Vanishing of the Dissipationless Spin Hall Effect in a Diffusive Two-Dimensional Electron Gas with Spin-Orbit Coupling

S. Y. Liu and X. L. Lei
Department of Physics, Shanghai Jilting University, 1954 Huashan Road, Shanghai 200030, China
(Dated: November 21, 2018)

We propose a nonequilibrium Green’s function approach to study the spin-Hall effect in a two-dimensional electron system with both the Rashba and Dresselhaus spin-orbit couplings. By taking into account the long-range electron-impurity scattering, the derived kinetic equations are solved numerically. It is found the vanishing of the total zero-temperature dissipationless spin-Hall effect, contributing from the intrinsic and disorder-mediated processes. This result has been examined in the wide ranges of spin-orbit coupling constants and electron density.

PACS numbers: 72.10.-d, 72.25.Dc, 73.50.Bk

In the growing field of spintronics, which aims to manipulation of spin degrees of freedom in semiconductors[1], the issue of creating spin polarization of carriers in nonmagnetic semiconductors with spin-orbit coupling by employing only the electric field has attracted a great deal of theoretical and experimental attention. More recently, the non-dissipative spin currents with the spin-polarization perpendicular to the flow direction and to the applied electric field have been predicted in p-doped bulk semiconductor[2] and two-dimensional (2D) systems with the Rashba[3] and Dresselhaus spin-orbit couplings[4, 5]. This dissipationless spin-Hall effect essentially arises from the dc-field-induced transition between the helicity bands: the dc field causes the variation of the electron momentum, but leaves its spin unchanged[6]. In experiment, the spin-Hall effects have been observed in n-doped bulk semiconductors[7] and two-dimensional hole gas[8].

The disorder effects on the spin-Hall current have been extensively investigated for the two-dimensional electron systems with Rashba spin-orbit coupling. In Refs. [2, 10, 11], it has been argued that the disorder can reduce the spin-Hall effect and the ballistic value of spin-Hall conductivity reaches for the weak scattering limit. However, based on the Kubo formalism, Inoue et al.[12], Dimitrova[13] and Chalaev and Loss[14] have shown that the spin-Hall effect disappears even when the disorder is weak. This conclusion has been confirmed in Refs. [15] and 16 by employing the spin-density matrix method and the Keldysh approach, respectively. Further, using the nonequilibrium Green’s function approach, Liu and Lei have demonstrated that, even for the case of long-range disorders, the dissipationless spin-Hall effect also vanishes[6]. Obviously, the nonexistence of dissipationless spin-Hall effect still holds for the Dresselhaus spin-orbit coupling, since it relates to the Rashba coupling through a global unitary transformation[4]. It is commonly believed that this complete cancellation of spin-Hall effect is not a consequence of any symmetries[17] and relates to the isotropy of the dispersion in the helicity basis.

It is very interesting to examine the spin-Hall effect for 2D electron systems with both the Rashba and Dresselhaus spin-orbit interactions. In such systems, the spin-orbit-coupled bands become anisotropic, leading to many interesting phenomena, such as the spin splitting[18], spin precession and relaxation[19, 20, 21, 22], and anisotropic transport[23], etc. In particular, when the Rashba coefficient is turned to be equal to the Dresselhaus one, such system can be used as the new type of spin field-effect transistor (SFET) operated in the nonballistic regime[21]. However, the spin-Hall effect in the 2D system with the Rashba and Dresselhaus spin-orbit couplings has been studied only in the ballistic regime[4, 5]. It has been demonstrated that the spin-Hall conductivity has a universal value, independent on the strength of the coupling, but its sign is determined by the relative ratio of the two couplings.

In this letter, we develop a nonequilibrium Green’s function approach to the spin-Hall effect in 2D electron systems with both the Rashba and Dresselhaus spin-orbit couplings. The electron-impurity scattering is considered in the self-consistent Born approximation and the obtained equations are solved numerically. We find that the dissipationless spin-Hall conductivity vanishes at zero temperature. This result contrasts with the general belief that the anisotropy can lead to the nonzero dissipationless spin-Hall conductivity in 2D electron system with spin-orbit coupling[12].

We consider an effective Hamiltonian for a two-dimensional electron of momentum $p = (p_x, p_y)$ and effective mass $m$

$$\tilde{H}_0 = \frac{p^2}{2m} + \tilde{H}_R + \tilde{H}_D,$$  \hspace{1cm} (1)

with the terms of Rashba spin-orbit coupling $\tilde{H}_R$[24],

$$\tilde{H}_R = \alpha(p_y \sigma_x - p_x \sigma_y),$$  \hspace{1cm} (2)

and Dresselhaus spin-orbit interaction[25],

$$\tilde{H}_D = \beta(p_y \sigma_y - p_x \sigma_x).$$ \hspace{1cm} (3)
Here, $\sigma \equiv (\sigma_x, \sigma_y, \sigma_z)$ are the Pauli matrices, and, the $\alpha$ and $\beta$ are the coupling constants. After taking the local unitary transformation

$$U(p) = \frac{1}{\sqrt{2}} \left( \begin{array}{cc} 1 & 1 \\ -e^{i\chi_p} & e^{i\chi_p} \end{array} \right)$$  \hspace{1cm} (4)$$

with the $\chi_p$ being

$$\chi_p = \arg(\alpha p_y - \beta p_x - i(\alpha p_x - \beta p_y)),$$  \hspace{1cm} (5)$$

we can diagonalize this Hamiltonian in the helicity basis $H = \text{diag}(\varepsilon_1(p), \varepsilon_2(p))$. The $\varepsilon(\mu)(\mu = 1, 2)$ are of the forms

$$\varepsilon(\mu)(p) = \frac{p^2}{2m} + (-1)^{\mu}\varepsilon_{RD},$$  \hspace{1cm} (6)$$

where, $\varepsilon_{RD} \equiv \sqrt{(\alpha p_y - \beta p_x)^2 + (\alpha p_x - \beta p_y)^2}.$

When the system is driven by a dc field $E$ applied along the $x$-direction, the $i$-direction-polarized spin current for electron with momentum $p$ can be defined as

$$J_y^i = \sum_p \frac{p_n}{2m} [\rho_{12}(p) + \rho_{21}(p)] = \sum_p \frac{p_n}{2m} \text{Re} \rho_{12}(p),$$  \hspace{1cm} (7)$$

where $\rho_{\mu\nu}(p)$ is the distribution function with the symmetry relation $\rho_{12}(p) = \rho_{21}(p)^\dagger$. Further, the spin-Hall conductivity is given by $\sigma_{SH} = J_y^i/E$.

To derive the kinetic equations for the 2D systems with the spin-orbit coupling, we follow the procedure described in Ref. [6]. We first carry out the Dyson equations for the real-space less Green’s functions in the spin basis. While the Fourier and the unitary transformation are performed, these equations reduce to the kinetic equations in the helicity basis.

\[
\left\{ \begin{array}{l}
p \frac{\partial}{\partial T} + iE \cdot \nabla_p \end{array} \right\} \gamma \left[ G^< + \frac{eE}{2} \nabla_p \chi_p [G^<, \sigma_x] - \varepsilon_{RD} [G^<, \sigma_z] = \Sigma^r G^< + \Sigma^< G^r - G^r \Sigma^< - G^< \Sigma^r, \right. \hspace{1cm} (8)$$

where, the self-energy for the electron-impurity interaction with scattering matrix $V(p - k)$ reads

$$\Sigma^{r,<}(p, \sigma, \mu, \nu) = \frac{1}{2} \lim_{\varepsilon \to 0^+} \sum_k |V(p - k)|^2 \left\{ \begin{array}{l}
a_1 G^{r,<} + a_2 \sigma_x G^{r,=} - i\alpha [\sigma_x, G^{r,=}] \end{array} \right\},$$  \hspace{1cm} (9)$$

with the center-of-mass and difference times $T$ and $t$ relating to the two times of the Green’s functions $t_1$ and $t_2$: $T = (t_1 + t_2)/2$, $t = t_1 - t_2$. In these equations, for shortness, the arguments $(k, T, t)$ of Green’s functions $G^{r,<}$ are dropped. $a_i (i = 1, 2, 3)$ are the factors associated with the directions of the momenta, $a_1 = 1 + \cos(\chi_p - \chi_k)$, $a_2 = 1 - \cos(\chi_p - \chi_k)$, $a_3 = \sin(\chi_p - \chi_k)$. We can see that in comparison with the case of only the Rashba term [6], here, the factor $a_i$ is dependent on the momentum angle complicatedly through $\chi_p$. This fact leads to the kinetic equations analytically unsolvable.

Further, we assume that the dc field is sufficiently weak and only the linear response in the steady state needs to
be considered. Otherwise, since we are just interested in the dissipationless spin-Hall effect, it is sufficient to treat the nondiagonal distribution functions in the lowest order of electron-impurity scattering, i.e. in the order of \(n_i^0\). Under these considerations, the solutions of the kinetic equations \(\rho_{i2}^{(1)}(p) = \rho_{21}^{(1)}(p) = \frac{e E \cdot \nabla p \chi(p)}{4 \epsilon_{RD}} \{ n_F[\varepsilon_1(p)] - n_F[\varepsilon_2(p)] \}, \) can be separated into two parts. The first part of solution associates with the driving term \(\frac{e E \cdot \nabla p \chi(p)}{4 \epsilon_{RD}} \) with \(\rho_0\) being the distribution function in equilibrium \(\rho_0(p) = \text{diag}[n_F(\varepsilon_1(p)), n_F(\varepsilon_2(p))]\), and is a nondiagonal matrix in the spin space. It can be written as

\[
\rho_{12}^{(1)}(p) = \rho_{21}^{(1)}(p) = \frac{e E \cdot \nabla p \chi(p)}{4 \epsilon_{RD}} \{ n_F[\varepsilon_1(p)] - n_F[\varepsilon_2(p)] \},
\]

and results in the nonzero spin-Hall conductivity at zero temperature \([1, 2]\)

\[
\sigma_{sH}^{(1)}(p) = \frac{e^2}{h} \frac{J^z_y}{E} = \begin{cases} -e^2/8\pi & \alpha > \beta \\ 0 & \alpha = \beta \\ e^2/8\pi & \alpha < \beta \end{cases}.
\]

The second part of solution relates to the transport process and the corresponding diagonal distribution functions should be determined from the coupled equations

\[
\frac{-\partial n_F(E)}{\partial E}{\bigg|}_{E=\varepsilon_\mu} \frac{\partial \varepsilon_\mu}{\partial p_\mu} = \pi \sum_k |V(p-k)|^2 \{ [\rho_{\mu\mu}^{(2)}(p) - \rho_{\mu\mu}^{(2)}(k)]a_1\Delta_{\mu\mu} + [\rho_{\mu\mu}^{(2)}(p) - \rho_{\mu\mu}^{(2)}(k)]a_2\Delta_{\mu\mu} \},
\]

with \(\mu = 1, 2\), \(\bar{\mu} = 3 - \mu\) and \(\Delta_{\mu\mu} \equiv \delta(\varepsilon_\mu(p) - \varepsilon_\nu(k))\). And then, the real part of nondiagonal distribution can be given by

\[
\text{Re} \rho_{12}^{(2)} = \text{Re} \rho_{21}^{(2)} = \frac{\pi}{2} \sum_{k=1,2} |V(p-k)|^2 a_3(-1)^\mu \{ \Delta_{\mu\mu}[\rho_{\mu\mu}^{(2)}(p) - \rho_{\mu\mu}^{(2)}(k)] - \Delta_{\bar{\mu}\bar{\mu}}[\rho_{\bar{\mu}\bar{\mu}}^{(2)}(p) - \rho_{\bar{\mu}\bar{\mu}}^{(2)}(k)] \}.
\]

This part of spin-Hall effect depends on the electron-impurity scattering and entirely vanishes in clean samples. However, it is of order of \(n_i^0\) and the disorder only plays an intermediate role. This disorder-mediated spin-Hall effect contributes from the electrons near the Fermi surface and, physically, can be understood as the following way. The electrons, participating in the longitudinal transport, also can be scattered by impurities. This process, which changes the electron momentum and conserves its spin, yields the transition between two spin-orbit-coupled bands in the helicity basis.

Unfortunately, the above coupled equations for diagonal distribution functions only can be resolved numerically. We perform a numerical evaluation for a GaAs-based heterojunction, where the electron-impurity scattering is assumed to be long-range \([30]\). The momentum integration is calculated by the Gauss-Legendre scheme. The linear system equations, arising from the discretization of the integral equations for diagonal distribution functions, are solved by using the singular value decomposition. Here, we concern on the zero-temperature spin-Hall effect. Therefore, only the distribution functions at the Fermi surface need to be carried out.

This part of spin-Hall conductivity corresponds to the intrinsic spin-Hall effect and agrees with the previous studies in the ballistic regime \([1, 2]\). Essentially, this dissipationless spin-Hall effect stems from the dc-field-induced transitions between two spin-orbit-coupled bands. When a dc field is applied to the 2D system with spin-orbit coupling, electron obtains an additional momentum, but its spin remains unchanged. Consequently, in the helicity basis, the transition between the spin-orbit-coupled bands occurs. It is obvious that all electrons in 2D systems join in this process.

The second part of solution relates to the transport process and the corresponding diagonal distribution functions should be determined from the coupled equations

\[
\frac{-\partial n_F(E)}{\partial E}{\bigg|}_{E=\varepsilon_\mu} \frac{\partial \varepsilon_\mu}{\partial p_\mu} = \pi \sum_k |V(p-k)|^2 \{ [\rho_{\mu\mu}^{(2)}(p) - \rho_{\mu\mu}^{(2)}(k)]a_1\Delta_{\mu\mu} + [\rho_{\mu\mu}^{(2)}(p) - \rho_{\mu\mu}^{(2)}(k)]a_2\Delta_{\mu\mu} \},
\]

with \(\mu = 1, 2\), \(\bar{\mu} = 3 - \mu\) and \(\Delta_{\mu\mu} \equiv \delta(\varepsilon_\mu(p) - \varepsilon_\nu(k))\). And then, the real part of nondiagonal distribution can be given by

\[
\text{Re} \rho_{12}^{(2)} = \text{Re} \rho_{21}^{(2)} = \frac{\pi}{2} \sum_{k=1,2} |V(p-k)|^2 a_3(-1)^\mu \{ \Delta_{\mu\mu}[\rho_{\mu\mu}^{(2)}(p) - \rho_{\mu\mu}^{(2)}(k)] - \Delta_{\bar{\mu}\bar{\mu}}[\rho_{\bar{\mu}\bar{\mu}}^{(2)}(p) - \rho_{\bar{\mu}\bar{\mu}}^{(2)}(k)] \}.
\]

This part of spin-Hall effect depends on the electron-impurity scattering and entirely vanishes in clean samples. However, it is of order of \(n_i^0\) and the disorder only plays an intermediate role. This disorder-mediated spin-Hall effect contributes from the electrons near the Fermi surface and, physically, can be understood as the following way. The electrons, participating in the longitudinal transport, also can be scattered by impurities. This process, which changes the electron momentum and conserves its spin, yields the transition between two spin-orbit-coupled bands in the helicity basis.

Unfortunately, the above coupled equations for diagonal distribution functions only can be resolved numerically. We perform a numerical evaluation for a GaAs-based heterojunction, where the electron-impurity scattering is assumed to be long-range \([30]\). The momentum integration is calculated by the Gauss-Legendre scheme. The linear system equations, arising from the discretization of the integral equations for diagonal distribution functions, are solved by using the singular value decomposition. Here, we concern on the zero-temperature spin-Hall effect. Therefore, only the distribution functions at the Fermi surface need to be carried out.

We know that, when only presence of the Rashba term, the dependence of the nondiagonal distribution function on the momentum angle can be expressed through the sine function \([3]\). However, in our case, the deviation from this rule is expected due to the anisotropic dispersion relations. In Fig. 1, we plot the real part of nondiagonal distribution functions, which could be observed by the experimental tools, such as Raman spectroscopy, angle resolved photoelectron spectroscopy (ARPES) etc. It can be seen that when the \(\beta\) closes to the value of \(\alpha\), a peak always appears near the angle \(\pi/4\) or \(5\pi/4\). With the ratio of \(\beta/\alpha\) approaching to the unit, the peak becomes more pronounced. The amplitude of \(\text{Re} \rho_{12}\) for the same value of \(\beta/\alpha\) increases with the parameter \(\alpha\). It is obvious that such structures near \(\pi/4\) and \(5\pi/4\) relate to the zero of \(\epsilon_{RD}\) at \(\alpha = \beta\), producing a jump in the electron velocity.

Further, the spin-Hall current has been carried out by substituting the \(\text{Re} \rho_{12}\) into Eq. \([7]\). The total spin-Hall current, coming from the above two parts of the kinetic equation solutions, is found to vanish. We have exam-
of spin-Hall current, associating with the transport, is expected to deviate from the linear rule. Hence, the non-dissipative spin-Hall effect can be found for the strength of dc field larger than about 1 kV/cm.

For 2D electron systems with both the Rashba and Dresselhaus spin-orbit couplings, we even have calculated the longitudinal spin conductivity $\sigma_{z,x}$, defined in Ref. [3]. Its value also has been found to be zero.

In summary, we construct a nonequilibrium Green’s function approach to the spin-Hall effect of 2D electron systems with both the Rashba and Dresselhaus spin-orbit interaction. Taking into account the long-range electron-impurity scattering, we have resolved the kinetic equations numerically. It is found that, although the dispersion relation of such systems exhibits an anisotropic character, the total dissipationless spin-hall conductivity vanishes.

One of the authors (SYL) gratefully acknowledge invaluable discussions with Drs. W. S. Liu, Y. Chen and W. Xu. This work was supported by the National Science Foundation of China, the Special Funds for Major State Basic Research Project, and the Youth Scientific Research Startup Foundation of SJTU.
[20] N. S. Averkiev and L. E. Golub, Phys. Rev. B 60, 15582 (1999); L. E. Golub, N. S. Averkiev, and M. Wilander, Nanotechnology 11, 215 (2000); A. A. Kiselev and K. W. Kim, Phys. Status Solidi B 221, 491 (2000).
[21] J. Schliemann, J. C. Egues, and D. Loss, Phys. Rev. Lett. 90, 146801 (2003).
[22] S. Saikin et al., J. Appl. Phys. 94, 1769 (2003); M. Shen et al. Lect. Notes Comp. Sci. 2668, 881 (2003); A. Lusakowski, J. Wrobel, and T. Dietl, Phys. Rev. B 68, 081201(R) (2003); R. Winkler, Phys. Rev. B 69, 045317 (2004).
[23] J. Schliemann and D. Loss, Phys. Rev. B 68, 165311 (2003).
[24] E. I. Rashba, Fiz. Tverd. Tela (Leningrad) 2, 1224 (1960) [Sov. Phys. Solid State 2, 1109 (1960)]; Y. A. Bychkov and E. I. Rashba, J. Phys. C 17, 6039 (1984).
[25] G. Dresselhaus, Phys. Rev. 100, 580 (1955).
[26] E. I. Rashba, Phys. Rev. B 68, 241315 (2003).
[27] H. Haug and A.-P. Jauho, Quantum Kinetics in Transport and Optics of Semiconductors (Springer, 1996).
[28] P. Lipavský, V. Špička, and B. Velicky, Phys. Rev. B 34, 6933 (1986).
[29] H. Haug, Phys. Status Solidi (b) 173, 139 (1992).
[30] T. Ando, A. B. Fowler, and F. Stern, Rev. Mod. Phys. 54, 437 (1982).