Design of Ultra-compact Graphene-based Superscatterers

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Abstract—The energy-momentum dispersion relation is a fundamental property of plasmonic systems. In this paper, we show that the method of dispersion engineering can be used for the design of ultra-compact graphene-based superscatterers. Based on the Bohr model, the dispersion relation of the equivalent planar waveguide is engineered to enhance the scattering cross section of a dielectric cylinder. Bohr conditions with different orders are fulfilled in multiple dispersion curves at the same resonant frequency. Thus the resonance peaks from the first and second order scattering terms are overlapped in the deep-subwavelength scale by delicately tuning the gap thickness between two graphene layers. Using this ultra-compact graphene-based superscatterer, the scattering cross section of the dielectric cylinder can be enhanced by five orders of magnitude.

Index Terms—Superscatterers, dispersion engineering, graphene, Mie scattering theory.

I. INTRODUCTION

SUPERSCATTERER is a device that can magnify the scattering cross section of a given object remarkably. This concept was first proposed based on the transformation optics approach, where the scattering cross section of a cylindrical perfect electric conductor (PEC) is enhanced by an anisotropic and inhomogeneous electromagnetic cover. Alternatively, for subwavelength objects, a superscatterer can be designed by the metal-dielectric layers, where the scattering cross section is magnified due to the enhancement of the localized surface plasmons. Besides, the concept of “surface superscatterers” is also proposed, where the scattering cross section of a deep-subwavelength dielectric cylinder is enhanced by a monolayer graphene sheet which is only one atom thick. Moreover, the scattering cross sections of subwavelength superscatterers can be further enhanced by overlapping the resonances from different scattering terms. Since compact superscatterers are more promising in the miniaturization and integration of plasmonic devices, it is quite necessary to design superscatterers by overlapping different resonance peaks in the deep-subwavelength scale.

This paper is organized as follows. In section II, the applicability of Bohr model to graphene-based structures is shown by taking the dielectric-graphene-air cylindrical structure as an example. In section III, based on the validation of Bohr model when multiple dispersion curves exist simultaneously, the resonance peaks from the first and second order scattering terms are overlapped by engineering the dispersion relation of the equivalent planar waveguide. In section IV, an ultra-compact graphene-based superscatterer is designed by dispersion engineering. Finally, section V is the conclusion.

II. BOHR MODEL FOR GRAPHENE BASED STRUCTURES

For the scattering of plasmonic structures, the scattering models can be related to their equivalent one dimensional...
planar waveguide models by Bohr model [21]. According to the Bohr model, if the phase accumulation along an enclosed optical path is an integral number of $2\pi$, namely if the Bohr condition

$$\int \beta dl = n \cdot 2\pi$$

is satisfied, the scattering cross sections of the plasmonic structures exhibit resonances. At the resonant frequencies, the scattering cross sections are significantly enhanced which demonstrate the occurrences of superscattering phenomena [11]. In Eq. (1), the integer $n$ is the order of resonance, and $\beta$ is the corresponding propagation constant of the plasmonic mode in equivalent planar waveguide. The integral (or the enclosed optical path) is calculated along the effective circumference of the plasmonic structures, since surface plasmons propagate along the surface between dielectric and plasmonic material with evanescent fields in the perpendicular directions. Specially, if the plasmonic structure is a cylinder or sphere, Bohr condition reduces to

$$\beta R_{\text{eff}} = n,$$  

where $R_{\text{eff}}$ is the effective radius of the cylindrical or spherical plasmonic structures. It is worth to note that, since the plasmonic field is highly localized on the interface and the penetration depth is usually small compared with the radius of the cylinder or sphere, the propagation constant $\beta$ of surface plasmons in a curved circumference is approximately equal to that in an equivalent planar waveguide. Meanwhile, the Bohr model is a phenomenological model and the effective radius is usually determined empirically.

Bohr model has been used to interpret the localized surface plasmons supported by layered metal-dielectric structures [21]. Since graphene is also one kind of plasmonic materials, in this section we show that different orders of superscattering supported by graphene-based structures can be interpreted by the Bohr model as well. Besides, due to the high confinement of graphene plasmons and the one atom thick graphene monolayer, graphene provides a suitable alternative to metal to design the ultra-compact superscatters in the deep-subwavelength scale [29]. In the calculation, graphene layer is modeled as a two dimensional conducting film [21]. The surface conductivity of graphene monolayer is calculated by the Kubo formula

$$\sigma_g(\omega, \mu_c, \tau, T) = \sigma_{\text{intra}} + \sigma_{\text{inter}},$$

where

$$\sigma_{\text{intra}} = \frac{i e^2}{\pi \hbar^2} \left[ \frac{\mu_c}{k_B T} + 2 \ln \left( e^{-\mu_c/k_B T} + 1 \right) \right]$$

and

$$\sigma_{\text{inter}} = \frac{i e^2}{\pi \hbar^2} \int_0^\infty \frac{f_d(-\varepsilon) - f_d(\varepsilon)}{(\omega + i\tau - 1)^2 - 4 (\varepsilon/\hbar)^2} d\varepsilon$$

are contributions from the intraband and interband transition, respectively. The inset is the simplified form. (a) Cross-sectional view of the layered dielectric-graphene-air cylindrical structure. The dielectric layers are denoted by the orange areas, and the graphene layer is denoted by the violet area. A $x$-polarized plane wave is incident normally from air onto the structure. The radius of the cylindrical graphene layer is $R = 250 \text{ nm}$. (b) Structure of the equivalent planar waveguide model, where the graphene layer (violet area) is separated by a semi-infinite dielectric medium (orange area) and the air (light gray area). The graphene plasmons propagate along the graphene surface in the direction indicated by an arrow, and $\beta$ is the corresponding propagation constant. (c) The normalized scattering cross sections (NSCSs) at different frequencies for the structure shown in (a), where the dashed green line denotes the total NSCS, while the solid black, red, and blue lines denote contributions from different orders of resonance. (d) Dispersion relation for the waveguide structure shown in (b). The two vertical dashed black lines indicate the first ($\beta R_{\text{eff}} = 1$) and second ($\beta R_{\text{eff}} = 2$) order Bohr conditions, respectively. The parameters are $\varepsilon_r = 1$, $\mu_r = 1$, $\mu_c = 0.35 \text{ eV}$, $\mu = 85 \text{ 600 cm}^2/(\text{V} \cdot \text{s})$, and $T = 300 \text{ K}$ for (a)-(d).
Refs. [13], [14], the localized electromagnetic fields interfere constructively, and the scattering cross sections are enhanced with resonance peaks accordingly.

For the scattering model in Fig. 1(a), we can calculate the normalized scattering cross section (NSCS) based on the Mie scattering theory [31], [32]. Detailed calculation shows that

\[ \text{NSCS} = \sum_{n=0}^{\infty} \delta_n |s_n|^2, \]  

(6)

where

\[ s_n = -\frac{J_n'(k_0 R) t_n - J_n(k_0 R) J_n'(k R)}{H_n^{(1)}(k_0 R) t_n - H_n^{(1)}(k_0 R) J_n'(k R)}, \]  

(7)

\[ t_n = \sqrt{\varepsilon_r} J_n(k R) + i \sigma_g \eta_0 J_n'(k R), \]

and 

\[ \delta_n = 1 \text{ for } n = 0 \]

\[ \delta_n = 2 \text{ for } n \neq 0, \]

\[ k_0 = \omega \sqrt{\varepsilon_0 \mu_0}. \]

The inset is the enlarged figure. Clearly, the total NSCS exhibits two resonance peaks at the resonant frequencies. However, this kind of resonances are caused by the resonance of a single scattering term, while the contributions from different scattering terms can be overlapped to further enhance the scattering cross sections [13], [14]. Due to the complexity of scattering models, the genetic algorithm has been used to optimize the superscattering of light where different resonance peaks are overlapped [20]. However, it still lacks an intuitive method to design the superscatters.

Based on the Bohr model, the scattering model is related to its equivalent planar waveguide model intuitively. Thus, it is possible to enhance the scattering cross sections by engineering the dispersion relations of the equivalent planar waveguides. According to Eqs. (1)-(2), different orders of Bohr conditions must be satisfied at the same frequency to overlap the resonance peaks. This requirement can be fulfilled in a single dispersion curve or multiple dispersion curves.

In Refs. [13], [14], different orders of Bohr conditions are satisfied in a single dispersion curve. The metal-dielectric-metal-dielectric-air planar waveguide supports a flat dispersion curve with a proper choice of the thickness of the dielectric layers. Thus, different scattering terms are overlapped and the scattering cross section is enhanced. Alternatively, in this paper we enhance the scattering cross sections of plasmonic structures from a different approach where Bohr conditions with different orders are fulfilled in multiple dispersion curves.

For simplicity, we consider a layered dielectric-graphene-dielectric-graphene-background cylindrical structure, as shown in Fig. 2(a). A \( x \)-polarized plane wave is incident normally from the background onto the structure. The background is a dielectric medium with the relative permittivity \( \varepsilon_b \) and relative permeability \( \mu_b = 1 \). The relative permittivities of the inner and outer dielectric layers are \( \varepsilon_1 \) and \( \varepsilon_2 \), respectively, and the relative permeabilities are \( \mu_1 = \mu_2 = 1 \). The radii of the inner and outer cylindrical graphene layers are \( R_1 \) and \( R_2 \), respectively. Besides, the parameters of the graphene layer are \( \mu_c = 0.35 \text{ eV}, \mu = 8.5 \times 10^6 \text{ cm}^2/(\text{V} \cdot \text{s}), \) and \( T = 300 \text{ K}. \) Similarly, for the scattering model in Fig. 2(a), the corresponding equivalent planar waveguide structure is shown in Fig. 2(b), where \( d = R_2 - R_1 \) is the gap thickness between two graphene layers.

In Refs. [13], [14], different orders of Bohr conditions are satisfied in a single dispersion curve. The metal-dielectric-metal-dielectric-air planar waveguide supports a flat dispersion curve with a proper choice of the thickness of the dielectric layers. Thus, different scattering terms are overlapped and the scattering cross section is enhanced. Alternatively, in this paper we enhance the scattering cross sections of plasmonic structures from a different approach where Bohr conditions with different orders are fulfilled in multiple dispersion curves.

For the scattering model in Fig. 2(a), the normalized scattering cross section (NSCS) is

\[ \text{NSCS} = \sum_{n=0}^{\infty} \delta_n |s_n|^2, \]  

(9)
The normalized scattering cross sections (NSCSs) at different frequencies for the structure shown in (a), where the dashed green line denotes the total NSCS, while the solid black, red, blue, and magenta lines denote contributions from $n = 0$, $n = 1$, $n = 2$, and $n = 3$ scattering terms, respectively. The inset is the enlarged figure. Note the NSCSs are expressed in the common logarithmic form. (d) Dispersion relation of graphene plasmons for the waveguide structure shown in (b). The three vertical dashed black lines indicate the first ($\beta_{\text{eff}} = 1$), second ($\beta_{\text{eff}} = 2$), and third ($\beta_{\text{eff}} = 3$) order Bohr conditions, respectively. The parameters are $\varepsilon_1 = \varepsilon_2 = \varepsilon_b = 1$, $\mu_1 = \mu_2 = \mu_c = 1$, $R_1 = 250 \text{ nm}$, $R_2 = 300 \text{ nm}$, $d = R_2 - R_1 = 50 \text{ nm}$, $\mu_c = 0.35 \text{ eV}$, $\mu = 85 \text{ 600 cm}^2/(\text{V} \cdot \text{s})$, and $T = 300 \text{ K}$ for (c) and (d).

where

\begin{align}
  s_n &= -\frac{J_2'(k_b R_2) q_n - \sqrt{\varepsilon_b} J_n(k_b R_2) r_n}{H_n^{(1)\prime}(k_b R_2) q_n - \sqrt{\varepsilon_b} H_n^{(1)}(k_b R_2) r_n}, \quad (10) \\
  q_n &= \sqrt{\varepsilon_2} \left[ J_n(k_2 R_2) + t_n H_n^{(1)\prime}(k_2 R_2) \right] + \text{i} \sigma_g \eta_2 r_n, \quad (11) \\
  r_n &= J_2'(k_2 R_2) + t_n H_n^{(1)\prime}(k_2 R_2), \quad (12) \\
  t_n &= -\frac{J_2'(k_2 R_1) p_n - \sqrt{\varepsilon_2} J_n(k_2 R_1) J_2'(k_1 R_1)}{H_n^{(1)\prime}(k_2 R_1) p_n - \sqrt{\varepsilon_2} H_n^{(1)}(k_2 R_1) J_n(k_1 R_1)}, \quad (13) \\
  p_n &= \sqrt{\varepsilon_1} J_n(k_1 R_1) + \text{i} \sigma_g \eta_1 J_n'(k_1 R_1), \quad (14)
\end{align}

and $k_1 = k_0 \sqrt{\varepsilon_1}$, $k_2 = k_0 \sqrt{\varepsilon_2}$, and $k_b = k_0 \sqrt{\varepsilon_b}$ are wavenumbers in the inner and outer dielectric layers, and the background, respectively \cite{31,32}. While for the waveguide model in Fig. 2(b), the dispersion relation of TM graphene plasmons is

\begin{align}
  e^{-2k_2 d} &= \frac{k_2}{k_2} \left( 1 + \text{i} \sigma_g \frac{k_0}{\omega \varepsilon_b} \right) + \frac{k_2}{k_2} \left( 1 + \text{i} \sigma_g \frac{k_0}{\omega \varepsilon_b} \right) + \frac{k_2}{k_2} \\
  &= \frac{k_2}{k_2} \left( 1 + \text{i} \sigma_g \frac{k_0}{\omega \varepsilon_b} \right) - \frac{k_2}{k_2} \left( 1 + \text{i} \sigma_g \frac{k_0}{\omega \varepsilon_b} \right) - \frac{k_2}{k_2} \left( 1 + \text{i} \sigma_g \frac{k_0}{\omega \varepsilon_b} \right). \quad (15)
\end{align}

Specically, when $\varepsilon_1 = \varepsilon_b$, this waveguide is a symmetric double-channel graphene plasmon waveguide, and the dispersion relation reduces to

\begin{equation}
  \tanh \left( \frac{k_2 d}{2} \right) = -\frac{k_2 \varepsilon_2}{k_2 \varepsilon_1 + 1 - \text{i} \sigma_g \frac{k_0}{\omega \varepsilon_b}} \quad (16)
\end{equation}

to the odd mode, and

\begin{equation}
  \tanh \left( \frac{k_2 d}{2} \right) = -\frac{k_2 \varepsilon_1}{k_2 \varepsilon_2 + 1 - \text{i} \sigma_g \frac{k_0}{\omega \varepsilon_b}} \quad (17)
\end{equation}

to the even mode \cite{35}. Note the odd and even modes are defined according to the parity of the tangential electric component $E_x$.

Based on Eqs. (3), we can tune the positions of the resonance peaks from different scattering terms by dispersion engineering. However, before the demonstration of dispersion engineering, first we need to validate the applicability of Bohr model when multiple dispersion curves exist simultaneously. We consider the simplest case with $\varepsilon_1 = \varepsilon_2 = \varepsilon_b = 1$, $R_1 = 250 \text{ nm}$, $R_2 = 300 \text{ nm}$, and $d = R_2 - R_1 = 50 \text{ nm}$. Under these parameters, Figs. 2(c) and (d) show the NSCSs at different frequencies and the dispersion relation, respectively. In Fig. 2(c), the dashed green line denotes the total NSCS, while the solid black, red, blue, and magenta lines denote contributions from $n = 0$, $n = 1$, $n = 2$, and $n = 3$ scattering terms, respectively. The inset is the enlarged figure. Clearly, the total NSCS exhibits four resonance peaks, where the peak at $f = 19.62 \text{ THz}$ is caused by the resonance of $n = 1$ scattering term, the peaks at $f = 11.40 \text{ THz}$ and $f = 26.70 \text{ THz}$ are caused by the resonances of $n = 2$ scattering term, and the peak at $f = 16.35 \text{ THz}$ is caused by the resonance of $n = 3$ scattering term, respectively. Note the resonance peaks of total NSCS at $f = 11.40 \text{ THz}$ and $f = 16.35 \text{ THz}$ are not pronounced since the contributions from $n = 2$ and $n = 3$ scattering terms at the corresponding resonant frequencies are small. For this scattering model, these resonances are related to the Bohr conditions with the effective radius $R_{\text{eff}} = (R_1 + R_2)/2$. This effective radius is determined because it corresponds to the bisector of the two graphene layers. As shown in Fig. 2(d), the first order Bohr condition $\beta_{\text{eff}} = 1$ at $f = 19.63 \text{ THz}$ corresponds to the resonance of $n = 1$ scattering term, the third order Bohr condition $\beta_{\text{eff}} = 3$ at $f = 16.35 \text{ THz}$ corresponds to the resonance of $n = 3$ scattering term, and the second order Bohr conditions $\beta_{\text{eff}} = 2$ at $f = 26.58 \text{ THz}$ for the even mode and $f = 11.40 \text{ THz}$ for the odd mode correspond to the resonances of $n = 2$ scattering term at $f = 11.40 \text{ THz}$ and $f = 26.70 \text{ THz}$, respectively. The calculation results from Bohr model agree well with that from the scattering model, which implies that Bohr model is still applicable when multiple dispersion curves exist simultaneously.

The idea of overlapping the resonance peaks by dispersion engineering is very straightforward. From Fig. 2(d), the dispersion curves of the even mode and odd mode are dependent on the gap thickness $d$. By delicately tuning the gap thickness, the first order Bohr condition for the even mode and the second order Bohr condition for the odd mode can be fulfilled at the same frequency. In the following calculations, the parameters of the graphene layer are $\mu_c = 1 \text{ eV}$, $\mu = 230 \text{ 000 cm}^2/(\text{V} \cdot \text{s})$. 


and \( T = 300 \text{ K} \) to reduce the optical loss of graphene. As shown in Fig. 3(a), when \( \varepsilon_1 = \varepsilon_2 = \varepsilon_b = 1 \) and the gap thickness \( d = 138.20 \text{ nm} \), the first order Bohr condition for the even mode and the second order Bohr condition for the odd mode are fulfilled at \( f = 35.26 \text{ THz} \). Then we can obtain the radii of the inner and outer graphene layers according to the Bohr condition, namely \( R_1 = 131.54 \text{ nm} \) and \( R_2 = 269.74 \text{ nm} \). However, under these parameters the resonance peaks from \( n = 1 \) and \( n = 2 \) are not overlapped at \( f = 35.26 \text{ THz} \), as shown in Fig. 3(b). This deviation is due to the invalidation of Bohr model. As shown in Fig. 3(a)-(b), Bohr model relates the scattering model to the waveguide model based on the assumption that the gap thickness \( d \) is smaller than the radius of the inner graphene layer. Under this assumption, the excited whispering-gallery-like modes can be approximated by the plasmonic modes in the equivalent one dimensional planar waveguide. For Fig. 3(b), since the gap thickness is nearly equal to the value of \( R_1 \), the resonance peaks deviate from the resonant frequency predicted by Bohr model.

In order to fulfill the Bohr condition, we let \( \varepsilon_1 = \varepsilon_b = 3 \), \( \varepsilon_2 = 1 \), and \( d = 44.38 \text{ nm} \). Note the background is not air for demonstration purpose. Similarly, as shown in Fig. 3(c), the first order Bohr condition for the even mode and the second order Bohr condition for the odd mode are are fulfilled at \( f = 27.04 \text{ THz} \), and we can obtain \( R_1 = 122.06 \text{ nm} \) and \( R_2 = 166.43 \text{ nm} \). Under these parameters, the resonance peaks from \( n = 1 \) and \( n = 2 \) are overlapped at \( f = 27.04 \text{ THz} \) with a small deviation, as shown in Fig. 3(d). Clearly, Bohr condition is valid since \( d < R_1 \) in this case. The above result can be optimized further using the simplex search method \cite{10}, where \( R_1 = 122.06 \text{ nm} \) and \( d = 44.38 \text{ nm} \) are set as the initial estimates. This optimization method can find the local optimum values starting at the initial estimates. Fig. 4 show the normalized scattering cross sections (NSCSs) at different frequencies under the optimized values of \( R_1 = 112.47 \text{ nm} \) and \( R_2 = 160.24 \text{ nm} \). Note the resonant frequency slightly changes to \( f = 28.23 \text{ THz} \). Thus the resonant peaks are overlapped by dispersion engineering based on the Bohr model. Note that the total normalized scattering cross section at \( f = 28.23 \text{ THz} \) is 2.02, which exceeds the single channel limit of a single scattering term \cite{13}. This implies that it is possible to design the ultra-compact graphene-based superscatterers from an intuitive view.

IV. DESIGN OF SUPERSCATTERERS

In the above section, the resonance peaks are overlapped based on the layered dielectric-graphene-dielectric-graphene-background cylindrical structures. For demonstration purpose, the background is chosen as the dielectric medium with \( \varepsilon_b = 3 \). Considering the practical applications, the background is the air with \( \varepsilon_b = 1 \). Thus it is necessary to take this restriction into account when designing the superscatterers. In this section, based on the structure shown in Fig. 2(a) with \( \varepsilon_b = 1 \), we design an ultra-compact graphene-based superscatterer by dispersion engineering.

Following the similar procedure, we let \( \varepsilon_1 = 6 \), \( \varepsilon_2 = 1.1 \), and \( \varepsilon_b = 1 \), and tune the gap thickness \( d \) between two graphene layers. When \( d = 63.48 \text{ nm} \), the first order Bohr condition is fulfilled at \( f = 26.30 \text{ THz} \), the second order Bohr condition is fulfilled at \( f = 21.90 \text{ THz} \), and the radii of the inner graphene layer is \( R_1 = 140.623 \text{ nm} \). Using the
simplex search method, we obtain the optimized values of $R_1 = 118.88$ nm and $R_2 = R_3 + d = 193.00$ nm. Fig. 5 shows the normalized scattering cross sections (NSCSs) at different frequencies, where the resonance peaks are overlapped at the resonant frequency $f = 26.77$ THz. Note that the normalized scattering cross section of a dielectric cylinder with $R = 118.88$ nm is 1.25 × 10⁻⁵ at $f = 26.77$ THz, as shown in Fig. 5. Although the total normalized scattering cross section of the superscatterer shown in Fig. 5 is 1.82 which is still under the single channel limit, the scattering cross section contributed by the overlapped resonance peaks has enhanced for five orders of magnitude. Meanwhile, the radius of the ultra-compact superscatterer is only 0.017$\lambda_0$, where $\lambda_0$ is the incident wavelength. Besides, the optimized values of the superscatterer can be easily obtained by the local optimization algorithm. Compared with the global optimization algorithms, the method of dispersion engineering provides an intuitive way to design the ultra-compact graphene-based superscatterers.

Since the plasmonic field is highly localized on the graphene surface, the superscattering phenomenon is sensitive to the optical loss of graphene. Fig. 6 shows the normalized scattering cross sections (NSCSs) at different frequencies for the designed superscatterer under different values of carrier mobility of graphene, where the red and blue lines denote contributions from $n = 1$ and $n = 2$ scattering terms, respectively. For simplicity, the curves for $n = 0$ and $n = 3$ are omitted. Note the NSCSs are expressed in the common logarithmic form. The parameters are $\varepsilon_1 = 6$, $\varepsilon_2 = 1.1$, $\varepsilon_b = 1$, $\mu_1 = \mu_2 = \mu_0 = 1$, $R_1 = 118.88$ nm, $R_2 = 193.00$ nm, $\mu_c = 1$ eV, $\mu = 230,000$ cm²/(V·s), and $T = 300$ K. As indicated by the dashed black line, the resonant frequency where the resonant peaks are overlapped is $f = 26.77$ THz. The inset shows the dispersion relation of graphene plasmons in the equivalent planar waveguide.

Finally, we compare our superscatterers designed by dispersion engineering with those designed using transformation optics. Compared with dispersion engineering, transformation optics is a more general method that can be used to control the scattering of a given object freely. Apart from enhancing the scattering by superscatterers, the scattering can also be suppressed to realize cloaks [17], [18]. Besides, transformation optics can also be used to design electrically-small antennas [39], [40]. These devices are hard to realize by dispersion engineering from an intuitive way. However, superscatterers designed by transformation optics are difficult to implement in experiments since the electromagnetic covers are made of anisotropic and inhomogeneous materials. In contrast, the method of dispersion engineering is more intuitive. More importantly, the superscatterers designed by dispersion engineering are more easier to implement where only isotropic and homogeneous materials are used.

V. CONCLUSIONS

In conclusion, based on the validation of Bohr model, we show that the method of dispersion engineering can be introduced to the design of ultra-compact graphene-based superscatterers. Since Bohr conditions with different orders are fulfilled in multiple dispersion curves at the same resonant frequency, the resonance peaks from the first and second order scattering terms are overlapped in the deep-subwavelength scale to further enhance the scattering cross sections by five orders of magnitude. Compared with the global optimization algorithms, our method is more intuitive in physics. Our work will provide theoretical guidance for the design of superscatterers and other scattering based devices, which have great potential applications in plasmonics.
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