A model for predicting the growth of *Eucalyptus globulus* seedling stands in Bolivia

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**Abstract**

*Eucalyptus globulus* is one of the most planted species in the Inter-Andean Valleys of Bolivia, where growing conditions are different from most places where eucalypts have been studied. This prevents a straightforward utilization of models fitted elsewhere. In this study a distance-independent individual-tree growth model for *E. globulus* plantations in Bolivia was developed based on data from 67 permanent sample plots. The model consists of sub-models for dominant height, tree diameter increment, height-diameter relationship and survival. According to model-based simulations, the mean annual increment with the optimal rotation length is about 13 m³ ha⁻¹ yr⁻¹ on medium-quality sites and 18 m³ ha⁻¹ yr⁻¹ on the best sites. A suitable rotation length for maximizing wood production is approximately 30 years on medium sites and 20 years on the most productive sites. The developed models provide valuable information for further studies on optimizing the management and evaluating alternative management regimes for the species.

**Key words:** plantations; individual-tree growth and yield; simulation; eucalypts; blue gum.

Introduction

Most of the growth and yield models that can be found in the literature for *Eucalyptus globulus* are for stand-level calculations (e.g. Tomé et al., 1995; García and Ruiz, 2003; Wang and Baker, 2007). Goodwin and Candy (1986) and Soares and Tomé (2003) developed individual-tree growth models for *E. globulus*. Compared to stand level models, individual-tree models have higher resolution (Pretzsch et al., 2002) and allow detailed stand description and flexible simulations. They predict how competition affects individual trees allowing a logical simulation of tree size differentiation within a stand. An individual-tree model can produce a similar output as lower resolution models (e.g. stand-level models) since stand characteristics can be calcu-
lated from the tree variables. In Bolivia, *E. globulus* plantations have been promoted since the 1960s. The singular Bolivian conditions (high altitudes, dry climate and small influence of the ocean) are different from most places where eucalypts have been studied, which prevents a straightforward utilization of models fitted elsewhere. Therefore, there is a need for Bolivia-specific equations that would enable reliable growth and yield prediction of *E. globulus* to support plantation management and planning. The aim of the research work summarized in this article was to develop a system of models that enables the simulation of *E. globulus* stand dynamics on an individual-tree basis.

### Material and methods

#### Data

The modelling data consisted of 67 permanent plots (400 m² each) measured in 16 sites (Table 1). All trees were measured for diameter and a variable number of trees also for height. There were 1-7 measurement intervals per plot, and the length of measurement interval was 1–7 years. The stand age ranged from 1 to 29 years. Elevation ranged from 2,180 to 3,590 m.

#### Dominant height modelling

Several algebraic difference models derived from Hossfeld (cited by Peschel, 1938), Korf (1939), Schumacher (1939), Chapman-Richards (Richards, 1959), Sloboda (1971) and McDill-Amateis (1992), fitted using non-linear mixed-effects analysis, were tested to describe dominant height development using a dataset of 272 pairs of dominant height-age observations. Dominant height was defined as the average height of the 100 thickest trees at breast height per hectare. Choice of the best model was based on statistical and biological criteria. The index age was taken as 20 years, since this is a reasonable rotation length for this species in the study area.

#### Diameter increment modelling

The model was fitted to the data (9,041 observations) using linear mixed-effects analysis. The purpose was to model the influence of tree size, site quality and competition on diameter increment. Each type of predictors (tree size, site, competition) was described by several variables that were tested during the model fitting. Transformations of predictors were also tested.

#### Height-diameter modelling

The fitting of the height-diameter model used 6,171 observations and was based on the Stoffels and Van Soest (1953) power equation modified by Tomé (1989), which constrains the model so that it passes through the point determined by dominant diameter and dominant height. The model was fitted using non-linear mixed-effects analysis.

#### Survival modelling

Since dead and cut trees could not be distinguished in the data, it was not possible to fit an individual-tree

| Table 1. Summary of the data used in modelling |
|-----------------------------------------------|
| **Minimum** | **Maximum** | **Mean** | **Std. Deviation** |
| Stand age, years | 1.0 | 29.0 | 10.1 | 6.6 |
| Site index, m | 7.9 | 26.9 | 18.4 | 3.1 |
| Dominant height, m | 0.7 | 26.2 | 11.9 | 6.7 |
| Dominant Diameter, cm | 0.5 | 37.2 | 8.5 | 6.1 |
| Stand basal area, m²/ha | <0.05 | 49.3 | 9.2 | 8.6 |
| Number of trees per hectare | 50 | 2250 | 1137 | 450 |
| Tree height, m | 1.3 | 29.0 | 8.8 | 5.5 |
| Diameter increment, cm | −1.3 | 6.5 | 0.7 | 0.5 |
| Diameter at breast height, cm | 0.5 | 37.2 | 9.2 | 5.8 |
| BAL, m²/ha | 0.0 | 46.5 | 5.7 | 6.2 |
| Time interval between measurements, years | 1.0 | 7.0 | 1.6 | 1.1 |
mortality model. Therefore, a model was fitted for the maximum number of trees per hectare (self-thinning limit), using the densest plots.

**Results**

**Dominant height and site index model**

The model termed Schumacher 1 (Eq. 1) was judged the best dominant height equation. This model was visually compared with the measured dominant height developments in the permanent sample plots (Fig. 1) and was regarded suitable. The dominant height model is therefore

\[
\hat{H}_{jk} = \exp \left[ \ln(H_{i,j}) + (3.698 + a_{i} + b_{j,k}) \left( \frac{1}{T_{i,j}} - \frac{1}{T_{j,k}} \right) \right]
\]

where \( H_{i,j} \) and \( \hat{H}_{j,k} \) are dominant height in measurement occasions \( j \) and \( j+1 \) (m), \( T_{i,j} \) and \( T_{j,k} \) are stand age in measurement occasions \( j \) and \( j+1 \) (years), \( a_{i,k} \) is random site effect, \( b_{j,k} \) and \( b_{2j,k} \) are random measurement effects (measurement \( j \) of site \( k \)). All predictors were significant (\( p < 0.05 \)). The variances of the random effects were \( a_{i,k} = 0.204 \), \( b_{j,k} = 1.200 \), \( b_{2j,k} = 0.002 \). The residual variance was 0.219.

**Diameter increment model**

The final model for diameter increment was as follows:

\[
\hat{d}_{ijk} = 1.206 \exp(y_{ijk})
\]

\[
y_{ijk} = -1.445 - 0.238 P_{jk} + 0.649 \ln(d_{ijk}) - 0.109 H_{dom,ij} + 0.0925 \ln(BAL_{ijk}) + 0.0552 S_{ij,k} + a_{i} + b_{j,k}
\]

where \( \hat{d} \) is diameter increment (cm), \( P \) is the length of the prediction period (years), \( d \) is dbh (cm), \( H_{dom} \) is dominant height (m), \( BAL \) is basal area in larger trees (m²/ha) and \( SI \) is site index (m). All predictors were significant (\( p < 0.05 \)). The variance of the site effect \( a_{i} \) was 0.051 and that of the measurement effect \( b_{j,k} \) was 0.089. The variance of the residual was 0.245. Plot effect was not significant. Multiplier 1.206 is the Snowdon (1991) correction factor for removing the bias caused by the logarithmic transformation of the predicted variable.

**Height-diameter model**

The model was as follows:

\[
\hat{h}_{ijkl} = H_{dom,ij} \times \left( \frac{d_{ijkl}}{D_{dom,ij}} \right)^{a_{i} + c_{ijkl} \left( \frac{1}{D_{mean}} - 1 \right) - b_{j,k} T_{ijkl}}
\]

where \( \hat{h} \) is predicted tree height (m), \( H_{dom} \) is dominant height (m), \( D_{dom} \) is dominant diameter, defined as the average diameter of the 100 thickest trees per hectare (cm), and \( T \) is stand age (years), \( a_{ij} \) is random site effect, \( b_{ijkl} \) and \( c_{ijkl} \) are random plot effects (plot \( k \) of stand \( l \)), \( c_{ijkl} \) and \( c_{2ijkl} \) are random tree effects (tree \( j \) of plot \( k \) of stand \( l \)). All predictors were significant (\( p < 0.05 \)). The variances of the random effects were \( a_{ij} = 0.008 \), \( b_{ijkl} = 0.012 \), \( c_{ijkl} = 0.013 \), \( c_{1ijkl} = 0.105 \), \( c_{2ijkl} = 0.098 \). The residual variance was 0.172.

**Survival model**

A self-thinning limit was fitted to the data but, due to the lack of dense plots, the fitted self-thinning limit was unrealistically low. Therefore, the slope parameter of the model was adjusted so that the maximum basal areas agree with observed maximum basal areas of tree plantations in the same area (Fig. 2). The adjusted model was:

\[
N_{max} = \exp(12.389 - 1.75 \ln(D_{mean}))
\]

where \( N_{max} \) is the self-thinning limit and \( D_{mean} \) is the quadratic mean diameter of the stand.
Discussion

This study presents a site index model, a height-diameter model, an individual-tree diameter increment model and a self-thinning model for *E. globulus* stands in Bolivia. The main equations provided in this article are mixed-effects models. This approach was applied in order to take into account the hierarchical structure of the data. All those hierarchical levels which explain a significant part of the variance as well as all the significant random parameters were included in the models. With this type of approach the site, plot and measurement effects can be included in prediction if additional measurements are available for the stands of interest. If random parameters are not used in prediction, mixed effect models result in higher residual variation than fixed-effect models. This is because fixed-effect models are fitted to minimize the residual variance around the prediction provided by the fixed parameters. In the current study, the increase in RSME, due to the use of mixed-effect modelling, was 0.3% for the dominant height model, 1.6% for the diameter increment model, and 1.9% for the height model.

The oldest plot in the modelling data was 29 years old and, hence, extrapolations beyond that age should be simulated with caution. Furthermore, an individual-tree mortality model would also be needed in order to increase the accuracy and soundness of the model-based projections. Simulations were run to show the way the models perform together and to provide additional information regarding the productivity and suitable rotation lengths for this species in Bolivia (Fig. 3). The volume equations fitted by Pohjonen and Pukkala (1990) for *E. globulus* in Ethiopia for similar ecological conditions (altitude, stand density and growth rate) as in Bolivia were used to calculate stem volumes. According to simulation results for the mean and current annual increments (MAI and CAI), an appropriate rotation length for low- and medium-quality sites when aiming at maximum wood production would be 50 and 30 years, respectively. In very good sites the rotation length could be shortened to 20 years. Mean annual increments up to about 18 m³ ha⁻¹ yr⁻¹ can be achieved in the best sites whereas, in medium and poor sites, the maximum mean annual increment is 13 m³ ha⁻¹ yr⁻¹ and 5 m³ ha⁻¹ yr⁻¹, respectively. The results derived from this study regarding the yield of *E. globulus* are in line with the annual increments reported elsewhere (10-15 m³ ha⁻¹ yr⁻¹) (Jacobs, 1981; Eldridge et al., 1993). However, the rotation lengths obtained for Bolivia are longer and the maximum mean annual increments are lower than those reported by Ugalde and Pérez (2001) for Latin American countries; 30 m³ ha⁻¹ yr⁻¹ in Argentina (Mangieri and Dimitri, 1961) or 44 m³ ha⁻¹ yr⁻¹ in Chile (INFOR, 1986). When comparing the productivity of *E. globulus* and *P. radiata* (the most planted exotic species in the study area) in Bolivia, eucalypts seem to be less productive since MAIs up to 25 m³ ha⁻¹ yr⁻¹ have been observed in the most productive pine stands. The optimal rotation lengths for maximum wood production would be 5-10 years longer for the pines (Guzmán et al., 2012) on the best sites. To our knowledge, this is the first study in

![Figure 2. Simulation of the self-thinning limit according to the fitted (solid line) and adjusted (dotted line) self-thinning models.](image)
Bolivia, and probably also in Latin America, providing tree-level growth models for *E. globulus* that enable simulations of stand dynamics on an individual-tree basis. The set of equations presented in this article provide valuable information for further studies on optimising the management and evaluating alternative management regimes for the species.

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