1. INTRODUCTION

Since the pioneering suggestion of Gell-Mann and Zweig, that hadrons are composed of fractionally charged constituents, the quark model has achieved a remarkable success. Chromo-electric charge (color) of \( s = 1/2 \) quarks, naturally leads to the concept of chromo-magnetic moments, which are responsible for the “hyperfine” splitting of hadronic masses. Quarks, possessing the electric charge, should also exhibit the “hyperfine” splitting of hadronic masses. Quarks, chromo-magnetic moments, which are responsible for the strong magnetic fields, behaviour of pseudoscalar \( J^P = 0^- \) mesons is discussed. We speculate on the existence of an induced magnetic moment of quark meson.

One is tempted to ask then, whether quark-antiquark states (mesons) also have magnetic moments. For charged \( J^P = 1^- \) mesons composed of different-flavour quark pairs \( q \bar{q} \) the answer is simple and positive: Yes, such mesons should have a magnetic moment. In the case of vector mesons with hidden flavour \( (J^P, Y, \phi) \) a more detailed quantum approach is necessary. For pseudoscalar \( J^P = 0^- \) mesons \( (\pi, K, \eta) \) the situation is even less clear. Can quantum system with zero angular momentum have a magnetic moment?

Based on the analogy with magnetic behavior of positronium \( (e^+e^-) \) and muonium \( (\mu^+\mu^-) \) triplet and singlet ground states, we investigate here magnetic moments and polarizability of vector mesons \( J^P = 1^- \) and magnetic polarizability of pseudoscalar mesons \( \eta, \eta' \) and \( \eta(547) \). Quenching of \( J^P = 0^- \) mesons in very strong magnetic fields (created in heavy ion collisions) naturally appears if our analogy with ortho-positronium is justified.

Can \( \eta \) Mesons Have a Magnetic Moment?\(^1\)

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Abstract—The response of pseudoscalar and vector mesons to strong magnetic fields is studied within a simple constituent quark model using analogy with bound states of Positronium. Magnetic moments of charged vector mesons \( K^0, D^0, B^0 \) are predicted and it is found that \( \eta \) mesons have magnetic polarizability. In extremely strong magnetic fields, behaviour of \( J^P = 1^- \) mesons is discussed. We speculate on the existence of an induced magnetic moment of \( \eta \) meson.

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1. INTRODUCTION

Assuming that each constituent quark has magnetic moment \( \mu_q = \hbar Q/2m^* \) (where \( m^* \) is the effective quark mass), magnetic moments of baryons can be calculated \(^1\) as: \( \mu_s = \Sigma \mu_q \) for \( s = 3/2 \) baryons (e.g. \( \Omega^- \) and \( \Lambda^{++}, \Lambda^+ \)); \( \mu_s = \mu_{u,c,b} \) for baryons of type \( \Lambda, \Lambda_c, \Lambda_b \) containing \( (ud) \) diquark and different quark \((s, c, b)\); and \( \mu_q = (4\mu_u - \mu_b)/3 \) for type \((q_d q_b)q_s \) baryons (e.g. \( n, p, \Xi \)). Magnetic moments \( \mu_q \) of quarks \( u, d, s \) and their effective masses inferred from the measured magnetic moments of hyperons, \( p \) and \( n \) are shown in the upper three rows of table.

Observing that effective \( (m^* \) and constituent masses \( m_q \) of \( s, d, u \) quarks are similar, one may predict magnetic moments of heavy \( c, b, t \) quarks for which the corresponding strong heavy hyperon magnetic moments are not measured.

3. MAGNETIC PROPERTIES OF MESONS

Bound states of quark-antiquark pairs (mesons) in the ground \( S \)-state can have parallel or antiparallel spins of their constituents resulting in vector \( (J^P = 1^-) \) or pseudoscalar \( (J^P = 0^-) \) mesons. We shall assume

| Quark | \( Q \) | \( \mu_q [\mu_B] \) | \( m^* [\text{MeV}] \) | \( m_M [\text{MeV}] \) |
|-------|-------|-----------------|-----------------|----------------|
| \( u \) | 2/3   | 1.852           | 338             | 350            |
| \( d \) | -1/3  | -0.972          | 322             | 370            |
| \( s \) | -1/3  | -0.613          | 510             | 500            |
| \( c \) | 2/3   | 0.404           | 1550            | 1600           |
| \( b \) | -1/3  | -0.066          | 4730            | 4770           |
| \( t \) | 2/3   | 0.004           | 172900          |                |

\(^1\) The article is published in the original.
here, that response of these mesons to strong magnetic fields is similar to behavior of positronium ($e^+e^-$) and muonium ($e^–\mu^+$) in the magnetic field. For example, triplet $S$-state of positronium (with parallel $e^+e^-$ spins) is analogous to the quantum state of vector mesons $\varphi(1020)$, $Y$ or $J/\Psi$ with $J^P=1^–$, while the singlet state of positronium (antiparallel $e^+e^-$ spins) resembles the structure of pseudoscalar $\eta_\rho$, $\eta_\rho$ mesons.

For vector mesons composed of unlike-flavour quark-antiquark pair with parallel spins e.g. $\rho^+(u\bar{d})$, $K^+(s\bar{u})$ or $D^+(c\bar{u})$, one can use analogy with muonium ($e^–\mu^+$) and add the magnetic moment of quark and antiquark $\mu_{q\bar{q}} = |\mu_q| + |\mu_{\bar{q}}|$. This approach gives $\mu = -2.82\mu_N$ and $2.46\mu_N$, $-1.37\mu_N$, $-1.02\mu_N$, $-1.92\mu_N$ for mesons $\rho^–(d\bar{u})$ and $K^{*+}(u\bar{s})$, $D^*+(c\bar{d})$, $D^{**}(s\bar{c})$, $B^*–(b\bar{u})$. Our obtained value $\mu_\rho = -2.82\mu_N$ agrees well with lattice calculations [3].

Using the analogy with triplet and singlet states of Positronium in magnetic field [4], one can predict that $J/\Psi$ and $\eta_c$ mesons do not have magnetic moment (see Fig. 1), which applies also to $Y(b\bar{b})$, $\phi(s\bar{s})$, $\eta_\rho$, and $\eta_\rho$ mesons. A possibility of the magnetic quenching of $J/\Psi$ decay (as observed for ortho-positronium [5]) is very interesting and it deserves a detailed study. Energy of the singlet muonium state in magnetic field behaves as [6]

$$E_{\mu^–e^+} = -\frac{\Delta E_{hf}}{2} \left[ 1 + \frac{B^2}{\Delta E_{hf}} \right],$$

which decreases as $E^– \approx -\Delta E_{hf}/2 - (\tilde{\mu} B)^2/\Delta E_{hf}$ for small $B$ fields (here $\tilde{\mu} = (|\mu_u| + |\mu_d|)/2$). Muonium singlet state thus achieves induced magnetic moment $\tilde{\mu}[B] = \tilde{\mu} B/\Delta E_{hf}$ and $\Delta E_{hf} = 1.8 \times 10^{-5}$ eV.

Replacing magnetic moments and masses of $e^–$ and $\mu^+$ in Eq. (1) by corresponding quark values (from table) and using $\Delta E_{hf} = 45.8$ MeV, one can predict also the magnetic behavior of $B^*–$ and $B^–(b\bar{u})$ mesons (see Fig. 2).

Two $(m_c = \pm 1)$ components of $B^*–$ triplet state do have magnetic moment $\tilde{\mu} = \pm(|\mu_u| + |\mu_d|)$, while $m_c = 0$ component of $B^*–$ and the meson $B^–(b\bar{u})$ have magnetic polarizability $\beta$. Using $\Delta E = -\beta B(2\pi/\mu_\rho)$ from [7] and comparing to Eq. (1) one has $\beta_\rho = \mu_\rho/2\pi(|\mu_u| - |\mu_d|)^2/4/\Delta E_{hf}$, which gives $\beta = 22.0 \times 10^{-4}$ fm$^3$ for $B^–$.

Quadratic energy response of $\eta_c$(2981) to magnetic field (Fig. 1) suggests magnetic polarizability $\beta = 1.78 \times 10^{-4}$ fm$^3$, obtained using $\Delta E_{hf} = 116.6$ MeV. For $\eta_c$(9391) one can predict $\beta = 0.08 \times 10^{-4}$ fm$^3$ (here $\Delta E_{hf} = 69.3$ MeV).

4. INDUCED MAGNETIC MOMENT OF $\eta$ MESON

We may now suggest, that $\eta$(547) also has magnetic polarizability, due to the similarity of its quantum structure with $\eta_c$. Assuming $\omega(782)$ meson to be spin-triplet partner of $\eta$(547) one has $\Delta E_{hf} = 235$ MeV and this gives magnetic polarizability $1.3 \times 10^{-4} < \beta < 4.6 \times 10^{-4}$ fm$^3$ for $\eta$(547), depending on the exact nature of its $(c\bar{u}u + c\bar{d}d\bar{d})$ quantum state. Analogously to positronium (see text below Eq. (1)), meson $\eta$(547) in magnetic fields should behave as having an induced magnetic moment $\tilde{\mu}[B] = \tilde{\mu} B/\Delta E_{hf}$ and $\Delta E_{hf} = 1.8 \times 10^{-5}$ eV. From nuclear physics we may expect, that in $H^3$ nucleus two neutrons form $S = 0$ scalar state (similar to singlet-positronium) and the magnetic moment of

**Fig. 1.** Energy of $\eta$ and $J/\Psi$ in very strong magnetic fields.

**Fig. 2.** Energy of $B^–(b\bar{u})$ and $B^*–$ in very strong magnetic fields.
H$^3$ is to be generated by the proton with $\mu_p = 2.79 \mu_N$. However, $\mu_{H^3} = 2.98 \mu_N$, which is 7% larger compared to $\mu_p$. Where does 7% increase come from?

Since two neutrons in $S = 0$ state are located in the magnetic field of the proton in H$^3$, one can speculate, that the bound state of two neutrons does have magnetic polarizability $\beta$, and that induced magnetic moment $\mu^*$ of scalar di-neutron contributes by 7% to the magnetic moment of H$^3$ nucleus.

If such picture is correct, then induced magnetic moment $\mu^*$ of $\eta(547)$ meson can produce its own magnetic field—it is real.

Consequently, also scalar diquarks in baryons, e.g. (ud) diquark in $\Lambda$, $\Lambda_c$ hyperons or Nucleon, may contribute to the total observed magnetic moment.

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