Spectral lags and the energy dependence of pulse width in gamma-ray bursts: contributions from the relativistic curvature effect

Rong-Feng Shen,*† Li-Ming Song and Zhuo Li

Particle Astrophysics Center, Institute of High Energy Physics, Chinese Academy of Sciences, Beijing 100039, China

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ABSTRACT

We compute the temporal profiles of the gamma-ray burst pulse in the four Burst and Transient Source Experiment (BATSE) Large Area Detector (LAD) discriminator energy channels, with the relativistic curvature effect of an expanding fireball being explicitly investigated. Assuming an intrinsic ‘Band’ shape spectrum and an intrinsic energy-independent emission profile, we show that merely the curvature effect can produce detectable spectral lags if the intrinsic pulse profile has a gradually decaying phase. We examine the spectral lag’s dependences on some physical parameters, such as the Lorentz factor $\Gamma$, the low-energy spectral index, $\alpha$, of the intrinsic spectrum, the duration of the intrinsic radiation $t_d'$ and the fireball radius $R$. It is shown that approximately the lag $\propto \Gamma^{-1}$ and $\propto t_d'$, and a spectrum with a more extruded shape (a larger $\alpha$) causes a larger lag. We find no dependence of the lag on $R$. Quantitatively, the lags produced from the curvature effect are marginally close to the observed ones, while larger lags require extreme physical parameter values, e.g. $\Gamma < 50$, or $\alpha > -0.5$. The curvature effect causes an energy-dependent pulse width distribution but the energy dependence of the pulse width we obtained is much weaker than the observed $W \propto E^{-0.4}$ one. This indicates that some intrinsic mechanism(s), other than the curvature effect, dominates the pulse narrowing of gamma-ray bursts.

Key words: relativity – gamma-rays: bursts – gamma-rays: theory.

1 INTRODUCTION

Cheng et al. (1995) were the first to analyse the spectral lag of gamma-ray bursts (GRBs), which is the time delay between the peaks in the Burst and Transient Source Experiment (BATSE) Large Area Detector (LAD) channel 1 (25–50 keV) and channel 3 (100–300 keV) light curves. Subsequently, several authors have carried out more analysis work on the GRB lags. Norris et al. (1996) and Norris, Marani & Bonnell (2000) found that the cross-correlation function lags between BATSE channel 1 and channel 3 photons tend to concentrate near $<100$ ms; for six bursts with known redshift, the $z$-corrected lags are distributed between 6 and 200 ms. Wu & Fenimore (2000) extended the analysis to very low energy (2 keV); they found that about 20 per cent of GRBs have detectable lags and that GRBs do not show larger lags at lower energy. A recent measurement by Chen et al. (2005) for the BATSE bursts shows that the majority of lags are below $\sim200$ ms, and that the histogram of the lags peaks around 30 ms. More intriguingly, Norris et al. (2000) found that, for those six bursts with known $z$, the peak luminosity is anticorrelated with the lags. This relationship provides a useful tool to estimate the distances of a large sample of GRBs by analysing their light curves.

Three theoretical explanations for the lag/luminosity relation have been proposed: for example, the relationship is due to the variation in line-of-sight velocity among bursts (Salmonson 2000); it is caused by the variation of the off-axis angle when viewing a narrow jet (Ioka & Nakamura 2001); or it is caused by radiation cooling – highly luminous burst cools fast and the lag will be short (Schaefer 2004). However, the problem of what mechanism(s) causes the spectral lags of GRBs remains unresolved. Salmonson (2000) did not explain the origin of the lag but assumed that it derives from some proper decay time-scale $\Delta t'$ in the rest frame of the emitter. In the model by Ioka & Nakamura (2001), the lag is caused by the far side of the emitting region producing lower-energy radiation after a longer light-travel time, for a narrow jet with viewing angles outside the cone of jet. However, their model requirements seem too stringent (see below). Schaefer (2004) proposed radiative cooling as the origin of the lag. The difficulty of this explanation is that in order to adjust the observed synchrotron cooling time-scale to be comparable with the lag time-scale, the magnetic field has to be $\sim7$ G, a value much below the strength required by most of the current models (e.g. Piran 1999).

Kocevski & Liang (2003) have assumed that the observed lag is the direct result of spectral evolution, another property of GRBs (Norris et al. 1986; Bhat et al. 1994). In particular, as the peak

*E-mail: rfshen@astro.as.utexas.edu
†Present address: Department of Astronomy, University of Texas at Austin, Austin, TX 78712, USA.

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energy of the GRB’s νFν spectrum decays through the four BATSE channels, the photon-flux peak in each individual channel will be shifted, probably producing the measured lag. From a sample of 19 GRBs, Kocevski & Liang (2003) found an empirical relation between the peak energy’s decaying rate and the GRB lag.

It is widely accepted that the gamma-rays come from a relativistically expanding fireball surface with Lorentz factor Γ > 100 (Lithwick & Sari 2001, and references therein). At some distance from the central source (e.g. R = 10^{12} ∼ 10^{14} cm; cf. Piran 1999), photons emitted from the region on the line of sight and those from the side region at an angle of θ ∼ 1/Γ with respect to the line of sight are Doppler-boosted by different factors and travel different distances to the observer. This is what we call the curvature effect. Comparing the radiation from the side region and that from the line-of-sight region, for the latter its photons are Doppler-boosted to higher energies and arrive at the observer earlier; its observed temporal structure will be boosted to be narrower.

The motivation of this paper is to see how the curvature effect will change the intrinsic pulse profile in different energy channels, and special interest is focused on whether merely the curvature can produce the spectral lags of the pulses. Except for the soft photon lags, the pulses in GRBs show another temporal property, i.e. the pulse narrowing at higher energy, or pulse width as a function of energy (Fenimore et al. 1995; Norris et al. 1996). We are also interested in probing the contributions of the curvature effect to these properties.

A fireball internal–external shocks model has emerged for the theoretical understanding of the origin of GRBs (Piran 1999, 2004). According to this model, GRBs are produced when an ultrarelativistic outflow dissipates its kinetic energy through the internal collisions within the outflow itself. The afterglow occurs when the flow is decelerated by shocks with the circumburst medium. This model has made many successful explanations and predictions to the observations of GRBs. Our investigation of the curvature effect will be based on the frame of the internal shock model, where the shock is generated from the colliding shells, and electrons are accelerated in the shock and radiate.

Ioka & Nakamura (2001) proposed a model in which a narrow jet is viewed at an off-axis angle to explain the lag/luminosity relation and the variability/luminosity relation. Their model successfully reproduces the lag/luminosity relation, while the lag is caused by the curvature effect of the jet, which increases with the off-axis angle. However, one substantial problem with this model, as pointed out by Schaefer (2004), is that it works with an exacting assumption that the jet opening angle always equals Γ^{-1}. Furthermore, Ioka & Nakamura (2001) only consider an instantaneous emission in the jet rest frame, which is a too simple assumption. Consequently, in their model there is no lag when the jet is viewed near-on-axis. Different from Ioka & Nakamura (2001), we have considered various rest-frame emission profiles and assumed an isotropically expanding radiation surface.

We present our model in Section 2, including the basic assumptions and formulae. The major results are presented in Section 3, based on which we give our conclusions and discussion in Section 4.

2 MODEL

2.1 Three time-scales

Three time-scales are involved in determining the temporal structures of pulses in GRBs: (i) cooling time-scale; (ii) hydrodynamic time-scale; (ii) angular spreading time-scale (Kobayashi, Piran & Sari 1997; Wu & Fenimore 2000).

In the synchrotron cooling model, the shock-accelerated electrons cool via synchrotron emission, and the electron’s average energy becomes smaller and the radiated power decays. As pointed out by Wu & Fenimore (2000), the standard internal-shock model gives an observed synchrotron cooling time-scale at a given photon energy as

\[ T_{\text{sys}}(h\nu) \sim 2 \times 10^{-6} \text{s} \, \epsilon_B^{3/4} \left( \frac{h\nu_{\text{obs}}}{100 \text{ keV}} \right)^{-1/2}, \]

where \( \epsilon_B \) is the equipartition parameter for the ratio of the magnetic energy density to the total internal energy density; for a typical value in the internal-shock model, e.g. \( \epsilon_B = 0.01 \) (\( B \sim 10^3 \text{ G} \)), the cooling time-scale is far shorter than the lag time-scale.

The hydrodynamic time-scale is related to the energizing of the electrons. In the internal-shock model, if we assume that the local microscopic acceleration of electrons is instantaneous, then the hydrodynamic time-scale is attributed to the shell-crossing time of the shock, \( T_{\text{dyn}} = \Delta / v_{\text{sh}} \), where \( \Delta \) is the shell width and \( v_{\text{sh}} \) is the shock velocity, both in the comoving frame of the upstream flow. We hardly know about \( \Delta \). However, we may assume that the shells are radially expanding, in the observer frame this time-scale is

\[ T_{\text{dyn}} \sim 1 \, \text{s} \, \beta_{\text{sh}}^{-1} \left( \frac{R}{10^{15} \text{ cm}} \right) \left( \frac{\Gamma}{100} \right)^{-2}, \]

where \( R \) is the radius at which the shell radiates, and \( \Gamma \) is the Lorentz factor of the shell (Ryde & Petrosian 2002). [Note that apart from the shock acceleration scenario, there can be other particle energizing mechanisms, e.g. magnetic field reconnection (Stern 1999), which may have a different hydrodynamic time-scale.]

The angular spreading time-scale is the delay between the arrival times of the photons emitted at the line-of-sight region and those emitted at the side region of the shell (Sari & Piran 1997). Because of the relativistic beaming of the moving radiating particles, only the emission from a narrow cone with an opening angle of \( \sim 1/\Gamma \) is observed. This gives a time-scale of the delay of

\[ T_{\text{ang}} = 1.7 \, \text{s} \left( \frac{R}{10^{15} \text{ cm}} \right) \left( \frac{\Gamma}{100} \right)^{-2}. \]

2.2 Assumptions

In this paper, different from Schaefer (2004), we assume that the cooling time-scale is much shorter than the other two time-scales, i.e. the accelerated particles radiate their energy rapidly. Thus, the rest-frame duration of the emission is determined by the hydrodynamic time-scale.

We consider a thin shell expanding with a relativistic speed, whose Lorentz factor is \( \Gamma \). The shell begins to radiate at radius \( R \). In the comoving frame of the shell, the radiation intensity of the shell surface \( I'(v', t') \) is assumed to be isotropic, and has an energy-independent time history \( f'(t') \). Note that all the quantities in the rest frame of the radiation surface are labelled with a prime note.

Band et al. (1993) found that the GRB spectra are well described at low energy by a power law with an exponential cut-off and by a steeper power law at high energy. The typical fitted value distributions for the low-energy spectral index (\( \alpha \)) is \(-1.5 \sim -0.3 \), the high-energy spectral index (\( \beta \)) is \(-2 \sim -3 \) and the peak energy (\( E_p \)) of the νFν spectrum is 100 ∼ 500 keV (see Preece et al. 2000). By modelling, Qin (2002) showed that the relativistic expanding of the
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2.3 Formulae

The radiation intensity in the observer’s frame is \( I(\mu, v, t) = I(\mu, v)f(t) \). It is connected with the rest-frame radiation intensity by

\[
I(\mu, v)f(t) = D^2(\mu)I'(v)f'(\Gamma')
\]

and

\[
v = v'D(\mu),
\]

where \( \mu = \cos \theta, \) and \( \theta \) is the angle of the concerned local radiation surface to the line of sight; \( D(\mu) = |\Gamma(1-\mu\beta)|^{-1} \) is the local relativistic Doppler factor respect to the observer, where \( \Gamma \) is the Lorentz factor of the radiation surface and \( \beta = \sqrt{1-\Gamma^{-2}} \).

We use \( t \) to refer to the photon emitting time and we use \( T \) to refer to the time that the photon arrives at the observer. We define that, at time \( t = 0 \), the first photons are emitted from the surface and, for simplicity, we also tune the arrival time \( T \) of the first photon emitted from the line-of-sight region (\( \mu = 1 \)) to be 0. Then the arrival time of a photon emitted from region \( \mu \) at time \( t \) is

\[
T = (1-\beta)t + (1-\mu)(R + \beta ct)/c.
\]

The first part of the right-hand side of the equation is caused by the motion of the shell, and the second part is due to the difference between the light travel distances of photons emitted from the line-of-sight region and from the side region. It can be rewritten as

\[
T = (1-\mu\beta)t + (1-\mu)\tau,
\]

where \( \tau = R/c \), which connects the photon’s arrival time, emitting time and emitting place in one equation.

At time \( T \), the observed flux comes from the photons emitted at the region \( 1 > \mu > \mu(T, t = 0) \), where the boundary \( \mu(T, t = 0) = 1 - T/\tau \), calculated from equation (3), is the place whose first photon is emitted with arrival time \( T \). The observed specific flux can be obtained by integrating the radiation intensity over this region

\[
F_{\nu}(T) = \int_{0}^{1-\tau/T} I_{\nu}(\mu, t)\mu R^2(t) d\mu.
\]

For a BATSE LAD discriminator channel \((v_a, v_b)\), the observed photon counts flux is

\[
n_{ab}(T) = \int_{v_a}^{v_b} F_{\nu}(T) dv.
\]

From equation (4), substituting the emitting time \( t \) in the integral with \( T \) and \( \mu \) using equation (3), \( n_{ab}(T) \) can be rewritten as

\[
n_{ab}(T) = \int_{0}^{1-\tau/T} f(T, \mu) d\mu D^2(\mu) \int_{v_a}^{v_b} \frac{I'(v)}{v} dv', \]

where \( R(t) = c(\Gamma + t(T, \mu)\beta) \) and equations (1) and (2) are used. So, we obtain the explicit formula that is used in our calculation.

2.4 Intrinsic time profile of the radiation

Here we introduce three cases of the intrinsic emission time profile.

(i) Rectangular profile. During a finite duration, the emissivity is constant, with the instantaneous rising phase and decaying phase:

\[
f(t) = \begin{cases} 1, & 0 < t < t_d \\ 0, & t > t_d \end{cases}
\]

In our assumptions \( t_d \) is related to the hydrodynamic time-scale discussed above, but note that \( t_d \) is defined in the observer’s frame, without taking into account the Doppler boosting (i.e. it is not the observed time-scale). In addition, we refer to \( t_d' \) as the \( t_d \) measured in the comoving frame of the shell.

(ii) One-sided exponential profile. The emissivity decays exponentially after an instantaneous rising phase:

\[
f(t) = \begin{cases} 0, & t < 0 \\ \exp(-t/t_d), & t > 0 \end{cases}
\]

(iii) Symmetric Gaussian profile. The emissivity has both a finite-time rising phase and a finite-time decaying phase

\[
f(t) = \begin{cases} 0, & t < 0 \\ \exp \left(-\frac{(t-1.5t_d)^2}{t_d} \right), & t > 0 \end{cases}
\]

where we introduce the coefficient \(-1.5\) in order that the emission starts at \( t = 0 \) with the radiation intensity of a tenth \((e^{-1.5} \approx 0.1)\) of its peak value.

3 RESULTS

3.1 Observed temporal profiles in different energy bands and time lags

Using the equations and the typical parameters introduced above, we calculate the observed pulse profiles in the four BATSE LAD energy channels, assuming three different intrinsic pulse profiles. The results are plotted in Figs 1, 2 and 3, respectively.

First, we calculate the pulse light curves in the four energy channels for the rectangular intrinsic radiation profile. It has a steady rising phase, followed by a distinct peak, as shown in Fig. 1. The rising of the flux is due to the expanding of the radiation surface. The rising is steady because the radiation intensity is constant during a finite time. The decay phase of the observed pulse is due to the angular spreading effect. The peak occurs when the intrinsic radiation begins to cease. For various parameter spaces (duration of the emission \( t_d = 10^3-10^5 \) s, radius \( R = 10^{13}-10^{15} \) cm, Lorentz factor \( \Gamma = 50-500 \)), we do not detect any time lag between the peaks observed at different energies in this case.

As for the one-sided exponential and the symmetric Gaussian emission profiles, their observed light curves calculated in the four energy channels are illustrated in Figs 2 and 3, respectively. For these two emission profiles, their peaks are more gradual. The maximum of the photon flux at higher energy arrives earlier than at lower energy. If we define the time lag as being the difference between the arrival times of the peaks in energy channels 1 and 3 or 4, the reproduced lags are quantitatively comparable to the observed ones (e.g. \( \sim 10^{-2} \) s if corrected for the cosmological time dilation; cf. Norris et al. 2000). Note that here we do not take into account the redshifts, \( z \), of the GRBs, which must make the observed lags \((1+z)\) times larger.
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Figure 1. Observed pulse from intrinsic rectangular time profile in four BATSE LAD energy channels: solid line, 25–50 keV; dashed line, 50–100 keV; dot-dashed line, 100–300 keV; dotted line, 300–1800 keV. The vertical coordinate is in arbitrary units. The peaks of the pulses at the four energies arrived simultaneously. $\Gamma = 100, \alpha = -0.8, \beta = -2.4, R = 3 \times 10^{14}$ cm, $t_d = 0.9 \times 10^4$ s. The rising phase comes from the expanding of the radiation surface and the rising time is determined by duration of the intrinsic radiation. The decay phase is due to the angular spreading effect.

Three of the parameters ($\Gamma, \alpha, \beta$) of the temporal profile are left free, while the intensity profile was fixed to a power-law in the rest frame. The rising phase comes from the expanding of the radiation surface, while the decay phase is due to the angular diffusion of radiation. The parameter $t_d$ is the duration of the intrinsic radiation. The model predictions are shown as the solid lines in the figure.

The intrinsic rectangular profile cannot produce the observed lags, because in this case the radiation diminishes immediately. In the observed pulse, the rising phase comes from the expanding of the radiation surface and the rising time is determined by duration of the intrinsic radiation ($\approx t_d/2\Gamma^2$); also see Qin et al. 2004.

The decay phase is due to the angular spreading effect. Hence, if the intrinsic radiation switches off immediately, the transition from the rising phase to the decay occurs abruptly and induces a sharp peak in the observed pulse. For this case, the immediate switch-off of the intrinsic radiation dilutes the relativistic curvature effect, and hence does not produce the peak lags.

3.2 Pulse width–energy relation

For the three intrinsic emission profiles, we find that the pulse observed in the high-energy channel is narrower than in the lower-energy channel, which is manifested by the fact that the FWHMs of the pulses in separated energy channels decrease with the energy in a power-law form. However, the pulse narrowing we obtained is less prominent than that observed in real GRBs, for which the pulse width decay power-law index $\sim -0.4$; while the pulse width decay index we obtained, for instance in the case of Fig. 2 (one-sided exponential decay emission profile), is $-0.13$.

Other than the ‘Band’ spectrum, we also used an alternative function – a low-energy power law plus the high-energy exponential cutoff at $E'_p$ – for the rest-frame spectrum. For the typical values of the parameters we have used, this changing of spectrum only narrows the channel 4 pulse by $\sim 6$ per cent, and hence hardly changes the slope of the pulse width versus the energy.

3.3 Lag dependence on other physical parameters

The spectral lag is an important observational property of the pulse in GRBs in that it may be used to derive the cosmological distribution of GRBs (Norris 2002) and to discriminate the internal shock signature and the external shock signature in the pulses (e.g. Hakkila & Giblin 2004). This motivates us to probe the dependences of the peak lags on other physical parameters of the simple model. We choose the symmetric Gaussian profile as the intrinsic emission profile, which includes an intrinsic rising phase. We alter the Lorentz factor $\Gamma$, the spectral parameters of the rest-frame emission ($\alpha, \beta$ and $E'_p$), the radius of the radiation surface $R$, and the rest-frame duration $t'_d$ of the intrinsic radiation, respectively, and see how the channel 1/3 and channel 1/4 peak lags vary with these changes.

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3.3.1 Lorentz factor

Fig. 4 shows that the lag decreases with the Lorentz factor following \( \text{Lag} \propto \Gamma^{-1} \). We think this is a natural outcome of the relativistic boosting of the time structure. Those pulses whose lags are larger may come from colliding shells with low Lorentz factors, according to the current standard models (e.g. Piran 1999).

3.3.2 Spectral parameters

Fig. 5 shows that the lag increases with the low-energy spectral index \( \alpha \) of the rest-frame spectrum of the pulse. A larger \( \alpha \) means more photons are concentrated around the peak energy \( E'_p \) of the \( \nu F'_\nu \) spectrum. Then, for a narrower spectrum, the curvature effect will work more effectively in producing the spectral lags. This conjecture is supported when we alter the high-energy spectral index \( \beta \). We found that a steeper high-energy power-law spectrum produces a larger lag, e.g. the channel 1/4 lag has a 14 per cent increase for \( \beta \) changing from \(-2.4\) to \(-3.0\). Compared with the channel 1/4 lag, the channel 1/3 lag has a weaker dependence upon \( \beta \).

The lag’s dependence on the observed break energy of the spectrum \( E_p \) is shown in Fig. 6. The lag has its maximum when \( E_p \) falls near the starting energy of the corresponding high-energy channel that is used in measuring the lag (i.e. \( \sim 100 \text{ keV} \) for channel 3, \( \sim 300 \text{ keV} \) for channel 4).

The above findings about the lag’s dependences on the spectral parameters are qualitatively consistent with the tendency observed in those long-lag wider-pulse bursts by Norris et al. (2005). They found that their long-lag (measured for channel 1/3) burst sample has, on average, lower \( E_p \) (centred around \( \sim 110 \text{ keV} \)), larger \( \alpha \) (harder low-energy power law) and smaller \( \beta \) (softer high-energy power law), than the bright burst sample analysed by Preece et al. (2000).

Substituting the ‘Band’ spectrum with an alternative one of a single power law plus an exponential high-energy cut-off causes no changes to the channel 1/3 lag, while the channel 1/4 lag has a \( \sim 40 \) per cent increase if the observed cut-off energy \( E_p \) is below 300 keV; for \( E_p > 300 \text{ keV} \), the increase of the channel 1/4 lag is much smaller.

3.3.3 Duration of the emission

We find that longer rest-frame duration \( t'_d \) of emission will cause larger lags, as shown in Fig. 7. This result may be associated with an observed tendency that wider pulses exhibit longer lags (Norris et al. 1996; Norris, Scargle & Bonnell 2001; Norris et al. 2005).

3.3.4 Radius of the radiation surface

The lag appears to be independent of \( R \), the radius of the radiation surface (see Fig. 8). We know \( R \) determines the angular spreading time-scale, \( T_{\text{ang}} = R/(2c) \). This result suggests that although in our model the angular spreading effect is a necessity in causing the lags, the peak lag in the pulses is not correlated with the angular spreading time-scales.
Figure 7. Symmetric Gaussian intrinsic pulse: lag dependence on $t'_d$, the rest-frame duration of the radiation. $\Gamma = 100, E'_p = 1.75$ keV, $\alpha = -0.8, \beta = -2.4, R = 5 \times 10^{13}$ cm.

Figure 8. Symmetric Gaussian intrinsic pulse: lag dependence on $R$, the radius of radiation surface. $\Gamma = 100, E'_p = 1.75$ keV, $\alpha = -0.8, \beta = -2.4, t'_d = 40$ s.

In addition, we find that the decrease of pulse width with the photon energy is dependent upon the low-energy spectral index of the radiation spectrum, as shown in Fig. 9.

4 CONCLUSIONS AND DISCUSSION

By assuming an intrinsic ‘Band’-shape spectrum and an exponential or Gaussian emission profile, we show that merely the curvature effect produces detectable soft lags in the GRB pulses. Therefore, the soft time lags can be a signature of the relativistic motion occurring in GRBs.

The observed channel 1/3 lags are typically distributed among $10^{-2}$–$10^{-1}$ s (Norris et al. 2000). For typical physical parameters, i.e. $\Gamma \approx 100, t'_d \approx 40$ s, $\alpha \approx -1$, as shown in Figs 4–8, the lags produced by the relativistic curvature effect are slightly above $10^{-2}$ s, marginally close to those observed, after considering the cosmological time dilation effect if a GRB redshift of 2 is assumed. To account for these observed larger lags (~0.1 s), it requires extreme physical parameter values, e.g. $\Gamma < 50$ or $\alpha > -0.5$.

We did not find any peak lag for a rectangular intrinsic emission profile, from which a straightforward conclusion regarding the radiation process of the pulse in GRBs can be obtained – the radiation intensity must have a decaying phase in order to produce the observed peak lags. The intrinsic decaying phase may be due to the variations associated with hydrodynamic processes, such as the decaying of emission caused by density or magnetic field attenuation as the shock moves through the shell.

We have investigated the possible dependences of the pulse peak lag upon other physical parameters of the kinematic model. We found the following: the lag is proportional to the inverse of the Lorentz factor; the lag is proportional to the duration of the intrinsic radiation $t'_d$; the lag is weakly dependent on $R$; the lag is larger when larger amount of energy is concentrated at $E'_p$ (larger $\alpha$ or smaller $\beta$).

The pulse width decreases with energy ($W \propto E^{-0.1-0.2}$), but not as fast as the observed ($W \propto E^{-0.4}$), although we found a faster decrease with a larger low-energy spectral index $\alpha$. There must be other energy-dependent narrowing mechanisms underlying. Similar to this conclusion, Dermer (2004) pointed out that other processes including adiabatic and radiative cooling, a non-uniform jet, or the external shock process, rather than the curvature effect, should be needed to explain the relationship between the measured peak photon energy $E'_p$ and the measured $\nu F_\nu$ flux at $E'_p$ in the decaying phase of a GRB pulse. They found that the curvature relationship does not agree with the observation (Borgonovo & Ryde 2001).

For simplicity, we have assumed a spherically symmetric radiation surface in this paper, although there is some observational evidence indicating that the GRB outflow may be collimated. Derived from the afterglow observations, the jet coming from the GRB central source generally has a half opening angle $\theta_j > \Gamma^{-1}$ (Frail et al. 2001). Assuming a jet geometry with the jet opening angle of $1^\circ$ or $4^\circ$ and the same parameter values ($\Gamma, R$ and $t_d$) used in the spherical geometry, we calculated the observed pulse shapes and the lags; they show no difference from those of the spherical geometry. The reason for this is as follows. Even in the case of
the isotropic radiation surface, the contribution of the flux from the outer side region where the observing angle $\theta$ is larger than $\Gamma^{-1}$ is relatively very small, because the local flux contribution from the radiation surface (i.e. the first-part integrand of equation 5 in Section 2) will decrease drastically with $\theta$ as $\propto (1 + \Gamma^2 \theta^2)^{-5}$ when $\theta$ is small.

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**REFERENCES**

Band D. et al., 1993, ApJ, 413, 281
Bhat P. N., Fishman G. J., Meegan C. A., Wilson R. B., Kouveliotou C., Paciesas W. S., Pendleton G. N., Schaefer B. E., 1994, ApJ, 426, 604
Borgonovo L., Ryde F., 2001, ApJ, 548, 770
Chen L., Lou Y.-Q., Wu M., Qu J.-L., Jia S.-M., Yang X.-J., 2005, ApJ, 619, 983
Cheng L. X., Ma Y. Q., Cheng K. S., Lu T., Zhou Y. Y., 1995, A&A, 300, 746
Dermer C. D., 2004, ApJ, 614, 284
Fenimore E. E., in’t Zand J. J. M., Norris J. P., Bonnell J. T., Nemiroff R. J., 1995, ApJ, 448, L101
Frail D. A. et al., 2001, ApJ, 562, L55
Hakkila J., Giblin T. W., 2004, ApJ, 610, 361
Ioka K., Nakamura T., 2001, ApJ, 554, L163
Kobayashi S., Piran T., Sari R., 1997, ApJ, 490, 92
Kocevski D., Liang E., 2003, ApJ, 594, 385
Lithwick Y., Sari R., 2001, ApJ, 555, 540
Norris J. P., 2002, ApJ, 579, 386
Norris J. P., Share G. H., Messina D. C., Dennis B. R., Desai U. D., Cline T. L., Matz S. M., Chupp E. L., 1986, ApJ, 301, 213
Norris J. P., Nemiroff R. J., Bonnell J. T., Scargle J. D., Kouveliotou C., Paciesas W. S., Meegan C. A., Fishman, G. J., 1996, ApJ, 459, 393
Norris J. P., Marani G. F., Bonnell J. T., 2000, ApJ, 534, 248
Norris J. P., Scargle J. D., Bonnell J. T., 2001, in Ritz S., Gehrels N., Shreader C. R., eds, AIP Conf. Proc. Vol. 587, Gamma 2001: Gamma-Ray Astrophysics. Am. Inst. Phys., New York, p. 176
Norris J. P., Bonnell J. T., Kazanas D., Scargle J. D., Hakkila J., Giblin T. W., 2005, ApJ, 627, 324
Piran T., 1999, Phys. Rep., 314, 575
Piran T., 2004, Rev. Mod. Phys., 76, 1143
Preece R. D., Briggs M. S., Mallozzi R. S., Pendleton G. N., Paciesas W. S., Band D. L., 2000, ApJS, 126, 19
Qin Y.-P., 2002, A&A, 396, 705
Qin Y.-P., Zhang Z.-B., Zhang F.-W., Cui X.-H., 2004, ApJ, 617, 439
Ryde F., Petrovian V., 2002, ApJ, 578, 290
Salmonson J. D., 2000, ApJ, 544, L115
Sari R., Piran T., 1997, ApJ, 485, 270
Schaefer B., 2004, ApJ, 602, 306
Stern B., 1999, in Poutanen J., Svensson R., eds, ASP Conf. Ser. Vol. 161. High Energy Processes in Accreting Black Holes. Astron. Soc. Pac., San Francisco, p. 277
Wu B., Fenimore E., 2000, ApJ, 535, L29

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