Collisionless reconnection: Mechanism of self-ignition in thin current sheets

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Abstract. The spontaneous onset of magnetic reconnection in thin collisionless current sheets is shown to result from a thermal-anisotropy driven magnetic Weibel-mode, generating seed-magnetic field X-points in the centre of the current layer.

Keywords. Reconnection, Weibel fields in thin current sheets, Weibel thermal level, Magnetospheric substorms

1 Introduction

The question of the role of Hall currents in reconnection is not resolved yet. Classically they are unimportant for the reconnection process proper.¹ Nevertheless below, when using numbers, we will for reasons of resolution refer to conditions in which Hall-electrons would more directly be involved. In principle, observation of the electron-inertial (‘electron-thermal-anisotropy driven magnetic Weibel-mode, generating seed-magnetic field X-points in the centre of the current layer.

In the present Letter we show that in an ideal current sheet ions and electrons become non-magnetic on their respective inertial scales \( \lambda_{i,e} = c/\omega_{i,e} \), where \( \omega_{i,e} = e\sqrt{N/\epsilon_0 m_{i,e}} \) are the plasma frequencies of ions and electrons, (classical) collisionless convective transport of magnetic fields into the current layer takes place up to a vertical distance \( z \sim \lambda_e \) from the centre of the current sheet. The region between \( \lambda_e \lesssim z \lesssim \lambda_i \) is known as the ‘Hall-current’ (Sonnerup 1979) or (mistakenly, as there is no diffusion present) ‘ion-diffusion’ region. Being a by-product of thinning of the current layer, the Hall currents are presumably not involved in the reconnection process proper. They close along the magnetic field by electrons that are accelerated in the oblique lower-hybrid-drift/modified-two-stream instability driven by magnetised Hall-electrons on the non-magnetic ion background thereby coupling the reconnection site to the auroral ionosphere (Treumann et al. 2009).

When speaking of a current sheet, we refer to ideal current sheets separating strictly antiparallel fields. The observational paradigm of a reconnecting current sheet is the magnetospheric tail-current sheet. This current sheet is not ideal in the above sense as it is embedded into a quasi-dipolar field which still might preserve a weak rudimentary (normal) magnetic field component \( B_z \) pointing northward. This \( B_z \) component re-magnetises the central-sheet electrons and affects the evolution of (collisionless) tearing modes (Galeev and Zelenyi 1975). Nevertheless below, when using numbers, we will for reasons of resolution refer to conditions in the magnetotail even though our theory might better apply to the magnetopause, interplanetary space or astrophysics. In principle, observation of the electron-inertial (‘electron-

¹The question of the role of Hall currents in reconnection is not resolved yet. Classically they are unimportant for the reconnection process. It is, however, not certain whether or not on the microscopic scales non-classical (quantum-Hall) effects are induced by the environmental conditions (einselection effects, see Zurek 2003) in which Hall-electrons would more directly be involved.
2 Magnetic field generation in the current layer

Unless a guide field is imposed from the outside, the inner current region \( z \lesssim \) few \( \lambda_e \) is about free of magnetic fields, while at the same time carries a (diamagnetic) current \( J_\perp \) perpendicular to the antiparallel magnetic fields to both sides of the current, caused (for instance in the geomagnetic tail current sheet or the Earth’s magnetopause) by a (macroscopic) electric potential drop \( \Delta U \) along the current.

For the understanding of the mechanism of reconnection it is of no interest how this potential drop is generated. This may happen when two magnetised collisionless plasmas of finite lateral extension collide. In the magnetotail current sheet the potential amounts to \( \Delta U \lesssim \) few \( 10 \) kV, and electron and ion temperatures are of the order of \( T_e \sim 0.1 \) keV and \( T_i \sim 1 \) keV, respectively. Electrons entering the centre of the current sheet accelerate along the current, thereby becoming the main current carriers here. Their high translational velocity \( V_e = \sqrt{e \Delta U / m_e} > v_e \) exceeds their thermal speed \( v_e = \sqrt{2T_e / m_e} \) providing conditions that are unstable against the Buneman two-stream instability (Buneman, 1958), a fast growing electrostatic instability with high frequency \( \omega_B \sim 0.03 \omega_e \) and large growth rate \( \gamma_B \sim \omega_B \) (cf., e.g., [Treumann and Baumjohann, 1997] p. 22). In the geomagnetic tail current sheet the growth rate amounts to \( \gamma_B \approx 1.7 \) kHz, corresponding to a growth time of \( \tau_B \approx 0.006 \) s.

The Buneman instability readily generates localised electrostatic structures (known as electron and ion phase space holes) which trap a substantial part of the electrons and heat them in the direction along the current drift velocity. Numerical simulations suggest that this process takes roughly 100-1000 plasma periods (Buneman, 1959; Newman et al., 2001), or few 10 e-folding times, in the magnetospheric tail \( \lesssim 0.1 \) s. In this process the instability shuts off itself by increasing the parallel electron temperature until \( v_{e\parallel} \sim V_e \). At the end of this very fast process the electrons develop a temperature anisotropy

\[
A = T_{\|}/T_{\perp} - 1 > 0
\]

with current-parallel temperature \( T_{\|} \geq T_{\perp} = T_e \) exceeding the initial electron temperature, roughly \( A \lesssim 1 \) in the magnetospheric tail current sheet. The subscripts \( \| \) and \( \perp \) refer to the respective directions of maximum and minimum electron temperatures, i.e. the two directions of the electron pressure tensor

\[
P_e = N[T_{\perp}e^1 + (T_{\|} - T_{\perp})V_eV_e/V_e^2]
\]

In this thermally anisotropic case the electrons obey a bi-Maxwellian equilibrium distribution function

\[
f_e(v_{\perp},v_{\|}) = \frac{(m_e/2\pi)^{\frac{3}{2}}}{T_{\perp}V_eT_{e\perp}} \exp \left[ -\frac{m_e v_{\perp}^2}{2T_{e\perp}} - \frac{m_e v_{\|}^2}{2T_{e\|}} \right]
\]

which, in a nonmagnetised plasma (like the inner current region \( z \lesssim \lambda_e \)) is unstable with respect to the family of Weibel instabilities (Weibel, 1959). These are very low (about zero) frequency (purely growing) electromagnetic instabilities which are capable of generating stationary magnetic fields that grow from thermal fluctuations (not requiring any magnetic dynamo mechanism). The linear electromagnetic dispersion relation of the plasma becomes

\[
(n^2 - \epsilon_{\perp})^2 \epsilon_{\perp} = 0
\]

where \( n = ke/\omega \) is the refraction index, and \( \omega \) is the frequency of the linear disturbance. The dielectric tensor has the two scalar components \( \epsilon_{\parallel} \ll (k,\omega), \epsilon_{\perp}(k,\omega) \) which are the longitudinal and transverse response functions, respectively. For our purposes it suffices to consider the electromagnetic (transverse) response buried in

\[
\epsilon_{\perp} \approx 1 - \frac{\omega_p^2}{\omega^2} \left(1 - (A + 1)[1 + \zeta Z(\zeta)]\right) - \frac{\omega_p^2}{\omega^2} = n^2
\]

where \( Z(\zeta) \) is the plasma dispersion function, \( \zeta = \omega/k_e v_{e\perp} \), and \( v_{e\perp} = \sqrt{2T_e/m_e} \) is the electron thermal speed perpendicular to the current. The Weibel instability grows in the plane perpendicular to the direction of higher velocity, which in our case has been assumed as the parallel direction. Hence, \( k = (k_x,0,k_z) = (k_\perp \sin \theta,0,k_\perp \cos \theta) \); in an extended medium there is no \( \theta \)-dependence, a point to which we will return later. The contribution of the resting ions has been retained for completeness; because of the smallness of the ion plasma frequency \( \omega_i \ll \omega_e \), being much less than the electron plasma frequency \( \omega_e \), it plays no role in the instability.

At zero real frequency \( \omega = i\gamma \) and \( A > 0 \) the right-hand side of Eq. (5) becomes the dispersion relation of the thermal-anisotropy driven Weibel mode (Weibel, 1959; Yoon and Davidson, 1987 and others). Instability \( \gamma(k_\perp) > 0 \) sets on at phase velocities \( \omega/k_\perp > v_{e\perp} \) for wavenumbers \( k_\perp < k_0 \),

\[
k_0 \lambda_e \approx \sqrt{A}
\]

with instability growth rate

\[
\frac{\gamma \omega_p}{\omega_e} \approx \sqrt{\frac{2}{\pi}} \frac{v_{e\perp}}{c} \frac{k_\perp}{k_0} (1 - \frac{k_\perp^2}{k_0^2})(A + 1)(k_0 \lambda_e)^3
\]

Weibel – or current filamentation instabilities, as they are sometimes called following Fried (1959) where a simple physical model of their mechanism was given early – have mostly been investigated in view of astrophysical applications in a relativistic approach.
vanishing at long wavelengths $k_{\perp} = 0$. The growth rate maximises at wavenumber $k_{\perp m} = k_0/\sqrt{3} = \lambda_e^{-1} \sqrt{A/3}$ (see Figure 1) where its value is

$$\gamma_{W,m} / \omega_e \approx 4 \sqrt{3} \left( \frac{A^{3/2}}{3\pi} \right) \left( A + 1 \right)$$

(8)

with $\Theta_e = T_{e,\perp} / m_e c^2$ the (ambient) temperature normalised to the rest energy of an electron. Numerically this expression yields for the maximum growth rate

$$\gamma_{W,m} \approx 34 \sqrt{n_{[cm^{-3}]} T_{e,\perp}[eV]} A^{2/3} (A+1) \text{ Hz}$$

(9)

Depending on the value of the anisotropy, this growth rate can be substantial. If $A > 1$, it grows as $\gamma \propto \sqrt{A^3}$, while for anisotropies $A < 1$ it grows like $\gamma \propto A^3$. In the tail plasma sheet we have $T_e \sim 100$ eV and $N \sim 1$ cm$^{-3}$. Then, even with $A \sim 0.1$ one finds quite a fast growth rate of $\gamma_{W,m} \lesssim 10$ Hz.

The important point is that even though the growth rate might not be extraordinarily large, it generates a magnetic field that has two components, $B_W = (B_x, 0, B_z)$, both being transverse to the initial current. The component $B_x$ is alternating between the directions parallel and antiparallel to the initial magnetic field outside the current layer, being directed $\pm \hat{x}$ while the other component is perpendicular to the current layer directed along $\pm \hat{z}$. This field modulates the current layer along $\hat{x}$ causing magnetic islands whose vertexes lie in the centre of the current layer. It thus provides seed-$X$ points which, if sufficiently large amplitude, will spontaneously ignite reconnection. The finite nonmagnetic current sheet width in $z$ imposes a limit $2\pi / k_z < 2\lambda_e$ which yields

$$k_z / k_x = \cot \theta \approx k_z / k_m > \pi \sqrt{3/4A}$$

(10)

the lower limit resulting from the restriction on $A > 3m_e/2m_i$ (see below). Thus the Weibel mode propagates at angles

$$\tan^{-1} \left[ \frac{\pi}{3} \sqrt{m_e/2m_i} \right] < \theta < \tan^{-1} \left[ \frac{\pi}{3} \sqrt{A/3} \right]$$

(11)

against $\hat{x}$. This is the maximum angle the wavevector assumes in the Weibel-field vertexes. For $A = 0.1$ and $A = 1$ this inclination angles are $0.3^\circ < \theta < 3.4^\circ$ and $\sim 11^\circ$, respectively. However, in addition, the Weibel mode can propagate in two directions $\pm \hat{x}$. The two cases are shown in Figure 1(a) when the propagation direction choses to be along the external field. In this case simple seed-X points in the current sheet are generated which will allow reconnection to evolve in the usual way. For the oppositely directed Weibel vertices shown in Figure 1(b), however, a multitude of additional reconnection sites are produced along $z \sim \pm \lambda_e$, and the current layer becomes highly unstable. Which is the most probable case can be decided only after a complete solution of the Weibel-unstable boundary value problem of the current layer.

3 Thermal fluctuation level

In order to infer how long it takes the instability to achieve substantial magnetic field amplitudes we need to estimate the magnetic thermal fluctuation level $\langle |b|^2 \rangle_{k,\omega=0}$ from where the Weibel instability starts growing in the presence of the electron pressure anisotropy (thermally fluctuating magnetic fields will be denoted by lower case letters). Magnetic thermal levels have recently been estimated [Yoon 2007b, Baumjohann et al. 2010]. From basic fluctuation theory (Sitenko 1967) the spectral energy density of the zero-frequency thermally-anisotropic Weibel mode can be written

$$\langle |b|^2 \rangle_{k_0} = \frac{\mu_0 c}{\omega_e} \frac{T_{e,\perp} k_\perp \lambda_e (A+1)^2}{[k_\perp^2 \lambda_e^2 - A + m_e/m_i]^2}$$

(12)

The 0-subscript refers to vanishing real frequency. Here the ion contribution has been retained. In the isotropic $A = 0$ and Weibel-stable $-2 < A < 0$ cases, the spectral energy density vanishes at $k_\perp \rightarrow 0, k_\perp \rightarrow \infty$ and, in a proton-electron plasma, maximises at $k_\perp \lambda_e \approx 0.013$. Its maximum value is

$$\langle |b|^2 \rangle_{k_0,m} = 8.25 \times 10^{-23} \sqrt{\frac{T_{e,\perp}[eV]}{n_{[cm^{-3}]}}} \frac{V^2s^3}{m}$$

(13)

One might note that for positive anisotropies the current sheet is not in equilibrium anymore, and the thermal fluctuations explode close to the boundary of the unstable domain for $k_\perp \lambda_e \sim \sqrt{A} \approx k_0 \lambda_e$ indicating onset of instability and phase transition.
3.1 Fastest growing Weibel mode

The Weibel instability chooses from this spectral energy density and supports the fastest growing wavenumber \( k_{\perp m} \). Inserting for \( k_{\perp m} \) the initial thermal level of the fastest growing mode becomes

\[
\langle |b|^2 \rangle_{k_{\perp m}0} \approx \frac{9\mu_0 m_e c^2}{4 \omega_e} \sqrt{\frac{\pi}{3}} \frac{\kappa c}{m_e c^2} \left( A + 1 \right)^2 \left( A + 2 \right)
\]

(14)

Since large thermal anisotropies are unrealistic, the cases of small \( A \ll 1 \) and large anisotropies \( A \sim 1 \) may be distinguished yielding the limiting initial levels

\[
\langle |b|^2 \rangle_{k_{\perp m}0} \approx \frac{\alpha \mu_0 m_e c^2}{A} \frac{\kappa c}{\omega_e} \sqrt{\frac{\pi}{3}} \frac{\kappa c}{m_e c^2}, \quad A > \frac{3 m_e}{2 m_i}
\]

(15)

with \( \alpha = 9/8 \) for \( A \ll 1 \), and \( \alpha = 3 \) for \( A \lesssim 1 \). Numerically:

\[
\langle |b|^2 \rangle_{k_{\perp m}0} \approx 8.8 \times 10^{-28} \alpha \frac{\kappa c}{A} \sqrt{\frac{T_e [eV]}{N_{[cm^{-3}]}}} \frac{V^2 s^3}{m}
\]

(16)

where the temperature is measured in eV, and the density is in \( cm^{-3} \). The numerical factor for the largest expected anisotropy \( A \sim 1, \alpha = 3 \) is \( \approx 2.63 \times 10^{-27} \).

Fig. 2. Wavenumber dependencies of the normalised fluctuation spectrum and normalised growth rates. The normalisation of the thermal fluctuation spectrum is to its maximum value given in Eq. [13]. Normalisation of the growth rate is to \( \gamma_0 = \omega_e \sqrt{\pi/2} (c/v_e). \)

Since the growth rate depends on anisotropy \( A \) it is given for the two cases \( A = 0.1, 1 \). Note the competition between growth rate and fluctuation level. At long wavelengths the high fluctuation level partially compensates for the low growth rate. The range of wavelength of interest in the magnetospheric tail is shown shaded. It centres around maximum thermal fluctuation level.

![Fig. 2. Wavenumber dependencies of the normalised fluctuation spectrum and normalised growth rates.](image)

Fig. 3. Normalised thermal fluctuation level at maximum growing Weibel wave number \( k_{\perp m} \), as function of Debye length \( \lambda_D \). The fluctuations are normalized to their value at \( T_e = 1 \ eV, N = 1 \ cm^{-3} \). The Debye normalisation is taken to the Debye length \( \lambda_{D1} \) at these numbers. As suggested by Fig. 2 the initial fluctuation level from where the maximum unstable Weibel mode grows decreases \( \sim A^{-1} \) because of its dependence on \( k_{\perp m} \), which increases as \( \sqrt{A} \). Since the spectral energy density of fluctuations decreases \( \sim k^{-3} \), the initial level of the fastest growing Weibel mode also decreases with growing anisotropy.

The unstable Weibel spectral energy density evolves according to

\[
\langle |B(t, k_{\perp m}, 0)|^2 \rangle \approx \langle |b|^2 \rangle_{k_{\perp m}0} \exp(2\gamma_{W,m} t) \quad (17)
\]

The growth time of the fastest growing mode follows from this expression as

\[
\tau_{W,m} \approx \frac{1}{2 \gamma_{W,m} \ln \langle |B(k_{\perp m}, \tau_{W,m})|^2 \rangle / \langle |b|^2 \rangle_{k_{\perp m}0}} \quad (18)
\]

The spectral energy density of a \( |B| = 1 \ nT \) magnetic field fluctuation is \( \langle |B_{inT}|^2 \rangle_{k_{0}} \approx 4.3 \times 10^{-12} \ V^2 s^3 / m \). This value may be used when estimating the time it needs for the maximum growing thermal-anisotropy driven Weibel mode in the magnetotail current sheet to grow up to a value comparable to the external (lobe) magnetic field \( B_0 \sim 50 \ nT \). If we take the growth rate in the range \( 1 \lesssim \gamma_{W,m} < 50 \ Hz \) which holds for \( 0.1 \lesssim A < 1 \), short growth times from thermal level to 1 nT fields of the order of

\[
\tau_{W,m} \gtrsim 0.1 \ s \quad (19)
\]

are obtained, corresponding to mostly a few seconds of growth time in the magnetospheric tail. Given the uncertainty of the numerical values used, this is not an unreasonable estimate of the length of the ignition phase that initiates reconnection in the tail current sheet, i.e. the time to produce
initial X-points which subsequently start reconnection. Typical times for the evolution of substorms following onset of reconnection range from minutes to few tens of minutes and depend on the connection of the magnetotail reconnection site to the response of the ionosphere.

3.2 Long wavelength Weibel modes

Fastest growth corresponds to very short wavelengths $k \lambda_e \lesssim \sqrt{A}/3$. There may, however, as well be competition between decreasing growth rate and increasing initial fluctuation level at long wavelengths as shown in Fig. 2.

Equation (12) suggests that the spectral energy density of thermal fluctuations for $k^2 \lambda_e^2 < A + m_e/m_i$ increases as $\sim k \lambda_e$. In isotropic plasma $A = 0$ this implies wavelengths $\lambda > 2\pi \lambda_e \sqrt{m_i/m_e} \approx 300 \lambda_e$. In the magnetospheric tail the wavelength of maximum thermal fluctuation level is thus $\lambda \sim 1500$ km. The spectral energy density in this long-wavelength range is given by Eq. (15).

At such wavelengths one can neglect the term $k \lambda_e/k_0$ in the expression for the Weibel growth rate. For small anisotropies $A < 1$ the growth rate becomes

$$\gamma_W \approx \frac{4Ac}{\lambda} \sqrt{\frac{\pi T_{e\perp}}{m_e c^2}}$$

Inserting the long-wavelength restriction on $\lambda$ yields

$$\gamma_W \ll 0.01 A \sqrt{\frac{\pi T_{e\perp}}{m_e c^2}} \omega_e \approx 1.4A \sqrt{T_e [eV] N_{cm^{-3}}} \text{ Hz}$$

The Weibel growth rate, for $A \sim 0.1$ and $T_e \sim 0.1$ keV, in this wavelength range is thus of the order of $\gamma_W \sim 0.1$ Hz, one order of magnitude less than at maximum growth, yielding exponential times $\tau_{\gamma_W}^{-1} \sim 1$ s and growth times $\tau_{\gamma_W} \sim 10$ s (see Fig. 5). This is the time a Weibel wavelength of $\lambda \sim 1500 - 3000$ km, i.e. roughly half one Earth radius, needs to grow from thermal level to an amplitude of 1 nT in the geomagnetic tail prior to onset of reconnection.

In anisotropic plasma $A \neq 0$ and we may relax the condition on the wavelength. In this case the mass ratio in the thermal fluctuation expression becomes unimportant for reasonably large $A \gg m_e/m_i$. Then long wavelengths imply that $k \lambda_e \ll \sqrt{A} = k_0 \lambda_e$ and

$$\langle |\mathbf{b}|^2 \rangle_{k_0} \approx \frac{\mu_0 m_e c^2}{\omega_e} \sqrt{\frac{\pi T_{e\perp}}{m_e c^2}} \frac{k \lambda_e}{A^2}$$

$$\ll \frac{\mu_0 m_e c^2}{\omega_e} \sqrt{\frac{\pi T_{e\perp}}{A^3 m_e c^2}} \approx 3.2 \times 10^{-24} A^{-2} \sqrt{T_e [eV]} \text{ V}^2 s^3 / m$$

With $T_{e\perp} = 100$ eV, and $A = 0.1$ and using the former expression for the growth rate, one correspondingly expects growth times from thermal level of the order of $\tau_W \sim 100$ s, between 1 and 2 min, for wavelengths of the order of $\lambda \sim 1000$ km $\gg 2\pi \lambda_e / \sqrt{A} \sim 110$ km.
4 Collisionless reconnection scenario

These estimates are sufficiently encouraging for developing a microscopic scenario for collisionless reconnection as follows: Assume a plane Harris current layer \( J_x = -J_0 \text{sech}^2(2z/\Delta) \), with \( \Delta \) the layer half-width, separating two (lobe) regions of antiparallel magnetic fields. The magnitude of the field changes as \( B_z(z) = B_0 \tanh(2z/\Delta) \). Let this current layer be (locally) compressed until its width shrinks to \( \Delta \sim \lambda_i \). In the ion-inertial region the ions become locally non-magnetic and are accelerated in \( -\hat{y} \) direction by the cross-field electric potential, carrying the pure ion Harris current. Electrons remain magnetised, transporting the magnetic field with inward velocity \( -E/B(z) \) thus giving rise to Hall currents (Sonnerup [1979]) which are restricted solely to the ion-inertial region and close along the magnetic field lines which connect them to the ionosphere (Treumann et al. [2009]). The \( z \)-dependence of the Hall currents is

\[
J_H(z) \approx eN_eB_z \frac{2B_0}{\sinh(4z/\Delta)} \left[ 1 - \Theta(|z| - |z_i|) \Theta(|z| - |z_e|) \right]
\]

where \( z_i, e \equiv \xi_i, e \lambda_i, e \) and \( 1 \lesssim \xi_i, e \in \mathbb{R} \) are rational numbers close to but larger than unity. Field line bending in reconnection is not taken into account here. Hall currents vanish in the centre of the current sheet at distances \( |z|/\xi \lesssim \lambda_e \) less than the electron inertial length \( \lambda_e = c/\omega_e \), where the electrons demagnetise. For the onset of reconnection they are thus of no importance.

The non-magnetic electrons in the central current sheet experience the cross-field potential \( \Delta U \), accelerate in \( +\hat{y} \) direction and become the primary carriers of the cross-tail current here. Accelerated to large cross-tail velocities \( V_e > v_e \), these electron currents excite the Buneman two-stream instability on growth times shorter than \( \tau_B < 10^{-5} \) s, a number holding in the magnetotail. The Buneman instability stabilizes within 0.01 < \( \tau \) < 0.1 s by heating the trapped electrons along \( \pm\hat{y} \) until \( \psi_e \sim V_e \). As a consequence the current sheet electrons develop a positive temperature anisotropy 0 < \( A < 1 \) which is sufficiently large to drive the Weibel mode unstable and result in the generation of a stationary magnetic Weibel-field \( B_W = (B_x, 0, B_z) \) in the current sheet with components in the \( (x, z) \)-plane perpendicular to both, the current flow and anisotropy directions. The fastest growing wavelength is \( \lambda_W \sim 2\pi \lambda_e \sqrt{3/\Delta} \).

In the magnetosipal tail current layer we have \( \lambda_e \approx 5.4/\sqrt{N_{\text{en}} - 1} \) km. The maximum growing wavelength of the Weibel magnetic vortices thus becomes shorter than \( \lambda_m < 180 \) km, the value obtained for a weak anisotropy \( A \sim 0.1 \). The time for this field to grow to values of the order of 1 nT (or a fraction of it) is of the order of one or few seconds.

It is usually claimed that the Weibel instability stabilizes when the electron gyroradius in the Weibel magnetic field becomes comparable to the Weibel wavelength. When the electrons are accelerated to \( > \) keV energies their gyroradius in a 1 nT magnetic field becomes the order of \( \sim 100 \) km, roughly the same order as the above estimated maximum wavelength. Thus the short wavelength Weibel field has sufficient time to grow to substantial values until it stabilizes self-consistently by deflection the current electrons. Prior to this the Weibel field has penetrated the current sheet forming vortices and vertexes which serve as seed-\( X \)-points for reconnection which may then proceed at about along the lines that were discussed long ago in an attempt of formulating a kinetic theory of collisionless reconnection by \textit{ad hoc} imposing a \( B_z \)-field on the current layer (Galeev and Zelenyi [1975] Sagdeev [1979]). This attempt led to the proposal of a scenario for the spontaneous onset of magnetospheric substorms. In the presence of \( B_z \neq 0 \) the current sheet is in a metastable state that goes spontaneously unstable. The main deficiency of this theory was the lack of any reason for the appearance of \( B_z \). Imposing it \textit{ad hoc} is the equivalent of igniting reconnection artificially.

What happens in the long wavelength regime? Here we have \( \lambda \gtrsim 300 \lambda_e \approx 1.5 \times 10^3 \) km. The growth time to observable/relevant amplitudes we found to be of the order of \( \tau_W \sim 10 \) s, which is not unreasonable for the processes going on there.

The short wavelength modes grow about ten times faster than the long wavelength modes but may not be of substantial importance for reconnection until they cascade inversely down to longer wavelength structures. This could be provided by the coalescence of magnetic islands which is strongest at short wavelengths. The Weibel instability in this case excites an entire spectrum of magnetic field structures in the current layer. In the geomagnetic tail both the short Weibel modes and the long wavelengths modes provide seed \( X \)-points on geophysically reasonable spatial and temporal scales.

5 Discussion and Conclusions

The present approach has so far only implicitly taken into account the narrow width of the non-magnetic central current region which imposes boundary conditions on the evolution of the Weibel mode. Continuity of \( \mathbf{B} \) at \( z = \pm \lambda_e \) implies a vanishing \( B_z \) here. As demonstrated, this imposes limits on wave number and propagation angles of the Weibel mode. In addition the presence of a boundary implies that \( B_z \) is either parallel or antiparallel. Clearly the parallel case is preferred as the antiparallel case generates small-scale current bifurcation. On the other hand, this is possible because the evolution of the Weibel mode is completely independent of the presence of the external field.

One thus distinguishes between two types of Weibel modes depending on their propagation directions \( \pm \hat{x} \). One of them (Figure 6a) just causes seed-\( X \)-points, the other (bi- or trifurcated) mode may lead to multiple – probably explosive – reconnection (see Figure 6b). The reason for an explo-
Fig. 6. Sketch of the electron-inertial region (width $\Delta z = 2\lambda_e$) around the centre of the current layer, embedded into the ion-inertial region. Hall-electrons carrying the Hall-current in the ion-inertial region enter the electron-inertial region (shown on one side of the current layer only) with isotropic temperature distribution, experience the electric field $E$, accelerate into $+\hat{y}$ direction. After being heated by the two-stream instability they develop a temperature anisotropy and excite magnetic Weibel-vortices (blue) along $\hat{x}$ the axes of which serve as seed-$X$ points for reconnection. The condition that $\Delta z = 0$ at $z = \pm\lambda_e$ allows for two types of Weibel vortices: (a) a symmetric (magnetically continuous) vortex-mode (same direction as the external fields $B_{0x}$), and a (b) anti-symmetric non-continuous vortex-mode (antiparallel to the external field) which yields tail-current bifurcation. The anti-symmetric mode gives rise to multiple reconstruction sites (shown in green colour) and presumably leads to explosive reconnection.

Hence, one suspects that the tail reconstruction structure is inherently three-dimensional putting all two-dimensional models in question.

In addition, the present theory refers to stationary ions. The ions that carry the tail current move into direction $-\hat{y}$. Hence, the Weibel structures and the resulting reconstruction sites as a whole move in the direction of the combined speed of the electron holes and current ions. Numerical simulations suggest (cf., e.g., [Newman et al. 2001]) that this direction is opposite to the electron flow velocity $V_e$, i.e. in the direction of ion flow. As a result one expects that the whole set of magnetotail-reconnection sites will displace slowly – roughly at translational velocity of the ion-sound speed $c_s \simeq \sqrt{T_e/m_i} \sim 200$ km/s – into $-\hat{y}$-direction, the direction of ion flow, which in the magnetotail is westward. This is in accord with observation of the initial westward displacement of substorm sources. Mapping along the stretched magnetic field lines into the ionosphere decreases this translational westward drift speed by about one order of magnitude.

In conclusion, it is the thermal-anisotropy driven Weibel instability which provides the magnetic field to penetrate the inner region of the current layer, generates a local normal field component $B_z$ and, by producing short wavelength magnetic vortices and vertexes, it may ignite reconnection on a time scale of tens of seconds to few minutes in the magnetotail. This is in rough agreement with observations, e.g. in the magnetotail, and is sufficiently short for initiating magnetospheric substorms.

One may ask what structure of the field and current layer is expected in the direction parallel to the current flow. This question cannot be answered without detailed analysis. However, one may argue that the structure along the current is determined by two facts: the mechanism of electron heating, and the dynamics of the ions. Electron heating occurs in electron holes which have (short) longitudinal extension of $\Delta y \sim 100\lambda_D$, where $\lambda_D$ is the Debye length. The heating scale is orders of magnitude longer including several to many phase space holes. However, though it is long, it is still microscopic. In the magnetotail current layer this length becomes the order of several $\sim 100$ km to few 1000 km only.
This logically consistent chain of processes provides a satisfactory mechanism for the spontaneous self-ignited onset of fast magnetic reconnection in thin collisionless current layers. Its numerical verification requires three-dimensional PIC simulations resolving the fully electromagnetic electron dynamics in the current layer.

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