Creation and utilization of straightness standard due to reciprocal measurement of linear stage

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Abstract. In general, profile measuring machine consists of a displacement sensor and movable mechanism. However, as the movable mechanism has profile errors, the output of displacement sensor includes the profile error of the movable mechanism. Therefore, it is necessary to distinguish the profile error of the movable mechanism from the output of the displacement sensor. So we propose a new way to distinguish the profile of object from movable profiles. The new method requires two linear stages and the displacement sensor. Two stages can move and locate independently, and displacements are measured at every location. The profile of object is calculated from the output of displacement sensor. Finally, the pre-known profile was measured by the proposed method, and the result is compared with the pre-known profile.

1. Introduction

In general, profile measuring machine consists of a displacement sensor and movable mechanism. It could measure the object profile by the amount of movement of the displacement sensor and output of the displacement sensor. However, as the movable mechanism has profile errors, the output of displacement sensor includes the profile error of the movable mechanism. Therefore, it is necessary to distinguish the profile error of the movable mechanism from the output of the displacement sensor.

In some previous works, the multi-point method and reversal method were studied as the method to distinguish the object profile from the profile error of the movable mechanism [1-2]. The multi-point method uses multi-sensors, and determines the relationship between the multi-sensors. But the multi-point method has the difficulty to adjust the zero positions of multi-sensors [3-4]. The reversal method measures the object profile by rotating the object 180 degrees after measuring the object profile. By using the measurement result before and after reversal, it is possible to separate the profile error of the movable mechanism from displacement sensor. But the reversal method requires the complicated mechanism and it becomes large scale equipment [5-8]. So we will propose a new way to distinguish the profiles of objects from movable profiles which does not require zero adjustment in this article and do not require extensive equipment or action. The new method requires two linear stages and the displacement sensor [9].

Two stages can move and locate independently. The profile errors of two stages and the object profile are calculated from the combination of the two stage locations and the output of displacement sensor in this method.

2. Principle
Figure 1 shows model of the profile measuring machine. Profile measuring machine consists of a displacement sensor and movable mechanism. If movable mechanism does not have any profile errors, the relationship of the object profile \( f(x) \) and output of the displacement sensor \( m(x) \) is expressed as the equation (1).

\[
f(x) = m(x) + ax + c
\]  
(1)

where \( ax + c \) is the slope and offset of movable mechanism from the object. When movable mechanism does not have any slope, this value shall be zero. It is possible to measure the object profile because the output of the displacement sensor agrees with the object profile. But, as shown in figure 2, the movable mechanism has some profile error. The relationship of the object profile \( f(x) \), profile error of movable mechanism \( g(x) \) and output of the displacement sensor \( m(x) \) is expressed as the equation (2).

\[
f(x) = g(x) + m(x) + ax + c
\]  
(2)

As shown in figure 2, when movable mechanism has some profile errors, the output of displacement sensor includes the profile error of the movable mechanism. Therefore, it is necessary to measure the profile of the movement axis \( g(x) \).

A new method to distinguish the object profile from the movable profile as shown in figure 3. The new method composed two linear stages and three displacement sensors. Object is put on the linear stage, and creates mechanism which can move the object on movable axis. Therefore, two stages can
move and locate independently. Positional relationship between object and displacement sensor on each of movable axes, the object profile \( f(u) \) can be expressed as equation (3).

\[
f(u) = g(x) + h(y) + mx + by + c \quad (3)
\]

where \( g(x) \) is profile error of movement axis at side of displacement sensor, \( h(y) \) is profile error of movement axis at side of object, \( m(x, y) \) is output of the displacement sensor measuring object profile. In this case, there is a relationship of \( u = x - y \), by obtaining the output of the displacement sensor at the any position of \( x \) and \( y \), it is possible to obtain multiple combinations. By discretizing \( x, y, \) and \( u \) as \( x = i\Delta, y = j\Delta \) and \( u = (i-j)\Delta \), where \( \Delta \) is a step of movement, equation (3) is discretized as equation (4).

\[
f_{i-j} = g_i + h_j + m_{i,j} + a \cdot i + b \cdot j + c \quad (4)
\]

where \( i = 0, \ldots, n, j = 0, \ldots, n. \)

In equation (4), \( f_{i-j}, g_i, h_j, a, b \) and \( c \) are unknown parameters and \( m_{i,j} \) is the output of the displacement sensor. Equation (4) is transformed to equation (5) by separating the terms including the unknown parameters or not including unknown parameters.

\[
f_{i-j} - g_i - h_j - a \cdot i - b \cdot j - c = m_{i,j} \quad (5)
\]

\( f_{i-j} - g_i - h_j - a \cdot i - b \cdot j - c \) is expressed as \( s_{i,j} \). The unknown parameters list is expressed as a vector \( d_o \).

\[
d_o = (f_{-n}, \ldots, f_n, g_0, \ldots, g_n, h_0, \ldots, h_n, a, b, c) \quad (6)
\]

As the coordinate system could be freely chosen, the both ends of the profile can be set to zero. So both ends of the profile and are removed from the unknown parameters vector and the determined unknown parameters vector is expressed as equation (7).

\[
d = (f_{1-n}, \ldots, f_{n-1}, g_1, \ldots, g_{n-1}, h_1, \ldots, h_{n-1}, a, b, c) \quad (7)
\]
Here, using Jacobian Matrix \( A \), equation (5) is expressed as equation (8).

\[
A \cdot d = m \tag{8}
\]

where \( A_{i(n+1)+j,k} = \frac{\partial s_{i,j}}{\partial d_k} \) and \( m \) is the vector of output of the displacement sensor.

As the result, it is possible to solve linear equation as shown in equation (9) which is deformed from equation (8).

\[
d = (A^t \cdot A)^{-1} \cdot A^t \cdot m \tag{9}
\]

3. Experiment

Figure 4 shows an experimental setup used in this study. Two linear stages are prepared and the displacement sensor are installed on one stage, the object with pre-known profile was installed on another stage. The specifications of the linear stage and displacement sensor are shown in table 1. The profile of object is shown in figure 5. In the experiment, the object is measured at 10000 measured points that 100 measurement points of the stage on which displacement sensor is installed and 100 measurement points of the stage on which is installed object.

![Figure 4. Experimental setup.](image)

![Figure 5. Workpiece with pre-known profile.](image)
The measurement procedure is the following:
1) The stage moves 100 steps in the \(x\)-direction and the displacements in are measured in each step. (One step is 20μm.)
2) The stage returns to the origin in \(x\)-direction.
3) The stage moves 1 step in \(y\)-direction. (One step is 20μm.)

The above procedure is iterated 100 times. So the displacement is measured at 10000 points. This experiment is repeated 3 times. The axis profiles are separated from the output of the displacement sensor according to equations (7), (8) and (9). Figure 6 shows the calculated result.

| Table 1. The specification of equipment. |
|----------------------------------------|
| **Linear stage**                        | **Displacement sensor**               |
| Resolution                              | Resolution                            |
| 50nm                                    | 500nm                                  |
| Repeatability                           | Beam diameter                          |
| ±300nm                                  | 150μm                                  |
| Positioning accuracy                    | Measuring range                        |
| Below 5μm                               | ±4mm                                   |
| Movable range                           | Sampling period                        |
| 20mm                                    | 200μs                                  |
| Guide type                              | Measurement method                     |
| Cross roller                            | Diffused reflection                    |

![Figure 6. Comparison between measured and calibrated profile.](image)

From figure 6, it was possible to separate the profile errors of two movement axes from output of the displacement sensor. Therefore, in this experiment, it can be said that the metrological frame of 20 μm interval was able to be constructed.

Once the profile of object and the movement axis profile are measured, the movement axis profile can be considered as the pre-known. So the metrological frame can be constructed as the pre-known profile.

4. Summary
In this study, the new method of separating the profile error of the two movable mechanism and object profile from measurement results is proposed. This method is confirmed in the experiment using the object with the pre-known profile. The experiment shows the proposed method is effective and useful.
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