Double magnetic resonance in the hyperfine structure of optically oriented alkali atoms with laser pumping

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Abstract. The paper is devoted to the investigation of mutual influence of low-frequency and microwave resonances in $^{87}$Rb vapor cell magnetometers system with laser pumping. One of the magnetometers was based on a low-frequency spin generator principle, while the second one was built as passive microwave spectrometer with a resonance frequency lock loop. The paper analyzes the frequency shifts in the low-frequency and microwave channels of the tandem of magnetometers associated with the simultaneous action of the resonant radio fields on the alkali atoms. Such effect is manifested in the frequency shifts of the spin generator for fixed changes in the amplitude of the microwave field.

1. Introduction

The phenomenon of radio-optical resonance is actively used in the development of the precision quantum devices, including the nuclear gyroscopes, as well as the various types of the atomic frequency standards [1-6] and the optically pumped quantum magnetometers [7-9]. The growing demands for stability and measuring accuracy of these devices have determined the search for new circuit solutions in which the semiconductor lasers are used as the pumping sources. Such sources have a higher efficiency, small dimensions and, in contrast to the ordinary spectral lamps, allow effective control of an intensity, a polarization, and a spectrum of the pump radiation. These advantages are of the fundamental importance in the development of the miniature quantum sensors installed in on-board equipment of the mobile placing. In this case, in the variant of the quantum devices based on the effect of coherent population trapping, their circuit solution makes it possible to eliminate the traditional component (microwave cavity) and an isotopic filter, which are necessary when lamp source is used for pumping [10].

In the development of the small-sized atomic clocks with laser pumping, the problem of minimizing the line width of the observed resonance arises, since the miniaturization of the construction requires an increase in the density of the working substance to obtain a sufficiently reliable signal. To achieve this goal, the so-called end resonance observed at transitions between the magnetic sublevels of the hyperfine structure with the extreme values of the atom total angular momentum projection can be implemented [11]. Unlike the radio-optical resonance at the 0-0 transition, used in quantum frequency standards, it is observed with a higher quality factor of the detected signal (the signal-to-noise ratio divided by the resonance line width). This makes it possible to minimize the size of the absorption cell, but requires the use of increased pressures of the inert buffer gas, which is necessary to reduce the rate of the atom diffusion to the walls of the cell. The authors consider an alternative version of the end resonance observed in $^{87}$Rb pairs placed in a small-sized absorption cell with an antirelaxation walls coating without a buffer gas [12], [13]. At the same time, the problem of identification the possibilities for the mutual compensation of the various components of the light shift with the aim of increasing the long-term stability of the quantum
magnetometers was solved. One of that magnetometers was performed on the principle of a low-frequency spin generator; the other, in the form of a passive radio spectrometer with of the resonance frequency self-tuning to the microwave radio-optical resonance. However, the authors did not consider the inevitable mutual influence of LF-microwave channels associated with the effect of a coherence circulation between the states of the atom hyperfine structure [14, 15]. To some extent, this gap is filled by the present work, which analyzes the frequency shifts in the LF and microwave channels of the tandem of magnetometers associated with the simultaneous action of the resonance radio fields on the alkali metal atoms and manifested in the frequency shifts of the spin generator for fixed changes in the amplitude of the microwave field.

2. **“Dressed” atom theory implementation for quantum magnetometers system**

The physical nature of such frequency shifts is associated with the coherence circulation between the magnetic sublevels of the alkali atoms “dressed” by the spin oscillator LF field. Absolute value of this shift strongly depends on the magnetic-dipole transition number and reaches its maximum at 0-0 microwave transition, which is commonly used in the atomic frequency standards.

In accordance with the quantum theory of the “dressed” atom, simultaneous action of the LF and microwave fields on the alkali atoms yields in multiphoton absorption and reemission of the LF field quants and the probing microwave field absorption [16]. The microwave transitions spectrum between the sublevels of the hyperfine structure takes the form of central and side groups of the satellites lines, separated at intervals equal to the frequency of the LF fields. Probability amplitudes of microwave transitions in the energy structure of the "dressed" atom determine the specific contribution of various magnetic-dipole transitions in the radio-optical resonance signal. In accordance with [10], the probability amplitude for such transitions is expressed through the matrix element of the operator of magnetic dipole moment as follows

\[
\langle F^*, m^* | M | F, m \rangle = \sum_{m_{1}, m_{1}^*} D_{m_{1}, m}^{F*} \cdot D_{m_{1}, m}^{F} J_{0} \left[ (m + m_{1}^*) \frac{\omega_{1}}{\omega} \right] \langle F^*, m_{1}^* | M | F, m_{1} \rangle \delta_{m_{1}, m_{1}^* + q} \quad (1)
\]

where \( D_{m_{1}, m}^{F*} \) and \( D_{m_{1}, m}^{F} \) – of the Wigner function, \( J_{0} \left[ (m + m_{1}^*) \frac{\omega_{1}}{\omega} \right] \) – Bessel functions \( \langle F^*, m_{1}^* | M | F, m_{1} \rangle \) –the matrix element of the magnetic-dipole transition, \( \delta_{m_{1}, m_{1}^* + q} \) – the Kronecker symbol, where the index \( q \) takes the value 0 or 1 depending on the direction of the vector of the RF field relative to the direction of the constant magnetic field, \( \omega_{1} \) characterizes an amplitude of the magnetic field \( H_{1} \) according to the equation \( \omega_{1} = \gamma H_{1} \) and \( \omega \) is the frequency the low-frequency RF field.

As an example, the possible microwave transitions (arrows) between the hyperfine structure sublevels of alkali "dressed" atom with a nuclear spin of 3/2 is shown in figure 1.

For example, at 0-0 and end resonances frequency transitions, indicated in table 1, are induced in the central series of the absorption spectrum for the magnetic microwave resonances of \(^{87}\)Rb “dressed” atoms. In the second column of the table the probabilities of the corresponding transitions are presented.
Figure 1. The spectrum of microwave transitions "dressed" alkali atom with a nuclear spin equals 3/2

Table 1. Microwave transitions probabilities atoms of alkali metals

| Rubidium 87 |  |
|---|---|
| F = 1, m_F ↔ F = 2, -m_F |  |
| F = 1, m_F = ± 1 ↔ F = 2, m_F = ± 1 | \( \frac{3}{64} \left[ J_0 \left( \frac{\omega_1}{\omega} \right) - J_0 \left( \frac{3 \omega_1}{\omega} \right) \right]^2 \) |
| F = 1, m_F = 0 ↔ F = 2, m_F = 0 | \( \frac{1}{64} \left[ J_0 \left( \frac{\omega_1}{\omega} \right) + 3 J_0 \left( \frac{3 \omega_1}{\omega} \right) \right]^2 \) |
The absorption spectrum of the "dressed" atoms, is shown in figure 1, meets the condition of the significant excess of low frequency field $\omega$ over Larmor frequency $\omega_0$. In the tandem magnetometers, the equality of these frequencies was observed and an overlap of the absorption spectra of the neighboring line groups of the satellites was registered. Since the amplitude of the self-generating magnetometer RF field $\omega_1$ was significantly less than its resonant frequency $\omega$ in the experiments, the dynamics of the observed frequency shifts was determined only by the center band of the absorption lines. In practice it is necessary to take into account the weight contribution of various

| $F=1, m_F \leftrightarrow F=2, m_F \pm 1$ |
|------------------------------------------|
| $F = 1, m_F = \pm 1 \leftrightarrow F = 2, m_F = \pm 2$ | $\frac{3}{64} \left[ J_0 \left( \frac{\omega_1}{\omega} \right) \right]^2$ |

Rubidium 85

| $F = 1, m_F \leftrightarrow F = 2, -m_F$ |
|------------------------------------------|
| $F = 2, m_F = \pm 2 \leftrightarrow F = 3, m_F = \mp 2$ | $\frac{5}{4} \left[ 5J_0 \left( \frac{\omega_1}{\omega} \right) - 8J_0 \left( 3 \frac{\omega_1}{\omega} \right) + 3J_0 \left( 5 \frac{\omega_1}{\omega} \right) \right]^2$ |
| $F = 2, m_F = \pm 1 \leftrightarrow F = 3, m_F = \mp 1$ | $\frac{1}{2} \left[ 6J_0 \left( \frac{\omega_1}{\omega} \right) + 9J_0 \left( 3 \frac{\omega_1}{\omega} \right) - 15J_0 \left( 5 \frac{\omega_1}{\omega} \right) \right]^2$ |
| $F = 2, m_F = 0 \leftrightarrow F = 3, m_F = 0$ | $9 \left[ 2J_0 \left( \frac{\omega_1}{\omega} \right) + J_0 \left( 3 \frac{\omega_1}{\omega} \right) + 5J_0 \left( 5 \frac{\omega_1}{\omega} \right) \right]^2$ |

| $F=2, m_F \leftrightarrow F=3, m_F \pm 1$ |
|------------------------------------------|
| $F = 2, m_F = \pm 2 \leftrightarrow F = 3, m_F = \pm 3$ | $\frac{15}{128} \left[ 3 + 4J_0 \left( 2 \frac{\omega_1}{\omega} \right) + J_0 \left( 4 \frac{\omega_1}{\omega} \right) \right]^2$ |

Cesium 133

| $F = 3, m_F \leftrightarrow F = 4, -m_F$ |
|------------------------------------------|
| $F = 3, m_F = \pm 3 \leftrightarrow F = 4, m_F = \mp 3$ | $7 \left[ 5J_0 \left( \frac{\omega_1}{\omega} \right) - 9J_0 \left( 3 \frac{\omega_1}{\omega} \right) + 5J_0 \left( 5 \frac{\omega_1}{\omega} \right) - J_0 \left( 7 \frac{\omega_1}{\omega} \right) \right]^2$ |
| $F = 3, m_F = \pm 2 \leftrightarrow F = 4, m_F = \mp 2$ | $3 \left[ 5J_0 \left( \frac{\omega_1}{\omega} \right) + 3J_0 \left( 3 \frac{\omega_1}{\omega} \right) - 15J_0 \left( 5 \frac{\omega_1}{\omega} \right) + 7J_0 \left( 7 \frac{\omega_1}{\omega} \right) \right]^2$ |
| $F = 3, m_F = \pm 1 \leftrightarrow F = 4, m_F = \mp 1$ | $15 \left[ 3J_0 \left( \frac{\omega_1}{\omega} \right) + J_0 \left( 3 \frac{\omega_1}{\omega} \right) + 3J_0 \left( 5 \frac{\omega_1}{\omega} \right) - 7J_0 \left( 7 \frac{\omega_1}{\omega} \right) \right]^2$ |
| $F = 3, m_F = 0 \leftrightarrow F = 4, m_F = 0$ | $9J_0 \left( \frac{\omega_1}{\omega} \right) + 15J_0 \left( 3 \frac{\omega_1}{\omega} \right) + 5J_0 \left( 5 \frac{\omega_1}{\omega} \right) + 35J_0 \left( 7 \frac{\omega_1}{\omega} \right)$ |

| $F=3, m_F \leftrightarrow F=4, m_F \pm 1$ |
|------------------------------------------|
| $F = 3, m_F = \pm 3 \leftrightarrow F = 4, m_F = \pm 4$ | $14 \left[ 10 + 15J_0 \left( 2 \frac{\omega_1}{\omega} \right) + 6J_0 \left( 4 \frac{\omega_1}{\omega} \right) + J_0 \left( 6 \frac{\omega_1}{\omega} \right) \right]^2$ |
magnetic-dipole transitions that depend on the difference of populations in the magnetic sublevels of the hyperfine structure. This parameter, in turn, is determined from the solution of the equations of motion for the diagonal elements of the density matrix.

In the case of using antirelaxation material coated $^{87}$Rb vapor cells with circularly polarized laser pumping source tuned to high-frequency transition of the D$_1$-line, these equations have the following form:

$$
\begin{align*}
\sigma_{11} &= 0.125 - 0.96\Gamma^*\sigma_{11} + 0.144\Gamma^*\sigma_{22} + 0.04\Gamma^*\sigma_{33} \\
\sigma_{22} &= 0.125 - 0.576\Gamma^*\sigma_{11} + 0.04\Gamma^*\sigma_{22} + 0.2\Gamma^*\sigma_{33} \\
\sigma_{33} &= 0.125 - 0.144\Gamma^*\sigma_{22} + 0.04\Gamma^*\sigma_{33} \\
\sigma_{44} &= 0.125 \\
\sigma_{55} &= 0.125 + 0.04\Gamma^*\sigma_{33} \\
\sigma_{66} &= 0.125 + 0.144\Gamma^*\sigma_{11} + 0.04\Gamma^*\sigma_{22} + 0.04\Gamma^*\sigma_{33} \\
\sigma_{77} &= 0.125 + 0.48\Gamma^*\sigma_{11} + 0.144\Gamma^*\sigma_{22} + 0.04\Gamma^*\sigma_{33} \\
\sigma_{88} &= 0.125 + 0.48\Gamma^*\sigma_{11} + 0.144\Gamma^*\sigma_{22}
\end{align*}
$$

where the index of the diagonal element of the density matrix corresponds to the number of magnetic sublevels of the rubidium atoms in the ground state, $\Gamma^*$ is expressed in relative units and is coupled with the speed of the pump (the ratio of the rate of pumping to the rate of dark relaxation). Figure 2 shows examples of dependencies of the probability of microwave transitions ($F=1, m_F \leftrightarrow F=2, m_F$) from the argument obtained based on the data of table 1 by solving the system of equations (2) for the corresponding difference between diagonal elements of the density matrix under various parameters $\Gamma^*$.

**Figure 2.** The transition probabilities for the double magnetic microwave resonance of atoms of rubidium-87 in a function of the relative amplitude of RF field spin generator.
3. Experiment

The experiments were provided in the conditions of a laser pumping on LF D1-line component of S_{1/2}-P_{3/2} transition from hyperfine structure ground state with $F_g = 2$ where the highest polarization of alkali atoms is reached. When the laser is tuned to a high-frequency component of HFS with $F_e = 1$ of the ground state doesn't give the positive yield because of feedback omission owing to low concentration of atoms on the observed level. A different situation takes place in the case of the D1-line laser pumping. In this case, as the elementary calculation of the density matrix diagonal elements difference demonstrates, it is possible to reach the higher level of the atoms polarization both on the microwave transition, and in the structure of the magnetic levels with $F_g = 2$ and $F_g = 1$. The experiment with the D1-line laser pumping the tandem of magnetometers worked both at LF components of electric-dipole transition of this line (transitions of $F_g = 2 \leftrightarrow F_e = 1$ and $F_g = 2 \leftrightarrow F_e = 2$) and at a short-wave component of $F_g = 1 \leftrightarrow F_e = 2$, though with considerably smaller signal-to-noise ratio. The last fact has a basic role in a choice of the optimal pumping mode which would allow to realize the lowest level of a flicker noise of Allan deviation [2]. The equality of the detected signals intensity in the microwave and LF channels of a magnetometers tandem is satisfied to such conditions as both of these channels have equivalent value. The mutual influence of the microwave and LF signals is shown as the changing in dynamics of their intensity and spectrum. As an example, the experimental recordings of the self-generating magnetometer signal and a microwave absorption spectrum of Rb$^{87}$ atoms are shown in figure 2, The record was carried out when the frequency of the microwave field was scanning.

The relative influence of LF and the microwave channels in magnetometers tandem was checked during the experiment in two modes: a) measurements of the frequency shift of the spin generator $\delta \nu_{SG}$ at the fixed detuning of the microwave frequency $\delta \nu_{UHF}$; and b) measurements of the frequency shift for the microwave resonance $\Delta \nu_{UHF}$ at the fixed detuning of the spin generator frequency $\Delta \nu_{SG}$ due to the change of the phase shift in the feedback circuit.

The experiment showed a significant difference in the magnetometers tandem LF and microwave channels interaction for operation modes “a” and “b”. For example, when the same artificial deviations $\delta \nu_{UHF}$ and $\Delta \nu_{SG}$ in magnetic field of 0.01 Oe $\delta \nu_{SG}$ with respect to $\Delta \nu_{UHF}$ was the order of magnitude of $10^2$. 
Figure 3. The recordings of the self-generating magnetometer signal and a microwave absorption spectrum of the optical-oriented $^{87}$Rb atoms: 1 – signal of the self-generating magnetometer, 2 – absorption signal of RF resonance in a case of the scanning frequency field of microwave range. The absorption signals R1 and R2 correspond to end resonances of transitions between UHS sublevels.

4. Conclusions
The presented results allow us to conclude that there is a significant mutual influence of the signals in quantum magnetometers tandem, one of which is constructed as a traditional self-generating magnetic resonance device. Such effect can be noticeably weakened by using of the rotating radio frequency field. In this case, it is possible to eliminate the non-resonance component of the field, and, hence, to eliminate the Bloch-Siegert frequency shift [17]. However, from our point of view, a good solution for this problem is the use of the modulated optical pumping in a self-generating circuit of the magnetometers tandem [18]. In this case, the errors induced by the radio-frequency field are eliminated, as well as the errors due to the light shifts of the resonance frequency, which are coupled with circularly polarized radiation of the lamp [19]. As a preliminary calculation shows, the distinctive feature of a self-generating magnetometer with modulated pumping is the absence of a shift in the resonance frequency associated with the action of the feedback loop, which allows one to predict its better long-term stability in comparison with the schemes using the radio-frequency field.
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