MEASUREMENTS AND DECOHERENCE

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Abstract. A pedagogical and reasonably self-contained introduction to the measurement problems in quantum mechanics and their partial solution by environment-induced decoherence (plus some other important aspects of decoherence) is given. The point that decoherence does not solve the measurement problems completely is clearly brought out. The relevance of interpretation of quantum mechanics in this context is briefly discussed.
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I. Introduction

During the past three and a half decades, there have been some important developments relating to the foundations of quantum mechanics (QM). The key development is the realisation that a realistic quantum system is not isolated; it is immersed in an environment with which it continuously interacts. An important effect of this interaction is decoherence — the destruction of phase correlations in certain superpositions of quantum states. Incorporation of this phenomenon in the quantum theoretic treatment of measurements serves to contribute substantially towards the solution of the measurement problem which is essentially the problem of understanding as to how the superposition of quantum states of (the measured system + the apparatus) resulting from the measurement-interaction between the two results eventually into a unique pointer state. This development has implications relating to the problem of understanding the emergence of the classical world in a quantum universe and to deeper issues like the objective reality (of objects and phenomena).

A large number of articles (original papers and reviews) and books relating to various aspects of the ‘decoherence program’ have already appeared in literature [1-13]. In these lectures, I shall briefly describe some important developments relating to this program.

II. Density Operators and Reduced Density Operators

In this section we shall cover some elementary background relating to the density operators which will be used throughout the article.

The trace of an operator $A$ in a Hilbert space $\mathcal{H}$ is defined as the sum of expectation values of $A$ with respect to an orthonormal basis:

$$\text{Tr} A = \sum_i <e_i|A|e_i>$$

( the right hand side is well defined only for a subclass of operators— the trace class operators); it is independent of the choice of the orthonormal basis.

A density operator (also called density matrix or statistical operator in quantum mechanics literature) $\rho$ is a self adjoint, positive operator of unit trace:

$$\rho^\dagger = \rho; \quad \rho \geq 0; \quad \text{Tr} \rho = 1.$$  

[An operator $A$ is positive if $<\psi|A|\psi> \geq 0$ for all $\psi \in \mathcal{H}$. Positivity, in fact, implies self-adjointness. The implied redundancy in the definition above,
however, is harmless. A convex combination (weighted sum) of density operators:

$$\rho = \sum_i p_i \rho_i; \quad p_i \geq 0; \quad \sum_i p_i = 1 \tag{1}$$

is a density operator.

Density operators in a quantum mechanical Hilbert space represent states. Those density operators which cannot be expressed as nontrivial convex combinations of other density operators [i.e. those not having more than one nonzero $p_i$ in Eq.(1)] represent pure states. These operators are of the form $\rho = |\psi><\psi|$ (projection operators on one dimensional subspaces of $\mathcal{H}$) and satisfy the condition $\rho^2 = \rho$. Other (non-pure) states are called mixed states or mixtures.

The expectation value of an observable (self-adjoint operator) $A$ in a state $\rho$ is given by $Tr(\rho A)$. With $\rho_i = |\psi_i><\psi_i|$ in Eq.(1), we have

$$Tr(\rho A) = \sum_i p_i Tr(\rho_i A) = \sum_i p_i <\psi_i|A|\psi_i>. \tag{2}$$

In Eq.(2), the averaging over the states $|\psi_i>$ reflects the irreducible probabilistic nature of quantum mechanics (QM); the second averaging with weights $p_i$ reflects the ignorance of the observer as treated in classical probability theory.

In the Schrödinger picture, evolution of states is given in terms of the unitary evolution operators $U(t', t)$ by

$$\rho(t') = U(t', t)\rho(t)U(t', t)^\dagger. \tag{3}$$

For $U(t', t) = exp[-iH(t' - t)/\hbar]$, it gives the von Neumann equation

$$i\hbar \frac{d\rho(t)}{dt} = [H, \rho(t)]. \tag{4}$$

Given the Hilbert spaces $\mathcal{H}_1$ and $\mathcal{H}_2$ for two quantum mechanical systems $S_1$ and $S_2$ respectively, the Hilbert space for the combined system is the tensor product $\mathcal{H} = \mathcal{H}_1 \otimes \mathcal{H}_2$. Denoting by $I_1$ and $I_2$ the unit operators on $\mathcal{H}_1$ and $\mathcal{H}_2$, the operators $A$ on $\mathcal{H}_1$ and $B$ on $\mathcal{H}_2$ are represented as, respectively, the operators $A \otimes I_2$ and $I_1 \otimes B$ on $\mathcal{H}_1 \otimes \mathcal{H}_2$. Given orthonormal bases $\{|e_i>\}$ in $\mathcal{H}_1$ and $\{|f_r>\}$ in $\mathcal{H}_2$, the family $\{|e_i> \otimes |f_r>\}$ constitutes an orthonormal basis in $\mathcal{H}_1 \otimes \mathcal{H}_2$. 

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Given an operator $A = A_1 \otimes A_2$ on $\mathcal{H}_1 \otimes \mathcal{H}_2$, the partial traces
$$Tr_1(A) = (\sum_i <e_i|A_1|e_i>)A_2, \quad Tr_2(A) = (\sum_r <f_r|A_2|f_r>)A_1$$
are operators on $\mathcal{H}_2$ and $\mathcal{H}_1$ respectively. These definitions are extended to general operators of $\mathcal{H}_1 \otimes \mathcal{H}_2$ (supposedly expressible as linear combinations of operators of the form $A_1 \otimes A_2$) by linearity. If $\rho$ is a density operator for the joint system on $\mathcal{H}_1 \otimes \mathcal{H}_2$, the operators
$$\rho_1 = Tr_2(\rho), \quad \rho_2 = Tr_1(\rho)$$
are referred to as the reduced density operators for the systems $S_1$ and $S_2$ respectively; they incorporate the effect of the interaction between the two systems (as reflected in the expectation values of the observables of the two systems) on $S_1$ and $S_2$ respectively in the following sense:
$$Tr[\rho(A \otimes I_2)] = Tr[\rho_1 A], \quad Tr[\rho(I_1 \otimes B)] = Tr[\rho_2 B].$$
Replacing $\rho_i$ (i=1,2) and $\rho$ in Eq.(5) by $\rho_i(t)$ and $\rho(t)$, the von Neumann equation for $\rho(t)$ gives the effective evolution equations [the so-called (generalized) master equations] for $\rho_1(t)$ and $\rho_2(t)$. In the presence of an interaction between $S_1$ and $S_2$, a unitary evolution of $\rho(t)$ generally leads to non-unitary evolutions for $\rho_1(t)$ and $\rho_2(t)$.

Note (1). The decomposition [see Eq.(1)] of a mixed state into pure states is generally not unique. (There is nothing paradoxical or surprising about it.)

(2). Given a pure state on $\mathcal{H}_1 \otimes \mathcal{H}_2$ which is a superposition of the form
$$|\psi > = \sum_{i,r} a_{ir} |e_i > \otimes |f_r >,$$
the corresponding reduced density operators $\rho_1$ and $\rho_2$ generally correspond to mixed states. For example
$$\rho_1 = Tr_2(|\psi <>|\psi >) = \sum_{i,j} a_{is} a_{js}^* |e_i > <> e_j |.$$
[Again, there is nothing paradoxical or surprising about it; the mixed state character of $\rho_1$ reflects the implicit ignorance of the precise (pure) state of $S_1$ when the joint system is given in the state (6).]

(3). On the Hilbert space $\mathcal{H}$ of a system $S$, a density operator of the form
$$\rho = \sum_i p_i |\psi_i > <> \psi_i |$$

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represents an ensemble of systems (each of which is a copy of S) which randomly occupy (pure) states $|\psi_i>$, the probability of occupation of the state $\psi$ (i.e. the fraction of systems being in the state $\psi_i$) being $p_i$. (4). Since every self-adjoint operator can be diagonalized, a reduced density operator also admits a decomposition of the form (8); however, it generally does not admit the interpretation in the note (3) above. (See Ref.[31,32].)

III. The Measurement Problem(s) in Quantum Mechanics

3.1 Measurements in traditional quantum mechanics; the problem of macroscopic superpositions [14,16-18,12,13]

Let us consider the situation in which a quantum mechanical system $S$ (the measured system; it is generally microscopic but need not always be so) with the associated Hilbert space $\mathcal{H}_S$ is given to be in a pure state $|\psi>$. We wish to perform a measurement of a physical quantity represented by the observable (self-adjoint operator) $F$ on $\mathcal{H}_S$. We assume, for simplicity, that $F$ has a discrete non-degenerate spectrum:

$$F|s_j> = \lambda_j|s_j>.$$  \hspace{1cm} (9)

It is assumed that we can experimentally prepare any linear superposition of the states $|s_j>$. The state $|\psi>$ will generally be such a superposition:

$$|\psi> = \sum_j c_j|s_j>.$$  \hspace{1cm} (10)

The measuring apparatus $A$ is a macroscopic system chosen such that a suitable ‘pointer variable’ associated with it takes values $a_j$ in one-to-one correspondence with the $\lambda_j$s. Different pointer positions are assumed to be macroscopically distinguishable.

Niels Bohr had advocated the use of classical physics for the treatment of the apparatus. There is, however, no consistent formalism to describe the interaction between a quantum and a classical system. John von Neumann emphasized that quantum mechanics being (supposedly) a universally applicable theory, every system is basically quantum mechanical. To have a consistent theory of measurement, we must, therefore, treat the apparatus $A$ quantum mechanically. Accordingly, we introduce a Hilbert space $\mathcal{H}_A$ for $A$ and assume that the pointer positions $a_j$ are represented by the states $|a_j>$ in this space.
In the conventional treatment \[14\], one treats the combined system \((S + A)\) as a closed system (ignoring the environment; we shall have to include it later) with the Hilbert space
\[ \mathcal{H} = \mathcal{H}_S \otimes \mathcal{H}_A. \]  
(11)
For simplicity, we shall write \(|\phi>\otimes|\chi>\) for the state \(|\phi>\otimes|\chi>\).

In his axiomatic scheme for QM, von Neumann postulated that there are two kinds of changes in quantum mechanical states:
(i) ‘Discontinuous, noncausal and instantaneous changes acting during experiments or measurements’ (processes of the first kind); these he called ‘arbitrary changes by measurement’. They are irreversible in nature.
(ii) ‘Continuous and causal changes in the course of time’ (processes of the second kind); these he called ‘automatic changes’. They are reversible.

Changes of type (ii) are described by the traditional unitary evolution operators \(U(t', t) = \exp[-iH(t' - t)/\hbar]\). Those of type (i) are described, in the context of an observable \(F\) as above, by the replacement of the density operator \(\rho\) of the system by
\[ \rho' = \sum_j P_j \rho P_j \]  
(12)
where \(P_j = |s_j><s_j|\). We shall refer to this as the reduction process. Under this process, the density operator \(\rho_\psi = |\psi><\psi|\) for the state (10) goes to
\[ \rho'_\psi = \sum_j P_j |\psi><\psi| P_j = \sum_j |c_j|^2 |s_j><s_j| \]  
(13)
which is a mixture of the states \(P_j = |s_j><s_j|\) with weights \(w_j = |c_j|^2\).

von Neumann regarded the measuring process as consisting of two stages:
(i) the interaction between the measured system and the apparatus; this is governed by the unitary evolution process;
(ii) the act of observation; this involves the reduction process.

Let \(|a>\) be the initial ready state of the apparatus and \(U = U(t_f, t_i)\) the evolution operator for the interacting system \((S + A)\) for the duration \([t_i, t_f]\) of the measurement. If the initial state of the system is one of the \(|s_j>\), the
pointer state after the measurement interaction is the corresponding \( |a_j > \); we describe this situation by the relation
\[
U(|s_j > |a >) = |s_j > |a_j >. \tag{14}
\]
Writing the total Hamiltonian of the system-apparatus combine as \( H = H_S + H_A + H_{\text{int}} \), a standard way of satisfying the condition (14) is to employ the interaction of the von Neumann form
\[
H_{\text{int}} = \sum_j |s_j < s_j| \otimes A_j \tag{15}
\]
where \( A_j \)s are operators acting on \( \mathcal{H}_A \), and stipulate that, during the interval \([t_i, t_f]\) of the measurement interaction, the interaction term in the Hamiltonian dominates over the other two terms (so that, effectively, \( H \simeq H_{\text{int}} \)). This gives eq(14) with
\[
|a_j > = e^{-iA_j(t_f - t_i) / \hbar} |a>. \tag{16}
\]
It must be emphasised, however, that Eq.(14) is a more general and clean way of describing the effective interaction between the system and apparatus. The operator \( U \) acts essentially like the \( S \)-operator (traditionally called \( S \)-matrix) of quantum field theory [12].

Linearity of the evolution operator now implies that, when the initial state is (1), we must have
\[
U(\sum_j c_j |s_j > |a>) = \sum_j c_j |s_j > |a_j > \equiv |\psi_f >. \tag{17}
\]
Note that the right hand side of Eq.(17) is a superposition of the quantum states of the macroscopic system \( (S + A) \). The operation represented by this equation is often referred to as premeasurement. This marks the completion of stage (i) of the measurement process.

In the second stage, the reduction process becomes operative which transforms the density operator
\[
|\psi_f > < \psi_f | = \sum_{j,k} c_j^* c_k (|s_k < s_j|)(|a_k < a_j|) \tag{18}
\]
to
\[
\rho^f_{S_A} = \sum_j P_j |\psi_f > < \psi_f |P_j \]
\[
= \sum_j |c_j|^2 (|s_j < s_j|)(|a_j < a_j|). \tag{19}
\]
This equation represents the joint state of system + apparatus at the completion of the measurement. It represents an ensemble of (system + apparatus systems) in which a fraction

\[ p_j = |c_j|^2 \]  

appears in the jth product state in the summand. The final state of the apparatus is supposed to indicate the measured value of F. One concludes, therefore, that, given the system in the state (10) and performing a measurement of the observable F,

(i) the measured values of the observable F are the random numbers \( \lambda_j \) with probabilities \( p_j \) given by eq(20);

(ii) when the measurement outcome is \( \lambda_j \), the final state of the system is \( |s_j> \).

The experimental verification of the prediction (i) consists in repeating this experiment a large number of times [with the same initial state (10)] and verifying (20) by invoking the frequency interpretation of probability. The prediction (ii) can be verified by repeating the experiment with the final state of the system in the previous experiment as the input state. Both predictions are in complete accord with experiment.

The main problem with the treatment of a quantum measurement given above is the ad hoc nature of the reduction [from Eq.(18) to (19)]. The so-called measurement problem in quantum mechanics is essentially the problem of arriving at the above-mentioned random outcomes without introducing anything ad hoc in the theoretical treatment. This means that one should either give a convincing dynamical explanation of the reduction process or else circumvent it.

To the question: ‘where and how does the reduction process (19) take place?’ von Neumann’s answer was that this is due to the involvement of human consciousness at the stage of actual observation of the outcome by the conscious observer. This proposal was later developed by London and Bauer [19] and Wigner [20]. This position, however, is not acceptable because the outcome of any experiment can be recorded by an appropriate device (say, a printer) and seen at convenience by a conscious observer. There is clearly no scope for the involvement of any subjective element in the explanation of the reduction process.

Our description of the measurement problem will be very much incomplete without a mention of Schrödinger’s cat[21]. To emphasize the awk-
wardness of the macroscopic superpositions in Eq.(17), Schrödinger introduced, in an experiment with two possible outcomes, a cat and a hypothetical device which would, in the event of one of the outcomes, kill the cat and leave it alive in the other case. One can now take the live and the dead states of the cat as the two pointer positions. The superpositions of Eq.(17) in this case would take the form

\[ c_1|s_1 > |LIVE\ CAT > + c_2|s_2 > |DEAD\ CAT > \]  

(21)
eq
eq
eq

eq
eq
eq

eq
eq
eq

eq

etc. It has now become a tradition to refer to macroscopic superpositions as *Schrödinger cat states* (or simply *cat states*).

### 3.2 Comparison with the classical case [32,17]

It is instructive to compare the quantum mechanical situation described above with the classical case. Following Zurek [22], we adopt a Dirac-like notation \(|\xi>\) for pure states of a classical system S with phase space \(\Gamma\) [\(\xi = (q,p)\) is a point of \(\Gamma\)]. A general (pure or mixed) state of S is represented by a density function \(\rho(\xi)\) on the phase space; pure states correspond to the \(\delta\)-function densities \(\rho_\xi(\xi') = \delta(\xi,\xi')\).

In classical mechanics (CM), there is no analogue of the superposition of pure states in quantum mechanics. The analogue of Eq.(1) of QM in CM is

\[ \rho(\xi) = \sum_i p_i \rho_i(\xi). \]  

(22)

(A convex comination of densities is a density.) The expansion of a mixed state in terms of pure states, however, is of the form

\[ \rho(\xi) = \int \rho(\xi')\rho_\xi(\xi')d\xi'. \]  

(23)

We shall formally consider (23) as a special case of (22) (with appropriate understanding about the ‘summations’).

Clearly, classical pure states correspond, not to vectors in Hilbert space, but to the projection operators:

\[ |.\} \leftrightarrow \{|.\} <\}. \]  

(24)

Given two systems \(S_1\) and \(S_2\) with phase spaces \(\Gamma_1\) and \(\Gamma_2\), a pure state of the combined system [a point in the phase space \(\Gamma_1 \times \Gamma_2\) (Cartesian product)] may be represented as \(|\zeta> = |\xi^{(1)}>\xi^{(2)}>\) with \(\xi^{(i)} \in \Gamma_i\) (i=1,2). The
time development of a state $|\zeta(0)\rangle = |\xi^{(1)}(0)\rangle|\xi^{(2)}(0)\rangle$ gives the unique state $|\zeta(t)\rangle = |\xi^{(1)}(t)\rangle|\xi^{(2)}(t)\rangle$ where $\xi^{(i)}(t)$ is the phase space trajectory of the initial point $\xi^{(i)}(0)$ under the Hamiltonian evolution of the system $S_i$ (which may involve interaction between the systems $S_1$ and $S_2$). Both classical and quantum evolutions, therefore, preserve purity of states.

In a measurement situation with $S_1 = S$ and $S_2 = A$ (now both considered as classical systems), we represent the (pure) states of the combined system as $|\zeta\rangle = |s\rangle|a\rangle$. As in the quantum case, there is a one-to-one correspondence between the states $|s_i\rangle$ of the system and the pointer states $|a_i\rangle$ of the apparatus. The classical analogue of Eq.(14) is (denoting the ‘ready’ state of the pointer by $|a_0\rangle$)

$$|s_j\rangle|a_0\rangle \rightarrow |s_j\rangle|a_j\rangle$$

where the arrow represents the classical evolution of the combined system during the interval $[t_i, t_f]$ of the measurement interaction. There is, however, no classical analogue of Eq.(17); instead, we have

$$\left(\sum_j p_j |s_j\rangle\right)|a_0\rangle \rightarrow \sum_j p_j |s_j\rangle|a_j\rangle.$$  \hspace{1cm} (26)

The right hand side has an obvious interpretation. There is no problem in classical mechanics analogous to the measurement problem in QM described above.

In the limit in which the left hand side of Eq.(26) goes to that of Eq.(25), the right hand side of Eq.(26) also goes to that of Eq.(25). Zurek refers to this as the complete information limit.

### 3.3 The preferred basis problem

Apart from the problem of macroscopic superpositions, there is another problem in quantum measurement theory: the preferred basis problem. This arises because the expansion in the final pre-measurement system-apparatus state $|\psi_f\rangle$ in Eq.(17) is generally not unique. If this state admits another expansion:

$$|\psi_f\rangle = \sum_k c_k'|s'_k\rangle|a'_k\rangle,$$  \hspace{1cm} (27)

then the same state $|\psi_f\rangle$ could correspond to measurement of any of the two observables

$$F = \sum_j \lambda_j |s_j\rangle < s_j|,$$  \hspace{1cm} \hspace{1cm} \hspace{1cm} $$F' = \sum_k \lambda'_k |s'_k\rangle < s'_k|.$$  \hspace{1cm} (28)
which may, in general, be even non-commuting as the example below shows. If the experimenter planned for a measurement of $F$, she/he cannot be sure that the pointer indicates a value of $F$ (and not that of $F'$).

Example. We take $S$ and $A$ to be two-state systems; the two Hilbert spaces are, therefore, two dimensional. Let

$$F = \sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \quad F' = \sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

and let $|z, \pm>, |x, \pm>$ be their respective eigenstates. Then we have, for example,

$$|\psi_f> \equiv \frac{1}{\sqrt{2}}[|z, +>_S |z, ->_A -|z, ->_S |z, +>_A]$$

$$= \frac{1}{\sqrt{2}}[|x, +>_S |x, ->_A -|x, ->_S |x, +>_A]. \quad (29)$$

IV. Decoherence

In this section, we shall present a brief but reasonably self contained account of some important aspects of decoherence.

4.1 Basic concepts about decoherence

Given a quantum mechanical state of some system $S'$ (we shall reserve the symbols $S$ and $A$ to denote the measured system and apparatus in a measurement context) which is a superposition of the form

$$|\phi> = \sum_j a_j |\phi_j> \quad (30)$$

the corresponding density operator is

$$\rho_\phi = \sum_{j,k} a_j a_k^* |\phi_j><\phi_k|. \quad (31)$$

The off-diagonal terms in eq(31) contain phase correlations between different terms in the superposition (30)(phases of the complex amplitudes $a_j$ appear only in these terms). Quite often, one is interested in getting rid of these terms [as, for example, was the situation in the previous section with $S' =$
$S + \mathcal{A}$ and $|\phi> = |\psi_f>$ of eq(17). \textit{Decoherence} is the general term employed to represent any phenomenon/process involving the disappearance (or strong suppression) of such off-diagonal terms.

When the system in question is a macroscopic system (for example, a measurement apparatus), a common occurrence is the \textit{environment-induced decoherence} which is caused by the interaction of the system with the environment in which it is located. This environment consists of two parts: (i) the external environment consisting of air molecules, photons etc around the system, and (ii) the internal environment which is explained below.

In theoretical work, a macroscopic system is generally described in terms of the so-called \textit{collective observables} [12] like the position of a pointer, angular position of a pendulum, etc. (The word ‘collective’ refers to the fact that these observables represent some property of the system as a whole rather than that of a few microscopic constituents.) The system, in its microscopic structure, consists of a large number of particles. Its fundamental observables are the canonical coordinate-momentum pairs, spin operators and operators for internal properties (like charge, isospin etc) for the constituent particles. Replacing these by an equivalent (in the sense that they generate the same algebra of observables) set of observables which includes the collective observables (and their conjugate partners) as a subset, then the observables in this set other than the collective observables represent a quantum system called the internal environment. The full quantum mechanical system representing the totality of the constituents of the macroscopic system is thus formally divided into a \textit{collective system} (represented by the collective observables and their conjugate partners) and the (internal) \textit{environment}.

The (system + environment ) combine is to be treated as a closed system. Since, strictly speaking, only the universe as a whole is a closed system, the external environment for a system should be the ‘rest of the universe’. One would like to have a suitable criterion for a smaller subsystem of this to be the effective external environment. A simple criterion is that, if one is considering a phenomenon involving a time interval of order $T$ (for example, in a Stern- Gerlach experiment, $T$ is of the order of the time taken by a typical atom in the beam from the source to the detector), then the particles (including photons) in a sphere of radius of the order of $cT$ (where $c$ is the velocity of light) around the system in question should be included in the external environment. The effect of including interaction with the environment in the dynamics of the system depends (apart from the nature of the interaction) essentially on the fact that the number of degrees of freedom in the environ-
ment is large. Some ambiguity in the identification of the environment is, therefore, of no consequence.

Inclusion of interaction with the environment in the quantum theoretic treatment of a system results in, besides decoherence effects, energy transfer between the system and the environment which generally leads to dissipative effects in the dynamics of the system; at macroscopic level, they manifest themselves as friction, viscosity etc. In a quantum theoretic treatment of macroscopic systems, therefore, decoherence and dissipation effects appear side-by-side.

4.2 Standard mechanism of environment-induced decoherence

The Hilbert space for the (system $S'$ plus the environment $E$) is the tensor product $\mathcal{H} = H_{S'} \otimes H_E$. The total hamiltonian for the combine may be written as (in obvious notation)

$$ H = H_{S'} \otimes I_E + I_{S'} \otimes H_E + H_{int}. $$

(32)

Dynamics of the combined system is given by the von Neumann equation (4). Effects of interaction with the environment on the dynamics of the system are described by the master equation for the reduced density operator $\rho_{S'} = Tr_E (\rho)$.

We shall assume $H_{int}$ to be of the von Neumann form [24] [see Eq.(15)]

$$ H_{int} = \sum_n |n><n| \otimes B_n. $$

(33)

As in the treatment of measurements in the previous section, we shall assume that $H_{int}$ dominates over the other two terms in Eq.(32). (As we shall see below, decoherence is a very fast process; effects of the first two terms during the extremely short time intervals involved are relatively insignificant.) Assuming the initial state of the combined system as $(\sum_n b_n |n> |\chi_0>)$ and proceeding as in the previous section, we have the following analogues of Eqs.(14), (17) and (18):

$$ |n> |\chi_0> \rightarrow e^{-iH_{int}t/\hbar} |n> |\chi_0> \equiv |n> |\chi_n(t)> $$

(34)

$$ |\phi(0)> \equiv (\sum_n b_n |n>) |\chi_0> \rightarrow \sum_n b_n |n> |\chi_n(t)> \equiv |\phi(t)> $$

(35)

$$ \rho(t) \equiv |\phi(t)> <\phi(t)| = \sum_{m,n} b_n b_m^* (|n><m|) \otimes (|\chi_n(t)> <\chi_m(t)|). $$

(36)
The reduced density operator for $S'$ corresponding to the density operator of Eq.(36) is
\[ \rho_{S'}(t) = Tr_E \rho(t) = \sum_{m,n} b_n b^*_m |n><m| < \chi_m(t) |\chi_n(t)>. \] (37)

Studies with fairly realistic models of the environment $E$ show that, for $t$ large compared to the decoherence time scale $\tau_d$ [which is usually very small; see, for example, Eq.(85)],
\[ <\chi_m(t)|\chi_n(t)> \rightarrow \delta_{mn}. \] (38)

This implies
\[ \rho_{S'}(t) \rightarrow \sum_n |b_n|^2 |n><n| \] (39)
which has no interference terms between the different $|n>$ states.

As a concrete example [3,11], let $S'$ be a two-state system with a basis $|u>, |d>$ (u for ‘up’, d for ‘down’) for $\mathcal{H}_{S'}$ and $E$ a system consisting of $N$ two-state systems ($N$ large) with basis $|u_k>, |d_k>$ ($k = 1,...,N$) for $\mathcal{H}_E$. We take
\[ H \simeq H_{int} = (|u><u| - |d><d|) \otimes \left[ \sum_{k=1}^N g_k (|u_k><u_k| - |d_k><d_k|) \otimes_{k' \neq k} I_{k'} \right]. \] (40)

The initial state
\[ |\phi(0)> = (a|u> + b|d>) \otimes_{k=1}^N (\alpha_k |u_k> + \beta_k |d_k>) \] (41)
evolves to
\[ |\phi(t)> = a|u> |e_u(t)> + b|d> |e_d(t)> \] (42)
where
\[ |e_u(t)> = |e_d(-t)> = \otimes_{k=1}^N (\alpha_k e^{i\gamma_k t} |u_k> + \beta_k e^{-i\gamma_k t} |d_k>). \] (43)
The relevant reduced density operator is
\[ \rho_{S'}(t) = Tr_E(|\phi(t)> <\phi(t)|) = |a|^2 |u><u| + |b|^2 |d><d| + z(t)ab^* |u><d| + z^*(t)a^*b|d><u| \] (44)
where

\[ z(t) = \langle e_u(t) | e_d(t) \rangle = \prod_{k=1}^{N} (|\alpha_k|^2 e^{i\omega_k t} + |\beta_k|^2 e^{-i\omega_k t}). \]  

(45)

This gives

\[ |z(t)|^2 = \prod_{k=1}^{N} \{1 + [(|\alpha_k|^2 - |\beta_k|^2)^2 - 1] \sin^2 2g_k t \}. \]  

(46)

We have \( z(0) = 1 \). For those initial environment states which make the square bracket in Eq.(46) vanish for each \( k \), \( z(t) = 1 \) for all \( t \) and the interference terms between \( |u> \) and \( |d> \) in Eq.(44) remain present for all times. For generic environments, however, these terms are generally nonzero. Note that

\[ |z(t)| = |\langle e_u(t) | e_d(t) \rangle| \leq 1. \]  

(47)

\[ <z(t)> \equiv \lim_{T \rightarrow \infty} T^{-1} \int_{t-T/2}^{t+T/2} z(t') dt' = 0. \]  

(48)

\[ <|z(t)|^2 | = 2^{-N} \prod_{k=1}^{N} [1 + (|\alpha_k|^2 - |\beta_k|^2)^2] \rightarrow 0 \text{ as } N \rightarrow \infty. \]  

(49)

This shows that, for large generic environments, the interference terms in Eq.(44) are strongly suppressed.

**4.3 Environment induced decoherence in measurements**

We now include interaction with the environment \( \mathcal{E} \) in the quantum mechanical treatment of measurements given in the previous section. Taking \( \mathcal{S}' = \mathcal{S} + \mathcal{A} \) in the treatment of section 4.2, we have

\[ \mathcal{H} = \mathcal{H}_S \otimes \mathcal{H}_A \otimes \mathcal{H}_E. \]  

(50)

Let the initial state of the environment be \( |e> \). We assume the initial state of the combined system \( (\mathcal{S} + \mathcal{A} + \mathcal{E}) \) to be

\[ |\phi(0) > = (\sum_j c_j |s_j>) |a> |e>. \]  

(51)

The measurement interaction between the system and the apparatus takes it to the state [see Eq.(17)]

\[ (\sum_j c_j |s_j> |a_j>) |e>. \]  

(52)
Interaction of the system $S' = S + A$ with the environment, acting as in section 4.2, takes the state (52 to [see Eq.(35)]

$$|\phi_f> = \sum_j c_j |s_j> |a_j> |e_j> .$$

(53)

This gives the density operator

$$\rho_{SAE}^f = |\phi_f><\phi_f| = \sum_{j,k} c_j^* c_k (|s_k><s_j|)(|a_k><a_j|)(|e_k><e_j|)$$

(54)

which gives the following reduced density operator for $S + A$ :

$$\rho_{SA}^f = Tr_E(\rho_{SAE}^f) = \sum_{j,k} c_j^* c_k (|s_k><s_j|)(|a_k><a_j|) <e_j|e_k> .$$

(55)

Assuming [see Eq.(38)]

$$<e_j|e_k> = \delta_{jk}$$

(56)

we have

$$\rho_{SA}^f = \sum_j |c_j|^2 (|s_j><s_j|)(|a_j><a_j|)$$

(57)

which is nothing but the density operator of Eq.(19) obtained from Eq.(18) by invoking the reduction process. Environment-induced decoherence, therefore, provides the dynamical mechanism for the reduction process.

4.4 Pointer basis of the quantum apparatus

Coupling to the environment also solves the preferred basis problem described in section 3.3. This is because, in contrast to the expansions of the form of Eq.(17) (which are generally non-unique), those of the form of Eq.(53) are unique. This is guaranteed by the tridecompositional uniqueness theorem [25-27, 11] stated below.

Tridecompositional uniqueness theorem : If a vector $|\psi> >$ in the Hilbert space $\mathcal{H}_1 \otimes \mathcal{H}_2 \otimes \mathcal{H}_3$ can be decomposed into the diagonal (“Schmidt”) form

$$|\psi> = \sum_i \alpha_i |\phi_i>_1 |\phi_i>_2 |\phi_i>_3$$

(58)

the expansion is unique provided that $\{|\phi_i>_1\}$ and $<\phi_i|_2$ are sets of linearly independent normalised vectors in $\mathcal{H}_1$ and $\mathcal{H}_2$ and that $\{|\phi_i>_3\}$ is a set of mutually noncollinear vectors in $\mathcal{H}_3$.  

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This can be generalized to an N-decomposition uniqueness theorem where \( N \geq 3 \).

This theorem, however, does not answer the question ‘Which basis is preferred?’ Zurek[2] provides the answer: The preferred pointer basis should be the basis (in \( \mathcal{H}_A \)) which contains a reliable record of the states of \( S \), i.e. the basis \( \{|s_j >|a_j >\rangle \} \) for which the correlated states \( |s_j >|a_j >\rangle \) are left undisturbed by the subsequent formation of correlations with the environment.

To illustrate this, we consider pointer basis in a simple bit-by-bit measurement [3]. Both, the system \( S \) and the apparatus \( A \) are assumed to be two-state systems and a measurement interaction between them is assumed to give the correlated state which admits the two different expansions of Eq.(29a,b). Let us identify the apparatus \( A \) of this equation with the two-state system \( S' \) of the example treated in section 4.2 taking

\[
|u > = |z, + >_A \quad \text{and} \quad |d > = |z, - >_A .
\]

Now, the interaction (40) between this apparatus and the environment \( E \) (consisting of \( N \) two-state systems) suppresses the interference terms between \( |u > \) and \( |d > \); these states (the ‘pointer states’), therefore, constitute a preferred basis (the ‘pointer basis’). Out of the two expansions in Eq.(29a,b), the one in (29a) represents the stable correlation between the system and apparatus and the measured observable is \( F = \sigma_z \).

This decoherence-induced selection of the preferred set of pointer states that remain stable in the presence of the environment is called environment-induced superselection or einselection.

In the example above, there is an effective superselection operative in \( \mathcal{H}_A \) (disallowing /suppressing superpositions of the states \( |u > \) and \( |d > \)). Any operator of the form

\[
G = \zeta_1 |u > < u | + \zeta_2 |d > < d |
\]

(pointer observable) with \( \zeta_i \) real and distinct acts as a superselection operator. We present a systematic treatment of such environment-induced superselection rules in the next subsection.

4.5 Environment-induced superselection rules

We come back to the \( S' + \mathcal{E} \) system of section 4.2 and treat it more systematically [3]. The Hilbert space is \( \mathcal{H} = \mathcal{H}_{S'} \otimes \mathcal{H}_E \) and we choose orthonormal
bases \{\ket{n}\} in \mathcal{H}_S' and \{\ket{e_j}\} in \mathcal{H}_E. The total Hamiltonian is taken as

\[ H = \sum_n \delta_n \ket{n}\bra{n} + \sum_j \epsilon_j \ket{e_j}\bra{e_j} + H_{S'E} \]  

(61)

where

\[ H_{S'E} = \sum_{n,j} \gamma_{nj} \ket{n}\bra{n} \otimes \ket{e_j}\bra{e_j} \]

which has a diagonal form with eigenvalues \(\gamma_{nj}\) associated with the eigenvectors \(\ket{n} \otimes \ket{e_j}\). (For a justification for the omission of off-diagonal terms of the form \(\ket{m}\bra{n} \otimes \ket{e_j}\bra{e_k}\), see [3].)

The initial state of the combined system is assumed to be a product state:

\[ \ket{\Phi(0)} = \ket{\phi_{S'}} \otimes \ket{\psi_{E}} = (\sum_n \alpha_n \ket{n}) (\sum_j \beta_j \ket{e_j}) \]  

(62)

It evolves, at time \(t\), to

\[ \ket{\Phi(t)} = \sum_{n,j} \alpha_n \beta_j \exp\left[-i(\delta_n + \epsilon_j + \gamma_{nj})t/\hbar\right] \ket{n} \otimes \ket{e_j}. \]  

(63)

The corresponding reduced density operator for \(S'\) is

\[ \rho_{S'}(t) = Tr_E(\ket{\Phi(t)}\bra{\Phi(t)}) = \sum_{m,n} \rho_{mn}(t) \ket{m} \bra{n} \]  

(64)

where (putting \(\hbar = 1\))

\[ \rho_{mn}(t) = \alpha_m \alpha_n^* e^{-i(\delta_m - \delta_n)t} \sum_k |\beta_k|^2 e^{-i(\gamma_{mk} - \gamma_{nk})t}. \]  

(65)

The diagonal terms are time-independent:

\[ \rho_{nn}(t) = |\alpha_n|^2 \sum_k |\beta_k|^2 = |\alpha_n|^2. \]  

(66)

The off-diagonal terms have time dependence in the two exponentials. The first one is a trivial ‘rotation’ (in the relevant complex plane); the more important is the second one contained in the ‘correlation amplitude’

\[ z_{mn}(t) = \sum_k |\beta_k|^2 e^{-i(\gamma_{mk} - \gamma_{nk})t} \]

\[ \equiv \sum_k p_k e^{-i\omega_{mn}^kt} \]  

(67)
where $p_k = |\beta_k|^2$ and $\omega_{mn}^{mk} = \gamma_{mk} - \gamma_{nk}$. Now, defining

$$< f(t) > = \lim_{T \to \infty} \frac{1}{T} \int_t^{t+T} f(t') dt'$$

(68)

we have

$$< z_{mn}(t) > = 0$$

(69)

and

$$< |z_{mn}(t)|^2 > = \sum_{k,k'} p_k p_{k'} \delta(\omega_{mn}^{mk}, \omega_{mn}^{k'})$$

(70)

Assuming, for simplicity, that all $\omega_{mn}^{mk}$ are distinct and that there are $N$ active states in the environment, we have the standard deviation $\Delta$ of the correlation amplitude given by

$$\Delta^2 = \sum_{k=1}^{N} p_k^2.$$  

(71)

Assuming that all the $p_k$ are approximately equal (with $p_k \sim N^{-1}$), we have

$$\delta \sim N^{-1/2}.$$  

(72)

Eqs. (69) and (72) show that large environments effectively damp out correlations between those states of the system which correspond to different eigenvalues of $H_{S'E}$.

There may, in general, be more than one vectors in $H_{S'}$ corresponding to the same eigenvalue $\gamma_{nj}$. These vectors span a subspace $H_r$ of $H_{S'}$. We have

$$H_{S'} = \bigoplus_{r} H_r.$$  

(73)

Only superpositions of states in a single $H_r$ are protected under the environmental monitoring. (Superpositions of states from more than one $H_r$ would have off-diagonal terms in the density operator, which are not allowed.) This implies the operation of a superselection rule.

A selfadjoint operator $A$ on $H_{S'}$ is an observable only if, acting on a physical state, it gives a physical state. This implies

$$|\phi > \in H_r \Rightarrow A|\phi > \in H_r \ for \ all \ r.$$  

(74)

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Observables which are (distinct) multiples of identity on the ‘coherent subspaces’ $\mathcal{H}_r$ are called superselection operators. They are of the form

$$\Lambda = \sum_r \zeta_r P_r$$

(75)

where $P_r$ is the projection operator on $\mathcal{H}_r$ and $\zeta_r$ are (distinct) real numbers. These are the observables which serve to distinguish different coherent subspaces. If the system $S'$ is to serve as an apparatus, distinctness of different pointer positions will be protected under environmental monitoring if different pointer positions correspond to vectors in different coherent subspaces. Such states will be eigenstates of $\Lambda$ corresponding to distinct eigenvalues. For this reason, observables of the form (75) are referred to as pointer observables in the decoherence-related literature. A basis in $\mathcal{H}_{S'}$ which consists of a pointer observable (and, therefore, all pointer observables) is called a pointer basis.

In a pointer basis, $H^{S'E}$ as well as all the $P_r$ are diagonal. It follows that

$$[\Lambda, H^{S'E}] = 0.$$  

(76)

[Being an operator equation, Eq.(76) is, of course, independent of the choice of basis.]

In practice, pointer observables relating to macroscopic apparatus are often position observables (whose eigenvalues correspond to pointer positions). This is related to the fact that typical interaction potentials are functions of position variables (and other operators like spin operators which commute with the position variables). For more details on this, see [3,6,7].

A question naturally arises whether the familiar superselection rules associated with electric charge and univalence [essentially $(-1)^{2J}$ where $J = \text{angular momentum}$] and possibly others could have their origin in the action of environment-induced decoherence at a deeper level. For a detailed treatment of this topic, we refer to Giulini [28].

4.6 Decoherence in a soluble model; decoherence time scale [29,5]

To illustrate some features of decoherence, we next consider a soluble model in one space dimension in which the system is a harmonic oscillator with position coordinate $q(t)$ and the environment is modelled as a scalar field $\phi(x,t)$. The action is

$$I = \int \int dt dx \left\{ \frac{1}{2} \left( \dot{\phi}^2 - \left( \frac{\partial \phi}{\partial x} \right)^2 \right) + \frac{1}{2} \delta(x)(m\ddot{q}^2 - \Omega_0^2 q^2 - \epsilon q \dot{\phi}) \right\}. \quad (77)$$
The time derivative coupling between the field and the oscillator is taken to ensure simple damping behaviour for the oscillator in the coupled system.

At time $t = 0$, the oscillator and the field are assumed to be uncorrelated:

$$\rho_{OF}(0) = \rho_O(0)\rho_F(0).$$

(78)

We shall be concerned with the reduced density operator $\rho = Tr_F \rho_{OF}$ of the oscillator. In the calculation of (a Fourier transform of) the density matrix elements $\rho(q, q', t)$, a logarithmic divergence appears which is tackled by introducing a high frequency cutoff $\Gamma$. The density operator for the field is taken to be the one corresponding to thermodynamic equilibrium at temperature $T$. In the high temperature limit

$$T >> \Gamma >> \text{max}(\gamma, \Omega)$$

(79)

where

$$\gamma = \frac{\epsilon^2}{4m}, \quad \Omega = \sqrt{\Omega_0^2 - \gamma^2}$$

(80)

the master equation for $\rho$ is

$$\frac{\partial}{\partial t}\rho(q, q', t) = \left\{ \frac{i}{\hbar} \left( \frac{\partial^2}{\partial q^2} - \frac{\partial^2}{\partial q'^2} - \Omega^2(q^2 - q'^2) \right) - \gamma(q - q')(\frac{\partial}{\partial q} - \frac{\partial}{\partial q'}) - \frac{2m\gamma k_BT}{\hbar^2}(q - q')^2 \right\} \rho(q, q', t).$$

(81)

In this equation, the term $\{\}$ is the von Neumann term $-i\hbar^{-1}[H_0, \rho]$ (with the bare frequency $\Omega_0$ of the oscillator replaced by $\Omega$). The second term on the right causes dissipation. The last ‘quantum diffusion’ term is the one that will be seen to be important in the context of decoherence.

Let us consider a Schrödinger cat state for the oscillator given by the wave function

$$\psi(q) = \frac{1}{\sqrt{2}}[\chi_+(q) + \chi_-(q)]$$

(82)

where

$$\chi_{\pm}(q) = \langle q | \chi_{\pm} = A e^{i \frac{q^2}{2}} \text{exp} \left[ -\frac{(q \pm \Delta q/2)^2}{4\delta^2} \right].$$
For wide separation between the peaks of the two Gaussian wave packets in the superposition \((82) (\Delta x >> \delta)\), the density matrix \(\rho(q, q') = \psi(q)\psi(q')^*\) has four peaks: two on the diagonal \((q = q')\) and two off-diagonal. The presence of the latter two peaks signifies quantum coherence; decoherence will be seen as vanishing/decay of these peaks. This decay is caused by the last term in Eq.(81). Noting that, for the off-diagonal peaks, \((q - q')^2 \simeq (\Delta q)^2\), we have, for \(\rho_{+-} = |\chi_+><\chi_-|\),

\[
\frac{d}{dt} \rho_{+-} \sim -\tau_D^{-1} \rho_{+-}
\]

(83)

where

\[
\tau_D = \tau_R \left[ \frac{\lambda_{dB}}{\Delta q} \right]^2.
\]

(84)

Here \(\tau_R = \gamma^{-1}\) is the relaxation time and \(\lambda_{dB} = \hbar (2mk_BT)^{-1/2}\) is the thermal de Broglie wave length.

For \(T = 300\) K, \(m = 1gm\), \(\Delta q = 1\) cm, we have

\[
\frac{\tau_D}{\tau_R} \sim 10^{-40}.
\]

(85)

Thus, even if the relaxation time \(\tau_R\) were of the order of the age of the universe \((\sim 10^{17}\) sec), we have \(\tau_D \sim 10^{-23}\) sec. For macroscopic systems, therefore, decoherence is an EXTREMELY FAST process. For microscopic systems (small \(m\) and \(\Delta q\)), \(\tau_D\) is relatively large. It is also large at low temperatures.

**4.7 Emergence of classicality [5]**

Classical behaviour of macroscopic systems that we normally come across must be explained in quantum mechanical terms because, as emphasized earlier, all systems in nature are quantum mechanical. (We live in a world with \(\hbar \neq 0\).) For this, it is not adequate to show that, in a certain limit, appropriate quantum mechanical equations go over to the familiar classical equations. Discussion of classical behaviour of macroscopic systems involves states as well as equations of motion. A typical macroscopic object (a coin, for example) as a quantum mechanical system, occupies a small subset of the possible states in the quantum mechanical Hilbert space of the constituent particles. [In particular, Schrödinger cat states (macroscopic superpositions)
are absent.] We have seen how, in the context of a simple model, decoherence leads to effective elimination of Schrödinger cat states. To show how the quantum mechanical equations of motion of such objects reduce to the classical ones, we shall consider the Wigner function of the oscillator of the previous section.

Wigner transform of a wave function $\psi(q)$ is defined as

$$W_{\psi}(q,p) = \frac{1}{2\pi\hbar} \int_{-\infty}^{\infty} ds e^{ips/\hbar} \psi(q - \frac{s}{2})\psi^*(q + \frac{s}{2}).$$ (86)

This object is real but not necessarily non-negative and therefore not a phase space probability density. We have, however,

$$\int_{-\infty}^{\infty} W_{\psi}(q,p) dp = |\psi(q)|^2, \quad \int_{-\infty}^{\infty} W_{\psi}(q,p) dq = |\tilde{\psi}(p)|^2$$ (87)

where $\tilde{\psi}$ is the Fourier transform of $\psi$.

For the minimum uncertainty wave packet

$$\psi(q) = \pi^{-\frac{1}{4}} \delta^{-\frac{1}{2}} \exp \left[ -\frac{(q-q_0)^2}{2\delta^2} + ip_0q \right]$$ (88)

$W_{\psi}$ is Gaussian in both $q$ and $p$ (and non-negative):

$$W_{\psi}(q,p) = \frac{1}{\pi\hbar} \exp \left[ -\frac{(q-q_0)^2}{2\delta^2} - \frac{(p-p_0)^2}{\hbar^2} \right].$$ (89)

It describes a system localised in both $q$ and $p$. This is the closest approximation to a point in phase space that a wave function can yield.

Note that, on the right hand side in Eq.(86), the dependence on $\psi$ is in terms of the corresponding density matrix. Generalizing this to a general density matrix $\rho(q,q')$, we have

$$W_{\rho}(q,p) = \frac{1}{2\pi\hbar} \int_{-\infty}^{\infty} e^{ips/\hbar} \rho(q - \frac{s}{2},q + \frac{s}{2}) ds.$$ (90)

Eq.(86) is a special case of (90) with $\rho(q,q') = \psi(q)\psi^*(q')$. For

$$\rho = \sum_{i=1}^{n} p_i |\psi_i><\psi_i|$$
where $p_i \geq 0$ and $|\psi_i>\,$ are minimum uncertainty wave packets, we have $W_\rho \geq 0$ and represents a probability density in phase space.

Combining Eqs(81) and (89) to obtain a ‘master equation’ for $W$ (dropping the subscript $\rho$) and replacing the harmonic oscillator potential by a general potential $V(q)$, we have

$$\frac{\partial W}{\partial t} = -\frac{p}{m} \frac{\partial W}{\partial q} + \frac{\partial V_r(q)}{\partial q} \frac{\partial W}{\partial p} + 2\gamma \frac{\partial}{\partial p} (pW) + D \frac{\partial^2 W}{\partial p^2}$$

(91)

where $V_r (= V + \text{extra terms})$ is the renormalized potential (recall the replacement of $\Omega_0$ by $\Omega$ in the previous subsection) and $D = 2m\gamma k_B T$.

The first two terms on the right in Eq.(90) can be written as the Poisson bracket $\{H, W\}$. For large $m$, $\gamma$ is small. When the last term has done its main job (of suppressing the off-diagonal terms in the density matrix), its action on the remaining (density operator)/(Wigner function) is negligible and we have, finally, the desired classical equation

$$\frac{\partial W}{\partial t} = \{H, W\}.$$ 

(92)

V. Does Decoherence Completely Solve the Measurement Problems?

The answer to the question posed above is closely tied up with the interpretation of quantum mechanics.

By interpretation of the formalism of a scientific theory, one essentially means the explanation/translation of the new concepts and terms used in the theory in the commonly accepted logical framework (if necessary, by appropriately extending the existing logical framework). The need for interpretation in quantum mechanics arises mainly because it is an inherently probabilistic theory whose probabilistic aspects do not admit the traditional ignorance interpretation. Its kinematical framework employing novel objects like (state vectors)/(wave functions) and operator observables gives rise to questions like the interpretation of $\psi$ (whether it represents a single system or an ensemble), the question of assignment of definite values to observables, the relationship of the formalism with objective reality, etc.

We shall be mainly concerned with the question: ‘Is it possible to consistently interpret the formalism of quantum mechanics so that, taking into
consideration decoherence-related developments, the measurement problems are completely solved?'

Let us have a quick look at the relevant decoherence-related developments. Interaction of a (typically macroscopic) system with the environment selects a preferred basis (in the quantum mechanical Hilbert space of the system) consisting of a set of robust (in the sense that they persist in the presence of continuous environmental monitoring) quasiclassical states characterised by eigenvalues of observables (approximately) commuting with the system-environment Hamiltonian. Since interactions are generally described in terms of position/configuration variables, the emerging quasiclassical properties generally involve localization of objects. The equations of motion for the class of quantum systems referred to above go over to the familiar deterministic classical equations. This serves to explain the appearance of determinate, objective (in the sense of the above-mentioned robustness) properties to a local observer.

One is now tempted to conclude that this (plus the specific results obtained in sections 4.3 and 4.4) must be adequate to explain, for all practical purposes, the observation of a unique stable pointer state at the end of a measurement. A critical look at the whole development (see Bub [25] and Adler [33] for a careful discussion and detailed references) shows, however, that the answer to the question posed above must be in the negative.

In section 4.3, the reduced density operator of system + apparatus is obtained by taking trace (over the environment) of the density operator of system + apparatus + environment. Zurek [5] interprets this as ignoring the uncontrolled and unmeasured degrees of freedom. This is supposed to be taken as similar to the procedure of deriving probability 1/2 for ‘heads’ as well as for ‘tails’ in the experiment of tossing a fair coin by averaging over the uncontrolled and unmeasured degrees of freedom of the environment of the coin.

The two procedures are, in fact, substantially different [25]. In the coin toss experiment, when, ignoring the environment, we claim that the probability of getting ‘heads’ in a particular toss of the coin is 1/2, we can also claim that we do in fact get either ‘heads’ or ‘tails’ on each particular toss. A definite outcome can be predicted if we take into consideration appropriate environmental parameters.

In the case of a quantum measurement, however, we cannot claim that, taking the environment into consideration, a definite outcome of the experiment will be predicted. In fact, taking the environment into account will
give us back the troublesome equation (53) from which we derived the mixed state (55) by tracing over the environment.

**What is/are the way(s) out?**

**One route: (Relative state)/(Many worlds) interpretation of quantum mechanics (Everett, DeWitt and others [34])**

One insists that the superposition (17) *is* the final outcome of the measurement. This equation is to be interpreted as a splitting of the state vector of (system + apparatus) into various branches (these are often called *Everett branches*) only one of which we observe.

This approach is very uneconomical and intuitively unappealing. Moreover, the preferred basis problem is not solved in this approach.

**Another route: Bohmian mechanics [35,36]**

In this approach, one has, apart from the wave function $\psi(q,t)$, the functions $q_\alpha(t)$ describing configuration space trajectories of the system. The wave function $\psi(q,t)$ serves as a guidance field for the motion of the trajectories $q(t)$ [essentially analogous to the way the Hamilton-Jacobi function $S(q,t)$ serves, in classical mechanics, as a guidance field for the system trajectories in configuration space].

The functions $q_\alpha(t)$ serve as ‘hidden variables’. They serve to pick up unique outcomes in measurement situations. [At any given time, $q(t)$ has a definite value. At the end of a measurement, the system trajectory is expected to be in any one of the various configuration domains corresponding to the different outcomes in the superposition (17).] Born rule probabilities emerge for the observer who cannot access the additional information contained in $q(t)$.

Problems with this approach: it fails in the relativistic domain and quantum field theory.

These two approaches are *interpretations* of quantum mechanics in the specific sense that, by design, they reproduce all physical predictions of the traditional (nonrelativistic) quantum mechanics and so are empirically indistinguishable from the orthodox theory, while changing the formalism so as to resolve some difficulties of measurement theory.

If we insist on having one world existing within the standard arena of states and operators in Hilbert space, we must inject new physics in the formalism of quantum mechanics (and should be willing to discard, if necessary, one or more of the assumptions made in the traditional formalism.)
An alternative: Dynamical collapse models (Ghirardi, Rimini and Weber [37], Pearle [38,39], Gisin [40] and Diosi [41])

In this approach, one abandons the assumption of a unitary evolution; this is replaced by a stochastic unitary one:

$$d\psi(t) = (A dt + B dW_t)\psi(t)$$

where $W_t$ is a Wiener process and $A$ and $B$ are suitably chosen operators. Heuristically, the idea is that, quantum mechanics may be modified by a low level universal noise, akin to Brownian motion (possibly arising from physics at the Planck scale) which, in certain situations, causes reduction of the state vector.

This approach reproduces the observed fact of discrete outcomes governed by Born rule probabilities. It predicts the maintenance of coherence where it is observed (superconductive tunnelling, ...) while predicting state vector reduction in measurement situations [42,43]. For more details, the reader is referred to the literature cited above.

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