Effective Description of a Gauge Field and a Tower of Massive Vector Resonances

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In this work we review an effective description of the interaction of a gauge field with a tower of massive vector fields by introducing a non-diagonal mass matrix in a gauge invariant way. Particular cases of the method with only one vector resonance have been used by the author elsewhere, nevertheless in this paper the method is developed in a general way and we proof its main features for an arbitrary number of vector resonances. Additionally, we show how to couple the vector resonances with fermions. We find that the method can be useful in order to describe the low energy phenomenology of scenarios like Kaluza-Klein resonances of usual gauge bosons or Technicolor vector resonances and detailed examples are provided.

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1. Introduction

There are some physical scenarios where gauge bosons interact and mix with massive vector bosons. Perhaps the best known example is the case of the photon, the rho and the omega (if isospin violation is allowed) in hadron physics. In fact, this is the original motivation for the development of the Vector Meson Dominance Hypothesis (VMD)(for a review see [1]). On the other hand, some extensions of the Standard Model include massive vector fields which mix with vector gauge bosons. In Technicolor models, for example, we find the technirho and the techniomega, which mix with the photon and the Z, and the color octet technirho, which mixes with the gluon. In Topcolor assisted Technicolor models the gluon mixes with four color octet technirhos and the so called colorons (see, for example [2]). Another important case is the interaction of the usual gauge bosons with

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(for example the gluon) with their Kaluza-Klein resonances in models with extra dimensions. Another recent example is Higgsless model. The expectation of finding these resonances at the LHC has strongly motivated the question about how to describe them in the context of a four-dimensional phenomenological model\cite{3}. For example, the so called Deconstruction Theory was first invented to describe the Kaluza-Klein resonance of the gluon \cite{3,4}. In this kind of models, a set of “hidden local symmetries” is introduced. The breaking down of the hidden symmetry is described by a non-linear sigma model and the massive gauge fields associated with the “hidden symmetries” are identified with the Kaluza-Klein resonances.

In this paper we review a general method for describing the interaction of a gauge field with a set of massive vector resonances based on the observed low energy local symmetries. This method can be useful as a first effective approach for studying new vector resonances. Particular cases of this method have been used for studying the phenomenology of the color-octet technirho \cite{5} and the axigluon \cite{6}. We build the method in a general way and we proof its more important property for an arbitrary number of resonances. Additionally, important particular cases are studied.

We have organized this work as follow. In section 2 we present the basic Lagrangian and its properties (including the coupling of the resonances with fermions). Section 3 is devoted to show some examples and applications. Finally, some conclusions are presented.

2. The Lagrangian

In order to fix ideas, let us consider an extension of QCD defined by a Lagrangian of the form:

\[ \mathcal{L} = -\frac{1}{4} \sum_{i} \tilde{F}_{a}^{i\mu\nu} \tilde{F}_{a}^{i\mu\nu} + \sum_{i,j} \frac{1}{2} A_{i}^{a} \mathcal{M}_{ij}^{2} A_{j}^{a\mu} \]  

with

\[ \tilde{F}_{a}^{i\mu\nu} = \partial_{\mu} A_{a}^{i\nu} - \partial_{\nu} A_{a}^{i\mu} - \tilde{g}_{i} f^{abc} A_{b}^{i\mu} A_{c}^{a\nu} \]  

where \( a \) is a group index and \( i = 1 \ldots N \) is the “flavor” of the vector field. In order to make the notation clear we have written explicitly the sum over the flavor indices. We assume that all the vector fields transform under the same group \( SU(3) \) as follow:

\[ \delta A_{a}^{i} = -f^{abc} A_{a}^{i\Lambda_{c}} - \frac{1}{\tilde{g}_{i}} \partial_{a} A_{a}^{i} \]  

From equations (1) and (3) we find that the variation of the Lagrangian is:

\[ \delta \mathcal{L} = \sum_{i,j} A_{i}^{a} \mathcal{M}_{ij}^{2} \frac{1}{\tilde{g}_{j}} \partial^{\mu} A_{a}^{i} \]  

Evidently, $\delta \mathcal{L}$ must vanish independently of the fields, so the following equation must be satisfied:

$$\sum_{j} M_{ij}^2 \frac{1}{g_j} = 0 \quad (5)$$

We can solve equation (5) for the diagonal term and we obtain:

$$M_i^2 \equiv M_{ii}^2 = -\bar{g}_i \sum_{j \neq i} M_{ij}^2 \frac{1}{g_j} \quad (6)$$

### 2.1. Transformation of the Physical Fields

Equation (5) shows us that the vector

$$v_1 = \frac{1}{\sqrt{\sum_i \frac{1}{g_i}}} \begin{pmatrix} \frac{1}{g_1} \\ \cdot \\ \cdot \\ \frac{1}{g_N} \end{pmatrix} \quad (7)$$

is an eigenvector of $\mathcal{M}^2$ with null eigenvalue. We want to emphasize that this is the necessary and sufficient condition to include a mass matrix for the gauge sector of a gauge invariant theory.

Let us now define a new field:

$$G_1 = g \sum_i A_i \quad (8)$$

where $g$ is defined by

$$\frac{1}{g^2} \equiv \sum_i \frac{1}{g_i} \quad (9)$$

It is clear that, by construction, $G_1$ is massless. Using equation (8) and (8) it is easy to see that $G_1$ transforms in the following way:

$$\delta G_1^a = -f^{abc} G_1^b \Lambda^c - \frac{1}{g} \partial \Lambda^a \quad (10)$$

That is, $G_1$ transforms as a true gauge field.

Because matrix $\mathcal{M}^2$ is hermitian, all other independent eigenvectors must be orthogonal to (7). Let us call such vectors as $v_i \ (i = 2 \ldots N)$. We can now define:

$$G_i = \sum_j v_{ij} A_j \quad (11)$$
with \( i = 2 \ldots N, j = 1 \ldots N \) and \( v_{ij} \) such that

\[
\sum_j v_{ij} \frac{1}{g_j} = 0
\]

(12)

Equation (12) represents the orthogonality condition with \( G_1 \). Using equation (3), we find that \( G_i \) transforms as:

\[
\delta G_i^a = -\sum_j v_{ij} f^{abc} A_j^b \Lambda^c - \sum_j v_{ij} \frac{1}{g_j} \partial \Lambda^a
\]

(13)

From (11) and (12) we obtain:

\[
\delta G_i^a = -f^{abc} G_i^b \Lambda^c \quad \text{with} \quad i = 2 \ldots N
\]

(14)

That means that all the massive resonances \( G_i \) \((i \neq 1)\) transform like matter fields in the adjoint representation.

### 2.2. Couplings with Fermions

Any realistic model must consider the coupling of vector fields with fermions. In gauge theories, this coupling is obtained in a minimal way by defining a covariant derivative. Nevertheless, in our case, we have too many vector fields transforming in a gauge-like way under the same group. We can, then, ask whether it is possible to build a gauge invariant Lagrangian that describe the interaction of fermions with all these vectors fields. We propose the following simple solution. Let us consider the Lagrangian:

\[
\mathcal{L} = \bar{\psi} (i \gamma^\mu D_\mu - m) \psi
\]

(15)

With \( D_\mu \) given by:

\[
D_\mu \equiv \partial_\mu + i \frac{\lambda^a}{2} \sum_j \tilde{g}_j f_j A_{j\mu}^a
\]

(16)

and

\[
\sum_i f_i = 1
\]

(17)

It is clear that the variation of each \( A_{j\mu}^a \) cancels a fraction \( f_j \) of the total variation coming from the kinetic term of the fermions. Equation (17)
ensures the complete gauge invariance of the Lagrangian\(^1\).

It is interesting to investigate how the field \(G_1\) couples to fermions. The first step is to realize that, after the change of basis, Lagrangian (15) takes the form:

\[
\mathcal{L} = \mathcal{L}_0 + \mathcal{L}_1 + \mathcal{L}_2
\]

where:

\[
\mathcal{L}_0 = \bar{\psi}(i\gamma^\mu \partial_\mu - m)\psi
\]

\[
\mathcal{L}_1 = -\xi_1 \bar{\psi} \frac{\lambda^a}{2} \gamma^\mu G^a_{1\mu}\psi
\]

\[
\mathcal{L}_2 = -\xi_j \bar{\psi} \frac{\lambda^a}{2} \gamma^\mu G^a_{j\mu}\psi \quad (j = 2 \ldots N)
\]

with \(\xi_i\) \((i = 1 \ldots N)\) are some coupling constants. It is easy to see, by equation (14), that \(\delta \mathcal{L}_2 = 0\). That means that, in order to have a gauge invariant Lagrangian, \(\delta \mathcal{L}_1\) must cancel the variation of the fermionic kinetic term. However, in virtue of (10), this cancellation only happens if \(\xi_1 = g\). That is, gauge invariance warrants that \(G_1\) couple to fermions as a true gauge boson independently of the value of \(f_i\) constants.

Conversely, we can start from the Lagrangian defined by equations (18)-(21) (with \(\xi_1 = g\)) and rotate the fields \(G_i\) by using equations (8) and (11). The resulting Lagrangian can be written as:

\[
\mathcal{L} = \bar{\psi}(i\gamma^\mu \partial_\mu - m)\psi - \bar{\psi} \frac{\lambda^a}{2} \gamma^\mu \psi \sum_i \left[ \frac{g}{g_i} + \sum_j \xi_j v_{ji} \right] A^a_{i\mu}
\]

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\(^1\) Perhaps a more rigorous but equivalent way to obtain our generalized covariant derivative is defining first an usual covariant derivative for each gauge-like field:

\[
D_{j\mu} = \partial_\mu + i\frac{\lambda^a}{2} \tilde{g}_j A^a_{j\mu}
\]

We can, then, construct the covariant derivative as a weighted sum:

\[
D_\mu = \sum_j f_j D_{j\mu}
\]

where the set of weights \(\{f_j\}\) satisfy equation (17) and can be viewed as a normalized discrete distribution. On the other hand, we can use \(D_{j\mu}\) to define formally the field strength tensor for each gauge-like field:

\[
i\tilde{g}_j \frac{\lambda^a}{2} F^a_{j\mu\nu} \equiv [D_{j\mu}, D_{j\nu}]
\]
If we make the definition:

\[ f_i \equiv \frac{1}{g_i} \left[ \frac{g}{\tilde{g}_i} + \sum_j \xi_j v_{ji} \right] \]  

(without sum in the repeated indexes) then we obtain exactly the Lagrangian defined by equations (15) and (16). On the other hand, if we sum all the \( f_i \) defined in (23) and we use equations (9) and (12) then we obtain the condition \( \sum_i f_i = 1 \).

3. Examples and Applications
3.1. Case \( N = 2 \)

The gauge sector of this case was partially studied in a previous work \[7\] on the gluon-color octet technirho mixing \footnote{In this case, however, additional dimension six terms must be taken into account in order to describe effects due to the technirho compositeness \[8\].} Nevertheless it is worth to review this important and simple case in order to illustrated the methods described here.

In this case the Lagrangian of the gauge sector can be written as:

\[ \mathcal{L} = -\frac{1}{4} F_{1 \mu \nu} F_{1}^{\mu \nu} - \frac{1}{4} F_{2 \mu \nu} F_{2}^{\mu \nu} + \frac{M^2}{2 \tilde{g}_2} (\tilde{g}_1 A_{1 \mu} - \tilde{g}_2 A_{2 \mu})^2 \]  

where \( M \) is a new mass scale present in the model. We want to emphasize this important point. It is an intrinsic feature of the class of model studied in this work the appearance of new mass scales that are compatible with gauge symmetry but remains unconstrained by it.

The mass matrix originated by this Lagrangian (which is in this case the most general mass matrix for the gauge sector compatible with gauge symmetry) is exactly diagonalizable. In fact, the fields which are mass eigenstates can be written as

\[ G_1 = A_1 \cos(\alpha) + A_2 \sin(\alpha) \]  
\[ G_2 = -A_1 \sin(\alpha) + A_2 \cos(\alpha) \]

where \( \sin(\alpha) = g/\tilde{g}_2 \) and \( g \) is, according to equation (9),

\[ g = \frac{\tilde{g}_1 \tilde{g}_2}{\sqrt{\tilde{g}_1^2 + \tilde{g}_2^2}}. \]

The mass of the physical states are:
so, we see that the Lagrangian describes, in fact, a massless gauge boson and a massive spin-one resonance in the adjoint representation.

Let us study, for a moment, the decay process $G_2 \rightarrow G_1 G_1$. The relevant part of the Lagrangian, written in terms of the physical fields, is:

$$
\mathcal{L}_{G_2 G_1 G_1} = \left( -g_1 \cos^2(\alpha) \sin(\alpha) + g_2 \cos(\alpha) \sin^2(\alpha) \right) f^{abc} \left\{ \partial_\mu G^a_{1 \nu} G^b_1 G^{c \nu} + \partial_\mu G^a_{2 \nu} G^b_1 G^{c \nu} + \partial_\mu G^a_{2 \nu} G^b_1 G^{c \nu} \right\} 
$$

(27)

It is clear that, due to the definition of $\alpha$, $\tilde{g}_1 \cos(\alpha) = \tilde{g}_2 \sin(\alpha)$. Using this identity, we can see that the coupling constant of the $G_2 G_1 G_1$ interactions vanishes exactly. This result is an important consequence of the gauge symmetry of the model.

Let us turn our attention to the coupling with fermions. When we write Lagrangian (15) in terms of physical fields we obtain:

$$
\mathcal{L} = \bar{\psi} \left[ i \gamma^\mu \partial_\mu - m - g_1^a \gamma^\mu G^a_1 G_1^\mu \frac{\lambda^a}{2} - \frac{(\tilde{g}_2 f_2 - \tilde{g}_1 f_1)}{\sqrt{\tilde{g}_1^2 + \tilde{g}_2^2}} \gamma^\mu G^a_2 G_2^\mu \frac{\lambda^a}{2} \right] \psi 
$$

(28)

Two interesting cases arise:

1. We can impose that $\tilde{g}_1 = \tilde{g}_2$ and $f_1 = f_2$. In this case, $G_2$ completely decouple from the fermionic sector. On the other hand, it is not difficult to see that $G_2$ interacts with the gauge boson ($G_1$) only through terms of the form $G_1 G_2 G_2$ and $G_1 G_1 G_2 G_2$ (In the case of the technirho other high dimensional operators must be taken into account due to its compositeness nature [8], but here we can assume that $G_2$ is an elementary field). All that mean that a symmetric choice of the constants makes $G_2$ to be stable.

2. Another interesting possibility come to us when we realize that the constants $f_1$ and $f_2$ can be different for different fermionic generations. We can for example choose that constants in such a way that $G_2$ decouple from the first two generations but couple strongly to the third one. In this way it may be possible to build an effective Topcolor model.
Finally, we must remark that Lagrangian (28) is a generalization of VMD. In fact, we obtain the traditional VMD result by choosing $f_2 = 0$ and $f_1 = 1$.

3.2. Case $N = 2$ with Symmetry Breaking

Let us consider again Lagrangian (24) but now we add a non-linear sigma model term:

$$v^2 \text{tr} \left[ D_\mu U^\dagger D^\mu U \right]$$

(29)

where $v$ is some energy scale at which the gauge symmetry spontaneously breaks down, and $D_\mu U$ is:

$$D_\mu U = \partial_\mu U - i \frac{g_1}{2} A_1 U - i \frac{g_2}{2} A_2 U$$

(30)

Notice that we have put in the covariant derivative $A_1$ as well as $A_2$ with equal relative weight. In the unitary gauge ($\langle U \rangle = 1$), the non-linear sigma model term produce additional contributions to the mass matrix. The resulting mass matrix can be exactly diagonalized but, as usual, in order to simplify our expressions we will consider only the case where $g_2 \gg g_1$. In this limits the mass eigenvalues are:

$$m_1^2 = \frac{2 g_1 v^2}{1 + \frac{g_2 v^2}{2M^2}}$$

$$m_2^2 = M^2 \left( 1 + \frac{g_2 v^2}{2M^2} \right)$$

Of course, the physical fields can be written again in the form given in equations (25) and (26), but now the mixing angle is given by:

$$\tan(\alpha) = \frac{g_1}{g_2} \left\{ \frac{1 - \frac{g_2 v^2}{2M^2}}{1 + \frac{g_2 v^2}{2M^2}} \right\}$$

(31)

Now we will consider again the decay $G_2 \to G_1 G_1$. This process is still described by Lagrangian (24) but this time $g_1 \cos(\alpha) \neq g_2 \sin(\alpha)$ and the coupling constant of this process doesn’t vanish. In fact, using the

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3 The model studied in [7] is based on the VMD hypothesis. That means that the vector resonance (the proto-technirho) couple to the quarks only through the mixing with the proto-gluon. Nevertheless Extended Technicolor must produce a direct (proto-)technirho-quark interaction. The formalism presented here allows us to include such interactions.
expression for \( \tan(\alpha) \) we wrote above, we can see that the Lagrangian (24) can be written as:

\[
\mathcal{L}_{G_2G_1} = - \frac{2g_i^2 v^2}{2M^2 + g_i v^2} \left( \frac{1 - \frac{g_i^2 v^2}{2M^2}}{1 + \frac{g_i^2 v^2}{2M^2}} \right)^2 f^{abc} \{ \partial_\mu G^a_{1\nu} G^b_{1\mu} G^c_{2\nu} + \\
+ \partial_\mu G^a_{1\nu} G^b_{2\mu} G^c_{1\nu} + \partial_\mu G^a_{2\nu} G^b_{1\mu} G^c_{1\nu} \} \tag{32}
\]

A similar phenomenon occurs when we couple the vector bosons to fermions. As we did above, we implement the minimal coupling defining a generalized covariant derivative. Thus, we write:

\[
\mathcal{L} = \bar{\psi} (i\gamma^\mu D_\mu - m) \psi \tag{33}
\]

with

\[
D_\mu = \partial_\mu + i \frac{\lambda^a}{2} (1 - f) \tilde{g}_1 A_{1\mu}^2 + i \frac{\lambda^a}{2} f \tilde{g}_2 A_{2\mu}^2 \tag{34}
\]

When we make the rotation to the physical basis, the interaction term between the fermions and the “would-be” gauge boson can be written as:

\[
[\tilde{g}_1 \cos(\alpha) + f (\tilde{g}_2 \sin(\alpha) - \tilde{g}_1 \cos(\alpha))] \bar{\psi} G_{1\mu}^a \frac{\lambda^a}{2} \gamma^\mu \psi \tag{35}
\]

Again, because of equation (31), \( \tilde{g}_2 \sin(\alpha) - \tilde{g}_1 \cos(\alpha) \neq 0 \). This means that, due to the symmetry breaking the coupling constant characteristic of the interaction between the fermions and the “would-be” gauge boson, depends on the free parameter \( f \).

### 3.3. First Neighbor Coupling on a Ring

Another interesting case is to study \( N \) vector bosons with first neighbor coupling on a ring. In this case, equation (5) reduce to:

\[
M_i^2 = - \frac{\tilde{g}_i}{\tilde{g}_{i-1}} M_{i-1}^2 - \frac{\tilde{g}_i}{\tilde{g}_{i+1}} M_{i+1}^2 \tag{36}
\]

(without sum in the repeated indexes) and we impose periodic boundary conditions:

\[
M_{10}^2 = M_{1N}^2 \tag{37}
\]

\[
M_{NN+1}^2 = M_{N1}^2 \tag{38}
\]
If additionally we assume that all the coupling constants are identical and $M_{ii}^2 = M_{i+1}^2 = k^2$ (with $k^2$ some arbitrary constant) we get:

$$M_i^2 = -2k^2$$  \hspace{1cm} (39)

Finally, the bosonic mass matrix has the form:

$$M^2 = k^2 \begin{pmatrix}
-2 & 1 & 0 & 0 & \cdots & 0 & 1 \\
1 & -2 & 1 & 0 & \cdots & 0 & 0 \\
0 & 1 & \ddots & & & & & \\
\vdots & & \ddots & & & & & \\
0 & \cdots & \cdots & \ddots & & & & \\
1 & 0 & \cdots & \cdots & \ddots & & & \\
& & & & & -2 & \end{pmatrix}$$  \hspace{1cm} (40)

It is well known that the eigenvalues of this mass matrix describe (for $N \gg 1$) a Kaluza-Klein like spectrum.

4. Conclusions

In this work we have studied a general method for describing the mixing of an arbitrary number of massive vector bosons with a gauge boson, and their interactions. The key idea is the introduction of a non-diagonal mass matrix in the gauge sector in a gauge invariant way. The coupling of all the vector bosons to fermions has been, also, treated in a consistent way. The resulting Lagrangian represents a generalization of the old idea of Vector Meson Dominance. We have shown that this method can be useful as a phenomenological description of a Kaluza-Klein-like tower of vector resonances. Other scenarios like Topcolor model can be also described.

Of course, the use of the method presented here aims to be only a low energy phenomenological description of the physics of vector resonances, but we think it can serve as a useful first approach if they are discovered at the LHC.

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