Electron Halo and Bound State Formation at Jellium Boundaries

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Abstract

Current research presents an innovative model of half-space plasmon excitations for electron gas of arbitrary degeneracy in an ambient jellium-like positive background. The linearized Schrödinger-Poisson system is used to derive effective coupled pseudo-force and damped pseudo-force system of second-order differential equations from which the state functions such as the electron probability density and electrostatic potential energy are calculated and the appropriate half-space equilibrium plasmon excitation wave-functions are constructed. Current model of half-space finite temperature electron plasmon reveals many interesting features not present in previous studies. This model benefits a dual length scale character of quantum plasmon excitations taking into account the detailed electrostatic interactions between single electrons and their collective entity in an unmagnetized arbitrary degeneracy electron gas. The interaction of these length scales is remarked to lead to the formation of well defined miniature periodic density fringes in the gas which are modulated over the envelop density pattern and causes the presence of an electron halo in front of the physical jellium boundary of the system. A novel attractive Lennard-Jones-like potential energy forms in front of the boundary for parametric density-temperature region relevant to the strongly doped N-type semiconductors as well as metallic surfaces. The later effect may appropriately explain the Casimir-Polder-like forces between parallel metallic plates in vacuum. Current model can be further extended to explore characteristic features of plasmon excitations in other geometries like spherical for investigation of microscopic interactions among nanoparticles or a better understanding of charged-particle metal-surface interactions. This model may also be used to study dynamic quantities such as the work function and surface polariton interactions in semiconductors and nanometallic compounds.

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I. INTRODUCTION

Plasmonics [1, 2] is a new interdisciplinary field with important applications in nano-electronics [3, 4], optoelectronics [5], and semiconductor integrated circuit industry [6–11]. Plasmons are elementary collective excitation of quantum plasmas which play an inevitable role in nature [12–18]. These entities provide ideal platform for fast THz device communications [19] and beyond where the conventional wired communications fail to operate. Quantum plasma regime ranges from doped semiconductors [20, 21] with moderately high electron number density at low temperature and high density metallic compounds up to high-temperature and density called warm dense matter (WDM) [22, 23]. Quantum effect arise naturally when the interspecies distances exceeds the thermal de Broglie wavelength [24]. In inertial confinement fusion experiment quantum effects vary in strength due to change in equation of state (EoS) of matter during shock compression causing increase in temperature and number density [25]. The change of EoS has one fundamental quantum effect on the faith of compact stellar object setting distinct limit on the mass of such entities [26]. Another quantum feature of charged particle ensemble is their complex electromagnetic collective interactions even when their average interparticle distances has not passed over the quantum limit. The later is because of mysterious nonlocal behavior of interactions acting via the Bohm’s quantum diffraction potential. This is why a dilute electron gas can lead to distinct quantum effect in Bragg diffraction phenomenon [27, 28]. The development of quantum plasmas had a long history with many pioneering contributions over the past century [29–38]. Because of the dominant quantum effects caused by the EoS of a Fermi gas semiclassical plasma theories which incorporate the quantum statistical pressure effects but ignore the electron nonlocality still lead to many interesting aspects [39–44]. However, many old quantum plasma theories based on the standard Thomas-Fermi assumptions which ignore the von Weizsacker correction to kinetic energy [45] as the main root to collective nonlocal aspects of an electron gas fail to capture the full essence of quantum many-body systems.

There has been an increasing interest in the quantum plasma theories over the recent decade due to their effective description of collective quantum features [46, 57]. Quantum kinetic [58], quantum (magneto)hydrodynamic [59] and gradient corrected density functional theory (DFT) has been used to investigate different aspects of many particle interactions.
Application of even the simplest one, i.e. quantum hydrodynamic approach, has led to discovery of many fascinating collective properties of plasmas which has not been discovered in old theories due to ignorance of quantum electron diffraction phenomenon. The quantum hydrodynamic theory has recently been cast into the form of well-known single particle Madelung fluid theory using the appropriate transformations on statistical quantities. The use of such framework, so-called Schrödinger-Poisson model \[60\], has the capability of studying many interesting linear and nonlinear features of plasmonic environments \[61–63\] with the least complexity. Recently, the pseudoforce system derived from linearized Schrödinger-Poisson model has been used to study some novel features of plasmon excitations \[64\]. This new theory has a fundamental property of taking into account the dual lengthscale nature of single-particle as well as collective interactions in a single picture. The use of linearized conventual quantum hydrodynamic model has recently to some controversies on the novel attractive potential between quantum screened ions due to the wavenumber scale mismatch \[65–74\]. It has been shown that energy dispersion of plasmon excitations consists of two main branches each representing different high and low phase-speed phenomena in plasmon excitations \[75–81\]. The attractive quantum screening potential obtained in linearized quantum hydrodynamic framework has also been found to be only the ground state plasmonic effect in Schrödinger-Poisson model.

II. THE MATHEMATICAL MODEL

In order to study the plasmon excitations in an arbitrary degenerate electrostatically interacting electron gas in an ambient jellium-like neutralizing fixed background positive ions, we use the following one-dimensional Schrödinger-Poisson model \[64\]

\[
\begin{align*}
\hbar \frac{\partial N}{\partial t} & = -\frac{\hbar^2}{2m} \frac{\partial^2 N}{\partial x^2} - e\phi N + \mu(n, T)N, \\
\frac{\partial^2 \phi}{\partial x^2} & = 4\pi e(|N|^2 - n_0),
\end{align*}
\]

in which \(N = \sqrt{n(x, t)} \exp[iS(x, t)/\hbar]\) is the state function of the spatiotemporal evolution in the jellium model with \(NN^* = n(x, t)\) characterizing the number density and \(v(x, t) = (1/m)\partial S(x, t)/\partial x\) speed of the electron fluid. Also, \(\phi\) is the collective electrostatic field due to all charges in the system and \(n_0\) is the background charge density. Moreover, \(\mu(n, T)\) is the chemical potential of the gas given through the generalized equation of state (EoS) for
isothermal electron gas with arbitrary degree of degeneracy.

For our case the electron EoS relates the fundamental thermodynamic quantities such as the statistical pressure and the number density to the chemical potential $\mu$ and the electron temperature $T$ as

$$n = -D \text{Li}_{3/2}[-\exp(\beta \mu)], \quad P = -\frac{D}{\beta} \text{Li}_{5/2}[-\exp(\beta \mu)],$$

(2)
in which $\beta = 1/k_B T$ and $D$ is called the effective electron density of states \cite{20}

$$D = \frac{2}{\Lambda^3} = 2 \left( \frac{m}{2\pi \beta \hbar^2} \right)^{3/2},$$

(3)

with the parameter $\Lambda$ being the electron thermal de Broglie wavelength. The EoS (2) is also conveniently written in a more compact way

$$P = \frac{n \text{Li}_{5/2}[-\exp(\beta \mu)]}{\beta \text{Li}_{3/2}[-\exp(\beta \mu)]},$$

(4)

where the polylogarithm function is defined in terms of the Fermi functions

$$\text{Li}_\nu(-e^z) = -\frac{1}{\Gamma(\nu)} \int_0^\infty \frac{x^{\nu-1}}{\exp(x - z) + 1} dx, \quad \nu > 0,$$

(5)

with $\Gamma$ being the ordinary gamma function.

Note that the EoS can be expanded in the limiting cases of non-degenerate (classical) and fully degenerate limits. For the fully degenerate isothermal electron gas, $z \gg 1$, one obtains the form $\lim_{z \to \infty} \text{Li}_\nu(-e^z) = -z^\nu/\Gamma(\nu + 1)$ while in the extreme opposite non-degenerate limit, $z \ll -1$, one gets $\text{Li}_\nu(-e^z) \approx -e^z$. It is evident that in the complete degeneracy limit (such as in metals) the equation of state becomes $P = (2/5)n k_B T_F$ in which $T_F = E_F/k_B$ is the electron Fermi temperature with $E_F = \hbar^2 (3\pi^2 n)^{2/3}/(2m)$ being the Fermi energy of the completely degenerate (zero temperature) electron gas and $\hbar$ is the scaled Planck constant whereas in the non-degenerate limit we retain $P = n k_B T$ for a classical dilute electron gas.

We intend to study the system \cite{11} in the linear limit for simplicity in current analysis. Such a simplified model while other minor interaction effects such as the electron exchange and correlations provides many interesting collective features of the electron system. Appropriately, normalized equations constitute a coupled time-independent system of coupled differential equation

$$\frac{d^2 \Psi(x)}{dx^2} + \Phi(x) + 2E \Psi(x) = 0,$$

(6a)

$$\frac{d^2 \Phi(x)}{dx^2} - \Psi(x) = 0,$$

(6b)
where it has been assumed that $N(x,t) = \psi(x)\psi(t)$ for the purpose of variables separation and $\Psi(x) = \psi(x)/\sqrt{n_0}$ and $\Phi(x) = e\phi(x)/2E_p$ with $E_p = \hbar\sqrt{4\pi e^2 n_0/m}$ is the plasmon energy of the system. Also, the $E = (\epsilon - \mu_0)/2E_p$ with $\epsilon$ being the eigenenergy of the system being defined through $\epsilon = \hbar\omega = i\hbar d\psi(t)/dt$ with $\omega$ being the eigenfrequency of the plasmon oscillations. Note that the space and time variables are also normalized respectively to the plasmon length $1/k_p$ with $k_p = \sqrt{2mE_p}/\hbar$ being the plasmon wavenumber and $\hbar/E_p$. Note also that we have used the Thomas-Fermi assumption for the chemical potential where the potential is supposed to be constant throughout the system in the linear approximation. However, for the Bose-Einstein gas or in a nonlinear charged system this quantity may vary considerably so that the local electrostatic potential can not cancel the variations in the chemical potential. The dual-tone solution to (6) is

$$
\begin{bmatrix}
\Phi_g(x) \\
\Psi_g(x)
\end{bmatrix} = \frac{1}{2\alpha} \begin{bmatrix}
\Psi_0 + k_2^2 \Phi_0 & - (\Psi_0 + k_1^2 \Phi_0) \\
-(\Phi_0 + k_1^2 \Psi_0) & \Phi_0 + k_2^2 \Psi_0
\end{bmatrix} \begin{bmatrix}
\cos(k_1 x) \\
\cos(k_2 x)
\end{bmatrix},
$$

(7)

where $\Phi_0$ and $\Psi_0$ denote the functional values at the origin and the characteristic wavenumbers $k_1$ and $k_2$ are given below

$$
k_1 = \sqrt{E - \alpha}, \quad k_2 = \sqrt{E + \alpha}, \quad \alpha = \sqrt{E^2 - 1}.
$$

(8)

The interesting complementarity-like relation $k_1 k_2 = 1$ characterizes the connection between the wave and particle aspects of excitations. The solution (8) leads to the generalized energy dispersion $E = (1 + k^4)/2k^2$ in which $E$ and $k$ are normalized to $2E_p$ and plasmon wavenumber, respectively. The dual-tone solution (7) interestingly reminds the de Broglie’s double solution proposed for the pilot-wave theory [82] in which a single electron can be assumed to be guided by the collective electrostatic interaction wave in the electron gas. The pilot wave theory has gained a renewed interest with many experimental support [83].

The one-dimensional model of plasmon excitations in electron gas (6) has been generalized to the damped pseudo-force system in order to include the charge screening effect or similar many-body problems [77].

$$
\begin{align*}
\frac{d^2\Psi(x)}{dx^2} + 2\xi \frac{d\Psi(x)}{dx} + \Phi(x) + 2E\Psi(x) &= 0, \\
\frac{d^2\Phi(x)}{dx^2} + 2\xi \frac{d\Phi(x)}{dx} - \Psi(x) &= 0,
\end{align*}
$$

(9a)

(9b)

where $\xi = k_{sc}/k_p$ with the normalized screening parameter $\xi^2 = (E_p/2n_0)\partial n/\partial \mu = (1/2\theta)\text{Li}_{1/2}[-\exp(2\mu/\theta)]/\text{Li}_{3/2}[-\exp(2\mu/\theta)]$ being the one-dimensional screening
wavenumber in the finite temperature Thomas-Fermi model \[77\] where \( \theta = T/T_p \) is the normalized temperature of the electron system with \( T_p = E_p/k_B \) being the characteristic plasmon temperature. The solution to damped pseudoforce system (9) is

\[
\Phi_d(x) = \frac{e^{-\xi x}}{2\alpha} \left\{ \begin{array}{l}
(k_2^2 \Phi_0 + \Psi_0) \left[ \cos(\beta_1 x) + \frac{\xi}{\beta_1} \sin(\beta_1 x) \right] - \\
(k_1^2 \Phi_0 + \Psi_0) \left[ \cos(\beta_2 x) + \frac{\xi}{\beta_2} \sin(\beta_2 x) \right]
\end{array} \right\},
\]

(10a)

\[
\Psi_d(x) = \frac{e^{-\xi x}}{2\alpha} \left\{ \begin{array}{l}
(\Phi_0 + k_2^2 \Psi_0) \left[ \cos(\beta_2 x) + \frac{\xi}{\beta_2} \sin(\beta_2 x) \right] - \\
(\Phi_0 + k_1^2 \Psi_0) \left[ \cos(\beta_1 x) + \frac{\xi}{\beta_1} \sin(\beta_1 x) \right]
\end{array} \right\},
\]

(10b)

where \( \beta_1 = \sqrt{k_1^2 - \xi^2} \) and \( \beta_2 = \sqrt{k_2^2 - \xi^2} \). The initial values \( \Psi(x) \) and \( \Phi(x) \) are related through a universal relation \( \Phi_0 = \gamma \Psi_0 \) with \( \gamma = 2\hbar \sqrt{\pi/m} \) at the boundary of two plasmonic environments [80].

Figure 1 shows variations of the characteristic plasmon parameters for the electron gas of arbitrary degeneracy. Figure 1(a) depicts the variations of normalized one-dimensional damping parameter \( \xi(\mu, \theta) \) in terms of the normalized chemical potential \( \mu \) for various values of the normalized plasmon temperature \( \theta \). It is remarked that with increase in the chemical potential of the gas the damping parameter decreases substantially. In the model of one-dimensional screened surface it might be interpreted that with increase in the degeneracy degree at given electron temperature the screening length increases while it seems unrealistic since when plasma density increases the Debye length always decreases. However, for the normalized screening parameter \( \xi \) with the scaling parameter as \( k_p \) which is directly related to the electron density the decrease in this parameter is convincing. It is also seen that as the normalized temperature increases variations in \( \xi \) over the chemical potential range decreases. The figure also shows that for the case of complete degeneracy the dependence of the normalized damping parameter to \( \theta \) becomes insignificant. Moreover, Fig. 1(b) shows the variation in plasmon energy in terms of electron number density in a logarithmic scale. It is remarked that the increase in plasmon energy becomes sharp as the degeneracy regimes \( (n_0 \simeq 10^{18} \text{cm}^{-3}) \) starts. For typical metals the plasmon energy amounts a few electron Volts. For instance, for cesium with the plasmon energy as low as \( E_p \simeq 2.9 \text{eV} \) can reach very high for aluminium with \( E_p \simeq 15 \text{eV} \). However, in order to compare the screening effect in these two metals the difference in their chemical potential which is their Fermi energy at zero-temperature limit \( (E_F^{Cs} = 1.59 \text{eV}, E_F^{Al} = 11.7 \text{eV}) \) must be taken into account. Figure
FIG. 1: (a) The variation of normalized screening (damping) parameter as a function of normalized chemical potential for different values of fractional electron temperature $\theta$. (b) Variation in the plasmon energy in terms of electron number density in logarithmic scale. (c) Variations of normalized electron temperature in terms of electron number density for different values of the electron temperature. (d) The plasmon length variation in terms of the electron number density. The increase in the thickness of curves in each plot is meant to represent an increase in the varied parameters above each panel.
1(c) depicts the variations in the fractional temperature $\theta$ as a function of the electron number density for various electron temperature values. It is seen that for any given electron temperature value the normalized electron temperature decreases significantly with increase in the electron number-density. The variation in this parameter gets even more significant when the electron temperature increases. It is remarked that in the complete degeneracy limit dependence of this parameter to electron temperature becomes insignificant. Figure 1(d) shows the variation in the plasmon length in terms of the electron number density in nanometer scale. It is remarked that this characteristic length decreases sharply with increase in the electron number density with values as high as few tenth of micrometers for semiconductors to as low as few nanometers in completely degenerate electrons in metallic compounds.

III. MODEL OF HALF-SPACE PLASMON EXCITATIONS

To this end, let us consider the case in which the electron gas resides in the half-space ($x < 0$). For simplicity we ignore the dynamics of ions in the jellium and their screening effect. Therefore, there is a sharp discontinuity at $x = 0$. For the half-space $x > 0$ there exists a small fraction of electron spill-out effect due to damped plasmon extension from the arbitrary degenerate electron gas at $x < 0$. Therefore, the complete solution of the system consists of $\Psi(x) = \Psi(x) + \Psi(x)$ and $\Phi(x) = \Phi(x) + \Phi(x)$ where $\Psi(x) = \Psi_d$ ($\Phi(x) = \Phi_d$) and $\Psi(x) = \Psi_g$ ($\Phi(x) = \Phi_g$).

Figure 2 shows the variations of normalized electron number density $n(x) = \Psi(x)\Psi^*(x)$ and electrostatic energy $\Phi(x)$ for the half-space electron gas for different values of the energy eigenvalues. It is clearly evident that the values and derivatives of functions match at the half-space boundary. As remarked in Figs. 2(a) and 2(b), for $x < 0$ there is a double-tone variation in the electron probability density, $\Psi(x)$ and consequently a complex profile in electron number density variations. Meanwhile in region $x > 0$ there is an exponential decrease in amplitude of density variations accompanied with rapid oscillation in the structure. In Fig. 2(b) the increase in energy eigenvalue leads to relative decrease in the density profile. Variations in the electrostatic energy of the system is shown in Figs. 2(c) and 2(d). It seems that the variations in $x < 0$ region of $\Phi(x)$ is more regular compared to that of $\Psi(x)$. The plasmonic oscillations are of double-wavenumber character with the smaller wavelength
FIG. 2: Profiles of electron number density and electrostatic potential energy for gas-vacuum half-space plasmon excitations at given energy eigenvalue given above each panel. The boundary at \( x = 0 \) separates the electron gas (shaded area) from the vacuum (unshaded region).

scale corresponding to single particle behavior and the larger one to collective behavior in the electron gas. It is therefore seen that the electrostatic energy contribution of the electron gas due to single particle excitations is comparable lower and is seen with small oscillations modulated over a larger pattern which is due to the collective excitations in the gas. However this is not the case for the probability density \( \Psi(x) \), hence, the number density profile
has a more complex character compared to those of the electrostatic energy. On the other hand, for \( x > 0 \) the electrostatic energy decay has approximately pure exponential decay without oscillations. In fact such oscillations exist but with very small amplitudes which are not detected in a large scale. Figure 2(d) reveals that unlike for the case of number density profiles increase of energy eigenvalue does not significantly alter the amplitude of electrostatic energy profile. However in the region \( x > 0 \) the decay rate for lower energy eigenvalue of Fig. 2(c) is slightly higher compared to that of high energy eigenvalue shown in Fig. 2(d).

The functional \( \Psi(x) \) and \( \Phi(x) \) provide sufficient information in order to obtain thermodynamic quantities at an equilibrium state. We need to know the plasmon energy statistical distribution and density of states (DoS) in order to obtain the statistical quantities as

\[
\langle \Phi(\mu, \theta, x) \rangle = \frac{\int_1^\infty \Phi(E, \mu, \theta, x) f(E, \mu, \theta) D(E) dE}{\int_1^\infty f(E, \mu, \theta) D(E) dE},
\]

(11a)

\[
\langle \Psi(\mu, \theta, x) \rangle = \frac{\int_1^\infty \Psi(E, \mu, \theta, x) f(E, \mu, \theta) D(E) dE}{\int_1^\infty f(E, \mu, \theta) D(E) dE},
\]

(11b)

\[
\langle n(\mu, \theta, x) \rangle = |\langle \Psi(\mu, \theta, x) \rangle|^2;
\]

(11c)

where \( f(E) = [\exp(2E/\theta) - 1]^{-1} \) is the Bose-Einstein occupation number for plasmon excitations and \( D(E) \) is the plasmon energy DoS given as [75]

\[
D(E) = \frac{V \sqrt{E(4E^2 - 3) + (4E^2 - 1) \sqrt{E^2 - 1}}}{2\pi^2 \sqrt{E^2 - 1}},
\]

(12)

in which \( V \) is the volume of the specimen which is taken to be unity in this analysis, for simplicity. Note that the DoS used here is different from that in Refs. [75, 77] because here we have used the plasmon dispersion \( E = (1 + k^4)/2k^2 \) in which \( E \) is normalized to \( 2E_p \) while in Refs. [75, 77] in calculation of DoS the dispersion \( E = (1 + k^4)/k^2 \) is used in which the energy is normalized to \( E_p \). Consequently, the lower integration limits differ here from Refs. [75, 77].

Figure 3 depicts the spatial distribution of normalized equilibrium electron number density and electrostatic energy with normalized chemical potential and temperature. Figure 3(a) shows the electron number density profile for given values of \( \mu = 0.3 \) and \( \theta = 0.2 \).
FIG. 3: Normalized and statistically averaged values of electron number density and electrostatic potential energy for gas-vacuum half-space plasmon excitations at the thermal equilibrium condition for given parameter values of $\theta$ and $\mu$. The jellium boundary at $x = 0$ separates the uniform positive charge background (shaded area) from the vacuum (unshaded region) in plots (a) and (c). The boundary $x = 0$ is the jellium edge before which the positive uniform static background charge takes place. This figure illustrates interesting features of half-space plasmon excitations at equilibrium state. It is remarked that before the jellium edge electrons are significantly depleted and consequently a plasmon boundary forms due to accumulation of
electrons near the boundary. The second feature which is quite unique to the system is the well-defined periodic density structure in the electron gas region \((x < 0)\) which is due to resonant interaction between single electron and collective excitations. Such feature has been shown to be characteristics of an electron gas confined in an infinite potential well \[64\].

Another remarkable feature is that beyond the jellium edge \((x > 0)\) small density hump appears which is usually explained as electron spill out effect. However, as it is evident the electron density in current analysis forms an electron halo rather than spill out due to the depleted region before the density hump after the boundary. The later is a very important feature with a far reaching consequences for bound state formation at the surface of perfectly conducting metallic elements, as will be discussed in the following. The envelop structure of density profile looks quite similar to those discussed in many literature \[84, 85\]. However, the periodic fine structure is quite unique for our model. Figure 3(b) depicts the electrostatic energy profile corresponding to values used in Fig. 3(a). It is seen that the periodic structure is also present for this case. Close to the jellium boundary the amplitude of electrostatic energy is slightly increases and after the boundary the energy monotonically decreases towards infinity. Figure 3(c) and 3(d) depict the normalized equilibrium profiles for increased value of fractional electron temperature. It is evident from Fig. 3(c) that the plasmon boundary gets higher and the electron depletion becomes more significant as \(\theta\) increases. Note that formation of a surface dipole just before the jellium edge. It is also remarked that electron halo density increases significantly due to increase of this parameter value. Quite similar trend appears for the electrostatic energy profile for elevated temperature \(\theta\).

Figure 4 depicts the scaled variations of equilibrium density and electrostatic energy after the physical boundary at \(x > 0\). Figure 4(a) shows that the density profile has a double peak structure with the smallest first peak which has more resemblance to electron spill-out is merged into the boundary while the second larger peak (electron halo) decreases towards infinity. The increase of the parameter \(\theta\) leads to significant increase in the amplitude of the electron halo as well as small peak near the surface with almost no effect on the position of the maximum amplitude. The scaled electrostatic energy profile for similar parameter variations as in Fig. 4(a) is depicted in Fig. 4(b). It is remarked that the electrostatic energy profile has a flat-top structure near the boundary with highest amplitude sharply dependent to the fractional temperature. Figures 4(c) and 4(d) show that these profiles agains the variations in normalized chemical potential of the electron gas. It is remarked that increase in the value
FIG. 4: The scaled variations of equilibrium electron density and electrostatic potential energy in the vacuum side for different values of the normalized electron temperature and chemical potential. The increase in the thickness of curves in each plot is meant to represent an increase in the varied parameter above each panel.

of the chemical potential with $\theta$ fixed leads to increase in the amplitude of electron halo density moving the maximum amplitude closer to the boundary. However, such variations in the chemical potential of the electron gas has insignificant effect on the small density peak. It is interesting that for higher values of $\mu$ which coincides with that of the metallic
compounds Friedel-like oscillations in density appearing in front of the electron halo. This feature is quite similar to charge screening in quantum plasmas. Effect of the change in normalized chemical potential on electrostatic energy profile for same parameter values as in Fig. 4(c) is shown in Fig. 4(d). It is seen that increase in the chemical potential leads to significant increase in potential decay-rate while for fully degenerate electron gas a fundamental feature emerges which is the formation of a pronounced bound state valley indication of a Lennard-Jones-like attractive potential and distances few ten to hundred nanometers away from the electron gas boundary. This feature also reflect quite similar features as charge screening in quantum plasmas. The existence of bound state at perfectly conducting surfaces along with the surface dipole effect may appropriately explain the well-known Casimir effect and less understood Casimir-Polder forces.

Figure 5 shows the detailed variations of the electrostatic potential valley features with the change in fractional temperature and chemical potential. Figure 5(a) shows that increase of the normalized temperature for fixed value of the chemical potential leads to significant decrease of depth of potential energy valley without altering the minimum position. On the other hand, Fig. 5(b) indicates that the increase of the chemical potential for fixed normalized electron gas temperature leads to increase in the depth of the attractive potential valley moving the potential minimum close to the boundary. Figures 5(c) and 5(d) show the variations of $\theta$ and $\mu$ with respect to the electron number density. In terms of $\theta$ and $\mu$ the electron number density can be written as $n_0(\mu, \theta) = n_p\theta^{6}\text{Li}_{3/2}[-\exp(2\mu/\theta)]^4$ with $n_p = 16e^6m^3/(\pi^3h^6) \approx 5.66 \times 10^{19}\text{cm}^{-3}$ being the characteristic plasmon number-density. According to Fig. 5(c) almost independent of $\theta$ the metallic density reaches for $\mu > 1$ region. It is therefore concluded that the attractive potential valley should be present for electron densities even relevant for strongly doped N-type semiconductors beside the known metals. It is further concluded from Fig. 5(d) that for very high values of $\theta$ the electron density approaches the fully degenerate limit where the attractive potential feature should be present.

IV. CONCLUSION

In current research we developed a theoretical model to study the half-space plasmonic excitations in an electron gas of arbitrary degeneracy. We used the linearized Schrödinger-
FIG. 5: (a) Changes in the attractive electrostatic energy valley with change in the fractional electron temperature. (b) Change in the attractive electrostatic energy valley with change in the normalized chemical potential of the electron gas. (c) Variation of the electron number density with normalized chemical potential for different values of electron number density. (d) Variation of the normalized electron temperature with normalized chemical potential for different values of normalized electron temperature. The increase in the thickness of curves in each plot is meant to represent an increase in the varied parameter above each panel.
Poisson system and reduced it to the coupled pseudoforce type second-order differential equations solution of which with and without damping effects is used to appropriately construct the state function of half-space electron gas excitations. Current dual-tone quantum theory appropriately captures the essential electrostatic interactions among single electrons and the collective entity and because of that supersedes many of previous studies which fail to consider such interactions. Many novel features of half-space plasmonic excitation were coined in this research. A remarkable Lennard-Jones-like attractive electrostatic potential energy valley was found in front of a physical jellium boundary of fully degenerate electron gas characteristics of the Casimir-like effect. Current study may also be extended to study the half-space plasmon excitations in other geometries such as spherical geometry in order to study the microscopic forces among nanoparticles.

V. DATA AVAILABILITY

The data that support the findings of this study are available from the corresponding author upon reasonable request.

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