Annihilation Contributions and CP Asymmetries

in $B^+ \to \pi^+ K^0, K^+ K^0$ and $B^0 \to K^0 \bar{K}^0$

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Abstract

Recently the branching ratios for $B^+ \to K^+ \bar{K}^0$ and $B^0 \to K^0 \bar{K}^0$ have been measured. Data indicate that the annihilation amplitudes in these decays are not zero. A non-zero annihilation amplitude plays an important role in CP violation for $B^+ \to \pi^+ K^0, K^+ \bar{K}^0$. Using the measured branching ratios for these decays, we show that there is an absolute bound of 5% for the size of CP asymmetry in $B^+ \to \pi^+ K^0$ from a relation between the amplitudes of these decays. The size of CP asymmetry in $B^+ \to K^+ \bar{K}^0$ can, however, be as large as 90%. Future experimental data will test these predictions.
Rare $B$ decays provide much information about the Standard Model (SM) of the strong and electroweak interaction. So far the SM is perfectly consistent with data on $B$ decays. With more data becoming available from Babar and Belle experiments, $B$ physics has entered an era of precision test. Not only stringent constraints on new physics beyond the SM have been derived, but also information about some of the more subtle aspects on the low energy strong interaction responsible for $B$ hadronization and related hadronic decay amplitudes has been extracted, in particular for $B$ to two $SU(3)$ octet pseudoscalars $PP[1, 2]$. In this work we study information about decay amplitudes and CP asymmetries using the recently measured branching ratios for $B^+ \rightarrow \pi^+ K^0, K^+ \bar{K}^0$ and $B^0 \rightarrow K^0 \bar{K}^0$.

There are several reasons why $B^+ \rightarrow \pi^+ K^0, K^+ \bar{K}^0$ and $B^0 \rightarrow K^0 \bar{K}^0$ decays are interesting to study[4]. The tree contributions are usually thought to be small for these decays since they come from the so called annihilation diagrams and are neglected in many of the previous studies. If the annihilation contributions are neglected, $B(B^+ \rightarrow \pi^+ K^0)/B(B^+ \rightarrow K^+ \bar{K}^0)$ (here the branching ratios are averaged over $B$ and $\bar{B}$ decays) is approximately equal to $|V_{ts}/V_{td}|^2$ which can be tested. A method using $B \rightarrow \pi K$ decays to determine the CP violating phase $\gamma$ in the Cabbibo-Kobayashi-Maskawa (CKM) matrix as discussed in Ref.[5] crucially depends on the assumption that the annihilation contribution to $B^+ \rightarrow \pi^+ K^0$ is zero. Therefore it is important to find out whether the annihilation contributions is zero or not. Using recent experimental data, we find that the annihilation contributions to rare $B$ decays may not be negligible after all. We also find that using a relation in the amplitudes for $B^+ \rightarrow \pi^+ K^0$ and $B^+ \rightarrow K^+ \bar{K}^0$, with non-zero annihilation contributions there is a relation for CP asymmetries in $B^+ \rightarrow \pi^+ K^0$ and $B^+ \rightarrow K^+ \bar{K}^0$. From this relation, we obtain an upper bound of 5% for the size of CP asymmetry in $B^+ \rightarrow \pi^+ K^0$. The size of CP asymmetry in $B^+ \rightarrow K^+ \bar{K}^0$ can, however, be as large as 90%. In the following we provide details.

In the SM the leading order decay amplitude for the charmless $B$ decays can be decomposed into two terms proportional to $V_{ub}^* V_{uq}$ and $V_{tb}^* V_{uq}$ respectively. For $B^+ \rightarrow \pi^+ K^0$, and $B^+ \rightarrow K^+ \bar{K}^0$, we have

$$A(B^+ \rightarrow \pi^+ K^0) = V_{ub}^* V_{us} T_s + V_{tb}^* V_{ts} P_s,$$

$$A(B^+ \rightarrow K^+ \bar{K}^0) = V_{ub}^* V_{ud} T_d + V_{tb}^* V_{td} P_d.$$  \hspace{1cm} (1)

In the SU(3) limit, $T_s = T_d$ and $P_s = P_d$. In terms of the diagram amplitudes discussed
in Ref. [6], one can parameterize them as $T_s = A$ and $P_s = P - P_{EW}^C/3$, where $P$, $P_{EW}$ and $A$ stand for penguin, electroweak penguin and annihilation amplitudes, respectively. When $SU(3)$ breaking effects are included, there are modifications. It is expected that a large fraction of the $SU(3)$ breaking effects may be absorbed into the relevant meson decay constants. This can be understood from the PQCD calculation for $B \rightarrow PP$ decay amplitudes [7]. In this approach quarks are viewed as partons forming the initial and final mesons and are convoluted with the relevant hadron light-cone wave functions which are normalized to their decay constants. To the leading order one would have $f_K T_d = f_\pi T_s$ and $f_K P_d = f_\pi P_s$. We can then further simplify the above decay amplitudes to

$$A(B^+ \rightarrow \pi^+ K^0) = P_s (V_{ub}^* V_{ts} a + V_{tb}^* V_{ts}),$$
$$A(B^+ \rightarrow K^+ \bar{K}^0) = \frac{f_K}{f_\pi} P_s (V_{ub}^* V_{ud} a + V_{tb}^* V_{td}),$$

where $a = T_s / P_s$ is a complex number. We parameterize it as $a = r e^{i \delta_r}$ for our later discussions.

For the above two processes only small annihilation amplitude $A$ contributes to terms proportional to $V_{ub} V_{us}^*$, they are usually neglected. If true, there are several interesting experimental consequences with one of them being that CP asymmetries in $B^+ \rightarrow \pi^+ K^0$, and $B^+ \rightarrow K^+ \bar{K}^0$ are zero, and also that $R = \bar{B}(B^+ \rightarrow \pi^+ K^0)/\bar{B}(B^+ \rightarrow K^+ \bar{K}^0)$ would give a good determination for $(f_\pi^2 / f_K^R)|V_{ts}|^2 / |V_{td}|^2$.

Recent experiments at the BaBar and Belle have measured branching ratios of these decays with $B(B^+ \rightarrow \pi^+ K^0) = (23.1 \pm 1.0) \times 10^{-6}$ and $\bar{B}(B^+ \rightarrow K^+ \bar{K}^0) = (1.36^{+0.29}_{-0.27}) \times 10^{-6}$. Using these numbers and $f_K / f_\pi = 1.198 \pm 0.003^{+0.016}_{-0.005}$ [8], we obtain,

$$\frac{|V_{td}|^2}{|V_{ts}|^2} = 0.041 \pm 0.009,$$

in accordance with best global fit value for the CKM parameters which gives $|V_{td}|^2 / |V_{ts}|^2 = 0.036$.

If the $SU(3)$ correction factor $f_K / f_\pi$ in eq. (2) is not included, one would obtain a value $|V_{td}|^2 / |V_{ts}|^2$ around 0.059 which is substantially away from the best global fit value. This supports the expectation that a large fraction of the $SU(3)$ breaking effects are taken care of by the light meson decay constants. We note that the central value of $|V_{td}|^2 / |V_{ts}|^2$ determined using $f_K^2 / f_\pi^2 R$ and using global fit deviate from each other. Of course it is too early to say there is a definitive difference due to large errors.
The available experimental data also allow us to extract more detailed information about the decay amplitudes. The amplitudes $T_{d,s}$ need not to be zero. In has been pointed out before that the data allow non-zero values for $T_{d,s}$. The discussion above also does not rule out this possibility. We would like to point out that there is another indication that the amplitude $A$ is not zero. This is from a comparison of the observed branching ratios for $B^+ \to K^+ \bar{K}^0$ and $B^0 \to K^0 \bar{K}^0$. One can write the amplitudes for these decays as

$$A(B^+ \to K^+ \bar{K}^0) = V_{ub}^* V_{ud} A + V_{tb}^* V_{td} (P - \frac{1}{3} P_{EW}^C),$$

$$A(B^0 \to K^0 \bar{K}^0) = V_{tb}^* V_{td} (P - \frac{1}{3} P_{EW}^C + PA).$$

(4)

Note that one needs to introduce a new amplitude $P_A$ which is called the penguin-annihilation amplitude, for $B^0 \to K^0 \bar{K}^0$. Since the Wilson coefficients involved in $P_A$ are smaller than those in $A$, $P_A$ is expected to be smaller than $A$ in size. If annihilation amplitudes $A$ and $P_A$ are negligibly small, one would obtain $S = (\tau_{B^+}/\tau_{B^0}) (\tilde{B}(B^+ \to K^+ \bar{K}^0)/\tilde{B}(B^0 \to K^0 \bar{K}^0)) = 1$. However, the measured branching ratios gives $S = 1.32 \pm 0.39$. The central value of $S$ is substantially away from 1. This indicates that the annihilation amplitude $A$ or $P_A$ (or both of them) maybe not zero. To have some detailed idea about what are the allowed ranges for $r$ and $\delta_r$, we consider the case with $P_A$ neglected in more detail. We have

$$S = 1 + r^2 \left| \frac{V_{ub}^* V_{ud}}{V_{tb}^* V_{td}} \right|^2 + 2r \cos \delta_r Re \left( \frac{V_{ub}^* V_{ud}}{V_{tb}^* V_{td}} \right).$$

(5)

Using the above we can obtain the allowed ranges for $r$ and $\delta_r$. In Fig. we show $r$ as a function of $\delta_r$ for several values of $S$. In obtaining Fig. we have treated the CKM matrix elements as known and used the most recent values given by the Particle Data Group with $\lambda = 0.2262$, $A = 0.815 \bar{\rho} = 0.235$ and $\bar{\eta} = 0.349$ (the corresponding sines of mixing angles and phase are: $s_{12} = 0.2262$, $s_{13} = 0.0039$, $s_{23} = 0.0417$ and $\delta_{13} = 0.9781$).

There are theoretical calculations trying to estimate the decay amplitudes for $B^+ \to \pi^+ K^0$ and $B^+ \to K^+ \bar{K}^0$. One of the popular methods used to estimate such contributions is the QCDF. In this method, the annihilation amplitudes have end point divergences which are usually indicated by a quantity $X_A = \int_0^1 dy/y$. In Ref. this is phenomenologically regulated by a cut off $\Lambda_b$ parameterized as $X_A = (1 + \rho_A e^{i\delta_A}) \ln(m_B/\Lambda_b)$ and $\rho_A < 1$. The resulting annihilation amplitude $A$ is in general complex. The typical value
from such a calculation gives $r$ of order 10% with a cut off $\Lambda_b \sim 0.5$ GeV. If a smaller cut off $\Lambda_b$ is used, $r$ can be larger. Owing to the uncertainties in treating the divergences, one cannot give a precise value for $a = re^{\delta_r}$. Therefore in our later discussions, we will not use theoretical value for $a$ but use allowed range from data. The important point is that one should not set $r$ to zero for precision studies. We now study some implications with non-zero annihilation contributions. Before carrying out a numerical analysis of the allowed ranges for $r$ and $\delta_r$, we study some implications for CP violation with non-zero values of $r$ and $\delta_r$. One consequence is that CP violating asymmetries $A_{CP}(B^+ \to \pi^+K^0, K^+\bar{K}^0)$ defined in the following will not be zero,

$$A_{CP}(B^+ \to \pi^+K^0) = \frac{B(B^- \to \pi^-\bar{K}^0) - B(B^+ \to \pi^+K^0)}{B(B^- \to \pi^-\bar{K}^0) + B(B^+ \to \pi^+K^0)} = -\frac{2r \sin \delta_r \text{Im}(V_{ub}^*V_{us}V_{tb}V_{ts})}{|V_{tb}^*V_{ts}|^2 + r^2|V_{ub}^*V_{us}|^2 + 2r \cos \delta \text{Re}(V_{ub}^*V_{us}V_{tb}V_{ts})},$$

$$A_{CP}(B^+ \to K^+\bar{K}^0) = \frac{B(B^- \to K^-\bar{K}^0) - B(B^+ \to K^+\bar{K}^0)}{B(B^- \to K^-\bar{K}^0) + B(B^+ \to K^+\bar{K}^0)} = -\frac{2r \sin \delta_r \text{Im}(V_{ub}^*V_{ud}V_{tb}V_{td}^*)}{|V_{tb}^*V_{td}|^2 + r^2|V_{ub}^*V_{ud}|^2 + 2r \cos \delta \text{Re}(V_{ub}^*V_{ud}V_{tb}V_{td}^*)}. \quad (6)$$

It has been pointed out before\cite{12} that, due to the property $\text{Im}(V_{ub}^*V_{us}V_{tb}V_{ts}^*) = -\text{Im}(V_{ub}^*V_{ud}V_{tb}V_{td}^*)$ of the CKM matrix, there are relations between rate differences $\Delta(PP) = \Gamma(\bar{B} \to \bar{P}\bar{P}) - \Gamma(B \to PP)$ for some $B \to PP$ decays. For $A_{CP}(B^+ \to \pi^+K^0)$ and $A_{CP}(B^+ \to K^+\bar{K}^0)$, we have

$$A_{CP}(B^+ \to \pi^+K^0) = -\frac{f_{\pi}^2}{f_K^2 R} A_{CP}(B^+ \to K^+\bar{K}^0). \quad (7)$$
Since the maximal size for $A_{CP}(B^+ \to K^+\bar{K}^0)$ can at most be 1, one immediately obtains an upper bound of 5% for the size of CP asymmetry in $B^+ \to \pi^+K^0$. The present data $0.009 \pm 0.025$ on $A_{CP}(B^+ \to \pi^+K^0)$ is consistent with the upper bound. Measurement of CP asymmetry in $B^+ \to \pi^+K^0$ therefore can test the relation in eq.(7). Using experimental data for $A_{CP}(B^+ \to \pi^+K^0)$, one would obtain, $A_{CP}(B^+ \to K^+\bar{K}^0) \approx -0.22 \pm 0.62$. This is to be compared with the data $0.12^{+0.05}_{-0.17}$ for $A_{CP}(B^+ \to K^+\bar{K}^0)$. Naively it seems that there is a potential problem since the central experimental data has opposite sign as that predicted. However, the error bar is still too large to draw a conclusion. Improved data on $B^+ \to K^+\bar{K}^0$ can provide more crucial information about hadronic parameters in $B$ decays.

To find out how large CP asymmetry in $B^+ \to K^+\bar{K}^0$ can be and how it is correlated to information from $B^+ \to \pi^+K^0$, more information is needed. Again we use the experimental data on the branching ratios and known values for the CKM matrix elements to constrain the allowed regions. Theoretically, we have

$$R = \frac{\bar{B}(B^+ \to \pi^+K^0)}{B(B^+ \to K^+\bar{K}^0)} = \frac{f^2_\pi |V_{ts}^*V_{tb}|^2 + r^2|V_{ub}^*V_{us}|^2 + 2r \cos \delta_r Re(V_{ub}^*V_{us}V_{td}V_{ts}^*)}{f^2_K |V_{td}^*V_{ub}|^2 + r^2|V_{us}^*V_{ud}|^2 + 2r \cos \delta_r Re(V_{ub}^*V_{ud}V_{td}V_{ts}^*)}.$$  \hspace{1cm} (8)$$

Comparing the above expression with data, we obtain the allowed ranges of the parameter space for $r$ and $\cos \delta_r$ in Fig[2]. We find that not the entire allowed one-$\sigma$ range of $R = 13.41 \sim 20.55$ has solutions for $r$ and $\delta_r$. With increased $R$ the allowed range becomes smaller and eventually shrinks to a point at $R = 19.44$. There are no solutions beyond that point. If future experimental data determine a $R$ larger than 19.44, it is a strong indication that there is new physics beyond the SM so that the relation in eq.(2) is badly broken. Within the one-$\sigma$ allowed range for $R$, there is a solution for $r = 0$ where $R = 18.65$. With more precise determination of $R$, one can have better information about the size of $r$. At present we see that there is a large allowed range for $r$ and $\delta_r$. Although large $r$ is unlikely from theoretical considerations, this possibility is not ruled out yet at present. We note that the allowed range is more restrictive than that shown in Fig[1] if both $R$ and $S$ are restricted to be within their one-$\sigma$ ranges. In the following discussions we will use the ranges for $r$ and $\delta_r$ constrained from $R$ instead that from $S$. The reason is two folds with one of them being that the constraint from $R$ is more stringent, and another being that in obtaining allowed ranges for $r$ and $\delta_r$ in Fig[1] $P_A$ has been neglected. A non-zero $P_A$ can change the ranges.

With the allowed ranges for $r$ and $\delta_r$ fixed, one can obtain the allowed range for CP asymmetry in $B^+ \to K^+\bar{K}^0$. We present the predicted CP asymmetry for $B^+ \to K^+\bar{K}^0$
FIG. 2: Allowed ranges for $r$ and $\delta$ for different values of $R = \hat{B}(B^+ \to \pi^+ K^0)/\hat{B}(B^+ \to K^+ \bar{K}^0)$.

in Fig.2. CP asymmetry for $B^+ \to \pi^+ K^0$ can be easily obtained from Fig.3 using eq. (7). We see that the size of CP asymmetry $A_{CP}(B^+ \to K^+ \bar{K}^0)$ becomes larger as $R$ decreases. At the lower end of the one-$\sigma$ allowed $R$, the size of $A_{CP}(B^+ \to K^+ \bar{K}^0)$ can be as large as 0.92. Consequently the size of $A_{CP}(B^+ \to \pi^+ K^0)$ can almost reach its upper bound of 0.05.

Measurement on CP asymmetries in these decay modes can provide valuable information about the hadronic parameters in $B$ decays.

FIG. 3: $A_{CP}(B^+ \to K^+ \bar{K}^0)$ as a function of $\delta$ for several allowed values $R$.

To summarize, in this work we have shown that the recently measured branching ratios for $B^+ \to K^+ \bar{K}^0$ and $B^0 \to K^0 \bar{K}^0$ indicate that the annihilation amplitudes in these decays may be not zero. This observation has important impacts on CP violation in $B^+ \to \pi^+ K^0, K^+ \bar{K}^0$. Using the measured branching ratios for these decays, we find that there is an absolute bound of 5% for the size of CP asymmetry in $B^+ \to \pi^+ K^0$ from a relation in the amplitudes of these decays. The size of CP asymmetry in $B^+ \to K^+ \bar{K}^0$ can, however, be as large as 90%.
Future experimental data will test these predictions.

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