In-medium Properties of $\Theta^+$ as a $K\pi N$ structure in Relativistic Mean Field Theory

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The properties of nuclear matter are discussed with the relativistic mean-field theory (RMF). Then, we use two models in studying the in-medium properties of $\Theta^+$: one is the point-like $\Theta^+$ in the usual RMF and the other is a $K\pi N$ structure for the pentaquark. It is found that the in-medium properties of $\Theta^+$ are dramatically modified by its internal structure. The effective mass of $\Theta^+$ in medium is, at normal nuclear density, about 1030 MeV in the point-like model, while it is about 1120 MeV in the model of $K\pi N$ pentaquark. The nuclear potential depth of $\Theta^+$ in the $K\pi N$ model is approximately $-37.5$ MeV, much shallower than $-90$ MeV in the usual point-like RMF model.

I. INTRODUCTION

The relativistic mean field theory (RMF) is one of the most popular methods in modern nuclear physics. It has been successful in describing the properties of ordinary nuclei/nuclear matter and hyper-nuclei/nuclear matter [1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15]. Appropriate effective meson-baryon interactions are essential to the RMF calculation. To describe the nuclear matter and/or finite nuclei, nonlinear self-interactions for $\sigma$ and $\omega$ mesons are introduced [15, 16]. In recent years, a number of effective interactions for meson-baryon couplings, e.g., the NL-Z [1], NL3 [4], NL-SH [20], TM1, and TM2 [21] etc., have been developed.

Given that RMF has been a popular model in describing the properties of ordinary nuclei/nuclear matter and hyper-nuclei/nuclear matter, we will study the in-medium properties of $\Theta^+$ within the framework of the relativistic mean field theory.

The pentaquark state $\Theta^+$ (1540) was first predicted by Diakonov et al. [22], attained much support in the following years [23, 24, 25, 26, 27, 28, 29], and finally listed in the review of particle physics [30]. Presently, experimental results on $\Theta^+$ are a little subtle (see Ref. [31] for a recent review), e.g., the newly published data by the CLAS collaboration turn out to be significantly different from the previous results [32]. The negative results have higher statistics and are quite convincing, but they may not completely wash away the evidence yet, or, in other words, the pentaquark is not quite dead. Just because of the uncertainties, it is necessary to study the in-medium properties of $\Theta^+$, which is helpful to look for signals in experiments to see whether it can exist as a bound state in nuclei.

II. NUCLEAR MATTER PROPERTIES IN RMF THEORY

In RMF, the effective Lagrangian density [17, 33, 34] can be written as

$$\mathcal{L} = \sum_B \left[ \bar{\psi}_B (i\gamma^\mu \partial_\mu - M_B) \psi_B - g_\sigma^B \psi_B \sigma \psi_B - g_\omega^B \psi_B \omega \psi_B - g_\rho^B \psi_B \rho \psi_B - \frac{1}{2} \partial^\mu \sigma \partial_\mu \sigma - \frac{1}{2} m_\sigma^2 \sigma^2 - \frac{1}{3} g_2 \sigma^3 - \frac{1}{4} g_3 \sigma^4 \right]$$

$$+ \frac{1}{2} \Omega_{\mu\nu} \Omega^{\mu\nu} + \frac{1}{2} m_\omega^2 \omega^\mu \omega_\mu - \frac{1}{4} R^\mu\nu R_{\mu\nu}$$
\[ + \frac{1}{2} m^2 \rho^\alpha \rho^\mu - \frac{1}{4} F_{\mu \nu} F^{\mu \nu} \]
\[ - e \bar{\Psi}_B \gamma^\mu A^\mu \frac{1}{2} (1 + \tau_3 B) \Psi_B \]
\begin{align*}
\Omega^{\mu \nu} &= \partial^\mu \omega^\nu - \partial^\nu \omega^\mu, \\
R^{\mu \nu} &= \partial^\mu \rho^\nu - \partial^\nu \rho^\mu, \\
F^{\mu \nu} &= \partial^\mu A^\nu - \partial^\nu A^\mu.
\end{align*}

The standard RMF Lagrangian involves baryons (\(\Psi_B\)), scalar mesons (\(\sigma\)), vector mesons (\(\omega, \rho\)), and photons (\(A_\mu\)). The sum on \(B\) is over protons, neutrons, hyperons or exotic baryon \(\Theta^+\).

The baryon mass is \(M_B\), while the masses of \(\sigma, \omega, \rho\) mesons are, respectively, \(m_\sigma, m_\omega, m_\rho\). \(g^B, g^\sigma, g^\rho\) are, respectively, the \(\sigma\)-baryon, \(\omega\)-baryon and \(\rho\)-baryon coupling constants. The Pauli matrices for baryons are written as \(\tau^a\) with \(\tau_3^B\) being the third component.

Using the mean-field approximation, i.e., replacing the meson fields by their mean values, and neglecting the coulomb field, we immediately have the equation of motion for baryons:

\[ (\gamma^\mu k^\nu - M_B^* - g^B \gamma^0 \omega_0 - g^\rho \gamma^0 \tau_3 \rho) \Psi_B = 0. \]

where
\[ M_B^* \equiv M_B + g^B \sigma_0 \]
is the effective mass of baryons. For infinite nuclear matter, the equations of motion for the mean-field values of the scalar and vector mesons, i.e., \(\sigma_0\) and \(\omega_0\), are given by

\[ m_\sigma^2 \sigma_0 + g_2 \sigma_0^2 + g_3 \sigma_0^3 = -g^B_\sigma \rho_s, \]
\[ m_\omega^2 \omega_0 = g^B_\rho \rho, \]

where \(\rho_s\) and \(\rho\) are the baryon scalar density and vector density, respectively, which are given by

\[ \rho_s \equiv \sum_B \langle \bar{\Psi}_B \gamma_0 \Psi_B \rangle = \frac{2}{(2\pi)^3} \sum_B \int_{0}^{k_F(B)} d\vec{k} (\vec{k}^2 + M_B^*^2)^{1/2}, \]

and

\[ \rho \equiv \sum_B \langle \bar{\Psi}_B \gamma_0 \gamma_5 \Psi_B \rangle = \sum_B \frac{k_F^3(B)}{3\pi^2}. \]

If, only one impurity baryon, e.g., \(\Theta^+\), is in symmetric infinite nuclear matter, the effect of impurity baryon on the mean field values can be neglected [43]. Then, Eqs. [4] and [5] are simplified, giving

\[ \rho_s = \langle \bar{\Psi}_N \gamma_5 \Psi_N \rangle = \frac{4}{(2\pi)^3} \int_{0}^{k_F} M_N^* \frac{d\vec{k}}{(\vec{k}^2 + M_N^*^2)^{1/2}}, \]
\[ = \frac{M_N^*}{\pi^2} \left[ k_F E_F^* - M_N^*^2 \ln \frac{k_F + E_F^*}{M_N^*} \right]. \]

and
\[ \rho = \langle \bar{\Psi}_N \gamma_5 \Psi_N \rangle = \frac{2k_F^3}{3\pi^2}. \]

where
\[ E_F^* = \left( M_N^*^2 + k_F^2 \right)^{1/2}, \quad k_F = \left( \frac{3}{2} \pi^2 \rho \right)^{1/3}. \]

| \(m_\sigma\) | \(m_\omega\) | \(g^N_\sigma\) | \(g^N_\rho\) | \(g^N_2\) | \(g^N_3\) |
|---|---|---|---|---|---|
| A | 526.059 | 783.0 | 10.444 | 12.945 | 4.383 | -6.9099 | -15.8337 |
| B | 508.194 | 782.501 | 10.217 | 12.868 | 4.474 | -10.434 | -28.885 |
| C | 550 | 783.0 | 9.55 | 11.67 |

TABLE I: Three sets of parameters. A, B, and C are, respectively, from NL-SH, NL3, and Ref. [46]. The masses are in MeV and the coupling \(g_2\) in fm\(^{-1}\). The mass of \(\rho\) mesons is \(m_\rho = 763.0\) MeV, and the nucleon mass is \(M_N = 939.0\) MeV for all the sets.

![FIG. 1: The scalar meson mean-field value as a function of the scalar density. The straight lines are for the results without nonlinear \(\sigma_0\) terms, and the curves correspond to the results with nonlinear \(\sigma_0\) terms. The dotted line/curve is the result for NL3, the solid curve is for the NL-SH parameter set, and the dashed line is for the parameters from Ref. [46].](image)

### A. The relation of \(\sigma_0 - \rho_s\)

Ignoring the nonlinear \(\sigma_0\) terms in Eq. [10], one then has a simple linear relation between the scalar mean field \(\sigma_0\) and the scalar density \(\rho_s\), i.e.,

\[ m_\sigma^2 \sigma_0 = g^N_\sigma \rho_s. \]

This approximation is often used to estimate some properties of nuclear matter. In fact, the nonlinear terms of \(\sigma_0\) are important to the RMF calculations, especially at dense matter. To see clearly the effects of the nonlinear
terms of $\sigma_0$, we plot, in Fig. 1, the mean-field value $\sigma_0$ as a function of the nuclear scalar density $\rho_s$, with and without nonlinear $\sigma_0$ terms.

In the calculations, we adopt three sets of parameters, respectively from NL-SH, NL3, and the parameters from Ref. [46], which are marked with A, B, and C in Tab. I. In Fig. 1, the dotted, solid, and dashed lines correspond, respectively, to these three sets of parameters. Obviously, the curves corresponding to the results with the nonlinear $\sigma_0$ terms are different from the results without the nonlinear terms of $\sigma_0$ (the straight lines). From Fig. 1, we also see the parameter dependence of the relation $\sigma_0 - \rho_s$, especially in the high-density region.

It is interesting to note that the scalar density $\rho_s$ has, with the nonlinear $\sigma_0$ terms, an upper limit ($\rho_{\text{max}} \simeq 1.6 \rho_0$ for NL3 set and $2.14 \rho_0$ for NL-SH set), which does not exist with the linear relation.

The mean-field value $\sigma_0$ is connected to the scalar density $\rho_s$ by Eq. 5, i.e.,

$$\rho_s = -\left( m^2_s \sigma_0 + g_2 \sigma_0^2 + g_3 \sigma_0^3 \right) / g^N_\sigma. \quad (16)$$

Therefore, for a given nucleon density $\rho$, we can first solve $\sigma_0$ from

$$m^2_s \sigma_0 + g_2 \sigma_0^2 + g_3 \sigma_0^3 = -g^N_\sigma \left( M_N + g^N_\sigma \sigma_0 \right)^3 f(x), \quad (17)$$

and then the scalar density $\rho_s$ can be calculated from Eq. (16) or (13).

Numerical results are shown in Fig. 2, where the dotted curve is the result without nonlinear $\sigma_0$ terms and the parameter set is from Ref. [46], the solid and dash-dotted curves are the results with the nonlinear $\sigma_0$ terms corresponding, respectively, to NL3 and NL-SH. One can see, from Fig. 2, that the differences among the three curves are nearly invisible in the region $\rho/\rho_0 < 1$, which indicates that the effect from nonlinear $\sigma_0$ terms on the scalar density $\rho_s$ is trivial, and there is little parameter dependence for scalar density $\rho_s$ in the region $\rho/\rho_0 < 1$. However, the differences become more and more obvious with increasing densities. Thus, the nonlinear $\sigma_0$ terms are more important for calculations at higher densities, and in the case, the scalar density is also sensitive to the parameter choice.

III. IN-MEDIUM PROPERTIES OF $\Theta^+$

A. The point-like $\Theta^+$ in RMF

The study of the hadron properties in nuclear matter is one of the most interesting topics in nuclear physics. The effective mass and nuclear potential of hadrons are two important aspects which should be studied. In the usual RMF framework, the effective mass of $\Theta^+$ is given by [14]

$$M_{\Theta^+} = M_{\Theta^+} + \frac{4}{3} g^N_\sigma \sigma_0, \quad (18)$$

and the nuclear potential for $\Theta^+$ is

$$U_{\Theta^+} = \frac{4}{3} g^N_\sigma \sigma_0 + \frac{4}{3} g^N_\omega \omega_0. \quad (19)$$

According to the analysis in Sec. III, the effective mass and nuclear potential of $\Theta^+$ as functions of $\rho$ can be obtained easily. The calculations from Eqs. (18) and (19) will be discussed later.

B. The $K\pi N$ structure of $\Theta^+$ in RMF

In the usual RMF model, the $\Theta^+$ is roughly regarded as a point-like particle in the calculation. However, it’s very likely that $\Theta^+$ is a bound state of quark aggregations such as diquark-triquark $\bar{u}d\bar{s})(ud)$ [38, 39] and $K\pi N$ molecule.
state. If the internal structure of $\Theta^+$ is considered, its properties may be very different. In the following, we will consider $\Theta^+$ as a $K\pi N$ molecule state and study its in-medium properties within the framework of RMF.

In the picture of QMC model, the mesons couple directly with the quarks in a nucleon. Similarly, we can also assume that the mesons couple directly with $K$, $\pi$, and $N$ in $\Theta^+$, given that we are considering $\Theta^+$ as a $K\pi N$ bound state. Or, in other words, the investigation of the interaction between $\Theta^+$ and nucleons in medium turns into the investigation of the interactions of $K\pi$, $\pi N$, and $NN$ in medium. In the calculation, we assume that the $\pi$ mesons, as Goldstone bosons, do not change their properties in the medium, i.e., we neglect the $\pi N$ interaction in medium as an approximation.

1. KN interactions in the nuclear medium

The NN interaction has been investigated in Sec. 11. Before investigating the in-medium properties of $\Theta^+$ with a $K\pi N$ structure, we will review the $KN$ in-medium interaction, which is essential to the study of $\Theta^+$ as a bound state in the nuclear medium.

Roughly speaking, there are two popularly used methods for the study of kaons in nuclear medium. One is RMF theory, and the other is chiral perturbation theory (ChPT).

a. RMF approach

Kaons can be incorporated into the RMF model by using kaon-nucleon interactions motivated by one meson exchange models. In the meson-exchange picture, the scalar and vector interaction between kaons and nucleons are mediated by the exchange of $\sigma$ and $\omega$ mesons. For symmetric nuclear matter, the simplest kaon-meson interaction Lagrangian is

$$\mathcal{L}_K = \partial_\mu \bar{K} \partial_\mu K - m_K^2 \bar{K} K - g_{\sigma K} m_K \bar{K} K \sigma - ig_{\omega K} (\bar{K} \partial_\mu K \omega^\mu - K \partial_\mu \bar{K} \omega^\mu) + (g_{\omega K} \omega^\mu)^2 \bar{K} K,$$

where $\sigma$ and $\omega^\mu$ are the scalar and vector fields, respectively. $g_{\sigma K}$ and $g_{\omega K}$ are the coupling constants between the kaon and the scalar and vector fields. The $\sigma$-K coupling constant is chosen from the SU(3) relation by assuming ideal mixing, i.e.,

$$2g_{\omega K} = 2g_{\pi\pi\pi} = 6.04.$$  \hspace{1cm} (21)

The $K\omega$ coupling constant can be obtained by fitting the experimental $KN$ scattering length by assuming ideal mixing,

$$g_{\sigma K} \approx 1.9 \sim 2.3.$$  \hspace{1cm} (22)

In the present work, we set $g_{\sigma K} = g^N_\sigma / 5$, which is in the range of $1.9 \sim 2.3$.

At the mean-field level, the equation of motion for kaons is

$$[\partial_\mu \partial^\mu + m_K^2 + g_{\sigma K} m_K \sigma_0 + 2g_{\omega K} \omega_0 \partial_\mu - (g_{\omega K} \omega)^2] K = 0,$$  \hspace{1cm} (23)

where $\sigma_0$ and $\omega_0$ are the mean-field value of the scalar and vector meson fields, respectively. Decomposing the kaon field into plane waves, we obtain the equation

$$- \omega^2 + k^2 + m_K^2 + g_{\sigma K} m_K \sigma_0 + 2g_{\omega K} \omega_0 \omega - (g_{\omega K} \omega)^2 = 0$$  \hspace{1cm} (24)

for the kaon (anti-kaon) energy $\omega$ and the momentum $k$. The energies of kaons and antikaons in nuclear medium are then given by

$$\omega_K = \left( m_K^2 + k^2 \right)^{1/2} + g_{\omega K} \omega_0,$$  \hspace{1cm} (25)

$$\omega_\bar{K} = \left( m_\bar{K}^2 + k^2 \right)^{1/2} - g_{\omega K} \omega_0,$$  \hspace{1cm} (26)

where the effective mass of kaons is

$$m_K^* = \sqrt{m_K^2 + g_{\sigma K} m_K \sigma_0}.$$  \hspace{1cm} (27)

From the in-medium dispersion relations and , the kaon/antikaon potential can be defined as

$$U_{K/\bar{K}} = \omega_{K/\bar{K}} - \sqrt{m_K^2 + k^2},$$  \hspace{1cm} (28)

b. Chiral approach

For comparison, we also introduce another method, the chiral perturbation approach in our calculations. For symmetric nuclear matter, the effect of isospin is neglected. Following Refs. 49, 53, the kaon-nucleon chiral Lagrangian is written as

$$\mathcal{L}^\text{chiral}_{KN} = - \frac{3i}{8f_K} \bar{\Psi}_N \gamma^\mu \Psi_N \left[ \bar{K} \partial_\mu K - (\partial_\mu \bar{K}) K \right]$$

$$+ \frac{\Sigma_{KN}}{f_K} \bar{\Psi}_N \Psi_N \bar{K} K$$

$$+ \frac{\delta}{f_K} \bar{\Psi}_N \Psi_N (\partial_\mu \bar{K} \partial^\mu K),$$  \hspace{1cm} (29)

where $f_K \approx 93$ MeV is the kaon decay constant and $\Sigma_{KN}$ is the KN sigma term. The first term corresponds to the Tomozawa-Weinberg vector interaction. The second term is the scalar interaction which will shift the effective mass of the kaon and the antikaon. The last term, which is sometimes called the off-shell term, modifies the scalar interaction. $\Sigma_{KN}$ is not known very well, in the original work, it was chosen to be $\Sigma_{KN} \approx 2m_\pi$ in accordance with the Born model. More recently, the value $\Sigma_{KN} \approx 450 \pm 30$ MeV is favored according to lattice gauge calculations. Thus, it may vary in the region from 270 MeV to 480 MeV. By fitting the KN scattering lengths, one can determine the constant

$$\tilde{D} = 0.33/m_K - \Sigma_{KN}/m_K^2.$$  \hspace{1cm} (30)
The equation of motion for kaon field in the mean-field approximation and in uniform matter reads

\[
\left[ \partial_\mu \partial^\mu + m_K^2 - \frac{\Sigma_{KN}}{f_K^2} \rho_s + \frac{\bar{D}}{f_K^2} \rho_s \partial_\mu \partial^\mu + 3i \frac{4f_K^3}{\rho_N^2} \rho_N \partial_0 \right] K = 0,
\]

where \( \rho_s = \langle \bar{\Psi}_N \Psi_N \rangle \) is the scalar density and \( \rho = \langle \bar{\Psi}_K \Psi_N \rangle \) is the vector density for nucleons. Plane wave decomposition of the equation of motion yields

\[
-\omega^2 + \bar{k}^2 + m_K^2 - \frac{\Sigma_{KN}}{f_K^2} \rho_s - \frac{\bar{D}}{f_K^2} \rho_s (\omega^2 - \bar{k}^2) - \frac{3}{4f_K^2} \omega \rho = 0.
\]

The kaon and antikaon energies in the nuclear medium are

\[
\omega_K = \left\{ \left[ (m_K^2 + \bar{k}^2) \left( 1 + \frac{\bar{D}}{f_K} \rho_s \right)^2 + 3 \frac{4f_K^3}{\rho_N^2} \rho_N \right]^{1/2} \right. \\
+ \frac{3}{8f_K^2} \rho \left. \right\} \left( 1 + \frac{\bar{D}}{f_K} \rho_s \right)^{-1},
\]

\[
\omega_{\bar{K}} = \left\{ \left[ (m_K^2 + \bar{k}^2) \left( 1 + \frac{\bar{D}}{f_K} \rho_s \right)^2 + 3 \frac{4f_K^3}{\rho_N^2} \rho_N \right]^{1/2} \right. \\
- \frac{3}{8f_K^2} \rho \left. \right\} \left( 1 + \frac{\bar{D}}{f_K} \rho_s \right)^{-1},
\]

where \( m_K^* \) is the kaon effective mass

\[
m_K^* = \sqrt{\left( m_K^2 - \frac{\Sigma_{KN}}{f_K^2} \rho_s \right) / \left( 1 + \frac{\bar{D}}{f_K} \rho_s \right)}.
\]

c. Results The kaon energy \( \omega_K \) at zero momentum \((k = 0)\) as a function of the nuclear density \( \rho \) is plotted in Fig. 3. For RMF approach, we adopt two sets of parameters: NL-SH and NL3. For ChPT method, we consider also three cases: \( \Sigma_{KN} = 270, 350, 450 \) MeV, and in the calculation we have used the relation \( \rho_s - \rho \) in RMF model with the NL-SH parameter set.

From Fig. 3 we hardly see any difference between all the curves in the low density region \( \rho < \rho_0 \). And in the region of \( \rho > \rho_0 \), the ChPT approach always gives larger kaon energy than RMF model. At \( \rho = \rho_0 \), the kaon energy is about \( \omega_K = 527 \) MeV, which is nearly independent of the models and parameter sets. The kaon potential \( U_K = 33 \) MeV for RMF model and \( U_K = 28 \sim 33 \) MeV for ChPT model at normal nuclear density, compatible with the prediction \( U_{opt} \approx 29 \) MeV in Ref. [43]. At high densities, the uncertainty of the results from different parameter sets within RMF model is \( \sim 10 \) MeV. At \( \rho = 3\rho_0 \), the kaon energy \( \omega_K = 630 - 640 \) MeV (\( \omega_K = 630 \) MeV for NL-Z in Ref. [43]) for RMF model, and \( \omega_K = 645 \sim 690 \) MeV (\( \omega_K = 630 \sim 670 \) MeV in Ref. [42]) for ChPT model. All the results are compatible with the calculation in Ref. [49] as a whole. Thus, they give us reliable bases for the following calculations and discussions.

2. Formulas for \( \Theta^+ \) with a \( K\pi N \) Structure in RMF

Now that the NN and KN interactions in nuclear matter have been known, it is convenient to investigate the \( K\pi N \) structure of \( \Theta^+ \) in the nuclear medium. The effective Lagrangian for \( \Theta^+ \) as a K\pi N bound state can be written as

\[
\mathcal{L}_{\Theta^+} \simeq \mathcal{L}_K + \mathcal{L}_N + \mathcal{L}_\pi + \mathcal{L}_0,
\]

where \( \mathcal{L}_K, \mathcal{L}_N \) and \( \mathcal{L}_\pi \) are the effective Lagrangian densities, respectively, for nucleons, kaons, and pions, and \( \mathcal{L}_0 \) is for the internal interaction of K\pi N system. Usually, the contributions from \( \mathcal{L}_\pi \) and \( \mathcal{L}_0 \) are small. Therefor, we do not explicitly consider them in the following.

In the RMF framework, the interactions between hadrons are mediated by the exchange of the scalar meson \( \sigma \) and vector meson \( \omega \). According to the above discussions, \( \mathcal{L}_N \) and \( \mathcal{L}_K \) for symmetric nuclear matter can be written as

\[
\mathcal{L}_N = \bar{\Psi}_N (i\gamma^\mu \partial_\mu - M_N) \Psi_N - g_{N}^N \bar{\Psi}_N \sigma \Psi_N \\
- g_{N}^N \bar{\Psi}_N \gamma^\mu \omega_\mu \Psi_N,
\]

\[
\mathcal{L}_K = \bar{\partial}_\mu K \partial^\mu K - m_K^2 KK - g_{\sigma K} m_K KK \sigma \\
- ig_{\omega K} (K \partial_\mu K \omega^\mu - K \partial^\mu K \omega^\mu) \\
+ (g_{\omega K} \omega^\mu)^2 K.
\]
Then, at the mean-field level, the in-medium energy of nucleons is given by

\[ E_N(p) = \sqrt{(M_N + g_N^N\sigma_0)^2 + p^2} + g_N^N\omega_0, \]  

(39)

and the in-medium energy of \( \Theta^+ \) is

\[ E_{\Theta^+} = E_N(p) + \omega_K + E_{\pi} + E_b, \]  

(40)

where \( E_\pi = \sqrt{m_\pi^2 + p^2} \) is the \( \pi \) energy and \( E_b \) is the bound energy of \( K\pi N \) system, neither changes in nuclear medium. At zero momentum, the in-medium \( \Theta^+ \) energy is given by

\[ E_{\Theta^+} = M_{\Theta^+}^* + g_N^N\omega_0 + g_\omega K\omega_0, \]  

(41)

where the effective mass of \( \Theta^+ \) in nuclear matter is

\[ M_{\Theta^+}^* = M_{\Theta^+} + g_N^N\sigma_0 + \sqrt{m_K^2 + g_\rho K m_K\sigma_0 - m_K}. \]  

(43)

The \( \Theta^+ \) potential is defined to be the energy difference between \( \Theta^+ \) in the medium and \( \Theta^+ \) in free space, i.e.,

\[ U^{K\pi N}_{\Theta}(\tilde{p}, \tilde{k}) = E_N(\tilde{p}) + \omega_K - \sqrt{M_{\Theta^+}^* + \tilde{p}^2} - \sqrt{m_K^2 + \tilde{k}^2}. \]  

(44)

At zero momentum (\( p = k = 0 \)), one then has

\[ U^{K\pi N}_{\Theta} = g_N^N\sigma_0 + g_\omega N\omega_0 + \left[ m_K^2 + g_\rho K m_K\sigma_0 \right]^{1/2} + g_\omega K\omega_0 - m_K. \]  

(45)

C. Effective mass of \( \Theta^+ \) in RMF

Now we can easily obtain the effective masses, respectively, for the point-like \( \Theta^+ \) and for the \( \Theta^+ \) with a \( K\pi N \) structure, according to the Eqs. (13) and (43). The results are given in Fig. 4 Two parameter sets, i.e., the NL-SH and NL3 are adopted in the calculations. It is found that, when \( \Theta^+ \) is regarded as a \( K\pi N \) bound state, its effective mass is dramatically enhanced. At normal nuclear density, the effective mass of \( K\pi N \) system is \( M_{\Theta^+}^* \approx 0.73 M_{\Theta^+} = 1120 \text{ MeV} \), much larger than that of the point-like \( \Theta^+ \) (\( M_{\Theta^+}^* \approx 0.67 M_{\Theta^+} = 1030 \text{ MeV} \)). The effective mass of \( \Theta^+ \) is enhanced by about 90 MeV (6% of \( M_{\Theta^+} \)) due to its internal structure. At higher densities, say \( \rho = 3 \rho_0 \), the effective mass of \( \Theta^+ \) with \( K\pi N \) structure is enhanced about 160 MeV compared with that of point-like \( \Theta^+ \).

The effective mass depends obviously on the parameter sets in the region of \( \rho > 1.5 \rho_0 \), the mass difference is about 10 ~ 30 MeV between the NL-SH and NL3 parameter sets. However, at lower densities \( \rho < 1.5 \rho_0 \), the difference is almost invisible between the two curves (NL-SH and NL3).

As a whole, the effective mass of \( \Theta^+ \) depends strongly on its internal structure. The difference is up to 90 MeV between \( K\pi N \) structure and point-like \( \Theta^+ \) at \( \rho = \rho_0 \). The effects on the effective mass for different parameter sets of the point-like \( \Theta^+ \) are much smaller than those of the \( K\pi N \) system.

D. Nuclear potential depth of \( \Theta^+ \) in RMF

The potential depth is another important aspect for understanding the interactions between \( \Theta^+ \) and nucleons. To see the \( \Theta^+ \) potential in nuclear matter clearly, the \( \Theta^+ \) potential of the \( K\pi N \) structure is plotted in Fig. 5. For comparison, the potential of the point-like \( \Theta^+ \) as a function of the nuclear density \( \rho \) is also plotted in the same figure.

From the figure, we can see that when \( \Theta^+ \) is considered as a \( K\pi N \) bound state, the potential depth becomes much shallower than that of point-like \( \Theta^+ \). At normal nuclear density, the potential depth of \( \Theta^+ \) with \( K\pi N \) structure is about \(-37.5 \text{ MeV} \), which is about 52 MeV shallower than that of the point-like \( \Theta^+ \). The potential depth of \( \Theta^+ \) with \( K\pi N \) structure at normal nuclear density is also shallower than the previous predictions in other models [35, 37]. Therefore, if \( \Theta^+ \) is indeed a \( K\pi N \) bound state, it should have a shallower in-medium potential in RMF. This is a quite different observation from the previous result of strong binding (several hundreds of MeV) in nuclei [33, 35].

On the other hand, we see, from the figure, that the results for NL3 and NL-SH are very similar in the region \( \rho < 1.2 \rho_0 \). However, there are strong parameters
dependence in the higher density region $\rho > 1.2\rho_0$.

IV. SUMMARY

Considering the $\Theta^+$ as a $K\pi N$ structure, we have calculated the effective mass and optical potential of $\Theta^+$ in the nuclear medium with both the RMF approach and the ChPT theory. We find that the potential depth is only $U_{\Theta^+} \simeq -37.5$ MeV at normal nuclear density, much shallower than $U_{\Theta^+} \simeq -90$ MeV for point-like $\Theta^+$. The effective mass of $\Theta^+$ with a $K\pi N$ structure is also dramatically enhanced, which is $M_{\Theta^+} \simeq 0.73M_{\Theta^+} = 1120$ MeV at normal density, 90 MeV larger than $M_{\Theta^+} \simeq 0.67M_{\Theta^+} = 1030$ MeV for point-like $\Theta^+$.

Of course, our results depend on parameters. Also, $\Theta^+$ may have other internal structures. Because different internal structure have different in-medium properties, further studies on the $\Theta^+$ in nuclear medium are needed to clarify uncertainties.

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