Inventory decision in a closed-loop supply chain with inspection, sorting, and waste disposal

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Abstract. The study of returned item inventory management in a closed-loop supply chain system has become an important issue in recent years. So far, investigations about inventory decision making in a closed-loop supply chain system have been confined to traditional forward and reverse oriented material flow supply chain. In this study, we propose an integrated inventory model consisting a supplier, a manufacturer, and a retailer where the manufacturer inspects all of the returned items collected from the customers and classifies them as recoverable or waste. Returned items that recovered through the remanufacturing process and the newly manufactured products are then used to meet the demand of the retailer. However, some recovered items which are not comparable to the ones in quality, classified as refurbished items, are sold to a secondary market at a reduced price. This study also suggests that the flow of returned items is controlled by a decision variable, namely an acceptance quality level of recoverable item in the system. We apply multiple remanufacturing cycle and multiple production cycle policy to the proposed model and give the corresponding iterative procedure to determine the optimal solutions. Further, numerical examples are presented for illustrative purpose.

1. Introduction

Recently, the study of returned item inventory management in a supply chain system has received considerable attention in the world of business and industry. Since improving sustainability was considered to be the key factor in achieving a successful supply chain [1], many companies try to apply the returned item inventory management in their system. According to [2] a supply chain strategy may be good for sustainable development while it enables manufacturing plant to rescue and recover many parts and components from used products through reverse logistics activities of remanufacturing and reuse. Closed-loop supply chain is a supply chain system that employs two types of inventory management, which is forward logistics management and reversed logistics management. Forward logistics management focuses on the flow of raw materials, finished products, and the flow of information from suppliers to end consumers. While reversed logistics management focuses on the backward flow of materials from customer to supplier (or alternate disposition) [3].

Nowadays, there are many manufacturing companies that implement the recovery process of the used items as one alternative to fulfil consumer demand. According to [4], product recovery described as a set of activities designed to reclaim value from a product at the end of its useful life (or for short EoL). Product recovery is carried out mainly for three reasons: (1) governmental regulations; (2) market requirements; and (3) hidden economic value of solid waste [4]. The principles of returned item recovery is currently mainly used for automobiles, computers, printers,
copiers, mobile phones, televisions, refrigerators, air conditioners, washing machines, tires and bulky products such as printed circuit boards [5].

According to [6] the recovery process performed by the manufacturer can be divided into several types, such as: repairing, refurbishing, remanufacturing, cannibalization, and recycling. Repairing makes the faulty products to working status. Refurbishing restores the quality of used products while extends their service-life. Remanufacturing reconditions a used product to as-good-as new state. While cannibalization recovers some of reusable parts from a used product and also recycles extracted materials and components from a used product [7]. Recently, there are many companies that implement more than one type of recovery process since the combination of those process will provide more profit to the companies.

The first inventory model which applies the principle of used product recovery was the model belonging to [8]. He proposed an EOQ model for repairable items under (1, R) policy which assumes that the recovery rate is instantaneous and no disposal cost. The inventory model was then developed by [9] that embodied a deterministic inventory model with finite recovery rate. Another extension of [8] was done by [10] who developed a single-product model with backorder. Koh, et al [11] and then developed the study of [9] by considering the capacity of the recovery process.

Other related study was done by [7] in which they consider more than one category of recovery process in the system. They consider inspection and sorting activities for two type of recovery process: remanufacturing and refurbishing, and assumes that remanufactured products will have the same level of quality as usual. Meanwhile, the refurbishing products will have lower quality and assumed to be sold on the secondary market at a lower price. Although the model accommodates more than one category of recovery process, the inventory model is less answer the real conditions possessed by most manufacturing companies. In fact, manufacturers generally conduct pre-inspection process to classify whether the recovery of used products can still be repaired or not, or in the other way, determine the acceptance quality level (AQL). The used products which have quality level lower than the AQL will be discarded and considered as waste, which in this case requires cost to be disposed. A model that has been applying the principles of waste disposal, or commonly called mixed policy strategy, is the model from [12] and [13]. Richter [12] concluded that the bang-bang/pure policy is optimal when compared to mixed (recovery and waste disposal) policy. In later study, it was observed that mixed strategy is optimal, rather than a pure strategy [13]. However, the study of [12] and [13] only consider one category of recovery process which is remanufacturing.

In addition to reverse logistics management, the manufacturing company should also consider forward logistics management which includes raw materials, finished products, and the flow of information from the supplier to the end consumer. Jauhari, et al [14] pointed that a traditional inventory management which focused on developing inventory policy in single-stage system was not suitable for managing inventories across the supply chain. Hence, management of forward logistics and reverse logistics should be done by the company in order to realize the balance of the system as well as to reduce the total cost significantly. The example of inventory model that consider the coordination of the players in the closed-loop supply chain system is the model by [15]. They developed an integrated retailer, manufacturer, supplier, and third party recycling dealer inventory system and manage to get the optimal solution for joint profit maximization on the integration of the existing supply chain systems. The model of [15] was then developed by [16] to the more general (1, R) and (P, 1) policies.

The model developed in this study combines three inventory models. The basic model of closed-loop supply chain that implements inventory integration between the parties i.e. suppliers, manufacturers, and retailers is adopted from a model belonging to [15]. The multiple type of recovery process is adopted from [7]. As for the waste disposal system, the quality dependant return rate, as well as multiple batch production cycle policy is adopted from [13].

2. Description of the inventory problem

In this study, we develop deterministic inventory model for the system of closed-loop supply chain which considering three parties: retailer, manufacturer, and supplier. In this system, orders from the retailer met by the manufacturer through the remanufacturing phase \(T_R\) and the regular production phase \(T_P\). In one period \(T\), there are \(R\) remanufacturing cycle and \(P\) regular production
cycles. Where in one remanufacturing cycle there are \( m \) shipments to retailers, and in a regular production cycle there are \( n \) shipments to retailers. This model assumes that shortage is not allowed and the products are shipped to arrive when the current product inventory level is zero.

There are two types of recovery process in this model, i.e. remanufacturing process and refurbishing. The remanufacturing process is a recovery process in which the used products are fully fixed to the same quality as before (as-good-as-new-one), at a cost of \( C_R \). Meanwhile, in refurbishing process, the used products are not fully fixed, so they can’t be as good as before, with a cost of \( C_f \). Remanufacturing products, with the proportion of \( 1-f \) from the total recoverable used item, are sold to major markets (primary market) at a price equal to the finished product, which are priced at \( P_r \). Whereas, products with a proportion of \( f \) refurbishing from the recoverable used item, are sold to the secondary market (secondary market) at a lower price (\( P_f < P_r \)). However, the refurbishable item proportion, \( f \), is assumed to be constant and it is meant that the model does not yet consider the real probabilistic condition in used item recovery process.

In this model, the acceptable quality level/AQL (\( q \)) is considered as decision variable and denoted as \( q \), where \( q \) is the portion of recoverable returned items (0<\( q \)<1). Then, 1-\( q \) is the proportion of items that are discarded and considered as a waste because it does not exceed the quality specifications. The discarded items then will be disposed at a cost of \( C_w \). The return rate of used/returned items, \( C \), is a function of \( q \) where \( C=C(q) \) and is a portion of the demand rate (\( D_m \)), i.e., (0<\( C(q)/D_m <1 \)). The function of \( C(q) \) itself adopted from [13] where the quality factor of the demand function is \( f_q = b e^{-\phi q} \) with 0<\( b < 1 \) and \( \phi > 1 \) are parameters.

![Figure 1](image.png)

Figure 1. Product flow of the integrated Closed-loop supply chain inventory system in an interval of length \( T \).

2.1 The parameters related to the retailer

- \( D_m \): the demand rate (unit/year), \( D_m > 0 \);
- \( A_r \): the fixed cost for the retailer ordering products ($/order);
- \( F_r \): the product inventory holding cost percentage per unit price per year for the retailer ($/year);
- \( P_c \): the unit product retail price for the retailer ($/unit);
- \( P_r \): the unit product wholesale price for the retailer ($/unit);
- \( T_r \): the length of the retailer ordering cycle (year);
- \( TIC_r \): the unit time cost for the retailer related to the product inventory ($/year);
- \( TP_r \): the unit time profit for the retailer related to the product inventory ($/year).
2.2 The parameters related to the manufacturer

- \( A_m \): the setup cost for each production batch ($/batch)
- \( A_R \): the setup cost for each remanufacture batch ($/batch)
- \( A_{mwa} \): the material ordering cost ($/order)
- \( F_M \): the manufacturer holding cost fraction for serviceable inventory \((0<F_M<1)\)
- \( F_{mwa} \): the manufacturer holding cost fraction for raw material inventory \((0<F_{mwa}<1)\)
- \( F_{rw} \): the manufacturer holding cost fraction for recoverable inventory \((0<F_{rw}<1)\)
- \( P_{rw} \): the manufacturer raw material price from supplier ($/unit)
- \( P_u \): the used item price from market ($/unit)
- \( P_f \): the refurbished item selling price ($/unit)
- \( T_{R1} \): the production period on each remanufacturing cycle (year)
- \( T_{R2} \): the non-production period on each remanufacturing cycle (year)
- \( T_P \): the production period on each regular production cycle (year)
- \( R \): the number of remanufacturing cycle on each model cycle \(T\) (positive integer)
- \( P \): the number of regular production cycle on each model cycle \(T\) (positive integer)
- \( m \): the number of delivery to retailer on remanufacturing cycle
- \( n \): the number of delivery to retailer on regular production cycle
- \( N \): the total number of delivery to retailer \((N=Rm+Pn, \text{where} \ N\text{ is positive integer})\)
- \( I_{P\text{max}} \): the maximum inventory level for finished product on remanufacturing cycle (unit)
- \( I_{P\text{max}} \): the maximum inventory level for finished product on regular production cycle (unit)
- \( I_{R\text{max}} \): the maximum inventory level for recoverable item (unit)
- \( I_{m\text{Wmax}} \): the maximum inventory level for raw material item (unit)
- \( M \): the fixed cost for processing all demand from end customer ($)
- \( p \): the annual production rate \((p>D, \text{unit/year})\)
- \( r \): the annual remanufacture rate \((r>D, \text{unit/year})\)
- \( \beta \): the multiple factor for annual production rate \((0<\beta<1)\)
- \( \gamma \): the multiple factor for annual remanufacture rate \((0<\gamma<1)\)
- \( f \): the constant percentage of recoverable item classified into the refurbishing category \((0<f<1)\)
- \( q \): the proportion of recoverable item in returned item – AQL \((0<q<1)\)
- \( C \): the annual return rate of used item (unit/year)
- \( C_f \): the refurbishing cost per unit product ($/unit)
- \( C_p \): the regular production cost per unit product ($/unit)
- \( C_R \): the remanufacturing cost per unit product ($/unit)
- \( C_w \): the waste disposal cost per unit product ($/unit)
- \( C_{insp} \): inspecting and sorting cost per unit product ($/unit)
- \( TIC_m \): the manufacturer total inventory cost on each cycle time \(T_r\) ($)
- \( TIC_{mR} \): the raw material total inventory cost on each cycle time \(T_r\) ($)
- \( TIC_{rw} \): the used item total inventory cost on each cycle time \(T_r\) ($)
- \( TR_m \): the manufacturer total revenue on each cycle time \(T_r\) ($)
- \( TP_m \): the supplier total profit on each cycle time \(T_r\) ($)

2.3 The parameters related to the supplier

- \( A_s \): the supplier fixed cost for each order ($/order)
- \( F_s \): the supplier annual holding cost fraction for serviceable inventory \((0=F_s<1)\)
- \( P_s \): the supplier purchasing cost ($/unit)
- \( I_{S\text{max}} \): the supplier maximum inventory level (unit)
- \( I \): the number of raw material shipment to manufacturer on each cycle
- \( L \): the fixed cost to process all kind of order ($)
- \( TIC_s \): the supplier total inventory cost on each cycle time \(T_r\) ($)
- \( TR_s \): the supplier total revenue on each cycle time \(T_r\) ($)
- \( TP_s \): the supplier total profit on each cycle time \(T_r\) ($)
2.4 Assumptions

- Demand rate is given and constant (deterministic).
- Lead time for shipment and lead time for production equal to zero.
- Shortage is not allowed.
- Production rate is always greater than demand rate ($p > D_m$).
- Remanufacture rate is always greater than demand rate ($r > D_m$).
- Return rate ($C$) depend on the value of $q$, while $C < D_m$.
- Remanufactured product have the same quality level as new product (as-good-as-new-one).
- There are no defective products on remanufacturing process, production process, and refurbishing.

3. Mathematical Model

The model developed in this section extends the models of [15] by adding some more considerations. The models of [15] implements the single remanufacturing cycle-single production cycle policy (for short $P(1,1)$). Yuan and Gao [16] are then develop the models of [15] to the more general $(1,R)$ and $(P,1)$ policies. However, those $(1,R)$ and $(P,1)$ policies may not be the optimal manufacturer policy. Therefore, this study will extend the work of [15] and [16] to the multiple remanufacturing cycle-multiple production cycle policy $P(P,R)$ with adding some consideration i.e. inspection, sorting, and waste disposal.

Figure 2 shows the inventory level of the retailer-manufacturer system which adopts the $(P,R)$ policy where $T$ is equal to $(R m + P n) T_r$. The average retailer inventory level is $D_m T_r/2$ and the unit time cost for the retailer related to the product inventory is:

$$TIC_r = \frac{A_r}{T_r} + \frac{D_m P_r}{T_r} + \frac{D_m F_r P_r T_r}{2}$$

(1)

The first term is the ordering cost ($OC_r$), the second term is the purchasing cost ($PC_r$), and the last term is the holding cost ($HC_r$). Then, the unit time profit for the retailer related to the product inventory can be expressed as follows:

$$TP_r = \frac{D_m P_c}{T_r} \cdot \frac{A_r}{T_r} - D_m P_r - \frac{D_m F_r P_r T_r}{2}$$

(2)

Figure 3 illustrates the integrated supplier-manufacturer inventory model. The inventory level of the manufacturer consisting recoverable inventory, serviceable inventory, dan raw material inventory with $(P,R)$ policy are also shown in Figure 3. Refer to [13], the return rate is dependant to the quality factor at the function:

$$C(q) = D_m (bc)^{q-w}$$

(3)
The unit time cost for the manufacturer related to the recoverable inventory is:

\[
TIC_{rw} = \frac{C_f D_m f R m}{(1-f)(R m + P n)} + \frac{C_{isc} \gamma D_m R m}{(1-f)q(R m + P n)} + \frac{D_m P_n R m}{(1-f)q(R m + P n)} + \frac{C_w D_m (1-q) R m}{(1-f)q(R m + P n)} + \frac{F_P x T r \gamma}{2(R m + P n)}
\]

The recoverable inventory cost for manufacturer consists of used item purchasing cost \(PC_{rw}\), inspection cost \(ISC_{rw}\), refurbish cost \(FC_{rw}\), waste disposal cost \(WDC_{rw}\) and recoverable inventory holding cost \(HC_{rw}\). While the unit time cost for the manufacturer related to the serviceable inventory is given by equation (5) below.

\[
TIC_{m} = \frac{A_R R + A_m P + M(R m + P n)}{T_A (R m + P n)} + \frac{C_p D_m P_n}{R m + P n} + \frac{C_p D_m R m}{R m + P n} + F_M \left(\frac{P_m R m + P_m P n}{R m + P n}\right) \times \left(\frac{R m + P n}{D_m T_r \gamma} \left[\frac{C_q (1-(R m + P n))}{D_m R} \right] + \frac{P_n (D_m T_n + D_m (1-C_q D_m) T_r (1-f)(R m + P n))}{R m + P n} \right)
\]

Where the first term is the setup cost for regular production and remanufacturing \(SC_m\), the second one is the holding cost \(HC_m\), the third one is the regular production cost \(PC_m\) and the last one is the remanufacturing cost \(RC_m\).

![Figure 3. The integrated supplier-manufacturer inventory model under \((P,R)\) policy in a model cycle \((R=2, P=2, l=2\) for this example).](image-url)
For the manufacturer raw material inventory, the unit time cost is given by:

\[
TIC_{rm} = \frac{A_{rw}P_l}{T_r (R_m + P_n)} + \frac{D_m P_{rm} P_n}{R_m + P_n} + F_{mw} P_{rm} \left( \frac{P D_m n^2 T_r \beta}{2 l (R_m + P_n)} \right)
\]  

(6)

Then, the unit time profit for the manufacturer obtained by subtracts unit time revenue by unit time cost below:

\[
TP_m = P_r D_m + P_f \left( \frac{D_m P_{rm} R_m}{(1-f)(R_m+P_n)} \right)
\]

\[
\frac{C_q D_m R_m}{R_m + P_n} - \frac{C_f D_m P_n}{(1-f)q(R_m+P_n)} (1-f)^2 \frac{D_m P_{rm} P_n}{R_m + P_n} + \frac{P (2 A_{mw} + 2 D_m l P_{rm} T_r n + D_m F_m P_{rm} P_n T_r 2 \beta n^2)}{2 l T_r (R_m + P_n)}
\]

\[
\frac{A_R R + A_m P + M (R_m + P_n)}{T_r (R_m + P_n)} - \frac{F_r P_u}{2 (R_m + P_n)} x 2 R^3 m (-C_f q + T_r (D_m + C_q (1 + (-2 + f) f))) m + C q T_r P_n (-2 (-1 - \gamma) m + P_n) + R_m T_r (-3 D_m + C_q (-1 - 3 + 3 \gamma f + \gamma^2) m + 2 C q (f + f r + T_r P_n))
\]

\[
- \frac{F_m T_r}{2 (R_m + P_n)} x (P_r R m + P_r P_n) (R m (D_m (-1 + 2 \beta)) + C (-1 + f) q (-1 + \gamma) m)
\]

\[
-(D_m - C_q) (-1 + \beta) m - P_n (D_m - 2 D_m \beta + C q (-1 + f + f r) m + (D_m - C q) (-1 + \beta) m)
\]

(7)

Based on supplier-manufacturer inventory level presented in figure 3, the supplier unit time inventory cost (TIC_s) consists of setup cost (SC_s), purchasing cost (PC_s), and holding cost (HC_s) which is given by:

\[
TIC_s = \frac{(A_s + l l) P}{T_r (R_m + P_n)} + \frac{D_m P_s P_n}{R_m + P_n} + \frac{D_m F_s P_s T_r \beta P n^2 (-1 + l)}{2 l (R_m + P_n)}
\]

(8)

The unit time profit is the unit time revenue subtracted by the unit time relevant inventory cost to the supplier, which is shown as follows:

\[
TP_s = \frac{P_{rm} P_n}{R_m + P_n} - \frac{(A_s + l l) P}{T_r (R_m + P_n)} + \frac{D_m P_s P_n}{R_m + P_n} + \frac{D_m F_s P_s T_r \beta P n^2 (-1 + l)}{2 l (R_m + P_n)}
\]

(9)

The joint annual profit is the unit time profits of the retailer, the manufacturer, and the supplier which is formulated by equation (10).

\[
JP(T_r, P, R, m, n, l, q) = TP_m + TP_s + TP_r
\]

(10)

The optimization problem is stated as

\[
\text{Max } JP(T_r, P, R, m, n, l, q), \quad \text{s.t. } T_r \geq 0
\]

\[
0 < q < l
\]

\[
R m D_m T_r = C q (1 - f) T_r (R m + P_n)
\]

(11)

\[
N = R m + P n
\]

(12)

\[
R, P, N, l = 1, 2, 3, \ldots
\]

4. Solution Procedure

The optimization problem is to determine the values of \( T_r, P, R, m, n, q \) and \( l \) that maximize \( JP(T_r, P, R, m, n, l, q) \). The joint profit function given in equation (10) can be optimized using the following solution procedure. However, the following solution procedure only gives the local optimization not the global optimization.

The annual remanufacturing quantity equals the annual remanufacturable used product rate as shown equation (11). Due to equation (12), we can obtain the function of \( m \) and \( n \) as follows.

\[
m = \frac{N C q (1 - f)}{D_m R}
\]

(13)

\[
n = \frac{N (1 - f) l C q D_m}{P}
\]

(14)
Substituting equation (13) and equation (14) into equation (10), we transform \( JP(T_r, P, R, m, n, q, l) \) into \( JP(T_r, P, R, N, q, l) \), i.e.:

\[
\begin{align*}
JP(T_r, P, R, N, q, l) &= \left[ 1/2 \left( \frac{2 C P_{\text{nc}}}{(\gamma-1)} + 2 D_m P_r + 2 D_m P_m + \frac{2 C P_a}{(\gamma-1)} - 2 C P_a \right) \right] - 2 C P_f q^2 (\gamma-1) + 2 C P_f \frac{q}{(\gamma-1)} \\
&+ \frac{1}{D_m^2} C F_r q \left( C(1-f)N \frac{q}{T_r} (D_m + 2 D_m^2 + C q (3 + 5 \gamma) q^2) \right) \\
&+ (D_m^2 \left((\gamma-1) N T_r \right) + \frac{1}{D_m^2} C F_r \frac{q}{(\gamma-1)} + \frac{1}{(\gamma-1)} N T_r + \frac{1}{(\gamma-1)} M N A_m + A_m P_m) \right] \frac{1}{2} \\
&+ R \left((\gamma-1) N (D_m + C q) \frac{q}{(\gamma-1)} + D_m (D_m^2 + 2 D_m^2 + 2 C q (\gamma-1) q^2) \right)
\end{align*}
\]

To obtain the optimal value of \( T_r \), that maximise the unit time joint profit in equation (15), let the derivative of equation (15) with respect to \( (\text{for short w.r.t.} T_r, \text{equal zero, then the optimal retailer ordering cycle} \ (T_r^*) \) is given below:

\[
T_r^*(R, P, N, q, l) = \left[ \frac{1}{2} D_m \left( D_m \left( F_r P_r + F_r P_m (1 + 2 \beta) + 2 C F_r (2 R + (1 + 2 \beta) + 2 R) \right) \left( A_m + A_r + (A_m + A_r + L) \right) \right) \right]^{1/2}
\]

The optimal value of \( N \) and \( l \) can be obtained by letting the derivative of equation (15) w.r.t. \( N \) and \( l \) equal zero and then substituting the optimal value of \( T_r^* \) in equation (16) to those equation respectively. Then we obtain the optimal values of \( N^* \) and \( l^* \) that maximise the unit time joint profit as given below.

\[
N^*(R, P, q, l) = \left[ \frac{1}{2} D_m \left( F_r P_r + F_r P_m (1 + 2 \beta) + 2 C F_r (2 R + (1 + 2 \beta) + 2 R) \right) \left( A_m + A_r + (A_m + A_r + L) \right) \right]^{1/2}
\]

The function of \( N^* \) and \( l^* \) in equation (17) and equation (18) are then used as an approximation to find the optimal solution of \( N^* \) and \( l^* \). As for the variable \( T_r^* \), \( T_r^* \) is used to calculate the value \( T_r^* \) at \( R^*, P^*, q^*, I^* \), and \( N^* \), and \( T_r^* \) that maximise \( JP(T_r, P, R, m, n, q, l) \) can be derived by these following procedure.

**Step 1** For \( q^{(0)}=1 \), determine the value of \( C(q) \) by solving equation (3).

**Step 2** Set \( q^{(0)}=1, P^{(0)}=1, \) and \( R^{(0)}=I \) then determine the value of \( I \) using equation (14). If the value of \( I \) obtained is not an integer, use the approximation \( I^{(n)}=\lceil I^{(n)} \rceil \) which states the starting point of the optimal value of \( I \).
Step 3 Use the value of \( l = \text{start} \) to compute the value of \( N^a \) by solving equation (13). If the value of \( N^a \) obtained is not an integer, use the approximation \( N^a = \lfloor N^a \rfloor \) which states the starting point of the optimal value of \( N \).

Step 4 Determine the value of \( T \) by solving equation (16), with \( R \in [1, N-R] \), \( P = P^a \) and \( q = q^b \), where \( a \) and \( b \) are iteration indices with \( a, b = 1, 2, 3, \ldots \).

Step 5 Derive the optimal value of \( N \) which satisfy the following inequality:
\[
JP(N-1,R^*,P^*,T^*) \leq JP(N,R^*,P^*,T^*) \geq JP(N+1,R^*,P^*,T^*)
\] (19)

Step 6 Repeat step 1 to 5 for the value of \( P^{(a-1)} = P^{(a)} + 1 \) to find the optimal value of \( P \) which satisfy the following inequality:
\[
JP(N,R^*,P^*,T^*) \leq JP(N,R^*,P^*,T^*) \geq JP(N,R^*,P^*,T^*)
\] (20)

Step 7 Derive the optimal value of \( l \) which satisfy the following inequality:
\[
JP(N,R^*,P^*,T^*,l) \leq JP(N,R^*,P^*,T^*,l) \geq JP(N,R^*,P^*,T^*,l)
\] (21)

Step 8 Repeat step 1 to 7 for the set value of \( q \) with the changes of \( q^{(b+1)} = q^{(b)} \times 0.01 \).

Step 9 Find the combination value of \( P^*, R^*, N^a, T^*, l^* \), and \( q^* \) that gives the maximum value of \( JP(N, R, P, l, T) \).

Step 10 Given a known \( N^a \), the optimal value of \( m^* \) and \( n^* \) can be obtained by equation (13) and equation (14). Hence the optimal values \( P^*, R^*, T^*, l^* \), and \( q^* \) are derived.

5. Numerical Example

This section presents numerical example to illustrate the behavior of the model \( P(R, P) \) presented herein. The parameters used to illustrate the concept are: \( D_n = 2000 \) units/year, \( A_r = $100/\)order, \( A_n = $350/\)collection, \( A_r = $200/\)order, \( A_w = $2000/\)batch, \( A_b = $2500/\)batch, \( P_r = $175/unit, P_m = $150/unit, P_n = $100/unit, P_w = $60/unit, P_r = $80/unit, P_m = $120/unit, F_r = 0.3/unit/year, F_m = 0.5/unit/year, F_w = 0.4/unit/year, F_r = 0.2/unit/year, M = $350/batch, L = $150/delivery, C_r = $20/unit, C_m = $5/unit, C_w = $1/unit, C_r = $23/unit, C_m = $15/unit, \( \gamma = 0.4, \beta = 0.6, \) and \( f = 0.3 \). Those parameters were adopted from [7], [13], and [15].

By using the solution procedure described in the previous section, the optimal value for each decision variable can be derived below.

### Table 1. The results obtained by the following numerical examples.

| \( q \) (dv) | \( l \) (dv) | \( P \) (dv) | \( R \) (dv) | \( N^a \) | \( T^* \) (dv) | \( TP_m \) | \( TP_r \) | \( TP_\text{l} \) | \( JP \) |
|---|---|---|---|---|---|---|---|---|---|
| 0.99 | 1 | 1 | 1 | 6 | 0.082 | $76,292 | $45,098 | $31,318 | $152,708 |
| 0.99 | 1 | 2 | 1 | 12 | 0.071 | $77,992 | $45,395 | $31,295 | $154,683 |
| 0.99 | 1 | 3 | 1 | 11 | 0.095 | $59,037 | $44,685 | $31,256 | $134,978 |
| 0.99 | 1 | 4 | 1 | 14 | 0.093 | $58,840 | $44,748 | $31,242 | $134,830 |
| 0.99 | 2 | 1 | 1 | 6 | 0.086 | $76,707 | $44,964 | $27,020 | $148,691 |
| 0.99 | 2 | 2 | 1 | 12 | 0.075 | $78,672 | $45,289 | $27,547 | $151,507 |
| 0.99 | 2 | 3 | 1 | 10 | 0.108 | $59,958 | $44,224 | $28,265 | $132,447 |
| 0.99 | 2 | 4 | 1 | 13 | 0.104 | $59,697 | $44,356 | $28,425 | $132,479 |
| 0.99 | 3 | 1 | 1 | 6 | 0.089 | $77,088 | $44,870 | $25,385 | $147,343 |
| 0.99 | 3 | 2 | 1 | 12 | 0.078 | $79,124 | $45,208 | $26,097 | $150,429 |
| 0.99 | 3 | 3 | 1 | 12 | 0.098 | $59,829 | $44,563 | $26,896 | $131,288 |
| 0.99 | 3 | 4 | 1 | 13 | 0.109 | $60,172 | $44,183 | $27,305 | $131,659 |
| 0.98 | 1 | 1 | 1 | 6 | 0.082 | $77,834 | $45,095 | $31,267 | $154,196 |
| 0.98 | 1 | 2 | 1 | 12 | 0.071 | $79,524 | $45,394 | $31,244 | $156,163 |
| 0.98 | 1 | 3 | 1 | 11 | 0.095 | $58,758 | $44,684 | $31,205 | $134,647 |
| 0.98 | 1 | 4 | 1 | 14 | 0.093 | $58,554 | $44,748 | $31,191 | $134,492 |
| 0.98 | 2 | 1 | 1 | 6 | 0.086 | $78,248 | $44,961 | $26,979 | $150,189 |
| 0.98 | 2 | 2 | 1 | 12 | 0.075 | $80,202 | $45,287 | $27,506 | $152,995 |
| 0.98 | 2 | 3 | 1 | 10 | 0.108 | $59,677 | $44,223 | $28,223 | $132,123 |
| 0.98 | 2 | 4 | 1 | 13 | 0.104 | $59,410 | $44,356 | $28,383 | $132,149 |
| 0.98 | 3 | 1 | 1 | 6 | 0.089 | $78,629 | $44,868 | $25,347 | $148,844 |
| 0.98 | 3 | 2 | 1 | 12 | 0.078 | $80,653 | $45,207 | $26,059 | $151,920 |
The optimal values of $P^*$, $R^*$, $N^*$, $\ell^*$, $q^*$, and $T^*$ satisfying the following solution procedure are 2, 1, 12, 1, 0.82, and 0.072 years. Hence, the value of $m^*$ and $n^*$ can be derived by solving equation (13) and equation (14) respectively. The optimal solution of the model using the following numerical example are presented in Table 2.

Table 2. The optimal solution of the model using the following numerical example.

| $q$ (dv) | $l$ (dv) | $P$ (dv) | $R$ (dv) | $N^*$ | $T^*$ (dv) | $TP_m$ | $TP_r$ | $TP_s$ | $JP$ |
|----------|----------|----------|----------|-------|------------|--------|--------|--------|------|
| 0.98     | 3        | 3        | 1        | 12    | 0.093      | $58,998$ | $44,756$ | $26,774$ | $130,969$ |
| 0.98     | 3        | 4        | 1        | 13    | 0.109      | $59,883$ | $44,182$ | $27,266$ | $131,332$ |
| 0.83     | 1        | 1        | 1        | 6     | 0.096      | $330,793$ | $44,612$ | $30,409$ | $405,813$ |
| **0.83** | **1**    | **2**    | **1**    | **12** | **0.071**  | **$330,878$** | **$45,390$** | **$30,234$** | **$406,504$** |
| 0.83     | 1        | 3        | 1        | 11    | 0.107      | $53,584$ | $44,226$ | $30,176$ | $127,987$ |
| 0.83     | 1        | 4        | 1        | 14    | 0.104      | $53,164$ | $44,342$ | $30,118$ | $127,623$ |
| 0.83     | 2        | 1        | 1        | 6     | 0.104      | $331,591$ | $44,333$ | $29,179$ | $405,103$ |
| 0.83     | 2        | 2        | 1        | 12    | 0.071      | $330,278$ | $45,390$ | $30,234$ | $405,904$ |
| 0.83     | 2        | 3        | 1        | 11    | 0.116      | $54,294$ | $43,916$ | $29,365$ | $127,575$ |
| 0.83     | 2        | 4        | 1        | 14    | 0.112      | $53,818$ | $44,050$ | $29,373$ | $127,241$ |
| 0.83     | 3        | 1        | 1        | 6     | 0.109      | $332,040$ | $44,166$ | $28,694$ | $404,899$ |
| 0.83     | 3        | 2        | 1        | 12    | 0.083      | $330,223$ | $44,712$ | $28,868$ | $403,803$ |
| 0.83     | 3        | 3        | 1        | 11    | 0.121      | $54,711$ | $43,699$ | $29,041$ | $127,451$ |
| 0.83     | 3        | 4        | 1        | 14    | 0.118      | $54,205$ | $43,840$ | $29,076$ | $127,120$ |
| 0.82     | 1        | 1        | 1        | 7     | 0.074      | $55,722$ | $45,327$ | $30,706$ | $131,756$ |
| 0.82     | 1        | 2        | 1        | 13    | 0.071      | $62,719$ | $45,388$ | $26,793$ | $135,080$ |
| 0.82     | 1        | 3        | 1        | 19    | 0.067      | $67,264$ | $45,485$ | $27,258$ | $140,007$ |
| 0.82     | 1        | 4        | 1        | 25    | 0.064      | $71,340$ | $45,553$ | $27,448$ | $144,342$ |
| 0.82     | 2        | 1        | 1        | 7     | 0.078      | $56,226$ | $45,421$ | $26,366$ | $127,813$ |
| 0.82     | 2        | 2        | 1        | 13    | 0.074      | $63,153$ | $45,318$ | $25,543$ | $134,015$ |
| 0.82     | 2        | 3        | 1        | 19    | 0.067      | $67,264$ | $45,485$ | $27,258$ | $140,007$ |
| 0.82     | 2        | 4        | 1        | 25    | 0.064      | $71,340$ | $45,553$ | $27,448$ | $144,342$ |
| 0.82     | 3        | 1        | 1        | 7     | 0.080      | $56,618$ | $45,147$ | $24,720$ | $126,485$ |
| 0.82     | 3        | 2        | 1        | 13    | 0.074      | $63,153$ | $45,318$ | $25,543$ | $134,015$ |
| 0.82     | 3        | 3        | 1        | 19    | 0.070      | $67,677$ | $45,422$ | $25,935$ | $139,035$ |
| 0.82     | 3        | 4        | 1        | 25    | 0.067      | $71,731$ | $45,498$ | $26,198$ | $143,427$ |

6. Conclusion

In this study, we analyze the closed-loop supply chain inventory problem considering inspection, sorting, and waste disposal under multiple remanufacturing and production cycle policy, or for short $R(P, P)$. Previous works on this problem only consider a traditional closed-loop supply chain with $(1, 1), (1, P), \text{and } (R, 1)$ policy. In this model, we assume that the used item return rate is dependant to the quality factor $(l_q)$ and is the portion of demand. Used items that collected from customers are kept in the recoverable inventory until it is time to start the inspection/sorting
and recovery process. There are two categories of used item recovery process considered in this model, i.e. remanufacturing and refurbishing. However, this model does not yet consider the real probabilistic condition in used item recovery process since the percentage of refurbishable item is still assumed to be constant.

From the perspective of the retailer, manufacturer, and the supplier, an optimal production and replenishment policy was formulated to maximize the joint profit. The optimal value of retailer cycle time, acceptable quality level, production cycle, remanufacturing cycle, number of shipments from manufacturer to the retailer under remanufacturing and regular production phase, and number of shipments from supplier to the manufacturer were determined. The result from numerical example shows that the model gives benefit to the retailer, the manufacturer, and the supplier respectively.

This study may be extended by increasing the number of used item classification to more than two categories (e.g. remanufacturable, recyclable, refurbishable, cannibalizable, repairable) and also considering the real probabilistic condition in used item recovery process. Another extension is to consider that the production, remanufacturing processes, and refurbishing processes are imperfect where defective items are either reworked or scrapped.

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