Composite Hashing for Data Stream Sketches

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Abstract—In rapid and massive data streams, it is often not possible to estimate the frequency of items with complete accuracy. To perform the operation in a reasonable amount of space and with sufficiently low latency, approximated methods are used. The most common ones are variations of the Count-Min sketch. By using multiple hash functions, they summarize massive streams in sub-linear space. In reality, data item ids or keys can be modular; e.g., a graph edge is represented by source and target node ids, a 32-bit IP address is composed of four 8-bit words, a web address consists of domain name, domain extension, path, and filename, among many others. In this paper, we investigate the modularity property of item keys, and systematically develop more accurate, composite hashing strategies, such as employing multiple independent hash functions that hash different modules in a key and their combinations separately, instead of hashing the entire key directly into the sketch.

However, our problem of finding the best hashing strategy is non-trivial, since there are exponential number of ways to combine the modules of a key before they can be hashed into the sketch. Moreover, given a fixed size allocated for the entire sketch, it is hard to find the optimal range of all hash functions that correspond to different modules and their combinations. We solve both these problems with extensive theoretical analysis, and perform thorough experiments with real-world datasets to demonstrate the accuracy and efficiency of our proposed method, MOD-Sketch.

Index Terms—Stream summary, sketch, Count-Min, composite hashing, modular keys.

I. INTRODUCTION

In many domains such as real-time IP traffic, telephone calls, text data from email/SMS/blog, web clicks and crawls, measurements from sensors and scientific experiments, rapid and massive volume of data are generated as a stream [10], [21]. In order to process fast streaming data, a growing number of applications relies on devices such as network interface cards, routers, switches, cell processors, FPGAs, and GPUs [25]; and usually these devices have very small on-chip memory. Whether in specialized hardware or in conventional architectures (e.g., static RAM, such as CPU cache), efficient processing of fast and large stream data requires creation of a succinct synopses in a single pass over that stream [8]. These summaries must be updated incrementally with incoming items. They should also support online query answering, e.g., estimating the frequency of an item in a long stream. Due to their smaller sizes compared to the original stream, it is often not possible to answer queries with complete accuracy — reducing summary size increases the efficiency, but also reduces the accuracy.

We study frequency estimation queries over data streams: Given a data item as a query, we estimate the frequency count of that item in the input stream. In the literature, sketch data structures [11], [9], [8], [6], [16], [20] have been employed for frequency estimation queries. By using multiple hash functions, they summarize massive data streams within a limited memory space. For example in Figure 1(a) we depict the Count-Min sketch, having $w$ such pairs of hash functions. To utilize same space, we ensure that $a \times b = h$. In this work, we investigate which design is better, and the optimal values of $a$ and $b$, given $h$.

In various settings, item keys are modular, i.e., consisting of multiple ordered parts. Many domains of stream data such as graph edges (e.g., web graphs, transportation networks, and social networks), IP addresses, telephone numbers, web addresses belong to this category. This enables us with more optimization scopes for hashing, such as one can hash different modules in a key and their combinations separately (i.e., composite hashing). In Figure 1(b) we illustrate an item key having modularity two, and the two parts are hashed via two independent hash functions separately, the first hash function with range $a$ and the second one with range $b$. To allocate the same amount of space as Count-Min, we ensure that $a \times b = h$, and the sketch (referred to as the MOD-Sketch, an abbreviation for modular sketch) uses $w$ such pairs of hash functions. Two immediate questions that arise are as follows: (1) How does one select the optimal values of $a$ and $b$? (2) Between Count-Min and MOD-Sketch, which one results in more accuracy?

Our problem is non-trivial — answers to both questions depend on underlying properties of the stream and parameters of the sketch. For example, consider the case: $a=b=\sqrt{h}$, and a stream of graph edges in the form $(x, y, f)$. Here, $(x, y)$ represents a directed edge from the source node $x$ to the target node $y$, with frequency $f$. Next, consider the edges having
the same source node. MOD-Sketch maps the source node to same hash values, resulting in more collisions across edges having the same source node. This problem is exacerbated when the number of source nodes is more than that of target nodes in the graph stream. However, such skewness also allows us to find better MOD-Sketch parameters, e.g., \( a > b \). Similar optimization, on the contrary, is not possible for Count-Min.

We find optimal values of \( a \) and \( b \) in a data-dependent manner: By sampling a small portion of the stream, and analyzing out-degrees of source nodes and in-degrees of target nodes in this sample. Furthermore, we decide the best choice between Count-Min and MOD-Sketch, again by sampling a small portion of the stream, and then based on the standard deviation of the values stored in different cells of these two sketches. We develop our solutions with theoretical guarantees, and demonstrate their performance with empirical results.

After studying the optimal hashing strategy for keys with modularity two, we focus on generalizations where keys can have higher modularity, e.g., a 32-bit IP address has modularity four (composed of four 8-bit words), thus opening the stage to a wider scope of optimizations. This problem is more complex due to two reasons. (1) There are exponential number of ways to combine the modules of a key before they can be hashed into the sketch. (2) Given a fixed \( h \), it is hard to find the optimal ranges of all hash functions that correspond to different modules and their combinations. We propose greedy heuristics, and empirically demonstrate that our method achieves higher accuracy compared to several baselines.

**Our contribution and roadmap.** We summarize our contributions as follows.

- We investigate the fundamental problem of constructing more accurate, composite hashing for sketches over data streams (Section \[III\]).
- We devise scalable, effective, and data-dependent solution for finding the optimal hashing ranges for different modules of an item key, with theoretical guarantees (Section \[IV\]).
- We further generalize our algorithm to find good-quality hashing strategies for keys with modularity higher than two (Section \[V\]).
- We perform detailed experiments to demonstrate the accuracy, efficiency, and throughput of our developed MOD-sketch, comparing it with several baselines including Count-Min \[9\] and Equal-Sketch \[19\], \[29\], while also demonstrating its generalizability by implementing MOD-Sketch on top of the FCM sketch \[30\] (Section \[VI\]).

## II. RELATED WORK

**Data stream sketches.** The problem of synopsis construction has been studied extensively \[8\] in the context of a variety of techniques such as sampling \[15\], \[5\] (including graph stream sampling, e.g., \[24\], \[2\]), wavelets \[18\], \[14\], histograms \[17\], sketches \[11\], \[9\], \[3\], \[6\], \[10\], \[20\], and counter-based methods, e.g., Space Saving \[22\] and Frequent \[7\]. Sketches and counter-based approaches are widely used for stream data summarization. Sketches are typically used for frequency estimation, which is the focus of this paper. Sketches keep approximate counts for all items, counter-based approaches maintain approximate counts only for the top-\(k\) frequent items.

Sketches bear similarities to Bloom filters \[4\] in that both employ hashing to summarize data; however, they differ in how they update the hash buckets and use these hashed data to derive estimates. Among various sketches \[9\], \[8\], \[6\], \[12\], \[13\], \[16\], \[20\], Count-Min \[9\] is widely studied, it achieves good update throughput in general, as well as very high accuracy on skewed distributions. Several approaches have been proposed to further improve its accuracy, e.g., frequency-aware hashing (FCM \[30\], gSketch \[32\], and ASketch \[26\]), and non-uniform counter sizes (Cold Filter \[34\]). However, they do not consider the modularity of item keys unlike ours.

In the domain of graph edge stream, Count-Min has been extended to gMatrix \[19\] and TCM \[29\], which separately hash the source and target node ids corresponding to an edge. These approaches follow composite hashing. However, unlike ours, they allocate the same hashing range to both source and target nodes. Moreover, they do not address the problem of hashing data items with modularity higher than two. Composite hashing over IP data (i.e., having modularity four) is discussed in \[23\]. Once again, they allocate the same hashing range to all 4 bytes of the IP address. For generality, in this paper we refer to these prior works as Equal-Sketch.

Based on detailed experiments, our proposed MOD-Sketch results in higher accuracy compared to Equal-Sketch.

**Data-dependent and other composite hashing.** Recently, data-driven learning methods for advanced hash functions (i.e., learning to hash) have become popular, particularly in the context of nearest neighbor queries (for a survey, see \[31\]). Composite hashing for nearest neighbor queries has been studied in \[23\]. While we select ranges of our composite hash functions in a data-dependent manner, our focus is sketches for data stream summarization. This is different from the existing work on learning to hash for nearest neighbor queries.

## III. PRELIMINARIES

The incoming data stream contains tuples \((i_1, f_1), (i_2, f_2), \ldots, (i_t, f_t), \ldots\). Here, \((i_t, f_t)\) denotes the arrival of the \(t\)-th tuple with item \(i_t\) having an associated positive count \(f_t\). In many applications, the value of \(f_t\) is set to one, though we assume an arbitrary positive count in order to retain the generality of our model. As an example, in a telecommunication application, the frequency count \(f_t\) may denote the number of seconds in the \(t\)-th phone conversation. An item may appear multiple times in the stream, i.e., it is possible that \(i_t = i_{t'}\), for \(t \neq t'\). When we issue a query, e.g., finding the frequency of an item \(i\), we are looking for the aggregate count of that item in the stream so far. While sketch-based methods including ours can be adapted for time-window queries \[1\], as well as for removal of items \[7\] (i.e., a negative-count-update can be performed in the same way as a positive-count-update, so long as the overall count of an item never becomes negative), we do not consider them in this work. Furthermore, we use the
notation \(i\) interchangeably to denote an item as well as its key, because a key uniquely identifies an item.

An item key is often modular, i.e., composed of \(n\) ordered parts: \(i = (x^{(1)}, x^{(2)}, \ldots, x^{(n)})\). We assume that every module \(x^{(j)}\) is drawn from a predefined set of integers. For example, in case of telecommunication network, each item is a communication (edge) from source to target users (nodes). The domains of source and target nodes are generally known apriori, and the edges between them arrive continuously in the stream.

In this paper, we shall discuss our framework, MOD-Sketch (an abbreviation for modular sketch) as a composite hashing strategy on top of the Count-Min, one of the most widely-studied sketches. Nevertheless, the MOD-Sketch framework is generic, and it can be applied in combination with several other sketches. In our experiments in Section VI, we also demonstrate the performance of MOD-Sketch when it is constructed with other underlying sketches. Next, for ease in the presentation, we introduce the standard Count-Min sketch.

A. Count-Min

In Count-Min, a hashing approach is employed to approximately maintain the frequency counts of a large number of distinct items in a stream (Figure 1(a)). We use \(w = \lceil \ln(1/\delta) \rceil\) pairwise independent hash functions, each of which maps onto uniformly random integers in the range \(h = [0, e/\epsilon]\), where \(\epsilon\) is the base of the natural logarithm, \(\epsilon\) and \(\delta\) are terms to define the error and the probabilistic error guarantee, respectively, which we shall introduce shortly. The data structure consists of a 2-dimensional array with \(h \times w\) cells of length \(h\) and width \(w\). Each hash function corresponds to one of \(w\) 1-dimensional arrays with \(h\) cells each. Next, consider a data stream with items drawn from a massive set of domain values. When an item is received, we apply each of the \(w\) hash functions to map onto a number in \([0 \ldots h - 1]\). The count of each of these \(w\) cells is incremented by 1. To estimate the count of an item, we determine the set of \(w\) cells to which each of the \(w\) hash-functions maps, and compute the minimum value among all these cells. Let \(c_i\) be the true value of the count being estimated. We note that the estimated count is at least equal to \(c_i\), since we are dealing with non-negative counts only, and there may be an over-estimation because of collisions in hash cells. It has been shown in [9] that for a data stream with \(L\) arrivals, the estimate is at most \(c_i + \epsilon \cdot L\) with probability at least \(1 - \delta\). In the event that the items have frequencies associated with them, we increment the corresponding count with the appropriate frequency. The same bounds hold in this case, except that we define \(L\) as the sum of the frequencies of the items received so far.

For Count-Min to be effective, the hash functions are required to be pairwise independent. Following the Count-Min work [9], we select modular hash functions as follows.

\[
H(i) = ((q \times i + r) \mod P) \mod h \quad (1)
\]

Here, \(P\) is a prime number larger than the maximum value of any key id \(i\), and \(h\) is the range of the hash function. We select \(q\) and \(r\) uniformly at random from the interval \((0, P - 1)\).

B. Hashing Keys with Modularity Two

We next introduce our problem for hashing keys with modularity two, i.e., \(i = (x^{(1)}, x^{(2)})\). The hashing problem for higher modularity keys will be discussed in Section VI.

As we demonstrated in Figure 1 there are two possible choices for hashing the keys with modularity two. (1) Concatenate two ordered integer modules, construct a single integer id, i.e., \(i = x^{(1)}x^{(2)}\), and hash it directly in the Count-Min. To distinguish between the keys such as \((1, 12)\) and \((11, 2)\), we first consider the domains of the modules before concatenating them. For example, if the domain of each module is the set of integers \(\in (0, 99)\), then \((1, 12)\) is concatenated as 0112, whereas \((11, 2)\) is concatenated as 1102. (2) Employ two independent hash functions that hash two modules of the key separately in MOD-Sketch. In total, MOD-Sketch uses \(w\) such pairs of hash functions. All \(2w\) hash functions need to be pairwise independent, which is ensured by modular hash functions as in Equation [1]. Let the range of the hash functions in Count-Min be \(h\), whereas for MOD-Sketch the ranges are \(a\) and \(b\), respectively. To ensure the same amount of space, we have: \(a \times b = h\). The other parameter, i.e., the number of hash functions, \(w\) remains the same for both sketches, as the probability of the error bound is determined by \(w\), whereas the actual error bound depends on the range of the hash functions, \(h, a,\) and \(b\).

Probabilistic accuracy guarantee. We now derive the accuracy guarantees for Count-Min and MOD-Sketch.

Theorem 1. Let the total frequency of items received so far in the stream be denoted by \(L\). Let \(Q(x^{(1)}, x^{(2)})\) be the true frequency of the item \(i = (x^{(1)}, x^{(2)})\). Let \(\epsilon \in (0, 1)\) be a very small fraction. Consider Count-Min with hash function range \(h\) and width \(w\). Then, with probability at least \(1 - (1/\epsilon)^w\), the estimated frequency \(Q(x^{(1)}, x^{(2)})\) is related to the true frequency by the following relationship:

\[
\frac{Q(x^{(1)}, x^{(2)})}{Q(x^{(1)}, x^{(2)})} \leq Q(x^{(1)}, x^{(2)}) \leq Q(x^{(1)}, x^{(2)}) + L\epsilon \quad (2)
\]

Proof. We note that \(Q(x^{(1)}, x^{(2)})\) is always an over-estimate on \(Q(x^{(1)}, x^{(2)})\), since all frequencies are assumed to be non-negative. Any incoming item is equally likely to map onto one of \(h\) cells of a Count-Min hash function. The probability that any incoming item maps onto a particular cell is given by \(1/h\). Therefore, the expected number of items which get mapped onto the cell \((A_k(x^{(1)}), B_k(x^{(2)}), k)\) is given by at most \(L/h\). Let the number of such items for the \(k\)-th hash function be denoted by the random variable \(R_k\). Then, by using the Markov inequality, we have:

\[
P(R_k > L \cdot \epsilon) \leq E[R_k]/(L \cdot \epsilon) \leq 1/(\epsilon h) \quad (3)
\]

For the estimate to violate Inequality [1] we require the above condition to be true for all \(k \in (1, w)\). The probability that this is true is given by at most \(1 - (1/\epsilon)^w\). The result follows.

For the above probability to be less than 1, we need the value of \(h > 1/\epsilon\). Furthermore, since \(w\) occurs in the exponent, the robustness of the above result can be magnified even for
By combining the three above inequalities with the following

\[ Q(x^{(1)}, x^{(2)}) \leq Q(x^{(1)}, x^{(2)}) \]
\[ \leq Q(x^{(1)}, x^{(2)}) + [L + O(\ast, x^{(2)}) \cdot b + O(x^{(1)}, \ast) \cdot a] \cdot \epsilon \quad (4) \]

**Proof.** Any incoming item, for which the ordered modules are neither \( x^{(1)} \) nor \( x^{(2)} \), is equally likely to map onto one of \( ab \) cells of a hash function. The probability that any incoming item maps onto a particular cell is given by \( 1/ab \). Therefore, the expected number of spurious items for which the modules are neither \( x^{(1)} \) nor \( x^{(2)} \), yet they get mapped onto the cell \( (A_k(x^{(1)}), B_k(x^{(2)}), k) \) is given by at most \( L/(ab) \). Let the number of such spurious items for the \( k \)-th hash function be denoted by the random variable \( R_k \). Then, by using the Markov inequality, we have:

\[ P(R_k > L \cdot \epsilon) \leq E[R_k]/(L \cdot \epsilon) \leq 1/(abe) \quad (5) \]

Next, we examine the case of spurious items for which the first module is \( x^{(1)} \). The number of such items is \( O(x^{(1)}, \ast) \) and the expected number of such items which map onto the entry \( (A_k(x^{(1)}), B_k(x^{(2)}), k) \) is given by \( O(x^{(1)}, \ast)/a \). Let \( U_k^{(1)} \) be the random variable representing the number of such items. Then, by using the Markov inequality, we get:

\[ P(U_k^{(1)} > O(x^{(1)}, \ast) \cdot a \cdot \epsilon) \leq E[U_k^{(1)}]/(O(x^{(1)}, \ast) \cdot a \cdot \epsilon) \leq 1/(abe) \quad (6) \]

Similarly, we denote by the random variable \( U_k^{(2)} \) the number of items for which the second module is \( x^{(2)} \). The number of such items is \( O(\ast, x^{(2)}) \) and the expected number of such items which map onto the entry \( (A_k(x^{(1)}), B_k(x^{(2)}), k) \) is given by \( O(\ast, x^{(2)})/a \). With Markov inequality, we have:

\[ P(U_k^{(2)} > O(\ast, x^{(2)}) \cdot b \cdot \epsilon) \leq E[U_k^{(2)}]/(O(\ast, x^{(2)}) \cdot b \cdot \epsilon) \leq 1/(abe) \quad (7) \]

By combining the three above inequalities with the following rule \( P(A \cup B \cup C) \leq P(A) + P(B) + P(C) \), we get:

\[ P(R_k + U_k^{(1)} + U_k^{(2)} > L \cdot \epsilon + O(\ast, x^{(2)}) \cdot b \cdot \epsilon + O(x^{(1)}, \ast) \cdot a \cdot \epsilon) \leq 3/(abe) \quad (8) \]

For the estimate to violate Inequality 8, we require the above condition to be true for all \( k \in (1, w) \). The probability that this is true is given by at most \( 1 - (3/abe)^w \). The result follows. \( \square \)

For the above probability to be less than 1, we require \( ab > 3/\epsilon \). As earlier, since \( w \) occurs in the exponent, the robustness of the above result can be magnified for modest values of \( w \).

C. Problem Statement

We state our problems in regards to hashing keys with modularity two as follows.

**Problem 1.** For a data stream and a pre-defined length \( h \), find the most accurate MOD-Sketch range parameters \( a \) and \( b \) such that \( a \times b = h \).

**Problem 2.** For a data stream and a pre-defined length \( h \), select the most accurate sketch between Count-Min and MOD-Sketch having the same size, i.e., \( a \times b = h \).

These are difficult problems as the entire stream may not be available for such computations, and there are several possible values for \( a \) and \( b \) (satisfying \( a \times b = h \)). We discuss our algorithms for solving Problems 1 and 2 in the next section.

IV. ALGORITHMS FOR MODULARITY TWO

A. Finding High-Quality MOD-Sketch Parameters

We set to find high-quality hashing ranges \( a \) and \( b \) for MOD-Sketch, by comparing its accuracy guarantee with that of a baseline technique. For the baseline method, we consider a special version of MOD-Sketch, referred to as the Equal-Sketch, where \( a = b = \sqrt{h} \). Given a fixed \( h \) and for a data stream, we select \( a \) and \( b \) such that the error of MOD-Sketch is as small as possible, compared to the error produced by Equal-Sketch. We note that the approach itself does not ensure finding optimal values of \( a \) and \( b \). However, based on our empirical results, hashing ranges \( a \) and \( b \) found in this manner are of high-quality, and the accuracy of MOD-Sketch, in fact, becomes comparable to that of an exhaustive method which experimentally finds the best choice of hash function ranges corresponding to two modules of the key.

**Theorem 3.** Consider an item \( i = (x^{(1)}, x^{(2)}) \) with its estimated frequency via MOD-Sketch and Equal-Sketch as \( Q(x^{(1)}, x^{(2)}) \) and \( Q_E(x^{(1)}, x^{(2)}) \), respectively. Let \( O(x^{(1)}, \ast) \) be the sum of frequencies of the items having the first module as \( x^{(1)} \), and \( O(\ast, x^{(2)}) \) the sum of frequencies of the items having the second module as \( x^{(2)} \). We denote by \( \alpha \) be the ratio: \( \alpha = O(x^{(1)}, \ast)/O(\ast, x^{(2)}) \), and by \( \beta \) the ratio: \( \beta = a/b \). Let \( \epsilon \in (0, 1) \) be a very small fraction. Then, with a high probability, \( Q(x^{(1)}, x^{(2)}) \) is smaller than \( Q_E(x^{(1)}, x^{(2)}) \) by the largest margin when \( \beta = 1/\alpha \). Formally,\

\[ P\left(Q_E(x^{(1)}, x^{(2)}) - Q(x^{(1)}, x^{(2)}) \leq O(x^{(1)}, \ast)/(\sqrt{h} - a)\epsilon + O(x^{(1)}, \ast)(\sqrt{h} - a)\epsilon \geq 1 - 2(3/abe)^w \right) \quad (9) \]

The difference of \( Q(x^{(1)}, x^{(2)}) \) from \( Q_E(x^{(1)}, x^{(2)}) \) is maximized when \( \beta = 1/\alpha \).

**Proof.** We denote by \( Q(x^{(1)}, x^{(2)}) \) the true frequency of the item \( i = (x^{(1)}, x^{(2)}) \). Let the total frequency of items received so far in the stream be \( L \). From Theorem 2 we get:
Theorem 3 helps us to compute the optimal values of \( \alpha \) and \( \beta \). Choice between MOD-Sketch & Count-Min is two times larger than times, and between the multi-set consisting of minimum, maximum, median, average, etc.) to compute the sample. Next, we employ an aggregate operator (e.g.,

\[
P(Q_E(x^{(1)}, x^{(2)}) - Q(x^{(1)}, x^{(2)}) \leq L\epsilon + O(\epsilon, \sigma_2^2) = 1/3)
\]

By combining the two above inequalities, we derive:

\[
P(Q(x^{(1)}, x^{(2)}) - Q(x^{(1)}, x^{(2)}) \leq L\epsilon + O(\delta, \epsilon^2) = 1/3)
\]

Furthermore, it can be shown with simple calculation that the difference \( O(\epsilon, \sigma_2^2) = 1/3 \) is two times larger than \( O(\epsilon, \sigma_2^2) = 1/3 \), then the optimal setting of \( \beta = 2/1 \). Hence, compared to an Equal-Sketch with \( a = 600 \), the optimal MOD-Sketch with \( a = 848 \) and \( b = 424 \) is expected to produce the least amount of error. Our result is intuitive, since \( O(x^{(1)}, *) < O(\epsilon, \sigma_2^2) \) suggests that there are generally large number of distinct source nodes as that of distinct target nodes, and this results in \( a > b \).

In Theorem 3, we compute the optimal value of \( \beta = a/b \) for a specific item \( i = (x^{(1)}, x^{(2)}) \) in the stream. For a data stream, we first sample a small portion of the incoming stream uniformly at random. For every item presented in our sample, we also estimate its related \( \alpha = O(x^{(1)}, *)/O(\epsilon, \sigma_2^2) \) from the sample. Next, we employ an aggregate operator (e.g., minimum, maximum, median, average, etc.) to compute the aggregated \( \alpha_{agg} \). We finally derive \( \beta \) as \( 1/\alpha_{agg} \). We illustrate our method with an example below. Based on our detailed empirical evaluation over several real-world stream datasets, we find that about 2~4% stream sample, together with the median aggregate of \( \alpha \), is generally sufficient to estimate a high-quality value of \( \beta \).

**Example 1.** Assume via uniform at random sampling, we obtained 3 items: (1, 2), (1, 3), (2, 3) with aggregated frequency 15, 5, 7, respectively. We compute the optimal values for these items as follows. \( \alpha(1, 2) = O(1, 2)/O(1, 2) = 18/13 \). In a similar way, we find \( \alpha(1, 3) = 18/12 \) and \( \alpha(2, 3) = 7/12 \). We next compute the median of these \( \alpha \) values, i.e., median of the multi-set consisting of 18/12 for 5 times, 18/13 for 13 times, and 7/12 for 7 times. Thus, \( \alpha_{agg} = 18/13 \), and \( \beta = 1/\alpha_{agg} = 13/18 \).

**B. Choice between MOD-Sketch & Count-Min**

We now turn our attention to Problem 4. For a data stream and a pre-defined length \( h \), select the most accurate sketch between Count-Min and the optimal MOD-Sketch having the same size, i.e., \( a \times b = h \). Unlike Theorem 3, we may not directly compare the error bounds of these two sketches, since the hashing strategies are different (i.e., Count-Min hashes concatenated keys, whereas MOD-Sketch performs composite hashing). We, therefore, consider a more practical approach by computing the standard deviation of the count values stored in different cells of these sketches. As we prove next in Theorem 4, the sketch with the smaller standard deviation generally results in less error for the frequency estimation.

**Theorem 4.** Consider two sketches \( S_1 \) and \( S_2 \) having the same size: \( h \times w \), such that the standard deviations of count values stored in different cells of these sketches are \( \sigma_1 \) and \( \sigma_2 \), respectively. Assume \( \sigma_2 > \sigma_1 \). Let us consider an item \( i \) with its estimated frequency via \( S_1 \) and \( S_2 \) as \( Q_1(i) \) and \( Q_2(i) \), respectively. Assume \( \delta > 0 \). Then, with probability at least \( 1 - 2/(1 + \delta^2) \), the following holds:

\[
Q_1(i) - Q_2(i) \leq \sigma_1 - \sigma_2 \cdot \delta
\]

**Proof.** Let the total frequency of items received so far in the stream be \( L \). Let the values stored in the cells of \( S_1 \) be denoted by the random variable \( R_1 \). The expected value of \( R_1 \) is \( L/h \), which is the mean value of the counts in the sketch. By using the Cantelli’s inequality, with \( \delta > 0 \), the possibility of the lower bound of \( R_1 \) can be measured as follows.

\[
P(R_1 - L/h \geq \sigma_1 - \sigma_2 \cdot \delta) \leq 1/(1 + \delta^2)
\]

Analogously, let the values stored in the cells of \( S_2 \) be denoted by the random variable \( R_2 \). The expected value of \( R_2 \) is \( L/h \). By employing the Cantelli’s inequality, we have:

\[
P(R_2 - L/h \geq \sigma_2 - \sigma_2 \cdot \delta) \leq 1/(1 + \delta^2)
\]

The above inequality can be written as follows.

\[
P(R_2 - L/h \leq \sigma_2 - \sigma_2 \cdot \delta) \geq 1 - 1/(1 + \delta^2)
\]

Combining the above two inequalities \( 14 \) and \( 16 \), we derive:

\[
P(R_1 - R_2 \leq \sigma_1 - \sigma_2 \cdot \delta) \geq 1 - 2/(1 + \delta^2)
\]

This completes the proof.

Hence, by comparing the variance of two sketches with the same \( h \) and \( w \) (but with different hashing techniques), one can predict which sketch would result in less frequency estimation error for the given data stream. In practice, we uniformly at random, sample only a small portion of the incoming stream, and store it in the two sketches having same \( h \times w \). Then, based on the standard deviation of the values stored in different cells of these sketches, we decide which sketch we shall use. The accuracy of our sampling-based approach can be theoretically guaranteed as follows.

**Theorem 5.** Consider a sample with size \( L_0 \) obtained uniformly at random, such that \( L_0 = L/p \), where \( L \) denotes the total frequency of items in the stream. Consider two sketches \( S_1 \) and \( S_2 \) having the same size: \( h \times w \). We store the sample in both sketches, and compute the standard deviations of count values in different cells of these sketches. Let the standard deviations be \( \sigma_1^2 \) and \( \sigma_2^2 \), respectively. Assume \( \sigma_2^2 > \sigma_1^2 \).

Considering an item \( i \), we would like to estimate its frequency
TABLE I: \(T(n)\) denotes \#ways the modules of a key with modularity \(n\) can be combined. \(T(n)\) increases at a higher rate than \(2^n\).

\[Q_1(i) - Q_2(i) \leq (\sigma_i^2 - \sigma_2^2) \cdot \delta \]  

\(\sigma_1 = \sigma_2\) because for any two keys of modularity \(n\), \(X_{\text{sketch}}(i)\) and \(Y_{\text{bell}}(i)\) have the same mean.

\[Q_1(i) - Q_2(i) \leq (\sigma_i^2 - \sigma_2^2) \cdot \delta \]  

Proof. We shall assume that \(n > 1\), since the base cases for \(n = 0\) and \(n = 1\) are already given. Let us consider the first module in order, i.e., \(x^{(1)}\). Clearly, the number of ways \(x^{(1)}\) remains as a separate module is \(T(n-1)\). Next, we consider the number of ways \(x^{(1)}\) can be combined with exactly one of the remaining modules. The number of remaining modules is \((n-1)\), and after \(x^{(1)}\) is combined with exactly one of the remaining modules, e.g., \(x^{(1)} \cdot x^{(2)}\), we have total \(T(n-2)\) possible ways. In general, when \(x^{(1)}\) is combined with exactly \(k\) of the remaining modules, where \(1 \leq k \leq n-1\), total number of possible ways is \((n-1)\!\!\cdot \!\! T(n-k-1)\). By combining all these counts as they are mutually exclusive, the theorem follows.

In summary, (1) we sample a small portion (about 2~4%) of the incoming stream having modularity of keys two. (2) Given a predefined \(h\), we find the optimal range parameters \(a, b\) of a MOD-Sketch by measuring \(\alpha_{\text{agg}}\) from this sample, as discussed in Section IV-A. We then store this sample in both Count-Min and MOD-Sketch, with \(a\) and \(b\) computed as above. Next, (3) we compute the standard deviations of the count values stored in different cells of these two sketches, and use the sketch that reports the smaller standard deviation. Furthermore, it is important to note that our theoretical framework for selecting between Count-Min and MOD-Sketch (Section IV-B) could be applied for keys having modularity even higher than two.

V. ALGORITHMS FOR MODULARITY > TWO

In this section, we consider the generalization of our problem for item keys with modularity higher than two. Two natural baselines for hashing such keys are: (1) Count-Min, i.e., concatenate \(n\) ordered modules of the key and hash it directly with a hash function of range \(h\), and (2) Equal-Sketch, i.e., separately hash \(n\) modules with \(n\) independent hash functions, each having equal range \((h)^1/n\). However, a more accurate MOD-Sketch could combine some modules of the key and hash them together, and could separately hash the remaining modules. Therefore, it raises the problem of how to design a more accurate MOD-Sketch for a given key with modularity higher than two, considering modules might be combined or kept separate as necessary.

A. An Exact Strategy

Let \(i = \{x^{(1)}, x^{(2)}, \ldots, x^{(n)}\}\) be a key of modularity \(n\). 

**Theorem 6.** Let us denote by \(T(n)\) the total number of ways one can combine different parts of a key having modularity \(n\). Then, we have the following relationship:

\[T(n) = \sum_{k=0}^{n-1} \binom{n-1}{k} \cdot C_k \cdot T(n-k-1)\]  

with base cases: \(T(0) = T(1) = 1\). \((n-1)\!\!\cdot \!\! C_k = \frac{(n-1)!}{k!(n-k-1)!}\).

Proof. We shall assume that \(n > 1\), since the base cases for \(n = 0\) and \(n = 1\) are already given. Let us consider the first module in order, i.e., \(x^{(1)}\). Clearly, the number of ways \(x^{(1)}\)

The mean values of the counts in both sketches are same, i.e., \(\mu_{\text{sketch}} = \mu_{\text{bell}}\). Thus, \(\text{MOD-Sketch}\) and \(\text{Count-Min}\) could be applied for keys having modularity more than two. Instead, we develop a more scalable, greedy approach.

B. Greedy Solution

Due to the high cost of enumerating all combinations, we develop an efficient heuristic to find a good quality solution to our problem. We discuss our algorithm in two phases.

1) Range Ratio Computation: Our first problem is: Given a total length \(h\) and a specific way of combining the modules, how shall we find the optimal range of all hash functions that correspond to different (combined) parts of the key? In particular, consider the modules of a key \(i = \{x^{(1)}, x^{(2)}, \ldots, x^{(n)}\}\), and a specific way of combining them, i.e., \(\{y^{(1)}, y^{(2)}, \ldots, y^{(m)}\}\), where each \(y^{(i)}\) is a singleton, or a combination of some modules that are hashed together. For \(k \neq j\), \(y^{(k)}\) and \(y^{(j)}\) do not have a common module. Moreover, \(y^{(k)}\) and \(y^{(j)}\) are separately hashed for all \(k \neq j\). Clearly, \(m \leq n\), and the key \(i\) is hashed using \(m\) independent hash functions. Given a length parameter \(h\), we aim at finding the optimal ranges \(a_1, a_2, \ldots, a_m\) of these hash functions, where \(a_1 \times a_2 \times \cdots \times a_m = h\).

Recall that for keys having modularity two, we developed our solution in Section IV-A. We now generalize our method for keys with modularity more than two with a recursive strategy as follows. We start by measuring the range ratio
Algorithm 1 Greedily find optimal hashing of keys w/ modularity $\geq 2$

Require: uniform stream sample with keys having modularity $n \geq 2$, sketch parameters $h, w$

Ensure: optimal MOD-Sketch configuration

1: $currentConfig \rightarrow$ first module
2: $k \rightarrow 1$
3: while $k \leq n$ do
4: for each $(n-k+1)$ hashing choices do
5: $m \rightarrow$ separate parts in the current choice; $m \leq k + 1$
6: find optimal ranges $\{y(1), y(2), \ldots, y(m-1)\}$ such that $y(1) \times y(2) \times \ldots \times y(m) = (h)$ (via Section V-B)
7: $end$ for
8: $CurrentConfig$ $\rightarrow$ find the best hashing option among $(n-k+1)$ choices (via Section IV-B)
9: $k \rightarrow k + 1$
10: end while
11: return $CurrentConfig$

between $a_m$ (corresponding to the last hash function) and the rest. Formally, we take the parts $y(1), y(2), \ldots, y(m-1)$ as a combined part, then we can calculate the ratio of ranges $\beta_m = a_m/(a_1, a_2, \ldots, a_{m-1})$, where $a_m \times a_1, a_2, \ldots, a_{m-1} = h$. We compute $\beta_m$ with our method in Section IV-A that is, $\beta_m = \frac{a_m}{a_1, a_2, \ldots, a_{m-1}}$. We measure $\alpha_m = O(\ast, \ast, \ast\ldots, \ast, y(m)/O(y(1), y(2), \ldots, y(m-1), \ast)$.

Next, we recursively calculate the ratio $\beta_{m-1} = a_{m-1}/a_{1, 2, \ldots, m-2}$ where $a_{m-1} \times a_1, a_2, \ldots, a_{m-2} = a_1, a_2, \ldots, a_{m-1}$. We repeat this process until we find the ratio $\beta_2 = a_2/a_1$.

Our recursive steps are illustrated in Figure 2. The time complexity to compute the optimal values of $a_1, a_2, \ldots, a_m$ by our method is $O(m)$, where $m \leq n$, $n$ being the number of modules. We next discuss the details of the greedy approach.

2) Greedy Algorithm: Our greedy algorithm works by traversing the search space in a depth-first manner until a path of length $n$ has been found covering all the $n$ modules of the key. In Figure 3, we demonstrate our method with keys of modularity 4, i.e., $i = \{x(1), x(2), x(3), x(4)\}$. We start with the first module in order, which is $x(1)$, and verify four possibilities, that is, hashing $x(1)$ separately (denoted by the dotted arrow between $x(1)$ and $x(2)$), or combining $x(1)$ with one of the remaining modules, $x(2)$, $x(3)$, or $x(4)$ (denoted by solid arrows between these nodes). Among these four possibilities, we greedily find the best strategy, e.g., in Figure 3(b), we already selected the best strategy as to hash $x(1)$ separately.

By traversing the dotted edge from $x(1)$ to $x(2)$, we arrive at the current module $x(2)$. At this intermediate stage, we have three choices: Hashing $x(2)$ separately (denoted by the dotted arrow between $x(2)$ and $x(3)$), or combining $x(2)$ with one of the remaining modules, $x(3)$ or $x(4)$ (denoted by solid arrows between these nodes). Again, we greedily find the best strategy, e.g., in Figure 3(c) we already selected the best strategy as to combine $x(2)$ and $x(4)$.

In particular, at any intermediate stage $k$ ($1 \leq k \leq n$), we computed the configuration with $k$ modules, and next we have to select from $(n-k+1)$ choices in a greedy manner. For example, assume in Figure 3(c), out of two hashing options, $x(3)$, and $x(4)$, we arrived at the best strategy as to hash $x(3)$ and $x(4)$. You have two options: Further combine $x(4)$ with $x(3)$, or keep them separate. We make a greedy selection between these two options, and finally terminate our method.

Our complete procedure is given in Algorithm 1. Note that by greedy selection, we only consider $\sum_{k=1}^{n}(n-k+1) = O(n^2)$ choices for combining the modules, as opposed to the exact number of ways $T(n)$ in which the modules can be combined. We have shown in Table II that $T(n)$ increases at a higher rate compared to $2^n$, and this demonstrates the scalability of our algorithm. Moreover, at any intermediate stage $k$, since we do not change the optimal configuration with $k$ modules obtained previously, some of the optimal range ratio estimations (i.e., computation of $\beta$ values required in Line 6, Algorithm 1) from earlier stages could be re-used.

For example, assume in Figure 3(c) out of two hashing options, the best one selected is to separate $x(4)$ and $x(3)$. Thus, the three separate parts of the key are $x(1)$, $x(2)x(4)$, and $x(3)$. To find the optimal hashing ranges due to these three parts by following our recursive method in Section IV-B, one needs to compute the range ratio of the parts $x(1)$ and $x(2)x(4)$. However, this ratio has already been computed in Figure 3(b) and therefore, can be re-used. Such re-using of range ratio estimation further improves our efficiency.

VI. EXPERIMENTAL RESULTS

A. Environment Setup

1) Datasets: We consider six real-world stream datasets from two sources. (1) Twitter Communication Stream: We obtain the Twitter graph dataset from https://snap.stanford.edu/, which consists of all public tweets during a 7-month period from June 1, 2009 to December 31, 2009. Each edge (directed) in our dataset represents a communication between two users in the form of a re-tweet. The edge frequency is defined as the number of communications between the corresponding source and target users. Clearly, every item (i.e., an edge) in this dataset has modularity two, consisting of a source and a target node. (2) IPv4 Trace Stream: We use two IP trace streams from the IPv4 Route /24 Topology dataset (http://www.caida.org/data/overview/), which contains the raw IPv4 team-probing data from January 1, 2008 to December 31, 2008 by CAIDA. We obtain two datasets of

| Datasets | Modularity | # Distinct Items | Agg. Item Frequency | Max. Item Frequency | Flat Stream (GB) | Compressed Stream (GB) |
|-----------------|------------|------------------|---------------------|--------------------|-----------------|------------------------|
| Twitter | 2 | 78,508,963 | 151 × 10^7 | 17,149 | 3.69 | 0.15 |
| IPv4-1#2 | 2 | 94,820,182 | 6 × 10^7 | 123,614 | 17,23 | 1.72 |
| IPv4-1#4 | 4 | 94,820,182 | 6 × 10^7 | 123,614 | 17,23 | 1.72 |
| IPv4-1#8 | 8 | 94,820,182 | 6 × 10^7 | 123,614 | 17,23 | 1.72 |
| IPv4-2#2 | 2 | 94,820,182 | 6 × 10^7 | 123,614 | 17,23 | 1.72 |
| IPv4-2#4 | 4 | 94,820,182 | 6 × 10^7 | 123,614 | 17,23 | 1.72 |
| IPv4-2#8 | 8 | 94,820,182 | 6 × 10^7 | 123,614 | 17,23 | 1.72 |

| TABLE II: Data stream characteristics and sizes | Datasets | #Source nodes | #Target nodes |
|-------------------------------------------------|----------|---------------|---------------|
| Twitter | 4,790,726 | 15,062,241 |
| IPv4-1#2 | 7,234,121 | 665,279 |
| IPv4-2#2 | 8,352,656 | 697,121 |

| TABLE III: Additional statistics for streams having modularity two example in Figure 3(c) | |

| Datasets | Source nodes | Target nodes | Distinct Items | Frequency | Latency |
|----------|--------------|--------------|----------------|-----------|---------|
| Twitter | 4,790,726 | 15,062,241 | 78,508,963 | 151 × 10^7 | 17,149 |
| IPv4-1#2 | 7,234,121 | 665,279 | 94,820,182 | 6 × 10^7 | 123,614 |
| IPv4-2#2 | 8,352,656 | 697,121 | 94,820,182 | 6 × 10^7 | 123,614 |
IP address streams from source-to-destination and source-to-respond traces, and refer to them as IPv4-1 and IPv4-2, respectively. Each item in both datasets contains two 4-bytes IP addresses. Therefore, both IPv4-1 and IPv4-2 have modularity eight, denoted as IPv4-1#8 and IPv4-2#8. Next, we generate modularity-two datasets from IPv4-1#8 and IPv4-2#8 by hashing each IP address to one integer id, thereby obtaining datasets IPv4-1#2 and IPv4-2#2. Similarly, we generate modularity-four datasets by hashing each IP address with two integer ids — the first integer corresponds to the first 2-bytes of the IP address, and the second one corresponds to the next 2-bytes of the IP address. In this way, we obtain another two datasets IPv4-1#4 and IPv4-2#4, having modularity four.

In Table III the flat stream size represents the total stream size that contains repetition of items. The compressed stream size, in contrast, is defined as the size of all distinct items that have nonzero frequency, along with their frequency counts. Both flat stream and compressed stream can answer our queries with complete accuracy. We shall demonstrate in our experiments that MOD-Sketch, though a small fraction of the flat and compressed stream representations, achieves reasonably high accuracy. In Table III we show additional statistics for our three datasets with modularity two.

2) Comparing methods: We compare MOD-Sketch with three following methods. (1) Count-Min [9], i.e., concatenate $n$ ordered modules of the key and hash it directly with a hash function of range $h$. We use $w$ such pairwise independent hash functions. (2) Equal-Sketch, i.e., separately hash $n$ modules with $n$ independent hash functions, each having range $(h)^{1/n}$. We use $w$ such pairs of hash functions. In the context of graph edge stream (i.e., having modularity two), similar methods were explored, e.g., TCM [29] and gMatrix [19]. (3) Exhaustive, i.e., empirically evaluate all $T(n)$ possibilities of combining the modules of a key (the exact value of $T(n)$ can be obtained from Theorem 6 and Table I). For each possibility, experimentally find the best choice of hash function ranges corresponding to different (combined) parts of the key. Finally, select the best option that minimizes the frequency estimation error. Clearly, the Exhaustive approach is very expensive, and it does not scale well beyond modularity $n > 4$.

3) System description: We implement our code in Python, and perform experiments on a single core of 2.40GHz Xeon server. Each experiment uses less than 1GB of the main memory, and our empirical results are averaged over 10 runs.

4) Evaluation metrics: We present the accuracy of frequency estimation queries using observed error [9], which is measured as the difference between the estimated frequency and the true frequency, accumulated over all the items queried. The observed error is expressed as a ratio over the aggregate true frequency of all the items queried.

$$\text{Observed error} = \frac{\sum_{i \in \text{Query}} |\text{estimated freq}(i) - \text{true freq}(i)|}{\sum_{i \in \text{Query}} \text{true freq}(i)}$$

We generate two types of query sets: (1) Top-$k$ query consists of the top-$k$ high frequency items. (2) Random-$k$ query consists of $k$ randomly selected items from the data stream.

B. Results for Modularity Two

In Figures 4 and 5 we show the accuracy of edge frequency queries over our datasets with modularity two, under varying $k$ of both top-$k$ and random-$k$ queries, and for different range parameter $h$. We notice that in all experiments, MOD-Sketch obtains smaller observed error compared to Count-Min and Equal-Sketch. As an example, in Twitter, MOD-Sketch with $a = 434$ and $b = 2304$ has a relatively lower observed error, compared to Count-Min of one combined range $h = 10^6$ and Equal-Sketch of two equal ranges $a = b = 10^5$. Compared to Exhaustive, which empirically finds the optimal parameters: $a = 470$ and $b = 2127$, the setting of MOD-Sketch is very similar. Further delving into the Twitter dataset in Table III we find that the distinct number of target nodes is more than that of source nodes. This justifies why our method MOD-Sketch, as well as Exhaustive report $b > a$ in the optimal setting. We find similar observations with IPv4-1#2 and IPv4-2#2.

While Exhaustive produces the least observed error, finding the optimal parameters experimentally by considering various combinations requires about 20 hours, which is not affordable (Figure 6). In comparison, MOD-Sketch requires a few minutes (5~20 minutes) to find good-quality $a$ and $b$ values that result in comparable accuracy with Exhaustive. Moreover, the observed error of MOD-Sketch reduces when the size of the data sample for its parameter estimation increases. We find that with about 2% of the stream, the observed error converges (Figure 5), therefore in our experiments, we use 2% of the stream for parameter estimation.

C. Results for Modularity $> 2$

We analyze the accuracy of edge frequency estimation over our datasets with modularity 4 and 8, and with varying $w$ (Figure 7). The observed error increases with higher modularity. However, the observed error of MOD-Sketch is always smaller than that of Equal-Sketch and Count-Min, and is also comparable to Exhaustive. In fact, for modularity 8, the observed error of MOD-Sketch is almost half of that due to Count-Min and Equal-Sketch.

Moreover, the running time to find the optimal parameters using Exhaustive is at least two orders of magnitude higher than that of MOD-Sketch for modularity 4. In fact, with modularity 8, Exhaustive does not complete within 100 hours (Figure 9). These results demonstrate the efficiency and scalability of MOD-Sketch over highly-modular data streams.

D. Stream Processing Throughput

The throughputs of MOD-Sketch, Count-Min, and Equal-Sketch are close when the modularity is small (Figure 8). For streams with higher modularity, the throughput of MOD-Sketch is lower than that of Count-Min, but higher than that of Equal-Sketch. This can be explained as follows. For an item with modularity $n$, Count-Min uses $w$ hash functions, whereas Equal-Sketch employs $nw$ hash functions. Since MOD-Sketch optimally combines some modules of the key, the number of hash functions used by it is less than $(n \times w)$, but more than $w$. In all our experiments, the throughput of MOD-Sketch varies between 30K~90K items second.
E. Generalizability

We evaluate the generalizability of MOD-Sketch by implementing it on top of the FCM sketch [30]. FCM improves the accuracy of Count-Min by employing different number of hash functions for high-frequency and low-frequency items, detected by an additional Misra-Gries counter [23]. FCM first applies two separate hash functions to compute an offset and a gap, which determines the subset of hash functions to be
used for hashing the item. Then, it utilizes the selected hash functions to hash the item into the sketch. We refer to our implementation of MOD-Sketch on top of FCM as FMOD (Figure 10). As expected, FCM reduces the observed error compared to Count-Min. However, our designed FMOD further reduces the observed error even compared to FCM. These results demonstrate the generalizability of MOD-Sketch.

F. Effectiveness of Median Aggregate

In Figure 11, we demonstrate the effectiveness of our median aggregate in computing the optimal setting of MOD-Sketch. Top-100 query, Twitter.

VII. CONCLUSIONS

We present MOD-Sketch to improve the accuracy of sketches by employing multiple independent hash functions that hash different modules in a key and their combinations separately. We develop scalable algorithms that sample a small portion of the stream, and find the optimal strategy to combine different modules of the key before they are hashed into the sketch. Furthermore, we compute good-quality hashing ranges for various combined parts of the key. Based on our empirical results over six real-world data streams, MOD-Sketch outperforms several baselines including Count-Min, and it can be used together with more sophisticated sketches, e.g., FCM to further improve the frequency estimation accuracy.

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