Some of the most prominent theoretical predictions of the modern times such as Unruh effect, Hawking radiation, gravity assisted particle creation, etc. stem support from the fact that particle content of a quantum field is observer-dependent. For example, the Unruh effect states that a uniformly accelerating observer visualizes the vacuum of an inertial observer as thermally populated. Till date none of these predictions have been experimentally realized because one needs extremely strong gravity (or acceleration) to bring such effects within existing experimental resolution. In this work, we show that a Post-Newtonian rotating atomic system inside a far-detuned cavity may experience strongly enhanced field content in the otherwise inertial vacuum. As a result, the spontaneous emission rate gets enhanced by many orders of magnitude with a shifted spectrum. Based on the current available technology, we propose an opto-mechanical setup which is capable of realizing such Unruh-like effects.

Random fluctuations in vacuum is a well known phenomenon in quantum field theory which manifests itself in various places such as Casimir effect, Hawking radiation, Schwinger effect etc. Typically of tiny strength, these random fluctuations can be amplified in extreme physical scenarios, e.g., strong gravitational field. Due to the equivalence principle, a similar amplification in these fluctuations can be observed also in highly accelerating frames. For example, the Unruh effect states that a uniformly accelerating observer perceives the inertial vacuum as thermally populated. Therefore, the particle content of a quantum field is observer-dependent.

An operational approach to detect the presence of particles in a field configuration is through a Unruh-Dewitt detector (UDD): a proposed quantum device in which a quantum mechanical system couples with a permeating quantum field and registers transitions if it sees the field in a non-vacuum state. Such a detector is put on a non-inertial trajectory, when the state of the field isvacuum according to inertial observers, and the number of “clicks” are observed. The discrepancy between the number of clicks registered by UDD in a non-inertial and inertial scenario characterizes the field content in non-inertial frames.

Despite a robust theoretical genesis, such non-inertial transitions remain experimentally unverified till date, primarily because any appreciable non-inertial effect would demand ridiculously high acceleration. For example, any observable thermal signature in the Unruh effect will require a uniform acceleration as large as $10^{26} m s^{-2}$. There have been attempts of enhancing such effects using techniques such as ultra intense LASERs, Penning trap and optical cavities obtaining high accelerations momentarily to witness any alteration in the response pattern of a quantum system. The approaches suggested for capturing the finite temperature effects of accelerating system range from monitoring thermal quivering to decay of accelerated protons or even radiation emission from accelerated systems. Also, other quantum features such as judicious selection of Fock states and utilizing geometric phase have been suggested to enhance the effect of non-inertial motion. Unfortunately, all these efforts remain far from being realized as of today. Any observation of such non-inertial distortion of the field content is not only important in itself but also substantiates our understanding of gravitational particle creation.

Constant acceleration is not the only way to observe the modified field content due to non-inertial motion. In principle, any non-inertial motion should modify the field content, compared to as viewed by inertial observers. For example, a circularly rotating UDD will also register a non-zero response. In this article, we propose a setup to facilitate the observation of the Unruh-type effect by measuring the change in the transition rates of a rotating atom inside an electromagnetic cavity. We show that the transition rate between suitably selected energy levels of an atom placed in a far detuned electromagnetic cavity can be significantly influenced by the non-inertial motion of the atom. We show that in the leading order Post-Newtonian limit, by choosing the parameters appropriately, the transition rates can be made directly sensitive to the non-inertial motion making our setup much more sensitive than setups proposed till date.

Consider an atom with two energy levels separated by energy $\Delta E = \hbar \Omega$. Due to its electronic configuration, the atom possesses a well defined electric dipole moment, say $\mu$, which will interact with electric field of...
the surrounding. Hence the interaction between the atom and the surrounding is \( H_I = -d^\mu E_\mu \), where \( E_\mu \) represents the electric field operator [22]. This interaction can cause a transition between the excited state and the ground state of the atom (spontaneous emission). The transition rate can be calculated using Fermi’s golden rule which relates the transition rate between two energy levels \( |\psi_i\rangle \) and \( |\psi_f\rangle \) in the presence of the interaction Hamiltonian \( H_I \) as [23]

\[
\Gamma = \frac{2\pi}{\hbar} |\langle \psi_f, \psi_f | H_I |\psi_i, \psi_i \rangle|^2 \sigma, \tag{1}
\]

where \( \sigma \) is the density of final states \( |\psi_f\rangle \) and \( |\phi_{i/f}\rangle \) is the initial/final state of the electric field. For an atom at rest interacting with the surrounding electric field set up in vacuum state (as per the laboratory inertial frame), transition rate \( \Gamma_0 = \Omega^3 d^2/3\pi\epsilon_0\hbar c^3 \), where \( d^2 = |\langle \psi_f | d |\psi_i \rangle|^2 \) (see appendix).

**Electric dipole coupling in the accelerated frame:** If an atom is set in an eternally accelerating motion, the rate of transition or the spontaneous emission both will expectedly become thermal [7]. In the Unruh effect, an atom moving with uniform acceleration \( a \) experiences a temperature \( T = \hbar a/2\pi k_B c \) which results in the modified transition rate \( \Gamma_T = \Gamma_0/(1 - \exp(-2\pi\epsilon_0\hbar/a)) \). Any appreciable effect of the acceleration induced temperature will require \( a \sim 2\pi\epsilon_0\hbar; \) however achieving such higher acceleration scales, for typical atomic transitions, is beyond the reach of current technology.

In order to observe the Unruh effect, one may consider reducing \( \Omega \) to very low values, such as radio frequencies (10\(^7\) Hz). Unfortunately, at such low frequencies, the natural spontaneous emission rates (\( \Gamma_0 \)) of typical atoms are so vanishingly small that any hope to see the non inertial modifications even at high accelerations becomes very feeble. If by any means we increase \( \Gamma_0 \) while keeping \( \Omega \) low then the non inertial effects also can possibly be observed efficiently. Electromagnetic cavity is a well known tool to achieve this goal [24]. The electromagnetic cavity modifies the boundary conditions for the surrounding field which leads to a change in density of states of the electric field modes. This in turn, alters the transition rate of the atom. In a cavity of volume \( V \) and quality factor \( Q \) the spontaneous emission rate at resonance is given by

\[
\Gamma_{\text{cav}} \approx \frac{d^2}{\epsilon_0\hbar V} \frac{1}{Q}, \tag{2}
\]

which is independent of the transition frequency \( \Omega \). Next we show that a rapidly rotating quantum system inside an electromagnetic cavity, displays an enhanced transition rate.

**UNRUH LIKE EFFECT INSIDE CAVITY**

The proposed setup consists of an atom rotating in a circular trajectory of radius \( R \) and frequency \( \omega \) inside an electromagnetic cavity. The direction of the atomic transition dipole \( d^\mu \) is taken to be orthogonal to the plane of rotation. The relevant expression for the spontaneous emission rate, if we do the expansion for the time averaged transition rate (over many cycles) in the leading order of \( \zeta \equiv R^2\omega^2/c^2 \), reads (see appendix)

\[
\Gamma_{\text{cav}} \sim \frac{d^2}{\epsilon_0\hbar V} \int_0^\infty dk \rho(k)\omega_k \left[ \frac{R^2}{2c^2} \left( \frac{\omega^2}{3} + \frac{\omega_k^2}{30} \right) \{\delta(\Omega - \omega - \omega_k) + \delta(\Omega - \omega + \omega_k)\} \right. \\
- \frac{1}{10} \frac{R^2\omega_k^2}{c^2} \delta(\Omega - \omega - \omega_k) + O \left( \sum_{n>1} \frac{R^{2n}\omega_k^{2n}}{c^{2n}} \delta(\Omega \pm n\omega - \omega_k) \right), \tag{3}
\]

with \( \Omega = (1 - \omega^2R^2/c^2)^{1/2} \approx \Omega \). where \( \rho(k) \) is the density of field states of the electromagnetic modes inside the cavity. In the case of \( \omega \gg \Omega \), the above expression can be approximated in the leading order of \( \zeta \) to

\[
\Gamma_c \sim \frac{d^2}{\epsilon_0\hbar V} \left[ \frac{\omega_c}{c} \rho(\Omega) + \frac{33\zeta}{60} \omega(\omega) \right]. \tag{4}
\]

The density of states inside the cavity is typically taken as a Lorentzian profile of width \( \omega_c/Q \)

\[
\rho(k) \sim \frac{(\omega_c/Q)}{(\omega_c/Q)^2 + (k - \omega_c)^2} \tag{5}
\]

with the quality factor \( Q \gg \omega_c \), and \( \omega_c \) being the normal mode frequency of the cavity. We see that if we put \( \omega_c \sim \Omega \), i. e., when the cavity is in resonance with the atomic transition, the first term in the above equation dominates and we obtain an expression close to the standard spontaneous rate inside the cavity as given in Eq. (2). However, if \( \omega_c \sim \omega \) then the second term of dominates the standard term by many orders, and can be approximated to

\[
\Gamma_c \sim \frac{33\zeta}{60} \frac{d^2}{\epsilon_0\hbar V} \frac{1}{Q}. \tag{6}
\]

If the parameters inside the cavity could be arranged to the values \( \zeta \sim 10^{-13}, d \sim 10^{-29}\text{m}, V \sim 10^{-14}\text{m}^3 \) with a quality factor \( Q \sim 10^8 \) we will have a transition rate of the order of \( \Gamma_c \sim 10^{-8}\text{s}^{-1} \). For a system of (say) \( 10^6 \) atoms we can expect hundreds of events per hour. As a
FIG. 1. Schematic diagram of the proposed experiment. A silica nanoparticle is levitated and spun at a frequency \( \omega \approx 5 \) GHz by a circularly polarized focused laser beam. The atoms placed on the silica particle feature a characteristic electric dipole allowed transition at frequency \( \Omega \approx 10^{-100} \) MHz. The nanoparticle is placed between the plates of a capacitor, which is part of a superconducting microwave LC resonator tuned at \( \omega_c \). The non-inertial motion of the atoms riding the silica particle will cause enhanced stimulated atomic transitions with mean frequency \( \omega + \Omega \approx \omega_c \). Emitted photons will be detected by a quantum limited microwave amplifier.

A comparison, in this far detuned region, the contribution from the inertial term is \( \Gamma_c \sim 10^{-10} s^{-1} \). Furthermore, this transition would be mediated by photons with frequency \( \omega_c \), which is very different from the natural transition frequency \( \Omega \). Observation of the photons with frequency \( \omega_c \) itself will be a testimony to the Unruh-like effect. In the following, we present experimental feasibility of our proposal.

**PROPOSED EXPERIMENTAL SET-UP**

A schematic diagram for the proposed experimental setup is shown in Fig. 1. In our setup, a silica particle (SiO\(_2\)) with radius \( R \approx 50 \) nm is trapped by optical gradient force in the focal spot of a focused optical beam which falls inside the sensitive volume of a lumped element high Q microwave cavity with frequency \( \omega_c \approx 5 \) GHz. The silica particle is then rotated with a frequency \( \omega \approx \omega_c \) by means of circularly polarized light [25, 26]. Suitable atoms with dipole allowed transition frequency \( \Omega \approx 10\text{MHz} \) are placed atop the silica particle. The non-inertially stimulated transitions, upconverted to \( \omega + \Omega \approx \omega_c \), are detected by a quantum limited microwave amplifier. The whole experimental setup can be divided into four main components: (a) Levitated opto-mechanical oscillator on which we place our atoms, (b) microwave cavity used to suppress undesired transition lines of the atom, (c) the appropriate atom with the desired range of transition frequency, and (d) photon detection. Below, we present details of each of the experimental components.

*Levitated opto-mechanical oscillator.* A silica nanoparticle with radius \( R \approx 50 \) nm will be trapped by the optical gradient force in the focal spot of a 1550 nm laser beam. Recent experiments have shown the ability to control the translational motion of trapped particles, for instance by parametric feedback cooling [27, 28], state preparation [29, 30], squeezing [31] and an unprecedented potential for ultra-precise force sensing [32, 33]. Beside the translational motion, different rotational motions have been experimentally demonstrated in levitated opto-mechanical systems, such as libration [25, 34], free rotation [35, 36] and precession [37] of non-spherical nanoparticles. The free rotation can be stabilized to utmost precision, \( Q_R = 10^{11} \) [38], and can reach very large frequencies in the GHz range [39, 40], only limited by the centrifugal damage threshold of the rotating particle. Here, we propose to use such high rotational frequency at the maximum experimentally demonstrated value \( \omega \approx 5 \) GHz in order to observe the stimulated emission by the non-inertial motion [41].

*Microwave cavity.* A lumped LC resonator will be used as a microwave cavity [42]. The particle will be placed inside the capacitor gap, as shown in Fig. 1. The capacitor could be made with the ends of two wires in front of each other, with the wires connected to a properly designed inductor. With a quasi-cubic geometry the microwave cavity could be designed with resonator mode volume as low as \( V \approx 10^{-14} \text{ m}^3 \). The gap between the capacitor plates (of the order of 20\( \mu \)m) will be large enough to accommodate the confining laser beam waist. The LC resonator will be cooled to cryogenic temperatures, to reduce the thermal spontaneous emission. For temperatures around milliKelvin, it is expected to be in the quantum regime. In addition, superconducting wiring is required for the cavity in order to achieve the highest possible electrical Q factor. Typical resonators made on thin films at GHz frequency feature Q factors up to \( 10^5 - 10^6 \) [43].

*Atoms.* The atoms required in this experiment must feature electric dipole allowed transitions with frequency of the order of tens of MHz. This can be achieved by considering two levels such as \( nS_{1/2} \) and \( nP_{1/2} \) of the Hydrogen atom, which are degenerate in Dirac theory and differ only by the Lamb shift which can be \( \sim 10 \) s of MHz. Another simple example is the hydrogen atom at large \( n \) and \( l \) quantum number. For instance, with \( n = 10 \) one finds electric dipole allowed transition between \( l = 9 \) and \( l = 8 \) with frequency of \( \sim 100 \) MHz.

*Photon detection.* The LC cavity will be monitored by an ultra-low noise microwave amplifier, weakly coupled to the LC cavity as usually done in circuit-QED technologies. A crucial benefit of a cryogenic system will be the availability of ultra-low noise microwave amplifiers for detecting the photons emitted in the cavity. Here, we take advantage of the cutting edge technology developed in the context of superconducting quantum technologies and circuit-QED, which are optimized for frequencies of the order of 5 GHz. Josephson Parametric Amplifiers

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
Conclusion

To summarize, we have shown that a rapidly rotating atom inside a far detuned cavity with experimentally feasible parameters displays strong signature of non-inertial motion through its spectral lines which are easily measurable with the current technology. We find that when the cavity is in resonance with the rotational frequency of the atom which is much larger than its transition frequency then the dominant contribution in the transition rate comes from the non inertial motion. This set-up provides a unique avenue where the prediction of non-inertial (and hence curved space) quantum field theory can be investigated in lab settings. Besides circuit-QED, these devices are already in use for detecting very rare emission of microwave photons in the context of dark matter axion detection [51].

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Appendix

Interaction With Electric Field

If the initial state of the field is vacuous and we integrate over all final states of the field we get

$$\Gamma = \int d\phi_f \frac{2\pi}{\hbar} |\langle \phi_f, \psi_f | H_{\text{in}} |0, \psi_i \rangle|^2 \sigma. \quad (7)$$

In the case of time dependent dipole interaction of electric field, the transition rate $\Gamma$ becomes,

$$\Gamma = \int dt_[-] |\langle \psi_f | d |\psi_i \rangle|^2 e^{i\omega t_[-]} \langle 0 | E^z(x) E^z(x') |0 \rangle, \quad (8)$$

with $t_- = x^0 - x^0$, while assuming the dipole points along the $z$-direction in the inertial frame. It is seen from this expression that the transition rate $\Gamma$ is proportional to the Fourier transform of the two-point correlation function of the electric field operator. An atom in vacuum interacts with the electromagnetic field modes of the vacuum in the ground state. The coupling between the electromagnetic modes and the atom is characterized by the interaction Hamiltonian $H_{\text{in}} = -\hat{d}_\mu E^\mu$, where $\hat{d}_\mu$ and $E^\mu$ are the atomic dipole and the electric field vectors in the rest frame of the atom. If the dipole moment is taken to be along the $z$-axis, then $H_{\text{in}} = -d E^z$ in the inertial frame, where $\vec{d}^2 = \hat{d}_\mu \hat{d}_\mu$ gives the magnitude of the atomic dipole. For the electric dipole moment pointing in the $z$-direction the two-point correlation function reads $[55]$

$$\langle 0 | E^z(x, t) E^z(x', t') |0 \rangle \propto \int \frac{d^3k \rho_f(k)}{(2\pi)^3} e^{-i\omega_k (t-t')} \frac{\omega_k}{2} \left( 1 - \frac{(k^2)^2}{k^2} \right) e^{ik \cdot (x(t) - x(t'))}, \quad (9)$$

where $\rho_f(k)$ is the density of states in free space. In the linear perturbation theory, we obtain the probability of transition from one state to another as the Fourier transform of the Wightman function w.r.t. the energy gap of the levels. Therefore the spontaneous emission rate will be obtained as

$$P_{1\to 0} = \frac{c}{\epsilon_0 \hbar} \int_{-\infty}^{\infty} d\tau d\tau' e^{i\Omega (\tau - \tau')} C'(\tau, \tau'), \quad (10)$$

where

$$C'(\tau, \tau') = \int d\phi_f \langle 0, \psi_i | \hat{d}_\mu(0) E^\mu_{\tau'}(\tau') |\phi_f, \psi_f \rangle \langle \phi_f, \psi_f | \hat{d}_\mu(0) E^\mu_{\tau}(\tau) |0, \psi_i \rangle. \quad (11)$$

When the atom is at rest, i.e. $\Delta x^i = 0$, using [9] the transition probability becomes

$$P_{1\to 0} = \frac{d^2}{\epsilon_0 \pi^2 \hbar c^3} \int_{-\infty}^{\infty} d\tau d\tau' \frac{e^{i\Omega (\tau - \tau')}}{(\tau - \tau')^4}. \quad (12)$$
Therefore, the spontaneous emission rate \( \Gamma_0 = dP_{1\rightarrow 0}/d\tau_+ \) in the rest frame reads (each \( \tau_- \) (or \( t_- \) in the later portions) appearing in the Wightman function is actually understood to be \( \tau_- - i\epsilon \) (or \( t_- - i\epsilon \)).)

\[
\Gamma_0 = \frac{d^2 e_0}{\pi^2 \hbar c^3} \int \frac{d^3\Omega}{\tau_-} \frac{d^2 \Omega}{\tau_+} = \frac{d^2 \Omega}{3\pi e_0 \hbar c^3}.
\]

(13)

Here we have changed the variables from \( \tau, \tau' \) to \( \tau_+ = (\tau + \tau')/2 \) and \( \tau_- = \tau - \tau' \).

Finally before we go to discuss the theoretical construction of the proposed set up, for completeness, we make note of a rather obvious point. If the atom is moving with velocity \( v \) in the lab frame, then the clocks of the atom and the lab are related by \( \tau = \gamma^{-1} t \) and \( (\tau - \tau')^2 = \gamma^{-2} (t - t')^2 = [(t - t')^2 - (x - x')^2] \), where \( \gamma = 1/\sqrt{1 - v^2/c^2}. \) This will yield the the spontaneous emission rate in the lab frame \( \Gamma_0 = \Gamma_0/\gamma. \) The same can be obtained from first writing the electric field two-point function and then transforming it as a tensor to the lab frame. We will do this exercise for a rotating atom put inside a cavity, in the lab frame. In this case the dipole is pointing in a direction (say \( y^- \)) which is orthogonal to the plane \((x - z)\) of the rotation.

The co-ordinates for a rotating particle inside the cavity, are given in the lab frame \((t, x, y, z)\) as

\[
x(t) = x_0 + R \cos \omega t, \; z(t) = z_0 + R \sin \omega t, \; y = 0.
\]

(14)

where \( R \) is the radius of rotation and \( \omega \) is the rotation speed. In a non-inertial frame co-rotating with the atom, the co-ordinates are related as

\[
x' = x - x_0 - R \cos \omega t, \; z' = z - z_0 - R \sin \omega t, \; y' = y.
\]

(15)

Thus, the proper time and the co-ordinate (lab) time are related as

\[
d\tau = \left(1 - \frac{\omega^2 R^2}{c^2}\right)^{1/2} d\tau
\]

leading to a line element

\[
ds^2 = -d\tau^2 + \sum_i (dx_i)^2 - \frac{2\omega R \sin \omega t(\tau)}{(1 - \omega^2 R^2)^{3/2}} d\tau dx' + \frac{2\omega R \cos \omega t(\tau)}{(1 - \omega^2 R^2)^{3/2}} d\tau dz'.
\]

(17)

Now, in the rest frame of the atom the interaction Hamiltonian is \( H_{\text{int}} = -g_{\mu\nu} \hat{d}^\mu E'^\nu \), where the primed quantities are the vectors as seen from the co-moving frame. Using the fact that the dipole 4-vector is orthogonal to the world line of the static observer (in the rest frame) i.e. \( g_{\mu\nu} d'^\mu u'^\nu = 0 \), for \( u'^\mu = (1, 0, 0, 0) \) we get \( d'^0 = 0 \) leading to the fact \( d'^2 = g_{\mu\nu} d'^\mu d'^\nu = (d'^0)^2 \). Since in the atom’s frame, the dipole is pointing in the \( y'\)-direction, the interaction Hamiltonian becomes

\[
H_{\text{int}} = -g_{\mu\nu} \hat{d}^\mu E'^\nu = -\hat{d}^0 E'^y.
\]

(18)

Thus, we can transform the interaction term to the lab frame

\[
H_{\text{int}} = \frac{-\hat{d}}{(1 - \omega^2 R^2)^{3/2}} \left[ E'^y - \frac{R\omega}{c} \sin \omega t c B^z - \frac{R\omega}{c} \cos \omega t c B^y \right].
\]

(19)

Therefore, in the first order perturbation theory we end up getting the two-point correlator of

\[
\langle 0 | E'^y(t) - \frac{R\omega}{c} \sin \omega t c B^z(t) - \frac{R\omega}{c} \cos \omega t c B^y | 0 \rangle \langle 0 | E'^{y'}(t') - \frac{R\omega}{c} \sin \omega t' c B^z(t') - \frac{R\omega}{c} \cos \omega t' c B^y(t') | 0 \rangle.
\]

(20)

We also transform the integration measure \( d\tau d\tau' \) in [10] using [16] and again go to the variables \( t_+ = (t + t')/2 \) and \( t_- = t - t' \). Since the various two point correlators of the electromagnetic fields in inertial vacuum \( \langle 0 | E^i B^j(t') | 0 \rangle \) depend on \( t_- \) the crossed terms of [20] vanish under \( t_+ \) integration. The relevant surviving correlators will be given as
Thus, obtaining $Q$ factor with $\bar{\Omega} = \rho$ where

$$
\langle 0 | E^y(t) E^y(t') | 0 \rangle = \int \frac{d^3 k \rho(k)}{(2\pi)^3 V} e^{-i\omega_k t} \frac{\omega_k}{2} \left( 1 - \frac{(k^y)^2}{k^2} \right) e^{i \mathbf{k} \cdot (\mathbf{x}(t) - \mathbf{x}(t'))},
$$

$$
\langle 0 | B^y(t) B^y(t') | 0 \rangle = \int \frac{d^3 k \rho(k)}{(2\pi)^3 V} e^{-i\omega_k t} \frac{\omega_k}{2c^2} \left( 1 - \frac{(k^z)^2}{k^2} \right) e^{i \mathbf{k} \cdot (\mathbf{x}(t) - \mathbf{x}(t'))},
$$

$$
\langle 0 | B^z(t) B^z(t') | 0 \rangle = \int \frac{d^3 k \rho(k)}{(2\pi)^3 V} e^{-i\omega_k t} \frac{\omega_k}{2c^2} \left( 1 - \frac{(k^y)^2}{k^2} \right) e^{i \mathbf{k} \cdot (\mathbf{x}(t) - \mathbf{x}(t'))},
$$

with $\rho(k)$ is the density of energy states inside the cavity and $V$ its volume. Lastly using the fact that the vector $\mathbf{x}(t) - \mathbf{x}(t')$ changes both direction and magnitude, $(x(t) - x(t'))^2 = 4R^2 \sin^2 \omega t / 2c^2$ over time and performing the $t_-$ integration we get

$$
\Gamma_{\text{cav}} \sim \frac{d^2}{\varepsilon_0 \hbar V} \int_0^\infty dk \rho(k) \omega_k \left[ \delta(\bar{\Omega} - \omega_k) + \frac{3 R^2}{2 c^2} \left( \frac{\omega^2}{3} + \frac{\omega_k^2}{30} \right) \right] \left[ \delta(\bar{\Omega} + \omega - \omega_k) + \delta(\bar{\Omega} - \omega - \omega_k) \right]
$$

$$
- \frac{1}{10} \frac{R^2 \omega_k^2}{c^2} \delta(\bar{\Omega} - \omega_k) + O \left( \sum_{n>1} \frac{R^2 \omega_k^2}{c^2} \delta(\bar{\Omega} \pm n \omega - \omega_k) \right),
$$

for a cavity tuned near rotational frequency $\omega$.

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5. H. B. G. Casimir and D. Polder, Physical Review 73, 360 (1948).
6. P. C. W. Davies, Nature 263, 377 (1976).
7. W. G. Unruh, Phys. Rev. D 14, 870 (1976).
8. N. D. Birrell and P. C. W. Davies, Quantum Fields in Curved Space (Cambridge Univ. Press, Cambridge, UK, 1984).
9. L. C. B. Crispino, A. Higuchi, and G. E. A. Matsas, Rev. Mod. Phys. 80, 787 (2008).
10. P. Chen and T. Tajima, Phys. Rev. Lett. 83, 256 (1999).
11. J. Rogers, Phys. Rev. Lett. 61, 2113 (1988).
12. M. O. Scully, V. V. Kocharovsky, A. Belavanin, E. Fry, and F. Capasso, Phys. Rev. Lett. 91, 243004 (2003).
13. G. Cozzella, A. G. Landulfo, G. E. Matsas, and D. A. Vanzella, Phys. Rev. Lett. 118, 161102 (2017).
14. A. Raval, B. L. Hu, and J. Anglin, Phys. Rev. D 53, 7003 (1996).
15. D. A. T. Vanzella and G. E. A. Matsas, Phys. Rev. Lett. 87, 151301 (2001).
16. A. Retzker, J. I. Cirac, M. B. Plenio, and B. Reznik, Phys. Rev. Lett. 101, 110402 (2008).
17. R. Schützhold, G. Schaller, and D. Habs, Phys. Rev. Lett. 97, 121302 (2006).
18. M. Aspachs, G. Adesso, and I. Fuentes, Phys. Rev. Lett. 105, 151301 (2010).
19. E. Martín-Martínez, I. Fuentes, and R. B. Mann, Phys. Rev. Lett. 107, 131301 (2011).
20. D. Kothawala and T. Padmanabhan, Phys. Lett. B 690, 201 (2010).
21. P. C. W. Davies, T. Dray, and C. A. Manogue, Phys. Rev. D 53, 4382 (1996).
22. J. Anandan, Phys. Rev. Lett. 85, 1354 (2000).
23. B. H. Bransden and C. J. Joachain, Physics of atoms and molecules (Pearson Education India, 2003).
24. E. M. Purcell, Physical Review 69, 61 (1946).
25. S. Kuhn, A. Kosloff, B. A. Stickler, F. Patolsky, K. Hornberger, M. Arndt, and J. Millen, Optica 4, 356 (2017).
[26] M. Toroš, M. Rashid, and H. Ulbricht, Physical Review A \textbf{98}, 053803 (2018).
[27] V. Jain, J. Gieseler, C. Moritz, C. Dellago, R. Quidant, and L. Novotny, Physical review letters \textbf{116}, 243601 (2016).
[28] J. Vovroš, M. Rashid, D. Hempston, J. Bateman, M. Paternostro, and H. Ulbricht, JOSA B \textbf{34}, 1421 (2017).
[29] M. Frimmer, J. Gieseler, and L. Novotny, Physical review letters \textbf{117}, 033602 (2016).
[30] A. Setter, M. Toroš, J. F. Ralph, and H. Ulbricht, Physical Review A \textbf{97}, 033803 (2018).
[31] M. Rashid, T. Tufarelli, J. Bateman, J. Vovroš, D. Hempston, M. Kim, and H. Ulbricht, Physical review letters \textbf{117}, 273601 (2016).
[32] A. A. Geraci, S. B. Papp, and J. Kitching, Physical review letters \textbf{105}, 101101 (2010).
[33] G. Ranjit, M. Cunningham, K. Casey, and A. A. Geraci, Physical Review A \textbf{93}, 053801 (2016).
[34] D. Hempston, J. Vovroš, M. Toroš, G. Winstone, M. Rashid, and H. Ulbricht, Applied Physics Letters \textbf{111}, 133111 (2017).
[35] G. Winstone, M. Rademacher, R. Bennett, S. Buhmann, and H. Ulbricht, arXiv preprint arXiv:1712.01426 (2017).
[36] E. Hebestreit, M. Frimmer, R. Reimann, and L. Novotny, Physical review letters \textbf{121}, 063602 (2018).
[37] C. Timberlake, M. Toroš, D. Hempston, G. Winstone, M. Rashid, and H. Ulbricht, Applied Physics Letters \textbf{114}, 023104 (2019).
[38] T. M. Hoang, Y. Ma, J. Ahn, J. Bang, F. Robicheaux, Z.-Q. Yin, and T. Li, Physical review letters \textbf{117}, 123604 (2016).
[39] Y. Arita, M. Mazilu, and K. Dholakia, Nature communications \textbf{4}, 2374 (2013).
[40] S. Kuhn, P. Asenbaum, A. Kosloff, M. Sclafani, B. A. Stickler, S. Nimmrichter, K. Hornberger, O. Cheshnovsky, F. Patolsky, and M. Arndt, Nano letters \textbf{15}, 5604 (2015).
[41] A. A. Rahman and P. Barker, Nature Photonics \textbf{11}, 634 (2017).
[42] M. Rashid, M. Toroš, A. Setter, and H. Ulbricht, Physical review letters \textbf{121}, 253601 (2018).
[43] S. Kuhn, B. A. Stickler, A. Kosloff, F. Patolsky, K. Hornberger, M. Arndt, and J. Millen, Nature communications \textbf{8}, 1670 (2017).
[44] R. Reimann, M. Doderer, E. Hebestreit, R. Diehl, M. Frimmer, D. Windey, F. Tebbenjohanns, and L. Novotny, Physical review letters \textbf{121}, 033602 (2018).
[45] J. Ahn, Z. Xu, J. Bang, Y.-H. Deng, T. M. Hoang, Q. Han, R.-M. Ma, and T. Li, Physical review letters \textbf{121}, 033603 (2018).
[46] A. Bienfait, J. J. Pla, Y. Kubo, X. Zhou, M. Stern, C. C. Lo, C. D. Weis, T. Schenkel, D. Vion, D. Esteve, J. J. L. Morton, and P. Bertet, Nature \textbf{531}, 74 (2016).
[47] H. Li, Y. Wang, L. Wei, P. Zhou, Q. Wei, C. Cao, Y. Fang, Y. Yu, and P. Wu, Chinese Science Bulletin \textbf{58}, 2413 (2013).
[48] M. M. R. V. E. M. L. F. D. E. P. R. J. S. S. M. G. H. D. N. Bergeal, F. Schackert, Nature \textbf{465}, 64 (2010).
[49] G. C. H. L. R. V. M. A. Castellanos-Beltran, K. D. Irwin and K. W. Lehnert, Nature Physics \textbf{4}, 929 (2008).
[50] R. Vijay, D. H. Slichter, and I. Siddiqi, Phys. Rev. Lett. \textbf{106}, 110502 (2011).
[51] L. Zhong, S. Al Kenany, K. M. Backes, B. M. Brubaker, S. B. Cahn, G. Carosi, Y. V. Gurevich, W. F. Kindel, S. K. Lamoreaux, K. W. Lehnert, S. M. Lewis, M. Mahon, R. H. Maruyama, D. A. Palken, N. M. Rapidis, J. R. Root, M. Simonovskia, T. M. Shokair, D. H. Speller, I. Urdanaran, and K. A. van Bibber, Phys. Rev. D \textbf{97}, 092001 (2018).
[52] D. Singleton and S. Wilburn, Phys. Rev. Lett. \textbf{107}, 081102 (2011).
[53] J. Steinhauer, Nat. Phys. \textbf{10}, 864 (2014).
[54] J. Steinhauer, Nat. Phys. \textbf{12}, 959 (2016).
[55] T. Padmanabhan, \textit{Quantum Field Theory : The Why, What and How} (Springer : Graduate texts in Physics, Springer, 2017).