Submitted to "Publications of the Astronomical Society of the Pacific"

A Generalized $K$ Correction for Type Ia Supernovae: Comparing $R$-band Photometry Beyond $z = 0.2$ with $B$, $V$, and $R$-band Nearby Photometry

Alex Kim, Ariel Goobar, and Saul Perlmutter

November 1995
A Generalized $K$ Correction for Type Ia Supernovae: Comparing $R$-band Photometry Beyond $z = 0.2$ with $B$, $V$, and $R$-band Nearby Photometry

Alex Kim, Ariel Goobar, and Saul Perlmutter

Physics Division
Lawrence Berkeley National Laboratory
University of California
Berkeley, California 94720

November 1995

1Center for Particle Astrophysics and Lawrence Berkeley National Laboratory, 50-232, University of California, Berkeley, CA 94720; agkim@lbl.gov

2Physics Department, Stockholm University, Box 6730, S-113 85 Stockholm, Sweden; ariel@physto.se

3Lawrence Berkeley National Laboratory, 50-232, and Center for Particle Astrophysics, University of California, Berkeley, CA 94720; saul@lbl.gov

This work was supported in part by the Director, Office of Energy Research, Office of High Energy and Nuclear Physics, Division of High Energy Physics, of the U.S. Department of Energy under Contract No. DE-AC03-76SF00098 and by the National Science Foundation (ADT-88909616) and the University of California.
ABSTRACT

Photometric measurements show that, as a group, nearby Type Ia supernovae follow similar light curves and reach similar peak magnitudes (Branch & Tammann 1992). Thus, these supernovae may serve as standard candles or calibrated candles at cosmological distances. Magnitudes of local and distant supernovae, both in the same filter band, are compared using a $K$ correction to account for the different spectral regions incident on that filter. A generalized approach compares magnitudes in different bands for the nearby and distant supernovae, bands that are selected to give sensitivity in corresponding regions of the unredshifted and redshifted spectra. Thus, $R$ magnitudes for supernovae at $z \approx 0.5$ are compared with $B$ magnitudes of local supernovae. We compute these generalized $K$ corrections over a range of redshifts and bandpass pairs and discuss their advantages over the traditional single-band $K$ correction. In particular, errors near maximum light can be kept below 0.05 mag out to at least $z = 0.6$, whereas the traditional $K$ correction is less accurate and can be difficult to determine beyond $z > 0.2$. 
1. Introduction

Given a homogeneous set of Type Ia supernovae and assuming no evolution, a perfect and complete catalog of Type Ia spectra can be used to calculate the apparent magnitudes at any redshift and for any particular date with respect to maximum light. In practice, this is not feasible due to several problems: (1) The available supernova spectra often have insufficient wavelength coverage to calculate broadband photometry and insufficient time coverage to track the quickly evolving supernova; (2) Many of the available spectra do not have the signal-to-noise ratio to calculate precise magnitudes; (3) Spectral miscalibrations can lead to large errors in magnitude determinations; (4) The filter transmission function and detector response function are not perfectly known; (5) Even within the subset of Type Ia's with remarkably similar spectra, there are minor differences that can lead to slight supernova-to-supernova variation in magnitude. More reliable magnitude calculations can be made using spectra and photometry together, photometry being less sensitive to most of the above problems than spectra. In this paper, we calculate and discuss the errors for a generalized \( K \) correction, an example of such a technique, with a preliminary analysis using three Type Ia supernova. (One "peculiar" was also examined for comparison purposes.) These \( K \) corrections are particularly important for use with supernovae at \( z > 0.2 \), which are now being discovered in systematic searching (e.g., Perlmutter et al. 1994, 1995). Analysis of standard \( K \) corrections for Type Ia supernovae, useful for lower redshifts, have been performed by Hamuy et al. (1993) and Leibundgut (1990).

2. A Generalized \( K \) Correction

The standard \( K \) correction, \( K_z \), is used to calculate the apparent magnitude in some "\( x \)" filter band of an object at redshift \( z \) according to the equation:
\( m_x(z, t) = M_x(t) + \mu(z) + K_x(z, t) \), where \( \mu \) is the distance modulus (based on luminosity distance) and \( M_x \) is the absolute \( x \) magnitude (we omit explicit time dependence in subsequent equations). The \( K \) correction relates nearby and distant magnitudes measured with the same filter:

\[
K_x = 2.5 \log(1 + z) + 2.5 \log \left( \frac{\int F(\lambda) S_x(\lambda) d\lambda}{\int F(\lambda/(1 + z)) S_x(\lambda) d\lambda} \right)
\]

(1)

where \( F(\lambda) \) is the spectral energy distribution at the source (in this case the supernova), and \( S_x(\lambda) \) is the \( x \)'th filter transmission (Oke & Sandage 1968).

We generalize this expression to handle different filters, adding a term that accounts for the differences in the zeropoints of the magnitude system:

\[
K_{xy} = -2.5 \log \left( \frac{\int Z(\lambda) S_x(\lambda) d\lambda}{\int Z(\lambda) S_y(\lambda) d\lambda} \right) + 2.5 \log(1 + z) + 2.5 \log \left( \frac{\int F(\lambda) S_x(\lambda) d\lambda}{\int F(\lambda/(1 + z)) S_y(\lambda) d\lambda} \right)
\]

(2)

\[
= -2.5 \log \left( \frac{\int Z(\lambda) S_x(\lambda) d\lambda}{\int Z(\lambda) S_y(\lambda) d\lambda} \right) + 2.5 \log \left( \frac{\int F(\lambda) S_x(\lambda) d\lambda}{\int F(\lambda/(1 + z)) S_y(\lambda) d\lambda} \right)
\]

where \( Z(\lambda) \) is an idealized stellar spectral energy distribution at \( z = 0 \) for which \( U = B = V = R = I = 0 \) in the photometric system being used. \( K_{xy} \) is thus defined so that \( m_y = M_x + \mu + K_{xy} \). If \( S_x \equiv S_y \), the first term drops out and this reduces to the standard \( K \) correction of Equation 1.

The second line of Equation 2 is a change of variables, \( \lambda' = \lambda/(1 + z) \), that makes it easier to understand the \( K_{xy} \) correction in the case \( S_y(\lambda(1 + z)) = S_x(\lambda) \), a situation approximated by the dashed lines in Figure 1. If the "blueshifted" \( y \)'th filter matches the \( x \)'th filter function the second term in this equation drops out, and one is left with the term accounting for the difference in zeropoints of the filters (this difference is the "color zeropoint"). In this case, spectral dependence on the correction is eliminated. Note that this cross-filter approach has previously been used for galaxy \( K \) corrections (e.g. Gunn 1978).
3. $K_{xy}$ Calculation

We calculate generalized $K$ corrections using Equation 2 with Bessell's (1990) color zeropoints and realizations of the Johnson-Cousins UBVRI filter system (Figure 1). The color zeropoints are expected to match real photometric color zeropoints to better than 0.01 mag (Hamuy et al. 1992 quotes ≤ 0.009, Bessell (private communication) quotes ≤ 0.005). We use the same sample of supernova spectra as Hamuy et al. (1993), excluding those of SN 1991T from the main analysis as it was spectroscopically peculiar (Filippenko et al. 1992; Phillips et al. 1992), and including spectra of SN 1981B, a supernova that has been frequently used as a template Type Ia.

Our full sample is presented in Table 1 and contains 29 spectra from epochs $-14 < t_{max}^R < 76$ days (in the supernova rest frame) after blue maximum for SN 1981B, SN 1990N, and SN 1992A. The SN 1981B data is from Branch et al. (1983), SN 1990N data is described in Leibundgut et al. (1991), and SN 1992A is in Suntzeff et al. (1995), and Kirshner et al. (1993). The SN 1981B spectra labeled by epoch (0) is a composite of four spectra from March 6-9 (Branch et al. 1983). The SN 1992A HST spectra from epoch 5 with a spectral range of 1650-4800 Å has been augmented by the CTIO spectra from epoch 6 as described in Kirshner et al. (1993); it is labeled epoch (5). The $K$ corrections were not calculated for cases in which the spectra did not cover at least 99% of the effective acceptance of the passband; these cases are labeled with ellipses in the tables.

Tables 2, 3, and 4 have $K_{xy}$ corrections for $x = B, V, R, y = R$ and redshifts spanning from 0 to 0.7 in increments of 0.025. Epochs are given in the supernova rest-frame. Note that $K_{RR}$ is just the standard $R$ band $K$ correction. Each column of data is for a single supernova spectrum; the three tables have different number of columns because the number of spectra with sufficient wavelength coverage to calculate each correction varies. In particular, Table 4 has much fewer entries because there are only a few spectra covering the
needed wavelength range to calculate $K_{RR}$. See Hamuy et al. (1993) for tables of $K_{BB}$ and $K_{VV}$; these $K$ corrections have poor spectral coverage at $z > 0.1$ for $K_{BB}$ and at $z > 0.3$ for $K_{VV}$ because at those redshifts the rarely observed near-UV region of the spectrum is redshifted into the relevant bands.

4. Error Estimates and Determination of Optimal Filter Pair

We consider the contributions of the following sources of error in the $K$ correction: numerical integration error, spectral measurement and calibration error, intrinsic supernova-to-supernova dispersion, instrumental effects, and zeropoint uncertainty. We perform all analyses for each individual redshift between 0 and 0.7 in steps of 0.025. We illustrate our analysis techniques by presenting results for the specific redshift of $z = 0.5$, although our final conclusions will be based on the full analysis for all redshifts. Note that the example of redshift $z = 0.5$ demonstrates both a good filter match case (for $K_{BR}$) and a poor filter match case (for $K_{VR}$).

The numerical integration is accurate to 0.005 mag and we are able to reproduce the standard $K$ corrections in $B$ and in $V$ of Hamuy et al. (1993) to that accuracy. A larger source of uncertainty comes from the noise and calibration error of the spectra themselves. Lacking prior spectral error information, we test each spectrum's error properties by calculating, for $z = 0$, $B - V$ colors on the subset of 24 spectra with sufficient coverage. (This test was also performed in Hamuy et al.) The differences between these $B - V$ colors and the photometrically observed colors form a Gaussian distribution with a sigma of 0.04 mag. These $B - V$ colors compare two spectral regions that have little overlap, while $K$ corrections that compare overlapping regions are less sensitive to large scale miscalibrations and therefore should have smaller error.

The rapid but smooth evolution of supernova spectra should make $K$ corrections
a smooth function on the scale of a few days. However, the data shows scatter from measurement and calibration error in the spectra. The good temporal sampling of SN 1992A allows us to make $K$ correction error estimates based on this scatter. We illustrate by considering the specific examples of $K_{BR}$ and $K_{VR}$ at $z = 0.5$, shown in Figures 2 and 3. (Unfortunately, there are insufficient data points to similarly consider $K_{RR}$.) We first study the subset of SN 1992A $K$ corrections calculated from spectra measured at a single telescope, the CTIO 1.0-m. This includes all SN 1992 spectra except the ones at epochs 5, 6, 9, 17, 46, and 76. We estimate the root-mean-square scatter of the data from a smooth curve to be $< 0.002$ for $K_{BR}$ and $< 0.02$ for $K_{VR}$; we take this to be the bound on the effects of spectral measurement errors. Considering the full sample of $K$ corrections for SN 1992A from all the telescopes, we find the range (not the root-mean-square) in scatter to be $\sim 0.004$ for $K_{BR}$ and $\sim 0.1$ for $K_{VR}$; we take these to be the bounds on systematic instrumental error.

Having studied errors involved in the $K$ corrections of a single Type Ia supernova, we now consider the uncertainties involved in constructing a single $K$ correction for this entire class of supernovae. We do this by examining systematic differences between the three supernovae in our sample. Again consider $K$ corrections for $z = 0.5$ plotted in Figures 2 and 3. The scatter at a given epoch in $K_{BR}$ and $K_{VR}$ is dominated by intrinsic supernova-to-supernova differences. These differences are understood as being due to the observed variance in these particular supernovae’s color evolution, particularly around 20 days after maximum when the supernovae have quickly reached their reddest color. The range of scatter is $\sim 0.015$ mag for $K_{BR}$ and $\sim 0.2$ mag for $K_{VR}$; for epochs before day 17, the range narrows to $< 0.002$ for $K_{BR}$ and $\sim 0.1$ mag for $K_{VR}$.

As a preliminary test of the variation in $K$ correction for Type Ia supernovae that are not “normal,” the $K$ corrections of the “peculiar” Type Ia SN 1991T were also
calculated (Ford et al. (1993) and Phillips et al. (1992) describe this supernova). Their scatter with respect to the normal supernovae's $K$ corrections was within the intrinsic supernova-to-supernova dispersion discussed above, even for epochs < 14, when the SN 1991T spectra least resembled "normal" Type Ia spectra. Despite its spectral peculiarities, SN 1991T's light curve shapes are similar to the other supernova light curves from our sample. Systematically different $K$ corrections are expected for supernovae with peculiar color evolution. $K$ corrections of other peculiar Type Ia's (e.g. SN 1991bg) will be needed to test this.

In the above discussions, no conclusions could be made on $K_{RR}$ error due to sparseness of data. However, error estimates can be made based on the range of the correction. Recalling Equation 2, we see that $K$ corrections are the sum of an overall offset due to the different filter zeropoints plus a spectrally dependent term. Smaller spectral terms will propagate smaller errors into the $K$ correction than larger spectral terms. In this analysis, the relative size of the spectral term is apparent in the spread over time of the $K$ correction, since it is the only time-dependent term. Comparing the spread over time from Figure 4(a) with that from Figure 2, we see that the spectral contribution in the standard single-filter $K$ correction, $K_{RR}$, is almost twice that of $K_{VR}$, showing that the errors in $K_{RR}$ are much larger than those of $K_{VR}$. This point is demonstrated more dramatically in Figure 4(a,b) where $K_{BR}$ and $K_{RR}$ are plotted on the same scale. It is clear that the scatter in $K_{BR}$ is much smaller than would be expected for $K_{RR}$.

The transformation between instrumental and standard magnitudes, i.e. the color correction, is based on the observation of standard stars. However, supernovae and stars are spectroscopically different and will generally require different color corrections. The application of the standard star color correction thus leads to supernova magnitude error. In order to examine this effect, we 'observe' a series of spectrophotometric standards to
determine the color correction by convolving stellar spectra from Gunn and Stryker's (1983) spectrophotometric atlas with an instrumental passband. Our instrumental passband, as plotted in Figure 5, is the effective Kitt Peak 4-m $R$ passband constructed from the quantum efficiency of the TK2B CCD camera, the atmospheric transmission at the Kitt Peak site, and the KP1466 Harris R filter function. Figure 6 plots the difference between true $R$ magnitudes and instrumentally determined $R$ magnitudes after color correction as a function of observed $V - R$, for supernovae at the same redshifts considered before ($0 \leq z \leq 0.7$). Magnitudes of supernovae bluer than $V - R = 1.0$ match to better than 0.02 mag but redder supernovae give systematically different magnitudes. These redder supernovae are generally observed at $z > 0.6$ although at epochs greater than 15 days, supernovae will have $V - R > 1.0$ at $z > 0.45$. Application of an additional correction, in addition to the standard color correction for instrumental systems, can yield the true standard magnitude to within 0.01 mag. A more detailed analysis of potential systematics will require a more complete spectral data set and a variety of instrumental transmission functions.

There is an additional error due to zeropoint uncertainty for $K_{BR}$ and $K_{VR}$ mag which does not effect $K_{RR}$. This is an advantage of the single-filter $K$ correction, since zeropoints cancel when comparing data in the same band. As discussed earlier, the size of this error is less than 0.01 mag.

Given these contributing sources of error, we can compare the overall uncertainties for the generalized and standard $K$ corrections. Once again, we illustrate with the case of $z = 0.5$. $K_{BR}$ and $K_{VR}$ have the same zeropoint uncertainty, however at this redshift $K_{BR}$ has smaller errors than $K_{VR}$ from all other sources. This includes measurement error, instrumental systematics, and supernova-to-supernova systematics. Although $K_{RR}$ has no zeropoint error, it is otherwise expected to have errors even larger than $K_{VR}$. We emphasize
that $K_{BR}$ has these advantages because at $z = 0.5$, $S_R(\lambda(1 + z)) \approx \text{constant} \times S_B(\lambda)$, as shown in Figure 1, minimizing the spectral dependence on the $K$ correction.

The case of $z = 0.5$ is important as an extreme in which it is possible to match filters. To illustrate the errors expected for other redshifts, Figure 7 also shows the calculated root-mean-square scatter for the group of $K$ corrections near peak magnitude, for SN 1992A at epochs -1 and 3, and SN 1981B at epoch 0. The root-mean-square scatter is minimized at redshifts where the filters best match, and monotonically worsens as one moves away from these redshifts. Note that the error can be kept below 0.04 mag by switching from the nearby $V$ photometry to the nearby $B$ photometry when comparing supernovae at $z > 0.36$.

In the preceding analyses, we have calculated the $K$ corrections using Equation 2, which is the integral of energy flux, for consistency with previous work. Actual photometric measurements are performed with detectors that are photon counters, not bolometers. Therefore the correct $K$ correction calculation to be used with measured photometric magnitudes is the integral of photon counts:

$$K_{xy}^{\text{counts}} = -2.5 \log \left( \frac{\int \lambda Z(\lambda)S_x(\lambda)d\lambda}{\int \lambda Z(\lambda)S_y(\lambda)d\lambda} \right) + 2.5 \log(1 + z) + 2.5 \log \left( \frac{\int \lambda F(\lambda)S_x(\lambda)d\lambda}{\int \lambda F(\lambda/(1 + z))S_y(\lambda)d\lambda} \right).$$

The single filter version of this equation, i.e. a photon-count standard $K$ correction, is given in Schneider, Gunn and Hoessel (1983). Table 5 lists $K_{BR}^{\text{counts}}$ corresponding to the same data and redshifts of Table 2. These tables give the preferred values to use for generalized $K$ corrections when comparing actual photometry measurements. Figure 8 shows the difference between the two $K$ corrections, $K_{BR}^{\text{energy}} - K_{BR}^{\text{counts}}$, as a function of redshift. For $z < 0.6$, the difference is less than 0.04 mag over all epochs considered.

5. Conclusions
We have considered a generalized $K$ correction as an alternative to the single-band $K$ correction for relating local and high-redshift supernova magnitudes. Error sizes depend on redshift and the filter pair combination chosen and reflect the size of the term that accounts for the different spectral regions observed in distant and local supernovae. Minimizing this term by matching filters to observe the same region reduces error and can make a generalized $K$ correction better than a single-band $K$ correction. Matching filters also reduces the wavelength range needed to perform $K$ corrections, making more of the available data usable for better temporal coverage and for studies in systematic differences in supernovae. Generally, error estimates and optimal filter pair determinations at any redshift can be made using a procedure similar to the one we have outlined for $z = 0.5$. Our analyses of $0 \leq z \leq 0.7$ show that combined statistical and systematic uncertainties in $K$ correction determinations are within 0.05 mag. Roughly, we find that for $z < 0.1$, $K_{RR}$ should be used, for $0.1 < z < 0.35$, $K_{VR}$ should be used, and for $0.35 < z < 0.7$, $K_{BR}$ should be used. By extension, objects at even higher redshifts ($> 0.7$) will have smaller $K$ correction error if measured in the $I$ band, i.e. if $K_{BI}$ is used.

These general results for the preferred observation bands at a given redshift to “match filters” will, of course, be independent of object. However, further studies based on more supernova spectra will improve the estimates of the generalized $K$ corrections, and help characterize the detailed dependence on supernova-to-supernova variation. In particular, it will be important to analyze more subluminous, red, and/or spectroscopically peculiar supernovae, and to search for any relation between $K$ correction and light curve shape.

We would like to thank B. Leibundgut, M. Bessell, D. Branch, R. Kron, B. Schmidt, and the Calan/Tololo Search Group (M. Phillips, N. Suntzeff, J. Maza, and M. Hamuy) for providing data and for helpful discussions. This work was supported in part by the National Science Foundation (ADT-88909616) and U. S. Department of Energy
(DE-AC03-76SF000098).
REFERENCES

Bessell, M. S. 1990, PASP, 102, 1181

Branch, D., Falk, S. W., McCall, M. L., Rybski, P., Uomoto, A. K., and Wills, B. J. 1983, ApJ, 270, 123

Branch, D., and Tammann, G. A. 1992, ARA&A, 30, 359

Filippenko, A. V., Richmond, M. W., Matheson, T., Dickinson, M., Shields, J. C., Burbidge, E. M., Cohen, R. D., Malkan, M. A., Nelson, B., and Pietz, J. 1992, ApJ, 384, L15

Ford, C. H., Herbst, W., Richmond, M. W., Baker, M. L., Filippenko, A. V., Treffers, R. R., Paik, Y., and Benson, P. J. 1993, AJ, 106, 1101

Gunn, J. E. 1978, in Observational Cosmology: 8th advanced course of the Swiss Society of Astronomy and Astrophysics, ed. A. Maeder, L. Martinet, and G. Tammann (Sauverny, Geneva)

Gunn, J. E. and Stryker, L. L. 1983, ApJS, 52, 121

Hamuy, M., Phillips, M. M., Wells, L. A., and Maza, J. 1993, PASP, 105, 787

Hamuy, M., Walker, A. R., Suntzeff, N. B., Gigoux, P., Heathcote, S. R., and Phillips, M. M. 1992, PASP, 104, 533

Kirshner, R. P. et al. 1993, ApJ, 415, 589

Kirshner, R. P., Blades, J. C., Branch, D., Chevalier, R. A., Fransson, C., Panagia, N., Wagoner, R. V., and Wheeler, J. C. 1988, HST Proposal SINS: The Supernova Intensive Study, unpublished
Leibundgut, B. 1988, PhD thesis, University of Basel

Leibundgut, B., Kirshner, R. P., Filippenko, A. V., Shields, J. C., Foltz, C. B., Phillips, M. M., and Sonneborn, G. 1991, ApJ, 371, L23

Leibundgut, B. 1990, A&AS, 229, 1

Leibundgut, B., Tammann, G. A., Cadonau, R., and Cerrito, D. 1991, A&AS, 89, 537

Oke, J. B., and Sandage, A. 1968, ApJ, 154, 21

Perlmutter, S., Pennypacker, C. R., Goldhaber, G., Goobar, A., Muller, R. A., Newberg, H. J. M., Desai, J., Kim, A. G., Kim, M. Y., Small, I. A., Boyle, B. J., Crawford, C. S., McMahon, R. G., Bunchkrl, P. S., Carter, D., Irwin, M. J., Terlevich, R. J., Ellis, R. S., Glazebrook, K., Couch, W. J., Mould, J. R., Small, T. A., and Abraham, R. G. 1995. ApJ, 440, L41.

Perlmutter, S., et al. 1994. International Astronomical Union Circular, nos. 5956 and 5958.

Phillips, M. M., Wells, L. A., Suntzeff, N. B., Hamuy, M., Leibundgut, B., Kirshner, R. P., and Foltz, C. B. 1992, AJ, 103, 1632

Schneider, D. P., Gunn, J. E., and Hoessel, J. G. 1983, ApJ, 264, 337

Suntzeff, N. B. 1994, Private communication

Suntzeff, N. B. 1995, In preparation

Vaughan, T., Branch, D., Miller, D., and Perlmutter, S. In press. 1995, ApJ

This manuscript was prepared with the AAS \LaTeX macros v3.0.
## Table 1

Selected Spectra of SNe Ia

| SN       | Epoch a | UT Date   | Observatory/Tel | Observer(s)   |
|----------|---------|-----------|-----------------|---------------|
| 1990N    | -14     | 1990 Jun. 26.17 | MMTO/MMT | Foltz         |
| 1990N    | -7      | 1990 Jul. 02.99 | CTIO/1.5-m | Phillips      |
| 1992A    | -5      | 1992 Jan. 14.11 | CTIO/1.0-m | Winge         |
| 1981B    | -1(a)   | 1981 Mar. 6   | McDonald/2.7-m | See Branch et al. (1983) |
| 1981B    | -1(b)   | 1981 Mar. 6   | McDonald/2.7-m | See Branch et al. (1983) |
| 1992A    | -1      | 1992 Jan. 18.13 | CTIO/1.0-m | Winge         |
| 1981B    | (0)b    | 1981 Mar. 7   | McDonald/2.7-m | See Branch et al. (1983) |
| 1981B    | 0       | 1981 Mar. 7   | McDonald/2.7-m | See Branch et al. (1983) |
| 1992A    | +3      | 1992 Jan. 22.04 | CTIO/1.0-m | Winge         |
| 1992A    | (+5)b   | 1992 Jan. 24  | HST            | SINS c        |
| 1992A    | +6      | 1992 Jan. 25.04 | CTIO/1.5-m | Smith/Winkler |
| 1992A    | +7      | 1992 Jan. 26.04 | CTIO/1.0-m | Winge         |
| 1990N    | +7      | 1990 Jul. 17  | Lick/3.0-m    | Shields/Filippenko |
| 1992A    | +9      | 1992 Jan. 28.04 | CTIO/1.5-m | Hamuy/Williams |
| 1992A    | +11     | 1992 Jan. 30.04 | CTIO/1.0-m | Winge         |
| 1990N    | +14     | 1990 Jul. 23.98 | CTIO/4.0-m | Phillips/Baldwin |
| 1992A    | +16     | 1992 Feb. 04.04 | CTIO/1.0-m | Winge         |
| 1981B    | +17     | 1981 Mar. 24  | McDonald/2.7-m | See Branch et al. (1983) |
| 1992A    | +17     | 1992 Feb. 05.04 | CTIO/4.0-m | Hamuy         |
| 1990N    | +17     | 1990 Jul. 27.16 | MMTO/MMT | Huchra        |
| 1981B    | +20     | 1981 Mar. 27  | McDonald/2.7-m | See Branch et al. (1983) |
| 1981B    | +24     | 1981 Mar. 31  | McDonald/2.7-m | See Branch et al. (1983) |
| 1992A    | +24     | 1992 Feb. 12.03 | CTIO/1.0-m | Winge         |
| 1992A    | +28     | 1992 Feb. 16.02 | CTIO/1.0-m | Winge         |
| 1981B    | +29     | 1981 Apr. 5   | McDonald/2.7-m | See Branch et al. (1983) |
| 1992A    | +37     | 1992 Feb. 25.01 | CTIO/1.0-m | Winge         |
| 1990N    | +38     | 1990 Aug. 16.98 | CTIO/1.5-m | Phillips      |
| 1992A    | +46     | 1992 Mar. 05.02 | CTIO/1.5-m | Phillips      |
| 1981B    | +49     | 1981 Apr. 25  | McDonald/2.1-m | See Branch et al. (1983) |
| 1981B    | +58     | 1981 May. 4   | McDonald/2.7-m | See Branch et al. (1983) |
| 1992A    | +76     | 1992 Apr. 04.02 | CTIO/4.0-m | Hamuy/Maza    |

aEpoch relative to $B$ maximum light.

bSee text for discussion.

cSupernova Intensive Study General Observer program. Spectra described in Kirshner et al. (1993)
| $\tau$ | $t_d^U = -14$ | $-7$ | $-5$ | $-1$ | (0) | 3 | (5) | 6 | 7 | 9 | 11 | 14 | 16 |
|-------|-------------|-----|-----|-----|-----|---|-----|---|---|---|----|----|----|
| 0.000 | 0.024       | ... | ... | ... | 0.107 | ... | ... | ... | ... | ... | ... | ... | ... |
| 0.025 | -0.042      | ... | ... | ... | 0.013 | ... | ... | ... | ... | ... | ... | ... | ... |
| 0.050 | -0.094      | ... | ... | ... | -0.043 | ... | ... | ... | ... | ... | ... | ... | ... |
| 0.075 | -0.143      | -0.128 | ... | ... | -0.087 | ... | ... | ... | ... | ... | ... | ... | ... |
| 0.100 | -0.199      | -0.180 | ... | ... | -0.146 | ... | ... | ... | ... | ... | ... | ... | ... |
| 0.125 | -0.256      | -0.223 | ... | ... | -0.202 | ... | ... | ... | ... | ... | ... | ... | ... |
| 0.150 | -0.296      | -0.268 | ... | ... | -0.230 | ... | ... | ... | ... | ... | ... | ... | ... |
| 0.175 | -0.320      | -0.271 | -0.356 | -0.365 | -0.245 | -0.418 | -0.497 | -0.488 | -0.572 | -0.560 | -0.640 | ... | ... |
| 0.200 | -0.345      | -0.303 | -0.372 | -0.379 | -0.265 | -0.435 | -0.424 | -0.503 | -0.584 | -0.577 | -0.654 | ... | ... |
| 0.225 | -0.376      | -0.342 | -0.398 | -0.408 | -0.306 | -0.467 | -0.463 | -0.538 | -0.616 | -0.618 | -0.695 | ... | ... |
| 0.250 | -0.414      | -0.388 | -0.437 | -0.452 | -0.367 | -0.512 | -0.516 | -0.584 | -0.658 | -0.665 | -0.740 | ... | ... |
| 0.275 | -0.454      | -0.427 | -0.474 | -0.490 | -0.421 | -0.545 | -0.550 | -0.609 | -0.676 | -0.682 | -0.748 | -0.795 | -1.082 |
| 0.300 | -0.488      | -0.455 | -0.499 | -0.513 | -0.453 | -0.559 | -0.562 | -0.614 | -0.675 | -0.679 | -0.741 | -0.776 | -1.039 |
| 0.325 | -0.510      | -0.474 | -0.514 | -0.525 | -0.471 | -0.567 | -0.574 | -0.619 | -0.675 | -0.680 | -0.738 | -0.763 | -0.997 |
| 0.350 | -0.532      | -0.498 | -0.540 | -0.552 | -0.502 | -0.592 | -0.603 | -0.639 | -0.686 | -0.691 | -0.740 | -0.756 | -0.955 |
| 0.375 | -0.568      | -0.539 | -0.583 | -0.594 | -0.552 | -0.628 | -0.636 | -0.665 | -0.699 | -0.702 | -0.738 | -0.750 | -0.911 |
| 0.400 | -0.615      | -0.592 | -0.629 | -0.635 | -0.606 | -0.659 | -0.663 | -0.684 | -0.707 | -0.708 | -0.733 | -0.742 | -0.867 |
| 0.425 | -0.661      | -0.644 | -0.662 | -0.666 | -0.649 | -0.682 | -0.685 | -0.699 | -0.714 | -0.715 | -0.732 | -0.743 | -0.832 |
| 0.450 | -0.696      | -0.691 | -0.695 | -0.698 | -0.691 | -0.707 | -0.709 | -0.717 | -0.726 | -0.728 | -0.739 | -0.750 | -0.803 |
| 0.475 | -0.716      | -0.717 | -0.716 | -0.714 | -0.712 | -0.714 | -0.711 | -0.716 | -0.722 | -0.725 | -0.729 | -0.737 | -0.761 |
| 0.500 | -0.715      | -0.721 | -0.709 | -0.702 | -0.701 | -0.695 | -0.689 | -0.692 | -0.694 | -0.700 | -0.698 | -0.702 | -0.704 |
| 0.525 | -0.693      | -0.706 | -0.684 | -0.679 | -0.676 | -0.668 | -0.663 | -0.662 | -0.670 | -0.659 | -0.665 | -0.644 | ... |
| 0.550 | -0.654      | -0.684 | -0.657 | -0.661 | -0.659 | -0.647 | -0.640 | -0.637 | -0.632 | -0.640 | ... | ... | ... |
| 0.575 | -0.607      | -0.678 | -0.637 | -0.650 | -0.652 | -0.631 | -0.619 | -0.611 | -0.600 | -0.607 | ... | ... | ... |
| 0.600 | -0.562      | -0.684 | ... | ... | -0.647 | ... | -0.593 | -0.580 | ... | -0.567 | ... | ... | ... |
| 0.625 | -0.522      | -0.689 | ... | ... | -0.634 | ... | -0.560 | -0.542 | ... | -0.519 | ... | ... | ... |
| 0.650 | -0.480      | -0.683 | ... | ... | -0.608 | ... | -0.518 | -0.494 | ... | -0.464 | ... | ... | ... |
| 0.675 | -0.427      | -0.663 | ... | ... | -0.571 | ... | -0.469 | ... | ... | -0.404 | ... | ... | ... |
| 0.700 | -0.363      | -0.631 | ... | ... | -0.525 | ... | -0.416 | ... | ... | -0.339 | ... | ... | ... |
| $z$  | $t_0^P = 17$ | 17 | 17 | 20 | 24 | 24 | 28 | 29 | 37 | 38 | 46 | 58 | 76 |
|------|--------------|----|----|----|----|----|----|----|----|----|----|----|----|
| 0.000 | -0.558       | ... | ... | -1.092 | -1.817 | ... | ... | ... | ... | ... | ... | ... | ... |
| 0.025 | -0.650       | ... | ... | -1.157 | -1.830 | ... | ... | ... | ... | ... | ... | ... | ... |
| 0.050 | -0.776       | ... | ... | -1.245 | -1.847 | ... | ... | ... | ... | ... | ... | ... | ... |
| 0.075 | -0.907       | ... | ... | -1.319 | -1.843 | ... | ... | ... | -1.504 | -1.237 | ... | ... | ... |
| 0.100 | -0.985       | ... | ... | -1.344 | -1.806 | ... | ... | ... | -1.480 | -1.238 | ... | ... | ... |
| 0.125 | -1.007       | -0.982 | -1.165 | -1.328 | -1.745 | ... | ... | ... | -1.438 | -1.219 | ... | -0.804 | ... |
| 0.150 | -1.018       | -0.989 | -1.161 | -1.308 | -1.686 | -1.428 | -1.405 | ... | ... | -1.405 | -1.205 | ... | -0.820 | ... |
| 0.175 | -1.016       | -0.982 | -1.149 | -1.278 | -1.618 | -1.394 | -1.376 | ... | -1.351 | -1.263 | -1.177 | ... | -0.820 | ... |
| 0.200 | -1.007       | -0.965 | -1.126 | -1.238 | -1.542 | -1.350 | -1.337 | ... | -1.305 | -1.213 | -1.145 | ... | -0.837 | ... |
| 0.225 | -1.032       | -0.967 | -1.123 | -1.224 | -1.485 | -1.316 | -1.300 | ... | -1.272 | -1.283 | -1.137 | ... | -0.894 | ... |
| 0.250 | -1.063       | -0.969 | -1.120 | -1.215 | -1.431 | -1.281 | -1.269 | ... | -1.236 | -1.253 | -1.123 | ... | -0.925 | ... |
| 0.275 | -1.051       | -0.939 | -1.085 | -1.172 | -1.353 | -1.226 | -1.212 | ... | -1.183 | -1.194 | -1.081 | ... | -0.913 | ... |
| 0.300 | -1.016       | -0.900 | -1.039 | -1.115 | -1.266 | -1.165 | -1.153 | -1.133 | -1.126 | -1.131 | -1.032 | ... | -0.895 | ... |
| 0.325 | -0.980       | -0.867 | -0.995 | -1.058 | -1.184 | -1.106 | -1.095 | -1.079 | -1.072 | -1.071 | -0.987 | ... | -0.879 | ... |
| 0.350 | -0.945       | -0.838 | -0.951 | -1.002 | -1.104 | -1.046 | -1.036 | -1.023 | -1.014 | -1.009 | -0.940 | ... | -0.857 | ... |
| 0.375 | -0.905       | -0.811 | -0.906 | -0.944 | -1.027 | -0.986 | -0.976 | -0.966 | -0.956 | -0.949 | -0.891 | ... | -0.829 | ... |
| 0.400 | -0.861       | -0.788 | -0.864 | -0.888 | -0.955 | -0.929 | -0.920 | -0.910 | -0.903 | -0.895 | -0.852 | ... | -0.807 | ... |
| 0.425 | -0.823       | -0.777 | -0.831 | -0.845 | -0.897 | -0.879 | -0.874 | -0.867 | -0.860 | -0.855 | -0.827 | ... | -0.794 | ... |
| 0.450 | -0.797       | -0.775 | -0.804 | -0.821 | -0.854 | -0.833 | -0.831 | -0.833 | -0.820 | -0.825 | -0.803 | ... | -0.769 | ... |
| 0.475 | -0.760       | -0.758 | -0.763 | -0.787 | -0.801 | -0.776 | -0.775 | -0.786 | -0.765 | -0.778 | -0.757 | ... | -0.718 | ... |
| 0.500 | -0.704       | -0.723 | -0.707 | -0.735 | -0.734 | -0.707 | -0.710 | -0.720 | -0.698 | -0.715 | -0.699 | ... | -0.659 | ... |
| 0.525 | -0.644       | -0.690 | -0.652 | -0.685 | -0.673 | -0.641 | -0.649 | ... | -0.637 | -0.659 | -0.650 | ... | -0.607 | ... |
| 0.550 | ...          | -0.600 | -0.602 | -0.645 | -0.615 | ... | ... | ... | ... | -0.611 | -0.604 | -0.585 | -0.550 | ... |
| 0.575 | ...          | -0.625 | -0.547 | -0.602 | -0.558 | ... | ... | ... | ... | -0.556 | -0.550 | -0.535 | -0.484 | ... |
| 0.600 | ...          | -0.587 | -0.486 | -0.554 | -0.498 | ... | ... | ... | ... | -0.496 | -0.491 | -0.477 | -0.411 | ... |
| 0.625 | ...          | -0.543 | -0.422 | ... | -0.436 | ... | ... | ... | ... | -0.424 | -0.428 | -0.410 | -0.331 | ... |
| 0.650 | ...          | -0.493 | -0.354 | ... | -0.368 | ... | ... | ... | ... | -0.342 | -0.363 | -0.337 | -0.246 | ... |
| 0.675 | ...          | -0.439 | -0.284 | ... | ... | ... | ... | ... | ... | -0.257 | -0.297 | -0.258 | -0.155 | ... |
| 0.700 | ...          | -0.380 | -0.209 | ... | ... | ... | ... | ... | ... | -0.167 | -0.227 | -0.172 | -0.055 | ... |
| \( z \) | \( t_0^V = -14 \) | -7 | -5 | -1 | -1 | 0 | (0) | (5) | 6 | 7 | 7 | 9 | 11 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 0.000 | 0.008 | ... | ... | ... | ... | 0.038 | ... | ... | ... | ... | ... | ... | -0.014 | ... |
| 0.025 | -0.058 | ... | ... | ... | ... | -0.055 | ... | ... | ... | -0.047 | -0.048 | ... | ... | ... |
| 0.050 | -0.110 | ... | ... | ... | ... | -0.111 | ... | ... | -0.105 | -0.106 | ... | ... | -0.143 | ... |
| 0.075 | -0.159 | -0.198 | ... | ... | ... | -0.156 | ... | ... | -0.164 | -0.164 | ... | ... | -0.194 | -0.114 |
| 0.100 | -0.215 | -0.249 | ... | ... | ... | -0.214 | ... | ... | -0.227 | -0.228 | ... | ... | -0.251 | -0.197 |
| 0.125 | -0.272 | -0.292 | ... | ... | ... | -0.270 | ... | ... | -0.272 | -0.272 | ... | ... | -0.284 | -0.243 |
| 0.150 | -0.311 | -0.317 | ... | ... | ... | -0.299 | ... | ... | -0.297 | -0.297 | ... | ... | -0.297 | -0.272 |
| 0.175 | -0.336 | -0.341 | -0.349 | ... | ... | -0.331 | -0.313 | ... | -0.319 | -0.308 | -0.306 | -0.303 | -0.288 | -0.285 |
| 0.200 | -0.361 | -0.373 | -0.366 | ... | ... | -0.345 | -0.334 | -0.323 | -0.326 | -0.323 | -0.310 | -0.306 | -0.305 | -0.299 |
| 0.225 | -0.392 | -0.412 | -0.392 | -0.369 | -0.371 | -0.375 | -0.374 | -0.362 | -0.369 | -0.365 | -0.358 | -0.343 | -0.333 | -0.345 | -0.340 |
| 0.250 | -0.430 | -0.458 | -0.431 | -0.426 | -0.430 | -0.418 | -0.426 | -0.421 | -0.414 | -0.417 | -0.404 | -0.384 | -0.369 | -0.393 | -0.386 |
| 0.275 | -0.470 | -0.497 | -0.468 | -0.476 | -0.481 | -0.456 | -0.490 | -0.471 | -0.447 | -0.451 | -0.430 | -0.403 | -0.388 | -0.410 | -0.394 |
| 0.300 | -0.504 | -0.525 | -0.493 | -0.504 | -0.512 | -0.479 | -0.521 | -0.499 | -0.460 | -0.464 | -0.434 | -0.402 | -0.388 | -0.407 | -0.387 |
| 0.325 | -0.526 | -0.543 | -0.507 | -0.518 | -0.529 | -0.492 | -0.539 | -0.512 | -0.468 | -0.476 | -0.439 | -0.401 | -0.388 | -0.408 | -0.384 |
| 0.350 | -0.548 | -0.567 | -0.534 | -0.540 | -0.557 | -0.518 | -0.570 | -0.540 | -0.493 | -0.504 | -0.459 | -0.413 | -0.400 | -0.419 | -0.385 |
| 0.375 | -0.584 | -0.609 | -0.577 | -0.582 | -0.607 | -0.560 | -0.621 | -0.586 | -0.529 | -0.537 | -0.485 | -0.426 | -0.421 | -0.430 | -0.384 |
| 0.400 | -0.631 | -0.662 | -0.622 | -0.629 | -0.662 | -0.692 | -0.674 | -0.636 | -0.560 | -0.565 | -0.505 | -0.434 | -0.440 | -0.435 | -0.378 |
| 0.425 | -0.677 | -0.714 | -0.656 | -0.663 | -0.705 | -0.633 | -0.718 | -0.669 | -0.583 | -0.586 | -0.519 | -0.440 | -0.454 | -0.442 | -0.378 |
| 0.450 | -0.712 | -0.760 | -0.689 | ... | ... | -0.664 | -0.759 | ... | -0.608 | -0.610 | -0.537 | -0.453 | ... | -0.456 | -0.385 |
| 0.475 | -0.732 | -0.787 | -0.709 | ... | ... | -0.580 | -0.781 | ... | -0.616 | -0.612 | -0.536 | -0.448 | ... | -0.453 | -0.375 |
| 0.500 | -0.731 | -0.791 | -0.703 | ... | ... | -0.668 | -0.769 | ... | -0.596 | -0.591 | -0.512 | -0.421 | ... | -0.428 | -0.344 |
| 0.525 | -0.709 | -0.775 | -0.675 | ... | ... | -0.645 | -0.745 | ... | -0.569 | -0.564 | -0.483 | -0.388 | ... | -0.397 | -0.304 |
| 0.550 | -0.669 | -0.754 | -0.651 | ... | ... | -0.628 | -0.727 | ... | -0.548 | -0.542 | -0.457 | -0.359 | ... | -0.368 | ... |
| 0.575 | -0.623 | -0.747 | -0.631 | ... | ... | -0.616 | -0.721 | ... | -0.532 | -0.520 | -0.431 | -0.327 | ... | -0.334 | ... |
| 0.600 | -0.578 | -0.754 | ... | ... | ... | -0.716 | ... | ... | -0.495 | -0.401 | ... | ... | -0.294 | ... |
| 0.625 | -0.538 | -0.759 | ... | ... | ... | -0.703 | ... | ... | -0.461 | -0.362 | ... | ... | -0.247 | ... |
| 0.650 | -0.496 | -0.753 | ... | ... | ... | -0.677 | ... | ... | -0.419 | -0.315 | ... | ... | -0.192 | ... |
| 0.675 | -0.443 | -0.732 | ... | ... | ... | -0.639 | ... | ... | -0.371 | ... | ... | -0.131 | ... | ... |
| 0.700 | -0.379 | -0.701 | ... | ... | ... | -0.594 | ... | ... | -0.317 | ... | ... | -0.066 | ... | ... |
| $t^2_p = 14$ | 16 | 17 | 17 | 17 | 20 | 24 | 24 | 28 | 37 | 38 | 46 | 49 | 76 |
|-------------|----|----|----|----|----|----|----|----|----|----|----|----|----|
| $z$ | SN 1990N | 1992A | 1981B | 1992A | 1990N | 1981B | 1992A | 1992A | 1992A | 1990N | 1992A | 1981B | 1992A | 1981B | 1992A |
| 0.000 | ... | ... | 0.222 | ... | ... | -0.072 | -0.532 | ... | ... | ... | ... | ... | ... | ... |
| 0.025 | ... | ... | 0.131 | ... | ... | -0.137 | -0.545 | ... | ... | ... | ... | ... | ... | ... |
| 0.050 | ... | ... | 0.004 | ... | ... | -0.226 | -0.562 | ... | ... | ... | ... | ... | ... | ... |
| 0.075 | ... | ... | -0.127 | ... | ... | -0.300 | -0.558 | ... | ... | -0.442 | -0.348 | ... | ... | ... |
| 0.100 | ... | ... | -0.205 | ... | ... | -0.324 | -0.521 | ... | ... | -0.418 | -0.349 | ... | ... | ... |
| 0.125 | ... | ... | -0.227 | -0.311 | -0.283 | -0.308 | -0.460 | ... | ... | -0.376 | -0.330 | ... | -0.272 | ... |
| 0.150 | ... | ... | -0.303 | -0.237 | -0.307 | -0.290 | -0.288 | -0.400 | -0.342 | -0.335 | ... | -0.343 | -0.316 | ... | -0.289 |
| 0.175 | ... | ... | -0.296 | -0.236 | -0.295 | -0.282 | -0.258 | -0.333 | -0.310 | -0.306 | -0.330 | -0.301 | -0.288 | -0.275 | -0.288 |
| 0.200 | ... | ... | -0.278 | -0.227 | -0.273 | -0.266 | -0.219 | -0.257 | -0.267 | -0.267 | -0.283 | -0.251 | -0.256 | -0.246 | -0.305 |
| 0.225 | ... | ... | -0.279 | -0.252 | -0.270 | -0.268 | -0.205 | -0.199 | -0.233 | -0.236 | -0.250 | -0.221 | -0.248 | -0.249 | -0.363 |
| 0.250 | ... | ... | -0.277 | -0.263 | -0.266 | -0.270 | -0.196 | -0.146 | -0.197 | -0.200 | -0.215 | -0.190 | -0.234 | -0.247 | -0.393 |
| 0.275 | -0.327 | -0.243 | -0.270 | -0.231 | -0.240 | -0.153 | -0.068 | -0.142 | -0.142 | -0.162 | -0.132 | -0.192 | -0.209 | -0.382 | ... |
| 0.300 | -0.308 | -0.200 | -0.236 | -0.185 | -0.201 | -0.095 | 0.019 | -0.082 | -0.083 | -0.105 | -0.069 | -0.143 | ... | -0.364 | ... |
| 0.325 | -0.295 | -0.158 | -0.200 | -0.141 | -0.168 | -0.038 | 0.102 | -0.023 | -0.025 | -0.050 | -0.009 | -0.097 | ... | -0.347 | ... |
| 0.350 | -0.288 | -0.116 | -0.165 | -0.097 | -0.139 | 0.018 | 0.181 | 0.037 | 0.034 | 0.007 | 0.053 | -0.050 | ... | -0.325 | ... |
| 0.375 | -0.281 | -0.072 | -0.125 | -0.052 | -0.112 | 0.075 | 0.258 | 0.098 | 0.094 | 0.065 | 0.113 | -0.002 | ... | -0.298 | ... |
| 0.400 | -0.274 | -0.029 | -0.080 | -0.010 | -0.089 | 0.132 | 0.339 | 0.155 | 0.150 | 0.118 | 0.168 | 0.037 | ... | -0.275 | ... |
| 0.425 | -0.275 | 0.067 | -0.043 | 0.023 | -0.078 | 0.174 | 0.388 | 0.205 | 0.196 | 0.161 | 0.207 | 0.062 | ... | -0.262 | ... |
| 0.450 | -0.282 | 0.036 | -0.017 | 0.050 | -0.076 | 0.199 | 0.432 | 0.250 | 0.239 | 0.201 | 0.237 | 0.086 | ... | -0.237 | ... |
| 0.475 | -0.269 | 0.078 | 0.020 | 0.091 | -0.059 | 0.232 | 0.484 | 0.308 | 0.295 | 0.256 | 0.284 | 0.133 | ... | -0.187 | ... |
| 0.500 | -0.234 | 0.135 | 0.076 | 0.147 | -0.024 | 0.285 | 0.552 | 0.376 | 0.360 | 0.323 | 0.347 | 0.190 | ... | -0.128 | ... |
| 0.525 | -0.197 | 0.195 | 0.137 | 0.202 | 0.010 | 0.335 | 0.616 | 0.442 | 0.421 | 0.384 | 0.403 | 0.239 | ... | -0.075 | ... |
| 0.550 | -0.163 | ... | ... | 0.252 | 0.039 | 0.374 | 0.670 | ... | ... | ... | 0.451 | 0.285 | ... | -0.018 | ... |
| 0.575 | -0.124 | ... | ... | 0.307 | 0.074 | 0.417 | 0.727 | ... | ... | ... | 0.506 | 0.339 | ... | 0.048 | ... |
| 0.600 | -0.081 | ... | ... | 0.368 | 0.112 | 0.466 | 0.787 | ... | ... | ... | 0.566 | 0.398 | ... | 0.121 | ... |
| 0.625 | -0.030 | ... | ... | 0.432 | 0.156 | 0.850 | ... | ... | ... | 0.638 | 0.462 | ... | 0.200 | ... | ... |
| 0.650 | 0.027 | ... | ... | 0.500 | 0.206 | ... | 0.918 | ... | ... | ... | 0.720 | 0.526 | ... | 0.286 | ... |
| 0.675 | 0.089 | ... | ... | 0.570 | 0.260 | ... | ... | ... | ... | 0.805 | 0.592 | ... | 0.377 | ... | ... |
| 0.700 | 0.158 | ... | ... | 0.645 | 0.319 | ... | ... | ... | ... | 0.895 | 0.662 | ... | 0.476 | ... | ... |
TABLE 4  
$K_{RR}$ for SNe Ia

| $z$   | $t_U^{k} = -14$ | 0 (0) | 7 | 17 | 20 | 24 |
|-------|-----------------|-------|---|----|----|----|
|       | SN 1990N | 1981B | 1990N | 1981B | 1981B | 1981B |
| 0.000 | 0.000     | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |
| 0.025 | -0.066    | -0.084 | -0.093 | -0.078 | -0.091 | -0.065 | -0.013 |
| 0.050 | -0.118    | ...   | -0.149 | -0.129 | -0.218 | -0.153 | -0.030 |
| 0.075 | -0.167    | ...   | -0.194 | -0.180 | -0.349 | -0.227 | -0.026 |
| 0.100 | -0.223    | ...   | -0.252 | -0.237 | -0.427 | -0.252 | 0.011  |
| 0.125 | -0.280    | ...   | -0.308 | -0.270 | -0.449 | -0.236 | 0.073  |
| 0.150 | -0.319    | ...   | -0.337 | -0.283 | -0.459 | -0.216 | 0.132  |
| 0.175 | -0.344    | ...   | -0.352 | -0.286 | -0.458 | -0.186 | 0.199  |
| 0.200 | -0.369    | ...   | -0.372 | -0.292 | -0.449 | -0.146 | 0.275  |
| 0.225 | -0.400    | ...   | -0.412 | -0.319 | -0.474 | -0.132 | 0.323  |
| 0.250 | -0.438    | ...   | -0.474 | -0.356 | -0.505 | -0.123 | 0.386  |
| 0.275 | -0.478    | ...   | -0.528 | -0.375 | -0.492 | -0.080 | 0.464  |
| 0.300 | -0.512    | ...   | -0.560 | -0.375 | -0.458 | -0.023 | 0.551  |
| 0.325 | -0.534    | ...   | -0.578 | -0.374 | -0.422 | 0.034  | 0.634  |
| 0.350 | -0.556    | ...   | -0.608 | -0.386 | -0.387 | 0.090  | 0.713  |
| 0.375 | -0.592    | ...   | -0.659 | -0.407 | -0.347 | 0.148  | 0.790  |
| 0.400 | -0.639    | ...   | -0.712 | -0.426 | -0.302 | 0.204  | 0.862  |
| 0.425 | -0.685    | ...   | -0.756 | -0.440 | -0.265 | 0.247  | 0.920  |
| 0.450 | -0.720    | ...   | -0.798 | ...   | -0.239 | 0.271  | 0.964  |
| 0.475 | -0.740    | ...   | -0.819 | ...   | -0.202 | 0.305  | 1.016  |
| 0.500 | -0.739    | ...   | -0.807 | ...   | -0.146 | 0.357  | 1.084  |
| 0.525 | -0.717    | ...   | -0.783 | ...   | -0.085 | 0.407  | 1.148  |
| 0.550 | -0.677    | ...   | -0.765 | ...   | ...   | 0.447  | 1.202  |
| 0.575 | -0.631    | ...   | -0.759 | ...   | ...   | 0.489  | 1.259  |
| 0.600 | -0.586    | ...   | -0.754 | ...   | ...   | 0.538  | 1.319  |
| 0.625 | -0.546    | ...   | -0.741 | ...   | ...   | ...    | 1.382  |
| 0.650 | -0.504    | ...   | -0.715 | ...   | ...   | ...    | 1.450  |
| 0.675 | -0.451    | ...   | -0.677 | ...   | ...   | ...    | ...    |
| 0.700 | -0.387    | ...   | -0.632 | ...   | ...   | ...    | ...    |
### Table 5

| $z$ | $t_E^{(0)}$ | -14 | -7 | -5 | -1 | (0) | 3 | (5) | 6 | 7 | 9 | 11 | 14 | 16 |
|-----|-------------|-----|----|----|----|-----|---|----|---|---|---|----|----|----|
| 0.000 | 0.019 | ... | ... | ... | 0.111 | ... | ... | ... | ... | ... | ... | ... | ... |
| 0.025 | -0.049 | ... | ... | ... | 0.016 | -0.136 | -0.215 | ... | ... | ... | ... | ... | ... |
| 0.050 | -0.103 | ... | ... | ... | -0.044 | -0.199 | -0.278 | ... | ... | ... | ... | ... | ... |
| 0.075 | -0.152 | -0.141 | ... | ... | -0.093 | -0.261 | -0.339 | ... | -0.371 | ... | ... | ... | ... |
| 0.100 | -0.208 | -0.193 | ... | ... | -0.153 | -0.325 | -0.404 | ... | -0.456 | ... | ... | ... | ... |
| 0.125 | -0.263 | -0.236 | ... | ... | -0.210 | -0.372 | -0.451 | ... | -0.506 | ... | ... | ... | ... |
| 0.150 | -0.304 | -0.263 | ... | ... | -0.242 | -0.402 | -0.481 | ... | -0.540 | -0.618 | ... | -1.134 |
| 0.175 | -0.331 | -0.288 | -0.375 | -0.383 | -0.261 | -0.431 | -0.417 | -0.495 | -0.577 | -0.651 | -0.698 | ... | -1.129 |
| 0.200 | -0.357 | -0.320 | -0.392 | -0.399 | -0.283 | -0.450 | -0.437 | -0.513 | -0.592 | -0.582 | -0.655 | ... | -1.119 |
| 0.225 | -0.388 | -0.358 | -0.417 | -0.427 | -0.323 | -0.481 | -0.475 | -0.548 | -0.624 | -0.621 | -0.695 | ... | -1.121 |
| 0.250 | -0.426 | -0.402 | -0.453 | -0.467 | -0.381 | -0.524 | -0.524 | -0.621 | -0.695 | ... | ... | ... | ... |
| 0.275 | -0.465 | -0.440 | -0.488 | -0.503 | -0.433 | -0.556 | -0.558 | -0.617 | -0.683 | -0.687 | -0.751 | -0.804 | -1.089 |
| 0.300 | -0.499 | -0.469 | -0.512 | -0.526 | -0.405 | -0.571 | -0.573 | -0.625 | -0.685 | -0.688 | -0.748 | -0.790 | -1.051 |
| 0.325 | -0.523 | -0.489 | -0.528 | -0.540 | -0.485 | -0.580 | -0.586 | -0.631 | -0.687 | -0.691 | -0.747 | -0.779 | -1.013 |
| 0.350 | -0.546 | -0.513 | -0.554 | -0.565 | -0.515 | -0.604 | -0.613 | -0.651 | -0.698 | -0.702 | -0.750 | -0.774 | -0.973 |
| 0.375 | -0.581 | -0.553 | -0.595 | -0.605 | -0.563 | -0.638 | -0.646 | -0.675 | -0.711 | -0.714 | -0.750 | -0.768 | -0.931 |
| 0.400 | -0.626 | -0.604 | -0.638 | -0.645 | -0.615 | -0.669 | -0.672 | -0.695 | -0.719 | -0.720 | -0.746 | -0.761 | -0.890 |
| 0.425 | -0.670 | -0.654 | -0.671 | -0.675 | -0.657 | -0.692 | -0.694 | -0.710 | -0.726 | -0.728 | -0.746 | -0.760 | -0.854 |
| 0.450 | -0.705 | -0.699 | -0.704 | -0.707 | -0.699 | -0.716 | -0.718 | -0.727 | -0.738 | -0.740 | -0.752 | -0.765 | -0.823 |
| 0.475 | -0.727 | -0.727 | -0.726 | -0.724 | -0.722 | -0.726 | -0.729 | -0.735 | -0.739 | -0.744 | -0.753 | -0.782 |
| 0.500 | -0.729 | -0.734 | -0.723 | -0.717 | -0.716 | -0.711 | -0.706 | -0.709 | -0.712 | -0.717 | -0.716 | -0.722 | -0.727 |
| 0.525 | -0.713 | -0.725 | -0.705 | -0.700 | -0.698 | -0.690 | -0.685 | -0.684 | -0.691 | -0.681 | -0.688 | -0.687 | ... |
| 0.550 | -0.681 | -0.709 | -0.683 | -0.687 | -0.685 | -0.672 | -0.667 | -0.663 | -0.665 | -0.665 | ... | -0.656 | ... |
| 0.575 | -0.642 | -0.705 | -0.668 | -0.678 | -0.682 | -0.659 | -0.648 | -0.640 | -0.629 | -0.635 | ... | -0.621 | ... |
| 0.600 | -0.604 | -0.712 | ... | ... | -0.679 | ... | -0.627 | -0.613 | ... | -0.599 | ... | -0.581 | ... |
| 0.625 | -0.568 | -0.718 | ... | ... | -0.699 | ... | -0.597 | -0.579 | ... | -0.556 | ... | -0.534 | ... |
| 0.650 | -0.531 | -0.715 | ... | ... | -0.647 | ... | -0.560 | -0.537 | ... | -0.506 | ... | -0.482 | ... |
| 0.675 | -0.484 | -0.698 | ... | ... | -0.615 | ... | -0.516 | ... | -0.451 | ... | -0.425 | ... | ... |
| 0.700 | -0.427 | -0.671 | ... | ... | -0.575 | ... | -0.468 | ... | -0.392 | ... | -0.362 | ... | ... |
| z   | $t_0^2 = 17$ | 17 | 17 | 20 | 24 | 24 | 28 | 29 | 37 | 38 | 46 | 58 | 76 |
|-----|-------------|----|----|----|----|----|----|----|----|----|----|----|----|
|     | SN 1981B   | 1990N | 1992A | 1981B | 1992A | 1981B | 1992A | 1981B | 1992A | 1990N | 1992A | 1981B | 1992A |
| 0.000 | -0.521 | ... | ... | -1.065 | -1.804 | ... | ... | ... | ... | ... | ... | ... | ... |
| 0.025 | -0.614 | ... | ... | -1.132 | -1.820 | ... | ... | ... | ... | ... | ... | ... | ... |
| 0.050 | -0.738 | ... | ... | -1.219 | -1.839 | ... | ... | ... | ... | ... | ... | ... | ... |
| 0.075 | -0.866 | ... | ... | -1.293 | -1.839 | ... | ... | ... | ... | ... | ... | ... | ... |
| 0.100 | -0.948 | ... | ... | -1.324 | -1.809 | ... | ... | ... | ... | ... | ... | ... | ... |
| 0.125 | -0.977 | -0.975 | -1.155 | -1.316 | -1.756 | ... | ... | ... | ... | ... | ... | ... | ... |
| 0.150 | -0.995 | -0.988 | -1.156 | -1.304 | -1.704 | -1.428 | -1.406 | ... | ... | ... | ... | ... | ... |
| 0.175 | -1.001 | -0.986 | -1.149 | -1.281 | -1.643 | -1.402 | -1.383 | ... | ... | ... | ... | ... | ... |
| 0.200 | -0.999 | -0.975 | -1.131 | -1.249 | -1.573 | -1.364 | -1.350 | ... | ... | ... | ... | ... | ... |
| 0.225 | -1.024 | -0.979 | -1.129 | -1.236 | -1.517 | -1.332 | -1.321 | ... | ... | ... | ... | ... | ... |
| 0.250 | -1.055 | -0.982 | -1.126 | -1.228 | -1.463 | -1.298 | -1.287 | ... | ... | ... | ... | ... | ... |
| 0.275 | -1.047 | -0.956 | -1.095 | -1.189 | -1.388 | -1.247 | -1.234 | ... | ... | ... | ... | ... | ... |
| 0.300 | -1.019 | -0.922 | -1.054 | -1.137 | -1.305 | -1.190 | -1.178 | -1.158 | -1.151 | -1.163 | -1.056 | ... | ... |
| 0.325 | -0.989 | -0.891 | -1.013 | -1.084 | -1.224 | -1.133 | -1.123 | -1.109 | -1.097 | -1.105 | -1.013 | ... | ... |
| 0.350 | -0.957 | -0.864 | -0.972 | -1.030 | -1.145 | -1.075 | -1.066 | -1.056 | -1.041 | -1.044 | -0.967 | ... | ... |
| 0.375 | -0.921 | -0.838 | -0.929 | -0.975 | -1.068 | -1.016 | -1.007 | -1.000 | -0.984 | -0.984 | -0.920 | ... | ... |
| 0.400 | -0.880 | -0.814 | -0.889 | -0.920 | -0.906 | -0.959 | -0.951 | -0.945 | -0.932 | -0.930 | -0.880 | ... | ... |
| 0.425 | -0.843 | -0.800 | -0.855 | -0.876 | -0.934 | -0.908 | -0.903 | -0.899 | -0.887 | -0.887 | -0.853 | ... | ... |
| 0.450 | -0.817 | -0.795 | -0.825 | -0.847 | -0.985 | -0.953 | -0.950 | -0.932 | -0.930 | -0.930 | -0.880 | ... | ... |
| 0.475 | -0.780 | -0.777 | -0.785 | -0.810 | -0.902 | -0.802 | -0.801 | -0.812 | -0.790 | -0.803 | -0.780 | ... | ... |
| 0.500 | -0.727 | -0.743 | -0.731 | -0.758 | -0.761 | -0.753 | -0.737 | -0.747 | -0.725 | -0.742 | -0.725 | ... | ... |
| 0.525 | -0.669 | -0.711 | -0.677 | -0.708 | -0.696 | -0.670 | -0.677 | ... | -0.665 | -0.686 | -0.676 | ... | ... |
| 0.550 | ... | -0.682 | -0.628 | -0.667 | -0.661 | ... | ... | ... | ... | -0.638 | -0.631 | -0.611 | -0.581 |
| 0.575 | ... | -0.649 | -0.575 | -0.624 | -0.585 | ... | ... | ... | ... | -0.585 | -0.580 | -0.565 | -0.520 |
| 0.600 | ... | -0.612 | -0.517 | -0.576 | -0.526 | ... | ... | ... | ... | -0.527 | -0.523 | -0.511 | -0.452 |
| 0.625 | ... | -0.570 | -0.455 | ... | -0.464 | ... | ... | ... | ... | -0.459 | -0.463 | -0.449 | -0.377 |
| 0.650 | ... | -0.524 | -0.391 | ... | -0.400 | ... | ... | ... | ... | -0.384 | -0.402 | -0.381 | -0.298 |
| 0.675 | ... | -0.474 | -0.324 | ... | ... | ... | ... | ... | ... | -0.305 | -0.340 | -0.307 | -0.212 |
| 0.700 | ... | -0.420 | -0.254 | ... | ... | ... | ... | ... | ... | -0.222 | -0.274 | -0.227 | -0.119 |
Fig. 1.—Bessell’s representations of the Johnson-Cousins $B$, $V$, and $R$ passband transmission functions, $S_B$, $S_V$, and $S_R$. The dotted lines represent the “blue-shifted” $R$ transmission function, $S_R(\lambda(1+z))$, at $z = 0.2$ and at $z = 0.5$. These $S_R(\lambda(1+z))$ transmission functions roughly match the $S_V(\lambda)$ and $S_B(\lambda)$ transmission functions for these values of $z$. 
Fig. 2.— $K_{VR}(z = 0.5)$ as a function of epoch for SN 1981B, SN 1990N, and SN 1992A.
Fig. 3.— $K_{BR}(z = 0.5)$ as a function of epoch for SN 1981B, SN 1990N, and SN 1992A.
Fig. 4.—(a) $K_{RR}(z = 0.5)$ (or the $R$ band $K$ correction) as a function of epoch for SN 1981B and SN 1990N. The available spectra of SN 1992A does not have sufficient coverage to make such a calculation. (b) $K_{BR}(z = 0.5)$ from Figure 3 plotted on the same scale as (a) to show the much smaller range, due to the close match of the $R$ filter at $z = 0.5$ with the $B$ filter at rest.
Fig. 5.— Comparison between the Bessell representation of $R$ and our constructed response of the KPNO $R$ as described in the text, normalized at peak transmission.
Fig. 6. — $K_{BR}^{\text{Bessell}} - K_{BR}^{\text{KPNO}}$ as a function of observed color for all redshifts.
Fig. 7.—The root-mean-square scatter of $K_{BR}$ (squares) and $K_{VR}$ (stars) for SN 1992A at epochs -1 and 3, and SN 1981B at epoch 0 as a function of redshift.
Fig. 8.— $K_{BR} - K_{BR}^{\text{counts}}$ as a function of redshift for epochs $-14 \leq t_0^B \leq 76$. 