Power flux in five layers slab waveguide with metamaterials

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Abstract. Five-layer slab a waveguide combined with a metamaterial has been studied. The central layers were the metamaterial and a normal material in the core of waveguide. The dispersion relation, field formulas, power formulas and the power flux formula has been derived. The waveguide property was controlled by changing the permittivity and permeability and thicknesses of the layers. The presence of metamaterial contributes to cut-off the mode TE_0 and other modes were propagated to backward. The stopping light was determined using the power flux value. However, the negative values of it refer to the backward direction of propagation. Furthermore the stopped light can be achieved in a certain value of power. Our results found it is good for different applications in communication and controlling.

1. Introduction
The speed of light is intriguing: it is fast beyond human imagination, and yet its finite value sets the ultimate speed limit for the communication of information. Recently, laboratory demonstrations have shown that light pulses can be greatly slowed down, and even effectively halted and re-started, using coherently prepared atomic media. The materials had a negative index or metamaterial can slow down the light even it possible to stop it. That material displayed for the first time theoretically in 1968 by Veselago [1, 2]. The metamaterials simultaneously have a negative permittivity and permeability of electric and magnetic field with abnormal optical properties which make it appropriate for unique applications particularly in the area of quantum information processing [3]. An ability to controllably decelerate, stop, store and regenerate / release optical pulses in a low-loss regime, will conceivably have important potential applications, ranging from quantum memories for photons and storage of light, to the realization of optical buffers for photonic communication networks [4]. The must abnormality properties are a negative refraction index, sub wavelength imaging, backward wave propagation, and reverse Doppler and Cherenkov effects [5-7]. The Unconventional electromagnetic properties of metamaterials are presented when paired with other materials at least one oppositely signed constitutive parameter. In other words, when a double negative (DNG) material is paired with a double Positive (DPS), epsilon-negative (ENG), or mu-negative (MNG) layer, remarkable wave propagation characteristics that may be absent [8, 9]. This paper is aimed to investigate the nonlinear properties of guided modes and the power flux propagation and dispersion properties of in a planar asymmetric waveguide.

2. Five Layers Slab-Waveguide with Metamaterials
A schematic diagram of a five-layer waveguide structure is illustrated in Figure (1). It composed of a lossless guiding layer of permittivity ε_3, permeability μ_3, and thickness d_1. The guiding film is sandwiched between two left hand material LHM layers of permittivity ε_4, permeability μ_4 and
thickness $d_1$ for the lower layer and permittivity $\varepsilon_1$, permeability $\mu_2$ and thickness $d_2$ for the upper layer as shown in Fig.(1). Also, the study obtains the characteristics of waveguide with central LHM layer that inserts between two normal layers. The three layers are sandwiched between a semi-infinite substrate with permittivity $\varepsilon_5$ and permeability $\mu_5$ and a semi-infinite cladding with permittivity $\varepsilon_1$ and permeability $\mu_1$.

![Figure 1. Schematic diagram of the five layers structure.](image)

3. Wave Equation of Slab-Waveguide

All analyses of the electromagnetic waves propagation are based on Maxwell’s equations which control the time dependence of the intensity of the electric and magnetic field respectively[10].

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} \quad (1a)$$

$$\nabla \times \mathbf{H} = -\frac{\partial \mathbf{D}}{\partial t} \quad (1b)$$

It is also possible to present the electric permittivity and the magnetic permeability to describe the response of the materials to an external electric and magnetic fields as[11, 12]

$$\mathbf{D} = \varepsilon \mathbf{E} \quad (2a)$$

$$\mathbf{B} = \mu \mathbf{H} \quad (2b)$$

The electric and magnetic fields for the TE waves propagating along the z-axis with angular frequency $\omega$ and wavenumber $\beta_z$ can express as [13]

$$\mathbf{E} = \begin{bmatrix} 0 \\ E_x \\ 0 \end{bmatrix} e^{i(\beta_z - \omega t)} \quad , \quad \mathbf{H} = \begin{bmatrix} H_z \\ 0 \\ H_x \end{bmatrix} e^{i(\beta_z - \omega t)}$$

(3)

For similar parameters, the electric and magnetic fields of TM waves propagating along the z-axis will be[14]

$$\mathbf{E} = \begin{bmatrix} E_y \\ E_x \\ 0 \end{bmatrix} e^{i(\beta_z - \omega t)} \quad , \quad \mathbf{H} = \begin{bmatrix} 0 \\ H_y \\ 0 \end{bmatrix} e^{i(\beta_z - \omega t)}$$

(4)

A homogenous medium is one for which the quantities $\varepsilon$, $\mu$, $\sigma$ are constants through the medium. The medium is isotropic if $\varepsilon$ is a scalar constant so that the electric field and the displacement vectors in everywhere the same direction. Substituting Eq.(2) into (1) and assuming that the dielectric waveguide is linear, isotropic and homogeneous, yields[15]
Using Eq. (3) into (5) and simplifying the result, yields

$$\vec{\nabla} \times \vec{E} = -\mu \frac{\partial \vec{H}}{\partial t}$$

$$\vec{v} \times \vec{H} = \varepsilon \frac{\partial \vec{E}}{\partial t}$$

(5a)

(5b)

Substituting Eqs. (6a) and (6b) into (6c), simplifying the result and using the fact

$$\mu \varepsilon r$$

Substituting Eq. (4) into (5) and using a similar manner for TM mode, one may obtain

$$\frac{\partial^2 E_y}{\partial x^2} + \left( k_0^2 \varepsilon_i \mu_i - \beta_z^2 \right) E_y = 0$$

(7)

where $\beta_z = k_0 \varepsilon_{\text{eff}}$ is the propagation constant along the longitudinal direction and $\varepsilon_{\text{eff}}$ is the effective index of the propagation mode. If we set $\varepsilon = \varepsilon_r e_0$ and $\mu = \mu_r \mu_0$, Eq. (7) will be

$$\frac{\partial^2 E_y}{\partial x^2} + \left( k_0^2 \varepsilon_i \mu_i - \beta_z^2 \right) E_y = 0$$

(8)

where $\varepsilon_i$ is the relative permittivity, $\mu_i$ is the relative permeability, $\varepsilon_i$ (where $i=1,2,3,4$ and 5) represents the relative permittivity of cladding, LHM and substrate, and $\mu_i$ the same range of $i$ denote the relative permeability of cladding, LHM, and substrate. The free space wavenumber is defined as $k_0 = \omega / c$.

Substituting Eq. (4) into (5) and using a similar manner for TM mode, one may obtain

$$\frac{\partial^2 H_y}{\partial x^2} + \left( k_0^2 \varepsilon_i \mu_i - \beta_z^2 \right) H_y = 0$$

(9)

4. Fields in the Five Layers Slab-Waveguide with Metamaterials

In order to model the present waveguides, Eqs. (8) and (9) must be solved to construct the fields $E_y$ and $H_y$ in different layers. Consequently, Eqs. (6) will be used to explain the other components of the electric and magnetic fields. The solutions of Eq. (8) in the five-layers are given by

$$E_{y1} = A e^{-k_1 (x-d_2)} \quad , \quad x > d_2$$

$$E_{y2} = B e^{-k_2 (x-d_1)} + C e^{-k_3 (x-d_1)} \quad , \quad d_2 > x > d_1$$

$$E_{y3} = D e^{-i k_4 x} + E e^{-i k_5 x} \quad , \quad d_1 > x > 0$$

$$E_{y4} = F e^{-k_4 x} + G e^{k_5 x} \quad , \quad 0 > x > -d_3$$

$$E_{y5} = H e^{k_5 (x+d_3)} \quad , \quad x < -d_3$$

(10a)

(10b)

(10c)

(10d)

(10e)

where

$$k_3 = \sqrt{k_0^2 \varepsilon_3 \mu_3 - \beta_z^2}$$

$$k_4 = \sqrt{k_0^2 \varepsilon_4 \mu_4 - \beta_z^2}$$

$$k_5 = \sqrt{k_0^2 \varepsilon_5 \mu_5 - \beta_z^2}$$

(11a)

(11b)

The parameters $A, B, C, D, E, F, G$ and $H$ represent the amplitudes of the waves in the different layers. The parameters $k_i$ represent the propagation constants in the $x$-direction for the five-layers. The magnetic field component $H_y$ can be calculated using Eq. (6a) as follows:
Where the coefficients \( A \) to \( H \). That is;

The determinant of the matrix coefficients in Eq.(14) must be zero to obtain the nonzero solutions of this equation is called the characteristic equation of TE modes. Using the following normalization while the component of the magnetic field \( H_z \) are calculated using Eq.(6b) as follows

\[
H_z = \frac{l}{\omega} \begin{cases} 
-k_1 \frac{A}{\mu_1} e^{-k_1(x - d_2)}, & x > d_2 \\
-k_2 \frac{B}{\mu_2} e^{-k_2(x - d_1)} + \frac{k_2}{\mu_2} \frac{C}{\mu_2} e^{-k_2(x - d_1)}, & d_2 > x > d_1 \\
-ik_2 D e^{-ik_3 x} + \frac{ik_4}{\mu_4} G e^{ik_4 x}, & d_1 > x > 0 \\
-k_4 \frac{F}{\mu_4} e^{-k_4 x} + \frac{k_4}{\mu_4} G e^{k_4 x}, & 0 > x > -d_3 \\
\frac{k_5}{\mu_5} H e^{k_5(x + d_3)}, & x < -d_3 
\end{cases}
\]

Matching the components of the tangential and normal magnetic fields at \( x = -d_3, 0, d_1, d_2 \), yields

\[
\begin{align*}
A - aB - \frac{1}{a} C &= 0 \\
A - \eta_1 aB + \frac{\eta_1}{a} C &= 0 \\
B + C - bD - \frac{1}{b} E &= 0 \\
B - C - i\eta_2 bD + \frac{i\eta_2}{b} E &= 0 \\
D + E - F - G &= 0 \\
D - E + i\eta_3 F - i\eta_3 G &= 0 \\
cF + \frac{1}{c} G - H &= 0 \\
cF - \frac{1}{c} G + \eta_4 H &= 0
\end{align*}
\]

where

\[
\eta_i = \frac{\mu_i k_{i+3}}{\mu_i k_{i+4}}, \quad i=1,2,3,4
\]

The determinant of the matrix coefficients in Eq.(14) must be zero to obtain the nonzero solutions of the coefficients \( A \) to \( H \). That is;

\[
k_3 d_1 = \tan^{-1} \left( \frac{\eta_3 k_1}{\mu_3^2 k_3} \right) + \tan^{-1} \left( \frac{\eta_4 k_2}{\mu_4^2 k_3} \right) + m\pi, \quad m = 1,2,3,\ldots\text{mode order}
\]

Where \( R = \frac{(1+\eta_1 T_3)(1+\eta_3 T_3)}{(\eta_4 T_3)(\eta_4 + T_3)} \)

This equation is called the characteristic equation of TE modes. Using the following normalization rules[16]
\[
V = k_0 d_1 \sqrt{n_3^2 - n_4^2}
\]
\[
p = n_{\text{eff}}^2 - n_4^2
\]
\[
\Delta = \frac{n_2^2 - n_4^2}{n_2^2 - n_4^2}
\]

where \( V \) denotes normalized frequency that increases with the increase of frequency \( p \) denotes the normalized propagation constant that has a value range \(-\infty \) to \( \infty \), when \( p=0 \), the guided mode vanish. \( n_{\text{eff}} \) is the effective refractive index \( n_{\text{eff}} = \beta / k_0 \). \( \Delta \) is a parameter depicts rate of asymmetric in a five layers waveguide.

When \( \Delta = 0 \), it means a symmetric waveguide. A new normalized dispersion equation can be obtained from these transformations:

\[
V = \frac{\tan^{-1} \left( \frac{\mu_2}{\mu_4 - p} \right) + \tan^{-1} \left( \frac{\mu_2 p}{\mu_4 - p} \right) + m}{\sqrt{1 - p}} \tag{17}
\]

and the propagation constants will be

\[
k_i = k_0 \sqrt{p n_3^2 + (1 - p) n_2^2 - n_4^2} \quad i=1,2,4,5
\]
\[
k_3 = k_0 \sqrt{n_3^2 + p n_2^2 - (1 - p) n_4^2}
\]

Now, depending on the above results, the parameters take the following expressions:

\[
\begin{aligned}
H &= 1 \\
F &= \frac{1 - \eta_4}{2c} \\
G &= \frac{1 + \eta_4}{2c} \\
D &= \frac{1}{2} \left( \frac{1 - \eta_2}{2} F + \frac{1 + i \eta_3}{2} G \right) \\
E &= \frac{1}{2} \left( \frac{1 + i \eta_3}{2} F + \frac{1 - \eta_2}{2} G \right) \\
B &= b \left( \frac{1 - \eta_2}{2} D + \frac{1 - i \eta_3}{2} E \right) \\
C &= b \left( \frac{1 - i \eta_3}{2} D + \frac{1 + i \eta_3}{2} E \right) \\
A &= a \left( \frac{1 + i \eta_3}{2} B + \frac{1}{a} \left( \frac{1 - \eta_3}{2} C \right) \right)
\end{aligned}
\tag{18}
\]

Using the result into Eqs.(10), yields

\[
E_{y1} = \left[ \frac{a}{2} \left( \frac{1 + i \eta_3}{2} B + \frac{1}{a} \left( \frac{1 - \eta_3}{2} C \right) \right) e^{-i k_1 (x-d_2)} \right]
\]
\[
E_{y2} = \left[ b \left( \frac{1 + i \eta_2}{2} D + \frac{1}{b} \left( \frac{1 - i \eta_2}{2} E \right) e^{-i k_2 (x-d_1)} + \left[ \frac{1 - i \eta_3}{2} F + \frac{1}{b} \left( \frac{1 + i \eta_3}{2} E \right) e^{i k_3 x} \right] \right] \right]
\]
\[
E_{y3} = \left[ \frac{1}{2c} \left( \frac{1 + i \eta_3}{2} B + \frac{1}{a} \left( \frac{1 - \eta_3}{2} C \right) \right) e^{i k_4 x} \right]
\]
\[
E_{y4} = \left[ \frac{1}{2c} \left( \frac{1 - i \eta_3}{2} D + \frac{1 - i \eta_3}{2} e^{i k_5 x} \right] \right]
\]
\[
E_{y5} = \left[ \frac{a}{2} \left( \frac{1 + i \eta_3}{2} B + \frac{1}{a} \left( \frac{1 - \eta_3}{2} C \right) \right) e^{-i k_5 x} \right]
\]

5. Energy Flux

Energy flux in waveguide considers as an important parameter, which may be calculated using the Poynting vector averaged over the period \( T \) and it is defined as \( S = \text{Re} \left[ \mathbf{E} \times \mathbf{H}^* \right] / 2 \). For monochromatic plan wave where the energy flux is directed along the z-axis, the transverse profile of
$S_z$ guided mode for given waveguide parameters can be constructed as $S_z = (\beta_z |E|^2 / 2)\mu_0$. It is possible to achieve power flux as an integral of the Poynting vector. There for the energy flux per unit length for TE mode is given by [16, 17]

$$p_{TE}^{tot} = \frac{\beta_z}{2\omega\mu_0} \int_{-d_2}^{d_2} |E|^2 dx = P_1^{TE} + P_2^{TE} + P_3^{TE} + P_4^{TE} + P_5^{TE}$$

(20)

Using Eqs.(19) and the definition of the power at each layer will be

$$P_1 = \frac{\beta_z}{4\omega\mu_1 k_1} |A|^2$$

(21a)

$$P_2 = \frac{\beta_z}{2\omega\mu_2} \left[(BC^* + CB^*)\Delta + \frac{\sinh k_2 \Delta}{k_2} \frac{a^2 |B|^2 + |C|^2}{a b} \right]$$

(21b)

$$P_3 = \frac{\beta_z}{2\omega\mu_3} \left[(D^2 + |E|^2)d_1 + \frac{\sinh k_3 d_1}{k_3} \frac{b^2 |D|^2 + |E|^2}{c} \right]$$

(21c)

$$P_4 = \frac{\beta_z}{2\omega\mu_4} \left[(F^* G^* + G F^*)d_3 + \frac{\sinh k_4 d_3}{k_4} \frac{c^2 |F|^2 + |G|^2}{c} \right]$$

(21d)

$$P_5 = \frac{\beta_z}{4\omega\mu_5 k_5}$$

(21e)

where $p_i^{TE}, i = 1, 2, 3, 4, 5$ are the powers in the indicated layers. It is noted that the powers in the cladding and substrate layers are always positive while the power in the LHM layer can be negative or positive. For TE or TM mode, to obtain the net power flux in waveguide, normalized power was used [12]. It is possible to illustrate normalized power flux as:

$$\bar{P}^i = \frac{p_i^{TE}}{|p_i^{TE}|} + p_i^{TM} + p_i^{TM}, \quad i = TE, TM$$

(22)

When $\bar{P}^i < 0$, the power flow of the guided mode is antiparallel with the direction of phase flow. The resulting wave is known as the backward wave. The opposite result can be obtained when $\bar{P}^i > 0$. The wave, in this case, is known as the forward wave.

6. Result and Discussion

Figure (2) illustrates the relationship between $V$ and $p$ for the first six modes in a waveguide of five layers with MM and another one without MM. Colors represent the different modes: TE0 to TE6. It should be noted here that the modes: TM has not been plotted here as they are attenuated with $TE$ and there is no need to repeat the discussion of the modes TM. The presence of the normal materials contributes to the creation of the mode TE0 which disappears when there are MM. In other words, Equation (17) does not produce the case n=0. This means that MM contribute to the cut-off of the mode TE1, which is impossible in case of the normal materials. The cut-off frequency for all modes decreases in the presence of MM [18]. On the other hand, it is observed that the slope of all normal waveguide curves is positive, which means that the forward propagation is always constant. When there is a MM, some curves have a negative slope and the slope of the mode TE6 is always negative. This means the possibility of a backward propagation. It is noted that the negative part of slope decreases as the mode order increases. In other words, lower-order modes contribute more to backward propagation.
Figure (3) illustrates the relationship between $V$ and $P$ for the first six modes using a normal central layer surrounded by two layers of MM. The different colors indicate different values of permeability $\mu_3$. In this figure, the section of each mode has been magnified to illustrate the idea of backward propagation. The cut-off frequency of each mode was not affected by the change of $\mu_3$, but curvature increases with increasing $\mu_3$. This indicates that backward propagation of each mode increases with increasing $\mu_3$. On the other hand, it is noticed that curvature decreases when the mode order increases[19].

Figure (4) shows the distribution of the electric field during waveguide layers of the modes $TE_1$ to $TE_6$. It is evident from the figure that the domains with the odd-order are asymmetric, but the domains with the even order are symmetric. On the other hand, the number of intersecting points with the x-axis equals the order. In general, the main field is within the central layer, and it attenuates through the surrounding layers. If the propagation constant is imaginary in a given layer, the field attenuates outward.

Figure 3. The enlarged view of the dispersion relation for different values of $\mu_3$, when $\varepsilon_1 = 1, \mu_1 = 1, \varepsilon_2 = -2, \mu_2 = -2, d_1 = 2\mu m, d_2 = 3\mu m$
If propagation is conventional or real, there is an opportunity for the mode to propagate in the
direction \( z \), on the surface separating the insulator and the conductor, or between MMs. Surface waves

can be noticed at \( x = 0.2 \) as the value of the domain \( E \) is of great value. Figure (5) shows the
relationship between the power flow and the cutting off frequency of the calibrated \( V \) at
\( d_1 = 2 \mu \text{m}, \; d_2 = 2.5 \mu \text{m}, \; \mu_2 = \mu_4 = 2 \) for the values \( \mu_3 = (-2.8, \; -2.5, \; -2.05) \).
of the modes \( TE^1 \) to \( TE^4 \). It is clear from the figure that \( TE^1 \) is different from the other modes, and
it tends to achieve a stopping of light with a decrease in the value of \( \mu_3 \). The approach of \( |\mu_3| \) from
\( \mu_2 \) is enable reaching a stop of the mode. This is one of the principles of waveguide work that the
coefficient of refraction between the core and the perimeter is slightly different. For the other modes, it
is possible to achieve a zero-power flow for all values \( \mu_3 \).

\begin{figure}
\centering
\includegraphics[width=\textwidth]{figure4.png}
\caption{The field distributions in five layers waveguide, when
\( \varepsilon_1 = 1, \mu_1 = 1, \varepsilon_2 = 2, \mu_2 = 2, \varepsilon_3 = -2.1, \mu_3 = -2.1, d_1 = 2\mu\text{m}, d_2 = 3\mu\text{m} \).}
\end{figure}

Nonetheless, the decrease in \( |\mu_3| \) achieves a higher calibration frequency. This behavior is
due to the negative inclination zones of the modes, which varies due to the change in the
values of \( \mu_3 \), the increase of that values result a negative slope. From the figure, it is also
noticed that the range \( V \) which results in a flow of power is different from one mode to
another. \( V \) (x-axis) is plotted in the first figure in the range (1-4), in the second figure, in the
range (5-7), and in the third figure, in the range (8-21). This completely corresponds to the
dispersion relationship painted in Figure (4.1).
Conclusions

By using the slab waveguide in any number of layers leads to cut-off the mode $\text{TE}_0$. The mode $\text{TE}_1$ is the first mode in the slab waveguide, that contains a metamaterial and it has a negative curved slope for all frequency range. All the other modes $\text{TE}_n (n \geq 2)$, have a range of frequency, which makes their tendency negative. That is; they may suffer from a backward propagation within this range. The negative range of all modes is affected by the values $\mu_2$ and $\mu_3$ where the negative range increases with increasing $|\mu_1|$ or decreasing $|\mu_3|$. The change of power through the waveguide depends on the presence of metamaterial layer, in which, it is negative and positive in the other layers. The power flow is affected by the value of $|\mu_2|$, $|\mu_3|$, $d_1$, and $d_2$. The mode $\text{TE}_1$ can achieve the stopping of light within a wide range of $V$ and $p$ unlike other modes, which achieve the stopping of light only at specific points.

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References

[1] Shalaev V M 2007 Nature photonics 1 41
[2] Taya S A and Qadoura I M 2013 Optik-International Journal for Light and Electron Optics 124 1431-6
[3] Kullab H M, Qadoura I M and Taya S A 2015
[4] Tsakmakidis K L and Hess O 2008 Advances in Slow and Fast Light: International Society for Optics and Photonics) 690405
[5] Taya S A, Kullab H M and Qadoura I M 2013 JOSA B 30 2008-13
[6] Grbic A and Eleftheriades G V 2003 Applied Physics Letters 82 1815-7
[7] Abadla M M and Taya S A 2014 Optik-International Journal for Light and Electron Optics 125 1401-5
[8] Engheta N and Ziolkowski R W 2006 Metamaterials: physics and engineering explorations: John Wiley & Sons
[9] Afanas’ev S, Sementsov D and Yakimov Y 2016 Optics Communications 369 164-70
[10] Kumar S, Kumari A and Pradhan B 2015 Optik 126 3706-12
[11] Abu Lebda S J 2016 Theoretical Analysis of Metamaterial–Multilayer Waveguide Structures for Solar Cells
[12] Shu W 2008
[13] Markos P and Soukoulis C M 2008 Wave propagation: from electrons to photonic crystals and left-handed materials: Princeton University Press)
[14] Mehjez E M 2008 Metamaterial Optical Waveguide Sensors
[15] Wartak M S, Tsakmakidis K L and Hess O 2011 Physics in Canada 67 30-4
[16] Al Juaidi A A 2016 Properties of graded index thin film waveguides comprising left-handed materials
[17] Baqir M and Choudhury P K 2012 Journal of Electromagnetic Waves and Applications 26 2165-75
[18] Lee C-H and Lee J 2012 Computers, Materials, & Continua 31 147-56
[19] Abadla M M and Taya S A 2017 Optik 131 562-73