DETECTION OF SIGNALS OF MOVING OBJECTS BASED ON THE TIME SELECTION METHOD

Abstract. To increase the efficiency of detecting moving objects in radiolocation, additional features are used, associated with the characteristics of trajectories. The authors assumed that trajectories are correlated, that allows extrapolation of the coordinate values taking into account their increments over the scanning period. The detection procedure consists of two stages. At the first, detection is carried out by the classical threshold method with a low threshold level, which provides a high probability of detection with high values of the probability of false alarms. At the same time uncertainty in the selection of object trajectory embedded in false trajectories arises. Due to the statistical independence of the coordinates of the false trajectories in comparison with the correlated coordinates of the object, the average duration of the first of them is less than the average duration of the second ones. This difference is used to solve the detection problem at the second stage based on the time-selection method. The obtained results allow estimation of the degree of gain in the probability of detection when using the proposed method.

Keywords: radar station, detection, trajectory selection, probability of detection, probability of false alarms

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Introduction. In radar, detection and tracking of moving objects is carried out consecutively, and the algorithms are based on the theory of statistical solutions [1]. At the detection stage, the Neyman–Pearson criterion is customarily used, whereby the false alarm probability $F$ is set, resulting in achieving the maximum probability of true detection $D$ of an object’s signal. To come up with a unique solution, low probabilities are selected, $F \sim 10^{-7} \div 10^{-5}$, when, in order to obtain high levels of $D$, sizable signal power to noise power ratios (SNR) have to be ensured, $\rho \sim 30 \div 40$ [2]. Low probability levels $D$ result in protracted interruptions of information about the coordinates of an object due to fading of signals, which may tangibly increase errors during the tracking phase.

In the early days of the development of the theory of radio location, it was suggested to solve the problem of detection and tracking jointly, which makes it possible to additionally use the trajectory characteristics of the object [3, 4]. The resultant algorithms turned out to be rather complicated, poorly suitable for practical uses. Lately, this idea has been resorted to the “track before detect” method which, essentially, consists in a recurrent sequence of formation of an a posteriori probability density function (PDF) of the presence of an object’s signal, taking into account the statistical characteristics of its motion [2, 5, 6]. This approach enables achieving a greater true detection probability, however, it is rather taxing on calculations. When using this approach, it is customary to employ the particle filtering [7] method for multipoint approximation of the PDF.

Formulating the task. In this work, it is suggested to use the empirical approach for cooperative detection and measurement, which implies resolving the detection task in two stages. During the first stage, classical detection of a signal by means of threshold is used, except in this case, instead of setting the false alarm probability $F$ as the source datum, a high value of the signal true detection probability $D$ is selected, $D \sim 0.8 \div 0.9$. This is enabled through a low level of the detection threshold, which results in high false alarm probabilities $F \sim 10^{-2} \div 10^{-1}$. During the second stage, based on the time selection method, minimizing the false alarm probability to the required level is achieved; however, the true detection probability also decreases, even though not as notably. In the time selection method suggested, information about the characteristics of the object motion is used, which makes it possible to achieve a greater pace in the decrease of $F$ than the pace of $D$ diminishing and, as a result, come up with a premium in comparison with the classical detection method.

Measurements in radar can be performed by scanning, periodically in time, the space by the range, range rate and angular coordinates. First, fast scanning of the object’s coordinates is carried out with $v_0$ interval, throughout which one can assume their values can be considered as constant. Next, it is reiterated with $T_0 \gg v_0$ period, the time within which the coordinates may change due to motion of the object. Scanning can be effected separately for each coordinate, or jointly.

A continuous, in terms of time, output signal of a receiver within the limits of $v_0$ interval is viewed as a mix of noise and useful signal, which is processed by a threshold device (quantizer) converting the continuous signal into a binary one. Further, we assume that the quantized signal resolution bins are much smaller than the interval $v_0$, which is why the binary signals on the coordinate axis $x$ can be viewed as point ones, including the object’s signals.

High probabilities of true detection are provided by a low level of the quantization threshold, whereby, at the expense of increasing the probabilities of false alarms, there emerge noisy point signals that, in binary representation, will be indistinguishable from the useful signals, which leads to ambiguity of detection. For a one-dimensional case, the result of the procedure of formation of point signals are presented in Figure 1.
In the Figure, the horizontal axis is an axis of continuous time \( t \) with the periods \( T_0 \). The discrete moments of time \( k = 1, 2, 3, \ldots \) are beginnings of the scanning intervals \( v_0 \), which, on the condition \( v_0 \ll T_0 \), are considered as points on axis \( t \). Values of the coordinates \( x \) of the random binary points are plotted on the vertical axis within the limits of \( x_{\min} \) to \( x_{\max} \). The trajectory of the object is shown by a dashed line where its crossings by vertical lines show the positions of the points of signals of the object \( x_{sk} \). Since the probability \( D \) of the object’s signal exceeding the quantization threshold is lesser than one, it testifies to dropouts of the object points due to fading of the useful signal. An example of such situation is shown in Figure 1 at the moment \( t = r \). At any \( k \)-th period, a random number of noise points emerges, as shown in the Figure.

The authors have used the assumption that the trajectory of an object represents a time-correlated function, whereas the positions of the noise points are statistically independent. The essence of the proposed time selection method consists in that, based on the results of measurements of the points’ coordinates, the trajectories of all the point signals are formed, both those of the object and the noise ones. By virtue of the assumption about the correlated nature of the coordinates of the object’s signal points, the average duration of sections of the object’s trajectory between the moments of fading of the useful signals may exceed, under certain conditions, that of the noise signals. Based on this, time selection of the object trajectory relative to noise trajectories is performed, and a decision about the detection is adopted.

**Trajectory shaping resultant from measurements.** Let us consider the model of a trajectory of motion of an object with the coordinates \( x_{sk}, k = 1, 2, \ldots \). Its main property is correlation in time, i.e. statistical dependence between the values of the coordinates at different moments of time. In this paper, correlation has been determined as follows: it was assumed that, at discrete moments of time, increments of the coordinates \( x_{sk} - x_{sk-1} \) cannot be greater than the assigned ones and are within the limits of the increment interval \( \Delta_x \). Note that it was considered that the values of increments in this domain are statistically independent random magnitudes with an even probability distribution law. In this case, the model of the trajectory \( x_{sk} \) can be presented as a first-order stochastic equation:

\[
x_{sk} = x_{sk-1} + n_k,
\]

where \( n_k \) is a discrete white noise with an even distribution law within the domain \( \Delta_x \). Such a model can be made more complicated by assuming that the value \( x_{sk} \) will depend not only on the previous value \( x_{sk-1} \), but also on the rate of change of the coordinates \( x_{sk-1} - x_{sk-2} \). A second-order equation of such a model will be determined by the correlation

\[
x_{sk} = x_{sk-1} + [x_{sk-1} - x_{sk-2}] + n_k = c_1 x_{sk-1} + c_2 x_{sk-2} + n_k,
\]

where \( c_1 = 2, c_2 = -1 \). Similarly, it is possible to take into account other, higher levels of increment, resulting in the \( R \)-th order stochastic equation:

\[
x_{sk} = \sum_{r=1}^{R} c_r x_{sk-r} + n_k = \tilde{x}_{sk-1} + n_k,
\]

where \( \tilde{x}_{sk-1} \) can be viewed as a forecast of the value of the coordinate per the scanning interval. The values \( \Delta_x \) and \( \tilde{x}_{sk-1} \) are the principal trajectory characteristics of the model of motion of the object.

Shaping the trajectory of motion resultant from measurements is performed as follows. At the moment of time \( k \), a scanning strobe \( \Delta \) is set relative to the positions of every point \( x_k \), whereby the domain of this strobe may or may not contain the points from the next moment of time \( k + 1 \). If such points do occur, trajectories are plotted from each value of \( x_k \) up to the corresponding elements of each point located within the strobe. In this manner, the trajectory of an object or of noise trajectories is initiated. At the ensuing moments of time, the procedure is reiterated for all the points, and here the following basic cases are possible.

In case the initial point is a noise one and the point in the next strobe again happens to be noise, a noise trajectory will be initiated, which will be interrupted if no points emerge in the subsequent strobe.

When an object’s point follows the noise one in the strobe, the noise trajectory will switch over to the trajectory of the object. If the duration of the noise trajectory is shorter than that of the object’s trajectory, it will not significantly affect further shaping of the object trajectory.

When both a noise point and an object point or several noise points emerge in the strobe after the noise point, then several trajectories will be shaped.
Thus, as a result of shaping trajectories at the detection stage, segments of both noise and object trajectories will show up in the domain of the coordinates changing. The basis of the method under consideration is to ensure that the average duration of a noise trajectory be shorter than the average duration of the object trajectory, which is achieved by taking into account the trajectory characteristics. The way to achieve this objective is by using time selection of the trajectories.

**Time selection of the trajectories.** Time selection is the second stage of detection, which consists in that only the trajectories reach the selector output whose duration is longer than \( l \). Since, resultant from measurements, the prognosticated values \( \tilde{x}_{sk-1} \) in (3) may be obtained with a random error, the dimensions of the tracking strobe \( \Delta \) have to be greater than the dimensions of the coordinate increments \( \Delta_c \). Those are determined by the number of resolution bins they contain. In case the error of the forecast exceeds the size \( \Delta \) with the probability \( P \), the probability of true detection of a signal in the strobe for the model of the object motion under consideration dwindles and becomes equal to \( D_0 = D(1 - P) \). Further, we assume that the values of the object signals, strobe-to-strobe, are statistically independent (fast fluctuations). The probability of statistically independent object points emerging in a row in \( g \) successive strobes is equal to \( D_0^g \). Departing from this, the PDF for the duration of \( g \) uninterrupted trajectory segments at the output of the selector \( w_g(\vartheta) \) can be obtained in the form of the Bernoulli equation

\[
w_g(\vartheta) = (1 - D_0)D_0^{g-1},
\]

where the normalization condition is taken into account \( \sum_{g=1}^{\infty} w_g(\vartheta) = 1 \). The average duration of continuous sections of the object trajectories at the quantizer output is \([8]\)

\[
\tau = \sum_{g=1}^{\infty} gw_g(\vartheta) = \frac{1}{1 - D_0}.
\]

The average duration of the intervals of intermission between the trajectory sections is found by substituting the value \( D_0 \) with \( 1 - D_0 \) in the expression (5), and is equal to

\[
\bar{\tau} = \frac{1}{D_0}.
\]

Using the formulas (5) and (6), the probability \( D_0 \) can be expressed as follows:

\[
D_0 = \frac{\tau}{\tau + \bar{\tau}},
\]

which corresponds to the relative average duration of the trajectory continuous sections.

Following the selection, the number of points along a continuous section of the trajectory decreases and is equal to the value \( f(\vartheta, l) \), expressed as follows:

\[
f(\vartheta, l) = \begin{cases} 0 & \text{for} \quad 1 \leq \vartheta < l; \\ \vartheta - l & \text{for} \quad \vartheta \geq l + 1, \end{cases}
\]

where \( l \) is an integer non-negative value. The average duration of the section, following the selection \( \tau_c \) is found using the formula

\[
\tau_c(l) = \sum_{g=1}^{\infty} f(\vartheta, l)w_g(\vartheta) = \frac{D_0^l}{1 - D_0} = \tau - \frac{D_0^l}{1 - D_0}.
\]

The average duration of a continuous section of the trajectory decreases by the value

\[
\delta_c(l) = \frac{1 - D_0}{1 - D_0}.
\]

By the same value will increase the average duration of intervals between continuous sections, which will become equal to

\[
\bar{\tau}_c(l) = \bar{\tau} + \delta_c(l) = \frac{1 - D_0^{l+1}}{D_0(1 - D_0)}.
\]
If the probability of a noise point appearing in a resolution bin is equal to $F$, the probability of at least one noise point appearing in the strobe $\Delta$ corresponds to $1 - (1 - F)^\Delta$, which represents the false alarm probability in the strobe without taking into account the selection. The PDF of the durations of continuous sections of the noise trajectory at the selector output shall look as (taking into account the normalizing)

$$w_n(\theta) = (1 - F)^\Delta \left[1 - (1 - F)^\Delta\right]^{\theta - 1}.$$  \hspace{1cm} (11)

The average duration of those sections is equal to

$$\varphi = \sum_{\theta=1}^{\infty} \theta w_n(\theta) = \frac{1}{(1 - F)^\Delta}. \hspace{1cm} (12)$$

The value of the average duration of intervals between continuous sections can be obtained by substituting the value $(1 - F)^\Delta$ with $1 - (1 - F)^\Delta$ in the formula (11). As a result, we obtain the expression

$$\varphi_l = \frac{1}{1 - (1 - F)^\Delta}. \hspace{1cm} (13)$$

By analogy with formula (7) we find the ratio for the probability of false alarms in the strobe in the absence of selection:

$$1 - (1 - F)^\Delta = \frac{\varphi}{\varphi + \varphi_l}. \hspace{1cm} (14)$$

At the selector output, the number of remaining points of a continuous section of the noise trajectory is determined by the function (8). As a result, their average duration will decrease compared to (12) and become equal to

$$\varphi_s(l) = \sum_{\theta=1}^{\infty} f(\theta, l)w_n(\theta) = \frac{\left[1 - (1 - F)^\Delta\right]^l}{(1 - F)^\Delta} = \varphi - \frac{1 - \left[1 - (1 - F)^\Delta\right]^l}{(1 - F)^\Delta}. \hspace{1cm} (15)$$

The average duration of the continuous sections of the noise trajectory will decrease by the value of

$$\delta_{\varphi}(l) = \frac{1 - \left[1 - (1 - F)^\Delta\right]^l}{(1 - F)^\Delta}. \hspace{1cm}$$

The average duration of intervals between continuous sections will increase by the same value, which will become equal to

$$\varphi_{\varphi}(l) = \varphi + \delta_{\varphi}(l) = \frac{1 - \left[1 - (1 - F)^\Delta\right]^l}{(1 - F)^\Delta}. \hspace{1cm} (16)$$

Further, the functions $\tau_s(l)$, $\tau_{\varphi}(l)$, $\varphi_s(l)$, $\varphi_{\varphi}(l)$ are used to find the true detection and false alarm probabilities in the strobe at the selector output.

**Detection probabilities at the selector output.** The true detection probability in the tracking strobe $D_s(l)$, by analogy with the formula (7), shall denote the relative value of the average duration of continuous sections of the object’s trajectory

$$D_s(l) = \frac{\tau_s(l)}{\tau_s(l) + \tau_{\varphi}(l)}. \hspace{1cm}$$

Once we substitute values from the formulas (9) and (10) into this correlation, we obtain the expression

$$D_s(l) = D_{s,l}^{l+1} = (D(1 - P))^{l+1}, l = 0, 1, 2... \hspace{1cm} (17)$$

In order to find the false alarm probability $F_s(l)$, by analogy with the formula (14), we obtain the formula

$$F_s(l) = \frac{\varphi_s(l)}{\varphi_s(l) + \varphi_{\varphi}(l)}. \hspace{1cm}$$
By substituting into it the values from (15) and (16), we obtain the correlation
\[ F_l = \left[ 1 - (1 - F)^\Delta \right]^{l+1}. \]

The expressions (17) and (18) define the nature of the change in signal detection probabilities in the
tracking strobe when using the time selection method.

**Comparing with the classic detection method.** In the absence of selection, the probabilities of
detection in the strobe are found from the formulas (17) and (18) for \( l = 0 \). Of interest is a comparative
assessment of the detection probabilities when using the classical method and the time selection method.

For the true detection probabilities, their ratio is found from the formula (17) and is equal to
\[ \alpha(l) = \frac{D_s(l)}{D_s(0)} = D_s(l) = (D(1 - P))^l. \]

For the false alarm probabilities, their ratio is found from the formula (18):
\[ \beta(l) = \frac{F_s(l)}{F_s(0)} = \left[ 1 - (1 - F)^\Delta \right]^l. \]

The values \( \alpha(l) \) and \( \beta(l) \) depend on the SNR \( \rho \) through the probabilities \( D \) and \( F \).

Let us consider the case when the PDF of the mix of the object’s signal and noise at the input of the
quantizer conforms to the Rayleigh law of distribution. It is known [1] that there is a connection between
the true detection probabilities \( D \), the false alarm probability \( F \) and the SNR \( \rho \), expressed as follows
\[ F = D^{1+\rho}. \]

We shall assign the dependencies (19) and (20) on the selection interval \( l \) and SNR \( \rho \) for the concrete
values: \( D = 0.8, P = 0, \Delta = 4, \rho = 15 \). Note that, in accordance with (21), the value of the false alarm
probability is \( F = 2.8 \cdot 10^{-2} \). For the SNR \( \rho = 5; 10; 15; 20 \) the graphs of the functions (19) and (20) are
presented in Figure 2.

![Figure 2](attachment:image.png)

*Figure 2. Functions of probabilities of detection with the time selection method compared with the classical method:
 a – correct detection \( \alpha(l) \), b – false alarms \( \beta(l) \)*

The course of these functions shows the degree of change in the detection probabilities when using
the time selection method, i.e., taking into account the characteristics of the object trajectory as compared
with the classical detection method.

**Modeling results.** For modeling, the trajectories were selected, corresponding to the model of motion
assigned by the equation (1) with the interval of increments \( \Delta = 4 \). For the first trajectory, the initial
value is set at the moment of time \( k = 1 \) and is equal to 0, for the second one at the moment \( k = 10 \) and is
equal to 20. The remaining parameters are taken from the preceding example with the interval of selection \( l = 2 \) and the SNR \( \rho = 15 \).
Figure 3 shows the positions of random point signals of objects and noise at discrete moments of time \( k = 1, \ldots, 50 \) at the output of the quantizer.

![Figure 3. Random point signals at the output of the quantizer](image)

In order to illustrate the nature of the trajectories of the objects, their points are connected by solid lines. In their absence, the points of objects and noises are indistinguishable. Interruptions between the sections of the trajectories correspond to the true detection probabilities \( D = 0.8 \). Figure 4 shows the results of processing of this data using the time selection method.

![Figure 4. The results of processing data from Figure 3 by time-selection method](image)

Resultant from the selection, the false alarm probabilities have decreased from the level \( F_s(0) = 2.8 \cdot 10^{-2} \) down to \( F_s(2) = 1.3 \cdot 10^{-3} \). From Figure 4, one can see that in the realization given, the trajectories of objects stand out which increase due to a decrease in the true detection probability \( D = 0.8 \) down to \( D_s(2) = 0.64 \). In some places, trajectories of the objects bifurcate by the onset of noise trajectories, but they are quickly cut off or merge with the object trajectories. In addition, there are five residual noise points and one noise trajectory with a duration of one period in the realization. The number of noise trajectories can be reduced by increasing the selection interval, however, this will increase the duration of interruptions in the trajectory.

At the next stage of tracking, the interruptions in the trajectories are filled through their extrapolation. The development of a filter-extrapolator algorithm is the subject of further research.
Conclusion. Detection, taking into account the characteristics of the trajectory of motion of an object, enables obtaining better performance in the detection of moving objects compared to the classical method. This is expressed, in particular, in the fact that with the same probabilities of true detection and signal-to-noise ratio, it is possible to ensure lower false alarms probabilities. This is achieved by using, at the second stage of detection, the method of time selection taking into account characteristics of the trajectory of the object. The problem is solved for the case of statistically independent fluctuations of the object’s signal (fast fluctuations) and statistically independent positions on the coordinate axis of false alarm signals. It is of interest to further develop the method for probability distributions of a wider class. Since, at the output of the selector, the true detection probability is lesser than one, there are discontinuities in the measurements of the object coordinates. It is advisable to conduct research on filtering and extrapolation of the rupturing processes with respect to the detection method reviewed.

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