(14)

\[
g(\omega) = \exp\left(-\frac{\omega^2}{\omega_M^2}\right) \rightarrow \Lambda(t) = \frac{1}{\omega_M \sqrt{\pi}} \left[ e^{-\frac{\omega_M^2 t^2}{4}} - 1 \right] + \frac{t}{2} \text{Erf}\left(\frac{\omega_M t}{2}\right)
\]

(15)

\[
g(\omega) = \exp\left(-\frac{\omega}{\omega_M}\right) \rightarrow \Lambda(t) = \frac{1}{\pi} \left[ t \arctan(\omega_M t) - \log(1 + \frac{\omega_M^2 t^2}{2}) \right] 
\]

(16)

\[
g(\omega) = \frac{\omega_M^2}{\omega^2 + \omega_M^2} \rightarrow \Lambda(t) = \frac{e^{-\omega_M t} + \omega_M t - 1}{2\omega_M}
\]

(17)

\[
g(\omega) = 1 \rightarrow \Lambda(t) = \frac{t}{2}
\]

(18)

The white noise limit is obtained for $\omega_M \rightarrow \infty$. Figure 5 shows that the behaviour of $\Lambda(t)$ at fixed cutoff frequency $\omega_M = 10^4$ varies only little for the different cutoff functions. We remark that the higher $\omega_M$, the smaller the difference among the functions. In the main text we opted for the Lorentzian cutoff because the associated $\Lambda(t)$ has a simpler analytic expression.
FIG. 5: Left figure shows $\Lambda(t)$ for different cutoff functions on the full range $t = 10^{-10} - 10^{-3}$ s for a (quite low) cutoff frequency $\omega_M = 10^4$ s$^{-1}$. For higher cutoffs the lines are almost undistinguishable. The right plot is a zoom of the range $t = 10^{-5} - 10^{-4}$ s.

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