A separable approximation for Skyrme interactions and charge-exchange excitations

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Abstract. A finite rank separable approximation for the quasiparticle random phase approximation with Skyrme interactions is applied to study properties of the Gamow-Teller resonances and the spin-dipole resonances in some spherical nuclei. It is shown that characteristics calculated within the suggested approach are in a good agreement with available experimental data.

1. Introduction
A study of the charge-exchange nuclear modes is an interesting problem not only from the nuclear structure point of view but it is very important for the nuclear astrophysics applications. Many versions of the self-consistent approach based on the Random Phase Approximation (RPA) or the quasiparticle RPA (QRPA) were developed during last years, see for example [1-10]. A comparison of such calculations with recent experimental data demonstrates that model mentioned above can not reproduced correctly the strength distribution of the spin-isospin resonances. It is necessary to take into account a coupling with more complex configurations that results in shifting some strength up [11, 12, 13]. Moreover the tensor forces can give an additional shift [10, 11]. It is much simpler to include a coupling in QRPA calculations if one uses separable forces [12, 13, 14]. This idea stimulated us to develop the finite rank approximation for the Skyrme interactions [15, 16] that enables one to perform calculations in the large configuration space. Applications of our method to study the low-lying states and giant resonances within the QRPA and beyond are given in [15, 16, 17, 18]. Recently, we have proposed an extension of our approach for the charge-exchange nuclear excitations [19]. Before to investigate effects of the coupling we need to be sure that the finite rank approximation for the Skyrme interactions is enough good to reproduce main characteristics of such nuclear modes.

In this report we describe briefly our method for the charge-exchange excitations and present our investigations of properties of the Gamow-Teller resonances (GTR) and the spin-dipole resonances (SDR) in some spherical nuclei.

2. Method of calculation
The starting point of the method is the HF-BCS calculation [23] of the parent ground state, where spherical symmetry is imposed on the quasiparticle wave functions. The continuous part
of the single-particle spectrum is discretized by diagonalizing the Skyrme HF Hamiltonian on a harmonic oscillator basis. We work in the quasiparticle representation defined by the canonical Bogoliubov transformation:

\[ a_{jm}^+ = u_j \alpha_{jm}^+ + (-1)^{j-m} v_j \alpha_{j-m} \]  

(1)

where \( jm \) denote the quantum numbers \( nljm \). We use the Skyrme interaction [20] in the p-h channel, while the pairing correlations are generated by the surface peaked density-dependent zero-range force

\[ V_{\text{pair}}(r_1, r_2) = V_0 \left( 1 - \frac{\rho(r_1)}{\rho_c} \right) \delta (r_1 - r_2) \]  

(2)

Here \( \rho(r_1) \) is the particle density in the coordinate space, \( \rho_c \) is equal to the nuclear saturation density, the strength \( V_0 \) is the parameter fixed to reproduce the odd-even mass difference of nuclei in the study region. In order to avoid divergences, it is necessary to introduce a cutoff in the single-particle space. This cutoff limits the active pairing space above the Fermi level. We used a procedure that is proposed in Ref. [16, 21].

The residual interaction in the p-h channel \( V_{\text{res}} \) can be obtained as the second derivative of the energy density functional with respect to the particle density \( \rho \). Following our previous paper [15] we simplify \( V_{\text{res}} \) by approximating it by its Landau-Migdal form. For Skyrme interactions all Landau parameters with \( l > 1 \) are zero and they are functions of the coordinate \( r \). We keep only the \( l = 0 \) terms in \( V_{\text{res}} \) and the expressions for \( F_0', G_0' \) in terms of the Skyrme force parameters can be found in Ref. [22]. The Coulomb and spin-orbit residual interactions are dropped. Therefore we can write the residual interaction in the isovector channel in the following form:

\[ V_{\text{res}}(r_1, r_2) = N_0^{-1} [F_0'(r_1) + G_0'(r_1) \sigma^{(1)} \cdot \sigma^{(2)}] \tau^{(1)} \tau^{(2)} \delta (r_1 - r_2) \]  

(3)

where \( \sigma^{(i)} \) and \( \tau^{(i)} \) are the spin and isospin operators, and \( N_0 = 2k_Fm^*/\pi^2\hbar^2 \) with \( k_F \) and \( m^* \) standing for the Fermi momentum and nucleon effective mass in nuclear matter.

In what follows we use the second quantized representation and \( \hat{V}_{\text{res}} \) can be written as:

\[ \hat{V}_{\text{res}} = \frac{1}{4} \sum_{1234} (V_{1234} - V_{1243}) : a_1^+ a_2^+ a_3 a_4 : = \hat{V}_M + \hat{V}_{SM} \]  

(4)

where \( a_1^+ \) (\( a_1 \)) is the particle creation (annihilation) operator and 1 denotes quantum numbers \( (n_1, l_1, j_1, m_1) \). Following our method [15, 16, 17, 18] the residual interactions can be reduced to a finite rank separable form:

\[ \hat{V}_M = -2 \sum_{JM} \sum_{k=1}^{N} \kappa_F^{(k)} : \hat{M}_J^{(k)+} \hat{M}_J^{(k)} : , \]  

(5)

\[ \hat{V}_{SM} = -2 \sum_{JM} \sum_{L,J \pm 1} \sum_{k=1}^{N} \kappa_G^{(k)} : \hat{S}_{LMJ}^{(k)+} \hat{S}_{LMJ}^{(k)} : , \]  

(6)

\[ \left( \begin{array}{c} \kappa_F^{(k)} \\ \kappa_G^{(k)} \end{array} \right) = -N_0^{-1} \frac{R w_k}{2 \bar{r}_k^2} \left( \begin{array}{c} F_0'(r_k) \\ G_0'(r_k) \end{array} \right) , \]  

(7)

where \( R \) is a large enough cutoff radius for a \( N \)-point integration Gauss formula with abscissas \( r_k \) and weights \( w_k \) [15].
The operators entering the normal products in Eqs. (5,6) are defined as follows:

\[
\hat{M}_{JM}^{(k)+} = (-1)^{J-M} \hat{j}^{-1} \sum_{mn} \langle j_n m_n j_p - m_p | JM \rangle \\
\times f_{j_n j_p}^{(k)} (-1)^{j_p - m_p} a_{j_n m_n}^+ a_{j_p m_p},
\]

\[
\hat{S}_{LJM}^{(k)+} = (-1)^{J-M} \hat{j}^{-1} \sum_{mn} \langle j_n m_n j_p - m_p | JM \rangle \\
\times g_{j_n j_p}^{(LJK)} (-1)^{j_p - m_p} a_{j_n m_n}^+ a_{j_p m_p},
\]

\[
f_{j_n j_p}^{(JK)} = u_{j_n}(r_k) u_{j_p}(r_k) \langle j_n || i^J Y_j || j_p \rangle,
\]

\[
g_{j_n j_p}^{(LJK)} = u_{j_n}(r_k) u_{j_p}(r_k) \langle j_n || i^L T_{LM} || j_p \rangle.
\]

In the above equations, \( \langle j_n || i^J Y_j || j_p \rangle \) is the reduced matrix element of the spherical harmonics \( Y_{JLM} \), \( \hat{j} = \sqrt{2J + 1} \), \( T_{LM}(\hat{r}, \sigma) = [Y_L \times Y_J]^M \). The radial wave functions \( u_j(r) \) are related to the HF single-particle wave functions,

\[
\phi_{l,m}(1) = \frac{u_i(r_1)}{r_1} \psi^m_{l,i}(\hat{r}_1, \sigma_1).
\]

We introduce the phonon creation operators

\[
Q_{JM}^+ = \sum_{j_n j_p} X_{j_n j_p} A^+(j_n j_p; JM) - (-1)^{J-M} Y_{j_n j_p} A(j_n j_p; J - M),
\]

\[
A^+(j_n j_p; JM) = \sum_{mn} \langle j_n m_n j_p m_p | JM \rangle \alpha_{j_n m_n}^+ \alpha_{j_p m_p}^+,
\]

where the index \( J \) denotes total angular momentum and \( M \) is its z-projection in the laboratory system. One assumes that the ground state is the QRPA phonon vacuum \( | 0 \rangle \) and the excited states are defined as \( Q_{JM}^+ | 0 \rangle \). Making use of the linearized equation-of-motion approach one can get the QRPA equations [23]:

\[
\begin{pmatrix}
A & B \\
-B & -A
\end{pmatrix}
\begin{pmatrix}
X \\
Y
\end{pmatrix}
= \omega
\begin{pmatrix}
X \\
Y
\end{pmatrix}.
\]
Figure 1. Gamow-Teller strength distribution for $^{132}$Sn. The calculation within the LM treatment for the p-h interaction (dashed lines), and the calculation with the full velocity-dependent terms of the p-h interaction (solid lines) are shown.

Table 1. Ikeda sum rule values. $S_-$, $S_+$ are the sum rule values of the $T_-$ and $T_+$ channels, respectively.

|          | $S_-$ | $S_+$ | $S_--S_+$ |
|----------|-------|-------|-----------|
| $^{90}$Zr | full  | 30.5  | 0.5       | 30.0     |
|          | LM    | 30.5  | 0.5       | 30.0     |
|          | Expt. | 28.0±1.6 | 1.0±0.3 | 27.0±1.6 |
| $^{132}$Sn | full  | 96.8  | 0.8       | 96.0     |
|          | LM    | 96.8  | 0.8       | 96.0     |

3. Results

In this work, we use the SGII parameterization [22] of the Skyrme force. As the first example we consider the Gamow-Teller (GT) excitations in $^{90}$Zr and $^{132}$Sn. We calculate the Ikeda sum rule for the $T_-$ and $T_+$ channels with the LM forces and compare with results of calculations with the full Skyrme interactions. Results of our calculations for $^{90}$Zr and $^{132}$Sn are shown in table 1. To compare with experimental data in $^{90}$Zr we use the same energy interval $E_x \leq 50$ MeV for the $T_-$ channel and $E_x \leq 32$ MeV for the $T_+$ channel [24, 25]. One can see that the LM approximation for the Skyrme forces reproduces experimental data for the Ikeda sum rules and gives the same results as calculations with the full Skyrme interactions. This conclusion is valid for the $^{132}$Sn case also.

The GT strength distributions in $^{132}$Sn that were calculated within different approaches are shown in the Fig. 1. To present our results we use the energy averaging procedure with the Lorentz weight function [14]. The Lorentzian smearing parameter is taken equal to 1 MeV. It seen from this figure that the GT strength distributions calculated with LM and Skyrme forces are similar but the GT resonance for the LM case is shifted down by 1 MeV. This is due to an approximation for the velocity-dependent terms for the LM forces. It is worthwhile to mention that, an inclusion of the tensor correlations can shift up to 10 % of the GT strength to the energy region above 30 MeV [10, 26, 27]. Moreover one needs to go beyond the QRPA[12] and such calculations are in progress now.

Let us consider the spin-dipole (SD) excitations. Results of our calculations for the SD states
with different values of the spin and parity for the $T_-$ channel in $^{48}$Ca are shown in Fig. 2. One can conclude that our approach reproduces the SD strength distribution for different states that are alike the full Skyrme force.

4. Conclusions
A finite rank separable approximation for the QRPA calculations with Skyrme interactions, which was proposed in our previous work, is applied to study the charge-exchange nuclear modes. The suggested approach enables one to reduce remarkably the dimensions of the matrices that must be inverted to perform nuclear structure calculations in very large configuration spaces. As an illustration we present results of our investigations for the Gamow-Teller and spin-dipole resonances in some spherical nuclei. A comparison of our results with experimental data and other theoretical calculations demonstrates that our method gives a good description

**Figure 2.** Strength distributions in the $T_-$ channel of the spin-dipole excitations in $^{48}$Ca. See the legend to Fig. 1 for details.
of properties of the charge-exchange excitations. An inclusion of the tensor terms of the Skyrme interaction and taking into account the coupling of the QRPA states with more complex components of the wave functions are in progress now.

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