Quantum discord: ‘discord’ between the whole and its constituent

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Abstract

Quantum discord, a measure of quantum correlation beyond entanglement, is initially defined as the discord between two classically equivalent while quantum discordant definitions of mutual information. In this paper, we report some new interpretations of discord which rely on the differences between measurement-induced effects on the local measured system and the whole system. Specifically, with proper quantitative definitions introduced in Buscemi \textit{et al} (2008 \textit{Phys. Rev. Lett.} \textbf{100} 210504), we find that quantum discord can be interpreted as the differences of measurement-induced disturbance or information gain on the local measured system and on the whole system. Combined with previous similar results based on measurement-induced entanglement and decoherence, our results provide a unified view on quantum discord.

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1. Introduction

In quantum theory, quantum systems can have correlations different from classical correlation. For example, quantum entanglement is a special quantum correlation which enables quantum teleportation, superdense coding, quantum key distribution and is considered as an important resource for quantum computation \cite{1}. Then, can quantum systems have other nonclassical correlations beyond entanglement that still provide advantages over their classical opponents? Ignited by one important algorithm named deterministic quantum computation with one qubit (DQC1) \cite{2}, which contains negligible amounts of entanglement during the whole computation process, much attention has been paid recently to answer the above question with different proposed nonclassical correlations. Among these nonclassical correlations, quantum discord attracts particular attention \cite{3}. 

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The definition of quantum discord [4, 5] for a bipartite state $\rho^{AB}$ comprises two different definitions of quantum mutual information which are extensions of two equivalent definitions of classical mutual information. Quantum mutual information,

$$F^{A:B}(\rho^{AB}) = [S(\rho^A) + S(\rho^B) - S(\rho^{AB})] = S(\rho^A) - S(\rho^{A|B}),$$

quantifies the total correlations in a bipartite state $\rho^{AB}$, where $S(\rho^{A|B}) = S(\rho^{AB}) - S(\rho^B)$ is the conditional entropy of the $A$ side. Alternative mutual information based on a general measurement $M^B$ on party $B$ is

$$J^{\rightarrow}_{M^B}(\rho^{AB}) = S(\rho^A) - \sum_m p(m)S(\rho^A_m) = S(\rho^A) - S(A|B_C),$$

where $\rho^A_m$ corresponds to $A$’s state conditioned on $B$’s measurement output $m$, and $S(A|B_C) = \sum_m p(m)S(\rho^A_m)$ gives $A$’s averaged entropy conditioned on $B$’s measurement outcomes. Since $J^{\rightarrow}_{M^B}(\rho^{AB})$ is the correlation obtained from $B$’s local measurement and classical communication from $B$ to $A$, it is considered as the classical correlation between $A$ and $B$. Subtracting classical correlation $J^{\rightarrow}_{M^B}(\rho^{AB})$ from the total correlation measure $F^{A:B}(\rho^{AB})$, we obtain a quantum correlation measure

$$F^{A:B}(\rho^{AB}) - J^{\rightarrow}_{M^B}(\rho^{AB}) = S(A|B_C) - S(A|B),$$

which is measurement dependent. Quantum discord is measurement independent and is obtained through minimizing the above difference or maximizing the classical correlation $J^{\rightarrow}_{M^B}(\rho^{AB})$ over all possible measurements,

$$D^{\rightarrow}(\rho^{AB}) = \min_{M^B} \left[ F^{A:B}(\rho^{AB}) - J^{\rightarrow}_{M^B}(\rho^{AB}) \right]$$

$$= [S(\rho^A) + S(\rho^B) - S(\rho^{AB})] - \max_{M^B} \left[ S(\rho^A) - \sum_m p(m)S(\rho^A_m) \right]$$

$$= S(\rho^B) - S(\rho^{AB}) + \min_{M^B} \sum_m p(m)S(\rho^A_m)$$

$$= \min_{M^B} [S(A|B_C) - S(A|B)]. \quad (1)$$

In order to attain the minimum in equation (1), we do not have to go through all general measurements; a set of rank-1 positive operator valued measurements (POVMs) of $B$ is enough. The reason is the concavity of conditional entropy over the convex set of POVMs and the minimum is attained on the extremal points of the set of POVMs, which are rank-1 [3, 6].

Studies on quantum discord have been developed quickly in recent years; for a comprehensive and insightful review we recommend [3]. In [7], through direct calculations, Datta et al showed that quantum discord scales with the calculation efficiency which provides the first quantitative evidence that quantum correlation beyond entanglement plays a role in the speedup associated with a quantum algorithm. Some interesting operational interpretations of quantum discord such as in terms of quantum state merging [8] were put forward in [9, 10]. In [11], quantum discord was related to the irreversibility of entanglement dilution and distillation. The dynamics of quantum discord were already discussed in [12, 13]. In [14, 15], quantum discord was linked to entanglement generation between the bipartite system and the measuring apparatus. We note that, besides entanglement generation, quantum measurement also introduces disturbance on the measured system and provides information gain which should have links with quantum discord. In this paper, we link quantum discord to all these measurement-induced effects through their differences on the measured subsystem and on the whole system, hence giving a unified view on quantum discord in terms of measurement-induced effects.
The organization of this paper is outlined as follows. Firstly, we introduce the information gain and disturbance of a quantum measurement defined by Buscemi et al [16]. These definitions satisfy an information–disturbance tradeoff relation and balance the information in quantum measurements. Secondly, we apply them to a local rank-1 POVM measurement on a bipartite system. It turns out that a general local rank-1 POVM has different disturbances or information gains on the local measured system and on the whole system. We will show that these differences are related to quantum discord and provide a unified view on quantum discord in terms of measurement-induced effects. Finally, we draw our conclusion.

2. Quantum measurement: quantitative tradeoff between disturbance and information gain

Quantum measurement provides information on the measured system; at the same time it also introduces disturbance and destroys the coherence in the measured system. The pursuit of proper definitions for information gain and measurement-induced disturbance which satisfy consistent information–disturbance tradeoff relation takes a long time. It is in [16] that, in terms of previous proposed concepts, Buscemi et al gave both definitions and obtained the global information balance for arbitrary measurements. As these definitions are essential for our discussion, we first give a detailed introduction of them.

A general measurement process $\mathcal{M}^B$ on the input system $B$, described by the input density matrix $\rho^B$ on the (finite-dimensional) Hilbert space $\mathcal{H}^B$, can be described as a collection of classical outcomes $X := \{m\}$, together with a set of completely positive (CP) maps $\{\mathcal{E}_m^B\}$, such that, when the outcome $m$ is observed with probability $p(m) = \text{tr}[\mathcal{E}_m^B(\rho^B)]$, the corresponding a posteriori state $\rho_m^B = \mathcal{E}_m^B(\rho^B)/p(m)$ is output by the apparatus. Generally speaking, we can think that the action of the measurement $\mathcal{M}^B$ on $\rho^B$ is given in average by the mapping

$$\mathcal{M}^B(\rho^B) := \sum_m p(m) \rho_m^B \otimes m^X := \rho^{B,X},$$

where $\{m^X\}$ is a set of orthonormal (hence perfectly distinguishable) vectors on the classical register space $X$ of outcomes. Such a measurement process can be realized through the following indirect measurement model [18]. First, an apparatus $Q$ with pure initial state $\phi^Q$ is introduced to interact with $B$ through a suitable unitary interaction $U_{BQ} : BQ \rightarrow B'Q' \cong BQ$. Subsequently, a particular measurement $\mathcal{M}^Q$, depending also on $U_{BQ}$, is performed on the apparatus $Q'$. If we further introduce a reference system $R$ purifying the input state as $\Psi^{RB}$, $\text{Tr}_R[\Psi^{RB}] = \rho^B$, the global tripartite state after the unitary interaction $U_{BQ}$ is

$$|\chi^{RQB}⟩ := (I^R \otimes U_{BQ})(|\Psi^{RB}⟩ \otimes |\phi^Q⟩).$$

The measurement on the apparatus $Q'$ can be chosen such that

$$(I^{RB} \otimes M^Q)(|\chi^{RQB}⟩) := \sum_m p(m) \gamma_m^{RBQ'} \otimes m^X := \rho^{RBQ'X}, \tag{2}$$

where $\{\gamma_m^{RBQ'}\}$ are pure states such that

$$\text{Tr}_Q[\gamma_m^{RBQ'}] = (I^R \otimes E_m^B)(\Psi^{RB})/p(m) := \rho_m^{RB}, \tag{3}$$

also $\text{Tr}_m[\rho_m^{RB}] = \rho_m^B$. In this indirect measurement model, the ancillary apparatus $Q$ helps to disclose the contributions of inaccessible degrees of freedom to the tradeoff between information gain and disturbance [16].

With the indirect measurement model (2), the information gain $\iota(\rho^B, \mathcal{M}^B)$ of measurement $\mathcal{M}^B$ on $\rho^B$ is defined in terms of quantum mutual information between reference $R$ and classical outcome $X$,

$$\iota(\rho^B, \mathcal{M}^B) := I^{RX}(\rho^{RX}), \tag{4}$$
Equation (4) shows that the information gain is usually better understood as being about the remote purifying system $R$, while $B$, correlated with $R$, represents just the information carrier that is measured. With the indirect measurement model (2), the quantum disturbance introduced by measurement $M^B$ is defined in terms of coherent information [17],

$$
\delta(\rho^B, M^B) := S(\rho^B) - I_{\text{coh}}^{R \rightarrow X}(\rho^{RX}),
$$

where coherent information is defined as $I^{A \rightarrow B}(\sigma^{AB}) := S(\sigma^B) - S(\sigma^{AB})$. Equation (5) shows that the measurement-induced disturbance is equal to the information flow into both the classical outputs $X$ and the internal degrees of freedom of the apparatus or environment $Q'$. In [16], it is proved that when $\delta(\rho^B, M^B)$ is infinitely small, it is always possible to introduce a set of recovering operations $\{E_{m}^{B}\}$ that can asymptotically correct the operations $\{\mathcal{E}_{m}^{B}\}$ performed on $B$ by the measurement and recover the quantum correlations between $R$ and $B$. This confirms the correctness of $\delta$ to measure the disturbance. The above two definitions provide a tradeoff relation between information gain and quantum disturbance of a quantum measurement,

$$
\epsilon(\rho^B, M^B) + \Delta(\rho^B, M^B) = \delta(\rho^B, M^B). \tag{6}
$$

Here, $\Delta(\rho^B, M^B) = I^{R \rightarrow Q'X}(\rho^{RO'X})$ measures the missing information in terms of the hidden correlations between $R$ and $B$, while $\Delta(\rho^B, M^B)$ represents just the information carried by $B$.

3. A unified view of quantum discord based on measurement-induced effects

Now we are ready to apply the above quantitative definitions of measurement-induced disturbance and information gain to a bipartite state to measure its quantum correlation. For a bipartite state $\rho^{AB}$ and a local measurement $M^B$ on $B$, we introduce a reference system $R$ purifying $\rho^{AB}$ to $\Psi^{RA}$ and an ancillary apparatus $Q$ for the indirect measurement model of $M^B$. Noting that the systems $R$ and $A$ purify $\rho^B$, we can directly write down the quantum disturbance of $M^B$ on $\rho^B$ and on $\rho^{AB}$, respectively,

$$
\delta(\rho^B, M^B) = S(\rho^B) - I_{\text{coh}}^{R \rightarrow X}(\rho^{RX}) = I^{R \rightarrow Q'X}(\rho^{RO'X}), \tag{7}
$$

$$
\delta(\rho^{AB}, M^B) = S(\rho^{AB}) - I_{\text{coh}}^{R \rightarrow X}(\rho^{RX}) = I^{R \rightarrow Q'X}(\rho^{RO'X}). \tag{8}
$$

Their difference is

$$
\delta(\rho^B, M^B) - \delta(\rho^{AB}, M^B)
\begin{align*}
&= S(\rho^B) - S(\rho^{AB}) + I_{\text{coh}}^{R \rightarrow X}(\rho^{RX}) - I_{\text{coh}}^{R \rightarrow X}(\rho^{RX}) \\
&= S(\rho^B) - S(\rho^{AB}) + S(\rho^{AB}) - S(\rho^{AB}) \\
&= S(\rho^B) - S(\rho^{AB}) + \sum p(m)[S(\rho^B) - S(\rho^B)] \\
&= I^{R \rightarrow Q'X}(\rho^{RO'X}) - I^{R \rightarrow Q'X}(\rho^{RO'X}) \\
&= I^{R \rightarrow Q'X}_{|R}(\rho^{RO'X}). \tag{9}
\end{align*}
$$

The difference between information gains by measurement $M^B$ on $B$ and on $AB$ is

$$
\epsilon(\rho^B, M^B) - \epsilon(\rho^{AB}, M^B)
\begin{align*}
&= I^{R \rightarrow X}(\rho^{RX}) - I^{R \rightarrow X}(\rho^{RX}) \\
&= S(\rho^B) - S(\rho^{AB}) + \sum p(m)[S(\rho^B) - S(\rho^B)]
\end{align*}
$$

4
\[
\begin{align*}
&= S(\rho^B) - S(\rho^{AB}) + \sum_m p(m) \left[ S(\rho_m^{AB}Q') - S(\rho_m^{B}Q') \right] \\
&= I^{A:X|R}(\rho^{RA,X}).
\end{align*}
\]

Strong subadditivity of von Neumann entropy implies that
\[
\delta(\rho^B, \mathcal{M}^B) - \delta(\rho^{AB}, \mathcal{M}^B) \geq \iota(\rho^B, \mathcal{M}^B) - \iota(\rho^{AB}, \mathcal{M}^B),
\]

which also follows from the conditional mutual information inequality \( I^{A:Q|X|R}(\rho^{RA,Q,X}) \geq I^{A:X|R}(\rho^{RA,X}) \).

For a general measurement \( \mathcal{M}^B \),
\[
S(A|B_C) - S(A|B) \geq \delta(\rho^B, \mathcal{M}^B) - \delta(\rho^{AB}, \mathcal{M}^B),
\]

which comes from the subadditivity of von Neumann entropy \( S(\rho^A) + S(\rho^B) \geq S(\rho^{AB}) \). However, if \( \mathcal{M}^B \) is a rank-1 POVM, its operator \( \Psi_m^B \) correlated with outcome \( m \) is proportional to a projector. Noting that \( \Upsilon^{\mathcal{R}AB}Q' \) is a pure state \([16]\), we have \( \rho_m^{AB} = \rho_m^A \otimes \Psi_m^B \) and
\[
S(A|B_C) - S(A|B) = \delta(\rho^B, \mathcal{M}^B) - \delta(\rho^{AB}, \mathcal{M}^B).
\]

Hence, we obtain the following expression for quantum discord in terms of measurement-induced disturbance:
\[
D^{-}(\rho^{AB}) = \min_{\mathcal{M}^B}[\delta(\rho^B, \mathcal{M}^B) - \delta(\rho^{AB}, \mathcal{M}^B)],
\]

where \( \mathcal{M}^B \) is chosen from rank-1 POVMs.

In addition, if we choose measurement \( \mathcal{N}^B \) from more restricted ‘good’ rank-1 POVM set that has zero \( \Delta(\rho^B, \mathcal{N}^B) \) in the tradeoff relation (6), then we have
\[
\iota(\rho^B, \mathcal{N}^B) - \iota(\rho^{AB}, \mathcal{N}^B) = \delta(\rho^B, \mathcal{N}^B) - \delta(\rho^{AB}, \mathcal{N}^B) = S(A|B_C) - S(A|B).
\]

For such kinds of measurements, the information gain is balanced with quantum disturbance. One kind of such measurement is ‘single-Kraus’ or ‘multiplicity free measurement with output states \( \rho_m^{AB}Q' = \rho_m^A \otimes \Psi_m^B \otimes \omega_m^{Q'} \) \([16]\). Therefore, in terms of measurement information gain, we obtain another expression for quantum discord,
\[
D^{-}(\rho^{AB}) = \min_{\mathcal{N}^B}[\iota(\rho^B, \mathcal{N}^B) - \iota(\rho^{AB}, \mathcal{N}^B)].
\]

Now we present some physical discussions on the above results. When \( A \) is only classically correlated with \( B \), it is reasonable to expect that measurement \( \mathcal{M}^B \) on \( B \) induces equal quantum disturbance on \( B \) locally and on \( AB \) as a whole, since \( A \) does not contribute to the quantum coherence of \( B \), nor quantum disturbance neither. However, when \( A \) and \( B \) have quantum correlations, the situation is different. To be explicit, let us assume \( R \) to be a reference system purifying \( A \) and \( B \). In terms of quantum coherence, \( S(\rho^B) \) quantifies the coherent interrelations between \( B \) and \( AR \); similarly, \( S(\rho^{AB}) \) quantifies the coherent interrelations between \( AB \) and \( R \). A measurement on \( B \) introduces a disturbance on both of them and we may say that the quantum coherence of \( \delta(\rho^B, \mathcal{M}^B) \) has been destroyed for \( B \); at the same time the quantum coherence of \( \delta(\rho^{AB}, \mathcal{M}^B) \) has been destroyed for \( AB \). When \( A \) and \( B \) are quantum mechanically correlated, \( A \) shares part of \( B \)'s coherent relations; this part of quantum coherence certainly experiences the quantum disturbance introduced by the measurement on \( B \). However, it does not exist in \( S(\rho^{AB}) \) and its disturbance naturally will not come up in \( \delta(\rho^{AB}, \mathcal{M}^B) \). In other words, for a general measurement \( \mathcal{M}^B \) on \( B \), its quantum disturbance \( S(A|B_C) - S(A|B) \) exists in \( S(\rho^B) \) but not in \( S(\rho^{AB}) \); therefore, it is contained in \( \delta(\rho^B, \mathcal{M}^B) \) but not in \( \delta(\rho^{AB}, \mathcal{M}^B) \). Similarly, for a good measurement \( \mathcal{N}^B \) on \( B \), there are information gains \( S(A|B_C) - S(A|B) \) in \( S(\rho^B) \) but not in \( S(\rho^{AB}) \). Equations (11) and (12) show that the quantum correlation quantified by quantum discord can be directly understood as the discrepancy of measurement-induced disturbance and information gain between the local measured subsystem and the whole system.
In fact, besides quantum disturbance and information gain, there is one other important effect of measurement; entanglement induced between the measuring apparatus and the system. For a bipartite system state $\rho^{AB}$, a general measurement $\mathcal{M}^B$ on $B$ induces entanglement between the measuring apparatus $M$ and $B$. Furthermore, if $A$ and $B$ have quantum correlations, the distillable entanglement between $M$ and $B$ is different from the distillable entanglement between $M$ and $AB$; their minimal discrepancy is equal to quantum discord [14],

$$D^r(\rho^{AB}) = \min_{\mathcal{M}^B} [E_D^{M|AB} - E_D^{M|B}],$$

where $E_D$ is the distillable entanglement.

Restricting measurements to rank-1 projective measurements, we will show that all the above three expressions of quantum discord (equations (11)–(13)) become equivalent. This point can be made clear with the following relation between conditional information and relative entropy:

$$S(A|B_c) - S(A|B) = D\left(\rho^{AB}|| \sum_m \Pi_m^B \rho^{AB} \Pi_m^B\right) - D\left(\rho^B|| \sum_m \Pi_m^B \rho^B \Pi_m^B\right),$$

where $D(\rho||\sigma) = \text{tr}(\rho \ln \rho) - \text{tr}(\rho \ln \sigma)$ is the relative entropy and $D\left(\rho^{AB}|| \sum_m \Pi_m^B \rho^{AB} \Pi_m^B\right)$, $D\left(\rho^B|| \sum_m \Pi_m^B \rho^B \Pi_m^B\right)$ correspond to the distillable entanglements $E_D^{M|AB}$ and $E_D^{M|B}$, respectively [14]. The definition of quantum disturbance (5) is in terms of coherent information which is closely related to decoherence. Therefore, it is possible to relate decoherence to quantum discord. In [19], for rank-1 projective measurements, Coles discusses the relation between quantum discord and decoherence through the following relation:

$$D \left(\rho^{AB}|| \sum_m \Pi_m^B \rho^{AB} \Pi_m^B\right) = S(\mathcal{X}|R),$$

where $R$ is the purifying system of $\rho^{AB}$ and conditional entropy $S(\mathcal{X}|R)$ quantifies the missing information from the purifying system $R$ which results in the decoherence of measurement $\{\Pi_m^B\}$. Furthermore, the equivalence between decoherence and information gain for rank-1 projective measurement can be found through the following relation:

$$S(\mathcal{X}|R) - S(\mathcal{X}|RA) = I^{RA,X}(\rho^{RA,X}) - I^{R,X}(\rho^{RX}).$$

In summary, restricting to rank-1 projective measurements $\mathcal{M}^B = \{\Pi_m^B\}$, we obtain the following equivalent expressions for quantum discord with different physical meanings:

$$D^r_{\mathcal{M}^B=\{\Pi_m^B\}}(\rho^{AB}) = \min_{\mathcal{M}^B=\{\Pi_m^B\}} [\delta(\rho^B, \mathcal{M}^B) - \delta(\rho^{AB}, \mathcal{M}^B)] \quad (14)$$

$$= \min_{\mathcal{M}^B=\{\Pi_m^B\}} [\delta(\rho^B, \mathcal{M}^B) - \delta(\rho^{AB}, \mathcal{M}^B)] \quad (15)$$

$$= \min_{\mathcal{M}^B=\{\Pi_m^B\}} [E_D^{M|AB} - E_D^{M|B}] \quad (16)$$

$$= \min_{\mathcal{M}^B=\{\Pi_m^B\}} [S(\mathcal{X}|R) - S(\mathcal{X}|RA)]. \quad (17)$$

For rank-1 projective measurements, our results coincide with the results given in [19]. However, it should be pointed out that our results also apply to rank-1 POVMs which are not covered in [19] but are needed for optimization of quantum discord [3]. It is interesting to note that, for the initial ‘discord’ which is defined as the difference between two discordant definitions of mutual information in the quantum case, the above four equations provide different physical meanings for ‘discord’ in terms of the difference between the whole and its constituents.
4. Conclusion

In this paper, we discuss the quantum correlation in terms of measurement-induced effects. The differences between measurement effects on the local measured subsystem and on the whole system are used to measure quantum correlation. It is shown that for rank-1 POVMs on one subsystem of a bipartite system, the minimal difference between the measurement-induced disturbance on the measured subsystem and on the whole system corresponds to quantum discord of the bipartite state. Similarly, the minimized difference between the information gain of the measured subsystem and the whole system over good rank-1 POVMs also corresponds to quantum discord. Combined with similar results in terms of measurement-induced entanglement and decoherence, our results provide a unified view on quantum discord in terms of measurement-induced effects.

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References

[1] Horodecki R, Horodecki P, Horodecki M and Horodecki K 2009 Rev. Mod. Phys. 81 865
[2] Knill E and Laflamme R 1998 Phys. Rev. Lett. 81 5672
[3] Modi K, Brodutch A, Cable H, Paterek T and Vedral V 2011 arXiv:1112.6238
[4] Ollivier H and Zurek W H 2001 Phys. Rev. Lett. 88 017901
[5] Henderson L and Vedral V 2001 J. Phys. A: Math. Gen. 34 6899
[6] Devetak I and Winter A 2004 IEEE Trans. Inf. Theory 50 3183
[7] Datta A, Shaji A and Caves C M 2008 Phys. Rev. Lett. 100 050502
[8] Horodecki M, Oppenheim J and Winter A 2005 Nature 436 673
[9] Cavalcanti D, Aolita L, Boixo S, Modi K, Piani M and Winter A 2011 Phys. Rev. A 83 032324
[10] Madhok V and Datta A 2011 Phys. Rev. A 83 032323
[11] Cornello M F, de Oliveira M C and Fanchini F F 2011 Phys. Rev. Lett. 107 020502
[12] Auccaise R, Celeri L C, Soares-Pinto D O, deAzevedo E R, Maziero J, Souza A M, Bonagamba T J, Sarthour R S, Oliveira I S and Serra R M 2011 Phys. Rev. Lett. 107 140403
[13] Streltsov A, Kampermann H and Bruß D 2011 Phys. Rev. Lett. 107 170502
[14] Streltsov A, Kampermann H and Bruß D 2011 Phys. Rev. Lett. 106 160401
[15] Piani M, Gharibian S, Adesso G, Calsamiglia J, Horodecki P and Winter A 2011 Phys. Rev. Lett. 106 220403
[16] Buscemi F, Hayashi M and Horodecki M 2008 Phys. Rev. Lett. 100 210504
[17] Schumacher B and Nielsen M A 1996 Phys. Rev. A 54 2629
[18] Lloyd S 1997 Phys. Rev. A 55 1613
[19] Ozawa M 1984 J. Math. Phys. 25 79
[20] Coles P J 2012 Phys. Rev. A 85 042103