Numerical simulation of the unsteady cavitating flow in a Francis turbine draft tube at Upper-Part-Load (UPL) conditions

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Abstract. At part-load conditions, Francis turbines experience the development of a precessing vortex rope in their draft tube (DT), rotating with a frequency between 0.2 and 0.5 times the runner frequency. It induces pressure pulsations at the precession frequency in the whole hydraulic system, undesirable vibrations and noise putting at risk the stability of the system and the lifetime of the machine components. In specific machines, a dramatic amplification of the noise and vibrations can be observed between 70% and 85% of the design conditions. In these cases, synchronous pressure pulsations with a high frequency, typically between 2 and 4 times the runner frequency, are observed, referred to as the so-called Upper-Part-Load (UPL) pulsations. These pulsations are induced by a self-excitation of the entire hydraulic system including the cavitation vortex rope at one of its high order eigenmodes. Besides, the cavitation vortex rope features an elliptical cross-section rotating around the vortex axis at a high frequency. In this manuscript, unsteady two-phase flow simulations using a Scaled-Adaptive-Simulation (SAS) turbulence model and ZGB cavitation model of a Francis turbine draft tube at 80% of the design condition are performed to clarify the mechanisms responsible for the formation of the elliptical shape of the vortex and the UPL pulsations. However, the numerical simulation with a constant inlet boundary condition cannot reproduce the UPL pulsation phenomenon since it is a self-excited oscillation at one eigenmode of the complete system, which is not considered. Therefore, the UPL pulsations component is extracted from the measured pressure fluctuations data by applying a band-pass filter and is then set as the inlet boundary condition in the numerical simulation. The flow field is therefore artificially excited which aims to confirm whether the same phenomenon can be observed and to compare with the flow field obtained with constant inlet boundary condition. The numerical simulations are validated by experimental results, and the wave propagation in the DT is clarified.

1. Introduction

Hydroelectric powerplants are increasingly operated in off-design conditions to adjust their production to the electricity network requirements and compensate for the intermittent electricity production by wind and solar sources. At part-load conditions with a low cavitation number, Francis turbines experience the development of a cavitation precessing vortex in their draft tube, rotating with a frequency between 0.2 and 0.4 times the runner frequency [1]. The pressure pulsation induced by the vortex rope propagates along the whole hydraulic system and goes along with undesirable vibrations and noise. Several researchers have reported that the amplitude of vibrations and noise may dramatically increase between 70% and 85% of the design conditions [2][3][4][5][6] in certain machines. This phenomenon is called Upper-Part-Load (UPL) instability and occurs only in presence of cavitation in the draft tube. It induces synchronous pressure pulsations at a high frequency, typically between 2 and 4 times the runner frequency [5][6]. A pressure node is commonly observed along the draft tube, characterized by a phase shift of π of the UPL pulsations component [8]. Moreover, the cross-section of
the cavitation vortex rope takes an elliptical shape and rotates around the vortex axis, while the whole vortical structure features a precession movement around the draft tube axis [5][6][7]. Dörfler investigated the mechanisms of this phenomenon by 1D model stability analysis and concluded that the UPL pulsations are caused by a self-excited instability of one higher eigenmode of the whole system [7]. However, the physical mechanisms of this phenomenon and notably the link between the elliptical cross-section of the vortex and UPL pulsations remain unclear. Guan et al. [8] investigated the UPL vortex rope behavior by unsteady Reynolds-Averaged Navier-Stokes (URANS) single-phase simulation. Although the cross-section of the vortex rope features an elliptical shape, high frequency pressure fluctuations induced by the UPL instability are not observed in their computational results.

This manuscript presents a preliminary numerical study of the UPL phenomenon based on a preliminary experimental study [6]. Unsteady one-phase flow simulation with constant boundary conditions (CBC), unsteady two-phase flow simulation with CBC and unsteady two-phase flow simulation with fluctuating boundary conditions (FBC) are performed to determine an appropriate method reproducing the UPL instability. The simulation with FBC aims to investigate the response of the DT flow fields to an artificial excitation of the whole system at the UPL pulsations frequency. Firstly, the specifications of three types of simulation are described. The pressure fluctuations in the DT are then investigated by Fast Fourier Transform (FFT) and cross-spectrum. The phase-shift of the UPL frequency component along the cross-section and the DT are finally clarified.

2. Test case

2.1. Francis turbine reduced scale model

A preliminary experimental study [6] was performed on a reduced scale model of a Francis turbine of specific speed \( N_s = 162 \) m-kW installed on Waseda University open-loop test apparatus, see figure 1. The investigated Francis turbine was designed by Takahashi et al. [9] and features an unshrouded runner, 20 stay-vanes (SV), 20 guide-vanes (GV) and 18 runner-vanes (RV). The Reynold’s number defined by the mean velocity at the turbine inlet of the design point is \( 6.30 \times 10^5 \).

The operating conditions are characterized by the speed coefficient \( N_{11} = N D_1 / \sqrt{H_e} \) and the discharge coefficient \( Q_{11} = Q / (D_1^2 \sqrt{H_e}) \), where \( N \), \( Q \), \( H_e \), \( D_1 \) are the rotational speed, discharge, effective head and the diameter of runner inlet. The test rig is operated in cavitation test mode, see figure 1. The downstream suction pump is used to decrease the pressure at the outlet of the draft tube and therefore to change the cavitation number \( \sigma \). Several dynamic pressure sensors are installed in the inlet pipe and draft tube, see figure 1 (from Section 0 to Section 6).
2.2. Preliminary experimental results

The tested operating conditions are shown in figure 2. Figure 3 shows the influence of the discharge coefficient on the peak-to-peak value of the pressure fluctuations measured in \( p_{11} \) in the cases of no-cavitation (\( \sigma = 0.55 \)) and cavitation (\( \sigma = 0.25 \)) conditions. The amplitude of the pressure fluctuations dramatically increases when the discharge coefficient is about 80% of the best-efficiency condition which is within the range of UPL conditions. In this range, the cavitation vortex rope features an elliptical cross-section rotating around the vortex axis and a movement of precession around the axis of the draft-tube, as shown in [6] and illustrated in figure 4. The period of self-rotation of the elliptical cross-section of the vortex is much shorter than the precession motion of the whole vortical structure.

![Figure 2. Operating conditions](image1)

![Figure 3. Influence of \( Q_{11} \) on the pressure fluctuations amplitude \( (N_{11}/N_{11}^{*} = 1) \)](image2)

![Figure 4. Example of the elliptical shape of the vortex rope \( (N_{11} = 62, Q_{11} = 0.626, \sigma = 0.18) \) [6]](image3)

The FFT of the pressure signals measured in \( p_{11} \) and \( p_{21} \) are shown in figure 5. The precession frequency of the vortex is equal to 0.3 times the runner frequency. UPL pulsations are observed at a frequency of about 2.7 times the runner frequency. The phase distribution of the UPL pressure pulsations along the draft tube is shown in figure 6: UPL pressure pulsations features a phase shift of \( \pi \) between the sections 1 and 2, indicating the presence of a pressure node.

![Figure 5. FFT spectra of pressure fluctuations](image4)

![Figure 6. Phase distribution of UPL pulsations component along the test rig and draft tube](image5)
3. Computational framework with ANSYS-CFX

The computations are performed by using the commercial code ANSYS-CFX 19.0 which solves the Navier-Stokes equations with the finite volume method. The computational domains include one periodic guide-vane (GV), the full runner domain and the complete draft tube (DT), see figure 7. Using only one GV aims at reducing the computational cost. Scale-Adaptive Simulation Shear Stress Transport (SAS SST) with bounded central difference scheme is used as turbulence model and the Zwart-Gerber-Belamri (ZGB) model based on the Rayleigh Plesset equation is used as cavitation model. The RMS residual target for convergence criteria in the simulations is set at $10^{-5}$. The mesh information is provided in table 1.

![Inlet and Outlet](image)

Figure 7. Computational domains

| Domain | Mesher  | Nodes   | Orthogonal Angle | Expansion Factor | Aspect Ratio |
|--------|---------|---------|------------------|------------------|-------------|
| GV     | Turbo Grid | 160344  | 72.6            | 6                | 271         |
| RV     | Turbo Grid | 3826170 | 7.3             | 10               | 344         |
| DT     | ICEM-CFD  | 8261751 | 20.3            | 39               | 18758       |

The operating points and their conditions are shown in table 2. Three different types of simulation with different boundary conditions are performed at the same $N_{11}/N_{11}^*$ and $Q_{11}/Q_{11}^*$. Cases 1 and 2 are simulated with constant mass flow rate. The differences between cases 1 and 2 are the value of the cavitation number $\sigma$: the cavitation model is applied only to case 2. For the case 3, a fluctuating total pressure is set as inlet boundary condition. The average total pressure at inlet was extracted from the computational results of case 2, whereas the fluctuating total pressure component was extracted from the experimental pressure signal of $p_{1,1}$ because the fluctuating pressure signal at GV inlet was not measured. Therefore, there is no comparison between the experimental and computational results about pressure amplitude in this paper. A band-pass filter was applied to the pressure signal to conserve only the UPL pressure fluctuations. The experimental pressure signals used to define the inlet boundary conditions of case 3 are shown in figure 8. The outlet BC of these three cases are all set as an “Opening” with “Entrainment Option”. The opening BC allows the fluid to cross the boundary surface in either direction [10]. Entrainment option allows the solver to calculate the flow direction based on the direction of the velocity field which is suitable for physical phenomena with recirculation zone. The time-step is set at 1 degree of runner rotation.
Table 2. Simulated operating points and conditions

| Case | \( N_{11}/N_{11}^* \) [-] | \( Q_{11}/Q_{11}^* \) [-] | \( \sigma \) [-] | Cavitation | Inlet BC |
|------|-----------------|-----------------|--------|-----------|----------|
| 1    | 1               | 0.84            | 0.55   | Without   | Constant mass flow rate |
| 2    | 1               | 0.84            | 0.11   | With     | Constant mass flow rate |
| 3    | 1               | 0.84            | 0.11   | With     | Fluctuating total pressure |

Figure 8. Pressure signals in time series at \( p_{1-1} \) and corresponding FFT spectra (black: raw signal; red: pass-band filtered signal)

4. Numerical results

4.1. FFT of pressure fluctuations

Figure 9 shows the pressure fluctuation spectrum at the \( p_{1-1} \) position for the CFD cases 1, 2 and 3. The frequency component corresponding to the vortex precession is observed in each case. The comparison of the frequency values between measurements and simulations is shown in table 3 to validate the numerical simulations. The deviation of the cavitation free simulation (case 1) and the cavitation simulation (case 2) with CBC is lower than 5%. The cavitation simulation (case 3) has higher deviation but it remains lower than 10%. Therefore, the accuracy of the numerical simulation is acceptable [11]. The pressure fluctuations amplitude due to PVC are higher in the cavitation free condition (case 1) than in cavitating conditions (cases 2&3), which agrees with the experimental results shown in figure 10. However, this should be confirmed again with more experimental data.

For case 1, the simulation is consistent with the experimental observation: UPL pulsations do not occur in cavitation-free conditions. Concerning case 2, i.e. two-phase flow with constant inlet boundary conditions, the results confirm that CFD simulation with constant boundary conditions cannot reproduce the UPL pulsations since this phenomenon is a self-excited oscillation of the whole system. In case 3, the UPL component artificially exciting the system from the inlet can be observed in the DT.

Figure 9. Pressure fluctuation spectrum at the \( p_{1-1} \) position from CFD
Table 3. Comparison of frequency values between measurements and simulations

|                | $f_{PVC}/f_{RV}$ | $f_{UPL}/f_{RV}$ |
|----------------|------------------|------------------|
| Measurement ($\sigma = 0.55$) | 0.264            | -                |
| Case1 ($\sigma = 0.55$)         | 0.277 (Deviation = 4.92%) | -                |
| Measurement ($\sigma = 0.11$)   | 0.259            | 1.38             |
| Case2 ($\sigma = 0.11$)         | 0.248 (Deviation = 4.25%) | -                |
| Case3 ($\sigma = 0.11$)         | 0.240 (Deviation = 7.53%) | 1.38             |

Figure 10. Pressure fluctuation amplitude at $f_{PVC}$ at the p1.1 position from measurements

4.2. Cross-spectrum of pressure fluctuations (CFD case 3)
The phase shift between the pressure fluctuations measured in the sections 1 and 4 and along the DT are investigated for the CFD case 3. The position of the monitor points is shown in figure 11. Figure 12 shows the cross-spectrum amplitude and phase shift between the positions 1&1, 1&2, 1&3 and 1&4 in sections 1 and 4. The frequency components corresponding to the precessing vortex rope and the “forced” UPL pulsations are highlighted in the figures displaying the cross-spectrum amplitude. The PVC component features a phase-shift in agreement with the spatial-position of each point. The UPL component has a zero phase-shift in one cross-section, which agrees with the synchronous nature of the UPL components observed in the experiments [6].

Figure 11. Position of the monitor points along the DT and the cross-section
Figure 12. Cross spectrum amplitude and phase shift at section 1 and 4

Figure 13 and 14 show the cross-spectrum amplitude and phase of the pressure fluctuations at $f_{UPL}$ along the DT, respectively. A comparison with experimental results is given for the phase distribution. The data at the position 1 in each section are used for the calculation. Both PVC and UPL components can be observed in figure 13. The amplitude of the PVC component increases and reaches its maximum between the sections 1 and 2. It decreases dramatically from section 3, indicating a collapse of the precessing vortex rope between the sections 2 and 3. The amplitude of the UPL decreases sharply from section 1 to 2 which is the area with the cavitation rope. After that, it decreases gradually. The phase-shift of the UPL component is equal to approximately zero all along the DT, which does not correspond to the experimental observation: the pressure node observed in the experiments is not reproduced by the simulation.

Figure 13. Cross-spectrum amplitude along the DT  Figure 14. Phase-shift along the DT compared with experimental results

4.3. Visualization of the vortical structure and vortex cross-section shape
The vortical structure in the DT is identified by the velocity invariant $Q$ which defines a vortex as a “connected fluid region with a positive second invariant of $\nabla u$ ” [12]. The area with a positive $Q$-value means that the vorticity magnitude is greater than the magnitude of the rate-of-strain. The volume
rendering of the $Q$-criterion in the DT cone is given in figure 14 to visualize the structure of the precessing vortex rope. In case 1, the vortex features a strong vorticity magnitude in its core (in red) and a vortex collapse can be observed before section 2 which is in agreement with the visualization results, see figure 4. In case 2, the vortex features a strong vorticity magnitude at the runner outlet and gradually loses its structure and collapses before the position of section 2 similarly to the case 1. In case 3, the vortical structure is similar to the case 2. The vorticity magnitude at the vortex centre is slightly stronger than in case 2. The position of the vortex collapse position is the same as in the cases 1 and 2.

The shape of the cavitation cross-sections below the runner is visualized by vapor volume fraction in figure 15 for the CFD cases 2 and 3. The cavitation cross-sections feature a well-formed elliptical shape in the case 2 but a torsional shape in the case 3.

![Figure 15. Volume rendering of Q-criterion in DT cone](image)

![Figure 16. Shape of cavitation cross-sections in the DT cone](image)
5. Conclusion
Three cases of SAS simulation are performed to determine a proper method to reproduce the UPL pulsations observed in Francis turbine draft tube. Finally, we obtain the conclusions listed below:

1. One-phase flow simulation and two-phase flow simulation with constant boundary conditions are not able to reproduce the UPL pulsations.
2. When the flow field is artificially excited by inlet boundary conditions fluctuating at the UPL pulsations frequency, the pressure field in the DT reacts with the same frequency.
3. The UPL frequency component has no phase-shift along the cross-sections of the DT, which means that this component is synchronous.
4. The UPL frequency component has almost zero phase-shift along the cross-section of the DT, which is not in agreement with the pressure node observed in the DT by experiments.
5. The vortex collapse position can be predicted in all 3 cases.

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