Monopole percolation in scalar QED

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Monopole Percolation was first introduced in the study of the non-compact lattice QED in both, the pure case and coupled to Higgs fields. Monopole percolation has been also observed coupled to the monopole condensation in the study of the pure gauge compact QED. We present here the results coming from the analysis of the role of the monopole percolation in the coupled gauge-higgs compact QED.

1. Introduction

One of the most challenging unsolved questions that arise in the application of lattice theory to particle physics is to understand the nature of the phase structure of lattice Quantum Electrodynamics.

Numerical simulations of pure gauge Lattice QED with periodic boundary conditions and using the compact gauge action $S = \beta \cos \theta$ show the existence of a first order phase transition [1]. This avoids a continuum limit for the theory. The introduction of matter fields seems not to change this behavior.

Dagotto Kogut and Kocić [2] formulated a version of QED using a non compact form of the gauge term $S = -\beta \theta^2$, i.e. keeping only the first term in the Taylor development. In this case the full theory exhibits a second order phase transition. Nevertheless, the pure gauge non-compact theory, being the action Gaussian, is simply trivial.

On the other hand, it has been recognized that the monopoles play a central role in the explanation of the phase structure of lattice QED. They produce disorder and give rise of the confinement via the dual superconductor mechanism. In this sense, the gauge vacuum behaves as a magnetic superconductor, (a monopole condensate). The knowledge of the behavior of monopoles in an actual simulation may help in the comprehension of the phase structure of QED.

2. Monopole percolation

In $d = 3$ monopoles are point-like excitations, while in $d = 4$ they are one-dimensional excitations. For this reason, its behavior in a finite lattice is far from being trivial.

An important observation was made by Kogut, Kocić and Hands [3]. They showed that in pure gauge non-compact QED monopoles percolate and satisfy the hyperscaling relations characteristic of an authentic second-order phase transition.

Baig, Fort and Kogut [4] showed that in the compact pure gauge theory, just over the phase transition point, monopoles condensate and also percolate. They pointed out that the strange behavior of this phase transition -its unexpected first order- can be related to the confluency of this two phenomena.

Furthermore, Baig, Fort, Kogut and Kim [5] showed that in the case of non-compact QED coupled to scalar Higgs fields, the monopole percolation phenomena -previously observed over the gauge line- actually propagate into the full $(\beta - \gamma)$ plane, (being $\gamma$ the gauge-Higgs coupling). Surprising, this monopole-percolation phenomenon is
decoupled from the phase transition line that separates the confined and the Higgs phases, a transition that is of second order, and logarithmically trivial.

3. Our analysis

We have performed a numerical simulation of the compact Lattice QED coupled to Higgs fields of unitary norm. We have reproduced some results about the phase diagram previously obtained by Alonso et al. [6], but measuring at the same time the behavior of monopoles – condensation and percolation. Results of this analysis are summarized in Fig. 1.

Figure 1. The phase diagram and the monopole percolation in SQED

The comparison of this two figures is very instructive. In the non-compact case, the monopole-percolation line remains decoupled from the phase transition besides in the compact case, the previously observed confluence of condensation-percolation-transition in the pure gauge case, still occurs in all the plane ($\beta - \gamma$).

An interesting result is that percolation occurs even when the transition line finishes. In this case, the behavior of all the percolation-related parameters change suddenly. Preliminary results from a finite-size scaling of the monopole susceptibility seems to suggest that the behavior is that of the pure bond percolation (as in the trivial non-compact case!)

4. Analysis of the results

- Energy histograms. In Fig. 2 we present the histograms of the energy for several values of the gauge coupling, near the $C_v$ peak, keeping always the Higgs coupling fixed $\gamma = 0.25$. Lattice size is $6^4$ and the number of lattice sweeps is 30,000 per point on a thermal cycle. The clear two-peaks structure of the histogram suggest that this phase transition line is of second order, like the pure gauge case. Fig. 3 collects the same measurements but now keeping $\gamma = 0.3$, i.e. just
above the end-point of the phase transition. The shadow of the phase transition is visible as a little flattening of the histogram, but the two-peaks structure has clearly disappeared.

\begin{figure}
\centering
\includegraphics[width=\textwidth]{figure4}
\caption{\(n_{\text{max}}/n_{\text{tot}}\) vs. gauge energy}
\end{figure}

\begin{figure}
\centering
\includegraphics[width=\textwidth]{figure5}
\caption{Susceptibility peak maximum along the percolation line}
\end{figure}

- **Monopole percolation.** In Fig. 4 we collect together the internal energy and the monopole-percolation parameter \(n_{\text{max}}/n_{\text{tot}}\) (number of sites in the larger cluster over the total number of connected sites) measured simultaneously over the line \(\gamma = 0.25\) that crosses the phase transition. Note that the discontinuity of both parameters occurs at the same point. Since monopole density also decreases suddenly at the same point, we can conclude, as in the pure gauge case, the concurrence of the three phenomena: Phase Transition, Monopole Condensation and Monopole Percolation.

- **Monopole susceptibility.** We have measured the monopole susceptibility for different values of the Higgs coupling in order to determine the monopole percolation line in the phase diagram. This line remains coupled to the phase-transition line up to its end-point (see Fig. 4). Above this point, the percolation-line continues approaching the vertical axis. An interesting observation (Fig. 5) is that the value of the maximum of the susceptibility changes suddenly over the end-point, i.e. precisely when the percolation decouples from condensation. Above the value of \(\gamma = 1\) the behavior of this peak with the lattice size is very similar to that observed in pure gauge non-compact QED when only pure percolation occurs.

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