Spatial coherence characterization of light generated from incoherent sources: an experimental study using digital micromirror devices

Tiago E. C. Magalhães\textsuperscript{a,b,*}, José M. Rebordão\textsuperscript{a,b}, Alexandre Cabral\textsuperscript{a,b}

\textsuperscript{a}Departamento de Física, Faculdade de Ciências, Universidade de Lisboa, Edifício C8, Campo Grande, PT1749-016 Lisboa, Portugal
\textsuperscript{b}Instituto de Astrofísica e Ciências do Espaço, Edifício C8, Campo Grande, PT1749-016 Lisboa, Portugal

Abstract

We present spatial coherence measurements of partially coherent light in the far-field of incoherent sources with an experimental setup based on the Thompson-Wolf and Partanen-Turunen-Tervo experiments, to be performed in the context of a possible solar coherence measurement space instrument. The optical setup consists on a telescope to collimate light from a source, to diffract it by a digital micromirror device implementing a Young double-aperture interferometer in retroreflection mode, and finally to image the source into a 2D sensor. Two multimode optical fibers with different diameters were used as incoherent sources and the results obtained for the spectral degree of coherence are compared to those expected from the van Cittert-Zernike theorem.

Keywords: Spatial coherence, Spatial light modulators, Statistical Optics

1. Introduction

The complex degree of coherence and the spectral degree of coherence are of critical importance to characterize spatial coherence in the space-time and space-frequency domains \cite{1}, respectively. They are usually measured with Young experiments, i.e., a double-aperture interferometer and, by varying the separation between the apertures, it is possible to measure these quantities through an interference pattern in the observation plane. One pioneer work in this field was authored by Thompson and Wolf \cite{2} in 1957. In their experiment, light from an incoherent source propagates in free space and acquires spatial coherence (explained by the van Cittert-Zernike theorem) before impinging on a double-aperture. The modulation of the interference pattern is analysed in order to retrieve the magnitude of the complex degree of coherence. However, to obtain a significant number of data points to estimate the complex degree of coherence, a large number of Young experiments must be performed with different separations between apertures and, in asymmetric cases, with different orientations (baselines) a composite aperture is therefore synthesized from each data point of the complex degree of coherence. Since 1957, several experimental methods have been proposed (see \cite{3} and references therein) to measure spatial coherence. For example, Gonzalez and co-workers \cite{4} suggested using multi-aperture non-redundant arrays as an alternative to implement simultaneously multiple Young experiments. Divitt et al. \cite{3} suggested using non-parallel slits, which have the advantage of measuring the spectral degree of coherence for broadband sources. In the same year, Partanen et al. \cite{5} used a digital micromirror device (DMD) \cite{6} to materialize the double-slit aperture to perform Young experiments for a multimode broad-area laser. Their method has the advantage of dynamically varying the separation between slits without manufacturing several apertures and is only limited by the dimensions and spatial resolution of the DMD. In 2017, experiments regarding the measurement of spatial coherence using DMDs for lensless imaging were reported \cite{7,8}. In this case, the authors used an incoherent light source in which light propagates a given distance until it is obstructed by an object. Spatial coherence of light diffracted by the object is measured using a DMD, and by applying back-propagation techniques, the object’s size and location can be retrieved.

A conceptual space-based instrument for the measurement of spatial coherence of structured light sources was recently proposed \cite{9}. In the design of this instrument, two DMDs are used in retroreflection mode to perform selective imaging and spatial coherence measurements. The scope of this work is to experimentally verify this configuration regarding the spatial coherence measurements by using a DMD in retroreflection. The polarization control used in \cite{9} is not needed in this case since no selective imaging will be performed. The experimental apparatus is based on both the Thompson-Wolf \cite{2} and Partanen-Turunen-Tervo \cite{5} experiments. The light source is placed at the back focal plane of a positive lens in order to generate a collimated beam impinging on a DMD. The latter is used to perform Young double-slit experiments and is positioned to enable retroreflection of the diffracted light.
The optical setup is compact, of interest for astronomical ground-based instruments which seek to measure spatial coherence of light in the optical domain. Studies on the space robustness and operational stability of DMDs, through environmental testing have been recently reported, anticipating their use in space. Our configuration is, in fact, envisaged for space-based optical instruments including balloon experiments. The conceptual instrument described in reference is an example of such configuration.

2. Experimental Setup

The central component of the experiment is a DMD in retroreflection mode, previously used for other applications. A DMD is a spatial light modulator, consisting of an \( N_x \times N_y \) array of squared micromirrors with two stable angular orientations, tilted by \( +\theta_m \) and \( -\theta_m \) (of the order of \( \sim 15 \) degrees), designated herein as "on" and "off" states, respectively. The rotation axis of individual micromirrors is usually in their diagonal. To ensure retroreflection from the "on" state micromirrors, one has to tilt the DMD by \(-\theta_m\) around the y-axis and by 45 degrees around the z-axis (see inset of Fig. 1). Some DMDs do not require the latter rotation since micromirrors are already in-plane rotated by this amount.

In our case, we are using Texas Instruments DLP7000 model, micromirrors are not rotated and a 45 degrees tilt is still needed. The DMD has 1024 \( \times \) 768 square micromirrors (width of \( \Delta = 13.68 \mu m \)) and the value of the tilt angle is \( \theta_m = 12 \) degrees. To the best of our knowledge, DMDs with a stable tilt angle of \( \theta_m = 0 \) do not exist, otherwise, one would not need to rotate the DMD around the y-axis. However, analog micromirror arrays can provide a zero degree tilt. For more information regarding DMDs and their applications, see [15].

The layout of the experimental setup is depicted in Fig. 1. A LED (Thorlabs M660FP1) with central wavelength \( \lambda_0 = 660 \) nm is coupled to an optical fiber. A collimating lens L1 with focal length \( f_1 \) is placed at a distance \( f_1 \) from the tip of the fiber (at plane A). Light passes through a beam splitter (BS) and reaches plane C where a DMD is located. The reflected light from the BS is lost out of the system. Nevertheless, in other instruments (e.g., afoocal telescopes), this light can serve other purposes, such as imaging and spectroscopy. The "on" state mirrors of the DMD build up a Young double-slit aperture with separation \( b \) as represented in the inset of Fig. 1. Light reflected from the "off" state mirrors is lost out of the system. To ensure retroreflection in the "on" state mirrors, the DMD is tilted around the z-axis by 45 degrees and around the y-axis by \( \theta_m \) degrees. After retroreflection, light is reflected in the BS and passes through lens L2 with focal length \( f_2 \) which creates a magnified image of the source at plane D, where a CCD camera is placed. The CCD (AVT Marlin F-080) has 1032 \( \times \) 778 pixels with pixel pitch of 4.65 \( \mu m \). Both the DMD and the CCD are connected to a computer which dynamically changes the Young slits separation in the DMD and measures the corresponding interference pattern. An array of images is then analysed in order to retrieve the spectral degree of coherence. If light reaching the DMD has some degree of spatial coherence for a given separation \( b \), an interference pattern is observed in the CCD, provided three conditions are fulfilled: (1) the CCD has adequate spatial resolution to resolve interference fringes, (2) the fringe pattern period is smaller than CCD dimensions and (3) the signal-to-noise ratio (SNR) is sufficient to quantify the amplitude of the fringes. Lenses have a direct impact on the results: L2 determines the period of the fringes in the CCD, while L1 is responsible for the range of spatial coherence of light that can be measured in plane C.

3. Theory

We will now briefly discuss the propagation of spatial coherence from plane A to plane C. For convenience, we will use the space-frequency approach, i.e., spatial coherence will be characterized by the spectral degree of coherence instead of the complex degree of coherence.

Let \( \rho \) be a circular, perfectly incoherent and quasi-monochromatic light source with diameter \( a \) and central frequency \( \omega_0 \), contained in plane A, as illustrated in Fig. 2 which represents the tip of the fiber in plane A. The propagation of the spectral degree of coherence from A to B can be described by the van Cittert-Zernike theorem. Assuming that the physical extent of light impinging on lens L1 is smaller than the lens aperture, we can neglect the pupil function and, therefore, we can ignore diffraction effects due to the lens finite size aperture. Since the light source is at the back focal plane of lens L1, the magnitude of the spectral degree of coherence \( \mu_C(|x_1 - x_2|, \omega) \)
at plane $C$ will be the same as in plane $B$ and is given by

$$|\mu_C(\mathbf{x}_1 - \mathbf{x}_2), \omega_0)| = 2 \left| J_1 \left( \frac{\omega_0 a |\mathbf{x}_1 - \mathbf{x}_2|}{2 cf_1} \right) \right| \times \frac{2cf_1}{\omega_0 a |\mathbf{x}_1 - \mathbf{x}_2|}, \quad (1)$$

where $\mathbf{x} = (x, y)$, $c$ is the speed of light in free space and $J_1$ is the first-order Bessel function of the first kind. Since we will perform measurements of spatial coherence as a function of the center-to-center separation $b$ of the slits, we will define $b$ such that

$$b = |\mathbf{x}_1 - \mathbf{x}_2|. \quad (2)$$

Thus, we can simplify the notation of Eq. (1) as

$$|\mu_C(b, \omega_0)| = 2 \frac{J_1 \left( \frac{\omega_0 a b}{2 cf_1} \right)}{\omega_0 a b / (2 cf_1)}. \quad (3)$$

Young double-slit experiments will be performed in plane $C$ using a DMD and interference fringes will be measured in plane $D$ by a CCD camera. The intensity $I(u)$ at plane $D$ is given by

$$I(u) = I_1(u) + I_2(u) + 2\sqrt{I_1(u)I_2(u)} \times |\mu_C(\mathbf{u}, \omega_0)| \cos \left[ \beta(b, \omega_0) - \frac{\omega_0 bv}{cf_2} \right], \quad (4)$$

where $\mathbf{u} = (u, v)$ and $\beta(b, \omega_0)$ is the phase of the spectral degree of coherence.

To observe interference fringes in plane $D$, one has to evaluate if the DMD has the appropriate dimensions or spatial resolution. The factor $a/f_1$ of Eq. (3) will determine the range of spatial coherence in plane $C$. To quantify the spatial coherence generated, we can define the effective correlation length $\sigma_\mu$ as the first zero position of the first-order Bessel function:

$$\sigma_\mu \approx 7.66 \frac{cf_1}{\omega_0 a}. \quad (5)$$

By choosing a given focal length $f_1$ for a light source with diameter $a$, one can estimate the number of data points that can be measured within the central lobe of the Bessel function.

4. Results

We performed experiments with two different multimode fibers, OF1 and OF2, with diameters 200 $\mu$m (ThorLabs, M75L01) and 50 $\mu$m (ThorLabs, M16L01), respectively. For fiber OF1, lenses L1 and L2 have focal lengths of $f_1 = 60$ mm and $f_2 = 250$ mm, respectively, while for fiber OF2, $f_1 = 60$ mm and $f_2 = 100$ mm. A photograph of the setup is shown in Fig. 3 for fiber OF2. The estimated effective correlation lengths for fibers OF1 and OF2 are $\sigma_\mu = 0.241$ mm and $\sigma_\mu = 0.966$ mm, respectively. Thus, more data points in the central lobe of the Bessel function will be measured for OF2 since the effective correlation length is larger.

For each separation $b$, the digital intensity pattern $I(u)$ is represented by a two-dimensional array. To extract the spectral degree of coherence from Eq. (4), we measured the intensity pattern of the double-slit, $I(u)$, and also for each slit, $I_1(u)$ and $I_2(u)$, and defined a new matrix $I_M(u)$:

$$I_M(u) = \frac{I(u) - I_1(u) - I_2(u)}{2 \sqrt{I_1(u)I_2(u)}} = |\mu_C(b, \omega_0)| \cos \left[ \beta(b, \omega_0) - \frac{\omega_0 bv}{cf_2} \right]. \quad (6)$$

The intensity pattern when all micromirrors are in the "off" state is also acquired and is subtracted from the other measurements. In this case, we noticed that a very faint pattern was observed in the CCD, possibly emerging from the tiny linear separations between micromirrors. To increase the SNR, we increased the integration time and the gain of the CCD. However, by increasing the gain, we noticed that the number of saturated pixels increased, and such pixels had to be discarded. We performed a binning.
process to further increase the SNR, which consists in combining pixels of the CCD in both horizontal and vertical directions. However, vertical binning reduces spatial resolution, which could be critical to retrieve the highest spatial frequencies of the spectral degree of coherence. With regard to horizontal binning, one must be careful since some columns may be noisier than others. The values of $I_M(u)$ are fitted using the following template:

$$g_1(y', b, A, B, C) = A \cos \left[ \frac{\omega_0 b (y' - B)}{c f_2} \right] + C, \quad (7)$$

where $A$ corresponds to the magnitude of spectral degree of coherence $\mu_C(b, \omega_0)$, for a given $b$, $B$ corresponds to the shift (dephasing) of the center of the diffraction pattern and $C$ accounts for background noise and for high gain effects on the CCD. $B$ is also determined using interferograms. Nonetheless, in the fitting process we allow for minor corrections, even though it is well constrained to a small set of CCD pixels. For fiber OF2, a nonlinear chirp term was added, since the frequency of the fringe pattern was not constant throughout $y'$ (most probably due to uncorrected lens distortion). The modified fitting function is

$$g_2(y', b, A, B, C) = A \cos \left[ \frac{\omega_0 b (y' - B)}{c f_2} \right] + D (y' - B)^2 + C, \quad (8)$$

where $D$ accounts for the nonlinear chirp. Note that $D$ is treated as a constant and not a variable of function $g_2$, since it is constant for all separations $b$.

Results for both fibers are presented in Fig. 4. Intensity interferograms in Figs. 4(a) and 4(b) have different frequencies for different separation between slits, $b$, as expected from Eq. (4). A sample of results for $I_M(u, b)$ for both fibers are represented in Figs. 4(c) and 4(d). The fit used for OF1 corresponds to the function $g_1$ of Eq. (7), while for OF2 corresponds to $g_2$ of Eq. (8). The magnitude spectral degree of coherence obtained from the previous fittings for each separation $b$ are represented in Figs. 4(c) and 4(d) for fibers OF1 and OF2, respectively. For fiber OF2, the estimated value for the nonlinear chirp was $D = 4.64$ m. The data from the magnitude of the spectral degree of coherence $|\mu_C(b, \omega_0)|$ was fitted using Eq. (3) in order to retrieve the diameter $a_1$ and $a_2$ of fibers OF1 and OF2, respectively. The values obtained were $a_1 = 192 \pm 2 \mu m$ and $a_2 = 54.3 \pm 0.6 \mu m$, with deviations of 4% and 9%, respectively, with respect to manufacturers’ specifications. Results are summarized in Table 1.

Spatial coherence measurements may take some minutes. The time depends on the number of interferograms and on the integration time of the CCD. If the light source intensity changes significantly while performing the measurement, the spectral degree of coherence cannot be retrieved correctly, which could be a problem to study some light sources. However, it is worthwhile studying new patterns of micromirrors on the DMD such as non-parallel slit by Divitt et al. [3], instead of the double-slit pattern, in order to retrieve the spectral degree of coherence faster and possibly with adequate spectral resolution for broadband sources. Another problem that can affect spatial coherence measurement is the tilting of the DMD around the $y$-axis by $-\theta_m$. This means that there will be different path lengths from opposite edges of the DMD, as pointed out by Partanen et al. [5]. In our case, since light is collimated, the difference in spatial coherence is not significant for the different edges of the DMD. Nevertheless, it still affects imaging and these phase effects are worth studying.

Table 1: Summary of results. The retrieved values for the fibers diameter do match well with values given by the manufacturer.

| Fibers | Focal length (mm) | Diameter ($\mu$m) |
|--------|------------------|------------------|
|        | $f_1$            | $f_2$            | theory | retrieved |
| OF1    | 60               | 250              | 200    | 192       |
| OF2    | 60               | 100              | 50     | 54.3      |
5. Conclusions

The magnitude of the spectral degree of coherence of partially coherent light generated by an incoherent source was measured using a setup based on the Thompson-Wolf and Partanen-Turunen-Tervo experiments with a digital micromirror device acting as a Young double-aperture interferometer in retroreflection mode. Light from a fiber tip, coupled to a LED, was used as light source. The van Cittert-Zernike theorem was used to model the propagation of partially coherent light and the values for the spatial coherence were compared to the theoretical ones.

To use this setup integrated in a payload of ground and space-based instruments, one would have to replace the fiber by an imaging lens in order have a secondary light source in plane $A$. Assuming that the imaged light source is incoherent, one would select lens L1 according to the spatial dimensions and distance to the source, using the van Cittert-Zernike theorem as a reference. Source reconstruction from spatial coherence measurements is an example of what these measurements can provide, as mentioned in Ref. [18]. We also point out that, by using selective imaging, one could image, in plane $A$, a region of interest to perform spatial coherence measurements. This is the basis of the conceptual instrument proposed recently [9]. Nonetheless, further studies are needed to evaluate phase problems when measuring spatial coherence with DMDs of light sources not modelled by the van Cittert-Zernike theorem, i.e., partially coherence sources. Finite-size aperture effects may also have to be taken into account in cases where these effects are of the order of magnitude of the spatial coherence of light at plane $B$ for a given separation $b$.

Acknowledgement

Tiago E. C. Magalhães acknowledges the support from Fundação para a Ciência e a Tecnologia (FCT, Portugal) through the grant PD/BD/105952/2015, under the FCT PD Program PhD::SPACE (PD/00040/2012). This work was supported by FCT/MCTES through national funds (PIDDAC) by this grant UID/FIS/04434/2019.

References

[1] E. Wolf, Introduction to the theory of coherence and polarization of light, Cambridge University Press, 2007.
[2] B. Thompson, E. Wolf, Two-beam interference with partially coherent light, JOSA 47 (1957) 895–902.
[3] S. Divitt, Z. J. Lapin, L. Novotny, Measuring the coherence functions using non-parallel double slits, Optics Express 22 (2014) 8277–8290.
[4] A. I. González, Y. Mejía, Nonredundant array of apertures to measure the spatial coherence in two dimensions with only one interferogram, JOSA A 28 (2011) 1107–1113.
[5] H. Partanen, J. Turunen, J. Tervo, Coherence measurement with digital micromirror device, Optics letters 39 (2014) 1034–1037.
[6] L. J. Hornbeck, Digital light processing for high-brightness high-resolution applications, in: Projection Displays III, volume 3013, International Society for Optics and Photonics, 1997, pp. 27–41.
[7] H. E. Kondakci, A. Beckus, A. E. Halawany, N. Mohammadian, G. K. Atia, A. F. Abouraddy, Coherence measurements of scattered incoherent light for lensless identification of an object’s location and size, Opt. Express 25 (2017) 13087–13100.
[8] A. El-Halawany, A. Beckus, H. E. Kondakci, M. Monroe, N. Mohammadian, G. K. Atia, A. F. Abouraddy, Incoherent lensless imaging via coherence back-propagation, Optics letters 42 (2017) 3089–3092.
[9] T. E. C. Magalhães, J. M. Rebordão, Spatial coherence mapping of structured sources: a flexible instrument for solar studies, Appl. Opt. 58 (2019) 8840–8851.
[10] K. Fourspring, Z. Ninkov, S. Heap, M. Roberto, A. Kim, Testing of digital micromirror devices for space-based applications, in: Emerging Digital Micromirror Device Based Systems and Applications V, volume 8618, International Society for Optics and Photonics, 2013, p. 86180B.
[11] A. Travinsky, D. Vorobiev, Z. Ninkov, A. Raismen, M. A. Quijada, S. A. Smee, J. A. Pellish, T. Schwartz, M. Robberto, S. Heap, et al., Evaluation of digital micromirror devices for use in space-based multiobject spectrometer application, Journal of Astronomical Telescopes, Instruments, and Systems 3 (2017) 035003.
[12] N. A. Riza, M. J. Mughal, Broadband optical equalizer using fault-tolerant digital micromirrors, Optics express 11 (2003) 1559–1565.
[13] J.-R. Park, W. Donaldson, R. Sobolewski, K. Kearney, Arbitrary wave profile generation of a laser using a digital micromirror device, in: Conference on Lasers and Electro-Optics, Optical Society of America, 2004, p. CThT19.
[14] Y. Song, R. M. Panas, J. B. Hopkins, A review of micromirror arrays, Precision Engineering 51 (2018) 729–761.
[15] Y.-X. Ren, R.-D. Lu, L. Gong, Tailoring light with a digital micromirror device, Annalen der Physik 527 (2015) 447–470.
[16] L. Mandel, E. Wolf, Optical Coherence and Quantum Optics, Cambridge University Press, 1995.
[17] J. W. Goodman, Statistical optics, John Wiley & Sons, 2015.
[18] A. Beckus, A. Tamasan, A. Dogariu, A. F. Abouraddy, G. K. Atia, On the inverse problem of source reconstruction from coherence measurements, JOSA A 35 (2018) 959–968.