Research on tidal harmonic analysis based on correlation observation error

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Abstract. The analysis and calculation of tides play a very important role in the correction of water level. The analysis and calculation accuracy are directly related to the correction of water level and the calculation of depth reference surface. For the problem that the observation error is not correlated in the tidal harmonic analysis, and the actual situation of the correlation of the observation error is taken into account. On the basis of the traditional harmonic analysis model, the cofactor matrix is derived by calculating the correlation coefficient sequence of different time intervals of residual water level, and the new observation weight matrix is obtained by inversion. The tidal harmonic analysis model based on the correlation of observation errors is derived and improved. The experimental analysis is carried out by tidal data of monthly adjustment and analysis, and the experimental results show that most of the main tidal harmonic constants are closer to the results of 2019, and the forecast tidal level is closer to the real tidal level. The improved method has achieved certain improvement effects.

1. Introduction

The tidal harmonic constant is the necessary data for tidal model construction, tidal forecasting, and determination of the sea level depth datum\cite{1-3}. In the hydrographic survey, the harmonic constant is obtained by tidal harmonic analysis of the measured water level data of the tide gauge station\cite{4}. In tidal harmonic analysis, the accuracy of the calculation of harmonic constants is often affected by factors such as observation errors, meteorological conditions at sea, and duration of observation\cite{5}. In order to improve the accuracy of tidal analysis calculations, many experts and scholars have done a lot of research on tidal data processing, tidal analysis and calculation\cite{6}. Based on the traditional tidal harmonic analysis, this paper considers the practical problem of observation error correlation, constructs a new observation weight matrix by using the time correlation of the residual water level, improves the traditional harmonic analysis method, and experimentally verifies its feasibility.

2. Construction of harmonic analysis model related to observation error

In the tidal analysis, the water level observed by the tide station is inevitably affected by non-tidal factors such as short-term meteorological changes, resulting in a non-tidal water level, the residual water level r(t), also known as the disturbance term, in the harmonic analysis\cite{5}. It is considered as an
observation error. Since the disturbance term is random, and for the convenience of processing, it is generally considered that the observation errors are irrelevant to each other, so the observation weight matrix is the unit matrix I. In fact, the residual water level (disturbance term) is not only random, but also affected by weather factors and the inertia of the ocean water body[7]. It has certain continuity in the time domain and the spatial domain within a certain range. Therefore, there must be a correlation between the observation errors, so the observation weight matrix should not be set to the unit array I.

2.1. Traditional harmonic analysis adjustment model
Assuming that \( J \) tidal waves are selected. To facilitate subsequent calculations, the measured tidal level is expressed as[8]:

\[
h(t) = \hat{h} + r = a_0 + \sum_{i=1}^{J} (x_i \cos \sigma_i t + y_i \sin \sigma_i t) + r
\]

(1)

In the formula, \( x_i = H_i \cos \theta_i, y_i = H_i \sin \theta_i \), in addition, \( \theta_i = g_i - v_{oi}, \) \( v_{oi} \) represents the phase of the tide at the \( i \) moment, which can be calculated by looking up the table, \( a_0, H_i, g_i \) represents the true value of the mean sea surface and the harmonic constant, and \( r \) is the observation error.

After adding the equation (1) to the intersection point factor \( f \) and the intersection correction angle \( u \), it is written as the error equation:

\[
v(t) = a_0 + \sum_{i=1}^{J} (f_i \cos(V_i(t) + u_i) \hat{x}_i + f_i \sin(V_i(t) + u_i) \hat{y}_i) - h(t)
\]

(2)

In the formula, \( v(t) \) is the Greenwich astronomical phase angle at time \( t \), and the meanings of the remaining parameters have the same meaning as the parameters of (1).

The water level observation error equations at all times are written in matrix form in the order of observation[9]:

\[
V = B \hat{Z} - L = \begin{bmatrix} B_1 & B_2 \end{bmatrix} \begin{bmatrix} X \ X^T \end{bmatrix} - L
\]

(3)

In the equation, \( V \) is the correction vector, expressed as \( V = [v(t_1) v(t_2) \cdots v(t_N)]^T \), \( B \) is the coefficient matrix of the observation equation, which is written as the block matrix form \( B = [B_1, B_2] \), \( \hat{Z} = \begin{bmatrix} X & Y \end{bmatrix} \), \( r \) is the parameter estimation vector according to the cosine component and the sine component decomposition, \( \hat{X} = \begin{bmatrix} \hat{x}_1 & \hat{x}_2 & \cdots & \hat{x}_j \end{bmatrix}^T \), \( \hat{Y} = \begin{bmatrix} \hat{y}_1 & \hat{y}_2 & \cdots & \hat{y}_j \end{bmatrix}^T \), \( L = [h(t_1) h(t_2) \cdots h(t_N)]^T \) are water level observation vector.

The expressions of \( B \) and \( C \) are:

\[
B_1 = \begin{bmatrix} 1 & f_1 \cos(V_1(t_1) + u_1) & \cdots & f_j \cos(V_j(t_1) + u_j) \\
1 & f_1 \cos(V_1(t_2) + u_1) & \cdots & f_j \cos(V_j(t_2) + u_j) \\
\vdots & \vdots & \ddots & \vdots \\
1 & f_1 \cos(V_1(t_N) + u_1) & \cdots & f_j \cos(V_j(t_N) + u_j) \\
\end{bmatrix}
\]

(4)

\[
B_2 = \begin{bmatrix} f_1 \sin(V_1(t_1) + u_1) & \cdots & f_j \sin(V_j(t_1) + u_j) \\
f_1 \sin(V_1(t_2) + u_1) & \cdots & f_j \sin(V_j(t_2) + u_j) \\
\vdots & \vdots & \ddots & \vdots \\
f_1 \sin(V_1(t_N) + u_1) & \cdots & f_j \sin(V_j(t_N) + u_j) \\
\end{bmatrix}
\]

(5)
Based on the least squares criterion, the specific model used for long-term tidal observation data is the indirect adjustment model. Usually, the water level observation error is independent and accurate at each time, so the least squares estimation expression of the parameter vector can be obtained:

$$\hat{Z} = (B^T B)^{-1} B^T L$$  \hfill (6)

For medium- and short-term tidal observation data, such as one-month observations, the specific model used is accompanied by a constraint adjustment model\[10\]. The method equation is:

$$\begin{align*}
N_{BB} \hat{Z} + C^T K_S - W &= 0 \\
C \hat{Z} + W_s &= 0
\end{align*}$$  \hfill (7)

Convert to matrix form as:

$$\begin{bmatrix} N_{BB} & C^T \\ C & 0 \end{bmatrix} \begin{bmatrix} \hat{Z} \\ K_S \end{bmatrix} = \begin{bmatrix} W \\ W_s \end{bmatrix}$$  \hfill (8)

In the equation, $N_{BB} = B^T B, W = B^T L, C$ is a coefficient matrix of constraints, and get the equation from the above formula:

$$\hat{Z} = (N_{BB}^{-1} - N_{BB}^{-1} C^T N_{CC}^{-1} C N_{BB}^{-1}) W - N_{BB}^{-1} C^T N_{CC}^{-1} C^T W_s$$  \hfill (9)

The purpose of the improvement in this section is to redefine the weight $P$ using the time continuity of the residual water level.

2.2. Construction of new observation weights

In view of the fact that the observed noise is small compared to the non-tidal water level, regardless of the noise, the residual water level and the measured water level minus the astronomical tide level and the average sea level value, the expression of the residual water level at time $t$ is:

$$r(t) = h(t) - a_0 - \sum_{j=1}^{J} f_j H_j \cos(V_j(t) + u_j - g_j)$$  \hfill (10)

Affected by the continuity of ocean water body inertia and meteorological factors, the residual water level has a correlation in the time domain, and the shorter the time, the larger the correlation coefficient\[11\]. By finding the correlation coefficient sequence of the residual water level at different time intervals (1h-23h) in one year, the co-factor matrix is constructed, and the new observation weight matrix is obtained by inverting the co-factor matrix, and then the new harmonic constant is obtained through the tidal harmonic analysis. Considering the practical significance, when the time interval exceeds 24 hours, the correlation of the residual water level almost disappears, so the time interval is set from 1h to 23h. The co-factor matrix is a sparse matrix in which the main diagonal is 1 and the sequence of residual water correlation coefficients is symmetrically paralleled by the main diagonal:

The residual water level sequence of one year observation data obtained by the above formula is the number of residual water levels, and the mathematical level of the residual water level is 0 for a sufficiently long time series. The residual water level variance and the medium error estimate can be calculated by the following formula: \[11\]

$$\sigma_r^2 = \frac{\sum_{j=1}^{N} r_j^2}{N-1}$$  \hfill (11)

The residual water level covariance with time interval $t=1$ can be calculated by:

$$\sigma_r(k) = \frac{\sum_{j=1}^{N-K} r_j r_{j+k}}{N-K-1}$$  \hfill (12)

All covariances are divided by the variance, and the correlation coefficient corresponding to different time intervals is obtained, that is, the cofactor series:
After the formula (8) is added to the weight matrix, it is:

\[ Q_r(k) = \rho(k) = \frac{\sigma_r(k)}{\sigma_r} \]

(3)

After the formula (8) is added to the weight matrix, it is:

\[ \hat{Z} = (B^TPB)^{-1}B^TPL \]

\[ N_{BB} = B^TPB \quad W = B^T p l \]

(4)

3. Data analysis

The text of your paper should be formatted as follows: In the results of the harmonic analysis of the observation data of 2019, stable results cannot be obtained for long-period tidal waves. Because the long-period tidal vibration reflects the seasonal characteristics of the sea surface, there is a certain randomness. The 11 major tidal harmonic constants obtained from the traditional annual and monthly analysis and the improved harmonic analysis are the same as those obtained in 2019. Harmonic constant results comparison.

3.1. Monthly adjustment analysis experiment

The data is the hourly water level observation data of Lianyungang in January 1993. First, according to the traditional method, the selected data is subjected to monthly adjustment and analysis, and the harmonic constants \( H, g \) of eleven major tides are obtained[12]. According to the amplitude of the partial tides \( K_1, O_1, M_2 \), after calculation: \( (H_{K1} + H_{O1}) / H_{M2} = 0.29 \), less than 0.5, it is determined that Lianyungang is a half-day tide port. Using the improved model derived from the analysis, the data is analyzed and calculated by monthly adjustment. Getting the remaining water level co-factor sequence of Lianyungang in 1993: \( Q_r, (23) \), and the values of \( Q_r, (23) \) are plotted. The horizontal axis of the curve represents the time interval in hours, and the vertical axis represents the correlation coefficient. As shown in Figure 1.

![Figure 1. Lianyungang 1993 water level co-factor curve](image)

| Tidal | Amplitude difference (H/ cm) | lag angle difference (g/°) |
|-------|-------------------------------|---------------------------|
|       | Traditional methods | Improved methods | Traditional methods | Improved methods |
| \( Q_1 \) | 0.7 | 1 | 26.3 | -2.6 |
| \( O_1 \) | -0.2 | -0.1 | 10 | 13.2 |
| \( P_1 \) | -0.6 | -0.8 | -0.4 | -0.6 |
The results of the amplitude difference of each tide are shown in Figure 2. The difference of the lag angle values of each tide is shown in Figure 3. Figure 2, Figure 3 The horizontal axis represents the tide, and the vertical axis of Figure 2 represents the amplitude difference in centimeters. The vertical axis of Figure 3 represents the difference in lag angle, in degrees. The blue histogram represents the difference between the results of the traditional monthly adjustment and the results of the 2019 harmonic analysis. The red histogram represents the difference between the monthly analysis after adding the non-unit weight matrix and the 2019 analysis.

In general, the amplitude results of most of the divergent tides are closer to the stable amplitude results of 2019. Among them, the amplitude improvement effect of the half-day tide is better than the full-day tide and shallow water tide. The lag angle of most tide are closer to 2019 years’ results, and the predicted tidal level is closer to true tidal level.
4. Experimental verification
In order to further study the influence degree of the harmonic constant of the improved method on the tidal level of the residual water level, the residual water level value (predicted tidal level value) is calculated according to the harmonic constant, and compared with the calculated value of the harmonic constant obtained by the conventional method.

4.1. Residual water level value extraction
The residual water level value is extracted according to the formula (9), and the residual water level value is extracted by monthly adjustment and analysis according to the improved method. The residual water level values obtained by the two methods of random interception for four days are shown in Figure. 4. The horizontal axis is time, the unit is hour, and the vertical axis represents the residual water level value in centimeters.

![Figure 4: 4-day residual water level obtained from the two month-analysis method](image)

The solid black line represents the residual water level curve obtained by the conventional method, and the red dotted line represents the residual water level curve obtained by the improved method. From the figure, the absolute value of most of the residual water level obtained after the weighting is smaller than the absolute value of the residual water level obtained by the traditional method, and it is not excluded that there is a case where the absolute value of the remaining water level is slightly larger than the absolute value of the conventional method.

4.2. Mean square error comparison of residual water level
Calculating the mean square error in the residual water level obtained by the two methods according to the formula (10). The calculation results are shown in Table 2.

|                             | Before improvement | After improvement |
|-----------------------------|--------------------|-------------------|
| Mean square error of overall data/cm | 21.4               | 21.1              |
| Mean square error intercept data/cm | 11.3               | 11.2              |

It is concluded in Table 2 that the error in the residual water level obtained by the improved monthly analysis is smaller than the error in the residual water level before the improvement. The residual water level is obtained by subtracting the predicted tide level from the measured water level. The smaller the error in the residual water level, the closer the forecast tide level is to the real tide level. Compared with the traditional method, using the harmonic constant obtained by the improved method, when forecasting the tide level, the forecast tide level can be closer to the real tide level.

4.3. Comparison between forecasting and measured tide level
In order to more clearly observe the closeness of the forecast tidal level and the measured tidal level, intercept the half-day forecast tidal level, and compare the forecast tidal level obtained by the two methods with the measured water level. The water level curve is shown as Figure. 5.
In Figure 5, the solid black line represents the measured tidal level, the blue dotted line represents the predicted tidal level obtained by the conventional method, and the red dotted line represents the predicted tidal level obtained by the improved method. It can be clearly seen from the figure that the monthly tidal level of the new observation weight matrix is more closely related to the measured tide level.

5. Conclusion

This paper gives a brief introduction to the evaluation model of traditional harmonic analysis. The analysis considers that the observation weight matrix in the harmonic analysis model is a unit matrix. This assumption may have a certain impact on the result, and it is necessary to construct a new observation weight matrix. Considering the correlation between observation errors, it is better to determine the monthly analysis method of the new observation weight matrix. The amplitude and lag angle of most of the tides are closer to the stable results obtained in 2019, and the mean square error in the residual water level is also reduced to some extent. The tidal level prediction is carried out by using the harmonic constant obtained by the improved method, and the predicted tidal level can be closer to the real tidal level. Therefore, the improved method is more feasible and effective, and has a strong practical significance.

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