AN EXPERIMENTAL AND NUMERICAL STUDY OF FACTORS INFLUENCING THE SCOUR DUE TO THE COLLISION OF WATER JET IN STILLING BASIN FLOOR

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ABSTRACT

This study based on the laboratory experiments and numerical analysis to study the configuration of the local scour downstream of hydraulic structure result by free water jet. This jet will have a high kinetic energy making it able to transport the sediment particles downstream of the impinging area, thereby forming a scour hole. The purpose of conducting this study was to evaluate the minimum distance of scour hole from the nozzle jet. One hundred and thirty-eight runs were carried out to investigate the local scour downstream the hydraulic structure under clear-water conditions with different discharges, tail water depths, and jet angles. Two types of jets were used, normal jet and dispersion jet. Results indicated that minimum distance of scour increase with increasing densimetric Froude number and jet velocity. The increase in tail-water depth decrease the distance to the scour hole until access to a critical depth at which the scour hole is largest possible after that the distance back to become large when the depth is overstepping the critical value. The optimum jet angle depends on the jet velocity, with a value of 20° for jet discharges 60 and 80 L/min and 40° for jet discharges 100 and 120 L/min. The dispersion jet technique where used to investigate its effects on the scour processes where its observed that the scour hole resulted small than the scour hole from case of normal jet. Also, the numerical simulation was used by (FLOW3D) software program to investigated from the result of the experimental work, the results showed an acceptable agreement with the experimental results with percentage of error equal to (% 6.28).

KEYWORDS: Local scour; Plunge pool; Free water jet; Numerical analysis; FLOW3D.
1. INTRODUCTION
When the excess water is discharged from a reservoir into the downstream water body, it sometimes flows in the form of a jet. The jet is an intensified flow, where water with high velocity discharge into fluid downstream where this fluid either be static or in moving. This high energy water jet can lift and transport the sediment particles downstream of the impinging area, thereby forming a scour hole. As well, the stability of the foundation of a hydraulic structure may undermine because of generating the local scour downstream of these structures due to of the flow in the form of turbulent water jets. Examples of these structures, local scour downstream of a flip bucket, vertical gates, spillways, weirs, drops, culvert, and downstream outlet. In all cases, the complete protection against scouring is often impractical and expensive. Therefore, it is important for hydraulic engineers understand the scour mechanism to find appropriate solutions and ways to guide and control the scouring process so as to reduce the risk of failure.

Some studies have been published on the scouring of cohesionless bed sediment downstream of hydraulic structure. Faruque et al., 2006, studied the influence of submergence on the scour processes due to three dimensional jet release by a square cross-section nozzle on a non-cohesive sand bed, three different tail-water depths, conforming to $2b_o$, $4b_o$ and $6b_o$ were selection for the study where $b_o$ is a nozzle width. Results indicated that the tail-water depth, densimetric Froude number $F_o$, and grain size-to nozzle size ratio, all has an effect on the scour hole extent caused by three-dimensional jet. The results also indicate that at very low tail-water depths, for $F_o<5$, the jet expansion ratio can have a significant effect. The maximum scour depth is not necessarily deeper at the lower range of submergences, where the depth of scour was large at higher tail-water depths. Also, it must be observed that the volume of scour hole is large at higher tail-water depths. Pagliara et al., 2006, studied the effect of many parameters on plunge pool scour of a totally incoherent rock surface. The angles of jet collision changed between 30 and 90°, the result showed that the scour hole depths increase with the jet angle whereas the ridge height decreases with increase the jet angle. Hamidifar, 2011, studied the local scour in noncohesive sediment downstream of a horizontal rigid apron. Two kinds of bed material were used in the experiments with median size diameters of 0.73 mm and 1.85 mm. For the scour hole, the maximum of scour depth, the location of maximum scour depth, the maximum length of scour, the maximum height of the ridge, and the location of maximum ridge height for all the experiments has been measured. Results showed that there are good correlations among scour hole factors. It was found that the maximum extension of the scour
hole, the maximum dune height, the maximum extension of the dune, will increase with the increasing of the maximum depth of scour. Ghodsian, et al., 2012, studied the local scour due to free fall jets. Four kinds of sediment mixture were used. The results showed that by increasing the geometric standard deviation of the particles, the scour hole parameters and the ridge height decrease. The decreases in the maximum scour depth and the height of downstream ridge by increasing the geometric standard deviation from 1.3 to 2.7 is about 32% and 45% respectively. Moreover, the scour hole length and width decrease by about 30% as the geometric standard deviation change from 1.3 to 2.7. Al Faruque and Hanna, 2014 studied the local scour by three-dimensional wall jet great by a nozzle with square cross section. Three various tail-water depths were used and the jets expansion ratio was larger than 10. It was noted that the coarsest particles are deposited towards the downstream end of the scour hole. In fact, the coarsest particles are existing on down slope of the ridge. Also they observed that the volume of scour hole increase with increases of time. However, for 72 hours of test, there is a large increase in scour volume when compared with time equal to 48 hours. Also the volume of scour hole be increasing with increasing tail-water depth where it’s larger for H/b₀ =6 compared with H/b₀ equal to 4 and 2 where H is a depth of water in the channel and b₀ is a nozzle width. The aim of the present study was to investigate the effects of the variables that concluded the rate of the discharge and the tail-water depth level and the angle of the jet on the configuration and location of scour hole.

2. EXPERIMENTAL WORK

The experiments were conducted using a locally made flume. The flume was made from galvanized iron with a long 4 m, width 0.8 m, and height 0.45 m as shown in Fig. 1. The flume contains a settling basin with 2.5 m long and 0.2 m height that filled with uniform sediment sand. At the end of the film, there is a gate with a length 0.6 m used to control the tail-water level above the sand bed by setting it to the required level and the water passed on it as a wear. The ground reservoir located at the end of the flume, this reservoir used to store and recirculating the water to the flume. A pump type streen (2 HP) used to draw the water from the ground reservoir to jet nozzle where the water jet is generated. The discharge is controlled by a flowmeter installed vertically after the pump to obtain the required flow at the jet pipe. The jet installation system consists of a circular iron pipe with 3.8 cm inner diameter and 30 cm long installed at the beginning of the experimental flume using a perforated metal plate at an altitude of 27 cm from the sand bed and it's fixed on a plate of iron by screws to control the angle of the jet. The configuration of the sour hole and the maximum depth of scour is taken by using a tool named “measurements mesh”. This tool made locally by a mold of wood with a
metal mesh. The mesh openings are in is a square shape with rib length is 5 cm as shown in Fig. 2. In order to find the limited scour time and adopt it in all tests, thirty preliminary tests were made at lowest tail-water and angle of a jet by using tow flow condition which it’s the maximum discharge 120 and 100 L/min. Maximum depth of scour is recorded after each run by using a measurement mesh and measurement tube at different time intervals, the scour depth has sharply increased during first 20 minute of the test duration and the development of scouring has become increases gradually and with deceleration. It was observed that the scour depth continued to increase but very slowly, so the time of the test runs was selected for two hours.

Two types of jets were used in the present study, normal jet NJ, and dispersion jet DJ. The classification of the jet depends on the type of a nozzle. The first type represents the natural state where the water extrudes from the nozzle freely, while the second type was made locally by inserting a twisted plate inside the pipe of the jet as shown in Fig. 3. The stuffing of the twisted plate in an iron pipe works to force the flowing water to move spirally inside the pipe which helps to spread the water when leaving the nozzle because of the characteristic of centrifugation generated in the flowing water. One hundred and twenty runs were conducted for NJ under different hydraulic condition which contains four discharges (60,80,100,120) L/min, six tail-water depths (5,7.5,10,12.5,15,17.5) cm, and five angles for the jet (10°,20°,30°,40°,50°). Also, additional eighteen runs were conducted, nine tests for each NJ and DJ under same hydraulic conditions, one angle of 30°, three tail-water depths (5, 10, 15) cm, and three various discharges (80,100,120) L/min.
3. DIMENSIONAL ANALYSIS

For clear water condition the distance of scour hole from the nozzle ($x_s$) result by free water jet is a function of following parameter AL-khateeb et.al, 2016.

$$x_s = f\{y_t, v, R, d_{50}, \varnothing, g, \rho, \rho_s, \mu, \sigma g, B, S, T\}$$

Where, $y_t = $ Tail-water depth; $v = $ Jet velocity, $R = $ hydraulic radius, $d_{50} = $ median sediment size, $\varnothing = $ angle of jet, $g = $ gravitational acceleration, $\rho = $ density of the fluid, $\rho_s = $ density of the sediment, $\mu = $ dynamic viscosity of the fluid, $\sigma g = $ standard deviation of particle size distribution, $B = $ channel width, $S = $ channel bed slope, $T = $ duration of test. By using dimensional analysis eq. (1) can be rewritten as:

$$\frac{x_s}{y_t} = f\{\frac{R}{y_t}, \varnothing, Re, \frac{\rho_s}{\rho}, \frac{d_{50}}{y_t}, Fr, \frac{B}{y_t}, \sigma g, S, \frac{\nu T}{y_t}\}$$
In equation (2), the effect of Re is neglected as the viscosity has a negligible effect on the scour processes due to the turbulence. Also the parameters $\frac{B}{y_t}$, $S_z$, $\sigma_g \frac{v_T}{y_t}$ can be neglected because their values are constant in all the tests. After these simplifications the functional relationship which describes scour parameter normalized with tail-water depth may be written as:

$$\frac{x_s}{y_t} = f\left(\frac{R}{y_t}, F_o, \Theta\right)$$

where $F_o$ is densimetric Froude number and equal to $v/\sqrt{gd_50\Delta \rho/\rho}$.

4. COMPUTATIONAL FLUID DYNAMICS (FLOW3D)

In the present study, the software FLOW3D program was used. The objective from the numerical simulation in this study is to compare between the experimental and numerical method for NJ state and to find out the compatibility between the two procedures. Then numerical simulation for rectangular jet (plane jet) was examined with aspect ratio of same cross section area of circular nozzle under same conditions of the jet. The results of numerical simulation for circular and plane jet was compared.

4.1. Numerical model

Many different physics options are available, only four options were selected is required to get a delicate simulation of the data where it’s required in this study, an acceleration of gravity, a sediment scour model with a median size particle diameter of 0.81 mm, the viscosity, and the turbulence options. The geometry of the model represented by a rectangular flume contains a barrier in the upstream to hold the water and the barrier contains circular hole launches water to the downstream in a form of a jet where can be controlled in the angle of the jet.

4.2. Meshing

In FLOW3D, for accurate solution, grid is one of the most important matter. Cell size and grid can influence both on the simulation time and the results accuracy. If the sizes of the cell are not optimal, geometry problems will happen as shown in Fig. 4, (FLOW3D manual, 2014). To avoid this problem, FAVOR option for mesh selection helps us to obtain delicate geometric shapes.

The mesh of FLOW3D software have a cell of cubic shape and considering one of the affecting factors on the simulation process, therefore, different cell sizes are selected as (20, 16, 12, 10 and 8) mm to identify the best cell size that satisfies the phenomenon conditions. The best
number of cells was 1,000,000 which the corresponding size of cells was 12 mm were used in this study when the comparison was made among the output results.

**Fig. 4. FAVOR option with different cell size.**

### 4.3. Boundary and initial conditions

In numerical flow analysis, one of the most important stages is the determination of the appropriate boundary conditions. The boundary condition should correspond to the physical condition of the problems duly, on rectangular mesh prism, there are five various types of boundaries were used as shown in **Fig. 5**. These boundaries are:

**Fig. 5. Boundary and initial conditions.**

1. **Upstream boundary (X-min):** specified pressure condition (P) used fluid elevation where the head of fluid upstream of the flume is determined based on the Bernoulli equation to get the required discharge for the jet. \[ v = \sqrt{2g \cdot y} \] where \( v \) = velocity of the water (m/s), \( y \) = hydraulic head (m).

2. **Downstream boundary (X-max):** An outflow condition (O).
3- Top boundary (Z-max): Specified Pressure condition (P) used a fluid fraction and it represent the atmospheric pressure.

4- Bottom boundary (Z-min): Wall condition (W) its represent the ground under the packed sediment.

5- Side boundary (Y-min, Y-max): Wall condition (W) its represent the wall of the flume.

5. RESULTS AND DISCUSSION

This section presents results and analysis of experimental tests that were carried out to investigate the influence of nozzle discharge, tail-water depth, angle of the jet, and type of jet on the scour hole geometry and the position of scour hole downstream hydraulic structure. It should be mentioned that by focusing the effect of any parameter, the same experimental conditions are held fixed. Thus, the only effect of that parameter can be specified.

5.1. Effect of discharge

The different values of discharge are plotted against the distance of scour hole from the nozzle for all values of tail water depth and angle of jet. Fig. 6 shown the effect of varying the discharge on the distance of scour hole from the nozzle for angle of jet equal to 30º.

![Fig. 6. Effect of discharge on distance of scour hole from the nozzle for various tail-water depths at angle of jet (Ø) = 20 º.](image)

It's clear from figure. s that the distance of scour from the nozzle increase with increasing the discharge for all tail water depths. The increase in discharge leads to an increase in the water velocity, which leads to increasing its kinetic energy and subsequently the minimum distance of scour hole from the nozzle.
For more elucidation to the influence of discharge 3d configuration shape of scour hole was drawn by surfer 10 program as shown in Fig. 7 and 8 for $Q = 60$ and 120 respectively.

**Fig. 7.** The contour map and 3D configuration of scour hole for the normal jet.

**Fig. 8.** The contour map and 3D configuration of scour hole for the normal jet.

### 5.2. Effect of tail-water depth

The influence of tail water depth on the position scour hole was investigated by using six levels of tail water depths. Fig. 9 show the effect of varying the tail-water depth on the distance of scour hole from the nozzle for all discharge and for the angle of jet equal to $30^\circ$. 
Fig. 9 shows that the less value for the distance of scour hole occurs when the value of the tail water is 10 and 12.5 cm. The value of the depth of the tail water at which the greatest value of scour hole volume is called the critical tail water depth and the change of the tail-water depth above or under the critical value caused decreases in the volume of scour hole as mentioned by (Aderibigbe and Rajaratnam, 1996) and (Ghodsian et al., 2006).

In the low tail-water level the volume of scour is small and that because of the dynamic movement of particles that happen under these conditions which represent by digging and refilling processes and the refilling accurate because of there is not enough flow to transfer particles away from the scour hole and this sequent digging and refilling processes at low tail-water depth confirmed by (Ghodsian et al., 2006).

The configuration of scour hole was recorded for many runs by measurement mesh and painted by surfer 10 to show the effect of tail-water depth on the scoudr hole. Figs. 10 and 11 show the difference in the scour hole for tow tail-water depth (7.5, 17.5) cm.

5.3. Effect of the angle of jet

The effect of the jet direction on the position of scour hole was studied by using 5 angels (10°, 20°, 30°, 40°, 50°). Fig. 12 shows the minimum distance of scour hole for all discharges and for tail water depths equal to 5 cm.
Fig. 10. The contour map and configuration of scour hole for the normal jet.

Fig. 11. The contour map and configuration of scour hole for the normal jet.

Fig. 12. Effect of angle of the jet on distance of scour hole from the nozzle for various discharge at tail water depth of 5 cm.
For each angle, the distance of scour increases with increasing the jet discharge. Generally, these figures explain that the best angle of the two jet discharge 60 and 80 l/min is 20° and sometimes 30° while the best angle for discharges 100 and 120 l/min is 40° and sometimes 30°.

At jet direction of 50°, the scour hole is clearly approaching to the foundation of hydraulic structure (the beginning of stilling basin), especially when using the discharge of 60 l/min.

6. COMPARISON OF NORMAL AND DISPERSION JETS

Table 1 show results of the comparison for the distance of scour from the beginning for 18 runs, nine runs for each jet type.

| Discharge (L/min) | Tail-water depth(cm) | The distance from the nozzle (cm) | Difference (cm) | decrease% |
|-------------------|----------------------|----------------------------------|----------------|----------|
|                   |                      | N_J | D_J |                |          |
| 80                | 5                    | 22  | 21  | 1              | 4.76     |
| 100               | 5                    | 26  | 24.5| 1.5            | 6.12     |
| 120               | 5                    | 31.5| 30  | 1.5            | 5.00     |
| 80                | 10                   | 19  | 19.5| 0.5            | -2.56    |
| 100               | 10                   | 25.5| 24  | 1.5            | 6.25     |
| 120               | 10                   | 31.5| 31  | 0.5            | 1.61     |
| 80                | 15                   | 24  | 22.5| 1.5            | 6.67     |
| 100               | 15                   | 32  | 30.5| 1.5            | 4.92     |
| 120               | 25                   | 36  | 34  | 2              | 5.88     |

Table 1 demonstrate that the distance of scour hole from the nozzle for dispersion jet was less than the distance which was achieved by normal jet but at a low rate.

The adverse effect in the case of dispersion jet that the shape of scour hole cannot be predicted, in addition to that there is amount of losses in the discharge of range between (3-6) % because of the twisted plate. Fig. 13 show the configuration of the scour hole due to dispersion jet.
7. FLOW3D RESULTS AND DISCUSSION

This section aims to compare results of the laboratory experiments of plunge pool results by free water jet were held previously in the laboratory with numerical simulation results of CFD code. To test the effectiveness of the numerical model, this numerical model was applied in similar conditions of the physical model. A twenty laboratory experiment was selected for testing it in the FLOW3D software program.

So, it explains how much compatibility in the position of scour hole result from FLOW3D software program and the position of scour hole in the laboratory work. The results and comparing of the position of scour hole in the FLOW3D and experimental data are shown in Table 2. The results below shows a good agreement between the experimental work and FLOW3D software program for the minimum distance of scour from the nozzle where the average of the error equal to % 6.28.

Also, the results of the distance of scour from the nozzle for experimental work and FLOW3D are plotted for more comparison as shown in Fig. 14.

Consequently, the results numerical simulation is very comparable to the results of experimental work especially in relation to the distance of the scour form the nozzle, thus the FLOW3D can be depended. Tested has been performed for plane jet under the same conditions and with the same cross-sectional area for rectangular slot with dimensions (2*5.7) cm for comparison between it and circular jet.
Table 2. The comparing of distance of scour from the nozzle between FLOW3D and experimental work.

| Run no | Exp ds (cm) | FLOW3D ds (cm) | Difference(cm) | Abs error % |
|--------|-------------|----------------|----------------|-------------|
| 41     | 12          | 12.5           | -0.5           | 4.16        |
| 42     | 13          | 14.5           | -1.5           | 11.5        |
| 43     | 12.5        | 14             | -1.5           | 12          |
| 44     | 12.5        | 13             | -0.5           | 4           |
| 45     | 10.5        | 11.5           | -1             | 9.52        |
| 46     | 19          | 19             | 0              | 0           |
| 47     | 19          | 20.5           | -1.5           | 7.89        |
| 48     | 20.5        | 21.5           | -1             | 4.87        |
| 49     | 19          | 20.5           | -1.5           | 7.89        |
| 50     | 16          | 17.5           | -1.5           | 9.37        |
| 51     | 25          | 23             | 2              | 8           |
| 52     | 27          | 26             | 1              | 3.7         |
| 53     | 28          | 25.5           | 2.5            | 8.9         |
| 54     | 27          | 30             | -3             | 11.11       |
| 55     | 24          | 22             | 2              | 8.33        |
| 56     | 30.5        | 28             | 2.5            | 8.19        |
| 57     | 36.5        | 34             | 2.5            | 6.84        |
| 58     | 37          | 36             | 1              | 2.7         |
| 59     | 36          | 35             | 1              | 2.77        |
| 60     | 32          | 30             | 2              | 6.25        |

Table 3: The comparing of distance of scour from the nozzle for FLOW3D between circular jet and plane jet.

| Run no | Cir jet xs (cm) | plane jet xs (cm) | Difference(cm) |
|--------|-----------------|-------------------|----------------|
| 41     | 12.5            | 15.5              | 3              |
| 42     | 14.5            | 15                | 0.5            |
| 43     | 14              | 14.5              | 0.5            |
| 44     | 13              | 15.5              | 2.5            |
| 45     | 11.5            | 13                | 1.5            |
| 46     | 19              | 20.5              | 1.5            |
| 47     | 20.5            | 20                | -0.5           |
| 48     | 21.5            | 20                | -1.5           |
| 49     | 20.5            | 21                | 0.5            |
| 50     | 17.5            | 18.5              | 1              |
| 51     | 23              | 22.5              | -0.5           |
| 52     | 26              | 25.5              | -0.5           |
| 53     | 25.5            | 28                | 2.5            |
| 54     | 30              | 31                | 1              |
| 55     | 22              | 23.5              | 1.5            |
| 56     | 28              | 27.5              | -0.5           |
| 57     | 34              | 33                | -1             |
| 58     | 36              | 36                | 0              |
| 59     | 35              | 38                | 3              |
| 60     | 30              | 32                | 2              |
Fig. 14. Comparison of the distance of the scour form the nozzle between experimental work and FLOW3D.

From the Table 3 it was observed that the results of distance of scour from the nozzle opening was approach to circular jet.

Fig. 15 shows the three dimensional fluid fraction output of numerical model for the scour hole configuration for test with (circular jet).

![Three dimensional fluid fraction output](image1)

Fig. 15. Three dimensional output for scour hole configuration result by circular jet (fraction of fluid).

Also, Fig. 16 show the fluid fraction three dimensional output of numerical model for velocity magnitude distribution for test with (plane jet).

![Three dimensional output](image2)
8. REGRESSION ANALYSIS

The computer package (SPSS v21.0) was used to develop the equations through a non-linear regression analysis.

For the distance of scour from the nozzle: Eq. (3) can be written as below:-

$$\frac{x_s}{y_t} = c_1 (F˳)^{c_2} \times \left(\frac{R}{y_t}\right)^{c_3} - c_4 \cos \theta$$

$$c_3 = 1.081 \quad c_4 = 0.104 \quad c_1 = 0.312 \quad c_2 = 1.549$$

So, the equation becomes:

$$\frac{x_s}{y_t} = 0.312(F˳)^{1.549} \times \left(\frac{R}{y_t}\right)^{1.081} - 0.104 \cos \theta$$

Another data was used to test the equations, and this by separating a twenty values from a total 120 values randomly by the program. The values of $\frac{x_s}{y_t}$ were calculated from the eq (5) a statistical comparison with the experimental data that separating was used to show the convergence of the predicted to observed records see Fig. 17. The value of $R^2$ was 0.99.
9. CONCLUSION

- The biggest part of the scour processes occurring in the first twenty minutes of the test duration.

- The minimum distance of scour from the nozzle increase with increasing densimetric Froude number, flow rate, velocity of the jet.

- There is no better angle for the jet in all cases where we found that each velocity jet favorites angle where cause maximum distance for scour, ranging between 20° for low velocity to 40° for high jet velocities, generally we found that the angle equal to 30° is the optimum angle.

- Dispersion technique for the water jet decreases the minimum distance of scour from the nozzle but in low percent.

- The Comparison of the results between the numerical and experimental models for the distance of scour hole from the nozzle shows a good agreement between them by 93.72%, and thus can be depended on the FLOW3D to inclusion of other variables.

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