Quantum features in statistical observations of “timeless” classical systems

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Abstract

We pursue the view that quantum theory may be an emergent structure related to large space-time scales. In particular, we consider classical Hamiltonian systems in which the intrinsic proper time evolution parameter is related through a probability distribution to the discrete physical time. This is motivated by studies of “timeless” reparametrization invariant models, where discrete physical time has recently been constructed based on coarse-graining local observables. Describing such deterministic classical systems with the help of path-integrals, primordial states can naturally be introduced which follow unitary quantum mechanical evolution in suitable limits.

Key words: reparametrization invariance, discrete time, emergent quantum theory
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Dedicated to Constantino Tsallis on occasion of his 60th birthday.

1 Introduction

Since its very beginnings, there have been speculations on the possibility of deriving quantum theory from more fundamental dynamical structures, possibly deterministic ones [1,2]. Famous is the discussion by Einstein, Podolsky and Rosen, interpreted as the need for a more complete fundamental theory. However, just as numerous have been attempts to prove no-go theorems prohibiting exactly such “fundamentalism”, culminating in the studies of Bell. The EPR paradox as well as the Bell inequalities have come under experimental scrutiny in recent years, confirming the predictions of quantum mechanics in laboratory experiments on scales very large compared to the Planck scale.

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However, to this day, the feasible experiments cannot rule out the possibility that quantum mechanics emerges as an effective theory only on sufficiently large scales and can indeed be based on more fundamental models.

Motivated by the clash between general relativity and quantum theory, 't Hooft has strongly argued in favour of model building in this context [3,4]. Various further arguments for deterministically induced quantum features have recently been proposed, for example, in Refs. [5,6,7,8,9], in the context of statistical systems, of considerations related to quantum gravity, and of matrix models, respectively. – Here I report on a large class of classical deterministic systems which show emergent quantum mechanical features [1]. This is based on recent work on time-reparametrization invariant models [10,11], where discrete physical time has been constructed. Essential aspects of what has been presented in my talk will be summarized, while the details may be found in the references.

1.1 Discrete physical time in “timeless” classical models

Analogous to common gauge theories, e.g. the standard model of particle physics, reparametrization invariant systems are invariant under a kind of gauge transformation. In the case of diffeomorphism invariant theories, such as general relativity or string theory, this amounts to invariance under general coordinate transformations. Considering time-reparametrization invariance only, this means invariance of the dynamics under arbitrary transformations:

\[ t \rightarrow t', \quad \text{with} \quad t \equiv f(t') \]

with \( f \) monotonous and differentiable. The corresponding constrained Lagrangian dynamics is discussed in Refs. [10,11] for the respective models.

Similarly as Gauss’ law in electrodynamics, for example, an important consequence of time-reparametrization invariance is the (weak) constraint that the Hamiltonian has to vanish (on the solutions of the equations of motion). Since the Hamiltonian commonly is the generator of time evolution, this has led to name such systems “timeless”. A Newtonian external time parameter does not exist. Problems arise when trying to quantize such a system, since the standard Schrödinger equation does not exist. Thus, the Wheeler-DeWitt equation, \( \hat{H}\ket{\psi} = 0 \), epitomizes the intrinsic problems of quantum gravity.

Numerous approaches have been tried to resolve this (in)famous “problem of time”. In distinction to others, I insist on a local description. Still, for a particle with time-reparametrization invariant dynamics, be it relativistic or nonrelativistic, one can define quasi-local observables which characterize the
evolution in a gauge invariant way \cite{10,11}. Essentially, some of the degrees of freedom of the system are employed to trigger a localized “detector”, which can be defined invariantly. It amounts to attributing to an observer the capability to count discrete events. Then, the detector counts present an observable measure of \textit{discrete physical time}.

This result can be understood differently by noting that a Poincaré section is invoked here, which reflects an ergodic if not periodic aspect of the dynamics – quite analogous to a pendulum which triggers a coincidence counter each “time” it passes through its equilibrium position. Reparameterization invariance strongly limits the information which can be extracted from it with respect to a complete trajectory. This is the reason that physical time based on local observations (clock readings) necessarily is discrete.

The possibility of a fundamentally discrete time (and possibly other discrete coordinates) has been explored before, ranging from an early realization of Lorentz symmetry in such a case to detailed explorations of its consequences and consistency in classical mechanics, quantum field theory, and general relativity. So far, however, no classical physical models implying such discreteness were proposed. \textit{Quantization as an additional step} – resulting in discreteness of coordinates in some cases – has always been performed as usual.

\subsection*{1.2 \textit{Does discrete time induce quantum mechanical features?}}

There are indications for a qualified ‘Yes’, answering this question. The findings of Refs. \cite{10,11} suggested that those discrete-time models can be mapped on a cellular automaton studied by ‘t Hooft \cite{3,4}. Employing the algebra of SU(2) generators, it has been shown that these models actually reproduce the quantum mechanical harmonic oscillator in a large-scale limit. The Ref. \cite{1} presents an attempt to show more generally that due to inaccessibility of globally complete information on trajectories of the system, the evolution of remaining degrees of freedom appears as in a quantum mechanical model when described in relation to the discrete physical time.

We may call this “stroboscopic” quantization: when a continuous physical time is not available but a discrete one is – like reading an analog clock under a stroboscopic light – then states of the system which fall in between subsequent clock “ticks” cannot be resolved. (Of course, evolution in the unphysical parameter time is continuous in the constrained Lagrangian models we refer to.) Such unresolved states form equivalence classes which can be identified with primordial Hilbert space states \cite{3,4,5,10,11}. The residual dynamics then leads the evolution of these states through discrete steps. Under favourable circumstances, this results in unitary quantum mechanical evolution.
Presently, incorporating the discreteness of time is simplified by relating the physical time $t$ via a probability distribution $P$ to the proper time $\tau$ of the equations of motion:

$$P(\tau; t) \equiv P(\tau - t) \equiv \exp\left(-S(\tau - t)\right), \quad \int d\tau \, P(\tau; t) = 1.$$ \hfill (2)

Note that the almost perfect clock described in this way does not age with physical time. Explicit examples of this can be found in Refs. [10,11], when the clock degrees of freedom evolve independently of the rest of the system, apart from the Hamiltonian constraint. Thus, while the study of fully coupled system-clock dynamics will be reported elsewhere, here corresponding back-reaction effects are assumed to be small, which characterizes a good clock. In a selfconsistent treatment, however, a closed system has to include its own clock, reflecting the experience of an observer in the universe.

There is no need for the construction of discrete physical time in ordinary mechanical systems or field theories, where time is an external parameter. However, assuming that truly fundamental theories will turn out to be diffeomorphism invariant, adding further the requirement that observables be quasi-local, then such an approach seems natural, which promises to lead to quantum mechanics as an emergent description or effective theory on the way.

2 From discrete time to evolution of primordial states

Introducing a functional description of classical mechanics, similarly as in Refs. [12,13], one recognizes the primordial state:

$$\langle \tau, \pi_a | t; t_0 \rangle \equiv \int d\tau' \int D\Phi \exp[i \int_{\tau'}^{\tau+t} d\tau'' L_J - S(\tau'' - t_0) + i\pi_a \varphi^a(\tau + t)],$$ \hfill (3)

and, similarly, the (complex conjugated) adjoint state, as useful basic entities for our present purposes [1]; $H$ stands for the presence of the Hamiltonian constraint and $\pi_a$ arise from exponentiating $\delta$-functions involving $\varphi^a$ at a fixed time; $t_0$ is the initial time. Furthermore, $D\Phi \equiv D\varphi D\lambda Dc D\bar{c}$ indicates the functional integration over all fields which enter the effective Lagrangian:

$$L_J \equiv \lambda_a \left( \partial_\tau \varphi^a - \omega^{ab} \partial_b H \right) + i\bar{c}_a \left( \delta^a_b \partial_\tau - \omega^{ac} \partial_c \partial_b H \right) c^b + J_a \varphi^a ,$$ \hfill (4)

where $J_a$ denotes an external source, $c^a, \bar{c}^a$ are anticommuting Grassmann variables, $\lambda_a$ is an auxiliary variable, and $\varphi^a, a = 1 \ldots 2n$ denote the classical phase space variables characterizing the system.
Generic states of this form, possibly including additional weights for different initial conditions of the paths contributing in Eq. (3), are employed to calculate all physical (time dependent) observables of the classical system. Considering observables which are function(al)s of the phase space variables \( \varphi \), we define and calculate:

\[
\langle O[\varphi]; t \rangle \equiv \int d\tau P(\tau; t)O[-i \frac{\delta}{\delta J(\tau)}]C[-i \frac{\delta}{\delta J(\tau)}] \log Z[J] |_{J=0} = Z^{-1} \int d\tau d\pi P(\tau)\langle t|\tau,\pi\rangle O[-i \partial_{\pi}]C[-i \partial_{\pi}] \langle \tau,\pi|t\rangle = \langle \Psi(t)|\hat{O}[\varphi]\hat{C}[\varphi]|\Psi(t)\rangle,
\]

where \( Z \equiv Z[0] \) is the generating functional and \( C \) the projector representing the Hamiltonian constraint, which are not discussed here [1], and where all states refer to \( J = 0 \) as well; here:

\[
\hat{O}[\varphi] \equiv O[\varphi], \quad \hat{C}[\varphi] \equiv C[\varphi], \quad \varphi \equiv -i \partial_{\pi},
\]

in “\( \tau, \pi \)-representation”. In Eq. (5), the final result is for a generic state \( |\Psi(t)\rangle \) \( (J = 0) \), with the scalar product to be evaluated as in the preceding expression. Thus, classical observables are represented by corresponding function(al)s of a momentum operator. Its expectation value at physical time \( t \) appears as a quantum mechanical expectation value, referring to the considered state and incorporating a \( P \)-weighted average over its “history” in proper time \( \tau \).

Now, generic states at physical times \( t \) and \( t + T \) can be related to each other, making use of the classical Liouville operator propagating phase space variables in \( \tau \) [1]. Taking the conserved Hamiltonian constraint into account, one obtains a discrete physical time Schrödinger equation:

\[
\langle \tau', \pi'|\Psi(t + T)\rangle = \int d\tau P(\tau) \exp[-i(\tau' + T - \tau)\hat{H}_Q(\pi', -i \partial_{\pi'})] \langle \tau, \pi|\Psi(t)\rangle,
\]

with the emergent Hamilton operator:

\[
\hat{H}_Q(\pi, -i \partial_{\pi}) \equiv -\pi \cdot \omega \cdot \frac{\partial}{\partial \varphi} H(\varphi)|_{\varphi=-i \partial_{\pi}},
\]

for a given classical Hamiltonian \( H \); here \( \omega \) is the usual symplectic matrix.

Equipped with the Hamiltonian \( \hat{H}_Q \), together with the operator \( \hat{C} \) projecting out the constraint subspace, the stationary state eigenvalue problem related
to Eq. (7) can be studied. – Due to the unusual structure of the Hamiltonian $\hat{H}_Q$, one finds in various examples that the generated spectrum is too rich and, in particular, includes notorious negative energy states. Thus, at first sight, the emergent models seem not acceptable, since they do not possess a stable groundstate. However, it can be shown that a regularization (by discretization) of the operators involved overcomes this problem.

Thus, while general principles guiding the regularization remain to be investigated, for an underlying harmonic oscillator model, one finds the quantum oscillator as the corresponding emergent model. Use has to be made of the freedom to choose an arbitrary phase of the regularized eigenfunctions, in order to recover the correct zeropoint energy (in the continuum limit). Three coupled quantum oscillators are found for an underlying relativistic particle model, where the coupling is a combined effect of the Minkowski metric and the Hamiltonian constraint [1].

Instead of embarking to further describe these models here, some general remarks seem in order. It is well known from ordinary quantum (field) theory that the harmonic oscillator is peculiar in many respects. Therefore, the reader should not be misled by the present results, namely that a quantum harmonic oscillator spectrum is obtained from an underlying classical harmonic oscillator model. In particular, it may appear as if this corresponds to “just another quantization method”, in line with canonical commutators, path-integral or stochastic quantization, etc. However, this is not the case.

It seems an accident of the harmonic system that the usual quantized energy spectrum results here. This is revealed by the fact, already demonstrated in Refs. [3,4,10,11], that localization with respect to the coordinate $q$ of the underlying classical model has nothing to do with localization with respect to the operator $\hat{q}$, which is introduced a posteriori when interpreting the emergent quantum Hamiltonian corresponding to said spectrum. Rather, such localized quantum (oscillator) states are widely spread over the $q$-space of the underlying classical model. Interestingly, depending on how the continuum limit is defined in a particular model, the $\hat{p}, \hat{q}$-commutator may suffer corrections [11], which interpolate between the classical limit (high energy) and usual quantum mechanics (low energy).

Therefore, generally, one cannot expect to find the usual quantized counterpart of a classical reparametrization invariant model in the present approach, based on discrete physical time. Here, the states are collections of classical trajectories, as defined in Eq. (3), and there is not the usual close correspondence, for example, via coherent states. On the other hand, in this way a necessary element of quantum mechanical nonlocality enters the description of the underlying reparametrization invariant classical system. Further related remarks may be found in Refs. [3,4,8].
To put it differently, the \textit{classical limit} of emergent quantum theories \textit{cannot} be expected to give back the underlying model. Instead, the emergent quantum model must be seen as coarse-grained large-scale description of an underlying deterministic, possibly dissipative classical dynamics. In itself it is to have a classical limit in accordance with the familiar correspondence principle.

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