Kaluza-Klein towers on orbifolds: divergences and anomalies

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Abstract

The ultra-violet behavior of Kaluza-Klein theories on a one dimensional orbifold is discussed. An extension of dimensional regularization that can be applied to a compact dimension is presented. Using this, the FI-tadpole is calculated in the effective KK theory resulting from compactifying supersymmetric theories in 5 dimensions.

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1 Introduction

Models with 5 dimensional global supersymmetry compactified on orbifolds may be good candidates for extensions of the standard model and may have interesting cosmological applications. The supersymmetry may give rise to many impressive ultra-violet properties while the orbifold compactification can produce to phenomenologically interesting particle spectra. One such setup proposed by Barbieri, Hall, Nomura (BHN) \[1\] has particular remarkable properties. This model has the low energy spectrum identical to the standard model including a single massless Higgs, obtained by compactifying a supersymmetric theory with vector and hyper multiplets on the orbifold \(S^1/\mathbb{Z}_2 \times \mathbb{Z}_2\) in 5 dimensions. In the following table the Kaluza-Klein spectrum of the BHN model is presented.

| spectrum | \(p_5/R\) |
|----------|------------|
| \(\psi_M, \phi_H, A^\mu\) | \(x\) |
| \(\phi_M, \psi_H, \lambda\) | \(x\) |
| \(\phi_M^c, \psi_H^c, \psi_\Sigma\) | \(x\) |
| \(\psi_M^c, \phi_H^c, \phi_\Sigma\) | \(x\) |

The parities dictate in which mode functions a given field has to be expanded. The field theory consists of a complex Higgs scalar \(\phi_H\), its Higgsino \(\psi_H\), the standard model fermions \(\psi_M\), their mirrors \(\psi_M^c\), the standard model gauge fields \(A_\mu\), the two gauginos \(\lambda, \psi_\Sigma\) and the sfermions \(\phi_M, \phi_M^c\). The 5th component of the gauge field \(A_5\) in 5 dimension and an additional scalar reside in \(\phi_\Sigma\). Finally the superscript \(c\) denotes independent charge conjugate states. These states form vector and hyper multiplets of the original supersymmetric theory.

One can raise various questions about the physical properties of such models with towers of KK states: Can we make sense of infinitely many fields (at the quantum level)? Can low energy anomalies arise? What happens in the limit of very large radius \(R \to \infty\)? Is a theory with KK excitations “better” behaved than the low-energy its theory?

It was claimed in the recent literature that these types of orbifold models have a extremely mild ultra-violet behavior \([2]\): the effective action was claimed to be finite at one loop or even to all orders in perturbation theory. Others \([3]\) raised objections to such strong (unmotivated) claims. In ref. \([4]\) it was shown that the BHN model has a quadratic divergence due to a Fayet-Iliopoulos (FI) term and therefore its UV behavior is similar to that of the ordinary standard model.

In this proceedings we report on this work using technique of dimensional regularization of a compact dimension introduced in ref. \([5]\). After this method is described, it is used to calculate the quadratically divergent FI term and
to confirm some naive intuition concerning anomalies in low-energy effective theories. The talk ends with a short conclusion.

2 Dimensional regularization of a compact dimension

Dimensional regularization in 4 uncompact dimensions is a very powerful and convenient regularization scheme [3] because it relies on properties of complex functions. It is universal in the sense that it can be applied to an arbitrary loop calculation. It respects all symmetries of the classical theory, except when this symmetry develops an anomaly at the quantum level.

Dimensional regularization has been used before in the connection with compact manifolds. For example, in ref. [7, 8, 9] it was combined with ζ-function regularization (see ref. [10] for a general review of this method) for the compact dimension. The crucial difference with the method introduced in [5] is that there are two independent regulators $D_4$ and $D_5$ for the 4 dimensional integration and the additional summation of KK momenta.

It is not possible to directly apply the standard dimensional regularization techniques to a field theory defined on a compact dimension. We describe a procedure [3] how this can be done in two steps: 1. rewrite the sum over KK excitations as a complex contour integral, 2. modify this integral by inserting a regulator function. For concreteness we consider the infinite sum

$$\sum_{n \geq 0} \frac{1}{p_4^2 + (2n)^2 R^{-2}} = \frac{1}{2} \sum_{n \in \mathbb{Z}} \frac{1}{p_4^2 + (2n)^2 R^{-2}} + \frac{1}{2} \frac{1}{p_4^2}. \quad (1)$$

Here we have used that the summation is symmetric under $n \to -n$. This can be represented as an integral along a (clockwise) contour $\equiv$ around the real axis [11, 12], given in the picture below,

$$\int_{\equiv} \frac{-dp_5 \mathcal{P}^{++}(p_5)}{2\pi i \, p_4^2 + p_5^2} = \int_{\ominus} \frac{dp_5 \mathcal{P}^{++}(p_5)}{2\pi i \, p_4^2 + p_5^2 + m^2}, \quad \mathcal{P}^{\pm\pm} = \frac{1}{2} \left( \pm \frac{1}{p_5} + \frac{\mp \pi R}{\tan \frac{1}{2} \pi Rp_5} \right), \quad (2)$$

where we have introduce the “pole functions” $\mathcal{P}^{\pm\pm}$. (In the computation of the Fayet-Iliopoulos term we use $\mathcal{P}^{--}$ as well.) By noticing that the function $1/(p_4^2 + (2n)^2 R^{-2})$ does not have a pole at infinity, it follows that this integral can be re-written as contour integral $\ominus$ over the upper and lower half plane with opposite orientation (anti-clockwise) to the $\equiv$ contour. (We have introduced an IR regulator mass $m$, which is needed to turn the sum into a contour integral by complex function analysis [3].) The figure below gives a schematic picture of this situation in the complex $p_5$-plane:
The dots ⋅ denote the KK masses $2n/R$; the poles of $\mathcal{P}^{++}$ and the $X$’s denote generic poles in the integrand: in this case at $±i\sqrt{p_4^2 + m^2}$. The regulated sum-integral is now defined by

$$
\int \Theta \, d^D p_5 \int d^{D_4} p_4 \frac{\mathcal{P}^{++}(p_5)}{p_4^2 + p_5^2 + m^2} \equiv \int \Theta \, \frac{dp_5}{2\pi i} \int_0^\infty dp_4 R_4(p_4) R_5(p_5) \frac{\mathcal{P}^{++}(p_5)}{p_4^2 + p_5^2 + m^2},
$$

with complex dimensions $D_4$ and $D_5$ that act as regulators. The regulator functions $R_4(p_4)$ and $R_5(p_5)$ are given by

$$
R_4(p_4) = \frac{2\pi \frac{1}{2} D_4}{\Gamma(\frac{1}{2} D_4)} p_4^{\frac{1}{2} D_4 - 4}, \quad R_5(p_5) = \frac{\pi \frac{1}{2} D_5}{\Gamma(\frac{1}{2} D_5)} p_5^{\frac{1}{2} D_5 - 1}.
$$

Here we have introduced two (arbitrary) renormalization scales $\mu_4$ and $\mu_5$. The regulator function $R_4$ is the standard one for dimensional regularization of 4 non-compact dimensions [13].

One of the crucial properties the regularization of the sum-integral in this way is that the classification of different types of divergences is independent of the regulator, but depends only on the spectrum of the KK tower encoded in the pole functions $\mathcal{P}^{±±}$. In fact, it can be shown that (for $\text{Im} \, p_5 > 0$) the pole functions consist of three parts [3]:

$$
\mathcal{P}^{±±}(p_5) = -\frac{1}{2} \pi R \pm \frac{1}{2} \frac{1}{p_5^2} \rho>(p_5)
$$

By inserting only the first part into the regulated sum-integral expression a cubic divergence arises as one would expect for an integral over this propagator in 5 dimensions. The second term represents just a single pole, hence there is no need for a separate regulator $D_5$ for the sum. Therefore we can safely put it to 1, and obtain a quadratic divergence as in 4 dimensions. The last term gives a finite contribution, because it can be shown that $\rho>$ exponentially suppressed for large complex momenta with $\text{Im} \, p_5 > 0$. (For $\text{Im} \, p_5 < 0$ a similar result can be derived.)

3 The Fayet-Iliopoulos term

In a (unbroken) supersymmetric field theory in 4 dimensions the FI-term is either quadratically divergent or vanishes at one loop. The diagram of the FI-contribution to the selfenergy of a scalar is given by:
The dotted line corresponds to the auxiliary field $D\parallel$ of the Abelian gauge multiplet in 4 dimensions. In [4] we have investigated what happens to the FI-term in the effective field theory coming from 5 dimensions with a mass spectrum of the complex scalars of the hyper multiplet on $S^1/Z_2 \times Z'_2$. We take the charges of these scalars such that $q_{n^+} = -q_{n^-} = 1$. Formally, the expression for the one loop contribution to the FI term reads

$$
\xi = \sum_{n,\alpha} g q^{\alpha\alpha}_n \int \frac{d^4p_4}{(2\pi)^4} \frac{1}{p_4^2 + (m_{n^\alpha})^2 + m^2},
$$

where $m_{n^\alpha} = 2n/R$ and the sum for $\alpha = +$ is over $n \geq 0$, while for $\alpha = -$ over $n > 0$. Using dimensional regularization we obtain

$$
\xi = g \int \frac{dD^4p_4}{(2\pi)^{D_4}} \int \frac{dD^5p_5}{2\pi i} \left\{ \frac{P^{++}(p_5)}{p_5^2 + p_5^2 + m^2} - \frac{P^{--}(p_5)}{p_4^2 + p_5^2 + m^2} \right\}.
$$

Substituting the expressions of the pole functions (2), gives exactly the same result as the regulated FI term for one massless complex scalar:

$$
\xi = g \int \frac{dD^4p_4}{(2\pi)^{D_4}} \int \frac{dD^5p_5}{2\pi i} \frac{1}{p_5^2 + p_5^2 + m^2} = g \int \frac{dD^4p_4}{(2\pi)^{D_4}} \frac{1}{p_4^2 + m^2}.
$$

Since it behaves as a single particle contribution we can safely take $D_5 = 1$ giving the 4 dimensional quadratically divergent expression. This result holds for any finite $R$, since it is independent of the radius $R$ of the compact dimension. Therefore, we conclude that it is also true in the limit $R \to \infty$. This signals that the orbifolding is not undone in this decompactification limit.

One may wonder whether this divergence may be canceled by other gauge corrections. In [4] it is shown that the other gauge contributions give a finite correction and can therefore never cancel this quadratic divergence. Heuristically, this is to be expected since the FI-term is the only diagram of all gauge correction to the selfenergy that is proportional to the trace of the charges in the loop.

For this Fayet-Iliopoulos contribution, an auxiliary field tadpole counter term has to be introduced. Such a counter term of course has to be consistent with the symmetries of the theory. On both branes we have at most $N = 1$ supersymmetry: the other supersymmetry vanishes there as it has the opposite parity. Therefore, on the branes $D$-terms can be added for the auxiliary fields that do not vanish. By a similar analysis on the effective field theory level one can indeed show that one obtains the same quadratic divergence for all $D\parallel_{2n}$. In the 5 dimensional picture this means that the divergences occur on the two branes only.
4 Low energy anomalies

Gauge anomalies render a theory to be inconsistent at the quantum level. Also in models with one extra dimension that are under investigation here, one has to address the question of low energy anomalies. Therefore in the effective field theory in 4 dimensions the triangle diagram

\[ \sum \psi_{A_a} A_{\mu} \psi \]

has to be considered with the sum over all (chiral) fermion species. This includes summing over all KK excitations since of course in the loop heavy virtual particles may run around.

The standard way of thinking about anomalies is that only the zero-mode fermions can contribute. Hence, if the massless fermionic spectrum is anomaly free, then no anomalies in the low-energy field theory can arise. In particular, the BHN model having the fermionic spectrum of the standard model is anomaly free. Using the dimensional regularization procedure of a compact dimension discussed above this naive expectation is indeed confirmed that there are no gauge anomalies in this model.

These anomaly are on the level of the low-energy effective field theory. In ref. [14] a more subtle form of anomaly is discussed which is localized at the both fixed points where opposite chiral states are projected out, while the integrated anomaly vanishes. However, it still has to be clarified how these fixed point anomalies exactly effect the low-energy physics.

5 Conclusion

In this talk we have discussed an extension of dimensional regularization that can be applied to a (factorizable) space-time that has one compact dimension, using complex contour integral to represent (divergent) sums. This contour integral can then be regularized by introducing a regulator function inspired by standard dimensional regularization. Having two regulators \((D_4, D_5)\), this regularization prescription treats the additional dimension without any prejudice. And in addition it leave the properties of the KK towers in tact, since they are encoded in regularization independent pole functions.

Using this method we showed that the tadpole contribution to a component of the auxiliary field is quadratically divergent and proportional to the sum of (hyper) charges of massless scalar fields in the BHN model. It is not difficult to identify the 5 and 4 dimensional divergent and finite contribution, exploiting the properties of the pole functions associated with the orbifold. The behavior of the FI-tadpole is very similar to the low-energy anomalies in the sense that they are both independent of the size of the extra dimension. In particular, even if we take \( R \to \infty \) we find the same expression due to the massless modes.
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