Supporting Condition Evaluation Method of Cable-Stayed Bridges Based on Modal Assurance Criterion

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Abstract. In order to identify the supporting condition of cable-stayed bridges with semi-floating system by using the measured mode shapes, this paper proposed an actual supporting condition evaluation method of cable-stayed bridges based on the indicator of modal assurance criterion (MAC). Firstly, the measured mode shapes and theoretical mode shapes under different supporting conditions could be obtained. Then the corresponding MAC indicator values were calculated, which would be compared to evaluate the supporting condition of bridges. The experiment results revealed that the MAC values between the measured mode shapes and theoretical mode shapes were in good agreement with MAC values above 0.9, which indicated that the actual structural support condition was consistent with the theoretical calculation condition. The experimental results of actual bridges proved that the proposed method in this paper is feasible and effectiveness.

Keywords. Mode shape, modal assurance criterion, cable-stayed bridge, supporting condition.

1. Introduction
The mode shapes of bridges are the inherent characteristics of bridge structures, which characterize the vibration form composed by each point of bridge structures during the vibration process. The mode shape represents the relative positions of each point during the vibration process and it is a dimensionless quantity [1]. Simultaneously, the mode shape of bridges can intuitively reflect the overall state of bridge structures, especially sensitive to the supports [2-4]. Therefore, the test and evaluation of mode shape are important in the detection of bridge structures.

In general, there are some differences between the actual completed bridge structures and the theoretically designed structures. The degree of difference between the two could be determined through the static and dynamic load tests of bridges, and then it is possible to judge whether the actual status of completed bridges meets the design requirements [5-7]. This is also the main means in the detection of highway and municipal bridges. The status of bridges would change to varying degrees as the service life of bridges increases. It is also necessary to determine whether the status of bridges meets the design requirements by using dynamic and static detection methods [8-9]. Among them, the dynamic detection method is the main testing means for bridge structures. Therefore, it has gradually become a research hotspot in recent years to study how to identify the actual mechanical status and damage of bridge structures through the measured dynamic characteristics. And many researchers have made a lot of efforts to study the bridge damage identification by using dynamic characteristics.
In this paper, the main purpose is to identify the supporting status by using the measured mode shapes of bridges and then presents an evaluation method of measured mode shapes based on modal assurance criterion (MAC) for the long-span cable-stayed bridges. The actual support condition of bridges could be evaluated intuitively by comparing the MAC indicator values of the measured mode shapes and theoretical mode shapes under different supporting conditions. Thus, the identification method of supporting condition change could be formed, which would provide a reference for the application of dynamic characteristics in the engineering to evaluate the bridge condition.

2. Testing Method of Mode Shapes
For small structures, the vibration mode shape test usually adopts known excitation and then collects response data, the mode shape could be identified by the relationship between the excitations and responses, which is called experimental modal test. Due to the particularity of bridge structures, it is often difficult to carry out the experimental modal test by using any known excitation. In actual engineering, only the output data of bridge vibration can be collected, and the structural modal parameters identified through the measured vibration response data are called operational modal analysis (OMA). OMA often requires advanced analysis methods for modal parameter identification, for which the commonly used methods mainly include transfer rate method, eigen system realization algorithm (ERA), stochastic subspace method (SSI) and so on [10].

At present, the pulsation test analysis is regarded as the commonly used modal test method in actual engineering. The random vibration of bridges under environmental excitation could be collected by the high-sensitivity vibration transducer, and then the structural vibration parameters would be identified by using the OMA modal identification method. The detailed testing identification process is shown in figure 1.

![Figure 1. The dynamic testing process of bridge structures.](image)

Based on the principle of obtaining the largest mode shape of bridge structures, the cross-section of the measurement arrangement is determined for on-site testing. Generally, the measurement points are arranged at the peak and valley of the tested mode shape, and some measurement points are also arranged points appropriately between the peak and valley points. It is worth noting that the positions of bridge supports are generally located at the nodes of mode shapes (i.e., the corresponding mode shape value is 0), where there is no vibration transducer arranged.

In actual engineering, the on-site acquisition time is required to be no less than 30 min in order to ensure the frequency resolution and improve signal-to-noise ratio through the spectrum analysis. Meanwhile, it is necessary to perform batch measurements while all the measurement points can not be tested at once due to the limit of the number of vibration transducer. For different batch measurements, one measurement point, except the node of mode shape, is selected as the fixed reference point, which could be a bridge to fit the modal values of all measurement points.
3. Evaluation Method of Mode Shapes Based on Modal Assurance Criterion

3.1. Modal Assurance Criterion (MAC)

The modal assurance criterion (MAC) is a commonly used method for evaluating the dynamic characteristics of structures, which is a statistical indicator that characterizes the correlation between two sets of mode vectors. The MAC represents the spatial intersection angle of two sets of modal vectors, which can be expressed by the following equations:

\[
\text{MAC} = \frac{\left( \{\phi_i\}^T \{\phi_j\} \right)^2}{\{\phi_i\}^T \{\phi_i\} \{\phi_j\}^T \{\phi_j\}},
\]

in which \(\{\phi_i\}\) and \(\{\phi_j\}\) are two sets of modal vectors, respectively.

The MAC value is bounded between 0 and 1, i.e., [0, 1]. When the MAC value is larger than 0.9, it indicates that the two sets of mode shapes are related. While if the MAC value is smaller than 0.05, it indicates that the two sets of mode shapes are completely uncorrelated. If the MAC value is between 0.05 and 0.9, it means that the two sets of mode shapes are partially related.

3.2. Evaluation Method of Measured Mode Shapes Based on MAC

As can be seen from the introduction of MAC in the section 3.1, MAC can be used to evaluate the correlation between two sets of mode shapes. In the dynamic characteristics test of bridge structures, a finite element analysis model could be established by using the bridge structure calculation to analyze the theoretical mode shapes of bridges \(\{\phi_i\}\). Then the actual measured mode shapes \(\{y_i\}\) of bridges are obtained through the pulsation test. After that, it can be intuitively determined whether the actual measured mode shapes \(\{y_i\}\) are related with the theoretical mode shapes of bridges \(\{\phi_i\}\) by calculating the MAC values of the measured mode shapes and theoretical mode shapes. At the same time, it could be also determined whether the vibration characteristics of the actual bridge structure meet the theoretical design requirements. The evaluation method can be used to evaluate the dynamic characteristics in dynamic tests of bridges:

1. If the MAC value between the measured mode shapes and theoretical mode shapes is larger than 0.9, it indicates that the measured mode shape is consistent with the theoretical mode shape and the overall dynamic characteristics of structure meet the design requirements.

2. If the MAC value between the measured mode shapes and theoretical mode shapes is smaller than 0.9, it indicates that there is a certain difference between the measured mode shapes and theoretical mode shapes and the actual bridge structure may be inconsistent or damaged.

The evaluation method and process of mode shape using the modal assurance criterion (MAC) are shown in figure 2.

[Figure 2. The evaluation process of mode shape based on MAC.]
4. Experimental Measurement and Verification of Actual Bridges

4.1. Bridge Overview

Figure 3 shows a three-span continuous high-low tower concrete cable-stayed bridge with the span of 598 m = (193 + 332 + 113) m. The main beam adopts the form of C50 prestressed concrete bilateral box, which is the beam plate cross section shown in figure 3.

The cable-stayed bridge with semi-floating system was supported vertically on five rows of supports and longitudinally unconstrained. After the bridge was built, it is suspected that the auxiliary pier supports may be on unseating. Therefore, the experimental testing and evaluation method of mode shape is used in this paper to identify whether the supports are on unseating.

4.2. The Measured Mode Shapes and Theoretical Mode Shapes

Before the pulsation test, the finite element analysis needs to be performed to and calculate the dynamic characteristics including natural frequency and mode shape of bridge structure. The software MIDAS CIVIL is usually adopted to establish a finite element model in order to analyze and calculate the structural dynamic characteristics of the main bridge. Then the theoretical natural frequency and mode shape of the bridge under intact condition and auxiliary pier supports on unseating could be calculated, and the calculation results including natural frequency and mode shape are listed in table 1 and shown in figure 4, respectively.

| Mode No. | Theoretical frequency under intact condition (Hz) | Theoretical frequency under supports unseating condition (Hz) |
|----------|-----------------------------------------------|----------------------------------------------------------|
| 2        | 0.337                                         | 0.292                                                  |
| 3        | 0.484                                         | 0.459                                                  |
| 4        | 0.660                                         | 0.658                                                  |

Figure 4. The second order theoretical mode shape.
According to the low mode shape characteristics of the cable-stayed bridge, the accelerometer sensors are arranged at the peak and valley of the tested mode shape in order to obtain the measured values of the mode shape, damping and frequency of the bridge structure. The accelerometer vibration transducers are arranged at the cable anchorages of the main bridge deck and the joints between the tower and beam, respectively. There are a total of 164 measurement points arranged for the cable-stayed bridge and the arrangement of measurement points is shown in figure 5. The measured natural frequency results are listed in table 2 and the corresponding mode shapes are shown in figure 6. It can be seen from the comparison between table 1 and table 2, it is impossible to intuitively determine whether the supports of the bridge are on unseating through the natural frequency. Thus, it needs the further judgment evaluation by using the mode shapes.

![Figure 5. The arrangement of accelerometer vibration transducers.](image)

### Table 2. The measured frequency results.

| Mode No. | Measured frequency (Hz) |
|----------|-------------------------|
| 2        | 0.293                   |
| 3        | 0.513                   |
| 4        | 0.781                   |

![Figure 6. The comparison between measured mode shapes and theoretical mode shapes: (a) the second mode shape, (b) the third mode shape, and (c) the fourth mode shape. Note: TValue1: theoretical mode shape under intact condition; TValue2: theoretical mode shape supports unseating condition; MValue: measured mode shape.](image)

### 4.3. Evaluation Results of Measured Mode Shapes Using MAC

Based on the theory of MAC as well as the results of theoretical and measured mode shapes under intact condition and supports unseating condition in the section 4.2, the MAC value between the theoretical and measured mode shapes under intact condition is marked as MAC₁ and the MAC value
between the theoretical and measured mode shapes under supports unseating condition is marked as MAC₂. The calculation results are shown in Table 3.

| Mode No. | MAC₁ values under intact condition | MAC₁ values under supports unseating condition |
|----------|-----------------------------------|-----------------------------------------------|
| 2        | 0.862                             | 0.991                                         |
| 3        | 0.637                             | 0.924                                         |
| 4        | 0.314                             | 0.918                                         |

From Table 3, it could be seen that the MAC₁ values of different mode orders between the theoretical and measured mode shapes under intact condition are smaller than 0.9, especially the MAC₁ value of the fourth mode has reached 0.314, which indicates that the actual structure does not match the theoretical design structure under intact condition. On the other hand, the MAC₂ values of different mode orders between the theoretical and measured mode shapes under supports unseating condition are larger than 0.9, which indicates that the actual structure is consistent with the theoretical design structure under supports unseating condition. Furthermore, the supporting condition of the cable-stayed bridge can be evaluated and determined based on the MAC values.

5. Conclusions
This paper proposed an actual supporting condition evaluation method of cable-stayed bridges based on the indicator of modal assurance criterion (MAC). Firstly, the measured mode shapes and theoretical mode shapes were obtained and the corresponding MAC indicator values were calculated for the further evaluation. Then the MAC values under different supporting conditions were used to intuitively determine the actual condition of bridge structure, which forms an evaluation method of dynamic characteristics based on modal assurance criterion (MAC). This evaluation method was also verified through an actual engineering. Conclusions can be achieved as follows:

(1) When the MAC values between the theoretical and measured mode shapes are larger than 0.9, the measured mode shapes are consistent with the theoretical mode shapes, which indicates that the overall dynamic characteristics of structure meet the design requirements.

(2) When the MAC values between the theoretical and measured mode shapes are smaller than 0.9, there are some differences between the theoretical and measured mode shapes, which means that the actual bridge structure may be inconsistent with the design requirements or damaged.

(3) In actual engineering, the actual condition of bridge structure could be intuitively determined by using the MAC values between the theoretical and measured mode shapes under different supporting conditions.

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Conflicts of Interest
The authors declare no conflict of interest.

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