Heavy Quark Potential and Mass Spectra of Heavy Mesons

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Abstract. The relativistic quark model is presented. The quark-antiquark potential for the Schrödinger-like equation is constructed with the account of retardation effects and one-loop radiative corrections. It consists of the one-gluon exchange part and the confining part which is the mixture of the Lorentz scalar and Lorentz vector contributions. The latter contains both the Dirac and Pauli terms. In the $v^2/c^2$ approximation the mass spectra of heavy quarkonia (charmonium and bottomonium) are calculated in good agreement with experiment. In the case of heavy-light mesons ($B$ and $D$) the light quark is treated completely relativistically and only the expansion in the inverse heavy quark mass is used. The mass spectra of the ground and excited states of $D, D_s, B, B_s$ mesons are calculated. They exhibit some features of the so-called “level inversion”. The obtained results are generally in accord with experimental data. Still there exist some discrepancies between measurements of different collaborations.

INTRODUCTION

The heavy flavour studies lie on the frontiers of elementary particle physics. The investigation of heavy meson mass spectra provides substantial information about the non-perturbative content of quantum chromodynamics (QCD) and fundamental parameters of the relativistic quark model. Since it is impossible to give here any kind of comprehensive review of the subject we present instead the consideration based mainly on two papers [1, 2]. They are extended to include recent experimental data and should be taken as an illustration of theoretical approaches.

RELATIVISTIC QUARK MODEL

In the quasipotential approach the meson is described by the wave function of the bound quark-antiquark state, which satisfies the quasipotential equation of the Schrödinger type in the centre-of-mass frame:

$$\left(\frac{b^2(M)}{2\mu_R} - \frac{p^2}{2\mu_R}\right)\Psi_M(p) = \int \frac{d^3q}{(2\pi)^3} V(p,q;M)\Psi_M(q),$$

where $\mu_R$ is the relativistic reduced mass

$$\mu_R = \frac{M^4 - (m_q - m_Q)^2}{4M^3},$$
and $b^2(M)$ denotes the on-mass-shell relative momentum squared

$$b^2(M) = \frac{[M^2 - (m_q + m_Q)^2][M^2 - (m_q - m_Q)^2]}{4M^2}.$$ 

The kernel $V(p,q;M)$ is the quasipotential operator of the quark-antiquark interaction. It is constructed with the help of the off-mass-shell scattering amplitude, projected onto the positive energy states.

We have assumed that the effective interaction is the sum of the usual one-gluon exchange term and the mixture of vector and scalar linear confining potentials. The quasipotential is then defined by

$$V(p,q;M) = \bar{u}_q(p)\bar{u}_Q(-p)\left\{4\alpha_s D_{\mu\nu}(k)\gamma^\mu q^\nu + V^V_{\text{conf}}(k)\Gamma^q_{\mu}\Gamma_{Q\mu} + V^S_{\text{conf}}(k)(p,q;M)\right\}u_q(q)u_Q(-q),$$

where $\alpha_s$ is the QCD coupling constant, $D_{\mu\nu}$ is the gluon propagator in the Coulomb gauge and $k = p - q$; the Dirac spinor is given by

$$u^\lambda(p) = \sqrt{\frac{\epsilon(p) + m}{2\epsilon(p)}} \left(\frac{1}{\epsilon(p) + m}\right) \chi^\lambda$$

with $\epsilon(p) = \sqrt{p^2 + m^2}$. The effective long-range vector vertex has the form

$$\Gamma_{\mu}(k) = \gamma_{\mu} + \frac{i\kappa}{2m}\sigma_{\mu\nu}k^\nu, \quad k^0 = 0,$$

where $\kappa$ is the nonperturbative anomalous chromomagnetic moment of quarks. Vector and scalar confining potentials in the nonrelativistic limit reduce to

$$V^V_{\text{conf}}(r) = (1-\epsilon)(Ar + B), \quad V^S_{\text{conf}}(r) = \epsilon( Ar + B),$$

reproducing $V_{\text{conf}}(r) = V^V_{\text{conf}}(r) + V^S_{\text{conf}}(r) = Ar + B$, where $\epsilon$ is the mixing coefficient. The potential parameters are as follows:

- $A = 0.18$ GeV$^2$, $B = -0.30$ GeV, $\alpha_s(m_c) = 0.32$, $\alpha_s(m_b) = 0.22$.

The constituent quark masses are

- $m_b = 4.88$ GeV, $m_s = 0.50$ GeV, $m_c = 1.55$ GeV, $m_u,d = 0.33$ GeV.

The value of the mixing parameter $\epsilon = -1$ is fixed by matching the heavy quark expansion and from the consideration of charmonium radiative decays ($J/\psi \to \eta_c\gamma$). The anomalous chromomagnetic quark moment $\kappa = -1$ is determined from the heavy quark expansion and from fitting the fine splitting of heavy quarkonia $^3P_J$ states. The long range chromomagnetic contribution to the potential is proportional to $(1 + \kappa)$ and vanishes for $\kappa = -1$ in accord with the flux tube model.
HEAVY QUARK-ANTIQUARK POTENTIAL

With the account of retardation effects and one loop radiative corrections the spin-independent potential reads as [1]

\[
V_{SI}(r) = -\frac{4}{3} \frac{\alpha_V(\mu^2)}{r} + Ar + B - \frac{4}{3} \frac{\beta_0 \alpha_s^2(\mu^2)}{2\pi} \frac{\ln(\mu r)}{r} + \frac{1}{8} \left( \frac{1}{m_a^2} + \frac{1}{m_b^2} \right) \Delta \left[ -\frac{4}{3} \frac{\alpha_V(\mu^2)}{r} - \frac{4}{3} \frac{\beta_0 \alpha_s^2(\mu^2)}{2\pi} \frac{\ln(\mu r)}{r} \right] + (1 - \varepsilon)(1 + 2\kappa)Ar + \frac{1}{2m_am_b} \left\{ \frac{4}{3} \frac{\alpha_V}{r} \left[ \frac{p^2 + (p \cdot r)^2}{r^2} \right] \right\}_W - \frac{4}{3} \frac{\beta_0 \alpha_s^2(\mu^2)}{2\pi} \left\{ \frac{p^2 \ln(\mu r)}{r} + \frac{(p \cdot r)^2}{r^2} \left( \frac{\ln(\mu r)}{r} - 1 \right) \right\}_W + \frac{1 - \varepsilon}{2m_am_b} - \frac{\varepsilon}{4} \left( \frac{1}{m_a^2} + \frac{1}{m_b^2} \right) \left\{ Ar \left[ \frac{p^2 - (p \cdot r)^2}{r^2} \right] \right\}_W - \frac{\varepsilon \lambda_s}{2} \left[ \frac{1}{2} \left( \frac{1}{m_a^2} + \frac{1}{m_b^2} \right) + \frac{1}{m_am_b} \right] \left\{ Ar \left[ \frac{p^2 + (p \cdot r)^2}{r^2} \right] \right\}_W + \frac{1}{4} \left( \frac{1}{m_a^2} + \frac{1}{m_b^2} \right) + \frac{1}{m_am_b} \right\} Bp^2,
\]

(3)

where

\[
\alpha_V(\mu^2) = \alpha_s(\mu^2) \left[ 1 + \left( \frac{a_1}{4} + \frac{\gamma_E \beta_0}{2} \right) \frac{\alpha_s(\mu^2)}{\pi} \right],
\]

\[
\alpha_s(\mu^2) = \frac{4\pi}{\beta_0 \ln(\mu^2/\Lambda^2)}, \quad a_1 = \frac{31}{3} - \frac{10}{9} n_f, \quad \beta_0 = 11 - \frac{2}{3} n_f.
\]

Here \( n_f \) is a number of flavours, \( \mu \) is a renormalization scale and the subscript \( W \) means the Weyl ordering of operators.

The spin-dependent part of the quark-antiquark potential for equal quark masses (\( m_a = m_b = m \)) is given by [1]

\[
V_{SD}(r) = a \mathbf{L} \cdot \mathbf{S} + b \left[ \frac{3}{r^2} (\mathbf{S}_a \cdot \mathbf{r})(\mathbf{S}_b \cdot \mathbf{r}) - (\mathbf{S}_a \cdot \mathbf{S}_b) \right] + c \mathbf{S}_a \cdot \mathbf{S}_b,
\]

(4)

\[
a = \frac{1}{2m^2} \left\{ \frac{4\alpha_s(\mu^2)}{r^3} \left[ 1 + \frac{\alpha_s(\mu^2)}{\pi} \left[ \frac{1}{18} n_f - \frac{1}{36} + \gamma_E \left( \frac{\beta_0}{2} - 2 \right) + \frac{\beta_0}{2} \ln \frac{\mu}{m} \right] \right] + \left( \frac{\beta_0}{2} - 2 \right) \ln(mr) \right\} - \frac{A}{r} + 4(1 + \kappa)(1 - \varepsilon) \frac{A}{r},
\]

\[
b = \frac{1}{3m^2} \left\{ \frac{4\alpha_s(\mu^2)}{r^3} \left[ 1 + \frac{\alpha_s(\mu^2)}{\pi} \left[ \frac{1}{6} n_f + \frac{25}{12} + \gamma_E \left( \frac{\beta_0}{2} - 3 \right) + \frac{\beta_0}{2} \ln \frac{\mu}{m} \right] \right] + \left( \frac{\beta_0}{2} - 3 \right) \ln(mr) \right\} + (1 + \kappa)^2(1 - \varepsilon) \frac{A}{r},
\]

\[
c = \frac{4}{3m^2} \left( \frac{8\pi \alpha_s(\mu^2)}{3} \left[ 1 + \frac{\alpha_s(\mu^2)}{\pi} \left( \frac{23}{12} - \frac{5}{18} n_f - \frac{3}{4} \ln 2 \right) \right] \delta^3(r) \right).
\]
\[ 
\begin{align*}
+ \frac{\alpha_s(\mu^2)}{\pi} \left[ -\frac{\beta_0}{8\pi} \gamma^2 \left( \frac{\ln(\mu/m)}{r} \right) + \frac{1}{\pi} \left( \frac{1}{12} n_f - \frac{1}{16} \right) \nabla^2 \left( \frac{\ln(mr + \gamma E)}{r} \right) \right] \\
+ (1 + \kappa)^2 (1 - \varepsilon) \frac{A}{r} \right) 
\end{align*}
\]

where \( L \) is the orbital momentum and \( S_{a,b}, S = S_a + S_b \) are the spin momenta. The total angular momentum is \( J = L + S \).

HEAVY QUARKONIUM MASS SPECTRA

The heavy quarkonium is similar to the positronium atom and its levels are usually specified by the notation \( n^{(2S+1)I_J} \), where \( n \) is the radial quantum number. The results of calculations of heavy quarkonium mass spectra on the basis of Eqs. (1), (3), (4) are presented in Tables 1 and 2 [1].

Recently the contribution of the finite \( c \)-quark mass to the bottomonium mass spectrum in one loop was considered in our model [4]. The correction to the \( b\bar{b} \) static poten-

TABLE 1. Charmonium Mass Spectrum (GeV).

| State \( (n^{(2S+1)I_J}) \) | Particle | Theory [1] | PDG(2000) | BES | CLEO | E835 |
|-----------------------------|----------|------------|-----------|-----|------|------|
| \( 1^3S_0 \) \( \eta_c \) | 2.979   | 2.979(18)   | 2.9763    | 2.9804 |
| \( 1^3S_1 \) \( J/\Psi \) | 3.096   | 3.09687(4)  |           |      |      |
| \( 1^3P_0 \) \( \chi_{c0} \) | 3.424   | 3.4150(8)   | 3.4141    | 3.4154 |
| \( 1^3P_1 \) \( \chi_{c1} \) | 3.510   | 3.51051(12) |           |      |      |
| \( 1^3P_2 \) \( \chi_{c2} \) | 3.556   | 3.55618(13) |           |      |      |
| \( 2^1S_0 \) \( \eta_c' \) | 3.583   | 3.594(5)    |           |      |      |
| \( 2^1S_1 \) \( \Psi' \) | 3.686   | 3.68596(9)  |           |      |      |
| \( 1^3D_1 \) | 3.798   | 3.7699(25)* |           |      |      |
| \( 1^3D_2 \) | 3.813   |             |           |      |      |
| \( 1^3D_3 \) | 3.815   |             |           |      |      |
| \( 2^3P_0 \) \( \chi_{c0} \) | 3.854   |             |           |      |      |
| \( 2^3P_1 \) \( \chi_{c1} \) | 3.929   |             |           |      |      |
| \( 2^3P_2 \) \( \chi_{c2} \) | 3.972   |             |           |      |      |
| \( 3^1S_0 \) \( \eta_c'' \) | 3.991   |             |           |      |      |
| \( 3^1S_1 \) \( \Psi'' \) | 4.088   | 4.040(10)*  |           |      |      |
| \( 2^3D_1 \) | 4.194   | 4.159(20)*  |           |      |      |
| \( 2^3D_2 \) | 4.215   |             |           |      |      |
| \( 2^3D_3 \) | 4.223   |             |           |      |      |

* Mixture of \( S \) and \( D \) states
TABLE 2. Bottomonium Mass Spectrum (GeV).

| State  | Particle | Theory | PDG(2000) | CLEO | MD1 |
|--------|----------|--------|----------|------|-----|
| $(n^2S^1L_J)$ | η_b | 9.400 | 9.46030(26) | 9.46051 | |
| $1^3S_1$ | Y | 9.460 | | | |
| $1^3P_0$ | Χ₀ | 9.864 | 9.8599(10) | 9.8600 | |
| $1^3P_1$ | Χ₁ | 9.892 | 9.8927(6) | 9.8937 | |
| $1^3P_2$ | Χ₂ | 9.912 | 9.9126(5) | 9.9119 | |
| $2^1S_0$ | η'₀ | 9.990 | | | |
| $2^3S_1$ | Υ' | 10.020 | 10.02326(31) | 10.0235 | |
| $1^3D_1$ |  | 10.151 | | | |
| $1^3D_2$ |  | 10.157 | | | |
| $1^3D_3$ |  | 10.160 | | | |
| $2^3P_0$ | Χ₀' | 10.232 | 10.2321(6) | | |
| $2^3P_1$ | Χ₁' | 10.253 | 10.2552(5) | | |
| $2^3P_2$ | Χ₂' | 10.267 | 10.2685(4) | | |
| $3^1S_0$ | η''₀ | 10.328 | | | |
| $3^3S_1$ | Υ'' | 10.355 | 10.3552(5) | | |
| $2^3D_1$ |  | 10.441 | | | |
| $2^3D_2$ |  | 10.446 | | | |
| $2^3D_3$ |  | 10.450 | | | |
| $3^3P_0$ | Χ₀'' | 10.498 | | | |
| $3^3P_1$ | Χ₁'' | 10.516 | | | |
| $3^3P_2$ | Χ₂'' | 10.529 | | | |
| $4^1S_0$ | η''''₀ | 10.578 | | | |
| $4^3S_1$ | Υ'''' | 10.604 | 10.5800(35) | | |

The potential due to $m_c \neq 0$ is approximately given by [5] ($a_0 = 5.2$, $\gamma_E = 0.5772\ldots$)

$$
\delta V(r) \approx -\frac{4}{9} \frac{\alpha^2}{\pi r} \left[ \ln(\sqrt{a_0 m_c} r) + \gamma_E + E_1(\sqrt{a_0 m_c} r) \right],
$$

$$
E_1(x) = \int_x^\infty \frac{dt}{t} e^{-t}.
$$

Averaging over solutions of Eq. (1) with the Cornell potential yields the following bottomonium mass shifts:

$$
\langle \delta V \rangle, \text{MeV} \quad -12 \quad -9.3 \quad -8.7 \quad -7.6 \quad -7.5 \quad -7.2.
$$

These shifts are within the estimates of theoretical uncertainties ($\sim 10$ MeV) and partially could be adsorbed either in the value of the constituent $b$ quark mass or in the constant term $B$ of the confining potential.
MASS SPECTRA OF B AND D MESONS

The $B$ and $D$ mesons are alike the hydrogen atom with the heavy quark $Q = b, c$ near the centre-of-mass and the light quark $q = u, d, s$ orbiting around it. Thus it is appropriate to make the expansion in the inverse heavy quark mass. In the limit of infinitely heavy quark ($m_Q \to \infty$) its mass and spin decouple and as a result heavy quark symmetry arises. In this limit the meson angular momentum is a sum of the orbital momentum $L$ and the light quark spin $S_q$ i.e. $J = L + S_q$. The $1/m_Q$ correction to the $Q\bar{q}$ potential depends on the heavy quark spin $S_Q$ and leads to the spin-spin interaction. The total angular momentum is $J = j + S_q$. Thus, for $S$-wave mesons there is a doublet of $j = 1/2$ states with $J^P = 0^-, 1^-$ ($P$ is the meson parity) which are degenerate in the limit $m_Q \to \infty$. For $P$-wave mesons there are similarly two doublets of initially degenerate states with $j = 1/2 (J^P = 0^+, 1^+)$ and $j = 3/2 (J^P = 1^+, 2^+)$. The $j = 1/2$ levels are expected to be broad because they decay in an $S$-wave, while the $j = 3/2$ levels should be narrow since they decay in a $D$-wave.

The heavy-light quark-antiquark potential in configuration space in the limit $m_Q \to \infty$ reads as [2] $(V_{\text{Coul}}(r) = -(4/3)\alpha_s/r, \hspace{0.5cm} \alpha_s = 0.5\text{ for } B, D; \hspace{0.5cm} \alpha_s = 0.45\text{ for } B_s, D_s)$

\[
V_{m_Q \to \infty}(r) = \frac{E_q + m_q}{2E_q} \left[ V_{\text{Coul}}(r) + V_{\text{conf}}(r) + \frac{1}{(E_q + m_q)^2} \left( \mathbf{p} [\tilde{V}_{\text{Coul}}(r) \right. \right.
\]
\[
\left. + V_{V}^{\prime}(r) - V_{V}^{\prime}(r)\mathbf{p} - \frac{E_q + m_q}{2m_q} \Delta V_{\text{conf}}^{\prime}(r) \right] 1 - (1 + \kappa) \right]
\]
\[
+ 2 \left( V_{\text{Coul}}(r) - V_{\text{conf}}^{\prime}(r) - V_{V}^{\prime}(r) \left[ \frac{E_q}{m_q} - 2(1 + \kappa) \frac{E_q + m_q}{2m_q} \right] \right) \mathbf{LS}_q \right] \right).
\]

The $1/m_Q$ correction to the potential (5) is given by [2]

\[
\delta V_{1/m_Q}(r) = \frac{1}{E_q m_Q} \left\{ \mathbf{p} [V_{\text{Coul}}(r) + V_{\text{conf}}^{\prime}(r)] \mathbf{p} + V_{\text{Coul}}^{\prime}(r) \frac{L^2}{2r} \right.
\]
\[
- \frac{1}{4} \Delta V_{\text{conf}}^{\prime}(r) + \left[ \frac{1}{r} V_{\text{Coul}}^{\prime}(r) + \frac{(1 + \kappa)}{r} V_{\text{conf}}^{\prime}(r) \right] \mathbf{L} \mathbf{S} \right.
\]
\[
+ \frac{1}{3} \left[ \frac{1}{r} V_{\text{Coul}}^{\prime}(r) - V_{\text{conf}}^{\prime}(r) + (1 + \kappa)^2 \left[ \frac{1}{r} V_{\text{conf}}^{\prime}(r) - V_{V}^{\prime}(r) \right] \right] \left[ \Delta V_{\text{Coul}}(r) + (1 + \kappa)^2 \Delta V_{\text{conf}}^{\prime}(r) \right] \mathbf{S}_Q \mathbf{S}_q \right\}.
\]

Here the prime denotes differentiation with respect to $r$, $L$ is the orbital momentum, $S_q$ and $S_Q$ are the spin operators of the light and heavy quarks, $S = S_q + S_Q$ is the total spin.

First Eq. (1) is solved numerically with the complete potential (5) and then the corrections (6) is treated perturbatively. The results of the calculation of heavy-light meson mass spectra are presented in Tables 3-6 and in Figs. 1-4 [2].

From Figs. 1-4 it follows that in the limit $m_Q \to \infty$ the $P$-wave doublet with $j = 1/2$ lies higher than the one with $j = 3/2$ (abnormal level ordering), i.e. these doublets are inverted. For finite $m_Q$ the hyperfine splitting of the doublet states makes this picture
TABLE 3. Mass Spectrum of $D$ Mesons (GeV).

| State | Particle | Theory [2] | PDG(2000) | CLEO | DELPHI |
|-------|----------|------------|-----------|------|--------|
| $1S_0$ | $D$      | 1.875      | 1.8693(5) |      |        |
| $1S_1$ | $D^*$    | 2.009      | 2.0100(5) |      |        |
| $1P_2$ | $D_2$    | 2.459      | 2.459(4)  |      |        |
| $1P_1$ | $D_1$    | 2.414      | 2.4222(18)|      |        |
| $1P_1$ | $D_1$    | 2.501      | 2.461$^{+0.041}_{-0.034}$ ± 0.01 ± 0.032 |      |        |
| $1P_0$ | $D_0$    | 2.438      |          |      |        |
| $2S_0$ | $D'$     | 2.579      |          |      |        |
| $2S_1$ | $D^{*'2}$| 2.629      |          |      |        |

TABLE 4. Mass Spectrum of $D_s$ Mesons (GeV).

| State | Particle | Theory [2] | PDG(2000) | FOCUS |
|-------|----------|------------|-----------|-------|
| $1S_0$ | $D_s$    | 1.981      | 1.9686(6) |       |
| $1S_1$ | $D^{*} _s$ | 2.111      | 2.1124(7) |       |
| $1P_2$ | $D^{*} _s$ | 2.560      | 2.5735(17)| 2.5673(13) |
| $1P_1$ | $D_{s1}$  | 2.515      | 2.5355(60)| 2.5351(6) |
| $1P_1$ | $D_{s1}$  | 2.569      |          |       |
| $1P_0$ | $D^{*} _{s0}$ | 2.508      |          |       |
| $2S_0$ | $D'$     | 2.670      |          |       |
| $2S_1$ | $D^{*'2}$| 2.716      |          |       |

much more complicated (some of the levels from different doublets overlap). Only for $B$ mesons the pure inversion is preserved.

In the limit $m_Q \to \infty$ heavy quark symmetry predicts simple relations between the spin-averaged masses of $B$ and $D$ states

$$\bar{M}_{B_1} - \bar{M}_{D_1} = \bar{M}_{B_{s1}} - \bar{M}_{D_{s1}} = \bar{M}_{B_s} - \bar{M}_{D_s} = \bar{M}_B - \bar{M}_D = m_b - m_c = 3.33 \text{ GeV}, \quad (7)$$

where $\bar{M}_{B_1} = (3M_{B_1} + 5M_{B_s})/8$, $\bar{M}_B = (M_B + 3M_{B^*})/4$, etc. From Tables 3-6 the following values (in GeV) for the calculated mass differences can be obtained

$$\bar{M}_{B_1} - \bar{M}_{D_1} \quad \bar{M}_{B_{s1}} - \bar{M}_{D_{s1}} \quad \bar{M}_{B_s} - \bar{M}_{D_s} \quad \bar{M}_B - \bar{M}_D$$

3.29 \quad 3.30 \quad 3.34 \quad 3.33

Thus relations (7) are satisfied with good accuracy.

The hyperfine mass splittings of the initially degenerate $P$ states

$$\Delta M_B \equiv M_{B_2} - M_{B_1}, \quad M_{B_1} - M_{B_0}; \quad \Delta M_D \equiv M_{D_2} - M_{D_1}, \quad M_{D_1} - M_{D_0}$$

should scale with heavy quark masses: $\Delta M_B = (m_c/m_b)\Delta M_D$ and the same for $B_s$ and $D_s$ mesons. Our model predictions for these splittings are displayed in Table 7 [2].
TABLE 5. Mass Spectrum of $B$ Mesons (GeV).

| State | Part. | Theor.[2] | PDG(2000) | OPAL | L3  | DELPHI | CDF  | ALEPH |
|-------|-------|-----------|-----------|------|-----|--------|------|-------|
| $1S_0$ | $B$ | 5.285 | 5.2790(5) |
| $1S_1$ | $B^*$ | 5.324 | 5.3250(6) |
| $1P_2$ | $B^*_2$ | 5.733 | 5.768(8)? | 5.732(21) | 5.739(13) |
| $1P_1$ | $B_1$ | 5.719 | 5.738(9) | 5.71(2) |
| $1P_1$ | $B_1^*$ | 5.757 | 5.670(16)? |
| $1P_0$ | $B_0^*$ | 5.738 | 5.839(14) |
| $2S_0$ | $B'$ | 5.883 |
| $2S_1$ | $B^{'*}$ | 5.988 | 5.90(2)? |

TABLE 6. Mass Spectrum of $B_s$ Mesons (GeV).

| State | Particle | Theory [2] | PDG(2000) | OPAL |
|-------|----------|------------|-----------|------|
| $1S_0$ | $B_s$ | 5.375 | 5.3696(24) |
| $1S_1$ | $B^*_s$ | 5.412 | 5.4166(35) |
| $1P_2$ | $B^*_2$ | 5.844 | 5.853(15) |
| $1P_1$ | $B^*_{s1}$ | 5.831 |
| $1P_1$ | $B_{s1}$ | 5.859 |
| $1P_0$ | $B^*_{s0}$ | 5.841 |
| $2S_0$ | $B'_s$ | 5.971 |
| $2S_1$ | $B^{'*}_s$ | 5.984 |

CONCLUSIONS

So one may conclude that for heavy quarkonium mass spectra most of theoretical calculations are generally in good agreement with high-precision experimental data. This makes possible an accurate determination of fundamental parameters governing the dynamics of the heavy $Q\bar{Q}$ interaction. On the other hand, for heavy-light meson mass spectra the situation is much more indefinite and unclear in both theory and experiment. Not all models predict the $P$ level inversion. The broad $j = 1/2$ levels are very poorly studied due to difficulties in their observation. Much further work is required to remove discrepancies between existing experimental data.

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TABLE 7. Hyperfine Splittings of $P$ Levels (MeV).

| States | $\Delta M_D$ | $\frac{m_b}{m_c} \Delta M_D$ | $\Delta M_B$ | $\frac{m_b}{m_c} \Delta M_{B_D}$ | $\Delta M_{B_s}$ |
|--------|--------------|-----------------------------|--------------|-------------------------------|-----------------|
| $1P_2 - 1P_1$ | 45 | 14 | 14 | 45 | 14 | 13 |
| $1P_1 - 1P_0$ | 63 | 20 | 19 | 61 | 19 | 18 |
FIGURE 1. The ordering pattern of \( D \) meson states. The mass scale is in GeV.

FIGURE 2. The ordering pattern of \( D_s \) meson states. The mass scale is in GeV.

REFERENCES

1. Ebert, D., Faustov, R. N., and Galkin, V. O., Phys. Rev. D 62, 034014-1-11 (2000).
2. Ebert, D., Faustov, R. N., and Galkin, V. O., Phys. Rev. D 57, 5663-5669 (1998); 59, 019902 (1999) (Erratum).
FIGURE 3. The ordering pattern of $B$ meson states. The mass scale is in GeV.

FIGURE 4. The ordering pattern of $B_s$ meson states. The mass scale is in GeV.

3. Particle Data Group (PDG), *Eur. Phys. J. C* 15, 1-878 (2000).
4. Ebert, D., Faustov, R. N., and Galkin, V. O., in preparation.
5. Melles, M., *Phys. Rev. D* 62, 074019-1-14 (2000).