Origin of the Fermi arcs in cuprates: a dual role of quasiparticle and pair excitations

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Abstract
Angle resolved photoemission spectroscopy (ARPES) measurements in cuprates have given key information on the temperature and angle dependence of the gap (d-wave order parameter, Fermi arcs and pseudogap). We show that these features can be understood in terms of a Bose condensation of interacting pairons (prefomed hole pairs which form in their local antiferromagnetic environment). Starting from the basic properties of the pairon wavefunction, we derive the corresponding k-space spectral function. The latter explains the variation of the ARPES spectra as a function of temperature and angle up to T*, the onset temperature of pairon formation. While Bose excitations dominate at the antinode, the fermion excitations dominate around the nodal direction, giving rise to the Fermi arcs at finite temperature. This dual role is the key feature distinguishing cuprate from conventional superconductivity.

Keywords: cuprates, Fermi arcs, ARPES, novel mechanisms, pseudogap

(Some figures may appear in colour only in the online journal)
In this article, we answer these questions in the framework of the condensation of preformed pairons [22]. To proceed, we calculate the spectral function for cuprates and directly compare the computed energy distribution curves (EDCs) to the ARPES measurements as a function of temperature and angle at the Fermi surface. We show that the condensation of pairons fully describes the continuous evolution of the ARPES spectra with temperature and angle, from the antinode to the node. Remarkably, the boson condensation is revealed in the antinodal (AN) direction where Bose–Einstein statistics dominate, whereas the fermion excitations dominate near the nodal (N) direction. At $T_c$ and above, the pseudogap is revealed by the incoherent pair excitations at the antinode, which coexist with the Fermi arcs around the nodes—a direct consequence of the composite character of pairons.

From pairons to Cooper pairs

In the framework of the $t – J$ Hamiltonian, it has been shown that two holes in an antiferromagnetic system form a bound state provided that the ratio $J/t$ is sufficiently large [23, 24]. In a recent work [22], we have extended this mechanism to a more realistic system with a large number of holes. In this scenario, pairs of holes are trapped in their antiferromagnetic environment, on the scale of the antiferromagnetic correlation length $\xi_{\text{AF}}$, forming pairons. This idea is strongly supported by the experimental finding that $\xi_{\text{AF}} \sim 1/\sqrt{p}$ [25], where $p$ is the number of holes per copper atom. Thus $\xi_{\text{AF}}$ varies as the distance between holes (or hole pairs), providing an immediate explanation for the linear variation of the antinodal gap, $\Delta_p$, with hole doping $p$ [22].

We start with a boson Hamiltonian corresponding to a gas of non-interacting pairons which, in absence of mutual interactions, describes the incoherent pseudogap state:

$$H_B = \sum_i \varepsilon_i b_i^\dagger b_i,$$  \hspace{1cm} (1)

where the operator $b_i^\dagger$ creates a given boson of energy $\varepsilon_i$.

In the two-particle center of mass the pairon wavefunction $\psi_i(\vec{r})$, (where $\vec{r} = \vec{r}_1 – \vec{r}_2$ is the distance between the two holes) is determined by the AF environment, which imposes its symmetry. As a result, $\psi_i(\vec{r})$ has to vanish along the lattice diagonal, which corresponds to the nodal direction in $k$-space (see figure 1). We thus take the wavefunction to be the generic form:

$$\psi_i(\vec{r}) = \frac{1}{\sqrt{4}} \left[ \varphi(\vec{r} – a \hat{x}) + \varphi(\vec{r} + a \hat{x}) 
- \varphi(\vec{r} – a \hat{y}) - \varphi(\vec{r} + a \hat{y}) \right],$$  \hspace{1cm} (2)

where $a$ is the lattice parameter, and $\varphi(\vec{r}) = e^{-\frac{\vec{r}^2}{2}} / \sqrt{2\pi b^2}$. The parameter $b$ fixes the spatial extension of the pairon wavefunction $\psi_i$ (see figure 1).

The pairon wavefunction $\psi_i$ and associated operator $b_i$ can equivalently be described by a superposition of delocalized Cooper pairs,

$$b_i = \sum_\vec{k} g_\vec{k}^i b_\vec{k},$$  \hspace{1cm} (3)

where the operator $b_\vec{k}^\dagger = c_\vec{k}^\dagger c_{\vec{k} – \vec{r}}$ creates a Cooper pair $| \vec{k} \uparrow, -\vec{k} \downarrow \rangle$. In this formulation, just as in the original Cooper-pair problem [2], the ground state of the system is constructed from pairs in the zero-momentum state. The weight $g_\vec{k}$ appearing in the sum is given by the Fourier transform of the wavefunction $g(k_x, k_y) = \int e^{i\vec{k} \cdot \vec{r}} \psi_i(\vec{r}) d^2 r$.

Taking the quantum average $b_i^\dagger b_i \approx \langle b_i^\dagger b_i \rangle = b_i^\dagger b_i$, and using the standard BCS expression $\langle b_i^\dagger b_i \rangle = \frac{\Delta^2_{\vec{k}}}{2E_F}$, we obtain the mean-field Hamiltonian:

$$H_{MF} = \sum_{\vec{k},i} \varepsilon_i c_{\vec{k} i}^\dagger c_{\vec{k} i} + \sum_{\vec{i},\vec{k}} \Delta_{\vec{k}}^i b_{\vec{k}}^\dagger b_{\vec{k} i}^\dagger + \sum_{\vec{i},\vec{k}} \Delta_{\vec{k}}^i b_{\vec{k}} b_{\vec{k} i},$$  \hspace{1cm} (4)

where the first term is the kinetic energy and the second is the pairing term. In the latter, the binding energy $\Delta_{\vec{k}}^i$ is determined by the self-consistent equation

$$\Delta_{\vec{k}}^i = \varepsilon_i \sum_{\vec{k}'} g_\vec{k'}^i g_{\vec{k}'}^i \left( \frac{\Delta_{\vec{k}}^i}{2E_F} \right).$$  \hspace{1cm} (5)

In the continuum limit the sum is replaced by an integral in the standard fashion. The latter expression bears a strong analogy with the BCS gap equation, however both the integration limits and the integrand involving $g_\vec{k}$ differ quantitatively.

Dropping the $i$ index, one can show that $g_\vec{k}$ takes the form:

$$g_\vec{k} \propto e^{-\frac{\Delta_{\vec{k}}^2}{E_F}} [\cos(k_x a) – \cos(k_y a)],$$  \hspace{1cm} (6)

which reveals both the extent of the $k$-states involved, and $g_{\vec{k}} \propto \cos(\theta)$ imposes the $d$-wave dependence of the gap parameter. Note that equations (4) and (5) imply the existence of quasiparticles of the form:

$$E_{\vec{k}} = \sqrt{\varepsilon^2_{\vec{k}} + \Delta_{\vec{k}}^2},$$

associated with degenerate pairs of binding energy $\varepsilon_i$. 

**Figure 1.** Pairon wavefunction (left panel) in the center of mass ($\vec{R} = 0$) and its Fourier transform (right panel). The Fermi surface, indicated by the red curve in the right panel, is calculated based on the accurate band structure proposed by Markiewicz et al [26]. In the right panel the antinodal direction (AN or $\theta = 0$) and the nodal direction (N or $\theta = \pi/4$) are indicated.

$\varepsilon_{\vec{k}}^2 + \Delta_{\vec{k}}^2.$
can be well approximated by \( \Delta g \Delta b_0 \). Assuming \( \epsilon_c \) is proportional to the pairon energy or pair binding energy, optimal doping. The satisfying conclusion is that the Cooper–\( \Gamma \)K), with the broadening parameter in equation (15) given by \( \Gamma = k_B T \). Note that we plot here the excited-state spectral function where the coherence factors are absent (see text).

Let us emphasize that the \( i \)-index is formally equivalent to a band index in the context of multi-band superconductivity [27]. However in our case, the \( i \)-index represents pairons of differing binding energies and the Hamiltonian (4) represents a non-superconducting state of independent Cooper pairs. In order to establish a macroscopic coherent state, a coupling between pairs of different energies is necessary [28–30]. Without the interaction term between pairons, no long range order is possible. Still, a gap in the density of states is present at the Fermi level without the characteristic peaks indicating phase coherence, see figure 2(b). Thus the Hamiltonian (4) provides a description of the main features of the pseudogap state.

Our numerical study of the self-consistent equation (5) shows that, for values of \( \epsilon_i \) in the relevant range for cuprates (\( \epsilon_i \sim 80–210 \) meV), \( \Delta_{i}^* \) can be well approximated by

\[
\Delta_{i}^* = c \epsilon_i g_i,
\]

where \( c \) is a constant. Taking the parameter \( b = a/4 \) in the calculation, we find \( c \approx 0.25 \) and an energy gap \( \sim 40 \) meV at optimal doping. The satisfying conclusion is that the Cooper-pair binding energy \( \Delta_{i}^* \) is proportional to the pairon energy \( \epsilon_i \), and thus both concepts are formally equivalent.

In the absence of pairon–pairon interactions, the system is in an incoherent state, the ‘Cooper glass’ [28–30]. The pairon energies are distributed with a pair density of states \( P_0(\Delta_i) \), characterized by the mean value \( \Delta_0 \) and dispersion \( \sigma_0 \). A convenient form is the Lorentzian:

\[
P_0(\Delta_i) \propto \frac{\sigma_0^2}{(\Delta_i - \Delta_0)^2 + \sigma_0^2}.
\]

As will be revealed in the spectral function, equations (4)–(8) describe the non-superconducting incoherent state giving rise to the pseudogap phenomena at the critical temperature and above.

**Pairon condensation**

As a result of the interaction between pairons, described by the additional coupling term

\[
H_{int} = \sum_{i \neq j} V_{ij} b_i^\dagger b_j,
\]

the system condenses in a homogeneous ground state where pairons lie in the same quantum state. All Cooper pairs are then characterized by a unique binding energy \( \Delta' = \Delta_p \). In the mean field approximation, \( \Delta_p \) is given by a self-consistent equation, which in the antinodal direction has the form [28]:

\[
\Delta_p = \Delta_0 - 2\beta P_0(\Delta_p) .
\]

It includes a pair-field term proportional to the average interaction energy \( \beta \). In previous work we showed that this interaction follows the critical dome and has the value \( \beta \sim 2k_B T_c \). Moreover, the pairon condensate model matches the phase diagram for a wide range of doping in terms of a single energy scale, \( J \), the exchange energy [22].

Using the gap equation (9), and the energy bands \( \epsilon_k \) of cuprates from Markiewicz et al [26], the spectral function of quasiparticles is obtained for any wavevector \( k \). Assuming nearly optimal doping (\( \Delta_p(0) = 40 \) meV, \( T_c = 92 \) K) the spectral intensity in the AN direction crossing \( k_F \) is shown in figure 2 (left panels) at three different temperatures. The \( k \)-sum of these spectra gives directly the associated quasiparticle density of states (DOS) relevant to tunneling (right panel).

A well-defined gap is visible at low temperature with a strong change of slope above the gap energy \( \Delta_g \) giving rise to the characteristic dip in the DOS (a), which has been widely observed by tunneling spectroscopy in cuprates (see [3] and references therein). At the critical temperature (b), a pseudogap persists in the DOS which finally disappears at a much higher temperature \( T^* \) (c). In the remainder of this work we focus on the EDCs with the wave vector on the Fermi surface to compare with the experiments.

**Pair densities**

We start with the low-temperature SC state where all pairs belong to the condensate. An essential concept of the model is that, upon rising temperature, pairs are excited out of the condensate without pair-breaking—a highly non-BCS feature.
Taking the relevant Bose–Einstein statistics with $\mu = 0$, and assuming $d$-wave pairing, the condensate angular density is given by:

$$n_{oc}(T, \theta) = n_0 - A(T, \theta) \int_{\Delta_p \cos(2\theta)}^{\infty} d\Delta_p P_0(\Delta_i) \times f_B(\Delta_i - \Delta_p \cos(2\theta), T),$$  

(10)

where $f_B(E) = (e^{E/kT} - 1)^{-1}$ is the Bose–Einstein distribution, $\delta$ is a low-energy cut-off [28] and $A(T, \theta)$ is a normalization factor to be discussed below. The constant $n_0$, directly proportional to the doping value, is assumed to be independent of $\theta$.

The integrated condensate density, $\int d\theta n_{oc}(T, \theta)/(2\pi)$, shown in figure 3, is a monotonically decreasing function of temperature (lower curve), due to pair excitations, and vanishes at $T_c$, as expected. Note that in this temperature range, the antinodal gap $\Delta_p(T)$ (upper curve), reflecting the total number of pairs, is practically constant up to $T_c$, in agreement with experiment. However for higher temperatures, $T > T_c$ the gap $\Delta_p(T)$ markedly decreases to finally vanish at $T^*$ as a result of pair dissociation [31].

The normalization factor $A(T, \theta)$ is determined using appropriate boundary conditions. We assume that, even at finite temperature (below $T_c$), the condensate density remains uniform: $n_{oc}(T, \theta) = n_{oc}(T)$. Furthermore, the temperature dependence of the antinodal gap $\Delta_p(T)$ (figure 3) is taken throughout this work as the BCS function, however with the ratio $\Delta_p(0)/k_B T^* = 2.2$ compatible with the gap equation (5).

Finally, imposing $n_{oc}(T_c) = 0$, we obtain a self-consistent form for $A(T, \theta)$ with the quadrature:

$$A(T, \theta)^{-1} = (1 - n_{oc}(T))^{-1} \times \int_{\Delta_p \cos(2\theta)}^{\infty} d\Delta_p P_0(\Delta_i) \times f_B(\Delta_i - \Delta_p \cos(2\theta), T).$$  

(11)

Note that the latter self-consistent equation implies the constraint of particle conservation at all temperatures.

Let us now consider the excited-pair and dissociated-pair densities. The latter dissociation phenomenon occurs if the pair binding energy is typically small compared to the thermal energy. As in BCS theory, this process is governed by the Fermi–Dirac distribution $f(E, T) = (e^{E/kT} + 1)^{-1}$ giving rise to the $\left[1 - \tanh \left(\frac{E}{kT}\right)\right]$ factor in the number of dissociated pairs:

$$n_{diss}(T, \theta) = A(T, \theta) \int_{\Delta_p \cos(2\theta)}^{\infty} d\Delta_p P_0(\Delta_i) \times f_B(\Delta_i - \Delta_p \cos(2\theta), T) \times \left[1 - \tanh \left(\frac{\Delta_i}{kT}\right)\right].$$  

(12)

In a similar way, one can write the excited pair density:

$$n_{ex}(T, \theta) = A(T, \theta) \int_{\Delta_p \cos(2\theta)}^{\infty} d\Delta_p P_0(\Delta_i) \times f_B(\Delta_i - \Delta_p \cos(2\theta), T) \times \tanh \left(\frac{\Delta_i}{kT}\right).$$  

(13)

The three densities must follow the sum rule:

$$n_{oc}(T) + n_{ex}(T, \theta) + n_{diss}(T, \theta) = n_0,$$  

(14)

which can be verified by inspection. Furthermore, they inherently capture the physical properties of the cuprate phase diagram, from $T = 0$ through $T_c$ up to $T^*$. It is remarkable that both Bose–Einstein and Fermi–Dirac statistics appear in $n_{ex}$ and $n_{diss}$—a direct consequence of the composite nature of the pairons.

**Spectral function**

Similarly as for the DOS [31], the spectral function $A(\vec{k}, E)$ can be expressed as a sum of three terms, the condensate spectral function $A_{cond}(\vec{k}, E)$, the excited pairs contribution $A_{ex}(\vec{k}, E)$ and finally the dissociated pairs term $A_{diss}(\vec{k}, E)$. The first term is essentially determined by the number of condensed pairs with energy $\Delta_p$, associated with the quasiparticles $E_{\vec{k}} = \sqrt{\epsilon_{\vec{k}}^2 + \Delta_\phi^2}$, where $\Delta_\phi$ is the condensate gap function [28]:

$$A_{cond}(\vec{k}, E) = \frac{-1}{\pi} \Im m \frac{n_{oc}(T)}{E - E_{\vec{k}} + i\Gamma}.$$  

(15)

$\Gamma$ is the standard parameter describing energy broadening.

In the latter equation we have neglected the coherence factors for occupied versus unoccupied states, which corresponds to the spectral function for excited states (both hole and electron-type). In this case, as shown by Schrieffer [32], for symmetric bands at the Fermi level, the coherence factors
disappear. Since we are concerned here with energies very close to the Fermi level, where the particle/hole asymmetry is small (i.e. with no additional self-energy), equation (15) provides a good approximation for the EDCs.

The excited-pair term of the spectral function results from thermal excitations of pairons out of the condensate:

\[ A_{\text{ex}}(\vec{k}, E) = -\frac{A(T, \theta)}{\pi} \Im \int \frac{d\Delta_p}{E - E_i + i\Gamma} \times f_B(\Delta^i - \Delta_p(T, T) \tanh \left( \frac{\Delta^i}{kT} \right) ) \]

Finally, the dissociated pair term is caused by the thermal dissociation of Cooper pairs into normal fermions of energy \( \epsilon_F \). It has the simple expression:

\[ A_{\text{diss}}(\vec{k}, E) = -\frac{A(T, \theta)}{\pi} \Im \int \frac{n_{\text{diss}}(\epsilon_F, \theta)}{E - \epsilon_F + i\Gamma} \]

The total spectral function at the Fermi energy \( A(\vec{k}_F, E) \), comparable to the measured EDC, is shown in figure 4 as a function of angle for four different temperatures, ranging from low \( T \) up to a temperature close to \( T^* \). The corresponding Fermi surface is indicated by the inset in each panel. Two values of the broadening parameter were considered. First a small value, \( \Gamma = 2 \text{ meV} \) to highlight the intrinsic spectral weight (solid lines). Secondly, choosing \( \Gamma = k_B T \) accounts for the thermal broadening (figure 4, dashed lines) present in experiments.

At low temperature, \( T = 45 \text{ K} \) (which is well below \( T^* \approx 200 \text{ K} \)), two well-defined Bogoliubov peaks are clearly visible in the spectra (figure 4(a)). The latter is maximum in the AN direction, at the energy \( \pm \Delta_p \), and decreases as a function of angle to vanish in the N direction in agreement with \( d \)-wave symmetry: \( \Delta_p \cos(2\theta) \). However, in the vicinity of the node, the spectra exhibit a peak at \( E = 0 \) which originates from dissociated pairs. Thus, at low temperature the spectral function \( A(\vec{k}_F, E = 0) \) is zero along the Fermi surface except around the N direction where a tiny arc is revealed (inset of figure 4(a)). This arc of normal states reduces to a ‘Fermi point’ at zero temperature, in agreement with experiments [5].

At the critical temperature (figure 4(b)), the gap closes well before the node at the critical angle \( \theta_c \) above which the peaks remain at the Fermi level. This effect constitutes the critical Fermi arc which, as will be explained below, results from pair breaking concomitant with thermally induced quasiparticle excitations. Even above \( T_c \), Bogoliubov coherence peaks are still present in the AN direction, due to excited pairs in the pseudogap state. As the temperature continues to increase, the Fermi arc progressively expands and finally the full Fermi surface is almost recovered at \( T = 190 \text{ K} \), which is close to \( T^* \approx 200 \text{ K} \) (d).

**Comparison to ARPES experiments**

Hashimoto et al [5] have done extensive ARPES studies of slightly underdoped Bi2Sr2CaCu2O8+δ (\( T_c = 92 \text{ K} \)). In figures 5 and 6, we compare the temperature dependence of the calculated EDC spectra at the anti-node (\( \theta = 0 \)) and near the node at the critical angle (\( \theta_c = 31^\circ \)) to the experimental spectra reported in [5]. The overall similarity between the two sets of spectra is striking. As discussed previously, in figure 5 one observes clear Bogoliubov peaks at \( \sim 40 \text{ meV} \) that remain virtually constant in energy up to \( T_c \) and then slowly close and vanish at the higher temperature \( T^* \). On the contrary, at the critical angle nearer the node (figure 6) the closing of the gap, initially of smaller value \( \sim 19 \text{ meV} \), is at \( T_c \). This Fermi arc formation is directly seen in figure 4(b). Further ARPES data confirms that the closing temperature of the Bogoliubov peaks is indeed a monotonic function of angle at the Fermi surface.

The temperature-dependent Fermi arc is thus governed by a single mechanism. Consider the natural hypothesis that each Cooper pair, as in the BCS theory [2], is subject to quasiparticle excitations as a function of temperature. Consequently, the gap amplitude \( \Delta(\theta) \) for a given pair must follow the BCS-type temperature dependence and close at a temperature such that \( \Delta(\theta)/k_B T \sim C \), where \( C \) is a constant of order 2.
A simple extension of the pairon gap equation (5) to finite temperature gives this result. From the available ARPES and tunneling data we find a good agreement using $C = 2.2$. We note that this value is slightly larger than the BCS value, $C_{\text{BCS}} \approx 1.76$, indicating strong coupling.

The overall interpretation of the angular and temperature evolution of the ARPES EDCs is illustrated in figure 7. At zero temperature, the condensate gap $\Delta_F$ has the standard $d$-wave symmetry, with the angular dependence $\cos(2\theta)$, closing in the N direction, the Fermi point observed in ARPES measurements.

As the temperature rises, some pairons are excited out of the condensate. In the AN direction, their concentration $n_{\text{ex}}(T, \theta = 0)$ is determined by Bose statistics, however their binding energy decreases with $\theta$ due to the $d$-wave symmetry and, for a given angle, $\Delta_F(T, \theta)$ decreases according to the BCS function. Therefore all the excited pairons with energy less than $2.2 \, k_B T$ will dissociate, leading to the Fermi arc of normal

Figure 5. Left panel: measured ARPES EDC spectra (symmetrized) at the Fermi surface as a function of temperature in the antinodal direction (data from Hashimoto et al [5]). Right panel: calculated spectral function at the Fermi surface $\mathcal{A}(\vec{k}_F, E)$ as a function of temperature in the antinodal direction. The broadening parameter is $\Gamma = k_B T$. Note that the incoherent background of the experimental spectrum is not accounted for. [5] (2014) (© 2018 Springer Nature Limited. All rights reserved). With permission of Springer.

Figure 6. Left panel: measured ARPES EDC spectra (symmetrized) at the Fermi surface as a function of temperature near the node ($\theta = 31^\circ$) (data from Hashimoto et al [5]). Right panel: calculated spectral function at the Fermi surface $\mathcal{A}(\vec{k}_F, E)$ as a function of temperature near the node. The broadening parameter is $\Gamma = k_B T$. [5] (2014) (© 2018 Springer Nature Limited. All rights reserved). With permission of Springer.
states in the region near the node (between 45° and the intercept between the $\Delta_p(T, \theta)$ curve with the abscissa in figure 7).

We stress that this nodal pair dissociation is due to quasi-particle excitations driven by the Fermi–Dirac statistics, as given explicitly in equations (12) and (13). At the critical temperature, the Fermi arc has an angular width $\theta_c$ to $\frac{\pi}{2} - \theta_c$ where $\theta_c$ satisfies the relation:

$$\Delta_p(0) \times \cos(2\theta_c) = 2.2 k_B T_c.$$  \hspace{1cm} (18)

While weakly bound pairs first dissociate near the node, pairs persist in the antinode and are progressively excited out of the condensate up to the critical temperature where $\Delta_p(T, \theta = 0)$ has hardly decreased and where $n_{pc}(T_c) = 0$ (see figure 7, red curve). Note that the pairs in the condensate do not directly contribute to the Fermi arcs, the consequence of our assumption that their concentration $n_{pc}(T)$ is independent of angle.

Above $T_c$, as the arc continues to extend away from the node, the gap progressively closes in the antinode as more and more pairs are being dissociated into quasiparticles. Finally, all pairs are dissociated near $T^*$, where the full Fermi surface is recovered. As discussed previously, this temperature is given empirically by the relation:

$$\Delta_p(0) = 2.2 k_B T^*,$$  \hspace{1cm} (19)

regardless of the doping. Therefore, using (18), we have the important relation

$$\cos(2\theta_c) = \frac{T_c}{T^*}.$$  \hspace{1cm} (20)

This equation should be valid for all doping values in the phase diagram. Thus the critical angle should satisfy:

cos(2$\theta_c$) = $f(p)$, where $f(p)$ is a unique function of the carrier concentration.

In the underdoped limit ($T^* \gg T_c$) the critical angle is close to $\pi/4$ indicating very small nodal Fermi arcs at $T_c$. Then the antinodal boson character dominates the phase transition. To the contrary, on the overdoped side ($T^* \sim T_c$) the Fermi arcs grow very rapidly with temperature indicating the coexistence of fermion and boson excitations, in good agreement with experiments [15, 17]. Thus, the cuprate phenomenology is in fact continuous as a function of doping while the simple relation (20) implies that the superconducting order, the pseudogap state and the Fermi arcs are intimately linked.

**Conclusion**

We have shown that ARPES measurements in cuprates can be understood in the context of a Bose–Einstein condensation of preformed pairs. The nature of these composite bosons, pairs of holes in their antiferromagnetic environment, or pairs, naturally imposes a d-wave symmetry of the order parameter and gives the correct energy scale of the phase transition, the exchange energy $J$.

Clear Bogoliubov peaks in ARPES and corresponding pseudogap in tunneling reveal the preformed pairs which, due to their strong binding energy, persist in the system at all temperatures below $T^*$. Below $T_c$, boson excitations dominate in the antinodal direction (strong pairing) while fermion excitations dominate near the node (weak pairing), giving rise to the Fermi arcs. Above $T_c$, the preformed pairs dissociate continuously, while the Fermi arcs grow at a rate which is doping dependent. Finally, the complete Fermi surface is recovered at $T^*$.

In conclusion, the superconducting state, the pseudogap state and the Fermi arcs are tied together as a consequence of a unique phenomenon, the pairs and their excitations.

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