P-brane solutions in IKKT IIB matrix theory

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Abstract

We find p-brane solutions to the recently proposed IKKT IIB matrix theory for all odd p. We also propose central charges for the p-branes.

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1 Introduction

Recently Banks et. al. [1] proposed that a non-perturbative formulation of M-theory is a certain gauged matrix quantum mechanics. This theory is $N = 1$ super $SU(N)$ Yang-Mills theory in the large $N$ limit dimensionally reduced to one dimension. This remarkable proposal has passed a series of tests which suggests that this may indeed be the correct formulation of the theory. Inspired by this proposal Ishibashi et. al. [2] have proposed a matrix theory (IKKT) in zero dimensions which purports to describe type IIB strings non-perturbatively.

One of the features of IKKT matrix theory is that it has manifest 10D Lorentz invariance in contrast to the BFSS description of M-theory which does not have manifest 11D Lorentz invariance since it is believed to describe M-theory in a light cone frame. The authors of [2] put their theory to a number of tests. They showed it supports D-string solutions to its equations of motion. They also calculated the force between two arbitrarily oriented static D-strings and showed that it reproduced the result known from supergravity.

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2 p-brane solutions

As is well known by now, the type IIB string theory has even dimensional Ramond-Ramond fields which can couple to odd dimensional p-branes. One of the predictions of string duality is the existence of these p-branes. In this note we show that for every odd integer $p$ there is a p-brane solution to the IKKT matrix model. We calculate its tension and propose central charges for each one of the p-branes.

We use the slightly modified matrix model proposed by Li \cite{3} in the following:

$$L = -\frac{1}{2\pi (\alpha')^2 g_s} \text{Tr} \left( \frac{1}{4} [A^\mu, A^\nu]^2 + \frac{1}{2} \bar{\psi} \gamma^\mu [A^\mu, \psi] \right) + \frac{\pi}{g_s} \text{Tr} 1. \quad (1)$$

Here the $A^\mu$ ($\mu = 0, ..., 9$) are hermitian $N \times N$ matrices and $\psi$ is a $N \times N$ matrix of Majorana-Weyl spinors in 10D. We have chosen a Euclidean formulation for simplicity. To convert to Minkowski space-time is a simple matter of changing the minus sign in the action to a plus sign and inserting a factor of $-1$ when raising or lowering the index 0. The equations of motion with the spinor set to zero are:

$$[A_\mu, [A_\mu, A_\nu]] = 0. \quad (2)$$

We can construct solutions of this equation such that

$$[A^\mu, A^\nu] \propto 1, \quad (3)$$

and thus commutes with all other matrices. Such solutions are, as shown in \cite{3}, BPS saturated (they are annihilated by half of the supersymmetry).

Consider the following solution to the above equations of motion:

$$A_{2k-2} = \otimes_{i=1}^{k-1} 1_i \otimes g_k \otimes_{l=k+1}^{(p+1)/2} 1_l,$$

$$A_{2k-1} = \otimes_{i=1}^{k-1} 1_i \otimes p_k \otimes_{l=k+1}^{(p+1)/2} 1_l, \text{ for } k = 1, ..., (p+1)/2$$

$$A_{2k-2} = A_{2k-1} = 0 \text{ for } k > (p+1)/2. \quad (4)$$

with

$$[g_k, p_k] = \frac{2\pi i}{n_k} T_k L_k. \quad (5)$$

It should be noted that these operators can only be expressed in terms of infinite matrices. This can easily be seen since the trace of a commutator of
finite matrices vanishes. However it is convenient to define such operators formally for finite \( n_k \) and then take the limit \( n_k \to \infty \). Here the \( i \)-th sector of the direct product has dimension \( n_i \). Hence we have:

\[
\prod_{i=1}^{(p+1)/2} n_i = N, \tag{6}
\]

and

\[
[A_{2k-2}, A_{2l-1}] = \delta_{kl} \frac{2\pi i}{n_k} T_k 1_{N \times N},
\]

\[
[A_{2k-2}, A_{2l-2}] = [A_{2k-1}, A_{2l-1}] = 0 \quad \tag{7}
\]

This solution clearly corresponds to the solution identified as a D-string in [2] for the particular case of \( p = 1 \).

We substitute the above solution for general \( p \) into the action to obtain:

\[
S_p = \pi N \alpha'^2 \sum_{k=1}^{p+1} \left( \frac{T_k L_k}{n_k} \right)^2 + \pi g_s N \quad \tag{8}
\]

We now follow [3] and take the large \( N \) limit so that:

\[
\frac{T_k L_k}{n_k} = c_p \alpha'
\]

where we have allowed arbitrary constants \( c_p \) for each \( p \). For finite \( N \) the system is compactified on a \( p + 1 \) dimensional torus so we can define the volume as:

\[
V_{p+1} = \prod_{k=1}^{p+1} T_k L_k = (c_p \alpha')^{p+1} N \quad \tag{10}
\]

So we can see that the action reduces to the simple form:

\[
S_p = \left( \frac{p+1}{2} c_p^2 + 1 \right) \frac{\pi}{g_s (c_p \alpha')^{p+1}} V_{p+1} \quad \tag{11}
\]

which we can interpret as the action for a \( p \)-brane with tension given by:

\[
T_p = \left( \frac{p+1}{2} c_p^2 + 1 \right) \frac{\pi}{g_s (c_p \alpha')^{p+1}} \quad \tag{12}
\]
Thus we have shown that the correct form of $p$-brane action can be obtained from the IKKT matrix model for all odd $p$. As we have already noted our solutions should be D-branes which couple to the Ramond-Ramond fields in type IIB string theory. This is seen to agree with the formula for the tension which has the expected $\frac{1}{g_s}$ dependence. These solutions also preserve half the number of supersymmetries as D-branes should. The powers of $\alpha'$ are also correctly produced. However, we cannot predict the exact numerical constants. We can fix the constants $c_p$ by comparing our result with the tension calculated from string theory \[5\]. In units where $\kappa = \gamma \alpha'^2 g_s$ (where $\kappa^2$ is 8 times the gravitational constant):

$$T_p = \frac{16\pi \frac{3}{5}}{\gamma g_s (4\pi^2 \alpha')^{\frac{p+1}{2}}}$$ (13)

In \[2\] $N$ was treated as a variable and the equation of motion for $N$ was used to determine a relation between the unspecified constants in the original action. Equivalently we can view this equation of motion as providing the correct scaling relation for $N$ when the action has been fully determined. A similar procedure here could provide the correct scaling relations for the $n_k$, determining $c_p$. However, this method does not seem to be consistent for $p > 1$. One solution to this may be to add higher order terms to the action. This is not called for if one believes the deduction of the action from the superstring action. However, if one views this action as coming from the Eguchi-Kawai reduction of 10D super-Yang-Mills such terms are to be expected. If suitable terms are introduced with undetermined coefficients then consistency of the tensions derived using also the equations of motion for the $n_k$ would give relations between these coefficients. In this paper we shall not add higher order terms to the action and so cannot consider equations of motion for the $n_k$. (However, these equations are valid and consistent for systems containing only D-strings.)

To show that it is not trivial to identify which $p$-brane(s) some configuration describes, consider the following:

$$A_0 = \begin{pmatrix} q_1 & 0 \\ 0 & 0 \end{pmatrix}$$
$$A_1 = \begin{pmatrix} p_1 & 0 \\ 0 & 0 \end{pmatrix}$$
$$A_2 = \begin{pmatrix} 0 & 0 \\ 0 & q_2 \end{pmatrix}$$
\[ A_3 = \begin{pmatrix} 0 & 0 \\ 0 & p_2 \end{pmatrix} \]  

(14)

where

\[
[q_k, p_k] = \frac{2\pi i}{n_k} T_k L_k 1_{n_k \times n_k}
\]

(15)

with all other commutators zero.

Superficially this looks similar to our solution for a 3-brane (for example it spans 4 dimensions). However, there is an important difference since the dimensions of the matrices are different, i.e. \( N = \text{Tr} 1 = n_1 + n_2 \) rather than \( N = n_1 n_2 \) for the 3-brane solution. A little consideration suggests that this solution is a sum of 2 separate solutions. If we substitute this solution into the action we will get the sum of 2 D-string actions rather than a 3-brane action. However, for general configurations it would not always be easy to decide what branes were present. In the next section we will define central charges which will at least allow easy determination of the highest dimensional brane present.

3 Proposed definitions for central charges

We have shown that there are solutions to the IKKT matrix model which correspond to D-\( p \)-branes for all odd \( p \). We know from general arguments that there should be corresponding central charges \( Z_{\mu \nu \ldots}^p \) with \( p \) indices. These central charges have the properties that they are anti-symmetric in all indices and they vanish if any index is 0 (corresponding to time). Thus we could define totally anti-symmetric objects with \( p + 1 \) indices \( C_{p+1} \) and make the identification:

\[
Z_{\mu \nu \ldots}^p = C_{p+1}^{\alpha \mu \nu \ldots}
\]

(16)

The central charges appear in the supersymmetry algebra but we don’t know how to construct the algebra for the IKKT matrix model. The algebra has been constructed for the BFSS matrix model [4] where it was possible to define conjugate momenta but the analogous construction does not seem possible in the IKKT model. However we will proceed by proposing objects with the correct properties. The main consideration is that we should define the objects \( C_{p+1} \) so that a \( p \)-brane has non-zero \( Z_p \) and also that \( Z_{p'} = 0 \) for all \( p' > p \). In this section we shall always assume \( p \) and \( p' \) to be odd integers.
The simplest guess is to define:

\[ C_{p+1}^{\mu_0 \mu_1 \cdots \mu_p} = A^{[\mu_0 A^{\mu_1} \cdots A^{\mu_p}]} \]  

(17)

where \([\ldots]\) means anti-symmetrise over all indices. We will now show that this proposed form leads to a consistent definition of central charges, at least for our D-brane solutions.

First we can show that a \(p\)-brane has \(Z_{p'} = 0\) for all \(p' > p\). This follows simply from the definition of \(C_{p'+1}\). If for some \(\mu\):

\[ [A^\mu, A^\nu] = 0 \]  

(18)

for all \(\nu\) then \(C_{p'+1}\) vanishes if it contains the index \(\mu\) since \(C_{p'+1}\) has an even number of indices. Also \(C_{p'+1}\) vanishes if it contains 2 indices the same. But our solutions obviously contain only \(p + 1\) fields \(A\) which do not commute with all other \(A\)'s. Therefore \(C_{p'+1} = 0\) and so \(Z_{p'} = 0\) for all \(p' > p\).

Now we can show that \(Z_p \neq 0\) for our \(p\)-brane solutions. First note that the totally anti-symmetric product of \(p + 1\) \(A^\mu\)'s can be expressed as a sum of products of \(\frac{1}{2}(p + 1)\) commutators \([A^\mu, A^\nu]\) (here we are using the fact that \(p\) is odd for all our solutions). It is now relatively simple to see that \(Z_p \neq 0\) for our solutions. For example if \(p = 1\):

\[ Z_1^\mu = [A^0, A^\mu]. \]  

(19)

So we have the non-zero component:

\[ Z_1^1 = 2\pi i c_1 \alpha' \]  

(20)

Similarly, with a little more effort, we can see that for \(p = 3\) the non-zero components of \(Z_3\) are given by:

\[ Z_3^{123} = [A^0, A^1][A^2, A^3] + [A^2, A^3][A^0, A^1] = -8\pi^2 (c_3 \alpha')^2. \]  

(21)

Of course we have not chosen any particular normalisation of \(Z_p\) since we couldn’t explicitly construct the central charges from the supersymmetry algebra. However, with the normalisation chosen above it should now be clear that our \(p\)-brane solutions have:

\[ Z_p^{12 \cdots p} = \left( \frac{p + 1}{2} \right) ! \left( 2\pi i c_p \alpha' \right)^{\frac{p+1}{2}} \]  

(22)

with the important point not being the actual normalisation but that \(Z_p \neq 0\).
Another observation is that our solutions carry non-zero central charge \( Z_{p'} \) for \( p' < p \). This appears to be unavoidable with our construction but is analogous to the observation \([4]\) that a construction of a longitudinal 5-brane in the BFSS matrix model carried non-zero \( Z_{2}^{12} \) and \( Z_{3}^{34} \). There the interpretation was that the longitudinal 5-brane was bound with two orthogonal infinite stacks of membranes (2-branes). A similar interpretation in this case would imply that every \( p \)-brane solution we have constructed is bound with an infinite number of \( p' \)-branes for all odd \( p' < p \). This is in a way obvious from the construction of the solutions where we can view any \( p \)-brane solution as a \((p - 2)\)-brane solution at every point of an orthogonal 2-dimensional space.

One feature which seems to be different in this case compared to the longitudinal 5-brane case in the BFSS model is that we don’t get the analogous orthogonal stacks of lower dimensional branes. For example our 3-brane solution has only \( Z_{1}^{1} \neq 0 \). This is due to the different formulations which treat time very differently.

One should note that a similar phenomenon occurs when there are non-zero world-volume gauge fields. In this case it was shown in \([5]\) that \( p \)-branes carry non-zero charge corresponding to lower dimensional branes. It should be noted that our \( Z_{p} \)'s are really central charge densities and are in general matrices rather than c-numbers. In order to calculate the true central charge we should take the trace of \( Z_{p} \). Apart from normalisation this is irrelevant in our examples but it is important in general. For example parallel D-branes can be represented by block-diagonal matrices and so the trace is necessary to count the central charge of the system as the sum over each individual D-brane (block diagonal).

Let us now return to the solution of eqs. \([14]\). Now it is easy to see that this solution has non-zero \( Z_{1}^{1} \) as we would expect for a D-string. Again we see the similarity with the 3-brane solution when we note that the only non-zero commutators are \([A^0, A^1]\) and \([A^2, A^3]\). But although these are constants they are not proportional to the identity in this case. This means that this solution breaks more than half the supersymmetry. It is also easy to see from the block diagonal structure of the matrices that \( Z_{3}^{123} = 0 \) and so this solution does not have the appropriate charge for a 3-brane.

We have shown how our definitions of central charge can even be used to identify what certain configurations correspond to. However, there is still a puzzle about the solution of section \([14]\). We have identified it as a configuration of 2 D-strings yet only one of them seems to have non-zero \( Z_{1} \). This is explained by the fact that we are working with a Euclidean metric.
This solution really corresponds to 2 completely orthogonal D-strings in Euclidean space. So when we identify one coordinate as time and rotate to Minkowski space-time, only one of the D-strings is physically sensible.

4 Conclusions

In this paper we have shown that the IKKT matrix model allows solutions which can be identified as D-$p$-branes for all odd $p$. We have done this by showing that the matrix model action reduces to the expected form for extended objects $S_p = T_p V_{p+1}$. Our solutions also preserve half the supersymmetry. We have defined central charges $Z_p$ which satisfy the general properties expected. The existence of $p$-branes for all odd $p$ can be seen as evidence for the conjecture that the IKKT matrix model really describes non-perturbative type IIB string theory.

We have chosen how to take the large $N$ limit by matching tensions to the expected tensions from string theory. However, it is still not clear why slightly different limits should be taken for different dimensional objects.

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Note added

As we were completing this paper the paper [7] appeared discussing similar issues.

References

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