Leptonic and semi-leptonic B decays

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We present results for the semi-leptonic and leptonic decay s of B mesons. These non-perturbative matrix elements are important for constraining the CKM matrix. Results are presented for the pseudoscalar and vector decay constants, as well as flavour breaking ratios and heavy quark symmetry relations. We consider the chiral and momentum dependence of the semi-leptonic form factors of the decay \( B \to \pi l \nu \) and the soft pion relation (SPR) on the lattice. These calculations were performed in the quenched approximation at two values of the coupling with non-perturbatively \( \mathcal{O}(a) \) improved action and currents.

1. INTRODUCTION

The determination of decay constants and form factors arising from the decay of B and D mesons is of great importance to phenomenology, in particular in constraining the Cabibbo-Kobayashi-Maskawa (CKM) matrix. These quantities can be calculated on the lattice.

The action used in this calculation is the \( \mathcal{O}(a) \) non-perturbatively improved action \([1]\), where \( c_{SW} \) and the coefficients needed to improve the currents, are determined non-perturbatively where possible. The calculation was done at two values of the coupling, \( \beta = 6.0 \) and \( \beta = 6.2 \). The simulation parameters are shown in Table 1. We simulate with three light quark masses around strange and four propagating heavy quarks around charm. Heavy Quark Symmetry (HQS) is used to motivate the form of the heavy extrapolation to the \( B \) scale. The light quark mass parameters (\( \kappa_{\text{crit}} \) etc) have been determined by the UKQCD collaboration \([2]\). The vector and axial currents used in this calculation are improved and renormalised as follows:

\[
V_{\mu}^R = Z_V(1 + b_V a m_q) \left( V_{\mu} + a c_V \partial^\nu T_{\mu \nu} \right) \quad (1)
\]

\[
A_{\mu}^R = Z_A(1 + b_A a m_q) \left( A_{\mu} + a c_A \partial^\nu P \right) \quad (2)
\]

2. DECAY CONSTANTS

The results for the decay constants can be found in \([3]\), as well as a discussion of the improvement coefficients, but we present the main results again. The definition of the vector decay constants differs from the pseudoscalar and so we give it here:

\[
\langle 0 | V_{\mu}^R(0) | V \rangle = i e_{\mu} \frac{M_i^2}{f_V} \quad (3)
\]

The decay constants were extracted from appropriate ratios of correlation functions, and then extrapolated linearly in the light quark mass to \( \kappa_n \) and \( \kappa_s \). The extrapolation to the \( B \) meson is motivated by HQS, of the form

\[
\Phi_i(M_i) \equiv C(M_i) f_i \sqrt{M_i} = \gamma_i \left( 1 + \frac{\delta_i}{M_i} + \frac{\eta_i}{M_i^2} \right) \quad (4)
\]

where \( i \) is \( P \) or \( V \) and the function \( C(M_i) \) contains the leading logarithmic \( \alpha_s \) corrections to the scaling relation.

The results for the decay constants are shown in Table 2. The large value of the decay constants is in part due to the choice of quantity used to set

\[1\) In this case \( f_i = M_V / f_V \)
the scale. \( f_B \) varies from 218 MeV using \( r_0 \) to set the scale to 186 using \( m_\rho \). This dominates the systematic uncertainty. The remaining systematic error comes from difference between \( \beta = 6.2 \) and \( \beta = 6.0 \), and choice of functional form for the heavy extrapolation (quadratic vs linear). According to HQS, the \( \Phi \) functions for the \( P \) and \( V \) decay constants should agree in the heavy quark limit. However, the \( \beta = 6.0 \) data fail to satisfy this constraint, which may be indicative of large discretisation errors on the coarser lattice.

To examine this further we considered, the KLM factor and the dispersion relation. At \( \beta = 6.2 \), the KLM factor was close to unity and the dispersion relation showed good continuum behaviour. However, at \( \beta = 6.0 \), the KLM factor has a finite effect, and the dispersion relation shows a deviation from the continuum version. These all suggest that the \( \beta = 6.0 \) data are being affected by \( \mathcal{O}(am_\rho^2) \) errors.

Table 2
The decay constants. The first error is statistical, the second is systematic.

| \( f \)  | \( V \)  |
|-------|--------|
| \( B \) | 218(5)\( ^+0 \)\( ^-41 \) MeV | 22.6(0.7)\( ^+4.4 \)\( ^-3.6 \) |
| \( D \) | 220(3)\( ^+21 \)\( ^-48 \) MeV | 7.5(0.1)\( ^+3.3 \)\( ^-3.8 \) |
| \( B_s \) | 242(4)\( ^+13 \)\( ^-48 \) MeV | 20.9(0.4)\( ^+4.2 \)\( ^-3.3 \) |
| \( D_s \) | 241(2)\( ^+13 \)\( ^-30 \) MeV | 7.3(0.1)\( ^+0.9 \)\( ^-0.4 \) |
| \( B/B_s \) | 1.11(0.01)\( ^+0.03 \)\( ^-0.03 \) | 0.92(0.01)\( ^+0.04 \)\( ^-0.04 \) |
| \( D/D_s \) | 1.09(0.001)\( ^+0.02 \)\( ^-0.02 \) | 0.98(0.01)\( ^+0.02 \)\( ^-0.02 \) |

3. RENORMALISATION

The matrix element of a \( P \rightarrow P \) transition can be parameterised in terms of two form factors,

\[
Z_V^\text{eff} \langle P(p\bar{q})|V_\mu|P(p\bar{q})\rangle^{\text{lat}} = f_+(q^2)(p_1 + p_2 - \Delta_{m^2})_\mu + f_0(q^2)\Delta_{M^2}q_\mu
\]

where \( q = p_1 - p_2 \) and \( \Delta_{m^2} \) is the difference of the square of the masses normalised by \( q^2 \). Recalling that the vector current is conserved, we know that \( f_+(0) \) must be one and therefore consider the forward, degenerate matrix element. Here, \( Z_V^\text{eff} \) can then be measured directly by taking the appropriate ratio of two- to three-point correlation functions. This can then be compared to \( Z_V(1 + b_V am_\rho^2) \) with the values of \( Z_V \) and \( b_V \) determined by [3], at the values of quark mass for which we measure \( Z_V^\text{eff} \). This is shown in figure [4].

![Figure 1](image.png)

Figure 1. The mass dependence of \( Z_V^\text{eff} \). The vertical line shows the charm quark mass as determined from the heavy-light pseudoscalar.

These data have already been shown for \( \beta = 6.2 \) in [3]. The ALPHA determinations of \( Z_V \) and \( b_V \) used chiral ward identities, and so used zero or very small quark mass. Our results show excellent agreement with the mass-independent scheme even at large quark mass.

4. THE SOFT PION RELATION

HQS can be used to relate the scalar form factor, \( f_0 \) to the ratio of the \( B \) to \( V \) pion decay constant,

\[
f_0(q^2_{\text{max}}) = \frac{f_B}{f_\pi}
\]

A recent review [3] found that the majority of lattice calculations found this relation violated. However, the key issue for the SPR on the lattice is the chiral extrapolation. This is complicated by the dependence of the form factors on \( q^2 \), itself heavily dependent on the light quark mass. In
this calculation the active light quark (A), which occurs in the heavy-light current and the spectator light quark (S) which takes no part in the weak decay have different mass. Hence,

\[ f = f(q_{(m_A, m_S)}; m_A, m_S) \]  

A Taylor expansion of \( q^2 \) shows \( q^2 \sim M_{P_S}^{\text{light}} \), and we know from PCAC that \( (M_{P_S}^{\text{light}})^2 \sim \bar{m} \) where the average quark mass \( \bar{m} = (m_A + m_S)/2 \). Thus the light quark mass dependence of the form factor is

\[ f(q^2, \bar{m}, m_S) = \alpha + \beta \bar{m}^{1/2} + \gamma \bar{m} + \delta m_S \]  

The second term in this expression makes the extrapolation difficult to control, as it is non-analytic and \( \partial \bar{m}^{1/2}/\partial \bar{m} \) blows up as we approach the chiral limit, in the region where we have no data.

Instead of extrapolating at fixed pion three-momentum we separate the \( q^2 \) and chiral behaviour of the form factor. We first interpolate in \( q^2 \) at fixed quark mass so that at each quark mass we have the form factors at the same momentum value, and then extrapolate at fixed \( q^2 \) without the troublesome second term. However, because we extrapolate at fixed \( q^2 \), we no longer have data at \( q^2_{\text{max}} \) for physical quark masses. This method has already been successfully employed to calculate the differential decay rate for \( B \to \pi \) at \( \beta = 6.2 \) in [3]. In this analysis, an extra-momentum channel is included \( (1 \to \sqrt{2} \gamma) \), and the range of \( q^2 \) altered to allow interpolation at fixed quark mass for both values of \( \beta \), in the same range of \( q^2 \).

The form of interpolating function is motivated by pole-dominance models.

\[ f_i(q^2) = \frac{f_i}{1 - q^2/M_i^2} \]  

with kinematic constraint \( f_0(0) = f_+(0) \) imposed. We have also tried other functions for the interpolation, including dipole/pole for \( f_+ \) and \( f_0 \) respectively and with or without the kinematic constraint. This is shown in figure 3. Also shown in figure 3 are other less well motivated functions, such as linear or quadratic in \( q^2 \). As we are interpolating any reasonable, smooth function should produce similar results.

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{fig2.pdf}
\caption{Interpolation functions for heaviest \( \kappa \) combinations at \( \beta = 6.0 \). The heavy quark has a mass around charm.}
\end{figure}

For the extrapolation to the \( B \) meson mass, we are again guided by HQS,

\[ C(M, M_B)f_i(v \cdot k)M_i^{n_i/2} = \varepsilon_i \left[ 1 + \frac{\zeta_i}{M} + \frac{\xi_i}{M^2} \right] \]  

where \( n_i = -1, +1 \) when \( i = +, 0 \) and \( v \cdot k \) is the kinematic variable,

\[ v \cdot k = \frac{M^2 + M_B^2 - q^2}{2M} \]  

The fixed \( q^2 \) method allows us to choose the \( q^2 \) values at each heavy quark mass such that \( v \cdot k \) remains constant during the heavy quark extrapolation.

We can now consider the momentum dependence of the form factors at the \( B \) scale. Combining HQS scaling relations and pole dominance models suggest the momentum dependence is a dipole for \( f_+ \) and a pole for \( f_0 \) (DPP). A more physical model has been suggested by Becirevic and Kaidalov (BK) [8]

\[ f_+(q^2) = \frac{c_B(1 - \alpha)}{(1 - q^2/M_B^2)(1 - \alpha q^2/M_B^2)} \]

\[ f_0(q^2) = \frac{c_B(1 - \alpha)}{(1 - q^2/\beta M_B^2)} \]  

\[ \beta = 6.0 \]

\[ \kappa_a = 0.1123 \]

\[ \kappa_b = 0.1344 \]
These are shown in figure 3. The value obtained for \( f_0(q_{\text{max}}^2) \) can then be compared to the value of \( f_B/f_\pi \). To try and estimate the systematic errors we considered the following: \( r_0 \) versus \( m_\rho \) to set the scale, \( \beta = 6.2 \) versus \( \beta = 6.0 \), different models for the fixed \( q^2 \) interpolation, quadratic vs linear heavy extrapolation. We see the SPR satisfied on the lattice, with large systematic errors, but with a tendency for \( f_0(q_{\text{max}}^2) \) to lie below \( f_B/f_\pi \).

We can also check to see if the extrapolations in \( q^2 \) and heavy quark mass commute. In figure 4 the heavy extrapolation of \( f_B/f_\pi \) and \( f_0(q_{\text{max}}^2) \) are shown after \( f_0(q^2) \) has been extrapolated with a pole model to \( q_{\text{max}}^2 \). Again we see the SPR is satisfied.

A recent calculation [9] using a NRQCD formulation of heavy quarks confirms the difficulties with the chiral extrapolations of the form factors and also adopts a fixed kinetic variable approach, which reduces their observed violation of the SPR. Another calculation [10] using the relativistic heavy Wilson formulation, and a fixed kinematic variable approach see the SPR satisfied if a quadratic heavy extrapolation is used.

The SPR is rather difficult to satisfy on the lattice because we have to extrapolate the light quark mass into a region where \( q^2 \) changes rapidly. This can be circumvented by using the fixed \( q^2 \) approach. However, lattice calculations can reliably determine the differential decay rate for \( B \rightarrow \pi \) away from \( q_{\text{max}}^2 \) where the phase space is non-zero to extract \( V_{ub} \). This work was supported by EPSRC grant GR/K41663 and PPARC grants PPA/G/S/1999/00022 and PPA/GS/1997/00655. CMM acknowledges PPARC grant PPA/P/S/1998/00255.

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