Nuclear Decay Parameter Oscillations as Possible Signal of Quantum Nonlinearity

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Abstract

Several experimental groups reported the evidence of multiple periodic modulations of nuclear decay constants which amplitudes are of the order $10^{-3}$ and periods of one year, 24 hours or about one month. We argue that such deviations from radioactive decay law can be described in nonlinear quantum mechanics framework, in which decay process obeys to nonlinear Shroedinger equation with Doebner-Goldin terms. Proposed corrections to Hamiltonian of quantum system interaction with gravitation field correspond to some emergent gravity theories, in particular, bilocal gravity model. Decay parameter variations under influence of Sun gravity, calculated in our model, agree well with experimental results for $\alpha$-decay life-time oscillations of Polonium isotopes.

1 Introduction

Natural radioactivity law is one of most fundamental laws of modern physics, in accordance with it, nuclear decay parameters are time-invariant and practically independent of environment [1]. However, some recent experiments have reported the evidence of periodic modulations of nuclear $\alpha$– and $\beta$–decay parameters of the order of $10^{-3}$ and with typical periods of one year, one day or about one month [2-8]. Possible mechanism of such decay parameter oscillations is still unclear, explanations proposed until now don’t look convincing [3]. Therefore, it’s sensible to start the effect analysis from reconsideration of nuclear decay fundamentals. In this paper, these oscillation effects studied in the framework of quantum-mechanical theory of unstable system decay. It follows from our analysis that standard quantum formalism can’t explain the observed parameter oscillations. However, it will be argued that nonlinear modifications of quantum mechanics, extensively studied in last years [9-12], in particular, nonlinear interactions of quantum systems with gravity, presumably can describe the decay parameter variations with the similar features.

First results, indicating the essential deviations from exponential $\beta$-decay rate dependence, were obtained during the precise measurement of $^{32}$Si isotope life-time [2]. Sinusoidal annual oscillations with the amplitude $6 \times 10^{-4}$ relative to total decay rate and maxima located at the end of February were registrated during 5 years of measurements. Since then, the annual oscillations of $\beta$-decay rate for different heavy nuclei from Ba to Ra were reported, for most of them the oscillation amplitude is of the order $5 \times 10^{-4}$ with its maximum on the
average at mid-February [3]. Annual oscillations of $^{238}\text{Pu}$ $\alpha$-decay rate with the amplitude of the order $10^{-3}$ also were reported [4]. Life-time of short-living $\alpha$-decayed isotopes $^{214}\text{Po}$, $^{213}\text{Po}$ was measured directly, the annual and daily oscillations with amplitude of the order $9 \times 10^{-4}$, with annual maxima at mid-March and daily maxima around 6 a.m. were found during five years of measurements [5]. Some other effects related to nucleus decay oscillations also described in the literature. Annual and daily oscillation periods were found in the studies of $^{239}\text{Pu}$ $\alpha$-decay statistics [7, 8]. Small oscillations of decay electron energy spectra with period 6 months were found in Tritium $\beta$-decay [6]. Some other $\beta$-decay experiments exclude any decay constant modulations as large as reported ones [14, 15].

Until now theoretical discussion of these results had quite restricted character. In particular, oscillations of $\beta$-decay rate was hypothized as anomalous interaction of Sun neutrino flux with nuclei or seasonal variations of fundamental constants [3]. Yet, neither of these hypothesis can explain $\alpha$-decay parameter oscillations of the same order, because nucleus $\alpha$-decay should be insensitive to neutrino flux or other electro-weak processes. Really, $\alpha$- and $\beta$-decays stipulated by nucleon strong and weak interactions correspondingly. Therefore, observations of parameter oscillations for both decay modes supposes that some universal mechanism independent of particular type of nuclear interactions induces the decay parameter oscillations.

Nowadays, the most universal microscopic theory is quantum mechanics (QM), so it’s worth to start the study of these effects from reminding quantum-mechanical description of nucleus decay [16]. In its framework, radioactive nucleus treated as the metastable quantum state, evolution of such states was the subject of many investigations and its principal features are now well understood [17]. However, due to serious mathematical difficulties, precise calculations of decay processes aren’t possible, and due to it, the simple semi-qualitative models are used. In particular, some decays modes of heavy nuclei can be effectively described as the quantum tunneling of decay products through the potential barrier constituted by nuclear shell and nucleus Coulomb field. The notorious example, is Gamow theory of $\alpha$-decay which describes successfully its main features, as well as some other decay modes [18, 19]. However, in its standard formulation, Gamow theory excludes any significant variations of decay parameters under influence of Sun gravity and similar factors. In this paper, it’s argued that such influence can appear, if one applies for $\alpha$-decay description the nonlinear modification of standard QM, which developed extensively in the last years [9, 10]. In particular, we shall use Doebner-Goldin nonlinear QM model for the description of gravity influence on nucleus decay parameters [11, 12]. Basing on its ansatz, Gamow $\alpha$-decay theory with nonlinear Hamiltonian corrections will be constructed, its model calculations compared with experimental results for $^{214}\text{Po}$ $\alpha$-decay life-time variations [5]. In this framework, observed influence of Sun gravity on nuclear decay parameters corresponds to some emergent gravity theories [13].

2 Nonlinear QM model

Interest to nonlinear evolution equations can be dated back to the early days of quantum physics, but at that time they appeared in effective theories describing the collective effects [16]. Now it’s acknowledged that nonlinear corrections to standard QM Hamiltonian can exist also at fundamental level [20, 21]. Significant progress in the studies of such nonlinear QM formalism was achieved in the 80s, marked by the seminal papers of Bialynicki-Birula
and Mycielcki (BM), and Weinberg [9, 10]. Since then, many variants of nonlinear QM formalism were considered in the literature (see [12] and refs. therein). Some experimental tests of QM nonlinearity were performed, but they didn’t have universal character, rather they tested Weinberg and BM models only [21].

Currently, there are two different approaches to the nature of QM nonlinearity. The main and historically first one supposes that dynamical nonlinearity is universal and generic property of quantum systems [9, 10]. In particular, it means that nonlinear evolution terms can influence their free motion, inducing soliton-like corrections to standard QM wave packet evolution [9]. Alternative concept of QM nonlinearity which can be called interactive, was proposed by Kibble, it postulates that free system evolution should be principally linear, so that nonlinear dynamics related exclusively to the system interactions with the fields or field self-interaction [22]. Until now, detailed calculations of such nonlinear effects were performed only for hard processes of particle production in the strong fields [23, 24], this formalism can’t be applied directly to nonrelativistic processes, like nuclear decay. Due to it, to describe the interaction of metastable state with external field, we’ll start from consideration of universal nonlinear models and develop their modification, which can incorporate the nonlinear particle-field and nucleus-field interactions at low energies.

In the universal approach to QM nonlinearity, it supposed that the particle evolution described by nonlinear Schroedinger equation of the form [16]

\[
i\hbar \partial_t \psi = -\frac{\hbar^2}{2m} \nabla^2 \psi + V(\vec{r}, t)\psi + F(\psi, \bar{\psi})\psi
\]  

(1)

where \( m \) is particle mass, \( V \) is external potential, \( F \) is arbitrary functional of system state. Currently, the most popular and elaborated nonlinear QM model of universal type is by Doebner and Goldin (DG) [11, 12], in its formalism, the simplest variant of nonlinear term

\[
F = \hbar^2 \lambda \left( \nabla^2 + \frac{\nabla |\psi|^2}{|\psi|^2} \right)
\]

(2)

where \( \lambda \) is imaginary constant. With the notation

\[
H_0 = -\frac{\hbar^2}{2m} \nabla^2 + V(\vec{r}, t)
\]

we abbreviate (1) to \( i\hbar \partial_t \psi = H_0 \psi + F\psi \). In fact, general DG model describes six-parameter family of nonlinear Hamiltonians, but the action of all its nonlinear terms on realistic quantum systems is similar to \( F \) of (2), hence for the start only this ansatz will be used in our calculations [12]. The choice of \( \lambda \) of (2) to be imaginary was prompted by results of nonrelativistic current algebras [11], but they doesn’t have mandatory character; below we’ll consider \( F \) terms both with imaginary and real \( \lambda \).

Main properties of eq. (1) for imaginary \( \lambda \) were studied in [11], they can be promptly extended on real \( \lambda \) and summarized for both cases as follows: (a) The probability is conserved. (b) The equation is homogeneous. (c) The equation is Euclidian- and time-translation invariant ( for \( V = 0 \)). (d) Noninteracting particle subsystem remain uncorrelated (separation property). Distinct values of \( \lambda \) can occur for different particle species. (e) Writing \( \langle Q \rangle = \int \bar{\psi} Q \psi d^3x \) for operator expectation value, in particular, since \( \int \bar{\psi} F \psi d^3x = 0 \), the energy functional for a solution of (1) is \( \langle i\hbar \partial_t \rangle = \langle H_0 \rangle \), this property is quite important for our model. In this framework, it follows for \( \vec{p} = -i\hbar \nabla \) and imaginary \( \lambda \),

\[
\frac{d}{dt} \langle \vec{r} \rangle = \langle \vec{p} \rangle \frac{\hbar}{m},
\]
whereas for real \( \lambda \),
\[
\frac{d}{dt} \langle \vec{r} \rangle = \frac{1 - 2\lambda m}{m} \langle \vec{p} \rangle
\]
For \( V = 0 \), plane waves \( \psi = \exp[i(k_0 \vec{r} - \omega t)] \) with \( \omega = E\hbar \), \( |k_0|^2 = 2mE/\hbar \) are solutions both for real and imaginary \( \lambda \). For real \( \lambda \), QM continuity equation for probability density \( \rho \) fulfilled, but density current acquires the form
\[
\vec{j} = \frac{\hbar(1 - 2\lambda m)}{2mi}(\bar{\psi} \nabla \psi - \psi \nabla \bar{\psi})
\]
For imaginary \( \lambda \) the continuity equation becomes of Fokker-Plank type [11].

As was mentioned above, the simple quantum model of metastable state decay describes it as the particle tunneling through the potential barrier with suitable parameters [16]. It’s natural to expect that for small \( |\lambda| \) the tunneling mechanism doesn’t change principally, and resulting difference from standard QM solutions is small. Hence the nonlinear solutions can be treated as the perturbations to linear solutions for the same system parameters. Exploiting this auxiliary assumption, we’ll find some unnormalised partial solutions of nonlinear problem. Basing on them, we’ll obtain the exact solution of nonlinear problem for practically important situations, which is independent of mentioned auxiliary assumption. To illustrate the influence of nonlinear DG term on particle tunneling, consider 1-dimensional plane wave tunneling through the potential barrier. Suppose that the rectangular barrier of the height \( V_0 \) located between \( x = 0 \) and \( x = a \), and the plane wave with energy \( E < V_0 \) spreads from \( x = -\infty \). Long-living metastable states appear for small transmission coefficient \( D \to 0 \), which corresponds to barrier width \( a \to \infty \) for fixed \( E, V_0 \). For example, for \(^{238}\text{U} \alpha\)-decay \( D \approx 10^{-37} \) [18]. We’ll study solutions of stationary equation (1), basing on its asymptotic in this limit. Standard QM stationary solution for \( x < 0 \)
\[
\psi_0(x) = \exp(ikx) + A \exp(-ikx)
\]
with \( k = \frac{1}{\hbar}(2mE)^{\frac{1}{2}} \); for \( a \to \infty \) it gives \( |A| \to 1 \), i.e. it describes nearly complete wave reflection from the barrier. Hence \( \psi_0 \) can be decomposed as \( \psi_0 = \psi_\infty + \psi_d \) where the asymptotic state \( \psi_\infty = 2\cos(kx - \alpha_0) \), \( \alpha_0 = \arctan \chi_0/k \) where
\[
\chi_0 = \frac{1}{\hbar}[2m(V_0 - E)]^{\frac{1}{2}}
\]
Then, \( \psi_d = A_d \exp(-ikx) \) where \( |A_d| \simeq \exp(-2\chi_0 a) \), i.e. is exponentially small. In distinction, for nonlinear equation
\[
E\psi = H_0\psi + F\psi
\]
the incoming and reflected waves suffer the rescattering, hence the stationary state \( \psi \neq \psi_0 \) for \( x < 0 \). In our case, for real \( \lambda \), the stationary solution can be obtained performing Ricatti transformation of solution of adjoined linear equation [11, 12]. Namely, for real solution \( \eta(x) \) of such Schrodinger equation the solution of corresponding nonlinear equation
\[
\psi = \eta \exp(\nu \ln \eta^2)
\]
where \( \nu = \gamma(1 - 4\gamma)^{-1} \) and \( \gamma = \lambda m \). Below for brevity such exponential ansatz is replaced by corresponding function rate. In particular, the asymptotic solution for \( x \leq 0 \) and \( a \to \infty \) can be written as
\[
\psi_N = A_N[\cos(qx - \alpha)]^{\frac{1-2\gamma}{1+4\gamma}}
where
\[ q = \frac{1}{\hbar} \frac{[2mE(1 - 4\gamma)]^{\frac{3}{2}}}{1 - 2\gamma} \]
and
\[ A_N = 2^{\frac{1}{2}} \pi^{\frac{1}{4}} \Gamma^{-\frac{1}{2}}(\omega) \]
where \( \omega = \frac{2 - 6\gamma}{1 - 4\gamma} \); for small \( |\gamma| \), \( \alpha \approx \alpha_0 \). Plainly, \( q \approx k + \alpha(\gamma^2) \), hence below for small \( |\gamma| \) they are taken to be equal. For imaginary \( \lambda \) the corresponding nonlinear transformation given in [11, 12], however, even for complete wave reflection from the barrier the consistent asymptotic solution for \( \psi_N \) doesn’t exist, because \( \psi_N \) phase singularities appear at its nodes. In this case, the linear QM solution \( \psi_\infty(x) \) for the same system parameters can be used as its approximation. For \( x > a \), both for real and imaginary \( \lambda \), the solution is
\[ \psi(x) = C_+ \exp(ikx) \approx C_+ \exp(iqx), \]
To calculate the tunneling parameters, we need to use \( \psi_N \) only for \( x \to 0 \), in this limit
\[ \psi_N(x) \approx A_N[1 + \frac{\gamma}{1 - 4\gamma} \cos(qx - \alpha)] \]
for finite but large \( a \) the correction to it can be taken to be equal to \( \psi_d \), i.e. \( \psi = \psi_N + \psi_d \). Next, it’s necessary to find the partial solution for \( 0 < x < a \) with \( \psi(a) \to 0 \) for \( a \to \infty \), which should be the main term of tunneling state. For real \( \lambda \), such solution of eq. (4) is
\[ \psi_1(x) = B_1 \exp(-\chi x) \]
where
\[ \chi = \frac{1}{\hbar} \frac{[2m(V_0 - E)]^{\frac{3}{2}}}{(1 - 4\gamma)^{\frac{1}{2}}} \]
Due to nonlinearity, the superpositions of two terms \( \psi_{1,2} \), in general, aren’t solutions. Analytic solutions, which correspond to such superpositions, exist in two cases only, defined by \( b_s = B_2/B_1 \) ratio. First, for imaginary \( b_s \) the solution is just \( \psi = \psi_1 + \psi_2 \); second, for \( b_s \) real
\[ \psi(x) = B_1^{\frac{1 - 2\gamma}{4\gamma}} [\exp(-\chi_b x) + b_s \exp(\chi_b x)]^{\frac{1 - 2\gamma}{1 - \chi_b}} \]
where
\[ \chi_b = \frac{1}{\hbar} \frac{[2m(V_0 - E)(1 - 4\gamma)]^{\frac{3}{2}}}{1 - 2\gamma} \]
Meanwhile, for typical \( \alpha \)-decay parameters \( E \approx V_0/2 \), it corresponds to \( \chi, q \) values such that \( \chi \approx q \), hence we’ll consider mainly such parameter range. It follows that for \( \chi = q \), there is the solution of nonlinear problem with \( \psi = \psi_1 + \psi_2 \) where
\[ B_1 = -\frac{2q(q - i\chi)}{(q^2 + \chi^2)^2 + (q - i\chi)^2 \exp(-2\chi a)} \]
and
\[ B_2 = B_1 \frac{\chi^2 - q^2 + 2iq\chi}{\chi^2 + q^2} \exp(-2\chi a) \]
so that \( b_s \) is imaginary. \( A_N, A_d, \alpha, \) and \( C^+ \) can be calculated from \( \psi \) continuity conditions for 0, \( a \). It’s notable that \( B_{1,2} \) dependence on \( \chi, q \) coincide with the formulas for linear Hamiltonian [16]. Denote

\[
d_s = \frac{q^2 - \chi^2}{q^2 + \chi^2}
\]

Then, for \( |d_s| \ll 1 \) the obtained \( \psi \) ansatz can be used as first order approximation for nonlinear problem with \( \chi \neq q \). If to denote the problem solution as \( \psi_s \), then the next order term \( \varphi^1 \) for \( \psi_s = \psi + \varphi^1 \) obeys to linear equation

\[
\left( \frac{1}{2} - \gamma \right) \nabla^2 \varphi^1 - m(V_0 - E + \lambda \chi^2)\varphi^1 + \frac{4\gamma \chi^2 d_s \exp(-2\lambda a)\psi}{\exp(-2\lambda x) + \exp(2\lambda x - 4\lambda a) - 2d_s \exp(-2\lambda a)} = 0
\]

For \( \psi \) obtained above, this equation can be solved exactly. Resulting \( \psi_s \) can be extended further as \( \psi_s = \psi + \varphi^1 + \varphi^2 + \ldots \) to higher \( \gamma \) rate \( \varphi^2, \varphi^3, \ldots \) terms, which also would obey to linear equations. In this case, the resulting transmission coefficient \( D_s \approx 2D_1 \), so that \( D_s \) also has the exponential dependence on \( \lambda \).

For imaginary \( \lambda \) and small \( \gamma \), the main term \( \psi_1 = B_1 \exp(-\chi_0 \varpi x) \) where

\[
\varpi = \frac{1 + 2m\lambda}{(1 + 4m^2|\chi|^2)^{\frac{1}{2}}}
\]

Transmission coefficient for main term is equal to \( D_1 = |B_1|^2 \exp(-2\chi_0 \nu a) \) where \( \nu = \text{Re} \varpi \). It supposes that \( D_1 \) dependence on \( \lambda \) is less pronounced than for real \( \lambda \). Then, the secondary term \( \psi_2 = B_2 \exp(\chi_0 \varpi x) \). Both for real and imaginary \( \lambda \), \( C^+ = \psi(a) \exp(-ika) \), which defines \( \psi(x) \) for \( x > a \). Analogous considerations permit to derive next order corrections \( \varphi^2, \varphi^3, \ldots \) to \( \psi \) for imaginary \( \lambda \).

It’s noteworthy that considered nonlinear Hamiltonian term \( F \) influences mainly the transitions between degenerate states, as property (e) demonstrates. Due to it, tunneling transmission coefficients and related decay rates are sensitive to the presence of nonlinear terms in evolution equation. Therefore, the experimental studies of such process parameters can be important method of quantum nonlinearity search.

### 3 \( \alpha \)-decay oscillation model

Gamow theory of nucleus \( \alpha \)-decay supposes that in the initial nuclei state, free \( \alpha \)-particle already exists inside the nucleus, but its total energy \( E \) is smaller than maximal height of potential barrier \( V(r) \) constituted by nuclear forces and Coulomb potential [18]. Hence \( \alpha \)-particle can leave nucleus volume only via quantum tunneling through this barrier. For real nucleus, The barrier potential isn’t rectangular, but has complicated form described by some function \( V(r) \) defined experimentally [18, 19]. In this case, to calculate transmission rate in our model, WKB approximation for Hamiltonian of eq. (1) with nonlinear term of eq. (2) was used [16]; its applicability to our nonlinear Hamiltonian can be easily checked. The calculations described here only for real \( \lambda \), for imaginary \( \lambda \) they are similar, it supposed now that, in principle, it can depend on time, i.e. \( \lambda = \lambda(t) \). In this ansatz, 3-dimensional \( \alpha \)-particle wave function reduced to \( \psi = \frac{1}{\sqrt{T}} \exp(i\sigma/h) \); function \( \sigma(r) \) can be decomposed in \( \hbar \) order \( \sigma = \sigma_0 - i\hbar \sigma_1 + \ldots \) [16]. Neglecting \( \sigma_1, \ldots \) and higher terms, the equation for main term \( \sigma_0 \)

\[
\left( \frac{1}{2m} - \lambda \right)[(\frac{\partial \sigma_0}{\partial r})^2 - i\hbar \frac{\partial \sigma_0}{\partial r^2}] - \lambda |\frac{\partial \sigma_0}{\partial r}|^2 = E - V(r)
\]
Given $\alpha$-particle energy $E$, one can find the distances $R_0, R_1$ from nucleus centre at which $V(R_{0,1}) = E$. Then, neglecting the term proportional to $\hbar$ this equation becomes

$$
\left( \frac{1}{2m} - \Lambda(r) \right) \left( \frac{\partial \sigma_0}{\partial r} \right)^2 = E - V(r)
$$

(9)

where $\Lambda(r) = 2\lambda$ for $R_0 \leq r \leq R_1$, $\Lambda(r) = 0$ for $r < R_0, r > R_1$. Its solution for $R_0 \leq r \leq R_1$ can be written as

$$
\psi = \frac{1}{r} \exp(i\sigma_0/\hbar) = \frac{C_r}{r} \exp\left[ -\frac{1}{\hbar} \int_{R_0}^r p(\epsilon) d\epsilon \right]
$$

where $C_r$ is normalization constant,

$$
p(\epsilon) = \frac{1}{\hbar} \left[ \frac{2m(V(\epsilon) - E)}{1 - 4\gamma} \right]^{1/2}
$$

where $\gamma(t) = m\lambda(t)$. Account of higher order $\sigma$ terms doesn’t change transmission coefficient which is equal to

$$
D = \exp\left[ -\frac{1}{\hbar} \int_{R_0}^{R_1} p(\epsilon) d\epsilon \right] = \exp\left[ -\frac{\phi}{(1 - 4\gamma)^{1/2}} \right]
$$

(10)

here $\phi$ is constant, whereas $\gamma$ can change in time. For imaginary $\lambda$ the calculations result in the same $D$ ansatz, but with

$$
p(r) = \frac{1}{\hbar} \left[ \frac{2m(V - E)}{1 + 4m^2|\lambda|^2} \right]^{1/2}
$$

To calculate the nucleus life-time, $D$ should be multiplied by the number of $\alpha$-particle kicks $n_d$ into potential barrier per second [18].

To study the decay parameter variations in external field, we’ll suppose now that non-linear Hamiltonian term $F$ depends on external field. In our model, such field is gravity, characterized by its potential $U(\vec{R}, t)$. In this case, $U$ should be accounted in evolution equation twice. First, $mU$ should be added to $H_0$, so that it changed to $H'_0 = H_0 + mU$; second, nonlinear $H$ term $F$ can depend on $U$ or some its derivatives. For minimal modification of DG model we’ll assume that for $F$ ansatz of (2) its possible dependence on external field is restricted to parameter $\lambda$ dependence: $\lambda = f(U)$, so now $\lambda$ isn’t constant, but the function of $\vec{R}$ and $t$. It supposed also that $f \to 0$ for $U \to 0$, so that the free particle evolution is linear.

Considered model doesn’t permit to derive $\lambda$ dependence on Sun gravity, but it can be obtained from its comparison with experimental results for $^{214}$Po $\alpha$-decay [5]. We’ll suppose that $\lambda$ is function of potential $U(\vec{R}_n, t)$ where $\vec{R}_n$ is nucleus coordinate in Sun reference frame (SRF). As follows from eq. (10) for small $\lambda$

$$
D \approx (1 + 2\phi \gamma) \exp(-\phi)
$$

For $^{214}$Po decay, its life-time $\tau_0 = 16.4 \times 10^{-6}$ sec, model estimate gives $\phi \approx 60$. For annual $\tau$ variation the best fit for 3 year exposition has main harmonics

$$
\tau_a(t) = \tau_0 \left[ 1 + A_a \sin(w_a(t + \varphi_a)) \right]
$$

where $t$ defined in days, $A_a = 9.8 \times 10^{-4}$, $w_a = 2\pi/365$, $\varphi_a = 174$ days [5]. Remind that Earth orbit is elliptic, the minimal distance from Sun is at about January 3 and maximal
at about July 5, maximal/minimal orbit radius difference is about $3 \times 10^{-2}$ [25]. Plainly, the minima and maxima of $U$ time derivative $\partial_t U$ will be located approximately in the middle between these dates, i.e. about April 5 and October 3, correspondingly. In general, this dependence described as

$$\partial_t U = K^a \sin w_a(t + \varphi_u)$$

here $\varphi_u = 185$ days, $K^a = 1.5 \, m^2/sec$, as the result, such model $\varphi_u$ value in a good agreement with experimental $\varphi_u$ value. Thereon, it means that the plausible data fit is $\lambda(t) = g\partial_t U$, where $g$ is interaction constant, which can be found from the data for $^{214}$Po decay. It follows from the assumed equality of oscillation amplitudes $A_a = 2\phi mgK^a$ that the resulting $g = .35 \times 10^{-8} \frac{sec^2}{m^2 MeV}$.

Another experimentally found harmonic corresponds to daily variations with best fit

$$\tau_d(t) = \tau_0[1 + A_d \sin w_d(t + \varphi_d)]$$

where $t$ defined in hours, $A_d = 8.3 \times 10^{-4}$, $w_d = 2\pi/24$, $\varphi_d = 12$ hours [5]. Such oscillation can be attributed to variation of Sun gravity due to daily lab. rotation around Earth axe. It’s easy to check that nucleus life-time dependence also coincides with $\partial_t U$ time dependence with high precision. Really, it described as

$$\partial_t U = K^e \sin w_d(t + \varphi_e)$$

with $\varphi_e = 12$ hours, $K^e = .9 \, m^2/sec$ [25]. It follows that $A_d = 2\phi mgK^e$; if to substitute in this equality $g$ value, calculated above, it gives $A_d = 5.5 \times 10^{-4}$, which is in a reasonable agreement with its experimental value. It’s possible also that $\tau_{a,d}$ can depend on some other orbit parameter or $U$ derivative, in particular, on lab. velocity in SRF or some absolute reference frame [5]; such options will be considered elsewhere.

4 Nonlinearity, Nonlocality and Causality

In this paper, we studied hypothetical nonlinear corrections to standard QM description of system interaction with external fields. It’s notable that in nonrelativistic QM, the ansatz for system interaction with massless fields, such as gravity and electromagnetism, is stipulated, in fact, by Bohr quantum-to-classical correspondence principle [20, 26]. However, in the last sixty years many new physical concepts were discovered, which presumably are independent of it. The illustrative example is, in our opinion, quantum chromodynamics, the theory derived just from experimental facts with no reference to correspondence principle [27]. In the same vein, there is no obvious prohibition on the existence of additional Hamiltonian terms for the system interaction with massless fields. Such terms can have strictly quantum origin and disappear in classical limit, their existence should be verified in dedicated experiments. To study their general features, we considered the simple nonrelativistic model, which includes the additional terms for the interaction of quantum systems with gravitational field. Account of these terms permits to describe with the good accuracy the annual and daily oscillations of $^{214}$Po $\alpha$-decay parameters observed in the experiment.

It was argued earlier that QM nonlinearity violates relativistic causality for multiparticle systems, in particular, it permits the superluminal signaling for EPR-Bohm pair states [28, 29]. However, this conclusion was objected and still disputed [21]. Plainly, these arguments would be even more controversial, if nonlinear effects exist only inside the field volume.
In particular, the metastable state in external field can be considered as the open system, yet it was shown that the superluminal signaling between such systems is impossible [30]. Moreover, heavy nucleus is strongly-bound system, so it’s unclear whether it’s possible to prepare the entangled state of two $\alpha$-particles located initially inside two different nuclei. It was shown that in our model the standard relation between average system momentum and velocity can be violated and differ from particle mass. However, because in our model this ratio depends on external gravitational field, then for Sun gravity influence it can depend on time of day or year season, hence its tests demand the special subtle experiments.

It was proposed by many authors that gravity is emergent (induced) theory and originates, in fact, from some nonlocal field theory (see [13] and refs therein). In this framework, it was supposed that gravity can be effectively described by multilocal (collective) field $\Phi_n(x_1, ..., x_n)$ or the array of such field modes $\{\Phi_1, ..., \Phi_n\}$. It was shown that bilocal scalar field $\Phi_2$ reproduces the classical gravity effects up to the second order [13]. Such bilocal field $\Phi_2$ presumably can interact with bilocal operators of massive fields, in particular, such field can be the nonrelativistic particle system. Such interaction doesn’t violate causality, if for the pair of separated quantum objects it influences only their bilocal observables of EPR-Bohm type [20, 28]. The simple example of such observable is the spin projection difference for two fermions.

In our phenomenological model, it assumed that in infrared limit the gravity effectively described by two terms $\{\Phi_1, \Phi_2\}$ where $\Phi_1 = U(\vec{r})$ is standard Newtonian potential. Denote as $\vec{r}_1$ the coordinate of $\alpha$-particle, $\vec{r}_2$ the coordinate of remnant nucleus centre of mass, and $\vec{r}_s = \vec{r}_1 - \vec{r}_2$. For considered $\alpha$-decay model, the joint state of remnant nucleus and $\alpha$-particle is entangled, their bilocal observable $\vec{r}_s$ is of EPR-Bohm type. It’s notable that it’s equivalent to the basic coordinates of bilocal field which described as $\Phi_2(\vec{R}_a, \vec{r}_s)$ where $\vec{R}_a = \frac{1}{2}(\vec{r}_1 + \vec{r}_2)$ [13]. If gravity field is local then for $|\vec{r}_s| >> |\vec{R}_n|$ it will act mainly on nucleus total state $\Psi(\vec{R}_n)$, its influence on on nucleus internal state $\psi(\vec{r}_s)$ will be negligible. Only bilocal field can change it, and as follows from our analysis of $^{214}$Po $\alpha$-decay data, it’s plausible that for bilocal scalar field $\Phi_2 \sim \partial_t U$. As was argued above, the nonlinear term should act on $\vec{r}_s$, hence assuming that $\Phi_2$ factorized from such operator, it follows

$$F \sim \partial_t U G(\frac{\partial}{\partial \vec{r}_s})$$

where $G$ is some operator function. In this case, the analogue of D-G ansatz will contain variable mutiplier $k_b \partial_t U$ in place of fixed $\lambda$, where $k_b$ is arbitrary constant. Then, in distinction from initial nonlinear term of eq. (2), the corresponding nonlinear term of our Hamiltonian for nucleus $\alpha$-decay becomes

$$F = k_b \partial_t U(\vec{R}_n)(\frac{\partial^2}{\partial \vec{r}_s^2} + \frac{1}{|\psi|^2} \frac{\partial \psi}{\partial \vec{r}_s} \frac{\partial \psi}{\partial \vec{r}_s})$$

where $k_b$ is an arbitrary constant. It means that $\Phi_2$ interaction with the nucleus described by nonlinear operator; as the result, $\alpha$-particle transmission coefficient $D(t)$ can oscillate around its constant value defined by Gamow theory. For large $|\vec{r}_s|$ it can be supposed that $\partial_t U(\vec{R}_n)$ should be replaced by

$$\Phi_2 = \frac{1}{2\pi} \{\partial_t U(\vec{r}_1)\partial_t U(\vec{r}_2)[\partial_t U(\vec{r}_1) + \partial_t U(\vec{r}_2)]\}^\frac{1}{2}$$  

(11)
As follows from equivalence principle, in lab. reference frame, located on Earth surface, Sun gravitation potential $U'(\vec{R}_c) \approx 0$, yet $\partial_t U' \neq 0$. It means that nuclear decay process violates equivalence principle, however, some theories of emergent (induced) gravity predict that it can be violated in quantum processes at small scale [13, 31]. In addition, other results for $^{214}\text{Po}$ $\alpha$-decay seems to support such conclusion. Namely, beside described lifetime oscillations, these data contains also harmonics with period 24 hours 50 minutes, which is equal to lunar day duration and so can be related to well-known moon gravity effects [5]. Studies in quantum gravity supposes that this theory can be similar to QFT with massless messenger called graviton ([13, 31] and refs. therein). Notorious example of massless messenger formalism gives QED, it’s well known yet that in its nonrelativistic limit there are some electromagnetic effects, like Casimir effect or Lamb shift, which can’t be described by Schrödinger equation, but only via accounting higher order QED terms [27]. It seems possible that the observed decay oscillations can have analogous origin corresponding to the nonzero infrared limit of some hypothetical quantum gravity terms, which can appear, in particular, if gravity is nonperturbative theory.

Considered QM nonlinearity supposedly has universal character, so beside nucleus decays, such temporary variations under influence of Sun gravity can be observed, in principle, for other systems in which metastable states and tunneling play important role. In particular, it can be some chemical reactions, molecular absorption by solids and liquids, etc. It’s worth to notice specially its possible role in some biological processes. Multiple publications indicate that some cosmophysical effects influence also biological system development and functioning (see [32, 33] and refs. therein). Of them, it’s worth to notice specially the observed influence of moon tide gravity variations $\delta g$ on seedling bioluminescence rate and tree stem diameter variations [34, 35]. There is no consistent explanation up to now how such small gravitational force variations $\delta g \sim 10^{-7}$ can seriously affect such subtle biological processes. Its worth to notice specially that bioluminescence data show the essential intensity dependence on $\delta g$ time derivative [35, 37]. Meanwhile, as was shown above, our model of gravitational field interaction with quantum systems predicts similar gravity influence on arbitrary quantum systems proportional to $U$ time derivatives. It’s established now that such bioluminescence stipulated mainly by biochemical reactions of protein oxidation [35, 36]. Hence it can be assumed that observed $\delta g$ time derivative dependence owed to such nonlinear gravity interaction with molecular states involved in these reactions.

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