Constraints on the Density and Internal Strength of 1I/Oumuamua

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Abstract

1I/Oumuamua was discovered by the Panoramic Survey Telescope and Rapid Response System (Pan-STARRS 1) on 2017 October 19. Unlike all previously discovered minor planets, this object was determined to have eccentricity $e > 1.0$, suggesting an interstellar origin. Since this discovery and within the limited window of opportunity, several photometric and spectroscopic studies of the object have been made. Using the measured light curve amplitudes and rotation periods we find that, under the assumption of a triaxial ellipsoid, a density range $1500 < \rho < 2800 \text{ kg m}^{-3}$ matches the observations and no significant cohesive strength is required. We also determine that an aspect ratio of $6 \pm 1:1$ is most likely after accounting for phase-angle effects and considering the potential effect of surface properties. This elongation is still remarkable, but less than some other estimates.

Key words: methods: statistical – minor planets, asteroids: individual (1I/Oumuamua) – techniques: photometric

1. Introduction

1I/2017U1 ('Oumuamua) was discovered by the Panoramic Survey Telescope and Rapid Response System (Pan-STARRS 1) in 2017 October and subsequently identified as the first "hyperbolic asteroid" (Williams et al. 2017). Its relatively faint H-magnitude along with its extreme orbital eccentricity means that it was only observable for a relatively short window of time. The current estimated detection rate of hyperbolic asteroids is estimated to be 0.2 yr$^{-1}$. When the Large Synoptic Survey Telescope begins operations, the detection rate will increase to 1 yr$^{-1}$ (Trilling et al. 2017).

Spectroscopic measurements of 1I suggest a surface comparable to that of cometary nuclei and outer solar system objects (Fitzsimmons et al. 2017; Masiero 2017). Light curve observations of the object suggest a highly elongated shape with reported values of $a/b = 5.9 \pm 1.0$ from Bolin et al. (2018) and $a/b = 10$ from Meech et al. (2017), assuming the object to be a triaxial ellipsoid. In either case this is an unusually elongated object compared to the known shapes and shape distributions of asteroids. Taking for instance the shape distribution for Main Belt Asteroids, we find that only 0.003% of objects are expected to be this elongated (McNeill et al. 2016).

The amplitude of a light curve is strongly affected by the phase angle of the observations. Increased shadowing and scattering effects at high phase angles cause light curve minima to appear fainter and hence the apparent light curve amplitude increases. This can lead to an overestimation of the elongation of an object. In Section 2 we correct for phase angle effects to derive the most likely shape of 1I. Then in Section 3 we use the assumption of fluid equilibrium to derive the internal cohesive strength of 1I. In Section 4 we discuss our results and some implications of our work.

2. Phase Angle Effects

Zappala et al. (1990) showed a linear relationship between the apparent amplitude of a light curve $A_{\text{obs}}$ and its actual amplitude $A(\alpha = 0^\circ)$ for phase angles $\alpha \leq 40^\circ$:

$$A(\alpha = 0^\circ) = \frac{A_{\text{obs}}}{1 + s\alpha}.$$  (1)

where $s$ is a taxonomy-dependent slope parameter. Table 1 shows a summary of the phase angles of the reported light curve observations of 1I. All of the reported light curve observations of 1I fall within this $\alpha$ domain and hence Equation (1) can be used to correct measured light curve amplitudes.

Meech et al. (2017) showed a light curve amplitude of approximately 2.5 mag: an incredibly large variation corresponding to an elongation of 10:1 if only geometric contributions are considered. From observations made several days later, Bolin et al. (2018) report a light curve with amplitude $A_{\text{obs}} = 2.05 \pm 0.53$ mag. Again assuming that all variation is purely due to geometric effects, this suggests an elongation between 4:1 and 11:1. At its upper end, this is in agreement with the result of Meech et al. (2017). Bolin et al. (2018) use Equation (1) to correct for the $s = 0.1$ phase angle of their observations and assume a slope parameter $s = 0.015 \text{ mag deg}^{-1}$. This yields a corrected amplitude $A(\alpha = 0^\circ) = 1.51 \pm 0.39$ mag, corresponding geometrically to an elongation between 3:1 and 6:1. We apply a similar correction to the Meech et al. (2017) $A_{\text{obs}}$, which was measured at $\alpha = 22^\circ$. This produces an amplitude $A(\alpha = 0^\circ) = 1.9$ mag, corresponding to an elongation of approximately 6:1.

Due to 1I’s flyby geometry, there have not been enough observations at different orbital geometries to allow light curve inversion to be carried out. To understand the uncertainties present in results derived from the data shown in Table 1, we use the Durech et al. (2010) light curve inversion technique code with a fixed spin pole latitude of $\beta = 90^\circ$. This model accounts for surface effects as well as geometric effects. In this case we find that the effects of scattering and/or limb darkening can affect the determined elongation of an object by approximately $a/b \pm 1$. This should be considered as an uncertainty in any stated elongation limits derived from a single light curve. Therefore, we consider our final result to be $a/b = 6 \pm 1$. 
3. Constraints on the Density and Cohesive Strength of ‘Oumuamua

3.1. Density

The critical rotation period of an object, $P_{\text{crit}}$, is defined at the point where the centrifugal force due to rotation is equal to the self-gravity of the object; if the asteroid spins up from this critical rotation period, then mass shedding will commence.

The potential at the surface of an ellipsoid can be represented as Equation (2), where the three $A_i$ functions are dimensionless parameters dependent upon the axis ratios of the body and are given in Equations (3)–(5).

$$
\Phi(a, b, c) = -\pi G \rho (A_0 - A_x a^2 - A_y b^2 - A_z c^2).
$$

By setting the acceleration at the tip of the object, i.e., $(x, y, z) = (a, 0, 0)$, to be equal to the centrifugal acceleration, we can determine the critical angular frequency at which the body will undergo rotational fission. This is given in Equation (6). This can be rearranged to give the critical rotation period, $P_{\text{crit}}$, given in Equation (7), where $\rho$ is the density in grams per cubic centimeter.

$$
\omega_{\text{crit}} = \sqrt{2 \pi G \rho A_x}.
$$

$$
P_{\text{crit}} = \frac{2.7 hr}{\sqrt{\rho A_x}}.
$$

For spherical objects this becomes $P_{\text{crit}} = \frac{3.3 hr}{\sqrt{\rho}}$. This value is taken by Bolin et al. (2018) and scaled according to the $a$ axis ratio. This allows them to determine a lower density limit for II assuming its rotation period to be equal to the critical rotation period, producing a value $\rho = 1000 \text{ kg m}^{-3}$. This assumption, however, only holds for spherical or near-spherical objects. Instead, we determine a more suitable equation for an elongated object assuming $a = 6$, giving $A_x = 0.086$. Substituting this into Equation (7), we find that for II and objects like it a better equation for the lower density limit is given by Equation (8)

$$
P_{\text{crit}} = \frac{9.21 hr}{\sqrt{\rho}}.
$$

Taking the rotation period determined by Meech et al. (2017), $P = 7.34 \text{ hr}$, we determine a lower density limit $\rho_{\text{lim}} = 1600 \text{ kg m}^{-3}$. This value only represents a lower limit for a cohesionless body, as it assumes that the rotation period is exactly equal to the critical spin rate of the body.

A more sophisticated approach is to assume that the object is a strengthless Jacobi ellipsoid approximating a rubble pile, a valid assumption for most asteroids with $D \geq 200 \text{ m}$. (This size is close to the estimated size of II so a different geophysical regime could apply, as discussed below.) The shape of a Jacobi ellipsoid required to generate a light curve of a given amplitude can be calculated by setting $a = 1$ and solving Equation (9) from Chandrasekhar (1969) for relative axis ratios $b$ and $c$:

$$
a^2 b^2 \int_0^\infty \frac{du}{(a^2 + u)(b^2 + u) \Delta} = c^2 \int_0^\infty \frac{du}{(c^2 + u) \Delta}
$$

where $\Delta$ is defined by

$$
\Delta^2 = (a^2 + u)(b^2 + u)(c^2 + u).
$$

When $abc$ is known and the angular rotation frequency of the asteroid, $\omega$, is also known, then the density may be estimated (also from Chandrasekhar 1969):

$$
\frac{\omega^2}{\pi G \rho} = 2abc \int_0^\infty \frac{udu}{(a^2 + u)(b^2 + u) \Delta}.
$$

Here $G$ is the gravitational constant, and $\rho$ is the density of the body in kg m$^{-3}$; it is assumed that the density of the object is constant throughout and that there is no internal strength. An object with an amplitude $A_{\text{obs}} = 1.9 \text{ mag}$ produces a best fit with $abc$ axis ratios 1:0.17:0.16 and a density of 1800 $< \rho < 2200 \text{ kg m}^{-3}$, consistent with the $\rho_{\text{lim}}$ determined previously.

3.2. Internal Strength

Jeans (1919) stated that strengthless ellipsoids where $b^2/a^2 = 0.44$ are potentially unstable; our solution has $b^2/a^2$ of around 0.17. They assumed the ellipsoid to be an incompressible fluid. Highly elongated fluid objects will settle into less-elongated equilibrium shapes due to fluid instability. There is a limit to how well a fluid ellipsoid approximates a rubble pile asteroid, as these will have some internal friction that will affect the equilibrium end-states of the object. Holsapple (2001) stated that for known asteroids assuming “a modest angle of friction” elongated shapes can be maintained and that these fluid instabilities are negligible. Therefore, we attempt to calculate the cohesive strength required using a simplified Drucker–Prager model (Holsapple 2004). The Drucker–Prager failure criterion is a model of the three-dimensional stresses within a geological material at its critical rotation state. The shear stresses on a body, $\tau$, in three orthogonal $x\tau\nu$ axes are dependent on the shape, density, and rotational properties of

\begin{table}[h]
\centering
\begin{tabular}{|c|c|c|}
\hline
Date & $\alpha$ & $A_{\text{obs}}$ (mag) & Telescope & References \\
\hline
2017 Oct 25 & 19° & 2.5 & VLT & Meech et al. (2017) \\
2017 Oct 26 & 21° & 2.5 & GS+VLT & Meech et al. (2017) \\
2017 Oct 27 & 22° & 2.5 & GS+CFHT+UKIRT & Meech et al. (2017) \\
2017 Oct 29 & 24° & >1.2 & APO & Bolin et al. (2018) \\
2017 Oct 30 & 24° & >1.5 & DCT & Knight et al. (2017) \\
\hline
\end{tabular}
\caption{Summary of the Phase Angles of Reported 1I/‘Oumuamua Light Curve Observations Considered Here}
\end{table}
the body (Holsapple 2007):

\[
\sigma_r = \left( \rho \omega^2 - 2 \pi \rho^2 G A_0 \right) \frac{a^2}{5} \tag{12}
\]

\[
\sigma_y = \left( \rho \omega^2 - 2 \pi \rho^2 G A_0 \right) \frac{b^2}{5} \tag{13}
\]

\[
\sigma_z = \left( -2 \pi \rho^2 G A_0 \right) \frac{c^2}{5}. \tag{14}
\]

The Drucker–Prager failure criterion is the point at which the object will undergo rotational fission and is given by

\[
\frac{1}{6} \left[ (\sigma_r - \sigma_y)^2 + (\sigma_r - \sigma_z)^2 + (\sigma_z - \sigma_y)^2 \right] 
\leq [k - s(\sigma_r + \sigma_y + \sigma_z)]^2 \tag{15}
\]

where \( k \) is the internal cohesive strength of the body and \( s \) is a slope parameter dependent on the assumed angle of friction, \( \phi \),

\[
s = \frac{2 \sin \phi}{\sqrt{3} (3 - \sin \phi)}. \tag{16}
\]

All asteroids modeled by Holsapple (2004) have an angle of friction, \( \phi_F \), with most having \( \phi_F < 15^\circ \). We assume the angle of friction for II in this case to be \( \phi = 15^\circ \).

Using a simple model based on this failure criterion we determine the required cohesive strength as a function of density for II using input parameters determined from the observations. We use a Monte Carlo numerical simulation to determine the required strengths for a range of synthetic objects generated using the estimates of size, shape, and rotation period of II and their associated uncertainties. This is repeated for a wide range of possible densities. For an object of II’s estimated size and elongation, and assuming sensible density estimates, we find that a cohesive strength of only a few Pascals is required—essentially no significant cohesive strength. This is in agreement with the result presented by Bolin et al. (2018).

Using Equation (15) it is possible to set constraints on the density of an object assuming zero cohesive strength, i.e., \( k = 0 \). Assuming an angle of friction \( \phi_F = 15^\circ \) and \( P = 7.34 \) hr we determined that for an object with II’s estimated shape a density range \( 1500 < \rho < 2800 \) kg m\(^{-3}\) is found. This density range is consistent with the assumption that II is a rubble pile.

4. Discussion and Caveats

The elongations determined for II are based upon the assumption that the light curve amplitude is entirely due to shape (not variations in surface reflectivity; see below) and that the object is being viewed equatorially. From the existing data, no period solution has been entirely agreed upon. Bannister et al. (2017) determined a period solution of 8.1 hr, while from a full light curve Meech et al. (2017) determined a period of 7.34 hr. It has been proposed by Fraser et al. (2018) and Drahus et al. (2017) that the seemingly variable rotation rate is due to non-principal axis rotation. This would explain why it has not been possible for a single period to fit all of the light curves. If this is the case, it is difficult to assess the validity of the assumption that the object was observed equator-on at its maximum amplitude. This means that the elongation is a lower limit and hence the density and strength estimates may be underestimated.

The V-shaped light curve minima observed from II could be an indication of binarity (Knight et al. 2017, Thirouin et al. 2017). From the observations obtained for the object to date, however, it is not possible to determine if this is the case. In the case of a binary system, the required density is effectively the same as that required for a Jacobi ellipsoid. The high amplitude of this asteroid’s light curve can be explained either with a single, highly elongated body of \( \frac{r}{a} \geq 6 \) or a binary system. Asteroids with light curve amplitudes similar to that displayed by II (e.g., (1620) Geographos) are generally explained using highly elongated single bodies, and we consider this to be the simpler and more likely option.

If the object is monolithic in nature, then the fluid approximations used here will be invalid. This requires a higher density due to lack of porosity, and the lower density limit estimate is still valid.

It is also possible to produce a large light curve amplitude if there is a significant variation in the albedo of the body across its surface. However, there are no minor planets known to have such a large variation. The only object in the Solar System with such a variation is the Saturnian moon Iapetus, which is highly variegated due to the fact that it sweeps up dust preferentially on one hemisphere.

The estimated density of 2000 kg m\(^{-3}\) is consistent with the previous average density estimates for asteroids (Carry 2012). This density is greater than that expected for cometary nuclei. The density value obtained is also less than that of most meteorite samples, suggesting that II must have some degree of porosity that supports a rubble pile assumption. The small required cohesive strength, of order several Pascals, suggests that if II is a rubble pile, it is effectively strengthless. Rubble pile asteroids have been determined to have possible strengths from zero to several hundred Pascals, so in this respect II is not unusual (Polishook et al. 2016). It is worth noting that the assumption that this object is a rubble pile is valid assuming \( D > 150 \) m (Pravec et al. 2002). For objects with a smaller diameter, a monolithic structure is more likely.

5. Conclusions

Using the reported light curve amplitudes and rotation periods of the hyperbolic asteroid 11/2017U1 (‘Oumuamua) and accounting for phase-amplitude effects, we determine a lower limit on the elongation of II of 6:1. Assuming a triaxial ellipsoid we constrain the plausible density range of this object to be \( 1500 < \rho < 2800 \) kg m\(^{-3}\); no significant cohesive strength is required at this density. It is possible to obtain a valid binary density solution for the system, but there is currently no evidence to favor this over the single-body explanation. These values are based on the assumption that the object was observed equatorially. If the object is tumbling, as proposed by Fraser et al. (2018) and Drahus et al. (2017), this is a more complicated scenario. As such, it should be emphasized that the elongation determined for the object is a lower limit, which may then lead to an understimation of the density and cohesive strength of the object.

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Software: Light curve inversion source code (Durech et al. 2010).
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