Modelling transients in high-order electrical circuits in Matlab Simulink

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Abstract. The relevance of the study is due to the need to improve and expand the possibilities of calculating electrical circuits in transients. As an object for research, linear electric circuits of the second and higher order with zero initial conditions are considered. The main goal of the research is the task of performing dynamic calculations when simulating transients in linear high-order circuits by the methods of structural simulation in the Matlab Simulink software environment. The main attention is paid to the issues of constructing structural diagrams of high-order electric circuit models. The basis for the description of processes is the equation, compiled according to the laws of Kirchhoff, linking the instantaneous values of currents and voltages of the elements of the electric circuit through their parameters. Examples of numerical modelling using variants of models of structural circuits of the electric circuit are given. The advantages of the method of simulation structural modelling in the calculation of dynamic circuits with respect to existing analytical methods for calculating transients in high-order circuits are also considered in this paper.

1. Introduction

The mathematical description and study of the features of the dynamic processes occurring in electrical circuits, electronic circuits diagram, and electrical devices represent a complex and time-consuming task [1]. In most cases, these studies are performed using modern software packages: Electronics Workbench, Multisim, MicroCAP using common approaches and mathematical methods for describing the studied objects [2–4].

In the theory and practice of studying complex objects, different approaches are used for obtaining the initial equations describing their dynamic state. The most general approach to the mathematical description of processes in dynamics is based on the compilation of systems of ordinary differential equations (ODEs), one of the ways to solve which is fully met by an interactive tool for modelling, simulation and analysis of dynamic systems Matlab Simulink [5–8].

Structural modelling instrument (Matlab Simulink) is used in the study of electric drives, semiconductor and control systems [9, 10].

Structural modelling instrument is also widely used in modelling of electromechanical systems for special purposes [11–14].

The investigations examined approaches to the calculation of transients in linear electric circuits of the second and higher order with zero initial conditions by the methods of structural modelling in Matlab Simulink [5–10]. The main attention is paid to the construction of detailed structural diagrams of electrical circuits used for simulation.
It should be noted that when solving dynamics problems, the studied mathematical model in the form of the differential equations system of the electric circuit is reflected in the form of the real structure of the object. This means that the calculation model is formed following the created structural diagram of the investigated electrical circuit.

It should also be pointed out that the use of structural modelling simplifies the calculation of a real system greatly. This is especially manifested in the analysis of transients in chains of the second and higher-order. In this case, channels for calculating the instantaneous values of currents, voltages and their derivatives, instantaneous powers are formed quite simply, and many other functions included in the Simulink application can also be used.

In this research, the fundamentals of the analysis of mathematical models using the methods of structural simulation are described in the example of the study of transients in linear electrical circuits of the second and higher-order.

The examples of building detailed structural diagrams in the Matlab Simulink software environment and recommendations on their use are given here.

2. Second-order electric circuit model

One possible example of a second-order chain is shown in Figure 1. The circuit is powered by a constant source of EMF (electro-motive force). It is required to set the current in the transient process due to a short circuit key with the following circuit parameters (Figure 1): \( E = 10 \text{ V} \); \( R_1 = 10 \Omega \); \( R_2 = 15 \Omega \); \( R_3 = 5 \Omega \); \( R_4 = 60 \Omega \); \( L = 1 \text{ mH} \); \( C = 10 \mu\text{F} \).

![Figure 1. Second-order electric circuit](image)

It is obvious (Figure 1) that the voltage across the capacitor and the current through the inductance before switching the key are zero. The system of equations written according to the laws of Kirchhoff regarding the instantaneous values of currents and voltages (Figure 1) can be represented as:

\[
\begin{align*}
\dot{i}_1 - \dot{i}_2 - \dot{i}_3 &= 0, \\
i_1(R_1 + R_2) + \frac{1}{C}\int \dot{i}_2 dt &= E, \\
i_3(R_3 + R_4) + L\frac{d\dot{i}_3}{dt} - \frac{1}{C}\int \dot{i}_2 dt &= 0.
\end{align*}
\]

The solution of the inhomogeneous system of second-order differential equations (1) with respect to the current \( i_3 \) is reduced to the calculation of the free and forced components of this current

\[
i_3 = i_{3fr} + i_{3fp},
\]

The calculated forced component for the current, taking into account the given parameters of the circuit, is determined as \( i_{3fr} = 0.1111 \text{ A} \).

The characteristic equation of the circuit (Figure 1)

\[
p^2 + 690 \cdot 10^2 p + 360 \cdot 10^6 = 0,
\]

where are the roots of the quadratic equation: \( p_1 = -57 \cdot 10^2 \text{s}^{-1} \); \( p_2 = -633 \cdot 10^2 \text{s}^{-1} \).
The initial conditions for the circuit relative to the current $i_3$ are: $i_3(0) = 0$, $\frac{di_3}{dt}(0^+) = 0$, and the free component of the current, taking into account the found values of the integration constants with respect to the general solution for the case of an aperiodic process

$$i_{ip} = -0.122e^{-57.10^2t} + 0.011e^{-633.10^2t} \text{ A}.$$  

Taking into account (2), the solution of the transient process relative to a given current value is obtained in a known form

$$i_3 = 0.111 - 0.122e^{-57.10^2t} + 0.011e^{-633.10^2t} \text{ A.} \quad (3)$$

We will obtain the solution of the problem by structural modelling methods in the Matlab Simulink environment. It should be recalled that the Simulink visual modelling system is a library of models of mathematical operations of signal conversion. The system of integro-differential equations (1) written according to Kirchhoff’s laws regarding the instantaneous values of quantities that determine the nature of the relationship between the current state of the system of its input and output signals will serve as a way of representing an analogue dynamic system.

Equations (1) for constructing a detailed structural diagram of the electric circuit (Figure 1) for the channels for calculating the current $i_1, i_2$ and voltage across the inductance should be presented in a different form:

$$
\begin{cases}
    i_2 = -i_3 + i_1, \\
    i_1 = \frac{1}{R_3 + R_2} \left(E - \frac{1}{C} \int i_2 dt\right), \\
    L \frac{di_3}{dt} = -i_s (R_s + R_t) + \frac{1}{C} \int i_2 dt.
\end{cases}
\quad (4)
$$

The resulting system (4) corresponds to a complete detailed structural diagram of the model of the circuit in question, in which each element of the mathematical operation corresponds to a certain link (Figure 2).

![Figure 2. Circuit diagram of the electric circuit of the second order in Matlab Simulink](image)

The results of the structural diagram operation of the electric circuit model in Matlab Simulink are shown on the screen of virtual oscillographs in the form of graphs of the current for the current $i_1, i_2, i_3$ and voltage $u_L$ (Figure 3).
Figure 3. Transient graph for currents $i_1$, $i_2$ and voltage on inductance $u_L$.

Transient graph for current $i_1$ (Figure 3) completely coincides with its solution, presented in the form of expression (3).

3. High-order circuit model
The undoubted advantage of the considered approach is the possibility of calculating transients in higher-order circuits. In Figure 4, a circuit with five reactive elements is shown as an example.

Figure 4. Fifth order electrical circuit diagram

The transient process in the circuit in Figure 4 is considered under zero initial conditions. The order of the circuit is determined by the total number of independent initial conditions defined by the number of reactive elements of the circuit. In the specific case (Figure 4), there are no independent nodes during the formation of the inductive subchain, and there are no independent circuits during the formation of the capacitive subchain. Therefore, the order of the chain is determined by the total number of independent initial conditions. In the example (Figure 4), the order of the circuit is five and is determined by the number of inductive and capacitive elements in which the initial charges and flux linkages can be independently set.
By analogy with the circuit diagram in Figure 1, the calculated system of equations written according to Kirchhoff’s laws with respect to the selected circuits and nodes for instantaneous values of currents and voltages (Figure 4) can be represented in the following form:

\[
\begin{align*}
    i_5 + i_y - i_3 &= 0, \\
    i_5 - i_b + i_1 - i_2 &= 0, \\
    i_1 - i_2 + i_b &= 0, \\
    i_b - i_7 - i_0 &= 0, \\
    i_1 R_1 + L_1 \frac{di_1}{dt} + i_2 R_2 + i_3 R_6 &= E, \\
    -i_2 R_2 - L_1 \frac{di_2}{dt} + i_3 R_3 + \frac{1}{C_1} \int i_s dt + i_s R_4 &= 0, \\
    L_2 \frac{di_6}{dt} + i_s R_3 - i_s R_2 &= -E, \\
    i_s R_6 - \frac{1}{C_2} \int i_s dt &= 0, \\
    \frac{1}{C_1} \int i_s dt - L_3 \frac{di_7}{dt} &= 0.
\end{align*}
\]

(5)

To construct a detailed structural diagram of an electric circuit in Matlab Simulink, equations of system (5) under the condition of equal currents \( i_5 = i_b \) it is advisable to present the equations in the following system:

\[
\begin{align*}
    i_1 &= -i_5 + i_b + i_2, \\
    i_5 &= i_1 + i_2 - i_3, \\
    i_b &= -i_7 + i_6, \\
    L_1 \frac{di_1}{dt} &= E - i_1 R_1 - i_2 R_2 - i_3 R_6, \\
    L_2 \frac{di_6}{dt} &= -E - R_s i_5 + i_1 R_1, \\
    i_3 R_6 - \frac{1}{C_1} \int i_s dt + i_3 R_3 + L_1 \frac{di_1}{dt}, \\
    i_2 R_2 - \frac{1}{C_2} \int i_s dt, \\
    \frac{1}{C_1} \int i_s dt - L_3 \frac{di_7}{dt}.
\end{align*}
\]

(6)

The system of equations (6) corresponds to the full detailed structural diagram of the model of the circuit under consideration (Figure 4), in which each element of the mathematical operation corresponds to a certain link (Figure 5).

As an example, Figure 6 shows graphs of the dependences of the instantaneous values of currents on time when the circuit is switched on to the EMF constant with the following circuit parameters (Figure 4): \( E = 60 \text{ V}; \ R_1 = 15 \text{ Om}; \ R_2 = 20 \text{ Om}; \ R_3 = 20 \text{ Om}; \ R_4 = 20 \text{ Om}; \ R_5 = 10 \text{ Om}; \ R_6 = 25 \text{ Om}; \ L_1 = 0.002 \text{ H}; \ L_2 = 0.005 \text{ H}; \ L_3 = 0.004 \text{ H}; \ C_1 = 100 \mu \text{F}; \ C_2 = 250 \mu \text{F}. \)
As an example, Figure 6 shows graphs of the dependences of the instantaneous values of currents on time when the circuit is switched on to the EMF constant with the following circuit parameters (Figure 4): $E = 60 \, \text{V}$; $R_1 = 15 \, \text{Om}$; $R_2 = 20 \, \text{Om}$; $R_3 = 20 \, \text{Om}$; $R_4 = 20 \, \text{Om}$; $R_5 = 10 \, \text{Om}$; $R_6 = 25 \, \Omega$; $L_1 = 0.002 \, \text{H}$; $L_2 = 0.005 \, \text{H}$; $L_3 = 0.004 \, \text{H}$; $C_1 = 100 \, \mu\text{F}$; $C_2 = 250 \, \mu\text{F}$.
4. Conclusion
The calculations using structural modelling methods in Matlab Simulink showed a great advantage of the method in the calculation of dynamic circuits in comparison with existing analytical approaches. In particular, these advantages are manifested in solving problems of analysis and synthesis of high-order dynamic circuits.

The use of numerical algorithms using the Simulink application of the Matlab program for calculating transients of chain models shows their high computational stability.

To prevent errors in the calculations, this applies in particular to high-order circuits; it is advisable to implement equations describing the state of the circuit model written in accordance with the theory of electrical circuits by algorithms using integration operations. The implementation of algorithms for such calculations using only integration operations is shown in Figure 2, Figure 5.

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