Minimizing the Installation Cost of Ground Stations in Satellite Networks: Complexity, Dynamic Programming and Approximation Algorithm

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Abstract—In this letter, we study the optimum selection of ground stations (GSs) in RF/optical satellite networks (SatNets) in order to minimize the overall installation cost under an outage probability requirement. First, we show that the optimization problem can be formulated as a binary-linear-programming problem, and then we give a formal proof of its NP-hardness. Furthermore, we design a dynamic-programming algorithm of pseudo-polynomial complexity with global optimization guarantee as well as an efficient (polynomial-time) approximation algorithm with provable performance guarantee on the distance of the achieved objective value from the global optimum. Finally, the performance of the proposed algorithms is verified through numerical simulations.

Index Terms—RF/Optical satellite networks, site diversity, outage probability, ground station selection, computational complexity, NP-hardness, combinatorial optimization, dynamic programming, approximation algorithm.

I. INTRODUCTION

THE availability of satellite networks (SatNets) is heavily affected by atmospheric impairments, especially rain in radio-frequency (RF) and clouds in optical SatNets. Site diversity technique is able to improve the network availability by mitigating the extremely high attenuation induced by rain and clouds [1]. An optimization method for selecting optical GSs is proposed in [2], taking into consideration the single-site availabilities and the spatial-correlation between sites as well. In [3], a joint optimization algorithm for the design of optical SatNets is presented, which is divided into two parts: the GS positioning and the backbone network optimization considering the optical fiber cost.

Moreover, [4] and [5] present low-complexity heuristic algorithms, which exploit the spatial correlation and the monthly variability of cloud coverage, in order to select the minimum number of GSs in optical SatNets with a geostationary (GEO) or a medium-earth-orbit (MEO) satellite, respectively. A multi-objective optimization approach that achieves various tradeoffs between availability, latency and cost is examined in [6], so as to determine the optimal location of optical GSs for low-earth-orbit (LEO) SatNets. In addition, as concerns the smart gateway diversity optimization in extremely-high-frequency (EHF) SatNets, [7] presents another multi-objective approach using genetic algorithms.

Recently, [8] provides an efficient gradient-projection method to select a given number of GSs maximizing the availability of free-space optical (FSO) SatNets. Finally, a branch-and-bound (B&B) algorithm with global optimization guarantee and low average-case complexity is developed in [9] to select the minimum number of GSs under availability requirements for each time period.

In this letter, we develop useful optimization algorithms for selecting GSs with the minimum installation cost satisfying an outage probability constraint. More specifically, the main contributions of this letter are summarized as follows:

• Mathematical formulation of the optimization problem in binary-linear-programming form with a rigorous proof of its computational complexity (NP-hardness).
• Design of a dynamic-programming algorithm with pseudo-polynomial complexity, which is theoretically guaranteed to find the global optimum.
• Design of a polynomial-time approximation algorithm with provable performance guarantee on the distance between the objective value of the achieved solution and the global optimum (thus achieving a reasonable performance-complexity tradeoff).

Unlike existing approaches that minimize just the number of GSs (cardinality minimization problem, assuming implicitly the same cost for each GS), the proposed algorithms minimize the overall installation cost allowing possibly different costs of GSs.

The remainder of this letter is organized as follows. Section II presents the formulation of the optimization problem with a theoretical proof of its NP-hardness. Subsequently, a global optimization algorithm using dynamic programming is given in Section III, while a polynomial-time approximation algorithm is presented in Section IV. Finally, Section V provides some numerical results and Section VI concludes this letter.

Mathematical notation: The set of positive integers is denoted by \( \mathbb{Z}_+ = \{1, 2, 3, \ldots \} \), while \( \mathbb{0}_K \) and \( \mathbb{1}_K \) are respectively the \( K \)-dimensional all-zeros and all-ones vectors. Moreover, \([\cdot]\) and \(\lceil\cdot\rceil\) stand for the floor and ceiling functions, respectively.

II. PROBLEM FORMULATION & NP-HARDNESS

Consider an RF/optical SatNet with site diversity, consisting of a GEO satellite and a network of geographically distributed GSs. In particular, \( \mathcal{K} = \{1, 2, \ldots, K\} \) denotes the set of candidate locations/sites for installing a GS (\( K \in \mathbb{Z}_+ \)). In addition, we assume that: 1) the network outage probability is defined as the probability of having all GSs in outage and 2) the distance between any two distinct locations is large enough so that the spatial correlation between sites can be ignored, without significant error on the calculation of network outage probability; this implies (approximately) independent weather conditions between the candidate locations.

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In this context, we study the minimization of the total installation cost of GSs satisfying a given outage probability requirement:

\[
\min_{S} \sum_{s \in S} c_s \quad (1a)
\]

subject to \( P_{\text{out}}(S) \leq P_{\text{th}} \) \( S \subseteq \mathcal{K} \) \( (1b) \)

where \( S \) is the set of selected locations, \( c_k \in \mathbb{Z}^+ \) denotes the cost of installing a GS at the \( k \)-th location, \( \forall k \in \mathcal{K} \) (without loss of generality, we assume that \( c_1 \leq c_2 \leq \cdots \leq c_k \); this requires an extra complexity of \( O(K \log K) \) for sorting the sites in ascending-cost order), \( P_{\text{out}}(S) = \prod_{k \in \mathcal{K}} p_k \) is the network outage probability achieved by the set \( S \), with \( p_k \in (0, 1] \) being the outage probability of a GS installed at the \( k \)-th location, \( \forall k \in \mathcal{K} \), and \( P_{\text{th}} \in (0, 1] \) is the (network) outage probability threshold. Note that in the special case where \( c_k = 1 \), \( \forall k \in \mathcal{K} \), we have a cardinality minimization problem.

Afterwards, by introducing the vector \( z = [z_1, z_2, \ldots, z_K] \) of binary (0/1) variables (\( z_k = 1 \) if and only if \( k \in S \)), we can equivalently formulate problem (1) as follows (note that \( \sum_{s \in S} c_s = \sum_{k \in \mathcal{K}} c_k z_k \) and \( P_{\text{out}}(S) = \prod_{k \in \mathcal{K}} (p_k)^{z_k} \)):

\[
\min_{z} f(z) = \sum_{k \in \mathcal{K}} c_k z_k \quad (2a)
\]

subject to \( \prod_{k \in \mathcal{K}} (p_k)^{z_k} \leq P_{\text{th}} \) \( z_k \in \{0, 1\}, \forall k \in \mathcal{K} \) \( (2b) \)

Exploiting the fact that \( x \leq y \iff \log(x) \leq \log(y) \), \( \forall x, y > 0 \), the constraint \( \prod_{k \in \mathcal{K}} (p_k)^{z_k} \leq P_{\text{th}} \) is equivalent to \( \sum_{k \in \mathcal{K}} z_k \log(p_k) \leq \log(P_{\text{th}}) \). Consequently, problem (2) can be written as a binary-linear-programming problem:

\[
\min_{z} f(z) = \sum_{k \in \mathcal{K}} c_k z_k \quad (3a)
\]

subject to \( \sum_{k \in \mathcal{K}} a_k z_k \geq b \) \( z_k \in \{0, 1\}, \forall k \in \mathcal{K} \) \( (3b) \)

where \( a_k = -\log(p_k) \geq 0, \forall k \in \mathcal{K} \), and \( b = -\log(P_{\text{th}}) \geq 0 \). Let \( \mathcal{F} = \{ z \in \{0, 1\}^K : \sum_{k \in \mathcal{K}} (p_k)^{z_k} \leq P_{\text{th}} \} \), or equivalently \( \mathcal{F} = \{ z \in \{0, 1\}^K : \sum_{k \in \mathcal{K}} a_k z_k \geq b \} \), be the feasible set and \( z^* = \arg \min_{z} \{ f(z) : z \in \mathcal{F} \} \) be a (globally) optimal solution of problem (2) (3). Since \( a_k \geq 0, \forall k \in \mathcal{K} \), we can easily prove the following necessary and sufficient feasibility condition: problem (2) (3) is feasible (i.e., \( \mathcal{F} \neq \emptyset \)) if and only if \( \sum_{k \in \mathcal{K}} p_k \leq P_{\text{th}} \) or, equivalently, \( \sum_{k \in \mathcal{K}} a_k \geq b \) (i.e., \( 1 \in \mathcal{F} \)).

Theorem 1 (NP-hardness): The binary-linear-programming problem (3) is NP-hard.

\[\text{Proof:} \text{ In order to prove the NP-hardness of problem (3), it is sufficient to show that a special case of this problem is NP-hard.}
\]

NP-hard. Firstly, let consider the 0-1 knapsack problem which is a well-known NP-hard problem [10]:

\[
\max_{x} \sum_{k \in \mathcal{K}} v_k x_k \quad (4a)
\]

subject to \( \sum_{k \in \mathcal{K}} w_k x_k \leq W \) \( x_k \in \{0, 1\}, \forall k \in \mathcal{K} \) \( (4b) \)

where \( W \in \mathbb{Z}^+ \) is the knapsack capacity, and \( v_k, w_k \in \mathbb{Z}^+ \) are the value and weight of the \( k \)-th item, respectively, \( \forall k \in \mathcal{K} \). Moreover, applying the polynomial-time, \( \Theta(k) \), variable transformation \( x_k = 1 - z_k \), \( \forall k \in \mathcal{K} \), we get the following equivalent problem:

\[
\min_{z} \sum_{k \in \mathcal{K}} v_k z_k \quad (5a)
\]

subject to \( \sum_{k \in \mathcal{K}} w_k z_k \geq W' \) \( z_k \in \{0, 1\}, \forall k \in \mathcal{K} \) \( (5b) \)

where \( W' = \sum_{k \in \mathcal{K}} w_k - W \). Without loss of generality, we can assume that the integer \( W' \geq 0 \); otherwise the optimal solution of problem (5) is trivially equal to \( 0 \). Obviously, the NP-hard problem (5) is a subcase of problem (3), and this completes the proof. \( \square \)

III. GLOBAL OPTIMIZATION USING DYNAMIC PROGRAMMING

Due to the fact that problem (3) is NP-hard, it cannot be (globally) solved in polynomial time unless P=NP. Nevertheless, we can use a powerful optimization technique, namely, dynamic programming (DP), in order to achieve the global minimum with pseudo-polynomial complexity.

DP performs an intelligent enumeration of all the feasible solutions, thus providing a global optimization guarantee. In particular, DP follows a bottom-up approach by decomposing the problem into “smaller” subproblems and combining their optimal solutions (using a recursive formula) in order to find an optimal solution to the original problem; this is known as the principle of optimality and such problems are said to have optimal substructure [10]. Furthermore, DP is a tabular method where each subproblem is solved only once and then its solution is stored in a table, so that it can be readily used (without re-computation) by “larger” problems when needed.

Let \( C \) be an integer upper bound on the optimum value of \( \text{problem} (3) \), i.e., \( f(z^*) \leq C, \forall z \in \{0, 1, \ldots, Z\} \) with \( Z = \sum_{k \in \mathcal{K}} c_k \) (this is the “worst” upper bound that can be used). In addition, we define the following bivariate function \( R(i, j) = \max_{z \in \{0, 1, \ldots, C\}} \left\{ \sum_{k \in \mathcal{K}} a_k z_k : \sum_{k \in \mathcal{K}} c_k z_k = j, \ z_I \in \{0, 1\}^I \right\} \) \( (6) \), where \( I = \{1, 2, \ldots, i\} \) and \( z_I = [z_1, z_2, \ldots, z_i] \), with \( i = 0 \Rightarrow I = \emptyset \) and \( \sum_{k \in I} a_k z_k = \sum_{k \in I} c_k z_k = 0 \). If this maximization problem is infeasible, then \( R(i, j) = -\infty \).

\[\text{Note that the optimum objective values of problems (4) and (5) differ only by a constant, i.e., } \sum_{k \in \mathcal{K}} v_k x_k = \sum_{k \in \mathcal{K}} v_k - \sum_{k \in \mathcal{K}} v_k z_k.\]
Theorem 2 (Computation of the global optimum): Assuming that problem \( \mathcal{P}(\mathcal{T}) \) is feasible, its global minimum can be found as follows: \( f(\mathbf{z}^*) = \min \{ j \in \mathbb{C}_0 : R(K,j) \geq b \} \).

Proof: Firstly, observe that when \( i = K \), we have \( \mathcal{I} = \mathcal{K} \) and \( \mathbf{z}_\mathcal{I} = \mathbf{z} \). Secondly, we know that \( f(\mathbf{z}^*) \in \mathbb{C}_0 \) and \( R(K,f(\mathbf{z}^*)) \geq \sum_{k \in \mathcal{K}} a_k z_k^* \geq b \). Now, suppose that \( f(\mathbf{z}^*) \neq j^* \), where \( j^* = \min \{ j \in \mathbb{C}_0 : R(K,j) \geq b \} \). Let examine two cases: 1) \( f(\mathbf{z}^*) < j^* \) and 2) \( f(\mathbf{z}^*) > j^* \). In the former case, we would have that \( R(K,f(\mathbf{z}^*)) < b \), which leads to a contradiction. Moreover, the latter case contradicts the global optimality of \( f(\mathbf{z}^*) \). Hence, \( f(\mathbf{z}^*) = j^* \) and Theorem 2 has been proven. \( \square \)

Subsequently, we partition the feasible set of problem (6), by setting \( z_1 = 0 \) and \( z_i = 1 \), respectively (note that \( \mathcal{I}\setminus\{i\} = \{1, 2, \ldots, i-1\} \)):

\[
\max_{\mathbf{z}_\mathcal{I}} \left\{ \sum_{k \in \mathcal{K}} a_k z_k : \sum_{k \in \mathcal{K}} c_k z_k = j, \ \mathbf{z}_\mathcal{I} \in \{0,1\}^i, \ z_i = 0 \right\} = \\
\max_{\mathbf{z}_\mathcal{I}(i)} \left\{ \sum_{k \in \mathcal{K}\setminus\{i\}} a_k z_k : \sum_{k \in \mathcal{K}\setminus\{i\}} c_k z_k = j, \ \mathbf{z}_\mathcal{I}(i) \in \{0,1\}^{i-1} \right\} = \\
R(i-1,j)
\]

(7)

\[
\max_{\mathbf{z}_\mathcal{I}} \left\{ \sum_{k \in \mathcal{K}} a_k z_k : \sum_{k \in \mathcal{K}} c_k z_k = j, \ \mathbf{z}_\mathcal{I} \in \{0,1\}^i, \ z_i = 1 \right\} = \\
= \max_{\mathbf{z}_\mathcal{I}(i)} \left\{ \sum_{k \in \mathcal{K}\setminus\{i\}} a_k z_k : \sum_{k \in \mathcal{K}\setminus\{i\}} c_k z_k = j - c_i, \ \mathbf{z}_\mathcal{I}(i) \in \{0,1\}^{i-1} \right\} = \\
a_i + R(i-1,j-c_i)
\]

(8)

Therefore, we have the following recursive formula \( \forall i \in \mathcal{K} = \{1,2, \ldots, K\} \) and \( \forall j \in \mathbb{C}_0 = \{0,1, \ldots, C\} \):

\[
R(i,j) = \begin{cases} 
\max \{ R(i-1,j), a_i + R(i-1,j-c_i) \}, & \text{if } j \geq c_i \\
R(i-1,j), & \text{otherwise}
\end{cases}
\]

with initial conditions: a) \( R(0,0) = 0 \) and b) \( R(0,j) = -\infty \), \( \forall j \in \mathbb{C} = \{1,2, \ldots, C\} \). Observe that if \( j < c_i \), then problem \( \mathcal{P}(\mathcal{U}) \) is definitely infeasible, so \( R(i-1,j-c_i) = -\infty \); this explains the 2nd branch in (9).

Algorithm 1 presents a DP procedure based on the previous analysis. First, we compute the coefficients \( [a_k]_{k \in \mathcal{K}} \) and \( b \) is \( \Theta(K) \). Moreover, the greedy method used to find an upper bound on the optimum value requires at most \( K \) iterations (since \( 1 \in \mathcal{F} \)), thus having \( O(K) \) complexity. In addition, the computation of the table \( R \) requires \( \Theta(KC) \) arithmetic operations in total. Finally, the computation of \( j^* \) requires \( O(C) \) comparisons, while the complexity of reconstructing/tracing the solution is \( \Theta(K) \) since it starts in row \( K \) of the table and moves up one row at each step. Ultimately, the overall complexity of Algorithm 1 is \( \Theta(KC) = O(K \mathcal{C}) = O(K^2 c_{\max}) \), because \( C \leq \mathcal{C} \leq K c_{\max} \) where \( c_{\max} = \max_{k \in \mathcal{K}}(c_k) = c_K \). As a result, the proposed DP algorithm has pseudo-polynomial time complexity [10].

Algorithm 1 is theoretically guaranteed to find a (globally) optimal solution.

**Complexity of Algorithm 1:** The complexity of computing the coefficients \( [a_k]_{k \in \mathcal{K}} \) and \( b \) is \( \Theta(K) \). Moreover, the greedy method used to find an upper bound on the optimum value requires at most \( K \) iterations (since \( 1 \in \mathcal{F} \)), thus having \( O(K) \) complexity. In addition, the computation of the table \( R \) requires \( \Theta(KC) \) arithmetic operations in total. Finally, the computation of \( j^* \) requires \( O(C) \) comparisons, while the complexity of reconstructing/tracing the solution is \( \Theta(K) \) since it starts in row \( K \) of the table and moves up one row at each step. Ultimately, the overall complexity of Algorithm 1 is \( \Theta(KC) = O(K \mathcal{C}) = O(K^2 c_{\max}) \), because \( C \leq \mathcal{C} \leq K c_{\max} \) where \( c_{\max} = \max_{k \in \mathcal{K}}(c_k) = c_K \). As a result, the proposed DP algorithm has pseudo-polynomial time complexity [10].

**Remark 1:** Strictly speaking, Algorithm 1 is an exponential-time algorithm, since the size of the input is upper bounded by \( O(K \log c_{\max}) = O(K \log \mathcal{C}) \), because \( c_{\max} \leq \mathcal{C} \). Nevertheless, under certain conditions, this algorithm is practical despite its exponential worst-case complexity. For example, if \( \mathcal{C} = O(K^d) \) for some constant \( d \geq 0 \) (which is usually the case in practice), then the running time of Algorithm 1 will be polynomial in \( K \).

**Remark 2:** In Algorithm 1, due to the fact that \( C = \sum_{k \in \mathcal{U}} c_k \) for some \( \mathcal{U} \subseteq \mathcal{K} \) depending on \( [a_k]_{k \in \mathcal{K}} \) and \( b \), we can divide all coefficients \( [c_k]_{k \in \mathcal{K}} \) with their greatest common divisor (i.e., \( c_k' = c_k / \zeta \), \( \forall k \in \mathcal{K} \), where
Algorithm 2 DP-based Approximation Algorithm (DPAA)

Input: \( K \in \mathbb{Z}_+ \), \( x = [c_1, c_2, \ldots, c_k] \in \mathbb{Z}_+^k \) where \( c_1 \leq c_2 \leq \cdots \leq c_K \), \( p = [p_1, p_2, \ldots, p_K] \in [0, 1]^K \), \( P_{\text{out}}^b \in (0, 1) \) with \( \prod_{k \in K} p_k \leq P_{\text{out}}^b \)\( > 0 \)

Output: \( \tilde{z} \in F \) such that \( f(\tilde{z}) \leq f(\tilde{x}) \leq f(z^*) + \min(\epsilon c_{\text{max}}, C) \)

1: \( \hat{\vartheta} := \epsilon c_{\text{max}}/K \), where \( c_{\text{max}} = \max_{k \in K} c_k = c_K \)
2: \( \hat{c}_k := [c_k/\hat{\vartheta}], \forall k \in K \)
3: Run Algorithm 1 with input \([K, \hat{c}, p, P_{\text{out}}^b]\) and return the optimal solution \( z^* \), where \( \hat{c} = [c_1, c_2, \ldots, c_k] \in \mathbb{Z}_+^k (c_1 \leq c_2 \leq \cdots \leq c_K) \)

\( \hat{\vartheta} = \gcd(c_1, c_2, \ldots, c_K) \in \mathbb{Z}_+ \) without altering the set of optimal solutions. In this way, the complexity of Algorithm 1 can be reduced, since \( C' = \sum_{u \in \mathbb{Z}_+^k} c'_u = C/\hat{\vartheta} \leq C \).

IV. POLYNOMIAL-TIME APPROXIMATION ALGORITHM

Subsequently, a practical and efficient (polynomial-time) approximation algorithm with provable performance guarantee is given. The design of the approximation algorithm is based on the idea of trading accuracy for running time, thus achieving a reasonable tradeoff between performance and complexity.

The approximation algorithm utilizes Algorithm 1 and is shown in Algorithm 2. Specifically, Algorithm 2 is similar to the fully polynomial-time approximation scheme (FPTAS) for the knapsack problem provided in [11], which is inspired by the work of Ibarra and Kim [12]. Moreover, note that \( \vartheta > 0 \), and therefore \( \hat{c}_k \in \mathbb{Z}_+, \forall k \in K \).

**Theorem 3** (Performance guarantee): Assuming that problem (2) is feasible, Algorithm 2 takes a parameter \( \epsilon > 0 \) as input and produces an approximate solution \( \tilde{z} \in F \) such that
\[
f(\tilde{z}) \leq f(\tilde{x}) \leq f(z^*) + \min(\epsilon c_{\text{max}}, C),
\]
due to the fact that \( x \leq [\tilde{x}] < x + 1 \), we have \( c_k/\hat{\vartheta} \leq c_k < c_k/\hat{\vartheta} + 1 \Rightarrow c_k \leq \hat{c}_k < c_k + \frac{\vartheta}{c_k} \). Also, let the function \( g(z) = \sum_{k \in \mathbb{Z}_+} \hat{c}_k z_k \). From \( \hat{c}_k < c_k + \vartheta \), we deduce that \( \hat{c}_k z_k \leq c_k z_k + \frac{\vartheta}{c_k} z_k, \forall k \in K \) (because \( z_k \geq 0 \)). By taking the sum for all \( k \in K \), we obtain
\[
\hat{\vartheta} g(z^*) \leq f(\tilde{x}) + \vartheta \sum_{k \in \mathbb{Z}_+} z_k^* \leq f(\tilde{z}) + \vartheta K = f(z^*) + \epsilon c_{\text{max}}.
\]
Since \( \tilde{z} \in F \), we conclude that \( g(\tilde{z}) \leq g(z^*) \Rightarrow g(\tilde{z}) \leq g(z^*) \) because \( \hat{\vartheta} > 0 \), and therefore \( \hat{\vartheta} g(\tilde{z}) \leq f(z^*) + \epsilon c_{\text{max}} \). In addition, from \( c_k \leq \hat{c}_k \Rightarrow c_k z_k \leq c_k z_k \), \( \forall k \in K \) (because \( z_k \geq 0 \)). By taking the sum for all \( k \in K \), once more, we get \( f(\tilde{z}) \leq \hat{\vartheta} g(\tilde{z}) \). Consequently, \( f(\tilde{z}) \leq f(z^*) + \epsilon c_{\text{max}} \). Afterwards, due to the fact that \( f(\tilde{z}) \) and \( f(z^*) \) are integers, we have \( f(\tilde{z}) - f(z^*) \leq \epsilon c_{\text{max}} \). Moreover, since \( f(\tilde{z}) \leq C \) and \( f(z^*) \geq 0 \), we obtain \( f(\tilde{z}) - f(z^*) \leq \epsilon c_{\text{max}} \). Hence, \( f(\tilde{z}) - f(z^*) \leq \min(\epsilon c_{\text{max}}, C) \), because it holds that: \( x \leq u \) and \( x \leq u \Rightarrow x \leq \min(u, v) \).

Furthermore, if \( 0 < \epsilon < 1/c_{\text{max}} \Rightarrow 0 < \epsilon c_{\text{max}} < 1 \Rightarrow \epsilon c_{\text{max}} = 0 \), and thus \( f(x) \leq f(\tilde{z}) \leq f(z^*) \Rightarrow f(\tilde{z}) = f(z^*) \). In other words, for any \( 0 < \epsilon < 1/c_{\text{max}} \), the approximation algorithm will be forced to produce an optimal solution.

**Complexity of Algorithm 2** The complexity of DPAA is mainly due to Algorithm 1, so it is \( O(K^2 c_{\text{max}}) = O(K^2 [K/\epsilon]) = O(K^3/\epsilon) \), where \( c_{\text{max}} = \max_{k \in K} \{c_k\} = [c_{\text{max}}/\epsilon] = [K/\epsilon] \). As a result, Algorithm 2 has polynomial complexity in \( K \) and \( 1/\epsilon \). Observe that, for any fixed \( \epsilon > 0 \), DPAA has cubic complexity \( O(K^3) \).

Finally, the performance and complexity of all optimization algorithms are summarized in Table I. The exhaustive search algorithm simply checks all subsets of \( K \) and selects that with the minimum objective value satisfying the outage probability constraint. Therefore, it requires \( \sum_{i=0}^{K} \binom{K}{i} \leq K \sum_{i=0}^{K} \binom{K}{i} = O(2^K K) \) arithmetic operations to find the global minimum.

### TABLE I

| Optimization Algorithm | Performance Guarantee | Computational Complexity |
|------------------------|-----------------------|-------------------------|
| Exhaustive Search      | Global Optimization   | \( O(2^K K) \)          |
| DP (Algorithm 1)       | Global Optimization   | \( O(K^3/\epsilon) \)   |
| DPAA (Algorithm 2)     | Global Optimization   | \( O(K^3 K) \)          |
| DPAA (Algorithm 2) with \( \epsilon = 0.1 \) | Global Optimization | \( O(K^3 K) \)          |
| DPAA (Algorithm 2) with \( \epsilon = 10 \)    | Global Optimization   | \( O(K^3 K) \)          |
| DPAA (Algorithm 2) with \( \epsilon = 20 \)   | Global Optimization   | \( O(K^3 K) \)          |

Fig. 1. Performance comparison between optimization algorithms.

V. NUMERICAL SIMULATIONS AND DISCUSSION

In this section, we examine the performance of the proposed optimization algorithms through numerical simulations. In particular, the following system parameters have been used: \( K = 25 \) and \( c_k = [k/5], \forall k \in K \) (\( C = \sum_{k \in K} c_k = 75 \) and \( c_{\text{max}} = 5 \)). Moreover, we generate 100 independent (feasible) optimization problems where the outage probabilities of GSs, \( p_k \) \( k \in K \), are uniformly distributed in the interval \((0.25, 0.75)\).

Fig. 1 illustrates the average installation cost, versus the outage probability threshold, achieved by a) the exhaustive search, b) DP (Algorithm 1), and c) DPAA (Algorithm 2) for different values of the parameter \( \epsilon \). More specifically, DP and
DPAA with $\epsilon = 0.1$ have identical performance with the exhaustive search; this is in agreement with the theory presented in the previous sections, since DP is a global optimization algorithm and DPAA is forced to produce an optimal solution when $0 < \epsilon < 1/\epsilon_{\text{max}} = 0.2$ (see Theorem 3). Furthermore, as expected, DPAA leads to higher installation cost (with lower complexity) by increasing the parameter $\epsilon$. Finally, it is interesting to note that, for $\epsilon \in \{10, 20, 30\}$, the actual distance of the objective value achieved by DPAA from the global minimum is much less than the absolute-error bound, i.e., $f(\tilde{z}^*) - f(z^*) \ll \min(\lfloor \epsilon \epsilon_{\text{max}} \rfloor, \overline{C})$.

VI. CONCLUSION

In this letter, we have dealt with the minimization of the installation cost of GSs in RF/optical SatNets satisfying an outage probability constraint. In particular, the examined problem has been theoretically proven to be NP-hard. Moreover, we have presented a global optimization algorithm with pseudo-polynomial complexity as well as a polynomial-time approximation algorithm with provable performance guarantee.

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