TESTING MODIFIED GRAVITY WITH GLOBULAR CLUSTER VELOCITY DISPERSIONS

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ABSTRACT

Globular clusters (GCs) in the Milky Way have characteristic velocity dispersions that are consistent with the predictions of Newtonian gravity, and may be at odds with Modified Newtonian Dynamics (MOND). We discuss a modified gravity (MOG) theory that successfully predicts galaxy rotation curves, galaxy cluster masses and velocity dispersions, lensing, and cosmological observations, yet produces predictions consistent with Newtonian theory for smaller systems, such as GCs. MOG produces velocity dispersion predictions for GCs that are independent of the distance from the Galactic center, which may not be the case for MOND. New observations of distant GCs may produce strong criteria that can be used to distinguish between competing gravitational theories.

Subject headings: dark matter — globular clusters: general — gravitation

Online material: color figures

1. INTRODUCTION

Modified Gravity (MOG; Moffat 2005, 2006a, 2006b; Moffat & Toth 2007c) is a fully covariant theory of gravity that is based on postulating the existence of a massive vector field, \( \phi_v \). The choice of a massive vector field is motivated by our desire to introduce a repulsive modification of the law of gravitation at short range. The vector field is coupled universally to matter. The theory yields a Yukawa-like modification of gravity with three constants: \( \mu \) and \( \omega \), which determine the coupling strength between the \( \phi_v \) field and matter, and a further constant \( \mu_0 \) that arises as a result of considering a vector field of nonzero mass, and controls the coupling range. In the most general case, these constants must be allowed to run. An approximate solution of the MOG field equations (Moffat & Toth 2007c) allows us to compute the values of \( \mu \) and \( \omega \) as functions of the source mass.

MOG has been used successfully to describe observational phenomena on astrophysical and cosmological scales without resorting to dark matter. The theory correctly predicts galaxy rotation curves (Brownstein & Moffat 2006a; Moffat & Toth 2007c), the mass and thermal profiles of clusters of galaxies (Brownstein & Moffat 2006b; Moffat & Toth 2007c), the merging of the two clusters (Bullet Cluster; Brownstein & Moffat 2006a, 2006b; Moffat & Toth 2007c), the acoustical peaks of the cosmic microwave background (Moffat 2005, 2006b; Moffat & Toth 2007c), in the form

\[
\frac{G_{N} M}{r^{2}} \{ 1 + \alpha [ 1 - (1 + \mu r) e^{-\mu r} ] \},
\]

where \( G_N \) is Newton’s gravitational constant.

In accordance with our recent results (Moffat & Toth 2007c), the parameters \( \alpha \) and \( \mu \) can now be predicted:

\[
\alpha = \frac{M}{\sqrt{M + C_1^2}} \left( \frac{G_{\infty}}{G_N} - 1 \right),
\]

\[
\mu = \frac{C_1^2}{\sqrt{M}}.
\]

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\[ G_\infty \simeq 20G_N, \]
\[ C'_1 \simeq 25,000 \, M_\odot^{1/2}, \]
\[ C'_2 \simeq 6250 \, M_\odot^{1/2} \, \text{kpc}^{-1}. \]

For even a large GC, with mass exceeding \(10^6 \, M_\odot\), the predicted values are
\[ \alpha \simeq 0.03, \]
\[ \mu \simeq (160 \, \text{pc})^{-1}. \]

Given the smallness of \(\alpha\) and the fact that \(\mu^{-1}\) is much larger than the GC radius, it is clear that our theory predicts Newtonian behavior for such GCs:
\[ a_{\text{MOND}} \simeq a_{\text{Newton}} = -\frac{G_NM}{r^2}. \]

For smaller GCs, the predictions are even closer to Newtonian values.

In contrast, the MOND acceleration \(a_{\text{MOND}}\) is given by the solution of the nonlinear equation
\[ a_{\text{MOND}}\left(\frac{\mu_{\text{MOND}}}{a_0}\right) = -\frac{G_NM}{r^2}, \]
where \(a_0 = 1.2 \times 10^{-8} \, \text{cm} \, \text{s}^{-2}\). The form of the function \(\mu(x)\) originally proposed by Milgrom (1983) is given by \(\mu(x) = x/\sqrt{1 + x^2}\); however, better fits and better asymptotic behavior are achieved using \(\mu(x) = x/(1 + x)\) (Famaey & Binney 2005), which yields the acceleration function
\[ a_{\text{MOND}} = -\frac{G_NM}{r^2} \left(1 + \sqrt{1 + \frac{a_0 r^2}{G_NM}}\right). \]

Regardless of the form of \(\mu(x)\) chosen, when the combined acceleration experienced by stars in a GC is below \(a_0\), MOND predicts dynamical behavior that is markedly different from the Newtonian prediction.

\[ 2.2. \text{The Jeans Equation} \]

In the spherically symmetric, nonrotating case the Jeans equation for the velocity dispersion \(\sigma(r)\) takes the form (see eq. [4-64a] in Binney & Tremaine 1987):
\[ \frac{\partial}{\partial r}\left(r^2 \sigma^2 \frac{\partial \Phi}{\partial r}\right) + \nu \frac{\partial \Phi}{\partial r} = 0, \]
where \(r\) is the radial distance from the GC center, \(\nu(r)\) is the number density distribution function, and \(\Phi(r)\) is the gravitational potential. If \(\nu(r)\) and \(\Phi(r)\) are known, the velocity dispersion can be obtained by direct integration. Using \(a(r) = \partial \Phi/\partial r\) and utilizing the fact that \(\lim_{r \to \infty} \sigma^2(r) = 0\), we get
\[ \sigma^2(r) = \frac{1}{\nu} \int_r^{\infty} \nu a(v') \, dv'. \]

The observed velocity dispersion of GCs is a function of the actual radial distance \(r\) but the projected distance \(R\) between the GC center and the star being observed. The velocity dispersion

\[ \sigma_{\text{LOS}}(R) \]

for stars observed along the line of sight (LOS) at projected distance \(R\) from the GC center is related to \(\sigma(r)\) as
\[ \sigma_{\text{LOS}}^2(R) = \left(\frac{\int_0^\infty \sigma^2(v)(\nu(y) \, dy)}{\int_0^\infty \nu(y) \, dy}\right)^2, \]
where
\[ y^2 = r^2 - R^2, \]
as can be verified by simple geometric reasoning. Eliminating \(y\), we can rewrite equation (14) as
\[ \sigma_{\text{LOS}}^2(R) = \left(\frac{\int_R^\infty \sigma^2(v)(\nu(r) \, vr) \, dr}{\int_R^\infty \nu(r) \, dr}\right)^2. \]

\[ 2.3. \text{Density Distribution} \]

Several models exist that can mimic the density distribution of a spherically symmetric set of stars. One particularly simple model is that of Hernquist (1990), which models the number density of stars as a function of radius as
\[ \nu_{\text{Hernquist}}(r) = \frac{N_0}{2\pi r(r + r_0)^3}, \]
where \(N\) is the total number of stars, and \(r_0\) is a characteristic radius.

Another, similar model is that of Jaffe (1983):
\[ \nu_{\text{Jaffe}}(r) = \frac{N_0}{4\pi r^2(r + r_0)^2}. \]

Without benefiting from a photometric profile of the globular cluster under investigation, there are no a priori reasons to prefer one model over another. We found that the choice of model does not have a significant impact on the conclusions we present; hence, we use Hernquist’s model consistently, but we note that similar results are obtained using alternate number density distribution functions.

\[ 3. \text{Observations and Predictions} \]

Velocity dispersion data for several GCs were recently published by Scarpa et al. (2007). We read velocity dispersion values and their standard deviations from Figures 1, 2, and 4 of Scarpa et al. (2007) for NGC 288, NGC 5139 (ω Centauri), NGC 6171 (M107), NGC 6341 (M92), NGC 7078 (M15), and NGC 7099

| Name          | \(R_0\)  | \(L\)   | \(r_0\)  | \(M/L\) |
|---------------|---------|---------|---------|---------|
| NGC 288       | 7.4     | 3.94 \times 10^4 | 2.9     | 4.38    |
| NGC 5139      | 6.4     | 1.04 \times 10^6  | 6.4     | 2.79    |
| NGC 6171      | 3.3     | 5.65 \times 10^4  | 5.0     | 2.54    |
| NGC 6341      | 9.6     | 1.51 \times 10^3  | 2.6     | 1.50    |
| NGC 7078      | 10.4    | 3.70 \times 10^3  | 3.2     | 0.85    |
| NGC 7099      | 7.1     | 7.45 \times 10^4  | 2.7     | 1.51    |
| AM 1          | 123.2   | 6.08 \times 10^3  | 17.7    | 2       |
| Pal 14        | 69.0    | 6.19 \times 10^3  | 24.7    | 2       |

\[ \text{Notes}. \text{— Data for AM 1 and Pal 14 are also included. The distance } R_0 \text{ from the galactic center, luminosity } L \text{ in units of solar luminosity, and the half-light radius } r_0 \text{ (pc) are given (Harris 1996). Mass-to-light ratios are estimated by fitting the velocity dispersion using the Hernquist model, except for AM 1 and Pal 14, for which } M/L = 2 \text{ was fixed.} \]
Some of the basic characteristics of these GCs are summarized in Table 1.

Using the Hernquist distribution as the number density distribution function for a spherically symmetric cluster of stars with isotropic velocity dispersion, we obtained very good fits to the velocity dispersion data (Fig. 1). These results also yield mass-to-light ratios in the range $0.8 < \frac{M}{L} < 4.4$ (Table 1), which are typical for globular clusters.

For these results, we used the Newtonian gravitational potential. These calculations are consistent with Newtonian theory,
MOG (given the smallness of the predicted value of the MOG $\alpha$ parameter and the large size of the parameter $\mu^{-1}$), and also MOND, as the GCs in question are located relatively near the Galactic center, and the Galactic acceleration is always greater than $a_0$.

The possible presence of dark matter does not appreciably alter these results either. A typical dark matter density for the galactic halo is $\sim 7.8 \times 10^{-3} M_\odot \text{pc}^{-3}$ ($\geq 0.3 \text{GeV cm}^{-3}$; see Sumner 2002), a density that is much smaller than the globular cluster’s stellar mass density.

The situation is different, however, in the case of MOND and globular clusters that are located a long distance away from the Galactic center. To quote Milgrom (1983): “We are then compelled to conclude that the internal dynamics of the open clusters embedded in the field of the Galaxy is different from that of a similar but isolated cluster.” For instance, Pal 14, located at 69 kpc from the Galactic center, would experience a Galactic acceleration of $\sim 2.3 \times 10^{-11} \text{ m s}^{-2}$, which is well within the MOND regime. As this is a low mass cluster of stars, its internal accelerations are also significantly below MOND’s $a_0$, except perhaps in the innermost regions of the cluster.

Two distant clusters, AM 1 and Pal 14, are currently the subject of an observational project by Kroupa et al. As the absolute luminosity of these GCs is known, using a (typical) value of $M/L \approx 2$ we can obtain a crude estimate of their mass, allowing us to apply the Jeans equation and derive a velocity dispersion profile using both Newtonian and MOND gravity. These predictions are shown in Figure 2.

4. DISCUSSION

For globular clusters with a mass of a few times $10^6 M_\odot$ or less, MOG predicts little or no observable deviation from Newtonian gravity. This is verified by our demonstration that a simple model, using a spherically symmetric distribution and no velocity anisotropy, can easily reproduce the velocity dispersion profiles of several diverse globular clusters with varying mass.

The predictions of neither Newtonian gravity nor MOG depend on the distance from the galactic center. The same remains true when dark matter is considered; although the density of dark matter may be a function of distance from the galactic center, at predicted dark matter densities, the amount of dark matter contained within a GC does not noticeably alter the dynamics of the cluster.

The situation is different for MOND: for a low-mass cluster, internal accelerations are below the MOND threshold of $a_0 \approx 1.2 \times 10^{-10} \text{ m s}^{-2}$, and if the cluster is far enough from the Galactic center, its Galactic acceleration is also below this value. For this reason, distant GCs may offer a unique method to distinguish observationally between MOG and MOND.

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