Implications of Isospin Conservation in $Λ_b$
Decays and Lifetime

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Abstract

We consider isospin predictions for the semi-leptonic and non-leptonic decays of the $Λ_b$ baryon. Isospin conservation of the strong interactions constrains the possible final states in $Λ_b$ decays. This leads in general to phase space enhancements in $Λ_b$ decays relative to $B$ meson decays for the same underlying quark transitions. Consequently the $Λ_b$ lifetime is smaller than the $B$ lifetime. Phase space enhancements in $Λ_b$ decays relative to $B$ decays can be understood in terms of hyperfine interactions in the bottom system.

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1 Introduction

Isospin conservation is a good approximate symmetry of the strong interactions and can be applied fruitfully in $\Lambda_b$ decays. The $\Lambda_b$ baryon is made of a heavy $b$ quark and a $ud$ light diquark system in a spin and isospin singlet state. Semi-leptonic $\Lambda_b$ decays involve the weak $b \to c$ transition without the involvement of the light quarks. The final hadronic decay products have to be in an isosinglet state as the weak current is an isoscalar. Strong interactions do not change the isospin state of the light diquark which combines with the $c$ quark to form the hadrons in the final state. Single particle hadronic states would therefore dominantly involve the ground and excited $\Lambda_c$ baryons. In non-leptonic $\Lambda_b$ decays the effective current\times current Hamiltonian gives rise to the following quark diagrams [1]: the internal and external W-emission diagrams, which result in the factorizable contribution, and the W-exchange diagrams which gives rise to the non-factorizable contribution. The W-annihilation diagram is absent in baryon decay and we neglect the penguin contributions. The contribution from the W-exchange diagram is expected to be small in $\Lambda_b$ decays. The final states in non-leptonic $\Lambda_b$ decays result from the isosinglet diquark combining with the final state quarks. For instance in the quark level transition $b \to c$ the diquark can combine with the $c$ quark in the final state. Hence final states like $\overline{D}s\Lambda_c$ are allowed but states like $\overline{D}s\Sigma_c$ are not.

As the quark mass becomes heavier many differences among the properties of spin–1/2 and spin–3/2 baryons and also among pseudo scalar and vector mesons containing a heavy quark are expected to become less pronounced [2]. As the quark mass increases it is expected that the lifetimes of particles containing one heavy quark will become very similar [3]. It is in the corrections to the lowest order in $\Lambda_{QCD}/M$ where models play a role.

In Heavy Quark Effective Theory (HQET) the lifetimes of the $\Lambda_b$ and the $B^0$ meson were expected to be the same in the heavy quark limit and just slightly different when certain quark scattering processes that could occur in the $\Lambda_b$ but not in the meson were included. These principally included (a) the “weak scattering” process, first invoked for the $\Lambda_c^+$ lifetime [4], and here of the form, $bu \to cd$, and (b), the so–called “Pauli interference” process $bd \to \bar{c}udd$ [3, 5]. The results of including these terms is a slight enhancement in the decay rate leading to $\tau(\Lambda_b)/\tau(B^0) \sim 0.9$, whereas the evaluation [7] of $\tau(\Lambda_b)$ is $1.24 \pm 0.08$ ps and $\tau(B^0) = 1.56 \pm 0.04$ ps gives a very much reduced fraction $\tau(\Lambda_b)/\tau(B^0) = 0.79 \pm 0.07$, or conversely a very much enhanced decay rate. (There is a recent CDF result [8] which would move this fraction higher than the world average to a value of $0.85 \pm 0.10 \pm 0.05$).
A possible explanation of the $\Lambda_b$ lifetime is an enhancement of the decay width, $\Delta \Gamma(\Lambda_b)$ from the $q - q$ scattering. This involves replacing the usual flux factor by $|\psi(0)|^2$, the wave function at the origin of the pair of quarks $bu$ in the $\Lambda_b$, (or the pair $bd$, for which the wave function is the same by isospin symmetry). This wave function at the origin naturally appears in hyperfine splitting [3]. Rosner [10] tried to account for the enhancement by changing the wave function $|\psi(0)_{bu}|^2$; this would also correlate with the surprisingly large hyperfine splitting suggested by the DELPHI group [11]. He was able to show that, under certain assumptions, there could be at most a $13 \pm 7\%$ increase of the amount needed to explain the decay rate of the $\Lambda_b$. In a more dramatic attempt to explain the lifetime problem it has been proposed [12] to allow the ratio $r = |\psi_{bq}^{\Lambda_b}(0)|^2/|\psi_{bq}^{B}(0)|^2$ to vary between $1/4$ and $4$. Clearly such a large variation would be ruled out by hyperfine relations.

Here we show that isospin conservation leads naturally to a phase space enhancement in $\Lambda_b$ decays relative to $B$ decays resulting in a shorter lifetime for $\Lambda_b$. As shown in [13] isospin conservation chooses the final state in $\Lambda_b$ decays with the lowest hyperfine energy. In $B$ decays the spectator quark can combine with the $c$ quark to form vector or pseudo scalar final states. The hyperfine energy in this case is averaged out resulting in a phase space advantage for the baryon transition over the meson transition.

In the following sections we study the isospin predictions in semi-leptonic and non-leptonic $\Lambda_b$ decays. We then show that phase space enhancements in $\Lambda_b$ decays relative to $B$ decays lead to shorter lifetime for $\Lambda_b$ relative to $B$.

2 Semi-Leptonic Decays

Semi-leptonic $\Lambda_b$ decay involves the quark level $b \to c$ transition due to an isoscalar current. The amplitude for the process can be written as

$$A = \langle X| J_\mu |\Lambda_b \rangle > L^\mu$$

(1)

where $J_\mu = \bar{c} \gamma_\mu (1 - \gamma_5)b$ is the isoscalar weak current and $L^\mu$ is the leptonic weak current. The final state $X$ has to be in an isosinglet state. In the heavy quark limit the light degrees of freedom in a hadron, the diquark in this case, have conserved isospin and angular momentum quantum numbers. Due to isospin conservation the light diquark in $\Lambda_b$ remains in an isosinglet state as it combines with the $c$ quark to generate the spectrum of final states. When the diquark combines with the $c$ quark it will form dominantly a $\Lambda$ type charmed baryon. The $\Lambda$ type baryon can be classified according to the quantum numbers carried.
by the light degrees of freedom. So the lowest state corresponds to the light degree having isospin $I_l = 0$ and spin $s_l = 0$. The diquark can be excited to a higher orbital angular momentum state with $L_l = 1$. This creates baryons with net spin $1/2$ and $3/2$ denoted by $\Lambda^*_1$ and $\Lambda^*_2$ or alternately as $\Lambda_c(2593)$ and $\Lambda_c(2625)$. Other final states that can populate the $X$ spectrum to a lesser fraction than single particle $\Lambda_c$ states are $D^0 p D^0 p\pi^0$ etc. Note isosinglet combinations like $\Sigma^{++} \pi^-$, $\Sigma^+_c \pi^0$ can only be the decay product of excited $\Lambda_c$ type baryons where the pion is emitted from the diquark changing it from an isosinglet to an isovector state.

Hence the decay $\Lambda_b \to X l \bar{\nu}$ should be dominantly $\Lambda_b \to \Lambda^{(*)} l \bar{\nu}$ where $\Lambda^{(*)}$ denotes the ground state or the excited $\Lambda_c$. Because the excited $\Lambda_c$ decays to the ground state $\Lambda_c$ we have the prediction

$$\Lambda_b \to X l \bar{\nu} \approx \Lambda_b \to \Lambda_c X l \bar{\nu}.$$  

We can therefore have the following decays for $\Lambda_b \to X l \bar{\nu}$:

- $\Lambda_b \to \Lambda_c l \bar{\nu}$. This is expected to be the dominant decay in the inclusive semi-leptonic $\Lambda_b$ decay. In the heavy quark limit at maximum $q^2$ where $q^2$ is the invariant energy of the $l\bar{\nu}$ system or equivalently at $\omega = v \cdot v' = 1$ where $v$ and $v'$ are the initial and final baryon velocities transitions to excited $\Lambda_c$ are suppressed by $1/m_b^2$. Model estimates of this branching fraction are between $7 - 8\%$ \cite{[14]} while estimate of $\Lambda_b \to \Lambda_c X l \bar{\nu}$ is around $(10 \pm 4)\%$ \cite{[7]}. This also indicates that $\Lambda_c \to \Lambda_c l \bar{\nu}$ dominates $\Lambda_b \to X l \bar{\nu}$.

$$\begin{align*}
\Lambda_b & \to \Lambda_c(2593, 2625) l \bar{\nu} \\
\Lambda_c(2593, 2625) & \to \Lambda_c \pi^+ \pi^- \\
& \to \Lambda_c \pi^0 \pi^0 \\
& \to \Sigma_c(2455)^{++} \pi^- \to \Lambda_c \pi^+ \pi^- \\
& \to \Sigma_c(2455)^0 \pi^+ \to \Lambda_c \pi^- \pi^+ \\
\end{align*}$$

From the above we see that if a $\Sigma_c$ is in the final state it must be associated with a $\pi$ and further the invariant mass of the $\Sigma_c - \pi$ or the $\Lambda_c \pi \pi$ system must be the mass of the excited $\Lambda_c$.

Finally our prediction is

$$\Lambda_b \to X l \bar{\nu} \approx \Lambda_b \to \Lambda_c X l \bar{\nu} = \Lambda_b \to \Lambda_c l \bar{\nu}, \Sigma_c \pi l \bar{\nu}, \Lambda_c \pi \pi l \bar{\nu}$$
In semi-leptonic $B$ decays the largest fraction of final states will involve the $D$ and the $D^*$ meson. When compared to the dominant decay $\Lambda_b \to \Lambda_c l \bar{\nu}$ there is a phase space advantage in $\Lambda_b$ decays relative to $B$ decays which results in a shorter lifetime for $\Lambda_b$ relative to $B$. We will discuss this more quantitatively in a later section.

3 Non-Leptonic Decays

Non-leptonic $\Lambda_b$ decays proceed through the underlying quark transitions $b \to c \bar{c} s'$ and $b \to c \bar{u} d'$ where $d' = d \cos \theta_c + s \sin \theta_c$ and $s' = -d \sin \theta_c + s \cos \theta_c$. We neglect the $b \to u$ and penguin transitions. As mentioned in the introduction non-leptonic transitions involve the $W$-emission and the $W$ exchange diagrams. From the study of $\Lambda_b$ lifetime, the $W$-exchange contribution relative to the spectator $b$ quark decay rate is of the order $32\pi^2 |\psi(0)|^2/m_b^3$. This is of the order unity in the case of charmed baryons [3, 13] (which has $m_c$ in place of $m_b$) and so is much suppressed in the case of $\Lambda_b$ baryons. Note the wave function at the origin, $\psi(0)$, is approximately same for the charm and bottom system. Hence in non-leptonic $\Lambda_b$ decays the $W$-exchange term will be small unlike in the case of charmed baryons. So factorization is expected to be a good approximation in the study of non-leptonic $\Lambda_b$ decays.

We now list the predictions for $\Lambda_b$ non-leptonic decays which follow from the conservation of the isospin quantum number of the light diquark in the $\Lambda_b$ baryon

- For $b \to c \bar{c} s$ transition the effective Hamiltonian is
  \[ H_W = c_1 \bar{c} b \bar{s} c + c_2 \bar{s} b \bar{c} c \]
  where $c_{1,2}$ are the Wilson’s coefficients and we have suppressed the color and Dirac index as well as the $\gamma_\mu(1 - \gamma_5)$ factors. Since the $W$-emission diagram, which is given by the factorization amplitude, is the dominant contribution here we can write the non-leptonic amplitude as
  \[ A[\Lambda_b \to XX'] = (c_1 + c_2/N_c) <X|\bar{c}b|\Lambda_b><X'|\bar{s}c|0> \]
  \[ A_s[\Lambda_b \to XX'] = (c_2 + c_1/N_c) <X|\bar{s}b|\Lambda_b><X'|\bar{c}c|0> \]
  where $A$ and $A_s$ are the color allowed and color suppressed amplitudes and $N_c$ is the number of colors. Now for the color allowed transition, from our analysis of the semi-leptonic decays, $X$ is mainly $\Lambda_c^{(*)}$. Hence some possible final states are $\Lambda_c D_s$, $\Sigma_c \bar{\pi} D_s$, $\Lambda_c \pi \bar{D}_s$. Note no single $\Sigma_c(\Sigma_c^{*})$ is possible in the final state unless accompanied by a
pion. For the color suppressed transitions we can have final states like \( \Lambda^{(*)} J/\psi(D\bar{D}) \), \( \Sigma(\Sigma^{*})\pi J/\psi(D\bar{D}) \). Again a single \( \Sigma(\Sigma^{*}) \) final state is not allowed unless accompanied by a pion. Also most of the time the \( \Sigma\pi \) system would be the decay product of an excited \( \Lambda \) and therefore would have an invariant mass of an excited \( \Lambda \).

In \( B \) decays the same Hamiltonian would generate \( D(D^{*}\bar{D}) \) final states in color allowed transitions and and as in the semi-leptonic case when we sum over all \( |X'\rangle \) states there will be an enhancement in the \( \Lambda_b \) width relative to the \( B \) width.

- The Cabibbo suppressed \( b \to cd\bar{d} \) color allowed transitions would give rise to the following possible final states \( \Lambda_c D^{(*)}, \Sigma_c \pi D^{(*)}, \Lambda_c \pi\pi D^{(*)} \). Color suppressed transitions would have states like \( N^{(*)} J/\psi(D\bar{D}), \Delta\pi J/\psi(D\bar{D}) \). A single \( \Delta \) in the final state is disallowed unless accompanied by a pion and in most cases the \( \Delta - \pi \) invariant mass would correspond to an excited nucleon.

- Cabibbo allowed \( b \to cd\bar{d} \) can lead to final states as \( \Lambda_c^{(*)}\pi(\rho), \Sigma_c\pi\pi(\rho), \Lambda_c\pi\pi\pi(\rho) \). Color suppressed decays will have final states as \( N^{(*)}D^0(D^{0*}), \Delta\pi D^0(D^{0*}) \).

- Cabibbo suppressed \( b \to c\bar{u}s \) can lead to final states as \( \Lambda_c^{(*)}K(K^*), \Sigma_c\pi K(K^*), \Lambda_c\pi\pi K(K^*) \). Color suppressed decays will have final states as \( \Lambda^{(*)}D^0(D^{0*}), \Delta\pi D^0(D^{0*}) \).

### 4 \( \Lambda_b \) Lifetime

Lifetimes of the \( \Lambda_b \) and \( B \) are calculated using the operator product expansion (O.P.E) to write the square of the decay amplitude as a series of local operators \([12]\). The expression for the lifetime can be arranged as an expansion in \( 1/m_b \). The inclusive rate calculated in this manner is expected to equal the inclusive rate by summing up individual exclusive modes by assumption of duality. The validity of duality has not been proved but it can be shown in a certain kinematic limit, the Shifman-Voloshin limit, defined by \( m_b, m_c >> (m_b - m_c) >> \Lambda_{QCD} \) that the inclusive rate calculated by the method of OPE gives the same result as summing up the exclusive modes which are saturated by \( B \to D + D^* \) in \( B \) decays and \( \Lambda_b \to \Lambda_c \) in \( \Lambda_b \) decays \([18, 19]\).

As mentioned in the introduction, in the leading order, the lifetimes of \( \Lambda_b \) and \( B \) are expected to be same if the OPE method is used in calculating the lifetimes. Spectator effects that distinguish between \( \Lambda_b \) and \( B \) only arise at order \( 1/m_b^3 \) and are not enough to
explain the observed $\Lambda_b$, $B$ lifetime ratio. In our analysis of exclusive semi-leptonic decays we found that $\Lambda_b$ goes dominantly to $\Lambda_c$ while $B$ goes to $D$ and $D^*$. The result is a phase space advantage in the baryon transition over the meson transition leading to an enhanced $\Lambda_b$ lifetime relative to the $B$ lifetime. We can calculate the inclusive rates by summing up the exclusive modes. For a quantitative estimate we use the following toy model for semileptonic decays: We assume that the $\Lambda_b$ goes only to $\Lambda_c$, that the $B$ goes to a statistical mixture $(3/4) \ D^*$ and $(1/4) \ D$ and that all transitions to higher states are small. In the SV limit, in the leading order, for semi-leptonic transition $H_1 \rightarrow H_2 l\bar{\nu}$ the decay rates go as $(H_1 - H_2)^5$. In our toy model we will assume that this behavior of the decay rate persists away from the SV limit also. Therefore the phase space for the $\Lambda_b$ decay is then given by the mass difference $\Lambda_b - \Lambda_c$ to the fifth power. The phase space for the $B$ decay is then given by the $B - D^*$ mass difference to the fifth power, weighted by a statistical factor of $(3/4)$ plus the $B - D$ mass difference to the fifth power, weighted by a statistical factor of $(1/4)$. It is interesting to note that in this toy model $\Gamma(B \rightarrow Dl\bar{\nu})/\Gamma(B \rightarrow D^*l\bar{\nu}) = 0.41$ which is very close to the experimental number 0.42 [7].

Including small corrections from neglected transitions we can write

$$
\Gamma_{SL}(\Lambda_b) = A(\Lambda_b - \Lambda_c)^5(1 + x_1) \tag{5}
$$

$$
\Gamma_{SL}(B) = A\left(\frac{1}{4}(B - D)^5 + \frac{3}{4}(B - D^*)^5\right)(1 + x_2) \tag{6}
$$

where $A$ is a constant involving the Fermi constant $G_F$ and $x_{1,2}$ are small corrections from neglected transitions. This well-defined model for semi-leptonic decays may be right or wrong, but its predictions are easily calculated and the basic assumptions can be easily tested when exclusive branching ratios into baryon final states including spin-excited baryons become available. We immediately obtain the following result for the ratio of semi-leptonic partial widths for $x_1 \approx x_2$:

$$
\frac{\Gamma(\Lambda_b)}{\Gamma(B)} = 1.07 \tag{7}
$$

In a toy model including only semi-leptonic modes this would give the ratio of the lifetimes

$$
\frac{\tau(\Lambda_b)}{\tau(B)} = 0.934 \tag{8}
$$

This shows a clear prediction of a significant enhancement of the $\Lambda_b$ partial semi-leptonic width in comparison with the $B$. The $\Lambda_b$ decay rate is enhanced by about 7%.

In the HQET picture of the hadrons, the heavy quark inside the hadron interacts with a complicated “brown muck”. In the case of the meson there is only a single “brown muck”
in a isospin $I = 1/2$ state while in the baryon the “brown muck” can be in a isospin $I = 0$
or $I = 1$ state corresponding to the $\Lambda$ and the $\Sigma$ baryon. In contrast to the meson hyperfine
mass splittings, which are always between states of the same isospin and decrease to zero
with the heavy quark mass, the $\Lambda - \Sigma$ splittings are between states of different isospins and
therefore separated from one another by isospin selection rules. Furthermore, they do not
decrease with heavy quark mass, but actually increase, and are expected in simple models
to approach a finite asymptotic value of 200 MeV with infinite heavy quark mass [20]. In
the standard HQET expansion this spin-isospin splitting is neglected and as we have shown
above, the effect of the spin-isospin splitting on phase space can be appreciable.

One can also find evidence of a similar enhancement in non-leptonic decays. Consider for
instance the quark transition $b \to c\bar{d}$. Considering only color allowed transitions we found
that for $\Lambda_b$ decays the final states are of the form $\Lambda_c^{(*)}X$ where $X = \pi \rho a_1 \eta \pi \ldots$. In the case
of $B$ decays the final states are dominantly $D(D^*)X$. If we now sum over the states $X$ then
in the leading order we have the effective transitions

$$\Lambda_b \to \Lambda_c^{(*)}\pi d$$

$$B \to D(D^*)\pi d$$

Here we have used the idea of duality in summing over the $X$ states. This maybe
reasonable because there are many hadronic channels and so summing over all the final
states will eliminate the bound state effects of the individual final states. We can then apply
the toy model for semi-leptonic decays considered above and we see that there is a phase
space enhancements for $\Lambda_b$ decays relative to $B$ decays. A similar treatment can also be
applied for other color allowed non-leptonic transitions taking proper care of the phase space
factors. For instance in $\Lambda_b \to \Lambda_c\pi d$ the invariant mass $M_X$ varies from $(\Lambda_b - \Lambda_c)$ to $(m_u + m_d)$
and for $\Lambda_b \to \Lambda_c\pi s$ the invariant mass $M_X$ varies from $(\Lambda_b - \Lambda_c)$ to $(m_c + m_s)$.

Note that in the traditional approach to calculating lifetimes using duality the isospin
selection rules are not taken into account. For instance the transitions due to $b \to c\bar{c}s$ are
$\Delta I = 0$ and so the final states in $\Lambda_b$ decays are rigorously required to be in an isoscalar
state while in $B$ decays only $I = 1/2$ states are allowed. Using duality, in the leading order,
both the $\Lambda_b$ and $B$ decays would be represented by the parton level process $b \to c\bar{c}s$. The
 dynamics of the two light quarks in the baryon and consequently the fine details of the
hadron spectrum is ignored. While it is conceivable that the arguments supporting quark-
hadron duality which neglect the fine details of the hadron spectrum maybe valid for mesons
it is likely to break down for baryons where there are two valence light quarks undergoing
very complicated non-perturbative QCD interactions. We note that the use of duality even in the case of the $B$ meson has been questioned recently [21].

As a concrete example consider the color suppressed $b \to c\pi s$ transition leading to the processes $B \to J/\psi X$ and $\Lambda_b \to J/\psi X$. Possible final states, as already mentioned before, for the $\Lambda_b$ decay are $J/\psi \Lambda^{(*)}$. In $B$ decays the corresponding final states are in $I = 1/2$ states and some possible final states are $J/\psi K^{(*)}, K\pi\pi$ [7]. If we used duality to sum over the $X$ states then in the leading order both $\Lambda_b$ and $B$ decaying to final state $J/\psi X$ could be represented by $b \to sJ/\psi$ and so the rates for both processes would be same. On the other hand isospin selects specific $X$ states. From measured rates in Particle Data Group [6] $X = \Lambda$ and $K, K^{*}, K\pi\pi$ for $\Lambda_b$ and $B$ decays. If we add the observed rates we find $\Gamma[B \to J/\psi X] \sim 6\Gamma[\Lambda_b \to J/\psi X]$. This appears to indicates a breakdown of duality unless there is also significant transition of the $\Lambda_b$ to excited $\Lambda$ which could show up as a $\Sigma\pi$ state.

Phase space enhancements in $\Lambda_b$ decays over $B$ decays can be understood in terms of hyperfine interactions [13, 16, 17]. In $B \to D^{*}$ decays there is a phase space disadvantage over $B \to D$ transition because of the higher $D^{*}$ mass but there is a spin phase space advantage by a factor of three for the $D^{*}$ in final state over the $D$ in the final state. So the hyperfine energy is averaged out in the $B \to D + D^{*}$ transition. In $\Lambda_b$ decays isospin conservation chooses final states with the lowest hyperfine energy. This added hyperfine energy is available for transition and leads to an phase space enhancement in $\Lambda_b$ decays over $B$ decays (A different argument [22] using the scaling of lifetimes as the inverse fifth power of hadronic rather than quark masses implicitly gives a larger phase space also). Phase space effects were also discussed in a different approach in Ref[23]. The lesson from our analysis is that the effect of phase space enhancements may be a key factor in understanding the lifetime difference between $\Lambda_b$ and $B$ hadron.

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