Magnetic-field–induced 3D-to-1D crossover in Sr$_{0.9}$La$_{0.1}$CuO$_2$

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Abstract - The effect of the magnetic field on the critical behavior of Sr$_{0.9}$La$_{0.1}$CuO$_2$ is explored in terms of reversible magnetization data. As the correlation length transverse to the magnetic field $H_i$, applied along the $i$-axis, cannot grow beyond the limiting magnetic length $L_{H_i} = (\Phi_0/(aH_i))^{1/2}$, related to the average distance between vortex lines, one expects a magnetic-field–induced finite-size effect. Invoking the scaling theory of critical phenomena we provide clear evidence for this effect. It implies that in type-II superconductors there is a 3D-to-1D crossover line $H_{ps}(T) = (\Phi_0/(a\xi_{\parallel}^0\xi_{\perp}^0))((1 - T/T_c)^{4/3}$ with $i \neq j \neq k$ and $\xi_{\parallel}^0,\xi_{\perp}^0$ denotes the critical amplitudes of the correlation length below $T_c$. Consequently, below $T_c$ and above $H_{ps}(T)$ superconductivity is confined to cylinders with diameter $L_{H_i} (1D)$. Accordingly, there is no continuous phase transition in the $(H, T)$-plane along the $H_c2$-lines as predicted by the mean-field treatment.

In this study we present and analyze magnetization data of the infinite-layer compound Sr$_{0.9}$La$_{0.1}$CuO$_2$ taken from Kim et al. [1]. Since near the zero-field transition thermal fluctuations are expected to dominate [2–5], Gaussian fluctuations point to a magnetic-field–induced 3D-to-1D crossover [6], whereby the effect of fluctuations is enhanced, it appears inevitable to take thermal fluctuations into account. Indeed, invoking the scaling theory of critical phenomena we show that the data are inconsistent with the traditional mean-field interpretation. On the contrary, we observe agreement with a magnetic-field–induced finite-size effect, whereupon the correlation length transverse to the magnetic field $H_i$, applied along the $i$-axis, cannot grow beyond the limiting magnetic length

$$L_{H_i} = (\Phi_0/(aH_i))^{1/2},$$

with $a \simeq 3.12$ [7]. $L_{H_i}$ is related to the average distance between vortex lines. Indeed, as the magnetic field increases, the density of vortex lines becomes greater, but this cannot continue indefinitely, the limit is roughly set on the proximity of vortex lines by the overlapping of their cores. This finite-size effect implies that in type II superconductors, superconductivity in a magnetic field is confined to cylinders with diameter $L_{H_i}$. Accordingly, below $T_c$ there is the 3D to 1D crossover line

$$H_{ps}(T) = (\Phi_0/(a\xi_{\parallel}^0\xi_{\perp}^0))((1 - T/T_c)^{4/3},$$

with $i \neq j \neq k$. $\xi_{\parallel}^0,\xi_{\perp}^0$ denotes the critical amplitudes of the correlation lengths below $T_c$ along the respective axis. It circumvents the occurrence of the continuous phase transition in the $(H, T)$-plane along the $H_c2$-lines predicted by the mean-field treatment [8]. Indeed, the relevance of thermal fluctuations already emerges from the reversible magnetization data shown in fig. 1. As a matter of fact, the typical mean-field behavior [8], whereby the magnetization scales below $T_c$ linearly with the magnetic field, does not emerge.
When thermal fluctuations dominate and the coupling to the charge is negligible the magnetization per unit volume, \(m = M/V\), adopts the scaling form \[2–5\]
\[
\frac{m}{TH^{1/2}} = \frac{-Q^\pm k_B \xi_{ab}}{\Phi_0^{3/2} \xi_c} F^\pm(z), \quad F^\pm(z) = z^{-1/2} \frac{dG^\pm}{dz},
\]
\[
z = x^{-1/2} = \left(\frac{\xi_0}{\xi_{ab}}\right)^2 |t|^{-2} H_c.
\]
(3)

\(Q\) is a universal constant and \(G^\pm(z)\) a universal scaling function of its argument, with \(G^\pm(z = 0) = 1\). \(\gamma = \xi_{ab}/\xi_c\) denotes the anisotropy, \(\xi_{ab}\) the zero-field in-plane correlation length and \(H_c\) the magnetic field applied along the \(c\)-axis. In terms of the variable \(x\) the scaling form (3) is similar to Prange’s [9] result for Gaussian fluctuations.

Approaching \(T_c\) the in-plane correlation length diverges as
\[
\xi_{ab} = \xi_{ab0} |t|^{-\nu}, \quad t = T/T_c - 1, \quad \pm = \text{sgn}(t).
\]
(4)

Supposing that 3D-\(xy\) fluctuations dominate the critical exponents are given by [10]
\[
\nu \approx 0.671 \pm 2/3, \quad \alpha = 2\nu - 3 \approx -0.013,
\]
and there are the universal critical amplitude relations \[2–5,10\]
\[
\frac{\xi_{ab0}^+}{\xi_{ab0}^-} = 2.21, \quad \frac{Q^-}{Q^+} \approx 11.5, \quad \frac{A^+}{A^-} = 1.07,
\]
(6)

and
\[
A^- \xi_{ab0}^- \xi_{c0}^- \approx A^- \left(\frac{\xi_{ab0}^-}{\xi_{c0}^-}\right)^3 \xi_{c0}^- = \frac{A^- \left(\xi_{ab0}^-\right)^3}{\gamma} = (R^-)^3, \quad R^- \approx 0.815,
\]
(7)

where \(A^\pm\) is the critical amplitude of the specific-heat singularity, defined as
\[
c = \frac{C}{k_B} = \frac{A^\pm}{\alpha} |t|^{-\alpha} + B,
\]
(8)

where \(B\) denotes the background. Furthermore, in the 3D-\(xy\) universality class \(T_c, \xi_0\) and the critical amplitude of the in-plane penetration depth \(\lambda_{ab0}\) are not independent but related by the universal relation \[2–5,10\]
\[
k_B T_c = \frac{\Phi_0^2}{16\pi^3} \frac{\xi_{c0}^-}{\lambda_{ab0}^2} = \frac{\Phi_0^2}{16\pi^3} \frac{\xi_{ab0}^-}{\gamma \lambda_{ab0}^2}.
\]
(9)

According to the scaling form (3) consistency with critical behavior requires that the data plotted as \(m/(TH^{1/2})\) vs. \(tH^{-1/2} \approx tH^{-3/4}\) should collapse near \(tH^{-3/4} \rightarrow 0\) on a single curve. Evidence for this collapse emerges from fig. 2 with \(T_c = 43.81\) K.

To check the estimate for \(T_c\) and to explore the magnetic-field-induced 3D to 1D crossover we invoke Maxwell’s relation
\[
\frac{\partial (C/T)}{\partial H_c} = \frac{\partial^2 M}{\partial T^2} |_{H_c}.
\]
(10)

\(T_c\) is determined from the scaling form (3) by the condition that for \(tH^{-3/4} \rightarrow 0\)
\[
\partial (C/T) = \frac{-k_B A^-}{2\alpha} (tH_{c0}^{-1} + \alpha) \frac{\partial f}{\partial x} = \frac{\partial^2 m}{\partial T^2}.
\]
(12)

In fig. 3 we depicted \(d^2m/dT^2\) vs. \(T\) for various magnetic fields \(H_c\). Apparently, the location \(T_p(H)\) and the height of the dip decrease with increasing magnetic field. Note that this dip differs drastically from the mean-field behavior where \(d^2m/dT^2 = 0\). Due to its finite depth, controlled by the magnetic-field–induced finite size effect, it differs from the reputed singularity at \(T_{c2}\), as obtained in the Gaussian approximation [9], as well. When scaled according to eq. (12) the data should then collapse on a single curve. From fig. 4, showing this plot in terms of \(H_c d^2m/dT^2\) vs. \(tH_{c0}^{-3/4}\), it is seen that this 3D-\(xy\) scaling behavior is reasonably well confirmed. The location of the dip determines the line
\[
t_p H_{c0}^{-3/4} = 0.0767 (10^{-3} \text{Oe}^{-3/4}).
\]
(13)
in the \((H_c, T)\)-plane where the 3D-to-1D crossover occurs. Along this line, rewritten in the form

\[
H_{pc}(T) = \frac{\Phi_0}{a} \left( \frac{1 - T}{T_c} \right) ^{4/3},
\]

the in-plane correlation length is limited by \(L_{Hc}\) (eq. (1)). From these equivalent relations and \(a = 3.12\) we derive for the critical amplitude of the in-plane correlation length the estimate

\[
\xi_{ab}^{-} = 46.5 \text{ Å}.
\]

This value is comparable to \(\xi_{ab}^{-} = 46.8 \text{ Å}\) for underdoped \(\text{YBa}_2\text{Cu}_3\text{O}_{6-\delta}\) with \(T_c = 41.5 \text{ K}\) [5] and \(\xi_{ab}^{-} = 52 \text{ Å}\) for \(\text{MgB}_2\) with \(T_c = 38.83 \text{ K}\) [11]. Invoking then the universal relation (9) we obtain with \(T_c = 43.81 \text{ K}\) and \(\gamma = 9\) [1] for the critical amplitude of the magnetic in-plane correlation length the value

\[
\lambda_{ab} = 2.72 \times 10^{-5} \text{ cm}.
\]

Unfortunately, the available magnetic penetration depth data do not enter the critical regime [12]. In any case the result for the Ginzburg parameter at criticality is \(\kappa_c = \lambda_{ab}/\xi_{ab}^{-} = 48.5\), which differs substantially from the mean-field estimate \(\kappa_c = 25.3\) [1].

To check the hitherto used value of \(T_c\) we invoke eq. (13) and the \(T_p^c\)'s taken from fig. 3 to determine \(T_c\). We obtain \(T_c \simeq 43.81 \text{ K}\), in agreement with our previous estimate. The resulting line \(H_{pc}(T)\) and the \(T_p^c\)'s taken from fig. 3, are shown in fig. 5. Below this line superconductivity occurs in 3D and above it is confined to cylinders of radius \(L_{Hc} = (\Phi_0/(aH_c))^{1/2}\) (1D).

We have shown that in \(\text{Sr}_{0.9}\text{La}_{0.1}\text{CuO}_2\) the fluctuation-dominated regime is experimentally accessible and uncovers remarkable consistency with 3D-\(xy\) critical behavior. There is, however, the magnetic-field-induced finite size effect. It implies that the correlation length transverse to the magnetic field \(H_c\) applied along the \(i\)-axis, cannot grow beyond the limiting magnetic length \(L_{Hc} = (\Phi_0/(aH_c))^{1/2}\), related to the average distance between vortex lines. Invoking the scaling theory of critical phenomena clear evidence for this finite-size effect has been provided. In type-II superconductors it comprises the 3D-to-1D crossover line \(H_{pc}(T) = (\Phi_0/(a\xi_{ab}^{-}\xi_{ab}^{c}))((1 - T/T_c)^{4/3}\text{)}\) with \(i \neq j \neq k\) and \(\xi_{0,0,0}^{-}\text{denoting the critical amplitude of the correlation length below } T_c\). As a result, below \(T_c\) and above \(H_{pc}(T)\) superconductivity is confined to cylinders with diameter \(L_{Hc}(1\text{D})\). Accordingly, there is no continuous phase transition in the \((H, T)\)-plane along the \(H_{pc}\)-lines as predicted by the mean-field treatment. Finally we note that the failure of the mean-field approximation in type-II superconductors was also observed in Monte Carlo simulations [13,14].

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