THE CONFORMAL UNIVERSE III:
Basic Mechanisms of Matter Generation

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Abstract
This is the last paper of a series of three on Conformal General Relativity. Here, the mechanisms of Higgs field production and baryogenesis during the acute stage of inflation are investigated. Quantum logical and quantum mechanical implications are discussed in advance to clarify the theoretical basis of the phenomenon. Higgs boson production is primarily explained by the release of energy following the decay of a conformal–invariant vacuum state (false–vacuum state) to a conformal–symmetry broken vacuum state (true vacuum state) falling down to lower and lower energy levels during spacetime dilation, to the exhaustion of geometry–to–matter energy transfer. Explicit solutions of Higgs field dynamics are analyzed and exemplified in numerical simulations. The coherent oscillation of Higgs–field amplitude, which starts immediately after the amplitude–jump from a zero to a non–zero value induced by geometry expansion, is also evidenced. The mechanism of baryogenesis is explained by the decay of a Higgs field multiplet interacting with the well–known triplet of fermion families of the Standard Model, through $SU(5)$–invariant Yukawa couplings satisfying the first three Sakharov conditions. The process is promoted by the inflation–induced spontaneous breaking of global $SU(5)$ symmetry down to $SU(3)\times U_Q(1)\times U_{B-L}(1)$, where $Q$ indicates electrical charge invariance and $B-L$ baryon–number minus lepton–number invariance.
# Contents

1. **The epistemic basis of the creation of the universe**
   1.1 Quantum–logical basis of the creation of the universe .................. 3
   1.2 Quantum–physical basis of matter generation ............................ 6
   1.3 False–vacuum to true–vacuum transition of a Higgs field .............. 8

2. **Higgs–field dynamics in conformally flat spacetime** ................. 10
   2.1 An almost exact determination of the scale–factor profile ............ 16
   2.2 Higgs field multiplets .......................................................... 18
   2.3 Global and local spontaneous breaking of internal symmetries . . 20

3. **A possible mechanism for baryogenesis** .................................. 22
   3.1 Sakharov conditions for baryogenesis in the framework of CGR .... 23
   3.2 Her Majesty $SU(5)$ .............................................................. 25
   3.3 $SU(5)$ Yukawa couplings and mass terms ............................... 28
   3.4 How $SU(5)$ symmetry can be broken down globally .................. 30
   3.5 Minimal Yukawa couplings and mass terms for mixed families .... 31
   3.6 Baryon currents and fermion fluid dynamics ............................ 34
   3.7 Inflationary mechanism of baryogenesis ................................... 37

References ....................................................................................... 40
1 The epistemic basis of the creation of the universe

In the introduction of Part I, a novel argument was given in support of the hypothesis that the universe originated from a quantum mechanical fluctuation of the vacuum state of an empty universe, which primed the spontaneous breaking of a perfect pre–primordial symmetry. The argument was that geometry and matter were generated from each other through a process of energy–momentum transfer from the former to the latter. In Parts I and II, proof was given, in the semi–classical approximation, that a process of this sort is conceivable on the basis of the spontaneous breakdown of a conformal–invariant field theory grounded on 4D spacetime, provided that the dilation field and one or more Higgs fields play strategic roles in energy–momentum transfer. However, in order for such hypotheses to make sense, from both a logical and physical standpoints, a few general considerations must be made.

1.1 Quantum–logical basis of the creation of the universe

The thesis of the spontaneous origin of the universe must be compatible with the following quantum–logical statements:

(1) Any entity, quantity or property can be said to really exist, provided that it can be directly observed or inferred from observable events or facts in the logical framework of a reliable theory. In turn, physical events or facts are regarded as having really occurred, provided that their effects produce permanent modifications of macroscopic–world states. This statement rules out that it makes sense to consider the existence of unobservable universes different from ours.

(2) In origin, before the occurrence of the primordial symmetry–breaking event, nothing existed really, but only virtually. That is, nothing else existed except the quantum–logical superposition of the vacuum states of all possible self–consistent conformal–invariant universes. The requirement of conformal invariance is important since, as argued at the beginning of Part II, it is strictly related to causality. The requirement of self–consistency is important, since it implies quantum–field renormalizability. Both requirements are
compatible with assuming the aprioristic quantum–logical coexistence of an infinite number of possible universes, differing from one another by the values of a finite number of adimensional parameters. This statement rules out the hyper–realistic but unprovable opinion that we are living in one of an infinite collection of actually existing but non–communicating universes.

(3) The spontaneous breaking of conformal symmetry may have been the only non–causal event which occurred in the universe. We may presume that the primordial spark which primed the history of universe was the anticipatory realization of a quantum fluctuation promoted by the perspective that the initial state of the universe had to evolve toward the production of living beings and, in the long term, of conscious observers (Anthropic Cosmological Principle [1]). Note that, if we add the even stronger hypothesis that the adimensional parameters of the incoming universe were optimized for maximum probability of conscious–observer existence, we arrive at the conclusion that such values are determined with infinite precision and the produced universe is unique.

Statement (1) is deeply rooted in the theoretical basis of quantum mechanics (QM), in which the concept of physical existence differs substantially from the corresponding concept of classical mechanics (CM). In CM, physical states are presumed to be governed by deterministic laws and to be insensitive to the effects of observations in principle, implying that they are one–to–one with their observable aspects. Consistent with this classic–logic view, what is usually meant as the virtual or the possible is regarded in CM as only existing in the metaphysical world of spirit.

By contrast, in QM, we are faced with the need to distinguish phenomena from inter–phenomena - in the sense devised by Reichenbach in 1944 [35] - in particular, real particles from virtual particles, both of which are assumed to exist physically, although in different ways and with different meanings. In addition, the phenomenological existence of events is explained as a product of observed–observer interactions, in ways and modes exclusively permitted by observational apparatuses, measurement devices and natural or artificial recording procedures. The observer should not be identified with the “abstract ego” of our mind - an expression coined
by the young von Neumann in 1932 [27] - but rather as the macroscopic world itself, regarded as an infinite thermodynamic system out of equilibrium, in which phase transitions and irreversible thermodynamic processes of unlimited complexity can take place.

Statement (2) implies the extension of the concept of quantum–logical possibility to a continuum of possible vacuum states. This concept is not admissible in elementary quantum logic (QL) [2] [34], which is the logic of physical systems with a finite number of degrees of freedom, whose states can always be described by the vectors of a single Hilbert space. Instead, it is admissible in the still poorly studied QL of open systems with infinite degrees of freedom [17], the states of which can only be described in the framework of an infinite direct product of Hilbert spaces. This product decomposes into a continuum of unitarily non–equivalent classes of unitarily equivalent Hilbert spaces, which represent an infinite collection of disjointed macroscopic worlds grounded on different vacuum states out of thermodynamic equilibrium (von Neumann, 1939 [28]; Hepp, 1972 [19]). It is a well–known theoretical result that the temporal evolution of these systems cannot be described by unitary transformations, as is the case in elementary QM, but rather by a dense sequence of algebraic dynamic maps [40], which represent a dense sequence of phase transitions accompanied by the absorption and emission of swarms of infinite infrared quanta of evanescent energy [22]. This point is of fundamental importance in explaining how and why the macroscopic world is the theater of information processes recursively progressing toward organization levels of indescribable complexity, which is essential for the existence of living beings (von Neumann–Burks, 1966 [29]).

Statement (3), no matter how astonishing it may appear, is necessary from a quantum–logical standpoint, since observers come to exist as substantial parts of the universe only after the symmetry–breaking event. Despite its teleological character, this is not a theological issue, but a perfectly physical one. In fact, cause–effect inversions due to massive gravitational ghosts may be observed in principle at sub–Planckian distances in super–high energy collisions, provided that they are not censured by the event–horizons of the small black holes created at the impact.
points of probe particles. This nicely fits the cosmic-censorship theorem of Penrose and Hawking, which states that, in very general conditions, which are very likely to be satisfied in the geometry of our universe, spacetime singularities are dressed in event horizons, with the possible exception of the singularity at the symmetry breaking event which primed the BingBang (Hawking & Penrose, 1979 [18]; Penrose, 1994 [33]). In other words, it is likely that the spontaneous breaking of conformal symmetry was the sole finalistic event which occurred in CGR history.

1.2 Quantum-physical basis of matter generation

The first idea, that huge and rapid volume expansion of a system may result in the creation of particles, was advanced by Takahashi and Umezawa in 1957 [38] [39] and later transferred to cosmology by Parker in 1969–1971 [31] [32]. It was then retrieved by Grib et al. in 1976 [14], and incorporated by Brout et al. in 1978 [3] into a conformal-theoretic approach to cosmology. To my knowledge, the last attempt to move along this line of thought was advanced by Englert in 1981 [7].

All such papers are very interesting and suggestive, and the present work drew great inspiration from them. Unfortunately, the theoretical demonstrations provided by their authors were often based on obscure arguments and even forced to reach the desired conclusions through inappropriate mathematical manipulations. The reason of their failure lies on the very simple concept that the energy content of an expanding system cannot change if the system is unconstrained. In short, these fascinating approaches simply provide us with instructive examples of excellent ideas supported by improper identifications. They were superseded by the more fruitful idea put forward by Frampton in 1976 [8] and Callan and Coleman in 1977 [4] [5], and transferred to cosmology by Guth (1981, 1993) [15] [16], according to which the basic mechanism of matter generation is the decay of one or more scalar fields from a false–vacuum state to a true–vacuum state of lower energy.

The approach presented in this paper differs substantially from all such views, in that the energy gap between the false vacuum state and the true vacuum state of the scalar field is assumed to be enormously amplified by the interaction of this field
with the dilation field responsible for spacetime expansion. As explained in Part I and in the last Section of Part II, this interaction binds the two fields together, so as to allow a huge energy–momentum transfer from geometry to matter. Were it not so, the decay mechanism devised by the last–mentioned authors would not suffice to explain the creation of the huge amount of matter that fills our universe.

Physical coherence requires matter generation to have taken place in four main conditions: (1) temporal continuity of the universe state, as suggested by the Schrödinger equation; (2) total energy–momentum conservation, guaranteed by the CGR equation; (3) spontaneous breakdown of conformal symmetry caused by a quantum fluctuation; (4) existence of “observers” with respect to which the original quantum–logical possibilities came to their actualization as physical phenomena.

Unfortunately, conditions (1) and (2) prohibit any sudden evolution of a Higgs field from a state of infinitesimal amplitude and energy–momentum to one of finite amplitude and positive energy–momentum. The point is that, in these initial conditions, the field is initially found precisely at the stagnation point of its potential energy, implying that the probability of a symmetry–breaking event remains infinitesimal during any finite elapse of time. How can we then explain the occurrence of this sort of nucleation of the cosmogenetic process?

There are two possible answers to this question. One is that nucleation took an infinite amount of time to occur, since the proper time taken by the initial state to descend from a point \( P \) close to the stagnation point 0 tends to become infinite as \( P \) approaches 0. This conclusion may be supported by the argument that an infinitesimal measure of probability times an infinite time interval may give 1. The main defect of this interpretation is that the state of the system should be regarded as that of a classical system, in which any possible effects of QM indetermination are ignored. The second possibility is that the spontaneous breaking of conformal symmetry entailed that of causality, since these properties are closely related.

In either case, however, the mean value of the energy–momentum tensor at proper time \( \tilde{\tau} = 0 \) had to be zero, i.e., that of the vacuum state before the symmetry–breaking event, since otherwise the postulate of physical continuity would be vio-
lated. The important point is that, some time after the occurrence of the nucleation process, the initial Minkowski vacuum is no longer the vacuum of the system, but rather the true physical state of the system. In other words, it becomes a false vacuum state, the true vacuum state being that in which the potential energy of the system is minimal.

The observer problem is even more intriguing, since observers come to exist as parts of the universe itself, only after the nucleation process. Here, it is important to consider that observations are irreversible thermodynamic processes accompanied by infrared emissions which continuously alter the vacuum state [22]. This is consistent with the hyperboloidal shapes of space-like surfaces, the boundaries of which extend to the infinite future, thus favoring the dispersion of infrared swarms to infinity, where the physical effects of observation processes therefore find their natural fate.

1.3 False–vacuum to true–vacuum transition of a Higgs field

Going back to the action integral for $\varphi$ on the Riemann manifold described in subsec.5.1 of Part II, we see that the potential–energy density term

$$U(x) = \frac{\lambda^2}{4} \left[ \varphi^2(x) - \frac{\mu^2}{\lambda^2} \frac{\sigma^2(x)}{\sigma_0^2} \right]^2 - \frac{R(x)}{12} \varphi^2(x),$$

has a minimum at

$$\varphi(x) = \varphi_0(x) \equiv \frac{\mu}{\lambda} \sqrt{\frac{\sigma^2(x)}{\sigma_0^2} + \frac{R(x)}{6\mu^2}}.$$

It is therefore evident that the true scalar field is more appropriately represented by the variation of $\varphi(x)$ from this minimum, i.e., by the difference

$$\eta(x) = \varphi(x) - \varphi_0(x).$$

Let us call this the true Higgs field. This is important in view of quantization since from a quantum logical standpoint, the quanta of a field are observable actualizations of virtual deviations from the fundamental state of the field. To clarify this point, we briefly digress on the physical state of the universe.

According to Heisenberg’s picture, the state of a system remains the same during the evolution of the system, while observables evolve in time. Consistent with this
view, we assume that the state of the universe just before the symmetry-breaking event, notably the Minkowskian vacuum state $|\Omega\rangle$ of the empty universe, also remained the state of the system after that event. In this transition, however, $|\Omega\rangle$ ceases to be the vacuum state of an empty world and begins to be the physical state of the evolving universe created by the symmetry-breaking event.

In addition, since one moment before the event $\langle \Omega | \varphi(x) | \Omega \rangle = 0$, we must assume by continuity that this equality also held one moment after the event, which means

$$\langle \Omega | \eta(\tau, \hat{x}) | \Omega \rangle = -\langle \Omega | \varphi_0(\tau, \hat{x}) | \Omega \rangle$$

for small $\tau$. This implies that the true Higgs field, as well as the dilation field and the Ricci scalar tensor, on which $\varphi_0(x)$ depends, are in general different from zero in state $|\Omega\rangle$ which, in the novel circumstances, is a false vacuum state.

Instead, the true vacuum state is a state $|\Omega_T(x)\rangle$, possibly depending on $x$, in which the VEV of the true Higgs field $\eta(x)$ is zero, i.e., $\langle \Omega_T(x) | \eta(x) | \Omega_T(x) \rangle = 0$.

The same concepts also apply to the Higgs field represented on the Cartan manifold, in which the potential energy term has the form

$$\tilde{U}(x) = \frac{\lambda^2}{4} \left[ \tilde{\varphi}^2(x) - \frac{\mu^2}{\lambda^2} \right] - \frac{\tilde{R}(x)}{12} \tilde{\varphi}^2(x),$$

which has its minimum at

$$\tilde{\varphi}(x) = \tilde{\varphi}_0(x) \equiv \frac{\mu}{\lambda} \sqrt{1 + \frac{\tilde{R}(x)}{6\mu^2}} = \frac{\mu}{\lambda} \sqrt{1 - \frac{\tilde{\Theta}^M(x)}{6\mu^2 M^2_{\text{rP}}}}.$$

Thus, the true Higgs field has the form $\tilde{\eta}(x) = \tilde{\varphi}(x) - \tilde{\varphi}_0(x)$. Since the contribution of the Ricci scalar is negligible, we simply have $\tilde{\varphi}_0(x) = \mu/\lambda$, and therefore the motion equation is

$$\tilde{D}^2 \tilde{\eta}(x) + \lambda^2 \left[ \tilde{\eta}(x) + \frac{\mu}{\lambda} \right] \left[ \tilde{\eta}^2(x) + 2 \frac{\mu}{\lambda} \tilde{\eta}(x) / \lambda \right] = 0.$$ 

After the inflation stage, the Riemann manifold picture converges to the Cartan manifold picture and therefore the motion equation of the true Higgs field becomes

$$D^2 \eta(x) + \lambda^2 \left[ \eta(x) + \frac{\mu}{\lambda} \right] \left[ \eta^2(x) + 2 \frac{\mu}{\lambda} \eta(x) / \lambda \right] = 0.$$ 

In the absence of other interactions, this describes a gas of mutually repelling scalar bosons of mass $\mu_H = \sqrt{2} \mu$, subjected to the action of the gravitational field.
2  Higgs–field dynamics in conformally flat spacetime

In this Section, we study the motion equations of a Higgs field interacting with the
dilation field, as described in Section 5 of Part II. We assume that, at the beginning,
and as long as local interactions do not enter into play, the universe is perfectly
homogenous and isotropic. Correspondingly, we assume that the fundamental tensor
is conformally flat, which implies that both $\sigma$ and $\phi$ depend only on $\tau$.

Let us start from the Riemann manifold picture. By variation of the simplified
action integral Eq.(44) of Part II subsec.(5.1), with respect to $\sigma(\tau)$, we obtain the
motion equation for the scale factor $s(\tau) = e^{\sigma(\tau)}$ as

$$\partial_\tau^2 s(\tau) + \frac{3}{\tau} \partial_\tau s(\tau) = \frac{\mu^2}{\sigma_0^2} \left[ \frac{\mu^2}{\lambda^2} s^2(\tau) - \varphi^2(\tau) \right] s(\tau), \quad \sigma_0^2 = 6M_r^2. \tag{1}$$

If $\varphi^2(\tau) \ll \mu^2 s^2(\tau)/\lambda^2$, the equation is approximated very well by

$$\partial_\tau^2 s_0(\tau) + \frac{3}{\tau} \partial_\tau s_0(\tau) = \frac{\mu^2}{\lambda^2 \sigma_0^2} s_0^3(\tau),$$

the general solution of which is

$$s_0(\tau) = \frac{s(0)}{1 - \tau^2/\tau_\infty^2}, \text{ where } s(0) \text{ is arbitrary and } \tau_\infty = \frac{s(0)\lambda^2 s_0}{\mu^2}. \tag{2}$$

This shows that, if $\varphi(0) = 0$, geometry expansion starts spontaneously and tends to
diverge at kinematic time $\tau = \tau_\infty$, which we call the dilation explosion time. This
certainly holds in the semi–classical approximation, if the initial values at $\tau = 0$ are
$\varphi = 0, \partial_\tau \varphi = 0$, as is evident considering that the motion equation for $\varphi$ is

$$\partial_\tau^2 \varphi(\tau) + \frac{3}{\tau} \partial_\tau \varphi(\tau) = \lambda^2 \left[ \frac{\mu^2}{\lambda^2} s^2(\tau) - \varphi^2(\tau) \right] \varphi(\tau). \tag{3}$$

Note that the left members of the equations for $s$ and $\varphi$ contain positive terms
proportional to the time derivative of the respective fields, acting as dissipative
terms which damp their time variations.

If the initial values of $\varphi$ and $\partial_\tau \varphi$ are very close to zero, the field amplitude takes
a long time to reach appreciable values, since $\varphi = 0$ is the stagnation point of the
potential energy density (Fig.1). If $\varphi$ and $\partial_\tau \varphi$ are not initially small and $\varphi < \mu/\lambda$, 
the field amplitude in any case becomes rapidly negligible, since the expansion factor term $3\tau^{-1}\partial_\tau \varphi$ acts as a strong damping agent. Numerical simulations showed that, even for moderately large initial values, $\varphi$ becomes first very small and then, at a critical time $\tau_c < \tau_\infty$, depending on the initial values, $\varphi$ and $\partial_\tau \varphi$ jump suddenly to

$$\varphi_c = \frac{\sqrt{2} \mu s(0)}{\lambda (1 - \tau^2/\tau_\infty^2)}, \quad (\partial_\tau \varphi)_c = \frac{\sqrt{8} \mu \tau_c}{\lambda \tau_\infty^2} \frac{s(0)}{(1 - \tau^2/\tau_\infty^2)^2},$$

which is precisely the value of $\varphi$ at which the potential energy is the same as for $\varphi = 0$ at $\tau = 0$. After the jump, $\varphi$ transits to an oscillatory regime of decreasing amplitude, while $s(\tau)$ tends to decelerate smoothly toward a sigmoidal profile.

![Fig.1. Potential energy profiles of Higgs boson field.](image)

**R**: On the Riemann manifold, the profile evolves in time with dilation field $\sigma$. At $\tau = 0$, the amplitude of scalar field $\varphi$ is very close to 0 and its energy is $U(0)$. At a critical time $\tau = \tau_c$, if the depth of the potential well is large enough, the amplitude of $\varphi$ jumps suddenly to $\varphi = \sqrt{2} \mu \sigma/\lambda \sigma_0$ with the same potential energy energy $U(0)$. It then moves back and forth with decreasing amplitude, while the potential energy is progressively transformed to kinetic energy. To facilitate the representation, the energy level of the system was shifted to the potential–energy minimum, consequently $U(\varphi)$ is represented as vanishing at $\varphi = \mu/\lambda$. **C**: Corresponding profile on the Cartan manifold. The profile shape remains unchanged and the field evolves toward its equilibrium value at $\tilde{\varphi} = \mu/\lambda$.

We can argue that, even if $\varphi$ and $\partial_\tau \varphi$ were infinitesimal at $\tau = 0$, the amplitude would jump to $\varphi_c$ as $\tau$ infinitely approaches $\tau_\infty$. In other terms, no matter how close to zero the initial state is, there is always a first instant $\tau_c + \varepsilon$, where $\varepsilon < \tau_\infty - \tau_c$, at which the jump suddenly occurs and the equality $\mu^2 s^2(\tau_c + \varepsilon) < \lambda^2 \varphi^2(\tau_c + \varepsilon)$ is
rapidly satisfied; afterwards, the curvature of the profile $s(\tau)$ changes sign.

Since the oscillation is extinguished rapidly, we can interpret this change as a transition from the false vacuum to the true vacuum, so that the former becomes the true physical state of the matter field. At the end of this process, after some kinematic time $\tau_q$, $\varphi(\tau)$ quite rapidly approaches its asymptotic value $\mu s(\tau)/\lambda$ and consequently the equation for the final portion $s_f(\tau)$ of $s(\tau)$ simplifies to

$$\partial^2_\tau s_f(\tau) + \frac{3}{\tau} \partial_\tau s_f(\tau) = 0,$$

the general solution of which, for $\tau_q < \tau < \infty$ with $s_f(\tau_q) = s_q$ and $s_f(\infty) = 1$, is

$$s_f(\tau) = s_q + \left(1 - s_q\right)\left(1 - \frac{\tau^2}{\tau_q^2}\right).$$

We see (Fig.2) that the scale factor profile is clamped between two curvilinear segments: at the initial end, by curve $s_0(\tau)$ from $\tau = 0$ to $\tau = \tau_c$, characterized by a positive curvature; at the asymptotic end by curve $s_f(\tau)$ from $\tau = \tau_q$ to $\tau \to \infty$, characterized by a negative curvature. The behavior of $s(\tau)$ in the joining region $\tau_c < \tau < \tau_q$ and the precise value of $s_q$, and consequently of $s(0)$, cannot be determined as easily, since it depends on the details of both $\varphi(\tau)$ and $s(\tau)$ dynamics.

It is thus clear that, in order to determine the exact profile of $s_0(\tau)$, we must know the solution of $s(\tau)$ in the joining region. Unfortunately, solving the twofold equation on the kinematic–time domain and on the Riemann manifold by standard numerical methods is complicated by time–grid problems arising from the fact that, in this representation, the VEV of $\varphi(\tau)$ depends strongly on the varying scale factor. Fortunately, however, it can be carried out more easily on the proper–time domain.
and Cartan manifold, where Eqs (1), (3) and kinematic time \( \tau \) are transformed into equations for \( \tilde{\phi}(\tilde{\tau}), \tilde{s}(\tilde{\tau}) \equiv s(\tau(\tilde{\tau})) \) and \( \tau(\tilde{\tau}) \) as functions of proper time \( \tilde{\tau} \), i.e., the equation system

\[
\begin{align*}
\partial_{\tilde{\tau}}^2 \tilde{s}(\tilde{\tau}) + \frac{1}{\tilde{s}(\tilde{\tau})} \left[ \partial_{\tilde{\tau}} \tilde{s}(\tilde{\tau}) \right]^2 + \frac{3}{\tilde{s}(\tilde{\tau})} \partial_{\tilde{\tau}} \tilde{s}(\tilde{\tau}) = \frac{2 \mu^2}{\sigma_0^2} \left[ \frac{\mu^2}{\lambda^2} - \tilde{\phi}^2(\tilde{\tau}) \right] \tilde{s}(\tilde{\tau}); \\
\partial_{\tilde{\tau}}^2 \tilde{\phi}(\tilde{\tau}) + 3 \left[ \frac{1}{\tau(\tilde{\tau})} + \frac{\partial_{\tilde{\tau}} \tilde{s}(\tilde{\tau})}{\tilde{s}(\tilde{\tau})} \right] \partial_{\tilde{\tau}} \tilde{\phi}(\tilde{\tau}) = \lambda^2 \left[ \frac{\mu^2}{\lambda^2} - \tilde{\phi}^2(\tilde{\tau}) \right] \tilde{\phi}(\tilde{\tau}); \\
\tau(\tilde{\tau}) = \int_0^{\tilde{\tau}} s(\tau') d\tau'.
\end{align*}
\]

For \( \tau < \tau_c \), we have \( s(\tau) = s_0(\tau) \), and therefore the third of Eqs.(4) can easily be obtained by integrating Eq.(2), which yields

\[
\tilde{\tau} = s(0) \frac{\tau_\infty}{2} \ln \frac{\tau_\infty + \tau}{\tau_\infty - \tau}; \quad \tau = \tau_\infty \tanh \frac{\tilde{\tau}}{s(0) \tau_\infty}; \quad \tilde{s}_0(\tilde{\tau}) = s(0) \cosh^2 \frac{\tilde{\tau}}{s(0) \tau_\infty}.
\]

Let us assume that the Higgs field on the Riemann manifold is initially in the fundamental state with values of \( \phi \) and \( \partial_\tau \phi \) very close to zero and all states of momentum \( k > 0 \) equal to 0. Thus, \( \phi \) depends only on \( \tau \) and remains in the fundamental state for \( \tau > 0 \).

The same occurs for \( \tilde{\phi} \) as a function of \( \tilde{\tau} \). The integration of Eqs.(4) was carried out by numerical methods in Matlab (The MathWorks, 2007) according to the Euler method with a time–grid of \( 10^6 \) points for non–realistic parameter values. Commented routines are freely available from the author upon request. The results are shown in Figs 3, 4, 5. These are illustrated in two different ways, in order to evidence the difference in Higgs–field oscillation profile on the Riemann and Cartan manifolds.
Fig. 4. An example of the time courses of scale factor $s(\tilde{\tau})$ (solid line, ordinates on right) and Higgs field amplitude on Riemann manifold $\varphi(\tilde{\tau})$ (gray area, ordinates on left) as functions of proper time, with unit of proper–time $\tilde{\tau}$ equal to critical time $\tilde{\tau}_c$. Gray area is actually a furiously oscillating profile with approximate frequency of 663 Hz in inverse critical–time units. These profiles were computed for the following values of parameters in Eqs. (4):

$$\mu = 1; \quad \lambda = 0.02; \quad \sigma_0 = 10^5; \quad \tau_c = 0.9 \tau_\infty.$$  

These profiles render the realistic ones very poorly, since the ratio $s(0)/\tilde{s}(\tilde{\tau}_\infty)$ $\sim 0.0158$ is enormously larger than the realistic one, by about 18 orders of magnitude, and the slope at $\tilde{\tau}_c = 1$ should be almost vertical. SFP = scale–factor profile; SFP$^0$ = scale-factor profile for $\varphi(\tilde{\tau}) = 0$.

On the Riemann manifold, Higgs–field oscillation is shown to adhere to scale expansion profile $SFP$, with amplitude $\varphi(\tau)$ oscillating up and down across its mean value $\mu \sigma(x)/\lambda \sigma_0$ (Fig. 4). The oscillation starts abruptly at critical time $\tau_c$, very close to dilation explosion time $\tau_\infty$, with $\varphi$ jumping from its minimum $\varphi(\tau_c = 0^–)$ of zero potential energy to its maximum $\varphi_m = \sqrt{2} \mu \sigma(\tau_c + 0^+)/\lambda \sigma_0$ of the same energy; the oscillation relative to mean value profile then decreases progressively with
potential–energy to kinetic–energy transfer. However, in order to avoid excessive profile deformations, all quantities are plotted as functions of proper–time $\tilde{\tau}$, with critical time $\tilde{\tau}_c$ as time unit, instead of kinematic time $\tau$, as would be more natural.

In the Cartan manifold representation, field amplitude $\tilde{\varphi}$ appears to oscillate up and down the fixed true VEV $\mu/\lambda$. In Fig.5, scale factor profile SFP as a function of proper time $\tilde{\tau}$ in critical time units $\tilde{\tau}_c$, as evolving in the Riemann manifold representation, is shown for comparison.
2.1 An almost exact determination of the scale–factor profile

Numerical simulations of $\varphi(\tilde{\tau})$ and $\tilde{\varphi}(\tilde{\tau})$ profiles showed that the proper–time interval of Higgs–field oscillation of appreciable amplitude, as a function of kinematic time $\tau$, shrinks more and more as $\tau_c$ moves closer and closer to $\tau_\infty$. Correspondingly, the direct–transition profile SFP_D shown in the Cartan manifold diagram (Fig.5) approaches closer and closer the true profile SFP. Since the order of magnitude of scale expansion ratio during inflation is estimated to be enormously large (e.g., about $10^{16}$ according to Brout et al. [3], so that $s(0) \approx 10^{-16}$), we conclude that the jump from geometry potential energy to Higgs–field potential energy and its subsequent relaxation to kinetic energy is a virtually instantaneous process - to be identified with the true BigBang - across which curves $s_0(\tau)$ and $s_f(\tau)$ join almost exactly and with continuity precisely at $\tau = \tau_c$ with $s_0(\tau_c) = s_f(\tau_c)$ and $s'_0(\tau_c) = s'_f(\tau_c)$ (where $s' \equiv \partial_\tau s$), from which, and using Eq.(2), we derive the equations

$$s(\tau) = \begin{cases} 
(1 - \frac{\tau_c^2}{\tau_\infty^2})^2 & \text{for } 0 < \tau < \tau_c; \\
1 - \frac{\tau_c^4}{\tau_\infty^2} & \text{for } \tau_c < \tau < \tau_\infty; 
\end{cases}$$

$$s(0) = s^2(\tau_c) = (1 - \frac{\tau_c^2}{\tau_\infty^2})^2; \quad s'(\tau_c) = 2 \frac{\tau_c}{\tau_\infty^2} \approx \frac{2}{\tau_\infty} = s(0) \mu^2 / \sqrt{2} \lambda \sigma_0. \quad (5)$$

Fig.6. Scale factor profile, as if time taken by false–vacuum to true–vacuum Higgs–field transition were relatively negligible (which happens if profile slope at critical time $\tau_c$ is very large). In this case, the whole profile is the join of an accelerated expansion, which precedes $\tau_c$, and a decelerated expansion, which follows $\tau_c$. A: profile as a function of kinematic time $\tau$. B: profile as a function of proper time $\tilde{\tau}$. Results are as in Brout et al. [3], but the interpretation is completely different.
Since in these conditions the critical value of kinematic time is virtually equal to the dilation–explosion time, we have

$$\tau_c \approx \tau_\infty = \frac{2\sqrt{2}\sigma_0}{s(0)\mu^2} = \frac{8\sqrt{3}M_{rP}}{s^2(\tau_c)\mu_H^2} \approx 2.6 \times 10^{17}$$

where we posed $M_{rP}/\mu_H = 1.87 \times 10^{16}$ with $\mu_H = \sqrt{2}\mu$ as true Higgs–boson mass, as explained at the end of Sec.(1.3). If we want to express $\tau_\infty$ in seconds, we must convert $\mu_H = 130$ GeV/c$^2$ to sec$^{-1}$, which is the approximate value of the Higgs–boson mass expected in the Standard Model. Using the conversion table

| Quantity | Value |
|----------|-------|
| 1 eV as mass: | $\times e^{-2}$ | $1.78 \times 10^{-36}$ kg |
| 1 eV$^{-1}$ as length: | $\times \hbar c$ | $1.97 \times 10^{-7}$ m |
| 1 eV$^{-1}$ as time: | $\times \hbar$ | $6.58 \times 10^{-16}$ sec |

we obtain

$$\mu_H = 1.3 \times 10^{11} \text{ eV} \approx 2.3 \times 10^{-25} \text{ kg}; \quad \tau_\infty \approx \frac{1.32 \times 10^{-10}}{s^2(\tilde{\tau}_c)} \text{ sec};$$

$$\tilde{\tau}_c \approx \frac{\tau_\infty}{2} s(\tau_c) \ln \frac{4}{s(\tau_c)} \approx 6.6 \times 10^{-11} \frac{4}{s(\tau_c)} \ln \frac{4}{s(\tau_c)} \text{ sec};$$

Assuming $\lambda^2 \approx 100^{-1}$, for the potential energy of the Higgs field at proper times $\tilde{\tau}_c$ and $\tilde{\tau} \gg \tilde{\tau}_c$, measured in kg/m$^3$, respectively, we find expressions

$$\tilde{U}(\tilde{\tau}_c) = \frac{\mu_H^2}{16\lambda^2} \approx \left(\frac{\mu_H \text{ as kg}}{\mu_H \text{ as m}^{-1}}\right)^3 = 4.15 \times 10^{30} \text{ kg/m}^3;$$

$$\tilde{U}(\tilde{\tau}) = \tilde{U}(\tilde{\tau}_c) \frac{\tilde{\tau}_c^3}{\tilde{\tau}^3} \approx \frac{16.4 \times 10^{-54}}{s^2(\tilde{\tau}_c)} \left[\ln \frac{4}{s(\tilde{\tau}_c)}\right]^3 \text{ kg/m}^3.$$

Assuming $\tilde{\tau} \approx 433 \times 10^{15}$ sec as the present proper–time age of the universe, for the present average energy density of the universe, we obtain the expression

$$\tilde{U}(\tilde{\tau}) \approx 14.6 \ln \frac{4}{s(\tilde{\tau}_c)^2} \times 10^{-54} \text{ kg/m}^4,$$

where $s(\tilde{\tau}_c)$ depends only on $s(0)$. Posing $\tilde{U}(\tilde{\tau}) \approx 9.9 \times 10^{-27}$ kg/m$^3$, in accordance with current cosmological estimates, we obtain the value $s(0)^{-1} = 2 \times 10^{14}$ for the expansion factor, hence $s(\tau_c) = s(\tilde{\tau}_c) = 10^{-7}/\sqrt{2}$. Note that, to obtain an expansion factor of the order of $10^{16}$, the true mass of the Higgs–boson field participating in the primordial inflation process should be $\mu_H \approx 130 \times 10/\sqrt{2} \approx 910$ GeV$^2/c^2.$
2.2 Higgs field multiplets

The conformal Lagrangian density of a single Higgs boson can be generalized to the case of a boson multiplet belonging to some symmetry group representation. Let us start with studying the simple case of a complex Higgs field on the Cartan manifold $\tilde{\varphi}(x) \equiv \tilde{\varphi}_R(x) + i \tilde{\varphi}_I(x)$, with global symmetry group $G = U(1)$. In this case, the Lagrangian density on the Cartan manifold has the form

$$\tilde{L}_{\tilde{\varphi}} = \frac{1}{2} (\tilde{D}_\mu \tilde{\varphi}^*) (\tilde{D}_\mu \tilde{\varphi}) - \frac{\lambda^2}{4} |\tilde{\varphi}|^2 - \frac{\tilde{R}}{12} |\tilde{\varphi}|^2,$$

(6)

where $\tilde{\varphi}^*$ is the complex conjugate of $\tilde{\varphi}$ and $|\tilde{\varphi}| \equiv \sqrt{\tilde{\varphi}_R^2 + \tilde{\varphi}_I^2}$ and $\tilde{R}$ the Ricci–Cartan scalar tensor. Posing $\tilde{\varphi}(x) = \tilde{\varphi}(x) e^{i \tilde{\theta}(x)}$, where $\tilde{\theta}(x)$ is a scalar field of dimension 0 representing the phase of $\tilde{\varphi}$, we can rewrite Eq.(6) as

$$\tilde{L}_{\tilde{\varphi}} = \frac{1}{2} \tilde{\varphi}^2 (\tilde{D}_\mu \tilde{\theta}) (\tilde{D}_\mu \tilde{\theta}) + \frac{1}{2} (\tilde{D}_\mu \tilde{\varphi}) (\tilde{D}_\mu \tilde{\varphi}) - \frac{\lambda^2}{4} \left( \tilde{\varphi}^2 - \frac{\mu^2}{\lambda^2} \right)^2 - \frac{\tilde{R}}{12} \tilde{\varphi}^2,$$

(7)

It is clear that, as $\tilde{\varphi}$ reaches its resting value $\mu/\lambda$, $\mu \tilde{\theta}/\lambda$ becomes a free field of mass zero and dimension -1, thus acquiring the properties of a Nambu–Goldstone (NG) boson capable of giving mass to a vector meson, via the Englert & Brout–Higgs–Kibble mechanism [21] [6] [20] or of breaking other symmetries [30].

Fig.7 shows the Mexican hat profile of 2D potential–energy density $\tilde{U}(\tilde{\varphi})$ as a function of complex variable $\tilde{\varphi}$. The circular contour at the base represents the set of points of minimum potential energy, conventionally assumed as zero. Clearly, field dynamics can be decomposed into a radial mode $\tilde{\varphi}(x)$ behaving as a unidimensional Higgs field of mass $\sqrt{2} \mu$, which oscillates around its VEV $\langle \Omega | \tilde{\varphi} | \Omega \rangle = \mu/\lambda$ at the bottom of the hat, and angular mode $\tilde{\theta}$, behaving as a massless field with kinetic energy density tending to become independent of $\tilde{\varphi}$, as $\tilde{\varphi} \to \mu/\lambda$. Fig.7. The Mexican hat: potential energy profile of a Higgs boson doublet on the Cartan manifold.
Passing to the Riemann manifold, $\varphi(x)$ is replaced by $\varphi(x) \equiv \varphi_R(x) + i \varphi_I(x) \equiv \varphi e^{i\theta(x)}$, with $\theta(x) = \tilde{\theta}(x)$, and Lagrangian density (7) takes the form
\[
L_{\varphi} = \frac{1}{2} \varphi^2 (D_{\mu} \theta)(D^\mu \theta) + \frac{1}{2} (D_{\mu} \varphi)(D^\mu \varphi) - \frac{\lambda^2}{4} \left( \varphi^2 - \frac{\mu^2}{\lambda^2} \frac{\sigma^2}{\sigma_0^2} \right)^2 - \frac{R}{12} \varphi^2.
\]
(8)

In this representation, the depth of the Mexican hat starts from the very small value $s^4(0) \mu^4/4\lambda^2 = \mu^4\sigma^4(0)/4\lambda^2\sigma_0^4$ and increases to $\mu^4/4\lambda^2$ at kinematic time $\tau = \infty$, i.e., by about $10^{60} - 10^{64}$ times, according to subsec.(2.1). However, since in the earliest expansion stage spacetime is conformally flat, we have $R = 0$ and, as discussed at the end of Part II, subsec.5.1, $\varphi$ and $\sigma$ also depend only on $\tau$. Hence, Eq.(8) simplifies to
\[
L_{\varphi} = \frac{1}{2} \varphi^2 (\partial_\tau \varphi)^2 + \frac{1}{2} (\partial_\tau \varphi)^2 - \frac{\lambda^2}{4} \left( \varphi^2 - \frac{\mu^2}{\lambda^2} \frac{\sigma^2}{\sigma_0^2} \right)^2.
\]
(9)

Let us assume that the Higgs field doublet interacts with fermion fields through conformal invariant Yukawa couplings of the form $\varphi F(\theta)$, where $F(\theta)$ is in general a sum of fermion bilinears with coefficients depending on $\theta$. Thus the action integral has the form
\[
A_{\varphi} = \int_0^\infty d\tau \int_{V_1} \tau^3 [L_{\varphi}(\tau, \hat{x}) + \varphi F(\theta)]
\]
from which we obtain the motion equation for $\varphi$
\[
\partial_\tau^2 \varphi + \frac{3}{\tau} \partial_\tau \varphi - (\partial_\tau \varphi)^2 + \lambda^2 \left( \varphi^2 - \frac{\mu^2}{\lambda^2} \frac{\sigma^2}{\sigma_0^2} \right) \varphi = F(\theta),
\]
(10)

which replaces the first of Eqs. (45) of Part II, subsec.5.1, and the motion equation for $\theta$
\[
\tau^{-3} \partial_\tau [\tau^3 \varphi^2 \partial_\tau \varphi] = \varphi^2 \left( \partial_\tau^2 \theta + \left( \frac{3}{\tau} + \partial_\tau \ln \varphi \right) \partial_\tau \theta \right) = \varphi \frac{\delta F(\theta)}{\delta \theta}.
\]
(11)

The generalization to a Higgs field multiplet belonging to the fundamental representation of $SU(N)$ is straightforward. Let
\[
\varphi = \{\varphi_1, \varphi_2, \ldots, \varphi_N\} = \varphi(x) \{c_1 e^{i\theta_1(x)}, c_2 e^{i\theta_2(x)}, \ldots, c_N e^{i\theta_N(x)}\},
\]
(12)

where $c_i \geq 0$ are real constants satisfying the equations $\sum_i c_i^2 = 1$ and $\varphi = \sqrt{\varphi^\dagger \varphi}$, is an $N$–plet of complex Higgs fields and $\varphi^\dagger$ the hermitian conjugate $N$–plet. Thus
\[
U(\varphi) = \frac{\lambda^2}{4} \left( \varphi^2 - \frac{\mu^2}{\lambda^2} \frac{\sigma^2}{\sigma_0^2} \right)^2.
\]
still represents the $SU(N)$–invariant potential energy density of $\varphi$ on the Riemann manifold. Assume that $\varphi$ interacts with a set of fermion fields through a term $\varphi F(\theta_i)$ formed of conformal invariant Yukawa couplings and mass–like terms. The complete conformally flat Lagrangian density of the Higgs field multiplet then is

$$L_\varphi = \frac{1}{2} \varphi^2 \sum_i c_i^2 (\partial_\tau \theta_i)^2 + \frac{1}{2} (\partial_\tau \varphi)^2 - \frac{\lambda^2}{4} \left( \varphi^2 - \frac{\mu^2}{\lambda^2} \sigma^2 \right)^2 + \varphi F(\theta_i).$$  \hspace{1cm} (13)

The vector of components $c_i$ represents the direction of spontaneous breakdown of $\varphi$ in the unitary space of its representation.

Generalizing the results obtained for the Lagrangian density of Eq.(9), we obtain the motion equation for $\varphi$

$$\frac{d^2 \varphi}{d\tau^2} + \frac{3}{\tau} \frac{d\varphi}{d\tau} - \sum_i c_i^2 (\partial_\tau \theta_i)^2 \varphi + \lambda^2 \left( \varphi^2 - \frac{\mu^2}{\lambda^2} \sigma^2 \right) \varphi - F(\theta_i) = 0,$$

and those for $\theta_i$

$$\tau^{-3} \partial_\tau \left[ \tau^3 \varphi^2 \partial_\tau \theta_i \right] = \varphi^2 \left[ \frac{d^2 \theta_i}{d\tau^2} + \left( \frac{3}{\tau} + \partial_\tau \ln \varphi \right) \partial_\tau \theta_i \right] = \varphi \frac{\delta F(\theta)}{\delta \theta_i}.$$  \hspace{1cm} (15)

### 2.3 Global and local spontaneous breaking of internal symmetries

The results given in the previous subsection deserve comment. At variance with the usual view of the spontaneous breakdown of internal symmetries, in which the vacuum expectation value of Higgs field amplitude $\varphi$ is a positive constant, in our view it is a dynamic quantity depending on kinematic time $\tau$, which approaches a constant value at large $\tau$. In addition, phases $\theta_i$ of Higgs field components $\varphi_i$ do not describe NG–boson fields, in the usual sense of the term, since they do not possess zero–mass propagators. Nevertheless, their energy spectra are gapless - were not so, the spontaneous breaking of $SU(N)$ symmetry could not take place. In other words, the symmetry breaks down globally (see Sec.2 of Part II).

This sort of symmetry breaking makes sense, provided that all physical quantities are homogeneous and isotropic. This is ensured by the strong viscous drag and dilation–field repulsion to which matter is subjected during the acute stage of inflation, as explained in Sec.4 of Part I. In these conditions, all currents depend
only on \( \tau \) and therefore zero–mass vector fields cannot be created. This regime of
global evolution of matter ceases after the acute expansion stage and the spontane-
ous breakdown of symmetries can occur locally, as expected, provided that zero–
mass gauge bosons are produced and a second Higgs field equipped with zero–mass
NG–bosons comes into play, giving mass to gauge bosons.

Thus, the inflation process develops in two main stages, the first characterized
by global–symmetry breaking and the second by local–symmetry breaking. During
the first stage, Higgs fields of type 1 and mass \( \mu \), interact with a set of fermion fields
through pure Yukawa couplings, so as to promote baryogenesis. During the second
stage, Higgs fields of type 2 and mass \( \mu' \), interact with gauge fields through gauge–
invariant couplings and, possibly with fermion fields through additional Yukawa
couplings, so as to produce the existing matter structure. Since the latter process
tends to destroy the global character of the former, we are led to assume that the
amplitude of Higgs fields of type 2 jumps to its maximum at a kinematic time
\( \tau_{c'} > \tau_c \), which is possible provided that \( \mu' < \mu \). Since the mass estimated for Higgs
fields of type 2 is about 130 GeV, we are led to assume \( \mu' > 130 \) GeV, which may
appreciably modify the profile of the scale factor and the conclusions of subsec.(2.1).

Due to Fermi–Dirac statistics, fermions cannot be at rest and are therefore parti-
tioned in energy levels of different momentum up to Fermi energy \( E_F \), which depends
on temperature and on the size of the cell - or composed object - in which they are
presumed to be confined. However, in agreement with the conclusions of subsecs.
4.3, 4.4 of Part I, we assume that, during the inflation stage, fermions of different
species are completely delocalized and dispersed within the infinite volume of space–
like hyperboloids, so that only their density is well–defined. In other words, they
form a homogeneous, isotropic and cold superconductive fluid with zero charge den-
sity and zero momentum density, which is at rest in the comoving–observer frame.

In the following, we describe of global symmetry breaking with special regard to
the problem of baryogenesis. We totally neglect the problem of the second local–
symmetry breakdown, for instance of \( SU(5) \) or \( SO(10) \) to \( SU(3)_C \times U(1)_{e.m.} \) [13]
[24], within the framework of CGR.
3 A possible mechanism for baryogenesis

The extraordinary results achieved by fundamental particle physics in the last half-century indicate very clearly that the winning strategy for theoretical invention, discovery and prediction was essentially respect to the conditions of quantum–field renormalizability and minimal non–trivial complexity of matter structure.

During the 1970s, the condition of spontaneous breakdown of all symmetries paired to triangle–anomaly cancellation, which is necessary for renormalizability, led theorists to select local symmetry group $G_{SM} = SU(3) \times SU(2) \times U(1)$, from a potentially infinite number of mathematical possibilities, as the algebraic structure of a theory, called the Standard Model (SM), capable of describing a minimal set of fundamental particles. This condition is fulfilled, provided that left–handed fermions belong to a reducible representation of $G_{SM}$ with five irreducible components $(u^i_L, d^i_L), (\bar{u}^i_L), (\bar{d}^i_L), (\nu, e^{-}_L), (e^{+}_L)$ - summing up to a total of 15 fields - and the right–handed ones to the complex–conjugate representations [12]. With this choice, generators $\tau_i$ of either representation satisfy the conditions $\text{Tr}(\tau_i \tau_j \tau_k) = 0$ for arbitrary $i, j, k$, as a consequence of which all triangle anomalies cancel, together with $\text{Tr}(\tau_i) = 0$, which is the condition in which the total electric charge of each multiplet is zero [10]. With these assignments, the set of all possible fermions is partitioned into families, or generations, isomorphic to the 15 + 15 fields indicated above.

Now, the requirement of gravitational field renormalizability led us to extend General Relativity (GR) to Conformal General Relativity (CGR), to the point at which we could predict four spacetime dimensions, spacetime geometry inflation accompanied by matter generation, Higgs field dynamics, the original homogeneity and isotropy of the universe, etc., as described so far in the present work.

However, the renormalizability criterion alone does not suffice to provide us with a realistic representation of the matter existing in our universe. The missing point is that the universe must contain stable aggregations of heavy particles, so as to permit the development of thermodynamic processes of indescribable complexity. We are clearly referring here to a mechanism capable of producing matter/anti-matter asymmetry, i.e., the mechanism for baryogenesis, or leptogenesis production.
3.1 Sakharov conditions for baryogenesis in the framework of CGR

Let us introduce the four general conditions for the occurrence of matter/anti–matter asymmetry, starting from an initial state of matter/anti–matter symmetry, as stated by Sakharov in 1967 [37]. A fifth condition is added for consistency with CGR.

1) **Violation of global baryon number** $B$. This is necessary, since we presume that the average baryon number is zero in origin.

2) **Violation of charge conjugation symmetry** $C$. This is necessary so that the interactions which produce more baryons than anti–baryons are not counter-balanced by interactions which produce more anti–baryons than baryons.

3) **Violation of PC symmetry** (parity–reversal $\times$ charge–conjugation). This is required, since otherwise equal numbers of left–handed baryons and right–handed anti–baryons, as well as left–handed anti–baryons and right–handed baryons, would be produced.

4) **Thermodynamic evolution of the universe from a temporary high–temperature state to a permanent low–temperature state**. Since $PCT$ is a discrete symmetry of the Hamiltonian and since $(PCT)B = -B(PCT)$, the thermal equilibrium value of $B$ would obey equation $\text{Tr}[Be^{-\beta H}] = \text{Tr}[(PCT)B e^{-\beta H}(PCT)] = \text{Tr}[(PCT)B(PCT)e^{-\beta H}] = -\text{Tr}[Be^{-\beta H}] = 0$. Therefore, in a persistent state of thermal equilibrium, matter/anti–matter asymmetry would disappear sooner or later, depending on temperature. This condition ultimately states that the reaction which generated baryon–antibaryon asymmetry occurred before the end of universe expansion and survived the thermodynamic stage of universe overheating.

5) **Spontaneous breakdown of matter/anti–matter symmetry immediately after the BigBang event**. Because of the very strong repulsive interaction among all massive particles during the rapid stage of inflationary expansion, the matter field generated by the BigBang persisted in a frozen state. During the subsequent decelerating stage, the temperature peaked, due to fermion–antifermion anni-
hilation, and then decreased because of free post–inflationary expansion. As explained in point 4), when this happens, the baryon charge of the universe can only decrease. We infer that the breakdown of matter/anti–matter symmetry to an excess of baryons and leptons occurred in coincidence with the BigBang event, which in our view was characterized by the sudden jump of the Higgs field amplitude to a maximum, followed by a violent oscillation of rapidly decreasing amplitude. Since in these conditions matter distribution on synchronous–observers’ frames was homogeneous and isotropic, the spontaneous breakdown of the symmetry may have been purely global.

All these conditions point to the question: what was the primary agent of matter/anti–matter symmetry breakdown? The answer is of course: one or more Nambu–Goldstone boson fields which, in order to satisfy Sakharov conditions, must bear baryon number, as well as electric charge, and interact with quarks and leptons through $PC$–violating coupling, either directly or indirectly, through the intermediation of a vector or pseudo–vector field. Clearly, this agent can only be one or more scalar or pseudo–scalar fields, otherwise Lorentz rotation invariance would be destroyed.

In any case, the agent can act only by mixing quarks and leptons. Also, although violating $C$, it must preserve the difference $D = B - L$ between global baryon number $B$ and global lepton number $L$, since otherwise the total electric charge of the universe would not be zero, in contrast with cosmological evidence and reasonable beliefs.

Remember that $B$ is defined as 1 for neutrons and protons and -1 for their anti–particles - consequently, 1/3 for quarks and -1/3 for anti–quarks - while $L$ is defined as 1 for electrons and neutrinos and -1 for their anti–particles.

A diverting aspect of this conclusion is that $PC$ violation requires quarks and leptons to be partitioned into $n \geq 3$ families, whose states are the direct sum of $n$ quark–lepton states linearly combined by an $n \times n$ unitary matrix of the Cabibbo–Kobayashi–Maskawa (CKM) type [23, 30]. No use to say how interesting is the fact that the number of known fermion families is precisely 3.
3.2 Her Majesty $SU(5)$

Before commencing our investigation of the possible mechanisms of baryogenesis, let us briefly digress to $SU(5)$ symmetry and $B - L$ number conservation.

As is well known, the Standard Model (SM) represents quarks and leptons as absolutely independent entities, and therefore nothing may be inferred about possible differences in their electrical properties. In particular, the reason why quarks have fractional charges of $\pm 1/3$ and $\pm 2/3$ and are ranged in three different colors was a mystery. This state of affairs was so distasteful to the masters of SM that the question of whether $G_{SM}$ could be extended to a wider, perhaps more predictive, symmetry group was soon raised [12]. However, the search for a Grand Unified Theory (GUT) was not only motivated by aesthetic reasons, but also by other pending problems, including that of baryogenesis.

The main guidelines for the search of such extensions were: (1) the extended symmetry group must be simple, or the direct product of isomorphic simple groups, because otherwise fundamental particles would still be partitioned into independent sets; (2) the fermion representations of the extended group must incorporate and possibly coincide with the five left–handed representations of $G_{SM}$ and their right–handed counterparts; (3) possible triangle–anomalies introduced by new gauge fields must cancel out. Of course, these assumptions imply that quarks may be transformed into leptons and vice versa, and that scalar and/or vector bosons coupled with quark anti–lepton or anti–quark lepton pairs may exist.

If we assume that neutrinos are massless, the first choice falls immediately on group $SU(5)$, the lower representations of which are 5, $\bar{5}$, 10 and $\bar{10}$, exactly fitting the $G_{SM}$ representations. This makes $SU(5)$ a privileged candidate, since no more fermions are predicted besides those already known. Quite spectacularly, group representations provide the colored triplets of quarks and anti–quarks with the expected electric charges. As a bonus, the current algebra deriving from this group contains a novel conservative current $J^D_\mu(x)$, whose global charge is precisely $D = B - L$. In view of the possible role of this property of $SU(5)$ in explaining matter/anti–matter asymmetry, this too makes this group a privileged extension of $G_{SM}$.
The critical point is that $J^D_\mu$ cannot be the source of a gauge field, for two main reasons. First, because there is no experimental evidence of zero–mass vector bosons other than photons; second, because such a gauge field would introduce an unwanted triangle anomaly into the theory, since trace $\text{Tr}(\tau^3_D)$, where $\tau_D$ is the Lie–algebra element associated with $J^D_\mu$, does not vanish \[12\]. By contrast, the equalities $\text{Tr}(\tau_D \tau_i \tau_j) = 0$ do hold for all generators $\tau_i$ of the Lie–algebra of $G_{SM}$ subgroup representations, which means that no extra anomalies are introduced, provided that the symmetry carried by $J^D_\mu$ is global.

Clearly, we are faced with a serious dilemma: we can either abandon the idea that $SU(5)$ is the right extension, or inquire whether locality is really necessary in the context of CGR and try to implement the baryogenesis mechanism by the spontaneous breakdown of a global symmetry.

In the context of CGR, the second horn of the dilemma seems to be the more promising. In Part I, we proved that, during the entire period of rapid universe expansion, spacetime keeps conformally flat and matter is uniformly and isotropically distributed over the 3D space–like surfaces of synchronized observers. In these conditions, all intensive quantities described on the Riemann manifold depend only on kinematic time $\tau$. Since all conservative charges are uniformly distributed, all vector currents, as well as their possible zero–mass gauge fields, vanish. Consequently, the symmetry associated with the conservation of current $J^D_\mu(x)$ may be regarded as perfectly global.

Having thus clarified the reason for preferring $SU(5)$ to other symmetries, in particular $SO(10)$ \[25\], let us consider what sort of Higgs fields $h_i$ may best serve the purpose of providing fermion mass and symmetry breakdown. Of course, these fields must belong to a representation of $SU(5)$ capable of forming $SU(5)$–invariant Yukawa couplings of the type $h^i_\dagger \bar{\psi}_R \Gamma^i_A \psi_L$ and $h_i \bar{\psi}_L \Gamma^i_B \psi_R$, where $\Gamma^i_A$ and $\Gamma^i_B$ are squared matrices provided with indices running on group–representation and fermion–family indices. Since both $\bar{\psi}_R$ and $\psi_L$ belong to a $\overline{5} \oplus 10$ representation and both $\bar{\psi}_L$ and $\psi_R$ to a $5 \oplus \overline{10}$ representation, $h^i_\dagger$ must belong to an irreducible component of representation $(\overline{5} \oplus 10) \otimes (5 \oplus 10)$ and $h_i$ to its h.c. counterpart. Using Young
tableaux \cite{36} \cite{24}, we find that decompositions into irreducible components are of the type $5 \otimes 5 = 10 \oplus 15$, $5 \otimes 10 = 5 \oplus 45$ and $10 \otimes 10 = 5 \oplus 45 \oplus 50$. Note that only multiplets 5 and 45 can provide the Lagrangian density with fermion–mass terms, since they occur in complex–conjugate pairs \cite{25}. These multiplets are respectively denoted as $\mathcal{H}_5 = \{ \mathcal{H}_a \}$ and $\mathcal{H}_{45} = \{ \mathcal{H}_{ab} \}$, with $\mathcal{H}_{ab}^a = - \mathcal{H}_{ab}^b$, $\sum_a \mathcal{H}_{ab}^a = 0$.

**NOTE:** In dealing with bilinears of the form $\bar{\psi}_a \psi_b$, where $a, b$ may comprise group–representation and fermion–family indices, it is convenient to separate a Dirac field $\psi$ into the left–handed component $\psi_L = \frac{1}{2} (1 + \gamma^5) \psi$ and its right–handed component $\psi_R = \frac{1}{2} (1 - \gamma^5) \psi$. In this representation, we have

$$\bar{\psi}_a \gamma^\mu \psi_b = \bar{\psi}_{Ra} \gamma^\mu \psi_{Lb} + \bar{\psi}_{La} \gamma^\mu \psi_{Rb}; \quad \bar{\psi}_a \psi_b = \bar{\psi}_{Ra} \psi_{Lb} + \bar{\psi}_{La} \psi_{Rb}; \quad (16)$$

$$\bar{\psi}_a \gamma^5 \gamma^\mu \psi_b = \bar{\psi}_{La} \gamma^\mu \psi_{Lb} - \bar{\psi}_{Ra} \gamma^\mu \psi_{Rb}; \quad \bar{\psi}_a \gamma^5 \psi_b = \bar{\psi}_{Ra} \psi_{Lb} - \bar{\psi}_{La} \psi_{Rb}. \quad (17)$$

Parity reversal $P$ and charge conjugation $C$ act on $\psi_{L,R}$ respectively as

$$\psi_{L,R}(x^0, \bar{x}) \overset{P}{\to} \gamma^0 \psi_{R,L}(x^0, -\bar{x}); \quad \psi_{L,R} \overset{C}{\to} C \psi_{R,L}; \quad \bar{\psi}_{L,R} \overset{C}{\to} \bar{\psi}_{R,L} C^{-1};$$

where $C$ is the Dirac matrix defined by $C^{-1} \gamma^\mu C = -(\gamma^\mu)^T \cite{24}$. In our representation of Dirac matrices we have $C = -C^\dagger = -C^{-1} = -C^T = i \gamma^2 \gamma^0$ [also see Part II, subsec.(1.4)]. According to these relations, we derive the identities

$$\psi_{R,L} = C \psi_{L,R}^T; \quad \bar{\psi}_{R,L} = -\psi_{L,R}^T C^{-1} = \psi_{L,R}^T C; \quad \psi_{L,R}^C = C \psi_{L,R}^T; \quad \bar{\psi}_{L,R}^C = \psi_{R,L}^T C; \quad (18)$$

showing that $\psi_R$ and $\psi_L$ may be equivalently expressed as functions of $\psi_L^C$ and $\psi_R^C$.

In particular, for vector and scalar bilinears of anti–commuting fields $\psi$ and $\chi$, we have the identities

$$\bar{\psi}_R \gamma^\mu \chi_R = -\chi_L \gamma^\mu \psi_L^C; \quad \bar{\psi}_L \gamma^\mu \chi_L = -\chi_R \gamma^\mu \psi_R^C; \quad \bar{\psi}_{R \chi L} = \chi_R \psi_L^C; \quad \bar{\psi}_{R \chi L} = \chi_R \psi_L^C; \quad (19)$$

where transposition combined with anti–commutation relations were used. Charge conjugation and hermitian conjugation act on the same bilinears as follows

$$\bar{\psi}_R \gamma^\mu \chi_R \overset{C}{\to} -\bar{\chi}_L \gamma^\mu \psi_R^C; \quad \bar{\psi}_{R \chi L} \overset{C}{\to} \bar{\chi}_R \psi_L \quad \text{and same with } R \leftrightarrow L; \quad (20)$$

$$\bar{\psi}_R \gamma^\mu \chi_R \overset{h.c.}{\to} \bar{\chi}_L \gamma^\mu \psi_L^C; \quad \bar{\psi}_{R \chi L} \overset{h.c.}{\to} \bar{\chi}_R \psi_R \quad \text{and same with } R \leftrightarrow L. \quad (21)$$

In Eqs. (20), transposition and anti–commutation relations were again used.
3.3 SU(5) Yukawa couplings and mass terms

Using conventions and notations established in the note of the previous subsection, let us indicate left–handed leptons and quarks simply as \( e, \nu, d_i, u_i \), where \( i \) is a 3–component color index, rather than as \( e_L, \nu_L, d_{Li}, u_{Li} \), and their (right–handed) anti–particles as \( e^c, \nu^c, d^c_i, u^c_i \), rather than as \( e^c_L, \nu^c_L, d^c_{Li}, u^c_{Li} \). All of them may have implicit 3–component family indices. It is known \(^{24} \) \(^{36} \) that Yukawa terms of \( SU(5) \) fermion representation must have the general form

\[
\psi^T_L C \chi^L_e \; + \; h.c. = \chi^L_T C \psi_L + h.c. = (\psi^c_R)_a \chi^L_e + h.c.; \tag{22}
\]

\[
\chi^L_T C \chi^L_d + h.c. = \chi^L_T C \chi^L_L + h.c. = (\chi^c_R)^{ab} \chi^L_d + h.c.. \tag{23}
\]

We take care not to confuse \( c \) as charge–conjugation with \( c \) as a superscript, where

\[
\psi_L = \begin{bmatrix} d^c_1 \\ d^c_2 \\ d^c_3 \\ e \\ -\nu \end{bmatrix}_L; \quad \chi_L = \begin{bmatrix} 0 & u^c_3 & -u^c_2 & -u_1 & -d_1 \\ -u^c_3 & 0 & u^c_1 & -u_2 & -d_2 \\ u^c_2 & -u^c_1 & 0 & -u_3 & -d_3 \\ u_1 & u_2 & u_3 & 0 & -e^c \\ d_1 & d_2 & d_3 & e^c & 0 \end{bmatrix}_L. \tag{24}
\]

the signs being chosen in order to provide correct mass terms \(^{24} \).

For a single family of fermions, the possible Yukawa couplings with Higgs fields of types \( H_5 \) and \( H_{45} \) are \( L_5 = g_d Y_d + g_u Y_u \) and \( L_{45} = g^\prime_d Y^\prime_d + g^\prime_u Y^\prime_u \), where

\[
Y_d = \psi^T_L C \chi^L_e H^1_b + h.c., \quad Y_u = \frac{1}{4} \epsilon_{abcde} \chi^T_L C \chi^L_d H^e + h.c., \tag{25}
\]

\[
Y^\prime_d = \psi^T_L C \chi^L_d (H^d_e)^{ab} + h.c., \quad Y^\prime_u = \frac{1}{4} \epsilon_{abcde} \chi^T_L C \chi^L_d H^d_e + h.c. \tag{26}
\]

where \( g_d \), \( g_u \), \( g^\prime_d \) and \( g^\prime_u \) are positive constants. However, for the sake of simplicity, only minimal coupling \( L_5 \) described by Eq.(25) is considered.

Let us simplify the algebra by introducing the shorthand notation

\[
\psi_a \equiv \psi_{La}; \quad \hat{\psi}_a \equiv \psi^T_{La} C; \quad \hat{\psi}^c \equiv \bar{\psi}_a; \quad \chi^{ab} \equiv \chi^L_e^{ab}; \quad \hat{\chi}^{ab} \equiv \chi^T_L C; \quad \hat{\chi}^{cab} = \hat{\chi}^{ab}_R. \tag{27}
\]

Combining these with Eqs.(18) (19), we find the following identities and relations

\[
\hat{\psi}^c \chi = \bar{\psi}_{RXL} \frac{h.c.}{h.c.} \bar{\chi}_L \psi_R; \quad \hat{\psi} \chi = \bar{\psi}_{RXL} \frac{h.c.}{h.c.} \bar{\chi}_L \psi_R; \quad \hat{\psi}^c \chi = \bar{\psi}_{RXL} \frac{h.c.}{h.c.} \bar{\chi}_L \psi_R. \tag{28}
\]

\[
\hat{\psi} \chi = \bar{\psi}_{RXL} \frac{h.c.}{h.c.} \bar{\chi}_L \psi_R; \quad \hat{\psi} \chi = \bar{\psi}_{RXL} \frac{h.c.}{h.c.} \bar{\chi}_L \psi_R. \tag{29}
\]
We can then rewrite Eqs. (22), (23) and (25) as follows:

$$\tilde{\psi}_a \chi^{bc} + h.c. = \chi^{bc} \tilde{\psi}_a + h.c. = (\bar{\psi}^*)_a \chi_L^{bc} + h.c.,$$

$$\chi^{ab} \chi^{cd} + h.c. = \chi^{cd} \chi^{ab} + h.c. = (\bar{\chi}^*)^{ab} \chi_L^{cd} + h.c.,$$

$$Y_d = \tilde{\psi}_a \chi^{ab} H_1^b + h.c., \quad Y_u = \frac{1}{4} \epsilon_{abcd} \chi^{ab} \chi^{cd} H^e + h.c. \quad (32)$$

Hence, with Eqs. (24) and (28), we obtain

$$\begin{align*}
Y_d &= H_i^1(\hat{e} \hat{d} - \hat{d} \hat{e} - \epsilon^{ijk} \hat{d}_j \hat{u}_k) - H_i^4(\hat{e} e + \hat{d} e) - H_i^5(\hat{d} d + \hat{e} e) + h.c., \quad (33) \\
Y_u &= -H_i(\hat{e} \hat{c} + \hat{e} c + \epsilon^{ijk} \hat{u}_j \hat{d}_k) + H_i^4(\hat{e} e + \hat{d} e) - H_i^5(\hat{d} d + \hat{e} e) + h.c. = -2H_i(\hat{e} \hat{c} e - \epsilon^{ijk} \hat{u}_j \hat{d}_k) + 2H_i^4 \hat{e} e - 2H_i^5 \hat{d} d + h.c., \quad (34)
\end{align*}$$

where summation over color indices is understood wherever they are omitted. It is evident that all $H_i^1$ carry electrical charges $\{1/3, 1/3, 1/3, 1/3, 1/3\}$ and all $H_i$ the opposite values. In Eq. (34) some of identities (28) were used.

To implement fermion interaction definitely with a Higgs field multiplet of the form $H_i = \varphi \ c_i \ e^{i \theta_i}$, as described in subsec.(2.2), we follow the criterion of maximal simplicity by assuming the direction of spontaneous breaking as follows:

$$H_i = \varphi \ c_i \ e^{i \theta_i}; \quad \text{for} \ i = 1, 2, 3; \quad H_4 = 0; \quad H_5 = \varphi \ c_m \ e^{i \theta_5}, \quad (35)$$

where $c_m = \sqrt{1 - 3 \ c_0^2} < 1$ and $\theta_5(x) = 0$, since this is a possible solution of Eq. (15) [subsec.(2.2)] when $\theta_5$ appears only in the kinetic term of the Higgs field Lagrangian density. As will be shown later, these assumptions ensure the conservation of the $B - L$ number. Lastly, using the second of Eqs. (29), resuming standard notation and carrying out appropriate substitutions and term rearrangements, we obtain the interaction Lagrangian on the Riemann manifold as $L = L_m + L_\theta$, where

$$\begin{align*}
L_m &= -\varphi \ c_m [g_d(\bar{e}e + \bar{d}d) + 2g_a \bar{u}u] ; \quad (36) \\
L_\theta &= \varphi \ c_0 \sum_i e^{i \theta_i} [g_d(\bar{u}_L^i c_R^i - \bar{d}_L^i \nu_R^i) - 2g_a \bar{u}_{R}^i c_L^i + \epsilon^{ijk}(g_d \bar{u}_{R}^i d_{Rk} - 2g_a \bar{u}_{R}^i d_{Lk})] + h.c. \quad (37)
\end{align*}$$

Maximal symmetry is clearly achieved for $g_u = g_d/2$. Mass term $L_m$ preserves all fermion numbers and is separately invariant under $C$ and $P$. By contrast, $L_\theta$ violates
the baryon number, since it mixes quarks with leptons, and also violates \( C \) since, by interchanging \( i \) with \(-i\) and using Eqs. (20) and (19), we obtain the same expression with interchanged \( L \) and \( R \). Hence, \( L_\theta \) is invariant under \( CP \).

Thus, of the Sakharov conditions listed in subsec.(3.1), the first two are fulfilled but the third is not. However, as we show in the following subsections, by extending the Yukawa couplings to three or more families of fermions with different mass, the third condition can also be fulfilled. It is therefore of paramount importance for the number of known fermion families to be precisely three.

3.4 How \( SU(5) \) symmetry can be broken down globally

Since \( H_i = \varphi c_\theta e^{i\theta}, u^i, d^i, \bar{u}^i, \bar{d}^i, \epsilon^{ijk} \bar{u}^k_i d_j \) and \( \epsilon^{ijk} \bar{d}^j_i u_k \) all belong to representation 3 of \( SU(3)_C \), whereas their hermitian conjugate counterparts belong to representation \( \bar{3} \), both \( L_m \) and \( L_\theta \), as well as all kinetic Lagrangian–density terms, are manifestly invariant under this group. Lagrangian density is also manifestly invariant under global gauge group \( U(1)_{\text{e.m.}} \), since all its terms carry zero electric charge.

Now note that the kinetic terms of total Lagrangian density and Yukawa couplings (25) and (26) are invariant under global symmetry group \( U_G(1) \) described by the transformations

\[
\psi_{L,R} \to e^{-i\lambda} \psi_{L,R}; \quad \chi_{L,R} \to e^{i\lambda} \chi_{L,R}; \quad H \to e^{-i2\lambda} H; \quad \text{with } H \in \mathcal{H}_5, \mathcal{H}_{45}.
\]

Unfortunately, however, due to the symmetry breaking stated by Eqs. (35), the Lagrangian density term (37) is not invariant under this group.

Both this symmetry and gauged symmetry \( U_Y(1) \), where \( Y \), defined as \( Y = Q - T_3 \) with \( T_3 \) the third isospin component, indicates the neutral electro–weak charge, is spontaneously broken by Higgs fields of the type introduced in Eqs. (37), (25). Since generators \( G \) of \( U_G(1) \) and \( Y \) of \( U_Y(1) \) commute, they can be linearly combined. With Higgs multiplet \( \mathcal{H}_5 \), they reed respectively \( G = \text{diag}( -2, -2, -2, -2, -2) \) and \( Y = \text{diag}(1/3, 1/3, 1/3, -1/2, -1/2) \), and therefore the linear combination \( D = (4Y - G)/5 \) reeds \( D = \text{diag}(2/3, 2/3, 2/3, 0, 0) \).

This means that all mass terms in Eqs. (42) are invariant under transformations carried out by unitary operators \( e^{icD} \in U_D(1) \), where \( c \) is a real parameter. Readers
can verify that the same property also holds with $H_{45}$ in place of $H_5$. As an exercise, we leave them to prove that $D$ reads $1/3$ on quarks, $-1/3$ on anti-quarks, $-1$ on leptons and $1$ on anti-leptons, leading us to the identification $D = B - L$, as anticipated in Section (3) and subsection (3.2), and check the self-consistency of the following table.

| Class $\rightarrow$ | $\psi_L$ | $\chi_L$ | $H$ |
|---------------------|----------|----------|-----|
| Charge ↓            | $d_{L_{i}}^i$ | $\nu_{L_{i}}$ | $e_{L_{i}}$ | $u_{L_{i}}$ | $d_{L_{i}}^c$ | $u_{L_{i}}^c$ | $H_{1,2,3}$ | $H_4$ | $H_5$ |
| $Y$                 | 1/3      | $-1/2$   | $-1/2$   | 1       | 1/6      | 1/6      | $-2/3$     | $1/3$ | $-1/2$ | $-1/2$ |
| $Q$                 | 1/3      | 0        | $-1$     | 1       | 2/3      | $-1/3$   | $-2/3$     | $1/3$ | 1     | 0     |
| $T_3$               | 0        | 1/2      | $-1/2$   | 0       | 1/2      | $-1/2$   | 0          | 0     | 3/2   | 1/2   |
| $G$                 | 3        | 3        | 3        | $-1$    | $-1$     | $-1$     | $-1$       | $-2$  | $-2$  | $-2$  |
| $B - L$             | $-1/3$   | $-1$     | $-1$     | 1       | 1/3      | 1/3      | $-1/3$     | 2/3  | 0     | 0     |

Table of global charges.

Of course, all charges read the opposite values on hermitian conjugate fields. Higgs fields $H_i$ described by Eqs. (35), breaks $SU(5)$ symmetry down to $SU(3)_C \times U(1)_{\text{e.m.}} \times U(1)_{B-L}$, which is indeed, as required, a maximal subgroup of $SU(5)$, as simple group–rank counting immediately confirms.

### 3.5 Minimal Yukawa couplings and mass terms for mixed families

To extend Yukawa couplings non-trivially to three fermion families, the scheme presented in the previous subsection must be expanded as follows. Let $f_\alpha$ ($\alpha = 1, 2, 3$) be a triplet of generally unphysical fermion families in standard notation, putatively $f_1 = \{e^0_\alpha, \nu^0_\alpha, u^0_\alpha, d^0_\alpha\}$, $f_2 = \{\mu^0_\alpha, \nu^0_{\mu_\alpha}, s^0_\alpha, c^0_\alpha\}$ and $f_3 = \{\tau^0_\alpha, \nu^0_{\tau_\alpha}, l^0_\alpha, b^0_\alpha\}$, where color indices are omitted, and denote by $\mathbf{f} = [f_1, f_2, f_3]^T$, where superscript $T$ indicates transposition, the ordered column formed by these fields, to be regarded as a 3D vector in the fermion–family space. For the sake of conciseness, we rename the three families $f_\alpha = [e^0_\alpha, \nu^0_\alpha, u^0_\alpha, d^0_\alpha]^T$.

Following the shorthand notation introduced in the previous subsection, we indicate by $\psi = [\psi_{L1}, \psi_{L2}, \psi_{L3}]^T$ the vector formed of the three families of left-handed
fermions, to be understood as three different instances of the column exemplified by the first of Eqs.\([24]\), and of \(\hat{\psi} = [\psi^T_{L1}C, \psi^T_{L2}C, \psi^T_{L3}C]\) its hat–conjugate counterpart. Similarly, we indicate by \(\chi = [\chi_{L1}, \chi_{L2}, \chi_{L3}]^T\) the vector formed of the three families of left–handed fermions exemplified by the second of Eqs.\([24]\), and of \(\hat{\chi} = [\chi^T_{L1}C, \chi^T_{L2}C, \chi^T_{L3}C]\) its hat–conjugate counterpart.

Now, let \(G_d \equiv [G^a_b]\) and \(G_u \equiv [G^a_b]\) be two \(3 \times 3\) complex matrices with superscripts \(\alpha, \beta\) as family indices. \(\hat{\psi}_a G_d \chi^{bc} \equiv G^a_b \hat{\psi}_a \chi^{bc}\) and \(\hat{\chi}^{ab} G_u \chi^{cd} \equiv G^a_u \chi^a \chi^b \chi^c \chi^d\) then represent the mixed Yukawa bilinears \(Y_d, Y_u\) which extend the Yukawa couplings in Eq.\([25]\) to the case of three fermion families. Thus, in shorthand notation, single–family couplings, times coupling constants, displayed in Eqs.\([32]\) are replaced by mixed couplings

\[
Y_d = \hat{\psi}_a G_d \chi^{ab} H^b \quad \text{and} \quad Y_u = \frac{1}{2} \epsilon_{abcde} \hat{\chi}^{ab} G_u \chi^{cd} H^e \quad \text{h.c.}
\]

In other words, coupling constants \(g_d\) and \(g_u\) are now replaced by \(G_d\) and \(G_u\).

Assuming that Higgs field components \(H_i\) and \(H^i\) have the properties established in the previous subsection, we replace Eq.\([36]\) with the extended mass term \(L_m = \varphi_c (F^d_m + F^u_m)\), where

\[
F^d_m = \hat{\psi}_a G_d \chi^{a5} + \text{h.c.} = e^L_R G_d e^c_R + \bar{e}^L_R G^L_d e^c_R + d^R R G_d d_L \quad \text{and} \quad \bar{d}^L R G^L_d d_R ;
\]

\[
F^u_m = \frac{1}{4} \epsilon_{abcd} \hat{\chi}^{ab} G_u \chi^{cd} + \text{h.c.} = 2 (\bar{u} R G_u u_L + \bar{u} L G^u R u_R).
\]

Note that we put \(e^{R,L}_R \equiv e^{R,L}_L\) instead of \(e^{R,L}_R\) because \(\hat{e} e^c \equiv \hat{e}^c e^+\), rather than \(\hat{e} e\), occurs in the mass term of Eq.\([33]\).

An important point regarding the second member of this equation is that, as \(\hat{\psi}_a \chi^5\) are all different and independent of each other, all matrix elements of \(G_d\) are involved in the first term, whereas only the symmetric part of \(G_u\) is involved in the second term, since \(G^a_u \epsilon_{abcd} \hat{\chi}^{ab} \chi^{cd} = G^a_u \epsilon_{abcd} \hat{\chi}^{ab} \chi^{cd} = G^a_u \epsilon_{abcd} \hat{\chi}^{ab} \chi^{cd}\). To prove this, equality \(C^T = -C\) and fermion anticommutation relations are used.

Also note that, since an arbitrary \(3 \times 3\) complex matrix \(G_d\) depends on 18 independent parameters, whereas an arbitrary \(3 \times 3\) unitary matrix depends on 9, the linear transformation of \(G_d\) into a diagonal matrix \(G_d \equiv \text{diag}(g^d_1, g^d_2, g^d_3)\), with \(0 < g^d_1 \leq g^d_2 \leq g^d_3\), requires two unitary operators, \(U_L\) and \(U_R\), the first acting
on $d_L, e_L^c$ and the second on $d_R, e_R^c$, so as to satisfy the equation $U_R^\dagger G_d U_L = \hat{G}_d$. Note that these operators are determined up to a common phase–matrix factor $\Phi_d = \text{diag}(e^{i\phi_1}, e^{i\phi_2}, e^{i\phi_3})$, so as $\Phi_d^* \hat{G}_d \Phi_d = \hat{G}_d$. Hence, of the 18 parameters available in $U_L$ and $U_R$, only 15 are necessary and sufficient to diagonalize a complex $3 \times 3$ matrix $G_d$ into $\hat{G}_d$ (assuming no degeneracy of eigenvalues).

In turn, $G_u$ too can be diagonalized by two unitary operators $V_L$ and $V_R$, respectively acting on $u_L$ and $u_R$, so as to satisfy the equation $V_R^\dagger G_u V_L = \text{diag}(g_1^u, g_2^u, g_3^u) \equiv \hat{G}_u$ (with $0 < g_1^u \leq g_2^u \leq g_3^u$). However, since in general $G_u$ is complex and symmetric, i.e., $G_u = G_u^T$, we can find an orthogonal operator $C$ so as to satisfy the equation $C^T G_u C = \Phi \hat{G}_d$, where $\Phi = \text{diag}(e^{i\lambda_1}, e^{i\lambda_2}, e^{i\lambda_3})$ is a diagonal phase–matrix. Thus, we can pose $V_L = C$ and $V_R = \Phi C$. We can view $C$, which is a real 3D rotation of the family space, as a generalized Cabibbo matrix.

As is evident from the structure of Eq. (39), redefining the $(3 \times 10)$–D family–array $\chi_L$ by performing the substitution $\chi_L \rightarrow U_L \chi_L$ (thus $d_L^c \rightarrow U_L d_L^c, e_L^c \rightarrow U_L e_L^c$), and the $(3 \times 5)$–D family–array $\psi_L$ by the substitution $\psi_L \rightarrow \chi_L U_R^\dagger$ (thus $\bar{d}_L \rightarrow \bar{d}_L U_R^\dagger, \bar{e}_L \rightarrow \bar{e}_L U_R^\dagger, \bar{\nu}_L \rightarrow \bar{\nu}_L U_R^\dagger$), is equivalent to replacing $G_d$ by $\hat{G}_d$ in Eq. (39) and replacing $G_u$ in Eq. (40) by another non–diagonal matrix of the same name.

The fermion–family triplet, interacting with quintuplet $H$, as defined in Eq. (35), with $G_d = \hat{G}_d$ but in general $G_u \neq \hat{G}_u$, may be written as [24]

$$\psi_L = \begin{bmatrix} d_1^c \\ d_2^c \\ d_3^c \\ e \\ -\nu \end{bmatrix}_L; \quad \chi_L = \begin{bmatrix} 0 & C\Phi^* u_3^c & -C\Phi^* u_2^c & -C u_1 & -d_1 \\ -C\Phi^* u_3^c & 0 & C\Phi^* u_1^c & -C u_2 & -d_2 \\ C\Phi^* u_2^c & -C\Phi^* u_1^c & 0 & -C u_3 & -d_3 \\ C u_1 & C u_2 & C u_3 & 0 & -e^c \\ d_1 & d_2 & d_3 & e^c & 0 \end{bmatrix}_L.$$ (41)

We can easily verify that the mixed mass term maintains the diagonal form

$$L_m = -\varphi c_m (\bar{e} \hat{G}_d e + \bar{d} \hat{G}_d d + 2 u \hat{G}_u u) \equiv -\varphi c_m N.$$ (42)

Now, let us generalize interaction Lagrangian density $L_\theta$ described by Eq. (37) to its extended counterpart, by posing $H_4 = H_5 = 0$ in Eqs. (38) and defining
Here, identities \( \hat{e} \equiv \overline{e}_R, \ u^i \equiv u^i_R, \ \hat{u}^i \equiv \overline{u}_R, \ \nu \equiv \overline{\nu}_R, \ d^c \equiv d^c_L, \ d_k \equiv d_{Lk}, \ d_j^c \equiv \overline{d}_{Rj}, \) and equalities \( C^T G_u = \hat{G}_u C^T \) and \( [\Phi^*, \hat{G}_u] = 0 \) were used. Thus, the generalized interaction Lagrangian density in its extensive form is

\[
L_\theta(\Phi) = \varphi c_\theta \sum_i e^{i\theta_i} \left( \overline{u}_L^i C^T \hat{G}_u \overline{e}_R^i - \overline{d}_L^i \hat{G}_d \nu_R^i - \epsilon^{ijk} \overline{u}_L^i \Phi^* C^T \hat{G}_u d_{Lj}^c \right) + \varphi c_\theta \sum_i \epsilon^{i\theta_i} \left( \overline{u}_L^i C^T \hat{G}_u \overline{e}_R^i - \epsilon^{ijk} \overline{u}_L^i \Phi^* \overline{G}_u d_{Lj}^c \right) - 2 \epsilon^{i\theta_i} \left( \overline{u}_L^i \Phi^* C^T \hat{G}_u \overline{e}_R^i - 2 \epsilon^{ijk} \overline{u}_L^i \Phi^* \overline{G}_u d_{Lj}^c \right) \equiv \varphi c_\theta \sum_i \left[ e^{i\theta_i} F_i(\Phi) + e^{-i\theta_i} F_i^*(\Phi) \right];
\]

which not only violates the same symmetries as \( L_\theta \) defined by Eq. (37), but also \( CP \).

With \( H_i \xrightarrow{C} H_i^\dagger \), i.e., \( \theta_i \xrightarrow{C} -\theta_i \), and the second of Eqs. (20) and then \( R \leftrightarrow L \), we find \( e^{i\theta_i} \xrightarrow{C} e^{-i\theta_i}, \ F_i(\Phi) \xrightarrow{C} F_i^*(\Phi^*), \ F_i^*(\Phi) \xrightarrow{C} F_i(\Phi^*), \) hence \( L_\theta(\Phi) \xrightarrow{C} L_\theta(\Phi^*). \)

Thus, provided that \( \Phi \neq 1, \ L_\theta(\Phi) \neq L_\theta(\Phi^*), \) and therefore the third Sakharov condition is also satisfied. As well known from QFT basics, \( CP \) is equivalent to time reversal, which means that \( L_\theta(\Phi^*) \) is the interaction Lagrangian density of the back-running system.

### 3.6 Baryon currents and fermion fluid dynamics

The fermion Lagrangian density for all fermion fields has the form

\[
L_F = i \sum_i \left( \overline{u}_L^i \overset{\gamma}{\partial} u_L^i + \overline{u}_R^i \overset{\gamma}{\partial} u_R^i + \overline{d}_L^i \overset{\gamma}{\partial} d_L^i + \overline{d}_R^i \overset{\gamma}{\partial} d_R^i + \overline{e}_L^i \overset{\gamma}{\partial} e_L^i + \overline{e}_R^i \overset{\gamma}{\partial} e_R^i + \overline{\nu}_L^i \overset{\gamma}{\partial} \nu_L^i \right) - \varphi c_\theta \sum_i \left( 2 \overline{u}_R^i \hat{G}_u u_R^i + 2 \overline{u}_L^i \hat{G}_d u_L^i + \overline{d}_R^i \hat{G}_d d_R^i + \overline{d}_L^i \hat{G}_d d_L^i + \overline{e}_R^i \hat{G}_d e_R^i + \overline{e}_L^i \hat{G}_d e_L^i + \overline{\nu}_R^i \hat{G}_d \nu_R^i + \overline{\nu}_L^i \hat{G}_d \nu_L^i \right) + \varphi c_\theta \sum_i \left[ e^{i\theta_i} F_i(\Phi) + e^{-i\theta_i} F_i^*(\Phi) \right],
\]

where \( \overset{\gamma}{\partial} \) is the covariant Dirac operator and \( F_i(\Phi) \) is defined in Eq. (45).
By variation of $L_F$ with respect to $\mu_i^L$ and $\bar{d}_i^R$ we obtain the motion equations

\begin{align}
\imath \partial u_i^L - 2 \varphi c_m G_a u_i^R &= - \frac{\partial L_\theta}{\partial u_i^L} = - \varphi c_\theta e^{i\theta} C^T \hat{G}_d e_{\hat{R}}^i; \quad (47) \\
\imath \partial u_i^R - 2 \varphi c_m G_a u_i^L &= - \frac{\partial L_\theta}{\partial u_i^R} = \varphi c_\theta e^{-i\theta} \Phi^C \hat{G}_d e_{\hat{L}}^i; \quad (48) \\
\imath \partial d_i^L - \varphi c_m G_{d} d_i^R &= - \frac{\partial L_\theta}{\partial d_i^L} = \varphi c_\theta (2 e^{-i\theta} \epsilon^{kji} \hat{G}_a u_{\hat{R}j} + e^{i\theta} \Phi^* e_{\hat{L}}^i); \quad (49) \\
\imath \partial d_i^R - \varphi c_m G_{d} d_i^L &= - \frac{\partial L_\theta}{\partial d_i^R} = 2 \varphi c_\theta e^{-i\theta} \epsilon^{jik} \hat{G}_d C \Phi^* u_{\hat{L}k}. \quad (50)
\end{align}

From these equations and their hermitian conjugates, and denoting by

$$D^\mu J^u_\mu \equiv D^\mu (\bar{u}_i^L \gamma_\mu u_i^L + \bar{u}_R^i \gamma_\mu u_R^i), \quad D^\mu J^d_\mu \equiv D^\mu (\bar{d}_i^L \gamma_\mu d_i^L + \bar{d}_R^i \gamma_\mu d_R^i)$$

the covariant divergences of quark currents, we obtain the remarkable formula

$$D^\mu J^{B_i}_\mu (\tau, \hat{x}) = \frac{\delta L_\theta(\Phi, \tau, \hat{x})}{\delta \theta_i(\tau)}; \quad (51)$$

where $J^{B_i}_\mu(\tau, \hat{x}) = J^{u}_\mu(\tau, \hat{x}) + J^{d}_\mu(\tau, \hat{x})$ are the baryon–number current densities of quarks of color $i$ as functions of hyperbolic coordinates.

Let us write $\varphi(\tau) = \eta(\tau) + \varphi_0(\tau)$, where $\eta(\tau)$ is the oscillating component of the Higgs amplitude (gray region in Fig.4) and $\varphi_0 = \mu s(\tau)/\lambda$, where $s(\tau)$ is the scale factor of the dilation field for $\tau > \tau_c$ (sigmoidal profile in Fig.4). Then $\eta(\tau) e^{i\theta_i(\tau)}$ represent the charged components of the Higgs field and $\varphi_0(\tau) e^{i\theta_i(\tau)}$ their backgrounds. With this notation, the mass term appearing on the right in Eq.(46), here abbreviated to $- \varphi c_m N$ as in Eq.(42), splits into background component $- \varphi_0 c_m N$ and oscillating component $- c_m \eta N$. The former provides fermions with masses augmenting like the scale factor; the latter works as the excitation factor for the production of particle–antiparticle pairs of massive fermions acting globally and coherently on the fermion field. Pair production starts abruptly at the moment of Higgs amplitude jump and decreases progressively on average for $\tau > \tau_c$ because of the progressive energy transfer from Higgs field to fermion field. Since $\eta$ is an oscillating combination of creation and annihilation operators, fermion densities themselves, as well as their energy densities, are engaged in an oscillatory regime.
To highlight a few details of this process, let us recall that, according to Fermi–Dirac statistics, $N$ free fermions of any species and given helicity, enclosed in a spherical box of volume $V_F$ at zero temperature, occupy all energy levels characteristic of the box, up to the level of Fermi energy $E_F = \sqrt{m^2 + p_N^2}$, where $m$ is the fermion mass and $p_N = (6\pi^2 N/V_F)^{1/3}$ the momentum of the highest level.

The total energy density of $N$ fermions of the same helicity in volume $V_F$ is thus $\varepsilon = E_T/V_F = V_F^{-1}\sum_{n>0}^{N}\sqrt{m^2 + (6\pi^2 n/V_F)^{2/3}}$. In our case, all fermions are completely delocalized and dispersed within the infinite volume of space–like hyperboloids, so that only fermion density $\rho_F = N/V_F$, for $V_F$ and $N$ going to infinity, is well–defined (and depends only on $\tau$, for the reasons given in subsec.s 4.3, 4.4, Part I). Thus, Fermi energy (in natural units) is $E_F = \sqrt{m^2 + (6\pi^2 \rho_F)^{2/3}}$ and the total energy density of the fermion population on the hyperboloid at $\tau$ is

$$\varepsilon(\tau) = \int_0^{\rho_F(\tau)} \sqrt{m^2(\tau) + (6\pi^2 \rho)^{2/3}} d\rho.$$

(52)

Because of the invariance of Lagrangian density under $SU(3)$, fermions of different colors have the same density and, because of $Q$ and $B – L$ conservation, the densities of these quantities maintain their initial zero values. Since all fermions are free, they form a homogeneous and isotropic superconducting fluid, in which massless gauge fields of any sort (gluons, photons, etc.) cannot exist. Instead, they may appear later, when the inflation rate attenuates to such an extent that the gravitational field causes the spontaneous breaking of matter homogeneity and uniformity, thus starting the epoch of local symmetry breakdown.

Under the global action of the Yukawa interaction terms of $L_\theta(\Phi)$, the composition of the fermion fluid is continuously rearranged by fermion–antifermion transitions, as well as by the continuous creation or annihilation of fermion–fermion and antifermion–antifermion pairs. However, because of $CP$ violation, forward and backward processes proceed at different rates. Thus, the entire process takes place irreversibly, also favored by the energy dissipation caused by neutrino production.

I we assume that transitions from fermions to antifermions are favored on average, relative to their inverse, an accumulation of fermions of some selected species at the expense of antifermions of other species takes place.
3.7 Inflationary mechanism of baryogenesis

Let $L_F(\tau, \hat{x})$ be the conformally flat Lagrangian density of the Higgs multiplet on the Riemann manifold introduced in Eq. (13) subsec. (2.2) completed with the addition of mass and Yukawa coupling terms $N(\tau, \hat{x})$ and $F_i(\Phi, \tau, \hat{x})$ defined in Eqs. (42) (43), and let $\bar{L}_F(\tau)$ be its average over unit volume $V_1$. We obtain

$$\bar{L}_\phi = \frac{1}{2} (\partial_\tau \phi)^2 + \frac{1}{2} \phi^2 \sum_i c_\theta^2 (\partial_\tau \theta_i)^2 + \frac{\lambda^2}{4} \left( \phi^2 - \mu^2 \rho^2 \right)^2 + \varphi \lambda \sum_i [e^{i\theta_i} \bar{F}_i(\Phi) + e^{-i\theta_i} \bar{F}_i^\dagger(\Phi)] - \varphi c_m \bar{N}, \quad (53)$$

where $s(\tau)$ is the scale factor and $\bar{N}(\tau)$ and $\bar{F}_i(\tau)$ are respectively the averages of $N(\tau, \hat{x})$ and $F_i(\Phi, \tau, \hat{x})$ over the unit volume.

Using Eq. (45), we easily derive the equation for Higgs field amplitude $\varphi$

$$\partial_\tau^2 \varphi + \frac{3}{\tau} \partial_\tau \varphi - \varphi \sum_i c_\theta^2 (\partial_\tau \theta_i)^2 + (\lambda^2 \varphi^2 - \mu^2 \rho^2) \varphi = c_\theta \sum_i [e^{i\theta_i} \bar{F}_i(\Phi) + e^{-i\theta_i} \bar{F}_i^\dagger(\Phi)] - c_m \bar{N}, \quad (54)$$

and the equations for $\theta_i$

$$\partial_\tau \frac{\delta \bar{L}_\phi}{\delta \partial_\tau \theta_i} = \frac{1}{\tau^3} \partial_\tau (\tau^3 \varphi^2 c_\theta^2 \partial_\tau \theta_i) = i c_\theta \varphi \left[ e^{i\theta_i} \bar{F}_i(\Phi) - e^{-i\theta_i} \bar{F}_i^\dagger(\Phi) \right] = \frac{1}{\tau} \partial_\tau (\tau^3 \rho_i^B), \quad (55)$$

from which we derive the equation $\rho_i^B = \varphi^2 c_\theta^2 \partial_\tau \theta_i$, which states the equality of baryon–charge density $\rho_i^B$ to the charge density of the Nöther current $\varphi^2 c_\theta^2 \partial_\tau \theta_i$ associated with transformation $\theta_i \to \theta_i + \alpha_i$, and the equation

$$\rho_i^B(\tau) = c_\theta^2 \varphi^2 (\tau) \partial_\tau \theta_i(\tau) = \frac{i c_\theta}{\tau^3} \int_{\tau_c}^{\tau} \left[ e^{i\theta_i(\tau)} \bar{F}_i(\Phi, \tau) - e^{-i\theta_i(\tau)} \bar{F}_i^\dagger(\Phi, \tau) \right] d\tau, \quad (56)$$

showing that $\rho_i^B(\tau) \to 0$ for $\tau \to +\infty$ - as expected, since the baryon charge is increasingly diluted as the volume expands. At the same limit, we have $\varphi \to \mu/\lambda$ and therefore $\partial_\tau \theta_i(\tau) \to \lambda^2 (c_\theta \mu)^{-2} \rho_i^B(\tau)$; therefore $\theta_i$ becomes constant and the kinetic energy term of $\theta_i$ in Eq. (53) disappears. Exploiting global gauge invariance, we can remove these constants, so that, for $\tau \to +\infty$, the mass and Yukawa terms in Eq. (53) become respectively

$$\bar{L}_m(\tau) = -c_m \frac{\mu}{\lambda} \bar{N}(\tau); \quad \bar{L}_\theta(\tau) = c_\theta \frac{\mu}{\lambda} \sum_i \left[ F_i(\Phi, \tau) + \bar{F}_i^\dagger(\Phi, \tau) \right].$$
Since at this limit $\bar{L}_\theta(\tau)$ is a linear combination of unit-volume averaged Yukawa-interaction terms proportional to kinematic-time factors of the form $f_\alpha^\pm(\tau) = \tau^{-3} e^{i\theta_\alpha \tau}$, where $\Omega_\alpha$ are sums of Fermi energies, integrals of the form

$$I_\theta(\tau) = \int_{\tau}^{+\infty} \tau^3 \bar{L}_\theta(\bar{\tau}) d\bar{\tau}$$

tend to vanish for large $\tau$. In other words, all unit-volume averaged Yukawa-interaction terms vanish at $\tau \to +\infty$.

Thus, at the end of the global expansion stage, all fermions decouple from the Higgs field multiplet and become free. However, provided that the integral on the second member of Eq.(56) does not vanish for $\tau \to +\infty$, numbers of baryons $dN_i^B = \tau^3 \rho_i^B(\hat{x})$ per unit of expanding volume $dV_i(\hat{x}) = \tau^3 dV_i(\hat{x})$, are in the meantime produced, and since fermions of different species have acquired different masses, the original $SU(5)$ symmetry is not restored but remains broken down to $SU(5)$ symmetry down to $SU(3)_C \times U(1)_{e.m.} \times U(1)_{B-L}$. With these results, we may presume that a second symmetry breaking process takes place toward the end of this stage, when gauge fields becomes effective and a new Higgs field triplet provided with zero-mass NG bosons jumps from the top of the Mexican hat to a point of its brim of the same energy (Fig.1R), giving mass to $Z$ and $W^\pm$ gauge bosons.

To analyze the process described above in detail, let us first translate Eq.(57) to the Heisenberg picture by replacing Yukawa terms $\varphi^i(\tau) \bar{F}_i(\Phi, \tau)$, where $\varphi^i = \varphi e^{i\theta_i}$, with their VEVs $\langle \Omega_F | \varphi^i(\tau) \bar{F}_i(\Phi, \tau) | \Omega_F \rangle$, where $| \Omega_F \rangle$ is the false vacuum state. Let us recall that $| \Omega_F \rangle$ becomes the physical state of the universe after the spontaneous breakdown of conformal symmetry. Since $| \Omega_F \rangle$ is presumed to be CP-invariant, according to the results reported at the end of subsec.(3.5), we find equality $\langle \Omega_F | \varphi^*\varphi(\tau) F_i^\dagger(\Phi, \tau) | \Omega_F \rangle = \langle \Omega_F | (CP)^{-1}[\varphi^*\varphi F_i^\dagger(\Phi, \tau)]CP | \Omega_F \rangle$ and hence

$$\langle \Omega_F | \varphi^*\varphi(\tau) F_i^\dagger(\Phi, \tau) | \Omega_F \rangle = \langle \Omega_F | \varphi^*\varphi(\tau) F_i^\dagger(\Phi, \tau) | \Omega_F \rangle.$$

Thus, the false VEV of Eq.(56) is

$$\langle \Omega_F | \rho_i^B(\tau) | \Omega_F \rangle = \frac{i c_a}{\tau^3} \int_{\tau}^{+\infty} \tau^3 \varphi(\tau) e^{i\theta_\alpha \tau} \langle \Omega_F | \bar{F}_i(\Phi, \bar{\tau}) - F_i(\Phi^*, \bar{\tau}) | \Omega_F \rangle d\bar{\tau},$$

displaying that baryogenesis does not take place if $L_\theta$ is CP-invariant.
In conclusion

In the context of SU(5) symmetry, it is commonly believed that the mechanism of baryogenesis may be explained by the decay of superheavy vector bosons belonging to a 24–multiplet interacting with a 24–multiplet of Higgs bosons and three families of SM fermions [23]. By contrast, in our view, the absolute homogeneity and isotropy of the matter field during the initial stage of inflation makes plausible a model in which the phenomenon is ascribed to the breakdown of SU(5) global symmetry embedded in the conformal–invariant environment during the initial stage of inflation. The model is based on SU(5)–invariant Yukawa couplings of SM fermions with a Higgs–boson quintuplet, which preserve Q and B − L but violate B, C and CP, precisely as required by the first three Sakharov conditions.

In this model, the charged Higgs–field triplet \( \varphi e^{i\theta_i} \), interacting with the non–diagonal fermion bilinears of Yukawa interaction terms, promotes the production of an excess of fermions over antifermions, while the oscillation of Higgs–field amplitude continues to feed the matter field with new fermion–antifermion pairs through mass like coupling with diagonal fermion bilinears. In these conditions, the matter field may be regarded as a cold superconducting fluid formed of free fermions and antifermions embedded in a Higgs field background, which makes the fourth Sakharov condition unnecessary, at least as far as the initial inflation stage is concerned. At the end of this process, SU(5) symmetry broken down to SU(3)_C \times U(1)_{e.m.} \times U(1)_{B−L} tends to become local under the action of the gravitational field. In these conditions, a second symmetry–breaking stage occurs, and the gauge and Higgs fields characteristic of Standard Model enter into play.

What percents of fermions and antifermions survives the end of the global inflation process described here depends on the constants on which \( L_m \) and \( L_\theta \) depend. I did not attempt to provide a calculation for this difficult problem, neither did I attempt to explain how the transition from the globally broken regime to the locally broken regime may take place. I merely hope that the problem I leave unsolved is sufficiently well posed to attract the attention of more skillful theorists.
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