Four Dimensional Supergravity from String Theory

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A derivation of N=1 supergravity action from string theory is presented. Starting from a Nambu-Goto bosonic string, matter field is introduced to obtain a superstring in four dimension. The excitation quanta of this string contain graviton and the gravitino. Using the principle of equivalence, the action in curved space time are found and the sum of them is the Deser-Zumino N=1 supergravity action. The energy tensor is Lorentz invariant due to supersymmetry.

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1. INTRODUCTION

There are only a few generally accepted ways to extract gravity from String Theory. One of them is to follow the method of Feynman [1] using gravity as a spin-2 field theory coupled to its energy momentum tensor. Parallel to this is also the work of Weinberg [2] who constructed a Hamiltonian in interaction picture and obtained the Einstein equation from the equations in Heisenberg picture. Another popular method is that of Callan, Fiedan et al [3] who looked for the conditions of consistency for space time to allow propagation of strings. Gravitation tensor also emerges in the zero slope limit of the string coupling parameter given in reference [4].

Here, matter field is first introduced in Nambu-Goto string in 26 dimensions through 11 vector Majorana spinors in the bosonic representation of $SO(3, 1)$. A superstring is constructed which was reported first in reference [5]. Details of tachyonlessness in mass spectrum, modular invariance and anomaly cancellation are given in reference [6]. The particle spectrum is very rich. The construction of the standard model particle spectrum has been reported in reference [7] and the gauge symmetry $SO(6) \otimes SO(5)$ generators along with the descent to three generations of standard model, i.e., $Z_3 \otimes SU_C(3) \otimes SU_L(2) \otimes U_Y(1)$ have also been reported [8].

The mass spectrum in the bosonic NS sector [9] as well as in the fermionic R sector [10] contain tachyons. Their self energies cancel and do not show up in the supersymmetric Fock space [6]. These tachyons are exceedingly useful in constructing zero mass ground states, which are physical and observable. In this paper, they are used to construct physical graviton and gravitino in section 3 and 4 respectively. In sections 5 and 6, action for them in curved space time are derived. In section 2, the quantisation, Virasoro algebra and physical state conditions are stated. It would be worthwhile to present the salient features of the superstring in this section.

Nambu-Goto action with open string coordinates $X^\mu(\sigma, \tau)$ in the world sheet $(\sigma, \tau)$ is

$$S = -\frac{1}{2\pi} \int d^2 \sigma \partial^\alpha X^\mu \partial_\alpha X_\mu, \quad \mu = 0, 1, 2, ..., 25$$

The central charge of this action is 26 and is cancelled by adding the action of the conformal ghost whose central charge is -26, independent of dimensionality. The theory is made anomaly free. It has been shown by Mandelstam [11] that two Majorana fermions in $1 + 1$ dimensions are equivalent to one real boson in the finite volume. It has also been shown that the equivalence is true on a finite interval or a circle like in Veneziano model if the string is anomaly free. Taking 44 Majorana fermions to form the matter fields and along with the four boson, one can write an anomaly free action

$$S = -\frac{1}{2\pi} \int d^2 \sigma \left[ \partial^\alpha X^\mu \partial_\alpha X_\mu - i \sum_{j=1}^{44} \bar{\phi}_j \rho^0 \partial_\alpha \phi_j \right]$$

with

$$\partial_\alpha = (\partial_\sigma, \partial_\tau), \quad \rho^0 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \quad \rho^1 = \begin{pmatrix} 0 & i \\ i & 0 \end{pmatrix} \text{ and } \bar{\phi} = \phi^\dagger \rho^0$$

Unfortunately the Lorentz invariance of the matter part does not hold good. However, there are also Majorana
fermions $\psi^\mu$ in the bosonic representation of SO(3, 1) so that the action (2) can be recast to read

$$S = -\frac{1}{2\pi} \int d^2\sigma \left[ \partial^\alpha X^\mu \partial_\alpha X_\mu - i \sum_{j=1}^{11} \bar{\psi}^{\mu,j} \rho^\alpha \partial_\alpha \psi_{\mu,j} \right]$$ (4)

The central charge is 26. However, this is not supersymmetric. The supersymmetric version is

$$S = -\frac{1}{2\pi} \int d^2\sigma \left[ \partial^\alpha X^\mu \partial_\alpha X_\mu - i \bar{\psi}^{\mu,j} \rho^\alpha \partial_\alpha \psi_{\mu,j} + i \bar{\phi}^{\mu,k} \rho^\alpha \partial_\alpha \phi_{\mu,k} \right]$$ (5)

Here $j$ runs from 1 to 6 and, the the positive and negative frequency parts are, as normal, $\psi^{\mu,j} = \psi^{(+)^{\mu,j}} + \psi^{(-)^{\mu,j}}$, whereas the index $k$ runs from 1 to 5 and $\phi^{\mu,k} = \phi^{(+)^{\mu,k}} + \phi^{(-)^{\mu,k}}$. The negative sign of the latter is absorbed in creation operators which is allowed by the choice of freedom of Majorana fermions.

To write the supersymmetric transformation of the $SO(6) \times SO(5)$ invariant action (5), one has to introduce arrays $(e^j, e^k)$ which are rows of ten zeros and only one 1 in the $j$th or $k$th place. $e^j e^j = 6$ and $e^k e^k = 5$. The upper index refers to a row and the lower to a column. The action of equation (5) is invariant under supersymmetric transformations

$$\delta X^\mu = \bar{\epsilon} (e^j \psi_j^{\mu} - e^k \phi_k^{\mu})$$ (6)

$$\delta \psi^{\mu,j} = -i \epsilon e^j \rho^\alpha \partial_\alpha X^\mu$$ (7)

and

$$\delta \phi^{\mu,k} = -i \epsilon e^k \rho^\alpha \partial_\alpha X^\mu$$ (8)

Here $\epsilon$ is a constant anticommuting spinor. In this action, there are only four bosonic coordinates but forty four fermionic modes. There should not be such mismatch.

The inconsistency should show up by making two successive SUSY transformations. Indeed the two such transformations lead to spatial translation with the coefficient $a^{\alpha} = 2i \bar{\epsilon}^1 \rho^\alpha \epsilon_2$, only if

$$\psi_j^{\mu} = e_j \Psi^{\mu} \quad \text{and} \quad \phi_k^{\mu} = e_k \Psi^{\mu}$$ (9)

The superpartner of $X^\mu$ is the $\Psi^{\mu}$ where,

$$\Psi^{\mu} = e^j \psi_j^{\mu} - e^k \phi_k^{\mu}$$ (10)

The action (5) is reduced to

$$S = -\frac{1}{2\pi} \int d^2\sigma \left[ \partial^\alpha X^\mu \partial_\alpha X_\mu - i \bar{\Psi} \rho^\alpha \partial_\alpha \Psi \right]$$ (11)

The bosonic and fermionic modes match. The $\Psi^{\mu}$ is not only the superpartner of $X^\mu$, but also emits the quanta of $\psi_j^{\mu}$ and $\phi_k^{\mu}$ while in the sites $j$ or $k$ of the array respectively. The theory is tachyonless in the physical Fock space, modular invariant with a vanishing partition function, ghost free and anomaly free. These features have been discussed, in detail, in references [5] and [6].

2. QUANTISATION, SUPER VIRASORO ALGEBRA AND PHYSICAL STATES

Quantisation is normal as usual.

Coordinates: $X^\mu(\sigma, \tau) = x^\mu + p^\mu \tau + i \sum_{n \neq 0} \frac{1}{n} \alpha_n^\mu e^{-in\sigma} \cos(n\sigma)$ (12)

or, $\partial_{\pm} X^\mu = \frac{1}{2} \sum_{n>0}^{+\infty} \alpha_n^\mu e^{-in(\tau \mp \sigma)}$ (13)
with \( [ \alpha_m^\mu, \alpha_n^\nu ] = m \, \delta_{m+n} \, \eta^{\mu\nu} \) \hspace{1cm} (14)

NS fermions: \( \psi_{\pm}^{\mu,j}(\sigma, \tau) = \frac{1}{\sqrt{2}} \sum_{r \in \mathbb{Z} + 1/2} b_r^{\mu,j} \, e^{-ir(\tau \pm \sigma)} \) \hspace{1cm} (15)

\( \phi_{\pm}^{\mu,k}(\sigma, \tau) = \frac{1}{\sqrt{2}} \sum_{r \in \mathbb{Z} + 1/2} b_r^{\mu,k} \, e^{-ir(\tau \pm \sigma)} \) \hspace{1cm} (16)

This is called the bosonic sector and the quanta obey the anticommutation relations

\( \{ b_r^{\mu,j}, b_{s}^{\nu,j'} \} = \delta_{r+s} \, \delta_{j,j'} \, \eta^{\mu\nu}, \quad \{ b_r^{\mu,k}, b_{s}^{\nu,k'} \} = \delta_{r+s} \, \delta_{k,k'} \, \eta^{\mu\nu} \) \hspace{1cm} (17)

R fermions: \( \psi_{\pm}^{\mu,j}(\sigma, \tau) = \frac{1}{\sqrt{2}} \sum_{m=-\infty}^{+\infty} d_m^{\mu,j} \, e^{-im(\tau \pm \sigma)} \) \hspace{1cm} (18)

\( \phi_{\pm}^{\mu,k}(\sigma, \tau) = \frac{1}{\sqrt{2}} \sum_{m=-\infty}^{+\infty} d_m^{\mu,k} \, e^{-im(\tau \pm \sigma)} \) \hspace{1cm} (19)

with \( \{ d_m^{\mu,j}, d_{m'}^{\nu,j'} \} = \delta_{m+m'} \, \delta_{j,j'} \, \eta^{\mu\nu} \) \hspace{1cm} (20)

and \( \{ d_m^{\mu,k}, d_{m'}^{\nu,k'} \} = \delta_{m+m'} \delta_{k,k'} \, \eta^{\mu\nu} \) \hspace{1cm} (21)

To construct admissible physical states, it is essential to give the super Virasoro generators [12]. In the notation of reference [4],

\[ L_m = \frac{1}{\pi} \int_{-\pi}^{\pi} d\sigma \, e^{im\pi} \, T_{++} \]

\[ = \frac{1}{2} \sum_{-\infty}^{+\infty} : \alpha_{-n} \cdot \alpha_{m+n} + \frac{1}{2} \sum_{r \in \mathbb{Z} + 1/2} \left( r + \frac{m}{2} \right) : (b_{-r} \cdot b_{r+m} - b_{-r} \cdot b_{r+m} ) : \quad \text{NS} \] \hspace{1cm} (22)

\[ = \frac{1}{2} \sum_{-\infty}^{+\infty} : \alpha_{-n} \cdot \alpha_{m+n} + \frac{1}{2} \sum_{-\infty}^{+\infty} \left( n + \frac{m}{2} \right) : (d_{-n} \cdot d_{m+n} - d_{-n} \cdot d_{m+n} ) : \quad \text{R} \] \hspace{1cm} (23)

\[ G_r = \frac{\sqrt{2}}{\pi} \int_{-\infty}^{+\infty} d\sigma \, e^{ir\sigma} \, J_+ = \sum_{-\infty}^{+\infty} \alpha_{-n} \cdot (e^j b_{n+r,j} - e^k b'_{n+r,k}) = \sum_{-\infty}^{+\infty} \alpha_{-n} \cdot B_{n+r} \quad \text{NS} \] \hspace{1cm} (24)

\[ F_r = \frac{\sqrt{2}}{\pi} \int_{-\infty}^{+\infty} d\sigma \, e^{ir\sigma} \, J_+ = \sum_{-\infty}^{+\infty} \alpha_{-n} \cdot (e^j d_{n+r,j} - e^k d'_{n+r,k}) = \sum_{-\infty}^{+\infty} \alpha_{-n} \cdot D_{n+r} \quad \text{R} \] \hspace{1cm} (25)

where \( B^{\mu}_{n+r} = e^j b^{\mu}_{n+r,j} - e^k b'_{n+r,k} \)
and \[ D_{n+r}^\mu = e^\beta d_{n+r,j}^\mu - e^k d_{n+r,k}^\mu \] (27)

The super Virasoro algebra is

\[ [L_m, L_n] = (m - n)L_{m+n} + \frac{c}{12}(m^3 - m) \delta_{m+n} \] (28)

\[ [L_m, G_r] = \left( \frac{m}{2} - r \right) G_{m+r} \] NS (29)

\[ \{G_r, G_s\} = 2L_{r+s} + \frac{c}{3}(r^2 - 1/4) \delta_{r+s} \] (30)

\[ [L_m, F_n] = \left( \frac{m}{2} - n \right) F_{m+n} \] R (31)

\[ \{F_m, F_n\} = 2L_{m+n} + \frac{c}{3}(m^2 - 1) \delta_{m+n} \] (32)

The central charge \( c \) for the action of equation(5) is 26.

In this superstring, there are four bosons (\( c_B = 4 \)), twenty two transverse fermions (\( c_T = 11 \)) spanned by two transverse superfermionic modes \( \Psi^{1,2} \) and usual conformal ghosts \( c_{FP} = -26 \). The light cone part of the action has \( c_{LC} = 11 \), which is exactly equivalent to the action of the superconformal ghosts \( (\beta, \gamma) \) as shown in reference[6]. They represent the superfermionic modes \( \Psi^{0,3} \) representation of the 22 longitudinal ghost modes \( (\psi_j^{0,3}, \phi_k^{0,3}) \). The total central charge is zero and the superstring is anomaly free.

The physical states are defined as

NS : \( (L_0 - 1)|\Phi > = 0, \quad L_m|\Phi > = 0, \quad G_r|\Phi > = 0, \quad r, m > 0 \) (33)

R : \( (L_0 - 1)|\psi > = (F_0^2 - 1)|\psi > = 0, \quad L_m|\psi > = 0, \quad F_m|\psi > = 0, \quad m > 0 \) (34)

Here, \( \beta \) is the spinor component index.

The mass spectra are obtained from \( L_0 \).

\[ \alpha' M_{NS}^2 = -1, -\frac{1}{2}, 0, \frac{1}{2}, 1, \frac{3}{2}, \ldots \] NS (35)

\[ \alpha' M^2_R = -1, 0, 1, 2, \ldots \] R (36)

The G.S.O. projection operators \( [1 + (-1)^F] \) on the states eliminates the half integral values. The tachyonic energy \( < 0 | (L_0 - 1)^{-1} | 0 > \) of the bosonic vacuum is cancelled by \(- < 0 | (F_0 + 1)^{-1} (F_0 - 1)^{-1} | 0 >_R \) of the Ramond sector. The negative sign is due to the fermionic loop. Such tadpole cancellations have also been noted by Chattarputi et al.[14]. To be more sure, consider the supersymmetric charge

\[ Q = \frac{1}{\pi} \int_0^\pi \rho^\alpha \rho^{\prime \alpha} \partial_\alpha X^\mu \Psi_\mu \, d\sigma \] (37)

and find the supersymmetry result

\[ \sum \{Q_\alpha^\dagger Q_\alpha\} = 2H, \quad \sum | Q_\alpha | \phi_0 > |^2 = 2 < \phi_0 | H | \phi_0 > \] (38)

The ground state is of zero energy. There are no tachyons. The physical mass spectrum in both sectors are integral numbers of Regge trajectory, \( \alpha' M^2 = 0, 1, 2, \ldots \).
However, the existence of the two tachyonic vacuum is important and crucial. These will be used to build ground states of zero energy in both NS and R sectors. Since

$$[L_0, \alpha_{-1}^\mu] = \alpha_{-1}^\mu$$

(39)

the state $$A^\mu = \alpha_{-1}^\mu \ket{0}$$ has zero energy, i.e., $$L_0 A^\mu = 0$$. Similarly, the product of the two $$b$$ (or $$b'$$) quanta

$$A_{jj'}^{\mu\nu} = b_{-1/2,j}^{\mu} b_{-1/2,j'}^{\nu} \ket{0}$$

(40)

has zero energy. The index $$j$$ runs from 1 to 6. The quanta specified by them belong to $$SO(6)$$ sector. The $$SO(6)$$ has a representation which are four component spinors. These will be needed to construct the gravitino from the Ramond sector tachyon. Supergravity will need the graviton and a simple choice is to choose $$SO(6)$$ sector with indices $$j$$ running from 1 to 6. This will be built from the NS bosonic tachyon-vacuum by applying two NS creation operators as in equation (40).

3. THE GRAVITON

The gravitons are the quanta of the gravitational field. At present there does not exist any complete and self consistent theory of gravitation. The reasons are nicely given by Weinberg\[15\], which will also be elucidated here. A tensor of the type $$A^{\mu\nu}$$ will, in general, contain a spin-0, a spin-1 and the spin-2. The last is identifiable with the graviton. The first problem is to write a tensor without the dilaton spin-0 or the anti-symmetric tensor spin-1.

$$A^{\mu\nu}$$ is not only massless but also physical, since $$G_{1/2}$$ operating on $$A^{\mu\nu}$$ i.e., $$G_{1/2} A^{\mu\nu}$$ kills one of the $$b_i^\dagger$$'s of $$A^{\mu\nu}$$ and the state becomes a half integral mass tachyonic vacuum. This can be projected out of the physical space by a GSO projection operation. The product of two $$b_i^\dagger b_j^\nu$$ is antisymmetric in $$j$$, $$j'$$ due to anticommutativity of the $$b_i^\dagger$$'s. So the state will exist if $$C_{ij}$$ is antisymmetric and will vanish if $$C_{ij}$$ is symmetric.

If the dilaton state is $$\Phi$$, then

$$g_{\mu\nu} \Phi = g_{\mu\nu} \sum_{ij} C_{ij} b_i^\dagger \lambda b_j^\nu \ket{0, p}$$

(41)

Due to anticommuting $$b$$'s,

$$\sum_{ij} C_{ij} b_i^\dagger \lambda b_j^\nu = - \sum_{ij} C_{ij} b_j^\nu \lambda b_i^\dagger = - \sum_{ij} C_{ji} b_j^\nu \lambda b_i^\dagger = 0$$, if $$C_{ij} = C_{ji}$$

(42)

Hence, the choice $$C_{ij} = C_{ji}$$ ensures that there are no dilatons and thus replaces the gauge choice $$h_{\mu\mu} = 0$$ The anti-symmetric tensorial zero mass state is

$$A^{\mu\nu} = \sum_{ij} C_{ij} (b_i^{\dagger \mu} b_j^\nu - b_i^{\dagger \nu} b_j^\mu) = -A^{\nu\mu}$$

(43)

Again, if $$C_{ij} = C_{ji}$$ then $$A^{\mu\nu} = -A^{\nu\mu} = 0$$. The symmetric traceless tensor, which is non zero, physical and of zero mass is

$$h^{\mu\nu}(p) = \sum_{ij} C_{ij} (b_i^{\dagger \mu} b_j^\nu + b_i^{\dagger \nu} b_j^\mu - 2 \eta^{\mu\nu} b_i^\lambda b_j^\lambda) \ket{0, p}$$

(44)

Even though $$C_{ij} = -C_{ji}$$, using GSO projection

$$\left(1 + (-1)^F\right) G_{1/2} h_{\mu\nu}(p) = \left(1 + (-1)^F\right) [G_{1/2}, h_{\mu\nu}(p)] = 0$$

(45)

This can be taken as the graviton. The commutator is

$$[h^{\mu\nu}(p), h^{\lambda\sigma}(q)] = f^{\mu\nu,\lambda\sigma} \ket{C} \delta^{(4)}(p - q)$$

(46)

where

$$f^{\mu\nu,\lambda\sigma} = g^{\mu\lambda} g^{\nu\sigma} + g^{\nu\lambda} g^{\mu\sigma} - g^{\mu\nu} g^{\lambda\sigma}$$

(47)
and $|C|^2$, besides $C_i$'s, also include normalization factors.

Formally one can go over to quantum field theory by defining the flat space time Fourier transform as

$$h_{\mu\nu}(x) = \frac{1}{2\pi^3} \int \frac{d^3p}{\sqrt{2p_0}} \left( h_{\mu\nu}(p)e^{ipx} + h^\dagger_{\mu\nu}(p)e^{-ipx} \right)$$

(48)

with the commutator

$$[h^{\mu\nu}(x), h^{\lambda\sigma}(y)] = \frac{1}{2\pi^3} \int \frac{d^3p}{2p_0} \left( e^{ip(x-y)} - e^{-ip(x-y)} \right) f^{\mu\nu,\lambda\sigma}$$

(49)

The Feynman propagator is

$$\Delta^{\mu\nu,\lambda\sigma}(x-y) = \langle 0 | T(h^{\mu\nu}(x)h^{\lambda\sigma}(y)) | 0 \rangle = \frac{1}{2\pi^4} \int d^4p \Delta^{\mu\nu,\lambda\sigma}_F(p)e^{ip(x-y)}$$

(50)

where

$$\Delta^{\mu\nu,\lambda\sigma}_F(p) = \frac{1}{2} f^{\mu\nu,\lambda\sigma} \frac{1}{p^2 - i\epsilon}$$

(51)

This is the Feynman propagator of the graviton in interaction representation.

Very unfortunately the tensor $h_{\mu\nu}(p)$ is not Lorentz invariant. A true Lorentz tensor would have helicities 0 and ±1 as well as ±2. Unless the 0 and ±1 are eliminated, they will enter into the calculation of Lorentz invariant amplitudes. It is essential that the dictation of the generally covariant content of general relativity, i.e., the quantum theory of gravitation should be Lorentz invariant. There is a method, pioneered by Feynman[1], where one starts out with manifestly Lorentz invariant calculational rules, and then add or modify them to prevent the unphysical particles with helicities 0 or ±1 in physical states. This has been carried out in a successful manner by Mandelstam, Deser and DeWitt[16].

A stringy solution is presented here which may be possible only in this new type of four dimensional superstring due to its supersymmetry content. Examine the action of Lorentz transformation $\Lambda_{\mu\nu}$ on $h_{\mu\nu}$[15]

$$h_{\mu\nu} \rightarrow \Lambda_\mu^\rho \Lambda_\nu^\sigma h_{\rho\sigma} + p_\mu \epsilon_\nu + p_\nu \epsilon_\mu$$

(52)

In string theory $\alpha_\mu^\mu = p^\mu$. So, writing

$$h_{\mu\nu}(p) = h_{\mu\nu} | 0, p > \rightarrow \Lambda_\mu^\rho \Lambda_\nu^\sigma h_{\rho\sigma} | 0, p > + O_{\mu\nu}(p)$$

(53)

where

$$O_{\mu\nu}(p) = (p_\mu \epsilon_\nu + p_\nu \epsilon_\mu) | 0, p >$$

(54)

with

$$O_{\mu\nu} = p_\mu \epsilon_\nu + p_\nu \epsilon_\mu$$

(55)

leads to

$$L_0 O_{\mu\nu}(p) = O_{\mu\nu}(p)$$

(56)

So the extra term generated by Lorentz transformation is tachyonic. This is not permissible in this superstring theory. Ramond sector states should cancel these additional tachyons due to supersymmetry and since $L_0 = F_0^2$,

$$O_{\mu\nu}(p) = L_0 O_{\mu\nu}(p) = F_0^2 O_{\mu\nu}(p)$$

(57)

in Ramond sector. So,

$$F_0 O_{\mu\nu}^\alpha(p) = \pm O_{\mu\nu}^\alpha(p)$$

(58)

In general, one can construct spinorial states $| 0 >_\alpha$ such that

$$F_0 | 0 >_\alpha = | 0 >_\alpha, \quad \alpha < 0 \quad | F_0 = -\alpha < 0 | \quad \text{and} \quad \sum_\alpha | 0 >_{\alpha\alpha} < 0 | = 1$$

(59)
\[ O_{\mu\nu}(p) = L_0 O_{\mu\nu} \mid 0, p > = \sum_\alpha F_0 \mid 0 >_{\alpha<0} \mid F_0 O_{\mu\nu} \mid 0, p > = -\sum \mid 0 >_{\alpha<0} \mid O_{\mu\nu} \mid 0, p > = -O_{\mu\nu}(p) = 0 \] (60)

This is like the tadpole-like cancellation of the vacuum energies of the two NS and R sectors. All tachyonic contributions will cancel out due to supersymmetry. Thus \( h_{\mu\nu}(p) \) of equation (44) is Lorentz invariant with spin-2 only. It should not dilute the Lorentz invariant rules in calculation of quantum transition amplitudes. This solves the most major problem of the quantum theory of gravitation.

4. GRAVITINO IN FLAT AND CURVED SPACE TIME

In four flat dimensions, the Rarita-Schwinger\[13\] equation for spin-3/2 objects is

\[ \epsilon^{\mu\nu\lambda\sigma} \gamma_5 \gamma_\lambda \partial_\nu \psi_{3/2}^\sigma = \left( g^{\mu\sigma} \gamma^\nu - g^{\mu\nu} \gamma^\sigma - \gamma^\mu \gamma^\nu \gamma^\sigma \right) \partial_\nu \psi_{3/2}^\sigma \] (61)

As shown by GSO\[13\], in off shell momentum space this is equivalent to the three equations

\[ p^\mu \psi_{3/2}^\mu = 0 \] (62)

\[ (\gamma \cdot p) \psi_{3/2}^\mu = 0 \] (63)

Conditions (62) and (63), when satisfied, ensure that there is no admixture of spin-1/2 component. Spin-1/2 component cannot be ruled out if only equations (62) are satisfied.

Let the Fock ground state above the Ramond tachyon \( \mid 0, p >_\alpha \) be

\[ \mid \phi_0 >= \alpha_{\alpha-1}^\mu \mid 0, p > u_\mu \] (64)

or

\[ \mid \phi_0 >= D_{\alpha-1}^\mu \mid 0, p > u_{1\mu} \] (65)

In both the options \( L_0 \mid \phi_0 >= 0 \), so that the ground state \( \mid \phi_0 > \) is massless. Physical state constraints \( L_1 \mid \phi_0 >= 0 \) and \( F_1 \mid \phi_0 >= 0 \) imply that

\[ p^\mu u_\mu = 0, \quad \gamma^\mu u_\mu = 0 \] (66)

Here, \( u_\mu \) is a mixture of spin-3/2 and spin-1/2. Since \( L_0 \mid \phi_0 > \) and \( L_0 = F_0^2 \), we have

\[ F_0 \mid \phi_0 >= 0 \] (67)

It is well known that \( D_{0}^\mu \sim \gamma^\mu \) as they satisfy the Dirac algebra\[4\]. Further, since \( \alpha_0^\mu = p^\mu \), we have

\[ p^\mu u_{1\mu} = 0 \quad \text{and} \quad \gamma^\mu u_{1\mu} = 0 \] (68)

\( u_\mu \) and \( u_{1\mu} \) are arbitrary spinors satisfying (66) and (68) respectively. Let us choose \( u_{1\mu} = \gamma^5 u_\mu \). Obviously

\[ \psi_{3/2}^\mu = (1 + \gamma_5) u_\mu \] (69)

satisfies (66) and (68) i.e.,

\[ \gamma^\mu \psi_{3/2}^\mu = 0, \quad \text{and} \quad p^\mu \psi_{3/2}^\mu = 0 \] (70)

Since \( F_0 \) is essentially \( \gamma \cdot p, F_0 \mid \phi_0 > \) gives us the necessary spin-1/2 admixture elimination condition

\[ F_0 \psi_{3/2}^\mu \sim (\gamma \cdot p) \psi_{3/2}^\mu = 0 \] (71)
So, $\psi^{3/2}$ is the vector spinor of the Rarita-Schwinger equation (57).

The general relativity curved space time equation can be found by invoking the principle of equivalence [17]. To every curved space time point on the manifold, a tangent space is constructed. The vector indices are $a, b, c, ...$ in this space. The vierbein matrices like $e^a_{\mu}$ take $x-$ space objects to the tangent space and vice versa.

$$e^a_{\mu} e^a_{\nu} = g_{\mu\nu}, \quad e^a_{\mu} e^{b\mu} = \delta^{ab} \tag{72}$$

For completely specifying the spin connections, we need $\omega^{ab}_{\mu}$, in terms of which we introduce the covariant derivative $D_\mu$ such that

$$D_\mu e^a_\lambda = 0 \tag{73}$$

With the definition of $\gamma$ matrices

$$\gamma^a e^a_{\mu} = \gamma^\mu \tag{74}$$

the covariant derivative of a spinor $\psi$ is

$$D_\mu \psi = \left( \partial_\mu + \omega^{ab}_{\mu} \sigma_{ab} \right) \psi \tag{75}$$

Here, $\sigma_{ab}$ is the antisymmetric product of two $\gamma$-matrices. The principle of equivalence is satisfied if the ordinary flat space derivative $\partial_\mu$ is replaced by $D_\mu$. The Rarita-Schwinger equation, in curved space time, is

$$\gamma_\mu \gamma^5 D_\sigma \psi^{3/2} \epsilon^{\mu\nu\sigma\rho} = 0 \tag{76}$$

The invariant action of the gravitino becomes [4]

$$S_{\text{gravitino}} = - i \frac{1}{2} \int d^4 x \bar{\psi}^{3/2} \gamma_\mu \gamma^5 D_\sigma \psi^{3/2} \epsilon^{\mu\nu\sigma\rho} \tag{77}$$

where $e = \sqrt{g}$.

5. GRAVITON AND GENERAL RELATIVITY

The massless tensor of the graviton field $h_{\mu\nu}(p)$, given by equation (44), satisfies $L_0 h_{\mu\nu}(p) = - p^2 h_{\mu\nu}(p) = 0$. It follows that, in flat space time,

$$\Box h_{\mu\nu}(x) = 0 \tag{78}$$

For simplicity and without loss of generality in flat space time, we take plane wave solutions,

$$h_{\mu\nu}(x) = h_{\mu\nu}(p)e^{ipx} \tag{79}$$

It is easy to derive

$$h^\lambda_\mu h^\rho_\nu = \delta^\mu_\nu \tag{80}$$

where the coefficients $C_{ij}$ of equation (44) have been adjusted to take care of the factors of $2\pi$, integers leading to the coefficient of $\delta^\mu_\nu$ to be the one in the r.h.s. of equation (80). In the tangent space,

$$h_{\mu\nu}(x) = e^a_{\mu} e^b_{\nu} T_{ab} \tag{81}$$

After some calculation [4], one gets

$$[D_\mu, D_\lambda] h^\lambda_\nu = e^a_{\nu} e^b_{\rho} R^{ac}_{\mu\lambda} T^{ab} \tag{82}$$

where the Riemannian curvature tensor $R^{ac}_{\mu\lambda}$ is

$$R^{ac}_{\mu\lambda} = \partial_\mu \omega^{ac}_\lambda + \omega^{ab}_\mu \omega^{bc}_\lambda - (\mu \leftrightarrow \lambda) \tag{83}$$
Inverse calculated from equation (82) is

\[ (D_{\mu}, D_{\lambda}) h^\lambda_\nu = R_{\mu\lambda} h^\lambda_\nu = 0 \]  

(84)

This is parallel transport equation. Using the normalisation relation (80), it is found that

\[ R_{\mu\nu} = h^\lambda_\nu R_{\mu\sigma} h^\lambda_\sigma = h^\lambda_\nu [D_{\mu}, D_{\sigma}] h^\lambda_\sigma \]  

(85)

A recipe to go to the curved space time has been succintly put by Misner, Thorne and Wheeler 17. “The laws of Physics written in abstract geometric form, differ in no way, whatsoever, between curved space time and flat space time, this is guaranteed by and, in fact, is a mere rewording of the equivalence principle.” In flat space time \( D_{\mu} = \partial_{\mu} \) and \([\partial_{\mu}, \partial_{\nu}] = 0\). Thus r.h.s. of equation (85) vanishes. So anywhere in the universe, with curved or flat space time

\[ R_{\mu\nu} = 0 \]  

(86)

This is the Einstein equation of general relativity deduced from this new construction of the superstring 18. The action of the graviton is

\[ S_{\text{graviton}} = -\frac{1}{2\kappa^2} \int d^4 x \, e R \]  

(87)

where

\[ R = g_{\mu\nu} R_{\mu\nu} \]  

(88)

and \( \kappa \) is the gravitational constant, whose square is proportional to the Newtonian constant.

6. SUPERGRAVITY ACTION AND CONCLUDING REMARKS

The \( N = 1 \) supergravity action \( S \) in this four dimensional string theory is the sum of \( S_{\text{gravitino}} \) and \( S_{\text{graviton}} \)

\[ S = -\frac{1}{2} \int d^4 x \, e \left( \frac{1}{\kappa^2} R + \bar{\psi}^{3/2}_\mu \gamma^\nu \gamma^5 D_{\sigma} \psi^{3/2}_\mu \epsilon^{\mu\nu\sigma\rho} \right) \]  

(89)

The local supergravity transformations that leave it invariant are 4

\[ \delta \psi^{3/2}_\mu = \frac{1}{\kappa} D_{\mu} \epsilon(x) \]  

(90)

and

\[ \delta^m_{\mu} = -\frac{i}{2} \epsilon(x) \gamma^m \psi^{3/2}_\mu \]  

(91)

This was first written down by Weiss and Zumino 18 in 1976. The application of supergravity to quantum gravity has not been pursued vigorously.

Unfortunately, besides the breakdown of Lorentz invariance due to the appearance of spin-0 and spin-1 objects, the theory of quantum gravity contains infinities from integrals over large virtual momenta. In quantum electrodynamics, there are only three types of divergences, and they can be absorbed in mass, charge and wave function normalisation factors. But in quantum gravity, there are many more infinities.

Most of these infinities are from from calculation of self energies. If they are tachyonic in nature, or even otherwise, they may be cancelled by the Ramond fermion contributions in supergravity. Much work needs to be done in this direction. Perhaps, one has successfully tackled the Lorentz invariance problem of the quantum gravity.

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