A Modified Double Yield Surface Model of Frozen Soil Considering Particle Breakage

Fei Luo¹,², *, FangYan Guo¹,², Siwen Cai¹,², Yuxin Liu¹,², Wei Wei¹,² and Lin Ji¹,²

¹ College of Civil Engineering, Sichuan Agricultural University, Dujiangyan, Sichuan 611830, China
² Sichuan Higher Education Engineering Research Center for Disaster Prevention and Mitigation of Village Construction, Dujiangyan, Sichuan 611830, China
*Corresponding author’s e-mail: 1447151511@qq.com

Abstract: As a basic phenomenon of granular materials under high stress conditions, particle breakage has an important influence on the strength and stress-strain relationship of materials. It is generally believed that that frozen soil undergoes significant particle breakage during the triaxial shearing process. Based on original double yield surface model, a modified double yield surface constitutive model which considered particle breakage has been proposed. Compared with the experimental data, the proposed double yield surface model can describe its strain softening characteristics, dilatancy, elastoplasticity and particle breakage characteristics.

1. Introduction
Double yield surface models have shown significant advantages in describing the soil dilatancy, stress path influence and stress history effects. Therefore, the importance of double yield surfaces has become a more widely accepted modelling method [1]. However, most of the parameters of the double yield surface model are not easily determined, and consequently not easily adopted in the industry. The Shen Zhujiang double yield surface model has been proposed for the analysis of several soft soil foundation projects which is simple in form and the parameters can be completely determined by three axis test [2]. The Shen zhujiang double yield surface model from different perspectives was modified to make the model more widely used by [3-5]. However, the abovementioned double yield surface models have all been proposed for unfrozen soils.

In this paper, an improved double yield surface model from the energy point of view which considers particle breakage is proposed.

2. The Double Yield Surface Model
The double yield surface model uses the two yield surfaces of the volume yield surface and the shear yield surface to describe the yield characteristics of the soil. The yield surface equation is as follows:

\[ \begin{align*}
    f_1 &= p^2 + r^2 q^2 \\
    f_2 &= q^2 / p
\end{align*} \]

(1)

where \( q \) is the generalized shear stress, \( p \) is the mean principal stress, and \( R \) and \( s \) are material parameters.

When the normality flow rule is adopted, the elastic-plastic stress-strain relationship can be expressed as
\[
\{\Delta \varepsilon\} = [D]^{-1} \{\Delta \sigma\} + A_1 \left\{ \frac{\partial f_1}{\partial \sigma} \right\} \Delta f_1 + A_2 \left\{ \frac{\partial f_2}{\partial \sigma} \right\} \Delta f_2
\]  
(2)

where \(A_1\) and \(A_2\) are the plastic coefficients that correspond to the yield surfaces \(f_1\) and \(f_2\), respectively.

In the conventional triaxial tests stress state, \(A_1\) and \(A_2\) obtained by (1) and (2) can be respectively expressed as

\[
A_1 = \frac{1}{4p^2} \left[ \frac{\eta \left( \frac{9}{E} - \frac{3\mu}{E} \frac{3}{G} \right) + 3s \left( \frac{3\mu}{E} - \frac{1}{B} \right)}{3(1 + 3\eta r^2)(s + \eta r^2)} \right]
\]

\[
A_2 = \frac{p^2q^2}{q^2s} \left[ \frac{\eta \left( \frac{9}{E} - \frac{3\mu}{E} \frac{3}{G} \right) - 3r^2\eta \left( \frac{3\mu}{E} - \frac{1}{B} \right)}{3(3s - \eta)(s + \eta r^2)} \right]
\]

where \(G\) and \(B\) are test parameters; \(E\) and \(\mu\) are the tangent modulus and the volume tangent modulus, respectively.

3. The Tangential Modulus \(E_t\)

The stress-strain relationship curves of geomaterials can be divided into strain hardening type and strain softening type according to geometric form. In order to better describe the softening characteristics of frozen soil, the stress-strain relationship in the modified Duncan–Chang model proposed by Lai et al. [6] are modified, and the modified expression is as follows:

\[
\sigma_1 - \sigma_3 = \frac{\varepsilon_1}{j\varepsilon_i^2 + i\varepsilon_i + g}
\]

\[
g = \frac{100}{E_0}
\]

\[
i = \frac{1}{\sigma_1 - \sigma_3} \frac{100}{E_0} \left[ \frac{1}{\sigma_1 - \sigma_3} \frac{2000}{E_0 \varepsilon_m} \frac{\varepsilon_m}{20\varepsilon_m} \right]
\]

\[
j = \frac{100}{E_0 \varepsilon_m^2} \left[ 0.8 + \frac{2\varepsilon_m}{\varepsilon_m} \right]
\]

we can obtain the following:

\[
E_t = \frac{-j\varepsilon_i^2 + g}{(j\varepsilon_i^2 + i\varepsilon_i + g)^2}
\]

4. The Volume Tangent Modulus \(\mu\)

During triaxial shearing, soil particles undergo deformation, slippage, rolling, and particle breakage. Ueng and Chen [7] analyzed the stress and deformation of granular materials under triaxial test conditions from the view of mesoscopic, and obtained the energy balance equation considering particle breakage which is as follows:

\[
\sigma_1 d\varepsilon_i + \sigma_3 (d\varepsilon_i - d\varepsilon) \tan^2 \left( \frac{\pi + \phi}{4} \right) = dE_b (1 + \sin \phi)
\]

where \(\sigma_1\) and \(\sigma_3\) are axial stress and lateral stress respectively, \(\varepsilon_1\), \(\varepsilon_3\) and \(\varepsilon_b\) are axial strain, bulk strain and generalized shear strain respectively, and \(\phi\) is the friction angle that does not consider the effect of dilatancy and particle breakage.
Without considering the elastic shear strain, (7) can be written as the following equation in the $p$-$q$ coordinate system:

$$p d \varepsilon_v + q d \varepsilon_v = p d \varepsilon_v^c + M_c p d \varepsilon_v + \frac{2q-3p}{9} M_c d \varepsilon_v^p + \frac{(3-M_c)(6+4M_c)}{3(6+M_c)} d E_B$$  \hspace{1cm} (8)

where $p$ and $q$ are mean stress and generalized shear stress respectively, and $M_c$ is the critical state stress ratio.

It was found that the elastic volume strain $\varepsilon_v^e$ can be calculated by using the expression suggested by El-Sohby [8]:

$$\varepsilon_v^e = k(p)^l$$  \hspace{1cm} (9)

where the test parameters $k$ and $l$ can be obtained by fitting the rebound test data. In this paper, $k=0.3936$, $l=-0.6366$.

It is recommended to describe the relationship between stress ratio and shear strain by the following formula:

$$M = \frac{q}{p} = A e^{-B\varepsilon_v^c} + C$$  \hspace{1cm} (10)

where $A$, $B$ and $C$ are test parameters.

Based on the above analysis, it is preliminarily assumed that the relationship between the friction angle and the stress ratio during loading is as follows:

$$\tan^2 \left( \frac{\pi + \phi}{4} \right) = \frac{1 + \sin \phi}{1 - \sin \phi} = \frac{3 + 2M}{3 - M}$$  \hspace{1cm} (11)

where the friction angle $\phi$ is a variable in the shearing process.

Therefore, (7) and (8) can be rewritten as follows:

$$\sigma d \varepsilon_v + \sigma_1 (d \varepsilon_v - d \varepsilon_v^c) \tan^2 \left( \frac{\pi + \phi}{4} \right) = d E_B (1 + \sin \phi)$$  \hspace{1cm} (12)

$$p d \varepsilon_v + q d \varepsilon_v = p d \varepsilon_v^c + M p d \varepsilon_v + \frac{2q-3p}{9} M d \varepsilon_v^p + \frac{(3-M)(6+4M)}{3(6+M)} d E_B$$  \hspace{1cm} (13)

The larger the confining pressure is, the greater the energy consumption of particle breakage would be. The relationship between particle breakage energy consumption and axial strain can be expressed as follows:

$$E_B = \frac{\varepsilon_v}{a\varepsilon_v^2 + b\varepsilon_v + c_1}$$  \hspace{1cm} (14)

where $a$, $b$ and $c_1$ are both test parameters.

Taking the derivative of $E_B$, we can obtain the following:

$$\frac{dE_B}{d\varepsilon_v} = \frac{ae_v^2 + (b-2a)\varepsilon_v + c_1 - b}{(ae_v^2 + b\varepsilon_v + c_1)^2}$$  \hspace{1cm} (15)

The volume tangent modulus $\mu_v$ is defined as follows:

$$\mu_v = \frac{d \varepsilon_v}{d \varepsilon_v^c}$$  \hspace{1cm} (16)

According to the test results of the frozen soil test, the modified dilatancy equation is (12), and the following formula can be obtained from (12) and (15):
\[
\mu_i = \frac{d\varepsilon_i}{d\sigma_i} = 1 - \frac{\frac{\sigma_i}{\sigma_3} - \frac{dE_B}{\sigma_3 d\varepsilon_i} (1 + \sin \varphi)}{\tan^2 \left( \frac{\pi}{4} + \frac{\varphi}{2} \right)}
\]

(17) contains the particle breakage energy parameter \(dE_B\), so the influence of particle breakage on the volume tangent modulus \(\mu_i\) is considered.

Let \(G = dE_B/d\varepsilon_i\), and Substituting (10) and (11) into (17), we can obtain:

\[
\mu_i = 1 - \frac{\left[ (6 + M) \sigma_1 - (6 + 4M) G \sigma_3 \right] (3 - M)}{(6 + M) (3 + 2M) \sigma_3}
\]

(18)

5. Test verification

The calculation results of the three models are compared with the test results in order to preliminarily validate the applicability of the proposed model and compare the fitting effect of the three models mentioned above. The comparison results from Fig. 1 and Fig. 2 show that the proposed model is more accurate than the original Shen Zhujiang double yield surface model and the correction model proposed by Zhang et al. when describing the stress-strain relationship of frozen soil. It is worth explaining that the applicability of the proposed model is only preliminarily verified on the condition that the temperature is -3 °C and the confining pressure is 8MPa. Furthermore, the application scope and parameter optimization of the proposed model should be studied in more detail.

![Fig.1 Deviatoric stress-strain strain relationship fitting results](image-url)
5. Conclusions
(1) The key parameters $E_t$ and $\mu_t$ of the Shen Zhujiang double yield surface model are corrected to form the proposed modified double yield surface model.
(2) Comparing with test results, it can be seen that the proposed model are closest to the test results.

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References
[1] Lade, P. V. (1977). “Elasto-plastic stress-strain theory for cohesionless soil with curved yield surfaces”. International Journal of Solids & Structures, Vol. 13, No. 11, pp. 1019-1035.
[2] Shen Z, J. (1990). “A new constitutive model for soils.” Proceedings of the 5th Chinese Conference on Soil Mechanics and Foundation Engineering, Architecture & Building Press, Beijing, China, pp.101-105.
[3] Zhang, B. Y., Jia, Y. A., & Zhang, Z. L. (2007). “Modified Rowe's dilatancy law of rockfill and
Shen Zhujiang's double yield surfaces elastoplastic model.” Chinese Journal of Geotechnical Engineering. Vol. 29, No. 10, pp. 1443-1448.

[4] Luo G., & Zhang J. M. (2004). “Improvement of Duncan-Chang nonlinear model and Shen Zhujiang’s elastoplastic model for granular soils.” Rock and Soil Mechanics, Vol. 25, No. 6, pp. 887-890.

[5] Wang, T. B., Chen, S. S., & Fu Z. Z. (2016). “Two modifications to the Shen Zhujiang's double yield surface model.” Journal of Tongji University. Vol. 44, No. 3, pp. 362-368.

[6] Lai, Y. M., Gao, Z. H., Zhang, S. J., & Chang, X. X. (2007). “Stress-strain relationships and nonlinear mohr strength criteria of frozen soily clay.” Chinese Journal of Rock Mechanics & Engineering, Vol. 50, No. 1, pp. 45-53.

[7] Ueng, T. S., & Chen, T. J. (2015). “Energy aspects of particle breakage in drained shear of soils.” Géotechnique, Vol. 50, No. 1, pp. 65-72.

[8] El-sohby, M. A. (1969). “Elastic behavior of soil.” Proc of Asce, Vol. 95, No. SM6, pp. 1393-1409.