N-BODY SIMULATIONS OF PLANETARY ACCRETION AROUND M DWARF STARS

Masahiro Ogihara and Shigeru Ida

Department of Earth and Planetary Sciences, Tokyo Institute of Technology, 2-12-1 Okayama, Meguro-ku, Tokyo 152-8551, Japan; ogihara@geo.titech.ac.jp

ABSTRACT

We have investigated planetary accretion from planetesimals in terrestrial planet regions inside the ice line around M dwarf stars through N-body simulations including tidal interactions with disk gas. Because of low luminosity of M dwarfs, habitable zones (HZs) are located in inner regions (~0.1 AU). In the close-in HZ, type-I migration and the orbital decay induced by eccentricity damping are efficient according to the high disk gas density in the small orbital radii. Since the orbital decay is terminated around the disk inner edge and the disk edge is close to the HZ, the protoplanets accumulated near the disk edge affect formation of planets in the HZ. Ice lines are also in relatively inner regions at ~0.3 AU. Due to the small orbital radii, icy protoplanets accrete rapidly and undergo type-I migration before disk depletion. The rapid orbital decay, the proximity of the disk inner edge, and large amount of inflow of icy protoplanets are characteristic in planetary accretion in terrestrial planet regions around M dwarfs. In the case of full efficiency of type-I migration predicted by the linear theory, we found that protoplanets that migrate to the vicinity of the host star undergo close scatterings and collisions, and four to six planets eventually remain in mutual mean-motion resonances and their orbits have small eccentricities (≤0.01) and they are stable both before and after disk gas decays. In the case of slow migration, the resonant capture is so efficient that densely packed ~40 small protoplanets remain in mutual mean-motion resonances. In this case, they start orbit crossing, after the disk gas decays and eccentricity damping due to tidal interaction with gas is no more effective. Through merging of the protoplanets, several planets in widely separated non-resonant orbits with relatively large eccentricities (~0.05) are formed. Thus, the final orbital configurations (separations, resonant or non-resonant, eccentricity, and distribution) of the terrestrial planets around M dwarfs sensitively depend on strength of type-I migration. We also found that large amount of water–ice is delivered by type-I migration from outer regions and final planets near the inner disk edge around M dwarfs are generally abundant in water–ice except for the innermost one that is shielded by the outer planets, unless type-I migration speed is reduced by a factor of more than 100 from that predicted by the linear theory.

Keywords: methods: N-body simulations – planetary systems – planetary systems: formation – planetary systems: protoplanetary disks – stars: low-mass, brown dwarfs

Online-only material: color figures

1. INTRODUCTION

Over 300 extrasolar planets have been discovered. Target stars for exoplanet search were mostly solar-type stars (F, G, and K dwarfs), although M dwarfs make up 70%–80% of all stars in the galactic disk. The low luminosity of M dwarfs is disadvantageous for high-dispersion spectroscopic observation, so that radial velocity surveys have not discovered large number of planets around M dwarfs. However, as improvement of spectroscopic observations, ground-based radial velocity surveys are revealing planetary systems around M dwarfs. Due to the low luminosity of M dwarfs, habitable zones (HZs), in which a planet with sufficient amount of atmosphere can sustain liquid water on its surface, are close to the host stars (Kasting et al. 1993). The proximity of the HZs to the host stars allows for detection of planets in HZs by current radial velocity observation. In fact, two planets with minimum masses below 10 M⊕ were discovered near the HZ in a triple planet system around an M star, GJ 581, with stellar mass M* = 0.31 M☉ (Udry et al. 2007). The habitability of these planets (GJ 581c,d) is vigorously under discussion theoretically. In addition, gravitational microlensing survey is suited for detection of M dwarf planets, because its detection efficiency is independent of stellar luminosity. Most of the planets detected by microlensing are orbiting M dwarfs. Recent radial velocity and microlensing observations show that Jupiter-mass gas giants are generally rare (e.g., Endl et al. 2006; Johnson et al. 2007), but Neptune-mass planets are rather abundant (e.g., Beaulieu et al. 2006), compared with solar-type stars.

GJ 436b is the planet that was discovered first among Neptune-mass planets (Butler et al. 2004). Transit observations revealed a planet’s radius, and its combination with radial velocity measurements permits a determination of the planet’s density. The evaluated internal density suggests that GJ 436b can be composed mainly of ice (Gillon et al. 2007; Deming et al. 2007), in spite of proximity to the host star. Ongoing and upcoming transit surveys using space telescopes such as Corot, Kepler, and TESS, besides ground-based transit surveys, are expected to reveal lower-mass exoplanets around M dwarfs.

“Core accretion” model (e.g., Hayashi et al. 1985) naturally accounts for the low abundance of gas giants around M dwarfs, because observationally inferred low disk mass around M dwarfs inhibits formation of cores large enough for runaway gas accretion (Laughlin et al. 2004; Ida & Lin 2005). The relatively high abundance of Neptune-mass planets could be accounted for by truncation of gas accretion at smaller planetary mass due to lower disk temperature (Ida & Lin 2005). Thus, the Monte Carlo calculation by Ida & Lin (2005) (at least qualitatively) explained the observed properties in the population of gas giants and Neptune-mass planets around M dwarfs. However, they did not predict abundance of habitable planets, because it is affected by type-I migration and detailed orbital configurations of close-in planets, which were not taken into account in their calculation. Ida & Lin (2008a) included the effect of type-I migration in the
similar calculation, but treatment of close-in planets was still too simple to discuss the abundance of habitable planets at $\sim$0.1 AU around M dwarfs.

$N$-body simulation is an efficient tool to address this issue. Since physical sizes of planetesimals occupy a larger fraction of their Hill radii in the terrestrial planet regions ($\sim$0.1 AU) around M dwarfs than around solar-type stars, strong gravitational scattering is suppressed, which reduces computational cost. Moreover, we can neglect perturbations from gas giants because they are rare in the M dwarf planetary systems, which also makes $N$-body simulation simple.

Raymond et al. (2007) performed a first $N$-body simulation of terrestrial planet formation from planetary embryos around low-mass stars. They found that the planets in a HZ may be too small to retain ocean because they assumed that disk surface density is proportional to the stellar mass and the disk model for $1 M_\odot$ is the minimum mass solar nebula (MMSN) model (Hayashi 1981). Under this assumption, the isolation mass of the planets is proportional to stellar mass (Section 2.3). In HZs, icy grains do not condense in the protoplanetary disk in which gas pressure is much smaller than the planetary atmospheric pressure. One of available sources for the water on the planets is delivery of icy planetesimals from the regions beyond the ice line (Morbidelli et al. 2000), although the possibility of forming H$_2$O through chemical interaction between the planetary magma ocean and primitive H$_2$ atmosphere is also pointed out (Ikoma & Genda 2006). Assuming the delivery hypothesis of origin of H$_2$O, Raymond et al. (2007) suggested that the planets in HZs around M dwarf stars are likely to be dry, since radial mixing and therefore water delivery are inefficient in the low-mass disks. Lissauer (2007) also pointed out the possible lack of large volatile inventories due to the large collision speeds of impacting comets and substantial mass loss of volatiles due to high activity and luminosities of young M dwarfs.

Although the simulations by Raymond et al. (2007) provided important insights into probability of habitable planets around M dwarfs, they neglected the effects of protoplanetary disk gas. Since in such inner regions, gas density is so high that migration due to gas drag and tidal interaction with a gas disk are efficient (see Section 2.3) and accretion timescale of terrestrial planets would be much shorter than disk lifetime (see Section 2.3), the effects of disk gas play important roles in the accretion of planets in HZs around M dwarfs and water delivery to them. We will point out in Section 3.5 that the planets in HZs are rather composed mainly of water–ice unless type-I migration speed is reduced by a factor of more than 100 from the linear theory or the migrating protoplanets are trapped at an inner boundary of dead zone (Kretke & Lin 2007; Ida & Lin 2008b; Kretke et al. 2009).

With type-I migration, the proximity of the HZ to the inner disk edge would play an important role for final configuration of planets in HZs, because type-I migration stops at the disk edge and planets would accumulate there. Terquem & Papaloizou (2007) performed N-body simulations of protoplanets undergoing type-I migration around solar-type stars. Their important finding is that the migrating protoplanets originally formed at $\sim$1 AU interact with the preceding protoplanets near the disk inner edge and finally two to five close-in planets remain in mutual mean-motion resonances. They found that the resonant configuration is maintained even after disk gas is removed or tidal interaction with the star is added. They only carried out simulations with type-I migration with full efficiency that is predicted by the linear theory (Ward 1986; Tanaka et al. 2002). We will show that the final orbital configuration is sensitively dependent of migration speed.

We thereby carry out N-body simulations including the effects of damping of orbital eccentricity, inclination, and semimajor axis (type-I migration) due to disk gas. Although Raymond et al. (2007) and Terquem & Papaloizou (2007) start their simulations from planetary embryos that have already grown to their isolation masses, our simulation starts from many small planetesimals, taking fully into account their gravitational interactions. Our calculations also cover a broad range of orbital radius from the disk inner edge to beyond the ice line. Since there is still uncertainty in the type-I migration speed, we perform both simulations with and without type-I migration.

In Section 2, we describe the disk model, the formulae of forces for aerodynamic and gravitational gas drag, and calculation methods. The results of N-body simulations of planetary accretion are shown in Section 3. In Section 3.4, we discuss water delivery to inner planets around M dwarfs. Section 4 is devoted to the conclusion and discussion.

2. MODEL AND CALCULATION METHODS

Here, we consider an MSV-type star with mass $M_\ast = 0.2 M_\odot$ and luminosity $L_\ast \simeq 0.01 L_\odot$, using a mass–luminosity relation ($L_\ast \propto M_\ast^2$). This relation roughly fits observational data for a range of 0.1–1 $M_\odot$ stars in main-sequence stages (e.g., Habets & Heintze 1981; Scalo et al. 2007). Although pre-main-sequence stages are relatively long for the low-mass stars and luminosity is relatively high during the pre-main-sequence stages, we will use HZs determined by the main-sequence radiation, because our main purpose is to clarify the dynamics and accretion process among protoplanets that have migrated to the regions near disk inner edges. The simulation with evolving HZs and ice lines due to the luminosity evolution is left to a future work.

We integrate the orbits of planetesimals, taking into account their merging by direct collisions, their gravitational interactions, aerodynamic gas drag, “gravitational drag” (damping of orbital eccentricity and inclination due to tidal interaction with a gas disk), and type-I migration. The models for surface densities of disk gas and an initial planetesimal swarm are explained in Section 2.1. Basic equations for orbital integration and initial conditions are presented in Section 2.2. Although the detailed expressions for the aerodynamic gas and gravitational drag forces are given in Appendices A and B, we summarize their characteristic timescales in Section 2.3, as well as the timescale of planetesimal accretion, which are useful to understand the results of the N-body simulations.

2.1. Disk Model

Following Ida & Lin (2004), we scale the gas surface density $\Sigma_g$ of disks as

$$\Sigma_g = 2400 f_g \left( \frac{r}{1 \text{AU}} \right)^{-3/2} \text{g cm}^{-2},$$

where $f_g$ is a scaling factor; $\Sigma_g$ is 1.4 times of those in the MMSN model if $f_g = 1$. Because current observations cannot strictly constrain the radial gradient of $\Sigma_g$, we here assume $f_g$ is constant with $r$ except for the inner edge. For disks around stars with $M_\ast \sim 1 M_\odot$, the observationally inferred averaged value of $f_g$ is $\sim 1$ although the values have dispersion of 2 orders of magnitude (see discussion in Ida & Lin 2004). We here adopt $f_g = 1$ for the disks around the $M_\ast = 0.2 M_\odot$ star. Although averaged $f_g$ may be several times smaller for these stars, $f_g = 1$ is within the 2 orders of magnitude dispersion. The relatively large value of $f_g$ is to study possible formation of habitable
planets, which are large enough to retain water on their surface, around low-mass stars. In Section 3.5, we will discuss how the results are affected if less-massive disks ($f_g \sim 0.2$), which may be averaged disks around these stars, are considered.

We assume the temperature distribution of an optically thin disk (Hayashi 1981),

$$T = 2.8 \times 10^2 f_g \left( \frac{r}{1 \text{ AU}} \right)^{-1/2} \left( \frac{L_*}{L_\circ} \right)^{1/4} \text{ K.} \tag{2}$$

Corresponding sound velocity $c_s$ and disk scale height $h$ are

$$c_s = 1.0 \times 10^5 \left( \frac{r}{1 \text{ AU}} \right)^{-1/4} \left( \frac{L_*}{L_\circ} \right)^{1/8} \text{ cm s}^{-1}, \tag{3}$$

$$h \simeq 4.7 \times 10^{-2} \left( \frac{r}{1 \text{ AU}} \right)^{5/4} \left( \frac{L_*}{L_\circ} \right)^{1/8} \left( \frac{M_*}{M_\circ} \right)^{-1/2} \text{ AU.} \tag{4}$$

In the current paradigm, T Tauri disks are truncated by the stellar magnetosphere within the corotation radius, with materials accreting along magnetic field lines onto high-latitude regions of the star. The corotation radius $r_{\text{corot}}$ is the radius at which the Keplerian orbital period in the disk equals the stellar rotation period ($P$),

$$r_{\text{corot}} = 0.04 \left( \frac{P}{3 \text{ days}} \right)^{2/3} \left( \frac{M_*}{M_\circ} \right)^{1/3} \text{ AU.} \tag{5}$$

Here, we set the location of disk inner edge at 0.05 AU. From Equation (4), the scale height of a disk at 0.05 AU around a star with $M_* = 0.2 M_\circ$ is $\sim 1.4 \times 10^{-3}$ AU. We assume that the gas surface density of the disk (equivalently $f_g$) smoothly vanishes at 0.05 AU with a hyperbolic tangent function with the width of $10^{-3}$ AU, which is comparable to $h$. When planets enter the inner cavity, they do not feel gas drag any more and their inward migration ceases because the drag and migration rates are proportional to disk gas surface density. In some runs, we reverse the direction of the migration near the edge according to positive pressure gradient there (Tanaka et al. 2002; Masset et al. 2006).

According to gas surface density given by Equation (1), we scale the surface density of a planetesimal disk with a scaling factor, $f_d$, as

$$\Sigma_d = 10 \eta_{\text{ice}} f_d \left( \frac{r}{1 \text{ AU}} \right)^{-3/2} \text{ g cm}^{-2}. \tag{6}$$

For solar metallicity, $f_d = f_g$, so we use $f_d = 1$. The factor $\eta_{\text{ice}}$ expresses the increase of solid materials due to ice condensation outside the “ice line.” The location of the ice line is determined in such a way that the disk temperature equals the ice condensation temperature. Assuming the condensation temperature of 170 K, the location of the ice line is derived from Equation (2):

$$r_{\text{ice}} \simeq 2.7 \left( \frac{L_*}{L_\circ} \right)^{1/2} \text{ AU.} \tag{7}$$

Since we consider stars with $L_* \simeq 0.01 M_\circ$, we set $r_{\text{ice}} = 0.3$ AU. In the MMSN model, $\eta_{\text{ice}} = 4.2$ at $r > r_{\text{ice}}$. Pollack et al. (1994) derived $\eta_{\text{ice}} \simeq 3$. In the standard set of our simulations, we adopt $\eta_{\text{ice}}$ as

$$\eta_{\text{ice}} = \begin{cases} 1 & [r < 0.3 \text{ AU}] \\ 3 & [0.3 \text{ AU} < r]. \end{cases} \tag{8}$$

But, in some runs, we adopt a higher value for $r_{\text{ice}}$ (see Section 3.3).

2.2. Orbital Integration and Initial Conditions

We integrate the orbits of planetesimals with fourth-order Hermite scheme (Makino & Aarseth 1992) and hierarchical individual time step (Makino 1991). The basic equations of motions of particle $k$ at $r_k$ in heliocentric coordinates are

$$\frac{d^2 r_k}{d t^2} = -G M_* \frac{r_k}{|r_k|^3} - \sum_{j \neq k} G M_j \frac{r_k - r_j}{|r_k - r_j|^3} + F_{\text{aero}} + F_{\text{damp}} + F_{\text{mig}}, \tag{9}$$

where $k, j = 1, 2, \ldots, $ the first term is gravitational force of the central star and the second term is mutual gravity between the bodies. We calculate the self gravity directly summing up interactions of all pairs on the special-purpose computer for $N$-body simulation, GRAPE-6. $F_{\text{aero}}$, $F_{\text{damp}}$, and $F_{\text{mig}}$ are specific forces due to aerodynamic gas drag, gravitational drag, and type-I migration, the detailed expressions of which are described in Appendices A and B. We neglect the indirect term since the total mass of the planetesimals is $\sim 10^{-4}$ times the mass of the central star.

When physical radii of two bodies overlap, perfect accretion is assumed. After the collision, a new body is created, conserving total mass and momentum of the two colliding bodies. The physical radius of a body is determined by its mass $M$ and internal density $\rho_p$ as

$$r_p = \left( \frac{3}{4 \pi} \frac{M}{\rho_p} \right)^{1/3}. \tag{10}$$

We adopt a realistic value 3 g cm$^{-3}$ for $\rho_p$.

Initially 5000 planetesimals are placed between 0.05 AU (disk inner edge) and 0.4 AU. To study accretion process of terrestrial planets inside the ice line in more detail, we initially set more bodies inside the ice line (3898 bodies with mass 2.3$\times$10$^{24}$ g) than outside it (1102 bodies with mass 6.5$\times$10$^{24}$ g) in the nominal simulations. The initial velocity dispersion of the bodies is set to be their escape velocity. The corresponding initial eccentricity and inclination are given by $e_{\text{esc}} = \sqrt{e^2 + i^2 v_K}$ with $e = 2i$ (Ida & Makino 1992). Table 1 shows simulation conditions for individual runs and final results. In the first set (setA) of runs (runA1–runA4), we include all the damping of $e$, $i$, and $a$ due to aerodynamic and gravitational drag and type-I migration, while type-I migration is neglected in the second set (setB: runB1–runB4). Because coagulation to final planets may be a stochastic process, for each set we performed four runs with different seeds for random numbers to generate initial angular distribution of planetesimals.

In addition to these sets, we also carried out runs with reversed torque of type-I migration near the disk inner edge (setC: runC1–runC2) and runs with $\eta_{\text{ice}} = 14$ at $r > r_{\text{ice}}$ (setD: runD1–runD4 with type-I migration, setE: runE1–runE4 without type-I migration), which are described in Section 3.3.

2.3. Characteristic Timescales

To understand the results of $N$-body simulations, we here summarize the timescales of gas and gravitational drag, type-I migration, and planetesimal accretion. The characteristic
damping timescale for orbital eccentricity (e) and inclination (i) of an isolated planetesimal due to aerodynamic gas drag is

\[ t_{aero} \sim \frac{\Delta u}{\dot{F}_{aero}} \simeq 3.4 \times 10^6 \left( \frac{((5/8)\epsilon^2 + (1/2)i^2 + \eta^2)^{1/2}}{0.01} \right)^{-1} \times \left( \frac{M}{M_\oplus} \right)^{1/3} \left( \frac{M_e}{0.2 M_\oplus} \right)^{-1/2} \left( \frac{\rho_p}{3 \text{ g cm}^{-3}} \right)^{2/3} \times \left( \frac{a}{1 \text{ AU}} \right)^{13/4} \text{ yr,} \] (11)

where \( \dot{F}_{aero} \) is specific drag force acting on the planetesimal (Equation (A1)) and \( a, M, \) and \( \rho_p \) are its semimajor axis, mass, and bulk density, respectively. The relative velocity between the planetesimals and disk gas, \( \Delta u \), is given by \( \simeq ((5/8)\epsilon^2 + (1/2)i^2 + \eta^2)^{1/2} v_K \), where \( v_K \) is Keplerian velocity. The velocity of disk gas \( v_{gas} \) is smaller than Kepler velocity \( v_K \) by a fraction (Equation (A2)),

\[ \eta \simeq \frac{v_K - v_{gas}}{v_K} = 2.8 \times 10^{-3} \left( \frac{r}{1 \text{ AU}} \right)^{1/2} \left( \frac{M_e}{0.2 M_\oplus} \right)^{-1} \times \left( \frac{L_e}{0.01 L_\oplus} \right)^{1/4}. \] (12)

The timescale for damping of semimajor axis (a) is

\[ t_{aero,a} = \frac{a}{\dot{a}} \sim \frac{t_{aero}}{2\eta} = 0.6 \times 10^9 \left( \frac{C_D}{0.5} \right)^{-1} \times f_g^{-1} \left( \frac{((5/8)\epsilon^2 + (1/2)i^2 + \eta^2)^{1/2}}{0.01} \right)^{-1} \]

At \( a \simeq 0.1 \text{ AU}, t_{aero,a} \sim 1.5 \times 10^6 f_g^{-1} \text{ yr} \) even for a Mars-mass planet \( (M \sim 0.1 M_\oplus) \), so that gas drag cannot be neglected even for protoplanets. For small planetesimals, the gas drag is more important.

A planet gravitationally perturbs the disk gas and excites density waves. The waves damp the eccentricity, the inclination (e.g., Ward 1993; Artyomnovich 1993), and the semimajor axis of the planet (e.g., Goldreich & Tremaine 1980; Ward 1986). The detailed expressions of gravitational gas drag forces \( F_{damp} \) and \( F_{mig} \) are given in Appendix B. Orbital eccentricities and inclinations are damped by both torques from inner and outer disks in a similar way to dynamical friction from planetesimals.

The damping timescales are (Tanaka & Ward 2004)

\[ t_{damp} = \frac{e}{\epsilon} \sim \left( \frac{M}{M_e} \right)^{-1} \left( \frac{\Sigma_e a^2}{M_e} \right)^{-1} \left( \frac{c_s}{v_{K}} \right)^4 \Omega_K^{-1} \] (14)

\[ \simeq 70 f_g^{-1} \left( \frac{M}{M_\oplus} \right)^{-1} \left( \frac{a}{1 \text{ AU}} \right)^2 \left( \frac{M_e}{0.2 M_\oplus} \right)^{-1/2} \left( \frac{L_e}{0.01 L_\oplus} \right)^{1/2} \text{ yr.} \] (15)

On the other hand, secular inward migration due to tidal interaction with disk gas that is known as “type-I migration” is caused by torque imbalance. The migration timescale is

\[ t_{mig} = -\frac{a}{\dot{a}} = \frac{1}{2.7 + 1.1 q} \left( \frac{M}{M_e} \right)^{-1} \left( \frac{\Sigma_e a^2}{M_e} \right)^{-1} \left( \frac{c_s}{v_{K}} \right)^2 \Omega_K^{-1} \] (16)
\[
\frac{dM}{dt} \sim \frac{1}{r_p} \left(1 + \frac{v_{\text{esc}}}{v_{\text{ran}}} \right) \Omega_K.',
\]

where \(v_{\text{ran}} \approx (e^2 + l^2)^{1/2} v_K\). With more detailed formula and \(v_{\text{ran}}\) that is determined by balance between scattering of the planetesimals by the protoplanet and gas drag to them, the accretion timescale is expressed as (Ida & Lin 2004)

\[
t_{\text{acc}} \approx 1.2 \times 10^6 \eta_{\text{ice}} \frac{f_d}{f_g} \frac{f_d^{2/3}}{f_g^{2/5}} \left(\frac{a}{1 \text{ AU}}\right)^{27/10} \left(\frac{M_*}{0.2 M_\odot}\right)^{1/3} \times \left(\frac{M_*}{0.2 M_\odot}\right)^{-1/6} \left(\frac{m}{10^{24} \text{ g}}\right)^{2/15} \left(\frac{\rho_p}{3 \text{ g cm}^{-3}}\right)^{1/3} \text{ yr.}
\]

The growth timescale of terrestrial planets around M dwarfs is shorter than that around solar-type stars because of small \(a\). Hence, it is reasonable to assume that full amount of disk gas exists during the terrestrial planet formation because depletion timescale of disks around M dwarfs may be \(\sim 10^4 \text{ yr}\) or more. On the other hand, the damping timescale are short in inner regions where the gas density is high. As stated in Section 1, the effects of disk gas cannot be neglected when we consider accretion of planets in HZs around M dwarfs.

The isolation mass, which is the mass of all the solid materials in a feeding zone of the protoplanets, is

\[
M_{\text{iso}} \sim 0.23 \eta_{\text{ice}} \frac{f_d^{3/2}}{f_g^{3/2}} \left(\frac{a}{1 \text{ AU}}\right)^{3/4} \left(\frac{\Delta a}{7.5 r_H}\right)^{3/2} \left(\frac{M_*}{0.2 M_\odot}\right)^{-1/2} M_\odot,
\]

where \(\Delta a\) is width of the feeding zone. The mutual Hill radius \(r_H\) is defined by

\[
r_H = \left(\frac{M_1 + M_2}{3 M_\odot}\right)^{1/3} \frac{M_1 a_1 + M_2 a_2}{M_1 + M_2},
\]

where \(M_1\) and \(M_2\) are masses of interacting bodies that we are concerned with and \(a_1\) and \(a_2\) are their semimajor axes. In Equation (20), \(M_1 = M_2 = M_{\text{iso}}\) is assumed. The value of \(\Delta a = 7.5 r_H\) is a typical value obtained by N-body simulation at \(a \approx 0.1 \text{ AU}\), which is slightly smaller than the value obtained at \(\sim 1 \text{ AU}\) (Kokubo & Ida 1998, 2002). The isolation mass is the maximum mass that the planet can acquire through “runaway/oligarchic” growth (accretion of planetesimals) before onset of orbit crossing and coagulation among the isolated protoplanets.

The migration timescale for the protoplanet with mass \(M_{\text{iso}}\) is

\[
t_{\text{mig,iso}} = 3.1 \times 10^4 \eta_{\text{ice}}^{-3/2} \frac{f_d^{3/2}}{f_g^{1/2}} \frac{f_d}{f_g} \left(\frac{a}{1 \text{ AU}}\right)^{3/4} \left(\frac{L_*}{0.01 L_\odot}\right)^{1/4} \text{ yr.}
\]

Its accretion timescale is

\[
t_{\text{acc,iso}} = 0.70 \times 10^6 \eta_{\text{ice}}^{-1/2} \frac{f_d}{f_g} \left(\frac{a}{1 \text{ AU}}\right)^{59/20} \times \left(\frac{M_*}{0.2 M_\odot}\right)^{-1/3} \left(\frac{\rho_p}{3 \text{ g cm}^{-3}}\right)^{1/3} \text{ yr.}
\]

At \(a \lesssim 0.3 \text{ AU}\) we that are concerned with in this paper, both \(t_{\text{mig,iso}}\) and \(t_{\text{acc,iso}}\) are significantly shorter than disk lifetime of \(\sim 10^8 \text{ yr}\) for \(f_g \sim f_d \sim 1\). This is also the case even for small-mass disks with the surface density five times smaller than that of the MMSN (\(f_d = f_g \approx 0.15\)), which Raymond et al. (2007) considered. Thus, type-I migration cannot be neglected even for the small-mass disks, unless nonlinear effects slow down or halt the migration significantly (Section 3.5).

### 3. RESULTS

#### 3.1. Runaway/Oligarchic Growth before Disk Gas Depletion

#### 3.1.1. Case with Type-I Migration

The result for a typical run including the effect of type-I migration (runA1) is shown in Figure 1. In the snapshots, the sizes of circles are proportional to the physical radii of the bodies, and \(T_K\) is the Kepler time at 0.1 AU around a 0.2 \(M_\odot\) star, which is \(\sim 0.071\) yr. Since the accretion timescale depends strongly on semimajor axis \(a\) (Equation (19)) and it is shorter for smaller \(a\), accumulation of planetesimals proceeds in an inside-out manner. Equation (19) for \(M \sim M_{\text{iso}} \sim 0.04\) \(M_\odot\) at 0.1 AU is \(10^3\) yrs = \(1.4 \times 10^4\) \(T_K\), which agrees with the result in Figure 1.

According to growth of a protoplanet, the type-I migration timescale becomes shorter, while the accretion timescale becomes longer. Thereby, the protoplanet starts migration when its mass exceeds the critical mass beyond which \(t_{\text{mig}} < t_{\text{acc}}\). From Equations (17) and (19), the critical mass is

\[
M_{\text{crit},\text{mig}} \approx 4.0 \times 10^{-2} \eta_{\text{ice}}^{3/4} \frac{f_d^{3/4}}{f_g^{9/20}} \left(\frac{a}{1 \text{ AU}}\right)^{-9/10} \times \left(\frac{M_*}{0.2 M_\odot}\right)^{1/2} \left(\frac{L_*}{0.01 L_\odot}\right)^{3/16} M_\odot.
\]

Because at \(\sim 0.1\) AU this value is close to the isolation mass, protoplanets start migration after they accrete most of planetesimals in their feeding zone. On the other hand, in outer regions, since \(M_{\text{crit},\text{mig}} < M_{\text{iso}}\), protoplanets start migration leaving behind large amount of planetesimals. Most of the initial mass is finally concentrated near the disk inner edge after \(10^5\) \(T_K\), leaving few planetesimals in outer regions. Although eccentricities of the bodies are excited by close encounters with neighbor bodies during the growth stage, final eccentricities of the planets are kept small (\(\lesssim 0.01\)) due to gravitational drag. Resonant effect can raise the eccentricities, however, gas drag damps them significantly because of the high gas surface density.
Figure 1. Time evolution of a system on the $a$–$e$ plane. The circles represent bodies and the radii of the circles are proportional to the physical radii of the bodies. Note that overlapping circles does not mean the planets actually overlap each other because the sizes of circles are expanded so as to be easy to see. The system initially consists of 5000 planetesimals. The numbers of bodies are 2970 ($1000T_K$), 2144 ($10^{5},000T_K$), 1862 ($20^{5},000T_K$), 1448 ($50^{5},000T_K$), 1107 ($100^{5},000T_K$), 390 ($500^{5},000T_K$), 170 ($1,000,000T_K$), and 26 ($5,000,000T_K$). $T_K$ is Keplerian time at 0.1 AU around a $0.2M_\odot$ star, which is $\sim 0.071$ yr. In the electronic version, bodies with $M > 0.01M_\oplus$, $M > 0.1M_\oplus$, and $M > M_\oplus$ are expressed with blue, red, and green circles, respectively.

Orbital evolution of runA1 is shown in Figure 2. In this figure, the most massive 30 planets at each time are plotted as circles (there are many other small bodies). Accretion takes place more rapidly in inner region than in outer region at the beginning of the simulation. The protoplanets with $M > M_{\text{crit,mig}}$ undergo inward type-I migration. Since $t_{\text{mig}}$ is shorter at smaller orbital radius, the migration is accelerated until the protoplanets reach the disk inner edge. The firstly migrated protoplanets interact with each other. After some merging, the survived protoplanets are captured in mutual mean-motion resonances. After that, successively migrated planets interact with outermost planets that have accumulated near the disk edge. The interaction mostly results in merging of the planets and the merger is again captured in a mean-motion resonance. Note that the number of remaining planets near the edge is almost constant (four to six planets) during this phase, although protoplanets that formed in outer regions migrate to interact with the close-in planets one after another. In this run, after $\sim 5 \times 10^6 T_K$, any more protoplanets which are large enough to affect the inner planets do not approach to the inner planetary system, so that this run ends up with a stable configuration consisting of six planets with $M > 0.01M_\oplus$ near the disk inner edge. Most of the final planets are pushed into the cavity at $a < 0.05$ AU, in which type-I migration is no more effective, by the outer planets that keep loosing angular momentum by type-I migration. The largest planet in the final state is the third

Figure 2. Evolution of semimajor axes of massive planets. At each time for runA1, 30 most massive planets are plotted. $T_K$ is Keplerian time at 0.1 AU around a $0.2M_\odot$ star. The circles represent bodies and their radii are proportional to the physical radii of the bodies.

(A color version of this figure is available in the online journal.)
innermost planet, the mass of which is $0.63 M_\oplus$. Note that this value is more than 10 times of the values of $M_{\text{crit, mig}}$ ($\sim 0.01$–$0.06 M_\oplus$ at 0.05–0.4 AU). This implies that coagulation near the disk edge is so efficient. The value of $0.63 M_\oplus$ is also more than 25 times larger than $M_{\text{iso}}$ ($\sim 0.024 M_\oplus$) at 0.05 AU. Without migration, such large planets cannot accrete at $\sim 0.05$ AU.

All the final six planets are trapped in first-order commensurability (mean-motion resonances) with the planets which lie next to the planets. For example, the innermost pair (a pair of the innermost and the second innermost planets) has 6:5 commensurability, and the second pair (a pair of the second and the third innermost planets) has 5:4 commensurability. The perturbations during one passage rapidly increase for $\Delta a \lesssim 5 r_{H}$ (Ida 1990). Since resonant trapping at $\Delta a > 5 r_{H}$ may not be resistant against perturbations from other bodies other than the pair, it is expected that the pair may be eventually trapped at mean-motion resonances close to $\Delta a \sim 5 r_{H}$. The obtained orbital separations of the $i$th pairs ($i = 1, 2, \ldots, 5$) are $\Delta a \simeq 7.3 r_{H}, 8.3 r_{H}, 6.8 r_{H}, 5.4 r_{H}$, and $8.7 r_{H}$, respectively. As will be shown in Section 3.2, this final orbital configuration is stable even if disk gas is removed.

All the runs in setA with the effect of type-I migration end in very similar results: most of the protoplanets (large planetesimals) pile up near the inner disk edge, and in final state, most of the planets are captured in mean-motion resonances after coagulation and close scattering among the protoplanets. The average number of the final planets in runA1–runA4 is $N = 5.0 \pm 0.71$, which is comparable to that obtained by Terquem & Papaloizou (2007). The average mass of the largest planet is $\overline{M}_{\text{max}} = 0.82 \pm 0.17 M_\oplus$, which is significantly larger than $M_{\text{iso}}$ at 0.05 AU and $M_{\text{crit, mig}}$ at 0.05–0.4 AU.

### 3.1.2. Case without Type-I Migration

A typical result excluding the effect of type-I migration in setB is shown in Figure 3. The figure shows snapshots of the system on the $a$–$e$ planes for runB1. Although the effect of type-I migration is not included, planets tend to migrate inward. The inward migration is induced by eccentricity damping by tidal interaction with disk gas. Since the angular momentum ($L = \sqrt{GM_\oplus a(1-e^2)}$) is almost conserved, damping of eccentricity yields damping of semimajor axis. The damping timescale of semimajor axis is roughly given by

\[
\tau_{\text{damp, a}} = \frac{a}{\dot{a}} = \frac{1}{2 e^2} = \frac{\tau_{\text{damp}}}{2 e^2} (25)
\]

\[
\simeq 0.7 \times 10^6 f_g^{-1} \left(\frac{0.01}{g}ight)^{-2} \left(\frac{M}{M_\oplus}\right)^{-1} \left(\frac{a}{1 \text{ AU}}\right)^2
\times \left(\frac{M_a}{0.2 M_\oplus}\right)^{-1/2} \left(\frac{L_a}{0.01 L_\odot}\right)^{1/2} \text{ yr}. (26)
\]

where Equation (15) was used. The values of $e$ are typically $\sim 0.01$ in our simulations, but they change with time and depend on locations. Compared to Equation (13), we find that the semimajor axis damping induced from the eccentricity damping is more effective than aerodynamic gas drag for $M \sim 0.01$–$1 M_\oplus$, even if the uncertainty in the values of $e$ is taken into account.
Nevertheless, this migration timescale is of the order of $10^5$ yrs to the case with 100 times reduced type-I migration speed. Since this timescale is about 100 times longer than that in setA. The average mass of the largest planet is $M_{\text{max}} = 0.20 \pm 0.033 M_\oplus$.

The critical planet mass beyond which $t_{\text{damp},a} < t_{\text{acc}}$ is

$$M_{\text{crit},\text{damp}} \simeq \frac{3}{4} \frac{n_\text{acc}}{f_g} \frac{a}{1 \text{AU}} \left( \frac{a}{0.01} \right)^{-3/2} \left( \frac{M_\star}{0.2 M_\odot} \right)^{-1/4} \left( \frac{L_\star}{0.01 L_\odot} \right)^{3/8} M_\odot. \quad (27)$$

This value is greater than the isolation mass (Equation (20)) at $a \lesssim 2$ AU, so that all the protoplanets in the simulations in this set grow up to the isolation mass before onset of migration. Hence, the maximum mass $M_{\text{max}}$ is roughly equal to $M_{\text{iso}}$, which is consistent with the value of $M_{\text{max}}$ obtained in our results.

The orbital separations are $\sim 5 - 6r_\odot$, which means that the planets are more packed than in setA. The larger number and smaller separations of the final planets than those in setA suggest that the final configuration could be unstable. As will be shown below, in setB, the final planets start orbit crossing immediately after the removal of disk gas.

### 3.2. Stability after Disk Gas Depletion

All the runs in setA with type-I migration and setB without it end with multiple planets. Because eccentricity damping due to gravitational drag is so strong that the systems of these multiple planets are orbitally stable (Iwasaki et al. 2002; Kominami & Ida 2002), although the minimum orbital separations in the systems are rather small ($\sim 5 - 7r_\odot$). However, disk gas should dissipate on timescales less than $10^7$ yr.

In the gas-free case, it is predicted that these planets may become unstable on timescales of $t_{\text{cross}} \sim 10^7 T_K \sim 10^2$ yr (Chambers et al. 1996). Note, however, that these timescales are for non-resonant planets. In our results, the final multiple planets are usually captured in mean-motion resonances that generally stabilize the systems. Actually, Terquem & Papaloizou (2007) found that the final multiple planets, which correspond to our results in runA1–runA4, are stable on timescales much longer than $t_{\text{cross}}$ even after removal of disk gas. We show similar results for runA1–runA4 below, but also show that the removal of disk gas makes the systems unstable for runB1–runB4.

To examine long-term stability, we take out the planets with masses larger than 0.01 $M_\odot$ from the final state of the runs, and integrate their orbits, neglecting many other smaller-mass planets for saving computational cost. As expected, we found that the system of runA1 is stable until $2 \times 10^7 T_K$ under the disk gas damping. To study the stability after the disk gas removal, we re-start the calculation from the final state of runA1 at $5 \times 10^6 T_K$ without the disk gas damping. We adiabatically adjust the orbital configuration to the gas-free condition, by decreasing $f_g$ as

$$f_g = \exp \left( -\frac{t - 5 \times 10^6 T_K}{10^4 \text{ yr}} \right). \quad (28)$$

The decay timescale of $10^4$ yr ($\sim 1.4 \times 10^5 T_K$) is long enough for the adjustment.

At the end of runA1 (Figure 2), six planets are remain with separations of $5 - 9r_\odot$. The orbital evolution after the disk gas removal of runA1 is shown in Figure 5 (the left panel). The figure shows that the eccentricities of the planets are kept less than 0.01 and the planets remain stable even after the gas removal. Although the minimum orbital separation is about $5r_\odot$, the resonant configuration stabilizes the system.

The other runs (runA2–runA4) in this set with type-I migration also show similar results: even after the disk gas removal
removal, orbital separations hardly change (the average orbital separation is $\overline{d} = 9.5 \pm 0.97 r_{H}$) and most of the planets keep their commensurate relationships until the end of simulations. The average orbital eccentricity is $\overline{e} = 0.0086 \pm 0.0061$. This is consistent with the result by Terquem & Papaloizou (2007).

The stability for setB without type-I migration is completely different. At the end of runB1 (Figure 4), 45 planets with masses larger than 0.01 $M_\oplus$ remain with the orbital separations of the planets are $5 \sim r_{H}$. Figure 5 (the right panel) shows the semimajor axis evolution of runB1 after the disk gas removal. Soon after the gas removal, the eccentricities are pumped up and the planets start orbit crossing. Note, however, that the planets do not exhibit global orbital crossing that the planets at $\sim 1$ AU around solar-type stars exhibit after disk gas removal (e.g., Kominami & Ida 2002). In terrestrial planet regions around M dwarfs ($\sim 0.1$ AU), physical radii of the planets $r_p$ relative to their Hill radii $r_H$ are larger than that in $\sim 1$ AU ($r_p/r_H \propto M_i^{1/3} a^{-1}$). Then, the eccentricities are pumped less highly and furthermore merging proceeds before the eccentricities are fully excited. Thus, the planets collide with only neighboring planets. Finally, nine planets with moderate eccentricities ($\sim 0.08$) are formed. The mass of the largest planet is 0.65 $M_\oplus$. All the commensurabilities are lost in the course of close encounters and the final planets are not trapped in mean-motion resonances at all. The orbital separations are $\sim 20 r_{H}$, which are large enough to be dynamically isolated from each other in the non-resonant configurations. Since relatively many planets ($\sim 40$) are formed with small orbital separations, all the runs in setB without type-I migration exhibit orbital crossing and merging of the planets after the disk gas removal, resulting in $\sim 10$–20 planets with the average eccentricity of $0.055 \pm 0.020$ and the average orbital separation of $19 \pm 2.2 r_{H}$ that lost commensurate relationships. The average mass of the largest planet is $\overline{M}_{\text{max}} = 0.50 \pm 0.097 M_\oplus$. The non-resonant orbital configurations with wider separations and larger eccentricities are the characteristic to setB without type-I migration (with 100 times reduced migration efficiency). The semimajor axes of the planets are not concentrated to the regions near the disk inner edge, in contrast to the results with type-I migration. Thus, type-I migration efficiency in inner disk regions regulates orbital configurations of close-in terrestrial planets.

### 3.3. Dependence on Boundary and Initial Conditions

Since we are concerned with planetesimal accretion near the disk inner edge, we study the effects of general relativity that is effective in the proximity of the host star and reserved type-I migration torque that occurs near the edge. We restarted the stability calculation in gas-free condition for the runA1–runA4, incorporating the relativistic effect directly into orbital integration. The detailed expression of post-Newtonian gravitational force from the host star is given in Appendix C. Although the relativistic effect causes the precession of the perihelion of short-period planets, we found that the resonant relationships are not changed by the relativistic effect and the systems stayed in a stable state. Terquem & Papaloizou (2007) found that the tidal dissipation does not affect the stability as well as the relativity.

It is argued that inward protoplanet migration can be halted before reaching the inner cavity, because the tidal torque from the disk is reversed due to inverse pressure gradient near the disk edge (Masset et al. 2006). Because the planets in the inward torque regions gain angular momentum from the disk gas, the inwardly migrating planets in outer regions cannot push the inner planets into the cavity, before the depletion of disk gas. Substituting the component of the gas surface density gradient at the inner edge in our model into $-q$ in Equation (16), we performed simulations in runC1 and runC2 (this effect was also investigated in Terquem & Papaloizou 2007). The orbital evolution for runC1 is shown in Figure 6. Although qualitative evolution is almost the same as the result without the reversed torque, the number of final planets is fewer than that obtained in runA1–runA4. Because the innermost planet cannot penetrate into the cavity, the orbital separation between the innermost planet and the second innermost one is small ($\sim 3.8 r_H$). However, the right panel of Figure 6 shows that the planets remain orbitally stable after the disk gas removal, because they keep being trapped in 5:4 and 9:8 resonances.

Around M dwarfs, the ice line is so close that significant amount of icy protoplanets are quickly formed and migrate to

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**Figure 5.** Orbital evolution of successive simulation of runA1 (left) and runB1 (right) in a gas-free environment, neglecting small bodies. In runB1, planets suffer giant impacts, which results in losing of commensurabilities. (A color version of this figure is available in the online journal.)
inner regions. The migrated icy protoplanets affect accretion of planets in terrestrial planet regions, in particular, they regulate the mass of the largest planet in final state. So far, we have adopted the ice condensation factor as $\eta_{\text{ice}} = 3$ in Equation (6). However, more enhanced $\eta_{\text{ice}}$ was proposed by several authors. Stevenson & Lunine (1988) proposed that sublimation of icy grains that have migrated to inside the ice line and the diffusion of the water vapor enhances the surface density of icy materials near the ice line up to $\eta_{\text{ice}} \sim 75$. Recent more detailed study (Ciesla & Cuzzi 2006) showed $\eta_{\text{ice}} \sim 10$. To highlight the effect of migrated icy protoplanets, we performed calculations with

$$\eta_{\text{ice}} = \begin{cases} 1 & [r < 0.3 \text{ AU}] \\ 14 & [0.3 \text{ AU} < r]. \end{cases}$$  (29)

The result including the effect of type-I migration (runD1) is shown in Figure 7. It is qualitatively similar to runA1 with $\eta_{\text{ice}} = 3$ at $r > 0.3$ AU: several planets are formed near the disk inner edge trapped in mutual mean-motion resonances before the disk removal. Even if disk gas is removed, the final planets are stable. We also did three additional runs with the same setting (runD2–runD4) and they show similar results. However, the average mass of the largest planet is $M_{\text{max}} = 3.6 \pm 0.29 M_\oplus$, which is significantly larger than that with $\eta_{\text{ice}} = 3$ (setA). This difference means that the planets mostly consist of migrated icy protoplanets. Because orbital separations must be greater than $\sim 5r_H$ for the planets to be stable and $r_H$ is proportional to $M_1^{1/3}$, the average number of final planets ($\bar{N} = 4.0 \pm 0.71$) is smaller than that in setA.

We also performed runs with $\eta_{\text{ice}} = 14$ that did not include type-I migration (setE; runE1–runE4). The result for runE1 is shown in Figure 8. Before the disk gas removal, the results in setE are similar to those in setB with $\eta_{\text{ice}} = 3$, except for the final maximum mass ($\bar{M}_{\text{max}} = 0.91 \pm 0.064 M_\oplus$) and number ($\bar{N} = 27 \pm 2.3$). In setB, resonant trapping was so efficient that $M_{\text{max}}$ was no other than $M_{\text{crit,mig}}$. In setE, the inner protoplanets cannot halt the migrated protoplanets from outer
regions, total mass of which is 1 order larger than the total mass of inner planets, and mergers and rearrangement occur in the inner regions. This results in the larger $M_{\text{max}}$ and smaller $N$ in setE than in setB.

So far, we have been using $\Sigma \propto r^{-1.5}$. We also did several simulations with less steep radial gradient, $\Sigma \propto r^{-0.5}$. Because of weaker dependence of type-I migration timescale on $r$ corresponding to the weaker $r$-dependence of $\Sigma$, we found that a few protoplanets that are trapped by each other migrate together, which McNeil et al. (2005) called a “convoy.” But, we also found that this feature does not affect the final orbital configurations of close-in planets before disk gas removal and their stability after the disk gas removal. The final orbital configurations depend on only type-I migration speed around 0.1 AU, because it determines the efficiency of resonant trapping.

### 3.4 Composition and Habitability

As stated in Section 1, delivery of icy planetesimals from the regions beyond the ice line is one of likely sources for the $\text{H}_2\text{O}$–water on planetary surface (Morbidelli et al. 2000; Robert 2001). Assuming this scenario, Raymond et al. (2007) suggested through $N$-body simulation neglecting type-I migration that the planets in HZs around M dwarfs are likely to be dry, since radial mixing is inefficient in the lower-mass disks.

Figure 9 shows $\text{H}_2\text{O}$–water mass fraction of the final planets in runA1 (the left panel) and runB1 (the right panel), using the following simple prescription for components of planetesimals that originated at $r$:

$$\frac{M_{\text{water}}}{M} = \frac{\eta_{\text{ice}} - 1}{\eta_{\text{ice}}} = \begin{cases} 0 & [r < r_{\text{ice}}] \\ 0.67 & [r > r_{\text{ice}}]. \end{cases}$$

Note that the initial mass beyond the ice line make up 41% of total mass in our calculation range, following the MMSN model (Equation (6)). The shaded regions in Figure 9 represent analytically estimated HZ (Kasting et al. 1993; Selsis et al. 2007). Note that the positional relationship between the disk inner edge and the HZ is not exact because the locations of disk inner edge are uncertain. In both runA1 and runB1, significant amount of water was delivered by planetary migration. As a result, the final planets are considerably “wet” except for the innermost planets that are shielded by outer planets. Thus, water delivery to the HZ is rather efficient around M dwarfs and the terrestrial planets would be rich in water. Even if the effect of type-I migration is fully retarded, water-rich protoplanets migrate inward by the eccentricity damping due to gravitational drag. The right panel of Figure 9 shows that water-rich planets in HZs may be usually formed for relatively slow migration.

### 3.5 Dependence on Disk Mass

So far, we have adopted $f_g = 1$ for protoplanetary disk in Equation (1) which is relatively large for the disks around $M_* = 0.2 M_\odot$ stars. The reduced computational cost due to the high $f_g$ allows us to carry out large enough number of runs for
the statistical arguments. Here, we discuss how the results can be changed if we consider less-massive disks for M dwarfs with $f_g \approx 0.2$, which may be averaged values (Raymond et al. 2007, adopted $f_g \simeq 0.15$). As described above, the results of N-body simulations are explained well by using the timescales derived in Section 2.3. We discuss results of planetary formation in less-massive disk by applying the timescales for smaller values of $f_g$.

We found that the migration speed regulates final orbital configurations of the close-in terrestrial planets. The migration is slower in less-massive disks. According to Equations (17) and (26), both the timescales of type-I migration and the migration induced from eccentricity damping are inversely proportional to the disk gas scaling factor $f_g$. Thus, in the less-massive disks, the final planets tend to have relatively large separations in non-resonant orbits because of the slower migration.

We found that the terrestrial planets around M dwarfs are generally water–ice rich by the relatively fast migration of icy protoplanets due to the relatively small radius (~0.3 AU) of the ice line. From Equations (24) and (27) with $f_g = f_a = 0.2$, the critical masses for retention against type-I migration and the migration induced from eccentricity damping are given, respectively, by

$$M_{\text{crit,mig}} \simeq 0.17 \left( \frac{\eta_{\text{ice}}}{3} \right)^{3/4} \left( \frac{f_a}{0.2} \right)^{3/4} \left( \frac{f_g}{0.2} \right)^{-9/20} \times \left( \frac{a}{0.3 \text{ AU}} \right)^{-9/10} M_{\oplus},$$

$$M_{\text{crit,damp}} \simeq 3 \left( \frac{\eta_{\text{ice}}}{3} \right)^{3/4} \left( \frac{f_a}{0.2} \right)^{3/4} \left( \frac{f_g}{0.2} \right)^{-9/20} \times \left( \frac{e}{0.01} \right)^{-3/2} \left( \frac{a}{0.3 \text{ AU}} \right)^{-21/40} M_{\oplus}. \quad (31)$$

The isolation mass is (Equation (20))

$$M_{\text{iso}} \simeq 0.24 \left( \frac{\eta_{\text{ice}}}{3} \right)^{3/2} \left( \frac{f_a}{0.2} \right)^{3/2} \left( \frac{a}{0.3 \text{ AU}} \right)^{3/4} M_{\oplus}. \quad (32)$$

Comparison of these masses shows that protoplanets just outside the ice line almost grow to the isolation mass before starting migration. Substituting the isolation mass into Equation (19), we obtain the accretion time as

$$t_{\text{acc}} \simeq 8.0 \times 10^4 \left( \frac{\eta_{\text{ice}}}{3} \right)^{-1/2} \left( \frac{f_a}{0.2} \right)^{-1/2} \left( \frac{f_g}{0.2} \right)^{-2/5} \times \left( \frac{a}{0.3 \text{ AU}} \right)^{59/20} \text{ yr}. \quad (33)$$

Similarly, substituting Equation (33) into Equations (17) and (26), we obtain the migration timescales as

$$t_{\text{mig}} \simeq 2.4 \times 10^4 \left( \frac{\eta_{\text{ice}}}{3} \right)^{-3/2} \left( \frac{f_a}{0.2} \right)^{-3/2} \left( \frac{f_g}{0.2} \right)^{-1} \left( \frac{a}{0.3 \text{ AU}} \right)^{3/4} \text{ yr}, \quad (34)$$

$$t_{\text{damp,a}} \simeq 1.3 \times 10^6 \left( \frac{\eta_{\text{ice}}}{3} \right)^{-3/2} \left( \frac{f_a}{0.2} \right)^{-3/2} \left( \frac{f_g}{0.2} \right)^{-1} \times \left( \frac{e}{0.01} \right)^2 \left( \frac{a}{0.3 \text{ AU}} \right)^{5/4} \text{ yr}, \quad (35)$$

where $t_{\text{mig}}$ is the type-I migration timescale and $t_{\text{damp,a}}$ is the timescale of the $e$-damping induced migration. Both $t_{\text{mig}}$ and $t_{\text{damp}}$ are significantly smaller than disk lifetimes of $\sim 10^6–10^7$ yr even for $f_g = f_d \sim 0.2$. Therefore, our finding that the close-in terrestrial planets around M dwarfs are rather “wet” is still valid for the averaged-mass disks with $f_g = f_d \sim 0.2$ around $M_\star \sim 0.2 M_\odot$.

Note, however, that the upper limit of migration timescale, $t_{\text{damp,a}}$, is comparable to the disk lifetimes. If the type-I migration timescale is elongated by a factor of more than 100, $t_{\text{mig}}$ is also shorter than or comparable to the disk lifetimes. Then, the transfer of water/icy materials by migrations of protoplanets is not efficient enough. If the migration barrier near the ice line (Kretke & Lin 2007; Ida & Lin 2008b; Kretke et al. 2009) is effective also for disks around M dwarfs, the transfer is inefficient irrespective of the reduction factor of type-I migration. Since Raymond et al. (2007) showed that the transfer of water/icy materials by scattering is also inefficient, the planets in HZs are not always rich in water–ice in these cases. Thus, whether the planets in HZs around M dwarfs are habitable or not may be strongly regulated by efficiency of type-I migration.

4. CONCLUSIONS AND DISCUSSION

We have investigated accretion of terrestrial planets from planetesimals around M dwarf stars through a set of N-body simulations, including the effects of disk gas. In general, accretion of terrestrial planets have two stages: runaway/oligarchic growth and following long-term giant impacts. Our simulations cover all the stages from initial 5000 planetesimals to the final planets that are stable for long time after possible giant impacts, fully including gravitational interactions of all the bodies.

Since M dwarfs are fainter than solar-type stars, both the HZs and ice lines are located in the proximity of central stars. Due to the proximity, accretion of terrestrial planets have different features around M dwarf than around solar-type stars that are caused by the three factors:

1. the effective damping by disk gas due to high gas density in inner regions,
2. the influence from inner protoplanets that have migrated toward the disk inner edge,
3. the influence from outer icy protoplanets that migrate into the terrestrial planet regions.

Regarding factor (1), it is noticed that higher disk gas density due to the proximity overwhelms expected smaller disk and planetary isolation masses around M dwarfs. Around M dwarfs, the disk mass and consequently, isolation mass of protoplanets may be generally smaller than those around solar-type stars. However, due to the higher disk gas density, planet–disk tidal interactions, that is, eccentricity (and inclination) damping and type-I migration are more efficient than those at ~1 AU around solar-type stars. Furthermore, accretion timescale in such regions is much shorter than disk lifetime ($\sim 10^6–10^7$ yrs). As a result, the eccentricity damping and type-I migration play important roles in architecture of terrestrial planets around M dwarfs. To highlight this effect, for N-body simulations, we adopted disks comparable to MMSN that may be relatively massive among disks around M dwarfs. Even in the case without type-I migration, the migration induced from eccentricity damping, which is 100 times slower than type-I migration, is still fast enough to bring protoplanets into the
terrestrial planet regions for such disks. The dominance of the effects of disk gas was discussed with analytical arguments (Sections 2.3 and 3.5).

Regarding factor (2), resonant trapping plays an important role. Because of the efficient type-I migration, many protoplanets migrate toward the disk inner edge and accumulate there, usually trapped in mutual mean-motion resonances. We set the disk inner edge to be 0.05 AU, while the HZ is around ~0.1 AU, which is only a factor 2 difference. In the slow migration case, in which resonant trapping is so efficient, trapped protoplanets line up through almost all the simulation regions (0.05–0.4 AU), while with full migration speed predicted by the linear theory, trapping is not so efficient that coagulation often occurs and the final planets tend to be concentrated in inner regions. In the former case, about 40 planets remain in the resonances before the disk gas removal. After the disk gas is removed, the orbits of the planets become unstable and giant impacts occur. As a result, widely spaced (~20r_H), non-resonant, multiple planets are formed with relatively high eccentricities (~0.05) between disk inner edge and outer region. On the other hand, in the full migration case, the trapped planets, the number of which is about 5, are stable even after the disk gas removal and closely packed (~5–10r_H), resonant planets remain in the proximity of the disk inner edge with low eccentricities (~0.01).

Therefore, we conclude that the migration speed is a key factor for final orbital configuration of close-in terrestrial planets around M dwarfs. The close-in planets that ongoing radial velocity surveys have discovered may support the slow migration. The three-planet system around Gl 581 with M_s ≳ 0.3 M_⊙ is composed of planet b (M_p sin i = 16 M_⊕, a = 0.041 AU), c (5 M_⊕, 0.073 AU), and d (7.5 M_⊕, 0.25 AU). They are widely spaced (Δa ∼ 21–47r_H) non-resonant planets that are consistent with our slow migration model, although this system was probably formed from heavier disk than the MMSN disk and may need the enhancement of surface density of icy materials near the ice line. Around solar-type stars, the radius of the ice line is larger, so the factor (3) may not be applied. However, the dependence of final configuration of close-in terrestrial planets on the migration speed can be applied to solar-type stars. The three-planet system around a K dwarf HD 40307 with M_s ≳ 0.77 M_⊙ consists of planet b (M_p sin i = 4.2 M_⊕, a = 0.047 AU), c (6.9 M_⊕, 0.081 AU), and d (9.2 M_⊕, 0.13 AU) (Mayor et al. 2009). They are also widely spaced (Δa ∼ 17–20r_H), non-resonant planets. This configuration is explained by setB in our calculation.

Note that the final state could be altered by other effects which we did not address. We will discuss the effects of random torques exerted by strong disk turbulence due to magneto-rotational instability in a separate paper. Two mechanisms have been proposed which lead to the excitation of eccentricities of the bodies and additional collisions between them: (1) during type-I migration of a gas giant planet, its mean-motion resonances (mainly 2:1 resonance) sweep the terrestrial planet region (e.g., Zhou et al. 2005), or (2) during the dispersal of the gas disk, secular resonances caused by the gas giant planet and the disk sweep through the inner orbits (e.g., Nagasawa et al. 2005). However, since gas giants are generally rare around M dwarfs, these mechanisms do not play an important role for terrestrial planets around M dwarfs.

Factor (3) is caused by the smaller radius of the ice line around M dwarfs. Less efficient planetesimal accretion due to the smaller disk surface density is overwhelmed by the faster accretion due to smaller radii of icy regions (r_H > 0.3 AU) than those around solar-type stars. Combined with factor (1), migration of the icy protoplanets into the terrestrial planet regions is so efficient that the largest final planets have significant mass of icy components except for the innermost one that is shielded from impacts of icy protoplanets. In the case of enhanced surface density of ice near the ice line, the largest final planets are mostly composed of ice but not rocks. It is interesting that the estimated bulk density for a 20 M_⊕ transiting planet, GJ 436b, around a M dwarf is consistent with water–ice (Gillon et al. 2007; Deming et al. 2007). Although the mass and number of final planets are affected by this factor as well as the inner boundary conditions for type-I migration and radial gradient of Σ_c (see Section 3.3), the stability of the resonantly trapped planets after the disk gas removal, that is, orbital separation, resonant or non-resonant configuration, eccentricities of final planets are determined only by migration speed.

For the disks with f_g that is several times smaller than that of our fiducial model (f_g = 1), which may be typical disks around M dwarfs, accretion and migration timescales are still much shorter than disk lifetime, so the formed close-in planets are abundant in water–ice. However, if type-I migration speed is much slower than that predicted by the linear theory or the migration is trapped near the ice line (Kretke & Lin 2007; Ida & Lin 2008b; Kretke et al. 2009), the final close-in terrestrial planets would be rocky due to the inefficient water delivery by the migration, because radial mixing of planetesimals is also inefficient around M dwarfs (Raymond et al. 2007). As Lissauer (2007) pointed out, the ice line evolves during relatively long pre-main-sequence phase of M dwarfs. We need to carry out detailed N-body simulations in low-mass disks, including the evolution of the ice line in a future work, to clarify the details on how wet are the planets in HZs around M dwarfs.

Our results around M dwarfs and calculations by Terquem & Papaloizou (2007) around G dwarfs suggest that existence of close-in relatively large terrestrial planets are robust. However, our solar system does not have any close-in planet inside 0.4 AU and large fraction of extrasolar planetary systems may not have the close-in super-Earths either. We will address this issue elsewhere.

Our conclusion is that the migration speed determines diversity of final orbital configuration of close-in terrestrial planets around M dwarfs through the stability of the planets trapped in mutual mean-motion resonances, so we will study more detailed dependence on the migration speed as well as the dependence on conditions of inner disk regions in a separate paper. Around M dwarfs, these planets could be in HZs. Characteristics of habitable planets around M dwarfs are significantly affected by the details of these formation mechanisms. Although these close-in planets are well inside HZs for solar-type stars, their formation mechanisms may be similar. Future observations of close-in terrestrial planets around M dwarfs as well as solar-type stars by radial velocity surveys from ground and transit surveys from space will constrain the migration efficiency and the quantitative features of inner disk regions.

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APPENDIX A

EXPRESSION FOR AERODYNAMICAL GAS DRAG FORCE

The aerodynamic gas drag force per unit mass is (Adachi et al. 1976)

\[ F_{\text{aero}} = -\frac{1}{2M} C_D \pi b^2 \rho_g \Delta u, \]  
(A1)

where \( C_D = 0.5 \) is the gas drag coefficient, \( r_p \) is the physical radius of the body, and \( M \) is its mass. The density of disk gas \( \rho_g \) is (Hayashi 1981)

\[ \rho_g = 2.0 \times 10^{-9} f_g \left( \frac{r}{1 \text{ AU}} \right)^{-11/4} \text{ g cm}^{-3}, \]  
(A2)

where \( \Delta u \) is the relative velocity of the body to the disk gas. Due to pressure gradient, the velocity of disk gas \( v_{\text{gas}} \) is smaller than Kepler velocity \( v_K \) by a fraction (Adachi et al. 1976)

\[ \eta \simeq \frac{v_K - v_{\text{gas}}}{v_K} = 1.8 \times 10^{-3} \left( \frac{r}{1 \text{ AU}} \right)^{1/2}, \]  
(A3)

where the temperature distribution of an optically thin disk given by Equation (2) is used.

APPENDIX B

EXPRESSION FOR GRAVITATIONAL GAS DRAG FORCE

Tanaka et al. (2002) and Tanaka & Ward (2004) derived the damping forces exerted on the planet, through three-dimensional linear calculation,

\[ F_{\text{damp}, r} = \left( \frac{M}{M_*} \right) \left( \frac{v_K}{c_s} \right)^4 \left( \frac{\Sigma r^2}{M_*} \right) \Omega \left( 2A_r' v_\theta + A_r' \frac{dz}{dr} \right), \]  
(B1)

\[ F_{\text{damp}, \theta} = \left( \frac{M}{M_*} \right) \left( \frac{v_K}{c_s} \right)^4 \left( \frac{\Sigma r^2}{M_*} \right) \Omega \left( 2A_\theta' v_r + A_\theta' z \frac{\Omega}{\Omega_K} \right), \]  
(B2)

\[ F_{\text{damp}, z} = \left( \frac{M}{M_*} \right) \left( \frac{v_K}{c_s} \right)^4 \left( \frac{\Sigma r^2}{M_*} \right) \Omega \left( A_z' v_r + A_z' z \Omega \right), \]  
(B3)

\[ F_{\text{mig}, r} = 0, \]  
(B4)

\[ F_{\text{mig}, \theta} = -2.17 \left( \frac{M}{M_*} \right) \left( \frac{v_K}{c_s} \right)^2 \left( \frac{\Sigma r^2}{M_*} \right) \Omega v_r, \]  
(B5)

\[ F_{\text{mig}, z} = 0, \]  
(B6)

where \( F_{\text{damp}} \) is the specific damping force for \( e \) and \( i \), \( F_{\text{mig}} \) is the specific damping force for \( a \), and \( \Omega \) is the Keplerian angular velocity. The coefficients are given by

\[ A_r' = 0.057 \quad A_\theta' = 0.176 \]  
\[ A_\theta' = -0.868 \quad A_z' = 0.325 \]  
\[ A_z' = -1.088 \quad A_z' = -0.871. \]
Pollack, J. B., Hollenbach, D., Beckwith, S., Simonelli, D. P., Roush, T., & Fong, W. 1994, ApJ, 421, 615
Raymond, S. N., Scalo, J., & Meadows, V. S. 2007, ApJ, 669, 606
Robert, F. 2001, Science, 293, 1056
Scalo, J., et al. 2007, Astrobiology, 7, 85
Selsis, F., Kasting, J. F., Levrard, B., Paillet, J., Ribas, I., & Delfosse, X. 2007, A&A, 476, 1373
Stevenson, D. J., & Lunine, J. I. 1988, Icarus, 75, 146
Tanaka, H., Takeuchi, T., & Ward, W. R. 2002, ApJ, 565, 1257
Tanaka, H., & Ward, W. R. 2004, ApJ, 602, 388
Terquem, C., & Papaloizou, J. C. B. 2007, ApJ, 654, 1110
Udry, S., et al. 2007, A&A, 469, L43
Ward, W. R. 1986, Icarus, 67, 164
Ward, W. R. 1993, Icarus, 106, 274
Zhou, J.-L., Aarseth, S. J., Lin, D. N. C., & Nagasawa, M. 2005, ApJ, 631, L85