Heuristic for Maximizing Grouping Efficiency in the Cell Formation Problem

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Abstract In our paper we consider the Cell Formation Problem in Group Technology with grouping efficiency as an objective function. We present a heuristic approach for obtaining high-quality solutions of the CFP. The suggested heuristic applies an improvement procedure to obtain solutions with high grouping efficiency. This procedure is repeated many times for randomly generated cell configurations. Our computational experiments are performed for popular benchmark instances taken from the literature with sizes from 10x20 to 50x150. Better solutions unknown before are found for 23 instances of the 24 considered.

1 Introduction

Flanders (1925) was the first who formulated the main ideas of the group technology. The notion of the Group Technology was introduced in Russia by Mitrofanov (1959), though his work was translated to English only in 1966 (Mitrofanov, 1966). One of the main problems stated by the Group Technology is the optimal formation of manufacturing cells, i.e. grouping of machines and parts into cells such that for every machine in a cell the number of the parts from this cell processed by this machine is maximized and the number of the parts from other cells processed by this machine is minimized. In other words the intra-cell loading of machines is maximized and simultaneously the inter-cell movement of parts is minimized. This problem is called the Cell Formation Problem (CFP). Burbidge (1961) suggested...
his Product Flow Analysis (PFA) approach for the CFP, and later popularized the Group Technology and the CFP in his book (Burbidge, 1975).

The CFP is NP-hard since it can be reduced to the clustering problem (Ghosh et al., 1996). That is why there is a great number of heuristic approaches for solving CFP and almost no exact ones. The first algorithms for solving the CFP were different clustering techniques. Array-based clustering methods find rows and columns permutations of the machine-part matrix in order to form a block-diagonal structure. These methods include: Bond Energy Algorithm (BEA) of McCormick et al. (1972), Rank Order Clustering (ROC) algorithm by King (1980), its improved version ROC2 by King & Nakornchai (1982), Direct Clustering Algorithm (DCA) of Chan & Milner (1982), Modified Rank Order Clustering (MODROC) algorithm by Chandrasekaran & Rajagopalan (1986), the Close Neighbor Algorithm (CAN) by Boe & Cheng (1991). Hierarchical clustering methods at first form several big cells, then divide each cell into smaller ones and so on gradually improving the value of the objective function. The most well-known methods are Single Linkage (McAuley, 1972), Average Linkage (Seifoddini & Wolfe, 1986) and Complete Linkage (Mosier, 1989) algorithms. Non-hierarchical clustering methods are iterative approaches which start from some initial partition and improve it iteratively. The two most successful are GRAFICS algorithm by Srinivasan & Narendran (1991) and ZODIAC algorithm by Chandrasekaran & Rajagopalan (1987). A number of works considered the CFP as a graph partitioning problem where machines are vertices of a graph. Rajagopalan & Batra (1975) used clique partitioning of the machines graph. Askin & Chiu (1990) implemented a heuristic partitioning algorithm to solve CFP. Ng (1993) and Ng (1996) suggested an algorithm based on the minimum spanning tree problem. Mathematical programming approaches are also very popular for the CFP. Since the objective function of the CFP is rather complicated from the mathematical programming point of view most of the researchers use some approximation model which is then solved exactly for small instances and heuristically for large. Kusiak (1987) formulated CFP via p-median model and solved several small-size CFP instances, Shtub (1989) used Generalized Assignment Problem as an approximation model, Won & Lee (2004) proposed a simplified p-median model for solving large CFP instances, Krushinsky & Goldengorin (2012) applied minimum k-cut problem to the CFP, Goldengorin et al. (2012) used p-median approximation model and solved it exactly by means of their pseudo-boolean approach including large CFP instances up to 50x150 instance. A number of meta-heuristics have been applied recently to the CFP. Most of these approaches can be related to genetic, simulated annealing, tabu search, and neural networks algorithms. Among them such works as: Goncalves & Resende (2004), Wu et al. (2007), Xambre & Vilariinho (2003), Lei & Wu (2006), Liang & Zolfaghari (1999), Yang & Yang (2008).

Our heuristic algorithm is based on sequential improvements of the solution. We modify the cell configuration by enlarging one cell and reducing another. The basic procedure of the algorithm has the following steps:

1. Generate a random cell configuration.
2. Improve the initial solution moving one row or column from one cell to another until the grouping efficiency is increasing.
3. Repeat steps 1-2 a predefined number of times (we use 2000 times for computational experiments in this paper).

The paper is organized as follows. In the next section we provide the Cell Formation Problem formulation. In section 3 we present our improvement heuristic that allows us to get good solutions by iterative modifications of cells which lead to increasing of the objective function. In section 4 we report our computational results and section 5 concludes the paper with a short summary.

2 The Cell Formation Problem

The CFP consists in an optimal grouping of the given machines and parts into cells. The input for this problem is given by \( m \) machines, \( p \) parts and a rectangular machine-part incidence matrix \( A = [a_{ij}] \), where \( a_{ij} = 1 \) if part \( j \) is processed on machine \( i \). The objective is to find an optimal number and configuration of rectangular cells (diagonal blocks in the machine-part matrix) and optimal grouping of rows (machines) and columns (parts) into these cells such that the number of zeros inside the chosen cells (voids) and the number of ones outside these cells (exceptions) are minimized. A concrete combination of rectangular cells in a solution (diagonal blocks in the machine-part matrix) we will call a cells configuration. Since it is usually not possible to minimize these two values simultaneously there have appeared a number of compound criteria trying to join it into one objective function. Some of them are presented below.

For example, we are given the machine-part matrix (Waghodekar & Sahu, 1984) shown in table 1. Two different solutions for this CFP are shown in tables 2 and 3. The left solution is better because it has less voids (3 against 4) and exceptions (4 against 5) than the right one. But one of its cells is a singleton – a cell which has less than two machines or parts. In some CFP formulations singletons are not allowed, so in this case this solution is not feasible. In this paper we consider the both cases (with allowed singletons and with not allowed) and when there is a solution with singletons found by the suggested heuristic better than without singletons we present the both solutions.

| \( m_1 \) | \( P_1 \) | \( P_2 \) | \( P_3 \) | \( P_4 \) | \( P_5 \) | \( P_6 \) | \( P_7 \) |
|---------|--------|--------|--------|--------|--------|--------|--------|
| \( m_1 \) | 1      | 0      | 0      | 1      | 1      | 1      | 1      |
| \( m_2 \) | 0      | 1      | 1      | 1      | 0      | 0      |        |
| \( m_3 \) | 0      | 0      | 1      | 1      | 1      | 0      |        |
| \( m_4 \) | 1      | 1      | 1      | 0      | 0      | 0      |        |
| \( m_5 \) | 0      | 1      | 0      | 1      | 1      | 0      |        |

Table 1: Machine-part 5x7 matrix from Waghodekar & Sahu (1984)

There are a number of different objective functions used for the CFP. The following four functions are the most widely used:
1. Grouping efficiency suggested by Chandrasekharan & Rajagopalan (1989):

\[ \eta = q \eta_1 + (1 - q) \eta_2, \quad (1) \]

where

\[ \eta_1 = \frac{n_1 - n_{1_{\text{out}}}}{n_1 - n_{1_{\text{out}}} + n_{0_{\text{in}}}} = \frac{n_{1_{\text{in}}}}{n_{0_{\text{in}}}}, \]

\[ \eta_2 = \frac{mp - n_1 - n_{1_{\text{in}}}}{mp - n_1 - n_{1_{\text{in}}} + n_{0_{\text{out}}}} = \frac{n_{0_{\text{out}}}}{n_{0_{\text{out}}}}, \]

\( \eta_1 \) – a ratio showing the intra-cell loading of machines (or the ratio of the number of ones in cells to the total number of elements in cells).

\( \eta_2 \) – a ratio inverse to the inter-cell movement of parts (or the ratio of the number of zeroes out of cells to the total number of elements out of cells).

\( q \) – a coefficient \((0 \leq q \leq 1)\) reflecting the weights of the machine loading and the inter-cell movement in the objective function. It is usually taken equal to \( \frac{1}{2} \), which means that it is equally important to maximize the machine loading and minimize the inter-cell movement.

\( n_1 \) – a number of ones in the machine-part matrix,

\( n_0 \) – a number of zeroes in the machine-part matrix,

\( n_{1_{\text{in}}} \) – a number of elements inside the cells,

\( n_{1_{\text{out}}} \) – a number of elements outside the cells,

\( n_{0_{\text{in}}} \) – a number of ones inside the cells,

\( n_{0_{\text{out}}} \) – a number of zeroes outside the cells.

2. Grouping efficacy suggested by Kumar & Chandrasekharan (1990):
Heuristic for Maximizing Grouping Efficiency in the Cell Formation Problem

\[ \tau = \frac{n_1 - n_{1}^{\text{out}}}{n_1 + n_{0}^{\text{out}}} = \frac{n_{1}^{\text{in}}}{n_1 + n_{0}^{\text{in}}} \]  
(2)

3. Group Capability Index (GCI) suggested by Hsu (1990):

\[ GCI = 1 - \frac{n_{1}^{\text{out}}}{n_1} = \frac{n_1 - n_{1}^{\text{out}}}{n_1} \]  
(3)

4. Number of exceptions (ones outside cells) and voids (zeroes inside cells):

\[ E + V = n_{1}^{\text{out}} + n_{0}^{\text{in}} \]  
(4)

The values of these objective functions for the solutions in tables 2 and 3 are shown below.

\[ \eta = \frac{1}{2} \cdot \frac{16}{19} + \frac{1}{2} \cdot \frac{12}{16} \approx 79.60\% \quad \eta = \frac{1}{2} \cdot \frac{15}{19} + \frac{1}{2} \cdot \frac{11}{16} \approx 73.85\% \]

\[ \tau = \frac{20 - 4}{20 + 3} \approx 69.57\% \quad \tau = \frac{20 - 5}{20 + 4} \approx 62.50\% \]

\[ GCI = \frac{20 - 4}{20} \approx 80.00\% \quad GCI = \frac{20 - 5}{20} \approx 75.00\% \]

\[ E + V = 4 + 3 = 7 \quad E + V = 5 + 4 = 9 \]

In this paper we use the grouping efficiency measure and compare our computational results with the results of Yang & Yang (2008) and Goldengorin et al. (2012).

The mathematical programming model of the CFP with the grouping efficiency objective function can be described using boolean variables \( x_{ik} \) and \( y_{jk} \). Variable \( x_{ik} \) takes value 1 if machine \( i \) belongs to cell \( k \) and takes value 0 otherwise. Similarly variable \( y_{jk} \) takes value 1 if part \( j \) belongs to cell \( k \) and takes value 0 otherwise. Machines index \( i \) takes values from 1 to \( m \) and parts index \( j \) from 1 to \( p \). Cells index \( k \) takes values from 1 to \( c \) because every cell should contain at least one machine and one part and so the number of cells cannot be greater than \( m \) and \( p \). Note that if a CFP solution has \( n \) cells then for \( k \) from \( n + 1 \) to \( c \) all variables \( x_{ik}, y_{jk} \) will be zero in this model. So we can consider that the CFP solution always has \( c \) cells, but some of them can be empty. The mathematical programming formulation is as follows.

\[ \max \left( \frac{n_{1}^{\text{in}}}{2n^{\text{in}}} + \frac{n_{0}^{\text{out}}}{2n^{\text{out}}} \right) \]  
(5)

where

\[ n^{\text{in}} = \sum_{k=1}^{c} \sum_{i=1}^{m} \sum_{j=1}^{p} x_{ik}y_{jk}, \quad n^{\text{out}} = mp - n^{\text{in}} \]

\[ n_{1}^{\text{in}} = \sum_{k=1}^{c} \sum_{i=1}^{m} \sum_{j=1}^{p} a_{ij}x_{ik}y_{jk}, \quad n_{0}^{\text{out}} = n_0 - (n^{\text{in}} - n_{1}^{\text{in}}) \]

subject to
The objective function (5) is the grouping efficiency in this model. Constraints (6) and (7) impose that every machine and every part belongs to some cell. Constraints (8) and (9) guarantee that every non-empty cell contains at least one machine and one part. Note that if singleton cells are not allowed then the right sides of inequalities (8) and (9) should have a coefficient of 2. All these constraints can be linearized in a standard way, but the objective function will still be fractional. That is why the exact solution of this problem presents considerable difficulties.

A cells configuration in the mathematical model is described by the number of machines $m_k$ and parts $p_k$ in every cell $k$.

$$m_k = \sum_{i=1}^{m} x_{ik}, \quad p_k = \sum_{j=1}^{p} y_{jk}$$

It is easy to see that when a cells configuration is fixed all the optimization criteria (1) - (4) become equivalent (proposition 1).

**Proposition 1.** If a cells configuration is fixed then objective functions (1) - (4): $\eta$, $\tau$, GCI, $E + V$ become equivalent and reach the optimal value on the same solutions.

**Proof.** When a cells configuration is fixed the following values are constant: $m_k$, $p_k$, $n_i$ and $n_0$ are always constant. The values of $n_{11}$ and $n_{00}$ are constant since $n^{in} = \sum_{k=1}^{c} m_k p_k$ and $n^{out} = mp - n^{in}$. So if we maximize the number of ones inside the cells $n_{11}$ then simultaneously $n_{01}^{in} = n^{in} - n_{11}^{in}$ is minimized, $n_{00}^{out} = n_0 - n_{00}^{in}$ is maximized, and $n_{11}^{out} = n_1 - n_{01}^{in}$ is minimized. This means that the grouping efficiency $\eta = q \frac{n_{11}^{in}}{n_1} + (1 - q) \frac{n_{00}^{out}}{n_0}$ is maximized, the grouping efficacy $\tau = \frac{n_{11}^{in}}{n_{11}^{in} + n_0}$ is maximized, the grouping capability index GCI $= 1 - \frac{n_{00}^{out}}{n_1}$ is maximized, and the number of exceptions plus voids $E + V = n_{01}^{out} + n_{00}^{in}$ is minimized simultaneously on the same optimal solution. $\square$
3 Algorithm description

The main function of our heuristic is presented by algorithm 1. First we call

Algorithm 1 Main function

function Solve()
    FINDOPTIMALCELLRANGE(MinCells, MaxCells)
    ConfigsNumber = 2000
    AllConfigs = GENERATECONFIGS(MinCells, MaxCells, ConfigsNumber)
    return CMHEURISTIC(AllConfigs)
end function

FINDOPTIMALCELLRANGE(MinCells, MaxCells) function that returns a potentially optimal range of cells - from MinCells to MaxCells. Then these values and ConfigsNumber (the number of cell configurations to be generated) are passed to GENERATECONFIGS(MinCells, MaxCells, ConfigsNumber) function which generates random cell configurations. The generated configurations AllConfigs are passed to CMHEURISTIC(AllConfigs) function which finds a high-quality solution for every cell configuration and then chooses the solution with the greatest efficiency value.

Algorithm 2 Procedure for finding the optimal cell range

function FINDOPTIMALCELLRANGE(MinCells, MaxCells)
    if (m > p) then
        minDimension = p
    else
        minDimension = m
    end if
    ConfigsNumber = 500
    Configs = GENERATECONFIGS(2, minDimension, ConfigsNumber)
    Solution = CMHEURISTIC(Configs)
    BestCells = GETCELLSNUMBER(Solution)
    MinCells = BestCells - [minDimension * 0.1] \[ ] - integer part
    MaxCells = BestCells + [minDimension * 0.1]
end function

In function FINDOPTIMALCELLRANGE( MinCells, MaxCells) (algorithm 2) we look over all the possible number of cells from 2 to maximal possible number of cells which is equal to min(m, p). For every number of cells in this interval we generate a fixed number of configurations (we use 500 in this paper) calling GENERATECONFIGS(2, minDimension, ConfigsNumber) and then use our CMHEURISTIC(Configs) to obtain a potentially optimal number of cells. But we consider not only one number of cells but together with its 10%-neighborhood [MinCells, MaxCells].
Algorithm 3 Configurations generation

```plaintext
function GENERATECONFIGS(MinCells, MaxCells, ConfigsNumber)
    Configs = ∅
    for cells = MinCell, MaxCells do
        Generated = GENERATECONFIGS(cells, ConfigsNumber)
        Configs = Configs ∪ Generated
    return Configs
end function
```

Function `GENERATECONFIGS(MinCells, MaxCells, ConfigsNumber)` (algorithm 3) returns a set of randomly generated cell configurations with a number of cells ranging from `MinCells` to `MaxCells`. We call `GENERATECONFIGSUNIFORM(cells, ConfigsNumber)` function which randomly selects with uniform distribution `ConfigsNumber` configurations from all possible cell configurations with the specified number of cells. Note that mathematically a cell configuration with `k` cells can be represented as an integer partition of `m` and `p` values into sums of `k` summands. We form a set of configurations for every number of cells and then join them.

Algorithm 4 CMHeuristic

```plaintext
function CMHEURISTIC(Configs)
    Best = 0
    for all config ∈ Configs do
        Solution = IMPROVESOLUTION(config)
        if Solution > Best then
            Best = Solution
        end if
    end for
    return Best
end function
```

Function `CMHEURISTIC(Configs)` (algorithm 4) gets a set of cell configurations and for each configuration runs an improvement algorithm to obtain a good solution. A solution includes a permuted machine-part matrix, a cell configuration, and the corresponding grouping efficiency value. The function chooses the best solution and returns it.

Improvement procedure `IMPROVESOLUTION(config, η_current)` (algorithm 5) works as follows. We consider all the machines and the parts in order to know if there is a machine or a part that we can move to another cell and improve the current efficiency `η_current`. First we consider moving of every part on all other cells and compute how the efficiency value changes. Here `η_{part, cell}` is the efficiency of the current solution where the part with index `part` is moved to the cell with index `cell`. This operation is performed for all the parts and the part with the maximum increase in efficiency `Δ_{parts}` is chosen. Then we repeat the same operations for all the machines. Finally, we compare the best part movement and the best machine movement and choose the
Algorithm 5 Solution improvement procedure

```plaintext
function IMPROVESOLUTION(config, ηcurrent)
    ηcurrent = GROUPINGEFFICIENCY(config)
    repeat
        PartFrom = 0
        PartTo = 0
        for part = 1, partsNumber do
            for cell = 1, cellsNumber do
                if (ηpart,cell > ηcurrent) then
                    Δparts = (ηpart,cell - ηcurrent)
                    PartFrom = GetPartCell(part)
                    PartTo = cell
                end if
            end for
        end for
        MachineFrom = 0
        MachineTo = 0
        for machine = 1, machinesNumber do
            for cell = 1, cellsNumber do
                if (ηmachine,cell > ηcurrent) then
                    Δmachines = (ηmachine,cell - ηcurrent)
                    MachineFrom = GETMACHINECELL(machine)
                    MachineTo = cell
                end if
            end for
        end for
        if Δparts > Δmachines then
            MOVEPART(PartFrom, PartTo)
        else
            MOVEMACHINE(MachineFrom, MachineTo)
        end if
    until Δ > 0
end function
```

one with the highest efficiency. This procedure is performed until any improvement is possible and after that we get the final solution.

The main idea of IMPROVESOLUTION(config, ηcurrent) is illustrated on Seifodini & Wolfe (1986) instance 8x12 (table 4). To compute the grouping efficiency for this solution we need to know the number of ones inside cells \( n_{in} \), the total number of elements inside cells \( n_{in} \), the number of zeros outside cells \( n_{out} \) and the number of elements outside cells \( n_{out} \). The grouping efficiency is then calculated by the following formula:

\[
\eta = q \cdot \frac{n_{in}}{n_{in}} + (1 - q) \cdot \frac{n_{out}}{n_{out}} = \frac{1}{2} \cdot \frac{20}{33} + \frac{1}{2} \cdot \frac{48}{63} \approx 68.4\%
\]

Looking at this solution (table 4) we can conclude that it is possible for example to move part 4 from the second cell to the first one. And this way the number of zeros inside cells decreases by 3 and the number of ones outside cells also decreases by
4. So it is profitable to attach column 4 to the first cell as it is shown on table 5. For the modified cells configuration we have:

$$\eta = \frac{1}{2} \cdot \frac{23}{33} + \frac{1}{2} \cdot \frac{51}{63} \approx 75.32\%$$

As a result the efficiency is increased almost for 7 percent. Computational results show that using such modifications could considerably improve the solution. The idea is to compute an increase in efficiency for each column and row when it is moved to another cell and then perform the modification corresponding to the maximal increase. For example, table 6 shows the maximal possible increase in efficiency for every row when it is moved to another cell.

### 4 Computational results

In all the experiments for determining a potentially optimal range of cells we use 500 random cell configurations for each cells number and for obtaining the final solution we use 2000 random configurations. An Intel Core i7 machine with 2.20 GHz CPU
Table 6: Maximal efficiency increase for each row

|   | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
|---|---|---|---|---|---|---|---|---|---|----|----|----|
| 1 | 1 | 1 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 2 | 1 | 0 | 1 | 1 | 1 | 1 | 1 | 1 | 0 | 1 | 0 | 0 |
| 3 | 0 | 0 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 0 | 0 | 0 |
| 4 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 1 | 1 | 0 | 0 |
| 5 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 1 | 0 | 0 | 0 |
| 6 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 0 | 1 | 0 | 0 |
| 7 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 |
| 8 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 |

and 8.00 Gb of memory is used in our experiments. We run our heuristic on 24 CFP benchmark instances taken from the literature. The sizes of the considered problems vary from 10x20 to 50x150. The computational results are presented in table 7. For every instance we make 50 algorithm runs and report minimum, average and maximum value of the grouping efficiency obtained by the suggested heuristic over these 50 runs. We compare our results with the best known values taken from Goldengorin et al. (2012) and Bhatnagar & Saddikuti (2010). We have found better solutions unknown before for 23 instances of the 24 considered. For CFP instance 6 we have found the same optimal solution with 100% of grouping efficiency as in Goldengorin et al. (2012). For CFP instance 1 the solution of Goldengorin et al. (2012) has some mistake. For this instance having a small size of 10x20 it can be proved that our solution is the global optimum applying an exact approach (Elbenani & Ferland, 2012) for the grouping efficiency objective and all the possible number of cells from 1 to 10.

5 Concluding remarks

In this paper we present a new heuristic algorithm for solving the CFP. The high quality of the solutions is achieved due to the enumeration of different numbers of cells and different cell configurations and applying our improvement procedure. Since the suggested heuristic works fast (the solution for one cell configuration is achieved in several milliseconds for any instance from 10x20 to 50x150) we apply it for thousands of different configurations. Thus a big variety of good solutions is covered by the algorithm and the best of them has high grouping efficiency.
| #  | Source                          | mxp | Efficiency value,% | Time, sec | Cells |
|----|--------------------------------|-----|--------------------|-----------|-------|
|    |                                |     | Bhatnagar & Saddikuti | Goldengorin et al. | Our Min | Avg | Max |
| 1  | Sandbothe (1998)               | 10x20 | 95.40              | 95.93 "a" | 95.66 95.66 95.66 | 95.99 95.99 95.99 | 95.99 95.99 95.99 | 0.36 7 |
| 2  | Ahi et al. (2009)              | 20x20 | 92.62              | 93.85     | 93.34 93.47 93.55 | 93.46 93.55 93.59 | 1.62 10 |
| 3  | Mosier & Taube (1985)          | 20x20 | 85.63              | 88.71     | 95.43 95.78 95.79 | 95.45 95.78 95.79 | 1.54 10 |
| 4  | Boe & Cheng (1991)             | 20x35 | 90.76              | 93.64     | 95.36 95.45 95.54 | 95.45 95.54 95.59 | 1.8 7 |
| 5  | Carrie (1973)                  | 20x35 | 98.82              | 100.00    | 100 100 100 | 100 100 100 | 1.8 7 |
| 6  | Ahi et al. (2009)              | 20x51 | 97.48              | 97.75     | 97.88 97.88 97.88 | 97.88 97.88 97.88 | 2.42 12 |
| 7  | Chandrasekharan & Rajagopalan (1989) | 20x40 | 95.33              | 97.48     | 97.75 97.75 97.76 | 97.75 97.75 97.76 | 2.56 12 |
| 8  | Chandrasekharan & Rajagopalan (1989) | 20x40 | 93.78              | 96.36     | 96.84 96.88 96.89 | 96.88 96.88 96.89 | 2.56 12 |
| 9  | Chandrasekharan & Rajagopalan (1989) | 20x40 | 89.92              | 94.32     | 96.11 96.16 96.21 | 96.16 96.16 96.21 | 3.3 15 |
| 10 | Chandrasekharan & Rajagopalan (1989) | 20x40 | 84.95              | 94.21     | 95.94 96.03 96.1 | 96.03 96.03 96.1 | 2.84 15 |
| 11 | Chandrasekharan & Rajagopalan (1989) | 20x40 | 85.06              | 92.32     | 95.85 95.9 95.95 | 95.9 95.9 95.95 | 2.76 15 |
| 12 | Chandrasekharan & Rajagopalan (1989) | 20x40 | 96.44              | 97.39     | 97.78 97.78 97.8 | 97.78 97.78 97.8 | 2.12 10 |
| 13 | Nair & Narendran (1996)        | 20x40 | 92.35              | 95.74     | 97.4 97.4 97.4 | 97.4 97.4 97.4 | 2.2 14 |
| 14 | Nair & Narendran (1996)        | 20x40 | 95.35              | 98.51     | 98.63 98.63 98.67 | 98.63 98.63 98.67 | 2.48 12 |
| 15 | Nair & Narendran (1996)        | 20x40 | 91.11              | 96.40     | 96.98 96.98 96.98 | 96.98 96.98 96.98 | 2.78 14 |
| 16 | Nair & Narendran (1996)        | 20x40 | 91.09              | 95.52     | 96.48 96.48 96.48 | 96.48 96.48 96.48 | 2.58 14 |
| 17 | Ahi et al. (2009)              | 20x35 | 93.43              | 93.82     | 94.81 94.85 94.86 | 94.85 94.85 94.86 | 2.46 10 |
| 18 | Yang & Yang (2008)             | 20x35 | 90.66              | 97.22     | 97.38 97.54 97.62 | 97.54 97.54 97.62 | 3.54 18 |
| 19 | Kumar & Vannelli (1987)        | 30x30 | 88.17              | 96.48     | 96.77 96.83 96.9 | 96.83 96.83 96.9 | 5.02 18 |
| 20 | Stantel (1985)                 | 30x30 | 93.18              | 94.62     | 95.37 95.84 96.27 | 95.84 95.84 96.27 | 13.1 25 |
| 21 | King & Nakornchai (1982)       | 30x90 | 89.72              | 98.06     | 98.18 98.18 98.13 | 98.18 98.18 98.13 | 16.88 17 |
| 22 | Chandrasekharan and Rajagopalan (1987) | 40x100 | 97.45              | 95.91     | 98.06 98.18 98.13 | 98.18 98.18 98.13 | 23.9 18 |
| 23 | Yang & Yang (2008)             | 46x105 | 90.98              | 95.20     | 96.16 96.18 96.29 | 96.18 96.18 96.29 | 51.66 24 |
| 24 | Liang & Zolfaghari (1999)      | 50x150 | 93.05              | 92.92     | 96.08 96.17 96.27 | 96.17 96.17 96.27 | 51.66 24 |

"a" This solution has a mistake.
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References

Askin, R. G., & Chiu, K. S. (1990). A graph partitioning procedure for machine assignment and cell formation in group technology. International Journal of Production Research, 28(8), 1555–1572, 1990.

Bouazza Elbenani & Jacques A. Ferland (2012). Manufacturing cell formation by state-space search. Montreal, Quebec. CIRRELT-2012-07, 14 pages.

Subrata Ghosh, Ambuj Mahanti, Rakesh Nagi & Dana S. Nau (1996). Cell Formation Problem Solved Exactly with the Dinkelbach Algorithm. Annals of Operations Research, 65(1), 35–54.

Boe, W., & Cheng, C. H. (1991). A close neighbor algorithm for designing cellular manufacturing systems. International Journal of Production Research, 29(10), 2097–2116.

Burbidge, J. L. (1961). The new approach to production. Prod. Eng., December, 3 - 19.

Burbidge, J. L. (1975). The introduction of group technology. New York: Wiley.

Carrie, S. (1973). Numerical taxonomy applied to group technology and plant layout. International Journal of Production Research, 11, 399–416.

Chan, H.M., Milner, D.A., (1982). Direct clustering algorithm for group formation in cellular manufacture. Journal of Manufacturing Systems 1 (1), 64–76.

Chandrasekaran, M.P., Rajagopalan, R., (1986). MODROC: An extension of rank order clustering of group technology. International Journal of Production Research 24 (5), 1221–1233.

Chandrasekaran, M. P., & Rajagopalan, R. (1987). ZODIAC—An algorithm for concurrent formation of part families and machine cells. International Journal of Production Research, 25(6), 835–850.

Chandrasekharan, M. P., & Rajagopalan, R. (1989). Groupability: Analysis of the properties of binary data matrices for group technology. International Journal of Production Research, 27(6), 1035–1052.

Cheng, C. H., Gupta, Y. P., Lee, W. H., & Wong, K. F. (1998). A TSP-based heuristic for forming machine groups and part families. International Journal of Production Research, 36(5), 1325–1337.

Flanders, R. E. (1925). Design manufacture and production control of a standard machine, Transactions of ASME, 46, 691-738.

Goncalves, J.F., Resende, M.G.C. (2004). An evolutionary algorithm for manufacturing cell formation. Computers & Industrial Engineering 47, 247–273.

Hsu, C. P. (1990). Similarity coefficient approaches to machine-component cell formation in cellular manufacturing: A comparative study. PhD Thesis, Department of Industrial and Manufacturing Engineering, University of Wisconsin Milwaukee.

King, J. R. (1980). Machine-component grouping in production flow analysis: An approach using a rank order clustering algorithm. International Journal of Production Research, 18(2), 213–232.

King, J. R., & Nakornchai, V. (1982). Machine-component group formation in group technology: Review and extension. International Journal of Production Research, 20(2), 117–133.

Krushinsky, D., Goldengorin, B. (2012). An exact model for cell formation in group technology. Computational Management Science. 9. 323-338. DOI 10.1007/s10287-012-0146-2.

Goldengorin, B., Krushinsky, D., Slomp, J. (2012). Flexible PMP approach for large size cell formation. Operations Research, 60(5), 1157–1166.

Kumar, K. R., Chandrasekharan, M. P. (1990). Grouping efficacy: A quantitative criterion for goodness of block diagonal forms of binary matrices in group technology. International Journal of Production Research, 28(2), 233–243.

Kumar KR, Vannelli A. (1987). Strategic subcontracting for efficient disaggregated manufacturing. International Journal of Production Research, 25(12), 1715–1728.

Kusiak, A. (1987). The generalized group technology concept. International Journal of Production Research, 25(4), 561–569.

Lei, D., Wu, Z., (2006). Tabu search for multiple-criteria manufacturing cell design. International Journal of Advanced Manufacturing Technology 28, 950–956.
Heuristic for Maximizing Grouping Efficiency in the Cell Formation Problem

Liang, M., Zolfaghari, S. (1999). Machine cell formation considering processing times and machine capacities: an ortho-synapse Hopfield neural network approach. Journal of Intelligent Manufacturing, 10, 437–447.

McAuley, J. (1972). Machine grouping for efficient production. Production Engineering, 51(2), 53–57.

McCormick, W.T., Schweitzer, P.J., White, T.W. (1972). Problem decomposition and data reorganization by a clustering technique. Operations Research 20(5), 993–1009.

Mitrofanov, S. P. (1959). Nauchnye osnovy grupyovoy tehnologii. Lenizdat, Leningrad, Russia, 1959. 435 pages. (in Russian).

Mitrofanov, S. P. (1966). The Scientific Principles of Group Technology. Boston Spa, Yorkshire: National Lending Library Translation. Translation of Mitrofanov (1959).

Mosier, C. T. (1989). An experiment investigating the application of clustering procedures and similarity coefficients to the GT machine cell formation problem. International Journal of Production Research, 27(10), 1811–1835.

Mosier, C. T., & Taube, L. (1985). Weighted similarity measure heuristics for the group technology machine clustering problem. OMEGA, 13(6), 577–583.

Ng, S. (1993). Worst-case analysis of an algorithm for cellular manufacturing. European Journal of Operational Research, 69(3), 384–398.

Ng, S. (1996). On the characterization and measure of machine cells in group technology. Operations Research, 44(5), 735–744.

Rajagopalan, R., & Batra, J. L. (1975). Design of cellular production systems: a graph-theoretic approach. International Journal of Production Research, 13(6), 567–579.

Seifoddini, H., & Wolfe, P. M. (1986). Application of the similarity coefficient method in group technology. IIE Transactions, 18(3), 271–277.

Shih, A. (1989). Modelling group technology cell formation as a generalized assignment problem. International Journal of Production Research, 27(5), 775–782.

Srinivasan, G., & Narendran, T. T. (1991). GRAFICS-A nonhierarchical clustering-algorithm for group technology. International Journal of Production Research, 29(3), 463–478.

Stanfel, L. (1985). Machine clustering for economic production. Engineering Costs and Production Economics, 9, 73-81.

Waghodekar, P. H., & Sahu, S. (1984). Machine-component cell formation in group technology MACE. International Journal of Production Research 22, 937-948.

Won, Y., Lee, K.C. (2004). Modified p-median approach for efficient GT cell formation. Computers and Industrial Engineering 46, 495–510.

Wu, X., Chao-Hsien, C., Wang, Y., Yan, W. (2007). A genetic algorithm for cellular manufacturing design and layout. European Journal of Operational Research 181, 156–167.

Xambre, A.R., Vilarinho, P.M. (2003). A simulated annealing approach for manufacturing cell formation with multiple identical machines. European Journal of Operational Research 151, 434–446.

Yang, M.-S. & Yang, J.-H. (2008). Machine-part cell formation in group technology using a modified ART1 method. Eur. J. Oper. Res., 188(1), 140-152.

Sandbothe R.A. (1998). Two observations on the grouping efficacy measure for goodness of block diagonal forms. Int J Prod Res 36: 3217-3222.

Ahi, A., Mir.B. Aryanezhad, B. Ashtiani, A. Makui. (2009). A novel approach to determine cell formation, intracellular machine layout and cell layout in the CMS problem based on TOPSIS method. Comput. Oper. Res. 36(5) 1478-1496

Nair, G.J.K. and Narendran, T.T. (1996). Grouping index: A new quantitative criterion for goodness of block-diagonal forms in group technology. Int J Prod Res 34(10): 2767-2782.

Bhatnagar, R., V. Saddikatti. (2010). Models for cellular manufacturing systems design: matching processing requirements and operator capabilities. J. Opl. Res. Soc. 61, 827-839