A microscopic approach to $^3$He scattering

Masakazu Toyokawa, Takuma Matsumoto, Koshio Minomo, and Masanobu Yahiro

1Department of Physics, Kyushu University, Fukuoka 812-8581, Japan
2Research Center for Nuclear Physics, Osaka University, Ibaraki 567-0047, Japan

(Dated: March 24, 2015)

We propose a practical folding model to describe $^3$He elastic scattering. In the model, $^3$He optical potentials are constructed by making the folding procedure twice. First the nucleon-target potential is evaluated by folding the Melbourne $g$-matrix with the target density and localizing the nonlocal potential with the Brieva-Rook method, and second the resulting local nucleon-target potential is folded with the $^3$He density. This double single-folding model well describes $^3$He elastic scattering from $^{208}$Ni and $^{209}$Pb targets in a wide incident-energy range from 30 MeV/nucleon to 150 MeV/nucleon with no adjustable parameter. Spin-orbit force effects on differential cross sections are found to be appreciable only at higher incident energies such as 150 MeV/nucleon. Three-nucleon breakup effects of $^3$He are investigated with the continuum discretized coupled-channels method and are found to be appreciable only at lower incident energies around 40 MeV/nucleon. Effects of knock-on exchange processes are also analyzed.

PACS numbers: 25.55.Ci, 24.10.Eq, 24.10.Ht

\section{I. INTRODUCTION}

Microscopic understanding of nucleon–nucleus (NA) and nucleus–nucleus (AA) optical potentials is a goal of nuclear physics. The optical potentials are not only essential quantities to describe the elastic scattering but also key inputs in calculations of the distorted-wave Born approximation (DWBA) and the continuum discretized coupled-channels method (CDCC) to describe inelastic scattering, breakup and transfer reactions.

The $g$-matrix folding model is a method of deriving the optical potentials microscopically. Here the $g$-matrix is an effective nucleon-nucleon (NN) interaction in nuclear matter and depends on the density $\rho$ of nuclear matter: $g = g(\rho)$. In the folding model, the optical potential is obtained by folding the $g$-matrix [4–14] with the target density $\rho_T$ for NA scattering and with $\rho_T$ and the projectile one $\rho_P$ for AA scattering; see for example Refs. [15–19] for the folding procedure. The model is now called the single-folding (SF) model for NA scattering and the double-folding (DF) model for AA scattering. For AA scattering, occasionally, the optical potential is obtained semi-microscopically by folding the phenomenological nucleon-target potential with $\rho_P$. This procedure is also called the SF model.

For NA elastic scattering, the SF model based on the Melbourne $g$-matrix [11] well reproduces the experimental data with no adjustable parameter. In the folding procedure, the value of $\rho$ in $g(\rho)$ is assumed to be a value of $\rho_T$ at the midpoint $r_m$ of interacting two nucleons: $\rho = \rho_T(r_m)$. This local-density approximation seems to be reasonable because of the success of the Melbourne $g$-matrix folding model.

The NA potential thus obtained is non-local, because knock-on exchange processes are taken into account in the folding procedure. However, it can be localized by the Brieva–Rook method [6] with good accuracy. The validity of the method is shown in Refs. [20, 21]. In fact, the local version of the folding potential agrees with the phenomenological NA optical potentials determined from the NA scattering data [22–25] particularly in the surface region important for the scattering [26].

The multiple NN collision series in AA scattering [27] is more complicated than in NA scattering [28, 29]. This makes microscopic description of AA scattering more difficult. In fact, the DF model has a basic problem. In nuclear matter calculations, in principle, the $g$-matrix should be derived by solving the scattering between a nucleon in a Fermi sphere and a nucleon in another Fermi sphere [30], but in practice it is evaluated by solving nucleon scattering on a single Fermi sphere. In addition to the single-sphere approximation, furthermore, in the folding procedure the density $\rho$ of the single Fermi sphere is assumed to be identical with the sum of $\rho_P$ and $\rho_T$ at the midpoint $r_m$ of interacting two nucleons:

$$\rho = \rho_P(r_m) + \rho_T(r_m).$$

The prescription is called the frozen-density approximation (FDA). The DF model with the FDA is referred to as the DF-FDA model in this paper.

In actual DF-FDA calculations, the magnitude of the folding potential is usually adjusted to the experimental data. It is then an important subject in future to clarify how good the DF-FDA model is. As a successful example, the DF-FDA model based on the Melbourne $g$-matrix well reproduced measured total reaction cross sections $\sigma_T$ for $^{12}$C scattering from stable nuclei at around 250 MeV/nucleon with no adjustable parameter [19, 31–35]. The DF-FDA model was then applied to measured $\sigma_T$ [32, 33] for neutron-rich Ne and Mg isotopes [19, 31–35]. This analysis leads to the result that $^{31}$Ne and $^{37}$Mg are deformed halo nuclei.

*toyokawa@phys.kyushu-u.ac.jp
As an alternative approach to the DF-FDA model, we can consider the model Hamiltonian

\[ H_{\text{eff}} = K_R + \sum_{i \in P} U_{iT} + h_P, \]

where \( K_R \) stands for the kinetic energy with respect to the relative coordinate \( \mathbf{R} \) between a projectile (P) and a target (T) and \( h_P \) is the intrinsic Hamiltonian of P. Here \( U_{iT} \) represents the interaction between T and the \( i \)th nucleon in P. As \( U_{iT} \), the phenomenological nucleon-target optical potential was used so far; see for example Refs. [1-3] for deuteron scattering. In this paper, meanwhile, \( U_{iT} \) is constructed microscopically by folding the Melbourne \( g \)-matrix with \( \rho_T \) and is localized by the Brieva-Rook method. The potential \( U_{iT} \) is then always obtainable even if no experimental data is available for nucleon-target scattering of interest. This is an advantage of the present approach from the previous one. Furthermore, this approach is consistent with the fact that a single Fermi sphere is considered in the \( g \)-matrix calculation for nuclear matter. In addition, one can treat projectile breakup with CDCC if necessary, since \( U_{iT} \) is localized. These are advantages of the present approach from the DF-FDA model. The model Hamiltonian (2) can be derived from the many-body Hamiltonian with reasonable approximations, as shown later in Sec. II.

It is known that the model Hamiltonian (2) well accounts for deuteron scattering, when deuteron breakup effects are properly taken into account with CDCC [1-3]. For \(^4\)He scattering, meanwhile, projectile-excitation effects are quite small since \(^4\)He is hardly excited. We can then expect that the scattering is described by the optical potential that is obtained by folding the local version of microscopic \( U_{iT} \) with the \(^4\)He density. We refer to this model as the double single-folding (DSF) model in this paper. Very recently, it was shown that the DSF model well accounts for differential elastic cross sections in this paper. Very recently, it was shown that the DSF model as the double single-folding (DSF) model for deuteron and \(^3\)He scattering by using the DSF model. Finally, we analyze effects of knock-on exchange processes on \(^4\)He scattering. For AA scattering, the processes make the microscopic optical potential nonlocal, but the processes are approximately treated in the DSF model, since \( U_{iT} \) is localized. We then investigate how the approximation affects \(^3\)He scattering.

In Sec. II, we derive the model Hamiltonian (2) from the many-body Hamiltonian with reasonable approximations, using the multiple scattering theory [27-29]. Brief explanation is made on the DSF and DF-FDA models. The explicit form of the spin-orbit potential between \(^4\)He and T is shown, and four-body CDCC is recapitulated. In Sec. III, numerical results are shown. Section IV is devoted to a summary.

II. MODEL BUILDING

We consider the scattering of P with mass number \( A_P \) from T with mass number \( A_T \). In principle, the scattering is described by the many-body Schrödinger equation

\[ \left[ K_R + h_T + h_P + \sum_{i \in P, j \in T} v_{ij} - E \right] \Psi^{(+)0} = 0 \]  

for the total wave function \( \Psi^{(+)0} \), where \( v_{ij} \) is the realistic NN interaction and \( h_T \) stands for the internal Hamiltonian of T. The total energy \( E \) is related to the incident energy \( E_{\text{in}} \) in the center of mass system as \( E = E_{\text{in}} + E_0(P) + E_0(T) \), where \( E_0(P) \) and \( E_0(T) \) are the ground-state energies of P and T, respectively. Following the multiple scattering theory [27-29], one can rewrite Eq. (3) into

\[ \left[ K_R + h_T + h_P + \frac{Y - 1}{Y} \sum_{i \in P, j \in T} \tau_{ij} - E \right] \hat{\Psi}^{(+)} = 0, \]

where \( \tau_{ij} \) denotes the effective NN interaction in nuclear medium and the factor \( Y = A_P A_T \) represents the number of the \( \tau_{ij} \) working between P and T. For AA scattering with \( Y \gg 1 \), the factor \( (Y - 1)/Y \) can be approximated into 1. When Eq. (4) is derived from Eq. (3), the antisymmetrization between nucleons in P and those in T are neglected. However, the antisymmetrization effects are well taken care of, if the \( \tau_{ij} \) are symmetrical with respect to the exchange of colliding nucleons [37, 38]. The \( \tau_{ij} \) is often replaced by the \( g \)-matrix \( (g_{ij}) \) in many applications, since both include nuclear medium effects.
Therefore, we reach the Schrödinger equation

$$[K_R + h_P + h_T + \sum_{i \in P,j \in T} g_{ij} - E]\Psi^{(+)} = 0. \quad (5)$$

In this work, we use the Melbourne g-matrix [11] as $g_{ij}$. As mentioned in Sec. I, $g(\rho)$ is evaluated in nuclear matter by solving nucleon scattering on a single Fermi sphere. For consistency with the nuclear-matter calculation, we consider the nucleon-target subsystem in the P+T system and assume

$$\rho = \rho_T(r_m) \quad (6)$$

as $\rho$ in $g(\rho)$. This procedure is referred to as the target-density approximation (TDA) in this paper.

In the TDA, $g(\rho_T)$ includes target-excitation effects approximately, but does not include projectile-excitation effects. We can then assume that $\Psi^{(+)} = \phi_0(T)\psi$, where $\phi_0(T)$ is the ground state of $T$ and $\psi$ describes the scattering of $P$ from $T$ in its ground state. Left-multiplying Eq. (5) by $\phi_0(T)$, we can get the Schrödinger equation

$$[H_{\text{eff}} - E_{\text{cm}}^\infty - c_0(P)] \psi = 0 \quad (7)$$

for $\psi$. Here the nucleon-target potential

$$U_{\text{TT}} = \langle \phi_0(T) | \sum_{j \in T} g_{ij}(\rho_T) | \phi_0(T) \rangle \quad (8)$$

is composed of the direct and knock-on exchange terms. The knock-on exchange process makes $U_{\text{TT}}$ nonlocal, but it can be localized with the Brieva–Rook method [6] based on the local semi-classical approximation with good accuracy [20]. In this approach, projectile excitations have to be treated explicitly by solving Eq. (7).

When projectile excitations are negligible, one can assume $\psi = \phi_0(P)\chi_0(R)$ with the ground state $\phi_0(P)$ of $P$ and the relative wave function $\chi_0(R)$. Equation (7) is then reduced to the Schrödinger equation

$$[K_R + U_{\text{DSF}}(R) - E_{\text{cm}}^\infty] \chi_0(R) = 0, \quad (9)$$

where the optical potential $U_{\text{DSF}}$ is obtained by

$$U_{\text{DSF}}(R) = \langle \phi_0(P) | \sum_{i \in P} U_{\text{TT}} | \phi_0(P) \rangle. \quad (10)$$

The potential $U_{\text{DSF}}$ is local, since $U_{\text{TT}}$ is localized. This folding procedure is the DSF. The difference between the DSF and SF models comes from the difference of $U_{\text{TT}}$. In the DSF model, $U_{\text{TT}}$ is a microscopic potential obtained by folding the Melbourne g-matrix with $\rho_T$, but $U_{\text{TT}}$ is a phenomenological nucleon-nucleon potential in the SF model. The Coulomb potential $U_{\text{ Coul}}$ is added to $U_{\text{DSF}}$ in actual calculations. It is reported in Ref. [36] that the DSF model works well for $^4$He scattering.

Another approach to AA scattering is the DF-FDA model. In this approach, the g-matrix $g(\rho_P + \rho_T)$ includes both projectile- and target-excitation effects approximately. The optical potential of P+T scattering is then obtained from Eq. (5) as

$$U_{\text{DF}}^{\text{FDA}}(R) = \langle \phi_0 | \sum_{i \in P,j \in T} g_{ij}(\rho_P + \rho_T) | \phi_0 \rangle \quad (11)$$

for $\phi_0 = \phi_0(P)\phi_0(T)$. The potential is also composed of the direct and knock-on exchange terms. The nonlocality coming from the knock-on exchange process can be localized by the Brieva–Rook method with good accuracy [21]. In actual calculations, $U_{\text{ Coul}}$ is added to the localized $U_{\text{DF}}^{\text{FDA}}$. This is the DF-FDA model.

Comparing Eq. (10) with Eq. (11), one can see that the difference between the DSF and DF-FDA potentials mainly comes from that between the TDA and the FDA; see Appendix A for the detail. Since $\rho_P + \rho_T > \rho_T$, nuclear-medium effects are smaller in the DSF potential than in the DF-FDA potential. This makes the DSF potential more attractive and more absorptive than the DF-FDA potential.

As for the central part of DSF and DF potentials, the explicit forms are summarized in Appendix A. In general, the $^3$He optical potential has the spin-orbit part in addition to the central part. General derivation of the spin-orbit part for AA scattering was shown in Ref. [39]. For $^3$He scattering, the spin-orbit part was calculated in Ref. [40, 41] for the DF model and in Ref. [41] for the SF model. Although adjustable parameters are introduced in the analyses, spin-orbit effects on differential cross sections are similar between the two models. We then derive the spin-orbit part of $^3$He optical potential only in the DSF model. In $^3$He, the two-proton subsystem is considered to be spin-singlet with good accuracy. Hence the spin-orbit force $U_{\text{DSF}}^{LS}(R)\ell \cdot s$ of $^3$He scattering at an incident energy $E_{\text{in}}$ in the laboratory system is obtained by folding the spin-orbit force $U_{\text{DSF}}^{LS}(r)\ell \cdot s$ of neutron scattering at an incident energy $E_{\text{in}}/3$ with the neutron wave function $\phi_n(t)$ in $^3$He, where $s$ is the neutron spin, $\ell$ (t) is the coordinate of neutron from the center of mass of $T$ ($^3$He) and $L$ (t) is the angular momentum with respect to $R$ (r). Assuming that $\phi_n(t)$ is an s-state, one can get

$$U_{\text{DSF}}^{LS}(R)\ell \cdot s = \int \phi_n(t)^* U_{\text{ DSF}}^{LS}(r)\ell \cdot s \phi_n(t) dt, \quad (12)$$

with

$$Z(R) = \int e^{iK \cdot R} \frac{1}{K} \frac{d\hat{\Phi}(K)}{dK} \hat{\Phi}(K) dK, \quad (13)$$

$$\hat{\Phi}(K) = \frac{1}{(2\pi)^3} \int e^{-iK \cdot r} U_{\text{DSF}}^{LS}(r) dr, \quad (14)$$

$$\hat{\phi}(t) = \int e^{iK \cdot t} \phi_n(t) dt. \quad (15)$$

where the explicit form of $U_{\text{DSF}}^{LS}(r)$ is shown in Ref. [12]. Finally, three-body breakup effects of $^3$He are taken into account with four-body CDCC [3]. Four-body dynamics of the $N+N+N+T$ system is treated in a model
where $\phi_\gamma(P)$ is the $\gamma$th eigenstate with an eigenenergy $\epsilon_\gamma(P)$ obtained by diagonalizing $h_F$ by the Gaussian basis functions and among the eigenstates the $\phi_{\gamma_m}(P)$ has the highest eigenenergy in the model space $\mathcal{P}$; note that $P$ is $^3$He in the present case. This model-space approximation reduces Eq. (7) to

$$
\mathcal{P}[H_{\text{eff}} - E_{\text{cm}}^m - \epsilon_0(P)]\mathcal{P}\psi = 0,
$$

where

$$
\mathcal{P}\psi = \sum_{\gamma=0}^{7m} \phi_\gamma(P)\chi_\gamma(R).
$$

This leads to the CDCC equation for $\chi_\gamma(R)$ as

$$
[E_{\text{cm}} - K_R - (\epsilon_\gamma(P) - \epsilon_0(P))]\chi_\gamma(R) = \sum_{\gamma'} \langle \phi_{\gamma'}(P) | \sum_{i\in P} U_{iT} | \phi_{\gamma}(P) \rangle \chi_{\gamma'}(R).
$$

As shown later in Sec. III, $^3$He breakup effects are appreciable only at lower incident energies around 40 MeV/nucleon and negligible at higher incident energies. Therefore, we estimate the breakup effects as simply as possible. For this purpose, we assume that $^3$He is a spinless particle, and take the Minnesota force [42] as the nucleon-nucleon interaction in $^3$He and introduce a three-body force to reproduce the binding energy of $^3$He. In CDCC calculations, $0^+$ and $2^+$ states are considered as breakup states, since the contribution of $1^-$ breakup states is confirmed to be negligibly small. For each of $0^+$ and $2^+$, we take 35 breakup states below 20 MeV. The model space spanned by the ground and breakup states is confirmed to give the convergence of CDCC solutions for the elastic and reaction cross sections.

### III. RESULTS

We consider $^3$He scattering from $^{58}\text{Ni}$ and $^{208}\text{Pb}$ targets in a wide incident-energy range of $30 \lesssim E_{\text{in}}/A_P \lesssim 150$ MeV, where $E_{\text{in}}$ is the incident energy in the laboratory system. First the scattering are analyzed with the DSF and DF-FDA models in which the spin-orbit force is not included. As the $^3$He density $(\rho_P)$, we consider the density calculated with the three-nucleon model mentioned in Sec. II and the phenomenological density determined from electron scattering [43]. For the latter, finite-size effects due to the proton charge are unfolded in the standard manner [44], and the neutron density is assumed to have the same geometry as the proton one. Both the densities yield almost the same differential cross section, so we will take the former density to compare results of the DSF model with those of CDCC later. The target density $\rho_T$ is calculated with the spherical Hartree-Fock (HF) method with the Gogny-D1S interaction [45] in which the spurious center-of-mass motion is removed with the standard procedure [19].

The Melbourne interaction is provided only up to $k_F = 1.5$ fm$^{-1}$ ($\rho = 1.3\rho_0$), where $k_F$ is the Fermi momentum and $\rho_0$ is the normal density. We then assume that the Melbourne interaction at $k_F > 1.5$ fm$^{-1}$ is the same as that at $k_F = 1.5$ fm$^{-1}$. This assumption does not affect any result of DSF calculations, since the $g$-matrix at $k_F > 1.5$ fm$^{-1}$ yields small effects only on differential cross sections at larger angles in DF-FDA calculations.

Figure 1 shows $\sigma_R$ for $^3$He scattering from $^{58}\text{Ni}$ and $^{208}\text{Pb}$ targets in a range of $E_{\text{in}}/A_P = 30$–$150$ MeV. The DSF model (circles) yields better agreement with the experimental data [46] than the DF-FDA model (squares). For $^{58}\text{Ni}$ target, the DSF model slightly underestimates the data around $E_{\text{in}}/A_P = 30$ MeV, but this underestimation is solved by projectile breakup effects, as shown later in Fig. 3(b).

![Figure 1](image)

FIG. 1: (Color online) Total reaction cross section $\sigma_R$ as a function of $E_{\text{in}}/A_P$ for (a) $^3\text{He}+^{58}\text{Ni}$ scattering and (b) $^3\text{He}+^{208}\text{Pb}$ scattering. The circles (squares) stand for results of the DSF (DF-FDA) model. The spin-orbit force is not included in both the models. The experimental data are taken from [46].

Figure 2 shows differential cross sections $d\sigma/d\Omega$ as a function of transfer momentum $q$ for $^{58}\text{Ni}$ and $^{208}\text{Pb}$ tar-
gets. For $E_{i\mathrm{n}}/A_P \approx 40$ and 150 MeV, the DSF model (solid line) definitely yields better agreement with the experimental data [47–51] than the DF-FDA model (dashed line). For $E_{i\mathrm{n}}/A_P = 72$ MeV, agreement with the data is comparable between the two models; more precisely, the DSF model is better than the DF-FDA model at $q < 2$ fm$^{-1}$, but the latter is superior to the former at $q > 2$ fm$^{-1}$. The overestimation of the DSF model at $q > 2$ fm$^{-1}$ comes from the fact that knock-on processes are treated only approximately in the DSF model, as discussed later in Fig. 5. The DSF model thus yield better description for $^3$He scattering than the DF-FDA model.

Figure 3 shows projectile-breakup and spin-orbit force effects on $d\sigma/d\Omega$ and $\sigma_R$ for $^3$He+$^58$Ni scattering. In panel (a) for $d\sigma/d\Omega$, the dot-dashed and dashed lines stand for results of the DSF model with and without the spin-orbit force respectively, while the solid lines denote results of CDCC in which the spin-orbit force is neglected. Hence the difference between the dashed and solid lines shows projectile-breakup effects, while that between the dashed and dot-dashed lines corresponds to spin-orbit force effects. Both the effects are small, but improve agreement with the data. More precisely, spin-orbit force effects are appreciable at higher incident energies around $E_{i\mathrm{n}}/A_P = 150$ MeV, but projectile-breakup effects are visible at lower incident energies such as $E_{i\mathrm{n}}/A_P = 40$ MeV. The present result on spin-orbit force effects is consistent with the previous one of Ref. [41].

In panel (b) for $\sigma_R$, the circles denote results of the DSF model without the spin-orbit force, while the triangles correspond to results of CDCC in which the spin-orbit force is neglected. In this panel, results of the DSF model with the spin-orbit force are not shown, since they agree with those of the DSF model without the spin-orbit force. This indicates that spin-orbit force effects are negligibly small. Projectile-breakup effects are shown by the difference between the triangle and the corresponding circle. The effects are more appreciable for $\sigma_R$ than for $d\sigma/d\Omega$. The effects improve agree with the experimental data particularly at low incident energies around $E_{i\mathrm{n}}/A_P = 30$ MeV.

Comparing Fig. 3 with Figs. 1 and 2, one can easily see that CDCC calculations based on the model Hamiltonian well describe $^3$He scattering and yield better agreement with the data than the DF-FDA model. The model Hamiltonian is thus good not only for deuteron and $^3$He scattering but also for $^3$He scattering.

Figure 4 shows $L$ dependence of the elastic $S$-matrix elements $S_L$. The filled (open) circles denote the $S_L$s calculated with the DSF model (CDCC), where the spin-orbit force is neglected. Projectile-breakup effects shown by the difference between the open and filled circles are sizable for large $L$ but not for small $L$. At large $L$, furthermore, the effects reduce the absolute value of $S_L$. Therefore the dynamical polarization potential generated by $^3$He breakup is strongly absorptive in the peripheral region of $T$. This is the reason why $^3$He breakup effects are more appreciable for $\sigma_R$ than for $d\sigma/d\Omega$.

In the DSF model, the nucleon-target potential $U_{i\mathrm{T}}$ is localized by the Brieva–Rook method. Now we discuss how the localization affects $^3$He scattering, although the localization is accurate for nucleon-target scattering itself as shown in Ref. [20]. When the nucleon-target potential is not localized, the potential between $P$ and $T$ is
obtained by

\begin{equation}
U_{DF}^{TDA} = \langle \Phi_0 | \sum_{i \in P, \mu \in T} g_{ij}(p_{\mu}) | \Phi_0 \rangle.
\end{equation}

This is nothing but the DF model with the TDA. This model is referred to as the DF-TDA model in this paper. The explicit form of DF-TDA potential is shown in Appendix A, together with the explicit forms of DF-FDA and DSF potentials.

In the DF-TDA model, the nonlocality of the folding potential comes from knock-on exchange processes between interacting two nucleons, one in P and the other in T. As a consequence of the exchange, the one-body densities \( \rho_P(\mathbf{r}_P) \) and \( \rho_T(\mathbf{r}_T) \) are changed into the corresponding mixed densities \( \tilde{\rho}_P(\mathbf{r}_P, \mathbf{r}_P - \mathbf{s}) \) and \( \tilde{\rho}_T(\mathbf{r}_T, \mathbf{r}_T + \mathbf{s}) \) as shown in Appendix A, where \( \mathbf{r}_P \) (\( \mathbf{r}_T \)) denotes the coordinate of nucleon in P (T) from the center of mass of P (T) and \( \mathbf{s} \) is the relative coordinate between interacting two nucleons. The one-body and mixed densities of P are defined by

\begin{align}
\rho_P^\mu(\mathbf{r}_P) &= \sum_a \phi^\mu_{P,a}(\mathbf{r}_P) \phi^{\*}_{P,a}(\mathbf{r}_P), \quad (21) \\
\tilde{\rho}_P^\mu(\mathbf{r}_P, \mathbf{r}_P - \mathbf{s}) &= \sum_a \phi^\mu_{P,a}(\mathbf{r}_P) \phi^{\*}_{P,a}(\mathbf{r}_P - \mathbf{s}) \quad (22)
\end{align}

with the single particle wave function \( \phi^\mu_{P,a}(\mathbf{r}) \) of P characterized with the quantum number \( a \), where \( \mu \) denotes either proton or neutron. One can take the same definition also for the one-body and mixed densities of T.

The knock-on exchange is properly treated in the DF-TDA model. The resulting nonlocal potential between P and T can be localized with high accuracy by the Brieva–Rook method based on the local semi-classical approximation [21]. In the DSF model, the change of \( \rho_T(\mathbf{r}_T) \) due to knock-on exchange is taken into account in the nucleon-target potential. However, the change of \( \rho_P(\mathbf{r}_P) \) is not treated as shown in Appendix A, since the nucleon-target potential is localized. Therefore, one can see how the change of \( \rho_P(\mathbf{r}_P) \) due to knock-on exchange affects \(^3\text{He}\) scattering, comparing results of the DSF model with those of the DF-TDA model.

Figure 5 shows differential cross sections calculated with the DSF, DF-TDA and DF-FDA models for (a) \(^3\text{He}+^{58}\text{Ni}\) scattering and (b) \(^3\text{He}+^{208}\text{Pb}\) scattering, where the spin-orbit force is neglected. The dashed (solid) lines correspond to results of the DF-TDA (DSF)
model. The difference between the dashed and solid lines shows effects of the $p\rho$ change due to knock-on exchange. The effects are small for higher incident energies such as $E_{in}/A_p = 150$ MeV. For lower incident energies less than $E_{in}/A_p = 100$ MeV, meanwhile, the effects appear at larger angles and improve agreement with the experimental data, particularly in $q \gtrsim 3$ fm$^{-1}$ for $^3\text{He}^+{^{58}}\text{Ni}$ scattering at $E_{in}/A_p = 40$ MeV and in $q \gtrsim 2$ fm$^{-1}$ for $^3\text{He}^+{^{58}}\text{Ni}$ scattering at $E_{in}/A_p = 72$ MeV. The DF-TDA model thus yields better agreement with the data than the DSF model particularly at the larger angles. Eventually, the DF-TDA model (dashed line) well reproduce the data in a wide incident-energy range from $E_{in}/A_p = 40$ MeV to 150 MeV, compared with the DF-FDA model (dotted-dashed line). For $\sigma_R$, furthermore, we have confirmed that the DF-TDA model yields almost the same result as the DSF model and hence better agreement with the data than DF-FDA model.

Finally, we compare DSF, DF-TDA and DF-FDA potentials in Fig. 6 for $^3\text{He}^+{^{58}}\text{Ni}$ scattering at $E_{in}/A_p = 40$ MeV. The FDA has stronger Pauli-blocking effects than the TDA because of $\rho_P + \rho_T \geq \rho_T$, so that the DF-FDA potential (dot-dashed line) is less attractive and less absorptive than the DF-TDA potential (dashed line). The DSF model (solid line) well simulates the DF-TDA potential in the peripheral region. This is the reason why the DSF model well simulates the DF-TDA model at forward angles of $q \lesssim 3$ fm$^{-1}$.

### IV. SUMMARY

We have investigated how good the model Hamiltonian (2) is for $^3\text{He}$ scattering. In the model Hamiltonian, the nucleon-target potential $U_{\text{TT}}$ is obtained by folding the Melbourne $g$-matrix with the target density and localizing the resultant nonlocal folding potential with the Brevi-Rook method. In many calculations done so far, the phenomenological nucleon optical potential was used as $U_{\text{TT}}$. As an advantage of the present approach from the previous one, the microscopic $U_{\text{TT}}$ is obtainable even for the nucleon+target system in which no experimental data is available. As an advantage of the present approach from the DF-FDA model, the model Hamiltonian is consistent with the fact that the single Fermi sphere is considered when the $g$-matrix is evaluated in nuclear matter. In the present approach, projectile excitation effects have to be treated explicitly, but this can be done with CDCC since $U_{\text{TT}}$ is localized.

The validity of the model Hamiltonian has been investigated in a wide incident-energy range of $E_{in}/A_p = 30$–150 MeV for heavier targets $^{58}\text{Ni}$ and $^{208}\text{Pb}$, since the $g$-matrix is considered to be more reliable for heavier targets. CDCC calculations well reproduce the experimental data with no adjustable parameter. The calculations show that $^3\text{He}$ breakup effects are appreciable only at lower incident energies around 40 MeV/nucleon and negligibly small at higher incident energies. Therefore the

![Figure 5](image_url) (Color online) Comparison of DSF, DF-TDA and DF-FDA models for differential cross sections in (a) $^3\text{He}^+{^{58}}\text{Ni}$ scattering and (b) $^3\text{He}^+{^{208}}\text{Pb}$ scattering. The differential cross section at each $E_{in}/A_p$ is multiplied by the factor shown in the panel. The dashed (dot-dashed) lines denote results of the DF-TDA (DF-FDA) model, while the solid lines stand for results of the DSF model. The spin-orbit force is not included in both the models. The experimental data are taken from Refs. [47–51].

The DSF model also well accounts for the experimental data. In fact, the DSF model yields better agreement with the data than the DF-FDA model. Thus we can propose that the DSF model is a practical model to describe $^3\text{He}$ scattering and can conclude that the model Hamiltonian is good not only for deuteron and $^4\text{He}$ scattering but also...
larger angles of \( q \gtrsim 2 \text{ fm}^{-1} \) and improves agreement with the experimental data there. When one is interested in the backward angles, the DF-TDA model is better than DSF model. For higher incident energies such as \( E_{\text{in}}/A_p = 150 \text{ MeV} \), meanwhile, the effects are small and hence the DSF model is as good as the DF-TDA model.

In conclusion, the model Hamiltonian (2) works well for scattering of \( 0s \)-shell nuclei such as deuteron, \(^3\text{He}\) and \(^3\text{He}\). As a future work, it is quite interesting to investigate whether the model Hamiltonian is good also for scattering of \( 0p \)-shell nuclei such as \(^{12}\text{C}\) and \(^{16}\text{O}\).

**Acknowledgments**

We thank K. Ogata and M. Kohno for useful discussions. This work is supported in part by Grant-in-Aid for Scientific Research (No. 26400278) from Japan Society for the Promotion of Science (JSPS).

**Appendix A: Explicit forms of DF-FDA, DF-TDA and DSF potentials**

Here we show the explicit forms of DF-FDA, DF-TDA and DSF potentials and discuss their relation. This is an overview of the previous discussion in Ref. [36].

The DF potential \( U_{\text{DF}} \) for \( P+T \) scattering at an incident energy \( E_{\text{in}} \) is composed of the direct and knock-on exchange terms, \( U_{\text{DF}}^{\text{DR}} \) and \( U_{\text{DF}}^{\text{EX}} \), defined by [15, 18, 19]

\[
U_{\text{DF}}^{\text{DR}}(\mathbf{R}) = \sum_{\mu,\nu} \int \rho_p(r_P) \rho_T(r_T) g_{\mu\nu}^{\text{DR}}(s;\rho_{\mu\nu}) d\mathbf{r}_P d\mathbf{r}_T, \quad (A1)
\]

\[
U_{\text{DF}}^{\text{EX}}(\mathbf{R}) = \sum_{\mu,\nu} \int \rho_{\mu}^s(r_P, r_T - s) \rho_T^s(r_T, r_T + s) g_{\mu\nu}^{\text{EX}}(s;\rho_{\mu\nu}) \exp[-i\mathbf{K}(\mathbf{R}) \cdot \mathbf{s}/M] d\mathbf{r}_P d\mathbf{r}_T, \quad (A2)
\]

where \( s = r_P - r_T + \mathbf{R} \) and \( \mu \) and \( \nu \) denote either proton or neutron. The non-local \( U_{\text{EX}} \) has been localized with the local semi-classical approximation [6], where the local momentum \( \mathbf{hK}(\mathbf{R}) \) between \( P \) and \( T \) is given by \( \mathbf{hK}(\mathbf{R}) = \sqrt{2M(E_{\text{in}} - U_{\text{DF}}(\mathbf{R}))} \) with the reduced mass \( M = A_P A_T/(A_P + A_T) \) of the P+T system. The direct and exchange parts, \( g_{\mu\nu}^{\text{DR}} \) and \( g_{\mu\nu}^{\text{EX}} \), of g-matrix depend on the density \( \rho_{\mu\nu} \) defined by

\[
\rho_{\mu\nu} = \rho_P^\mu(r_P - s/2) + \rho_T^\nu(r_T + s/2) \quad (A3)
\]

in the FDA and

\[
\rho_{\mu\nu} = \rho_P^\mu(r_T + s/2) \quad (A4)
\]

in the TDA. The difference between the DF-FDA and DF-TDA stems from only that between Eqs. (A3) and (A4).
For N+T scattering at an incident energy $E^N_{in}$, the SF potentials $U_{\mu}$ for proton ($\mu = -1/2$) and neutron ($\mu = 1/2$) scattering also consist of the direct and knock-on exchange terms, $U^{DR}_{\mu}$ and $U^{EX}_{\mu}$, defined by [20]

$$U^{DR}_{\mu}(r_\mu) = \sum_\nu \int \rho^\mu_P(r_P)g^{\mu\nu}_\mu(s; \rho_{\mu\nu})dr_P,$$  

(A5)

$$U^{EX}_{\mu}(r_\mu) = \sum_\nu \int \rho^\mu_P(r_P)g^{\mu\nu}_\mu(s; \rho_{\mu\nu})\exp[-iK_{\mu}(r_\mu) \cdot s]dr_P,$$  

(A6)

for $s = r_\mu - r_T$, where $r_\mu$ is the coordinate of an incident nucleon N from the center of mass of T and the local momentum $hK_{\mu}(r_\mu)$ is given by $hK_{\mu}(r_\mu) \equiv \sqrt{2\mu_{NT}(E^N_{in} - U_{\mu}(r_\mu))}$ for the reduced mass $\mu_{NT}$ of the N+T system.

The DSF potential $U_{DSF}$ is obtained by folding the nucleon-target potentials $U^{DR}_{\mu} + U^{EX}_{\mu}$ with the projectile density $\rho^\mu_P$. The direct and knock-on exchange parts are then given by

$$U^{\mu}_{DSF}(R) = \sum_\mu \int \rho^\mu_P(r_P)U^{DR}_{\mu}(R + r_P)dr_P,$$  

$$= \sum_\mu \int \rho^\mu_P(r_P)\rho^\mu_P(r_T)g^{\mu\mu}(s; \rho_{\mu\nu})dr_Pdr_T, \quad (A7)$$

$$U^{EX}_{DSF}(R) = \sum_\mu \int \rho^\mu_P(r_P)U^{EX}_{\mu}(R + r_P)dr_P,$$  

$$= \sum_\mu \int \rho^\mu_P(r_P)\rho^\mu_P(r_T)g^{\mu\mu}(s; \rho_{\mu\nu})\exp[-iK_{\mu}(r_\mu) \cdot s/\mu]dr_Pdr_T, \quad (A8)$$

Now we consider heavy targets satisfying $A_T \gg A_P > 1$ for simplicity. The $g$ matrix depends on an energy of nucleon in P. The energy is $E^T_{in}$ for N+T scattering and $E^N_{in}/A_P$ for P+T scattering. We can then find that $U^{DR}_{DSF - TDA} = U^{DR}_{DSF}$ when $E^N_{in} = E^N_{in}/A_P$. In the peripheral region of T important for elastic scattering, the local momenta $hK_{\mu}(r_\mu)$ and $hK(R)$ are close to their asymptotic values, $hK_{\mu}(\infty)$ and $hK(\infty)$, respectively. When $E^N_{in} = E^N_{in}/A_P$, the asymptotic values satisfy

$$K_{\mu}(\infty) = K(\infty)/M.$$

Therefore we see that $U^{DR}_{DSF - TDA} = U^{DR}_{DSF}$ if $\rho^\mu_P(r_P, r_P - s)$ is identical with $\rho^\mu_P(r_P)$. The approximation $\rho^\mu_P(r_P, r_P - s) \approx \rho^\mu_P(r_P)$ is known to be good in the peripheral region of T [20].

[1] M. Kamimura, M. Yahiro, Y. Iseri, Y. Sakuragi, H. Kameyama, and M. Kawai, Prog. Theor. Phys. Suppl. 89, 1 (1986).

[2] N. Austern, Y. Iseri, M. Kamimura, M. Kawai, G. Rawitscher, and M. Yahiro, Phys. Rep. 154, 125 (1987).

[3] M. Yahiro, K. Ogata, T. Matsumoto, and K. Minomo, Prog. Theor. Exp. Phys. 2012, 01A206 (2012).

[4] G. Bertsch, J. Borysowicz, H. McManus, and W. G. Love, Nucl. Phys. A 284, 399 (1977).

[5] J.-P. Jeukenne, A. Lejeune, and C. Mahaux, Phys. Rev. C 16, 80 (1977);

J.-P. Jeukenne, A. Lejeune, and C. Mahaux, Phys. Rep. 25, 83 (1976).

[6] F. A. Brieva and J. R. Rook, Nucl. Phys. A 291, 299 (1977); ibid. 291, 317 (1977); ibid. 297, 206 (1978).

[7] G. R. Satchler and W. G. Love, Phys. Rep. 55, 183 (1979).

[8] G. R. Satchler, “Direct Nuclear Reactions”, Oxford University Press, (1983).

[9] N. Yamaguchi, S. Nagata, and T. Matsuda, Prog. Theor. Phys. 70, 459 (1983); N. Yamaguchi, S. Nagata, and J. Michiyama, Prog. Theor. Phys. 76, 1289 (1986).

[10] L. Rikus, K. Nakano, and H. V. Von Geramb, Nucl. Phys. A 414, 413 (1984); L. Rikus, and H. V. Von Geramb, Nucl. Phys. A 426, 496 (1984).

[11] K. Amos, P. J. Dortmans, H. V. Von Geramb, S. Karagulidis, and J. Raynal, in Advances in Nuclear Physics, edited by J. W. Negele and E. Vogt (Plenum, New York, 2000) Vol. 25, p. 275.

[12] T. Furumoto, Y. Sakuragi, and Y. Yamamoto, Phys. Rev. C 78, 044610 (2008).

[13] Y. Yamamoto, T. Furumoto, N. Yasutake, and Th. A. Rijken, Phys. Rev. C 88, 022801 (2013).

[14] S. M. Salem and W. Haider, J. Phys. G 28, 1313 (2002).

[15] B. Sinha, Phys. Rep. 20, 1 (1975).

B. Sinha and S. A. Moseskovski, Phys. Lett. B81, 289.
(1979).

[16] H. F. Arellano, F. A. Brieva, and W. G. Love, Phys. Rev. C 52, 301 (1995).

[17] D. T. Khoa, W. von Oertzen, H. G. Bohlen, and S. Ohkubo, J. Phys. G 34, R111 (2007).

[18] T. Furumoto, Y. Sakuragi, and Y. Yamamoto, Phys. Rev. C 82, 044612 (2010).

[19] T. Sumi et al., Phys. Rev. C 85, 064613 (2012).

[20] K. Minomo, K. Ogata, M. Kohno, Y. R. Shimizu, and M. Yahiro, J. Phys. G 37, 085011 (2010) [arXiv:0911.1184 [nucl-th]].

[21] K. Hagino, T. Takehi, and N. Takigawa, Phys. Rev. C 74 (2006), 037601.

[22] A. J. Koning and J. P. Delaroche, Nucl. Phys. A 713 231 (2003).

[23] S. Hama, B. C. Clark, E. D. Cooper, H. S. Sherif, and R. L. Mercer, Phys. Rev. C 41, 2737 (1990).

[24] E. D. Cooper, S. Hama, B. C. Clark, and R. L. Mercer, Phys. Rev. C 47, 297 (1993).

[25] C. M. Perey and F. G. Perey, At. Data Nucl. Data Tables 17, 1 (1976).

[26] M. Toyokawa, K. Minomo, and M. Yahiro, Phys. Rev. C 88, 054602 (2013).

[27] M. Yahiro, K. Minomo, K. Ogata, and M. Kawai, Prog. Theor. Phys. 120, 767 (2008).

[28] K. M. Watson, Phys. Rev. 89, 575 (1953).

[29] A. K. Kerman, H. McManus, and R. M. Thaler, Ann. Phys. 8, 551 (1959).

[30] T. Izumoto, S. Krewald, and A. Faessler, Nucl. Phys. A 341, 319 (1980).

[31] S. Watanabe et al., Phys. Rev. C 89, 044610 (2014).

[32] M. Takechi et al., Phys. Lett. B 707, 357 (2010).

[33] M. Takechi et al., Phys. Rev. C 90, 061305(R) (2014).

[34] K. Minomo, T. Sumi, M. Kimura, K. Ogata, Y. R. Shimizu, and M. Yahiro, Phys. Rev. C 84, 034602 (2011).

[35] K. Minomo, T. Sumi, M. Kimura, K. Ogata, Y. R. Shimizu, and M. Yahiro, Phys. Rev. Lett. 108, 052503 (2012).

[36] K. Egashira, K. Minomo, M. Toyokawa, T. Matsumoto and M. Yahiro, Phys. Rev. C 89, 064611 (2014).

[37] G. Takeda and K. M. Watson, Phys. Rev. 97, 1336(1955).

[38] A. Picklesimer and R. M. Thaler, Phys. Rev. C23, 42(1981).

[39] F. Petrovich, R. J. Philpott, A. W. Carpenter and J. A. Carr, Nucl. Phys. A 425, 609 (1984).

[40] J. Cook, Nucl. Phys. A 465, 207 (1987).

[41] Y. Sakuragi, M. Katuma, Nucl. Instrum. Methods Phys. Res. A 402, 347 (1998).

[42] Y. C. Tang, M. LeMere, and D. R. Thompson, Phys. Rep. 47, 167 (1978).

[43] H. de Vries, C. W. de Jager, and C. de Vries, At. Data Nucl. Data Tables 36, 495 (1987).

[44] R. P. Singhai, M. W. S. Macauley, and P. K. A. De Witt Huberts, Nucl. Instr. and Meth. 148, 113 (1978).

[45] J. F. Berger, M. Girod, and D. Gogny, Comput. Phys. Commun. 63, 365 (1991).

[46] A. Ingemarsson et al., Nucl. Phys. A 696, 3 (2001)

[47] M. Hyakutake et al., Nucl. Phys. A 333, 1 (1980).

[48] N. Willis, I. Brillaud, Y. Le Bornec, B. Tatischeff, and G. Duhamel, Nucl. Phys. A 204, 454 (1973).

[49] J. Kamiya et al., Phys. Rev. C 67, 064612 (2003).

[50] A. Djalois, J. -P. Didelez, A. Galonsky, and W. Oelert, Nucl. Phys. A 306, 221 (1978).

[51] T. Yamagata et al., Nucl. Phys. A 589, 425 (1995).