Dynamical coupled-channels approach to electroweak meson productions on nucleon and deuteron

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Introduction
Why electroweak meson productions on nucleon and deuteron?

- **Baryon spectroscopy**: Identification of N* and extracting their properties (mass, width) Combined with hadron models and LQCD → understanding QCD in nonperturbative regime

- **Basis of understanding nuclear target reactions**:
  -- medium effects on propagation of mesons and N* in nuclear matter
  -- neutrino-nucleus reactions for extracting neutrino oscillation parameters

Why dynamical model approach?

- **Physical insight into the reactions**: Understanding reaction mechanisms and dynamical contents of resonances in terms of hadronic degrees of freedom

- **Application to nuclear target reactions**
  Dynamical meson-baryon reaction model is compatible well with multiple scattering theory for many-body system
Plan of this talk

Overview of our recent activity with dynamical coupled-channels (DCC) approach

- **DCC model for electroweak meson productions on single nucleon**
  -- DCC analysis of $\gamma N, \pi N \rightarrow \pi N, \pi \pi N, \eta N, K\Lambda, K\Sigma$ data
  -- Extension to finite $Q^2$ and neutrino reactions

- **Application of DCC model to deuteron-target processes**
  -- Extraction of neutron-target observables from $\gamma d \rightarrow \pi NN$
  -- Novel method to extract $\eta$-nucleon scattering length from $\gamma d \rightarrow \eta pn$
  -- Extraction of neutron-neutron scattering length from $\gamma d \rightarrow \pi^+ nn$
Dynamical coupled-channels model for

\[ \gamma N, \pi N \rightarrow \pi N, \pi\pi N, \eta N, K\Lambda, K\Sigma \]

Kamano, SXN, Lee, Sato, PRC 88 035209 (2013); 94 015201 (2016)
Meson photo-productions in resonance region

Data for $\gamma p \rightarrow X$

We develop a DCC model to describe these reactions

- Several nucleon resonances form characteristic peaks
- $2\pi$ production is comparable to $1\pi$
- $\eta, K$ productions (multi-channel couplings are important physics)
Dynamical coupled-channels model

ANL-Osaka Collaboration: Kamano, SXN, Lee, Sato, PRC 88 (2013); 94 (2016)

Coupled-channel
Lippmann-Schwinger eq.

\[ T_{\beta,\alpha}(p_\beta, p_\alpha; W) = V_{\beta,\alpha}(p_\beta, p_\alpha; W) + \int dp_\gamma p^2 V_{\beta,\gamma}(p_\beta, p_\alpha; W) G_{\gamma}(p; W) T_{\gamma,\alpha}(p, p_\alpha; W) \]

\[ \{\alpha, \beta, \gamma\} = \{\pi N, \pi\pi N, \eta N, K\Lambda, K\Sigma, \pi\Delta, \rho N, \sigma N\} \]

\( V_{\beta,\alpha} \): Resonance excitation + non-resonant meson-exchange mechanisms

Channel-couplings required by unitarity \((\pi N, \eta N, K\Lambda, K\Sigma\) stable channels) \(\rho N, \sigma N, \pi\Delta \leftrightarrow \pi\pi N\) channels

Features

2 \(\pi\) production mechanisms

Bare resonance states are dressed by meson clouds \(\rightarrow\) nucleon resonances

Developed through analyzing \(\gamma N, \pi N \rightarrow \pi N, \eta N, K\Lambda, K\Sigma\) data (\(\sim 27,000\) data pts.)
Partial wave amplitudes of $\pi N$ scattering

Kamano, Nakamura, Lee, Sato, PRC 88 (2013)

Previous model (fitted to $\pi N \rightarrow \pi N$ data only) [PRC76 065201 (2007)]

Data: SAID $\pi N$ amplitude

$|I| = \frac{3}{2}$
\[ \gamma p \rightarrow \pi^0 p \]

\[ \frac{d\sigma}{d\Omega} \text{ for } W < 2.1 \text{ GeV} \]

Comparison of DCC model with data

Reasonable fit to data in the whole resonance region
All ANL-Osaka DCC analysis results and partial wave amplitudes are collected in:

https://www.phy.anl.gov/theory/research/anl-osaka-pwa/

Contact Person: T.-S. H. Lee (tshlee@anl.gov)
Extension to finite $Q^2$ region and neutrino reactions

SXN, H. Kamano, T. Sato, PRD 92, 074024 (2015)
Motivation

High precision neutrino oscillation experiments (T2K, DUNE, ...)

Goal: discover CP violation in lepton sector, determine neutrino mass hierarchy

They need precise neutrino-nucleus pion production model! (~5% level)

Essential ingredient of neutrino-nucleus model: elementary neutrino-nucleon model

We develop this
Strategy to tackle neutrino-nucleon reactions in resonance region

Vector current

Good amount of data available to determine the vector $N \rightarrow N^*$ transition form factors

1. EM couplings and $Q^2$-dependence $\leftrightarrow$ Analysis of electron-induced reaction data (1$\pi$ production and inclusive)

2. Isospin separation: $V_{CC} = V_{isovector} = V_{EM}^{proton} - V_{EM}^{neutron}$

Both proton- and neutron-target electron data need to be analyzed
Strategy to tackle neutrino-nucleon reactions in resonance region

Axial current

(neutrino-nucleon) data are scarce (some useful data available for $\Delta(1232)$ only)

→ Guiding principle to derive the axial current: PCAC relation with $\pi N$ reaction amplitude

$$< X | q \cdot A(Q^2 \sim 0) | N > \sim i f_\pi < X | T | \pi N >$$

$$g_{AN \rightarrow N^*}(Q^2 \sim 0) \propto g_{\pi N \rightarrow N^*}$$  (phase is also fixed)

Both $\pi N \rightarrow \pi N$ and $AN \rightarrow \pi N$ reaction models need to be developed consistently with PCAC

done only in the DCC model

Common procedure: $\Gamma(N^* \rightarrow \pi N)$ from PDG

$$g_{AN \rightarrow N^*}(Q^2 \sim 0) \propto |g_{\pi N \rightarrow N^*}|$$

No control of phase

$Q^2$-dependence needs assumption for lack of data $\rightarrow$ dipole form factors with cutoff $\sim 1$ GeV
Single $\pi$ production in electron-proton scattering

$\sigma_T + \varepsilon \sigma_L$ for $Q^2=0.40 \text{ (GeV/c)}^2$ and $W=1.1 - 1.68 \text{ GeV}$

$p(e,e'\pi^0)p$ $p(e,e'\pi^+)n$
• Reasonable fit to data for application to neutrino interactions

• Important $2\pi$ contributions for high $W$ region
Comparison: Neutrino-nucleon pion production data

$\nu_\mu p \rightarrow \mu^- \pi^+ p$

$\nu_\mu n \rightarrow \mu^- \pi^0 p$

$\nu_\mu p \rightarrow \mu^- \pi^+ \pi^0 p$

DCC model prediction is consistent with data, and still has flexibility $(ANN^*(Q^2))$

First model for 2 $\pi$ production in resonance region

The DCC model is being implemented into neutrino-oscillation analysis!
DCC approach to electroweak meson productions on the deuteron
Model for \( \gamma d \rightarrow \pi NN \)

Multiple scattering theory truncated at the first-order rescattering

**Impulse**

\[
\gamma 
\rightarrow \pi N \rightarrow N
\]

**NN rescattering**

\[
\gamma N \rightarrow \pi N, \quad \pi N \rightarrow \pi N \\
T_{NN}, \text{ deuteron w.f.}
\]

\( T_{NN} \) amplitude \( \rightarrow \) DCC model (Kamano et al., PRC94 (2016))

\( \pi N \) rescattering

\( \gamma N \rightarrow \pi N, \quad \pi N \rightarrow \pi N \) amplitude \( \rightarrow \) CD-Bonn potential (Machleidt et al., PRC 63 (2001))
\( \gamma d \rightarrow \pi NN \): model predictions and data

- Large NN FSI effect for \( \pi^0 \) productions
  \( \leftrightarrow \) NN and deuteron wave fn. are orthogonal
- FSI effects are small for \( \pi^- \) productions
- Reasonable agreement with data

Data: EPJA 6, 309 (1999); 10, 365 (2001) for \( \gamma d \rightarrow \pi^0 pn \)

NPB 65, 158 (1973) for \( \gamma d \rightarrow \pi^- pp \)
Comparison with data for $\eta$ angular distribution in $\gamma d \rightarrow \eta X$

$E_\gamma = 775$ MeV

Impulse  
Full  

Data: Phys. Lett. B 358 40 (1995)

Prediction excellently agrees with data

$\eta N \rightarrow \eta N$ rescattering is important for describing backward $\eta$ production
Extraction of neutron-target observables from $\gamma d \rightarrow \pi N N$

SXN, PRC 98 042201(R) (2018)
Why neutron target data?

\( \gamma^{(*)} \; p \rightarrow N^* \; \text{AND} \; \gamma^{(*)} \; n \rightarrow N^* \) form factors \( \Rightarrow \) Isospin structure of \( \gamma^{(*)} \; N \rightarrow N^* \) form factors

- Interesting quantities for understanding hadron structures
- Necessary for application to neutrino-induced reactions

To obtain \( \gamma^{(*)} \; 'n' \rightarrow \pi N, \eta N, KY \) data, \textit{Deuteron is the primary target}; we need to understand:

\[ Q : \text{How to extract } \sigma(\gamma^{(*)} \; 'n' \rightarrow \pi N) \text{ from } \sigma(\gamma^{(*)} \; d \rightarrow \pi NN) ? \]

Common (and practical) procedure

- Kinematical cuts \( \Rightarrow \) select quasi-free \( \gamma^{(*)} \; 'n' \rightarrow \pi N \) events (supposedly FSI free)

Concerns: FSI and/or cuts could distort \( \sigma(\gamma^{(*)} \; 'n' \rightarrow \pi N) \) from free one? \textcolor{red}{We address this}
Q: How FSI and kinematical cuts could distort $\gamma 'n'$ observables ($d\sigma/d\Omega$, $\Sigma$, $E$, $G$)?

DCC elementary amplitudes for $\gamma n \rightarrow \pi N$ and $\gamma p \rightarrow \pi N$

Installed in $\gamma d \rightarrow \pi NN$ model

$\sigma (\gamma d \rightarrow \pi NN)$

kinematical cuts

Free $\sigma (\gamma n \rightarrow \pi N)$

Comparison

Quasi-free $\sigma (\gamma 'n' \rightarrow \pi N)$
Q: How FSI and kinematical cuts could distort $\gamma \ 'n' \ $observables ($d\sigma/d\Omega, \Sigma, E, G$)?

DCC elementary amplitudes for $\gamma n \to \pi N$ and $\gamma p \to \pi N$ installed in $\gamma d \to \pi NN$ model.

kinematical cuts

Quasi-free $\sigma (\gamma 'n' \to \pi N)$

comparison

Kinematical cuts of recent CLAS@JLab analyses [PRC 86 (2012), PRC 96 (2017), PRL 118 (2017)]

Additional cut by A2@MAMI: $W_{\text{min}} < W < W_{\text{max}}$

CLAS@JLab inferred $W$ assuming initial neutron at rest

$\iff$ The validity will be examined
\[ \frac{d\sigma}{d\Omega_\pi} \text{ for } \gamma ' n' \rightarrow \pi^0 n \]

W-cut included

- Significant FSI effects reduce \( \frac{d\sigma}{d\Omega_\pi} \); \( \pi N \) and NN FSI are comparably important
- Kinematical cuts cannot remove FSI effect \( \rightarrow \) FSI correction necessary
- Good agreement with FSI effects estimated by A2@MAMI (first theoretical explanation)

A2@MAMI Data
[PRL 112, 142001 (2014)]

\( \Delta \) : before FSI correction
\( \Delta \) : after FSI correction

\( W = 1210 \text{ MeV} \)
\( E_\gamma = 300 \text{ MeV} \)

\( W = 1350 \text{ MeV (x 5)} \)
\( E_\gamma = 500 \text{ MeV} \)

\( W = 1500 \text{ MeV (x 10)} \)
\( E_\gamma = 720 \text{ MeV} \)
Extraction without W-cut

Significant difference in Δ(1232)-region (W~1.2 GeV) caused by Fermi motion

Important to apply W-cut to suppress problematic Fermi motion effect

→ extracting observables accurately
Low-energy $\eta$-nucleon interaction studied with $\gamma d \rightarrow \eta pn$

SXN, H. Kamano, T. Ishikawa, PRC 96, 042201(R) (2017)
$\eta N$ scattering length ($a_{\eta N}$)

\[
S_{\eta N \rightarrow \eta N} = \eta e^{2i\delta} = 1 + 2iF_{\eta N} \quad F_{\eta N} \approx \frac{p}{1 + \frac{r}{a} p^2 - ip} \quad a : \text{scattering length}
\]

\[
r : \text{effective range}
\]

- Governs low-energy $\eta N$ scattering $\rightarrow$ basic feature of meson-baryon dynamics
- Important relevance to the existence of $\eta$-mesic nuclei

But not well-determined yet

**Status**

Several coupled-channels analyses of $\pi N \rightarrow \pi N$, $\pi N \rightarrow \eta N$ and $pn \rightarrow d\eta$ data

\[
\text{Re}[a_{\eta N}] = 0.2 \sim 1.1 \text{ fm} \quad \text{(rather scattered)}
\]

\[
\text{Im}[a_{\eta N}] = 0.2 \sim 0.3 \text{ fm} \quad \text{(better constrained due to optical theorem)}
\]

$a_{\eta N}$ has been extracted from indirect information $\rightarrow$ model dependence

Need process that sensitively probes $\eta N \rightarrow \eta N$, with other mechanisms suppressed
Ongoing experiment at ELPH@Tohoku Univ.

\[ \gamma d \rightarrow \eta n p \text{ at } E_\gamma \sim 0.95 \text{ GeV} \text{ and proton detected at } 0^\circ \text{ (ELPH kinematics)} \]

Ideal kinematical setting for extracting $\eta N$ scattering length

- $\eta$ is produced almost at rest $\Rightarrow$ strong $\eta n \rightarrow \eta n$ rescattering is expected
- $\pi n \rightarrow \eta n$ and $NN$ rescatterings (background) are expected to be suppressed

Model needed to extract $a_{\eta N}$ from data

$\Rightarrow$ DCC-based $\gamma d \rightarrow \eta pn$ model

This work addresses

- $\gamma d \rightarrow \eta n p$ at ELPH kinematics
- Sensitivity of ELPH exp. to $a_{\eta N}, r_{\eta N}$
\( \gamma d \rightarrow \eta n p \) at ELPH kinematics

- Impulse current dominates
- \( \eta n \) rescattering (~ s-wave) gives sizable (-40% ~ +20%) contribution
- Small \( \pi N \rightarrow \eta N \) rescattering \rightarrow controllable contribution (data exist)

- \( NN \) rescattering negligible \rightarrow more multiple rescattering negligible

\( E_\gamma = 950 \text{ MeV}, \ \theta_p \sim 0^\circ \)

\( \frac{d^3\sigma}{dM_{\eta n} d\Omega_p} (\mu b/\text{MeV sr}) \)

\( M_{\eta n} \) (GeV)
Study sensitivity of $\gamma d \rightarrow \eta n p$ to $a_{\eta N}$ and $r_{\eta N}$

$\eta N$ rescattering

DCC amplitude $\rightarrow$ $T_{\eta N \rightarrow \eta N} \propto \frac{1}{1 + \frac{r_{\eta N}}{2} p^2 - ip}$

Vary $a_{\eta N}$ and $r_{\eta N}$

$\rightarrow$ how sensitively $\gamma d \rightarrow \eta n p$ cross sections at ELPH kinematics change
Re$[a_{\eta N}]$-dependence of $\gamma d \rightarrow \eta n p$ at ELPH kinematics

$E_\gamma = 950$ MeV, $\theta_p \sim 0^\circ$

ELPH measures both

$\sigma(\gamma d \rightarrow \eta n p)$ & $\sigma(\gamma p \rightarrow \eta p)$

$\rightarrow R_{\text{exp}}$ will be measured

5% precision measurement of $R_{\text{exp}} \rightarrow \Delta(\text{Re}[a_{\eta N}]) \sim 0.2$ fm (current: $\Delta(\text{Re}[a_{\eta N}]) \approx 1.3$ fm)

$\gamma d \rightarrow \eta n p$ data at ELPH kinematics can significantly better determine Re$[a_{\eta N}]$
Extraction of neutron-neutron scattering length from

$$\gamma d \rightarrow \pi^+ nn$$

SXN, H. Kamano, T. Ishikawa, in progress
NN scattering lengths

**Status**

\[
p \cot \delta \approx -\frac{1}{a} + \frac{1}{2} rp^2
\]

\[
\begin{align*}
nn: & \quad a = -18.9 \pm 0.4 \text{ fm} \\
np: & \quad a = -23.740 \pm 0.020 \text{ fm} \\
pp: & \quad a = -17.3 \pm 0.4 \text{ fm} \quad \leftarrow \text{error of EM correction}
\end{align*}
\]

**Importance of precise determination of** \(a_{nn}\)

- Charge symmetry breaking (CSB): \(a_{nn} \neq a_{pp}\) \leftarrow \text{quark mass, EM effects (Standard Model)}
  \[\rho^0-\omega\text{ mixing (hadronic model)}\]

- Input for nuclear structure calculations
  - High-precision ab-initio few-body calculations \((A \lesssim 12)\)
  - Existence of tetraneutron ?
Previous determinations of $a_{nn}$

\[
\begin{align*}
da \rightarrow nnp & \\
a_{nn} &= -16.1 \pm 0.4 \text{ fm} \quad \text{from } d (n, np) n \quad \text{PRL 85, 1190 (2000)} \\
a_{nn} &= -16.5 \pm 0.9 \text{ fm} \quad \text{from } d (n, p) nn \quad \text{PRC 74, 014001 (2006)} \\
a_{nn} &= -18.7 \pm 0.7 \text{ fm} \quad \text{from } d (n, nnp) \quad \text{PRC 73, 034001 (2006)}
\end{align*}
\]

\[
\begin{align*}
\pi^- d \rightarrow nn\gamma & \\
a_{nn} &= -18.59 \pm 0.40 \text{ fm} \quad \text{from } d (\pi^-, n\gamma) n \quad \text{PLB 444, 252 (1998)}
\end{align*}
\]

Current standard

**Difficulties:** complex three-body problem has to be under control

-- three-body force, Faddeev calculation for $nd \rightarrow nnp$

-- possible three-body $\pi NN$ dynamical effect in $\pi^- d \rightarrow nn\gamma$

Similar $pp \rightarrow \gamma pp$ data not explained yet

E.S. Konobeevski, et al., arXiv: 1703.00519
Experimental proposal by T. Ishikawa

\[ \gamma d \rightarrow \pi^+ nn \] measurement at Mainz electron facility

Theoretical task is to identify optimal kinematics (\( E_\gamma \) and \( \theta_\pi \)) for extracting \( a_{nn} \):

- neutron-neutron relative kinetic energy is small
- pion-neutron FSI is suppressed
Work in progress!

\( E_\gamma = 200 \text{ MeV} \)
\( \theta_\pi = 0^\circ \)

\[ p_\pi (\text{MeV}) \]

\begin{align*}
\text{CD-Bonn} & (a = -18.9 \text{fm}) \\
\text{Nijmegen I} & (a = -17.3 \text{fm}) \\
\text{Reid93} & (a = -17.3 \text{fm})
\end{align*}

High-precision measurement is required: 0.1 MeV resolution of pion momentum

\( \rightarrow \) Mainz electron facility can do
Summary
Summary

Overview of our recent activity with dynamical coupled-channels (DCC) approach

• DCC model for electroweak meson productions on single nucleon
  -- DCC analysis of $\gamma N, \pi N \rightarrow \pi N, \pi\pi N, \eta N, K\Lambda, K\Sigma$ data ($W < 2.1$ GeV)
  -- Extension to finite $Q^2$ and neutrino reactions

• Application of DCC model to deuteron-target processes
  -- Extraction of neutron-target observables from $\gamma d \rightarrow \pi NN$
  -- Novel method to extract $\eta$-nucleon scattering length from $\gamma d \rightarrow \eta pn$
  -- Extraction of neutron-neutron scattering length from $\gamma d \rightarrow \pi^+ nn$
**Future perspective**

Hyperon-nucleon interaction from studying $\gamma d \Rightarrow K^+ YN$ data (ELPH experiment)

- **Impulse**
  - $\gamma$ to $K^+ YN$

- **YN rescattering**
  - $\gamma N \Rightarrow \pi N, KY$, $\pi N \Rightarrow KY$, $KN \Rightarrow KN$
  - $T_{YN}$

- **Meson-baryon rescattering**
  - $\pi, K^+ \Rightarrow K^+$

DCC-based model would be useful for this purpose

- $\gamma N \Rightarrow \pi N, KY$, $\pi N \Rightarrow KY$, $KN \Rightarrow KN$ amplitudes
- $T_{YN}$

- DCC model (already available)
- Nijmegen, Juelich, chiral, etc.
BACKUP
$\gamma n \rightarrow \pi^0 n$

$\frac{d\sigma}{d\Omega}$ for $W < 2$ GeV

- **New Fit**: Recent MAMI data included
- **Previous Fit**: PRL 112, 142001 (2014)

$\gamma n \rightarrow \pi^- p$

- **New Fit**: Recent JLab data included
- **Previous Fit**: PRC 96, 035204 (2017)
\( \pi N \) FSI effect on spectator momentum distribution

\[ \gamma d \rightarrow \pi^0 pn \]

- Before cut, net \( \pi N \) FSI effect is small; enhancement and reduction cancel
- After cut, net \( \pi N \) FSI effect reduces the spectrum
$d\sigma/d\Omega_\pi$ for $\gamma' n' \rightarrow \pi^0 n$

W-cut included

- Significant FSI effects reduce $d\sigma/d\Omega_\pi$; $\pi N$ and $NN$ FSI are comparably important
- Kinematical cuts cannot remove FSI effect → FSI correction necessary
- Larger FSI effects for smaller $E_\gamma$
\( \frac{d\sigma}{d\Omega_\pi} \) for \( \gamma' n' \rightarrow \pi^- p \)

**W-cut included**

- **Triangle:** impulse + NN FSI
- **Filled Circle:** impulse + NN FSI + \( \pi N \) FSI
- **Dashed Line:** free \( \sigma(\gamma n \rightarrow \pi N) \)

**Graphs:**
- **Left:** \( W = 1210 \text{ MeV} \), \( E_\gamma = 300 \text{ MeV} \)
- **Middle Left:** \( W = 1350 \text{ MeV} \) (x 3), \( E_\gamma = 500 \text{ MeV} \)
- **Middle Right:** \( W = 1500 \text{ MeV} \) (x 3), \( E_\gamma = 720 \text{ MeV} \)
- **Right:** \( W = 1660 \text{ MeV} \), \( E_\gamma = 1000 \text{ MeV} \)
- **Middle:** \( W = 1780 \text{ MeV} \) (x 2), \( E_\gamma = 1220 \text{ MeV} \)
- **Right:** \( W = 1900 \text{ MeV} \) (x 2), \( E_\gamma = 1460 \text{ MeV} \)

- **Notes:**
  - \( NN (\pi N) \) FSI reduce \( \frac{d\sigma}{d\Omega_\pi} \) of forward (backward) pion
  - Kinematical cuts cannot remove the FSI effect \( \rightarrow \) FSI correction necessary
  - Larger FSI effects for smaller \( E_\gamma \)
Comparison with Data : FSI effect

\[ \gamma 'n' \rightarrow \pi^0 n \]

\[ \begin{align*}
W &= 1210 \text{ MeV} \\
E_\gamma &= 300 \text{ MeV}
\end{align*} \]

\[ \begin{align*}
W &= 1350 \text{ MeV (x 5)} \\
E_\gamma &= 500 \text{ MeV}
\end{align*} \]

\[ \begin{align*}
W &= 1500 \text{ MeV (x 10)} \\
E_\gamma &= 720 \text{ MeV}
\end{align*} \]

\[ \begin{align*}
W &= 1660 \text{ MeV} \\
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\end{align*} \]

\[ \begin{align*}
W &= 1780 \text{ MeV} \\
E_\gamma &= 1220 \text{ MeV}
\end{align*} \]

\[ \begin{align*}
W &= 1900 \text{ MeV} \\
E_\gamma &= 1460 \text{ MeV}
\end{align*} \]

\[ \begin{align*}
\sigma(\gamma n \rightarrow \pi^0 n) &= \sigma(\gamma p \rightarrow \pi^0 p) \\
\text{free } \sigma(\gamma n \rightarrow \pi^0 n) &= \text{free } \sigma(\gamma p \rightarrow \pi^0 p)
\end{align*} \]

\[ \begin{align*}
\text{A2@MAMI Data} \rightarrow \\
\text{[PRL 112, 142001 (2014)]} \rightarrow
\end{align*} \]

\[ \begin{align*}
1360 \text{ MeV} \rightarrow \\
1480 \text{ MeV}
\end{align*} \]

- Good agreement with FSI effects estimated by A2@MAMI (first theoretical explanation)

- A2@MAMI analysis assumed \( \frac{\sigma(\gamma'n' \rightarrow \pi^0 n)}{\text{free } \sigma(\gamma n \rightarrow \pi^0 n)} = \frac{\sigma(\gamma'p' \rightarrow \pi^0 p)}{\text{free } \sigma(\gamma p \rightarrow \pi^0 p)} \) not theoretical estimate
DCC (Dynamical Coupled-Channel) model

Coupled-channel Lippmann-Schwinger equation for meson-baryon scattering

\[ T_{ab} = V_{ab} + \sum_c V_{ac} G_c T_{cb} \]

\[ G_c = \text{for stable channels} \]

\[ \pi \quad \text{for unstable channels} \]

\[ \Delta \]

Matsuyama et al., Phys. Rep. 439, 193 (2007)
Kamano et al., PRC 88, 035209 (2013)
DCC (Dynamical Coupled-Channel) model

By solving the LS equation, coupled-channel unitarity is fully taken into account.

Kamano et al., PRC 88, 035209 (2013)

Coupled-channel Lippmann-Schwinger equation for meson-baryon scattering

\[ T_{ab} = V_{ab} + \sum_c V_{ac} G_c T_{cb} \]

In addition, \( \gamma N \) channel is included perturbatively.
Model predictions for $E$ of $\gamma d \rightarrow \pi N N$

Polarization observable $E$:

$$E \equiv \frac{\sigma_{++} - \sigma_{--}}{\sigma_{++} + \sigma_{--}}$$

$$\sigma_{+\pm} \equiv \sigma(s_{\gamma}^z = +1, s_d^z = \pm 1)$$

$\gamma d \rightarrow \pi^0 pn$

- FSI effects are smaller than $d\sigma/d\Omega_{\pi}$, but still visible
  - not completely cancelled out in the ratio

\[ E_{\gamma} = 500 \text{ MeV} \]

\[ E_{\gamma} = 720 \text{ MeV} \]
Model predictions for \( E \) of \( \gamma d \rightarrow \pi N N \)

Polarization observable \( E \):
\[
E \equiv \frac{\sigma_+ - \sigma_-}{\sigma_+ + \sigma_-}
\]
\( \sigma_{\pm} \equiv \sigma(s_{\gamma} = +1, s_{d} = \pm 1) \)

\( \gamma d \rightarrow \pi^- pp \)

\( E_{\gamma} = 500 \text{ MeV} \)

\( E_{\gamma} = 720 \text{ MeV} \)

- FSI effects are small as the same for \( d\sigma/d\Omega_{\pi} \)
How to extract \( \sigma(\gamma 'n' \rightarrow \pi^0 n) \) from \( \sigma(\gamma d \rightarrow \pi^0 np) \)

First we need to establish a formula to relate \( \sigma(\gamma 'n' \rightarrow \pi^0 n) \) and \( \sigma(\gamma d \rightarrow \pi^0 np) \)

**Ideal situation**: only this mechanism contributes →

No FSI, No exchange terms, No deuteron D-wave

“quasi-free mechanism”

\[
\frac{d^2 \sigma(\gamma d \rightarrow \pi^0 np; E_\gamma)}{dW \, d \cos \theta^*_\pi} = \phi(W) \frac{\tilde{E}_\gamma}{E_\gamma} \frac{d \sigma(\gamma n \rightarrow \pi^0 n; W)}{d \cos \theta^*_\pi}
\]

\( W : \pi^0 n \) invariant mass \quad \cos \theta^*_\pi \equiv \hat{q}_\gamma \cdot \hat{k}_\pi \) in \( \pi^0-n \) CM frame

\( \phi(W) \): Probability of a photon of \( E_\gamma \) interacting with the nucleon with \( W \)

\[
\phi(W) \equiv \int dp_p^3 \, \delta(W - W(\vec{p}_p, E_\gamma)) \left| \psi_d(\vec{p}_p) \right|^2, \quad W(\vec{p}_p, E_\gamma) = \sqrt{(E_\gamma + m_d - E_p(\vec{p}_p))^2 - (\vec{q}_\gamma - \vec{p}_p)^2}
\]
How to extract \( \sigma(\gamma 'n' \rightarrow \pi^0 n) \) from \( \sigma(\gamma d \rightarrow \pi^0 np) \)

In reality, photon hits the other nucleon and FSI contribute
→ kinematical cuts are applied to remove them

Counterpart from our \( \gamma d \) model:

\[
\frac{d^2 \sigma(\gamma d \rightarrow \pi^0 np)}{dW \, d\cos \theta^*_\pi} \bigg|^{\text{cuts}}
\]

Assuming that this is solely from quasi-free contribution integrated over the same phase-space

\[
\frac{d^2 \sigma(\gamma d \rightarrow \pi^0 np)}{dW \, d\cos \theta^*_\pi} \bigg|^{\text{cuts}} = \phi_{\text{mod}}(W) \frac{\tilde{E}_\gamma}{E_\gamma} \frac{d\sigma(\gamma n \rightarrow \pi^0 n)}{d\cos \theta^*_\pi}, \quad \phi_{\text{mod}}(W) \equiv \int^{\text{cuts}} dp_p^3 \, \delta(W - W(\vec{p}_p, E_\gamma)) \left| \psi_d(\vec{p}_p) \right|^2
\]

Formula to extract \( \sigma(\gamma 'n' \rightarrow \pi^0 n) \) from \( \sigma(\gamma d \rightarrow \pi^0 np) \) from either experiment or model

Examine within our model how much \( \sigma(\gamma 'n' \rightarrow \pi^0 n) \) deviates from ‘free’ \( \sigma(\gamma n \rightarrow \pi^0 n) \) due to FSI
Kinematical cuts

For extracting $\gamma 'n' \rightarrow \pi^- p$ from $\gamma d \rightarrow \pi^- pp$, we use:

| Ref. | $[1] \ (d\sigma/d\Omega)$ | $[2] \ (d\sigma/d\Omega)$ | $[3] \ (E)$ |
|------|--------------------------|--------------------------|---------------------|
| $E_\gamma$ (MeV) | 301 – 455 | 445 – 2510 | 700 - 2400 |
| $\pi^-$ momentum (MeV) | $> 80$ | $> 100$ | $> 400$ |
| Faster proton momentum (MeV) | $> 270$ | $> 360$ | $> 400$ |
| Slower proton momentum (MeV) | $< 270$ | $< 200$ | $< 100$ |

$$\Delta \phi_\pi = |\phi_\pi - \phi_{\text{faster proton}}|$$

- $- 160^\circ < \Delta \phi < 200^\circ$

For extracting $\gamma 'n' \rightarrow \pi^0 n$ from $\gamma d \rightarrow \pi^0 pn$, the same cuts are used after modification:

“$\pi^-$” $\rightarrow$ “$\pi^0$”, “Faster proton” $\rightarrow$ “neutron”, “Slower proton” $\rightarrow$ “proton”

NOTE: Different kinematical cuts were used in A2@MAMI analysis on $\gamma n \rightarrow \pi^0 n$
Check the extraction method in ideal case

Q: With quasi-free mechanism only, examine if the formula returns $\sigma(\gamma ' n' \rightarrow \pi N)$ that agree with ‘free’ $\sigma(\gamma n \rightarrow \pi N)$

\[
\frac{d^2\sigma(\gamma d \rightarrow \pi^0 np)}{dW \, d\cos\theta^*_\pi} \bigg|_{\text{cuts}} = \phi_{\text{mod}}(W) \frac{\bar{E}_\gamma}{E_\gamma} \frac{d\sigma(\gamma n \rightarrow \pi^0 n)}{d\cos\theta^*_\pi}
\]

- : extracted $\sigma(\gamma ' n' \rightarrow \pi N)$
- : free $\sigma(\gamma n \rightarrow \pi N)$

Note:
* Phase-space integrals for $\sigma(\gamma d)$ are done with Monte-Carlo method to easily implement any cuts; statistical error bars are very small
* Forward pion productions are invisible (left figures) due to the kinematical cuts

Free $\sigma(\gamma n \rightarrow \pi N)$ are well reproduced as expected for the ideal case

Now go on to realistic case!
Comparison of FSI effects on: 
\[ \gamma 'n' \rightarrow \pi^0 n \quad \text{and} \quad \gamma 'p' \rightarrow \pi^0 p \]

- FSI effects on \( \gamma 'n' \rightarrow \pi^0 n \) and \( \gamma 'p' \rightarrow \pi^0 p \) are generally similar (a few % difference)
  \[ \Rightarrow \text{Same FSI effects are assumed in A2@MAMI analysis [PRL 112 (2014)]} \]
- But sometimes more different
(More) common extraction method

Quasi-free mechanism only; no FSI, no exchange terms

\[ \Sigma \]

\[ E \]

\[ G \]

Reasons for deviations from free ones

- **E** at \( E_\gamma = 300 \text{ MeV}, \cos \theta > 0.5 \): nucleon-at-rest kinematics is largely restricted by cuts; contributions are from different \( W \)

- **G** at \( E_\gamma = 1 \text{ GeV} \): Average non-monotonic \( W \)-dependence \( \Rightarrow \) \( W \)-dependence not cancelled
(More) common extraction method

Extraction formula used so far in this presentation

\[
\frac{d^2\sigma(\gamma d \to \pi^0 np; E'_\gamma)}{dW\, d\cos\theta^*_\pi} = \phi_{\text{mod}}(W) \frac{\tilde{E}'_\gamma}{E'_\gamma} \frac{d\sigma(\gamma n \to \pi^0 n; W)}{d\cos\theta^*_\pi}, \quad \phi_{\text{mod}}(W) \equiv \int Cuts dp^3_p \, \delta(W - W(\vec{p}_p, E'_\gamma)) \left| \psi_d(\vec{p}_p) \right|^2
\]

Integrate both sides with respect to \( W \), assuming dominant contribution from nucleon-at-rest in deuteron and/or fairly weak \( W \)-dependence

(More) commonly used extraction formula

\[
\frac{d^2\sigma(\gamma d \to \pi^0 np; E'_\gamma)}{d\cos\theta^*_\pi} \bigg|_{\text{cuts}} = \frac{d\sigma(\gamma n \to \pi^0 n; W)}{d\cos\theta^*_\pi} \int Cuts dp^3_p \, \frac{\tilde{E}'_\gamma}{E'_\gamma} \left| \psi_d(\vec{p}_p) \right|^2, \quad \tilde{W}^2 = (E'_\gamma + m_N)^2 - E'_\gamma^2
\]

Used in recent CLAS@JLab analyses:  PRC 96 (2017) for \( d\sigma/d\Omega_\pi \)  
PRL 118 (2017) for \( E \)

also in theoretical analysis  Tarasov et al., PRC 84, 035203 (2011)

Examine the validity of this formula without \( W \)-cut
Predicted $\pi N \rightarrow \pi\pi N$ total cross sections with our DCC model

\[ \pi^+ p \rightarrow \pi^+ \pi^+ n \]
\[ \pi p \rightarrow \pi^+ \pi^- n \]
\[ \pi p \rightarrow \pi^0 \pi^0 p \]
\[ \pi^- p \rightarrow \pi^- \pi^- n \]
\[ \pi^- p \rightarrow \pi^- \pi^0 n \]

Kamano, PRC88(2013)045208
Kamano, Julia-Diaz, Lee, Matsuyama, Sato PR79(2008)025206
**Re[\(r_{\eta N}\)]-dependence of \(\gamma d \rightarrow \eta n p\) at ELPH kinematics**

\[
\text{ELPH measures both } \sigma(\gamma d \rightarrow \eta n p) \text{ and } \sigma(\gamma p \rightarrow \eta p) \rightarrow R_{\text{exp}} \text{ will be measured}
\]

\[
E_{\gamma} = 950 \text{ MeV, } \theta_p \sim 0^\circ
\]

\[
R_{\text{th}} \equiv \frac{d^3\sigma_{\text{full}}}{dM_{\eta n}d\Omega_p} / \frac{d^3\sigma_{\text{IA}}}{dM_{\eta n}d\Omega_p}
\]

\(\gamma d \rightarrow \eta n p\) at ELPH exp. kinematics has sensitivity to \(\text{Re}[r_{\eta N}]\)

5% precision measurement of \(R_{\text{exp}} \rightarrow \Delta(\text{Re}[r_{\eta N}]) \sim 1 \text{ fm} \) (current: \(\Delta(\text{Re}[r_{\eta N}]) \approx 7 \text{ fm}\)
FSI corrections to neutrino-nucleon pion production cross sections from deuteron data

SXN, H. Kamano, T. Sato, PRD 99, 031301(R) (2019)
ALL neutrino-nucleon models fit this data by adjusting axial $N \rightarrow \Delta(1232)$

But, $\nu_\mu p \rightarrow \mu^- \pi^+ p$ data were extracted from $\nu_\mu d \rightarrow \mu^- \pi^+ p n$ data

Need care about corrections from nuclear effects (FSI) for the precision in question!

$\rightarrow$ We address this problem
Spectator momentum distribution for $\nu_\mu d \rightarrow \mu^- \pi^+ p n$

Minimal information to extract $\nu_\mu N \rightarrow \mu \pi N$ cross sections

Contribution from other nucleon (spectator) is expected to be small in small $p_s$ region

Convoluted cross section ($\tilde{\sigma}$):

$$\frac{d\tilde{\sigma}_\alpha(E_\nu)}{dp_s} = p_s^2 \int d\Omega_{p_s} \sigma_{\alpha}(E_\nu) |\Psi_d(\vec{p}_s)|^2$$

$\alpha = \nu_\mu N \rightarrow \mu^- \pi^+ N$

$\Psi_d$ : deuteron w.f.

= Quasi-free contribution
Spectator momentum distribution for $\nu_\mu d \rightarrow \mu^- \pi^+ p n$

**FSI effect**

![Graphs showing spectator momentum distribution for different FSI scenarios and comparison with naive expectation.]

**Naïve expectation**: FSI affects high $p_s$ region, leaving small $p_s$ region unchanged

**Reality**: FSI significantly reduces spectrum in small $p_s$ (quasi-free peak) region

large NN FSI effect $\leftrightarrow$ orthogonality between NN scattering state and deuteron

FSI effect is small for $\nu_\mu d \rightarrow \mu^- \pi^0 p p$ spectator momentum distribution
FSI and Fermi-motion corrections on $\nu_\mu N \rightarrow \mu^- \pi^+ N$ cross section data

Future impact → Significantly improved $\nu_\mu N \rightarrow \mu^- \pi^+ N$ model for oscillation experiments