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To cite this article: D Pietras et al 2018 IOP Conf. Ser.: Mater. Sci. Eng. 416 012085

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Impact of Exploitation Flaws on Load Capacity of Tempered Glass Stairs Assessed by Numerical Analysis with XFEM

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Abstract. Nowadays the structures made of tempered glass are widespread in civil engineering. The most popular are building facades and self-supporting stairs, where the glass is applied to carry the service loadings. In structures such as stairs the exploitation process leads to surface damage. The flaws emerged on the surface cause significant reduction of the structure load capacity. Therefore, it is purposeful to acquire the knowledge about the impact of flaws on glass strength to increase the safety of civil structures. The main aim of this paper is to assess the load capacity of the tempered glass slab with a flaw on the surface which is the major element of stairs made of glass. In order to do this a new numerical model of the one-step stair was built in ABAQUS. The residual stress was calculated by modelling of the quenching process. An initial flaw was introduced to the stair model after the tempering process. The conducted calculations in bending state of considered slab indicated that the flaw 30 x 2 x 0.3 mm in cantilever beam 1000 x 300 x 10 mm could be treated as critical.

1 Introduction

Each part of the building construction is exposed to wear due to environmental influence or attrition. The glass is sensitive to small cracks which are the exploitation result. When the part made of glass is carrying the loadings, the knowledge of response to applied loadings is important to increase the safety. This paper aims to calculate the real strength of tempered glass construction parts which contain the flaws. The glass fracture toughness was analysed in [1]. The method to improve mechanical properties of soda-lime-silica glass introduces the residual stress by tempering or ion exchange. Since the desired surface compressive stress generated in chemical methods has relatively small depth [2], in the civil engineering the most popular is tempering process. The phenomena occurring in the material during the tempering process were presented in [3-5].

The tempered glass stairs usually are constructed as laminar. Analytical approach to analyse laminated glass elements was presented in [6, 7]. The finite elements to analyse the layered glass plates were presented in [8]. The surface tolerance of tempered glass is worthless, due to this fact the development of methods to assess the strength of construction parts with a scratched surfaces are desired by construction industry. The analyses in this paper were performed in macro-scale, example of analyses in another scales are presented in [9-11]. The crack sensitivity of building materials may be analysed by using of artificial neural networks [12]. The analyses of behaviour of popular building materials in shear mode is the subject of test of many researchers [13].

2 Materials and methods

The soda-silica-lime glass which is the most popular in civil engineering applications was considered. The numerical analyses were conducted in ABAQUS finite element method. The dimensions of analysed glass stair were 1000 mm x 300 mm x 10 mm (figure 1). Homogenous slab was considered.
The residual stress was introduced by modelling of the tempering process. The user subroutine was programmed to consider the viscous and temperature dependent properties of glass in cooling process. The detailed data about calculation of residual stress due to tempering process is shown in [3-5]. The boundary conditions during the tempering process are presented in figure 1.

The modelling of the tempering process was divided into two parts:
- analysis of the heat flow during the cooling process of considered slab,
- stress analysis for viscous material.

![Figure 1. Boundary conditions during the tempering process.](image)

The initial state was modelled as a stress free body at 670°C. The analysis was carried out using finite elements of the DC3D8 type in the HEAT TRANSFER step type with the option TRANSIENT. The cooling process of the slab by air with temperature 20°C was analysed. A heat transfer coefficient was assumed to be equal to \( h_1 = 200 \text{ W/m}^2\text{K} \) for surfaces perpendicular to the direction of heat flow and 100 \( \text{W/m}^2\text{K} \) for parallel surfaces. It was assumed that the cooling device was blowing the air from top and bottom (along \( x_1 \) axis, figure 1). As a signal to complete the analysis, the temperature change was chosen by less than 0.02°C in the next time step. The maximum temperature change in the next step was set to 40°C. The initial time was set to \( 10^{-6} \text{ s} \). The cooling time that equalled 583.659 s was obtained. The solution was found in 71 increments.

The previously obtained temperature fields in time domain were imported to the analysis of the stress state made with the VISCO step type. During tempering the considered slab was supported by locking the displacements in \( x_1 \) direction in bottom surface as well as for displacements on edges A and B in \( x_2 \) and \( x_3 \) directions, respectively (figure 1). The tolerance parameter of deformation resulting from creep parameter was assumed to be 0.005. The obtained residual stress was imported in a scratched stair analysis.

To calculate the stress intensity factors the Extended Finite Element Method [XFEM] was used. The considered flaw had a rectangular cross-section and dimensions of 30 mm x 0.3 mm, which was based on the observation of the acceptable flaws declared by many manufacturers. The dimensions of considered flaw were much smaller than the size of the analysed stair, due to this fact the submodelling method was applied. The element being analysed had a flaw in the centre of the panel perpendicular to the length of the element (figure 2). A single flaw was considered to obtain a clear result of the defect in the glass slab. The size of the mesh of finite elements was equal to 3 mm for general model and 0.3 mm for submodel was assessed in mesh study analysis made for simple cases and compared to reference values [14]. It is well known that considered crack position is not critical, but these the crack position was chosen due to high probability occurrence of the exploitation flaws in this area. The mesh of the submodel was 3 times densified in the crack vicinity.
The boundary conditions of considered bending state are shown in figure 2. The mechanical load $F$ was applied as the SURFACE TRACTION to avoid the undesirable concentrations.

![Figure 2. Boundary conditions in bending state.](image)

The residual stress state in volume element of the considered stair is shown in figure 3. The residual stress tensor component $\sigma_{11}$ has small values, due to this fact it can be considered as plane stress. After the superposition of the stress state caused by bending in upper surface stress state may be represented by figure 4. When the stress tensor component $\sigma_{22}$ takes positive values the tension state in direction $x_2$ emerges. In tensioned surface the crack is able to grow until the slab failure. As shown in figure 4, when $\sigma_{22}$ reaches positive values the element begins to work in complex stress state. Therefore, the fracture analyses have to take into account the mix-mode model of fracture.

![Figure 3. Residual stress state near the surface of the slab.](image)  
![Figure 4. Stress state on the surface of the slab in bending conditions.](image)

To calculate the stress intensity factors the linear fracture mechanics theory was applied. The contact between the flaw surfaces was omitted because of its high thickness (2 mm). The mix-mode fracture criterion for soda-lime-silica glass was taken from [15].
\[
\frac{K_d}{K_c} \left( \frac{K_L}{K_c} \right) = -7.371 \left( \frac{K_L}{K_c} \right)^5 + 12.980 \left( \frac{K_L}{K_c} \right)^4 - \\
+ 7.927 \left( \frac{K_L}{K_c} \right)^3 + 1.231 \left( \frac{K_L}{K_c} \right)^2 - 0.073 \left( \frac{K_L}{K_c} \right) + 1.160
\]

(1)

3 Results and discussion

3.1 Residual stresses

The temperature timing during the tempering process is shown in figure 5. The cooling of the glass slab is faster on the surface than inside, which is in accordance with the practice. The cooling process duration to room temperature is approximately 300 s.

![Figure 5. Temperature timing during tempering process.](image)

The development of residual stress is shown in figure 6. The stress is increasing nonlinearly during the whole cooling process.

![Figure 6. The residual stress growth during the tempering process.](image)

Derived stress tensor components \( \sigma_{11} \) and \( \sigma_{22} \) distribution across the considered slab is shown in figure 7.
3.2 Bending state of the considered stair

In table 1 the achieved results of numerical analyses were collected. In the calculations were taken into account the residual stress and applied force $F$ simultaneously. The considered crack shape is a cuboid 30x2x0.3 mm, therefore the crack front is a rectangle. The calculated stress intensity factors are the mean values along the crack front. The sign in the value of $K_{I}$ and $K_{III}$ means the direction of shear state. The negative values of $K_{I}$ mean the compression state. When the $K_{I}$ has negative values the crack propagation is impossible.

| $F$ [kN] | $K_{I}$ [MPa√m] | $K_{II}$ [MPa√m] | $K_{III}$ [MPa√m] |
|----------|-----------------|-----------------|------------------|
| 1.35     | 3.845           | -0.034          | -0.002           |
| 1.65     | 7.816           | 0.435           | 0.002            |

On the basis of table 1 the ratios $K_{I}$ to $K_{c}$ and $K_{II}$ to $K_{c}$ were calculated. The relation of ratio $K_{II}/K_{c} (K_{I}/K_{c})$ can be described by linear function as follows:

$$\frac{K_{II}}{K_{c}} = 0.118 \left(\frac{K_{I}}{K_{c}}\right) - 0.642.$$  \hspace{1cm} (2)

The above relation is plotted in figure 8. The blue curve with squares is the fracture law described by formula (1). The slight red line is the response of the considered structure to applied loadings which is described by formula (2). The intersection point of the blue curve and red straight line means the crack growth.
Figure 8. Comparison of response of considered slab and fracture criterion.

The intersection point calculated by comparison of the formulas (1) and (2) is:

$$\left( \frac{K_I}{K_c} \right) = 0.881. \quad (3)$$

The maximal value of $K_I$ after consideration the impact of shearing mode is equal to $0.881 K_{II}$. The critical value of stress intensity factor $K_{IC}$ for glass used in civil engineering was estimated in [16] as equal to $0.76 \text{ MPa} \cdot \sqrt{\text{m}}$. The relation between $K_I$ and applied force $F$ based on table 1 is presented in figure 9.

Figure 9. Assessment of critical value of force $F$(see figure 2)

Dependence between $F$ and $K_I$ can be interpolated by using linear regression:

$$K_I(F) = 418.5F - 443.4. \quad (4)$$

When the $K_I$ reaches the critical value $K_c = 0.881 \cdot 0.76 \text{ MPa} \cdot \sqrt{\text{m}}$, the force $F$ takes the critical value $F_{c,\text{cracked}} = 1.11\text{kN}$. The critical value $F$ of uncracked slab in the middle span calculated by using simple Timoshenko’s beam theory in the middle span of the beam can be found on the base of the following equation:
\[ \frac{0.5 \cdot 6}{bh^2} LF_{c\text{, uncracked}} - \sigma_{\text{res}}^{\text{max}} \leq 65\text{MPa}. \]  \hspace{1cm} (5)

where \( \sigma_{\text{res}}^{\text{max}} = 98.4\text{MPa} \) are the residual stress in the surface of the slab. \( F_{c\text{, uncracked}} = 1.634 \text{kN} \). The reduction of maximal load after the slab stretching is 32.1%.

4 Conclusions

On the basis of performed numerical analyses the following conclusions can be drawn:

- The impact of complex stress state in tempered glass to reduce the fracture toughness is 12\%. (formula 3). Therefore the predictions of the critical forces in tempered glass constructions carried out by means of simplified numerical models are inaccurate.
- The scratch with depth that equals 0.3 mm and length 30 mm reduces the load capacity of considered glass slab by 32.1%.

The above formulated model can be adopted to similar problems solution for other engineering materials used in aerospace, mechanical and civil engineering, e.g. [17 – 23].

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Acknowledgement

This work was financially supported by Ministry of Science and Higher Education within the statutory research number S/20/2018.