Controllable optical bistability in a cavity optomechanical system with a Bose–Einstein condensate

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Abstract
The optical bistability (OB) in a two-mode optomechanical system with a Bose–Einstein condensate (BEC) is studied. By investigating the behavior of steady state solutions, we show that how OB develops in the system for a certain range of cavity-pump detunings and pump amplitudes. We then investigate the effects of the decay rate of the cavity photons and coupling strength between the cavity and the BEC as well as the pump-atom detuning on the optical behaviour of the system. We find that one can control the OB threshold and width of the bistability curve via adjusting properly the coupling strength and the detuning. By applying Routh–Hurwitz criterion, we then derive stability conditions for different branches of the OB curve. Moreover, by introducing an effective potential for the system, a simple physical interpretation is obtained.

Keywords: optical bistability, optomechanical system, Bose–Einstein condensate

(Some figures may appear in colour only in the online journal)
Berkeley [15] and ETH Zürich [12] have found OB in systems comprising of vapours of ultracold atoms trapped in optical cavities. Unlike an optomechanical system consisting of an empty cavity with a movable end mirror, the photon numbers in the cavity with a BEC are usually low and even below unity which is desirable for applications such as optical communication and quantum computation.

On the other hand, because of the practical applications of the OB such as all-optical switches, optical transistors and optical memory elements controlling the OB seems to be crucial for the prominent applications. However, from an experimental point of view due to lack of controllability, it has limited applications. In recent years, some attempts have appeared both theoretically and experimentally in order to control the OB. Joshi et al [28], manipulated the OB by changing the intensity and frequency detuning of the coupling field. Almost a year later, Chang and colleagues [29] showed controllable shift of the threshold points of the OB induced by two suitable tuned fields. By controlling this shift, all-optical flip-flop and storage of optical pulse signals were implemented. Recently, the OB of an ultracold atomic ensemble located in an optical cavity was investigated and it was shown that a transverse pumping field could be used to control the bistable behaviour [30]. In another study, the OB was investigated for BECs of atoms in a driven optical cavity with Kerr medium and it was shown that the OB can be controlled by adjusting Kerr interaction between the photons [31]. More recently, Safaei et al [32] have considered two-component BEC in a one-dimensional optical cavity, while parallel and transverse laser fields are applied to the system and have observed simultaneous and mutual bistability of the cavity field and magnetization of the two modes for different values of transverse and parallel pump strengths, under the discrete mode approximation. Moreover, they have investigated the effects of the parallel pump and atom cavity coupling on bistable behavior of the system. In this paper, we investigate effects of the system parameters, i.e. decay rate of the cavity photons, coupling strength between the cavity and the BEC as well as the pump-atom detuning, on the OB in an optomechanical system with a BEC. Controlling the OB is done by simply changing the coupling strength between the cavity and the BEC and the pump-atom detuning without necessity of another field nor Kerr medium which can be much more efficient than the previous ones.

We also present our model that describes a cigar-shaped BEC confined along the axis of an optical ultrahigh-finesse Fabry–Perot cavity and the cavity is driven by an external pump laser. We then investigate the behavior of steady state solutions and show that how this system exhibits the behavior of bistability for a certain range of cavity-pump detunings and pump amplitudes. Noting that, self-organization is another novel phenomena arising in systems of BEC and high-finesse optical cavity, when the condensate atoms are driven from the side by a laser field, directed perpendicularly to the cavity axis. In reference [18], it was shown that the steady-state of the condensate is either the homogeneous distribution or a \( \lambda \) -periodic ordered pattern, which is the quantum analogue of the classical self-organization phase transition exists for BEC. Pump threshold for self-organization was determined at \( T = 0 \), which is analogous to the one obtained in the case of a thermal classical gas. Most recently, the onset of self-organization has been experimentally investigated in a BEC coupled to an optical resonator [33].

This paper is organized as follows. In section 2, we introduce the Hamiltonian of the system consisting of a BEC trapped in a cavity in the discrete-mode approximation. Then in section 3, we focus on the conditions for bistability, the effects of the corresponding parameters of the system on the OB and controllability of the OB characteristics via adjusting properly the coupling strength and the detuning, which may lead to promising applications such as all-optical switching. We also investigate the stability of the different branches of the OB curve by applying Routh–Hurwitz criterion and find the minimum value of the threshold intensity. Moreover, we investigate the dynamic properties by introducing an effective potential for the system.

2. Formalism of the optomechanical system

The system under consideration is depicted schematically in figure 1. We consider a cigar-shaped BEC of \( N ^{87} \text{Rb} \) atoms trapped in an optical ultrahigh-finesse Fabry–Perot cavity. The coupled BEC-cavity system is driven by a pump laser with frequency \( \omega_p \) and amplitude \( |E_p| = \frac{\gamma P_p}{\sqrt{\hbar \omega_p}} \), which is a function of laser power \( P_p \) and decay rate of the cavity photons \( \kappa \). In the large detuning between the pump laser frequency and the atomic resonance, \( \Delta_\omega = \omega_p - \omega_a \), we can eliminate the excited state of the atoms and the Hamiltonian describing the coupled BEC-cavity system reads [34]

\[
\hat{H}_{\text{eff}} = -\frac{\hbar^2}{2m} \frac{\partial^2}{\partial x^2} + V_{\text{ext}}(x) + \hbar U_0 \cos^2(kx) \hat{a}^\dagger \hat{a} \right) \hat{\Psi}(x) \right| \text{d}x + \hat{H}_{\text{vac}} + \hbar \Delta \hat{a}^\dagger \hat{a} + i\hbar E_p (\hat{a}^\dagger - \hat{a}^\dagger) \tag{1}
\]

Here \( \hat{a}^\dagger (\hat{a}) \) denotes the creation (annihilation) operator of the cavity photons with mode function \( \cos(kx) \) and wave vector \( k = \frac{\omega_p}{\lambda} \). The atomic field operator for the creation of an atom in the ground state with mass \( m \) at position \( x \) is given by \( \hat{\Psi}(x) \). As seen in equation (1), the atom-cavity photon interaction provides a dynamic and quantized potential lattice \( U_0 \cos^2(kx) \hat{a}^\dagger \hat{a} \) for the atoms. Also, \( U_0 = \frac{\gamma P_p}{\sqrt{\hbar \omega_p}} \) denotes the potential depth for a single photon with the atom-photon coupling constant \( g_0 \). Cavity frequency is detuned from the
pump laser field by $\Delta = \omega_c - \omega_p$. Moreover, $V_{\text{ext}}(x)$ and $\hat{H}_{A-A}$ are the external trapping potential and atom–atom interaction, respectively. Applying the discrete-mode approximation [35], we can expand the atomic field operator as

$$\hat{\Psi}(x) = \varphi_0 \hat{a}_0 + \varphi_2 \hat{a}_2.$$  \hfill (2)

Where $\varphi_0 = 1$ and $\varphi_2 = \sqrt{2} \cos(2k x)$ are two spatial modes and $\hat{a}_0$ and $\hat{a}_2$ denote the corresponding annihilation operators. For weakly interacting atoms in a shallow external trapping potential, after applying the Bogoliubov approximation and in a rotating frame, the Hamiltonian describing the coupled BEC-cavity reduces to

$$\hat{H} = 4k \omega_{\text{rec}} \hat{\xi}_2 \ddot{x}_2 + \hbar \Delta \hat{a}^\dagger \hat{a} + \hbar g (\hat{c}_2^\dagger + \hat{c}_2) \hat{a} \ddot{a} + \hbar E_p (\hat{a}^\dagger - \hat{a}).$$  \hfill (3)

Here, $4k \omega_{\text{rec}} = \hbar^2 k^2$ is the recoil energy. Also, $\Delta_c = \Delta + \frac{\hbar \omega_{\text{rec}}}{2a}$ and $g = \frac{\omega}{2 \sqrt{N} \sqrt{3}}$ stand for the shifted cavity-pump detuning and the effective coupling strength between the cavity and the BEC, respectively. Moreover, the last term in the Hamiltonian, describes the classical pump light input.

From equation (3), Langevin equations for the optomechanical system are given by

$$\frac{d}{dt} \hat{a}(t) = (-i \Delta_c - \frac{k}{2}) \hat{a}(t) - i g \sqrt{2} \hat{X}(t) \hat{a}(t) + E_p, \quad (4)$$

$$\frac{d}{dt} \hat{X}(t) = 4 \omega_{\text{rec}} \hat{P}(t), \quad (5)$$

$$\frac{d}{dt} \hat{P}(t) = -4 \omega_{\text{rec}} \hat{X}(t) - \gamma \hat{P}(t) - \sqrt{2} g \hat{a} \hat{a}^\dagger \hat{a}(t). \quad (6)$$

We have introduced the position and momentum operators of the BEC, $\hat{X} = \frac{\hat{c}_1^\dagger + \hat{c}_2^\dagger}{\sqrt{2}}$ and $\hat{P} = \frac{\hat{c}_2^\dagger - \hat{c}_1^\dagger}{i \sqrt{2} a}$ as well as the damping rate of the atomic excited state $\gamma_{\text{in}}$.

Assuming $\hat{a}(t) = \hat{a} + \delta \hat{a}(t)$, $\hat{X}(t) = \bar{x} + \delta \hat{X}(t)$ and $\hat{P}(t) = \bar{P} + \delta \hat{P}(t)$, we first find the self-consistent steady state solutions

$$\bar{a} = \frac{E_p}{\sqrt{2} \bar{x} + \frac{\kappa}{2} + i g \sqrt{2}}, \quad (7)$$

$$\bar{x} = \frac{-g \sqrt{2} |\bar{a}|^2}{4 \omega_{\text{rec}}}, \quad (8)$$

here $\bar{a}$ and $\bar{x}$ stand for the interactivity field and the mechanical position. Moreover, $\delta \hat{a}$ and $\delta \hat{X}$ are the fluctuations around these steady states. Combining equations (7) and (8), leads

$$n = \frac{E_p^2}{\left(\frac{\kappa}{2}\right)^2 + \left(\Delta_c - \frac{g^2}{2 \omega_{\text{rec}}} n\right)^2}, \quad (9)$$

where $n = |\bar{a}|^2$ is the number of photons. Rearranging equation (9), the resulting equation is

$$n \left(\frac{\kappa}{2}\right)^2 + \left(\Delta_c - \frac{g^2}{2 \omega_{\text{rec}}} n\right)^2 = E_p^2. \quad (10)$$

Setting equation (10) to zero, gives a third-order polynomial equation of the form

$$n^3 + a_2 n^2 + a_1 n + a_0 = 0, \quad (11)$$

where

$$b = \frac{g^4}{4 \omega_{\text{rec}}^2}, \quad a_2 = -\frac{4 \Delta \omega_{\text{rec}}}{g^2}, \quad a_1 = \frac{\kappa^2 + \Delta_c^2}{2 b}, \quad a_0 = -\frac{E_p^2}{b} . \quad (12)$$

By noting the fact that the steady state photon numbers exhibit OB for a certain range of cavity-pump detuning by increasing the amplitude of the pump field, in the following we are going to calculate the critical values of cavity-pump detuning and pump amplitude. So the system first becomes bistable at a single value of the detuning, denoted by $\Delta_{\text{cr}}$ and the critical pump amplitude, at which the bistability at $\Delta_{\text{cr}}$ occurs, is denoted by $E_{\text{cr}}$. In order to calculate the critical values, let us have a look at equation (11). The critical points of the third-order polynomial equation are obtained by setting derivative of the equation equal to zero and are given by $n = -\frac{a_1 + \sqrt{a_1^2 - 3a_2}}{3}$. In the case where $\sqrt{a_1^2 - 3a_2}$ is positive, the function has a local minimum and a local maximum. Equating $\sqrt{a_1^2 - 3a_2}$ to zero so that the function has only a critical point, the critical cavity-pump detuning is given by

$$\Delta_{\text{cr}} = \sqrt[3]{-\frac{2}{k}}, \quad (13)$$

and we define the critical pump amplitude, at which the bistability at $\Delta_{\text{cr}}$ occurs, as

$$E_{\text{cr}} = \sqrt{\frac{4 \kappa^2 \omega_{\text{rec}}}{6 \sqrt{3} g^2}}. \quad (14)$$

In the following, by using the ansatz and retaining only first order terms in the small quantities $\delta \hat{a}$, $\delta \hat{X}$, $\delta \hat{a}^\dagger$ and $\delta \hat{P}$, one gets the linearized quantum Langevin equations for the fluctuation operators

$$\frac{d}{dt} \delta \hat{a}(t) = (-i \Delta_c - \frac{k}{2}) \delta \hat{a}(t) - i g \sqrt{2} \delta \hat{X}(t), \quad (15)$$

$$\frac{d}{dt} \delta \hat{X}(t) = \omega_{\text{in}} \delta \hat{P}(t), \quad (16)$$

$$\frac{d}{dt} \delta \hat{P}(t) = -\omega_{\text{in}} \delta \hat{X}(t) - \gamma \delta \hat{P}(t) - \sqrt{2} g (\hat{a} \delta \hat{a}^\dagger(t) + \hat{a}^\dagger \delta \hat{a}(t)). \quad (17)$$

where $\Delta_c = \Delta + \sqrt{2} \bar{x}$ and $\omega_{\text{in}} = 4 \omega_{\text{rec}}$.

3. Results and discussion

In the beginning of this section, we present the results based on the numerical solutions of equation (9) and focus on the conditions for bistability. In figure 2, we show how the photon numbers depend on the shifted cavity-pump detuning for given
amplitudes of the pump $E_p$. Experimentally used parameters are $\kappa = 2\pi \times 1.3$ MHz, $N = 1.2 \times 10^5$, $\Delta_0 = 2\pi \times 32$ GHz, $g_0 = 2\pi \times 10.9$ MHz and $\omega_m = 2\pi \times 15.2$ KHz [12]. It can be seen from equation (9) that when $E_p$ is sufficiently small, the photon numbers are also small, so we can neglect $n^2$ and we have a symmetrical lorentzian curve centred at $\Delta_c = 0$. In figure 2(b), it is shown that by increasing the amplitude of the pump field, the lorentzian curve is largely unchanged; however, the curve becomes more and more asymmetric and its only maximum moves to the right. But as $E_p$ reaches a critical value, which is obtained from equation (14), the nature of the curve changes so that it has an infinite slope at $\Delta = \Delta_{cr}$, according to figure 2(c). For amplitudes beyond the critical value, equation (9) has three real roots, corresponding to the branch CD in figure 2(d) where the multiple solutions indicate the bistable behaviour. The bistable behaviour can also be seen from the OB hysteresis curve, shown in figure 3. As figure 3(a) shows, the numerical result for critical pump amplitude presents a good agreement with the analytical one, equation (14). For the detuning above the critical value by increasing the amplitude of the pump field system goes to the bistability branch, according to figure 3(b). Consider that amplitude increases from zero gradually; the photon numbers initially follow the lower stable branch. When the amplitude increases to the first bistable point, it jumps to the upper stable branch and if the amplitude is increased further, continues to follow that branch. Now if we start decreasing the amplitude, the steady state photon numbers remain on the upper stable

Figure 2. Steady state photon numbers as a function of the shifted cavity-pump detuning for the four different amplitude of the pump field, $E_p = 0$ (a), $E_p = 0.5$ MHz (b), $E_p = 0.783$ MHz (c) and $E_p = 1.5$ MHz (d). Other used parameters are $\kappa = 2\pi \times 1.3$ KHz, $N = 1.2 \times 10^5$, $\Delta_0 = 2\pi \times 32$ GHz, $g_0 = 2\pi \times 10.9$ MHz and $\omega_m = 2\pi \times 15.2$ KHz. Noting that the critical values of cavity-pump detuning ($\Delta_{cr}$) and pump amplitude ($E_{cr}$) are obtained from equations (13) and (14), respectively.

Figure 3. S-shaped curve for the steady state photon numbers as a function of the amplitude of the pump for $\Delta_c = 7$ MHz (a) and $\Delta_c = 11$ MHz (b). Other parameters are the same as in figure 2.
branch at first; however, by decreasing the amplitude even further, when it reaches the second bistable point, it jumps down to the lower stable branch and continues to decrease along that branch for further decrease of the amplitude. We note that placing the BEC inside an optical cavity allows for OB at extremely low cavity photon numbers; however, an empty optomechanical system does not exhibit the behaviour.

First, we are going to examine the effects of the decay rate of the cavity photons on the OB. Figure 4 depicts steady state photon numbers as a function of amplitude of the pump for various decay rates. Here, we can observe that by increasing the decay rate, width of the bistability curve decreases. In addition, the OB threshold increase by increasing the value of the decay rate. By further increasing the decay rate, the OB may be eliminated. In order to clarify the behaviour of the OB curve with variation of the decay rate, we present the figure 5 considering the same parameters used for figure 3(b).

In what follows, we are mainly focused on controllability of the OB. we will investigate the effects of the coupling strength between the cavity and the BEC on the OB threshold and the width of the bistability curve. In reference [13], the authors have shown the possibility of controlling the coupling strength by locating the BEC anywhere within the cavity, so we can introduce the coupling strength as a control parameter. Steady state photon numbers as a function of the amplitude of the pump and for different coupling strengths are plotted in figure 6. Other parameters are the same as in figure 3(b). As is seen, increasing the coupling strength, i.e. \( g_0 \), causes the system to become bistable at lower amplitudes and also decreases the width of the bistability curve. Therefore, this control parameter can provide the possibility of realizing a controllable optical switch. Moreover, the coupling strength allows for the OB at extremely low photon numbers as well as fast and low-threshold optical switches.

Then we show that optical behaviour is sensitive to the detuning between the pump laser field and the atomic resonance. This means that the detuning may act as another control parameter. Figure 7 depicts steady state photon numbers as a function of the amplitude of the pump for \( \Delta_1 = 2\pi \times 30 \text{ GHz} \) (solid line), \( \Delta_2 = 2\pi \times 40 \text{ GHz} \) (dashed line) and \( \Delta_3 = 2\pi \times 50 \text{ GHz} \) (dotted line). Other parameters are the same as those in figure 3(b).

As the OB threshold intensity is a function of the corresponding parameters, we minimize it over the range of experimental data [12, 13] and find the minimum threshold intensity
of the OB, $E_{\text{res}} = 1$ MHz and the corresponding laser power $P_{\text{res}} = 3 \times 10^{-14}$ W.

In the following, we are going to study the stability of the different branches of the OB curve. Applying the Routh–Hurwitz criterion to the equations (15)–(17), we can get the stability conditions

$$s_1 = \kappa_m \left\{ \left[ \frac{\kappa^2}{4} + (\omega_m - \Delta_c)^2 \right] \left[ \frac{\kappa^2}{4} + (\omega_m + \Delta_c)^2 \right] + \gamma_m \left[ (\gamma_m + \kappa) \left( \frac{\kappa^2}{4} + \Delta_c^2 \right) + \kappa \omega_m^2 \right] \right\}$$

$$+ 4 g^2 n \omega_m \Delta_c (\gamma_m + \kappa)^2 > 0,$$

$$s_2 = \omega_m \left( \frac{\kappa^2}{4} + \Delta_c^2 \right) - 4 g^2 n \Delta_c > 0.$$  

For the positive detuning, $\Delta_c > 0$, the first condition is always satisfied. In figure 8, we see that for the middle branch, $s_2$ is negative and so the steady states on the region are unstable. It is worth mentioning that the condition is consistent with the fact that the branches with negative slope are always unstable.

In what follows, we will try to obtain the physical interpretation of the condition. Note that experimentally [12, 13], the decay rate of the cavity photons is often two orders of magnitude faster than the mechanical motion of the condensate, so we safely assume that the cavity field follows the condensate adiabatically. Setting $\frac{d}{dt} \delta a = 0$ in equation (15), one gets:

$$\delta a(t) = -\frac{ig \sqrt{2}}{i \Delta_c + \frac{\omega_m}{2}} \delta X(t),$$

$$\delta a^*(t) = -\frac{ig \sqrt{2}}{-i \Delta_c + \frac{\omega_m}{2}} \delta X(t),$$

$$\frac{d}{dt} \delta \dot{P}(t) = -(\omega_m - \frac{4 g^2 n \Delta_c}{\Delta_c^2 + \omega_m^2}) \delta \ddot{X}(t) - \gamma_m \delta \dot{P}(t),$$

$$\frac{d^2}{dt^2} \delta \ddot{X}(t) + \gamma_m \frac{d}{dt} \delta \dot{X}(t) + (\omega_m - \frac{4 g^2 n \omega_m \Delta_c}{\Delta_c^2 + \omega_m^2}) \delta \dot{X}(t) = 0.$$  

It can be seen that the coefficient of $\delta \ddot{X}$ in third term in equation (22), corresponds to a restoring force. On the other hand, from the second condition, equation (19), we can see that the mechanical oscillator is stable if the restoring force is positive. In the middle branch, the third coefficient in the equation (22) is negative and so it is no longer a restoring force leading to the unstable behaviour. The phenomenon can be also viewed as optical spring effect [36, 38]. It is worth to mention that except here, we do not make an adiabatic approximation.

Considering that OB can be well understood in the framework of the simple double-well potential model, so we proceed to obtaining the effective potential. Putting the interactivity field into equations (5) and (6), we get the evolution of the displacement for the BEC

$$\frac{d^2}{dt^2} X(t) = -\omega_m^2 X(t) - \sqrt{2} g \omega_m n,$$  

and the potential is given by

$$V(X) = \frac{1}{2} \omega_m^2 X^2 + \frac{\omega_m E_p^2}{k/2} \arctan \left( \frac{\Delta_c + \Delta_e X}{k/2} \right).$$

As is seen, we have a sum of the harmonic potential and inverse tangent terms. The latter may lead to two minima and the double-well like potential.

Next, we investigate the dynamic properties by the effective potential which is plotted in figure 9 as a function of the displacement and for different pump amplitudes. The potential with $E_p = 200$ KHz is shown as the solid line. It gets the form of the harmonic potential and has a single minimum close to the origin, corresponding to the stable lower branch. The dashed line corresponds to the case $E_p = 1.1$ MHz which lies in the OB region. So, the potential is a double-well with three extrema, two minimum and one maximum. The first minimum located in the vicinity of the origin, corresponding to the lower branch of OB and the other one, is far away from the origin and corresponds to the upper branch. The maximum also represents the unstable branch. For $E_p = 2$ MHz, the inverse tangent term gives just a shift to the potential and so it has a new minimum at a larger displacement corresponding to the purely upper stable branch.
The values are $E_\phi = 200$ KHz (solid line), $E_\phi = 1.1$ MHz (dashed line) and $E_\phi = 2$ MHz (dotted line). The top and the bottom curve have only one minimum, while the middle one has two minimum and one maximum. The inset shows a zoom-in of the middle curve. Parameters values are the same as in figure 3(b).

4. Conclusion

We investigated the OB in a system consisting of a BEC inside an optical cavity. It was first shown that how the bistability develops in this cavity optomechanical system for a certain range of cavity-pump detunings and pump amplitudes. Further, the effects of the decay rate of the cavity photons and coupling strength between the cavity and the BEC as well as the pump-atom detuning on the optical behaviour of the system were investigated and it was demonstrated that one can control the OB threshold and the width of the bistability curve by means of the coupling strength and the pump-atom detuning. To the best of our knowledge, our work is the first of its kind to investigate the effects of the system parameters on the optical behaviour in a two-mode optomechanical system with a BEC. Another additional advantage is that extremely low cavity photon numbers exhibit bistable behaviour which is absent for previous systems. Also, the system parameters allow for a considerable reduction of the threshold and this phenomenon has potential applications in optical switches at very low intensities. Moreover, width of the bistability curve may be altered by changing the parameters allowing all-optical flip-flop and storage of optical pulse signals. We then analyzed the stability of the different branches of the OB curve by applying the Routh–Hurwitz criterion as well as offering a physical interpretation for these conditions. Finally, we derived an effective potential in order to have an intuitive picture.

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