Heterogeneous Power-Splitting Based Two-Way DF Relaying with Non-Linear Energy Harvesting

Li Qin Shi\(^1\), Wen Chi Cheng\(^1\), Ying Hui Ye\(^1\), Hai Lin Zhang\(^1\), and Rose Qing Yang Hu\(^2\)
\(^1\)State Key Laboratory of Integrated Services Networks, Xidian University, China
\(^2\)Department of Electrical and Computer Engineering, Utah State University, USA

Abstract—Simultaneous wireless information and power transfer (SWIPT) has been recognized as a promising approach to improving the performance of energy constrained networks. In this paper, we investigate a SWIPT based three-step two-way decode-and-forward (DF) relay network with a non-linear energy harvester equipped at the relay. As most existing works require instantaneous channel state information (CSI) while CSI is not fully utilized when designing power splitting (PS) schemes, there exists an opportunity for enhancement by exploiting CSI for PS design. To this end, we propose a novel heterogeneous PS scheme, where the PS ratios are dynamically changed according to instantaneous channel gains. In particular, we derive the closed-form expressions of the optimal PS ratios to maximize the capacity of the investigated network and analyze the outage probability with the optimal dynamic PS ratios based on the non-linear energy harvesting (EH) model. The results provide valuable insights into the effect of various system parameters, such as transmit power of the source, source transmission rate, and source to relay distance on the performance of the investigated network. The results show that our proposed PS scheme outperforms the existing schemes.

Index Terms—Simultaneous wireless information and power transfer, two-way decode-and-forward relay, dynamic heterogeneous power splitting, non-linear energy harvesting.

I. INTRODUCTION

Recently, simultaneous wireless information and power transfer (SWIPT) has emerged as an appealing approach to prolonging the lifetime of energy-constrained networks, e.g., relay networks\(^1\), wireless sensor networks\(^3\), cooperative non-orthogonal multiple access networks\(^4\), D2D assisted cellular networks\(^5\), by harvesting energy from radio frequency (RF) signals. Of particular interest is integrating SWIPT with relay networks, which not only extends the wireless transmission range, but also prolongs the operating time of the energy-constrained relay nodes\(^\text{6}\). Compared with one-way relaying, two-way relaying, which can be performed in two steps or three steps, can offer a more efficient use of the available resources by allowing two destination nodes to exchange information with each other. Regarding this consideration, increasing attention has been paid to the SWIPT based two-way relay networks (TWRNs), where wireless signals are either switched in the time domain or split in the power domain to facilitate SWIPT, i.e., time switching (TS) scheme and power splitting (PS) scheme.

Some studies on the design of TS/PS scheme for two-step TWRNs\(^7\),\(^8\) have been hitherto reported. The authors of\(^7\) studied the optimal TS/PS scheme for amplify-and-forward (AF) and decode-and-forward (DF) based TWRNs. It was shown that at high signal-to-noise (SNR) ratio, the PS scheme can achieve a larger sum rate than the TS scheme. The authors of\(^8\) proposed a resource allocation strategy, which jointly optimizes the time allocation ratio and the PS/TS ratio, to minimize the outage probability of DF based TWRNs.

Since the low complexity of hardware is very vital to energy-constrained networks, three-step two-way relaying has attracted extensive research interests\(^9\),\(^10\). Based on the TS receiver, three wireless power transfer policies have been proposed to maximize the capacity\(^9\). A static equal PS scheme has been developed to maximize the overall outage capacity for three-step AF TWRNs\(^10\), where the PS ratio is determined by the statistical channel state information (CSI). The outage capacity can be improved by adopting a dynamic PS scheme, because the PS ratio can be adaptive to the instantaneous CSI instead of to the statistical CSI. For this reason, the dynamic equal PS scheme was further developed\(^11\). Recently, the three-step AF relay has also been extended to the DF relay system\(^12\), where the upper and lower bounds of the outage probability with respect to the static equal PS scheme were studied. Note that although the instantaneous CSI is required at both destinations to perform successive interference cancellation (SIC)\(^12\), it is not used in determining the PS ratio. Moreover, as the channel gains between the source nodes and the relay are both heterogeneous and instantaneously changing, a PS scheme based on both heterogeneous and instantaneous CSI can achieve more efficient transmission than the equal PS scheme.

Motivated by the reasons stated above, we propose a dynamic heterogeneous PS scheme, where the PS ratio for each link can be dynamically adjusted based on its instantaneous CSI, and apply it into the SWIPT based three-step DF TWRNs. Unlike the works mentioned above\(^9\)–\(^12\), we consider a non-linear energy harvesting (EH) model\(^13\) instead of the conventional linear one and study the outage capacity of the proposed scheme in the investigated network.

The main contributions of this paper are summarized as follows.

- We propose a novel dynamic heterogeneous PS scheme to maximize the capacity of SWIPT based DF TWRNs with the non-linear EH model and derive the closed-form expressions for the optimal PS ratios. Compared with the scheme in\(^12\), the proposed scheme is more flexible and can achieve better performance.
- We analyze the outage capacity with the optimal PS ratios, as an effort to know how much performance gain the
designed scheme could offer in the investigated network. Simulation results verify the correctness of the derived results and demonstrate that the proposed PS scheme can significantly improve the capacity of the investigated system as compared with the existing schemes.

The remainder of this paper is organized as follows. The system model is provided in Section II. In Section III, we propose a novel dynamic heterogeneous PS scheme to maximize the capacity of SWIPT based DF TWRNs with the non-linear EH model and analyze the corresponding outage performance. Simulation results are provided in Section IV, followed by conclusions in Section V.

II. SYSTEM MODEL

As shown in Fig. 1, we consider a three-step two-way DF relay network, where two destination nodes A and B exchange information via an energy-constrained relay node R. Each node is equipped with a single antenna and works in the half-duplex mode. There is no direct link between A and B due to severe path loss and shadowing. The path loss model is given by $|h_i|^2d_i^{-\alpha}$ ($i = A$ or B), where $h_i$ is the $i$-R channel coefficient, $d_i$ is the $i$-R distance, and $\alpha$ is the path loss exponent. All the channels are assumed to undergo independent identically distributed (i.i.d) quasi-static Rayleigh fading and all the channels are assumed to be reciprocal. Note that the use of such channels can be found widely in prior research in this field \cite{5,12,14}. Let $T$ denote the total transmission block which can be divided into three time slots. Let $\beta \in (0, 0.5)$ be the time proportion for $R$ to harvest energy and decode signals from $A$ or $B$. The transmission time for $A$ or $B$ to $R$ is $\beta T$. After receiving signal from $i$ ($i = A$ or B), $R$ splits it into two parts with ratio $\rho_i$ with one part used for energy harvesting and the other part used for information processing. In the remaining block time $(1-2\beta)T$, $R$ decodes the signals and forwards them to $A$ and $B$.

At the first or the second time slot with $\beta T$, $A$ or $B$ transmits the signal $s_A$ or $s_B$ to $R$ and the received RF signal from $i$ ($i = A$ or B) at $R$ is given by

$$y_{iR} = h_i\sqrt{P_i d_i^{-\alpha}} s_i + n_{iR}, \quad (1)$$

where $P_i$ denotes the transmit power of $i$, $\mathbb{E}\{ |s_i|^2 \} = 1$ and $n_{iR} \sim \mathcal{CN}(0, \sigma_i^2)$ is the additive white Gaussian noise (AWGN).

Thus, the received power from $i$ at $R$ before the third time slot is given by

$$P_{iRF} = \rho_i P_i |h_i|^2d_i^{-\alpha}. \quad (2)$$

Since the conventional linear EH model cannot capture the practical EH circuit due to the nonlinearity of the diodes, inductors and capacitors \cite{15}, we employ a more practical non-linear EH model in \cite{13}, which has been verified by comparing with measurement data from \cite{16} and \cite{17}. Compared with the non-linear EH model in \cite{15}, the model in \cite{13}, namely piecewise linear EH model, is more mathematically tractable, and able to provide sufficient precision by selecting the proper number of segments (see Fig. 2 in \cite{13}). According to the piecewise linear EH model in \cite{13}, the harvested power $P_{iH}$ from $i$ can be modelled as

$$P_{iH} = \begin{cases} \end{cases}
= \begin{cases} 0, & P_{iRF} < P_{iH}^{th}; \\
 a_j P_{iRF} + b_j, & P_{iRF} \in [P_{iH}^{th}, P_{iH}^{th+1}], j = 1, \cdots, N-1; \\
 P_{m}, & P_{iRF} > P_{m}^{th}, \end{cases} \quad (3)$$

where $P_{iH} = \{P_{iH}^{th}, 1 \leq j \leq N \}$ are the thresholds on $P_{iRF}$ for $N + 1$ linear segments\footnote{Note that the $P_{iH}^{th}$ represents the power sensitivity for the EH circuits.} $a_j$ and $b_j$ are the slope and the intercept for the linear function in the $j$-th segment, respectively, and $P_{m}$ denotes the maximum harvestable power when the circuit is saturated.

Then, the total harvested energy is\footnote{If $i = A$, $\bar{i} = B$; if $i = B$, $\bar{i} = A$.}

$$E_{\text{total}} = \beta T (P_{iH} + P_{iH}^{\overline{\text{th}}}) = \beta T \left( a_j \rho_i P_i |h_i|^2 d_i^{-\alpha} + a_k \rho_i P_i |h_i|^2 d_i^{-\alpha} + b_k + b_j \right), \quad (4)$$

where $j, k \in \{0, \cdots, N\}$ denote to which segment $P_{iRF}$ or $P_{iH}^{th}$ belongs. Let the segment with $P_{iRF} < P_{iH}^{th}$ be the $0$-th segment and the segment with $P_{iRF} > P_{iH}^{th}$ be the $N$-th segment. Based on Eq. (3), we have $a_0 = b_0 = a_N = 0$, $b_N = P_m$. According to \cite{13}, both $\{a_j\}_{j=1}^{N-1}$ and $\{b_j\}_{j=1}^{N-1}$ are obtained by linear regression to minimize the difference with the practical EH circuit. Thus, $a_j$ and $b_j$ in $j$-th ($j = 1, \cdots, N-1$) segment are given by

$$a_j = \frac{\sum_{i=1}^{n} (x_i - \bar{x})(y_i - \bar{y})}{\sum_{i=1}^{n} (x_i - \bar{x})^2} = \frac{\sum_{i=1}^{n} x_i y_i - n \bar{x} \bar{y}}{\sum_{i=1}^{n} x_i^2 - n \bar{x}^2};$$
$$b_j = \bar{y} - a_j \bar{x}, \quad (5)$$

where $\{(x_1, y_1), (x_2, y_2), \cdots, (x_{n-1}, y_{n-1}), (x_n, y_n)\}$ denote the experimental data in the $j$-th segment, $\bar{x} = \sum_{i=1}^{n} x_i / n$, and $\bar{y} = \sum_{i=1}^{n} y_i / n$.

For the information processing, the received SNR for decoding $s_i$ is

$$\gamma_{iR} = \frac{P_i |h_i|^2 (1 - \rho_i)}{d_i^\alpha \sigma_i^2}. \quad (6)$$

Fig. 1. System model of the three-step two-way DF relay network.
Let \( \bar{s}_A \) and \( \bar{s}_B \) denote the decoded signals for \( A \) and \( B \) during the first and the second time slots, respectively. In the third time slot, \( R \) combines \( \bar{s}_A \) and \( \bar{s}_B \) and broadcasts the normalized signal \( s_R = \frac{s_A + s_B}{\sqrt{2}} \) to both \( A \) and \( B \) with the harvested energy \( E_{\text{total}} \). Then the received signal at \( i \) is given by

\[
y_{Ri} = h_i \sqrt{P_{Ri}d_i^{-\alpha}} s_R + n_{Ri},
\]

where \( P_{Ri} = \frac{E_{\text{total}}}{(1-2\beta)^2} \) is the transmit power at \( R \), \( n_{Ri} = \bar{n}_{Ri} \sim \mathcal{CN}(0, \sigma^2_{Ri}) \) is the AWGN caused by the receiving antenna at \( i \), (a) follows by using SIC due to the fact that the CSI and other system parameters are available at \( i \), and \( \bar{i} \) denotes the index of the other destination node.

For analytical simplicity, we assume \( P_A = P_B = P \) and \( \sigma^2_{AR} = \sigma^2_{BR} = \sigma^2_{RA} = \sigma^2_{RB} = \sigma^2 \). Based on Eq. (7), the end-to-end SNR of the link \( i \rightarrow \bar{i} \) is given by

\[
\gamma_{Ri} = \frac{P_{Pi}|h_i|^2}{2d_i^\alpha \sigma^2} = a_{\bar{i}} \rho_i P|h_i|^4 X d_i^{-2\alpha} + X d_i^{-\alpha} (a_k \rho_i P|h_i|^2 |h_i|^2 d_i^{-\alpha} + (b_k + b_j)|h_i|^2),
\]

where \( X = \frac{\sigma}{2(1-2\beta)\sigma^2} \).

### III. Performance Analysis

In this section, we first derive the closed-form expression for the optimal dynamic heterogeneous PS ratios, \( \rho^*_A \) and \( \rho^*_B \), respectively, to maximize the capacity of the system. Then, an analytical expression of the outage probability with \( \rho^*_A \) and \( \rho^*_B \) under the piecewise linear EH model is provided.

#### A. Dynamic Heterogeneous PS Scheme

Let \( P(\cdot) \) denote the probability. Let \( P_{\text{out}}^i \) be the outage probability at node \( i \). For a predefined threshold \( \gamma_{th} \), \( P_{\text{out}}^i \) is given by

\[
P_{\text{out}}^i = \begin{cases} P(\gamma_{Ri} < \gamma_{th}) + P(\gamma_{Ri} < \gamma_{th}, \gamma_{RA} \geq \gamma_{th}), \\ P(\gamma_{RA} < \gamma_{th}) \\ P(\gamma_{RA} < \gamma_{th}, \gamma_{RB} \geq \gamma_{th}), \\ P(\gamma_{RB} < \gamma_{th}) \\ P(\gamma_{RB} < \gamma_{th}, \gamma_{RA} \geq \gamma_{th}) \\ P(\gamma_{th} < \gamma_{th}, \gamma_{RB} < \gamma_{th}, \gamma_{RA} < \gamma_{th}) \end{cases}
\]

where \( P_1 \) is the outage probability at the relay and \( P_2 \) is the outage probability at the destination node \( i \).

According to [12], [18], the capacity of the system can be calculated as

\[
C_{\text{total}} = \left( 2 - P_{\text{out}}^A - P_{\text{out}}^B \right) UT \times \min(\beta, 1 - 2\beta)
\]

where \( U = \log_2(1 + \gamma_{th}) \) is the source transmission rate of nodes \( A \), \( B \), and \( R \). Then we formulate the optimization problem to maximize the capacity of the system as

\[
P1: \max_{\rho_A, \rho_B} C_{\text{total}} \quad \text{s.t.} \quad 0 \leq \rho_i < 1, \ i \in \{A, B\}.
\]

Based on Eq. (9), the optimization problem can be transformed into

\[
P2: \max_{\rho_A, \rho_B} & P(\gamma_{RA} \geq \gamma_{th}) + P(\gamma_{RB} \geq \gamma_{th}) \quad \text{s.t.} \quad 0 \leq \rho_i \leq \max \left\{ 1 - \frac{\gamma_{th} d_i^\alpha \sigma^2}{P|h_i|^4}, 0 \right\}, \ i \in \{A, B\}.
\]

Note that \( P_1 = 1 \) always holds when \( 1 - \frac{\gamma_{th} d_i^\alpha \sigma^2}{P|h_i|^4} < 0 \). Since both \( \gamma_{RA} \) and \( \gamma_{RB} \) increase with the increasing of \( \rho \), with a given \( \rho^*_\gamma \), it is readily seen that the optimal solution to \( P2 \) can be obtained when \( \rho_A = \max \left\{ 1 - \frac{\gamma_{th} d_i^\alpha \sigma^2}{P|h_i|^4}, 0 \right\} \) and \( \rho_B = \max \left\{ 1 - \frac{\gamma_{th} d_i^\alpha \sigma^2}{P|h_i|^4}, 0 \right\} \). Thus, the optimal dynamic PS ratio \( \rho^*_i \) is given by

\[
\rho^*_i = \max \left\{ 1 - \frac{\gamma_{th} d_i^\alpha \sigma^2}{P|h_i|^4}, 0 \right\}.
\]

#### B. End-to-End Outage Probability with \( \rho^*_i \)

Based on Eq. (9), \( P_{\text{out}}^B \) can be expressed as

\[
P_{\text{out}}^B = \begin{cases} P(\gamma_{RA} < \gamma_{th}) + P(\gamma_{RB} < \gamma_{th}, \gamma_{AR} \geq \gamma_{th}) \\ P(\gamma_{RA} < \gamma_{th}, \gamma_{RB} \geq \gamma_{th}) \\ P(\gamma_{RB} < \gamma_{th}) \end{cases}
\]

where \( P_3 \) is the outage probability at relay \( R \) and \( P_2 \) is the outage probability at destination \( B \). Substituting the optimal PS ratios in Eq. (13) into Eq. (14), we have

\[
P_{321} = \begin{cases} P(\gamma_{RA} < \gamma_{th}) \geq \frac{\gamma_{th} d_{A} d_{B} \sigma^2}{P} \quad \omega, 1 - \exp \left( -\frac{\omega d_{A}^2}{\lambda_A} \right) \end{cases}
\]

where (b) holds due to \( |h_A|^2 \sim \exp \left( \frac{1}{\lambda_A} \right) \) and \( \omega = \frac{\gamma_{th} d_{A} d_{B} \sigma^2}{P} \).

There are two cases for the value of \( \rho^*_B \), which are 0 for the case with \( 1 - \frac{\gamma_{th} d_{A} d_{B} \sigma^2}{P} < 0 \) and \( 1 - \frac{\gamma_{th} d_{A} d_{B} \sigma^2}{P} > 0 \) for the other case. Combining with the piecewise linear EH model in Eq. (3), there are \( N \) plus 1 pairs of values for \( (a_j, b_k) \). Thus, \( P_{32} \) is given by

\[
P_{32} = \sum_{k=0}^{N} P_{321}^k + \sum_{j=0}^{N} P_{322}^j.
\]

where \( P_{321}^k \) is the part of \( P_{32} \) where the energy harvester operates in the \( k \)-th linear region for \( P_A \) with \( \rho_B = 0 \) and \( P_{322}^j \) is the part of \( P_{32} \) where the energy harvester operates in the \( k \)-th linear region for \( P_A \) and the \( j \)-th linear region for \( P_B \) with \( \rho_B = 1 - \frac{\gamma_{th} d_{A} d_{B} \sigma^2}{P} \).

Based on the above two cases, if \( |h_B|^2 \leq \omega d_{B} \) is satisfied, we have \( P_{321}^k \) as

\[
P_{321}^k = \begin{cases} P(\gamma_{RA} \geq \gamma_{th}) + P(\gamma_{RB} \geq \gamma_{th}) \quad \text{s.t.} \quad 0 \leq \rho_i \leq \max \left\{ 1 - \frac{\gamma_{th} d_i^\alpha \sigma^2}{P|h_i|^4}, 0 \right\}, \ i \in \{A, B\}.
\]

where \( Y_{A1} = \frac{\gamma_{th} d_{A} d_{B} \sigma^2}{P} \) and \( Y_{A3} = \omega (a_k + a_j - \frac{b_k + b_j}{P}) d_{A} \).

If \( |h_B|^2 \geq \omega d_{B} \), \( P_{322}^j \) are

\[
P_{322}^j = \begin{cases} P(a_k \omega d_{A} < a_k |h_A|^2 < \frac{Y_{A1}}{|h_B|^2} + Y_{A2} |h_B|^2 + Y_{A3} |h_B|^2, |h_B|^2 < \omega d_{B}) \end{cases}
\]

where \( Y_{A2} = \frac{a_k d_{A}^2}{|h_B|^2} \).
1) The derivation of $P_{21}^k$. Based on the value of $k$ and the piecewise linear EH model in Eq. (3), there are three cases for $P_{321}^k$ as follows.

Case I: When $k = 0$, we have $a_k = b_k = 0$ and $P_{1bf} < P_{1th}$. Combining with $\|h_B\|^2 < \omega d_B^2$ is given by

$$P_{321}^k = P\left(0 < Y^{A1} - \omega d_A^2 < \|h_B\|^2 < (\theta + \omega) d_B^2 \right)$$

$$\approx \left[ 1 - \exp\left(-\frac{\omega d_B^2}{\lambda_B} \right) \right] \left[ \exp\left(-\frac{\omega d_A^2}{\lambda_A} \right) - \exp\left(-\frac{(\theta + \omega) d_B^2}{\lambda_B} \right) \right],$$

where (c) holds due to $|h_B|^2 \sim \exp\left(\frac{\omega}{\lambda_B}\right)$ and $\theta_0 = \frac{P_{th}}{\lambda_B}$.

Case II: When $k = N$, we have $a_k = b_k = P_m$. Let $\theta_k = \frac{P_m}{\lambda_B^2}$. Then $|h_B|^2 > (\omega + \theta_k) d_A^2$ and $P_{321}^N$ is given by

$$P_{321}^N = P\left(0 < Y^{A1} - \omega d_A^2 < |h_A|^2 < (\omega + \theta_k) d_A^2, \|h_B\|^2 < \omega d_B^2 \right)$$

$$= \exp\left(-\frac{(\omega + \theta_k) d_A^2}{\lambda_A} \right) \left[ 1 - \exp\left(-\frac{\omega d_B^2}{\lambda_B} \right) \right],$$

where $\delta^0 = \min\left\{ \frac{\gamma}{\lambda_B^2}, \omega d_A \right\}$.

Case III: When $k \in \{1,...,N-1\}$, we have $a_k \neq 0$ and $(\omega + \theta_k) d_A^2 \leq |h_A|^2 \leq (\omega + \theta_{k+1}) d_A^2$. Based on Eq. (17), $P_{321}^k$ is given by

$$P_{321}^k = P\left(|h_B|^2 < \omega d_B^2, (\omega + \theta_k) d_A^2 \leq |h_A|^2 \leq (\omega + \theta_{k+1}) d_A^2 \right)$$

$$\approx \int_0^{\delta^k_{\max}} \int_0^{\delta^k_{\max}} \phi_{\max}(x) \left[ \exp\left(-\frac{\omega d_B^2}{\lambda_B} \right) \right] \left[ \exp\left(-\frac{\omega d_A^2}{\lambda_A} \right) \right] d\lambda d\gamma$$

$$= \exp\left(-\frac{(\omega + \theta_k) d_A^2}{\lambda_A} \right) \left[ 1 - \exp\left(-\frac{\omega d_B^2}{\lambda_B} \right) \right],$$

where (d) holds from $y = |h_B|^2$ and $x = |h_A|^2$ and

$$\phi_{\max}(x) = \max\left\{ \min\left( (\omega + \theta_{k+1}) d_A, \frac{Y^{A1}}{\alpha_k \lambda_A} + \omega d_A^2 \right), (\omega + \theta_k) d_A \right\}$$

$$\delta_{\min}^k = \min\left( \frac{Y^{A1}}{\alpha_k \lambda_A \kappa_m}, \omega d_B^2 \right) \delta_{\min}^k$$

Note that it is difficult to obtain the accurate closed-form expression for $P_{321}^k$ with $Y^{A1} > 0$ due to the integral $\int_1^{\delta^k_{\max}} \exp\left(z_1 x + \frac{\omega z_2}{2}\right) dx$ with any value of $z_1$ and $z_2 \neq 0$. Fortunately, we can use Gaussian-Chebyshev quadrature to find an approximation for $P_{321}^k$. According to [14], Gaussian-Chebyshev quadrature is defined as

$$\int_{-1}^{1} f(x) \, d\xi \approx \sum_{j=1}^{N} w_j \sqrt{1 - z_j^2} f(\xi_j), \quad w_j = \frac{2}{K},$$

$$z_j = \cos \frac{2j - 1}{2K} \pi, \quad \xi_j = \frac{1}{2} \left( z_j + \sqrt{1 - z_j^2} \right).$$

Thus, $\Xi$ can be calculated as

$$\Xi = \pi (\delta_{\max} - \delta_{\min}) \sum_{m=1}^{M} 1 - \nu_m^2 \exp\left(-\frac{Y^{A1}}{\alpha_k \lambda_A \kappa_m} - \frac{K_m}{\lambda_B} \right),$$

where $M$ is a parameter that determines the tradeoff between complexity and accuracy, $\nu_m = \cos \frac{2m - 1}{2M} \pi$, and $\kappa_m = \frac{(\delta_{\max} - \delta_{\min}) \rho_m}{\Xi}$. Note that a larger $M$ results in a higher accuracy while a moderate yet acceptable accuracy can be realized at a small $M$. This is verified in our simulation results. Based on Eqs. (19), (20), and (21), the approximation of $P_{21}$ can be obtained.

2) The derivation of $P_{22}^k$. Likewise, we derive the expression of $P_{22}^k$ as follows. Given the values of $j$ and $k$, $P_{22}^{j,k}$ is expressed as follows.

Case I: When $j = 0$ with $\rho_B^j = 1 - \frac{\gamma \omega d_B^2}{P_{th} \lambda_B^2}$, we have $a_j = b_j = 0$ and $\|h_B\|^2 < (\theta + \omega) d_B^2$. Based on Eq. (18), the expression of $P_{22}^{0,k}$ is given by

$$P_{22}^{0,k} = P\left(0 < a_k |h_A|^2 < Y^{A1} - \omega d_A^2 \|h_B\|^2 < \omega d_B^2 \leq (\theta + \omega) d_B^2 \right).$$

Similarly, when $k = 0$, we have

$$P_{22}^{0,0} = \left[ \exp\left(-\frac{\omega d_B^2}{\lambda_B} \right) - \exp\left(-\frac{(\theta + \omega) d_B^2}{\lambda_B} \right) \right] \left[ \exp\left(-\frac{\omega d_B^2}{\lambda_B} \right) - \exp\left(-\frac{(\theta + \omega) d_B^2}{\lambda_B} \right) \right].$$

When $k \in \{1,...,N-1\}$, $P_{22}^{k,k}$ can be calculated as

$$P_{22}^{k,k} \approx \exp\left(-\frac{(\omega + \theta_k) d_A^2}{\lambda_A} \right) \left[ \exp\left(-\frac{\omega d_B^2}{\lambda_B} \right) - \exp\left(-\frac{(\theta + \omega) d_B^2}{\lambda_B} \right) \right]$$

$$\times \exp\left(-\frac{\omega d_B^2}{\lambda_B} \right) - \exp\left(-\frac{(\theta + \omega) d_B^2}{\lambda_B} \right),$$

where

$$\delta_{\min}^k = \min\left( \frac{Y^{A1}}{\alpha_k \lambda_A \kappa_m}, \omega d_B^2 \right) \delta_{\min}^k$$

$$\delta_{\max}^k = \max\left( \frac{Y^{A1}}{\alpha_k \lambda_A \kappa_m}, \omega d_B^2 \right) \delta_{\max}^k$$

$$\kappa_m = \frac{\delta_{\max}^k - \delta_{\min}^k}{\pi (\delta_{\max} - \delta_{\min})} \rho_m = \frac{\delta_{\max}^k - \delta_{\min}^k}{\pi (\delta_{\max} - \delta_{\min})} \rho_m.$$

When $k = N$, $P_{321}^{N,k}$ is given by

$$P_{321}^{N,k} = \left[ \exp\left(-\frac{(\omega + \theta_k) d_A^2}{\lambda_A} \right) \left[ \exp\left(-\frac{\omega d_B^2}{\lambda_B} \right) - \exp\left(-\frac{(\theta + \omega) d_B^2}{\lambda_B} \right) \right] \right] \left[ \exp\left(-\frac{\omega d_B^2}{\lambda_B} \right) - \exp\left(-\frac{(\theta + \omega) d_B^2}{\lambda_B} \right) \right]$$

where

$$\delta_{\max}^N = \max\left( \frac{Y^{A1}}{\alpha_k \lambda_A \kappa_m}, \omega d_B^2 \right) \delta_{\max}^N$$

and

$$\kappa_m = \frac{\delta_{\max}^N - \delta_{\min}^N}{\pi (\delta_{\max} - \delta_{\min})} \rho_m = \frac{\delta_{\max}^N - \delta_{\min}^N}{\pi (\delta_{\max} - \delta_{\min})} \rho_m.$$
If $k \in \{1, \ldots, N-1\}$, $P_{322}^{N,k}$ can be computed as
\[ P_{322}^{N,k} \approx \exp\left(-\frac{d_A^P}{\lambda_A} \left((\varpi + \theta_N) d_B^P\right)\right) - \exp\left(-\frac{d_A^P}{\lambda_A} \delta_{\text{max}}^{N,k}\right) \times \exp\left(-\frac{d_A^P}{\lambda_A} + \frac{\pi(\delta_{\text{max}}^{N,k} - \delta_{\text{min}}^{N,k})}{2M\lambda_B}\right) \times \sum_{m=1}^{M} \sqrt{1 - \nu_m^2} \exp\left(-\frac{Y_{A1}}{\alpha_0 k_{m,n,k}}\right), \tag{27}\]

where
\[ \delta_{\text{max}}^{N,k} = \max \left\{ \frac{Y_{A1}}{\alpha_0 d_B^P, (\varpi + \theta_N) d_B^P}\right\}, \]
\[ \delta_{\text{min}}^{N,k} = \max \left\{ \frac{Y_{A1}}{\alpha_0 d_B^P, (\varpi + \theta_N) d_B^P}\right\}, \]
\[ \kappa_m = \frac{\delta_{\text{max}}^{N,k} + \delta_{\text{min}}^{N,k}}{2\nu_m}. \]

If $k = N$, $P_{322}^{N,N}$ is given by
\[ P_{322}^{N,N} = \exp\left(-\frac{(\varpi + \theta_N) d_B^P}{\lambda_A}\right) \times \exp\left(-\frac{\pi(\delta_{\text{max}}^{N,N} - \delta_{\text{min}}^{N,N})}{2M\lambda_B}\right) \times \sum_{m=1}^{M} \sqrt{1 - \nu_m^2} \exp\left(-\frac{Y_{A1}}{\alpha_0 k_{m,N,N}}\right), \tag{28}\]

where $\delta_{\text{max}}^{N,N} = \max \left\{ \frac{Y_{A1}}{\alpha_0 d_B^P, (\varpi + \theta_N) d_B^P}\right\}$.

**Case 3:** When the energy harvester for $P_{1}^{\text{th}}$ works in the $j$-th linear region with $j \in \{1, \ldots, N-1\}$, we have $(\varpi + \theta_j) d_B^P \leq |h_B|^2 \leq (\varpi + \theta_{j+1}) d_B^P$.

(1) For the case $P_{A1}^P < P_{1}^1$, we have $k = 0$. Based on Eq. [18], there are two cases for $P_{322}^{0}$.

If $\Delta_{j,0} = (Y_{A1}^3)^2 - 4Y_{A1}^2 Y_{A2}^3 < 0$, there is no $|h_B|^2$ that satisfies $Y_{A2}^3 |h_B|^2 + Y_{A1} = 0$ and $P_{322}^{0} = 0$ always holds.

If $\Delta_{j,0} \geq 0$, there are two solutions to the equation $Y_{A2}^3 |h_B|^2 + Y_{A1} = 0$, which are
\[ x_{j,0}^1, x_{j,0}^2 = \min \left\{ \frac{-Y_{A1}^3 - \sqrt{\Delta_{j,0}}}{2Y_{A2}^3}, \frac{-Y_{A1}^3 + \sqrt{\Delta_{j,0}}}{2Y_{A2}^3} \right\}. \]

Combining with the condition that $(\varpi + \theta_j) d_B^P \leq |h_B|^2 \leq (\varpi + \theta_{j+1}) d_B^P$, $P_{322}^{0}$ is given by
\[ P_{322}^{0} = \mathbb{P}\left((\varpi + \theta_j) d_B^P \leq |h_B|^2 \leq (\varpi + \theta_{j+1}) d_B^P\right) \times \exp\left(-\frac{\pi(\delta_{\text{max}}^{0} - \delta_{\text{min}}^{0})}{2M\lambda_B}\right) \times \sum_{m=1}^{M} \sqrt{1 - \nu_m^2} \exp\left(-\frac{x_{j,0}^1}{\alpha_0 k_{m,0}}\right), \tag{29}\]

where
\[ \delta_{\text{max}}^{0} = \min \left\{ \frac{(\varpi + \theta_j) d_B^P, x_{j,0}^1}, \frac{(\varpi + \theta_{j+1}) d_B^P, x_{j,0}^2} \right\}, \]
\[ \delta_{\text{min}}^{0} = \max \left\{ \frac{(\varpi + \theta_j) d_B^P, x_{j,0}^1}, \frac{(\varpi + \theta_{j+1}) d_B^P, x_{j,0}^2} \right\}. \]

(2) For the case $P_{A1}^P \in [P_{1}^j, P_{1}^{j+1})$ with $j \in \{1, \ldots, N-1\}$, $P_{322}^{j,k}$ with $j, k \in \{1, \ldots, N-1\}$ is given by
\[ P_{322}^{j,k} = \mathbb{P}\left((\varpi + \theta_j) d_B^P \leq |h_B|^2 \leq (\varpi + \theta_{j+1} + \varpi) d_B^P\right) \times \exp\left(-\frac{\pi(\delta_{\text{max}}^{j,k} - \delta_{\text{min}}^{j,k})}{2M\lambda_B}\right) \times \sum_{m=1}^{M} \sqrt{1 - \nu_m^2} \exp\left(-\frac{x_{j,k}^1}{\alpha_0 k_{m,j,k}}\right), \tag{30}\]

where
\[ x_{j,k}^1 = \min \left\{ \frac{Y_{A1}^3 - \sqrt{\Delta_{j,k}}}{2Y_{A2}^3}, \frac{Y_{A1}^3 + \sqrt{\Delta_{j,k}}}{2Y_{A2}^3} \right\}. \]

By using Gaussian-???Chebyshev quadrature, the approximation of $P_{322}^{j,k}$ is given by
\[ P_{322}^{j,k} \approx \exp\left(-\frac{(\varpi + \theta_j) d_B^P}{\lambda_A}\right) - \exp\left(-\frac{(\varpi + \theta_{j+1}) d_B^P}{\lambda_B}\right) \times \exp\left(-\frac{\pi(\delta_{\text{max}}^{j,k} - \delta_{\text{min}}^{j,k})}{2M\lambda_B}\right) \times \sum_{m=1}^{M} \sqrt{1 - \nu_m^2} \exp\left(-\frac{x_{j,k}^1}{\alpha_0 k_{m,j,k}}\right), \tag{31}\]

where $\kappa_{m,j,k} = \frac{(\varpi + \theta_{j+1} - \varpi) d_B^P}{\lambda_B} + \frac{(\varpi + \theta_{j+1} + \varpi) d_B^P}{\lambda_B}$.

(3) For the case $P_{A1}^P > P_{1}^N$, we have $k = N$ and $a_k = 0, b_k = P_{m}^N$. Based on Eq. [18], the value of $P_{322}^{j,N}$ depends on $\Delta_{j,N} = (Y_{A1}^3)^2 - 4Y_{A1}^2 Y_{A2}^3$. If $\Delta_{j,N} < 0$, similar to $P_{322}^{j,0}$, we have $P_{322}^{j,N} = 0$. If $\Delta_{j,N} \geq 0$, $P_{322}^{j,N}$ is given by
\[ P_{322}^{j,N} = \mathbb{P}\left(|h_A|^2 \geq (\varpi + \theta_N) d_B^P, \delta_{\text{max}}^{j,N} \leq |h_B|^2 \leq \delta_{\text{min}}^{j,N}\right) \times \exp\left(-\frac{(\varpi + \theta_N) d_B^P}{\lambda_B}\right) - \exp\left(-\frac{\delta_{\text{max}}^{j,N}}{\lambda_B}\right). \tag{32}\]

where
\[ \delta_{\text{max}}^{j,N} = \min \left\{ \frac{(\varpi + \theta_j) d_B^P, x_{j,0}^1}, \frac{(\varpi + \theta_{j+1}) d_B^P, x_{j,0}^2} \right\}, \]
\[ \delta_{\text{min}}^{j,N} = \max \left\{ \frac{(\varpi + \theta_j) d_B^P, x_{j,0}^1}, \frac{(\varpi + \theta_{j+1}) d_B^P, x_{j,0}^2} \right\}. \]

Based on Eq. [16], the approximation of $P_{B}^H$ can be obtained. Similarly, $P_{A_{\text{out}}}^H$ can be obtained by the same way. Based on $P_{A_{\text{out}}}^H$ and $P_{B_{\text{out}}}^H$, the capacity of the system $C_{\text{total}}$ can be determined.

**IV. Simulations**

In this section, we validate the performance of the proposed scheme and the derived outage probability via a $1 \times 10^6$ Monte-Carlo simulations. Unless otherwise specified, the simulation parameters are set as follows: $d_A = 15$ meters, $d_B = 10$ meters. The distances for source-relay and relay-destination links reflects the total communications distance ranges from several meters to tens of meters.

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3The distances for source-relay and relay-destination links reflects the total communications distance ranges from several meters to tens of meters.
and $\sigma^2 = -90$ dBm. The transmission rate is assumed as $U = 3$ bit/s/Hz and the corresponding SNR threshold $\gamma_{th}$ is $2^U - 1$. The transmit power of the source is set to be $10$ mW. We employ the piecewise linear EH model with $N = 4$, where $P_{th} = [10, 57.68, 230.06, 100]$ uW, $\{a_k\}_3^4 = [0.3899, 0.6967, 0.0127]$, $\{b_k\}_3^4 = [-1.6613, -19.1737, 108.2778]$ uW and $P_m = 250$ uW. The accuracy of this model is verified by comparing it with the experimental data in [16].

Figure 2 plots the outage probability at $B$ with the piecewise linear EH model achieved by the proposed scheme versus the transmit power with $\alpha = 2, 2.7$, and $3$, respectively. The theoretical results with different $M$ are computed based on Eq. (14), Eq. (15), and Eq. (16). It can be observed that the theoretical results match perfectly with Monte Carlo simulation results, which verifies the accuracy of the theoretical results. Besides, it can also be seen that a small $M$ (e.g. $M = 10$) is sufficient to provide an accurate $P_{out}$. Another observation is that the outage probability at node $B$ converges to the error floors when the transmit power $P$ keeps increasing, which is the main difference from the outage behaviors with the linear energy harvesting model. This is due to the fact that the harvested energy from $A$ or $B$ is constrained to $P_m$ when $P$ is large enough.

Figure 3 plots the capacity as a function of $P$ with three PS schemes: the proposed scheme, the existing static equal scheme in [12], and the random PS scheme in [2]. For the random PS scheme, the PS ratio follows a uniform distribution over the closed interval $[0, 1]$. It can be observed that the capacity increases with the increase of $P$ and converges to the maximum value when $P$ is large enough, which perfectly matches the results in Fig. 2. Another observation is that the proposed scheme has a higher capacity than the existing schemes in [2] and [12]. The reason is that the proposed scheme can provide more flexibility and effectively utilize the instantaneous CSI.

Figure 4 plots the capacity for three PS schemes versus the transmission rate $U$. It can be seen that the capacity increases first, reaches the peak value, and then decreases. This is because that the outage probability at $A$ or $B$ goes up with
the increase of $U$ and the influence of the outage probability becomes the dominant factor to the capacity when $U$ is large enough. As shown in this figure, we can also see that the proposed scheme can provide a significant performance gain over the existing schemes. Fig. 5 compares the capacity of various PS schemes as a function of $d_A$. It is assumed that $d_A + d_B = 25$. Given a fixed $d_A$, $d_B$ can be computed as $25 - d_A$. It can be observed that with the increase of $d_A$, the capacity decreases, reaches the minimum value and then increases. This is because that the total harvested energy is higher when the relay is closer to either of the nodes. Besides, we can see that the proposed scheme is superior to the existing schemes in terms of capacity.

V. CONCLUSIONS

In this paper, we have proposed a dynamic heterogeneous PS scheme to maximize the capacity of SWIPT based three-step DF TWRNs with a non-linear EH model. Specifically, by considering the heterogeneous instantaneous channel gains between the destination nodes and the relay, we have derived the closed-form expression of the optimal PS ratio for each link. Based on the optimal PS ratios, we have derived an analytical expression for the optimal outage probability under the non-linear EH model. Simulation results have verified the correctness of the derived outage probability and shown that the proposed PS scheme can achieve a higher capacity compared with the existing schemes.

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