Nonlinear Dissipative System Mathematical Equations in the Multi-regression Model of Information-based Teaching

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Abstract

The advancement of Chinese education informatisation construction has injected new vitality into the development of Chinese educational technology in the new era and brought new challenges to the development of Chinese educational technology. Nonlinear dissipative structure theory has been a necessary enlightenment for the development of education informatisation. Based on the theory of nonlinear dissipative structure, the paper explores the relationship between the theory and education and teaching. It constructs a diversified regression calculation model of the information-based teaching ecology. Finally, it points out the strategies and ways to apply the dissipative structure theory to improve information-based teaching.

Keywords: Nonlinear dissipative system, mathematical equations, teaching ecology, diversified regression model, education informatisation

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1 Introduction

Big data and cloud computing technologies have realised extensive storage of knowledge and efficient processing of data. The Internet provides an environment for knowledge-sharing and efficient intercommunication. Artificial intelligence technology allows people to discover the commonalities behind things more and better as well as realise the personalisation of services. Information technology has a far-reaching impact on all walks of life, not only as an efficient tool but also as a natural ‘lifestyle’ and ‘work method’ to integrate people in all fields.

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The research on education ecology mainly uses ecological theories and methods to study education and teaching problems. It uses ecological systems and functions, ecological balance, system evolution and other viewpoints and principles to study education laws and teach development [1]. Ecological research methods have the characteristics of highlighting systematicity, openness, synergy and nonlinear influence. Therefore, it can better explain the relationship between elements in education informatisation, the mutual influence between elements and the path of system development. This research uses Prigogine's dissipative system theory to explore the evolutionary laws. The thesis uses this theory to help the ecological deduction and innovative development of education informatisation in the new stage.

2 Theoretical basis

2.1 Dissipative system theory

Dissipative structure theory was first used to study non-equilibrium thermodynamics. Then, non-equilibrium statistical physics is used to reveal the relevant characteristics of complex systems with the in-depth study of natural and social research. An essential point of this theory is the self-organising nature of the system [2]. The entire system may undergo abrupt changes through fluctuations. This new system structure needs to continuously exchange material and energy with the outside world to maintain its orderly structure and, at the same time, have a particular ability to resist external interference. The structure with this property is named ‘dissipative structure’ by Prigogine.

The dissipative stochastic nonlinear Schrödinger equation is as follows: Consider the following one-dimensional dissipative stochastic nonlinear Schrödinger equation with additive noise

\[
\begin{align*}
&du - i(\Delta u + iau + \lambda |u|^2 u)dt = \varepsilon dW = \\
&u(t, 0) = u(t, 1) = 0, \quad t \geq 0, \quad x \in [0, 1] \subset Ru(0, x) = u_0(x)
\end{align*}
\] (1)

Here \(\lambda = \pm 1\), the dissipation factor \(a > 0, \varepsilon\) is the noise scale. The complex-valued Wiener process \(W = W_1 + iW_2\) is defined in the filter probability space \((\Omega, F, \{F_t\}_{t \geq 0}, P)\) and \(W\) has the following Karhunen-Loève expansion

\[
W(t, x) = \sum_{k=0}^{\infty} Qe_k(x)\beta_k(t) = \sum_{k=0}^{\infty} \sqrt{\lambda_k}e_k(x)\beta_k(t), \quad t \geq 0, \quad x \in [0, 1]
\] (2)

Among them \(Q\) is the linear positive definite operator on \(L^2 = L^2(0, 1)\) and \(\Delta\) is commutative with the \(\Delta\) operator. It satisfies that \(Qe_k = \sqrt{\lambda_k}e_k, \{e_k\}_{k \geq 1}\) is the eigenvector of the homogeneous \(\Delta\) operator. \(\beta_k = \beta_k^1 + i\beta_k^2\) and \(\{\beta_k^j\}_{k \geq 1, i=1,2}\) is a family of independent and identically distributed real-valued standard Brownian motions. We modify Equation (1) as follows. Define space-time white noise \(\tilde{\chi} = \frac{dW}{dt}\), set \(u = p + iq, \tilde{\chi} = \chi_1 + i\chi_2\), and \(p, q, \chi_1 = \frac{dW_1}{dt}\) and \(\chi_2 = \frac{dW_2}{dt}\) are real-valued random processes, then Equation (1) can be rewritten as

\[
\begin{align*}
&p_t + q_{sx} + ap + \lambda(p^2 + q^2)q = \varepsilon \chi_1, \\
&q_t + p_{sx} - aq + \lambda(p^2 + q^2)p = \varepsilon \chi_2
\end{align*}
\] (3)

Then using \(v = p_x, w = q_x, z = (p, q, v, w)^T\) the above equation can be transformed into a compact form

\[
Md\tilde{z} + Kd\tilde{z} = -aM\tilde{z}dt + \nabla S_0(\tilde{z})dt + \nabla S_1(\tilde{z}) \circ dW_1 + \nabla S_2(\tilde{z}) \circ dW_2
\] (4)

Among them

\[
M = \begin{pmatrix}
0 & -1 & 0 & 0 \\
1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0
\end{pmatrix}, \quad K = \begin{pmatrix}
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1 \\
-1 & 0 & 0 & 0 \\
0 & -1 & 0 & 0
\end{pmatrix}
\] (5)
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\begin{equation}
S_0(z) = -\frac{\lambda}{4}(p^2 + q^2)^2 - \frac{1}{2}(v^2 + w^2), \quad S_1(z) = \epsilon q, \quad S_2(z) = -\epsilon p
\end{equation}

\( S_i(z) \) means that the equation is established under the meaning of Stratonovich-type stochastic integral.

2.2 Relevant research on the strategy of informatisation teaching ecology construction

Informatisation teaching ecology is defined as the sum of the processes [3]. The research process of information-based teaching ecology and the development process of teaching application under an information technology environment reciprocate to each other. To develop teaching ecology with the help of information technology, researchers study the renewal and iteration of information teaching ecology from the physical environment and humanistic environment of classroom teaching. According to the change of teaching equipment, four stages of the development of the teaching environment are proposed: the first stage represented by blackboard or writing whiteboard, the second stage is the computer combined with projection template, the third stage is the computer combined with interactive electronic whiteboard and the fourth is the computer combined with multiple Tap of the touch screen. Scholars took the star C teaching application to state the transformation of teaching resources, teaching processes and teaching methods under the support of the fourth-generation teaching environment.

3 Self-organised information teaching ecological model

The teaching ecosystem has the dual functions of student development and teacher professional growth. It can be seen from the self-organising development law of informatisation teaching ecology that teachers’ innovation and community development are the keys to the transformation of informatisation ecology [4]. Based on this principle, this paper combines the UGBS concept of the integration of information technology and education and teaching proposed by Yang Zongkai based on the ecological circle structure characteristics. It proposes a dual ecological model of information-based teaching ecology that represents the self-organised form. It is shown in Figure 1.

![Fig. 1 The structure and external environment of the informatisation teaching ecology](image)
3.1 Ecological structure of informatisation teaching

Informatisation classroom is the venue for the development of informatisation teaching. It is the principal place for teacher-student interaction and student-student interaction. In the information classroom space, teachers use students as the main object to transform the object. The cultivation of students in the information classroom is an obvious function of the information teaching ecology. Its essence is a process of objectification of the subject. In the classroom teaching ecological layer, teachers transform the material flow, energy flow and information flow in the internal and external environment into the knowledge flow, ability flow and value flow of students through multiple teaching activities [5]. The teacher community teaching is based on a particular organisational environment and organisational activities and comprises teachers with different knowledge structures, teaching abilities, thinking styles and behaviour styles. Community is the group resonance of teachers. This is an effective form of professional learning, communication, reflection and a way for teachers’ collective emotions to exist.

3.2 Interaction between binary subsystems

The intelligent classroom and the teacher community are mutually interdependent and affect each other. Moreover, the dual development of subject and object in the process of ecological practice is realised through the dual integration of ‘teaching’ and ‘research’. A smart classroom is the basis of activities of the teacher community. Teachers mainly use teaching activities to understand and promote students’ cognition, thinking and physical and mental development. Informatised classroom is the primary manifestation of teachers’ professional values, teaching ideas and teaching skills, and it is also the source of teachers’ professional reflections [6]. The teacher community outside the classroom provides teachers with a flow of information and energy for professional growth.

On the other hand, teachers in the classroom provide a steady stream of negative entropy for the growth and development of students. Students are the main targets of teachers’ practical activities and provide primary problem streams for teachers’ professional development and provide spiritual support for professional growth. It can be seen that the cultivation of students and the development of teachers are mutually the input and output, forming a dual power system.

3.3 The external environment of informatisation teaching ecology and its supporting mechanism

Each organisation unit represents different professional forces in the cooperation and division of labour to form a community to construct an informatisation teaching ecology. As the supporting system of the informatisation teaching ecology, the external ecological environment has a complex and overlapping influence on the development and evolution of the ecology [7]. This is considered as the information technology material environment, information teaching operation and maintenance environment, policy and system environment for ecological development and the professional information and energy for the ecological input of teaching behaviour evaluation and teaching research through professional exchanges. Thus, the internal and external environments of the information-based teaching ecology form a broader material and humanistic ecological circle and professionals from various systems integrate to form a mutually beneficial and symbiotic ecological relationship.

4 Ecological diagnosis model and application

The thesis establishes a dual ecological model of information-based teaching ecology in the form of self-organisation. This article analyses the core elements of informatisation teaching ecology from the target level, element level, variable level and state-level [8]. The information teaching ecology’s positive entropy and negative entropy index system is constructed by combining divergence and convergence. This research applies it to
the informatisation teaching ecology measurement of H Province. It can diagnose the development of the infor-
matisation teaching ecology in the province. It provides essential and suggestions for the further development
of educational informatisation.

4.1 Establishment of positive and negative entropy index system

The positive entropy index system of information teaching ecology mainly includes three elements of mutual
influence among smart classrooms, teacher community and subsystems, as shown in Figure 2.

![Fig. 2 The positive entropy index system of information teaching ecology](image1)

The negative entropy index system of information teaching ecology is constructed from the three elements
of information technology material environment, information teaching operation and maintenance environment
and policy and institutional environment. The details are shown in Figure 3.

![Fig. 3 Ecological negative entropy index system of informatisation teaching](image2)

4.2 Ecological diagnosis model based on the dissipative structure

The thesis uses the Brussels model to quantitatively analyse the dynamic evolution of the dissipative structure
of the information teaching ecology. Usually, the mathematical conditions are solved to get the dynamic critical
value condition for forming the dissipative structure: the negative entropy $B$ of the system is more significant than $1 + A^2$. This article escapes the model based on the characteristics of the information teaching ecology [9]. We set the positive entropy of the information teaching ecology to be equal to $A$ and the negative entropy to be equal to $B$. It can be confirmed that the basis for discriminating the dissipative structure of the information-based teaching ecology is:

$$|B| = \begin{cases} < 1 + A^2, \text{Non-dissipative structure} \\ = 1 + A^2, \text{Critical structure} \\ > 1 + A^2, \text{Dissipative structure} \end{cases} \quad (7)$$

Equation (7) shows that only when the negative entropy of the information-based teaching ecology is large, and the positive entropy is small, the dissipative structure will be formed to promote the continuous evolution of the information-based teaching ecology.

4.3 Calculation formula of positive and negative entropy

First, we can calculate the entropy value of an index system with $m$ variable layers and $n$ evaluation levels according to Boltzmann’s formula. The formula for calculating the entropy value of the $i$ evaluation index is as follows:

$$e_i = k \sum_{j=1}^{n} p_{ij} \ln p_{ij} \quad (8)$$

Among them, $k$ is a constant, usually $k_A = -1/\ln n$ is used for the calculation of positive entropy and $k_B = 2/\ln n \circ p_{ij}$ is used for calculation of negative entropy. From formula (8), it can be concluded that the value range of $e_{Ai}$ is usually $[0, 1]$. $e_{Ai}$ The closer it is to 0, the more stable the internal structure of the ecology and the opposite, the more disorderly. The value range of $e_{Bi}$ is $[-2, 0]$. When $e_{Bi}$ is closer to $-2$, it indicates that the external ecological environment contributes to the ordering of the ecological interior and vice versa. Secondly, the entropy weight of the $i$ evaluation index can be calculated according to the entropy value of the index. The entropy weight calculation formulas of the $i$ evaluation index in positive and negative entropy are shown in formulas (9) and (10), respectively.

$$\omega_{Ai} = \frac{1 - e_{Ai}}{m - \sum_{i=1}^{m} e_{Ai}} \quad (9)$$

$$\omega_{Bi} = \frac{2 + e_{Bi}}{2m + \sum_{i=1}^{m} e_{Bi}} \quad (10)$$

Further, the positive entropy value $E_A$ and negative entropy value $E_B$ of the information teaching ecology can be obtained based on the overall entropy value calculation formula (10) of the index system.

$$E = \sum_{i=1}^{m} e_i \omega_i \quad (11)$$

4.4 Analysis of empirical results

This article selects 41 digital demonstration schools in H Province as samples for empirical analysis. Based on the informatisation teaching ecological indicator system, two sets of questionnaires for schools and teachers are compiled. It is further divided into five grades $n_A = n_B = 5$ based on the results of the questionnaire. At the same time, $m_A = 12, m_B = 16$ can be known from the evaluation index system. Based on formulas (8)–(11), the information teaching ecology’s positive and negative entropy values are obtained. The details are shown in Tables 1 and 2.

It can be seen from the table that the total entropy of the informatisation teaching ecology of 41 digital demonstration schools in H province indicates that the overall entropy is in a relatively orderly state. Thus,
Table 1  Ecological positive entropy of informatisation teaching

| Target layer                  | Variable                                      | Entropy weight | Entropy weight |
|-------------------------------|-----------------------------------------------|----------------|----------------|
| The positive entropy of information teaching ecology 0.4402 | Digital Resource Management                   | 0.4907         | 0.0840         |
|                               | Informationised teaching methods               | 0.5326         | 0.0771         |
|                               | Teaching model innovation                      | 0.7066         | 0.0485         |
|                               | Learning test and evaluation                   | 0.4085         | 0.0976         |
|                               | Information Exchange Platform                  | 0.5489         | 0.0744         |
|                               | Online collaborative teaching and research     | 0.4973         | 0.0829         |
|                               | Online information sharing                     | 0.6673         | 0.0549         |
|                               | Teaching demonstration and training            | 0.7220         | 0.0458         |
|                               | Teacher information awareness                  | 0.0712         | 0.1532         |
|                               | Teacher information knowledge                  | 0.4415         | 0.0921         |
|                               | Teacher development                            | 0.4085         | 0.0976         |
|                               | Student literacy                               | 0.4425         | 0.0920         |

Table 2  Ecological negative entropy of informatisation teaching

| Target layer                  | Variable                                      | Entropy weight | Entropy weight |
|-------------------------------|-----------------------------------------------|----------------|----------------|
| Informatisation teaching ecological negative entropy −0.8811 | Campus network construction                   | −0.6133        | 0.1214         |
|                               | Data centre management                        | −0.7946        | 0.1055         |
|                               | Teacher and student electronic terminal       | −0.7969        | 0.1053         |
|                               | Digital teaching space                        | −1.6168        | 0.0336         |
|                               | Innovation space                             | −1.7051        | 0.0258         |
|                               | Digital Resource Library                      | −1.8275        | 0.0151         |
|                               | Teacher Teaching System                       | −1.6098        | 0.0342         |
|                               | Teacher Teaching and Research System          | −1.5495        | 0.0394         |
|                               | School Management System                      | −1.5140        | 0.0426         |
|                               | School Evaluation System                      | −1.8976        | 0.009          |
|                               | Home-school exchange platform                  | −1.6586        | 0.0299         |
|                               | Information team structure                    | −1.0444        | 0.0837         |
|                               | Guarantee and incentive system                | −0.2422        | 0.1539         |
|                               | Informatisation funding investment            | −0.7707        | 0.1076         |
|                               | Campus network construction                   | −1.3191        | 0.0596         |
|                               | Data centre management                        | −1.6182        | 0.0334         |

the existing external environment of the information teaching ecology promotes the continuous ordering of smart classrooms, teacher communities and their influence. At the same time, \(|B| = 0.8811 < 1 + A^2 = 1.1938\) can be known according to the discriminant formula of the dissipative structure. This shows that the current information teaching ecology is not in a state of the dissipative structure. That is, it has failed to cause a change in the information teaching ecology.

From the above results, it can be seen that the average entropy value of the positive entropy index of the informatisation teaching ecology is 0.4946 [10]. On the other hand, the relatively high entropy values of teaching model innovation (0.7060), teaching demonstration and training (0.7220) and online information sharing (0.6673) indicate that there is still an imbalance in the development of teaching model innovation, demonstration and guidance between schools and teachers. This requires the further improvement and promotion of teaching innovation model exploration and demonstration leading practice. This is also a vital issue in the realisation of
educational informatisation from 1.0 to 2.0.

On the other hand, in the negative entropy index of the information technology teaching ecology, the absolute values of the negative entropy of the campus network construction (−0.6133), data centre management (−0.7946) and teacher-student electronic terminal (−0.7969) are low in the material environment elements of information technology, while the values of digital teaching space (−1.6168) and innovation space (−1.7051) are relatively high. This is because the development of essential equipment such as networks, data centres and terminals in each school is more consistent and has reached the requirements of digital campus construction. However, there are differences in constructing a more prosperous digital environment, and each school can make breakthroughs in the information infrastructure. This provides a particular foundation for the reform of the information teaching ecology [11]. The relatively high absolute value of negative entropy of related indicators of information-based teaching operation and maintenance environment elements can effectively inject entropy flow into the information-based teaching ecology. On the other hand, the absolute value of negative entropy of the overall indicators is low in terms of policy and institutional environmental elements, especially the guarantee and incentive system (−0.2422) indicator. At present, each unit has the same level of school management rules and regulations, teaching incentive policies and resource co-construction and sharing mechanisms. Be assured. However, formulating an interconnected system for the overall development of the school that is suitable for education informatisation 2.0 needs to be innovated and explored.

5 Related policies and recommendations

5.1 Change the organisational structure and lay a solid foundation for innovation

The reshaping of the teaching ecology is a comprehensive adjustment process of human resources, materials, systems and information. Note that the positioning of the value of organisational philosophy is first reflected in the organisational structure. So implement decentralisation and proper distributed management on management decision-making power. Exploratory problems often require group decision-making and the participation of all teachers can significantly stimulate and enhance the initiative and enthusiasm of teachers [12]. The transformation of management from administrative order to service management requires corresponding structural changes. Build intercommunication channels conducive to the efficient transmission of information while maintaining the flat and simplified organisational structure and enhancing the organisation’s collaboration characteristics and decision-making benefits. This is conducive to the rapid deployment of school human resources and materials and promotes the flow and effective conversion of information and energy in the ecology.

5.2 Create a research atmosphere and enhance the sense of innovation

Innovative organisations are organisations that can guide and encourage innovative behaviours. First of all, cultivating an organisational atmosphere for expressing different opinions and tolerating new ideas and new ideas should be done. It requires a relatively loose mechanism environment and a democratic and equal decision-making mechanism to ensure the participation of teachers in learning and research. Secondly, it takes both operational guidance and active encouragement into consideration [13]. Finally, while launching a wealth of information-based teaching and research activities, it encourages innovative ideas and behaviours of informal organisations and pays attention to the psychological achievements of teachers.

Strengthen school-to-school exchanges, promote mutual learning of advanced educational management experience and explore suitable operating mechanisms between schools to form a cooperative model that can effectively implement, evaluate, feedback and adjust. Provide opportunities for in-depth exchanges between teachers through foreign cooperation, including class management, education and teaching, teaching, research and scientific research. Provide opportunities for in-depth exchanges between students through foreign cooperation, including mutual learning and mutual exchanges. It will bring more resources and opportunities to the respective schools through foreign cooperation, bringing advantages in school management, education, teaching,
campus culture, research and scientific research, exchanges, and learning.

6 Conclusion

The information-based teaching ecosystem must achieve a self-organising state. The key to maintaining the sustained vitality and innovation of the dual ecosystem is to enhance teachers' research awareness. Education informatisation 2.0 stage requires teachers to pursue teaching innovation based on informatisation teaching competence. The innovation of education often starts from organisational management and leadership reform and should take the formation of corresponding organisational culture as a sign of maturity. Based on the theory of dissipative systems, this research explores the laws of ecological development. It proposes to develop new teaching relationships from the following three aspects; develop a digital teacher community and create a cultural atmosphere so that the environment stimulates everyone to innovate and explore.

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References
[1] Han, F., Dong, H., Wang, Z., & Li, G. Local design of distributed $H_\infty$-consensus filtering over sensor networks under multiplicative noises and deception attacks. International Journal of Robust and Nonlinear Control., 2019. 29(8): 2296-2314
[2] Çitil, H. Important Notes for a Fuzzy Boundary Value Problem. Applied Mathematics and Nonlinear Sciences., 2019. 4(2): 305-314
[3] Josheski, D., Karamazova, E. & Apostolov, M. Shapley-Folkman-Lyapunov theorem, and Asymmetric First price auctions. Applied Mathematics and Nonlinear Sciences., 2019. 4(2): 331-350
[4] El-Attar, S. I., Hendy, M. H., & Ezzat, M. A. On phase-lag Green–Naghdi theory without energy dissipation for electro-thermoelasticity including heat sources. Mechanics Based Design of Structures and Machines., 2019. 47(6): 769-786
[5] Chen, Z., Su, G., Ju, J. W., & Jiang, J. Experimental study on energy dissipation of fragments during rockburst. Bulletin of Engineering Geology and the Environment., 2019. 78(7): 5369-5386
[6] Lau, H. C., & Holtzman, B. K. “Measures of Dissipation in Viscoelastic Media” Extended: Toward Continuous Characterization Across Very Broad Geophysical Time Scales. Geophysical Research Letters., 2019. 46(16): 9544-9553
[7] Li, B., Yin, J., Liu, X., Wu, H., Li, J., Li, X., & Guo, W. Probing van der Waals interactions at two-dimensional heterointerfaces. Nature nanotechnology., 2019. 14(6): 567-572
[8] Yi, K., Liu, Z., Lu, Z., Zhang, J., & Sun, Z. Transfer and dissipation of strain energy in surrounding rock of deep roadway considering strain softening and dilatancy. Energy Science & Engineering., 2021. 9(1): 27-39
[9] Aldawody, D. A., Hendy, M. H., & Ezzat, M. A. Fractional Green–Naghdi theory for thermoelectric MHD. Waves in Random and Complex Media., 2019. 29(4): 631-644
[10] Huang, W., Wang, P., Lv, L., Wang, L., & Wang, H. H. An inventive high-performance computing electronic information system for professional postgraduate training. International Journal of Computers and Applications., 2020. 42(4): 422-428
[11] Kim, T. W., & Park, T. S. Size effect on compressible flow and heat transfer in microtube with rarefaction and viscous dissipation. Numerical Heat Transfer, Part A: Applications., 2019. 76(11): 871-888
[12] Tobin, R. G., Lacy, S. J., Crissman, S., Haddad, N., Wentink, O., & Seeley, L. Where does energy go when it’s “Gone”? Promoting understanding of energy dissipation. American Journal of Physics., 2019. 87(7): 569-576
[13] Malakar, D., & Geete, A. Application of entropy and entransy concepts to design shell and tube type surface condenser at different 4L/D ratios for Maral Overseas Ltd. International Journal of Ambient Energy., 2020. 41(7): 813-822