Light-front quantum chromodynamics may lead to an accurate constituent approximation for the low-energy properties of hadrons. This requires a cutoff that violates explicit gauge invariance and Lorentz covariance, leading to the calculation of a renormalized QCD Hamiltonian using a similarity renormalization group. Renormalization repairs broken symmetries, and in light-front field theory it moves dynamical effects that usually require large numbers of partons to few-body effective interactions. This has been shown to work in QED through dominant contributions to the Lamb shift. In QCD logarithmic confinement arises at second order, and initial bound state calculations produce reasonable results.

1 Motivation and Outline

Quantum chromodynamics (QCD) is the fundamental theory of the strong interaction, but our understanding of QCD and our ability to use it to solve low-energy problems where the interactions are truly strong falls far short of our accomplishments in the study of quantum electrodynamics (QED). Both are gauge theories, but the fact that gluons carry color charge drastically complicates QCD. Couplings in QED are weak at low energies and photons are nearly free, so that the interactions their exchange produces are readily approximated. As a result low energy bound states can be accurately described using a small number of constituents whose interactions appear in the Hamiltonian at second order, and the vacuum has no effect on bound states in light-front QED.

The study of QCD and strong interaction phenomenology leads to little hope of deriving similar approximations. Interactions in QCD are strong at low energies and the color-charged gluons interact strongly while mediating interactions. The vacuum is supposed to have a complicated structure to which conferences are devoted. Its structure is assumed to be responsible for essential aspects of the theory such as confinement and chiral symmetry breaking in all widely respected treatments of the theory, and it is hard to imagine an accurate description of individual hadrons that does not also include a complicated description of the vacuum in which they reside. Nonetheless, we advocate an approach in which an accurate description of hadrons resembles the accurate description of atoms in QED.

We start with the heretical conjecture that a constituent picture of
hadrons can be derived from QCD. This conjecture guides our calculations, but the approach I describe is completely fixed by QCD and the constituent picture will fail if it is inadequate.

If a constituent approximation is accurate, we can study the low-energy properties of hadrons (e.g., mesons) by solving a relativistic Schrödinger equation:

\[ H_\Lambda | \Psi_\Lambda \rangle = E | \Psi_\Lambda \rangle, \quad (1) \]

with,

\[ | \Psi_\Lambda \rangle = \phi^{\Lambda}_{\bar{q}q} | q\bar{q} \rangle + \phi^{\Lambda}_{\bar{q}qg} | q\bar{q}g \rangle + \cdots, \quad (2) \]

where I use shorthand notation for the Fock space components of the state. The exact state vector includes an infinite number of terms. In a constituent approximation we truncate this series, adding terms to improve the approximation. We derive the hamiltonian from QCD, so we must allow for the possibility of constituent gluons. I have indicated that the hamiltonian and the state both depend on a cutoff, \( \Lambda \), which is critical for the approximation.

This approach has no chance of working without a renormalization scheme tailored to light-front hamiltonian field theory. Much of our work has focused on the development of such a renormalization scheme.

Consider the conditions under which it might be possible to truncate the above Fock space series without making an arbitrarily large error in the eigenvalue. I focus on the eigenvalue, because it is certainly not possible to approximate all observable properties of hadrons (e.g., wee parton structure functions) this way. For this approximation to be valid, all many-body states must approximately decouple from the dominant few-body components.

We know that even in perturbation theory, high energy many-body states do not decouple from few-body states. In fact, the errors from simply discarding high energy states are infinite. In second-order perturbation theory, for example, high energy photons contribute an arbitrarily large shift to the mass of an electron. This second-order effect is illustrated in Fig. 1. The solution to this problem is well-known, renormalization. Renormalization moves the effects of high energy components in the state to effective interactions in the hamiltonian.

It is difficult to see how a constituent approximation can emerge in a hamiltonian calculation using any regularization scheme without a cutoff that

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\(^a\text{The description of light hadrons requires refinements to the simple approximations I describe, because of chiral symmetry breaking.}\)
Figure 1. The second-order shift in the self-energy of a bare electron due to mixing with electron-photon states.

either removes high energy degrees of freedom or removes direct couplings between low and high energy degrees of freedom.

In the best case scenario we expect the cutoff to act like a resolution. If the cutoff is increased to an arbitrarily large value, the resolution increases and instead of seeing a few constituents we resolve the substructure of the constituents and the few-body approximation breaks down. As the cutoff is lowered, this substructure is removed from the state vectors, and the renormalization procedure replaces it with effective interactions in the Hamiltonian. Any “cutoff” that does not remove this substructure from the states is of no use to us.

This point is well-illustrated by the QED calculations discussed below. There is a window into which the cutoff must be lowered for the constituent approximation to work. If the cutoff is raised atomic states start to include photons, and as the cutoff is raised further they start to include additional photons and electron-positron pairs. After the cutoff is lowered to a value that can be self-consistently determined \textit{a-posteriort}, photons and pairs are removed from the states and replaced by the Coulomb interaction and relativistic corrections in the Hamiltonian. The cutoff cannot be lowered too far using a perturbative renormalization group, hence the window.

Thus, if we remove high energy degrees of freedom, or coupling to high energy degrees of freedom, we should encounter self-energy corrections leading to effective one-body operators, vertex corrections leading to effective vertices, and exchange effects leading to explicit many-body interactions not found in the canonical Hamiltonian. We naively expect these operators to be local when acting on low energy states, because simple uncertainty principle arguments indicate that high energy virtual particles cannot propagate very far.
Figure 2. Mixing of low-energy few-body states with high-energy many-body states alters the dispersion relation for single particles, alters emission and absorption vertices, and produces approximately local few-body interactions.

Unfortunately these arguments break down in light-front coordinates, and at best we can maintain transverse locality.

In this article I give a brief overview of the new renormalization techniques we employ, and in a separate article in these Proceedings Sérgio Szpigel and I illustrate them using the two-dimensional δ-function potential.

Low energy many-body states do not typically decouple from low energy few-body states. The worst of these low energy many-body states is the vacuum. This is what drives us to use light-front coordinates. Fig. 3 shows a pair of particles being produced out of the vacuum in equal-time coordinates \( t \) and \( z \). The transverse components \( x \) and \( y \) are not shown, because they are the same in equal-time and light-front coordinates. The figure also shows the light-front time axis,

\[
x^+ = t + z ,
\]

and the light-front longitudinal spatial axis,

\[
x^- = t - z .
\]

In equal-time coordinates it is kinematically possible for virtual pairs to be produced from the vacuum, as long as their momenta sum to zero so that three-momentum is conserved. Because of this, the state vector for a proton includes an arbitrarily large number of particles that are disconnected from the proton. The only constraint imposed by relativity is that particle velocities be less than or equal to that of light.
In light-front coordinates, however, we see that all allowed trajectories lie in the first quadrant. In other words, light-front longitudinal momentum, \( p^+ \) (conjugate to \( x^- \) since \( a \cdot b = \frac{1}{2}(a^+b^- + a^-b^+) - a_\perp \cdot b_\perp \)), is always positive,

\[
p^+ \geq 0.
\]

(5)

We exclude particle modes with \( p^+ = 0 \), forcing the vacuum to be trivial because it is the only state with \( p^+ = 0 \). Moreover, the light-front energy of a free particle of mass \( m \) is

\[
p^- = \frac{p^2 + m^2}{p^+}.
\]

(6)

This implies that all free particles with zero longitudinal momentum have infinite energy, unless their mass and transverse momentum are zero.

Is the vacuum really trivial? What about confinement? What about chiral symmetry breaking? What about instantons? What about the job security of theorists who study the vacuum?

I simply discard all \( p^+ = 0 \) degrees of freedom and replace their effects using a renormalization procedure that does not require any explicit reference to zero modes. Thus the vacuum in our formalism is trivial. We are forced to work in the “hidden symmetry phase” of the theory, and to introduce effective interactions that reproduce all effects associated with the vacuum in other formalisms. The simplest example of this approach is provided by a scalar field theory with spontaneous symmetry breaking. It is possible to
shift the scalar field and deal explicitly with a theory containing symmetry
breaking interactions. In the simplest case $\phi^3$ is the only relevant or marginal
symmetry breaking interaction, and one can simply tune this coupling to the
value corresponding to spontaneous rather than explicit symmetry breaking.

The use of a symmetry-breaking cutoff and the removal of zero-modes
leads to a large number of operators not found in the canonical QCD Hamiltonian.
This complicates the renormalization procedure, but it may lead to
tremendous simplifications in the final nonperturbative problem. For exam-
ple, few-body operators must produce confinement manifestly!

Confinement cannot require particle creation and annihilation, flux tubes,
etc. This is easily seen using a variational argument. Consider a color neutral
quark-antiquark pair that are separated by a distance $R$, which is slowly
increased to infinity. Moreover, to see the simplest form of confinement assume
that there are no light quarks, so that the energy should increase indefinitely
as they are separated if the theory possesses confinement. At each separation
the gluon components of the state adjust themselves to minimize the energy.
But this means that the expectation value of the Hamiltonian for a state
with no gluons must exceed the energy of the state with gluons, and therefore
must diverge even more rapidly than the energy of the true ground state. This
means that there must be a two-body confining interaction in the Hamiltonian.
If the renormalization procedure is unable to produce such confining two-body
interactions, it is invalid.

1.1 Simple Strategy

I want to outline a conceptually simple strategy for bound state calculations.
The first step is to use a perturbative similarity renormalization group and
coupling coherence to find the renormalized Hamiltonian as an expansion
in powers of the canonical coupling:

$$H^\Lambda = h_0 + g_1^\Lambda h_1^\Lambda + g_2^\Lambda h_2^\Lambda + \cdots.$$  (7)

We compute this series to a finite order and to date have not required any
ad hoc assumptions to uniquely fix the Hamiltonian. No operators are added
to the Hamiltonian by hand, so it is completely determined by the underlying
theory to this order. This step is illustrated in a separate article in these
Proceedings.

The second step is to employ bound state perturbation theory to solve
the eigenvalue problem. The complete Hamiltonian contains every interaction
(although each is cut off) contained in the canonical Hamiltonian, and many
more. We separate the hamiltonian,

$$H^\Lambda = H^\Lambda_0 + \mathcal{V}^\Lambda,$$

(8)
treating $H^\Lambda_0$ nonperturbatively and computing the effects of $\mathcal{V}^\Lambda$ in bound state perturbation theory. We must choose $H^\Lambda_0$ and $\Lambda$ so that $H^\Lambda_0$ is tractable and to minimize corrections from higher orders of $\mathcal{V}^\Lambda$ within a constituent approximation.

If a constituent approximation is valid after $\Lambda$ is lowered to a critical value that must be determined, we may be able to move all particle creation and annihilation to $\mathcal{V}^\Lambda$. $H^\Lambda_0$ includes many-body interactions that do not change particle number, and these interactions should be primarily responsible for the constituent bound state structure.

There are several obvious flaws in this strategy. Chiral symmetry-breaking operators, which must be included in the hamiltonian since we work entirely in the hidden symmetry phase of the theory, do not appear at any finite order in the coupling. There is only one relevant chiral symmetry breaking operator, and it appears in the canonical hamiltonian when quarks are massive (spin-flip gluon emission by quarks) although it can acquire non-canonical dependence on longitudinal momenta since there is no longitudinal locality. This operator must simply be added if quarks are massless and tuned to fit spectra or fixed by a non-perturbative renormalization procedure. In addition, there are perturbative errors in the strengths of all operators. We know from simple scaling arguments that when $\Lambda$ is in the perturbative scaling regime:

- small errors in relevant operators exponentiate in the output,
- small errors in marginal operators produce comparable errors in output,
- small errors in irrelevant operators tend to decrease exponentially in the output.

This means that even if a relevant operator appears (e.g., a constituent quark or gluon mass operator), we may need to tune its strength rather than use its perturbative value to obtain reasonable results. We have not had to do this, but we have recently studied some of the effects of tuning a gluon mass operator.

To date this strategy has produced well-known results in QED through the Lamb shift, and reasonable results for heavy quark bound states in QCD. All of these calculations rely on a nonrelativistic reduction of the effective hamiltonian, which leads to drastic simplifications. Glueball
calculations are being completed and will be the first fully relativistic QCD calculations in this approach.

The best place to begin the study of light-front field theory and to see how a constituent approximation can arise in light-front gauge theories is the Schwinger model, massless QED in $1 + 1$ dimensions. This model can be solved analytically, and its physical content is remarkably simple. There is only one physical particle, a massive neutral scalar particle with no self-interactions. The Fock space content of the physical states depends crucially on the coordinate system and gauge, and it is only in light-front coordinates that a simple constituent picture emerges. In light-front field theory the physical particle is a bound state of a single electron-positron pair, with a wave function that is constant in the longitudinal fraction carried by either particle.

The Schwinger model does not display the renormalization problems that must be solved in QED$_{3+1}$ and QCD$_{3+1}$. Before turning to these theories, I discuss the renormalization machinery that has been developed for light-front hamiltonians.

## 2 Light-Front Renormalization Group

In $3 + 1$ dimensions we must introduce a cutoff, $\Lambda$, and we never perform explicit bound state calculations with $\Lambda$ anywhere near its continuum limit. In fact, we want to let $\Lambda$ become as small as possible. In my opinion, any strategy for solving light-front QCD that requires the cutoff to explicitly approach infinity in the nonperturbative part of the calculation is useless. Therefore, we must set up and solve

$$
P^-_\Lambda | \Psi_\Lambda(P) \rangle = \frac{P^2 + M^2}{P^+} | \Psi_\Lambda(P) \rangle .
$$

Physical results, such as the mass, $M$, can not depend on the arbitrary cutoff, $\Lambda$. This means that $P^-_\Lambda$ and $| \Psi_\Lambda \rangle$ must depend on the cutoff in such a way that $\langle \Psi_\Lambda | P^-_\Lambda | \Psi_\Lambda \rangle$ does not. Wilson based the derivation of his renormalization group on this observation, and we modify Wilson’s renormalization group to compute $P^-_\Lambda$.

It is difficult to even talk about how the hamiltonian depends on the cutoff without having a means of changing the cutoff. If we can change the cutoff, we can explicitly watch the hamiltonian’s cutoff dependence change and fix its cutoff dependence by insisting that this change satisfy certain requirements (e.g., that the limit in which the cutoff is taken to infinity exists).
Figure 4. Two ways to run a cutoff on free energy. In (a) a cutoff on the magnitude of the energy is lowered from the solid to the dashed lines, with problems resulting from the removed shaded region. In (b) a cutoff on how far off diagonal matrix elements appear is lowered from the dashed to the solid lines.

We introduce an operator that changes the cutoff,

$$H(\Lambda_1) = T[H(\Lambda_0)],$$

where I assume that $\Lambda_1 < \Lambda_0$. To simplify the notation, I let $H(\Lambda_1) = H_r$. To renormalize the hamiltonian we study the properties of the transformation.

Fig. 4 displays two generic cutoffs that might be used. Traditionally theorists have used cutoffs that remove high energy states, as shown in Figure 4a. This is the type of cutoff Wilson employed in his initial work and I have studied its use in light-front field theory. When a cutoff on energies is reduced, all effects of couplings eliminated must be moved to effective operators. When these effective operators are computed perturbatively they involve products of matrix elements divided by energy denominators. Expressions closely resemble those encountered in standard perturbation theory, with the second-order operator involving terms of the form

$$\delta V_{ij} \sim \frac{\langle \phi_i | V | \phi_k \rangle \langle \phi_k | V | \phi_j \rangle}{\epsilon_i - \epsilon_k}.\tag{11}$$

This new effective interaction replaces missing couplings, so the states $\phi_i$ and $\phi_j$ are retained and the state $\phi_k$ is one of the states removed. The problem comes from the shaded, lower right-hand corner of the matrix, where the energy denominator vanishes for states at the corner of the remaining matrix. In this corner we should use nearly degenerate perturbation theory rather than...
perturbation theory, but to do this requires solving high energy many-body problems nonperturbatively before solving the low energy few-body problems.

An alternative cutoff, which does not actually remove any states and that can be run by a similarity transformation[^l] is shown in Fig. 4b. This cutoff removes couplings between states whose free energy differs by more than the cutoff. The advantage of this cutoff is that the effective operators resulting from it contain energy denominators that are never smaller than the cutoff, so that a perturbative approximation for the effective hamiltonian may work well.

Before discussing the similarity transformation, which runs a cutoff on off-diagonal matrix elements, I need to introduce the classification of operators as relevant, marginal, and irrelevant. This classification is not necessary to use the similarity renormalization group, but it is necessary for the definition of coupling coherence, and it is essential for a full understanding of the renormalization group. It starts with the definition of a fixed point, which is a hamiltonian that is left invariant by the transformation,

\[ H^* = T[H^*] . \] (12)

Obviously a fixed point is a renormalized hamiltonian, because the cutoff fixed point hamiltonian is identical to the fixed point hamiltonian with infinite cutoff.

Consider the immediate ‘neighborhood’ of the fixed point, and assume that the trajectory remains in this neighborhood. This assumption must be justified \textit{a posteriori}, but if it is true we should write

\[ H_l = H^* + \delta H_l , \] (13)

and consider the trajectory of small deviations \( \delta H_l \).

As long as \( \delta H_l \) is ‘sufficiently small,’ we can use a perturbative expansion in powers of \( \delta H_l \), which leads us to consider

\[ \delta H_{l+1} = L \cdot \delta H_l + N[\delta H_l] . \] (14)

Here \( L \) is the linear approximation of the full transformation in the neighborhood of the fixed point, and \( N[\delta H_l] \) contains all contributions to \( \delta H_{l+1} \) of \( \mathcal{O}(\delta H_l^2) \) and higher.

The object of the renormalization group calculation is to compute trajectories and this requires a representation for \( \delta H_l \). The problem of computing trajectories is one of the most common in physics, and a convenient basis

[^l]: In deference to the original work I call this a similarity transformation even though in all cases of interest to us it is a unitary transformation.
for the representation of $\delta H_l$ is provided by the eigenoperators of $L$, since $L$ dominates the transformation near the fixed point. These eigenoperators and their eigenvalues are found by solving

$$L \cdot O_m = \lambda_m O_m .$$

(15)

For light-front field theory this linear transformation is a scaling of the transverse coordinate, the eigenoperators are products of field operators and transverse derivatives, and the eigenvalues are determined by the transverse dimension of the operator. All operators can include both powers and inverse powers of longitudinal derivatives because there is no longitudinal locality.

Using the eigenoperators of $L$ as a basis we can represent $\delta H_l$,

$$\delta H_l = \sum_{m \in R} \mu_m O_m + \sum_{m \in M} g_m O_m + \sum_{m \in I} w_m O_m .$$

(16)

Here the operators $O_m$ with $m \in R$ are relevant (i.e., $\lambda_m > 1$), the operators $O_m$ with $m \in M$ are marginal (i.e., $\lambda_m = 1$), and the operators with $m \in I$ are irrelevant (i.e., $\lambda_m < 1$). The motivation behind this nomenclature is made clear by considering repeated application of $L$, which causes the relevant operators to grow exponentially, the marginal operators to remain unchanged in strength, and the irrelevant operators to decrease in magnitude exponentially.

### 2.1 Similarity transformation

Stan Glazek and Ken Wilson studied the problem of small energy denominators that typically appear in effective interactions and realized that a similarity transformation that runs a different form of cutoff (as discussed above) avoids this problem. Independently, Wegner developed a similarity transformation that is easier to use than those first studied by Glazek and Wilson. Recently Walhout has developed a transformation that may be well-suited to analytical and higher-order calculations.

Many details and a simple example of the use of Wegner’s similarity transformation can be found elsewhere in these Proceedings. Here I summarize the details needed to complete QED and QCD calculations through second order. Consider a hamiltonian, $H_s = h + V_s$, where $h$ is a free hamiltonian that may contain mass terms. The dependence of $H$ on the cutoff, where $s$ is the inverse of the cutoff squared, is given by

$$\frac{dH_s}{ds} = [H_s, [H_s, h]].$$

(17)
This equation is most easily analyzed in terms of matrix elements between eigenstates of $h$,

$$h|\phi_i\rangle = \epsilon_i|\phi_i\rangle . \quad (18)$$

A reduced interaction, $\mathbf{V}_s$, can be defined,

$$V_{sij} = e^{-s\Delta_{ij}^2} V_{sij} , \quad (19)$$

where $\Delta_{ij} = \epsilon_i - \epsilon_j$. A gaussian cutoff factor that clearly forces the interactions towards the diagonal is isolated, and the equation for $\mathbf{V}_s$ starts at second order,

$$\frac{dV_{sij}}{ds} = -\sum_k (\Delta_{ik} + \Delta_{jk}) V_{sik} V_{skj} e^{-2s\Delta_{ik}\Delta_{jk}} , \quad (20)$$

where I use $\Delta_{ij}^2 - \Delta_{ik}^2 - \Delta_{jk}^2 = -2\Delta_{ik}\Delta_{jk}$.

This is a first-order differential equation and it is exact. Its solution is completely determined when a complete set of boundary conditions are specified. In its simplest form the boundary conditions would be given by specifying $H_{s0}$ using a single value of $s_0$ for all matrix elements, with $s_0 = 0$ corresponding to an infinite cutoff. As discussed in the simple $\delta$-function example, divergences prevent us from using $s_0 = 0$ to specify boundary conditions for relevant and marginal operators, but we are free to choose different values of $s_0$ for different operators and this freedom is crucial. The boundary conditions are given by coupling coherence, which I discuss in the next section.

For QED and QCD we can approximate $\mathbf{V}_s$ using a perturbative expansion in powers of a single running coupling constant,

$$\mathbf{V}_s = g_s V^{(1)} + g_s^2 V^{(2)} + \cdots . \quad (21)$$

In both QED and QCD the coupling does not begin to run until third order, and the first order term, $V^{(1)}$, is given by the canonical hamiltonian and has no dependence on $s$ other than that of the running coupling. To second order we have

$$\frac{dV^{(2)}_{sij}}{ds} = -\sum_k (\Delta_{ik} + \Delta_{jk}) V^{(1)}_{ik} V^{(1)}_{kj} e^{-2s\Delta_{ik}\Delta_{jk}} . \quad (22)$$

Integrating from $s_0$ to $s$, we obtain

$$V^{(2)}_{sij} = \frac{1}{2} \sum_k V^{(1)}_{ik} V^{(1)}_{kj} \left( \frac{1}{\Delta_{ik}} + \frac{1}{\Delta_{jk}} \right) \times$$

$$\times \left[ e^{-2s_0\Delta_{ik}\Delta_{jk}} - e^{-2s\Delta_{ik}\Delta_{jk}} \right] . \quad (23)$$
This simple equation is sufficient to compute the renormalized QED and QCD hamiltonians through second order the canonical couplings.

In order to use the concept of fixed points in the similarity renormalization group, we need to modify the transformation slightly so that it is possible to get a fixed point with interactions. The free hamiltonian $h$ is obviously a fixed point, but Eq. (20) clearly shows that the linearized transformation is given by Eq. (19). If we want to have fixed points for which $V_s$ is not zero, we need to modify the transformation so that the gaussian cutoff factor in Eq. (13) does not change as $s$ changes. This is most easily accomplished in the traditional manner by rescaling variables so that the eigenvalues in Eq. (19) absorb the change in $s$. For example, in massless light-front field theory,

$$\epsilon_i = p^-_i = \frac{p_1^2}{p^+_1}.$$  

We can absorb a change in $s$ by rescaling all transverse momenta, and this leads to an operator classification based on the transverse engineering dimension of operators. There are other interesting possibilities, but the result is that transverse local operators (i.e., no inverse powers of transverse momenta) that vanish when all transverse momenta are taken to zero are irrelevant. The electron-photon and quark-gluon couplings are marginal, except for a spin-flip piece that is relevant and breaks light-front chiral symmetry.

The interesting new feature of light-front field theory that is not encountered in equal-time or euclidean field theory is that there is no longitudinal locality, so that the longitudinal dimension of operators does not affect their classification. A relevant or marginal operator can contain a function of longitudinal momentum fractions, and these functions inevitably appear in renormalized light-front hamiltonians. This means that there are effectively an infinite number of relevant and marginal operators in renormalized light-front hamiltonians, and such a situation is usually regarded as a disaster for renormalization group treatments because it indicates that there may be an infinite number of free physical parameters. Ken Wilson and I developed coupling coherence to deal with this problem.

2.2 Coupling coherence

The basic mathematical idea behind coupling coherence was first formulated by Oehme, Sibold, and Zimmerman. They were interested in field theories where many couplings appear, such as the standard model, and wanted to find some means of reducing the number of couplings.

The puzzle that led us to the same results is how to reconcile our knowl-
edge from covariant formulations of QCD that only one running coupling constant characterizes the renormalized theory with the appearance of new counterterms and functions required by the light-front formulation. What happens in perturbation theory when there are effectively an infinite number of relevant and marginal operators? In particular, does the solution of the perturbative renormalization group equations require an infinite number of independent counterterms (i.e., independent functions of the cutoff)? Coupling coherence provides the conditions under which a finite number of running variables determines the renormalization group trajectory of the renormalized Hamiltonian. To leading nontrivial orders these conditions are satisfied by the counterterms introduced to restore Lorentz covariance in scalar field theory and gauge invariance in light-front gauge theories. In fact, the conditions can apparently be used to determine all counterterms in the Hamiltonian, including relevant and marginal operators that contain functions of longitudinal momentum fractions; and with no direct reference to Lorentz covariance, this symmetry seems to be restored to observables.

A coupling-coherent Hamiltonian is analogous to a fixed point Hamiltonian, but instead of reproducing itself exactly it reproduces itself in form with a limited number of independent running couplings. If $g_{\Lambda}$ is the only independent coupling in a theory, in a coupling-coherent Hamiltonian all other couplings are invariant functions of $g_{\Lambda}$, $f_i(g_{\Lambda})$. The extra couplings $f_i(g_{\Lambda})$ depend on the cutoff only through their dependence on the running coupling $g_{\Lambda}$, and in general we demand $f_i(0) = 0$. This boundary condition on the dependent couplings is motivated in our calculations by the fact that it is the combination of the cutoff and the interactions that force us to add the counterterms we seek, so the counterterms should vanish when the interactions are turned off.

Let me illustrate the idea with a simple example with a finite number of relevant and marginal operators ab initio, and use coupling coherence to discover when only one or two of these may independently run with the cutoff. Such conditions are met when an underlying symmetry exists, although this is not necessary.

Consider a theory in which two scalar fields interact,

$$V(\phi) = \frac{\lambda_1}{4!} \phi_1^4 + \frac{\lambda_2}{4!} \phi_2^4 + \frac{\lambda_3}{4!} \phi_1^2 \phi_2^2.$$  \hspace{1cm} (25)

Under what conditions are there fewer than three independent running coupling constants? We can use a simple cutoff on Euclidean momenta, $q^2 < \Lambda^2$. 

\hspace{1cm}
Letting $t = \ln(\Lambda/\Lambda_0)$, the Gell-Mann–Low equations are
\[ \frac{\partial \lambda_1}{\partial t} = 3\zeta \lambda_1^2 + \frac{1}{12} \zeta \lambda_2^3 + \mathcal{O}(2 \text{ loop}), \quad (26) \]
\[ \frac{\partial \lambda_2}{\partial t} = 3\zeta \lambda_2^2 + \frac{1}{12} \zeta \lambda_3^3 + \mathcal{O}(2 \text{ loop}), \quad (27) \]
\[ \frac{\partial \lambda_3}{\partial t} = \frac{2}{3} \zeta \lambda_3^2 + \zeta \lambda_1 \lambda_3 + \zeta \lambda_2 \lambda_3 + \mathcal{O}(2 \text{ loop}); \quad (28) \]
where $\zeta = \hbar/(16\pi^2)$. It is not important at this point to understand how these equations are derived.

Assume that there is only one independent variable, $\lambda = \lambda_1$, so that $\lambda_2$ and $\lambda_3$ are functions of $\lambda$. In this case Eqs. (27) and (28) become,
\[ \left( 3\tilde{\lambda}^2 + \frac{1}{12} \lambda_3^3 \right) \frac{\partial \lambda_2}{\partial \lambda} = 3\lambda_2^2 + \frac{1}{12} \lambda_3^3, \quad (29) \]
\[ \left( 3\tilde{\lambda}^2 + \frac{1}{12} \lambda_3^3 \right) \frac{\partial \lambda_3}{\partial \lambda} = \frac{2}{3} \lambda_3^2 + \lambda \lambda_3 + \lambda_2 \lambda_3. \quad (30) \]
The only non-trivial solutions are $\lambda_2 = \lambda$, and either $\lambda_3 = 2\lambda$ or $\lambda_3 = 6\lambda$. If $\lambda_3 = 2\lambda$,
\[ V(\phi) = \frac{\lambda}{4!} \left( \phi_1^2 + \phi_2^2 \right)^2, \quad (31) \]
and we find the $O(2)$ symmetric theory. If $\lambda_3 = 6\lambda$,
\[ V(\phi) = \frac{\lambda}{2 \cdot 4!} \left[ (\phi_1 + \phi_2)^4 + (\phi_1 - \phi_2)^4 \right], \quad (32) \]
and we find two decoupled scalar fields. Therefore, $\lambda_2$ and $\lambda_3$ do not run independently with the cutoff if there is a symmetry that relates their strength to $\lambda_1$.

The condition that a limited number of variables run with the cutoff does not only reveal symmetries broken by the regulator, it may also be used to uncover symmetries that are broken by the vacuum. For example, it is straightforward to show that in a scalar theory with a $\phi^3$ coupling, this coupling can be fixed as a function of the $\phi^2$ and $\phi^4$ couplings only if the symmetry is spontaneously broken rather than explicitly broken.

For the QED and QCD calculations, I need to compute the hamiltonian to second order, while the canonical coupling runs at third order. In this case we can use Eq. (23), with $s_0 \to \infty$ for relevant operators and $s_0 \to 0$ for
irrelevant operators, and with the bare coupling $e$ replaced by the running coupling $e_s$ or $e_\Lambda$. The more interesting case of marginal operators can be avoided at first.

3 Light-Front QED and QCD

Various forms of the canonical light-front QED and QCD Hamiltonians can be found in several articles. Following Brodsky and Lepage, I have displayed these Hamiltonians elsewhere.

In light-cone gauge and using light-front coordinates, it is possible to explicitly eliminate all unphysical degrees of freedom and write the Hamiltonian in terms of two-component fermions and transverse gluons. Any ambiguities in the procedure that come from the zero-mode problem or normal-ordering are resolved by coupling coherence, so the renormalized Hamiltonian is apparently uniquely determined order-by-order in the running coupling.

3.1 Light-front QED

In this section I follow the strategy outlined in the first section to compute the positronium spectrum. I outline the calculation through the leading order Bohr results and indicate how higher order calculations proceed.

The first step is to compute a renormalized cutoff Hamiltonian as a power series in the running coupling $e_\Lambda$,

$$H_\Lambda = h_0 + e_\Lambda h_1 + e_\Lambda^2 h_2 + \cdots + e_\Lambda^N h_N .$$

Having obtained the Hamiltonian to some order in $e_\Lambda$, the next step is to split it into two parts,

$$H_\Lambda = H_0^\Lambda + V^\Lambda .$$

$H_0^\Lambda$ must be accurately solved non-perturbatively, producing a zeroth order approximation for the eigenvalues and eigenstates. The greatest ambiguities in the calculation appear in the choice of $H_0^\Lambda$, which requires one of science’s most powerful computational tools, trial and error.

In QED and QCD I conjecture that for sufficiently small $\Lambda$ all interactions in $H_0^\Lambda$ preserve particle number, with all interactions that involve particle creation and annihilation in $V^\Lambda$. Corrections from $V^\Lambda$ are then computed in bound state perturbation theory.

Since $H_0^\Lambda$ is assumed to include interactions that preserve particle number, the zeroth order positronium ground state is a pure electron-positron state. We only need one- and two-body interactions; i.e., the electron self-energy
and the electron-positron interaction. The hamiltonian is computed to second order using Eq. (23). We must specify $s_0$ in Eq. (23), which corresponds to the inverse cutoff squared at which boundary conditions are placed on the hamiltonian. Coupling coherence leads to the prescription that $s_0 \to 0$ for irrelevant operators and $s_0 \to \infty$ for relevant operators.

Bare electron mixing with electron-photon states leads to a self-energy (see Fig. 1):

$$\Sigma^\Lambda_{coh}(p) = \frac{e^2}{8\pi^2 p^+} \left\{ 2y\Lambda^2 \ln \left( \frac{y^2\Lambda^2}{(y\Lambda^2 + m^2)\epsilon} \right) - \frac{3}{2}y\Lambda^2 + \frac{1}{2} \frac{ym^2\Lambda^2}{y\Lambda^2 + m^2} \right. $$

$$+ \left. 3m^2 \ln \left( \frac{m^2}{y\Lambda^2 + m^2} \right) \right\} + \mathcal{O}(\epsilon/y) ; \quad (35)$$

where $y$ is the fraction of longitudinal momentum carried by the electron, $y = p^+/P^+$. To simplify the discussion I have replaced the gaussian cutoff factors that appear in all integrals with step functions, and completed the integrals analytically using $1/\sqrt{s} = \Lambda^2/P^+$. It is possible to produce such step function cutoffs with a similarity transformation, but this leads to pathologies at higher order. More importantly, I have been forced to introduce a second cutoff,

$$xp^+ > \epsilon P^+ , \quad (36)$$

because there is a logarithmic divergence in the loop longitudinal momentum integration even with the gaussian cutoff in place. This second cutoff must be taken to zero and no new counterterms can be added to the hamiltonian, so all divergences must cancel before it is taken to zero.

We have no choice about whether this divergent operator is in the hamiltonian if we use coupling coherence. We can only choose between putting it in $\mathcal{H}_0^\Lambda$ or in $\mathcal{V}^\Lambda$. I make different choices in QED and QCD, and the arguments are based on physics.

The divergent electron ‘mass’ is a complete lie. We encounter a term proportional to $e^2\Lambda^2 \ln(1/\epsilon)/P^+$ when the scale is $\Lambda$; however, we can reduce this scale as far as we please in perturbation theory. Photons are massless, so the electron continues to dress itself with small-x photons to arbitrarily small $\Lambda$. Since I believe that this divergent self-energy is exactly canceled by mixing with small-x photons, and that this mixing is perturbative in QED, I simply put it in $\mathcal{V}^\Lambda$.

There are two time-ordered diagrams involving photon exchange between an electron with initial momentum $p_1$ and final momentum $p_2$, and a positron
with initial momentum $k_1$ and final momentum $k_2$. These are shown in Fig. 5, along with the instantaneous exchange diagram. I refer the reader to longer articles where details are given and concentrate here on the essential results.

Photon exchange above the cutoff produces an effective interaction that cancels the instantaneous photon exchange operator in the canonical hamiltonian, replacing it with a Coulomb interaction and magnetic interactions that are partially responsible for fine structure. Instantaneous photon exchange below the cutoff remains, and as is discussed in the section on QCD it produces a long-range confining interaction. In QED this long range interaction is exactly cancelled by further exchange of massless low energy photons in bound state perturbation theory. In QCD the long range interaction also acts on gluons, blocking such a cancellation and producing the essential difference between QED and QCD in this approach.

This means that we can concentrate on photon exchange above the cutoff to leading order, which still leaves us with a complicated operator in light-front coordinates. In order to present an analytic analysis I make assumptions that are justified a posteriori. First I assume that the electron and positron momenta can be arbitrarily large, but that in low-lying states their relative momenta satisfy

$$|p_\perp - k_\perp| \sim \alpha m, \quad (37)$$

$$|p^+ - k^+| \sim \alpha (p^+ + k^+) . \quad (38)$$
This allows us to use power counting to evaluate the perturbative strength of operators for small coupling.

Given these order of magnitude estimates for momenta, we can drastically simplify all of the operators in the Hamiltonian. At this point we can complete the zeroth order analysis of positronium using the state, 

$$| \Psi(P) \rangle = \sum_{\sigma \lambda} \int \frac{dp^+ dp^\perp}{16\pi^3 p^+} \frac{dk^+ dk^\perp}{16\pi^3 k^+} \sqrt{p^+ k^+} 16\pi^3 \delta^3(P - p - k) \phi(p, \sigma; k, \lambda) b^\dagger(p, \sigma) d^\dagger(k, \lambda) |0\rangle ,$$  

(39)

where $\phi(p, \sigma; k, \lambda)$ is the wave function for the relative motion of the electron and positron, with the center-of-mass momentum being $P$. We need to choose the longitudinal momentum appearing in the cutoff, and I use the natural scale $P^+$.

If we want to find a cutoff for which the ground state is dominated by the electron-positron component of the wave function, we need the cutoff to remove the important part of the electron-positron-photon phase space. Since the exchanged photon energy is typically $O(\alpha m^2)$, we need:

$$\Lambda^2 < \alpha m^2 .$$  

(40)

On the other hand, we cannot allow the cutoff to remove the region of the electron-positron phase space from which the wave function receives most of its strength. This requires

$$\Lambda^2 > \alpha^2 m^2 .$$  

(41)

For cutoffs that satisfy $\alpha m^2 > \Lambda^2 > \alpha^2 m^2$, the bound state equation can be simplified to:

$$-E \phi(k_1) = \frac{k_1^2}{m} \phi(k_1) - \alpha \int \frac{d^3k_2}{(2\pi)^3} \frac{1}{(k_1^2 - k_2^2)^2} \phi(k_2) ,$$  

(42)

where a simple change of variables has replaced longitudinal momentum fractions with a z-component of momentum, making the system’s nonrelativistic dynamics manifest. The cutoffs drop out to leading order, leaving us with the familiar nonrelativistic Schrödinger equation for positronium in momentum space. The solution is

$$\phi(k) = \frac{N}{(k^2 + mE)^2} ,$$  

(43)

$$E = \frac{1}{4} \alpha^2 m .$$  

(44)
\[ N \] is a normalization constant. This is the Bohr energy for the ground state of positronium, and it is obvious that the entire nonrelativistic spectrum is reproduced to leading order.

Beyond this leading order result the calculations become much more interesting, and in any Hamiltonian formulation they rapidly become complicated. The leading correction to the binding energy is \( O(\alpha^4) \), and producing these corrections is a much more serious test of the renormalization procedure. We have shown that the fine structure of positronium is correctly reproduced when the first- and second-order corrections from bound state perturbation theory are added. This is a formidable calculation, because the exact Coulomb bound and scattering states appear in second-order bound state perturbation theory.

A complete calculation of the Lamb shift in hydrogen would also require a fourth-order similarity calculation of the Hamiltonian; however, the dominant contribution to the Lamb shift that was first computed by Bethe can be computed using a Hamiltonian determined to \( O(\alpha) \). In our calculation a Bloch transformation was used rather than a similarity transformation because the Bloch transformation is simpler and small energy denominator problems can be avoided in analytical QED calculations.

The primary obstacle to using our light-front strategy for precision QED calculations is algebraic complexity. We have successfully used QED as a testing ground for this strategy, but these calculations can be done much more conveniently using other methods. The theory for which we believe our methods are best suited is QCD.

### 3.2 Light-front QCD

We only require the QCD Hamiltonian determined to \( O(\alpha) \) to discuss a simple confinement mechanism that appears naturally in light-front QCD and to complete reasonable zeroth order calculations for heavy quark bound states. To this order the QCD Hamiltonian in the quark-antiquark sector is almost identical to the QED Hamiltonian in the electron-positron sector. Of course the QCD Hamiltonian differs significantly from the QED Hamiltonian in other sectors, and this is essential for justifying my choice of \( H_0^A \) for non-perturbative calculations.

The basic strategy for doing a sequence of (hopefully) increasingly accurate QCD bound state calculations is almost identical to the strategy for doing QED calculations. Find an expansion for \( H^A \) in powers of the QCD coupling constant to a finite order. Divide the Hamiltonian into a non-perturbative part, \( H_0^A \), and a perturbative part, \( V^A \). The division is based on the physical
argument that adding a parton in an intermediate state should require more energy than indicated by the free hamiltonian, and that as a result these states ‘freeze out’ as the cutoff approaches $\Lambda_{QCD}$. When this happens the evolution of the hamiltonian as the cutoff is lowered further changes qualitatively, and operators that were consistently canceled over an infinite number of scales also freeze, so that their effects in the few parton sectors can be studied directly. A one-body operator and a two-body operator arise in this fashion, and serve to confine both quarks and gluons.

The simple confinement mechanism I outline is certainly not the final story, but it may be the seed for the full confinement mechanism. One of the most serious problems we face when looking for non-perturbative effects such as confinement is that the search itself depends on the effect. A candidate mechanism must be found and then shown to produce itself self-consistently as the cutoff is lowered towards $\Lambda_{QCD}$.

Once we find a candidate confinement mechanism, it is possible to study heavy quark bound states with little modification of the QED strategy. Of course the results in QCD differ from those in QED because of the new choice of $\mathcal{H}_0$, and in higher orders because of the gluon interactions.

When we compute the QCD hamiltonian to $\mathcal{O}(\alpha)$, several significant new features appear. First are the familiar gluon interactions. In addition to the many gluon interactions found in the canonical hamiltonian, there are modifications to the instantaneous gluon exchange interactions, just as there were modifications to the electron-positron interaction. For example, a Coulomb interaction automatically arises at short distances. In addition the gluon self-energy differs drastically from the photon self-energy.

The photon develops a self-energy because it mixes with electron-positron pairs, and this self energy is $\mathcal{O}(\alpha \Lambda^2/P^+)$). When the cutoff is lowered below $4m^2$, this mass term dies exponentially because it is no longer possible to produce electron-positron pairs. For all cutoffs the small bare photon self-energy is exactly canceled by mixing with pairs below the cutoff. I do not go through the calculation, but because the gluon also mixes with gluon pairs in QCD, the gluon self-energy acquires an infrared divergence, just as the electron did in QED. In QCD both the quark and gluon self-energies are proportional to $\alpha \Lambda^2 \ln(1/\epsilon)/P^+$, where $\epsilon$ is the secondary cutoff on parton longitudinal momenta introduced in the last section. This means that even when the primary cutoff $\Lambda^2$ is finite, the energy of a single quark or a single gluon is infinite, because we are supposed to let $\epsilon \to 0$.

In QED I argued that the bare electron self-energy is a complete lie, because the bare electron mixes with photons carrying arbitrarily small longitudinal momenta to cancel this bare self-energy and produce a finite mass
physical electron. However, in QCD there is no reason to believe that this perturbative mixing continues to arbitrarily small cutoffs. There are no massless gluons in the world. In this case, the free QCD Hamiltonian is a complete lie and cannot be trusted at low energies.

On the other hand, coupling coherence gives us no choice about the quark and gluon self-energies as computed in perturbation theory. The question is not whether large self-energies appear in the Hamiltonian. The question is whether these self-energies are canceled by mixing with low energy multi-gluon states. As the cutoff approaches $\Lambda_{QCD}$, I speculate that these cancellations cease to occur because perturbation theory breaks down and a mass gap between states with and without extra gluons appears.

But if the quark and gluon self-energies diverge, and the divergences cannot be canceled by mixing between sectors with an increasingly large number of partons, how is it possible to obtain finite mass hadrons? The parton-parton interaction also diverges, and the infrared divergence in the two-body interaction exactly cancels the infrared divergence in the one-body operator for color singlet states.

Of course, the cancellation of infrared divergences is not enough to obtain confinement. The cancellation is exact regardless of the relative motion of the partons in a color singlet state, and confinement requires a residual interaction. The $\mathcal{O}(\alpha)$ QCD Hamiltonian contains a logarithmic potential in both longitudinal and transverse directions. There is no rigorous demonstration that the confining interaction is linear, and a logarithmic potential is of interest phenomenologically for heavy quark bound states. I would be delighted if a better light-front calculation produces a linear potential, but this may not be necessary even for successful light hadron calculations.

The calculation of how the quark self-energy changes when a similarity transformation lowers the cutoff on energy transfer is almost identical to the electron self-energy calculation. We find the one-body operator required by coupling coherence,

$$\Sigma^{\Lambda}_{coh}(p) = \frac{g^2 C_F}{8\pi^2 p^+} \left[ 2y^2 \ln \left( \frac{y^2 \Lambda^2}{(y^2 + m^2)\epsilon} \right) - \frac{3}{2} y^2 \Lambda^2 + \frac{1}{2} y m^2 \Lambda^2 + 3m^2 \ln \left( \frac{m^2}{y^2 + m^2} \right) \right] + \mathcal{O}(\epsilon/y) , \tag{45}$$

where $C_F = (N^2 - 1)/(2N)$ for a SU(N) gauge theory.

The calculation of the quark-antiquark interaction required by coupling coherence is also nearly identical to the QED calculation. Just as in QED the coupling coherent interaction induced by gluon exchange above the cutoff
partially cancels instantaneous gluon exchange. For the discussion of con-
finement the part of $V_{\text{coh}}$ that remains is not important, because it produces
the short range part of the Coulomb interaction. However, the part of the
instantaneous interaction that is not canceled is

$$\tilde{V}_{\text{instant}} = -8 g_{\Lambda}^2 C_F \sqrt{p_1^+ p_2^+ k_1^- k_2^-} \left( \frac{1}{q^+} \right)^2 \delta_{\sigma_1 \sigma_2} \delta_{\lambda_1 \lambda_2}$$

\[ \times \theta(|p_1^- - p_2^-| - \epsilon P^+) \]

\[ \times \exp \left[ -2 s(p_1^- - p_2^- - q^-)(k_2^- - k_1^- - q^-) \right] . \] (46)

Note that this interaction contains a cutoff that projects onto exchange
energies below the cutoff, because the interaction has been screened by gluon
exchange above the cutoffs. This interaction can become important at long
distances, if parton exchange below the cutoff is dynamically suppressed. In
QED I argued that this singular long range interaction is exactly canceled by
photon exchange below the cutoff, because such exchange is not suppressed
no matter how low the cutoff becomes. Photons are massless and experience
no significant interactions, so they are exchanged to arbitrarily low energies
as effectively free photons. This cannot be the case for gluons.

For the discussion of confinement, place the most singular parts of the
quark self-energy and the quark-antiquark interaction in $H_{\Lambda_0}$. To see that
all infrared divergences cancel and that the residual long range interaction is
logarithmic, study the matrix element of these operators for a quark-antiquark
state,

$$|\Psi(P)\rangle = \sum_{\sigma \lambda} \sum_{rs} \int \frac{dp^+ d^2p_\perp dk^+ d^2k_\perp}{16 \pi^3 p^+ 16 \pi^3 k^+} \sqrt{p^+ k^+} 16 \pi^3 \delta^3(P - p - k) \phi(p, \sigma; r, k, \lambda, s) b^{\dagger\nu}(p, \sigma) d^{\dagger\nu}(k, \lambda) |0\rangle , \quad (47)$$

where $r$ and $s$ are color indices and $\phi$ is a color singlet.

When the expectation value of the hamiltonian is taken using this state
there are divergences as $\epsilon \to 0$ in both the expectation value of the self-energy
and the surviving piece of instantaneous gluon exchange. These divergences
cancel exactly for any color-singlet state. The cancellation resembles what
happens in the Schwinger model. If the state is a color octet the divergences
are both positive and cannot cancel. Since the cancellation occurs in the
matrix element, we can let $\epsilon \to 0$ before diagonalizing $H_{\Lambda_0}$.

The fact that the divergences cancel exactly does not indicate that con-
finement occurs. This requires the residual interactions to diverge at large
distances. It is easily shown that for large longitudinal separations the interaction becomes:

\[ V(x^-) = \frac{g_A^2 C_F \Lambda^2}{2\pi^2} \ln(|x^-|) . \]  

(48)

At large transverse separations it becomes:

\[ V(x_\perp) = \frac{g_A^2 C_F \Lambda^2}{\pi^2} \ln(|x_\perp|) . \]  

(49)

The strength of the long-range logarithmic potential is not spherically symmetrical in these coordinates, with the potential being larger in the transverse than in the longitudinal direction. Of course, there is no reason to demand that the potential be rotationally symmetric in these coordinates, because rotations are dynamical and are supposed to alter the Fock space composition of states in addition to rotating the state in a given Fock space sector.

Had we computed the quark-gluon or gluon-gluon interaction, we would find essentially the same residual long range two-body interaction in every Fock space sector, although the strengths would differ because different color operators appear. In QCD gluons have a divergent self-energy and experience divergent long range interactions with other partons if we use coupling coherence. In this sense, the assumption that gluon exchange below some cutoff is suppressed is consistent with the hamiltonian that results from this assumption. To show that gluon exchange is suppressed when \( \Lambda \to \Lambda_{QCD} \), rather than some other scale (i.e., zero as in QED), a non-perturbative calculation of gluon exchange is required. This same confinement mechanism appears using the similarity transformation developed by Wallhout.

I provide only a brief summary of our heavy quark bound state calculations and refer the reader to the original articles for details. We follow the strategy that has been successfully applied to QED, with modifications suggested by the fact that gluons experience a confining interaction.

For heavy quark bound states we can simplify the hamiltonian by making a nonrelativistic reduction and solving a Schrödinger equation. We must then choose values for \( \Lambda \), \( \alpha \), and \( M \). These should be chosen differently for bottomonium and charmonium. The cutoff for which the constituent approximation works well depends on the constituent mass, as in QED where it is obviously different for positronium and muonium. In order to fit the ground state and first two excited states of charmonium, we use \( \Lambda = 2.5GeV \), \( \alpha = 0.53 \), \( M_c = 1.6GeV \). In order to fit these states in bottomonium we use \( \Lambda = 4.9GeV \), \( \alpha = 0.4 \), and \( M_b = 4.8GeV \). Violations of rotational invariance from the remaining parts of the potential are only about 10%, and we
expect corrections from higher Fock state components to be at least of this magnitude for the couplings we use.

These calculations show that the approach is reasonable, but they are not yet very convincing. There are a host of additional calculations that must be done before the success of this approach can be judged.

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