Building topological devices through emerging robust helical surface states

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Abstract

We propose a nonlocal manipulation method to build topological devices through emerging robust helical surface states in $Z_2 = 0$ topological systems. Specifically, in a ribbon of $Z_2 = 0$ Bernevig-Hughes-Zhang (BHZ) model with finite-size effect, if magnetic impurities are doped on the top (bottom) edge, the edge states on the bottom (top) edge can be altered according to the strengths and directions of these magnetic impurities. Consequently, the backscattering between the emerging robust helical edge states and gapped normal edge states due to finite-size confinement is also changed, which makes the system alternate between a good one-channel conductor and a good insulator. This effect allows for fabricating topological devices with a high on-off ratio. Furthermore, this proposal can be generalized to three-dimensional (3D) models and more realistic Cd$_3$As$_2$ type Dirac semimetals.

1. Introduction

The $Z_2 = 1$ quantum spin Hall states and the consequent three-dimensional (3D) strong topological insulators (STIs) have attracted great interest since the success generated from their theoretical predictions and experimental observations \cite{1–7}. One of the most important reasons for this is that there exist helical surface states characterized by a full insulating gap and protected by time-reversal symmetry \cite{8, 9}. Notably, those surface states are spin-momentum locked and exhibit a robustness against nonmagnetic impurities, which makes the $Z_2 = 1$ STI an ideal candidate for energy-saving topological devices.

However, there still exist great challenges in the application of the $Z_2 = 1$ STI due to their own intrinsic characteristics. Specifically, as an electric device, the on and off status should be controlled, which means the system can be alternated between a conductor and an insulator through an external field. Since the backscattering is suppressed with nonmagnetic impurities, the $Z_2 = 1$ STI is born a perfect conductor. Yet for this reason, it is difficult to realize the off status in the $Z_2 = 1$ STI. Nevertheless, people have undergone great effort, in such aspects as gapping/proximity/heterostructure, to reach the off status in the STI recently \cite{10, 11}. Besides, by now, the most usual manipulation method is to dope magnetic impurities or apply an electromagnetic field \cite{12–15}, which all encounter the same problems. First, both surfaces of the topological system should be doped in order to induce backscattering, which also makes the system difficult to return to a conductor. Second, since the backscattering only exists where the magnetic impurities or external fields are induced, the surface states can only be controlled locally. However, experimentally, topological insulators usually grow on a substrate, which means that any manipulation on a substrate surface becomes nearly impossible due to the difficulty of doping there. Consequently, a nonlocal method to control the surface states close to the substrate in order to build topological devices becomes rather an important topic.
Not long ago, theorists proposed a new kind of $Z_2 = 0$ topological system [16–19]. Considering the finite-size effect, the $Z_2 = 0$ topological system with proper size inherits the transport properties of the $Z_2 = 1$ STI, as well as unique emerging robust helical surface states due to the finite-size confinement. Here, the words ‘robust’ and ‘emerging’ mean those helical surface states which are robust against nonmagnetic disorder and which only exist in a finite-size system compared to the traditional one [16]. Recently, Zhu et al found the sign of these $Z_2 = 0$ emerging robust helical surface states in epitaxial Bi(111) thin films [20]. Moreover, in contrast to the $Z_2 = 1$ STI, there also exists an unexpected phenomenon that the weak anti-localization (WAL) peak suddenly disappears after doping magnetic Co on the top surface of thin Bi film. This experiment implies that the surface states on the bottom of the thin $Z_2 = 0$ topological system can be manipulated through a nonlocal method on the top surface, which may have potential application for building topological devices.

In this work, we propose a new nonlocal method to build topological devices through the emerging robust helical edge or surface states in $Z_2 = 0$ topological systems. The schematic diagram for our proposed system is shown in figure 1. First, we demonstrate the basic principle of our proposed method in a narrow ribbon described by a two-dimensional (2D) $Z_2 = 0$ anisotropic Bernevig-Hughes-Zhang (BHZ) model. Without magnetic doping, we find that there exists one pair of emerging robust helical edge states and one pair of gapped normal edge states due to the finite-size confinement. Therefore, the system exhibits conducting characteristics. However, if the top (bottom) of the 2D BHZ model is doped with magnetic impurities, the gapped normal edge states on the bottom (top) gradually become gapless with increasing magnetization. Thus, for moderate magnetization on the top (bottom) edge, the system remains a good conductor since there is no backscattering on the bottom (top) edge. When magnetization becomes sufficiently strong, the Anderson disorder can cause strong backscattering on both edges, resulting in insulating characteristics for the system. Using the Landauer–Büttiker formula, we verify the above physical pictures by the two-terminal conductance $G$, which starts from $2e^2/h$, then maintains at the value of $e^2/h$, and finally falls to zero. This novel phenomenon enables us to nonlocally manipulate the on and off status in a $Z_2 = 0$ topological system by changing the strengths or directions of the magnetic impurities on the top surface.

2. Results and discussion

2.1. 2D BHZ model and 3D Wilson–Dirac model

We first take the anisotropic BHZ model, with the basis $(A_1, B_1, A_1, B_1)$ in a square lattice as an example [21, 22]. The four-band tight-binding Hamiltonian in the momentum representation reads:
$\mathcal{H}(\vec{k}) = \begin{pmatrix} H(\vec{k}) & 0 \\ 0 & H^*(\vec{k}) \end{pmatrix}$

$H(\vec{k}) = \tau_z \left( m - m_x - m_y + m_x \cos k_x + m_y \cos k_y \right)$

\[ + \tau_x v_x \sin k_x + \tau_y v_y \sin k_y \]

(1)

where $m$ determines the band gap, $v_{x,y}$ reflects the Fermi velocity, and $m_{x,y}$ represents the hopping amplitude between nearest-neighbor sites along the $x$ and $y$ directions, respectively. $\tau_{x,y,z}$ are Pauli matrices representing different orbits. During our calculation, the values of these parameters are chosen as flows:

$m = 1.64, m_x = 0.8, m_y = 1.2, v_{x,y} = 3$. We consider the ribbon geometry, and the width along the $y$ direction is chosen as $L_y = 30a$ ($a$ is the lattice constant) in order to induce the finite-size effect.

In this work, our plan of magnetic doping is to dope a soft magnetic stripe on one edge or surface of the sample (see figure 1). Therefore, the strengths and directions of the magnetic impurities are entirely determined by the external magnetic field. We consider the effect of magnetic impurities in the $x$ direction by an adding term $M_z \sigma_z \otimes \tau_0$ on the $y = 0$ edge of the ribbon, thus the spin can flip from $A_1$ to $A_2$, or from $B_1$ to $B_2$. Actually, even though it will be difficult to accurately control the magnetization strength $M_z$ of the soft magnetic stripe in the experiment, an external magnetic field rotating in an $x-z$ direction will also work. This is because $M_z$ is determined by $M \cos \theta$, and the existence of $M_z$ has little influence on the final result. Actually, the role of magnetic impurities in our manuscript is just to introduce an effective exchange field and cause a hysteresis loop, which can also be done by the van Vleck magnetism as well [23–25].

Without magnetic impurities, the energy band of the system is drawn in figure 2(a), which is doubly degenerated due to the spin degeneration. There exist two pairs of gapless edge states and two pairs of gapped edge states locating at $k_x = 0$ and $k_x = \pi$, respectively. To be specific, the gapless edge states around $k_x = 0$ are robust with a very small penetration length, and are named as emerging robust helical edge states. Meanwhile the gapped edge states around $k_x = \pi$ result from the hybridization with the very large penetration length, and are named as hybridized gapped normal edge states. Actually, because of the large hybridization gap in $k_x = \pi$ helical edge states, the transport properties of $k_x = 0$ helical edge states exhibit emerging robustness in a narrow system [16]. When the Fermi energy is set $E_F = 0.08$ as the black dashed line shows in figure 2(a), the corresponding probability density $|\Psi(y)|^2$ is drawn in figure 2(d). It is obvious that the gapless edge states around $k_x = 0$ mostly locate at two edges.

The energy band with magnetic impurity strength $M_z = 1$ is shown in figure 2(b). We find that one pair of the previously double-degenerate edge states around $k_x = 0$ opens a small gap, while the other one remains closed. Moreover, one of the originally double-degenerate energy gaps around $k_x = \pi$ becomes smaller. In figure 2(e), we find that the only existing edge states are the emerging robust helical ones locating at $y = 30a$.

With a continuing increment of the impurity strength, the above tendency exhibits much more explicitly. If the impurity strength reaches as strong as $M_z = 2.1$, the smaller gap in figure 2(b) around $k_x = \pi$ becomes nearly gapless in figure 2(c). In figure 2(f), the corresponding probability density $|\Psi(y)|^2$ tells us that the only existing edge states are those locating around $y = 30a$. Specifically, the emerging robust helical edge state colored by blue mostly locates around $y = 30a$, while the normal edge state colored by red extends to the inner space, which means it still exhibits properties of bulk states.

In order to quantitatively analyze the physical pictures of energy bands, we further study the transport properties of this system, and calculate the conductance $G$ of this model with nonmagnetic disorder in a two-terminal system. The nonmagnetic disorder exists inevitably in the realistic system and is modeled by the Anderson disorder with random potential uniformly distributed in $[-w/2, w/2]$, where $w$ is the disorder strength. The distance between the two leads is $3000a$ and the Fermi energy of the central region is $E_F = 0.03$. In order to simulate metallic leads at the two terminals, their widths along the $y$ direction and the Fermi energies are chosen as $N_L = N_R = 200a$ and $E_L = E_R = 2.3$, respectively.

Considering zero temperature, the two-terminal conductance of the system can be calculated from the Landauer–Büttiker formula: $G = T_{LR}(E_F)$, where $T_{LR}(\epsilon) = \text{Tr} [\Pi_L G_1 \Gamma_{\uparrow}(\epsilon) \Pi_R G_2]$, and the Green functions $G_{i}(\epsilon) = [G_{i}(\epsilon)]^* = 1/\epsilon - H_{\text{center}} - \Sigma_{\text{fl}}^{(0)} - \Sigma_{\text{fl}}^{(1)}]$. Here, $\Sigma_{\text{fl}}^{(0)}$ is the retarded self-energy due to the coupling to the lead, and $H_{\text{center}}$ is the Hamiltonian in the center region [26, 27].

We first show how the conductance varies with the strength of magnetic impurities $M_z$ in figure 3(a). For the red line without Anderson disorder, the conductance undergoes a transition from $G = 2e^2/h$ to $e^2/h$, then back to $2e^2/h$. The second plateau of $G = 2e^2/h$ shakes vigorously, which is the result of resonance in the Fabry–Pérot cavity. Briefly, those different quantized values can be explained by the number of occupied

7 As we can see in figure 2(f), the edge states colored by red still show properties of bulk states. Therefore, they must obey the resonance conditions in the Fabry–Pérot cavity: $k_x L = n\pi (n = 0, \pm 1 \ldots)$. This means that the longer the $L$ is, the more vigorous oscillations will be found.
states at the Fermi level, as shown in figure 2(a). Moreover, the physical pictures behind the whole transition processes can be better understood with the help of the schematic diagrams from figures 3(b)–(d). First, as shown in figure 3(b), due to the finite-size effect, there exist one pair of emerging robust helical edge states and one pair of gapped normal edge states [16], while the gapped normal ones have no contribution to the conductance. Therefore, the conductance exhibits \( G = 2e^2/h \) at the beginning. Then, in figure 3(c), if we slightly dope a little amount of magnetic impurities in the \( x \) direction on the \( y = 0 \) edge, which will induce the spin-flip scattering between the counter propagating emerging robust helical edge states on \( y = 0 \), only the emerging robust helical edge states on \( y = L_y \) contribute to the conductance. Thus, the conductance falls to \( G = e^2/h \) fast and then remains unchanged. At last, in figure 3(d), when \( M_x \) is sufficiently strong, the gapped normal edge states on \( y = 0 \) owns a relatively large gap, and the coupling between the gapped normal edge states on \( y = 0 \) and \( y = L_y \) is really weak. Therefore, the originally gapped normal edge states on \( y = L_y \) become gapless and could make a contribution to the conductance. As a result, the conductance gets back to \( G = 2e^2/h \).

In this work, we neglect the effect of edge roughness. However, considering the inevitable impurities in reality, the system shows exotic transport properties, which is the central result of this paper. If the strength of the Anderson disorder is not zero, as shown by the green, blue and brown lines in figure 3(a), the conductance never returns to \( G = 2e^2/h \), but continues falling to nearly zero with strong \( M_x \).

8 We also calculate a 2D BHZ sample with weak edge roughness. According to our calculation results, the conductance \( G \) exhibits the same as shown in figure 3(a). Moreover, it is 3D materials that are really applicable in experiments. As far as we know, the molecular beam epitaxy technology can grow 3D samples whose surfaces exhibit smoothness even at the atomic scale, and such smoothness exists in extended systems with micron size. Due to the high qualities of these 3D samples, there is nearly no roughness at their surfaces. Therefore, we think it is fine for us to consider only bulk disorders and neglect the effect of edge roughness.
difference can be explained by the backscattering between the emerging robust helical edge states and the normal ones on \( y = 30a \) caused by the Anderson disorder. The existence of this backscattering is guaranteed by the physical meaning of the \( Z_2 \) topological invariant: gapless helical edge states with an even number are not robust against disorder. Besides, for the green line, when the impurity strength approaches \( M_x = 1.5 \), we find the conductance exhibits a minimum value during the process of decreasing. This critical point is actually also the transition point from \( G = \frac{e^2}{h} \) to \( 2 \frac{e^2}{h} \) without disorder, which means the Fermi energy just touches the bottom of the upper energy band around \( k_x = \pi \). In this case, the backscattering between the \( k_x = 0 \) and \( k_x = \pi \) is permitted. Therefore, there exist more backscattering channels shown in figure 3(d), and the backscattering between the emerging robust helical edge states and the gapped normal ones on \( y = 30a \) is somehow enhanced. As a result, the conductance at this point exhibits a smaller value than the surrounding points.

Next, let us fix the magnetic impurity strength \( M_x \), and study how the conductance \( G \) varies with the Anderson disorder strength \( w \). In figure 3(e), it is obvious that the conductance with \( M_x = 0.5 \) exhibits stronger robustness than that with \( M_x = 2 \), which is consistent with figure 3(a). Moreover, for instance, if the disorder strength is chosen as \( w = 2 \), the ‘on’ status of the red line with \( M_x = 0.5 \) exhibits \( G \approx 0.87 \frac{e^2}{h} \), while the ‘off’ status of the blue line with \( M_x = 2 \) exhibits \( G \approx 0.03 \frac{e^2}{h} \). Therefore, the on-off ratio can be achieved at about 30:1 in this case. Until now, we have found that by altering the strength of the magnetic impurities on one edge, a
$Z_2 = 0$ BHZ model with emerging robust helical edge states can be tuned from a good one-channel conductor to a good insulator, which can be applied to build topological devices.

However, in some cases, it may not be convenient to alter the external field, and the strength of the magnetic impurities $M_x$ is fixed. Therefore, let us consider a situation that the direction of doped magnetic impurities is not ordered but in a random arrangement. For simplicity, we just assume that $\vec{M}$ is either in an $x$ or $z$ direction. During our calculation, $M_x^2 + M_z^2 = M_{\text{total}}^2$, $M_x \in [-M_{\text{total}}, M_{\text{total}}]$, and $M_{\text{total}} \in [0, M]^\circ$. The red line of figure 3(f) shows how the conductance varies with the magnetic impurity strength $M$ when $w = 1.5$. To have a better comparison, a similar $w = 1.5$ situation with magnetic impurities ordered in the $x$ direction is also plotted by a blue line. As we can see, when $M = 2$, the conductance of the red line still maintains at $G \approx 0.8 e^2/h$ compared with $G \approx 0.1 e^2/h$ of the blue line when $M_x = 2$, which can be regarded as the on and off status of the current. In experiments, this kind of random can be realized in many ways, such as by heating and so on. Therefore, topological devices built by the $Z_2 = 0$ 2D BHZ model with emerging robust helical edge states can also be tuned from a good one-channel conductor to a good insulator by altering the directions of magnetic impurities on one edge.

To further show the advantages of the above proposals, we also consider a 2D $Z_2 = 1$ BHZ model by changing the parameter $m$ in equation (1) from 1.64 to 1 and $L_y$ still maintains 30a. As shown in figure 4(a), no matter whether there exists Anderson disorder or not, the conductance never falls to zero, but keeps $G = e^2/h$ all the time. Compared with the $Z_2 = 0$ topological system in figure 4(b), the $Z_2 = 1$ STI cannot turn off the current, which means that it is not suitable to build topological devices. Moreover, we also calculate a $Z_2 = 0$ 2D BHZ model without a finite-size effect by expanding $L_y$ from 30a to 200a. The red line in figure 4(c) represents how the conductance $G$ varies with the magnetic impurity strength $M_x$ without Anderson disorder, and there exists two characteristic quantum plateaus $G = 3e^2/h$ and $G = 2e^2/h$. In figure 4(d), we show how the conductance on these two plateaus varies with the Anderson disorder strength $w$. The red line and the blue line represent $M_x = 0.1$ and $M_x = 1$, respectively. Unlike figure 3(e), the main feature of figure 4(d) is that the conductances of these two lines start decreasing at the beginning, and fall to zero nearly with the same $M_x$. This feature tells us that one cannot get a high on-off ratio for any disorder strength $w$ or magnetic impurity strength $M_x$. The reason for this is that there are always two pairs of gapless helical edge states around $y = L_y$, and the Anderson disorder can cause the backscattering between them (the helical edge states around $k_x = 0$ are no longer emerging robust ones now). Therefore, if we want to switch the current on or off in order to build topological devices just by tuning the strength of doped magnetic impurities on one edge,
the existence of the $Z_2 = 0$ emerging robust helical edge states due to finite-size confinement is the necessary condition.

Similar phenomenon also exists in 3D $Z_2 = 0$ topological systems, and we take the anisotropic Wilson–Dirac type model as an example [28–33]. The Hamiltonian in a cubic lattice reads:

$$\mathcal{H}(\vec{k}) = m(\vec{k}) \sigma_0 \otimes \tau_z + \sum_\alpha \nu_\alpha \sin k_\alpha \sigma_0 \otimes \tau_x$$

$$m(\vec{k}) = m + \sum_\alpha m_\alpha (\cos k_\alpha - 1)$$

(2)

where $\sigma$ are Pauli matrices in spin space, and $\alpha = x, y, z$. Parameters $\tau$, $m$, $\nu_\alpha$, $m_\alpha$ have the same meanings as the 2D model of equation (1). The width along the $z$ direction $L_z$ is chosen as $L_z = 15a$, and the magnetic impurities are doped in the $z$ direction on the $z = 0$ side. During the calculation, related parameters are chosen as follows: $m = 2.26$, $m_x = 0.9$, $m_y = 1.1$, $m_z = 0.8$, $\nu_x, \nu_y = 1.5$. In figures 5(a)–(c), we show how the 3D energy band varies with the strength of doped magnetic impurities $M_z = 0, 1$ and 2, respectively. There exist two nonequivalent Dirac points $A(C)$ and $B(D)$, and the blue and pink curves represent the two energy bands nearest to the Dirac points. In figure 5(a), the energy bands locating around $A(C)$ are gapless, because they are the emerging robust helical surface states. However, the appearance of the gaps on $B(D)$ tells us these surface states are the gapped normal ones due to finite-size confinement. In figure 5(b), $M_z = 1$, we find that the pink curves on $A(C)$, which were gapless at first, now open a small gap, because there exists the backscattering between the counter propagating emerging robust helical surface states on $z = 0$ induced by the magnetic impurities. At last, in figure 5(c), when the strength of the magnetic impurities is sufficiently strong, all blue curves at $A(C)$ and $B(D)$ become gapless. Briefly, the Dirac points $A(C)$ and $B(D)$ correspond to $k_z = 0$ and $k_z = \pi$ in the 2D BHZ model, respectively. In addition, all the physical pictures of figures 5(a)–(c) are exactly the same as those shown in figures 2(a)–(c), respectively. The above results calculated in the 3D topological system is consistent with what Zhu et al found in epitaxial Bi(111) thin films [20]. Specifically, they found that the WAL peak remains when a 0.5 ML of nonmagnetic Cu is introduced to the top surface of the sample. This is because of the existence of the emerging robust helical surface states on the top and bottom surfaces. In contrast, when they add a 0.25 ML of magnetic Co, the WAL peak is completely gone. This is because the magnetic Co on the top surface has broken the emerging robust helical edge states on the bottom surface through the inner interaction. In this case, the emerging robust helical edge states on the bottom surface are not protected by time-reversal symmetry, and could be easily scattered due to the Anderson disorder.

By now, we have afforded two kinds of nonlocal methods to build topological devices in $Z_2 = 0$ topological systems. The first one is to alter the strengths of the doped magnetic impurities. Specifically, we first dope a soft–magnetic film at the top surface of a thin $Z_2 = 0$ topological system grown on a substrate. Then, if we only apply a weak magnetic field on the soft–magnetic film, the whole device is a good conductor. However, if the soft–magnetic film is magnetized sufficiently strong, the whole device turns into an insulator. The second method is to alter the directions of doped magnetic impurities. If the doped magnetic film is magnetized in order with moderate strength, the whole device is an insulator. However, if the magnetic direction of doped magnetic film is in a random arrangement, such as by heating and so on, the whole system gets back to being a conductor. Since the above methods exhibit a large degree of manipulation and are all nonlocal ones on one edge or surface of the sample, it shows great advantages to build topological devices through emerging robust helical surface states in $Z_2 = 0$ topological systems compared with a traditional $Z_2 = 1$ STI.

**Figure 5.** (a)–(c) correspond to the surface energy bands of the Wilson–Dirac model with different strengths of magnetic impurities: $M_z = 0, 1$ and 2, respectively. As $M_z$ increases, one of the two doubly degenerated gapless surface states on $A(C)$ opens a small gap, while one of the two doubly degenerated gapped surface states on $B(D)$ gradually becomes gapless.
2.2. Extension to real materials

Based on the above two simple models, we have in principle shown the nonlocal manipulation methods to build topological devices with emerging robust helical surface states in $Z_2 = 0$ topological systems. In this section, let us further extend our proposal to more realistic materials. Though there are several 3D realistic material candidates [20, 34], we restrict our discussion to 2D cases to avoid the huge computational requirements.

Motivated by the recent theoretical prediction and experimental realization of Dirac semimetal Cd$_3$As$_2$ [35–37], we consider a Cd$_3$As$_2$ stripe whose effective low-energy Hamiltonian $H(k)$ reads as [35]:

$$H(k) = \epsilon_0(k) + \begin{pmatrix} M(k) & A(k) & D(k) & B^*(k) \\ A(k) & -M(k) & B^*(k) & 0 \\ D(k) & B(k) & M(k) & -A(k) \\ B(k) & 0 & -A(k) & -M(k) \end{pmatrix}$$

where $\epsilon_0(k) = C_0 + C_1 k_x^2 + C_2 (k_x^2 + k_y^2)$, $k_{\pm} = k_x \pm i k_y$, and $M(k) = M_0 - M_1 k_x^2 - M_2 (k_x^2 + k_y^2)$ with parameters $M_0, M_1, M_2 < 0$. Since $B(k) = (\alpha k_x + \beta k_y) k_x^2$, $B(k)$ can be neglected compared with the $Dk_{\pm}$ term if we only consider the expansion up to $O(k^2)$ [35, 38].

In this paper, $C_0, C_1$ and $C_2$ are chosen as zero, which can simplify our numerical calculation but has no influence on the physical properties of this model. In fact, the main results got from this approximation satisfy realistic Cd$_3$As$_2$ materials, though they are not quite true in experiment. Other related parameters are chosen as follows: $M_0 = -0.44, M_1 = -0.5, A = 1.2, D = 0.55$. Considering 3D stripe geometry in which $L_x = 6a$ and $L_y = 20a$, the above $\hat{p}$ Hamiltonian can be rewritten in tight-binding form, and the energy bands of Cd$_3$As$_2$ are drawn in figures 6(a)–(c). Compared with figures 2(a)–(c), the two Dirac points $k_x = 0$ and $k_x = \pi$ in the BHZ model both locate at $k_x = 0$ with two inverted bands [35]. In figure 6(a), there is no doped magnetic impurity. The two pairs of gapless surface states represent the emerging robust helical ones, and the other two with a visible gap represent the gapped normal ones. Taking the Fermi energy as $E_F = 0.014$, we obtain the distribution of the...
Figure 7. (a) The red and blue lines stand for the conductance $G$ versus $M_x$ in Cd$_3$As$_2$ for the two Anderson disorder strengths $w = 0$ and $w = 0.5$, respectively. (b) The red and blue lines stand for the conductance $G$ vs $w$ in Cd$_3$As$_2$ for the two fixed magnetic impurity strengths $M_x = 0.2$ and $0.8$, respectively.

emerging robust helical surface states in figure 6(d), and it has been integrated along the $z$ axis. As we can see, the red line and blue line seem degenerated, which originates from the coupling between the emerging robust helical surface states on $y = 0$ and $y = 20a$. Next, a little amount of magnetic impurities in the $x$ direction ($M_x \sigma_x \otimes \tau_0$) are doped on $y = 0$. In figure 6(b), we find that one of the two pairs of emerging robust helical surface states opens a small gap, while one of the two pairs of gapped normal surface states tends to close. In this case, the only existing gapless surface states are the emerging robust helical ones concentrating on $y = 20a$ as shown in figure 6(e). Finally, when the strength of magnetic impurities is strong enough, such as shown in figure 6(c) with $M_x = 0.9$, there exist two pairs of gapless surface states consisting of one pair of emerging robust helical surface states and one pair of normal surface states. The probability density distribution in this case is shown in figure 6(f). The blue and red lines represent the emerging robust helical surface states and the normal ones, respectively. We find only the surface states concentrating around $y = 20a$ exist now. In fact, every characteristic in figures 6(a)–(f) of Cd$_3$As$_2$ owns its correspondence in figures 3(a)–(f) of the 2D BHZ model, and the physical pictures between these two models are exactly the same.

Finally, we also calculate the conductance of Cd$_3$As$_2$ with Anderson-type disorder in a two-terminal system. The Fermi energies of the two leads and the central part are $E_l = E_R = 0.5$ and $E_C = 0.01$, respectively. Without loss of generality, the disorder is only added on the $y = 20a$ boundary\(^{10}\). Figure 7(a) tells us how the conductance varies with the strength of magnetic impurities $M_x$. In the red line, there is no Anderson disorder and the distance between the two leads is $4 \times 10^2a$. Similar to the 2D BHZ model, the conductance falls to a plateau of $G = e^2/h$ from the beginning $G = 2e^2/h$ rapidly, and then goes back to $G = 2e^2/h$. However, if the disorder strength is chosen as $w = 0.5$ shown by the blue line, the conductance decreases from $G \approx 2e^2/h$ to $G \approx e^2/h$, and never goes back to $G = 2e^2/h$ but continues falling to nearly zero. Similar to the green line in figure 3(a), a critical point around $M_x = 0.6$ also exists, but exhibits much more explicitly than that which occurs in the BHZ model. According to the conductance behavior under the moderate disorder strength $w = 0.5$, we can claim the two-terminal device is in an ‘on’ status when $M_x \in [0.1, 0.3]$ and in an ‘off’ status when $M_x > 0.55$. In order to better understand the performance of the topological device, we also show how the conductance varies with the Anderson disorder strength $w$ and with the fixed magnetic impurity strength $M_x$ in figure 7(b). When the device is in the claimed ‘off’ status (such as $M_x = 0.8$), the conductance $G$ quickly falls to zero with the increasing disorder strength $w$. In contrast, when the device is in the claimed ‘on’ status (such as $M_x = 0.2$), the conductance nearly maintains at $G = e^2/h$, no matter how the disorder strength $w$ or the length of the strip changes. According to figure 7(b), a relatively high on-off ratio of about 10 : 1 is obtained when $w > 0.4$, which is actually a large advance compared with traditional $Z_2 = 1$ topological devices. Moreover, the stronger the disorder strength is and the longer the stripe is, the higher the on-off ratio which can be obtained. Nevertheless, the disorder strength cannot be too strong in case of the breaking of the ‘on’ status. In fact, figure 7(b) is consistent with figure 3(e), which is also the result of the finite-size effect and the key to switch on or off the current by doping magnetic impurities on one surface.

To summarize, all conclusions obtained from the 2D BHZ model can also be found in Cd$_3$As$_2$ type Dirac semimetals, which generalizes our proposal concerning building new topological devices to a much broader prospect. Moreover, we need to clarify that it is not the intention of this work to prove that a $Z_2 = 1$ phase and a $Z_2 = 0$ phase are the same academically. However, in view of practical application and tunability, a $Z_2 = 0$.
phase can exhibit similar or even better transport properties compared with a $Z_2 = 1$ phase under certain circumstances, which can be used to build new topological devices.

3. Conclusion

In conclusion, we show that in $Z_2 = 0$ topological systems, one can switch on and off the current nonlocally just by doping magnetic impurities on one edge or surface of the sample, which enables us to build topological devices through emerging robust helical surface states in $Z_2 = 0$ topological systems. This proposal is first demonstrated in a 2D $Z_2 = 0$ anisotropic BHZ model and a 3D Wilson–Dirac type model in detail. Notably, it is also generalized to realistic Cd$_3$As$_2$ type Dirac semimetals. Since the manipulation methods proposed here are all nonlocal ones with a large degree of manipulation, the $Z_2 = 0$ topological system with emerging robust helical surface states exhibits much brighter prospects in building topological devices than the traditional $Z_2 = 1$ STI.

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