The usability of discrete representation of holograms

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The usability of discrete representation of holograms

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Abstract. In this paper, we analyze the existing methods of digital representation of images and fields, based on calculating digital holograms of various objects. We studied discrete Fourier, cosine and Fresnel transform are compared, advantages and disadvantages of each. Cases of the use of each transformation are suggested. We estimated the quality of reconstructed images using objective metrics such as signal-to-noise ratio and root-mean-square error. The computational complexity of the considered discrete transformations is analyzed.

1. Introduction

Photonics systems are actively developed at the present time. This is expressed in the constant increase in their quality and information productivity [1]. Digital methods of obtaining and processing images are widely used [2, 3]. Therefore, the key tasks in the process of information exchange are improving the quality of information transmitting process and media capacity. Modern systems and electronic computing resources have made it possible to solve the problem of the formation and recording of images of large information capacity [4] and high quality [5] for many applied problems. However, there is a problem of correct processing of a huge amount of information contained in such images: the problem of processing interferograms [6], holograms, etc.

If we consider a hologram as an object of digital processing, then we must take into account certain limitations in the construction of digital representations of analog fields and their transforms. Images and analog fields are described by large data sets when they are digitally processed. Therefore, it is necessary to develop and apply effective theories for realizations of transformation algorithms [7].

In this paper, we will focus on the three major transformations: discrete Fourier transform (DFT), discrete cosine transform (DCT), discrete Fresnel transform (DFrT) [8, 9], because these transformations underlie the algorithms for compression and transmission of digital data. They are also well known and studied as a basis for computer-generated holograms. Our goal is to compare these transformations and analyze their applicability for some photonics problems. We hope to provide a good amount of comparative data about the transformations for digital holography, holograms generation and image processing. The evaluation of the effectiveness of the method of digital processing of halftone objects for the problems of optics and photonics is based on an analysis of the existing methods of digital representation of images and fields. We estimate the computational complexity of the algorithms, the possibilities of embedding them in digital technology, and suggest the optimal cases of use. Quantitative characteristics of the methods will be specified: errors in the discrete representation, signal-to-noise ratio and others.
2. Initial data
In this paper we will consider an object as a function of two variables, defined at the points of the finite raster. The original object is represented as a matrix of size $M \times N$:

$$I_o = [I_o(m,n) | m = 1, M; n = 1, N]$$

(1)

here $I_o(m,n)$ — the value of the pixel attribute (intensity), depending on the color model of the image representation.

For the comparative analysis of the effectiveness of algorithms for the synthesis of digital holograms based on discrete transformations we picked object "Lena" with various bit depth from 0 to 8.

Figure 1 covers the several objects with 1, 3 and 8 bits of halftones.

![Figure 1. Objects for creating holograms (512x512 pixels).](image)

3. Analysis of discrete transforms

3.1. Discrete Fourier Transform
The discrete Fourier transform is the decomposition of a finite sequence of real values into a finite sequence of complex values. As a result of synthesizing the hologram using this transformation, it is possible to obtain a digital Fourier hologram, which can be either amplitude or phase. This type of hologram has a distinctive feature, namely, the initial redundancy, which arises because expansion of the complex part in the exponential form and its representation in the form of trigonometric functions (cosine and sine).

In addition to this feature, information about the entire object is distributed evenly over the array (matrix) of the computer-generated Fourier hologram (CGH), therefore each hologram point carries information about the entire object, which provides a high degree of noise immunity.

In digital computer synthesis only the real values of the spectrum of a two-dimensional discrete Fourier transform are used:

$$U_1(p,q) = real \sum_{m=0}^{M-1} \sum_{n=0}^{N-1} I_o(m,n) \cdot \exp \left[-j2\pi \left(\frac{m \cdot p}{2M} + \frac{n \cdot q}{2N}\right)\right].$$

(2)

To restore an object without loss, it is necessary to satisfy the conditions of the Kotelnikov theorem (Nyquist-Shannon). Therefore, the size of the hologram must be at least $2M \times 2N$. When synthesizing digital holograms, regardless of the type of discrete transformation chosen, it is necessary to bring the dynamic range to the interval $[0, 255]$, which will allow to avoid additional quantization and sampling losses.

$$H(p,q) = round \left\{ \frac{U_1(p,q) - \min[U_1(p,q)]}{\max[U_1(p,q)] - \min[U_1(p,q)]]} \right\}.$$ 

(3)

It is known that the Fourier hologram has the property of complex conjugation symmetry [10]. This property is the key in cases where it is necessary to display an axially symmetric symbolic information, for example, in holographic sights [11]. This effect is not taken into account in the analysis in this paper.

Although digital Fourier holograms are now well studied [12] and in this paper we using well known algorithms, there remains a great potential for the application of digital processing methods
[10] to improve the quality of recoverable information and increase the capacity and other characteristics of this method.

3.2. Discrete Cosine Transform

The discrete cosine transform (DCT) is the decomposition of a sequence of a finite number of points into a sum of cosine functions that oscillate at different frequencies [13]. DCT is a kind of discrete Fourier transform, but the algorithm is based only on real values. In DCT, an object is considered as a set of spatial waves for which a one-dimensional discrete transformation is carried out along the different axes (X and Y), and the intensity value of the corresponding pixel of the image is plotted along the Z axis. After using the DCT at the output, we obtain a spectral interpretation of this object. This is because each spatial wave decomposes into a set of harmonics of different orders.

In the general case, the expression for DCT has the form:

$$U_1(p, q) = C(p)C(q) \sum_{m=0}^{N-1} \sum_{n=0}^{N-1} I_0(m, n) \cos \left( \frac{(2m+1)p\pi}{2N} \right) \cos \left( \frac{(2n+1)q\pi}{2N} \right).$$

$$C(i) = \begin{cases} \frac{1}{\sqrt{N}}, & i = 0 \\ \frac{2}{\sqrt{N}}, & i \neq 0 \end{cases}, \quad i, j = 0, \ldots, N-1. \quad (4)$$

By taking the DCT we obtain an image, which can be divided into the three components: high frequency (FH), middle frequency (FM) and low frequency (FL) components. The low frequency components contain the most information about the encoded object itself, and the high frequency parts of image are almost equal to zero and therefore barely visible figure 9. This method can be used to apply watermarking using different components [14, 15].

In the resulting matrix in figure 2b seen that most of the information about the watermark is concentrated in a small number of FL (upper left corner). Such a concentration of significant coefficients allows us to reduce the physical size of the hologram while minimizing the loss of information. Disadvantage here is, that by damaging this FL-part of holograms physically or digitally, we are unable to reconstruct the encoded object.

For computer-generated holograms, DCT has several advantages over against Fourier transform. It allows to exclude the axial symmetry, which provides a two-fold reduction in redundancy. Reduction of redundancy corresponds to the redistribution of the energy in the reconstructed image of the watermark [16, 17]. DCT is the basis for JPEG encoding-decoding, so it’s useful for big images and fields encoding. Due to JPEG-compression abilities cosine holograms can be very effectively compressed [18, 19, and 20].

However, as we mentioned before, cosine holograms have a significant drawback: information about the encoded object is concentrated in the area FL and FM. Therefore, this type of hologram has a low level of noise and damage immunity. The disadvantage of DCT also can be called a large number of noise when working with objects with a small resolution.
It should be noted that the use of the discrete cosine transform provides an accelerated calculation of cosine holograms due to a reduction in the number of operations compared to a discrete Fourier transform. This advantage of DCT is widely known and is used in many practical problems in which high processing speed is required in combination with preservation of qualitative data representation.

3.3. Discrete Fresnel Transform

One of the possible methods for creating digital hologram elements is to use the discrete Fresnel transform. Because of the high computational complexity, this algorithm is used less often than FFT or DCT: in the calculation of the Fresnel integral in discrete form, in general, a construction of 4 cycles is required [20, 21]. There are also simplified calculation algorithms, including those using the fast Fourier transform [22], which in turn inevitably lead to errors in the synthesis of holograms. Nevertheless, even so, Fresnel holograms have some advantages over Fourier transform and cosine holograms. We will describe these advantages further.

In the general case, the diffraction Fresnel integral has the form

$$\mathcal{U}(x,y) = \frac{\exp\{j k z\}}{j k z} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} U(p,q) \exp\left\{j \frac{\pi}{\lambda z} \left[(x-p)^2 + (y-q)^2\right]\right\} \, dp \, dq.$$  \hspace{1cm} (5)

The kernel of the integral can be represented as follows

$$\int U(x,y) \exp\left\{j \frac{\pi}{\lambda z} \left[(x-p)^2 + (y-q)^2\right]\right\} \, dp \, dq = \exp\left\{-j k (p^2 + q^2)\right\} \text{FFT}\left\{ U(x,y) \exp\left\{j k (x^2 + y^2)\right\}\right\}. \hspace{1cm} (6)$$

Realization of the Fresnel integral via FFT makes it possible to simplify the algorithm and accelerate the calculation of the discrete Fresnel transform. The final expression for calculating the discrete Fresnel transform takes the form

$$U_1(p,q) = \frac{\exp\{j k z\}}{j k z} \exp\left\{-j k (p^2 + q^2\right\} \text{FFT}\left\{ U(x,y) \exp\left\{j k (x^2 + y^2)\right\}\right\}. \hspace{1cm} (7)$$

The hologram is reconstructed using formulas obtained from the inverse Fresnel integral in a similar way.

Fresnel holograms work well with high-resolution images, contour and high-contrast images, whereas for low-resolution images it is preferable to use a Fourier transform. A distinctive feature of the Fresnel CGH is the possibility of keeping the hologram dimensions equal to the size of the encoded object, which is impossible in the case of Fourier transformer analysis.

4. Estimation of the accuracy of discrete transformations in the synthesis of holograms

There are two types of metrics to measure image quality: subjective and objective [23]. For subjective quality metrics human observers estimate quality of reconstructed image. This method is based on accumulation of statistics, i.e. subjective rating, given whether the reconstructed image is good looking for human eye or not. Every observer will give its own subjective assessments, and the resulting mean rating may be not optimal to describe the quality of image. Therefore, automatic and objective algorithms for evaluating the quality parameters of the objects under study are more preferable.

Objective image quality assessments can be divided into the different categories [24]. The first category includes difference based measures like mean squared error (MSE), root mean square error (RMSE) and the peak signal-to-noise ratio (PSNR) [25]; the second category is human visual system (HVS) [26] based that includes structure similarity measures like mean structural similarity index (MS-SIM) [27], visual information fidelity in pixel domain (VIFP), structure & hue similarity (SHSIM)
[28], and edge similarity based measures. The HVS based methods take advantage of the known characteristics of HVS [29], and measure image quality by estimating perceived error.

In this paper, we estimate the quality of reconstructed images only using objective methods. To compare the quality of the reconstruction of holograms created using the transformations described above, we used the PSNR, RMSE, and the average SNR of the image which determined according to the following formulas

\[
\text{PSNR} = 10 \log_{10} \left( N \cdot \frac{\max(I_r)}{\sum_{m,n}[I_r(m,n) - I_o(m,n)]^2} \right),
\]

\[
\text{RMSE} = \sqrt{\frac{\sum_{m,n}[I_r(m,n) - I_o(m,n)]^2}{(M-1)(N-1)}},
\]

\[
\text{SNR} = 10 \log_{10} \left( N \cdot \frac{\text{mean}(I_r)}{\sum_{m,n}[I_r(m,n) - I_o(m,n)]^2} \right).
\]

Here \(I_r(m,n)\) is reconstructed image, and \(I_o(m,n)\) is reference object.

In figure 3, the most interesting are SNR and RMS: the signal-to-noise ratio in practice shows the emerging noise on a black background, which reduces the contrast of the reconstructed image. The RMS estimates not only the error in determining the value of the amplitude in a particular pixel, but also indirectly indicates errors in the transmission of the position of the object. It can be seen from the figure 3 that the best quality of the recovery is provided by the Fourier CGH, which correlates with the visual perception of the reconstructed images. It can also be argued that the cosine transform is better suited for encoding images in grayscale than the Fresnel transform. The lower value of the signal-to-noise ratio for this image is due to the bright region (noise) in the upper left corner, typical for the cosine transformation. While the RMS shows that the error in the determination of the intensity is less than in the case of the Fresnel transform.

The DCT and DFrT are slightly more suitable for transmission of binary images, as they show higher SNR and lower RMSE values in 1 bit area. Although these transformations show lower results for halftone images, they still can be used for holographic purposes: the SNR value of 20 shows that images are usable.

The DCT algorithm we used in this paper shows inconclusive results. But they can be compensated through the simplicity and ease of use of this method for image encoding and transmission.

Also we can conclude that both of DCT and DFrT need to be modified to equally compare with DFT.

5. Conclusion

The article considered various methods of synthesizing digital holograms, based on discrete transformations: Fourier, Fresnel and cosine. We obtained the holograms and reconstructed images for each method. A comparative analysis of the quality of information transmission and reconstruction was carried out using these transforms in the synthesis of holograms. The analysis showed that, depending on the type of problem being solved and the required characteristics of the CGH, each of the methods considered can have a number of advantages over others, i.e. its use will be preferred. It is also worth noting that the advantage of frequency embedding methods is their relatively good resistance to compression and noise. The disadvantage is the high computational complexity and weak stability of some methods to geometric transformations (scaling, rotation, etc.) and impediments to reconstruction.

For further work we will focus on studying non-tradition digital holograms generation methods, described in the introduction of this paper.
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