Mean transverse momentum as a mass composition estimator of cosmic rays

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Abstract

Determination of mass composition of high energy cosmic rays is one of the greatest challenge in modern astrophysics. All of previous methods for finding the mass composition of primary cosmic rays in a surface array require at least two independent measurements (e.g. muon and electron components) of extensive air showers (EAS). Here a new statistical parameter is introduced which can be used to determine the mass composition of vertical downward cosmic rays in a simple surface array. The main advantage of the introduced parameter is that it does not need two independent measurements and can be used in a simple surface array which does not have muon detectors.

Keywords — Cosmic ray; Extensive air showers; Mass compositions

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1 Introduction

Since the beginning of cosmic ray researches, one of the most important questions has been about their nature. After it became clear that most of them are ionized nucleus of common atoms, this question was changed to a question how frequent are individual nucleus in the primary cosmic rays composition. Now, the mass composition of primary cosmic rays is very important for the interpretation of different astrophysical processes and for acceleration mechanisms of ultra high energy cosmic rays. Up to energies of about $10^{14}$ eV, balloon-borne or satellite-borne experiments directly measured the mass composition of primaries with negligible uncertainties [11, 7]. At higher energies, direct measurement of mass composition is not possible and mass composition estimation is only possible through extensive air showers which is generated by primary cosmic rays. Propagation of an extensive air shower through atmosphere includes lots of intrinsic fluctuations and so ground based experiments can not access the mass composition of every single primary. Strictly speaking, no EAS experiment measures the mass content of primaries. Instead, they measure one or more of primaries’ mass sensitive parameters of EASs generated by primaries. The events can then be interpreted in terms of primary mass by a comparison to air shower simulations using different hadronic interaction models. Mass composition estimation of primaries (MCEP) can be achieved either in observation of longitudinal development of EAS through atmosphere or simultaneous measurement of the electromagnetic and muonic component of EASs in a surface array. Where, in the former case, people usually determine the EAS maximum development through atmosphere which is a relatively good mass sensitive parameter. From the EAS maximum development measurements, two observables are derived: $<X_{\text{max}}>$, the mean depth of EAS maximum development in g/cm$^2$ and its standard deviation, $\sigma(X_{\text{max}})$, which are related to $\ln A$ and $\sigma(\ln A)$, where A is the atomic mass of the primary. EAS simulations shows that proton and iron induced EASs maximum development are expected to differ by around 100 g/cm$^2$. However, unfortunately even extreme mass content like proton and iron have considerable overlap [21]. Currently, direct observation of the EAS maximum development is only possible by fluorescence light telescopes. At the moment, there are two active experiments that use fluorescence telescopes for MCEP: The Pierre Auger observatory [1] and the Telescope Array (TA) [22]. For the Auger Observatory, EAS maximum depth is determined with a resolution of about 25 g/cm$^2$ at low energies decreasing down to about 15 g/cm$^2$ above $10^{18}$ eV. For the latter, the resolution is better than 40 g/cm$^2$ at energy of $10^{16}$ eV and at higher energies, the accuracy
performs better and reaches to 20 g/cm$^2$ [23].

Also, non-imaging Cherenkov Telescope can be used for determination of the EAS maximum. It can be deduced either from the measurement of the pulse width at 400 m from the core or from the ratio of the photon densities at two different distances from the shower core [24]. Recently, the Low Frequency Array, LOFAR [26], a radio telescope consisting of thousands of crossed dipoles, reported a radio measurements of EAS maximum with a high resolution, a mean uncertainty of 16 g/cm$^2$ [9]. The situation will be even better with the new proposed installations. For the example of the Square Kilometre Array (SKA) which will be operating in Australia in 2023, it is claimed to reach the resolution of 6 g/cm$^2$ in determination of EAS maximum [20]. If that is the case, this results in a better mass spectroscopy than ever done before. Although with such a high resolution in determination of $X_{\text{max}}$, a far better mass spectroscopy is possible, single event mass determination is nevertheless impossible.

Another method for the MCEP is the measurement of particle densities at a surface array. Unlike the observation of EAS maximum development which takes places in a wide range of Earth’s atmosphere, surface arrays measure EAS profile only in a single plane and so is more susceptible to the EAS fluctuations. Nevertheless, surface arrays are far more common than telescopes, air Cherenkov detectors and recently developed radio detection techniques. Furthermore, unlike telescopes or air Cherenkov detectors, they have a full duty cycle. So MCEP in surface arrays are still widely used.

The most common method for the MCEP in a surface array is the estimation of electron and muon numbers. While the sum of electron and muon numbers at the ground relate to the energy content of an EAS, the ratio of muons number to electrons number is an indication of the primary mass content. The most widely used technique for electron-muon discrimination in a surface array is simultaneous use of unshielded and shielded scintillation detectors (e.g. AGASA [12], CASA-MIA [8], EAS-TOP [4], GRAPES [18], KASCADE [5], KASCADE-Grande [6], Maket-ANI [14], GAMMA [16], and Yakutsk [3]). The Auger observatory uses Cherenkov tanks which also enables a limited muon identification [2].

More recently, Canal et al. in their simulations found that the ratio $r_{\mu e} = n_\mu/(E_{\text{em}}/0.5\text{ MeV})$ of the muons number to the energy of electromagnetic component of an EAS at the ground level is a good measure for the MCEP. They reported that those vertical EASs with only a value of $r_{\mu e}$ between 0.5 and 3 enable us to reach a 98% efficiency for discrimination of proton from iron primaries [10].

Another method for the MCEP is the analysis of lateral distribution of charged particles of EASs. Most of lateral distribution functions (LDF) which
are in use for energy estimation of EASs have an age parameter which is an estimation of EASs maximum development depth. EASs of heavy primaries reach their maximum development depth at higher altitudes than EASs of lighter primaries. So the LDF of heavier primaries are flatter and age parameter has higher values than lighter primaries. However, the precision of this method for the MCEP is lower than other methods (for a review of MCEP methods, see [21]).

In this paper, a new parameter is introduced which enable a good MCEP for vertical EASs. The new parameter can be used in a simple surface array that lacks muon detectors and only measures charged particles in general.

2 Mean transverse momentum as a mass composition estimator

From the new results of LHC, it is evident that the more multiplicity of secondary particles of a reaction, the higher transverse momentum per particle of the reactions products (e.g. see [13]).

On the other hand, we conclude from extensive simulations of EASs which conducted by different research groups that the higher the mass composition of primaries, the higher the multiplicity of secondary particles (see chapter 3 of [17]).

The overall result of the above discussion is that when the primary particle is heavier, the generated secondary particles have higher transverse momentum. So if we could determine the transverse momentum per particles (\(\langle P_T \rangle\)) of an EAS, we may estimate the primary’s rough mass composition.

In order to test this hypothesis, some CORSIKA [19] simulated EASs have been generated whose general properties are summarized in table 1. The primary particles of these EASs are protons or irons. For each type and energy, 10000 EASs have been separately generated.

According to table 1 there are 8 different combinations of types and energies altogether. So the number of all generated EASs are 80000. For the vertical
| Specifications                      | Values                                      |
|------------------------------------|---------------------------------------------|
| energy of primaries                | \( E = 100, 200, 300 \text{ and } 400 \text{TeV} \) |
| type of primaries                  | proton and iron                             |
| zenith angle of primaries          | \( \theta = 0^\circ \)                     |
| geographical longitude             | 51 E                                        |
| geographical latitude              | 35 N                                        |
| altitude                           | 1200 m                                      |
| earth magnetic field \((B_x)\)     | 28.1 \( \mu \) T                           |
| earth magnetic field \((B_z)\)     | 38.4 \( \mu \) T                           |
| low energy hadronic model          | Fluka 2011.2b [15]                         |
| high energy hadronic model         | QGSJETIII-04 [23]                          |

Table 1: EASs’ specifications. Other specifications are CORSIKA default values.

EASs, \( \langle P_T \rangle \) is calculated from the following equation:

\[
\langle P_T^e \rangle = \frac{1}{N_e} \sum_{i=1}^{N_e} \sqrt{p_{xi}^2 + p_{yi}^2} \tag{1}
\]

\[
\langle P_T^\mu \rangle = \frac{1}{N^\mu} \sum_{i=1}^{N^\mu} \sqrt{p_{xi}^2 + p_{yi}^2} \tag{2}
\]

\[
\langle P_T \rangle = \frac{1}{N} \sum_{i=1}^{N} \sqrt{p_{xi}^2 + p_{yi}^2} = \frac{N_e}{N} \langle P_T^e \rangle + \frac{N^\mu}{N} \langle P_T^\mu \rangle \tag{3}
\]

where \( p_{xi} \) and \( p_{yi} \) are the components of horizontal momentum of secondary particles.

Figure 1 shows the distribution of the \( \langle P_T \rangle \) for EASs of table 1 on the ground level. As can be seen in this figure, iron primaries are distinguished from protons based on their charged secondary particles’ \( \langle P_T \rangle \)’s. Actually \( \langle P_T \rangle \) of charged secondary particles are higher for iron EASs than proton EASs which is consistent with our expectation. Based on part b and c of figure 1 it is evident that the good separation of \( \langle P_T \rangle \) distributions for all charged particles, can not be seen for muonic and electronic components separately. Though at first glance it may seem strange, examining equation 3 shows that the contribution of coefficients of \( N_e/N \) and \( N^\mu/N \) should also be taken into account. It should also be apparent that \( \langle P_T \rangle \) for electronic component is one order of magnitude smaller than \( \langle P_T \rangle \) for muonic component. This difference is due to more interaction of electrons with air nucleus along their path compared to the muons.
Figure 1: The distribution curves represent the $\langle P_T \rangle$ for secondary particles of vertical simulated EASs. The energies and types of primaries are shown in the margins.

In order to evaluate the effect of each term of equation 3 on $\langle P_T \rangle$ separately, their distributions have been depicted in figure 2. As is evident, $N_\mu \langle P_T^\mu \rangle / N$
Figure 2

(a) $N_e \langle P_T \rangle / N$ distributions.

(b) $N_\mu \langle P_T \rangle / N$ distributions

distribution has a far better separation than $N_e \langle P_T \rangle / N$ distribution. So, the separation of $\langle P_T \rangle$ is mainly due to the contribution of muonic term. The electron contribution has negligible effect on the separation.

Next, we should evaluate the effect of $N_\mu / N$ on the separation. Figure 3a shows the results of the $N_\mu / N$ distribution. A comparison of this figure and figure 1c convince us that the $N_\mu / N$ coefficient has the most important role in the MCEP of the $\langle P_T \rangle$. Figure 3b also shows the results of $N_\mu / N_e$ for reference.

Up to now, we only have done a qualitative investigation of the MCEP parameters. However, a quantitative comparison will be more informative. Also, the precisions have not been investigated with respect to the energy of EASs.

In order to study the resolution of $\langle P_T \rangle$ and $N_\mu / N$ for the MCEP, we intro-
duce the resolution coefficient with the following equation:

$$ R = \frac{|m_P - m_{Fe}|}{\sigma_P + \sigma_{Fe}} $$

(4)

where $m$ is the mean value of a distribution and $\sigma$ is the standard deviation. Actually, resolution coefficient $R$ is a measure of distance between two distributions’ averages as well as the narrowness of the distributions. The merit of such a dimensionless parameter is that it checks not only the separation of the two distributions’ averages but also the mixing amount of them.

As can be seen in table 2, both of $\langle P_T \rangle$ and $N_\mu/N$ have almost the same precision. Although, $\langle P_T \rangle$ has slightly less precision, it has an obvious advantage: It only uses the information of all charged particles and do not need muon specialized detectors. Also according to table 2, $\langle P_T \rangle$ and $N_\mu/N$ have better results than the $N_\mu/N_e$. 

Figure 3
Table 2: Resolution coefficients $R$ of MCEP parameters.

| Energy  | 100 TeV | 200 TeV | 300 TeV | 400 TeV |
|---------|---------|---------|---------|---------|
| $\langle P_T \rangle$ | 2.17    | 2.29    | 2.35    | 2.37    |
| $N_\mu/N$ | 2.21    | 2.32    | 2.37    | 2.38    |
| $N_\mu/N_e$ | 2.06    | 2.18    | 2.26    | 2.30    |
| $\langle d_T \rangle$ | 2.02    | 2.05    | 2.079   | 2.066   |

3 Mean transverse distance from axis

Although, $\langle P_T \rangle$ is a useful parameter for the MCEP, it cannot be directly obtained by the current surface arrays in use (A surface array of hodoscopes is probably appropriate for this purpose). Therefore, we should find a new parameter which has some correlation with the above-mentioned parameters and can be obtained from the information provided by a common surface array.

Although the propagation of an EAS in the atmosphere has lots of stochastic fluctuations, obviously when the $\langle P_T \rangle$ of an EAS is large, its secondary particles have a higher chance to be found at larger distances from axis. So, a new variable of an EAS which is correlated to $\langle P_T \rangle$ is the mean transverse distance of particles from the axis ($\langle d_T \rangle$):

$$\langle d_T \rangle = \frac{1}{N} \sum_{i=1}^{N} d_i,$$

where $d_i$ is the distance of $i$th secondary particle from the axis.

Table 3: Correlation coefficient between $\langle P_T \rangle$ and $\langle d_T \rangle$
Table 3 shows the correlation coefficient between $\langle P_T \rangle$ and $\langle d_T \rangle$ for the EASs of table 1. This correlation coefficient is given by the following equation:

$$\rho_{Pd} = \frac{\text{cov}(\langle P_T \rangle, \langle d_T \rangle)}{\sigma_P \sigma_d},$$

$$\text{cov}(\langle P_T \rangle, \langle d_T \rangle) = E[(\langle P_T \rangle - \mu_P)(\langle d_T \rangle - \mu_d)]$$

$$= \frac{1}{N_{EASs}} \sum_{i=1}^{N_{EASs}} (\langle P_T \rangle_i - \mu_P)(\langle d_T \rangle_i - \mu_d),$$

$$\mu_P = E[\langle P_T \rangle] = \frac{1}{N_{EASs}} \sum_{i=1}^{N_{EASs}} \langle P_T \rangle_i,$$

$$\mu_d = E[\langle d_T \rangle] = \frac{1}{N_{EASs}} \sum_{i=1}^{N_{EASs}} \langle d_T \rangle_i,$$

$$\sigma_P^2 = \frac{1}{N_{EASs}} \sum_{i=1}^{N_{EASs}} (\langle P_T \rangle_i - \mu_P)^2,$$

$$\sigma_d^2 = \frac{1}{N_{EASs}} \sum_{i=1}^{N_{EASs}} (\langle d_T \rangle_i - \mu_d)^2$$

where $\langle P_T \rangle_i$ and $\langle d_T \rangle_i$ are the mean transverse momentum and mean transverse distance of $i$th EAS, the $E$ means the ensemble average over all the EASs, $\mu_P$ and $\mu_d$ are the ensemble average of $\langle P_T \rangle$ and $\langle d_T \rangle$ of all EASs, respectively and $N_{EASs}$ parameter is the total number of EASs that is here 10000.

As is evident from table 3, the correlation between $\langle P_T \rangle$ and $\langle d_T \rangle$ is nearly perfect. So $\langle d_T \rangle$ can be safely used instead of $\langle P_T \rangle$ for MCEP. Figure 4 shows $\langle d_T \rangle$ distribution for all charged particles.

![Figure 4: $\langle d_T \rangle$ distribution for all charged particles.](image-url)
the distribution of $\langle d_T \rangle$. It is apparent from this figure that $\langle d_T \rangle$ is a suitable parameter for the MCEP, as it was anticipated.

As we can see from figure 4, $\langle d_T \rangle$ can also discriminate between P and Fe with a relatively high resolution. Another interesting feature in figure 4 is that the distribution of $\langle d_T \rangle$ becomes sharper with increasing energy of the primaries (especially for the irons). Also the $\langle d_T \rangle$ approaches to lower (but distinct) values for irons than protons.

Table 2 also shows the $R$ coefficient for the distributions of figure 4. As can be seen from this table, this parameter have better results in the higher energies.

4 Conclusions

In this paper a new parameter has been introduced which can be applied for the mass composition estimation of vertical primary cosmic rays. This parameter is the transverse momentum per secondary particles of an EAS, $\langle P_T \rangle$. In an obvious manner, we concluded that $\langle P_T \rangle$ can be reduced to $N_\mu/N$ parameter for the MCEP with a negligible loss of information. It is also shown that a related parameter to $\langle P_T \rangle$ which can be directly measured in a simple surface array without muonic specialized detectors, is the mean transverse distance from axis, $\langle d_T \rangle$. It is also evident that the distributions of these parameters are sharper in higher energies.

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