Signature Quantization in Fading CDMA With Limited Feedback

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Abstract

In this paper, we analyze the performance of a signature quantization scheme for reverse-link Direct Sequence (DS) Code Division Multiple Access (CDMA). Assuming perfect estimates of channel and interference covariance, the receiver selects the signature that maximizes signal-to-interference plus noise ratio (SINR) for a desired user from a signature codebook. The codebook index corresponding to the optimal signature is then relayed to the user with finite number of bits via a feedback channel. Previously, we showed that a Random Vector Quantization (RVQ) codebook, which contains independent isotropically distributed vectors, is optimal (i.e., maximizes SINR) in a large system limit in which number of interfering users, processing gain, and feedback bits tend to infinity with fixed ratios. Dai et al. have analyzed the large system SINR for a matched filter with nonfading channel. Here we extend the results to linear minimum mean squared error (MMSE) receiver and multipath fading channel. Numerical examples show that the derived large system results give a good approximation to the performance of finite-size system.

Index Terms

Random Vector Quantization, large system limit, signature quantization, limited feedback, multipath fading, CDMA.

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I. INTRODUCTION

A user performance in Direct Sequence (DS)- Code Division Multiple Access (CDMA) depends on a signature code, which can be optimized to increase signal-to-interference plus noise ratio (SINR). Several works in the literature [1]–[7] have investigated a joint transmitter-receiver signature optimization and showed that a performance difference between optimized and random signatures can be substantial. However, adapting signature adds more complexity and requires knowledge of channel and interference covariance at both the transmitter and receiver. All aforementioned works assume that perfect estimates of channel and interference covariance are available. This assumption, especially at the transmitter, is not practical.

A receiver typically estimates channel coefficients and interference covariance from pilot signals during training period. The accuracy of the estimation increases with amount of available pilots. The transmitter, on the other hand, is usually unable to directly estimate the forward channel. However, channel information may be obtained from the receiver via a feedback channel. Thus, accuracy of channel information at the transmitter depends on available feedback rate, which is normally low. In recent years, many researchers [8]–[15] have proposed feedback schemes in which the receiver computes and quantizes the optimal signature and relays the quantized coefficients to the transmitter via a low-rate feedback channel. References [10]–[15] consider multiantenna systems where spatial signatures are optimized and quantized. Here our interest is signature quantization in DS-CDMA and its performance, which depends largely on quantization codebook and available feedback rate.

The signature codebook is known a priori at both the transmitter and receiver. With $B$ feedback bits, the receiver selects the signature vector, which maximizes the instantaneous SINR, from $2^B$-signature codebook and relays the corresponding index to the transmitter via an error-free feedback channel. References [8], [16] proposed a Random Vector Quantization (RVQ) codebook, which consists of independent isotropically distributed vectors and showed that the RVQ codebook is optimal (i.e., maximize the SINR over all codebooks) in a large system limit in which number of users $K$, processing gain $N$, and feedback bits $B$ tend to infinity with fixed $\bar{K} = K/N$ and $\bar{B} = B/N$. The upper bound on asymptotic SINR for single-user matched filter was derived in [8]. Reference [8] also considered a minimum mean square error (MMSE) receiver and derived an approximation for a large system SINR. The large system performance...
was shown to predict the performance of a finite-size system well for small $\bar{B}$.

Recently, [9] derives the exact expression of a large system SINR for RVQ with a matched filter and ideal nonfading channel. (Similar results for the performance of RVQ in multiantenna system were derived in [12].) Here we extend the results for matched filter to multipath fading channel and arbitrary transmit power across users. We apply similar techniques used in [9], [12] to derive expressions for asymptotic SINR with linear MMSE receiver. For an MMSE receiver, we first consider a nonfading channel and derive an exact expression for a large system SINR, which is a function of $\bar{K}$ and $\bar{B}$. We remark that the expressions for MMSE receiver are not trivial extensions of [9], [12]. Comparison between the large system SINR and the approximation derived in [8], which over-estimates the performance for large $\bar{B}$, is shown. Numerical examples show that the large system results estimate the performance of the finite-size system well. From examples shown, one feedback bit per signature element achieves close to the performance with unlimited feedback.

II. System Model

We consider a discrete-time reverse-link synchronous DS-CDMA in which there are $K$ users and processing gain $N$. The $N \times 1$ received vector is given by

$$ r = \sum_{k=1}^{K} \sqrt{A_k} H_k s_k b_k + n \quad (1) $$

where $\sqrt{A_k}$ is the amplitude of user $k$, $H_k$ is the $N \times N$ channel matrix for user $k$, $s_k$ is the $N \times 1$ signature vector for user $k$, $b_k$ is the transmitted symbol for user $k$, and $n$ is the additive white Gaussian noise with zero mean and covariance $\sigma_n^2 I$. For ideal nonfading channel, $H_k = I$. For frequency-selective channel, we assume that the symbol duration is much longer than the delay spread and, thus, we discard any inter-symbol interference. Assuming that each
user traverses $L$ Rayleigh fading paths, we have

$$H_k = \begin{bmatrix}
h_{k,1} & 0 & \cdots & 0 & 0 & \cdots & 0 \\
\vdots & h_{k,1} & \vdots & 0 & \vdots \\
h_{k,L} & \ddots & \vdots & 0 & \vdots \\
0 & h_{k,L} & h_{k,1} & 0 & \cdots & 0 \\
\vdots & 0 & \ddots & h_{k,1} & 0 & \ddots & 0 \\
0 & \vdots & h_{k,L} & \ddots & 0 \\
0 & 0 & \cdots & 0 & h_{k,L} & \cdots & h_{k,1}
\end{bmatrix}$$  \hspace{1cm} (2)$$

where fading gains for user $k$, $h_{k,1}, \ldots, h_{k,L}$, are independent complex Gaussian random variables with zero mean and variances $E|h_{k,1}|^2, \ldots, E|h_{k,L}|^2$, respectively. For flat fading channel ($L = 1$), $H_k = h_{k,1}I$.

The receiver applies linear filter on the received signal to obtain the received symbol. We consider both matched filter and linear MMSE receivers and assume, without loss of generality, that user 1 is the user of interest. The matched filter for user 1 is given by

$$c_1 = \hat{s}_1$$  \hspace{1cm} (3)$$

where we let $\hat{s}_k \triangleq H_k s_k$, which is the effective signature for user $k$. The matched filter is simple and can be a performance benchmark for a more complex receiver. The associated SINR is given by

$$\gamma = \frac{|\sqrt{A_1} c_1^\dagger \hat{s}_1|^2}{c_1^\dagger R_1 c_1} = \frac{A_1(s_1^\dagger H_1^\dagger H_1 s_1)^2}{s_1^\dagger H_1^\dagger R_1 H_1 s_1}$$  \hspace{1cm} (4)$$

where the interference-plus-noise covariance

$$R_1 = E[r_1 r_1^\dagger]$$  \hspace{1cm} (5)$$

where expectation is over transmitted symbols and noise and

$$r_1 = r - \sqrt{A_1} H_1 s_1 b_1 = \sum_{k=2}^{K} \sqrt{A_k} H_k s_k b_k + n.$$  \hspace{1cm} (6)$$

Assuming that $b_k$’s are independent and identically distributed (i.i.d.) with zero mean and unit variance, we have

$$R_1 = \sum_{k=2}^{K} A_k \hat{s}_k \hat{s}_k^\dagger + \sigma_n^2 I = \hat{S}_1 A_1 \hat{S}_1^\dagger + \sigma_n^2 I,$$  \hspace{1cm} (7)$$
where $\tilde{S}_1$ is the $N \times (K - 1)$ effective signature matrix whose columns consist of $\tilde{s}_k$, $\forall k \neq 1$ and $A_1$ is the $(K - 1) \times (K - 1)$ where diagonal matrix whose diagonal entries are $A_2, \ldots, A_K$.

On the other hand, a linear MMSE receive filter for user 1 is given by

$$c_1 = R^{-1} \tilde{s}_1$$

(8)

where the received covariance

$$R = E[rr^\dagger] = \sum_{k=1}^{K} A_k \tilde{s}_k \tilde{s}_k^\dagger + \sigma_n^2 I,$$

(9)

assuming that $b_k$’s are (i.i.d.) with zero mean and unit variance. Similar to the matched filter, we can compute the SINR for user 1 given by

$$\beta = A_1 s_1^\dagger H_1^\dagger R_1^{-1} H_1 s_1$$

(10)

where matrix inversion lemma was used to simplify the expression. A linear MMSE receiver is shown to be robust in suppressing multiple-access interference [17]. We note that, for given $R_1$ and $H_1$, the SINR for user 1 is a function of the signature $s_1$ for both receivers.

The receiver, which is assumed to have a perfect estimate of the interference covariance $R_1$ and channel matrix $H_1$, can optimize the signature for the desired user to maximize the received SINR. Ideally, the receiver sends the optimal signature back to user 1 via a feedback channel and the user changes the signature, accordingly. Practically, a feedback channel has limited rate and thus, the receiver can only relay finite number of feedback bits to the user. (We assume that the feedback does not incur any errors.) With $B$ bits, the receiver selects the signature from a signature set or codebook containing $2^B$ signatures. This codebook is designed \textit{a priori}, and is known at both the user and receiver. The performance of the optimized user depends on the codebook. Several work [8], [10]–[14], [18] focused on codebook design and analyzed the associated performance. (All except [8] are in context of spatial signature in a multiantenna channel.) In this work, we analyze the performance of a Random Vector Quantization (RVQ) codebook proposed by [8], [16]. RVQ codebook

$$\mathcal{V} = \{v_1, \ldots, v_{2^B}\}$$

(11)

in which $v_j$’s are independent isotropically distributed with unit norm ($\|v_j\| = 1$). In other words, signature vectors in RVQ codebook are uniformly distributed on a surface of an $N$-dimensional unit sphere. In [8], [9], [12], RVQ was shown to maximize SINR over all quantization codebooks.
in a large system limit to be defined. Although RVQ is optimal in a large system limit [8], it was shown to perform close to the optimal codebook designed for a finite-size system [19].

Given the codebook $\mathcal{V}$, the receiver selects

$$s_1 = \arg \max_{v_j \in \mathcal{V}} \text{SINR}(v_j).$$  \hspace{1cm} (12)

The index of the optimal signature vector is relayed to user 1 via a feedback channel. We are interested in analyzing the corresponding SINR, which is a function of available feedback bits, for both matched filter and MMSE receiver.

III. LARGE SYSTEM PERFORMANCE

A. Matched filter

We first consider ideal nonfading channel ($H_i = I$ for all $i$) for which the optimal signature that maximizes SINR also minimizes the interference. Given the RVQ codebook $\mathcal{V}$, the optimal signature is given by

$$s_1 = \arg \min_{v_j \in \mathcal{V}} \{I(v_j) \triangleq v_j^\dagger R_1 v_j\}$$ \hspace{1cm} (13)

where $I$ is the instantaneous interference power. Since $v_j$’s in RVQ codebook are $i.i.d.$, the corresponding $I(v_j)$’s are also $i.i.d.$ and thus, the associated interference averaged over codebook is given by

$$E_{\mathcal{V}}[I(v_1), \ldots, I(v_{2^B}) | R_1] = 2^B \int_0^\infty x [G_{I|R_1}(x)]^{2^B-1} g_{I|R_1}(x) \, dx$$ \hspace{1cm} (14)

where $G_{I|R_1}(\cdot)$ and $g_{I|R_1}(\cdot)$ are cumulative distribution function (cdf) and probability density function (pdf) for $I(v_j)$, respectively. It is difficult to evaluate (14) for any finite $N$, $K$, and $B$. It was shown that the interference power converges to a deterministic value in a large system limit in which $K$, $N$, and $B$ all tend to infinity with fixed normalized load $\bar{K} = K/N$ and normalized feedback bits $\bar{B} = B/N$ [8], [9]. Applying theory of extreme order statistics [20] similar to [8], the large system interference power with fading channel is given by

$$I_{\text{rvq}}^\infty = \lim_{(N,K,B) \to \infty} G_{I|R_1}^{-1}(2^{-B})$$ \hspace{1cm} (15)

where the empirical eigenvalue distribution of $R_1$ converges almost surely to a nonrandom limit as $(N,K) \to \infty$ with fixed $K/N$. Rearranging (15) gives

$$\lim_{(N,K,B) \to \infty, z \to I_{\text{rvq}}^\infty} [G_{I|R_1}(z)]^\frac{1}{2^B} = 2^{-B}$$ \hspace{1cm} (16)
References [9], [12] show that

$$
\lim_{\substack{(N,K,B) \to \infty \\\ \ \ \ \ \ \ \ \ \ z \to I_{rvq}^\infty}} [G_{1|R_i}(z)]^{1/2} = \exp\{ - \Psi(\rho^*, I_{rvq}^\infty) \} \tag{17}
$$

where

$$
\Psi(\rho, I_{rvq}^\infty) = \int \log(1 + \rho(\lambda - I_{rvq}^\infty)) g_{R_i}(\lambda) \, d\lambda, \tag{18}
$$

$$
\rho^* = \arg \max_{0 < \rho < \frac{1}{I_{rvq}^\infty} \cdot \min} \Psi(\rho, I_{rvq}^\infty), \tag{19}
$$

and $g_{R_i}(\cdot)$ is an asymptotic eigenvalue density for $R_i$. Equating (16) and (17), we have that $I_{rvq}^\infty$ satisfies

$$
\Psi(\rho^*, I_{rvq}^\infty) = \bar{B} \log(2). \tag{20}
$$

With (7), Eq. (18) becomes

$$
\Psi(\rho, I_{rvq}^\infty) = \int \log(1 + \rho(\lambda + \sigma^2_n - I_{rvq}^\infty)) g_{S_1A_1S_1^\dagger}(\lambda) \, d\lambda. \tag{21}
$$

$$
= \log(1 + \rho\sigma^2_n - \rho I_{rvq}^\infty) + \int \log(1 + \xi \lambda) g_{S_1A_1S_1^\dagger}(\lambda) \, d\lambda \tag{22}
$$

$$
= \log(1 + \rho\sigma^2_n - \rho I_{rvq}^\infty) + \nu_{S_1A_1S_1^\dagger}(\xi) \tag{23}
$$

where

$$
\xi \equiv \frac{\rho}{1 + \rho \sigma^2_n - \rho I_{rvq}^\infty} \tag{24}
$$

and $\nu_{S_1A_1S_1^\dagger}(\cdot)$ is the Shannon transform for asymptotic eigenvalue distribution for $S_1A_1S_1^\dagger$. Reference [21] defines the Shannon transform for a density function $f_X(\cdot)$ as follows

$$
\nu_X(\gamma) = \int \log(1 + \gamma x) f_X(x) \, dx. \tag{25}
$$

Suppose $s_k$, $2 \leq k \leq K$, has independent complex Gaussian entries with zero mean and variance $1/N$ ($\|s_k\| \to 1$). The eigenvalue distribution for $S_1S_1^\dagger$ converges to a deterministic function as $N, K \to \infty$ with fixed $\bar{K}$ [22] and we assume that empirical distribution of $A_2, \ldots, A_K$ converges to a limit. It is shown that [21]

$$
\nu_{S_1A_1S_1^\dagger}(w) = \bar{K} \nu_{A_1}(w\eta_{S_1A_1S_1^\dagger}(w)) - \log(\eta_{S_1A_1S_1^\dagger}(w)) + \eta_{S_1A_1S_1^\dagger}(w) - 1 \tag{26}
$$
where $\eta_{S_1A_1S_1^†}(\cdot)$ is the $\eta$-transform for the asymptotic eigenvalue distribution for $S_1A_1S_1^†$ and the $\eta$-transform for a distribution for random variable $X$ is defined in [21] as follows
\[
\eta_X(\gamma) = \int_1^{1+\gamma x} f_X(x) \, dx.
\] (27)

With earlier assumption on the distribution for $S_1$, [21] shows that $\eta_{S_1A_1S_1^†}(x)$ is the solution to the following fixed point equation
\[
\bar{K} = \frac{1 - \eta_{S_1A_1S_1^†}(x)}{1 - \eta_{A_1}(x)\eta_{S_1A_1S_1^†}(x)}.
\] (28)

Combining (20), (23) (26), and (28), we have the first result.

**Theorem 1:** The large system interference power $I_{rvq}^\infty$ at the output of single-user matched filter satisfies the following equation
\[
\max_{0 < \rho < \frac{1}{\alpha_{rvq} - \lambda_{min}}} \{\log(1 + \rho \sigma^2_n - \rho I_{rvq}^\infty) + \bar{K}\nu_{A_1}(\xi(\Theta(\xi))) - \log \Theta(\xi) + \Theta(\xi) - 1\} = \bar{B} \log(2)
\] (29)

where $\Theta(x)$ is the solution to the following fixed point equation
\[
\bar{K} = \frac{1 - \Theta(x)}{1 - \eta_{A_1}(x)\Theta(x)}
\] (30)

and $\xi$ is given by (24).

Solving for $I_{rvq}^\infty$ requires numerical solution in most cases. However, for equal power allocation ($A_1 = A_2 = \cdots = A_K$), the explicit expression for $I_{rvq}^\infty$ was obtained by [9] as follows.

**Corollary 1 ([9]):** Let $\bar{B}^* = \frac{-K \log(1 - \frac{1}{\sqrt{K}}) - \sqrt{K}}{\log(2)}$ for $K > 1$. For $K > 1$ and $\bar{B} > \bar{B}^*$,
\[
I_{rvq}^\infty = \sigma^2_n + (1 - \sqrt{K})^2 + \sqrt{K}(1 - \frac{1}{\sqrt{K}})^{1-K} \exp(-\sqrt{K} - \bar{B} \log(2)).
\] (31)

Otherwise, $I_{rvq}^\infty = Q + \sigma^2_n$ where $Q$ satisfies the following equation
\[
Q = \bar{K} e^{(Q-K)/K} 2^{-B/K}.
\] (32)

Thus, interference power decreases exponentially with the normalized feedback bits and is near a single-user performance with only few feedback bits per processing gain. The associated SINR for user 1 in a large system limit is then given by
\[
\gamma_{rvq}^\infty = \frac{1}{I_{rvq}^\infty}.
\] (33)
If the interfering signatures are orthogonal \((S_1^\dagger S_1 = I)\) with equal power allocation, the eigenvalue distribution is given by for \(K < 1\)

\[
g_{S_1A_1S_1^\dagger}(\lambda) = \bar{K}\delta(\lambda - 1) + (1 - \bar{K})\delta(\lambda).
\] (34)

Evaluating (20) with the distribution in (34), we obtain the following result.

**Theorem 2:** The large system interference power \(I_{rvq}^\infty\) for orthogonal interfering signatures with equal power allocation satisfies the following fixed-point equation

\[
(I_{rvq}^\infty - \sigma_n^2)^\bar{K}(1 + \sigma_n^2 - I_{rvq}^\infty)^{1-\bar{K}} = \bar{K}(1 - \bar{K})^{1-\bar{K}}2^{-\bar{B}}
\] (35)

for \(0 < \bar{K} < 1\).

For \(\bar{K} \approx 1\), we obtain the following approximation

\[
I_{rvq}^\infty \approx \sigma_n^2 + \bar{K}2^{-\bar{B}}.
\] (36)

Here we see clearly that the interference power decreases exponentially with the normalized feedback bits.

For a flat-fading channel \((L = 1)\), we can combine the channel gain \(|h_{k,1}|^2\) for user \(k\) with its transmit power \(A_k\). That is the diagonal matrix \(A_1 = \text{diag}\{||h_{2,1}|^2A_2, \ldots ,|h_{K,1}|^2A_K\}\) whose empirical distribution converges to a nonrandom limit. With an asymptotic distribution for diagonal entries of \(A_1\), we can apply (20) - (30) to obtain the output SINR \(\gamma_{rvq}^\infty\).

For frequency-selective fading, the signal of each users is assumed to propagate \(L\) discrete chip-spaced paths. The channel matrix for user \(k\) is shown in (2). First, we assume that \(L\) is finite and does not grow with \(N\). Thus, the number of paths per processing gain \(L = L/N \rightarrow 0\).

To compute \(I_{rvq}^\infty\), we require the asymptotic eigenvalue distribution of \(R_1\) (7). Reference [23] showed that the asymptotic eigenvalue distribution of \(R_1\) with \(L\)-path channels (2) equals that of \(R_1\) with flat-fading channels and \(A_1 = \text{diag}\{A_2(\sum_{l=1}^{L} |h_{2,l}|^2), \ldots ,A_K(\sum_{l=1}^{L} |h_{K,l}|^2)\}\).

Thus, a multipath interferer is asymptotically equivalent to a single-path interferer with combined gain of \(\sum_{l=1}^{L} |h_{k,l}|^2\). For \(L \rightarrow \infty\) with fixed \(L/N\), the same result applies as long as \(\lim_{L \rightarrow \infty} \sum_{l=1}^{L} E|h_{k,l}|^2 < \infty\), for all \(k\).

The associated SINR at the output of matched filter with fading channel is given by

\[
\gamma_{rvq}^\infty = \frac{A_1\alpha_1}{I_{rvq}^\infty}
\] (37)

where \(\sum_{l=1}^{L} |h_{1,l}|^2 \rightarrow \alpha_1\) and \(I_{rvq}^\infty\) is obtained by Theorem 1.
B. Linear MMSE Receiver

The SINR with the optimal signature averaged over the RVQ codebook is given by

\[
E_{\mathbf{V}}[\max\{\beta(\mathbf{v}_1), \ldots, \beta(\mathbf{v}_{2^B})\} | \mathbf{R}_1, \mathbf{H}_1] = 2^B \int_0^\infty x [F_{\beta|\mathbf{R}_1,\mathbf{H}_1}(x)]^{2^B-1} f_{\beta|\mathbf{R}_1,\mathbf{H}_1}(x) \, dx \tag{38}
\]

where \( f_{\beta|\mathbf{R}_1,\mathbf{H}_1}(\cdot) \) and \( F_{\beta|\mathbf{R}_1,\mathbf{H}_1}(\cdot) \) be pdf and cdf for the output SINR \( \beta(\mathbf{v}_j) \), respectively. Similar to the matched filter, computing (38) for finite parameters are difficult. Taking the large system limit as \( N, K, B \to \infty \) with fixed ratios, the SINR converges to a deterministic value

\[
\beta_{\text{rvq}}^\infty = \lim_{(N,K,B) \to \infty} E_{\mathbf{V}}[\max\{\beta_1, \ldots, \beta_{2^B}\} | \mathbf{R}_1, \mathbf{H}_1] \tag{39}
\]

\[
= \lim_{(N,K,B) \to \infty} F_{\beta|\mathbf{R}_1,\mathbf{H}_1}^{-1}(1 - 2^{-B}), \tag{40}
\]

which can be shown by applying theory of extreme order statistics [20]. Reference [8] derived the approximation for \( \beta_{\text{rvq}}^\infty \) by approximating cdf for \( \beta(\mathbf{v}_j) \) to be Gaussian. The approximation is a function of \( \bar{K}, \bar{B}, \) and \( \sigma_n^2 \) and is good for small \( \bar{B} \). For large \( \bar{B} \), it over-estimates the actual performance. In this section, we derive exact expressions for \( \beta_{\text{rvq}}^\infty \).

We first consider the ideal channel (\( \mathbf{H}_k = \mathbf{I}, \forall k \)). We rearrange (40) to obtain

\[
\lim_{(N,K,B) \to \infty} \frac{1}{z \to \beta_{\text{rvq}}^\infty} [1 - F_{\beta|\mathbf{R}_1}(z)]^\frac{1}{N} = 2^{-B}. \tag{41}
\]

Similar to [9], [12], it can be shown that

\[
\lim_{(N,K,B) \to \infty} \frac{1}{z \to \beta_{\text{rvq}}^\infty} [1 - F_{\beta|\mathbf{R}_1}(z)]^\frac{1}{N} = \exp\{-\Phi(\rho^*, \beta_{\text{rvq}}^\infty)\} \tag{42}
\]

where

\[
\Phi(\rho, \beta_{\text{rvq}}^\infty) = \int \log(1 + \rho(\beta_{\text{rvq}}^\infty - \frac{A_1}{\tau + \sigma_n^2})) f_{\mathbf{S}_1\mathbf{A}_1\mathbf{S}_1^\dagger}(\tau) \, d\tau, \tag{43}
\]

\[
\rho^* = \arg \max_{\rho < \rho_{\text{max}} - \beta_{\text{rvq}}^\infty} \Phi(\rho, \beta_{\text{rvq}}^\infty), \tag{44}
\]

\( f_{\mathbf{S}_1\mathbf{A}_1\mathbf{S}_1^\dagger}(\cdot) \) is the asymptotic eigenvalue density for \( \mathbf{S}_1\mathbf{A}_1\mathbf{S}_1^\dagger \), \( \mathbf{S}_1 \) is the \( N \times (K - 1) \) signature matrix whose columns are \( \mathbf{s}_2, \ldots, \mathbf{s}_K \), and \( \beta_{\text{rvq}}^\infty \) is the asymptotic maximum eigenvalue of \( A_1 \mathbf{R}_1^{-1} \) and corresponds to the SINR with infinite feedback (\( \bar{B} \to \infty \)).

Combining (41) and (42), \( \beta_{\text{rvq}}^\infty \) satisfies the following fixed-point equation

\[
\Phi(\rho^*, \beta_{\text{rvq}}^\infty) = \bar{B} \log(2). \tag{45}
\]
To evaluate $\Phi(\rho^*, \beta_{rvq}^\infty)$, we rewrite (43) as follows

$$
\Phi(\rho, \beta_{rvq}^\infty) = \int \log(1 + \zeta \tau) f_{S_1 A_1 S_1^\dagger}(\tau) \, d\tau - \int \log(1 + \frac{1}{\sigma_n^2} \tau) f_{S_1 A_1 S_1^\dagger}(\tau) \, d\tau + \log(1 + \rho(\beta_{rvq}^\infty - A_1^2/\sigma_n^2))
$$

(46)

$$
= \nu_{S_1 A_1 S_1^\dagger}(\zeta) - \nu_{S_1 A_1 S_1^\dagger}(1/\sigma_n^2) + \log(1 + \rho(\beta_{rvq}^\infty - A_1^2/\sigma_n^2))
$$

(47)

where

$$
\zeta \triangleq \frac{1 + \rho \beta_{rvq}^\infty}{\sigma_n^2 + \rho \beta_{rvq}^\infty \sigma_n^2 - \rho A_1}
$$

(49)

and $\nu_{S_1 A_1 S_1^\dagger}(\cdot)$ is the Shannon transform for asymptotic eigenvalue distribution for $S_1 A_1 S_1^\dagger$.

With similar steps used to derive Theorem 1, we obtain the following theorem.

**Theorem 3:** For $\bar{B}$, the large system SINR $\beta_{rvq}^\infty$ is given by

$$
\max_{0 < \rho < \bar{B}_{max}} \{ \log(1 + \rho(\beta_{rvq}^\infty - A_1^2/\sigma_n^2)) + \bar{K} \nu_{A_1}(\zeta \Theta(\zeta)) - \bar{K} \nu_{A_1}(\sigma_n^{-2} \Theta(\sigma_n^{-2})) - \log(\Theta(\zeta)) + \log(\Theta(\sigma_n^{-2})) + \Theta(\zeta) - \Theta(\sigma_n^{-2}) \} = \bar{B} \log(2)
$$

(50)

where $\zeta$ and $\Theta(x)$ are given by (49) and (30), respectively.

For an equal-power ($A_1 = A_2 = . . . = A_K$) system, we can simplify expression for $\beta_{rvq}^\infty$ as follows.

**Corollary 2:** We assume ideal channels, i.i.d. interfering signatures, and equal transmitted power across users. For $\bar{K} \leq 1$, $\beta_{rvq}^\infty$ satisfies the following equation

$$
\log\left( \frac{\bar{K}}{1 - \beta_{rvq}^\infty \sigma_n^2} - \frac{1}{\beta_{rvq}^\infty} \right) + (1 - \bar{K}) \log\left( \frac{p}{\sigma_n^2} \right)
$$

$$
+ \bar{K} \log\left( \frac{w(p)}{w(\sigma_n^2)} \right) - (1 - \bar{K}) \log\left( \frac{1 - v(p)}{1 - v(\sigma_n^2)} \right)
$$

$$
- v(p) + v(\sigma_n^2) = \bar{B} \log(2)
$$

(51)

where

$$
w(x) = \frac{1}{2} (1 + \bar{K} + x + \sqrt{(1 + \bar{K} + x)^2 - 4\bar{K}})
$$

(52)

$$
v(x) = \frac{1}{2} (1 + \bar{K} + x - \sqrt{(1 + \bar{K} + x)^2 - 4\bar{K}})
$$

(53)
and
\[ p = \frac{1 - \beta_{\text{eq}}^2\sigma_n^2}{K\beta_{\text{eq}}^2 - 1 + \beta_{\text{eq}}^2\sigma_n^2} - \frac{1}{\beta_{\text{eq}}^2} + \sigma_n^2. \]  

(54)

For \( \bar{K} > 1 \) and \( \bar{B} \leq \bar{B}^* \), \( \beta_{\text{eq}} \) satisfies the following equation
\[
\log\left(\frac{\bar{K}}{1 - \beta_{\text{eq}}^2\sigma_n^2} - \frac{1}{\beta_{\text{eq}}^2}\right) + \log\left(\frac{w(p)}{w(\sigma_n^2)}\right) \\
- (\bar{K} - 1) \log\left(\frac{\bar{K} - v(p)}{\bar{K} - v(\sigma_n^2)}\right) - v(p) + v(\sigma_n^2) = \bar{B} \log(2) \]  

(55)

where
\[
\bar{B}^* = \frac{1}{\log(2)} \{\log(\bar{K} - \sqrt{\bar{K} + \sigma_n^2}) + \bar{K} \log(\sqrt{\bar{K}}) \\
- \bar{K} \log(\sqrt{\bar{K}} - 1) - \sqrt{\bar{K}} - \log(w(\sigma_n^2)) + (\bar{K} - 1) \log(1 - \frac{v(\sigma_n^2)}{\bar{K}}) + v(\sigma_n^2)\}. \]  

(56)

For \( \bar{K} > 1 \) and \( \bar{B} > \bar{B}^* \),
\[
\beta_{\text{eq}} = \beta_{\text{max}}^\infty (1 - 2^{-B}) \left\{\exp\{\frac{1}{2} \bar{K} \log(\bar{K}) - (\bar{K} - 1) \log\left(\frac{\bar{K}\sqrt{\bar{K} - \bar{K}}}{\bar{K} - v(\sigma_n^2)}\right) - \log(w(\sigma_n^2)) + v(\sigma_n^2) - \sqrt{\bar{K}}\}\} \}. \]  

(57)

The derivation is shown in the Appendix.

We can also derive the SINR when the interfering signatures are orthogonal. Substituting the corresponding eigenvalue distribution (34) into (47) and simplifying give the following result.

Theorem 4: For orthogonal set of interfering signature with \( 0 < \bar{K} < 1 \) and equal power allocation, the large system SINR \( \beta_{\text{eq}}^\infty \) is the solution of the following fixed-point equation
\[
(A_1 - \beta_{\text{eq}}^\infty\sigma_n^2)^K (\beta_{\text{eq}}^\infty - (A_1 - \beta_{\text{eq}}^\infty\sigma_n^2))^{1-K} = \left(\frac{A_1 \bar{K}}{1 + \sigma_n^2}\right)^\bar{K} \left(\frac{A_1 (1 - \bar{K})}{\sigma_n^2}\right)^{1-\bar{K}} 2^{-\bar{B}}. \]  

(58)

For a system with heavy load (\( \bar{K} \approx 1 \)), we have
\[
\beta_{\text{eq}}^\infty \approx \frac{A_1}{\sigma_n^2} - \frac{A_1 \bar{K}}{\sigma_n^2 (1 + \sigma_n^2)} 2^{-\bar{B}}. \]  

(59)

The first term on the right-hand side of (59) is the single-user performance and thus, the performance with the MMSE receiver also increases exponentially with \( \bar{B} \), which is the same as the that of matched filter, which is a much simpler receiver.
Similar to the matched filter, a multipath fading is asymptotically equivalent to a single fading path with combined gain of $\sum_{i=1}^{L} |h_{k,l}|^2$. Combining the fading gain with transmitted power, we obtain a new $A_1$. We assume that the distribution for a diagonal elements of $A_1$ converges to a deterministic function given that $A_k < \infty$ and $\sum_{i=1}^{L} |h_{k,l}|^2 < \infty$ for all $k$.

IV. Numerical Results

Fig. 1 shows the asymptotic SINR for MMSE receiver in Corollary 2 versus normalized feedback bit $\bar{B}$ with different normalized loads $\bar{K} = 0.25, 0.5, 1, 1.25$. As expected, the SINR increases with normalized feedback and decreases with normalized load. For $\bar{K} = 0.25$, RVQ achieves close to the single-user performance with approximately $\bar{B} = 0.5$ (0.5 bits per processing gain or degree of freedom). As number of interfering users increases, amount of feedback required also increases to achieve a target SINR. For example, $\bar{B} = 3$ is needed for system with $\bar{K} = 1$ to achieve close to the single-user performance. We also compare the asymptotic results with simulation results marked by pluses in Fig. 1. We note that the large system results predict the performance of finite-size systems ($N = 12$) well. As $N$ increases, the gap between the simulation and analytical results is expected to be closing. RVQ codebook requires an exhaustive search to locate the optimal signature. The search complexity increases exponentially with feedback bits $B$. (For $\bar{B} = 3$, number of entries in RVQ codebook is $2^{36}$.) Thus, we do not have simulation results for a large $B$.

Fig. 2 shows the large system performance of both receivers, which is obtained from Theorems 1, 2 with different distributions for interfering signatures. Interfering signatures either have i.i.d. Gaussian elements or are orthogonal. From the figure, the system with independent Gaussian signatures performs a bit better than that with orthogonal signatures for both linear receivers. The difference is more pronounced in the matched filter. Which distribution for interfering signatures gives the maximum performance is an interesting open problem.

In Fig. 3 we compare the asymptotic SINR for MMSE receiver in Corollary 2 with the approximation derived in [8] for $\bar{K} = 0.75$ and SNR = 10 dB. Also shown is the simulation results with $N = 12$. The large system SINR is closer to the simulated performance than the approximation. We also show the RVQ performance of a matched filter in Corollary 1 [9] with that of MMSE receiver. The performance difference can be substantial for small to moderate $\bar{B}$. With 1 feedback bit per degree of freedom, the MMSE receiver outperforms a matched filter by
as much as 30%. However, an MMSE filter is more complex than a matched filter. Therefore, there is a performance tradeoff between feedback and receiver complexity.

We also simulated a multipath fading channel in which each user’s signal transverses 2 paths with different gains ($E|h_{k,1}|^2 = 0.9$ and $E|h_{k,2}|^2 = 0.1, \forall k$). Furthermore, $K$ interfering users are divided into 2 groups. $K_1$ users transmit signal with $A_k = P_1$ while $K_2$ users with $A_k = P_2$. This scenario may follow from a system with differentiated quality of service. We obtain the large system SINR from Theorem 3 with the asymptotic distribution of $A_1$

$$f_{A_1}(a) = \frac{\bar{K}_1}{K}\delta(a - P_1) + \frac{\bar{K}_2}{K}\delta(a - P_2)$$

(60)

where normalized loads $\bar{K}_1 = K_1/N$ and $\bar{K}_2 = K_2/N$. Both the large system and corresponding simulated results with $\bar{K}_1 = \bar{K}_2 = 0.25$ and different sets of $P_1$ and $P_2$ are shown in Fig. 4. The large system performance closely approximates the performance of the system with $N = 32$. As $N$ grows, the performance of a finite-size system will converge to that of the large system. In this example, reducing the transmit power of one group of users by 20 dB ($P_2$ from 10 to 0.1) decreases the required feedback to achieve 0.5 dB away from the single-user performance by $\bar{B} = 0.4$.

V. CONCLUSIONS

We have shown expressions for a large system SINR for RVQ with both matched filter and linear MMSE receiver. The SINR is a function of normalized load (number of users per degree of freedom) and normalized feedback bit (number of feedback bit per degree of freedom). Both ideal nonfading channel and multipath fading channel were considered. The SINR of the quantized signature for both receivers increases exponentially with $\bar{B}$. For a small load, RVQ achieves close to the single-user performance with only fraction of feedback bit per quantized signature coefficient. We compared performance of the MMSE receiver with that of matched filter derived in [9] and showed that the performance gap is large for small $\bar{B}$. The simpler matched filter requires more feedback to achieve a target SINR than the MMSE receiver does.

In this work, we assume that the receiver can estimate channel and interference covariances perfectly. In practice, a very accurate channel estimation is achieved by a large amount of training. How the performance of RVQ is affected by imperfect channel estimate at the receiver (or limited training) was studied by [24], [25]. Here we consider signature quantization for a
single user. Future work includes performance analysis of group of users with RVQ-quantized signatures.

APPENDIX

PROOF OF COROLLARY 2

We rewrite (43) as follows

\[ \Phi(\rho, \beta_{rvq}^\infty) = \int \log(1 + \rho(\beta_{rvq}^\infty - \frac{1}{x + \sigma_n^2})) f_{S_iS_i^t}(x) \, dx \]  
\[ = \int \log(x + \sigma_n^2 + \rho(\beta_{rvq}^\infty(x + \sigma_n^2) - 1)) f_{S_iS_i^t}(x) \, dx - \int \log(x + \sigma_n^2) f_{S_iS_i^t}(x) \, dx \]  
\[ (61) \]

where

\[ f_{S_iS_i^t}(x) = \frac{\sqrt{(x-a)(b-x)}}{2\pi x} \quad \text{for} \quad a \leq x \leq b, \]  
\[ (63) \]

where \( a = (1 - \sqrt{K})^2 \) and \( b = (1 + \sqrt{K})^2 \) for \( K > 1 \).

To determine \( \rho^* \), we take the first derivative of (62) with respect to \( \rho \) given by

\[ \frac{d\Phi(\rho, \beta_{rvq}^\infty)}{d\rho} = \frac{1}{\rho} - \frac{1}{\rho(\beta_{rvq}^\infty + 1)} - \frac{1}{(\rho\beta_{rvq}^\infty + 1)^2} \int_{S_f(y)} \frac{1}{x-y} f_{S_iS_i^t}(x) \, dx \]  
\[ (64) \]

where \( S_f(\cdot) \) is the Stieltjes transform of \( f_{S_iS_i^t}(\cdot) \) and

\[ y = \frac{\rho}{\rho\beta_{rvq}^\infty + 1} - \sigma_n^2. \]  
\[ (65) \]

We solve for \( \rho^* \) (or equivalently \( y^* \)) by setting (64) to zero and obtain

\[ S_f(y^*) = (\rho^*\beta_{rvq}^\infty + 1)\beta_{rvq}^\infty. \]  
\[ (66) \]

Substituting the Stieltjes transform of \( g(\cdot) \) and using the change of variable from (65) in (66) give

\[ -1 + K - y^* \pm \sqrt{(y^*)^2 - 2(K+1)y^* + (K-1)^2} = \frac{\beta_{rvq}^\infty}{2y^*} \]  
\[ (67) \]

Simplifying (67) gives

\[ y^* = \frac{(1 - \beta_{rvq}^\infty(K - 1 + \sigma_n^2))(1 - \beta_{rvq}^\infty\sigma_n^2)}{\beta_{rvq}^\infty(1 - \beta_{rvq}^\infty(K + \sigma_n^2))}. \]  
\[ (68) \]

With change of variable (65), we obtain

\[ \rho^* = \frac{K}{\beta_{rvq}^\infty(1 - \beta_{rvq}^\infty\sigma_n^2)} - \frac{1}{(\beta_{rvq}^\infty)^2} = \frac{1}{\beta_{rvq}^\infty}. \]  
\[ (69) \]
To show that \( \rho^* \) achieves the maximum, we compute the second derivative of \( \Phi(\rho, \beta_{rvq}^\infty) \) with respect to \( \rho \):

\[
\frac{d^2 \Phi(\rho, \beta_{rvq}^\infty)}{d\rho^2} = -\int_a^b \frac{(\beta_{rvq}^\infty (x + \sigma_n^2) - 1)^2}{(x + \sigma_n^2 + \rho(\beta_{rvq}^\infty (x + \sigma_n^2) - 1))^2} f_{S_1S_1^t}(x) \, dx
\]

(70)

\[
\leq 0. \quad (71)
\]

For large enough \( \beta_{rvq}^\infty \geq \beta_{rvq}^* \), \( \rho^* \) can exceed \( 1/(\beta_{max}^\infty - \beta_{rvq}^\infty) \). To determine \( \beta_{rvq}^\infty \), we set

\[
\frac{1}{\beta_{rvq}^\infty(1 - \beta_{rvq}^\infty \sigma_n^2)} - \frac{1}{(\beta_{rvq}^\infty)^2} = \frac{1}{\beta_{max}^\infty - \beta_{rvq}^\infty}. \quad (72)
\]

Simplifying (72) gives the following quadratic equation

\[
[K + \sigma_n^2 - \bar{\beta} \sigma_n^2] \beta_{rvq}^\infty + [(1 - K - \sigma_n^2) \bar{\beta} - 1] \beta_{rvq}^\infty + \beta_{max}^\infty = 0. \quad (73)
\]

Solving (73) gives the only solution

\[
\beta_{rvq}^\infty = \frac{K - \sqrt{K + \sigma_n^2}}{(K - \sqrt{K})^2 + 2\sigma_n^2(K - \sqrt{K}) + \sigma_n^4}. \quad (74)
\]

Thus,

\[
\rho^* = \begin{cases} \frac{K}{\beta_{rvq}^\infty(1 - \beta_{rvq}^\infty \sigma_n^2)} - \frac{1}{(\beta_{rvq}^\infty)^2}, & \bar{\beta} \leq \beta_{rvq}^\infty \leq \beta_{rvq}^* \\ \frac{1}{\beta_{max}^\infty - \beta_{rvq}^\infty}, & \beta_{rvq}^\infty > \beta_{rvq}^* \end{cases}. \quad (75)
\]

Substituting \( \rho = \rho^* \) in (62) and rearranging give

\[
\Phi(\rho^*, \beta_{rvq}^\infty) = \log(\rho^* \beta_{rvq}^\infty + 1) + \int_a^b \log(x + \sigma_n^2 - \frac{\rho^*}{\rho^* \beta_{rvq}^\infty + 1}) \, dx
\]

(76)

\[
- \int_a^b \log(x + \sigma_n^2) \, dx.
\]

First, we consider the case where \( \bar{\beta} \leq \beta_{rvq}^\infty \leq \beta_{rvq}^* \). Substituting \( \rho^* \) into the first term in (76) gives

\[
\log(\rho^* \beta_{rvq}^\infty + 1) = \log(\frac{K}{1 - \beta_{rvq}^\infty \sigma_n^2} - \frac{1}{\beta_{rvq}^\infty}). \quad (77)
\]

To evaluate the two integrals in (76), we apply the following lemma.

**Lemma 1 (26):** For \( K \geq 1 \),

\[
\int_a^b \log(x + \alpha) \, dx = \log(w(\alpha)) - (K - 1) \log(1 - \frac{1}{K} v(\alpha)) - v(\alpha) \quad (78)
\]
where
\[ w(\alpha) = \frac{1}{2} (1 + K + \alpha + \sqrt{(1 + K + \alpha)^2 - 4K}), \tag{79} \]
\[ v(\alpha) = \frac{1}{2} (1 + K + \alpha - \sqrt{(1 + K + \alpha)^2 - 4K}). \tag{80} \]

Using Lemma 1 and (77), we can evaluate (76) for \( \beta \leq \beta_{rvq}^\infty \)
\[
\Phi(\rho^*, \beta_{rvq}^\infty) = \log\left( \frac{K}{1 - \beta_{rvq}^\infty \sigma_n^2} \right) + \log\left( \frac{w(\rho)}{w(\sigma_n^2)} \right) - (K - 1) \log\left( \frac{K - v(p)}{K - v(\sigma_n^2)} \right) - v(p) + v(\sigma_n^2) \tag{81} \]

where
\[
p = \frac{1 - \beta_{rvq}^\infty \sigma_n^2}{K \beta_{rvq}^\infty - 1 + \beta_{rvq}^\infty \sigma_n^2} \tag{82} \]

Next we evaluate \( \Phi(\rho^*, \beta_{rvq}^\infty) \) for \( \beta_{rvq}^\infty > \beta_{rvq}^\infty \).

Substituting the value of \( \rho^* \) from (75) gives
\[
\log(\rho^* \beta_{rvq}^\infty + 1) = \log(\beta_{max}^\infty) - \log(\beta_{max}^\infty - \beta_{rvq}^\infty) \tag{83} \]

and
\[
\sigma_n^2 - \frac{\rho^*}{\rho^* \beta_{rvq}^\infty + 1} = \sigma_n^2 - \frac{1}{\beta_{max}^\infty} = -(1 - \sqrt{K})^2. \tag{84} \]

Substituting (84) into the second term in (76) and applying Lemma 1 gives
\[
\int_a^b \log(x + \sigma_n^2 - \frac{\rho^*}{\rho^* \beta_{rvq}^\infty + 1}) f_{S_1 S_1}^*(x) \, dx = \frac{1}{2} K \log(K) - (K - 1) \log(\sqrt{K} - 1) - \sqrt{K}. \tag{85} \]

Thus, for \( \beta_{rvq}^\infty > \beta_{rvq}^\infty \),
\[
\Phi(\rho^*, \beta_{rvq}^\infty) = \log(\beta_{max}^\infty) - \log(\beta_{max}^\infty - \beta_{rvq}^\infty) + \frac{1}{2} K \log(K) - (K - 1) \log(\sqrt{K} - 1) - \sqrt{K} - \log(w(\sigma_n^2)) + (K - 1) \log(1 - \frac{1}{K} v(\sigma_n^2)) + v(\sigma_n^2). \tag{86} \]

Also \( \Phi(\rho^*, \beta_{rvq}^\infty) = \tilde{B} \log(2) \). We can explicitly solve for \( \beta_{rvq}^\infty \) as follows
\[
\beta_{rvq}^\infty = \beta_{max}^\infty (1 - 2^{-\tilde{B}} [\exp\{\frac{1}{2} K \log(K) - (K - 1) \log(\frac{K \sqrt{K} - K}{K - v(\sigma_n^2)}) - \log(w(\sigma_n^2)) + v(\sigma_n^2) - \sqrt{K})\}]). \tag{87} \]

To solve \( \tilde{B}^* \) which is corresponding to \( \beta_{rvq}^\infty \) (74), we substitute \( \beta_{rvq}^\infty \) in (87).
REFERENCES

[1] P. Rapajic and B. Vucetic, “Linear adaptive transmitter-receiver structures for asynchronous CDMA systems,” *European Trans. on Telecommun.*, vol. 6, no. 1, pp. 21–28, Jan-Feb 1995.

[2] S. Ulukus and R. D. Yates, “Iterative construction of optimum sequence sets in synchronous CDMA systems,” *IEEE Trans. Inf. Theory*, vol. 47, no. 5, pp. 1989–1998, Jul. 2001.

[3] T. F. Wong and T. M. Lok, “Transmitter adaptation in multicode DS-CDMA systems,” *IEEE J. Sel. Areas Commun.*, vol. 19, no. 1, pp. 69–82, Jan. 2001.

[4] G. S. Rajappan and M. L. Honig, “Signature sequence adaptation for DS-CDMA with multipath,” *IEEE J. Sel. Areas Commun.*, vol. 20, no. 2, pp. 384–395, Feb. 2002.

[5] P. Viswanath, V. Anantharam, and D. N. C. Tse, “Optimal sequences, power control and capacity of spread-spectrum systems with multiuser linear receivers,” *IEEE Trans. Inf. Theory*, vol. 45, no. 6, pp. 1968–1983, Sep. 1999.

[6] ———, “Optimal sequences and sum capacity of synchronous CDMA systems,” *IEEE Trans. Inf. Theory*, vol. 45, no. 6, pp. 1984–1991, Sep. 1999.

[7] C. Rose, S. Ulukus, and R. D. Yates, “Wireless systems and interference avoidance,” *IEEE Trans. Wireless Commun.*, vol. 1, no. 3, pp. 415–428, Jul. 2002.

[8] W. Santipach and M. L. Honig, “Signature optimization for CDMA with limited feedback,” *IEEE Trans. Inf. Theory*, vol. 51, no. 10, pp. 3475–3492, Oct. 2005.

[9] W. Dai, Y. Liu, and B. Rider, “The effect of finite rate feedback on CDMA signature optimization and MIMO beamforming vector selection,” *IEEE Trans. Inf. Theory*, vol. 55, no. 8, pp. 3651–3669, Aug. 2009.

[10] D. J. Love and R. W. Heath, Jr., “Grassmannian beamforming for multiple-input multiple-output wireless systems,” *IEEE Trans. Inf. Theory*, vol. 49, no. 10, pp. 2735–2745, Oct. 2003.

[11] K. K. Mukkavilli, A. Sabharwal, E. Erkip, and B. Aazhang, “On beamforming with finite rate feedback in multiple antenna systems,” *IEEE Trans. Inf. Theory*, vol. 49, no. 10, pp. 2562–2579, Oct. 2003.

[12] W. Santipach and M. L. Honig, “Capacity of a multiple-antenna fading channel with a quantized precoding matrix,” *IEEE Trans. Inf. Theory*, vol. 55, no. 3, pp. 1218–1234, Mar. 2009.

[13] J. C. Roh and B. D. Rao, “Transmit beamforming in multiple-antenna systems with finite rate feedback: A VQ-based approach,” *IEEE Trans. Inf. Theory*, vol. 52, no. 3, pp. 1101–1112, Mar. 2006.

[14] V. K. N. Lau, Y. Liu, and T.-A. Chen, “On the design of MIMO block-fading channels with feedback-link capacity constraint,” *IEEE Trans. Commun.*, vol. 52, no. 1, pp. 62–70, Jan. 2004.

[15] S. Zhou, Z. Wang, and G. B. Giannakis, “Performance analysis for transmit-beamforming with finite-rate feedback,” in *Proc. Conf. on Inform. Sciences and Systems*, Princeton, NJ, Mar. 2004, pp. 880–885.

[16] W. Santipach and M. L. Honig, “Signature optimization for DS-CDMA with limited feedback,” in *Proc. IEEE Int. Symp. on Spread-Spectrum Tech. and Appl. (ISSSTA)*, Prague, Czech Republic, Sep. 2002, pp. 180–184.

[17] U. Madhow and M. L. Honig, “MMSE interference suppression for direct-sequence spread-spectrum CDMA,” *IEEE Trans. Commun.*, vol. 42, no. 12, pp. 3178–3188, Dec. 1994.

[18] A. Narula, M. J. Lopez, M. D. Trott, and G. W. Wornell, “Efficient use of side information in multiple antenna data transmission over fading channels,” *IEEE J. Sel. Areas Commun.*, vol. 16, no. 8, pp. 1423–1436, Oct. 1998.

[19] D. J. Love, R. W. Heath, Jr., W. Santipach, and M. L. Honig, “What is the value of limited feedback for MIMO channels?” *IEEE Commun. Mag.*, vol. 42, no. 10, pp. 54–59, Oct. 2004.

[20] J. Galambos, *The Asymptotic Theory of Extreme Order Statistics*, 2nd ed. Robert E. Krieger, 1987.
[21] A. M. Tulino and S. Verdú, “Random matrix theory and wireless communications,” *Foundations and Trends in Communications and Information Theory*, vol. 1, no. 1, pp. 1–182, 2004.

[22] V. A. Marčenko and L. A. Pastur, “Distribution of eigenvalues for some sets of random matrices,” *Math. USSR-Sbornik*, vol. 1, pp. 457–483, 1967.

[23] L. Li, A. M. Tulino, and S. Verdú, “Design of reduced-rank MMSE multiuser detectors using random matrix methods,” *IEEE Trans. Inf. Theory*, vol. 50, no. 6, pp. 986–1008, Jun. 2004.

[24] W. Santipach and M. L. Honig, “Capacity of beamforming with limited training and feedback,” in *Proc. IEEE Int. Symp. on Inform. Theory (ISIT)*, Seattle, WA, Jul. 2006.

[25] ——, “Optimization of training and feedback for beamforming over a MIMO channel,” in *Proc. IEEE Wireless Commun. and Networking Conf. (WCNC)*, Hong Kong, China, Mar. 2007.

[26] P. B. Rapajic and D. Popescu, “Information capacity of a random signature multiple-input multiple-output channel,” *IEEE Trans. Commun.*, vol. 48, no. 8, pp. 1245–1248, Aug. 2000.
Fig. 1. Shown is a large system SINR for MMSE receiver versus normalized feedback bit $\bar{B}$ with different normalized loads $\bar{K} = 0.25, 0.5, 1, 1.25$ and SNR = 5 dB.
Fig. 2. Shown are large system SINR’s for different distributions of interfering signatures with $\bar{K} = 0.5$ and SNR $= 8$ dB.
Fig. 3. The large system SINR for MMSE receiver is compared with the approximation derived in [8] and the large system SINR for matched filter [9]. Also shown is the simulation result for $N = 12$, $\bar{K} = 0.75$ and SNR = 10 dB.
Fig. 4. A large system SINR for MMSE receiver and multipath fading with two groups of users is shown with simulation results. SNR = 5 dB, number of paths $L = 2$ for all users, and $\bar{K} = 0.5$. 