Magnetoresistance of a 2D electron gas caused by electron interactions in the transition from the diffusive to the ballistic regime.

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On a high-mobility 2D electron gas we have observed, in strong magnetic fields ($\omega_c \tau > 1$), a parabolic negative magnetoresistance caused by electron-electron interactions in the regime of $k_B T/\hbar \sim 1$, which is the transition from the diffusive to the ballistic regime. From the temperature dependence of this magnetoresistance the interaction correction to the conductivity $\delta \sigma_{xx}^{ee}(T)$ is obtained in the situation of a long-range fluctuation potential and strong magnetic field. The results are compared with predictions of the new theory of interaction-induced magnetoresistance.

Electron-electron interaction (EEI) corrections to the Drude conductivity $\sigma _0$ of 2D systems have been intensively studied over two decades. These studies were based on the theory of interactions in the diffusive regime, $k_B T/\hbar < 1$ \([1]\). Physically this condition implies that the effective interaction time, $\hbar/k_B T$, is larger than the momentum relaxation time $\tau$ and therefore the two interacting electrons experience scattering by many impurities. In the ballistic regime, $k_B T/\hbar > 1$, electrons interact when scattered by a single impurity. A theory of the interaction correction for such a case was only recently developed \([4]\), and there have already been several experimental attempts to apply it to the conductance of high-mobility (large $\tau$) semiconductor structures \([3, 4, 5, 6, 7, 8]\). An essential feature of this theory is that the impurities are treated as point-like scatterers – the condition which is not satisfied in structures where the impurities are separated from the 2D channel by an undoped spacer (unless the spacer is thick enough for the background impurities to dominate the scattering). There is then a question of how the interaction correction in the ballistic regime manifests itself in a smooth fluctuation potential.

Introducing a long-range scattering potential is expected to suppress the interaction correction in the ballistic regime considered in \([2]\). This correction is caused by electron back-scattering, but in the case of a smooth potential the backscattering is significantly reduced. However, as shown in \([4, 14]\), applying a strong magnetic field increases the probability of an electron to return back and restores the interaction correction.

Experimentally, the interaction correction $\delta \sigma_{xx}^{ee}(T)$ is usually obtained from the temperature dependence of the conductance, where it has to be separated from the interference correction $\delta \sigma_{xx}^{WL}(T)$ caused by the weak localization (WL) effect \([1]\), as well as a possible classical contribution from phonon scattering. It has been shown however that in the diffusive regime there is an elegant method of detecting $\delta \sigma_{xx}^{ee}(T)$ from the perpendicular magnetoresistance \([2]\). The interaction correction gives rise to the parabolic negative magnetoresistance (NMR), expressed by the following relation at $\omega_c \tau > 1$:

$$\rho_{xx} = \frac{1}{\sigma_0} + \frac{1}{\sigma_0^2} (\mu^2 B^2) \delta \sigma_{xx}^{ee}(T),$$

(1)

where $\mu$ is the electron mobility. This relation is derived by converting the conductivity tensor into the resistivity tensor and using the fact that in the diffusive regime the Hall conductivity is not affected by interactions: $\delta \sigma_{xy}^{ee}(T) = 0$. Another essential specific of the diffusive regime used in its derivation is that $\delta \sigma_{xx}^{ee}(T)$ and $\delta \sigma_{xy}^{ee}(T)$ are not changed with strong magnetic field applied \([13]\).

There have been several experiments where this method of extracting the interaction correction was used \([12, 14, 15]\). However, apart from the experiment \([14]\) on low-mobility structures, these experiments were in fact performed not in the diffusive but ballistic regime, where Eq. 1 is not justified. The extracted quantity $\delta \sigma_{xx}^{ee}(T)$ should be neither a logarithmic correction in the diffusive regime \([1]\), nor a linear correction in the ballistic regime \([2]\) derived for classically small magnetic fields.

The new theory \([10]\) considers the interactions in the ballistic and intermediate regimes in strong fields $\omega_c \tau > 1$. It shows that interactions in systems with a long-range potential will also produce a parabolic magnetoresistance described by Eq. 1. According to this theory, strong magnetic field not only restores the interaction correction, but also provides the required condition for Eq. 1: $\delta \sigma_{xy}^{ee}(T)/\sigma_{xy} < \delta \sigma_{xx}^{ee}(T)/\sigma_{xx}$. The theory gives a distinct prediction for the magnitude of the magnetoresistance $\delta \sigma_{xx}^{ee}(T)$ in Eq. 1. The aim of this work is to study the magnetoresistance caused by electron-electron interactions in the intermediate regime, in a structure with long-range fluctuation potential, and to compare the results with the prediction of \([10]\).

The sample is a standard modulation doped n-GaAs heterostructure. The doped layer is separated from the conducting channel by an undoped spacer $d = 20$ nm. The wafer has been mesa etched to a Hall bar pattern. Measurements have been performed by a standard 4-terminal method with a current of $0.4 - 2$ nA. The electron mobility changes in the range $0.42 \times 10^5 - 5.5 \times$
$10^5\text{cm}^2/\text{Vs}$ with increasing carrier density, which is lower than that of the samples in [12]. The range of electron densities is from 0.46$\times 10^{11}$ cm$^{-2}$ to 2$\times 10^{11}$ cm$^{-2}$. The parameter $k_B T/\hbar$ in our experiment is varied from 0.04 to 3.8. (This value is 3.3–33 in [14], 1.7–5.6 in [14] and 0.004–0.18 in [14]).

The magnetoresistance of the 2DEG with $n = 6.8 \times 10^{10}$ cm$^{-2}$ is shown in Fig. 1(a). To analyse the data in terms of theory [10], we have to prove first that the experimental conditions satisfy the theoretical approximations. Firstly, in the measured electron density range the $k_F d$ value varies from 1.2 to 2.2, which proves that the fluctuation potential, with the correlation length $d$, is indeed long-range. This is further supported by an estimation of the ratio of the momentum relaxation time to the quantum time found from the magnitude of the Shubnikov–de Haas oscillations: $\tau/\tau_q$ varies from 24 at the highest to 4 at the lowest electron density. Secondly, the magnetoresistance is analysed in the magnetic field range $\omega_c \tau > 1$. Thirdly, the gated structure is suitable for the study of interaction effects as the gate is separated from the 2DEG by 70 nm, while the separation between electrons is in the range from 13 nm to 26 nm and therefore interactions are not screened by the metallic gate.

One can see in Fig. 1 that the negative magnetoresistance exhibits a sharp change in small fields, followed by a parabolic dependence in higher fields. The sharp change is caused by the WL effect which is suppressed by magnetic field. We will analyse the parabolic magnetoresistance in the range of fields well above the ‘transport’ magnetic field $B_{tr} = \hbar/4D\tau \sim 0.013T$, to make sure that the WL contribution to the negative magnetoresistance is negligible. On the other hand, the magnetic field should not be too large, in order to prevent the development of the positive magnetoresistance caused by the Zeeman effect on the interaction correction [1]. This condition is also satisfied in our experiments where $g\mu_B B/k_BT \lesssim 1$. The contribution of this effect to the measured magnetoresistance can be estimated, from the theory of interactions in the diffusive regime [1] and in the ballistic regime with point scatterers [3], as being less than 1% in the entire range of studied fields and thus can be neglected.

After the initial rapid change, the magnetoresistance develops a parabolic dependence, Fig. 1 (b). It is temperature dependent and shows immediately the qualitative features of the model [1], Fig. 1(c). A flat region can be seen at small fields, which is a clear indication that the long-range potential at small field suppresses the interaction correction which is then restored by larger fields. In accordance with the theory, the flat region is better seen at higher temperatures. Another feature is seen in Fig. 1(d) - with increasing temperature the magnetoresistance becomes positive (in the model this is the result of the Hartree term becoming dominant over the exchange term at higher temperatures).

Before proceeding with the analysis of the strength of the magnetoresistance at different temperatures and extracting the temperature dependent EEI correction, we would like to note that there is also a classical contribution to the parabolic negative magnetoresistance in high-mobility structures which can also be seen in strong fields [10, 14]. This NMR originates from the fact that the magnetic field makes it possible for electrons to ‘cycle’ around impurities and become localised, and hence do not contribute to the conductance. The magnitude of this effect is very different for short-range and long-range fluctuation potentials. In the case of short range scatterers the classical magnetoresistance has the form [10, 14, 15]:

$$\rho_{xx} = \rho_0 \frac{1 - p}{1 + \frac{p^2}{(\omega_c \tau)^2}}$$

where $p = e\mu_B (-2\pi/\omega_c \tau)$ is the fraction of cycling electrons, and $\omega_c$ is the cyclotron frequency. Fig. 2 shows the comparison of the prediction by [10] with experimental results at the intermediate electron densities $n = 5.7 \times 10^{10}$ cm$^{-2}$ and $n = 9.0 \times 10^{10}$ cm$^{-2}$. At high densities the magnitude of the classical effect expected for short-range scatterers is much stronger than the measured magnetoresistance. Furthermore, the direction of its change with varying density (different $\tau$) would be opposite to the experimental situation. This proves again that in the studied samples we are dealing with long-range rather than short-range scattering. It is the presence of a smooth potential which significantly decreases the classical NMR [17], since the ‘cycling’ electron trajectories will now become the trajectories ‘wandering’ in the potential landscape. For the moment we assume that the contribution of the classical magnetoresistance is negligible and attribute all magnetoresistance to the interaction correction $\delta\sigma_{xx}^{ee}(T)$ (later we will come back to this question).

We plot the resistivity as a function of $B^2$ and from the slope of the straight line obtain $\delta\sigma_{xx}^{ee}(T)$. Fig. 3 shows the temperature dependence of $\delta\sigma_{xx}^{ee}$ at different electron densities, where experimental points concentrate around one curve. This curve becomes rather close to the interaction correction in the exchange channel [14] if one makes a vertical shift of the theoretical curve by $\Delta\sigma = -0.07e^2/h$. We want to emphasise that apart from this small shift there are no adjustable parameters involved in the analysis. It is interesting to note that the interaction correction $\sigma_{xx}^{ee}(T)$ found from the temperature dependence of the conductance at $B = 0$ is defined with an accuracy of a constant contributing to a renormalised value of the momentum relaxation time. In the method of quadratic magnetoresistance, this constant does not contribute to the magnetoresistance and no shift is allowed in comparing the results with the theory. We will discuss below a possible physical origin of this additional, temperature independent contribution to the quadratic magnetoresistance.
The theoretical curve plotted in Fig. 3 is given by the following expression that describes electron interactions in the exchange channel:

\[
\frac{\delta \rho_{xx}(B)}{\rho_0} = \frac{(\omega \tau)^2}{\pi k_F l} G_F(T \tau),
\]  

(3)

where \(k_F\) is the Fermi wave number and \(l\) is the mean free path. The function \(G_F(x)\) has the asymptotes \(G_F(x \ll 1) \approx -\ln x + \text{const}\) and \(G_F(x \gg 1) \approx (c_0/2)x^{-1/2}\), with \(c_0 \approx 0.276\).

Measurements at different electron densities have enabled us to cover a broad range of the parameter \(T \tau\) and detect the specific features of this dependence. It shows a logarithmic behaviour at small \(T \tau\), followed by a rapid disappearance of the interaction correction at higher temperatures. The results show good agreement with the expected ‘saturation’ at \(T \tau \approx 0.5\), which means that the turning effect of magnetic field is reduced with increasing temperature and electrons have less chance to return back in the long-range potential.

Note that if one naively compared the obtained dependence \(\sigma_{xx}^c(T)\) with the value calculated in [2] (for point-like scatterers in zero field), a striking difference would be seen, Fig. 3 (dotted line). The latter has a linear asymptote at \(T \tau \gg 1\) and does not show any saturation.

It is important to emphasise that the comparison was made with the contribution from the exchange channel only, however it is known that there is another (Hartree) term in interactions controlled by the Fermi liquid interaction parameter \(F_0^s\). Comparing the total interaction correction (exchange plus Hartree [11]) with the experiment shows that the Hartree contribution is much smaller than the exchange contribution. It can be seen in Fig. 3 that within the experimental error the magnitude of the parameter \(F_0^s\) in our case cannot be larger than 0.1–0.2. The value of \(F_0^s\) depends on the parameter \(r_s = 1/(\pi n)^{1/2} a_B\), which is the ratio of the Coulomb and kinetic energy of electrons. The value of the Fermi liquid parameter is only known for \(r_s < 1\):

\[
F_0^s = -\frac{1}{2\pi} \frac{\tau_s}{\sqrt{2 - \tau_s^2}} \ln \left(\frac{\sqrt{2} + \sqrt{2 - \tau_s^2}}{\sqrt{2} - \sqrt{2 - \tau_s^2}}\right).
\]  

We plot this value in Fig. 4, together with results of two recent experiments where it was determined, in different 2D structures, at large \(r_s\). (The latter results were obtained on systems with little effect of the smooth potential, using analysis based on theory [4].) The overall trend on Fig. 4 indicates that in our range of \(r_s = 1.2 - 2.6\) it is reasonable to expect the value of \(F_0^s\) to be \(\approx -0.15\).

Let us now return to the observed shift in Fig. 3. We believe that it is caused by a contribution from the classical NMR [17], known to be \(T\)-independent, which we completely ignored earlier because the long-range potential suppresses the mechanism described by Eq. 2. Quantitatively, the parabolic classical negative magnetoresistance depends on the ratio of the short- and long-range scattering times, \(\tau_s/\tau_L\). For the case of \(\tau_L > \tau_s\), the magnetoresistance is given by the following relation [17]:

\[
\delta \rho_{xx}/\rho_0 = -\omega_0^2/\omega_1^2,
\]  

(4)

where \(\omega_0 = (2\pi n_s)^{1/2} v_F (2l_s/l_L)^{1/4}\), with \(n_s\) the concentration of the strong scatterers, \(l_s\) and \(l_L\) the mean free paths for strong and smooth potential scatterers, respectively. Increasing the contribution of long-range scattering (increasing this ratio) significantly suppresses the classical magnetoresistance. We can estimate the value of \(\tau_L\) using the expression for \(\tau\) in the case of remote donor scattering [2]:

\[
\frac{1}{\tau} = \frac{(2\pi n_s^{2D})}{m(k_F a_B)^3},
\]

for \(n_s^{2D} \approx 3 \times 10^{14} \text{cm}^{-2}\) these estimations give \(\tau_L \approx 6 \text{ps}\) and \(\tau_s \approx 10 \text{ps}\), which are close to the momentum relaxation time \(\tau \approx 6.8 \text{ps}\). An estimate using Eq. 4 shows that this effect can indeed account for the experimentally observed shift. A more accurate comparison is, however, complicated due to uncertainties in the values of \(\tau_s\) and \(\tau_L\) and the fact that Eq. 4 is, strictly speaking, only valid for \(\tau_L \gg \tau_s\).

In conclusion, we have studied the magnetoresistance of a high-mobility 2D electron gas in a GaAs/AlGaAs heterostructure where the electron scattering is determined by a long-range fluctuation potential. In classically strong magnetic fields we have observed negative magnetoresistance, which is parabolic and temperature dependent. We have shown that this magnetoresistance originates from the electron-electron correction to the conductance. We have extracted this correction and demonstrated that it is well described by the recent theory of interactions in the regime which is intermediate between the diffusive and ballistic regimes.

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Figure captions:
Fig. 1. (a) Longitudinal resistivity versus magnetic field for electron density $n = 6.8 \times 10^{10}$ cm$^{-2}$ at different temperatures: $T = 0.2, 0.8, 1.2$ K. (b) The same data presented as a function of $B^2$. (c) Zoomed-in region of $\rho_{xx}(B)$ from (a) at $T = 1.2$ K, showing a flat region at small fields. (d) $\rho_{xx}$ versus $B^2$ for another density, $n = 9 \times 10^{10}$ cm$^{-2}$, at $T = 0.4, 1.2, 4.2$ K, showing a transition from negative to positive magnetoresistance.

Fig. 2. Solid lines: Negative magnetoresistance for the momentum relaxation time $\tau = 6.8$ ps and $\tau = 2.3$ ps. Dashed lines: NMR introduced by the classical effect [16], with $\tau = 6.8$ and 2.3 ps.

Fig. 3. Conductivity correction due to interactions, obtained experimentally at different electron densities $n = 0.46, 0.57, 0.68, 0.90, 1.2, 1.4, 2.0 \times 10^{11}$ cm$^{-2}$ (different symbols for different densities, circles correspond to the lowest density). Solid line - theoretical prediction for the correction due to exchange interaction, shifted by $-0.07e^2/h$. Dashed line - theory for the total correction which includes exchange (Fock) and Hartree interactions with Fermi-liquid parameter $F_\sigma^* = -0.15$. Dotted line shows the result of the interaction theory for point-like scatterers in the transition from the diffusive to the ballistic regime (exchange term at zero magnetic field). Inset: the same results presented in the logarithmic scales.

Fig. 4. The dependence of the Fermi liquid parameter on $r_s$. Dashed box indicates an approximate range of $r_s$ and $F_\sigma^*$ for the structures in this work. Open squares: results from Ref.[3] for hole gas in p-GaAs heterostructures. Solid squares: results from Ref.[4] for electron gas in Si MOSFETs. Solid line is the theoretical curve for small $r_s$. [2].

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(a)\[\rho_{xx} \text{ (Ohm)}\]

(b) $\rho_{xx} \text{ (Ohm)}$

(c) $B(T)$

(d) $B^2(T^2)$

- $T = 0.2 \text{ K}$
- $T = 0.4 \text{ K}$
- $T = 1.2 \text{ K}$
- $T = 4.2 \text{ K}$
\[ \tau = 2.25 \text{ ps} \]

\[ \tau = 6.82 \text{ ps} \]
\( \delta \sigma^{ee} (e^2/h) \)

\( \tau \)

Total,
\( F_0^\sigma = -0.15 \)

Exchange
