SWENet: A Physics-Informed Deep Neural Network (PINN) for Shear Wave Elastography

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Abstract—Shear wave elastography (SWE) enables the measurement of elastic properties of soft materials in a non-invasive manner and finds broad applications in various disciplines. The state-of-the-art SWE methods rely on the measurement of local shear wave speeds to infer material parameters and suffer from wave diffraction when applied to soft materials with strong heterogeneity. In the present study, we overcome this challenge by proposing a physics-informed neural network (PINN)-based SWE (SWENet) method. The spatial variation of elastic properties of inhomogeneous materials has been introduced in the governing equations, which are encoded in SWENet as loss functions. Snapshots of wave motions have been used to train neural networks, and during this course, the elastic properties within a region of interest illuminated by shear waves are inferred simultaneously. We performed finite element simulations, tissue-mimicking phantom experiments, and ex vivo experiments to validate the method. Our results show that the shear moduli of soft composites consisting of matrix and inclusions of several millimeters in cross-section dimensions with either regular or irregular geometries can be identified with excellent accuracy. The advantages of the SWENet over conventional SWE methods consist of using more features of the wave motions and enabling seamless integration of multi-source data in the inverse analysis. Given the advantages of SWENet, it may find broad applications where full wave fields get involved to infer heterogeneous mechanical properties, such as identifying small solid tumors with ultrasound SWE, and differentiating gray and white matters of the brain with magnetic resonance elastography.

Index Terms—Shear wave elastography, deep learning, physics-informed neural network, inverse problem, phantom and ex vivo experiments, tumor biomechanics.

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I. INTRODUCTION

MOST soft materials including soft biological tissues have large Poisson ratios and small shear moduli. These features determine that the velocity of a pressure wave in them is much faster than that of a shear wave. For a long time, the characteristics of reflection and scattering of MHz pressure waves (i.e., ultrasonic waves) in soft materials have been used in medical imaging (i.e., ultrasound imaging [1]) to probe anatomical abnormalities in vivo, which is of clinical importance for diagnoses of various diseases. Complementarily, the slow propagation speeds of shear waves that are closely related to tissue stiffness enable a new imaging modality targeting biomechanics, i.e., shear wave elastography (SWE), which has received considerable attention across different disciplines during the past two decades [2], [3], [4].

SWE enables the measurement of mechanical parameters (e.g., linear elastic, viscoelastic, and hyperelastic parameters) of soft tissues in vivo and finds broad clinical applications because the occurrence and development of diseases are usually accompanied by the variations of their mechanical properties [5], [6]. For example, SWE has been used to stage liver fibrosis [7], detect artery stiffening [8], and differentiate malignant tumors from benign ones [9], [10]. Considering the complexity of the constitutive behavior of soft tissues and diverse clinical applications, different SWE methods have been developed in the literature [3], [11], [12], [13], [14], [15], [16]. A common feature in these methods is that shear wave velocities are the main inputs of the inverse analysis to infer tissue mechanical properties. It is of notice that most soft tissues are inhomogeneous and wave reflection and scattering may occur; in this case, an accurate measurement of wave velocities and further infer the mechanical properties are by no means trivial. A previous study demonstrated strong size effects observed in SWE of solid cylindrical inclusions given by conventional SWE methods [4], in which large errors occur when the cross-section dimension of a phantom tumor is smaller than ~1 cm. To overcome this limitation, it is necessary and important to include more features of elastic waves in inhomogeneous soft materials, not just wave velocities, in inverse analysis.

Deep learning (DL) relies on an improved artificial neural network, consisting of multiple processing layers to learn the representative features of data, and has been demonstrated to be useful for medical image analysis [17], [18]. Being able to include the spatial and temporal features of wave motion in training, DL-based SWE has the potential to provide a more accurate evaluation of the spatial distribution of tissue
mechanical properties in comparison with conventional SWE. Indeed, efforts towards this direction have been made in recent years [19], [20], [21], [22], [23]. Given that sufficient dataset required for training a deep neural network is difficult to achieve in vivo, numerical simulations, phantom experiments [19], [20], [21] and ex vivo experiments [24], [25], [26], [27] have been used to generate the required training dataset. However, the accuracy of a DL-based SWE trained in this way largely depends on the extent to which the features of elastic waves in target solids, e.g., in soft tissues in vivo, can be captured by the simulations or phantom experiments. Considering the difficulties encountered in SWE in achieving large datasets, in this study, we propose a deep neural network, SWENet, based on physics-informed neural networks (PINN), to perform image reconstruction in SWE. PINN encodes the governing equations of a physical problem, e.g., partial differential equations (PDE), as a part of the neural network training and has emerged as a useful tool to solve both direct and inverse problems [28]. Very recent studies have demonstrated that PINN is promising for performing full-waveform inversion in geophysics from limited data [29].

Unlike the data achieved in geophysics, stimulus utilized in SWE to generate elastic waves can be applied internally and/or externally in a programmable manner, resulting in multi-source data and desired wave fields required for a reliable inverse analysis. Multi-source data can be easily integrated with PINN, which enables an approach to improving the performance of SWE in probing small inclusions as demonstrated in this study.

The paper is organized as follows. In section II, governing equations for the wave motions in an inhomogeneous soft material are first briefly described, which are subsequently encoded in the SWENet. To demonstrate the usefulness of the proposed method, finite element analysis, tissue-mimicking phantom experiments, and ex vivo experiments have been performed to generate the datasets used in training the networks and inferring the material parameters. Section III shows the results of learning hidden elastic properties of inhomogeneous soft materials from wave propagation given by numerical simulation, phantom experiments, and ex vivo experiments. In section IV, salient features of SWENet and speeding up SWENet by transfer learning have been discussed. Section V gives the concluding remarks.

II. MATERIALS AND METHODS

A. Wave Motion in Elastic Soft Materials

The wave equation that will be encoded into the deep neural network is simply described here. The equation of motion that governs elastic wave propagation is

\[ \frac{\partial^2 \sigma_{ij}}{\partial x_j} = \rho \frac{\partial^2 u_i}{\partial t^2}, \]  

(1)

where \( \sigma_{ij} \) is the Cauchy stress tensor, \( \rho \) is the mass density, \( u_i \) is the displacement, \( x_i \) \((i = 1, 2, 3)\) and \( t \) are spatial coordinates and time, \( j \) is a dummy index following Einstein summation convention, respectively. Here, we consider isotropic incompressible materials for which the constitutive relation is given by

\[ \sigma_{ij} = \mu \left( \frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right) - p_0 \delta_{ij}, \]  

(2)

where \( \mu \) is the shear modulus; in general, it varies with spatial coordinates \( \mu = \mu(x_i) \). \( \delta_{ij} \) denotes Kronecker delta. \( p_0 \) is a Lagrange multiplier associated with the incompressible constraint, i.e.,

\[ \frac{\partial u_k}{\partial x_k} = 0. \]  

(3)

Inserting Eq. (2) into Eq. (1) yields

\[ \rho \frac{\partial^2 u_i}{\partial t^2} = \mu \frac{\partial^2 u_i}{\partial x_j \partial x_j} + \frac{\partial \mu}{\partial x_j} \frac{\partial u_i}{\partial x_j} + \frac{\partial}{\partial x_j} \left( \frac{\partial u_i}{\partial x_j} \right) - \frac{\partial p_0}{\partial x_j}. \]  

(4)

Equation (4) denotes the wave equation expressed in terms of displacements. For SWE, the particle velocity \( u_i \) varies with coordinates but is time-independent. And more importantly, we can take the time derivative of Eq. (4) and get the wave equation expressed in terms of \( v_i \). For wave motions in \((x_1, x_2)\), the wave equations can be written as

\[ \begin{align*}
\mu \frac{\partial^2 v_1}{\partial x_1^2} + \frac{\partial^2 v_1}{\partial x_2^2} + 2 \frac{\partial \mu}{\partial x_1} \frac{\partial v_1}{\partial x_1} + \frac{\partial \mu}{\partial x_2} \frac{\partial v_1}{\partial x_2} + \frac{\partial}{\partial x_1} \left( \frac{\partial v_1}{\partial x_1} \right) - \frac{\partial p_0}{\partial x_1} & = 0, \\
\mu \frac{\partial^2 v_2}{\partial x_1^2} + \frac{\partial^2 v_2}{\partial x_2^2} + 2 \frac{\partial \mu}{\partial x_2} \frac{\partial v_2}{\partial x_2} + \frac{\partial \mu}{\partial x_1} \frac{\partial v_2}{\partial x_1} + \frac{\partial}{\partial x_2} \left( \frac{\partial v_2}{\partial x_2} \right) - \frac{\partial p_0}{\partial x_2} & = 0.
\end{align*} \]

(5)

where \( p = \frac{\partial p_0}{\partial t}, \ p_i = \frac{\partial p}{\partial x_i}. \) And the updated incompressibility constraint reads

\[ v_{1,1} + v_{2,2} = 0. \]  

(6)

Here we introduce a stream function \( \psi = \psi(x_1, x_2, t) \) that satisfies

\[ v_1 = \frac{\partial \psi}{\partial x_2}, \ v_2 = -\frac{\partial \psi}{\partial x_1}, \]  

(7)

which makes Eq. (6) automatically satisfied.

The wave motions are completely described by \( \psi \) and \( p \); both are functions of spatial coordinates and time. In contrast, the shear modulus \( \mu \) varies with coordinates but is time-independent. We therefore will use separate neural networks for \( \psi \) and \( p \), and \( \mu \).

B. SWENet: A Physics-Informed Deep Neural Network for Shear Wave Elastography

Here we propose SWENet, which consists primarily of two deep neural networks, as shown in Fig. 1. In brief, neural network 1 (NN1) takes the spatial coordinates \((x_1, x_2)\) and time \( t \) as inputs. Whereas neural network 2 (NN2) takes \((x_1, x_2)\) as input. The outputs of the neural networks are enforced to match the data and satisfy the physical constraints (i.e., the wave equations) after training. Therefore, the output of NN1 will closely approximate \( \psi \) and \( p \). And more importantly, the NN2 will infer the shear modulus in the region-of-interest (ROI) \( \mu_{ROI} \), the ultimate goal of SWE.
which aims to minimize a loss function during the training process based on the back-propagation algorithm [31]. The weights and biases will be identified through a stochastic gradient descent (SGD) optimization. The weights for the input layer, hidden layers, and the output layer, respectively. In this study, tanh is utilized as the activation function. It can be proved that the transformation can approximate any measurable functions [30]. The weights and biases will be identified through a training process based on the back-propagation algorithm [31], which aims to minimize a loss function $\mathcal{L}$.

As shown in Fig. 1, the ROI is a subdomain of the full field with an arbitrary shape; any points within ROI must have been illuminated by the traveling shear waves in order to infer $\mu_{\text{ROI}}$. For simplicity, we took square ROIs for all the studies. We took fully connected feed-forward architectures for NN1 and NN2, as suggested in previous studies on PINN [28], [29]. Denote $L$ ($L > 2$) the number of layers (an input layer, an output layer, and $L - 2$ hidden layers) and $N_l$ the number of neurons for the $l$-th layer ($l = 1, 2, \ldots, L$). Taking the NN1 as an example (see Table I), $L = 10$ and the numbers of neurons for the hidden layers are identical $N_l = 40$ ($l = 2, \ldots, 9$). A fully connected feed-forward neural network is a transformation from the input $x$ to the output $y$

\[
\begin{align*}
\hat{x}^{(1)} &= x \\
\hat{x}^{(l)} &= \tanh(W^{l}x^{(l-1)} + b^l), \quad 1 < l < L \\
y &= W^{L}x^{(L-1)} + b^{L},
\end{align*}
\]

where $W^{l} \in \mathbb{R}^{N_l \times N_{l-1}}$ and $b^{l} \in \mathbb{R}^{N_l}$ denote the weights and biases for the $l$-th layer (no weights and biases for the input layer), respectively. In this study, $\tanh(x) = (e^x - e^{-x})/(e^x + e^{-x})$ is utilized as the activation function. It can be proved that the transformation can approximate any measurable functions [30]. The weights and biases will be identified through a training process based on the back-propagation algorithm [31], which aims to minimize a loss function $\mathcal{L}$.

In SWENet, $\mathcal{L}$ primarily consists of two parts, i.e., the data-driven part and the physic-informed part that encodes the wave equations. As $\psi$ is the output of NN1, $v_1$ and $v_2$ can be derived according to Eq. (7) by automatic differentiation of $\psi$ (see Fig. 1). The data-driven part $\mathcal{L}_{\text{Data}}$ is defined as

\[
\mathcal{L}_{\text{Data}} = |v_2 - v_2^*|^2, \quad \text{(9)}
\]

where $v_2^*$ denotes experimental data. ‘|’ ‘|’ denotes the L2 norm. The physics informed part is $\mathcal{L}_{\text{PDE}} = \mathcal{L}_{\text{PDE1}} + \mathcal{L}_{\text{PDE2}}$, where

\[
\begin{align*}
\mathcal{L}_{\text{PDE1}} &= |\mu(\partial^2 v_1/\partial x_1^2) + \partial^2 v_1/\partial x_2^2) + 2\partial \mu/\partial x_1 \partial v_1/\partial x_1 + \partial \mu/\partial x_2 \partial v_1/\partial x_2 - \partial^2 v_1/\partial (x_1^2) - \partial^2 v_1/\partial (x_2^2)|^2, \quad \text{(10)}
\end{align*}
\]

and

\[
\begin{align*}
\mathcal{L}_{\text{PDE2}} &= |\mu(\partial^2 v_2/\partial x_1^2) + \partial^2 v_2/\partial x_2^2) + 2\partial \mu/\partial x_1 \partial v_2/\partial x_1 + \partial \mu/\partial x_2 \partial v_2/\partial x_2 - \partial^2 v_1/\partial (x_1^2) - \partial^2 v_1/\partial (x_2^2)|^2. \quad \text{(11)}
\end{align*}
\]

It is worth noting that $v_1$ has not been introduced into $\mathcal{L}_{\text{Data}}$ because currently only $v_2^*$ is available in experiments. It will be straightforward to incorporate $v_1$ into $\mathcal{L}_{\text{Data}}$ if $v_1^*$ is measurable, and we expect an improved performance of SWENet from introducing additional experimental data $v_1^*$.

![Fig. 1. Architecture of the SWENet.](image)

**TABLE I**

| NN   | Input variables | Output variables | Hidden layers | Neurons of hidden layers (simulation dataset) | Neurons of hidden layers (experiment dataset) |
|------|----------------|-----------------|--------------|-----------------------------------------------|-----------------------------------------------|
| NN1  | 3              | 2               | 8            | 40                                            | 40                                            |
| NN2  | 2              | 1               | 4            | 10                                            | 20                                            |

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(see Sec.III-A). To balance the contributions of data and physics, we define
\[
\mathcal{L} = \frac{1}{M} \lambda_{PDE} \mathcal{L}_{PDE} + \frac{1}{M} \lambda_{Data} \mathcal{L}_{Data}
\]  
(12)

where \(\lambda_{PDE}\) and \(\lambda_{Data}\) are hyperparameters that should be optimized, especially when noisy data are involved. \(M\) denotes the number of the datapoints used for training, i.e., batch size, \(M = M_{i} + M_{o}\), where \(M_{i}\) and \(M_{o}\) denote the numbers of datapoints taken from inside and outside of the ROI, respectively. The datapoints in the batch are randomly chosen from the spatiotemporal space \((x_{1} \times x_{2} \times t)\) and will be updated every 1,000 epochs.

To terminate the training when the loss function reaches convergence, we define \(N_{0} = 8 \times 10^{4}\) and \(\varepsilon = 10^{-4}\), as shown in Fig. 1. The number of epochs needed for convergence primarily depends on the size of the network, batch size, learning rate (default value, 0.001), etc. For the network used in this study, we find the loss function gradually approaches to the convergence value when the epoch number is larger than \(8 \times 10^{4}\). The loss function shows strong oscillations when approaching the convergence value. A big \(\varepsilon\) such as \(10^{-3}\) for the stopping criteria may be triggered by the oscillations before reaching the convergence. Here we choose \(\varepsilon = 10^{-4}\), which is usually triggered when the epoch number is larger than \(N_{0}\) if the default learning rate is used, but can also be triggered earlier if a larger learning rate (e.g., 0.005) is used. For the latter case, the loss function has already reached convergence and thus the trigger for stopping works properly.

**C. Transfer Learning to Speed Up the Training of SWENet**

Different from the data-driven deep learning approaches, SWENet is re-trained and infers the material parameters simultaneously when a new dataset is obtained from an experiment. Retraining on every dataset largely reduces the quantity of training data needed to avoid overfitting but makes the prediction process less efficient. We thus consider transfer learning to speed the training [32], [33]. In general, the weights and biases of NN1 and NN2 in the SWENet are initialized randomly, making the training of SWENet a time-consuming process. To speed up the training, we can perform transfer learning by utilizing the weights and biases of a well-trained SWENet. Please note it is unnecessary for the new dataset to have the same size as the well-trained one. In fact, when performing SWE measurements, the size and location of the ROI should be determined according to the size and location of a real tumor. Fewer iterations are needed to train SWENet for a new dataset if it is more similar to the well-trained one. In this study, SWENet is built on the deep learning framework Keras, which is based on Tensorflow 2.10.0 (cudna 11.2, cuDNN 8.1). The computer is equipped with 4 NVIDIA GeForce RTX 2080 Ti GPUs. For each case, every 100 epochs takes \(\sim 60\) s using one GPU.

**D. Time of Flight**

For comparison, we also performed inverse analysis on each dataset by the time-of-flight (ToF) algorithm [24]. In this method, the time delay of the shear waves traveling at a given distance is computed. And therefore, the shear wave speed can be estimated. A robust algorithm to compute the time delay has previously been proposed by Wiens and Bradley [34], and the codes for this algorithm are available freely.\(^1\) In this study, we developed our codes to execute the ToF inversions based on this code. Briefly, the key parameter in this algorithm is the number of delays (NoD). The NoD depends on SNR and shear wavelength (or shear modulus). A small NoD guarantees a better spatial resolution. However, an over-small NoD will produce a map with many speckles. So we gradually enlarged the NoD from 2 until the speckles could be suppressed. Usually, the delay will be larger for data with lower SNR. Here NoD is set to be 2 for the simulation datasets, 10 for simulation datasets with artificial white noises. As for the experimental dataset, a spatial averaging filter (kernel 8\(\times\)8) was used on the IQ data to improve the inference results, and the number of delays is set to be 20 for the filtered experimental datasets (276\(\times\)135 pixel). Besides, the directional filter proposed previously [24] was applied to the spatiotemporal data before the ToF algorithm was implemented. All the ToF inversions were performed on Matlab R2020b (The Mathworks, Natick, MA, USA).

**E. Finite Element Simulations**

We performed finite element simulations to produce simulation data that were used to validate SWENet. All the simulations were performed with the help of Abaqus/standard (Abaqus 6.14, Dassault Systèmes®). We created a two-dimensional square domain that was large enough to avoid wave reflections at the boundaries. The constitutive model for the material was incompressible neo-Hookean. The density \(\rho\) was a constant 1000 kg/m\(^3\) throughout the model. For illustration, in all numerical examples and phantom experiments performed below, the shear modulus outside the ROI is supposed to be a known constant \(\mu_{0}\) that can be measured, for example, by shear wave speed. Then the shear modulus for the full field can be expressed as
\[
\mu(x_{1}, x_{2}) = \begin{cases} 
\mu_{ROI}, & (x_{1}, x_{2}) \in ROI \\
\mu_{0}, & \text{otherwise} 
\end{cases}
\]  
(13)

The ROI was a subdomain within the model that shear waves would completely illuminate.

In simulations, we set the size of the square ROI to be \(8 \times 8\) mm\(^2\). To describe spatial distributions of \(\mu_{ROI}\) analytically, we defined a linearly temperature-dependent shear modulus and prescribe an analytical temperature field. Briefly, the shear modulus linearly varied from 4 kPa to 16 kPa when the temperature \(T\) increased from 0 to 1 °C. For the first simulation study where a circular inclusion was involved, the analytical expression (the Sigmoid function) for \(T\) was
\[
T = 1 - \frac{1}{1 + e^{-A_{0}(T-r_{0})}},
\]  
(14)

\(^1\) see https://www.mathworks.com/matlabcentral/fileexchange/25210-subsample-delay-estimation.
where \( r = \sqrt{(x_1 - x_1')^2 + (x_2 - x_2')^2} \), \((x_1', x_2') = (0, 0)\) is the center of the inclusion, \( r_0 = 1.5 \text{ mm} \) is the radius of circular inclusion, and \( A_0 = 6 \) is a constant parameter. To demonstrate transfer learning, we performed additional simulations by changing the peak value of shear modulus from 9 kPa, \((x_1', x_2')\) and \( r_0 \) to \((-0.5, 0.5)\) and 2.5 mm, and \((0.5, -0.5)\) and 2 mm, respectively, to generate two new datasets. Then the two new datasets were fed to the pre-trained SWENet.

For the second simulation where a malignant tumor is modeled, we utilized the superimposition of two Sigmoid functions to describe \( T \), as described in a previous study [35], [36].

\[
T = 1 - \frac{1}{1 + e^{-A_1(d_1 - 1)}} \times \frac{1}{1 + e^{-A_2(d_2 - 1)}}
\]

\[
d_1 = \sqrt{\frac{(x_1 - x_1')^2}{a_1^2} + \frac{(x_2 - x_2')^2}{b_1^2}}
\]

\[
d_2 = \sqrt{\frac{(x_1 - x_1')^2}{a_2^2} + \frac{(x_2 - x_2')^2}{b_2^2}}
\]

(15)

where \((x_1', x_2') = (-0.6, 0)\) and \((x_1', x_2') = (1.6, 0.6)\) are the coordinates for the centers of the ellipses, \((a_1, b_1, a_2, b_2) = (2.01, 1.50, 1.16, 0.95)\) are the geometric constants of the ellipse, and \((A_1, A_2) = (9, 6)\) are constant parameters.

The simulations were performed in the time domain. To generate shear waves, in the first analysis step, we applied a vertically moving body force \( f \),

\[
f(x_1, x_2, t) = f_0 \exp\left[-\frac{(x_1 - x_{10})^2}{2r_1^2} - \frac{(x_2 - x_{20} - v_d t)^2}{2r_2^2}\right],
\]

(16)

where \( f_0 = [0, f_0] \) is a vector that specifies the magnitude and direction of the peak body force, respectively. We took a small value for \( f_0 = 0.001 \text{ N/mm}^2 \) to ensure a small amplitude for the shear waves. \( v_d \) denotes the moving speed of the force, which was set to be \( \sim 40 \sqrt{\rho / \rho} \). The greater force moving speed than these of the shear waves gives rise to the elastic Cherenkov effect, resulting in a Mach cone for the shear waves [37]. The parameters \( r_1 = 0.14 \text{ mm} \) and \( r_2 = 1.1 \text{ mm} \) specify the spatial distribution, and the coordinate \((x_{10}, x_{20})\) specifies the location of the body force. The duration of the body force was about 0.1 ms. The moving body force simulates the acoustic radiation force (ARF) generated by an ultrasound sound beam that successively focuses on different sides of the ROI, respectively. When the ARF was imposed on the left/right side of the ROI, shear waves propagating to the right-hand/left-hand direction would illuminate the ROI.

We used a uniform mesh with an element size of \( \sim 0.05 \times 0.05 \text{ mm}^2 \). The element type was CPE8RH in Abaqus (plane strain, 8-node biquadratic, hybrid with linear pressure and reduced integration). The convergence of simulations results was checked by decreasing the element size and finding the change in the results is smaller than 1%. After the simulation, we down-sampled the results based on a uniform grid size of 0.05 \times 0.1 \text{ mm}^2, making the dataset consistent with our experiments.

F. Tissue-Mimicking Phantom Experiments

In experiments, we performed SWE on a gelatin phantom. Following the protocol described in our previous paper [38], we prepared a phantom consisting of a soft matrix and a stiffer inclusion. Briefly, we prepared the matrix and the inclusion separately, each with different concentrations of carrageenan to tune the stiffness.

Taking the preparation of the inclusion as an example, we dissolved gelatin (AR, 1kg, Greagent, Shanghai Titan Scientific Co., Ltd, China) with a weight fraction of 5% and carrageenan (C804872-1kg, Shanghai Macklin Biochemical Co., Ltd, China) with a weight fraction of 0.2% into 80 °C water, and then we added cellulose (Type 40, C875044-1kg, Shanghai Macklin Biochemical Co., Ltd, China) with a weight fraction of 2% into the solution to serve as ultrasound scatters. We put the solution into a designated mold and then cured the solution in a 4 °C fridge for 6 h. Similarly, the softer matrix was prepared, with the weight fractions of carrageenan and cellulose to be 0.1% and 1%, respectively.

Our experimental system was built on a Verasonics Vantage 64LE (Verasonics Inc., Kirkland, WA, USA) with an L9-4 transducer (central frequency 7 MHz, 128 elements, pitch 0.3 mm). Shear waves were generated by focused ultrasound beams using 32 elements (P3 voltage in Verasonics ~25 V, aperture size ~10 mm, and uniform apodization). Six ultrasound beams (the duration of each beam was 0.042 ms) successively focused at 7.1 mm to 22.6 mm deep, resulting in a moving ARF. The ARFs were applied to the left and right sides of the ROI, respectively, to illuminate the ROI.

After the excitation, the system switched to the plane wave imaging mode working at a frame rate of 10 kHz to acquire the shear wave propagation. In this mode, all the 128 elements (P3 voltage in Verasonics ~25 V, aperture size ~40 mm, uniform apodization) were used to transmit, but only the 64 elements at the center of the transducer were used to receive. We simply used the delay and sum beamforming for the plane wave imaging. The in-phase and quadrature (IQ) data were analyzed offline to get the particle velocity \( v_s \) using the Loupas’ estimator [39]. We expect the average of the measurements can improve the signal-to-noise ratio (SNR). Therefore, we performed thirty successive measurements, and the average of the measurements was taken as the training data.

The training data for SWENet was obtained in the cross-section of the cylindrical inclusion. However, to estimate the elastic properties phantom, we also measured the shear wave speeds in the inclusion and the softer matrix, respectively, in the longitudinal view, i.e., shear waves propagate along the axial direction. The group velocity of the elastic waves in the inclusion is approximately equal to that of the bulk shear waves because the waveguiding effect is negligible owing to a relatively large inclusion diameter (8~9 mm thickness); and in this case, the shear moduli of the inclusion
and the matrix were calculated using \( \mu = \rho v^2 \), where \( v \) denotes the shear wave speed.

For comparison, we also performed SWE measurements on the phantoms with an Aixplorer ultrasound machine (SuperSonic Imagine, Aix-en-Provence, France). The default settings for Abdominal/muscle were used for the measurements.

**G. Ex Vivo Experiments on Tumors**

To demonstrate the performance of SWENet on real tumor data, we performed ex vivo experiments with a tumor harvested from a mouse. The Laboratory Animal Welfare and Ethical Review Committee of Institute of Analysis and Testing, Beijing Academy of Science and Technology, approved all animal protocols, and the procedures used here followed these approved protocols (#221205-SWDWF-001).

Human Colon Cancer Cells (HCT 116, \( 3 \times 10^6 \) tumor cell) were subcutaneously injected into the back of the right forelimb of female nude mice (4 weeks old, 15~20g, BALB/c Nude Mouse, Beijing Vital River Laboratory Animal Technology Co., Ltd.). All tumors were allowed to grow until their volume was between 400 and 600 mm\(^3\). HCT116 tumors reached the desired volume range with 12~14 days.

All tumors were surgically removed and encased in a gelatin block as described previously [25], [26], [27] to minimize the impact of breathing and cardiac motion on shear wave imaging. After exposure, the tumor was mounted inside a homogenous block of gelatin, which consisted of 5% by weight gelatin (AR, 1kg, Greagent, Shanghai Titan Scientific Co., Ltd, China), 0.1% by weight carrageenan (C804872-1kg, Shanghai Macklin Biochemical Co., Ltd, China), and 0.8% by weight cellulose (Type 40, C875044-1kg, Shanghai Macklin Biochemical Co., Ltd, China). In short, the molten solution was prepared and cooled to a constant temperature (~25°C). Then, the tumor was hung by an acupuncture needle (~0.3 mm in diameter) and suspended inside the 11 × 11 × 4.7 cm\(^3\) homogenous block of gelatin, until solidifying.

When performing SWE, the tumor was positioned in the center of the square ROI. The side lengths of the square field of view (FOV) and the ROI for mechanical property inversion were 70 wavelengths (~15 mm) and 55 wavelengths (~12 mm), respectively. The ARF was first imposed on the left side of the ROI. The duration of the measurement was 8 ms. We performed thirty successive measurements and took the average of the data to improve the signal-to-noise ratio (SNR). The experiments were repeated with the ARF imposed on the right side of the ROI.

We hypothesize that tumor stiffness is correlated with collagen density. To test the hypothesis, Masson staining was performed on the excised tumor after the SWE measurements. The sample was cut along the imaging plane to expose the tumor. Then the tumor was formalin-fixed and embedded in paraffin for thin sectioning (2 \( \mu \)m) along the imaging plane and stained with Masson trichrome stain. Slide scanning was performed on an Axio Scan.Z1 (Zeiss, Oberkochen, Germany).

**H. Synthetic Data for Shear Waves Excited by Multiple Sources**

The data of a transient shear wave is sparse in the spatiotemporal domain. We thus check whether the shear waves resulting from multiple wave sources can improve the accuracy of the inference. To demonstrate, we applied the moving ARF to the left and right sides of the ROI, respectively. Then the multi-source data were obtained by superposing the two datasets. The multi-source data generated in this way may differ from those generated by a simultaneous multi-source excitation because the interaction between shear waves has been ignored. Since the amplitudes of the shear waves used for SWE are small, the interaction is reasonably negligible.

**III. RESULTS**

**A. Learning Hidden Elastic Heterogeneities From Traveling Shear Waves With SWENet**

We first show how SWENet infers hidden elasticity from traveling shear waves. In this case, the simulation for a circular inclusion was utilized for demonstration. Figure 2a shows representative snapshots (\( t = 0 \) ms, 1.5 ms, 3 ms, and 4.5 ms) of the simulation data (denoted by \( v_2^s \)). The dashed square indicates the ROI. The waves propagating to the right-hand direction illuminate the inclusion, and then the planar wavefront is distorted by elastic heterogeneity. In the training, the batch sizes of the NN1 and NN2 were \( M_i = 1.2 \times 10^4 \) and \( M_o = 2 \times 10^4 \), respectively. We tried different weight ratios \( \lambda_{PDE}/\lambda_{Data} \) (0.1, 1, 10) and found it has negligible effects on the inference for this dataset. Here we present the results obtained when \( \lambda_{PDE} = 1 \) and \( \lambda_{Data} = 0.1 \).

Figure 2b depicts the evolution of each loss term (\( L_{PDE} \) and \( L_{Data} \)). At the first ~ 2 × 10\(^3\) epochs, \( L_{Data} \) decreases dramatically but \( L_{PDE} \) increases, due to the low contribution of \( L_{PDE} \) to the total loss function. After 2 × 10\(^3\) epochs, \( \lambda_{PDE} L_{PDE} \approx \lambda_{Data} L_{Data} \), so the two-term decrease simultaneously with similar speeds. The total loss reaches a plateau after about 7 × 10\(^4\) epochs. Therefore, we stopped the training at 8 × 10\(^5\) epochs. The total loss is reduced by more than four orders of magnitude after the training. In Fig. 2a, we show \( v_2 \) output by the NN1 and the comparison with the training data \( v_2^s \). The relative error, defined as \( |(v_2 - v_2^s)/\max(v_2^s)| \) for each snapshot, is less than 3%, indicating an excellent fit to the training data. Notably, despite the fact that we didn’t use \( v_2^s \) in the training as this data is not available for ultrasound SWE, the NN1 correctly retrieves \( v_1 \), with good accuracy (relative error less than 10%, see Fig. 2e). The ability of NN1 to retrieve \( v_1 \) should ascribe to the incompressible constraint (see Eq. (6)), revealing the benefits of incorporating physical information into deep neural networks.

SWENet also learns the hidden elastic heterogeneity within the ROI. As the NN2 is a surrogate model of the shear modulus in the ROI, it has been trained to enforce the wave...
Fig. 2. Learning hidden elasticity from traveling shear waves with SWENet. (a) First row, the snapshots of the training data (vertical component of the particle velocity, $v^*$). The dashed square shows the ROI. Scalebar, 5 mm. Second row, $v_2$ output by NN1. Third row, the relative error between $v^*$ and $v_2$. (b) Evolutions of the physics-informed (PDE) and data-driven (Data) terms in the loss function and their sum. (c) First row, the snapshots of $v^*_1$ that are used to evaluate the accuracy of $v_1$ (second row) predicted by NN1. The third row shows the relative error between $v^*_1$ and $v_1$. (d) The maps of the true shear modulus ($\mu^*$), and the inference by SWENet and ToF. The plots show comparisons of the shear moduli profiles along $x_1$ and $x_2$ axes.

equations satisfied. Figure 2d shows the shear modulus output by the NN2, which is in excellent agreement with the shear modulus ($\mu^*$) used for the simulation. To make a quantitative comparison, we also plot the transverse profiles of the shear moduli through the centers of the maps. The maximum relative error between the two curves is less than 8%.

For this simple case, we find the SWENet greatly outperforms the traditional inversion method used in SWE, i.e., the ToF algorithm. As shown in Fig. 2d, the shape of the inclusion identified by ToF deviates from a circular shape, and a fake wake emerges, due to the diffraction of the shear waves. In fact, wave reflections and diffractions are primary factors that may lead to bias in the ToF [24], [40], making the inference of complex elastic heterogeneities from traveling waves extremely difficult.

We also find the introduction of $v^*_1$ can further improve the inference. To quantify, we calculate the root mean square error (RMSE) between $\mu$ and $\mu^*$. The RMSE drops from 0.2180 to 0.2061 after introducing additional data for $v^*_1$.

B. Transfer Learning of the SWENet

Despite the success in identifying elastic heterogeneity, the training of the SWENet from randomly initialized biases and weights takes over $7 \times 10^4$ epochs, which is a time-consuming process. Figure 3 shows the training of SWENet can be sped up by transfer learning. The two new datasets utilized here are shown in Fig. 3a (see Materials and Methods for details). We fed the two new datasets to the pre-trained SWENet, respectively, and then performed the training process to minimize the loss function. Figure 3b plots the loss functions (blue lines) of the transfer learning. For comparison, we also plot the loss functions (gray lines) for the two new datasets when trained on randomly initialized SWENet. To achieve three
orders of magnitude decrease in the loss function, it takes about $2 \times 10^3$ epochs for the pre-trained SWENet, whereas ten times more epochs for the randomly initialized SWENet. Figure 3c shows the shear modulus inferred by the pre-trained SWENet during the transfer learning. Since the weights and biases have been reloaded, the shear modulus output by the pre-trained SWENet is similar to that shown in Fig. 2d at the beginning of the training (epoch = 1). After about $10^4$ epochs, the shear modulus inferred by the pre-trained SWENet do not vary significantly and show excellent agreement with the ground truth. The correctly identified shear moduli by fewer training epochs thus suggest that transferring is an effective way to speed up the training of SWENet.

C. Effect of Data Noise on the Inferring Material Parameters With SWENet

We proceed to study the effect of data noise on the performance of SWENet. To this end, we introduced white noises to the simulation data. To quantify, we define the signal-to-noise ratio (SNR) as $20 \times \log_{10} \left[ \frac{\max(\|v^*_2\|)}{\sigma} \right]$, where $\max(\|v^*_2\|)$ is the maximum value of the signal and $\sigma$ is the standard deviation of the noise floor. According to this definition, we find the SNR of our phantom experiments is about 30 dB. For illustration, we introduced 30 dB SNR to the simulation data, as shown in Fig. 4a. For noisy data, we find the inference in the shear modulus shows dependency on the hyperparameter $\lambda_{PDE}/\lambda_{Data}$. Generally speaking, the weight for PDE should be larger to filter out noises. Figure 4b depicts $v_2$ (snapshot at $t = 4.5$ ms) output by the NN1 after $8 \times 10^4$ epochs for different weight ratios $\lambda_{PDE}/\lambda_{Data}$ ($10^1$, $10^2$, $10^3$, and $10^4$). The noises have been learned when the weight of data is large. However, $v_2$ becomes much smoother as the weight of the physical constraint is enhanced, suggesting that the physical constraint is an effective spatial filter. Figure 4c shows the shear modulus $\mu$ inferred by SWENet. To evaluate the accuracy of $\mu$, we calculated the RMSE between $\mu$ and $\mu^*$, as shown in Fig. 4d. The plot suggests an optimal ratio $\lambda_{PDE}/\lambda_{Data} \approx 10^3$, which results in the minimal RMSE. We realize the optimal ratio should be primarily ascribed to the trade-off between computation cost and inference performance. On the one hand, a smaller weight for physics results in a larger convergence speed. Therefore, the shear modulus inferred by the SWENet does not vary significantly before the training has been terminated (at $6 \times 10^4$ epochs). However, the fast convergence does not guarantee good inference due to the
noise learned. On the other hand, increasing the number of the weight for physics dramatically slows the training, leading to a poor performance when the training is terminated at the given number of epochs. Actually, for the case with the largest physical weight ($\lambda_{PDE}/\lambda_{Data} = 10^4$), the convergence of inference hasn’t reached at $8 \times 10^4$ epochs, and the shear modulus inferred by SWENet can be slightly improved with more iterations.

While we expect the optimal ratio is also a function of the SNR, we find for 20 dB, 30 dB, and 40 dB data, $\lambda_{PDE}/\lambda_{Data} \approx 10^3$ among $\{10^1, 10^2, 10^3, 10^4\}$ results in the best performance for SWENet. Figure 4d shows the identified shear moduli by SWENet (first row) and ToF (second row) for data with different noise levels. Apparently, the engagement of the noises results in speckles in the shear modulus map obtained by ToF. The plot presents the corresponding RMSE for the results identified by SWENet. For each SNR, we synthesized three datasets by adding random noises to the same simulation. We find the inferences don’t vary significantly. Therefore, the standard deviations are not reported here. These data collectively suggest that SWENet can filter out noises thanks to the physical constraints and successively identify the hidden elastic heterogeneity. However, the noises will deteriorate the accuracy of SWENet, which underscores the importance of high-quality data.

D. Integrating Multi-Source Data With SWENet

As the data for a transient shear wave is sparse in spatiotemporal space, we expect integrating multiple shear waves will improve the performance of SWENet. In fact, acoustic radiation force (ARF) that can be programmed both spatially and temporally is frequently used to induce shear waves in soft biological tissues in state-of-the-art SWE measurements. While ARF enables a flexible way to introduce multiple source shear waves in the same spatiotemporal space, it remains to be a challenge to infer the elasticity from this data [41].
Here we show that multi-source data given by programmed ARFs can be seamlessly integrated into SWENet to achieve more accurate results in the inverse analysis. The ability to seamlessly integrate multi-source data is thus a significant advantage of SWENet.

Figure 5 shows the identification of a simulated tumor (see Materials and Methods for details) from multi-source data. In our simulations, the tumor was illuminated by ARFs applied on the left (L) and right (R) sides of the ROI, respectively, as shown in Fig. 5a. The multi-source data was then generated by the superposition of the two simulations. The distribution of the shear modulus for the tumor is shown in Fig. 5b. The data from single (left and right stimuli) and multi-source were fed to the SWENet to perform inversions. Fig. 5c. shows the modulus identified from each dataset. The shear moduli inferred by ToF are provided for comparison. For each dataset, it is obvious that the SWENet outperforms the ToF because the latter suffers heavily from reflection and diffraction of shear waves introduced by the irregular shape of the tumor. Importantly, we find the shear modulus inferred from the merged data matches the ground truth best. The profiles of the identified shear moduli along the four paths (see Fig. 5b) indicate excellent agreement with the ground truth.

E. Elastogram Reconstruction for Tissue-Mimicking Phantom With SWENet

Here we performed SWENet on the phantom experimental data. Figure 6b outlines the shape (red dashed lines) of the inclusion, which mimics the simulations shown in Fig. 5. We integrated the data resulting from the left and right stimuli. As shown in Fig. 6a, the triangles indicate the two shear wavefronts. According to the definition (see Sec.III-C), we get the SNR of the phantom experimental data is ∼29.8 dB. The average of thirty measurements results in a ∼0.1 dB improvement. Since the SNR has not been improved significantly, a single measurement is adequate to perform the inversion when short acquisition time is highly favorable. We fed the averaged data to SWENet and set $\lambda_{PDE} = 1$, $\lambda_{Data} = 10^{-4}$, $M_i = 1.5 \times 10^4$, and $M_o = 5 \times 10^3$. The large weight for PDE was used to suppress the noise, which resulted in the smooth wave fields as shown in the second row of Fig. 6a. Figure 6c shows the maps of the shear moduli and the profiles through the representative paths depicted in Fig. 6c. The dashed line denotes the ground truth obtained from bulk shear wave speeds, i.e., $1.95 \pm 0.02$ m/s in the softer matrix and $4.81 \pm 0.08$ m/s in the stiffer inclusion (see Methods). The SWENet precisely identifies the locations and shape of the inclusion. We find the Aixplorer underestimates the stiffness of the inclusion (see Lines 1 and 2 in Fig. 6c), in line with our previous observations [38]. Differently, SWENet overestimates the stiffness of the inclusion at the center and shows slow yet smooth transitions at the edges of the inclusion. The overestimation and slow transition in contrast to simulations shown in Fig. 2d should be ascribed to the viscosity of the phantom that filters out the high-frequency shear waves. For the smaller inclusion, SWENet and Aixplorer give consistent results (Lines 3 and 4 in Fig. 6c), both slightly underestimating...
Fig. 6. Elastogram reconstruction for a tissue-mimicking phantom based on SWENet. (a) First row, snapshots of the merged experimental data (L+R). Second row, $v_2^2$ output from the NN1 after training on the merged data. The blue and red triangles indicate the locations of the wavefronts that correspond to L and R stimuli, respectively. (b) Photo of the phantom. The black dashed square shows the ROI. The red dashed circle highlights the location of the stiff cylindrical inclusion. The white dashed line shows the profiles of interest. (c) The maps of the shear moduli inferred by SWENet, Aixplorer, and ToF. The solid lines in the plot show the profiles of the shear moduli through the two centers of inclusion. The dashed line shows the reference values of the shear moduli obtained from bulk shear wave speeds.

Overall, SWENet has a comparable performance as Aixplorer on this specific sample though our experimental setup was built on the Vantage 64LE system (see Methods) of which the number of channels is half of the Aixplorer. This may be attributed to the reason that more experimental information was used in SWENet in comparison with conventional SWE methods.

F. Elastogram Reconstruction for Ex Vivo Tumors

Figure 7 shows the experiment performed on the ex vivo tumor. Likely because the tumor has similar acoustic properties as the matrix, the tumor cannot be clearly identified from the grayscale B-mode image shown in Fig. 7a. To better visualize the tumor, we present a photo of the cross-section of the tumor, which is obtained after the SWE measurements. We fed SWENet with the integrated data obtained from left and right stimuli, as shown in Fig. 7b, and set $\lambda \rho D E = 1$, $\lambda \text{Data} = 10^{-4}$, $M_l = 2 \times 10^4$, and $M_o = 10^4$. The existence of the tumor apparently distorts the wavefronts (see the triangles in the figure). Interestingly, we find SWENet correctly identifies the tumor. For comparison, we outline the shape of the tumor with a dashed line, as shown in Fig. 7c. The shear modulus within the dashed line is significantly higher than the matrix, which provides much better contrast for the tumor and the matrix than the ultrasonic image. In contrast, the shear modulus map reconstructed by ToF shows numerous speckles and does not correspond well with the profile of the embedded tumor.

The shear modulus of the tumor shows a non-uniform distribution. To justify, we make a comparison with Masson’s trichrome stain of the tumor. The Masson’s trichrome stain helps to visualize the collagen density (blue). The white areas in the tumor, however, indicate the fluidic tissues that cannot be strained. We find the region with higher collagen density and less fluidic tissues (upper right corner) corresponds well with the region with a higher shear modulus, which verifies the effectiveness of SWENet in elastogram reconstruction for real tumors.

IV. DISCUSSION

Our study demonstrates that SWENet enables inferring the spatial distribution of elastic properties from traveling shear waves with higher accuracy in comparison with conventional SWE methods relying on the measurement of shear wave velocities. Identification of material parameters from traveling...
waves is a typical inverse problem. There are several merits in SWENet in dealing with such an inverse problem. First, SWENet makes full use of the data to infer the elastic properties instead of merely calculating the local shear wave speed such as with the time-of-flight algorithm. Alternatively, the full waveform inversion (FWI) implemented through the adjoint-state algorithm can also make full use of the data [42].

Despite a quantitative comparison for the efficiency between SWENet and FWI is not possible as we do not have complete information about the model studied by Arnal et al., we perform SWENet on a similar model, as shown in Fig. 8. Both the inferences from SWENet and FWI agree well with the ground truth. Differently, FWI needs much fewer iterations (∼25 times), but each iteration takes a longer time to solve the direct problems. To speed up, Arnal et al. have built FWI on the linear acoustic equation, instead of the complete vector wave equation encoded in SWENet. Technically, the advantage of SWENet over FWI is that the size and shape of the geometry can be flexibly chosen for practical use [43]. Second, SWENet encodes the governing equations of wave motion in loss functions, which essentially acts as a regularization term to constrain the space of admissible solutions. In contrast, conventional regularization approaches usually suffer from the drawback of lacking physical meanings. Third, compared to the supervised learning-based methods, the PINN-based SWENet doesn’t require a vast dataset for training and therefore retains the flexibility to choose different ways to generate shear waves and designate the shapes of the ROI. Finally, SWENet allows us to integrate multi-source data in the inverse analysis, which is particularly useful for the elastography of a soft material for which the spatial distribution of elastic modulus is complex.

Regarding the ability to integrate multiple-source data, the examples we show in this study are only based on two shear waves propagating along transverse directions. We expect the inference in the longitudinal direction will significantly benefit from additional tilt plane shear waves, which are possible by focusing the ultrasound beams along tilt paths. Although integrating multiple-source shear waves help infer complex hidden elasticities, we note there is a fundamental limitation...
Fig. 9. Effect of viscoelasticity on SWENet. (a) First row, the snapshots of the training data (vertical component of the particle velocity, $v_2^*$). The dashed square shows the ROI. Scalebar, 5 mm. Second row, $v_2$ output by NN1. Third row, the relative error between $v_2^*$ and $v_2$. (b) Evolutions of the physics-informed (PDE) and data-driven (Data) terms in the loss function and their sum. (c) First row, the snapshots of $v_1^*$ that are used to evaluate the accuracy of $v_1$ (second row) predicted by NN1. The third row shows the relative error between $v_1^*$ and $v_1$. (d) The maps of the true shear modulus ($\mu^*$), and the inference by SWENet and ToF. The plots show comparisons of the shear moduli profiles along $x_1$ and $x_2$ axes.

on the spatial resolution of SWE, primarily determined by the wavelength of the shear wave. We envision the idea to integrate multiple-source data can easily be extended to integrating multiple-fidelity data obtained from different imaging modalities, such as ultrasound SWE, magnetic resonance elastography [44], and optical coherence elastography [45], an approach to probe multiscale mechanical properties in situ.

This study subjects the following limitations. First, considering only the wave motion information in the 2D imaging plane can be obtained in our experiments, the 2D problem is considered in the present version of SWENet. It is straightforward to extend SWENet to 3D by introducing an input layer with an additional neuron for $x_3$ and encoding the three-dimensional wave equations into the network. Developing 3D shear wave elastography is promising and has received considerable attention in recent years [46]. We envision SWENet can be useful for image reconstruction of 3D SWE, where complex wave reflection and diffraction get involved. This issue deserves further investigation. Second, the potential of SWENet in biomechanical characterization of inhomogeneous soft materials including tumors has been demonstrated preliminarily in this study by means of finite element simulations, phantom experiments, and ex vivo experiments. However, in vivo experiments need to be performed to further demonstrate the usefulness of SWENet in clinics. Third, wave motion equations for elastic soft materials are encoded in SWENet. In practice, tissue viscoelastic deformation may induce dispersion and attenuation of shear waves. To illustrate the effect of tissue viscoelasticity, we introduce the Voigt model [47] ($\mu_m = 4$ kPa and $\eta_m = 0.15$ Pa·s for the matrix, $\mu_i = 16$ kPa and $\eta_i = 0.6$ Pa·s for the inclusion) to the simulation of the circular inclusion shown in Fig. 2. A direction comparison between Figs. 2a and c and Figs. 9a and c clearly shows the dispersion and attenuation introduced by viscoelasticity. To test the robustness, we studied the performance of SWENet on the viscoelastic data.
According to Fig. 9b, the PDE term in the loss function decreases with a rather low speed, which should be attributed to the mismatch between the wave equation encoded in SWENet and the simulation. Despite this mismatch, we find the shear modulus identified by SWENet agrees well with the storage modulus used for simulation, greatly outperforming the ToF method (NoD is 4). Along the \(x_1\) axis, the profile of the shear modulus shows a sharper rising edge but a slower falling edge (see Fig. 9d) because the viscoelastic dispersion widens the wavefronts and thus decreases the spatial resolution. This example shows the robustness of SWENet in dealing with viscoelastic data. In the future, we aim to encode viscoelastic wave equations into SWENet, for example, the fractional viscoelastic model that is demonstrated to be able to capture the power-law dispersion of soft biological tissues. However, how to infer additional viscoelastic parameters by training the network with the experimental data given by a proper design requires further effort.

V. CONCLUSION

SWE of inhomogeneous soft materials remains a challenging issue due to the complexity of wave fields and their obscured correlation with material parameters. In the present study, we overcome this challenge by proposing a physics-informed neural network (PINN)-based SWE (SWENet) method considering the merits of PINN in solving an inverse problem. The spatial variation of elastic properties of inhomogeneous materials has been defined in governing equations, which are encoded in PINN as loss functions. The particle velocities in vertical direction inside a local region that are measurable in practical experiments have been used to train the neural networks. The trained neural network enables the full-wave inversion and inferring the spatial distribution of elastic properties with high spatial resolution. Finite element simulations, tissue-mimicking phantom experiments and ex vivo experiments have been performed to validate the method. Moreover, the effect of data noise, a seamless integration of multi-source data in the inverse analysis and speeding up of the SWENet with transferring have been addressed, and the results facilitate the use of SWENet in practice. Although tissue-mimicking phantom experiments performed in this study have demonstrated the applicability of SWENet in the mechanical characterization of inhomogeneous soft materials, its use in clinics, for instance, its application to imaging elastic properties of nerves in vivo and differentiating small malignant tumors from benign ones by quantitatively measuring their distinct stiffnesses, deserves further efforts.

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