Alignment function as a new manifestation of Boer-Mulders-like functions

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We present a new $k_{\perp}$-dependent function which can be manifested in the Drell-Yan (SIDIS)-like processes. This function resembles the well-known Boer-Mulders function, but in contrast it describes the transverse motion of partons inside the hadron due to the collective alignment of quark spin vectors rather than the quark spin asymmetry.

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The hadron-hadron matrix element of quark-gluon operators,

$$\Phi_{\alpha\lambda}(k) = \int (d^4 z) e^{+i(kz)} \langle P,S|\psi_0(0)|0;\bar{z}_A^{(\pm)}|P,S\rangle (1)$$

and its general parametrization involving the hadron spin vector as one of Lorentz structures plays an important role for the study of different spin characteristics in the Drell-Yan (SIDIS)-like processes (see, for example, \textsuperscript{[1]}(\textsuperscript{4})). In (\textsuperscript{1}), the underlined Greek indices correspond to the open spinor indices; $\{0;\bar{z}^{(\pm)}\}$ stands for the future- and past-pointed Wilson line (WL). Throughout the paper, we use the standard notations for the plus and minus light-cone directions, while the two dimensional transverse vectors of Minkowski space are denoted by the Greek indices.

Among all parametrizing functions, the $k_{\perp}$-dependent Boer-Mulders (BM) function introduced in \textsuperscript{[4]} and associated with the $\sigma^{\alpha\lambda}$ projection of $\Phi(k)$ in Eqn. \textsuperscript{[1]} can be singled out as a function which describes the transverse spin asymmetry of quarks inside the unpolarized hadron. In other words, the BM-function characterizes the inner spin structures of quarks with the help of quark spin asymmetry. Indeed, for the BM-function $h_{\perp}^1(x,k_{\perp}^2)$ contribution, we have the following representation after the factorization procedure applied to the corresponding semi-inclusive deep-inelastic-scattering (SIDIS) \textsuperscript{[3]}

$$\Phi^{[\sigma^{\alpha\lambda}]}(x,k_{\perp}^2) \sim \mathcal{P}^{q1/N}(x,k_{\perp}) - \mathcal{P}^{q1/N}(x,k_{\perp})$$

$$= \frac{\tilde{k}_{\perp}}{m_q} \sin(\phi_5 - \phi_5) h_{\perp}^1(x,k_{\perp}^2),$$

where $\mathcal{P}^{q1/N}(x,k_{\perp})$ denotes the probability to find the transverse polarized quark inside the unpolarized hadron, $\phi_5 - \phi_5$ defines the angle between $\tilde{k}_{\perp}$ and $\bar{s}_{\perp}$ being denoted as the quark transverse momentum and quark transverse spin vector (in two dimensional Euclidian space) respectively. The expression $|\tilde{k}_{\perp}| \sin(\phi_5 - \phi_5)$ in Eqn. \textsuperscript{[2]} stems from the vector product $\tilde{k}_{\perp} \wedge \bar{s}_{\perp}$ provided $|\bar{s}_{\perp}| = 1$.

The aim of our paper is to study a new manifestation of $k_{\perp}$-dependent parameterizing function of the matrix element \textsuperscript{[2]} which can be associated with the inner transverse quark motion generated by the collective spin alignment rather than the quark spin asymmetry. In other words, the function we consider in the paper is related to the probability $\mathcal{P}^{q1/N}(x,k_{\perp})$ defined by the operator $[\psi^{(\uparrow)}\gamma^\nu\gamma^\lambda\psi^{(\downarrow)}]$ involving the projections $\psi^{(\uparrow)} = 1/2(1 \pm \gamma_5)\psi$. Here, $(i = 1) \leftrightarrow (x)$, $(i = 2) \leftrightarrow (y)$ and $x, y$ are the polarization axes.

We begin with the well-known $k_{\perp}$-dependent function $f_1$ which parameterizes the unpolarized hadron matrix element of quark operator involving $\gamma^\nu$ as

$$\Phi^{[\gamma^\nu]}(k) = P^+ f_1(x,k_{\perp}^2,(k_{\perp}P_{\perp})) =$$

$$P^+(k_{\perp}P_{\perp}) f_1(x,k_{\perp}^2) + \left\{ \text{terms of (}\frac{k_{\perp}P_{\perp}}{n} n = 0, n \geq 2 \right\},$$

where $k = (x\gamma^\nu,k_{\perp})$ and $f_1(x,k_{\perp}^2,(k_{\perp}P_{\perp}))$ has been decomposed into the powers of $(k_{\perp}P_{\perp})$. For brevity, all corresponding normalization and dimensionful pre-factors have been absorbed in the definitions of integration measures or in the definitions of parametrizing functions. Notice that $P_{\perp}$ is substantially non-zero within the Collins-Soper (CS) frame of Drell-Yan-like (DY) processes or in the hadron production in semi-inclusive processes which is basically similar to DY-process. The matrix elements in Eqns. \textsuperscript{[2]} and \textsuperscript{[3]} are usually treated within the Heisenberg representation (H-representation). For our goals, we mainly adhere the interaction representation (I-representation). It is worth to notice that the Lorentz structures which parameterize the corresponding correlators are related to the spinor lines appearing in consideration at the given order of the coupling constant.

If we perform the Fourier transforms of the quark operators in \textsuperscript{[3]}, we obtain that

$$\int (d^4 k) e^{-i(kz)} \Phi^{[\gamma^\nu]}(k) =$$

$$\int d\mu_{\lambda,\lambda'}^{[\gamma^\nu]}(k_1,k_2) \langle P,S|b_{\lambda'}^* (k_1)b_{\lambda'}^*(k_2)|P,S\rangle,$$
where
d_{\mu\lambda}^{\gamma^+}(k_1, k_2) = (d^4k_1)(d^4k_2)e^{-i(k_2\cdot k_1)}[\tilde{u}^{(\lambda)}(k_1)\gamma^+ u^{(\lambda)}(k_2)],\tag{5}
and
\[\langle P, S|b^\lambda_k(k_1)b^{\lambda'}_k(k_2)|P, S\rangle^H = \langle P, S|b^\lambda_k(k_1)b^{\lambda'}_k(k_2)|S|P, S\rangle^H = \delta^{(4)}(k_1 - k_2)\mathcal{M}_{\mu\lambda}(k_1^2, k_2^2, (k_1, k_2), |P|),\tag{6}\]
where $S$ is the S-matrix operator and $b^\pm$ are the quark creation and annihilation operators.

In what follows we omit the spin state indices $\lambda$ and $\lambda'$ and we do not pay an attention on the difference between contravariant and covariant vectors unless it may lead to a confusion.

Taking into account the delta function of (6), we can see that the spinor line formed by $[\tilde{u}(k)\gamma^+ u(k)]$ results in the appearance of $k^+ \sim P^+$ in the most trivial case where spin polarizations and interactions are absent.

Now, we are going over to the second order of strong interactions within the interaction representation. We get that
\[
\langle \phi^{[\gamma^+]}(\gamma^+) \rangle^{(2)} = \langle P, S|T \bar{\psi}(0)\gamma^+ \psi(z)S^{(2)}_{QCD}|P, S\rangle = \int (d^4k)e^{-i(k_2\cdot k_1)}\int (d^4\ell)\Delta(k^2)\int (d^4\bar{k})
\times\mathcal{M}(k^2, \ell^2, k^2, \ldots)
\times[\tilde{u}(k)\gamma^+ k\tilde{\gamma}_A u(k - \ell)][\tilde{u}(\bar{k})\gamma_A^\dagger u(\bar{k} + \ell)],\tag{7}\]
where $S^{(2)}_{QCD}$ denotes the S-matrix operator at the order of $g^2$.

In Eqn. (7) the quark and gluon propagators read
\[
S(k) = \frac{k\Delta(k^2)}{2}, \quad D^\perp_{\mu\nu}\ell = \frac{-ig_\perp_{\mu\nu}\Delta(k^2)}{2},
\Delta(k^2) = \frac{1}{k^2 + i\epsilon}, \quad \bar{k} = (k\gamma)\tag{8}\]
and the amplitude $\mathcal{M}$ is given by
\[
\mathcal{M}(k^2, \ell^2, k^2, \ldots) \delta(k_1 + k_2 - k - k_4) = \langle P, S|b^+ (k_1)b^- (k_2)b^+ (k_3)b^- (k_4)|P, S\rangle.\tag{9}\]

We single out the region where $|\ell| \ll |k|, |\bar{k}|$ and as a result we obtain that
\[
\langle \phi^{[\gamma^+]}(\gamma^+) \rangle^{(2)} \sim \int (d^4k)e^{-i(k_2\cdot k_1)}\Delta^2(k^2)[\tilde{u}(k)\gamma^+ k\tilde{\gamma}_A u(k)]
\times\int (d^4\bar{k})[\tilde{u}(\bar{k})\gamma_A^\dagger u(\bar{k})]\int (d^4\ell)\Delta(k^2)\mathcal{M}(k^2, \ell^2, k^2, \ldots).\tag{10}\]

The next stage is to transform the spinor lines of this expression. For the first spinor line we write
\[
[\tilde{u}(k)\gamma^+ k\tilde{\gamma}_A u(k)] = \epsilon^{+\alpha\beta\mu\lambda}[\tilde{u}(k)\gamma^\beta u(k)] + (axial)
\Rightarrow k^\alpha\tilde{u}(k)\gamma^\beta u(k) + (\text{other terms}),\tag{11}\]
where the following notation has been used
\[
\epsilon^{\mu_1\mu_2\mu_3\mu_4} = \frac{1}{4}\text{tr}[\gamma^{\mu_1}\gamma^{\mu_2}\gamma^{\mu_3}\gamma^{\mu_4}].\tag{12}\]

The second spinor line can be considered with the help of the covariant (invariant) integration given by
\[
k^\alpha_1\int (d^4\tilde{k})[\tilde{u}(\tilde{k})\gamma^\beta u(\tilde{k})] \cdot \mathcal{M}(\tilde{k}^2, (\tilde{k}P), \ldots) =
k^\alpha_1\int (d^4\tilde{k})k^\alpha_1 \cdot \mathcal{M}(\tilde{k}^2, (\tilde{k}P), \ldots) =
\langle k_1P_\perp | (d^4\tilde{k}) (k^\alpha_1 P_\perp) | \mathcal{M}(\tilde{k}^2, (\tilde{k}P), \ldots) \rangle.\tag{13}\]

Using (11) and (13), one can see that the form of the r.h.s. of (10) coincides with the parametrization of (5) at $g^2$-order. Indeed, we have the following
\[
P^+(k_1P_\perp)f_1^{(1)}(x; k_1^2) \sim
\langle \tilde{u}(k)\gamma^+ u(k) \rangle \int (d^4\bar{k})[\tilde{u}(\bar{k})\bar{k}u(\bar{k})]
\times\int (d^4\ell)\Delta(k^2)\times\mathcal{M}(k^2, \ell^2, k^2, \ldots).\tag{14}\]

In other words, the parametrization of (5) with the Lorentz combination $P^+(k_1P_\perp)$ is related to the two spinor lines
\[
[\tilde{u}(k)\gamma^+ u(k)] \Rightarrow k^+ \sim P^+, \quad [\tilde{u}(\bar{k})\bar{k}u(\bar{k})] \Rightarrow (k_1P_\perp)\tag{15}\]
at $g^2$-order.

In the region of $|\bar{k}| \sim |k|$, two spinor lines of (14) can be transformed into the other spinor lines with the help of Fierz transformations. Using its general form (see, [5])
\[
\frac{1}{4}\int [\tilde{u}(c)\Gamma_1 u^{(b)}][\tilde{u}(c)\Gamma_2 u^{(d)}]
\times\frac{1}{4}\sum_{A,R_1,R_2}\left\{\frac{1}{4}\text{tr}[\Gamma_1 A \Gamma_1 R_1]\right\}\left\{\frac{1}{4}\text{tr}[\Gamma_2 A \Gamma_2 R_2]\right\}
\times[\tilde{u}(c)\Gamma_1 u^{(b)}] \times[\tilde{u}(c)\Gamma_2 u^{(d)}] =\tag{16}\]
with $O_1 = \gamma^+ \gamma_\perp^1 \gamma_\perp^2$, $O_2 = 1$, $\Gamma_1 = \gamma_\perp^1$, $\Gamma_2 = \gamma_\perp^2 = \gamma_\perp^3$, we obtain that
\[
[\tilde{u}(\gamma_\perp^1)(k)\gamma^+ \gamma_\perp^1 \gamma_\perp^2 \gamma_\perp^1 u(\gamma_\perp^1)(k)] = C[\tilde{u}(\gamma_\perp^1)(k)\gamma^+ \gamma_\perp^1 \gamma_\perp^2 \gamma_\perp^1 u(\gamma_\perp^1)(k)].\tag{17}\]

Here, we assume that, for the fixed indices, $i \neq j$, the coefficient $C$ is given by
\[
C = \frac{1}{16}\text{tr}[\gamma^+_{\perp} \gamma^+ \gamma_\perp^1 \gamma_\perp^2 \gamma_\perp^1].\tag{18}\]

Eqn. (17) can be readily inverted in order to get the following representation of Eqn. (14)
\[
[\tilde{u}(\gamma_\perp^1)(k)\gamma^+ \gamma_\perp^1 \gamma_\perp^2 \gamma_\perp^1 u(\gamma_\perp^1)(k)] \int (d^4\tilde{k})\int (d^4\ell)
\times\Delta(k^2)\Delta(k^2) \Rightarrow
k^+ e^{+\perp P_\perp} f^{(1)}(1; x; k_1^2)\tag{19}\]
On the r.h.s. of (19), we write down the complex $i$ explicitly in order to stress that it stems from the trace of four $\gamma$-matrices with $\gamma_\perp^3$. 

Let us note that Eqn. [19] contains the Lorentz structure with the quark polarization vector \( s_\perp \) and it explicitly shows that the function \( \tilde{f}^{(1)}_1(x; k_\perp^2) \) must appear in the parametrization of the hadron matrix element, i.e. (omitting the WL)

\[
\Phi^{[1]}(k) = \int (d^4z)e^{i(kz)} \langle P,S|\psi(0)\gamma^+\psi(z)|P,S\rangle |_{k^+ = xP^+} = ie^{+P_\perp s_\perp} \tilde{f}^{(1)}_1(x; k_\perp^2) + ... \tag{20}
\]

This observation is our principal result which reveals the existence of a new transverse-momentum dependent function \( \tilde{f}^{(1)}_1(x; k_\perp^2) \). We also observe that Lorentz structure tensor, \( \epsilon^{+P_\perp s_\perp} \), associated with our function (see [20]) resembles the Sivers structure, \( \epsilon^{+P_\perp S_\perp} \), where the nucleon spin vector \( S_\perp \) is replaced by the quark spin vector \( s_\perp \). However, despite this similarity the Sivers function and the introduced function \( \tilde{f}^{(1)}_1(x; k_\perp^2) \) have totally different physical meaning.

We now comment about the hermitian (complex) conjugation property of a new function \( \tilde{f}^{(1)}_1(x; k_\perp^2) \). We note that the hermitian conjugation of Eqn. [20] implemented in a naive way suggests that its parameterizing function \( \tilde{f}^{(1)}_1(x; k_\perp^2) \) should be an imaginary function. However, the situation is more subtle if the final (initial) quark-hadron interactions have been taken into account. Indeed, the standard forward (\( \langle P,S|out \rangle \neq \langle P,S|in + inter, \rangle \)) which leaves room for the generation of \( \kappa_\perp \)-dependence of \( f_1(x; k_\perp^2) \) has totally different physical meaning.

As a last comment, we note that the structure function \( \tilde{f}^{(1)}_1(x; k_\perp^2) \) of Eqn. [20], roughly speaking, coincides with the function \( \tilde{g}_1(x; k_\perp^2) \) provided (a) we decipher the hadron matrix element of quark-(gluon) operators at least up to the order of \( \tilde{g}_1 \); (b) we focus on the regime of \( l \ll |\vec{k}| \sim |k| \) at this order; (c) the occurred four spinors generated by two spinor lines are aligned along the same transverse direction. Thus, at the order of \( g^2 \), we observe the regime where \( \kappa_\perp \)-dependence (or the transverse motion of quarks inside the hadron) has been generated by the quark spin alignment along the transverse direction. From the mechanical point of view, it resembles the deviation of alike-rotated balls from the straightforward motion.

We are now in a position to discuss the possible relation of \( \tilde{f}^{(1)}_1(x; k_\perp^2) \) with the gluon pole contributions where the presence of the WL in Eqn. [20] is important. For that, we dwell on the meson-nucleon DY-process with the unpolarized nucleons and with the substantial \( \kappa_\perp \)-dependence of parametrizing functions. We consider the following process

\[
M(P_1) + N(P_2) \rightarrow \gamma'(q) + \bar{q}(K) + X(P_X) \rightarrow \bar{q}(K) + \ell(l_1) + \bar{\ell}(l_2) + X(P_X), \tag{21}
\]

where \( M \) and \( N \) denote the initial meson and nucleon, respectively. Here, the virtual photon producing the lepton pair \( (l_1 + l_2 = q) \) has a large mass squared \( (q^2 = Q^2 > 0) \) (see [11] for the detail description of kinematics) and we analyse the process (21) in the CS-frame. Our study can be readily extended to the hadron production in semi-inclusive process with a large transverse momentum of the produced meson.

The differential cross section can be presented in the following symbolical form

\[
d\sigma \sim L_{\mu\nu}^U \bar{\psi}_{U\mu} \psi_{U\nu}, \tag{22}
\]

where \( L_{\mu\nu} \) is the corresponding leptonic tensor and \( \bar{\psi}_{U\mu} \) stands for the hadronic tensor. The superscript \( U \) in Eqn. (22) indicates that we consider the case of unpolarized leptons and hadrons.

The analogous process has been already investigated in [11]. Here, we use the final expression for the gauge-invariant hadron tensor that takes the form of

\[
\bar{\psi}_{U\mu} = \int d^2q_\perp \bar{\psi}_{U\mu} \equiv \langle \bar{P}\bar{S}|out \rangle = \int (dx_1) (dy) \delta(x_1 P_1^+ - q^+) \delta(y P_2^+ - q^-) \times \left\{ \int d^2k_{1\perp}^{\perp} \tilde{f}^{(1)}_1(y; k_{1\perp}^2) \right\} \times \left\{ \int (dx_2) \bar{B}(x_1, x_2) \frac{i e^{P_1^+ x_1 + P_2^+ x_2} V_{\perp}}{P_1^+ P_2^-} \frac{P_1^+}{y} + \frac{P_2^-}{x_1} \right\}, \tag{23}
\]

where the Lorentz tensor \( V_{\perp} \) means the meson transverse momentum \( P_1^+ \) for the pion case and the transverse polarization vector \( e^+ \) for the rho-meson case. The function \( \bar{B}(x_1, x_2) \) is defined as in Eqns. (19) and (20) of [11]

\[
\bar{B}(x_1, x_2) = \frac{1}{2} \frac{\Phi^{[2]}(x_1, \bar{k}_{1\perp}^+)}{x_2 - x_1 - i\epsilon} \tag{24}
\]

and it possesses the pole as \( x_1 = x_2 \) which has been regulated by the complex prescription emanated from the corresponding integral representation of theta function (see [12] for details). In (24), we also introduce the short notation as (here, \( x_{21} = x_2 - x_1 \))

\[
\Phi^{[2]}(x_1, \bar{k}_{1\perp}^+) \equiv \int (d^2\bar{k}_{1\perp}^+) F(\bar{k}_{1\perp}^+) \times \int (d\ell d^2\bar{\ell}_{\perp}^\perp) \Delta(\ell^+, \bar{\ell}_{\perp}^\perp) G(\bar{k}_{1\perp}^+, \ell^+ \bar{\ell}_{\perp}^\perp), \tag{25}
\]

\[
\Delta(\ell^+, \bar{\ell}_{\perp}^\perp) = \frac{1}{\ell^+ - \bar{\ell}_{\perp}^\perp (2x_{21} P_{1\perp}^+) + i\text{sign}(x_{21})0}. \tag{26}
\]

Whereas the meson \( \kappa_\perp \)-dependent distribution functions are
given by

\[ \Phi^{(2)}_{sly}(x_2, k_1^\perp)P_1^\perp = \int (d\eta_1^\perp-d^2\eta_1^\perp) e^{i\epsilon_1^\perp P^\perp_1 \eta_1^\perp} + (P_1^\perp-k_1^\perp) \eta_1^\perp \times \langle 0 | \Psi(\eta_1^\perp, \eta_1^\perp) \gamma^+ \Psi(0) | P_1^\perp \rangle \]  

(27)

\[ \Phi^{(1)}_{(1)}(x_1, k_1^\perp)P_1^\perp = \int (d\xi^\perp - d^2\xi^\perp) e^{-i\epsilon_1^\perp P^\perp_1 \xi^\perp - i\epsilon_1^\perp k^\perp_1 \xi^\perp} \times \langle P_1^\perp | \Psi(\xi^\perp, \xi^\perp) \gamma^+ \Psi(0) | 0 \rangle \]  

(28)

In [22] the nucleon function \( f_1^{(1)}(y, k_1^\perp) \) parametrizes the following matrix element

\[ \sum_x \int (d\eta_2^\perp-d^2\eta_2^\perp) e^{-i\epsilon_2^\perp P^\perp_2 \eta_2^\perp - i\epsilon_2^\perp k_2^\perp \eta_2^\perp} \times \langle P_2^\perp | \langle \Psi(0) | P_2^\perp | \Psi(\eta_2^\perp, \eta_2^\perp) \rangle \gamma^+ | P_2^\perp \rangle = i e^{-i\epsilon_2^\perp k_2^\perp \eta_2^\perp} \mathcal{F}_1(y, k_1^\perp) \]  

(29)

We remind that for the nucleon correlator the dominant direction is defined by the minus light-cone direction in contrast to the meson distribution functions where the dominant light-cone direction corresponds to the plus direction [11].

Since \( f_1^{(1)}(y, k_1^\perp) \) is an imaginary function as discussed above, and while \( \tilde{B}(x_1, x_2) \) is a complex function, the unpolarized lepton (real valued) tensor singles out the combination given by

\[ \{ \Im f_1^{(1)}(y, k_1^\perp) \} \bigtimes \{ \Re \bar{B}(x_1, x_2) \} \]  

(30)

in order to compensate the complex \( i \) in parametrization.

Calculating the corresponding differential cross section, within the CS-frame [23], we obtain that

\[ d\sigma \sim \mathcal{L}_{\mu
u} \mathcal{L}^{\mu\nu} \]  

\[ \frac{1}{x_B y_B} \int d^2k_1^\perp \Im f_1^{(1)}(y_B, k_1^\perp) \int (d\eta_1^\perp) \Re \bar{B}(x_B, x_2) \times \frac{2}{x_B y_B} (\ell_1^\perp \cdot z) (\ell_1^\perp \cdot v^\perp) s_\perp \wedge \bar{P}_2^\perp, \]  

(31)

where

\[ 2(\ell_1 \cdot z) = -Q^2 \cos \theta, \]  

\[ s_\perp \wedge \bar{P}_2^\perp = | \bar{P}_2^\perp | \sin(\phi_2 - \phi_P), \]  

(32)

Here, \( \phi_2 \) and \( \phi_P \) imply the azimuthal angles of the quark spin vector and the nucleon momentum.

To conclude, in the paper we have introduced the new \( k_\perp \)-dependent function \( f_1^{(1)}(x, k_1^\perp) \) which describes the transverse quark motion by the quark alignment along the fixed transverse direction. The introduced function \( f_1^{(1)}(x, k_1^\perp) \) can be considered as a “in-between” function of the Sivers and Boer-Mulders functions. Indeed, the Lorentz tensor accompanying our function is similar to the analogous tensor at the Sivers function, however it describes the polarized quark effects inside the unpolarized nucleon like the Bour-Mulders function.

We have shown that, to the second order of strong interactions, the new parametrizing function \( f_1^{(1)}(x, k_1^\perp) \) can be related to the function \( f_1^{(1)}(x, k_1^\perp) \) of [3] imposing the condition \( \ell \ll |k| \sim |k| \) which corresponds to the regime where the appeared two spinor lines are interacting by exchanging of soft gluon. Moreover, the occurred four spinors generated by two spinor lines have the polarizations aligned along the same transverse direction. In physical terms, the \( k_\perp \)-dependent function \( f_1^{(1)}(x, k_1^\perp) \) which describes the regime where \( k_\perp \)-dependence (or the transverse motion of quark inside the hadron) has been entirely generated by the quark spin alignment. We have mentioned the possible application of our function in the connection with the gluon pole contributions to the unpolarized differential cross section corresponding to the pion-nucleon DY-like (or similar to SIDIS-like) process.

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