MULTIPARTICLE PRODUCTION IN THE SOFT LIMIT AND QCD COHERENCE

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The production of gluons in a jet is considered in limited phase space, either with a cut in transverse momentum with respect to the jet axis $k_{\perp} < k_{\perp}^{\text{cut}}$ or with a cut in absolute momentum $|\vec{k}| < k^{\text{cut}}$. It is shown in the perturbative QCD calculations that in the soft limit $k_{\perp}^{\text{cut}} \to 0$ the multiplicity distribution becomes Poissonian, whereas for $k^{\text{cut}} \to 0$ the distribution remains non-Poissonian. The Poissonian limit is a consequence of the soft gluon coherence in the genuine multiparticle correlations. We also investigate how incoherent hadronization processes could possibly modify the predictions for small cut-off parameters.

1 Introduction

Multiparticle production remains an interesting testing ground for the predictions of perturbative QCD and also for the models of colour confinement. An approach of great simplicity is based on the idea of duality between the properties of the hadronic and partonic final state. This became first quantitative for the particle energy spectra which – when calculated in the modified leading log approximation (MLLA) – provided a good description of the data ("Local Parton Hadron Duality"). This poses the question whether the perturbative calculations are relevant to a larger class of observables including the soft particle production. Quite a number of different observables have been calculated and the comparison with experiment is encouraging.

The standard hadronization models and the duality picture are comparable in the first phase of the jet evolution where secondary partons are produced by bremsstrahlung and pair production processes down to a relative scale of about 1 GeV. At this scale nonperturbative processes may take over as they are described in hadronization models. Alternatively, the perturbative phase may be evolved further down towards lower scales of a few 100 MeV before the cascade is terminated by a cut-off in transverse momentum

$$k_{\perp} \geq Q_0. \quad (1)$$

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This non-perturbative cut-off represents the influence of the confinement: only if a gluon is radiated with larger transverse momentum it gives rise to the production of an extra particle. The production of hadrons may then be rather similar to the production of partons and in fact the data on hadron and jet multiplicities suggest taking the number of partons and hadrons to be equal.

In order to test this duality picture further it is particularly important to investigate the final stage of the jet evolution at scales around and even below 1 GeV. Can the decay processes involving resonance decays be described in an average sense by a perturbative partonic cascade?

To approach this question it is interesting to study the peculiar aspects of perturbative QCD in the soft region, in particular:
1. the running of the coupling $\alpha_s(k_\perp)$ with the transverse momentum
2. the soft gluon coherence which limits the emission angle of a soft gluon by the angle of the nearest colour connected parton.

Some support for the running coupling at small scales is suggested by the rapid rise of multiplicity under small scale variations at small absolute scales. Support for the coherence phenomenon at small scales can be drawn from the observation of an essentially energy independent yield of soft particles (below a momentum of $\sim 200$ MeV) over the full energy range explored in $e^+e^-$ annihilation.

In this talk I will report about a recent analysis of genuine multipartice correlations in the soft limit carried out in collaboration with S. Lupia and J. Wosiek. The question is whether the peculiar predictions of the perturbative theory are followed by hadrons in this novel situation. As a consequence of the soft gluon coherence, the gluons emitted with small transverse momentum from the primary parton in the jet – which typically requires small angles – cannot radiate again. Therefore in this limit the gluon emission resembles the photon emission in QED in Born approximation and one expects besides the energy independent bremsstrahlung with a flat rapidity plateau an independent multiparticle emission and this corresponds to a Poissonian distribution of the emitted soft particles.

2 Definition of Observables and Methods of calculation

In order to study the multiparticle phenomena in the soft limit we introduce the multiplicity moments in restricted phase space

$$f_c^{(q)}(C,Q) = \int_{\Gamma_c(C,Q)} \rho^{(q)}(k_1\ldots k_q;Q)dk_1\ldots dk_q, \quad (2)$$
where \( \rho^{(q)} \) denotes the \( q \)-particle inclusive distribution in the momenta \( k_i \) at energy scale \( Q \) and the phase space integration is restricted by \( \Gamma_c(C, Q) \) with a cut variable \( C \). Two types of cuts to be applied to all particles in the final state have been discussed: the momentum cut \( |\vec{k}_i| < k_{\text{cut}} \), and the transverse momentum cut \( k_{\perp,i} < k_{\text{cut}}^{\perp} \), for \( i = 1, \ldots, q \). Clearly, the moments (2) determine the multiplicity distribution of particles produced in the restricted phase space, and as such they provide a more differential characteristics than the global quantities. The latter ones are retained for the maximal \( k_{\text{cut}} \) and \( k_{\text{cut}}^{\perp} \) at a given \( Q \).

The theoretical analysis in perturbative QCD has been carried out in two ways. First, analytical results have been obtained within the double logarithmic approximation (DLA) in which only the contributions of the leading soft and collinear singularities are kept. These results can serve to provide the correct qualitative picture and to test the influence of the colour coherence and the running coupling on the results. The DLA for the cut moments has been worked out completely and the results can be obtained perturbatively to any order in the coupling.

It is known since a while that the global multiplicity moments are not reproduced by the DLA results quantitatively. More recently it has been shown that these moments are not well reproduced by the NLO results, neither nor by the NNLO calculations although a clear improvement is then obtained. Agreement with the data is found finally in a numerical solution of the evolution equations. In order to obtain realistic estimates of the new cut moments we have therefore derived also the predictions from a Monte Carlo program (ARIADNE) which is built on similar procedures than the analytical calculations.

3 Analytical results on Multiplicity Moments

3.1 Results in the soft limit

The theoretical analysis starts from the equations for the generating functionals for the quark and gluon jets; then by the appropriate differentiation one obtains the equations for cumulant and factorial moments which are solved and the moments integrated over the regions of phase space required.

In the case of small cut-off parameters which is of primary interest to us various simplifications in the analytical analysis can be applied: first, it is enough to take into account the terms of lowest order in the coupling, moreover, in this limit, the solutions for running coupling approach those for fixed coupling in which case close expressions are obtained. We present here only the main results and refer to the original paper for the derivation.

Let’s consider first the cylindrical phase space with small cut-off \( k_{\perp,\text{cut}} \). For
the normalized cumulant moments we find

\[ K^{(q)}(X_\perp, Y) = \frac{qf_{q-1}}{2q-1} \left( \frac{X_\perp}{Y} \right)^{q-1} \text{ for } X_\perp \ll \lambda, Y. \]  

(3)

in terms of the logarithmic variables

\[ X_\perp = \ln \frac{k_{\perp \text{cut}}}{Q_0}, \quad Y = \ln \frac{P\Theta}{Q_0}, \quad \lambda = \ln \frac{Q_0}{\Lambda}, \]  

(4)

for a jet of momentum \( P \) and half opening angle \( \Theta \). The numbers \( f_q \) are to be calculated recursively \( (f_0 = 1, f_1 = \frac{1}{2}, f_2 = \frac{1}{3}, \ldots ) \). One observes that the moments \( K^{(q)} \) decrease quickly with the order \( q \) corresponding to what has been called “linked pair ansatz” for multiparticle correlations. For the normalized factorial moments \( F^{(2)} = 1 + K^{(2)} = \frac{<n(n-1)>}{<n>} \) one obtains for small cut-off \( X_\perp \)

\[ F^{(q)}(X_\perp, Y) \approx 1 + q(q-1)X_\perp / 6Y. \]  

(5)

So we obtain the remarkable result that for small transverse momentum cut-off all factorial moments approach unity and therefore the multiplicity distribution becomes Poissonian. This is a consequence of the dominance of the single soft gluon emission at small \( k_\perp \), i.e., the absence of branching processes with secondary gluon emissions. This behaviour is just analogous to the usual QED bremsstrahlung and follows from the coherence of the soft gluon radiation and the angular ordering condition \( \theta \) which limits the angles of the secondary particle emission by the (typically small) emission angle of the first gluon. Namely, if this angular ordering condition is suppressed artificially the moments remain constant with \( X_\perp \) and the multiplicity distribution remains non-Poissonian.

On the other hand, if we carry out the same calculations for the spherically cut moments we obtain quite a different behaviour: the moments stay constant down to small cut-off parameters \( X = \ln \frac{k_{\text{cut}}}{Q_0} \) as

\[ K^{(q)}(X, Y) = 2^{q-1}f_{q-1}/(2q-1). \]  

(6)

Consequently, soft gluons with limited momentum \( k \) have essentially a non-Poissonian multiplicity distribution, while those with limited \( k_\perp \) follow a Poissonian one. These calculations can be carried out for running \( \alpha_s \) as well and the results approach those of fixed \( \alpha_s \) in the soft limit as expected on general grounds.
3.2 General Solution in the DLA

The multiplicity moments to arbitrary order in the coupling $\alpha_s$ can be obtained by solving evolution equations in the full range of the cut-off’s $0 < X, X_\perp < Y$. The results reduce to the solutions for small cut-off discussed above and approach correctly the known results for the global moments in full phase space.

The evolution equations for the unnormalized cumulant moments $c^{(q)} = K^{(q)} \bar{n}^{q}$ in case of spherical momentum cut read

$$c^{(q)}_{sph}(X, Y) = \int_0^X dy \gamma_0^2(y) [q f^{(q-1)}(y) + f^{(q)}(y)](X - y)
+ \int_0^X dx \int_x^{Y - X + x} dy \gamma_0^2(y) f^{(q)}_{sph}(x, y)$$

(7)

where $f^{(q)}(Y)$ refer to the known global unnormalized factorial moments and $\gamma_0^2 = 6\alpha_s/\pi$. From this equation the cut moments can be calculated recursively to any precision.

The moments in cylindrical phase space can be obtained from those in spherical phase space using the equation

$$c^{(q)}_{cyl}(X_\perp, Y) = c^{(q)}(X_\perp) +
(Y - X_\perp) \int_0^{X_\perp} \gamma_0^2(y) [q f^{(q-1)}(y) + f^{(q)}(y)] dy
+ \int_{X_\perp}^Y (Y - y) \gamma_0^2(y) f^{(q)}_{sph}(X_\perp, y) dy.$$ 

(8)

The results from this calculation up to $O(\alpha_s^8)$ are shown in Fig. 1 in the full range of cut-off parameters $X, X_\perp$. The figure displays the very different behaviour of the moments in spherical and cylindrical phase space with the latter ones approaching unity for $X_\perp \rightarrow 0$ corresponding to the Poisson limit.

4 Monte Carlo results

We complement our analytical DLA calculations by a Monte Carlo computation of the multiplicity moments. Our aim is to clarify whether the qualitative features of our DLA calculations are verified and to obtain quantitative predictions at the parton level. We have chosen the ARIADNE program\[4] which has a similar procedure in evolving the parton cascade and in the final cutoff\[5] which can be chosen independently of the QCD scale $\Lambda$. By comparing the
Monte Carlo results with the data on the total multiplicities and global moments in $e^+e^-$ annihilation we adjust the parameters of the program $Q_0 = 0.2$ GeV and $\lambda = 0.015$. With these parameters we obtain the parton level predictions for the cylindrical moments in Fig. 2a. The moments clearly show the decrease towards unity for decreasing cut parameter $k_{\text{cut}}$ in qualitative agreement with our analytical calculation. The absolute size, however, is lower in the Monte Carlo than in the analytical calculation which neglect various finite energy effects. The moments for the spherical cut (not shown) keep some larger non-zero values at small cut-off. We take these findings as confirmation of our main analytical results on the very different behaviour of the two families of moments and of the approach to the Poissonian limit of the cylindrical moments.

Furthermore, we studied the effect of hadronization on the considered observables. The idea of Local Parton Hadron Duality has been originally proposed for the single inclusive distributions. The study of the moments
Figure 2: (a) cut moments of order 2 (diamonds), 3 (crosses) and 4 (squares) in one hemisphere defined through the thrust axis as a function of $k_{\perp}^{\text{cut}}$ as predicted by ARIADNE at parton level with parameters readjusted; (b) same as in (a), but at hadron level with default values of the parameters and string fragmentation.
considered here allows for a novel test of these ideas in the case of genuine multiparticle correlations. In Fig. 2b we show the results of the ARIA DNE program with string hadronization using the standard parameters. There is the remarkable prediction that the moments first decrease with decreasing $k_{\perp}^{\text{cut}}$ as in the parton level calculation but for $k_{\perp}^{\text{cut}} < 0.8$ GeV the moments rise again and do not approach the Poissonian limit.

5 Conclusions and Outlook to Other Processes

We have shown that perturbative QCD predicts the different behaviour of multiplicity moments in cylindrical and spherical phase space regions. The approach to a Poissonian in the soft limit in case of the cylindrical cut is a consequence of the soft gluon coherence. This behaviour is also supported by the Monte Carlo calculation which takes into account the most important non-leading effects. The Poissonian multiplicity corresponds to the independent emission of soft gluons distributed with a flat rapidity plateau in close similarity to the QED Bremsstrahlung. On the other hand, hadronization effects may distort the predicted behaviour. It will therefore be very interesting to study these moments experimentally and to find out whether, or, to what extent the perturbative predictions survive the hadronization process in this case of genuine multiparticle correlations.

Our calculations apply to quark and gluon jets in hard collisions. If the (untriggered) high energy $pp$ collisions proceed with semihard gluon exchange and subsequent gluon bremsstrahlung a similar behaviour is expected and the moments with cylindrical cut should approach a Poissonian distribution. On the other hand, in collisions of heavy nuclei, if there is a thermalization process with quark gluon plasma formation, the angular ordering in the cascade process is destroyed and no Poissonian limit is obtained. In this way the multiplicity moments could provide a new probe of thermalization.

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