Charged-Current Leptoproduction of D-Mesons in the Variable Flavor Scheme

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Abstract

We present formulae for the momentum ($z$) distributions of D-mesons produced in neutrino deep-inelastic scattering off strange partons. The expressions are derived within the variable flavor scheme of Aivazis et al. (ACOT-scheme), which is extended from its fully inclusive formulation to one-hadron inclusive leptoproduction. The dependence of the results on the assumed strange quark mass $m_s$ is investigated and the $m_s \to 0$ limit is compared to the corresponding MS results. The importance of $\mathcal{O}(\alpha_s)$ quark-initiated corrections is demonstrated for the $m_s = 0$ case.
The momentum \((z)\) distributions of D-mesons from the fragmentation of charm quarks produced in neutrino deep-inelastic scattering (DIS) have been used recently to determine the strange quark distribution of the nucleon \(s(x, Q^2)\) at leading order (LO) \([1]\) and next-to-leading order (NLO) \([2]\). A proper QCD calculation of this quantity requires the convolution of a perturbative hard scattering charm production cross section with a nonperturbative \(c \rightarrow D\) fragmentation function \(D_c(z)\) leading at \(\mathcal{O}(\alpha_s)\) to the breaking of factorization in Bjorken-\(x\) and \(z\) as is well known for light quarks \([3]\). So far experimental analyses have assumed a factorized cross section even at NLO \([2]\). This shortcoming has been pointed out in \([4]\) and the hard scattering convolution kernels needed for a correct and complete NLO analysis have been calculated in the \(\overline{\text{MS}}\) scheme with three massless flavors \((u, d, s)\) using dimensional regularization. In the experimental NLO analysis in \([2]\) the variable flavor scheme (VFS) of Aivazis, Collins, Olness and Tung (ACOT) \([5]\) for heavy flavor leptoproduction has been utilized. In this formalism one considers, in addition to the quark scattering (QS) process, e.g. \(W^+ s \rightarrow c\), the contribution from the gluon fusion (GF) process \(W^+ g \rightarrow c \bar{s}\) with its full \(m_s\)-dependence. The collinear logarithm which is already contained in the renormalized \(s(x, Q^2)\) is subtracted off numerically. The quark–initiated contributions from the subprocess \(W^+ s \rightarrow cg\) (together with virtual corrections) which were included in the complete NLO (\(\overline{\text{MS}}\)) analysis in \([4]\) are usually neglected in the ACOT–formalism. The ACOT–formalism has been formulated explicitly only for fully inclusive leptoproduction \([3]\). It is the main purpose of this article to fill the gap and provide the expressions needed for a correct calculation of one-hadron (D-meson) inclusive leptoproduction also in this formalism.

In the following we will stick closely to the ACOT formalism as formulated in \([3]\) except that we are not working in the helicity basis but prefer the standard tensor basis implying the usual structure functions \(F_{i=1,2,3}\). We are not considering kinematical effects arising from an initial state quark mass in the \(W^+ s \rightarrow c\) quark scattering contribution, i.e., \(s(x, Q^2)\) represents massless initial state strange quarks. This latter choice must be consistently kept in the subtraction term \([3]\) to be identified below from the \(m_s \rightarrow 0\) limit of the \(W^+ g \rightarrow c \bar{s}\) gluon fusion contribution. The fully massive partonic matrix elements
have been calculated for the general boson-gluon-fusion process $Bg \to \bar{Q}_1 Q_2$ in where $B = \gamma^*, W^\pm, Z$. When they are convoluted with a nonperturbative gluon distribution $g(x, \mu^2)$ and a fragmentation function $D_{Q_2}(z)$, one obtains the GF part of the hadronic structure function $F_1(x, z, Q^2)$ describing the momentum ($z$) distribution of a hadron $H$ containing the heavy quark $Q_2$:

$$F_{1,3}^{GF}(x, z, Q^2) = \int_{ax}^{1} \frac{dx'}{x'} \int_{\max[z, \zeta_{min}(x/x')]}^{\zeta_{max}(x/x')} \frac{d\zeta}{\zeta} g(x', \mu^2) f_{1,3}(\frac{x}{x'}, \frac{z}{\zeta}, Q^2) D_{Q_2}(\frac{z}{\zeta})$$

$$F_2^{GF}(x, z, Q^2) = \int_{ax}^{1} \frac{dx'}{x'} \int_{\max[z, \zeta_{min}(x/x')]}^{\zeta_{max}(x/x')} \frac{d\zeta}{\zeta} x' g(x', \mu^2) f_2(\frac{x}{x'}, \frac{z}{\zeta}, Q^2) D_{Q_2}(\frac{z}{\zeta})$$

(1)

with the fractional momentum variables $z = p_H \cdot p_N / q \cdot p_N$ and $\zeta = p_{Q_2} \cdot p_N / q \cdot p_N, p_N$ and $q$ being the momentum of the nucleon and the the virtual boson, respectively. The structure functions $F_i(x, z, Q^2)$ generalize the usual fully inclusive structure functions $F_i(x, Q^2)$, if one considers one-hadron (H) inclusive leptoproduction. The partonic structure functions $f_i(x', \zeta, Q^2)$ are given by

$$f_{i=1,2,3}(x', \zeta, Q^2) = \frac{\alpha_s(\mu^2)}{\pi} \left[ \frac{A_i}{(1-\zeta)^2} + \frac{B_i}{\zeta^2} + \frac{C_i}{1-\zeta} + \frac{D_i}{\zeta} + E_i \right]$$

(2)

with

$$A_1 (x', Q^2) = q_+ \frac{x'^2}{4} \frac{m_1^2}{Q^2} \left( 1 + \frac{\Delta m^2}{Q^2} - \frac{q_-}{q_+} \frac{2m_1m_2}{Q^2} \right)$$

$$C_1 (x', Q^2) = q_+ \frac{1}{4} \left[ \frac{1}{2} - x'(1 - x') - \frac{\Delta m^2 x'}{Q^2} (1 - 2x') + \left( \frac{\Delta m^2 x'}{Q^2} \right)^2 \right]$$

$$+ \frac{q_-}{q_+} \frac{m_1 m_2}{Q^2} \left( 2x' (1 - x' - \frac{m_1^2 + m_2^2}{Q^2}) \right)$$

$$E_1 (x', Q^2) = q_+ \frac{1}{4} \left( -1 + 2x' - 2x'^2 \right)$$

$$A_2 (x', Q^2) = q_+ x' \left[ x'^2 \frac{m_1^2}{Q^2} \left( \frac{1}{2} \left( \frac{\Delta m^2}{Q^2} \right)^2 + \frac{\Delta m^2 - m_1^2}{Q^2} + \frac{1}{2} \right) \right]$$

$$C_2 (x', Q^2) = q_+ \frac{x'}{4} \left[ 1 - 2x'(1 - x') + \frac{m_1^2 Q^2}{\Delta m^2} \left( 1 + 8x' - 18x'^2 \right) \right]$$

$$+ \frac{m_2^2}{Q^2} \left( 1 - 4x' + 6x'^2 \right) - \frac{m_1^2 + m_2^4}{Q^4} \left( 2x'(1 - 3x') + \frac{m_1^2 m_2^2}{Q^4} \right) 4x'(1 - 5x')$$

$$+ \frac{\Delta m^4 \Delta m^2}{Q^6} 2x'^2 - \frac{q_-}{q_+} \frac{2m_1 m_2}{Q^2} \right]$$
\[ E_2 (x', Q^2) = q_+ x' \left[ -\frac{1}{2} + 3x'(1 - x') \right] \]

\[ A_3 (x', Q^2) = R_q m_1^2 x^2 \frac{\Delta m^2 + Q^2}{Q^4} \]

\[ C_3 (x', Q^2) = R_q \left[ \frac{1}{2} - x'(1 - x') - \frac{\Delta m^2}{Q^2} x'(1 - 2x') + \frac{\Delta m^4}{Q^4} x^2 \right] \]

\[ E_3 (x', Q^2) = 0 \]

\[ B_{i=1,2,3} (x', Q^2) = \pm A_i (x', Q^2) [m_1 \leftrightarrow m_2] \]

\[ D_{i=1,2,3} (x', Q^2) = \pm C_i (x', Q^2) [m_1 \leftrightarrow m_2] \]

where \( \Delta m^n \equiv m_2^n - m_1^n \), \( m_{1,2} \) being the mass of the heavy quark \( Q_{1,2} \). The kinematical boundaries of phase space in the convolutions in eq. (1) are

\[ ax = \left[ 1 + \frac{(m_1 + m_2)^2}{Q^2} \right] x, \quad \zeta_{\text{min, max}}(x') = \frac{1}{2} \left[ 1 + \frac{\Delta m^2}{Q^2} \frac{x'(1 - 2x')}{1 - x'} \pm v \bar{v} \right] \]

with \( v^2 = 1 - \frac{(m_1 + m_2)^2}{Q^2} \frac{x'}{1 - x'} \), \( \bar{v}^2 = 1 - \frac{(m_1 - m_2)^2}{Q^2} \frac{x'}{1 - x'} \). The vector (V) and axialvector (A) couplings of the \( \gamma_\mu (V - A_\gamma_5) \) quark current enter via \( q_\pm = V^2 \pm A^2 \), \( R_q = VA \). If the partonic structure functions in eq. (2) are integrated over \( \zeta \) the well known inclusive structure functions [8] for heavy flavor production are recovered:

\[ \int_{\zeta_{\text{min}}(x')}^{\zeta_{\text{max}}(x')} d\zeta f_{i=1,2,3}(x', \zeta; Q^2) = \pm f_i(x', Q^2) \]

where the \( f_i(x', Q^2) \) can be found in [8].

In the following we will consider the special case of charged current charm production, i.e., \( m_1 = m_s, m_2 = m_c \) \( (q_\pm = 2, 0; R_q = 1 \) assuming a vanishing Cabibbo angle). Of course, all formulae below can be trivially adjusted to the general case of eqs. (1,2). The \( m_s \to 0 \) limit of the partonic structure functions in eq. (2) is obtained by keeping terms up to \( \mathcal{O}(m_s^2) \) in the \( A_i, C_i \) and in \( \zeta_{\text{max}} \) due to the singularity of the phase space integration stemming from \( \zeta \to 1 \). One obtains

\[ \lim_{m_s \to 0} \frac{\pi}{\alpha_s} \frac{f_i(x', \zeta, Q^2)}{m_s} = c_i H_{i}^{g} \left( \frac{x'}{\lambda}, \zeta, m_s^2, \lambda \right) \]

\[ = c_i \delta(1 - \zeta) P_{qg}^{(0)} \left( \frac{x'}{\lambda} \right) \ln \frac{Q^2 + m_c^2}{m_s^2} + \mathcal{O}(m_s^0) \]

\[ \text{(5)} \]
where $P_{qg}^{(0)}(x') = \frac{1}{2}(x'^2 + (1 - x')^2)$, $\lambda = Q^2/(Q^2 + m_c^2)$, $c_1 = 1/2$, $c_2 = x'/\lambda$, $c_3 = 1$ and the $H_i^g$ are the same as the dimensionally regularized $\overline{\text{MS}}$ ($m_s = 0$) gluonic coefficient functions obtained in [4]. The $c_i$ arise from different normalizations of the $f_i$ and the $H_i^g$ and are such that the infrared–safe subtracted [see below eq.(7)] convolutions in eq. (1) converge towards the corresponding ones in [4] as $m_s \to 0$ if one realizes that \[ x'/\lambda = \xi', \quad x/\lambda = \xi \equiv x(1 + m_c^2/Q^2). \] Taking also the limit $m_c \to 0$ in eq. (5) gives –besides the collinear logarithm already present in eq. (5)– finite expressions which agree [10] with the massless results of [3].

In the ACOT formalism the GF convolutions in eq. (1) coexist with the Born level quark scattering contributions

$$F_{iQS}(x, z, Q^2) = k_i s(\xi, \mu^2) D_c(z), \quad k_i = 1, 2, \quad 1, 2, \quad \xi, \quad 2.$$  

The overlap between the QS and the GF contributions is removed by introducing a subtraction term (SUB) [5] which is obtained from the massless limit in eq. (5)

$$F_{iSUB} = k_i \frac{\alpha_s(\mu^2)}{2\pi} \ln \frac{\mu^2}{m_s^2} \left[ \int_1^{\xi} \frac{dx'}{x'} g(x', \mu^2) P_{qg}^{(0)} \left( \frac{\xi}{x'} \right) \right] D_c(z).$$  

The complete $O(\alpha_s)$ structure functions for the $z$ distribution of charmed hadrons (i.e., dominantly D-mesons) produced in charged current DIS are then given in the ACOT formalism [4] by

$$F_{iACOT} = F_{iQS} - F_{iSUB} + F_{iGF}. \quad (7)$$

In Fig. 1 we show the structure function $F_{2ACOT}$ at experimentally relevant values of $x$ and $Q^2$ for several finite choices of $m_s$ together with the asymptotic $m_s \to 0$ limit. For $D_c$ we use a Peterson fragmentation function [11]

$$D_c(z) = N \left\{ z \left[ 1 - z^{-1} - \varepsilon_c/(1 - z) \right]^2 \right\}^{-1}$$  

with $\varepsilon_c = 0.06$ [12, 13] normalized to $\int_0^1 dz D_c(z) = 1$ and we employ the GRV94(HO) parton distributions [14] with $m_c = 1.5$ GeV. Our choice of the factorization scale is $\mu^2 = Q^2 + m_c^2$ which ensures that there is no large $\ln(Q^2 + m_c^2)/\mu^2$ present in the difference GF–SUB. As can be seen from Fig. 1 the effects of a finite strange mass are small and converge rapidly towards the massless $\overline{\text{MS}}$ limit provided $m_s \lesssim 200$ MeV as is usually assumed [2].
In Fig. 2 we show the effects of adding the quark–initiated $\mathcal{O}(\alpha_s)$ correction from the process $W^+s \rightarrow cg$ (together with virtual corrections) to the asymptotic ($m_s \rightarrow 0$) $F_2^{ACOT}$, employing the CTEQ4(\overline{\text{MS}}) densities \cite{15} with $m_c = 1.6$ GeV. The $\mathcal{O}(\alpha_s)$ quark contribution is usually neglected in the ACOT formalism since it is assumed to be effectively suppressed by one order of $\alpha_s$ with respect to the gluon fusion contribution due to $s(x, \mu^2)/g(x, \mu^2) = \mathcal{O}(\alpha_s)$. To check this assumption for the quantity $F_2(x, z, Q^2)$ we show, besides the full result, the contributions from the different subprocesses (using again $\mu^2 = Q^2 + m_s^2$). The $W^+g \rightarrow c\bar{s}$ contribution corresponds to GF–SUB in eq. (7). The quark–initiated $\mathcal{O}(\alpha_s)$ contribution has been calculated in the \overline{MS} scheme according to \cite{4} which is consistent with the asymptotic gluon–initiated correction in the ACOT scheme due to eq. (5). It can be seen that the quark–initiated correction is comparable in size to the gluon–initiated correction around the maximum of $F_2$. Since most of the experimentally measured D-mesons originate from this region the $\mathcal{O}(\alpha_s)$ quark contributions should not be neglected in a complete NLO calculation. It would be worthwhile to calculate these diagrams also in the ACOT scheme to study finite $m_s$ effects as has been done in this article for the $\mathcal{O}(\alpha_s)$ gluon contributions.

To summarize we have given formulae which extend the ACOT scheme \cite{5} for the leptoproduction of heavy quarks from its fully inclusive formulation to one-hadron inclusive leptoproduction. We have applied this formulation to D-meson production in charged current DIS and studied finite $m_s$ corrections to the asymptotic $m_s \rightarrow 0$ limit. The corrections turned out to be small for reasonable choices of $m_s \lesssim 200$ MeV and we have shown that the $m_s \rightarrow 0$ limit reproduces the dimensionally regularized $\overline{\text{MS}}$ ($m_s = 0$) gluonic coefficient functions \cite{4}. Furthermore we have investigated the quark–initiated $\mathcal{O}(\alpha_s)$ corrections for $m_s = 0$ using the relevant $\overline{\text{MS}}$ fermionic coefficient functions \cite{4}. The latter corrections turned out to be numerically important at experimentally relevant values of $x$ and $Q^2$ \cite{4} and should be included in a complete NLO calculation of charged current leptoproduction of D-mesons.
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$q_\pm = V\gamma V^Z \pm A\gamma A^Z$, $R_q = 1/2(V\gamma A^Z + V^Z A\gamma)$.

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$m_c \to 0$ limit that differs from the massless results in [3] by terms $\sim \delta(1 - x')$. The  
difference is due to $\zeta_{min} \to 1$ as $\xi' \to 1$ such that terms $\sim \delta(1 - \xi')$ are automatically  
$\sim \delta(1 - \zeta)$ [4] if $m_c \neq 0$.

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Figure Captions

Fig. 1 The structure function $F_2^{ACOT}(x, z, Q^2)$ as defined in eq. (7) using the GRV94(HO) parton densities [14] with $m_c = 1.5$ GeV and a Peterson fragmentation function [11] with $\varepsilon_c = 0.06$. Several finite choices for $m_s$ are shown as well as the asymptotic $m_s \to 0$ limit.

Fig. 2 The structure function $F_2(x, z, Q^2)$ for charged current leptoproduction of D-mesons at $\mathcal{O}(\alpha_s)$ for $m_s = 0$ using the CTEQ4(MS) parton distributions [15] and a Peterson fragmentation function [11] with $\varepsilon_c = 0.06$. The full $\mathcal{O}(\alpha_s)$ result is shown as well as the individual contributions from the distinct quark– and gluon–initiated processes.
\[ F_{2}^{\text{ACOT}}(x,z,Q^{2}) \]

**Fig. 1**

- **x = 0.015**
  - \[ Q^{2} = 2.4 \text{ GeV}^2 \]

- **x = 0.125**
  - \[ Q^{2} = 17.9 \text{ GeV}^2 \]

- **m_s [MeV]**
  - \( 
    \begin{array}{c}
    \text{---} \\
    0 \\
    \ldots \\
    100 \\
    \ldots \\
    300 \\
    \ldots \\
    500 \\
    \ldots
    \end{array} 
  \)
$x = 0.015$

$Q^2 = 2.4 \text{ GeV}^2$

$m_s = 0$

$F_2(x, z, Q^2)$

$z = 0.125$

$Q^2 = 17.9 \text{ GeV}^2$

Fig. 2