Effect of Delay Approximation of various orders on the open loop response of a Delayed Process Model

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Abstract. In present analysis, response of a selected delayed first order process model is compared for various orders of approximation of dead time present in the transfer function model. Effect of different orders of approximation of dead time is analysed on the open loop step response of the system in terms of percentage change in original and dead time approximated system models. For this purpose, consistency parameter in a headbox of a paper machine is selected as a process model as it closely approximates to a first order delayed process model.

Keywords. Delayed Control Process, Pade Approximations, First Order Plus Dead Time (FOPDT), Second Order Plus Dead Time (SOPDT)

1. Introduction

The primary function of the head box is to uniformly distribute the stock on the next element of wet end of paper machine, called wire. This is situated just below the opening of the head box as a continuous wire mesh, moving at a constant speed such that when stock falls on it, the water is drained out to give shape of loose wet sheet of fibre which goes to the press section. The speed of the moving wire mesh should be so adjusted in relation to the falling rate of stock on it from head box such that a uniform sheet of a particular grade is formed. Paper machine Headbox is the heart of the paper making process [1]. For high speed machine or for better property development, twin wire or hybrid wire are used instead of open wire.

The paper machine is the last part of the long chain of processes for making paper. It is an important subsystem at the end of the entire paper making system, consisting of the raw material preparation steps to paper finishing stage. Process flow in paper machine is shown in Fig.1.
One of the most important process variables in the pulp and paper industry is consistency of pulp and paper stock [2, 3]. Consistency is defined as the percentage, by weight, of dry fibrous material in any combination of pulp and water or stock and water. It is majorly affected by the dilution water [4]. It is calculated by the following formula:

\[ C = \frac{F}{W} \times 100 \quad (1) \]

Where, 
- \( C \) = the consistency of pulp expressed in per cent
- \( W \) = the total weight of a particular amount of pulp or stock slurry
- \( F \) = the weight of fibrous material in that amount of pulp or stock slurry.

Consistencies of less than 1% are usually considered low, those greater than 6% high. Most development of consistency control has been in ranges between these two values.

**Figure 1. Process flow in paper machine**

2. Literature Review

The control system design is achieved through the mathematical modeling of the process. The designed controller is also modified and checked for most of the processes specially the delayed processes (both FOPDT and SOPDT) [5, 6], for robustness [7] and various other parameters [8]. There are various techniques to this such as rationalisation of the model [8], Prediction based [9], IMC-PID [10] and various others [11, 12]. There have been various control strategies that are implemented to Paper Making Process (specially head box) from a very long span of time such as MPC [13], Level control [14], Fuzzy clustering [15] and IMC [16] etc.

3. Methodology

The simplest pole-zero approximation is the first order Pade approximation.

\[ e^{-\theta s} \approx G_1(s) = \frac{1 - \frac{\theta}{2} s}{2} \]

(2)

Above equation is called the first order Pade approximation. The second order Pade approximation:
The transfer function for the selected consistency parameter of a headbox in a paper machine has been taken from [17] is:

\[
e^{-\theta k} \approx G_2(s) = 1 - \frac{\theta k}{2} + \frac{\theta_2 s_2}{12} \quad (3)
\]

This process transfer function is approximated for its delay element of various orders.

4. Result and Discussion

Once we have approximated the selected transfer function using Pade approximation technique for first, second, third, fifth, fifteenth and twentieth orders, their step responses are achieved with the help of MATLAB software.

Fig. 2 shows the comparative step responses for transfer function between original transfer function and its first order approximation of dead time. Fig. 3 shows the comparative step responses for transfer function between its first order and fifth order approximated transfer function models.

![Figure 2. Comparison of Step responses for first order Pade approximation and without approximation.](image-url)
Figure 3. Comparison between step response for first order and fifth order Pade approximation of consistency transfer function.

Figure 4. Comparison between step response for fifth order and fifteen order Pade approximation of consistency transfer function.
Figure 5. Comparison between step response for first order and fifteen order Pade approximation of consistency transfer function.

Figure 6. Comparison between fifteenth order and twentieth order with important characteristics.
Fig. 4 shows the comparative step responses for transfer function between its fifth order and fifteenth order approximated transfer function models. The step responses resemble very closely, except for few seconds of starting. Fig. 5 shows the comparative step responses for transfer function between its first order and fifteenth order approximated transfer function models. The step responses resemble very closely, after 10 seconds onwards.

Fig. 6 shows the comparative step responses for transfer function between its fifteenth order and twentieth order approximated transfer function models. The step responses resemble very closely throughout the time span for which responses are shown. Table 1 shows the important step response characteristics, rise time and settling time for different orders of Pade approximations for selected consistency parameter transfer function.

| Order of Pade approximation | Rise time, s | Settling time, s | Percentage error in rise time in comparison to without approximation | Percentage error in settling time in comparison to without approximation |
|-----------------------------|--------------|------------------|---------------------------------------------------------------------|---------------------------------------------------------------------|
| Without dead time approximation | 8.44         | 21.9             | 0                                                                   | 0                                                                   |
| With 1st order approximation  | 11.5         | 23.0             | 36.25                                                              | 5                                                                   |
| With second order approximation | 9.39         | 21.3             | 11.25                                                              | 2.73                                                                |
| With third order approximation | 8.83         | 21.6             | 4.62                                                               | 1.36                                                                |
| With fifth order approximation | 8.58         | 21.7             | 1.65                                                               | 0.913                                                               |
| With fifteenth order approximation | 8.42         | 21.8             | 0.236                                                              | 0.45                                                                |
| With twentieth order approximation | 8.42        | 21.8             | 0.236                                                              | 0.45                                                                |

5. Conclusion

In the case of twentieth order approximation of dead time, the response is exactly same as that of fifteenth order. With increase in order of the delay approximation, the response does not improve but the complexity increases in the transfer function mathematical expression. There must be a compromise made between the order of approximation between model perfection and the order of the dead time Pade approximation. From present simulation example, fifth order approximation is an optimal compromise
between model perfection and the complexity in mathematical transfer function model representation and there is no need for further increase in order of approximation.

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