Geometric synthesis of the folding bridge mechanism

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Abstract. In London there is a folding bridge composed of eight platforms articulated between them. One platform is fixed to base (a platform from one end). The platforms have the shape of isosceles trapezoid. When the bridge is folded the large bases of the trapezoids form an octagon. The driving mechanism of the bridge is composed by motor dyads (active RRTaR groups) and RRR dyads. For the eight platform bridge the maximum piston stroke is less than the platform height (trapeze height). In this case, the connections between the hydraulic cylinders and the other elements are usually made, i.e. at the ends of the cylinder. If we want to make a folding bridge with fewer moving platforms, there are situations when the piston stroke is greater than the height of the trapeze. In this case must be performed the geometrical synthesis of the mechanism such that the folded bridge forms desired geometric figures (square, pentagon etc.).

1. Introduction
There are different constructions, generally called bridges, which are made for crossing watercourses. These constructions can be fixed or movable. The fixed bridges have a simple structure, but they do not allow the passage of boats exceeding a certain height. The folding bridges remove this disadvantage. Many such constructions have been made in the world, such as: Rolling Bridge, England [1], Slauerhoffbrug, Netherlands [2], Scale Lane Footbridge, England [3], Rhyl Foryd Harbor Bridge [4] etc. Each of the mentioned constructions has advantages and disadvantages. The folding bridge in London has the great advantage that it can be easily folded being shaped like a regular polygon (octagon). At any moment during the folding, the bridge can be stopped in a random position, as ordered.

Figure 1 shows the kinematic scheme of the folding bridge in London, in working position, and Figure 2 shows the kinematic diagram of the same bridge, in a folded position. As shown in Figures 1 and 2 the folding bridge has eight platforms, one of them being fixed to the base (end platform).

![Figure 1. The kinematic scheme of the folding bridge with seven platform in working position](image-url)
In this paper it is presented the synthesis of the drive mechanism’s dimensions of the platforms so that, at the folding, to obtain the geometrical figure under the optimal conditions of the hydraulic actuators.

2. The geometrical synthesis of the folding bridge mechanisms

After folding, the bridges can form regular polygons, such as: square, pentagon, hexagon etc. In order to obtain these geometrical figures, it is imperative to be well-established relationships between the dimensions of the platforms and the lengths of the mechanism’s elements.

Given the width of the watercourses over which these folding bridges are built, the number of mobile platforms can be established, as well as their length. The mobile platforms have the shape of some isosceles trapezoids. Figure 3 shows the geometrical scheme of a folded bridge with eight platforms, one of them being fixed. Figure 4 shows the diagram of a platform within the bridge and the dimensions.
Figure 5 shows the working position (initial position) of two platforms (one of them being fixed) of a folding bridge. Figure 6 shows the folded position of two mobile platforms.

![Figure 5. The kinematic scheme of the folding bridge with one mobile platform in working position](image1)

![Figure 6. The kinematic scheme of the folding bridge with two mobile platforms in folded position](image2)

From figures 5 and 6 it is observed that, for a certain regular polygon which is realized by folded bridge, it can be determined the $\beta$ angle at the base of the isosceles trapezoid.

In order to determine the other dimensions of the platforms as well as the dimension of the kinematic elements of the bridge drive mechanisms there are also considered:

- $n_p$ – the total number of platforms;
- $B$ – the large base of the isosceles trapezoid;
- $d_{cil}$ – the diameter of the hydraulic cylinder;
- $\Delta_j$ – the distance between the cylinder generator and the platform;
- $r_v$ – the distance between the tip of the isosceles triangle and $B_1$ point, in the extended position of the cylinder.

For the construction of the bridge platforms, as well as the drive mechanisms, the followings should be known:

- $\alpha$ – the tilt angle of the first platform (in case of total folding of the bridge);
- $\beta$ – the base angle of the isosceles trapezoid;
- $l$ – the length tilt side of the trapezoid;
- $b$ – the length of the small base of the isosceles trapezoid;
- $h$ – the height of the isosceles triangle;
- $l_{tr}$ – the length of the side isosceles triangle;
- $h_p$ – the height of the platform;
- $A_1B_1$ – the length of element 1 ($B_1E_1 = A_1B_1$);
- $C_p$ – hydraulic cylinder stroke.
There can be determined the elements mentioned above: \( \alpha = 2\pi / n_p \), \( \beta = (\pi - \alpha) / 2 \), 
\( h = B / 2 \cdot \tan \beta \), \( l_{tr} = B / 2 \cdot \cos \beta \).

In order to determine other dimensions of the trapeze, as well as the lengths of the elements of the driving mechanism of the bridge, it is necessary to write the relations between the parameters that characterize the respective bridge.

For the hydraulic cylinder stroke, when the platform is folded to the maximum position, the following relation can be written (Figure 6):

\[
V_{tr} = l_{tr} - r_v
\]  

The distance between the hydraulic cylinders’ joints, in the initial phase (bridge in working position), is considered equal to the height of the isosceles trapezoid, so the following relation can be written \( S_0 = h_p \). The height of the isosceles trapezoid is a basic size in bridge construction because the initial length of the hydraulic cylinder corresponds to this size.

Then the relations between the dimension of the platform and the elements of the mechanism can be established, depending on the height of the isosceles trapezoid \( h_p \).

The relation that gives the length of the hydraulic cylinder after the bridge folding operation is:

\[
S_{cil} = S_0 + C_p
\]  

Using relations (1) and (2) we obtain:

\[
S_{cil} = l_{tr} - r_v
\]  

where
\( l = h_p / \sin \beta \) - represents the length of tilt part of trapezoid;
\( b = B - 2h_p / \tan \beta \) - represents the length of small base of isosceles trapezoid.

The length \( A_1B_1 \) is given by the relation (in the working position of the bridge it is considered \( A_1B_1 \) in the horizontal position):

\[
A_1B_1 = h_p / \tan \beta + \Delta X
\]  

Using the generalized Pythagorean Theorem in the triangle \( B_1E_1E'_1 \) it results (Figure 6):

\[
(A_1B_1)^2 = (B_1E'_1)^2 + \Delta X^2 - 2B_1E' \cdot \Delta X \cos \beta
\]  

The distance between the axis of the hydraulic cylinder and the tilt side of the platform is:

\[
\Delta l = d_{cil} / 2 + \Delta j
\]  

The distance from the point of joint of the hydraulic cylinder to the intersection of the base with the tilt side of the platform is:

\[
\Delta X = \Delta l / \sin \beta
\]  

To determine the height of the isosceles trapezoid of the platform it is written the connection relation:

\[
B_1E'_1 + l = h_p + C_p
\]
from which it is obtained $B_1E'_1 = h_p + C_p - h_p / \sin \beta$. Using the relation (1) the size $B_1E'_1$ is obtained:

$$B_1E'_1 = B \tan \beta - \frac{h_p}{\sin \beta} - r_v$$

(9)

Considering the initial data, as well as the relations (5), (6), (7) and (9), it is obtained:

$$h_p^2 - 2 \frac{l_w - r_v}{\sin \beta} h_p + (l_w - r_v)(l_w - r_v - 2 \frac{\Delta_I}{\tan \beta}) = 0$$

(10)

or else:

$$a_0 \cdot h_p^2 - 2a_1 h_p + a_2 = 0$$

(11)

where: $a_0 = 1$; $a_1 = \frac{l_w - r_v}{\sin \beta}$; $a_2 = (l_w - r_v)(l_w - r_v - 2 \frac{\Delta_I}{\tan \beta})$

From relation (11) the height of the isosceles trapezoid corresponding to the height of the platform is obtained:

$$h_p = a_1 \pm \sqrt{a_1^2 - a_0 \cdot a_2}$$

(12)

From the relation (12) is maintained the solution:

$$h_p = a_1 - \sqrt{a_1^2 - a_0 \cdot a_2}$$

(13)

After determining the height of the isosceles trapezoid, there are calculated the length and the stroke of the piston:

$$A_1B_1 = \frac{h_p}{\tan \beta} + \frac{\Delta_I}{\sin \beta}$$

$$C_p = l_w - r_v - h_p$$

(14)

(15)

The distance between the points $E_1A_1$ is given by the relation: $E_1A = b - 2\Delta_X$. The length of the small base of the trapezoid is: $b = B - 2h_p / \tan \beta$ .

3. Case study example of calculation:

The paper presents the synthesis of the dimensions of the kinematic elements of the drive mechanism and of the platforms for three, four, five, six and seven folding mobile platforms.

At complete folding, the following geometrical figures are formed square, pentagon, hexagon, heptagon and octagon.

To achieve the synthesis program there were considered as known the following:

- $B$ - the length bridge platforms;
- $n_p$ - the number of regular polygons;
- $r_v$ - the radius of the circle where are the extremities of piston rod on which the ends of the rods of the driving cylinders are located, at the time of the total folding of the bridge.

Figures 7, 8, 9, 10 and 11 presents the dimensions of the kinematic elements of the mechanisms, as well as the dimensions of the platforms. The kinematic schemes of the folding bridges with three, four, five, six and seven movable platforms are presented. The kinematic schemes from those figures were
obtained following the synthesis elaborated by the authors and the animation of the respective mechanisms, constructed using Matlab.

SQUARE, \(rv = 0.1000 \text{ m}\)

| B   | A1B1 | hp  | Cp  | alfa | beta |
|-----|------|-----|-----|------|------|
| 1.24000 | 0.36373 | 0.23645 | 0.54036 | 1.57080 | 0.78540 |

| b       | l      | delX  | delL  | E1A2 | d.cyl |
|---------|--------|-------|-------|------|-------|
| 0.76710 | 0.33439 | 0.12728 | 0.09000 | 0.51254 | 0.15000 |

Figure 7. Results for square folding bridge

PENTAGON, \(rv = 0.1300 \text{ m}\)

| B   | A1B1 | hp  | Cp  | alfa | beta |
|-----|------|-----|-----|------|------|
| 1.24000 | 0.39209 | 0.38655 | 0.53826 | 1.25664 | 0.94248 |

| b       | l      | delX  | delL  | E1A2 | d.cyl |
|---------|--------|-------|-------|------|-------|
| 0.67831 | 0.47780 | 0.11125 | 0.09000 | 0.45582 | 0.15000 |

Figure 8. Results for pentagon folding bridge

HEXAGON, \(rv = 0.1500 \text{ m}\)

| B   | A1B1 | hp  | Cp  | alfa | beta |
|-----|------|-----|-----|------|------|
| 1.24000 | 0.41856 | 0.54496 | 0.54504 | 1.04720 | 1.04720 |

| b       | l      | delX  | delL  | E1A2 | d.cyl |
|---------|--------|-------|-------|------|-------|
| 0.61073 | 0.62927 | 0.10392 | 0.09000 | 0.40288 | 0.15000 |

Figure 9. Results for hexagon folding bridge
4. Conclusions
The paper covered the geometric synthesis of a folding bridge. Depending on the width of the watercourse on which it is mounted, the folding bridge may have one or more mobile platforms. When completely folded, the bridges shape becomes a regular polygon (square, pentagon, hexagon etc.). For folding bridges with one or two movable platforms the values of kinematic elements are obtained from the geometric synthesis of a folding bridge with three or more mobile platforms, depending on the position of the folding platforms. For example, if it is requested to build a folding bridge with a movable platform, at vertical folding position (the angle $\alpha=\pi/2$), the constructive values are obtained from the synthesis of a folding bridge with three movable platforms, which at full fold forms a square.

5. References
[1] https://www.youtube.com/watch?v=x0Dj7XA77hw.
[2] https://www.youtube.com/watch?v=fllpbQ4LhiMY&t=85s.
[3] https://www.youtube.com/watch?v=AtjCUihUvZc.
[4] https://www.youtube.com/watch?v=rOx_ZGLRgy4.
[5] Artobolevski I.I. *Théorie des Mécanismes et des Machines (Theory of Mechanisms and Machines)*. Moscow: Ed. Mir, 1977. 453 p. (In French).
[6] Demidovich B., Maron A.I. Éléments de calcul numérique. 2nd edition. Moscow: Ed. Mir, 1987. 717 p. (In French).

[7] Dorn W.S., Mc Cracken D.D. Metode numerice cu programe în FORTRAN IV, București: Editura Tehnică, 1976. 468 p. (In Romanian).

[8] Duca C., Buium Fl., Pârăoanu G. Mecanisme, Iași: Editura Gh. Asachi, 2003. 481 p. (In Romanian).

[9] Moise V., Maican E., Moise Șt. I. Metode numerice. Aplicații în MATLAB, București: Ed. Bren, 2016. 292 p. (In Romanian).

[10] Moise V., Sînteca mecanismelor plane cu bare articulate. Aplicații în MATLAB, București: Editura Printech, 2018.282 p. (In Romanian).

[11] Pelecudi Chr. Precizia mecanismelor, București: Editura Academiei Republicii Socialiste Romania, 1975. 398 p. (In Romanian).

[12] Pelecudi Chr., Maroș D., Merticaru V., Pandre N., Simionescu I. Mecanisme, București: Editura Didactică și Pedagogică, 1985. 394 p. (In Romanian).

[13] Simionescu I., Moise V. Mecanisme, București: Editura Tehnică. 1999, 238 p. (In Romanian).