Theoretical studies of the THz compression of low-to-medium energy electron pulses and the single-shot stamping of electron–THz timing jitter

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Abstract
The recent development of optical control of electron pulses brings new opportunities and methodologies in the fields of light–electron interaction and ultrafast electron diffraction (UED)/microscopy. Here, by a comprehensive theoretical study, we present a scheme to compress the longitudinal duration of low (\(\leq 1\) keV) to medium energy (1–70 keV) electron pulses by the electric field of a THz wave, together with a novel shot-by-shot jitter correction approach by using the magnetic field from the same wave. Our theoretical simulations suggest the compression of the electron pulse duration to a few femtoseconds and even sub-femtosecond. A comprehensive analysis based on typical UED patterns indicates a sub-femtosecond precision of the jitter correction approach. We stress that the energy independence of Coulomb interaction in the compression and the compact structure of THz device lay the foundation of the compression of low energy electron pulses. The combination of the THz compression of the electron pulse and the electron–THz jitter correction opens a way to improve the overall temporal resolution to attosecond for ultrafast electron probes with low to medium energies and high charge number per pulse, and therefore, it will boost the ultrafast detection of transient structural dynamics in surface science and atomically thin film systems.

1. Introduction
Tailoring the properties of free electron pulses with THz and optical fields has sparked broad interest recently in the fields of shaping free-electron quantum states [1, 2], electron acceleration [3–10], and ultrafast electron diffraction (UED)/microscopy [11–17]. Compared to the widely used radiofrequency (RF) techniques, THz and optical field can reach much higher field strength and therefore, be more compact in space, as demonstrated in the dielectric laser accelerator [5–8] and the THz accelerator [3, 4]. For UED/microscopy, the most significant advantage of using THz/optical field to control free electron pulses is the much lower timing jitter between the driving field and the electron pulses, in contrast to RF techniques [18–24]. The peak-to-peak timing jitter is usually on the order of 10 fs for the THz field [11, 25] and attoseconds for the optical field [14], while the typical jitter for the RF field is \(~50\) fs [22, 26]. Recently, by using optical field to compress single-electron pulses, the attosecond synchronization and the attosecond electron pulse trains are achieved, which enables an overall attosecond temporal resolution for
UED/microscopy [1, 14, 15, 27]. However, to probe ultrafast structural/electronic dynamics which are irreversible or require a low repetition rate of pumping [28–33], isolated ultrashort electron pulses with high charge number are needed. To compress such a pulse, the half wavelength of the compression electric field must be longer than the pre-compressed pulse length, and a uniform field distribution over the transverse size of the electron pulse (typically 100 to 200 μm) is required. In this regard, THz field with a medium size wavelength between the optical and the RF field, is more suitable [3, 11]. However, for the THz compression of electron pulses, the synchronization jitter on the order of 10 fs still impedes achieving an attosecond temporal resolution and more work need to be done to solve this issue.

So far, most work focuses on compressing electron pulses with energies ranging from tens of keV to several MeV [1, 3, 11–17], while the compression of lower energy electron pulses has not been studied yet. In the community of UED, the interest in the low to medium energy UED with electronic kinetic energy of a few 100 eV to a few keV, has dramatically increased recently. The nano-tip [34, 35] or flat photocathode [36–38] based low to medium energy UED has been demonstrated, in which, a temporal resolution of tens to hundreds of femtoseconds has been achieved by lowering the electron number in a single pulse and shortening the propagation distance between the photocathode and the sample. Due to the large scattering cross section and the high damage threshold to samples, using low energy electron pulses as probes is beneficial for studying structural dynamics in atomically thin films [39] on surface [40, 41], such as the transient processes in the twistronics [42–45] and the high temperature exciton condensation states [46, 47]. Further improving the temporal resolution without sacrificing the electron number of the low energy electron pulse is the key for these UEDs to apply to broader research fields.

In this work, we present a comprehensive theoretical study of the THz compression of low (⩽1 keV) and medium energy (1–70 keV) electron pulses with a moderate charge number (10^5–10^6 electrons per pulse). The simulation results show that the compressed pulses can reach a few femtoseconds to sub-femtosecond pulse duration. In addition, we propose a novel approach to stamp the electron–THz timing jitter shot-by-shot, by simultaneously recording the modulation of a reference electron pulse by the magnetic field from the same THz pulse used for compression. A sub-fs accuracy of this jitter stamping approach is demonstrated by a comprehensive analysis based on some typical UED diffraction patterns. Our results demonstrate the potential to improve the overall temporal resolution to attosecond for ultrafast electron probes with low to medium electron energies, providing new opportunities in a broad range of researches on ultrafast structural dynamics.

2. Schemes for the THz compression of low to medium energy electron pulses and the electron–THz timing jitter correction

Before a quantitative analysis, we firstly discuss the three physical foundations for the THz compression of the low to medium energy electron pulses with a moderate charge number per pulse. First, we evaluate the impact of Coulomb interaction (CI) in the compression of low energy electron pulses. In our previous work, we propose the energy independence of CI in electron pulse compression [48]. The relativistic effect diminishes the Coulomb force \( F_c \) as \( F_c \propto \beta^2 \gamma^{-3} \) (see page 296 in reference [49]), where \( \beta \) is the average velocity of electrons and \( \gamma \) is relativistic Lorentz factor, so low energy electron pulses will suffer stronger Coulomb repulsion force during the compression. However, the temporal focal length \( s \) is proportional to the kinetic energy of electron pulses as \( s \propto \beta^2 \gamma^{-3} \) [48]. Then, the CI which integrates Coulomb force within the temporal focal length, i.e. CI = \( \int F_c \, dz \), is energy-independent. In other words, the eV electron pulses suffer the same amount of CI as that of MeV electron pulses in the compression process. This energy independence of CI indicates that the compressed low to medium energy electron pulses hold the same beam quality as that of the conventional compressed 100 keV and MeV electron pulses. Second, the temporal focal length \( s \) for low to medium energy electron pulses is on the scale of a few millimeters to sub-millimeter, much shorter than that of high energy electron pulses (centimeters to meters). And therefore, a more compact millimeter-size compression device, shorter than \( s \), is required [50, 51]. The recent developed ultra-compact THz devices can fulfill this requirement [3, 4, 52, 53]. Third, the impact of CI on diffraction pattern in the post-specimen region can be neglected in UED systems with a compression unit. Because the enlarged energy chirp by the compression field will extend the electron pulse duration very quickly after the temporal focus, quenching the CI induced distortion in diffraction patterns [54] to a negligible level.

The schematic of a UED system with our proposed THz compression unit is shown in figure 1(a). One femtosecond laser pulse is split into three pulses to generate photoelectron pulses, few-cycle THz pulses and to excite the sample as pump, respectively. The photoelectron pulses, including one probe pulse and one reference pulse, are accelerated by a DC field to a kinetic energy of 0.5–70 keV, and then interact with a THz standing field. The device housing such THz field is based on a recently demonstrated parallel-plate
Figure 1. Schematic illustration of the THz compression of low to medium energy electron pulses and the snapshot of electron–THz timing jitter. (a) A femtosecond laser pulse is split and used to generate photoelectron pulses, few-cycle THz pulses and to excite the sample. The 3D field is a spatial distribution of the THz electric standing field and the below is the time-dependent THz field when the electron pulse passes through. The probe electron pulse accelerated by the DC field passes through the THz field region with coherent constructive electric field $E$ and then is diffracted by the sample at the temporal focus position. Besides the forward scattering in the transmission mode, the compressed low energy electron pulses can be used in the back reflection mode. The reference electron pulse passes through the THz field region with coherent constructive magnetic field $H$ and the induced modulation of the beam transverse distribution is used for jitter stamping. The schematic of the THz waveguide based on a tapered PPWG is shown in the dotted square. (b) The illustration of the jitter stamping by the THz magnetic standing field. The center position shift in case 1 and the transverse spot size change in case 2 are proposed to stamp the jitter between the reference electron pulse and the THz field.

waveguide (PPWG) structure (or horn structure) [12, 52, 55, 56] which can produce standing electric and magnetic waves with an intrinsic $\pi/2$ phase difference between them (note that the phase difference does not need to be exact $\pi/2$ for the subsequent simulation and design). The schematic of a tapered PPWG structure that can focus THz pulses and give rise to a uniform field distribution across the gap is shown in figure 1(a). In the simulation, a continuous THz standing field with a central frequency of 0.3 THz is generated between two parallel plates by CST studio (see appendix A for the 3D structure of the parallel plates), mimicking the uniform field distribution across the gap in such a PPWG [52]. The impact of the entrance and the exit aperture on the plates for electron pulses passing through, is negligible to the compression (see appendix A for the simulation results). For an experimental implementation, the tunable single-cycle or multicycle narrowband THz field used in the simulations can be generated by existing methods [11, 17, 57, 58]. As shown in figure 1(a), the probe electron pulse passes through the position of the parallel plates with coherent constructive THz electric field (i.e. the antinode position) at the zero crossing phase (indicated by the dashed blue arrow), imprinting an energy chirp on the probe beam for the subsequent pulse compression. The compressed electron pulse can work either in the transmission mode for low to medium energy electron pulse to study atomically thin and thicker films, or in the reflection mode for low energy electron pulse to study ultrafast surface dynamics. The propagation and the compression of electron pulses are simulated by general particle tracer (GPT) with a 3D space charge model [59]. The 3D THz field, as shown in figure 1(a), is generated by CST studio and imported into GPT to simulate the THz control of electron pulses. In order for an easy experimental implementation, no magnetic lens is used in this THz compression based UED system, similar to those compact UED designs [37, 60, 61]. And the beam qualities are still good enough to give decent diffraction patterns in such a design (see appendix B for more details).
To reach a sub-femtosecond temporal resolution, both the pulse duration and the jitter correction need to be on the sub-femtosecond timescale. The microscale suppression of the CI [48] limits the minimum achievable compressed pulse duration, so a feasible solution is to lower the electron number to a level of $10^3$–$10^4$ per pulse. However, lowering electron number degenerates the accuracy of most shot-by-shot jitter stamping approaches, such as methods proposed in references [16, 48, 62], due to inadequate signal-to-noise ratio. Here, we propose a novel jitter correction approach to remove this restriction and gain a higher jitter stamping accuracy. As shown in figure 1(a), a reference electron pulse passes through the position of the parallel plates with coherent constructive (or destructive) THz magnetic field at the zero crossing phase (indicated by the dashed orange arrow), then the transverse distribution of the reference pulse is modulated due to the timing jitter between the reference pulse and the THz magnetic field. Since the probe and reference electron pulses are derived from the same laser pulse, this timing jitter for the reference pulse equals the timing jitter for the probe pulse. A schematic illustration of the jitter stamping approach is shown in figure 1(b). The spatial modulation of the reference pulse in transverse direction can be either the center position shift in case 1 or the transverse spot size change in case 2 (see more details on the magnetic field distribution and the spatial modulation regime in appendix A). The combination of the probe pulse compression and the jitter stamping by the reference pulse provides a feasible way toward sub-femtosecond temporal resolution. In contrast to the recent studies using THz streaking to stamp the jitter between the RF field and the electron pulses in MeV UED [16, 17], the jitter correction approach in this work stamps the intrinsic jitter between the THz compression field and the electron pulses. Moreover, the spatial separation of the probe pulse and the reference pulse indicates that this jitter correction approach is noninvasive without disturbing the diffraction patterns.

3. Simulations for the THz compression of low to medium energy electron pulses

Simulation results for the THz compression of electron pulses with 0.5 keV, 1 keV, 2 keV, 5 keV and 70 keV kinetic energies are summarized as five cases in table 1. The initial parameters of electron pulses and the THz fields in the simulation are chosen from the commonly used or readily accessible values in the UED community. Electron distributions of compressed and uncompressed electron pulses are shown in the appendix C. Note that the simulation results listed in table 1 aim at convenient experimental implementation, and more details about the compression regime and compression results on specific experimental requirements are discussed in subsequent figures 2 and 3.

In the case of the compression of 1 keV electron pulse, i.e. case 2 in table 1, the temporal evolution of the electron pulse duration and the transverse spot size are displayed in figure 2(a). The electron pulse is compressed to $\sim 10$ fs by the THz field with a field strength of $\sim 1.0$ kV mm$^{-1}$, while the transverse spot size expands continuously which indicates that the transverse divergence is not impacted by the longitudinal compression. The dependence of the compressed pulse duration on the phase jitter of the THz field is shown in figure 2(b). The peak-to-peak jitter of the THz field is on the order of tens of femtoseconds [11, 25], and such jitter changes the compressed pulse duration in less than 1 fs, which is much smaller than the few femtoseconds pulse duration and therefore can be neglected. The linear ratio between the phase shift of the THz field and the arrival time change (i.e. the red spots) of electron pulses at the sample position is 1.22 as shown in figure 2(b), indicating that the CI during compression is dynamically suppressed similar as that in the compression of high energy electron pulses [20]. The inset in figure 2(b) displays the schematic of the interaction between the THz electric field and the electron pulse. The 1 keV electron pulse enters the THz field at the entrance phase of 1.10 rad and passes through the 160 $\mu$m interaction region (see table 1) within 2.6 THz cycles. The interaction of low energy electron pulses with multicycle THz fields aims to lengthen the interaction region to a few 100 microns, a size easier for the THz waveguide to be fabricated with conventional machining techniques [12, 52]. The compression scheme is the same as that in the compression of high energy electron pulses, i.e. the tails electrons gain more energy than the head electrons from the compression field then catch up the head electrons at the temporal focus. By increasing the THz field strength, both the pulse duration and the temporal focus length are decreased as shown in figure 2(c). A balance between the compressed pulse duration and the temporal focus length is needed preventing a too short temporal focus for a practical experimental implementation. In this work, a temporal focus length no less than 1 mm is designed for an easy experimental implementation [37, 60]. The minimum pulse duration in figure 2(c) is around 4 fs rms. A further compression is possible by breaking the limit from the microscale suppression of the CI [48] and the high order nonlinear term of the compression field [63], which is beyond the scope of this work. The impact of the entrance and exit apertures on the THz device to the compression is examined and no significant impact is observed as shown in the appendix A.

The inset in figure 2(c) shows a sinusoidal modulation of the momentum chirp (MC) of an electron pulse by the phase of the THz field. The momentum changes of the head and tail electrons (i.e. $\Delta p_{\text{head}}$ and $\Delta p_{\text{tail}}$)
Table 1. Initial parameters and simulation results for the THz compression of low to medium energy electron pulses.

| Parameters                                      | Case 1         | Case 2         | Case 3         | Case 4         | Case 5         |
|------------------------------------------------|----------------|----------------|----------------|----------------|----------------|
| Electron pulse energy                          | 0.5 keV        | 1 keV          | 2 keV          | 5 keV          | 70 keV         |
| Initial pulse duration (RMS)                   | 15 fs          | 15 fs          | 15 fs          | 15 fs          | 15 fs          |
| Initial beam radius (RMS)                      | 58 μm          | 58 μm          | 58 μm          | 66 μm          | 58 μm          |
| Initial energy spread                          | 0.1 eV         | 0.1 eV         | 0.1 eV         | 0.1 eV         | 0.1 eV         |
| Electrons per pulse                            | 5 k            | 5 k            | 10 k           | 10 k           | 50 k           |
| Accelerating field strength                    | 2.5 keV mm⁻¹   | 2.5 keV mm⁻¹   | 2.0 keV mm⁻¹   | 2.5 keV mm⁻¹   | 7.0 keV mm⁻¹   |
| Pulse duration before compression (RMS)        | 120 fs         | 120 fs         | 120 fs         | 120 fs         | 120 fs         |
| Beam radius before compression                 | 62 μm          | 61 μm          | 61 μm          | 70 μm          | 70 μm          |
| Energy spread before compression               | 0.048%         | 0.12%          | 0.08%          | 0.078%         | 0.12%          |
| Distance from photocathode to THz field        | 2.4 mm         | 2.4 mm         | 3 mm           | 5 mm           | 20 mm          |
| THz frequency                                  | 0.3 THz        | 0.3 THz        | 0.3 THz        | 0.3 THz        | 0.3 THz        |
| THz–e interaction distance                    | 150 μm         | 160 μm         | 150 μm         | 200 μm         | 200 μm         |
| THz field strength                             | 0.6 kV mm⁻¹    | 1.3 kV mm⁻¹    | 1.4 kV mm⁻¹    | 1.9 kV mm⁻¹    | 9.6 kV mm⁻¹    |
| Compressed electron pulse duration (RMS)       | 17.1 fs        | 7.4 fs         | 8.6 fs         | 5.0 fs         | 3.7 fs         |
| Temporal focal length                          | 1.10 mm        | 1.06 mm        | 2.28 mm        | 3.70 mm        | 14.9 mm        |
| Transverse spot size (RMS) at temporal focus   | 67 μm          | 63 μm          | 65 μm          | 74 μm          | 66 μm          |
| Energy spread after compression                | 0.46%          | 0.70%          | 0.66%          | 0.40%          | 0.47%          |
| Divergence after compression                   | 8.40 mrad      | 6.78 mrad      | 4.79 mrad      | 1.61 mrad      | 0.96 mrad      |

Figure 2. Simulation results for the THz compression of 1 keV electron pulse under the circumstance of case 2 in Table 1 unless adjustments are indicated. (a) Temporal evolutions of the pulse duration (circles) and the transverse beam size (stars) as a function of the longitudinal position for three different electrons per pulse. (b) The dependence of the compressed pulse duration (black and green circles) and the arrival time at the sample position (red circles) on the phase shift of the THz field. The black circles are pulse durations at the fixed temporal focus position determined by the simulation with zero phase shift. The green circles are minimum pulse durations at varied temporal focus position which depend on the phase shift. The inset is the schematic illustration of the 1 keV electron pulse entering the THz field at the entrance phase of 1.10 rad and passing through the 160 μm interaction region within 2.6 cycles of the THz field. (c) The dependence of the compressed pulse duration and the temporal focus length on the THz field strength. The inset is the calculated dependence of the gained MC on the entrance phase of the THz field (see the analytical model in the text). (d) The dependence of the pulse duration and the temporal focus length on the THz-electron interaction distance. The inset is the calculated dependence of the MC on the number of THz cycles interacting with the electron pulse (see the analytical model in the text).
Δp_{tail} gain from the THz field can be estimated as:

\[ Δp_{\text{head}} = e * A * \int_{0}^{t_{E-THz}} \sin(\omega t + \varphi) dt \]

\[ Δp_{\text{tail}} = e * A * \int_{t_{\text{dur}}}^{t_{E-THz} + t_{\text{dur}}} \sin(\omega t + \varphi) dt, \]

where \( e \) is the electronic charge of the electron, \( A \) is the amplitude of the THz field, \( t_{\text{dur}} \) is the duration of the electron pulse before compression, \( t_{E-THz} \) is the timescale of the electron pulse passing through the THz field, \( \varphi \) is the phase of THz field when electron beam enters. Then the overall MC gained from the THz field is \( Δp = Δp_{\text{head}} - Δp_{\text{tail}} \). The phase of ~1.0 rad in the analytical model for maximal MC in the inset of figure 2(c) coincides with the optimized entrance phase of 1.10 rad for maximal pulse compression obtained in the simulation. Based on this analytical model, the inset in figure 2(d) displays the modulation of the MC by changing the THz field cycles interacting with electron pulses. In order to compress the electron pulse, the head electrons will be slowed down and the tail electrons will be speeded up after interacting with the THz field, then generally half of the period of the THz field is used for achieving a maximum energy chirp \([18, 20, 21]\). However, the short wavelength of the THz field enables a lengthened interaction period by adding an integer cycle. As demonstrated in the inset of figure 2(d), the 0.5 THz cycle, 1.5 THz cycle, 2.5 THz cycle give the same maximal energy chirp and therefore the same compression ratio. Note that the speed of the low energy electron pulse (<1 keV) is <1.8 × 10\(^7\) m s\(^{-1}\), much smaller than the speed of the THz field. If the low energy electron pulse interacts with 0.5 (2.5) THz cycle, the THz-electron interaction distance is ~30 μm (~160 μm). In this case, to lower the difficulty in fabricating waveguides, we choose the 2.6 THz cycle to perform simulations with GPT. The pulse duration and the temporal focus length as a function of the THz-electron interaction distance, are shown in figure 2(d). The compressed
pulse duration is found to be very sensitive to the interaction distance between the THz field and the electron pulse, so a calculation of the required MC based on the above presented analytical model is necessary before an experimental design and implementation.

The ultra-compact structure and the cost efficiency of the THz waveguide in comparing with conventional RF technique makes the THz field an excellent alternative in compression of medium to high energy electron pulses. In figure 3, we present the simulation results for the compression of 70 keV electron pulses. The minimum pulse duration of 1.4 fs (rms) with $1 \times 10^4$ electrons in single pulse is obtained as shown in figure 3(a). Further compression reaching a sub-fs duration is achievable by further decreasing the electron number and optimizing the electron–THz interaction. Note that, concomitant with the longitudinal compression, the THz magnetic field gives rise to a focusing of the transverse pulse size as shown in figure 3(a). Though the magnetic field is coherently destructive at the position where the electric field is coherently constructive, the small wavelength of the THz field and the velocity dependent Lorentz force make the 70 keV electrons feel the magnetic field. In order to examine the impact of the transverse focusing on the diffraction pattern, we simulate the dependence of the pulse size at the phosphor screen position (0.4 m away from the photocathode) on the phase jitter of the THz magnetic field, and the results are summarized in figure 3(d). The ~0.5 \( \mu m \) transverse spot size change within $\pm 40$ fs phase jitter suggests a negligible impact of the phase jitter on the peak intensity of diffraction spots/rings in UED experiments. The dependence of the compressed pulse duration and the temporal focus length on the THz electric field strength and the electron numbers are shown in figure 3(b). The temporal focus length is roughly 10–20 mm and readily accessible for the experimental implementation. The compressed pulse duration and the temporal focus length as a function of the THz-electron interaction distance are shown in figure 3(c). The inset in figure 3(c) displays the schematic of the interaction between the THz electric field and the electron pulse. The 70 keV electron pulse enters the THz field at the entrance phase of 1.70 rad and passes through the 200 \( \mu m \) interaction region (see table 1) within 0.42 THz cycles.

4. Single-shot stamping of the electron–THz timing jitter

To achieve a sub-fs temporal resolution in the THz compression based UED/microscopy, the timing jitter between the electron pulse and the THz field must be stamped and corrected shot by shot. Here we propose a novel approach, as depicted in figure 1(b), to stamp the electron–THz timing jitter. If no jitter, the reference pulse will pass through the THz magnetic field at the phase crossing the zero, suffering no modulation in the beam position and the transverse spot size. However, the THz phase jitter will shift the passing through phase and modulate the transverse spot size and the transverse center position of the reference pulse (see appendix A for more detailed information). Thus, by tracking these two parameter changes of the reference beam, the corresponding electron–THz timing jitter can be determined.

Specifically, this novel approach can be carried out in two ways: either tracing the beam size change or monitoring the beam position change. We examined the feasibility and accuracy of these two methods by a combination of simulations and analyzing experimental diffraction patterns. The subsequent simulations are based on the compression condition in case 5 in table 1. The schematic in figure 4(a) is the jitter stamping by recording the transverse spot size change of the reference pulse induced by the THz magnetic field. The accuracy in measuring the spot size change of the reference pulse is determined through fitting the FWHM of the (200) reflection in static diffraction patterns of MoTe$_2$ using a Gaussian function. More information about the experimental system can be found elsewhere [60]. As shown in figure 4(a), the accuracy of the fit over 20 images is 0.09 pixels, i.e. 1.4 \( \mu m \) (15.6 \( \mu m \) per pixel of the detector), which is nearly the same as the allowed detection of spot width change in reference [11]. A cosinusoidal modulation of the transverse spot size of the reference pulse generated by tuning the phase of the THz magnetic field is displayed in figure 4(b) (left). The spot size change is characterized at a position of 0.4 m away from the THz field. The linear part, marked by a dotted ellipse, is used for the jitter stamping. The linear change of the transverse spot size on the phase jitter of the THz magnetic field is shown in figure 4(b) (right). The linear ratio in combination with the accuracy in fitting the spot size in figure 4(a) suggests a precision of 1.77 fs for the field strength of 10.5 kV mm$^{-1}$ and 0.83 fs for the field strength of 21.0 kV mm$^{-1}$ in stamping the jitter. The precision can be further improved by enlarging the spot size change, such as lengthening the distance between the THz field and the phosphor screen.

Note that the above method aims at stamping the jitter shot-by-shot, however, the pulse-to-pulse electron number change can also contribute to the spot size change and degenerate the precision of the jitter stamping. As shown in the appendix D, the linear ratio of $\sim 0.8 \mu m$ fs$^{-1}$ retains as the electron number changes, whereas the absolute spot size changes $\sim 24 \mu m$ with every 10% change of the electron number. Fortunately, the electron number change can be corrected by counting the intensity change of the beam at the phosphor screen, therefore, the electron number fluctuation induced degeneration can be monitored.
Figure 4. Snapshot of the electron–THz timing jitter by tracing the transverse spot size change of the reference electron pulse induced by the THz magnetic field. (a) Schematic illustration of the jitter stamping approach. The accuracy for determining the spot size of the reference pulse is estimated by fitting the FWHM of the (200) spot in typical static diffraction patterns of MoTe\(_2\) by a Gaussian function. The accuracy of the fitting is 0.09 pixels (15.6 \(\mu\)m per pixel of the detector). (b) Simulation results for the THz field induced modulation of the transverse spot size under the circumstance of case 5 in table 1. (left) A cosinusoidal modulation of the transverse spot size by tuning the phase of the THz magnetic field. (right) The linear change of the spot size as a function of the phase jitter of the THz magnetic field. The linear ratio and the accuracy for fitting the spot size in (a) indicate a sub-fs precision in the jitter stamping.

and corrected. In order to evaluate the detectable electron number change, we measure the intensity fluctuation of the brightest spot (200) in single crystalline diffraction patterns of MoTe\(_2\), as shown in the appendix E. The measured peak-to-peak intensity fluctuation is \(\sim 0.2\%\), i.e. the upper limit of the electron number change can be measured in experiments is 0.2\%. Then the measurable electron spot size change induced by electron number fluctuation is \(\sim 0.5\ \mu\)m. Based on the linear ratio of 0.79 \(\mu\)m fs\(^{-1}\) in figure 4(b), the degeneration of the jitter stamping precision by the electron number fluctuation can be suppressed to sub-fs. In table 2 in the appendix D, the ratio of the spot size change to the phase jitter is retained as the electron number increases, so much more electrons in the reference pulse can be used to further improve the signal-to-noise ratio and the precision of the jitter stamping.

The schematic for the snapshot of timing jitter by recording the center position shift of the reference beam is shown in figure 5(a). The cosinusoidal modulation of the center position of the reference pulse induced by tuning the phase of the THz magnetic field is shown in figure 5(b) (top right). The position calculating the center position shift is 0.4 m away from the THz field position. The linear part marked by a dotted ellipse is used for the jitter stamping. The linear change of the center position on the phase jitter in the range of \(\pm 60\) fs is shown in figure 5(b) (bottom). The electron pulse pointing instability will degenerate the precision in measuring the center position shift. In UED experimental system, the electron pulse pointing instability is \(\sim 10\ \mu\)m in the single pulse regime [16, 62, 64] and 1.56 \(\mu\)m in accumulation regime [11]. The linear ratio in figure 5(b) (bottom right) in combination with the pointing instability implies a precision of 1.66 fs (with the THz field strength of 21.0 kV mm\(^{-1}\)) in the single shot mode and 0.26 fs (with the THz field strength of 21.0 kV mm\(^{-1}\)) in the accumulation mode in stamping the jitter. The electron
Figure 5. Snapshot of the electron–THz timing jitter by tracing the transverse center position shift of the reference electron pulse induced by the THz magnetic field. The reference pulse passes through the position with the coherent constructive magnetic field, giving rise to a net deflection of the pulse away from the original propagation direction. (a) Schematic illustration of the jitter stamping approach. (b) Under the circumstance of case 5 in table 1, (top right) a cosinusoidal modulation of the center position of the reference pulse by tuning the phase of the THz magnetic field, and (bottom right) the linear change of center positions as a function of the phase jitter of the THz magnetic field. The linear ratio and the pointing stability of 10 μm in reference [16] indicate a precision of 1.66 fs with the 21.0 kV mm$^{-1}$ field strength. (c) The center shift of the reference pulse as a function of the center position shift of the THz magnetic field under the circumstance of case 5 in table 1. The zero position is the spatial center of the coherent constructive THz electric field and the –0.3 mm position is the spatial center of the coherent constructive THz magnetic field.

pulse center shift as a function of the center position change of the THz magnetic field is shown in figure 5(c), which indicates that a well alignment of the reference beam passing through the spatial center of the magnetic field is required for a high precision jitter stamping. Lengthening the distance between the THz field and the phosphor screen will decrease the impact of the beam pointing instability and further improve the precision of the jitter stamping to sub-fs.

5. Conclusions

In this work, we present a comprehensive theoretical study of the electron pulse compression by the THz field, extending to low beam energy region which is not regularly accessed by the conventional RF technique. We discuss the physical foundation for the THz compression of low-to-medium energy electron pulses, and carry out precise numerical simulations and analytical analyses, which pave the way for the further experimental implementation. Our results show that for electron pulse with beam energy less than 1 keV and thousands of electrons per pulse, a sub-10 fs rms pulse duration can be routinely maintained. These results reaffirm our earlier finding of energy independence of CI [48], suggesting that the compressed low energy electron pulses hold the same beam quality as that of the compressed 100 keV and MeV electron pulses. For the THz compression of medium energy electron pulses, a minimum 1.4 fs rms pulse duration can be realized and even sub-fs pulse duration is achievable by reducing the number of electrons per pulse to below 10$^4$. In addition, we propose a novel approach to stamp the THz-electron synchronization jitter on a shot by shot scheme and a sub-fs precision is determined through a comprehensive analysis on typical UED diffraction patterns. This jitter stamping approach is also applicable in UED and UEM experiments using optical field to control electron pulses. Our work shows an overall sub-fs temporal resolution of UED
with a compressed single pulse, which is comparable to that of attosecond pulse trains by optical field compression [1, 14]. Such compressed single pulse based UED with the electron number per pulse several orders of magnitude larger than that of attosecond pulse trains, is a more generic case in exploring laser–matter interaction.

**Contributions**

Yingpeng Qi devised the project and conceived the presented ideas. Lele Yang performed the numerical simulation under the supervision of Yingpeng Qi. Yingpeng Qi performed the theoretical analysis and collected the experimental data. The paper was written by Yingpeng Qi with contributions from all the authors.

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**Data availability**

The data that support the findings of this study are available from the corresponding author upon reasonable request.

**Appendix A. THz magnetic standing field in a THz device**

See figure A.

**Appendix B. The dependence of the scattering angle and the divergence of the free propagating electron pulse on the kinetic energy and the electron number per pulse**

In the schematic of the THz compression of low to medium energy UED in figure 1(a) and subsequent simulations, no magnetic lens is used. Instead, a compact structure from electron source to the sample is taken for an easy experimental implementation, which is similar to the compact concept in conventional UED system [37, 60, 61]. High reciprocal space resolution or $s$-resolution is required in UED systems to identify the ring or spot features in diffraction patterns and track changes of their intensity, position, and width with high precision. With the electron number of $10^3$ to $10^5$, the divergence of the free propagating electron pulse and the scattering angle reflected by the (111) plane of the aluminum film in the energy range of 0.5 keV–70 keV are show in figure B(a) in appendix B. In figure B(b), the ratio between the scattering angle and the divergence indicates that the divergence is smaller than the scattering angles by one order of magnitude, which is comparable to the beam quality in conventional keV and MeV UED [22]. Importantly, as demonstrated by subsequent simulation results in figures 2(a) and 3(a), the compression will not change the divergence of the electron pulse with a moderate charge number. So the compact design for the THz compression of low to medium energy in figure 1(a) is effective and reliable to keep high quality of diffraction patterns. A focusing lens after the sample can be placed to optimize the diffraction patterns [37, 60, 61] if needed. The compact design in compression based UED will improve the compression ability by suppressing the nonlinear curvature of the compression field [48, 63].

**Appendix C. Propagation time of electrons in a single pulse at the positions before compression and at the temporal focus position**

See figure C.
Figure A. (a) Schematic illustration of the THz device used in the simulation. For the simulation results in the main text, the THz standing field is generated between two ideal parallel plates. For a practical THz device, an aperture on parallel plates for the electron pulse passing through is required. In order to explore the impact of the aperture to the compression, we carry out the following simulations with and without apertures. (b) Simulation results of electron pulse compression with and without the entrance and exit apertures. The simulation is carried out based on the circumstance of case 2 in Table 1. Note that no significant impact of the aperture to the compression results is observed. The aperture size of 0.4 mm is large enough for all electrons passing through. (c) THz magnetic standing field in the THz device. The region A with the coherent constructive magnetic field will deflect the reference electron beam in transverse direction, and the region B with the destructive magnetic field will focus (or defocus) the reference electron beam in transverse direction. Both the deflection and the focusing of the reference electron pulse can be detected to stamp the timing jitter between the electron pulse and the THz field. The corresponding strength of the electric field is 9.6 kV mm\(^{-1}\).

Figure B. (a) The dependence of the scattering angle (green circles) and the divergence of the free propagating electron pulse (hexagons) on the kinetic energy and the electron number per pulse. The scattering angles are calculated based on the (111) reflection of the aluminum film. (b) The ratio between the scattering vector and the divergence as a function of the kinetic energy of electron pulse with 5 k electrons per pulse (the 70 keV electron pulse includes 50 k electrons).
Figure C. Propagation time of electrons VS \( Bz (v/c) \) in a single pulse at the positions before compression ((a) and (c)) and at the temporal focus position ((b) and (d)). (a) and (b) Correspond to the circumstance of case 2 in table 1, i.e. the kinetic energy is 1 keV. (c) and (d) Correspond to the circumstance of case 5 in table 1, i.e. the kinetic energy is 70 keV. The calculated std of the propagation time gives the electron pulse duration.

Figure D. The transverse spot size change of the electron pulse as a function of the phase jitter of THz magnetic field and the fluctuation of electron number per pulse. The linear ratio as electron number changes from 40 k to 60 k keeps the same value, while the absolute pulse size changes 24 \( \mu \text{m} \) with every 10% change of the electron number per pulse.

Appendix D. The dependence of the transverse spot size of the electron pulse on the phase jitter of THz magnetic field and the fluctuation of electron number per pulse

See figure D and table 2.

Appendix E. Evaluation of the detectable intensity change

See figure E.
**Table 2.** Dependence of the pulse size change on the number of electrons per pulse.

| Number of electrons per pulse (k) | Pulse size change (μm fs⁻¹) |
|----------------------------------|-----------------------------|
| 50                               | 0.80                        |
| 100                              | 0.84                        |
| 200                              | 0.89                        |
| 500                              | 0.92                        |

*Figure E.* We select the brightest spot (200) in the diffraction pattern of MoTe₂ to evaluate the detectable intensity change, which can be linked to the precision counting the electron number change. (a) The diffraction pattern of MoTe₂. (b) (left) The temporal resolved intensity change of the (200) reflection induced by the femtosecond laser. (right) The intensity change at negative time delays. The peak-to-peak intensity fluctuation is ∼0.2%, i.e. the electron number change of 0.2% can be measured in experiments.

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