Heuristic Approaches to Solve E-Scooter Assignment Problem

MAHMOUD MASOUD1, MOHAMMED ELHENAWY1, MOHAMMED H. ALMANNAA2, SHI QIANG LIU3, SEBASTIEN GLASER1, AND ANDRY RAKOTONIRAINY1

1Centre for Accident Research and Road Safety - Queensland (CARRS-Q), Queensland University of Technology, Brisbane, QLD 4000, Australia
2Civil Engineering Department, King Saud University, Riyadh 11451, Saudi Arabia
3School of Economics and Management, Fuzhou University, Fuzhou 350108, China

Corresponding author: Mahmoud Masoud (mahmoud.masoud@qut.edu.au)

ABSTRACT Nowadays, rapid urbanization causes a wide-range of congestion and pollution in megacities worldwide, which bears an urgent need for micromobility solutions such as electric scooters (e-scooter). Many e-scooter firms use freelancers to charge the scooter where they compete to collect and charge the e-scooters at their homes. This competition leads the chargers to travel long distances to collect e-scooters. In this paper, we developed a mixed-integer linear programming (MILP) model for a real-world e-scooter-Chargers Allocation (ESCA) problem. The proposed model allocates the e-scooters to the chargers with an emphasis on minimizing the chargers’ average travelled distance to collect the e-scooters. The MILP returns optimal solutions in most cases; however, the ESCA is identified as a generalized assignment problem which classifies as an NP-complete combinatorial optimization problem. Moreover, we modelled the charging problem as a game between two sets of disjoint players, namely e-scooters and chargers. Then we adapted the college admission algorithm (ACA) to solve the ESCA problem. For the sake of comparison, we applied the black hole optimizer (BHO) algorithm to solve this problem using small and medium cases. The experimental results show that the ACA solutions are close to the optimal solutions obtained by the MILP. Furthermore, the BHO solutions are not as good as the ACA solutions, but the ACA solution consumes more time to solve large-scale real cases. Based on the obtained results, we recommend applying the ACA1 to find the near-optimal solution for large-scale instances, as the MILP is inapplicable to find the exact solution in comparison.

INDEX TERMS Micromobility modes, e-scooter-chargers allocation, mixed-integer linear programming, heuristic.

I. INTRODUCTION Nowadays, transportation systems are an important part of human activities. In recent years the dependency of people on the transportation system has increased. Currently, an average of 40 per cent of the world’s population spends at least 1 hour on the road each day. As the dependency of people on transportation systems increases, these systems face several challenges. Congestion is one of these challenges facing transportation systems and has a direct impact on people. Firstly, congestion has an environmental impact where it increases fuel consumption and consequently, air pollution [1]. Congestion is also one of the major factors influencing emissions. It increases the emission by 40% at 45 km/h [2].

Lastly, congestion increases travel time and wastes commuters’ time. The annual hours spent in road congestion in the UK, Belgium, Italy, Luxembourg and Greece are 45.73, 39.37, 37.36, 36.88 and 36.28 respectively; thus, costing the EU nearly EUR 100 billion, or 1% of the EU’s annual GDP. The situation in Australia is no better than Europe, average driving speeds in the big Australian cities have significantly dropped. For example, the speeds in Sydney, Brisbane and Melbourne fell by 3.6 per cent, 3.7 per cent and 8.2 per cent respectively. Moreover, congestion cost the Australian economy $16.5 billion in 2015 and is expected to reach between $27.7 and $37.3 billion by 2030 [3].

The above numbers suggest that there is a real need to find solutions to reduce traffic congestion and enhance transportation safety. Most of the traditional solutions tend to build new infrastructure. As a result, these solutions need land resources...
ogy and promoted a new model where e-scooters are made charged to become a challenge for the operators. Consequently, many companies took advantage of technology, and e-scooters to the chargers and the state of practice is based as it is the station-less system. Since 2017, e-scooter sharing systems similarly as the BSS, yet it provides more flexibility to users remarkably in 2018, as shown in Figure 1. This system works as the peer-to-peer arrangement between the e-scooter company and the charger. Although this gig economy offers flexibility and freedom to the chargers, there is no guarantee for minimum wage and maximum hours. To the best of our knowledge, we are among the first to propose a solution to this problem. So, there are no previous published papers that address such an e-scooters/chargers assignment. In this paper, we hypothesize that if we produce an optimal assignment of the e-scooters to the chargers then, the competition and possible physical violence will be eliminated. Where we assumed that, the proposed assignment algorithm/model will run on a central computer at the e-scooters’ operator data center. This central computer is fed with the locations of the e-scooters needing charging and the chargers.

Using locations of the e-scooters and the chargers the distance matrix between e-scooters and chargers can be calculated using google API or any similar software. Consequently the assignment problem is solved and each group of e-scooters (i.e. unique IDs) are assigned to a charger (i.e. unique ID as well). Therefore, the central computer communicate with the charger mobile application and give it the e-scooters IDs and locations. Now it is the responsibility of charger to travel to the e-scooters locations. Once he reaches an e-scooter location he scans the e-scooter ID using his mobile application. Then, the application matches the e-scooter ID with the assigned e-scooters IDs. If the application found a match then it unlocks the e-scooter and the charger take it for charging otherwise it displays a message informing him this is the wrong e-scooter. So, there is no competition since the application only unlock the assigned e-scooter and the chargers know no one else can take his e-scooter and the chargers know no one else can take his e-scooter.

Moreover, it can reduce the cost of the e-scooters’ charging and the renting cost at the same time, and increase the hourly rate of the charger. To confirm our hypothesis, the e-scooter-Chargers Allocation (ESCA) Problem is formulated as a mixed-integer linear programming (MILP) model. Because of the time complexity of the MILP, we adopted two other

![Figure 1. Comparison of trips taken by station-based, dockless, and scooter systems (source: https://nacto.org/shared-micromobility-2018/).](image-url)
algorithms, namely the ACA and the BHO, which has polynomial complexity [12], [13]. Consequently, we compared the solutions of the ESCA using the three approaches and recommended ACA for large instances.

The remainder of the paper is organized as follows. The mathematical formulation is given in Section II. Heuristic algorithms to solve the proposed problem are developed in Section III. The experimental work is presented in Section IV. Finally, the concluding remarks are drawn in Section V.

II. MATHEMATICAL FORMULATION

The assignment problem is common in the fields related to the network flow theory. The standard assignment problem is to determine an optimal assignment of assigning \( n \) jobs and \( n \) individuals using the cost or profit as an objective function. In this problem, any individual can be assigned to perform any job, including some costs and profit that may vary depending on individual-job assignment. In addition, each has a budget, and the sum of tasks assigned to it cannot exceed this budget. It is necessary to find an assignment in which all individuals do not exceed their budget, and the total profit of the assignment is maximized. In this paper, we adopt an extended assignment problem to be applicable for the e-scooters and chargers assignment problem by assigning each charger (individual) to some e-scooters (jobs) under a group of constraints.

The capacitated facility location (CFL) problem is a classical optimization problem for determining the potential sites in order to minimize costs, where demands at several points must be satisfied by the established facilities under limited capacity. The local search techniques and hybrid algorithms have been proposed to solve the CFL problem for many real applications [20], [22], [23]. Furthermore, the k-median problem is another example of facility location problems, where seeking to establish \( k \) facilities, without considering fixed costs. The k-median problem is applicable for many real applications and heuristic algorithms are used to obtain good solutions. [20], [21], [24].

The ESCA problem is formulated mathematically as a MILP model to optimize the number of chargers, the e-scooter allocation for each charger and freelance chargers’ location with a particular emphasis on the associated costs. The extended model will consider the optimal e-scooters assignment for each charger to minimize the total cost. The charger is joined at locations with high charging potential based on demand and distance between charger and e-scooter. In the proposed model, optimizing the number of chargers in the system can lead to increase the hourly rate of each charger. For that reason, we added a penalty term in the objective function to restrict the number of new chargers that will join to meet the requirements for the number of e-scooters. Figure 2 shows the proposed framework of the ESCA based on different types of data and optimization methods to minimizing the associated costs and solving the allocation problem for small and large-scale size real cases. The proposed framework includes five stages that inherited in data Centre, operator management and best assignment dataset. The data centre includes data collection, data cleaning and filtering, while best assignment dataset includes output and final solutions, and minimum associated costs. The operator is managing the data processing with the proposed models and methodologies.

In this paper, we suppose that there is one charger which starts the service at a location. Multiple chargers can be deployed from different close points but not the same point at a location.

Also, we assumed that the e-scooter operators only provide six chargers per person hence the maximum number a person can collect is six. So if a certain location (block) has more than six e-scooters need charging the model will deploy more than one charger as shown in the illustration below in Figure 3.

Consider the complete graph \( G = (N, A) \), where \( N \) is the set of nodes with two subsets \( V \) and \( S \); \( V \subseteq N \) and \( S \subseteq N \). Set \( A \) is the edge set of graph \( G \), where \( A \subseteq N \times N \). \( G \) is adapted
to be the route network, and \( N \) is the number of locations. \( V \) is the total number of e-scooters in the system and \( S \) is the total number of chargers in the system. Optimizing the number of chargers and their locations with producing an optimal e-scooters assignment to a charger is the main aim of this model. Let \( z_{ik} \in \{0,1\} \) be 1 if e-scooter located at node \( i \) is allocated to a charger located at node \( k \) and 0 otherwise. Based on \( N \) of set nodes (Locations), \( y_{kk} \) means a specific location is assigned for a charger. In particular, \( y_{kk} = 1 \) implies that the location at node \( k \) is selected as a charger. We assumed in the proposed model that one charger will be deployed from different close points (locations), but not the same point.

Indices and Sets

- \( V \): Total number of e-scooters in the system
- \( i \): Index of an e-scooter; \( i = 1 \ldots V \)
- \( S \): Total number of chargers in the system
- \( k \): Index of a charger; \( k = 1 \ldots S \)

Parameters

- \( d_{ik} \): Distance between e-scooter \( i \) and charger \( k \); \( k = 1 \ldots S \)
- \( D_k \): Maximum number of e-scooters assigned to the charger; \( k = 1 \ldots S \)
- \( b_k \): Penalty of using one charger \( k \); \( k = 1 \ldots S \)
- \( B \): Total investment for joining all chargers
- \( f \): Total cost for the given system

Decision Variables

- \( z_{ik} \):
  \[
  z_{ik} = \begin{cases} 
    1 : & \text{e-scooter has been assigned to charger } k; \\
    0 : & \text{otherwise} \\
  \end{cases} \quad i = 1 \ldots V \text{ and } k = 1 \ldots S
  \]

- \( y_{kk} \):
  \[
  y_{kk} = \begin{cases} 
    1 : & \text{charger } k \text{ is ready to work at a specific location}; k = 1 \ldots S \\
    0 : & \text{otherwise} \\
  \end{cases}
  \]

Objective function

The objective function in Equation (1) is to minimize the associated costs for the e-scooter charging. The developed equation demonstrates two terms of costs; the distances from the e-scooter location to the charger location, and the penalties of joining new chargers. The joining penalties term has been identified in the objective function to optimise number of chargers in the systems and increase the number of assigned e-scooters for each charger considering the maximum number of assigned e-scooters for each charger. Consequently, optimizing the number of chargers can lead to an increase in the hourly rate for a charger who are already in the system by assigning mostly the maximum number of e-scooters is (6) that has been identified in advance based on practices. In the objective function, the costs included the distances and penalty of adding each charger. We chose the magnitude of the \( b_k \) such that it is close to the longest link in the graph. Thus we do not need to do normalization.

\[
f = \text{Min} \left( \sum_{i=1}^{V} \sum_{k=1}^{S} d_{ik} z_{ik} + \sum_{k=1}^{S} b_k y_{kk} \right)
\]  

Constraints

- Constraints (2) impose that each e-scooter \( i \) is assigned to exactly one charger \( k \):

\[
\sum_{k=1}^{S} z_{ik} = 1 \quad \forall i \in V.
\]

- Constraints (3) ensure that no e-scooter \( i \) is assigned to a charger \( k \) unless a charger is ready to work at a specific location.

\[
z_{ik} \leq y_{kk} \quad \forall i \in V, k \in S,
\]

- Constraints (4) ensure that the total costs of joining \( S \) chargers are less than the total investment \( B \), where the cost of each charger \( k \) is \( b_k \). The penalty of joining new charger, \( b_k \) is given in advance.

\[
\sum_{k=1}^{S} b_k y_{kk} \leq B
\]

- Constraints (5) ensure the assigned number of e-scooters to open charger \( k \) is less than the maximum number of assigned e-scooters for this charger. In the next sections, the maximum number of scooters will be found based on the historical data for Brisbane city. In this case, the maximum assigned number of e-scooters for each charger is six to give a fair hourly rate for chargers based on practices.

\[
\sum_{i=1}^{V} z_{ik} \leq D_k y_{kk} \quad \forall k \in S.
\]

Constraints (6) ensure that the assignment decision will result in either no assignment, \( Z_{ik} = 0 \), or assignment, \( z_{ik} = 1 \).

\[
z_{ik} \in \{0,1\} \quad \forall i \in V, k \in S,
\]

III. METHODS

The developed MILP model is solved to find the exact solution of the ESCA problem using the Branch and Bound method [14]. However, as the ESCA problem is NP-hard, the mathematical programming will be used to solve small-size cases but intractable to handle large-size case studies. As a result, two algorithms are proposed to solve large-size instances. The exact solution obtained by the MILP model will be used to test the performance of the other proposed algorithms. The real case study will be solved using the developed MILP model, which we restricted its running time to a maximum of two hours. We compare solutions obtained from the MILP to the ACA and BHO solutions.

The hyper-parameters of the Heuristic algorithms is critical, time consuming and may change based on the size of the
problem and distribution of distances. So that we chose the heuristics which has small number of hyper-parameters.

In the case of the ACA, there is no hyper-parameters at all which make it a great choice for us. In the case of the BHO, the number of the hyper-parameters is small which made it easy for us to find a good values for the hyper-parameters.

**A. ACA ALGORITHM**

The ACA algorithm is developed based on the college admission (CA) algorithm to formulate the ESCA problem. The CA problem is many to one matching algorithm [13]. In the context of ESCA Problem, there are two sets of players, namely the chargers (colleges) and the e-scooters (students) that need to be matched, as shown in Figure Each charger builds a preference list (ranked list) of the e-scooters based on the distance, where the most preferred e-scooter is the nearest one.

Similarly, each e-scooter builds a preference list of chargers based on the distance. However, the e-scooters operator can choose any criteria such as the charging price per e-scooter requested by the charger. The best-qualified e-scooters are offered to charge first, followed by the lesser-qualified e-scooters. The ACA algorithm finds a stable matching solution through a series of iterations, as described in Table 1. At each iteration, the chargers offer to charge the best-qualified e-scooters, and the e-scooters have to reply by either deferred accepting the offer or not. Deferred accepting means that the e-scooter will “holds” the most preferred proposal (deferred acceptance) if it receives more than one proposal. In other words, the e-scooter uses its own preference list to compare the incoming proposals from the chargers and holds its most preferred proposal. In the next iteration, if that e-scooter received a better proposal than the current match (deferred acceptance), it breaks the tie with the old match and holds the better proposal. At the end of the iteration, some e-scooters have temporary assigned to chargers, and others do not. Chargers then update their list accordingly in the next iteration and offer to charge to e-scooters who did not receive an offer in the previous iterations, regardless of whether they are assigned to other chargers or not. The e-scooters’ lists do not change, but e-scooters can change their decision at each iteration if they are offered to charge service with a better charger. The algorithm continues iterating until it reaches a stable matching solution.

The idea of stability of the solution which means that there is no blocking pair of (charger C, e-scooter b). If there are two e-scooters a and b who are assigned to chargers C and D, respectively, although b prefers C to D and C prefers b to a then the pair (charger C, e-scooter b) is blocking and the solution is unstable.

In ACA, the stable matching solution is not a guarantee to be optimal. However the solution is still good. The advantage of the ACA algorithm:

- It has \( O(VS) \), where \( V \) is the total number of e-scooters in the system and \( S \) is the total number of chargers.
- This algorithm allows each charger and each e-scooter to have its preference list. Each preference list could be built based on different criteria. For example, in future work we will allow e-scooters to build their preference list of chargers based on how much they are asking to charge an e-scooter. At the same time we will allow chargers to build their preference list of e-scooters based on distance.
- It does not suffer from comparing different quantities such as money and distance because at the end the matching is done based on the preference list which is rank-based list.

Recall that, the complexity of this matching algorithm is \( (VS) \) given that the preference lists are constructed. This algorithm can consider any other factors during constructing preference lists such as the charging costs and the battery level. Moreover, in the case of \( S > V/6 \), we can use the above algorithm to select a good subset of chargers and assign the e-scooters to this subset, where we assume the charger can collect six e-scooters. The cardinality of this subset of chargers is \( \lceil V/6 \rceil \) where \( \lceil \cdot \rceil \) is the ceiling function maps \( V/6 \) to the least integer greater than or equal to \( V/6 \). We can do the subset selection and assignment by running the above algorithm using the full set of the chargers until the best match achieved. Consequently, we sort the charger based on the number of assigned e-scooters and remove that charger that has the least number of assigned e-scooter. Then, we run the algorithm again using the new reduced set of chargers. We
repeat the matching and removing steps until we reach the desired several chargers.

**B. OTHER RECOMMENDATIONS**

The BHO algorithm is an optimization algorithm inspired by the black holes found in deep space [15]. The first step of the BHO is selecting a large number of solutions at random points in the search space. The algorithm evaluates the initial solutions using the objective function, naming the best solution the “black hole” and the other solutions “stars.” At each iteration, the algorithm moves the stars randomly toward the black hole and re-evaluates the cost associated with each new star. If a star has a cost better than the black hole, it becomes the new black hole, and the old black hole turns into a star. If a star gets close to the black hole, it is removed from the solutions population, and another one is randomly generated. The BHO keeps iterating until a stopping criterion is reached.

In this paper, we used the BHO to assign the e-scooters to the chargers. We divide the set of e-scooters $G$ into smaller sets $\{g_1, g_2, \ldots, g_p\}$, where $g_1 \cup g_2 \cup \ldots \cup g_p = G$ and $|g_i| \leq 6$ for $\forall i$. The number of the small subset equals or less than the number of chargers. The algorithm is initialized by randomly generating a large number of initial solutions. Each initial solution is a random permutation of the e-scooters. If the number of e-scooters is less than the number of the chargers multiplied by six, then each initial solution will be padded with zeros at random locations. Such that the length of each initial solution will be six multiplied by the number of the chargers. We evaluate each solution by dividing it into $p$ contiguous sub-vectors, where the length of each sub-vector equals six. The cost of each solution is computed using objective function (1) that includes two costs; the distances from the e-scooter location to the charger location, and the costs of joining new chargers. The main procedure of the BHO is detailed below in Table 2.

| 1. | Input number of e-scooters, $V$ |
| 2. | Input number of Chargers $S$ |
| 3. | Generate an initial number of solutions, $M$ |
| 4. | Set $j=1$ |
| 5. | While ($i \leq M$) |
| 5.1 | Select charger $i = 1, i \in S$ |
| 5.2 | While ($i \leq S$) |
| 5.2.1 | Generate initial assignment $g_i$ for charger $i$, where $|g_i| \leq 6$ for $\forall i \in S$ |
| 5.2.2 | If $|g_i| = n < 6$, then |
| 5.2.2.1 | Complete the set $g_i$ by number of zeroes equals ($6-n$) |
| 5.3 | Set $G = G \cup g_i$, $\forall i \in S$ |
| 5.4 | Set $i = i + 1$ |
| 5.5 | Go to Step 5.2 |
| 6. | Evaluate M solutions and select the best (black hole) |
| 6.1 | Calculate the objective function for each solution $j, j \leq M$ |
| 6.2 | Select the solution $j^*$ that has the lowest value of the objective function |
| 6.3 | Set the Black Hole, BH = $j^*$ and the Stars $= M = j^*$ |
| 6.4 | Stop if $\sum_{t=1}^{T} \frac{1}{t} < \epsilon$ where $f^{t+1}$ is the objective function (1) at iteration $t = 1$ |
| 7. | Move Stars toward $j^*$ |
| 7.1 | Identify the set of positions $q$ on $ST_j$ such that $j^*(i) \neq ST_j(i)$ |
| 7.2 | Randomly choose one element of the set $q$ and change $ST_j$ as shown in Figure 5. |
| 7.3 | Measure the difference between the star $ST_j$ and the $j^*; j \neq j^*$ |
| 7.3.1 | $Difference(j^*, ST_j) = \sum_{i=1}^{n} l_{j^*(i)}$, where $l_{j^*(i)}$ is the indicator function |
| 7.3.2 | $l_{j^*(i)} = \begin{cases} 1 & j^*(i) \neq ST_j(i) \\ 0 & j^*(i) = ST_j(i) \end{cases}$ and $ST_j(i)$ are the e-scooter IDs or zero at the position $i$ in the $j^*(i)$ solution and $ST_j$ respectively. |
| 7.4 | Replace the star $ST_j$ with a randomly generated star if $D(BH, ST_j) < 3$. |
| 8. | Go To Step 6. |

The BHO assigns the black hole role to the solution that has the lowest cost, while other solutions become stars. Then the following is applied to each star, Figure 5:

![Figure 5. Illustration of the modification of the $ST_j$ solution to converge towards the black hole solution [16].](image-url)
After the four steps of the BHO are applied to each star in the population, all the costs are re-evaluated and a new black hole, having the lowest cost, is selected for the next iteration. The BHO continues until the stopping criterion is satisfied.

After BHO stop, we examine the best solution, where we may have two sets of chargers; a complete and incomplete set.

**Definition 1:** a complete set of chargers is a set that includes at most one charger who is assigned less than six e-scooters.

**Definition 2:** an incomplete set of chargers is a set that includes more than one chargers who is assigned less than six e-scooters.

If the best solution doesn’t have an incomplete set, then this solution is the final one. Otherwise, we need to reduce the number of chargers by keeping the complete chargers of the solutions and try all possible swaps of the e-scooters between the incomplete chargers. In this way, we create from the best solution many other solutions which will be the initial solutions for another BHO run to find a fine-tuned solution.

In the BHO algorithm, a grid search approach to set up these hyper parameters such as the size of the initial population. A grid search approach iterates through BHO parameter combination to find the best parameters to use. Although grid-searching is simple but it can be computationally expensive if the number of the hyper-parameters is large.

**C. ALGORITHMS ACA1 AND HYBRID ACA-BHO FOR LARGE INSTANCES**

As shown in the experimental work section, the proposed solution methods of the mathematical model (MILP) provide the best results in most cases. However, the generalized assignment problem is identified as NP-complete problem combinatorial optimization problem, [19]. So for large scale instances, the model implementations take longer time than the time accepted by the e-scooter operator. On the other hand, the adapted college admission algorithm returns a good solution in most of the instances. Moreover, in the case of large instances, its computational time is relatively smaller than the mathematical model. The drawback of the ACA is selecting a subset of chargers by running the ACA algorithm to find the stable match between the e-scooters and the chargers then eliminates the charger which has the least number of assigned e-scooters. This procedure continued until the required number of chargers is achieved. For example, if there are 90 e-scooters need charging and there are 30 chargers available. Because each charger can collect up to six e-scooters. So, we need 15 chargers out of the 30 chargers. In order to choose the 15 chargers and assign the e-scooters to them, we start by matching the 90 e-scooters to 30 chargers then we eliminate the charger which has the minimum number of e-scooters. Consequently, we match the 90 e-scooters to the remaining 29 chargers and again eliminate one charger. Therefore, we alternate between matching and eliminating until we reach the required number of chargers and their assigned e-scooters. In this example, we run the matching 16 times to reach the assignment solution. For the instances where we have a larger number of chargers \( M \) to choose from them a smaller set of chargers \( m \), we proposed a modification to the ACA1 as shown in Table 3.

**TABLE 3. ACA1 for the ESCA.**

1. Input number of e-scooters, \( V \)
2. Input number of Chargers \( S \)
3. Set \( i = 1 \) and \( N \) is the maximum number of the proposed iterations
4. While \( (i \leq N) \)
5. Choose randomly \( s \) chargers; \( s \leq S \) and \( s = \lfloor V/6 \rfloor \)
6. Recall ACA algorithm
   6.1 Assign the e-scooters to the randomly Chosen \( s \) chargers; \( s \leq S \)
   6.2 Calculate the distance a charger \( i \in s \) travel to collect his assigned e-scooters
7. Save the cost of each charger \( i \in s \)
8. End While
9. Calculate the average cost for each of the \( S \) chargers
10. Sort the \( S \) chargers in ascending order based on the average cost
11. Choose the first \( s \) ranked chargers
12. Assign the e-scooters to the \( s \) selected chargers using the ACA

ACA1 estimates the expected marginal cost of each charger in step nine and then finds the best \( s \) chargers based on this estimated cost. Finally, in step twelve, we run the matching algorithm once to find the final matching/assignment of the e-scooters.

For the sake of completeness, we proposed hybrid ACA-BHA, which uses the BHA to solve large instances with a simple modification. This modification is improving some of the initial solutions at the hope this will lead to a better final solution. Table 4 shows the hybrid ACA-BHA

**TABLE 4. The hybrid ACA-BHA for the ESCA.**

1. Input number of e-scooters, \( V \)
2. Input number of Chargers \( S \)
3. Set \( i = 1 \) and \( N \) is the maximum number of the proposed iterations
4. While \( (i \leq N) \)
5. Choose randomly \( s \) chargers; \( s \leq S \) and \( s = \lfloor V/6 \rfloor \)
6. Recall ACA algorithm
   6.1 Assign the e-scooters to the randomly Chosen \( s \) chargers; \( s \leq S \)
   6.2 Calculate the distance a charger \( i \in s \) travel to collect his assigned e-scooters
7. Save the cost of each charger \( i \in s \)
8. End While
9. Augment the initial solutions obtained in the above steps by some random solutions
10. Recall the BHA to search the space using the initial solutions to find the best possible assignment solution

**IV. EXPERIMENTAL WORK**

To compare the model with the college-based algorithm and the black hole based algorithm, we used two different datasets. The first dataset is simulated data where we randomly generate the coordinates of the e-scooters and the chargers according to uniform distribution inside a square that has an area of 25 km\(^2\). In the simulation data, the distances between e-scooters and chargers are Euclidean distances.
The second dataset consists of 12 small and medium-size benchmark instances and ten large size benchmark instances [17], [18]. The benchmark instances are real-world instances and the distances between the e-scooters and chargers are calculated using road network.

In each instance, we assume the following:

1. There is only one e-scooter at each location.
2. The chargers are initially located at the e-scooters.
   In other words, if we assume there are M chargers at different locations, then the first M e-scooters are positioned at the same chargers’ locations.

The experimental work consists of three steps. In the first step, we compared the solution of the proposed mathematical model and the other two algorithms using the simulated dataset. The comparison is made in terms of the total travelled distance of all the chargers, and the average travelled distance of the chargers to collect six e-scooters. In the second step, based on the comparison measures, we selected the best two out of the MILP, ACA and BHO. Then we tested the selected methods using the small and medium-size instances using the same measures plus the running time. In the third step, we again tested the two selected methods plus algorithm 1 and algorithm two using the large instances hoping that algorithm 1 and algorithm 2 run in less time and return good quality solutions. The PC used to solve these simulated and real-world instances has Intel® Core™ i7-8650 CPU @ 1.90 GHz 2.11 GHz and 16.0 GB RAM. The MILP was solved using the “intlinprog” MATLAB function. We coded the other algorithms in MATLAB.

**A. THE SIMULATED DATASET**

We simulated four scenarios. In the first scenario, the number of e-scooters needing charging is 120, and the number of chargers is 20. In the second, third and fourth scenarios, we increase the number of chargers while keeping the number of e-scooters unchanged. In all scenarios, the allowed maximum number of collected e-scooters per charger is six that have been identified to give a fair hourly rate for chargers based on practices.

To capture the stochastic nature of the average travelled distance, each scenario has been repeated 100 times, and the histogram and the total travelled distance by all chargers and mean of the travelled distance for a charger are estimated. We should mention that in the current state of practice, there is no assignment and a charger can visit an e-scooter’s location and do not find it because another charger collected it before him. On the other side using the proposed assignment model and algorithms fulfill that;

There is no competition between chargers at the time of collecting e-scooters, and hence the charger only visited six locations to collect the e-scooters.
The proposed assignment approaches are based on distance, and hence, the six assigned e-scooters are clustered, and average travelled distance will be reduced.

We calculated the total, and the average travel distance for the three proposed approaches for each scenario for each of the 100 randomly generated a dataset of the e-scooters and chargers locations.

Figure 6 shows the histograms of the total travelled distance for the four scenarios. As shown in the figure, in each scenario, the BHO histogram is well separated from the MILP algorithm while the ACA histogram is close to the MILP histogram.

Table 5 shows compare the three approaches. It is obvious that increasing the number of chargers improves the total travelled distance because the program has more options. For the black hole based algorithm, this trend is not as clear as in the case of the mathematical model and the college admission based algorithm.

As shown in Table 5, ACA algorithm solutions are close to the solutions of the MILP model using the “intlinprog” algorithm.
TABLE 8. Comparison of MILP, ACA and the two algorithms using large size benchmark instances.

|            | ACA | AACL | Hybrid ACA-BHO | MILP |
|------------|-----|------|----------------|------|
| **MILP**   |     |      |                |      |
| Brussels   | 10  | 8.0  | 8.41           | 5.21 |
| Milan      | 10  | 9.56 | 15.16          | 9.21 |
| Berlin     | 10  | 8.1  | 13.5           | 5.21 |
| Rome       | 10  | 12.5 | 12.5           | 5.21 |
| Madrid     | 10  | 8.3  | 22.88          | 5.21 |
| Paris      | 10  | 12.5 | 12.5           | 5.21 |
| Tokyo      | 10  | 8.3  | 22.88          | 5.21 |
| Sydney     | 10  | 22.88| 22.88          | 5.21 |
| **MILP**   |     |      |                |      |
| Brussels   | 10  | 8.0  | 8.41           | 5.21 |
| Milan      | 10  | 9.56 | 15.16          | 9.21 |
| Berlin     | 10  | 8.1  | 13.5           | 5.21 |
| Rome       | 10  | 12.5 | 12.5           | 5.21 |
| Madrid     | 10  | 8.3  | 22.88          | 5.21 |
| Paris      | 10  | 12.5 | 12.5           | 5.21 |
| Tokyo      | 10  | 8.3  | 22.88          | 5.21 |
| Sydney     | 10  | 22.88| 22.88          | 5.21 |
| **MILP**   |     |      |                |      |
| Brussels   | 10  | 8.0  | 8.41           | 5.21 |
| Milan      | 10  | 9.56 | 15.16          | 9.21 |
| Berlin     | 10  | 8.1  | 13.5           | 5.21 |
| Rome       | 10  | 12.5 | 12.5           | 5.21 |
| Madrid     | 10  | 8.3  | 22.88          | 5.21 |
| Paris      | 10  | 12.5 | 12.5           | 5.21 |
| Tokyo      | 10  | 8.3  | 22.88          | 5.21 |
| Sydney     | 10  | 22.88| 22.88          | 5.21 |

MATLAB function. As a result, the ACA and MILP techniques will be used in the Section below to solve real-world benchmark instances at a different number of chargers.

B. THE SIMULATED DATASET

In this section, we used 12 real-world benchmark instances. The size of the instances varies from 13 to 116.

TABLE shows the same trend. Increasing the number of the available chargers gives a better solution because we have the chance of selecting a better subset of chargers. Furthermore, we observed that the solution obtained using the ACA algorithm is close to the MILP solution. So that ACA would provide good solutions for the prospective large instances based on its polynomial complexity.

C. THE SIMULATED DATASET

In this section, we used ten real-world benchmark instances. The size of the instances varies from 150 to 564. TABLE 8 shows a comparison of the four approaches.

From this table we observe the following:

1- The “intlinprog” MATLAB function researches its preset maximum running time in many instances before researching the optimal solution.

2- For many large instances, ACA found good solutions in a short time compared to the function solving the MILP.

3- In some instances, ACA found a better solution better than the MILP solver. This is because the MILP is solved based on MATLAB heuristic function, (i.e. “intlinprog”). The overall algorithm of this function depends on using cut generation to tighten the LP relaxation of MILP, then using the branch-and-bound heuristic to find an upper bound on the objective function and feasible solution. The branch and bound methods produce a sequence of sub-problems that try to converge to a solution of the MILP. The sub-problems provide a sequence of upper and lower bounds on the solution of the given objective function. The first upper bound is any feasible solution, and the first lower bound is the solution to the relaxed problem.

4- The running time (i.e. CPU time) of MILP for small cases is shorter than the proposed algorithm; however, the MILP starts to increase exponentially more than the proposed algorithms for large scale size problems. So, the proposed algorithms are more suitable in case of solving large size sale problems.

5- Algorithm 1 is good in terms of time compared to ACA. Its running time for very large instances is better than the ACA, but its solution quality is not as good as the ACA.

6- Algorithm 2 is not an option to solve this problem because of its solution quality and running time.

In order to get a better idea of how these algorithms are performing in terms of the total travelled distance in Table, 175102

VOLUME 7, 2019
we did statistical comparison between these algorithms. The analysis should consider the dependency between the observations. In other words, we should realize that the total distances of different algorithms solutions for the same instance are correlated. Therefore, we used a generalized linear mixed-effects model to explain the variability of the total distance as a function of the algorithm used and the number of available chargers. We used three indicator variables to code the four algorithms. We set MILP as the baseline.

As shown in Table 6:
1. All factors (i.e. ACA, ACA1, Hybrid ACA-BHO and the Number of chargers) are statistically significant.
2. Keeping the algorithm unchanged and increasing the number of charger by one reduce the total distance by 2.96 km.
3. Keeping the number of chargers unchanged and changing the algorithm from MILP to ACA, ACA1, Hybrid ACA-BHO increases the total distance by 43 km, 58 km and 101 km respectively.

For the sake of completeness, we used google maps to generate a toy example. Consequently, we used Google directions API to calculate the distance matrix of the chargers and e-scooter.

Finally, we solved this toy example using the mathematical model and visualized the optimal assignment based on the calculated distance matrix returned by Google directions API in Figure 7.

As shown in the paper and the statistical analysis above ACA provides good solutions in a short time. Recall that:

1. The e-scooters/chargers assignment needs solution more than once a day
2. The key point here is finding a good solution in a short time to this e-scooters/chargers assignment problem.
3. ACA is based on the college admission algorithm which is hyper-parameters free algorithm. So it is easy for the operator to run it.
4. As defined by [13] “A stable assignment is called optimal if every applicant is at least as well off under it as under any other stable assignment.”. Moreover, the mentioned “Thus the principles of stability and optimality will, when the existence questions are settled, lead us to a unique “best” method of assignment”. So based on that, we will advise the operator to Run ACA to find the first stable solution then break the tie between one pair and find another stable solution. The operator repeats this breaking and matching several times to find many stable solutions and then find the best.

V. CONCLUSION

This paper investigates the ESCA problem to minimize the associated costs by optimizing the e-scooter allocation for each charger and chargers location. The MILP formulation model is developed with two adapted algorithms to solve the ESCA problem. The ESCA problem is an NP-hard combinatorial optimization problem. The proposed MILP was formulated to solve the small case study and compare with the two adapted algorithms (i.e., ACA and BHO) for testing the performance of these two algorithms. The results of the simulated dataset show that the ACA solutions are better than the BHO in solving medium- and large-scale real-world instances. These instances vary in size from 150 to 564. Moreover, we proposed a method to speed up the ACA and yield good quality solutions as well. In computational experiments, 22 small-scale instances are used for the comparisons of the exact solutions by MILP with those of the ACA algorithm, which points out that ACA1 is more efficient than other algorithms based the accuracy of the solutions and implementation times.

Successful application of this approach can help the e-scooter companies to meet the customer’s demand with considering the renting cost, and at the same time, increase the hourly rate of the charger. Future research will investigate:

1. How to generalize the model to ensure it is broadly applicable based on the collected real data from too many areas in Australia such as Queensland.
2. How to find the good solution (i.e. near optimal solution) for large e-scooters operators who may need to solve this problem for thousands of e-scooters
3. The current solution to the problem does not consider the inaccuracy of the e-scooters positions so we will work on improving the solutions by developing a scholastic approach considering the uncertainties in e-scooters positions.
4. Our solution assumes that the chargers will accept the assignment solution which maybe not the case. Thus a fuzzy logic-based solution will be developed to address the uncertainty of accepting the solution by the chargers.

REFERENCES

[1] J. Shawe-Taylor, T. De Bie, and N. Cristianini, “Data mining, data fusion and information management,” IEE Proc. Intell. Transp. Syst., vol. 153, no. 3, pp. 221–229, Sep. 2006.
[2] M. Barth and K. Boriboonsomsin, “Real-world carbon dioxide impacts of traffic congestion,” Transp. Res. Rec., vol. 2058, no. 1, pp. 163–171, 2008.
[3] Australian Automobile Association. (2018). Road Congestion in Australia. [Online]. Available: https://www.aaa.asn.au/wp-content/uploads/2018/10/AAA-Congestion-Report-2018-FINAL.pdf
[4] H. Dediu, “Uber trip distances are heavily skewed toward the low end. I’m quite surprised. The implications are profound. [Log-normal Approximations US private car vs. Uber],” Tech. Rep., 2019.
[5] C. Williams, “Australia’s commuting distance: Cities and regions,” Dept. Infrastruct. Regional Develop., Bur. Infrastruct., 2015.
[6] P. DeMaio, “Bike-sharing: History, impacts, models of provision, and future,” J. Public Transp., vol. 12, no. 4, p. 3, 2009.
[7] E. Howe and B. Bock, “Global scootersharing market report 2018,” InnoZ-Innov. Centre Mobility Societal Change, Tech. Rep., 2018.
[8] A. J. Hawkins. (2019). Electric Scooter Charging is a Cutthroat Business, and Lime Wants to Fix That. [Online]. Available: https://www.theverge.com/2019/3/15/18267128/lime-electric-scooter-charging-juicers-harvesting-business
[9] K. Elkins. (2019). You Can Make Hundreds of Dollars a Day Charging Electric Scooters-Here’s How. [Online]. Available: https://www.cnbc.com/2019/05/22/how-to-make-money-charging-electric-scooters.html
MAHMOUD MASOUD received the M.Phil. degree in applied mathematics—operations research and decision support systems from Cairo University (CU) and the Ph.D. degree in operations research and mathematical sciences from the School of Mathematical Sciences, Queensland University of Technology (QUT). He is currently a Research Associate with the Centre for Accident Research and Road Safety—Queensland (CARRS-Q), Queensland University of Technology (QUT). He has extensive experience, as a Research Associate, in many industrial projects as a part of effective teamwork at the Centre for Tropical Crops and Bio-commodities (CTCB), School of the Mathematical Sciences, and CARRS-Q, QUT, Brisbane. This team constructed industrial linkages with big industrial organizations, such as EY and MLA (beef supply chain projects), owners of the Australian Miles (Biomass and Bioenergy assessment—sugar cane projects), and Brisbane Royal Hospital (health system project). He has a wide range of experience in academic research and industrial projects with more than 22 refereed journal and conference papers and industrial reports.

MOHAMMED ELHENAWY received the Ph.D. degree in computer engineering from Virginia Tech (VT). He worked, for three years, as a Post-doctoral Research at the Virginia Tech Transportation Institute (VTTI), Blacksburg, VA, USA. He is currently a Research Fellow with the Center for Accident Research and Road Safety—Queensland (CARRS-Q), Queensland University of Technology. He has authored or coauthored over 40 ITS related articles. His research interests include machine learning, statistical learning, game theory, and their application in intelligent transportation systems (ITS) and cooperative intelligent transportation systems (C-ITS).

SHI QIANG LIU received the two bachelor’s and one master’s degrees from the Harbin Institute of Technology, which is one of the top nine universities in China, the master’s degree in industrial and systems engineering from the National University of Singapore, and the Ph.D. degree in operations research from the School of Mathematical Sciences, Queensland University of Technology (QUT). He was working, in Australia for over 12 years, as a Senior Research Scientist with CRC ORE and as a Research Fellow with the Decision Science Discipline, Queensland University of Technology (QUT), Brisbane, Australia. He worked as a Design Engineer and Software Engineer in Singapore for three years. He is currently a Professor with the title of Min-Hang Scholar at the School of Economics and Management, Fuzhou University, China. He has published over 40 refereed articles, most of which were published in leading journals, including Transportation Science, Decision Support Systems, Expert Systems with Applications, International Journal of Production Economics, International Journal of Production Research, Computers & Operations Research, Computers & Industrial Engineering, Journal of Heuristics, Engineering Optimization, Journal of the Operational Research Society, Flexible Services and Manufacturing Journal, Optimization Letters, and Advances in Engineering Software. He also has solid background in scheduling theory and applications, metaheuristics, and combinatorial optimization. He received the Dean’s Award from the School of Mathematical Sciences, QUT, for his Academic Excellence, and the New Outstanding Researcher Medal from the Australian Society for Operations Research.

[10] A. Conti. (2019). Someone’s Going to Get Killed Charging Those E-Scooters. [Online]. Available: https://www.vice.com/en_us/article/wjmx8y/someones-going-to-get-killed-charging-those-e-scooters-juicing-limes
[11] S. Emerson. (2018). The Gig Economy Workers Who Power The Scooter Ridesharing Crazed for a Pittance. [Online]. Available: https://www.vice.com/en_us/article/zm8vz8/the-gig-economy-workers-who-power-the-scooter-ridesharing-crazed-for-a-pittance
[12] R. Soto, “Adaptive black hole algorithm for solving the set covering problem,” Math. Problems Eng., vol. 2018, Oct. 2018, Art. no. 2183214.
[13] D. Gale and L. S. Shapley, “College admissions and the stability of marriage,” Amer. Math. Monthly, vol. 69, no. 1, pp. 9–15, Jan. 1962.
[14] E. Balas and P. Toth, “Branch and bound methods for the traveling salesman problem,” Carnegie-Mellon Univ. Pittsburgh Pa Manage. Sci. Res. Group, Tech. Rep., 1983.
[15] A. Hatamlou, “Black hole: A new heuristic optimization approach for data clustering,” Inf. Sci., vol. 222, pp. 175–184, Feb. 2012.
[16] M. Elhenawy, Y. Bichiou, and H. Rakha, “A heuristic algorithm for rebalancing large-scale bike sharing systems using multiple trucks,” in Proc. 98th Annu. Meeting Transp. Res. Board, 2019, p. 16.
[17] M. Dell’Amico, E. Hadjicostantinou, M. Iori, and S. Novellani, “The bike sharing rebalancing problem: Mathematical formulations and benchmark instances,” Omega, vol. 45, pp. 7–19, Jun. 2014.
[18] M. Dell’Amico, M. Iori, S. Novellani, and T. Stützle, “A destroy and repair algorithm for the bike sharing rebalancing problem,” Comput. Oper. Res., vol. 71, pp. 149–162, Jul. 2016.
[19] I. H. Osman, “Heuristics for the generalised assignment problem: Simulated annealing and tabu search approaches,” Oper.-Res.-Spektrum, vol. 17, no. 4, pp. 211–225, 1995.
[20] V. Arya, N. Garg, R. Khandekar, A. Meyerson, K. Munagala, and V. Pandit, “Local search heuristics for K-median and facility location problems,” SIAM J. Comput., vol. 33, no. 3, pp. 544–562, 2004.
[21] M. Charikar and S. Guha, “Improved combinatorial algorithms for the facility location and K-median problems,” in Proc. 40th Annu. Symp. Found. Comput. Sci., Oct. 1999, pp. 378–388.
[22] S. Raghavan, M. Sahin, and F. S. Salman, “The capacitated mobile facility location problem,” Eur. J. Oper. Res., vol. 277, no. 2, pp. 507–520, 2019.
[23] Z. Yang, H. Chen, F. Chu, and N. Wang, “An effective hybrid approach to the two-stage capacitated facility location problem,” Eur. J. Oper. Res., vol. 275, no. 2, pp. 467–480, 2019.
[24] I. Vasilyev, A. V. Ushakov, N. Maltugueva, and A. Sforza, “An effective heuristic for large-scale fault-tolerant K-median problem,” Soft Comput., vol. 23, no. 9, pp. 2959–2967, 2019.
SEBASTIEN GLASER received the Ph.D. degree in automatic and control (defining a driving assistance system in interaction with the driver), in 2004. He is currently a Professor with CARRS-Q, where he focuses on a safe and sustainable development/deployment of automated driving system in interaction with other road users (drivers, cyclists, and pedestrians). He worked as a Researcher in the development of connected and automated vehicles (CAV). He was involved in several European Union initiatives (EU FP6, such as SAFESPOT on V2I communication) and in French National Research Agency (ANR) initiative (such as PARTAGE on shared control between the driver and the vehicle). Since 2009, he has worked across academia and industrial sectors and held senior researcher positions. He led the French ABV project, gathering eight academic and industrial partners, to develop a CAV solution at low speed. He has created, with Dominique Gruyer, CIVITEC, which commercialized the research outputs on virtual environment and simulation and is currently a part of the ESI Group. He has been the Deputy Director and the Director of the LIVIC (a Research Unit of IFSTTAR, the French Institute of Science and Technology for Transport, Spatial Planning, Development, and Networks), from 2012 to 2015, and a Project Leader of the VEDECOM (Public Private Partnership Research Institute), from 2015 to 2017, developing the autonomous vehicle prototypes. He was involved in French (ANR) and European (FP7 and H2020) initiatives on AV development, test, and evaluation. He was also leading the technological roadmap task force in France for the AV.

ANDRY RAKOTONIRAINY is currently the Director of the CARRS-Q and Intelligent Transport System Human Factor Research Program with CARRS-Q. He has 20 years of research and management experience in computer science and brings advanced expertise in road safety and ITS. He has established the CARRS-Q Advanced Driving Simulator Laboratory, which was the most advanced driving simulator facility in Australia. He has been proactive in investigating the use of existing and emerging ITS from multiple disciplines. It incorporates disciplines, such as computer science, mathematics, human factors, engineering, psychology, and sociology. His research has made extensive use of driving simulators, traffic simulators, and instrumented vehicles for developing system prototypes, assessing cost benefits, understanding human errors, and evaluating system deployment. His research on ITS has received numerous competitive grants and generated extensive interest from road safety stakeholders.

* * *