Entanglement Entropy for time dependent two dimensional holographic superconductor

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We studied entanglement entropy for a time dependent two dimensional holographic superconductor. We showed that the conserved charge of the system plays the role of the critical parameter to have condensation.

Keywords: AdS/CFT Correspondence; holography and condensed matter physics (AdS/CMT).

1. Introduction

Entanglement exists in the core of many aspects of quantum field theory and statistical mechanics and it is therefore desirable to understand its structure as well as possible using gauge-gravity duality\textsuperscript{1-4}. One attempt to improve our understanding of entanglement is the study of our ability to perform calculations on asymptotic Anti de Sitter bulk models. This is due to the celebrated anti de Sitter/conformal field theory (AdS/CFT) conjecture, which it relates the weakly coupled gravitational theory in bulk to the strongly coupled quantum theory in a flat boundary\textsuperscript{5}. This conjecture provides a framework to study the role of entanglement entropy of a quantum system in terms of minimal surfaces\textsuperscript{6,7}.

In\textsuperscript{6,7} it was shown that the holographic entanglement entropy (HEE) of a quantum system in boundary is defined as the entropy of a region of space $\tilde{A}$ and its complement on the minimal surfaces in $AdS_{d+1}$ using gauge-gravity duality\textsuperscript{6,7}:

$$S_{\tilde{A}} \equiv S_{\text{HEE}} = \frac{\text{Area}(\gamma_{\tilde{A}})}{4G_{d+1}}. \quad (1)$$
For time-independent backgrounds we need to compute the minimal area of a region in bulk with the same boundary $\partial A$ with the quantum system in boundary.

Another surprising development is that there exist a time dependent formulation of HEE using extremal surfaces instead of minimal ones. This approach used to study HEE in the time-dependent Janus background. The technique is to replace "minimal" with "extremal" surfaces:

$$ S_{\text{time-dependent}}^A \equiv S_{\text{HEE}} = \text{ext} \left[ \frac{\text{Area}(\gamma_A)}{4G_{d+1}} \right]. $$

In case of multiple extremal surfaces, we should select the extremal surface with the minimum area included in them.

Here we discuss the issue of HEE for two dimensional time dependent holographic superconductors, and show that in this regime the HEE and phase transition can be achieved.

### 2. Phase diagram of 2D holographic superconductors

The bulk action with an Abelian gauge field $A$ in the presence of a generally massive scalar field $\Psi$ with charge $q$ and mass $m$ was selected in order to determine the two dimensional holographic superconductor, and scalar field and gauge fields as follo:

$$ D_{\mu}D^\mu \Psi - m^2 \Psi = 0, $$

$$ \nabla_{\mu}F^{\mu\nu} = i[\Psi^* D^\nu \Psi - \Psi (D^\nu \Psi)^*]. $$

The advantage of using AdS/CFT set up was that the dual CFT temperature could be obtained as the Hawking temperature of the horizon (denoted by $z_h$) by the following formula:

$$ T = \frac{1}{2\pi z_h}. $$

The AdS boundary was located at $z = 0$, which provided initial data for scalar field and of a physical sources for the scalar operator $\mathcal{O}$. Because the
conformal scaling was used for field theory, conformal dimension was defined using two conformal dimensions $\Delta_{\pm} = 1 \pm \sqrt{1 + m^2 l^2}$. In this report we study the case of $m^2 l^2 = 0, \Delta = \Delta_+ = 2$

The vacuum expectation value (VEV) of the corresponding boundary quantum field theory operators basically can be computed using the following variations of the effective and renormalized action $\delta S_{\text{ren}}$:

$$\langle J^\nu \rangle = \frac{\delta S_{\text{ren}}}{\delta a^\nu} = \lim_{z \to 0} \sqrt{-g} q^2 F_{z^\nu},$$

$$\langle O \rangle = \frac{\delta S_{\text{ren}}}{\delta \phi} = \lim_{z \to 0} \left[ z \sqrt{-g} \frac{1}{l q^2} (D^z \Psi)^\ast - \frac{z \sqrt{-g}}{l q^2} \Psi^\ast \right],$$

where $S_{\text{ren}}$ is the renormalized action:

$$S_{\text{ren}} = S - \frac{1}{l q^2} \int_B \sqrt{-\gamma} |\Psi|^2,$$

and by dot we mean derivative with respect to the time coordinate $t$.

3. Calculation of holographic entanglement entropy

An alternative form for metric could be found using the coordinate transformation $y = t + 2 \tanh^{-1} z$ in the metric on AdS boundary. Under this transformation the metric becomes flat which is necessary to construct CFT. The new non static (time dependent) form of metric is given by the following:

$$ds^2 = \frac{l^2}{\tanh^2 \left( \frac{y}{2} \right)} \left[ dx^2 - \cosh^{-2} \left( \frac{t-y}{2} \right) dt dy \right].$$

In this coordinate $y$, the black hole horizon $z_h = 1$ maps to $y = \infty$ and the conformal (AdS) boundary $z = 0$ maps to the $y - t = 0$. Note that, so far the metric on conformal boundary is manifested as flat in the form $ds^2 \sim dx^2 - dt^2$. We now calculate entanglement entropy given by Eq. (2)

3.1. Extremal areas in connected phase

For connected surfaces, we should regularized HEE per length $l$ as a function of the $\{u(y_\infty), c\}$. It was shown that the area per entangled length $\frac{A}{l}$ is a monotonic-increasing function. This quantity shows that the system remains on a regular phase of matter for $T > T_c$ or equivalently for $u(y_\infty) \geq 15.5$. Regular attendance at these non superconducting phase has proved numerically. Boundary conditions and regular tiny parameter $c$ will help
to keep normal phase for longer. A more interesting observation is that the hardenability to form normal phase becomes more harder with the increase of entropy $A$.

### 3.2. Extremal areas in disconnected phase

In disconnected phase, we should compute the extremal area of the disconnected surfaces as a function of boundary coordinates $(t, x)$. We show that the conserved charge $J$ of the associated Euler-Lagrange system of the Eqs. plays the role of the critical parameter. A numerical computation shows that when $J = \frac{1}{2}$, the normalized area per length $A$ is a monotonic-decreasing function. Near $J_c \approx 0.58\pm 0.02$ we can simplify write the time evolution of the extremal path as the following $\dot{u} = T_1 + T_2(J - 0.5)$ where $T_i$ are functions of $u$. In this case with disconnected areas, we detect that the extremal area per length is always decreasing. It produces a regular phase of matter for $J > J_c$. Furthermore we can show that the HEE is a linear function of total length $y$. The simple physical reason backs to the emergence of new extra degrees of freedom in small values of belt length (small sizes).

### 4. Summary

We investigated holographic entanglement entropy of dual quantum systems using a time dependent version of AdS spacetime. By a numerical computation we demonstrated that HEE is a monotonic-decreasing function. It is always decreasing. It produces a regular phase of matter for a critical value $J > J_c$. We showed that $\log(S_{\text{HEE}})$ is proportional to the numbers of degrees of freedom (dof) of the system $\mathcal{N}$. Just as the phenomena of sudden condensate, we observed an emergence of new dof in system $\mathcal{N}$. The HEE growing and exhibiting its activity in accordance with thermodynamic laws. Furthermore we demonstrate that the HEE as function of $J$ and $u(y_\infty)$ when time $y$ is growing up and for large values of $J$ reaches a local maxima. So, system undergoes a normal thermodynamically time evolution according to the second law. Consequently the arrow of time never changes.

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