

Stückelberg Supersymmetry Breaking and Mediation

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We show that multiple Abelian sectors with Stückelberg mass-mixing generically break supersymmetry at tree-level and naturally mediate the effect to induce soft parameters. Moreover, Stückelberg superfield, as a compensator, adds new interaction terms in the Kähler potential and the superpotential so that supersymmetry is broken while gauge symmetries could remain preserved. We studied a minimal model with two Abelian factors with charged matter and one Stückelberg superfield thus only a diagonal gauge supermultiplet is massive. If light matter is charged under one Abelian group then supersymmetry is broken via D-term given a non-zero genuine FI parameter. If light matter is doubly charged then supersymmetry is broken through mixed D and F-terms without a need for an FI parameter which is favored in supergravity/superstring theories. Moreover, mass matrices receive universal charge-independent contributions so that supertrace constraint is alleviated.

Dedicated to Hessamaddin Arfaei on his 70th birthday.

Introduction

Additional Abelian gauge boson has long been considered as a natural extension of the Standard Model (SM) or its supersymmetric version. It is predicted from (and vastly ubiquitous in) string compactifications/D-brane constructions and grand unified theories. There are huge number of models and we refer to [1–3] for a review of an extensive literature. On the other hand, there are strong motivations to assume that there exists secluded sectors form the Standard Model sector. To experimentally verify this common lore, we need to thoroughly study channels through which sectors communicate. Sectors accommodating Abelian group factors interact through efficient portals such as kinetic mixing [4] and mass mixing [5], through the Stückelberg mechanism [6,7]. Moreover, the phenomenology of the Minimal Supersymmetric Standard Model (MSSM) with additional $U(1)$’s is very rich. The experimental signatures of the kinetic mixing between the MSSM hypercharge and an extra $U(1)$ have been extensively studied before, see [8–10]. The mass mixing among $U(1)$’s has also been studied in great details in [11–15]. Similar studies were performed in [16,17] where mixing is lifted from the hypercharge to another $U(1)$ in the MSSM sector.

On the other hand in a supersymmetric framework, finding a mechanism for supersymmetry (SUSY) breaking and computing dimensionful soft parameters are at the core of model building. Phenomenologically, supersymmetry is broken in a sector with no direct coupling to the MSSM fields. A mediation mechanism is thus needed the nature of which fixes the pattern of soft parameters and low energy phenomenology (see [18]).

In this paper we show that multiple $U(1)$’s with unsuppressed Stückelberg mass-mixing operator actively participate in tree-level SUSY breaking and mediating it effect to soft parameters. Generically, SUSY is broken through D and F-terms when there is at least two $U(1)$’s. In this framework, SUSY breaks while preserving gauge symmetries. It helps to make mass scales independent and hierarchical. New interaction terms, which is made possible by Stückelberg compensation, in the Kähler potential and superpotential stabilize matter fields at the origin even without superpotential mass parameters. In fact, there are universal contributions to the charged scalar fields which can take over charge-dependent sources. Furthermore, they help to alleviate the supertrace constraint so that the mass spectrum after SUSY breaking is phenomenologically acceptable.

The structure of this paper is as follows: In the next section we present the framework with emphasis on its distinctive features including sources for SUSY breaking and induced soft parameters. We use the general forms of the Kähler potential and superpotential. Then we study supersymmetry breaking and compute the mass spectrum in a simple model with two Abelian gauge groups with charged light matter and one Stückelberg field. Finally we summarize in the last section with prospects for future a work with applications to the MSSM.

The Framework

To present the general idea, we consider $n_V$ Abelian sectors with vector superfields $V^a$ and $n_S$ chiral superfields $S^m$ in non-linear representation of gauge groups. Besides gauge-invariant kinetic mixing $f^{ab}W^a_iW^b_i$, where $W^a_i = D_i^aD_iV^a$, Abelian sectors can mix via $n_S$ portals in the Kähler function as

$$K \supset M^2 \exp[2\alpha^a V^a + \beta^m (S + S^I) m M^{-1}].$$

(1)

Apparently, it is invariant under gauge transformations

$$V^{(a)} \rightarrow V^{(a)} + \Lambda^{(a)} + \Lambda^{(I)},$$

$$\beta^m S^m \rightarrow \beta^m S^m - 2M\alpha^a\Lambda^a,$$

(2)

where $\alpha$ and $\beta$ are positive constants. Apparently, it is a sizable portal with unsuppressed interactions among different sectors. We apply an exponential parametrization as it naturally appears from the canonical Kähler potential of a UV completion above scale $M$. In the following for brevity, we may set $M = 1$ which can be easily restored on dimensional analysis.
Integrating (1) over full superspace gives
\[ e^{-\beta \cdot s} \mathcal{L} - (M^a_a A^a_a + \beta^m \partial_m \sigma^m)^2 + M^2 \alpha^a D^a + M \beta^m \alpha^a (\psi^m \lambda^a + \text{h.c.}) + \beta^m \beta^a F^m F^{*n}. \] (3)
where \( s \) and \( \sigma \) are the real and imaginary parts of the lowest component of \( S \) respectively. It provides masses to \( n_S \) vector fields through the Stueckelberg mechanism without breaking gauge invariance and spoiling unitarity. It also gives supersymmetric Dirac masses to \( n_S \) gauginos. Moreover, it induces an effective field-dependent Fayet-Iliopoulos (FI) parameter besides a potentially genuine one. As is shown in the next section, if one adds the FI terms \( \mathcal{K} \supset \xi^a V^a \) with at least one non-zero negative parameter \( \xi \), then for \( n_V > 1 \), SUSY is generically broken via \( D \)-term. There is no flat direction and the moduli \( s \) are stabilized.

Furthermore, the real operator in (1), can be applied in the Kähler potential to mediate SUSY breaking and induce soft parameter. For instance, the following term
\[ \mathcal{K} \supset c_A \text{tr}(W^A W^A + \text{h.c.}) \exp[2 \alpha^a V^a + \beta^m (S + S^\dagger)^m M^{-1}], \] (4)
where \( A \) runs over all Abelian and non-Abelian gauge superfields, gives soft Majorana masses to gauginos as
\[ M_{1/2,A} = c_A e^{\beta \cdot s} (\alpha^a D^a + M^{-2} \beta^m \beta^a F^m F^{*n}). \] (5)
For Abelian gauginos, it is in addition to the supersymmetric Dirac mass. Moreover, the canonical Kähler potential with (1) as follows
\[ \mathcal{K} \supset c_i (\phi_i e^{2 \sigma^a V^a}) \phi_i \exp[2 \alpha^a V^a + \beta^m (S + S^\dagger)^m M^{-1}], \] (6)
induces soft SUSY breaking scalar masses as
\[ M_{0,i}^2 \supset c_i e^{\beta \cdot s} [(q_i g^a + \alpha^a)^2 D^a + M^{-2} \beta^m \beta^a F^m F^{*n}]. \] (7)
It must be added to the conventional tree-level SUSY breaking mass \( M_0^2 \supset q_i^2 g^a D^a \).

Interestingly, one immediate consequence of the above coupling is that the supertrace constraint can be lifted. SUSY breaking at tree-level in a globally supersymmetric theory with canonical Kähler potential implies that
\[ \text{str} M^2 \equiv (-1)^{2s} (2s + 1) M^2_s = -q_i g^a D^a \] [19]. It is not favored phenomenologically as it predicts light fermions. However in this framework, we find that the supertrace is modified by the above coupling as
\[ \text{str} M^2 = -[q_i^2 g^a (1 + c_i e^{\beta \cdot s})] + c_i e^{\beta \cdot s} \alpha^a D^a. \] (8)
Apparently, there is a charge-dependent source so that it can be made positive-definite over the full spectrum (which does not need to vanish here on anomaly arguments) or any arbitrary multiplet of particles. Moreover for the same reason, one can get the MSSM hypercharge FI parameter to break supersymmetry at tree-level through non-zero \( D \)-term. One can choose parameters in the Kähler potential so that all scalar directions are non-tachyonic and SUSY breaking minimum occurs at zero field values. Normally, tachyonic directions are removed by superpotential mass terms which was not possible in the MSSM. Thus, D-term SUSY breaking was not an option in the MSSM without breaking electromagnetic or color symmetry.

Finally, the rest of soft parameters, like holomorphic scalar mass and a-terms, are induced by using the holomorphic superpotential through
\[ \mathcal{K} \supset c(W + \text{h.c.}) \exp[2 \alpha^a V^a + \beta^m (S + S^\dagger)^m M^{-1}], \] (9) which gives
\[ \mathcal{L} \supset e^{\beta \cdot s} (\alpha^a D^a + M^{-2} \beta^m \beta^a F^m F^{*n}). \] (10)
It also generates (suppressed) masses for chiral fermions
\[ M_{1/2,i} \supset c M^{-1} e^{\beta \cdot s} \beta^a F^{*n}. \] (11)

In the rest for later phenomenological purpose and without lose of generality, we consider a model with two Abelian sectors \( U(1)_X \) and \( U(1)_Y \) (in the following \( a = X, Y \) and one Stückelberg superfield \( S \) so only a diagonal gauge boson is massive. We call \( U(1)_Y \) the hypercharge group and demand it is not broken alongside SUSY breaking. Then, the Stückelberg Kähler function is parametrized as follows
\[ K_{St} = M^2 \exp[2 \alpha^a V^a + (S + S^\dagger)] M^{-1}. \] (12)

Moreover, we assume that there are chiral superfields \( \phi_i, i = 1, \ldots, N_f \) of charges \( q_i^a \). Besides, they can be in some representation \( R \) of non-Abelian group \( G \) that must be taken into account when we make invariants. We neglect this in this work. It is interesting to note that the Stückelberg superfields, as a compensator, can be applied to make new singlets. If the chiral superfield \( \phi_i \) is doubly charged and \( \mathcal{O} [\phi_i] \) is a composite operator of non-zero charges \( q_i^X, q_i^Y \) then
\[ \mathcal{O} [\phi_i] e^{-\gamma \sigma^S} = \phi_i \cdots \phi_j e^{-\gamma \sigma^{+\cdots+\gamma_1}S}, \] (13)
is a singlet of \( U(1)_X \times U(1)_Y \) for \( \gamma_S = q_i^X g^X \alpha_X^{-1} = q_i^Y g^Y \alpha_Y^{-1} \). [Otherwise, for arbitrary parameters or the case of singly charged \( \phi_i \), the above operator is invariant under a diagonal subgroup \( U(1)_D \) for \( \Lambda_X = -c \Lambda_Y \) where \( c = (-q_i^X g^X + \gamma \alpha_O V^S)(-q_i^Y g^Y + \gamma \alpha_O V^S) \). Then, the general Kähler potential is represented as follows
\[ \mathcal{K} = \mathcal{K} [\phi_i e^{2 \alpha^a V^a}, \phi_i e^{-\gamma \sigma^S} \phi_i e^{-\gamma \sigma^S} e^{2 \alpha^a V^a + (S + S^\dagger)}]. \] (14)
In passing, we note that the following term in the Kähler
\[ \mathcal{K} \supset \tilde{c}_i e^{-\gamma_i \sigma^S} e^{-\gamma_i \sigma^S} e^{2 \alpha^a V^a + (S + S^\dagger)}, \] (15)
further adds the following charge-independent contribution to the weighted sum \[ \text{str} M^2 \supset -\tilde{c} e^{(1-\gamma) s} \alpha^a D^a. \] (16)
Finally, the general holomorphic superpotential is parametrized as

$$W = \mathbf{W} [\phi_j, \phi_i e^{-\gamma_S}] = \mathbf{W} [\phi_j] + (\phi_i e^{-\gamma_S})^n.$$  \hspace{1cm} (17)

We insist on $U(1)_X \times U(1)_Y$ invariant operators and therefore terms involving $[\phi_i]$ are present only if there are doubly-charged matter fields.

Generally, the dynamics in the (Abelian) gauge sectors is accounted for through

$$\mathcal{L} \supset \int d^2 \theta (\mathbf{f}^{ab} \mathbf{W} + \mathbf{W}^a \cdot \mathbf{W}^b + \text{h.c.}),$$  \hspace{1cm} (18)

where $\mathbf{f}^{ab} = \frac{1}{8 \pi i} \left( \frac{\tau^a}{\epsilon} - \frac{\tau^b}{\epsilon} \right)$ and $\tau$ is the complexified coupling $(\theta/2\pi) + i(4\pi/g^2)$. Moreover, we can add the following to the action

$$\mathcal{L} \supset \int d^2 \theta d^2 \theta \mathbf{h}_{ab} (\mathbf{W}^a \cdot \mathbf{W}^b + \text{h.c.}) \mathbf{K}.$$  \hspace{1cm} (19)

In the above $f$ and $h$ are symmetric matrices in the field space. The kinetic mixing can be removed by diagonalizing $f$, however, the matrices $f$ and $h$ could not be made into diagonal form simultaneously. Gauge dynamics in the non-Abelian sectors follows straightforwardly.

The total action can be read from superspace integration of the above superpotential and the Kähler potential [20]. We are interested in SUSY breaking minima and the spectrum of particles around them. Therefore, it is useful to find the mass matrices. We let $I$ run over all chiral superfields including $\mathbf{W}$. The scalar, fermion and auxiliary components of the latter includes $-\chi^a \chi^b, -\chi^a D^b$ and $D^a D^b$ and we explicitly use in the mass matrices below).

The fermionic mass matrix can be read from fermion bilinear terms in the total action as follows

\begin{align*}
[M_{1/2}]_{1J} &= W_{1J} + g_{KLI} F_{1K} F^{*K} + g_{1J} h_{ab} D^a D^b, \\
[M_{1/2}]_{ia} &= -i\sqrt{2} c_i - \frac{1}{\sqrt{2}} f_{ab} g_{1I} W_i F^{*a} + 2g_{1I} h_{ab} F^{*a} D^b, \\
[M_{1/2}]_{ab} &= -\frac{1}{2} f_{ab} W_i F^i - 2h_{ab} K^a D^b + 2h_{ab} g_{1I} F^{*a} D^b, \hspace{1cm} (20)
\end{align*}

which has both supersymmetric and supersymmetry breaking sources. It is an $(N_f + N_S + N_V) \times (N_f + N_S + N_V)$ matrix which mixes chiral fermion-gaugino spinors in flavor basis.

In the above $g_{1I} = K_{1I}$ and the Killing potential $\mathbf{K}^a$ can be determined through $[20]

\begin{align*}
g_{1I} \mathbf{Y}^{aIJ} &= i \mathbf{K}^a_{1I}, \quad \text{and} \quad g_{1I} \mathbf{Y}^{aJI} &= -i \mathbf{K}^a_{1J} \hspace{1cm} (21)
\end{align*}

The Killing vectors $\mathbf{Y}^a$ are fixed via gauge transformations $\delta \mathbf{Y}^a = \mathbf{Y}^a \epsilon^c$. It also takes care of (19) if one lets $I$ run over all chiral superfields including $\mathbf{W}$. Here, the gauge transformations are as follows

$$\delta s = -\frac{1}{2} (M \alpha^a c^a + \delta \phi^i = -i g q_i \phi^i \epsilon, \hspace{1cm} (22)$$

The scalar mass matrix, which receives supersymmetric and non-supersymmetric contributions, is determined from the scalar potential as

\begin{align*}
[M^2]_{IJ} &= g_{KLI} g^{KN} F_{LI} F^{NK} + g_{J} g^{N} W_{N} W_{J} F^{NK} + g_{KLI} W_{K} W_{L} F^{NK} \\
&+ g_{KLI} W^{K}_{L} W^{*}_{L} + \frac{1}{2} \lambda^{a} \mathbf{K}^a_{J} + \mathbf{K}^a_{J} \mathbf{D}^a, \\
[M_0]_{IJ} &= (g_{KLI} W_{K} W_{L} + g_{J} W_{J} + g_{J} W^{*}_{J}) F^{NK} \\
&+ g_{KLI} F^{K} F^{*L} + \frac{1}{2} \lambda^{a} \mathbf{K}^a_{J} + \mathbf{K}^a_{J} \mathbf{D}^a + f_{ab} D^a D^b W_{I} W_{J} \hspace{1cm} (23)
\end{align*}

Finally, the mass matrix of vector fields is given by

$$[M^2]_{ab} = 2 g^{J} \mathbf{K}^a \mathbf{K}^b, \hspace{1cm} (24)$$

The auxiliary fields $F^I$ and $D^a$ can be integrated out from the action by using their equation of motion

$$g_{1I} F^{I} = \frac{1}{2} g_{1I} \mathbf{K}^a \mathbf{K}^b = W_{J} + \frac{1}{4} f_{ab} W_{J} \mathbf{K}^a \mathbf{K}^b = 0\hspace{1cm} (25)$$

$$\text{Re}(f_{ab} f_{ab} W_{0}) D^b + \frac{1}{2} \sqrt{2} \left( i f^{ac} W_{J} \mathbf{K}^a \mathbf{K}^c + \mathbf{K}^a \mathbf{K}^c \right) = 0 \hspace{1cm} (26)$$

Then, the scalar potential reads as

$$V = g^{J} W_{J} W_{J} + \frac{1}{2} f_{ab} f_{ab} \mathbf{K}^a \mathbf{K}^b, \hspace{1cm} (27)$$

from which we determine the scalar mass matrix

$$M_{0}^2 = \left( \begin{array}{cc}
V_{I I} & V_{IJ} \\
V_{IJ} & V_{JJ}
\end{array} \right). \hspace{1cm} (28)$$

The fermion mass matrix is written as follows

\begin{align*}
[M_{1/2}]_{IJ} &= W_{IJ} - \Gamma_{IJ} \mathbf{K}^a \mathbf{K}^b, \\
[M_{1/2}]_{I a} &= -i\sqrt{2} \lambda_{a} + \frac{1}{\sqrt{2}} \lambda^{a} f_{bc} f_{ab} + i\sqrt{2} g_{1I} F^{I} F^{J} \mathbf{K}^b, \\
[M_{1/2}]_{a b} &= \frac{1}{2} f_{ab} g^{I} W_{I} \mathbf{K}^a \mathbf{K}^b \hspace{1cm} (29)
\end{align*}

Using above, the supertrace is computed as follows

$$\text{str} M^2 = 2 g^{J} V_{J} + 6 g^{J} \mathbf{K}^a \mathbf{K}^b = -2 \delta^{ab} M_{1/2ab} M_{1/2ab} \hspace{1cm} (30)$$

SUSY Breaking in a Generic Model

In order to study SUSY breaking vacua and compute the mass spectrum we consider matter fields $\phi_{\pm}$ of charges $(\pm, \pm g_{Y} \alpha_{X}/g_{Y} \alpha_{Y})$ under $U(1)_{X} \times U(1)_{Y}$. The general superpotential is given by

$$W = \lambda_{p \pm} (\phi_{\pm} e^{\pm \gamma_S})^{p} + \mu_{a} (\phi_{+} \phi_{-})^{n}, \hspace{1cm} (31)$$
where $\gamma = g_Y \alpha_Y^{-1} = g_X g_Y \alpha_X^{-1}$ and $\lambda_p, \mu_n$ are coefficients of mass dimension $3 - p, 3 - 2n$ respectively. The leading order terms in the Kähler potential is as follows

$$K \supset [M^2 + c_{1\pm} \phi_{1\pm} e^{2 \gamma S} + c_{2\pm} \phi_{2\pm} e^{2 \gamma S(1 \pm 1)}]$$

where $c_n$ are coefficients of dimension $2 - n$. As can be explicitly seen, new interaction terms are possible through Stückelberg compensation. Terms including $\gamma$ is only possible when there are doubly charged fields. Otherwise, they are absent. As mentioned above, in this study we confine ourselves to minima where the charged matter do not receive vacuum expectation value, $\langle \phi_{\pm} \rangle = 0$, so that the hypercharge symmetry is preserved when SUSY is broken. It helps making SUSY breaking and hypercharge breaking scales independent for later application.

Before studying the general case of non-zero $\gamma$, for $g_X = 0$ the leading order modification to the Kähler potential is as follows

$$K \supset (c_{2\pm} \phi_{2\pm} e^{2 \gamma S} + c_{2\pm} \phi_{-} + h.c. + M^2)\times e^{2 \alpha X V_X + 2 \alpha Y V_Y + S + S^\dagger}.$$  

Given this, the D-terms can be found as

$$D^a = \xi_{\alpha}(s) + \phi_{\gamma} \rho [\pm g_Y \delta^a_\gamma (1 + c_{2\pm} e^{s}) + c_{2\pm} \alpha^a e^{s}] + (c_{2\pm} \phi_{+} + h.c.) \alpha^a e^{s},$$

where $\xi_{\alpha}(s) = \xi_{\alpha} + M^2 \alpha e^{s}$. In particular, we find that

$$\mathcal{V} \supset \phi_{\pm} \phi_{\pm} [\xi_{\alpha}(s) (\pm g_Y + c_{2\pm} e^{s}) + c_{2\pm} \alpha Y e^{s}] + \xi_{\gamma}(s) c_{2\pm} \alpha X e^{s} + m^2],$$

where in the above $m$ is the superpotential mass parameter. There is no tachyonic direction along either of scalar fields if the square bracket is positive definite. It is interesting to note that even if gauge symmetry would not allow the mass parameter in the superpotential, as in the MSSM for lepton and quark superfields, there are other charge-independent contributions to the scalars effective masses so that neither is tachyonic. In this case $\langle \phi_{\pm} \rangle = 0$ is a stable minimum. Around this minimum, the scalar potential is read as

$$2V = \xi_{\alpha}(s) + \xi_{\gamma}(s).$$

Generically, equations $\xi_{\alpha}(s_0) = 0$ and $\xi_{\gamma}(s_0) = 0$ cannot be simultaneously solved and SUSY is generically broken at tree-level via D-term. If both genuine FI parameter are zero, then $s$ has a run-away behavior and vacuum energy asymptotically goes to zero as $s \to -\infty$. Otherwise, using $V_\alpha = 0$, the minimum along $s$ is at

$$s_0 = M \log[-M^{-2} (\alpha_X \xi_X + \alpha_Y \xi_Y)/(\alpha_X^2 + \alpha_Y^2)].$$

Consistent solution is found if (given that $\alpha^a > 0$) at least one the FI parameters is non-zero and negative so that $\alpha_X \xi_X + \alpha_Y \xi_Y < 0$. The vacuum energy is found as

$$2V_0 = (\alpha_X \xi_Y - \alpha_Y \xi_X)^2/(\alpha_X^2 + \alpha_Y^2).$$

Now we study the case of doubly charged matter where $g_X = g_Y \alpha_X/g_X \alpha_Y$. Then all gauge-invariant terms in (31) and (32) must be taken into account. From the superpotential we find that

$$W_\pm = \lambda_p pe^{\gamma S} (\phi_{\pm} e^{\gamma S})^{p-1} + \mu_n n (\phi_{+} \phi_{-})^{-n}.$$  

The D-terms are derived from the action or by computing the Killing potential using the Kähler potential. They are found as follows

$$-D^a = [\xi_{\alpha} + M^2 \alpha^a e^{S + S^\dagger}]$$

$$+ c_{1\pm} \alpha^a (\phi_{\pm} e^{(1\pm 1)S + S^\dagger} + h.c.)$$

$$+ \alpha^a (\phi_{\pm} e^{(1\pm 1)S + S^\dagger} + h.c.)$$

$$+ \alpha e^{S + S^\dagger} (c_{2\pm} \phi_{+} + h.c.)$$

$$+ \phi_{\pm} \phi_{\pm} [q^a g^a (1 + c_{2\pm} e^{S + S^\dagger}) + h.c.]$$

$$+ \alpha (c_{2\pm} e^{S + S^\dagger} + c_{2\pm} e^{(1\pm 1)S + S^\dagger}].$$

The genuine FI parameters $\epsilon^a$ can be interpreted as integration constants from (27). We compute the scalar potential using (27) and we find that the effective scalar masses

$$M^2_{\alpha, \gamma} = \xi_{\alpha}(s) [q^a g^a (1 + c_{2\pm} e^{S + S^\dagger}) + \alpha^a (c_{2\pm} e^{S + S^\dagger} + c_{2\pm} e^{(1\pm 1)S + S^\dagger}) + 2\lambda(s),$$

where $\lambda(s) = \lambda^2_{1\pm} e^{\gamma S} + \lambda^2_{2\pm} e^{\gamma S}$. Apparently, we cannot solve $\xi_{\alpha}(s_0) = 0$ and $\xi_{\gamma}(s_0) = 0$ simultaneously. Thus, SUSY is broken at tree-level through mixed F and D-terms (21) (22). However in this case, the result is completely independent of the genuine FI parameters; they can be zero (which is favored in supergravity/superstring theory) or non-zero of either sign. There is a point in the vast parameter space that for negative genuine FI parameters D-terms are zero and SUSY is broken via pure F-term. In fact due to the F-term contribution to the scalar potential, $V_\alpha = 0$ has a solution and a stable minimum exists along $s$ direction for any set of parameters. These results are also confirmed by conducting a numerical analysis using Mathematica.

Now we compute the mass spectrum around the SUSY breaking hypercharge preserving vacuum. The components of the $5 \times 5$ mass matrix of fermions are as follows

$$[M_{1/2}]_{\pm, \pm} \supset \lambda_{2\pm} e^{2\gamma S} + g_{\pm, \pm} F_{\pm} + g_{\pm, \pm} F_{\pm}$$

$$+ g_{\pm, \pm} h_{ab} D^a D^b,$$

$$[M_{1/2}]_{\pm, \pm} \supset \mu + g_{\pm, \pm} F_{\pm} + g_{\pm, \pm} F_{\pm} + g_{\pm, \pm} h_{ab} D^a D^b,$$

$$[M_{1/2}]_{\pm, \pm} \supset 2\gamma c_{\pm} e^{\gamma S}$$

$$+ g_{\pm, \pm} F_{\pm} + g_{\pm, \pm} F_{\pm} + g_{\pm, \pm} h_{ab} D^a D^b.$$
Finally, the mass matrix of vector bosons is

$$\begin{align*}
[M_{1/2}]_{ss} & \supset g_{\pm s, s}F_{\pm}^* + g_{ss,s}F_{ss}, \\
[M_{1/2}]_{as} & \supset -4i\sqrt{2}c_{\alpha\epsilon}s e^{\gamma s} - \frac{i}{\sqrt{2}}(f_{ab} - 4h_{ab})D_{b}\epsilon e^{\gamma s}, \\
[M_{1/2}]_{as} & \supset M_{\alpha\epsilon}s e^{\gamma s}, \\
[M_{1/2}]_{ab} & \supset -\frac{1}{2}(f_{ab} - 4h_{ab})c_{\pm}F_{\pm}^* e^{\gamma s} - 2h_{ab}M_{\alpha\epsilon}e^{\gamma D_{\epsilon}}.
\end{align*}$$

(44)

The components of the scalar mass matrix are

$$\begin{align*}
[M_0]_{\pm \mp} & \supset (\pm g_{\alpha\epsilon}s^{(e + c \epsilon \gamma s)}D_{\alpha} + M_{\alpha\epsilon}e^{(2\epsilon \gamma s)} + g_{\mp}W_{\pm}F_{\mp} + g_{\pm}W_{\pm}F_{\pm} + g_{\mp}W_{\mp}W_{\pm}, \\
[M_0]_{\pm \pm} & \supset (\mp g_{\alpha\epsilon}s^{(e + c \epsilon \gamma s)}D_{\alpha} + M_{\alpha\epsilon}e^{(2\epsilon \gamma s)} + g_{\mp}W_{\pm}F_{\mp} + g_{\pm}W_{\pm}F_{\pm} + g_{\mp}W_{\mp}W_{\pm}, \\
[M_0]_{ss} & \supset e^{\alpha\epsilon}D_{\alpha} + M_{\alpha\epsilon}e^{2\epsilon \gamma s} + g_{\pm}W_{\pm}F_{\pm} + g_{\mp}W_{\pm}F_{\mp} + g_{\pm}W_{\pm}F_{\mp}.
\end{align*}$$

(45)

Finally, the mass matrix of vector bosons is $[M_0]_{ab} = M_{\alpha\epsilon}e^{\gamma s}$. In the above, non-zero F and D terms are as

$$\begin{align*}
F_{\pm} & = \lambda_{\pm}g_{\pm}^{\pm \epsilon}e^{\gamma s}, \\
F_{ss} & = \lambda_{s}g_{ss}^{\pm \epsilon}e^{\gamma s}, \\
D_{\alpha} & = -\xi_{\alpha} - M_{\alpha\epsilon}e^{\gamma s},
\end{align*}$$

(46)

and the elements of the inverse metric are found from the orthonormality conditions.

**Conclusion**

In this work we observed that multiple Abelian sectors which are mixed through S"uckelberg mass-term generically break SUSY. It was possible to achieve this without gauge symmetry breaking. We computed the mass spectrum around SUSY breaking vacuum. We demonstrated that, due to new K"ahler and superpotential interactions, the supertrace constraint could be lifted.

One would immediately like to embed the MSSM in this framework to study its phenomenological consequences. The hypercharge can mix with an extra $U(1)$ via a single S"uckelberg field so that one gauge multiplet is massive and the other remains massless at low energies. One can assume that the MSSM chiral multiplets are not charged under the extra $U(1)$. Then, SUSY is broken in the visible sector by the D-term via genuine FI parameter. Although there is no superpotential mass parameter, that happens without gauge symmetry breaking. Moreover, an acceptable spectrum could be reached. On the other hand, if the MSSM chiral fields are doubly charged then new interaction terms can be added. In particular, right-handed leptons, as singlet of non-Abelian factors, play a special role in the superpotential. In this case SUSY is broken via mixed F and D-terms and no genuine FI parameter is needed. We expect that the phenomenology of the S"uckelberg MSSM in this framework is very rich and we postpone the detailed study to a work in preparation.

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