Photonic forces in the near field of statistically homogeneous fluctuating sources

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Electromagnetic sources, as e.g. lasers, antennas, diffusers or thermal sources, produce a wavefield that interacts with objects to transfer them its momentum. We show that the photonic force exerted on a small particle in the near field of a planar statistically homogeneous fluctuating source uniquely depends and acts along the coordinate perpendicular to its surface. The gradient part of this force is contributed by only the evanescent components of the emitted field, its sign being opposite to that of the real part of the particle polarizability. The non-conservative force part is uniquely due to the propagating components, being repulsive and constant. Also, the source coherence length adds a degree of freedom since it largely affects these forces. The excitation of plasmons in the source surface drastically enhances the gradient force. Hence, partially coherent wavefields from fluctuating sources constitute new concepts for particle manipulation at the subwavelength scale.

PACS numbers: 42.50.Wk, 87.80.Cc, 42.25.Kb, 05.40.-a

I. INTRODUCTION

Photonic forces are increasingly studied due to their potential in many disciplines ranging from physics and chemistry to biology [1, 3]. Of special importance is the manipulation of dipolar particles, understood as those for which the incident wave excites their first electric and/or magnetic Mie coefficients [4, 5]. Extensive studies done on light from quasi-coherent sources show that it exerts mechanical action on these particles through both their conservative (gradient) and non-conservative components, allowing the design of optical tweezers which interact with objects to transfer them its momentum. We show that the photonic force exerted on a small particle in the near field of a planar statistically homogeneous fluctuating source uniquely depends and acts along the coordinate perpendicular to its surface. The gradient part of this force is contributed by only the evanescent components of the emitted field, its sign being opposite to that of the real part of the particle polarizability. The non-conservative force part is uniquely due to the propagating components, being repulsive and constant. Also, the source coherence length adds a degree of freedom since it largely affects these forces. The excitation of plasmons in the source surface drastically enhances the gradient force. Hence, partially coherent wavefields from fluctuating sources constitute new concepts for particle manipulation at the subwavelength scale.

II. FLUCTUATING OPTICAL FORCES

Let us consider a fluctuating source emitting from the plane \( z = 0 \), (see Fig. 1). We shall assume that the radiated random field is described by an ensemble which is stationary, then we may work in the space-frequency domain [18] so that its electric vector is expressed at frequency \( \omega \) as an angular spectrum of plane waves propagating throughout the half-space \( z > 0 \) [12, 28]:

\[
E(r, \omega) = \int_{-\infty}^{\infty} \mathbf{e}(ks_j, \omega) e^{i\mathbf{s}_j \cdot \mathbf{r}} d^2\mathbf{s}_j \tag{1}
\]

where \( k = \omega/c \), \( c \) being the speed of light in vacuum. The propagation vector \( \mathbf{k} = k \mathbf{s} \) is expressed as \( \mathbf{k} = k(\mathbf{s}_\perp, s_z) \), so that \( \mathbf{s}_\perp = (s_x, s_y) \) are the transversal components of \( \mathbf{s} \) and \( s_z = \sqrt{1 - |\mathbf{s}_\perp|^2} \), \( (|\mathbf{s}_\perp|^2 \leq 1) \), for homogeneous or propagating waves, and \( s_z = i\sqrt{|\mathbf{s}_\perp|^2 - 1} \), \( (|\mathbf{s}_\perp|^2 > 1) \), for evanescent components.

Let a dipolar particle with dynamic electric polarizability \( \alpha_e \), be placed in the source vicinity. Being \( \mathbf{p} = \alpha_e \mathbf{E} \) the dipole moment induced in the particle by the \( \mathbf{E} \) field, the \( ith \) Cartesian component \( (i = 1, 2, 3) \) of the mean force that the emitted wavefield exerts on it at frequency \( \omega \) is \( \mathbb{E} [\mathbf{E} \cdot \alpha_e \mathbf{E}] \) \( \mathbb{E} \) being the expectation operator.

\[
F_i (r, \omega) = \frac{1}{2} \mathbb{E} \left\{ \alpha_e \langle E_i^{\ast} \partial_j E_j \rangle \right\} \\
= \frac{1}{4} \mathbb{E} \alpha_e \partial_i \langle E_i^{\ast} E_j \rangle + \frac{1}{2} \alpha_e \Im \left\{ \langle E_i^{\ast} \partial_i E_j \rangle \right\} \\
= F_i^{\text{grad}} (r, \omega) + F_i^{\text{nc}} (r, \omega), (i, j = 1, 2, 3), \tag{2}
\]
expressed as the sum of a conservative, or gradient force, \( F_{i}^{\text{grad}} \) proportional to \( R_{\alpha} e \) and a non-conservative term \( F_{i}^{\text{nc}} \) proportional to \( 3\alpha_{e} e \). \( R \) and \( 3 \) stand for real ad imaginary parts, respectively. The symbol * denotes complex conjugate. The angular brackets mean ensemble average. Einstein’s convention of omitting the sum symbol \( \sum_{j=1}^{3} \) on the repeated index \( j \) has been used.

On introducing Eq. (1) into (2) one obtains

\[
F_{i}^{\text{grad}} (r, \omega) = -\frac{k}{4} R_{\alpha} e \int_{-\infty}^{\infty} \text{Tr} A_{jk}^{(e)} (ks_{\perp}, ks'_{\perp}, \omega) \times (s_{i}' - s_{i}) e^{-ik(s'-s)\cdot r} d^2 s_{\perp} d^2 s'_{\perp},
\]

(3)

\[
F_{i}^{\text{nc}} (r, \omega) = \frac{1}{2} 3\alpha_{e} 3 \left\{ i k \int_{-\infty}^{\infty} \text{Tr} A_{jk}^{(e)} (ks_{\perp}, ks'_{\perp}, \omega) \times s_{i}' e^{-ik(s'-s)\cdot r} d^2 s_{\perp} d^2 s'_{\perp} \right\},
\]

(4)

where \( A_{jk}^{(e)} \) and \( F_{i}^{\text{nc}} \) are the gradient and non-conservative components of the electric field, respectively.

Analogously, from Eq. (1) one also derives for the non-conservative term

\[
\delta^{(2)} \text{ representing the two-dimensional Dirac-delta function.}
\]

On introducing the above \( \delta \)-function expression for \( A_{jk}^{(e)} (ks_{\perp}, ks'_{\perp}, \omega) \) into Eqs. (3) and (4) one straightforwardly obtains for the gradient force

\[
F_{i}^{\text{grad}} (z, \omega) = \int_{|s_{\perp}|^2 > 1} \delta^{(0)} (|s_{\perp}|^2 - 1) \times \delta^{(2)} (s_{\perp} - s_{\perp}', \omega)
\]

\[
\times (s_{i}' - s_{i}) e^{-2k \sqrt{|s_{\perp}'|^2 - 1} z} d^2 s_{\perp}',
\]

(6)

The subindex in the integral of Eq. (6) means that the integration only extends to the non-radiative region because the difference vector \( s' - s \) in Eq. (6) is clearly zero for propagating waves, \( |s_{\perp}|^2 \leq 1 \). Therefore the radiative components of the field emitted by statistically homogeneous sources do not contribute to the gradient force, which only depends on the evanescent components, \( |s_{\perp}|^2 > 1 \), for which \( s' - s = (0, 0, s_{z}' - s_{z}) \). Hence this force only exists in the near field, and depends on the distance \( z \) of the particle to the source, having solely \( z \)-component normal to its surface. In addition, this force is attractive or repulsive depending on the sign of \( R_{\alpha} e \). Small particles with relative permittivity \( \epsilon > 1 \) have \( R_{\alpha} > 0 \) out of resonance and thus \( F_{i}^{\text{grad}} (z, \omega) \) will drag them towards the source. Conversely, near a resonance \( R_{\alpha} \) may be negative \( \epsilon < 1 \), thus this force being repulsive. However, further study is required in this latter case, since then the particle strongly scatters the field emitted by the source, and the analysis developed here should not be exact due to multiple scattering of the radiation between the source and the particle. Hence it is shown that the gradient force near a statistically homogeneous source is entirely of non-radiative nature and may work as a tractor force troubled by the signs of \( \alpha_{e} \).

A. Statistically homogeneous sources. Gradient and non-conservative forces

Let us address the wide variety of statistically homogeneous sources. Then their electric cross-spectral density tensor

\[
\tilde{E}_{ij} (r_1, r_2, \omega) = \langle E_i^* (r_1) E_j (r_2) \rangle
\]

in the source plane \( z = 0 \) is

\[
\tilde{E}_{ij}^{(0)} (\rho_1, \rho_2, \omega) = \delta_{ij} \delta_{\rho_1, \rho_2} \delta_{\omega, \omega_0}, \quad \rho = \rho_2 - \rho_1, \quad \omega = \omega_0.
\]

It is well-known that

\[
A_{jk}^{(e)} (ks_{\perp}, ks'_{\perp}, \omega) = k^{2} \tilde{E}_{jk} (ks_{\perp}, ks'_{\perp}, \omega),
\]

where \( \tilde{E}_{jk} (ks_{\perp}, ks'_{\perp}, \omega) \) is the four-dimensional inverse Fourier transform of \( \tilde{E}_{ij}^{(0)} (\rho_1, \rho_2, \omega) \). In addition, it was proven that for a homogeneous source the components of the electric angular correlation tensor are \( \delta \)-correlated as

\[
A_{jk}^{(c)} (ks_{\perp}, ks'_{\perp}, \omega) = k^{4} \delta^{(2)} (|s_{\perp} - s'_{\perp}|, \omega)
\]

\[
\times \tilde{e}_{jk}^{(0)} (k s_{\perp}, \omega),
\]

(5)
conservative force $F_{i}^{nc}$ a dependence on $z$ only:

$$F_{i}^{nc}(z, \omega) = F_{i,h}^{nc}(z, \omega) + F_{i,ev}^{nc}(z, \omega)$$

$$= \frac{k^3}{2} \Im \alpha \Im \left\{ i \int_{|\mathbf{s}_z|^2 \leq 1} \mathcal{E}_{jk}^{(0)}(k \mathbf{s}_z, \omega) s_i d^2 \mathbf{s}_z \right\}$$

Thus, while $F_{i,h}^{nc}(z, \omega) > 0$ is constant throughout $z > 0$, as so is the spectrum $S^{(0)}(\omega)$ propagating into $z > 0$ [22], the evanescent waves do not contribute to the non-conservative force $F_{i}^{nc}(r, \omega)$.

In summary there are therefore two force components acting on the particle: $F_{z, ev}^{nc}(z, \omega)$ and $F_{z, h}^{nc}(z, \omega)$, perfectly distinguishable from each other since the former is due to the non-radiative plane wave components of the emitted field, whereas to the latter only the radiative components contribute. As the distance from the particle to the source plane grows to values $z > \lambda$, $F_{z, ev}^{grad}(z, \omega)$ tends to zero due to its evanescent wave composition. Nevertheless, as we shall see, the source coherence length $\sigma$ plays an important role on these contributions.

The integration of Eqs. (6) and (9) using the Gaussian spectral degree of coherence, quoted before: $\mu(\rho, \omega) = \exp \left[ -\rho^2 / 2\sigma^2 \right]$, leads to an analytical expression for the gradient and for the non-conservative force. For the latter, Eq. (7) yields the proportion of radiation pressure and curl components for unpolarized emission. This calculation is straightforwardly done on making: $s_x = s_\perp \cos \phi$, $s_y = s_\perp \sin \phi$, and leads to

$$F_{z}^{grad}(z, \omega) = \Re \alpha S^{(0)}(\omega) e^{-\frac{k^2 \sigma^2}{\sigma^2}} \left\{ s_z \right\}$$

$$- \frac{1}{2} \int_{\Omega} e^{-\frac{k^2 \sigma^2}{\sigma^2}} \operatorname{erf}(k\sigma/\sqrt{2})]$$

where $\operatorname{erf}(x) = 1 - \operatorname{erf}(x)$, $\operatorname{erf}(x)$ being the error function: $\operatorname{erf}(x) = 2/\sqrt{\pi} \int_{t}^{x} e^{-t^2} dt$, and $\operatorname{erf}(x)$ is a positive real function defined as $\operatorname{erf}(x) = \operatorname{erf}(ix)/i$.

**B. The curl force**

It is well-known [20, 30] that the non-conservative part of the force $F_{i}^{nc}$ is the sum of a scattering force, or radiation pressure

$$F_{i}^{nc} = (k/2) \Re \alpha S^{(0)}(\omega) E \times B^*$$

$$= (1/2) \Re \alpha S \left\{ \langle E^* \partial_j E_j \rangle - \langle E^* \partial_j E_j \rangle \right\},$$

given by the averaged field Poynting vector, plus the curl of an electric spin density:

$$F_{i}^{nc, curl} = (1/2) \Re \alpha S \langle E^* \cdot \nabla E \rangle$$

$$= (1/2) \Re \alpha S \langle E^*_j \partial_j E_i \rangle.$$
non-conservative force is:
\[ F_{i}^{nc,pr} = \frac{k^3}{2} 3 \alpha_e \int_{|s| \leq 1} \left[ \frac{\text{Tr} \hat{\epsilon}_{jk}(k s, \omega) s_i - \hat{\epsilon}_{ij}(k s, \omega) s_j}{3F^{(0)}(k s) - F^{(0)}(k s)} \right] s_i d^2 s \]
\[ = \frac{k^3}{2} 3 \alpha_e \int_{|s| \leq 1} \hat{F}^{(0)}(k s) s_i d^2 s = F_{i}^{nc,pr} \]

since the azimuthal angle integrations when \( s_i \) is either \( s_x \) or \( s_y \) is zero.

In a similar manner, the curl force contribution \( F_{i}^{nc,curl} \) to \( F_{i}^{nc} \) is
\[ F_{i}^{nc,curl} = \frac{k^3}{2} 3 \alpha_e \int_{|s| \leq 1} \hat{\epsilon}_{ji}(k s, \omega) s_j d^2 s \]
\[ = \frac{k^3}{2} 3 \alpha_e \int_{|s| \leq 1} \hat{F}^{(0)}(k s) s_j d^2 s = F_{i}^{nc,curl} \]

Namely, for unpolarized radiation:
\[ F_{z}^{nc,pr} = 2F_{z}^{nc,curl} = \frac{2}{3} F_{z}^{nc} \]

III. EXCITATION OF SURFACE PLASMON POLARITONS. NUMERICAL RESULTS

Without loss of generality, we shall also address surface plasmon polaritons (SPPs), excited on the source plane \( z = 0 \). Let this be gold for example, choosing for instance \( \lambda = 459.9\text{nm} \), its permittivity is \( \varepsilon = -2.546 + i3.37 \). The SPP wave vector \( k s = k s^{\text{SPP}} = \pm k \sqrt{\varepsilon} \) corresponds to a pole of the Fresnel coefficient, (either on reflection or on transmission depending on the set-up configuration), \( R(k s, \omega) \). Then it is easy to obtain that the former equations (6) and (7) are valid on substituting \( \hat{\epsilon}_{jk}^{(0)}(k s, \omega) \) by \( \hat{\epsilon}_{jk}^{(0)}(k s, \omega) |R(k s, \omega)|^2 \).

Figure 2 shows the attractive gradient optical force due to evanescent components for two random sources: one without and one with excited SPPs (cf. Fig. 2a and Fig. 2b, respectively). The normalized value \( F_{z}^{\text{grad}}(z, \omega) / (k S^{(0)}(\omega) R\alpha_e/2\pi) \) is represented in arbitrary units. As predicted by Eq. (6), the gradient force drags the particle towards the source plane; (notice that since this normalization does not include \( R\alpha_e \), it does not contain an eventual negative value of this quantity). In both figures we observe its exponential increase as the distance \( z \) of the particle to the source decreases. Nevertheless, this force is mainly governed by the coherence length \( \sigma \). For \( \sigma = \lambda/8 \), the magnitude of this force is maximum, but we observe that around \( \sigma = \lambda/2 \), (blue line), and beyond, there is an important decrease, with values between \( 10^{-2} \) and \( 10^{-3} \) in the magnitude.

![FIG. 2. Pulling gradient optical force due to evanescent components. a. Gradient force in arbitrary units (a.u.) versus distance to the source \( z/\lambda \) for different values of the source coherence length \( \sigma \). b. The same force when SPPs are excited in the source. A significant decrease of the magnitude of this force is clearly seen as \( \sigma \) grows about \( \sigma = \lambda/2 \).](image-url)
if SPPs are excited, an increment on the distance \( \Delta z \) is produced where the gradient component cannot be neglected, (compare Figs. 3a and 3b). The enhancement of the near field intensity due to SPPs resonances then implies a longer-range of the this latter pulling force.

III. CONCLUSIONS

We have reported a new area of study on photonic effects exerted on small particles by discussing near field effects due to fluctuating sources. This opens new perspectives on subwavelength effects and manipulation in such general physics cases that range from light propagation through the turbulent atmosphere \( [20] \), to speckle patterns from a large variety of statistical sources, also including scatterers, optical diffusers \( [37–39] \), as well as thermal or blackbody sources, which opens new possibilities at the subwavelength scale, particularly at the nanoscale. We have seen that in the large variety of statistically stationary and homogeneous sources, only the evanescent components contribute to the gradient forces, while the non-conservative part that contains radiation pressure and curl forces is due solely to emitted propagating components. Hence the subwavelength information is encoded in the gradient forces. Same numerical examples were given for statistically isotropic unpolarized emitted wavefields, showing the important effect that the source coherence length has on these forces, specially on the gradient component. Also the excitation of surface waves importantly enhances the magnitude of these forces. We expect that these findings stimulate experiments and applications by this novel particle manipulation scenario.

ACKNOWLEDGEMENTS

The authors acknowledge support from the Spanish Ministerio de Ciencia e Innovacin (MICINN) through the Consolider NanoLight CSD2007-00046 and FIS2009-1540-01 research grants. J. M. Auñón thanks a scholarship from MICINN.

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