Hard Electroproduction of Hybrid Mesons

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We estimate the sizeable cross section for deep exclusive electroproduction of an exotic $J^{PC} = 1^{-+}$ hybrid meson in the Bjorken regime. The production amplitude scales like the one for usual meson electroproduction, i.e. as $1/Q^2$. This is due to the non-vanishing leading twist distribution amplitude for the hybrid meson, which may be normalized thanks to its relation to the energy momentum tensor and to the QCD sum rules technique. The hard amplitude is considered up to next-to-leading order in $\alpha_S$ and we explore the consequences of fixing the renormalization scale ambiguity through the BLM procedure.

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1 Introduction

Within quantum chromodynamics, hadrons are described in terms of quarks, anti-quarks and gluons. The usual, well-known, mesons are supposed to contain quarks and anti-quarks as valence degrees of freedom while gluons play the role of carrier of interaction, i.e. they remain hidden in a background. On the other hand, QCD does not prohibit the existence of the explicit gluonic degree of freedom in the form of a vibrating flux tube, for instance. The states where the $q\bar{q}g$ and $gg$ configurations are dominating, hybrids and glueballs, are of fundamental importance to understand the dynamics of quark confinement and the nonperturbative sector of quantum chromodynamics.

The study of these hadrons outside the constituent quark models, namely exotic hybrids, is the main reason of the present paper. We investigate how hybrid mesons with $J^{PC} = 1^{-+}$ may be studied through the so-called hard reactions.
We focus on deep exclusive meson electroproduction (see, for instance [3]) which is well described in the framework of the collinear approximation where generalized parton distributions (GPDs) [4] and distribution amplitudes [5] describe the nonperturbative parts of a factorized amplitude [6].

2 Hybrid meson production amplitude

We propose to study the exotic hybrid meson by means of its deep exclusive electroproduction, i.e.
\[ e(k_1) + N(p_1) \rightarrow e(k_2) + H(p) + N(p_2), \] (1)
where we will concentrate on the subprocess:
\[ \gamma^*(q) + N(p_1) \rightarrow H_L(p) + N(p_2) \] (2)
when the baryon is scattered at small angle. This process is a hard exclusive reaction due to the transferred momentum \( Q^2 \) is large (Bjorken regime). Within this regime, a factorization theorem is valid, at the leading twist level, which claims that a partonic subprocess part described in perturbative QCD (pQCD) can be detached from universal soft parts, which are generalized parton distributions and meson distribution amplitudes. Below we will analyze in more details how this factorization theorem applies to the process under study.

Let us fix the kinematics of the deep electroproduction process. We are interested in the scaling regime where the virtuality of the photon \( Q^2 = -q^2 \) is large and scales with the energy of the process. We denote by \( p_1 \) (\( p_2 \)) the momentum of the incoming (outgoing) nucleon, while \( p \) is the momentum of the longitudinally polarized hybrid meson of mass \( M_H \). We construct the average momentum \( \overline{p} \) and transferred momentum \( \Delta \):
\[ \overline{p} = \frac{p_2 + p_1}{2}, \quad \Delta = p_2 - p_1, \quad \Delta^2 = t. \] (3)

With two light-cone vectors: \( n^* \cdot n = 0, n^* \cdot n = 1 \), the Sudakov decompositions for all the relevant momenta take the form:
\[ \Delta_\mu = -2\xi n^*_\mu + \xi M^2 n_\mu + \Delta_\mu^T, \quad \Delta^T \cdot n = \Delta^T \cdot n^* = 0, \]
\[ \overline{p}_\mu = n^*_\mu + \frac{M^2}{2} n_\mu, \quad \overline{p}^2 = M^2, \quad \xi \leq \frac{\sqrt{-\Delta^2}}{2M} \leq 1, \]
\[ q_\mu = -2\tilde{\xi} n^*_\mu + \frac{Q^2}{4\tilde{\xi}} n_\mu, \]
\[ p_\mu = q_\mu - \Delta_\mu = 2(\xi - \tilde{\xi}) n^*_\mu + \left( \frac{Q^2}{4\tilde{\xi}} - \xi M^2 \right) n_\mu - \Delta_\mu^T. \] (4)

Here, the parameters \( \xi \) and \( \tilde{\xi} \) are related by
\[ M_H^2 = 4(\xi - \tilde{\xi}) \left( \frac{Q^2}{4\tilde{\xi}} - \xi M^2 \right) + \Delta_\mu^T. \] (5)
The leading order amplitude for the process (1) is

\[ A(q) = \frac{e\pi\alpha_s f_H C_F}{\sqrt{2N_c Q}} [e_u H_{uu} - e_d H_{dd}] \gamma^{(H, -)}, \]  

where

\[ H_{ff}^\pm = \frac{1}{2} \int_1^{-1} dx \left[ \overline{U}(p_2) \gamma_\mu U(p_1) H_{ff}(x) + \overline{U}(p_2) i\gamma_\mu \gamma_5 U(p_1) E_{ff}(x) \right] \]

\[ \left[ \frac{1}{x + \xi - ie} \pm \frac{1}{x - \xi + ie} \right], \quad \gamma^{(M, \pm)} = \int_0^1 dy \phi^M(y) \left[ \frac{1}{y} \pm \frac{1}{1 - y} \right]. \]  

Here, functions \( H \) and \( E \) are standard leading twist GPD’s and their properties are fairly well-known. In (7), we include the definition of \( H_{ff}^\pm \) and \( \gamma^{(M, \pm)} \) which will be useful for the comparison with the \( \rho \) meson case. The hybrid meson distribution amplitude is a new object and we will carefully study it in the next subsection.

We will now consider the properties of the hybrid meson distribution amplitude (see also [7]–[9]). The Fourier transform of the hybrid meson –to–vacuum matrix element of the bilocal vector quark operator may be written as

\[ \langle H_L(p, 0) | \bar{\psi}(-z/2) \gamma_\mu \psi(z/2) | 0 \rangle = if_H M_H e^{(0)}_{L \mu} \int_0^1 dy e^{i(\bar{y} - y)p \cdot z/2} \phi^H_L(y), \]  

where \( e^{(0)}_{L \mu} = (e^{(0)} \cdot z)/(p \cdot z) p_\mu \) and \( \bar{y} = 1 - y \) and \( H \) denotes the isovector triplet of hybrid mesons; \( f_H \) denotes a dimensionful coupling constant of the hybrid meson, so that \( \phi^H \) is dimensionless.

In [3], we imply the path-ordered gluonic exponential along the straight line connecting the initial and final points \([z_1; z_2]\) which provides the gauge invariance for bilocal operator and equals unity in a light-like (axial) gauge. For simplicity of notation we shall omit the index \( L \) from the hybrid meson distribution amplitude.

Although exotic quantum numbers like \( J^{PC} = 1^- \) are forbidden in the quark model, it does not prevent the leading twist correlation function from being non zero. The basis of the argument is that the non-locality of the quark correlator opens the possibility of getting such a hybrid state, because of dynamical gluonic degrees of freedom arising from the Wilson line (more details can be found in [3]).

## 3 Cross-sections for hybrid meson electroproduction

The unpolarized cross section corresponding to the reaction (2) is defined by

\[ \frac{d\sigma_L}{dt} = \frac{1}{16\pi(\hat{s} - m_H^2)} \lambda(\hat{s}, -Q^2, m_H^2) \sum_{pol.} |A(q)|^2, \]  

\[ ^1 \) The flux factor is chosen as in [10].
where the amplitude $A(q)$ is determined by (6); $s$, $t$ are the usual Mandelstam variables and $m_N$ is the nucleon mass. The function $\lambda$ is standardly defined by

$$\lambda^2(x, y, z) = x^2 + y^2 + z^2 - 2xy - 2xz - 2yz.$$  \hspace{1cm} (10)

To calculate the cross section (9), we need to model the corresponding GPD’s. We apply the Radyushkin model [11] where the function $H$, see (6), is expressed with the help of double distributions $F^q(x, y; t)$. To get prediction for the cross sections we need to fix the renormalization scales. In order to estimate theoretical uncertainties of this procedure we fix the scale $\mu_R^2$ in two different ways: firstly, in the naive way, by assuming $\mu_R^2 = Q^2$, and secondly, by applying the BLM prescription [15]. The BLM procedure, which is discussed in details in [9], leads to the following values of the renormalization scales:

$$\mu_R^2 = e^{-4.9}Q^2, \quad \text{for } \rho \text{ meson},$$

$$\mu_R^2 = e^{-5.13}Q^2, \quad \text{for } H \text{ meson}. \hspace{1cm} (11)$$

for the case $\xi = 0.2$ (or $x_B \approx 0.33$). These renormalization scales have rather small magnitudes. This has a tendency to enlarge the cross sections but may endanger the validity of the perturbative approach. However, it is possible that the coupling constant $\alpha_S$ stays below unity and the perturbative theory does not suffer from the IR divergencies. We will use the Shirkov and Solovtsov’s ansatz [16] where the
analytic running coupling constant takes the form:

\[ \alpha_s^n(\mu^2_R) = \frac{4\pi}{\beta_0} \left[ \frac{1}{\ln \mu^2_R/\Lambda^2_{QCD}} + \frac{\Lambda^2_{QCD}}{\Lambda^2_{QCD} - \mu^2_R} \right]. \] (12)

Here \( \Lambda_{QCD} \) is the standard scale parameter in QCD. The second term in (12) assures the absence of a ghost pole at \( \mu^2_R = \Lambda^2_{QCD} \) and has a nonperturbative source. Detailed discussion on this point may be found in [17] and references therein.

Recently, in [10] the role of power corrections due to the intrinsic transverse momentum of partons (the kinematical higher twist) has been investigated. In that approach the inclusion of the intrinsic transverse momentum dependence results in a rather strong effect on the differential cross-section before the scaling regime is achieved. In [10], the renormalization scale \( \mu^2_R \) is defined by the gluon virtuality so that the scale is a function of parton fractions flowing into the corresponding gluon propagator. On Fig. we present our results for the differential cross section of the hybrid meson electroproduction compared to the \( \rho \) meson electroproduction, using the BLM scales. We can see that the hybrid cross section is rather sizeable in comparison with the corresponding \( \rho \) meson cross section. One can see that in the region \( Q^2 \sim 5 - 10 \text{ GeV}^2 \) the size of the \( \rho \) meson cross section obtained with the inclusion of transverse momentum effects is very close to the analogous cross section computed with the BLM scale and without the intrinsic transverse momentum dependence. On the other hand, for higher values of \( Q^2 \) the leading order amplitude computed with the BLM scale fixing is falling faster that the corresponding amplitude derived in Ref. [10], whereas for smaller values of \( Q^2 \) it is larger than that prediction.

4 Conclusion

In conclusion, we have calculated the leading twist contribution to exotic hybrid meson with \( J^{PC} = 1^{--} \) electroproduction amplitude in the deep exclusive region. The resulting order of magnitude is somewhat smaller than the \( \rho \) electroproduction but similar to the \( \pi \) electroproduction. The obtained cross section is sizeable and should be measurable at dedicated experiments at JLab, Hermes or Compass. We made a systematic comparison with the non-exotic vector meson production. To take into account NLO corrections, the differential cross-sections for these processes have been computed using the BLM prescription for the renormalization scale. In the case of \( \rho \) production, our estimate is not far from a previous one which took into account kinematical higher twist corrections. In the region of small \( Q^2 \) higher twist contributions should be carefully studied and included. Note that they have already been considered in the case of deeply virtual Compton scattering [18] where their presence was dictated by gauge invariance, and for transversely polarized vector mesons [19] where the leading twist component vanishes. We leave this study for future works.

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