SWIPT using Hybrid ARQ over Time Varying Channels

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Abstract

In this work, we consider a class of wireless powered devices employing Hybrid Automatic Repeat reQuest (HARQ) to ensure reliable end-to-end communications over a time-varying channel. By using Simultaneous Wireless Information and Power Transfer (SWIPT), the perpetually powered transmitter transmits information bearing RF signal to the receiver. The receiver has no power source, and relies on the energy harvested from the received RF blocks from the transmitter. At each incoming RF signal block, the receiver splits the incoming RF signal between the energy harvester and information decoder so that the message is decoded with the least number of re-transmissions. We cast the problem in a Markovian framework, and we show that the optimal policy minimizing the expected number of re-transmissions utilizes the incoming RF signal to either exclusively harvest energy or to accumulate mutual information. By doing so, we reduce the problem which is a two dimensional uncountable state Markov chain (MC) into a two dimensional countable state MC and thus, enable a tractable solution for the optimal policy. For independent and identically distributed channels, we also prove that practically simple-to-implement policies such as harvest-first-store-later type of policies are optimal. However, for the case of time-correlated channels, we demonstrate that the statistical knowledge of the channel state may significantly improve the performance when compared to those of the simple-to-implement policies.

I. INTRODUCTION

In traditional networks, wireless nodes are powered by limited capacity batteries which should be regularly charged or replaced. Energy harvesting has been recognized as a promising solution.

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to replenish batteries without using any physical connections for charging. In simultaneous wireless information and power transfer (SWIPT), the incoming RF signal is used for both energy harvesting and decoding of information bits. The inherent challenge of energy harvesting (EH) is the stochastic nature of the EH process, which dictates the amount and availability of harvested energy that is beyond the control of system designers. However, recent advances in wireless power transfer via RF signal has the potential to replenish the energy of the remote devices and provide control to the network administrators.

SWIPT is a concept first introduced by Varshney in [1], where the same information bearing signal is also used for power transfer. In [1], the rates at which energy and reliable information can be transferred over a single point-to-point noisy link was characterized. This result was later extended to frequency-selective channels with additive white Gaussian noise (AWGN) in [2]. Note that with the technology available today, it is not possible to use the same wireless signal for both harvesting energy as well as carrying information due to practical circuit limitations [3]. In [4], the authors examined separated and co-located information and energy receiver architectures in a multiple-input multiple-output (MIMO) wireless broadcast system. In a separated architecture, both receivers have separate antennas. Due to space limitations, an EH device may not accommodate two separate antennas, which dictates the sharing of the same antenna by the two receivers. In the more practical co-located receiver architecture, the incoming RF signal needs to be shared by the two receivers which arises a resource allocation problem. In the literature, two main classes of policies are investigated to share the RF signal among the EH and information decoding receivers. In the so called time-switching (TS) policy, the incoming RF signal is entirely utilized either for energy harvesting or for information decoding purposes. To achieve this, a time switcher feeds the received signal from the antenna either to the information decoder or to the energy harvesting circuit. Meanwhile, in a so called power splitting (PS) policy the incoming signal is first split into two streams by a power splitter, in which the incoming RF power is proportionally utilized by both the energy harvester and the information decoder. In [5], the authors proposed integrated and separated information and energy receivers. In [6], the optimal power splitting policy at the receiver was characterized to balance various trade-offs between the maximum ergodic capacity and the maximum average harvested energy in a single-input-single-output system. The optimal TS policy for an EH receiver is characterized for a point-to-point link over a narrow band flat-fading channel in [7].

In inherently error-prone wireless communications systems, re-transmissions triggered by
decoding errors, have a major impact on the energy consumption of wireless devices. Hybrid automatic repeat request (HARQ) schemes are frequently used in order to reduce the number of re-transmissions by employing various channel coding techniques [8]. Nevertheless, this comes at the expense of extra processing time and energy associated with the enhanced error-correction decoders. A receiver employing HARQ encounters two major energy consuming operations: (1) sampling or Analog-to-Digital Conversion (ADC), which includes all RF front-end processing, and (2) decoding. The energy consumption attributed to sampling, quantization and decoding plays a critical role in energy-constrained networks which makes their study a non-trivial problem. The work in [9] investigated the performance of HARQ over an RF-energy harvesting point-to-point link, where the power transfer occurs over the downlink and the information transfer over the uplink. The authors studied the use of a TS policy when two HARQ mechanisms are used for information transfer; Simple HARQ (SH) and HARQ with Chase Combining (CC) [10]. More recently, [11] studied the performance of HARQ in RF energy harvesting receivers, where heuristic TS policies are proposed to reduce the number of re-transmissions.

In this work, we consider a point-to-point link where a transmitter employs HARQ to deliver a message reliably to the receiver. The receiver has no energy source, and thus, it relies on harvesting the RF energy from the same signal bearing information. The channel is time-varying where the amount of the energy harvested and information collected varies depending on the quality of the channel. The receiver aims to split the incoming RF signal between energy and information receivers so that the expected number of re-transmissions is minimized. In our work, we do not assume the availability of the channel state information (CSI) at the receiver, which was the case in [12]. Due to the time and energy cost, the acquisition of CSI in EH networks is challenging. Some interesting ideas along this line, such as limited CSI feedback, have been discussed in [13]. Also, unlike [11], we aim to determine optimal policies achieving the minimum number of re-transmissions. Our main contributions in this paper are summarized as follows:

- We develop a Markovian framework to calculate and minimize the expected number of re-transmissions starting from any initial state for independent and identically distributed (i.i.d.), and for time-correlated channels, respectively.
- For i.i.d. channels, we prove that there is an optimal TS policy. We also show that the optimal policy is not unique, but there is a class of simple-to-implement policies with the same performance. For example, a harvest-first-store-later type of policy is an optimal
policy, which lends itself for a rather simple implementation.

- For a time-correlated channel, we once again show that there is an optimal TS policy. We develop a low complexity algorithm to determine the TS decision for each state of the receiver. Note that unlike the i.i.d. case, a simple policy such as harvest-first-store-later is no longer optimal for correlated channels as demonstrated by our numerical analysis.

- We utilize the TS structure of the optimal policy to reduce the problem from a two dimensional *uncountable* state MC into a two dimensional *countable* state MC.

- We provide extensive numerical simulations to verify the analytical results established in the paper.

### II. Related Works

A recent work in [14] demonstrated energy transfer over conventional IEEE 802.11 wireless networks to power low-power sensor and devices using the off the shelf hardwares without significantly comprising the network performance. Early works on wireless energy transfer [15] considered a point-to-point single antenna communication system and studied its rate-energy trade-off. Single antenna systems are extended to single-input-multiple-output (SIMO) in [6], multiple-input-single-output (MISO) in [16] and multiple-input-multiple-output (MIMO) system in [17].

Note that in an energy-constrained network, EH devices harvest energy only in minuscule amounts (orders of $\mu W$s), and thus, the energy consumption of receiver circuitry to perform simple sampling and decoding can no longer be neglected. The authors in [20] addressed the energy consumption of sampling and decoding operations over a point-to-point link where the receiver harvests energy at a constant rate. In [21, 22], a decision-theoretic approach is developed to optimally manage the transmit energy of an EH transmitter transmitting to an EH receiver, where both the transmitter and the receiver harvests energy independently from a Bernoulli energy source. The receiver uses selective sampling (SS) and informs the transmitter about the SS information and its delayed battery state by feedback. Based on this feedback, the transmitter adjusts its transmission policy to minimize the packet error probability.

Meanwhile, in [23], the performance of different HARQ schemes for an EH receiver harvesting energy from a deterministic energy source with a constant energy rate was studied. In [24], the impact of the battery’s internal resistance at the receiver was analyzed for an EH receiver with imperfect battery, with the aim of maximizing the amount of information decoded by
the EH receiver. While ignoring the sampling energy cost at the receiver, [25] investigates the performance of TS policies to maximize the amount of information decoded at the receiver operating over a binary symmetric channel (BSC), by optimizing the fraction of time used for harvesting energy and for extracting information. For an EH transmitter and an EH receiver pair both harvesting ambient environmental energy with possible spatial correlation, [26] addresses the problem of outage minimization over a fading wireless channel with ACK-based re-transmission scheme by optimizing the power allocation at the transmitter. In [27], an EH transmitter regulates the transmitted number of bits to minimize the loss of bits due to energy starvation at the receiver. In [28], an offline algorithm that knows of energy harvested in the future slots was investigated for maximizing the throughput with decoding costs for three different network settings, i.e., single EH transmitter-receiver pair; two EH transmitters and an EH receiver; and an EH transmitter, an EH relay, and an EH receiver. A two-way energy transfer scheme between an EH transmitter and an EH receiver harvesting energy from a common energy source was studied in [29] to, first, maximize the throughput over an AWGN channel model with deterministic energy source and, second, to minimize the outage probability over a fading channel with a stochastic energy source. For an EH transmitter and an EH receiver, the optimal power allocation for the transmitter, and scheduling for the receiver are studied and asymptotically near optimal distributed policies, maximizing the long-term average Gaussian channel rate, were developed in [30]. The problem of throughput optimization for an EH receiver operating in a multi-access network was studied in [31] where the receiver takes samples from the incoming RF signal to calculate the probability of a collision event and based on that decides to either utilize the incoming RF energy to replenish its battery or to extract information bits.

Differently from the available literature, we study the reliability of transmission by an HARQ mechanism in a SWIPT scenario, over time varying channels with unknown CSI and by considering an accurate model of energy consumption of the EH receiver. We develop a novel Markovian framework for the analysis which facilitates characterizing the optimal decision at any given time. A major contribution of this work is that we prove by beginning from a general setting that there exists an optimal TS policy that minimizes the number of re-transmissions. This finding enables a tractable optimal solution by reducing a two dimensional uncountable state MC into a countable state MC. In particular, for i.i.d. channels, we show that policies such as harvest-first-store-later are optimal enabling simple to implement optimal policies suitable for low power EH devices. However, for the case of correlated channels, we show that an intelligent algorithm
that utilizes the correlation information of the channel states, can significantly outperform those simple-to-implement policies.

III. SYSTEM MODEL AND PRELIMINARIES

A. Channel Model and Receiver Architecture

Consider a point-to-point time varying wireless link between a transmitter-receiver pair. The wireless channel is modeled according to a two-state block fading model where the states are GOOD and BAD. Note that the two-state channel process is an approximation of a more general multi-state time varying channel, where each state of the channel supports a maximum transmission rate. Here, we employ two-state channel process due to its analytical tractability. Let \( G_t \in \{0, 1\} \) be the state of the channel at time slot \( t \) where a BAD state is denoted by 0 and a GOOD state is denoted by 1. Let \( g_i \) be the instantaneous complex channel gain for \( i = 0, 1 \). The channel state information (CSI) is neither available at the transmitter nor at the receiver due to high computational and energy costs of transmitting and receiving a pilot signal necessary for measuring the CSI.

We consider a communication scheme where the transmitter is connected to a power source with an unlimited energy supply. The receiver is equipped with separate rectifier circuit for energy harvesting and a transceiver for information decoding (ID), both connected to the same antenna as in Figure 1. A single antenna enables a compact EH device where EH and ID circuits observe the same channel. The incoming RF signal is fed to EH and ID circuits according to a time switching (TS) or a power splitting (PS) policy. Figure 1a illustrates a TS architecture where the incoming RF signal is either fed into the EH or ID, whereas in Figure 1b, we demonstrate a PS architecture in which the received RF signal is split into two streams with varying signal powers and fed into both the EH and ID circuits.
Time is slotted and each slot has a length of $N$ channel uses. We assume that $N$ is sufficiently large so that we can apply information theoretic arguments. The instantaneous achievable rate of the receiver is the maximum achievable mutual information between the output symbols of the transmitter and input symbols at the receiver. Let us denote the achievable rate of the receiver by $R(t)$ at time $t$. As $N \to \infty$, $R(t)$ approaches the Shannon rate, and it can be computed as:

$$R(t) = \log(1 + P|g(t)|^2),$$

(1)

where $g(t) \in \{g_0, g_1\}$ is the channel gain at time $t$ and $P$ is the noise-normalized transmit power of the transmitter. We assume that the transmitter power is fixed and known to the receiver. Let $R_1$ and $R_2$ be the achievable rates corresponding to channel states GOOD and BAD, respectively. In particular,

$$R_1 = \log(1 + P|g_1|^2),$$

(2)

$$R_2 = \log(1 + P|g_0|^2).$$

(3)

The instantaneous channel states are not known a priori so we employ an HARQ scheme with incremental redundancy (IR) for providing reliability [32]. In the following, we give a brief overview of HARQ-IR.

B. A Brief Overview of HARQ

HARQ is a well known method to provide reliable point to point communications [32]. There are several types of HARQ implementations, e.g., simple HARQ, HARQ with Chase Combining (CC), repetition time diversity and incremental redundancy (IR). In our system, we consider HARQ-IR due to its superior throughput performance compared to other alternatives as well as its robustness against the absence of CSI [33]. Note that since the instantaneous CSI is not available, the transmitter cannot adapt the code rates according to a particular channel gain. Let us denote a message of the transmitter by $W \in \{1, 2, \ldots, 2^{NC}\}$ where, $C$ denotes the rate of the information. Every incoming transport layer message into the transmitter is encoded by using a mother code of length $MN$ channel uses. The encoded message, $x$, is divided into $M$ blocks, each of length $N$ channel uses, with a variable redundancy and it is represented by $x = [x^1, \ldots, x^M]$. Let us assume that $x^1$ is transmitted at $t_1$. If $x^1$ is successfully decoded, then the receiver sends a 1-bit, error-free, zero-delay, Acknowledgement (ACK) message, otherwise, the transmitter times out after waiting a certain time period. In case of no ACK received, the transmitter transmits
At time slot $t_2$ and the receiver combines the previous block $x^1$ with $x^2$. This procedure is repeated until the receiver accumulates $C$ bits of mutual information or maximum blocks of information, $M$, is sent. We assume that, $M$ is chosen sufficiently large so that the probability of decoding failure, due to exceeding the maximum number of re-transmissions, is approximately equal to zero. With HARQ-IR scheme, after $r$ re-transmissions, the amount of accumulated mutual information at the receiver is $\sum_{k=1}^r R(t_k)$. The receiver, given that it has sufficient energy, can perform a successful decoding attempt after $r$ re-transmissions, if the amount of accumulated mutual information exceeds the information rate of the transmitted message, i.e., $\sum_{k=1}^r R(t_k) \geq C$. We assume that each message is encoded at rate $R_1$, i.e., $C = R_1$ so that a transmission in a GOOD channel state carries all the information needed for decoding.

C. Energy Harvesting and Consumption Model

In the following, we assume that the receiver has a sufficiently large battery and memory, so there is no energy or information overflow. The receiver utilizes a power splitting policy, where $\rho(t) \in [0, 1]$ denotes the power splitting parameter at the beginning of time slot $t$. Note that $\rho(t) = 0$ indicates that the received signal is used solely for mutual information accumulation, and $\rho(t) = 1$ indicates that the received signal is used solely for harvesting energy. Any value of $\rho(t)$ between 0 and 1 refers to the case where the received signal is used for both harvesting energy and mutual information accumulation. Note that TS is a special case of PS with $\rho(t) \in \{0, 1\}$.

We incorporate a simplified energy harvesting model, which facilitates the formulation of a tractable optimization problem. In this model, the receiver harvests a maximum of $e \geq 1$ energy units in the GOOD channel state and zero units during the BAD channel state.\footnote{The maximum energy is harvested if the received signal is completely directed to the energy harvester, i.e., $\rho(t) = 1$.} The reason that no energy can be harvested during a BAD channel state is because in a typical EH device there are two stages\footnote{Note that this assumption is practically reasonable, since a time slot is typically defined as the duration of time necessary for transmission of a single information packet.}: a rectifier stage that converts the incoming alternating current (AC) radio signals into direct current (DC); and a DC-DC converter that boosts the converted DC signal to a higher DC voltage value. The main limitation in an energy harvester is that every DC-DC converter has a minimum input voltage threshold below which it cannot operate. Hence, when the channel is in a BAD state, the input voltage is below the threshold of the DC-DC converter.
so no energy can be harvested. Even though the receiver cannot harvest any RF energy in a BAD channel state, it can still accumulate mutual information since ID circuit operates at a lower power sensitivity, e.g., $-10$ dBm for EH and $-60$ dBm for ID circuits [34].

The energy consumption of HARQ was recently investigated in [35]. The energy is consumed at the start up of the receiver, during decoding, for operating passband receiver elements (low-noise amplifiers, mixers, filters, frequency synthesizers, etc.), and for providing feedback to the transmitter. In order to develop a tractable analytical optimization model, we combine the individual costs of energy into two parameters only: the receiver consumes $E_d \geq 1$ energy units for a decoding attempt and 1-energy unit for each mutual information accumulation event per time slot, i.e., operating the passband receiver elements.

IV. THE MINIMUM EXPECTED NUMBER OF RE-TRANSMISSIONS FOR I.I.D. CHANNELS

In this section, we calculate the minimum expected number of re-transmissions needed for successful decoding for time varying channels. We first consider an i.i.d. channel, and in Section VI, we will investigate the system under a time correlated channel model. Note that the receiver requires at least $E_d$ units of energy and $R_1$ bits of information before it can successfully decode the transmitted packet. Our objective is to optimally determine a scheduling policy $\rho(t)$ so that the transmission is successfully decoded with a minimum delay at the receiver. We formally define $\rho(t)$ next.

Definition. A scheduling policy $\pi = (\rho(1),\rho(2),\ldots,)$ is a sequence of decision rules as such the kth element of $\pi$ determines the power splitting ratio at kth time slot based on the observed system state $(b, m)$ at the beginning of this time-slot for $t \in \{1,2,\ldots\}$. Similarly, a tail scheduling policy $\pi_t = (\rho(t),\rho(t+1),\ldots)$ is a sequence of decision rules that determines the power splitting ratios for the time slots from $t$ to $\infty$.

The probability that the channel is in GOOD state is $\lambda$, i.e., $P[G_t = 1] = \lambda$. The problem can be mathematically modeled as a two-state Markov chain (MC). Let the states of the MC be $(b, m)$, where $b$ is the total residual battery level and $m$ is the total accumulated mutual information normalized by $R_2$. For clarity of presentation, in the rest of the paper we assume that $R_2 = 1$. Note that the decision of the scheduling policy is blind to the channel state, but it depends on $(b, m)$.

\footnote{One energy unit is normalized to the energy cost of operating the RF transceiver circuit during one time slot.}
A. Markov Decision Process (MDP) Formulation

At any given time $t$, the next state of the system only depends on the current state, $(b, m)$, and the power split ratio $\rho(t)$. Hence, we can utilize the theory of the MDP to formulate the problem. Let $f^\pi(t) \in \{0, 1\}$ be an indicator function taking a value of 0 if the message can be decoded at the end of slot $t$ under policy $\pi$, and a value of 1 otherwise. Then, the optimization problem we aim to solve is given as,

$$\min_{\pi} \sum_{t=0}^{\infty} f^\pi(t).$$

(4)

Let $V^\pi(b, m)$ be the expected discounted reward with initial state $S_0 = (0, 0)$ under policy $\pi$ with discount factor $\beta \in [0, 1)$. The expected discounted reward has the following expression

$$V^\pi(b, m) = \mathbb{E}^\pi \left[ \sum_{t=0}^{\infty} \beta^t U(S_t, \rho(t)) | S_0 = (b, m) \right],$$

(5)

where $\mathbb{E}^\pi$ is the expectation with respect to the policy $\pi$, $t$ is the time index, $\rho(t) \in [0, 1]$ is the action chosen at time $t$, and $U(S_t, \rho(t))$ is the instantaneous reward acquired when the current state is $S_t$.

In the rest of the paper, we use $\rho(t)$ and $\rho(b, m)$ interchangeably by assuming that at time slot $t$, the system is at state $(b, m)$. The battery is recharged with incoming RF signal depending on the value of the power split ratio $\rho(t)$. Meanwhile, one unit of energy is consumed in order to accumulate non-zero bits of mutual information. Hence, the evolution of the battery state is characterized as follows:

$$B(t) = \begin{cases} B(t-1) + \rho(t)e - 1_{\rho(t) \neq 1}, & \text{if } G_t = 1 \\ B(t-1) - 1_{\rho(t) \neq 1}, & \text{if } G_t = 0 \end{cases},$$

(6)

where $1_{\rho(t) \neq 1} = 0$, if $\rho(t) = 1$, and $1_{\rho(t) \neq 1} = 1$, otherwise.

According to (2) and (3), the transmit power is equal to $P = \frac{2^{\frac{K}{|g_1|^2}}}{|g_1|^2} = \frac{2^{\frac{K}{|g_0|^2}}}{|g_0|^2}$. At the power splitter, $1 - \rho(t)$ portion of the received power is directed into the ID, so the maximum achievable mutual information accumulation is:

$$R(t) = \log(1 + g(t)P(1 - \rho(t)))$$

(7)
Inserting the value of $P$ in (7) for GOOD and BAD channel states gives the mutual information accumulation in these states respectively for a given power splitting ratio $\rho$ as

$$R^H(\rho) = \log(\rho + (1 - \rho)2^{R_1}),$$

(8)

$$R^L(\rho) = \log(\rho + (1 - \rho)2^{R_2}).$$

(9)

Thus, the accumulated mutual information, $I(t)$, evolves as:

$$I(t) = \begin{cases} 
\min(I(t-1) + R^H(\rho(t)), R_1), & \text{if } G_t = 1 \\
\min(I(t-1) + R^L(\rho(t)), R_1), & \text{if } G_t = 0
\end{cases}.$$  

(10)

The instantaneous reward is zero if the message can be correctly decoded, and it is minus one otherwise. Note that the decoding operation is successful if and only if the accumulated mutual information is above a certain threshold, and the battery level is sufficient to decode the message. Hence, the instantaneous reward is given as follows:

$$U(S_t, \rho(t)) = \begin{cases} 
0, & \text{if } B_t \geq E_d, \text{ and } I(t) = R_1 \\
-1, & \text{otherwise}
\end{cases}.$$  

(11)

Define the value function $V(b, m)$ as

$$V(b, m) = \max_\pi V^\pi(b, m), \forall b \in [0, \infty), \forall m \in [0, R_1].$$  

(12)

The value function $V(b, m)$ satisfies the Bellman equation

$$V(b, m) = \max_{0 \leq \rho \leq 1} V_\rho(b, m),$$  

(13)

where $V_\rho(b, m)$ is the cost incurred by taking action $\rho$ when the state is $(b, m)$ and is given by

$$V_\rho(b, m) = U((b, m), \rho) + \beta \mathbb{E} \left[ V(\hat{b}, \hat{m}) | S_0 = (0, 0) \right],$$  

(14)

where $(\hat{b}, \hat{m})$ is the next visited state and the expectation is over the distribution of the next state. The use of expected discounted reward allows us to obtain a tractable solution, and one can gain insights into the optimal policy when $\beta$ is close to 1. Value iteration algorithm (VIA) is a standard tool to solve the Bellman equations in (13). However, this problem suffers from the curse of dimensionality as it can be seen from (6) and (10) that the problem is a two dimensional uncountable state MDP with continuous actions at every state. Also, letting $\beta \to 1$, to approximate the average reward, extremely slows the algorithm to the point of infeasibility [36]. Hence, due to the extreme complexity of the numerical methods, in the following, we opt for analytical methods and propose a novel approach to gain insights into the structure of the optimal policy.
B. Absorbing Markov Chain Formulation

The MC describing the operation of our system is an absorbing MC, where all states except those \((b,m)\) where \(b \geq E_d\), and \(m \geq R_1\) are transient states. The absorbing states are those where the receiver has both sufficient energy and information accumulated to correctly decode. In an absorbing MC, the expected number of steps taken before being absorbed in an absorbing state characterizes the mean time to absorption. Hence, the mean time to absorption starting from a given transient state \((b, m)\) provides the number of re-transmissions until successful decoding when the battery has \(b\) units of energy and the memory contains \(m\) bits of information. It should be noted that the receiver is blind to the CSI before choosing the power splitting ratio. However, after it decides to sample the incoming RF signal for mutual information accumulation, the amount of the information in the sampled portion of the RF signal is revealed to the receiver.

In a finite absorbing chain, starting from a transient state, the chain makes a finite number of visits to some transient states before its eventual absorption into one of the absorbing states. Hence the mean time to absorption of the chain, starting from transient state \(i\) initially, is the sum of the expected numbers of visits made to transient states. In the following, we perform first-step analysis, by conditioning on the first step the chain makes after moving away from a given initial state to obtain the mean time to absorption. Let \(k_{b,m}\) be the expected number of transitions needed to hit an absorbing state when the MC starts from state \((b, m)\).

Let us first consider the trivial case when the battery has less than one unit of energy, i.e., \(b < 1\), in which case the receiver has no option but harvest incoming RF signal. In this case, the mean time to absorption starting from an initial state \((b, m)\) is

\[
k_{b,m} = 1 + \lambda k_{b+e,m} + (1 - \lambda)k_{b,m} = \frac{1}{\lambda} + k_{b+e,m}, \quad \text{if } b < 1. \tag{15}
\]

Note that in (15), one slot is needed to harvest energy, and depending on the channel state in that slot, the battery state either transitions to \(b + e\) or remains the same. Similarly, if the amount of accumulated mutual information is \(R_1\), there is no point in further accumulating mutual information since the receiver has sufficient mutual information to decode the incoming packet. Hence,

\[
k_{b,m} = 1 + \lambda k_{b+e,m} + (1 - \lambda)k_{b,m} = \frac{1}{\lambda} + k_{b+e,m}, \quad \text{if } m = R_1. \tag{16}
\]
The following lemma plays an important role in establishing the structure of the optimal policy.

**Lemma 1.** For any $E_d - i \cdot e \leq b < E_d - (i - 1) \cdot e$ such that $i = 1, \ldots, E_d$, given that $m = R_1$, the mean time to absorption is given by, $k_{b,R_1} = \frac{i}{\lambda}$.

**Proof.** The proof is by induction.

1) Base case: Let us consider the smallest possible value for $i$, i.e., $i = 1$, such that $E_d - e \leq b < E_d$. Note that since $m = R_1$, the optimal decision is to use incoming RF signal only for harvesting energy, i.e., $\rho^*(b, R_1) = 1$. Thus, we get

$$k_{b,R_1} = 1 + \lambda k_{b+e,R_1} + (1 - \lambda) k_{b,R_1}.$$  \hspace{1cm} (17)

For $E_d - e \leq b < E_d$, if the channel is GOOD then the MC transitions into state $(b + e, R_1)$, which is an absorbing state, so $k_{b+e,R_1} = 0$. Hence, $k_{b,R_1} = \frac{1}{\lambda}$ and thus, the lemma holds for $i = 1$.

2) Induction step: assume that the lemma is true for some $i = n$, i.e., $k_{b,R_1} = \frac{n}{\lambda}$ for $E_d - n \cdot e \leq b < E_d - (n - 1) \cdot e$.

3) Proof for case $i = n + 1$: Let us calculate the mean time to absorption for the case $n + 1$:

$$k_{b,R_1} = 1 + \lambda k_{b+e,R_1} + (1 - \lambda) k_{b,R_1}, \text{ for } E_d - (n + 1)e \leq b < E_d - nl,$$  \hspace{1cm} (18)

which reduces to $k_{b,R_1} = \frac{n+1}{\lambda}$ for $E_d - (n + 1) \cdot e \leq b < E_d - n \cdot e$.

Thus, the lemma holds by induction.

We will use Lemma 1 to show that the optimal policy minimizing the mean time to absorption does not need to split the incoming RF signal. In order to show this, let us define two tail policies $\pi^i_t = (a_i, \pi_{t+1})$, $i = split, no - split$ taking different actions, $a_i$ in the current slot, but following the same set of actions, $\pi_{t+1}$ afterwards\(^4\). Let policy $\pi^{split}_t = (\rho, \pi_{t+1})$ be a tail policy that always splits the incoming RF energy, i.e., $0 < \rho < 1$, except when $B(t) < 1$ or $I(t) = R_1$, when it only harvests energy. Assume that the state of the system is $(b, m)$ at time slot $t$. Then, the mean time to absorption for tail policy $\pi^{split}_t$ is:

$$k^{\pi^{split}}_{b,m} = 1 + \lambda k_{b-1+\rho e,m+R^l} + (1 - \lambda) k_{b-1,m+R^r},$$  \hspace{1cm} (19)

\(^4\)Note that $(a_i, \pi_{t+1})$ defines a tail policy obtained by concatenating action $a_i$ in the current slot with tail policy $\pi_{t+1}$.
where \( k_{x,y} \) is the mean time to absorption of policy \( \pi_{t+1} \) beginning at state \((x,y)\). Note that with probability \( \lambda \) the channel is in GOOD state, and thus, \( \rho \cdot e \) units of energy is harvested. However, one unit of energy is spent by operating the transceiver to accumulate \( R^H(\rho) \) bits of mutual information. Meanwhile, with probability \( 1 - \lambda \) the channel is in BAD state, and no energy is harvested, but the transceiver still consumes one unit of energy to accumulate \( R^L(\rho) \) bits of mutual information.

Under tail policy \( \pi_t^{no\text{-}split} \) the RF signal is never split at time slot \( t \), but rather, it is completely used for mutual information accumulation except when \( B(t) < 1 \) or \( I(t) = R_1 \) when it harvests energy only. In a similar way as before, we may calculate \( k_{b,m}^{\pi_{no\text{-}split}} \) as follows:

\[
k_{b,m}^{\pi_{no\text{-}split}} = 1 + \lambda k_{b-1,R_1} + (1 - \lambda) k_{b-1,m+1}.
\]

(20)

**Theorem 1.** There exists an optimal time switching (TS) policy minimizing the number of re-transmissions until successful decoding which only harvests energy or accumulates information at every time slot.

**Proof.** Assume that at time slot \( t \) the system is at state \((b,m)\). Consider policy \( \pi^{split} \) which always chooses \( 0 < \rho < 1 \). Hence, it follows that \( R^H(\rho) < R_1 \), \( R^L(\rho) < 1 \) and, from (10), we have \( I(t) \leq R_1 \). Also, it is easy to verify that for any \( b \), we have \( k_{b,m_1} \leq k_{b,m_2} \) whenever \( m_1 \geq m_2 \). Thus, a lower bound on \( k_{b,m}^{\pi^{split}} \) in (19) can be established as,

\[
k_{b,m}^{\pi^{split}} \geq 1 + \lambda k_{b-1,R_1} + (1 - \lambda) k_{b-1,m+1}.
\]

(21)

Furthermore, since \( b - 1 < b - 1 + \rho \cdot e < b - 1 + e \), from Lemma 1 we know that \( k_{b-1+\rho e,R_1} = k_{b-1,R_1} \). Hence, the lower bound in (21) is exactly the same as \( k_{b,m}^{\pi_{no\text{-}split}} \) given in (20), i.e.,

\[
k_{b,m}^{\pi_{no\text{-}split}} \leq k_{b,m}^{\pi^{split}}.
\]

Theorem 1 proves that a time switching (TS) policy can achieve the minimum number of re-transmissions. Hence, in the latter part of the paper, we focus on characterizing the optimal TS policy by determining the optimal switching decision for each state of the MC. Also recall that TS is a special case of a PS policy where the only power splitting ratios a TS policy can choose are \( \rho = 0 \) and \( \rho = 1 \). Therefore, the state space of the discrete MC associated with the optimal TS policy is \( b = 0, 1, \ldots, \infty \), and \( m = 0, 1, \ldots, R_1 \).

---

5 We assume that the energy harvesting circuit is generating energy linearly proportional to the energy of the incoming RF signal.

6 Note that in the original problem the states of the MC is \([0, \infty) \times [0, R_1]\).
Remark. Theorem 1 plays an important role in simplifying the original problem by reducing the two dimensional uncountable state MDP with continuous action space into a two dimensional countable state MDP with binary decision space. This significantly reduces the complexity of numerical methods such as VIA. However, as we shall see in the following, absorbing MC framework even results in less complex algorithm, suitable for EH devices. Hence, in the rest of the paper, we will study the problem in the absorbing MC framework.

Since the class of policies we are interested in does not observe the channel, but make a decision based only on \((b, m)\), the time of the decision is irrelevant. Hence, stochastically time \(t\) and \(t+1\) are identical given \((b, m)\). Therefore, in the rest of the paper we will omit the time index and optimize for any given state \((b, m)\). Define \(\pi^*\) as the optimal policy minimizing the mean time to absorption beginning at any given state \((b, m)\). Let \(k_{b,m}^*\) be the minimum mean time to absorption obtained by policy \(\pi^*\). Define the tail policy \(\pi_i(b, m) = (i, \pi^*(\cdot, \cdot))\) for \(i = 0, 1\) such that it chooses \(\rho = i\) at state \((b, m)\) but follows policy \(\pi^*\) after transitioning into the new state \((\cdot, \cdot)\). Let \(k_{b,m}^i\) be the mean time to absorption of policy \(\pi_i\), \(i = 0, 1\). We can characterize \(k_{b,m}^0\) and \(k_{b,m}^1\) as follows:

\[
k_{b,m}^0 = 1 + \lambda k_{b-1,R_1} + (1 - \lambda)k_{b-1,m+1}
\]

\[
k_{b,m}^1 = 1 + \lambda k_{b+e,m} + (1 - \lambda)k_{b,m}
= \frac{1}{\lambda} + k_{b+e,m}
\]

(22)

(23)

Note that by evaluating and then comparing the values of \(k_{b,m}^0\) and \(k_{b,m}^1\), at all possible states \((b, m)\) for \(b = 0, 1, \ldots, \infty\), and \(m = 0, 1, \ldots, R_1\), one can obtain the optimal policy \(\pi^*\) and its associated \(k_{b,m}^*\).

Theorem 2. For states \((b, m) = (E_d + j, R_1 - j)\) for \(j = 1, 2, \ldots, R_1\), the minimum mean time to absorption, \(k_{b,m}^*\) is given by

\[
k_{E_d + j, R_1 - j} = \frac{j}{(1 - \lambda)^{j-1}}
\]

(24)

Furthermore, \(k_{b,R_1 - j}^* = k_{E_d + j, R_1 - j}^*\) for \(b = E_d + j + 1, E_d + j + 2, \ldots\).

Note that the mean time to absorption calculated in Lemma 2 is the smallest possible value, i.e., \(k_{b,R_1}^* = k_{b,b}^*\) for \(b = 0, 1, \ldots, E_d - 1\).
Proof. The proof is given in Appendix A.

Theorem 2 states that if the receiver has $R_1 - n$ bits of mutual information accumulated and more than $E_d + n$ units of energy in its battery, then it should use the incoming RF signal power for mutual information accumulation only. For any given state $(b, m)$, we exploit Lemma 1 and Theorem 2 to develop Algorithm 1 for calculating the minimum mean time to absorption, $k_{b,m}^\pi^*$, and the optimal TS decision at every state.

The idea of Algorithm 1 is to use Lemma 1 and Theorem 2 as boundary conditions and to recursively calculate the mean time to absorption $k_{b,m}^0$ and $k_{b,m}^1$ starting from $(b, m) = (E_d, R_1 - 1)$. Note that $k_{E_d,R_1-1}^0$ and $k_{E_d,R_1-1}^1$ depend on the values of $k_{E_d-1,R_1}^\pi^*$ and $k_{E_d+1,R_1-1}^\pi^*$, which are obtained in the initialization step, and the optimal TS decision at state $(E_d, R_1 - 1)$ is given by $\arg\min_{i \in \{0, 1\}} k_{b,m}^i$. The procedure in Algorithm 1 continues by decrementing the value of $b$ by 1 at each iteration, until $b = 0$ at which time the value of $m$ is decremented by 1, $b$ is initialized to $E_d + n$ and the procedure is repeated. The aforementioned order to span the states of the MC ensures that at each iteration the mean time to absorption can be calculated from the values determined in the previous iterations.

Algorithm 1 Calculating the minimum mean time to absorption for an i.i.d. channel

1: Initialize $k_{b,R_1}^\pi^*$ for $b = 0, \ldots, E_d - 1$ using Lemma 1
2: Initialize $k_{E_d+j,R_1-j}^\pi^*$ for $j = 1, \ldots, R_1$ using Theorem 2
3: $n \leftarrow 0$
4: for $m = R_1 - 1 : 0$ do
5:     for $b = E_d + n : 0$ do
6:         Calculate $k_{b,m}^0$, $k_{b,m}^1$ from (22) and (23), respectively.
7:         $k_{b,m}^\pi^* = \min\left(k_{b,m}^0, k_{b,m}^1\right)$
8:         $\rho^*(b,m) = \arg\min_{i \in \{0, 1\}} k_{b,m}^i$ for $i = 0, 1$
9:     $n \leftarrow n + 1$

Theorem 3. Algorithm 1 calculates the minimum mean time to absorption starting from an arbitrary state $(b, m)$ for which $b = 0, \ldots, \infty$ and $m = 0, 1, \ldots, R_1$.

Proof. The proof is given in Appendix B.

□
V. Optimal Class of Policies for i.i.d. Channels

In the previous section, we have given a procedure to obtain the optimal scheduling decision of a TS policy, once we established that there exists a TS policy achieving the minimum number of re-transmissions. In this section, we formally determine the optimal class of scheduling policies minimizing the number of re-transmissions until successful decoding. In the following, we obtain our analytical results for \( e = 1 \) and \( R_2 = 1 \). However, our analysis holds in general for different values of \( e \) and \( R_2 \), which is demonstrated by the numerical results presented in Section VII.

The following theorem states that once the battery has sufficient charge to decode the packet, it is better to use the incoming RF signal only for information accumulation.

**Theorem 4.** If \( b = E_d + 1, E_d + 2, \ldots \), the optimal decision is to accumulate mutual information, i.e., \( \rho^*(b, m) = 0 \) for \( b = E_d + 1, E_d + 2, \ldots \).

**Proof.** The proof is given in Appendix C.

Since it is optimal to accumulate mutual information whenever \( b = E_d + 1, E_d + 2, \ldots \) (i.e., \( \rho^*(b, m) = 0 \) for all \( m = 0, \ldots, R_1 \) and \( b = E_d + 1, E_d + 2, \ldots \)), we can calculate \( k_{b,m}^\pi = k_{b,m}^\pi_0 \) for those states. Using (22), we have:

\[
k_{E_d+i,R_1−1−i}^{\pi^*} = 1 + (1−\lambda)k_{E_d+i−1,R_1−i+1}^{\pi^*} \quad \text{for } j = 1, \ldots, R_1 − 1, \quad i = 1, \ldots, R_1 − j
\]  

(25)

Using the recursion in (25), it is possible to show that:

\[
k_{E_d+i,R_1−1−i}^{\pi^*} = \frac{1−(1−\lambda)^i}{\lambda} + (1−\lambda)^i k_{E_d,R_1−j}^{\pi^*}
\]  

(26)

Note that in order to calculate the minimum mean time to absorption using (26), one needs to know the values of \( k_{E_d,R_1−j}^{\pi^*}, j = 1, \ldots, R_1 − 1 \). However, from (22), we know that \( k_{E_d,R_1−j}^{\pi^*} \) depends on the unknown values of \( k_{E_d−1,R_1−j+1}^{\pi^*} \), so it is not possible to compare \( k_{E_d,R_1−j}^{\pi^*} \) and \( k_{E_d,R_1−j}^{\pi^*} \) just yet.

Hence, let us first calculate \( k_{E_d,R_1−1}^{\pi^*} \) and \( k_{E_d,R_1−1}^{\pi^*} \). Using Lemma 1 and 2, as well as (22) and (23), it can be seen that

\[
k_{E_d,R_1−1}^{\pi^*} = k_{E_d,R_1−1}^{\pi^*} = 1 + \frac{1}{\lambda}.
\]  

(27)

Note that whenever the receiver is at state \((E_d, R_1−1)\), the decision to choose either energy harvesting or mutual information accumulation, does not alter the mean time to absorption at that specific state. The following theorem generalizes this observation to other states as well.
**Theorem 5.** At state \((b, m)\) where \(b = 1,2,\ldots,E_d,\) and \(m = 0,1,\ldots,R_1-1,\) any time switching decision is optimal.

**Proof.** The proof is given in Appendix D.

**Theorem 6.** Optimal policy, \(\pi^*\), satisfies the following properties.

1) If \(b = 0\) or \(m = R_1\), it chooses \(\rho = 1\).
2) If \(b = E_d + 1, E_d + 2,\ldots\), it chooses \(\rho = 0\).
3) If \(b = 1,2,\ldots,E_d\) and \(m = 0,1,\ldots,R_1-1,\) chooses either \(\rho = 0\) or \(\rho = 1\).

**Proof.** The proof of the theorem is straightforward and proceeds as follows:

1) When \(b = 0\), the receiver has no energy to activate the RF transceiver and should first recharge its battery. When \(m = R_1\), the receiver collected sufficient mutual information to decode, but needs energy to perform the decoding operation. Hence, it harvests energy.
2) This part of the theorem is proven in Theorems 2 and 4.
3) Theorem 5 states that whenever \( b = 1, 2, \ldots, E_d \), and \( m = 0, 1, \ldots R_1 - 1 \), then \( k_{b,m}^0 = k_{b,m}^1 \).

Consider a policy \( \beta \) which satisfies part 1 and 2 of the theorem. Whenever \( b = 1, 2, \ldots, E_d \) and \( m = 0, 1, \ldots R_1 - 1 \), the policy chooses \( \rho = 0 \) with probability \( p \). The mean time to absorption of policy \( \beta \), \( k_{b,m}^\beta \), can be calculated as follows

\[
k_{b,m}^\beta = pk_{b,m}^0 + (1-p)k_{b,m}^1 = k_{b,m}^0 = k_{b,m}^1
\]

(28)

Theorem 6 is illustrated in Fig. 2. Simple examples of such optimal policies that belong to the optimal family of policies characterized in Theorem 6 are:

- **Battery First (BF):** the receiver harvests energy until it acquires \( E_d \) units of energy and then starts accumulating the mutual information.
- **Information First (IF):** the receiver always accumulates mutual information unless \( b = 0 \) or \( m = R_1 \).
- **Coin Toss (CT):** the receiver harvests energy when \( b = 0 \) or \( m = R_1 \), while it accumulates mutual information when \( b = E_d + 1, E_d + 2, \ldots \). Otherwise, it tosses a fair coin to choose between harvesting energy or accumulating mutual information.

Given that we now know the optimal splitting decision \( \rho^* \) at every state, we can obtain the minimum number of re-transmissions as follows.

**Theorem 7.** For \( b = 1, 2, \ldots, E_d \) and \( m = 0, 1, \ldots R_1 - 1 \), the minimum mean time to absorption, when \( R_2 = 1 \) and \( e = 1 \), is

\[
k_{E_d-i, R_1-j}^\pi = 2 - j + (1 - \lambda) f_j(\lambda) + (1 - \lambda)^{j-1} + \frac{i+j}{\lambda}, \quad j = 1, \ldots, R_1, \quad i = 0, \ldots, E_d, \quad (29)
\]

where

\[
f_{j+1}(\lambda) = 2 - j + (1 - \lambda) f_j(\lambda) \quad j = 2, \ldots, R_1,
\]

\[
f_1(\lambda) = \frac{-1}{1 - \lambda}.
\]

(30)

**Proof.** The proof is given in Appendix E.

VI. THE MINIMUM EXPECTED NUMBER OF RE-TRANSMISSIONS FOR A CORRELATED CHANNEL

In many wireless systems, the wireless channel cannot be modeled as an i.i.d. channel. In this section, we investigate optimal scheduling policies under a time-correlated channel model. Our
analysis for a correlated channel follows a similar approach to our analysis for i.i.d. channels. However, due to correlation between the subsequent channel states, the receiver can improve its decision by incorporating its knowledge of the current state. Even under this model, we prove that a TS policy is optimal. We prove this by modeling the system by a three dimensional MC with an additional state of the realized channel state in the current slot.

Let $G_t$ be the state of the channel at time slot $t$, which is modeled as a one-dimensional MC with two states: GOOD state denoted by 1, and BAD state denoted by 0. At time $t = 0$, i.e., when transmission begins, we have no information of the channel, but for $t > 0$. The transition probabilities are given by $P[G_t = 1|G_{t-1} = 1] = \lambda_1$ and $P[G_t = 1|G_{t-1} = 0] = \lambda_0$. The channel is not observable by the receiver before it makes a decision, and thus, its decision is only based on its battery level, the mutual information accumulated as in the case of i.i.d. channel and also its belief about the channel state. Note that due to time correlation, the previous state of the channel provides information about the current channel state to the receiver. Hence, although once again we model the system as a MC, this time the state space of MC is extended where the states are $(b, m, G)$. Let $G \in \{0, 1\}$ be the previous state of the channel.

The resulting MC is still an absorbing MC, and the mean time to absorption is equivalent to the minimum expected number of re-transmissions until successful decoding. Define $\pi^*$ as the optimal policy minimizing the mean time to absorption at any given state $(b, m, G)$. Let $k_{\pi^*}^*$ be the mean time to absorption obtained by policy $\pi^*$ at state $(b, m, G)$. Similar to i.i.d. channel case, we first prove that there exists an optimal TS policy minimizing the number of re-transmissions.

**Lemma 2.** For any $E_d - i \cdot e \leq b < E_d - (i - 1) \cdot e$ such that $i = 1, \ldots, E_d$, and given that $m = R_1$, the minimum mean time to absorption is given by

$$k_{b,R_1,1}^{\pi^*} = \frac{1 + \lambda_0 - \lambda_1}{\lambda_0} \quad i = 1, \ldots, E_d, \quad (31)$$

$$k_{b,R_1,0}^{\pi^*} = \frac{1}{\lambda_0} + (i - 1) \frac{1 + \lambda_0 - \lambda_1}{\lambda_0} \quad i = 1, \ldots, E_d. \quad (32)$$

**Proof.** The proof is given in Appendix F.

**Lemma 2** plays an important part in proving that the optimal policy minimizing the mean time to absorption does not split the incoming RF signal even when the channel states are correlated. Let $\pi^{split}(b, m, G) = (\rho, \pi^*(\hat{b}, \hat{m}, \hat{G}))$, $0 < \rho < 1$ be a tail policy that splits the RF signal at

\[ ^8 \text{Note that the receiver becomes aware of the channel state after it decides to sample the incoming RF signal.} \]
state \((b, m, G)\) but follows \(\pi^*\) after transitioning to the new state \((\hat{b}, \hat{m}, \hat{G})\) except when \(b < 1\) or \(m = R_1\) where it only harvests energy. The mean time to absorption for policy \(\pi^\text{split}\) is calculated as follows:

\[
k_{b,m,0}^{\text{split}} = 1 + \lambda_0 k_{b-1,m+1,0}^{\pi^*} + (1 - \lambda_0) k_{b-1,m+1,0}^{\pi^*},
\]
\[
k_{b,m,1}^{\text{split}} = 1 + \lambda_1 k_{b-1,m+1,0}^{\pi^*} + (1 - \lambda_1) k_{b-1,m+1,0}^{\pi^*}.
\]

Furthermore, let \(\pi^\text{no-split}\) be another tail policy concatenated with \(\pi^*\) which never splits the RF energy but rather it accumulates mutual information at all states except where \(b < 1\) or \(m = R_1\) where it harvests energy. One can calculate \(k_{b,m,G}^{\text{no-split}}\) as follows

\[
k_{b,m,0}^{\text{no-split}} = 1 + \lambda_0 k_{b-1,R_1,0}^{\pi^*} + (1 - \lambda_0) k_{b-1,m+1,0}^{\pi^*},
\]
\[
k_{b,m,1}^{\text{no-split}} = 1 + \lambda_1 k_{b-1,R_1,1}^{\pi^*} + (1 - \lambda_1) k_{b-1,m+1,0}^{\pi^*}.
\]

**Theorem 8.** Splitting the incoming RF energy is not optimal when the channel states are correlated.

**Proof.** The proof is given in Appendix G. \qed

Theorem 8 proves that the optimal policy should either choose energy harvesting or information accumulation at any given state \((b, m, G)\). Therefore, MC associated with the optimal strategy has discrete states in which \(b = 0, 1, \ldots, \infty\), \(m = 0, 1, \ldots, R_1\) and \(G = 0, 1\). Define the tail policy \(\pi^i(b,m,G) = (i, \pi^*(\hat{b}, \hat{m}, \hat{G}))\), \(i = 0, 1\) that chooses \(\rho = i\) at state \((b, m, G)\) but follows policy \(\pi^*\) after transitioning into the new state \((\hat{b}, \hat{m}, \hat{G})\). Let \(k_{b,m,G}^{\pi^i}\) be the mean time to absorption of policy \(\pi^i(b,m,G)\), \(i = 0, 1\). We can calculate \(k_{b,m,G}^{\pi^0}\) and \(k_{b,m,G}^{\pi^1}\) as follows:

\[
k_{b,m,0}^{\pi^0} = 1 + \lambda_0 k_{b+1,m+1,1}^{\pi^*} + (1 - \lambda_0) k_{b+1,m+1,0}^{\pi^*},
\]
\[
k_{b,m,1}^{\pi^0} = 1 + \lambda_1 k_{b+1,m+1,1}^{\pi^*} + (1 - \lambda_1) k_{b+1,m+1,0}^{\pi^*},
\]
\[
k_{b,m,0}^{\pi^1} = 1 + \lambda_0 k_{b+1,m+1,0}^{\pi^*} + (1 - \lambda_0) k_{b+1,m+1,1}^{\pi^*},
\]
\[
k_{b,m,1}^{\pi^1} = 1 + \lambda_1 k_{b+1,m+1,0}^{\pi^*} + (1 - \lambda_1) k_{b+1,m+1,1}^{\pi^*}.
\]
Similar to the outline of the Theorem 2 in the following, we consider states \((b, m, G) = (E_d + j, R_1 - j, G)\) for \(j = 1, \ldots, R_1\) and derive the optimal strategy for those states. We have,

\[
k_{E_d+j,R_1-j,0}^* = 1 + \lambda_0 k_{E_d+j-1,R_1-1,1}^* + (1 - \lambda_0) k_{E_d+j-1,R_1-j+1,0}^*
\]

\[
= 1 + (1 - \lambda_0) k_{E_d+j-1,R_1-j+1,0}^*.
\]

(41)

\[
k_{E_d+j,R_1-j,1}^* = 1 + \lambda_0 k_{E_d+j-1,R_1-1,1}^* + (1 - \lambda_1) k_{E_d+j-1,R_1-j,0}^*
\]

\[
= 1 + (1 - \lambda_1) k_{E_d+j-1,R_1-j,0}^*.
\]

(42)

\[
k_{E_d+j,R_1-j,0}^* = 1 + \lambda_0 k_{E_d+j+1,R_1-1,1}^* + (1 - \lambda_0) k_{E_d+j+1,R_1-j,0}^*
\]

\[
= 1 + (1 - \lambda_0) k_{E_d+j+1,R_1-j,0}^*.
\]

(43)

\[
k_{E_d+j,R_1-j,1}^* = 1 + \lambda_1 k_{E_d+j+1,R_1-1,1}^* + (1 - \lambda_1) k_{E_d+j+1,R_1-j,0}^*.
\]

(44)

We need the following lemma in order to derive the optimal strategy for states \((E_d + j, R_1 - j, G)\).

**Lemma 3.** The optimal strategy in states \((E_d + j, R_1 - j, G)\) for \(j = 1, \ldots, R_1\) and \(G = 0, 1\) is to accumulate mutual information \((\rho^*(E_d + j, R_1 - j, G) = 0)\) and also \(k_{E_d+j,R_1-j,G}^* = k_{E_d+j,R_1-j,G}^0\) for \(b = E_d + j + 1, E_d + j + 2, \ldots\).

**Proof.** The proof is given in Appendix **H**.  

Now that we know the optimal policy for states \((E_d + j, R_1 - j, G)\), we can calculate the minimum mean time to absorption for those state as follows:

\[
k_{E_d+j,R_1-j,0}^* = k_{E_d+j,R_1-j,0}^0 = \sum_{i=1}^{j} (1 - \lambda_0)^{i-1} \quad j = 1, \ldots, R_1,
\]

(45)

\[
k_{E_d+j,R_1-j,1}^* = k_{E_d+j,R_1-j,1}^0 = 1 + (1 - \lambda_1) \sum_{i=1}^{j-1} (1 - \lambda_0)^{i-1} \quad j = 2, \ldots, R_1,
\]

(46)

\[
k_{E_d+j,R_1-j,1}^* = 1 \quad j = 1.
\]

(47)

Algorithm 2 calculates the \(k_{b,m,G}^*\) and the corresponding \(\rho^*\) for any \(b, m,\) and \(G\). Proving the optimality of Algorithm 2 is similar to the outline of the optimality proof of Algorithm 1 and hence it is omitted here. Note that the knowledge of the the previous channel state, \(G\), enables the receiver to be able to fully utilize the information yielded by the correlation. However, it also results in four coupled equations, (37)-(40), over numerous states which makes the analysis extremely hard. For this reason, we omit full characterization of the structure of the optimal policy. Nevertheless, note that Algorithm 2 provides a recursive method to determine the optimal TS decisions for each state \((b,m,G)\). In fact, we use these optimal decisions in the Section VII to calculate the minimum number of re-transmissions.
Algorithm 2 Calculating the minimum mean time to absorption for correlated channel

1: Initialize $k_{b,R_1,G}^{\pi^0}$ for $b = 0, \ldots, E_d - 1$ using (31) and (32).
2: Initialize $k_{E_d+j,R_1-j,G}^{\pi^1}$ for $j = 1, \ldots, R_1$ using (45), (46) and (47).
3: $n \leftarrow 0$
4: for $m = R_1 - 1 : 0$ do
5: for $b = E_d + n : 0$ do
6: Calculate $k_{b,m,G}^{\pi^0}$ for $G = 0, 1$ using (37) and (38), respectively.
7: Calculate $k_{b,m,G}^{\pi^1}$ for $G = 0, 1$ using (39) and (40), respectively.
8: $k_{b,m,G}^{\pi^*} = \min\left(k_{b,m,G}^{\pi^0}, k_{b,m,G}^{\pi^1}\right)$.
9: $\rho^*(b,m,G) = \arg\min_i k_{b,m,G}^{\pi^i}$ for $i = 0, 1$
10: $n \leftarrow n + 1$

VII. Numerical Results

In this section, we provide numerical evidence to support the analytical results established in the paper. We will divide our attention to validate the optimal policy for i.i.d. and correlated channel models. We use a simple ARQ mechanism as a baseline for understanding the performance merits of the HARQ mechanism. In the following, we formally define the simple ARQ scheme for i.i.d. and correlated channels.

A. Simple ARQ

In simple ARQ, the packet is transmitted successfully whenever the channel is in a GOOD state and the receiver has sufficient energy to decode the packet. Otherwise, the receiver drops the packet and awaits re-transmissions. When the receiver employs simple ARQ, before any decoding attempt, it has to make sure that its battery has at least $E_d + 1$ units of energy. Otherwise, after consuming 1 unit of energy for sampling, it will not have sufficient energy to decode the data packet and it will drop the packet. It is easy to prove that the optimal simple ARQ policy minimizing the mean time to absorption first harvests $E_d + 1$ units of energy and then attempts decoding. If the decoding attempt is not successful, it harvests energy until its battery state reaches $E_d + 1$ units again before attempting to decode.
B. i.i.d. Channel States

In this section, we evaluate the minimum mean time to absorption obtained from Algorithm 1, and compare it to that of the following three simple policies. The studied policies are as follows: i) Battery First (BF), ii) Information First (IF), and iii) Coin Toss (CT). Also, we compare the performance of the receiver equipped with HARQ mechanism with the case of a receiver equipped with simple ARQ mechanism. We determine the mean number of re-transmissions by Monte Carlo simulations, and compare them with that of analytical calculation described in Algorithm 1. Note that Monte Carlo simulations provide only sample mean time which is a random variable. The mean of this random variable is equal to the mean time to absorption and its variance decreases with the number of samples and becomes zero only if the number of iterations go to infinity. Hence, we expect to see small differences between the results obtained by the Monte Carlo simulations and analytical results, which is the reason why some policies have slightly smaller mean time to absorption than the optimal analytical value.

Table I summarizes the mean time to absorption for $R_1 = 10$, $e = 1$, $\lambda = 0.5$ and $E_d = 5$ with respect to $R_2$ associated with different policies. For IF, BF, CT and simple ARQ policies, we run Monte Carlo simulations for $10^7$ iterations and evaluate the sample mean. It can be seen from Table I that all policies have almost the same performance. This observation confirms our major finding in Theorem 5 that the optimal policy achieving the minimum mean time to absorption is not unique.

Table II summarizes the mean time to absorption for $R_2 = 5$, $R_1 = 10$, $E_d = 5$ and $e = 2$ with respect to $\lambda$ is summarized in Table II. As expected, it can be seen that the mean time to absorption decreases as the channel quality improves. Also, the performance gap between the HARQ and simple ARQ mechanism becomes smaller as the channel quality improves. This is because as the channel quality improves, the probability of harvesting energy and accumulating $R_1$ bits of mutual information also increases. Finally, the mean time to absorption for $R_2 = 5$, $R_1 = 10$, $E_d = 10$ and $\lambda = 0.3$ with respect to $e$ is summarized in Table III. We observe that the mean time to absorption is approximately the same for all policies and it is decreasing with respect to the amount of harvested energy, $e$.

The results presented in Table I, II and III confirm our theoretical results that, indeed, the optimal policy harvests energy whenever $b = 0$ or $m = R_1$ and accumulates mutual information whenever $b > E_d$. For the rest of the states it does not matter what the receiver does, as long
Table I: Mean time to absorption for \( R_1 = 10, \ e = 1 \) and \( E_d = 5 \) vs. \( R_2 \)

| \( R_2 \) | Optimal analytical | Optimal Monte-Carlo | BF | IF | CT | Simple ARQ |
|---|---|---|---|---|---|---|
| 1 | 15.9941 | 15.9910 | 15.9938 | 15.9966 | 15.9992 |
| 2 | 15.8125 | 15.8103 | 15.8116 | 15.8140 | 15.9992 |
| 3 | 15.6250 | 15.6235 | 15.6259 | 15.6266 | 15.9992 |
| 4 | 15.2500 | 15.2490 | 15.2504 | 15.2508 | 16.0006 |
| 5 | 14.5000 | 14.4992 | 14.4999 | 14.5020 | 16.0007 |
| 6 | 14.5000 | 14.5001 | 14.4995 | 14.5007 | 15.9995 |
| 7 | 14.5000 | 14.5000 | 14.4993 | 14.5009 | 15.9996 |
| 8 | 14.4999 | 14.4983 | 14.4993 | 14.4984 | 14.5000 |
| 9 | 14.4999 | 14.4999 | 14.4993 | 14.4984 | 14.5000 |

Table II: Mean time to absorption for \( R_1 = 10, \ R_2 = 5, \ e = 2 \) and \( E_d = 5 \) vs. \( \lambda \)

| \( \lambda \) | Optimal analytical | Optimal Monte-Carlo | BF | IF | CT | Simple ARQ |
|---|---|---|---|---|---|---|
| 0.1 | 40.9000 | 40.8904 | 40.8920 | 40.8978 | 40.8961 | 87.3286 |
| 0.2 | 20.8000 | 20.7979 | 20.7962 | 20.7960 | 20.8006 | 31.1145 |
| 0.3 | 14.0333 | 14.0320 | 14.0337 | 14.0331 | 14.0333 | 17.9077 |
| 0.4 | 10.6000 | 10.5985 | 10.6002 | 10.5991 | 10.5973 | 12.3428 |
| 0.5 | 8.5000 | 8.4989 | 8.4995 | 8.5002 | 8.4986 | 9.3310 |
| 0.6 | 7.0667 | 7.0659 | 7.0666 | 7.0667 | 7.0665 | 7.4591 |
| 0.7 | 6.0143 | 6.0140 | 6.0153 | 6.0140 | 6.0137 | 6.1846 |
| 0.8 | 5.2000 | 5.1999 | 5.1998 | 5.1999 | 5.2001 | 5.2607 |
| 0.9 | 4.5444 | 4.5443 | 4.5445 | 4.5443 | 4.5444 | 4.5568 |

Table III: Mean time to absorption for \( R_1 = 10, \ R_2 = 5, \ \lambda = 0.3 \) and \( E_d = 10 \) vs. \( e \)

| \( e \) | Optimal analytical | Optimal Monte-Carlo | BF | IF | CT | Simple ARQ |
|---|---|---|---|---|---|---|
| 1 | 40.7000 | 40.6956 | 40.8920 | 40.8978 | 40.8961 | 47.7832 |
| 2 | 21.7000 | 21.6999 | 20.7962 | 20.7960 | 20.8006 | 26.5340 |
| 3 | 15.0333 | 15.0320 | 14.0337 | 14.0331 | 14.0333 | 19.1515 |
| 4 | 11.7000 | 11.5985 | 10.6002 | 10.5991 | 10.5973 | 15.4839 |
| 5 | 11.7000 | 11.5985 | 10.6002 | 10.5991 | 10.5973 | 14.0076 |
| 6 | 8.3667 | 8.3648 | 7.0666 | 7.0667 | 7.0665 | 11.8479 |
| 7 | 8.3667 | 8.3651 | 6.0140 | 6.0132 | 6.0137 | 10.8730 |
| 8 | 8.3667 | 8.3675 | 5.1999 | 5.1998 | 5.2001 | 10.4191 |
| 9 | 8.3667 | 8.3670 | 4.5443 | 4.5443 | 4.5444 | 10.2021 |

C. Correlated Channel

In this section, we investigate the performance of the optimal policy presented in Algorithm 2 for the case of correlated channel and compare its performance to the three baseline policies that employ HARQ mechanism as well as a simple ARQ mechanism. We also consider a randomized policy, which we call Bernoulli policy which harvests energy with probability, \( p \), unless its battery does not split the received RF signal.

as, it does not split the received RF signal.
state is less than one unit or it has accumulated sufficient mutual information during when it solely harvests energy. In the following, we study the effects of the encoding rate, the time correlation, and the EH rate. Note that the mean time to absorption is determined by calculating $k_{b,m,0}$ and $k_{b,m,1}$ and then averaging them with respect to the steady-state distribution of the channel states, i.e., $k_{b,m} = \phi(0)k_{b,m,0} + \phi(0)k_{b,m,1}$, where $\phi(0) = 1 - \phi(1) = \frac{1-\lambda_1}{1+\lambda_0-\lambda_1}$.

**Remark.** Note that, in this section, we do not calculate the mean time to absorption by Algorithm 2 (i.e., $k_{b,m,0}^\pi$ and $k_{b,m,1}^\pi$). Instead, we use the optimal TS decisions dictated by Algorithm 2 for each state $(b,m,G)$ to determine the mean time to absorption by Monte-Carlo simulations. This is because both methods yield the same mean time to absorption for the optimal policy and illustrating both on the same figure distinctly is not possible.

To investigate the effect of the encoding rate on the mean time to absorption, we set the simulation parameters as $R_1 = 10$, $e = 1$, $E_d = 5$ and $p = 0.1$. The mean time to absorption with respect to $R_2$, for negatively and positively correlated channel states, are depicted in Figures 3a and 3b respectively. Unlike the i.i.d. case the knowledge of the channel state makes a significant difference in the performance of the proposed optimal policy as compared to the baseline policies. Hence, when the channel is correlated, a simple scheduling policy is not sufficient to achieve a low number of re-transmissions.

Next, we study the effect of the channel quality and the correlation on the mean time to absorption. We set $R_1 = 10$, $e = 1$, $E_d = 5$ and $p = 0.1$. We fix $\lambda_1 = 0.2$ and by varying $\lambda_0$, we calculate the mean time to absorption as illustrated in Figure 4a. Similarly, we fix $\lambda_0 = 0.2$ and by varying $\lambda_1$, we calculate the mean time to absorption by the aforementioned baseline policies and illustrate the results in Figure 4b. Note that when the channel is negatively correlated, as in Figure 4b, the gap between the optimal policy and the baseline policies is high. However, when the channel is positively correlated, as in Figure 4b, the gap disappears as $\lambda_1$ increases. This is because, when the channel is positively correlated, the channel tends to stay in the same state for a longer time before changing its state. On the contrary, in negatively correlated channel states, the channel is more likely to change its state at any time. This rapid change in state transition in the case of negatively correlated channel states requires a more adaptive policy rather than the case of the positively correlated channel state which rarely changes its state. Thus, the performance gain of Algorithm 2 is more evident in negatively correlated channels.

Finally the effect of EH rate, $e$, on the mean time to absorption for negatively and positively
Figure 3: The effect of the encoding rate on the minimum expected number of re-transmissions for $R_1 = 10$, $e = 1$, $E_d = 5$ and $p = 0.1$.

Figure 4: The effect of the channel quality and correlation on the minimum expected number of re-transmissions for $R_1 = 10$, $R_2 = 3$, $e = 1$, $E_d = 5$ and $p = 0.1$. 
Figure 5: The effect of the EH rate on the minimum expected number of re-transmissions for $R_1 = 10$, $R_2 = 5$, $E_d = 10$ and $p = 0.1$.

correlated channel states is depicted in Figure 5a and 5b respectively. The results are obtained by setting $R_1 = 10$, $R_2 = 5$, $E_d = 10$, $p = 0.1$, $\lambda_0 = 0.7$ and $\lambda_1 = 0.2$ for negatively correlated channel states; and $\lambda_0 = 0.2$ and $\lambda_1 = 0.7$ for positively correlated channel states. We, again, observe that the optimal policy outperforms the baseline policies and the performance gain is more evident for negatively correlated channel states for the same reason we provided for the results in Figure 4.

It should be noted that when the channel states are correlated, the knowledge about the future channel states plays a major role in making decision about the power splitting ratio. On the contrary, when the channel states evolve i.i.d. over time, there exist a class of optimal policies instead of a single optimal policy.

VIII. CONCLUSION

We analyzed a point-to-point wireless link employing HARQ for reliable transmission, where the receiver can only empower itself via the transmitter’s RF signal. We modeled the problem of optimal power splitting using a Markovian framework, and developed an optimal algorithm achieving the minimum mean time to absorption for both time varying i.i.d. and correlated channels. We developed computationally inexpensive algorithms to calculate the minimum mean time to absorption and optimize the power splitting ratio starting at any arbitrary state.
We proved that the optimal policy in case of i.i.d. channel states is not unique, and indeed the optimal policy belongs to the optimal family of policies. For correlated channel, we observed that it is only possible to achieve the optimal performance by intelligently utilizing the information offered by channel’s correlation information. Finally, we numerically validated the analytical results established in the paper by providing extensive number of simulations.

**REFERENCES**

[1] L. R. Varshney, “Transporting information and energy simultaneously,” in *2008 IEEE International Symposium on Information Theory*, 2008, pp. 1612–1616.

[2] P. Grover and A. Sahai, “Shannon meets tesla: Wireless information and power transfer,” in *ISIT*, 2010, pp. 2363–2367.

[3] K. Huang and E. Larsson, “Simultaneous information and power transfer for broadband wireless systems,” *IEEE Transactions on Signal Processing*, vol. 61, no. 23, pp. 5972–5986, Dec 2013.

[4] R. Zhang and C. K. Ho, “Mimo broadcasting for simultaneous wireless information and power transfer,” *IEEE Transactions on Wireless Communications*, vol. 12, no. 5, pp. 1989–2001, 2013.

[5] X. Zhou, R. Zhang, and C. K. Ho, “Wireless information and power transfer: Architecture design and rate-energy tradeoff,” *IEEE Transactions on Communications*, vol. 61, no. 11, pp. 4754–4767, 2013.

[6] L. Liu, R. Zhang, and K. C. Chua, “Wireless information and power transfer: A dynamic power splitting approach,” *IEEE Transactions on Communications*, vol. 61, no. 9, pp. 3990–4001, Sept. 2013.

[7] Liu, Liang and Zhang, Rui and Chua, Kee-Chaing, “Wireless information transfer with opportunistic energy harvesting,” *IEEE Transactions on Wireless Communications*, vol. 12, no. 1, pp. 288–300, 2013.

[8] J. Huang, R. A. Berry, and M. L. Honig, “Wireless scheduling with hybrid arq,” *IEEE Transactions on Wireless Communications*, vol. 4, no. 6, pp. 2801–2810, Nov 2005.

[9] F. A. de Witt, R. D. Souza, and G. Brante, “On the performance of hybrid arq schemes for uplink information transmission with wireless power transfer in the downlink,” in *2014 IFIP Wireless Days (WD)*. IEEE, 2014, pp. 1–6.
[10] H. Chen, R. G. Maunder, and L. Hanzo, “A survey and tutorial on low-complexity turbo coding techniques and a holistic hybrid ARQ design example,” IEEE Communications Surveys & Tutorials, vol. 15, no. 4, pp. 1546–1566, Fourth 2013.

[11] M. Zohdy, T. ElBatt, M. Nafie, and O. Ercetin, “RF energy harvesting in wireless networks with ARQ,” in 2016 IEEE Globecom Workshops (GC Wkshps), Dec 2016, pp. 1–6.

[12] B. Makki, T. Svensson, and M. Zorzi, “Wireless energy and information transmission using feedback: Infinite and finite block-length analysis,” IEEE Transactions on Communications, vol. 64, no. 12, pp. 5304–5318, Dec 2016.

[13] Y. Zeng, B. Clerckx, and R. Zhang, “Communications and signals design for wireless power transmission,” IEEE Transactions on Communications, vol. 65, no. 5, pp. 2264–2290, May 2017.

[14] V. Talla, B. Kellogg, B. Ransford, S. Naderiparizi, J. R. Smith, and S. Gollakota, “Powering the next billion devices with Wi-Fi,” Commun. ACM, vol. 60, no. 3, pp. 83–91, Feb. 2017. [Online]. Available: http://doi.acm.org/10.1145/3041059

[15] X. Zhou, R. Zhang, and C. K. Ho, “Wireless information and power transfer: Architecture design and rate-energy tradeoff,” IEEE Transactions on Communications, vol. 61, no. 11, pp. 4754–4767, November 2013.

[16] C. Shen, W. C. Li, and T. H. Chang, “Wireless information and energy transfer in multi-antenna interference channel,” IEEE Transactions on Signal Processing, vol. 62, no. 23, pp. 6249–6264, Dec 2014.

[17] J. Park and B. Clerckx, “Transmission strategies for joint wireless information and energy transfer in a two-user MIMO interference channel,” in 2013 IEEE International Conference on Communications Workshops (ICC), June 2013, pp. 591–595.

[18] A. A. Nasir, X. Zhou, S. Durrani, and R. A. Kennedy, “Wireless-powered relays in cooperative communications: Time-switching relaying protocols and throughput analysis,” IEEE Transactions on Communications, vol. 63, no. 5, pp. 1607–1622, May 2015.

[19] H. Liang, C. Zhong, X. Chen, H. A. Suraweera, and Z. Zhang, “Multi-antenna SWIPT relaying systems: Impact of antenna correlation and channel state information,” in 2016 IEEE Global Communications Conference (GLOBECOM), Dec 2016, pp. 1–6.

[20] R. D. Yates and H. Mahdavi-Doost, “Energy harvesting receivers: Packet sampling and decoding policies,” IEEE Journal on Selected Areas in Communications, vol. 33, no. 3, pp. 558–570, March 2015.
[21] A. Yadav, M. Goonewardena, W. Ajib, and H. Elbiaze, “Novel retransmission scheme for energy harvesting transmitter and receiver,” in 2015 IEEE International Conference on Communications (ICC), June 2015, pp. 3198–3203.

[22] A. Yadav, M. Goonewardena, W. Ajib, O. A. Dobre, and H. Elbiaze, “Energy management for energy harvesting wireless sensors with adaptive retransmission,” IEEE Transactions on Communications, vol. PP, no. 99, pp. 1–1, 2017.

[23] H. Mahdavi-Doost and R. D. Yates, “Hybrid arq in block-fading channels with an energy harvesting receiver,” in 2015 IEEE International Symposium on Information Theory (ISIT), June 2015, pp. 1144–1148.

[24] Z. Ni, R. V. Bhat, and M. Motani, “Performance of energy-harvesting receivers with batteries having internal resistance,” in 2017 IEEE Wireless Communications and Networking Conference Workshops (WCNCW), March 2017, pp. 1–6.

[25] Z. Ni and M. Motani, “Transmission schemes and performance analysis for time-switching energy harvesting receivers,” in 2016 IEEE International Conference on Communications (ICC), May 2016, pp. 1–6.

[26] S. Zhou, T. Chen, W. Chen, and Z. Niu, “Outage minimization for a fading wireless link with energy harvesting transmitter and receiver,” IEEE Journal on Selected Areas in Communications, vol. 33, no. 3, pp. 496–511, March 2015.

[27] K. Saidi, W. Ajib, and M. Boukadoum, “Adaptive transmitter load size using receiver harvested energy prediction by kalman filter,” in 2016 10th International Symposium on Communication Systems, Networks and Digital Signal Processing (CSNDSP), July 2016, pp. 1–5.

[28] A. Arafa and S. Ulukus, “Optimal policies for wireless networks with energy harvesting transmitters and receivers: Effects of decoding costs,” IEEE Journal on Selected Areas in Communications, vol. 33, no. 12, pp. 2611–2625, Dec 2015.

[29] W. Ni and X. Dong, “Energy harvesting wireless communications with energy cooperation between transmitter and receiver,” IEEE Transactions on Communications, vol. 63, no. 4, pp. 1457–1469, April 2015.

[30] M. K. Sharma, C. R. Murthy, and R. Vaze, “On distributed power control for uncoordinated dual energy harvesting links: Performance bounds and near-optimal policies,” in 2017 15th International Symposium on Modeling and Optimization in Mobile, Ad Hoc, and Wireless Networks (WiOpt), May 2017, pp. 1–8.
[31] Y. Sarikaya and O. Ercetin, “Self-sufficient receiver with wireless energy transfer in a multi-access network,” *IEEE Wireless Communications Letters*, vol. 6, no. 4, pp. 442–445, Aug 2017.

[32] S. B. Wicker, *Error control systems for digital communication and storage*. Prentice hall Englewood Cliffs, 1995, vol. 1.

[33] N. Varnica, E. Soljanin, and P. Whiting, “Ldpc code ensembles for incremental redundancy hybrid arq,” in *Proceedings. International Symposium on Information Theory, 2005. ISIT 2005.*, Sept 2005, pp. 995–999.

[34] X. Lu, P. Wang, D. Niyato, D. I. Kim, and Z. Han, “Wireless networks with rf energy harvesting: A contemporary survey,” *IEEE Communications Surveys Tutorials*, vol. 17, no. 2, pp. 757–789, Secondquarter 2015.

[35] F. Rosas, R. D. Souza, M. E. Pellenz, C. Oberli, G. Brante, M. Verhelst, and S. Pollin, “Optimizing the code rate of energy-constrained wireless communications with harq,” *IEEE Transactions on Wireless Communications*, vol. 15, no. 1, pp. 191–205, 2016.

[36] M. S. H. Abad, O. Ercetin, and D. Gündüz, “Channel sensing and communication over a time-correlated channel with an energy harvesting transmitter,” *IEEE Transactions on Green Communications and Networking*, vol. PP, no. 99, pp. 1–1, 2017.

**APPENDIX A**

**PROOF OF THEOREM 2**

The proof is by induction. For the base case consider the initial case when $j = 1$ so that $b = E_d + 1, E_d + 2, \ldots$ and $m = R_1 - 1$. We have

$$k^{\pi_0}_{E_d+1,R_1-1} = 1 + \lambda k^{\pi^*}_{E_d,R_1} + (1 - \lambda)k^{\pi^*}_{E_d,R_1} = 1,$$

$$k^{\pi_0}_{E_d+1,R_1-1} = \frac{1}{\lambda} + k^{\pi^*}_{E_d+e+1,m} > k^{\pi_0}_{E_d+1,R_1-1}. \quad (49)$$

Note that when $b = E_d + 1, E_d + 2, \ldots$, by choosing $\rho = 0$, regardless of the channel state, the next state, $(b - 1, R_1)$, is an absorbing state so $k^{\pi_0}_{b,R_1-1} = 1$. Thus, the lemma holds for $j = 1$. In the induction step assume that the theorem holds for $j = n - 1$, i.e., $k^{\pi^*}_{b,R_1-n+1} = k^{\pi_0}_{E_d+n-1,R_1-n+1} = \ldots$
\[ \sum_{i=1}^{n-1} (1 - \lambda)^{i-1} \text{ for } b = E_d + n - 1, E_d + n, \ldots \] Now, we prove that the claim is also true for \( j = n \).

\[ k_{E_d+n,R_1-n}^0 = 1 + (1 - \lambda) k_{E_d+n-1,R_1-n+1}^0 \]
\[ = 1 + \sum_{i=1}^{n-1} (1 - \lambda)^i \]
\[ = \sum_{i=1}^{n} (1 - \lambda)^{i-1}, \quad (50) \]

\[ k_{E_d+n,R_1-n}^1 = \frac{1}{\lambda} + k_{E_d+n+e,R_1-n}^0 > \frac{1}{\lambda} + k_{E_d+n+e,R_1-n+1}^0 \]
\[ = \frac{1}{\lambda} + k_{E_d+n-1,R_1-n+1}^0 \]
\[ = \frac{1}{\lambda} + \frac{1 - (1 - \lambda)^{n-1}}{\lambda} \quad (51) \]

Furthermore,

\[ k_{E_d+n,R_1-n}^0 = \frac{1 - (1 - \lambda)^n}{\lambda} \]
\[ = 1 + (1 - \lambda) \frac{1 - (1 - \lambda)^{n-1}}{\lambda} < k_{E_d+n,R_1-n}^1 \quad (52) \]

For the last part of the proof, we need to show that \( k_{b,R_1-n}^0 = k_{E_d+n,R_1-n}^0 \) for \( b = E_d + n + 1, E_d + n + 2, \ldots \) We may write:

\[ k_{b,R_1-n}^* = 1 + (1 - \lambda) k_{b-1,R_1-n+1}^0 \]
\[ = 1 + (1 - \lambda) k_{E_d+n-1,R_1-n+1}^0 = k_{E_d+n,R_1-n}^0 \quad (53) \]

**APPENDIX B**

**PROOF OF THEOREM 3**

In Lemma 1 we characterized the minimum mean time to absorption for all states \((b, R_1)\), for \( b = 0, \ldots, E_d - 1 \). Also, in Theorem 2 we characterized the minimum mean time to absorption for states, \((b, R_1 - j)\) where, \( b = E_d + j, E_d + j + 1, \ldots \) and \( j = 1, \ldots, R_1 \). Furthermore, Theorem 1 proves that at any state \((b, m)\), the receiver should either choose to harvest energy or accumulate mutual information. Note that the iterations are ordered in Algorithm 1 (line 4-8) so that \( k_{b,m}^0 \) and \( k_{b,m}^1 \) only depend on \( k_{b-1,R_1}^*, k_{b-1,m+1}^*, \) and \( k_{b+1,m}^* \) which are obtained at the previous rounds of the algorithm.
APPENDIX C

PROOF OF THEOREM 4

We need to show that \( k^0_{E_d+j-i,R_1-j} \leq k^1_{E_d+j-i,R_1-j} \) for all \( j = 1, \ldots, R_1 \) and \( i = 0, 1, \ldots, j - 1 \). The proof is by induction. For the base case, we need to show that the theorem holds for \( i = 0 \) and all \( j = 1, \ldots, R_1 \). We know from Theorem 2 that \( k^0_{E_d+j,R_1-j} < k^1_{E_d+j,R_1-j} \) and, hence, the theorem is true for \( i = 0 \) and all \( j = 1, \ldots, R_1 \). Next, in the induction step, assume that the theorem is true for \( i = n \) and all \( j = 1, \ldots, R_1 \) i.e., \( k^0_{E_d+j-n,R_1-j} < k^1_{E_d+j-n,R_1-j} \). We need to show that the theorem also holds for \( i = n + 1 \) and all \( j = 1, \ldots, R_1 \).

\[
k^1_{E_d+j-(n+1),R_1-j} = \frac{1}{\lambda} + k^1_{E_d+j-n,R_1-j}
\]

\[
= \frac{1}{\lambda} + k^0_{E_d+j-n,R_1-j} \tag{54a}
\]

\[
= \frac{1}{\lambda} + 1 + (1 - \lambda)k^*_{E_d+(j-1)-n,R_1-(j-1)} \tag{54b}
\]

where \( 54a \) follows because of the induction hypothesis, i.e., \( k^0_{E_d+j-n,R_1-j} < k^1_{E_d+j-n,R_1-j} \). Also,

\[
k^0_{E_d+j-(n+1),R_1-j} = 1 + (1 - \lambda)k^*_{E_d+j-(n+1)-1,R_1-j+1} \leq 1 + (1 - \lambda)k^1_{E_d+j-(n+1)-1,R_1-j+1} \tag{55a}
\]

\[
= \frac{1}{\lambda} + (1 - \lambda)k^*_{E_d+(j-1)-n,R_1-(j-1)} \tag{55b}
\]

where \( 55a \) is due to \( k^*_{x,y} = \min(k^0_{x,y}, k^1_{x,y}) \). From \( 54a \) and \( 55b \), we have \( k^0_{E_d+j-(n+1),R_1-j} \leq k^1_{E_d+j-(n+1),R_1-j} - 1 \), which in turn proves the following inequality:

\[
k^0_{E_d+j-(n+1),R_1-j} < k^1_{E_d+j-(n+1),R_1-j} \tag{56}
\]

Since the theorem is also true for \( i = n + 1 \) and all \( j \), by induction, the theorem holds for all \( j = 1, \ldots, R_1 \) and \( i = 0, 1, \ldots, j - 1 \).

APPENDIX D

PROOF OF THEOREM 5

We have to show that \( k^0_{i,R_1-j} = k^1_{i,R_1-j} \) for \( i = 1, \ldots, E_d \) and \( j = 1, \ldots, R_1 \). The outline of the induction proof is as follows:

- For the base case we show that \( k^0_{i,R_1-1} = k^1_{i,R_1-1} \) for all \( i = 1, \ldots, E_d \).
• In the induction step, we assume the lemma is true for \( j = n \) and all \( i = 1, \ldots, E_d \).

• Using the induction step, we prove that the theorem also holds for \( j = n + 1 \) and all \( i = 1, \ldots, E_d \).

Let us consider the base case of \( j = 1 \). From (27), we know that the theorem holds for \( i = E_d \), i.e., \( k_{i,R_1}^{E_d, R_1} = k_{i,R_1}^{E_d, R_1-1} \). Assume that \( k_{i,R_1}^{E_d, R_1-1} = k_{i,R_1}^{E_d, R_1-1} \) and calculate:

\[
\begin{align*}
k_{i-1,R_1-1}^{E_d, R_1-1} &= \frac{1}{\lambda} + k_{i,R_1}^{E_d, R_1-1} \\
&= \frac{1}{\lambda} + k_{i,R_1}^{E_d, R_1-1} \\
&= \frac{1}{\lambda} + 1 + k_{i-1,R_1}^{E_d, R_1-1} \\
&= 1 + \frac{E_d - i + 2}{\lambda} \tag{57}
\end{align*}
\]

Hence, \( k_{i-1,R_1-1}^{E_d, R_1-1} = k_{i,R_1}^{E_d, R_1-1} \) and the theorem holds for \( j = 1 \) and all \( i = 1, \ldots, E_d \). Next, for the induction step assume that the theorem is true for \( j = n \) and all values of \( i = 1, \ldots, E_d \), i.e., \( k_{i,R_1}^{E_d, R_1} = k_{i,R_1}^{E_d, R_1-1} \). To show that the theorem is also true for \( j = n+1 \) and all values of \( i = 1, \ldots, E_d \), we have to start by first showing that the theorem holds for the state \((E_d, n+1)\) and work our way to show that it also holds for all states \((i, n+1)\). Let us calculate and compare values of \( k_{E_d,R_1}^{E_d,R_1-(n+1)} \) and \( k_{E_d,R_1}^{E_d,R_1-(n+1)} \):

\[
\begin{align*}
k_{E_d,R_1}^{E_d,R_1-(n+1)} &= \frac{1}{\lambda} + k_{E_d+1,R_1-(n+1)}^{E_d,R_1} \\
&= \frac{1}{\lambda} + k_{E_d+1,R_1-(n+1)}^{E_d,R_1} \\
&= \frac{1}{\lambda} + 1 + (1 - \lambda)k_{E_d,R_1-n}^{E_d,R_1-n} \tag{59}
\end{align*}
\]

\[
\begin{align*}
k_{E_d,R_1}^{E_d,R_1-(n+1)} &= 1 + (1 - \lambda)k_{E_d-1,R_1-n}^{E_d,R_1-1} + \lambda k_{E_d-1,R_1}^{E_d,R_1-1} \\
&= 1 + (1 - \lambda)k_{E_d-1,R_1-n}^{E_d,R_1-1} \\
&= 1 + (1 - \lambda)(\frac{1}{\lambda} + k_{E_d,R_1-n}^{E_d,R_1-n}) + 1 \\
&= 1 + \frac{1}{\lambda} + (1 - \lambda)k_{E_d,R_1-n}^{E_d,R_1-n} \tag{61}
\end{align*}
\]

Thus, \( k_{E_d,R_1}^{E_d,R_1-(n+1)} = k_{E_d,R_1}^{E_d,R_1-(n+1)} \). Next, we assume that \( k_{i,R_1}^{E_d,R_1-(n+1)} = k_{i,R_1}^{E_d,R_1-(n+1)} \) and prove that
\[ k_{i-1,R_1-(n+1)}^{\pi_0} = k_{i-1,R_1-(n+1)}^{\pi_1}. \] We have:

\[
k_{i-1,R_1-(n+1)}^{\pi_0} = 1 + (1 - \lambda)k_{i-2,R_1-n}^{\pi^*} + \lambda k_{i-2,R_1}^{\pi^*}
\]
\[
= 1 + (1 - \lambda)k_{i-2,R_1-n}^{\pi_1} + E_d - i + 2
\]
\[
= 1 + (1 - \lambda)\left(\frac{1}{\lambda} + k_{i-1,R_1-n}^{\pi^*}\right) + E_d - i + 2
\]
\[
= \frac{1}{\lambda} + E_d - i + 2 + (1 - \lambda)k_{i-1,R_1-n}^{\pi^*}.
\]

(62)

\[
k_{i-1,R_1-(n+1)}^{\pi_1} = \frac{1}{\lambda} + k_{i,R_1-(n+1)}^{\pi^*}
\]
\[
= \frac{1}{\lambda} + k_{i,R_1-(n+1)}^{\pi^*}
\]
\[
= \frac{1}{\lambda} + 1 + (1 - \lambda)k_{i-1,R_1-n}^{\pi^*} + \lambda k_{i-1,R_1}^{\pi^*}
\]
\[
= \frac{1}{\lambda} + 1 + (1 - \lambda)k_{i-1,R_1-n}^{\pi^*} + E_d - i + 1 = k_{i-1,R_1-(n+1)}^{\pi_0}.
\]

(63)

Hence, the theorem holds for \( j = n + 1 \) and all \( i = 1, \ldots, E_d \). Therefore, the theorem is true by induction.

**Appendix E**

**Proof of Theorem 7**

We have proven in Theorem 5 that, for the states of the MC associated with the theorem, either \( \rho = 0 \) or \( \rho = 1 \) is optimal. However, it is easier to calculate \( k_{b,m}^{\pi_0} \). Let us calculate \( k_{E_d-i,R_1-1}^{\pi^0} \) for all \( i = 0, \ldots, E_d \) using (22).

\[
k_{E_d-i,R_1-1}^{\pi_0} = 1 + (1 - \lambda)k_{E_d-i-1,R_1}^{\pi^0} + \lambda k_{E_d-i-1,R_1}^{\pi^0}
\]
\[
= 1 + \frac{i+1}{\lambda}, \quad i = 0, \ldots, E_d - 1,
\]

(64)

\[
k_{0,R_1-1}^{\pi_1} = \frac{1}{\lambda} + k_{1,R_1-1}^{\pi^*}
\]
\[
= 1 + \frac{E_d + 1}{\lambda}.
\]

(65)
Hence, \( k_{E_d-i,R_1-1}^0 = 1 + \frac{i+1}{\lambda} \) for \( i = 0, \ldots, E_d \). By comparing \( k_{E_d-i,R_1-1}^0 \) and (29), it is easy to verify that \( f_1(\lambda) = \frac{1}{1-\lambda} \). Next, consider calculating \( k_{E_d-i,R_1-(j+1)}^0 \) using (22) for \( i = 0, \ldots, E_d-1 \):

\[
k_{E_d-i,R_1-(j+1)}^0 = 1 + (1 - \lambda) k_{E_d-i-1,R_1-j}^* + \lambda k_{E_d-i-1,R_1}^*
\]

\[= 2 + i + (1 - \lambda) (2 - j + (1 - \lambda) f_j(\lambda) + (1 - \lambda)^j - 1 + \frac{i + j + 1}{\lambda})
\]

\[= 1 - j + (1 - \lambda) (2 - j + (1 - \lambda) f_j(\lambda)) + (1 - \lambda)^j - 1 + \frac{i + j + 1}{\lambda}
\]  

(66)

We can, alternatively, calculate \( k_{E_d-i,R_1-(j+1)}^0 \) using (29) as follows:

\[
k_{E_d-i,R_1-(j+1)}^0 = 2 - (j + 1) + (1 - \lambda) f_{j+1}(\lambda) + (1 - \lambda)^j - 1 + \frac{i + j + 1}{\lambda}
\]  

(67)

By comparing (66) and (67) we conclude that \( f_{j+1}(\lambda) = 2 - j + (1 - \lambda) f_j(\lambda) \). Finally, we need to show that \( k_{0,R_1-j}^0 \) is governed by (29) and (30).

\[
k_{0,R_1-j}^* = k_{0,R_1-j}^1 + \frac{1}{\lambda} = k_{1,R_1-j}^* + \frac{1}{\lambda} = 2 - j + (1 - \lambda) f_j(\lambda) + (1 - \lambda)^j - 1 + \frac{E_d - 1 + j}{\lambda}
\]  

(68)

**APPENDIX F**

**PROOF OF LEMMA 2**

The proof is by induction. Let us consider \( i = 1 \) as the base case such that \( E_d - e \leq b < E_d \). Note that since \( m = R_1 \), the optimal decision is to harvest energy, i.e., \( \rho = 1 \). We have:

\[
k_{b,R_1,0}^* = 1 + \lambda_0 k_{b+e,R_1,1}^* + (1 - \lambda_0) k_{b,R_1,0}^* = \frac{1}{\lambda_0},
\]

(69)

\[
k_{b,R_1,1}^* = 1 + \lambda_1 k_{b+e,R_1,1}^* + (1 - \lambda_1) k_{b,R_1,0}^* = \frac{1 + \lambda_0 - \lambda_1}{\lambda_0}.
\]

(70)

Hence lemma holds for \( i = 1 \). Next, for induction step assume that the lemma is true for \( i = n \), i.e., \( k_{b,R_1,0}^* = \frac{1}{\lambda_0} + (n - 1) \frac{1 + \lambda_0 - \lambda_1}{\lambda_0} \) and \( k_{b,R_1,1}^* = n \frac{1 + \lambda_0 - \lambda_1}{\lambda_0} \) for \( E_d - n \cdot e \leq b < E_d - (n - 1) \cdot e \). Let us consider the case \( n + 1 \):

\[
k_{b,R_1,0}^* = 1 + \lambda_0 k_{b+e,R_1,1}^* + (1 - \lambda_0) k_{b,R_1,0}^*
\]

\[= \frac{1}{\lambda_0} + k_{b+e,R_1,1}^*
\]

\[= \frac{1}{\lambda_0} + n + \frac{1 + \lambda_0 - \lambda_1}{\lambda_0}
\]

for \( E_d - (n + 1) e \leq b < E_d - nl \).
Consider policy $\pi^{\text{split}}$ with mean time to absorption as defined by (33) and (34). Since for policy $\pi^{\text{split}}$, we have assumed that $0 < \rho < 1$, it follows that $R^U(\rho) < R_1$, $R^L(\rho) < 1$ and also $m \leq R_1$. Also, it is easy to verify that $k_{b,1,G}^{\pi^{\text{split}}} \leq k_{b,2,G}^{\pi^{\text{split}}}$ for $G = 0, 1$ whenever $I_1 \geq I_2$. Thus, lower bounds for $k_{b,m,0}^{\pi^{\text{split}}}$ in (33) and $k_{b,m,1}^{\pi^{\text{split}}}$ in (34) can be written as:

$$k_{b,m,0}^{\pi^{\text{split}}} \geq 1 + \lambda_0 k_{b-1,m+1,0}^{\pi^{\text{split}}}, \quad \text{(71)}$$

$$k_{b,m,1}^{\pi^{\text{split}}} \geq 1 + \lambda_1 k_{b-1,m+1,0}^{\pi^{\text{split}}}, \quad \text{(72)}$$

Furthermore, since $b - 1 < b - 1 + \rho e < b + e - 1$, from Lemma 2 we know that $k_{b-1,R_1,G}^{\pi^{\text{split}}} = k_{b-1,R_1,G}^{\pi^{\text{split}}}$ for $G = 0, 1$. Hence, $k_{b,m,0}^{\pi^{\text{split}}}$ for $G = 0, 1$ achieves the lower bound in (71) and (72), respectively. Accordingly $k_{b,m,G}^{\pi^{\text{split}}} \leq k_{b,m,G}^{\pi^{\text{split}}}$ for $G = 0, 1$.

**APPENDIX H**

**PROOF OF LEMMA 3**

The proof is by induction. Let us first consider $(E_d + j, R_1 - j, 0)$. For $j = 1$ we have

$$k_{E_d+1,R_1-1,0}^{\pi^0} = 1 + (1 - \lambda_0)k_{E_d,R_1,0}^{\pi^*} = 1 \quad \text{(73)}$$

$$k_{E_d+1,R_1-1,0}^{\pi^1} = 1 + \lambda_0 k_{E_d+1+e,R_1-1,1}^{\pi^*} + (1 - \lambda_0)k_{E_d+1,R_1-1,0}^{\pi^*} > 1, \quad \text{(74)}$$

where it also follows that $k_{b,R_1-1,0}^{\pi^0} = 1 + (1 - \lambda_0)k_{b-1,R_1,0}^{\pi^*}$ for $b = E_d + 1, E_d + 2, \ldots$. Hence the theorem holds for $j = 1$. Let us assume that the theorem holds for $j = n - 1$ i.e., $k_{b,R_1-n+1,0}^{\pi^*} =$
Next we will prove the lemma for states $k_{E_d+n, R_1-n+1, 0}$ for $b = E_d + n, E_d + n + 1, \ldots$. We have

\[
k_{E_d+n,R_1-n,0} = 1 + (1 - \lambda_0)k_{E_d+n-1,R_1-n+1,0}^0,
\]

\[
= 1 + (1 - \lambda_0)k_{E_d+n-1,R_1-n+1,0}^0,
\]

\[
k_{E_d+n,R_1-n,0} = 1 + \lambda_0k_{E_d+n+e,R_1-n+1} + (1 - \lambda_0)k_{E_d+n,R_1-n,0}
\]

\[
> 1 + (1 - \lambda_0)k_{E_d+n,R_1-n,0}
\]

\[
\geq 1 + (1 - \lambda_0)k_{E_d+n,R_1-n+1} = 1 + (1 - \lambda_0)k_{E_d+n,R_1-n,0}.
\]

(75)

And we need to show that $k_{b,R_1-n,0}^0 = k_{E_d+n,R_1-n,0}^0$ for $b = E_d + n + 1, E_d + n + 2, \ldots$. We have

\[
k_{b,R_1-n,0} = 1 + (1 - \lambda_0)k_{b-1,R_1-n+1,0}^0
\]

\[
= 1 + (1 - \lambda_0)k_{b-1,R_1-n+1,0}^0 = k_{E_d+n,R_1-n,0}.
\]

(77)

Next we will prove the lemma for states $(E_d + j, R_1 - j, 1)$. Let us consider the base case $j = 1$.

\[
k_{E_d+1,R_1-1,1} = 1 + (1 - \lambda_1)k_{E_d,R_1,0}^1 = 1
\]

(78)

\[
k_{E_d+1,R_1-1,1} = 1 + \lambda_1k_{E_d+1+e,R_1-1,1} + (1 - \lambda_1)k_{E_d+1,R_1-1,0} > 1.
\]

(79)

Also, $k_{b,R_1-1,1} = 1 + (1 - \lambda_1)k_{b-1,R_1,0} = 1$ for $b = E_d + 1, E_d + 2, \ldots$. Hence the lemma holds for $j = 1$. Let us assume that the lemma holds for $j = n - 1$ i.e. $k_{b,R_1-n+1,1} = k_{E_d+n-1,R_1-n+1,0}$ for $b = E_d + n, E_d + n + 1, \ldots$. We have

\[
k_{E_d+n,R_1-n,1} = 1 + \lambda_1k_{E_d+n+e,R_1-n+1} + (1 - \lambda_1)k_{E_d+n,R_1-n,0}
\]

\[
> 1 + (1 - \lambda_1)k_{E_d+n,R_1-n,0}
\]

\[
\geq 1 + (1 - \lambda_1)k_{E_d+n,R_1-n+1} = 1 + (1 - \lambda_1)k_{E_d+n,R_1-n,0}.
\]

(80)

Finally we conclude the proof by showing that $k_{b,R_1-n,1}^0 = k_{E_d+n,R_1-n,1}^0$ for $b = E_d + n + 1, E_d + n + 2, \ldots$. We have

\[
k_{b,R_1-n,1} = 1 + (1 - \lambda_1)k_{b-1,R_1-n+1,0}^0
\]

\[
= 1 + (1 - \lambda_0)k_{E_d+n-1,R_1-n+1,0}^0 = k_{E_d+n,R_1-n,1}^0.
\]

(81)