Trapped cold atoms represent a new frontier for the study of many body phases, due to higher versatility if compared to ordinary condensed matter systems [1]. Confining an atomic gas to live in a 2D geometry represents a benchmark for testing quantum Hall effects (QHE), where fast rotation for bosons plays formally the same role as the magnetic field for electrons confined in etrojunctions [2]. Strictly speaking, the analogy with the QHE is attained when the centrifugal force equals the radial trapping force, recovering the big degeneracy of the lowest Landau level (LLL). Nonetheless, some phases can be stabilized keeping the rotation below this limit [3].

One of the most intriguing states appearing in the QHE is the Pfaffian, first introduced by Moore and Read [4] in the context of paired Hall states. Successively, it was proposed as a candidate for describing the filling factor $\nu = 5/2$ in fermionic QHE, although the question is still under debate and the Pfaffian state has not been observed so far. However, this special wavefunction captures the attention of the scientific community, since it has the peculiar property of non-Abelian braiding statistics of its anyonic excitations. This feature discriminates the Pfaffian from other QHE states (like the Laughlin) and makes it attractive in the context of topological quantum computation (TQC) [5,6].

The aim of this Letter is to propose and investigate a novel method for preparing and stabilizing the Pfaffian state with high fidelity in rapidly rotating 2D traps containing a small number of bosons. The goal is achieved by strongly increasing 3-body loss processes, which suppress superpositions of three particles while permitting pairing. This filtering mechanism gives rise to reasonably small losses if the system is initialized with the right angular momentum. We discuss some methods for tuning 3-body interactions independently of 2-body collisions.

We propose a scheme for preparing and stabilizing the Pfaffian state with high fidelity in rapidly rotating 2D traps, which suppress superpositions of three particles while permitting pairing. This filtering mechanism gives rise to reasonably small losses if the system is initialized with the right angular momentum. We discuss some methods for tuning 3-body interactions independently of 2-body collisions.

The paper is organized as follows. First, we briefly recall the QHE regime requirements for cold bosons and the physics of the LLL, which is dominated by the nature of interactions. Hereby we demonstrate the different behavior of 2- and 3-body contact potential under density rescaling in 2D. The ground state (GS) diagram in the case of conservative 3-body scattering is presented, with particular attention to the Pfaffian-like sector. This motivates the need for an effective 2-body repulsion. The dissipative projection mechanism is then proven without 2-body collisions, and investigated numerically in their presence. Finally we suggest some experimental signatures and a method to prepare the initial state.

At low energy, a realization of an effective 2D system is obtained by setting the longitudinal trap frequency much bigger than the transverse one, $\omega_{\perp} \gg \omega_{\parallel}$, in order to freeze longitudinal motion in the GS. The filling factor is defined as $\nu = N/l_{\max}$ with $l_{\max}$ the maximum angular momentum occupied by single particles. In the frame rotating at angular speed $\Omega z$, the single body Hamiltonian in the trap can be written as

$$H_{\text{trap}} = \left( \frac{p^2 + A}{2m} \right) + \frac{m}{2}(\Omega^2 - \Omega^2)(x^2 + y^2),$$

with $A = m\Omega^2z \times \vec{r}$. In the limit of centrifugal deconfinement $\Omega \rightarrow \omega$, only the Coriolis force is remaining and the system is formally equivalent to bosons of charge $q$ in uniform magnetic field $\vec{B} = (2m\Omega^2 \bar{z})$. The one body eigenfunctions in the LLL take a simple form when written in terms of the complex coordinate $\tilde{z} = (x + i\bar{y})/\xi$, with $\xi = \sqrt{\hbar/m\omega_{\parallel}}$

$$\psi_n(z) = \frac{1}{\sqrt{\pi n!}} e^{-|z|^2/2},$$

and have energies $E_n = n\hbar(\omega - \Omega) = l_n \delta\omega$ where $l_n$ are the angular momenta projections. Other Landau levels (LL) are separated by a gap $\sim 2\omega$, and the LLL restriction is claimed to be even more valid for higher-$L$ QHE states, due to their stronger correlations [7]. All the numerical calculations presented below have been checked by including the first LL and verifying that occupation is small there.

The physics in the LLL is dominated by the nature and the strength of interactions, which drive the system into different filling factors [2]. In the context of cold bosonic gases in 2D, 2-particles interactions can be modeled by contact potentials

$$H_2 = g_2^{2D} \sum_{i < j} \delta^{(2)}(\vec{x}_i - \vec{x}_j),$$

with $g_2^{2D} = \sqrt{8\pi\hbar\omega_\perp^2a/\xi_\perp}$, being $a$ the s-wave scattering length in 3D and $\xi_\perp = \sqrt{\hbar/m\omega_\perp}$ the longitudinal trap size.
L belongs to the sector of
thus energy

\begin{equation}
\mathcal{H}_3 = g_3^{2D} \sum_{i<j<k} \delta^{(2)}(\vec{x}_i - \vec{x}_j) \delta^{(2)}(\vec{x}_j - \vec{x}_k),
\end{equation}

neglecting for the moment their physical origin. Let us call K2
and K3 the kernel of 2-body Eq.(1) and 3-body Eq.(2) term,
respectively. Inside K2, the Laughlin state with \( v = 1/2 \)

\[ \Psi_{\text{Lau}} = \prod_{i<j}(z_i - z_j)^2, \]

has the lowest total angular momentum \( L_{\text{Lau}} = N(N-1) \), and
thus energy \( E_{\text{Lau}} = L_{\text{Lau}} \delta \omega \). As usual, in (3) and in the sub-
sequent \( \text{QHE} \) wavefunctions we omit the ubiquitous exponential
and normalization factors. Of course, \( K_2 \subset K_3 \) and (3) is also
annihilated by the 3-body interaction, despite not being the
GS. The lowest \( L \) state in K3 is indeed the Pfaffian \(4\)

\[ \Psi_{\text{pt}} = \text{Pf} \left( \frac{1}{z_i - z_j} \right) \prod_{i<j}(z_i - z_j), \]

with \( L_{\text{pt}} = N(N-2)/2 \) and \( v = 1 \). The prefactor \( \text{Pf} \left( \frac{1}{z_i - z_j} \right) \)
makes possible the superposition of pairs, and it is formally
equivalent to a projected \( p+i \) BCS wavefunction \(16,17\).

The interplay between 2- and 3-body terms and the imper-
fect matching \( \delta \omega > 0 \) of trapping and rotation frequencies
enriches the phase diagram, by stabilizing other states aside \(3,4,5\). The sequence of GSs has been computed numer-
cally by exact diagonalization in the case of elastic 3-body
repulsion; the result is presented in Fig.1 for \( N = 6 \), in a trunc-
cated LLL single particle basis that contains all the angular
momenta up to \( l_{\text{max}} = 2(N-1) \). This is enough to describe ex-
actly \( \Psi_{\text{Lau}} \), hence suitable also for the description of states with
lower angular momenta. In the LLL, central contact potentials
\(12\) can be expressed in terms of a single pseudopotential
acting only on zero relative angular momenta \(18\). Thus
their sum can be recast in a very compact form \(19\)

\[ \mathcal{H}_{\text{int}} = \mathcal{H}_2 + \mathcal{H}_3 = \sum_{n=2,3} \sum_{l} g_{nl} d_n^\dagger d_n \]

where \( d_n \) is an annihilation operator of \( n \) particles with total
angular momentum projection \( l \). The GS energy \( E_0(L) \) is a
monotone nonincreasing function of \( L \). The global GS simply
belongs to the sector of \( L \) that minimizes the quantity \( E_0(L) +
L^2 \delta \omega \). Notice that the separation lines in Fig. 1 are
nearly straight. In absence of the 3-body term, \( c_3 = 0 \), the GS of \( \mathcal{H}_2 \)
with \( L = L_{\text{pt}} = 12 \) that we indicate as \( |\psi_0^{(2)}(12)\rangle \) for brevity
- is unique and stable for a narrow interval of \( 6 \delta \omega \). The state
\( |\psi_0^{(2)}(12)\rangle \) and the Pfaffian state \(4\) share some similarities,
since they have the same angular momentum and their relative
fidelity is \( F = |\langle \psi_0^{(2)}| \psi_{\text{pt}}^{(2)}(12) \rangle |^2 \approx 0.803 \). The most important
feature of 3-body interaction is the enlargement of both the
stability interval and the fidelity, up to \( F \approx 0.991 \), for \( c_3 = 1 \).

The previous analysis urges us to design a mechanism for
independent tuning of 2- and 3-body terms, a task we tackle
with the following dimensional argument. In accordance
with the fact that \( \delta^{(2)} \) has dimensions of an inverse squared length \( [E^{-1/2}] \),
the coupling \( g_3^{2D} \) in \(1\) has the dimensions of an energy times a squared length. So, under rescaling \( E \rightarrow
E/\hbar \omega, \ell \rightarrow \ell/\xi \)
d the adimensional coupling \( c_3 = \sqrt{8 \pi a/\xi} \)
does not depend anymore on the planar trap frequency. In
other words, for 2-body collisions \(2D\) are special since den-
sity does not discriminate between weakly and strongly in-
teracting regime. Along the same line, the adimensional 3-
body coupling scales as the inverse of the effective trap area,
i.e. \( c_3 = g_3^{2D} \xi^{-1}/\hbar \omega = (g_3^{2D} m/\hbar^2) \xi^{-2} \). This suggests that the
relative importance of 3-body collisions can be boosted by a
squeeze of the \(2D\) trap, which increases the density.

Unfortunately, elastic collisions involving 3-body processes
are rather rare in nature. The most prominent 3-body collision
process known in physics of bosonic condensates is due to
recombination \(20\). The formation of a biatomic molecule is
assisted by a third particle that assures energy-momentum
conservation. The recombination rate displays a rich behav-
or as a function of the 2-body scattering length \( a \), typically
vented via Feshbach resonances. For \( a > 0 \) it shows a universal
behaviour \( \sim a^5 \) \(22\), whereas for \( a < 0 \) some genuine 3-body
resonances are appearing due to Efimov trimer states \(24\). In-
interestingly, 3-body processes are still present even in absence
of 2-body scattering (\( a = 0 \)). This nonlinear features allows
to tune \( c_3/c_2 \), in conjuction with the squeezing mechanism
exposed above. Typically, recombination is considered an un-
wanted effect in experiments with condensates since it yields
to severe 3-body losses. Nonetheless, strong dissipation has
been exploited successfully to induce strong 2-body correla-
tions in the \(1D\) Tonks-Girardeau gas, as observed in recent
experiments \(7,8\). Moreover, 3-body dissipation has been pro-
posed for obtaining a dimer superfluid phase in \(1D\) attrac-
tive boson-Hubbard models \(9\). On the same line, we propose
to use strong 3-body recombination rate to filter out the
Pfaffian state \(4\), with high fidelity and paying the small price

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{fig1}
\caption{GS phase diagram for \( N = 6 \) particles and \( c_2 = 1/6 \). Different
colors correspond to different angular momenta of the global GS.
We identify the Laughlin state \( (v = 1/2) \) with \( L = N(N-1) = 30 \),
the Pfaffian \( (v = 1) \) with \( L = N(N-2)/2 = 12 \), the single vortex
\( L = N \) and finally \( L = 0 \).}
\end{figure}
of moderate losses.

The Markovian dynamics of the system is described by a Lindblad master equation for the density matrix $\rho$

$$\dot{\rho} = -\frac{i}{\hbar} [\mathcal{H}_{\text{eff}} \rho - \rho \mathcal{H}_{\text{eff}}] + \sum \gamma_l d^\dagger_l d^l \rho + \frac{i}{2} \gamma_l d^\dagger_l d^\dagger_l \rho d^l,$$

where $\gamma_l$ is the rate of decay in the channel of 3-body total angular momentum $l$. The effective non-hermitian Hamiltonian is given by

$$\mathcal{H}_{\text{eff}} = \mathcal{H}_{\text{trap}} + \mathcal{H}_2 - \frac{i}{2} \sum_l \gamma_l d^\dagger_l d^l = \mathcal{H}_{\text{trap}} + \mathcal{H}_2 - \frac{i}{2} \mathcal{H}_3.$$

In the quantum jump approach [22, 23], loss events are assumed to be rare and are ideally detected by performing very frequent measurements. This allows to describe the non-unitary evolution of the system in terms of wave functions, through the equation $i\hbar \psi(t) = \mathcal{H}_{\text{eff}} \psi(t)$. The reduction of $||\psi(t)||^2$ gives the probability of having suffered from a jump, i.e. a 3-body loss, in $[0,t]$. Actually, we are interested only in lossless samples and we discard the cases where losses occur, since generally they lead to projection onto excited states. Experimentally, the discarding procedure is made possible by using post-selection of the samples, after measuring the number of particles. In the limit case $c_2 = 0$, the dynamics is governed only by the dissipative term and $|\psi(t)| = \exp(-t\mathcal{H}_3)|\psi(0)|$.

Due to the positiveness of $\mathcal{H}_3$, this evolution projects onto $K_3$ for long enough times, whose scale is set by $1/c_3$.

Let us assume now to have prepared several copies of a few body system with predominant 2-body collisions in the GS at angular momentum $L_{\text{Pf}}$. We will discuss later the experimental feasibility of this assumption. At $t = 0$, strong 3-body dissipation is suddenly switched on by squeezing the trap. The ideal situation is accomplished by turning off completely $c_2$ while having $c_3$ not too small. In this case the filtering produces the Pfaffian state since it is unique in $K_3$ for $L_{\text{Pf}}$. This works provided the starting $|\psi_0^{(2)}(L_{\text{Pf}})|$ has a sizable overlap with the Pfaffian, which indeed occurs for a moderate $N$.

Possibly, a more feasible experimental procedure would avoid to switch off $c_2$ at $t = 0$. The presence of 2-body collisions contrasts the formation of pairing contained in the Pfaffian. However, we expect that for $c_3/c_2 \gg 1$ the perfect projection is almost recovered. In order to check this statement, we have numerically simulated the evolution for $N = 6$ by using a fourth-order Runge-Kutta algorithm. In the left panel of Fig.2 the population of trajectories unaffected by jumps and the fidelity with the exact Pfaffian state are plotted as a function of time for different values of $c_3$. It emerges the following scenario: strong enough dissipative rate yields to intense losses up to a time after which the population is substantially stationary. The stabilization of losses is a signal that the filtering procedure has converged to a state very close to the Pfaffian one, as witnessed by the saturation of fidelity to a value close to 1. This behaviour may be interpreted as a sort of Quantum Zeno effect, where strong dissipation freezes the system in $K_3$, suppressing losses [7]. The typical time we have to wait for reaching the threshold of 25% losses is shown in the right panel of Fig.2. As expected, losses are not completely suppressed for long times, since the Pfaffian is not an eigenstate of $\mathcal{H}_3$.

The considerable improvement in the reproduction of the Pfaffian state, with infidelity sinking from 20% to almost 0.05%, though relevant by itself, does not exhaust all the importance of the proposed scheme. Indeed the flurry of interest about this state is mainly related to its zero-energy excitations, that have non-Abelian braiding properties and may constitute the basic ingredients for TQC [6]. We now show that our filtering procedure allows for the production and manipulation of the typical “half-flux” quasiholes

$$\Psi_{\text{Pf}+2\text{holes}} = \text{Pf} \left( \frac{(z_i-w_1)(z_j-w_2)}{z_i-z_j} + (i \leftrightarrow j) \prod_{i<j}(z_i-z_j) \right),$$

otherwise inaccessible with only rotation and conservative 2-body interactions. The quasihole identification and motion is indeed possible only in presence of an appropriate gap protected subspace, i.e. states in $K_3$ with $L > L_{\text{Pf}}$. Such states are obviously steady states of the 3-body dissipation, and the quasi-constant gap guarantees that the filtering have almost the same speed and neatness for all of them. Then, starting from having filtered the Pfaffian out of an initial state, it would be in principle possible to engineer some excitation scheme being sure to lie inside $K_3$. However, due to the 80% similarity between the GS $|\psi_0^{(2)}(L_{\text{Pf}})|$ of $\mathcal{H}_3$ and the Pfaffian $|\psi_{\text{Pf}}|$, one might be tempted to speculate an approximate scheme for quasiholes with a similar precision. This is not the case, since the degeneracy of the quasihole subspace is completely spoiled out, to the point we cannot speak about a manifold protected by a gap. For testing our assertion, we choose the state (8) with quasiholes located at the center, i.e. $w_1 = w_2 = 0$, living in the sector $L = L_{\text{Pf}} + N$. By expanding (8) in terms of the eigenvectors of $\mathcal{H}_3$, we discover that this peculiar state has a sizable overlap with several excited states of $\mathcal{H}_3$, whose energies are spread over an interval of the order of $c_2$. 

Figure 2: Left panel: fidelity with the Pfaffian and surviving population as a function of time, after switching on dissipation $c_3$ on the initial state $|\psi_0^{(2)}(12)|$. The fidelity reaches 0.9994 for $c_3 = 10$. Right panel: threshold time for having lost 25% of population after switching the dissipation $c_3$. 

- $H_{\text{trap}}$: The effective non-hermitian Hamiltonian
- $L_{\text{Pf}}$: Angular momentum of the Pfaffian
- $c_2$: Dissipative rate for 3-body loss
- $c_3$: Dissipative rate for 2-body loss
- $|\psi_0^{(2)}(L_{\text{Pf}})|$: Initial state after 3-body dissipation
- $|\psi_{\text{Pf}}|$: Pfaffian state
For completeness, we want to discuss about the experimental detection of the Pfaffian state, that naturally follows the preparation. Of course, a great evidence is provided by the suppression of losses, that signals the absence of local 3-body superpositions. The probe is conclusive after a measurement of $L$ and $N$, since the unicity of the Pfaffian in K3. The Pfaffian is also expected to have an increased pairing with respect to $|\psi_0(12)\rangle$. As a matter of fact, the expectation value of the 2-body contact potential $\langle H_2 \rangle$ is respectively $3.13 c_2$ and $3.31 c_2$, with a difference of about 5%, maybe not striking enough to be resolved in a clean way in experiments.

In the previous study, we assume that before applying the filtering via dissipation, it is possible to initialize the system in $|\psi_0(2)(L_{\ell})\rangle$. To obtain a given filling factor $\nu$ it is necessary to apply a rotation to the ultracold gas. The widely used experimental technique consists on stirring the condensate [24, 25]. In the rotating frame, it is equivalent to induce a small quadrupole deformation to the trap $H \rightarrow H + H_\ell$, with

$$ H_\ell = \varepsilon (x^2 - y^2), $$

where $\varepsilon$ is meant to be small for avoiding heating and coupling with higher LLs. The term $\varepsilon$ couples single particle states with angular momenta differing by 2. A possible route now is to find an adiabatic path in the plane $(\delta \omega, \varepsilon)$ connecting an already realized state, such as the single vortex GS $|\psi_0(2)(N)\rangle$, to the GS in the desired sector $|\psi_0(2)(L_{\ell})\rangle$. Once again we resort to the case of $N = 6$ and diagonalize the problem in the full LLL Hilbert space in order to draw the map of the first excitation gap [12], as displayed in Fig.3. A good path would try to avoid regions with a small gap, since they are roughly associated to a slow down of the adiabatic evolution. We have studied numerically the time evolution along the path marked by the white arrows in Fig.3 with an adaptive method that adjusts the parameter speeds according to the adiabatic condition $\langle \psi_0 | H | \psi_m \rangle \ll |E_m - E_0|^{2/m} \neq 0$. At the end of the path, it is possible to reach a fidelity of 99.7% with the GS at $L = 12$. The price to pay is a evolution time $t \approx \alpha^{-1}$ which is quite long, but still comparable with the typical time of such experiments in traps. The reason is that for reaching high angular momenta, it seems unavoidable to cross regions with a small gap. The way how Fig.5 scales with $N$ is not clear, and it may be the case that the mean-field and QHE regimes are separated by a quantum phase transition. This fact could explain why so far experiments failed in reaching the QHE regime [12, 24, 25]. However, for a small number of particles like in our case, the gap is still sizable and we expect experiments with the single holes of optical lattices [12] to succeed in the next future. Any other scheme of GS preparation would be equally good to provide the starting point for our procedure.

It is remarkable that the filtering strategy is working best if $c_3$ is turned on after the preparation of $|\psi_0(2)(L_{\ell})\rangle$. Our simulations indicate that switching on $c_3$ slowly or along the path for preparing $|\psi_0(2)(L_{\ell})\rangle$ produce more losses. Finally, we emphasize that starting the adiabatic preparation from states with higher $L$, like $\Psi_{\text{Lau}}$, needs evolution times which are too long, due to crossing of regions with very small gaps.

In conclusion, we propose a feasible filtering scheme for the realization and stabilization of Pfaffian state and excitations. Once the present considerable efforts about preparing a system in the desired angular momentum would achieve their goal, any other requirement is far within the present technologies. We showed that a tuning mechanism for increasing the relative importance of 3-body losses relies indeed on squeezing the harmonic trap. Experiments in this direction have been done recently [10]. Reducing the magnitude of 2-body collisions, by Feshbach resonance techniques, further enhances the filtering procedure. The almost perfect cancellation of $c_3$ have been obtained in Ref.[11]. Detailed calculations and discussions are included in a forthcoming work [19].

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