Search for New Physics in Rare B Decays

The OPAL Collaboration

Abstract

A search for the decay $B \rightarrow K \pi K$ was performed using data collected by the OPAL detector at LEP. These decays are strongly suppressed in the Standard Model but could occur with a higher branching ratio in supersymmetric models, especially in those with R-parity violating couplings. No evidence for a signal was observed and a 90% confidence level upper limit of $1.29 \times 10^{-4}$ was set for the branching ratio.

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1 Introduction

Rare b decays offer an opportunity to discover new physics beyond the Standard Model (SM). Many studies have been done in recent years to predict flavor changing neutral current (FCNC) processes, both within the SM and beyond \[1\]. One of these FCNC induced processes, \(b \rightarrow s \bar{s}d\), has been measured \[2\] and the branching ratio found to be consistent with the SM prediction \[3\]. However, significant uncertainties still remain in both the theoretically predicted branching ratio and the measurement. Thus it is hard to conclude if this process shows signs of new physics \[4\]. This is also true in most of the channels such as \(b \rightarrow s \bar{s}d\) and \(b \rightarrow s \bar{s}d\), due to theoretical uncertainties.

The process \(b \rightarrow s \bar{s}d\), induced by a box diagram, is predicted to be exceedingly small in the SM (Figure 1(a)), of the order of \(10^{-11}\) \[7\]. However, in the minimal supersymmetric standard model (MSSM) \[8\], this transition can be induced by the squark-gaugino (or higgsino) box diagram (Figure 1(b)) at a level of \(10^{-7} \sim 10^{-8}\). An alternative mechanism for this channel in supersymmetric models is through R-parity violating couplings \[9\] (Figure 1(c)). These two possibilities appear to be the only ones that will produce significant enhancement of this decay within supersymmetric models \[7\]. Two higgs doublet models could also induce this decay at branching ratios significantly larger than in the Standard Model, for a certain range of the parameters involved \[10\].

Typical exclusive processes of \(b \rightarrow s \bar{s}d\) include \(B \rightarrow K K^0\), which are difficult to separate from the standard penguin process \(b \rightarrow d \bar{s}s\). Although the interference of these two sources of the final state is crucial in the study of phenomena such as CP violation, this channel is not suitable for a direct search for new physics. However, the decay \(B \rightarrow K K^+\), either as a direct three-body decay or through a \(K^3\)-like resonance, is a clear signature of this process. This document describes the first search for the decay \(B \rightarrow K K^+\).

![Figure 1: b \rightarrow s \bar{s}d transition (a) SM, (b) MSSM, (c) MSSM with R-parity violating coupling.](image-url)

\(^1\)charge conjugation is assumed throughout this paper
2 Hadronic Event Selection and Simulation

We used data collected at LEP by the OPAL detector [1] between 1990 and 1995 running at center-of-mass energies in the vicinity of the Z\(^0\) peak. Hadronic Z\(^0\) decays were selected using the number of charged tracks and the visible energy in each event as in Reference [12]. This selection yielded 4.41 million hadronic events.

Monte Carlo events were used to determine the selection efficiency, for training of an artificial neural network (ANN) used in selecting the final event sample (Section 3), and for the determination of some of the systematic uncertainties (Section 4). To determine the selection efficiency we generated 100 000 Monte Carlo events of the process Z\(^0\rightarrow b\bar{b}\) where one of the b quarks hadronised into a B meson which subsequently decayed to K\(^-\)K\(^+\). One sample was generated according to three-body decay phase space, while another sample was generated with angular distribution as expected from a weak decay matrix element. In addition, we generated samples in which the B meson decayed via an intermediate K\(^\ast\) resonance, K\(^\ast\)(892), K\(^\ast\)(1680) and K\(^\ast\)(2045) resonances were chosen as they all decay into K\(^-\)K\(^+\) and cover the entire spectrum of K\(^\ast\) resonances.

For optimisation of the selection of events and for some of the studies of systematic uncertainties, we generated 4 million 5-flavour hadronic Z\(^0\) decays (referred to as q\(q\bar{q}\) Monte Carlo). All these samples were generated with the JETSET 7.4 Monte Carlo program [13] with parameters tuned to the OPAL data [14]. The heavy quark fragmentation was parametrised by the fragmentation function of Peterson et al. [15], and all samples were processed with the OPAL detector simulation package [16].

3 Analysis Procedure

In each event, charged tracks and electromagnetic clusters not associated to a charged track were combined into jets, using the JADE algorithm with the E0 recombination scheme [17]. Within this algorithm jets are defined by \(y_{\text{cut}} = 0.04\); where \(y_{\text{cut}}\) is defined in Reference [17].

The primary vertex of the event was reconstructed using the charged tracks in the event and the knowledge of the position and spread of the e\(^+\)e\(^-\) collision point.

We searched the hadronic event sample for the decay B\(^-\)\rightarrow K\(^-\)K\(^+\) by combining three charged tracks to form a B meson candidate. All three track combinations were considered. All tracks were required to have a momentum of at least 2 GeV/c and to be in the same jet. Two of the tracks were required to have the same charge and were assigned the mass of a kaon. A third track, with an opposite charge, was assumed to be the pion. Tracks were required to satisfy selection criteria based on the measured rate of energy loss due to ionisation (dE=dx) [18] as listed in Table 1. These dE=dx selection criteria are 44% efficient while rejecting 98.5% of the background.

The three tracks were fitted to a common vertex and the decay length, the distance from the e\(^+\)e\(^-\) interaction point to the reconstructed secondary vertex, was calculated. Candidates where the secondary vertex is in the hemisphere opposite to the candidate’s jet were rejected. This criterion left 55% of the remaining background events, but kept 96% of the signal events.

Since the hadronic data sample consisted mostly of non-\(b\bar{b}\) events, we suppressed these events by means of a b-tagging algorithm, based on reconstructed displaced secondary vertices. An artificial neural network with inputs based on decay length significance, vertex multiplicity and invariant mass information [19] was used to select vertices with a high probability of coming from b hadron decays. Events were accepted if any of the jets were tagged by the neural network.
The b-tagging selection was found to be 79% efficient, while rejecting 80% of the remaining background.

Table 1: Summary of dE = dx selection criteria: dE = dx (K) is the difference between the measured value of the ionisation energy loss in the jet chamber and the expected value for (K) and represents the expected standard deviation of the distribution. N_{dE=dx} is the number of jet chamber hits used for dE = dx. The dE = dx cuts were chosen based on probability values.

| Selection | dE = dx | dE = dx_{K} | dE = dx_{p} | N_{dE=dx} |
|-----------|---------|-------------|-------------|-----------|
|           | < 1.29  | > 1.15      | > 0         | > 20      |

The background estimation was based on a combinatorial background with no second-order polynomial to the invariant mass of the background, obtained by releasing the ANN cut, and then normalising the shape to the mass side-bands of Figure 3 (4{5 GeV=c^2 and 5.6{6 GeV=c^2). Monte Carlo studies indicated that the background shape is not altered by this.
Figure 2: Input variables to the artificial neural network (a-f) and output (g). The solid line represents the q̅q Monte Carlo while the dots represent the data. The dashed line shows the distribution of B → K^0(892)K^0 Monte Carlo events. The arrow in (g) shows the cut value. All variables are normalised and are plotted after appropriate transformation to the range [0, 1].
Figure 3: Invariant mass distribution of the $K^- K^+$ candidates after all selection criteria were applied a) via intermediate $K^*$ resonance and b) with direct production. The dots represent the data, the solid line shows the expected signal shape from Monte Carlo events after all the selection criteria were applied with arbitrary normalisation, and the dashed line is the expected background.
procedure. Alternatively, we repeated this procedure by releasing each of the selection criteria separately and by obtaining the shape from Monte Carlo. All the alternative tests gave a consistent result. We also took the number of events within the signal region in each of the above cases and scaled it to the appropriate sample size. Here too, all estimates were consistent.

As we are setting upper limits, the conservative approach is to estimate the number of signal events using the lowest background estimate. While the fitted background shown in Figure 3 (a) gave 18.8 events in the signal region, the lowest estimate was 17.5 events. The respective numbers for the direct production case were 14.8 and 14.1 events.

3.2 Limit Determination

The above numbers were used to determine \( N^{90} \), the 90% C.L. upper limit on the number of signal events. Using the formalism of reference [21], we obtained \( N^{90} = 7.8 \) and \( N^{90} = 7.4 \) events for the resonant/direct decay, respectively.

To calculate an upper limit on the branching ratio we used:

\[
Br(B \rightarrow K K^+) \frac{N^{90}}{N_B};
\]

where \( N_B \) is the number of charged \( B \) mesons in the sample and \( \epsilon \) is the efficiency for Monte Carlo simulated events of the process \( B \rightarrow K K^+ \) to survive the selection procedure. With 4.41 million hadronic \( Z^0 \) decays, using \( Br(B \rightarrow K K^+) = 0.397^{+0.018}_{-0.022} \) and \( b_B = 0.2170 \pm 0.0009 \) [22], we obtained \( N_B = 759800 \pm 34600 \pm 42200 \).

The conservative approach when setting upper limits is to use the model giving the lowest efficiency for the signal. If one assumes resonance production, then the lowest efficiency, 8.11% 0.19%, is obtained when assuming the signal decay channel is via \( K^70 \) (892): The lowest efficiency for non-resonant decay, obtained with a phase space particle distribution, was found to be 11.3% 0.2%.

4 Systematic Uncertainties

Systematic uncertainties may arise from the limited accuracy with which \( N_B \) is known from the uncertainty in the simulation used to determine the efficiency and from the background estimation.

Modelling of \( dE = dx \)

To estimate the uncertainty arising from the modelling of the \( dE = dx \) selection criteria, we compared the efficiency of the \( dE = dx \) cuts in Monte-Carlo-simulated events and in data. We took advantage of the abundance and relative ease of reconstruction of \( D^{*+} \) mesons, and exploited them for testing the systematic uncertainties associated with the \( dE = dx \) selection criteria. We searched for \( D^{*+} \) mesons via their decay into a \( D^0 \) and a \( K^+ \), where the \( D^0 \) decays via a \( K^+ \). To enhance the signal to background ratio we required the momentum of the \( D^{*+} \) candidate to be larger than 15 GeV/c; the \( D^{*+} \) decay vertex to be at least 50 m away from the interaction point; and the helicity angle, \( \gamma \), between the kaon momentum in the \( D \) rest frame and the \( D \) direction in the laboratory frame to satisfy \( \cos \gamma < 0.7 \). Background estimation, after applying these selection criteria, was done as in [23]. To avoid possible uncertainty due to the difference in the momentum spectrum of the \( D^{*+} \) products with respect to the momentum spectrum of
tracks from the process $B^+ K^+ K^-$, we reweighted the $dE/dx$ selection efficiency as a function of the track momenta. By applying the $dE/dx$ criteria used to select the kaon we obtained an efficiency of 56.2% in data and 57.0% in Monte Carlo. The respective values for the pion selection efficiency are 75.0% and 76.8%. Combining all numbers, the relative uncertainty on the signal efficiency associated with the $dE/dx$ cuts is estimated at 3.7%.

Artificial neural network uncertainty

The agreement is good between the data and the Monte Carlo, in all of the input variables to the ANN (Figure 3). However, the Monte Carlo simulation compared in that figure represents the background and is not used in setting the upper limit. The simulation used to set the limit is that of signal events and thus, signal input variables should be compared. This is not possible for the decay $B^+ K^+ K^-$. Therefore, once again we made use of the $D^{\pm}$ signal. We compared the kaon and pion momentum distribution, the $D^{\pm}$ momentum and fraction of energy from the jet’s energy, the decay length and the vertex probability. Good agreement was achieved between the data and Monte Carlo as shown in Figure 3. Events were reweighted as a function of the track momenta to reflect the signal spectrum as in the $dE/dx$ uncertainty section, and the ANN output was evaluated. In order to assign a systematic uncertainty to the efficiency of the ANN, we took the difference between the fraction of $D^{\pm}$ events passing the ANN cut in the data to that in the Monte Carlo. We obtained an uncertainty of 4.1%. In addition, we took the difference in the ANN efficiency obtained for the signal involving different $K^0$ resonances. This difference was found to be 2.1% and the overall uncertainty assigned for this source was 4.6%.

B hadron lifetime and decay multiplicity

The probability to reconstruct the signal $B$ meson from also depends on the efficiency to reconstruct secondary vertices in both hemispheres. This in turn is sensitive to the charged decay multiplicity and lifetime of the $B$ hadrons. The Monte Carlo was reweighted to reflect the measured multiplicities and lifetimes [22]. The uncertainty on these figures gave an uncertainty of 1.1% and 1.3%, respectively, on the selection efficiency.

Detector modelling

The resolution of the tracking devices has an effect on the efficiency. The simulated resolutions were varied by 10% relative to the values that optimally describe the data following the studies in [24]. The analysis was repeated and the efficiency estimation was recalculated. This source contributed an uncertainty of 1.2%.

Fragmentation modelling

The heavy-quark fragmentation was simulated using the function of Peterson et al. [15]. The heavy-quark fragmentation model parameter was varied to change the mean scaled energy of weakly-decaying bottom hadrons within the experimental range: $\langle x \rangle_b = 0.702 \pm 0.008$ [22]. This change resulted in a 2% change in the efficiency. In addition, the heavy-quark fragmentation model was changed to that suggested by Collins and Spiller [23] and to that of Kartvelishvili et al. [24], with parameters tuned according to Reference [27]. No significant change in the resulting efficiency was observed.
Figure 4: Comparison of ANN input variables for $D^+$ candidates. The solid line represents the $q\bar{q}$ Monte Carlo and the dots are the data.

OPAL
Background estimation uncertainty

The uncertainty on the fitted shape parameters and on the normalisation gave an uncertainty on the background estimate. The different techniques used to estimate the background resulted in consistent estimates with a small standard deviation (about 1 event). However, since we used the lowest background estimate, these uncertainties were not taken into account as they were smaller than the difference between the mean background estimate and the one used.

5 Results

Combining all sources of systematic uncertainties mentioned above, as well as the statistical uncertainty in determining the efficiency and the uncertainty on $N_{b}$; we obtained an uncertainty of 8.4% on the denominator of Equation 1. This uncertainty was incorporated according to the method outlined in reference [28]. With $N_{90}^{res} = 7.8$ events and $N_{90}^{no\, res} = 7.4$ events we obtained:

\[
\begin{align*}
\text{Br}(B \to K K^+) & \quad 1.29 \times 10^{-4} \quad @ \quad 90\% \quad C.L.: \\
\text{Br}(B \to K K^+) \quad \text{non resonance} & \quad 8.79 \times 10^{-5} \quad @ \quad 90\% \quad C.L.: 
\end{align*}
\]

6 Summary

We have searched for the decay of charged B mesons to $K K^+$. This decay channel is strongly suppressed in the Standard Model, but may be large in $R$-parity violating models. Hence, this decay mode may serve as a probe for new physics beyond the Standard Model. No evidence has been observed for such a decay. Upper limits on the branching ratio have been set of $1.29 \times 10^{-4}$; or of $8.79 \times 10^{-5}$ if one assumes that the decay is not via a $K^0$ resonance, both at 90% confidence level.

Using these limits, and the estimate $\frac{B \to K K^+}{b! \, \text{had}} = \frac{1}{4} [4]$, we can put new limits on the contribution of $R$-parity violating couplings in this process. Starting from Equation 9 of reference [7],

\[
\frac{q}{j} \begin{pmatrix} n_{31} \alpha_{n_{31}} \alpha_{n_{21}} \text{d} + j \begin{pmatrix} n_{21} \alpha_{n_{21}} \alpha_{n_{23}} \text{d} \end{pmatrix} \end{pmatrix} < 5.9 \times 10^{-4},
\]

which can be compared to the existing limit of 0.1 obtained from $b! \, \overline{s} \overline{d}$ [7] and neutrino mass calculation [29].

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