Probabilistic analysis of offshore geotechnical site investigation in a homogeneous stiff clay deposit

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Abstract. Unlike most onshore geotechnical projects, conducting offshore site investigations is often difficult and technically challenging due to the harsh ocean environment. Hence, an important cost consideration is minimizing the number of soundings for an offshore project while ensuring reliable and safe designs. This study examines the possibility of reducing the number of soundings at site by using practical numerical tools to predict cone tip resistance ($q_t$) at unsampled locations. Prediction errors were quantified within a probabilistic framework. Two prediction approaches, i.e. 2D linear regression and Kriging, were adopted. Site investigation data from an existing offshore wind farm with homogeneous stiff clay deposit were collected and analyzed. Result shows that in a homogeneous soil deposit, it is possible to use as little as 20% of the available data to predict the remaining 80% $q_t$ profiles with small error. Monopile deformation under typical design load was also calculated using shear strength derived from the $q_t$ profile at each wind turbine location. The small spatial variation of monopile deformation suggests that a reduction in the number of geotechnical soundings for sites with uniform soil conditions would not significantly impact the foundation design.

1. Introduction

Offshore geotechnical site investigation is very important in soil characterization and foundation design. However, offshore site investigation usually requires expensive special vessels and equipment and is time consuming. Hence it is desired to minimize the total number of soundings (e.g., boreholes, in situ testing, etc.) for a site investigation program while ensuring foundation safety. To achieve this goal, it is necessary to study the possibility of predicting soil properties at unsampled locations with satisfactory accuracy, which can help reduce the number of soundings. Many studies have proposed geostatistical or probabilistic models to predict soil properties at unsampled locations (e.g. [1][2][3][4][5][6][7][8][9][10]). However, there is only a limited number of studies focused on probabilistic analysis of offshore geotechnical site investigation data (e.g. [9][10]).

This study focuses on applying practical numerical tools to carry out prediction of soil properties at unsampled locations using cone tip resistance ($q_t$) measured from piezocone penetration test (CPTU) in an offshore homogeneous stiff clay deposit. Two-dimensional linear regression and Kriging approach [11] were used for prediction of slope and intercept to construct linear $q_t$ profile. To examine the effectiveness of each approach, a Monte Carlo framework was created to randomly sample the number of CPTU soundings used in prediction. The prediction error at each CPTU location was computed by introducing a concept of root mean square error ratio. The relationship between number of soundings and prediction error was studied in detail. Lastly, monopile design examples were
analyzed and monopile deformation at each wind turbine location was calculated to evaluate spatial variation of pile deformation and its sensitivity to number of soundings. The proposed probabilistic framework can help decision making during preliminary design process when limited number of soundings are available and assumptions of soil profiles at unsampled locations need to be made for preliminary foundation design.

2. Interpretation of CPTU data

2.1. Greater Gabbard wind farm
Greater Gabbard wind farm is a 504 MW wind farm in UK and located 23 kilometers off the coast of Suffolk. It covers an area of 146 km². The major soil deposit at Greater Gabbard wind farm is stiff London Clay. The layout of Piezocone Cone Penetration Test (CPTU) at Greater Gabbard wind farm site [12] is shown in figure 1. For convenience only relative coordinates are shown. The total number of CPTU soundings is 96. Typically, each wind turbine location has one CPTU sounding. As shown in figure 1, the distance between two nearest-neighbor CPTU soundings is quite uniform and varies from 800 m to 1100 m.

![Figure 1. CPTU layout at Greater Gabbard (Note: squared data points are locations of qₜ examples shown in Figure 2).](image)

2.2. Soil behavior type index (Iₑ)
Piezocone Cone Penetration Test (CPTU) generally measures three important parameters, i.e. cone tip resistance, sleeve friction and excess pore water pressure. Using these parameters, the soil behavior type index (Iₑ) at each depth can be computed using equations proposed by Robertson [13]. The soil behavior type index (Iₑ) is an indicator of soil mechanical behavior derived from CPTU parameters using empirical equations. A detailed description of soil types corresponding to various soil behavior type index (Iₑ) is shown in table 1. The critical value of Iₑ that differentiates clayey soils from sandy soils is 2.6.
Table 1. Soil behavior type (SBT) chart (modified from [13]).

| Zone | Soil Behavior Type                                      | $I_c$  |
|------|--------------------------------------------------------|--------|
| 1    | Sensitive, fine grained                                | N/A    |
| 2    | Organic soils – clay                                   | > 3.60 |
| 3    | Clays – silty clay to clay                             | 2.95 – 3.60 |
| 4    | Silt mixtures – clayey silt to silty clay              | 2.60 – 2.95 |
| 5    | Sand mixtures – silty sand to sandy silt               | 2.05 – 2.60 |
| 6    | Sands – clean sand to silty sand                       | 1.31 – 2.05 |
| 7    | Gravelly sand to dense sand                            | < 1.31 |
| 8    | Very stiff sand to clayey sand*                        | N/A    |
| 9    | Very stiff, fine grained*                              | N/A    |

* heavily overconsolidated or cemented

Figure 2 shows three example $q_t$ profiles. The location of these CPTU examples is highlighted in black squares in figure 1. The average CPTU penetration depth is 20 m. In each subplot of these CPTU profiles, the data points are plotted with different marker types based on the $I_c$ values calculated, thus the variation of soil behavior type through depth is easily identified. It can be seen that the majority of $I_c$ is greater than 2.6 and the $q_t$ profiles have small variations, indicating that the soil deposit is fairly homogeneous. All three $q_t$ profiles have obvious linear increasing trend with an increasing rate equal to 0.11 to 0.12 MPa/m. There are a few exceptions in soil behavior type where sandy material is presented at shallow depth, e.g. CPTU No. 4 and No. 52 at depth less than 1 m. However, as these sandy deposits are very thin and they only occur at a few CPTU locations, they were not considered in the subsequent analyses and generally would not be of concern for deep foundation design. Also shown in figure 2 is that 1D linear regression was performed to each $q_t$ profile. The slope and intercept values obtained for each $q_t$ profile completely describe the trend of each $q_t$ profile through depth.

2.3. Slope and intercept values of linearized $q_t$ profile

In practice, 1D linear regression to $q_t$ profile is frequently used by engineers as a first step to interpret soil properties and derive soil strength parameters versus depth for foundation design. As noted above, soil strength parameters for design can be estimated using CPTU parameters. To take into account soil uncertainty over the wind farm, usually a linear lower bound and a linear upper bound of soil properties (e.g. undrained shear strength) are proposed based on variability of data and engineer’s experience [14][15]. Both the upper and lower bounds are often required in foundation design. For example, the upper bound of soil undrained shear strength is necessary for calculation of required drivability for pile installation while the lower bound is needed for calculation of pile capacity. It is obvious that the best fit, the lower and upper bounds of $q_t$ profile can be fully described using respective slope and intercept values. In the following sections, the slope and intercept values of 1D linear regression to $q_t$ profile will be analyzed using different prediction models and the uncertainties of prediction by each model will be quantified.

Figure 3 shows the spatial distribution of slope values obtained from 1D linear regression of $q_t$ profiles at Greater Gabbard wind farm. The number of data points in Greater Gabbard is 96. As shown in figure 3, the mean of slope is 0.11 MPa/m while the standard deviation is only 0.02 MPa/m, indicating again that the soil deposits are very homogeneous across the site. The mean and standard deviation of intercept at Greater Gabbard wind farm are respectively 1.62 MPa and 0.29 MPa.
Figure 2. Examples of 1D linear regression to $q_t$ profiles.

Figure 3. Spatial distribution of slope values of 1D linear regression to $q_t$. 
3. Probabilistic analysis of $q_I$ prediction

In this section 2D linear regression and Kriging are used for prediction of slope and intercept values at "unsampled" locations, which have actual CPTU soundings but were excluded in the prediction analysis and used for verification only. The predicted slope and intercept values are then used to construct linear $q_I$ profiles. The prediction models used are straightforward and can be easily implemented in practice.

3.1. Introduction of probabilistic framework

A probabilistic framework is proposed to study the influence of number of CPTU soundings on prediction accuracy of linearized $q_I$ profile at unsampled locations. In order to quantify the accuracy of prediction models, the CPTU data are divided into two sections, i.e. training datasets and testing datasets. The training datasets are used to construct the prediction models while the testing datasets are used for calculation of prediction error. The probabilistic framework that incorporates prediction model with Monte Carlo simulation is described in the following section.

Assuming the total number of $q_I$ datasets is $n$, then within each Monte Carlo simulation,
1. Perform 1D linear regression to all $q_I$ profiles to obtain best linear fit $y$ and corresponding slope and intercept values. This step needs to be carried out only once.
2. Randomly divide $q_I$ datasets into $m$ training datasets and $(n-m)$ testing datasets.
3. As shown in figure 4a, calculate root mean square errors ($RMSE_a$) between best linear fit $y$ and actual $q_I$ using equation (1) in testing datasets.

$$RMSE_a = \left[ \frac{1}{K} \sum_{i=1}^{K} (y_i - q_{ti})^2 \right]^{1/2}$$  \hspace{1cm} (1)

where $i$ represents counter of data points that linearly increases with depth, $q_{ti}$ is the measured $q_I$ value at point $i$, $y_i$ is the best linear fit value at point $i$, and $K$ is the total number of data points in each $q_I$ profile.
4. Construct prediction models using slope and intercept values in training datasets.
5. Estimate slope and intercept values using previously constructed prediction models, and
   a. compute linear $q_I$ profiles ($yp$) for testing datasets (figure 4b).
   b. compute lower and upper bounds (i.e. $yp_L$ and $yp_U$) for testing datasets. As shown in figure 4c, the lower bound $yp_L$ is calculated using lower bound values of slope and intercept computed using ($\mu-\sigma$), where $\mu$ is the predicted mean of slope or intercept and $\sigma$ is the standard deviation of slope or intercept values in training datasets. Similarly, the upper bound $yp_U$ is calculated using ($\mu+\sigma$).
6. Calculate root mean square error ($RMSE_{p1}$) between $yp$ and actual $q_I$ profiles in testing datasets using equation (2)

$$RMSE_{p1} = \left[ \frac{1}{K} \sum_{i=1}^{K} (yp_i - q_{ti})^2 \right]^{1/2}$$  \hspace{1cm} (2)

where $yp_i$ is the predicted $q_I$ value at point $i$.
7. Calculate root mean square error ($RMSE_{p2}$) between the area bounded by $yp_L$ and $yp_U$ and actual $q_I$ profiles in testing datasets using equation (3) as shown in figure 4c

$$RMSE_{p2} = \left[ \frac{1}{K} \sum_{i=1}^{K} \varepsilon_i^2 \right]^{1/2}$$  \hspace{1cm} (3)

if $yp_L > q_{ti}$, $\varepsilon_i = yp_L - q_{ti}$;
if $yp_U < q_{ti}$, $\varepsilon_i = yp_U - q_{ti}$;
if $yp_L \leq q_{ti} \leq yp_U$, $\varepsilon_i = 0$.

where $yp_L$ and $yp_U$ are the predicted lower and upper bounds of $q_I$ at point $i$, respectively. It is worth mentioning that when the actual measurement falls within the predicted bounds, the error is taken as zero. Hence, the overall predicted error ($RMSE_{p2}$) could be smaller than the error in best fit ($RMSE_a$).
8. Normalized $RMSE_{p1,2}$ to $RMSE_a$ to obtain root mean square error ratio ($RMSE_r$) for testing datasets as the following

$$RMSE_r = \frac{RMSE_{p1,2}}{RMSE_a}$$  \hspace{1cm} (4)
After 200 Monte Carlo simulations, summarize the mean and standard deviation of $\text{RMSE}_r$. From equation (4) it can be seen that $\text{RMSE}_r$ measures the difference between predicted linear profile and best fit linear profile.

![Figure 4](image-url)

**Figure 4.** Definition of prediction error calculated with respect to best linear fit, predicted linear mean and predicted area bounded by upper and lower bounds.

### 3.2. 2D linear regression

Two-dimensional linear regression is the simplest model to predict parameters of interests at unsampled locations in a three-dimensional space. For illustration purpose, 2D linear regression to the slope values from all 96 corrected cone tip resistance ($q_t$) datasets at Greater Gabbard wind farm is presented in figure 5. As mentioned previously, the mean of slope is 0.11 MPa/m and the standard deviation of slope is 0.02 MPa/m. In addition to the best fit plane obtained from 2D linear regression (medium plane in figure 5), upper and lower bounds of the prediction are also plotted. Over 77% of the data points fall within the bounds. Clearly the slope parameter has small variation across the site. The same approach was also applied to intercept values. Linear $q_t$ profile was then constructed and $\text{RMSE}_r$ at each CPTU location was calculated.

The influence of number of soundings on prediction error ($\text{RMSE}_r$) using 2D linear regression in Greater Gabbard wind farm was studied using the proposed probabilistic approach. Results of Greater Gabbard wind farm in figure 6 shows that when the medium plane is used for prediction of slope and intercept parameters, $\text{RMSE}_r$ decreases with increasing number of soundings but remains greater than 1. On the other hand, the $\text{RMSE}_r$ can be smaller than 1 when upper and lower bounds are considered. This is because $\text{RMSE}_r$ is taken as zero when data points fall within predicted bounds, as shown in figure 4c. Both curves in figure 6 show that the change of $\text{RMSE}_r$ remains insignificant as the number of soundings becomes greater than (say) 20, which is much smaller than the total number of
soundings. Hence, the 2D linear regression approach is very effective in predicting $q_t$ profile using small number of datasets and achieving good accuracy. Although this seems promising, it should be reminded that Greater Gabbard wind farm has homogeneous soil deposit, and the effectiveness of 2D linear regression approach in complex soil conditions requires more research, which is beyond the scope of this paper.

![Figure 5](image1.png)

**Figure 5.** Example of 2D linear regression (medium plane, upper and lower bounds) to slope values. (Note: black dots are data points out of bounds)

![Figure 6](image2.png)

**Figure 6.** Influence of number of soundings on mean of $RMSE_r$ using 2D linear regression.

### 3.3. Kriging

Kriging approach is a geostatistical tool frequently used in geotechnical community for prediction of soil properties at location of interest using limited data (e.g. [5][6][2]). It provides estimation of both mean and standard deviation of a random variable. The major difference between Kriging [11] and previous approach is that Kriging considers spatial correlation structure of soil. The mean trend determined using 2D linear regression was first removed before analysis. This step is usually known as
detrending and is necessary when data show observable spatial trend. Kriging was then applied to the residuals. Assuming the residuals were stationary, method of moment was applied to estimate the scale of fluctuation of slope and intercept, respectively. The estimated values of scale of fluctuation for slope and intercept were 400 m and 1074 m for Greater Gabbard wind farm. Since slope and intercept values were derived from 1D linear regression, they are in fact spatially averaged parameters. Hence, the scale of fluctuation of these parameters are much larger compared to horizontal scale of fluctuations of soil properties reported by [16][17][18][19], which are in the order of 10 ~ 100 m.

Figure 7a shows the predicted mean of slope by Kriging using the whole dataset. Theoretically, Kriging estimation is exact at training data location, meaning that the Kriging surface will pass through the training data points. However, it can be noticed that some data points are not exactly resting on the surface. This is due to the numerical inconsistency between the coordinates of the regularly spaced grid used to generate Kriging results and the irregularly spaced coordinates of the actual CPTU locations. Nevertheless, this minor inconsistency does not affect the interpretation. The standard deviation of slope by Kriging for Greater Gabbard wind farm, as shown in figure 7b, is essentially zero at CPTU locations and gradually reach maximum values of 0.016 MPa/m as it moves further away from the CPTU locations.

Figure 7. Kriging estimation of slope using whole dataset: (a) mean of slope; (b) standard deviation of slope.
The influence of a number of CPTU soundings on mean of $RMSE_r$ using Kriging is presented in figure 8. The results of 2D linear regression in figure 6 are also plotted for comparison purpose. It shows that in all cases Kriging and 2D linear regression give very similar results. This is because the scale of fluctuations of slope and intercept values are comparable or even smaller than the nearest-neighbor distance between CPTU locations. In other words, the autocorrelation structure among data is not strong due to large separation distance. Therefore, in this case 2D linear regression performs better in terms of effectiveness and simplicity.

4. Spatial variation of monopile deformation under design load

It is of interest to see the spatial variation of pile foundation deformation under typical design load in this homogeneous stiff clay deposit. In this section, it is assumed that monopiles with uniform size with length of 35 m and diameter of 7.2 m were constructed in all CPTU locations. However, the diameter was slightly adjusted to ensure the majority of the pile head rotations over the site are less than 0.25°, which is a rotation limit suggested by DNV standard [20]. The wall thickness was assumed 0.078 m, which meets the minimum wall thickness criterion suggested by the API standard [21]. All piles are assumed to be subjected to a combined loading condition with horizontal force $H = 16$ MN and moment $M = 562$ MN·m at the pile head [22]. A finite element program coupled with $p$-$y$ curves for stiff clay proposed by [23] was developed to estimate pile response according to the API standard. Figure 9 shows the analysis results for CPTU No. 1, where the pile was loaded incrementally until design load scenario. The ultimate pile head lateral displacement and rotation are 0.034 m and 0.24°, respectively. The pile deformation analysis results for the whole site are shown in figure 10. The mean and standard deviation of pile lateral displacement are 0.030 m and 0.004 m, while the mean and standard deviation of pile head rotation are 0.22° and 0.01°, respectively. Clearly, the spatial variation of pile head lateral displacement and rotation are both very small (COV ≃ 10%), which suggests that a reduction in the number of soundings is possible in the homogeneous soil deposit.
Figure 9. Example of finite element analysis result using p-y curve for CPTU No. 1.
5. Conclusion

Geotechnical site characterization is an essential task for offshore wind turbine foundation design. The large spatial coverage of offshore wind farm poses a challenge to the determination and prediction of soil properties at unsampled locations. Hence, it is desired to quantify the uncertainty in offshore geotechnical site characterization data to aid decision makers. This paper developed a probabilistic framework that combines two prediction models and Monte Carlo simulation to examine the prediction error of associated models. Site investigation data from an existing offshore wind farm were collected and analyzed. The following conclusions can be drawn:

1. The 2D linear regression model and Kriging are able to predict 80% of the $q_t$ profile with sufficient accuracy using only 20% of the CPTU datasets for the case study used which consists of a homogeneous soil deposit. Increasing the number of CPTU soundings does not necessarily reduce prediction error. This result provides engineers with confidence to carry out preliminary foundation design using limited early-stage site investigation data, which may not align with final wind turbine locations. However, the exact percentage of possible reduction will likely depend on the variability of the site and the influence of reduction of soundings on foundation design needs further study.

2. The 2D linear regression approach performs equally accurate compared to Kriging in most circumstances, suggesting that the autocorrelation between soil properties is negligible due to large separation of CPTU soundings.

3. In terms of effectiveness and accuracy, 2D linear regression is the preferred approach to implement in practice for homogeneous soil cases.

4. In the homogeneous soil deposit studied, the pile deformation over the site with uniform pile dimension varied little under typical design loads. Therefore, using fewer soundings to derive soil properties is unlikely to pose significant risk to pile safety in foundation design.
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