I. INTRODUCTION

In a recent paper [1], two of us proposed a general scenario for unification of dark matter and inflation into a single field. The key ingredient is the survival of a residual amount of the inflaton field’s energy density, which undergoes coherent oscillations and can serve as a cold dark matter candidate. In the context of the string landscape, one can further argue for a non-zero vacuum energy of this field, thus unifying inflation, dark matter and dark energy into a single fundamental field.

We construct an explicit scenario whereby the same material driving inflation in the early Universe can comprise dark matter in the present Universe, using a simple quadratic potential. Following inflation and preheating, the density of inflaton/dark matter particles is reduced to the observed level by a period of thermal inflation, of a duration already invoked in the literature for other reasons. Within the context of the string landscape, one can further argue for a non-zero vacuum energy of this field, thus unifying inflation, dark matter and dark energy into a single fundamental field.

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II. COSMOLOGICAL EVOLUTION

For definiteness, we consider throughout the model of Ref. [1] where the inflaton $\phi$ has potential $V_0 + \frac{1}{2} m^2 \phi^2$. Here $V_0$ has the small value needed to explain the observed dark energy density, and otherwise does not play a significant role. For sufficiently large $|\phi| \gtrsim m_{\text{Pl}}$, this potential drives inflation and produces density perturbations in agreement with observations provided $m \approx 10^{-6} m_{\text{Pl}}$. Subsequently $H \ll m$ at all times, where $H$ is the Hubble parameter, and the $\phi$ field oscillates rapidly on the Hubble timescale. Such an oscillating field behaves as cold dark matter, both in the redshifting of the mean density $\rho_{\phi} \propto a^{-3}$ and in the evolution of perturbations.

Unless some mechanism exists to reduce the energy density of the oscillating field, and indeed to transform some of it into conventional material, it is not possible to recover a satisfactory Big Bang cosmology. The original resolution was reheating — the complete transfer of energy from the inflaton via single-particle decays. Later coherent decays, known as preheating [2, 3, 4, 5], were invoked as well. Such decays may be extremely efficient when the inflaton oscillations are large, but if the only interactions present are annihilations, the process will necessarily shut off once the density reduces. This led Kofman, Linde, and Starobinsky [2, 4] to propose that the residual field could act as dark matter, but in fact detailed calculations [1] show the relic abundance is far too high under standard assumptions. It has usually thus been considered that preheating is followed by a period of reheating leading to complete decay of the inflaton field.

Having recognized that an inefficient reheating is a main concern in our unification scenario, the authors in Refs. [7, 8] suggest that plasma mass effects [10] could

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1 This holds for the original four-legs interaction studied in the preheating literature [2, 3, 4], though a complete decay of the inflaton can be obtained from the introduction of other couplings [6].

2 There exists a sneutrino (which is a scalar field) unification model for inflation and dark matter [5, 6], with similar properties to our phenomenological model; under certain conditions, our approach also applies to it.
provide the mechanism for an incomplete reheating after inflation. The idea is that the decay of the inflaton field is kinematically forbidden in part of the reheating phase. However, the inflaton field is free to decay once it becomes subdominant with respect to the radiation fluid (see the paragraph after Eq. (9) in Ref. [11]), so thermal masses cannot be thought of by itself as a mechanism for incomplete reheating.

We can consider three main possibilities for reducing this excess abundance, while leaving a relic level of oscillations capable of acting as cold dark matter. The first is to modify the shape of the inflaton potential. However it is easy to show that the required level of post-inflationary oscillations is too small for such a modification to work; inflation must end long before the field is near enough the minimum to give the right abundance. This approach is therefore fruitless. The second possibility is to modify the reheating process so that it leaves a relic abundance level; this was the approach of Ref. [1], who chose a phenomenological form for the decay rate intended to correspond to particles which only had annihilation routes rather than decay routes, thus permitting incomplete reduction of the inflaton oscillations. However fine-tuning of the decay rate, unmotivated by fundamental theory, is required to make this scenario work.

In this paper we consider a third possibility, which appears more attractive and natural, which is to consider a brief period of inflation at lower energy densities. Such a period, often called thermal inflation [11] [12] [13], was introduced in order to remove possible relic abundance problems left over by the original high-energy inflation period. This second period would be too short to imprint any new large-scale perturbations, but would reduce the abundance of any relic particles compared to the ultimate radiation background. An oscillating scalar field would have its density reduced by this mechanism.

For future reference, Fig. 1 shows a schematic of the Universe’s evolution for our proposed scenario. As we will show, the required reduction, assuming a period of preheating after inflation but no reheating, is achieved provided thermal inflation lasts for around 12 e-foldings. This is in agreement with the duration already suggested in the literature [11] [12].

III. A DETAILED SCENARIO

We first revisit the calculation of the dark matter mass-per-photon for our scalar field, which ultimately gives the strongest constraint on the parameters of our model.

Let us denote by \( t_* \) the time after which the required Hot Big Bang (HBB) cosmology is recovered\(^3\); hence, the averaged scalar field energy density will be given by \( \rho_\phi = m^2 \phi_0^2 a^3 / a^3 \) for \( t > t_* \). Hereafter, all quantities with an asterisk denote values at time \( t_* \).

As in Ref. [1], we define the scalar field dark matter mass-per-photon as \( \xi_{dm} \equiv \rho_\phi / n_\gamma \), and we assume expansion at constant entropy implying that \( \xi_{dm} / g_* \) remains constant where \( g_\ast \) is the entropic degrees of freedom, usually very similar to the relativistic degrees of freedom that we denote here by \( g_\ast [14] \). It is straightforward to show that, for any time \( t > t_* \),

\[
\frac{\xi_{dm}}{m_{Pl}} = \frac{\pi^2}{2 \zeta(3)} \frac{g_\ast(T_*)}{g_\ast(T)} \frac{m^2 \phi_0^2}{m_{Pl}^2 m_{Pl}^2 T_*^3},
\]

where \( T \) is the temperature of the Universe, as measured from the relativistic particles in thermal equilibrium at time \( t \).

Eq. (1) contains two free parameters, which are the scalar field \( \phi_0 \) and the temperature \( T_\ast \) at the beginning of the HBB; equivalently, we shall call this the time at the end of reheating. It is then necessary to predict the aforementioned values and determine whether they can match the observed value of \( \xi_{dm} \).

In the early Universe, there is first a stage of slow-roll inflation, at the end of which the inflaton field value is \( \phi_{end} \approx 0.28 m_{Pl} \). Then a preheating stage starts in which part of the inflaton energy density is converted into relativistic degrees of freedom. We assume the simplest model of preheating [5], in which the inflaton field is coupled to a massless scalar field \( \chi \) through the four-legs

\[^3\text{Notice that the meaning of } t_*, \text{ is changed with respect to Ref. [1], where it was intended to denote the time at which the equality } H = m \text{ was achieved.} \]
interaction term

$$V_{\text{int}} = \frac{g^2}{2} \phi^2 \chi^2,$$

(2)

where $10^{-10} < g^2 < 10^{-5}$ is the typically-considered range for the coupling constant.

The preheating process ends once the inflaton amplitude is of the order $\phi_{pr} \approx m/g$, at which point the ratio between relativistic ($\chi$ and $\phi$ quanta) and non-relativistic degrees of freedom (coherent oscillations of $\phi$) $\rho_r/\rho_\phi$ is of order of a few $[15]$. In such a case, we cannot expect a prolonged radiation-dominated era after preheating; rather, we expect the appearance of a matter-dominated era just a few e-folds after the end of preheating when the coherent $\phi$ field comes back into domination.

Alternative coupling terms in the potential, such as three-leg decay interactions, can lead to a complete decay of the inflaton field $[4]$: that would also happen in cases where the inflaton field is coupled to fermionic fields $[4]$. However, we do not allow such couplings for the inflaton field in our model, for instance by presuming that the $Z_2$ symmetry $\phi \leftrightarrow -\phi$ is (almost) exact. The $\phi$ field therefore survives right through to the present; however if the radiation simply redshifts away as normal its density will be far too low relative to that of $\phi$ by the present.

Instead, our proposal is that the subsequent evolution of the Universe raises the radiation energy density back above that of the $\phi$ field, so as to re-establish a standard Hot Big Bang evolution. After the preheating process, the energy density of the Universe is composed of relativistic particles and non-relativistic matter represented by the coherent oscillations of the inflaton field. We now assume that there is a second scalar field, initially part of the relativistic thermal bath, that will drive thermal inflation in a later stage. This second field, known as the flaton field, has thermal corrections to its potential which trap it in a false vacuum with energy density denoted by $V$. A hat will be used henceforth to denote quantities related to the flaton field.

According to the standard picture of thermal inflation $[11, 12, 13]$, an inflationary stage starts once the false vacuum energy dominates over the radiation fluid; this happens once the temperature of the latter is $T < 1/V^{1/4}$. Thermal inflation then ends once the thermal corrections to the potential are insufficient to oppose the underlying symmetry-breaking (SB) potential, so that the thermal inflaton can escape from its false vacuum and undergoes an SB transition. This happens once the temperature of the Universe is below the flaton mass scale, $T < m_{pr}$.

In our scenario the sequence is a little different, as seen in Fig. 1 because the flaton density is initially sub-dominant to the oscillating $\phi$ field. However after some interval the $\phi$ density falls below it and thermal inflation starts; sometime afterwards the SB transition then takes place.

After the preheating process the inflaton field redshifts as cold dark matter, $\phi \propto a^{-3/2}$, and we can calculate the total dilution of the inflaton field from the end of pre-heating up to the SB process. The square of the inflaton field at the end of thermal inflation is given by

$$\phi_{SB}^2 = \phi_{pr}^2 \left(\frac{a_{pr}}{a_{SB}}\right)^3 = \phi_{pr}^2 \frac{g_s(T_{pr})}{g_s(T_{SB})} \left(\frac{m_{pr}}{m_{SB}}\right)^3.$$

(3)

To obtain the above equation we are assuming both entropy conservation and that the radiation fluid is in thermal equilibrium. $T_{pr}$ and $T_{SB}$ are the values of the temperature at the end of the preheating stage and at the SB process, respectively; likewise, $\phi_{pr}$ and $a_{SB}$ are the corresponding values of the scale factor.

Once thermal equilibrium is attained at the end of preheating $[5, 15]$, the usual formula for the temperature of the radiation fluid applies, $\rho_r / \rho_\phi \simeq (\pi^2 / 30) g(E_{T_{pr}}) T_{pr}^4$. Recalling that $\rho_r / \rho_{\phi, pr} \simeq T_{pr}^4$, then $T_{pr} \simeq (30 / \pi^2)^{1/4} g^{-1/2} g_s^{-1/4}(T_{pr}) m^4$. Thus, from Eq. (3), the total dilution of the inflaton field is largely determined by the mass scales of the inflationary fields,

$$\phi_{SB}^2 \simeq \frac{3^{3/2}}{30^{3/4}} \frac{g_s(T_{SB}) (g(E_{T_{pr}}) T_{pr}^4 m_{pr}^3)}{g(E_{T_{pr}}) T_{pr}^4} m^3 m^2.$$

(4)

The last process is the reheating of the Universe at the end of thermal inflation. We shall assume that each flaton particle decays at a single-particle decay rate $\Gamma$, which is a new free parameter in our phenomenological approach. In principle the value of $\Gamma$ can be estimated in terms of $m_{pr}$ and $V[12]$, as we discuss later.

The Universe is reheated when $\Gamma \simeq H_*$, where $H_*$ is the Hubble rate at the beginning of the HBB. In between, the Universe is dominated by the energy density of the oscillating flaton field (which redshifts as $a^{-3}$), so that the change in the scale factor is given by

$$a_{SB}^3 = H_{SB}^2 T_{SB}^2 \simeq \frac{3 m_{pr} T_{SB}^2}{8 \pi V}.$$

(5)

The inflaton field is further affected by this expansion as well, so that we get

$$\frac{\phi_{SB}^2}{m_{pr}^2} = \frac{\phi_{SB}^2}{m_{pr}^2} \phi_{SB}^2 \simeq \phi_{SB}^2 \frac{3 \Gamma^2}{8 \pi V},$$

(6)

where $\phi_{SB}^2$ is given in Eq. (4). Finally, the reheating temperature $T_*$ is estimated to be $[4]$

$$T_* \simeq (90 / 8 \pi^3)^{1/4} \sqrt{g_{E^{-1/4}(T_*)}} \sqrt{m_{pr} \Gamma}.$$

(7)

We are now in a position to use the dark matter constraint from Eq. (4), which now takes the form

$$\frac{\xi_{\text{dm}}}{m_{pr}} \simeq \frac{3 \pi}{16 \zeta(3)} \frac{g_s(T_{SB}) g_s(T)}{g_s(T_{pr})} \frac{g(E_{T_{pr}}) T_{pr}^{3/4}(T_*) g(E_{T_{pr}}) T_{pr}^{3/4}(T_{SB})}{g(E_{T_{pr}}) T_{pr}^{3/4}(T_{pr})} \times \left(\frac{3}{8 \pi}\right)^{1/4} g^{-1/2} g_s^{-1/4} \left(\frac{m_{pr}}{V^{1/4}}\right) \frac{m_{pr}}{V^{1/4}}.$$

(8)

4 Incidentally, thermal inflation can resolve the relic abundance troubles, e.g. the gravitino, that such a high temperature $T_{pr} \sim m \simeq 10^{13}$ GeV may lead to $[16]$. 


The measured value of the current dark matter mass per photon is $\xi_{\text{CDM}} = 2.4 \times 10^{-26}$, using values from WMAP5 [17]. We shall take that $g_0(T) \approx g_\text{B}(T) \approx 100$ for temperatures $T \geq T_\text{X}$, and $g_\text{B}(T_0) = 3.9$, where ‘0’ indicates present values; Eq. (5) then becomes

$$g^{-1/2} \frac{m}{\bar{m}} \left( \frac{\bar{m}}{\bar{V}^{1/4}} \right)^4 \sqrt{\frac{\Gamma}{m_{\text{Pl}}} \approx 1.4 \times 10^{-29}}. \quad (9)$$

We define the number of e-folds of thermal inflation as $N_{\text{TI}} \equiv \ln(\bar{V}^{1/4}/\bar{m})$, whereas we denote the number of e-folds between the end of thermal inflation and the completion of reheating, from Eq. (5), as

$$N_{\text{reh}} \equiv \frac{1}{3} \ln \frac{8\pi V}{3m_{\text{Pl}}^2 \Gamma^2}. \quad (10)$$

Thus, an equivalent expression for Eq. (9) is, in terms of the above-defined e-folding numbers,

$$N_{\text{TI}} + \frac{1}{4} N_{\text{reh}} \approx 18 - \ln g^{1/6}, \quad (11)$$

where we have used $m/m_{\text{Pl}} \approx 10^{-6}$. For the expected range $10^{-10} < g^2 < 10^{-5}$, the last term on the righthand side contributes one to two extra e-folds.

Equation (11) is our main result, giving the duration of thermal inflation and subsequent reheating required to give a viable Universal history. We now investigate how achievable it is. The only genuinely free parameter of our model is the decay width $\Gamma$, which determines $N_{\text{reh}}$ (the dependence on $g$ over its expected range is modest). The reason is that thermal inflation parameters are expected to lie in more or less definite ranges of energy [11]. The mass of the flaton field should be of the order of $\bar{m} \approx 10^7$ to $10^8$ GeV, and on general grounds we expect $\bar{V}^{1/4} \approx 10^7$ to $10^8$ GeV, so that $N_{\text{TI}} \approx 11$ with an uncertainty of one or two in either direction. This could be increased by having more than one period of thermal inflation, but we do not need this.

The decay width is sandwiched by two limits: that the decay should take place after thermal inflation is complete, $\Gamma < H_{\text{TH}} \approx 10^{-24} m_{\text{Pl}}$, and that it should be complete before the run-up to nucleosynthesis begins at around $10$ MeV, requiring $\Gamma > 10^{-14} m_{\text{Pl}}$. Figure 2 shows the required value of $\Gamma$ to satisfy the observational constraint [11], as a function of $V$ and for some different values of $N_{\text{TI}}$. We see that the nucleosynthesis constraint can readily be satisfied provided $N_{\text{TI}}$ and $V$ are large enough, and that suitable values lie well within the expected range.

Actually, one can argue that the typical decay width of flaton particles is of the form $\Gamma \approx 10^{-2} \bar{m}(\bar{V}/\bar{m})^{4/3}$. If we plot this in combination with Eq. (11) in Fig. 2 we find that the favoured flaton parameters are $\bar{m} \approx 10^8$ GeV and $V^{1/4} \approx 10^8$ GeV.

We therefore conclude that thermal inflation, with properties already well established in the literature, can indeed dilute the inflaton density sufficiently that it can act as dark matter.

![Fig. 2](image-url)
can be achieved. Moreover, the amount of thermal inflation needed to achieve this is pretty much the amount already taken as standard in the literature, for completely different reasons.

Further, since the thermal inflation scenario comes quite close to failing the nucleosynthesis constraint, it is clear that less drastic modifications to early Universe dynamics, such as a protracted period of matter domination due to temporary domination by some long-lived massive particle species, would not be sufficient to achieve our goals. Extra periods of early Universe inflation appear essential.

It is of course not so attractive that we have had to invoke a second period of inflation, in order to unify the first type of inflaton with dark matter. But at least the thermal inflaton is more grounded in conventional particle physics, specifically supersymmetry. Additionally, even conventional high-scale inflation models may too need thermal inflation in order to solve extra relic abundance problems such as the gravitino \[16\].

The scenario that we have described is based around the quadratic potential, but the construction is of course more general and can be applied to a wide range of inflation models. Indeed, at least within the context of the string landscape, the quadratic potential is actually quite unattractive as its form has to hold over field values many times greater than the (reduced) Planck mass, which is the scale on which we expect the potential to have features \[18\]. It may be much more plausible to consider inflation as occurring near a hilltop \[19\] between neighbouring minima in the landscape; we anticipate the calculation going through more or less as in this paper, but perhaps different in the fine numerical details (for instance, Eq. \[9\] depends on the inflaton mass, whose value depends on the shape of the potential during inflation).

Another reason to consider different potentials is that thermal inflation significantly reduces the number of inflationary e-folds corresponding to the present horizon, perhaps to 40 rather than the usual 50 to 60 \[20\]. This forces the predictions for the observables \(n\) and \(r\) further from the slow-roll limit \(n = 1\) and \(r = 0\), and WMAP5 is starting to exert significant observational pressure against the quadratic potential for low e-folding numbers \[21\]. While this is not yet conclusive, it certainly motivates study of potentials which can produce smaller values of \(r\).

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