The problem of phaseless aperture synthesis is of current interest in phase-unstable VLBI with a small number of elements when either the use of closure phases is not possible (a two-element interferometer) or their quality and number are not enough for acceptable image reconstruction by standard adaptive calibration methods. Therefore, we discuss the problem of unique image reconstruction only from the spectrum magnitude of a source. We suggest an efficient method for phaseless VLBI mapping of compact extragalactic radio sources. This method is based on the reconstruction of the spectrum magnitude for a source on the entire UV-plane from the measured visibility magnitude on a limited set of points and the reconstruction of the sought-for image of the source by Fienup’s method from the spectrum magnitude reconstructed at the first stage. We present the results of our mapping of the extragalactic radio source 2200+420 using astrometric and geodetic observations on a global VLBI array. Particular attention is given to studying the capabilities of a two-element interferometer in connection with the putting into operation of a Russian-made radio interferometer based on Quasar RT-32 radio telescopes.

Key words: astronomical observing techniques, devices and instruments.
INTRODUCTION

The situation with uncertainty in measuring the phase due to ionospheric and tropospheric inhomogeneity is typical of phase-unstable interferometry. Phase errors, let alone the fact that the phase is unknown, significantly restrict the signal and image processing quality (Oppenheim and Lim 1981). Many studies that are systematized in the monographs by Stark (1987) and Thompson et al. (1986) are devoted to the influence of phase errors.

In VLBI, adaptive calibration methods, including hybrid and self-calibration methods that directly or indirectly use closure phases (Cornwell and Fomalont 1999; Thompson et al. 1986), are traditionally used to solve the phase retrieval problem. The number of equations for closure phases depends on the number of interferometer elements $N$ and is $(N - 1)(N - 2)/2$ at each instant in time. The larger the number of interferometer elements and the more the visibility measurements on each baseline, the higher the phase retrieval accuracy. In contrast, at a small number of elements and a small number of measurements, the number of equations for closure phases may prove to be insufficient for high-quality phase retrieval. For a two-element ($N = 2$) interferometer, there are no equations for closure phases at all.

Phase retrieval errors can give rise to spurious features. Thus, for compact extragalactic radio sources with a typical "core+jet" structure, a spurious symmetric counterjet can appear on the map. We encounter such a situation, for example, when mapping using astrometric and geodetic observations on a global VLBI array (IRIS, NEOS, etc.), as these are distinguished by relatively poor UV coverage (Bajkova et al. 1997).

The necessity of mapping based on astrometric data arises from the need to improve the coordinates of reference sources by taking into account their structure on milliarcsecond (mas) angular scales (Bajkova 2002a). Clearly, because of their spectral and temporal variability, the mapping of sources and the astrometric reduction should be performed using the same data. In addition, since the observations are regular (each source has been observed once a week for many years), they are also of considerable interest in astrophysics, providing unique data for investigating the structural evolution of extragalactic sources on long time scales (Pyatunina et al. 1998).

Thus, apart from adaptive calibration methods, phaseless mapping methods using only the visibility magnitude can also be invoked to overcome the phase uncertainty in VLBI, provided that the signal-to-noise is high enough for the application of reconstruction algorithms (see the section titled Reconstruction Accuracy).

Interest in phaseless VLBI mapping is also being aroused by the fact that the Institute of Applied Astronomy (Russian Academy of Sciences) has recently put into operation a two-element interferometer based on RT-32 radio telescopes of the Quasar VLBI project at the Svetloe Observatory near St.-Petersburg and at the Zelenchuk Observatory in the North Caucasus and that the first preliminary mapping results have already been obtained (Bajkova 2002b).

In this paper, we present the results of our study of the capabilities of a two-element interferometer designed to map sources that can be represented as a set of compact components. The results of this study can also be applied to ground-based-spaceborne radio interferometers with the high orbit of a space station (Finkelstein and Bajkova 1990).

The problem of aperture image synthesis without invoking phase information was first
considered and solved by Baldwin and Warner (1976, 1978). However, these authors suggested methods that were applicable only in special cases where a source could be represented as a limited number of point components and, therefore, were not widely used in VLBI.

Here, our goal is to develop a more universal and efficient phaseless image synthesis method that can be used for the VLBI mapping of extragalactic radio sources, including those with a “core+jet” structure, that, apart from compact components (core), also contain extended components (jet); to study the capabilities of a two-element interferometer; and to demonstrate the potentialities of the suggested methods by mapping the well-known source 2200+420 using astrometric and geodetic observations on a global VLBI array as an example.

THE UNIQUENESS OF THE SOLUTION

In the most general formulation where constraints are imposed only on the spectrum magnitude, the problem of reconstructing the function has an infinite set of solutions. Indeed, any function that has a given spectrum magnitude and an arbitrary spectral phase satisfies these constraints, and, if at least one solution is known, the other can be obtained by convolving this solution with a function that has an arbitrary phase and a spectrum magnitude equal to unity at all frequencies.

However, a significant narrowing of the set of solutions is possible for certain constraints imposed on the function being reconstructed in the spatial domain (Stark 1987). One of these is the constraint imposed on the spatial extent of an object; i.e., the sought-for function must have a finite carrier. Another severe constraint in the spatial domain is the requirement that the solution be real and nonnegative. Below, in solving the phase retrieval problem, we assume that the sought-for function satisfies these constraints; i.e., it is real and nonnegative and has a finite extent.

The finite extent of an object (the finiteness of the function) ensures that the Fourier spectrum is analytic in accordance with the Wiener-Paley theorem (Khurgin and Yakovlev 1971). As a result, this spectrum can be reconstructed from the known part of it, which is used to reconstruct images from the visibility function measured on a limited set of points in the UV plane. If we determine the class of equivalent functions to within a linear shift and reversal of the argument (rotation through 180°), then all of the functions that belong to this class have the same spectrum magnitude. The solution of the phase retrieval problem is assumed to be unique if it was determined to within the class of equivalent functions.

In the case of one-dimensional functions, even these severe constraints on finiteness and nonnegativity do not guarantee a unique reconstruction from the spectrum magnitude. As the dimensionality of the function increases ($n \geq 2$), a unique (to within the class of equivalent functions) solution becomes possible, except for the degenerate cases defined on the set of measure zero (Bruck and Sodin 1979; Hayes 1982). This follows from the qualitative difference between the properties of the $z$-transformations of one-dimensional and multidimensional sequences.

For a unique solution to exist, the $z$-transformation must be irreducible, which is not achievable in principle in the one-dimensional case and almost always holds in the multidimensional case. Since we deal with two-dimensional images in VLBI, we assume that the solution of the phase retrieval problem exists and is unique. However, the existence of a unique solution does not yet guarantee that the retrieval algorithms converge. The papers
by Gerchberg and Saxton (1972) and Fienup (1978) are of greatest importance in developing the theory and algorithms of solving the phase retrieval problem. Fienup’s algorithm and its modifications aimed at speeding up the convergence to the required solution (Fienup 1982) are most efficient in terms of their applications. Based on numerous simulations, we choose a hybrid input-output algorithm modified to ensure convergence (by specifying the initial approximation) and to prevent stagnation due to computational effects from the entire set of various modifications of Fienup’s algorithm (Bajkova 1994, 1996).

A MODEL FOR THE STRUCTURE OF COMPACT EXTRAGALACTIC RADIO SOURCES

Before turning to a description of the suggested phaseless mapping method, let us consider simplified models for the structure of compact extragalactic radio sources and their spectrum magnitudes. The simplest approximation of a “core+jet” source structure is a model that consists of two point components. This model will be useful below when considering a two-element interferometer. Let the brightest component be located at the phase center of the map and be the core of the source (the optically thick base of the jet) and the second component (the optically thinner feature of the jet) have a lower brightness and be located at an angular distance of $r$ mas with the position angle $\theta$ reckoned from north to south through east (clockwise). We will consider the relative brightnesses by assuming the core brightness to be equal to unity. Let the brightness of the second component be $A < 1$.

The complex visibility function of such a source when its individual components are represented as $\delta$-functions is

$$V_1(u, v) = 1 + A \exp(2\pi i (u\xi_o + v\eta_o)),$$

where $\xi_o = r \sin \theta, \eta_o = r \cos \theta$ are the rectangular coordinates of the source.

The square of the visibility magnitude is

$$|V_1(u, v)|^2 = 1 + A^2 + 2A \cos \phi,$$

where $\phi = 2\pi (u\xi_o + v\eta_o)$.

Clearly, since $\cos \phi$ is even, the source that is symmetrical to the original source relative to the map center will have the same visibility magnitude. Therefore, image reconstruction from the visibility magnitude alone is possible to within rotation through 180°.

For a symmetrical source with brightness $B$ of the symmetrical components and coordinates $(\xi_o, \eta_o), (-\xi_o, -\eta_o)$, the visibility magnitude is

$$|V_2(u, v)| = |1 + 2B \cos \phi|.$$

Clearly, function (2) has singularities at zeros, and it cannot have an analytic continuation to the entire complex plane. In this case, the condition for the spectrum magnitude being analytic (the existence of all derivatives) is satisfied for $B < 1/2$, because the following representation is valid:

$$|V_2(u, v)| = 1 + 2B \cos \phi.$$
Below, we are concerned only with the cases where the spectrum magnitude of the source is an analytic function; this is achieved only when the flux from the central component dominates over that from the remaining components of the source. This condition is almost always satisfied for real sources. A deviation from this condition is possible only in the case of baseline-by-baseline data editing.

The square of the visibility magnitude is

\[ |V_2(u, v)|^2 = 1 + 4B \cos \phi + 4B^2 \cos^2 \phi. \]  (4)

The maximum and minimum values of (2) and (4) are reached at \( \cos \phi = 1 \) and \( \cos \phi = -1 \), respectively. For \( A = 2B \), the minima and maxima of (2) and (4) coincide. The difference between the squares of the visibility magnitudes for the two-component and the corresponding three-component sources under the condition \( A = 2B \) is

\[ \Delta(V^2(u, v)) = A^2 - A^2 \cos^2 \phi = A^2 \sin^2 \phi. \]

Clearly, a symmetrical object can be distinguished from an asymmetric object only due to this difference; the higher the relative brightness of the jet component \( A \), the more reliable the uniqueness of the solution.

Let us now complicate the source model to a three-component one with relative brightnesses \( A_1 \) and \( A_2 \) of the compact jet features and coordinates \((\xi_1, \eta_1)\) and \((\xi_2, \eta_2)\), respectively. The square of the visibility magnitude for this model can be represented as the sum

\[ |V(u, v)|^2 = 1 + (A_1^2 + 2A_1 \cos \phi_1) + (A_2^2 + 2A_2 \cos \phi_2) + 2A_1A_2 \cos(\phi_1 - \phi_2), \]  (5)

where \( \phi_1 = 2\pi(u\xi_1 + v\eta_1) \), \( \phi_2 = 2\pi(u\xi_2 + v\eta_2) \).

The first term on the right-hand side of (5) represents the core, the second term represents the first component of the jet, the third term represents the second component of the jet, and the fourth term represents the mutual orientation of the second and third components that gives rise to additional components in the autocorrelation function with coordinates equal to the difference between the coordinates of the first and the second components of the jet.

It is easy to show that in the general case of an \( N \)-component source,

\[ |V(u, v)|^2 = 1 + \sum_{i=1}^{N-1} A_i^2 + 2 \sum_{i=1}^{N-1} A_i \cos \phi_i + 2 \sum_{i=1}^{N-2} \sum_{j=i+1}^{N-1} A_iA_j \cos(\phi_i - \phi_j), \]  (6)

where \( \phi_i = 2\pi(u\xi_i + v\eta_i) \).

In the case of extended features, the source is represented as a set of point sources specified in each pixel. The visibility function of the corresponding symmetrical (relative to the center) source with relative brightnesses \( B_i = A_i/2 \) of the components is

\[ |V(u, v)| = 1 + \sum_{i=1}^{N-1} 2B_i \cos \phi_i. \]  (7)

Discarding the terms of the second order of smallness from (6) for \( A_i \ll 1 \) (which is valid for most of the compact radio sources with the core much brighter than the components of
the jet), we can roughly represent the visibility magnitude when the flux from the central component dominates (for $\sum_{i=1}^{N-1} A_i < 1$) as

$$|V(u, v)| \approx 1 + \sum_{i=1}^{N-1} A_i \cos \phi_i. \quad (8)$$

As we see, expressions (7) and (8) are identical, to within the discarded terms of the higher order of smallness. These terms are important in providing unique reconstruction of the mutual orientation of the jet components and in suppressing the symmetric counterjet that is present on the intermediate map with a zero spectral phase (see the next section).

As follows from expression (8), to the first approximation, each feature of an $N$-component source introduces a harmonic to the visibility magnitude whose amplitude, frequency, and phase are determined by the brightness, the distance from the center, and the position angle of the component, respectively.

**DESCRIPTION OF THE METHOD**

The algorithms for image reconstruction from the spectrum magnitude of an object, the most efficient of which are Fienup’s algorithm and its various modifications, require knowledge of the entire input two-dimensional sequence of sampled points. In VLBI, however, the visibility function is measured only on a limited set of points in the $UV$-plane, revealing large unfilled areas and a diffraction-limited constraint. Therefore, prior reconstruction of the visibility magnitude on the entire $UV$-plane from a limited data set is required to successfully use existing reconstruction algorithms like Fienup’s algorithm.

Thus, the suggested phaseless aperture synthesis method consists of the following steps: (1) prior reconstruction of the visibility magnitude (the object’s spectrum) on the entire $UV$-plane, and (2) reconstruction of the sought-for image using Fienup’s algorithm or its modifications (Fienup 1978, 1982; Bajkova 1994, 1996) from the spectrum magnitude reconstructed at the first step of the method. The first step is performed through the reconstruction of an intermediate image that satisfies the measured visibility magnitude and a zero phase. Clearly, the intermediate image is symmetric relative to the phase center of the map. The Fourier transform of the image obtained yields the spectrum magnitude of the source extrapolated to the region of the $UV$-plane where there are no measurements.

Recall that Fienup’s algorithm is an iterative process of the passage from the spatial domain of an object to the spatial frequency domain and back (using the direct and inverse Fourier transforms) in an effort to use the input constraints on the spectrum magnitude of the object in the frequency domain and the constraints on its nonnegativity and finite extent in the spatial domain. The finite carrier within which the source is expected to be localized should be specified in the form of a rectangle centered at the coordinate origin, because the structure of the source may turn out to be symmetric. Fienup et al. (1982) estimated the size of the carrier from the autocorrelation function equal to the inverse Fourier transform of the square of the spectrum magnitude for the object. Very important advantages of Fienup’s algorithm are its high stability against noise (Sanz and Huang 1983) (stability means that a small change in input data causes the solution to change only slightly) and high speed (due to the application of fast Fourier transform algorithms) compared to other algorithms. The various modifications of Fienup’s algorithm aimed at increasing the reliability of its
convergence were also developed by Bajkova (1994, 1996). These papers are devoted to the synthesis of
the initial approximation that increases the reliability of the algorithm convergence and to the prevention of
the algorithm stagnation due to computational effects.

The intermediate image can be reconstructed from the visibility magnitude by the standard method of
analytic continuation of the spectrum using the non-linear CLEAN deconvolution procedures or the
maximum entropy method (MEM) (Cornwell et al. 1999). Therefore, an important requirement for the
visibility magnitude is its analyticity. As we noted in the previous section, this condition is satisfied for
most of the compact extragalactic radio sources if the flux from the central compact component dominates
over the flux from the remaining fainter components.

Note an important point concerning the use of the maximum entropy method. In contrast to the
CLEAN method: the solution based on the standard MEM is strictly positive, while the sought-for
intermediate image with a zero spectral phase is generally alternating, taking on both positive and
negative values; in addition, it is characterized by a wider effective carrier. For clarity, let us show this
using the simple two-component model as an example.

Let us represent the visibility magnitude (1) for a two-component source as the following
expansion into a series:

\[ |V_1(u, v)| = \sqrt{1 + 2A \cos \phi + A^2} = \sqrt{(1 + A \cos \phi)^2 + A^2 \sin^2 \phi} = \]

\[ = (1 + A \cos \phi) \sqrt{1 + x^2} = (1 + A \cos \phi)(1 + \frac{x^2}{2} - \frac{1}{4} \cdot \frac{1}{2} \cdot \frac{1}{4} \cdot \frac{3}{6} x^6 - \frac{1}{2} \cdot \frac{1}{4} \cdot \frac{3}{6} \cdot \frac{5}{8} x^8 + ...), \]

where \( x^2 = A^2 \sin^2 \phi/(1 + A \cos \phi)^2 < 1. \)

Disregarding the terms higher than the fourth order of smallness and expanding \( 1/(1 + A \cos \phi) \) into a
Taylor series, we obtain the following representation in the form of an infinite trigonometric series with
rapidly decreasing terms:

\[ |V_1(u, v)| \approx (1 + A \cos \phi) + \frac{A^2}{2} \sin^2 \phi(1 - A \cos \phi + A^2 \cos^2 \phi - A^3 \cos^3 \phi + ...) = \]

\[ = (1 + \frac{A^2}{4} + \frac{A^4}{16}) + (A - \frac{A^3}{8} - \frac{A^5}{16}) \cos \phi - \frac{A^2}{4} \cos 2\phi + \]

\[ + \left( \frac{A^3}{8} + \frac{A^5}{32} \right) \cos 3\phi - \frac{A^4}{16} \cos 4\phi + \frac{A^5}{32} \cos 5\phi - .... \] (9)

Clearly, each coefficient of the cosine in expansion (9) with a multiple argument \( n\phi \) in the
spatial domain is twice the amplitude of the point component and the component symmetric to it (relative
to the center) with coordinates \((n\xi, n\eta)\) and \((-n\xi, -n\eta)\), respectively (see also (3)).
The coefficients of the cosines with odd and even arguments are positive and negative, respectively,
suggesting that the image is alternating.

It is easy to show that, in general, the intermediate image that corresponds to the spectrum magnitude of
a finite object and a zero spectral phase is also an alternating function with a theoretically unbounded
carrier (since \( n = \infty \)). In this case, the effective carrier of
the intermediate image is much wider than the carrier of the object, which must be properly taken into account when reconstructing the intermediate image.

Thus, the positive definiteness of the intermediate image is indicative of a zero phase and, hence, a symmetry of the object, while the alternating property is indicative of a nonzero phase and, accordingly, an asymmetry of the object relative to the coordinate origin. Therefore, in general, since we have no prior knowledge of whether the sought-for source is symmetric, it would be improper to use the standard MEM with positive output to reconstruct the intermediate image. A MEM modification suitable for obtaining both positive and alternating solutions is required. A generalized MEM (GMEM) suitable for reconstructing functions of any form was developed by Bajkova (1992). In addition, in contrast to the MEM, the GMEM yields an unbiased solution and is more stable against noise in the data (Bajkova 2000).

It should be emphasized that the combination of the alternating property and spatial unboundedness of the intermediate image, on the one hand, and the constraints on the nonnegativity and finite sizes of the object carrier, on the other hand, ensure that the reconstruction of the sought-for image is unique.

Below, we present simulation results that confirm our conclusions. Let us turn to Fig.1, which shows the following: (a) the source model that consists of two Gaussian components (the central component represents the core, and the second fainter and extended component represents the jet); (b) the ”dirty” image that corresponds to incomplete UV-coverage whose Fourier transform is equal to the visibility magnitude at the measured points and to zero at the points without measurements (the visibility function was generated with a signal-to-noise ratio of \( \approx 10 \)); (c) the intermediate image with a zero spectral phase reconstructed from the measured visibility function using the standard MEM with positive output; (d) the intermediate image with a zero spectral phase reconstructed using the GMEM with alternating output; (e) and (f) the images reconstructed using Fienup’s algorithm from the previous MEM and GMEM images, respectively. On all maps, the lower level of the contour line corresponds to 0.2% of the peak value.

As we see from the maps, the GMEM properly intermediate-reconstructed the intermediate image with both positive and negative values, which subsequently allowed us to accurately reconstruct the structure of the source and, hence, its spectral phase (to within rotation through 180°) using Fienup’s algorithm. In contrast, the standard MEM algorithm with positive output yielded no desirable result: at the output of Fienup’s algorithm, the structure of the source was found to be nearly symmetric (the spectral phase was virtually unreconstructed), which is in conflict with the original model (a). Thus, at the first step of our mapping method, only algorithms with alternating output, either CLEAN or GMEM, should be used to reconstruct the intermediate image with a zero spectral phase.

An advantage of the suggested phaseless mapping method over the methods by Baldwin and Warner (1976,1978) is the possibility of reconstructing sources that can be represented not only as a finite set of point components, but also sources with extended features, which makes the suggested method more efficient.

THE RECONSTRUCTION ACCURACY

For a given reconstruction method, the accuracy of the images being obtained depends
on the following three major factors: (1) the structure of the source being mapped, (2) the UV filling (information on the various spatial frequencies of the source), and (3) the accuracy of the input data.

Clearly, the simpler the structure of a source at a given resolution (e.g., several (2 or 3) compact components with a low dynamic range), the less stringent the requirements for the UV filling and the signal-to-noise ratio of the measurements. The more complex the structure of a source (the presence of extended features against the background of compact features, a large number of components, and a high dynamic range), the more complete the filling of the spatial frequency domain (the presence of a low-frequency domain) and the higher the signal-to-noise ratio. General analytical expressions for estimating the image reconstruction accuracy are very difficult to derive by nonlinear methods, because the nonlinearity of the methods leads to nonlinear image distortions. Therefore, it would be appropriate to study the influence of various factors on the reconstruction accuracy by mathematical simulation for a large number of sources with various degrees of complexity of their structure, various UV fillings, and various specified data accuracies.

The accuracy of reconstructing the sought-for image by Fienup’s method depends significantly on the accuracy of reconstructing the intermediate image or, in other words, on the spectrum magnitude of the sought-for image over the entire spatial frequency domain. Although Fienup’s algorithms are highly stable against noise, significant distortions of the input data may lead to unpredictable nonlinear image distortions (the coordinates and amplitudes of the source’s components), because the spectrum is no longer analytic. Distortions arise, because the spatial boundaries specified in Fienup’s algorithm within which the solution is sought cease to correspond to the image being reconstructed. In these cases, the sizes of the carrier should be increased to reduce the distortions of the coordinates of the components. Our simulations of Fienup’s hybrid input-output algorithms when the structure of sources is approximated by simple models show that the minimum signal-to-noise ratio of the input sampled points of the spectrum magnitude that does noticeably distort the coordinates of the compact components of the source, when the coordinates of the peak values of the components are taken as their coordinates, is about 5. In this case, the errors in the data were simulated as additive noise with a uniform distribution.

To achieve the required data quality at the output of the first step of the phaseless mapping method for typical “core+jet” structures of compact extra-galactic radio sources and the UV filling obtained on VLBI arrays with a small number of elements (the cases that are the subject of discussion in this paper), the visibility magnitude must be measured with a signal-to-noise ratio of no less than 10. This accuracy is needed to reconstruct not only the positively defined, but also the negatively defined components of the intermediate image (see (9) for a two-component source), which are relatively small, but play a major role in suppressing the counterjets. As a result, we reconstruct the amplitudes of the two or three brightest components of the source with an accuracy of no less than 10% without any apparent distortions of their coordinates (for the mapping results of the source 2200+420, see below) and with satisfactory suppression of the symmetric counterjet. This is the minimum sufficient condition for solving the restricted problem of studying the structural evolution of compact extragalactic radio sources using interferometers with a small number of elements. Under the conditions of this problem, the situation where the coordinates of the components would be noticeably and unpredictably distorted because of the nonlinearity of the reconstruction algorithms is totally unacceptable. In reality, the visibility magnitude in VLBI can
also be measured with a higher signal-to-noise ratio. Therefore, below, we consider examples of image reconstruction where the data are generated with a signal-to-noise ratio of no less than 10. When the real data are processed, the sampled points of the visibility function with unacceptable noise parameters can always be discarded during prior data editing.

**PHASELESS MAPPING OF THE SOURCE 2200+420 USING MULTIBASELINE OBSERVATIONS**

The goal of this section is to demonstrate the potentialities of phaseless mapping using the well-known radio source 2200+420 (BL Lacertae), which exhibits several compact bright features on milliarcsecond angular scales, as an example. Parameters of this source can be found in the NASA/IPAC Extragalactic Database (NED) [25]. This source belongs to the class of BL Lacertae objects, being its brightest representative. It exhibits long-period variability of both flux and structure. It has been studied quite well. The structure of the source can be judged, for example, by the VLBA maps [25] obtained over the period 1996–2000 from 15-GHz (\( \lambda = 2 \text{ cm} \)) observations.

We constructed five maps over the period 1996-2000 using the observations of the International Astrometric and Geodetic Programs (NEOS) on the global International VLBI Array at a frequency of 8.2 GHz (\( \lambda = 3.5 \text{ cm} \)).

The mapping was performed by using two independent packages: QUASAR VLBImager developed by the author at the Institute of Applied Astronomy (Russian Academy of Sciences) and CalTech’s DIFMAP.

The parameters of the observations and the synthesized maps are given in the table. It lists the dates and frequencies of the observations, the global VLBI stations involved in the experiment, the number of measurements, and parameters of the maps (peak fluxes and the parameters of the Gaussian beam with which the solution was convolved).

The maps obtained by the phaseless method (QUASAR VLBImager) described above and by the self-calibration method in terms of differential mapping (DIFMAP) are shown in Fig.2. In these maps and those shown below, the minimum level of the contour line corresponds to 1% of the peak value.

Let us turn to the figure. The images along the rows pertain to different dates of observations. The first column ((a) – (e)) gives the UV fillings; the second column ((f) – (j)) gives the intermediate images obtained from a given visibility magnitude and a zero spectral phase (some of them ((f), (i)) were obtained by using the GMEM, while others ((g), (h), (j)) were obtained by the CLEAN method); and the third column ((k) – (o)) gives the sought-for images reconstructed from the previous images by using Fienup’s algorithm. The images obtained show the structure of the source that is immediately adjacent to the core (within 2 to 3 mas) and allows its evolution with time to be traced. We see individual components of the jet at position angles in the range (-170°, -180°) whose brightnesses and positions change from date to date. Our maps qualitatively agree with the 2-cm VLBA maps see [26]).

The fourth column of Fig.2 ((p) – (t)) gives the maps obtained by the adaptive calibration method using the DIFMAP package (the scale of these maps is half the scale of the previous ones).

Analysis of images ((p) – (t)) shows that it is not always justifiable to use equations for closure phases. Thus, for example, we see a symmetric counterjet on maps (p) and (q), which,
as follows from the higher-visibility quality astrophysical VLBA maps (Fig. 2), should not be there. The presence of a counterjet is indicative of incomplete spectral phase retrieval, which shows the spurious quasi-symmetry of the source. As a result, the strong component of the jet was not fully reconstructed; it broke down into two parts, distributing the brightness between them. Thus, here, we clearly show a case where using unreliable phase information may prove to be more dangerous than its retrieval from the visibility magnitude measured with a sufficient accuracy.

A comparison (given the beam size) of maps \((m)\) and \((n)\) with maps \((r)\) and \((s)\), respectively, shows good agreement (see also the peak fluxes in the table), implying that the visibility function was measured reliably in these cases. A comparison of map \((o)\) with map \((t)\) again argues for the phaseless mapping that resolved not one, but two components of the jet present on the maps for the two previous dates.

### Table. Parameters of the observations and the synthesized maps for the source 2200 +420

| Date dd/mm/yy | Frequency (MHz) | VLBI stations | Number of UV-points | Package "QVImager" Peak flux Jy/beam | FWHM Jy/beam | Package "DIFMAP" Peak flux Jy/beam | FWHM Jy/beam |
|--------------|-----------------|---------------|---------------------|-------------------------------------|--------------|-------------------------------------|--------------|
| 26/03/96     | 8210.99         | GKWNyN20F     | 103                 | 1.62 \(0.63 \times 0.63\)           | 1.30 \(0.56 \times 0.53\) 34.3° | 1.57 \(0.51 \times 0.57\) -1.6° |
| 03/09/96     | 8182.99         | GKN20WFNy     | 127                 | 1.48 \(0.63 \times 0.63\)           | 1.82 \(0.55 \times 0.54\) 40.4° |  |
| 12/11/96     | 8182.99         | FGN20WKNy     | 100                 | 1.79 \(0.63 \times 0.63\)           |  |
| 26/08/97     | 8182.99         | KWAN20y       | 83                  | 0.72 \(0.63 \times 0.63\)           | 0.89 \(0.80 \times 0.51\) -33.0° |  |
| 06/10/98     | 8182.99         | KN20GyWFyN     | 115                 | 1.31 \(0.63 \times 0.63\)           | 1.04 \(0.56 \times 0.43\) -15.3° |  |
| 26/03/96     | 8210.99         | GK             | 21                  | 1.10 \(0.63 \times 0.63\)           |  |  |

Abbreviated names of the stations: \(A\) – Algopark, \(F\) – Fortleza, \(G\) – Gilcreek, \(G_g\) – GGAO7108, \(K\) – Kokee, \(N_{20}\) – NRAO20, \(N_y\) – NyAlesund, \(W\) – Wettzell.

The latter result can be explained as follows. If the visibility magnitude has been measured with a sufficiently high accuracy, then the intermediate image of the source with a zero spectral phase will contain all of the structural features and their mirror features relative to the phase center of the map. The accuracy of reconstructing the coordinates of the source’s components depends only on the accuracy of measuring the visibility magnitude. The subsequent image reconstruction using Fienup’s algorithm leads to an approximately twofold enhancement of the structural components with correct coordinates, as we see from a comparison of the maps in the middle and the last columns of Fig. 2. In contrast, using distorted or insufficient phase information in adaptive calibration methods may lead to an unpredictable distortion of the source’s structure, both the brightness and the coordinates of its individual components. In this case, the reconstruction accuracy depends not only on the accuracy of the visibility magnitude, but also on the degree of distortion of the phase information.

**Mapping of Compact Radio Sources**
ON A TWO-ELEMENT INTERFEROMETER

During the Earth’s rotation, the interferometer baseline vector describes an ellipse whose coordinates on the UV-plane are defined by the equation (Thompson et al. 1986)

\[
\frac{u^2}{(L_x^2 + L_y^2)} + \frac{(v - v_o)^2}{(L_x^2 + L_y^2) \sin^2 \delta_o} = 1,
\]

(10)

where \(L_x, L_y\) – are the components of the baseline vector \(P_{ij}(u, v, w)\) along the equatorial coordinate axes, \(\delta_o\) is the declination of the phase center of the source, and \(v_o = w \cos \delta_o\). Below, we assume that the coordinates \(u, v, w\) are measured in units of wave-lengths.

In investigating a single-baseline interferometer for mapping, it would be appropriate to use the approximation of a model source by a set of point ponents. This approximation is justifiable, because, in this case, the spectrum magnitude is extrapolated to higher frequencies from the values specified only on one curve (10); image reconstruction in the form of a set of very compact features in the limit of point components corresponds to this case. Therefore, sources with this type of structure are most suitable for observations on a two-element interferometer. If, however, the source at a given resolution of the instrument reveals extended features, they will be represented in the reconstructed maps as compact components with coordinates at the points of maximum brightness.

Clearly, the parameters \(A, r, \theta\) uniquely, to within rotation through \(180^\circ\), determine the one-dimensional visibility function on curve (10). Let us illustrate how variations of the source’s parameters affect the form of the visibility magnitude using a specific example.

In Fig. 3, the left column shows the images of model sources with the following parameters of the compact component of the jet:

(a) \(A = 0.4, \ r = 6.0\ \text{mas}, \ \theta = 38^\circ\),
(b) \(A = 0.1, \ r = 6.0\ \text{mas}, \ \theta = 38^\circ\),
(c) \(A = 0.4, \ r = 2.0\ \text{mas}, \ \theta = 38^\circ\),
(d) \(A = 0.4, \ r = 6.0\ \text{mas}, \ \theta = 68^\circ\),
(e) \(B = 0.2, \ r = 6.0\ \text{mas}, \ \theta_1 = 38^\circ, \ \theta_2 = 218^\circ\).

All of the changes in the form of the visibility magnitude determined on the Svetloe-Zelenchuk baseline for a source with a declination of \(\delta = 73.5^\circ\) are shown in Fig. 4. The corresponding UV-coverage is shown in Fig. 5(a).

In all panels of Fig. 4, the pluses indicate the visibility magnitude of the reference source (time is plotted along the horizontal axis) shown in Fig. 3a; the crosses indicate the visibility magnitude of the source with a changed value of a particular parameter. Thus, we can see from Figs. (3a – 3d) how changes in brightness \(A\) of the component, in distance \(r\) from the map center, in position angle \(\theta\), and the separation of the component into two symmetric parts with equal brightnesses, respectively, affect the visibility function.

More specifically, a decrease in the brightness of the component causes a decrease in the modulation of the visibility magnitude; a decrease in the distance of the component from the center causes a decrease in the modulation frequency; a change in the position angle causes a linear shift in the visibility function with time; and the separation of the jet component into two symmetric parts with equal brightnesses causes an increase in the visibility magnitude by approximately \(A^2 \sin^2 \phi/2(1 + A \cos \phi)^2\).
Let us now present the results of our reconstruction of the two-component sources shown in Fig. 3 from visibility magnitude data generated with a signal-to-noise ratio of \( \approx 10 \) on the Svetloe-Zelenchuk baseline. The right column of Fig. 3 shows the final images reconstructed by using the GMEM and Fienup’s algorithm. Analysis of our results indicates that the brightnesses of the components were reconstructed with an accuracy up to 10\%, and the coordinates of the components were reconstructed without apparent distortions.

Thus, for two-component sources, mapping using single-baseline data yields a reconstruction quality that may well be acceptable for solving a number of problems. One of these problems might be, for example, investigation of the motion of the brightest components of a source on long time scales; another problem can be allowance for the influence of the structure in astrometric reduction, because the components that are brightest and farthest from the core introduce the largest error in determining the coordinates of reference sources (Bajkova 2002a).

Clearly, the more complete and accurate the determination of the visibility magnitude on an interferometer baseline, the higher the accuracy of reconstructing the structure of a source by using the methods of the analytic continuation of the spectrum. In practice, the reconstruction problem is solved with a limited accuracy, because the data are discrete and contain measurement errors, causing the necessary analyticity condition to be violated. Fortunately, however, as we showed above, acceptable image estimates can be obtained by using reconstruction methods (GMEM, Fienup’s algorithm) that are stable against noise. In this case, the brightest components are reconstructed reliably. Thus, data with a signal-to-noise ratio of \( \approx 10 \) yield maps that generally have two or three very bright, well-reconstructed components.

Below, we present the results of yet another simulation (Fig. 5) that confirm our conclusions. Figure 5a shows the diurnal UV-coverage that corresponds to the Svetloe-Zelenchuk interferometer and the declination of the source 0212+735. Figure 5(b) shows the four-component model of 0212+735 that was roughly estimated from existing VLBA maps (see footnote 2). Figure 5(c) shows the intermediate GMEM map reconstructed from the visibility magnitude that was measured with a signal-to-noise ratio of \( \approx 10 \). Figure 5(d) presents the final reconstruction result obtained by Fienup’s method. A comparison of the original model (Fig. 5(b)) and the reconstructed map (Fig. 5(d)) indicates that we have been able to reconstruct three of the four components; the resolution of the component near the core was not quite as good as that on the original map. The fourth (faintest and farthest) component was not reconstructed because of the visibility errors in which the contribution from this component was lost (in fact, the third and fourth components merged together). Clearly, as the accuracy of measuring the visibility magnitude increases, the accuracy of reconstructing individual components can also increase. However, as follows from our results, we can obtain maps of acceptable quality for solving the limited range of problems outlined above even at a relatively low signal-to-noise ratio.

PHASELESS MAPPING OF THE SOURCE 2200+420 USING SINGLE-BASELINE OBSERVATIONS

Let us now present the results of our mapping of the source 2200+420 using data obtained only on one interferometer baseline (Fig. 6). To this end, we separated out the observations
on the Gilcreek-Kokee baseline from the geodetic and astrometric on March 26, 1996. The corresponding $UV$-coverage is shown in Fig. 6(a). Parameters of the observations and the output map are given in the last row of the table.

The image of the source reconstructed using the GMEM and Fienup’s algorithm is shown in Fig. 6(b). The extent to which the input and output data agree can be judged from Fig. 6(c). In this figure, the plus signs indicate the measured visibility magnitude (time is plotted along the horizontal axis), and the crosses indicate the visibility magnitude that corresponds to the reconstructed image. As we see from Fig. 6(b), we were able to reconstruct only the most prominent two-component structure of the source, in close agreement with the reasoning given in the previous section.

The derived relative fluxes from the components are close to the fluxes from the components of image $(k)$ in Fig. 2. Thus, we conclude that mapping using single-baseline data may prove to be quite acceptable in solving the restricted problem of studying the structural evolution of sources that consist of several bright compact features.

**CONCLUSIONS**

Despite the existence of efficient adaptive calibration methods that use closure phases directly or indirectly (Cornwell and Fomalont 1999) and the existence of powerful processing packages that perform them (AIPS, DIFMAP, ASL [27]), it seems to be of considerable interest to develop and use alternative phaseless mapping methods for the following reasons:

1. The spectral phase is such an important characteristic of the image that using erroneous or insufficient phase information (at a small number of baselines) is more dangerous than restoring it from the spectrum magnitude measured with sufficient accuracy. Such situations are not uncommon for VLBI observations that are not directly intended for astrophysical mapping, but provide invaluable data for studying the structural evolution of sources on long time scales.

2. The phase can be uniquely retrieved in principle from the spectrum magnitude for multidimensional ($\geq 2$) images with a finite carrier.

3. Since the spectra of finite functions are analytic, the entire function can be reconstructed from the known part of it, which is important for VLBI.

4. There are reliable numerical methods and reconstruction algorithms that are stable against noise in the data.

5. The relative simplicity of the structure of compact extragalactic radio sources on milliarcsecond angular scales ensures that our phaseless mapping method converges reliably and rapidly.

6. Available Russian-made instruments with a small number of elements (in the limit, two-element ones) can be used to solve a limited range of problems (e.g., to study the evolution of the brightest components of extragalactic radio sources).

However, it is important to emphasize that we did not set the goal of contrasting the suggested phaseless method with the traditional VLBI mapping methods that use partial phase information (equations for closure phases). The methods that use any properly measured phase information are always better than the phaseless methods. In this paper, we suggest using the phaseless methods as alternatives to the existing methods that use phase information only in the following two cases: (1) when the equations for closure phases are
insufficient or unavailable (a two-element interferometer) and (2) the available phase information is unreliable, and the visibility magnitudes were measured with a sufficient accuracy. In particular, the suggested method can be used for mapping based on data from an intensity interferometer.

Thus, we have considered an efficient phaseless mapping method that was tested both on models and on real VLBI observations. We have presented the results of our study of a two-element interferometer, which are of current interest even now in connection with the putting into operation of a Russian-made instrument based on Quasar RT-32 radio telescopes (Bajkova 2002b).

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Fig.1. Simulation of phaseless image reconstruction. Comparison of the reconstruction results when using the MEM and the GMEM (see the text).
Fig. 2. Mapping of the source 2200 +420 based on astrometric and geodetic data from a global VLBI array using the phaseless (QUASAR VLBIImager) self-calibration (DIFMAP) methods (see the text).
Fig.3. Simulation of phaseless mapping for a two-component source with various parameters on a two-element interferometer (see the text).
Fig. 4. Influence of the parameters of a two-component source on the form of the visibility magnitude on an interferometer baseline (see the text).
Fig. 5. Mapping simulation for the four-component source 0212+735 based on observations on the Svetloe-Zelenchuk interferometer (see the text).
Fig. 6. Phaseless mapping of the source 2200+420 using the astrometric and geodetic VLBI observations of March 26, 1996, on the Gilcreek-Kokee baseline (see the text).