Giant Gravitons and a Correspondence Principle

Vijay Balasubramanian\textsuperscript{*} and Asad Naqvi\textsuperscript{†}

David Rittenhouse Laboratories, University of Pennsylvania
Philadelphia, PA 19104, U.S.A.

Abstract

We propose a correspondence between the physics of certain small charge black holes in AdS\textsubscript{k} × S\textsuperscript{l} and large charge black holes in AdS\textsubscript{l} × S\textsuperscript{k}. The curvature singularities of these solutions arise, following Myers and Tafjord, from a condensate of giant gravitons. When the number of condensed giants \(N_g\) is much greater than the number of background branes \(N\), we propose that the system has an equivalent description in terms of \(N_g\) giant gravitons condensed in a background created by \(N_g\) branes. Our primary evidence is an exact correspondence between gravitational entropy formulae of small and large charge solutions in different dimensions.

1 Introduction

String theory enjoys a number of duality symmetries that relate the physics of apparently different theories in different dimensions. A particularly striking illustration of such a symmetry is the AdS/CFT duality \cite{1}, which relates gravity on AdS space to a CFT on the AdS boundary. In this note we propose a correspondence between the physics of large charge black holes in AdS\textsubscript{k} × S\textsuperscript{l} and small charge black holes in AdS\textsubscript{l} × S\textsuperscript{k}.

The black holes in question are electrically charged under \(U(1)\) gauge fields arising from Kaluza-Klein reduction of 10d or 11d supergravity on a sphere. Myers and Tafjord \cite{2} showed that the curvature singularities of such charged solutions of AdS\textsubscript{5} × S\textsuperscript{5} gravity can be understood as condensates of giant gravitons \cite{3} on S\textsuperscript{5}. In Sections 2 and 3 we extend the work of Myers and Tafjord \cite{2} to charged black holes in the four and seven dimensional gauged supergravities. Lifting the solutions to eleven dimensions \cite{4} as asymptotically AdS\textsubscript{k} × S\textsuperscript{l} spaces reveals that their singularities arise from condensation of giant gravitons on S\textsuperscript{l}. In four and seven dimensions there are four and two possible \(U(1)\) charges that the black holes can carry, while the five dimensional solutions in \cite{3} can carry three charges. Each of the different charges

\begin{itemize}
\item \textsuperscript{*}vijay@endive.hep.upenn.edu
\item \textsuperscript{†}naqvi@rutabaga.hep.upenn.edu
\end{itemize}
arises from a different species of giant graviton. In all cases, the single charge BPS solution has a horizon that coincides with the curvature singularity and adding some energy creates a solution with finite gravitational entropy.

As is well known, \( \text{AdS}_4 \times S^7 \), \( \text{AdS}_5 \times S^5 \) and \( \text{AdS}_7 \times S^4 \) arise in string theory as the near horizon geometries of stacks of M2, D3 or M5-branes. Typically, we take the near-horizon limit of a flat brane and find the Poincaré patch of \( \text{AdS}_k \), and a duality follows with the low energy world-volume CFT of the corresponding planar \( k-2 \) brane. The charged black holes studied here are in global \( \text{AdS} \) space, so we do not have such a near-horizon construction. Nevertheless, these spaces are dual to a \( k-2 \) brane CFT on a sphere. Hence we will speak of \( N \) spherical “background” branes creating the spacetime. As described above, the single charge black hole solutions of these theories can be interpreted as condensates of giant gravitons which are themselves spherical branes moving on the sphere factor of \( \text{AdS}_k \times S^l \). The giants of \( \text{AdS}_4 \), \( \text{AdS}_5 \) and \( \text{AdS}_7 \) are spherical M5, D3 and M2 branes respectively. Near any one of these branes, the geometry should locally be \( \text{AdS}_l \times S^k \). When the number of giants is very large, the background spacetime should be dominated by the presence of the giants rather than the presence of background branes. This leads to an intriguing hypothesis: \textit{When the number of condensed giants} \( N_g \) \textit{is much greater than the number of background branes} \( N \), \textit{the system has an equivalent description in terms of} \( N \) \textit{giant gravitons condensed in a background created by} \( N_g \) \textit{branes}.

In Section 4 we find evidence for such a correspondence by examining the thermodynamics of near-BPS, single charge black holes of \( \text{AdS}_k \times S^l \). We measure entropies and temperatures in terms of the number of condensed giants, and find an exact match between the large charge black holes in \( \text{AdS}_k \times S^l \) and the small charge black holes in \( \text{AdS}_l \times S^k \) when the number of giants is exchanged with the number of background branes while holding energies and the AdS scale fixed. Implications of our findings and directions forward are discussed in Section 5.

## 2 Four dimensions

The Kaluza-Klein compactification of M-theory on \( S^7 \) can be truncated consistently to \( SO(8) \) gauged \( \mathcal{N} = 8 \) supergravity in four dimensions. We are interested in black holes charged under the maximal Abelian subgroup \( U(1)^4 \). There is a truncation of the full \( \mathcal{N} = 8 \), \( SO(8) \) theory to a \( U(1)^4 \), \( \mathcal{N} = 2 \) theory, for which the bosonic fields are the metric \( g_{\mu\nu} \), four \( U(1) \) gauge fields \( A_i \), three scalars and three pseudo-scalars (the pseudo-scalars will be set to zero in this note). This theory admits four-charge

\footnote{However, see \cite{footnote} for global \( \text{AdS}_3 \) arising from the near-horizon limit of the spinning D1-D5 string. It would be interesting if a similar construction could be carried out in higher dimensions from the near horizon limit of spinning branes.}
AdS black hole solutions, given by [3]:

\[
\begin{align*}
    \frac{ds_4^2}{4} &= -(H_1 H_2 H_3 H_4)^{-1/2} f \, dt^2 + (H_1 H_2 H_3 H_4)^{1/2}(f^{-1}dr^2 + r^2d\Omega_2^2), \\
    X_i &= H_i^{-1}(H_1 H_2 H_3 H_4)^{1/4}, \quad A^i = \frac{\tilde{q}_i}{r + q_i} \, dt, \quad (i = 1 \cdots 4)
\end{align*}
\] (1)

where

\[
f = 1 - \frac{\mu}{r} + \frac{r^2}{L_4^2}(H_1 H_2 H_3 H_4), \quad H_i = 1 + \frac{q_i}{r}.
\]

The four \(X_i\) satisfy the relation

\[
X_1 X_2 X_3 X_4 = 1,
\]

parameterizing three physical scalars, while \(\mu\) is a SUSY breaking parameter. The BPS solution occurs when \(\mu = 0\). The physical \(U(1)\) charges \(\tilde{q}_i\) are given in terms of \(q_i\) as

\[
\tilde{q}_i = \sqrt{q_i(\mu + q_i)}.
\] (2)

The mass of this black hole is:

\[
M_4 = \frac{1}{4G_4}(2\mu + \sum_{i=1}^{4} q_i) \equiv \frac{1}{4G_4} \sum_{i=1}^{4} q_i + 2 \delta M_4,
\] (3)

where \(2\delta M_4\) measures the deviation from the BPS limit. Black holes carrying a charge under one of the four \(U(1)\)’s have a curvature singularity surrounded by horizon, which shrinks to zero area as \(\mu \to 0\). The multi-charge solutions are qualitatively different. They have a critical value, \(\mu = \mu_{\text{crit}}\), below which the horizon disappears giving a nakedly singular space. For \(\mu > \mu_{\text{crit}}\), there is a regular black hole horizon with finite entropy and a non-zero temperature. As \(\mu \to \mu_{\text{crit}}\) from above, the horizon approaches a finite limiting area. This critical black hole has zero temperature, but finite entropy [7].

**Lifting to 11D supergravity:** The general Kaluza-Klein ansatz that lifts any solution of 4D \(\mathcal{N} = 2, U(1)^4\) gauged SUGRA to a solution of 11D supergravity was given in [4]. The charged black hole solution [4] lifts to

\[
\begin{align*}
    \frac{ds_{11}^2}{4} &= \Delta^{2/3}\left((H_1 H_2 H_3 H_4)^{-1/2} f \, dt^2 + (f^{-1}dr^2 + r^2d\Omega_2^2)\right) \\
    &\quad + 4\Delta^{-1/3} \sum_i H_i \left(L_4^2 \, d\mu_i^2 + \mu_i^2 \, (L_4 d\phi_i + \frac{\tilde{q}_i}{r + q_i} \, dt)^2\right),
\end{align*}
\] (4)

where \(\sum \mu_i^2 = 1\), \(\Delta = (H_1 H_2 H_3 H_4) \sum_{i=1}^{4}(\mu_i^2/H_i)\), and \(d\Omega_2^2 = \sin^2 \alpha_1 d\alpha_1 d\alpha_2\) is the volume element on a unit two-sphere. The solution is asymptotically \(\text{AdS}_4 \times S^7\) where the AdS and sphere length scales are \(L_4\) and \(2L_4\) respectively. The sphere is parameterized by \(\phi_i\) and \(\mu_i\) as

\[
d\Omega_7^2 = \sum_i (d\mu_i^2 + \mu_i^2 d\phi_i^2),
\] (5)
and the four $\mu_i$ are expressed in terms of three angles as

$$
\mu_1 = \cos \theta_1, \quad \mu_2 = \sin \theta_1 \cos \theta_2, \quad \mu_3 = \sin \theta_1 \sin \theta_2 \cos \theta_3, \quad \text{and} \quad \mu_4 = \sin \theta_1 \sin \theta_2 \sin \theta_3.
$$

The lift of the black hole also has a four form field strength $F^{(4)} = dB^{(3)}$ with

$$
B^{(3)} = -\frac{r^3}{L_4} \Delta dt \wedge d\Omega_2^2 - L_4 \sum_{i=1}^4 q_i \mu_i^2 (L_4 d\phi_i - dt) \wedge d\Omega_2^2.
$$

**Interpretation as condensed giants:** The lifted 11 D black hole solutions have curvature singularities localized in AdS$_4$ and distributed all over $S^7$. These singularities are even naked in the multi-charge cases for small $\mu$. In the analogous AdS$_5 \times S^5$ solutions, Myers and Tafjord [2] argued that the singularity could be understood as a condensate of giant gravitons, by showing that the 5-form flux near the singularity in their solution was consistent with presence of a distribution of spherical D3-branes on $S^5$. Then, using the charge-mass relation of BPS giant gravitons they showed that the BPS solutions have an ADM mass that is also consistent with a source that is a condensate of giant gravitons [2].

In our case, the relevant brane source will be a distribution of giant gravitons on $S^7$. These are spherical M5 branes occupying an $S^5$ of the $S^7$ and carrying angular momentum along one direction of the sphere [3]. We will start with the BPS case ($\mu = 0$) and a single non-zero charge ($q_1$) and then state the results for the general case.

Being spherical branes, our giants correspond to M5 brane dipoles. They locally excite the seven-form field strength which can be detected by integrating the dual four-form over a surface enclosing a part of the M5-sphere. A closed surface transverse to the M5-brane spans the angular coordinates of AdS$_4$ and the directions on $S^7$ that are transverse to the M5-brane. The relevant four-form component is therefore

$$
F_{\theta_1, \phi, \alpha_1, \alpha_2}^{(4)} = 4q_1 L_4^2 \sin \theta_1 \cos \theta_1 \sin \alpha_1.
$$

Integrating this form over the angular coordinates transverse to the giant gravitons ($\alpha_{1,2}$ in AdS and $\phi_1$ and $\theta_1$ on the sphere) at any fixed $r$ and $t$ gives a net flux which is proportional to the number of enclosed giants. Following [2, 3], giant gravitons exciting these four-form components are moving in the $\phi$ direction of $S^7$ and localized along the $\theta$ direction. We can express this number in terms of 11d Planck length $l_p$ and $N$ which counts the number of units of background 4-form flux that are present independently of the giant gravitons, or equivalently the number of M2-branes whose near-horizon limit yields AdS$_4 \times S^7$ with AdS length scale $L_4 = l_p (\frac{1}{2} \pi^2 N)^{1/6}$. Then, with our conventions,

$$
16\pi G_{11} T_5 n_1 = \int d\theta_1 d\phi_1 d\alpha_1 d\alpha_2 F_{\theta_1, \phi, \alpha_1, \alpha_2}^{(4)},
$$

4
where $G_{11} = 16\pi^7\ell_p^9$ and $T_5 = \frac{2\pi}{(2\pi\ell_p)^7}$ is the M5 brane tension. By dropping the integration over $\theta$, we obtain the distribution of giant gravitons in $\theta$:

\[
\frac{dn_1}{d\theta_1} = \frac{N^{1/2}}{8\sqrt{2}\pi^2 L_4^3} \int F_{\theta_1\phi_1\alpha_1\alpha_2} d\phi_1 d^2\alpha = 2N^{1/2} \frac{q_1}{\sqrt{2}L_4} \sin 2\theta_1. \quad (9)
\]

Integrating over $\theta_1$ gives the total number of giants,

\[
n_1 = \int_0^{\pi/2} d\theta_1 \frac{dn_1}{d\theta_1} = \sqrt{2N} \frac{q_1}{L_4}. \quad (10)
\]

Treating a single giant graviton as a probe in a background AdS$_4 \times S^7$ geometry, one finds that a giant located at $\theta_1$ has a radius $L_4$ sin $\theta_1$ and carries an angular momentum along the $\phi_1$ direction equal to $N \sin^4 \theta_1$ [5]. Using this relation, we expect that the total angular momentum of the condensate of giants (9) is

\[
P_{\phi_1} = \int_0^{\pi/2} d\theta_1 \frac{dn_1}{d\theta_1} N \sin^4 \theta_1 = \frac{2N^{3/2} q_1}{3\sqrt{2} L_4}. \quad (11)
\]

Likewise, using the energy-momentum relation of BPS giant gravitons, we expect that a condensate with distribution (9) has a total energy

\[
E = \frac{P_{\phi_1}}{2L} = \frac{N^{3/2} q_1}{3\sqrt{2} L_4^2}. \quad (12)
\]

This agrees beautifully with the ADM mass (3)

\[
M_4 = \frac{1}{4G_4 q_1} = \frac{N^{3/2} q_1}{3\sqrt{2} L_4^2}, \quad (13)
\]

where we have used $G_4 = G_{11}/\text{Vol}(S^7) = \frac{3G_{11}}{128\pi^6 L_4^3}$.

We can extend this analysis to the multi-charge black hole solutions with arbitrary $q_i$. The generic BPS solution is nakedly singular. The singularity arises from condensation of sets of giant gravitons, separately moving along the different $\phi_i$. It is convenient to use the radius of the giant graviton moving along $\phi_i$ as a coordinate $\rho_i = 2L_4 \sqrt{1 - \mu_i^2}$. Then, an analysis like the one given above shows that the distribution of each set of giant gravitons is

\[
\frac{dn_i}{d\rho_i} = 2\sqrt{2N} \frac{q_i}{L_4^3} \rho_i. \quad (14)
\]

The giant graviton of radius $\rho_i$ carries an angular momentum $N\rho_i^4/L_4^4$. The total angular momentum carried by each set of giant gravitons is

\[
P_i = \frac{2N^{3/2} q_i}{3\sqrt{2} L_4}. \quad (15)
\]

Likewise, the total energy of the giant gravitons $E = \sum P_i/2L_4$ is in agreement with the ADM mass of the multi-charge black hole.
Beyond BPS: So far we have considered the BPS solutions with $\mu = 0$. As a result, we were able to compute the energy of a condensate of probe giant gravitons following the distribution (9) and this exactly matched the gravitational mass of the fully back-reacted spacetime solution. Repeating the analysis in the non-supersymmetric case when $\mu > 0$, we find that the net giant number density, the net number of giants and their net momentum are simply given by replacing $q_i$ in (9), (10) and (11) by the physical charge $\tilde{q}_i$ (2) of the non-extremal solution. However, replacing $q_i$ by $\tilde{q}_i$ in the giant energy formula (12) does not reproduce the spacetime mass (3) because the non-extremality can produce additional fluctuations of the giant gravitons as well as giant-anti-giant pairs at any fixed net momentum (11).

3 Seven dimensions

In an analysis parallel to the one above, we can consider the $N = 4$, $SO(5)$ gauged supergravity in 7 dimensions, arising from dimensional reduction and consistent truncation of 11d SUGRA on $S^4$. This theory can be further truncated to $N = 2$, $U(1)^2$ gauged SUGRA, where the only bosonic fields retained are the metric, two gauge potentials and two scalars [4]. The charged black hole solutions in this theory are

$$ds_7^2 = -(H_1 H_2)^{-4/5} f dt^2 + (H_1 H_2)^{1/5} (f^{-1} dr^2 + r^2 d\Omega_5^2),$$
$$X_i = H_i^{-1} (H_1 H_2)^{2/5}, \quad A^i = \frac{\tilde{q}_i}{r^4 + \tilde{q}_i} dt,$$

and

$$f = 1 - \frac{\mu}{r^4} + \frac{r^2}{L_7^2} (H_1 H_2), \quad H_i = 1 + \frac{q_i}{r^4}, \quad i = 1, 2.$$  (16)

Here $\mu$ is a SUSY-breaking parameter and the $\tilde{q}_i$ are defined as in (2). The mass of this solution is

$$M_7 = \frac{\pi^2}{4G_7} \left( \frac{5\mu}{4} + q_1 + q_2 \right) = \frac{\pi^2}{4G_7} (q_1 + q_2) + 5\delta M_7,$$  (17)

where $\delta M_7$ measures the deviation from the BPS limit. Using the general Kaluza-Klein ansatz to lift to a solution of 11d SUGRA [4], the charged black hole solution (16) becomes

$$ds_{11}^2 = \Delta^{1/3} \left( (H_1 H_2)^{-1} f dt^2 + (f^{-1} dr^2 + r^2 d\Omega_5^2) \right)$$
$$+ \frac{1}{4} \Delta^{-2/3} \left[ d\mu_0^2 + \sum_{i=1}^{2} H_i (L_7^2 d\mu_i^2 + \mu_i^2 (L_7 d\phi_i + \frac{\tilde{q}_i}{r^4 + q_i} dt)^2) \right],$$

This is not in general a consistent truncation but is so for solutions of the form considered here [4].
where
\[ \Delta \equiv (H_1 H_2) \left( \mu_0^2 + \sum_{i=1}^{2} \frac{\mu_i^2}{H_i} \right), \quad \mu_0 \equiv \sin \theta_1 \sin \theta_2, \quad \mu_1 \equiv \cos \theta_1, \quad \mu_2 \equiv \sin \theta_1 \cos \theta_2, \]
and \( L_7^2 = 4l_p^2 (\pi N)^{2/3} \). This solution is asymptotically AdS_7 × S^4 with scales \( L_7 \) and \( L_7/2 \) respectively. \( N \) counts the number of units of background 7-form flux, or equivalently, the number of M5-branes whose near-horizon limit is AdS_7 × S^4 with AdS_7 scale \( L_7 \). First consider the BPS solution (\( \mu = 0 \)). Following the reasoning in [2] and the previous section, the relevant 7-form component is
\[ F^{(7)}_{\rho_1 \phi_0 \alpha_1 \alpha_2 \alpha_3 \alpha_4} = 4q_i \rho_i \sin^4 \alpha_1 \sin^3 \alpha_2 \sin^2 \alpha_3 \sin \alpha_4, \]
(20)
where \( \rho_i \equiv \frac{L_7}{2} \sqrt{1 - \mu_i^2} \). The density of giant gravitons is
\[ \frac{dn_i}{d\rho_i} = \frac{1}{16\pi G_1 T_2} \int F^{(7)}_{\rho_1 \phi_0 \alpha_1 \alpha_2 \alpha_3 \alpha_4} d\phi_i d^{5}\alpha = 8N^2 q_i L_7^6 \rho_i. \]
(21)
The total number of giant gravitons in each set is
\[ n_i = \int_0^{L/2} d\rho_i 8N^2 q_i L_7^6 \rho_i = N^2 q_i L_7^4. \]
(22)
Giant gravitons of radius \( \rho_i \) carry angular momentum \( \frac{2N}{L_7} \rho_i \) [3] and therefore the total angular momentum carried by each set of giant gravitons is
\[ P_{\phi_i} = \int_0^{L/2} d\rho_i \frac{dn_i}{d\rho_i} \frac{N}{L_7} \rho_i = \frac{2N^3 q_i}{3} L_7^4. \]
(23)
Likewise, using the energy-momentum relation for giants, the total energy of the condensate of giants is
\[ E = \frac{P_{\phi_1} + P_{\phi_2}}{L_7/2} = \frac{4N^3}{3} \frac{q_1 + q_2}{L_7^5}. \]
(24)
As before, these energy and momentum calculations are performed while treating giant gravitons as probes in a background AdS_7 × S^4 geometry. Nevertheless, the total energy agrees exactly with the spacetime mass of the solution (18)
\[ M_7 = \frac{\pi^2}{4G_7} (q_1 + q_2) = \frac{4N^3}{3} \frac{q_1 + q_2}{L_7^5}. \]
(25)
where we have used \( G_7 = 6G_{11}/(\pi^2 L_7^4) \). In contrast, the non-supersymmetric solutions with \( \mu > 0 \) have a net condensate of giants in which \( q_i \) in (21) and (23) is replaced by the physical charge \( \tilde{q}_i \) defined as in (4). A similar replacement in the giant energy formula (24) will not reproduce the mass (18) because at positive \( \mu \) additional excitations of the giants as well as brane-anti-brane pairs may be present.
4 Entropy

Above we have generalized the calculation of Myers and Tafjord [2] in 5d to interpret certain charged black holes and curvature singularities of 4d and 7d gauged supergravity as condensates of giant gravitons. Giant gravitons are themselves spherical M2, D3 and M5 branes, and so we expect that after including gravitational backreaction the geometry very close to the surface of a giant should be locally AdS$_t \times S^k$. In particular, near the M5-brane giants of the AdS$_4 \times S^7$ space, the geometry should be locally AdS$_7 \times S^4$. Likewise, near the M2-brane giants of AdS$_7 \times S^4$ the local geometry should be AdS$_4 \times S^7$. Finally, near the D3-brane giants of AdS$_5 \times S^5$ the local geometry should again be AdS$_5 \times S^5$, but the AdS scale should be determined by $N_g$, the number of giants, rather than $N$, the number of background branes. This suggests a simple correspondence principle that should hold when a single species of giant is condensed: When the number of giants in AdS$_k \times S^l$ is sufficiently large, the physics can be equivalently described by a dual AdS$_t \times S^k$ in which the giants and the background branes have exchanged roles.

If this correspondence is correct, the thermodynamics of the large charge black holes of AdS$_k \times S^l$ and the small charge black holes of AdS$_t \times S^k$ should be equivalent. In the near-BPS limit we expect it to imply equality of black hole entropies and temperatures. Below, we test this by expressing the entropy and temperature of near-BPS, single charge black holes of AdS$_k \times S^l$ in terms of the number of condensed giants $N_g$, the number of background branes $N$ and the energy above extremality $\delta M$.

AdS$_5 \times S^5$: The previous sections have discussed the charged black hole solutions of AdS$_7$ and AdS$_4$ but here we begin by analyzing the entropy of the AdS$_5$ black holes whose interpretation in terms of giant gravitons was given in [2]. In detail, the $\mathcal{N} = 8$, SO(6) gauged SUGRA arising from consistent truncation of 10d, IIB SUGRA compactified on $S^5$ has a further truncation to 5d $\mathcal{N} = 2$, $U(1)^3$ gauged SUGRA. The black hole solutions of this theory are [8]

\begin{align}
    ds_5^2 &= -(H_1 H_2 H_3)^{-2/3} f\,dt^2 + (H_1 H_2 H_3)^{1/3}(f^{-1} dr^2 + r^2 d\Omega_3^2), \\
    X_i &= H_i^{-1} (H_1 H_2 H_3)^{1/3}, \quad A^i = \frac{q_i}{r + q_i} dt, \quad (i = 1 \cdots 4) \tag{26}
\end{align}

where

\begin{align}
    f &= 1 - \frac{\mu}{r^2} + \frac{\mu r^2}{L_5^2} (H_1 H_2 H_3), \quad H_i = 1 + \frac{q_i}{r^2}. \tag{27}
\end{align}

The solution has a mass

\begin{align}
    M_5 &= \frac{\pi}{4G_5} \frac{3}{2} \pi \mu + \sum q_i = \frac{\pi}{4G_5} \sum q_i + 3 \delta M_5. \tag{28}
\end{align}
Here $3\delta M_5$ measures the additional mass over the BPS limit produced by turning on $\mu$. In the single charge BPS case ($\mu = 0$) Myers and Tafjord showed that the singularity contains

$$N_g = N \frac{q}{L_5^2}$$

(29)
giants where $L_5^4 = 4\pi g_s N l_s^4$ and $N$ is the number of background units of 5-form flux, or equivalently the number of 3-branes whose near-horizon limit gives AdS$_5$ with scale size $L_5$. Beyond the BPS limit ($\mu > 0$), the number of giants is given by replacing $q$ in (29) by the physical charge $\tilde{q}$ as in (2). The horizon of the single charge black holes occurs at

$$r_h = \frac{1}{2} \sqrt{-2(L_5^2 + q) + 2\sqrt{(L_5^2 + q)^2 + 4\mu L_5^2}}.$$  

(30)

We will consider near-BPS limits in which $\mu \ll q$ and compute the horizon entropy in the two cases $q \ll L_5^2$ and $q \gg L_5^2$, or equivalently $N_g \ll N$ and $N_g \gg N$. Gravitational entropy of a 5d horizon is given by

$$S = \frac{A}{4G_5} = \frac{\Omega_3}{4G_5} r_h^2 \sqrt{r_h^2 + q},$$

(31)

where $r_h$ is the location of the horizon, $G_5 = \frac{G_{10}}{\pi L_5^5}$ and $G_{10} = 8\pi^6 g_s^2 l_s^8$ are the 5d and 10d Newton constants. In terms of the number of giants we find that

| Limits         | $r_h$   | $S = \frac{A}{4G_5}$ |
|----------------|---------|----------------------|
| $\mu \ll q$, $N_g \ll N$ | $\sqrt{\mu}$ | $S = 4\pi (L_5 \delta M_5) \sqrt{\frac{N_g}{N}}$ |
| $\mu \ll q$, $N_g \gg N$ | $r_h = L_5 \sqrt{\mu/q}$ | $S = 4\pi (L_5 \delta M_5) \sqrt{\frac{N_g}{N}}$ |

(32)

Notice that in the small and large charge limits, the entropy formula interchanges the roles of the giant gravitons ($N_g$) and the D3-branes ($N$) whose near-horizon created the background geometry, if we hold $L_5$ and $\delta M_5$ fixed.

**AdS$_4 \times S^7$:** We will study single charge 4d black holes (1) for which the horizon occurs when

$$f = 1 - \frac{\mu}{r} + \frac{q r}{L_4^2} + \frac{r^2}{L_4^2} = 0.$$  

(33)

As above we will compare the entropy of small charge ($q \ll L_4$) and large charge ($q \gg L_4$) black holes. From (11), the number of giants $N_g$ satisfies $N_g \ll \sqrt{N}$ and $N_g \gg \sqrt{N}$ in these cases. We will study the near-BPS solutions for which $\mu \ll q$, but in the large charge case we will need to separately consider the regimes $\mu \ll L_4^2/q$ ($\mu L_4 \ll \sqrt{N}$) and $L_4^2/q \ll \mu \ll q$ ($\sqrt{N} \ll \mu L_4 \ll \frac{N_g}{\sqrt{N}}$). The gravitational entropy is

$$S = \frac{A}{4G_4} = \frac{\Omega_2}{4G_4} r_h^2 \sqrt{1 + \frac{q}{r_h}},$$

(34)

$^{3}\Omega_D = \frac{2\pi^{(d+1)/2}}{\Gamma((d+1)/2)}$ is the volume of the unit D-sphere.
where $r_h$ is the location of the horizon. Then, using $G_4 = G_{11}/\text{Vol}(S^7)$ and $G_{11}$ as in Sec. 2, we obtain,

\[
\begin{array}{|c|c|c|}
\hline
\text{Limits} & r_h & S = \frac{A}{4G_4} \\
\hline
\frac{\mu}{L_4} \ll \frac{N_g}{\sqrt{N}}, \ N_g \ll \sqrt{N} & \mu & \sqrt{48\pi^2} \left( L_4 \delta M_4 \right)^{3/2} \frac{\sqrt{N_g}}{N} \\
\frac{\mu}{L_4} \ll \frac{N_g}{\sqrt{N}}, \ N_g \gg \sqrt{N} & \mu & \sqrt{48\pi^2} \left( L_4 \delta M_4 \right)^{3/2} \frac{\sqrt{N_g}}{N} \\
\frac{N_g}{\sqrt{N}} \ll \frac{\mu}{L_4} \ll \frac{N_g}{\sqrt{N}}, \ N_g \gg \sqrt{N} & L_4 \sqrt{\mu/q} & 4\pi \frac{1}{3\sqrt{\mu}} \left( L_4 \delta M_4 \right)^{3/4} \left( \frac{N}{{N_g}^{\frac{1}{2}}} \right) \\
\hline
\end{array}
\]

(35)

Here $\delta M_4$ is a measure of energy over the BPS limit as in (3). Unlike the AdS\textsubscript{5} case the entropies for the small and large charge cases do not seem to be related to each other. However, we will see a remarkable fact below – the large charge AdS\textsubscript{4} black hole entropy is related to the small charge AdS\textsubscript{7} entropy and vice versa.

**AdS\textsubscript{7} $\times$ S\textsuperscript{4}:** The horizon of the single charge 7d black hole (16) occurs when

\[
1 - \frac{\mu}{r^4} + \frac{r^2}{L_7^2} + \frac{q}{r^2 L_7^2} = 0. 
\]

(36)

We will consider small charge ($q \ll L_7^2$) and large charge ($q \gg L_7^2$) limits, which imply $N_g \ll N^2$ and $N_g \gg N^2$ respectively from (22). As above, we will study the near-BPS ($\mu \ll q$) limit, but in the small charge case we must separately consider the cases ($q/L_7^2 \ll \mu \ll q \left( \frac{N^2}{N_g} \ll \frac{\mu}{L_7^2} \ll \frac{N_g}{N^2} \right)$ and $\mu \ll (q/L_7^2)^2 \left( \frac{\mu}{L_7^2} \ll \frac{N^2}{N_g} \right)$. The gravitational entropy is given by

\[
S = \frac{A}{4G_7} = \frac{\Omega_5}{4G_7} \sqrt{1 + \frac{q}{r_h^4} \frac{r_h^5}}. 
\]

(37)

Using $G_7 = G_{11}/\text{Vol}(S^4)$ and $G_{11}$ as in Sec. 3,

\[
\begin{array}{|c|c|c|}
\hline
\text{Limits} & r_h & S = \frac{A}{4G_7} \\
\hline
\frac{\mu}{L_7^2} \ll \frac{N^2}{N_g}, \ N_g \gg N^2 & L_7 \sqrt{\mu/q} & \sqrt{48\pi^2} \left( L_7 \delta M_7 \right)^{3/2} \frac{\sqrt{N}}{N_g} \\
\frac{\mu}{L_7^2} \ll \frac{N^2}{N_g}, \ N_g \ll N^2 & L_7 \sqrt{\mu/q} & \sqrt{48\pi^2} \left( L_7 \delta M_7 \right)^{3/2} \frac{\sqrt{N}}{N_g} \\
\frac{N^2}{N_g} \ll \frac{\mu}{L_7^2} \ll \frac{N^2}{N_g}, \ N_g \ll N^2 & \mu^{1/4} & 4\pi \frac{1}{3\sqrt{\mu}} \left( L_7 \delta M_7 \right)^{3/4} \left( \frac{N}{{N_g}^{\frac{1}{2}}} \right) \\
\hline
\end{array}
\]

(38)

Here $\delta M_7$ is a measure of energy over the BPS limit as in (18).

### 4.1 A thermodynamic correspondence

The entropy formulae derived above were necessarily functions of $\delta M$, the energy above extremality. However, since entropy is itself dimensionless, this energy always appears in the dimensionless combination $L \delta M$, where $L$ is the AdS scale. The appearance of this combination, which is natural from the point of the gravitational
solution, also has a natural interpretation from the microscopic point of view. Das, Jevicki and Mathur have shown that a giant graviton has a discrete spectrum that is determined by the AdS scale $L$ and is independent of all other length scales including the radius of the giant $\delta M$. This remarkable fact, coupled with the appearance of $L \delta M$ in the above entropy formulae, suggests that the thermodynamics of the charged black holes studied here can be microscopically explained in terms of the fluctuations of giant gravitons that make up the singularity. In effect, the giant gravitons appear to play a role for our black holes analogous to the role of the D1-D5 string in the classic Strominger-Vafa analysis of black hole entropy in string theory [10].

Above we argued that when the number of giants in the solution is large there ought to be a correspondence exchanging the roles of the giant gravitons and the background branes creating the AdS spacetime. Essentially, this should relate the entropies of large charge black holes in AdS$_l \times S^k$ and small charge black holes in AdS$_k \times S^l$. The results presented in (32), (35) and (38) are a striking confirmation of this fact. In AdS$_5$ the entropy at large $N_g$ is obtained from the small $N_g$ result by exchanging $N$ and $N_g$ while holding the dimensionless energy $L_5 \delta M_5$ fixed. Similarly, the large $N_g$ entropy in AdS$_4$ reproduces the small $N_g$ entropy in AdS$_7$ if we hold the dimensionless energy fixed ($L_4 \delta M_4 = L_7 \delta M_7$) while exchanging $N_g$ and $N$. The numerical factors match exactly, as do the limits on the supersymmetry breaking parameter $\mu$.

We can derive the temperature of our solutions from the thermodynamic relation $\beta = 1/T = dS/dE$. By requiring that the temperatures match between corresponding solutions in AdS$_l \times S^k$ and AdS$_k \times S^l$ we find that while exchanging $N$ and $N_g$ we should separately hold $\delta M$ and the supergravity length scale $L$ constant. Since $L$ can be expressed in terms of the number of background branes and the fundamental length scales $l_5 = g_s^{1/4} \ell_s$ or the Planck length $l_p$, this requires us to rescale $l_5$ or $l_p$ when $N_g$ and $N$ are exchanged. In particular, $N_g \gg N$ solutions in AdS$_5$ with a given value of $l_5 = g_s^{1/4} \ell_s$ are related to the $N_g \ll N$ solutions with $\tilde{l}_5 = l_5 (N/N_g) \ll l_5$. Similarly, the $N_g \gg \sqrt{N}$ solutions in AdS$_4$ with a given Planck length $l_p$ are related to $N_g \ll N^2$ spaces in AdS$_7$ with $\tilde{l}_p = l_p (1/2)^{7/6} (N/N_g)^{1/6} \ll l_p$. Finally, the $N_g \gg N^2$ solutions in AdS$_7$ with a given $l_p$ are related to $N_g \ll \sqrt{N}$ black holes in AdS$_4$ with $\tilde{l}_p = l_p (2)^{7/6} (N^2/N_g)^{1/6} \ll l_p$. In all cases, the new fundamental length scale is much smaller than the original scale. While this will be important for a quantum mechanical analysis of our proposed correspondence, it is not relevant for the supergravity analysis of this article since only the scale $L$ appears in the spacetime solutions.

5 Discussion: A correspondence principle?

We have presented evidence from supergravity that there is a correspondence between the physics of certain large charge black holes in AdS$_l \times S^k$ and small charge black
holes in AdS\(_k \times S^l\). It would be interesting to extend the correspondence that we suggest to a full quantum mechanical duality.

To this end, it is important to understand how the charged black holes discussed here are represented in the CFT dual to AdS space. In \([11, 12]\) it was shown that non-normalizable bulk modes (boundary conditions) map to CFT couplings, while normalizable supergravity lumps give rise to VEVs for CFT operators. The operators relevant to condensation of giant gravitons were identified in \([13]\) as subdeterminants of products of CFT fields. Presumably, the black holes we have studied are related to superselection sectors of the CFT in which these operators have VEVs.

The CFT state corresponding to large \(N_g\) would not be well-approximated by the planar limit. (See \([13]\) for a discussion of the combinatorial explosion in perturbative calculations involving giant gravitons.) In the bulk spacetime non-planar diagrams correspond to string loop corrections. The correspondence we are proposing could be understood in AdS\(_5\) as stating that these loop corrections re-sum to give another description in which the SU\((N)\) gauge symmetry of the dual CFT is exchanged with an SU\((N_g)\) symmetry arising from the condensed giants.

A brief glance at the entropy formulae in \((32), (35)\) and \((38)\) and the relation \(1/T = dS/dE\) shows that the black holes that we are examining have unusual thermodynamic relations. For example, in the limits we examined in \((32)\) the temperature is independent of the energy. In fact, some of the solutions we have considered here have thermodynamic, and possibly dynamical, instabilities \([7, 14]\). We expect that such solutions with large charges in AdS\(_k \times S^l\) will correspond to unstable small charge solutions in AdS\(_l \times S^k\) and that the instabilities can be related to each other. Understanding this will be important for the goal of relating various features of the gravitational solutions to the physics of the giant gravitons making up the singularities.

Acknowledgements
We thank Micha Berkooz and Matt Strassler for useful discussions. This work was supported by DOE grant DOE-FG02-95ER40893.

References

[1] J. Maldacena, “The large \(N\) limit of superconformal field theories and supergravity,” Adv. Theor. Math. Phys. 2, 231 (1998) [Int. J. Theor. Phys. 38, 1113 (1998)] [hep-th/9711200].

[2] R. C. Myers and O. Tafjord, “Superstars and giant gravitons,” JHEP 0111, 009 (2001) [hep-th/0109127].
[3] R. C. Myers, “Dielectric-branes,” JHEP 9912, 022 (1999) [hep-th/9910053]; J. McGreevy, L. Susskind and N. Toumbas, “Invasion of the giant gravitons from anti-de Sitter space,” JHEP 0006, 008 (2000) [hep-th/0003073]; M. T. Grisaru, R. C. Myers and O. Tafjord, “SUSY and Goliath,” JHEP 0008, 040 (2000) [hep-th/0008013]; A. Hashimoto, S. Hirano and N. Itzhaki, “Large branes in AdS and their field theory dual,” JHEP 0008, 051 (2000) [hep-th/0008016].

[4] M. Cvetic et al., “Embedding AdS black holes in ten and eleven dimensions,” Nucl. Phys. B 558, 96 (1999) [hep-th/9903214].

[5] V. Balasubramanian, J. de Boer, E. Keski-Vakkuri and S. F. Ross, “Supersymmetric conical defects: Towards a string theoretic description of black hole formation,” Phys. Rev. D 64, 064011 (2001) [hep-th/0011217]; J. Maldacena and L. Maoz, “De-singularization by rotation,” [hep-th/0012025].

[6] M. J. Duff and J. T. Liu, “Anti-de Sitter black holes in gauged N = 8 supergravity,” Nucl. Phys. B 554, 237 (1999) [hep-th/9901149], W. A. Sabra, “Anti-de Sitter BPS black holes in N = 2 gauged supergravity,” Phys. Lett. B 458, 36 (1999) [hep-th/9903143].

[7] A. Chamblin, R. Emparan, C. V. Johnson and R. C. Myers, “Charged AdS black holes and catastrophic holography,” Phys. Rev. D 60, 064018 (1999) [hep-th/9902170]; M. Cvetic and S. S. Gubser, “Phases of R-charged black holes, spinning branes and strongly coupled gauge theories,” JHEP 9904, 024 (1999) [hep-th/9903195].

[8] K. Behrndt, A. H. Chamseddine and W. A. Sabra, “BPS black holes in N = 2 five dimensional AdS supergravity,” Phys. Lett. B 442, 97 (1998) [hep-th/9807187]; K. Behrndt, M. Cvetic and W. A. Sabra, “Non-extreme black holes of five dimensional N = 2 AdS supergravity,” Nucl. Phys. B 553, 317 (1999) [hep-th/9810227].

[9] S. R. Das, A. Jevicki and S. D. Mathur, “Vibration modes of giant gravitons,” Phys. Rev. D 63, 024013 (2001) [hep-th/0009019].

[10] A. Strominger and C. Vafa, “Microscopic Origin of the Bekenstein-Hawking Entropy,” Phys. Lett. B 379, 99 (1996) [hep-th/9601029].

[11] S. S. Gubser, I. R. Klebanov and A. M. Polyakov, “Gauge theory correlators from non-critical string theory,” Phys. Lett. B 428, 105 (1998) [hep-th/9802109]; E. Witten, “Anti-de Sitter space and holography,” Adv. Theor. Math. Phys. 2, 253 (1998) [hep-th/9802150].

[12] V. Balasubramanian, P. Kraus and A. E. Lawrence, “Bulk vs. boundary dynamics in anti-de Sitter spacetime,” Phys. Rev. D 59, 046003 (1999) [hep-th/9805171]; V. Balasubramanian, P. Kraus, A. E. Lawrence and S. P. Trivedi,
“Holographic probes of anti-de Sitter space-times,” Phys. Rev. D 59, 104021 (1999) [hep-th/9808017].

[13] V. Balasubramanian, M. Berkooz, A. Naqvi and M. J. Strassler, “Giant gravitons in conformal field theory,” [hep-th/0107119].

[14] R. Gregory and R. Laflamme, “Black Strings And P-Branes Are Unstable,” Phys. Rev. Lett. 70, 2837 (1993) [hep-th/9301052]; S. S. Gubser and I. Mittra, “Instability of charged black holes in anti-de Sitter space,” [hep-th/0009126]; H. S. Reall, “Classical and thermodynamic stability of black branes,” Phys. Rev. D 64, 044005 (2001) [hep-th/0104071], S. Yoon, “Black hole dynamics from thermodynamics in anti-de Sitter space,” [hep-th/0106286].