Spin detection at elevated temperatures using a driven double quantum dot

G. Giavaras, 1 J. Wabnig, 1, 2 B. W. Lovett, 1 J. H. Jefferson, 3 and G. A. D. Briggs 1

1Department of Materials, University of Oxford, Oxford OX1 3PH, UK
2Cavendish Laboratory, Department of Physics, University of Cambridge, Cambridge CB3 0HE, UK
3QinetiQ, St. Andrews Road, Malvern WR14 3PS, UK

We consider a double quantum dot in the Pauli blockade regime interacting with a nearby single spin. We show that under microwave irradiation the average electron occupations of the dots exhibit resonances that are sensitive to the state of the nearby spin. The system thus acts as a spin meter for the nearby spin. We investigate the conditions for a non-demolition read-out of the spin and find that the meter works at temperatures comparable to the dot charging energy and sensitivity is mainly limited by the intradot spin relaxation.

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I. INTRODUCTION

Electron spins in semiconductors and molecular systems are good candidates for qubits due to their relatively long coherence times. Manipulation of single spins and controlled interaction between pairs of spins are essential ingredients for quantum information processing. Single spin rotation has been demonstrated in electrostatically defined quantum dots using the electron spin resonance technique. 1 Coherent manipulation of a pair of qubits, giving rise to entanglement, has also been achieved in a semiconductor double dot (DD) device based on fast electrical pulses and operating the dots in the spin blockade regime. 2 Single spin rotations together with entanglement generation in principle enable universal quantum operations. A further operation for a quantum processor is spin detection, which is essential for projection of the quantum state after computation and read-out of the result. This is the main focus of this paper.

Single spin detection is also important for future spintronic devices in general, and various electrical and optical schemes have been proposed and demonstrated. For example Elzerman et al. demonstrated experimentally a single-shot read-out of a quantum dot spin using a spin to charge conversion technique, 3 while Rugar et al. employed magnetic resonance force microscopy to probe the state of a single spin. 4 Other schemes for spin read-out involve open quantum dots with an inhomogeneous Zeeman splitting and closed DD systems which are coupled to quantum point contacts. 5, 6 It has been shown theoretically that the dc-electrical current and shot noise through the dots or the point contacts can provide valuable information about the spin state, the energy spectrum and the relevant decoherence rates. 5, 6 However, for optically non-active molecular spins no reliable read-out scheme exists.

In this work we consider a single spin (target spin) that interacts with the spins of two tunnel-coupled quantum dots, as shown schematically in Fig. 1. We use microwave irradiation to control the spin dynamics of both, and demonstrate how to probe its state by monitoring the average electron occupation on one of the two dots. The interaction between the target spin and the spins on the DD induces an effective Zeeman splitting that is different in each dot. Also, the sign of the Zeeman splitting depends on the orientation of the target spin. This target-spin dependent asymmetry of the Zeeman splitting makes it possible to rotate only one of dot spins and thus results in a target-spin dependent lifting of the Pauli blockade. We show this lifting of the blockade as a change in the average dot occupation, which can be measured by a charge detector.

In order to realize a non-demolition spin measurement, we consider a system where the target spin is spin detected, which is essential for projection of the spin state after computation and read-out of the result. This is the main focus of this paper.

FIG. 1: Schematic illustration of the proposed system for a single spin detection. Two tunnel-coupled quantum dots are connected to leads enabling current to pass through. A nearby spin interacts with the spins on dot 1 and this interaction induces a different Zeeman splitting in the two dots. In an external magnetic field and under microwave irradiation the spins on the two dots can be rotated with a Rabi frequency Ω and the average electron occupation exhibits resonances which are sensitive to the state of the nearby spin.

A charge detector is capacitively coupled to dot 1 and is used to monitor a change in the current through the double dot, relative to the incident microwave field, and the change in the average electron occupation at node 1 and node 2 with respect to the incident microwave field leads to the Zeeman splitting, which depends on the orientation of the nearby spin and the interaction strength J.
the spins on the DD and the target spin must have different Zeeman splitting, most likely to be achieved by different $g$-factors through $g$-factor engineering or choice of materials. A general spin-spin interaction always contains so-called spin-flip terms, which are suppressed when the difference in the Zeeman splitting of the dot electrons and the target spin is larger than the spin-spin interaction strength.

A theoretical investigation has shown that a single quantum dot works as a spin meter, but the proposed device has the drawback of operating only at low temperatures, comparable to the energy scale set by the Zeeman splitting. In this article we show that the driven DD spin detector works at much higher temperatures, comparable to the dot charging energy and we find that the main limitation to its sensitivity is intradot spin relaxation. Thus we present a scheme that allows the spin state of a nearby spin to be probed noninvasively in a single shot, using different electron occupations on the double dot for spin up and spin down orientations as a basic readout mechanism.

In the following section we introduce our model and discuss technical details of the solution. In Sec. [III] we show how the double dot acts as a spin meter and explore how interdot hopping, microwave intensity, temperature, spin-spin interaction strength and spin relaxation influence its performance. We conclude in Sec. [IV] by discussing the operation of the spin meter for experimentally accessible parameters.

II. PHYSICAL MODEL

The total system consists of the DD, a nearby spin, metallic leads and a bosonic heat bath. This system is modelled by the Hamiltonian

$$H_{tot} = H_S + H_{leads} + H_T + H_B + H_{SB},$$

where $H_S$ models the DD and the spin, $H_{leads}(H_B)$ models the leads (heat bath) and $H_T(H_{SB})$ models the interaction between the leads (heat bath) and the DD. Specifically, for the DD and the nearby target spin we write the Hamiltonian as

$$H_S = H_{DD} + H_M + H_I,$$

where $H_{DD}$ is a Hubbard Hamiltonian describing the DD, $H_M$ is due to the applied magnetic fields and $H_I$ models the interaction of the nearby spin with the DD system. For the DD we have

$$H_{DD} = \sum_{i} \varepsilon_i n_i - \gamma \sum_{\sigma} (c_{1\sigma}^\dagger c_{2\sigma} + c_{2\sigma}^\dagger c_{1\sigma})$$

$$+ U \sum_{i=1}^{2} n_{i\uparrow} n_{i\downarrow} + V n_1 n_2,$$

that allows up to 2 electrons per dot. The number operator is $n_i = \sum_{\sigma} n_{i\sigma} = \sum_{\sigma} c_{i\sigma}^\dagger c_{i\sigma}$ for dot $i = \{1, 2\}$ and spin $\sigma = \{\uparrow, \downarrow\}$. The operator $c_{i\sigma}^\dagger$ ($c_{i\sigma}$) creates (annihilates) an electron on dot $i$ with on-site energy $\varepsilon_i$. $\gamma$ is the tunnel coupling between the two dots, $U$ is the charging energy (intradot Coulomb energy) and $V$ the interdot Coulomb energy. The Hamiltonian part due to the applied magnetic fields, that breaks the spin degeneracy, is

$$H_M = \sum_{i=0}^{2} \frac{\Delta_i}{2} \sigma_i^z + \sum_{i=1}^{2} \hbar \Omega \cos(\omega_0 t) \sigma_i^x,$$

where $i = 0$ refers to the target spin and the spin operators are defined in the standard way $\sigma_i = \sum_{\sigma} c_{i\sigma}^\dagger \sigma_{\sigma\sigma}^\prime c_{i\sigma}$, with $\sigma$ being the vector of the $2 \times 2$ Pauli matrices. $\Delta_i = g_i \mu_B B_i$ is the Zeeman splitting due to a static magnetic field $B_i$ along $z$, and a $g$-factor $g_i$. $\Omega$ is the Rabi frequency and $\omega_0$ the frequency of the oscillating magnetic field along $x$. For a single spin the oscillating magnetic field rotates the $z$-component of the spin with frequency $\Omega$ when $\Delta = \hbar \omega_0$. We have ignored the effect of the oscillating field on the target spin which is a good approximation for narrow-band radiation that is only resonant with the spins on the DD (or alternatively only with the target spin), a condition that can be achieved for example by engineering different $g$-factors in the dots and the target spin.

Moreover, we assume that the target spin interacts only with dot 1, although the basic idea can be extended to the most general case when the target spin interacts with both dots. As shown below our scheme is still efficient provided that the strength of the interaction between the target spin and each dot is different, a condition that is typically satisfied. We consider an Ising interaction between dot $i = 1$ and the target spin $i = 0$ of the form

$$H_I = J \frac{1}{2} \sigma_0^x \sigma_1^z,$$

with $J$ being the strength of the interaction. $J$ mainly depends on the distance of dot 1 from the target spin as well as the actual size of the dot and the target spin. Physical values for $J$ for a purely dipolar interaction are within the range of a few MHz as shown in Ref. 7. This form of interaction is justified when there is negligible tunnel-coupling between dot 1 and the nearby spin so that a good approximation hopping can be ignored. In addition, spin-flip processes are weak due to the Zeeman splitting induced by the static magnetic field and thus neglected. This is a good approximation when the difference in the Zeeman splittings between the target spin and the DD spins is much larger than the interaction strength $J$. Under these conditions the Ising interaction Eq. (4) is a reasonable choice and leads to a non-demolition measurement.

The choice of Ising interaction dictates that the combination of quantum dot system and target electron has to be specifically tailored to realise this non-demolition measurement. The necessary regime of parameters might
be difficult to realise in a gate-defined quantum dot system, e.g. in GaAs, but arises quite naturally in carbon nanotube dots probing a molecular spin. For example with a typical dipole-dipole spin interaction strength of 5 MHz, a typical difference in Zeeman splitting of about 5% and a typical EPR Zeeman splitting of 10 MHz (see also Ref. 7) we arrive at a ratio of coupling strength to relaxation rate, thus making a non-demolition measurement possible. The left and right leads are described by a Hamiltonian of the form $H_{\text{leads}} = \sum_{\ell k\sigma} t_{\ell k} \sigma^\dagger \ell \sigma_{k\sigma}$, while $d_\ell^\dagger (d_\ell)$ creates (destroys) an electron in lead $\ell = \{L, R\}$ with momentum $\mathbf{k}$, spin $\sigma$ and energy $\epsilon_\ell \sigma$. The interaction between the dots and the leads is given by the tunneling Hamiltonian

$$H_T = \sum_{k\sigma} (t_L c^\dagger_{1\sigma} d_L^{k\sigma} + t_R c^\dagger_{2\sigma} d_R^{k\sigma}) + \text{H.c.},$$

where $t_L (t_R)$ is the tunnel coupling between dot 1(2) and lead $L (R)$ and we consider the symmetric case where $t_L = t_R$.

To take into account spin relaxation we have considered a generic bosonic bath that is modelled as a set of harmonic oscillators and is described by the Hamiltonian $H_B = \sum_j \hbar \omega_j a^\dagger_j a_j + \sum_j \hbar \omega_{2j} a^\dagger_{2j} a_{2j}$. We have assumed that each quantum dot is coupled to an independent bosonic bath and there are no environment-induced correlations between the two dots. The operators $a^\dagger_{1j}$ ($a_{1j}$) create (destroy) a boson in mode $j$ and similarly for $a^\dagger_{2j}$ ($a_{2j}$), while $\omega_{1j}$ are the corresponding frequencies of the bath modes. The interaction between the baths and the spins of the DD is given by the general model Hamiltonian

$$H_{SB} = \sigma^\dagger_i \sum_j \Lambda_{i1j} a^\dagger_{1j} + \sigma^\dagger_2 \sum_j \Lambda_{2j} a^\dagger_{2j} + \text{H.c.},$$

where the spin-flip operators are $\sigma^\dagger_i = c^\dagger_i c_i\dagger$ and $\Lambda_{i1j} (\Lambda_{2j})$ is the coupling constant between dot 1(2) and the $j$th mode of the corresponding bath. $H_{SB}$ allows spin-flip processes for electrons in the DD via energy exchange with the bath which, as shown in the next section, leads to a leakage current. We consider spin relaxation only in the DD since for the target spin an upper limit to its relaxation rate is set implicitly by the coupling to the leads. In our scheme the relaxation rate of the target spin has to be smaller than the electron tunneling rate.
from the leads to the DD to ensure a measurable change in the DD occupations before spin relaxation.

To investigate the electron occupation of the system we employ a master equation approach and derive an equation of motion for the reduced density matrix, \( \rho \), for the system of interest that consists of the DD and the target spin. The occupation probabilities are given by the diagonal elements of \( \rho \). It is convenient to eliminate the time dependence from the system Hamiltonian \( H_S^0 \) and for this reason we perform a rotating wave approximation. This approximation is well-justified only for weak driving, i.e., when \( \Omega \ll \omega_0 \) as in our system. In the rotating frame an arbitrary system operator \( \mathcal{K} \) is transformed as \( U^\dagger \mathcal{K} U_z \) with the unitary operator \( U_z = \exp(-i\sigma^z\omega_0t/2) \) and \( \sigma^z = \sum_{i=0}^2 \sigma_i^z \).

Starting with the total density matrix, \( \chi \), and within the standard Born and Markov approximations we derive an equation of motion for \( \rho \) by tracing over the leads and bosonic bath degrees of freedom, i.e., \( \rho = \text{Tr}_E\{\chi\} \) where \( \text{Tr}_E\{\cdots\} \) means trace over the environmental degrees of freedom. In the rotating frame and having performed a rotating wave approximation the density matrix \( \rho \) satisfies the equation of motion

\[
\dot{\rho}(t) = \mathcal{L}_S \rho(t) + \mathcal{L}_{leads} \rho(t) + \mathcal{L}_B \rho(t),
\]

(8)

with the free evolution term

\[
\mathcal{L}_S \rho(t) = -\frac{i}{\hbar} [H_S^0, \rho(t)],
\]

and the terms due to the electronic leads

\[
\mathcal{L}_{leads} \rho(t) = -\frac{1}{\hbar^2} \text{Tr}_E \left\{ \int_0^\infty dt \left[ H_T(t), \left[ U(\tau) H_T(t-\tau) U^\dagger(\tau), \rho(t) \otimes \rho_{leads} \right] \right] \right\},
\]

and the bosonic bath

\[
\mathcal{L}_B \rho(t) = -\frac{1}{\hbar^2} \text{Tr}_E \left\{ \int_0^\infty dt \left[ H_{SB}(t), \left[ V(\tau) H_{SB}(t-\tau) V^\dagger(\tau), \rho(t) \otimes \rho_{B} \right] \right] \right\}.
\]

The operators are \( U(\tau) = \exp[-i(H_S^0 + H_{leads})\tau/\hbar] \) and \( V(\tau) = \exp[-i(H_S^0 + H_B)\tau/\hbar] \), with \( \rho_{leads}, \rho_{B} \) being the equilibrium density matrix for the leads and the bosonic bath respectively. The time dependent operators are \( c_i(t) = c_i \exp(-i\omega_0 t/2) \), \( c_{i_L}(t) = c_{i_L} \exp(+i\omega_0 t/2) \) and the Hamiltonian \( H_S^0 \) depends on the nearby spin, \( \sigma = +i(1-1) \) for spin up(down), i.e.,

\[
H_S^0 = H_{DD} + \delta_0 \frac{1}{2} \sigma^z + \frac{\sigma^z}{2} \frac{\sigma^z}{2} + \sum_{i=1}^2 \frac{\hbar\Omega}{2} \sigma^z_i, \quad (9)
\]

where we have introduced the magnetic field detuning \( \delta_0 = \Delta_1 - \hbar\omega_0 \).

For the numerical calculations we write Eq. (5) in the energy basis. This results in a system of 256 coupled equations for all the matrix elements of \( \rho \) which is solved numerically taking into account the normalisation condition for the diagonal elements, \( \sum_{i=1}^8 \rho_{ii} = 1 \). We are interested in the steady state, \( \rho_{st} \), that corresponds to \( \dot{\rho} = 0 \) in Eq. (5). The quantity of interest is the average electron occupation of the DD, for example of dot 1, that is calculated as \( N_1 = \text{Tr}\{n_1\rho_{st}\} \).

In the next section we present the basic results and explain the influence of various system parameters on the average electron occupation of the DD.

### III. RESULTS AND DISCUSSION

Before we examine the influence of microwave radiation we have to make a choice for the operating regime of the DD. DD systems and their physical response are highly tunable by adjusting the gate voltages and the source-drain bias voltage in the leads. A regime which is easily accessible and has attracted a lot of interest is the Pauli spin blockade regime which has been demonstrated experimentally in various systems such as AlGaAs/GaAs and Si/Ge double quantum dots as well as carbon nanotube dots. In this regime one electron is confined in each dot and the three triplet states are almost equally and fully populated. In the absence of spin relaxation and microwaves the (1,1) triplet state is blocked from moving on to a (0,2) state by the Pauli exclusion principle \((n, m)\) denotes a charge state with \(n(m)\) electrons on dot 1(2)). Thus the electrical current as a function of the source-drain bias is suppressed.

For a fixed source-drain bias in the spin blockade regime a change in the occupations of the two dots can occur and a microwave-induced current can flow provided that the two dots have a different Zeeman splitting. In this case the oscillating magnetic field in combination with a static field induces coherent spin rotations that mix two-electron states and current flows through the transport cycle \((0,1) \to (1,1) \to (0,2) \to (0,1)\). When the two dots have the same Zeeman splitting the spins in the two dots rotate at the same rate in the triplet subspace and therefore the average occupation remains fixed and current does not flow. In this case only spin relaxation can give rise to a change in the occupation of the dots.

Inspection of the \( \sigma^z \) terms in Hamiltonian Eq. (6) shows that the interaction of the nearby spin with the spins on dot 1 induces an effective Zeeman asymmetry between the two dots of order \( J \) that depends on the orientation of the nearby spin. This suggests that a microwave-induced change in the occupation of the DD could take place and reveal information about the spin state when the dot parameters are adjusted to the spin blockade regime.

A Zeeman asymmetry can in principle arise due to intrinsic factors as in the case where the two coupled dots have different \( g \)-factors leading to \( \Delta_1 \neq \Delta_2 \) which could make the spin detection difficult. In Ref. 17 we have shown how to detect a magnetic field gradient and/or a difference in the \( g \)-factors in the absence of the nearby
and specifically we choose \( E(1,1) - E(0,2) = 0 \), with \( E(n,m) \) the energy of the charge state \((n,m)\). For a practical realization this configuration could be achieved via adjusting the gate voltages that define the confining potential of the double dot. We choose for the interdot Coulomb energy \( V = U/2 \) and for the on-site energies \( \varepsilon_1 = -U/2 \) and \( \varepsilon_2 = -U \). The bias voltage is \( V_{sd} = (\mu_L - \mu_R)/e = U/2e \) and is applied symmetrically, thus \( \mu_L = U/4 \) and \( \mu_R = -U/4 \), with \( \mu_L (\mu_R) \) being the chemical potential of the left (right) lead. When the interdot hopping satisfies \( \gamma \ll U \) and the temperature \( k_B T \lesssim U/80 \) the current as a function of source-drain bias is suppressed due to spin blockade and each dot contains a single electron.

Figure 3(a) shows the average electron occupation of dot 1 as a function of the magnetic field detuning \( \delta_2 \) for the two possible states of the target spin. We consider no spin relaxation and therefore we set \( A_{1,j} = A_{2,j} = 0 \) in Eq. (7). The dot occupation exhibits resonances (peaks) due to intradot spin rotations induced by the oscillating magnetic field and interdot tunneling, and it is constant close to unity far from the resonances due to spin blockade. For each spin configuration there are in total four peaks whose positions depend on \( \gamma \) and \( J \). The two outer peaks correspond to resonances R1 and R3 (see Fig. 2) and we would expect a third resonance at \( \hbar \omega_0 = (\Delta_1 + \Delta_2) / 2 \), corresponding to R2, but this resonance is split by an antiresonance, resulting in the two inner peaks. In terms of spin dependent detuning the condition for the antiresonance is \( |\delta_1 \pm J| = |\delta_2| \). This very symmetric situation together with the microwave driving leads to the emergence of an eigenstate of the system with purely \((1,1)\) triplet components, thus leading to spin blockade. It can be shown from the steady-state occupations that for this detuning the \( |S_{02}⟩ = |0,↑↓⟩ \) state is unoccupied and the current is suppressed. Away from the symmetry point, when \( \kappa > \gamma \hbar \Omega / J \), defining the distance from the symmetry point as \( \kappa = |\delta_1 \pm J| - |\delta_2| \), the eigenstate is no longer a pure triplet and current can flow, resulting in the two inner peaks left and right of the antiresonance. We can try to gain an intuitive understanding of the outer peak positions. Naively one would expect the outer peaks to appear when either the spin in dot 1 is on resonance or the spin in dot 2 is on resonance. However, for finite interdot hopping the outer peaks are somewhat shifted from the positions that one would expect for independent spins since intradot spin rotations take place with interdot hopping. As a result the shift is large when \( \gamma \) is large. Interdot hopping leads to delocalization of the electron spins via the resonant coherent transitions \( |↑,↓⟩ \leftrightarrow |0,↑⟩ \) and \( |↓,↑⟩ \leftrightarrow |0,↑⟩ \), with an amplitude proportional to \( \gamma \), that populate the \( |0,↓⟩ \) state and thus lead to a change in the populations. In addition, the populations of the \( |↑,↓⟩ \) and \( |↓,↑⟩ \) states are unequal leading to a mixing of the \((1,1)\) singlet and the \( S_z = 0 \) triplet that depends on the magnitude of \( J \) and \( \gamma \). When \( J \) is small the two outer peaks overlap with the inner peaks as shown for example in Fig. 3(b),
are suppressed and the microwaves have no significant effect resulting in a rather small resonant change. Therefore, probing a small $J$ needs a small interdot hopping $\gamma$.

To quantify this effect we show in Fig. 4(a) the relative occupation on resonance as a function of $J$ for a fixed interdot hopping and for different temperatures when the nearby spin is up. A spin down results in the same behaviour. The relative occupation of dot 1, $N'_1$, is calculated as $N'_1 = N^0_1 - N^p_1$ where $N^p_1$ is the occupation on resonance, i.e., the value of the occupation at the strongest peak, and $N^0_1$ is the occupation off resonance (background occupation). The weak feature in the curves at $J \sim 2\gamma$ is due to the outer peaks (R1 and R3 regions in Fig. 2) becoming distinguishable from the inner peaks (R2 region). For temperatures $k_B T \lesssim U/80$ the background occupation is fixed, $N^0_1 \sim 1$, due to spin blockade and hence the relative occupation is essentially temperature independent. In this regime there is to good approximation one electron in each dot in the $|T_{\pm}\rangle$ states. With increasing temperature spin blockade is gradually lifted and the background occupation increases. All one- and two-electron states acquire a finite population and even three-electron states, for instance $(1,2)$, become occupied and have to be included in the dynamics of the density matrix. This happens since the lengthening tail of the Fermi-Dirac distribution of the lead electrons leads to the opening of additional transport channels. The exact temperature dependence of $N'_1$ depends on various factors such as coupling to the leads, spin relaxation rate, as well as the applied source-drain bias. Even though this dependence may not be monotonic in all cases for high enough temperatures ($k_B T \gtrsim U$) the resonances cannot be clearly resolved and $N'_1 \sim 0$. From Fig. 4(a) we conclude that the DD detector has a higher temperature range of operation compared with a single dot, since the charging energy is the relevant energy scale. A similar increase in operating temperature has been predicted for an undriven DD read-out of a charge qubit. In a spin read-out situation we are not only interested in the height of the resonant peaks, but we want to distinguish two target spin states. Thus the figure of merit for a spin read-out has to be the maximum difference in population for target spin up and target spin down. In Fig. 4(b) we plot the maximum difference $\Delta N'_1$ of the spin up and down occupations as a function of $J$ and for a fixed interdot hopping. For $J \gtrsim 2\gamma$ the maximum difference occurs at the inner peak of the spin up (down) occupation with $\delta_2 < 0$ ($\delta_2 > 0$). The results indicate that a large difference can be induced making possible the discrimination between spin up and down states.

As shown above the achievable difference in dot occupation depends on a range of parameters. Figure 3 shows a contour plot of the average occupation as a function of detuning and interdot hopping for a fixed spin interaction strength. The occupation exhibits a distinct resonant pattern for both spin up and down and further it enables the two possible outcomes to be distinguished in a range.
of interdot hopping. Our calculations confirm that this is a robust behaviour that occurs for other values of \( J \) in the range \( \sim (10^{-7} - 10^{-6})U \). However, as explained above for \( \gamma \gg J \) the occupation peaks decrease and this could make the spin detection relatively difficult.

In addition to the temperature effect the background average occupation increases due to spin relaxation and as a result the microwave-induced resonances cannot be clearly resolved since the relative occupation in both dots drops. Spin relaxation and decoherence will also influence the peak height of the resonances, since they inhibit coherent spin rotations. Spin-flip processes which take place because of the interaction of the DD spins with the bosonic bath described by Eq. (7) allow incoherent transitions between two-electron states, for example \( | \uparrow, \uparrow \rangle \leftrightarrow | \downarrow, \uparrow \rangle, | \uparrow, \downarrow \rangle \) which in turn populate the \( |S_{92}\rangle \) state, lifting the spin blockade, and thus increasing the background occupation. This happens even in the absence of the nearby spin, i.e., when \( J = 0 \) though spin blockade can still be recovered as shown in Ref. 16 depending on the spin relaxation rate and the coupling to the leads.

To examine the effect of spin relaxation on the driven DD spin detector we have calculated the change in population for various spin relaxations rates \( \gamma_s \). Results are shown in Fig. 6 for the relative occupation of the strongest (outer) peaks as a function of hopping when the nearby spin is up (the same behaviour results for spin down). We have taken \( \gamma_s = \pi |A|^2 D(\delta \epsilon)[2n(\delta \epsilon, T) - 1]/\hbar \), with the Bose function \( n(\delta \epsilon, T) = [\exp(\delta \epsilon/k_B T) - 1]^{-1} \), and \( \delta \epsilon = \hbar \omega_0 \). \( D \) is the density of states for the bosonic bath that is taken constant and also \( |A| = |A_{1j}| = |A_{2j}| \) in Eq. (7). This expression for \( \gamma_s \) can be derived by assuming a single spin with Zeeman splitting \( \delta \epsilon \) coupled to a bosonic bath at temperature \( T \) which we assume to be the same as the temperature in the leads. We focus on the most interesting experimental regime in which the spin relaxation rate is much smaller than the tunneling rate through the double dot and the system weakly deviates from the spin-blockade regime. As seen in Fig. 7 the effect of spin relaxation becomes important as \( \gamma_s \) increases which in turn leads to a decrease in the relative occupation. The sensitivity of the spin detector is limited by the minimum detectable change in the occupations. The optimum resolution can be achieved when the driving is efficient. This happens when the Rabi frequency, which is controlled by the intensity of the oscillating field, is larger than the spin relaxation rate of the dot spins and the target spin, as well as the tunneling rate through the DD.

IV. CONCLUSIONS

In summary, we have suggested an electrical scheme to probe a single spin which makes use of two serially tunnel-coupled quantum dots connected to metallic leads. The spin is located at some distance from the dots, which has to be smaller that the typical interdot separation, and the total system is in a static magnetic field under the application of a microwave magnetic field. The spin interacts with the spins on the dots and this interaction results in an effective Zeeman splitting that is different in the two dots. Due to an electron spin resonance effect the electron occupations of the dots exhibit resonances which reveal information about the state of the nearby spin. In particular, the ac-driven DD spin detector provides an explicit signal in the induced occupations for both spin orientations and enables the spin state to be probed noninvasively in a single shot provided that the target spin has a different g-factor from the DD system, a condition that is typically satisfied.

We identified a range of parameters for which the system can operate and analysed how we can tune its sensitivity with the interdot hopping and intensity of the microwave field that defines the Rabi frequency for spin rotations. The operation of the DD detector depends on a lifting of a Pauli spin blockade and therefore it can operate at much higher temperatures than the single dot which is limited to temperatures comparable to the Zeeman energy. For instance for a charging energy of 10 meV and interaction strength in the range \( \sim 5 \) MHz the resonances survive up to temperatures of a few tens of Kelvin. To achieve a similar operating temperature with a single dot, magnetic fields of a few tens of Tesla and correspondingly microwave frequencies of several hundreds of GHz would be necessary, conditions which are available only in specialised laboratories. The sensitivity of the detector is limited by internal spin relaxation which essentially leads to a small change in the occupations and as a consequence the resonances cannot be clearly resolved. For an efficient read-out the tunnelling rates from dots to leads and the microwave-induced Rabi frequency have to be larger than all the relevant spin relaxation rates.

Finally, the change in population effected by the mi-
crowave field has to be seen in relation to the relaxation time of the target spin. On one hand, the spin read-out has to be finished within the relaxation time of the target spin, otherwise random spin flips will obscure the result. On the other hand, for a given change in population, $\Delta N_i$, of dot $i = 1, 2$ a certain number of electrons, $n$, has to pass through the double dot and be counted by the charge detector. To achieve reliable statistics the requirement $n > 1/\Delta N_i^2$ has to be fulfilled, since then fluctuations in the average number are smaller than the change in population that we want to distinguish. At a given tunneling rate $\Gamma$ through the device this determines the minimum time of the measurement $T_m = n/\Gamma = 1/(\Delta N_i^2 \Gamma) < T_1$, which must be smaller than the spin relaxation time of the target spin. We arrive therefore at a minimum tunneling rate through the dot: $\Gamma > 1/(\Delta N_i^2 T_1)$. However, we are not free to increase the tunneling rate arbitrarily; once the tunneling rate approaches the Rabi frequency the resonance peaks in the population disappear. By demanding $10\Gamma < \Omega$ we obtain the minimum resolvable change in population as $\Delta N_i = \sqrt{10/(\Omega T_1)} \approx 10^{-9}$.

We conclude by estimating the feasibility of measuring the state of a molecular spin system, for example Sc@C$_8$2, which can be coupled with a carbon nanotube double quantum dot. Such carbon based systems are promising candidates for quantum information processing and around several Kelvin have a $T_1 \approx 1$ s (Refs. 20, and 21). Then with a Rabi frequency of 10 MHz we arrive at a minimum resolvable change in population of $\Delta N_i = 0.001$. An interdot hopping of $\gamma = 10$ MHz and a spin-spin interaction of $J \approx 5$ MHz would lead to $\Delta N_i \approx 0.07$ in the case of no spin decoherence. With a spin decoherence rate of $\gamma_s = 1$ MHz this reduces to $\Delta N_i \approx 0.007$, indicating that a single spin read-out with realistic parameters at liquid helium temperatures is feasible.

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