Robust finite-time synchronization of uncertain chaotic systems: application on Duffing-Holmes system and chaos gyro

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ABSTRACT
This paper considers the finite-time synchronization of chaotic systems in the presence of model uncertainties and/or external disturbances. The synchronization happens between the two nonlinear master and slave systems. Control law is designed in such a way that the state variables of the slave system follow the state variables of the master system in the presence of uncertainties and external disturbances. In order to design a robust finite-time controller, first, a novel terminal sliding surface is proposed and its finite-time convergence to zero is analytically proved. Then a terminal sliding mode controller is designed which can conquer the uncertainties and guarantees the finite-time stability of the sliding motion equations. In this regard, a theorem is proposed and according to the Lyapunov approach it is proved that the synchronization happens in finite-time. Additionally, in order to show the applicability of the proposed controller, it is applied on two practical systems, the Duffing–Holmes system and chaotic gyroscope system. Computer simulations verify the theoretical results and also display the effective performance of the proposed controller.

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Chaotic systems; finite-time control; robust synchronization; terminal sliding mode; Lyapunov stability

1. Introduction
Fractal properties of the motion in phase-plane, extremely sensitive to initial conditions and system parameter changes, broad-band Fourier power spectrum and strange attractors are the special characteristics of the chaotic systems. Due to these features, synchronization of chaotic systems has found many applications in image encryption (Wen, Zeng, Huang, Meng, & Yao, 2015), secure communication (Zhang, An, & Zhang, 2013), genetic oscillators (Aloli, Ren, Al-Mazrooei, Elaiw, & Cao, 2015) and so on. The master and slave systems are the fundamental frameworks for synchronization. In this structure, the state trajectories of the slave system should track the state trajectories of the master system. Until now, a wide variety of approaches have been proposed for the synchronization of chaotic systems which include adaptive control (Mohammadpour & Binazadeh, 2018a; Yau & Chen, 2007), observer-based control (Mohammadpour & Binazadeh, 2017, 2018b), sliding mode control (Li, Liao, & Li, 2011), backstepping control (Xing-Yuan & Hao, 2013), active control (Cai, Jing, & Zhang, 2010), fractional-order control (Velmurugan & Rakkiyappan, 2016), fuzzy control approach (Lin & Wang, 2011) and so on.

However, most of the above-mentioned papers have been focused on asymptotic synchronization of chaotic systems. In a practical engineering process, it has more potential to synchronize the chaotic systems in the finite-time. Finite-time stability means that the system trajectories converge to the equilibrium point in finite-time (Binazadeh & Shafiei, 2014a; Wang, Wang, Wei, & Alsaaadi, 2017; Zhang, Wang, Zou, & Fang, 2017). Moreover, the finite-time control techniques have demonstrated better robustness and disturbance rejection properties (Binazadeh, 2016).

Recently, some researchers have paid their attentions to synchronization of chaotic systems in finite-time. Finite-time control techniques have been proposed for synchronization of some practical chaotic systems by Sun, Wu, Bai, Wei, and Sun (2016); Liu, Li, and Feng (2016) and Du, Li, and Qian (2011). Moreover, Mobayen (2016) studied finite-time synchronization of chaotic systems with matched and unmatched uncertainties via LMI technique. A control Lyapunov function (CLF) scheme has also been reported for finite-time synchronization of chaotic systems (Wang, Han, Xie, & Zhang, 2009; Yu, 2010). The synchronization of uncertain fractional-order chaotic systems in finite-time has been studied by Li and Zhang (2016) and Kengne, Tchitnga, Mezatio, Fomethe, and Litak (2017). Abdurahman, Jiang, and Teng (2016) studied finite-time synchronization for a class of fuzzy neural networks with time-varying delays. However, to the best knowledge of the authors, the problem of robust
finite-time synchronization of chaotic systems in the presence of model uncertainties and/or external disturbances via terminal sliding mode and with estimation of the settling-times has not been investigated yet. This issue is studied in this paper.

The sliding mode control (SMC) is an useful and effective control scheme to deal with uncertainties and/or disturbances (Chenarani & Binazadeh, 2017; Yousefi & Binazadeh, 2016). The basic SMC cannot guarantee the convergence of the states of the closed-loop system to the equilibrium point in finite-time. In order to tackle this problem, the terminal sliding mode (TSM) control technique has been proposed to address the finite-time stabilization issue (Binazadeh & Shafiei, 2014b; Rath, Defoort, Karimi, & Veluvolu, 2017).

In this paper a novel TSM controller is designed for robust finite-time synchronization of chaotic systems in the presence of model uncertainties and/or external disturbances. For this purpose, first, the dynamical equations of synchronization error between master and slave systems are established. Then the terminal sliding surface and the TSM control law are proposed to guarantee the finite-time occurrence of reaching phase and finite-time stability of motion equations on the sliding surface. In this regard, a theorem is given and proved by Lyapunov approach. Additionally, in order to display the applicability of the proposed method, it is applied on two practical examples: the Duffing–Holmes system and the gyroscope system. Computer simulations verify the theoretical results and demonstrate the effective performance of the proposed robust control law in finite-time synchronization in spite of existence of model uncertainties and external disturbances.

2. Problem statement

Consider the following dynamical systems:

Master system:

\[ \dot{x}_i(t) = x_{i+1}(t) \quad i = 1, 2, \ldots, n - 1 \]
\[ \dot{x}_n(t) = f(x, t) + \Delta f(x, t) + d_1(t) \]  
(1)

Slave system:

\[ \dot{y}_i(t) = y_{i+1}(t) \quad i = 1, 2, \ldots, n - 1 \]
\[ \dot{y}_n(t) = g(y, t) + \Delta g(y, t) + d_2(t) + u(t) \]  
(2)

where \( x(t) \) and \( y(t) \in \mathbb{R}^n \) are the state vectors of master and slave systems, \( \Delta f(x, t) \) & \( \Delta g(z, t) \in \mathbb{R} \) are uncertain terms representing the un-modeled dynamics or structural variation of the master and slave systems, \( d_1(t) \) & \( d_2(t) \in \mathbb{R} \) present external disturbances and \( f(x, t) \) & \( g(y, t) \in \mathbb{R} \) are known continuously nonlinear Lipschitz functions which are appeared in master and slave systems, respectively. Additionally, \( u(t) \in \mathbb{R} \) is the input of the slave system which may be used to synchronize the slave system with the master system.

**Assumption 1:** The external disturbances \( d_1(t) \) and \( d_2(t) \) are absolutely bounds and satisfy:

\[ |d_1(t) - d_2(t)| < d \]  
(3)

where \( d \) is the known positive constant.

**Assumption 2:** The uncertain terms \( \Delta f(x, t) \) and \( \Delta g(y, t) \) are bounded such that:

\[ |\Delta f(x, t)| < \lambda_1, \quad |\Delta g(y, t)| < \lambda_2 \]  
(4)

As a result, one has:

\[ |\Delta f(x, t) - \Delta g(y, t)| < \lambda \]  
(5)

where \( \lambda_1, \lambda_2 \) and \( \lambda \) are the known positive constants. The main purpose of this paper is design a suitable robust nonsingular TSM controller for synchronization of two chaotic systems (1) and (2) in finite-time.

3. Design of the finite-time controller

In this section, first, the dynamical equations of the synchronization error are introduced. Then, the control law \( u(t) \) is designed which guarantees the robust finite-time stability of the synchronization error. For this purpose, the vector of synchronization error between the master and slave systems is defined as \( e(t) = [e_1(t), \ldots, e_n(t)]^T \in \mathbb{R}^n \) where \( e_i(t) = x_i(t) - y_i(t), i = 1, 2, \ldots, n \). From Equations (1) and (2), the dynamical equations of the synchronization error are obtained as follows:

\[ \dot{e}_i(t) = e_{i+1}(t) \quad i = 1, 2, \ldots, n - 1 \]
\[ \dot{e}_n(t) = f(x, t) - g(y, t) + \Delta f(x, t) - \Delta g(y, t) + d_1(t) - d_2(t) - u(t) \]  
(6)

**Definition 1:** Consider the system (6). If there exist a constant time \( T_0 = T_0(e(0)) > 0 \) such that \( \lim_{t \rightarrow T_0} ||e(t)|| = 0 \) and \( ||e(t)|| \equiv 0 \) for \( t \geq T_0 \), then the synchronization error will converge to zero in the finite-time \( T_0 \) or equivalently the chaotic systems (1) and (2) will synchronize in the finite-time \( T_0 \).

In the following, the TSM approach is used to design a robust control law which ensures synchronization objective. Design of TSM control law is divided into two phases: first, choosing an adequate terminal sliding surface to achieve the control objective. Second, design of a discontinuous control law which forces the system trajectories...
to reach the terminal sliding surface in finite-time and stay on it. In this paper, the terminal sliding surface is chosen as follows:

\[ s(t) = e_n(t) + \int_0^t \sum_{i=1}^n (c_i \text{sgn}(e_i(t))|e_i(t)|^q)dt \tag{7} \]

where \( q \in (0, 1) \) is a scalar and \( c_i \) are positive constants. When the system operates in the sliding mode, it satisfies the following equations:

\[ e_n(t) + \int_0^t \sum_{i=1}^n (c_i \text{sgn}(e_i(t))|e_i(t)|^q)dt = 0 \tag{8} \]

Maintaining the system’s trajectories on \( s(t) = 0 \) results in:

\[ \dot{e}_n(t) = -\sum_{i=1}^n (c_i \text{sgn}(e_i(t))|e_i(t)|^q) \tag{9} \]

In the theory of sliding mode, when the sliding surface is attractive, the states converge to the surface in finite-time and stay on it. In such situation, for finite-time stabilization, the motion equation on the sliding surface should be finite-time stable. Considering (9); then the dynamical equations of the synchronization error on the sliding surface \( s(t) = 0 \) are as follows:

\[ \dot{e}_i(t) = e_{i+1}(t) \quad i = 1, 2, \ldots, n-1 \]

\[ \dot{e}_n(t) = -\sum_{i=1}^n (c_i \text{sgn}(e_i(t))|e_i(t)|^q) \tag{10} \]

In what follows, the finite-time stability of the system (10) is proved.

**Theorem 1:** The motion equations (10) on the given sliding surface (7) is finite-time stable if \( c_i > 0 \) are chosen such that the polynomial \( \lambda^n + c_n \lambda^{n-1} + \ldots + c_2 \lambda + c_1 \) is Hurwitz.

**Proof:** See (Bhat & Bernstein, 2005).

In the following, the appropriate control law is designed.

**Theorem 2:** Consider system (6). The TSM controller (11) guarantees that the trajectories of the synchronization error will converge to the proposed sliding surface \( s(t) = 0 \) in finite-time.

\[ u(t) = f(x, t) - g(y, t) + \sum_{i=1}^n (\text{sgn}(e_i(t))|e_i(t)|^q) + k \text{sgn}(s(t)) \tag{11} \]

where \( k \) is a positive constant and \( k > \lambda + d \).

**Proof:** Considering Equations (6) and (7), the time derivative of \( s(t) \) is obtained as follows:

\[ \dot{s}(t) = \dot{e}_n(t) + \sum_{i=1}^n (\text{sgn}(e_i(t))|e_i(t)|^q) \]

\[ = (f(x, t) - g(y, t) + \Delta f(x, t) - \Delta g(y, t) + d_1(t) - d_2(t) - u(t) + \sum_{i=1}^n (\text{sgn}(e_i(t))|e_i(t)|^q) \tag{12} \]

Substituting (11) into (12), yields:

\[ \dot{s}(t) = \delta(x, t) - k \text{sgn}(s(t)) \tag{13} \]

where \( \delta(x, t) = \Delta f(x, t) - \Delta g(y, t) + d_1(t) - d_2(t) \) and based on Assumptions 1 and 2 \( |\delta(x, t)| \leq \lambda + d \). Define a Lyapunov function as follows

\[ V_2(t) = \frac{1}{2} s^2(t) \tag{14} \]

Taking the time derivative of \( V_2(t) \), one has:

\[ \dot{V}_2(t) = s(t) \dot{s}(t) \]

\[ = s(t)(\delta(x, t) - k \text{sgn}(s(t))) \]

\[ \leq |s(t)||\delta(x, t)| - ks(t) \text{sgn}(s(t)) \]

\[ \leq -k(\lambda + d)|s(t)| \leq k_0 \]

Therefore, \( s \dot{s} < -k_0 |s| \) where \( k_0 \) is a positive constant. For \( s(t_0) < 0 \), this inequality leads to \( s \geq k_0 \), and for \( s(t_0) > 0 \), it leads to \( s \leq -k_0 \); consequently, it is guaranteed that terminal sliding surface \( s(t) \) becomes zero in a finite reaching-time \( t_f \leq (|s(t_0)|/k_0) + t_0 \). Therefore, the finite-time convergence of trajectories of system (6) to \( s(t) = 0 \) is also proved. 

![Figure 1](image-url) Figure 1. Phase trajectories of the chaotic Duffing-Holmes system.
4. Practical design examples

In this section, the proposed controller is applied on two practical systems: the Duffing–Holmes system and gyroscope system. Since, the \( \text{sgn}(\cdot) \) term in the control input (11) produces chattering phenomena, for chattering reduction \( \tanh(\cdot) \) can be used instead of the \( \text{sgn}(\cdot) \) function (Hakimi & Binazadeh, 2017). In this section the sign function is approximated by the tangent hyperbolic function to alleviate chattering.

4.1. Example 1: Duffing-Holmes system

The Duffing-Holmes system is a nonlinear dynamical system exhibiting complex and chaotic behaviour (Fang, Li, Li, & Li, 2013). Figure 1 shows the phase trajectories of this system.

Considering the following equations of the master system:

\[
\begin{align*}
\dot{x}_1 &= x_2 \\
\dot{x}_2 &= -p_1 x_1 - p_2 x_2 - x_1^3 + h \cos(\omega t) + \Delta f(x, t) + d_1(t) \\
\end{align*}
\]

and the slave system as:

\[
\begin{align*}
\dot{y}_1 &= y_2 \\
\dot{y}_2 &= -p_1 y_1 - p_2 y_2 - y_1^3 + h \cos(\omega t) \\
&\quad + \Delta g(y, t) + d_2(t) + u(t) \\
\end{align*}
\]

where \( \Delta f(x, t), \Delta g(y, t) \) and \( d_1(t), d_2(t) \) are uncertainties and external disturbances. The parameter values are

\[
\begin{align*}
p_1 &= -1, p_2 = 0.25, h = 0.3, \omega = 1. \quad \text{For simulation, the}
\end{align*}
\]

**Figure 2.** Time-responses of synchronization errors without applying the proposed controller.

**Figure 3.** Time-responses of the synchronization errors by applying the proposed controller.
proposed controller (11) is applied for synchronization between two chaotic systems (16) and (17) with considering:
\[ q = 0.95, k = 10, \Delta f(x, t) = -0.1 \sin (x_2), \]
\[ \Delta g(y, t) = 0.2 \cos (\pi y_1), d_1(t) = 0.3 \cos (5t), d_2(t) = 0.4 \sin (t) \]

\[ \dot{x}_1 = x_2 \]  

4.2. Example 2: Chaotic Gyroscope System

The gyroscope system is an important dynamical system that has major applications in navigation, space engineering and etc. In (Behjameh, Delavari, & Vali, 2014), the following model for a gyroscope system has been introduced which has a chaotic behaviour with the coefficients \( \beta_1 = 35.5, \alpha^2 = 100, \beta_0 = 1, k_1 = 0.5, k_2 = 0.05, \omega = 2. \)
Figure 6. Time-responses of state variables of the master and slave systems by applying the proposed controller: (a) $x_1(t), y_1(t)$; (b) $x_2(t), y_2(t)$.

\[
\dot{x}_2 = -k_1 x_2 - k_2 x_2^3 - \alpha^2 \frac{(1 - \cos x_1)^2}{\sin^2 x_1} + (\beta_0 + \beta_1 \sin \omega t) \sin x_1 + \Delta f(x, t) + d_1(t) \quad (19)
\]

\[
\dot{y}_1 = y_2
\]

where $\Delta f(x, t)$ and $d_1(t)$ are model uncertainties and external disturbances, respectively. The chaotic behaviour of system (19) is illustrated in Figure 7.

Consider (19) as the equations of master system, the dynamical equations of the slave system are considered as follows:

\[
\dot{y}_2 = -k_1 y_2 - k_2 y_2^3 - \alpha^2 \frac{(1 - \cos y_1)^2}{\sin^2 y_1} + (\beta_0 + \beta_1 \sin \omega t) \sin y_1 + \Delta g(y, t) + d_2(t) + u(t) \quad (20)
\]

where $\Delta g(y, t)$ and $d_2(t)$ present the model uncertainties and external disturbance in the slave system. Moreover, $u(t)$ is the control input. For simulations, the proposed controller (11) is applied for finite-time synchronization between two chaotic systems (19) and (20) with considering:

\[ q = 0.8, k = 10, \Delta f(x, t) = 0, \Delta g(y, t) = 0.6 \sin (y_1), \]
\[ d_1(t) = 0.4 \sin(\pi t), \quad d_2(t) = 0.2 \cos(\pi t) \quad (21) \]

The time-responses of synchronization errors with and without the proposed controller are illustrated in Figure 8 which verify the performance of the proposed controller. The time-responses of the terminal sliding surface and the control input are shown in Figures 9 and 10, respectively. As it is seen, \( s(t) \) converges to zero in finite-time.

5. Conclusions

This paper studied the problem of finite-time synchronization between two chaotic systems with parameter uncertainty and external disturbances. For this purpose, first, the dynamical equations of the synchronization
error were constructed. Then, a novel terminal sliding surface was introduced and the nonsingular terminal sliding mode control law was designed. On the basis of Lyapunov approach, two theorems were presented to guarantee the occurrence of the reaching and sliding phases in finite-time. Finally, the proposed method was used to design robust controllers for two practical examples: chaotic Duffing–Holmes system and chaotic gyroscope system. Computer simulations showed the efficiency of the proposed controller in robust finite-time synchronization between the master and slave systems.

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No potential conflict of interest was reported by the authors.

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