A Non-Linear Improved Double-Integral Sliding Mode Controller (IDI-SMC) for Modeling and Simulation of 6 (DOF) Quadrotor system

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Abstract. Due to recent technological achievements, modeling and simulation of Unmanned Aerial Vehicles (UAVs) such as Quadrotor contracted great potential. In this paper, the Improved Double-Integral Sliding Mode Controller (IDI-SMC) scheme is presented to cope with the problem like chattering and tracking. Under-actuated six Degree Of Freedom (DOF) Quadrotor model is formulated through Newton-Euler-Lagrange (NEL) equations. The idea of design (IDI-SMC) is to implement Double Sliding Mode control with proper coefficient scheduling for switching surface with Hurwitz stability criteria. Furthermore, introducing the strong condition of the sliding surface made the results and Simulations became more efficient with Hurwitz stability for both translational and rotational motion of the 6 DOF Quadrotor system is achieved to show the effectiveness of the proposed design controller.

1. Introduction
In recent decades, the importance of unmanned air vehicles (UAVs) in intelligence robot system engineering, especially in control system engineering, has been increased manifold. Compared to other traditional UAVs, Quadrotor optimization is an interesting area and plays a very vital role in flight control due to several advantages [1]. The concept of Quadrotor was first given by Breguet-Riched in 1907. The quadrotor has been extensively considered in many domains. Quadrotor has numerous military and civil applications for having advantages of its smaller size, quadrotor used in military operations like surveillance, trajectory planning, navigation, searching and rescuing. Quadrotor can also be used in civil tasks like aerial photography, production in agriculture plants, surface, and transmissions line inspection, accurate poisoning and altitude control.
The contributions of researchers with several control techniques have been proposed for controlling of Quadrotor. For implementation and simulations various linear, nonlinear and robust controllers have been studied. PID is used by Bouabdallah et al [2] for controlling Quadrotor which only deals with little disturbance. Li a Young also proposed a PID controller in the pole place technique considering with low speed of propellers [3]. For inner loop (poison) and outer loop (altitude) control, Hongping Liu used PID and PD named PID-PD controller respectively but it needs to online gain regulation [4]. Bilinear modeling and attitude control of quadrotor, Chee Hwee [5] Seah used a robust bilinear controller. To avoid noise, Model Reference Adaptive Control (MRAC) and PID control have been used [6]. Recently, Aws Abdul sahram comes with a Nonlinear PID controller for Quadrotor to ensure Hurwitz stability [7]. Linear parameter varying LPV-LQR control techniques are proposed by Carlos Trapiello which only preferred for a small quadrotor [8]. For the control and design of translational position and heading, T.L Wang proposed a state feedback controller respectively [9] with the existence of little noise and small algorithm error. Zhi and Li [10] proposed a Lyapunov based SMC algorithm for a quadrotor. For trajectory tracking, a robust SMC controller integrated with Human Machine Interface (HMI) has been implemented by M.Reinoso [11]. High and varying payload causes quadrotor to unstable and creates wind disturbance. An Adaptive fractional Sliding Mode Control technique based upon Backstepping has been simulated by Mohsen [12]. This scheme has the ability to successfully attenuate wind disturbance and cope with undesirable fluctuations in control angles and poisons due to its inertia. Through results and study, the ideal path trajectory has been achieved with minor transient changes. Recently, the discrete-time Sliding Mode Control technique (DSMC) has been proposed by J.Xiong for attitude and position control after continuous to discrete transformation [13]. Integral Backstepping Sliding Mode Controller (IBS-SMC) with an external disturbance procedure has been applied [14]. The result shows this algorithm robust to disturbance as well as a good tracking problem. Recently, Feedback Linearization (FL) and Backstepping techniques have been proposed by M.A-Vvallego [15]. First, by reducing the system model by FL and after implementing Leader-Follower (LF) based and integrated with Backstepping controller which resulted asymptotic stability for a quadrotor. To achieve asymptotically stability for desired states trajectories nonlinear feedback controller is applied with neglecting high order Lie derivatives [16]. A strong robust based sliding mode and fuzzy controllers for inner loop and Feedback linearization for outer loop has been introduced [17].

In this paper, Improved Double Integral Sliding Mode control (IDI-SMC) is presented which strongly robust to uncertainties with proper gain scheduling and to tackle chattering. In several controller coefficients and gains are designed by simulations. Here introducing proper gain called switching surface coefficient by using of Eigenvalue which based upon Hurwitz criteria. Furthermore, for a sliding mode controller, the proper and strong sliding surface is introduced to ensure the surface remains to zero so that trajectories stay on it.

The rest of this paper has been arranged as follows. In section 2, modeling of simulation derived through Newton’s Euler equations. Mathematical modeling of three different control techniques has been presented in sections 3. In section 4, results verified through simulations have been obtained. In last, the conclusion and final discussion have been discussed briefly.

2. Mathematical Modeling

In this section, the Quadrotor model was developed by using Newton-Euler formulations. A Cross like the structure of Quadrotor having four propellers with a control system placed at the center shown in Fig [1]. The power of the propeller supplied by DC motor. Two of them (1, 3) propellers move the same direction (anti-clockwise) and the other two move in clockwise directions to neutralize or cancel the effect of thrust force. A 6 DOF [ϕ, θ, ψ, x, y, z] Quadrotor model is basically Under-actuated in nature with six states and only four inputs which make it difficult to flight control [2]. The system is sub-divided into two parts which are rotationally and translationally parts having states roll (ϕ), pitch
(θ) and yaw (ψ) with position (x y z) respectively. The control diagram altitude (x y z) and heading and attitude with heading are shown in Fig [1]. Tracking, stabilizing, and orientation of Quadrotor can be achieved by thrust force and angular torque.

The quadrotor model is divided into two parts. Rotational dynamics roll, yaw, and pitch (ϕ, θ, ψ) and translational dynamics (x,y,z) have been formulated around X, Y, and Z axes respectively. The rotation matrix R is given by

\[
R = \begin{pmatrix}
\cos(\theta) & \sin(\theta)
\sin(\phi)\cos(\theta) & -\sin(\phi)\sin(\theta) + \cos(\phi)\cos(\psi) & \cos(\phi)\sin(\psi)
\cos(\phi)\cos(\psi) & \cos(\phi)\sin(\psi) & \sin(\phi)
\end{pmatrix}
\]

where \( c\theta = \cos(\theta) \) and \( s\theta = \sin(\theta) \) (1)

2.1. Rotational Equations Dynamics

Rotational equations roll, pitch and yaw (ϕ, θ, and ψ) have been derived from body inertia frame (x, y, and z) with the help of N-Euler equation the total torque is given as:

\[
\dot{\mathbf{w}} + \mathbf{w} \times \mathbf{I} \dot{\mathbf{w}} + \mathbf{M}_G = \mathbf{\tau}
\]

where \( I = \text{Diagonal inertia matrix}, \ w \) is Angular velocity , \( M_G \) is Gyroscopic moments and \( \tau \) is Total torque

The inertia matrix contains diagonal entries with their product is equal to zero having moments of inertia \( I_{xx}, I_{yy} \) and \( I_{zz} \)

\[
I = \begin{pmatrix}
I_{xx} & 0 & 0 \\
0 & I_{yy} & 0 \\
0 & 0 & I_{zz}
\end{pmatrix}
\]

(3)

From these moments rotation is produced from rotors and creates a force called aerodynamics forces or simply called lift forces. The forces and moments are shown in equation (4)

\[
\mathbf{M} = \frac{1}{2} \rho A C_r r^2 \Omega_j \Rightarrow K_j \Omega_j \quad ; \quad \mathbf{F} = \frac{1}{2} \rho A C_r r^2 \Omega_j \Rightarrow K_j \Omega_j
\]

(4)

Where \( K_f \) and \( K_m \) are the constants, \( \rho \) is Air density, \( C_r \) is Aerodynamic coefficient and \( \Omega_j \) is \( j \)th Angular velocity. The total moments about x,y, and z-axis has been calculated as

\[
\mathbf{\tau} = \begin{pmatrix}
M_x = IK_f (\Omega_1^2 - \Omega_2^2) \\
M_y = IK_f (\Omega_2^2 - \Omega_3^2) \\
M_z = K_m (\Omega_1^2 - \Omega_2^2 + \Omega_3^2 - \Omega_4^2)
\end{pmatrix}
\]

(5)

2.2. Translational Quadrotor Equations Dynamics

Translational equations of motions can be derived from using Newton’s second law according to earth inertia frame

\[
\mathbf{F} = \mathbf{ma} \Rightarrow m \ddot{\mathbf{r}} = 0 + \mathbf{RF}_b
\]

(6)
where \( r = [x \ y \ z]^T \) is Quadratic distance, \( m \) is Quadrotor mass while \( F_g \) is a force without gravitational. Let we have a state vector \( X \) and input vector \( U \) can be defined as:

\[
X = \begin{bmatrix} x_1 & x_2 & x_3 & x_4 & x_5 & x_6 & x_7 & x_8 & x_9 & x_{10} & x_{11} & x_{12} \\ \phi & \dot{\phi} & \theta & \dot{\theta} & \psi & \dot{\psi} & \dot{x} & \dot{y} & \dot{z} & \ddot{z} \end{bmatrix}^T
\]

\[
\begin{align*}
U_1 &= K_f(\Omega_1^2 + \Omega_2^2 + \Omega_3^2 + \Omega_4^2) \\
U_2 &= K_f(-\Omega_2^2 + \Omega_3^2) \\
U_3 &= K_f(\Omega_3^2 - \Omega_1^2) \\
U_4 &= K_f(\Omega_4^2 - \Omega_2^2 + \Omega_3^2 - \Omega_1^2)
\end{align*}
\]  

(7)

Substituting equations (7) to (5) through mapping state vector \( X \) we get total torque from rotational and translational equations of motion and simplifying we get State Space model shown in Eq. (9)

\[
\begin{align*}
\ddot{\phi} &= b_2 U_2 - a_2 x_2 \Omega_x + a_4 x_4 x_6 \\
\ddot{\theta} &= b_3 U_3 - a_4 x_2 \Omega_x + a_5 x_2 x_6 \\
\ddot{\psi} &= a_3 x_2 x_4 + b_3 U_4 \\
\ddot{x} &= -\frac{U_1}{m} (\sin x_1 \sin x_3 + \cos x_1 \cos x_3 x_1) \\
\ddot{y} &= -\frac{U_1}{m} (\cos x_1 \sin x_3 - \cos x_3 x_1) \\
\ddot{z} &= g - \frac{U_1}{m} (\cos x_1 \cos x_3)
\end{align*}
\]  

(8)

3. Controller Design

The proposed scheme has been implemented in three different phases for modeling and simulation of Quadrotor. First, one Sliding mode control which deals with uncertainties but produces chattering reaching to steady-state. To deal this further leads to building a double integral sliding mode controller in which Chattering successfully has been reduced. The proposed scheme has been implemented in three different phases for modeling and simulation of Quadrotor. Furthermore, for proper gain scheduling parameterization of sliding surface has been introduced by comparing sliding surface constants gains to the system given Eigenvalue to achieve Hurwitz stability this technique named Improved Double Integral Sliding Mode Controller (IDI-SMC).

\[
\begin{align*}
\dot{x}_1 &= \dot{\phi} = x_2 \\
\dot{x}_2 &= \dot{\phi} = b_2 U_2 - a_2 x_2 \Omega_x + a_4 x_4 x_6 \\
\dot{x}_3 &= \dot{\theta} = x_4 \\
\dot{x}_4 &= \dot{\theta} = b_3 U_3 - a_4 x_2 \Omega_x + a_5 x_2 x_6 \\
\dot{x}_5 &= \dot{\psi} = x_6 \\
\dot{x}_6 &= \dot{\psi} = a_3 x_2 x_4 + b_3 U_4 \\
\dot{x}_7 &= \dot{z} = x_8 \\
\dot{x}_8 &= \dot{z} = g - \frac{U_1}{m} (\cos x_1 \cos x_3) \\
\dot{x}_9 &= \dot{y} = x_9 \\
\dot{x}_{10} &= \dot{y} = \frac{U_1}{m} (\sin x_1 \sin x_3 + \cos x_1 \cos x_3 x_1) \\
\dot{x}_{11} &= \dot{y} = x_{11} \\
\dot{x}_{12} &= \dot{y} = \frac{U_1}{m} (\cos x_1 \sin x_3 - \cos x_3 x_1)
\end{align*}
\]  

(9)
Let general state space equation is:

\[ X(t)^n = f(x,t) + g(x,t)u \]  

(10)

Where \( X(t) \) the state is vector and \( u \) is control input. For designing of sliding mode controller, two steps are necessary. Step 1: The choice of the sliding surface and step 2: The verification of control law.

Generally, we have a sliding surface like in equation (21).

\[ S(x) = c e + \dot{e} \]  

(11)

Where \( c \) is a positive constant value and \( e \) is an error which is \( e = x - x_d \). For control and verification, Lyapunov function and from theory \( V(s) \) should be positive and \( \dot{V}(s) \) should be negative definite for stability criteria described below

\[ V(s,x,t) = \frac{1}{2} S^2(x,t) \text{ and } \dot{V}(s,x,t) = \dot{V}(s) = SS - < \eta | S > \]  

(12)

Where \( \eta \), \( K_1 \) and \( K_2 \) are a positive value (\( K_2, \eta, K_1 > 0 \)). After this it obtained two control law called switching and equivalent terms:

\[ u = u_i + u_s \text{ and } u_s = -K_s sign(s) - K_c(s) \text{ where } sign(s) = \begin{cases} -1 & \text{if } s < 0 \\ 1 & \text{if } s > 0 \end{cases} \]  

(13)

3.1. Sliding Mode Controller for Quadrotor

Now for controlling of row angle let we have to design errors and Sliding surface

\[ V(s,x,t) = \frac{1}{2} S^2(x,t) \text{ and } \dot{V}(s,x,t) = \dot{V}(s) = SS - < \eta | S > \]  

(14)

From equation (14) we get control law for row angle which is given in equation (25).

\[ U_i = \frac{1}{b_0} \left[ K_s sign(s) + K_c(s) + c_i(\dot{\phi}_d - \dot{\phi}) + \dot{\phi}_d + a_i \ddot{\Omega} - a_i \ddot{\phi} \dot{\psi} \right] \]  

(15)

Similarly, for control inputs for Pitch, Yaw and Altitude \([U_3, U_4, U_1]\) have been given in the equation. (16). The

the phases of the sliding surface have been shown in Figure-2.

\[ \frac{1}{b_0} \left[ K_s sign(s) + K_c(s) + c_i(\dot{\theta}_d - \dot{\theta}) + \dot{\theta}_d + a_i \ddot{\Omega} - a_i \dot{\theta} \dot{\psi} \right] \]

\[ \frac{1}{b_3} \left[ K_s sign(s) + K_c(s) + c_i(\dot{\psi}_d - \psi) + \dot{\psi}_d + a_i \dot{\psi} \right] \]

\[ \frac{m}{\cos \phi \cos \theta} \left[ K_s sign(s) + K_c(s) + c_i(\dot{z}_d - \dot{z}) - \dot{z} + g \right] \]

(16)

A double integral sliding mode controller has been used in this technique for the removal of the steady-state error. To tackle chattering across steady-state integral control also has been preferred. To drive control law mathematical modeling has been given below.
Figure 1. Configuration and schematic of the quadrotor.

Figure 2. Sliding Phases

Let for Roll angle we can drive control law for Roll angle with their error.

$$\dot{x}_i = x_i \Rightarrow \dot{x}_i = x_i x_i a_i - x_i \Omega_i a_i + b U_2$$

$$e_1 = x_1 - x_{1_{ref}}, \quad e_2 = \int e_1 dt \quad e_3 = e_2$$

$$\dot{\hat{S}}_i = (c_1 + 1)(x_i - \hat{x}_{i_{ref}}) + (c_2 + 1)e_i + c_3 e_2 + x_i x_i a_i - x_i \Omega_i a_i + b U_2 - \hat{x}_{i_{ref}}$$

$$\dot{\hat{S}}_i = -K_i \text{sign}(s) - K_i(s)$$

Now from Eq. (18), (19) a control law has been developed for Double Sliding Mode Controller shown in Eq. (20).

$$U_2 = \frac{1}{b_1} \left[ -(c_1 + 1)(x_2 - \hat{x}_{2_{ref}}) - (c_2 + 1)e_2 - c_2 e_2 - x_2 x_2 a_2 + x_2 \Omega_2 a_2 + \hat{x}_{2_{ref}} - K_i \text{sign}(s) - K_i(s) \right]$$

Similarly, for control inputs for Pitch, Yaw and Altitude $U_3, U_4, U_5$ have been given in equations (21)

$$U_3 = \frac{1}{b_2} \left[ -(c_1 + 1)(x_3 - \hat{x}_{3_{ref}}) - (c_2 + 1)e_3 - c_2 e_3 - x_3 x_3 a_3 - x_3 \Omega_3 a_3 + \hat{x}_{3_{ref}} - K_i \text{sign}(s) - K_i(s) \right]$$

$$U_4 = \frac{1}{b_3} \left[ -(c_1 + 1)(x_4 - \hat{x}_{4_{ref}}) - (c_2 + 1)e_4 - c_2 e_4 - x_4 x_4 a_4 - \hat{x}_{4_{ref}} - K_i \text{sign}(s) - K_i(s) \right]$$

$$U_5 = \frac{m}{\cos x_1 \cos x_3} \left[ -(c_1 + 1)(x_5 - \hat{x}_{5_{ref}}) - c_2 e_5 - (c_2 + 1)e_5 - g + \hat{x}_{5_{ref}} - K_i \text{sign}(s) - K_i(s) \right]$$
3.2. Determination of Switching Surface coefficient

To determine the switching surface coefficient $S$ and $\dot{S}$ should be taken as zero. The sliding surface and errors have given below

$$S = a_1e_1 + a_2e_2 + a_3e_3, \quad e_1 = x_1 - x_{1,ref}, \quad e_2 = \int e_1 dt, \quad e_3 = \int \left( \int e_1 dt \right) dt$$  \hspace{1cm} (22)

When $S$ taken as zero we get error

$$e_1 = -\frac{a_2}{a_1}e_2 - \frac{a_3}{a_1}e_3 \Rightarrow -\frac{a_2}{a_1} \int x_1 - x_{1,ref} dt - \frac{a_3}{a_1} \int \left( \int x_1 - x_{1,ref} dt \right) dt$$  \hspace{1cm} (23)

Considering $y_1 = e_1$, $y_2 = e_2$ and $y_3 = e_3$ we get

$$\dot{y}_1 = y_2 \Rightarrow \dot{y}_2 = \frac{a_2}{a_1}y_1 - \frac{a_3}{a_1}y_1$$  \hspace{1cm} (24)

After linearizing with Tayler series expansion around equilibrium point we get set of the equation as

$$\begin{bmatrix} \dot{y}_1 \\ \dot{y}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -\frac{a_2}{a_1} & -\frac{a_3}{a_1} \end{bmatrix} \lambda^2 + \frac{a_2}{a_1} \lambda + \frac{a_3}{a_1}$$  \hspace{1cm} (25)

According to Hurwitz’s stability criteria, a matrix is stable if real parts of Eigen-values lie on the left half-plane. The coefficient of the surface will be obtained by using $\lambda I - A$ comparing with desirable stable points such as $(\lambda + 1)(\lambda + 2)$ and assuming $a_i = 1$ we get a sliding surface coefficient.

$$\frac{a_2}{a_1} = 3, \quad \frac{a_3}{a_1} = 2, \quad a_3 = 1$$  \hspace{1cm} (26)

4. Results and Simulations

In this section, Simulations of proposed scheme controller and other controller has been done in the MATLAB/SIMULINK environment. In Fig. (3), the graphs of the roll ($\phi$), pitch ($\theta$) and yaw ($\psi$) with Altitude have been shown to differentiate these controllers respectively Green, blue and red line shown for SMC, Double Integral SMC and Improved Double Integral Sliding Mode Control for Row, angels respectively. As from Figure Simple Sliding Mode controller shows overshoot with a steady-state error and it produces chattering. In the blue, the chattering is successfully removed by the improved double integral sliding mode controller. The graph which is for IDI-SMC has been shown in the red line. The effectiveness of the proposed graph. The reference trajectory for a roll ($\phi$), pitch ($\theta$) and yaw ($\psi$) are taken as $5^\circ$ where Altitude reference is taken as $2^\circ$. In all cases, it has been seen that the SMC controller is good but due to the sliding surface it produces chattering around steady-state. As the integral is preferred for reducing steady-state error to achieve asymptotically stability so applying double integral SMC the chattering has been successfully reduced In the presence of three different control techniques, IDI-SMC based trajectories show better performance.

Figure 3(a). SMC, DI-SMC, and IDI-SMC response for the Row angle
The performance of altitude control has been shown in Fig.3(d) which shows that if Euler angles are not controlled directly, altitude still remains stable. These Schemes have been verified by these simulations. As the integral is preferred for reducing steady-state error to active asymptotically stability so applying double integral SMC the chattering has been successfully reduced. Furthermore, after the parameterizations of sliding surface gain, comprehensive results have been achieved. It can be clearly seen that all states of the quadrotor are converged to their desired states while all control errors converge to zeros shown in Fig (4).
Figure 4. Control inputs and errors

Table 1. Quadrotor Parameters

| Parameter | Value   | Unit   |
|-----------|---------|--------|
| $I_{xx}$  | 7.5e-3  | kg.m$^2$ |
| $I_{yy}$  | 7.5e-3  | kg.m$^2$ |
| $I_{zz}$  | 1.4e-2  | kg.m$^2$ |
| $I$       | 0.24    | M      |
| $J_r$     | 6e-5    | kg.m$^2$ |
| m         | 0.65    | kg     |
| $K_f$     | 3.13e-5 | Ns$^2$ |
| $K_m$     | 7.5e-7  | Nm s$^2$ |
| $K_r$     | diag(0.1,0.1,0.15) | Nm s |

5. Conclusion
In this paper, we presented an efficient controller technique for a quadrotor. Firstly, start with the development of a dynamic model we design Sliding Mode Controller for a quadrotor. Steady-state errors of the control system have been eliminated by augmented improved double integral action taking the strong condition of the sliding surface. Furthermore rather than taking assumption we introduce parameterization technique through Eigen-values made this scheme valuable. In this way, strong controller techniques have been developed. The simulations results proved satisfactory results addressed by IDI-SMC and proved the effectiveness of the proposed scheme. By further improvements and designing of parameters of sliding surface, the control effort could be less and control will be more valuable.

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