Four Distances for Circular Intuitionistic Fuzzy Sets

Krassimir Atanassov * and Evgeniy Marinov †

Bioinformatics and Mathematical Modelling Department, Institute of Biophysics and Biomedical Engineering, Bulgarian Academy of Sciences, 105 Acad. G. Bonchev Str., 1113 Sofia, Bulgaria; evgeniy.iv.marinov@gmail.com
* Correspondence: krat@bas.bg
† These authors contributed equally to this work.

Abstract: In the paper, for the first time, four distances for Circular Intuitionistic Fuzzy Sets (C-IFSs) are defined. These sets are extensions of the standard IFS that are extensions of Zadeh’s fuzzy sets. As it is shown, the distances for the C-IFS are different than those for the standard IFSs. At the moment, they do not have analogues in fuzzy sets theory. Examples, comparing the proposed distances, are given and some ideas for further research are formulated.

Keywords: distance; intuitionistic fuzzy set; circular intuitionistic fuzzy set

MSC: 03E72

1. Introduction

The concept of an Intuitionistic Fuzzy Set (IFS, see [1,2]) (introduced in 1983) was one of the first in time extensions of L (Zadeh’s fuzzy set [3]). On the other hand, the IFS is also an object of different extensions. One extensions is the Circular IFS (C-IFS, see [4]). It is defined as follows.

Let us have a fixed universe $E$ and its subset $A$. The set $A_r = \{ \langle x, \mu_A(x), \nu_A(x); r \rangle \mid x \in E \}$, where $0 \leq \mu_A(x) + \nu_A(x) \leq 1$ and $r \in [0, \sqrt{2}]$ is a radius of the circle around each element $x \in E$, is called a C-IFS and functions $\mu_A : E \rightarrow [0, 1]$ and $\nu_A : E \rightarrow [0, 1]$ represent the degree of membership (validity, etc.) and non-membership (non-validity, etc.) of element $x \in E$ to a fixed set $A \subseteq E$. Now, we can define also function $\pi_A : E \rightarrow [0, 1]$ by means of $\pi_A(x) = 1 - \mu_A(x) - \nu_A(x)$ and it corresponds to degree of indeterminacy (uncertainty, etc.). Let us remark that in [4], the radius $r$ was defined to take values from the interval $[0,1]$. Here, we extended the region of $r$ values to be $[0, \sqrt{2}]$ because we would like the points with center $\langle 0, 1 \rangle$ and $\langle 1, 0 \rangle$ to be able to cover the whole IFS triangle, which can be valid only if $r \geq \sqrt{2}$. In future, it would be appropriate using the herewith presented form of the C-IFS definition.

When $r = 0$, the C-IFS is transformed to an IFS. On the other hand, when $r > 0$, the C-IFS is an object that is different from the ordinary IFS. In reality, in ordinary IFS theory, there is a way to represent the existence of circles around the elements of universe $E$ (see Figures 1 and 2).

![Figure 1. Geometrical interpretation of an element of an IFS.](image-url)
Figure 2. Geometric representation of different circular IFSs onto the IFS interpretation triangle.

A metric (topological) space can be thought of as a very basic space that satisfies a few axioms. The ability to measure and compare distances between elements of a set is often crucial, and it provides more structure than general topological space possesses (see, [5, 6]).

When we refer to the elements or “points” of the underlying set, we do not necessarily refer to geometrical points, although this is how most of us usually visualize them. They may be objects of any type, such as sequences, functions, images, sounds, signals, decisions, etc.

**Definition 1 ([5, 6]).** A metric on a set $X$ is a function $d : X \times X \to \mathbb{R}$ with the following properties:
1. $d(x, y) \geq 0$ for all $x, y \in X$, and equality holds if and only if (iff) $x = y$.
2. $d(x, y) = d(y, x)$ for all $x, y \in X$ (symmetry).
3. $d(x, z) \leq d(x, y) + d(y, z)$ for all $x, y, z \in X$ (the triangle inequality).

We call $d(x, y)$ the distance between $x$ and $y$, and the pair $(X, d)$ a metric space.

It is evident that $d$ has the properties we expect when we measure a distance between points in rigid geometry. Let us now introduce two from the most popular metrics in $\mathbb{R}^n$, for any positive number $n$.

**Definition 2 ([6, 7]).** Taking any $x = (x_1, \ldots, x_n), y = (y_1, \ldots, y_n) \in \mathbb{R}^n$, let us define:
1. Euclidean metric:
   
   \[ d_2(x, y) = \sqrt{\sum_{i=1}^{n} (x_i - y_i)^2} \]

2. Manhattan (Hamming) metric:

   \[ d_1(x, y) = \sum_{i=1}^{n} |x_i - y_i| \]

In the present paper, for the first time, we will introduce distances over two C-IFSs. Ideas for norms, metrics and distances over IFSs were originally introduced in [8] and described in more details in [9], where the first two distances were given. The next two distances, which are extensions of the first two, were introduced in [10] by E. Szmidt and J. Kacprzyk. In [2], these two distances were called after their names. Later, a lot of other distances were introduced over IFSs (see, e.g., [7, 11–39]).

The first four distances over IFSs are the following.
Let us have two Intuitionistic Fuzzy Sets (IFSs; see [2, 9]) \( A \) and \( B \):
\[
A = \{ (x, \mu_A(x), \nu_A(x)) | x \in E \}, \\
B = \{ (x, \mu_B(x), \nu_B(x)) | x \in E \},
\]
where \( \mu_A, \nu_A, \mu_B, \nu_B : E \to [0, 1] \) and \( \mu_A(x) + \nu_A(x) \leq 1, \mu_B(x) + \nu_B(x) \leq 1 \) for each \( x \in E \).
Let everywhere below \( C_E \) be the cardinality of universe \( E \). In [2, 9] the following distances are described:
\[
H_2(A, B) = \frac{1}{2C_E} \sum_{x \in E} (|\mu_A(x) - \mu_B(x)| + |\nu_A(x) - \nu_B(x)|),
\]
(intuitionistic fuzzy Hamming distance) \( (1) \)
\[
E_2(A, B) = \sqrt{\frac{1}{2C_E} \sum_{x \in E} ((\mu_A(x) - \mu_B(x))^2 + (\nu_A(x) - \nu_B(x))^2)},
\]
(intuitionistic fuzzy Euclidean distance) \( (2) \)
\[
H_3(A, B) = \frac{1}{2C_E} \sum_{x \in E} (|\mu_A(x) - \mu_B(x)| + |\nu_A(x) - \nu_B(x)| + |\pi_A(x) - \pi_B(x)|),
\]
(Szmidt and Kacprzyk’s form of intuitionistic fuzzy Hamming distance) \( (3) \)
\[
E_3(A, B) = \sqrt{\frac{1}{2C_E} \sum_{x \in E} ((\mu_A(x) - \mu_B(x))^2 + (\nu_A(x) - \nu_B(x))^2 + (\pi_A(x) - \pi_B(x))^2)},
\]
(Szmidt and Kacprzyk’s form of intuitionistic fuzzy Euclidean distance) \( (4) \)

2. Definitions of the First Four Distances over C-IFSs

Here, we introduce the following four distances for C-IFS that are modifications of distances (1)–(4) as follows:
\[
H_2(A, B) = \frac{1}{2} \left( \frac{|r_A - r_B|}{\sqrt{2}} + \frac{1}{2C_E} \sum_{x \in E} (|\mu_A(x) - \mu_B(x)| + |\nu_A(x) - \nu_B(x)|) \right)
\]
(intuitionistic fuzzy Hamming distance) \( (5) \)
\[
E_2(A, B) = \frac{1}{2} \left( \frac{|r_A - r_B|}{\sqrt{2}} + \sqrt{\frac{1}{2C_E} \sum_{x \in E} ((\mu_A(x) - \mu_B(x))^2 + (\nu_A(x) - \nu_B(x))^2)} \right)
\]
(intuitionistic fuzzy Euclidean distance) \( (6) \)
\[
H_3(A, B) = \frac{1}{2} \left( \frac{|r_A - r_B|}{\sqrt{2}} + \frac{1}{2C_E} \sum_{x \in E} (|\mu_A(x) - \mu_B(x)| + |\nu_A(x) - \nu_B(x)| + |\pi_A(x) - \pi_B(x)|) \right)
\]
(Szmidt and Kacprzyk’s form of intuitionistic fuzzy Hamming distance) \( (7) \)
\[
E_3(A, B) = \frac{1}{2} \left( \frac{|r_A - r_B|}{\sqrt{2}} + \sqrt{\frac{1}{2C_E} \sum_{x \in E} ((\mu_A(x) - \mu_B(x))^2 + (\nu_A(x) - \nu_B(x))^2 + (\pi_A(x) - \pi_B(x))^2)} \right)
\]
(Szmidt and Kacprzyk’s form of intuitionistic fuzzy Euclidean distance) \( (8) \)

Obviously, if \( A \) and \( B \) are standard IFS, i.e., \( r_A = r_B = 0 \), the new distances coincide with (1)–(4).
Let $C_{IFS}(E)$ and $IFS(E)$ be the sets of all C-IFSs and of all IFSs over the universe $E$, respectively. As we mentioned above, $A \in IFS(E)$ iff $A \in C - IFS(E)$ and $r_A = 0$.

**Theorem 1.** For any $A_{r_A}, B_{r_B} \in C\text{-IFS}(E)$, that is $A, B \in IFS(E)$ where $r_A, r_B \in [0, \sqrt{2}]$, the expressions (5)–(8) are well-defined metrics (distances).

**Proof.** We need to show that the formulas $H_2(A_{r_A}, B_{r_B}), H_3(A_{r_A}, B_{r_B}), E_2(A_{r_A}, B_{r_B}), E_3(A_{r_A}, B_{r_B})$ stated in expressions (5)–(8) obey the three axioms for a metric from Definition 1.

As it has already been shown, the expressions stated in (1)–(4) are well-defined distances in $IFS(E)$. Let us take $D$ to be any one of $H_2, H_3, E_2$ or $E_3$. Therefore, $D$ is a metric in $IFS(E)$.

Since it is obvious from the definition of C-IFSs, $A_{r_A} = B_{r_B}$ in $C - IFS(E)$ iff $A = B$ in $IFS(E)$ and $r_A = r_B$. But $A = B$ in $IFS(E)$ iff $D(A, B) = 0$ and the sum of two non-negative numbers is 0 iff both numbers are equal to 0, therefore the validity of the first axiom for a distance is proved.

The validity of the second axiom is obvious since $D$ is symmetric.

In order to show the validity of the third axiom, let us take a third C-IFS $C_{r_C}$ and show that the triangle property

$$D(A_{r_A}, C_{r_C}) \leq D(A_{r_A}, B_{r_B}) + D(B_{r_B}, C_{r_C})$$  

holds.

We know that

$$D(A, C) \leq D(A, B) + D(B, C)$$

holds for the IFSs $A$, $B$ and $C$. From well-known inequality $|x| + |y| \geq |x + y|$ for three real numbers it follows that

$$|r_C - r_A| \leq |r_B - r_A| + |r_C - r_B|$$

for all choices of $r_A, r_B, r_C \in [0, \sqrt{2}]$. Therefore, summing up both sides of the last two inequality expressions (10) and (11), the validity of (9), i.e., the third axiom for distance holds.

**Remark 1.** From the definition of $H_2, H_3$ and $E_2, E_3$ and since for any two IFS, $A, B$ and $x \in E: |\pi_B(x) - \pi_A(x)| \geq 0$, the following inequalities are valid.

- $H_2(A, B) \leq H_3(A, B)$
- $E_2(A, B) \leq E_3(A, B)$

3. **Numerical Example**

A numerical example of an IFS with $E = \{0, 1, 2\}$, $C_E = 3$ and $A_{r_A}, B_{r_B} \in C - IFS(E)$ is depicted on Figure 3. For these IFSs, $A$ and $B$, we have that $r_A = 0.1$ and $r_B = 0.06$ and the degrees of the corresponding elements $x$ from the universe $E$ are given in Table 1.

| $x \in E$ | $\mu_A(x)$ | $\nu_A(x)$ | $\pi_A(x)$ | $\mu_B(x)$ | $\nu_B(x)$ | $\pi_B(x)$ |
|-----------|------------|------------|------------|------------|------------|------------|
| 0         | 0.321      | 0.070      | 0.609      | 0.241      | 0.049      | 0.710      |
| 1         | 0.099      | 0.102      | 0.799      | 0.075      | 0.071      | 0.854      |
| 2         | 0.200      | 0.699      | 0.101      | 0.150      | 0.489      | 0.361      |
Let us consider the example of the two IFS, $A_{r_A}$ and $B_{r_B}$ from the previous section. A simple computation applying the corresponding formulas and the concrete values for the arbitrary chosen $A_{r_A}, B_{r_B}$ shows that

$$\frac{|r_A - r_B|}{\sqrt{2}} = \frac{|0.1 - 0.06|}{\sqrt{2}} = 0.0283,$$

$$H_2(A, B) = 0.069 \text{ and } E_2(A, B) = 0.096,$$

$$H_3(A, B) = 0.092 \text{ and } E_3(A, B) = 0.123.$$

Hence we conclude that,

$$H_2(A_{r_A}, B_{r_B}) = \frac{1}{2} \left( \frac{|r_A - r_B|}{\sqrt{2}} + H_2(A, B) \right) = 0.049,$$

$$E_2(A_{r_A}, B_{r_B}) = \frac{1}{2} \left( \frac{|r_A - r_B|}{\sqrt{2}} + E_2(A, B) \right) = 0.062,$$

and

$$H_3(A_{r_A}, B_{r_B}) = \frac{1}{2} \left( \frac{|r_A - r_B|}{\sqrt{2}} + H_3(A, B) \right) = 0.083,$$

$$E_3(A_{r_A}, B_{r_B}) = \frac{1}{2} \left( \frac{|r_A - r_B|}{\sqrt{2}} + E_3(A, B) \right) = 0.089.$$

The results from the comparison of the four different distances are shown in Table 2.

| $D$     | $H_2(A_{r_A}, B_{r_B})$ | $E_2(A_{r_A}, B_{r_B})$ | $H_3(A_{r_A}, B_{r_B})$ | $E_3(A_{r_A}, B_{r_B})$ |
|---------|-------------------------|-------------------------|-------------------------|-------------------------|
| $D(A_{r_A}, B_{r_B})$ | 0.049                   | 0.062                   | 0.083                   | 0.089                   |
The reader may compare the values of the different distances from $A$ and $B$ with Remark 1. The plot from Figure 3 is taken from a software implementation for computation and visualization in an interactive mode of C-IFS and different distances for them. In a future research, the authors will go into more detail introducing multiple distances of C-IFSs and presenting a software implementation about them. Figure 3 was created with the Python’s library Matplotlib and it shows what happens if we fix $A$, $B$ and $r_A$ and change the length of $r_B$. In future, new distances for C-IFS will be introduced. Each distance over ordinary IFSs can be transformed for the case of C-IFS by the method discussed in Section 2.

4. Result and Discussion

Below we show that each one of the distances satisfies the triangular inequality for three arbitrary C-IFS. Let us consider $A_{r_A}, B_{r_B} \in C\text{-}IFS(E)$ from Section 3 and take another $C_{r_C} \in C\text{-}IFS(E)$ with degrees of the corresponding elements $x$ from the universe $E$ given in Table 3.

Table 3. Degrees of the element $x$ in $C_{r_C}$.

| $x \in E$ | $\mu_C(x)$ | $\nu_C(x)$ | $\pi_C(x)$ |
|-----------|-------------|-------------|-------------|
| 0         | 0.2411      | 0.2523      | 0.5069      |
| 1         | 0.0746      | 0.3413      | 0.584       |
| 2         | 0.1501      | 0.7293      | 0.120       |

Applying the formulas from Section 2 as in the previous Section we obtain the values for the different distances between all combinations of pairs from $\{A_{r_A}, B_{r_B}, C_{r_C}\} \in C\text{-}IFS(E)$ given in Table 4.

Table 4. Values of the distances $H_2, E_2, H_3, E_3$.

| $D$                  | $H_2$ | $E_2$ | $H_3$ | $E_3$ |
|----------------------|-------|-------|-------|-------|
| $D(A_{r_A}, C_{r_C})$ | 0.058 | 0.072 | 0.086 | 0.088 |
| $D(A_{r_A}, B_{r_B})$ | 0.049 | 0.062 | 0.083 | 0.089 |
| $D(B_{r_B}, C_{r_C})$ | 0.067 | 0.092 | 0.126 | 0.127 |

For the above table if for any of the columns if we pick up an arbitrary permutation of the row indices let the corresponding values be $a, b, c$. Then it can be easily checked that $a \leq b + c$ which exactly shows the validity of the triangular inequality of any of the proposed distances for the C-IFSs $A_{r_A}, B_{r_B}, C_{r_C}$. As an example, let us take the column $H_3$ and the permutation of the row indices 3, 1, 2, then $a = 0.126, b = 0.086, c = 0.083$ and $0.126 < 0.086 + 0.083 = 0.169$.

5. Conclusions

The present paper is the second one devoted to C-IFS. Here, for the first time, distances for C-IFS were introduced. The introduced distances could be applied in diverse areas where objects and processes can be evaluated in more detail compared to an ordinary IFS. In future, new distances over C-IFSs will be introduced and some of their properties will be studied. In the meantime, the concept of an C-IFS was extended to Elliptic IFS (E-IFS) [40]. The present and other distances will be re-formulated for the E-IFSs.

Author Contributions: All authors have participated in equal measure in the conceptualization, preparation, editing and final layout of the presented work. Conceptualization, K.A.; Formal analysis, K.A.; Investigation, E.M.; Methodology, K.A.; Software, E.M.; Supervision, K.A.; Visualization, E.M.
Writing—original draft, K.A. and E.M. All authors have read and agreed to the published version of the manuscript.

**Funding:** This research was funded by the National Science Fund of Bulgaria under Grant Ref. No. DN-02-10/2016.

**Institutional Review Board Statement:** Not applicable.

**Informed Consent Statement:** Not applicable.

**Data Availability Statement:** Not applicable.

**Conflicts of Interest:** The authors declare no conflict of interest.

**References**

1. Atanassov, K. Intuitionistic fuzzy sets. In *VII ITKR's Session*; Deposited in Central Sci.—Techn. Library of Bulg. Acad. of Sci., 1697/84; Sofia, Bulgaria, June 1983. (In Bulgarian).
2. Atanassov, K. *On Intuitionistic Fuzzy Sets Theory*; Springer: Berlin, Germany, 2012.
3. Zadeh, L. Fuzzy sets. *Inf. Control* **1965**, 8, 338–353.
4. Atanassov, K. Circular intuitionistic fuzzy sets. *J. Intell. Fuzzy Syst.*, **2020**, 39, 5981–5986.
5. Adams, C.; Franzosa, R. *Introduction to Topology—Pure and Applied*; Pearson Prentice Hall: Hoboken, NJ, USA, 2008.
6. Kuratowski, K. *Topology*; Academic Press: New York, NY, USA; London, UK, 1966; Volume I.
7. Marinov, E.; Szmidt, E.; Kacprzyk, J.; Tsvetkov, R. A modified weighted Hausdorff distance between intuitionistic fuzzy sets. In Proceedings of the 2012 6th IEEE International Conference Intelligent Systems, Sofia, Bulgaria, 6–8 September 2012; pp. 138–141.
8. Atanassov K. Norms and metrics over intuitionistic fuzzy sets. *BUSEFAL* **1993**, 55, 11–20.
9. Atanassov, K. *Intuitionistic Fuzzy Sets: Theory and Applications*; Springer: Heidelberg, Germany, 1999.
10. Szmidt, E.; Kacprzyk, J. Distances between intuitionistic fuzzy sets. *Fuzzy Sets Syst.* **2000**, 114, 505–518.
11. Abdullah, L.; Ismail, W. K. W. Hamming Distance in Intuitionistic Fuzzy Sets and Interval Valued Intuitionistic Fuzzy Sets: A Comparative Analysis. *Adv. Comput. Math. Its Appl.* **2012**, 1, 7–11.
12. Atanassov, K. A new approach to the distances between intuitionistic fuzzy sets. In *Information Processing and Management of Uncertainty in Knowledge-Based Systems*; Hullermeier, E., Kruse, R., Hoffmann, F., Eds.; Springer: Heidelberg, Germany, 2010; pp. 581–590.
13. Atanassov, K.; Vassilev, P.; Tsvetkov, R. *Intuitionistic Fuzzy Sets, Measures and Integrals*; Academic Publishing House “Prof. M. Drinov” Sofia, Bulgaria, 2013.
14. Gong, Z.; Xu, X.; Yang, Y.; Zhou, Y.; Zhang, H. The spherical distance for intuitionistic fuzzy sets and its application in decision analysis. *Technol. Econ. Dev. Econ.* **2016**, 22, 393–415.
15. Han, J.; Yang, Z.; Sun, X.; Xu, G. Chordal distance and non-Archimedean chordal distance between Atanassov’s intuitionistic fuzzy sets. *J. Intell. Fuzzy Syst.*, **2017**, 33, 3889–3894.
16. Hatzimichailidis, A. G.; Papakostas, G. A.; Kaburlasos, V. G. A Novel Distance Measure of Intuitionistic Fuzzy Sets and Its Application to Pattern Recognition Problems. *Int. J. Intell. Syst.* **2012**, 27, 396–409.
17. Ke, D.; Song, Y.; Quan, W. New distance measure for Atanassov’s intuitionistic fuzzy sets and its application in decision making. *Symmetry* **2018**, 10, 429.
18. Luo, D.; Xiao, J. Distance and Similarity between Intuitionistic Fuzzy Sets. In Proceedings of the 2013 IEEE International Conference on Mechanical and Automation Engineering (MAEE), Jiujang, China, 21–23 July 2013; pp. 157–160.
19. Luo, M.; Zhao, R. A distance measure between intuitionistic fuzzy sets and its application in medical diagnosis. *Artif. Intell. Med.* **2018**, 89, 34–39.
20. Luo, X.; Li, W. Novel Distance Measure Between Intuitionistic Fuzzy Sets. *Xitong Fangzhen Xuebao/J. Syst. Simul.* **2017**, 29, 2360–2372.
21. Ngan, R. T.; Son, L. H.; Cuong, B. C.; Ali, M. H-max distance measure of intuitionistic fuzzy sets in decision making. *Appl. Soft Comput. J.* **2018**, 69, 393–425.
22. Papakostas, G. A.; Hatzimichailidis, A. G.; Kaburlasos, V. G. Distance and similarity measures between intuitionistic fuzzy sets: A comparative analysis from a pattern recognition point of view. *Pattern Recognit. Lett.* **2013**, 34, 609–1622.
23. Peng, D.-H.; Gao, C.-Y.; Gao, Z.-F. Generalized hesitant fuzzy synergetic weighted distance measures and their application to multiple criteria decision-making. *Appl. Math. Model.* **2013**, 37, 5837–5850.
24. Szmidt, E. *Distances and Similarities in Intuitionistic Fuzzy Sets*; Studies in Fuzziness and Soft Computing; Springer: Berlin/Heidelberg, Germany, 2014; Volume 307.
25. Szmidt, E.; Kacprzyk, J. A concept of similarity for intuitionistic fuzzy sets and its use in group decision making. In Proceedings of 2004 IEEE International Conference on Fuzzy Systems, Budapest, Hungary, 25–29 July 2004; Volume 2, pp. 1129–1134.
26. Szmidt, E.; Kacprzyk, J. Similarity of intuitionistic fuzzy sets and the Jaccard coefficient. In Proceedings of Tenth International Conference IPMU’2004, Perugia, Italy, 4–9 July 2004; Volume 2, pp. 1405–1412.
27. Szmidt, E.; Kacprzyk, J. New measures of entropy for intuitionistic fuzzy sets. *Notes Intuit. Fuzzy Sets* **2005**, *11*, 12–20.
28. Szmidt, E.; Kacprzyk, J. Distances between intuitionistic fuzzy sets and their applications in reasoning. In *Computational Intelligence for Modelling and Prediction*; Halgamuge, S., Wang, L., Eds.; Springer: Berlin, Germany, 2005; pp. 101–116.
29. Szmidt, E.; Kacprzyk, J. A new concept of a similarity measure for intuitionistic fuzzy sets and its use in group decision making. *Lect. Notes Artif. Intell.* **2005**, *3558*, 272–282.
30. Szmidt, E.; Kacprzyk, J. Similarity measures for intuitionistic fuzzy sets. In *Issues in the Representation and Processing of Uncertain Imprecise Information: Fuzzy Sets, Intuitionistic Fuzzy Sets, Generalized Nets, and Related Topics*; Atanassov, K., Kacprzyk, J., Krawczak, M., Szmidt, E., Eds.; Akademicka Oficyna Wydawnictwo EXIT: Warsaw, Poland, 2005; pp. 355–372.
31. Szmidt, E.; Kacprzyk, J. Distances between intuitionistic fuzzy sets: Straightforward approaches may not work. In Proceedings of the 3rd International IEEE Conference “Intelligent Systems” (IS’06), London, UK, 4–6 September 2006; pp. 716–721.
32. Szmidt, E.; Kacprzyk, J. Analysis of similarity measures for Atanassov’s intuitionistic fuzzy sets. In Proceedings of the IFSA/EUSFLAT 2009, Lisboa, Portugal, 20–24 July 2009; pp. 1416–1421.
33. Szmidt, E.; Kacprzyk, J. Some remarks on the Hausdorff distance between Atanassov’s intuitionistic fuzzy sets. In Proceedings of the EUROFUSE Workshop’09, Preferences Modelling and Decision Analysis, Pamplona, Spain, 16–18 September 2009; Public University of Navarra: Pamplona, Spain, 2009; pp. 311–316.
34. Xu, C. Improvement of the distance between intuitionistic fuzzy sets and its applications. *J. Intell. Fuzzy Syst.* **2017**, *33*, 1563–1575.
35. Xue, W.; Xian, S.; Dong, Y. A Novel Intuitionistic Fuzzy Induced Ordered Weighted Euclidean Distance Operator and Its Application for Group Decision Making. *Int. J. Intell. Syst.* **2017**, *32*, 739–753.
36. Yang, Y.; Chiclana, F. Consistency of 2D and 3D distances of intuitionistic fuzzy sets. *Expert Syst. Appl.* **2012**, *39*, 8665–8670.
37. Zeng, S. Some intuitionistic fuzzy weighted distance measures and their application to group decision making. *Group Decis. Negot.* **2013**, *22*, 281–298.
38. Zhang, H. Entropy for intuitionistic fuzzy sets based on distance and intuitionistic index. *Int. J. Uncertain., Fuzziness Knowlege-Based Syst.* **2013**, *21*, 139–155.
39. Zhang, Q.; Huang, Y.; Xing, H.; Liu, F. Distance measure, information entropy and inclusion measure of intuitionistic fuzzy sets and their relations. *Int. J. Adv. Comput. Technol.* **2012**, *4*, 480–487.
40. Atanassov, K. *Elliptic Intuitionistic Fuzzy Sets*; Comptes Rendus de l’Académie Bulgare des Sciences; Academic Publishing House “Prof. M. Drinov”: Sofia, Bulgaria, 2021; Volume 74, (to appear).