The influence of the turbulent perturbation scale on pre-stellar core fragmentation and disc formation

S. Walch, A. P. Whitworth and P. Girichidis

School of Physics & Astronomy, Cardiff University, 5 The Parade, Cardiff CF24 3AA, Wales
Institut für Theoretische Astrophysik, Albert-Ueberle-Str. 2, 69120 Heidelberg, Germany

Accepted 2011 August 30. Received 2011 August 22; in original form 2011 April 11

ABSTRACT

The collapse of weakly turbulent pre-stellar cores is a critical stage in the process of star formation. Being highly non-linear and stochastic, the outcome of collapse can only be explored theoretically by performing large ensembles of numerical simulations. Standard practice is to quantify the initial turbulent velocity field in a core in terms of the amount of turbulent energy (or some equivalent) and the exponent in the power spectrum ($n \equiv -\frac{d\log P_k}{d\log k}$). In this paper, we present a numerical study of the influence of the details of the turbulent velocity field on the collapse of an isolated, weakly turbulent, low-mass pre-stellar core. We show that, as long as $n \gtrsim 3$ (as is usually assumed), a more critical parameter than $n$ is the maximum wavelength in the turbulent velocity field, $\lambda_{\text{MAX}}$. This is because $\lambda_{\text{MAX}}$ carries most of the turbulent energy, and thereby influences both the amount and spatial coherence of the angular momentum in the core. We show that the formation of dense filaments during collapse depends critically on $\lambda_{\text{MAX}}$, and we explain this finding using a force balance analysis. We also show that the core has only a high probability of fragmenting if $\lambda_{\text{MAX}} > R_{\text{CORE}}/2$ (where $R_{\text{CORE}}$ is the core radius), the dominant mode of fragmentation involves the formation and break-up of filaments and although small protostellar discs (with radius $R_{\text{DISC}} \lesssim 20 \text{ au}$) form routinely, more extended discs are rare. In turbulent, low-mass cores of the type we simulate here, the formation of large, fragmenting protostellar discs is suppressed by early fragmentation in the filaments.

Key words: hydrodynamics – turbulence – circumstellar matter – stars: formation – stars: low-mass.

1 INTRODUCTION

Although there is strong observational evidence that circumstellar discs are formed during the early (Class 0 and Class I) phases of protostellar evolution, estimates of their masses and extents are uncertain by up to a factor of 10 (Jørgensen et al. 2009). Since protostellar discs that are sufficiently massive and extended to fragment might be an important site for forming low-mass stars, brown dwarfs and planetary mass objects (e.g. Stamatellos et al. 2011), it is important to understand the circumstances under which such discs can form.

In this paper, we study the formation of protostellar discs in collapsing, weakly turbulent cores and evaluate the influence of the scale of turbulent perturbations and the net core angular momentum. In contrast to models based on rigid rotation (Walch et al. 2009; Machida, Inutsuka & Matsumoto 2010; Machida & Matsumoto 2011), it has never been demonstrated unambiguously that the net angular momentum in a turbulent core (Dib et al. 2010) significantly affects the size of the protostellar disc that it spawns. Rather, previous numerical models (e.g. Goodwin, Whitworth & Ward-Thompson 2004a; Walch et al. 2010) find no correlation between core angular momentum and the disc size, suggesting that the disc size is determined by details of the specific turbulent velocity field. However, previous work (Klessen, Heitsch & Mac Low 2000; Fisher 2004; Goodwin et al. 2004a; Goodwin, Whitworth & Ward-Thompson 2004b, 2006; Matzner & Levin 2005; Kratter & Matzner et al. 2009; Machida, Inutsuka & Matsumoto 2010; Machida & Matsumoto 2011), of length-scales, and not in the more restrictive sense of fully developed turbulence, in which energy is injected on large length-scales and cascades through a large inertial range of length-scales, before being dissipated on much smaller length-scales (e.g. Kritsuk et al. 2007). Here, turbulence is simply a device for seeding a star-forming cloud or core with the fluctuations that will eventually lead to fragmentation; the term is routinely used in this loose sense by those who simulate collapse and fragmentation.
2006; Krumholz, Klein & McKee 2007; Attwood et al. 2009; Walsh et al. 2010) has not investigated fully the parameter space used to initialize turbulent pre-stellar cores.

A random, Gaussian, turbulent velocity field is characterized – in a statistical sense – by five parameters. (i) Some measure of the total amount of turbulent energy, for example, the rms turbulent velocity, \( v_{\text{rms}} \), or the mean turbulent Mach number, \( \mathcal{M} \). Observations (Barranco & Goodman 1998; Goodman et al. 1998) suggest that turbulent velocities in low-mass cores are approximately sonic \((\mathcal{M} \sim 1)\). (ii) The partition of energy between solenoidal and compressive modes (e.g. Federrath, Klessen & Schmidt 2008), where the statistical equilibrium distribution is 2:1 solenoidal to compressive. (iii) The slope of the turbulent velocity power spectrum \( n \), where \( P_k \propto k^{-n} \), and \( n \) is typically between 3 and 4 (Burkert & Bodenheimer 2000, note that here \( n \) is defined so that – for example – the Kolmogorov scaling index corresponds to \( n = 11/3 \)). (iv) The wavelength of the largest turbulent perturbation, \( \lambda_{\text{MAX}} \). (v) The wavelength of the smallest turbulent perturbation \( \lambda_{\text{MIN}} \). Previous studies of low-mass core collapse – and of star cluster formation from the collapse of larger, more turbulent molecular cloud cores, e.g., Bate, Bonnell & Bromm (2003), Bonnell, Bate & Vine (2003), Bate (2009a, b) – have not explicitly specified the last two parameters \( \lambda_{\text{MAX}}, \lambda_{\text{MIN}} \).

In this paper, we evaluate their influence in more detail.

As long as (a) the power spectrum is sufficiently steep, \( n \gtrsim 3 \), and (b) the inertial range of the initial turbulent velocity field, \( \lambda_{\text{MAX}}/\lambda_{\text{MIN}} = \lambda_{\text{MAX}}/\lambda_{\text{MIN}} \), is sufficiently large, \( \lambda_{\text{MIN}} \) is unimportant, since very little turbulent power is invested in the shortest wavelengths and it is divided between many modes. However, \( \lambda_{\text{MAX}} \) has a major impact on core collapse, fragmentation and disc formation. For the low levels of turbulence typical of low-mass cores, dynamical filament fragmentation requires \( \lambda_{\text{MAX}} \gtrsim R_{\text{CORE}} \); fragmentation is very rare when \( \lambda_{\text{MAX}} \lesssim R_{\text{CORE}}/2 \). In addition, core angular momenta and the radii of protostellar discs both increase with increasing \( \lambda_{\text{MAX}} \). These differences arise because large-scale turbulence promotes the formation of large, coherent filaments. Such filaments not only fragment, but also deliver streams of material with disparate specific angular momenta into the centre of the core, where this material then forms large discs.

The structure of the paper is as follows. In Section 2, we describe the numerical method. We present the results of our simulations in Section 3, discuss the results in Section 4, compare with previous work in Section 5 and summarize the main conclusions in Section 6.

2 INITIAL CONDITIONS AND NUMERICAL METHOD

2.1 Turbulence

Turbulent cores have irregular internal velocity fields (Belloche, André & Motte 2001; André et al. 2007; Maruta et al. 2010), and these velocities result in a net angular momentum (Goldsmith & Arquilla 1985; Goodman et al. 1993; Dubinski, Narayan & Phillips 1995; Barranco & Goodman 1998; Caselli et al. 2002). Jijina, Myers & Adams (1999) estimate that a low-mass core typically has a ratio of turbulent-to-gravitational energy in the range \( 0 < \gamma_{\text{TURB}} \lesssim 0.5 \) and a mean specific angular momentum \( J_{\text{CORE}} \sim 10^{21} \text{ cm}^2\text{ s}^{-1} \). Burkert & Bodenheimer (2000) have shown that these features can be reproduced if the turbulence has a power spectrum of the form \( P_k \propto k^{-n} \) with \( n = 3 \) or 4, and this finding has been employed by several authors (e.g. Fisher 2004; Matzner & Levin 2005; Kratter &
value of Simpson (2010). Our particular choice of parameters results in the core being highly supercritical, with \( \alpha_{\text{THERM}} + \gamma_{\text{TURB}} = 0.027 \). Hence, the results may only pertain to such highly supercritical cores. We will explore the consequences of adopting larger values of \( \alpha_{\text{THERM}} \) and/or \( \gamma_{\text{TURB}} \) in a subsequent paper.

### 2.3 Gravity and hydrodynamics

We use the {	extsc{seren}} SPH code (Hubber et al. 2011), which is parallelized using {	extsc{openmp}} and designed for star formation simulations. It has been extensively tested and applied to a wide range of problems (e.g. Bisbas et al. 2009, 2011; Stamatellos & Whitworth 2010; Stamatellos et al. 2011; Walch et al. 2011). It includes both the traditional SPH formulation (Monaghan 1992) and the more recent \textit{grand-h} SPH formulation (Price & Monaghan 2004), which we use in this paper. To solve the SPH equations, we employ the symplectic second-order Leapfrog-KDK integrator, in conjunction with a block time-stepping scheme. We invoke additional features within the basic SPH algorithm such as the Balsara viscosity switch (Balsara 1995) to reduce artificial shear viscosity. We use a Barnes–Hut octal-spatial tree (Barnes & Hut 1986) with the \textsc{gadget}-style multipole acceptance criterion (Springel, Yoshida & White 2001). Each SPH particle has a mass of \( m_{\text{SPH}} = 10^3 \, M_{\odot} \), resulting in a mass resolution of \( 10^{-3} \, M_{\odot} \) (a Jupiter mass). For random seed 500, we demonstrate convergence by re-simulating the fiducial setup using 10 times as many SPH particles, i.e. with \( m_{\text{SPH}} = 10^{-2} \, M_{\odot} \). These runs are referred to as the ‘\_hr’ simulations. Gravitationally bound condensations that form in the core are replaced with sinks, resulting in a mass resolution of \( 10^{-3} \, M_{\odot} \) (\( \rho_{\text{sink}} = 10^{-9} \, g \, cm^{-3} \)) and, subsequently grow by accretion, using a new algorithm (Walch et al. 2011) that (a) ensures excellent numerical convergence and (b) broadcasts the angular momentum of the accreted material to the surrounding gas (rather than assimilating it, which would be non-physical).

### 2.4 Energy equation and radiative transfer

We use the radiative diffusion approximation of Stamatellos et al. (2007) (RAD-WS method) to solve the energy equation and evaluate radiative transfer effects. The RAD-WS method uses the density, \( \rho_i \), temperature, \( T_i \), and gravitational potential, \( \psi_i \), of an SPH particle \( i \) to estimate a mean column density, \( \Sigma_i = \Sigma(\rho_i, \psi_i) \), and a mean optical depth, \( \tau_i = \tau(\rho_i, T_i, \psi_i) \), through which the particle cools and heats. This optical depth includes contributions from dust, lines and free–free processes, and accounts for the variation in the opacity in the cooler, less dense material that is presumed to surround particle \( i \). The net radiative heating rate for the particle \( i \) is then

\[
\frac{d\epsilon_i}{dt}_{\text{RAD}} = \frac{4 \sigma_{SB} (T_i^4 - T^4)}{\Sigma_i (\tau_i + \tau_i^{-1})}.
\]

The positive term on the right-hand side represents heating by the background radiation field, and ensures that the gas and dust cannot cool radiatively below the background radiation temperature \( T_0 \), which we set to \( T_0 = 7 \, K \). The energy equation then takes account of compressional heating, viscous heating, radiative heating by the background and radiative cooling. It has been extensively tested against detailed numerical (Boss & Bodenheimer 1979; Boss & Myhill 1992; Masunaga & Inutsuka 2000; Whitehouse & Bate 2006) and analytical results (Spiegel 1957; Hubeny 1990), and performs well in both the optically thin and optically thick regimes.

### 3 RESULTS

#### 3.1 Core angular momenta

As \( k_{\text{MIN}} \) increases, the coherence lengths of the most energetic turbulent modes (\( \lesssim \lambda_{\text{MAX}}/2 \approx R_{\text{CORE}}/k_{\text{MIN}} \)) decrease. Since these modes are uncorrelated, the specific angular momentum,

\[
j_{\text{CORE}} = \frac{1}{N_{\text{SPH}}} \sum_{i=1}^{N_{\text{SPH}}} |r_i \times v_i|,
\]

which is compounded by contributions from all the different modes, also decreases in magnitude. Fig. 1 shows the variation of \( j_{\text{CORE}} \) with \( k_{\text{MIN}} \) at \( \tau = 0 \). For each value of \( k_{\text{MIN}} \) we simulate five different realizations by invoking five different seeds, and this produces an \( \sim 0.5 \, \text{dex} \) spread in \( j_{\text{CORE}} \). However, there is an underlying systematic variation that, in the interval \( 1 \lesssim k_{\text{MIN}} \lesssim 4 \), can be approximated by

\[
j_{\text{CORE}} \sim 4 \times 10^{19} \, \text{cm}^2 \, \text{s}^{-1} \, k^{-2}_{\text{MIN}}.
\]

At smaller \( k_{\text{MIN}} \), \( j_{\text{CORE}} \) approaches the maximum value consistent with the amount of turbulent energy (see below). In addition, much of the turbulent energy is invested in modes of such long wavelength that it results in bulk motion of the core, rather than intrinsic spin. The overall range of values is \( 10^{17.5} \, \text{cm}^2 \, \text{s}^{-1} \lesssim j_{\text{CORE}} \lesssim 10^{20.2} \, \text{cm}^2 \, \text{s}^{-1} \).

We note that for a critical Bonnor–Ebert sphere, the ratio of rotational-to-gravitational energy is given by

\[
\beta_{\text{ROT}} = (j_{\text{CORE}}/j_{\text{MIN}})^2,
\]

where \( j_{\text{MIN}} = 0.644 (G M_{\text{CORE}} R_{\text{CORE}})^{1/2} \). For A-MM8, \( j_{\text{MIN}} = 1.6 \times 10^{21} \, \text{cm}^2 \, \text{s}^{-1} \), and hence

\[
\beta_{\text{ROT}} = \left( \frac{j_{\text{CORE}}}{1.6 \times 10^{21} \, \text{cm}^2 \, \text{s}^{-1}} \right)^2.
\]

Since \( \beta_{\text{ROT}} \lesssim \gamma_{\text{TURB}} = 0.01 \) (the rotational energy cannot exceed the turbulent energy), there is a maximum specific angular momentum \( \lambda_{\text{MAX}} = \gamma_{\text{TURB}} j_{\text{MIN}} = 1.6 \times 10^{20} \, \text{cm}^2 \, \text{s}^{-1} \). Evidently, this rather small \( \lambda_{\text{MAX}} \) is a consequence of small \( R_{\text{CORE}} \) (observational estimates of \( j \) tend to derive from more extended cores) and small \( \gamma_{\text{TURB}} \).

![Figure 1](https://academic.oup.com/mnras/article-abstract/419/1/760/1006807)  

**Figure 1.** Magnitude of the specific core angular momentum \( j_{\text{CORE}} \) as a function of \( k_{\text{MIN}} \) (lower abscissa) or equivalently \( \lambda_{\text{MAX}}/R_{\text{CORE}} \) (upper abscissa). Results obtained with the same seed but different \( k_{\text{MIN}} \) are connected with dashed lines. The thick black line is the best fit to the data in the interval \( 1 \lesssim k_{\text{MIN}} \lesssim 4 \) (see equation 4).
Figure 2. Montage of false-colour images of the density on the mid-planes of the protostellar discs formed in the entire ensemble of simulations of A-MM8. Each column corresponds to a different value of $k_{\text{MIN}}$ (from left to right: $1/2$, $1$, $2$ and $4$), and hence to a different $\lambda_{\text{MAX}} = 2R_{\text{CORE}}/k_{\text{MIN}}$ (from left to right: $4R_{\text{CORE}}, 2R_{\text{CORE}}, R_{\text{CORE}}$ and $R_{\text{CORE}}/2$). Each row corresponds to a different seed for generating the initial turbulent velocity field. The false-colour encodes the same range of density on all plots: $10^{-15}$–$10^{-11}$ g cm$^{-3}$. However, the linear sizes of the frames are different for different seeds, varying from 100 to 400 au. Black dots mark the positions of sink particles.
### 3.2 Disc-density distributions

At $t_{50}$, the time at which 50 per cent of the initial core mass has been converted into protostars, we define a Cartesian frame of reference, $(x_{IF}, y_{IF}, z_{IF})$, in which the $z_{IF}$-axis is aligned with the largest principal moment of inertia of the remaining dense ($\rho > 10^{-12}$ g cm$^{-3}$) material. The algorithm for doing this is described in Appendix A. Provided there is a single dominant primary disc, $z_{IF}$ is then aligned with its rotation axis. Fig. 2 displays a montage of false-colour images of the density on the $z_{IF} = 0$ plane for the entire ensemble of simulations of A-MM8. Each column of images corresponds to a different value of $k_{MIN}$ and each row to a different seed. From these plots, we see that (a) the sizes of discs tend to decrease with increasing $k_{MIN}$ and (b) that for $k_{MIN} = 1$ and 2, multiple protostars usually form, whereas for $k_{MIN} = 1/2$ and 4, only single stars are formed.

### 3.3 Resolution study

We re-simulate three setups for seed 500 ($k_{MIN} = 1/2, 1$ and 4) with ten times higher resolution (run 500_hr), i.e. a total of 1 280 000 particles and $m_{SPH} = 10^{-6}$. The results of these simulations are shown in Fig. 3. With regard to the density profiles and the disc radii, we find remarkably good convergence between low- and high-resolution runs (see also Table 1). The disc masses are also in reasonable agreement, considering the fact that the end times are slightly different. For $k_{MIN} = 1/2$, we could not follow the simulation until 50 per cent of the core mass collapsed into the sink because of CPU time limitations. Therefore, the runs are mistimed and the disc masses cannot be strictly compared in this case. For $k_{MIN} = 1$, the sink masses grow a bit quicker in the high-resolution run ($t_{50} = 14.2$ kyr instead of $t_{50} = 15.2$ kyr) leading to a smaller disc mass of $0.14 \, M_\odot$ rather than $0.31 \, M_\odot$. This difference may be caused by the different fragmentation properties as in run 500_hr only two instead of three sink particles form. Despite the different number of sink particles, the disc-density distributions and the ‘system masses’ within the two identifiable, individual condensations (see Figs 2 and 3) are very similar, i.e. $M_\star = 0.323$ and 0.318 for 500_hr and $M_\star = 0.36$ and (0.27 + 0.01) = 0.28 for 500. For $k_{MIN} = 4$, we find good agreement of all interesting quantities. Overall, the results of our study are only weakly dependent on resolution.

### 3.4 Global disc properties and scaling relations

| $k_{MIN}$ | SEED | $t_{50}$ (kyr) | $M_{DISC}$ ($M_\odot$) | $R_{DISC}$ (AU) | $M_\star$ ($M_\odot$) |
|-----------|------|----------------|------------------------|-----------------|-------------------|
| 1/2 | 200 | 16.2 | 0.39 | 30 | 0.64 |
| 300 | 14.7 | 0.26 | 17 | 0.64 |
| 400 | 20.2 | 0.57 | 94 | 0.64 |
| 500 | 20.3 | 0.53 | 47 | 0.64 |
| 500_hr | 16.0 | 0.59 | 45 | 0.41 |
| 600 | 26.0 | 0.59 | 100 | 0.64 |
| 1 | 200 | 16.2 | 0.27 | 16 | 0.628/0.012 |
| 300 | 15.3 | 0.30 | 16 | 0.64 |
| 400 | 15.9 | 0.37 | 40 | 0.430/0.210 |
| 500 | 15.2 | 0.31 | 30 | 0.360/0.270/0.010 |
| 500_hr | 14.2 | 0.14 | 30 | 0.323/0.318 |
| 600 | 15.3 | 0.26 | 25 | 0.310/0.190/0.140 |
| 2 | 200 | 14.3 | 0.10 | 15 | 0.588/0.052 |
| 300 | 14.6 | 0.15 | 8 | 0.320/0.320 |
| 400 | 14.1 | 0.14 | 17 | 0.330/0.163/0.147 |
| 500 | 14.2 | 0.19 | 15 | 0.616/0.024 |
| 600 | 14.6 | 0.25 | 20 | 0.250/0.207/0.183 |
| 4 | 200 | 13.9 | 0.05 | 8 | 0.64 |
| 300 | 13.8 | 0.10 | 6 | 0.64 |
| 400 | 13.9 | 0.12 | 9 | 0.64 |
| 500 | 13.8 | 0.10 | 9 | 0.64 |
| 500_hr | 13.7 | 0.16 | 9 | 0.64 |
| 600 | 13.8 | 0.11 | 6 | 0.64 |

There are many types of disc: circumstellar, circum-binary and circum-system (i.e. enclosing higher order systems). Here, we focus our discussion on the primary accretion disc, which is the most massive and extended disc in the simulation (at $t_{50}$), and always surrounds the protostar with the highest final mass – although this is not always the first protostar to form. The typical primary disc is rather...
Influence of the turbulent scale on disc formation

In general, more extended discs are more massive, and the most extended discs display signs of gravitational instability. However, this only results in the formation of spiral arms and the transport of angular momentum; there are no protostars formed by disc fragmentation. Table 1 lists for each simulation, the value of $k_{\text{MIN}}$, the seed used to generate the turbulent velocity field, the time to convert half the core mass into stars, $t_{50}$, the mass and radius of the primary disc at this time, $M_{\text{DISC}}$ and $R_{\text{DISC}}$, and the masses of the protostars formed. Fig. 4 shows the dependences of $R_{\text{DISC}}$ and $M_{\text{DISC}}$ on $k_{\text{MIN}}$ and $j_{\text{CORE}}$; and Fig. 5 shows the correlation between $M_{\text{DISC}}$ and $R_{\text{DISC}}$. The linear fits on these figures, and their uncertainties, obtained by $\chi^2$ minimization, are

\begin{align}
R_{\text{DISC}} &\simeq 26(\pm 2) \text{ au} \ k_{\text{MIN}}^{-0.86(\pm 0.13)}, \\
R_{\text{DISC}} &\simeq 16(\pm 2) \text{ au} \ \left(\frac{j_{\text{CORE}}}{10^{19} \text{ cm}^2 \text{s}^{-1}}\right)^{0.40(\pm 0.06)}, \\
M_{\text{DISC}} &\simeq 0.28(\pm 0.02) \, M_\odot \ k_{\text{MIN}}^{-0.73(\pm 0.09)}, \\
M_{\text{DISC}} &\simeq 0.19(\pm 0.01) \, M_\odot \ \left(\frac{j_{\text{CORE}}}{10^{19} \text{ cm}^2 \text{s}^{-1}}\right)^{0.36(\pm 0.03)}, \\
M_{\text{DISC}} &\simeq 0.30(\pm 0.03) \, M_\odot \ \left(\frac{R_{\text{DISC}}}{30 \text{ au}}\right)^{0.74(\pm 0.09)}.
\end{align}

4 DISCUSSION

4.1 Filament formation

Large-scale filaments play a critical role in core fragmentation and in the formation of the primary disc. First, large-scale filaments

Figure 4. Disc radius, $R_{\text{DISC}}$ and the disc mass, $M_{\text{DISC}}$, as a function of the minimum turbulent wavenumber, $k_{\text{MIN}}$, and specific angular momentum $j_{\text{CORE}}$.

Figure 5. The correlation of the disc mass, $M_{\text{DISC}}$, with the disc radius, $R_{\text{DISC}}$. 

© 2011 The Authors, MNRAS 419, 760–770
Monthly Notices of the Royal Astronomical Society © 2011 RAS
Figure 6. Left-hand column: the minimum value of the ratio of centrifugal-to-gravitational potential force, $F_C/F_G$, found along the line of sight, for the initial conditions of the simulations with seed 200 and different $k_{\text{MIN}}$. For $k_{\text{MIN}} = 1/2$ we find coherent features, whereas the distribution of small $F_C/F_G$ becomes increasingly random with increasing $k_{\text{MIN}}$. Right-hand column: the maximum density along the line of sight after 15 kyr of evolution for the simulations with seed 200 and different $k_{\text{MIN}}$. Filaments have formed where $F_C/F_G$ was initially small.
provide alternative sites (alternative to the centre of the mass of the core) where material can converge and form secondary protostars. Secondly, large-scale filaments deliver large parcels of material with disparate angular momenta into the centre, where they accumulate in a large disc around the primary protostar. Fig. 6 demonstrates that large-scale filaments are generated by turbulence with small $k_{\text{MIN}}$

The right-hand column of Fig. 6 shows false-colour images of the maximum density on lines of sight parallel to the $z_{\text{sp}}$ axis, at time $t = 15$ kyr, for different values of $k_{\text{MIN}}$. All the simulations presented in Fig. 6 derive from the same seed, but the results obtained with other seeds are statistically similar. We see that for $k_{\text{MIN}} = 1/2$ the high-density gas ($n \gtrsim 10^6$ cm$^{-3}$) is concentrated in large-scale filaments. However, as $k_{\text{MIN}}$ is increased, the strength and coherence of the filaments decline, and by $k_{\text{MIN}} = 4$ there are no notable filaments. Filament formation can be understood in terms of the forces shaping the core. We neglect the pressure force, $F_p$, since $F_p$ is initially smooth and focuses on the ratio of centrifugal-to-gravitational force $F_c/F_G$. For each SPH particle, $i$, we compute $F_c^i = |r_i \times v_i|^2/|r_i|^2$ and $F_G^i = GM(|r_i|)/|r_i|^2$, where $r_i$ and $v_i$ are the position and velocity of particle $i$ relative to the centre of mass, and $M(|r_i|)$ is the mass interior to radius $|r_i|$. Parcels of gas with low centrifugal support, i.e. small $F_c/F_G$, collapse first, and neighbouring parcels are then drawn into the regions they vacate, creating preferred accretion streams, i.e. filaments. The left-hand column of Fig. 6 shows false-colour images of the minimum value of $F_G/F_c^i$ found on each line of sight. For $k_{\text{MIN}} = 1/2$, there are well-defined structures with low $F_c/F_G$ that can be related to the filaments illustrated in the corresponding right-hand image. However, as $k_{\text{MIN}}$ is increased, structures with low $F_c/F_G$ become increasingly small and incoherent.

A complex of filamentary structures on scales of 1000 au, very similar to our case with small $k_{\text{MIN}}$, has recently been observed in the envelopes of Class 0 cores by Tobin et al. (2010) using Spitzer. Tobin et al. (2010) note that this complex envelope structure is spatially distinct from possible outflow cavities and explicitly suggest that it results from the collapse of pre-stellar cores with initial non-equilibrium structures.

### 4.2 Protostellar multiplicity

From Table 1, it appears that cores with $k_{\text{MIN}} = 1/2$ and 4 only spawn single stars, whereas cores with $k_{\text{MIN}} = 1$ and 2 tend to spawn two or three stars. In other words, multiple systems are formed only if $\lambda_{\text{MAX}}/R_{\text{CORE}} \sim 1–2$.

This is illustrated in panel A of Fig. 7, where we plot the mean number of stars formed, and its variance, against $k_{\text{MIN}}$. There are no instances of disc fragmentation. In all cases, where secondary stars form, they form by dynamical filament fragmentation. Even where two (or three) stars end up in a close binary (triple) system with a circum-binary (circum-system) disc, the components have always formed by filament fragmentation, with their own independent circumstellar discs, and then fallen into the centre and captured one another. This result was already suggested by the results of Walch et al. (2010), but the current study places it on a quantitative footing.

Panel B of Fig. 7 shows that by $t_{50}$ there is a wide range of disc sizes around single stars, but the discs around stars in multiple systems have been truncated by mutual tidal interactions. When $k_{\text{MIN}} = 1/2$, the filaments do not fragment because they fall into the centre very rapidly and consequently the material in them is stretched. On arrival at the centre, much of this material is initially parked in a massive extended disc around the primary protostar. However, despite being massive and extended, this disc does not fragment. Repeated perturbations due to irregular infall from the filaments (a) maintain a relatively high-velocity dispersion in the disc (and hence a high Toomre $Q$ parameter), (b) excite density waves that transport angular momentum by gravitational torques, thereby facilitating accretion on to the central primary star, and (c) shear proto-condensations apart. In a similar vein, Hayfield et al. (2011) have recently shown that discs in binary systems tend to be more stable towards fragmentation than discs around single stars.

When $k_{\text{MIN}} = 4$, there are no significant filaments, so dynamical fragmentation is suppressed. The discs that form around the primary protostar are too small to fragment.

We stress that these results are for a specific low-mass, low-turbulence core. The critical value of $k_{\text{MIN}}$, above which fragmentation is inhibited, probably increases with increasing core mass, since a small, low-contrast filament is more likely to fragment if it is more massive. Also, we might expect more fragmentation for increased levels of turbulence.
5 Comparison with Previous Work

There have been several other studies of the collapse and fragmentation of low-mass, turbulent cores.

Goodwin et al. (2004a, b) simulate the collapse and fragmentation of cores having mass $M_{\text{CORE}} = 5.4 \, M_\odot$ and radius $R_{\text{CORE}} = 50,000$ au, modelled with a Plummer-like density profile. These are SPH simulations, using a barotropic equation of state and sink particles. Different levels of turbulence are considered, $0.01 \leq \gamma_{\text{TURB}} \leq 0.25$, with $P_{\text{i}} \propto k^{-4}$, $k_{\text{MIN}} = 1$ and purely solenoidal modes; for each case many realizations are performed. In contrast to the simulations presented here (which result in fragmentation for $\gamma_{\text{TURB}} = 0.01$, provided $1 \lesssim k_{\text{MIN}} \lesssim 2$), Goodwin et al. (2004a, b) obtain fragmentation only when $\gamma_{\text{TURB}} \gtrsim 0.05$. This is due to the fact that their cores have a much higher level of thermal support than ours, $0.30 \lesssim \alpha_{\text{THERM}} \lesssim 0.45$, so the gas is less readily compressed, and to the fact that solenoidal modes produce less compression than the thermal mix of solenoidal and compressive modes that we use.

In a related study Goodwin et al. (2006) extend the study with $\gamma_{\text{TURB}} = 0.10$ to different turbulent power spectra, $P_{\text{i}} \propto k^{-n}$ with $n = 3, 4$ and 5. They find that with higher $n$ (i.e. a higher concentration of power at long wavelengths), there is more fragmentation and the protostars formed have somewhat lower masses.

These simulations have been repeated by Attwood et al. (2009), with the same initial conditions, but solving the energy equation and treating the associated transport of cooling radiation, instead of using a barotropic equation of state. The main differences in the results are that (i) fragmentation is more efficient (larger numbers of protostars are formed) and (ii) the binary systems have shorter periods, higher eccentricities and smaller mass ratios.

Walch et al. (2010) use SPH to simulate the collapse and fragmentation of cores having mass $M_{\text{CORE}} = 6.1 \, M_\odot$ and radius $R_{\text{CORE}} = 17,000$ au, modelled as marginally supercritical Bonnor-Ebert spheres ($\xi_B = 6.9$, density increased by 10 per cent). The cores have an initial ratio of thermal-to-gravitational energy $\alpha_{\text{THERM}} = 0.74$ and are contained by an external pressure $P_{\text{EXT}} = 9 \times 10^{-11}$ erg cm$^{-3}$. The turbulent velocity field is characterised by a mean Mach number $M = 1$ (i.e. trans-sonic turbulence), a power spectrum $P_{\text{i}} \propto k^{-4}$, $k_{\text{MIN}} = 1$ and a thermal mix of solenoidal and compressive modes. A large ensemble of cores is generated, and from these a representative subset, having specific angular momentum spanning the range $0.1 \lesssim (J_{\text{CORE}}/10^{31} \, \text{cm}^2 \, \text{s}^{-1}) \lesssim 2.7$, is extracted and evolved. The energy equation is solved using the molecular-line cooling rates of Neufeld, Lepp & Melnick (1995). As in the simulations presented here, Walch et al. (2010) find that dynamical filament fragmentation dominates over disc fragmentation. However, the gas in their simulations is much hotter (because dust cooling is not included), so the discs that form are more swollen. In addition, since they do not use sink particles, they are unable to follow the simulations to the point where multiple protostars with circumstellar, binary-circular and circumbinary-disc systems are formed.

Offner, Klein & McKee (2008) simulate the collapse and fragmentation of turbulent cores, using adaptive mesh refinement (AMR) and a barotropic equation of state. Their cores are produced in a large-scale simulation of a collapsing molecular cloud, with either driven or decaying turbulence. Individual cores are then followed at higher resolution. Offner et al. (2008) find that simulations with decaying turbulence form on average more low-mass protostars than simulations with driven turbulence. Offner et al. (2009) simulate fragmentation in a turbulent box, using AMR and solving the energy equation. They show that radiative feedback from the forming protostars inhibits disc fragmentation, thereby reducing the number of low-mass multiple systems formed (see also Bate 2009c). Further analysis of these results (Offner et al. 2010) suggests that dynamical filament fragmentation is the dominant mechanism forming low-mass stars and binary systems, rather than disc fragmentation. The material in filaments is sufficiently far from protostellar radiation sources to keep cool and fragment, whereas the material in discs around newly formed protostars is close and gets heated up so that it does not fragment.

However, Stamatellos, Whitworth & Hubber (2011) show that, if accretion on to a protostar is episodic (as is believed to be the case), the luminosity is also episodic, and the duty cycle has sufficiently long low-luminosity periods for the outer parts of a massive accretion disc to cool down and fragment. Disc fragmentation may therefore still be a viable mechanism for forming brown dwarfs.

6 Conclusions

We have performed an ensemble of SPH self-gravitating radiation-hydrodynamic simulations to demonstrate that – in low-mass turbulent cores – the largest wavelength in the turbulent spectrum has a critical bearing on the outcome of collapse and fragmentation. Specifically, if all other parameters (the initial critical Bonnor-Ebert density profile, $\alpha_{\text{THERM}} = 0.017$, $\gamma_{\text{TURB}} = 0.010$, $n = -\text{dln} \left( P_{\text{i}} \right)/\text{dln} \left( k \right) = 4$) are held fixed and $\lambda_{\text{MAX}}$ is varied,

(i) the mean specific angular momentum of the core increases approximately as the square of the the largest wavelength, $J_{\text{CORE}} \sim \lambda_{\text{MAX}}^2$ for $1/2 \lesssim \lambda_{\text{MAX}}/R_{\text{CORE}} \lesssim 2$;

(ii) filaments form in the regions where the centrifugal support is weakest and therefore the material collapses fastest;

(iii) the size and coherence of filaments therefore increase with increasing $\lambda_{\text{MAX}}$;

(iv) dynamical filament fragmentation (cf. Offner et al. 2010) is the dominant (only) fragmentation mechanism;

(v) and hence fragmentation and multiple star formation only occur for $1/2 \lesssim \lambda_{\text{MAX}}/R_{\text{CORE}} \lesssim 2$;

(vi) the primary (i.e. most massive) protostellar disc has mass and radius which scale approximately as $M_{\text{DISC}} \sim \lambda_{\text{MAX}}^{1/4}$ and $R_{\text{DISC}} \sim \lambda_{\text{MAX}}^{1/2}$;

(vii) massive extended discs form (for large $\lambda_{\text{MAX}}$) where the filamentary inflows deliver material with disparate specific angular momentum;

(viii) but these discs do not fragment because the inflowing material maintains a large velocity dispersion and therefore the gravitational modes excited in the disc are only strong enough to redistribute angular momentum and facilitate accretion on to the central protostar.

The global parameters of a core do not completely specify the initial conditions for a simulation. In particular, the initial turbulent velocity field is stochastic. Consequently, there is considerable variance amongst different realizations of the same parameter set and the conclusions listed above should be interpreted as statistical.

Acknowledgments

We thank the anonymous referee for a thorough and constructive report which helped us to improve the original version of this paper.
We acknowledge the support of the Marie Curie RTN ‘CONSTELATION’ (MRTN-CT-2006-035890). APW further acknowledges the support of Grant ST/H001530/1 from the UK Science and Technology Facilities Council. The simulations have been carried out on the ARCCA SRIF-3 cluster MERLIN in Cardiff.

REFERENCES

André P., Belloche A., Motte F., Peretto N., 2007, A&A, 472, 519
Attwood R. E., Goodwin S. P., Stamatellos D., Whitworth A. P., 2009, A&A, 495, 201
Balsara D. S., 1995, J. Computational Phys., 121, 357
Barnes J., Hut P., 1986, Nat, 324, 446
Barranco J. A., Goodman A. A., 1998, ApJ, 504, 207
Bate M. R., 2009a, MNRAS, 392, 590
Bate M. R., 2009b, MNRAS, 397, 232
Bate M. R., 2009c, MNRAS, 392, 1363
Bate M. R., Bonnell I. A., Bromm V., 2003, MNRAS, 339, 577
Belloche A., André P., Motte F., 2001, in Montmerle T., André P., eds, ASP Conf. Ser. Vol. 243, From Darkness to Light: Origin and Evolution of Young Stellar Clusters. Astron. Soc. Pac., San Francisco, p. 313
Bisbas T. G., Wünsch R., Whitworth A. P., Hubber D. A., 2009, A&A, 497, 649
Bisbas T. G., Wünsch R., Whitworth A. P., Hubber D. A., Walch S., 2011, ApJ, 736, 142
Bonnell I. A., Bate M. R., Vine S. G., 2003, MNRAS, 343, 413
Boss A. P., Bodenheimer P., 1979, ApJ, 234, 289
Bosch A. P., Myhill E. A., 1992, ApJS, 83, 311
Burkert A., Bodenheimer P., 2000, ApJ, 543, 822
Caselli P., Benson P. J., Myers P. C., Tafalla M., 2002, ApJ, 572, 238
Dib S., Hennebelle P., Fineda J. E., Csengeri T., Bontemps S., Audit E., Goodman A. A., 2010, ApJ, 723, 425
Dubinski J., Narayan R., Phillips T. G., 1995, ApJ, 448, 226
Federrath C., Klessen R. S., Schmidt W., 2008, ApJ, 688, L79
Fisher R. T., 2004, ApJ, 600, 769
Goldsmith P. F., Arquilla R., 1985, in Black D. C., Matthews M. S., eds, Protostars and Planets II: Rotation in Dark Clouds. Univ. Arizona, Tucson, p. 137
Goodman A. A., Benson P. J., Fuller G. A., Myers P. C., 1993, ApJ, 406, 528
Goodman A. A., Barranco J. A., Wilner D. J., Heyer M. H., 1998, ApJ, 504, 223
Goodwin S. P., Whitworth A. P., Ward-Thompson D., 2004a, A&A, 414, 633
Goodwin S. P., Whitworth A. P., Ward-Thompson D., 2004b, A&A, 423, 169
Goodwin S. P., Whitworth A. P., Ward-Thompson D., 2006, A&A, 452, 487
Hayfield T., Mayor L., Wisdom J., Boley A. C., 2011, MNRAS, doi:10.1111/j.1365-2966.2011.19371.x
Hubber D. A., Bally C. P., McLeod A., Whitworth A. P., 2011, A&A, 529, A27
Hubeny I., 1990, ApJ, 351, 632
Jijina J., Myers P. C., Adams F. C., 1999, ApJS, 125, 161
Jørgensen J. K., van Dishoeck E. F., Visser R., Bourke T. L., Wilner D. J., Lommen D., Hogerheijde M. R., Myers P. C., 2009, A&A, 507, 861
Klessen R. S., Heitsch F., Mac Low M., 2000, ApJ, 535, 887
Kratter K. M., Matzner C. D., 2006, MNRAS, 373, 1563
Kratter K. M., Matzner C. D., Krumholz M. R., 2008, ApJ, 681, 375
Kritsuk A. G., Norman M. L., Padoan P., Wagner R., 2007, ApJ, 665, 416
Krumholz M. R., Klein R. I., McKee C. F., 2007, ApJ, 656, 959
Machida M. N., Matsumoto T., 2011, MNRAS, 413, 2767
Machida M. N., Inutsuka S., Matsumoto T., 2010, ApJ, 724, 1006
Maruta H., Nakamura F., Nishi R., Ikeda N., Kitamura Y., 2010, ApJ, 714, 680
Masunaga H., Inutsuka S., 2000, ApJ, 531, 350
Matzner C. D., Levin Y., 2005, ApJ, 628, 817

© 2011 The Authors, MNRAS 419, 760–770
Monthly Notices of the Royal Astronomical Society © 2011 RAS

APPENDIX A: DISC DEFINITION

In order to objectively identify discs and evaluate their masses and radii, we apply the following procedure. First, we isolate all the material with density $\rho > 10^{-12}$ g cm$^{-3}$. Secondly, we compute the moment of inertia tensor for this material, and thereby define a new local co-ordinate system using the principal axes of inertia, $x_P$, $y_P$, $z_P$ (where the subscript ‘$P$’ stands for inertial frame). The properties of the dense material are now analysed relative to this new coordinate system. In particular, $z_P$ is allocated to the largest principal moment of inertia, and therefore if the material is in a disc, $z_P$ is its rotation axis.

In order to ascertain whether there is a disc, we compute the logarithmic density profile along each of the axes of inertia and smooth these profiles using a box-car-averaging technique. For a
disc, the profiles along $x_F$ and $y_F$ are very similar to one another, and the third, along $z_F$ is significantly steeper and less extended.

Because discs are embedded in, and grow from, filaments, we locate the edge of the disc at the first point along the $x_F$- and $y_F$-axes where the density is below $\rho_{\text{THRESH}} = 10^{-14} \text{ g cm}^{-3}$ and the second derivative of the logarithmic density profile is zero, $d^2 \log_{10} \rho / d (\log_{10} r)^2 = 0$. Fig. A1 shows the profiles along the principal axes for the disc formed in simulation with $k_{\text{MIN}} = 1/2$ and seed 400. This is the disc illustrated in the left-hand column of the middle row of Fig. 2. All the discs in Fig. 2 have been identified in this way and are viewed face-on down $z_F$ in their local inertial frame.

The dotted vertical lines in Fig. A1 mark the radii at which the density first falls below $\rho_{\text{THRESH}}$, at 88 au on $x_F$ and at 95 au on $y_F$, respectively. We identify the edge of the disc where the second derivative of the logarithmic density next falls to zero. This gives a mean radius of $R_{\text{DISC}} = 94$ au, in this case. The density typically drops very steeply inside this radius, and therefore the resulting estimate of the disc mass ($M_{\text{DISC}} = 0.57 \text{ M}_\odot$ in this case) is robust.

This paper has been typeset from a \TeX/\LaTeX file prepared by the author.