We describe a model for pion production off nucleons and coherent pions from nuclei induced by neutrinos in the 1 GeV energy regime. Besides the dominant $\Delta$ pole contribution, it takes into account the effect of background terms required by chiral symmetry. Moreover, the model uses a reduced nucleon-to-$\Delta$ resonance axial coupling, which leads to coherent pion production cross sections around a factor two smaller than most of the previous theoretical estimates. Nuclear effects like medium corrections on the $\Delta$ propagator and final pion distortion are included.

Keywords: Neutrino scattering; Coherent pion production; Axial coupling of the $\Delta$(1232) resonance.

I. INTRODUCTION

Neutrinos have been in the forefront of research in particle and nuclear physics for a long time. One of these fields is the study of pion production off nuclei induced by neutrinos. A proper understanding of this process is necessary in the analysis of the present generation of precision neutrino oscillation experiments. For instance, the $\pi^0$ produced in neutral currents (NC) is the most important $\nu_\mu$-induced background to experiments like MiniBoone[1] that are trying to measure $\nu_\mu \rightarrow \nu_e$ oscillations in the neutrino energy range around 1 GeV. Also of importance is the background that appears from $\pi^+$ charged current (CC) production in $\nu_\mu \rightarrow \nu_x$ disappearance searches like T2K[2]. Moreover the pion is strongly coupled to the $\Delta$(1232) resonance, and neutrino scattering is presently the best way to access to the axial nucleon-$\Delta$ transition couplings. The most complete information in this regard comes from the bubble chamber data of ANL[3, 4] and BNL[5, 6] where the target was cooled deuterium. However, in present oscillation experiments the target for neutrino interaction is finite nuclei, for instance $^{12}$C (mineral oil in MiniBoone) or $^{16}$O (water target in T2K). This introduces sizable many-body effects that are difficult to disentangle from the genuine single nucleon response to the neutrino probe.

In Sec. II of these paper we describe a phenomenological model[7] for pion production induced by neutrino scattering off free nucleons. This model takes into account non-resonant background processes that are usually neglected. These non-resonant processes are determined by chiral symmetry and thus do not introduce free parameters. We then perform a fit of the axial $N$-$\Delta$ parameters and discuss a possible violation of the non-diagonal Goldberger-Treiman relation. In Sec. III we extend the model to describe the CC coherent process in which the final nucleus is left in its ground state. We will further try to discuss how the coherent reaction can show some light on the values of the axial
II. SINGLE NUCLEON PION PRODUCTION

Here we review the model for the free nucleon reaction

\[ \nu_l(k) + N \rightarrow l^-(k') + N' + \pi^+(k_\pi) \]  

(1)

as introduced in Ref. \[7\]. This model considers the dominant \( \Delta \) pole mechanism in which the neutrino excites a \( \Delta(1232) \) resonance that subsequently decays into \( N\pi \). In our model we have also included non-resonant background terms as required by chiral symmetry, see Fig. 1. Some previous works\[12–14\] also considered background terms, though they were not consistent with the chiral counting.

The vector part of the interaction \( N-\Delta \) can be related to the electromagnetic current by imposing conservation of the vector current. For photon induced reactions extensive experimental data exist and in Ref. 15 they were employed to fit the vector current couplings. We shall use this fit in our work. Unfortunately the axial \( N-\Delta \) current is not so well studied. The usual approach is to parameterize the interaction in terms of four form factors \( C_3^{\pm,4,5,6}(q^2) \). One can assume partial conservation of the axial current (PCAC) and obtain the relation \( C_3^A = C_5^A M^2/(m_\pi^2 - q^2) \), with \( M \) the nucleon mass. Furthermore, one can deduce 16 from dispersion relations the following conditions: \( C_3^A(q^2) = 0 \) and \( C_4^A = -C_5^A/4 \). Thus we are left with only a free form factor, the dominant one \( C_5^A(q^2) \). A different number of parameterizations have been proposed for this form factor, nevertheless the experimental data are quite limited and thus a simple dipole form

\[ C_5^A(q^2) = \frac{C_5^A(0)}{(1 - q^2/M_5^A)^2} \]  

(2)

should be enough. In order to keep the axial transition radius in the range 0.7–0.8 fm one expect the axial mass to have a value of around \( M_\Delta \sim 0.85–1.0 \text{ GeV} \). Furthermore one can assume the well known Goldberger-Treiman relation (GTR) for \( \pi NN \) coupling to be also valid for the \( \pi N\Delta \) coupling and thus obtain

\[ C_5^A(0) = \sqrt{2} f_{\pi} \frac{f^*}{m_\pi} = 1.2, \]  

(3)

where \( f_\pi = 93 \text{ MeV} \) is the pion decay constant and \( f^* = 2.2 \), the \( \pi N\Delta \) coupling. Unfortunately, there are no constraints from Chiral Perturbation Theory 17 and the lattice QCD calculations 18 are still inconclusive.

Most of the approaches in the literature assume \( \Delta \) dominance, that is, only include the first two diagrams in Fig. 1. We improve this situation by including non-resonant contributions 9 required by chiral symmetry. In addition to the \( \Delta(1232) \) pole (\( \Delta P \)) (first row) mechanism the model includes background terms required by chiral symmetry: nucleon (second row) pole terms contact and pion pole contribution (third row) and pion-in-flight term. We calculate them by using the SU(2) non-linear \( \sigma \) model Lagrangian. The only parameters in the theory are the pion and nucleon masses and the pion decay constant. All other couplings are completely fixed by the theory, so no new parameter is introduced.

We found (see Fig. 2) that these background terms produced significant effects in all channels, namely an enhancement of about 10% in the cross section that resulted in a disagreement with the ANL data. As a result we had to readjust the strength of the dominant \( \Delta \) pole contribution. The least known ingredients of the model are the axial nucleon-to-\( \Delta \) transition form factors, of which \( C_5^A \) gives the largest contribution. This strongly suggested a refit of that form factor to the experimental data, which we did by fitting the flux-averaged \( \nu_{\mu}p \rightarrow \mu^-\pi^+ \) ANL \( q^2 \)-differential cross section for pion-nucleon invariant masses \[52\] \( W < 1.4 \text{ GeV} \). The obtained parameters were

\[ C_5^A(0) = 0.87 \pm 0.08, \quad M_\Delta = 0.985 \pm 0.082 \text{ GeV} \]  

(4)

with a \( \chi^2/\text{dof} = 0.4 \) and a correlation coefficient \( r = -0.85 \), that amounts to a 30% reduction of the GTR prediction. Thus, our full model leads to an overall better description of the data for one-pion production reactions off the nucleon. This reduction of the \( C_5^A(0) \) value is consistent with recent results in lattice QCD 18 and quark models 19.

Recently other fits have been proposed. For instance, in Ref. 20 they keep the GTR but introduce a non-dipole form factor with additional parameters. As in the ANL data the relevant phase space is around \( q^2 = 0.1 \text{ GeV}^2 \), they could keep the GTR at the cost of having a large dependence on \( q^2 \) for the form factor, that yields a large, somehow unphysical, axial transition radius of around 1.4 fm. Furthermore neither statistical errors nor correlation factors are
FIG. 1: Set of diagrams for the model for the $W^+ N \rightarrow N' \pi$ reaction.

FIG. 2: Flux averaged $q^2$-differential $\nu_p p \rightarrow \mu^- p \pi^+$ cross section for the ANL (left) and BNL (right). Dashed lines stand for the contribution of the $\Delta P$ mechanism with the GTR assumption for $C_A^A$. We also plot results with the full model of Fig. 1 assuming GTR (dashed-dotted) and with our best fit parameters, Eq. (4).

given in that reference. Another analysis[21] raised new questions, namely the effect of deuterium wave function on the cross section and the flux uncertainties in the ANL and BNL data. The authors of this latter work took into account both effects, though we believe that their statistical analysis is not quite robust (see discussion in Ref. [11]). Furthermore they only took into account the dominant $\Delta$ contribution. The inclusion of deuterium wave function reduces the cross section about an 8%, so this somehow compensates the neglect of the non-resonant background, and they obtained a best fit of $C_A^A(0) = 1.19 \pm 0.08$ in agreement with the GTR assumption.

Recently[11] we have improved our fit of Ref. [7], improving the lines suggested in Graczyk et al. In summary in this new fit

1: all diagrams in Fig. 1 are included;

2: the fitted data are the full ANL data set and the BNL total cross sections at the three lowest neutrino energies (we neglect higher energies where the effect of higher resonances beyond the $\Delta(1232)$ must be addressed; the BNL $q^2$-differential cross sections were not taken into account as they are not normalized);

3: deuterium wave function effects were introduced following the prescription of Ref. [22];

4: the form factors $C_3^A$ and $C_4^A$ were tentatively included in one fit, though the data were found to be quite insensitive to their values so we decided to stick to the Adler’s assumption; and

5: the uncertainties in ANL and BNL flux normalization are introduced as fully correlated systematic errors.
In this way we obtain a best fit of $C_5^A(0) = 1.00 \pm 0.11$ and $M_A = 0.93 \pm 0.07$ GeV with a goodness of fit value of $\chi^2 \text{d.o.f.} = 0.42$. Thus we observe a violation of the off diagonal Goldberger-Treiman relation at the level of 2$\sigma$.

### III. THE COHERENT REACTION

Here we describe our model$^9$ for the coherent reaction

$$\nu_l(k) + A_Z|_{gs}(p_A) \rightarrow l^- (k') + A_Z|_{gs}(p'_{A}) + \pi^+(k_{\pi})$$

(5)

where the target nucleus $A_Z$ is left in the ground state ($gs$). To calculate the amplitude of this process we sum over all individual nucleon wave functions, which are modelled by a Fermi gas in local density approximation. The individual nucleon amplitudes are modelled following the model of the previous section, using the fit of Eq. 4 for the individual nucleon wave functions, which are modelled by a Fermi gas in local density approximation. The individual nucleon wave functions are modelled by a Fermi gas in local density approximation.

We thus consider the wave function of the pion to be the outgoing solution to the Klein-Gordon equation with a microscopic optical potential$^{24}$ whose imaginary part takes into account the inelastic interactions of the pion with the nucleus, that thus disappear from the coherent channel. We must emphasize here that solving the Klein-Gordon equation is the correct way of describing the distortion of the outgoing pion. Other approaches use either a Monte Carlo simulation$^{25}$ or include an attenuation factor fitted to the pion nuclei scattering cross section$^{26}$. The first procedure, though physically sound, can be a bit misleading as it includes in its cross section processes (like quasi-elastic scattered pions) that do not leave the nucleus in its ground state, thus are not coherent. This kind of models are used in the analysis of MiniBoone experiment, thus making a bit messy the direct comparison between theoretical models and experimental results (see discussion in Ref. $^9$). The second approach is an oversimplification, as the pion-nucleus scattering is quite a different process from the neutrino induced pion production. As pion nucleus interaction is governed by the strong interaction, the incoming pion interacts strongly with the nuclear surface, thus the pion nucleus cross section is quite insensitive to the details of the nuclear core. On the other hand, neutrino scattering is a weak process, dominated by nuclear density, so pions are mostly produced in the deep, high density regions of the nucleus. The physics of pion interaction is thus quite different in pion scattering off nuclei and pion production by electroweak probes.

In left panel of Fig. 3 we show the pion momentum distribution for CC coherent pion production, in the peak energy region of the T2K experiment. Including $\Delta$ in-medium self-energy (long-dashed line) reduces the PWIA results (short-dashed line). Further inclusion of pion distortion (full model, solid line) reduces the cross section, and the peak is shifted towards lower energies, reflecting the strong absorption and the higher probability of a quasi-elastic collisions of the pion in the $\Delta$ kinematical region. The total cross section reduction is around 60%. Similar nuclear effects were already studied in Refs. $^{27, 28}$. However, the authors of these references neglected the nucleon momenta in the Dirac spinors. The effect of this approximation (nucleons at rest, dotted line) results in a $\sim 15\%$ decrease of the total cross section. In the right panel of Fig. 3 we show the pion angular distribution with respect to the incoming neutrino direction. The reaction is thus highly forward peaked, as expected due to the nucleus form factor. The angular distribution profile keeps its forward peaked behavior after introduction of nuclear medium effects. Furthermore we corrected some numerical errors in the mentioned papers. However one must be aware that this model does not take...
into account the non-localities in the $\Delta$ propagation. We believe this effect is partially taken into account in an effective fashion by our treatment of the $\Delta$ in nuclear medium; nevertheless further studies would be interesting.

In Table II we compare our model with present results of K2K experiment and show our predictions for MiniBoone and T2K. Our prediction, subject to sizable uncertainties, lies well below the K2K upper bound, due to the use of a low value for $C_\mu^A(0)$, while our prediction for the $\nu_\mu$ NC MiniBoone cross section is notably smaller than that given in the PhD thesis of J. L. Raaf. However, we believe (see discussion in Refs. [9, 10] that the MiniBoone analysis might importantly overestimate this cross section, not only because some of the $\pi^+$s which undergo FSI collisions are accounted for instead of being removed, but also because a possible mis-match between the absolute normalisation of the background and coherent yields. Note that the K2K and MiniBoone results seems somehow incompatible with the approximate relation $\sigma_{CC} \approx 2\sigma_{NC}$, which would be expected from $\Delta$ dominance and neglecting finite muon mass effects.

Acknowledgments

M. V. acknowledges a Postdoctoral Fellowship from the Japanese Society for the Promotion of Science (JSPS). Research supported by DGI contracts FIS2008-01143, FIS2006-03438, FPA2007-65748 and CSD2007-00042, JCyL contracts SA016A07 and GR12, Generalitat Valenciana contract PROMETEO/2009/0090 and by EU HadronPhysics2 contract 227431.

| Reaction | Experiment | $\sigma(10^{-40} \text{ cm}^2)$ | $\sigma(10^{-40} \text{ cm}^2)$ | Exp |
|----------|------------|-------------------------------|-------------------------------|-----|
| $\nu_\mu + ^{12}\text{C}$ | K2K | $< 7.7$ | $3.33$ | $7.7 \pm 1.6 \pm 3.6$ |
| $\nu_\mu + ^{12}\text{C}$ | MiniBoone | 4.46 | 
| $\nu_\mu + ^{12}\text{O}$ | T2K | 4.19 | 
| $\nu_\mu + ^{12}\text{C}$ | T2K | 3.54 | 

TABLE I: Total cross sections for the coherent process. We neglect the highest 10% of the energy spectrum.
[28] L. Alvarez-Ruso, L. S. Geng and M. J. Vicente Vacas, *Phys. Rev.* **C76**, p. 068501 (2007).
[29] T. Leitner, U. Mosel and S. Winkelmann, *Phys. Rev.* **C79**, p. 057601 (2009).
[30] M. Hasegawa et al., *Phys. Rev. Lett.* **95**, p. 252301 (2005).
[31] J. L. Raaf, Ph. D. Thesis. FERMILAB-THESIS-2005-20.
[32] This cut was introduced in order to avoid the effects of resonances higher than the $\Delta(1232)$. 