The Modeling of Fluid Flow and Heat Transfer in Mold Filling

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A mathematical representation has been developed for the filling of cylindrical molds, using a bottom pouring arrangement. In the model allowance has been made for thermal natural convection, solidification and for the presence of the free surface. Turbulence was represented by assigning artificially enhanced parametric values to the viscosity and the thermal conductivity. The results have shown that the temperature profiles are markedly non-uniform, and that solidification would commence quite soon after the pouring process has been initiated. For the conditions considered, the free surface disturbances were not very great, although quite high free surface velocities were predicted by the model.

KEY WORDS: modeling; mold filling; free surfaces; solidification; turbulence fluid mechanics; heat transfer; natural convection.

1. Introduction

The progressively tighter specifications for the properties of cast materials have prompted a great deal of research into the study of solidification and segregation phenomena, and this research is well-documented. It has long been recognized that the way molds are filled, and the flow behavior of the molten metal supplied, may play an important role in determining the ultimate quality of the cast material. However, up to the present time very little work has been published on mold-filling phenomena.

This state of affairs is readily explained by the fact that mold filling is inherently a very complex process which involves the combination of "free boundary" problems with heat transfer and solidification. While free boundary problems have been extensively studied by other members of the engineering community, notably ocean engineers in connection with wave phenomena, and by civil engineers in connection with percolation through dams, these problems have been relatively unfamiliar to the metallurgical community.

A notable exception to this rule has been the pioneering work of Stoehr who has studied the filling of simple, idealized mold shapes for isothermal situations. Other relevant work has been reported by Apelian, who has examined the spreading of droplets upon impacting on a solid surface in rapid solidification processing.

The work to be described in this paper is a first step in the study of complex mold-filling problems, where account will be taken of fluid flow, free surface behavior and the initial stages of solidification. Our intention in presenting this material is to illustrate the potential utility of these techniques for tackling the more complex real-life situations, and also to indicate the practical difficulties involved in this approach. We shall consider a relatively simple cylindrical mold shape, but these considerations could, of course, be applied to more complex geometries.

2. Formulation

Let us consider a cylindrical mold into which molten metal is introduced through a bottom pouring arrangement, as shown in Fig. 1. The entering metal will form a liquid body in the mold, with a progressively rising free surface as the filling proceeds. This free surface will be disturbed by the incoming metal stream, resulting in surface waves; furthermore, the body of the expanding metal pool will undergo (turbulent) recirculating motion as a result of the incoming metal stream. Concurrently with these flow phe-

![Fig. 1. A schematic view of a bottom pouring arrangement.](image-url)
nomena, and affected by them, the metal will transfer
hit to the cold walls, and eventually a solidified shell
will form.

In developing a mathematical representation of
this system, the following assumptions will be made:

(1) While the flow is turbulent, we shall use the
form of the laminar Navier–Stokes equations, but with
an artificially enhanced viscosity. The enhancement
factor chosen was 50 or 100. Such a procedure has
been used extensively by previous investigators in the
materials processing field, e.g., in the modeling of
aluminum electrolysis cells or the behavior of argon
stirred ladles.

(2) The free surface behavior of the system was
represented by the SOLA-VOF technique, which is
essentially a modified marker and cell method. The
principle upon which the technique is based is
discussed in detail in references.\textsuperscript{11} A brief
description would be to state that the region near the free surface
is divided into cells. Upon establishing a mass balance
over these cells, the upper boundary may expand or contract in an appropriate
time-dependent manner, while also satisfying a force balance.

(3) In representing convective heat transfer, the
usual differential, convective thermal energy balance
equation was employed, again allowing for turbulent
behavior by postulating that the turbulent Prandtl
number was unity. The top free surface was assumed
to be insulated (certainly an oversimplification),
and transient heat conduction through the mold walls
was postulated as the thermal boundary condition at the
mold–metal interface. It should be noted that the
effect of thermal buoyancy has been included in these
calculations.

(4) Solidification was represented by writing down
the usual enthalpy balance for the mushy region.

(5) As the initial condition we considered the mold
for the solidification of the molten metal.

(6) The boundary conditions are given as:

\begin{align*}
T &= T_{\text{cast}} \quad \text{at } 0 \leq R < R_m \quad z = 0 \\
T &= T_{\text{mold}} \quad \text{at } R_0 \leq R \leq R_m \quad z = 0 \\
T &= T_{\text{wall}} \quad \text{at } R_0 \leq R \leq R_m , z = H_m \\
\frac{\partial T}{\partial z} &= 0 \quad \text{at } 0 \leq z \leq H_m \\
\frac{\partial T}{\partial r} &= 0 \quad \text{at } r = 0 \quad 0 \leq z \leq H_m \\
q &= -\frac{k(T-T_m)}{\sqrt{\pi a t}} \quad \text{at } r = R_m \quad 0 \leq z \leq H_m
\end{align*}

where \( K \) is the drag coefficient introduced by Hirt, to
represent flow in the mushy region.

\begin{align*}
(\text{Conservation of thermal energy}) : \quad & \frac{\partial T}{\partial t} + U \frac{\partial T}{\partial r} + V \frac{\partial T}{\partial z} = a \omega \left[ \frac{\partial^2 T}{\partial r^2} + \frac{1}{r} \frac{\partial T}{\partial r} + \frac{\partial^2 T}{\partial z^2} \right] + \frac{\partial H}{\partial t} \frac{\partial f}{\partial t} \quad \text{...}(4)
\end{align*}

\begin{align*}
K \text{ depends only on temperature, and is expressed as:} \quad & K = \begin{cases}
0 & (T < T_0) \\
k_{\text{max}} \frac{T_0 - T}{T_0 - T_m} & (T_0 \leq T \leq T_k) \\
\infty & (T_k < T) 
\end{cases} \quad \text{...}(5)
\end{align*}

K_{\text{max}} \text{ in this study is set to } 10^5.

\begin{align*}
\rho &= \rho_0 [1 - \beta(T - T_0)] \quad \text{...}(6)
\end{align*}

It is assumed that \( f_r \) is proportional to temperature
(esimal of the mushy region), and is given by Eq. (7):

\begin{align*}
f_r &= \begin{cases}
1 & (T_k < T) \\
\frac{T - T_m}{T_0 - T_m} & (T_k \leq T \leq T_l) \\
0 & (T < T_k)
\end{cases} \quad \text{...}(7)
\end{align*}

\begin{align*}
&U = 0, \quad V = V_{\text{inlet}} \quad \text{at } 0 \leq R < R_m \quad z = 0 \\
&U = 0, \quad V = 0 \quad \text{at } R_0 \leq R \leq R_m \quad z = 0 \\
&U = 0, \quad \frac{\partial V}{\partial r} = 0 \quad \text{at } r = 0 \quad 0 \leq z \leq H_m \\
&U = 0, \quad V = 0 \quad \text{at } r = R_m \quad 0 \leq z \leq H_m
\end{align*}

i.e., the velocity was specified at the inlet,
zero velocities were specified at all the solid
surfaces, and radial symmetry was observed.

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The boundary conditions on the velocity at the free surface are written in detail in reference. Let us briefly comment on the appropriateness of these boundary conditions. Postulating zero velocities at the solid surfaces and specifying the inlet velocity is a standard procedure. The assumption that the free surface was insulated was made as a first approximation and could be readily relaxed. By the same token, the postulate of transient heat conduction into the mold walls is also an oversimplification, because it neglects the effect of an air gap. It was, however, thought desirable to make these simplifications in the first instance, because we sought insight regarding the effect of heat and fluid flow and solidification in the presence of a free surface; under these conditions, the introduction of the additional parameters characterizing the air gap and the free surface heat loss could have clouded the issue. These factors can be readily incorporated into subsequent modeling efforts.

3. Method of Solution

The method of solution adopted was as follows: The SOLA-VOF, a public-domain computational algorithm, was used for the free surface position and the velocity fields within the system. These velocity fields were then substituted into the differential thermal energy balance equation to calculate the temperature fields and hence the extent of solidification, when solidification had taken place. Once solidification had occurred, the shape of the molten domain had to be adjusted accordingly; this adjustment followed a procedure recommended by Hirt.

The input parameters used in the calculations are summarized in Table 1. It is seen that we considered an ingot mold of 40 cm in diameter and an inlet nozzle of 8 cm in diameter, with a "standard" teeming velocity of 20 cm/s through the inlet nozzle, which gave a nominal surface rising velocity of 0.8 cm/s. Calculations were conducted for 60 s of actual time corresponding to 56 cm of ingot height. The metal was cast with a 70°C superheat, and the initial wall temperature was set at 100°C. The working metal was a steel with liquidus temperature of 1450°C, and solidus temperature of 1400°C; the mold wall was cast iron.

Twenty radial grid points were employed, while the number of vertical grid points was time-dependent, starting at 4 and increasing up to 14 as the mold filling progressed. At this level of grid points employed, the results were no longer grid-sensitive. The time steps taken were generally less than 0.005 s. A typical calculation required about 24 h on a MicroVAX II digital computer. So this procedure places quite high demands on computer time.

4. Computed Results and Discussion

The following four figures, displaying the computed streamline patterns and temperature profiles, illustrate the sensitivity of the results to the assumptions made. Here, Fig. 2 depicts the behavior of the system if molecular transport properties were employed, and no allowance is made either for solidification or for natural convection.

Fig. 3 shows the identical system, but where allowance is made for turbulence by increasing the viscosity and the thermal conductivity by a factor of 100.

The input parameters for Fig. 4 are identical to those employed in Fig. 3, but an allowance has been made for thermal natural convection also; finally, Fig. 5 represents the effect of solidification in the presence of turbulent flow and natural convection.

Examination of these four figures is instructive because it is clearly seen that one must make an allowance for turbulent behavior. More specifically, when molecular thermal properties are being used, as in the computation of Fig. 2, the local temperature gradients will be much steeper and the overall rate of heat extraction will be much slower than in the cases depicted in Figs. 3 to 5 where an allowance has been made for turbulence through an artificially enhanced thermal diffusivity. The flow will, of course be turbulent from purely hydrodynamic considerations. The effect of natural convection appears to be less significant, at least during the filling period. Once filling has been completed, natural convection will have to become the dominant force in determining the ensuing circulation pattern.

A further examination of Fig. 5 reveals that solidification will have commenced even during the filling period and, as perhaps expected, this will occur initially where the mold wall is in contact with the most stagnant regions. One should note that the numerical values chosen for the turbulent viscosity and thermal conductivity are quite critical, as will be illustrated subsequently.

Let us now examine the actual mold-filling process for the conditions that were depicted in Fig. 5, that is with an allowance for turbulence, natural convection

Table 1. Input parameters.

| Case | Dm (cm) | Dn (cm) | V_inlet (cm/s) | T_mold (°C) | V_surf (cm/s) | μ_att (g/cm/s) | α_eff (cm²/s) | β (°C⁻¹) | ISOL* |
|------|---------|---------|---------------|------------|---------------|---------------|---------------|--------|------|
| A    | 40.0    | 8.0     | 20.0          | 1280       | 0.80          | 0.007         | 0.0285        | 0.0    | 0    |
| B    | 40.0    | 8.0     | 20.0          | 1280       | 0.80          | 0.007         | 0.0285        | 0.0    | 0    |
| C    | 40.0    | 8.0     | 20.0          | 1280       | 0.80          | 0.007         | 0.0285        | 0.0    | 0    |
| D    | 40.0    | 8.0     | 20.0          | 1280       | 0.80          | 0.007         | 0.0285        | 0.0    | 0    |
| E    | 40.0    | 6.0     | 35.6          | 1280       | 0.80          | 0.007         | 0.0285        | 0.0    | 1    |
| F    | 40.0    | 6.0     | 35.6          | 1280       | 0.80          | 3.35          | 0.484         | 0.0    | 1    |

* ISOL 0: without solidification, 1: with solidification.
Fig. 2. The computed normalized streamline pattern and temperature profile at 60 s, neglecting natural convection, solidification and the effect of turbulence (i.e., using the atomic properties for the viscosity and the thermal conductivity (A)).

Fig. 3. The computed normalized streamline pattern and temperature profile at 60 s allowing for turbulence by enhancing \( \mu \) and \( k \) by a factor of 100 but neglecting natural convection and solidification.

Fig. 4. The computed normalized streamline pattern and temperature profile at 60 s for (C), allowing for turbulence and for natural convection but neglecting the effect of solidification.

Fig. 5. The computed normalized streamline pattern and temperature profile at 60 s for (D), allowing for turbulence, natural convection and solidification.

Fig. 6 shows the computed normalized streamline maps at various times during the mold-filling process.

The characteristic recirculating flow pattern is readily evident; it is seen that the eye of the circulation moves upwards as the mold filling progresses. It should be remarked that while the free surface appears flat on
these plots, this is in fact not so. However, as will be shown subsequently, for the conditions specified the scale of the waves or disturbances is small compared to the size of the system.

Fig. 7 shows the computed temperature profiles, which indicate the expected hot region, in the vicinity of the incoming stream, and the onset of solidification in the quiescent corner region.

While the streamline patterns do provide a feel for the nature of the circulation, the absolute values of the velocity may also be helpful in assessing the behavior of the system. Figs. 8 and 9 show the axial distribution of the radial velocity component. The numerical values seen here may gain some perspective upon considering that the inlet velocity was 20 cm/s. Inspection of Fig. 9 clearly indicates the strong radial outflow as the incoming stream approaches the free surface.

Fig. 10 shows the computed isotherms for the system for a case when no allowance is made for solidification. It is seen that the temperature profiles are different; the system appears to be somewhat colder compared to the previously given Fig. 7; this is reasonable because no latent heat of solidification has been released under these latter conditions. Comparison of Figs. 7 and 10 further shows that it would be difficult to deduce local solidification rates, unless specific allowance is made for such behavior.

Let us now turn our attention to the free surface behavior of the system. Fig. 11 shows the radial free surface velocities both in the presence and in the absence of solidification. As perhaps expected, the solidification will not have a noticeable effect on these phenomena during the early stages of the process.

One would normally expect waves at the free surface of the melt. Fig. 12 shows the shape of the distorted free surface as mold filling progresses. These results indicate relatively small free surface deformations, which tend to diminish as the mold filling proceeds. This behavior is reasonable, because one would expect a smaller disturbance by the incoming stream for greater metal depth levels. We note, fur-
thermore, that one of the reasons for employing bottom pouring in practice is that it is generally thought that the free surface disturbances are minimized under these conditions. The calculations presented here tend to support this contention.

Since the ultimate objective of this work is to address the question of solidification and how this affected by the fluid motion in the system, let us examine two additional variations on the previously shown Fig. 5, which depicts the extent of solidification once the metal height has reached about 60 cm.

Fig. 13 shows the effect of a somewhat smaller inlet nozzle (but at the same metal mass flow rate), while Fig. 14 represents the effect of reducing the effective conductivity employed by a factor of 2.

A comparison of Figs. 5, 13 and 14 clearly shows that the initial solidification rate is much more affected by the turbulence level in the system (as represented by the value the effective thermal conductivity) than by changes in the inlet nozzle diameter. More specifically, as seen in Fig. 14, corresponding to case F, that is a lower thermal diffusivity, solidification will progress rather more rapidly. This is consistent with physical reasoning, because a lower thermal diffusivity (which is of course still 50 times the atomic value) will provide a lower rate of heat transfer to the melt-solid interface, thus will allow more rapid local rate of heat extraction.

In all fairness one should remark, though, that the turbulence levels in the system and the inlet nozzle diameter are not totally uncoupled; nonetheless, the results seem to indicate that the flow conditions in the bulk (and of course near the solid surfaces) would seem to play a more important role than the free surface phenomena, at least for the conditions considered here.

5. Conclusions

A mathematical representation has been developed for filling a simple cylindrical mold, using a bottom pouring arrangement in which an allowance has been made for the presence of a free surface, thermal natural convection, flow in the two-phase region, and solidification. The modeling of thermal natural convection and solidification are new developments which have not been discussed in the open literature up to the present time.

A key result of the present work is the demonstration that such problems may now be quite readily tackled without having to resort to supercomputers.

The principal findings of the work may be summarized as follows:

(1) For the conditions considered, i.e., bottom pouring with relatively small inlet velocities, the surface disturbances were relatively minor and it is not
clear whether these surface disturbances had a significant effect on the flow field within the mold, some distance from the free surface.

(2) The computed temperature fields were markedly non-uniform, and were obviously affected by the flow and also by the solidification process. This clearly points to the need for performing modeling studies of this type; the assumption of a homogeneous temperature and a well-mixed fluid would not have been correct.

(3) Solidification was found to commence quite rapidly, once the molten metal entered the mold; the extent of this initial solidification was found to depend quite critically on the flow field within the system and on the turbulence levels (more precisely, the parametric value chosen for the effective thermal conduc-
Fig. 13. The effect of the inlet velocity on the streamline and temperature profiles: (E)—change in the inlet nozzle diameter.

Fig. 14. The effect of the effective thermal diffusivity on the streamline and temperature profiles: (E).

g: gravitational acceleration (cm/s²)
H_fus: height of free surface (cm)
ΔH: heat of fusion (cal/g)
K: drag coefficient (r⁻¹)
κ: thermal conductivity of the mold (cal/cm·°C·s)
P: pressure (dyne/cm²)
r: distance from the axis of the mold (cm)
R_m: radius of the mold (cm)
R_n: radius of the inlet nozzle (cm)
q: heat flux (cal/cm²·s)
T: temperature (°C)
T_m: casting temperature (°C)
T_L, T_s: liquidus, solidus temperature (°C)
T_{ref}: reference value of T (°C)
T_0: initial temperature of the mold (°C)
t: time (s)
U: r-component of velocity (cm/s)
V: z-component of velocity (cm/s)
V_{inlet}: inlet velocity (cm/s)
V_{surf}: nominal velocity of the rising free surface (cm/s)
Z: distance from the bottom surface of the mold (cm)
\alpha_{eff}: thermal diffusivity (cm²/s)
\alpha_e: thermal diffusivity of the mold (cm²/s)
\beta: thermal expansion coefficient (°C⁻¹)
ρ: density (g/cm³)
\rho_0: reference value of ρ (g/cm³)
μ: viscosity (cm²/g·s)

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Nomenclature

C_p: specific heat (cal/g·°C)
D_m: diameter of the mold (cm)
D_n: diameter of the inlet nozzle (cm)
f_L: volume fraction of liquid (—)

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