UNIFORMLY CONVEX SPIRAL FUNCTIONS AND UNIFORMLY SPIRALLIKE FUNCTION ASSOCIATED WITH PASCAL DISTRIBUTION SERIES

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ABSTRACT. The aim of this paper is to find the necessary and sufficient conditions and inclusion relations for Pascal distribution series to be in the classes $\mathcal{SP}_p(\alpha, \beta)$ and $\mathcal{UCV}_p(\alpha, \beta)$ of uniformly spirallike functions. Further, we consider properties of a special function related to Pascal distribution series. Several corollaries and consequences of the main results are also considered.

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1. INTRODUCTION AND DEFINITIONS

Let $\mathcal{A}$ denote the class of functions of the form

$$f(z) = z + \sum_{n=2}^{\infty} a_n z^n,$$

which are analytic in the open unit disk $\mathbb{U} = \{z \in \mathbb{C} : |z| < 1\}$ and normalized by the conditions $f(0) = 0 = f'(0) - 1$. Further, let $\mathcal{T}$ be a subclass of $\mathcal{A}$ consisting of functions of the form,

$$f(z) = z - \sum_{n=2}^{\infty} |a_n| z^n, \quad z \in \mathbb{U}.$$

A function $f \in \mathcal{A}$ is spirallike if

$$\Re \left( e^{-i\alpha} \frac{zf'(z)}{f(z)} \right) > 0,$$

for some $\alpha$ with $|\alpha| < \pi/2$ and for all $z \in \mathbb{U}$. Also $f(z)$ is convex spirallike if $zf'(z)$ is spirallike.

In [23], Selvaraj and Geetha introduced the following subclasses of uniformly spirallike and convex spirallike functions.

**Definition 1.1.** A function $f$ of the form (1) is said to be in the class $\mathcal{SP}_p(\alpha, \beta)$ if it satisfies the following condition:

$$\Re \left\{ e^{-i\alpha} \left( \frac{zf'(z)}{f(z)} \right) \right\} > \left| \frac{zf'(z)}{f'(z)} - 1 \right| + \beta \quad (|\alpha| < \pi/2 ; 0 \leq \beta < 1)$$

and $f \in \mathcal{UCV}_p(\alpha, \beta)$ if and only if $zf'(z) \in \mathcal{SP}_p(\alpha, \beta)$. 
We write
\[ TSP_p(\alpha, \beta) = SP_p(\alpha, \beta) \cap T \]
and
\[ UCT_p(\alpha, \beta) = UCV_p(\alpha, \beta) \cap T. \]

In particular, we note that \( SP_p(\alpha, 0) = SP_p(\alpha) \) and \( UCV_p(\alpha, 0) = UCV_p(\alpha) \), the classes of uniformly spirallike and uniformly convex spirallike were introduced by Ravichandran et al. [20]. For \( \alpha = 0 \), the classes \( UCV_p(\alpha) \) and \( SP_p(\alpha) \) respectively, reduces to the classes \( UCV \) and \( SP \) introduced and studied by Ronning [22]. For more interesting developments of some related subclasses of uniformly spirallike and uniformly convex spirallike, the readers may be referred to the works of Frasin [5, 6], Goodman [9, 10], Tariq Al-Hawary and Frasin [13], Kanas and Wisniowska [11, 12] and Ronning [21, 22].

A function \( f \in A \) is said to be in the class \( R_\tau(A, B), \tau \in C\{0\} \), if it satisfies the inequality
\[
\left| \left( \frac{f'(z) - 1}{(A - B)\tau - B[f'(z) - 1]} \right) \right| < 1, \quad z \in U.
\]

This class was introduced by Dixit and Pal [3].

A variable \( x \) is said to be \textit{Pascal distribution} if it takes the values \( 0, 1, 2, 3, \ldots \) with probabilities \( (1 - q)^m, \frac{qm(1 - q)^m}{1!}, \frac{q^2m(m+1)(1 - q)^m}{2!}, \frac{q^3m(m+1)(m+2)(1 - q)^m}{3!}, \ldots \) respectively, where \( q \) and \( m \) are called the parameters, and thus
\[
P(x = k) = \binom{k + m - 1}{m - 1} q^k (1 - q)^m, \quad k = 0, 1, 2, 3, \ldots.
\]

Very recently, El-Deeb [4] introduced a power series whose coefficients are probabilities of Pascal distribution
\[
\Psi^m_q(z) = z + \sum_{n=2}^\infty \binom{n + m - 2}{m - 1} \cdot q^{n-1}(1 - q)^m z^n, \quad z \in U \quad (3)
\]
where \( m \geq 1; 0 \leq q \leq 1 \) and we note that, by ratio test the radius of convergence of above series is infinity. We also define the series
\[
\Phi^m_q(z) = 2z - \Psi^m_q(z) = z - \sum_{n=2}^\infty \binom{n + m - 2}{m - 1} \cdot q^{n-1}(1 - q)^m z^n, \quad z \in U. \quad (4)
\]

Now, we considered the linear operator
\[
\mathcal{I}^m_q(z) : A \rightarrow A
\]
defined by the convolution or Hadamard product
\[
\mathcal{I}^m_q f(z) = \Psi^m_q(z) * f(z) = z + \sum_{n=2}^\infty \binom{n + m - 2}{m - 1} \cdot q^{n-1}(1 - q)^m a_n z^n, \quad z \in U \quad (5)
\]
Motivated by several earlier results on connections between various subclasses of analytic and univalent functions by using hypergeometric functions (see for example, [2, 8, 14, 24, 25]) and by the recent investigations (see for example, [1, 7, 17, 18, 19, 15, 16]), in the present paper we determine the necessary and sufficient conditions for $\Phi_q^m(z)$ to be in our classes $T_{SP}^p(\alpha, \beta)$ and $UCT_p(\alpha, \beta)$ and connections of these subclasses with $R^\tau(A, B)$. Finally, we give conditions for the function $G_q^m f(z) = \int_0^z \Phi_q^m(t) \frac{dt}{t} \in$ belonging to the above classes.

To establish our main results, we need the following Lemmas.

**Lemma 1.2.** [23] A function $f$ of the form (2) is in $T_{SP}^p(\alpha, \beta)$ if and only if it satisfies

$$\sum_{n=2}^{\infty} (2n - \cos \alpha - \beta) |a_n| \leq \cos \alpha - \beta \quad (|\alpha| < \pi/2 ; 0 \leq \beta < 1).$$

(6)

In particular, when $\beta = 0$, we obtain a necessary and sufficient condition for a function $f$ of the form (2) to be in the class $T_{SP}^p(\alpha)$ is that

$$\sum_{n=2}^{\infty} (2n - \cos \alpha) |a_n| \leq \cos \alpha \quad (|\alpha| < \pi/2).$$

(7)

**Lemma 1.3.** [23] A function $f$ of the form (2) is in $UCT_p(\alpha, \beta)$ if and only if it satisfies

$$\sum_{n=2}^{\infty} n(2n - \cos \alpha - \beta) |a_n| \leq \cos \alpha - \beta \quad (|\alpha| < \pi/2 ; 0 \leq \beta < 1).$$

(8)

In particular, when $\beta = 0$, we obtain a necessary and sufficient condition for a function $f$ of the form (2) to be in the class $UCT_p(\alpha)$ is that

$$\sum_{n=2}^{\infty} n(2n - \cos \alpha) |a_n| \leq \cos \alpha \quad (|\alpha| < \pi/2).$$

(9)

**Lemma 1.4.** [3] If $f \in R^\tau(A, B)$ is of the form (1), then

$$|a_n| \leq (A - B) \frac{|\tau|}{n}, \quad n \in \mathbb{N}\{1\}.$$

The result is sharp.
2. The necessary and sufficient conditions

For convenience throughout in the sequel, we use the following identities that hold at least for \(m \geq 2\) and \(0 \leq q < 1\):

\[
\begin{align*}
\sum_{n=0}^{\infty} \binom{n + m - 1}{m - 1} \cdot q^n &= \frac{1}{(1-q)^m} \\
\sum_{n=0}^{\infty} \binom{n + m - 2}{m - 2} \cdot q^n &= \frac{1}{(1-q)^{m-1}} \\
\sum_{n=0}^{\infty} \binom{n + m}{m} \cdot q^n &= \frac{1}{(1-q)^{m+1}} \\
\sum_{n=0}^{\infty} \binom{n + m + 1}{m + 1} \cdot q^n &= \frac{1}{(1-q)^{m+2}}
\end{align*}
\]

By simple calculation we get the following:

\[
\begin{align*}
\sum_{n=2}^{\infty} \binom{n + m - 2}{m - 1} q^{n-1} &= \sum_{n=0}^{\infty} \binom{n + m - 1}{m - 1} q^n - 1 \\
\sum_{n=2}^{\infty} (n-1) \binom{n + m - 2}{m - 1} q^{n-1} &= q m \sum_{n=0}^{\infty} \binom{n + m}{m} q^n.
\end{align*}
\]

and

\[
\sum_{n=2}^{\infty} (n-1)(n-2) \binom{n + m - 2}{m - 1} q^{n-1} = q^2 m(m+1) \sum_{n=0}^{\infty} \binom{n + m + 1}{m + 1} q^n.
\]

Unless otherwise mentioned, we shall assume in this paper that \(|\alpha| < \pi/2\), \(0 \leq \beta < 1\) while \(m \geq 1\) and \(0 \leq q < 1\).

First we obtain the necessary and sufficient conditions for \(\Phi^m_q\) to be in the class \(TSP_p(\alpha, \beta)\).

**Theorem 2.1.** We have \(\Phi^m_q \in TSP_p(\alpha, \beta)\) if and only if

\[
\frac{2q m}{1-q} + (2 - \cos \alpha - \beta) [1 - (1-q)^m] \leq \cos \alpha - \beta.
\]

**Proof.** Since

\[
\Phi^m_q(z) = z - \sum_{n=2}^{\infty} \binom{n + m - 2}{m - 1} \cdot q^{n-1}(1-q)^m z^n
\]

in view of Lemma 1.2, it suffices to show that

\[
\sum_{n=2}^{\infty} (2n - \cos \alpha - \beta) \binom{n + m - 2}{m - 1} \cdot q^{n-1}(1-q)^m \leq \cos \alpha - \beta.
\]

Writing

\[n = (n-1) + 1\]
in (16) we have

\[ \sum_{n=2}^{\infty} (2n - \cos \alpha - \beta) \binom{n + m - 2}{m - 1} \cdot q^{n-1}(1 - q)^m \]

\[ = \sum_{n=2}^{\infty} (2(n - 1) + 2 - \cos \alpha - \beta) \binom{n + m - 2}{m - 1} \cdot q^{n-1}(1 - q)^m \]

\[ = 2 \sum_{n=2}^{\infty} (n - 1) \binom{n + m - 2}{m - 1} \cdot q^{n-1}(1 - q)^m + (2 - \cos \alpha - \beta) \sum_{n=2}^{\infty} \binom{n + m - 2}{m - 1} \cdot q^{n-1}(1 - q)^m \]

\[ = 2q \cdot m(1 - q)^m \sum_{n=0}^{\infty} \binom{n + m}{m} q^n + (2 - \cos \alpha - \beta)(1 - q)^m \left[ \sum_{n=0}^{\infty} \binom{n + m - 1}{m - 1} q^n - 1 \right] \]

\[ = \frac{2q \cdot m}{1 - q} + (2 - \cos \alpha - \beta) \left[ 1 - (1 - q)^m \right]. \]

But this last expression is bounded above by \( \cos \alpha - \beta \) if and only if (14) holds. □

**Theorem 2.2.** We have \( \Phi_q^m \in \mathcal{UCT}_p(\alpha, \beta) \) if and only if

\[ \frac{2q^2 \cdot m(m + 1)}{(1 - q)^2} + (6 - \cos \alpha - \beta) \frac{q \cdot m}{1 - q} + (2 - \cos \alpha - \beta) \left[ 1 - (1 - q)^m \right] \leq \cos \alpha - \beta. \quad (17) \]

**Proof.** In view of Lemma 1.3, we must show that

\[ \sum_{n=2}^{\infty} n(2n - \cos \alpha - \beta) \binom{n + m - 2}{m - 1} \cdot q^{n-1}(1 - q)^m \leq \cos \alpha - \beta. \quad (18) \]

Writing

\[ n = (n - 1) + 1 \]

and

\[ n^2 = (n - 1)(n - 2) + 3(n - 1) + 1 \]
in (18)

\[
\sum_{n=2}^{\infty} n(2n - \cos \alpha - \beta) \binom{n+m-2}{m-1} \cdot q^{n-1} (1 - q)^m e^{-m}
\]

\[= \sum_{n=2}^{\infty} (2(n-1)(n-2) + 6(n-1)) \binom{n+m-2}{m-1} \cdot q^{n-1} (1 - q)^m
\]

\[+ \sum_{n=2}^{\infty} (n-1)(-\cos \alpha - \beta) \binom{n+m-2}{m-1} \cdot q^{n-1} (1 - q)^m
\]

\[+ \sum_{n=2}^{\infty} (2 - \cos \alpha - \beta) \binom{n+m-2}{m-1} \cdot q^{n-1} (1 - q)^m
\]

\[= 2q^2 m(m+1)(1 - q)^m \sum_{n=0}^{\infty} \binom{n+m+1}{m+1} q^n + (6 - \cos \alpha - \beta) q m(1-q)^m \sum_{n=0}^{\infty} \binom{n+m}{m} q^n
\]

\[+ (2 - \cos \alpha - \beta)(1 - q)^m \left[ \sum_{n=0}^{\infty} \binom{n+m-1}{m-1} q^n - 1 \right]
\]

\[= 2q^2 m(m+1)(1 - q)^m \frac{1}{(1-q)^{m+2}} + (6 - \cos \alpha - \beta) q m(1-q)^m \frac{1}{(1-q)^{m+1}}
\]

\[+ (2 - \cos \alpha - \beta)(1 - q)^m \left[ \frac{1}{(1-q)^m} - 1 \right]
\]

\[= \frac{2q^2 m(m+1)}{(1-q)^2} + (6 - \cos \alpha - \beta) \frac{q m}{1-q} + (2 - \cos \alpha - \beta) [1 - (1-q)^m]
\]

Therefore, we see that the last expression is bounded above by \(\cos \alpha - \beta\) if (17) is satisfied. \(\square\)

3. Inclusion Properties

Making use of Lemma 1.4, we will study the action of the Pascal distribution series on the class \(\mathcal{T}_p^{\alpha, \beta}\).

**Theorem 3.1.** Let \(m > 1\). If \(f \in \mathcal{R}^\tau(A, B)\), then \(\mathcal{T}_q^m f(z) \in \mathcal{T}_p^{\alpha, \beta}\) if

\[
(A-B)\tau \left[ 2 \left[ 1 - (1-q)^m \right] - \frac{\cos \alpha + \beta}{q(m-1)} \left[ (1-q) - (1-q)^m - q(m-1)(1-q)^m \right] \right] \leq \cos \alpha - \beta.
\]

(19)

**Proof.** In view of Lemma 1.2, it suffices to show that

\[
\sum_{n=2}^{\infty} (2n - \cos \alpha - \beta) \binom{n+m-2}{m-1} \cdot q^{n-1} (1 - q)^m |a_n| \leq \cos \alpha - \beta.
\]

Since \(f \in \mathcal{R}^\tau(A, B)\), then by Lemma 1.4, we have

\[
|a_n| \leq \frac{(A-B)\tau}{n}.
\]

(20)
Thus, we have
\[
\sum_{n=2}^{\infty} (2n - \cos \alpha - \beta) \binom{n + m - 2}{m - 1} \cdot q^{n-1} (1 - q)^m |a_n|
\]
\[
\leq (A - B) |\tau| \left[ \sum_{n=2}^{\infty} \frac{1}{n} (2n - \cos \alpha - \beta) \binom{n + m - 2}{m - 1} \cdot q^{n-1} (1 - q)^m \right]
\]
\[
= (A - B) |\tau| (1 - q)^m \left[ 2 \sum_{n=2}^{\infty} \binom{n + m - 2}{m - 1} q^{n-1} - (\cos \alpha + \beta) \sum_{n=2}^{\infty} \frac{1}{n} \binom{n + m - 2}{m - 1} \cdot q^{n-1} \right]
\]
\[
= (A - B) |\tau| (1 - q)^m \left[ 2 \sum_{n=0}^{\infty} \binom{n + m - 2}{m - 2} q^n - (m - 1)q \right]
\]
\[
- \frac{\cos \alpha + \beta}{q(m-1)} \left[ \sum_{n=0}^{\infty} \binom{n + m - 2}{m - 2} q^n - (m - 1)q \right]
\]
\[
= (A - B) |\tau| \left[ 2 \left[ 1 - (1 - q)^m \right] - \frac{\cos \alpha + \beta}{q(m-1)} \left[ (1 - q) - (1 - q)^m - q(m-1)(1 - q)^m \right] \right]
\]
But this last expression is bounded by \( \cos \alpha - \beta \), if (19) holds. This completes the proof of Theorem 3.1.

Applying Lemma 1.3 and using the same technique as in the proof of Theorem 3.1, we have the following result.

**Theorem 3.2.** If \( f \in R^t(A, B) \), then \( I_q^m f \) is in \( UCT_p(\alpha, \beta) \) if and only if the inequality (14) holds.

4. Properties of a special function

**Theorem 4.1.** If the function \( G_q^m \) is given by
\[
G_q^m(z) := \int_0^z \frac{\Phi_q^m(t)}{t} dt, \quad z \in \mathbb{U},
\]
then \( G_q^m \in UCT_p(\alpha, \beta) \) if and only if the inequality (14) holds.

**Proof.** Since
\[
G_q^m(z) = z - \sum_{n=2}^{\infty} \binom{n + m - 2}{m - 1} q^{n-1} (1 - q)^m \frac{z^n}{n}, \quad z \in \mathbb{U},
\]
then by Lemma 1.3 we need only to show that
\[
\sum_{n=2}^{\infty} n(2n - \cos \alpha - \beta) \times \frac{1}{n} \binom{n + m - 2}{m - 1} q^{n-1} (1 - q)^m \leq \cos \alpha - \beta,
\]
or, equivalently
\[
\sum_{n=2}^{\infty} (2n - \cos \alpha - \beta) \binom{n + m - 2}{m - 1} \cdot q^{n-1} (1 - q)^m \leq \cos \alpha - \beta.
\]
The remaining part of the proof of Theorem 4.1 is similar to that of Theorem 2.1, and so we omit the details.

**Theorem 4.2.** If \( m > 1 \), then the function \( \mathcal{G}_q^m \in \mathcal{TSP}_p(\alpha, \beta) \) if and only if
\[
2 \left[ 1 - (1 - q)^m \right] - \frac{\cos \alpha + \beta}{q(m-1)} [(1-q) - (1-q)^m - q(m-1)(1-q)^m] \leq \cos \alpha - \beta.
\]

The proof of Theorem 4.2 is lines similar to the proof of Theorem 4.1, so we omitted the proof of Theorem 4.2.

5. Corollaries and consequences

By specializing the parameter \( \beta = 0 \) in Theorems 2.1–4.2, we obtain the following corollaries.

**Corollary 5.1.** We have \( \Phi_q^m \in \mathcal{TSP}_p(\alpha) \) if and only if
\[
\frac{2q}{1-q} m + (2 - \cos \alpha) [1 - (1-q)^m] \leq \cos \alpha.
\]  

**Corollary 5.2.** We have \( \Phi_q^m \in \mathcal{UCT}_p(\alpha) \) if and only if
\[
\frac{2q^2}{(1-q)^2} m(m+1) + \frac{6 - \cos \alpha}{q} \frac{m}{1-q} + (2 - \cos \alpha) [1 - (1-q)^m] \leq \cos \alpha.
\]

**Corollary 5.3.** Let \( m > 1 \). If \( f \in \mathcal{R}^r(A, B) \), then \( I_q^m f \in \mathcal{TSP}_p(\alpha) \) if
\[
(A-B)|\mathcal{R}^r(A, B)| \left[ 2 \left[ 1 - (1 - q)^m \right] - \frac{\cos \alpha}{q(m-1)} [(1-q) - (1-q)^m - q(m-1)(1-q)^m] \right] \leq \cos \alpha.
\]

**Corollary 5.4.** If \( f \in \mathcal{R}^r(A, B) \), then \( I_q^m f \in \mathcal{UCT}_p(\alpha) \) if
\[
(A-B)|\mathcal{R}^r(A, B)| \left( \frac{2q}{1-q} m + (2 - \cos \alpha) [1 - (1-q)^m] \right) \leq \cos \alpha.
\]

**Corollary 5.5.** The function \( \mathcal{G}_q^m \in \mathcal{UCT}_p(\alpha) \) if and only if the inequality (24) holds.

**Corollary 5.6.** If \( m > 1 \), then the function \( \mathcal{G}_q^m \in \mathcal{TSP}_p(\alpha) \) if and only if
\[
2 \left[ 1 - (1 - q)^m \right] - \frac{\cos \alpha}{q(m-1)} [(1-q) - (1-q)^m - q(m-1)(1-q)^m] \leq \cos \alpha.
\]

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