Saturation Physics in Heavy Ion Collisions

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Abstract. We discuss expectations of saturation physics for various observables in heavy ion collisions. We show how simple saturation-inspired assumptions about particle production in heavy ion collisions lead to Kharzeev-Levin-Nardi model. Comparing this model to RHIC data on particle multiplicities we conclude that saturation effects may play an important role in particle production and dynamics at the early stages of $Au-Au$ collisions already at RHIC energies. We then estimate the contribution of the initial state two-particle azimuthal correlations to elliptic flow observable $v_2$ in $Au-Au$ collisions by constructing a lower bound on these non-flow effects based on $v_2$ obtained from the analysis of proton-proton ($pp$) collisions.

INTRODUCTION

Saturation/Color Glass Condensate physics is based on the observation that the small-$x$ wave functions of ultrarelativistic hadrons and nuclei are characterized by a hard scale $Q_s$, known as the saturation scale [1, 2, 3, 4]. The scale $Q_s$ arises due to saturation of partonic densities at small-$x$ and is an increasing function of energy and atomic number of the nucleus [5, 6, 4]. This large scale makes the strong coupling constant small $\alpha_s(Q_s) \ll 1$ leading to dominance of the classical gluonic fields in all high energy processes [4, 7]. Gluon production in high energy collisions is given by the classical field of the scattering color charges [8]. Corresponding gluon production cross section was found for $pA$ collisions in [9] and the effects of quantum evolution [5] were included in it in [10]. The gluon production cross section for heavy ion collisions ($AA$) at the classical level has been studied both numerically [11] and analytically [12]. Since it is quite not clear at present how to include the effects of nonlinear quantum evolution [5] in the results of [11,12], one has to construct models to describe the actual rapidity-dependent data produced in heavy ion collisions. Below we are going to show how some of these models, based on rather basic properties of saturation physics, provide a reasonably good description of RHIC data.

PARTICLE MULTIPLICITY FROM SATURATION MODELS

Multiplicity at Mid-Rapidity Versus Centrality

Classical field $A_\mu \sim 1/g$ leads to produced gluon multiplicity

$$\frac{dN}{d^2k d^2b dy} \sim \langle A_\mu A_\mu \rangle \sim \frac{1}{\alpha_s}.$$  \hspace{1cm} (1)
Gluon transverse momentum spectrum described by a single scale $Q_s$ can be written as

$$\frac{dN}{d^2k d^2b dy} = \frac{1}{\alpha_s} f\left(\frac{k_\perp}{Q_s}\right)$$  \hspace{1cm} (2)$$

with some unknown function $f\left(\frac{k_\perp}{Q_s}\right)$ to be determined by actual calculations. Integrating over $k$ and $b$ yields

$$\frac{dN}{dy} = \text{const} \frac{1}{\alpha_s} \pi R^2 Q_s^2.$$ \hspace{1cm} (3)$$

where the value of the constant is determined from $f\left(\frac{k_\perp}{Q_s}\right)$. Following \[13, 14\] we assume that the scale for the coupling constant in Eq. (3) is set by $Q_s$. (This step, of course, goes beyond the classical limit and assumes that Eq. (3) is valid even when running coupling corrections are included.) Then, as $R^2 \sim A^{2/3}$ and if $Q_s^2 \sim A^{1/3}$ \[4, 7\], together with running coupling

$$\alpha_s(Q_s) = \frac{1}{b \ln \frac{Q_s^2}{A^2}} \sim \frac{1}{\ln A}$$ \hspace{1cm} (4)$$

we conclude from Eq. (5) that

$$\frac{1}{A} \frac{dN}{d\eta} \sim \ln A.$$ \hspace{1cm} (5)$$

For heavy ion experiments at different collision centralities we substitute $A$ by the number of participants $N_{\text{part}}$ so that Eq. (6) becomes

$$\frac{1}{N_{\text{part}}} \frac{dN}{d\eta} \sim \ln N_{\text{part}}.$$ \hspace{1cm} (6)$$

Eq. (6) allowed the authors of \[13\] to correctly predict the particle multiplicity at mid-rapidity at RHIC as a function of centrality at $\sqrt{s} = 130$ GeV. A fit of particle multiplicity at other values of rapidity at $\sqrt{s} = 130$ GeV taken from \[14\] is shown in Fig. 1.

![Figure 1](image-url)
Multiplicity as a Function of Energy

Eq. (3) allows one to test whether the scaling of total particle multiplicity with energy is consistent with saturation/Color Glass predictions. Using the fact that $Q_s^2 \sim 1/x_B^\lambda$ in Eq. (3) and, for the moment, dropping the slower $Q_s$-dependence in $\alpha_s$ leads to

$$\frac{dN}{d\eta}(\sqrt{s_1}) = \left(\frac{\sqrt{s_1}}{\sqrt{s_2}}\right)^\lambda \frac{dN}{d\eta}(\sqrt{s_2})$$

(7)

Using PHOBOS data for total charge multiplicity at $\sqrt{s} = 130$ GeV for most central collisions

$$\frac{dN}{d\eta}(\sqrt{s} = 130\text{GeV}) = 555 \pm 12(\text{stat}) \pm 35(\text{syst})$$

(8)

together with $\lambda = 0.25 \div 0.3$ obtained in [15] by analyzing HERA data, Kharzeev and Levin [14] predicted the total charge multiplicity at $\sqrt{s} = 200$ GeV to be

$$\frac{dN}{d\eta}(\sqrt{s} = 200\text{GeV}) = 616 \div 634$$

(9)

in agreement with the later measured PHOBOS result

$$\frac{dN}{d\eta}(\sqrt{s} = 200\text{GeV}) = 650 \pm 35(\text{stat}).$$

(10)

$dN/d\eta$ Versus $\eta$ and $N_{\text{part}}$

Describing rapidity distribution of the produced particles requires a little more modeling. Assuming $k_T$-factorization for particle production cross section

![Figure 2. Saturation model fit of the PHOBOS data on total charged particle multiplicity as a function of rapidity and centrality at $\sqrt{s} = 130$ GeV taken from [14].](image-url)
\[
\frac{d\sigma^{AA}}{d^2k\ dy} = \frac{2\alpha_s}{C_F} \frac{1}{k^2} \int d^2q \phi_A(q) \phi_A(k-q)
\]  

(11)

along with saturation-inspired unintegrated gluon distribution functions \((\phi_A(k) \sim \alpha_s/k^2)\) if \(k_\perp > Q_S\) and \(\phi_A(k) \sim S_{\perp}/\alpha_s\) if \(k_\perp < Q_S\) the authors of [14] produced an impressive fit of the PHOBOS data on charged particle multiplicities as functions of rapidity and centrality at \(\sqrt{s} = 130\) GeV shown in Fig. 2. The predictions made in [14] for similar multiplicity data at \(\sqrt{s} = 200\) GeV were also in good agreement with the later published BRAHMS data.

Phenomenological success of the saturation models presented above does not contradict the possibility of strong final state interactions leading to formation of quark-gluon plasma. As was argued in [16], thermalization in the saturation framework would not introduce any fundamentally new scales leaving Eqs. (3) and (6) practically unchanged. Late stage interactions are also not very likely to significantly modify the rapidity distribution of Fig. 2 due to causality constraints.

**NON-FLOW CONTRIBUTION TO V_2**

The contribution of non-flow two-particle azimuthal correlations from the early stages of heavy ion collisions to the elliptic flow observable \(v_2\) has been estimated in [17] using a saturation-inspired model of particle correlations. Here we are going to construct a model-independent lower bound on these non-flow effects using \(v_2\) extracted from the analysis of \(pp\) data. We start with the definition of \(v_2(p_T)\) for \(pp\) corrected for uncertainty in the “reaction plane” definition (or, equivalently, defined through the two-particle correlation functions, such that \(v_2(p_T) = \langle \cos(\phi_p - \phi_k) \rangle\))

\[
v_2^{pp}(p_T) = \frac{\int d^2k \frac{dN_{pp_{corr}}}{d^3p\ dy} \cos(\phi_p - \phi_k) \left( \frac{dN_{pp}}{dy_p} \frac{dN_{pp}}{dy_k} + \frac{dN_{pp_{corr}}}{dy_p} \frac{dN_{pp_{corr}}}{dy_k} \right)}{\int d^2p d^2k \frac{dN_{pp}}{d^3p\ dy} \cos(\phi_p - \phi_k)}. \tag{12}
\]

If we assume that the relative magnitude of the correlated terms in Eq. (12) compared to uncorrelated ones is roughly the same for all \(p_T\), we can drop the former compared to the latter finding that Eq. (12) is approximately bounded from above by

\[
v_2^{pp}(p_T) \leq \frac{\int d^2k \frac{dN_{pp_{corr}}}{d^3p\ dy} \cos(\phi_p - \phi_k) \left( \frac{dN_{pp}}{dy_p} \frac{dN_{pp}}{dy_k} \right)}{\int d^2p d^2k \frac{dN_{pp}}{d^3p\ dy} \cos(\phi_p - \phi_k)}. \tag{13}
\]

We want to estimate the contribution to \(v_2^{AA}(p_T)\) of the non-flow correlations of the same physical origin as the ones giving rise to \(v_2^{pp}(p_T)\) in Eq. (12). The contribution is

\[
v_2^{AA}(p_T)|_{\text{non-flow}} = \frac{\int d^2k \frac{dN_{AA_{corr}}}{d^3p\ dy} \cos(\phi_p - \phi_k) \left( \frac{dN_{AA}}{dy_p} \frac{dN_{AA}}{dy_k} \right)}{\int d^2p d^2k \frac{dN_{AA}}{d^3p\ dy} \cos(\phi_p - \phi_k)}. \tag{14}
\]
Using the fact that, approximately, both in saturation models and in the data \(dN/dy \sim N_{\text{part}}\), we rewrite the right hand side of Eq. (13) as

\[
v_2^{pp}(p_T) \leq v_2^{AA}(p_T)_{\text{non-flow}} \sqrt{\frac{N_{AA}}{N_{pp}}},
\]

with \(N_{AA}\) and \(N_{pp}\) total particle multiplicities in AA and pp collisions, proportional to the average number of participants involved. Inverting Eq. (15) we obtain a lower bound

\[
v_2^{AA}(p_T)_{\text{non-flow}} \geq v_2^{pp}(p_T) \sqrt{\frac{N_{pp}}{N_{AA}}},
\]

Preliminary analysis of pp data yields \(v_2^{pp}(p_T) \approx 1\) at high-\(p_T\). To get a lower bound we use \(N_{AA}^{\text{part}} = 394\) and \(N_{pp}^{\text{part}} = 2\) in Eq. (16) obtaining \(v_2^{AA}(p_T)_{\text{non-flow}} \geq 7\%\).

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