Correlated Multiphoton Holes

I. Afek, O. Ambar, and Y. Silberberg

Department of Physics of Complex Systems, Weizmann Institute of Science, Rehovot 76100, Israel (Dated: June 15, 2010)

We generate bipartite states of light which exhibit an absence of multiphoton coincidence events between two modes amid a constant background flux. These ‘correlated photon holes’ are produced by mixing a coherent state and relatively weak spontaneous parametric down-conversion using a balanced beamsplitter. Correlated holes with arbitrarily high photon numbers may be obtained by adjusting the relative phase and amplitude of the inputs. We measure states of up to five photons and verify their nonclassicality. The scheme provides a route for observation of high-photon-number nonclassical correlations without requiring intense quantum resources.

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Introduction.— The generation of multiphoton entangled states has motivated a large body of experimental work in quantum optics \[1, 2\]. The workhorse in such experiments has been spontaneous parametric down-conversion (SPDC), which allows generation of entangled photon pairs \[2\]. In recent years there has been an on going effort to create nonclassical states with more and more photons the highest value being a six photon graph state \[2\] using three SPDC pairs. Scaling up the number of photons in such schemes, however, is challenging since they rely on multiple emissions of SPDC pairs. This could be accomplished using state of the art, high intensity sources to pump the SPDC \[2\].

Recently, the inverse of SPDC, namely the process in which a photon pair is missing amid a constant background flux, has been demonstrated experimentally \[3\]. This state has been dubbed an ‘entangled photon hole’ \[1\]. Here we generalize this concept to more than two photons by creating two-mode states in which the probability for arbitrary photon numbers, \(N_1, N_2\), to arrive simultaneously in the respective modes is zero, where choosing \(N_1 = N_2 = 1\) corresponds to the case of entangled photon holes \[1, 3\]. We refer to the generated states as ‘correlated photon holes’ (CPH). As in the two photon case \[2\], our scheme involves the mixing of SPDC and coherent light. Interestingly, the larger \(N_1, N_2\) the higher the relative weight of the coherent light implying that our scheme may be implemented at high photon numbers with very modest SPDC fluxes. Boosting up the security of quantum cryptography with states similar to those generated here has been studied theoretically \[4, 10\].

Theoretical scheme.— To date, a handful of photon counting experiments have utilized interference of coherent light and SPDC in a configuration sensitive to the relative phase. The SPDC in these was produced in either a single pass geometry \[2, 11, 13\] or an OPO \[14\]. With the exception of our recent demonstration of ‘high-NOON’ states \[11\], these experiments focused on two photon correlations. Here we generate another class of high-photon-number states which emerges naturally in this type of interference.

Consider a 50/50 beam-splitter fed by a coherent state, \(|\alpha\rangle_a\), in one input port and collinear degenerate SPDC, \(|\xi\rangle_b\), in the other (see Fig. 1a). The input states are defined in the conventional way \[15\]

\[
|\alpha\rangle = \sum_{n=0}^{\infty} e^{-\frac{1}{2} |\alpha|^2} \frac{\alpha^n}{\sqrt{n!}} |n\rangle, \quad \alpha = |\alpha| e^{i \phi_{cs}}
\]

\[
|\xi\rangle = \frac{1}{\sqrt{\cosh r}} \sum_{m=0}^{\infty} (-1)^m \sqrt{\frac{2m!}{2^m m!}} (\tanh r)^m |2m\rangle,
\]

where the phase of \(|\xi\rangle\) has been set arbitrarily to zero leaving the relative phase of the two inputs to be determined by \(\phi_{cs}\). We denote the pair amplitude ratio of the coherent state and SPDC by

\[
\gamma = |\alpha|^2 / r.
\]

In physical terms \(\gamma^2\) is the two photon probability of the classical source divided by that of the quantum source (in the limit \(r, |\alpha| \ll 1\). The larger the value of \(\gamma\), the higher the relative weight of the classical resources. We denote the path entangled state at BS output modes \(c, d\) (Fig. 1b) by \(|\psi_{out}\rangle_{c,d}\). The amplitude for \(N_1, N_2\) photons simultaneously in the BS output modes is then given by

\[
A_{N_1,N_2} = \langle N_1, N_2 | \psi_{out} \rangle_{c,d}.
\]

In the absence of SPDC this is simply

\[
A_{N_1,N_2} = e^{-|\alpha/2|^2} (\alpha/2)^{N_1+N_2} / \sqrt{N_1! N_2!},
\]

which is non-zero for all values of \(N_1, N_2\). By adding SPDC it is possible to cancel \(A_{N_1,N_2}\) for arbitrary values of \(N_2\) by correctly adjusting \(\gamma\) and \(\phi_{cs}\). In the limit \(r \ll 1\), there are \(|(N_1 + N_2)/2|\) distinct solutions to the equation \(A_{N_1,N_2} = 0\) for any choice of \(N_1, N_2\). For example for \(N_1, N_2 = 1, 1\) we choose \(\gamma = 1\) and \(\phi_{cs} = \pi/2\). This produces an absence of 1, 1 coincidence events at the BS outputs which is similar to the previously studied case.
number resolving detection is performed using multiple single-mode PMF and a 3nm FWHM band-pass (BP) filter. Photon components: long-pass filter (LPF), short-pass filter (SPF).

Spatial and spectral modes are matched using the (single-mode) PMF and a 3nm FWHM band-pass (BP) filter. Measurement of multi-photon coincidence is performed using photon number resolving detectors. Additional components: long-pass filter (LPF), short-pass filter (SPF).

During the experiment, photons are generated using a source, and their spatial and spectral modes are matched using a single-mode PMF and a 3nm FWHM band-pass (BP) filter. Photon number resolving detection is performed using multiple single photon counting modules (SPCM, Perkin Elmer). Additional components: long-pass filter (LPF), short-pass filter (SPF).

FIG. 2: (Color online) Measurement of a 2,2 correlated photon hole. (a) (left y-axis) Coincidence events with two photons in D1 and two photons in D2 as a function of relative phase, $\phi_{cs}$, between the coherent state and the SPDC (see Fig. 1). The minima correspond to phases in which the 2,2 coincidence is canceled implying a correlated photon hole. The solid line is a theoretical calculation taking into account the overall setup transmission and detector positive-operator-valued-measures $|\psi_{cs}|$. The visibility is 94.9%. Error bars indicate ±σ statistical uncertainty. Dashed line indicates the classical bound for this measurement, see Eq. (3) and the preceding discussion. All points below this line (shaded area) exhibit non-classical behavior. The two arrows indicate the position of the correlated photon holes. (b), (c) (right y-axis) The single counts in detectors D1, D2 as a function of $\phi_{cs}$ which exhibit virtually no phase dependence, straight solid line is a guide to the eye.

Experimental setup and results.—Our setup (Fig. 1) is similar to the one we have used for generation of NOON states with high photon numbers in a recent experiment. The scheme requires generation of SPDC and coherent light with common spatial and spectral modes. The beams are prepared in perpendicular linear polarizations ($\hat{H}, \hat{V}$) and overlapped using a polarizing beam-splitter cube (PBS). The phase between the beams, $\phi_{cs}$, is controlled before the PBS using a liquid crystal (LC) phase retarder. The BS of Fig. 1a is then implemented in a collinear geometry using a polarization maintaining fiber with axes aligned at ±45° ($\hat{X}, \hat{Y}$). Multiphoton coincidences with $N_1, N_2 = 2,2$ photons arriving simultaneously in detectors D1, D2 respectively are measured as a function of $\phi_{cs}$ using an array of avalanche photodiodes. We ap-
Nonclassical properties.— To quantify the nonclassicality of the generated states we derive a classical bound for Glauber’s $n$th order, equal-time, correlation function $g^{(n)}(\tau = 0)$. Note that we chose $\tau = 0$ since we are interested in photons arriving to the detectors at the same time. This correlation function is proportional to the $n$ photon coincidence signal of a single spatial mode divided by the single count raised to the $n$th power. For thermal light, which exhibits bunching, $g^{(n)}(0) = n!$ and for a coherent state $g^{(n)}(0) = 1$ [13, 17]. In the following we show that the result for a coherent state is actually the lower bound for an arbitrary classical state i.e. a state with an arbitrary nonnegative well behaved $P$ function [18]. The inequality was implied in Glauber’s work on high-order correlation functions of coherent fields [18] and is derived here somewhat differently,

$$g^{(n)}(\tau = 0) = \frac{\text{Tr} \left\{ \hat{\rho} \hat{E}^{-(\tau)^n} \hat{E}^{(+\tau)^n} \right\}}{\text{Tr} \left\{ \hat{\rho} \hat{E}^{-(+\tau)^n} \right\}^n}$$

$$= \frac{\int P(\alpha)|\alpha|^{2n}d^2\alpha}{(\int P(\alpha)|\alpha|^2d^2\alpha)^n} \geq 1,$$

where the last inequality follows immediately from a multidimensional form of Jensen’s inequality [19, 20]. We note that for $n = 2$ this inequality is a well known result of the Schwarz inequality [15]. Correlated photon holes require a bound for the two-point equal-time Glauber correlation function $g^{(m,n)}(x_1, x_2; \tau = 0)$, where $x_1, x_2$ correspond to the BS output modes (Fig 1a). The derivation is based on the single mode result, Eq. 4, and requires the assumption that the two mode $P$ function describing the BS outputs is separable i.e. $P_{1,2}(\alpha, \beta) = P_1(\alpha)P_2(\beta)$, a condition which is satisfied by all classical two mode gaussian states [21]. Using this assumption the inequality follows immediately from the single mode result applied to each of the modes independently.
We used Eq. (6) to calculate the shaded nonclassical areas in Figs. 2 and 3. For the four photon case (Fig. 2) we substitute \( m = 2, n = 2 \). The classical bound is violated by 23.92 and 21.17 standard deviations at the phases for which the photon holes are created. For the five photon case (Fig. 3) we substitute \( m = 5, n = 0 \). The bound is violated by 25.5 standard deviations at the five photon hole.

**Conclusion.**— Mixing coherent light and SPDC is typically done in conjunction with homodyne detection [15] using a ‘macroscopic’ local oscillator as the coherent state. Adopting this paradigm in the ‘few photon’ regime and using number resolving detectors brings to light a rich structure exhibiting various nonclassical signatures. This has enabled us to create NOON states [11] in a recent work and correlated photon holes here. Extending the present work to higher photon numbers can be done using essentially the same setup, requiring only additional detectors to enable higher coincidence measurements. This does not entail a larger SPDC flux since relatively more of the photons originate from the coherent state which is practically unlimited in intensity, providing experimental simplification. As in the case of NOON states however, the visibility of interference is eventually limited by the overall setup transmission (currently 12.5%), determined by accounting for all sources of photon loss including the detector efficiency. The transmission could be improved by using high purity SPDC sources which can be spectrally mode matched to coherent states without requiring a bandpass filter [22, 24]. Improved single mode fiber coupling of the photon pairs and high efficiency photon number resolving detectors [23, 26] would also be beneficial. Reaching a transmission of 25% would allow measuring states with ten photons in a setup similar to ours.

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