Higgs Boson Masses in the MSSM with General Soft Breaking

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Abstract

The operators that break supersymmetry can be holomorphic or non-holomorphic in structure. The latter do not pose any problem for gauge hierarchy and are soft provided that the particle spectrum does not contain any gauge singlets. In minimal supersymmetric model (MSSM) we discuss the impact of non-holomorphic soft-breaking terms on the Higgs sector. We find that non-holomorphic operators can cause significant changes as are best exhibited by the correlation between the masses of the charginos and Higgs bosons.
I. INTRODUCTION

In general, breakdown of global supersymmetry is parameterized by a set of operators with dimensionality less than four. Each operator thus comes with an associated mass scale, which must fall in the TeV domain if supersymmetry is the correct description of Nature beyond Fermi energies. The operators that break the supersymmetry must be soft \textit{i.e.} quadratic divergences must not be regenerated. The mass terms for scalars as well as their trilinear couplings are soft operators \cite{1}. However, the most general list of supersymmetry breaking operators involve novel structures beyond these. Indeed, trilinear couplings, for example, can have both $A\phi\phi\phi$ type holomorphic structure as well as $A'\phi^*\phi\phi$ type non-holomorphic structure. There is nothing wrong in considering the non-holomorphic structures since they are perfectly soft if there are no gauge singlets in the theory \cite{2}. In this sense, MSSM provides a perfect arena for analyzing consequences of the non-holomorphic soft-breaking terms.

In this work we study the impact of non-holomorphic soft-breaking terms on the Higgs sector of the MSSM. We will analyze Higgs sector in conjunction with the chargino sector so as to single out the effects of non-holomorphic trilinear couplings from the $\mu$ parameter. Since these sectors are two of the prime concerns of experiments at the LHC, we expect that our results will be testable in near future.

The holomorphic soft terms in the MSSM involve

$$-\mathcal{L}_{\text{soft}} \ni \bar{\tilde{Q}} \cdot H_u h_t A_t \tilde{U} + \bar{\tilde{Q}} \cdot H_d h_b A_b \tilde{D} + \bar{\tilde{L}} \cdot H_d h_\tau A_\tau \tilde{E} + \text{h.c.}$$

(1)

in addition to the mass terms of scalars and gauginos. In writing this equation, we have included only the third generation of sfermions with trilinear couplings $A_{t,b,\tau}$.

As mentioned before, as has been shown explicitly in \cite{2,3}, in supersymmetric theories which do not have pure gauge singlets in their particle spectrum, the holomorphic supersymmetry breaking terms do not necessarily represent the most general set of soft-breaking operators. Namely, the non-holomorphic operators can also be regarded as soft symmetry breaking operators as long as the theory does not contain any gauge singlet chiral superfields. Being free of any gauge singlets, the soft breaking sector of the MSSM can be enlarged to cover

$$-\mathcal{L}'_{\text{soft}} = \bar{\bar{\tilde{Q}}} \cdot H_d^c h_t A'_t \tilde{U} + \bar{\bar{\tilde{Q}}} \cdot H_u^c h_b A'_b \tilde{D} + \bar{\bar{\tilde{L}}} \cdot H_u^c h_\tau A'_\tau \tilde{E} + \text{h.c.}$$

(2)
in addition to (1). Here $A'_{t,b,\tau}$ are non-holomorphic trilinear couplings which do not need to bear any relationship to the holomorphic ones in $A_{u,d,e}$ in (1).

The trilinear couplings in (1) and (2) break supersymmetry in a soft fashion, that is, in a way without regenerating quadratic divergences, and hence, both set of operators must be included in phenomenology of low-energy supersymmetry [1, 4].

Our concern in this letter is the Higgs sector. At low $\tan \beta (\tan \beta \leq 30)$, radiative effects in the Higgs sector drive mainly from the top (s)quarks since other fermions are too light to have significant Yukawa interactions. Thus, radiative corrections to Higgs sector [5, 6, 7] may be computed by considering an effective potential approach [6, 7] with the top quark and scalar top quark loops. Their field-dependent mass-squareds are given by

$$m^2_t(H) = h^2_t |H^0_u|^2,$$

(3)

for top quarks,

$$M^2_t(H) = \begin{pmatrix} m^2_{tL} + m^2_t - \frac{1}{4} \left( g_2^2 - \frac{1}{3} g_Y^2 \right) (|H^0_u|^2 - |H^0_d|^2) & h_t A_t^* H^0_u - h_t (\mu + A_t^*) H^0_d \\ h_t A_t H^0_u - h_t (\mu + A_t^*) H^0_d^* & m^2_{tR} + m^2_t - \frac{1}{3} g_Y^2 (|H^0_u|^2 - |H^0_d|^2) \end{pmatrix},$$

(4)

for top squarks. This very expression for stop masses clearly shows that entire effect of including the non-holomorphic trilinear coupling $A'_{t}$ is to shift the $\mu$ parameter as

$$\mu \to \mu + A'_{t},$$

(5)

which implies that all the effects of scalar top quarks on the Higgs sector, as described in detail in [5, 6, 7] for holomorphic soft terms, remain intact except that the $\mu$ parameter is not as it is in the superpotential yet it is the shifted one in (5).

Consequently, effects of the non-holomorphic trilinear coupling $A'$ parameter can be disentangled from those of the $\mu$ parameter if $\mu$ is known from an independent source. Clearly, an independent knowledge of $\mu$ can be obtained from neutralino or chargino sectors either via direct searches or via indirect bounds from certain observables. A readily recalled observable is $b \to s\gamma$ decay [9, 10, 11, 12]. In addition one can consider bounds from EDMs or muon $g - 2$ and suchlike but for purposes of obtaining a simple yet direct constraint on $\mu - A'_{t}$ relationship $b \to s\gamma$ decay suffices.
We first detail radiative corrections to Higgs boson masses in light of (3,4) and then revisit the $b \rightarrow s\gamma$ decay in order to show how $A'_t$ can be extracted from the bulk of the MSSM parameters. As mentioned before, the Higgs boson masses depend on $\mu + A'_t$ not $A'_t$ in isolation. In fact, the lightest Higgs boson mass reads as [5, 6, 7]

$$m_h^2 \simeq M_Z^2 + \frac{3g_2^2m_t^4}{8\pi^2M_W^2} \left[ \ln \left( \frac{M_S^2}{m_t^2} \right) + \frac{X_t^2}{M_S^2} \left( 1 - \frac{X_t^2}{12M_S^2} \right) \right]$$  \hspace{1cm} (6)$$

where the mean stop mass-squared

$$M_S^2 = \frac{1}{2}(m_{t_1}^2 + m_{t_2}^2)$$  \hspace{1cm} (7)$$
is independent of $A'_t$ while the left-right mixing term is $X'_t = A_t - (\mu + A'_t) \cot \beta$. Notice that the MSSM result [8] is recovered by setting $A'_t \rightarrow 0$. For a clearer view of the impact of $A'_t$ on the Higgs boson mass, one notes that the upper bound shift of the lightest Higgs boson mass can be expressed as

$$\Delta m_h^2 \propto \mu A'_t \cot \beta$$  \hspace{1cm} (8)$$
in which we have neglected the higher order cot $\beta$ terms since their effects will be negligible for the chosen $\tan \beta$ value, and the exact expression can easily be derived from (6). This shift may vary from a few MeVs (for low values of $|A'_t|$) up to tens of GeVs depending on the input parameters. This is an important aspect since it modifies the upper bound of the Higgs boson mass, and in case a Higgs signal below 130 GeV is not observed at the LHC, it provides an explanation for higher values of $m_h$ already in the MSSM (without resorting to NMSSM or U(1)$'$ models which generically yield higher values for $m_h$).

The rare radiative decay $b \rightarrow s\gamma$ provides an excellent arena for hunting the new physics effects since its characteristic mass scale, the $b$ quark mass $m_b$, admits direct application of perturbative QCD [9, 10, 11, 12]. Moreover, experimental precision has increased over the years at the level of essentially confirming the SM result [12, 13]. Therefore, the branching ratio of this decay is expected to place rather stringent limits on the sparticle contributions, and thus, provide an almost unique way of determining the allowed ranges of $A'_t$. The reason behind this observation is that $b \rightarrow s\gamma$ decay is sensitive to both $\mu$ (via chargino exchange) and $\mu + A'_t$ (via the stop exchange) as illustrated in Fig. 1. Therefore, one has both $\mu$ and $\mu + A'_t$ at hand.
FIG. 1: The stop–chargino exchange contribution to $b \to s\gamma$ decay (photon can be coupled to any charged line). While the stop mixing is directly sensitive to $\mu + A'_t$ the chargino exchange involves mass of the charged Higgsino, the $\mu$ parameter. This process thus involves both $\mu$ itself and $\mu + A'_t$ leading thus disentangling of $A'_t$ from rest of the parameters.

simultaneously and thus it becomes possible to disentangle $A'_t$ effects from rest of the soft masses. In fact, from the form of the chargino mass matrix

$$m_\chi = \begin{pmatrix} M_2 & M_W \sqrt{2} \sin \beta \\ M_W \sqrt{2} \cos \beta & \mu \end{pmatrix}$$

(9)

one observes that wino and higgsino components mix due to electroweak breaking (denoted by a cross on the horizontal line inside the loop), and higgsino mass $\mu$ enters the branching ratio in isolation. Unlike chargino sector, as suggested by Fig. 1, the stop left-right mixing (denoted by a cross on the dashed arc in the loop) depends explicitly on $\mu + A'_t$ as seen also from (4). The simultaneous $\mu$ and $\mu + A'_t$ dependencies of $b \to s\gamma$ decay, as depicted in Fig. 1, results thus in a distinction between $\mu$ and $\mu + A'_t$, which would not be possible by an analysis of the Higgs sector alone.

Depicted in Fig.2 is the impact of the different $A'_t$ values on certain parameters respecting $b \to s\gamma$ restrictions. The numerical results herein correspond to a specific choice of the parameters

$$M_1 = 140, \ M_2 = 280, \ M_3 = 1000, \ M_A = 500, \ A_t = -1600, \ m_{t_L} = 1000, \ m_{t_R} = 200,$$

(10)
all in GeV. We fix $\tan \beta = 5$ which is theoretically in accessible region \cite{14} and do not consider higher $\tan \beta$ values since large $\tan \beta$ values reduce $A'_t$ effects in this regime as also can be seen from the left-right mixing entry of (4). These parameter values are chosen judiciously such that $m_h$ and $\tan \beta$ agree with the LEP II lower bound of $m_h \geq 114$ GeV and $\tan \beta > 2$ when $A'_t$ vanishes \cite{14}. This choice will help in revealing the effects of $A'_t$ in a transparent way. We will see that typically large negative values of $A'_t$ leads to observable changes where how large it should be depends, of course, on the characteristic scale of soft mass parameters in (4).

Fig.2 (a) shows our color convention and how $m_h$ depends on $A'_t$. It is seen that $m_h$ just agrees with the LEP bound when $A'_t$ is small in magnitude. However, as it grows in negative direction up to $-2.5$ TeV the Higgs boson mass gets gradually shifted towards the 135 GeV borderline. This clearly shows that
a measurement of the Higgs boson mass can imply strikingly different parameter space than one would expect naively from a restricted set of soft-breaking terms given in (I). In addition, the horizontal behavior of the curves in Fig.2 (a) is due to the allowed range of $\mu$ parameter by the $b \to s\gamma$ bound. That is, the $\mu$ parameter takes on different values for each selection of $A'_t$ determined via the $b \to s\gamma$ restriction. This is also reflected in Fig.2 (c).

Shown in Fig. 2 (b) is the mass splitting between the CP-odd and CP-even Higgs bosons vs. the lightest Higgs boson mass. With the presence of non-holomorphic trilinear coupling $A'_t$ the CP-odd and CP-even higgs bosons may not be degenerate as they are in the MSSM. It is clear that, for each value of $A'_t$ a corresponding splitting $\sim 3.5$ GeV can exist. For small values of $A'_t$ the $\mu$ parameter falls in a rather narrow band, that is, bigger the $A'_t$ (in negative direction) larger the range of $\mu$ parameter. This increase in the mass splitting can be measured at the ILC if not at the LHC.

Depicted in Fig. 2 (c) is the dependence of $m_h$ on $\mu$ parameter for different values of $A'_t$. At low $A'_t$ the $\mu$ parameter is preferred to be $-1$ TeV for $m_h$ to agree with the experiment. However, as $A'_t$ grows to large negative values the $\mu$ parameter takes on its mirror symmetric value; $\mu = 1$ TeV. This large swing in the allowed range of $\mu$ stems solely from the dependence of the stop masses in (4) on $\mu$ and $A'_t$ where $b \to s\gamma$ does not allow their sum to exceed a certain threshold due to rather narrow band of values left to new physics contributions [12, 13].

Finally, shown in Fig. 2 (d) is the variation of $m_h$ with the lighter chargino mass $m_{\chi^\pm}$ as $A'_t$ varies. One notices how their relationship is modified at large negative $A'_t$ via especially the region at large $m_h$. Indeed, as $A'_t$ grows to large negative values the Higgs boson mass is shifted towards 130 GeV border wherein change of $m_{\chi^\pm}$ with $m_h$ is rather sharp. It is clear that both these masses are measurable at the LHC, and their interdependence can be guiding pivot if the model under concern is a minimal one based on (1) or a more general one based on (1) and (2) This of course requires a fit of the experimental data to model parameters.

In principle, a full experiment on chargino and neutralino masses must determine $M_2, M_1, \mu$ and $\tan \beta$ in a way independent of what happens in the sfermion sector. Experimentally, however, realization of this statement can be quite non-trivial; in particular, one might need to determine final states containing only
neutralinos or only charginos or neutralinos and charginos [15]. An extraction of $A'_t$ then follows from constructing relations like the ones illustrated in Fig. 2.

In conclusion, the soft breaking sector of the MSSM must in general be extended to include also $\lambda$, and their inclusion can have an important impact on various observables. In particular, they influence radiative corrections to Higgs boson masses, and their size can be examined within the LHC data by forming a cross correlation among Higgs boson mass and other observables. In particular, as illustrated in Fig. 2 (d), a simultaneous knowledge of chargino and Higgs boson masses enables one to search for $A'_t$ effects after a fit to the model parameters. The results advocated here could have important implications for a global analysis of the LHC data.

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