Implicit Multi-Agent Coordination at Unsignalized Intersections via Topological Inference

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Abstract—We focus on scenarios in which multiple rational, non-communicating agents navigate in close proximity, such as unsignalized street intersections. In these situations, decentralized coordination to achieve safe and efficient motion demands nuanced implicit communication between the agents. Often, the spatial structure of such environments constrains multi-agent trajectories to belong to a finite set of modes, each corresponding to a distinct spatiotemporal topology. Our key insight is that empowering agents with a model of this structure can enable effective coordination through implicit communication, realized via intent signals encoded in agents’ actions. In this paper, we do so by representing modes of joint behavior as topological braids. We design a decentralized planning framework that incorporates a mechanism for inferring the emerging braid in the decision-making process. By executing actions that minimize the uncertainty over the upcoming braid, agents converge rapidly to a consensus over a joint collision avoidance strategy despite lacking knowledge of the destinations of others and accurate models of their behaviors. We validate our approach with a case study in a four-way unsignalized intersection involving a series of challenging multi-agent scenarios. Our findings show that our model reduces frequency of collisions by at least 65% over a set of explicit trajectory prediction baselines, while maintaining comparable efficiency.

I. INTRODUCTION

Although real-world navigation environments such as street intersections often feature a significant amount of spatial structure (e.g., sidewalks, dedicated lanes etc), they do not always feature mechanisms for organizing traffic flows temporally (e.g., a street intersection in which the traffic lights are not working, cars violating traffic rules etc). The lack of temporal structure may result in highly unpredictable dynamics which may in turn yield inefficient and unsafe interactions among agents. In driving scenarios, cars may make use of dedicated means of signaling, such as turn signals, horns, or even gestures and verbal communication to reduce uncertainty and negotiate a joint plan. However, on many occasions, human distraction, design limitations, hardware or software failure may prohibit the use of the aforementioned signaling modalities, yielding catastrophic results. For reference, in the United States, during the year 2018, 43.7% of all motor vehicle crashes occurred at intersections (2,943,717 out of 6,734,416 incidents). Out of these, 8,245 incidents involved fatalities, representing the 24.5% of all fatal crashes for the same year (out of a total of 33,654 fatal crashes) [26].

Motivated by these facts, and by the projected rise of autonomous vehicles [2], we focus on scenarios involving multiple rational, non-communicating but perfectly-observing agents that are navigating in close proximity in a decentralized manner. A typical such example is the unsignalized four-way intersection of Fig. 1 in which multiple cars navigate between different sides. Unsignalized intersections have been part of the standards for crash avoidance research for decades [25]. The lack of explicit communication in these domains results in high
uncertainty about the unfolding dynamics (agents’ intended destinations, trajectories, behavior models etc), which makes decision making challenging. Our key insight is that the spatial structure of the environment constrains the collective behavior of rational agents to belong to a finite set of modes, each corresponding to a topologically distinct system behavior. We encode this domain knowledge into our approach by explicitly modeling these modes as topological braids \cite{1, 4}. This enables us to construct a probabilistic inference mechanism that symbolically reasons about the emerging braid, which allows an artificial agent to understand the likelihood and quality of all possible collision-free ways they can traverse the intersection. Based on this mechanism, we design a decentralized navigation planning framework that selects actions towards minimizing uncertainty over an emerging mode of behavior (see Fig. 5). By collectively contributing to uncertainty reduction through their actions, the system of agents converge rapidly to a state of consensus, which results in safe executions despite the lack of explicit communication and signaling.

In summary, we make the following contributions:

(a) We introduce a formal mathematical model that captures salient features of joint behavior at street intersections with multiple agents. This model enables us to compactly represent modes of multi-agent behaviors as symbols corresponding to topological braids \cite{4, 21}.

(b) We construct a novel probabilistic inference mechanism that connects past system behaviors to likely modes of future system behavior. Reasoning about modes has the potential of relaxing the prediction problem. Under mild assumptions on agents’ behavior (no U-turns or changing intentions), the space of modes is significantly smaller than the space of trajectories.

(c) We conduct an empirical study in which we compare our framework against a set of baselines that reason directly over the space of trajectories. We demonstrate that our framework enables multiple (2-4) non-communicating agents to coordinate implicitly and follow significantly safer paths (at least 65% fewer collisions) across a series of challenging scenarios. Our findings suggest that incorporating topological features in the decision making process of non-communicating agents enables effective coordination even in the absence of explicit communication.

(d) We illustrate the power of low-dimensional control actions in communicating complex multi-dimensional events, such as strategies of collision avoidance. We show that under the assumption of rationality on agents’ decision making (no incentive of actively pursuing collisions, and goal-driven otherwise), coarse prediction models could prove sufficient for collision avoidance, while exhibiting acceptably efficient behaviors.

II. RELATED WORK

Intersections are notoriously challenging, as they typically involve negotiation and coordination among multiple agents, often in the absence of explicit communication. Developing autonomous systems capable of making safe decisions under such settings remains a challenge, leading to extensive research on the design of prediction, planning and control techniques.

Ensuring safety while maintaining efficiency is the key objective driving research. Pierson et al. \cite{28} introduce a congestion cost quantifying the risk of collision and use it to plan within desired risk level sets for safe lane changes in congested highways. McGill et al. \cite{23} present a probabilistic framework for automated crossing of unsignalized intersections under occlusions and faulty perception, which was shown to result in safe behaviors in real-world experiments on miniature racecars. Isele et al. \cite{13} learn a policy for crossing unsignalized intersections under occlusions using deep reinforcement learning and show how it outperforms selected rule-based baselines. Finally, Okamoto et al. \cite{27} plan safe maneuvers at intersections by combining data-driven models for local and global vehicle interaction prediction.

A set of works model the problem of crossing an intersection using tools from belief-space planning. For instance, Bandyopadhyay et al. \cite{3}, use a Mixed Observability Markov Decision Process (MOMDP) to plan safe human intention-aware maneuvers in real-world vehicle-pedestrian interaction scenarios. Their approach has also been shown to enable safe merging in T-junction intersections \cite{50}. Bouton et al. \cite{5} plan safe and efficient maneuvers for merging in unsignalized intersections using a partially observable Markov decision process model (POMDP) solved via a Monte Carlo sampling-based method. Hubmann et al. \cite{11} also propose a POMDP-based planner that incorporates uncertainty related to sensor noise besides intentions.

A class of works integrates a series of prosocial metrics on top of intention prediction towards reinforcing vehicle coordination. Sadigh et al. \cite{29} plan intent-expressive maneuvers that reinforce safe and efficient coordination between autonomous and human-driven cars at intersections and highways in a series of experiments on a driving simulator. Similarly, Lazar
et al. [17] plan optimal lane changes that reinforce prosocial behaviors such as platooning, yielding increased capacity in congested highways. Buckman et al. [6] plan prosocial vehicle rearrangements that result in reduced system delays in a centrally managed signalized intersection, using a social psychology metric. Also within the centralized domain, Miculescu and Karaman [24] present a control framework inspired by polling systems that provides safety and efficiency guarantees for continuous car flows crossing an unsignalized intersection.

Finally, a series of works have focused on developing tools for testing and validating approaches for autonomous navigation in realistic scenarios involving traffic at intersections. For instance, Tian et al. [34] model traffic at unsignalized intersections using tools from game theory and propose a verification testbed for autonomous navigation algorithms. Similarly, Liebenwein et al. [18] propose a framework for safety verification of driving controllers based on compositional and contract-based principles, and validate it through a case study on a realistic road network. Gu et al. [9] plan humanlike behaviors at intersections involving vehicle-pedestrian traffic using a data-driven model. Finally, Hsu et al. [10], also focusing on vehicle-pedestrian interactions at intersections, explore how velocity signals generated by Markov decision processes affect interaction dynamics.

While existing literature focuses on the computational machinery for robust decision making under uncertainty, this paper identifies two key components that to the best of our knowledge have not been thoroughly studied in this domain: a) a salient mathematical representation that captures critical features of multi-agent collision avoidance in intersection scenarios; b) a pipeline that leverages the implicit communication phenomena arising naturally while multiple agents navigate in a shared environment. Our insight is that effective incorporation of such features into the decision-making process of rational agents may enable efficient coordination among them, despite the absence of explicit communication.

In this paper, we formally model the structure of joint decision-making at street intersections with a model that makes use of the topological braid representation [3]. This model builds upon and extends past work on the use of braided structures as modes of collision-free navigation behaviors [19,22] by (1) providing a more rigorous mathematical presentation; (2) adapting to the structured domain of street intersections; (3) enabling a natural Bayesian formulation. This formulation enables rational agents to coordinate implicitly by encoding intentions into their actions. This is realized via an information-theoretic framework that casts legibility [7] as uncertainty reduction over a space relevant to the domain of multi-agent navigation. The approach is motivated by recent works focusing on human-robot coordination [15,32].

III. PROBLEM STATEMENT

Consider the unsignalized street intersection of Fig. [30], where $n \geq 1$ agents are navigating. Denote by $q_i = (x_i, y_i, \theta_i) \in \mathcal{Q} \subseteq SE(2)$ the state of agent $i \in N = \{1, \ldots, n\}$ with respect to (wrt) a fixed reference frame, defined by a basis $(\hat{x}, \hat{y}, \hat{t})$. Agent $i$ is a dynamical system $\dot{q}_i = \phi(q_i, u_i)$ following standard car dynamics [16]. Agent $i$ starts from an initial state $s_i \in \mathcal{Q}$, lying on a side of the intersection, and moves towards a final--unknown to others--state $d_i \in \mathcal{Q}$ lying on a different side. Agent $i$ does so by tracking a path $\tau_i : I \rightarrow \mathcal{Q}$, for which it holds that $\tau_i(0) = s_i$ and $\tau_i(1) = d_i$, where $I = [0,1]$ is a path parametrization. Observing the complete system state $Q = (q_1, \ldots, q_n) \in \mathcal{Q}^n$, agent $i$ tracks $\tau_i$ by executing a policy $\pi_i : \mathcal{Q}^n \rightarrow \mathcal{U}_i$, generating actions $u_i^* \in \mathcal{U}_i$ (speed and steering angle), satisfying a specification:

$$u_i^* = \arg \min_{u_i \in \mathcal{U}_i} w_i C_d(u_i) + (1 - w_i) C_c(u_i),$$

where $\mathcal{U}_i \subseteq \mathbb{R} \times \mathbb{S}$ is a space of controls, $C_d : \mathcal{U}_i \rightarrow \mathbb{R}$ represents the distance cost-to-go and $C_c : \mathcal{U}_i \rightarrow \mathbb{R}$ the collision cost of taking an action in consideration $u_i$, and $w_i$ is a weight--unknown to other agents--describing agent $i$’s personal compromise over the two costs. Agent $i$ is not aware of the intended path $\tau_j$, the destination $d_j$ or the exact policy $\pi_j$ of agent $j \neq i \in N$ but assumes that any agent $j \neq i \in N$ is rational, in the sense that they also optimize for $C_d$ and $C_c$. Our goal is to design decentralized policies $\pi_i$ that enable agents to coordinate safe intersection crossings while following time-efficient trajectories under uncertainty.

IV. PLANNING WITH TOPOLOGICAL INFERENCE

The foundation of our approach lies in the observation that a constrained environment such as a street intersection couples the control decisions of rational agents. This coupling constrains collision-free multi-agent trajectories to belong to a set of modes, each corresponding to a distinct equivalence class of executions with identical topological properties. Our key insight is that explicitly reasoning about these modes during execution: (a) relaxes the inference problem, under the assumption that agents are acting rationally; (b) enables agents to understand and represent potential solutions to the coupled collision-avoidance problem despite their uncertainty over the intentions or the policies of others. In this paper, we show that the modes of joint behavior at intersections can be modeled as topological braids [11,24]. We then design an inference mechanism that predicts future braids given observations of past trajectories, and describe a policy generating uncertainty-minimizing actions to enable coordination among non-communicating agents.

A. Joint Behavior at Street Intersections

The complete sequence of controls that agent $i$ executes by tracking $\tau_i$ with the policy $\pi_i$, under the dynamics $\phi_i$, results in a time-parametrized trajectory $\xi_i : [0, t_\infty) \rightarrow \mathcal{Q}$, where $t_\infty$ corresponds to the end of the execution (the time at which the last agent reaches its destination—we assume that agents that reached their destinations earlier, remain stationary there until $t_\infty$). Following their individual policies, at time $t \in [0, t_\infty]$, the system of agents executes a control profile $U = (u_1, u_2, \ldots, u_n) \in \mathcal{U}$, where $\mathcal{U} = \mathcal{U}_1 \times \mathcal{U}_2 \times \cdots \times \mathcal{U}_n$ is the joint space of controls. Collectively, the complete
sequence of control profiles that the system of agents executes from time \( t = 0 \) to time \( t = t_\infty \) to track the system path \( T = (\tau_1, \tau_2, \ldots, \tau_n) \in \mathcal{T} \) (where \( \mathcal{T} \) represents the set of system paths) results in a time-parametrized system trajectory \( \Xi = (\xi_1, \ldots, \xi_n) : [0, t_\infty] \rightarrow \mathbb{Q}^n \). We assume that all agents remain at their destinations until the end of the execution \( t_\infty \), i.e., until the last agent reaches its destination, at which time the execution is terminated.

Depending on the relationship among the time parametrizations of agents’ individual trajectories, the system trajectory \( \Xi \) may exhibit different topological properties. These properties are indicative of the joint behavior of the system of agents, as they capture the succession with which agents traverse the intersection, e.g., which agent passed first/second, left/right.

We classify system trajectories into a set of modes, each corresponding to an equivalence class of topologically equivalent joint behaviors, represented as a topological braid [1, 4].

B. Topological Braids

Braids are topological objects with geometric and algebraic descriptions, which enable us to abstract collective behaviors into symbols.

From a geometric point of view, a braid (or \( n \)-braid) can be defined as a tuple \( (f_1, \ldots, f_n) \), where \( f_i : [0, 1] \rightarrow \mathbb{R}^2 \), \( i \in N \), is a function describing a curve, monotonically increasing in the \( t \) direction, called a braid, such that \( f_i(0) = (i, 0) \), and \( f_i(1) = (p(i), 0) \), where \( p : N \rightarrow N \) is a permutation in the symmetric group \( N_n \), and such that \( f_i(t) \neq f_j(t) \) \( \forall t \in (0, 1) \) and \( j \neq i \in N \). The set of the isotopy classes of \( n \)-braids, along with the composition operation forms a group, called the braid group on \( n \) strands, denoted as \( B_n \). By definition, the composition of two braids \( b_i = (f_1, \ldots, f_n) \), \( b_j = (g_1, \ldots, g_n) \), is also a braid \( b_h = b_f \cdot b_y = (f_1, \ldots, h_n) \), comprising a set of \( n \) curves, defined as:

\[
h_i(t) = \begin{cases} f_i(2t), & t \in [0, \frac{1}{2}] \\ g_j(2t-1), & t \in \left[\frac{1}{2}, 1\right]. \end{cases}
\]

where \( j = p(i) \) ensures proper indexing.

From an algebraic point of view, following Artin’s presentation [1], the braid group \( B_n \) can be generated from a set of \( n - 1 \) primitive braids, \( \sigma_1, \ldots, \sigma_{n-1} \), called generators (see Fig. 3), that satisfy the following relations:

\[
\sigma_i \sigma_j = \sigma_j \sigma_i, \quad |j - i| > 1, \quad \sigma_i \sigma_{i+1} \sigma_i = \sigma_{i+1} \sigma_i \sigma_{i+1}, \quad \forall i. \tag{3}
\]

A generator \( \sigma_i \), \( i \in \{1, 2, \ldots, n-1\} \), is a braid \( b_f = (f_1, \ldots, f_n) \), for which it holds that \( f_i(1) = (p_i(i), 0) \), where \( p_i : N \rightarrow N \) is an adjacent transposition (a permutation swapping only two adjacent elements) swapping the elements \( i \) and \( i + 1 \), and \( (f_i(t_c) - f_{i+1}(t_c)) \cdot \hat{y} > 0 \), at unique moment \( t_c \) that \( (f_i(t_c) - f_{i+1}(t_c)) \cdot \hat{x} = 0 \). The inverse of \( \sigma_i \), denoted as \( \sigma_i^{-1} \), is the braid \( b_f' = (f_1, \ldots, f_n) \), for which it holds that \( f_i'(1) = p_i(i) \), and \( (f_i'(t_c) - f_{i+1}(t_c)) \cdot \hat{y} < 0 \), at the unique moment \( t_c' = (f_i(t_c') - f_{i+1}(t_c')) \cdot \hat{x} = 0 \). The identity element, \( 1 \), is defined by the trivial permutation \( e \) which fixes all elements of the set \( N \). Any braid can be written as a product of generators and generator inverses. This product is commonly referred to as a braid word.

C. Modes of Joint Behavior as Topological Braids

Consider the tuple \( \Xi = (\xi_1, \ldots, \xi_n) \) containing the trajectories of \( n \) agents, describing their motion as they traverse the intersection of Sec. III. Denote by \( \hat{n} \) a unit vector parallel to the plane \( \hat{z} \cdot \hat{n} \). Let us define a set of functions \( \xi^+_i : [-\epsilon, t_\infty + \epsilon] \rightarrow \mathbb{R}^2, i \in N \), where \( \epsilon \) is a small positive number, such that:

\[
\xi^+_i(t) = \begin{cases} (\xi_i(0) \cdot \hat{n}) \hat{n}, & t = -\epsilon \\
(\xi_i(t) \cdot \hat{x}, \hat{y}), & t \in [0, t_\infty] \\
(\xi_i(1) \cdot \hat{n}) \hat{n}, & t = t_\infty + \epsilon. \end{cases}
\]

The function \( \xi^+_i \) corresponds to the trajectory of agent \( i \), augmented at times \( t = -\epsilon \) and \( t = t_\infty + \epsilon \) with the projections of agents’ initial and final states onto a selected line defined by \( \hat{n} \). The collection \( \{\xi^+_1, \ldots, \xi^+_n\} \) forms a braid \( \beta_\xi \), which is topologically equivalent to the entanglement of agents’ trajectories throughout the execution. In this sense, the topological properties of a system trajectory can be characterized following the theory of braids. In fact, given a system trajectory \( \Xi \), and a selected projection vector \( \hat{n} \), a corresponding braid word may be extracted by taking the projection of \( \Xi \) on the plane \( \hat{n} \cdot \hat{t} \), labeling the emerging crossings as generators, and placing them in temporal order [21]. Thus, the braids formalism serves both as an abstraction of a past execution but also as a tool to enumerate future behaviors.

D. Topological Inference

At time \( t \in [0, t_\infty) \), agent \( i \), having access to the complete system state history so far, \( \Xi \), maintains a belief \( bel_i(\beta_i) = P(\beta_i|\Xi) \) over the braid \( \beta_i \in B_n \) that describes the topology of the emerging (future) system trajectory \( \Xi' = \Xi_{t \rightarrow \infty} \), defined wrt a vector \( \hat{n} \). The emerging braid is heavily dependent on agents’ intended system path \( T \). To capture this dependency, we marginalize over \( T_i \subseteq \mathcal{T} \), the set of system paths for which agent \( i \) follows its intended path:

\[
bel_i = P(\beta_i|\Xi) = \sum_{T \in T_i} P(\beta_i|\Xi, T)P(T|\Xi). \tag{6}
\]

For a given path \( T \), different braids could possibly emerge, depending on the path tracking behavior of agents. To capture
paths of others, since agent $i$ where the product only considers the probabilities over the computation of the control profile prediction as:

$$P(\beta_i|\Xi, T) = \sum_{U \in \mathcal{U}} P(\beta_i|\Xi, U, T)P(U|\Xi, T).$$  \tag{7}$$

Substituting to eq. (6), we get:

$$\text{bel}_i = \sum_{\tau_i} \left\{ \sum_{U} P(\beta_i|\Xi, U, T)P(U|\Xi, T) \right\} P(T|\Xi).$$  \tag{8}$$

The outlined mechanism combines a local action selection model $P(U|\Xi, T)$ with a model of intent inference $P(T|\Xi)$ and a global behavior prediction model $P(\beta_i|\Xi, U, T)$.

The intention of agent $j \neq i$ over a path $\tau_j$ is conditionally independent of the intention of any other agent, given the past system trajectory $\Xi$. The probability over the path intention of agent $j$ does not depend on the trajectories of others. Thus, we simplify the computation of the system path prediction as:

$$P(T|\Xi) = \prod_{j \neq i} P(\tau_j|\xi_j),$$  \tag{9}$$

where the product only considers the probabilities over the paths of others, since agent $i$ is certain about its own path.

Similarly, since agents select a control input independently, without having access to the policies of others, we decompose the computation of the control profile prediction as:

$$P(U|\Xi, T) = \prod_{i=1}^{n} P(u_i|\Xi, T),$$  \tag{10}$$

where the distribution $P(u_i|\Xi, T)$ represents the control that agent $i$ executes to make progress along its path $\tau_i$ incorporating considerations such as preferred navigation velocity.

The model of inference of eq. (8) focuses on topology prediction, without considerations about collision avoidance. To filter out unsafe braids, we redefine eq. (8) by incorporating a model of collision prediction. Define by $c$ a boolean random variable representing the event that $\Xi'$, the emerging future trajectory contains collisions (true for a collision, false for no-collision). Denote by $\tilde{\beta} = (\beta_i, \neg c)$ the joint event that $\Xi'$ is both topologically equivalent (ambient-isotopic) to a braid $\beta_i \in B_n$, and not in collision, i.e., $c$ is false. Then the probability that belief $\beta_i$ is true can be computed as:

$$\text{bel}_i = \sum_{T_i} \left\{ \sum_{U} P(\tilde{\beta}_i|\Xi, U, T)P(U|\Xi, T) \right\} P(T|\Xi).$$  \tag{11}$$

The occurrence of a collision is conditionally independent of the emerging braid given the state history, the current control profile and the intended system path; thus, we may compute their joint distribution as:

$$P(\tilde{\beta}_i|\Xi, U, T) = P(\tilde{\beta}_i, \neg c|\Xi, U, T)$$
$$= P(\neg c|\Xi, U, T, \beta_i)P(\tilde{\beta}_i|\Xi, U, T)$$
$$= (1 - P(c|\Xi, U, T, \beta_i))P(\beta_i|\Xi, U, T).$$  \tag{12}$$

E. Decision Making

We design a policy that generates uncertainty-reducing actions by directly minimizing the Information Entropy of the distribution over braids eq. (12). The lower the Entropy is, the lower the uncertainty, and thus the closer agents are to a consensus over a braid. Agent $i$ is interested in the recovery of a collision-free braid from the set $\tilde{\beta}_i$; thus, it monitors the state of consensus by computing:

$$H(\tilde{\beta}_i) = -\sum_{B_n} P(\tilde{\beta}_i|\Xi) \log P(\tilde{\beta}_i|\Xi),$$  \tag{13}$$

where $(\tilde{\beta}_i|\Xi)$ is recovered using eq. (12). In order to contribute towards reducing this uncertainty, agent $i$ selects actions (velocities) that minimize the entropy:

$$u_i = \arg \min_{u_i} H(\tilde{\beta}_i).$$  \tag{14}$$

Fig. 5: Illustration of the decision-making scheme. At every cycle, the ego agent forward simulates a set of distinct futures, classifies them into topological outcomes, and selects the action that minimizes the uncertainty over such outcomes.
The selection of the Information Entropy as a decision-making cost illustrates our insight that the specific emerging braid is not important, as long as it is collision-free, and predictable by others. It should be noted that our goal in this work is to study the convergence to multi-agent consensus through the use of topological features in a general-purpose framework. It is possible to extend the framework by considering alternative control policies (e.g., Model Predictive Control (MPC))

V. APPLICATION

We employ our decision-making mechanism in a simulated study on an unsignaled intersection with multiple cars. Our setup is the 4-way symmetric intersection of Fig. 2. The intersection has lanes of length 50 m and width 3.6 m, whereas each car has a length of 4.7 m and a width of 1.7 m. Any side a is connected to any different side b ≠ a with a unique, publicly known legal path τ_{ab}, lying along the middle of the lane. We assume that any agent i ∈ N that attempts to reach side b from side a will attempt to track this path, τ_{ab}. Tracking is implemented with a simple proportional controller that follows the linearized car kinematics (14). The main decision variable of our scheme is the speed with which an agent tracks its path; given that speed, the tracking controller outputs a control input u_i which is immediately executed.

Each agent follows a path out of three options (left, right, or straight); thus |T_i| = 3^3 = 27. Path tracking is split in two parts: (a) the negotiation part, which corresponds to the initial straight-path part of the intersection (denoted as Q_i^{neg} for agent i), within which the agent attempts to reach a consensus with others wrt a joint strategy of collision avoidance; (b) the execution part, which corresponds to the rest of the path (denoted as Q_i^{exec} for agent i), within which the agent tracks the remainder of its path, by maintaining a constant speed. This decision emphasizes the importance of proactive negotiation during the first portion, and provides a natural metric of quality—the count of collisions during the execution part.

A. Models

We assume that agent i has no knowledge of the path τ_j of any other agent j ≠ i ∈ N while j is in the negotiation stage. However, we assume that τ_j becomes immediately obvious when agent j enters the intersection:

\[ P(τ_j | ξ_j(t)) = P(τ_j | q_j) = \begin{cases} \frac{1}{m} & \text{for } q_j \in Q_i^{neg} \\ 1 & \text{for } q_j \in Q_i^{exec} \end{cases}, \]

where q_j = ξ_j(t) is agent j’s current state, and m = 3 is the number of paths that agent j selects from.

We assume that the action space of all agents comprises two speeds, a high speed and a low speed. We further assume that all agents prefer the higher speed, and that in the beginning of the execution, they start with the high speed. We express this preference in the probability \( P(u_j | ξ_j, τ_j). \) In the following simulations we assume that agents prefer the high speed with higher probability (sampled uniformly from the range [0.6, 0.8]) over the low speed. We also assume that each agent assumes that others have the same exact preferences over speeds, i.e., they do not know the true preferences of others.

For the computation of the braid and collision probabilities, agent i generates a set of system trajectory rollouts. In particular, for each path set \( T \in T_i \) and each control profile \( U \in U \), a system trajectory \( Ξ \) is generated by linearly projecting forward all agents from the current system state \( Ξ(t) \) towards \( T \) with a constant speed \( U \). From each trajectory, we extract a corresponding braid word \( β_i \) (as described in Sec. IV-C using BraidLab), and the minimum inter-agent distance \( d_{\text{min}} \). Upon completing all rollouts, we have constructed a set \( B \subset B_n \) comprising the set of possible braids that could emerge in the remainder of the execution. Each braid \( β^* \) is then evaluated as:

\[ P(β_i = β^* | Ξ, U, T) = \begin{cases} 1 & \text{if } β_i = β^* \\ 0 & \text{otherwise} \end{cases}, \]

This model acts as a switch that determines which rollouts should be considered for each braid found at the simulation stage. Finally, we model the probability of a collision \( P(c | Ξ, U, T, β_i) \) with the following sigmoid model:

\[ P(c | Ξ, U, T, β_i) = \frac{1}{1 + e^{α(d_{\text{min}} - δ)}} \]

where \( α \) controls the rate of change of the function and \( δ \) denotes a threshold distance beyond which collision is imminent. According to this model, the smaller \( d_{\text{min}} \) is, it is exponentially more likely to have a collision.

B. Evaluation

We define three scenarios, involving 2, 3, and 4 agents respectively. For each scenario, we generate a set of experiments by varying agents’ speed preferences. We execute each experiment under 5 conditions, each corresponding to a different algorithm that the agents run. We then measure performance by looking at the frequency of collisions and the maximum experiment time per scenario and condition.

1) Scenarios: S1: Two agents, starting from the bottom and the right sides of the intersection, are moving straight towards the top and left sides respectively. They both draw speeds from \( U_{s1} \) containing 12 evenly spaced speeds within [5, 10] (m/s). We generate 144 experiments corresponding to the Cartesian product \( U_{s1}^2 \).

S2: Three agents, starting from the bottom, right and top are moving straight towards the top, left and bottom sides respectively. They draw speeds from \( U_{s2} \) containing 5 evenly spaced speeds within the range [5, 10] (m/s). We generate 125 experiments corresponding to \( U_{s2}^3 \).

S3: Four agents, starting from the bottom, right, top, and left, are moving straight towards the top, left, bottom, and right sides respectively. They draw speeds from \( U_{s3} \), containing 3 evenly spaced speeds within the range [5, 10] (m/s). We generate 81 experiments corresponding to \( U_{s3}^4 \).

2) Conditions: C1: Agents track their desired paths with their desired speeds, without avoiding collisions with each other. This condition serves as a characterization of the intensity of the multi-agent encounters at the intersection for each scenario.
Fig. 6: Performance evaluation: Fig. 6a and Fig. 6b depict collision frequency and experiment time for S1 (2 agents), computed over 144 experiments; Fig. 6c and Fig. 6d depict collision frequency and experiment time for S2 (3 agents), computed over 125 experiments; Fig. 6e and Fig. 6f depict collision frequency and experiment time for S3 (4 agents), computed over 81 experiments. Bars correspond to conditions; error bars indicate standard deviations and 25/75 percentiles in the collision frequency and time charts respectively.
reasoning about these modes enables a rational agent to anticipate the effect of its actions on system behavior. Our policy outputs local actions of global outlook that contribute towards reducing uncertainty over the emerging mode. Collectively, this results in implicit coordination, reflected in the reduced collision frequency of C2, C3. To illustrate this point, Fig. 7 depicts a comparative qualitative example of the behaviors generated by our policy. For the same experiment from S3 (run in the symmetric intersection of Fig. 2a), we observe that C2 agents (Fig. 7a) quickly converge to a clear order of intersection crossings as a result of their proactive decision making. On the other hand, C4 agents (Fig. 7b), lacking the ability of modeling the complex multi-agent dynamics, appear unable to coordinate their crossings and end up colliding.

Our findings may have broader implications about the value of topological features for multi-agent navigation. Reasoning about a bounded set of modes could enable significant computation speedup compared to naively reasoning about the space of trajectories. For reference, from the perspective of an agent $i$, the space of possible 4-agent trajectories over an horizon of $H = 10$ time steps, assuming a control space $\mathcal{U}^i = 10^4$ has size $S_i = |\mathcal{U}|^H = 27 \cdot 10,000^{10}$. The space of braids that could be practically possible for any $n$-agent scenario could be bounded to $S_b = (2n - 2)^D + 1$, where $D$ is the maximum number of generators appearing in a braid word. For a 4-agent scenario with $D = 5$ (the average value across our 4-agent experiments), this number would be $S_b = 7,777$.

VI. DISCUSSION

Although braids have the potential of significantly compressing the space of outcomes, and thus relaxing inference, in this paper we did not leverage the projected computation gains, as we conditioned our belief on the control profile and the system path (see eq. (8)). Ongoing work involves learning a distribution over the space of braids from a dataset of intersection scenarios. Reasoning directly over braids during execution will enable the outlined computation speedups and allow for scaling to more complex scenes.

Furthermore, although the considered scenario captures the main features of an unsignalized intersection, the setup is deliberately simplified to facilitate the extraction of foundational insights. Moving forward, we plan on incorporating heterogeneous agents in our scenarios, such as cars running different policies or pedestrians. We also plan on validating our approach with real-world hardware experiments on a miniature robotic racecar (e.g., [31]).

Finally, our evaluation setup was based on an ablation study, specifically chosen to illustrate the benefits of incorporating topological features in the inference mechanism. Although we did not compare our approach against baselines from the literature, we see our framework as a significant complement and extension of alternative approaches. Topological features could augment and improve the performance of existing belief-space approaches [5], reinforcement learning techniques [12] or prosocial control frameworks [17, 29].
ACKNOWLEDGMENT

This work was partially funded by the National Science Foundation NRI (award IIS-1748582).

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