Flavour structure, flavour symmetry and supersymmetry

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We discuss the role played by the horizontal flavour symmetry in supersymmetric theories. In particular, we consider the horizontal symmetry $SU(3)_H$ between the three fermion families and show how this concept can help in explaining the fermion mass spectrum and mixing pattern in the context of SUSY GUTs.

1. Introduction

One of the most obscure sides of particle physics concerns the fermion flavor structure. This is a complex problem with different aspects questioning the origin of the mass spectrum and mixing pattern of quarks and leptons (including neutrinos) and CP-violation, as well as the suppression of flavor changing (FC) neutral currents, the strong CP-problem, etc. Presently, thanks to the new data from the atmospheric and solar neutrino experiments, the flavor problem is getting more intriguing. On the one hand, the experimental data hint to a hierarchical neutrino mass spectrum, similarly to the case of the charged leptons and quarks. On the other hand, the lepton mixing pattern strongly differs from that of the quarks. In particular, the 2-3 lepton mixing angle is nearly maximal, $\theta_{23}^\ell \simeq 45^\circ$ in contrast with the analogous quark mixing angle, $\theta_{23}^q \simeq 2^\circ$.

The concept of supersymmetry per se does not help in understanding the fermion flavor structure, and in addition it creates another problem, the so called supersymmetric flavor problem, related to the sfermion mass and mixing pattern.

In the MSSM the fermion sector consists of chiral superfields containing the quark and lepton species of three families: $q_i = (u, d)_i$, $\bar{u}_i$, $\bar{d}_i$, $l_i = (\nu, e)_i$ and $\bar{e}_i$ ($i = 1, 2, 3$). The charged fermion masses emerge from the Yukawa terms in the superpotential:

$$W_{Yuk} = Y_u^{ij} \bar{u}_i q_j H_2 + Y_d^{ij} \bar{d}_i q_j H_1 + Y_e^{ij} \bar{e}_i l_j H_1 \quad (1)$$

where $H_{1,2}$ are the Higgs doublets, with the vacuum expectation values (VEVs) $v_{1,2}$ breaking the electroweak symmetry, $(v_1^2 + v_2^2)^{1/2} = v = 174$ GeV. The $3 \times 3$ Yukawa matrices $Y_{u,d,e}$ are not constrained by any symmetry property and thus remain arbitrary\footnote{The phenomenologically dangerous R-violating terms can be suppressed by R-parity. In other terms, one can impose the matter parity $Z_2$ under which the matter superfields change the sign while the Higgs ones are invariant.}

The second aspect of the flavour problem, specific of SUSY, questions the sfermion mass and mixing pattern which is determined by the soft SUSY breaking (SSB) terms. These include trilinear $A$-terms:

$$\mathcal{L}_A = A_u^{ij} \bar{u}_i q_j H_2 + A_d^{ij} \bar{d}_i q_j H_1 + A_e^{ij} \bar{e}_i l_j H_1 \quad (2)$$

(the tilde labels sfermions) and soft mass terms:

$$\mathcal{L}_m = \sum_f \tilde{f}_i^\dagger (m_f^2)^{ij} \tilde{f}_j, \quad (f = q, \bar{u}, \bar{d}, l, \bar{e}) \quad (3)$$

where $A_{u,d,e}$ and $m_f^2$ are $3 \times 3$ matrices with dimensional parameters. Theoretical arguments...
based on the Higgs mass stability imply that the typical mass scale $\tilde{m}$ of these terms should be of order 100 GeV, maybe up to TeV. The SSB terms have no a priori relation with the Yukawa constants $Y_{u,d,e}$. Hence, one expects that the splitting between the fermion mass eigenstates be large, of order $\tilde{m}$, and in addition the fermion mixing angles controlling the coupling with fermions and neutral gauginos be also large. This situation gives rise to dramatic contributions to FC and CP-violating processes. For example, the decay rate $\mu \rightarrow e + \gamma$ or the CP-violating parameters $\varepsilon_K$ and $\varepsilon'_K$ in $K^0 - \bar{K}^0$ system would much exceed the experimental bounds unless $\tilde{m}$ is larger than 10-100 TeV. In this case, however, the advantage of supersymmetry in stabilizing the Higgs mass would be lost. Thus, experimental limits on FC processes impose severe constraints on the mass and mixing pattern of the yet undiscovered squarks and sleptons.

As far as neutrinos are concerned, there is no renormalizable term that can generate their masses. However, the Majorana masses of neutrinos can emerge from the lepton-number violating higher order operator cutoff by some large scale $M$, e.g. the grand unification or Planck scale $\tilde{M}$.

$$\frac{1}{M} Y_{u,d,e} \bar{l}_i l_j H^2_+ \quad Y_{\nu} = Y_{\nu}^T$$

Any known mechanism for the neutrino masses reduces to this effective operator. E.g., in the ‘seesaw’ scheme it is obtained after integrating out heavy-neutral fermions with Majorana masses $\sim M$. Hence, modulo Yukawa coupling constants, the charged fermion masses are $\sim v$ while the neutrino masses are $\sim v^2/M$ which makes it clear why the latter are so small. However, the matrix $Y_{\nu}$ remains arbitrary.

The concept of the grand unification provides more constraints on the Yukawa matrices and in this way opens up some possibility for predictive schemes of quark and lepton masses. In the $SU(5)$ model all fermion states are unified within 10-plets $\bar{t}_i = (\bar{u}, q, \bar{e})_i$, and 5-plets $\bar{f}_i = (\bar{d}, l)_i$. The minimal structure of the Yukawa terms is the following

$$G^{ij} \bar{f}_i t_j H + G^{ij}_{\alpha} \bar{t}_i t_j H + \frac{1}{M} G_{\nu}^{ij} \bar{f}_i \bar{f}_j H^2$$

where $G_{\alpha}$ and $G_{\nu}$ should be symmetric while the form of $G$ is not restricted. After the $SU(5)$ symmetry breaking, these terms reduce to the standard couplings in $[\Phi]$ with $Y_d = Y_d^T = G$. This implies that the Yukawa eigenvalues are degenerate between the down quarks and charged leptons of the same generations. Although this prediction for the largest eigenvalues, the $b - \tau$ Yukawa unification, is a remarkable success of the $SU(5)$ theory, it is completely wrong for the light generations.

The spontaneous breaking of $SU(5)$ to the Standard Model by the adjoint superfield $\Phi$ (24-plet) can be used to remove that unrealistic degeneracy between down-quark and charged leptons. The Yukawa coupling matrices can be thought as operators depending on $\Phi$, i.e. $G = G(\Phi)$, $G_{\alpha} = G_{\alpha}(\Phi)$ etc. and hence understood as expansion series, e.g.

$$G^{ij}(\Phi) \bar{f}_i t_j H = G^{ij}_0 \bar{f}_i t_j H + G^{ij}_1 \frac{\Phi}{M} \bar{f}_i t_j H + \ldots$$

where $M$ is some cutoff scale. The tensor product $24 \times 5$ contains both $\mathbf{5}$ and $\mathbf{45}$ channels and thus can provide different Clebsch factors for the Yukawa entries between the quark and lepton states of light generations. Clearly, such higher order operators can be obtained by integrating out some heavy fermion states with masses of order $M$ just like in the (neutrino) seesaw mechanism.

In this way, the concept of GUT provides a more appealing framework for understanding the fermion mass and mixing structures. However, at the same time it makes more difficult the supersymmetric flavor problem. Namely, in the MSSM context natural suppression of the flavor-changing phenomena can be achieved by the SSB terms universality at the Planck scale, which can be motivated in the context of supergravity scenarios. However, in the SUSY GUT frames this idea becomes insufficient – the physics above the GUT scale does not decouple and can strongly violate the SSB terms universality at lower scales. In generic SUSY GUTS, the decoupling of heavy states would lead to big non-universal terms that can cause dangerous flavor-changing contributions and thus pose a serious challenge to the
SUSY GUT concept.

An attractive approach to both flavor problems – fermion and sfermion – is to invoke the idea of horizontal inter-family symmetry. Several models based on $U(1)$ or $U(2)$ family symmetries have been considered in literature. However, the chiral $U(3)_H$ or its non-abelian part $SU(3)_H$ unifying all fermion generations in horizontal triplets seems to be the most natural candidate for describing the family triplication. In this paper we demonstrate its power to provide a coherent picture for the fermion and sfermion masses, to explain the origin of the fermion mass spectrum and mixing structure, and to naturally solve the supersymmetric flavor problem.

In general, to construct realistic scenarios, we must require that in the horizontal symmetry limit the fermions remain massless, so that they can acquire mass only after the horizontal symmetry breaking. In this way, the fermion mass and mixing pattern could reflect the VEV pattern of the Higgs scalars leading to the spontaneous breaking of the horizontal symmetry. In other words, the horizontal symmetry should have chiral character. Clearly, the horizontal symmetry $SU(3)_H$ can be the appropriate chiral symmetry, unlike its $SU(2)_H$ sub-group.

Observe that, in the limit of massless fermions the standard model has a large global chiral symmetry, $U(3)^5 = U(3)_q \times U(3)_{\bar{q}} \times U(3)_{d} \times U(3)_{\bar{d}} \times U(3)_{e}$, separately transforming the quark and lepton superfields of the three families $q_i = (u, d)_i$, $\bar{u}_i$, $\bar{d}_i$, $l_i = (\nu, e)_i$ and $\bar{e}_i$ ($i = 1, 2, 3$). The Yukawa terms explicitly break this symmetry. We can suppose, however, that the Yukawa couplings emerge as a result of the spontaneous breaking of this symmetry. Such a situation will be considered in sect. 2. We will show that in this case we can obtain a natural fermion mass hierarchy and mixing pattern.

However, the maximal flavor symmetry $U(3)^5$ cannot be implemented in the SUSY GUT context. Namely, in the $SU(5)$ model the fermions of each generation are unified into two multiplets $t \sim 10$ and $f \sim 5$, so that the maximal global chiral symmetry reduces to $U(3)^2 = U(3)_{\nu} \times U(3)_{\bar{\nu}}$. In the $SO(10)$ model all fermions of each family fit into one multiplet $\psi \sim 16$, therefore the flavor symmetry reduces to $U(3)_H$. The latter is still chiral, since now all left handed fermions transform as 3 and the right handed ones as $\bar{3}$. Therefore, as far as the GUT framework is concerned, it would be most natural to consider the horizontal symmetry $U(3)_H$ or even only its non-abelian factor $SU(3)_H$. In addition it could be the local gauge symmetry emerging from some more fundamental theory on the same grounds as the GUT symmetry itself.

We consider $SU(3)_H$ as horizontal symmetry group in sect. 3. There, the key points of our discussion can be summarised in the following way:

- The spontaneous breaking features of $SU(3)_H$ turn the Yukawa constants of the low energy theory (MSSM) into dynamical degrees of freedom and fix the inter-family hierarchy in a natural way. Namely, the third generation becomes heavy ($Y_i \sim 1$), while the second and first ones become lighter by successively increasing powers of small parameters. We also show how to achieve the desired structure of the horizontal symmetry breaking VEVs.

- The adjoint Higgs (24-plet) which provides the gauge symmetry breaking $SU(5) \rightarrow SU(3) \times SU(2) \times U(1)$, should be used in the Yukawa operators to remove the unrealistic degeneracy between the down-quark and charged leptons.

- Most probably, the horizontal symmetry should exhibit the analogous breaking pattern $SU(3)_H \rightarrow SU(2)_H \times U(1)_H$ by the adjoint Higgs (octet), with $SU(2)_H$ acting between the light (first and second) fermion generations. This could naturally reproduce the observed pattern of the quark and lepton mixings.

2. Maximal family symmetry $SU(3)^5$

Let us consider the global flavor symmetry $SU(3)^3 = SU(3)_q \times SU(3)_{\bar{q}} \times SU(3)_{d}$ of the quark sector. The quark superfields transform as

$$q_i = \left( \begin{array}{c} u \\ d \end{array} \right)_i \sim (3, 1, 1), \quad \bar{u}_j \sim (1, 3, 1), \quad \bar{d}_k \sim (1, 1, 3),$$

(7)

In the context of TeV scale gravity theories with extra dimensions, the chiral flavour symmetries of this type can be helpful for suppressing FC phenomena.
\( i, j, k = 1, 2, 3 \) are family indices. The quark masses emerge from the effective operators \([13]\):

\[
\frac{X_{ij}^3}{M} \bar{u}_j q_i H_2 + \frac{X_{ki}^3}{M} \bar{d}_k q_i H_1
\]

where \( X_u \sim (3, 3, 1) \) and \( X_d \sim (3, 1, 3) \) are the horizontal Higgs superfields in the mixed representations of \( SU(3)_X \), and \( M \) is a cutoff scale (= flavor scale). In the context of the renormalizable theory, these effective operators can emerge by integrating out some extra heavy vector-like matter superfields \([4]\), e.g. the weak isosinglets \( U, \bar{U} \) and \( D, \bar{D} \) having the same color and electric charges as \( u, \bar{u} \) and \( d, \bar{d} \) transforming in the following representations of \( SU(3)_X \):

\[
U_i, D_i \sim (3, 1, 1), \quad \bar{U}^i, \bar{D}^i \sim (\bar{3}, 1, 1).
\]

The latter can get masses from the VEV of some scalar \( \Sigma \), which can be a singlet or an octet of \( SU(3)_Y \). \( \Sigma \sim (8, 1, 1) \). On the other hand, they can mix with the light states via the superfields \( X_{u,d} \) and \( H_{1,2} \). The relevant superpotential reads:

\[
\bar{u}U X_u + \Sigma U \bar{U} + \bar{U} q H_2 + \bar{d} D X_d + \Sigma D \bar{D} + \bar{D} q H_1
\]

where order one constants are understood at each coupling. After the fields \( X_{u,d} \) and \( \Sigma \) get large VEVs the Yukawa matrices get the form

\[
\left( \begin{array}{ccc}
0 & \mathbf{X}_u \\
H_2 & \mathbf{M}_{U} \\
\end{array} \right), \quad \left( \begin{array}{ccc}
0 & \mathbf{X}_d \\
H_1 & \mathbf{M}_{D} \\
\end{array} \right)
\]

where \( \mathbf{X}_{u,d} \sim \langle X_{u,d} \rangle \) and \( \mathbf{M}_{U,D} \sim \langle \Sigma \rangle \). The the effective operators \([4]\) emerge after integrating out the heavy states in the so called seesaw limit \( \mathbf{X}_{u,d} \lesssim M \). Namely, diagonalizing the matrices \([11]\), we see that the states \( \bar{u}, \bar{U} \) and \( \bar{d}, \bar{D} \) are mixed so that the actual light states which couple to \( q \) via \( H_1 \) and \( H_2 \) become:

\[
\bar{u}' \simeq \bar{u} + \mathbf{X}_u \mathbf{M}_{U}^{-1} \bar{U}, \quad \bar{d}' \simeq \bar{d} + \mathbf{X}_d \mathbf{M}_{D}^{-1} \bar{D}.
\]

Therefore, the Yukawa constants of the Standard Model are nothing but

\[
\mathbf{Y}_u = \mathbf{X}_u \mathbf{M}_{U}^{-1}, \quad \mathbf{Y}_d = \mathbf{X}_d \mathbf{M}_{D}^{-1}.
\]

The heavy fermion mass matrices are \( SU(3) \) invariant, \( \mathbf{M}_{U,D} \sim M \), if \( \Sigma \) is a singlet, or \( \mathbf{M}_{U,D} \sim \lambda_8 \mathbf{M} \), if \( \Sigma \) is an octet with a VEV towards the \( \lambda_8 \) generator of \( SU(3)_Y \). In either case they have rather democratic structure and cannot give rise to the fermion mass hierarchy. Thus, the Yukawa matrices will reflect the form of the horizontal symmetry breaking pattern by the VEVs of \( X_{u,d} \).

Let us consider now the VEV pattern for the horizontal Higgs superfields. In order to be able to write superpotential terms for \( X_{u,d} \), one has to introduce also the superfields in the conjugated representations \( \bar{X}_u \sim (3, 3, 1) \) and \( \bar{X}_d \sim (3, 1, 3) \). The latter do not couple to the fermion sector and just play the role of spectators in the fermion mass generation. The most general renormalizable superpotential does not contain, because of \( SU(3)_X \) symmetry, any mixed term between \( X_u \) and \( X_d \), and so it has a separable form \( W = W(X_u) + W(X_d) \), where:

\[
W(X_u) = \mu_u X_u \bar{X}_u + X_u^3 + \bar{X}_u^3,
\]

\[
W(X_d) = \mu_d X_d \bar{X}_d + X_d^3 + \bar{X}_d^3.
\]

Details about superpotentials of these type are given in Appendix. By means of bi-unitary transformations the VEV of \( X_u \) can be always chosen in the diagonal form, \( X_u = \mathbf{X}_u^D = \text{diag}(x_1^u, x_2^u, x_3^u) \), while \( X_d = \mathbf{X}_d^D V^\dagger \), \( X_d = (x_1^d, x_2^d, x_3^d) \), where the unitary matrix \( V \) defines the relative orientation of the two matrices \( \mathbf{X}_u \) and \( \mathbf{X}_d \) in the \( SU(3)_Y \) space and it is nothing but the CKM mixing matrix of the quarks, \( V = V_{\text{CKM}} \).

It is shown in Appendix that in the supersymmetric limit only the product of all three eigenvalues of \( X_u \) \( (X_d) \) \( x_1^u x_2^u x_3^u \) \( (x_1^d x_2^d x_3^d) \) can be fixed, but not each single eigenvalue. The vacuum degeneracy should be removed by the SSB terms. Then for a certain range of the coupling constants the largest eigenvalues of these fields \( (x_3^u, x_3^d) \) can be of order of the cutoff scale \( M \) while the others are smaller.

The above can be interpreted in the following manner. With respect to operators like \([4]\), the MSSM Yukawa constants as well as the CKM mixing angles become dynamical degrees of freedom. In particular, the first operator in \([8]\) implies

\[
\mathbf{Y}_u = \text{diag}(Y_u, Y_c, Y_t) \sim \frac{1}{M} \text{diag}(x_1^u, x_2^u, x_3^u).
\]
In the exact supersymmetric limit the values of the constants \( Y_{u,d,t} \) are not fixed – they have flat directions where only their product is fixed \( Y_u Y_d Y_t = (\mu_u/M)^3 \). However, the SSB terms could naturally fix the Yukawa constants so that \( Y_t \sim 1 \). (For related discussion, see also ref. \[14\].) The same is true for the constants \( Y_{d,s,b} \). In this way, we can naturally obtain the following hierarchy for the up and down quark Yukawa eigenvalues:

\[
\begin{align*}
Y_t : Y_e : Y_u &\sim 1 : \varepsilon_u : \varepsilon_u^2, \quad \varepsilon_u = \mu_u/M \\
Y_b : Y_e : Y_d &\sim 1 : \varepsilon_d : \varepsilon_d^2, \quad \varepsilon_d = \mu_d/M
\end{align*}
\] (16)

which for \( \varepsilon_u \sim 1/200 \) and \( \varepsilon_d \sim 1/20 \) well describes the observed spectrum of quark masses.

Now what about the CKM mixing angles ? In the SUSY limit the latter are flat directions as far as the superpotential is ‘separable’ as in (14) and so the VEV orientation of \( X_u \) and \( X_d \) remains arbitrary. However, also this degeneracy can be removed by the SSB terms. In particular, we consider the following effective D-term like operators \[13\]:

\[
\frac{1}{M^2} \int d^4z \bar{z} \left[ \lambda \text{Tr}(X_u^\dagger X_u X_d^\dagger X_d) + \lambda_1 \text{Tr}(X_u^\dagger X_u X_d^\dagger X_d) + \lambda_2 \text{Tr}(X_u^\dagger X_u X_d^\dagger X_d) \right]
\] (17)

where \( z = \bar{m}\theta^2, \bar{z} = \bar{m}\bar{\theta}^2 \) are supersymmetry breaking spurions, \( \bar{m} \) being a typical SUSY breaking mass. Clearly, such effective operators always emerge in the loop corrections after the supersymmetry breaking. If \( \Sigma \) is a singlet, then the VEVs of \( X_u \) and \( X_d \) are aligned in the \( SU(3)_q \) space and thus no CKM mixing can show up. However, if \( \Sigma \) is an octet of \( SU(3)_q \), then, for positive \( \lambda \)'s, there is a parameter range for which the VEVs of \( X_u \) and \( X_d \) are not anymore aligned and so nonzero CKM mixings are generated \[13\].

We have to remark that in this model the pattern of the CKM mixing angles is not related to the hierarchy of quark mass eigenvalues, and in general they should be large. The 1-2 mixing angle is indeed of order 1, \( s_{12} \gtrsim 0.22 \), while the 2-3 mixing is small, \( s_{23} \simeq 0.04 \), for which some fine tuning of the parameters is required \[13\].

Interestingly, then the third mixing angle is predicted as \( s_{13} \simeq (m_s/m_b)^3 (s_{12}/s_{23}) \), in a good agreement with the

We consider now the squark mass and mixing pattern in this model. By \( SU(3)_H \) symmetry reasons, the soft mass terms of the states \( q, \bar{u}, \bar{d} \) as well as those of the heavy states \( U, \bar{U}, D, \bar{D} \) are degenerate between families, while the trilinear A-terms have a structure proportional to the Yukawa couplings \[14\]. Therefore, after integrating out the heavy states, the pattern of the soft mass terms \[3\] should be the following. The states \( q = (u,d) \) do not mix with the heavy fermions, so the soft masses of the left-handed squarks maintain the \( SU(3)_H \) degeneracy (at the decoupling scale \( M \)). As for the states \( \bar{u} \) and \( \bar{d} \), they mix with the heavy states according to \[2\], so that the soft mass terms of the right-handed squarks should look like:

\[
\begin{align*}
\mathbf{m}_u^2 &= \tilde{m}_{1u}^2 + \tilde{m}_{2u}^2 Y_u Y_u^\dagger + \tilde{m}_{3u}^2 (Y_u Y_u^\dagger)^2 \\
\mathbf{m}_d^2 &= \tilde{m}_{1d}^2 + \tilde{m}_{2d}^2 Y_d Y_d^\dagger + \tilde{m}_{3d}^2 (Y_d Y_d^\dagger)^2
\end{align*}
\] (18)

where the overall factors are of order \( \tilde{m}^2 \). Hence, the latter are not degenerate, but they are fully aligned to the Yukawa matrices \( Y_u \) and \( Y_d \) respectively. In similar way, one can easily see that the trilinear terms \( A_{u,d} \) \[2\] are also fully aligned to the matrices \( Y_{u,d} \). Thus, in this theory no FC contributions emerge at the flavor scale \( M \). Clearly, the initial conditions for the SSB terms are different from the universal ones usually adopted in the MSSM. For computing the SSB terms at the electroweak scale, the above expressions are to be evolved down by the renormalization group equations. However, all FC effects will remain under control and the observable FC rates should be of the same order as in the “universal” MSSM.

Similar considerations can be applied to the lepton sector, for which the maximal chiral flavor symmetry is \( SU(3)_L = SU(3)_1 \times SU(3)_e \):

\[
l_i = \left( \begin{array}{c} \nu_i \\ e \end{array} \right) \sim (3, 1), \quad \bar{e}_a \sim (1, 3).
\] (19)

Therefore, the effective operators for the charged lepton and neutrino masses are

\[
\frac{X_{ei}^i}{M} \bar{e}_a l_i H_1 + \frac{X_{\nu i}^i}{M^2} \nu_i l_i H_2^2,
\] (20)

experimental data \[3\].
where $X_c \sim (3,3)$ and $X_\nu \sim (6,1)$ are horizontal Higgs superfields. Once more, these operators emerge from the decoupling of the heavy charged leptons $E, \bar{E}$ and neutral leptons $N, \bar{N}$ (right-handed neutrinos) in the following representations of $SU(3)_c^2$:

$$E_i, N_i \sim (3, 1), \quad \bar{E}^i, \bar{N}^i \sim (\bar{3}, 1).$$

(21)

The relevant superpotential terms read:

$$\bar{e}IX_c + \bar{E}I + \Sigma E\bar{E} + N^2 X_\nu + \bar{N}H_2 + \Sigma N\bar{N}$$

where $\Sigma$ is some scalar which can be a singlet or octet of $SU(3)_c$. Therefore, for the lepton Yukawa matrices we obtain:

$$Y_e = X_e M_e^{-1}, \quad Y_\nu/M = M_N^{-1}X_\nu M_N^{-1}$$

(23)

where $X_{e,\nu} = \langle X_{e,\nu} \rangle$, and $M_{E,N} \sim (\Sigma)$ are the mass matrices of the heavy states. Once again, the VEV of $X_\nu$ can be always chosen in the diagonal form, $X_\nu = X_\nu^D$ while $X_e = X_e^L V_l$, where the unitary matrix $V_l$ defines the relative orientation of the two matrices $X_\nu$ and $X_e$ in the $SU(3)_l$ space and it is related to the neutrino mixing matrix as $V_l = V_{MNS}$. As in the case of quarks, the hierarchy between the mass eigenstates can find a natural origin in the solution of the Higgs superpotential for $X_\nu$ and $X_e$, while the mixing pattern will be fixed by SSB terms analogous to (17). As we remarked above, the large neutrino mixing angles can be obtained in this situation in a rather generic case.

3. The $SU(5) \times SU(3)_H$ model

Let us consider now the grand unification case. In $SU(5)$ model the fermions of each generation are unified into two multiplets, 10-plets $t = (u, d, \ell)$ and 5-plets $f = (d, l)$. We now consider the horizontal symmetry $SU(3)_H$ which unifies the three fermion families as:

$$f_i \sim (\bar{5}, 3), \quad t_i \sim (10, 3),$$

(24)

($i = 1, 2, 3$ is a $SU(3)_H$ index), while the Higgs superfields are singlets of $SU(3)_H$, $H \sim (5, 1)$ and $\bar{H} \sim (\bar{5}, 1)$.

Since the fermion bilinears transform as $3 \times 3 = 3 + 6$, their “standard” Yukawa couplings to the Higgses are forbidden by the horizontal symmetry. Hence, the fermion masses can be induced only by higher order operators involving a set of “horizontal” Higgs superfields $X^{ij}$ in two-index representations of $SU(3)_H$: symmetric $X^{ij} \equiv S^{ij} \sim (1, 6)$ and antisymmetric $X^{ij} \equiv A^{ij} \sim (1, 3)$.

$$S^{ij} \frac{M_t t_i H + S^{ij} + A^{ij}}{M_\eta} \bar{f}_i t_j H + \frac{S^{ij}}{M_\eta} \bar{f}_i \bar{f}_j H^2$$

(25)

where $M$ is some large scale (flavor scale). In this way, the fermion mass hierarchy can be naturally linked to the hierarchy of the horizontal symmetry breaking scales [11,12]. Needless to say, because of the $SU(5)$ symmetry, the antisymmetric Higgses $A$ can participate only in second term.

In particular, let us assume that the horizontal Higgses include a sextet $S$ and one or more triplets $A$. Without loss of generality, the VEV of $S$ can be taken diagonal:

$$\langle S^{ij} \rangle = \begin{pmatrix} S_1 & 0 & 0 \\ 0 & S_2 & 0 \\ 0 & 0 & S_3 \end{pmatrix}, \quad S_1 \gg S_2 \gg S_3,$$

(26)

while the triplet $A^{ij} \equiv \varepsilon^{ijk} A_k$ can in general have the VEVs towards all three components:

$$\langle A^{ij} \rangle = \begin{pmatrix} 0 & A_3 & A_2 \\ -A_3 & 0 & A_1 \\ -A_2 & -A_1 & 0 \end{pmatrix}, \quad A_1 > A_2 > A_3.$$ (27)

Therefore, in the low-energy limit the operators (26) reduce to the Yukawa couplings which in terms of the dimensionless VEVs $S = \langle S \rangle / M$ and $A = \langle A \rangle / M$ read as:

$$Y_u = S, \quad Y_d, Y_e^T = \rho S + A, \quad Y_\nu = \eta S$$

(28)

where $\rho$ and $\eta$ are proportionality coefficients related to different coupling constants in (25). This predictive texture, so called Stech ansatz [11,18].

5 The theory may also contain conjugated Higgses $\bar{X}_{ij}$ in representations $\bar{S} \sim (1, 6)$ and $\bar{A} \sim (1, 3)$. These usually are needed for writing non-trivial superpotential terms in order to generate the horizontal VEVs (see next Section). These fields, however, do not couple to the fermions (25) and thus do not contribute to their masses.

6 The alert reader will notice our improper language: we often use for brevity and simplicity ‘sextet’ (or ‘triplet’) also when we deal with the ‘anti’-representation.
is completely excluded on the phenomenological grounds. However, its realistic modifications are possible as discussed below.

In view of the renormalizable theory, the operators (25) can be obtained as a result of integrating out some heavy fermion states with mass of order $M$. It is natural to assume that this scale itself emerges from the VEVs of some fields which can be in singlet or octet representation of $SU(3)_H$. In this case, all terms in the superpotential are trilinear terms, and one can impose a discrete $R$ symmetry under which all superfields as well as the superpotential changes sign. In particular, the operators (25) can be obtained by integrating out the following heavy states:

$$T^i \sim (10, 3), \quad \overline{T}_i \sim (\overline{10}, 3)$$
$$\overline{F} \sim (5, 3), \quad F_i \sim (5, 3)$$
$$N^i \sim (1, 3), \quad \overline{N}_i \sim (1, 3)$$

from the following superpotential terms:

$$W_T = i T H + f \overline{H} T + \Sigma I \overline{T} + 3 T \overline{H},$$
$$W_F = f F A + \Sigma F \overline{T} + \overline{F} T H,$$
$$W_N = f N H + \Sigma N \overline{N} + S \overline{N}^2. \quad (29)$$

If $\Sigma$ is a singlet, then one immediately obtains the ansatz (28). However, we can assume that $\Sigma$ contains also the $SU(3)_H$ octet with the VEV towards the $\Lambda_8$ component, and in addition the antisymmetric scalars $A$ contain also the adjoint of $SU(5)$, $A \sim (24, 3)$. Moreover, the form of the superpotential (30) can be motivated by some additional symmetries (for example discrete symmetries), which differently transform $S$ and $A$.

Let us now consider the superpotential of the horizontal Higgses $S$ and $A$. The invariance under additional discrete symmetries, can easily force the superpotential of these fields to have a ‘separable’ form, $W = W(S) + W(A)$. In addition, the discrete $R$ symmetry dictates superpotentials of the form:

$$W(S) = Z(\Lambda^2 - S \overline{S}) + Z^3 + S^3 + \overline{S}^3 \quad (31)$$
$$W(A) = Z'(\Lambda^2 - A \overline{A}), \quad (32)$$

where $Z$ and $Z'$ are some singlet superfields. Clearly, if $Z$ has a non-zero VEV, there is a solution when the sextet $S$ has a diagonal VEV with non-zero eigenvalues $(S) = \text{diag}(S_1, S_2, S_3)$. In this case, the term $Z S \overline{S}$ plays the role of the mass term $\mu S \overline{S}$. Similarly, from the term $W(A)$ the field $A$ gets a non-zero VEV which orientation with respect to that of $S$ will be determined by the SSB terms pattern.

We see that $Y_t \sim 1$ implies $S_1 \sim M$, close to cutoff scale, which can naturally arise from the Higgs sector. Similarly one can expect that also $Y_{b, \tau} \sim 1$ which would require large $\tan\beta$ regime. However, in realistic schemes also moderate $\tan\beta$ can be naturally accommodated [21]. The flavor scale in the theory can be consistently thought to be close to the GUT scale $M_G \sim 10^{16}$ GeV.

In conclusion, this theoretical background allows us to motivate the following Yukawa matrices [21]:

$$Y_u = S, \quad Y_d = \rho S + b^{-1} A_d,$$
$$Y_{\nu} = \eta b^{-1} S, \quad Y_{\nu}^T = \rho S + b^{-1} A_{\nu}, \quad (33)$$

where $A_{e,d}$ are antisymmetric matrices with 24-plet dependent entries inducing different Clebsch factors for the down quarks and charged leptons, and $b = \text{Diag}(1, 1, b)$, where $b$ is an asymmetry parameter induced by the $SU(3)_H$ symmetry breaking due to the interplay of the singlet and octet VEVs. Clearly the above pattern represents an extension of the Stech-like texture considered in [21]. A careful analysis proves that the above Yukawa pattern provides a successful description of fermion masses and mixing angles (for details see [21]). Alternatively, in another set up, we could obtain a different predictive pattern:

$$Y_u = S, \quad Y_d = b^{-1}(\rho S + A_d),$$
$$Y_{\nu} = \eta b^{-1} S, \quad Y_{\nu}^T = b^{-1}(\rho S + A_{\nu}), \quad (34)$$

Both these patterns have a remarkable property. Namely, they offer the key relation to understand the complementary mixing pattern of quarks and leptons. The origin of this relation in fact can be traced to the coincidence of the Yukawa matrices $Y_d = Y_{\nu}^T$ in the minimal $SU(5)$ theory. In the textures (33) and (34) this relation is not exact, but is fulfilled with the accuracy of the different
Clebsch factors in $A_d$ and $A_e$. Explicitly this means that the 2-3 mixing angles in the quark and lepton sectors are, respectively:

$$\tan \theta_{23}^q \simeq b^{-1/2} \sqrt{m_t / m_b} \quad \text{and} \quad \tan \theta_{23}^l \simeq b^{1/2} \sqrt{m_{\tau} / m_\tau}$$

which can be correctly fixed for $b \sim 10$. From here the following product rule is obtained [20]:

$$\tan \theta_{23}^q \tan \theta_{23}^l \simeq \left( \frac{m_t m_\tau}{m_b m_{\tau}} \right)^{1/2}$$

This product rule indeed works remarkably well.

It demonstrates a ‘see-saw’ correspondence between the lepton and quark mixing angles and tells us that whenever the neutrino mixing is large, $\tan \theta_{23}^l \sim 1$, the quark mixing angle comes out small and in the correct range, $\tan \theta_{23}^q \sim 0.04$.

Let us remark, however, that the patterns considered above rely on the fact that the sextet $S$ has a VEV with non-zero eigenvalues $\langle S \rangle = \text{diag}(S_1, S_2, S_3)$. Such a solution of the Higgs superpotential, with the hierarchy $S_3 \gg S_2 \gg S_1$, is indeed possible if the horizontal symmetry $SU(3)_H$ is a global symmetry. However, this solution disappears if $SU(3)_H$ is a local gauge symmetry, since it is not compatible with vanishing gauge $D$-terms of $SU(3)_H$.

In this case, however, we have to resort to the solution $\langle S \rangle = \langle S \rangle = \text{diag}(0, 0, S)$, which is instead compatible with the $SU(3)_H$ $D$-term flatness (see Appendix). As for triplet fields, the most general VEV pattern is (see Appendix):

$$\langle A \rangle = \langle \bar{A} \rangle = \begin{pmatrix} 0 & 0 & 0 \\ -A_3 & 0 & A_1 \\ 0 & -A_1 & 0 \end{pmatrix}.$$ (37)

So, we can start from the effective operators

$$
\begin{align*}
\left( \frac{S_{ij}^{ij} + A_{ij}^{ij}}{M^2} \right) t_i t_j H, & \quad \left( \frac{S_{ij}^{ij} + A_{ij}^{ij}}{M^2} \right) \bar{t}_i \bar{t}_j \bar{H}, \\
\left( \frac{S_{ij}^{ij} + A_{ij}^{ij}}{M^2} \right) \bar{t}_i \bar{t}_j H^2 & \quad \left( \frac{S_{ij}^{ij} + A_{ij}^{ij}}{M^2} \right) t_i t_j \bar{H},
\end{align*}
$$

which can be induced by integrating out heavy states from the appropriate Yukawa superpotential. In this case we find the following Yukawa texture [17]:

$$Y_f = \begin{pmatrix} 0 & A_{3f} & 0 \\ -A_{3f} & 0 & A_{1f}^l \\ 0 & -A_{1f} & S_f \end{pmatrix}, \quad f = u, d, e, \nu(39)$$

which resembles the familiar Fritzsch ansatz [13].

The latter in fact corresponds to the particular case $A_{1f}^l = A_{1f}$, which can be obtained if there is no $SU(3)_H$ breaking by the octet representation, i.e. the scalar $\Sigma$ is a singlet [12].

This situation is now completely excluded by the experimental data. However, for the case $A_{1f}^l \neq A_{1f}$, one can achieve a very good description of the quark and neutrino mass and mixing pattern (this is accounted by the 2-3 asymmetry parameter $b$ explicitly shown in eq. (33)). Notice, that as long as the octet $\Sigma$ participates in the $SU(3)_H$ breaking with VEV along $\lambda_8$ generator and so $M \propto \mathbb{1} + \lambda_8$, also the antisymmetric rep. A contributes to the neutrino mass matrix by filling the 23, 32 entries in a symmetric way [20].

We can take a more general approach and consider effective operators which also incorporate the $SU(5)$ adjoint Higgs $\Phi$:

$$
\begin{align*}
\left( \frac{X_{ij}^{ij}}{M} + \frac{X_{ij}^{ij} \Phi}{M^2} + \frac{X_{ij}^{ij} \Phi^2}{M^3} \right) t_i t_j H, & \quad \left( \frac{X_{ij}^{ij}}{M} + \frac{X_{ij}^{ij} \Phi}{M^2} + \frac{X_{ij}^{ij} \Phi^2}{M^3} \right) \bar{t}_i \bar{t}_j \bar{H}, \\
\left( \frac{X_{ij}^{ij}}{M} + \frac{X_{ij}^{ij} \Phi}{M^2} + \frac{X_{ij}^{ij} \Phi^2}{M^3} \right) \bar{t}_i \bar{t}_j H^2 & \quad \left( \frac{X_{ij}^{ij}}{M} + \frac{X_{ij}^{ij} \Phi}{M^2} + \frac{X_{ij}^{ij} \Phi^2}{M^3} \right) t_i t_j \bar{H},
\end{align*}
$$

where $X_{0,1,2}$ are the horizontal scalars, which can be symmetric or antisymmetric. (Here, for simplicity, we only consider the charged fermion sector.) This pattern can be motivated by some additional symmetry reasons, which differently transform the horizontal Higgses $X_{0,1,2}$. E.g. one can consider a $Z_3$ symmetry acting on the superfields as $\Psi \to \Psi \exp(i \frac{2\pi}{3} Q)\Phi$, and take the corresponding charges as $Q(X_0) = 0$, $Q(X_1) = 1$, $Q(X_2) = 2$ and $Q(\Phi) = -1$.

The underlying renormalizable superpotential can be:

$$t TH + \bar{t} H \bar{H} + \Sigma T \bar{T} + X_0 \bar{T} t$$

Notice that in this case the neutrino mass matrix can have only 33 non-zero element $S_{ij}$. 

where $\Sigma, \Sigma_1, \Sigma_2$ are some superfields in singlet or octet representations of $SU(3)_H$ (in the following, singlets will be denoted as $I$ and octet as $\Sigma$), with VEVs of order $M$. These VEVs can emerge from the Higgs superpotential including linear terms $\Lambda_k^2 I_k$ for the singlets, with $\Lambda_k \sim M$, and all possible trilinear terms consistent with the symmetry (among which there are also those like $I\Phi^2$ and $\Phi^3$). In this way, all singlet and adjoint fields can get order $M$ VEVs. There is also the possibility to generate some of these VEVs by means of the anomalous $U(1)_A$ symmetry and in this case the scale $M$ comes out to be slightly bigger than the grand unification scale $M_G \approx 10^{16}$ GeV.

The observed pattern of the fermion masses clearly requires that the "leading" horizontal scalar should be a $SU(3)_H$ sextet, $X_0 = S_0$. As for other fields $X_{1,2}$, these can be sextets or triplets. In the latter case we obtain the Fritzsch-like textures ($[19]$) already considered above. It is interesting to consider also the case when these fields are sextets, $S_{1,2}$ and the effective operators read as:

$$\left( \frac{S^{ij}_0}{M} + \frac{S^{ij}_1 \Phi}{M^2} + \frac{S^{ij}_2 \Phi^2}{M^3} \right) t_i t_j H,$$

$$\left( \frac{S^{ij}_0}{M} + \frac{S^{ij}_1 \Phi}{M^2} + \frac{S^{ij}_2 \Phi^2}{M^3} \right) \bar{t}_i \bar{t}_j \bar{H}. \quad (42)$$

We now turn to the superpotential of the horizontal Higgses. Due to the different quantum numbers of $S_{0,1,2}$ with respect to additional discrete symmetries the superpotential of these fields is ‘separable’ like, $W = \sum W(S_k)$, where

$$W(S_k) = Z_k(A_k^2 - S_k \tilde{S}_k) + S_k^3 + S_k^3. \quad (43)$$

In this case, each of the VEVs $\langle S_k \rangle$ can have only one non-zero eigenvalue. However these VEVs can have non-trivial orientations with respect to each other in the $SU(3)_H$ space, with generically large angles determined by the SSB terms:

$$\langle S_0 \rangle \propto P_0 = \text{diag}(0,0,1), \quad \langle S_1 \rangle \propto U_1^3 P_0 U_1, \quad \langle S_2 \rangle \propto U_2^3 P_0 U_2,$$

where $U_{1,2}$ are $SU(3)_H$ unitary matrices reflecting the relative orientation. Then we obtain Yukawa matrices of the following structure:

$$Y_f = P_0 + \varepsilon_f P_1 + \varepsilon_f^2 P_2, \quad f = u, d, e \quad (45)$$

where $P_{0,1,2}$ are rank-1 matrices with $O(1)$ elements which can be chosen as$^{[10]}$

$$P_0 = (0,0,1)^T \bullet (0,0,1), \quad P_1 = (0,c,s)^T \bullet (0,c,s), \quad P_2 = (x,y,z)^T \bullet (x,y,z), \quad (46)$$

and $\varepsilon_f \sim \langle \Phi \rangle / M$ are the Clebsch factors projected out from the couplings of $\Phi$ with the different types of fermions. Yukawa matrices of such structures have been considered in ref. [23], and earlier, in the context of radiative mass generation mechanism, in ref. [24].

All these considerations not only lead to a general understanding of the fermion mass and mixing pattern, but can also lead to the predictive schemes. In addition, the schemes arising from the heavy fermion exchanges ($[14]$), exhibit the remarkable alignment of the sfermion mass matrices to the Yukawa terms, and thus are natural as far as the supersymmetric flavor problem is concerned ($[17]$).

To conclude, we add the following remark. The horizontal symmetry may guarantee the R-parity. The $SU(3)_H$ symmetry does not work for this, though in certain context it could suppress some R-violating terms $[24]$. However, the automatic R-parity can be achieved in the context of the horizontal symmetry $SU(4)_H$ $[26]$.

4. Appendix: Horizontal VEV structures

Consider the following superpotential including the superfields $S = S^{ij}$ and $\tilde{S} = \tilde{S}^{ij}$:

$$W_S = -\mu S \tilde{S} + S^3 + \tilde{S}^3, \quad (47)$$

$^{[10]}$ Alternatively, the sextets $S_k$ can be regarded as ‘composite’ fields obtained from tensor products of the triplets $A_k$, i.e. $S_k^{ij} = A_k^i A_k^j$, having VEVs $\langle A_0 \rangle \propto (0,0,1)$, $\langle A_1 \rangle \propto (0,c,s)$, $\langle A_2 \rangle \propto (x,y,z)$ $[24]$.  

where $\mu$ is some mass parameter, $S^3 = \frac{1}{2} \varepsilon_{ijk} \varepsilon_{a b e} S^a S^b S^c R$ (similarly for $\bar{S}^3$), and order one coupling constants are absorbed. Observe also that this superpotential is manifestly invariant under $Z_3$ symmetry: $S \to \exp(i \frac{2 \pi}{3}) S$ and $\bar{S} \to \exp(-i \frac{2 \pi}{3}) \bar{S}$. Without loss of generality, the VEV of $S$ can be chosen in the diagonal form, $\langle S \rangle = \text{Diag}(S_1, S_2, S_3)$. Then the condition of vanishing $F$-terms $F_S, F_{\bar{S}} = 0$ implies that $\langle \bar{S} \rangle$ is also diagonal, $\langle \bar{S} \rangle = \text{Diag}(\bar{S}_1, \bar{S}_2, \bar{S}_3)$, and that $S_i S_j = \mu \varepsilon_{ijk} S^k, \quad \bar{S}_i \bar{S}_j = \mu \varepsilon_{ijk} \bar{S}_k$. (48)

So, in the exact supersymmetric limit the VEV pattern of $S$ and $\bar{S}$ is not fixed unambiguously and there are flat directions representing a two-parameter vacuum valley. In other words, the six equations (48) reduce to four conditions:

$$S_1 S^3 = S_2 S^2 = S_3 S^1 = \mu^2, \quad S_1 S_2 S_3 = \mu^3 \quad (49)$$

while the others are trivially fulfilled (e.g. equation $S^1 S^2 S^3 = \mu^3$ follows from eqs. (48)). Thus, in principle the eigenvalues $S_{1,2,3}$ can be different from each other, say $S_3 > S_2 > S_1$. Then eqs. (48) imply that $S_{1,2,3}$ should have an inverse hierarchy, $S^3 < S^2 < S^1$. More precisely, we have $S^1 : S^2 : S^3 = S_{1}^{-1} : S_{2}^{-1} : S_{3}^{-1}$.

The flat directions of the VEVs are lifted by the soft SUSY breaking D-like terms:

$$L = -\int d^4 \theta \bar{z} \left[ \alpha \text{Tr} S^1 S^3 + \frac{\beta}{M^2} \text{Tr} S^4 S^2 + \frac{\gamma}{M^2} \text{Tr} S^1 S^2 S^3 + \ldots \right] \quad (50)$$

having a similar form also for $\bar{S}$. Here $z = m \theta^2, \bar{z} = m \bar{\theta}^2$ are supersymmetry breaking spurions, with $m \sim 1 \text{ TeV}$. The cutoff scale $M$ is taken as the flavor scale, i.e. the same as in the superpotential (47), and we assume that $M > \mu$. The stability of the scalar potential associated with (50) implies that $\beta > 0$ and $\gamma > -\beta$, whereas $\alpha$ can be positive or negative. In the former case the minimization of the potential, under the conditions (48), would imply that $S_1 = S_2 = S_3 = \mu$, i.e. no hierarchy between the fermion families. In the latter case, however, the largest eigenvalue of $S$ and $\bar{S}$, respectively $S_3$ and $\bar{S}_1$, grow up above the typical VEV size $\mu$ and reach values of the order of the cutoff scale $M$:

$$S_3, \bar{S}_1 \approx \left( \frac{\alpha}{2(\beta + \gamma)} \right)^{1/2} M \sim M. \quad (51)$$

Then it follows from (48) that

$$S_2, \bar{S}_2 \sim \mu, \quad S_1, \bar{S}_3 = \frac{\mu^2}{\bar{S}_3} \sim \varepsilon^2 S_3, \quad (52)$$

where $\varepsilon \sim \mu / M$.

Let us now consider the variant (51) of the superpotential (47). In this case we have two solutions:

(i) $\langle Z \rangle \neq 0$: Clearly, in this case the $F$-term conditions are the same as in (48) apart from the fact that the mass scale $\mu$ should be substituted by the Z’s VEV, $\mu = \langle Z \rangle$. The latter is then fixed as $\langle Z \rangle = \Lambda$ by the condition $F_Z = 0$. Thus we still have flat directions which will be stabilized by the soft SUSY breaking terms in order to obtain a hierarchy of the eigenvalues $S_3 : S_2 : S_1 \sim 1 : \varepsilon : \varepsilon^2$, where $\varepsilon = \Lambda / M$. This solution exists if $SU(3)_H$ is a global symmetry, however it is no more valid if $SU(3)_H$ is local. The reason is simple: the inverse hierarchy of the VEVs $S_{1,2,3}$ and $\bar{S}_{1,2,3}$ is not compatible with the D-flatness condition of the gauge terms $D_a = \sum_n X_n^a X_{n,3}^{a,3}$, where $X_a$ are $SU(3)_H$ generators, $a = 1, \ldots, 8$, unless $S_3 = \bar{S}_2 = S_1$. This solution, however, is in contrast with the fermion mass hierarchy.

(ii) $\langle Z \rangle = 0$: In this case the condition $F_Z = 0$ tells us that $\sum S_i S_3^3 = \Lambda^2$, while the conditions $F_{S,\bar{S}} = 0$ yield $S_i S_j = 0$ and $\bar{S}_i \bar{S}_j = 0$. Therefore, the VEVs of $S, \bar{S}$ can only have one non-zero eigenvalue, which can be e.g. $S_3$ and $\bar{S}_3$, so that $S_3^3 = \Lambda^2$. Clearly, this solution is compatible with the D-flatness condition. The requirement $D_a = 0$ simply fixes $S_3 = \bar{S}_3 = \Lambda$.

Thus, in the case of local $SU(3)_H$ we obtain the non-degenerate solution $\langle S \rangle = \langle \bar{S} \rangle = \bar{S}_3 = S_3 = \Lambda$.

11 In principle, we could appeal to some extra “spector” superfields in different representations of $SU(3)_H$ with VEVs oriented so that to cancel the contributions of $\langle S \rangle$ in the $SU(3)_H$ gauge D-terms. In this way, not very appealing though, the hierarchical VEV solution could be consistent also in the case of local $SU(3)_H$. 
A·diag(0, 0, 1). In this case the operators like \( S \) involving \( S \) can only induce the third-generation masses and hence \( A \sim M \) is needed, which is quite a natural assumption, to obtain order one Yukawa couplings.

Now consider the superpotential \( W \) for the anti-symmetric Higgs superfields \( A \). In the exact supersymmetric limit the ground state has a continuous degeneracy (flat direction) related to unitary transformations \( A \rightarrow UA \) with \( U \subset SU(3)_H \). In other terms, the superpotential \( W = W_S + W_A \) has an accidental global symmetry \( SU(3)_S \times SU(3)_A \), with two \( SU(3) \) factors independently transforming the horizontal superfields \( S \) and \( A \).

Similarly to the case of the Higgs fields \( S, \bar{S} \), for the solution with \( \langle Z \rangle = 0 \) the conditions \( F_A, F_{\bar{A}} = 0 \) can only fix the values of the holomorphic invariant \( AA = \Lambda^2 \), while the \( D \)-term flatness requires \( \langle A \rangle = \langle \bar{A} \rangle \). By unitary transformation \( A \rightarrow UA \) \((U \subset SU(3)_A)\), one can choose a basis where the VEV of \( A \) points towards the first and third components. The relative VEV orientation between \( S \) and \( A \) should be fixed from the soft \( D \)-like terms:

\[
\frac{1}{M^2} \int d^4 \theta \bar{z} \bar{z} \left[ \alpha' \text{Tr}(S^\dagger S A^\dagger A) + \beta' \text{Tr}(S^\dagger S \Sigma^\dagger \Sigma) + \gamma' \text{Tr}(A^\dagger A \Sigma^\dagger \Sigma) \right].
\]

with \( z = \bar{m} \theta^2, \bar{z} = \bar{m} \bar{\theta}^2 \). Namely, for \( \alpha', \beta', \gamma' \) all positive, one can have the triplet VEV structure in eq. \((\ref{eq:32})\) with \( A_1 \) and \( A_3 \) both non-zero. For more details on the horizontal VEV structures, see also \cite{27}.

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