Fuzzy sequencing problem with uncertain processing time

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Abstract. In this paper, we suggested a new approach to obtained an optimal sequence for fuzzy sequencing problem with trapezoidal fuzzy number. Using new fuzzy arithmetic operation and ranking method, an improved algorithm is proposed to obtain the fuzzy total elapsed time for processing all the jobs without converting problem into classical form. The approach is efficiently illustrated with a numerical example.

1. Introduction

A sequencing problem specifies the optimal sequence (order) in which ‘n’ jobs to be processed by ‘m’ machine. It determines an appropriate order for a service of jobs to be done on a finite number of service facilities, so as to minimise the total time taken for finishing all the jobs. A sequencing problem could involve, jobs in a manufacturing plant, aircraft waiting for landing and clearance, maintenance scheduling in a factory, programmes to be run on a computer, customer in bank and so-on.

Johnson proposed an exact algorithm to get the minimum make span for two machine problem in 1954. The Johnson algorithm has also been extended to m-machine problem with make span as the objective. But it is difficult to apply those conventional approaches to real world problem.

In real life situation the information available is of imprecise nature and there is an uncertainty present in the problem. In order to tackle this uncertainty the concept of fuzzy sets can be used as an important decision making tool. These imprecise data may be represented by fuzzy numbers. The index of fuzzy set was introduced by Zadeh [11] in 1965.

K. Kripa et al [5] have discussed an algorithm to obtain an optimal sequence of a fuzzy sequencing problem with generalised triangular fuzzy numbers by converting fuzzy sequencing problem into crisp sequencing problem. S. U. Malini et al [7] proposed a ranking to solve fuzzy sequencing problem with octagonal fuzzy number. K. Selvakumari et al [10] framed the fuzzy sequencing problem...
where processing time taken as octagonal fuzzy number and pentagonal fuzzy number and solved by robust ranking method. LaxminarayanSahoo [6] used yagers ranking index method to fuzzy job sequencing problem, where processing time being taken as trapezoidal fuzzy numbers. R. Jansi et al [3] used Pascal’s algorithm to obtain optimal sequence of the fuzzy sequencing problem with trapezoidal fuzzy number.

In this paper, we proposed a new algorithm to find the optimal sequence for the fuzzy sequencing problem with trapezoidal fuzzy number, without converting in to a classical problem. In section 2, we review the basic definitions, ranking of trapezoidal fuzzy number and arithmetic operation of trapezoidal fuzzy number. In section 3, we proposed an improved algorithm to find the optimal sequence and total elapsed time of the sequencing problem with trapezoidal fuzzy number. In section 4, a numerical problem is provided to show the competence of methodology.

2. Preliminaries
In this section, we represent the idea about the fuzzy numbers and definitions which involved in the research work. Fuzzy set theory have been defined by Zadeh [11] and the fundamental concepts are given by Dubois and Prade [1] are applied to fuzzy environment.

Definition 2.1
A fuzzy set $\tilde{A}$ in $X$ is a set of ordered pair defined by, $\tilde{A}(x) = \{(x, \mu_{\tilde{A}}(x)) : x \in X, \mu_{\tilde{A}}(x) \in [0,1]\}$ where $\mu_{\tilde{A}}(x)$ is a membership function.

Definition 2.2
A fuzzy set $\tilde{A}$ defined on a set of real number $R$ is said to be a fuzzy number, if its membership function $\tilde{A} : R \rightarrow [0,1]$ has the following characteristic:

1) $\tilde{A}$ is convex (i.e.,) $\tilde{A} \{\lambda x_1 + (1-\lambda)x_2\} > \min \{\tilde{A}(x_1), \tilde{A}(x_2)\}$ for all $x_1, x_2 \in R$ and $\lambda \in [0,1]$

2) $\tilde{A}$ is normal, i.e., there exist an $x \in R$ such that $\tilde{A}(x) = 1$

3) $\tilde{A}$ is piecewise continuous.

Definition 2.3
A fuzzy number $\tilde{A}$ is a trapezoidal fuzzy number denoted by $\tilde{A} = (a_1, a_2, a_3, a_4)$ where $a_1, a_2, a_3, a_4$ are real numbers and its membership function $\tilde{A}(x)$ is given below,

$$\tilde{A}(x) = \begin{cases} 
\frac{x-a_1}{a_2-a_1}, & \text{if } a_1 \leq x \leq a_2 \\
1, & \text{if } a_2 \leq x \leq a_3 \\
\frac{a_4-x}{a_4-a_3}, & \text{if } a_3 \leq x \leq a_4 \\
0, & \text{otherwise}
\end{cases}$$
In this paper, the trapezoidal fuzzy number is represented as, \( \tilde{A} = (a_1,a_2,a_3,a_4) = (m,w,a^*,a_*) \)
where \( m = \frac{(a_1 + a_3)}{2} \) and \( w = \frac{(a_3 - a_2)}{2} \) are the midpoint and width of the core \([a_2,a_3]\) respectively.

Also \( a_* = (a_2 - a_1) \) and \( a^* = (a_4 - a_3) \) represents the left and right spreads respectively of the trapezoidal fuzzy number \( \tilde{A} = (a_1,a_2,a_3,a_4) \).

### 2.1. Ranking on Trapezoidal Fuzzy Number

The ranking function \( \forall: \mathcal{F}(R) \rightarrow R \) is defined by its graded mean as
\[
\mathcal{R}(\tilde{A}) = \left[ \frac{(a_2 + a_3) + (a_* - a^*)}{4} \right],
\]
where \( T(R) \) is the set of all trapezoidal fuzzy numbers defined on \( R \). A trapezoidal fuzzy number \( \tilde{A} = (a_1,a_2,a_3,a_4) \) in \( T(R) \) is said to be positive, if and only if \( \forall(\tilde{A}) > 0 \) and is denoted by \( \tilde{A} \succcurlyeq \tilde{0} \), for any two trapezoidal fuzzy numbers \( \tilde{A} = (a_1,a_2,a_3,a_4) \) and \( \tilde{B} = (b_1,b_2,b_3,b_4) \) in \( T(R) \), we have the following comparison:

(i). \( \tilde{A} \succeq \tilde{B} \) if and only if \( \mathcal{R}(\tilde{A}) \geq \mathcal{R}(\tilde{B}) \)

(ii). \( \tilde{A} \preceq \tilde{B} \) if and only if \( \mathcal{R}(\tilde{A}) \leq \mathcal{R}(\tilde{B}) \)

(iii). \( \tilde{A} \approx \tilde{B} \) if and only if \( \mathcal{R}(\tilde{A}) = \mathcal{R}(\tilde{B}) \)

### 2.2. Arithmetic Operation on Trapezoidal Fuzzy Number

For arbitrary trapezoidal fuzzy numbers \( \tilde{A} = (m(\tilde{A}),w(\tilde{A}),a_*,a^*) \) and \( \tilde{B} = (m(\tilde{B}),w(\tilde{B}),a_*,a^*) \) and \( * = (+,-,\times,\div) \), the arithmetic operations on the trapezoidal fuzzy numbers are defined by, \[
\tilde{A} * \tilde{B} = \left( m(\tilde{A}) \circ m(\tilde{B}), w(\tilde{A}) \lor w(\tilde{B}), a_+ \lor a_-, a^+ \lor b^+ \right)
\]
\[
= \left( m(\tilde{A}) \circ m(\tilde{B}), \max\{w(\tilde{A}),w(\tilde{B})\}, \max\{a_+\lor b_-, a^-\lor b^+\} \right)
\]

In particular, for any two trapezoidal fuzzy numbers, \( \tilde{A} = (m(\tilde{A}),w(\tilde{A}),a_*,a^*) \) and \( \tilde{B} = (m(\tilde{B}),w(\tilde{B}),b_*,b^*) \) we define,
Addition: \[ \tilde{A} + \tilde{B} = \left\{ \begin{array}{l}
\max \{a_*, b_*\}, \max \{a^*, b^*\} \\
m(a) + m(b), \max \{w(a), w(b)\}
\end{array} \right. \]

Subtraction: \[ \tilde{A} - \tilde{B} = \left\{ \begin{array}{l}
\max \{a_*, b_*\}, \max \{a^*, b^*\} \\
m(a) - m(b), \max \{w(a), w(b)\}
\end{array} \right. \]

Multiplication: \[ \tilde{A} \times \tilde{B} = \left\{ \begin{array}{l}
\max \{a_*, b_*\}, \max \{a^*, b^*\} \\
m(a) \times m(b), \max \{w(a), w(b)\}
\end{array} \right. \]

Division: \[ \tilde{A} \div \tilde{B} = \left\{ \begin{array}{l}
\max \{a_*, b_*\}, \max \{a^*, b^*\} \\
m(a) \div m(b), \max \{w(a), w(b)\}
\end{array} \right. \]

3. Fuzzy Sequencing Problem

A suitable order in which the number of jobs can be assigned to a finite number of service facilities is called sequencing. The jobs or items are the primary stimulus for sequencing. There should be a certain number of jobs say ‘n’ to be processed or sequenced. A machine is characterised by a certain processing capability of facility through which a job must pass before it is completed in the shop. There must be certain number of machines say ‘k’ to be used for processing the jobs. In reality, the duration has vagueness or uncertainty, which is managed by fuzzy set theory Zadeh [11]. The sequencing problem with uncertain processing time is termed as fuzzy sequencing problem. Using fuzzy set theory with arithmetic operations, a new algorithm is proposed to sequence the jobs to be processed in various machines, with minimum total processing time. The assumptions for the classical problem are also applicable for fuzzy sequencing problem.

3.1 Notations

- \(d_{ij}\): Processing duration for job i on machine j.
- T: Total elapsed time for processing all the jobs.
- \(I_{ij}\): Idle time on machine j from the end of job (i-1) to the start of job i.

3.2 Proposed Algorithm

Let there be n jobs, each of which is to be processed through m machines \(M_1, M_2, \ldots, M_m\) in the order \(M_1, M_2, \ldots, M_m\). Let \(\tilde{d}_{ij}\) be the fuzzy processing time to complete the \(i^{th}\) job on \(j^{th}\) machine. The optimal solution can be obtained if either or both of the following condition hold good,

i) \(\min \{\tilde{d}_{ij}\} \leq \max \{\tilde{d}_{ij}\} : j=2,3,\ldots,m-1 \) and/or

ii) \(\min \{\tilde{d}_{mj}\} \leq \max \{\tilde{d}_{ij}\} : j=2,3,\ldots,m-1 \).

That is the minimum fuzzy processing time on machines \(M_1\) and \(M_m\) is as great as the maximum fuzzy processing time on any of the remaining (m-1) machines. If either or both these conditions hold, then proceed with the steps of the algorithm.

In particular, we consider n jobs each of which is to be processed through 3 machines \(M_1, M_2, M_3\). The condition for optimality is, minimum fuzzy processing time on machines \(M_1\) and \(M_3\) should be greater than or equal to the maximum fuzzy processing time on machine \(M_2\).
The algorithm is summarized as follows:

**STEP 1**: Observe the fuzzy processing times of the given jobs on all three machines. If either or both the conditions mentioned above hold, then go to step 2 otherwise the algorithm fails.

**STEP 2**: Introducing two mythical machines R and L such that,

\[
\tilde{d}_{iR} = \tilde{d}_{i1} + \tilde{d}_{i2} + \cdots + \tilde{d}_{im} \quad \text{for } i = 1, 2, \ldots, n
\]

\[
\tilde{d}_{iL} = \tilde{d}_{i2} + \tilde{d}_{i3} + \cdots + \tilde{d}_{im} \quad \text{for } i = 1, 2, \ldots, n
\]

**STEP 3**: Examine the fuzzy processing time \(\tilde{T}_{i1}\) and \(\tilde{T}_{i2}\) for \(i = 1, 2, \ldots, n\) of the transferred two machine problem and identify \(\min(\tilde{T}_{i1}, \tilde{T}_{i2})\).

**STEP 4**: If the minimum fuzzy processing time appears in the first machine then place the job in the first available position in the sequence. If the minimum fuzzy processing time appears in the second machine then place the job at the last position in the sequence. If there is a tie then select the job corresponding to the largest job subscript and place it in the right position in the sequence.

**STEP 5**: Cross off assigned jobs from the table and repeat steps 3 & 4, until all the jobs have been assigned. Therefore, the obtained sequence is the optimal sequence (order) in which the jobs are to be processed on all the machines.

**STEP 6**: The total elapsed time to process all the jobs through the machines, is the time when the \(n^{th}\) job in a sequence finishes on last machine.

4. **Numerical Example**

Consider a fuzzy sequencing problem with the processing time as trapezoidal fuzzy numbers are given in the following table.

| Jobs / Machine | \(M_1\)    | \(M_2\)    | \(M_3\)    |
|----------------|------------|------------|------------|
| 1.             | (4, 6, 8, 14) | (2, 4, 5, 9) | (2, 3, 4, 7) |
| 2.             | (7, 8, 10, 15) | (3, 6, 7, 8) | (5, 9, 10, 12) |
| 3.             | (3, 6, 7, 8) | (0, 1, 2, 5) | (4, 6, 8, 14) |
| 4.             | (3, 7, 8, 10) | (1, 2, 4, 5) | (3, 6, 7, 8) |
| 5.             | (8, 10, 12, 14) | (2, 3, 4, 7) | (2, 4, 5, 9) |

Table 2: Trapezoidal fuzzy number is represented as \(\left(m(\tilde{a}), w(\tilde{a}), a_1, a^*\right)\)
Table 3: Three machine problems is converted into two Machine problem

|   | J1     | J2     | J3     | J4     | J5     |
|---|--------|--------|--------|--------|--------|
| R | (7,1,2,6) | (4.5,0.5,2,4) | (3.5,0.5,1,3) |        |        |
| L | (9,1,1,5) | (6.5,0.5,3,1) | (9.5,0.5,4,2) |        |        |
|   | (6.5,0.5,3,1) | (1.5,0.5,1,3) | (7,1,2,6) |        |        |
|   | (7.5,0.5,4,2) | (3,1,1,1) | (6.5,0.5,3,1) |        |        |
|   | (11,1,2,2) | (3.5,0.5,1,3) | (4.5,0.5,2,4) |        |        |

Table 4: Fuzzy optimal sequence

|   | J3 | J2 | J4 | J1 | J5 |
|---|----|----|----|----|----|

Table 5: Fuzzy total elapsed time

| Jobs | Machine M1 | | Machine M2 | | Machine M3 | |
|---|---|---|---|---|---|---|
| J1 | Time In | (0, 0, 0, 0) | Time Out | (6.5, 0.5, 3, 1) | | Time In | (8, 0.5, 3, 3) | Time Out | (15, 1, 3, 6) | |
| J2 | Time In | (6.5, 0.5, 3, 1) | Time Out | (15.5, 1, 3, 5) | | Time In | (22, 1, 3, 5) | Time Out | (31.5, 1, 4, 5) | |
| J3 | Time In | (15.5, 1, 3, 5) | Time Out | (23, 1, 4, 5) | | Time In | (26, 1, 4, 5) | Time Out | (38, 1, 4, 5) | |
| J4 | Time In | (23, 1, 4, 5) | Time Out | (30, 1, 4, 6) | | Time In | (34.5, 1, 4, 6) | Time Out | (41.5, 1, 4, 5) | |
| J5 | Time In | (30, 1, 4, 6) | Time Out | (41, 1, 4, 6) | | Time In | (44.5, 1, 4, 6) | Time Out | (49, 1, 4, 6) | |

4.1 Results

Therefore, total elapsed time for processing all jobs through the machines = (49, 1, 4, 6)

The total elapsed time in terms of trapezoidal fuzzy number $ (a_1, a_2, a_3, a_4 ) $ = (44, 48, 50, 56)

Idle time on machine $ M_1 $ = (8, 1, 4, 6)

Idle time on machine $ M_2 $ = (30, 1, 4, 6)

Idle time on machine $ M_3 $ = (18, 1, 4, 6)

But the total elapsed time for processing all jobs through the machines obtained by LaxminarayanSahoo [10] is 51 hours, where as we obtain the result as (49, 1, 4, 6), by using the ranking function we get 49.5 hours. Hence the proposed algorithm and the defined fuzzy arithmetic operation reduced the time taken to process all the jobs, which is more efficient than the existing one.
5. Conclusion

In this paper, we consider the processing duration as trapezoidal fuzzy number for a fuzzy sequencing problem. We proposed an improved algorithm, to obtain an optimal sequence and total elapsed time to process all jobs through machines, without converting the problem as a classical sequencing problem. The numerical illustration clearly shows that, the proposed methodology is more efficient than the existing method.

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