Effect of Fourth Generation $CP$ Phase on $b \to s$ Transitions

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We study the effect of a sequential fourth generation on $b \to s \gamma$, $B_s$ mixing, $B \to K^{(*)} \ell^+\ell^-$, and $\phi K_S$, taking into account the presence of a new $CP$ phase in quark mixing. We find that the effects via electromagnetic and strong penguins, such as in $b \to s \gamma$ or $CP$ violation in $B \to \phi K_S$, are rather mild, but impact via box or $Z$ penguins is quite significant. Not only can one have much larger than expected $B_s$ mixing, even if it is measured near the present limit, mixing dependent $CP$ violation could become maximal. For electroweak penguin modes, the fourth generation can provide the enhancement that perhaps may be needed, while the $m_{t\ell}$-dependence of the forward-backward asymmetry in $B \to K^{(*)} \ell^+\ell^-$ can further probe for the fourth generation. These effects can be studied in detail at the B factories and the Tevatron.

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I. INTRODUCTION

The source of $CP$ violation within the Standard Model lies in the Cabibbo-Kobayashi-Maskawa (CKM) quark mixing matrix, where a unique KM phase emerges with 3 generations of quarks and leptons. Despite its great success, there are reasons to believe that the 3 generation Standard Model (SM, or SM3) may be incomplete. For example, the generation structure is not understood, while recent observations of neutrino oscillations point towards an enlarged neutrino sector [1]. A simple enlargement is to add a sequential fourth generation (SM4); with additional mixing elements, one crucial aspect is that the source for $CP$ violation is no longer unique.

The B factories at KEK and SLAC have turned on with a splash, and $CP$ violation has been observed in $B^0 \to J/\psi K_S$ decay modes that is consistent with SM3. Both experiments have recently observed the $B \to K^{(*)} \ell^+\ell^-$ [2-5] decay mode, while the Belle Collaboration has just reported the observation of the inclusive $B \to X_s \ell^+\ell^-$ mode [6]. Though a bit on the high side, the results are again not inconsistent with SM3. However, there may be a hint for new physics in the mixing dependent $CP$ violation in $B^0 \to \phi K_S$ decay [7]. At any rate, together with many related loop-induced processes, we have entered an age where the uniqueness of the 3 generation SM phase can start to be checked.

The $B \to K^{(*)} \ell^+\ell^-$ type of decays are highly suppressed in the SM because they occur only via electroweak penguin processes. The first measurement of such processes through $b \to s \gamma$ was performed some years ago by CLEO [8]. The measured branching ratio has been used to constrain the Wilson coefficient $|C_7^{\text{eff}}|$. The measurement of $B \to (K, K^*)\ell^+\ell^-$ and $X_s \ell^+\ell^-$ modes can provide information on $C_9^{\text{eff}}$ and $C_{10}^{\text{eff}}$, as well as sign information on $C_7^{\text{eff}}$. The contributions from fourth generation to rare decays have been extensively studied [10-13], where the measured $b \to s \gamma$ decay rate has been used [13] to put stringent constraints on the additional CKM matrix elements. However, most of these studies neglect the effect of new $CP$ phases by taking $\lambda_\nu^t \equiv V^\dagger_{tb}V^\dagger_{t\nu}V_{\nu b}^*$ to be real. The $b \to s \gamma$ constraint then implies that, for fixed $t'$ quark mass, $\lambda_\nu^t$ can take on only one of two values.

In this paper, we study the effect of the fourth generation on $b \to s$ transitions by including the $CP$ phase in $V_{tb}^*V_{t\nu}$. We use the updated measured value of $b \to s \gamma$ [15] and the lower bound on $B^0_s-\bar{B}^0_s$ mixing, to put constraint on fourth generation parameters. The effect of the allowed region of parameter space on $b \to s$ transitions are compared with Belle and BaBar measurements. We further discuss the forward-backward asymmetry of $B \to (K, K^*)\ell^+\ell^-$, and indirect $CP$ violation in $B_s-B_s$ mixings and $B^0 \to \phi K_S$ decay. We stress that we focus on $V_{ts}^*V_{t\nu}$ only, in anticipation of new data. Thus, only one additional new $CP$ phase is introduced. The $K^{(*)} \to \pi^+\nu\bar{\nu}$ process, $B_d$ mixing and its $CP$ phase etc., involve $V_{td}^*V_{t\nu}$, $V_{td}V_{t\nu}$ respectively, which are quite independent from the $CP$ phase information contained in $V_{tb}^*V_{t\nu}$.

II. STRATEGY

The $b \to s \gamma$ process provides a stringent constraint on new physics, while the nonobservation of $B_s-B_s$ mixing also puts constraints on a fourth generation. We shall discuss these two traditional constraints before turning to more recent experimental observations.

A. The $b \to s \gamma$ Constraint

With a sequential fourth generation, the Wilson coefficients $C_7$ and $C_8$ receive contributions from $t'$ quark loop, which we will denote as $C_7^{\text{new}}$. Because a sequential fourth generation couples in a similar way to the photon and W, the effective Hamiltonian relevant for $b \to s \gamma$ decay has the following form:
\[ \mathcal{H}_{\text{eff}} = \frac{4GF}{\sqrt{2}} \sum_{i=1}^{8} [\lambda_i C_{i}^{\text{SM}}(\mu) + \lambda_i C_i^{\text{new}}(\mu)] O_i(\mu), \]

(1)

with \( \lambda_f = V_f^{*}V_{f6} \), and the \( O_i \)s are just as in SM. The unitarity of the \( 4 \times 4 \) CKM matrix leads to: \( \lambda_8 + \lambda_7 + \lambda_{t} = 0 \). Taking into account that \( \lambda_8 \) is very small compared to the others, one has \([10,11]\):

\[ \lambda_t \equiv -\lambda_c - \lambda'_{t}, \]

(2)

and \( \lambda_c = V_{cs}^{*}V_{cb} \) is real by convention. It follows that

\[ \lambda_c C_{8,7}^{\text{SM}} + \lambda_t C_{7,8}^{\text{SM}} = -\lambda_c C_{7,8}^{\text{SM}} + \lambda_t (C_{7,8}^{\text{SM}} - C_{7,8}^{\text{SM}}), \]

(3)

where the first term corresponds to the usual SM contribution. It is clear that, for \( m_{t'} \rightarrow m_t \) and \( \lambda_{t'} \rightarrow 0 \) limits, the \( \lambda_t (C_{7,8}^{\text{SM}} - C_{7,8}^{\text{SM}}) \) term vanishes, as required by the GIM mechanism. We parameterize

\[ \lambda_{t'} \equiv r_s e^{i\phi_s}, \]

(4)

where \( \phi_s \) is a new CP violating phase.

The branching ratio of \( b \rightarrow s\gamma \) is computed by using formulas given in [16]. Using \( B(B \rightarrow X_s e\nu_e) = 10.4\% \), \( \lambda_c = 0.04 \), we find \( B(B \rightarrow X_s\gamma) \approx 3.26 \times 10^{-4} \) in SM. The present world average for \( B(B \rightarrow X_s\gamma) \) rate is \([15]\),

\[ B(B \rightarrow X_s\gamma) = (3.3 \pm 0.4) \times 10^{-4}. \]

(5)

We will keep the \( b \rightarrow s\gamma \) branching ratio in the 1σ range of (2.9–3.7) \times 10^{-4} in the presence of fourth generation.

We note that the CDF collaboration excludes \([17]\) the \( b' \) quark in the mass range of 100 GeV < \( m_{t'} < 199 \) GeV at 95% C.L., if \( B(b' \rightarrow bZ) = 100\% \). * Precise EW data provide stringent constraint on the fourth generation: the splitting between \( t' \) and \( b' \) is severely constrained by \( |m_{s} - m_{t'}| \leq m_{Z} \) \([15,18,19]\). Assuming that \( m_{t'} > m_{s} \), we conclude that the \( t' \) should be heavier than \( \approx 199 \) GeV + \( m_{Z} \). In our analysis we will consider only \( t' \) heavier than 250 GeV.

In Fig. 1 we give a contour plot of \( B(B \rightarrow X_s\gamma) \) in \( (\Phi_s, r_s) \) plane for \( m_{t'} = 250 \) and 350 GeV. We see that the CP phase \( \Phi_s \) does make considerable impact on the range of \( B(B \rightarrow X_s\gamma) \), but one is not particularly sensitive to \( m_{t'} \). In general, for both \( m_{t'} = 250 \) and 350 GeV cases, \( r_s \) cannot be much larger than \( 0.02 \) if one wishes to keep \( B(B \rightarrow X_s\gamma) \) in the range of (2.9–3.7) \times 10^{-4}. But for \( \Phi_s \sim \pi/2, 3\pi/2, \) when \( V_{t'}^{*}V_{t6} \) is largely imaginary, \( r_s \) can take on much larger values; for these cases the \( t' \) contribution adds only in quadrature to SM contribution, hence much more accommodating. In this sense, our approach is different from the approach of \([10,13]\) where \( \lambda_{t'} \) is taken as real, and can take on only one of two values for a fixed \( m_{t'} \).

Since the chiral structure is the same as in SM, and since a heavy state such as \( t' \) does not bring in extra absorptive parts, CP asymmetry in \( b \rightarrow s\gamma \) remains small. However, the possibility of sizable CP odd \( t' \) contributions allowed by \( b \rightarrow s\gamma \), while not easily distinguished in the \( b \rightarrow s\gamma \) process itself, can make impact on CP observables in other \( b \rightarrow s \) transitions, as we shall see.

### B. \( B_{s}^{0} - \bar{B}_{s}^{0} \) Mixing Constraint

Because of strong \( m_{t'} \) dependence, the fourth generation contribution can easily impact on \( B_{s}^{0} - \bar{B}_{s}^{0} \) mixing and its CP phase \( \Phi_{B_{s}} \), and can be accessible soon at the Tevatron Run II.

We use the definition of \( \Delta m_{B_{s}} = 2|M_{12}^{B}| \) and \( M_{12}^{B} = |M_{12}^{B}|e^{i\phi_{B_{s}}} \), where \( M_{12}^{B} \) is given by \([20]\):

\[ M_{12}^{B} = \frac{G_{F}^{2}M_{W}^{2}}{12\pi^{2}} M_{B_{s}}B_{B_{s}}f_{B_{s}} \left\{ \lambda_{s}^{2} \Phi_{0}(x_{W}^{2}) + \eta'\lambda_{s}^{2}\Phi_{0}(x_{W}^{2}) \right\} \]

(6)

where \( \eta = 0.55 \) is the QCD correction factor. Taking into account the threshold effect from \( b' \) quark, we find \( \eta' = \alpha_{s}(m_{t'})^{3/4}(\alpha_{s}(m_{t'})^{3/4}(\alpha_{s}(m_{t'})^{3/4}(\alpha_{s}(m_{t'})^{3/4}(\alpha_{s}(m_{t'})^{3/4}(\alpha_{s}(m_{t'})^{3/4}(\alpha_{s}(m_{t'})^{3/4}(\alpha_{s}(m_{t'})^{3/4}(\alpha_{s}(m_{t'})))\]

(7)

\[ \Phi_{0}(x, y) = \left\{ \begin{array}{ccc} \frac{1}{1 - y} \left[ \frac{1}{4} + \frac{3}{2} - \frac{1}{(y - 1)^{2}} \right] & \log y \\ + \frac{1}{1 - y} \left[ \frac{1}{4} + \frac{3}{2} - \frac{1}{(y - 1)^{2}} \right] & \log x \\ + \frac{3}{4} - \frac{1}{(y - 1)^{2}} & \left\{ \begin{array}{ccc} \frac{1}{4} & \frac{1}{2} & \frac{1}{4} \end{array} \right\} \\ \end{array} \right. \]

(8)

\[ \eta' = \eta' \]

\[ \sqrt{m_{t'}} = 168 \text{ GeV}, \text{ which gives } \Delta m_{B_{s}} \approx 17.0 \text{ ps}^{-1} \]

\[ \text{FIG. 1. Contours for } B(B \rightarrow X_s\gamma)(\times 10^{4}) \text{ in the } (\Phi_s, r_s) \text{ plane, for } m_{t'} = 250 \text{ (350) GeV on the left (right).} \]
vanishing $\sin 2\Phi^{SM}_{B_s}$. For illustration, in Fig. 2 we give a contour plot of $\Delta m_{B_s}$ in $\Phi_s$-$r_s$ plane, for $m_{B_s} = 250$ and 350 GeV. The result of Fig. 1 is overlaid as dashed lines for comparison. We see that $\Delta m_{B_s}$ can reach 90 ps$^{-1}$ for $r_s \approx 0.02$ and $\Phi_s \approx \pi$, but for heavier $m_{B_s}$, $\mathcal{B}(B \to X_s \gamma)$ will reach beyond $3.7 \times 10^{-4}$ and become less likely. For $\Phi_s \sim \pi/2$, $3\pi/2$, where $b \to s\gamma$ is more tolerant, $\Delta m_{B_s}$ can get greatly enhanced. On the other hand, the experimental lower bound of $\Delta m_{B_s} > 14.9$ ps$^{-1}$ implies that the region where $0 \leq r_s \leq 0.03$ and $\cos \Phi_s > 0$ is disfavored, which rules out a region allowed by $b \to s\gamma$. We now see that the allowed parameter space is larger for the $m_{B_s} = 250$ GeV case, i.e. for heavier $m_{B_s}$, $r_s$ can only take on smaller values. This is because the $m_{B_s}$ dependence for $\Delta m_{B_s}$ is rather strong, but is much weaker in case of $b \to s\gamma$.

We stress that the $\Phi_s \sim \pi/2$, $3\pi/2$ cases are much more forgiving because the $CP$ phase for $b \to s$ transitions is practically real in SM3, hence a purely imaginary fourth generation contribution adds only in quadrature. We thus turn to study the possible impact of a fourth generation, especially through its new $CP$ phase, on other $b \to s$ transitions.

III. $CP$ VIOLATION IN $B$ MESON MIXING

Here we mean $B^0 - \bar{B}^0$ mixing. A fourth generation can affect $B^0_d - \bar{B}^0_d$ mixing and its $CP$ phase through $V_{td}^* V_{tb}$. Since there is no apparent deviation from SM3 [15] expectations, we ignore it. While enhanced $\Delta m_{B_s}$ can easily evade present bounds, of particular interest is whether and how a fourth generation could affect the mixing-dependent $CP$ violating observable $\sin 2\Phi_{B_s}$, which is analogous to the recently measured $CP$ phase in $B^0_d - \bar{B}^0_d$ mixing amplitude. We plot $\Delta m_{B_s}$ and $\sin 2\Phi_{B_s}$ vs. $\Phi_s$ in Figs. 3 and 4, for $m_{B_s} = 250$ and 350 GeV, respectively, for several values of $r_s$. In general, the interference between SM3 and fourth generation effects is constructive for $\cos \Phi_s < 0$, and can enhance $\Delta m_{B_s}$ up to 80–90 ps$^{-1}$ for large $r_s \approx 0.02$, as we have seen already in Fig. 2. Conversely, destructive interference occurs for $\cos \Phi_s > 0$, but this would give $\Delta m_{B_s} < \Delta m_{B_s}^{SM}$ hence disfavored by the present lower bound. We are thus more interested in the constructive interference scenario, or when $\Phi_s \sim \pi/2, 3\pi/2$, where the $b \to s\gamma$ and $\Delta m_{B_s}$ constraints are more forgiving.

We illustrate the $m_{B_s} = 250$ GeV case with a larger range of $r_s$ values. This not only allows a greatly enhanced $\Delta m_{B_s}$, it makes $\sin 2\Phi_{B_s}$ very interesting. We see from Fig. 3 that the $\sin 2\Phi_{B_s}$ structure is rich: for $r_s > 0.01$, $\sin 2\Phi_{B_s}$ can reach $\pm 100\%$. This can be traced to the dependence of $M_{12}$ on $\Phi_s$. We see from Eq. (6) that the $\lambda^2_i, \lambda_U$, and $\lambda^*_i$-indep. terms imply that $M_{12}$ takes the following form:

$$M_{12} = |M_{12}| e^{i\Phi_{B_s}} \approx r^2_s e^{2i\Phi_s} A + r_s e^{i\Phi_s} B + C$$

where $A$ and $B$ are explicit functions of $m_t$ and $m_{B_s}$ and $C$ is the usual SM3 contribution. For large enough $r_s$, the $r^2_s e^{2i\Phi_s}$ term dominates, and one has $e^{2i\Phi_s}$ modulation. But as $r_s$ decreases, the quadratic term becomes unimportant, and one is only subject to $e^{i\Phi_s}$ modulation arising from the interference term.

Thus, for $r_s$ as small as 0.002 (compare $|\lambda_s| = |V_{ts}^* V_{tb}| \approx 0.04$), $\Delta m_{B_s}$ is close to SM3 value, but $\sin 2\Phi_{B_s}$ can already reach $\pm 25\%$ near $\Phi_s = \pi/2, 3\pi/2$. For $r_s = 0.01$, $\Phi_s$ is restricted by the $\Delta m_{B_s}$ lower bound to the range of $70^\circ \lesssim \Phi_s \lesssim 300^\circ$, where $\sin 2\Phi_{B_s}$ can take on extremal values of $\mp 100\%$ near the boundaries. This is very interesting, for if $\Delta m_{B_s}$ is measured soon near present bounds, we may find the associated $CP$ violating phase to be maximal! For $r_s = 0.02$, the allowed range for $\Phi_s$ is slightly larger than (roughly $50^\circ \lesssim \Phi_s \lesssim 310^\circ$) the $r_s = 0.01$ case. The maximal $\sin 2\Phi_{B_s}$ now occurs close to $\Phi_s = \pi/2, 3\pi/2$ since the fourth generation effect overwhelms the SM3 top quark effect. In all these cases, $\sin 2\Phi_{B_s}$ vanishes for $\Phi_s = \pi$ when one has maximal constructive interference in $\Delta m_{B_s}$.

It should be noted that, for purely imaginary $e^{i\Phi_s}$, $r_s$ could be considerably larger than 0.02, leading to very large $\Delta m_{B_s}$ and maximal $\sin 2\Phi_{B_s}$. The measurement of such cases, however, are more difficult.

For the heavier $m_{B_s} = 350$ GeV scenario of Fig. 4, $b \to s\gamma$ puts a slightly tighter constraint, hence we have illustrated the $\Phi_s$ dependence of $\Delta m_{B_s}$ and $\sin 2\Phi_{B_s}$ for $r_s$ only up to 0.005. For this parameter space, the $r^2_s e^{2i\Phi_s}$ term in Eq. (9) is subdominant, and one largely has $e^{i\Phi_s}$ modulation. The case is similar to the low $r_s$ discussion of Fig. 3. Once again, for $\Phi_s \sim \pi/2, 3\pi/2$, $r_s$ could be much larger than the illustrated range, and $\sin 2\Phi_{B_s}$ would become maximal.
Belle experiment in 2001, giving first observation of $\bar{B}$ of 78 fb$^{-1}$ confirmed \cite{5} by the BaBar collaboration with a dataset 10$^{10}$ where a similar decomposition as in Eq. (3) can be made.

where a similar decomposition as in Eq. (3) can be made. The strong $m_t$ dependence \cite{11} makes the electroweak penguin a particularly good probe for the presence of fourth generation. Fourth generation contributions to $B \rightarrow (K,K^*)\ell^+\ell^-$ have been studied in Refs. \cite{11,25}. These studies, however, took $\lambda_t$ to be real, which would then be constrained by $b \rightarrow s\gamma$ to two specific $|\lambda_t|$ values for a given $m_{t\nu}$ \cite{13} (i.e. for $\Phi_s = 0$ and $\pi$). As we pointed out, allowing for a $CP$ phase in $\lambda_t$, for fixed $m_{t\nu}$ a large region in the ($\Phi_s, r_s$) plane is actually still allowed. In Fig. 5 we plot the non-resonant branching ratio for $B \rightarrow K\ell^+\ell^-$ and $B \rightarrow K^*\ell^+\ell^-$ vs. $CP$ phase $\Phi_s$ for several values of $r_s$ and $m_{t\nu}$. The horizontal bands give the SM range taking into account form factor uncertainties. The pattern is similar to $\Delta m_B$: for cos $\Phi_s > 0$ we have destructive interference, and the branching fraction is below SM; for cos $\Phi_s < 0$ we have constructive interference and the branching ratio is largest for $\Phi_s = \pi$.

We see that the non-resonant $B(B \rightarrow K\ell^+\ell^-)$ can reach approximately 2 to 3 times the SM value depending on $m_{t\nu}$, $r_s$, and $\Phi_s$. As stated, such enhancement may become really called for if the NNLO result \cite{24} of $B(B \rightarrow K\ell^+\ell^-) = (0.35 \pm 0.12) \times 10^{-6}$ is borne out. In any case, a possible factor of 2-3 enhancement over SM is still within the experimental range of Eq. (10), but

\begin{equation}
B(B \rightarrow K\ell^+\ell^-) = (0.58^{+0.17}_{-0.15} \pm 0.06) \times 10^{-6},
\end{equation}

at 5.4$\sigma$ significance. These results have been confirmed \cite{5} by the BaBar collaboration with a dataset of 78 fb$^{-1}$, giving $B(B \rightarrow K\ell^+\ell^-) = (0.78^{+0.24}_{-0.18}) \times 10^{-6}$, consistent with an earlier value of $(0.84^{+0.30}_{-0.18}) \times 10^{-6}$ at 56.4 fb$^{-1}$; the published BaBar upper limit in 2001, however, gives $B(B \rightarrow K\ell^+\ell^-) < 0.51 \times 10^{-6}$. The latest upper limits for $B(B \rightarrow K\ell^+\ell^-)$ are $1.4 \times 10^{-6}$ for Belle and, for BaBar, $3.0 \times 10^{-6}$, or (1.68$-0.58 \pm 0.28$) $\times 10^{-6}$ at 2.8$\sigma$ significance.

Given the volatility, including the muon efficiency problem of BaBar, we shall not combine the above values. However, while $B(B \rightarrow K\ell^+\ell^-) \sim (0.6-0.8) \times 10^{-6}$ is not inconsistent with SM predictions \cite{21}, it is somewhat on the high side, as we shall see. Similarly, the new Belle observation \cite{6} of inclusive $B \rightarrow X_s\ell^+\ell^-$ decay,

\begin{equation}
B(B \rightarrow X_s\ell^+\ell^-) = (6.1 \pm 1.4^{+1.3}_{-1.1}) \times 10^{-6},
\end{equation}

at 5.4$\sigma$ significance, is also a bit on the high side.

We apply the same approach we have introduced for $b \rightarrow s\gamma$. Using the notations and formulas from Ref. \cite{21}, the effective Hamiltonian for $b \rightarrow s\ell^+\ell^-$ is given by

$$ H_{\text{eff}} = \frac{4G_F}{\sqrt{2}} \sum_{i=1}^{10} [\lambda_i C_i^{SM}(\mu) + \lambda_{t\nu} C_i^{new}(\mu)] O_i(\mu), $$

where a similar decomposition as in Eq. (3) can be made. We use the Wilson coefficients $C_i^{SM}$ calculated in naive dimensional regularization \cite{22}. The analytic expressions for all Wilson coefficients can be found in Ref. \cite{23}. For simplicity, we will not take into account long-distance effects from real $c\bar{c}$ intermediate states.

We work with the $\overline{MS}$ bottom mass evaluated at $\mu = 5$ GeV. At the NLO, $m_b(\mu) = m_b^\text{pole}(1 - \frac{\alpha_s}{\pi}(1 - \frac{2}{3}\log(m_b^\text{pole}/\mu)))$ where $m_b^\text{pole} = 4.8$ GeV is the $b$ quark pole mass. For other parameters, we take 1.3 GeV as the charm quark mass, $\overline{m}_{c}(m_t) = 168$ GeV, $m_K = 0.497$ GeV, $m_{K^*} = 0.896$ GeV, $m_B = 5.28$ GeV, $|\lambda_s| = |V_{23}V_{cb}| = 0.04$ and $\sin^2\theta_W = 0.223$. To compute the $B \rightarrow (K,K^*)\ell^+\ell^-$ rates, we need the $B$ and $B \rightarrow K^*$ transition form factors, where we take the values from QCD sum rules on the light-cone (LCSR) \cite{21}.
the case of \( r_s \approx 0.02 \) with \( m_{\ell^+} = 350 \) GeV may have too large a branching ratio for \( \Phi_s \approx \pi \), hence \( |V_{\ell s} V_{\ell s}| \) and \( m_r \) should not be simultaneously too large.

Thus, the electroweak penguin modes have similar power as the \( \Delta m_{B_s} \) bound, which was illustrated in Fig. 2, in probing fourth generation at present. But it may improve rapidly once the experiments converge. We note that there may be some trouble with the \( B(B \rightarrow K^*\ell^+\ell^-)/B(B \rightarrow K^*\ell^+\ell^-) \) ratio between theory and experiment. This may be either an experimental problem (fluctuation), or a problem with form factors. For this reason, the inclusive \( B \rightarrow X_i \ell^+\ell^- \) may be more interesting, since it suffers less from hadronic uncertainties. We illustrate by contour plot in Fig. 6 the non-resonant contribution to \( B(B \rightarrow X_i \ell^+\ell^-) \times 10^3 \) in the \((\Phi_s, r_s)\) plane, for \( m_{\ell^\pm} = 250 \) and 350 GeV. As one can see, in both cases, \( \cos \Phi_s > 0 \) reduces \( B \rightarrow X_i \ell^+\ell^- \) to be less than 5.88 \times 10^{-3} and is less favored, while for \( \pi/2 < \Phi_s < 3\pi/2 \) the branching ratio is enhanced and grows with \( r_s \). The behavior is similar to \( B_s \) mixing. We note that the effect of fourth generation on \( B \rightarrow X_i \ell^+\ell^- \) has been studied recently in [14]. The constraints we obtain on \( r_s \) from \( B \rightarrow X_i \ell^+\ell^- \) are consistent with Ref. [14].

The dilepton invariant mass spectrum does not have further information content beyond the rates themselves. With proper rescaling, the shape is the same as in SM.

Let us now discuss the forward-backward asymmetry (FBA) of \( B \rightarrow K^*\ell^+\ell^- \). The usual definition is [21]

\[
\frac{dA_{FB}}{ds} = -\int_{0}^{s} \frac{d\lambda}{ds} \frac{d^{2}\Gamma}{duds} + \int_{-\lambda(s)}^{0} \frac{d\lambda}{ds} \frac{d^{2}\Gamma}{duds},
\]

where \( \lambda(s) = \sqrt{1 - 4m_{\ell^\pm}^2/s} \lambda \) and \( \lambda = \lambda(1, m_{K^*}, \hat{s}) = 1 + m_{K^*}^2 + \hat{s}^2 - 2s - 2m_{K^*}^2(1 + \hat{s}) \) with \( m_r^2 = m^2/M_B^2 \).

From the experimental point of view the normalized FBA (\( \tilde{A}_{FB} \)) is more useful and is defined by:

\[
\frac{d\tilde{A}_{FB}}{ds} = \frac{dA_{FB}}{ds} / \frac{d\Gamma}{ds}.
\]

Note that the FBA for \( B \rightarrow K^*\ell^+\ell^- \) vanishes.

In the SM, the FBA for \( B \rightarrow K^*\ell^+\ell^- \) is positive for \( \hat{s} < 0.1 \) with values going up to \( \approx 0.1 \), vanishes at \( \hat{s}_0 = 0.1 \) and turns negative for \( 0.7 > \hat{s} > 0.1 \) with FBA values going down to \( \approx -0.4 \). This reflects the interference between the \( b \rightarrow s \gamma^* \) vs. \( b \rightarrow s Z^* \) mediated processes. The presence of a fourth generation affects the latter more strongly and hence can change the pattern. We are interested in the parameter space where the shape of FBA change completely from that of SM.

In Fig. 7, we give contour plots for normalized FBA, for \( \Phi_s = 0 \), in the \((\hat{s}, r_s)\) plane for \( m_{\ell^\pm} = 350 \) GeV (left) and for \( (\hat{s}, m_{\ell^\pm}) \) plane for \( r_s = 0.02 \) (right). We see that FBA can vanish not only for \( \hat{s} = 0.1 \) but also in full range of \( 0.7 > \hat{s} > 0.1 \). In the left plot this happens for \( r_s^0 \approx 0.018 \) and \( m_{\ell^\pm} = 350 \) GeV while in the right plot this seems to happen for \( m_{\ell^\pm}^0 \approx 335 \) GeV and \( r_s = 0.02 \). From left contour, note that for \( r_s > r_s^0 \), the FBA is negative for \( \hat{s} > 0.1 \) exactly as in the SM, but for \( r_s < r_s^0 \), the FBA remains positive even for \( \hat{s} > 0.1 \). The same observation can be made for \( \hat{s} > 0.1 \) in the right contour: the FBA is negative for \( m_{\ell^\pm} < m_{\ell^\pm}^0 \) but positive for \( m_{\ell^\pm} > m_{\ell^\pm}^0 \), and FBA can reach +0.4 or more.

In Fig. 8, we illustrate the effect of the \( CP \) phase \( \Phi_s \) for normalized FBA in the \((\hat{s}, \Phi_s)\) plane for \( m_{\ell^\pm} = 250 \) GeV, \( r_s = 0.04 \) (left), and for \( m_{\ell^\pm} = 350 \) GeV, \( r_s = 0.02 \) (right). From the left plot, one can see that FBA has a constant sign (negative) for large \( \hat{s} > 0.1 \) and can become larger than \(-0.3 \) for large \( CP \) phase and large \( \hat{s} \). In the right plot the FBA can flip sign for \( \Phi_s \approx 0.5 \) and for values of \( \hat{s} > 0.1 \).
V. CP VIOLATION IN CHARMLESS B DECAYS

Although it is too early to conclude [26], both Belle and BaBar presented [7] some hints for mixing-dependent CP violation in the charmless $B^0 \to \phi K_S$ decay mode, with the two experiments agreeing in sign. It is intriguing that the sign is opposite that of $B^0 \to J/\psi K_S$. We do not advocate that new physics is already called for, but illustrate the impact of a fourth generation. It is known that the CP asymmetry in both $B^0 \to \phi K_S$ and $J/\psi K_S$ should measure the same quantity $\sin(2\phi)$, where $\phi = \phi_{K_S}$. A particularly interesting aspect of the existence of a fourth generation is that for $m_{t^\prime} < m_t$, the two experiments agreeing in sign. It is intriguing if we use the definition of $\Lambda$ defined above

$$\Lambda = e^{-2i\phi_1} \frac{\mathcal{A}}{\bar{\mathcal{A}}} = e^{-2i(\phi_1 + \theta)} \frac{\mathcal{A}}{\bar{\mathcal{A}}}$$

where we have used $\frac{\mathcal{A}}{\bar{\mathcal{A}}} = e^{-i2\theta} |A/|A|\) and $|A/|A|\) the CP-violating or CP-conserving, or $|A/|A|\) is similar to SM, and it is not possible to generate large $|A/|A|\) CP-violating $|A/|A|\) in the literature have not emphasized $|A/|A|\) $|A/|A|\) CP. The effective Hamiltonian for $|A/|A|\) = 1 transition can be written as [27]:

$$\mathcal{H}_{\text{eff}} = \frac{G_F}{\sqrt{2}} \left\{ \lambda_u (C_1 O_1 + C_2 O_2^\prime) + \lambda_c (C_1 O_1^* + C_2 O_2^\prime) \right\} \sum_{i=3}^{10} \left| \lambda_i C_i^\text{SM}(\mu) + \lambda_i C_i^\text{new}(\mu) |O_i(\mu) \right| .$$

(15)

For the SM part, we have used the next-to-leading order formulas given in [27] for the Wilson coefficients, while for the fourth generation effect in $C_i^\text{new}$, we have used the leading order analytic formulas given in [29].

Following the notation of Ref. [27], in the factorization approach, the amplitude of both $B^- \to K^- \phi$ and $B^0 \to K^0 \phi$ is proportional to $V_{tb}V_{ts}^\ast (a_3 + a_4 + a_5 - 1/2(a_7 + a_9 + a_{10}))$. The exact form of the factorizable hadronic matrix element is irrelevant for us since we are interested only in CP asymmetry observables in SM4, and the ratio of branching ratios in SM3 and SM4. The coefficients $a_i$ are defined as: $a_i = C_i^{\text{eff}} + 1/N_C C_i^{\text{HF}}$ for odd, even $i$.

The CP asymmetry $\sin(2\Phi_{K_S})$ is defined as:

$$\sin(2\Phi_{K_S}) = -\frac{2 \text{ Im } \Lambda}{1 + |\Lambda|^2},$$

(16)

where $\Lambda = e^{-2i\phi_1} \frac{\mathcal{A}}{\bar{\mathcal{A}}}$ with $\bar{\mathcal{A}}$ being the CP-conjugate amplitude of $\mathcal{A}$. We define also the following ratio:

$$R_{\phi K_S} = \frac{A_{\text{SM4}}(B^0 \to \phi K_S)}{A_{\text{SM3}}(B^0 \to \phi K_S)}.$$  

(17)

In our analysis we use the following set of parameters: $\mu = 2.5$ GeV, $m_t = 168$ GeV, $m_b = 4.88$ GeV, $k^2 = m_t^2/2$, $\alpha = 1/128$ and $s_W^2 = 0.223$ and $\alpha_s(M_W) = 0.118$. In SM3 (i.e. without 4th generation) we find that $\frac{\mathcal{A}}{\bar{\mathcal{A}}} = 0.979$ which is close to 1, and which means that $\sin(2\Phi_{K_S}) \approx \sin(2\phi_1) = \sin(2\Phi_{J/\psi K_S})$. In our numerical evaluation we take $\phi_1$ close to $24^\circ$ which reproduce the world average of $\sin(2\phi_1) = 0.734$ coming from $B^0 \to J/\psi K_S$.

In SM4, the ratio $\frac{\mathcal{A}}{\bar{\mathcal{A}}}$ has additional CP phase, which we denote as $2\theta$, that arises from our CP phase $\Phi_s$. So if we use the definition of $\Lambda$ defined above above

VI. DISCUSSION AND CONCLUSION

A particularly interesting aspect of the existence of a fourth sequential generation is the presence of new CP violating phases. Restricting ourselves to $b \to s$ transitions, besides the strength of $|V_{ts}||V_{tb}| \equiv r_s$, one has a unique new CP phase, arg $V_{ts}V_{tb} \equiv \Phi_s$. Existing work in the literature have not emphasized $\Phi_s \neq 0$, $\pi$ situation, hence have overconstrained the possible impact of the fourth generation. In the present work, we have tried to cover as much ground as possible, and we find a rather enriched phenomenology for $b \to s$ transitions, for both CP violating or CP-indep. observables.

The $b \to s \gamma$ process is not very sensitive to the presence of a fourth generation, since the structure is rather similar to SM, and it is not possible to generate large CP asymmetry because of the chiral structure. It is therefore especially accommodating for $\Phi_s = \pi/2, 3\pi/2, \ldots$ when...
fourth generation contribution adds only in quadrature in rate. These are the most interesting situations, since $CP$ violating effects elsewhere are the largest.

For example, the $B_s$ mixing and $CP$ phase $\sin 2\Phi_{B_s}$ are very interesting probes of fourth generation effects. While some part of parameter space allowed by $b \to s\gamma$ ($\cos \Phi_s \gtrsim 0$) is ruled out, $\Delta m_{B_s}$ can be much larger than SM3 expectations. Any value for $\sin 2\Phi_{B_s}$ between $-1$ and $+1$ is in principle possible. What may be interesting is that, even for $r_s$ rather small, hence $\Delta m_{B_s}$ just above present bound, $\sin 2\Phi_{B_s}$ could still be sizable. Thus, Tevatron Run II data is eagerly awaited.

The electroweak penguin modes $B \to K(\pi)\ell^+\ell^-$ and $X_s\ell^+\ell^-$ have recently been observed, and the rates tend to be a little on the high side, especially when compared to more recent NNLO results. Again, the best case seems to be for $\Phi_s \sim \pi/2, 3\pi/2$, when fourth generation effects add mildly in quadrature. Although the dilepton spectrum, and roughly speaking the direct $CP$ asymmetries as well (analogous to $b \to s\gamma$), are not much affected, the forward-backward asymmetry, $A_{FB}$, can be drastically affected. This is because the $\gamma^*$ effect is far less sensitive to fourth generation effects than the $Z^*$ effect, so the $s = m_{\ell^+\ell^-}/m_B^2$ dependence of $A_{FB}$ is a sensitive probe of the interference between the two terms. Thus, while we need the Belle and BaBar measured rates to converge better, in the long run the forward-backward asymmetry may provide a very interesting probe of the fourth generation.

Finally, we studied the mixing dependent $CP$ violation in the $B \to \phi K_S$ charmless decay, since some hint for beyond SM appeared here recently. We found the effect of fourth generation to be insignificant here. This is because the strong penguin, even more so than $b \to s\gamma$, depends very mildly on fourth generation, while the “harder” $C_9$ term, analogous to the electroweak penguin contribution to $B \to K^{(*)}\ell\ell$, is subdominant. Thus, even with its new source of $CP$ violation, it is unlikely for the fourth generation to change the sign of $\sin 2\Phi_{\phi K_S}$ with respect to $\sin 2\Phi_{J/\psi K_S}$ in the $B_d$ system.

The impact of the fourth generation is most prominent in $B_s$ mixing and electroweak penguin $b \to s$ transitions, but mild in electromagnetic and strong penguins. We have found that the fourth generation, even with $|V_{ts}V_{tb}|$ not far below $|V_{cs}V_{cb}| \approx 0.04$, could be lurking with purely imaginary KM phase.

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