DOES THE LIGHT AND BROAD $\sigma(500)$ EXIST?

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ABSTRACT

The lightest scalar and pseudoscalar nonets are discussed within the framework of the broken $U_3 \times U_3$ linear sigma model, and it is shown that already at the tree level this model works remarkably well predicting scalar masses and couplings not far from present experimental values, when all parameters are fixed from the pseudoscalar masses and decay constants. The linear $\sigma$ model is the simplest way to implement chiral symmetry together with the broken SU3 of the quark model, and its success in understanding experiment is comparable to that of the naive quark model for the heavier multiplets. It is argued that this strongly suggests that the light and very broad $\sigma$ resonance exists near 500 MeV.

1 Introduction

In this talk\footnote{Invited talk at the Workshop on Hadron Spectroscopy, March 8-13, 1999 Frascati, Italy. To appear in the Frascati Physics Series.} I shall discuss mainly the light and broad $\sigma$, which was picked by Matts Roos and myself\footnote{Invited talk at the Workshop on Hadron Spectroscopy, March 8-13, 1999 Frascati, Italy. To appear in the Frascati Physics Series.} from the particle data group wastebasket 4 years ago, having been there for over 20 years. Today an increasing number of papers, many of which have been reported at this meeting\footnote{Invited talk at the Workshop on Hadron Spectroscopy, March 8-13, 1999 Frascati, Italy. To appear in the Frascati Physics Series.}, are quoting its parameters, with a pole position near 500-i250 MeV (See the table 1).
Table 1: \( \sigma \) pole position.

| Reference               | pole position (MeV)                 |
|-------------------------|------------------------------------|
| Kaminski et al.         | \( 532 \pm 12 - i(259 \pm 7) \)    |
| Locher et al.           | \( 424 - i213 \)                   |
| Harada et al.           | \( \approx 500 - i250 \)           |
| Lucio et al.            | \( \approx 400 - i200 \)           |
| Ishida et al.           | \( 602 \pm 26 - i(196 \pm 27) \)   |
| Kaminski et al.         | \( 537 \pm 20 - i(250 \pm 17) \)   |
| Oller et al.            | \( 469.5 - i178.6 \)               |
| Törnqvist et al.        | \( 470 - i250 \)                   |
| Amsler et al.           | \( 1100 - i300 \)                  |
| Amsler et al.           | \( 400 - i500 \)                   |
| Janssen et al.          | \( 387 - i305 \)                   |
| Achasov et al.          | \( 525 - i269 \)                   |
| Zou et al.              | \( 370 - i356 \)                   |
| Zou et al.              | \( 408 - i342 \)                   |
| Au et al.               | \( 870 - i370 \)                   |
| Beveren et al.          | \( 470 - i208 \)                   |
| Estabrooks              | \( 750 \pm 50 - i(450 \pm 50) \)   |
| Protopescu et al.       | \( 660 \pm 100 - i(320 \pm 70) \)  |
| Basdevant et al.        | \( 650 - i370 \)                   |
| Scadron et al.          | \( \approx 500 - i250 \)           |
| Lucio et al.            | \( 600^{+200}_{-100} - i350 \)     |
| Igi et al.              | \( \approx 760 \)                  |

An important question today is: Does this broad resonance really exist? And if so, what is its nature, together with the other light scalar mesons in the 1 GeV region? The naive quark model (NQM), which works reasonably well for the vectors and heavier multiplets, definitely fails for the scalars taken as \( \sigma(500), f_0(980), a_0(980) \) and \( K_0^*(1420) \).

We all believe the vectors \( (\rho, \omega, \phi, K^*) \) and heavier well established multiplets are \( q\bar{q} \) states because with a few parameters, such as an equally spaced bare mass spectrum, a small OZI rule violating parameter and SU3\(_f\) related couplings, we can describe the masses, widths and couplings of the whole nonet. If we had data only on the \( \rho(770) \) we could not conclude that it is \( q\bar{q} \). But with the successful SU3\(_f\)
predictions for the whole nonet we strongly believe it is $q\bar{q}$.

The same is true for the $\sigma$. No single analysis of the $\pi\pi$ S-wave, however refined, could ever decide on what is the nature of the $\sigma$. Even the decision as to whether it really exists, cannot be done using data on the $\pi\pi$ S-wave alone, since there are inherent, model dependent, ambiguities as to how to continue analytically to a pole which is far from the physical region, as is the case for the broad $\sigma$.

It is also obvious why the NQM fails for the scalars: Chiral symmetry is absent in the NQM, but is crucial for the scalars. Chiral symmetry is believed to be broken in the vacuum, and two of the scalars ($\sigma$ and $f_0$) have the same quantum numbers as the vacuum. Thus to understand the scalar nonet in the same way as we believe we understand the vectors, and to make a sensible comparison with experiment, one must include chiral symmetry in addition to flavour symmetry in the quark model. The simplest such chiral quark model is the linear $U_3 \times U_3$ sigma model with 3 flavours.

Then we can treat both the scalar and pseudoscalar nonets simultaneously, and on the same footing, getting automatically small masses for the pseudoscalar octet, and symmetry breaking through the vacuum expectation values (VEV’s) of the scalar fields.

As an extra bonus we have in principle a renormalizable theory, i.e. “unitarity corrections” are calculable. In fact, in the flavour symmetric limit the unitarity corrections can be thought to be already included into the mass parameters of the theory, once the original 4-5 parameters are replaced by the 4 physical masses for the singlet and octet $0^{-+}$ and $0^{++}$ masses and the $\sigma$ VEV.

Unfortunately this over 30 years old model \cite{7} has had very few phenomenological applications. An important exception is the intensive efforts of M. Scadron and collaborators.

## 2 The Linear sigma model with 3 flavours

The well known linear sigma model \cite{7} generalized to 3 flavours with complete scalar ($s_a$) and pseudoscalar ($p_a$) nonets has at the tree-level the Lagrangian the same flavour and chiral symmetries as massless QCD. The $U_3 \times U_3$ Lagrangian with a symmetry breaking term $L_{SB}$ is

$$L = \frac{1}{2} \text{Tr}[\partial_\mu \Sigma \partial_\mu \Sigma^\dagger] - \frac{1}{2} \mu^2 \text{Tr}[\Sigma \Sigma^\dagger] - \lambda \text{Tr}[\Sigma \Sigma^\dagger \Sigma \Sigma^\dagger] - \lambda' (\text{Tr}[\Sigma \Sigma^\dagger])^2 + L_{SB}. \quad (1)$$

Here $\Sigma$ is a $3 \times 3$ complex matrix, $\Sigma = S + iP = \sum_{a=0}^{8} (s_a + ip_a) \lambda_a / \sqrt{2}$, in which $\lambda_a$ are the Gell-Mann matrices, normalized as $\text{Tr}[\lambda_a \lambda_b] = 2 \delta_{ab}$, and where for
the singlet \( \lambda_0 = (2/N_f)^{1/2} \mathbf{1} \) is included. Each meson in Eq. (1) has a definite SU3\(_f\) symmetry content, which in the quark model means that it has the same flavour structure as a \( q\bar{q} \) meson. Thus the fields \( s_a \) and \( p_a \) and potential terms in Eq. (1) can be given a conventional quark line structure \( \otimes \).

The symmetry breaking terms are most simply:

\[
L_{SB} = \epsilon_\sigma \sigma_{u\bar{u}+d\bar{d}} + \epsilon_{s\bar{s}} \sigma_{s\bar{s}} + c[\det \Sigma + \det \Sigma^\dagger],
\]

which give the pseudoscalars mass and break the flavour and \( U_A(1) \) symmetries. The small parameters \( \epsilon_i \) can be expressed in terms of the pion and kaon decay constants and masses:

\[
\epsilon_\sigma = m_\pi^2 f_\pi, \quad \epsilon_{s\bar{s}} = (2m_K^2 f_K - m_\pi^2 f_\pi)/\sqrt{2}.
\]

My fit to the scalars with the unitarized quark model (UQM) \( \otimes \) is essentially a unitarization of eq.\( \otimes \) with \( \lambda \approx 16 \) and \( \lambda' = 0 \), and with the main symmetry breaking generated by putting the pseudoscalar masses at their physical values. The model was used as an effective theory with a symmetric smooth 3-momentum cutoff 0.54 GeV/c given by a gaussian form factor. Such a form factor is natural, since physical mesons are of course not pointlike, but have finite size of 0.7-0.8 fm. The fit included the Adler zeroes which follow from eq.1, but only approximate crossing symmetry.

Eq. (1) without \( L_{SB} \) is clearly invariant under \( \Sigma \rightarrow U_L \Sigma U_R^\dagger \) of U3\( \times \)U3. After shifting the flavourless scalar fields by the VEV’s (\( \Sigma \rightarrow \Sigma + V \)) to the minimum of the potential, the scalars aquire masses and also the pseudoscalars obtain a (small) mass because of \( L_{SB} \). Then the \( \lambda \) and \( \lambda' \) terms generate trilinear \( spp \) and \( sss \) couplings, in addition to those coming from the \( U_A(1) \) symmetry breaking determinant term. The \( \lambda \) term, which turns out to be the largest, obeys the OZI rule, while the small \( \lambda' \) term, and \( c \) violate this rule.

3 Tree-level masses and couplings.

It is an ideal problem for a symbolic program like Maple V to calculate the predicted masses, and couplings from the Lagrangian, which has 6 parameters \( \mu, \lambda, \lambda', c \) and the VEV parameters \( u = d \) and \( s \), which define a diagonal matrix for the flavourless meson VEV’s: \( V = \text{diag}[u, d, s] \). These are at the tree level related to the pion and kaon decay constants through \( u = d = \langle \sigma_{u\bar{u}+d\bar{d}} \rangle / \sqrt{2} = f_\pi / \sqrt{2} \) (assuming isospin exact) and \( s = \langle \sigma_{s\bar{s}} \rangle = (2f_K - f_\pi)/\sqrt{2} \): One finds denoting the often occurring combination \( \mu^2 + 4\lambda'(u^2 + d^2 + s^2) \) by \( \tilde{\mu}^2 \), and expressing the flavourless mass matrices in the ideally mixed frame:
Table 2: Predicted masses in MeV and mixing angles for two values of the $\lambda'$ parameter. The asterix means that $m_\pi, m_K$ and $m_\eta^2 + m_\eta'^2$ are fixed by experiment together with $f_\pi$ and $f_K$.

| Quantity     | Model $\lambda' = 1$ | Model $\lambda' = 3.75$ | Experiment |
|--------------|----------------------|--------------------------|------------|
| $m_\pi$      | 137$^*$              | 137$^*$                  | 137        |
| $m_K$        | 495$^*$              | 495$^*$                  | 495        |
| $m_\eta$     | 538$^*$              | 538$^*$                  | 547.3      |
| $m_\eta'$    | 963$^*$              | 963$^*$                  | 957.8      |
| $\Theta_\eta^{'-\text{singlet}}$ | -5.0$^0$           | -5.0$^0$                 | (-16.5$\pm$6.5)$^0$ |
| $m_{a0}$     | 1028                 | 1028                     | 983        |
| $m_\kappa$   | 1123                 | 1123                     | 1430       |
| $m_\sigma$   | 651                  | 619                      | 400-1200   |
| $m_{f_0}$    | 1229                 | 1188                     | 980        |
| $\Theta_\sigma^{'-\text{singlet}}$ | 21.9$^0$           | 32.3$^0$                 | (28-18.5)$^0$ |

$$
\begin{align*}
  m_{2+}^2 & = \bar{\mu}^2 + 4\lambda(u^2 + d^2 - ud) + 2cs \\
  m_{K^+}^2 & = \bar{\mu}^2 + 4\lambda(u^2 + s^2 - su) + 2cd \\
  m_{\eta}^2 & = \text{diag} \left( \begin{array}{cc} \bar{\mu}^2 + 2\lambda(u^2 + d^2) - 2cs & -c\sqrt{2}(u + d) \\ -c\sqrt{2}(u + d) & \bar{\mu}^2 + 4\bar{s}^2 \end{array} \right) \\
  m_{\eta'}^2 & = \bar{\mu}^2 + 4\lambda(u^2 + d^2 + ud) - 2cs \\
  m_{20}^2 & = \bar{\mu}^2 + 4\lambda(u^2 + s^2 + su) - 2cd \\
  m_\sigma^2 & = \text{diag} \left( \begin{array}{cc} \bar{\mu}^2 + 4\lambda'(u + d)^2 + 6\lambda(u^2 + d^2) + 2cs & (4\lambda's + c)\sqrt{2}(u + d) \\ (4\lambda's + c)\sqrt{2}(u + d) & \bar{\mu}^2 + 8\lambda's^2 + 12\lambda s^2 \end{array} \right) \\
  m_{f_0}^2 & = \text{diag} \left( \begin{array}{cc} \bar{\mu}^2 + 4\lambda'(u + d)^2 + 6\lambda(u^2 + d^2) + 2cs & (4\lambda's + c)\sqrt{2}(u + d) \\ (4\lambda's + c)\sqrt{2}(u + d) & \bar{\mu}^2 + 8\lambda's^2 + 12\lambda s^2 \end{array} \right)
\end{align*}
$$

Now we fix 5 of the 6 parameters except $\lambda'$ by the 5 experimental quantities $m_\pi, m_K, m_\eta^2 + m_\eta'^2, f_\pi = 92.42$ MeV and $f_K = 113$ MeV $^3$. One finds at the tree level $\lambda = 11.57$, $\bar{\mu}^2 = 0.1424$ GeV$^2$, $c = -1701$ MeV, $u = d = 65.35$ MeV and $s = 94.45$ MeV. The remaining $\lambda'$ parameter affects only the $\sigma$ and $f_0$ masses and their trilinear couplings. This dependence is rather weak for the masses, but is very sensitive to the couplings as it changes the ideal mixing angle for the scalars. It turns out below that this must be small to fit the tri-linear couplings. By putting $\lambda' = 1$ one gets a reasonable compromise for most of these couplings. With $\lambda' \approx 3.75$ one almost cancels the OZI rule breaking coming from the determinant term, and the scalar mixing becomes near ideal (for $\lambda' = -c/(4s) = 4.5$ the cancellation is exact).
As can be seen from Table 2 the predictions are not far from the experimental values. Considering that one expects from our previous analysis\cite{1} that unitarity corrections can easily be more than 20%\footnote{\cite{1}}, and should go in the right direction, one must conclude that these results are even better than expected.

The trilinear coupling constants follow from the Lagrangian, and are at the tree level:

\[ g_{\sigma\pi\pi} = \cos \phi^d_S(m^2_\sigma - m^2_\pi)/f_\pi \]

\[ g_{\sigma K^+K^-} = -\sqrt{3} \sin(\phi^d_S - 35.26^\circ)(m^2_\sigma - m^2_K)/(2f_K) \]

\[ g_{f_0\pi\pi} = \sin \phi^d_S(m^2_{f_0} - m^2_\pi)/f_\pi \]

\[ g_{f_0K^+K^-} = \sqrt{3} \cos(\phi^d_S - 35.26^\circ)(m^2_{f_0} - m^2_K)/(2f_K) \]

\[ g_{a_0\pi\eta} = \cos \phi^d_P(m^2_{a_0} - m^2_\eta)/f_\pi \]

\[ g_{a_0\pi\eta'} = \sin \phi^d_P(m^2_{a_0} - m^2_\eta)/f_\pi \]

\[ g_{a_0K^+K^-} = (m^2_{a_0} - m^2_K)/f_K \]

\[ g_{\kappa\eta\bar{\pi}^0} = (m^2_\kappa - m^2_\pi)/(\sqrt{2}f_K) \]

\[ g_{\kappa K^+\eta} = -\sqrt{3} \sin(\phi^d_P - 35.26^\circ)(m^2_\kappa - m^2_\eta)/(2f_K) \]

\[ g_{\kappa K^+\eta'} = \sqrt{3} \cos(\phi^d_P - 35.26^\circ)(m^2_\kappa - m^2_\eta)/(2f_K) \].

Here \( \phi^d_P = 54.73^\circ + \Theta^\eta'_{\text{singlet}} \) i.e. the angle between \( \bar{s}s \) and \( \eta' \) and \( \phi^d_S = -35.26^\circ + \Theta^\sigma_{\text{singlet}} \), i.e. the angle between \( \bar{u}u + \bar{d}d \) and \( \sigma \).

In some of the channels of table 3 the resonance is below threshold and the widths vanish at the resonance mass. However, the coupling constants have then been determined through a loop diagram from \( \phi \to \gamma \pi \pi \) and \( \phi \to \gamma \pi \eta \) (albeit in a somewhat model dependent way) by the Novosibirsk group. For channels where the phase space is large, it is important that one includes a form factor related to the finite size of physical mesons. In the quark pair creation \((^3P_0)\) model a radius of 0.8 fm leads to a gaussian form factor, as in the formula below, where \( k_0 \approx 0.56 \text{ GeV/c} \) (as was found in the UQM\cite{1}). Thus the widths are computed from the formula:

\[ \Gamma(m) = \frac{1}{8\pi} \sum_{\text{isospin}} \frac{g_i^2}{m^2} \frac{k_{cm}(m)}{e^{-|k_{cm}(m)/k_0|^2}}. \]

As can be seen from table 3 most of the couplings are not far from experiment. The main exception is the \( f_0 \to \pi \pi \) coupling and width, but this is extremely sensitive to the ideal mixing angle. If one choses \( \lambda' = 3.75 \) this mixing angle nearly vanishes \( (\phi^d_S = -3.0^\circ) \) together with the \( f_0 \to \pi \pi \) coupling (c.f. eq.4). From our experience with the UQM\cite{1} the couplings, when unitarized, are very sensitive to especially the nearby \( K\bar{K} \) threshold. Similarly the \( a_0 \to \pi \eta \) peak width is reduced,
Table 3: Predicted couplings $\sum_i \frac{g_i^2}{4\pi}$ (in GeV²), when $\lambda' = 1$, compared with experiment and predicted widths with experiment (in MeV). (We have used isospin invariance to get the sum over charge channels, when data is for one channel only.) The predicted $f_0 \to \pi\pi$ width is extremely sensitive to the value of $\lambda'$ (for $\lambda' = 3.75$ it nearly vanishes) and unitarity effects as discussed in the text.

| Process | $\sum_i \frac{g_i^2}{4\pi}$ in model | $\sum_i \frac{g_i^2}{4\pi}$ in experiment | $\sum_i \Gamma_i$ in model | $\sum_i \Gamma_i$ in experiment |
|---------|--------------------------------------|----------------------------------------|--------------------------|--------------------------------|
| $\kappa^+ \to K^0\pi^+ + K^+\pi^0$ | 7.22 | - | 678 | 278 $\pm$ 23 [4] |
| $\kappa^+ \to K^+\eta$ | 0.28 | $\approx 0$ | 0 | 0 |
| $\sigma \to \pi^+\pi^- + \pi^0\pi^0$ | 2.17 | 1.95 [4] | 574 | 0 |
| $\sigma \to K^+K^- + K^0\bar{K}^0$ | 0.16 | 0.004 [4] | 0 | 0 |
| $f_0 \to \pi^+\pi^- + \pi^0\pi^0$ | 1.67 | 0.765$^{+0.20}_{-0.14}$ [10] | see text | 40 - 100 [3] |
| $f_0 \to K^+K^- + K^0\bar{K}^0$ | 6.54 | 4.26$^{+1.78}_{-1.12}$ [10] | 0 | 0 |
| $a_0^+ \to \pi^+\eta$ | 2.29 | 0.57 [10] | 273 see text | 50 - 100 [3] |
| $a_0^+ \to K^+\bar{K}^0$ | 2.05 | 1.34$^{+0.36}_{-0.28}$ [10] | 0 | 0 |

because of the $K\bar{K}$ threshold, by up to a factor 5. Therefore one cannot expect that the tree level couplings should agree better with data than what those of Table 3 do. In fact, I was myself astonished by the fact that the agreement turned out to be this good. After all, this is a very strong coupling model ($\lambda = 11.57$, leading to large $g_2^2/4\pi$) and higher order effects should be important.

4 Conclusions.

In summary, I find that the linear sigma model with three flavours works, at the tree level, much better than expected. It works, in my opinion, just as well as the naive quark model works for the heavier nonets. One should of course include higher order effects, i.e., the model should be unitarized phenomenologically, e.g., along the lines of the UQM [3], whereby a more detailed data comparison becomes meaningful.

Those working on chiral perturbation theory and nonlinear sigma models usually point out that the linear model does not predict all low energy constants correctly. However, one should remember that the energy regions of validity are different for the two approaches. Chiral perturbation theory usually breaks down when one approaches the first scalar resonance. The linear sigma model, on the other hand, includes the scalars from the start and can be a reasonable interpolating model in the intermediate energy region near 1 GeV, where QCD is too difficult to solve.
These results strongly favour the interpretation that the $\sigma(500)$, $a_0(980)$, $f_0(980)$, $K^*_0(1430)$ belong to the same nonet, and that they are the chiral partners of the $\pi$, $\eta$, $K$, $\eta'$. If the latter are believed to be unitarized $q\bar{q}$ states, so are the light scalars $\sigma(500)$, $a_0(980)$, $f_0(980)$, $K^*_0(1430)$, and the broad $\sigma(500)$ should be interpreted as an existing resonance.

The $\sigma$ is a very important hadron indeed, as is evident in the sigma model, because this is the boson which gives the constituent quarks most of their mass and thereby it gives also the light hadrons most of their mass. It is the Higgs boson of strong interactions.

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