Realization of Stable Trajectory in Mass Measurement System Using Relay Feedback of Velocity and Restoring Force Compensation

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Abstract: A mass measurement system using relay feedback of velocity was developed. In this system, the velocity of the object was fed back through a relay with hysteresis. Although the efficacy of the proposed method has been already confirmed experimentally, it was subject to a drift mainly because only the velocity was fed back. To prevent such a drift, a spring element was added to produce restoring force. However, smaller drifts still occurred, and the trajectory of the object fluctuated. To eliminate such fluctuations, intermediate control is introduced. It succeeds to eliminate fluctuation of the trajectory in the developed apparatus.

Key Words: mass measurement, limit cycle, relay feedback, pulse width modulation, stiffness.

1. Introduction

Various weightless or micro-gravity experiments have been carried out in the International Space Station (ISS) [1]. Various unique phenomena that cannot be observed on earth, have been disclosed in the experiments. The production of new materials and medicines in space is expected to start in the future. Mass measurement under weightless conditions will be necessary to perform such production.

Scales and balances have been used for mass measurement since the ancient past [2]. However, they are useless in space because they rely on the gravitational acceleration on earth. Therefore, unconventional methods are necessary to measure mass under weightless conditions. Various methods have already been proposed [3],[4].

(1) Measure the natural frequency of a spring-mass system [5],[6].
(2) Apply the frequency-controlled method [7].
(3) Apply the concept of the dynamic measurement method [8].
(4) Measure the centrifugal force in rotating an object [9], [10].
(5) Use a dynamic vibration absorber as a measurement device [11]–[13].
(6) Apply the law of conservation of momentum [14]–[16].
(7) Measure the frequency or period of a limit cycle induced in a relay feedback system [17]–[19].
(8) Apply the law of conservation of energy [20].
(9) Generate harmonic acceleration [21].

This paper targets the seventh method mainly because of the simplicity in the mechanism and the expected insensitivity to disturbance. Measurement systems using this method are classified into two types. One contains an on-off relay with hysteresis and feeds back the velocity of the measurement object [17]. The other contains an on-off relay with a dead zone and feeds back the displacement of the measurement object [18]. The advantage of the former over the latter is the simplicity of period measurement. While the on-state and off-state periods must be individually measured in the latter, a simple period or frequency measurement is sufficient in the former. It is favorable in constructing a measurement system. This paper, therefore, focuses on the former.

In this system, however, the object easily drifts during measurement because only the velocity is fed back. It may cause some difficulty in achieving measurement stably. To avoid such a drift, a spring-like element was added in the developed apparatus [17]. However, it is found experimentally that the object still drifts even with such restoring force compensation.

In this work, a simple intermediate control is proposed to prevent such a drift. The efficacy of the proposed control will be demonstrated experimentally.

2. Principles of Measurement

Figure 1 shows a physical model and a block diagram of the proposed mass measurement system. It is made up of four elements:

- actuator for moving an object to be measured,
- sensor for detecting the velocity of the object,
- controller for producing switching signals,
- amplifier for driving the actuator.

The operation of the measurement system is shown in Fig. 2. The force \( F(t) \) produced by the actuator is switched in relation to the velocity \( v \) of the object by a relay with hysteresis. When the force applied to the mass is positive, the velocity of the object increases. When the velocity reaches a preset threshold value \( v_0 \), the applied force is switched from \( F_0 \) to \( -F_0 \). Then the velocity decreases. When the velocity reaches the lower threshold value \( -v_0 \), the applied force is switched from \( -F_0 \) to \( F_0 \). When the actuator is controlled in these ways, a limit cycle appears as shown in Fig. 2.

When the applied force is \( F_0 \), the equation of motion is given
by
\[ \frac{dv(t)}{dt} = F_0. \] (1)

Solving Eq. (1) with \( v(0) = -v_0 \) leads to
\[ 2v_0 = \frac{F_0}{m} \cdot T_1, \] (2)

where \( T_1 \) is the period in which \( F(t) = F_0 \). From Eq. (2), we get
\[ T_1 = \frac{2v_0}{F_0} m. \] (3)

The period in which \( F(t) = -F_0 \) is obtained in a similar way as
\[ T_2 = \frac{2v_0}{F_0} m. \] (4)

Apparently,
\[ T_1 = T_2. \] (5)

The period of the limit cycle \( T \) is given by
\[ T = T_1 + T_2 = \frac{4mv_0}{F_0}. \] (6)

From Eq. (6), we get
\[ m = \frac{F_0}{4v_0} T. \] (7)

Therefore, the mass of the measurement object can be determined by measuring \( T \).

3. Estimation Considering Stiffness

In the original configuration shown by Fig. 1, only the velocity is fed back so that the drift of the moving part is unavoidable in actual measurement as shown in Fig. 3 where the velocity signal to be fed back included an offset [19]. Therefore, a spring was added in parallel with the actuator for avoiding the drift as shown by Fig. 4 [17]. The estimation equation including this effect is presented according to [22].

It is assumed that the applied force is switched when
\[ v(t) = \pm v_0 \quad \text{and} \quad x(t) = 0. \] (8)

The latter condition of switching at zero was not assumed in [22]. However, it is critical to derive a correct estimation equation. Figure 5 shows the assumed trajectory in the phase plane. When the applied force is \( F_0 \), the equation of motion becomes
\[ m\ddot{x} = F_0 - kx, \] (9)

where \( k \) is the stiffness of the spring. Solving Eq. (9) with \( x(0) = 0 \) and \( \dot{x}(0) = -v_0 \) leads to
\[ x(t) = \frac{\alpha}{\omega^2} - \frac{\alpha}{\omega^2} \cos \omega t - \frac{v_0}{\omega} \sin \omega t, \] (10)

\[ v(t) = \frac{\alpha}{\omega} \sin \omega t - v_0 \cos \omega t, \] (11)

where
\[ \omega = \sqrt{\frac{k}{m}}, \] (12)

\[ \alpha = \frac{F_0}{m}. \] (13)
Considering $v(T_1) = v_0$, we can derive the following equation from Eq. (11):

$$v_0 = \frac{\alpha}{\omega} \tan \frac{\omega T_1}{2}.$$  

(14)

This equation can be derived also from Eq. (10) with $x(T_1) = 0$. When the applied force is $-F_0$, the equation of motion becomes

$$m \ddot{x} = -F_0 - kx.$$  

(15)

Solving Eq. (15) with $x(0) = 0$ and $\dot{x}(0) = v_0$ leads to

$$x(t) = -\frac{\alpha}{\omega} \sin \omega t + v_0 \frac{\omega}{2} \cos \omega t,$$  

(16)

$$v(t) = -\frac{\alpha}{\omega} \sin \omega t + v_0 \cos \omega t.$$  

(17)

Considering $v(T_2) = -v_0$, we can derive the following equation from Eq. (17):

$$v_0 = \frac{\alpha}{\omega} \tan \frac{\omega T_2}{2}.$$  

(18)

It is obvious from Eqs. (14) and (18) that

$$T_1 = T_2.$$  

(19)

The period of the limit cycle $T(= T_1 + T_2)$ satisfies

$$\tan \left( \frac{T}{4} \sqrt{\frac{k}{m}} \right) = \frac{v_0}{F_0} \sqrt{km}.$$  

(20)

To express $m$ in an analytical form explicitly, we approximate the tangent as

$$\tan \left( \frac{T}{4} \sqrt{\frac{k}{m}} \right) \approx \frac{T}{4} \sqrt{\frac{k}{m}} + \frac{1}{3} \left( \frac{T}{4} \sqrt{\frac{k}{m}} \right)^3.$$  

(21)

From Eqs. (20) and (21), we get

$$m^2 - m_0 m - \frac{1}{48} kT^2 m_0 = 0,$$  

(22)

where $m_0$ is the mass estimated according to Eq. (7), which is considered to be obtained by neglecting the stiffness. The positive solution of Eq. (22) is given by

$$m = \frac{m_0}{2} \left( 1 + \sqrt{1 + \frac{1}{12} \left( \frac{k}{m_0} \right) T^2} \right) = m_0 + \frac{1}{48} kT^2.$$  

(23)

Equation (23) indicates that the estimated mass would be smaller than the actual value by $kT^2/48$ if the original estimating Eq. (7) were used.

4. Experiment

4.1 Measurement Device

Figure 6 shows a picture and a schematic drawing of an apparatus developed for experimental study [19]. It has two voice coil motors (VCM’s). One of them (VCM 1) is for acting force on the object, and the other (VCM 2) is for adding some disturbance on the object originally; here it is operated to add stiffness by feeding back the displacement. Each VCM has a mover with a coil wound about a shaft. The shaft is guided to move in a straight line by three linear air bearing to minimize friction during the reciprocal motion. The movers are connected with a mechanical coupling.

The velocity of the mover is detected with a laser vibrometer to minimize the delay of detection [19]. The displacement of the mover is detected with a laser displacement meter. The control algorithms are implemented with a digital controller. The signals produced by the vibrometer and the displacement sensor are inputted to the controller through A/D converters. The controller generates a binary command signal ($\pm I_0$) so as to simulate an on-off relay with hysteresis whose width is given by $\pm v_0$. This signal is sent to a power amplifier driving VCM 1. The force is switched to be $\pm F_0$ alternatively. Another control signal is also generated to operate VCM 2 as a spring by feeding back the displacement of the mover (virtual spring). The equivalent stiffness is designated by $k$. VCM 2 is driven by another power amplifier according to this signal.
4.2 Experimental Result

In the following experiment, $F_0$, $v_0$, and $k$ were set as $F_0 = 0.705$ N, $v_0 = 0.025$ m/s, $k = 557.3$ N/m, mainly because the linearity of measurement was best with these parameters in preliminary experiments. The trajectories of the mover in the phase plane are shown in Fig. 7. It is found that the trajectories vary gradually due to drift even though restoring force is generated by the virtual spring. Such behavior may deteriorate the measurement accuracy because it causes one problem related to the physical limitation. The stroke of the linear bearing is limited (approximately 0.01 m). To measure the period accurately, averaging is necessary, which requires rather long measurement time; such measurement may be interrupted due to the drift. In addition, the suspension characteristics slightly vary according to the position; the resisting force near the ends is larger than that at the middle; it may make the motion of the mover slower and cause measurement error. Therefore, it is better to keep the position of the mover around the middle of the linear bearing.

In the next subsection, the reason for such a drift will be investigated. Then, intermediate control will be introduced in the next section.

4.3 Study on Force Balance

The position at which the forces acting on the mover are balanced (balance point) is unique without the force switching control. This position corresponds to the natural-length state of the spring if a mechanical spring is used. In the case of virtual spring, it corresponds to the zero displacement ($x(t) = 0$). However, when the force of the actuator is switched according to the control law as shown by Fig. 2, any position between $-F_0/k$ and $F_0/k$ can be a balance point in average. This follows the principle of the pulse width modulation (PWM). The balance point $x_a$ satisfies

$$\frac{F_0 T_2 - T_1}{T} = k x_a,$$

where the left-hand side represents the average force acting on the mover.

Equation (24) indicates that any point can be a balance point in average. To support this inference, an open-loop experiment is conducted in which the force of the actuator is switched from $F_0/k$ to $-F_0/k$ or vice versa periodically as a function of time, that is, regardless of velocity. The ratio of the period $T_1$ when $F(t) = F_0$ and the period $T_2$ when $F(t) = -F_0$ are varied. Figure 8 shows the phase trajectories for various ratios of $T_1$ to $T$. The experimental conditions are

$$F_0 = 0.290 \text{ N}, k = 557.3 \text{ N/m}, T = 0.050 \text{ s}.$$ 

It indicates that the balance point in average depends on the ratio of $T_1$ to $T$. When $T_1/T$ is less than 0.5, the average force given by the left-hand side of Eq. (24) becomes negative so that the balance point $x_a$ is also negative. In contrast, when $T_1/T$ is more than 0.5, the balance point becomes positive.

This result also indicates that $T_1$ and $T_2$ are different when the switching positions are not at zero during mass measurement. In such a case, the estimation formula given by (20) should be modified.

Then, a closed-loop experiment in which the force is switched as a function of velocity, is conducted where the experimental conditions are

$$F_0 = 0.705 \text{ N}, v_0 = 0.025 \text{ m/s}, k = 557.3 \text{ N/m}.$$ 

Figure 9 shows the time series of the displacement and the balance point calculated according to

$$x_a = \frac{F_0}{k} \cdot \frac{T_1 - T_2}{T_1 + T_2},$$

(25)
Fig. 9 Time series of the displacement of the mover and the balance point calculated based on $T_1$ and $T_2$.

where $T_1$ and $T_2$ are measured values. It demonstrates that the drift of the balance point corresponds to that of the displacement.

5. Stabilization of Trajectory

5.1 Introduction of Intermediate Control

The cause of the drift is the fluctuation of the switching position $x$ at which the force $F(t)$ is switched from $F_0$ to $-F_0$ or vice versa. In the theoretical analysis, it is assumed to be zero as given by Eq. (8). The cause of such fluctuation is still unclear. Damping and/or friction in the linear bearing are possible causes. However, the complete elimination of such disturbances is impossible or at least very difficult technically. Therefore, a simple control law will be introduced to remove the drift.

Figure 10 shows the modified trajectory. When the velocity reaches the threshold value, for example, but the displacement $x(t) = -x$ is negative, $F(t)$ is switched from $F_0$ to zero instead of $-F_0$; it is kept until the displacement becomes zero. When the displacement reaches zero, $F(t)$ is switched from zero to $-F_0$. Meanwhile, the velocity reaches $v_1$ that is slightly larger than $v_0$ because the mass is accelerated by the restoring force of the spring. After $F(t)$ is switched to $-F_0$, the mass is decelerated by this force in addition to the restoring force. Thereby, the displacement $x_2$ at which the velocity returns to $v_0$ is expected to be less than $x_1$. According to the law of the conservation of energy, the following equation is derived.

\[
\frac{1}{2}kx_1^2 + \frac{1}{2}mv_0^2 = \frac{1}{2}kx_2^2 + \frac{1}{2}mv_0^2 + F_0x_2.
\]

\[
\therefore x_2^2 + 2dx_2 - x_1^2 = 0,
\]

where

\[
d = \frac{F_0}{k}.
\]

Assuming $x_2 > 0$ and $x_1 << d$, $x_2$ is obtained as

\[
x_2 = d + \sqrt{d^2 + x_1^2} \approx \frac{1}{2} \frac{x_1^2}{d}.
\]

\[
\therefore \frac{x_2}{x_1} \approx \frac{x_1}{2d}.
\]

Equation (28) indicates that the displacement at which the intermediate control starts converges to zero rapidly when $x_1 << d$. This condition can be easily achieved by starting the operation when the mass is in the neighborhood of the zero position ($x(t) = 0$).

It is to be mentioned that both displacement and velocity must be detected to realize the intermediate control. It would make the measurement system complicated and costly. One method of avoiding the use of two sensors is to install a displacement sensor solely and estimate the velocity by differentiating the output of the displacement sensor. This method enables the implementation of the intermediate control without complication and higher cost concerning hardware. However, such indirect measurement can cause some delay in detecting the velocity. An estimation formula considering such a delay has been discussed in [19].

5.2 Experimental Results

Figure 11 shows the time-series of displacement, velocity, and applied force when the intermediate control is operated. It demonstrates that the force acting on the mover is kept to be zero while $v(t) \geq v_0$ and $x(t) < 0$ as the assumed trajectory shown by Fig. 10. Figure 12 shows the trajectories during 100 periods under the modified control. Virtually no fluctuation is observed.

Figure 13 and Table 1 show the measurement results when
mass is estimated without and with the intermediate control. In the latter, the period during which $F(t) = 0$ is not included in calculating the period of the limit cycle. The relation between the estimated mass $m_e$ and actual mass $m_a$ is fitted to a straight line by the least-squares method as

$$m_e = 1.023m_a - 0.0062 \text{ kg},$$  \hfill (29)

in the former and

$$m_e = 1.022m_a - 0.0057 \text{ kg},$$  \hfill (30)
in the latter. It is found that the measurement accuracy is a little improved by introducing the intermediate control.

However, the improvement by the intermediate control is not remarkable. To improve the measurement accuracy, other factors should be considered. For example, the accuracy of the period measurement is also important. We think that one method of simplifying the problem is to make the actual behavior of the measurement system follow the theoretically predicted one. Therefore, the intermediate control is introduced to make the actual trajectory approach the assumed one shown by Fig. 5 even though it can make the system complicated and costly.

Meanwhile, the mass measurement range is 0.38 kg to 0.5 kg in this study. The upper limit of the measured mass is mainly determined by the load capacity of the air bearings in this apparatus. In space, however, it is unnecessary to support the gravitational force of the measured mass so that such limitation may be determined by another factor. In addition, the design of the support mechanism will be more flexible, which may allow us to use a simpler and lighter mechanism other than air bearing. For example, only leaf springs may be sufficient to guide the measured mass. It may enable smaller mass measurement by the proposed method.

### 6. Conclusion

A mass measurement system that uses a relay with hysteresis and feeds back the velocity of the object was subject to a drift even though a spring-like element added to produce restoring force. An intermediate control was introduced to make the trajectory stable. The efficacy of the proposed trajectory control
method was demonstrated experimentally.

The actual trajectory is almost the same as the assumed one in deriving the estimating equation. Therefore, measurement is expected to follow the theoretical predictions. However, the improvement of measurement accuracy was not remarkable. Further improvement is under way.

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