A Novel Method for Forecasting Surface Wind Speed using Wind-direction based on Hierarchical Markov Model

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**ABSTRACT**

This article presents a new method for detecting heterogeneities in wind data set to predict wind speed based on the well-known Hidden Markov Model (HMM). In the proposed method, the HMM categorizes the wind time series into some groups in which each group represents a wind regime. Each regime uses an internal first-order Markov Chain (MC) for forecasting, and the combination of all regimes outputs generates the final wind speed forecast. The model proposed in this study is called “Hierarchical Markov Model”. The first layer detects and separates wind regimes as heterogeneous groups of wind data by the use of wind direction data, based on HMM, and the second layer forecasts the wind speed using MC. The proposed model is implemented and tested using real data. Its effectiveness in terms of temporal stationary index is compared with that of a first-order MC-based method. The results showed that more than 70% improvement can be achieved in wind speed prediction by the proposed method. Moreover, it gives a probability distribution function of wind speed prediction, which is sharper than the one obtained with the first-order MC; means that more precise prediction.

**NOMENCLATURE**

| Symbol | Description |
|--------|-------------|
| $p$    | Transition probability matrix |
| $P_r$  | Probability value |
| $M_t$  | The sequence of random variables |
| $n_{i,j}$ | Probability of transition from state $i$ to state $j$ |
| $n_{i,j}$ | Number of transitions from state $i$ to state $j$ |
| $w_t$  | Wind speed (m/s) |
| $v_t$  | Quantized wind speed |
| $\beta$ | Temporal stationary |
| $T$    | Number of time intervals |
| $k$    | Number of Markov states |
| $L_T$  | Likelihood |
| $\alpha$ | Initial distribution |
| $x_t$  | Number of successes |
| $n_t$  | Number of experiments at time $t$ |
| $\pi_j$ | Probability of success |
| $m$    | Number of multinomial-HMM states |
| $q$    | Number of quantized levels of HMM |
| $D_j$  | Quantized wind direction |
| $R$    | Wind regime |
| $d_j$  | Wind direction |
| $V_i$  | Wind speed state vector |

**1. INTRODUCTION**

The wind is one of the most important atmospheric phenomena due to its influence on many aspects of human life. Decision-making, in many ways, is directly dependent on the wind. Examples include urban air pollution management, wind-power-plant generation, maritime and air transport, tourism, and sports. Wind speed forecasting has an essential role in wind-power-plant generation and operation because the generated power depends directly on wind speed if it is between two upper and lower thresholds [1]. So, this issue has been the subject of intense research. For example, comprehensive studies have been conducted on the wind by Keyhani et al. [2] to investigate wind energy potential as a power generation source.

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In meteorology, using numerical weather prediction (NWP) models is a common method of wind forecasting. NWP models use physical models of the atmosphere and oceans to predict the weather based on current weather conditions. These models, which are mainly used for large-scale phenomena, provide accurate results in long-term forecasts [3]. As discussed by Han et al. [4], statistical post-processing of NWP ensembles improved the results. They compared and analyzed the statistical post-processing methods, including bias-corrected and probabilistic forecasts of wind speed to provide more accurate weather information. However, NWPs are not efficient in terms of high computational volume in short-term and very short-term forecasts. In such cases, statistical models are preferred [5].

Short-term wind speed models are built upon either probability distribution theory or time series analysis approach. In probability distribution models, it is assumed that the wind speed follows a specific probability distribution such as Weibull. In these methods, the parameters of the probability distribution function are estimated, usually based on historical data. Since these historical data do not always obey one type of distribution, considering a particular distribution may cause a significant error in these conditions. Autoregressive Moving Average (ARMA) and Markov Chain (MC) are two methods in the time series analysis group. MCs have been used in many applications for wind speed modelling. The main feature of these models is their ability to incorporate both statistical and temporal characteristics of wind speed. In contrast to ARMA, MCs (despite their simplicity) can model time-dependent wind characteristics. Recent studies have shown that the simplicity of MCs makes them a valuable tool in modelling. However, MCs are not capable of modelling wind characteristics at high frequencies [6]. Wind speed time series had been found as long-term correlated statistics [7]. Therefore, Statistical methods have been used comprehensively to improve wind field simulation and wind forecasting. Liu et al. [8] presented an improved wind field simulations by the non-Gaussian Least Square model by precisely and stably simulating velocity skewness and kurtosis.

Based on Markov Model, several studies have been carried out for forecasting weather parameters like temperature and wind. Shamshad et al. [9] compared the results of the first and second-order MC for wind forecasting and showed that the second-order MC does not improve the results so much.

Tagliaferi et al. [6] showed the Nested Markov Chain (NMC) for wind modelling improves MC’s accuracy and temporal correlation without excessively increasing computational time. The NMC can be considered as an extended MC such that each state itself is a standalone MC process. In this model, the time series is generated using an internal MC. Here, non-Markovian models can also be used to generate the inner layer time series in this process. In some studies, the semi-Markov model has been used to improve the accuracy and autocorrelation of standard MCs [10]. This is because of the model’s ability to save past transitions through an auxiliary random process. The characteristics of this model are as follows. First, the time step is not constant; second, the random variable may have any distribution; and third, the duration of being in each state affects the transition probability.

The wind is affected by other meteorological parameters such as air temperature, air pressure and relative humidity. These parameters have cyclic change over a day and a season. As a result, wind changes have a daily periodic regularity [11, 12] or a seasonal period [13]. Xie et al. [14] presented a non-homogeneous Markov Chain (NHMC) wind speed model to develop a more accurate method for modelling wind speed time series by considering seasonal and daily changes for wind speed.

Ailliot and Monbet [15] used the Markov Switching Auto-Regressive (MSAR) model to describe the time series of the wind. In this method, some self-recursive models were used to describe the temporal behaviour of wind speed. Switching between these models is controlled by HMM. Also, non-homogeneous hidden Markov-switching models for wind time series were used by Ailliot et al. [16].

Generally, the aforementioned researches did not separate the behaviour of different wind regimes in order to achieve better forecasting results. Most of Markov-based methods use the MC only for predicting wind speed. In reality, detecting and separation of wind regimes is an important issue. This issue cannot be achieved through MCs.

Major efforts have been invested in finding a way to accommodate heterogeneous groups with distinct probability distribution function in wind direction time-series to produce a sharper and more accurate probabilistic wind speed forecast. For this aim, HMM can be utilized as a powerful tool. The aim of this article is to take the hidden Markov model to detect heterogeneities in wind data set from which wind regimes can be extracted. States of the model are in fact wind regimes, and the subsystems of each regime (HMM state) is a six-state MC denoting the specific wind speed interval. This hierarchal model gives us the probability of having a wind speed within a specific interval in a specific regime in the near future.

This paper is structured as follows. Following this introduction, the first-order MC and its stationary evaluation are briefly explained in section 2. The theoretical concept of mixed models is described, and HMM is introduced. Based on the described features, a new method for predicting wind speed is proposed based on HMM, and an evaluation method is developed based
on temporal stationary test. Having applied this method to real data, the results are compared with those of the first-order MC in section 3 and the advantages of the proposed model are described. Considerations for real-time applications are also discussed in this section. Section 4 concludes the paper with a summary and gives an outlook on future works.

2. MATERIAL AND METHODS

2.1. Markov Theory

The various versions of Markov models have been used in different engineering fields. MCs are used for short-term predictions. Semi-Markov models have been commonly used where state transition probabilities in systems are time-dependant. HMMs are used to find heterogeneities in data sequences and time-series, i.e., for behaviour recognition [17].

In this article, HMM is taken to detect and separate wind regimes as heterogenic groups of wind data. Each regime is, in fact, a state of this HMM. Members of each group are then modelled with an MC for wind prediction. In the following, the Markov chain and hidden Markov theorems are briefly explained.

2.1.1. First Order Markov Chain

A sequence of random variables (time-series) \( \{M_t; t \in \mathbb{N}\} \) is an MC if for all \( t \in \mathbb{N} \) it satisfies the property \( P_r(M_{t+1}|M_t, ..., M_2) = P_r(M_{t+1}|M_t) \). It means that considering the history of the process up to time \( t \) is equivalent to considering the most recent value \( M_t \).

In a first-order \( k \)-state MC, the transition probability matrix \( \Gamma \) has a size of \( k \times k \) represented as Equation (1):

\[
\Gamma = \begin{bmatrix}
Y_{1,1} & Y_{1,2} & \cdots & Y_{1,k} \\
Y_{2,1} & Y_{2,2} & \cdots & Y_{2,k} \\
\vdots & \vdots & \ddots & \vdots \\
Y_{k,1} & Y_{k,2} & \cdots & Y_{k,k}
\end{bmatrix}
\] (1)

where \( Y_{i,j} \) represents the probability of transition from state \( i \) to state \( j \). This probability is calculated as Equation (2):

\[
Y_{i,j} = \frac{n_{i,j}}{\sum_j n_{i,j}}
\] (2)

where \( n_{i,j} \) is the number of transitions from state \( i \) to state \( j \) in the whole time-series. Partitioning the whole range of wind speed to several equal or unequal levels gives the states of the representing MC of the system. A simple way is to take all partitions equal, except the last one, and to partition the whole range into some levels (here, for example, 6) as Equation (3):

\[
v_1 = i \quad \text{if} \quad 5(t-1) \leq \omega_t < 5i \quad , \quad i = 1, 2, 3, 4, 5
\]
\[
v_6 = 6 \quad \text{if} \quad 5i \leq \omega_t \quad , \quad i = 6
\] (3)

where \( \omega_t \) is wind speed (m/s) and \( v_t \) is the quantized wind speed. Probabilistic wind speed prediction in the next step can be declared easily using Equation (4).

\[
v_t = v_0, \gamma
\] (4)

where \( v_0 \) is the initial condition vector, indicating the current wind state. For example, if six levels are considered for wind speed, the vector \( v_0 = [0 \ 1 \ 0 \ 0 \ 0 \ 0] \) shows that the system is currently in state 2. In this case, according to Equation (4), \( v_1 \) shows the probability of each state in the next step. This method is only valid when the Markov process is not time-dependent (i.e. it is stationary).

A common way to test the temporal stationary attribute of an MC of a time-series is dividing the time-series into two or more parts and designing an MC for each part. The primary Markov chain is stationary if the transition probability matrices of each part’s MC are nearly equal. One way for evaluation of the temporal stationary conditions, parameter \( \beta \) is defined by Equation (5) [9].

\[
\beta = 2 \sum_i \sum_j n_{i,j}(t) \ln \left( \frac{Y_{i,j}(t)}{\gamma_{i,j}} \right) \quad , \quad t = 1, 2, \ldots, T
\] (5)

where \( T \) is the number of time intervals, \( k \) is the total number of states, and \( n_{i,j}(t) \) and \( Y_{i,j}(t) \) are the numbers of occurrence and probability of transitions from state \( i \) to state \( j \), respectively.

Such an MC is stationary if \( \beta \) has a \( \chi^2 \) distribution with \( k(k-1)(T-1) \) degrees of freedom. It is stationary in a 5% confidence interval if \( \beta < \chi^2(5\%, \text{Degrees of Freedom}) \).

2.1.2. Hidden Markov Model

Hidden Markov model is a well-known model for univariate or multivariable time series and is specially used for modelling discrete series such as group series or counting series.

Hidden Markov Model \( \{X_t; t \in \mathbb{N}\} \) is a special kind of dependent mixed models. Considering \( M^{(t)} \) and \( X^{(t)} \) as histories from 1 to \( t \), this model is expressed as Equation (6).

\[
P_r(M_t|M^{(t-1)}) = P_r(M_t|M_{t-1}), \quad t = 2, 3, \ldots
\]
\[
P_r(X_t|X^{(t-1)}, M^{(t)}) = P_r(X_t|M_t), \quad t \in N
\] (6)

The model has two parts. One is ‘parameter process’ \( \{M_t; t = 1, 2, \ldots\} \) which is unobserved and satisfies Markov property. The other is ‘state-dependent process’ \( \{X_t; t = 1, 2, \ldots\} \), where the distribution of \( X_t \) depends only on the current state \( M_t \) and is not dependent on previous states or observations. \( \{X_t\} \) is an \( m \)-state HMM, if the Markov chain \( \{M_t\} \) has \( m \) states. Whenever the model stays in one of these states, the distribution of the model will be its corresponding \( X_t \); i.e., \( \{X_t; t = 1, 2, \ldots\} \). This structure is expressed with the directional graph of Figure 1.
Likelihood of an m states HMM ($L_T$) for the data sequence ($x_1, x_2, ..., x_T$) with initial distribution $\alpha$ and transition probability matrix $\Gamma$, is expressed by Equation (7) where $P(x_T)$ is the probability of occurrence of $x_T$ and $1$ is a row vector of ones.

$$L_T = \alpha P(x_1) P(x_2) ... P(x_T) 1'$$  \hspace{1cm} (7)

2. 1. 3. Multinomial-HMM

When the output of an experiment is binary (success or failure), the binomial model is used to calculate the probability of success in a certain number of experiments. Binomial probabilities are expressed by Equation (8).

$$p_i(x_t) = \binom{n_t}{x_t} \pi_i^n(1-\pi_i)^{n_t-x_t}$$  \hspace{1cm} (8)

where $x_t$ is the number of successes, $n_t$ the number of experiments at time $t$ and $\pi_i$ is the probability of success. For instance, if “Tails” is the success in tossing a coin, and in the first experiment there are five times toss with two “Tails” outcome, then $n_1=5$ and $x_1=2$.

“Binomial-HMM” Model is a type of HMM in which the observed values $\{x_t: t = 1, ... , T\}$, are the number of successes in $n_1, n_2, ..., n_T$ independent Bernoulli experiments. A model with $m$ states has $m$ values for success probability $\pi_i$ (i indicates each of the states).

A “multinomial-HMM” is the extended model of binomial-HMM. In this case, it is assumed that there are $q$ possible outcomes in each experiment, where $q > 2$. Therefore, the number of observed results is $q$ times the previous case and is: $\{x_t: t = 1, ..., T; j = 1, ..., q\}$ and $x_{t1} + x_{t2} + ... + x_{tq} = n_t$ ($n_t$ is the number of experiments at time $t$). For example, in a 6-state dice throw, if the dice is thrown seven times in the first experiment and the number “3” comes twice, then $n_1 = 7$ and $x_{13} = 2$. The vector $X_t$, which contains all observations at time $t$, can be written as Equation (9).

$$X_t = (x_{t1}, x_{t2}, ..., x_{tq})$$  \hspace{1cm} (9)

For an m-state “multinomial-HMM” model, there is a $m \times m$ transition probability matrix $\Gamma$. Each of the $m$ states is associated with a multinomial distribution, and each of these multinomial distributions has $q$ unknown probabilities (emission), which for state $i$, denoted by $\pi_{i1}, \pi_{i2} ... \pi_{iq}$.

An important case of the multinomial–HMM is obtained by considering $n_t = 1$ for all $t$. In this case, as $\sum_{k=1}^{q} x_{tk} = 1$, in the vector $X_t$ with dimension $q$, one of the elements is “1”, and the rest is “0”. Considering:

$$X_t = (0, ..., 0, 1, 0, ..., 0)$$

Defining $\pi(j) = diag(\pi_{1j}, ..., \pi_{mj})$, the likelihood of observed groups $j_1, j_2, ..., j_T$ at times $1, 2, ..., T$ is expressed as Equation (10) [18].

$$L_T = \alpha \pi(j_1) \Gamma \pi(j_2) ... \pi(j_T) 1'$$  \hspace{1cm} (10)

2. 2. A Novel Method for Wind Forecasting

In this section, a novel wind speed forecasting method is presented based on a hierarchical Markov model. This is a two-layer model, in which the top layer is a Multinomial-HMM. In this layer, each state, representing a particular wind regime, is a first-order MC for which each state represents a specific wind speed interval. Figure 2 shows the basic structure of the proposed model.

The proposed method is suitable for very short-term wind forecasting (from a few seconds up to a few hours). Hourly forecasts of wind direction and speed are widely used for daily public planning. Meanwhile, shorter times are usually employed, for example, in wind turbine management to maximize the receivable power and prevent damage to equipment [19-21].

This kind of forecasting also is useful for aircraft landing and take-off processes.

2. 2. 1. Method Implementation

The implementation of the proposed method, results in predicting probability values for wind speed, leading to improved results of the well-known first-order MC.

Figure 3 illustrates the proposed method. The method includes the following three phases:

- Phase A: HMM definition and parameter estimation
- Phase B: Regime separation
- Phase C: Online forecasting
Each phase has some steps as the following:

**Phase A:**

a. In this step, wind speed and direction time series at a given interval is gathered and pre-processed to produce training data for estimating model parameters. Like other data-driven methods, the completeness and proper sequence of collected data must be checked. The purpose of this step is to ensure the validity of the wind time series. For better consistency, data series should be examined and, in case of missing, should be recovered based on an interpolation algorithm. When the wind speed is zero, the wind direction is assumed invalid and deleted from the data set. The observed wind direction data should be treated in a way that any single observed wind direction is coded from 1 to 16, depending on lying in one of the 16 conventional directions N, NNE, ..., NNW. Thus, there are 16 possible outcomes for any single direction value i.e. $D_1, D_2, ..., D_{16}$.

b. In this step, the maximum likelihood problem for the "multinomial-HMM" model problem based on the well-known Baum-welch algorithm on wind direction time series is solved such that to determine the model parameters to maximize Equation (10). These parameters are the transition probability matrix between m states ($P_{mm}$) and the probability corresponding to each of $D_1, D_2, ..., D_{16}$ in each state (emissions). Considering m=2, each direction $D_j$ is associated with two probability values: the probability of occurrence in state one $\pi_{1j}$ and the probability of occurrence in state two $\pi_{2j}$.

**Phase B:**

c. In this step, the wind time series is separated into two parts according to probability corresponding to each of 16 directions. Any direction that is more likely to occur in the first state falls into the first part, and any direction that is more likely to occur in the second state falls into the second one. Equation (11) shows the procedure.

$$
\text{if } \pi_{1j} > \pi_{2j} \text{ then } \\
D_j \in R_1 \\
\text{else } D_j \in R_2,
$$

$i = 1, ..., q$

This is used to separate wind time series into two groups $R_1$ and $R_2$ according to wind direction, which is named them as "wind regime" from now on.

d. In this step, the transition probability matrix of the first-order MC is calculated.

According to Equations (1) and (2) represent wind speed time series in each regime.

**Phase C:**

e. In this step, online wind speed and direction are measured, and the most probable regime is chosen according to the direction.

f. In the last step, wind speed is predicted based on current wind speed and the first-order MC corresponding to the dominant regime and also considering transition probability between regimes.

The above six steps can also be carried out for more numbers of regimes.

2.2.2 Method Evaluation Approach

Generally, the most important criterion to evaluate Markov models is to compare the transition probability matrix obtained from the test data with the transient probability matrix obtained from the training data. The more the similarity between these two transition probability matrices is, the more successful the forecasting model will be. "Temporal Stationary" is one of the most practical comparison tools.

![Proposed Flowchart for Implementation of HMM Model](image)

**Figure 3. Proposed Flowchart for Implementation of HMM Model, a. Flowchart Part I (Modeling), b. Flowchart Part II (Forecast)**
Using the proposed method, the temporal stationary value for the whole model is calculated as follows. By choosing T=2, the Equation (5) is simplified as Equation (12), and the degree of freedom is equal to k(k − 1).

$$\beta = 2 \sum_{i=1}^{k} (n_{ij}(1) \ln \left( \frac{P_{ij}(1)}{y_{ij}} \right) + n_{ij}(2) \ln \left( \frac{P_{ij}(2)}{y_{ij}} \right))$$

(12)

As the wind time series has been partitioned into two regimes, according to step C of the proposed method, temporal stationary of each regime is calculated by Equation (12). The value of $\beta_{Extended}$ can then be computed from the expanding Equation (13).

$$\beta_{Extended} = \frac{n^{(1)} \beta_1 + n^{(2)} \beta_2}{n^{(1)} + n^{(2)}}$$

(13)

where $\beta_1$ and $\beta_2$ are the temporal stationary values of the first and the second groups, respectively, and $n^{(r)}$ is the number of the existing elements in regime r, $r = 1, 2$.

To compare the performance of the proposed model against the first-order MC, it is necessary to calculate and compare their $\beta$ values for wind time-series data. The smaller value indicates a better (near to reality) forecast.

To calculate the temporal stationary index of first-order MC for two-time series $\{M_1, M_2, ..., M_n\}$ and $\{N_1, N_2, ..., N_n\}$, it is enough to find $\gamma_{ij}(1)$ for the first time-series and $\gamma_{ij}(2)$ for the second time-series according to Equation (2). The $\gamma_{ij}$ is then calculated according to Equation (2) for the time-series $\{M_1, M_2, ..., M_n, N_1, N_2, ..., N_n\}$. Now the value of $\beta$ can be calculated according to Equation (12). To calculate temporal stationary of the proposed method, each of the time-series $\{M_1, M_2, ..., M_n\}$ and $\{N_1, N_2, ..., N_n\}$ should be split into groups according to the defined HMM regimens. Group one of the first time-series and group one of the second time-series are used to calculate $\beta_1$ and send groups of them are used to calculate $\beta_2$. Then having the number of elements each group, it is easy to calculate the temporal stationary of the proposed method by Equation (12).

2. 3. Implementation of The Proposed Forecasting Method

In this section, the proposed method is implemented and examined by real wind time series. The results are compared to the first-order MC. It is assumed that the inner layer of each regime is a first-order MC. The states are defined as described in Equation (3).

2. 3. 1. Field Data

The measured values of wind speed and direction near the runway 29 of Imam Khomeini International Airport (IKIA) were taken as test data. Direction and speed of wind in this site are measured and saved every 30 seconds by an anemometer that is installed at the top of a 10 meters height mast. The installed anemometer is of WMT700 type with a measuring range of 0.01 to 75 m/s and accuracy of ±0.1 m/s or 2 % of reading, whichever is greater. Ultrasonic sensors provide reliable data due to their extremely low measuring threshold, good stability, accuracy and their ability to operate in harsh and cold climates [22].

The mast is placed at coordinates 35° 24' 54" N, 51° 9’ 46" E close to the runway as shown in Figure 4. Statistical data for four years (2013 to 2016) was gathered. It was seen that the ratio of missing data to all data is less than 0.1% in the entire collection period in our data set. The collected data are used by the proposed method, and the results are gathered to study. Previous studies on the wind in Tehran was carried out based on a 3-hour period measured long-term wind speed data of meteorological station in Mehrabad airport in Tehran [2]. An Alternative data source would be remote sensing systems that gather the data remotely over a wide area as reported in literature [23, 24] for precipitation.

2. 3. 2. Parameters Estimation

The two-state HMM is fed with eight sets of monthly test data (The wind speed of March and July of the year 2013 to the year 2016). Eight sets of parameters are presented in in Tables 1 and 2, which correspond to the eight sets of these monthly data.

Table 1 shows that the transition probability matrices for the same month of different years are more similar than the values of the other months.

From Table 2, it is observed that 16 probabilities associated with each state in March 2013 ($\pi_{ij}$ and $\pi_{j2}$) are almost similar to other March months, while they differ from values of July months. Also, those values are approximately similar in all July months.

![Figure 4. Position of runway anemometer of IKIA](image-url)
In Table 2, the probability values are shown by factor $x 1000$ for readability. For instance, in the first row, the numbers 0 and 40 in columns “March 2013” show that the north wind has a probability of 0.004 in regime 1 (i.e., in regime 1 northern wind does not exist) and 0.04 in regime 2. It can be concluded that every northern wind belongs to regime 2.

The results of Table 2 also are presented in Figures 5 and 6 in a bar graph form. As expected, the probability values for the same two months are not quite the same. But they are much closer than two months apart.

### 2.3.3 Regime Separation

Using Equation (11), in March 2013, directions 1, 2, 3, 4, 15 and 16 are assigned to regime 2. The remaining directions are assigned to regime one as Equation (14).

$$R_1 = \{5, 6, 7, 8, 9, 10, 11, 12, 13, 14\},$$
$$R_2 = \{1, 2, 3, 4, 15, 16\},$$

The sets $R_1$ and $R_2$ are used to separate March 2013 wind speed time series into two groups according to the corresponding direction. A first-order MC now can be used to forecast wind speed in each of the two separated groups. The transition between pre-quantized levels of wind speed is now modelled as a first-order MC. Moreover, the transition probability matrix for each group ($\Gamma_1$ and $\Gamma_2$) is calculated as Equation (2). These matrices will be used later for wind forecasting.

### 2.3.4 Online Forecasting

Now, as the wind time series of March 2013 has been used for parameter estimation, wind forecasting for March is feasible.
Since sets $R_1$ and $R_2$ are known in Equation (14), it is easy to find the current dominant regime as long as the wind direction can be measured online. For example, the wind direction of $93^\circ$ is coded as 4 (as yields in the 4th zone of 16 conventional directions). So $R_2$ will be the dominant regime. In this case, for the next sample, $R_2$ will remain dominant with probability $\gamma_{22} = 0.97$ and dominant regime will be changed from $R_2$ to $R_1$ with probability $\gamma_{21} = 0.03$ according to Table 1.

As described in the previous section, $\Gamma_1$ is the transition probability matrix of the first-order MC of wind speed in regime 1. Regarding Equation (15), if $R_1$ is dominant regime in current and next steps, $V_t \times \Gamma_1$ is vector of probabilistic wind forecasting in the next step, when $V_t$ representing the vector of current wind speed. Considering the possibility of transitions between regimes, the forecasting of wind speed in the next step would be as Equation (15),

$$
\begin{align*}
\text{if } d_t \in R_1 & \quad V_{t+1} = \gamma_{11} \cdot V_t \times \Gamma_1 + \gamma_{12} \cdot V_t \times \Gamma_2 \\
\text{if } d_t \in R_2 & \quad V_{t+1} = \gamma_{21} \cdot V_t \times \Gamma_1 + \gamma_{22} \cdot V_t \times \Gamma_2
\end{align*}
$$

where $\gamma_{ij}, i, j = 1, 2$ are the elements of regime transition probability matrix and $d_t$ is the measured current wind direction. $V_t$ is the current vector of wind speed; for example, it is $[0\ 1\ 0\ 0\ 0\ 0]$ if there are six quantized levels for wind, and if the wind speed yields currently in the second level, $V_{t+1}$ denotes the forecasting vector of the wind speed distribution in the next time step. Finally, $\Gamma_i, i = 1, 2$ are wind speed transition probability matrices in regimes 1 and 2.

Considering the possibility of transitions between regimes as Equation (17),

$$
\begin{align*}
\begin{bmatrix}
V^1_{t+1} \\
V^2_{t+1} \\
\vdots \\
V^m_{t+1}
\end{bmatrix}
= \gamma \cdot \text{diag}(V_t) \cdot 
\begin{bmatrix}
\Gamma_1 \\
\Gamma_2 \\
\vdots \\
\Gamma_m
\end{bmatrix}
\end{align*}
$$

where $\Gamma$ is a $2 \times 2$ matrix whose elements are $\gamma_{11}$ to $\gamma_{22}$. Equation (16) can be expanded for more number of regimes as Equation (17).

### 3. RESULTS

The proposed method has the following two advantages compared with the first-order MC.

- Temporal stationary improvement.
- Regimes identification

#### 3.1. Temporal Stationary Improvement

The first-order MC is the base of several extended methods that have been introduced by researchers for wind forecasting [9]. Extensions generally have been presented in the form of adding inner layers in the form of MC, ARMA, or semi-Markov process to the main MC [6, 10, 14]. In all of these works, a first-order MC in conjunction with some auxiliary process was utilized to
develop a more accurate method for modelling wind speed time-series. The proposed method in this article presents a new method for using wind direction time-series as extra information to enrich the MCs for modelling the wind speed time-series. In other words, the proposed method can be used as the base method instead of first-order MC in all of the presented complex methods as listed above.

To evaluate the efficiency of the proposed method, the results of the proposed method are compared with the results of the first-order MC, which introduced by Shamshad et al. [9] using real data.

The wind data of four consecutive years at IKIA wind station are processed using the first-order MC, and the temporal stationary index of two similar months is obtained in two consecutive and non-consecutive years. The results are presented in Table 3. Same processing was carried out using the proposed method with two, three and four regimes. The results are presented in Tables A1, A2 and A3 of the appendix as well as Figure 8. Obtaining temporal stationary index, for two similar months of two years as mentioned before, means using the one month as the training data and the other one as the testing data.

Comparing the results shows that the proposed method improves the temporal stationary index in most cases. For example, the first number on the top left of Table 3 (i.e., 144), shows that the temporal stationary value is 144 for January 2013 and January 2014 when the first-order MC is used. Meanwhile, it is 97.1 when the proposed method is used with two states (m=2), according to Table A1 of the appendix. According to Tables A2 and A3 of the appendix, the temporal stationary value is 81.97 when m=3 and 74.81 when m=4.

Inspecting the results show the proposed method improves temporal stationary value in about 70% of cases when m=2, 80% of cases when m=3 and 85% of cases when m=4 against the first-order MC.

Figure 8 shows the summarized results of the comparison of Table 3 and Tables A1 to A3 of the appendix. Percentage of improvement of the proposed method with the different number of regimes is shown in Figure 8. It is seen that the proposed method gives better temporal stationary index at least in 50% of cases in some months, while 100% improvement is not entirely rare. Improvement is better for larger values of m. However, increasing the number of states increases the computational cost. The computational cost of implementation of HMM has a proportional relation to the computational cost of the calculation of likelihood, as presented in Equation (7). The calculation of likelihood needs $Tm^2$ operations, where $T$ is the number of elements of time-series of observation, and $m$ is the number of regimes. As the $T$ is constant, the computational cost ratio of m regime separation to first-order MC (m=1) is $m^2$.

Figure 9 shows the percentage of improvement vs computational cost when the number of regimes increases to 4. It shows that increasing the number of regimes improves the results and increase the

| Month/Year | 13-14 | 13-15 | 13-16 | 14-15 | 14-16 | 15-16 |
|------------|-------|-------|-------|-------|-------|-------|
| 1          | 144   | 187   | 215   | 199   | 363   | 78    |
| 2          | 152   | 61    | 95    | 209   | 149   | 75    |
| 3          | 83    | 87    | 147   | 57    | 170   | 115   |
| 4          | 48    | 91    | 99    | 99    | 147   | 126   |
| 5          | 243   | 259   | 99    | 126   | 165   | 194   |
| 6          | 107   | 143   | 187   | 80    | 61    | 90    |
| 7          | 107   | 381   | 65    | 323   | 223   | 278   |
| 8          | 245   | 231   | 883   | 196   | 728   | 467   |
| 9          | 40    | 245   | 157   | 212   | 131   | 241   |
| 10         | 168   | 143   | 88    | 34    | 203   | 125   |
| 11         | 126   | 184   | 215   | 65    | 246   | 177   |
| 12         | 5     | 124   | 155   | 135   | 171   | 42    |

**TABLE 3.** Temporal stationary values of the first-order MC for two identical months of two different years.

![Figure 8](image_url)  
**Figure 8.** Percentage of improvement by the proposed method with 2, 3 and 4 regimes against first-order MC in different months.

![Figure 9](image_url)  
**Figure 9.** Improvement percentage vs ratio of the computational cost when the number of regimes increases.
computational cost. Figure 9 shows that investing computational power four times more, with respect to the first-order MC results in the improvement percentage of 70% for six times model run. Improvement percentage for investing nine times (when m=3) and sixteen times (when m=4) more computational power with respect to the first-order MC are also shown in Figure 9. It is clear that increasing computational power doesn’t offer the main changes in improvement percentage for a higher value of m. Figure 9 also shows that the effect of increasing the value of m from 2 to 3 in winter is much more than the summer.

3.2. Regimes Identification

To see how this approach works for regime identification, the wind speed in March 2014 is forecasted a) with the first-order MC (results showed in Figure 10) and b) with the proposed method with m=2 (results showed in Figure 11 and Figure 12).

To define states of the first-order MC (Equation (3)), the wind speed is quantized into six levels, each taken as a Markov state. Each of the six graphs in Figure 10 shows the probability of wind level in the next step. The top-left graph in this figure shows this probability when the current level of wind is 1; the middle top graph shows that value when the current level of wind is 2, and so on. The same order (from left to right and then from top to bottom) was considered for Figures 11 and 12. In Figures 11 and 12, it is assumed that the dominant regime is currently 1 and 2, respectively.

This example shows that by applying the proposed model, the single probability distribution of Figure 10 is separated into two probability distributions, as shown in Figures 11 and 12. Now, the dominant regime can be determined by online measuring of wind direction from which the relevant probability distribution is selected (Figures 11 or 12). This separation increases the concentration of the predictive distributions. For example, Figure 12 shows that levels 4 and 5 of wind speed have zero occurrence probability in regime 2, while the occurrence probability for these two levels of wind speed is not zero in regime 1 (Figure 11). Also, it is seen that level 3 of wind speed in regime one will remain unchanged or goes to level 4 of wind speed (Figure 11), while in regime two it will remain unchanged or goes to level 2 (Figure 11). This kind of distinction is one of the advantages of the proposed method.

According to long-term observations, there are two dominant wind directions at the IKIA weather station. One direction is from mountain-to-plain at night, which flows from the northern elevations of Tehran to the desert in the direction from 270 to 360°. The other is desert wind toward the mountain, in the opposite direction from about 90 to 180°.

This phenomena matches with the output of the proposed method and is well visible in the results of the
separation of dual regimes performed for all months over the four years. In winter, when large-scale phenomena are mainly active, in addition to the two aforementioned wind directions due to local conditions, there is one more dominant wind direction from the south-west. This causes to have better results in three regimes separation in winter times against summer times.

4. CONCLUSIONS

This article presented a model for wind speed forecasting based on the classification of a wind data set into some groups (regimes). Each regime has specific statistical behaviour. Actually, the model has two cascade layers, so it is called “Hierarchical Markov Model”. The first layer of the model detects and separates wind regimes as heterogenic groups of wind data by using wind direction data, based on HMM, and the second layer forecasts the wind speed under this regime using MC.

The model was implemented and tested with four years historical data belong to IKIA, and its results compared with those of a first-order MC-based method. For this comparison, two indices: temporal stationary and Probability Distribution Function (PDF) shape of the wind speed forecast, were used. The comparison results showed that the proposed method improves temporal stationary index (improves prediction accuracy) against the first-order MC in at least 70% of cases. Moreover, wind regimes identified by the proposed method match the long-term observations of local experts. Indeed, our method gives a PDF which is sharper than the one obtained with the first order MC for forecasted wind speeds; means that more precise prediction.

The proposed method can be used as a suitable tool for very short-term wind forecasting in aircraft landing and takeoff process, planning of wind power plants, and so on. The main restriction of the proposed method is its incapability to forecast rare events such as strong wind which doesn’t have enough frequency to affect the element of MCs. It would be an area of future research to deploy the remote sensing instruments to capture the rare extreme events before the entrance to the target zone and enrich the forecasts.

5. REFERENCES

1. Zhanxin, Y., Fang, Z., Lixiong, X., Hongjun, L., Dapeng, X., Junnan, L., Yu, D. and Yales, L., “Investigation on equivalent trans-utilization mode and benefit of wind energy”, International Journal of Engineering, Transactions A: Basics, Vol. 31, No. 10, (2018), 1708-1714. doi:10.5829/ijpe.2018.31.10a.13.

2. Keyhani, A., Ghasemi-Varnamkhasti, M., Khanali, M. and Abbaszadeh, R., “An assessment of wind energy potential as a power generation source in the capital of iran, tehran”, Energy, Vol. 35, No. 1, (2010), 188-201. doi: 10.1016/j.energy.2009.09.009.

3. Lei, M., Shiyan, L., Chuanwen, J., Hongling, L. and Yan, Z., “A review on the forecasting of wind speed and generated power”, Renewable and Sustainable Energy Reviews, Vol. 13, No. 4, (2009), 915-920. doi: 10.1016/j.rser.2008.02.002.

4. Han, K., Choi, J. and Kim, C., “Comparison of statistical post-processing methods for probabilistic wind speed forecasting”, Asia-Pacific Journal of Atmospheric Sciences, Vol. 54, No. 1, (2018), 91-101. doi: 10.1007/s13143-017-0062-z.

5. Wang, X., Guo, P. and Huang, X., “A review of wind power forecasting models”, Energy procedia, Vol. 12, (2011), 770-778. doi: 10.1016/j.egypro.2011.10.103.

6. Tagliaferri, F., Hayes, B., Viola, I. and Djokić, S., “Wind modelling with nested markov chains”, Journal of Wind Engineering and Industrial Aerodynamics, Vol. 157, (2016), 118-124. doi: 10.1016/j.jweia.2016.08.009.

7. Liu, L. and Hu, F., “Long-term correlations and extreme wind speed estimations”, Advances in Atmospheric Sciences, Vol. 36, No. 10, (2019), 1121-1128. doi: 10.1007/s00376-019-9031-z.

8. Liu, C., Fu, L., Yang, D., Miller, D.R. and Wang, J., “Non-gaussian lagrangian stochastic model for wind field simulation in the surface layer”, Advances in Atmospheric Sciences, Vol. 37, No. 1, (2020), 90-104. doi: 10.1007/s00376-019-9052-7.

9. Shamshad, A., Bawadi, M., Hussin, W.W., Majid, T. and Sanusi, S., “First and second order markov chain models for synthetic generation of wind speed time series”, Energy, Vol. 30, No. 5, (2005), 693-708. doi: 10.1016/j.energy.2004.05.026.

10. D’Amico, G., Petroni, F. and Practico, F., “Wind speed modeled as an indexed semi-markov process”, Environmetrics, Vol. 24, No. 6, (2013), 367-376. doi: 10.1002/env.2215.

11. Suomalainen, K., Silva, C., Ferrão, P. and Connors, S., “Synthetic wind speed scenarios including diurnal effects: Implications for wind power dimensioning”, Energy, Vol. 37, No. 1, (2012), 41-50. doi: 10.1016/j.energy.2011.08.001.

12. Scholz, T., Lopes, V.V. and Estanqueiro, A., “A cyclic time-dependent markov process to model daily patterns in wind turbine power production”, Energy, Vol. 67, (2014), 557-568. doi: 10.1016/j.energy.2013.12.071.

13. Karatepe, S. and Coscarelli, K.W., “Wind speed estimation: Incorporating seasonal data using markov chain models”, International Scholarly Research Notices, Vol. 2013, (2013), doi:10.1155/2013/3657437.

14. Xie, K., Liao, Q., Tai, H.-M. and Hu, B., “Non-homogeneous markov wind speed time series model considering daily and seasonal variation characteristics”, IEEE Transactions on Sustainable Energy, Vol. 8, No. 3, (2017), 1281-1290. doi:10.1109/TSTE.2017.2675445.

15. Alliot, P. and Monbet, V., “Markov-switching autoregressive models for wind time series”, Environmental Modelling & Software, Vol. 30, (2012), 92-101. doi: 10.1016/j.envsoft.2011.10.011.

16. Alliot, P., Bessac, J., Monbet, V. and Pene, F., “Non-homogeneous hidden markov-switching models for wind time series”, Journal of Statistical Planning and Inference, Vol. 160, (2015), 75-88. doi: 10.1016/j.jspi.2014.12.005.

17. Seyedarabi, H. and Feizi, A., “Application of combined local object based features and cluster fusion for the behaviors recognition and detection of abnormal behaviors”, International Journal of Engineering, Transactions B: Applications, Vol. 28, No. 11, (2015), 1597-1604. doi: 10.5829/dosij.ije.2015.28.11b.07.

18. Zucchini, W., MacDonald, I.L. and Langrock, R., “Hidden markov models for time series: An introduction using r, CRC press, (2017). doi: 10.1201/9781420010893.
6. APPENDIX

The wind data of four consecutive years at IKIA wind station are processed using the proposed method and the temporal stationary index of two similar months is obtained in two consecutive and non-consecutive years. The results are presented in Tables A1, A2 and A3.

### TABLE A1. Temporal stationary values of the proposed method for two identical months of two different years. m=2

| Month/Year | 13-14 | 13-15 | 13-16 | 14-15 | 14-16 | 15-16 |
|------------|-------|-------|-------|-------|-------|-------|
| 1          | 97.17 | 124.8 | 129.6 | 113.4 | 179.4 | 76.96 |
| 2          | 129.5 | 71.22 | 91.37 | 159.4 | 156.1 | 78.91 |
| 3          | 77.7  | 111.9 | 117.9 | 94.2  | 146.1 | 81.99 |
| 4          | 66.27 | 93.85 | 114.8 | 79.54 | 97.44 | 110.6 |
| 5          | 175.1 | 207.1 | 102.1 | 99.8  | 115.3 | 147.1 |
| 6          | 124.4 | 145.5 | 173.8 | 81.6  | 79.97 | 87.26 |
| 7          | 59.01 | 232.5 | 109   | 244.3 | 133.5 | 163.3 |
| 8          | 201.1 | 166   | 430.9 | 328.8 | 412.7 | 208.4 |
| 9          | 55.25 | 189.6 | 132.1 | 176.5 | 142.6 | 115.1 |
| 10         | 163.9 | 120.2 | 75.27 | 90.36 | 205   | 119.4 |
| 11         | 111.5 | 134.6 | 125.4 | 62.05 | 187.6 | 145.5 |
| 12         | 2.5   | 134   | 126.4 | 134.1 | 138.9 | 79   |

In Tables A1-A3, white cells show the cases for which, the proposed method improves the temporal stationary against first-order Markov chain (Table 3). Gray cells indicate the cases with no change, and dashed cells show the cases for which the proposed method worsens the temporal stationary compared to the first-order MC.

### TABLE A2. Temporal stationary values of the proposed method for two identical months of two different years. m=3

| Month/Year | 13-14 | 13-15 | 13-16 | 14-15 | 14-16 | 15-16 |
|------------|-------|-------|-------|-------|-------|-------|
| 1          | 81.77 | 128.1 | 122.1 | 107.7 | 175.5 | 75.08 |
| 2          | 109.1 | 62.73 | 93.84 | 132.1 | 129.4 | 98.67 |
| 3          | 63.82 | 78.59 | 108.5 | 82.07 | 117.9 | 82.58 |
| 4          | 69.89 | 94.3  | 103.3 | 70.71 | 99.06 | 91.28 |
| 5          | 132.1 | 166.8 | 93.37 | 89.65 | 80.78 | 134.6 |
| 6          | 120.5 | 133.9 | 153.9 | 91.22 | 83.38 | 82.49 |
| 7          | 62.79 | 207.8 | 102   | 218.4 | 129.8 | 153.7 |
| 8          | 185.5 | 150.5 | 378.7 | 196.4 | 344.9 | 181.5 |
| 9          | 51.82 | 216.7 | 111.1 | 166.5 | 113.8 | 143.7 |
| 10         | 116.3 | 102.9 | 59.96 | 111   | 131.8 | 88.49 |
| 11         | 75.91 | 82.19 | 101.4 | 51.27 | 135   | 102.8 |
| 12         | 1.898 | 114.1 | 90.36 | 121.6 | 99.11 | 68.33 |

### TABLE A3. Temporal stationary values of the proposed method for two identical months of two different years. m=4

| Month/Year | 13-14 | 13-15 | 13-16 | 14-15 | 14-16 | 15-16 |
|------------|-------|-------|-------|-------|-------|-------|
| 1          | 74.81 | 117.4 | 111.2 | 104.8 | 162.4 | 64.98 |
| 2          | 105.1 | 63.30 | 88.04 | 146.9 | 93.13 | 93.34 |
| 3          | 58.62 | 73.22 | 108   | 83.61 | 120.5 | 79.89 |
| 4          | 57.73 | 91.25 | 98.27 | 69.53 | 95.82 | 84.92 |
| 5          | 130.6 | 168.1 | 83.21 | 86.35 | 71.63 | 122.3 |
| 6          | 97.03 | 96.97 | 109.7 | 82.47 | 74.72 | 69.94 |
| 7          | 61.7  | 214.3 | 108   | 205.1 | 122.1 | 146.9 |
| 8          | 178   | 136.9 | 342.5 | 178.2 | 286.7 | 173.1 |
| 9          | 60.38 | 124.1 | 94.48 | 134.6 | 104.8 | 129.5 |
| 10         | 116.5 | 99.36 | 60.72 | 77.21 | 140   | 88.74 |
| 11         | 70.52 | 78.95 | 99.69 | 49.77 | 127.3 | 97.08 |
| 12         | 1.839 | 111.6 | 82.42 | 118.9 | 90.87 | 69.08 |
چکیده

این مقاله روشی جدید را با استفاده از مدل پنهان مارکوف (HMM) که به خوبی شناخته شده است، برای آشکارسازی ناهمگونی‌های موجود در داده‌های سری زمانی باد ارائه می‌دهد. در روش ارائه شده، سری زمانی باد را به گروه‌هایی که هر گروه نشان‌دهنده یک رژیم باد هستند، تقسیم کرده و در داخل هر گروه، یک زنجیره مارکوف مرتبه اول برای پیش‌بینی سرعت باد در هر گروه استفاده می‌شود. با استفاده از این مدل، ساختار سلسله مراتب مارکوف نامیده می‌شود و لایه اول را با مدل HMM استفاده می‌کند و لایه دوم را با استفاده از زنجیره مارکوف پیش‌بینی می‌کند. این روش توان مهیا مقدار سرعت باد را در هر گروه را با مدل ساده‌تر زنجیره مارکوف مرتبه اول تولید می‌کند. نتایج نشان می‌دهد که هنگام استفاده از این روش، بهبود در پیش‌بینی سرعت باد در بیش از 70 درصد موارد دارند. علاوه بر آن، پیش‌بینی‌های تابع توزیع احتمال سرعت باد را در زمان‌های مختلفتر و با ترکیب اصلاح‌های مدل پیش‌بینی دقیق‌تر است.