The duality-invariant gaugino condensation with or without massive matter fields is re-analysed, taking into account the dependence of the string threshold corrections on the moduli fields and recent results concerning one-loop corrected Kähler potentials. The scalar potential of the theory for a generic superpotential is also calculated.
The understanding of supersymmetry breaking is crucial to make contact between four-dimensional superstrings and low-energy physics. Recently, the duality-invariant gaugino condensation mechanism [1-3] provided substantial information about non-perturbative supersymmetry breaking within the context of low-energy effective string Lagrangians. In particular, important insight in the related problem of the dynamical determination of the moduli parameters, i.e. the lifting of the huge vacuum degeneracy of string compactifications, was gained. These results are essentially based on the field-theory one-loop running of the effective gauge coupling constant in the hidden sector, focusing in addition on the moduli-dependent string threshold effects [4], which are due to the presence of heavy string modes. This analysis, although leading to qualitatively correct results in most of the known cases, appears to be slightly incomplete, since loop corrections to the Kähler potential [5] were not taken into account. Specifically we will discuss the duality-invariant gaugino condensation mechanism in a pure Yang–Mills hidden sector for orbifold compactifications [6]. We will also calculate the scalar potential of the theory for a generic superpotential using the one-loop Kähler function. As a result we will find that the use of the loop corrected Kähler potential can be regarded as a simple redefinition of the tree-level gauge coupling constant. In turn this redefinition will not qualitatively affect the previous results about the dynamical supersymmetry breaking and the determination of the moduli parameters, except when those moduli are associated with complex planes rotated by all orbifold twists. In these cases, to achieve the final goal that all moduli acquire expectation values dynamically, determining the size of the compactified space, one should also include [7], [8] the moduli-dependent (twisted) Yukawa couplings as an extra piece in the superpotential. Finally, we will discuss duality-invariant gaugino condensation in hidden sectors with massive matter fields. This was previously studied in refs. [9] and [10] only for untwisted matter fields and without taking into account the above-mentioned corrections. As for the pure gauge case, the qualitative results are not affected, except for the fact that for twisted massive matter fields the moduli-dependent Yukawa couplings explicitly appear in the gaugino condensate.

Let us start by recalling some recent results on the effective $N = 1$ supergravity Lagrangian for orbifold compactifications. Apart from the gravitational
supermultiplet and gauge vector supermultiplets of the gauge group $G$, we first consider as the relevant massless string degrees of freedom the dilaton–axion chiral field $S$ and the three internal moduli fields $T_i$ ($i = 1, 2, 3$), whose real parts determine the sizes of the three underlying two-dimensional complex planes and whose imaginary parts are given by three internal axion fields: $T_i = R_i^2 + i B_i$. These three moduli fields are present in any Abelian $\mathbb{Z}_M$ and $\mathbb{Z}_M \times \mathbb{Z}_N$ orbifold compactification. Finally we consider also additional matter fields $C_\alpha$ in the $R_\alpha$ representation of $G$, which can be either massless or massive (depending on the form of the superpotential to be discussed in the following).

The couplings of the chiral fields are determined by the real function $G(\phi, \bar{\phi})$ [11], which is a combination of the Kähler potential and the holomorphic superpotential: $G(\phi, \bar{\phi}) = K(\phi, \bar{\phi}) + \log |W(\phi)|^2$. At string tree level, the Kähler potential for $S, T$ and $C_\alpha$ has the simple form (at lowest order in the matter fields) [12], [13]: $K_{\text{tree}} = - \log(S + \bar{S}) - \sum_{i=1}^3 \log(T_i + \bar{T}_i) + \sum_{\alpha} C_\alpha \bar{C}_\alpha \prod_{i=1}^3 (T_i + \bar{T}_i)^{n_{i\alpha}}$. The lowest order superpotential is cubic in the matter fields: $W_{\text{tree}} = h_{\alpha\beta\gamma}(T_i) C_\alpha C_\beta C_\gamma$, where the moduli-dependent functions $h_{\alpha\beta\gamma}(T_i)$ [14] are called Yukawa couplings. Specifically, the Yukawa couplings $h_{\alpha\beta\gamma}$ are moduli-dependent functions if all fields $C$ are in the twisted sectors of the orbifold, otherwise $h_{\alpha\beta\gamma} = \text{const}$. For all matter fields being untwisted, the Yukawa couplings are non-zero if the fields belong to different complex planes. Finally, the couplings of the chiral fields to gauge vector fields is determined by the gauge kinetic function $f(\phi)$. At string tree level this function is just given by the $S$-field, $f = S$, such that the tree level gauge coupling constant of $G$ is determined by the vacuum expectation value of the dilaton field (we assume the Kac–Moody level to be one): $\frac{1}{g^2_{\text{tree}}} = \frac{S + \bar{S}}{2}$.

As discussed in ref. [15] the underlying (super) conformal field theories are invariant under the target space modular transformations acting on the complex moduli fields $T_i$ as $T_i \rightarrow \frac{a_i T_i - ib_i}{c_i T_i + d_i}$, with $a_i, b_i, c_i, d_i \in \mathbb{Z}$, $a_i d_i - b_i c_i = 1$. These transformations include the well-known duality transformations $R_i \rightarrow \frac{1}{R_i}$ as well as discrete shifts of the axion fields $B_i$. Thus, up to permutation symmetries, the generalized target duality symmetries $\Gamma$ are described by the product of three modular groups: $\Gamma = [SL(2, \mathbb{Z})]^3$. Also the matter fields transform in general non-trivially under target space modular transformations like (up to a possible
constant matrix) [16]:
\[ C_\alpha \rightarrow C_\alpha \prod_{i=1}^{3} (ic_i T_i + d_i)^{n_i^\alpha}. \quad (1) \]

Thus the numbers \( n_i^\alpha \) are called the modular weights of the matter fields. Specifically, untwisted matter fields associated to the \( j \)th complex plane, \( C^\text{untw} = C^j \), have modular weights \( n_i^\alpha = -\delta_j^i \). A detailed discussion about the range of values of the \( n_i \) corresponding to various types of twisted matter fields can be found in refs. [13], [17].

Since the spectrum and all interactions of the underlying conformal field theories are target space modular-invariant at each order in string perturbation theory, the effective string action has to be modular-invariant as well. As discussed in refs. [18] and [16] the requirement of target space modular invariance puts strong constraints on the form of the low-energy supergravity action involving the moduli fields \( T_i \), providing a link to the theory of automorphic functions. Since at tree level the S-field is invariant under target space modular transformations, the change of the Kähler potential under these transformations has the form
\[ K_\text{tree} \rightarrow K_\text{tree} + \sum_{i=1}^{3} \log |ic_i T_i + d_i|^2. \quad (2) \]

In order to obtain invariant matter couplings, the superpotential then has to transform as (up to a field independent phase) [18]
\[ W_\text{tree} \rightarrow \frac{W_\text{tree}}{\prod_{i=1}^{3} (ic_i T_i + d_i)}. \quad (3) \]

Therefore the Yukawa couplings have to transform as (up to possible constant matrices)
\[ h_{\alpha \beta \gamma}(T_i) \rightarrow h_{\alpha \beta \gamma}(T_i) \prod_{i=1}^{3} (ic_i T_i + d_i)^{(-1-n_i^\alpha - n_i^\beta - n_i^\gamma)}. \quad (4) \]

As mentioned in the introduction, loop corrections will modify in general the Kähler potential and the gauge coupling constant. However at finite order
in perturbation theory the superpotential is expected to be unchanged. If this non-renormalization theorem holds, it can be concluded that the modular transformation rule eq.(3) holds in every order of perturbation theory. Consequently, also the Kähler potential has to transform in any order of perturbation theory as is displayed in eq.(2). Furthermore, these transformation rules will still hold when taking into account the non-perturbative modification of the superpotential due to the gaugino condensate. Now let us discuss the one-loop modification of the Kähler potential and the gauge coupling constant for the case of a pure Yang–Mills gauge theory. Specifically, the one-loop Kähler potential is given by [5]

\[
K_{1\text{-loop}} = - \log \left[ (S + \bar{S}) - \frac{1}{8\pi^2} \sum_{i=1}^{3} \delta^i_{GS} \log(T_i + \bar{T}_i) \right] - \sum_{i=1}^{3} \log(T_i + \bar{T}_i)
\]

(5)

\[
= - \log Y - \sum_{i=1}^{3} \log(T_i + \bar{T}_i).
\]

\(K_{1\text{-loop}}\) now leads to a mixing between the \(S\) and the \(T_i\) fields. This one-loop mixing term with coefficient \(\delta^i_{GS}\) generalizes the Green–Schwarz mechanism [19] and cancels anomalies of the underlying non-linear \(\sigma\)-model [5], [20], [21], which are described by triangle diagrams with two external gauge bosons and several external moduli fields \(T_i\). As we will see in the following, the function \(Y = S + \bar{S} - \frac{1}{8\pi^2} \sum_{i=1}^{3} \delta^i_{GS} \log(T_i + \bar{T}_i)\) can be regarded as the redefined string (gauge) coupling constant at the unifying string scale \(M_{\text{string}}\): \(Y = \frac{2}{g^2_{\text{string}}}\). This fact will turn out to be important for the discussion of the gaugino condensation.

Now, for the one-loop Kähler potential to transform in the required way (2) under target space duality transformations the dilaton has to acquire a non-trivial modular transformation behaviour at the one-loop level [5]:

\[
S \rightarrow S - \frac{1}{8\pi^2} \sum_{i=1}^{3} \delta^i_{GS} \log(i c_i T_i + d_i).
\]

(6)

Next consider the one-loop corrections to the gauge coupling constant (up to
a small field independent contribution:

\[ \frac{1}{g_1^{2-\text{loop}}(\mu)} = \frac{Y}{2} + \frac{b_0}{16\pi^2} \log \frac{M_{\text{string}}^2}{\mu^2} \]

\[ \quad - \frac{1}{16\pi^2} \sum_{i=1}^{3} \left( \frac{b_0}{3} - \delta^i_{\text{GS}} \right) \log |\eta(T_i)|^4. \]  

(7)

Here \( b_0 \) is the usual \( N = 1 \) \( \beta \)-function coefficient, \( b_0 = -3C(G) \), where \( C(G) \) is the quadratic Casimir of the gauge group \( G \). The moduli-dependent last term in eq. (7) has two different sources. (Up to this term the gauge coupling constant at \( \mu = M_{\text{string}} \) is given by \( Y/2 \), which motivates the previous definition \( g_{\text{string}}^{-2} = Y/2 \).) First, at one loop the massless gauginos lead to the non-harmonic term proportional to \( \sum_{i=1}^{3} b_0/3 \log(T_i + \bar{T}_i) \). This term is directly related to the already mentioned \( \sigma \)-model anomalies [5],[20],[21]. Specifically, the \( \sigma \)-model anomalies and also the target space modular anomalies associated to a modulus \( T_i \) are completely cancelled by the Green–Schwarz counter term, which follows from eq. (5), if all orbifold twists act non-trivially on the corresponding \( i \)th complex plane of the underlying six-torus. Then one has \( \delta^i_{\text{GS}} = b_0/3 \). Examples for this situation are the \( \mathbb{Z}_3 \) and \( \mathbb{Z}_7 \) orbifolds, where \( \delta^i_{\text{GS}} = b_0/3 \) for all three complex planes or the \( \mathbb{Z}_4 \) orbifolds with \( \delta^i_{\text{GS}} = b_0/3 \) for two of the three complex planes.

On the other hand there is in principle no reason that the modular and \( \sigma \)-model anomalies have to be completely cancelled by the Green–Schwarz mechanism. In fact, in general one has \( \delta^i_{\text{GS}} \neq b_0/3 \) as it is for example true for \( T_3 \) of the \( \mathbb{Z}_4 \) orbifold and also for all three \( T_i \) considering general \( \mathbb{Z}_M \times \mathbb{Z}_N \) orbifolds. In those cases there is a second moduli-dependent contribution to the gauge coupling constant. This contribution describes just the one-loop threshold effects [4] of the massive string excitations (momentum and winding states) and is proportional to \( \sum (b_0/3 - \delta^i_{\text{GS}}) \log |\eta(T_i)|^4 \) [22],[1],[5],[23], where \( \eta(T) \) is the well-known Dedekind function. For the case of complete Green–Schwarz anomaly cancellation, the contribution from the massive states is absent since the relevant massive spectrum is then organized into \( N = 4 \) supermultiplets. Now the remaining target space modular anomaly is exactly removed by the threshold contributions of the massive states, i.e. the expression (7) is explicitly target space modular-invariant.
Taking into account only the threshold piece of the massive states, one recognizes that the one-loop gauge coupling constant is given by the real part of a holomorphic gauge kinetic function of the following form:

\[ f_{1\text{-loop}} = S - \frac{1}{8\pi^2} \sum_{i=1}^{3} \left( \frac{b_0}{3} - \delta_{i\text{GS}} \right) \log \eta(T_i)^2. \]  

(8)

It is important to stress that this gauge kinetic function does not get further renormalized beyond one loop and that it is therefore an exact expression at all orders [24],[23].

For some purposes it will turn out to be convenient to perform a holomorphic field redefinition to use a target space modular-invariant dilaton field \( S' \) defined as follows [5]:

\[ S' = S + \frac{1}{8\pi^2} \sum_{i=1}^{3} \delta_{i\text{GS}} \log \eta(T_i)^2. \]  

(9)

Then the string coupling constant \( Y \) looks like \( Y = S' + S' - \frac{1}{8\pi^2} \sum_{i=1}^{3} \delta_{i\text{GS}} \log |(T_i + \bar{T}_i)| \eta(T_i)^4 \), and the holomorphic one-loop gauge kinetic function takes the form \( f_{1\text{-loop}} = S' - \frac{1}{8\pi^2} \sum_{i=1}^{3} \frac{b_0}{3} \log \eta(T_i)^2. \)

Now let us apply the above results and analyse the modular-invariant gaugino condensation mechanism first in a hidden sector, which is described by a pure Yang–Mills gauge theory with gauge group \( G \). Examples of pure gauge hidden sectors can be found in the literature. For instance, the \( E_8' \) gauge group for the standard embedding of \( Z_M \) and \( Z_M \times Z_N \) orbifolds is most well known. Also \( Z_7, Z_4 \) models with \( SU(3) \times SU(2) \times U(1)^5 \) observable sector and pure \( E_6, SU(3) \times SU(3) \) gauge hidden sectors respectively were constructed [25]. There are two consistent approaches for a dynamical description of the duality-invariant condensation of gauginos, namely the effective Lagrangian approach [2], which contains the gauge-singlet gaugino bound state as a dynamical degree of freedom, and the effective superpotential approach [1] where the gaugino condensate has been replaced by its vacuum expectation value. Both approaches have been shown to be equivalent in refs.[8],[9],[10], and the effective non-perturbative su-
perpotential is derived as

\[ W_{\text{np}} \sim e^{\frac{24\pi^2}{b_0}} f. \]  

(10)

Using eq.(8), the effective superpotential for the case of a pure Yang–Mills gauge group has the form

\[ W_{\text{np}} \sim \frac{e^{\frac{24\pi^2}{b_0} S}}{\prod_{i=1}^{3} [\eta(T_i)]^{(2 - \frac{2}{b_0} \delta \eta_{GS})}}. \]  

(11)

It is important to stress that the effective non-perturbative superpotential entirely follows from the holomorphic gauge kinetic function whose moduli dependence originates from the threshold effects of the massive states, whereas the non-harmonic contribution of the massless field does not enter the holomorphic superpotential. Since the gauge kinetic function does not get renormalized beyond one-loop, one has obtained an all order expression for the effective superpotential. It is also important to remark that the superpotential (11) has exactly the correct modular transformation behaviour (3) due to the transformation rules of the Dedekind function and the dilaton field eq.(6). This observation provides very strong confidence in the non-perturbative validity of target space modular-invariance. The correct modular transformation behaviour becomes even more transparent when using the modular-invariant dilaton field \( S' \) and the corresponding gauge kinetic function \( f \). Then the superpotential has an universal moduli dependence,

\[ W_{\text{np}} \sim \frac{e^{\frac{24\pi^2}{b_0} S'}}{\prod_{i=1}^{3} \eta(T_i)^2}, \]  

(12)

which is exactly of the form discussed in refs.[18] and [1] upon identification of three moduli \( T_i \). Of course, the physical implications will not depend on which version of the non-perturbative superpotential is used to describe the gaugino condensate, as we will show in the following.

For analysing supersymmetry breaking it is necessary to determine the scalar potential of the theory. The one-loop contribution to the Kähler function in eq.(5) implies that the formula for the scalar potential studied up to now [1] must be modified. In particular, taking into account the new Kähler metric, which leads
to mixing between the dilaton and the moduli, one obtains for the scalar potential

$$V = \frac{1}{Y \prod_{i=1}^{3} (T_i + \bar{T}_i)} \left\{ |W - Y W_S|^2 + \sum_{i=1}^{3} \frac{Y}{8\pi^2 \delta_{GS}^i} |(W - \frac{1}{8\pi^2 \delta_{GS}^i} W_S) - (T_i + \bar{T}_i) W_{T_i}|^2 - 3|W|^2 \right\},$$

where \( W_\phi = \partial W/\partial \phi, \phi = S, T_i \). The above formula can be applied for any superpotential. For the case of just one gaugino condensate in a pure Yang–Mills hidden sector, the superpotential eq.(11) leads to the following scalar potential:

$$V_{\text{np}} = \frac{e^{\frac{24\pi^2}{b_0} Y}}{Y \prod_{i=1}^{3} |\eta(T_i)^2(T_i + \bar{T}_i)^{1/2}|^{2-\frac{2}{b_0} \delta_{GS}^i}} \left\{ |1 - \frac{24\pi^2}{b_0} Y|^2 + \sum_{i=1}^{3} \frac{Y}{16\pi^2 \delta_{GS}^i} (2 - \frac{6}{b_0} \delta_{GS}^i)^2 \frac{(T_i + \bar{T}_i)^2}{|\hat{G}_2(T_i)|^2} - 3 \right\},$$

where \( \hat{G}_2(T_i) = -\frac{2\pi}{T_i + \bar{T}_i} - \frac{4\pi}{\eta(T_i) \partial \eta(T_i)/\partial T_i} \).

This result has various interesting aspects to be discussed. Using the variables \( Y \) and \( T \), the scalar potential looks very similar to the potential in [1], which was derived without taking into account the loop modifications of the Kähler potential. (The potential of [1] is rederived for \( \delta_{GS}^i = 0 \) and \( T = T_1 = T_2 = T_3 \).) Moreover, if \( \delta_{GS}^i \neq b_0/3 \) this potential exhibits a minimum at \( T_i = O(1) \), thus dynamically determining the size of the \( i \)th two-dimensional complex plane. On the other hand it is easy to realize that if some moduli do not appear in the superpotential (11) (i.e. \( \delta_{GS}^i = b_0/3 \)), which is the case for \( \mathbb{Z}_M \) orbifolds, then the scalar potential is flat with respect to \( T_i \). (This non-surprising behaviour is not automatic but it arises only after defining the variables \( Y \) and \( T \), since there is a non-trivial \( T_i \)-dependence in the Kähler potential.) Thus for the \( \mathbb{Z}_3 \) and \( \mathbb{Z}_7 \) orbifolds the scalar potential possesses no moduli-dependence at all. The flatness of the potential with respect to the moduli of completely rotated planes opens in principle the possibility of having a large internal radius as discussed in ref. [26]. However there is still the alternative mechanism for the final goal that all the moduli dynamically acquire vacuum expectation values, determining the size
of the compactified space, through the non-trivial $T_i$ dependence of the (twisted) Yukawa couplings as pointed out in refs. [7] and [8]. As it was shown in ref. [27], only if orbifold twists (involved in a particular coupling) act non-trivially on the $i^{th}$ complex plane, the moduli $T_i$ appear in the final expression of the Yukawa coupling. These are precisely the moduli that never appear in the string threshold corrections and hence in the non-perturbative superpotential obtained from the gaugino condensation. So there is a kind of completeness in the role of the different moduli in the perturbative and non-perturbative parts of the superpotential.

Also note that although the "dilaton" $Y$ appears in the second term of eq. (14), the scalar potential unfortunately exhibits no minimum with respect to $Y$. Therefore the situation is not improved concerning the run-away dilaton behaviour after the inclusion of the loop effects into the Kähler potential, and additional dynamics, as for example several gaugino condensates [28], [8], [10], [29] or $S$-field duality [30], is needed to overcome this serious obstacle. As already indicated, the use of the superpotential (12) with the universal $T_i$ dependence does not alter at all the above conclusions. Now the Kähler potential contains the field $S'$ and some additional moduli dependence. The corresponding scalar potential becomes

$$V = \frac{1}{Y \prod_{i=1}^{3} (T_i + \bar{T}_i)} \left\{ |W - Y W_{S'}|^2 - 3|W|^2 + \sum_{i=1}^{3} \frac{Y}{1 - \frac{1}{8\pi^2} \delta_{GS}} \right\}.$$  

Using the superpotential (12) one ends up again exactly with the scalar potential (14).

Let us now turn to the case with massive hidden matter [9], [10], [31]. In fact, it is known that the existence of hidden matter is the most general situation and occurs in all promising string constructions [32], [33].

* It is worth noticing [8], [10], [29] that in order to fix the dilaton to a realistic value by using the two gaugino condensation mechanism, the existence of hidden matter is crucial.
[SO(10)]' and three 16's hidden matter representations was constructed. In ref. [34] two $\mathbb{Z}_7$ models with the $SU(3) \times SU(2) \times U(1)^5$ observable gauge group and $SO(10)$, $SU(5) \times SU(3)$ hidden sectors with hidden matter $10$, $3(\bar{5} + \bar{5}) + 7(3 + \bar{3})$ respectively were constructed. Also a $\mathbb{Z}_4$ model with $E_6 \times SU(2)$ observable sector and $E_7$ hidden gauge group plus $2(56)$ hidden matter can be found in ref. [35].

Specifically, we consider gauge non-singlet chiral hidden matter fields $Q_\beta$, which become massive through the coupling to gauge singlet chiral fields $A_\alpha$. These fields have in any order of string perturbation theory a completely flat potential such that $\langle A_\alpha \rangle$ is a perturbatively undetermined parameter of the theory. Examples for this kind of flat directions are Wilson line moduli. Thus we consider the following trilinear perturbative superpotential:

$$ W = h_{\alpha\beta\gamma}(T_i)A_\alpha Q_\beta Q_\gamma. $$

The fields $A_\alpha$, $Q_\beta$ and $Q_\gamma$ have modular weights $n^i_{A_{\alpha}}$, $n^i_{Q_{\beta}}$, $n^i_{Q_{\gamma}}$, respectively. Thus the intermediate masses for the fields $Q_\beta$ and $Q_\gamma$ are given by

$$ M_I = |\partial^2 W/\partial Q_\beta \partial Q_\gamma| \cdot M_{\text{string}} = |h_{\alpha\beta\gamma}(T_i)A_\alpha| \cdot M_{\text{string}}. $$

† Remember that $h_{\alpha\beta\gamma}$ is moduli dependent if all fields $A_\alpha$, $Q_\beta$ and $Q_\gamma$ are in the twisted sector of the orbifold. Otherwise it is constant.

It also happens in several cases that the fields $A$ give masses $M_I$ to some vector fields $V$ and additional chiral fields $Q'$ through $D$-term couplings. (The fields $Q'$ build the longitudinal components of the massive vector bosons $V$.) Then the vector fields $V$ become massless for $A = 0$, leading to an enlargement of the gauge group. The breaking of the enlarged gauge group $G'$ down to $G$ by the vacuum expectation values of the moduli fields is the so-called stringy Higgs effect [36]. An especially interesting situation arises when considering orbifolds with the $i^{\text{th}}$ complex plane rotated by all orbifold twists and with an associated untwisted modulus $A_i$. Then one can show that, analogous to the modulus $T_i$, the (untwisted) massive string spectrum with $A_i$-dependent masses is $N = 4$ supersymmetric. Specifically, the fields $Q$ and $Q'$ are also in the untwisted sector of the orbifold with $R_Q = R_{Q'} = R_V$, and there are twice as many chiral fields $Q$ as fields $Q'$, i.e. $N_Q = 2N_{Q'} = 2N_V$. (The modular weights are for example distributed in the following way: $n^i_{A_{A_i}} = n^i_{Q_{Q'}} = (-1, 0, 0)$, $n^i_{Q_{Q'}} = (0, -1, 0)$,

† The physical mass of the normalized fields has some additional, non-analytic, $T_i$ dependence.
\[ n^i_{Q_3} = (0, 0, -1). \] Then the fields \( Q, Q' \) and \( V \) build perfect \( N = 4 \) vector supermultiplets. It follows that in this case the “mass” field \( A \) does not enter the non-perturbative superpotential as we will discuss now.

Let us analyse the case of intermediate masses higher than the condensation scale \( \Lambda \), i.e. \( \Lambda < M_I << M_{\text{string}} \). The one-loop hidden gauge coupling constant is of the form

\[
\frac{1}{g_{1-\text{loop}}^2(\mu)} = \frac{Y}{2} + \frac{b_0}{16\pi^2} \log \frac{|h_{\alpha\beta\gamma}(T_i)A_{\alpha}|^2 M_{\text{string}}^2}{\mu^2} - \frac{b_1}{16\pi^2} \log |h_{\alpha\beta\gamma}(T_i)A_{\alpha}|^2 \]

\[
- \frac{1}{16\pi^2} \sum_{i=1}^{3} (b_0/3 - \delta^i_{GS}) \log(T_i + \bar{T}_i) - \frac{1}{16\pi^2} \sum_{i=1}^{3} (b_i' - \delta^i_{GS}) \log |\eta(T_i)|^4.
\]

(16)

Here \( b_0 = -3C(G) \) is the \( N = 1 \) \( \beta \)-function coefficient where all matter fields are decoupled. On the other hand, \( b_1 \) describes the running between \( M_I \) and \( M_{\text{string}} \) where all fields contribute and is therefore given by \( b_1 = -3C(G) - 3 \sum_Q T(R_Q) + \sum_{Q'} T(R_{Q'}) \). The threshold corrections* due to the massive momentum and winding states are determined by the coefficient \( b_i' \), where all massless gauginos of \( G \) as well as the massive fields \( Q \) and possibly \( Q' \) and \( V \) “contribute” in the following way [20],[5]:

\[
b_i' = -C(G) - \sum_Q T(R_Q) + \sum_{Q'} T(R_{Q'})(1 + 2n^i_{Q}) + \sum_{Q'} T(R_{Q'})(1 + 2n^i_{Q'}).\]

Note that the \( \sigma \)-model anomalies, related to \( \log(T_i + \bar{T}_i) \), are only generated by the contribution of the massless gauginos of \( G \). However the coupling constant \( g(\mu)^{-2} \) is still target space modular-invariant due to the non-trivial transformation property of \( h_{\alpha\beta\gamma}(T_i)A_{\alpha} \). This follows immediately if there are no states \( Q' \) and \( V \). Otherwise the requirement of target space modular-invariance imposes the following restriction on the additional states:

\[
\sum_Q T(R_Q)(-2 - 3n^i_{Q_3} - 3n^i_{Q_3}) + \sum_{Q'} T(R_{Q'})(n^i_{Q_3} + n^i_{Q_3} - 2n^i_{Q'}) = 0. \quad (17)
\]

The fact that the massive matter fields have the same effect as the massless

* Strictly speaking, the mass of the momentum and winding states should not only depend on the moduli \( T_i \) but also on the field \( A \). Therefore the Dedekind function in eq.(16) provides an expression for the determinant of the heavy field mass matrix, which is only valid for \( M_I << M_{\text{string}} \). For arbitrary values of \( A, \eta(T_i) \) should be replaced by a more general automorphic function, which involves both \( T_i \) and \( A \) but still transforms under target modular transformations like \( \eta(T_i) \).
matter fields with respect to modular transformations was already noted in ref. [37] in the context of strong CP violation.

Consider also the special case of a completely rotated plane $i$ with an associated untwisted modulus $A_i$. Then the chiral fields $Q$ and $Q'$, being also untwisted, build together with the vectors $V N = 4$ vector supermultiplets, and one therefore obtains $b_1 = b_0$. This implies that the running coupling constant does not depend on $A_i$. In the same way it follows that $b'_i = -C(G) = b_0/3$. This can be explicitly checked in several examples [38]. Thus for planes which are rotated by all orbifold twists we still have $b'_i = b_0/3 = \delta_{iGS}$ as in the pure Yang–Mills case.

The gauge kinetic function, including the holomorphic contribution of the massive states to the gauge coupling constant, takes the following form:

$$f_{1-\text{loop}} = S + \frac{b_0 - b_1}{8\pi^2} \log(h_{\alpha\beta\gamma}(T_i)A_\alpha) - \frac{1}{8\pi^2} \sum_{i=1}^{3} (b'_i - \delta_{iGS}) \log \eta(T_i)^2. \quad (18)$$

Using the same arguments as in the pure Yang–Mills case, the non-perturbative superpotential is given by $W_{np} \sim e^{\frac{24\pi^2}{b_0}} f$. Then we obtain with eq. (18)‡

$$W_{np} \sim e^{\frac{24\pi^2}{b_0} S[h_{\alpha\beta\gamma}(T_i)A_\alpha]^{3(b_0-b_1)/b_0}} \frac{\prod_{i=1}^{3} \eta(T_i)^{6(b'_i-\delta_{iGS})/b_0}}{\prod_{i=1}^{3} \eta(T_i)^{6(b'_i-\delta_{iGS})/b_0}}. \quad (19)$$

The above superpotential has the correct modular transformation behaviour (3) due to the transformation rules of the Dedekind function, dilaton and matter fields eqs.(6),(1) and Yukawa couplings eq.(4). It is important to stress the fact that when the matter representations acquire mass through twisted Yukawa couplings (i.e. $h_{\alpha\beta\gamma} \neq \text{const}$), the whole set of moduli appears in the gaugino condensation superpotential (19). Therefore even for moduli $T_i$ with the $i^{th}$ plane rotated by all orbifold twists, i.e. $b'_i = \delta_{iGS}$, there will be no flat directions. On the other hand, for the particular case of a completely rotated plane with an associated untwisted field $A_i$, both $T_i$ and $A_i$ do not appear in $W_{np}$ since $b_0 = b_1 = 3b'_i = 3\delta_{iGS}$ as discussed before.

‡ For the case $M_i < \Lambda$ one obtains the same form for the non-perturbative superpotential including $Q_\beta Q_\gamma$ bound states into the effective Lagrangian.
In order to analyse supersymmetry breaking, one has to include in the Kähler potential the contribution of the matter fields. Let us study the case of untwisted matter $C_i$, where the string tree level Kähler potential is known at all orders in $C_i$, $K_{\text{tree}} = - \log(S + \bar{S}) - \sum_{i=1}^{3} \log X_i$, where $X_i = T_i + \bar{T}_i - |C_i|^2$. Therefore it is very plausible to assume that the one-loop modification is given by

$$K_{1-\text{loop}} = - \log Y - \sum_{i=1}^{3} \log X_i,$$

(20)

with $Y = S + \bar{S} - \frac{1}{8\pi^2} \sum_{i=1}^{3} \delta^l_{GS} \log X_i$. $K_{1-\text{loop}}$ leads to mixing between the dilaton, $T_i$ and $C_i$ fields, and one obtains for the scalar potential

$$V = \frac{1}{Y \prod_{i=1}^{3} X_i} \left\{ |W - Y W S|^2 + \sum_{i=1}^{3} \frac{Y}{Y - \frac{1}{8\pi^2} \delta^l_{GS}} \right\} \left\{ (W - \frac{1}{8\pi^2} \delta^l_{GS} W S) - X_i W T_i|^2 + X_i |W C_i + \bar{C}_i W T_i|^2 - 3|W|^2 \right\},$$

(21)

The above formula can be applied for any superpotential. Let us study the case of untwisted matter $Q_j, Q_k$ coupled to $A_i$ ($i \neq j \neq k \neq i$). For the case of just one gaugino condensate with hidden matter fields, the superpotential eq.(19) leads to the following scalar potential:

$$V_{\text{np}} = \frac{e^{2\pi^2 Y} |A_i|^{6(b_0 - b_1)/b_0}}{Y \prod_{l=1}^{3} |\eta(T_l)|^{12(b'_l - \delta^l_{GS})/b_0 X_l^{(1- \frac{1}{8\pi^2} \delta^l_{GS})}} \left\{ (1 - \frac{24\pi^2}{b_0} Y)^2 - 3 \right. $$

$$+ \sum_{l=1}^{3} \frac{Y}{Y - \frac{1}{8\pi^2} \delta^l_{GS}} |(1 - \frac{3\delta^l_{GS}}{b_0}) + X_l \frac{6}{b_0} (b'_l - \delta^l_{GS}) \frac{\partial \eta(T_l)}{\partial T_l} |^2$$

$$+ \frac{Y}{Y - \frac{1}{8\pi^2} \delta^l_{GS}} X_l \left| 3(1 - \frac{b_1}{b_0}) - |A_i|^2 \frac{6}{b_0} (b'_l - \delta^l_{GS}) \frac{\partial \eta(T_l)}{\partial T_l} \right|^2 \right\},$$

(22)

where $X_l = (T_l + \bar{T}_l)$ for $l \neq i$. As discussed in ref.[9], by minimizing this potential with respect to $T_l$ ($j = 1, 2, 3$) and $A_i$, one generically obtains that the vacuum expectation values of all fields $T_l, A_i$ are dynamically determined for

\footnote{For an orbifold example with all the hidden matter in the untwisted sector, see ref. [35].}
generic values of the parameters \(b_0, b_1, b'_l\) and \(\delta'_{GS}\). The only exception is again provided if the complex plane \(i\) is rotated by all orbifold twists. Then \(T_i\) and \(A_i\) do not appear in the scalar potential and take arbitrary vacuum expectation values. Also note that the “dilaton” \(Y\) still possesses the run-away behaviour as in the pure Yang–Mills case.

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