Local Flattening of the Fermi Surface and Quantum Oscillations in the Magnetoacoustic Response of a Metal

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In the present work we theoretically analyze the effect of the Fermi surface local geometry on quantum oscillations in the velocity of an acoustic wave travelling in metal across a strong magnetic field. We show that local flattenings of the Fermi surface could cause significant amplification of quantum oscillations. This occurs due to enhancement of commensurability oscillations modulating the quantum oscillations in the electron density of states on the Fermi surface. The amplification in the quantum oscillations could be revealed at fitting directions of the magnetic field.

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Oscillations in observables arising due to quantization of conduction electrons motion in strong magnetic fields are well known. They were repeatedly observed in experiments and used as a tool in studies of electron characteristics of metals. At present, these effects are extensively used to study geometrical characteristics of Fermi surfaces in various high-temperature superconductors. Quantum oscillations are specified with contributions from effective cross sections of the Fermi surface.

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The Fermi surfaces in various high-temperature superconductors are well known. They were repeatedly observed in experiments and used as a tool in studies of electron characteristics of metals. At present, these effects are extensively used to study geometrical characteristics of Fermi surfaces in various high-temperature superconductors. Quantum oscillations are specified with contributions from effective cross sections of the Fermi surface.

It is likely that FSs of some metals include locally flattened segments. Even slight distortion of a Fermi sphere due to the effect of crystalline fields results in the FS flattening at some points. There is an experimental evidence that points of flattening exist on the FSs of cadmium, zinc and potassium. When the FS is flattened at some points, this also may bring along an enhancement in the number of effective electrons and amplify the oscillations. To the best of our knowledge, the effect of local flattening of the FS on quantum oscillations was never studied before. We analyze this effect in the present work. As shown below, quantum oscillations could be noticeably strengthened due to the effect of locally flattened segments of the FS, and this could help to locate such segments (if any) in the experiments. The latter brings extra informations concerning fine geometrical features of the FSs. These informations are especially interesting for flattened pieces on the FSs (even small in area) could strongly contribute to the nesting function affecting the strength of the electron-phonon coupling and, consequently, the superconducting temperature in relevant metals.

To make the presented results more specific we apply our analysis to study the effects of the FS local geometry on the quantum oscillations in the velocity of ultrasound waves travelling in a metal. When an ultrasound wave propagates in a metal the crystalline lattice is periodically deformed. It gives rise to electric fields which influence the electrons. Besides, the periodical deformations of the lattice cause changes in the electronic spectrum. Here, we omit these deformation corrections to simplify further calculations.

To proceed we adopt the following energy-momentum relation:

\[ E(p) = \frac{p_1^2}{2m_1} \left( \frac{p_x^2 + p_y^2}{p_1^2} \right)^{l-1} + \left( \frac{p_x}{p_2} \right)^2 \frac{p_2^2}{2m_2}. \]  (1)

This energy-momentum relation could be reasonably justified within a nearly free electron approximation as shown in the recent work. The corresponding FS is a lens whose radius and thickness are \( p_1 \) and \( p_2 \), respectively. For \( l = 1 \) the energy-momentum relation (1) describes an ellipsoidal FS. In this case \( m_1 \) and \( m_2 \) are the principal values of the effective mass tensor.

The Gaussian curvature at any point of the lens corresponding to (1) is given by the expression

\[ K(p) = \frac{l}{m_1 v^2} \left( \frac{p_1^2 + p_2^2}{p_1^2} \right)^{l-1} \times \left[ \left( v_y^2 + v_z^2 \right) \frac{\partial v_x}{\partial p_x} + v_z^2 l(2l - 1) \frac{p_1^2 + p_2^2}{m_1} \right] \]  (2)

For \( l > 1 \), the Gaussian curvature \( K(p_x, p_y, p_z) \) turns zero at the points \((\pm p_2, 0, 0)\) coinciding with the vertices...
of the lens. In view of axial symmetry of the lens, the
curvatures of both principal cross-sections turn zero at
these points, so the vertices are the points of flattening
of the FS. The lens could be a part of a multiply con-
ected FS. For example, electron lenses are included in
the FSs of cadmium and zinc, and there are grounds to
believe that they are flattened. As usual, we assume the
magnetic field to be directed along the z axis, so, the
flattening points at the vertices of the lens belong to
an effective cross section.

We consider a longitudinal ultrasound wave travelling
along the y axis with frequency ω and wave vector q =
(0, q, 0). An expression for the wave vector of a sound
wave can be written down as follows:

\[ q = \omega / s + \Delta q. \quad (3) \]

Here, s is the speed of sound in the absence of the mag-
netic field, and the correction \( \Delta q \) determines the velocity
shift \( \Delta s \) and the attenuation rate \( \Gamma \):

\[ \frac{\Delta q}{q} = \frac{\Delta s}{s} + \frac{i \Gamma}{2 q}. \quad (4) \]

This correction is the sum of two terms. The first term
\( \Delta q_1 \) describes geometrical oscillations in the ultrasonic
attenuation rate and the velocity shift. Such oscillations
are very well known in conventional metals (See [1], as
well as in two-dimensional electron systems [9, 10, 11].

The effect is controlled with classical magnetotransport
mechanisms. Geometric oscillations arise due to the pe-
riodic reproduction of commensurability between the di-
ameters of the electron cyclotron orbits and the space
period of the external disturbance. In our case the latter
is the electric field accompanying the ultrasound wave,
while it propagates in a metal. Using the general equa-
tions, for the magnetoacoustic response of a metal [12],
we can write out the following expression for the con-
tribution from the lens to \( \Delta q_1 \):

\[ \frac{\Delta q_1}{q} = - \frac{\omega}{\rho_m s^2} \frac{N^2}{g} \int dp_z m_{\perp} (p_z) \sum_k \frac{n_{-k}(p_z, -q)n_k(p_z, q)}{\omega + i/\tau - k\Omega(p_z)}. \quad (5) \]

Here, \( \rho_m \) is the density of matter in the lattice; \( N \) is
the electron concentration; \( g \) is the electron density of
states on the FS in the absence of the external magnetic
field; \( m_{\perp}, \Omega \) are the cyclotron mass and the cyclotron fre-
cquency for the electrons associated with the lens; \( \tau \) is the
scattering time for electrons; and the quantity \( n_k(p_z, q) \)
is the Fourier transform in the expansion in terms of an
azimuthal angle Φ specifying the electron position at the
cyclotron orbit:

\[ n_k(p_z, q) = \frac{1}{2\pi} \int_0^{2\pi} d\Phi \exp \left[ ik\Phi - \frac{i q}{\Omega} \int_0^{\Phi} v_y(p_z, \Phi') d\Phi' \right]. \quad (6) \]

Another term \( \Delta q_2 \) originates from the quantization of
the orbital motion of electrons in strong magnetic fields,
and describes quantum oscillations in the velocity shift.
Assuming that the cyclotron quantum \( h\Omega \) is small com-
pared to the chemical potential of electrons \( \zeta \) (\( \zeta^{-1} =
(h\Omega/\zeta)^{1/2} \ll 1 \)), we obtain:

\[ \frac{\Delta q_2}{q} = - \frac{1}{2\rho_m s^2} \frac{N^2}{g} n(-q)n(q) \Delta \quad (7) \]

where \( n(q) = n_0(0, q) \), and the function \( \Delta \) gives the con-
tribution of the electron lens to the quantum oscillations
of the electron DOS:

\[ \Delta = \frac{1}{\gamma} \sum_{r=1}^{\infty} \frac{(-1)^r}{\sqrt{r}} \psi_r(\theta) \cos \left( \frac{r \epsilon A_{ex}}{h|e| B} - \frac{\pi}{4} \right) \cos \left( \frac{\pi r Q_0}{\Omega} \right). \quad (8) \]

Here, \( \psi_r(\theta) = r \theta / \sinh \theta; \theta = 2\pi^2 T/h\Omega_{ex}; T \) is
the temperature expressed in units of energy, \( h\Omega_{ex} \) is the
splitting energy; and \( \Omega_{ex} = \Omega(0) \).

At very strong magnetic fields when the characteris-
tic diameter of the cyclotron orbit \( 2R \) is smaller than
the wavelength of the sound \( (QR < 1) \), we can go to
the limit \( q \to 0 \) in the expression (6), and we get
\( n_{\pm k}(p_z, \pm q)|_{q=0} = \delta_{k0} \). Then the semiclassical correction
\( \Delta q_1 \) becomes independent of the magnetic field, and
magnetic oscillations are fully described with Eq.(7). These
are ordinary quantum oscillations originating from the
electron DOS oscillations. In moderately strong but still
quantizing magnetic field \( (QR \gg 1) \), magnetic oscillations
reveal more complex structure.

To show this we evaluate the Fourier transform (6) us-
ing the stationary phase method. The main contribu-
tion to the integrals over \( \Phi \) in the Eq. (6) comes from the
stationary points at the cyclotron orbit where electrons
move in parallel with the electric field created by the
acoustic wave. In the chosen geometry these points are
the vertices of the lens \((\pm p_z, 0, 0)\), so we have [13, 14]:

\[ n_{\pm k}(p_z, \pm q) = \frac{a(l)}{(qR)^{1/2}} \exp \left[ \pm i q R \pm \frac{i \pi k}{2} \right] \]

\[ \times \cos \left[ q R - \frac{\pi k}{2} + \frac{\pi}{4l} \right] \quad (9) \]

where the cyclotron radius \( R \) depends on \( p_z \), and
the constant \( a(l) \) is given by:

\[ a_l = \frac{\alpha}{2\pi l} \Gamma \left( \frac{1}{2l} \right) \sqrt{\frac{m_1 m_2}{m^2}}. \quad (10) \]

Here, the dimensionless factor \( \alpha \) takes on values of the
order of unity, \( \Gamma(1/2l) \) is the gamma function.

Substituting the approximation (9) into the expres-
sions (5), and (7) we obtain:

\[ \frac{\Delta q}{q} = \frac{\Delta q_1}{q} + \frac{\Delta q_2}{q} = \frac{1}{4\rho_m s^2} \frac{N^2}{g} Y_1(q, \omega). \quad (11) \]
The function $Y_l(q, \omega)$ introduced in Eq.(11) has the form:

$$Y_l(q, \omega) = \frac{a^2(l)}{(qR_{ex})^l} \left\{ V(\omega) + W(\omega) \cos \left[ 2qR_{ex} + \frac{\pi}{2l} \right] + 2 \cos \left[ 2qR_{ex} + \frac{\pi}{4l} \right] \Delta \right\}$$

(12)

where

$$V(\omega) = \frac{i\pi \omega}{\Omega_{ex}} \coth \left[ \frac{\pi}{\Omega_{ex} \tau} (1 - i\omega \tau) \right],$$

(13)

$$W(\omega) = \frac{i\pi \omega}{\Omega_{ex}} \sinh^{-1} \left[ \frac{\pi}{\Omega_{ex} \tau} (1 - i\omega \tau) \right].$$

(14)

There are two oscillating terms included in the expression (12) for $Y_l(q, \omega)$. The first term originates from the semiclassical dynamical correction $\Delta q_{\perp}$ and depicts commensurability oscillations. The second term gives quantum oscillations superimposed with the geometrical oscillations. The superposition of these two kinds of magnetic oscillations was studied for two-dimensional electron systems [15, 16]. Here, we showed that the same effect occurs in conventional metals.

It follows from (12) that when the FS is flattened in the neighborhoods of the points corresponding to the stationary points at the cyclotron orbit, the magnetic oscillations are noticeably amplified. In particular, when the ultrasound wave travels across the external magnetic field, the magnitude of quantum oscillations in the velocity shift $\Delta s/s$ is usually small compared to that of DOS oscillations due to the small factor $(qR_{ex})^{-1}$, as shown in the top panels of the Fig. 1. However, when the FS includes locally flattened segments, the modulated quantum oscillations could reach the same order of magnitude as the DOS oscillations (see the bottom panels of the Fig.1). So the increase in the number of effective electrons originating from the FS local flattening is too small to directly change the DOS oscillations. Nevertheless, it could make an effect on quantum oscillations in the observables by means of amplification of geometrical oscillations modulating the latter.

The amplification of commensurability oscillations due to local geometry of the FS can be observed only for a definite choice of the magnetic field direction with respect to the symmetry axes of the crystal lattice. When the magnetic field is tilted away from such direction the influence of the point of flattening vanishes and the amplitude of the oscillations decreases. Therefore, the amplification of geometric oscillations should exhibit a pronounced dependence on the direction of the magnetic field. Such dependencies were repeatedly discussed for quasi-two-dimensional organic metals [2].

In summary, in the present work we show that local flattening of the FS could noticeably influence the magnitude of quantum oscillations in the observables in conventional 3D metals. Unlike the known direct effect of nearly cylindrical segments, the effect of points of flattening on the FS occurs due to amplification of commensurability oscillations modulating DOS quantum oscillations. The effect could be observed in experiments for some particular directions of the magnetic field provided that the external disturbance propagates across the latter. When revealed, this effect could be helpful to discover locations of flattened segments on the FSs. This could be an interesting contribution to the fermiology of high-temperature superconducting materials.

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